Computer Algebra Independent Integration Tests

Summer 2024

6-Hyperbolic-functions/6.1-Hyperbolic-sine/296-6.1.5

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Contents

CHAPTER 1

INTRODUCTION

This report gives the result of running the computer algebra independent integration test.The download section in on the main webpage contains links to download the problems in plain text format used for all CAS systems. The number of integrals in this report is [**369**]. This is test number [296].

1.1 Listing of CAS systems tested

The following are the CAS systems tested:

1. Mathematica 14 (January 9, 2024) on windows 10 pro.
2. Rubi 4.17.3 (Sept 25, 2023) on Mathematica 14 on windows 10m pro.
3. Maple 2024 (March 1, 2024) on windows 10 pro.
4. Maxima 5.47 (June 1, 2023) using Lisp SBCL 2.4.0 on Linux Manjaro 23.1.2 KDE via sagemath 10.3.
5. FriCAS 1.3.10 built with sbcl 2.3.11 (January 10, 2024) on Linux Manjaro 23.1.2 KDE via sagemath 10.3.
6. Giac/Xcas 1.9.0-99 on Linux via sagemath 10.3.
7. Sympy 1.12 using Python 3.11.6 (Nov 14 2023, 09:36:21) [GCC 13.2.1 20230801] on Linux Manjaro 23.1.2 KDE.
8. Mupad using Matlab 2021a with Symbolic Math Toolbox Version 8.7 on windows 10.
9. Reduce CSL rev 6687 (January 9, 2024) on Linux Manjaro 23.1.2 KDE.

Maxima and Fricas and Giac are called using Sagemath. This was done using Sagemath `integrate` command by changing the name of the algorithm to use the different CAS systems.

Sympy was run directly in Python not via sagemath.

Reduce was called directly.

1.2 Results

Important note: A number of problems in this test suite have no antiderivative in closed form. This means the antiderivative of these integrals can not be expressed in terms of elementary, special functions or `Hypergeometric2F1` functions. `RootSum` and `RootOf` are not allowed. If a CAS returns the above integral unevaluated within the time limit, then the result is counted as passed and assigned an A grade.

However, if CAS times out, then it is assigned an F grade even if the integral is not integrable, as this implies CAS could not determine that the integral is not integrable in the time limit.

If a CAS returns an antiderivative to such an integral, it is assigned an A grade automatically and this special result is listed in the introduction section of each individual test report to make it easy to identify as this can be important result to investigate.

The results given in in the table below reflects the above.

System	% solved	% Failed
Rubi	100.00 (369)	0.00 (0)
Mathematica	100.00 (369)	0.00 (0)
Fricas	95.66 (353)	4.34 (16)
Maple	90.51 (334)	9.49 (35)
Giac	75.88 (280)	24.12 (89)
Maxima	72.09 (266)	27.91 (103)
Mupad	59.89 (221)	40.11 (148)
Reduce	39.84 (147)	60.16 (222)
Sympy	32.79 (121)	67.21 (248)

Table 1.1: Percentage solved for each CAS

The table below gives additional break down of the grading of quality of the antiderivatives generated by each CAS. The grading is given using the letters A,B,C and F with A being the best quality. The grading is accomplished by comparing the antiderivative generated with the optimal antiderivatives included in the test suite. The following table describes the meaning of these grades.

grade	description
A	Integral was solved and antiderivative is optimal in quality and leaf size.
B	Integral was solved and antiderivative is optimal in quality but leaf size is larger than twice the optimal antiderivatives leaf size.
C	Integral was solved and antiderivative is non-optimal in quality. This can be due to one or more of the following reasons 1. antiderivative contains a hypergeometric function and the optimal antiderivative does not. 2. antiderivative contains a special function and the optimal antiderivative does not. 3. antiderivative contains the imaginary unit and the optimal antiderivative does not.
F	Integral was not solved. Either the integral was returned unevaluated within the time limit, or it timed out, or CAS hanged or crashed or an exception was raised.

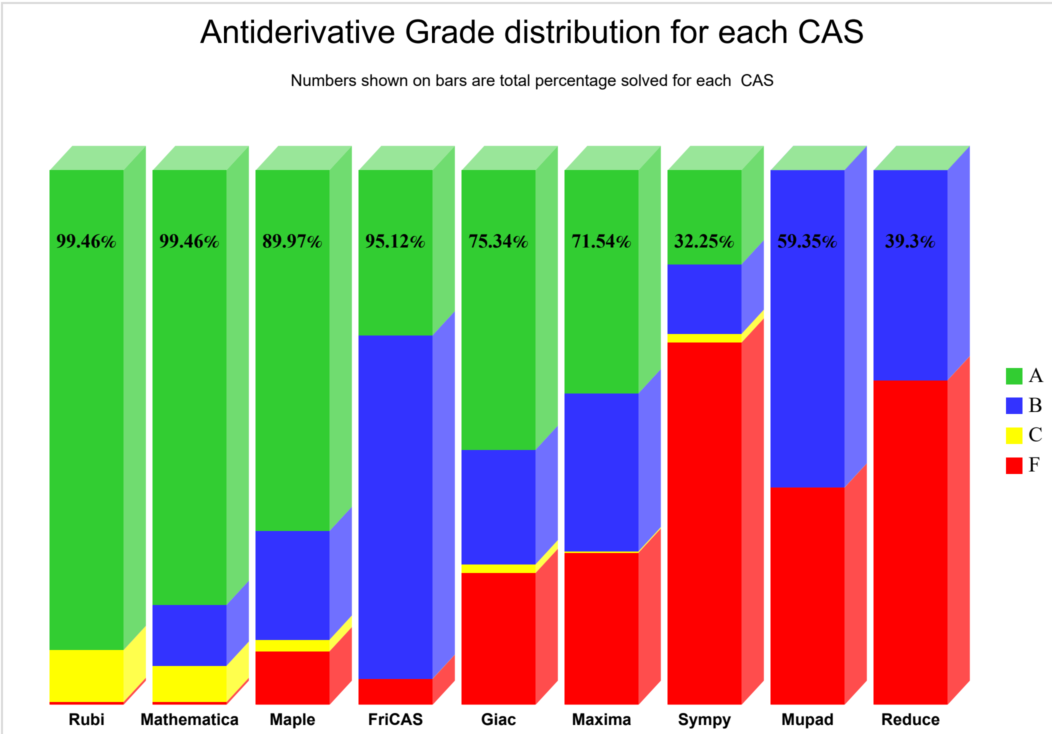
Table 1.2: Description of grading applied to integration result

Grading is implemented for all CAS systems. Based on the above, the following table summarizes the grading for this test suite.

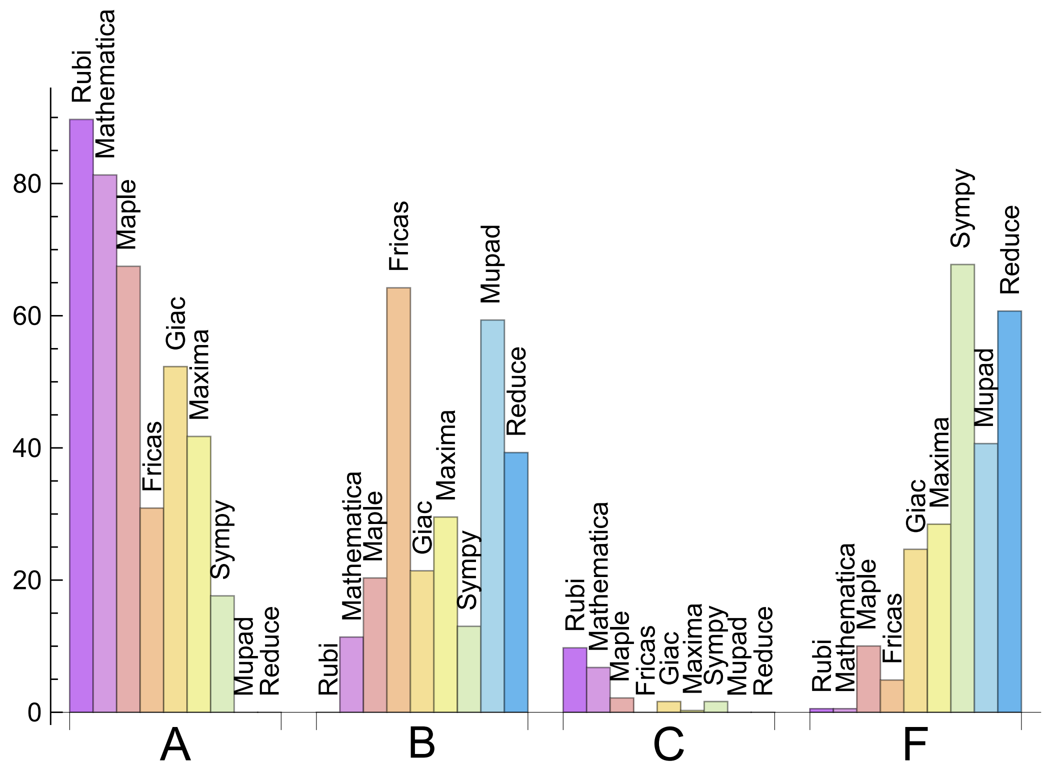
System	% A grade	% B grade	% C grade	% F grade
Rubi	89.702	0.000	9.756	0.542
Mathematica	81.301	11.382	6.775	0.542
Maple	67.480	20.325	2.168	10.027
Giac	52.304	21.409	1.626	24.661
Maxima	41.734	29.539	0.271	28.455
Fricas	30.894	64.228	0.000	4.878
Sympy	17.615	13.008	1.626	67.751
Mupad	0.000	59.350	0.000	40.650
Reduce	0.000	39.295	0.000	60.705

Table 1.3: Antiderivative Grade distribution of each CAS

The following is a Bar chart illustration of the data in the above table.



The figure below compares the grades of the CAS systems.



The following table shows the distribution of the different types of failures for each CAS. There are 3 types failures. The first is when CAS returns the input within the time limit, which means it could not solve it. This is the typical failure and given as **F**.

The second failure is due to time out. CAS could not solve the integral within the 3 minutes time limit which is assigned. This is assigned **F(-1)**.

The third is due to an exception generated, indicated as **F(-2)**. This most likely indicates an interface problem between sagemath and the CAS (applicable only to FriCAS, Maxima and Giac) or it could be an indication of an internal error in the CAS itself. This type of error requires more investigation to determine the cause.

System	Number failed	Percentage normal failure	Percentage timeout failure	Percentage exception failure
Rubi	0	0.00	0.00	0.00
Mathematica	0	0.00	0.00	0.00
Fricas	16	100.00	0.00	0.00
Maple	35	100.00	0.00	0.00
Giac	89	100.00	0.00	0.00
Maxima	103	98.06	0.00	1.94
Mupad	148	0.00	100.00	0.00
Reduce	222	100.00	0.00	0.00
Sympy	248	77.82	22.18	0.00

Table 1.4: Failure statistics for each CAS

1.3 Time and leaf size Performance

The table below summarizes the performance of each CAS system in terms of time used and leaf size of results.

Mean size is the average leaf size produced by the CAS (before any normalization). The Normalized mean is relative to the mean size of the optimal anti-derivative given in the input files.

For example, if CAS has **Normalized mean** of 3, then the mean size of its leaf size is 3 times as large as the mean size of the optimal leaf size.

Median size is value of leaf size where half the values are larger than this and half are smaller (before any normalization). i.e. The Middle value.

Similarly the **Normalized median** is relative to the median leaf size of the optimal.

For example, if a CAS has Normalized median of 1.2, then its median is 1.2 as large as the median leaf size of the optimal.

System	Mean time (sec)
Maxima	0.08
Fricas	0.11
Reduce	0.16
Giac	0.40
Rubi	0.44
Mathematica	0.58
Mupad	1.77
Sympy	2.55
Maple	8.12

Table 1.5: Time performance for each CAS

System	Mean size	Normalized mean	Median size	Normalized median
Rubi	89.34	1.06	70.00	1.00
Mathematica	114.36	1.35	72.00	1.00
Sympy	125.33	2.22	58.00	1.61
Maple	135.07	1.43	80.00	1.20
Maxima	137.14	1.97	94.00	1.50
Mupad	173.40	2.49	74.00	1.61
Giac	184.61	1.79	75.00	1.33
Reduce	296.30	2.81	98.00	1.88
Fricas	420.63	3.95	156.00	2.14

Table 1.6: Leaf size performance for each CAS

1.4 Performance based on number of rules Rubi used

This section shows how each CAS performed based on the number of rules Rubi needed to solve the same integral. One diagram is given for each CAS.

On the y axis is the percentage solved which Rubi itself needed the number of rules given the x axis. These plots show that as more rules are needed then most CAS system percentage of solving decreases which indicates the integral is becoming more complicated to solve.

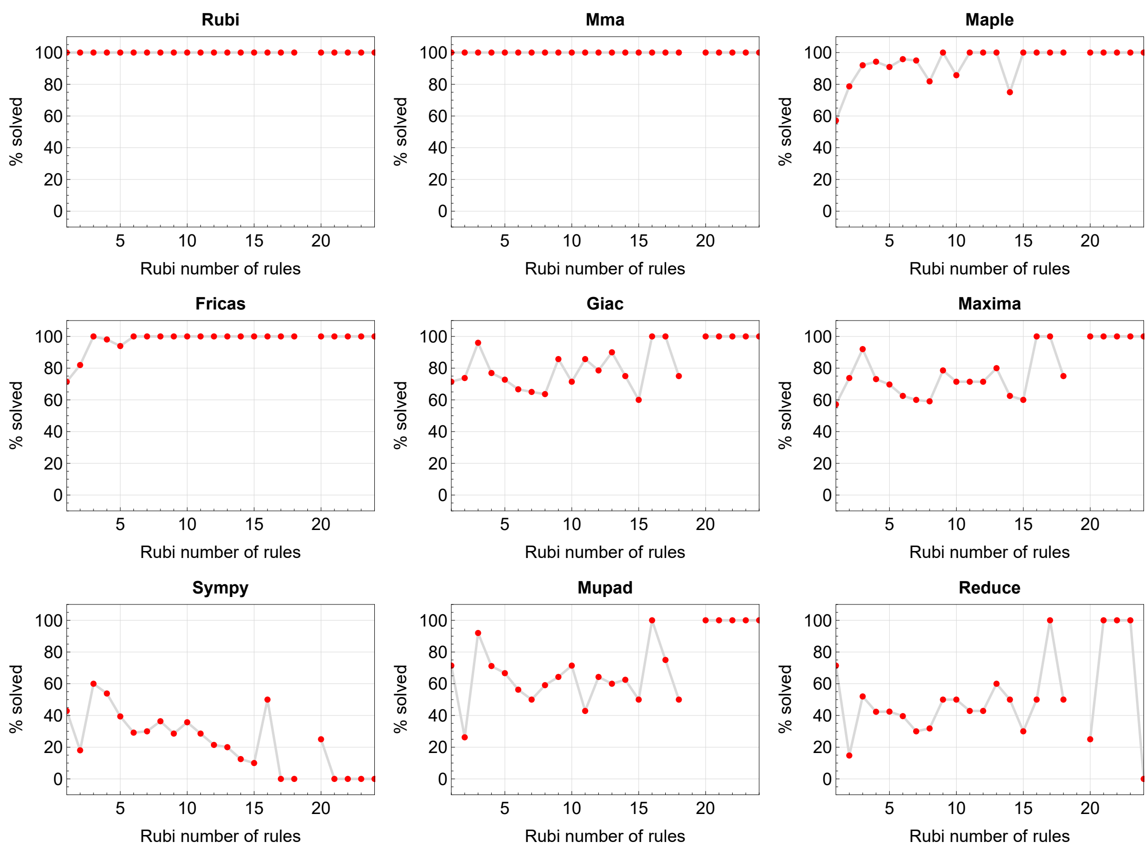


Figure 1.1: Solving statistics per number of Rubi rules used

1.5 Performance based on number of steps Rubi used

This section shows how each CAS performed based on the number of steps Rubi needed to solve the same integral. Note that the number of steps Rubi needed can be much higher than the number of rules, as the same rule could be used more than once.

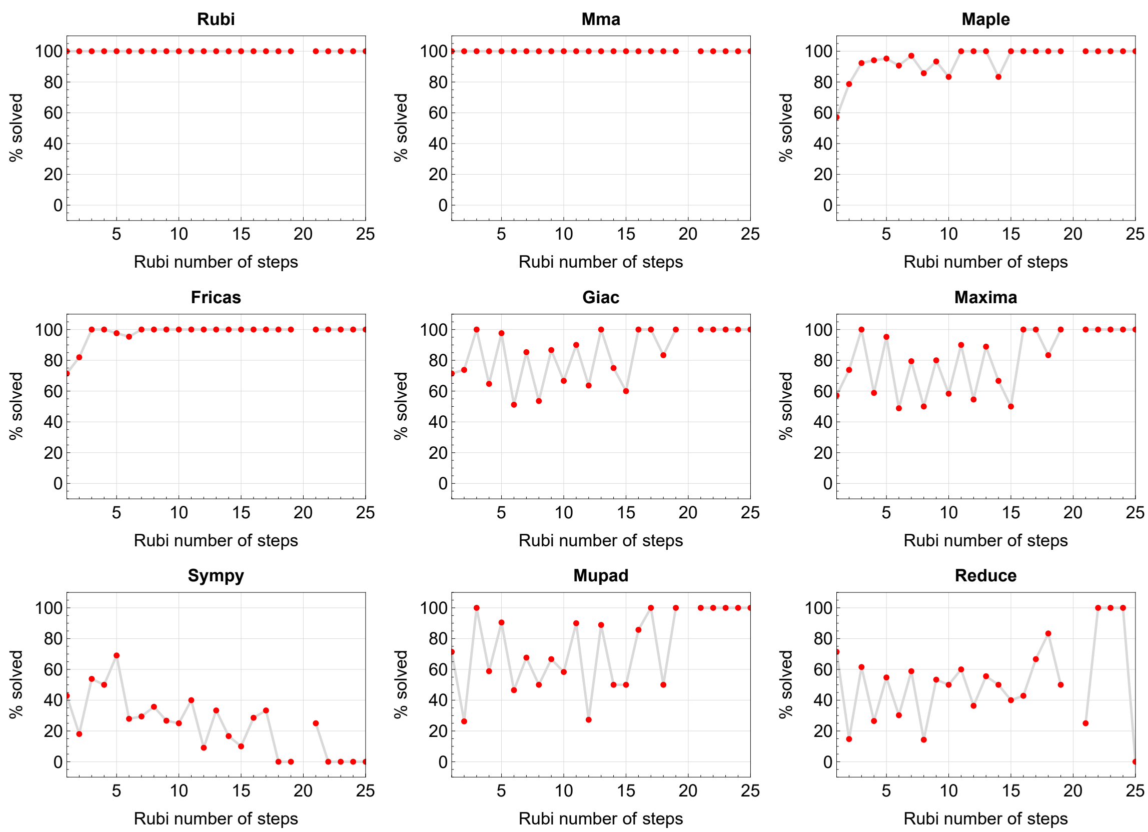


Figure 1.2: Solving statistics per number of Rubi steps used

The above diagram show that the precentage of solved intergals decreases for most CAS systems as the number of steps increases. As expected, for integrals that required less steps by Rubi, CAS systems had more success which indicates the integral was not as hard to solve. As Rubi needed more steps to solve the integral, the solved percentage decreased for most CAS systems which indicates the integral is becoming harder to solve.

1.6 Solved integrals histogram based on leaf size of result

The following shows the distribution of solved integrals for each CAS system based on leaf size of the antiderivatives produced by each CAS. It shows that most integrals solved produced leaf size less than about 100 to 150. The bin size used is 40.

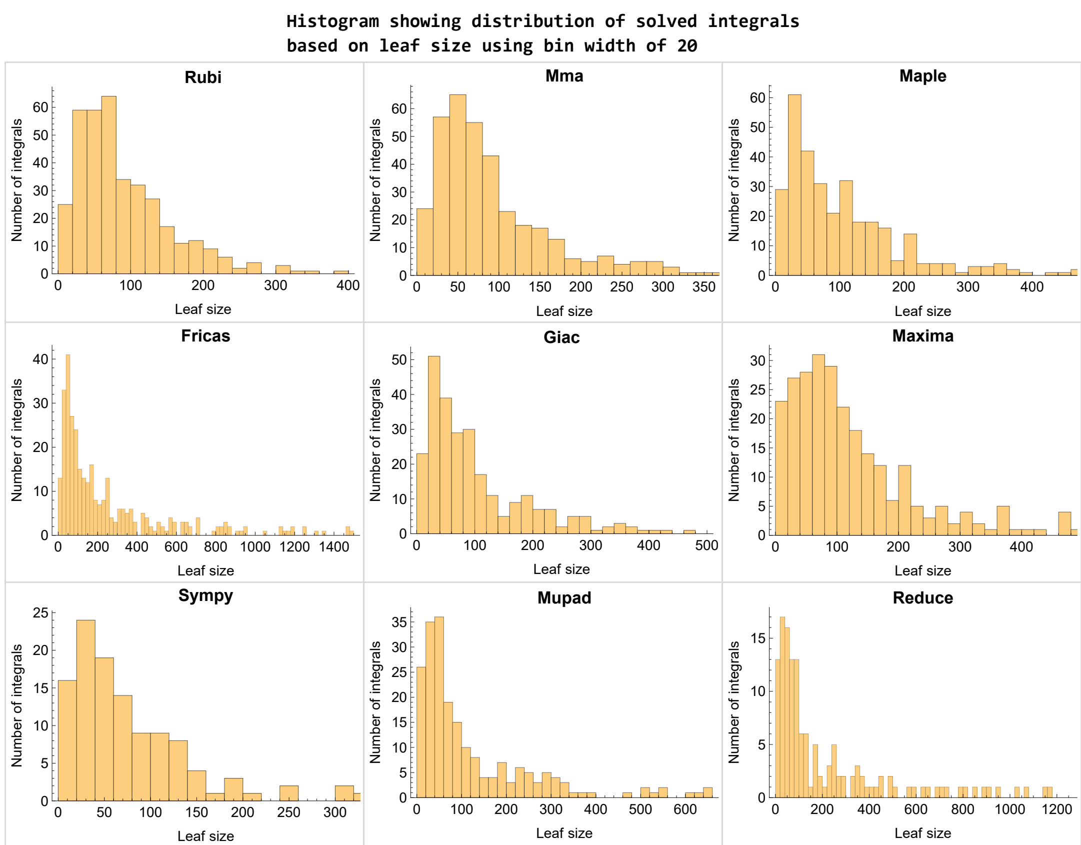


Figure 1.3: Solved integrals based on leaf size distribution

1.7 Solved integrals histogram based on CPU time used

The following shows the distribution of solved integrals for each CAS system based on CPU time used in seconds. The bin size used is 0.1 second.

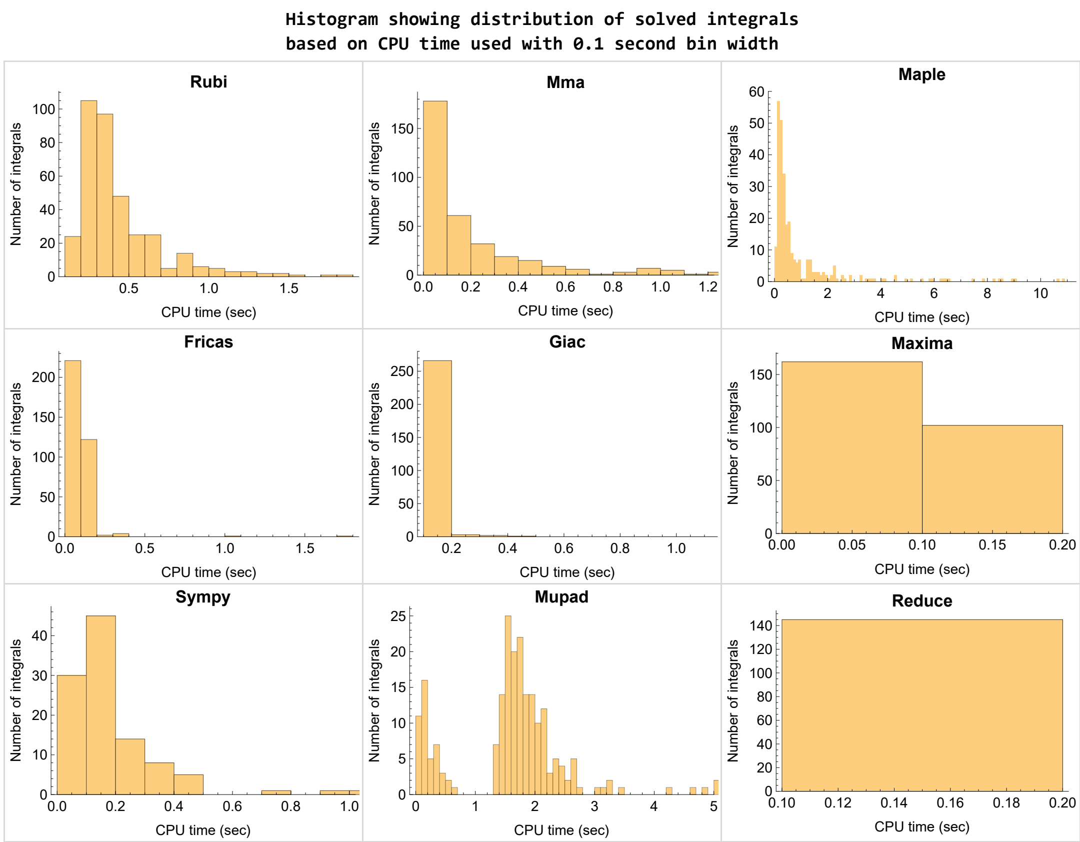


Figure 1.4: Solved integrals histogram based on CPU time used

1.8 Leaf size vs. CPU time used

The following shows the relation between the CPU time used to solve an integral and the leaf size of the antiderivative.

The result for Fricas, Maxima and Giac is shifted more to the right than the other CAS system due to the use of sagemath to call them, which causes an initial slight delay in the timing to start the integration due to overhead of starting a new process each time. This should also be taken into account when looking at the timing of these three CAS systems. Direct calls not using sagemath would result in faster timings, but current implementation uses sagemath as this makes testing much easier to do.

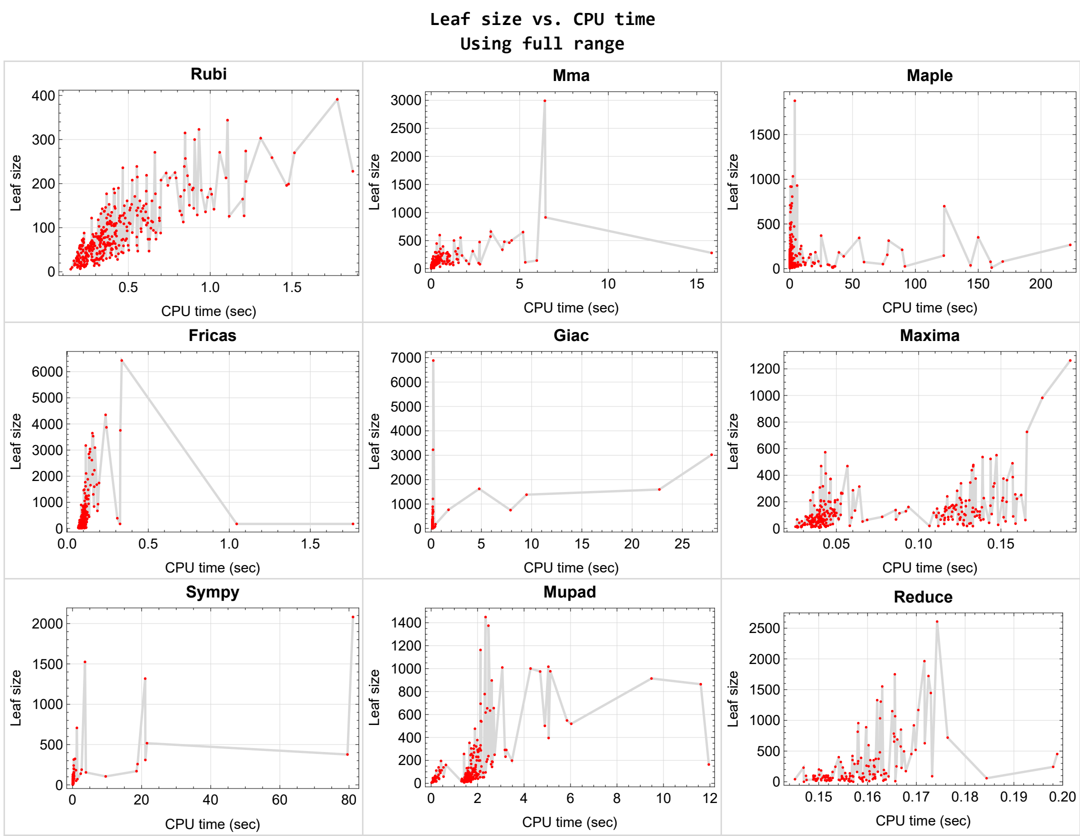


Figure 1.5: Leaf size vs. CPU time. Full range

1.9 list of integrals with no known antiderivative

$\{264, 265\}$

1.10 List of integrals solved by CAS but has no known antiderivative

Rubi $\{\}$

Mathematica $\{\}$

Maple $\{\}$

Maxima $\{\}$

Fricas $\{\}$

Sympy $\{\}$

Giac $\{\}$

Reduce $\{\}$

Mupad $\{\}$

1.11 list of integrals solved by CAS but failed verification

The following are integrals solved by CAS but the verification phase failed to verify the anti-derivative produced is correct. This does not necessarily mean that the anti-derivative is wrong as additional methods of verification might be needed, or more time is needed (3 minutes time limit was used). These integrals are listed here to make it possible to do further investigation to determine why the result could not be verified.

Rubi $\{101, 102, 103, 104, 253, 254, 255, 256, 285, 286, 302, 316, 329, 330\}$

Mathematica $\{124, 125, 190, 300, 356, 362, 363, 364\}$

Maple $\{332\}$

Maxima Verification phase not currently implemented.

Fricas Verification phase not currently implemented.

Sympy Verification phase not currently implemented.

Giac Verification phase not currently implemented.

Reduce Verification phase not currently implemented.

Mupad Verification phase not currently implemented.

1.12 Timing

The command `AbsoluteTiming[]` was used in Mathematica to obtain the elapsed time for each integrate call. In Maple, the command `Usage` was used as in the following example

```
cpu_time := Usage(assign ('result_of_int',int(expr,x)),output='realtime'
```

For all other CAS systems, the elapsed time to complete each integral was found by taking the difference between the time after the call completed from the time before the call was made. This was done using Python's `time.time()` call.

All elapsed times shown are in seconds. A time limit of 3 CPU minutes was used for each integral. If the integrate command did not complete within this time limit, the integral was aborted and considered to have failed and assigned an F grade. The time used by failed integrals due to time out was not counted in the final statistics.

1.13 Verification

A verification phase was applied on the result of integration for `Rubi` and `Mathematica`.

Future version of this report will implement verification for the other CAS systems. For the integrals whose result was not run through a verification phase, it is assumed that the antiderivative was correct.

Verification phase also had 3 minutes time out. An integral whose result was not verified could still be correct, but further investigation is needed on those integrals. These integrals were marked in the summary table below and also in each integral separate section so they are easy to identify and locate.

1.14 Important notes about some of the results

Important note about Maxima results

Since tests were run in a batch mode, and using an automated script, then any integral where Maxima needed an interactive response from the user to answer a question during the evaluation of the integral will fail.

The exception raised is `ValueError`. Therefore Maxima results is lower than what would result if Maxima was run directly and each question was answered correctly.

The percentage of such failures were not counted for each test file, but for an example, for the `Timofeev` test file, there were about 14 such integrals out of total 705, or about 2 percent. This percentage can be higher or lower depending on the specific input test file.

Such integrals can be identified by looking at the output of the integration in each section for Maxima. The exception message will indicate the cause of error.

Maxima integrate was run using SageMath with the following settings set by default

```
'besselexpand : true'
'display2d : false'
'domain : complex'
'keepfloat : true'
'load(to_poly_solve)'
'load(simplify_sum)'
'load(abs_integrate)' 'load(diag)'
```

SageMath automatic loading of Maxima `abs_integrate` was found to cause some problems. So the following code was added to disable this effect.

```
from sage.interfaces.maxima_lib import maxima_lib
maxima_lib.set('extra_definite_integration_methods', '[]')
maxima_lib.set('extra_integration_methods', '[]')
```

See `https://ask.sagemath.org/question/43088/integrate-results-that-are-different-from-using-maxima/` for reference.

Important note about FriCAS result

There were few integrals which failed due to SageMath interface and not because FriCAS system could not do the integration.

These will fail With error `Exception raised: NotImplementedError`.

The number of such cases seems to be very small. About 1 or 2 percent of all integrals. These can be identified by looking at the exception message given in the result.

Important note about finding leaf size of antiderivative

For Mathematica, Rubi, and Maple, the builtin system function `LeafSize` was used to find the leaf size of each antiderivative.

The other CAS systems (SageMath and Sympy) do not have special builtin function for this purpose at this time. Therefore the leaf size for Fricas and Sympy antiderivative was determined using the following function, thanks to user `slelievre` at `https://ask.sagemath.org/question/57123/could-we-have-a-leaf_count-function-in-base-sagemath/`

```
def tree_size(expr):
    r"""
    Return the tree size of this expression.
    """
    if expr not in SR:
        # deal with lists, tuples, vectors
        return 1 + sum(tree_size(a) for a in expr)
    expr = SR(expr)
    x, aa = expr.operator(), expr.operands()
    if x is None:
        return 1
    else:
        return 1 + sum(tree_size(a) for a in aa)
```

For Sympy, which was called directly from Python, the following code was used to obtain the leafsize of its result

```
try:
  # 1.7 is a fudge factor since it is low side from actual leaf count
  leafCount = round(1.7*count_ops(anti))

  except Exception as ee:
        leafCount =1
```

Important note about Mupad results

Matlab's symbolic toolbox does not have a leaf count function to measure the size of the antiderivative. Maple was used to determine the leaf size of Mupad output by post processing Mupad result.

Currently no grading of the antiderivative for Mupad is implemented. If it can integrate the problem, it was assigned a B grade automatically as a placeholder. In the future, when grading function is implemented for Mupad, the tests will be rerun again.

The following is an example of using Matlab's symbolic toolbox (Mupad) to solve an integral

```
integrand = evalin(symengine,'cos(x)*sin(x)')
the_variable = evalin(symengine,'x')
anti = int(integrand,the_variable)
```

Which gives `sin(x)^2/2`

1.15 Current tree layout of integration tests

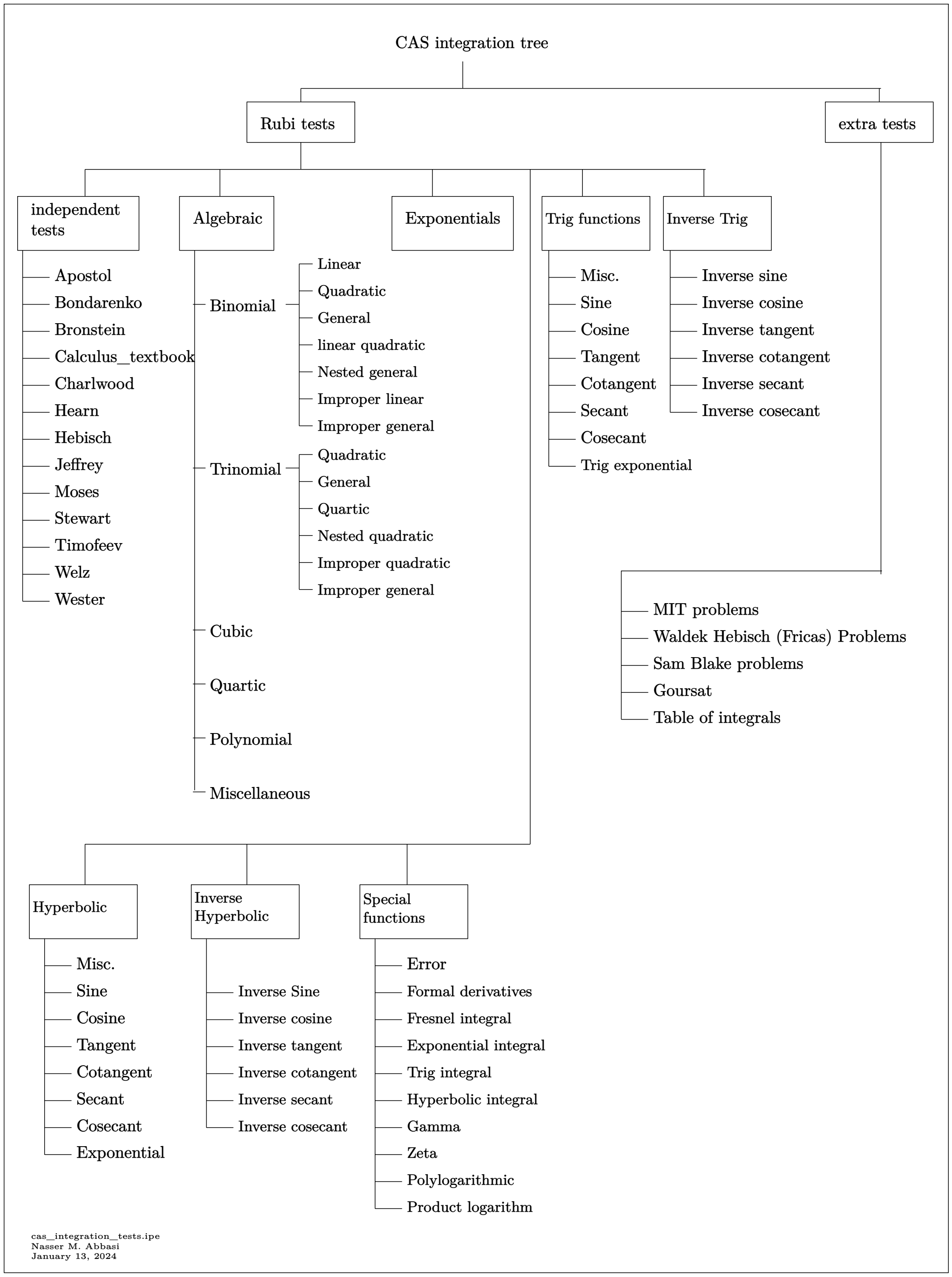
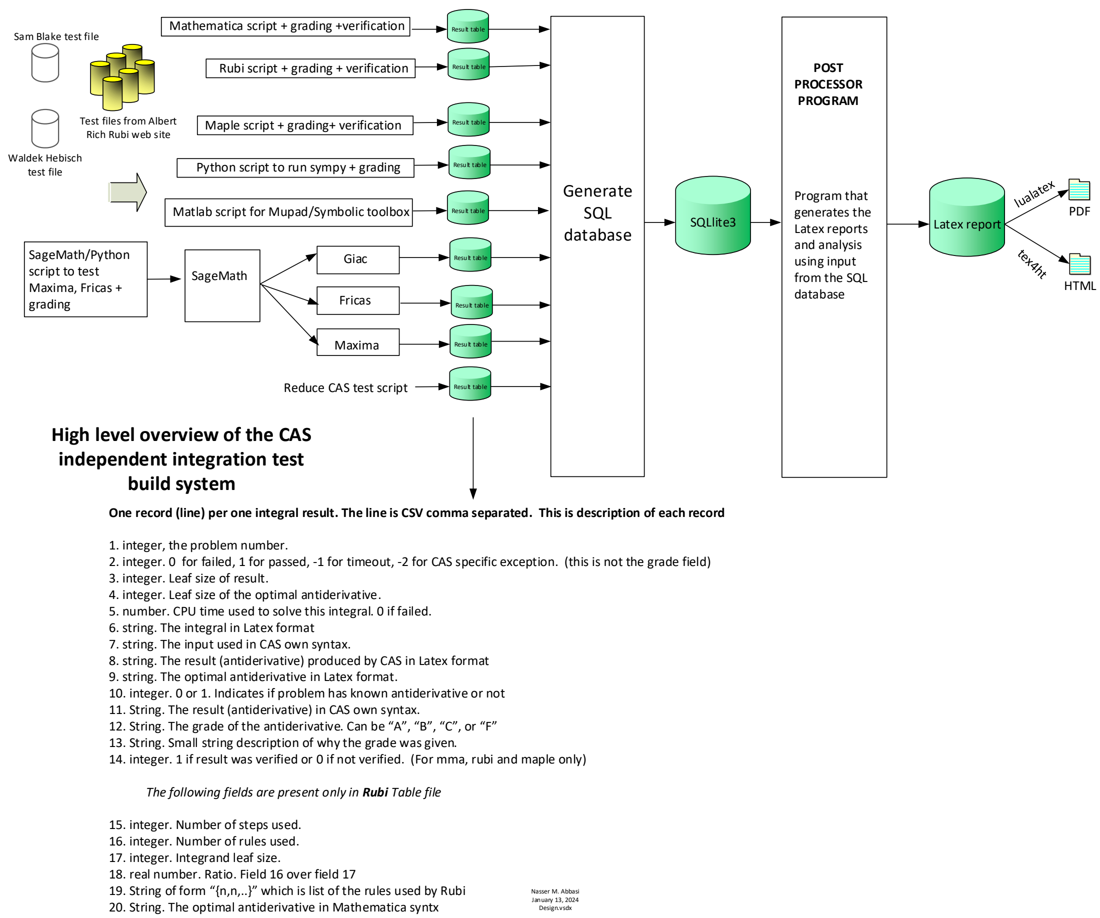


Figure 1.6: CAS integration tests tree

1.16 Design of the test system

The following diagram gives a high level view of the current test build system.



CHAPTER 2

DETAILED SUMMARY TABLES OF RESULTS

2.1 List of integrals sorted by grade for each CAS

Rubi

A grade { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 82, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 196, 197, 198, 199, 201, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 219, 220, 221, 222, 223, 224, 225, 226, 227, 228, 229, 230, 231, 232, 234, 236, 237, 238, 239, 240, 242, 244, 245, 246, 247, 248, 249, 250, 251, 253, 254, 255, 256, 257, 258, 259, 260, 262, 263, 266, 267, 268, 269, 270, 271, 272, 273, 274, 275, 276, 277, 278, 279, 280, 281, 282, 283, 284, 285, 286, 287, 288, 290, 301, 302, 303, 304, 305, 306, 307, 308, 309, 310, 311, 312, 313, 314, 315, 316, 317, 318, 319, 320, 321, 322, 323, 324, 325, 326, 327, 328, 329, 330, 331, 332, 333, 334, 335, 336, 337, 338, 339, 340, 341, 342, 343, 344, 345, 346, 347, 348, 349, 350, 351, 352, 353, 354, 355, 356, 357, 358, 359, 360, 361, 362, 363, 364, 365, 366, 367, 368, 369 }

B grade { }

C grade { 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 83, 84, 85, 86, 87, 156, 157, 200, 202, 233, 235, 241, 243, 252, 261, 289, 291, 292, 293, 294, 295, 296, 297, 298, 299, 300 }

F normal fail { }

F(-1) timedout fail { }

F(-2) exception fail { }

Mma

A grade { 2, 3, 4, 5, 6, 7, 8, 10, 11, 12, 14, 15, 16, 18, 19, 20, 22, 23, 24, 26, 27, 28, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 41, 44, 45, 46, 47, 49, 50, 51, 52, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 149, 150, 151, 152, 153, 154, 155, 156, 157, 159, 161, 163, 165, 166, 167, 168, 169, 170, 172, 173, 174, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 189, 191, 193, 195, 196, 197, 199, 201, 203, 204, 205, 206, 207, 209, 211, 212, 214, 216, 219, 221, 222, 224, 226, 228, 230, 232, 233, 234, 235, 236, 238, 240, 241, 242, 243, 244, 245, 246, 247, 250, 251, 252, 253, 254, 255, 256, 257, 258, 259, 260, 261, 262, 263, 266, 267, 268, 269, 270, 271, 272, 273, 275, 276, 277, 278, 279, 281, 282, 283, 286, 287, 288, 289, 290, 291, 292, 293, 294, 301, 302, 303, 304, 305, 306, 307, 308, 309, 310, 311, 312, 313, 314, 315, 318, 319, 322, 323, 324, 325, 326, 328, 329, 330, 331, 332, 333, 334, 335, 336, 337, 338, 339, 340, 341, 342, 343, 344, 345, 346, 347, 348, 349, 350, 351, 352, 353, 354, 355, 356, 357, 358, 359, 360, 361, 362, 363, 364, 366, 367, 369 }

B grade { 1, 40, 42, 43, 48, 53, 54, 55, 68, 92, 93, 94, 95, 115, 158, 160, 162, 164, 171, 175, 194, 198, 208, 210, 213, 215, 217, 218, 220, 223, 225, 227, 248, 274, 295, 296, 297, 298, 299, 300, 327, 365 }

C grade { 9, 13, 17, 21, 25, 29, 148, 188, 190, 192, 200, 202, 229, 231, 237, 239, 249, 280, 284, 285, 316, 317, 320, 321, 368 }

F normal fail { }

F(-1) timedout fail { }

F(-2) exception fail { }

Maple

A grade { 1, 2, 3, 4, 5, 6, 7, 9, 13, 15, 17, 19, 21, 23, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 66, 67, 68, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 111, 115, 116, 117, 118, 119, 120, 121, 122, 129, 130, 133, 134, 136, 140, 141, 142, 145, 152, 153, 155, 156, 157, 159, 161, 163, 165, 166, 167, 168, 169, 172, 173, 174, 175, 176, 177, 178, 179, 181, 182, 183, 184, 185, 186, 187, 189, 190, 191, 192, 193, 194, 195, 196, 197, 200, 201, 202, 203, 204, 205, 206, 207, 210, 211, 212, 213, 214, 218, 219, 220, 221, 222, 223, 224, 226, 228, 229, 230, 231, 232, 233, 234, 235, 236, 237, 238, 239, 240, 241, 242, 243, 246, 247, 248, 249, 250, 251, 252, 253, 254, 260, 266, 267, 268, 269, 274, 275, 276, 277, 278, 280, 282, 284, 289, 290, 291, 292, 293, 294, 301, 302, 303, 304, 305, 306, 307, 308, 309, 310, 311, 313, 314, 315, 317, 318, 319, 322, 323, 324, 329, 330, 331, 333, 334, 335, 336, 337, 338, 340, 341, 342, 343, 344, 345, 346, 347, 348, 349, 350, 351, 352, 353, 354, 355, 356, 357, 358, 359, 360, 361, 362, 363, 364, 365, 366, 367, 368, 369 }

B grade { 8, 10, 11, 12, 14, 16, 18, 20, 22, 24, 25, 26, 27, 28, 29, 30, 64, 65, 69, 70, 71, 72, 73, 103, 104, 105, 106, 107, 108, 109, 110, 123, 126, 127, 128, 131, 132, 135, 137, 138, 139, 143, 144, 154, 158, 160, 162, 164, 170, 171, 180, 188, 198, 199, 208, 209, 215, 216, 217, 225, 227, 255, 256, 257, 258, 259, 279, 281, 283, 295, 296, 297, 298, 299, 300 }

C grade { 244, 245, 312, 316, 320, 321, 332, 339 }

F normal fail { 31, 32, 33, 34, 35, 36, 37, 38, 39, 112, 113, 114, 124, 125, 146, 147, 148, 149, 150, 151, 261, 262, 263, 270, 271, 272, 273, 285, 286, 287, 288, 325, 326, 327, 328 }

F(-1) timedout fail { }

F(-2) exception fail { }

Fricas

A grade { 1, 2, 3, 4, 6, 9, 10, 11, 17, 18, 19, 23, 25, 27, 28, 40, 41, 43, 51, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 88, 89, 92, 93, 96, 97, 98, 99, 100, 108, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 134, 136, 137, 148, 149, 167, 171, 173, 175, 176, 178, 182, 185, 194, 212, 231, 232, 247, 248, 260, 266, 267, 268, 269, 270, 271, 274, 275, 276, 277, 280, 281, 282, 285, 286, 287, 288, 289, 290, 295, 298, 301, 303, 305, 311, 316, 320, 329, 330, 331, 332, 336, 337, 338, 339, 340, 341, 349, 366, 367 }

B grade { 5, 7, 8, 12, 13, 14, 15, 16, 20, 21, 22, 24, 26, 29, 30, 42, 44, 45, 46, 47, 48, 49, 50, 52, 53, 54, 55, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 90, 91, 94, 95, 101, 102, 103, 104, 105, 106, 107, 109, 110, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 135, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 168, 169, 170, 172, 174, 177, 179, 180, 181, 183, 184, 186, 187, 188, 189, 190, 191, 192, 193, 195, 196, 197, 198, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 213, 214, 215, 216, 217, 218, 219, 220, 221, 222, 223, 224, 225, 226, 227, 228, 229, 230, 233, 234, 235, 236, 237, 238, 239, 240, 241, 242, 243, 244, 245, 246, 249, 250, 251, 252, 253, 254, 255, 256, 257, 258, 259, 272, 273, 278, 279, 283, 284, 291, 292, 293, 294, 296, 297, 299, 300, 302, 304, 306, 307, 308, 309, 310, 312, 313, 314, 315, 317, 318, 319, 321, 322, 323, 324, 333, 334, 335, 342, 343, 344, 345, 346, 347, 348, 350, 351, 352, 353, 354, 355, 356, 357, 358, 359, 360, 361, 362, 363, 364, 365, 368, 369 }

C grade { }

F normal fail { 31, 32, 33, 34, 35, 36, 37, 38, 39, 261, 262, 263, 325, 326, 327, 328 }

F(-1) timedout fail { }

F(-2) exception fail { }

Maxima

A grade { 1, 2, 4, 6, 40, 41, 43, 44, 48, 49, 50, 56, 57, 60, 61, 72, 73, 74, 75, 76, 77, 79, 80, 81, 82, 84, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 115, 119, 136, 140, 141, 142, 143, 144, 145, 152, 153, 154, 155, 165, 171, 173, 175, 176, 181, 183, 184, 186, 190, 192, 193, 194, 195, 196, 202, 203, 204, 229, 230, 231, 233, 238, 239, 246, 247, 248, 249, 250, 251, 260, 266, 267, 268, 269, 270, 271, 272, 273, 274, 275, 277, 301, 302, 303, 304, 305, 306, 308, 310, 311, 312, 313, 314, 315, 316, 317, 318, 319, 320, 321, 322, 323, 324, 329, 330, 331, 332, 333, 334, 337, 338, 340, 341, 342, 343, 344, 345, 346, 347, 348, 349, 350, 351, 352, 353, 354, 355, 356, 357, 358, 359, 360, 361, 362, 363, 364, 365, 366, 367 }

B grade { 3, 5, 42, 45, 46, 47, 51, 52, 53, 54, 55, 58, 59, 62, 63, 78, 83, 85, 86, 87, 103, 104, 116, 117, 118, 120, 121, 122, 129, 130, 131, 132, 133, 134, 135, 156, 157, 158, 159, 160, 161, 162, 163, 164, 166, 167, 168, 169, 170, 172, 174, 177, 178, 179, 180, 182, 185, 187, 188, 189, 191, 197, 198, 199, 200, 201, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 219, 220, 221, 222, 223, 224, 225, 226, 227, 228, 232, 234, 235, 236, 237, 240, 241, 242, 243, 252, 253, 254, 255, 256, 276, 278, 307, 309, 335 }

C grade { 339 }

F normal fail { 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 64, 65, 66, 67, 68, 69, 70, 71, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 123, 124, 125, 126, 127, 128, 137, 138, 139, 146, 147, 148, 149, 150, 151, 244, 245, 257, 258, 259, 261, 262, 263, 279, 280, 281, 282, 283, 284, 285, 286, 287, 288, 289, 290, 291, 292, 293, 294, 295, 296, 297, 298, 299, 300, 325, 326, 327, 328, 368 }

F(-1) timedout fail { }

F(-2) exception fail { 336, 369 }

Giac

A grade { 2, 4, 6, 40, 41, 42, 43, 44, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 67, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 101, 102, 103, 115, 116, 117, 118, 119, 120, 121, 122, 129, 130, 133, 134, 136, 143, 144, 145, 152, 153, 154, 155, 156, 157, 167, 171, 173, 175, 176, 178, 180, 181, 182, 183, 184, 185, 186, 187, 189, 190, 191, 192, 193, 194, 195, 197, 202, 203, 205, 210, 212, 220, 222, 226, 228, 230, 231, 232, 233, 236, 238, 241, 243, 246, 247, 248, 249, 250, 251, 252, 253, 254, 260, 266, 267, 276, 277, 278, 285, 287, 288, 301, 302, 303, 304, 305, 306, 307, 308, 309, 310, 311, 313, 314, 315, 316, 317, 318, 319, 320, 321, 329, 330, 331, 332, 333, 334, 335, 336, 337, 338, 340, 341, 342, 344, 345, 347, 348, 349, 350, 351, 352, 353, 354, 355, 356, 357, 358, 359, 360, 361, 362, 363, 364, 365, 366, 367 }

B grade { 1, 3, 5, 45, 100, 104, 131, 132, 135, 140, 141, 142, 158, 159, 160, 161, 162, 163, 164, 165, 166, 168, 169, 170, 172, 174, 177, 179, 188, 196, 198, 199, 200, 201, 204, 206, 207, 208, 209, 211, 213, 214, 215, 216, 217, 218, 219, 221, 223, 224, 225, 227, 229, 234, 235, 237, 239, 240, 242, 255, 256, 268, 269, 270, 271, 272, 273, 274, 275, 289, 290, 291, 295, 296, 297, 298, 299, 300, 312 }

C grade { 322, 323, 324, 339, 343, 346 }

F normal fail { 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 64, 65, 66, 68, 69, 70, 71, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 123, 124, 125, 126, 127, 128, 137, 138, 139, 146, 147, 148, 149, 150, 151, 244, 245, 257, 258, 259, 261, 262, 263, 279, 280, 281, 282, 283, 284, 286, 292, 293, 294, 325, 326, 327, 328, 368, 369 }

F(-1) timedout fail { }

F(-2) exception fail { }

Mupad

A grade { }

B grade { 1, 2, 3, 4, 5, 6, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 68, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 115, 116, 117, 118, 119, 120, 121, 122, 129, 130, 133, 134, 135, 136, 142, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 196, 197, 198, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 219, 220, 221, 222, 223, 224, 225, 226, 227, 228, 229, 230, 231, 232, 233, 234, 235, 236, 237, 238, 239, 240, 241, 242, 243, 246, 247, 248, 249, 250, 251, 252, 253, 254, 266, 267, 268, 269, 270, 271, 272, 273, 274, 275, 276, 277, 278, 301, 302, 303, 304, 305, 306, 307, 308, 309, 310, 311, 312, 313, 314, 315, 316, 317, 318, 319, 320, 321, 322, 323, 324, 331, 333, 334, 335, 336, 366, 367 }

C grade { }

F normal fail { }

F(-1) timedout fail { 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 64, 65, 66, 67, 69, 70, 71, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 123, 124, 125, 126, 127, 128, 131, 132, 137, 138, 139, 140, 141, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 244, 245, 255, 256, 257, 258, 259, 260, 261, 262, 263, 279, 280, 281, 282, 283, 284, 285, 286, 287, 288, 289, 290, 291, 292, 293, 294, 295, 296, 297, 298, 299, 300, 325, 326, 327, 328, 329, 330, 332, 337, 338, 339, 340, 341, 342, 343, 344, 345, 346, 347, 348, 349, 350, 351, 352, 353, 354, 355, 356, 357, 358, 359, 360, 361, 362, 363, 364, 365, 368, 369 }

F(-2) exception fail { }

Sympy

A grade { 1, 3, 5, 40, 41, 42, 43, 48, 49, 50, 51, 56, 57, 58, 59, 60, 61, 62, 63, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 115, 116, 117, 118, 119, 120, 121, 122, 134, 136, 142, 165, 171, 173, 175, 181, 184, 185, 193, 203, 211, 212, 220, 221, 224, 247, 248, 278, 311, 315, 319, 366, 367 }

B grade { 2, 4, 6, 102, 158, 159, 160, 161, 162, 163, 164, 172, 174, 176, 182, 183, 186, 192, 201, 208, 209, 210, 213, 214, 215, 216, 217, 218, 219, 222, 223, 225, 226, 227, 274, 276, 301, 302, 303, 304, 310, 314, 318, 322, 323, 324, 331, 336 }

C grade { 75, 101, 129, 133, 246, 253 }

F normal fail { 8, 9, 10, 11, 12, 13, 14, 16, 17, 18, 19, 20, 21, 25, 26, 27, 28, 29, 31, 32, 33, 34, 35, 36, 37, 38, 39, 44, 45, 46, 47, 52, 53, 54, 64, 65, 67, 68, 69, 70, 76, 77, 78, 79, 84, 85, 86, 87, 106, 107, 108, 109, 110, 111, 113, 114, 123, 124, 127, 128, 137, 140, 141, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 166, 167, 168, 169, 170, 177, 178, 179, 180, 194, 195, 196, 197, 198, 199, 204, 205, 206, 207, 228, 229, 230, 231, 232, 233, 234, 235, 236, 237, 238, 239, 240, 241, 242, 243, 244, 245, 249, 250, 251, 252, 257, 258, 259, 260, 261, 262, 263, 266, 267, 268, 269, 270, 271, 275, 277, 280, 281, 282, 283, 286, 287, 289, 290, 292, 293, 295, 298, 305, 306, 307, 308, 309, 312, 313, 316, 317, 320, 321, 325, 326, 327, 328, 332, 333, 337, 338, 339, 340, 341, 342, 343, 344, 345, 346, 347, 348, 349, 350, 351, 352, 353, 354, 355, 356, 357, 358, 359, 360, 361, 362, 363, 364, 368, 369 }

F(-1) timedout fail { 7, 15, 22, 23, 24, 30, 55, 66, 71, 72, 73, 74, 80, 81, 82, 83, 103, 104, 105, 112, 125, 126, 130, 131, 132, 135, 138, 139, 187, 188, 189, 190, 191, 200, 202, 254, 255, 256, 272, 273, 279, 284, 285, 288, 291, 294, 296, 297, 299, 300, 329, 330, 334, 335, 365 }

F(-2) exception fail { }

Reduce

A grade { }

B grade { 1, 2, 3, 4, 5, 6, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 92, 96, 97, 98, 99, 100, 101, 102, 103, 104, 129, 130, 131, 132, 133, 134, 135, 136, 140, 141, 142, 143, 144, 145, 152, 153, 154, 155, 156, 157, 165, 187, 188, 189, 190, 191, 192, 193, 194, 195, 196, 197, 198, 199, 200, 201, 202, 203, 204, 205, 206, 207, 228, 229, 230, 231, 232, 233, 234, 235, 236, 237, 238, 239, 240, 241, 242, 243, 246, 249, 250, 251, 252, 253, 254, 255, 256, 266, 267, 268, 269, 270, 271, 272, 273, 274, 275, 276, 277, 278, 301, 302, 303, 304, 305, 306, 307, 308, 309, 310, 311, 312, 313, 314, 315, 316, 317, 318, 319, 320, 321, 324, 329, 330, 331, 332, 333, 334, 335, 336, 366, 367 }

C grade { }

F normal fail { 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 89, 90, 91, 93, 94, 95, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 137, 138, 139, 146, 147, 148, 149, 150, 151, 158, 159, 160, 161, 162, 163, 164, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 186, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 219, 220, 221, 222, 223, 224, 225, 226, 227, 244, 245, 247, 248, 257, 258, 259, 260, 261, 262, 263, 279, 280, 281, 282, 283, 284, 285, 286, 287, 288, 289, 290, 291, 292, 293, 294, 295, 296, 297, 298, 299, 300, 322, 323, 325, 326, 327, 328, 337, 338, 339, 340, 341, 342, 343, 344, 345, 346, 347, 348, 349, 350, 351, 352, 353, 354, 355, 356, 357, 358, 359, 360, 361, 362, 363, 364, 365, 368, 369 }

F(-1) timedout fail { }

F(-2) exception fail { }

2.2 Detailed conclusion table per each integral for all CAS systems

Detailed conclusion table per each integral is given by the table below. The elapsed time is in seconds. For failed result it is given as **F(-1)** if the failure was due to timeout. It is given as **F(-2)** if the failure was due to an exception being raised, which could indicate a bug in the system. If the failure was due to integral not being evaluated within the time limit, then it is given as **F**.

In this table, the column `N.S.` means **normalized size** and is defined as $\frac{\text{antiderivative leaf size}}{\text{optimal antiderivative leaf size}}$. To make the table fit the page, the name `Mathematica` was abbreviated to `MMA`.

Problem 1	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	A	A	A	A	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	10	10	21	11	10	10	12	26	10	10
N.S.	1	1.00	2.10	1.10	1.00	1.00	1.20	2.60	1.00	1.00
time (sec)	N/A	0.156	0.003	0.094	0.027	0.081	0.055	0.114	0.166	0.053

Problem 2	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	25	25	23	20	32	23	46	32	43	18
N.S.	1	1.00	0.92	0.80	1.28	0.92	1.84	1.28	1.72	0.72
time (sec)	N/A	0.167	0.011	0.198	0.034	0.079	0.083	0.117	0.161	1.306

Problem 3	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	A	A	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	27	24	29	23	54	38	36	54	53	24
N.S.	1	0.89	1.07	0.85	2.00	1.41	1.33	2.00	1.96	0.89
time (sec)	N/A	0.184	0.004	0.400	0.029	0.079	0.146	0.120	0.162	1.319

Problem 4	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	46	51	33	31	60	49	95	60	67	32
N.S.	1	1.11	0.72	0.67	1.30	1.07	2.07	1.30	1.46	0.70
time (sec)	N/A	0.242	0.036	0.566	0.035	0.078	0.185	0.123	0.159	0.087

Problem 5	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	B	A	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	41	35	44	33	82	79	58	82	79	31
N.S.	1	0.85	1.07	0.80	2.00	1.93	1.41	2.00	1.93	0.76
time (sec)	N/A	0.188	0.029	0.972	0.039	0.088	0.253	0.121	0.162	1.323

Problem 6	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	67	77	43	42	86	90	139	88	91	42
N.S.	1	1.15	0.64	0.63	1.28	1.34	2.07	1.31	1.36	0.63
time (sec)	N/A	0.328	0.045	1.395	0.032	0.093	0.343	0.118	0.156	0.147

Problem 7	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	B	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	103	108	75	116	0	326	0	0	18	0
N.S.	1	1.05	0.73	1.13	0.00	3.17	0.00	0.00	0.17	0.00
time (sec)	N/A	0.411	0.105	0.210	0.000	0.094	0.000	0.000	0.160	0.000

Problem 8	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F	B	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	80	80	68	164	0	202	0	0	18	0
N.S.	1	1.00	0.85	2.05	0.00	2.52	0.00	0.00	0.22	0.00
time (sec)	N/A	0.317	0.057	0.174	0.000	0.099	0.000	0.000	0.163	0.000

Problem 9	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	80	80	83	100	0	103	0	0	16	0
N.S.	1	1.00	1.04	1.25	0.00	1.29	0.00	0.00	0.20	0.00
time (sec)	N/A	0.321	0.068	0.153	0.000	0.094	0.000	0.000	0.164	0.000

Problem 10	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	54	54	50	108	0	37	0	0	9	0
N.S.	1	1.00	0.93	2.00	0.00	0.69	0.00	0.00	0.17	0.00
time (sec)	N/A	0.239	0.073	0.220	0.000	0.083	0.000	0.000	0.161	0.000

Problem 11	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	54	54	48	87	0	24	0	0	18	0
N.S.	1	1.00	0.89	1.61	0.00	0.44	0.00	0.00	0.33	0.00
time (sec)	N/A	0.240	0.087	0.138	0.000	0.115	0.000	0.000	0.158	0.000

Problem 12	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F	B	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	76	76	57	154	0	152	0	0	18	0
N.S.	1	1.00	0.75	2.03	0.00	2.00	0.00	0.00	0.24	0.00
time (sec)	N/A	0.313	0.044	0.148	0.000	0.110	0.000	0.000	0.159	0.000

Problem 13	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	B	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	80	80	86	101	0	314	0	0	18	0
N.S.	1	1.00	1.08	1.26	0.00	3.92	0.00	0.00	0.22	0.00
time (sec)	N/A	0.310	0.059	0.149	0.000	0.089	0.000	0.000	0.158	0.000

Problem 14	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F	B	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	103	104	73	192	0	621	0	0	18	0
N.S.	1	1.01	0.71	1.86	0.00	6.03	0.00	0.00	0.17	0.00
time (sec)	N/A	0.412	0.121	0.155	0.000	0.103	0.000	0.000	0.156	0.000

Problem 15	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	B	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	116	122	76	122	0	385	0	0	24	0
N.S.	1	1.05	0.66	1.05	0.00	3.32	0.00	0.00	0.21	0.00
time (sec)	N/A	0.445	0.206	0.325	0.000	0.116	0.000	0.000	0.159	0.000

Problem 16	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F	B	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	88	88	68	170	0	243	0	0	24	0
N.S.	1	1.00	0.77	1.93	0.00	2.76	0.00	0.00	0.27	0.00
time (sec)	N/A	0.334	0.095	0.271	0.000	0.094	0.000	0.000	0.173	0.000

Problem 17	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	88	88	88	106	0	112	0	0	20	0
N.S.	1	1.00	1.00	1.20	0.00	1.27	0.00	0.00	0.23	0.00
time (sec)	N/A	0.337	0.103	0.225	0.000	0.096	0.000	0.000	0.164	0.000

Problem 18	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	56	56	52	111	0	43	0	0	12	0
N.S.	1	1.00	0.93	1.98	0.00	0.77	0.00	0.00	0.21	0.00
time (sec)	N/A	0.259	0.041	0.336	0.000	0.089	0.000	0.000	0.165	0.000

Problem 19	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	56	56	54	89	0	27	0	0	24	0
N.S.	1	1.00	0.96	1.59	0.00	0.48	0.00	0.00	0.43	0.00
time (sec)	N/A	0.249	0.033	0.194	0.000	0.085	0.000	0.000	0.163	0.000

Problem 20	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F	B	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	86	86	62	159	0	156	0	0	24	0
N.S.	1	1.00	0.72	1.85	0.00	1.81	0.00	0.00	0.28	0.00
time (sec)	N/A	0.344	0.052	0.233	0.000	0.086	0.000	0.000	0.162	0.000

Problem 21	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	B	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	90	90	84	114	0	319	0	0	24	0
N.S.	1	1.00	0.93	1.27	0.00	3.54	0.00	0.00	0.27	0.00
time (sec)	N/A	0.346	0.075	0.228	0.000	0.089	0.000	0.000	0.160	0.000

Problem 22	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F	B	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	118	122	79	205	0	624	0	0	24	0
N.S.	1	1.03	0.67	1.74	0.00	5.29	0.00	0.00	0.20	0.00
time (sec)	N/A	0.481	0.124	0.242	0.000	0.094	0.000	0.000	0.161	0.000

Problem 23	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	91	96	65	122	0	101	0	0	23	0
N.S.	1	1.05	0.71	1.34	0.00	1.11	0.00	0.00	0.25	0.00
time (sec)	N/A	0.367	0.123	0.230	0.000	0.090	0.000	0.000	0.162	0.000

Problem 24	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F	B	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	62	62	55	169	0	91	0	0	22	0
N.S.	1	1.00	0.89	2.73	0.00	1.47	0.00	0.00	0.35	0.00
time (sec)	N/A	0.273	0.050	0.222	0.000	0.099	0.000	0.000	0.163	0.000

Problem 25	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	B	F	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	62	62	94	104	0	76	0	0	20	0
N.S.	1	1.00	1.52	1.68	0.00	1.23	0.00	0.00	0.32	0.00
time (sec)	N/A	0.288	0.101	0.235	0.000	0.097	0.000	0.000	0.168	0.000

Problem 26	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F	B	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	30	30	28	91	0	51	0	0	12	0
N.S.	1	1.00	0.93	3.03	0.00	1.70	0.00	0.00	0.40	0.00
time (sec)	N/A	0.186	0.022	0.306	0.000	0.091	0.000	0.000	0.156	0.000

Problem 27	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	30	30	28	68	0	17	0	0	23	0
N.S.	1	1.00	0.93	2.27	0.00	0.57	0.00	0.00	0.77	0.00
time (sec)	N/A	0.183	0.026	0.155	0.000	0.107	0.000	0.000	0.161	0.000

Problem 28	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	58	58	50	159	0	80	0	0	22	0
N.S.	1	1.00	0.86	2.74	0.00	1.38	0.00	0.00	0.38	0.00
time (sec)	N/A	0.272	0.075	0.203	0.000	0.095	0.000	0.000	0.166	0.000

Problem 29	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	B	F	B	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	62	62	83	113	0	122	0	0	22	0
N.S.	1	1.00	1.34	1.82	0.00	1.97	0.00	0.00	0.35	0.00
time (sec)	N/A	0.278	0.068	0.196	0.000	0.101	0.000	0.000	0.171	0.000

Problem 30	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F	B	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	91	92	80	204	0	165	0	0	21	0
N.S.	1	1.01	0.88	2.24	0.00	1.81	0.00	0.00	0.23	0.00
time (sec)	N/A	0.359	0.093	0.198	0.000	0.089	0.000	0.000	0.159	0.000

Problem 31	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	60	60	57	0	0	0	0	0	14	0
N.S.	1	1.00	0.95	0.00	0.00	0.00	0.00	0.00	0.23	0.00
time (sec)	N/A	0.212	0.040	0.000	0.000	0.000	0.000	0.000	0.167	0.000

Problem 32	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	60	60	57	0	0	0	0	0	14	0
N.S.	1	1.00	0.95	0.00	0.00	0.00	0.00	0.00	0.23	0.00
time (sec)	N/A	0.221	0.027	0.000	0.000	0.000	0.000	0.000	0.163	0.000

Problem 33	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	60	60	57	0	0	0	0	0	14	0
N.S.	1	1.00	0.95	0.00	0.00	0.00	0.00	0.00	0.23	0.00
time (sec)	N/A	0.205	0.026	0.000	0.000	0.000	0.000	0.000	0.164	0.000

Problem 34	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	60	60	57	0	0	0	0	0	14	0
N.S.	1	1.00	0.95	0.00	0.00	0.00	0.00	0.00	0.23	0.00
time (sec)	N/A	0.209	0.029	0.000	0.000	0.000	0.000	0.000	0.156	0.000

Problem 35	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	58	58	55	0	0	0	0	0	14	0
N.S.	1	1.00	0.95	0.00	0.00	0.00	0.00	0.00	0.24	0.00
time (sec)	N/A	0.202	0.029	0.000	0.000	0.000	0.000	0.000	0.163	0.000

Problem 36	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	58	58	55	0	0	0	0	0	14	0
N.S.	1	1.00	0.95	0.00	0.00	0.00	0.00	0.00	0.24	0.00
time (sec)	N/A	0.218	0.028	0.000	0.000	0.000	0.000	0.000	0.168	0.000

Problem 37	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	70	70	65	0	0	0	0	0	14	0
N.S.	1	1.00	0.93	0.00	0.00	0.00	0.00	0.00	0.20	0.00
time (sec)	N/A	0.220	0.033	0.000	0.000	0.000	0.000	0.000	0.157	0.000

Problem 38	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	72	72	67	0	0	0	0	0	14	0
N.S.	1	1.00	0.93	0.00	0.00	0.00	0.00	0.00	0.19	0.00
time (sec)	N/A	0.217	0.029	0.000	0.000	0.000	0.000	0.000	0.157	0.000

Problem 39	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	72	72	67	0	0	0	0	0	17	0
N.S.	1	1.00	0.93	0.00	0.00	0.00	0.00	0.00	0.24	0.00
time (sec)	N/A	0.221	0.030	0.000	0.000	0.000	0.000	0.000	0.157	0.000

Problem 40	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	A	A	A	A	A	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	46	53	134	51	59	67	58	50	13	50
N.S.	1	1.15	2.91	1.11	1.28	1.46	1.26	1.09	0.28	1.09
time (sec)	N/A	0.397	0.135	0.755	0.036	0.091	0.113	0.121	0.168	1.813

Problem 41	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	36	40	41	39	45	55	41	38	13	38
N.S.	1	1.11	1.14	1.08	1.25	1.53	1.14	1.06	0.36	1.06
time (sec)	N/A	0.274	0.089	0.459	0.035	0.092	0.096	0.117	0.161	1.734

Problem 42	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	A	B	B	A	A	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	22	30	79	25	33	37	20	26	17	24
N.S.	1	1.36	3.59	1.14	1.50	1.68	0.91	1.18	0.77	1.09
time (sec)	N/A	0.326	0.091	0.267	0.031	0.108	0.082	0.125	0.155	1.710

Problem 43	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	A	A	A	A	A	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	14	18	43	13	12	16	8	10	13	12
N.S.	1	1.29	3.07	0.93	0.86	1.14	0.57	0.71	0.93	0.86
time (sec)	N/A	0.247	0.046	0.169	0.039	0.083	0.054	0.123	0.165	1.712

Problem 44	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	B	F	A	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	19	24	30	21	29	33	0	24	11	35
N.S.	1	1.26	1.58	1.11	1.53	1.74	0.00	1.26	0.58	1.84
time (sec)	N/A	0.287	0.018	0.272	0.035	0.083	0.000	0.120	0.158	1.807

Problem 45	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	B	F	B	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	23	30	36	35	51	77	0	44	13	51
N.S.	1	1.30	1.57	1.52	2.22	3.35	0.00	1.91	0.57	2.22
time (sec)	N/A	0.366	0.032	0.392	0.041	0.087	0.000	0.123	0.153	1.943

Problem 46	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	B	F	A	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	37	45	49	53	75	126	0	51	13	70
N.S.	1	1.22	1.32	1.43	2.03	3.41	0.00	1.38	0.35	1.89
time (sec)	N/A	0.458	0.141	0.526	0.036	0.101	0.000	0.134	0.164	1.877

Problem 47	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	B	F	A	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	47	59	53	71	103	174	0	58	13	85
N.S.	1	1.26	1.13	1.51	2.19	3.70	0.00	1.23	0.28	1.81
time (sec)	N/A	0.470	0.170	0.708	0.035	0.080	0.000	0.117	0.166	1.966

Problem 48	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	A	A	B	A	A	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	58	63	147	52	71	89	70	50	20	97
N.S.	1	1.09	2.53	0.90	1.22	1.53	1.21	0.86	0.34	1.67
time (sec)	N/A	0.393	0.135	0.928	0.042	0.090	0.122	0.123	0.156	1.772

Problem 49	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	B	A	A	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	44	60	45	38	59	74	54	38	20	79
N.S.	1	1.36	1.02	0.86	1.34	1.68	1.23	0.86	0.45	1.80
time (sec)	N/A	0.552	0.092	0.569	0.043	0.095	0.099	0.112	0.162	1.761

Problem 50	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	B	A	A	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	32	43	55	26	40	50	41	22	20	71
N.S.	1	1.34	1.72	0.81	1.25	1.56	1.28	0.69	0.62	2.22
time (sec)	N/A	0.392	0.108	0.432	0.031	0.079	0.082	0.132	0.160	1.727

Problem 51	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	A	A	A	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	31	41	22	23	81	32	37	20	18	25
N.S.	1	1.32	0.71	0.74	2.61	1.03	1.19	0.65	0.58	0.81
time (sec)	N/A	0.274	0.009	0.268	0.040	0.073	0.079	0.130	0.163	1.681

Problem 52	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	B	F	A	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	34	51	26	36	55	78	0	34	18	41
N.S.	1	1.50	0.76	1.06	1.62	2.29	0.00	1.00	0.53	1.21
time (sec)	N/A	0.403	0.030	0.477	0.039	0.080	0.000	0.122	0.158	0.333

Problem 53	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	A	B	B	F	A	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	42	47	88	58	79	130	0	46	20	85
N.S.	1	1.12	2.10	1.38	1.88	3.10	0.00	1.10	0.48	2.02
time (sec)	N/A	0.557	1.110	0.682	0.033	0.104	0.000	0.127	0.159	1.953

Problem 54	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	A	B	B	F	A	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	58	74	140	72	105	174	0	59	20	79
N.S.	1	1.28	2.41	1.24	1.81	3.00	0.00	1.02	0.34	1.36
time (sec)	N/A	0.669	1.019	0.963	0.046	0.090	0.000	0.117	0.159	2.042

Problem 55	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	A	B	B	F(-1)	A	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	64	88	143	88	127	226	0	84	20	189
N.S.	1	1.38	2.23	1.38	1.98	3.53	0.00	1.31	0.31	2.95
time (sec)	N/A	0.697	1.979	1.300	0.044	0.090	0.000	0.125	0.157	2.388

Problem 56	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	27	27	42	18	20	16	15	15	14	17
N.S.	1	1.00	1.56	0.67	0.74	0.59	0.56	0.56	0.52	0.63
time (sec)	N/A	0.197	0.163	0.139	0.045	0.076	0.079	0.123	0.162	0.225

Problem 57	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	59	59	61	28	94	50	61	25	25	29
N.S.	1	1.00	1.03	0.47	1.59	0.85	1.03	0.42	0.42	0.49
time (sec)	N/A	0.295	0.246	0.246	0.038	0.088	0.132	0.124	0.161	1.719

Problem 58	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	A	A	A	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	88	93	81	40	211	85	109	36	37	40
N.S.	1	1.06	0.92	0.45	2.40	0.97	1.24	0.41	0.42	0.45
time (sec)	N/A	0.340	0.248	0.301	0.041	0.091	0.242	0.122	0.181	2.017

Problem 59	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	A	A	A	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	117	127	87	51	372	120	155	47	44	53
N.S.	1	1.09	0.74	0.44	3.18	1.03	1.32	0.40	0.38	0.45
time (sec)	N/A	0.454	0.166	0.365	0.046	0.088	0.335	0.119	0.181	2.273

Problem 60	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	27	27	42	18	20	16	17	15	16	17
N.S.	1	1.00	1.56	0.67	0.74	0.59	0.63	0.56	0.59	0.63
time (sec)	N/A	0.187	0.173	0.165	0.044	0.089	0.104	0.124	0.159	0.169

Problem 61	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	59	59	59	28	94	50	61	25	25	29
N.S.	1	1.00	1.00	0.47	1.59	0.85	1.03	0.42	0.42	0.49
time (sec)	N/A	0.258	0.156	0.243	0.048	0.076	0.164	0.121	0.157	1.723

Problem 62	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	A	A	A	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	88	93	81	40	211	85	109	36	35	40
N.S.	1	1.06	0.92	0.45	2.40	0.97	1.24	0.41	0.40	0.45
time (sec)	N/A	0.356	0.288	0.287	0.034	0.084	0.302	0.118	0.169	1.905

Problem 63	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	A	A	A	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	117	127	87	51	372	120	155	47	44	52
N.S.	1	1.09	0.74	0.44	3.18	1.03	1.32	0.40	0.38	0.44
time (sec)	N/A	0.449	0.169	0.363	0.046	0.073	0.423	0.123	0.181	2.226

Problem 64	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	57	65	75	108	0	76	0	0	52	0
N.S.	1	1.14	1.32	1.89	0.00	1.33	0.00	0.00	0.91	0.00
time (sec)	N/A	0.286	0.228	0.697	0.000	0.087	0.000	0.000	0.163	0.000

Problem 65	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	57	65	76	108	0	76	0	0	53	0
N.S.	1	1.14	1.33	1.89	0.00	1.33	0.00	0.00	0.93	0.00
time (sec)	N/A	0.291	0.238	0.783	0.000	0.082	0.000	0.000	0.164	0.000

Problem 66	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	104	108	145	93	0	101	0	0	67	0
N.S.	1	1.04	1.39	0.89	0.00	0.97	0.00	0.00	0.64	0.00
time (sec)	N/A	0.371	5.964	0.342	0.000	0.079	0.000	0.000	0.172	0.000

Problem 67	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	F	A	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	69	69	113	74	0	63	0	59	40	0
N.S.	1	1.00	1.64	1.07	0.00	0.91	0.00	0.86	0.58	0.00
time (sec)	N/A	0.280	5.314	0.274	0.000	0.079	0.000	0.209	0.164	0.000

Problem 68	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	A	F	A	F	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	31	31	74	55	0	27	0	0	16	53
N.S.	1	1.00	2.39	1.77	0.00	0.87	0.00	0.00	0.52	1.71
time (sec)	N/A	0.191	0.030	0.331	0.000	0.080	0.000	0.000	0.158	2.108

Problem 69	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F	B	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	52	52	84	127	0	93	0	0	68	0
N.S.	1	1.00	1.62	2.44	0.00	1.79	0.00	0.00	1.31	0.00
time (sec)	N/A	0.206	0.147	0.793	0.000	0.091	0.000	0.000	0.161	0.000

Problem 70	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F	B	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	87	87	156	176	0	235	0	0	104	0
N.S.	1	1.00	1.79	2.02	0.00	2.70	0.00	0.00	1.20	0.00
time (sec)	N/A	0.303	0.256	0.261	0.000	0.090	0.000	0.000	0.169	0.000

Problem 71	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F	B	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	122	127	210	218	0	348	0	0	67	0
N.S.	1	1.04	1.72	1.79	0.00	2.85	0.00	0.00	0.55	0.00
time (sec)	N/A	0.391	0.242	0.269	0.000	0.099	0.000	0.000	0.179	0.000

Problem 72	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	C	A	B	A	B	F(-1)	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	108	138	105	201	158	799	0	156	243	199
N.S.	1	1.28	0.97	1.86	1.46	7.40	0.00	1.44	2.25	1.84
time (sec)	N/A	0.815	1.544	0.402	0.119	0.100	0.000	0.130	0.198	2.106

Problem 73	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	C	A	B	A	B	F(-1)	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	82	106	82	152	118	459	0	117	173	159
N.S.	1	1.29	1.00	1.85	1.44	5.60	0.00	1.43	2.11	1.94
time (sec)	N/A	0.610	0.865	0.273	0.109	0.100	0.000	0.132	0.168	1.921

Problem 74	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	C	A	A	A	B	F(-1)	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	57	72	61	92	84	238	0	86	76	129
N.S.	1	1.26	1.07	1.61	1.47	4.18	0.00	1.51	1.33	2.26
time (sec)	N/A	0.459	0.227	0.157	0.111	0.097	0.000	0.134	0.159	1.832

Problem 75	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	C	A	A	A	B	C	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	47	61	52	63	65	134	170	67	58	99
N.S.	1	1.30	1.11	1.34	1.38	2.85	3.62	1.43	1.23	2.11
time (sec)	N/A	0.325	0.027	0.092	0.125	0.111	18.517	0.137	0.184	1.841

Problem 76	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	C	A	A	A	B	F	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	50	63	67	49	83	156	0	82	90	287
N.S.	1	1.26	1.34	0.98	1.66	3.12	0.00	1.64	1.80	5.74
time (sec)	N/A	0.380	0.278	0.156	0.125	0.114	0.000	0.126	0.173	2.008

Problem 77	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	C	A	A	A	B	F	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	59	75	90	73	100	345	0	98	242	292
N.S.	1	1.27	1.53	1.24	1.69	5.85	0.00	1.66	4.10	4.95
time (sec)	N/A	0.489	0.667	0.194	0.115	0.109	0.000	0.128	0.161	2.004

Problem 78	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	C	A	A	B	B	F	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	81	113	136	108	154	929	0	137	519	617
N.S.	1	1.40	1.68	1.33	1.90	11.47	0.00	1.69	6.41	7.62
time (sec)	N/A	0.836	0.619	0.254	0.118	0.190	0.000	0.133	0.170	2.345

Problem 79	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	C	A	A	A	B	F	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	109	142	211	151	194	1676	0	171	735	694
N.S.	1	1.30	1.94	1.39	1.78	15.38	0.00	1.57	6.74	6.37
time (sec)	N/A	1.023	0.927	0.345	0.126	0.182	0.000	0.129	0.166	2.131

Problem 80	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	C	A	A	A	B	F(-1)	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	162	188	118	218	256	1769	0	235	719	305
N.S.	1	1.16	0.73	1.35	1.58	10.92	0.00	1.45	4.44	1.88
time (sec)	N/A	1.002	0.943	0.498	0.159	0.111	0.000	0.135	0.176	2.102

Problem 81	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	C	A	A	A	B	F(-1)	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	115	146	95	161	208	1053	0	184	574	274
N.S.	1	1.27	0.83	1.40	1.81	9.16	0.00	1.60	4.99	2.38
time (sec)	N/A	0.692	0.397	0.308	0.129	0.108	0.000	0.135	0.167	2.016

Problem 82	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	B	F(-1)	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	83	111	86	127	149	521	0	131	452	228
N.S.	1	1.34	1.04	1.53	1.80	6.28	0.00	1.58	5.45	2.75
time (sec)	N/A	0.505	0.186	0.182	0.111	0.092	0.000	0.136	0.169	2.026

Problem 83	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	C	A	A	B	B	F(-1)	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	60	74	68	97	117	341	0	99	226	142
N.S.	1	1.23	1.13	1.62	1.95	5.68	0.00	1.65	3.77	2.37
time (sec)	N/A	0.335	0.120	0.152	0.112	0.084	0.000	0.129	0.167	1.916

Problem 84	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	C	A	A	A	B	F	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	85	122	100	115	162	672	0	142	627	1001
N.S.	1	1.44	1.18	1.35	1.91	7.91	0.00	1.67	7.38	11.78
time (sec)	N/A	0.621	0.425	0.259	0.116	0.186	0.000	0.136	0.172	4.280

Problem 85	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	C	A	A	B	B	F	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	115	144	128	141	251	1740	0	205	1168	1017
N.S.	1	1.25	1.11	1.23	2.18	15.13	0.00	1.78	10.16	8.84
time (sec)	N/A	0.900	0.744	0.316	0.162	0.196	0.000	0.127	0.170	5.048

Problem 86	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	C	A	A	B	B	F	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	158	196	174	175	363	3754	0	203	1964	977
N.S.	1	1.24	1.10	1.11	2.30	23.76	0.00	1.28	12.43	6.18
time (sec)	N/A	1.467	0.909	0.402	0.154	0.328	0.000	0.132	0.172	5.130

Problem 87	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	C	A	A	B	B	F	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	198	228	232	219	477	6430	0	236	2608	975
N.S.	1	1.15	1.17	1.11	2.41	32.47	0.00	1.19	13.17	4.92
time (sec)	N/A	1.872	1.061	0.500	0.133	0.336	0.000	0.129	0.174	4.693

Problem 88	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	53	50	81	36	19	28	31	32	18	39
N.S.	1	0.94	1.53	0.68	0.36	0.53	0.58	0.60	0.34	0.74
time (sec)	N/A	0.223	0.120	0.306	0.107	0.079	0.234	0.125	0.163	0.345

Problem 89	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	82	82	142	74	79	103	75	67	27	106
N.S.	1	1.00	1.73	0.90	0.96	1.26	0.91	0.82	0.33	1.29
time (sec)	N/A	0.334	0.356	0.317	0.116	0.084	0.294	0.125	0.162	2.115

Problem 90	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	B	A	A	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	111	116	204	100	124	193	138	89	38	147
N.S.	1	1.05	1.84	0.90	1.12	1.74	1.24	0.80	0.34	1.32
time (sec)	N/A	0.445	0.516	0.569	0.133	0.112	0.347	0.124	0.187	2.167

Problem 91	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	B	A	A	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	140	150	265	123	167	283	197	111	46	237
N.S.	1	1.07	1.89	0.88	1.19	2.02	1.41	0.79	0.33	1.69
time (sec)	N/A	0.594	0.528	0.678	0.129	0.096	0.412	0.121	0.181	2.467

Problem 92	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	37	37	171	32	36	26	31	28	33	32
N.S.	1	1.00	4.62	0.86	0.97	0.70	0.84	0.76	0.89	0.86
time (sec)	N/A	0.200	0.119	0.276	0.159	0.089	0.154	0.122	0.163	1.725

Problem 93	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	A	A	A	A	A	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	66	71	183	73	64	103	82	65	27	102
N.S.	1	1.08	2.77	1.11	0.97	1.56	1.24	0.98	0.41	1.55
time (sec)	N/A	0.278	0.340	0.334	0.116	0.100	0.200	0.120	0.163	2.104

Problem 94	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	A	A	B	A	A	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	95	105	277	96	108	193	141	87	38	143
N.S.	1	1.11	2.92	1.01	1.14	2.03	1.48	0.92	0.40	1.51
time (sec)	N/A	0.398	0.686	0.463	0.144	0.098	0.283	0.115	0.182	2.622

Problem 95	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	A	A	B	A	A	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	124	139	308	119	152	283	202	109	46	232
N.S.	1	1.12	2.48	0.96	1.23	2.28	1.63	0.88	0.37	1.87
time (sec)	N/A	0.550	1.376	0.581	0.121	0.100	0.351	0.129	0.185	3.202

Problem 96	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	183	196	138	155	272	223	314	269	354	160
N.S.	1	1.07	0.75	0.85	1.49	1.22	1.72	1.47	1.93	0.87
time (sec)	N/A	0.741	0.901	77.634	0.036	0.090	0.351	0.119	0.157	0.638

Problem 97	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	137	145	108	119	182	146	240	200	251	114
N.S.	1	1.06	0.79	0.87	1.33	1.07	1.75	1.46	1.83	0.83
time (sec)	N/A	0.512	0.462	10.640	0.043	0.076	0.333	0.131	0.166	0.366

Problem 98	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	92	95	71	77	115	91	128	135	161	75
N.S.	1	1.03	0.77	0.84	1.25	0.99	1.39	1.47	1.75	0.82
time (sec)	N/A	0.336	0.236	159.816	0.045	0.076	0.178	0.114	0.153	1.558

Problem 99	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	52	52	48	48	55	46	78	76	96	41
N.S.	1	1.00	0.92	0.92	1.06	0.88	1.50	1.46	1.85	0.79
time (sec)	N/A	0.209	0.134	0.502	0.033	0.070	0.146	0.122	0.157	1.557

Problem 100	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	15	15	26	16	15	17	17	31	17	15
N.S.	1	1.00	1.73	1.07	1.00	1.13	1.13	2.07	1.13	1.00
time (sec)	N/A	0.170	0.032	0.148	0.026	0.085	0.082	0.120	0.152	1.501

Problem 101	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	B	C	A	B	B
verified	N/A	No	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	44	42	52	43	67	162	155	67	49	55
N.S.	1	0.95	1.18	0.98	1.52	3.68	3.52	1.52	1.11	1.25
time (sec)	N/A	0.245	0.054	0.129	0.121	0.081	3.873	0.132	0.159	1.811

Problem 102	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	B	B	A	B	B
verified	N/A	No	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	79	77	85	118	138	423	2082	119	291	200
N.S.	1	0.97	1.08	1.49	1.75	5.35	26.35	1.51	3.68	2.53
time (sec)	N/A	0.350	0.216	0.197	0.117	0.094	81.164	0.125	0.157	1.966

Problem 103	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	B	B	F(-1)	A	B	F(-1)
verified	N/A	No	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	127	137	117	280	315	1347	0	231	1033	0
N.S.	1	1.08	0.92	2.20	2.48	10.61	0.00	1.82	8.13	0.00
time (sec)	N/A	0.577	0.228	0.361	0.145	0.108	0.000	0.126	0.163	0.000

Problem 104	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	B	B	F(-1)	B	B	F(-1)
verified	N/A	No	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	174	198	159	494	551	2934	0	357	1722	0
N.S.	1	1.14	0.91	2.84	3.17	16.86	0.00	2.05	9.90	0.00
time (sec)	N/A	0.874	0.497	0.771	0.147	0.140	0.000	0.141	0.172	0.000

Problem 105	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F	B	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	179	185	178	917	0	452	0	0	48	0
N.S.	1	1.03	0.99	5.12	0.00	2.53	0.00	0.00	0.27	0.00
time (sec)	N/A	0.946	0.306	1.408	0.000	0.098	0.000	0.000	0.156	0.000

Problem 106	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F	B	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	150	151	139	676	0	257	0	0	26	0
N.S.	1	1.01	0.93	4.51	0.00	1.71	0.00	0.00	0.17	0.00
time (sec)	N/A	0.873	0.254	0.886	0.000	0.089	0.000	0.000	0.155	0.000

Problem 107	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F	B	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	60	60	65	262	0	174	0	0	9	0
N.S.	1	1.00	1.08	4.37	0.00	2.90	0.00	0.00	0.15	0.00
time (sec)	N/A	0.387	0.131	0.933	0.000	0.089	0.000	0.000	0.152	0.000

Problem 108	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	60	60	60	125	0	61	0	0	18	0
N.S.	1	1.00	1.00	2.08	0.00	1.02	0.00	0.00	0.30	0.00
time (sec)	N/A	0.443	0.132	0.254	0.000	0.079	0.000	0.000	0.152	0.000

Problem 109	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F	B	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	94	94	81	456	0	376	0	0	30	0
N.S.	1	1.00	0.86	4.85	0.00	4.00	0.00	0.00	0.32	0.00
time (sec)	N/A	0.653	0.114	0.335	0.000	0.103	0.000	0.000	0.149	0.000

Problem 110	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F	B	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	197	205	166	438	0	1198	0	0	42	0
N.S.	1	1.04	0.84	2.22	0.00	6.08	0.00	0.00	0.21	0.00
time (sec)	N/A	1.219	0.458	0.455	0.000	0.119	0.000	0.000	0.151	0.000

Problem 111	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	128	128	101	218	0	174	0	0	20	0
N.S.	1	1.00	0.79	1.70	0.00	1.36	0.00	0.00	0.16	0.00
time (sec)	N/A	0.555	0.300	0.398	0.000	0.088	0.000	0.000	0.148	0.000

Problem 112	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	A	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	112	109	100	0	0	126	0	0	100	0
N.S.	1	0.97	0.89	0.00	0.00	1.12	0.00	0.00	0.89	0.00
time (sec)	N/A	0.472	2.682	0.000	0.000	0.087	0.000	0.000	0.168	0.000

Problem 113	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	81	81	83	0	0	82	0	0	62	0
N.S.	1	1.00	1.02	0.00	0.00	1.01	0.00	0.00	0.77	0.00
time (sec)	N/A	0.377	1.065	0.000	0.000	0.078	0.000	0.000	0.161	0.000

Problem 114	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	48	52	66	0	0	49	0	0	29	0
N.S.	1	1.08	1.38	0.00	0.00	1.02	0.00	0.00	0.60	0.00
time (sec)	N/A	0.287	0.145	0.000	0.000	0.081	0.000	0.000	0.154	0.000

Problem 115	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	A	A	A	A	A	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	23	25	53	26	26	23	15	17	26	21
N.S.	1	1.09	2.30	1.13	1.13	1.00	0.65	0.74	1.13	0.91
time (sec)	N/A	0.249	0.180	0.220	0.040	0.103	0.075	0.121	0.151	0.120

Problem 116	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	A	A	A	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	43	47	32	36	141	43	53	32	38	39
N.S.	1	1.09	0.74	0.84	3.28	1.00	1.23	0.74	0.88	0.91
time (sec)	N/A	0.263	0.025	0.306	0.040	0.080	0.169	0.118	0.150	1.518

Problem 117	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	A	A	A	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	68	66	50	51	267	70	82	46	54	52
N.S.	1	0.97	0.74	0.75	3.93	1.03	1.21	0.68	0.79	0.76
time (sec)	N/A	0.373	0.033	0.444	0.046	0.075	0.256	0.117	0.158	1.740

Problem 118	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	A	A	A	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	91	89	67	66	469	95	110	60	64	66
N.S.	1	0.98	0.74	0.73	5.15	1.04	1.21	0.66	0.70	0.73
time (sec)	N/A	0.667	0.039	0.654	0.041	0.080	0.415	0.123	0.166	1.994

Problem 119	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	27	28	34	27	27	24	15	18	32	21
N.S.	1	1.04	1.26	1.00	1.00	0.89	0.56	0.67	1.19	0.78
time (sec)	N/A	0.434	0.501	0.171	0.030	0.076	0.080	0.124	0.154	0.112

Problem 120	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	A	A	A	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	49	53	32	36	141	43	51	32	38	37
N.S.	1	1.08	0.65	0.73	2.88	0.88	1.04	0.65	0.78	0.76
time (sec)	N/A	0.427	0.022	0.296	0.049	0.080	0.145	0.119	0.154	1.503

Problem 121	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	A	A	A	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	76	74	49	51	267	70	82	46	52	52
N.S.	1	0.97	0.64	0.67	3.51	0.92	1.08	0.61	0.68	0.68
time (sec)	N/A	0.620	0.080	0.441	0.053	0.082	0.251	0.119	0.156	1.732

Problem 122	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	A	A	A	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	101	99	63	66	469	95	109	60	64	68
N.S.	1	0.98	0.62	0.65	4.64	0.94	1.08	0.59	0.63	0.67
time (sec)	N/A	0.638	0.038	0.586	0.057	0.093	0.438	0.120	0.161	1.960

Problem 123	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F	B	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	66	67	85	126	0	188	0	0	99	0
N.S.	1	1.02	1.29	1.91	0.00	2.85	0.00	0.00	1.50	0.00
time (sec)	N/A	0.330	0.211	1.221	0.000	0.091	0.000	0.000	0.151	0.000

Problem 124	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	B	F	F	F	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	79	79	105	0	0	264	0	0	139	0
N.S.	1	1.00	1.33	0.00	0.00	3.34	0.00	0.00	1.76	0.00
time (sec)	N/A	0.316	0.336	0.000	0.000	0.092	0.000	0.000	0.161	0.000

Problem 125	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	B	F(-1)	F	F	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	110	110	184	0	0	347	0	0	94	0
N.S.	1	1.00	1.67	0.00	0.00	3.15	0.00	0.00	0.85	0.00
time (sec)	N/A	0.427	0.315	0.000	0.000	0.105	0.000	0.000	0.153	0.000

Problem 126	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F	B	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	259	270	241	1878	0	1121	0	0	70	0
N.S.	1	1.04	0.93	7.25	0.00	4.33	0.00	0.00	0.27	0.00
time (sec)	N/A	1.515	0.591	4.022	0.000	0.130	0.000	0.000	0.160	0.000

Problem 127	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F	B	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	207	213	196	1033	0	623	0	0	48	0
N.S.	1	1.03	0.95	4.99	0.00	3.01	0.00	0.00	0.23	0.00
time (sec)	N/A	1.097	0.410	2.500	0.000	0.124	0.000	0.000	0.152	0.000

Problem 128	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F	B	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	164	165	151	731	0	318	0	0	26	0
N.S.	1	1.01	0.92	4.46	0.00	1.94	0.00	0.00	0.16	0.00
time (sec)	N/A	1.198	0.307	1.658	0.000	0.102	0.000	0.000	0.150	0.000

Problem 129	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	B	C	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	55	60	61	72	124	147	309	75	1	269
N.S.	1	1.09	1.11	1.31	2.25	2.67	5.62	1.36	0.02	4.89
time (sec)	N/A	0.498	0.141	0.139	0.134	0.103	21.072	0.129	0.147	1.849

Problem 130	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	B	F(-1)	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	74	79	82	113	229	444	0	119	42	223
N.S.	1	1.07	1.11	1.53	3.09	6.00	0.00	1.61	0.57	3.01
time (sec)	N/A	0.373	0.136	0.184	0.125	0.135	0.000	0.129	0.145	1.791

Problem 131	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	B	B	F(-1)	B	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	128	145	131	314	537	1614	0	279	230	0
N.S.	1	1.13	1.02	2.45	4.20	12.61	0.00	2.18	1.80	0.00
time (sec)	N/A	0.559	0.205	0.307	0.139	0.114	0.000	0.141	0.147	0.000

Problem 132	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	B	B	F(-1)	B	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	187	218	189	633	982	3870	0	477	815	0
N.S.	1	1.17	1.01	3.39	5.25	20.70	0.00	2.55	4.36	0.00
time (sec)	N/A	0.861	0.311	0.523	0.175	0.242	0.000	0.148	0.158	0.000

Problem 133	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	B	C	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	60	65	66	80	128	154	258	82	97	331
N.S.	1	1.08	1.10	1.33	2.13	2.57	4.30	1.37	1.62	5.52
time (sec)	N/A	0.329	0.105	0.162	0.116	0.103	18.818	0.133	0.149	2.129

Problem 134	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	6	6	6	7	128	6	3	6	1	6
N.S.	1	1.00	1.00	1.17	21.33	1.00	0.50	1.00	0.17	1.00
time (sec)	N/A	0.148	0.000	0.053	0.136	0.074	0.102	0.124	0.157	0.021

Problem 135	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	B	B	F(-1)	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	12	12	12	26	230	58	0	30	29	49
N.S.	1	1.00	1.00	2.17	19.17	4.83	0.00	2.50	2.42	4.08
time (sec)	N/A	0.208	0.156	0.179	0.145	0.073	0.000	0.135	0.160	1.480

Problem 136	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	34	36	28	33	34	42	51	33	30	48
N.S.	1	1.06	0.82	0.97	1.00	1.24	1.50	0.97	0.88	1.41
time (sec)	N/A	0.257	0.068	0.111	0.116	0.087	1.040	0.132	0.148	1.537

Problem 137	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	136	136	109	266	0	184	0	0	9	0
N.S.	1	1.00	0.80	1.96	0.00	1.35	0.00	0.00	0.07	0.00
time (sec)	N/A	0.606	0.382	1.126	0.000	0.092	0.000	0.000	0.147	0.000

Problem 138	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F	B	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	176	185	159	517	0	597	0	0	18	0
N.S.	1	1.05	0.90	2.94	0.00	3.39	0.00	0.00	0.10	0.00
time (sec)	N/A	0.842	0.488	1.289	0.000	0.107	0.000	0.000	0.149	0.000

Problem 139	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F	B	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	251	259	236	806	0	2069	0	0	30	0
N.S.	1	1.03	0.94	3.21	0.00	8.24	0.00	0.00	0.12	0.00
time (sec)	N/A	1.378	0.601	1.944	0.000	0.157	0.000	0.000	0.164	0.000

Problem 140	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	B	F	B	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	53	56	36	32	53	511	0	102	51	0
N.S.	1	1.06	0.68	0.60	1.00	9.64	0.00	1.92	0.96	0.00
time (sec)	N/A	0.423	0.057	0.439	0.153	0.120	0.000	0.125	0.159	0.000

Problem 141	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	B	F	B	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	34	34	26	24	35	226	0	70	33	0
N.S.	1	1.00	0.76	0.71	1.03	6.65	0.00	2.06	0.97	0.00
time (sec)	N/A	0.337	0.052	0.115	0.127	0.094	0.000	0.123	0.154	0.000

Problem 142	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	B	A	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	13	13	13	15	17	71	15	34	5	21
N.S.	1	1.00	1.00	1.15	1.31	5.46	1.15	2.62	0.38	1.62
time (sec)	N/A	0.239	0.022	0.109	0.133	0.106	0.152	0.119	0.156	1.403

Problem 143	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	A	B	F	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	17	17	17	49	24	110	0	1	21	0
N.S.	1	1.00	1.00	2.88	1.41	6.47	0.00	0.06	1.24	0.00
time (sec)	N/A	0.229	0.005	0.120	0.127	0.087	0.000	0.125	0.155	0.000

Problem 144	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	A	B	F	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	42	42	53	71	62	327	0	37	101	0
N.S.	1	1.00	1.26	1.69	1.48	7.79	0.00	0.88	2.40	0.00
time (sec)	N/A	0.307	0.065	0.149	0.127	0.094	0.000	0.126	0.149	0.000

Problem 145	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	B	F	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	61	69	77	89	96	875	0	52	184	0
N.S.	1	1.13	1.26	1.46	1.57	14.34	0.00	0.85	3.02	0.00
time (sec)	N/A	0.410	0.156	0.140	0.135	0.104	0.000	0.128	0.150	0.000

Problem 146	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	B	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	135	121	67	0	0	802	0	0	16	0
N.S.	1	0.90	0.50	0.00	0.00	5.94	0.00	0.00	0.12	0.00
time (sec)	N/A	0.588	0.150	0.000	0.000	0.112	0.000	0.000	0.147	0.000

Problem 147	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	B	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	83	85	57	0	0	305	0	0	14	0
N.S.	1	1.02	0.69	0.00	0.00	3.67	0.00	0.00	0.17	0.00
time (sec)	N/A	0.419	0.063	0.000	0.000	0.098	0.000	0.000	0.148	0.000

Problem 148	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	F	F	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	62	67	60	0	0	55	0	0	11	0
N.S.	1	1.08	0.97	0.00	0.00	0.89	0.00	0.00	0.18	0.00
time (sec)	N/A	0.351	0.080	0.000	0.000	0.096	0.000	0.000	0.154	0.000

Problem 149	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	60	63	42	0	0	84	0	0	16	0
N.S.	1	1.05	0.70	0.00	0.00	1.40	0.00	0.00	0.27	0.00
time (sec)	N/A	0.347	0.036	0.000	0.000	0.083	0.000	0.000	0.151	0.000

Problem 150	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	B	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	87	87	53	0	0	561	0	0	16	0
N.S.	1	1.00	0.61	0.00	0.00	6.45	0.00	0.00	0.18	0.00
time (sec)	N/A	0.420	0.074	0.000	0.000	0.103	0.000	0.000	0.152	0.000

Problem 151	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	B	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	135	117	69	0	0	1477	0	0	16	0
N.S.	1	0.87	0.51	0.00	0.00	10.94	0.00	0.00	0.12	0.00
time (sec)	N/A	0.592	0.159	0.000	0.000	0.129	0.000	0.000	0.151	0.000

Problem 152	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	B	F	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	132	93	53	171	100	1597	0	76	87	0
N.S.	1	0.70	0.40	1.30	0.76	12.10	0.00	0.58	0.66	0.00
time (sec)	N/A	0.492	0.134	7.424	0.137	0.167	0.000	0.124	0.151	0.000

Problem 153	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	B	F	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	78	61	38	125	63	659	0	50	55	0
N.S.	1	0.78	0.49	1.60	0.81	8.45	0.00	0.64	0.71	0.00
time (sec)	N/A	0.358	0.085	0.222	0.165	0.111	0.000	0.131	0.150	0.000

Problem 154	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	A	B	F	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	36	30	24	84	27	180	0	26	26	0
N.S.	1	0.83	0.67	2.33	0.75	5.00	0.00	0.72	0.72	0.00
time (sec)	N/A	0.227	0.048	0.243	0.148	0.080	0.000	0.119	0.153	0.000

Problem 155	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	B	F	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	16	16	16	29	18	122	0	13	21	38
N.S.	1	1.00	1.00	1.81	1.12	7.62	0.00	0.81	1.31	2.38
time (sec)	N/A	0.242	0.027	0.226	0.129	0.077	0.000	0.125	0.149	1.330

Problem 156	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	C	A	A	B	B	F	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	68	48	34	48	171	1163	0	27	60	48
N.S.	1	0.71	0.50	0.71	2.51	17.10	0.00	0.40	0.88	0.71
time (sec)	N/A	0.261	0.051	0.260	0.144	0.114	0.000	0.132	0.155	1.469

Problem 157	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	C	A	A	B	B	F	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	118	68	47	60	467	3093	0	39	102	256
N.S.	1	0.58	0.40	0.51	3.96	26.21	0.00	0.33	0.86	2.17
time (sec)	N/A	0.273	0.066	0.254	0.133	0.171	0.000	0.122	0.152	1.416

Problem 158	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	B	B	B	B	B	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	50	57	219	186	90	91	124	86	13	93
N.S.	1	1.14	4.38	3.72	1.80	1.82	2.48	1.72	0.26	1.86
time (sec)	N/A	0.359	0.118	0.092	0.031	0.093	0.205	0.126	0.150	1.648

Problem 159	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	B	B	B	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	43	43	42	39	75	72	100	71	13	77
N.S.	1	1.00	0.98	0.91	1.74	1.67	2.33	1.65	0.30	1.79
time (sec)	N/A	0.238	0.024	0.078	0.051	0.083	0.181	0.125	0.153	1.533

Problem 160	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	B	B	B	B	B	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	38	42	131	138	66	67	82	62	13	67
N.S.	1	1.11	3.45	3.63	1.74	1.76	2.16	1.63	0.34	1.76
time (sec)	N/A	0.325	0.165	0.080	0.032	0.092	0.148	0.121	0.153	1.483

Problem 161	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	B	B	B	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	33	33	28	26	51	48	63	47	13	51
N.S.	1	1.00	0.85	0.79	1.55	1.45	1.91	1.42	0.39	1.55
time (sec)	N/A	0.237	0.017	91.653	0.042	0.097	0.141	0.122	0.153	1.439

Problem 162	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	B	B	B	B	B	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	26	27	93	42	42	41	48	38	13	41
N.S.	1	1.04	3.58	1.62	1.62	1.58	1.85	1.46	0.50	1.58
time (sec)	N/A	0.275	0.117	12.163	0.042	0.097	0.099	0.123	0.166	0.133

Problem 163	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	B	B	B	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	14	14	12	13	27	22	27	23	13	31
N.S.	1	1.00	0.86	0.93	1.93	1.57	1.93	1.64	0.93	2.21
time (sec)	N/A	0.217	0.009	3.791	0.031	0.084	0.089	0.124	0.153	1.430

Problem 164	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	B	B	B	B	B	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	8	8	34	16	14	17	14	14	13	7
N.S.	1	1.00	4.25	2.00	1.75	2.12	1.75	1.75	1.62	0.88
time (sec)	N/A	0.207	0.034	1.272	0.035	0.093	0.084	0.118	0.148	1.428

Problem 165	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	B	A	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	7	7	7	7	5	11	8	11	5	10
N.S.	1	1.00	1.00	1.00	0.71	1.57	1.14	1.57	0.71	1.43
time (sec)	N/A	0.206	0.006	0.378	0.040	0.091	0.052	0.121	0.159	0.078

Problem 166	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	B	F	B	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	24	24	18	30	41	52	0	51	11	46
N.S.	1	1.00	0.75	1.25	1.71	2.17	0.00	2.12	0.46	1.92
time (sec)	N/A	0.232	0.019	1.507	0.044	0.100	0.000	0.117	0.155	0.211

Problem 167	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	A	F	A	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	25	25	22	24	53	26	0	29	13	63
N.S.	1	1.00	0.88	0.96	2.12	1.04	0.00	1.16	0.52	2.52
time (sec)	N/A	0.291	0.027	5.172	0.034	0.068	0.000	0.121	0.156	1.516

Problem 168	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	B	F	B	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	52	52	61	63	92	143	0	92	13	115
N.S.	1	1.00	1.17	1.21	1.77	2.75	0.00	1.77	0.25	2.21
time (sec)	N/A	0.271	0.033	19.833	0.040	0.114	0.000	0.121	0.158	1.845

Problem 169	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	B	F	B	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	37	40	35	37	205	62	0	53	13	231
N.S.	1	1.08	0.95	1.00	5.54	1.68	0.00	1.43	0.35	6.24
time (sec)	N/A	0.309	0.051	143.934	0.041	0.072	0.000	0.127	0.157	1.947

Problem 170	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	B	B	F	B	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	80	80	94	137	140	245	0	118	13	249
N.S.	1	1.00	1.18	1.71	1.75	3.06	0.00	1.48	0.16	3.11
time (sec)	N/A	0.288	0.051	0.079	0.042	0.109	0.000	0.129	0.155	2.732

Problem 171	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	B	A	A	A	A	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	40	50	121	112	54	55	65	50	20	54
N.S.	1	1.25	3.02	2.80	1.35	1.38	1.62	1.25	0.50	1.35
time (sec)	N/A	0.396	0.140	0.086	0.035	0.091	0.144	0.128	0.152	0.150

Problem 172	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	B	B	B	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	14	14	18	12	39	34	44	35	20	37
N.S.	1	1.00	1.29	0.86	2.79	2.43	3.14	2.50	1.43	2.64
time (sec)	N/A	0.222	0.015	160.729	0.034	0.091	0.126	0.120	0.153	0.100

Problem 173	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	30	31	46	29	30	31	29	26	20	28
N.S.	1	1.03	1.53	0.97	1.00	1.03	0.97	0.87	0.67	0.93
time (sec)	N/A	0.306	0.055	17.561	0.032	0.079	0.094	0.123	0.161	1.321

Problem 174	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	B	B	B	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	14	14	14	25	23	26	26	21	20	24
N.S.	1	1.00	1.00	1.79	1.64	1.86	1.86	1.50	1.43	1.71
time (sec)	N/A	0.209	0.012	6.559	0.038	0.095	0.126	0.121	0.159	1.391

Problem 175	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	A	A	A	A	A	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	14	14	69	13	12	16	8	10	20	12
N.S.	1	1.00	4.93	0.93	0.86	1.14	0.57	0.71	1.43	0.86
time (sec)	N/A	0.191	0.047	2.231	0.044	0.089	0.087	0.121	0.161	1.399

Problem 176	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	A	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	10	10	10	10	8	16	19	10	18	12
N.S.	1	1.00	1.00	1.00	0.80	1.60	1.90	1.00	1.80	1.20
time (sec)	N/A	0.189	0.019	1.889	0.026	0.071	0.070	0.121	0.159	1.413

Problem 177	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	B	F	B	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	34	34	26	45	70	103	0	66	18	86
N.S.	1	1.00	0.76	1.32	2.06	3.03	0.00	1.94	0.53	2.53
time (sec)	N/A	0.222	0.034	8.444	0.041	0.086	0.000	0.134	0.156	1.672

Problem 178	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	A	F	A	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	37	48	31	30	117	44	0	41	20	109
N.S.	1	1.30	0.84	0.81	3.16	1.19	0.00	1.11	0.54	2.95
time (sec)	N/A	0.348	0.019	24.708	0.043	0.079	0.000	0.126	0.158	1.579

Problem 179	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	B	F	B	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	60	60	68	80	120	201	0	105	20	198
N.S.	1	1.00	1.13	1.33	2.00	3.35	0.00	1.75	0.33	3.30
time (sec)	N/A	0.254	0.035	169.529	0.049	0.091	0.000	0.132	0.154	2.057

Problem 180	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	B	B	F	A	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	49	65	47	116	317	80	0	65	20	139
N.S.	1	1.33	0.96	2.37	6.47	1.63	0.00	1.33	0.41	2.84
time (sec)	N/A	0.380	0.030	0.085	0.042	0.090	0.000	0.132	0.165	0.533

Problem 181	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	B	A	A	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	28	28	47	26	33	50	31	21	324	41
N.S.	1	1.00	1.68	0.93	1.18	1.79	1.11	0.75	11.57	1.46
time (sec)	N/A	0.225	0.070	35.277	0.048	0.109	0.107	0.122	0.179	0.186

Problem 182	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	A	B	A	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	20	20	16	19	53	28	34	16	338	19
N.S.	1	1.00	0.80	0.95	2.65	1.40	1.70	0.80	16.90	0.95
time (sec)	N/A	0.203	0.045	33.895	0.037	0.079	0.108	0.125	0.179	1.599

Problem 183	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	B	B	A	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	16	16	14	13	10	30	37	12	28	16
N.S.	1	1.00	0.88	0.81	0.62	1.88	2.31	0.75	1.75	1.00
time (sec)	N/A	0.184	0.038	33.938	0.041	0.079	0.098	0.124	0.166	1.556

Problem 184	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	B	A	A	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	26	28	43	26	33	50	31	21	325	39
N.S.	1	1.08	1.65	1.00	1.27	1.92	1.19	0.81	12.50	1.50
time (sec)	N/A	0.221	0.069	36.039	0.042	0.083	0.120	0.114	0.181	0.170

Problem 185	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	A	A	A	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	20	20	16	19	53	28	34	16	338	19
N.S.	1	1.00	0.80	0.95	2.65	1.40	1.70	0.80	16.90	0.95
time (sec)	N/A	0.202	0.048	34.632	0.041	0.085	0.096	0.127	0.171	1.550

Problem 186	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	B	B	A	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	16	16	14	13	10	30	36	12	26	16
N.S.	1	1.00	0.88	0.81	0.62	1.88	2.25	0.75	1.62	1.00
time (sec)	N/A	0.188	0.037	34.516	0.035	0.071	0.088	0.120	0.158	0.214

Problem 187	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	B	F(-1)	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	138	138	121	146	308	2105	0	254	452	287
N.S.	1	1.00	0.88	1.06	2.23	15.25	0.00	1.84	3.28	2.08
time (sec)	N/A	0.354	0.115	122.621	0.040	0.116	0.000	0.135	0.199	2.143

Problem 188	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	B	B	B	F(-1)	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	145	169	463	344	283	1486	0	288	373	302
N.S.	1	1.17	3.19	2.37	1.95	10.25	0.00	1.99	2.57	2.08
time (sec)	N/A	0.984	4.413	55.250	0.123	0.106	0.000	0.148	0.162	2.081

Problem 189	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	B	F(-1)	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	81	80	76	82	180	865	0	139	261	169
N.S.	1	0.99	0.94	1.01	2.22	10.68	0.00	1.72	3.22	2.09
time (sec)	N/A	0.290	0.062	22.773	0.043	0.117	0.000	0.125	0.161	1.723

Problem 190	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	A	B	F(-1)	A	B	B
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	97	113	553	162	170	569	0	168	192	200
N.S.	1	1.16	5.70	1.67	1.75	5.87	0.00	1.73	1.98	2.06
time (sec)	N/A	0.649	1.660	8.224	0.120	0.102	0.000	0.154	0.163	1.709

Problem 191	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	B	F(-1)	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	38	38	38	37	81	221	0	61	119	77
N.S.	1	1.00	1.00	0.97	2.13	5.82	0.00	1.61	3.13	2.03
time (sec)	N/A	0.249	0.030	2.891	0.041	0.088	0.000	0.130	0.161	1.479

Problem 192	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	A	B	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	54	64	396	93	81	171	377	83	62	87
N.S.	1	1.19	7.33	1.72	1.50	3.17	6.98	1.54	1.15	1.61
time (sec)	N/A	0.419	0.669	0.928	0.137	0.089	79.563	0.139	0.161	1.488

Problem 193	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	B	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	11	11	11	12	11	27	14	22	11	11
N.S.	1	1.00	1.00	1.09	1.00	2.45	1.27	2.00	1.00	1.00
time (sec)	N/A	0.231	0.001	0.244	0.029	0.086	0.091	0.129	0.160	0.063

Problem 194	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	A	A	A	F	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	48	59	99	64	66	57	0	89	49	93
N.S.	1	1.23	2.06	1.33	1.38	1.19	0.00	1.85	1.02	1.94
time (sec)	N/A	0.294	0.068	1.957	0.113	0.089	0.000	0.136	0.156	2.241

Problem 195	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	B	F	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	59	64	67	71	89	259	0	87	160	321
N.S.	1	1.08	1.14	1.20	1.51	4.39	0.00	1.47	2.71	5.44
time (sec)	N/A	0.388	0.141	6.454	0.120	0.089	0.000	0.142	0.158	1.941

Problem 196	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	B	F	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	87	135	77	161	159	652	0	214	352	291
N.S.	1	1.55	0.89	1.85	1.83	7.49	0.00	2.46	4.05	3.34
time (sec)	N/A	0.398	0.131	16.361	0.138	0.115	0.000	0.136	0.161	3.176

Problem 197	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	B	F	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	100	116	102	182	230	1142	0	180	480	634
N.S.	1	1.16	1.02	1.82	2.30	11.42	0.00	1.80	4.80	6.34
time (sec)	N/A	0.691	0.264	39.137	0.151	0.120	0.000	0.139	0.163	2.538

Problem 198	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	B	B	B	F	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	135	215	284	313	345	2707	0	369	850	548
N.S.	1	1.59	2.10	2.32	2.56	20.05	0.00	2.73	6.30	4.06
time (sec)	N/A	0.555	0.308	78.745	0.131	0.137	0.000	0.114	0.167	5.847

Problem 199	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	B	B	F	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	146	176	146	350	438	3175	0	323	919	1010
N.S.	1	1.21	1.00	2.40	3.00	21.75	0.00	2.21	6.29	6.92
time (sec)	N/A	1.009	0.352	150.023	0.132	0.115	0.000	0.134	0.170	3.064

Problem 200	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	C	C	A	B	B	F(-1)	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	94	117	660	161	176	833	0	178	296	256
N.S.	1	1.24	7.02	1.71	1.87	8.86	0.00	1.89	3.15	2.72
time (sec)	N/A	0.600	3.371	16.702	0.121	0.103	0.000	0.138	0.158	1.737

Problem 201	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	B	B	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	40	37	37	41	102	370	133	82	62	60
N.S.	1	0.92	0.92	1.02	2.55	9.25	3.32	2.05	1.55	1.50
time (sec)	N/A	0.249	0.046	5.970	0.049	0.121	0.368	0.126	0.152	1.566

Problem 202	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	C	C	A	A	B	F(-1)	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	62	80	502	101	100	362	0	97	143	132
N.S.	1	1.29	8.10	1.63	1.61	5.84	0.00	1.56	2.31	2.13
time (sec)	N/A	0.400	1.288	2.208	0.128	0.108	0.000	0.134	0.155	1.538

Problem 203	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	B	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	13	13	13	14	13	51	19	22	14	14
N.S.	1	1.00	1.00	1.08	1.00	3.92	1.46	1.69	1.08	1.08
time (sec)	N/A	0.197	0.015	1.388	0.036	0.079	0.276	0.128	0.157	0.105

Problem 204	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	B	F	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	79	103	121	123	149	383	0	186	336	186
N.S.	1	1.30	1.53	1.56	1.89	4.85	0.00	2.35	4.25	2.35
time (sec)	N/A	0.320	0.292	15.282	0.116	0.110	0.000	0.131	0.158	2.643

Problem 205	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	B	F	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	93	109	94	138	215	802	0	167	421	302
N.S.	1	1.17	1.01	1.48	2.31	8.62	0.00	1.80	4.53	3.25
time (sec)	N/A	0.511	0.180	42.934	0.126	0.103	0.000	0.132	0.157	1.833

Problem 206	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	B	F	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	136	179	260	211	375	2615	0	295	1150	519
N.S.	1	1.32	1.91	1.55	2.76	19.23	0.00	2.17	8.46	3.82
time (sec)	N/A	0.416	1.444	89.439	0.135	0.152	0.000	0.137	0.165	6.031

Problem 207	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	B	F	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	144	171	137	266	490	3044	0	287	1067	476
N.S.	1	1.19	0.95	1.85	3.40	21.14	0.00	1.99	7.41	3.31
time (sec)	N/A	0.819	0.325	223.296	0.157	0.141	0.000	0.141	0.166	1.887

Problem 208	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	B	B	B	B	B	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	31	31	96	63	413	86	107	53	13	231
N.S.	1	1.00	3.10	2.03	13.32	2.77	3.45	1.71	0.42	7.45
time (sec)	N/A	0.335	0.128	4.168	0.044	0.070	0.116	0.124	0.158	2.021

Problem 209	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	B	B	B	B	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	36	41	42	67	95	151	99	92	13	113
N.S.	1	1.14	1.17	1.86	2.64	4.19	2.75	2.56	0.36	3.14
time (sec)	N/A	0.439	0.061	2.549	0.034	0.088	0.163	0.123	0.157	0.455

Problem 210	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	A	B	B	B	A	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	23	30	67	37	109	38	46	29	13	80
N.S.	1	1.30	2.91	1.61	4.74	1.65	2.00	1.26	0.57	3.48
time (sec)	N/A	0.334	0.085	1.539	0.040	0.075	0.078	0.108	0.158	1.513

Problem 211	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	B	A	B	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	26	26	20	31	42	55	32	53	11	29
N.S.	1	1.00	0.77	1.19	1.62	2.12	1.23	2.04	0.42	1.12
time (sec)	N/A	0.345	0.016	0.972	0.027	0.098	0.076	0.118	0.156	0.178

Problem 212	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	A	A	A	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	19	23	17	21	28	17	19	23	11	24
N.S.	1	1.21	0.89	1.11	1.47	0.89	1.00	1.21	0.58	1.26
time (sec)	N/A	0.199	0.008	0.911	0.046	0.106	0.101	0.126	0.155	1.477

Problem 213	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	A	B	B	B	B	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	12	14	41	23	27	37	22	24	13	28
N.S.	1	1.17	3.42	1.92	2.25	3.08	1.83	2.00	1.08	2.33
time (sec)	N/A	0.291	0.067	1.565	0.043	0.102	0.083	0.116	0.156	0.181

Problem 214	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	B	B	B	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	15	15	15	24	67	31	32	23	13	25
N.S.	1	1.00	1.00	1.60	4.47	2.07	2.13	1.53	0.87	1.67
time (sec)	N/A	0.321	0.009	2.800	0.026	0.098	0.076	0.122	0.160	1.442

Problem 215	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	B	B	B	B	B	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	26	35	111	46	61	90	61	44	13	74
N.S.	1	1.35	4.27	1.77	2.35	3.46	2.35	1.69	0.50	2.85
time (sec)	N/A	0.423	0.051	4.918	0.041	0.099	0.118	0.120	0.158	0.322

Problem 216	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	B	B	B	B	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	23	32	33	45	205	63	70	51	13	44
N.S.	1	1.39	1.43	1.96	8.91	2.74	3.04	2.22	0.57	1.91
time (sec)	N/A	0.387	0.009	7.820	0.042	0.098	0.105	0.124	0.160	1.479

Problem 217	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	B	B	B	B	B	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	36	49	175	65	91	144	100	62	13	124
N.S.	1	1.36	4.86	1.81	2.53	4.00	2.78	1.72	0.36	3.44
time (sec)	N/A	0.502	0.075	12.507	0.046	0.107	0.143	0.124	0.159	1.804

Problem 218	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	A	B	B	B	B	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	47	47	112	69	573	104	128	65	20	395
N.S.	1	1.00	2.38	1.47	12.19	2.21	2.72	1.38	0.43	8.40
time (sec)	N/A	0.341	0.154	18.533	0.043	0.076	0.135	0.122	0.157	5.057

Problem 219	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	B	B	B	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	66	66	52	76	115	197	129	102	20	209
N.S.	1	1.00	0.79	1.15	1.74	2.98	1.95	1.55	0.30	3.17
time (sec)	N/A	0.287	0.093	10.868	0.047	0.086	0.140	0.121	0.160	2.423

Problem 220	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	A	B	B	A	A	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	37	37	84	43	197	56	66	41	20	139
N.S.	1	1.00	2.27	1.16	5.32	1.51	1.78	1.11	0.54	3.76
time (sec)	N/A	0.345	0.117	6.236	0.049	0.078	0.102	0.123	0.153	0.323

Problem 221	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	B	A	B	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	36	38	29	36	61	94	58	66	18	99
N.S.	1	1.06	0.81	1.00	1.69	2.61	1.61	1.83	0.50	2.75
time (sec)	N/A	0.252	0.024	3.613	0.043	0.081	0.111	0.124	0.157	1.791

Problem 222	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	B	B	A	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	25	31	25	31	48	54	36	33	18	49
N.S.	1	1.24	1.00	1.24	1.92	2.16	1.44	1.32	0.72	1.96
time (sec)	N/A	0.239	0.021	2.240	0.040	0.092	0.100	0.123	0.158	0.313

Problem 223	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	A	B	B	B	B	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	26	26	66	35	53	78	49	48	20	60
N.S.	1	1.00	2.54	1.35	2.04	3.00	1.88	1.85	0.77	2.31
time (sec)	N/A	0.250	0.155	6.348	0.035	0.082	0.118	0.131	0.162	1.634

Problem 224	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	B	A	B	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	29	35	29	45	63	70	53	54	20	56
N.S.	1	1.21	1.00	1.55	2.17	2.41	1.83	1.86	0.69	1.93
time (sec)	N/A	0.255	0.013	12.178	0.037	0.099	0.120	0.122	0.158	0.272

Problem 225	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	B	B	B	B	B	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	28	28	107	56	67	100	66	50	20	111
N.S.	1	1.00	3.82	2.00	2.39	3.57	2.36	1.79	0.71	3.96
time (sec)	N/A	0.306	0.081	19.579	0.047	0.110	0.135	0.124	0.159	0.221

Problem 226	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	B	B	A	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	27	27	27	41	171	59	65	38	20	40
N.S.	1	1.00	1.00	1.52	6.33	2.19	2.41	1.41	0.74	1.48
time (sec)	N/A	0.225	0.010	31.440	0.040	0.074	0.114	0.124	0.155	1.502

Problem 227	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	B	B	B	B	B	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	48	48	175	74	103	160	114	74	20	246
N.S.	1	1.00	3.65	1.54	2.15	3.33	2.38	1.54	0.42	5.12
time (sec)	N/A	0.279	0.098	59.099	0.039	0.104	0.184	0.128	0.157	1.697

Problem 228	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	B	F	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	124	129	108	171	241	1199	0	197	496	654
N.S.	1	1.04	0.87	1.38	1.94	9.67	0.00	1.59	4.00	5.27
time (sec)	N/A	0.825	0.281	1.264	0.117	0.114	0.000	0.136	0.167	2.415

Problem 229	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	A	B	F	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	88	122	153	166	160	655	0	211	352	291
N.S.	1	1.39	1.74	1.89	1.82	7.44	0.00	2.40	4.00	3.31
time (sec)	N/A	0.367	0.127	0.831	0.126	0.114	0.000	0.136	0.161	3.237

Problem 230	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	B	F	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	69	74	69	84	89	257	0	87	164	330
N.S.	1	1.07	1.00	1.22	1.29	3.72	0.00	1.26	2.38	4.78
time (sec)	N/A	0.467	0.165	0.479	0.134	0.092	0.000	0.139	0.162	1.855

Problem 231	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	A	A	F	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	48	54	56	73	66	57	0	89	49	95
N.S.	1	1.12	1.17	1.52	1.38	1.19	0.00	1.85	1.02	1.98
time (sec)	N/A	0.259	0.043	0.333	0.118	0.089	0.000	0.125	0.159	2.345

Problem 232	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	A	F	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	20	22	20	33	46	40	0	39	37	195
N.S.	1	1.10	1.00	1.65	2.30	2.00	0.00	1.95	1.85	9.75
time (sec)	N/A	0.212	0.009	0.431	0.042	0.088	0.000	0.131	0.164	0.426

Problem 233	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	C	A	A	A	B	F	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	56	74	89	81	97	228	0	95	137	304
N.S.	1	1.32	1.59	1.45	1.73	4.07	0.00	1.70	2.45	5.43
time (sec)	N/A	0.630	0.224	0.823	0.111	0.100	0.000	0.132	0.159	1.725

Problem 234	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	B	F	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	52	54	45	98	116	427	0	125	374	1163
N.S.	1	1.04	0.87	1.88	2.23	8.21	0.00	2.40	7.19	22.37
time (sec)	N/A	0.309	0.043	1.277	0.050	0.104	0.000	0.140	0.162	2.127

Problem 235	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	C	A	A	B	B	F	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	108	129	197	169	212	1303	0	194	654	778
N.S.	1	1.19	1.82	1.56	1.96	12.06	0.00	1.80	6.06	7.20
time (sec)	N/A	0.921	0.352	2.201	0.131	0.161	0.000	0.137	0.165	2.306

Problem 236	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	B	F	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	224	224	144	245	523	3534	0	292	1445	543
N.S.	1	1.00	0.64	1.09	2.33	15.78	0.00	1.30	6.45	2.42
time (sec)	N/A	0.731	0.288	3.234	0.144	0.160	0.000	0.147	0.173	2.141

Problem 237	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	B	B	F	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	135	177	150	237	375	2850	0	307	1305	501
N.S.	1	1.31	1.11	1.76	2.78	21.11	0.00	2.27	9.67	3.71
time (sec)	N/A	0.665	0.474	2.176	0.152	0.134	0.000	0.142	0.163	4.895

Problem 238	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	B	F	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	144	144	100	142	223	900	0	181	596	377
N.S.	1	1.00	0.69	0.99	1.55	6.25	0.00	1.26	4.14	2.62
time (sec)	N/A	0.587	0.209	1.232	0.160	0.086	0.000	0.141	0.160	1.990

Problem 239	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	A	B	F	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	85	109	146	136	155	423	0	199	401	190
N.S.	1	1.28	1.72	1.60	1.82	4.98	0.00	2.34	4.72	2.24
time (sec)	N/A	0.384	0.167	0.882	0.133	0.094	0.000	0.130	0.154	2.626

Problem 240	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	B	F	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	32	34	27	57	75	158	0	75	171	240
N.S.	1	1.06	0.84	1.78	2.34	4.94	0.00	2.34	5.34	7.50
time (sec)	N/A	0.278	0.036	1.444	0.039	0.094	0.000	0.130	0.160	1.801

Problem 241	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	C	A	A	B	B	F	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	80	126	112	118	165	1257	0	148	888	897
N.S.	1	1.58	1.40	1.48	2.06	15.71	0.00	1.85	11.10	11.21
time (sec)	N/A	1.116	0.406	3.510	0.122	0.132	0.000	0.133	0.160	2.610

Problem 242	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	B	F	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	76	78	73	142	202	1463	0	190	955	1375
N.S.	1	1.03	0.96	1.87	2.66	19.25	0.00	2.50	12.57	18.09
time (sec)	N/A	0.358	0.133	5.522	0.045	0.102	0.000	0.135	0.158	2.467

Problem 243	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	C	A	A	B	B	F	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	159	199	235	221	339	3648	0	242	1552	1450
N.S.	1	1.25	1.48	1.39	2.13	22.94	0.00	1.52	9.76	9.12
time (sec)	N/A	1.479	0.586	8.506	0.126	0.155	0.000	0.139	0.163	2.344

Problem 244	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	F	B	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	37	37	37	21	0	390	0	0	12	0
N.S.	1	1.00	1.00	0.57	0.00	10.54	0.00	0.00	0.32	0.00
time (sec)	N/A	0.243	0.016	0.293	0.000	0.309	0.000	0.000	0.154	0.000

Problem 245	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	F	B	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	24	24	24	17	0	370	0	0	20	0
N.S.	1	1.00	1.00	0.71	0.00	15.42	0.00	0.00	0.83	0.00
time (sec)	N/A	0.250	0.011	0.200	0.000	0.123	0.000	0.000	0.157	0.000

Problem 246	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	B	C	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	51	51	59	49	68	170	517	87	101	198
N.S.	1	1.00	1.16	0.96	1.33	3.33	10.14	1.71	1.98	3.88
time (sec)	N/A	0.358	0.074	0.641	0.139	0.091	21.542	0.123	0.152	3.480

Problem 247	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	25	25	48	23	19	36	20	22	18	24
N.S.	1	1.00	1.92	0.92	0.76	1.44	0.80	0.88	0.72	0.96
time (sec)	N/A	0.282	0.122	0.583	0.038	0.092	0.087	0.121	0.157	0.133

Problem 248	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	A	A	A	A	A	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	27	27	81	24	20	35	20	21	24	23
N.S.	1	1.00	3.00	0.89	0.74	1.30	0.74	0.78	0.89	0.85
time (sec)	N/A	0.297	0.347	0.559	0.058	0.117	0.090	0.117	0.159	0.117

Problem 249	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	A	B	F	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	89	89	149	117	125	172	0	123	87	914
N.S.	1	1.00	1.67	1.31	1.40	1.93	0.00	1.38	0.98	10.27
time (sec)	N/A	0.418	0.582	0.637	0.120	1.046	0.000	0.143	0.156	9.484

Problem 250	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	B	F	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	60	60	65	73	106	183	0	102	139	164
N.S.	1	1.00	1.08	1.22	1.77	3.05	0.00	1.70	2.32	2.73
time (sec)	N/A	0.379	1.250	0.651	0.124	0.111	0.000	0.140	0.164	11.952

Problem 251	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	B	F	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	89	89	93	102	125	172	0	123	88	864
N.S.	1	1.00	1.04	1.15	1.40	1.93	0.00	1.38	0.99	9.71
time (sec)	N/A	0.524	0.247	2.191	0.126	1.764	0.000	0.134	0.164	11.604

Problem 252	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	C	A	A	B	B	F	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	58	71	77	58	141	172	0	90	130	539
N.S.	1	1.22	1.33	1.00	2.43	2.97	0.00	1.55	2.24	9.29
time (sec)	N/A	0.482	1.050	0.535	0.122	0.326	0.000	0.136	0.163	2.164

Problem 253	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	B	C	A	B	B
verified	N/A	No	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	81	79	85	111	176	249	1318	127	25	656
N.S.	1	0.98	1.05	1.37	2.17	3.07	16.27	1.57	0.31	8.10
time (sec)	N/A	0.551	2.135	1.368	0.123	0.097	21.025	0.143	0.159	2.696

Problem 254	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	B	F(-1)	A	B	B
verified	N/A	No	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	113	111	113	151	339	570	0	170	253	279
N.S.	1	0.98	1.00	1.34	3.00	5.04	0.00	1.50	2.24	2.47
time (sec)	N/A	0.618	1.224	4.586	0.146	0.103	0.000	0.151	0.160	2.045

Problem 255	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	B	B	F(-1)	B	B	F(-1)
verified	N/A	No	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	180	190	170	370	726	1880	0	405	689	0
N.S.	1	1.06	0.94	2.06	4.03	10.44	0.00	2.25	3.83	0.00
time (sec)	N/A	0.897	1.445	24.941	0.166	0.127	0.000	0.170	0.166	0.000

Problem 256	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	B	B	F(-1)	B	B	F(-1)
verified	N/A	No	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	250	274	235	699	1263	4350	0	685	1750	0
N.S.	1	1.10	0.94	2.80	5.05	17.40	0.00	2.74	7.00	0.00
time (sec)	N/A	1.217	1.742	122.986	0.192	0.237	0.000	0.200	0.166	0.000

Problem 257	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F	B	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	439	391	598	919	0	1655	0	0	16	0
N.S.	1	0.89	1.36	2.09	0.00	3.77	0.00	0.00	0.04	0.00
time (sec)	N/A	1.777	0.484	0.570	0.000	0.144	0.000	0.000	0.162	0.000

Problem 258	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F	B	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	327	303	448	710	0	1247	0	0	16	0
N.S.	1	0.93	1.37	2.17	0.00	3.81	0.00	0.00	0.05	0.00
time (sec)	N/A	1.309	0.311	0.246	0.000	0.117	0.000	0.000	0.157	0.000

Problem 259	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F	B	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	215	213	294	505	0	837	0	0	14	0
N.S.	1	0.99	1.37	2.35	0.00	3.89	0.00	0.00	0.07	0.00
time (sec)	N/A	0.791	0.228	0.209	0.000	0.166	0.000	0.000	0.157	0.000

Problem 260	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	A	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	47	47	41	47	42	67	0	43	35	0
N.S.	1	1.00	0.87	1.00	0.89	1.43	0.00	0.91	0.74	0.00
time (sec)	N/A	0.626	0.050	3.207	0.143	0.081	0.000	0.125	0.157	0.000

Problem 261	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	C	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	58	63	55	0	0	0	0	0	35	0
N.S.	1	1.09	0.95	0.00	0.00	0.00	0.00	0.00	0.60	0.00
time (sec)	N/A	0.356	0.054	0.000	0.000	0.000	0.000	0.000	0.326	0.000

Problem 262	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	58	57	57	0	0	0	0	0	35	0
N.S.	1	0.98	0.98	0.00	0.00	0.00	0.00	0.00	0.60	0.00
time (sec)	N/A	0.328	0.027	0.000	0.000	0.000	0.000	0.000	0.172	0.000

Problem 263	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	26	26	26	0	0	0	0	0	33	0
N.S.	1	1.00	1.00	0.00	0.00	0.00	0.00	0.00	1.27	0.00
time (sec)	N/A	0.286	0.023	0.000	0.000	0.000	0.000	0.000	0.174	0.000

Problem 264	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	34	34	36	30	33	32	29	32	33	35
N.S.	1	1.00	1.06	0.88	0.97	0.94	0.85	0.94	0.97	1.03
time (sec)	N/A	0.284	11.924	0.140	0.193	0.085	1.428	0.204	0.184	1.537

Problem 265	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	36	36	38	32	191	34	31	34	35	35
N.S.	1	1.00	1.06	0.89	5.31	0.94	0.86	0.94	0.97	0.97
time (sec)	N/A	0.311	38.988	0.164	0.256	0.098	1.427	0.272	0.223	1.620

Problem 266	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	54	54	41	42	52	44	0	47	41	43
N.S.	1	1.00	0.76	0.78	0.96	0.81	0.00	0.87	0.76	0.80
time (sec)	N/A	0.200	0.041	0.470	0.066	0.093	0.000	0.118	0.155	1.533

Problem 267	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	88	88	55	58	67	91	0	169	123	53
N.S.	1	1.00	0.62	0.66	0.76	1.03	0.00	1.92	1.40	0.60
time (sec)	N/A	0.229	0.069	1.854	0.086	0.086	0.000	0.136	0.160	1.502

Problem 268	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	149	143	120	254	114	200	0	665	323	93
N.S.	1	0.96	0.81	1.70	0.77	1.34	0.00	4.46	2.17	0.62
time (sec)	N/A	0.340	0.309	9.096	0.088	0.093	0.000	0.155	0.155	1.574

Problem 269	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	191	177	167	184	129	294	0	777	390	102
N.S.	1	0.93	0.87	0.96	0.68	1.54	0.00	4.07	2.04	0.53
time (sec)	N/A	0.367	0.284	20.380	0.092	0.095	0.000	0.154	0.159	1.602

Problem 270	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	A	A	F	B	B	B
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	73	74	54	0	64	98	0	235	66	56
N.S.	1	1.01	0.74	0.00	0.88	1.34	0.00	3.22	0.90	0.77
time (sec)	N/A	0.227	0.085	0.000	0.069	0.085	0.000	0.140	0.155	1.640

Problem 271	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	A	A	F	B	B	B
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	120	122	89	0	87	248	0	758	262	74
N.S.	1	1.02	0.74	0.00	0.72	2.07	0.00	6.32	2.18	0.62
time (sec)	N/A	0.275	0.177	0.000	0.078	0.106	0.000	0.170	0.160	1.677

Problem 272	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	A	B	F(-1)	B	B	B
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	203	188	292	0	138	585	0	3225	784	118
N.S.	1	0.93	1.44	0.00	0.68	2.88	0.00	15.89	3.86	0.58
time (sec)	N/A	0.411	0.916	0.000	0.086	0.119	0.000	0.185	0.165	1.786

Problem 273	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	A	B	F(-1)	B	B	B
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	266	236	311	0	161	1125	0	6884	1329	136
N.S.	1	0.89	1.17	0.00	0.61	4.23	0.00	25.88	5.00	0.51
time (sec)	N/A	0.466	2.348	0.000	0.094	0.121	0.000	0.215	0.162	1.718

Problem 274	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	A	A	A	B	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	18	18	37	19	18	19	34	40	18	18
N.S.	1	1.00	2.06	1.06	1.00	1.06	1.89	2.22	1.00	1.00
time (sec)	N/A	0.212	0.006	0.449	0.029	0.088	0.250	0.118	0.156	1.524

Problem 275	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	39	44	36	30	49	40	0	81	71	32
N.S.	1	1.13	0.92	0.77	1.26	1.03	0.00	2.08	1.82	0.82
time (sec)	N/A	0.241	0.016	1.754	0.038	0.095	0.000	0.130	0.158	1.557

Problem 276	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	43	37	45	36	86	65	71	81	85	37
N.S.	1	0.86	1.05	0.84	2.00	1.51	1.65	1.88	1.98	0.86
time (sec)	N/A	0.253	0.008	8.925	0.035	0.092	1.411	0.121	0.157	1.575

Problem 277	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	73	80	51	46	93	84	0	116	107	51
N.S.	1	1.10	0.70	0.63	1.27	1.15	0.00	1.59	1.47	0.70
time (sec)	N/A	0.301	0.028	30.217	0.043	0.096	0.000	0.132	0.163	1.651

Problem 278	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	B	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	65	53	68	51	130	130	105	115	122	49
N.S.	1	0.82	1.05	0.78	2.00	2.00	1.62	1.77	1.88	0.75
time (sec)	N/A	0.258	0.009	74.141	0.040	0.083	9.588	0.129	0.161	1.705

Problem 279	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F	B	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	109	109	96	227	0	331	0	0	31	0
N.S.	1	1.00	0.88	2.08	0.00	3.04	0.00	0.00	0.28	0.00
time (sec)	N/A	0.391	0.053	2.038	0.000	0.100	0.000	0.000	0.157	0.000

Problem 280	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	109	109	114	143	0	171	0	0	29	0
N.S.	1	1.00	1.05	1.31	0.00	1.57	0.00	0.00	0.27	0.00
time (sec)	N/A	0.383	0.084	0.204	0.000	0.099	0.000	0.000	0.153	0.000

Problem 281	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	70	72	68	146	0	58	0	0	18	0
N.S.	1	1.03	0.97	2.09	0.00	0.83	0.00	0.00	0.26	0.00
time (sec)	N/A	0.337	0.018	0.194	0.000	0.083	0.000	0.000	0.153	0.000

Problem 282	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	70	72	66	120	0	39	0	0	31	0
N.S.	1	1.03	0.94	1.71	0.00	0.56	0.00	0.00	0.44	0.00
time (sec)	N/A	0.323	0.021	0.175	0.000	0.092	0.000	0.000	0.152	0.000

Problem 283	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F	B	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	105	105	80	212	0	246	0	0	31	0
N.S.	1	1.00	0.76	2.02	0.00	2.34	0.00	0.00	0.30	0.00
time (sec)	N/A	0.420	0.033	0.196	0.000	0.097	0.000	0.000	0.155	0.000

Problem 284	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	B	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	109	109	122	144	0	504	0	0	31	0
N.S.	1	1.00	1.12	1.32	0.00	4.62	0.00	0.00	0.28	0.00
time (sec)	N/A	0.417	0.082	0.194	0.000	0.132	0.000	0.000	0.153	0.000

Problem 285	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	F	F	A	F(-1)	A	F	F(-1)
verified	N/A	No	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	209	208	86	0	0	162	0	171	97	0
N.S.	1	1.00	0.41	0.00	0.00	0.78	0.00	0.82	0.46	0.00
time (sec)	N/A	0.524	0.209	0.000	0.000	0.086	0.000	0.433	0.189	0.000

Problem 286	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	A	F	F	F	F(-1)
verified	N/A	No	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	103	112	74	0	0	117	0	0	80	0
N.S.	1	1.09	0.72	0.00	0.00	1.14	0.00	0.00	0.78	0.00
time (sec)	N/A	0.397	0.172	0.000	0.000	0.082	0.000	0.000	0.160	0.000

Problem 287	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	A	F	A	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	43	43	61	0	0	68	0	41	40	0
N.S.	1	1.00	1.42	0.00	0.00	1.58	0.00	0.95	0.93	0.00
time (sec)	N/A	0.468	0.083	0.000	0.000	0.101	0.000	0.314	0.153	0.000

Problem 288	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	A	F(-1)	A	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	89	144	121	0	0	128	0	81	40	0
N.S.	1	1.62	1.36	0.00	0.00	1.44	0.00	0.91	0.45	0.00
time (sec)	N/A	0.420	0.147	0.000	0.000	0.097	0.000	0.390	0.155	0.000

Problem 289	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	C	A	A	F	A	F	B	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	36	42	36	38	0	48	0	102	434	0
N.S.	1	1.17	1.00	1.06	0.00	1.33	0.00	2.83	12.06	0.00
time (sec)	N/A	0.351	0.017	0.351	0.000	0.112	0.000	0.138	0.156	0.000

Problem 290	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	F	B	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	39	38	37	50	0	73	0	130	490	0
N.S.	1	0.97	0.95	1.28	0.00	1.87	0.00	3.33	12.56	0.00
time (sec)	N/A	0.398	0.028	0.345	0.000	0.084	0.000	0.153	0.159	0.000

Problem 291	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	C	A	A	F	B	F(-1)	B	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	59	64	54	74	0	118	0	211	904	0
N.S.	1	1.08	0.92	1.25	0.00	2.00	0.00	3.58	15.32	0.00
time (sec)	N/A	0.404	0.041	0.516	0.000	0.100	0.000	0.165	0.162	0.000

Problem 292	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	C	A	A	F	B	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	74	91	93	113	0	253	0	0	442	0
N.S.	1	1.23	1.26	1.53	0.00	3.42	0.00	0.00	5.97	0.00
time (sec)	N/A	0.578	0.162	0.316	0.000	0.098	0.000	0.000	0.154	0.000

Problem 293	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	C	A	A	F	B	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	80	98	115	120	0	277	0	0	499	0
N.S.	1	1.22	1.44	1.50	0.00	3.46	0.00	0.00	6.24	0.00
time (sec)	N/A	0.577	0.214	2.253	0.000	0.095	0.000	0.000	0.157	0.000

Problem 294	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	C	A	A	F	B	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	143	150	232	250	0	701	0	0	922	0
N.S.	1	1.05	1.62	1.75	0.00	4.90	0.00	0.00	6.45	0.00
time (sec)	N/A	0.475	0.401	1.842	0.000	0.101	0.000	0.000	0.167	0.000

Problem 295	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	C	B	B	F	A	F	B	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	101	115	302	347	0	171	0	764	1337	0
N.S.	1	1.14	2.99	3.44	0.00	1.69	0.00	7.56	13.24	0.00
time (sec)	N/A	0.641	0.450	0.392	0.000	0.090	0.000	1.726	0.174	0.000

Problem 296	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	C	B	B	F	B	F(-1)	B	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	107	122	475	358	0	370	0	749	1575	0
N.S.	1	1.14	4.44	3.35	0.00	3.46	0.00	7.00	14.72	0.00
time (sec)	N/A	0.689	2.734	2.613	0.000	0.108	0.000	7.899	0.173	0.000

Problem 297	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	C	B	B	F	B	F(-1)	B	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	194	186	651	700	0	717	0	1383	0	0
N.S.	1	0.96	3.36	3.61	0.00	3.70	0.00	7.13	0.00	0.00
time (sec)	N/A	0.891	5.186	1.325	0.000	0.109	0.000	9.492	0.222	0.000

Problem 298	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	C	B	B	F	A	F	B	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	121	127	363	463	0	202	0	1624	0	0
N.S.	1	1.05	3.00	3.83	0.00	1.67	0.00	13.42	0.00	0.00
time (sec)	N/A	1.207	1.517	0.609	0.000	0.084	0.000	4.785	0.193	0.000

Problem 299	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	C	B	B	F	B	F(-1)	B	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	129	136	572	472	0	477	0	1596	0	0
N.S.	1	1.05	4.43	3.66	0.00	3.70	0.00	12.37	0.00	0.00
time (sec)	N/A	0.971	3.352	3.480	0.000	0.088	0.000	22.716	0.192	0.000

Problem 300	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	C	B	B	F	B	F(-1)	B	F	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	226	208	913	930	0	942	0	3021	0	0
N.S.	1	0.92	4.04	4.12	0.00	4.17	0.00	13.37	0.00	0.00
time (sec)	N/A	0.699	6.457	5.834	0.000	0.132	0.000	27.914	0.252	0.000

Problem 301	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	83	71	62	45	68	113	139	60	67	58
N.S.	1	0.86	0.75	0.54	0.82	1.36	1.67	0.72	0.81	0.70
time (sec)	N/A	0.222	0.031	1.364	0.044	0.111	2.366	0.123	0.151	0.535

Problem 302	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	B	B	A	B	B
verified	N/A	No	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	57	47	45	47	53	95	182	57	55	42
N.S.	1	0.82	0.79	0.82	0.93	1.67	3.19	1.00	0.96	0.74
time (sec)	N/A	0.223	0.026	0.555	0.034	0.079	0.990	0.123	0.149	1.826

Problem 303	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	49	43	39	35	40	54	78	34	38	36
N.S.	1	0.88	0.80	0.71	0.82	1.10	1.59	0.69	0.78	0.73
time (sec)	N/A	0.212	0.017	0.249	0.039	0.099	0.476	0.120	0.150	1.622

Problem 304	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	B	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	23	32	23	19	24	50	80	24	38	18
N.S.	1	1.39	1.00	0.83	1.04	2.17	3.48	1.04	1.65	0.78
time (sec)	N/A	0.208	0.010	0.103	0.035	0.080	0.228	0.120	0.149	0.072

Problem 305	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	19	19	19	17	27	30	0	24	25	16
N.S.	1	1.00	1.00	0.89	1.42	1.58	0.00	1.26	1.32	0.84
time (sec)	N/A	0.173	0.010	0.083	0.038	0.082	0.000	0.120	0.154	0.073

Problem 306	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	B	F	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	42	45	37	25	52	157	0	48	93	52
N.S.	1	1.07	0.88	0.60	1.24	3.74	0.00	1.14	2.21	1.24
time (sec)	N/A	0.189	0.046	0.185	0.035	0.083	0.000	0.125	0.155	1.608

Problem 307	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	B	F	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	31	31	29	24	68	88	0	31	41	31
N.S.	1	1.00	0.94	0.77	2.19	2.84	0.00	1.00	1.32	1.00
time (sec)	N/A	0.181	0.014	1.275	0.040	0.082	0.000	0.128	0.150	1.577

Problem 308	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	B	F	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	101	112	75	37	100	705	0	75	233	135
N.S.	1	1.11	0.74	0.37	0.99	6.98	0.00	0.74	2.31	1.34
time (sec)	N/A	0.373	0.049	1.746	0.043	0.093	0.000	0.117	0.155	1.604

Problem 309	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	B	F	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	66	66	44	35	172	233	0	42	81	42
N.S.	1	1.00	0.67	0.53	2.61	3.53	0.00	0.64	1.23	0.64
time (sec)	N/A	0.377	0.027	2.336	0.037	0.076	0.000	0.139	0.152	1.615

Problem 310	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	B	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	26	28	26	17	17	47	42	17	25	17
N.S.	1	1.08	1.00	0.65	0.65	1.81	1.62	0.65	0.96	0.65
time (sec)	N/A	0.323	0.012	0.306	0.026	0.083	0.185	0.122	0.151	0.090

Problem 311	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	19	19	16	14	13	26	20	13	18	12
N.S.	1	1.00	0.84	0.74	0.68	1.37	1.05	0.68	0.95	0.63
time (sec)	N/A	0.309	0.008	0.111	0.029	0.091	0.120	0.123	0.152	0.058

Problem 312	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	A	B	F	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	11	19	11	34	18	25	0	19	21	26
N.S.	1	1.73	1.00	3.09	1.64	2.27	0.00	1.73	1.91	2.36
time (sec)	N/A	0.296	0.008	0.204	0.112	0.102	0.000	0.120	0.150	0.177

Problem 313	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	B	F	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	32	42	32	24	32	182	0	33	74	36
N.S.	1	1.31	1.00	0.75	1.00	5.69	0.00	1.03	2.31	1.12
time (sec)	N/A	0.328	0.025	0.196	0.124	0.084	0.000	0.121	0.152	1.761

Problem 314	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	B	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	26	28	26	17	17	67	42	17	25	17
N.S.	1	1.08	1.00	0.65	0.65	2.58	1.62	0.65	0.96	0.65
time (sec)	N/A	0.317	0.013	0.278	0.043	0.106	0.204	0.119	0.152	1.604

Problem 315	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	B	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	19	23	16	14	13	40	20	13	18	12
N.S.	1	1.21	0.84	0.74	0.68	2.11	1.05	0.68	0.95	0.63
time (sec)	N/A	0.232	0.008	0.191	0.025	0.073	0.114	0.128	0.149	0.058

Problem 316	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	C	A	A	F	A	B	B
verified	N/A	No	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	54	56	22	79	73	83	0	43	79	65
N.S.	1	1.04	0.41	1.46	1.35	1.54	0.00	0.80	1.46	1.20
time (sec)	N/A	0.261	0.011	0.174	0.125	0.092	0.000	0.115	0.149	0.176

Problem 317	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	A	B	F	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	91	118	34	116	85	560	0	86	201	91
N.S.	1	1.30	0.37	1.27	0.93	6.15	0.00	0.95	2.21	1.00
time (sec)	N/A	0.316	0.020	0.293	0.118	0.106	0.000	0.125	0.150	0.403

Problem 318	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	B	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	26	28	26	17	17	87	42	17	25	17
N.S.	1	1.08	1.00	0.65	0.65	3.35	1.62	0.65	0.96	0.65
time (sec)	N/A	0.194	0.014	0.346	0.036	0.077	0.186	0.124	0.151	1.603

Problem 319	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	B	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	19	23	19	14	13	46	20	13	18	13
N.S.	1	1.21	1.00	0.74	0.68	2.42	1.05	0.68	0.95	0.68
time (sec)	N/A	0.196	0.009	0.129	0.037	0.096	0.104	0.122	0.150	1.545

Problem 320	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	C	A	A	F	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	88	130	22	56	95	116	0	96	98	106
N.S.	1	1.48	0.25	0.64	1.08	1.32	0.00	1.09	1.11	1.20
time (sec)	N/A	0.376	0.010	0.203	0.114	0.090	0.000	0.119	0.147	0.372

Problem 321	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	C	A	B	F	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	106	153	34	68	107	855	0	108	245	120
N.S.	1	1.44	0.32	0.64	1.01	8.07	0.00	1.02	2.31	1.13
time (sec)	N/A	0.382	0.021	0.262	0.128	0.114	0.000	0.110	0.153	1.906

Problem 322	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	B	B	C	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	202	190	157	147	134	2228	1525	1211	23	166
N.S.	1	0.94	0.78	0.73	0.66	11.03	7.55	6.00	0.11	0.82
time (sec)	N/A	0.439	0.473	1.358	0.061	0.168	3.606	0.174	0.150	2.581

Problem 323	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	B	B	C	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	132	132	86	90	94	703	707	890	23	97
N.S.	1	1.00	0.65	0.68	0.71	5.33	5.36	6.74	0.17	0.73
time (sec)	N/A	0.332	0.117	0.537	0.050	0.094	1.260	0.154	0.152	1.912

Problem 324	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	B	B	C	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	75	75	50	51	63	244	323	598	51	73
N.S.	1	1.00	0.67	0.68	0.84	3.25	4.31	7.97	0.68	0.97
time (sec)	N/A	0.225	0.064	0.161	0.041	0.121	0.720	0.144	0.156	1.702

Problem 325	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	66	66	79	0	0	0	0	0	21	0
N.S.	1	1.00	1.20	0.00	0.00	0.00	0.00	0.00	0.32	0.00
time (sec)	N/A	0.227	2.755	0.000	0.000	0.000	0.000	0.000	0.154	0.000

Problem 326	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	68	68	87	0	0	0	0	0	23	0
N.S.	1	1.00	1.28	0.00	0.00	0.00	0.00	0.00	0.34	0.00
time (sec)	N/A	0.226	0.820	0.000	0.000	0.000	0.000	0.000	0.155	0.000

Problem 327	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	122	137	281	0	0	0	0	0	23	0
N.S.	1	1.12	2.30	0.00	0.00	0.00	0.00	0.00	0.19	0.00
time (sec)	N/A	0.352	15.836	0.000	0.000	0.000	0.000	0.000	0.154	0.000

Problem 328	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	131	146	163	0	0	0	0	0	23	0
N.S.	1	1.11	1.24	0.00	0.00	0.00	0.00	0.00	0.18	0.00
time (sec)	N/A	0.361	1.506	0.000	0.000	0.000	0.000	0.000	0.151	0.000

Problem 329	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F(-1)	A	B	F(-1)
verified	N/A	No	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	250	102	109	326	90	218	0	266	97	0
N.S.	1	0.41	0.44	1.30	0.36	0.87	0.00	1.06	0.39	0.00
time (sec)	N/A	0.490	0.074	4.518	0.157	0.087	0.000	0.159	0.151	0.000

Problem 330	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F(-1)	A	B	F(-1)
verified	N/A	No	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	162	74	76	216	62	126	0	193	67	0
N.S.	1	0.46	0.47	1.33	0.38	0.78	0.00	1.19	0.41	0.00
time (sec)	N/A	0.386	0.044	0.208	0.138	0.086	0.000	0.140	0.151	0.000

Problem 331	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	74	61	48	106	36	66	189	75	55	77
N.S.	1	0.82	0.65	1.43	0.49	0.89	2.55	1.01	0.74	1.04
time (sec)	N/A	0.369	0.028	0.205	0.131	0.086	2.640	0.132	0.152	1.651

Problem 332	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	A	A	F	A	B	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	46	46	44	29	39	42	0	46	46	0
N.S.	1	1.00	0.96	0.63	0.85	0.91	0.00	1.00	1.00	0.00
time (sec)	N/A	0.325	0.036	0.132	0.132	0.101	0.000	0.133	0.149	0.000

Problem 333	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	B	F	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	58	58	46	69	84	121	0	62	50	76
N.S.	1	1.00	0.79	1.19	1.45	2.09	0.00	1.07	0.86	1.31
time (sec)	N/A	0.364	0.051	0.203	0.152	0.081	0.000	0.139	0.155	1.627

Problem 334	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	B	F(-1)	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	147	93	72	80	209	315	0	75	96	89
N.S.	1	0.63	0.49	0.54	1.42	2.14	0.00	0.51	0.65	0.61
time (sec)	N/A	0.426	0.049	0.209	0.151	0.076	0.000	0.141	0.156	1.640

Problem 335	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	B	F(-1)	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	199	113	84	91	386	592	0	88	138	353
N.S.	1	0.57	0.42	0.46	1.94	2.97	0.00	0.44	0.69	1.77
time (sec)	N/A	0.452	0.058	0.214	0.157	0.109	0.000	0.143	0.156	1.651

Problem 336	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F(-2)	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	41	41	28	28	0	42	99	32	28	45
N.S.	1	1.00	0.68	0.68	0.00	1.02	2.41	0.78	0.68	1.10
time (sec)	N/A	0.194	0.035	0.157	0.000	0.085	0.295	0.120	0.152	1.611

Problem 337	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	A	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	85	85	80	72	65	103	0	73	14	0
N.S.	1	1.00	0.94	0.85	0.76	1.21	0.00	0.86	0.16	0.00
time (sec)	N/A	0.282	0.065	0.209	0.043	0.078	0.000	0.124	0.149	0.000

Problem 338	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	A	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	101	101	92	97	81	129	0	91	17	0
N.S.	1	1.00	0.91	0.96	0.80	1.28	0.00	0.90	0.17	0.00
time (sec)	N/A	0.348	0.117	0.338	0.037	0.101	0.000	0.119	0.150	0.000

Problem 339	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	C	A	F	C	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	65	65	51	52	45	72	0	45	14	0
N.S.	1	1.00	0.78	0.80	0.69	1.11	0.00	0.69	0.22	0.00
time (sec)	N/A	0.268	0.049	0.170	0.038	0.078	0.000	0.121	0.158	0.000

Problem 340	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	A	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	65	65	72	48	47	75	0	49	16	0
N.S.	1	1.00	1.11	0.74	0.72	1.15	0.00	0.75	0.25	0.00
time (sec)	N/A	0.288	0.078	0.421	0.036	0.080	0.000	0.128	0.153	0.000

Problem 341	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	A	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	115	115	123	105	89	164	0	101	19	0
N.S.	1	1.00	1.07	0.91	0.77	1.43	0.00	0.88	0.17	0.00
time (sec)	N/A	0.388	0.268	0.444	0.038	0.079	0.000	0.130	0.153	0.000

Problem 342	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	B	F	A	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	110	110	103	100	90	213	0	106	20	0
N.S.	1	1.00	0.94	0.91	0.82	1.94	0.00	0.96	0.18	0.00
time (sec)	N/A	0.370	0.097	0.140	0.041	0.087	0.000	0.133	0.151	0.000

Problem 343	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	B	F	C	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	148	148	149	126	127	278	0	356	22	0
N.S.	1	1.00	1.01	0.85	0.86	1.88	0.00	2.41	0.15	0.00
time (sec)	N/A	0.416	0.511	0.347	0.137	0.089	0.000	0.142	0.154	0.000

Problem 344	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	B	F	A	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	239	239	287	207	200	445	0	223	22	0
N.S.	1	1.00	1.20	0.87	0.84	1.86	0.00	0.93	0.09	0.00
time (sec)	N/A	0.552	0.279	0.890	0.153	0.085	0.000	0.140	0.151	0.000

Problem 345	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	B	F	A	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	115	115	124	126	102	253	0	132	23	0
N.S.	1	1.00	1.08	1.10	0.89	2.20	0.00	1.15	0.20	0.00
time (sec)	N/A	0.445	0.190	0.182	0.053	0.082	0.000	0.126	0.154	0.000

Problem 346	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	B	F	C	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	161	161	220	158	143	334	0	388	25	0
N.S.	1	1.00	1.37	0.98	0.89	2.07	0.00	2.41	0.16	0.00
time (sec)	N/A	0.570	0.421	0.580	0.135	0.086	0.000	0.146	0.157	0.000

Problem 347	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	B	F	A	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	257	257	354	265	228	541	0	281	25	0
N.S.	1	1.00	1.38	1.03	0.89	2.11	0.00	1.09	0.10	0.00
time (sec)	N/A	0.848	0.528	1.688	0.140	0.095	0.000	0.152	0.154	0.000

Problem 348	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	B	F	A	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	133	133	104	117	105	217	0	132	20	0
N.S.	1	1.00	0.78	0.88	0.79	1.63	0.00	0.99	0.15	0.00
time (sec)	N/A	0.475	0.115	0.117	0.042	0.089	0.000	0.123	0.152	0.000

Problem 349	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	A	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	161	161	131	139	131	244	0	150	22	0
N.S.	1	1.00	0.81	0.86	0.81	1.52	0.00	0.93	0.14	0.00
time (sec)	N/A	0.492	0.174	0.283	0.044	0.089	0.000	0.133	0.151	0.000

Problem 350	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	B	F	A	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	271	271	214	234	211	427	0	264	22	0
N.S.	1	1.00	0.79	0.86	0.78	1.58	0.00	0.97	0.08	0.00
time (sec)	N/A	0.662	0.295	0.795	0.049	0.101	0.000	0.135	0.153	0.000

Problem 351	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	B	F	A	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	81	81	76	70	69	146	0	75	22	0
N.S.	1	1.00	0.94	0.86	0.85	1.80	0.00	0.93	0.27	0.00
time (sec)	N/A	0.427	0.235	0.118	0.051	0.086	0.000	0.136	0.155	0.000

Problem 352	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	B	F	A	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	128	128	179	101	100	254	0	107	24	0
N.S.	1	1.00	1.40	0.79	0.78	1.98	0.00	0.84	0.19	0.00
time (sec)	N/A	0.468	0.355	0.308	0.049	0.099	0.000	0.136	0.160	0.000

Problem 353	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	B	F	A	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	171	171	272	144	143	492	0	155	24	0
N.S.	1	1.00	1.59	0.84	0.84	2.88	0.00	0.91	0.14	0.00
time (sec)	N/A	0.540	0.852	0.795	0.048	0.115	0.000	0.148	0.166	0.000

Problem 354	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	B	F	A	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	140	140	166	147	127	322	0	172	25	0
N.S.	1	1.00	1.19	1.05	0.91	2.30	0.00	1.23	0.18	0.00
time (sec)	N/A	0.508	0.483	0.169	0.040	0.092	0.000	0.128	0.159	0.000

Problem 355	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	B	F	A	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	183	183	258	177	161	422	0	198	27	0
N.S.	1	1.00	1.41	0.97	0.88	2.31	0.00	1.08	0.15	0.00
time (sec)	N/A	0.609	0.979	0.532	0.041	0.109	0.000	0.138	0.156	0.000

Problem 356	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	B	F	A	F	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	300	300	480	302	263	848	0	352	27	0
N.S.	1	1.00	1.60	1.01	0.88	2.83	0.00	1.17	0.09	0.00
time (sec)	N/A	0.905	4.133	1.439	0.053	0.121	0.000	0.151	0.158	0.000

Problem 357	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	B	F	A	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	153	153	135	162	129	263	0	167	24	0
N.S.	1	1.00	0.88	1.06	0.84	1.72	0.00	1.09	0.16	0.00
time (sec)	N/A	0.520	0.201	0.135	0.048	0.105	0.000	0.131	0.159	0.000

Problem 358	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	B	F	A	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	219	219	183	210	185	343	0	223	26	0
N.S.	1	1.00	0.84	0.96	0.84	1.57	0.00	1.02	0.12	0.00
time (sec)	N/A	0.611	0.394	0.339	0.053	0.096	0.000	0.147	0.158	0.000

Problem 359	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	B	F	A	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	315	315	263	326	263	527	0	339	26	0
N.S.	1	1.00	0.83	1.03	0.83	1.67	0.00	1.08	0.08	0.00
time (sec)	N/A	0.846	0.632	0.868	0.054	0.124	0.000	0.146	0.157	0.000

Problem 360	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	B	F	A	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	154	154	179	160	139	325	0	181	26	0
N.S.	1	1.00	1.16	1.04	0.90	2.11	0.00	1.18	0.17	0.00
time (sec)	N/A	0.609	0.526	0.154	0.038	0.119	0.000	0.130	0.155	0.000

Problem 361	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	B	F	A	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	225	225	257	217	199	466	0	239	28	0
N.S.	1	1.00	1.14	0.96	0.88	2.07	0.00	1.06	0.12	0.00
time (sec)	N/A	0.785	1.462	0.388	0.046	0.103	0.000	0.147	0.155	0.000

Problem 362	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	B	F	A	F	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	323	323	503	326	287	852	0	369	28	0
N.S.	1	1.00	1.56	1.01	0.89	2.64	0.00	1.14	0.09	0.00
time (sec)	N/A	0.931	4.543	1.060	0.060	0.096	0.000	0.153	0.159	0.000

Problem 363	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	B	F	A	F	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	161	161	252	186	151	363	0	207	29	0
N.S.	1	1.00	1.57	1.16	0.94	2.25	0.00	1.29	0.18	0.00
time (sec)	N/A	0.644	0.980	0.186	0.047	0.121	0.000	0.134	0.155	0.000

Problem 364	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	B	F	A	F	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	239	239	339	249	215	516	0	271	31	0
N.S.	1	1.00	1.42	1.04	0.90	2.16	0.00	1.13	0.13	0.00
time (sec)	N/A	0.842	4.020	0.595	0.051	0.093	0.000	0.140	0.155	0.000

Problem 365	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	A	A	B	F(-1)	A	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	344	344	2991	384	315	940	0	427	31	0
N.S.	1	1.00	8.69	1.12	0.92	2.73	0.00	1.24	0.09	0.00
time (sec)	N/A	1.106	6.421	1.630	0.064	0.112	0.000	0.159	0.153	0.000

Problem 366	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	30	30	30	25	35	22	41	35	62	24
N.S.	1	1.00	1.00	0.83	1.17	0.73	1.37	1.17	2.07	0.80
time (sec)	N/A	0.242	0.036	0.697	0.034	0.074	0.089	0.115	0.153	1.514

Problem 367	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	56	56	48	52	81	58	85	75	114	46
N.S.	1	1.00	0.86	0.93	1.45	1.04	1.52	1.34	2.04	0.82
time (sec)	N/A	0.282	0.049	0.041	0.059	0.076	0.146	0.126	0.150	0.077

Problem 368	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	B	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	213	213	167	212	0	316	0	0	18	0
N.S.	1	1.00	0.78	1.00	0.00	1.48	0.00	0.00	0.08	0.00
time (sec)	N/A	0.753	0.485	0.194	0.000	0.097	0.000	0.000	0.152	0.000

Problem 369	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F(-2)	B	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	271	271	220	376	0	671	0	0	21	0
N.S.	1	1.00	0.81	1.39	0.00	2.48	0.00	0.00	0.08	0.00
time (sec)	N/A	1.058	1.027	0.367	0.000	0.111	0.000	0.000	0.156	0.000

2.3 Detailed conclusion table specific for Rubi results

The following table is specific to Rubi only. It gives additional statistics for each integral. the column **steps** is the number of steps used by Rubi to obtain the antiderivative. The **rules** column is the number of unique rules used. The **integrand size** column is the leaf size of the integrand. Finally the ratio $\frac{\text{number of rules}}{\text{integrand size}}$ is also given. The larger this ratio is, the harder the integral is to solve. In this test file, problem number [54] had the largest ratio of [1.8461499999999996]

Table 2.1: Rubi specific breakdown of results for each integral

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
1	A	3	3	1.00	6	0.500
2	A	4	4	1.00	8	0.500
3	A	5	4	0.89	8	0.500
4	A	7	7	1.11	8	0.875
5	A	5	4	0.85	8	0.500
6	A	10	10	1.15	8	1.250
7	A	8	8	1.05	10	0.800
8	A	6	6	1.00	10	0.600
9	A	6	6	1.00	10	0.600
10	A	4	4	1.00	10	0.400
11	A	4	4	1.00	10	0.400
12	A	6	6	1.00	10	0.600
13	A	6	6	1.00	10	0.600
14	A	8	8	1.01	10	0.800
15	A	8	8	1.05	12	0.667
16	A	6	6	1.00	12	0.500
17	A	6	6	1.00	12	0.500
18	A	4	4	1.00	12	0.333
19	A	4	4	1.00	12	0.333
20	A	6	6	1.00	12	0.500
21	A	6	6	1.00	12	0.500

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
22	A	8	8	1.03	12	0.667
23	A	6	6	1.05	14	0.429
24	A	4	4	1.00	14	0.286
25	A	4	4	1.00	14	0.286
26	A	2	2	1.00	14	0.143
27	A	2	2	1.00	14	0.143
28	A	4	4	1.00	14	0.286
29	A	4	4	1.00	14	0.286
30	A	6	6	1.01	14	0.429
31	A	2	2	1.00	12	0.167
32	A	2	2	1.00	12	0.167
33	A	2	2	1.00	12	0.167
34	A	2	2	1.00	12	0.167
35	A	2	2	1.00	12	0.167
36	A	2	2	1.00	12	0.167
37	A	2	2	1.00	10	0.200
38	A	2	2	1.00	12	0.167
39	A	2	2	1.00	12	0.167
40	A	16	15	1.15	13	1.154
41	A	8	8	1.11	13	0.615
42	A	10	10	1.36	13	0.769
43	A	6	6	1.29	11	0.545
44	A	9	9	1.26	11	0.818
45	A	16	15	1.30	13	1.154
46	A	21	20	1.22	13	1.538
47	A	16	15	1.26	13	1.154
48	A	9	9	1.09	13	0.692
49	A	16	16	1.36	13	1.231
50	A	9	9	1.34	13	0.692
51	A	6	6	1.32	11	0.545
52	A	12	12	1.50	11	1.091
53	A	21	20	1.12	13	1.538

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
54	A	25	24	1.28	13	1.846
55	A	19	18	1.38	13	1.385
56	A	2	2	1.00	14	0.143
57	A	4	4	1.00	14	0.286
58	A	6	6	1.06	14	0.429
59	A	8	8	1.09	14	0.571
60	A	2	2	1.00	14	0.143
61	A	4	4	1.00	14	0.286
62	A	6	6	1.06	14	0.429
63	A	8	8	1.09	14	0.571
64	A	7	6	1.14	16	0.375
65	A	7	6	1.14	16	0.375
66	A	6	6	1.04	17	0.353
67	A	4	4	1.00	17	0.235
68	A	2	2	1.00	17	0.118
69	A	4	3	1.00	17	0.176
70	A	6	5	1.00	17	0.294
71	A	8	7	1.04	17	0.412
72	C	17	16	1.28	13	1.231
73	C	13	12	1.29	13	0.923
74	C	13	12	1.26	13	0.923
75	C	8	7	1.30	11	0.636
76	C	11	10	1.26	11	0.909
77	C	15	14	1.27	13	1.077
78	C	19	18	1.40	13	1.385
79	C	21	20	1.30	13	1.538
80	C	17	16	1.16	13	1.231
81	C	13	12	1.27	13	0.923
82	A	11	10	1.34	13	0.769
83	C	10	9	1.23	11	0.818
84	C	15	14	1.44	11	1.273
85	C	18	17	1.25	13	1.308

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
86	C	23	22	1.24	13	1.692
87	C	24	23	1.15	13	1.769
88	A	5	4	0.94	14	0.286
89	A	8	7	1.00	14	0.500
90	A	11	10	1.05	14	0.714
91	A	14	13	1.07	14	0.929
92	A	2	2	1.00	14	0.143
93	A	5	5	1.08	14	0.357
94	A	8	8	1.11	14	0.571
95	A	11	11	1.12	14	0.786
96	A	8	8	1.07	12	0.667
97	A	6	6	1.06	12	0.500
98	A	4	4	1.03	12	0.333
99	A	2	2	1.00	12	0.167
100	A	1	1	1.00	10	0.100
101	A	5	4	0.95	12	0.333
102	A	9	8	0.97	12	0.667
103	A	12	11	1.08	12	0.917
104	A	14	13	1.14	12	1.083
105	A	15	15	1.03	10	1.500
106	A	12	12	1.01	10	1.200
107	A	4	4	1.00	10	0.400
108	A	4	4	1.00	10	0.400
109	A	7	7	1.00	10	0.700
110	A	15	15	1.04	10	1.500
111	A	10	10	1.00	13	0.769
112	A	8	8	0.97	20	0.400
113	A	6	6	1.00	20	0.300
114	A	4	4	1.08	20	0.200
115	A	4	4	1.09	15	0.267
116	A	4	4	1.09	15	0.267
117	A	6	6	0.97	15	0.400

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
118	A	8	8	0.98	15	0.533
119	A	4	4	1.04	17	0.235
120	A	4	4	1.08	17	0.235
121	A	6	6	0.97	17	0.353
122	A	8	8	0.98	17	0.471
123	A	6	5	1.02	20	0.250
124	A	6	5	1.00	20	0.250
125	A	8	7	1.00	20	0.350
126	A	18	18	1.04	17	1.059
127	A	15	15	1.03	17	0.882
128	A	12	12	1.01	17	0.706
129	A	7	6	1.09	15	0.400
130	A	9	8	1.07	15	0.533
131	A	12	11	1.13	15	0.733
132	A	14	13	1.17	15	0.867
133	A	7	6	1.08	20	0.300
134	A	2	2	1.00	20	0.100
135	A	3	3	1.00	16	0.188
136	A	4	4	1.06	13	0.308
137	A	9	9	1.00	17	0.529
138	A	12	12	1.05	17	0.706
139	A	15	15	1.03	17	0.882
140	A	9	9	1.06	10	0.900
141	A	7	7	1.00	10	0.700
142	A	5	5	1.00	10	0.500
143	A	5	5	1.00	10	0.500
144	A	7	7	1.00	10	0.700
145	A	9	9	1.13	10	0.900
146	A	14	14	0.90	10	1.400
147	A	10	10	1.02	10	1.000
148	A	8	8	1.08	10	0.800
149	A	8	8	1.05	10	0.800

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
150	A	10	10	1.00	10	1.000
151	A	14	14	0.87	10	1.400
152	A	18	18	0.70	10	1.800
153	A	12	12	0.78	10	1.200
154	A	6	6	0.83	10	0.600
155	A	7	6	1.00	10	0.600
156	C	7	6	0.71	10	0.600
157	C	7	6	0.58	10	0.600
158	A	9	9	1.14	13	0.692
159	A	5	4	1.00	13	0.308
160	A	7	7	1.11	13	0.538
161	A	5	4	1.00	13	0.308
162	A	5	5	1.04	13	0.385
163	A	4	3	1.00	13	0.231
164	A	3	3	1.00	13	0.231
165	A	4	3	1.00	11	0.273
166	A	5	4	1.00	11	0.364
167	A	6	5	1.00	13	0.385
168	A	5	4	1.00	13	0.308
169	A	6	5	1.08	13	0.385
170	A	5	4	1.00	13	0.308
171	A	7	7	1.25	13	0.538
172	A	4	3	1.00	13	0.231
173	A	5	5	1.03	13	0.385
174	A	5	4	1.00	13	0.308
175	A	3	3	1.00	13	0.231
176	A	4	3	1.00	11	0.273
177	A	5	4	1.00	11	0.364
178	A	8	7	1.30	13	0.538
179	A	5	4	1.00	13	0.308
180	A	8	7	1.33	13	0.538
181	A	5	4	1.00	15	0.267

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
182	A	2	2	1.00	15	0.133
183	A	4	3	1.00	13	0.231
184	A	5	4	1.08	15	0.267
185	A	2	2	1.00	15	0.133
186	A	4	3	1.00	13	0.231
187	A	6	5	1.00	13	0.385
188	A	16	15	1.17	13	1.154
189	A	5	4	0.99	13	0.308
190	A	13	12	1.16	13	0.923
191	A	6	5	1.00	13	0.385
192	A	10	9	1.19	13	0.692
193	A	4	3	1.00	11	0.273
194	A	8	7	1.23	11	0.636
195	A	9	8	1.08	13	0.615
196	A	7	6	1.55	13	0.462
197	A	11	10	1.16	13	0.769
198	A	10	9	1.59	13	0.692
199	A	14	13	1.21	13	1.000
200	C	14	13	1.24	13	1.000
201	A	6	5	0.92	13	0.385
202	C	11	10	1.29	13	0.769
203	A	4	3	1.00	11	0.273
204	A	7	6	1.30	11	0.545
205	A	11	10	1.17	13	0.769
206	A	7	6	1.32	13	0.462
207	A	14	13	1.19	13	1.000
208	A	11	10	1.00	13	0.769
209	A	17	16	1.14	13	1.231
210	A	14	13	1.30	13	1.000
211	A	15	14	1.00	11	1.273
212	A	8	7	1.21	11	0.636
213	A	13	12	1.17	13	0.923

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
214	A	12	11	1.00	13	0.846
215	A	13	12	1.35	13	0.923
216	A	13	12	1.39	13	0.923
217	A	21	20	1.36	13	1.538
218	A	3	3	1.00	13	0.231
219	A	7	6	1.00	13	0.462
220	A	5	5	1.00	13	0.385
221	A	7	6	1.06	11	0.545
222	A	7	6	1.24	11	0.545
223	A	5	5	1.00	13	0.385
224	A	7	6	1.21	13	0.462
225	A	5	5	1.00	13	0.385
226	A	7	6	1.00	13	0.462
227	A	5	5	1.00	13	0.385
228	A	23	22	1.04	13	1.692
229	A	9	8	1.39	13	0.615
230	A	14	13	1.07	13	1.000
231	A	10	9	1.12	11	0.818
232	A	7	6	1.10	11	0.545
233	C	18	17	1.32	13	1.308
234	A	7	6	1.04	13	0.462
235	C	18	17	1.19	13	1.308
236	A	3	3	1.00	13	0.231
237	A	8	7	1.31	13	0.538
238	A	4	4	1.00	13	0.308
239	A	9	8	1.28	11	0.727
240	A	6	5	1.06	11	0.455
241	C	22	21	1.58	13	1.615
242	A	7	6	1.03	13	0.462
243	C	22	21	1.25	13	1.615
244	A	7	6	1.00	13	0.462
245	A	6	5	1.00	13	0.385

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	number of rules / integrand leaf size
246	A	3	3	1.00	15	0.200
247	A	3	3	1.00	15	0.200
248	A	3	3	1.00	17	0.176
249	A	3	3	1.00	15	0.200
250	A	3	3	1.00	15	0.200
251	A	5	5	1.00	15	0.333
252	C	15	14	1.22	15	0.933
253	A	11	10	0.98	31	0.323
254	A	13	12	0.98	31	0.387
255	A	16	15	1.06	31	0.484
256	A	18	17	1.10	31	0.548
257	A	11	10	0.89	14	0.714
258	A	10	9	0.93	14	0.643
259	A	9	8	0.99	12	0.667
260	A	2	2	1.00	20	0.100
261	C	6	5	1.09	36	0.139
262	A	6	5	0.98	36	0.139
263	A	5	4	1.00	34	0.118
264	N/A	5	0	1.00	34	0.000
265	N/A	5	0	1.00	36	0.000
266	A	1	1	1.00	11	0.091
267	A	2	2	1.00	13	0.154
268	A	2	2	0.96	13	0.154
269	A	3	3	0.93	13	0.231
270	A	1	1	1.01	15	0.067
271	A	2	2	1.02	17	0.118
272	A	2	2	0.93	17	0.118
273	A	3	3	0.89	17	0.176
274	A	5	4	1.00	15	0.267
275	A	6	5	1.13	17	0.294
276	A	6	5	0.86	17	0.294
277	A	9	8	1.10	17	0.471

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
278	A	6	5	0.82	17	0.294
279	A	8	7	1.00	19	0.368
280	A	8	7	1.00	19	0.368
281	A	6	5	1.03	19	0.263
282	A	6	5	1.03	19	0.263
283	A	8	7	1.00	19	0.368
284	A	8	7	1.00	19	0.368
285	A	9	8	1.00	18	0.444
286	A	7	6	1.09	18	0.333
287	A	4	3	1.00	18	0.167
288	A	5	4	1.62	18	0.222
289	C	8	7	1.17	10	0.700
290	A	10	9	0.97	12	0.750
291	C	7	6	1.08	12	0.500
292	C	12	11	1.23	11	1.000
293	C	14	13	1.22	13	1.000
294	C	6	5	1.05	13	0.385
295	C	12	11	1.14	14	0.786
296	C	14	13	1.14	16	0.812
297	C	6	5	0.96	16	0.312
298	C	13	12	1.05	17	0.706
299	C	15	14	1.05	19	0.737
300	C	7	6	0.92	19	0.316
301	A	5	4	0.86	16	0.250
302	A	6	5	0.82	16	0.312
303	A	5	4	0.88	16	0.250
304	A	5	4	1.39	14	0.286
305	A	4	3	1.00	14	0.214
306	A	5	4	1.07	16	0.250
307	A	4	3	1.00	16	0.188
308	A	7	6	1.11	16	0.375
309	A	6	5	1.00	16	0.312

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
310	A	5	4	1.08	10	0.400
311	A	5	4	1.00	8	0.500
312	A	6	5	1.73	8	0.625
313	A	7	6	1.31	10	0.600
314	A	5	4	1.08	10	0.400
315	A	5	4	1.21	8	0.500
316	A	10	9	1.04	8	1.125
317	A	12	11	1.30	10	1.100
318	A	5	4	1.08	10	0.400
319	A	5	4	1.21	8	0.500
320	A	15	14	1.48	8	1.750
321	A	16	15	1.44	10	1.500
322	A	2	2	0.94	18	0.111
323	A	2	2	1.00	18	0.111
324	A	1	1	1.00	16	0.062
325	A	1	1	1.00	16	0.062
326	A	1	1	1.00	18	0.056
327	A	2	2	1.12	18	0.111
328	A	2	2	1.11	18	0.111
329	A	7	6	0.41	25	0.240
330	A	7	6	0.46	25	0.240
331	A	6	5	0.82	25	0.200
332	A	5	4	1.00	25	0.160
333	A	5	4	1.00	25	0.160
334	A	7	6	0.63	25	0.240
335	A	7	6	0.57	25	0.240
336	A	1	1	1.00	10	0.100
337	A	2	2	1.00	12	0.167
338	A	2	2	1.00	15	0.133
339	A	2	2	1.00	12	0.167
340	A	2	2	1.00	14	0.143
341	A	2	2	1.00	17	0.118

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
342	A	2	2	1.00	16	0.125
343	A	2	2	1.00	18	0.111
344	A	2	2	1.00	18	0.111
345	A	2	2	1.00	19	0.105
346	A	2	2	1.00	21	0.095
347	A	2	2	1.00	21	0.095
348	A	2	2	1.00	16	0.125
349	A	2	2	1.00	18	0.111
350	A	2	2	1.00	18	0.111
351	A	2	2	1.00	18	0.111
352	A	2	2	1.00	20	0.100
353	A	2	2	1.00	20	0.100
354	A	2	2	1.00	21	0.095
355	A	2	2	1.00	23	0.087
356	A	2	2	1.00	23	0.087
357	A	2	2	1.00	19	0.105
358	A	2	2	1.00	21	0.095
359	A	2	2	1.00	21	0.095
360	A	2	2	1.00	21	0.095
361	A	2	2	1.00	23	0.087
362	A	2	2	1.00	23	0.087
363	A	2	2	1.00	24	0.083
364	A	2	2	1.00	26	0.077
365	A	2	2	1.00	26	0.077
366	A	2	2	1.00	6	0.333
367	A	2	2	1.00	6	0.333
368	A	2	2	1.00	16	0.125
369	A	2	2	1.00	19	0.105

CHAPTER 3

Listing of integrals

3.1 $\int \sinh(a+bx)\,dx$

Optimal result

Integrand size = 6, antiderivative size = 10

$$\int \sinh(a+bx)\,dx=\frac{\cosh(a+bx)}{b}$$

output
```
cosh(b*x+a)/b
```

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 21 vs. 2(10) = 20.

Time = 0.00 (sec) , antiderivative size = 21, normalized size of antiderivative = 2.10

$$\int \sinh(a+bx)\,dx=\frac{\cosh(a)\cosh(bx)}{b}+\frac{\sinh(a)\sinh(bx)}{b}$$

input
```
Integrate[Sinh[a + b*x],x]
```

output
```
(Cosh[a]*Cosh[b*x])/b + (Sinh[a]*Sinh[b*x])/b
```

Rubi [A] (verified)

Time = 0.16 (sec) , antiderivative size = 10, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$, Rules used = {3042, 26, 3118}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sinh(a+bx)\,dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int -i\sin(ia+ibx)dx$$

$$\Big\downarrow \text{ 26}$$

$$-i\int \sin(ia+ibx)dx$$

$$\Big\downarrow \text{ 3118}$$

$$\frac{\cosh(a+bx)}{b}$$

input
```
Int[Sinh[a + b*x],x]
```

output
```
Cosh[a + b*x]/b
```

Defintions of rubi rules used

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3118
```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Maple [A] (verified)

Time = 0.09 (sec) , antiderivative size = 11, normalized size of antiderivative = 1.10

method	result	size
derivativedivides	$\frac{\cosh(bx+a)}{b}$	11
default	$\frac{\cosh(bx+a)}{b}$	11
orering	$\frac{\cosh(bx+a)}{b}$	11
parallelrisch	$\frac{1+\cosh(bx+a)}{b}$	13
risch	$\frac{e^{bx+a}}{2b}+\frac{e^{-bx-a}}{2b}$	27
meijerg	$-\frac{\sqrt{\pi}\,\cosh(a)\left(\frac{1}{\sqrt{\pi}}-\frac{\cosh(bx)}{\sqrt{\pi}}\right)}{b}+\frac{\sinh(a)\sinh(bx)}{b}$	35

input
```
int(sinh(b*x+a),x,method=_RETURNVERBOSE)
```

output
```
cosh(b*x+a)/b
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 10, normalized size of antiderivative = 1.00

$$\int \sinh(a+bx)\,dx=\frac{\cosh(bx+a)}{b}$$

input
```
integrate(sinh(b*x+a),x, algorithm="fricas")
```

output
```
cosh(b*x + a)/b
```

Sympy [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.20

$$\int \sinh(a+bx)\,dx = \begin{cases} \frac{\cosh(a+bx)}{b} & \text{for } b \neq 0 \\ x\sinh(a) & \text{otherwise} \end{cases}$$

input
```
integrate(sinh(b*x+a),x)
```

output
```
Piecewise((cosh(a + b*x)/b, Ne(b, 0)), (x*sinh(a), True))
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 10, normalized size of antiderivative = 1.00

$$\int \sinh(a+bx)\,dx = \frac{\cosh(bx+a)}{b}$$

input
```
integrate(sinh(b*x+a),x, algorithm="maxima")
```

output
```
cosh(b*x + a)/b
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 26 vs. $2(10) = 20$.

Time = 0.11 (sec) , antiderivative size = 26, normalized size of antiderivative = 2.60

$$\int \sinh(a+bx)\,dx = \frac{e^{(bx+a)}}{2\,b} + \frac{e^{(-bx-a)}}{2\,b}$$

input
```
integrate(sinh(b*x+a),x, algorithm="giac")
```

output
```
1/2*e^(b*x + a)/b + 1/2*e^(-b*x - a)/b
```

Mupad [B] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 10, normalized size of antiderivative = 1.00

$$\int \sinh(a+bx)\,dx=\frac{\cosh(a+b\,x)}{b}$$

input
```
int(sinh(a + b*x),x)
```

output
```
cosh(a + b*x)/b
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 10, normalized size of antiderivative = 1.00

$$\int \sinh(a+bx)\,dx=\frac{\cosh(bx+a)}{b}$$

input
```
int(sinh(b*x+a),x)
```

output
```
cosh(a + b*x)/b
```

3.2 $\int \sinh^2(a+bx)\,dx$

Optimal result

Integrand size = 8, antiderivative size = 25

$$\int \sinh^2(a+bx)\,dx=-\frac{x}{2}+\frac{\cosh(a+bx)\sinh(a+bx)}{2b}$$

output
```
-1/2*x+1/2*cosh(b*x+a)*sinh(b*x+a)/b
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.92

$$\int \sinh^2(a+bx)\,dx=\frac{-2(a+bx)+\sinh(2(a+bx))}{4b}$$

input
```
Integrate[Sinh[a + b*x]^2,x]
```

output
```
(-2*(a + b*x) + Sinh[2*(a + b*x)])/(4*b)
```

Rubi [A] (verified)

Time = 0.17 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$, Rules used = {3042, 25, 3115, 24}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sinh^2(a+bx)\,dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int -\sin(ia+ibx)^2dx$$

$$\Big\downarrow \text{ 25}$$

$$-\int \sin(ia+ibx)^2dx$$

$$\Big\downarrow \text{ 3115}$$

$$\frac{\sinh(a+bx)\cosh(a+bx)}{2b}-\frac{\int 1dx}{2}$$

$$\Big\downarrow \text{ 24}$$

$$\frac{\sinh(a+bx)\cosh(a+bx)}{2b}-\frac{x}{2}$$

input
```
Int[Sinh[a + b*x]^2,x]
```

output
```
-1/2*x + (Cosh[a + b*x]*Sinh[a + b*x])/(2*b)
```

Defintions of rubi rules used

rule 24
```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3115
```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Simp[b^2*((n - 1)/n)   Int[(b*Sin
[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[
2*n]
```

Maple [A] (verified)

Time = 0.20 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.80

method	result	size
parallelrisch	$\frac{-2bx+\sinh(2bx+2a)}{4b}$	20
derivativedivides	$\frac{\frac{\cosh(bx+a)\sinh(bx+a)}{2}-\frac{bx}{2}-\frac{a}{2}}{b}$	27
default	$\frac{\frac{\cosh(bx+a)\sinh(bx+a)}{2}-\frac{bx}{2}-\frac{a}{2}}{b}$	27
risch	$-\frac{x}{2}+\frac{e^{2bx+2a}}{8b}-\frac{e^{-2bx-2a}}{8b}$	33
orering	$x\sinh\left(bx+a\right)^{2}+\frac{\cosh(bx+a)\sinh(bx+a)}{2b}-\frac{x\left(2b^{2}\cosh(bx+a)^{2}+2\sinh(bx+a)^{2}b^{2}\right)}{4b^{2}}$	62

input
```
int(sinh(b*x+a)^2,x,method=_RETURNVERBOSE)
```

output
```
1/4*(-2*b*x+sinh(2*b*x+2*a))/b
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.92

$$\int \sinh^2(a+bx)\,dx = -\frac{bx-\cosh(bx+a)\sinh(bx+a)}{2\,b}$$

input
```
integrate(sinh(b*x+a)^2,x, algorithm="fricas")
```

output
```
-1/2*(b*x - cosh(b*x + a)*sinh(b*x + a))/b
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 46 vs. 2(19) = 38.

Time = 0.08 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.84

$$\int \sinh^2(a+bx)\,dx = \begin{cases} \frac{x\sinh^2(a+bx)}{2} - \frac{x\cosh^2(a+bx)}{2} + \frac{\sinh(a+bx)\cosh(a+bx)}{2b} & \text{for } b \neq 0 \\ x\sinh^2(a) & \text{otherwise} \end{cases}$$

input
```
integrate(sinh(b*x+a)**2,x)
```

output
```
Piecewise((x*sinh(a + b*x)**2/2 - x*cosh(a + b*x)**2/2 + sinh(a + b*x)*cos
h(a + b*x)/(2*b), Ne(b, 0)), (x*sinh(a)**2, True))
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.28

$$\int \sinh^2(a+bx)\,dx = -\frac{1}{2}\,x + \frac{e^{(2\,bx+2\,a)}}{8\,b} - \frac{e^{(-2\,bx-2\,a)}}{8\,b}$$

input
```
integrate(sinh(b*x+a)^2,x, algorithm="maxima")
```

output
```
-1/2*x + 1/8*e^(2*b*x + 2*a)/b - 1/8*e^(-2*b*x - 2*a)/b
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.28

$$\int \sinh^2(a+bx)\,dx = -\frac{1}{2}\,x + \frac{e^{(2\,bx+2\,a)}}{8\,b} - \frac{e^{(-2\,bx-2\,a)}}{8\,b}$$

input
```
integrate(sinh(b*x+a)^2,x, algorithm="giac")
```

output
```
-1/2*x + 1/8*e^(2*b*x + 2*a)/b - 1/8*e^(-2*b*x - 2*a)/b
```

Mupad [B] (verification not implemented)

Time = 1.31 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.72

$$\int \sinh^2(a+bx)\,dx = \frac{\sinh(2\,a+2\,b\,x)}{4\,b} - \frac{x}{2}$$

input
```
int(sinh(a + b*x)^2,x)
```

output
```
sinh(2*a + 2*b*x)/(4*b) - x/2
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 43, normalized size of antiderivative = 1.72

$$\int \sinh^2(a+bx)\,dx = \frac{e^{4bx+4a} - 4e^{2bx+2a}bx - 1}{8e^{2bx+2a}b}$$

input
```
int(sinh(b*x+a)^2,x)
```

output
```
(e**(4*a + 4*b*x) - 4*e**(2*a + 2*b*x)*b*x - 1)/(8*e**(2*a + 2*b*x)*b)
```

3.3 $\int \sinh^3(a+bx)\,dx$

Optimal result

Integrand size = 8, antiderivative size = 27

$$\int \sinh^3(a+bx)\,dx = -\frac{\cosh(a+bx)}{b} + \frac{\cosh^3(a+bx)}{3b}$$

output
```
-cosh(b*x+a)/b+1/3*cosh(b*x+a)^3/b
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.07

$$\int \sinh^3(a+bx)\,dx = -\frac{3\cosh(a+bx)}{4b} + \frac{\cosh(3(a+bx))}{12b}$$

input
```
Integrate[Sinh[a + b*x]^3,x]
```

output
```
(-3*Cosh[a + b*x])/(4*b) + Cosh[3*(a + b*x)]/(12*b)
```

Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.89, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {3042, 26, 3113, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sinh^3(a+bx)\,dx$$
$$\Big\downarrow \ 3042$$
$$\int i\sin(ia+ibx)^3dx$$
$$\Big\downarrow \ 26$$
$$i\int \sin(ia+ibx)^3dx$$
$$\Big\downarrow \ 3113$$
$$-\frac{\int \left(1-\cosh^2(a+bx)\right)d\cosh(a+bx)}{b}$$
$$\Big\downarrow \ 2009$$
$$-\frac{\cosh(a+bx)-\frac{1}{3}\cosh^3(a+bx)}{b}$$

input
```
Int[Sinh[a + b*x]^3,x]
```

output
```
-((Cosh[a + b*x] - Cosh[a + b*x]^3/3)/b)
```

Defintions of rubi rules used

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3113
```
Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Simp[-d^(-1)   Subst[Int[Exp
and[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x]
 && IGtQ[(n - 1)/2, 0]
```

Maple [A] (verified)

Time = 0.40 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.85

method	result	size
derivativedivides	$\frac{\left(-\frac{2}{3}+\frac{\sinh(bx+a)^2}{3}\right)\cosh(bx+a)}{b}$	23
default	$\frac{\left(-\frac{2}{3}+\frac{\sinh(bx+a)^2}{3}\right)\cosh(bx+a)}{b}$	23
parallelrisch	$\frac{-8+\cosh(3bx+3a)-9\cosh(bx+a)}{12b}$	25
risch	$\frac{e^{3bx+3a}}{24b}-\frac{3\,e^{bx+a}}{8b}-\frac{3\,e^{-bx-a}}{8b}+\frac{e^{-3bx-3a}}{24b}$	55
orering	$\frac{10\sinh(bx+a)^2\cosh(bx+a)}{3b}-\frac{6b^3\cosh(bx+a)^3+21\sinh(bx+a)^2b^3\cosh(bx+a)}{9b^4}$	59

input
```
int(sinh(b*x+a)^3,x,method=_RETURNVERBOSE)
```

output
```
1/b*(-2/3+1/3*sinh(b*x+a)^2)*cosh(b*x+a)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 38, normalized size of antiderivative = 1.41

$$\int \sinh^3(a+bx)\,dx = \frac{\cosh\left(bx+a\right)^3 + 3\,\cosh\left(bx+a\right)\sinh\left(bx+a\right)^2 - 9\,\cosh\left(bx+a\right)}{12\,b}$$

input
```
integrate(sinh(b*x+a)^3,x, algorithm="fricas")
```

output
```
1/12*(cosh(b*x + a)^3 + 3*cosh(b*x + a)*sinh(b*x + a)^2 - 9*cosh(b*x + a))
/b
```

Sympy [A] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.33

$$\int \sinh^3(a+bx)\,dx = \begin{cases} \frac{\sinh^2\left(a+bx\right)\cosh\left(a+bx\right)}{b} - \frac{2\cosh^3\left(a+bx\right)}{3b} & \text{for } b \neq 0 \\ x\sinh^3\left(a\right) & \text{otherwise} \end{cases}$$

input
```
integrate(sinh(b*x+a)**3,x)
```

output
```
Piecewise((sinh(a + b*x)**2*cosh(a + b*x)/b - 2*cosh(a + b*x)**3/(3*b), Ne
(b, 0)), (x*sinh(a)**3, True))
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 54 vs. 2(25) = 50.

Time = 0.03 (sec) , antiderivative size = 54, normalized size of antiderivative = 2.00

$$\int \sinh^3(a+bx)\,dx = \frac{e^{(3\,bx+3\,a)}}{24\,b} - \frac{3\,e^{(bx+a)}}{8\,b} - \frac{3\,e^{(-bx-a)}}{8\,b} + \frac{e^{(-3\,bx-3\,a)}}{24\,b}$$

input
```
integrate(sinh(b*x+a)^3,x, algorithm="maxima")
```

output
```
1/24*e^(3*b*x + 3*a)/b - 3/8*e^(b*x + a)/b - 3/8*e^(-b*x - a)/b + 1/24*e^(
-3*b*x - 3*a)/b
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 54 vs. $2(25) = 50$.

Time = 0.12 (sec) , antiderivative size = 54, normalized size of antiderivative = 2.00

$$\int \sinh^3(a+bx)\,dx = \frac{e^{(3\,bx+3\,a)}}{24\,b} - \frac{3\,e^{(bx+a)}}{8\,b} - \frac{3\,e^{(-bx-a)}}{8\,b} + \frac{e^{(-3\,bx-3\,a)}}{24\,b}$$

input
```
integrate(sinh(b*x+a)^3,x, algorithm="giac")
```

output
```
1/24*e^(3*b*x + 3*a)/b - 3/8*e^(b*x + a)/b - 3/8*e^(-b*x - a)/b + 1/24*e^(
-3*b*x - 3*a)/b
```

Mupad [B] (verification not implemented)

Time = 1.32 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.89

$$\int \sinh^3(a+bx)\,dx = -\frac{3\cosh(a+b\,x) - \cosh(a+b\,x)^3}{3\,b}$$

input
```
int(sinh(a + b*x)^3,x)
```

output
```
-(3*cosh(a + b*x) - cosh(a + b*x)^3)/(3*b)
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 53, normalized size of antiderivative = 1.96

$$\int \sinh^3(a+bx)\,dx = \frac{e^{6bx+6a} - 9e^{4bx+4a} - 9e^{2bx+2a} + 1}{24e^{3bx+3a}b}$$

input
```
int(sinh(b*x+a)^3,x)
```

output
```
(e**(6*a + 6*b*x) - 9*e**(4*a + 4*b*x) - 9*e**(2*a + 2*b*x) + 1)/(24*e**(3
*a + 3*b*x)*b)
```

3.4 $\int \sinh^4(a+bx)\,dx$

Optimal result

Integrand size = 8, antiderivative size = 46

$$\int \sinh^4(a+bx)\,dx = \frac{3x}{8} - \frac{3\cosh(a+bx)\sinh(a+bx)}{8b} + \frac{\cosh(a+bx)\sinh^3(a+bx)}{4b}$$

output
```
3/8*x-3/8*cosh(b*x+a)*sinh(b*x+a)/b+1/4*cosh(b*x+a)*sinh(b*x+a)^3/b
```

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.72

$$\int \sinh^4(a+bx)\,dx = \frac{12(a+bx) - 8\sinh(2(a+bx)) + \sinh(4(a+bx))}{32b}$$

input
```
Integrate[Sinh[a + b*x]^4,x]
```

output
```
(12*(a + b*x) - 8*Sinh[2*(a + b*x)] + Sinh[4*(a + b*x)])/(32*b)
```

Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 51, normalized size of antiderivative = 1.11, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.875, Rules used = {3042, 3115, 25, 3042, 25, 3115, 24}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sinh^4(a+bx)\,dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \sin(ia+ibx)^4dx$$

$$\Big\downarrow \text{ 3115}$$

$$\frac{3}{4}\int -\sinh^2(a+bx)dx+\frac{\sinh^3(a+bx)\cosh(a+bx)}{4b}$$

$$\Big\downarrow \text{ 25}$$

$$\frac{\sinh^3(a+bx)\cosh(a+bx)}{4b}-\frac{3}{4}\int \sinh^2(a+bx)dx$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{\sinh^3(a+bx)\cosh(a+bx)}{4b}-\frac{3}{4}\int -\sin(ia+ibx)^2dx$$

$$\Big\downarrow \text{ 25}$$

$$\frac{\sinh^3(a+bx)\cosh(a+bx)}{4b}+\frac{3}{4}\int \sin(ia+ibx)^2dx$$

$$\Big\downarrow \text{ 3115}$$

$$\frac{3}{4}\left(\frac{\int 1dx}{2}-\frac{\sinh(a+bx)\cosh(a+bx)}{2b}\right)+\frac{\sinh^3(a+bx)\cosh(a+bx)}{4b}$$

$$\Big\downarrow \text{ 24}$$

$$\frac{\sinh^3(a+bx)\cosh(a+bx)}{4b}+\frac{3}{4}\left(\frac{x}{2}-\frac{\sinh(a+bx)\cosh(a+bx)}{2b}\right)$$

input
```
Int[Sinh[a + b*x]^4,x]
```

output
```
(Cosh[a + b*x]*Sinh[a + b*x]^3)/(4*b) + (3*(x/2 - (Cosh[a + b*x]*Sinh[a +
b*x])/(2*b)))/4
```

Defintions of rubi rules used

rule 24
```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3115
```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Simp[b^2*((n - 1)/n)   Int[(b*Sin
[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[
2*n]
```

Maple [A] (verified)

Time = 0.57 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.67

method	result
parallelrisch	$\frac{12bx+\sinh(4bx+4a)-8\sinh(2bx+2a)}{32b}$
derivativedivides	$\frac{\left(\frac{\sinh(bx+a)^3}{4}-\frac{3\sinh(bx+a)}{8}\right)\cosh(bx+a)+\frac{3bx}{8}+\frac{3a}{8}}{b}$
default	$\frac{\left(\frac{\sinh(bx+a)^3}{4}-\frac{3\sinh(bx+a)}{8}\right)\cosh(bx+a)+\frac{3bx}{8}+\frac{3a}{8}}{b}$
risch	$\frac{3x}{8}+\frac{e^{4bx+4a}}{64b}-\frac{e^{2bx+2a}}{8b}+\frac{e^{-2bx-2a}}{8b}-\frac{e^{-4bx-4a}}{64b}$
orering	$x\sinh(bx+a)^4+\frac{5\cosh(bx+a)\sinh(bx+a)^3}{4b}-\frac{5x\left(4\sinh(bx+a)^4b^2+12\cosh(bx+a)^2\sinh(bx+a)^2b^2\right)}{16b^2}-$ 4(

input
```
int(sinh(b*x+a)^4,x,method=_RETURNVERBOSE)
```

output
```
1/32*(12*b*x+sinh(4*b*x+4*a)-8*sinh(2*b*x+2*a))/b
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 49, normalized size of antiderivative = 1.07

$$\int \sinh^4(a+bx)\,dx = \frac{\cosh(bx+a)\sinh(bx+a)^3+3\,bx+\left(\cosh(bx+a)^3-4\,\cosh(bx+a)\right)\sinh(bx+a)}{8\,b}$$

input
```
integrate(sinh(b*x+a)^4,x, algorithm="fricas")
```

output
```
1/8*(cosh(b*x + a)*sinh(b*x + a)^3 + 3*b*x + (cosh(b*x + a)^3 - 4*cosh(b*x
 + a))*sinh(b*x + a))/b
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 95 vs. 2(41) = 82.

Time = 0.18 (sec) , antiderivative size = 95, normalized size of antiderivative = 2.07

$$\int \sinh^4(a+bx)\,dx = \begin{cases} \frac{3x\sinh^4(a+bx)}{8} - \frac{3x\sinh^2(a+bx)\cosh^2(a+bx)}{4} + \frac{3x\cosh^4(a+bx)}{8} + \frac{5\sinh^3(a+bx)\cosh(a+bx)}{8b} - \frac{3\sinh(a+bx)\cosh^3(a+bx)}{8b} & \text{fo} \\ x\sinh^4(a) & \text{o} \end{cases}$$

input
```
integrate(sinh(b*x+a)**4,x)
```

output
```
Piecewise((3*x*sinh(a + b*x)**4/8 - 3*x*sinh(a + b*x)**2*cosh(a + b*x)**2/
4 + 3*x*cosh(a + b*x)**4/8 + 5*sinh(a + b*x)**3*cosh(a + b*x)/(8*b) - 3*si
nh(a + b*x)*cosh(a + b*x)**3/(8*b), Ne(b, 0)), (x*sinh(a)**4, True))
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 60, normalized size of antiderivative = 1.30

$$\int \sinh^4(a+bx)\,dx = \frac{3}{8}\,x + \frac{e^{(4\,bx+4\,a)}}{64\,b} - \frac{e^{(2\,bx+2\,a)}}{8\,b} + \frac{e^{(-2\,bx-2\,a)}}{8\,b} - \frac{e^{(-4\,bx-4\,a)}}{64\,b}$$

input
```
integrate(sinh(b*x+a)^4,x, algorithm="maxima")
```

output
```
3/8*x + 1/64*e^(4*b*x + 4*a)/b - 1/8*e^(2*b*x + 2*a)/b + 1/8*e^(-2*b*x - 2
*a)/b - 1/64*e^(-4*b*x - 4*a)/b
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 60, normalized size of antiderivative = 1.30

$$\int \sinh^4(a+bx)\,dx = \frac{3}{8}\,x + \frac{e^{(4\,bx+4\,a)}}{64\,b} - \frac{e^{(2\,bx+2\,a)}}{8\,b} + \frac{e^{(-2\,bx-2\,a)}}{8\,b} - \frac{e^{(-4\,bx-4\,a)}}{64\,b}$$

input
```
integrate(sinh(b*x+a)^4,x, algorithm="giac")
```

output
```
3/8*x + 1/64*e^(4*b*x + 4*a)/b - 1/8*e^(2*b*x + 2*a)/b + 1/8*e^(-2*b*x - 2
*a)/b - 1/64*e^(-4*b*x - 4*a)/b
```

Mupad [B] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.70

$$\int \sinh^4(a+bx)\,dx = \frac{3\,x}{8} - \frac{\frac{\sinh(2\,a+2\,b\,x)}{4} - \frac{\sinh(4\,a+4\,b\,x)}{32}}{b}$$

input
```
int(sinh(a + b*x)^4,x)
```

output
```
(3*x)/8 - (sinh(2*a + 2*b*x)/4 - sinh(4*a + 4*b*x)/32)/b
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 67, normalized size of antiderivative = 1.46

$$\int \sinh^4(a+bx)\,dx = \frac{e^{8bx+8a} - 8e^{6bx+6a} + 24e^{4bx+4a}bx + 8e^{2bx+2a} - 1}{64e^{4bx+4a}b}$$

input
```
int(sinh(b*x+a)^4,x)
```

output
```
(e**(8*a + 8*b*x) - 8*e**(6*a + 6*b*x) + 24*e**(4*a + 4*b*x)*b*x + 8*e**(2
*a + 2*b*x) - 1)/(64*e**(4*a + 4*b*x)*b)
```

3.5 $\int \sinh^5(a+bx)\,dx$

Optimal result

Integrand size = 8, antiderivative size = 41

$$\int \sinh^5(a+bx)\,dx=\frac{\cosh(a+bx)}{b}-\frac{2\cosh^3(a+bx)}{3b}+\frac{\cosh^5(a+bx)}{5b}$$

output
```
cosh(b*x+a)/b-2/3*cosh(b*x+a)^3/b+1/5*cosh(b*x+a)^5/b
```

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 44, normalized size of antiderivative = 1.07

$$\int \sinh^5(a+bx)\,dx=\frac{5\cosh(a+bx)}{8b}-\frac{5\cosh(3(a+bx))}{48b}+\frac{\cosh(5(a+bx))}{80b}$$

input
```
Integrate[Sinh[a + b*x]^5,x]
```

output
```
(5*Cosh[a + b*x])/(8*b) - (5*Cosh[3*(a + b*x)])/(48*b) + Cosh[5*(a + b*x)]
/(80*b)
```

Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.85, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {3042, 26, 3113, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sinh^5(a+bx)\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int -i\sin(ia+ibx)^5dx$$

$$\Big\downarrow \text{26}$$

$$-i\int \sin(ia+ibx)^5dx$$

$$\Big\downarrow \text{3113}$$

$$\frac{\int \left(\cosh^4(a+bx)-2\cosh^2(a+bx)+1\right)d\cosh(a+bx)}{b}$$

$$\Big\downarrow \text{2009}$$

$$\frac{\frac{1}{5}\cosh^5(a+bx)-\frac{2}{3}\cosh^3(a+bx)+\cosh(a+bx)}{b}$$

input
```
Int[Sinh[a + b*x]^5,x]
```

output
```
(Cosh[a + b*x] - (2*Cosh[a + b*x]^3)/3 + Cosh[a + b*x]^5/5)/b
```

Defintions of rubi rules used

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3113
```
Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Simp[-d^(-1)   Subst[Int[Exp
and[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x]
 && IGtQ[(n - 1)/2, 0]
```

Maple [A] (verified)

Time = 0.97 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.80

method	result
derivativedivides	$\frac{\left(\frac{8}{15}+\frac{\sinh(bx+a)^4}{5}-\frac{4\sinh(bx+a)^2}{15}\right)\cosh(bx+a)}{b}$
default	$\frac{\left(\frac{8}{15}+\frac{\sinh(bx+a)^4}{5}-\frac{4\sinh(bx+a)^2}{15}\right)\cosh(bx+a)}{b}$
parallelrisch	$\frac{128-25\cosh(3bx+3a)+150\cosh(bx+a)+3\cosh(5bx+5a)}{240b}$
risch	$\frac{e^{5bx+5a}}{160b}-\frac{5\,e^{3bx+3a}}{96b}+\frac{5\,e^{bx+a}}{16b}+\frac{5\,e^{-bx-a}}{16b}-\frac{5\,e^{-3bx-3a}}{96b}+\frac{e^{-5bx-5a}}{160b}$
orering	$\frac{259\sinh(bx+a)^4\cosh(bx+a)}{45b}-\frac{7\left(60\sinh(bx+a)^2b^3\cosh(bx+a)^3+65\sinh(bx+a)^4b^3\cosh(bx+a)\right)}{45b^4}+\frac{120b^5\cosh(b}{}$

input
```
int(sinh(b*x+a)^5,x,method=_RETURNVERBOSE)
```

output
```
1/b*(8/15+1/5*sinh(b*x+a)^4-4/15*sinh(b*x+a)^2)*cosh(b*x+a)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 79 vs. $2(37) = 74$.

Time = 0.09 (sec) , antiderivative size = 79, normalized size of antiderivative = 1.93

$$\int \sinh^5(a+bx)\,dx = \frac{3\cosh(bx+a)^5 + 15\cosh(bx+a)\sinh(bx+a)^4 - 25\cosh(bx+a)^3 + 15\left(2\cosh(bx+a)^3 - 5\cosh(\right.}{240\,b}$$

input
```
integrate(sinh(b*x+a)^5,x, algorithm="fricas")
```

output
```
1/240*(3*cosh(b*x + a)^5 + 15*cosh(b*x + a)*sinh(b*x + a)^4 - 25*cosh(b*x
+ a)^3 + 15*(2*cosh(b*x + a)^3 - 5*cosh(b*x + a))*sinh(b*x + a)^2 + 150*co
sh(b*x + a))/b
```

Sympy [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 58, normalized size of antiderivative = 1.41

$$\int \sinh^5(a+bx)\,dx = \begin{cases} \frac{\sinh^4(a+bx)\cosh(a+bx)}{b} - \frac{4\sinh^2(a+bx)\cosh^3(a+bx)}{3b} + \frac{8\cosh^5(a+bx)}{15b} & \text{for } b \neq 0 \\ x\sinh^5(a) & \text{otherwise} \end{cases}$$

input
```
integrate(sinh(b*x+a)**5,x)
```

output
```
Piecewise((sinh(a + b*x)**4*cosh(a + b*x)/b - 4*sinh(a + b*x)**2*cosh(a +
b*x)**3/(3*b) + 8*cosh(a + b*x)**5/(15*b), Ne(b, 0)), (x*sinh(a)**5, True)
)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 82 vs. 2(37) = 74.

Time = 0.04 (sec) , antiderivative size = 82, normalized size of antiderivative = 2.00

$$\int \sinh^5(a+bx)\,dx = \frac{e^{(5\,bx+5\,a)}}{160\,b} - \frac{5\,e^{(3\,bx+3\,a)}}{96\,b} + \frac{5\,e^{(bx+a)}}{16\,b} + \frac{5\,e^{(-bx-a)}}{16\,b} - \frac{5\,e^{(-3\,bx-3\,a)}}{96\,b} + \frac{e^{(-5\,bx-5\,a)}}{160\,b}$$

input
```
integrate(sinh(b*x+a)^5,x, algorithm="maxima")
```

output
```
1/160*e^(5*b*x + 5*a)/b - 5/96*e^(3*b*x + 3*a)/b + 5/16*e^(b*x + a)/b + 5/
16*e^(-b*x - a)/b - 5/96*e^(-3*b*x - 3*a)/b + 1/160*e^(-5*b*x - 5*a)/b
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 82 vs. 2(37) = 74.

Time = 0.12 (sec) , antiderivative size = 82, normalized size of antiderivative = 2.00

$$\int \sinh^5(a+bx)\,dx = \frac{e^{(5\,bx+5\,a)}}{160\,b} - \frac{5\,e^{(3\,bx+3\,a)}}{96\,b} + \frac{5\,e^{(bx+a)}}{16\,b} + \frac{5\,e^{(-bx-a)}}{16\,b} - \frac{5\,e^{(-3\,bx-3\,a)}}{96\,b} + \frac{e^{(-5\,bx-5\,a)}}{160\,b}$$

input
```
integrate(sinh(b*x+a)^5,x, algorithm="giac")
```

output
```
1/160*e^(5*b*x + 5*a)/b - 5/96*e^(3*b*x + 3*a)/b + 5/16*e^(b*x + a)/b + 5/
16*e^(-b*x - a)/b - 5/96*e^(-3*b*x - 3*a)/b + 1/160*e^(-5*b*x - 5*a)/b
```

Mupad [B] (verification not implemented)

Time = 1.32 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.76

$$\int \sinh^5(a+bx)\,dx = \frac{\frac{\cosh(a+b\,x)^5}{5} - \frac{2\cosh(a+b\,x)^3}{3} + \cosh(a+b\,x)}{b}$$

input
```
int(sinh(a + b*x)^5,x)
```

output
```
(cosh(a + b*x) - (2*cosh(a + b*x)^3)/3 + cosh(a + b*x)^5/5)/b
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 79, normalized size of antiderivative = 1.93

$$\int \sinh^5(a+bx)\,dx = \frac{3e^{10bx+10a} - 25e^{8bx+8a} + 150e^{6bx+6a} + 150e^{4bx+4a} - 25e^{2bx+2a} + 3}{480e^{5bx+5a}b}$$

input
```
int(sinh(b*x+a)^5,x)
```

output
```
(3*e**(10*a + 10*b*x) - 25*e**(8*a + 8*b*x) + 150*e**(6*a + 6*b*x) + 150*e
**(4*a + 4*b*x) - 25*e**(2*a + 2*b*x) + 3)/(480*e**(5*a + 5*b*x)*b)
```

3.6 $\int \sinh^6(a+bx)\,dx$

Optimal result

Integrand size = 8, antiderivative size = 67

$$\int \sinh^6(a+bx)\,dx = -\frac{5x}{16}+\frac{5\cosh(a+bx)\sinh(a+bx)}{16b} - \frac{5\cosh(a+bx)\sinh^3(a+bx)}{24b}+\frac{\cosh(a+bx)\sinh^5(a+bx)}{6b}$$

output
```
-5/16*x+5/16*cosh(b*x+a)*sinh(b*x+a)/b-5/24*cosh(b*x+a)*sinh(b*x+a)^3/b+1/
6*cosh(b*x+a)*sinh(b*x+a)^5/b
```

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 43, normalized size of antiderivative = 0.64

$$\int \sinh^6(a+bx)\,dx = \frac{-60a-60bx+45\sinh(2(a+bx))-9\sinh(4(a+bx))+\sinh(6(a+bx))}{192b}$$

input
```
Integrate[Sinh[a + b*x]^6,x]
```

output
```
(-60*a - 60*b*x + 45*Sinh[2*(a + b*x)] - 9*Sinh[4*(a + b*x)] + Sinh[6*(a +
 b*x)])/(192*b)
```

Rubi [A] (verified)

Time = 0.33 (sec) , antiderivative size = 77, normalized size of antiderivative = 1.15, number of steps used = 10, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 1.250$, Rules used = {3042, 25, 3115, 3042, 3115, 25, 3042, 25, 3115, 24}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sinh^6(a+bx)\,dx$$

$$\Big\downarrow \; 3042$$

$$\int -\sin(ia+ibx)^6dx$$

$$\Big\downarrow \; 25$$

$$-\int \sin(ia+ibx)^6dx$$

$$\Big\downarrow \; 3115$$

$$\frac{\sinh^5(a+bx)\cosh(a+bx)}{6b}-\frac{5}{6}\int \sinh^4(a+bx)dx$$

$$\Big\downarrow \; 3042$$

$$\frac{\sinh^5(a+bx)\cosh(a+bx)}{6b}-\frac{5}{6}\int \sin(ia+ibx)^4dx$$

$$\Big\downarrow \; 3115$$

$$\frac{\sinh^5(a+bx)\cosh(a+bx)}{6b}-\frac{5}{6}\left(\frac{3}{4}\int -\sinh^2(a+bx)dx+\frac{\sinh^3(a+bx)\cosh(a+bx)}{4b}\right)$$

$$\Big\downarrow \; 25$$

$$\frac{\sinh^5(a+bx)\cosh(a+bx)}{6b}-\frac{5}{6}\left(\frac{\sinh^3(a+bx)\cosh(a+bx)}{4b}-\frac{3}{4}\int \sinh^2(a+bx)dx\right)$$

$$\Big\downarrow \; 3042$$

$$\frac{\sinh^5(a+bx)\cosh(a+bx)}{6b}-\frac{5}{6}\left(\frac{\sinh^3(a+bx)\cosh(a+bx)}{4b}-\frac{3}{4}\int -\sin(ia+ibx)^2dx\right)$$

$$\Big\downarrow \text{25}$$

$$\frac{\sinh^5(a+bx)\cosh(a+bx)}{6b}-\frac{5}{6}\left(\frac{\sinh^3(a+bx)\cosh(a+bx)}{4b}+\frac{3}{4}\int \sin(ia+ibx)^2dx\right)$$

$$\Big\downarrow \text{3115}$$

$$\frac{\sinh^5(a+bx)\cosh(a+bx)}{6b}-\frac{5}{6}\left(\frac{3}{4}\left(\frac{\int 1dx}{2}-\frac{\sinh(a+bx)\cosh(a+bx)}{2b}\right)+\frac{\sinh^3(a+bx)\cosh(a+bx)}{4b}\right)$$

$$\Big\downarrow \text{24}$$

$$\frac{\sinh^5(a+bx)\cosh(a+bx)}{6b}-\frac{5}{6}\left(\frac{\sinh^3(a+bx)\cosh(a+bx)}{4b}+\frac{3}{4}\left(\frac{x}{2}-\frac{\sinh(a+bx)\cosh(a+bx)}{2b}\right)\right)$$

input
```
Int[Sinh[a + b*x]^6,x]
```

output
```
(Cosh[a + b*x]*Sinh[a + b*x]^5)/(6*b) - (5*((Cosh[a + b*x]*Sinh[a + b*x]^3
)/(4*b) + (3*(x/2 - (Cosh[a + b*x]*Sinh[a + b*x])/(2*b)))/4))/6
```

Defintions of rubi rules used

rule 24
```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3115
```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Simp[b^2*((n - 1)/n)   Int[(b*Sin
[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[
2*n]
```

Maple [A] (verified)

Time = 1.40 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.63

method	result
parallelrisch	$\frac{-60bx+\sinh(6bx+6a)-9\sinh(4bx+4a)+45\sinh(2bx+2a)}{192b}$
derivativedivides	$\frac{\left(\frac{\sinh(bx+a)^5}{6}-\frac{5\sinh(bx+a)^3}{24}+\frac{5\sinh(bx+a)}{16}\right)\cosh(bx+a)-\frac{5bx}{16}-\frac{5a}{16}}{b}$
default	$\frac{\left(\frac{\sinh(bx+a)^5}{6}-\frac{5\sinh(bx+a)^3}{24}+\frac{5\sinh(bx+a)}{16}\right)\cosh(bx+a)-\frac{5bx}{16}-\frac{5a}{16}}{b}$
risch	$-\frac{5x}{16}+\frac{e^{6bx+6a}}{384b}-\frac{3\,e^{4bx+4a}}{128b}+\frac{15\,e^{2bx+2a}}{128b}-\frac{15\,e^{-2bx-2a}}{128b}+\frac{3\,e^{-4bx-4a}}{128b}-\frac{e^{-6bx-6a}}{384b}$
orering	$x\sinh(bx+a)^6+\frac{49\cosh(bx+a)\sinh(bx+a)^5}{24b}-\frac{49x\left(6b^2\sinh(bx+a)^6+30\cosh(bx+a)^2\sinh(bx+a)^4b^2\right)}{144b^2}-$

input
```
int(sinh(b*x+a)^6,x,method=_RETURNVERBOSE)
```

output
```
1/192*(-60*b*x+sinh(6*b*x+6*a)-9*sinh(4*b*x+4*a)+45*sinh(2*b*x+2*a))/b
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 90, normalized size of antiderivative = 1.34

$$\int \sinh^6(a+bx)\,dx$$
$$=\frac{3\cosh(bx+a)\sinh(bx+a)^5+2\left(5\cosh(bx+a)^3-9\cosh(bx+a)\right)\sinh(bx+a)^3-30\,bx+3\left(\cosh\right.}{96\,b}$$

input
```
integrate(sinh(b*x+a)^6,x, algorithm="fricas")
```

output
```
1/96*(3*cosh(b*x + a)*sinh(b*x + a)^5 + 2*(5*cosh(b*x + a)^3 - 9*cosh(b*x
+ a))*sinh(b*x + a)^3 - 30*b*x + 3*(cosh(b*x + a)^5 - 6*cosh(b*x + a)^3 +
15*cosh(b*x + a))*sinh(b*x + a))/b
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 139 vs. $2(61) = 122$.

Time = 0.34 (sec) , antiderivative size = 139, normalized size of antiderivative = 2.07

$$\int \sinh^6(a+bx)\,dx$$
$$= \begin{cases} \frac{5x\sinh^6(a+bx)}{16} - \frac{15x\sinh^4(a+bx)\cosh^2(a+bx)}{16} + \frac{15x\sinh^2(a+bx)\cosh^4(a+bx)}{16} - \frac{5x\cosh^6(a+bx)}{16} + \frac{11\sinh^5(a+bx)\cosh(a+bx)}{16b} \\ x\sinh^6(a) \end{cases}$$

input
```
integrate(sinh(b*x+a)**6,x)
```

output
```
Piecewise((5*x*sinh(a + b*x)**6/16 - 15*x*sinh(a + b*x)**4*cosh(a + b*x)**
2/16 + 15*x*sinh(a + b*x)**2*cosh(a + b*x)**4/16 - 5*x*cosh(a + b*x)**6/16
 + 11*sinh(a + b*x)**5*cosh(a + b*x)/(16*b) - 5*sinh(a + b*x)**3*cosh(a +
b*x)**3/(6*b) + 5*sinh(a + b*x)*cosh(a + b*x)**5/(16*b), Ne(b, 0)), (x*sin
h(a)**6, True))
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 86, normalized size of antiderivative = 1.28

$$\int \sinh^6(a+bx)\,dx = -\frac{\left(9\,e^{(-2\,bx-2\,a)} - 45\,e^{(-4\,bx-4\,a)} - 1\right)e^{(6\,bx+6\,a)}}{384\,b} - \frac{5\,(bx+a)}{16\,b} - \frac{45\,e^{(-2\,bx-2\,a)} - 9\,e^{(-4\,bx-4\,a)} + e^{(-6\,bx-6\,a)}}{384\,b}$$

input
```
integrate(sinh(b*x+a)^6,x, algorithm="maxima")
```

output
```
-1/384*(9*e^(-2*b*x - 2*a) - 45*e^(-4*b*x - 4*a) - 1)*e^(6*b*x + 6*a)/b -
5/16*(b*x + a)/b - 1/384*(45*e^(-2*b*x - 2*a) - 9*e^(-4*b*x - 4*a) + e^(-6
*b*x - 6*a))/b
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 88, normalized size of antiderivative = 1.31

$$\int \sinh^6(a+bx)\,dx = -\frac{5}{16}\,x + \frac{e^{(6\,bx+6\,a)}}{384\,b} - \frac{3\,e^{(4\,bx+4\,a)}}{128\,b} + \frac{15\,e^{(2\,bx+2\,a)}}{128\,b} - \frac{15\,e^{(-2\,bx-2\,a)}}{128\,b} + \frac{3\,e^{(-4\,bx-4\,a)}}{128\,b} - \frac{e^{(-6\,bx-6\,a)}}{384\,b}$$

input
```
integrate(sinh(b*x+a)^6,x, algorithm="giac")
```

output
```
-5/16*x + 1/384*e^(6*b*x + 6*a)/b - 3/128*e^(4*b*x + 4*a)/b + 15/128*e^(2*
b*x + 2*a)/b - 15/128*e^(-2*b*x - 2*a)/b + 3/128*e^(-4*b*x - 4*a)/b - 1/38
4*e^(-6*b*x - 6*a)/b
```

Mupad [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.63

$$\int \sinh^6(a+bx)\,dx = \frac{\frac{15\sinh(2\,a+2\,b\,x)}{64} - \frac{3\sinh(4\,a+4\,b\,x)}{64} + \frac{\sinh(6\,a+6\,b\,x)}{192}}{b} - \frac{5\,x}{16}$$

input
```
int(sinh(a + b*x)^6,x)
```

output
```
((15*sinh(2*a + 2*b*x))/64 - (3*sinh(4*a + 4*b*x))/64 + sinh(6*a + 6*b*x)/
192)/b - (5*x)/16
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 91, normalized size of antiderivative = 1.36

$$\int \sinh^6(a+bx)\,dx = \frac{e^{12bx+12a}-9e^{10bx+10a}+45e^{8bx+8a}-120e^{6bx+6a}bx-45e^{4bx+4a}+9e^{2bx+2a}-1}{384e^{6bx+6a}b}$$

input
```
int(sinh(b*x+a)^6,x)
```

output
```
(e**(12*a + 12*b*x) - 9*e**(10*a + 10*b*x) + 45*e**(8*a + 8*b*x) - 120*e**
(6*a + 6*b*x)*b*x - 45*e**(4*a + 4*b*x) + 9*e**(2*a + 2*b*x) - 1)/(384*e**
(6*a + 6*b*x)*b)
```

3.7 $\int \sinh^{\frac{7}{2}}(a+bx)\,dx$

Optimal result

Integrand size = 10, antiderivative size = 103

$$\int \sinh^{\frac{7}{2}}(a+bx)\,dx = -\frac{10i\,\text{EllipticF}\left(\frac{1}{2}\left(ia-\frac{\pi}{2}+ibx\right),2\right)\sqrt{i\sinh(a+bx)}}{21b\sqrt{\sinh(a+bx)}} - \frac{10\cosh(a+bx)\sqrt{\sinh(a+bx)}}{21b} + \frac{2\cosh(a+bx)\sinh^{\frac{5}{2}}(a+bx)}{7b}$$

output
```
-10/21*I*InverseJacobiAM(1/2*I*a-1/4*Pi+1/2*I*b*x,2^(1/2))*(I*sinh(b*x+a))
^(1/2)/b/sinh(b*x+a)^(1/2)-10/21*cosh(b*x+a)*sinh(b*x+a)^(1/2)/b+2/7*cosh(
b*x+a)*sinh(b*x+a)^(5/2)/b
```

Mathematica [A] (verified)

Time = 0.11 (sec) , antiderivative size = 75, normalized size of antiderivative = 0.73

$$\int \sinh^{\frac{7}{2}}(a+bx)\,dx = \frac{40i\,\text{EllipticF}\left(\frac{1}{4}(-2ia+\pi-2ibx),2\right)\sqrt{i\sinh(a+bx)}-26\sinh(2(a+bx))+3\sinh(4(a+bx))}{84b\sqrt{\sinh(a+bx)}}$$

input
```
Integrate[Sinh[a + b*x]^(7/2),x]
```

output
```
((40*I)*EllipticF[((-2*I)*a + Pi - (2*I)*b*x)/4, 2]*Sqrt[I*Sinh[a + b*x]]
- 26*Sinh[2*(a + b*x)] + 3*Sinh[4*(a + b*x)])/(84*b*Sqrt[Sinh[a + b*x]])
```

Rubi [A] (verified)

Time = 0.41 (sec) , antiderivative size = 108, normalized size of antiderivative = 1.05, number of steps used = 8, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.800$, Rules used = {3042, 3115, 3042, 3115, 3042, 3121, 3042, 3120}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sinh^{\frac{7}{2}}(a+bx)\,dx$$

$$\Big\downarrow \ 3042$$

$$\int (-i\sin(ia+ibx))^{7/2}dx$$

$$\Big\downarrow \ 3115$$

$$\frac{2\sinh^{\frac{5}{2}}(a+bx)\cosh(a+bx)}{7b}-\frac{5}{7}\int \sinh^{\frac{3}{2}}(a+bx)dx$$

$$\Big\downarrow \ 3042$$

$$\frac{2\sinh^{\frac{5}{2}}(a+bx)\cosh(a+bx)}{7b}-\frac{5}{7}\int (-i\sin(ia+ibx))^{3/2}dx$$

$$\Big\downarrow \ 3115$$

$$\frac{2\sinh^{\frac{5}{2}}(a+bx)\cosh(a+bx)}{7b}-\frac{5}{7}\left(\frac{2\sqrt{\sinh(a+bx)}\cosh(a+bx)}{3b}-\frac{1}{3}\int \frac{1}{\sqrt{\sinh(a+bx)}}dx\right)$$

$$\Big\downarrow \ 3042$$

$$\frac{2\sinh^{\frac{5}{2}}(a+bx)\cosh(a+bx)}{7b}-\frac{5}{7}\left(\frac{2\sqrt{\sinh(a+bx)}\cosh(a+bx)}{3b}-\frac{1}{3}\int \frac{1}{\sqrt{-i\sin(ia+ibx)}}dx\right)$$

$$\Big\downarrow \ 3121$$

$$\frac{2\sinh^{\frac{5}{2}}(a+bx)\cosh(a+bx)}{7b}-\frac{5}{7}\left(\frac{2\sqrt{\sinh(a+bx)}\cosh(a+bx)}{3b}-\frac{\sqrt{i\sinh(a+bx)}\int\frac{1}{\sqrt{i\sinh(a+bx)}}dx}{3\sqrt{\sinh(a+bx)}}\right)$$

$$\Big\downarrow \text{3042}$$

$$\frac{2\sinh^{\frac{5}{2}}(a+bx)\cosh(a+bx)}{7b}-\frac{5}{7}\left(\frac{2\sqrt{\sinh(a+bx)}\cosh(a+bx)}{3b}-\frac{\sqrt{i\sinh(a+bx)}\int\frac{1}{\sqrt{\sin(ia+ibx)}}dx}{3\sqrt{\sinh(a+bx)}}\right)$$

$$\Big\downarrow \text{3120}$$

$$\frac{2\sinh^{\frac{5}{2}}(a+bx)\cosh(a+bx)}{7b}-\frac{5}{7}\left(\frac{2\sqrt{\sinh(a+bx)}\cosh(a+bx)}{3b}+\frac{2i\sqrt{i\sinh(a+bx)}\,\text{EllipticF}\left(\frac{1}{2}\left(ia+ibx-\frac{\pi}{2}\right),2\right)}{3b\sqrt{\sinh(a+bx)}}\right)$$

input
```
Int[Sinh[a + b*x]^(7/2),x]
```

output
```
(-5*((((2*I)/3)*EllipticF[(I*a - Pi/2 + I*b*x)/2, 2]*Sqrt[I*Sinh[a + b*x]]
)/(b*Sqrt[Sinh[a + b*x]]) + (2*Cosh[a + b*x]*Sqrt[Sinh[a + b*x]])/(3*b)))/
7 + (2*Cosh[a + b*x]*Sinh[a + b*x]^(5/2))/(7*b)
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3115
```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Simp[b^2*((n - 1)/n)   Int[(b*Sin
[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[
2*n]
```

rule 3120
```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3121
```
Int[((b_)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*Sin[c + d*x])
^n/Sin[c + d*x]^n   Int[Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] && Lt
Q[-1, n, 1] && IntegerQ[2*n]
```

Maple [A] (verified)

Time = 0.21 (sec) , antiderivative size = 116, normalized size of antiderivative = 1.13

method	result
default	$\frac{\frac{5i\sqrt{1-i\sinh(bx+a)}\,\sqrt{2}\,\sqrt{1+i\sinh(bx+a)}\,\sqrt{i\sinh(bx+a)}\,\operatorname{EllipticF}\left(\sqrt{1-i\sinh(bx+a)},\frac{\sqrt{2}}{2}\right)}{21}+\frac{2\cosh(bx+a)^4\sinh(bx+a)}{7}-\frac{16\cosh(bx+a)^2\sinh(bx+}{21}}{\cosh(bx+a)\sqrt{\sinh(bx+a)}\,b}$

input
```
int(sinh(b*x+a)^(7/2),x,method=_RETURNVERBOSE)
```

output
```
(5/21*I*(1-I*sinh(b*x+a))^(1/2)*2^(1/2)*(1+I*sinh(b*x+a))^(1/2)*(I*sinh(b*
x+a))^(1/2)*EllipticF((1-I*sinh(b*x+a))^(1/2),1/2*2^(1/2))+2/7*cosh(b*x+a)
^4*sinh(b*x+a)-16/21*cosh(b*x+a)^2*sinh(b*x+a))/cosh(b*x+a)/sinh(b*x+a)^(1
/2)/b
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 326 vs. 2(77) = 154.

Time = 0.09 (sec) , antiderivative size = 326, normalized size of antiderivative = 3.17

$$\int \sinh^{\frac{7}{2}}(a+bx)\,dx$$
$$= \frac{40\left(\sqrt{2}\cosh\left(bx+a\right)^3+3\sqrt{2}\cosh\left(bx+a\right)^2\sinh\left(bx+a\right)+3\sqrt{2}\cosh\left(bx+a\right)\sinh\left(bx+a\right)^2+\sqrt{2}\sinh}{}$$

input
```
integrate(sinh(b*x+a)^(7/2),x, algorithm="fricas")
```

output
```
1/84*(40*(sqrt(2)*cosh(b*x + a)^3 + 3*sqrt(2)*cosh(b*x + a)^2*sinh(b*x + a
) + 3*sqrt(2)*cosh(b*x + a)*sinh(b*x + a)^2 + sqrt(2)*sinh(b*x + a)^3)*wei
erstrassPInverse(4, 0, cosh(b*x + a) + sinh(b*x + a)) + (3*cosh(b*x + a)^6
 + 18*cosh(b*x + a)*sinh(b*x + a)^5 + 3*sinh(b*x + a)^6 + (45*cosh(b*x + a
)^2 - 23)*sinh(b*x + a)^4 - 23*cosh(b*x + a)^4 + 4*(15*cosh(b*x + a)^3 - 2
3*cosh(b*x + a))*sinh(b*x + a)^3 + (45*cosh(b*x + a)^4 - 138*cosh(b*x + a)
^2 - 23)*sinh(b*x + a)^2 - 23*cosh(b*x + a)^2 + 2*(9*cosh(b*x + a)^5 - 46*
cosh(b*x + a)^3 - 23*cosh(b*x + a))*sinh(b*x + a) + 3)*sqrt(sinh(b*x + a))
)/(b*cosh(b*x + a)^3 + 3*b*cosh(b*x + a)^2*sinh(b*x + a) + 3*b*cosh(b*x +
a)*sinh(b*x + a)^2 + b*sinh(b*x + a)^3)
```

Sympy [F(-1)]

Timed out.

$$\int \sinh^{\frac{7}{2}}(a+bx)\,dx = \text{Timed out}$$

input
```
integrate(sinh(b*x+a)**(7/2),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \sinh^{\frac{7}{2}}(a+bx)\,dx = \int \sinh\left(bx+a\right)^{\frac{7}{2}}\,dx$$

input
```
integrate(sinh(b*x+a)^(7/2),x, algorithm="maxima")
```

output
```
integrate(sinh(b*x + a)^(7/2), x)
```

Giac [F]

$$\int \sinh^{\frac{7}{2}}(a+bx)\,dx = \int \sinh(bx+a)^{\frac{7}{2}}\,dx$$

input
```
integrate(sinh(b*x+a)^(7/2),x, algorithm="giac")
```

output
```
integrate(sinh(b*x + a)^(7/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \sinh^{\frac{7}{2}}(a+bx)\,dx = \int \sinh(a+b\,x)^{7/2}\,dx$$

input
```
int(sinh(a + b*x)^(7/2),x)
```

output
```
int(sinh(a + b*x)^(7/2), x)
```

Reduce [F]

$$\int \sinh^{\frac{7}{2}}(a+bx)\,dx = \int \sqrt{\sinh(bx+a)}\,\sinh(bx+a)^{3}\,dx$$

input
```
int(sinh(b*x+a)^(7/2),x)
```

output
```
int(sqrt(sinh(a + b*x))*sinh(a + b*x)**3,x)
```

3.8 $\int \sinh^{\frac{5}{2}}(a+bx)\,dx$

Optimal result

Integrand size = 10, antiderivative size = 80

$$\int \sinh^{\frac{5}{2}}(a+bx)\,dx = \frac{6iE\left(\frac{1}{2}\left(ia-\frac{\pi}{2}+ibx\right)\middle|2\right)\sqrt{\sinh(a+bx)}}{5b\sqrt{i\sinh(a+bx)}} + \frac{2\cosh(a+bx)\sinh^{\frac{3}{2}}(a+bx)}{5b}$$

output
```
-6/5*I*EllipticE(cos(1/2*I*a+1/4*Pi+1/2*I*b*x),2^(1/2))*sinh(b*x+a)^(1/2)/
b/(I*sinh(b*x+a))^(1/2)+2/5*cosh(b*x+a)*sinh(b*x+a)^(3/2)/b
```

Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 68, normalized size of antiderivative = 0.85

$$\int \sinh^{\frac{5}{2}}(a+bx)\,dx = \frac{-6E\left(\frac{1}{4}(-2ia+\pi-2ibx)\middle|2\right)\sqrt{i\sinh(a+bx)}+\sinh(a+bx)\sinh(2(a+bx))}{5b\sqrt{\sinh(a+bx)}}$$

input
```
Integrate[Sinh[a + b*x]^(5/2),x]
```

output
```
(-6*EllipticE[((-2*I)*a + Pi - (2*I)*b*x)/4, 2]*Sqrt[I*Sinh[a + b*x]] + Si
nh[a + b*x]*Sinh[2*(a + b*x)])/(5*b*Sqrt[Sinh[a + b*x]])
```

Rubi [A] (verified)

Time = 0.32 (sec) , antiderivative size = 80, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.600$, Rules used = {3042, 3115, 3042, 3121, 3042, 3119}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sinh^{\frac{5}{2}}(a+bx)\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int (-i\sin(ia+ibx))^{5/2}dx$$

$$\Big\downarrow \text{3115}$$

$$\frac{2\sinh^{\frac{3}{2}}(a+bx)\cosh(a+bx)}{5b}-\frac{3}{5}\int \sqrt{\sinh(a+bx)}dx$$

$$\Big\downarrow \text{3042}$$

$$\frac{2\sinh^{\frac{3}{2}}(a+bx)\cosh(a+bx)}{5b}-\frac{3}{5}\int \sqrt{-i\sin(ia+ibx)}dx$$

$$\Big\downarrow \text{3121}$$

$$\frac{2\sinh^{\frac{3}{2}}(a+bx)\cosh(a+bx)}{5b}-\frac{3\sqrt{\sinh(a+bx)}\int \sqrt{i\sinh(a+bx)}dx}{5\sqrt{i\sinh(a+bx)}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{2\sinh^{\frac{3}{2}}(a+bx)\cosh(a+bx)}{5b}-\frac{3\sqrt{\sinh(a+bx)}\int \sqrt{\sin(ia+ibx)}dx}{5\sqrt{i\sinh(a+bx)}}$$

$$\Big\downarrow \text{3119}$$

$$\frac{2\sinh^{\frac{3}{2}}(a+bx)\cosh(a+bx)}{5b}+\frac{6i\sqrt{\sinh(a+bx)}E\left(\frac{1}{2}\left(ia+ibx-\frac{\pi}{2}\right)\middle|2\right)}{5b\sqrt{i\sinh(a+bx)}}$$

input
```
Int[Sinh[a + b*x]^(5/2),x]
```

output
```
(((6*I)/5)*EllipticE[(I*a - Pi/2 + I*b*x)/2, 2]*Sqrt[Sinh[a + b*x]])/(b*Sq
rt[I*Sinh[a + b*x]]) + (2*Cosh[a + b*x]*Sinh[a + b*x]^(3/2))/(5*b)
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3115
```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Simp[b^2*((n - 1)/n)   Int[(b*Sin
[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[
2*n]
```

rule 3119
```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3121
```
Int[((b_)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*Sin[c + d*x])
^n/Sin[c + d*x]^n   Int[Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] && Lt
Q[-1, n, 1] && IntegerQ[2*n]
```

Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 163 vs. $2(63) = 126$.

Time = 0.17 (sec) , antiderivative size = 164, normalized size of antiderivative = 2.05

method	result
default	$\frac{-\frac{6\sqrt{1-i\sinh(bx+a)}\,\sqrt{2}\,\sqrt{1+i\sinh(bx+a)}\,\sqrt{i\sinh(bx+a)}\ \mathrm{EllipticE}\left(\sqrt{1-i\sinh(bx+a)},\frac{\sqrt{2}}{2}\right)}{5}+\frac{3\sqrt{1-i\sinh(bx+a)}\,\sqrt{2}\,\sqrt{1+i\sinh(bx+a)}\,\sqrt{i\sinh(bx}{5}}{\cosh(bx+a)\sqrt{\sinh(bx+a)}\,b}$

input
```
int(sinh(b*x+a)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
(-6/5*(1-I*sinh(b*x+a))^(1/2)*2^(1/2)*(1+I*sinh(b*x+a))^(1/2)*(I*sinh(b*x+
a))^(1/2)*EllipticE((1-I*sinh(b*x+a))^(1/2),1/2*2^(1/2))+3/5*(1-I*sinh(b*x
+a))^(1/2)*2^(1/2)*(1+I*sinh(b*x+a))^(1/2)*(I*sinh(b*x+a))^(1/2)*EllipticF
((1-I*sinh(b*x+a))^(1/2),1/2*2^(1/2))+2/5*cosh(b*x+a)^4-2/5*cosh(b*x+a)^2)
/cosh(b*x+a)/sinh(b*x+a)^(1/2)/b
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 202 vs. $2(59) = 118$.

Time = 0.10 (sec) , antiderivative size = 202, normalized size of antiderivative = 2.52

$$\int \sinh^{\frac{5}{2}}(a+bx)\,dx$$
$$= \frac{12\left(\sqrt{2}\cosh\left(bx+a\right)^2 + 2\sqrt{2}\cosh\left(bx+a\right)\sinh\left(bx+a\right) + \sqrt{2}\sinh\left(bx+a\right)^2\right)\text{weierstrassZeta}(4,0,\text{weie}}{}$$

input
```
integrate(sinh(b*x+a)^(5/2),x, algorithm="fricas")
```

output
```
1/10*(12*(sqrt(2)*cosh(b*x + a)^2 + 2*sqrt(2)*cosh(b*x + a)*sinh(b*x + a)
+ sqrt(2)*sinh(b*x + a)^2)*weierstrassZeta(4, 0, weierstrassPInverse(4, 0,
 cosh(b*x + a) + sinh(b*x + a))) + (cosh(b*x + a)^4 + 4*cosh(b*x + a)*sinh
(b*x + a)^3 + sinh(b*x + a)^4 + 6*(cosh(b*x + a)^2 + 2)*sinh(b*x + a)^2 +
12*cosh(b*x + a)^2 + 4*(cosh(b*x + a)^3 + 6*cosh(b*x + a))*sinh(b*x + a) -
 1)*sqrt(sinh(b*x + a)))/(b*cosh(b*x + a)^2 + 2*b*cosh(b*x + a)*sinh(b*x +
 a) + b*sinh(b*x + a)^2)
```

Sympy [F]

$$\int \sinh^{\frac{5}{2}}(a+bx)\,dx = \int \sinh^{\frac{5}{2}}(a+bx)\,dx$$

input
```
integrate(sinh(b*x+a)**(5/2),x)
```

output
```
Integral(sinh(a + b*x)**(5/2), x)
```

Maxima [F]

$$\int \sinh^{\frac{5}{2}}(a+bx)\,dx = \int \sinh(bx+a)^{\frac{5}{2}}\,dx$$

input
```
integrate(sinh(b*x+a)^(5/2),x, algorithm="maxima")
```

output
```
integrate(sinh(b*x + a)^(5/2), x)
```

Giac [F]

$$\int \sinh^{\frac{5}{2}}(a+bx)\,dx = \int \sinh(bx+a)^{\frac{5}{2}}\,dx$$

input
```
integrate(sinh(b*x+a)^(5/2),x, algorithm="giac")
```

output
```
integrate(sinh(b*x + a)^(5/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \sinh^{\frac{5}{2}}(a+bx)\,dx = \int \sinh(a+b\,x)^{5/2}\,dx$$

input
```
int(sinh(a + b*x)^(5/2),x)
```

output
```
int(sinh(a + b*x)^(5/2), x)
```

Reduce [F]

$$\int \sinh^{\frac{5}{2}}(a+bx)\,dx = \int \sqrt{\sinh(bx+a)}\,\sinh(bx+a)^2\,dx$$

input
```
int(sinh(b*x+a)^(5/2),x)
```

output
```
int(sqrt(sinh(a + b*x))*sinh(a + b*x)**2,x)
```

3.9 $\int \sinh^{\frac{3}{2}}(a+bx)\,dx$

Optimal result

Integrand size = 10, antiderivative size = 80

$$\int \sinh^{\frac{3}{2}}(a+bx)\,dx = \frac{2i\,\text{EllipticF}\left(\frac{1}{2}\left(ia-\frac{\pi}{2}+ibx\right),2\right)\sqrt{i\sinh(a+bx)}}{3b\sqrt{\sinh(a+bx)}} + \frac{2\cosh(a+bx)\sqrt{\sinh(a+bx)}}{3b}$$

output
```
2/3*I*InverseJacobiAM(1/2*I*a-1/4*Pi+1/2*I*b*x,2^(1/2))*(I*sinh(b*x+a))^(1
/2)/b/sinh(b*x+a)^(1/2)+2/3*cosh(b*x+a)*sinh(b*x+a)^(1/2)/b
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 0.07 (sec) , antiderivative size = 83, normalized size of antiderivative = 1.04

$$\int \sinh^{\frac{3}{2}}(a+bx)\,dx$$

$$= \frac{\sinh(2(a+bx)) - 2\,\text{Hypergeometric2F1}\left(\frac{1}{4},\frac{1}{2},\frac{5}{4},\cosh(2(a+bx))+\sinh(2(a+bx))\right)\sqrt{1-\cosh(2a+2}}{3b\sqrt{\sinh(a+bx)}}$$

input
```
Integrate[Sinh[a + b*x]^(3/2),x]
```

output
```
(Sinh[2*(a + b*x)] - 2*Hypergeometric2F1[1/4, 1/2, 5/4, Cosh[2*(a + b*x)]
+ Sinh[2*(a + b*x)]]*Sqrt[1 - Cosh[2*a + 2*b*x] - Sinh[2*a + 2*b*x]])/(3*b
*Sqrt[Sinh[a + b*x]])
```

Rubi [A] (verified)

Time = 0.32 (sec) , antiderivative size = 80, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.600$, Rules used = {3042, 3115, 3042, 3121, 3042, 3120}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sinh^{\frac{3}{2}}(a+bx)\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int (-i\sin(ia+ibx))^{3/2}dx$$

$$\Big\downarrow \text{3115}$$

$$\frac{2\sqrt{\sinh(a+bx)}\cosh(a+bx)}{3b}-\frac{1}{3}\int \frac{1}{\sqrt{\sinh(a+bx)}}dx$$

$$\Big\downarrow \text{3042}$$

$$\frac{2\sqrt{\sinh(a+bx)}\cosh(a+bx)}{3b}-\frac{1}{3}\int \frac{1}{\sqrt{-i\sin(ia+ibx)}}dx$$

$$\Big\downarrow \text{3121}$$

$$\frac{2\sqrt{\sinh(a+bx)}\cosh(a+bx)}{3b}-\frac{\sqrt{i\sinh(a+bx)}\int \frac{1}{\sqrt{i\sinh(a+bx)}}dx}{3\sqrt{\sinh(a+bx)}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{2\sqrt{\sinh(a+bx)}\cosh(a+bx)}{3b}-\frac{\sqrt{i\sinh(a+bx)}\int \frac{1}{\sqrt{\sin(ia+ibx)}}dx}{3\sqrt{\sinh(a+bx)}}$$

$$\Big\downarrow \text{3120}$$

$$\frac{2\sqrt{\sinh(a+bx)}\cosh(a+bx)}{3b}+\frac{2i\sqrt{i\sinh(a+bx)}\,\text{EllipticF}\left(\frac{1}{2}\left(ia+ibx-\frac{\pi}{2}\right),2\right)}{3b\sqrt{\sinh(a+bx)}}$$

input
```
Int[Sinh[a + b*x]^(3/2),x]
```

output
```
(((2*I)/3)*EllipticF[(I*a - Pi/2 + I*b*x)/2, 2]*Sqrt[I*Sinh[a + b*x]])/(b*
Sqrt[Sinh[a + b*x]]) + (2*Cosh[a + b*x]*Sqrt[Sinh[a + b*x]])/(3*b)
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3115
```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Simp[b^2*((n - 1)/n)   Int[(b*Sin
[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[
2*n]
```

rule 3120
```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3121
```
Int[((b_)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*Sin[c + d*x])
^n/Sin[c + d*x]^n   Int[Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] && Lt
Q[-1, n, 1] && IntegerQ[2*n]
```

Maple [A] (verified)

Time = 0.15 (sec) , antiderivative size = 100, normalized size of antiderivative = 1.25

method	result	size
default	$\frac{-\frac{i\sqrt{1-i\sinh(bx+a)}\,\sqrt{2}\,\sqrt{1+i\sinh(bx+a)}\,\sqrt{i\sinh(bx+a)}\,\operatorname{EllipticF}\left(\sqrt{1-i\sinh(bx+a)},\frac{\sqrt{2}}{2}\right)}{3}+\frac{2\cosh(bx+a)^{2}\sinh(bx+a)}{3}}{\cosh(bx+a)\sqrt{\sinh(bx+a)}\,b}$	100

input
```
int(sinh(b*x+a)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
(-1/3*I*(1-I*sinh(b*x+a))^(1/2)*2^(1/2)*(1+I*sinh(b*x+a))^(1/2)*(I*sinh(b*
x+a))^(1/2)*EllipticF((1-I*sinh(b*x+a))^(1/2),1/2*2^(1/2))+2/3*cosh(b*x+a)
^2*sinh(b*x+a))/cosh(b*x+a)/sinh(b*x+a)^(1/2)/b
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 103, normalized size of antiderivative = 1.29

$$\int \sinh^{\frac{3}{2}}(a+bx)\,dx = -\frac{2\left(\sqrt{2}\cosh\left(bx+a\right)+\sqrt{2}\sinh\left(bx+a\right)\right)\text{weierstrassPInverse}(4,0,\cosh\left(bx+a\right)+\sinh\left(bx+a\right))-(\text{co}}{3\left(b\cosh\left(bx+a\right)+b\sinh\left(b\right.\right.}$$

input
```
integrate(sinh(b*x+a)^(3/2),x, algorithm="fricas")
```

output
```
-1/3*(2*(sqrt(2)*cosh(b*x + a) + sqrt(2)*sinh(b*x + a))*weierstrassPInvers
e(4, 0, cosh(b*x + a) + sinh(b*x + a)) - (cosh(b*x + a)^2 + 2*cosh(b*x + a
)*sinh(b*x + a) + sinh(b*x + a)^2 + 1)*sqrt(sinh(b*x + a)))/(b*cosh(b*x +
a) + b*sinh(b*x + a))
```

Sympy [F]

$$\int \sinh^{\frac{3}{2}}(a+bx)\,dx = \int \sinh^{\frac{3}{2}}(a+bx)\,dx$$

input
```
integrate(sinh(b*x+a)**(3/2),x)
```

output
```
Integral(sinh(a + b*x)**(3/2), x)
```

Maxima [F]

$$\int \sinh^{\frac{3}{2}}(a+bx)\,dx = \int \sinh(bx+a)^{\frac{3}{2}}\,dx$$

input
```
integrate(sinh(b*x+a)^(3/2),x, algorithm="maxima")
```

output
```
integrate(sinh(b*x + a)^(3/2), x)
```

Giac [F]

$$\int \sinh^{\frac{3}{2}}(a+bx)\,dx = \int \sinh(bx+a)^{\frac{3}{2}}\,dx$$

input
```
integrate(sinh(b*x+a)^(3/2),x, algorithm="giac")
```

output
```
integrate(sinh(b*x + a)^(3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \sinh^{\frac{3}{2}}(a+bx)\,dx = \int \sinh(a+b\,x)^{3/2}\,dx$$

input
```
int(sinh(a + b*x)^(3/2),x)
```

output
```
int(sinh(a + b*x)^(3/2), x)
```

Reduce [F]

$$\int \sinh^{\frac{3}{2}}(a+bx)\,dx = \int \sqrt{\sinh(bx+a)}\,\sinh(bx+a)\,dx$$

input
```
int(sinh(b*x+a)^(3/2),x)
```

output
```
int(sqrt(sinh(a + b*x))*sinh(a + b*x),x)
```

3.10 $\int \sqrt{\sinh(a+bx)}\,dx$

Optimal result

Integrand size = 10, antiderivative size = 54

$$\int \sqrt{\sinh(a+bx)}\,dx = -\frac{2iE\left(\frac{1}{2}\left(ia-\frac{\pi}{2}+ibx\right)\middle|2\right)\sqrt{\sinh(a+bx)}}{b\sqrt{i\sinh(a+bx)}}$$

output
```
2*I*EllipticE(cos(1/2*I*a+1/4*Pi+1/2*I*b*x),2^(1/2))*sinh(b*x+a)^(1/2)/b/(
I*sinh(b*x+a))^(1/2)
```

Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 50, normalized size of antiderivative = 0.93

$$\int \sqrt{\sinh(a+bx)}\,dx = \frac{2E\left(\frac{1}{2}\left(\frac{\pi}{2}-i(a+bx)\right)\middle|2\right)\sqrt{i\sinh(a+bx)}}{b\sqrt{\sinh(a+bx)}}$$

input
```
Integrate[Sqrt[Sinh[a + b*x]],x]
```

output
```
(2*EllipticE[(Pi/2 - I*(a + b*x))/2, 2]*Sqrt[I*Sinh[a + b*x]])/(b*Sqrt[Sin
h[a + b*x]])
```

Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 54, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.400$, Rules used = {3042, 3121, 3042, 3119}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{\sinh(a+bx)}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \sqrt{-i\sin(ia+ibx)}dx$$

$$\Big\downarrow \text{3121}$$

$$\frac{\sqrt{\sinh(a+bx)}\int \sqrt{i\sinh(a+bx)}dx}{\sqrt{i\sinh(a+bx)}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\sqrt{\sinh(a+bx)}\int \sqrt{\sin(ia+ibx)}dx}{\sqrt{i\sinh(a+bx)}}$$

$$\Big\downarrow \text{3119}$$

$$-\frac{2i\sqrt{\sinh(a+bx)}E\left(\frac{1}{2}\left(ia+ibx-\frac{\pi}{2}\right)\middle|2\right)}{b\sqrt{i\sinh(a+bx)}}$$

input
```
Int[Sqrt[Sinh[a + b*x]],x]
```

output
```
((-2*I)*EllipticE[(I*a - Pi/2 + I*b*x)/2, 2]*Sqrt[Sinh[a + b*x]])/(b*Sqrt[
I*Sinh[a + b*x]])
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3119
```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3121
```
Int[((b_)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*Sin[c + d*x])
^n/Sin[c + d*x]^n   Int[Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] && Lt
Q[-1, n, 1] && IntegerQ[2*n]
```

Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 107 vs. $2(43) = 86$.

Time = 0.22 (sec) , antiderivative size = 108, normalized size of antiderivative = 2.00

method	result
default	$\frac{\sqrt{-i(\sinh(bx+a)+i)}\,\sqrt{2}\,\sqrt{-i(-\sinh(bx+a)+i)}\,\sqrt{i\sinh(bx+a)}\,\left(2\,\mathrm{EllipticE}\left(\sqrt{1-i\sinh(bx+a)},\frac{\sqrt{2}}{2}\right)-\mathrm{EllipticF}\left(\sqrt{1-i\sinh(bx+a}\right.\right.}{\cosh(bx+a)\sqrt{\sinh(bx+a)}\,b}$
risch	$\frac{\sqrt{2}\,\sqrt{(e^{2bx+2a}-1)\,e^{-bx-a}}}{b}-\frac{\left(\frac{2\,e^{2bx+2a}-2}{\sqrt{(e^{2bx+2a}-1)\,e^{bx+a}}}-\frac{\sqrt{e^{bx+a}+1}\,\sqrt{-2\,e^{bx+a}+2}\,\sqrt{-e^{bx+a}}\,\left(-2\,\mathrm{EllipticE}\left(\sqrt{e^{bx+a}+1},\frac{\sqrt{2}}{2}\right)+\mathrm{EllipticF}\left(\right.\right.}{\sqrt{e^{3bx+3a}-e^{bx+a}}}\right.}{b(e^{2bx+2a}-1)}$

input
```
int(sinh(b*x+a)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
(-I*(sinh(b*x+a)+I))^(1/2)*2^(1/2)*(-I*(-sinh(b*x+a)+I))^(1/2)*(I*sinh(b*x
+a))^(1/2)*(2*EllipticE((1-I*sinh(b*x+a))^(1/2),1/2*2^(1/2))-EllipticF((1-
I*sinh(b*x+a))^(1/2),1/2*2^(1/2)))/cosh(b*x+a)/sinh(b*x+a)^(1/2)/b
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.69

$$\int \sqrt{\sinh(a+bx)}\,dx = -\frac{2\left(\sqrt{2}\text{weierstrassZeta}(4,0,\text{weierstrassPInverse}(4,0,\cosh{(bx+a)}+\sinh{(bx+a)}))+\sqrt{\sinh{(bx+a)}}\right)}{b}$$

input
```
integrate(sinh(b*x+a)^(1/2),x, algorithm="fricas")
```

output
```
-2*(sqrt(2)*weierstrassZeta(4, 0, weierstrassPInverse(4, 0, cosh(b*x + a)
+ sinh(b*x + a))) + sqrt(sinh(b*x + a)))/b
```

Sympy [F]

$$\int \sqrt{\sinh(a+bx)}\,dx = \int \sqrt{\sinh{(a+bx)}}\,dx$$

input
```
integrate(sinh(b*x+a)**(1/2),x)
```

output
```
Integral(sqrt(sinh(a + b*x)), x)
```

Maxima [F]

$$\int \sqrt{\sinh(a+bx)}\,dx = \int \sqrt{\sinh{(bx+a)}}\,dx$$

input
```
integrate(sinh(b*x+a)^(1/2),x, algorithm="maxima")
```

output
```
integrate(sqrt(sinh(b*x + a)), x)
```

Giac [F]

$$\int \sqrt{\sinh(a+bx)}\,dx = \int \sqrt{\sinh{(bx+a)}}\,dx$$

input
```
integrate(sinh(b*x+a)^(1/2),x, algorithm="giac")
```

output
```
integrate(sqrt(sinh(b*x + a)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \sqrt{\sinh(a+bx)}\,dx = \int \sqrt{\sinh{(a+b\,x)}}\,dx$$

input
```
int(sinh(a + b*x)^(1/2),x)
```

output
```
int(sinh(a + b*x)^(1/2), x)
```

Reduce [F]

$$\int \sqrt{\sinh(a+bx)}\,dx = \int \sqrt{\sinh{(bx+a)}}dx$$

input
```
int(sinh(b*x+a)^(1/2),x)
```

output
```
int(sqrt(sinh(a + b*x)),x)
```

3.11 $\int \frac{1}{\sqrt{\sinh(a+bx)}}\,dx$

Optimal result

Integrand size = 10, antiderivative size = 54

$$\int \frac{1}{\sqrt{\sinh(a+bx)}}\,dx = -\frac{2i\,\text{EllipticF}\left(\frac{1}{2}\left(ia-\frac{\pi}{2}+ibx\right),2\right)\sqrt{i\sinh(a+bx)}}{b\sqrt{\sinh(a+bx)}}$$

output
```
-2*I*InverseJacobiAM(1/2*I*a-1/4*Pi+1/2*I*b*x,2^(1/2))*(I*sinh(b*x+a))^(1/
2)/b/sinh(b*x+a)^(1/2)
```

Mathematica [A] (verified)

Time = 0.09 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.89

$$\int \frac{1}{\sqrt{\sinh(a+bx)}}\,dx = -\frac{2\,\text{EllipticF}\left(\frac{1}{4}(-2ia+\pi-2ibx),2\right)\sqrt{\sinh(a+bx)}}{b\sqrt{i\sinh(a+bx)}}$$

input
```
Integrate[1/Sqrt[Sinh[a + b*x]],x]
```

output
```
(-2*EllipticF[((-2*I)*a + Pi - (2*I)*b*x)/4, 2]*Sqrt[Sinh[a + b*x]])/(b*Sq
rt[I*Sinh[a + b*x]])
```

Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 54, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.400, Rules used = {3042, 3121, 3042, 3120}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt{\sinh(a+bx)}}\,dx$$
$$\Big\downarrow \text{3042}$$
$$\int \frac{1}{\sqrt{-i\sin(ia+ibx)}}dx$$
$$\Big\downarrow \text{3121}$$
$$\frac{\sqrt{i\sinh(a+bx)}\int \frac{1}{\sqrt{i\sinh(a+bx)}}dx}{\sqrt{\sinh(a+bx)}}$$
$$\Big\downarrow \text{3042}$$
$$\frac{\sqrt{i\sinh(a+bx)}\int \frac{1}{\sqrt{\sin(ia+ibx)}}dx}{\sqrt{\sinh(a+bx)}}$$
$$\Big\downarrow \text{3120}$$
$$-\frac{2i\sqrt{i\sinh(a+bx)}\,\text{EllipticF}\left(\frac{1}{2}\left(ia+ibx-\frac{\pi}{2}\right),2\right)}{b\sqrt{\sinh(a+bx)}}$$

input
```
Int[1/Sqrt[Sinh[a + b*x]],x]
```

output
```
((-2*I)*EllipticF[(I*a - Pi/2 + I*b*x)/2, 2]*Sqrt[I*Sinh[a + b*x]])/(b*Sqr
t[Sinh[a + b*x]])
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3120
```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3121
```
Int[((b_)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*Sin[c + d*x])
^n/Sin[c + d*x]^n   Int[Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] && Lt
Q[-1, n, 1] && IntegerQ[2*n]
```

Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 86 vs. $2(42) = 84$.

Time $= 0.14$ (sec) , antiderivative size $= 87$, normalized size of antiderivative $= 1.61$

method	result	size
default	$\frac{i\sqrt{-i(\sinh(bx+a)+i)}\,\sqrt{2}\,\sqrt{-i(-\sinh(bx+a)+i)}\,\sqrt{i\sinh(bx+a)}\;\mathrm{EllipticF}\left(\sqrt{-i(\sinh(bx+a)+i)},\frac{\sqrt{2}}{2}\right)}{\cosh(bx+a)\sqrt{\sinh(bx+a)}\,b}$	87

input
```
int(1/sinh(b*x+a)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
I*(-I*(sinh(b*x+a)+I))^(1/2)*2^(1/2)*(-I*(-sinh(b*x+a)+I))^(1/2)*(I*sinh(b
*x+a))^(1/2)*EllipticF((-I*(sinh(b*x+a)+I))^(1/2),1/2*2^(1/2))/cosh(b*x+a)
/sinh(b*x+a)^(1/2)/b
```

Fricas [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.44

$$\int \frac{1}{\sqrt{\sinh(a+bx)}}\,dx = \frac{2\sqrt{2}\text{weierstrassPInverse}(4,0,\cosh{(bx+a)}+\sinh{(bx+a)})}{b}$$

input
```
integrate(1/sinh(b*x+a)^(1/2),x, algorithm="fricas")
```

output
```
2*sqrt(2)*weierstrassPInverse(4, 0, cosh(b*x + a) + sinh(b*x + a))/b
```

Sympy [F]

$$\int \frac{1}{\sqrt{\sinh(a+bx)}}\,dx = \int \frac{1}{\sqrt{\sinh{(a+bx)}}}\,dx$$

input
```
integrate(1/sinh(b*x+a)**(1/2),x)
```

output
```
Integral(1/sqrt(sinh(a + b*x)), x)
```

Maxima [F]

$$\int \frac{1}{\sqrt{\sinh(a+bx)}}\,dx = \int \frac{1}{\sqrt{\sinh{(bx+a)}}}\,dx$$

input
```
integrate(1/sinh(b*x+a)^(1/2),x, algorithm="maxima")
```

output
```
integrate(1/sqrt(sinh(b*x + a)), x)
```

Giac [F]

$$\int \frac{1}{\sqrt{\sinh(a+bx)}}\,dx = \int \frac{1}{\sqrt{\sinh\left(bx+a\right)}}\,dx$$

input
```
integrate(1/sinh(b*x+a)^(1/2),x, algorithm="giac")
```

output
```
integrate(1/sqrt(sinh(b*x + a)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt{\sinh(a+bx)}}\,dx = \int \frac{1}{\sqrt{\sinh\left(a+b\,x\right)}}\,dx$$

input
```
int(1/sinh(a + b*x)^(1/2),x)
```

output
```
int(1/sinh(a + b*x)^(1/2), x)
```

Reduce [F]

$$\int \frac{1}{\sqrt{\sinh(a+bx)}}\,dx = \int \frac{\sqrt{\sinh\left(bx+a\right)}}{\sinh\left(bx+a\right)}dx$$

input
```
int(1/sinh(b*x+a)^(1/2),x)
```

output
```
int(sqrt(sinh(a + b*x))/sinh(a + b*x),x)
```

3.12 $\int \frac{1}{\sinh^{\frac{3}{2}}(a+bx)}\,dx$

Optimal result

Integrand size = 10, antiderivative size = 76

$$\int \frac{1}{\sinh^{\frac{3}{2}}(a+bx)}\,dx=-\frac{2\cosh(a+bx)}{b\sqrt{\sinh(a+bx)}}-\frac{2iE\left(\frac{1}{2}\left(ia-\frac{\pi}{2}+ibx\right)\middle|2\right)\sqrt{\sinh(a+bx)}}{b\sqrt{i\sinh(a+bx)}}$$

output
```
-2*cosh(b*x+a)/b/sinh(b*x+a)^(1/2)+2*I*EllipticE(cos(1/2*I*a+1/4*Pi+1/2*I*
b*x),2^(1/2))*sinh(b*x+a)^(1/2)/b/(I*sinh(b*x+a))^(1/2)
```

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.75

$$\int \frac{1}{\sinh^{\frac{3}{2}}(a+bx)}\,dx=-\frac{2\left(\cosh(a+bx)-E\left(\frac{1}{4}(-2ia+\pi-2ibx)\middle|2\right)\sqrt{i\sinh(a+bx)}\right)}{b\sqrt{\sinh(a+bx)}}$$

input
```
Integrate[Sinh[a + b*x]^(-3/2),x]
```

output
```
(-2*(Cosh[a + b*x] - EllipticE[((-2*I)*a + Pi - (2*I)*b*x)/4, 2]*Sqrt[I*Si
nh[a + b*x]]))/(b*Sqrt[Sinh[a + b*x]])
```

Rubi [A] (verified)

Time = 0.31 (sec) , antiderivative size = 76, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.600, Rules used = {3042, 3116, 3042, 3121, 3042, 3119}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sinh^{\frac{3}{2}}(a+bx)}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{(-i\sin(ia+ibx))^{3/2}}dx$$

$$\Big\downarrow \text{3116}$$

$$\int \sqrt{\sinh(a+bx)}dx-\frac{2\cosh(a+bx)}{b\sqrt{\sinh(a+bx)}}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{2\cosh(a+bx)}{b\sqrt{\sinh(a+bx)}}+\int \sqrt{-i\sin(ia+ibx)}dx$$

$$\Big\downarrow \text{3121}$$

$$-\frac{2\cosh(a+bx)}{b\sqrt{\sinh(a+bx)}}+\frac{\sqrt{\sinh(a+bx)}\int \sqrt{i\sinh(a+bx)}dx}{\sqrt{i\sinh(a+bx)}}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{2\cosh(a+bx)}{b\sqrt{\sinh(a+bx)}}+\frac{\sqrt{\sinh(a+bx)}\int \sqrt{\sin(ia+ibx)}dx}{\sqrt{i\sinh(a+bx)}}$$

$$\Big\downarrow \text{3119}$$

$$-\frac{2\cosh(a+bx)}{b\sqrt{\sinh(a+bx)}}-\frac{2i\sqrt{\sinh(a+bx)}E\left(\frac{1}{2}\left(ia+ibx-\frac{\pi}{2}\right)|2\right)}{b\sqrt{i\sinh(a+bx)}}$$

input
```
Int[Sinh[a + b*x]^(-3/2),x]
```

output
```
(-2*Cosh[a + b*x])/(b*Sqrt[Sinh[a + b*x]]) - ((2*I)*EllipticE[(I*a - Pi/2
+ I*b*x)/2, 2]*Sqrt[Sinh[a + b*x]])/(b*Sqrt[I*Sinh[a + b*x]])
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3116
```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1))), x] + Simp[(n + 2)/(b^2*(n + 1))   I
nt[(b*Sin[c + d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] &&
 IntegerQ[2*n]
```

rule 3119
```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3121
```
Int[((b_)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*Sin[c + d*x])
^n/Sin[c + d*x]^n   Int[Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] && Lt
Q[-1, n, 1] && IntegerQ[2*n]
```

Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 153 vs. $2(63) = 126$.

Time = 0.15 (sec) , antiderivative size = 154, normalized size of antiderivative = 2.03

method	result
default	$\frac{2\sqrt{1-i\sinh(bx+a)}\,\sqrt{2}\,\sqrt{1+i\sinh(bx+a)}\,\sqrt{i\sinh(bx+a)}\,\operatorname{EllipticE}\left(\sqrt{1-i\sinh(bx+a)},\frac{\sqrt{2}}{2}\right)-\sqrt{1-i\sinh(bx+a)}\,\sqrt{2}\,\sqrt{1+i\sinh(bx+}}{\cosh(bx+a)\sqrt{\sinh(bx+a)}\,b}$

input
```
int(1/sinh(b*x+a)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
(2*(1-I*sinh(b*x+a))^(1/2)*2^(1/2)*(1+I*sinh(b*x+a))^(1/2)*(I*sinh(b*x+a))
^(1/2)*EllipticE((1-I*sinh(b*x+a))^(1/2),1/2*2^(1/2))-(1-I*sinh(b*x+a))^(1
/2)*2^(1/2)*(1+I*sinh(b*x+a))^(1/2)*(I*sinh(b*x+a))^(1/2)*EllipticF((1-I*s
inh(b*x+a))^(1/2),1/2*2^(1/2))-2*cosh(b*x+a)^2)/cosh(b*x+a)/sinh(b*x+a)^(1
/2)/b
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 152 vs. $2(59) = 118$.

Time = 0.11 (sec) , antiderivative size = 152, normalized size of antiderivative = 2.00

$$\int \frac{1}{\sinh^{\frac{3}{2}}(a+bx)}\,dx = -\frac{2\left(\left(\sqrt{2}\cosh\left(bx+a\right)^2+2\sqrt{2}\cosh\left(bx+a\right)\sinh\left(bx+a\right)+\sqrt{2}\sinh\left(bx+a\right)^2-\sqrt{2}\right)\text{weierstrassZeta}(\cdots}{b\cosh\left(bx+\cdots\right.}$$

input
```
integrate(1/sinh(b*x+a)^(3/2),x, algorithm="fricas")
```

output
```
-2*((sqrt(2)*cosh(b*x + a)^2 + 2*sqrt(2)*cosh(b*x + a)*sinh(b*x + a) + sqr
t(2)*sinh(b*x + a)^2 - sqrt(2))*weierstrassZeta(4, 0, weierstrassPInverse(
4, 0, cosh(b*x + a) + sinh(b*x + a))) + 2*(cosh(b*x + a)^2 + 2*cosh(b*x +
a)*sinh(b*x + a) + sinh(b*x + a)^2)*sqrt(sinh(b*x + a)))/(b*cosh(b*x + a)^
2 + 2*b*cosh(b*x + a)*sinh(b*x + a) + b*sinh(b*x + a)^2 - b)
```

Sympy [F]

$$\int \frac{1}{\sinh^{\frac{3}{2}}(a+bx)}\,dx = \int \frac{1}{\sinh^{\frac{3}{2}}(a+bx)}\,dx$$

input
```
integrate(1/sinh(b*x+a)**(3/2),x)
```

output
```
Integral(sinh(a + b*x)**(-3/2), x)
```

Maxima [F]

$$\int \frac{1}{\sinh^{\frac{3}{2}}(a+bx)}\,dx = \int \frac{1}{\sinh\left(bx+a\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/sinh(b*x+a)^(3/2),x, algorithm="maxima")
```

output
```
integrate(sinh(b*x + a)^(-3/2), x)
```

Giac [F]

$$\int \frac{1}{\sinh^{\frac{3}{2}}(a+bx)}\,dx = \int \frac{1}{\sinh\left(bx+a\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/sinh(b*x+a)^(3/2),x, algorithm="giac")
```

output
```
integrate(sinh(b*x + a)^(-3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\sinh^{\frac{3}{2}}(a+bx)}\,dx = \int \frac{1}{\sinh\left(a+b\,x\right)^{3/2}}\,dx$$

input
```
int(1/sinh(a + b*x)^(3/2),x)
```

output
```
int(1/sinh(a + b*x)^(3/2), x)
```

Reduce [F]

$$\int \frac{1}{\sinh^{\frac{3}{2}}(a+bx)}\,dx = \int \frac{\sqrt{\sinh(bx+a)}}{\sinh(bx+a)^2}dx$$

input
```
int(1/sinh(b*x+a)^(3/2),x)
```

output
```
int(sqrt(sinh(a + b*x))/sinh(a + b*x)**2,x)
```

3.13 $\int \frac{1}{\sinh^{\frac{5}{2}}(a+bx)}\,dx$

Optimal result

Integrand size = 10, antiderivative size = 80

$$\int \frac{1}{\sinh^{\frac{5}{2}}(a+bx)}\,dx = -\frac{2\cosh(a+bx)}{3b\sinh^{\frac{3}{2}}(a+bx)} + \frac{2i\,\text{EllipticF}\left(\frac{1}{2}\left(ia-\frac{\pi}{2}+ibx\right),2\right)\sqrt{i\sinh(a+bx)}}{3b\sqrt{\sinh(a+bx)}}$$

output
```
-2/3*cosh(b*x+a)/b/sinh(b*x+a)^(3/2)+2/3*I*InverseJacobiAM(1/2*I*a-1/4*Pi+
1/2*I*b*x,2^(1/2))*(I*sinh(b*x+a))^(1/2)/b/sinh(b*x+a)^(1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 0.06 (sec) , antiderivative size = 86, normalized size of antiderivative = 1.08

$$\int \frac{1}{\sinh^{\frac{5}{2}}(a+bx)}\,dx = -\frac{2\left(\cosh(a+bx)+\text{Hypergeometric2F1}\left(\frac{1}{4},\frac{1}{2},\frac{5}{4},\cosh(2(a+bx))+\sinh(2(a+bx))\right)\sinh(a+bx)\sqrt{1-\right.}{3b\sinh^{\frac{3}{2}}(a+bx)}$$

input
```
Integrate[Sinh[a + b*x]^(-5/2),x]
```

output
```
(-2*(Cosh[a + b*x] + Hypergeometric2F1[1/4, 1/2, 5/4, Cosh[2*(a + b*x)] +
Sinh[2*(a + b*x)]]*Sinh[a + b*x]*Sqrt[1 - Cosh[2*a + 2*b*x] - Sinh[2*a + 2
*b*x]]))/(3*b*Sinh[a + b*x]^(3/2))
```

Rubi [A] (verified)

Time = 0.31 (sec) , antiderivative size = 80, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.600$, Rules used = {3042, 3116, 3042, 3121, 3042, 3120}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sinh^{\frac{5}{2}}(a+bx)}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{(-i\sin(ia+ibx))^{5/2}}dx$$

$$\Big\downarrow \text{3116}$$

$$-\frac{1}{3}\int \frac{1}{\sqrt{\sinh(a+bx)}}dx-\frac{2\cosh(a+bx)}{3b\sinh^{\frac{3}{2}}(a+bx)}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{2\cosh(a+bx)}{3b\sinh^{\frac{3}{2}}(a+bx)}-\frac{1}{3}\int \frac{1}{\sqrt{-i\sin(ia+ibx)}}dx$$

$$\Big\downarrow \text{3121}$$

$$-\frac{2\cosh(a+bx)}{3b\sinh^{\frac{3}{2}}(a+bx)}-\frac{\sqrt{i\sinh(a+bx)}\int \frac{1}{\sqrt{i\sinh(a+bx)}}dx}{3\sqrt{\sinh(a+bx)}}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{2\cosh(a+bx)}{3b\sinh^{\frac{3}{2}}(a+bx)}-\frac{\sqrt{i\sinh(a+bx)}\int \frac{1}{\sqrt{\sin(ia+ibx)}}dx}{3\sqrt{\sinh(a+bx)}}$$

$$\Big\downarrow \text{3120}$$

$$-\frac{2\cosh(a+bx)}{3b\sinh^{\frac{3}{2}}(a+bx)}+\frac{2i\sqrt{i\sinh(a+bx)}\,\text{EllipticF}\left(\frac{1}{2}\left(ia+ibx-\frac{\pi}{2}\right),2\right)}{3b\sqrt{\sinh(a+bx)}}$$

input
```
Int[Sinh[a + b*x]^(-5/2),x]
```

output
```
(-2*Cosh[a + b*x])/(3*b*Sinh[a + b*x]^(3/2)) + (((2*I)/3)*EllipticF[(I*a -
 Pi/2 + I*b*x)/2, 2]*Sqrt[I*Sinh[a + b*x]])/(b*Sqrt[Sinh[a + b*x]])
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3116
```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1))), x] + Simp[(n + 2)/(b^2*(n + 1))   I
nt[(b*Sin[c + d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] &&
 IntegerQ[2*n]
```

rule 3120
```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3121
```
Int[((b_)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*Sin[c + d*x])
^n/Sin[c + d*x]^n   Int[Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] && Lt
Q[-1, n, 1] && IntegerQ[2*n]
```

Maple [A] (verified)

Time = 0.15 (sec) , antiderivative size = 101, normalized size of antiderivative = 1.26

method	result	size
default	$-\frac{i\sqrt{1-i\sinh(bx+a)}\,\sqrt{2}\,\sqrt{1+i\sinh(bx+a)}\,\sqrt{i\sinh(bx+a)}\,\operatorname{EllipticF}\left(\sqrt{1-i\sinh(bx+a)},\frac{\sqrt{2}}{2}\right)\sinh(bx+a)+2\cosh(bx+a)^2}{3\sinh(bx+a)^{\frac{3}{2}}\cosh(bx+a)b}$	101

input
```
int(1/sinh(b*x+a)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
-1/3/sinh(b*x+a)^(3/2)*(I*(1-I*sinh(b*x+a))^(1/2)*2^(1/2)*(1+I*sinh(b*x+a)
)^(1/2)*(I*sinh(b*x+a))^(1/2)*EllipticF((1-I*sinh(b*x+a))^(1/2),1/2*2^(1/2
))*sinh(b*x+a)+2*cosh(b*x+a)^2)/cosh(b*x+a)/b
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 314 vs. $2(58) = 116$.

Time = 0.09 (sec) , antiderivative size = 314, normalized size of antiderivative = 3.92

$$\int \frac{1}{\sinh^{\frac{5}{2}}(a+bx)}\,dx = -\frac{2\left(\left(\sqrt{2}\cosh\left(bx+a\right)^{4}+4\sqrt{2}\cosh\left(bx+a\right)\sinh\left(bx+a\right)^{3}+\sqrt{2}\sinh\left(bx+a\right)^{4}+2\left(3\sqrt{2}\cosh\left(bx+a\right.\right.\right.\right.}{}$$

input
```
integrate(1/sinh(b*x+a)^(5/2),x, algorithm="fricas")
```

output
```
-2/3*((sqrt(2)*cosh(b*x + a)^4 + 4*sqrt(2)*cosh(b*x + a)*sinh(b*x + a)^3 +
 sqrt(2)*sinh(b*x + a)^4 + 2*(3*sqrt(2)*cosh(b*x + a)^2 - sqrt(2))*sinh(b*
x + a)^2 - 2*sqrt(2)*cosh(b*x + a)^2 + 4*(sqrt(2)*cosh(b*x + a)^3 - sqrt(2
)*cosh(b*x + a))*sinh(b*x + a) + sqrt(2))*weierstrassPInverse(4, 0, cosh(b
*x + a) + sinh(b*x + a)) + 2*(cosh(b*x + a)^3 + 3*cosh(b*x + a)*sinh(b*x +
 a)^2 + sinh(b*x + a)^3 + (3*cosh(b*x + a)^2 + 1)*sinh(b*x + a) + cosh(b*x
 + a))*sqrt(sinh(b*x + a)))/(b*cosh(b*x + a)^4 + 4*b*cosh(b*x + a)*sinh(b*
x + a)^3 + b*sinh(b*x + a)^4 - 2*b*cosh(b*x + a)^2 + 2*(3*b*cosh(b*x + a)^
2 - b)*sinh(b*x + a)^2 + 4*(b*cosh(b*x + a)^3 - b*cosh(b*x + a))*sinh(b*x
+ a) + b)
```

Sympy [F]

$$\int \frac{1}{\sinh^{\frac{5}{2}}(a+bx)}\,dx = \int \frac{1}{\sinh^{\frac{5}{2}}(a+bx)}\,dx$$

input
```
integrate(1/sinh(b*x+a)**(5/2),x)
```

output
```
Integral(sinh(a + b*x)**(-5/2), x)
```

Maxima [F]

$$\int \frac{1}{\sinh^{\frac{5}{2}}(a+bx)}\,dx = \int \frac{1}{\sinh(bx+a)^{\frac{5}{2}}}\,dx$$

input
```
integrate(1/sinh(b*x+a)^(5/2),x, algorithm="maxima")
```

output
```
integrate(sinh(b*x + a)^(-5/2), x)
```

Giac [F]

$$\int \frac{1}{\sinh^{\frac{5}{2}}(a+bx)}\,dx = \int \frac{1}{\sinh\left(bx+a\right)^{\frac{5}{2}}}\,dx$$

input
```
integrate(1/sinh(b*x+a)^(5/2),x, algorithm="giac")
```

output
```
integrate(sinh(b*x + a)^(-5/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\sinh^{\frac{5}{2}}(a+bx)}\,dx = \int \frac{1}{\sinh\left(a+b\,x\right)^{5/2}}\,dx$$

input
```
int(1/sinh(a + b*x)^(5/2),x)
```

output
```
int(1/sinh(a + b*x)^(5/2), x)
```

Reduce [F]

$$\int \frac{1}{\sinh^{\frac{5}{2}}(a+bx)}\,dx = \int \frac{\sqrt{\sinh\left(bx+a\right)}}{\sinh\left(bx+a\right)^{3}}\,dx$$

input
```
int(1/sinh(b*x+a)^(5/2),x)
```

output
```
int(sqrt(sinh(a + b*x))/sinh(a + b*x)**3,x)
```

3.14 $\int \frac{1}{\sinh^{\frac{7}{2}}(a+bx)}\,dx$

Optimal result

Integrand size = 10, antiderivative size = 103

$$\int \frac{1}{\sinh^{\frac{7}{2}}(a+bx)}\,dx = -\frac{2\cosh(a+bx)}{5b\sinh^{\frac{5}{2}}(a+bx)} + \frac{6\cosh(a+bx)}{5b\sqrt{\sinh(a+bx)}} + \frac{6iE\left(\frac{1}{2}\left(ia-\frac{\pi}{2}+ibx\right)\middle|2\right)\sqrt{\sinh(a+bx)}}{5b\sqrt{i\sinh(a+bx)}}$$

output
```
-2/5*cosh(b*x+a)/b/sinh(b*x+a)^(5/2)+6/5*cosh(b*x+a)/b/sinh(b*x+a)^(1/2)-6
/5*I*EllipticE(cos(1/2*I*a+1/4*Pi+1/2*I*b*x),2^(1/2))*sinh(b*x+a)^(1/2)/b/
(I*sinh(b*x+a))^(1/2)
```

Mathematica [A] (verified)

Time = 0.12 (sec) , antiderivative size = 73, normalized size of antiderivative = 0.71

$$\int \frac{1}{\sinh^{\frac{7}{2}}(a+bx)}\,dx = \frac{-2\coth(a+bx)+6iE\left(\frac{1}{4}(-2ia+\pi-2ibx)\middle|2\right)(i\sinh(a+bx))^{3/2}+3\sinh(2(a+bx))}{5b\sinh^{\frac{3}{2}}(a+bx)}$$

input
```
Integrate[Sinh[a + b*x]^(-7/2),x]
```

output
```
(-2*Coth[a + b*x] + (6*I)*EllipticE[((-2*I)*a + Pi - (2*I)*b*x)/4, 2]*(I*S
inh[a + b*x])^(3/2) + 3*Sinh[2*(a + b*x)])/(5*b*Sinh[a + b*x]^(3/2))
```

Rubi [A] (verified)

Time = 0.41 (sec) , antiderivative size = 104, normalized size of antiderivative = 1.01, number of steps used = 8, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.800$, Rules used = {3042, 3116, 3042, 3116, 3042, 3121, 3042, 3119}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sinh^{\frac{7}{2}}(a+bx)}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{(-i\sin(ia+ibx))^{7/2}}dx$$

$$\Big\downarrow \text{3116}$$

$$-\frac{3}{5}\int \frac{1}{\sinh^{\frac{3}{2}}(a+bx)}dx-\frac{2\cosh(a+bx)}{5b\sinh^{\frac{5}{2}}(a+bx)}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{2\cosh(a+bx)}{5b\sinh^{\frac{5}{2}}(a+bx)}-\frac{3}{5}\int \frac{1}{(-i\sin(ia+ibx))^{3/2}}dx$$

$$\Big\downarrow \text{3116}$$

$$-\frac{3}{5}\left(\int \sqrt{\sinh(a+bx)}dx-\frac{2\cosh(a+bx)}{b\sqrt{\sinh(a+bx)}}\right)-\frac{2\cosh(a+bx)}{5b\sinh^{\frac{5}{2}}(a+bx)}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{2\cosh(a+bx)}{5b\sinh^{\frac{5}{2}}(a+bx)}-\frac{3}{5}\left(-\frac{2\cosh(a+bx)}{b\sqrt{\sinh(a+bx)}}+\int \sqrt{-i\sin(ia+ibx)}dx\right)$$

$$\Big\downarrow \text{3121}$$

$$-\frac{2\cosh(a+bx)}{5b\sinh^{\frac{5}{2}}(a+bx)}-\frac{3}{5}\left(-\frac{2\cosh(a+bx)}{b\sqrt{\sinh(a+bx)}}+\frac{\sqrt{\sinh(a+bx)}\int \sqrt{i\sinh(a+bx)}dx}{\sqrt{i\sinh(a+bx)}}\right)$$

$$\Big\downarrow \; 3042$$

$$-\frac{2\cosh(a+bx)}{5b\sinh^{\frac{5}{2}}(a+bx)}-\frac{3}{5}\left(-\frac{2\cosh(a+bx)}{b\sqrt{\sinh(a+bx)}}+\frac{\sqrt{\sinh(a+bx)}\int\sqrt{\sin(ia+ibx)}dx}{\sqrt{i\sinh(a+bx)}}\right)$$

$$\Big\downarrow \; 3119$$

$$-\frac{2\cosh(a+bx)}{5b\sinh^{\frac{5}{2}}(a+bx)}-\frac{3}{5}\left(-\frac{2\cosh(a+bx)}{b\sqrt{\sinh(a+bx)}}-\frac{2i\sqrt{\sinh(a+bx)}E\left(\frac{1}{2}\left(ia+ibx-\frac{\pi}{2}\right)|2\right)}{b\sqrt{i\sinh(a+bx)}}\right)$$

input
```
Int[Sinh[a + b*x]^(-7/2),x]
```

output
```
(-3*((-2*Cosh[a + b*x])/(b*Sqrt[Sinh[a + b*x]]) - ((2*I)*EllipticE[(I*a -
Pi/2 + I*b*x)/2, 2]*Sqrt[Sinh[a + b*x]])/(b*Sqrt[I*Sinh[a + b*x]])))/5 - (
2*Cosh[a + b*x])/(5*b*Sinh[a + b*x]^(5/2))
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3116
```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1))), x] + Simp[(n + 2)/(b^2*(n + 1))   I
nt[(b*Sin[c + d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] &&
 IntegerQ[2*n]
```

rule 3119
```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3121
```
Int[((b_)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*Sin[c + d*x])
^n/Sin[c + d*x]^n   Int[Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] && Lt
Q[-1, n, 1] && IntegerQ[2*n]
```

Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 191 vs. $2(82) = 164$.

Time = 0.16 (sec) , antiderivative size = 192, normalized size of antiderivative = 1.86

method	result
default	$-\frac{6\sqrt{-i(\sinh(bx+a)+i)}\,\sqrt{2}\,\sqrt{-i(-\sinh(bx+a)+i)}\,\sqrt{i\sinh(bx+a)}\,\sinh(bx+a)^2\,\mathrm{EllipticE}\left(\sqrt{-i(\sinh(bx+a)+i)},\frac{\sqrt{2}}{2}\right)-3\sqrt{-i(\sinh}{5\sinh(b}$

input
```
int(1/sinh(b*x+a)^(7/2),x,method=_RETURNVERBOSE)
```

output
```
-1/5/sinh(b*x+a)^(5/2)*(6*(-I*(sinh(b*x+a)+I))^(1/2)*2^(1/2)*(-I*(-sinh(b*
x+a)+I))^(1/2)*(I*sinh(b*x+a))^(1/2)*sinh(b*x+a)^2*EllipticE((-I*(sinh(b*x
+a)+I))^(1/2),1/2*2^(1/2))-3*(-I*(sinh(b*x+a)+I))^(1/2)*2^(1/2)*(-I*(-sinh
(b*x+a)+I))^(1/2)*(I*sinh(b*x+a))^(1/2)*sinh(b*x+a)^2*EllipticF((-I*(sinh(
b*x+a)+I))^(1/2),1/2*2^(1/2))-6*sinh(b*x+a)^4-4*sinh(b*x+a)^2+2)/cosh(b*x+
a)/b
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 621 vs. $2(78) = 156$.

Time = 0.10 (sec) , antiderivative size = 621, normalized size of antiderivative = 6.03

$$\int \frac{1}{\sinh^{\frac{7}{2}}(a+bx)}\,dx = \text{Too large to display}$$

input
```
integrate(1/sinh(b*x+a)^(7/2),x, algorithm="fricas")
```

output
```
2/5*(3*(sqrt(2)*cosh(b*x + a)^6 + 6*sqrt(2)*cosh(b*x + a)*sinh(b*x + a)^5
+ sqrt(2)*sinh(b*x + a)^6 + 3*(5*sqrt(2)*cosh(b*x + a)^2 - sqrt(2))*sinh(b
*x + a)^4 - 3*sqrt(2)*cosh(b*x + a)^4 + 4*(5*sqrt(2)*cosh(b*x + a)^3 - 3*s
qrt(2)*cosh(b*x + a))*sinh(b*x + a)^3 + 3*(5*sqrt(2)*cosh(b*x + a)^4 - 6*s
qrt(2)*cosh(b*x + a)^2 + sqrt(2))*sinh(b*x + a)^2 + 3*sqrt(2)*cosh(b*x + a
)^2 + 6*(sqrt(2)*cosh(b*x + a)^5 - 2*sqrt(2)*cosh(b*x + a)^3 + sqrt(2)*cos
h(b*x + a))*sinh(b*x + a) - sqrt(2))*weierstrassZeta(4, 0, weierstrassPInv
erse(4, 0, cosh(b*x + a) + sinh(b*x + a))) + 2*(3*cosh(b*x + a)^6 + 18*cos
h(b*x + a)*sinh(b*x + a)^5 + 3*sinh(b*x + a)^6 + (45*cosh(b*x + a)^2 - 8)*
sinh(b*x + a)^4 - 8*cosh(b*x + a)^4 + 4*(15*cosh(b*x + a)^3 - 8*cosh(b*x +
 a))*sinh(b*x + a)^3 + (45*cosh(b*x + a)^4 - 48*cosh(b*x + a)^2 + 1)*sinh(
b*x + a)^2 + cosh(b*x + a)^2 + 2*(9*cosh(b*x + a)^5 - 16*cosh(b*x + a)^3 +
 cosh(b*x + a))*sinh(b*x + a))*sqrt(sinh(b*x + a)))/(b*cosh(b*x + a)^6 + 6
*b*cosh(b*x + a)*sinh(b*x + a)^5 + b*sinh(b*x + a)^6 - 3*b*cosh(b*x + a)^4
 + 3*(5*b*cosh(b*x + a)^2 - b)*sinh(b*x + a)^4 + 4*(5*b*cosh(b*x + a)^3 -
3*b*cosh(b*x + a))*sinh(b*x + a)^3 + 3*b*cosh(b*x + a)^2 + 3*(5*b*cosh(b*x
 + a)^4 - 6*b*cosh(b*x + a)^2 + b)*sinh(b*x + a)^2 + 6*(b*cosh(b*x + a)^5
- 2*b*cosh(b*x + a)^3 + b*cosh(b*x + a))*sinh(b*x + a) - b)
```

Sympy [F]

$$\int \frac{1}{\sinh^{\frac{7}{2}}(a+bx)}\,dx = \int \frac{1}{\sinh^{\frac{7}{2}}(a+bx)}\,dx$$

input
```
integrate(1/sinh(b*x+a)**(7/2),x)
```

output
```
Integral(sinh(a + b*x)**(-7/2), x)
```

Maxima [F]

$$\int \frac{1}{\sinh^{\frac{7}{2}}(a+bx)}\,dx = \int \frac{1}{\sinh\left(bx+a\right)^{\frac{7}{2}}}\,dx$$

input
```
integrate(1/sinh(b*x+a)^(7/2),x, algorithm="maxima")
```

output
```
integrate(sinh(b*x + a)^(-7/2), x)
```

Giac [F]

$$\int \frac{1}{\sinh^{\frac{7}{2}}(a+bx)}\,dx = \int \frac{1}{\sinh\left(bx+a\right)^{\frac{7}{2}}}\,dx$$

input
```
integrate(1/sinh(b*x+a)^(7/2),x, algorithm="giac")
```

output
```
integrate(sinh(b*x + a)^(-7/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\sinh^{\frac{7}{2}}(a+bx)}\,dx = \int \frac{1}{\sinh\left(a+b\,x\right)^{7/2}}\,dx$$

input
```
int(1/sinh(a + b*x)^(7/2),x)
```

output
```
int(1/sinh(a + b*x)^(7/2), x)
```

Reduce [F]

$$\int \frac{1}{\sinh^{\frac{7}{2}}(a+bx)}\,dx = \int \frac{\sqrt{\sinh(bx+a)}}{\sinh(bx+a)^4}dx$$

input
```
int(1/sinh(b*x+a)^(7/2),x)
```

output
```
int(sqrt(sinh(a + b*x))/sinh(a + b*x)**4,x)
```

3.15 $\int (b\sinh(c+dx))^{7/2}\,dx$

Optimal result

Integrand size = 12, antiderivative size = 116

$$\int (b\sinh(c+dx))^{7/2}\,dx = -\frac{10ib^4\,\text{EllipticF}\left(\frac{1}{2}\left(ic-\frac{\pi}{2}+idx\right),2\right)\sqrt{i\sinh(c+dx)}}{21d\sqrt{b\sinh(c+dx)}} - \frac{10b^3\cosh(c+dx)\sqrt{b\sinh(c+dx)}}{21d} + \frac{2b\cosh(c+dx)(b\sinh(c+dx))^{5/2}}{7d}$$

output
```
-10/21*I*b^4*InverseJacobiAM(1/2*I*c-1/4*Pi+1/2*I*d*x,2^(1/2))*(I*sinh(d*x
+c))^(1/2)/d/(b*sinh(d*x+c))^(1/2)-10/21*b^3*cosh(d*x+c)*(b*sinh(d*x+c))^(
1/2)/d+2/7*b*cosh(d*x+c)*(b*sinh(d*x+c))^(5/2)/d
```

Mathematica [A] (verified)

Time = 0.21 (sec) , antiderivative size = 76, normalized size of antiderivative = 0.66

$$\int (b\sinh(c+dx))^{7/2}\,dx = \frac{b^3\left(-23\cosh(c+dx)+3\cosh(3(c+dx))-\frac{20\,\text{EllipticF}\left(\frac{1}{4}(-2ic+\pi-2idx),2\right)}{\sqrt{i\sinh(c+dx)}}\right)\sqrt{b\sinh(c+dx)}}{42d}$$

input
```
Integrate[(b*Sinh[c + d*x])^(7/2),x]
```

output
```
(b^3*(-23*Cosh[c + d*x] + 3*Cosh[3*(c + d*x)] - (20*EllipticF[((-2*I)*c +
Pi - (2*I)*d*x)/4, 2])/Sqrt[I*Sinh[c + d*x]])*Sqrt[b*Sinh[c + d*x]])/(42*d
)
```

Rubi [A] (verified)

Time = 0.45 (sec) , antiderivative size = 122, normalized size of antiderivative = 1.05, number of steps used = 8, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.667$, Rules used = {3042, 3115, 3042, 3115, 3042, 3121, 3042, 3120}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (b\sinh(c+dx))^{7/2}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int (-ib\sin(ic+idx))^{7/2}dx$$

$$\Big\downarrow \text{3115}$$

$$\frac{2b\cosh(c+dx)(b\sinh(c+dx))^{5/2}}{7d}-\frac{5}{7}b^2\int (b\sinh(c+dx))^{3/2}dx$$

$$\Big\downarrow \text{3042}$$

$$\frac{2b\cosh(c+dx)(b\sinh(c+dx))^{5/2}}{7d}-\frac{5}{7}b^2\int (-ib\sin(ic+idx))^{3/2}dx$$

$$\Big\downarrow \text{3115}$$

$$\frac{2b\cosh(c+dx)(b\sinh(c+dx))^{5/2}}{7d}-\frac{5}{7}b^2\left(\frac{2b\cosh(c+dx)\sqrt{b\sinh(c+dx)}}{3d}-\frac{1}{3}b^2\int \frac{1}{\sqrt{b\sinh(c+dx)}}dx\right)$$

$$\Big\downarrow \text{3042}$$

$$\frac{2b\cosh(c+dx)(b\sinh(c+dx))^{5/2}}{7d}-\frac{5}{7}b^2\left(\frac{2b\cosh(c+dx)\sqrt{b\sinh(c+dx)}}{3d}-\frac{1}{3}b^2\int \frac{1}{\sqrt{-ib\sin(ic+idx)}}dx\right)$$

$$\Big\downarrow \text{3121}$$

$$\frac{2b\cosh(c+dx)(b\sinh(c+dx))^{5/2}}{7d}-\frac{5}{7}b^2\left(\frac{2b\cosh(c+dx)\sqrt{b\sinh(c+dx)}}{3d}-\frac{b^2\sqrt{i\sinh(c+dx)}\int\frac{1}{\sqrt{i\sinh(c+dx)}}dx}{3\sqrt{b\sinh(c+dx)}}\right)$$

$$\Big\downarrow \text{3042}$$

$$\frac{2b\cosh(c+dx)(b\sinh(c+dx))^{5/2}}{7d}-\frac{5}{7}b^2\left(\frac{2b\cosh(c+dx)\sqrt{b\sinh(c+dx)}}{3d}-\frac{b^2\sqrt{i\sinh(c+dx)}\int\frac{1}{\sqrt{\sin(ic+idx)}}dx}{3\sqrt{b\sinh(c+dx)}}\right)$$

$$\Big\downarrow \text{3120}$$

$$\frac{2b\cosh(c+dx)(b\sinh(c+dx))^{5/2}}{7d}-\frac{5}{7}b^2\left(\frac{2b\cosh(c+dx)\sqrt{b\sinh(c+dx)}}{3d}+\frac{2ib^2\sqrt{i\sinh(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}\left(ic+idx-\frac{\pi}{2}\right),2\right)}{3d\sqrt{b\sinh(c+dx)}}\right)$$

input
```
Int[(b*Sinh[c + d*x])^(7/2),x]
```

output
```
(2*b*Cosh[c + d*x]*(b*Sinh[c + d*x])^(5/2))/(7*d) - (5*b^2*((((2*I)/3)*b^2
*EllipticF[(I*c - Pi/2 + I*d*x)/2, 2]*Sqrt[I*Sinh[c + d*x]])/(d*Sqrt[b*Sin
h[c + d*x]]) + (2*b*Cosh[c + d*x]*Sqrt[b*Sinh[c + d*x]])/(3*d)))/7
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3115
```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Simp[b^2*((n - 1)/n)   Int[(b*Sin
[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[
2*n]
```

rule 3120
```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3121
```
Int[((b_)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*Sin[c + d*x])
^n/Sin[c + d*x]^n   Int[Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] && Lt
Q[-1, n, 1] && IntegerQ[2*n]
```

Maple [A] (verified)

Time = 0.32 (sec) , antiderivative size = 122, normalized size of antiderivative = 1.05

method	result
default	$\frac{b^4\left(5i\sqrt{-i\sinh(dx+c)+1}\,\sqrt{2}\,\sqrt{1+i\sinh(dx+c)}\,\sqrt{i\sinh(dx+c)}\;\mathrm{EllipticF}\left(\sqrt{-i\sinh(dx+c)+1},\frac{\sqrt{2}}{2}\right)+6\cosh(dx+c)^4\sinh(dx+c)-1\right.}{21\cosh(dx+c)\sqrt{b\sinh(dx+c)}\,d}$

input
```
int((b*sinh(d*x+c))^(7/2),x,method=_RETURNVERBOSE)
```

output
```
1/21*b^4*(5*I*(-I*sinh(d*x+c)+1)^(1/2)*2^(1/2)*(1+I*sinh(d*x+c))^(1/2)*(I*
sinh(d*x+c))^(1/2)*EllipticF((-I*sinh(d*x+c)+1)^(1/2),1/2*2^(1/2))+6*cosh(
d*x+c)^4*sinh(d*x+c)-16*sinh(d*x+c)*cosh(d*x+c)^2)/cosh(d*x+c)/(b*sinh(d*x
+c))^(1/2)/d
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 385 vs. 2(90) = 180.

Time = 0.12 (sec) , antiderivative size = 385, normalized size of antiderivative = 3.32

$$\int(b\sinh(c+dx))^{7/2}\,dx = \frac{80\sqrt{\frac{1}{2}}\left(b^3\cosh(dx+c)^3+3\,b^3\cosh(dx+c)^2\sinh(dx+c)+3\,b^3\cosh(dx+c)\sinh(dx+c)\right.}{}$$

input
```
integrate((b*sinh(d*x+c))^(7/2),x, algorithm="fricas")
```

output
```
1/84*(80*sqrt(1/2)*(b^3*cosh(d*x + c)^3 + 3*b^3*cosh(d*x + c)^2*sinh(d*x +
 c) + 3*b^3*cosh(d*x + c)*sinh(d*x + c)^2 + b^3*sinh(d*x + c)^3)*sqrt(b)*w
eierstrassPInverse(4, 0, cosh(d*x + c) + sinh(d*x + c)) + (3*b^3*cosh(d*x
+ c)^6 + 18*b^3*cosh(d*x + c)*sinh(d*x + c)^5 + 3*b^3*sinh(d*x + c)^6 - 23
*b^3*cosh(d*x + c)^4 - 23*b^3*cosh(d*x + c)^2 + (45*b^3*cosh(d*x + c)^2 -
23*b^3)*sinh(d*x + c)^4 + 4*(15*b^3*cosh(d*x + c)^3 - 23*b^3*cosh(d*x + c)
)*sinh(d*x + c)^3 + 3*b^3 + (45*b^3*cosh(d*x + c)^4 - 138*b^3*cosh(d*x + c
)^2 - 23*b^3)*sinh(d*x + c)^2 + 2*(9*b^3*cosh(d*x + c)^5 - 46*b^3*cosh(d*x
 + c)^3 - 23*b^3*cosh(d*x + c))*sinh(d*x + c))*sqrt(b*sinh(d*x + c)))/(d*c
osh(d*x + c)^3 + 3*d*cosh(d*x + c)^2*sinh(d*x + c) + 3*d*cosh(d*x + c)*sin
h(d*x + c)^2 + d*sinh(d*x + c)^3)
```

Sympy [F(-1)]

Timed out.

$$\int (b\sinh(c+dx))^{7/2}\,dx=\text{Timed out}$$

input
```
integrate((b*sinh(d*x+c))**(7/2),x)
```

output
```
Timed out
```

Maxima [F]

$$\int (b\sinh(c+dx))^{7/2}\,dx=\int (b\sinh(dx+c))^{\frac{7}{2}}\,dx$$

input
```
integrate((b*sinh(d*x+c))^(7/2),x, algorithm="maxima")
```

output
```
integrate((b*sinh(d*x + c))^(7/2), x)
```

Giac [F]

$$\int (b\sinh(c+dx))^{7/2}\,dx = \int (b\sinh(dx+c))^{\frac{7}{2}}\,dx$$

input
```
integrate((b*sinh(d*x+c))^(7/2),x, algorithm="giac")
```

output
```
integrate((b*sinh(d*x + c))^(7/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int (b\sinh(c+dx))^{7/2}\,dx = \int (b\sinh(c+d\,x))^{7/2}\,dx$$

input
```
int((b*sinh(c + d*x))^(7/2),x)
```

output
```
int((b*sinh(c + d*x))^(7/2), x)
```

Reduce [F]

$$\int (b\sinh(c+dx))^{7/2}\,dx = \sqrt{b}\left(\int \sqrt{\sinh(dx+c)}\,\sinh(dx+c)^3\,dx\right)b^3$$

input
```
int((b*sinh(d*x+c))^(7/2),x)
```

output
```
sqrt(b)*int(sqrt(sinh(c + d*x))*sinh(c + d*x)**3,x)*b**3
```

3.16 $\int (b\sinh (c+dx))^{5/2}\,dx$

Optimal result

Integrand size = 12, antiderivative size = 88

$$\int (b\sinh (c+dx))^{5/2}\,dx=\frac{6ib^2E\left(\frac{1}{2}\left(ic-\frac{\pi}{2}+idx\right)\middle|2\right)\sqrt{b\sinh (c+dx)}}{5d\sqrt{i\sinh (c+dx)}}+\frac{2b\cosh (c+dx)(b\sinh (c+dx))^{3/2}}{5d}$$

output
```
-6/5*I*b^2*EllipticE(cos(1/2*I*c+1/4*Pi+1/2*I*d*x),2^(1/2))*(b*sinh(d*x+c)
)^(1/2)/d/(I*sinh(d*x+c))^(1/2)+2/5*b*cosh(d*x+c)*(b*sinh(d*x+c))^(3/2)/d
```

Mathematica [A] (verified)

Time = 0.10 (sec) , antiderivative size = 68, normalized size of antiderivative = 0.77

$$\int (b\sinh (c+dx))^{5/2}\,dx=\frac{b^2\sqrt{b\sinh (c+dx)}\left(-\frac{6iE\left(\frac{1}{4}(-2ic+\pi -2idx)|2\right)}{\sqrt{i\sinh (c+dx)}}+\sinh (2(c+dx))\right)}{5d}$$

input
```
Integrate[(b*Sinh[c + d*x])^(5/2),x]
```

output
```
(b^2*Sqrt[b*Sinh[c + d*x]]*(((-6*I)*EllipticE[((-2*I)*c + Pi - (2*I)*d*x)/
4, 2])/Sqrt[I*Sinh[c + d*x]] + Sinh[2*(c + d*x)]))/(5*d)
```

Rubi [A] (verified)

Time = 0.33 (sec) , antiderivative size = 88, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {3042, 3115, 3042, 3121, 3042, 3119}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (b\sinh(c+dx))^{5/2}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int (-ib\sin(ic+idx))^{5/2}dx$$

$$\Big\downarrow \text{3115}$$

$$\frac{2b\cosh(c+dx)(b\sinh(c+dx))^{3/2}}{5d}-\frac{3}{5}b^2\int \sqrt{b\sinh(c+dx)}dx$$

$$\Big\downarrow \text{3042}$$

$$\frac{2b\cosh(c+dx)(b\sinh(c+dx))^{3/2}}{5d}-\frac{3}{5}b^2\int \sqrt{-ib\sin(ic+idx)}dx$$

$$\Big\downarrow \text{3121}$$

$$\frac{2b\cosh(c+dx)(b\sinh(c+dx))^{3/2}}{5d}-\frac{3b^2\sqrt{b\sinh(c+dx)}\int \sqrt{i\sinh(c+dx)}dx}{5\sqrt{i\sinh(c+dx)}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{2b\cosh(c+dx)(b\sinh(c+dx))^{3/2}}{5d}-\frac{3b^2\sqrt{b\sinh(c+dx)}\int \sqrt{\sin(ic+idx)}dx}{5\sqrt{i\sinh(c+dx)}}$$

$$\Big\downarrow \text{3119}$$

$$\frac{2b\cosh(c+dx)(b\sinh(c+dx))^{3/2}}{5d}+\frac{6ib^2E\left(\frac{1}{2}\left(ic+idx-\frac{\pi}{2}\right)\middle|2\right)\sqrt{b\sinh(c+dx)}}{5d\sqrt{i\sinh(c+dx)}}$$

input
```
Int[(b*Sinh[c + d*x])^(5/2),x]
```

output
```
(((6*I)/5)*b^2*EllipticE[(I*c - Pi/2 + I*d*x)/2, 2]*Sqrt[b*Sinh[c + d*x]])
/(d*Sqrt[I*Sinh[c + d*x]]) + (2*b*Cosh[c + d*x]*(b*Sinh[c + d*x])^(3/2))/(
5*d)
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3115
```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Simp[b^2*((n - 1)/n)   Int[(b*Sin
[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[
2*n]
```

rule 3119
```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3121
```
Int[((b_)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*Sin[c + d*x])
^n/Sin[c + d*x]^n   Int[Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] && Lt
Q[-1, n, 1] && IntegerQ[2*n]
```

Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 169 vs. $2(71) = 142$.

Time = 0.27 (sec) , antiderivative size = 170, normalized size of antiderivative = 1.93

method	result
default	$-\frac{b^3\left(6\sqrt{-i\sinh(dx+c)+1}\,\sqrt{2}\,\sqrt{1+i\sinh(dx+c)}\,\sqrt{i\sinh(dx+c)}\ \text{EllipticE}\left(\sqrt{-i\sinh(dx+c)+1},\frac{\sqrt{2}}{2}\right)-3\sqrt{-i\sinh(dx+c)+1}\,\sqrt{2}\,\sqrt{}\right.}{5\cosh(dx+c)\sqrt{b\sinh(dx+c)}}$

input
```
int((b*sinh(d*x+c))^(5/2),x,method=_RETURNVERBOSE)
```

output
```
-1/5*b^3*(6*(-I*sinh(d*x+c)+1)^(1/2)*2^(1/2)*(1+I*sinh(d*x+c))^(1/2)*(I*si
nh(d*x+c))^(1/2)*EllipticE((-I*sinh(d*x+c)+1)^(1/2),1/2*2^(1/2))-3*(-I*sin
h(d*x+c)+1)^(1/2)*2^(1/2)*(1+I*sinh(d*x+c))^(1/2)*(I*sinh(d*x+c))^(1/2)*El
lipticF((-I*sinh(d*x+c)+1)^(1/2),1/2*2^(1/2))-2*cosh(d*x+c)^4+2*cosh(d*x+c
)^2)/cosh(d*x+c)/(b*sinh(d*x+c))^(1/2)/d
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 243 vs. $2(67) = 134$.

Time = 0.09 (sec) , antiderivative size = 243, normalized size of antiderivative = 2.76

$$\int (b\sinh(c+dx))^{5/2}\,dx = \frac{24\sqrt{\frac{1}{2}}\left(b^2\cosh(dx+c)^2 + 2\,b^2\cosh(dx+c)\sinh(dx+c) + b^2\sinh(dx+c)^2\right)\sqrt{b}\text{weierstrass}}{}$$

input
```
integrate((b*sinh(d*x+c))^(5/2),x, algorithm="fricas")
```

output
```
1/10*(24*sqrt(1/2)*(b^2*cosh(d*x + c)^2 + 2*b^2*cosh(d*x + c)*sinh(d*x + c
) + b^2*sinh(d*x + c)^2)*sqrt(b)*weierstrassZeta(4, 0, weierstrassPInverse
(4, 0, cosh(d*x + c) + sinh(d*x + c))) + (b^2*cosh(d*x + c)^4 + 4*b^2*cosh
(d*x + c)*sinh(d*x + c)^3 + b^2*sinh(d*x + c)^4 + 12*b^2*cosh(d*x + c)^2 +
 6*(b^2*cosh(d*x + c)^2 + 2*b^2)*sinh(d*x + c)^2 - b^2 + 4*(b^2*cosh(d*x +
 c)^3 + 6*b^2*cosh(d*x + c))*sinh(d*x + c))*sqrt(b*sinh(d*x + c)))/(d*cosh
(d*x + c)^2 + 2*d*cosh(d*x + c)*sinh(d*x + c) + d*sinh(d*x + c)^2)
```

Sympy [F]

$$\int (b\sinh(c+dx))^{5/2}\,dx = \int (b\sinh(c+dx))^{\frac{5}{2}}\,dx$$

input
```
integrate((b*sinh(d*x+c))**(5/2),x)
```

output
```
Integral((b*sinh(c + d*x))**(5/2), x)
```

Maxima [F]

$$\int (b\sinh(c+dx))^{5/2}\,dx = \int (b\sinh(dx+c))^{\frac{5}{2}}\,dx$$

input
```
integrate((b*sinh(d*x+c))^(5/2),x, algorithm="maxima")
```

output
```
integrate((b*sinh(d*x + c))^(5/2), x)
```

Giac [F]

$$\int (b\sinh(c+dx))^{5/2}\,dx = \int (b\sinh(dx+c))^{\frac{5}{2}}\,dx$$

input
```
integrate((b*sinh(d*x+c))^(5/2),x, algorithm="giac")
```

output
```
integrate((b*sinh(d*x + c))^(5/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int (b\sinh(c+dx))^{5/2}\,dx = \int (b\sinh(c+d\,x))^{5/2}\,dx$$

input
```
int((b*sinh(c + d*x))^(5/2),x)
```

output
```
int((b*sinh(c + d*x))^(5/2), x)
```

Reduce [F]

$$\int (b\sinh(c+dx))^{5/2}\,dx = \sqrt{b}\left(\int \sqrt{\sinh(dx+c)}\,\sinh(dx+c)^2\,dx\right)b^2$$

input
```
int((b*sinh(d*x+c))^(5/2),x)
```

output
```
sqrt(b)*int(sqrt(sinh(c + d*x))*sinh(c + d*x)**2,x)*b**2
```

3.17 $\int (b\sinh(c+dx))^{3/2}\,dx$

Optimal result

Integrand size = 12, antiderivative size = 88

$$\int (b\sinh(c+dx))^{3/2}\,dx = \frac{2ib^2 \operatorname{EllipticF}\left(\frac{1}{2}\left(ic-\frac{\pi}{2}+idx\right),2\right)\sqrt{i\sinh(c+dx)}}{3d\sqrt{b\sinh(c+dx)}} + \frac{2b\cosh(c+dx)\sqrt{b\sinh(c+dx)}}{3d}$$

output
```
2/3*I*b^2*InverseJacobiAM(1/2*I*c-1/4*Pi+1/2*I*d*x,2^(1/2))*(I*sinh(d*x+c)
)^(1/2)/d/(b*sinh(d*x+c))^(1/2)+2/3*b*cosh(d*x+c)*(b*sinh(d*x+c))^(1/2)/d
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 0.10 (sec) , antiderivative size = 88, normalized size of antiderivative = 1.00

$$\int (b\sinh(c+dx))^{3/2}\,dx = \frac{b^2\Big(\sinh(2(c+dx)) - 2\,\text{Hypergeometric2F1}\left(\frac{1}{4},\frac{1}{2},\frac{5}{4},\cosh(2(c+dx))+\sinh(2(c+dx))\right)\sqrt{1}}{3d\sqrt{b\sinh(c+dx)}}$$

input
```
Integrate[(b*Sinh[c + d*x])^(3/2),x]
```

output
```
(b^2*(Sinh[2*(c + d*x)] - 2*Hypergeometric2F1[1/4, 1/2, 5/4, Cosh[2*(c + d
*x)] + Sinh[2*(c + d*x)]]*Sqrt[1 - Cosh[2*c + 2*d*x] - Sinh[2*c + 2*d*x]])
)/(3*d*Sqrt[b*Sinh[c + d*x]])
```

Rubi [A] (verified)

Time = 0.34 (sec) , antiderivative size = 88, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$, Rules used = {3042, 3115, 3042, 3121, 3042, 3120}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (b\sinh(c+dx))^{3/2}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int (-ib\sin(ic+idx))^{3/2}dx$$

$$\Big\downarrow \text{3115}$$

$$\frac{2b\cosh(c+dx)\sqrt{b\sinh(c+dx)}}{3d}-\frac{1}{3}b^2\int \frac{1}{\sqrt{b\sinh(c+dx)}}dx$$

$$\Big\downarrow \text{3042}$$

$$\frac{2b\cosh(c+dx)\sqrt{b\sinh(c+dx)}}{3d}-\frac{1}{3}b^2\int \frac{1}{\sqrt{-ib\sin(ic+idx)}}dx$$

$$\Big\downarrow \text{3121}$$

$$\frac{2b\cosh(c+dx)\sqrt{b\sinh(c+dx)}}{3d}-\frac{b^2\sqrt{i\sinh(c+dx)}\int \frac{1}{\sqrt{i\sinh(c+dx)}}dx}{3\sqrt{b\sinh(c+dx)}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{2b\cosh(c+dx)\sqrt{b\sinh(c+dx)}}{3d}-\frac{b^2\sqrt{i\sinh(c+dx)}\int \frac{1}{\sqrt{\sin(ic+idx)}}dx}{3\sqrt{b\sinh(c+dx)}}$$

$$\Big\downarrow \text{3120}$$

$$\frac{2b\cosh(c+dx)\sqrt{b\sinh(c+dx)}}{3d}+\frac{2ib^2\sqrt{i\sinh(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}\left(ic+idx-\frac{\pi}{2}\right),2\right)}{3d\sqrt{b\sinh(c+dx)}}$$

input
```
Int[(b*Sinh[c + d*x])^(3/2),x]
```

output
```
(((2*I)/3)*b^2*EllipticF[(I*c - Pi/2 + I*d*x)/2, 2]*Sqrt[I*Sinh[c + d*x]])
/(d*Sqrt[b*Sinh[c + d*x]]) + (2*b*Cosh[c + d*x]*Sqrt[b*Sinh[c + d*x]])/(3*
d)
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3115
```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Simp[b^2*((n - 1)/n)   Int[(b*Sin
[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[
2*n]
```

rule 3120
```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3121
```
Int[((b_)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*Sin[c + d*x])
^n/Sin[c + d*x]^n   Int[Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] && Lt
Q[-1, n, 1] && IntegerQ[2*n]
```

Maple [A] (verified)

Time = 0.22 (sec) , antiderivative size = 106, normalized size of antiderivative = 1.20

method	result
default	$-\frac{b^2\left(i\sqrt{-i\sinh(dx+c)+1}\,\sqrt{2}\,\sqrt{1+i\sinh(dx+c)}\,\sqrt{i\sinh(dx+c)}\,\operatorname{EllipticF}\left(\sqrt{-i\sinh(dx+c)+1},\frac{\sqrt{2}}{2}\right)-2\sinh(dx+c)\cosh(dx+c)^2\right)}{3\cosh(dx+c)\sqrt{b\sinh(dx+c)}\,d}$

input
```
int((b*sinh(d*x+c))^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-1/3*b^2*(I*(-I*sinh(d*x+c)+1)^(1/2)*2^(1/2)*(1+I*sinh(d*x+c))^(1/2)*(I*si
nh(d*x+c))^(1/2)*EllipticF((-I*sinh(d*x+c)+1)^(1/2),1/2*2^(1/2))-2*sinh(d*
x+c)*cosh(d*x+c)^2)/cosh(d*x+c)/(b*sinh(d*x+c))^(1/2)/d
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 112, normalized size of antiderivative = 1.27

$$\int (b\sinh(c+dx))^{3/2}\,dx =$$

$$-\frac{4\sqrt{\frac{1}{2}}(b\cosh(dx+c)+b\sinh(dx+c))\sqrt{b}\text{weierstrassPInverse}(4,0,\cosh(dx+c)+\sinh(dx+c))-(b\,c}{3\,(d\cosh(dx+c)+d\sinh(}$$

input
```
integrate((b*sinh(d*x+c))^(3/2),x, algorithm="fricas")
```

output
```
-1/3*(4*sqrt(1/2)*(b*cosh(d*x + c) + b*sinh(d*x + c))*sqrt(b)*weierstrassP
Inverse(4, 0, cosh(d*x + c) + sinh(d*x + c)) - (b*cosh(d*x + c)^2 + 2*b*co
sh(d*x + c)*sinh(d*x + c) + b*sinh(d*x + c)^2 + b)*sqrt(b*sinh(d*x + c)))/
(d*cosh(d*x + c) + d*sinh(d*x + c))
```

Sympy [F]

$$\int (b\sinh(c+dx))^{3/2}\,dx = \int (b\sinh{(c+dx)})^{\frac{3}{2}}\,dx$$

input
```
integrate((b*sinh(d*x+c))**(3/2),x)
```

output
```
Integral((b*sinh(c + d*x))**(3/2), x)
```

Maxima [F]

$$\int (b\sinh(c+dx))^{3/2}\,dx = \int (b\sinh{(dx+c)})^{\frac{3}{2}}\,dx$$

input
```
integrate((b*sinh(d*x+c))^(3/2),x, algorithm="maxima")
```

output
```
integrate((b*sinh(d*x + c))^(3/2), x)
```

Giac [F]

$$\int (b\sinh(c+dx))^{3/2}\,dx = \int (b\sinh{(dx+c)})^{\frac{3}{2}}\,dx$$

input
```
integrate((b*sinh(d*x+c))^(3/2),x, algorithm="giac")
```

output
```
integrate((b*sinh(d*x + c))^(3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int (b\sinh(c+dx))^{3/2}\,dx = \int (b\sinh(c+d\,x))^{3/2}\,dx$$

input
```
int((b*sinh(c + d*x))^(3/2),x)
```

output
```
int((b*sinh(c + d*x))^(3/2), x)
```

Reduce [F]

$$\int (b\sinh(c+dx))^{3/2}\,dx = \sqrt{b}\left(\int \sqrt{\sinh(dx+c)}\,\sinh(dx+c)\,dx\right)b$$

input
```
int((b*sinh(d*x+c))^(3/2),x)
```

output
```
sqrt(b)*int(sqrt(sinh(c + d*x))*sinh(c + d*x),x)*b
```

3.18 $\int \sqrt{b\sinh(c+dx)}\,dx$

Optimal result

Integrand size = 12, antiderivative size = 56

$$\int \sqrt{b\sinh(c+dx)}\,dx = -\frac{2iE\left(\frac{1}{2}\left(ic-\frac{\pi}{2}+idx\right)\middle|2\right)\sqrt{b\sinh(c+dx)}}{d\sqrt{i\sinh(c+dx)}}$$

output
```
2*I*EllipticE(cos(1/2*I*c+1/4*Pi+1/2*I*d*x),2^(1/2))*(b*sinh(d*x+c))^(1/2)
/d/(I*sinh(d*x+c))^(1/2)
```

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 52, normalized size of antiderivative = 0.93

$$\int \sqrt{b\sinh(c+dx)}\,dx = \frac{2iE\left(\frac{1}{4}(-2ic+\pi-2idx)\middle|2\right)\sqrt{b\sinh(c+dx)}}{d\sqrt{i\sinh(c+dx)}}$$

input
```
Integrate[Sqrt[b*Sinh[c + d*x]],x]
```

output
```
((2*I)*EllipticE[((-2*I)*c + Pi - (2*I)*d*x)/4, 2]*Sqrt[b*Sinh[c + d*x]])/
(d*Sqrt[I*Sinh[c + d*x]])
```

Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 56, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {3042, 3121, 3042, 3119}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{b\sinh(c+dx)}\,dx$$

$$\Big\downarrow \; 3042$$

$$\int \sqrt{-ib\sin(ic+idx)}dx$$

$$\Big\downarrow \; 3121$$

$$\frac{\sqrt{b\sinh(c+dx)}\int \sqrt{i\sinh(c+dx)}dx}{\sqrt{i\sinh(c+dx)}}$$

$$\Big\downarrow \; 3042$$

$$\frac{\sqrt{b\sinh(c+dx)}\int \sqrt{\sin(ic+idx)}dx}{\sqrt{i\sinh(c+dx)}}$$

$$\Big\downarrow \; 3119$$

$$-\frac{2iE\left(\frac{1}{2}\left(ic+idx-\frac{\pi}{2}\right)\middle|2\right)\sqrt{b\sinh(c+dx)}}{d\sqrt{i\sinh(c+dx)}}$$

input
```
Int[Sqrt[b*Sinh[c + d*x]],x]
```

output
```
((-2*I)*EllipticE[(I*c - Pi/2 + I*d*x)/2, 2]*Sqrt[b*Sinh[c + d*x]])/(d*Sqr
t[I*Sinh[c + d*x]])
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3119
```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3121
```
Int[((b_)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*Sin[c + d*x])
^n/Sin[c + d*x]^n   Int[Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] && Lt
Q[-1, n, 1] && IntegerQ[2*n]
```

Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 110 vs. $2(45) = 90$.

Time = 0.34 (sec) , antiderivative size = 111, normalized size of antiderivative = 1.98

method	result
default	$\frac{b\sqrt{-i(\sinh(dx+c)+i)}\,\sqrt{2}\,\sqrt{-i(i-\sinh(dx+c))}\,\sqrt{i\sinh(dx+c)}\,(2\,\text{EllipticE}(\sqrt{-i\sinh(dx+c)+1},\frac{\sqrt{2}}{2})-\text{EllipticF}(\sqrt{-i\sinh(dx+c)}}{\cosh(dx+c)\sqrt{b\sinh(dx+c)}\,d}$
risch	$\frac{\sqrt{2}\,\sqrt{b(e^{2dx+2c}-1)e^{-dx-c}}}{d}-\frac{(\frac{2b\,e^{2dx+2c}-2b}{b\sqrt{e^{dx+c}(b\,e^{2dx+2c}-b)}}-\frac{\sqrt{e^{dx+c}+1}\,\sqrt{-2\,e^{dx+c}+2}\,\sqrt{-e^{dx+c}}\,(-2\,\text{EllipticE}(\sqrt{e^{dx+c}+1},\frac{\sqrt{2}}{2})+\text{Ellipti}}{\sqrt{b\,e^{3dx+3c}-b\,e^{dx+c}}}}{d(e^{2dx+2c}-1)}$

input
```
int((b*sinh(d*x+c))^(1/2),x,method=_RETURNVERBOSE)
```

output
```
b*(-I*(sinh(d*x+c)+I))^(1/2)*2^(1/2)*(-I*(I-sinh(d*x+c)))^(1/2)*(I*sinh(d*
x+c))^(1/2)*(2*EllipticE((-I*sinh(d*x+c)+1)^(1/2),1/2*2^(1/2))-EllipticF((
-I*sinh(d*x+c)+1)^(1/2),1/2*2^(1/2)))/cosh(d*x+c)/(b*sinh(d*x+c))^(1/2)/d
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 43, normalized size of antiderivative = 0.77

$$\int \sqrt{b\sinh(c+dx)}\,dx = -\frac{2\left(2\sqrt{\frac{1}{2}}\sqrt{b}\text{weierstrassZeta}(4,0,\text{weierstrassPInverse}(4,0,\cosh{(dx+c)}+\sinh{(dx+c)}))+\sqrt{b\sinh{(}}\right.}{d}$$

input
```
integrate((b*sinh(d*x+c))^(1/2),x, algorithm="fricas")
```

output
```
-2*(2*sqrt(1/2)*sqrt(b)*weierstrassZeta(4, 0, weierstrassPInverse(4, 0, co
sh(d*x + c) + sinh(d*x + c))) + sqrt(b*sinh(d*x + c)))/d
```

Sympy [F]

$$\int \sqrt{b\sinh(c+dx)}\,dx = \int \sqrt{b\sinh{(c+dx)}}\,dx$$

input
```
integrate((b*sinh(d*x+c))**(1/2),x)
```

output
```
Integral(sqrt(b*sinh(c + d*x)), x)
```

Maxima [F]

$$\int \sqrt{b\sinh(c+dx)}\,dx = \int \sqrt{b\sinh{(dx+c)}}\,dx$$

input
```
integrate((b*sinh(d*x+c))^(1/2),x, algorithm="maxima")
```

output
```
integrate(sqrt(b*sinh(d*x + c)), x)
```

Giac [F]

$$\int \sqrt{b\sinh(c+dx)}\,dx = \int \sqrt{b\sinh(dx+c)}\,dx$$

input
```
integrate((b*sinh(d*x+c))^(1/2),x, algorithm="giac")
```

output
```
integrate(sqrt(b*sinh(d*x + c)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \sqrt{b\sinh(c+dx)}\,dx = \int \sqrt{b\sinh(c+d\,x)}\,dx$$

input
```
int((b*sinh(c + d*x))^(1/2),x)
```

output
```
int((b*sinh(c + d*x))^(1/2), x)
```

Reduce [F]

$$\int \sqrt{b\sinh(c+dx)}\,dx = \sqrt{b}\left(\int \sqrt{\sinh(dx+c)}dx\right)$$

input
```
int((b*sinh(d*x+c))^(1/2),x)
```

output
```
sqrt(b)*int(sqrt(sinh(c + d*x)),x)
```

3.19 $\int \frac{1}{\sqrt{b\sinh(c+dx)}}\,dx$

Optimal result

Integrand size = 12, antiderivative size = 56

$$\int \frac{1}{\sqrt{b\sinh(c+dx)}}\,dx = -\frac{2i\,\text{EllipticF}\left(\frac{1}{2}\left(ic-\frac{\pi}{2}+idx\right),2\right)\sqrt{i\sinh(c+dx)}}{d\sqrt{b\sinh(c+dx)}}$$

output
```
-2*I*InverseJacobiAM(1/2*I*c-1/4*Pi+1/2*I*d*x,2^(1/2))*(I*sinh(d*x+c))^(1/
2)/d/(b*sinh(d*x+c))^(1/2)
```

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.96

$$\int \frac{1}{\sqrt{b\sinh(c+dx)}}\,dx = \frac{2i\,\text{EllipticF}\left(\frac{1}{2}\left(\frac{\pi}{2}-i(c+dx)\right),2\right)\sqrt{i\sinh(c+dx)}}{d\sqrt{b\sinh(c+dx)}}$$

input
```
Integrate[1/Sqrt[b*Sinh[c + d*x]],x]
```

output
```
((2*I)*EllipticF[(Pi/2 - I*(c + d*x))/2, 2]*Sqrt[I*Sinh[c + d*x]])/(d*Sqrt
[b*Sinh[c + d*x]])
```

Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 56, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {3042, 3121, 3042, 3120}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt{b\sinh(c+dx)}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{\sqrt{-ib\sin(ic+idx)}}dx$$

$$\Big\downarrow \text{3121}$$

$$\frac{\sqrt{i\sinh(c+dx)}\int \frac{1}{\sqrt{i\sinh(c+dx)}}dx}{\sqrt{b\sinh(c+dx)}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\sqrt{i\sinh(c+dx)}\int \frac{1}{\sqrt{\sin(ic+idx)}}dx}{\sqrt{b\sinh(c+dx)}}$$

$$\Big\downarrow \text{3120}$$

$$-\frac{2i\sqrt{i\sinh(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}\left(ic+idx-\frac{\pi}{2}\right),2\right)}{d\sqrt{b\sinh(c+dx)}}$$

input
```
Int[1/Sqrt[b*Sinh[c + d*x]],x]
```

output
```
((-2*I)*EllipticF[(I*c - Pi/2 + I*d*x)/2, 2]*Sqrt[I*Sinh[c + d*x]])/(d*Sqr
t[b*Sinh[c + d*x]])
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3120
```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3121
```
Int[((b_)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*Sin[c + d*x])
^n/Sin[c + d*x]^n   Int[Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] && Lt
Q[-1, n, 1] && IntegerQ[2*n]
```

Maple [A] (verified)

Time = 0.19 (sec) , antiderivative size = 89, normalized size of antiderivative = 1.59

method	result	size
default	$\frac{i\sqrt{-i(\sinh(dx+c)+i)}\,\sqrt{2}\,\sqrt{-i(i-\sinh(dx+c))}\,\sqrt{i\sinh(dx+c)}\ \mathrm{EllipticF}\left(\sqrt{-i(\sinh(dx+c)+i)},\frac{\sqrt{2}}{2}\right)}{\cosh(dx+c)\sqrt{b\sinh(dx+c)}\,d}$	89

input
```
int(1/(b*sinh(d*x+c))^(1/2),x,method=_RETURNVERBOSE)
```

output
```
I*(-I*(sinh(d*x+c)+I))^(1/2)*2^(1/2)*(-I*(I-sinh(d*x+c)))^(1/2)*(I*sinh(d*
x+c))^(1/2)*EllipticF((-I*(sinh(d*x+c)+I))^(1/2),1/2*2^(1/2))/cosh(d*x+c)/
(b*sinh(d*x+c))^(1/2)/d
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.48

$$\int \frac{1}{\sqrt{b\sinh(c+dx)}}\,dx = \frac{4\sqrt{\frac{1}{2}}\mathrm{weierstrassPInverse}(4,0,\cosh\left(dx+c\right)+\sinh\left(dx+c\right))}{\sqrt{b}d}$$

input
```
integrate(1/(b*sinh(d*x+c))^(1/2),x, algorithm="fricas")
```

output
```
4*sqrt(1/2)*weierstrassPInverse(4, 0, cosh(d*x + c) + sinh(d*x + c))/(sqrt
(b)*d)
```

Sympy [F]

$$\int \frac{1}{\sqrt{b\sinh(c+dx)}}\,dx=\int \frac{1}{\sqrt{b\sinh{(c+dx)}}}\,dx$$

input
```
integrate(1/(b*sinh(d*x+c))**(1/2),x)
```

output
```
Integral(1/sqrt(b*sinh(c + d*x)), x)
```

Maxima [F]

$$\int \frac{1}{\sqrt{b\sinh(c+dx)}}\,dx=\int \frac{1}{\sqrt{b\sinh{(dx+c)}}}\,dx$$

input
```
integrate(1/(b*sinh(d*x+c))^(1/2),x, algorithm="maxima")
```

output
```
integrate(1/sqrt(b*sinh(d*x + c)), x)
```

Giac [F]

$$\int \frac{1}{\sqrt{b\sinh(c+dx)}}\,dx=\int \frac{1}{\sqrt{b\sinh{(dx+c)}}}\,dx$$

input
```
integrate(1/(b*sinh(d*x+c))^(1/2),x, algorithm="giac")
```

output
```
integrate(1/sqrt(b*sinh(d*x + c)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt{b\sinh(c+dx)}}\,dx = \int \frac{1}{\sqrt{b\sinh\left(c+d\,x\right)}}\,dx$$

input
```
int(1/(b*sinh(c + d*x))^(1/2),x)
```

output
```
int(1/(b*sinh(c + d*x))^(1/2), x)
```

Reduce [F]

$$\int \frac{1}{\sqrt{b\sinh(c+dx)}}\,dx = \frac{\sqrt{b}\left(\int \frac{\sqrt{\sinh(dx+c)}}{\sinh(dx+c)}dx\right)}{b}$$

input
```
int(1/(b*sinh(d*x+c))^(1/2),x)
```

output
```
(sqrt(b)*int(sqrt(sinh(c + d*x))/sinh(c + d*x),x))/b
```

3.20 $\int \frac{1}{(b\sinh(c+dx))^{3/2}}\,dx$

Optimal result

Integrand size = 12, antiderivative size = 86

$$\int \frac{1}{(b\sinh(c+dx))^{3/2}}\,dx = -\frac{2\cosh(c+dx)}{bd\sqrt{b\sinh(c+dx)}} - \frac{2iE\left(\frac{1}{2}\left(ic-\frac{\pi}{2}+idx\right)\middle|2\right)\sqrt{b\sinh(c+dx)}}{b^2d\sqrt{i\sinh(c+dx)}}$$

output
```
-2*cosh(d*x+c)/b/d/(b*sinh(d*x+c))^(1/2)+2*I*EllipticE(cos(1/2*I*c+1/4*Pi+
1/2*I*d*x),2^(1/2))*(b*sinh(d*x+c))^(1/2)/b^2/d/(I*sinh(d*x+c))^(1/2)
```

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 62, normalized size of antiderivative = 0.72

$$\int \frac{1}{(b\sinh(c+dx))^{3/2}}\,dx = -\frac{2\left(\cosh(c+dx)-E\left(\frac{1}{4}(-2ic+\pi-2idx)\middle|2\right)\sqrt{i\sinh(c+dx)}\right)}{bd\sqrt{b\sinh(c+dx)}}$$

input
```
Integrate[(b*Sinh[c + d*x])^(-3/2),x]
```

output
```
(-2*(Cosh[c + d*x] - EllipticE[((-2*I)*c + Pi - (2*I)*d*x)/4, 2]*Sqrt[I*Si
nh[c + d*x]]))/(b*d*Sqrt[b*Sinh[c + d*x]])
```

Rubi [A] (verified)

Time = 0.34 (sec) , antiderivative size = 86, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$, Rules used = {3042, 3116, 3042, 3121, 3042, 3119}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(b\sinh(c+dx))^{3/2}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{(-ib\sin(ic+idx))^{3/2}}dx$$

$$\Big\downarrow \text{3116}$$

$$\frac{\int \sqrt{b\sinh(c+dx)}dx}{b^2}-\frac{2\cosh(c+dx)}{bd\sqrt{b\sinh(c+dx)}}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{2\cosh(c+dx)}{bd\sqrt{b\sinh(c+dx)}}+\frac{\int \sqrt{-ib\sin(ic+idx)}dx}{b^2}$$

$$\Big\downarrow \text{3121}$$

$$-\frac{2\cosh(c+dx)}{bd\sqrt{b\sinh(c+dx)}}+\frac{\sqrt{b\sinh(c+dx)}\int \sqrt{i\sinh(c+dx)}dx}{b^2\sqrt{i\sinh(c+dx)}}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{2\cosh(c+dx)}{bd\sqrt{b\sinh(c+dx)}}+\frac{\sqrt{b\sinh(c+dx)}\int \sqrt{\sin(ic+idx)}dx}{b^2\sqrt{i\sinh(c+dx)}}$$

$$\Big\downarrow \text{3119}$$

$$-\frac{2\cosh(c+dx)}{bd\sqrt{b\sinh(c+dx)}}-\frac{2iE\left(\frac{1}{2}\left(ic+idx-\frac{\pi}{2}\right)\middle|2\right)\sqrt{b\sinh(c+dx)}}{b^2d\sqrt{i\sinh(c+dx)}}$$

input
```
Int[(b*Sinh[c + d*x])^(-3/2),x]
```

output
```
(-2*Cosh[c + d*x])/(b*d*Sqrt[b*Sinh[c + d*x]]) - ((2*I)*EllipticE[(I*c - P
i/2 + I*d*x)/2, 2]*Sqrt[b*Sinh[c + d*x]])/(b^2*d*Sqrt[I*Sinh[c + d*x]])
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3116
```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1))), x] + Simp[(n + 2)/(b^2*(n + 1))   I
nt[(b*Sin[c + d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] &&
 IntegerQ[2*n]
```

rule 3119
```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3121
```
Int[((b_)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*Sin[c + d*x])
^n/Sin[c + d*x]^n   Int[Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] && Lt
Q[-1, n, 1] && IntegerQ[2*n]
```

Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 158 vs. $2(73) = 146$.

Time = 0.23 (sec) , antiderivative size = 159, normalized size of antiderivative = 1.85

method	result
default	$\frac{2\sqrt{-i\sinh(dx+c)+1}\,\sqrt{2}\,\sqrt{1+i\sinh(dx+c)}\,\sqrt{i\sinh(dx+c)}\,\operatorname{EllipticE}\left(\sqrt{-i\sinh(dx+c)+1},\frac{\sqrt{2}}{2}\right)-\sqrt{-i\sinh(dx+c)+1}\,\sqrt{2}\,\sqrt{1+i\sin}}{b\cosh(dx+c)\sqrt{b\sinh(dx+c)}\,d}$

input
```
int(1/(b*sinh(d*x+c))^(3/2),x,method=_RETURNVERBOSE)
```

output
```
(2*(-I*sinh(d*x+c)+1)^(1/2)*2^(1/2)*(1+I*sinh(d*x+c))^(1/2)*(I*sinh(d*x+c)
)^(1/2)*EllipticE((-I*sinh(d*x+c)+1)^(1/2),1/2*2^(1/2))-(-I*sinh(d*x+c)+1)
^(1/2)*2^(1/2)*(1+I*sinh(d*x+c))^(1/2)*(I*sinh(d*x+c))^(1/2)*EllipticF((-I
*sinh(d*x+c)+1)^(1/2),1/2*2^(1/2))-2*cosh(d*x+c)^2)/b/cosh(d*x+c)/(b*sinh(
d*x+c))^(1/2)/d
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 156 vs. $2(69) = 138$.

Time = 0.09 (sec) , antiderivative size = 156, normalized size of antiderivative = 1.81

$$\int \frac{1}{(b\sinh(c+dx))^{3/2}}\,dx =$$

$$-\frac{4\left(\sqrt{\frac{1}{2}}\left(\cosh\left(dx+c\right)^{2}+2\,\cosh\left(dx+c\right)\sinh\left(dx+c\right)+\sinh\left(dx+c\right)^{2}-1\right)\sqrt{b}\mathrm{weierstrassZeta}(4,0,\mathrm{weie}}{b^{2}d\cosh\left(dx+c\right)^{2}+2\,b}$$

input
```
integrate(1/(b*sinh(d*x+c))^(3/2),x, algorithm="fricas")
```

output
```
-4*(sqrt(1/2)*(cosh(d*x + c)^2 + 2*cosh(d*x + c)*sinh(d*x + c) + sinh(d*x
+ c)^2 - 1)*sqrt(b)*weierstrassZeta(4, 0, weierstrassPInverse(4, 0, cosh(d
*x + c) + sinh(d*x + c))) + (cosh(d*x + c)^2 + 2*cosh(d*x + c)*sinh(d*x +
c) + sinh(d*x + c)^2)*sqrt(b*sinh(d*x + c)))/(b^2*d*cosh(d*x + c)^2 + 2*b^
2*d*cosh(d*x + c)*sinh(d*x + c) + b^2*d*sinh(d*x + c)^2 - b^2*d)
```

Sympy [F]

$$\int \frac{1}{(b\sinh(c+dx))^{3/2}}\,dx = \int \frac{1}{\left(b\sinh\left(c+dx\right)\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/(b*sinh(d*x+c))**(3/2),x)
```

output
```
Integral((b*sinh(c + d*x))**(-3/2), x)
```

Maxima [F]

$$\int \frac{1}{(b\sinh(c+dx))^{3/2}}\,dx = \int \frac{1}{(b\sinh(dx+c))^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/(b*sinh(d*x+c))^(3/2),x, algorithm="maxima")
```

output
```
integrate((b*sinh(d*x + c))^(-3/2), x)
```

Giac [F]

$$\int \frac{1}{(b\sinh(c+dx))^{3/2}}\,dx = \int \frac{1}{(b\sinh(dx+c))^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/(b*sinh(d*x+c))^(3/2),x, algorithm="giac")
```

output
```
integrate((b*sinh(d*x + c))^(-3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(b\sinh(c+dx))^{3/2}}\,dx = \int \frac{1}{(b\sinh(c+d\,x))^{3/2}}\,dx$$

input
```
int(1/(b*sinh(c + d*x))^(3/2),x)
```

output
```
int(1/(b*sinh(c + d*x))^(3/2), x)
```

Reduce [F]

$$\int \frac{1}{(b\sinh(c+dx))^{3/2}}\,dx = \frac{\sqrt{b}\left(\int \frac{\sqrt{\sinh(dx+c)}}{\sinh(dx+c)^2}dx\right)}{b^2}$$

input
```
int(1/(b*sinh(d*x+c))^(3/2),x)
```

output
```
(sqrt(b)*int(sqrt(sinh(c + d*x))/sinh(c + d*x)**2,x))/b**2
```

3.21 $\int \frac{1}{(b\sinh(c+dx))^{5/2}}\,dx$

Optimal result

Integrand size = 12, antiderivative size = 90

$$\int \frac{1}{(b\sinh(c+dx))^{5/2}}\,dx = -\frac{2\cosh(c+dx)}{3bd(b\sinh(c+dx))^{3/2}} + \frac{2i\,\mathrm{EllipticF}\left(\frac{1}{2}\left(ic-\frac{\pi}{2}+idx\right),2\right)\sqrt{i\sinh(c+dx)}}{3b^2d\sqrt{b\sinh(c+dx)}}$$

output
```
-2/3*cosh(d*x+c)/b/d/(b*sinh(d*x+c))^(3/2)+2/3*I*InverseJacobiAM(1/2*I*c-1
/4*Pi+1/2*I*d*x,2^(1/2))*(I*sinh(d*x+c))^(1/2)/b^2/d/(b*sinh(d*x+c))^(1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 0.07 (sec) , antiderivative size = 84, normalized size of antiderivative = 0.93

$$\int \frac{1}{(b\sinh(c+dx))^{5/2}}\,dx = -\frac{2\left(\coth(c+dx)+\sqrt{2}\,\mathrm{Hypergeometric2F1}\left(\frac{1}{4},\frac{1}{2},\frac{5}{4},\cosh(2(c+dx))+\sinh(2(c+dx))\right)\sqrt{-\left((1+\coth(c\right.}}{3b^2d\sqrt{b\sinh(c+dx)}}$$

input
```
Integrate[(b*Sinh[c + d*x])^(-5/2),x]
```

output
```
(-2*(Coth[c + d*x] + Sqrt[2]*Hypergeometric2F1[1/4, 1/2, 5/4, Cosh[2*(c +
d*x)] + Sinh[2*(c + d*x)]]*Sqrt[-((1 + Coth[c + d*x])*Sinh[c + d*x]^2)]))/
(3*b^2*d*Sqrt[b*Sinh[c + d*x]])
```

Rubi [A] (verified)

Time = 0.35 (sec) , antiderivative size = 90, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$, Rules used = {3042, 3116, 3042, 3121, 3042, 3120}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(b\sinh(c+dx))^{5/2}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{(-ib\sin(ic+idx))^{5/2}}dx$$

$$\Big\downarrow \text{3116}$$

$$-\frac{\int \frac{1}{\sqrt{b\sinh(c+dx)}}dx}{3b^2}-\frac{2\cosh(c+dx)}{3bd(b\sinh(c+dx))^{3/2}}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{2\cosh(c+dx)}{3bd(b\sinh(c+dx))^{3/2}}-\frac{\int \frac{1}{\sqrt{-ib\sin(ic+idx)}}dx}{3b^2}$$

$$\Big\downarrow \text{3121}$$

$$-\frac{2\cosh(c+dx)}{3bd(b\sinh(c+dx))^{3/2}}-\frac{\sqrt{i\sinh(c+dx)}\int \frac{1}{\sqrt{i\sinh(c+dx)}}dx}{3b^2\sqrt{b\sinh(c+dx)}}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{2\cosh(c+dx)}{3bd(b\sinh(c+dx))^{3/2}}-\frac{\sqrt{i\sinh(c+dx)}\int \frac{1}{\sqrt{\sin(ic+idx)}}dx}{3b^2\sqrt{b\sinh(c+dx)}}$$

$$\Big\downarrow \text{3120}$$

$$-\frac{2\cosh(c+dx)}{3bd(b\sinh(c+dx))^{3/2}}+\frac{2i\sqrt{i\sinh(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}\left(ic+idx-\frac{\pi}{2}\right),2\right)}{3b^2d\sqrt{b\sinh(c+dx)}}$$

input
```
Int[(b*Sinh[c + d*x])^(-5/2),x]
```

output
```
(-2*Cosh[c + d*x])/(3*b*d*(b*Sinh[c + d*x])^(3/2)) + (((2*I)/3)*EllipticF[
(I*c - Pi/2 + I*d*x)/2, 2]*Sqrt[I*Sinh[c + d*x]])/(b^2*d*Sqrt[b*Sinh[c + d
*x]])
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3116
```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1))), x] + Simp[(n + 2)/(b^2*(n + 1))   I
nt[(b*Sin[c + d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] &&
 IntegerQ[2*n]
```

rule 3120
```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3121
```
Int[((b_)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*Sin[c + d*x])
^n/Sin[c + d*x]^n   Int[Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] && Lt
Q[-1, n, 1] && IntegerQ[2*n]
```

Maple [A] (verified)

Time = 0.23 (sec) , antiderivative size = 114, normalized size of antiderivative = 1.27

method	result	s
default	$-\frac{i\sqrt{-i\sinh(dx+c)+1}\,\sqrt{2}\,\sqrt{1+i\sinh(dx+c)}\,\sqrt{i\sinh(dx+c)}\,\operatorname{EllipticF}\left(\sqrt{-i\sinh(dx+c)+1},\frac{\sqrt{2}}{2}\right)\sinh(dx+c)+2\cosh(dx+c)^2}{3b^2\sinh(dx+c)\cosh(dx+c)\sqrt{b\sinh(dx+c)}\,d}$	1

input
```
int(1/(b*sinh(d*x+c))^(5/2),x,method=_RETURNVERBOSE)
```

output
```
-1/3/b^2/sinh(d*x+c)*(I*(-I*sinh(d*x+c)+1)^(1/2)*2^(1/2)*(1+I*sinh(d*x+c))
^(1/2)*(I*sinh(d*x+c))^(1/2)*EllipticF((-I*sinh(d*x+c)+1)^(1/2),1/2*2^(1/2
))*sinh(d*x+c)+2*cosh(d*x+c)^2)/cosh(d*x+c)/(b*sinh(d*x+c))^(1/2)/d
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 319 vs. 2(68) = 136.

Time = 0.09 (sec) , antiderivative size = 319, normalized size of antiderivative = 3.54

$$\int \frac{1}{(b\sinh(c+dx))^{5/2}}\,dx =$$

$$-\frac{4\left(\sqrt{\frac{1}{2}}\left(\cosh(dx+c)^4+4\cosh(dx+c)\sinh(dx+c)^3+\sinh(dx+c)^4+2\left(3\cosh(dx+c)^2-1\right)\sinh(\right.\right.}{3\left(b^3d\cosh(dx\right.}$$

input
```
integrate(1/(b*sinh(d*x+c))^(5/2),x, algorithm="fricas")
```

output
```
-4/3*(sqrt(1/2)*(cosh(d*x + c)^4 + 4*cosh(d*x + c)*sinh(d*x + c)^3 + sinh(
d*x + c)^4 + 2*(3*cosh(d*x + c)^2 - 1)*sinh(d*x + c)^2 - 2*cosh(d*x + c)^2
 + 4*(cosh(d*x + c)^3 - cosh(d*x + c))*sinh(d*x + c) + 1)*sqrt(b)*weierstr
assPInverse(4, 0, cosh(d*x + c) + sinh(d*x + c)) + (cosh(d*x + c)^3 + 3*co
sh(d*x + c)*sinh(d*x + c)^2 + sinh(d*x + c)^3 + (3*cosh(d*x + c)^2 + 1)*si
nh(d*x + c) + cosh(d*x + c))*sqrt(b*sinh(d*x + c)))/(b^3*d*cosh(d*x + c)^4
 + 4*b^3*d*cosh(d*x + c)*sinh(d*x + c)^3 + b^3*d*sinh(d*x + c)^4 - 2*b^3*d
*cosh(d*x + c)^2 + b^3*d + 2*(3*b^3*d*cosh(d*x + c)^2 - b^3*d)*sinh(d*x +
c)^2 + 4*(b^3*d*cosh(d*x + c)^3 - b^3*d*cosh(d*x + c))*sinh(d*x + c))
```

Sympy [F]

$$\int \frac{1}{(b\sinh(c+dx))^{5/2}}\,dx = \int \frac{1}{(b\sinh{(c+dx)})^{\frac{5}{2}}}\,dx$$

input
```
integrate(1/(b*sinh(d*x+c))**(5/2),x)
```

output
```
Integral((b*sinh(c + d*x))**(-5/2), x)
```

Maxima [F]

$$\int \frac{1}{(b\sinh(c+dx))^{5/2}}\,dx = \int \frac{1}{(b\sinh{(dx+c)})^{\frac{5}{2}}}\,dx$$

input
```
integrate(1/(b*sinh(d*x+c))^(5/2),x, algorithm="maxima")
```

output
```
integrate((b*sinh(d*x + c))^(-5/2), x)
```

Giac [F]

$$\int \frac{1}{(b\sinh(c+dx))^{5/2}}\,dx = \int \frac{1}{(b\sinh{(dx+c)})^{\frac{5}{2}}}\,dx$$

input
```
integrate(1/(b*sinh(d*x+c))^(5/2),x, algorithm="giac")
```

output
```
integrate((b*sinh(d*x + c))^(-5/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(b\sinh(c+dx))^{5/2}}\,dx = \int \frac{1}{\left(b\sinh\left(c+d\,x\right)\right)^{5/2}}\,dx$$

input
```
int(1/(b*sinh(c + d*x))^(5/2),x)
```

output
```
int(1/(b*sinh(c + d*x))^(5/2), x)
```

Reduce [F]

$$\int \frac{1}{(b\sinh(c+dx))^{5/2}}\,dx = \frac{\sqrt{b}\left(\int \frac{\sqrt{\sinh(dx+c)}}{\sinh(dx+c)^3}dx\right)}{b^3}$$

input
```
int(1/(b*sinh(d*x+c))^(5/2),x)
```

output
```
(sqrt(b)*int(sqrt(sinh(c + d*x))/sinh(c + d*x)**3,x))/b**3
```

3.22 $\int \frac{1}{(b\sinh(c+dx))^{7/2}}\,dx$

Optimal result

Integrand size = 12, antiderivative size = 118

$$\int \frac{1}{(b\sinh(c+dx))^{7/2}}\,dx = -\frac{2\cosh(c+dx)}{5bd(b\sinh(c+dx))^{5/2}} + \frac{6\cosh(c+dx)}{5b^3d\sqrt{b\sinh(c+dx)}} + \frac{6iE\left(\frac{1}{2}\left(ic-\frac{\pi}{2}+idx\right)\middle|2\right)\sqrt{b\sinh(c+dx)}}{5b^4d\sqrt{i\sinh(c+dx)}}$$

output
```
-2/5*cosh(d*x+c)/b/d/(b*sinh(d*x+c))^(5/2)+6/5*cosh(d*x+c)/b^3/d/(b*sinh(d
*x+c))^(1/2)-6/5*I*EllipticE(cos(1/2*I*c+1/4*Pi+1/2*I*d*x),2^(1/2))*(b*sin
h(d*x+c))^(1/2)/b^4/d/(I*sinh(d*x+c))^(1/2)
```

Mathematica [A] (verified)

Time = 0.12 (sec) , antiderivative size = 79, normalized size of antiderivative = 0.67

$$\int \frac{1}{(b\sinh(c+dx))^{7/2}}\,dx = -\frac{2\left(-3\cosh(c+dx)+\coth(c+dx)\operatorname{csch}(c+dx)+3E\left(\frac{1}{4}(-2ic+\pi-2idx)\middle|2\right)\sqrt{i\sinh(c+dx)}\right)}{5b^3d\sqrt{b\sinh(c+dx)}}$$

input
```
Integrate[(b*Sinh[c + d*x])^(-7/2),x]
```

output
```
(-2*(-3*Cosh[c + d*x] + Coth[c + d*x]*Csch[c + d*x] + 3*EllipticE[((-2*I)*
c + Pi - (2*I)*d*x)/4, 2]*Sqrt[I*Sinh[c + d*x]]))/(5*b^3*d*Sqrt[b*Sinh[c +
 d*x]])
```

Rubi [A] (verified)

Time = 0.48 (sec) , antiderivative size = 122, normalized size of antiderivative = 1.03, number of steps used = 8, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.667, Rules used = {3042, 3116, 3042, 3116, 3042, 3121, 3042, 3119}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(b\sinh(c+dx))^{7/2}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{(-ib\sin(ic+idx))^{7/2}}dx$$

$$\Big\downarrow \text{3116}$$

$$-\frac{3\int \frac{1}{(b\sinh(c+dx))^{3/2}}dx}{5b^2}-\frac{2\cosh(c+dx)}{5bd(b\sinh(c+dx))^{5/2}}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{2\cosh(c+dx)}{5bd(b\sinh(c+dx))^{5/2}}-\frac{3\int \frac{1}{(-ib\sin(ic+idx))^{3/2}}dx}{5b^2}$$

$$\Big\downarrow \text{3116}$$

$$-\frac{3\left(\frac{\int \sqrt{b\sinh(c+dx)}dx}{b^2}-\frac{2\cosh(c+dx)}{bd\sqrt{b\sinh(c+dx)}}\right)}{5b^2}-\frac{2\cosh(c+dx)}{5bd(b\sinh(c+dx))^{5/2}}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{2\cosh(c+dx)}{5bd(b\sinh(c+dx))^{5/2}}-\frac{3\left(-\frac{2\cosh(c+dx)}{bd\sqrt{b\sinh(c+dx)}}+\frac{\int \sqrt{-ib\sin(ic+idx)}dx}{b^2}\right)}{5b^2}$$

$$\Big\downarrow \text{3121}$$

$$-\frac{2\cosh(c+dx)}{5bd(b\sinh(c+dx))^{5/2}}-\frac{3\left(-\frac{2\cosh(c+dx)}{bd\sqrt{b\sinh(c+dx)}}+\frac{\sqrt{b\sinh(c+dx)}\int\sqrt{i\sinh(c+dx)}dx}{b^2\sqrt{i\sinh(c+dx)}}\right)}{5b^2}$$

$$\Big\downarrow \text{ 3042}$$

$$-\frac{2\cosh(c+dx)}{5bd(b\sinh(c+dx))^{5/2}}-\frac{3\left(-\frac{2\cosh(c+dx)}{bd\sqrt{b\sinh(c+dx)}}+\frac{\sqrt{b\sinh(c+dx)}\int\sqrt{\sin(ic+idx)}dx}{b^2\sqrt{i\sinh(c+dx)}}\right)}{5b^2}$$

$$\Big\downarrow \text{ 3119}$$

$$-\frac{2\cosh(c+dx)}{5bd(b\sinh(c+dx))^{5/2}}-\frac{3\left(-\frac{2\cosh(c+dx)}{bd\sqrt{b\sinh(c+dx)}}-\frac{2iE\left(\frac{1}{2}\left(ic+idx-\frac{\pi}{2}\right)|2\right)\sqrt{b\sinh(c+dx)}}{b^2d\sqrt{i\sinh(c+dx)}}\right)}{5b^2}$$

input
```
Int[(b*Sinh[c + d*x])^(-7/2),x]
```

output
```
(-2*Cosh[c + d*x])/(5*b*d*(b*Sinh[c + d*x])^(5/2)) - (3*((-2*Cosh[c + d*x]
)/(b*d*Sqrt[b*Sinh[c + d*x]]) - ((2*I)*EllipticE[(I*c - Pi/2 + I*d*x)/2, 2
]*Sqrt[b*Sinh[c + d*x]])/(b^2*d*Sqrt[I*Sinh[c + d*x]])))/(5*b^2)
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3116
```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1))), x] + Simp[(n + 2)/(b^2*(n + 1))   I
nt[(b*Sin[c + d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] &&
 IntegerQ[2*n]
```

rule 3119
```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3121
```
Int[((b_)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*Sin[c + d*x])
^n/Sin[c + d*x]^n   Int[Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] && Lt
Q[-1, n, 1] && IntegerQ[2*n]
```

Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 204 vs. $2(97) = 194$.

Time = 0.24 (sec) , antiderivative size = 205, normalized size of antiderivative = 1.74

method	result
default	$-\frac{6\sqrt{-i(\sinh(dx+c)+i)}\,\sqrt{2}\,\sqrt{-i(i-\sinh(dx+c))}\,\sqrt{i\sinh(dx+c)}\,\sinh(dx+c)^2\,\mathrm{EllipticE}\left(\sqrt{-i(\sinh(dx+c)+i)},\frac{\sqrt{2}}{2}\right)-3\sqrt{-i(\sinh(}}{5b^3\sinh(dx+c)^2\,\mathrm{co}}$

input
```
int(1/(b*sinh(d*x+c))^(7/2),x,method=_RETURNVERBOSE)
```

output
```
-1/5/b^3/sinh(d*x+c)^2*(6*(-I*(sinh(d*x+c)+I))^(1/2)*2^(1/2)*(-I*(I-sinh(d
*x+c)))^(1/2)*(I*sinh(d*x+c))^(1/2)*sinh(d*x+c)^2*EllipticE((-I*(sinh(d*x+
c)+I))^(1/2),1/2*2^(1/2))-3*(-I*(sinh(d*x+c)+I))^(1/2)*2^(1/2)*(-I*(I-sinh
(d*x+c)))^(1/2)*(I*sinh(d*x+c))^(1/2)*sinh(d*x+c)^2*EllipticF((-I*(sinh(d*
x+c)+I))^(1/2),1/2*2^(1/2))-6*sinh(d*x+c)^4-4*sinh(d*x+c)^2+2)/cosh(d*x+c)
/(b*sinh(d*x+c))^(1/2)/d
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 624 vs. $2(93) = 186$.

Time = 0.09 (sec) , antiderivative size = 624, normalized size of antiderivative = 5.29

$$\int \frac{1}{(b\sinh(c+dx))^{7/2}}\,dx = \text{Too large to display}$$

input
```
integrate(1/(b*sinh(d*x+c))^(7/2),x, algorithm="fricas")
```

output
```
4/5*(3*sqrt(1/2)*(cosh(d*x + c)^6 + 6*cosh(d*x + c)*sinh(d*x + c)^5 + sinh
(d*x + c)^6 + 3*(5*cosh(d*x + c)^2 - 1)*sinh(d*x + c)^4 - 3*cosh(d*x + c)^
4 + 4*(5*cosh(d*x + c)^3 - 3*cosh(d*x + c))*sinh(d*x + c)^3 + 3*(5*cosh(d*
x + c)^4 - 6*cosh(d*x + c)^2 + 1)*sinh(d*x + c)^2 + 3*cosh(d*x + c)^2 + 6*
(cosh(d*x + c)^5 - 2*cosh(d*x + c)^3 + cosh(d*x + c))*sinh(d*x + c) - 1)*s
qrt(b)*weierstrassZeta(4, 0, weierstrassPInverse(4, 0, cosh(d*x + c) + sin
h(d*x + c))) + (3*cosh(d*x + c)^6 + 18*cosh(d*x + c)*sinh(d*x + c)^5 + 3*s
inh(d*x + c)^6 + (45*cosh(d*x + c)^2 - 8)*sinh(d*x + c)^4 - 8*cosh(d*x + c
)^4 + 4*(15*cosh(d*x + c)^3 - 8*cosh(d*x + c))*sinh(d*x + c)^3 + (45*cosh(
d*x + c)^4 - 48*cosh(d*x + c)^2 + 1)*sinh(d*x + c)^2 + cosh(d*x + c)^2 + 2
*(9*cosh(d*x + c)^5 - 16*cosh(d*x + c)^3 + cosh(d*x + c))*sinh(d*x + c))*s
qrt(b*sinh(d*x + c)))/(b^4*d*cosh(d*x + c)^6 + 6*b^4*d*cosh(d*x + c)*sinh(
d*x + c)^5 + b^4*d*sinh(d*x + c)^6 - 3*b^4*d*cosh(d*x + c)^4 + 3*b^4*d*cos
h(d*x + c)^2 - b^4*d + 3*(5*b^4*d*cosh(d*x + c)^2 - b^4*d)*sinh(d*x + c)^4
 + 4*(5*b^4*d*cosh(d*x + c)^3 - 3*b^4*d*cosh(d*x + c))*sinh(d*x + c)^3 + 3
*(5*b^4*d*cosh(d*x + c)^4 - 6*b^4*d*cosh(d*x + c)^2 + b^4*d)*sinh(d*x + c)
^2 + 6*(b^4*d*cosh(d*x + c)^5 - 2*b^4*d*cosh(d*x + c)^3 + b^4*d*cosh(d*x +
 c))*sinh(d*x + c))
```

Sympy [F(-1)]

Timed out.

$$\int \frac{1}{(b\sinh(c+dx))^{7/2}}\,dx = \text{Timed out}$$

input
```
integrate(1/(b*sinh(d*x+c))**(7/2),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{1}{(b\sinh(c+dx))^{7/2}}\,dx = \int \frac{1}{(b\sinh(dx+c))^{\frac{7}{2}}}\,dx$$

input
```
integrate(1/(b*sinh(d*x+c))^(7/2),x, algorithm="maxima")
```

output
```
integrate((b*sinh(d*x + c))^(-7/2), x)
```

Giac [F]

$$\int \frac{1}{(b\sinh(c+dx))^{7/2}}\,dx = \int \frac{1}{(b\sinh(dx+c))^{\frac{7}{2}}}\,dx$$

input
```
integrate(1/(b*sinh(d*x+c))^(7/2),x, algorithm="giac")
```

output
```
integrate((b*sinh(d*x + c))^(-7/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(b\sinh(c+dx))^{7/2}}\,dx = \int \frac{1}{(b\sinh(c+d\,x))^{7/2}}\,dx$$

input
```
int(1/(b*sinh(c + d*x))^(7/2),x)
```

output
```
int(1/(b*sinh(c + d*x))^(7/2), x)
```

Reduce [F]

$$\int \frac{1}{(b\sinh(c+dx))^{7/2}}\,dx = \frac{\sqrt{b}\left(\int \frac{\sqrt{\sinh(dx+c)}}{\sinh(dx+c)^4}dx\right)}{b^4}$$

input
```
int(1/(b*sinh(d*x+c))^(7/2),x)
```

output
```
(sqrt(b)*int(sqrt(sinh(c + d*x))/sinh(c + d*x)**4,x))/b**4
```

3.23 $\int (i \sinh(c+dx))^{7/2} \, dx$

Optimal result

Integrand size = 14, antiderivative size = 91

$$\int (i \sinh(c+dx))^{7/2} \, dx = -\frac{10i \operatorname{EllipticF}\left(\frac{1}{2}\left(ic-\frac{\pi}{2}+idx\right),2\right)}{21d} + \frac{10i \cosh(c+dx)\sqrt{i \sinh(c+dx)}}{21d} + \frac{2i \cosh(c+dx)(i \sinh(c+dx))^{5/2}}{7d}$$

output
```
-10/21*I*InverseJacobiAM(1/2*I*c-1/4*Pi+1/2*I*d*x,2^(1/2))/d+10/21*I*cosh(
d*x+c)*(I*sinh(d*x+c))^(1/2)/d+2/7*I*cosh(d*x+c)*(I*sinh(d*x+c))^(5/2)/d
```

Mathematica [A] (verified)

Time = 0.12 (sec) , antiderivative size = 65, normalized size of antiderivative = 0.71

$$\int (i \sinh(c+dx))^{7/2} \, dx = \frac{i\Big(20 \operatorname{EllipticF}\left(\frac{1}{4}(-2ic+\pi-2idx),2\right)+(23\cosh(c+dx)-3\cosh(3(c+dx)))\sqrt{i\sinh(c+}}{42d}$$

input
```
Integrate[(I*Sinh[c + d*x])^(7/2),x]
```

output
```
((I/42)*(20*EllipticF[((-2*I)*c + Pi - (2*I)*d*x)/4, 2] + (23*Cosh[c + d*x
] - 3*Cosh[3*(c + d*x)])*Sqrt[I*Sinh[c + d*x]]))/d
```

Rubi [A] (verified)

Time = 0.37 (sec) , antiderivative size = 96, normalized size of antiderivative = 1.05, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.429$, Rules used = {3042, 3115, 3042, 3115, 3042, 3120}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (i\sinh(c+dx))^{7/2}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \sin(ic+idx)^{7/2}dx$$

$$\Big\downarrow \text{3115}$$

$$\frac{5}{7}\int (i\sinh(c+dx))^{3/2}dx+\frac{2i(i\sinh(c+dx))^{5/2}\cosh(c+dx)}{7d}$$

$$\Big\downarrow \text{3042}$$

$$\frac{5}{7}\int \sin(ic+idx)^{3/2}dx+\frac{2i(i\sinh(c+dx))^{5/2}\cosh(c+dx)}{7d}$$

$$\Big\downarrow \text{3115}$$

$$\frac{5}{7}\left(\frac{1}{3}\int \frac{1}{\sqrt{i\sinh(c+dx)}}dx+\frac{2i\sqrt{i\sinh(c+dx)}\cosh(c+dx)}{3d}\right)+\frac{2i(i\sinh(c+dx))^{5/2}\cosh(c+dx)}{7d}$$

$$\Big\downarrow \text{3042}$$

$$\frac{5}{7}\left(\frac{1}{3}\int \frac{1}{\sqrt{\sin(ic+idx)}}dx+\frac{2i\sqrt{i\sinh(c+dx)}\cosh(c+dx)}{3d}\right)+\frac{2i(i\sinh(c+dx))^{5/2}\cosh(c+dx)}{7d}$$

$$\Big\downarrow \text{3120}$$

$$\frac{2i(i\sinh(c+dx))^{5/2}\cosh(c+dx)}{7d}+\frac{5}{7}\left(\frac{2i\sqrt{i\sinh(c+dx)}\cosh(c+dx)}{3d}-\frac{2i\,\text{EllipticF}\left(\frac{1}{2}\left(ic+idx-\frac{\pi}{2}\right),2\right)}{3d}\right)$$

input
```
Int[(I*Sinh[c + d*x])^(7/2),x]
```

output
```
(5*((((-2*I)/3)*EllipticF[(I*c - Pi/2 + I*d*x)/2, 2])/d + (((2*I)/3)*Cosh[
c + d*x]*Sqrt[I*Sinh[c + d*x]])/d))/7 + (((2*I)/7)*Cosh[c + d*x]*(I*Sinh[c
 + d*x])^(5/2))/d
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3115
```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Simp[b^2*((n - 1)/n)   Int[(b*Sin
[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[
2*n]
```

rule 3120
```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Maple [A] (verified)

Time = 0.23 (sec) , antiderivative size = 122, normalized size of antiderivative = 1.34

method	result
default	$-\frac{i\left(6i\cosh(dx+c)^4\sinh(dx+c)-5\sqrt{-i\sinh(dx+c)+1}\,\sqrt{2}\,\sqrt{1+i\sinh(dx+c)}\,\sqrt{i\sinh(dx+c)}\,\text{EllipticF}\left(\sqrt{-i\sinh(dx+c)+1},\frac{\sqrt{2}}{2}\right)-\right.}{21\cosh(dx+c)\sqrt{i\sinh(dx+c)}\,d}$

input
```
int((I*sinh(d*x+c))^(7/2),x,method=_RETURNVERBOSE)
```

output
```
-1/21*I*(6*I*cosh(d*x+c)^4*sinh(d*x+c)-5*(-I*sinh(d*x+c)+1)^(1/2)*2^(1/2)*
(1+I*sinh(d*x+c))^(1/2)*(I*sinh(d*x+c))^(1/2)*EllipticF((-I*sinh(d*x+c)+1)
^(1/2),1/2*2^(1/2))-16*I*cosh(d*x+c)^2*sinh(d*x+c))/cosh(d*x+c)/(I*sinh(d*
x+c))^(1/2)/d
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 101, normalized size of antiderivative = 1.11

$$\int (i\sinh(c+dx))^{7/2}\,dx = \frac{\left(\sqrt{\frac{1}{2}}\left(-3i\,e^{(6\,dx+6\,c)}+23i\,e^{(4\,dx+4\,c)}+23i\,e^{(2\,dx+2\,c)}-3i\right)\sqrt{i\,e^{(2\,dx+2\,c)}-i}e^{\left(-\frac{1}{2}\,dx-\frac{1}{2}\,c\right)}-80i\,\sqrt{\right.}{84\,d}$$

input
```
integrate((I*sinh(d*x+c))^(7/2),x, algorithm="fricas")
```

output
```
1/84*(sqrt(1/2)*(-3*I*e^(6*d*x + 6*c) + 23*I*e^(4*d*x + 4*c) + 23*I*e^(2*d
*x + 2*c) - 3*I)*sqrt(I*e^(2*d*x + 2*c) - I)*e^(-1/2*d*x - 1/2*c) - 80*I*s
qrt(1/2*I)*e^(3*d*x + 3*c)*weierstrassPInverse(4, 0, e^(d*x + c)))*e^(-3*d
*x - 3*c)/d
```

Sympy [F(-1)]

Timed out.

$$\int (i\sinh(c+dx))^{7/2}\,dx = \text{Timed out}$$

input
```
integrate((I*sinh(d*x+c))**(7/2),x)
```

output
```
Timed out
```

Maxima [F]

$$\int (i\sinh(c+dx))^{7/2}\,dx = \int (i\,\sinh(dx+c))^{\frac{7}{2}}\,dx$$

input
```
integrate((I*sinh(d*x+c))^(7/2),x, algorithm="maxima")
```

output
```
integrate((I*sinh(d*x + c))^(7/2), x)
```

Giac [F]

$$\int (i\sinh(c+dx))^{7/2}\,dx = \int (i\,\sinh(dx+c))^{\frac{7}{2}}\,dx$$

input
```
integrate((I*sinh(d*x+c))^(7/2),x, algorithm="giac")
```

output
```
integrate((I*sinh(d*x + c))^(7/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int (i\sinh(c+dx))^{7/2}\,dx = \int (\sinh(c+d\,x)\ 1\mathrm{i})^{7/2}\,dx$$

input
```
int((sinh(c + d*x)*1i)^(7/2),x)
```

output
```
int((sinh(c + d*x)*1i)^(7/2), x)
```

Reduce [F]

$$\int (i\sinh(c+dx))^{7/2}\,dx = -\sqrt{i}\left(\int \sqrt{\sinh(dx+c)}\,\sinh(dx+c)^3\,dx\right)i$$

input
```
int((I*sinh(d*x+c))^(7/2),x)
```

output
```
- sqrt(i)*int(sqrt(sinh(c + d*x))*sinh(c + d*x)**3,x)*i
```

3.24 $\int (i \sinh(c+dx))^{5/2}\,dx$

Optimal result

Integrand size = 14, antiderivative size = 62

$$\int (i\sinh(c+dx))^{5/2}\,dx=-\frac{6iE\left(\frac{1}{2}\left(ic-\frac{\pi}{2}+idx\right)\middle|2\right)}{5d}+\frac{2i\cosh(c+dx)(i\sinh(c+dx))^{3/2}}{5d}$$

output
```
6/5*I*EllipticE(cos(1/2*I*c+1/4*Pi+1/2*I*d*x),2^(1/2))/d+2/5*I*cosh(d*x+c)
*(I*sinh(d*x+c))^(3/2)/d
```

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 55, normalized size of antiderivative = 0.89

$$\int (i\sinh(c+dx))^{5/2}\,dx=\frac{6iE\left(\frac{1}{4}(-2ic+\pi-2idx)\middle|2\right)-\sqrt{i\sinh(c+dx)}\sinh(2(c+dx))}{5d}$$

input
```
Integrate[(I*Sinh[c + d*x])^(5/2),x]
```

output
```
((6*I)*EllipticE[((-2*I)*c + Pi - (2*I)*d*x)/4, 2] - Sqrt[I*Sinh[c + d*x]]
*Sinh[2*(c + d*x)])/(5*d)
```

Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 62, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {3042, 3115, 3042, 3119}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (i\sinh(c+dx))^{5/2}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \sin(ic+idx)^{5/2}dx$$

$$\Big\downarrow \text{3115}$$

$$\frac{3}{5}\int \sqrt{i\sinh(c+dx)}dx+\frac{2i(i\sinh(c+dx))^{3/2}\cosh(c+dx)}{5d}$$

$$\Big\downarrow \text{3042}$$

$$\frac{3}{5}\int \sqrt{\sin(ic+idx)}dx+\frac{2i(i\sinh(c+dx))^{3/2}\cosh(c+dx)}{5d}$$

$$\Big\downarrow \text{3119}$$

$$\frac{2i(i\sinh(c+dx))^{3/2}\cosh(c+dx)}{5d}-\frac{6iE\left(\frac{1}{2}\left(ic+idx-\frac{\pi}{2}\right)\middle|2\right)}{5d}$$

input
```
Int[(I*Sinh[c + d*x])^(5/2),x]
```

output
```
(((-6*I)/5)*EllipticE[(I*c - Pi/2 + I*d*x)/2, 2])/d + (((2*I)/5)*Cosh[c +
d*x]*(I*Sinh[c + d*x])^(3/2))/d
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3115
```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Simp[b^2*((n - 1)/n)   Int[(b*Sin
[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[
2*n]
```

rule 3119
```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 168 vs. $2(48) = 96$.

Time = 0.22 (sec) , antiderivative size = 169, normalized size of antiderivative = 2.73

method	result
default	$-\frac{i\left(3\sqrt{-i\sinh(dx+c)+1}\,\sqrt{2}\,\sqrt{1+i\sinh(dx+c)}\,\sqrt{i\sinh(dx+c)}\,\text{EllipticF}\left(\sqrt{-i\sinh(dx+c)+1},\frac{\sqrt{2}}{2}\right)-6\sqrt{-i\sinh(dx+c)+1}\,\sqrt{2}\,\sqrt{1}\right.}{5\cosh(dx+c)\sqrt{i\sinh(dx+c)}}$

input
```
int((I*sinh(d*x+c))^(5/2),x,method=_RETURNVERBOSE)
```

output
```
-1/5*I*(3*(-I*sinh(d*x+c)+1)^(1/2)*2^(1/2)*(1+I*sinh(d*x+c))^(1/2)*(I*sinh
(d*x+c))^(1/2)*EllipticF((-I*sinh(d*x+c)+1)^(1/2),1/2*2^(1/2))-6*(-I*sinh(
d*x+c)+1)^(1/2)*2^(1/2)*(1+I*sinh(d*x+c))^(1/2)*(I*sinh(d*x+c))^(1/2)*Elli
pticE((-I*sinh(d*x+c)+1)^(1/2),1/2*2^(1/2))+2*cosh(d*x+c)^4-2*cosh(d*x+c)^
2)/cosh(d*x+c)/(I*sinh(d*x+c))^(1/2)/d
```

Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 91 vs. 2(43) = 86.

Time = 0.10 (sec) , antiderivative size = 91, normalized size of antiderivative = 1.47

$$\int (i\sinh(c+dx))^{5/2}\,dx = -\frac{\left(\sqrt{\frac{1}{2}}\left(e^{(4\,dx+4\,c)}+12\,e^{(2\,dx+2\,c)}-1\right)\sqrt{i\,e^{(2\,dx+2\,c)}-i}e^{\left(-\frac{1}{2}\,dx-\frac{1}{2}\,c\right)}+24\,\sqrt{\frac{1}{2}i}e^{(2\,dx+2\,c)}\text{weierstrassZeta}\left(4,0,\text{we}\right.\right.}{10\,d}$$

input
```
integrate((I*sinh(d*x+c))^(5/2),x, algorithm="fricas")
```

output
```
-1/10*(sqrt(1/2)*(e^(4*d*x + 4*c) + 12*e^(2*d*x + 2*c) - 1)*sqrt(I*e^(2*d*
x + 2*c) - I)*e^(-1/2*d*x - 1/2*c) + 24*sqrt(1/2*I)*e^(2*d*x + 2*c)*weiers
trassZeta(4, 0, weierstrassPInverse(4, 0, e^(d*x + c))))*e^(-2*d*x - 2*c)/
d
```

Sympy [F(-1)]

Timed out.

$$\int (i\sinh(c+dx))^{5/2}\,dx = \text{Timed out}$$

input
```
integrate((I*sinh(d*x+c))**(5/2),x)
```

output
```
Timed out
```

Maxima [F]

$$\int (i\sinh(c+dx))^{5/2}\,dx = \int (i\,\sinh\left(dx+c\right))^{\frac{5}{2}}\,dx$$

input
```
integrate((I*sinh(d*x+c))^(5/2),x, algorithm="maxima")
```

output
```
integrate((I*sinh(d*x + c))^(5/2), x)
```

Giac [F]

$$\int (i\sinh(c+dx))^{5/2}\,dx = \int (i\,\sinh\left(dx+c\right))^{\frac{5}{2}}\,dx$$

input
```
integrate((I*sinh(d*x+c))^(5/2),x, algorithm="giac")
```

output
```
integrate((I*sinh(d*x + c))^(5/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int (i\sinh(c+dx))^{5/2}\,dx = \int (\sinh(c+d\,x)\ 1\mathrm{i})^{5/2}\,dx$$

input
```
int((sinh(c + d*x)*1i)^(5/2),x)
```

output
```
int((sinh(c + d*x)*1i)^(5/2), x)
```

Reduce [F]

$$\int (i\sinh(c+dx))^{5/2}\,dx = -\sqrt{i}\left(\int \sqrt{\sinh(dx+c)}\,\sinh(dx+c)^2\,dx\right)$$

input
```
int((I*sinh(d*x+c))^(5/2),x)
```

output
```
- sqrt(i)*int(sqrt(sinh(c + d*x))*sinh(c + d*x)**2,x)
```

3.25 $\int (i\sinh(c+dx))^{3/2}\,dx$

Optimal result

Integrand size = 14, antiderivative size = 62

$$\int (i\sinh(c+dx))^{3/2}\,dx = -\frac{2i\,\text{EllipticF}\left(\frac{1}{2}\left(ic-\frac{\pi}{2}+idx\right),2\right)}{3d} + \frac{2i\cosh(c+dx)\sqrt{i\sinh(c+dx)}}{3d}$$

output
```
-2/3*I*InverseJacobiAM(1/2*I*c-1/4*Pi+1/2*I*d*x,2^(1/2))/d+2/3*I*cosh(d*x+
c)*(I*sinh(d*x+c))^(1/2)/d
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 0.10 (sec) , antiderivative size = 94, normalized size of antiderivative = 1.52

$$\int (i\sinh(c+dx))^{3/2}\,dx = -\frac{2i\sqrt{i\sinh(c+dx)}\left(-\cosh(c+dx)+\operatorname{csch}(c+dx)\,\text{Hypergeometric2F1}\left(\frac{1}{4},\frac{1}{2},\frac{5}{4},\cosh(2(c+dx))+\sinh(2\right.\right.}{3d}$$

input
```
Integrate[(I*Sinh[c + d*x])^(3/2),x]
```

output
```
(((-2*I)/3)*Sqrt[I*Sinh[c + d*x]]*(-Cosh[c + d*x] + Csch[c + d*x]*Hypergeo
metric2F1[1/4, 1/2, 5/4, Cosh[2*(c + d*x)] + Sinh[2*(c + d*x)]]*Sqrt[1 - C
osh[2*c + 2*d*x] - Sinh[2*c + 2*d*x]]))/d
```

Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 62, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$, Rules used = {3042, 3115, 3042, 3120}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (i\sinh(c+dx))^{3/2}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \sin(ic+idx)^{3/2}dx$$

$$\Big\downarrow \text{3115}$$

$$\frac{1}{3}\int \frac{1}{\sqrt{i\sinh(c+dx)}}dx+\frac{2i\sqrt{i\sinh(c+dx)}\cosh(c+dx)}{3d}$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{3}\int \frac{1}{\sqrt{\sin(ic+idx)}}dx+\frac{2i\sqrt{i\sinh(c+dx)}\cosh(c+dx)}{3d}$$

$$\Big\downarrow \text{3120}$$

$$\frac{2i\sqrt{i\sinh(c+dx)}\cosh(c+dx)}{3d}-\frac{2i\,\text{EllipticF}\left(\frac{1}{2}\left(ic+idx-\frac{\pi}{2}\right),2\right)}{3d}$$

input
```
Int[(I*Sinh[c + d*x])^(3/2),x]
```

output
```
(((-2*I)/3)*EllipticF[(I*c - Pi/2 + I*d*x)/2, 2])/d + (((2*I)/3)*Cosh[c +
d*x]*Sqrt[I*Sinh[c + d*x]])/d
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3115
```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Simp[b^2*((n - 1)/n)   Int[(b*Sin
[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[
2*n]
```

rule 3120
```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 103 vs. 2(47) = 94.

Time = 0.24 (sec) , antiderivative size = 104, normalized size of antiderivative = 1.68

method	result
default	$\frac{i\left(\sqrt{-i\sinh(dx+c)+1}\,\sqrt{2}\,\sqrt{1+i\sinh(dx+c)}\,\sqrt{i\sinh(dx+c)}\,\operatorname{EllipticF}\left(\sqrt{-i\sinh(dx+c)+1},\frac{\sqrt{2}}{2}\right)+2i\cosh(dx+c)^{2}\sinh(dx+c)\right)}{3\cosh(dx+c)\sqrt{i\sinh(dx+c)}\,d}$

input
```
int((I*sinh(d*x+c))^(3/2),x,method=_RETURNVERBOSE)
```

output
```
1/3*I*((-I*sinh(d*x+c)+1)^(1/2)*2^(1/2)*(1+I*sinh(d*x+c))^(1/2)*(I*sinh(d*
x+c))^(1/2)*EllipticF((-I*sinh(d*x+c)+1)^(1/2),1/2*2^(1/2))+2*I*cosh(d*x+c
)^2*sinh(d*x+c))/cosh(d*x+c)/(I*sinh(d*x+c))^(1/2)/d
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 76, normalized size of antiderivative = 1.23

$$\int (i\sinh(c+dx))^{3/2}\,dx = \frac{\left(\sqrt{\frac{1}{2}}\left(i\,e^{(2\,dx+2\,c)}+i\right)\sqrt{i\,e^{(2\,dx+2\,c)}-i}e^{\left(-\frac{1}{2}\,dx-\frac{1}{2}\,c\right)}-4i\,\sqrt{\frac{1}{2}i}e^{(dx+c)}\mathrm{weierstrassPInverse}\left(4,0,e^{(c}\right.\right.}{3\,d}$$

input
```
integrate((I*sinh(d*x+c))^(3/2),x, algorithm="fricas")
```

output
```
1/3*(sqrt(1/2)*(I*e^(2*d*x + 2*c) + I)*sqrt(I*e^(2*d*x + 2*c) - I)*e^(-1/2
*d*x - 1/2*c) - 4*I*sqrt(1/2*I)*e^(d*x + c)*weierstrassPInverse(4, 0, e^(d
*x + c)))*e^(-d*x - c)/d
```

Sympy [F]

$$\int (i\sinh(c+dx))^{3/2}\,dx = \int (i\sinh(c+dx))^{\frac{3}{2}}\,dx$$

input
```
integrate((I*sinh(d*x+c))**(3/2),x)
```

output
```
Integral((I*sinh(c + d*x))**(3/2), x)
```

Maxima [F]

$$\int (i\sinh(c+dx))^{3/2}\,dx = \int (i\,\sinh(dx+c))^{\frac{3}{2}}\,dx$$

input
```
integrate((I*sinh(d*x+c))^(3/2),x, algorithm="maxima")
```

output
```
integrate((I*sinh(d*x + c))^(3/2), x)
```

Giac [F]

$$\int (i\sinh(c+dx))^{3/2}\,dx = \int (i\,\sinh(dx+c))^{\frac{3}{2}}\,dx$$

input
```
integrate((I*sinh(d*x+c))^(3/2),x, algorithm="giac")
```

output
```
integrate((I*sinh(d*x + c))^(3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int (i\sinh(c+dx))^{3/2}\,dx = \int (\sinh(c+d\,x)\;1\mathrm{i})^{3/2}\,dx$$

input
```
int((sinh(c + d*x)*1i)^(3/2),x)
```

output
```
int((sinh(c + d*x)*1i)^(3/2), x)
```

Reduce [F]

$$\int (i\sinh(c+dx))^{3/2}\,dx = \sqrt{i}\left(\int \sqrt{\sinh(dx+c)}\,\sinh(dx+c)\,dx\right)i$$

input
```
int((I*sinh(d*x+c))^(3/2),x)
```

output
```
sqrt(i)*int(sqrt(sinh(c + d*x))*sinh(c + d*x),x)*i
```

3.26 $\int \sqrt{i \sinh (c+d x)} \, dx$

Optimal result

Integrand size = 14, antiderivative size = 30

$$\int \sqrt{i \sinh (c+d x)} \, dx = -\frac{2 i E\left(\frac{1}{2}\left(i c-\frac{\pi}{2}+i d x\right) \middle| 2\right)}{d}$$

output
```
2*I*EllipticE(cos(1/2*I*c+1/4*Pi+1/2*I*d*x),2^(1/2))/d
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.93

$$\int \sqrt{i \sinh (c+d x)} \, dx = \frac{2 i E\left(\frac{1}{2}\left(\frac{\pi}{2}-i(c+d x)\right) \middle| 2\right)}{d}$$

input
```
Integrate[Sqrt[I*Sinh[c + d*x]],x]
```

output
```
((2*I)*EllipticE[(Pi/2 - I*(c + d*x))/2, 2])/d
```

Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {3042, 3119}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{i \sinh(c+dx)}\,dx$$
$$\Big\downarrow \text{3042}$$
$$\int \sqrt{\sin(ic+idx)}dx$$
$$\Big\downarrow \text{3119}$$
$$-\frac{2iE\left(\frac{1}{2}\left(ic+idx-\frac{\pi}{2}\right)\middle|2\right)}{d}$$

input
```
Int[Sqrt[I*Sinh[c + d*x]],x]
```

output
```
((-2*I)*EllipticE[(I*c - Pi/2 + I*d*x)/2, 2])/d
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3119
```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 90 vs. $2(24) = 48$.

Time = 0.31 (sec) , antiderivative size = 91, normalized size of antiderivative = 3.03

method	result
default	$\frac{i\sqrt{-i(\sinh(dx+c)+i)}\,\sqrt{2}\,\sqrt{-i(i-\sinh(dx+c))}\left(2\,\mathrm{EllipticE}\left(\sqrt{-i\sinh(dx+c)+1},\frac{\sqrt{2}}{2}\right)-\mathrm{EllipticF}\left(\sqrt{-i\sinh(dx+c)+1},\frac{\sqrt{2}}{2}\right)\right)}{\cosh(dx+c)d}$
risch	$\frac{\sqrt{2}\,\sqrt{i\left(\mathrm{e}^{2dx+2c}-1\right)\mathrm{e}^{-dx-c}}}{d}-\frac{\left(-\frac{2i\left(-i+i\mathrm{e}^{2dx+2c}\right)}{\sqrt{\mathrm{e}^{dx+c}\left(-i+i\mathrm{e}^{2dx+2c}\right)}}-\frac{\sqrt{\mathrm{e}^{dx+c}+1}\,\sqrt{-2\,\mathrm{e}^{dx+c}+2}\,\sqrt{-\mathrm{e}^{dx+c}}\,\left(-2\,\mathrm{EllipticE}\left(\sqrt{\mathrm{e}^{dx+c}+1},\frac{\sqrt{2}}{2}\right)+\mathrm{Ellip}\right.}{\sqrt{i\mathrm{e}^{3dx+3c}-i\mathrm{e}^{dx+c}}}}{d\left(\mathrm{e}^{2dx+2c}-1\right)}$

input
```
int((I*sinh(d*x+c))^(1/2),x,method=_RETURNVERBOSE)
```

output
```
I*(-I*(sinh(d*x+c)+I))^(1/2)*2^(1/2)*(-I*(I-sinh(d*x+c)))^(1/2)*(2*Ellipti
cE((-I*sinh(d*x+c)+1)^(1/2),1/2*2^(1/2))-EllipticF((-I*sinh(d*x+c)+1)^(1/2
),1/2*2^(1/2)))/cosh(d*x+c)/d
```

Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 51 vs. $2(21) = 42$.

Time = 0.09 (sec) , antiderivative size = 51, normalized size of antiderivative = 1.70

$$\int\sqrt{i\sinh(c+dx)}\,dx =$$
$$-\frac{2\left(\sqrt{\frac{1}{2}}\sqrt{i\,e^{(2\,dx+2\,c)}-i}e^{\left(-\frac{1}{2}\,dx-\frac{1}{2}\,c\right)}+2\,\sqrt{\frac{1}{2}}i\mathrm{weierstrassZeta}\left(4,0,\mathrm{weierstrassPInverse}\left(4,0,e^{(dx+c)}\right)\right)\right)}{d}$$

input
```
integrate((I*sinh(d*x+c))^(1/2),x, algorithm="fricas")
```

output
```
-2*(sqrt(1/2)*sqrt(I*e^(2*d*x + 2*c) - I)*e^(-1/2*d*x - 1/2*c) + 2*sqrt(1/
2*I)*weierstrassZeta(4, 0, weierstrassPInverse(4, 0, e^(d*x + c))))/d
```

Sympy [F]

$$\int \sqrt{i\sinh(c+dx)}\,dx = \int \sqrt{i\sinh\left(c+dx\right)}\,dx$$

input
```
integrate((I*sinh(d*x+c))**(1/2),x)
```

output
```
Integral(sqrt(I*sinh(c + d*x)), x)
```

Maxima [F]

$$\int \sqrt{i\sinh(c+dx)}\,dx = \int \sqrt{i\,\sinh\left(dx+c\right)}\,dx$$

input
```
integrate((I*sinh(d*x+c))^(1/2),x, algorithm="maxima")
```

output
```
integrate(sqrt(I*sinh(d*x + c)), x)
```

Giac [F]

$$\int \sqrt{i\sinh(c+dx)}\,dx = \int \sqrt{i\,\sinh\left(dx+c\right)}\,dx$$

input
```
integrate((I*sinh(d*x+c))^(1/2),x, algorithm="giac")
```

output
```
integrate(sqrt(I*sinh(d*x + c)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \sqrt{i\sinh(c+dx)}\,dx = \int \sqrt{\sinh\left(c+d\,x\right)\,1\mathrm{i}}\,dx$$

input
```
int((sinh(c + d*x)*1i)^(1/2),x)
```

output
```
int((sinh(c + d*x)*1i)^(1/2), x)
```

Reduce [F]

$$\int \sqrt{i\sinh(c+dx)}\,dx = \sqrt{i}\left(\int \sqrt{\sinh\left(dx+c\right)}dx\right)$$

input
```
int((I*sinh(d*x+c))^(1/2),x)
```

output
```
sqrt(i)*int(sqrt(sinh(c + d*x)),x)
```

3.27 $\int \frac{1}{\sqrt{i \sinh(c+dx)}} \, dx$

Optimal result

Integrand size = 14, antiderivative size = 30

$$\int \frac{1}{\sqrt{i \sinh(c+dx)}} \, dx = -\frac{2i \operatorname{EllipticF}\left(\frac{1}{2}\left(ic - \frac{\pi}{2} + idx\right), 2\right)}{d}$$

output
```
-2*I*InverseJacobiAM(1/2*I*c-1/4*Pi+1/2*I*d*x,2^(1/2))/d
```

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.93

$$\int \frac{1}{\sqrt{i \sinh(c+dx)}} \, dx = \frac{2i \operatorname{EllipticF}\left(\frac{1}{2}\left(\frac{\pi}{2} - i(c+dx)\right), 2\right)}{d}$$

input
```
Integrate[1/Sqrt[I*Sinh[c + d*x]],x]
```

output
```
((2*I)*EllipticF[(Pi/2 - I*(c + d*x))/2, 2])/d
```

Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {3042, 3120}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt{i\sinh(c+dx)}}dx$$
$$\Big\downarrow \text{3042}$$
$$\int \frac{1}{\sqrt{\sin(ic+idx)}}dx$$
$$\Big\downarrow \text{3120}$$
$$-\frac{2i\,\text{EllipticF}\left(\frac{1}{2}\left(ic+idx-\frac{\pi}{2}\right),2\right)}{d}$$

input
```
Int[1/Sqrt[I*Sinh[c + d*x]],x]
```

output
```
((-2*I)*EllipticF[(I*c - Pi/2 + I*d*x)/2, 2])/d
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3120
```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 67 vs. 2(23) = 46.

Time = 0.16 (sec) , antiderivative size = 68, normalized size of antiderivative = 2.27

method	result	size
default	$\frac{i\sqrt{-i(\sinh(dx+c)+i)}\,\sqrt{2}\,\sqrt{-i(i-\sinh(dx+c))}\,\operatorname{EllipticF}\left(\sqrt{-i(\sinh(dx+c)+i)},\frac{\sqrt{2}}{2}\right)}{\cosh(dx+c)d}$	68

input
```
int(1/(I*sinh(d*x+c))^(1/2),x,method=_RETURNVERBOSE)
```

output
```
I*(-I*(sinh(d*x+c)+I))^(1/2)*2^(1/2)*(-I*(I-sinh(d*x+c)))^(1/2)*EllipticF(
(-I*(sinh(d*x+c)+I))^(1/2),1/2*2^(1/2))/cosh(d*x+c)/d
```

Fricas [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.57

$$\int \frac{1}{\sqrt{i\sinh(c+dx)}}\,dx = -\frac{4i\sqrt{\frac{1}{2}i}\text{weierstrassPInverse}\left(4,0,e^{(dx+c)}\right)}{d}$$

input
```
integrate(1/(I*sinh(d*x+c))^(1/2),x, algorithm="fricas")
```

output
```
-4*I*sqrt(1/2*I)*weierstrassPInverse(4, 0, e^(d*x + c))/d
```

Sympy [F]

$$\int \frac{1}{\sqrt{i\sinh(c+dx)}}\,dx = \int \frac{1}{\sqrt{i\sinh{(c+dx)}}}\,dx$$

input
```
integrate(1/(I*sinh(d*x+c))**(1/2),x)
```

output
```
Integral(1/sqrt(I*sinh(c + d*x)), x)
```

Maxima [F]

$$\int \frac{1}{\sqrt{i\sinh(c+dx)}}\,dx = \int \frac{1}{\sqrt{i\,\sinh\left(dx+c\right)}}\,dx$$

input
```
integrate(1/(I*sinh(d*x+c))^(1/2),x, algorithm="maxima")
```

output
```
integrate(1/sqrt(I*sinh(d*x + c)), x)
```

Giac [F]

$$\int \frac{1}{\sqrt{i\sinh(c+dx)}}\,dx = \int \frac{1}{\sqrt{i\,\sinh\left(dx+c\right)}}\,dx$$

input
```
integrate(1/(I*sinh(d*x+c))^(1/2),x, algorithm="giac")
```

output
```
integrate(1/sqrt(I*sinh(d*x + c)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt{i\sinh(c+dx)}}\,dx = \int \frac{1}{\sqrt{\sinh\left(c+d\,x\right)\,1{}\mathrm{i}}}\,dx$$

input
```
int(1/(sinh(c + d*x)*1i)^(1/2),x)
```

output
```
int(1/(sinh(c + d*x)*1i)^(1/2), x)
```

Reduce [F]

$$\int \frac{1}{\sqrt{i\sinh(c+dx)}}\,dx = -\sqrt{i}\left(\int \frac{\sqrt{\sinh(dx+c)}}{\sinh(dx+c)}dx\right)i$$

input
```
int(1/(I*sinh(d*x+c))^(1/2),x)
```

output
```
- sqrt(i)*int(sqrt(sinh(c + d*x))/sinh(c + d*x),x)*i
```

3.28 $\int \frac{1}{(i\sinh(c+dx))^{3/2}}\,dx$

Optimal result

Integrand size = 14, antiderivative size = 58

$$\int \frac{1}{(i\sinh(c+dx))^{3/2}}\,dx=\frac{2iE\left(\frac{1}{2}\left(ic-\frac{\pi}{2}+idx\right)\middle|2\right)}{d}+\frac{2i\cosh(c+dx)}{d\sqrt{i\sinh(c+dx)}}$$

output
```
-2*I*EllipticE(cos(1/2*I*c+1/4*Pi+1/2*I*d*x),2^(1/2))/d+2*I*cosh(d*x+c)/d/
(I*sinh(d*x+c))^(1/2)
```

Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 50, normalized size of antiderivative = 0.86

$$\int \frac{1}{(i\sinh(c+dx))^{3/2}}\,dx=\frac{2\left(-iE\left(\frac{1}{4}(-2ic+\pi-2idx)\middle|2\right)+\coth(c+dx)\sqrt{i\sinh(c+dx)}\right)}{d}$$

input
```
Integrate[(I*Sinh[c + d*x])^(-3/2),x]
```

output
```
(2*((-I)*EllipticE[((-2*I)*c + Pi - (2*I)*d*x)/4, 2] + Coth[c + d*x]*Sqrt[
I*Sinh[c + d*x]]))/d
```

Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 58, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$, Rules used = {3042, 3116, 3042, 3119}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(i\sinh(c+dx))^{3/2}}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{\sin(ic+idx)^{3/2}}dx$$

$$\Big\downarrow \text{3116}$$

$$\frac{2i\cosh(c+dx)}{d\sqrt{i\sinh(c+dx)}}-\int \sqrt{i\sinh(c+dx)}dx$$

$$\Big\downarrow \text{3042}$$

$$\frac{2i\cosh(c+dx)}{d\sqrt{i\sinh(c+dx)}}-\int \sqrt{\sin(ic+idx)}dx$$

$$\Big\downarrow \text{3119}$$

$$\frac{2iE\left(\frac{1}{2}\left(ic+idx-\frac{\pi}{2}\right)\middle|2\right)}{d}+\frac{2i\cosh(c+dx)}{d\sqrt{i\sinh(c+dx)}}$$

input
```
Int[(I*Sinh[c + d*x])^(-3/2),x]
```

output
```
((2*I)*EllipticE[(I*c - Pi/2 + I*d*x)/2, 2])/d + ((2*I)*Cosh[c + d*x])/(d*
Sqrt[I*Sinh[c + d*x]])
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3116
```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1))), x] + Simp[(n + 2)/(b^2*(n + 1))   I
nt[(b*Sin[c + d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] &&
 IntegerQ[2*n]
```

rule 3119
```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 158 vs. $2(48) = 96$.

Time = 0.20 (sec) , antiderivative size = 159, normalized size of antiderivative = 2.74

method	result
default	$-\frac{i\left(2\sqrt{-i\sinh(dx+c)+1}\,\sqrt{2}\,\sqrt{1+i\sinh(dx+c)}\,\sqrt{i\sinh(dx+c)}\,\operatorname{EllipticE}\left(\sqrt{-i\sinh(dx+c)+1},\frac{\sqrt{2}}{2}\right)-\sqrt{-i\sinh(dx+c)+1}\,\sqrt{2}\,\sqrt{1+}\right.}{\cosh(dx+c)\sqrt{i\sinh(dx+c)}\,d}$

input
```
int(1/(I*sinh(d*x+c))^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-I*(2*(-I*sinh(d*x+c)+1)^(1/2)*2^(1/2)*(1+I*sinh(d*x+c))^(1/2)*(I*sinh(d*x
+c))^(1/2)*EllipticE((-I*sinh(d*x+c)+1)^(1/2),1/2*2^(1/2))-(-I*sinh(d*x+c)
+1)^(1/2)*2^(1/2)*(1+I*sinh(d*x+c))^(1/2)*(I*sinh(d*x+c))^(1/2)*EllipticF(
(-I*sinh(d*x+c)+1)^(1/2),1/2*2^(1/2))-2*cosh(d*x+c)^2)/cosh(d*x+c)/(I*sinh
(d*x+c))^(1/2)/d
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 80, normalized size of antiderivative = 1.38

$$\int \frac{1}{(i\sinh(c+dx))^{3/2}}\,dx = \frac{4\left(\sqrt{\frac{1}{2}}\sqrt{i\,e^{(2\,dx+2\,c)}-i}e^{\left(\frac{3}{2}\,dx+\frac{3}{2}\,c\right)}+\left(\sqrt{\frac{1}{2}i}e^{(2\,dx+2\,c)}-\sqrt{\frac{1}{2}i}\right)\mathrm{weierstrassZeta}\left(4,0\right.}{de^{(2\,dx+2\,c)}-d}$$

input
```
integrate(1/(I*sinh(d*x+c))^(3/2),x, algorithm="fricas")
```

output
```
4*(sqrt(1/2)*sqrt(I*e^(2*d*x + 2*c) - I)*e^(3/2*d*x + 3/2*c) + (sqrt(1/2*I
)*e^(2*d*x + 2*c) - sqrt(1/2*I))*weierstrassZeta(4, 0, weierstrassPInverse
(4, 0, e^(d*x + c))))/(d*e^(2*d*x + 2*c) - d)
```

Sympy [F]

$$\int \frac{1}{(i\sinh(c+dx))^{3/2}}\,dx = \int \frac{1}{(i\sinh{(c+dx)})^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/(I*sinh(d*x+c))**(3/2),x)
```

output
```
Integral((I*sinh(c + d*x))**(-3/2), x)
```

Maxima [F]

$$\int \frac{1}{(i\sinh(c+dx))^{3/2}}\,dx = \int \frac{1}{(i\,\sinh{(dx+c)})^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/(I*sinh(d*x+c))^(3/2),x, algorithm="maxima")
```

output
```
integrate((I*sinh(d*x + c))^(-3/2), x)
```

Giac [F]

$$\int \frac{1}{(i\sinh(c+dx))^{3/2}}\,dx = \int \frac{1}{(i\,\sinh{(dx+c)})^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/(I*sinh(d*x+c))^(3/2),x, algorithm="giac")
```

output
```
integrate((I*sinh(d*x + c))^(-3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(i\sinh(c+dx))^{3/2}}\,dx = \int \frac{1}{(\sinh{(c+d\,x)}\ 1\mathrm{i})^{3/2}}\,dx$$

input
```
int(1/(sinh(c + d*x)*1i)^(3/2),x)
```

output
```
int(1/(sinh(c + d*x)*1i)^(3/2), x)
```

Reduce [F]

$$\int \frac{1}{(i\sinh(c+dx))^{3/2}}\,dx = -\sqrt{i}\left(\int \frac{\sqrt{\sinh{(dx+c)}}}{\sinh{(dx+c)}^{2}}dx\right)$$

input
```
int(1/(I*sinh(d*x+c))^(3/2),x)
```

output
```
- sqrt(i)*int(sqrt(sinh(c + d*x))/sinh(c + d*x)**2,x)
```

3.29 $\int \frac{1}{(i\sinh(c+dx))^{5/2}}\,dx$

Optimal result

Integrand size = 14, antiderivative size = 62

$$\int \frac{1}{(i\sinh(c+dx))^{5/2}}\,dx = -\frac{2i\,\text{EllipticF}\left(\frac{1}{2}\left(ic-\frac{\pi}{2}+idx\right),2\right)}{3d}+\frac{2i\cosh(c+dx)}{3d(i\sinh(c+dx))^{3/2}}$$

output
```
-2/3*I*InverseJacobiAM(1/2*I*c-1/4*Pi+1/2*I*d*x,2^(1/2))/d+2/3*I*cosh(d*x+
c)/d/(I*sinh(d*x+c))^(3/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 0.07 (sec) , antiderivative size = 83, normalized size of antiderivative = 1.34

$$\int \frac{1}{(i\sinh(c+dx))^{5/2}}\,dx = \frac{2\left(\coth(c+dx)+\sqrt{2}\,\text{Hypergeometric2F1}\left(\frac{1}{4},\frac{1}{2},\frac{5}{4},\cosh(2(c+dx))+\sinh(2(c\right.\right.}{3d\sqrt{i\sinh(c+dx)}}$$

input
```
Integrate[(I*Sinh[c + d*x])^(-5/2),x]
```

output
```
(2*(Coth[c + d*x] + Sqrt[2]*Hypergeometric2F1[1/4, 1/2, 5/4, Cosh[2*(c + d
*x)] + Sinh[2*(c + d*x)]]*Sqrt[-((1 + Coth[c + d*x])*Sinh[c + d*x]^2)]))/(
3*d*Sqrt[I*Sinh[c + d*x]])
```

Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 62, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$, Rules used = {3042, 3116, 3042, 3120}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(i\sinh(c+dx))^{5/2}}\,dx$$

$$\Big\downarrow \; 3042$$

$$\int \frac{1}{\sin(ic+idx)^{5/2}}dx$$

$$\Big\downarrow \; 3116$$

$$\frac{1}{3}\int \frac{1}{\sqrt{i\sinh(c+dx)}}dx+\frac{2i\cosh(c+dx)}{3d(i\sinh(c+dx))^{3/2}}$$

$$\Big\downarrow \; 3042$$

$$\frac{1}{3}\int \frac{1}{\sqrt{\sin(ic+idx)}}dx+\frac{2i\cosh(c+dx)}{3d(i\sinh(c+dx))^{3/2}}$$

$$\Big\downarrow \; 3120$$

$$\frac{2i\cosh(c+dx)}{3d(i\sinh(c+dx))^{3/2}}-\frac{2i\,\text{EllipticF}\left(\frac{1}{2}\left(ic+idx-\frac{\pi}{2}\right),2\right)}{3d}$$

input
```
Int[(I*Sinh[c + d*x])^(-5/2),x]
```

output
```
(((-2*I)/3)*EllipticF[(I*c - Pi/2 + I*d*x)/2, 2])/d + (((2*I)/3)*Cosh[c +
d*x])/(d*(I*Sinh[c + d*x])^(3/2))
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3116
```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1))), x] + Simp[(n + 2)/(b^2*(n + 1))   I
nt[(b*Sin[c + d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] &&
 IntegerQ[2*n]
```

rule 3120
```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 112 vs. $2(47) = 94$.

Time = 0.20 (sec) , antiderivative size = 113, normalized size of antiderivative = 1.82

method	result
default	$-\frac{i\left(-\sqrt{-i\sinh(dx+c)+1}\,\sqrt{2}\,\sqrt{1+i\sinh(dx+c)}\,\sqrt{i\sinh(dx+c)}\;\mathrm{EllipticF}\left(\sqrt{-i\sinh(dx+c)+1},\frac{\sqrt{2}}{2}\right)\sinh(dx+c)+2i\cosh(dx+c)^{2}\right)}{3\sinh(dx+c)\cosh(dx+c)\sqrt{i\sinh(dx+c)}\,d}$

input
```
int(1/(I*sinh(d*x+c))^(5/2),x,method=_RETURNVERBOSE)
```

output
```
-1/3*I/sinh(d*x+c)*(-(-I*sinh(d*x+c)+1)^(1/2)*2^(1/2)*(1+I*sinh(d*x+c))^(1
/2)*(I*sinh(d*x+c))^(1/2)*EllipticF((-I*sinh(d*x+c)+1)^(1/2),1/2*2^(1/2))*
sinh(d*x+c)+2*I*cosh(d*x+c)^2)/cosh(d*x+c)/(I*sinh(d*x+c))^(1/2)/d
```

Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 122 vs. $2(42) = 84$.

Time = 0.10 (sec) , antiderivative size = 122, normalized size of antiderivative = 1.97

$$\int \frac{1}{(i\sinh(c+dx))^{5/2}}\,dx =$$
$$-\frac{4\left(\sqrt{\frac{1}{2}}\left(i\,e^{(3\,dx+3\,c)}+i\,e^{(dx+c)}\right)\sqrt{i\,e^{(2\,dx+2\,c)}-i}e^{\left(-\frac{1}{2}\,dx-\frac{1}{2}\,c\right)}+\left(i\,\sqrt{\frac{1}{2}}ie^{(4\,dx+4\,c)}-2i\,\sqrt{\frac{1}{2}}ie^{(2\,dx+2\,c)}+i\,\sqrt{\frac{1}{2}i}\right)\right.}{3\left(de^{(4\,dx+4\,c)}-2\,de^{(2\,dx+2\,c)}+d\right)}$$

input
```
integrate(1/(I*sinh(d*x+c))^(5/2),x, algorithm="fricas")
```

output
```
-4/3*(sqrt(1/2)*(I*e^(3*d*x + 3*c) + I*e^(d*x + c))*sqrt(I*e^(2*d*x + 2*c)
 - I)*e^(-1/2*d*x - 1/2*c) + (I*sqrt(1/2*I)*e^(4*d*x + 4*c) - 2*I*sqrt(1/2
*I)*e^(2*d*x + 2*c) + I*sqrt(1/2*I))*weierstrassPInverse(4, 0, e^(d*x + c)
))/(d*e^(4*d*x + 4*c) - 2*d*e^(2*d*x + 2*c) + d)
```

Sympy [F]

$$\int \frac{1}{(i\sinh(c+dx))^{5/2}}\,dx = \int \frac{1}{(i\sinh{(c+dx)})^{\frac{5}{2}}}\,dx$$

input
```
integrate(1/(I*sinh(d*x+c))**(5/2),x)
```

output
```
Integral((I*sinh(c + d*x))**(-5/2), x)
```

Maxima [F]

$$\int \frac{1}{(i\sinh(c+dx))^{5/2}}\,dx = \int \frac{1}{(i\,\sinh(dx+c))^{\frac{5}{2}}}\,dx$$

input
```
integrate(1/(I*sinh(d*x+c))^(5/2),x, algorithm="maxima")
```

output
```
integrate((I*sinh(d*x + c))^(-5/2), x)
```

Giac [F]

$$\int \frac{1}{(i\sinh(c+dx))^{5/2}}\,dx = \int \frac{1}{(i\,\sinh(dx+c))^{\frac{5}{2}}}\,dx$$

input
```
integrate(1/(I*sinh(d*x+c))^(5/2),x, algorithm="giac")
```

output
```
integrate((I*sinh(d*x + c))^(-5/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(i\sinh(c+dx))^{5/2}}\,dx = \int \frac{1}{(\sinh(c+d\,x)\ 1\mathrm{i})^{5/2}}\,dx$$

input
```
int(1/(sinh(c + d*x)*1i)^(5/2),x)
```

output
```
int(1/(sinh(c + d*x)*1i)^(5/2), x)
```

Reduce [F]

$$\int \frac{1}{(i\sinh(c+dx))^{5/2}}\,dx = \sqrt{i}\left(\int \frac{\sqrt{\sinh\left(dx+c\right)}}{\sinh\left(dx+c\right)^{3}}dx\right)i$$

input
```
int(1/(I*sinh(d*x+c))^(5/2),x)
```

output
```
sqrt(i)*int(sqrt(sinh(c + d*x))/sinh(c + d*x)**3,x)*i
```

3.30 $\int \frac{1}{(i\sinh(c+dx))^{7/2}}\,dx$

Optimal result

Integrand size = 14, antiderivative size = 91

$$\int \frac{1}{(i\sinh(c+dx))^{7/2}}\,dx = \frac{6iE\left(\frac{1}{2}\left(ic-\frac{\pi}{2}+idx\right)\middle|2\right)}{5d} + \frac{2i\cosh(c+dx)}{5d(i\sinh(c+dx))^{5/2}} + \frac{6i\cosh(c+dx)}{5d\sqrt{i\sinh(c+dx)}}$$

output
```
-6/5*I*EllipticE(cos(1/2*I*c+1/4*Pi+1/2*I*d*x),2^(1/2))/d+2/5*I*cosh(d*x+c
)/d/(I*sinh(d*x+c))^(5/2)+6/5*I*cosh(d*x+c)/d/(I*sinh(d*x+c))^(1/2)
```

Mathematica [A] (verified)

Time = 0.09 (sec) , antiderivative size = 80, normalized size of antiderivative = 0.88

$$\int \frac{1}{(i\sinh(c+dx))^{7/2}}\,dx = -\frac{2i\left(-3\cosh(c+dx)+\coth(c+dx)\operatorname{csch}(c+dx)+3E\left(\frac{1}{4}(-2ic+\pi-2idx)\middle|2\right)\sqrt{i\sinh(c+dx)}\right)}{5d\sqrt{i\sinh(c+dx)}}$$

input
```
Integrate[(I*Sinh[c + d*x])^(-7/2),x]
```

output
```
(((-2*I)/5)*(-3*Cosh[c + d*x] + Coth[c + d*x]*Csch[c + d*x] + 3*EllipticE[
((-2*I)*c + Pi - (2*I)*d*x)/4, 2]*Sqrt[I*Sinh[c + d*x]]))/(d*Sqrt[I*Sinh[c
 + d*x]])
```

Rubi [A] (verified)

Time = 0.36 (sec) , antiderivative size = 92, normalized size of antiderivative = 1.01, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.429$, Rules used = {3042, 3116, 3042, 3116, 3042, 3119}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(i\sinh(c+dx))^{7/2}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{\sin(ic+idx)^{7/2}}dx$$

$$\Big\downarrow \text{3116}$$

$$\frac{3}{5}\int \frac{1}{(i\sinh(c+dx))^{3/2}}dx+\frac{2i\cosh(c+dx)}{5d(i\sinh(c+dx))^{5/2}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{3}{5}\int \frac{1}{\sin(ic+idx)^{3/2}}dx+\frac{2i\cosh(c+dx)}{5d(i\sinh(c+dx))^{5/2}}$$

$$\Big\downarrow \text{3116}$$

$$\frac{3}{5}\left(\frac{2i\cosh(c+dx)}{d\sqrt{i\sinh(c+dx)}}-\int \sqrt{i\sinh(c+dx)}dx\right)+\frac{2i\cosh(c+dx)}{5d(i\sinh(c+dx))^{5/2}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{3}{5}\left(\frac{2i\cosh(c+dx)}{d\sqrt{i\sinh(c+dx)}}-\int \sqrt{\sin(ic+idx)}dx\right)+\frac{2i\cosh(c+dx)}{5d(i\sinh(c+dx))^{5/2}}$$

$$\Big\downarrow \text{3119}$$

$$\frac{2i\cosh(c+dx)}{5d(i\sinh(c+dx))^{5/2}}+\frac{3}{5}\left(\frac{2iE\left(\frac{1}{2}\left(ic+idx-\frac{\pi}{2}\right)\middle|2\right)}{d}+\frac{2i\cosh(c+dx)}{d\sqrt{i\sinh(c+dx)}}\right)$$

input
```
Int[(I*Sinh[c + d*x])^(-7/2),x]
```

output
```
(3*(((2*I)*EllipticE[(I*c - Pi/2 + I*d*x)/2, 2])/d + ((2*I)*Cosh[c + d*x])
/(d*Sqrt[I*Sinh[c + d*x]])))/5 + (((2*I)/5)*Cosh[c + d*x])/(d*(I*Sinh[c +
d*x])^(5/2))
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3116
```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1))), x] + Simp[(n + 2)/(b^2*(n + 1))   I
nt[(b*Sin[c + d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] &&
 IntegerQ[2*n]
```

rule 3119
```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 203 vs. $2(71) = 142$.

Time = 0.20 (sec) , antiderivative size = 204, normalized size of antiderivative = 2.24

method	result
default	$-\frac{i\left(6\sqrt{-i(\sinh(dx+c)+i)}\,\sqrt{2}\,\sqrt{-i(i-\sinh(dx+c))}\,\sqrt{i\sinh(dx+c)}\,\sinh(dx+c)^{2}\,\mathrm{EllipticE}\left(\sqrt{-i(\sinh(dx+c)+i)},\frac{\sqrt{2}}{2}\right)-3\sqrt{-i(\sinh}\right.}{5\sinh(dx+c)^{2}\,\mathrm{c}}$

input
```
int(1/(I*sinh(d*x+c))^(7/2),x,method=_RETURNVERBOSE)
```

output
```
-1/5*I/sinh(d*x+c)^2*(6*(-I*(sinh(d*x+c)+I))^(1/2)*2^(1/2)*(-I*(I-sinh(d*x
+c)))^(1/2)*(I*sinh(d*x+c))^(1/2)*sinh(d*x+c)^2*EllipticE((-I*(sinh(d*x+c)
+I))^(1/2),1/2*2^(1/2))-3*(-I*(sinh(d*x+c)+I))^(1/2)*2^(1/2)*(-I*(I-sinh(d
*x+c)))^(1/2)*(I*sinh(d*x+c))^(1/2)*sinh(d*x+c)^2*EllipticF((-I*(sinh(d*x+
c)+I))^(1/2),1/2*2^(1/2))-6*sinh(d*x+c)^4-4*sinh(d*x+c)^2+2)/cosh(d*x+c)/(
I*sinh(d*x+c))^(1/2)/d
```

Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 165 vs. 2(64) = 128.

Time = 0.09 (sec) , antiderivative size = 165, normalized size of antiderivative = 1.81

$$\int \frac{1}{(i\sinh(c+dx))^{7/2}}\,dx = \frac{4\left(\sqrt{\frac{1}{2}}\left(3\,e^{(6\,dx+6\,c)} - 8\,e^{(4\,dx+4\,c)} + e^{(2\,dx+2\,c)}\right)\sqrt{i\,e^{(2\,dx+2\,c)} - i}e^{\left(-\frac{1}{2}\,dx-\frac{1}{2}\,c\right)} + 3\left(\sqrt{\right.\right.}{5\,(de^{(6}}$$

input
```
integrate(1/(I*sinh(d*x+c))^(7/2),x, algorithm="fricas")
```

output
```
4/5*(sqrt(1/2)*(3*e^(6*d*x + 6*c) - 8*e^(4*d*x + 4*c) + e^(2*d*x + 2*c))*s
qrt(I*e^(2*d*x + 2*c) - I)*e^(-1/2*d*x - 1/2*c) + 3*(sqrt(1/2*I)*e^(6*d*x
+ 6*c) - 3*sqrt(1/2*I)*e^(4*d*x + 4*c) + 3*sqrt(1/2*I)*e^(2*d*x + 2*c) - s
qrt(1/2*I))*weierstrassZeta(4, 0, weierstrassPInverse(4, 0, e^(d*x + c))))
/(d*e^(6*d*x + 6*c) - 3*d*e^(4*d*x + 4*c) + 3*d*e^(2*d*x + 2*c) - d)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{1}{(i\sinh(c+dx))^{7/2}}\,dx = \text{Timed out}$$

input
```
integrate(1/(I*sinh(d*x+c))**(7/2),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{1}{(i\sinh(c+dx))^{7/2}}\,dx = \int \frac{1}{\left(i\,\sinh\left(dx+c\right)\right)^{\frac{7}{2}}}\,dx$$

input
```
integrate(1/(I*sinh(d*x+c))^(7/2),x, algorithm="maxima")
```

output
```
integrate((I*sinh(d*x + c))^(-7/2), x)
```

Giac [F]

$$\int \frac{1}{(i\sinh(c+dx))^{7/2}}\,dx = \int \frac{1}{\left(i\,\sinh\left(dx+c\right)\right)^{\frac{7}{2}}}\,dx$$

input
```
integrate(1/(I*sinh(d*x+c))^(7/2),x, algorithm="giac")
```

output
```
integrate((I*sinh(d*x + c))^(-7/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(i\sinh(c+dx))^{7/2}}\,dx = \int \frac{1}{\left(\sinh\left(c+d\,x\right)\,1\mathrm{i}\right)^{7/2}}\,dx$$

input
```
int(1/(sinh(c + d*x)*1i)^(7/2),x)
```

output
```
int(1/(sinh(c + d*x)*1i)^(7/2), x)
```

Reduce [F]

$$\int \frac{1}{(i\sinh(c+dx))^{7/2}}\,dx = \sqrt{i}\left(\int \frac{\sqrt{\sinh(dx+c)}}{\sinh(dx+c)^4}dx\right)$$

input
```
int(1/(I*sinh(d*x+c))^(7/2),x)
```

output
```
sqrt(i)*int(sqrt(sinh(c + d*x))/sinh(c + d*x)**4,x)
```

3.31 $\int (b\sinh(c+dx))^{4/3}\,dx$

Optimal result

Integrand size = 12, antiderivative size = 60

$$\int (b\sinh(c+dx))^{4/3}\,dx = \frac{3\cosh(c+dx)\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{7}{6},\frac{13}{6},-\sinh^2(c+dx)\right)(b\sinh(c+dx))^{7/3}}{7bd\sqrt{\cosh^2(c+dx)}}$$

output
```
3/7*cosh(d*x+c)*hypergeom([1/2, 7/6],[13/6],-sinh(d*x+c)^2)*(b*sinh(d*x+c)
)^(7/3)/b/d/(cosh(d*x+c)^2)^(1/2)
```

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.95

$$\int (b\sinh(c+dx))^{4/3}\,dx = \frac{3\sqrt{\cosh^2(c+dx)}\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{7}{6},\frac{13}{6},-\sinh^2(c+dx)\right)(b\sinh(c+dx))^{4/3}\tanh(c -}{7d}$$

input
```
Integrate[(b*Sinh[c + d*x])^(4/3),x]
```

output
```
(3*Sqrt[Cosh[c + d*x]^2]*Hypergeometric2F1[1/2, 7/6, 13/6, -Sinh[c + d*x]^
2]*(b*Sinh[c + d*x])^(4/3)*Tanh[c + d*x])/(7*d)
```

Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 60, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {3042, 3122}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (b\sinh(c+dx))^{4/3}\,dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int (-ib\sin(ic+idx))^{4/3}dx$$

$$\Big\downarrow \text{ 3122}$$

$$\frac{3\cosh(c+dx)(b\sinh(c+dx))^{7/3}\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{7}{6},\frac{13}{6},-\sinh^2(c+dx)\right)}{7bd\sqrt{\cosh^2(c+dx)}}$$

input
```
Int[(b*Sinh[c + d*x])^(4/3),x]
```

output
```
(3*Cosh[c + d*x]*Hypergeometric2F1[1/2, 7/6, 13/6, -Sinh[c + d*x]^2]*(b*Si
nh[c + d*x])^(7/3))/(7*b*d*Sqrt[Cosh[c + d*x]^2])
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3122
```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]))*Hypergeometric2
F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c + d*x]^2], x] /; FreeQ[{b, c, d, n}, x]
 && !IntegerQ[2*n]
```

Maple [F]

$$\int (b\sinh (dx+c))^{\frac{4}{3}}dx$$

input
```
int((b*sinh(d*x+c))^(4/3),x)
```

output
```
int((b*sinh(d*x+c))^(4/3),x)
```

Fricas [F]

$$\int (b\sinh(c+dx))^{4/3}\,dx=\int (b\sinh (dx+c))^{\frac{4}{3}}\,dx$$

input
```
integrate((b*sinh(d*x+c))^(4/3),x, algorithm="fricas")
```

output
```
integral((b*sinh(d*x + c))^(1/3)*b*sinh(d*x + c), x)
```

Sympy [F]

$$\int (b\sinh(c+dx))^{4/3}\,dx = \int (b\sinh(c+dx))^{\frac{4}{3}}\,dx$$

input
```
integrate((b*sinh(d*x+c))**(4/3),x)
```

output
```
Integral((b*sinh(c + d*x))**(4/3), x)
```

Maxima [F]

$$\int (b\sinh(c+dx))^{4/3}\,dx = \int (b\sinh(dx+c))^{\frac{4}{3}}\,dx$$

input
```
integrate((b*sinh(d*x+c))^(4/3),x, algorithm="maxima")
```

output
```
integrate((b*sinh(d*x + c))^(4/3), x)
```

Giac [F]

$$\int (b\sinh(c+dx))^{4/3}\,dx = \int (b\sinh(dx+c))^{\frac{4}{3}}\,dx$$

input
```
integrate((b*sinh(d*x+c))^(4/3),x, algorithm="giac")
```

output
```
integrate((b*sinh(d*x + c))^(4/3), x)
```

Mupad [F(-1)]

Timed out.

$$\int (b\sinh(c+dx))^{4/3}\,dx = \int (b\sinh(c+d\,x))^{4/3}\,dx$$

input
```
int((b*sinh(c + d*x))^(4/3),x)
```

output
```
int((b*sinh(c + d*x))^(4/3), x)
```

Reduce [F]

$$\int (b\sinh(c+dx))^{4/3}\,dx = b^{\frac{4}{3}}\left(\int \sinh\left(dx+c\right)^{\frac{4}{3}}dx\right)$$

input
```
int((b*sinh(d*x+c))^(4/3),x)
```

output
```
b**(1/3)*int(sinh(c + d*x)**(1/3)*sinh(c + d*x),x)*b
```

3.32 $\int (b\sinh(c+dx))^{2/3}\,dx$

Optimal result

Integrand size = 12, antiderivative size = 60

$$\int (b\sinh(c+dx))^{2/3}\,dx = \frac{3\cosh(c+dx)\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{5}{6},\frac{11}{6},-\sinh^2(c+dx)\right)(b\sinh(c+dx))^{5/3}}{5bd\sqrt{\cosh^2(c+dx)}}$$

output
```
3/5*cosh(d*x+c)*hypergeom([1/2, 5/6],[11/6],-sinh(d*x+c)^2)*(b*sinh(d*x+c)
)^(5/3)/b/d/(cosh(d*x+c)^2)^(1/2)
```

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.95

$$\int (b\sinh(c+dx))^{2/3}\,dx = \frac{3\sqrt{\cosh^2(c+dx)}\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{5}{6},\frac{11}{6},-\sinh^2(c+dx)\right)(b\sinh(c+dx))^{2/3}\tanh(c -}{5d}$$

input
```
Integrate[(b*Sinh[c + d*x])^(2/3),x]
```

output
```
(3*Sqrt[Cosh[c + d*x]^2]*Hypergeometric2F1[1/2, 5/6, 11/6, -Sinh[c + d*x]^
2]*(b*Sinh[c + d*x])^(2/3)*Tanh[c + d*x])/(5*d)
```

Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 60, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {3042, 3122}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (b\sinh(c+dx))^{2/3}\,dx$$

$$\Big\downarrow \; 3042$$

$$\int (-ib\sin(ic+idx))^{2/3}dx$$

$$\Big\downarrow \; 3122$$

$$\frac{3\cosh(c+dx)(b\sinh(c+dx))^{5/3}\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{5}{6},\frac{11}{6},-\sinh^2(c+dx)\right)}{5bd\sqrt{\cosh^2(c+dx)}}$$

input
```
Int[(b*Sinh[c + d*x])^(2/3),x]
```

output
```
(3*Cosh[c + d*x]*Hypergeometric2F1[1/2, 5/6, 11/6, -Sinh[c + d*x]^2]*(b*Si
nh[c + d*x])^(5/3))/(5*b*d*Sqrt[Cosh[c + d*x]^2])
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3122
```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]))*Hypergeometric2
F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c + d*x]^2], x] /; FreeQ[{b, c, d, n}, x]
 &&  !IntegerQ[2*n]
```

Maple [F]

$$\int (b\sinh (dx+c))^{\frac{2}{3}} dx$$

input
```
int((b*sinh(d*x+c))^(2/3),x)
```

output
```
int((b*sinh(d*x+c))^(2/3),x)
```

Fricas [F]

$$\int (b\sinh(c+dx))^{2/3} dx = \int (b\sinh (dx+c))^{\frac{2}{3}} dx$$

input
```
integrate((b*sinh(d*x+c))^(2/3),x, algorithm="fricas")
```

output
```
integral((b*sinh(d*x + c))^(2/3), x)
```

Sympy [F]

$$\int (b\sinh(c+dx))^{2/3}\,dx = \int (b\sinh{(c+dx)})^{\frac{2}{3}}\,dx$$

input
```
integrate((b*sinh(d*x+c))**(2/3),x)
```

output
```
Integral((b*sinh(c + d*x))**(2/3), x)
```

Maxima [F]

$$\int (b\sinh(c+dx))^{2/3}\,dx = \int (b\sinh{(dx+c)})^{\frac{2}{3}}\,dx$$

input
```
integrate((b*sinh(d*x+c))^(2/3),x, algorithm="maxima")
```

output
```
integrate((b*sinh(d*x + c))^(2/3), x)
```

Giac [F]

$$\int (b\sinh(c+dx))^{2/3}\,dx = \int (b\sinh{(dx+c)})^{\frac{2}{3}}\,dx$$

input
```
integrate((b*sinh(d*x+c))^(2/3),x, algorithm="giac")
```

output
```
integrate((b*sinh(d*x + c))^(2/3), x)
```

Mupad [F(-1)]

Timed out.

$$\int (b\sinh(c+dx))^{2/3}\,dx = \int (b\sinh(c+d\,x))^{2/3}\,dx$$

input
```
int((b*sinh(c + d*x))^(2/3),x)
```

output
```
int((b*sinh(c + d*x))^(2/3), x)
```

Reduce [F]

$$\int (b\sinh(c+dx))^{2/3}\,dx = b^{\frac{2}{3}}\left(\int \sinh\left(dx+c\right)^{\frac{2}{3}}dx\right)$$

input
```
int((b*sinh(d*x+c))^(2/3),x)
```

output
```
b**(2/3)*int(sinh(c + d*x)**(2/3),x)
```

3.33 $\int \sqrt[3]{b \sinh(c+dx)}\,dx$

Optimal result

Integrand size = 12, antiderivative size = 60

$$\int \sqrt[3]{b \sinh(c+dx)}\,dx = \frac{3\cosh(c+dx)\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{2}{3},\frac{5}{3},-\sinh^2(c+dx)\right)(b\sinh(c+dx))^{4/3}}{4bd\sqrt{\cosh^2(c+dx)}}$$

output
```
3/4*cosh(d*x+c)*hypergeom([1/2, 2/3],[5/3],-sinh(d*x+c)^2)*(b*sinh(d*x+c))
^(4/3)/b/d/(cosh(d*x+c)^2)^(1/2)
```

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.95

$$\int \sqrt[3]{b \sinh(c+dx)}\,dx = \frac{3\sqrt{\cosh^2(c+dx)}\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{2}{3},\frac{5}{3},-\sinh^2(c+dx)\right)\sqrt[3]{b\sinh(c+dx)}\tanh(c+dx)}{4d}$$

input
```
Integrate[(b*Sinh[c + d*x])^(1/3),x]
```

output
```
(3*Sqrt[Cosh[c + d*x]^2]*Hypergeometric2F1[1/2, 2/3, 5/3, -Sinh[c + d*x]^2
]*(b*Sinh[c + d*x])^(1/3)*Tanh[c + d*x])/(4*d)
```

Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 60, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {3042, 3122}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt[3]{b\sinh(c+dx)}\,dx$$

$$\Big\downarrow \; 3042$$

$$\int \sqrt[3]{-ib\sin(ic+idx)}dx$$

$$\Big\downarrow \; 3122$$

$$\frac{3\cosh(c+dx)(b\sinh(c+dx))^{4/3}\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{2}{3},\frac{5}{3},-\sinh^2(c+dx)\right)}{4bd\sqrt{\cosh^2(c+dx)}}$$

input
```
Int[(b*Sinh[c + d*x])^(1/3),x]
```

output
```
(3*Cosh[c + d*x]*Hypergeometric2F1[1/2, 2/3, 5/3, -Sinh[c + d*x]^2]*(b*Sin
h[c + d*x])^(4/3))/(4*b*d*Sqrt[Cosh[c + d*x]^2])
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3122
```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]))*Hypergeometric2
F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c + d*x]^2], x] /; FreeQ[{b, c, d, n}, x]
 && !IntegerQ[2*n]
```

Maple [F]

$$\int (b\sinh (dx+c))^{\frac{1}{3}} dx$$

input
```
int((b*sinh(d*x+c))^(1/3),x)
```

output
```
int((b*sinh(d*x+c))^(1/3),x)
```

Fricas [F]

$$\int \sqrt[3]{b\sinh(c+dx)}\, dx = \int (b\sinh (dx+c))^{\frac{1}{3}}\, dx$$

input
```
integrate((b*sinh(d*x+c))^(1/3),x, algorithm="fricas")
```

output
```
integral((b*sinh(d*x + c))^(1/3), x)
```

Sympy [F]

$$\int \sqrt[3]{b\sinh(c+dx)}\,dx = \int \sqrt[3]{b\sinh{(c+dx)}}\,dx$$

input
```
integrate((b*sinh(d*x+c))**(1/3),x)
```

output
```
Integral((b*sinh(c + d*x))**(1/3), x)
```

Maxima [F]

$$\int \sqrt[3]{b\sinh(c+dx)}\,dx = \int (b\sinh{(dx+c)})^{\frac{1}{3}}\,dx$$

input
```
integrate((b*sinh(d*x+c))^(1/3),x, algorithm="maxima")
```

output
```
integrate((b*sinh(d*x + c))^(1/3), x)
```

Giac [F]

$$\int \sqrt[3]{b\sinh(c+dx)}\,dx = \int (b\sinh{(dx+c)})^{\frac{1}{3}}\,dx$$

input
```
integrate((b*sinh(d*x+c))^(1/3),x, algorithm="giac")
```

output
```
integrate((b*sinh(d*x + c))^(1/3), x)
```

Mupad [F(-1)]

Timed out.

$$\int \sqrt[3]{b\sinh(c+dx)}\,dx = \int (b\sinh(c+d\,x))^{1/3}\,dx$$

input
```
int((b*sinh(c + d*x))^(1/3),x)
```

output
```
int((b*sinh(c + d*x))^(1/3), x)
```

Reduce [F]

$$\int \sqrt[3]{b\sinh(c+dx)}\,dx = b^{\frac{1}{3}}\left(\int \sinh(dx+c)^{\frac{1}{3}}\,dx\right)$$

input
```
int((b*sinh(d*x+c))^(1/3),x)
```

output
```
b**(1/3)*int(sinh(c + d*x)**(1/3),x)
```

3.34 $\int \frac{1}{\sqrt[3]{b\sinh(c+dx)}}\,dx$

Optimal result

Integrand size = 12, antiderivative size = 60

$$\int \frac{1}{\sqrt[3]{b\sinh(c+dx)}}\,dx = \frac{3\cosh(c+dx)\,\text{Hypergeometric2F1}\left(\frac{1}{3},\frac{1}{2},\frac{4}{3},-\sinh^2(c+dx)\right)(b\sinh(c+dx))^{2/3}}{2bd\sqrt{\cosh^2(c+dx)}}$$

output
```
3/2*cosh(d*x+c)*hypergeom([1/3, 1/2],[4/3],-sinh(d*x+c)^2)*(b*sinh(d*x+c))
^(2/3)/b/d/(cosh(d*x+c)^2)^(1/2)
```

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.95

$$\int \frac{1}{\sqrt[3]{b\sinh(c+dx)}}\,dx = \frac{3\sqrt{\cosh^2(c+dx)}\,\text{Hypergeometric2F1}\left(\frac{1}{3},\frac{1}{2},\frac{4}{3},-\sinh^2(c+dx)\right)\tanh(c+dx)}{2d\sqrt[3]{b\sinh(c+dx)}}$$

input
```
Integrate[(b*Sinh[c + d*x])^(-1/3),x]
```

output
```
(3*Sqrt[Cosh[c + d*x]^2]*Hypergeometric2F1[1/3, 1/2, 4/3, -Sinh[c + d*x]^2
]*Tanh[c + d*x])/(2*d*(b*Sinh[c + d*x])^(1/3))
```

Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 60, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {3042, 3122}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt[3]{b\sinh(c+dx)}}dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \frac{1}{\sqrt[3]{-ib\sin(ic+idx)}}dx$$

$$\Big\downarrow \text{ 3122}$$

$$\frac{3\cosh(c+dx)(b\sinh(c+dx))^{2/3}\text{Hypergeometric2F1}\left(\frac{1}{3},\frac{1}{2},\frac{4}{3},-\sinh^2(c+dx)\right)}{2bd\sqrt{\cosh^2(c+dx)}}$$

input
```
Int[(b*Sinh[c + d*x])^(-1/3),x]
```

output
```
(3*Cosh[c + d*x]*Hypergeometric2F1[1/3, 1/2, 4/3, -Sinh[c + d*x]^2]*(b*Sin
h[c + d*x])^(2/3))/(2*b*d*Sqrt[Cosh[c + d*x]^2])
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3122
```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]))*Hypergeometric2
F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c + d*x]^2], x] /; FreeQ[{b, c, d, n}, x]
 && !IntegerQ[2*n]
```

Maple [F]

$$\int \frac{1}{(b\sinh (dx+c))^{\frac{1}{3}}}dx$$

input
```
int(1/(b*sinh(d*x+c))^(1/3),x)
```

output
```
int(1/(b*sinh(d*x+c))^(1/3),x)
```

Fricas [F]

$$\int \frac{1}{\sqrt[3]{b\sinh(c+dx)}}\,dx = \int \frac{1}{(b\sinh (dx+c))^{\frac{1}{3}}}\,dx$$

input
```
integrate(1/(b*sinh(d*x+c))^(1/3),x, algorithm="fricas")
```

output
```
integral((b*sinh(d*x + c))^(2/3)/(b*sinh(d*x + c)), x)
```

Sympy [F]

$$\int \frac{1}{\sqrt[3]{b\sinh(c+dx)}}\,dx = \int \frac{1}{\sqrt[3]{b\sinh\left(c+dx\right)}}\,dx$$

input
```
integrate(1/(b*sinh(d*x+c))**(1/3),x)
```

output
```
Integral((b*sinh(c + d*x))**(-1/3), x)
```

Maxima [F]

$$\int \frac{1}{\sqrt[3]{b\sinh(c+dx)}}\,dx = \int \frac{1}{\left(b\sinh\left(dx+c\right)\right)^{\frac{1}{3}}}\,dx$$

input
```
integrate(1/(b*sinh(d*x+c))^(1/3),x, algorithm="maxima")
```

output
```
integrate((b*sinh(d*x + c))^(-1/3), x)
```

Giac [F]

$$\int \frac{1}{\sqrt[3]{b\sinh(c+dx)}}\,dx = \int \frac{1}{\left(b\sinh\left(dx+c\right)\right)^{\frac{1}{3}}}\,dx$$

input
```
integrate(1/(b*sinh(d*x+c))^(1/3),x, algorithm="giac")
```

output
```
integrate((b*sinh(d*x + c))^(-1/3), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt[3]{b\sinh(c+dx)}}\,dx = \int \frac{1}{\left(b\sinh\left(c+d\,x\right)\right)^{1/3}}\,dx$$

input
```
int(1/(b*sinh(c + d*x))^(1/3),x)
```

output
```
int(1/(b*sinh(c + d*x))^(1/3), x)
```

Reduce [F]

$$\int \frac{1}{\sqrt[3]{b\sinh(c+dx)}}\,dx = \frac{\int \frac{1}{\sinh(dx+c)^{\frac{1}{3}}}dx}{b^{\frac{1}{3}}}$$

input
```
int(1/(b*sinh(d*x+c))^(1/3),x)
```

output
```
int(1/sinh(c + d*x)**(1/3),x)/b**(1/3)
```

3.35 $\int \frac{1}{(b\sinh(c+dx))^{2/3}}\,dx$

Optimal result

Integrand size = 12, antiderivative size = 58

$$\int \frac{1}{(b\sinh(c+dx))^{2/3}}\,dx=\frac{3\cosh(c+dx)\,\text{Hypergeometric2F1}\left(\frac{1}{6},\frac{1}{2},\frac{7}{6},-\sinh^2(c+dx)\right)\sqrt[3]{b\sinh(c+dx)}}{bd\sqrt{\cosh^2(c+dx)}}$$

output
```
3*cosh(d*x+c)*hypergeom([1/6, 1/2],[7/6],-sinh(d*x+c)^2)*(b*sinh(d*x+c))^(
1/3)/b/d/(cosh(d*x+c)^2)^(1/2)
```

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 55, normalized size of antiderivative = 0.95

$$\int \frac{1}{(b\sinh(c+dx))^{2/3}}\,dx=\frac{3\sqrt{\cosh^2(c+dx)}\,\text{Hypergeometric2F1}\left(\frac{1}{6},\frac{1}{2},\frac{7}{6},-\sinh^2(c+dx)\right)\tanh(c+dx)}{d(b\sinh(c+dx))^{2/3}}$$

input
```
Integrate[(b*Sinh[c + d*x])^(-2/3),x]
```

output
```
(3*Sqrt[Cosh[c + d*x]^2]*Hypergeometric2F1[1/6, 1/2, 7/6, -Sinh[c + d*x]^2
]*Tanh[c + d*x])/(d*(b*Sinh[c + d*x])^(2/3))
```

Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 58, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {3042, 3122}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(b\sinh(c+dx))^{2/3}}\,dx$$
$$\Big\downarrow \; 3042$$
$$\int \frac{1}{(-ib\sin(ic+idx))^{2/3}}dx$$
$$\Big\downarrow \; 3122$$
$$\frac{3\cosh(c+dx)\sqrt[3]{b\sinh(c+dx)}\,\text{Hypergeometric2F1}\left(\frac{1}{6},\frac{1}{2},\frac{7}{6},-\sinh^2(c+dx)\right)}{bd\sqrt{\cosh^2(c+dx)}}$$

input
```
Int[(b*Sinh[c + d*x])^(-2/3),x]
```

output
```
(3*Cosh[c + d*x]*Hypergeometric2F1[1/6, 1/2, 7/6, -Sinh[c + d*x]^2]*(b*Sin
h[c + d*x])^(1/3))/(b*d*Sqrt[Cosh[c + d*x]^2])
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3122
```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]))*Hypergeometric2
F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c + d*x]^2], x] /; FreeQ[{b, c, d, n}, x]
 && !IntegerQ[2*n]
```

Maple [F]

$$\int \frac{1}{(b\sinh{(dx+c)})^{\frac{2}{3}}}dx$$

input
```
int(1/(b*sinh(d*x+c))^(2/3),x)
```

output
```
int(1/(b*sinh(d*x+c))^(2/3),x)
```

Fricas [F]

$$\int \frac{1}{(b\sinh(c+dx))^{2/3}}\,dx = \int \frac{1}{(b\sinh{(dx+c)})^{\frac{2}{3}}}\,dx$$

input
```
integrate(1/(b*sinh(d*x+c))^(2/3),x, algorithm="fricas")
```

output
```
integral((b*sinh(d*x + c))^(1/3)/(b*sinh(d*x + c)), x)
```

Sympy [F]

$$\int \frac{1}{(b\sinh(c+dx))^{2/3}}\,dx = \int \frac{1}{(b\sinh{(c+dx)})^{\frac{2}{3}}}\,dx$$

input
```
integrate(1/(b*sinh(d*x+c))**(2/3),x)
```

output
```
Integral((b*sinh(c + d*x))**(-2/3), x)
```

Maxima [F]

$$\int \frac{1}{(b\sinh(c+dx))^{2/3}}\,dx = \int \frac{1}{(b\sinh(dx+c))^{\frac{2}{3}}}\,dx$$

input
```
integrate(1/(b*sinh(d*x+c))^(2/3),x, algorithm="maxima")
```

output
```
integrate((b*sinh(d*x + c))^(-2/3), x)
```

Giac [F]

$$\int \frac{1}{(b\sinh(c+dx))^{2/3}}\,dx = \int \frac{1}{(b\sinh(dx+c))^{\frac{2}{3}}}\,dx$$

input
```
integrate(1/(b*sinh(d*x+c))^(2/3),x, algorithm="giac")
```

output
```
integrate((b*sinh(d*x + c))^(-2/3), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(b\sinh(c+dx))^{2/3}}\,dx = \int \frac{1}{(b\sinh(c+d\,x))^{2/3}}\,dx$$

input
```
int(1/(b*sinh(c + d*x))^(2/3),x)
```

output
```
int(1/(b*sinh(c + d*x))^(2/3), x)
```

Reduce [F]

$$\int \frac{1}{(b\sinh(c+dx))^{2/3}}\,dx = \frac{\int \frac{1}{\sinh(dx+c)^{\frac{2}{3}}}dx}{b^{\frac{2}{3}}}$$

input
```
int(1/(b*sinh(d*x+c))^(2/3),x)
```

output
```
int(1/sinh(c + d*x)**(2/3),x)/b**(2/3)
```

3.36 $\int \frac{1}{(b\sinh(c+dx))^{4/3}}\,dx$

Optimal result

Integrand size = 12, antiderivative size = 58

$$\int \frac{1}{(b\sinh(c+dx))^{4/3}}\,dx = -\frac{3\cosh(c+dx)\,\text{Hypergeometric2F1}\left(-\frac{1}{6},\frac{1}{2},\frac{5}{6},-\sinh^2(c+dx)\right)}{bd\sqrt{\cosh^2(c+dx)}\sqrt[3]{b\sinh(c+dx)}}$$

output
```
-3*cosh(d*x+c)*hypergeom([-1/6, 1/2],[5/6],-sinh(d*x+c)^2)/b/d/(cosh(d*x+c
)^2)^(1/2)/(b*sinh(d*x+c))^(1/3)
```

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 55, normalized size of antiderivative = 0.95

$$\int \frac{1}{(b\sinh(c+dx))^{4/3}}\,dx = -\frac{3\sqrt{\cosh^2(c+dx)}\,\text{Hypergeometric2F1}\left(-\frac{1}{6},\frac{1}{2},\frac{5}{6},-\sinh^2(c+dx)\right)\tanh(c+dx)}{d(b\sinh(c+dx))^{4/3}}$$

input
```
Integrate[(b*Sinh[c + d*x])^(-4/3),x]
```

output
```
(-3*Sqrt[Cosh[c + d*x]^2]*Hypergeometric2F1[-1/6, 1/2, 5/6, -Sinh[c + d*x]
^2]*Tanh[c + d*x])/(d*(b*Sinh[c + d*x])^(4/3))
```

Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 58, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {3042, 3122}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(b\sinh(c+dx))^{4/3}}dx$$
$$\Big\downarrow \text{3042}$$
$$\int \frac{1}{(-ib\sin(ic+idx))^{4/3}}dx$$
$$\Big\downarrow \text{3122}$$
$$-\frac{3\cosh(c+dx)\,\text{Hypergeometric2F1}\left(-\frac{1}{6},\frac{1}{2},\frac{5}{6},-\sinh^2(c+dx)\right)}{bd\sqrt{\cosh^2(c+dx)}\sqrt[3]{b\sinh(c+dx)}}$$

input
```
Int[(b*Sinh[c + d*x])^(-4/3),x]
```

output
```
(-3*Cosh[c + d*x]*Hypergeometric2F1[-1/6, 1/2, 5/6, -Sinh[c + d*x]^2])/(b*
d*Sqrt[Cosh[c + d*x]^2]*(b*Sinh[c + d*x])^(1/3))
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3122
```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]))*Hypergeometric2
F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c + d*x]^2], x] /; FreeQ[{b, c, d, n}, x]
 && !IntegerQ[2*n]
```

Maple [F]

$$\int \frac{1}{(b\sinh(dx+c))^{\frac{4}{3}}}dx$$

input
```
int(1/(b*sinh(d*x+c))^(4/3),x)
```

output
```
int(1/(b*sinh(d*x+c))^(4/3),x)
```

Fricas [F]

$$\int \frac{1}{(b\sinh(c+dx))^{4/3}}\,dx = \int \frac{1}{(b\sinh(dx+c))^{\frac{4}{3}}}\,dx$$

input
```
integrate(1/(b*sinh(d*x+c))^(4/3),x, algorithm="fricas")
```

output
```
integral((b*sinh(d*x + c))^(2/3)/(b^2*sinh(d*x + c)^2), x)
```

Sympy [F]

$$\int \frac{1}{(b\sinh(c+dx))^{4/3}}\,dx = \int \frac{1}{(b\sinh{(c+dx)})^{\frac{4}{3}}}\,dx$$

input
```
integrate(1/(b*sinh(d*x+c))**(4/3),x)
```

output
```
Integral((b*sinh(c + d*x))**(-4/3), x)
```

Maxima [F]

$$\int \frac{1}{(b\sinh(c+dx))^{4/3}}\,dx = \int \frac{1}{(b\sinh{(dx+c)})^{\frac{4}{3}}}\,dx$$

input
```
integrate(1/(b*sinh(d*x+c))^(4/3),x, algorithm="maxima")
```

output
```
integrate((b*sinh(d*x + c))^(-4/3), x)
```

Giac [F]

$$\int \frac{1}{(b\sinh(c+dx))^{4/3}}\,dx = \int \frac{1}{(b\sinh{(dx+c)})^{\frac{4}{3}}}\,dx$$

input
```
integrate(1/(b*sinh(d*x+c))^(4/3),x, algorithm="giac")
```

output
```
integrate((b*sinh(d*x + c))^(-4/3), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(b\sinh(c+dx))^{4/3}}\,dx = \int \frac{1}{\left(b\sinh\left(c+d\,x\right)\right)^{4/3}}\,dx$$

input
```
int(1/(b*sinh(c + d*x))^(4/3),x)
```

output
```
int(1/(b*sinh(c + d*x))^(4/3), x)
```

Reduce [F]

$$\int \frac{1}{(b\sinh(c+dx))^{4/3}}\,dx = \frac{\int \frac{1}{\sinh(dx+c)^{\frac{4}{3}}}dx}{b^{\frac{4}{3}}}$$

input
```
int(1/(b*sinh(d*x+c))^(4/3),x)
```

output
```
int(1/(sinh(c + d*x)**(1/3)*sinh(c + d*x)),x)/(b**(1/3)*b)
```

3.37 $\int (b\sinh(c+dx))^n\,dx$

Optimal result

Integrand size = 10, antiderivative size = 70

$$\int (b\sinh(c+dx))^n\,dx = \frac{\cosh(c+dx)\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{1+n}{2},\frac{3+n}{2},-\sinh^2(c+dx)\right)(b\sinh(c+dx))^{1+n}}{bd(1+n)\sqrt{\cosh^2(c+dx)}}$$

output
```
cosh(d*x+c)*hypergeom([1/2, 1/2+1/2*n],[3/2+1/2*n],-sinh(d*x+c)^2)*(b*sinh
(d*x+c))^(1+n)/b/d/(1+n)/(cosh(d*x+c)^2)^(1/2)
```

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 65, normalized size of antiderivative = 0.93

$$\int (b\sinh(c+dx))^n\,dx = \frac{\sqrt{\cosh^2(c+dx)}\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{1+n}{2},\frac{3+n}{2},-\sinh^2(c+dx)\right)(b\sinh(c+dx))^n\tanh(c+dx)}{d(1+n)}$$

input
```
Integrate[(b*Sinh[c + d*x])^n,x]
```

output
```
(Sqrt[Cosh[c + d*x]^2]*Hypergeometric2F1[1/2, (1 + n)/2, (3 + n)/2, -Sinh[
c + d*x]^2]*(b*Sinh[c + d*x])^n*Tanh[c + d*x])/(d*(1 + n))
```

Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 70, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {3042, 3122}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (b\sinh(c+dx))^n\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int (-ib\sin(ic+idx))^n dx$$

$$\Big\downarrow \text{3122}$$

$$\frac{\cosh(c+dx)(b\sinh(c+dx))^{n+1}\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{n+1}{2},\frac{n+3}{2},-\sinh^2(c+dx)\right)}{bd(n+1)\sqrt{\cosh^2(c+dx)}}$$

input
```
Int[(b*Sinh[c + d*x])^n,x]
```

output
```
(Cosh[c + d*x]*Hypergeometric2F1[1/2, (1 + n)/2, (3 + n)/2, -Sinh[c + d*x]
^2]*(b*Sinh[c + d*x])^(1 + n))/(b*d*(1 + n)*Sqrt[Cosh[c + d*x]^2])
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3122
```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]))*Hypergeometric2
F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c + d*x]^2], x] /; FreeQ[{b, c, d, n}, x]
 && !IntegerQ[2*n]
```

Maple [F]

$$\int (b\sinh (dx+c))^{n} dx$$

input
```
int((b*sinh(d*x+c))^n,x)
```

output
```
int((b*sinh(d*x+c))^n,x)
```

Fricas [F]

$$\int (b\sinh(c+dx))^n\,dx = \int (b\sinh (dx+c))^{n}\,dx$$

input
```
integrate((b*sinh(d*x+c))^n,x, algorithm="fricas")
```

output
```
integral((b*sinh(d*x + c))^n, x)
```

Sympy [F]

$$\int (b\sinh(c+dx))^n \, dx = \int (b\sinh{(c+dx)})^n \, dx$$

input
```
integrate((b*sinh(d*x+c))**n,x)
```

output
```
Integral((b*sinh(c + d*x))**n, x)
```

Maxima [F]

$$\int (b\sinh(c+dx))^n \, dx = \int (b\sinh{(dx+c)})^n \, dx$$

input
```
integrate((b*sinh(d*x+c))^n,x, algorithm="maxima")
```

output
```
integrate((b*sinh(d*x + c))^n, x)
```

Giac [F]

$$\int (b\sinh(c+dx))^n \, dx = \int (b\sinh{(dx+c)})^n \, dx$$

input
```
integrate((b*sinh(d*x+c))^n,x, algorithm="giac")
```

output
```
integrate((b*sinh(d*x + c))^n, x)
```

Mupad [F(-1)]

Timed out.

$$\int (b\sinh(c+dx))^n\,dx=\int (b\sinh(c+d\,x))^n\,dx$$

input
```
int((b*sinh(c + d*x))^n,x)
```

output
```
int((b*sinh(c + d*x))^n, x)
```

Reduce [F]

$$\int (b\sinh(c+dx))^n\,dx=b^n\left(\int \sinh(dx+c)^n\,dx\right)$$

input
```
int((b*sinh(d*x+c))^n,x)
```

output
```
b**n*int(sinh(c + d*x)**n,x)
```

3.38 $\int (i\sinh(c+dx))^n\,dx$

Optimal result

Integrand size = 12, antiderivative size = 72

$$\int (i\sinh(c+dx))^n\,dx = -\frac{i\cosh(c+dx)\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{1+n}{2},\frac{3+n}{2},-\sinh^2(c+dx)\right)(i\sinh(c+dx))^{1+n}}{d(1+n)\sqrt{\cosh^2(c+dx)}}$$

output
```
-I*cosh(d*x+c)*hypergeom([1/2, 1/2+1/2*n],[3/2+1/2*n],-sinh(d*x+c)^2)*(I*s
inh(d*x+c))^(1+n)/d/(1+n)/(cosh(d*x+c)^2)^(1/2)
```

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 67, normalized size of antiderivative = 0.93

$$\int (i\sinh(c+dx))^n\,dx = \frac{\sqrt{\cosh^2(c+dx)}\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{1+n}{2},\frac{3+n}{2},-\sinh^2(c+dx)\right)(i\sinh(c+dx))^n\tanh(c+dx)}{d(1+n)}$$

input
```
Integrate[(I*Sinh[c + d*x])^n,x]
```

output
```
(Sqrt[Cosh[c + d*x]^2]*Hypergeometric2F1[1/2, (1 + n)/2, (3 + n)/2, -Sinh[
c + d*x]^2]*(I*Sinh[c + d*x])^n*Tanh[c + d*x])/(d*(1 + n))
```

Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 72, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {3042, 3122}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (i\sinh(c+dx))^n\,dx$$

$$\Big\downarrow \; 3042$$

$$\int \sin(ic+idx)^n dx$$

$$\Big\downarrow \; 3122$$

$$-\frac{i\cosh(c+dx)(i\sinh(c+dx))^{n+1}\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{n+1}{2},\frac{n+3}{2},-\sinh^2(c+dx)\right)}{d(n+1)\sqrt{\cosh^2(c+dx)}}$$

input
```
Int[(I*Sinh[c + d*x])^n,x]
```

output
```
((-I)*Cosh[c + d*x]*Hypergeometric2F1[1/2, (1 + n)/2, (3 + n)/2, -Sinh[c +
 d*x]^2]*(I*Sinh[c + d*x])^(1 + n))/(d*(1 + n)*Sqrt[Cosh[c + d*x]^2])
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3122
```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]))*Hypergeometric2
F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c + d*x]^2], x] /; FreeQ[{b, c, d, n}, x]
 &&  !IntegerQ[2*n]
```

Maple [F]

$$\int (i \sinh (dx + c))^n \, dx$$

input
```
int((I*sinh(d*x+c))^n,x)
```

output
```
int((I*sinh(d*x+c))^n,x)
```

Fricas [F]

$$\int (i \sinh(c + dx))^n \, dx = \int (i \sinh (dx + c))^n \, dx$$

input
```
integrate((I*sinh(d*x+c))^n,x, algorithm="fricas")
```

output
```
integral((1/2*(I*e^(2*d*x + 2*c) - I)*e^(-d*x - c))^n, x)
```

Sympy [F]

$$\int (i\sinh(c+dx))^n\,dx = \int (i\sinh{(c+dx)})^n\;dx$$

input
```
integrate((I*sinh(d*x+c))**n,x)
```

output
```
Integral((I*sinh(c + d*x))**n, x)
```

Maxima [F]

$$\int (i\sinh(c+dx))^n\,dx = \int (i\,\sinh{(dx+c)})^n\;dx$$

input
```
integrate((I*sinh(d*x+c))^n,x, algorithm="maxima")
```

output
```
integrate((I*sinh(d*x + c))^n, x)
```

Giac [F]

$$\int (i\sinh(c+dx))^n\,dx = \int (i\,\sinh{(dx+c)})^n\;dx$$

input
```
integrate((I*sinh(d*x+c))^n,x, algorithm="giac")
```

output
```
integrate((I*sinh(d*x + c))^n, x)
```

Mupad [F(-1)]

Timed out.

$$\int (i\sinh(c+dx))^n\,dx = \int (\sinh(c+d\,x)\ 1\mathrm{i})^n\,dx$$

input
```
int((sinh(c + d*x)*1i)^n,x)
```

output
```
int((sinh(c + d*x)*1i)^n, x)
```

Reduce [F]

$$\int (i\sinh(c+dx))^n\,dx = i^n\left(\int \sinh(dx+c)^n\,dx\right)$$

input
```
int((I*sinh(d*x+c))^n,x)
```

output
```
i**n*int(sinh(c + d*x)**n,x)
```

3.39 $\int (-i\sinh(c+dx))^n\,dx$

Optimal result

Integrand size = 12, antiderivative size = 72

$$\int (-i\sinh(c+dx))^n\,dx = \frac{i\cosh(c+dx)\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{1+n}{2},\frac{3+n}{2},-\sinh^2(c+dx)\right)(-i\sinh(c+dx))^{1+n}}{d(1+n)\sqrt{\cosh^2(c+dx)}}$$

output
```
I*cosh(d*x+c)*hypergeom([1/2, 1/2+1/2*n],[3/2+1/2*n],-sinh(d*x+c)^2)*(-I*s
inh(d*x+c))^(1+n)/d/(1+n)/(cosh(d*x+c)^2)^(1/2)
```

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 67, normalized size of antiderivative = 0.93

$$\int (-i\sinh(c+dx))^n\,dx = \frac{\sqrt{\cosh^2(c+dx)}\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{1+n}{2},\frac{3+n}{2},-\sinh^2(c+dx)\right)(-i\sinh(c+dx))^n\tanh(c+dx)}{d(1+n)}$$

input
```
Integrate[((-I)*Sinh[c + d*x])^n,x]
```

output
```
(Sqrt[Cosh[c + d*x]^2]*Hypergeometric2F1[1/2, (1 + n)/2, (3 + n)/2, -Sinh[
c + d*x]^2]*((-I)*Sinh[c + d*x])^n*Tanh[c + d*x])/(d*(1 + n))
```

Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 72, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {3042, 3122}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (-i\sinh(c+dx))^n\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int (-\sin(ic+idx))^n dx$$

$$\Big\downarrow \text{3122}$$

$$\frac{i\cosh(c+dx)(-i\sinh(c+dx))^{n+1}\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{n+1}{2},\frac{n+3}{2},-\sinh^2(c+dx)\right)}{d(n+1)\sqrt{\cosh^2(c+dx)}}$$

input
```
Int[((-I)*Sinh[c + d*x])^n,x]
```

output
```
(I*Cosh[c + d*x]*Hypergeometric2F1[1/2, (1 + n)/2, (3 + n)/2, -Sinh[c + d*
x]^2]*((-I)*Sinh[c + d*x])^(1 + n))/(d*(1 + n)*Sqrt[Cosh[c + d*x]^2])
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3122
```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]))*Hypergeometric2
F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c + d*x]^2], x] /; FreeQ[{b, c, d, n}, x]
 && !IntegerQ[2*n]
```

Maple [F]

$$\int (-i \sinh (dx+c))^n dx$$

input
```
int((-I*sinh(d*x+c))^n,x)
```

output
```
int((-I*sinh(d*x+c))^n,x)
```

Fricas [F]

$$\int (-i \sinh(c+dx))^n dx = \int (-i \sinh (dx+c))^n dx$$

input
```
integrate((-I*sinh(d*x+c))^n,x, algorithm="fricas")
```

output
```
integral((1/2*(-I*e^(2*d*x + 2*c) + I)*e^(-d*x - c))^n, x)
```

Sympy [F]

$$\int (-i\sinh(c+dx))^n\,dx = \int (-i\sinh{(c+dx)})^n\,dx$$

input
```
integrate((-I*sinh(d*x+c))**n,x)
```

output
```
Integral((-I*sinh(c + d*x))**n, x)
```

Maxima [F]

$$\int (-i\sinh(c+dx))^n\,dx = \int (-i\,\sinh{(dx+c)})^n\,dx$$

input
```
integrate((-I*sinh(d*x+c))^n,x, algorithm="maxima")
```

output
```
integrate((-I*sinh(d*x + c))^n, x)
```

Giac [F]

$$\int (-i\sinh(c+dx))^n\,dx = \int (-i\,\sinh{(dx+c)})^n\,dx$$

input
```
integrate((-I*sinh(d*x+c))^n,x, algorithm="giac")
```

output
```
integrate((-I*sinh(d*x + c))^n, x)
```

Mupad [F(-1)]

Timed out.

$$\int (-i\sinh(c+dx))^n\,dx = \int (-\sinh(c+d\,x)\ 1\mathrm{i})^n\,dx$$

input
```
int((-sinh(c + d*x)*1i)^n,x)
```

output
```
int((-sinh(c + d*x)*1i)^n, x)
```

Reduce [F]

$$\int (-i\sinh(c+dx))^n\,dx = i^n(-1)^n\left(\int \sinh(dx+c)^n\,dx\right)$$

input
```
int((-I*sinh(d*x+c))^n,x)
```

output
```
i**n*( - 1)**n*int(sinh(c + d*x)**n,x)
```

3.40 $\int \frac{\sinh^4(x)}{i+\sinh(x)}\,dx$

Optimal result

Integrand size = 13, antiderivative size = 46

$$\int \frac{\sinh^4(x)}{i+\sinh(x)}\,dx = \frac{3ix}{2} - 4\cosh(x) + \frac{4\cosh^3(x)}{3} - \frac{3}{2}i\cosh(x)\sinh(x) - \frac{\cosh(x)\sinh^3(x)}{i+\sinh(x)}$$

output
```
3/2*I*x-4*cosh(x)+4/3*cosh(x)^3-3/2*I*cosh(x)*sinh(x)-cosh(x)*sinh(x)^3/(I
+sinh(x))
```

Mathematica [B] (verified)

Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 134 vs. 2(46) = 92.

Time = 0.14 (sec) , antiderivative size = 134, normalized size of antiderivative = 2.91

$$\int \frac{\sinh^4(x)}{i+\sinh(x)}\,dx$$
$$= \frac{\cosh(x)\left(-16i\left(\arcsin\left(\frac{\sqrt{1-i\sinh(x)}}{\sqrt{2}}\right)+\sqrt{\cosh^2(x)}\right)-\left(16\arcsin\left(\frac{\sqrt{1-i\sinh(x)}}{\sqrt{2}}\right)+7\sqrt{\cosh^2(x)}\right)\sinh(x\right.}{6\sqrt{\cosh^2(x)}(i+\sinh(x}$$

input
```
Integrate[Sinh[x]^4/(I + Sinh[x]),x]
```

output
```
(Cosh[x]*((-16*I)*(ArcSin[Sqrt[1 - I*Sinh[x]]/Sqrt[2]] + Sqrt[Cosh[x]^2])
- (16*ArcSin[Sqrt[1 - I*Sinh[x]]/Sqrt[2]] + 7*Sqrt[Cosh[x]^2])*Sinh[x] - I
*Sqrt[Cosh[x]^2]*Sinh[x]^2 + 2*Sqrt[Cosh[x]^2]*Sinh[x]^3 + I*ArcSinh[Sinh[
x]]*(I + Sinh[x])))/(6*Sqrt[Cosh[x]^2]*(I + Sinh[x]))
```

Rubi [A] (verified)

Time = 0.40 (sec) , antiderivative size = 53, normalized size of antiderivative = 1.15, number of steps used = 16, number of rules used = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 1.154$, Rules used = {3042, 3246, 3042, 25, 26, 3227, 25, 26, 3042, 25, 26, 3113, 2009, 3115, 24}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sinh^4(x)}{\sinh(x)+i}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\sin(ix)^4}{i-i\sin(ix)}dx$$

$$\Big\downarrow \text{3246}$$

$$-\int (3i-4\sinh(x))\sinh^2(x)dx-\frac{\sinh^3(x)\cosh(x)}{\sinh(x)+i}$$

$$\Big\downarrow \text{3042}$$

$$-\int -\left((4i\sin(ix)+3i)\sin(ix)^2\right)dx-\frac{\sinh^3(x)\cosh(x)}{\sinh(x)+i}$$

$$\Big\downarrow \text{25}$$

$$\int i\sin(ix)^2(4\sin(ix)+3)dx-\frac{\sinh^3(x)\cosh(x)}{\sinh(x)+i}$$

$$\Big\downarrow \text{26}$$

$$i\int \sin(ix)^2(4\sin(ix)+3)dx-\frac{\sinh^3(x)\cosh(x)}{\sinh(x)+i}$$

$$\Big\downarrow \text{3227}$$

$$i\left(3\int -\sinh^2(x)dx+4\int -i\sinh^3(x)dx\right)-\frac{\sinh^3(x)\cosh(x)}{\sinh(x)+i}$$

$$\Big\downarrow \text{25}$$

$$i\left(-3\int \sinh^2(x)dx+4\int -i\sinh^3(x)dx\right)-\frac{\sinh^3(x)\cosh(x)}{\sinh(x)+i}$$

$$\Big\downarrow \text{26}$$

$$i\left(-3\int \sinh^2(x)dx-4i\int \sinh^3(x)dx\right)-\frac{\sinh^3(x)\cosh(x)}{\sinh(x)+i}$$

$$\Big\downarrow \text{3042}$$

$$i\left(-3\int -\sin(ix)^2dx-4i\int i\sin(ix)^3dx\right)-\frac{\sinh^3(x)\cosh(x)}{\sinh(x)+i}$$

$$\Big\downarrow \text{25}$$

$$i\left(3\int \sin(ix)^2dx-4i\int i\sin(ix)^3dx\right)-\frac{\sinh^3(x)\cosh(x)}{\sinh(x)+i}$$

$$\Big\downarrow \text{26}$$

$$i\left(3\int \sin(ix)^2dx+4\int \sin(ix)^3dx\right)-\frac{\sinh^3(x)\cosh(x)}{\sinh(x)+i}$$

$$\Big\downarrow \text{3113}$$

$$i\left(3\int \sin(ix)^2dx+4i\int \left(1-\cosh^2(x)\right)d\cosh(x)\right)-\frac{\sinh^3(x)\cosh(x)}{\sinh(x)+i}$$

$$\Big\downarrow \text{2009}$$

$$i\left(3\int \sin(ix)^2dx+4i\left(\cosh(x)-\frac{\cosh^3(x)}{3}\right)\right)-\frac{\sinh^3(x)\cosh(x)}{\sinh(x)+i}$$

$$\Big\downarrow \text{3115}$$

$$i\left(3\left(\frac{\int 1dx}{2}-\frac{1}{2}\sinh(x)\cosh(x)\right)+4i\left(\cosh(x)-\frac{\cosh^3(x)}{3}\right)\right)-\frac{\sinh^3(x)\cosh(x)}{\sinh(x)+i}$$

$$\Big\downarrow \text{24}$$

$$i\left(3\left(\frac{x}{2}-\frac{1}{2}\sinh(x)\cosh(x)\right)+4i\left(\cosh(x)-\frac{\cosh^3(x)}{3}\right)\right)-\frac{\sinh^3(x)\cosh(x)}{\sinh(x)+i}$$

input
```
Int[Sinh[x]^4/(I + Sinh[x]),x]
```

output
```
-((Cosh[x]*Sinh[x]^3)/(I + Sinh[x])) + I*((4*I)*(Cosh[x] - Cosh[x]^3/3) +
3*(x/2 - (Cosh[x]*Sinh[x])/2))
```

Defintions of rubi rules used

rule 24
```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3113
```
Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Simp[-d^(-1)   Subst[Int[Exp
and[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x]
 && IGtQ[(n - 1)/2, 0]
```

rule 3115
```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Simp[b^2*((n - 1)/n)   Int[(b*Sin
[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[
2*n]
```

rule 3227
```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Simp[c   Int[(b*Sin[e + f*x])^m, x], x] + Simp[d/b   Int
[(b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

rule 3246
```
Int[((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_)/((a_) + (b_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Simp[(-(b*c - a*d))*Cos[e + f*x]*((c + d*Sin[e +
 f*x])^(n - 1)/(a*f*(a + b*Sin[e + f*x]))), x] - Simp[d/(a*b)   Int[(c + d*
Sin[e + f*x])^(n - 2)*Simp[b*d*(n - 1) - a*c*n + (b*c*(n - 1) - a*d*n)*Sin[
e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] &
& EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[n, 1] && (IntegerQ[2*n] ||
EqQ[c, 0])
```

Maple [A] (verified)

Time = 0.76 (sec) , antiderivative size = 51, normalized size of antiderivative = 1.11

method	result
risch	$\frac{3ix}{2}+\frac{\mathrm{e}^{3x}}{24}-\frac{i\mathrm{e}^{2x}}{8}-\frac{7\,\mathrm{e}^{x}}{8}-\frac{7\,\mathrm{e}^{-x}}{8}+\frac{i\mathrm{e}^{-2x}}{8}+\frac{\mathrm{e}^{-3x}}{24}-\frac{2}{\mathrm{e}^{x}+i}$
default	$-\frac{2i}{\tanh\left(\frac{x}{2}\right)+i}-\frac{3i\ln\left(\tanh\left(\frac{x}{2}\right)-1\right)}{2}+\frac{\frac{3}{2}-\frac{i}{2}}{\tanh\left(\frac{x}{2}\right)-1}+\frac{-\frac{1}{2}-\frac{i}{2}}{\left(\tanh\left(\frac{x}{2}\right)-1\right)^{2}}-\frac{1}{3\left(\tanh\left(\frac{x}{2}\right)-1\right)^{3}}+\frac{3i\ln\left(\tanh\left(\frac{x}{2}\right)+1\right)}{2}+$
parallelrisch	$\frac{\left(-36i\sinh\left(\frac{x}{2}\right)+36\cosh\left(\frac{x}{2}\right)\right)\ln\left(\tanh\left(\frac{x}{2}\right)-1\right)+\left(36i\sinh\left(\frac{x}{2}\right)-36\cosh\left(\frac{x}{2}\right)\right)\ln\left(\tanh\left(\frac{x}{2}\right)+1\right)-85i\cosh\left(\frac{x}{2}\right)-18i\cosh\left(\frac{3x}{2}\right)-2i}{24i\cosh\left(\frac{x}{2}\right)+24\sinh\left(\frac{x}{2}\right)}$

input
```
int(sinh(x)^4/(I+sinh(x)),x,method=_RETURNVERBOSE)
```

output
```
3/2*I*x+1/24*exp(x)^3-1/8*I*exp(x)^2-7/8*exp(x)-7/8/exp(x)+1/8*I/exp(x)^2+
1/24/exp(x)^3-2/(exp(x)+I)
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 67, normalized size of antiderivative = 1.46

$$\int \frac{\sinh^4(x)}{i+\sinh(x)}\,dx = -\frac{3\,(-12i\,x+7i)e^{(4\,x)}+3\,(12\,x+23)e^{(3\,x)}-e^{(7\,x)}+2i\,e^{(6\,x)}+18\,e^{(5\,x)}+18i\,e^{(2\,x)}+2\,e^{x}-i}{24\,\left(e^{(4\,x)}+i\,e^{(3\,x)}\right)}$$

input
```
integrate(sinh(x)^4/(I+sinh(x)),x, algorithm="fricas")
```

output
```
-1/24*(3*(-12*I*x + 7*I)*e^(4*x) + 3*(12*x + 23)*e^(3*x) - e^(7*x) + 2*I*e
^(6*x) + 18*e^(5*x) + 18*I*e^(2*x) + 2*e^x - I)/(e^(4*x) + I*e^(3*x))
```

Sympy [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 58, normalized size of antiderivative = 1.26

$$\int \frac{\sinh^4(x)}{i+\sinh(x)}\,dx = \frac{3ix}{2}+\frac{e^{3x}}{24}-\frac{ie^{2x}}{8}-\frac{7e^{x}}{8}-\frac{7e^{-x}}{8}+\frac{ie^{-2x}}{8}+\frac{e^{-3x}}{24}-\frac{2}{e^{x}+i}$$

input
```
integrate(sinh(x)**4/(I+sinh(x)),x)
```

output
```
3*I*x/2 + exp(3*x)/24 - I*exp(2*x)/8 - 7*exp(x)/8 - 7*exp(-x)/8 + I*exp(-2
*x)/8 + exp(-3*x)/24 - 2/(exp(x) + I)
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 59, normalized size of antiderivative = 1.28

$$\int \frac{\sinh^4(x)}{i+\sinh(x)}\,dx = \frac{3}{2}i\,x-\frac{2\,e^{(-x)}-18i\,e^{(-2\,x)}+69\,e^{(-3\,x)}+i}{8\,\left(-3i\,e^{(-3\,x)}+3\,e^{(-4\,x)}\right)}-\frac{7}{8}\,e^{(-x)}+\frac{1}{8}i\,e^{(-2\,x)}+\frac{1}{24}\,e^{(-3\,x)}$$

input
```
integrate(sinh(x)^4/(I+sinh(x)),x, algorithm="maxima")
```

output
```
3/2*I*x - 1/8*(2*e^(-x) - 18*I*e^(-2*x) + 69*e^(-3*x) + I)/(-3*I*e^(-3*x)
+ 3*e^(-4*x)) - 7/8*e^(-x) + 1/8*I*e^(-2*x) + 1/24*e^(-3*x)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 50, normalized size of antiderivative = 1.09

$$\int \frac{\sinh^4(x)}{i+\sinh(x)}\,dx = \frac{3}{2}i\,x - \frac{\left(69\,e^{(3\,x)} + 18i\,e^{(2\,x)} + 2\,e^{x} - i\right)e^{(-3\,x)}}{24\,(e^{x}+i)} + \frac{1}{24}\,e^{(3\,x)} - \frac{1}{8}i\,e^{(2\,x)} - \frac{7}{8}\,e^{x}$$

input
```
integrate(sinh(x)^4/(I+sinh(x)),x, algorithm="giac")
```

output
```
3/2*I*x - 1/24*(69*e^(3*x) + 18*I*e^(2*x) + 2*e^x - I)*e^(-3*x)/(e^x + I)
+ 1/24*e^(3*x) - 1/8*I*e^(2*x) - 7/8*e^x
```

Mupad [B] (verification not implemented)

Time = 1.81 (sec) , antiderivative size = 50, normalized size of antiderivative = 1.09

$$\int \frac{\sinh^4(x)}{i+\sinh(x)}\,dx = \frac{x\,3\mathrm{i}}{2} - \frac{7\,\mathrm{e}^{-x}}{8} + \frac{\mathrm{e}^{-2\,x}\,1\mathrm{i}}{8} - \frac{\mathrm{e}^{2\,x}\,1\mathrm{i}}{8} + \frac{\mathrm{e}^{-3\,x}}{24} + \frac{\mathrm{e}^{3\,x}}{24} - \frac{7\,\mathrm{e}^{x}}{8} - \frac{2}{\mathrm{e}^{x}+1\mathrm{i}}$$

input
```
int(sinh(x)^4/(sinh(x) + 1i),x)
```

output
```
(x*3i)/2 - (7*exp(-x))/8 + (exp(-2*x)*1i)/8 - (exp(2*x)*1i)/8 + exp(-3*x)/
24 + exp(3*x)/24 - (7*exp(x))/8 - 2/(exp(x) + 1i)
```

Reduce [F]

$$\int \frac{\sinh^4(x)}{i+\sinh(x)}\,dx=\int \frac{\sinh{(x)}^4}{\sinh{(x)}+i}dx$$

input
```
int(sinh(x)^4/(I+sinh(x)),x)
```

output
```
int(sinh(x)**4/(sinh(x) + i),x)
```

3.41 $\int \frac{\sinh^3(x)}{i+\sinh(x)}\,dx$

Optimal result

Integrand size = 13, antiderivative size = 36

$$\int \frac{\sinh^3(x)}{i+\sinh(x)}\,dx = -\frac{3x}{2} - 2i\cosh(x) + \frac{3}{2}\cosh(x)\sinh(x) - \frac{\cosh(x)\sinh^2(x)}{i+\sinh(x)}$$

output
```
-3/2*x-2*I*cosh(x)+3/2*cosh(x)*sinh(x)-cosh(x)*sinh(x)^2/(I+sinh(x))
```

Mathematica [A] (verified)

Time = 0.09 (sec) , antiderivative size = 41, normalized size of antiderivative = 1.14

$$\int \frac{\sinh^3(x)}{i+\sinh(x)}\,dx = \frac{1}{2}\cosh(x)\left(-\frac{3\text{arcsinh}(\sinh(x))}{\sqrt{\cosh^2(x)}} + \frac{4 - i\sinh(x) + \sinh^2(x)}{i+\sinh(x)}\right)$$

input
```
Integrate[Sinh[x]^3/(I + Sinh[x]),x]
```

output
```
(Cosh[x]*((-3*ArcSinh[Sinh[x]])/Sqrt[Cosh[x]^2] + (4 - I*Sinh[x] + Sinh[x]
^2)/(I + Sinh[x])))/2
```

Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 40, normalized size of antiderivative = 1.11, number of steps used = 8, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.615$, Rules used = {3042, 26, 26, 3246, 26, 3042, 26, 3213}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sinh^3(x)}{\sinh(x)+i}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{i\sin(ix)^3}{i-i\sin(ix)}dx$$

$$\Big\downarrow \text{26}$$

$$i\int -\frac{i\sin(ix)^3}{1-\sin(ix)}dx$$

$$\Big\downarrow \text{26}$$

$$\int \frac{\sin(ix)^3}{1-\sin(ix)}dx$$

$$\Big\downarrow \text{3246}$$

$$\int -i(3i\sinh(x)+2)\sinh(x)dx+\frac{i\sinh^2(x)\cosh(x)}{1-i\sinh(x)}$$

$$\Big\downarrow \text{26}$$

$$\frac{i\sinh^2(x)\cosh(x)}{1-i\sinh(x)}-i\int (3i\sinh(x)+2)\sinh(x)dx$$

$$\Big\downarrow \text{3042}$$

$$\frac{i\sinh^2(x)\cosh(x)}{1-i\sinh(x)}-i\int -i\sin(ix)(3\sin(ix)+2)dx$$

$$\Big\downarrow \text{26}$$

$$\frac{i\sinh^2(x)\cosh(x)}{1-i\sinh(x)}-\int \sin(ix)(3\sin(ix)+2)dx$$

$$\Big\downarrow \text{3213}$$

$$-\frac{3x}{2}-2i\cosh(x)+\frac{i\sinh^2(x)\cosh(x)}{1-i\sinh(x)}+\frac{3}{2}\sinh(x)\cosh(x)$$

input
```
Int[Sinh[x]^3/(I + Sinh[x]),x]
```

output
```
(-3*x)/2 - (2*I)*Cosh[x] + (3*Cosh[x]*Sinh[x])/2 + (I*Cosh[x]*Sinh[x]^2)/(
1 - I*Sinh[x])
```

Defintions of rubi rules used

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3213
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)
*(x_)]), x_Symbol] :> Simp[(2*a*c + b*d)*(x/2), x] + (-Simp[(b*c + a*d)*(Co
s[e + f*x]/f), x] - Simp[b*d*Cos[e + f*x]*(Sin[e + f*x]/(2*f)), x]) /; Free
Q[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

rule 3246
```
Int[((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_)/((a_) + (b_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Simp[(-(b*c - a*d))*Cos[e + f*x]*((c + d*Sin[e +
 f*x])^(n - 1)/(a*f*(a + b*Sin[e + f*x]))), x] - Simp[d/(a*b)   Int[(c + d*
Sin[e + f*x])^(n - 2)*Simp[b*d*(n - 1) - a*c*n + (b*c*(n - 1) - a*d*n)*Sin[
e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] &
& EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[n, 1] && (IntegerQ[2*n] ||
EqQ[c, 0])
```

Maple [A] (verified)

Time = 0.46 (sec) , antiderivative size = 39, normalized size of antiderivative = 1.08

method	result
risch	$-\frac{3x}{2}+\frac{e^{2x}}{8}-\frac{ie^{x}}{2}-\frac{ie^{-x}}{2}-\frac{e^{-2x}}{8}-\frac{2i}{e^{x}+i}$
default	$\frac{2}{\tanh(\frac{x}{2})+i}+\frac{\frac{1}{2}-i}{\tanh(\frac{x}{2})+1}-\frac{1}{2(\tanh(\frac{x}{2})+1)^{2}}-\frac{3\ln(\tanh(\frac{x}{2})+1)}{2}+\frac{\frac{1}{2}+i}{\tanh(\frac{x}{2})-1}+\frac{1}{2(\tanh(\frac{x}{2})-1)^{2}}+\frac{3\ln(\tanh(}{2}$
parallelrisch	$\frac{(12i\cosh(\frac{x}{2})+12\sinh(\frac{x}{2}))\ln(\tanh(\frac{x}{2})-1)+(-12i\cosh(\frac{x}{2})-12\sinh(\frac{x}{2}))\ln(\tanh(\frac{x}{2})+1)+16i\sinh(\frac{x}{2})-3i\sinh(\frac{3x}{2})+i\sinh}{8i\cosh(\frac{x}{2})+8\sinh(\frac{x}{2})}$

input
```
int(sinh(x)^3/(I+sinh(x)),x,method=_RETURNVERBOSE)
```

output
```
-3/2*x+1/8*exp(x)^2-1/2*I*exp(x)-1/2*I/exp(x)-1/8/exp(x)^2-2*I/(exp(x)+I)
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 55, normalized size of antiderivative = 1.53

$$\int \frac{\sinh^3(x)}{i+\sinh(x)}\,dx = -\frac{4\,(3\,x-1)e^{(3\,x)}+4\,(3i\,x+5i)e^{(2\,x)}-e^{(5\,x)}+3i\,e^{(4\,x)}-3\,e^{x}+i}{8\,\left(e^{(3\,x)}+i\,e^{(2\,x)}\right)}$$

input
```
integrate(sinh(x)^3/(I+sinh(x)),x, algorithm="fricas")
```

output
```
-1/8*(4*(3*x - 1)*e^(3*x) + 4*(3*I*x + 5*I)*e^(2*x) - e^(5*x) + 3*I*e^(4*x
) - 3*e^x + I)/(e^(3*x) + I*e^(2*x))
```

Sympy [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 41, normalized size of antiderivative = 1.14

$$\int \frac{\sinh^3(x)}{i+\sinh(x)}\,dx = -\frac{3x}{2}+\frac{e^{2x}}{8}-\frac{ie^{x}}{2}-\frac{ie^{-x}}{2}-\frac{e^{-2x}}{8}-\frac{2i}{e^{x}+i}$$

input
```
integrate(sinh(x)**3/(I+sinh(x)),x)
```

output
```
-3*x/2 + exp(2*x)/8 - I*exp(x)/2 - I*exp(-x)/2 - exp(-2*x)/8 - 2*I/(exp(x)
 + I)
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 45, normalized size of antiderivative = 1.25

$$\int \frac{\sinh^3(x)}{i+\sinh(x)}\,dx = -\frac{3}{2}\,x - \frac{3\,e^{(-x)} + 20i\,e^{(-2\,x)} + i}{8\left(-i\,e^{(-2\,x)} + e^{(-3\,x)}\right)} - \frac{1}{2}i\,e^{(-x)} - \frac{1}{8}\,e^{(-2\,x)}$$

input
```
integrate(sinh(x)^3/(I+sinh(x)),x, algorithm="maxima")
```

output
```
-3/2*x - 1/8*(3*e^(-x) + 20*I*e^(-2*x) + I)/(-I*e^(-2*x) + e^(-3*x)) - 1/2
*I*e^(-x) - 1/8*e^(-2*x)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 38, normalized size of antiderivative = 1.06

$$\int \frac{\sinh^3(x)}{i+\sinh(x)}\,dx = -\frac{3}{2}\,x - \frac{\left(20i\,e^{(2\,x)} - 3\,e^{x} + i\right)e^{(-2\,x)}}{8\left(e^{x} + i\right)} + \frac{1}{8}\,e^{(2\,x)} - \frac{1}{2}i\,e^{x}$$

input
```
integrate(sinh(x)^3/(I+sinh(x)),x, algorithm="giac")
```

output
```
-3/2*x - 1/8*(20*I*e^(2*x) - 3*e^x + I)*e^(-2*x)/(e^x + I) + 1/8*e^(2*x) -
 1/2*I*e^x
```

Mupad [B] (verification not implemented)

Time = 1.73 (sec) , antiderivative size = 38, normalized size of antiderivative = 1.06

$$\int \frac{\sinh^3(x)}{i+\sinh(x)}\,dx = \frac{e^{2x}}{8} - \frac{e^{-x}\,1i}{2} - \frac{e^{-2x}}{8} - \frac{3x}{2} - \frac{e^{x}\,1i}{2} - \frac{2i}{e^{x}+1i}$$

input
```
int(sinh(x)^3/(sinh(x) + 1i),x)
```

output
```
exp(2*x)/8 - (exp(-x)*1i)/2 - exp(-2*x)/8 - (3*x)/2 - (exp(x)*1i)/2 - 2i/(
exp(x) + 1i)
```

Reduce [F]

$$\int \frac{\sinh^3(x)}{i+\sinh(x)}\,dx = \int \frac{\sinh(x)^3}{\sinh(x)+i}dx$$

input
```
int(sinh(x)^3/(I+sinh(x)),x)
```

output
```
int(sinh(x)**3/(sinh(x) + i),x)
```

3.42 $\int \frac{\sinh^2(x)}{i+\sinh(x)} dx$

Optimal result

Integrand size = 13, antiderivative size = 22

$$\int \frac{\sinh^2(x)}{i+\sinh(x)} dx = -ix + \cosh(x) + \frac{i\cosh(x)}{i+\sinh(x)}$$

output
```
-I*x+cosh(x)+I*cosh(x)/(I+sinh(x))
```

Mathematica [B] (verified)

Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 79 vs. 2(22) = 44.

Time = 0.09 (sec) , antiderivative size = 79, normalized size of antiderivative = 3.59

$$\int \frac{\sinh^2(x)}{i+\sinh(x)} dx = \frac{\cosh(x)\left(2i + \frac{2i\arcsin\left(\frac{\sqrt{1-i\sinh(x)}}{\sqrt{2}}\right)}{\sqrt{\cosh^2(x)}} + \sinh(x) + \frac{2\arcsin\left(\frac{\sqrt{1-i\sinh(x)}}{\sqrt{2}}\right)\sinh(x)}{\sqrt{\cosh^2(x)}}\right)}{i+\sinh(x)}$$

input
```
Integrate[Sinh[x]^2/(I + Sinh[x]),x]
```

output
```
(Cosh[x]*(2*I + ((2*I)*ArcSin[Sqrt[1 - I*Sinh[x]]/Sqrt[2]])/Sqrt[Cosh[x]^2
] + Sinh[x] + (2*ArcSin[Sqrt[1 - I*Sinh[x]]/Sqrt[2]]*Sinh[x])/Sqrt[Cosh[x]
^2]))/(I + Sinh[x])
```

Rubi [A] (verified)

Time = 0.33 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.36, number of steps used = 10, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.769$, Rules used = {3042, 25, 26, 3225, 26, 3042, 26, 3214, 3042, 3127}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sinh^2(x)}{\sinh(x)+i}dx$$
$$\Big\downarrow \text{3042}$$
$$\int -\frac{\sin(ix)^2}{i-i\sin(ix)}dx$$
$$\Big\downarrow \text{25}$$
$$-\int -\frac{i\sin(ix)^2}{1-\sin(ix)}dx$$
$$\Big\downarrow \text{26}$$
$$i\int \frac{\sin(ix)^2}{1-\sin(ix)}dx$$
$$\Big\downarrow \text{3225}$$
$$i\left(-\int -\frac{i\sinh(x)}{1-i\sinh(x)}dx-i\cosh(x)\right)$$
$$\Big\downarrow \text{26}$$
$$i\left(i\int \frac{\sinh(x)}{1-i\sinh(x)}dx-i\cosh(x)\right)$$
$$\Big\downarrow \text{3042}$$
$$i\left(i\int -\frac{i\sin(ix)}{1-\sin(ix)}dx-i\cosh(x)\right)$$

$$\Big\downarrow \text{26}$$

$$i\left(\int \frac{\sin(ix)}{1-\sin(ix)}dx - i\cosh(x)\right)$$

$$\Big\downarrow \text{3214}$$

$$i\left(\int \frac{1}{1-i\sinh(x)}dx - x - i\cosh(x)\right)$$

$$\Big\downarrow \text{3042}$$

$$i\left(\int \frac{1}{1-\sin(ix)}dx - x - i\cosh(x)\right)$$

$$\Big\downarrow \text{3127}$$

$$i\left(-x - i\cosh(x) - \frac{i\cosh(x)}{1-i\sinh(x)}\right)$$

input
```
Int[Sinh[x]^2/(I + Sinh[x]),x]
```

output
```
I*(-x - I*Cosh[x] - (I*Cosh[x])/(1 - I*Sinh[x]))
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3127
```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> Simp[-Cos[c +
 d*x]/(d*(b + a*Sin[c + d*x])), x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b
^2, 0]
```

rule 3214
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])/((c_.) + (d_.)*sin[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[b*(x/d), x] - Simp[(b*c - a*d)/d   Int[1/(c + d
*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

rule 3225
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^2/((c_.) + (d_.)*sin[(e_.) + (f
_.)*(x_)]), x_Symbol] :> Simp[(-b^2)*(Cos[e + f*x]/(d*f)), x] + Simp[1/d
Int[Simp[a^2*d - b*(b*c - 2*a*d)*Sin[e + f*x], x]/(c + d*Sin[e + f*x]), x],
 x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Maple [A] (verified)

Time = 0.27 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.14

method	result	size
risch	$-ix+\frac{{\rm e}^{x}}{2}+\frac{{\rm e}^{-x}}{2}+\frac{2}{{\rm e}^{x}+i}$	25
default	$i\ln\left(\tanh\left(\frac{x}{2}\right)-1\right)-\frac{1}{\tanh\left(\frac{x}{2}\right)-1}+\frac{2i}{\tanh\left(\frac{x}{2}\right)+i}-i\ln\left(\tanh\left(\frac{x}{2}\right)+1\right)+\frac{1}{\tanh\left(\frac{x}{2}\right)+1}$	52
parallelrisch	$\frac{(2x+7i)\cosh\left(\frac{x}{2}\right)-2ix\sinh\left(\frac{x}{2}\right)+i\cosh\left(\frac{3x}{2}\right)+\sinh\left(\frac{x}{2}\right)+\sinh\left(\frac{3x}{2}\right)}{2i\cosh\left(\frac{x}{2}\right)+2\sinh\left(\frac{x}{2}\right)}$	53

input
```
int(sinh(x)^2/(I+sinh(x)),x,method=_RETURNVERBOSE)
```

output
```
-I*x+1/2*exp(x)+1/2*exp(-x)+2/(exp(x)+I)
```

Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 37 vs. 2(16) = 32.

Time = 0.11 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.68

$$\int\frac{\sinh^2(x)}{i+\sinh(x)}\,dx=\frac{(-2i\,x+i)e^{(2\,x)}+(2\,x+5)e^{x}+e^{(3\,x)}+i}{2\,\left(e^{(2\,x)}+i\,e^{x}\right)}$$

input
```
integrate(sinh(x)^2/(I+sinh(x)),x, algorithm="fricas")
```

output
```
1/2*((-2*I*x + I)*e^(2*x) + (2*x + 5)*e^x + e^(3*x) + I)/(e^(2*x) + I*e^x)
```

Sympy [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.91

$$\int \frac{\sinh^2(x)}{i+\sinh(x)}\,dx = -ix + \frac{e^x}{2} + \frac{e^{-x}}{2} + \frac{2}{e^x+i}$$

input
```
integrate(sinh(x)**2/(I+sinh(x)),x)
```

output
```
-I*x + exp(x)/2 + exp(-x)/2 + 2/(exp(x) + I)
```

Maxima [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 33 vs. 2(16) = 32.

Time = 0.03 (sec) , antiderivative size = 33, normalized size of antiderivative = 1.50

$$\int \frac{\sinh^2(x)}{i+\sinh(x)}\,dx = -i\,x + \frac{5\,e^{(-x)} - i}{2\,\left(-i\,e^{(-x)} + e^{(-2\,x)}\right)} + \frac{1}{2}\,e^{(-x)}$$

input
```
integrate(sinh(x)^2/(I+sinh(x)),x, algorithm="maxima")
```

output
```
-I*x + 1/2*(5*e^(-x) - I)/(-I*e^(-x) + e^(-2*x)) + 1/2*e^(-x)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.18

$$\int \frac{\sinh^2(x)}{i+\sinh(x)}\,dx = -i\,x + \frac{(5\,e^x+i)e^{(-x)}}{2\,(e^x+i)} + \frac{1}{2}\,e^x$$

input
```
integrate(sinh(x)^2/(I+sinh(x)),x, algorithm="giac")
```

output
```
-I*x + 1/2*(5*e^x + I)*e^(-x)/(e^x + I) + 1/2*e^x
```

Mupad [B] (verification not implemented)

Time = 1.71 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.09

$$\int \frac{\sinh^2(x)}{i+\sinh(x)}\,dx = \frac{e^{-x}}{2} - x\,1i + \frac{e^x}{2} + \frac{2}{e^x+1i}$$

input
```
int(sinh(x)^2/(sinh(x) + 1i),x)
```

output
```
exp(-x)/2 - x*1i + exp(x)/2 + 2/(exp(x) + 1i)
```

Reduce [F]

$$\int \frac{\sinh^2(x)}{i+\sinh(x)}\,dx = \cosh(x) - \left(\int \frac{1}{\sinh(x)+i}dx\right) - ix$$

input
```
int(sinh(x)^2/(I+sinh(x)),x)
```

output
```
cosh(x) - int(1/(sinh(x) + i),x) - i*x
```

3.43 $\int \frac{\sinh(x)}{i+\sinh(x)}\,dx$

Optimal result

Integrand size = 11, antiderivative size = 14

$$\int \frac{\sinh(x)}{i+\sinh(x)}\,dx = x-\frac{\cosh(x)}{i+\sinh(x)}$$

output
```
x-cosh(x)/(I+sinh(x))
```

Mathematica [B] (verified)

Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 43 vs. 2(14) = 28.

Time = 0.05 (sec) , antiderivative size = 43, normalized size of antiderivative = 3.07

$$\int \frac{\sinh(x)}{i+\sinh(x)}\,dx = i\operatorname{sech}(x)\left(1+2\arcsin\left(\frac{\sqrt{1-i\sinh(x)}}{\sqrt{2}}\right)\sqrt{\cosh^2(x)}+i\sinh(x)\right)$$

input
```
Integrate[Sinh[x]/(I + Sinh[x]),x]
```

output
```
I*Sech[x]*(1 + 2*ArcSin[Sqrt[1 - I*Sinh[x]]/Sqrt[2]]*Sqrt[Cosh[x]^2] + I*S
inh[x])
```

Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.29, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.545, Rules used = {3042, 26, 26, 3214, 3042, 3127}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sinh(x)}{\sinh(x)+i} \, dx$$
$$\Big\downarrow \text{3042}$$
$$\int -\frac{i\sin(ix)}{i-i\sin(ix)}dx$$
$$\Big\downarrow \text{26}$$
$$-i\int -\frac{i\sin(ix)}{1-\sin(ix)}dx$$
$$\Big\downarrow \text{26}$$
$$-\int \frac{\sin(ix)}{1-\sin(ix)}dx$$
$$\Big\downarrow \text{3214}$$
$$x-\int \frac{1}{1-i\sinh(x)}dx$$
$$\Big\downarrow \text{3042}$$
$$x-\int \frac{1}{1-\sin(ix)}dx$$
$$\Big\downarrow \text{3127}$$
$$x+\frac{i\cosh(x)}{1-i\sinh(x)}$$

input
```
Int[Sinh[x]/(I + Sinh[x]),x]
```

output
```
x + (I*Cosh[x])/(1 - I*Sinh[x])
```

Defintions of rubi rules used

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3127
```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> Simp[-Cos[c +
 d*x]/(d*(b + a*Sin[c + d*x])), x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b
^2, 0]
```

rule 3214
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])/((c_.) + (d_.)*sin[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[b*(x/d), x] - Simp[(b*c - a*d)/d   Int[1/(c + d
*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Maple [A] (verified)

Time = 0.17 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.93

method	result	size
risch	$x+\frac{2i}{{\mathrm e}^{x}+i}$	13
parallelrisch	$\frac{-2+i x+\tanh\left(\frac{x}{2}\right) x}{\tanh\left(\frac{x}{2}\right)+i}$	23
default	$\ln\left(\tanh\left(\frac{x}{2}\right)+1\right)-\ln\left(\tanh\left(\frac{x}{2}\right)-1\right)-\frac{2}{\tanh\left(\frac{x}{2}\right)+i}$	29

input
```
int(sinh(x)/(I+sinh(x)),x,method=_RETURNVERBOSE)
```

output
```
x+2*I/(exp(x)+I)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.14

$$\int \frac{\sinh(x)}{i+\sinh(x)}\,dx = \frac{xe^x + i\,x + 2i}{e^x + i}$$

input
```
integrate(sinh(x)/(I+sinh(x)),x, algorithm="fricas")
```

output
```
(x*e^x + I*x + 2*I)/(e^x + I)
```

Sympy [A] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 8, normalized size of antiderivative = 0.57

$$\int \frac{\sinh(x)}{i+\sinh(x)}\,dx = x + \frac{2i}{e^x + i}$$

input
```
integrate(sinh(x)/(I+sinh(x)),x)
```

output
```
x + 2*I/(exp(x) + I)
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.86

$$\int \frac{\sinh(x)}{i+\sinh(x)}\,dx = x + \frac{2i}{e^{(-x)} - i}$$

input
```
integrate(sinh(x)/(I+sinh(x)),x, algorithm="maxima")
```

output
```
x + 2*I/(e^(-x) - I)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.71

$$\int \frac{\sinh(x)}{i+\sinh(x)}\,dx = x + \frac{2i}{e^x+i}$$

input
```
integrate(sinh(x)/(I+sinh(x)),x, algorithm="giac")
```

output
```
x + 2*I/(e^x + I)
```

Mupad [B] (verification not implemented)

Time = 1.71 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.86

$$\int \frac{\sinh(x)}{i+\sinh(x)}\,dx = x + \frac{2\mathrm{i}}{\mathrm{e}^x+1\mathrm{i}}$$

input
```
int(sinh(x)/(sinh(x) + 1i),x)
```

output
```
x + 2i/(exp(x) + 1i)
```

Reduce [F]

$$\int \frac{\sinh(x)}{i+\sinh(x)}\,dx = -\left(\int \frac{1}{\sinh(x)+i}dx\right)i + x$$

input
```
int(sinh(x)/(I+sinh(x)),x)
```

output
```
- int(1/(sinh(x) + i),x)*i + x
```

3.44 $\int \frac{\operatorname{csch}(x)}{i+\sinh(x)}\,dx$

Optimal result

Integrand size = 11, antiderivative size = 19

$$\int \frac{\operatorname{csch}(x)}{i+\sinh(x)}\,dx = i\operatorname{arctanh}(\cosh(x)) + \frac{\cosh(x)}{i+\sinh(x)}$$

output
```
I*arctanh(cosh(x))+cosh(x)/(I+sinh(x))
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.58

$$\int \frac{\operatorname{csch}(x)}{i+\sinh(x)}\,dx = \operatorname{sech}(x)\left(-i + i\operatorname{arctanh}\left(\sqrt{\cosh^2(x)}\right)\sqrt{\cosh^2(x)} + \sinh(x)\right)$$

input
```
Integrate[Csch[x]/(I + Sinh[x]),x]
```

output
```
Sech[x]*(-I + I*ArcTanh[Sqrt[Cosh[x]^2]]*Sqrt[Cosh[x]^2] + Sinh[x])
```

Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.26, number of steps used = 9, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.818, Rules used = {3042, 26, 26, 3226, 26, 3042, 26, 3127, 4257}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\text{csch}(x)}{\sinh(x)+i}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{i}{(i-i\sin(ix))\sin(ix)}dx$$

$$\Big\downarrow \text{26}$$

$$i\int -\frac{i}{(1-\sin(ix))\sin(ix)}dx$$

$$\Big\downarrow \text{26}$$

$$\int \frac{1}{(1-\sin(ix))\sin(ix)}dx$$

$$\Big\downarrow \text{3226}$$

$$\int \frac{1}{1-i\sinh(x)}dx+\int -i\text{csch}(x)dx$$

$$\Big\downarrow \text{26}$$

$$\int \frac{1}{1-i\sinh(x)}dx-i\int \text{csch}(x)dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{1-\sin(ix)}dx-i\int i\csc(ix)dx$$

$$\Big\downarrow \text{26}$$

$$\int \frac{1}{1-\sin(ix)}dx+\int \csc(ix)dx$$

$$\Big\downarrow \text{3127}$$

$$\int \csc(ix)dx-\frac{i\cosh(x)}{1-i\sinh(x)}$$

$$\Big\downarrow \text{4257}$$

$$i\text{arctanh}(\cosh(x)) - \frac{i\cosh(x)}{1 - i\sinh(x)}$$

input
```
Int[Csch[x]/(I + Sinh[x]),x]
```

output
```
I*ArcTanh[Cosh[x]] - (I*Cosh[x])/(1 - I*Sinh[x])
```

Defintions of rubi rules used

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3127
```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> Simp[-Cos[c +
 d*x]/(d*(b + a*Sin[c + d*x])), x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b
^2, 0]
```

rule 3226
```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (
f_.)*(x_)])), x_Symbol] :> Simp[b/(b*c - a*d)   Int[1/(a + b*Sin[e + f*x]),
 x], x] - Simp[d/(b*c - a*d)   Int[1/(c + d*Sin[e + f*x]), x], x] /; FreeQ[
{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

rule 4257
```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Maple [A] (verified)

Time = 0.27 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.11

method	result	size
default	$-i \ln\left(\tanh\left(\frac{x}{2}\right)\right)+\frac{2}{\tanh\left(\frac{x}{2}\right)+i}$	21
risch	$-\frac{2i}{\mathrm{e}^{x}+i}-i\ln\left(\mathrm{e}^{x}-1\right)+i\ln\left(\mathrm{e}^{x}+1\right)$	28
parallelrisch	$-\frac{-2+\ln\left(\tanh\left(\frac{x}{2}\right)\right)\left(i\tanh\left(\frac{x}{2}\right)-1\right)}{\tanh\left(\frac{x}{2}\right)+i}$	29

input
```
int(csch(x)/(I+sinh(x)),x,method=_RETURNVERBOSE)
```

output
```
-I*ln(tanh(1/2*x))+2/(tanh(1/2*x)+I)
```

Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 33 vs. 2(15) = 30.

Time = 0.08 (sec) , antiderivative size = 33, normalized size of antiderivative = 1.74

$$\int \frac{\operatorname{csch}(x)}{i+\sinh(x)}\,dx=\frac{(i\,e^{x}-1)\log\left(e^{x}+1\right)+(-i\,e^{x}+1)\log\left(e^{x}-1\right)-2i}{e^{x}+i}$$

input
```
integrate(csch(x)/(I+sinh(x)),x, algorithm="fricas")
```

output
```
((I*e^x - 1)*log(e^x + 1) + (-I*e^x + 1)*log(e^x - 1) - 2*I)/(e^x + I)
```

Sympy [F]

$$\int \frac{\operatorname{csch}(x)}{i+\sinh(x)}\,dx = \int \frac{\operatorname{csch}(x)}{\sinh(x)+i}\,dx$$

input
```
integrate(csch(x)/(I+sinh(x)),x)
```

output
```
Integral(csch(x)/(sinh(x) + I), x)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.53

$$\int \frac{\operatorname{csch}(x)}{i+\sinh(x)}\,dx = -\frac{2i}{e^{(-x)}-i} + i\log\left(e^{(-x)}+1\right) - i\log\left(e^{(-x)}-1\right)$$

input
```
integrate(csch(x)/(I+sinh(x)),x, algorithm="maxima")
```

output
```
-2*I/(e^(-x) - I) + I*log(e^(-x) + 1) - I*log(e^(-x) - 1)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.26

$$\int \frac{\operatorname{csch}(x)}{i+\sinh(x)}\,dx = -\frac{2i}{e^{x}+i} + i\log\left(e^{x}+1\right) - i\log\left(|e^{x}-1|\right)$$

input
```
integrate(csch(x)/(I+sinh(x)),x, algorithm="giac")
```

output
```
-2*I/(e^x + I) + I*log(e^x + 1) - I*log(abs(e^x - 1))
```

Mupad [B] (verification not implemented)

Time = 1.81 (sec) , antiderivative size = 35, normalized size of antiderivative = 1.84

$$\int \frac{\mathrm{csch}(x)}{i+\sinh(x)}\,dx = -\ln\left(\mathrm{e}^x\,2\mathrm{i}-2\mathrm{i}\right)\,1\mathrm{i}+\ln\left(\mathrm{e}^x\,2\mathrm{i}+2\mathrm{i}\right)\,1\mathrm{i}-\frac{2\mathrm{i}}{\mathrm{e}^x+1\mathrm{i}}$$

input
```
int(1/(sinh(x)*(sinh(x) + 1i)),x)
```

output
```
log(exp(x)*2i + 2i)*1i - log(exp(x)*2i - 2i)*1i - 2i/(exp(x) + 1i)
```

Reduce [F]

$$\int \frac{\mathrm{csch}(x)}{i+\sinh(x)}\,dx = \int \frac{\mathrm{csch}(x)}{\sinh(x)+i}dx$$

input
```
int(csch(x)/(I+sinh(x)),x)
```

output
```
int(csch(x)/(sinh(x) + i),x)
```

3.45 $\int \frac{\operatorname{csch}^2(x)}{i+\sinh(x)}\,dx$

Optimal result

Integrand size = 13, antiderivative size = 23

$$\int \frac{\operatorname{csch}^2(x)}{i+\sinh(x)}\,dx = -\operatorname{arctanh}(\cosh(x)) + 2i\coth(x) + \frac{\coth(x)}{i+\sinh(x)}$$

output
```
-arctanh(cosh(x))+2*I*coth(x)+coth(x)/(I+sinh(x))
```

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.57

$$\int \frac{\operatorname{csch}^2(x)}{i+\sinh(x)}\,dx = \operatorname{sech}(x)\left(1-\operatorname{arctanh}\left(\sqrt{\cosh^2(x)}\right)\sqrt{\cosh^2(x)}+i\operatorname{csch}(x)+2i\sinh(x)\right)$$

input
```
Integrate[Csch[x]^2/(I + Sinh[x]),x]
```

output
```
Sech[x]*(1 - ArcTanh[Sqrt[Cosh[x]^2]]*Sqrt[Cosh[x]^2] + I*Csch[x] + (2*I)*
Sinh[x])
```

Rubi [A] (verified)

Time = 0.37 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.30, number of steps used = 16, number of rules used = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 1.154, Rules used = {3042, 25, 26, 3247, 3042, 25, 3227, 25, 26, 3042, 25, 26, 4254, 24, 4257}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\text{csch}^2(x)}{\sinh(x)+i}dx$$

$$\Big\downarrow \ 3042$$

$$\int -\frac{1}{(i-i\sin(ix))\sin(ix)^2}dx$$

$$\Big\downarrow \ 25$$

$$-\int -\frac{i}{(1-\sin(ix))\sin(ix)^2}dx$$

$$\Big\downarrow \ 26$$

$$i\int \frac{1}{(1-\sin(ix))\sin(ix)^2}dx$$

$$\Big\downarrow \ 3247$$

$$i\left(-\int \text{csch}^2(x)(i\sinh(x)+2)dx-\frac{\coth(x)}{1-i\sinh(x)}\right)$$

$$\Big\downarrow \ 3042$$

$$i\left(-\int -\frac{\sin(ix)+2}{\sin(ix)^2}dx-\frac{\coth(x)}{1-i\sinh(x)}\right)$$

$$\Big\downarrow \ 25$$

$$i\left(\int \frac{\sin(ix)+2}{\sin(ix)^2}dx-\frac{\coth(x)}{1-i\sinh(x)}\right)$$

$$\Big\downarrow \ 3227$$

$$i\left(2\int -\text{csch}^2(x)dx+\int -i\text{csch}(x)dx-\frac{\coth(x)}{1-i\sinh(x)}\right)$$

$$\Big\downarrow \ 25$$

$$i\left(-2\int \operatorname{csch}^2(x)dx+\int -i\operatorname{csch}(x)dx-\frac{\coth(x)}{1-i\sinh(x)}\right)$$

$\Big\downarrow$ 26

$$i\left(-2\int \operatorname{csch}^2(x)dx-i\int \operatorname{csch}(x)dx-\frac{\coth(x)}{1-i\sinh(x)}\right)$$

$\Big\downarrow$ 3042

$$i\left(-i\int i\csc(ix)dx-2\int -\csc(ix)^2dx-\frac{\coth(x)}{1-i\sinh(x)}\right)$$

$\Big\downarrow$ 25

$$i\left(-i\int i\csc(ix)dx+2\int \csc(ix)^2dx-\frac{\coth(x)}{1-i\sinh(x)}\right)$$

$\Big\downarrow$ 26

$$i\left(\int \csc(ix)dx+2\int \csc(ix)^2dx-\frac{\coth(x)}{1-i\sinh(x)}\right)$$

$\Big\downarrow$ 4254

$$i\left(\int \csc(ix)dx+2i\int 1d(-i\coth(x))-\frac{\coth(x)}{1-i\sinh(x)}\right)$$

$\Big\downarrow$ 24

$$i\left(\int \csc(ix)dx+2\coth(x)-\frac{\coth(x)}{1-i\sinh(x)}\right)$$

$\Big\downarrow$ 4257

$$i\left(i\text{arctanh}(\cosh(x))+2\coth(x)-\frac{\coth(x)}{1-i\sinh(x)}\right)$$

input
```
Int[Csch[x]^2/(I + Sinh[x]),x]
```

output
```
I*(I*ArcTanh[Cosh[x]] + 2*Coth[x] - Coth[x]/(1 - I*Sinh[x]))
```

Defintions of rubi rules used

rule 24
```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3227
```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Simp[c   Int[(b*Sin[e + f*x])^m, x], x] + Simp[d/b   Int
[(b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

rule 3247
```
Int[((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_)/((a_) + (b_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Simp[(-b^2)*Cos[e + f*x]*((c + d*Sin[e + f*x])^(
n + 1)/(a*f*(b*c - a*d)*(a + b*Sin[e + f*x]))), x] + Simp[d/(a*(b*c - a*d))
   Int[(c + d*Sin[e + f*x])^n*(a*n - b*(n + 1)*Sin[e + f*x]), x], x] /; Fre
eQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[
c^2 - d^2, 0] && LtQ[n, 0] && (IntegerQ[2*n] || EqQ[c, 0])
```

rule 4254
```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Simp[-d^(-1)   Subst[Int[Exp
andIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
 d}, x] && IGtQ[n/2, 0]
```

rule 4257
```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Maple [A] (verified)

Time = 0.39 (sec) , antiderivative size = 35, normalized size of antiderivative = 1.52

method	result	size
default	$\frac{i\tanh\left(\frac{x}{2}\right)}{2}+\frac{i}{2\tanh\left(\frac{x}{2}\right)}+\ln\left(\tanh\left(\frac{x}{2}\right)\right)+\frac{2i}{\tanh\left(\frac{x}{2}\right)+i}$	35
risch	$\frac{-4+2i\,\mathrm{e}^{x}+2\,\mathrm{e}^{2x}}{\left(\mathrm{e}^{2x}-1\right)\left(\mathrm{e}^{x}+i\right)}+\ln\left(\mathrm{e}^{x}-1\right)-\ln\left(\mathrm{e}^{x}+1\right)$	42
parallelrisch	$\frac{\left(2\tanh\left(\frac{x}{2}\right)+2i\right)\ln\left(\tanh\left(\frac{x}{2}\right)\right)+i\tanh\left(\frac{x}{2}\right)^{2}+6i-\coth\left(\frac{x}{2}\right)}{2\tanh\left(\frac{x}{2}\right)+2i}$	46

input
```
int(csch(x)^2/(I+sinh(x)),x,method=_RETURNVERBOSE)
```

output
```
1/2*I*tanh(1/2*x)+1/2*I/tanh(1/2*x)+ln(tanh(1/2*x))+2*I/(tanh(1/2*x)+I)
```

Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 77 vs. 2(19) = 38.

Time = 0.09 (sec) , antiderivative size = 77, normalized size of antiderivative = 3.35

$$\int \frac{\operatorname{csch}^2(x)}{i+\sinh(x)}\,dx = -\frac{\left(e^{(3x)}+i\,e^{(2x)}-e^{x}-i\right)\log\left(e^{x}+1\right)-\left(e^{(3x)}+i\,e^{(2x)}-e^{x}-i\right)\log\left(e^{x}-1\right)-2\,e^{(2x)}-2i\,e^{x}+4}{e^{(3x)}+i\,e^{(2x)}-e^{x}-i}$$

input
```
integrate(csch(x)^2/(I+sinh(x)),x, algorithm="fricas")
```

output
```
-((e^(3*x) + I*e^(2*x) - e^x - I)*log(e^x + 1) - (e^(3*x) + I*e^(2*x) - e^
x - I)*log(e^x - 1) - 2*e^(2*x) - 2*I*e^x + 4)/(e^(3*x) + I*e^(2*x) - e^x
- I)
```

Sympy [F]

$$\int \frac{\operatorname{csch}^2(x)}{i+\sinh(x)}\,dx = \int \frac{\operatorname{csch}^2(x)}{\sinh(x)+i}\,dx$$

input
```
integrate(csch(x)**2/(I+sinh(x)),x)
```

output
```
Integral(csch(x)**2/(sinh(x) + I), x)
```

Maxima [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 51 vs. $2(19) = 38$.

Time = 0.04 (sec) , antiderivative size = 51, normalized size of antiderivative = 2.22

$$\int \frac{\operatorname{csch}^2(x)}{i+\sinh(x)}\,dx = -\frac{2\left(-i\,e^{(-x)}+e^{(-2\,x)}-2\right)}{e^{(-x)}+i\,e^{(-2\,x)}-e^{(-3\,x)}-i} - \log\left(e^{(-x)}+1\right) + \log\left(e^{(-x)}-1\right)$$

input
```
integrate(csch(x)^2/(I+sinh(x)),x, algorithm="maxima")
```

output
```
-2*(-I*e^(-x) + e^(-2*x) - 2)/(e^(-x) + I*e^(-2*x) - e^(-3*x) - I) - log(e
^(-x) + 1) + log(e^(-x) - 1)
```

Giac [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 44 vs. $2(19) = 38$.

Time = 0.12 (sec) , antiderivative size = 44, normalized size of antiderivative = 1.91

$$\int \frac{\operatorname{csch}^2(x)}{i+\sinh(x)}\,dx = \frac{2\left(e^{(2\,x)}+i\,e^{x}-2\right)}{e^{(3\,x)}+i\,e^{(2\,x)}-e^{x}-i} - \log\left(e^{x}+1\right) + \log\left(|e^{x}-1|\right)$$

input
```
integrate(csch(x)^2/(I+sinh(x)),x, algorithm="giac")
```

output
```
2*(e^(2*x) + I*e^x - 2)/(e^(3*x) + I*e^(2*x) - e^x - I) - log(e^x + 1) + l
og(abs(e^x - 1))
```

Mupad [B] (verification not implemented)

Time = 1.94 (sec) , antiderivative size = 51, normalized size of antiderivative = 2.22

$$\int \frac{\operatorname{csch}^2(x)}{i+\sinh(x)}\,dx = \ln\left(2-2\,\mathrm{e}^{x}\right)-\ln\left(-2\,\mathrm{e}^{x}-2\right)+\frac{2\,\mathrm{e}^{2\,x}-4+\mathrm{e}^{x}\,2\mathrm{i}}{\mathrm{e}^{2\,x}\,1\mathrm{i}+\mathrm{e}^{3\,x}-\mathrm{e}^{x}-\mathrm{i}}$$

input
```
int(1/(sinh(x)^2*(sinh(x) + 1i)),x)
```

output
```
log(2 - 2*exp(x)) - log(- 2*exp(x) - 2) + (2*exp(2*x) + exp(x)*2i - 4)/(ex
p(2*x)*1i + exp(3*x) - exp(x) - 1i)
```

Reduce [F]

$$\int \frac{\operatorname{csch}^2(x)}{i+\sinh(x)}\,dx = \int \frac{\operatorname{csch}(x)^2}{\sinh(x)+i}dx$$

input
```
int(csch(x)^2/(I+sinh(x)),x)
```

output
```
int(csch(x)**2/(sinh(x) + i),x)
```

3.46 $\int \frac{\operatorname{csch}^3(x)}{i+\sinh(x)} \, dx$

Optimal result

Integrand size = 13, antiderivative size = 37

$$\int \frac{\operatorname{csch}^3(x)}{i+\sinh(x)} \, dx = -\frac{3}{2} i \operatorname{arctanh}(\cosh(x)) - 2\coth(x) + \frac{3}{2} i \coth(x)\operatorname{csch}(x) + \frac{\coth(x)\operatorname{csch}(x)}{i+\sinh(x)}$$

output
```
-3/2*I*arctanh(cosh(x))-2*coth(x)+3/2*I*coth(x)*csch(x)+coth(x)*csch(x)/(I
+sinh(x))
```

Mathematica [A] (verified)

Time = 0.14 (sec) , antiderivative size = 49, normalized size of antiderivative = 1.32

$$\int \frac{\operatorname{csch}^3(x)}{i+\sinh(x)} \, dx = \frac{1}{2} i \left(4i + 3\operatorname{csch}(x) - 3\operatorname{arctanh}\left(\sqrt{\cosh^2(x)}\right) \sqrt{\cosh^2(x)}\operatorname{csch}(x) + 2i\operatorname{csch}^2(x) + \operatorname{csch}^3(x)\right) \tanh(x)$$

input
```
Integrate[Csch[x]^3/(I + Sinh[x]),x]
```

output
```
(I/2)*(4*I + 3*Csch[x] - 3*ArcTanh[Sqrt[Cosh[x]^2]]*Sqrt[Cosh[x]^2]*Csch[x
] + (2*I)*Csch[x]^2 + Csch[x]^3)*Tanh[x]
```

Rubi [A] (verified)

Time = 0.46 (sec) , antiderivative size = 45, normalized size of antiderivative = 1.22, number of steps used = 21, number of rules used = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 1.538, Rules used = {3042, 26, 26, 3247, 26, 3042, 26, 3227, 25, 26, 3042, 25, 26, 4254, 24, 4255, 26, 3042, 26, 4257}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\text{csch}^3(x)}{\sinh(x)+i} \, dx$$

$$\Big\downarrow \text{3042}$$

$$\int -\frac{i}{(i-i\sin(ix))\sin(ix)^3}dx$$

$$\Big\downarrow \text{26}$$

$$-i\int -\frac{i}{(1-\sin(ix))\sin(ix)^3}dx$$

$$\Big\downarrow \text{26}$$

$$-\int \frac{1}{(1-\sin(ix))\sin(ix)^3}dx$$

$$\Big\downarrow \text{3247}$$

$$\int -i\text{csch}^3(x)(2i\sinh(x)+3)dx-\frac{i\coth(x)\text{csch}(x)}{1-i\sinh(x)}$$

$$\Big\downarrow \text{26}$$

$$-i\int \text{csch}^3(x)(2i\sinh(x)+3)dx-\frac{i\coth(x)\text{csch}(x)}{1-i\sinh(x)}$$

$$\Big\downarrow \text{3042}$$

$$-i\int -\frac{i(2\sin(ix)+3)}{\sin(ix)^3}dx-\frac{i\coth(x)\text{csch}(x)}{1-i\sinh(x)}$$

$$\Big\downarrow \text{26}$$

$$-\int \frac{2\sin(ix)+3}{\sin(ix)^3}dx-\frac{i\coth(x)\text{csch}(x)}{1-i\sinh(x)}$$

$$\Big\downarrow \text{3227}$$

$$-3\int i\text{csch}^3(x)dx-2\int -\text{csch}^2(x)dx-\frac{i\coth(x)\text{csch}(x)}{1-i\sinh(x)}$$

$$\Big\downarrow \text{25}$$

$$-3\int i\text{csch}^3(x)dx+2\int \text{csch}^2(x)dx-\frac{i\coth(x)\text{csch}(x)}{1-i\sinh(x)}$$

$$\Big\downarrow \text{26}$$

$$-3i\int \text{csch}^3(x)dx+2\int \text{csch}^2(x)dx-\frac{i\coth(x)\text{csch}(x)}{1-i\sinh(x)}$$

$$\Big\downarrow \text{3042}$$

$$2\int -\csc(ix)^2dx-3i\int -i\csc(ix)^3dx-\frac{i\coth(x)\text{csch}(x)}{1-i\sinh(x)}$$

$$\Big\downarrow \text{25}$$

$$-2\int \csc(ix)^2dx-3i\int -i\csc(ix)^3dx-\frac{i\coth(x)\text{csch}(x)}{1-i\sinh(x)}$$

$$\Big\downarrow \text{26}$$

$$-2\int \csc(ix)^2dx-3\int \csc(ix)^3dx-\frac{i\coth(x)\text{csch}(x)}{1-i\sinh(x)}$$

$$\Big\downarrow \text{4254}$$

$$-3\int \csc(ix)^3dx-2i\int 1d(-i\coth(x))-\frac{i\coth(x)\text{csch}(x)}{1-i\sinh(x)}$$

$$\Big\downarrow \text{24}$$

$$-3\int \csc(ix)^3dx-2\coth(x)-\frac{i\coth(x)\text{csch}(x)}{1-i\sinh(x)}$$

$$\Big\downarrow \text{4255}$$

$$-3\left(\frac{1}{2}\int -i\text{csch}(x)dx-\frac{1}{2}i\coth(x)\text{csch}(x)\right)-2\coth(x)-\frac{i\coth(x)\text{csch}(x)}{1-i\sinh(x)}$$

$$\Big\downarrow \text{26}$$

$$-3\left(-\frac{1}{2}i\int \text{csch}(x)dx-\frac{1}{2}i\coth(x)\text{csch}(x)\right)-2\coth(x)-\frac{i\coth(x)\text{csch}(x)}{1-i\sinh(x)}$$

$$\Big\downarrow \text{3042}$$

$$-3\left(-\frac{1}{2}i\int i\csc(ix)dx-\frac{1}{2}i\coth(x)\text{csch}(x)\right)-2\coth(x)-\frac{i\coth(x)\text{csch}(x)}{1-i\sinh(x)}$$

$$\Big\downarrow \text{26}$$

$$-3\left(\frac{1}{2}\int \csc(ix)dx-\frac{1}{2}i\coth(x)\text{csch}(x)\right)-2\coth(x)-\frac{i\coth(x)\text{csch}(x)}{1-i\sinh(x)}$$

$$\Big\downarrow \text{4257}$$

$$-3\left(\frac{1}{2}i\text{arctanh}(\cosh(x))-\frac{1}{2}i\coth(x)\text{csch}(x)\right)-2\coth(x)-\frac{i\coth(x)\text{csch}(x)}{1-i\sinh(x)}$$

input
```
Int[Csch[x]^3/(I + Sinh[x]),x]
```

output
```
-2*Coth[x] - 3*((I/2)*ArcTanh[Cosh[x]] - (I/2)*Coth[x]*Csch[x]) - (I*Coth[
x]*Csch[x])/(1 - I*Sinh[x])
```

Defintions of rubi rules used

rule 24
```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3227
```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Simp[c   Int[(b*Sin[e + f*x])^m, x], x] + Simp[d/b   Int
[(b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

rule 3247
```
Int[((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_)/((a_) + (b_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Simp[(-b^2)*Cos[e + f*x]*((c + d*Sin[e + f*x])^(
n + 1)/(a*f*(b*c - a*d)*(a + b*Sin[e + f*x]))), x] + Simp[d/(a*(b*c - a*d))
   Int[(c + d*Sin[e + f*x])^n*(a*n - b*(n + 1)*Sin[e + f*x]), x], x] /; Fre
eQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[
c^2 - d^2, 0] && LtQ[n, 0] && (IntegerQ[2*n] || EqQ[c, 0])
```

rule 4254
```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Simp[-d^(-1)   Subst[Int[Exp
andIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
 d}, x] && IGtQ[n/2, 0]
```

rule 4255
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Csc[c + d*x])^(n - 1)/(d*(n - 1))), x] + Simp[b^2*((n - 2)/(n - 1))
  Int[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
&& IntegerQ[2*n]
```

rule 4257
```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Maple [A] (verified)

Time = 0.53 (sec) , antiderivative size = 53, normalized size of antiderivative = 1.43

method	result	size
default	$-\frac{\tanh\left(\frac{x}{2}\right)}{2}-\frac{i\tanh\left(\frac{x}{2}\right)^{2}}{8}-\frac{2}{\tanh\left(\frac{x}{2}\right)+i}+\frac{i}{8\tanh\left(\frac{x}{2}\right)^{2}}+\frac{3i\ln\left(\tanh\left(\frac{x}{2}\right)\right)}{2}-\frac{1}{2\tanh\left(\frac{x}{2}\right)}$	53
risch	$\frac{i\left(3\,\mathrm{e}^{4x}-5\,\mathrm{e}^{2x}+3i\mathrm{e}^{3x}+4-i\mathrm{e}^{x}\right)}{\left(\mathrm{e}^{2x}-1\right)^{2}\left(\mathrm{e}^{x}+i\right)}+\frac{3i\ln\left(\mathrm{e}^{x}-1\right)}{2}-\frac{3i\ln\left(\mathrm{e}^{x}+1\right)}{2}$	62
parallelrisch	$\frac{\left(12i\tanh\left(\frac{x}{2}\right)-12\right)\ln\left(\tanh\left(\frac{x}{2}\right)\right)-i\tanh\left(\frac{x}{2}\right)^{3}-3i\coth\left(\frac{x}{2}\right)-\coth\left(\frac{x}{2}\right)^{2}-3\tanh\left(\frac{x}{2}\right)^{2}-24}{8\tanh\left(\frac{x}{2}\right)+8i}$	62

input
```
int(csch(x)^3/(I+sinh(x)),x,method=_RETURNVERBOSE)
```

output
```
-1/2*tanh(1/2*x)-1/8*I*tanh(1/2*x)^2-2/(tanh(1/2*x)+I)+1/8*I/tanh(1/2*x)^2
+3/2*I*ln(tanh(1/2*x))-1/2/tanh(1/2*x)
```

Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 126 vs. $2(27) = 54$.

Time = 0.10 (sec) , antiderivative size = 126, normalized size of antiderivative = 3.41

$$\int \frac{\operatorname{csch}^3(x)}{i+\sinh(x)}\,dx = -\frac{3\left(i\,e^{(5\,x)} - e^{(4\,x)} - 2i\,e^{(3\,x)} + 2\,e^{(2\,x)} + i\,e^{x} - 1\right)\log\left(e^{x}+1\right) + 3\left(-i\,e^{(5\,x)} + e^{(4\,x)} + 2i\,e^{(3\,x)} - 2\,e^{(2\,x)} - \cdots}{2\left(e^{(5\,x)} + i\,e^{(4\,x)} - 2\,e^{(3\,x)} - 2i\,e^{(2\,x)} + e^{x} + \cdots\right.}$$

input
```
integrate(csch(x)^3/(I+sinh(x)),x, algorithm="fricas")
```

output
```
-1/2*(3*(I*e^(5*x) - e^(4*x) - 2*I*e^(3*x) + 2*e^(2*x) + I*e^x - 1)*log(e^
x + 1) + 3*(-I*e^(5*x) + e^(4*x) + 2*I*e^(3*x) - 2*e^(2*x) - I*e^x + 1)*lo
g(e^x - 1) - 6*I*e^(4*x) + 6*e^(3*x) + 10*I*e^(2*x) - 2*e^x - 8*I)/(e^(5*x
) + I*e^(4*x) - 2*e^(3*x) - 2*I*e^(2*x) + e^x + I)
```

Sympy [F]

$$\int \frac{\operatorname{csch}^3(x)}{i+\sinh(x)}\,dx = \int \frac{\operatorname{csch}^3(x)}{\sinh(x)+i}\,dx$$

input
```
integrate(csch(x)**3/(I+sinh(x)),x)
```

output
```
Integral(csch(x)**3/(sinh(x) + I), x)
```

Maxima [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 75 vs. 2(27) = 54.

Time = 0.04 (sec) , antiderivative size = 75, normalized size of antiderivative = 2.03

$$\int \frac{\operatorname{csch}^3(x)}{i+\sinh(x)}\,dx = -\frac{e^{(-x)}+5i\,e^{(-2\,x)}-3\,e^{(-3\,x)}-3i\,e^{(-4\,x)}-4i}{e^{(-x)}+2i\,e^{(-2\,x)}-2\,e^{(-3\,x)}-i\,e^{(-4\,x)}+e^{(-5\,x)}-i} - \frac{3}{2}i\,\log\left(e^{(-x)}+1\right)+\frac{3}{2}i\,\log\left(e^{(-x)}-1\right)$$

input
```
integrate(csch(x)^3/(I+sinh(x)),x, algorithm="maxima")
```

output
```
-(e^(-x) + 5*I*e^(-2*x) - 3*e^(-3*x) - 3*I*e^(-4*x) - 4*I)/(e^(-x) + 2*I*e
^(-2*x) - 2*e^(-3*x) - I*e^(-4*x) + e^(-5*x) - I) - 3/2*I*log(e^(-x) + 1)
+ 3/2*I*log(e^(-x) - 1)
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 51, normalized size of antiderivative = 1.38

$$\int \frac{\operatorname{csch}^3(x)}{i+\sinh(x)}\,dx = \frac{i\,e^{(3\,x)}-2\,e^{(2\,x)}+i\,e^{x}+2}{\left(e^{(2\,x)}-1\right)^2}+\frac{2i}{e^x+i}-\frac{3}{2}i\,\log\left(e^x+1\right)+\frac{3}{2}i\,\log\left(|e^x-1|\right)$$

input
```
integrate(csch(x)^3/(I+sinh(x)),x, algorithm="giac")
```

output
```
(I*e^(3*x) - 2*e^(2*x) + I*e^x + 2)/(e^(2*x) - 1)^2 + 2*I/(e^x + I) - 3/2*
I*log(e^x + 1) + 3/2*I*log(abs(e^x - 1))
```

Mupad [B] (verification not implemented)

Time = 1.88 (sec) , antiderivative size = 70, normalized size of antiderivative = 1.89

$$\int \frac{\operatorname{csch}^3(x)}{i+\sinh(x)}\,dx = -\frac{\ln\left(-\mathrm{e}^x\,3\mathrm{i}-3\mathrm{i}\right)\,3\mathrm{i}}{2}+\frac{\ln\left(-\mathrm{e}^x\,3\mathrm{i}+3\mathrm{i}\right)\,3\mathrm{i}}{2} +\frac{2\mathrm{i}}{\mathrm{e}^x+1\mathrm{i}}+\frac{\mathrm{e}^x\,2\mathrm{i}}{\mathrm{e}^{4\,x}-2\,\mathrm{e}^{2\,x}+1}+\frac{-2+\mathrm{e}^x\,1\mathrm{i}}{\mathrm{e}^{2\,x}-1}$$

input
```
int(1/(sinh(x)^3*(sinh(x) + 1i)),x)
```

output
```
(log(3i - exp(x)*3i)*3i)/2 - (log(- exp(x)*3i - 3i)*3i)/2 + 2i/(exp(x) + 1
i) + (exp(x)*2i)/(exp(4*x) - 2*exp(2*x) + 1) + (exp(x)*1i - 2)/(exp(2*x) -
 1)
```

Reduce [F]

$$\int \frac{\operatorname{csch}^3(x)}{i+\sinh(x)}\,dx = \int \frac{\operatorname{csch}(x)^3}{\sinh(x)+i}dx$$

input
```
int(csch(x)^3/(I+sinh(x)),x)
```

output
```
int(csch(x)**3/(sinh(x) + i),x)
```

3.47 $\int \frac{\operatorname{csch}^4(x)}{i+\sinh(x)}\,dx$

Optimal result

Integrand size = 13, antiderivative size = 47

$$\int \frac{\operatorname{csch}^4(x)}{i+\sinh(x)}\,dx = \frac{3}{2}\operatorname{arctanh}(\cosh(x)) - 4i\coth(x) + \frac{4}{3}i\coth^3(x) - \frac{3}{2}\coth(x)\operatorname{csch}(x) + \frac{\coth(x)\operatorname{csch}^2(x)}{i+\sinh(x)}$$

output
```
3/2*arctanh(cosh(x))-4*I*coth(x)+4/3*I*coth(x)^3-3/2*coth(x)*csch(x)+coth(
x)*csch(x)^2/(I+sinh(x))
```

Mathematica [A] (verified)

Time = 0.17 (sec) , antiderivative size = 53, normalized size of antiderivative = 1.13

$$\int \frac{\operatorname{csch}^4(x)}{i+\sinh(x)}\,dx = \frac{1}{6}\operatorname{sech}(x)\left(-9+9\operatorname{arctanh}\left(\sqrt{\cosh^2(x)}\right)\sqrt{\cosh^2(x)} - 8i\operatorname{csch}(x) - 3\operatorname{csch}^2(x) + 2i\operatorname{csch}^3(x) - 16i\sinh(x)\right)$$

input
```
Integrate[Csch[x]^4/(I + Sinh[x]),x]
```

output
```
(Sech[x]*(-9 + 9*ArcTanh[Sqrt[Cosh[x]^2]]*Sqrt[Cosh[x]^2] - (8*I)*Csch[x]
- 3*Csch[x]^2 + (2*I)*Csch[x]^3 - (16*I)*Sinh[x]))/6
```

Rubi [A] (verified)

Time = 0.47 (sec) , antiderivative size = 59, normalized size of antiderivative = 1.26, number of steps used = 16, number of rules used = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 1.154, Rules used = {3042, 3247, 25, 3042, 3227, 26, 3042, 26, 4254, 2009, 4255, 26, 3042, 26, 4257}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\operatorname{csch}^4(x)}{\sinh(x)+i}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{(i-i\sin(ix))\sin(ix)^4}dx$$

$$\Big\downarrow \text{3247}$$

$$\int -\operatorname{csch}^4(x)(4i-3\sinh(x))dx+\frac{\coth(x)\operatorname{csch}^2(x)}{\sinh(x)+i}$$

$$\Big\downarrow \text{25}$$

$$\frac{\coth(x)\operatorname{csch}^2(x)}{\sinh(x)+i}-\int \operatorname{csch}^4(x)(4i-3\sinh(x))dx$$

$$\Big\downarrow \text{3042}$$

$$\frac{\coth(x)\operatorname{csch}^2(x)}{\sinh(x)+i}-\int \frac{3i\sin(ix)+4i}{\sin(ix)^4}dx$$

$$\Big\downarrow \text{3227}$$

$$-4i\int \operatorname{csch}^4(x)dx-3i\int i\operatorname{csch}^3(x)dx+\frac{\coth(x)\operatorname{csch}^2(x)}{\sinh(x)+i}$$

$$\Big\downarrow \text{26}$$

$$-4i\int \operatorname{csch}^4(x)dx+3\int \operatorname{csch}^3(x)dx+\frac{\coth(x)\operatorname{csch}^2(x)}{\sinh(x)+i}$$

$$\Big\downarrow \text{3042}$$

$$3\int -i\csc(ix)^3dx-4i\int \csc(ix)^4dx+\frac{\coth(x)\text{csch}^2(x)}{\sinh(x)+i}$$

$$\Big\downarrow \; 26$$

$$-3i\int \csc(ix)^3dx-4i\int \csc(ix)^4dx+\frac{\coth(x)\text{csch}^2(x)}{\sinh(x)+i}$$

$$\Big\downarrow \; 4254$$

$$-3i\int \csc(ix)^3dx+4\int \left(1-\coth^2(x)\right)d(-i\coth(x))+\frac{\coth(x)\text{csch}^2(x)}{\sinh(x)+i}$$

$$\Big\downarrow \; 2009$$

$$-3i\int \csc(ix)^3dx+4\left(\frac{1}{3}i\coth^3(x)-i\coth(x)\right)+\frac{\coth(x)\text{csch}^2(x)}{\sinh(x)+i}$$

$$\Big\downarrow \; 4255$$

$$-3i\left(\frac{1}{2}\int -i\text{csch}(x)dx-\frac{1}{2}i\coth(x)\text{csch}(x)\right)+4\left(\frac{1}{3}i\coth^3(x)-i\coth(x)\right)+\frac{\coth(x)\text{csch}^2(x)}{\sinh(x)+i}$$

$$\Big\downarrow \; 26$$

$$-3i\left(-\frac{1}{2}i\int \text{csch}(x)dx-\frac{1}{2}i\coth(x)\text{csch}(x)\right)+4\left(\frac{1}{3}i\coth^3(x)-i\coth(x)\right)+\frac{\coth(x)\text{csch}^2(x)}{\sinh(x)+i}$$

$$\Big\downarrow \; 3042$$

$$-3i\left(-\frac{1}{2}i\int i\csc(ix)dx-\frac{1}{2}i\coth(x)\text{csch}(x)\right)+4\left(\frac{1}{3}i\coth^3(x)-i\coth(x)\right)+\frac{\coth(x)\text{csch}^2(x)}{\sinh(x)+i}$$

$$\Big\downarrow \; 26$$

$$-3i\left(\frac{1}{2}\int \csc(ix)dx-\frac{1}{2}i\coth(x)\text{csch}(x)\right)+4\left(\frac{1}{3}i\coth^3(x)-i\coth(x)\right)+\frac{\coth(x)\text{csch}^2(x)}{\sinh(x)+i}$$

$$\Big\downarrow \; 4257$$

$$-3i\left(\frac{1}{2}i\text{arctanh}(\cosh(x))-\frac{1}{2}i\coth(x)\text{csch}(x)\right)+4\left(\frac{1}{3}i\coth^3(x)-i\coth(x)\right)+\frac{\coth(x)\text{csch}^2(x)}{\sinh(x)+i}$$

input
```
Int[Csch[x]^4/(I + Sinh[x]),x]
```

output
```
4*((-I)*Coth[x] + (I/3)*Coth[x]^3) - (3*I)*((I/2)*ArcTanh[Cosh[x]] - (I/2)
*Coth[x]*Csch[x]) + (Coth[x]*Csch[x]^2)/(I + Sinh[x])
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3227
```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Simp[c   Int[(b*Sin[e + f*x])^m, x], x] + Simp[d/b   Int
[(b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

rule 3247
```
Int[((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_)/((a_) + (b_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Simp[(-b^2)*Cos[e + f*x]*((c + d*Sin[e + f*x])^(
n + 1)/(a*f*(b*c - a*d)*(a + b*Sin[e + f*x]))), x] + Simp[d/(a*(b*c - a*d))
   Int[(c + d*Sin[e + f*x])^n*(a*n - b*(n + 1)*Sin[e + f*x]), x], x] /; Fre
eQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[
c^2 - d^2, 0] && LtQ[n, 0] && (IntegerQ[2*n] || EqQ[c, 0])
```

rule 4254
```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Simp[-d^(-1)   Subst[Int[Exp
andIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
 d}, x] && IGtQ[n/2, 0]
```

rule 4255
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Csc[c + d*x])^(n - 1)/(d*(n - 1))), x] + Simp[b^2*((n - 2)/(n - 1))
  Int[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
&& IntegerQ[2*n]
```

rule 4257
```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Maple [A] (verified)

Time = 0.71 (sec) , antiderivative size = 71, normalized size of antiderivative = 1.51

method	result
default	$-\frac{7i\tanh\left(\frac{x}{2}\right)}{8}+\frac{i\tanh\left(\frac{x}{2}\right)^{3}}{24}+\frac{\tanh\left(\frac{x}{2}\right)^{2}}{8}+\frac{i}{24\tanh\left(\frac{x}{2}\right)^{3}}-\frac{7i}{8\tanh\left(\frac{x}{2}\right)}-\frac{1}{8\tanh\left(\frac{x}{2}\right)^{2}}-\frac{3\ln\left(\tanh\left(\frac{x}{2}\right)\right)}{2}-\frac{2i}{\tanh\left(\frac{x}{2}\right.}$
risch	$-\frac{9i\mathrm{e}^{5x}-24\,\mathrm{e}^{4x}+9\,\mathrm{e}^{6x}-24i\mathrm{e}^{3x}+39\,\mathrm{e}^{2x}+7i\mathrm{e}^{x}-16}{3(\mathrm{e}^{2x}-1)^{3}(\mathrm{e}^{x}+i)}+\frac{3\ln(\mathrm{e}^{x}+1)}{2}-\frac{3\ln(\mathrm{e}^{x}-1)}{2}$
parallelrisch	$\frac{\left(-36\tanh\left(\frac{x}{2}\right)-36i\right)\ln\left(\tanh\left(\frac{x}{2}\right)\right)+i\tanh\left(\frac{x}{2}\right)^{4}-2i\coth\left(\frac{x}{2}\right)^{2}-18i\tanh\left(\frac{x}{2}\right)^{2}-\coth\left(\frac{x}{2}\right)^{3}+2\tanh\left(\frac{x}{2}\right)^{3}-90i+18\coth\left(\frac{x}{2}\right)}{24\tanh\left(\frac{x}{2}\right)+24i}$

input
```
int(csch(x)^4/(I+sinh(x)),x,method=_RETURNVERBOSE)
```

output
```
-7/8*I*tanh(1/2*x)+1/24*I*tanh(1/2*x)^3+1/8*tanh(1/2*x)^2+1/24*I/tanh(1/2*
x)^3-7/8*I/tanh(1/2*x)-1/8/tanh(1/2*x)^2-3/2*ln(tanh(1/2*x))-2*I/(tanh(1/2
*x)+I)
```

Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 174 vs. 2(35) = 70.

Time = 0.08 (sec) , antiderivative size = 174, normalized size of antiderivative = 3.70

$$\int \frac{\operatorname{csch}^4(x)}{i+\sinh(x)}\,dx$$
$$=\frac{9\left(e^{(7x)}+i\,e^{(6x)}-3\,e^{(5x)}-3i\,e^{(4x)}+3\,e^{(3x)}+3i\,e^{(2x)}-e^{x}-i\right)\log\left(e^{x}+1\right)-9\left(e^{(7x)}+i\,e^{(6x)}-3\,e^{(5x}\right.}{6\left(e^{(7x)}+i\,e^{(6x)}-3\,e^{(5x}\right.}$$

input
```
integrate(csch(x)^4/(I+sinh(x)),x, algorithm="fricas")
```

output
```
1/6*(9*(e^(7*x) + I*e^(6*x) - 3*e^(5*x) - 3*I*e^(4*x) + 3*e^(3*x) + 3*I*e^
(2*x) - e^x - I)*log(e^x + 1) - 9*(e^(7*x) + I*e^(6*x) - 3*e^(5*x) - 3*I*e
^(4*x) + 3*e^(3*x) + 3*I*e^(2*x) - e^x - I)*log(e^x - 1) - 18*e^(6*x) - 18
*I*e^(5*x) + 48*e^(4*x) + 48*I*e^(3*x) - 78*e^(2*x) - 14*I*e^x + 32)/(e^(7
*x) + I*e^(6*x) - 3*e^(5*x) - 3*I*e^(4*x) + 3*e^(3*x) + 3*I*e^(2*x) - e^x
- I)
```

Sympy [F]

$$\int \frac{\operatorname{csch}^4(x)}{i+\sinh(x)}\,dx=\int \frac{\operatorname{csch}^4(x)}{\sinh(x)+i}\,dx$$

input
```
integrate(csch(x)**4/(I+sinh(x)),x)
```

output
```
Integral(csch(x)**4/(sinh(x) + I), x)
```

Maxima [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 103 vs. 2(35) = 70.

Time = 0.04 (sec) , antiderivative size = 103, normalized size of antiderivative = 2.19

$$\int \frac{\operatorname{csch}^4(x)}{i+\sinh(x)}\,dx = \frac{-7i\,e^{(-x)}+39\,e^{(-2\,x)}+24i\,e^{(-3\,x)}-24\,e^{(-4\,x)}-9i\,e^{(-5\,x)}+9\,e^{(-6\,x)}-16}{3\left(e^{(-x)}+3i\,e^{(-2\,x)}-3\,e^{(-3\,x)}-3i\,e^{(-4\,x)}+3\,e^{(-5\,x)}+i\,e^{(-6\,x)}-e^{(-7\,x)}-i\right)} + \frac{3}{2}\log\left(e^{(-x)}+1\right)-\frac{3}{2}\log\left(e^{(-x)}-1\right)$$

input
```
integrate(csch(x)^4/(I+sinh(x)),x, algorithm="maxima")
```

output
```
1/3*(-7*I*e^(-x) + 39*e^(-2*x) + 24*I*e^(-3*x) - 24*e^(-4*x) - 9*I*e^(-5*x
) + 9*e^(-6*x) - 16)/(e^(-x) + 3*I*e^(-2*x) - 3*e^(-3*x) - 3*I*e^(-4*x) +
3*e^(-5*x) + I*e^(-6*x) - e^(-7*x) - I) + 3/2*log(e^(-x) + 1) - 3/2*log(e^
(-x) - 1)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 58, normalized size of antiderivative = 1.23

$$\int \frac{\operatorname{csch}^4(x)}{i+\sinh(x)}\,dx = -\frac{2}{e^x+i} - \frac{3\,e^{(5\,x)} + 6i\,e^{(4\,x)} - 24i\,e^{(2\,x)} - 3\,e^x + 10i}{3\,\left(e^{(2\,x)}-1\right)^3} + \frac{3}{2}\,\log\left(e^x+1\right) - \frac{3}{2}\,\log\left(|e^x-1|\right)$$

input
```
integrate(csch(x)^4/(I+sinh(x)),x, algorithm="giac")
```

output
```
-2/(e^x + I) - 1/3*(3*e^(5*x) + 6*I*e^(4*x) - 24*I*e^(2*x) - 3*e^x + 10*I)
/(e^(2*x) - 1)^3 + 3/2*log(e^x + 1) - 3/2*log(abs(e^x - 1))
```

Mupad [B] (verification not implemented)

Time = 1.97 (sec) , antiderivative size = 85, normalized size of antiderivative = 1.81

$$\int \frac{\operatorname{csch}^4(x)}{i+\sinh(x)}\,dx = \frac{3\,\ln\left(3\,e^x+3\right)}{2} - \frac{3\,\ln\left(3\,e^x-3\right)}{2} - \frac{e^x}{e^{2\,x}-1} - \frac{2\,e^x}{\left(e^{2\,x}-1\right)^2} - \frac{2}{e^x+1i} - \frac{2i}{e^{2\,x}-1} + \frac{4i}{\left(e^{2\,x}-1\right)^2} + \frac{8i}{3\,\left(e^{2\,x}-1\right)^3}$$

input
```
int(1/(sinh(x)^4*(sinh(x) + 1i)),x)
```

output
```
(3*log(3*exp(x) + 3))/2 - (3*log(3*exp(x) - 3))/2 - exp(x)/(exp(2*x) - 1)
- (2*exp(x))/(exp(2*x) - 1)^2 - 2/(exp(x) + 1i) - 2i/(exp(2*x) - 1) + 4i/(
exp(2*x) - 1)^2 + 8i/(3*(exp(2*x) - 1)^3)
```

Reduce [F]

$$\int \frac{\operatorname{csch}^4(x)}{i+\sinh(x)}\,dx = \int \frac{\operatorname{csch}(x)^4}{\sinh(x)+i}dx$$

input
```
int(csch(x)^4/(I+sinh(x)),x)
```

output
```
int(csch(x)**4/(sinh(x) + i),x)
```

3.48 $\int \frac{\sinh^4(x)}{(i+\sinh(x))^2} dx$

Optimal result

Integrand size = 13, antiderivative size = 58

$$\int \frac{\sinh^4(x)}{(i+\sinh(x))^2} dx = -\frac{7x}{2} - \frac{16}{3} i \cosh(x) + \frac{7}{2} \cosh(x) \sinh(x) - \frac{\cosh(x) \sinh^3(x)}{3(i+\sinh(x))^2} - \frac{8 \cosh(x) \sinh^2(x)}{3(i+\sinh(x))}$$

output
```
-7/2*x-16/3*I*cosh(x)+7/2*cosh(x)*sinh(x)-1/3*cosh(x)*sinh(x)^3/(I+sinh(x)
)^2-8*cosh(x)*sinh(x)^2/(3*I+3*sinh(x))
```

Mathematica [B] (verified)

Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 147 vs. 2(58) = 116.

Time = 0.13 (sec) , antiderivative size = 147, normalized size of antiderivative = 2.53

$$\int \frac{\sinh^4(x)}{(i+\sinh(x))^2}\,dx = \frac{5i\cosh(x)}{6(1-i\sinh(x))^2} - \frac{31i\cosh(x)}{6(1-i\sinh(x))} - \frac{i\sqrt{2}\cosh(x)\sqrt{1+\frac{1}{2}(-1+i\sinh(x))}}{\sqrt{1+i\sinh(x)}} - \frac{7i\arcsin\left(\frac{\sqrt{1-i\sinh(x)}}{\sqrt{2}}\right)\cosh(x)}{\sqrt{1-i\sinh(x)}\sqrt{1+i\sinh(x)}} - \frac{\cosh(x)\sinh^3(x)}{2(1-i\sinh(x))^2}$$

input
```
Integrate[Sinh[x]^4/(I + Sinh[x])^2,x]
```

output
```
(((5*I)/6)*Cosh[x])/(1 - I*Sinh[x])^2 - (((31*I)/6)*Cosh[x])/(1 - I*Sinh[x
]) - (I*Sqrt[2]*Cosh[x]*Sqrt[1 + (-1 + I*Sinh[x])/2])/Sqrt[1 + I*Sinh[x]]
- ((7*I)*ArcSin[Sqrt[1 - I*Sinh[x]]/Sqrt[2]]*Cosh[x])/(Sqrt[1 - I*Sinh[x]]
*Sqrt[1 + I*Sinh[x]]) - (Cosh[x]*Sinh[x]^3)/(2*(1 - I*Sinh[x])^2)
```

Rubi [A] (verified)

Time = 0.39 (sec) , antiderivative size = 63, normalized size of antiderivative = 1.09, number of steps used = 9, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.692, Rules used = {3042, 3244, 3042, 25, 3456, 26, 3042, 26, 3213}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sinh^4(x)}{(\sinh(x)+i)^2}\,dx$$

$$\Big\downarrow \; 3042$$

$$\int \frac{\sin(ix)^4}{(i-i\sin(ix))^2}dx$$

$$\Big\downarrow \text{3244}$$

$$-\frac{1}{3}\int \frac{(3i-5\sinh(x))\sinh^2(x)}{\sinh(x)+i}dx-\frac{\sinh^3(x)\cosh(x)}{3(\sinh(x)+i)^2}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{1}{3}\int -\frac{(5i\sin(ix)+3i)\sin(ix)^2}{i-i\sin(ix)}dx-\frac{\sinh^3(x)\cosh(x)}{3(\sinh(x)+i)^2}$$

$$\Big\downarrow \text{25}$$

$$\frac{1}{3}\int \frac{\sin(ix)^2(5\sin(ix)+3)}{1-\sin(ix)}dx-\frac{\sinh^3(x)\cosh(x)}{3(\sinh(x)+i)^2}$$

$$\Big\downarrow \text{3456}$$

$$\frac{1}{3}\left(\frac{8i\sinh^2(x)\cosh(x)}{1-i\sinh(x)}-\int i(21i\sinh(x)+16)\sinh(x)dx\right)-\frac{\sinh^3(x)\cosh(x)}{3(\sinh(x)+i)^2}$$

$$\Big\downarrow \text{26}$$

$$\frac{1}{3}\left(\frac{8i\sinh^2(x)\cosh(x)}{1-i\sinh(x)}-i\int (21i\sinh(x)+16)\sinh(x)dx\right)-\frac{\sinh^3(x)\cosh(x)}{3(\sinh(x)+i)^2}$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{3}\left(\frac{8i\sinh^2(x)\cosh(x)}{1-i\sinh(x)}-i\int -i\sin(ix)(21\sin(ix)+16)dx\right)-\frac{\sinh^3(x)\cosh(x)}{3(\sinh(x)+i)^2}$$

$$\Big\downarrow \text{26}$$

$$\frac{1}{3}\left(\frac{8i\sinh^2(x)\cosh(x)}{1-i\sinh(x)}-\int \sin(ix)(21\sin(ix)+16)dx\right)-\frac{\sinh^3(x)\cosh(x)}{3(\sinh(x)+i)^2}$$

$$\Big\downarrow \text{3213}$$

$$\frac{1}{3}\left(-\frac{21x}{2}-16i\cosh(x)+\frac{8i\sinh^2(x)\cosh(x)}{1-i\sinh(x)}+\frac{21}{2}\sinh(x)\cosh(x)\right)-\frac{\sinh^3(x)\cosh(x)}{3(\sinh(x)+i)^2}$$

input
```
Int[Sinh[x]^4/(I + Sinh[x])^2,x]
```

output
```
-1/3*(Cosh[x]*Sinh[x]^3)/(I + Sinh[x])^2 + ((-21*x)/2 - (16*I)*Cosh[x] + (
21*Cosh[x]*Sinh[x])/2 + ((8*I)*Cosh[x]*Sinh[x]^2)/(1 - I*Sinh[x]))/3
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3213
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)
*(x_)]), x_Symbol] :> Simp[(2*a*c + b*d)*(x/2), x] + (-Simp[(b*c + a*d)*(Co
s[e + f*x]/f), x] - Simp[b*d*Cos[e + f*x]*(Sin[e + f*x]/(2*f)), x]) /; Free
Q[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

rule 3244
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*c - a*d)*Cos[e + f*x]*(a + b*Sin[e
+ f*x])^m*((c + d*Sin[e + f*x])^(n - 1)/(a*f*(2*m + 1))), x] + Simp[1/(a*b*
(2*m + 1))   Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^(n - 2)*
Simp[b*(c^2*(m + 1) + d^2*(n - 1)) + a*c*d*(m - n + 1) + d*(a*d*(m - n + 1)
 + b*c*(m + n))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] &&
 NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -1]
&& GtQ[n, 1] && (IntegersQ[2*m, 2*n] || (IntegerQ[m] && EqQ[c, 0]))
```

rule 3456
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*((c + d*Sin[e + f*x])^n/(
a*f*(2*m + 1))), x] - Simp[1/(a*b*(2*m + 1))   Int[(a + b*Sin[e + f*x])^(m
+ 1)*(c + d*Sin[e + f*x])^(n - 1)*Simp[A*(a*d*n - b*c*(m + 1)) - B*(a*c*m +
 b*d*n) - d*(a*B*(m - n) + A*b*(m + n + 1))*Sin[e + f*x], x], x], x] /; Fre
eQ[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] &
& NeQ[c^2 - d^2, 0] && LtQ[m, -2^(-1)] && GtQ[n, 0] && IntegerQ[2*m] && (In
tegerQ[2*n] || EqQ[c, 0])
```

Maple [A] (verified)

Time = 0.93 (sec) , antiderivative size = 52, normalized size of antiderivative = 0.90

method	result
risch	$-\frac{7x}{2}+\frac{e^{2x}}{8}-ie^{x}-ie^{-x}-\frac{e^{-2x}}{8}-\frac{2i(21ie^{x}+12\,e^{2x}-11)}{3(e^{x}+i)^{3}}$
default	$\frac{2i}{(\tanh(\frac{x}{2})+i)^{2}}+\frac{4}{3(\tanh(\frac{x}{2})+i)^{3}}+\frac{6}{\tanh(\frac{x}{2})+i}+\frac{\frac{1}{2}+2i}{\tanh(\frac{x}{2})-1}+\frac{1}{2(\tanh(\frac{x}{2})-1)^{2}}+\frac{7\ln(\tanh(\frac{x}{2})-1)}{2}+\frac{\frac{1}{2}-2i}{\tanh(\frac{x}{2})}$
parallelrisch	$\frac{84\ln(\tanh(\frac{x}{2})-1)(i\sinh(\frac{3x}{2})+3i\sinh(\frac{x}{2})+\cosh(\frac{3x}{2})-3\cosh(\frac{x}{2}))+84(-i\sinh(\frac{3x}{2})-3i\sinh(\frac{x}{2})-\cosh(\frac{3x}{2})+3\cosh(\frac{x}{2}))\ln(}{24i\sinh(\frac{3x}{2})+72i\sinh(\frac{x}{2}}$

input
```
int(sinh(x)^4/(I+sinh(x))^2,x,method=_RETURNVERBOSE)
```

output
```
-7/2*x+1/8*exp(x)^2-I*exp(x)-I/exp(x)-1/8/exp(x)^2-2/3*I*(21*I*exp(x)+12*e
xp(x)^2-11)/(exp(x)+I)^3
```

Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 89 vs. 2(42) = 84.

Time = 0.09 (sec) , antiderivative size = 89, normalized size of antiderivative = 1.53

$$\int \frac{\sinh^4(x)}{(i+\sinh(x))^2}\,dx =$$
$$-\frac{21\,(4\,x-3)e^{(5\,x)}+21\,(12i\,x+7i)e^{(4\,x)}-3\,(84\,x+127)e^{(3\,x)}-(84i\,x+239i)e^{(2\,x)}-3\,e^{(7\,x)}+15i\,e^{(6\,x)}}{24\,(e^{(5\,x)}+3i\,e^{(4\,x)}-3\,e^{(3\,x)}-i\,e^{(2\,x)})}$$

input
```
integrate(sinh(x)^4/(I+sinh(x))^2,x, algorithm="fricas")
```

output
```
-1/24*(21*(4*x - 3)*e^(5*x) + 21*(12*I*x + 7*I)*e^(4*x) - 3*(84*x + 127)*e
^(3*x) - (84*I*x + 239*I)*e^(2*x) - 3*e^(7*x) + 15*I*e^(6*x) + 15*e^x - 3*
I)/(e^(5*x) + 3*I*e^(4*x) - 3*e^(3*x) - I*e^(2*x))
```

Sympy [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 70, normalized size of antiderivative = 1.21

$$\int \frac{\sinh^4(x)}{(i+\sinh(x))^2}\,dx = -\frac{7x}{2} + \frac{-24ie^{2x}+42e^x+22i}{3e^{3x}+9ie^{2x}-9e^x-3i} + \frac{e^{2x}}{8} - ie^x - ie^{-x} - \frac{e^{-2x}}{8}$$

input
```
integrate(sinh(x)**4/(I+sinh(x))**2,x)
```

output
```
-7*x/2 + (-24*I*exp(2*x) + 42*exp(x) + 22*I)/(3*exp(3*x) + 9*I*exp(2*x) -
9*exp(x) - 3*I) + exp(2*x)/8 - I*exp(x) - I*exp(-x) - exp(-2*x)/8
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 71, normalized size of antiderivative = 1.22

$$\int \frac{\sinh^4(x)}{(i+\sinh(x))^2}\,dx = -\frac{7}{2}\,x + \frac{15\,e^{(-x)} + 239i\,e^{(-2\,x)} - 405\,e^{(-3\,x)} - 216i\,e^{(-4\,x)} + 3i}{8\,(3i\,e^{(-2\,x)} - 9\,e^{(-3\,x)} - 9i\,e^{(-4\,x)} + 3\,e^{(-5\,x)})} - i\,e^{(-x)} - \frac{1}{8}\,e^{(-2\,x)}$$

input
```
integrate(sinh(x)^4/(I+sinh(x))^2,x, algorithm="maxima")
```

output
```
-7/2*x + 1/8*(15*e^(-x) + 239*I*e^(-2*x) - 405*e^(-3*x) - 216*I*e^(-4*x) +
 3*I)/(3*I*e^(-2*x) - 9*e^(-3*x) - 9*I*e^(-4*x) + 3*e^(-5*x)) - I*e^(-x) -
 1/8*e^(-2*x)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 50, normalized size of antiderivative = 0.86

$$\int \frac{\sinh^4(x)}{(i+\sinh(x))^2}\,dx = -\frac{7}{2}\,x - \frac{\left(216i\,e^{(4\,x)} - 405\,e^{(3\,x)} - 239i\,e^{(2\,x)} + 15\,e^x - 3i\right)e^{(-2\,x)}}{24\,(e^x+i)^3} + \frac{1}{8}\,e^{(2\,x)} - i\,e^x$$

input
```
integrate(sinh(x)^4/(I+sinh(x))^2,x, algorithm="giac")
```

output
```
-7/2*x - 1/24*(216*I*e^(4*x) - 405*e^(3*x) - 239*I*e^(2*x) + 15*e^x - 3*I)
*e^(-2*x)/(e^x + I)^3 + 1/8*e^(2*x) - I*e^x
```

Mupad [B] (verification not implemented)

Time = 1.77 (sec) , antiderivative size = 97, normalized size of antiderivative = 1.67

$$\int \frac{\sinh^4(x)}{(i+\sinh(x))^2}\,dx = \frac{e^{2x}}{8} - e^{-x}\,1\mathrm{i} - \frac{e^{-2x}}{8} - \frac{7x}{2} - e^{x}\,1\mathrm{i} - \frac{-2+\frac{e^{x}\,8\mathrm{i}}{3}}{e^{2x}-1+e^{x}\,2\mathrm{i}} + \frac{4\,e^{x} - \frac{e^{2x}\,8\mathrm{i}}{3} + \frac{8}{3}\mathrm{i}}{e^{2x}\,3\mathrm{i} + e^{3x} - 3\,e^{x} - \mathrm{i}} - \frac{8\mathrm{i}}{3\,(e^{x}+1\mathrm{i})}$$

input
```
int(sinh(x)^4/(sinh(x) + 1i)^2,x)
```

output
```
exp(2*x)/8 - exp(-x)*1i - exp(-2*x)/8 - (7*x)/2 - exp(x)*1i - ((exp(x)*8i)
/3 - 2)/(exp(2*x) + exp(x)*2i - 1) + (4*exp(x) - (exp(2*x)*8i)/3 + 8i/3)/(
exp(2*x)*3i + exp(3*x) - 3*exp(x) - 1i) - 8i/(3*(exp(x) + 1i))
```

Reduce [F]

$$\int \frac{\sinh^4(x)}{(i+\sinh(x))^2}\,dx = \int \frac{\sinh(x)^4}{\sinh(x)^2 + 2\sinh(x)\,i - 1}dx$$

input
```
int(sinh(x)^4/(I+sinh(x))^2,x)
```

output
```
int(sinh(x)**4/(sinh(x)**2 + 2*sinh(x)*i - 1),x)
```

3.49 $\int \frac{\sinh^3(x)}{(i+\sinh(x))^2} \, dx$

Optimal result

Integrand size = 13, antiderivative size = 44

$$\int \frac{\sinh^3(x)}{(i+\sinh(x))^2} \, dx = -2ix + \frac{4\cosh(x)}{3} - \frac{\cosh(x)\sinh^2(x)}{3(i+\sinh(x))^2} + \frac{2i\cosh(x)}{i+\sinh(x)}$$

output
```
-2*I*x+4/3*cosh(x)-1/3*cosh(x)*sinh(x)^2/(I+sinh(x))^2+2*I*cosh(x)/(I+sinh
(x))
```

Mathematica [A] (verified)

Time = 0.09 (sec) , antiderivative size = 45, normalized size of antiderivative = 1.02

$$\int \frac{\sinh^3(x)}{(i+\sinh(x))^2} \, dx = \frac{1}{3}\cosh(x)\left(-\frac{6i\text{arcsinh}(\sinh(x))}{\sqrt{\cosh^2(x)}} + \frac{-10+14i\sinh(x)+3\sinh^2(x)}{(i+\sinh(x))^2}\right)$$

input
```
Integrate[Sinh[x]^3/(I + Sinh[x])^2,x]
```

output
```
(Cosh[x]*(((-6*I)*ArcSinh[Sinh[x]])/Sqrt[Cosh[x]^2] + (-10 + (14*I)*Sinh[x
] + 3*Sinh[x]^2)/(I + Sinh[x])^2))/3
```

Rubi [A] (verified)

Time = 0.55 (sec) , antiderivative size = 60, normalized size of antiderivative = 1.36, number of steps used = 16, number of rules used = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 1.231$, Rules used = {3042, 26, 25, 3244, 27, 3042, 26, 3447, 3042, 3502, 27, 3042, 26, 3214, 3042, 3127}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sinh^3(x)}{(\sinh(x)+i)^2}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{i\sin(ix)^3}{(i-i\sin(ix))^2}dx$$

$$\Big\downarrow \text{26}$$

$$i\int -\frac{\sin(ix)^3}{(1-\sin(ix))^2}dx$$

$$\Big\downarrow \text{25}$$

$$-i\int \frac{\sin(ix)^3}{(1-\sin(ix))^2}dx$$

$$\Big\downarrow \text{3244}$$

$$-i\left(\frac{1}{3}\int -\frac{2i(2i\sinh(x)+1)\sinh(x)}{1-i\sinh(x)}dx+\frac{i\sinh^2(x)\cosh(x)}{3(1-i\sinh(x))^2}\right)$$

$$\Big\downarrow \text{27}$$

$$-i\left(\frac{i\sinh^2(x)\cosh(x)}{3(1-i\sinh(x))^2}-\frac{2}{3}i\int \frac{(2i\sinh(x)+1)\sinh(x)}{1-i\sinh(x)}dx\right)$$

$$\Big\downarrow \text{3042}$$

$$-i\left(\frac{i\sinh^2(x)\cosh(x)}{3(1-i\sinh(x))^2}-\frac{2}{3}i\int -\frac{i\sin(ix)(2\sin(ix)+1)}{1-\sin(ix)}dx\right)$$

$$\Big\downarrow \text{26}$$

$$-i\left(\frac{i\sinh^2(x)\cosh(x)}{3(1-i\sinh(x))^2}-\frac{2}{3}\int\frac{\sin(ix)(2\sin(ix)+1)}{1-\sin(ix)}dx\right)$$

$$\Big\downarrow \text{3447}$$

$$-i\left(\frac{i\sinh^2(x)\cosh(x)}{3(1-i\sinh(x))^2}-\frac{2}{3}\int\frac{i\sinh(x)-2\sinh^2(x)}{1-i\sinh(x)}dx\right)$$

$$\Big\downarrow \text{3042}$$

$$-i\left(\frac{i\sinh^2(x)\cosh(x)}{3(1-i\sinh(x))^2}-\frac{2}{3}\int\frac{2\sin(ix)^2+\sin(ix)}{1-\sin(ix)}dx\right)$$

$$\Big\downarrow \text{3502}$$

$$-i\left(\frac{i\sinh^2(x)\cosh(x)}{3(1-i\sinh(x))^2}-\frac{2}{3}\left(-\int-\frac{3i\sinh(x)}{1-i\sinh(x)}dx-2i\cosh(x)\right)\right)$$

$$\Big\downarrow \text{27}$$

$$-i\left(\frac{i\sinh^2(x)\cosh(x)}{3(1-i\sinh(x))^2}-\frac{2}{3}\left(3i\int\frac{\sinh(x)}{1-i\sinh(x)}dx-2i\cosh(x)\right)\right)$$

$$\Big\downarrow \text{3042}$$

$$-i\left(\frac{i\sinh^2(x)\cosh(x)}{3(1-i\sinh(x))^2}-\frac{2}{3}\left(3i\int-\frac{i\sin(ix)}{1-\sin(ix)}dx-2i\cosh(x)\right)\right)$$

$$\Big\downarrow \text{26}$$

$$-i\left(\frac{i\sinh^2(x)\cosh(x)}{3(1-i\sinh(x))^2}-\frac{2}{3}\left(3\int\frac{\sin(ix)}{1-\sin(ix)}dx-2i\cosh(x)\right)\right)$$

$$\Big\downarrow \text{3214}$$

$$-i\left(\frac{i\sinh^2(x)\cosh(x)}{3(1-i\sinh(x))^2}-\frac{2}{3}\left(3\left(-x+\int\frac{1}{1-i\sinh(x)}dx\right)-2i\cosh(x)\right)\right)$$

$$\Big\downarrow \text{3042}$$

$$-i\left(\frac{i\sinh^2(x)\cosh(x)}{3(1-i\sinh(x))^2}-\frac{2}{3}\left(3\left(-x+\int\frac{1}{1-\sin(ix)}dx\right)-2i\cosh(x)\right)\right)$$

$$\Big\downarrow \text{3127}$$

$$-i\left(\frac{i\sinh^2(x)\cosh(x)}{3(1-i\sinh(x))^2}-\frac{2}{3}\left(3\left(-x-\frac{i\cosh(x)}{1-i\sinh(x)}\right)-2i\cosh(x)\right)\right)$$

input
```
Int[Sinh[x]^3/(I + Sinh[x])^2,x]
```

output
```
(-I)*((-2*((-2*I)*Cosh[x] + 3*(-x - (I*Cosh[x])/(1 - I*Sinh[x]))))/3 + ((I
/3)*Cosh[x]*Sinh[x]^2)/(1 - I*Sinh[x])^2)
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3127
```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> Simp[-Cos[c +
 d*x]/(d*(b + a*Sin[c + d*x])), x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b
^2, 0]
```

rule 3214
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])/((c_.) + (d_.)*sin[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[b*(x/d), x] - Simp[(b*c - a*d)/d   Int[1/(c + d
*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

rule 3244
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*c - a*d)*Cos[e + f*x]*(a + b*Sin[e
+ f*x])^m*((c + d*Sin[e + f*x])^(n - 1)/(a*f*(2*m + 1))), x] + Simp[1/(a*b*
(2*m + 1))   Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^(n - 2)*
Simp[b*(c^2*(m + 1) + d^2*(n - 1)) + a*c*d*(m - n + 1) + d*(a*d*(m - n + 1)
 + b*c*(m + n))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] &&
 NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -1]
&& GtQ[n, 1] && (IntegersQ[2*m, 2*n] || (IntegerQ[m] && EqQ[c, 0]))
```

rule 3447
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Int[(a
 + b*Sin[e + f*x])^m*(A*c + (B*c + A*d)*Sin[e + f*x] + B*d*Sin[e + f*x]^2),
 x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0]
```

rule 3502
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-C)*Co
s[e + f*x]*((a + b*Sin[e + f*x])^(m + 1)/(b*f*(m + 2))), x] + Simp[1/(b*(m
+ 2))   Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m
 + 2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x]
 &&  !LtQ[m, -1]
```

Maple [A] (verified)

Time = 0.57 (sec) , antiderivative size = 38, normalized size of antiderivative = 0.86

method	result
risch	$-2ix+\frac{{\mathrm e}^{x}}{2}+\frac{{\mathrm e}^{-x}}{2}+\frac{10i{\mathrm e}^{x}+6\,{\mathrm e}^{2x}-\frac{16}{3}}{\left({\mathrm e}^{x}+i\right)^{3}}$
default	$\frac{4i}{3\left(\tanh\left(\frac{x}{2}\right)+i\right)^{3}}+\frac{4i}{\tanh\left(\frac{x}{2}\right)+i}-\frac{2}{\left(\tanh\left(\frac{x}{2}\right)+i\right)^{2}}-2i\ln\left(\tanh\left(\frac{x}{2}\right)+1\right)+\frac{1}{\tanh\left(\frac{x}{2}\right)+1}+2i\ln\left(\tanh\left(\frac{x}{2}\right)\right.$
parallelrisch	$\frac{\left(-36i\cosh\left(\frac{x}{2}\right)+12i\cosh\left(\frac{3x}{2}\right)-12\sinh\left(\frac{3x}{2}\right)-36\sinh\left(\frac{x}{2}\right)\right)\ln\left(\tanh\left(\frac{x}{2}\right)-1\right)+\left(36i\cosh\left(\frac{x}{2}\right)-12i\cosh\left(\frac{3x}{2}\right)+36\sinh\left(\frac{x}{2}\right)+12\sin\right.}{6i\sinh\left(\frac{3x}{2}\right)+18i\sinh\left(\frac{x}{2}\right)+6\cosh\left(\right.}$

input
```
int(sinh(x)^3/(I+sinh(x))^2,x,method=_RETURNVERBOSE)
```

output
```
-2*I*x+1/2*exp(x)+1/2*exp(-x)+2/3*(15*I*exp(x)+9*exp(2*x)-8)/(exp(x)+I)^3
```

Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 74 vs. $2(32) = 64$.

Time = 0.10 (sec) , antiderivative size = 74, normalized size of antiderivative = 1.68

$$\int \frac{\sinh^3(x)}{(i+\sinh(x))^2} \, dx = -\frac{3\,(4i\,x-3i)e^{(4\,x)}-6\,(6\,x+5)e^{(3\,x)}+6\,(-6i\,x-11i)e^{(2\,x)}+(12\,x+41)e^{x}-3\,e^{(5\,x)}+3i}{6\,\left(e^{(4\,x)}+3i\,e^{(3\,x)}-3\,e^{(2\,x)}-i\,e^{x}\right)}$$

input
```
integrate(sinh(x)^3/(I+sinh(x))^2,x, algorithm="fricas")
```

output
```
-1/6*(3*(4*I*x - 3*I)*e^(4*x) - 6*(6*x + 5)*e^(3*x) + 6*(-6*I*x - 11*I)*e^
(2*x) + (12*x + 41)*e^x - 3*e^(5*x) + 3*I)/(e^(4*x) + 3*I*e^(3*x) - 3*e^(2
*x) - I*e^x)
```

Sympy [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 54, normalized size of antiderivative = 1.23

$$\int \frac{\sinh^3(x)}{(i+\sinh(x))^2} \, dx = -2ix + \frac{18e^{2x}+30ie^{x}-16}{3e^{3x}+9ie^{2x}-9e^{x}-3i} + \frac{e^{x}}{2} + \frac{e^{-x}}{2}$$

input
```
integrate(sinh(x)**3/(I+sinh(x))**2,x)
```

output
```
-2*I*x + (18*exp(2*x) + 30*I*exp(x) - 16)/(3*exp(3*x) + 9*I*exp(2*x) - 9*e
xp(x) - 3*I) + exp(x)/2 + exp(-x)/2
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 59, normalized size of antiderivative = 1.34

$$\int \frac{\sinh^3(x)}{(i+\sinh(x))^2}\,dx = -2i\,x - \frac{41\,e^{(-x)} + 69i\,e^{(-2\,x)} - 39\,e^{(-3\,x)} - 3i}{2\,(3i\,e^{(-x)} - 9\,e^{(-2\,x)} - 9i\,e^{(-3\,x)} + 3\,e^{(-4\,x)})} + \frac{1}{2}\,e^{(-x)}$$

input
```
integrate(sinh(x)^3/(I+sinh(x))^2,x, algorithm="maxima")
```

output
```
-2*I*x - 1/2*(41*e^(-x) + 69*I*e^(-2*x) - 39*e^(-3*x) - 3*I)/(3*I*e^(-x) -
 9*e^(-2*x) - 9*I*e^(-3*x) + 3*e^(-4*x)) + 1/2*e^(-x)
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 38, normalized size of antiderivative = 0.86

$$\int \frac{\sinh^3(x)}{(i+\sinh(x))^2}\,dx = -2i\,x + \frac{(39\,e^{(3\,x)} + 69i\,e^{(2\,x)} - 41\,e^{x} - 3i)e^{(-x)}}{6\,(e^{x} + i)^3} + \frac{1}{2}\,e^{x}$$

input
```
integrate(sinh(x)^3/(I+sinh(x))^2,x, algorithm="giac")
```

output
```
-2*I*x + 1/6*(39*e^(3*x) + 69*I*e^(2*x) - 41*e^x - 3*I)*e^(-x)/(e^x + I)^3
 + 1/2*e^x
```

Mupad [B] (verification not implemented)

Time = 1.76 (sec) , antiderivative size = 79, normalized size of antiderivative = 1.80

$$\int \frac{\sinh^3(x)}{(i+\sinh(x))^2}\,dx = \frac{e^{-x}}{2} - x\,2i + \frac{e^{x}}{2} + \frac{2\,e^{x} + \frac{4}{3}i}{e^{2\,x} - 1 + e^{x}\,2i} + \frac{2\,e^{2\,x} - 2 + \frac{e^{x}\,8i}{3}}{e^{2\,x}\,3i + e^{3\,x} - 3\,e^{x} - i} + \frac{2}{e^{x} + 1i}$$

input
```
int(sinh(x)^3/(sinh(x) + 1i)^2,x)
```

output
```
exp(-x)/2 - x*2i + exp(x)/2 + (2*exp(x) + 4i/3)/(exp(2*x) + exp(x)*2i - 1)
 + (2*exp(2*x) + (exp(x)*8i)/3 - 2)/(exp(2*x)*3i + exp(3*x) - 3*exp(x) - 1
i) + 2/(exp(x) + 1i)
```

Reduce [F]

$$\int \frac{\sinh^3(x)}{(i+\sinh(x))^2}\,dx = \int \frac{\sinh\left(x\right)^3}{\sinh\left(x\right)^2+2\sinh\left(x\right)i-1}dx$$

input
```
int(sinh(x)^3/(I+sinh(x))^2,x)
```

output
```
int(sinh(x)**3/(sinh(x)**2 + 2*sinh(x)*i - 1),x)
```

3.50 $\int \frac{\sinh^2(x)}{(i+\sinh(x))^2}\,dx$

Optimal result

Integrand size = 13, antiderivative size = 32

$$\int \frac{\sinh^2(x)}{(i+\sinh(x))^2}\,dx = x+\frac{i\cosh(x)}{3(i+\sinh(x))^2}-\frac{5\cosh(x)}{3(i+\sinh(x))}$$

output
```
x+1/3*I*cosh(x)/(I+sinh(x))^2-5*cosh(x)/(3*I+3*sinh(x))
```

Mathematica [A] (verified)

Time = 0.11 (sec) , antiderivative size = 55, normalized size of antiderivative = 1.72

$$\int \frac{\sinh^2(x)}{(i+\sinh(x))^2}\,dx = -\frac{1}{3}i\cosh(x)\left(-\frac{6\arcsin\left(\frac{\sqrt{1-i\sinh(x)}}{\sqrt{2}}\right)}{\sqrt{\cosh^2(x)}}+\frac{4-5i\sinh(x)}{(i+\sinh(x))^2}\right)$$

input
```
Integrate[Sinh[x]^2/(I + Sinh[x])^2,x]
```

output
```
(-1/3*I)*Cosh[x]*((-6*ArcSin[Sqrt[1 - I*Sinh[x]]/Sqrt[2]])/Sqrt[Cosh[x]^2]
 + (4 - (5*I)*Sinh[x])/(I + Sinh[x])^2)
```

Rubi [A] (verified)

Time = 0.39 (sec) , antiderivative size = 43, normalized size of antiderivative = 1.34, number of steps used = 9, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.692, Rules used = {3042, 25, 25, 3237, 25, 3042, 3214, 3042, 3127}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sinh^2(x)}{(\sinh(x)+i)^2} \, dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int -\frac{\sin(ix)^2}{(i-i\sin(ix))^2}dx$$

$$\Big\downarrow \text{ 25}$$

$$-\int -\frac{\sin(ix)^2}{(1-\sin(ix))^2}dx$$

$$\Big\downarrow \text{ 25}$$

$$\int \frac{\sin(ix)^2}{(1-\sin(ix))^2}dx$$

$$\Big\downarrow \text{ 3237}$$

$$\frac{1}{3}\int -\frac{3i\sinh(x)+2}{1-i\sinh(x)}dx-\frac{i\cosh(x)}{3(1-i\sinh(x))^2}$$

$$\Big\downarrow \text{ 25}$$

$$-\frac{1}{3}\int \frac{3i\sinh(x)+2}{1-i\sinh(x)}dx-\frac{i\cosh(x)}{3(1-i\sinh(x))^2}$$

$$\Big\downarrow \text{ 3042}$$

$$-\frac{1}{3}\int \frac{3\sin(ix)+2}{1-\sin(ix)}dx-\frac{i\cosh(x)}{3(1-i\sinh(x))^2}$$

$$\Big\downarrow \text{ 3214}$$

$$\frac{1}{3}\left(3x-5\int \frac{1}{1-i\sinh(x)}dx\right)-\frac{i\cosh(x)}{3(1-i\sinh(x))^2}$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{1}{3}\left(3x-5\int \frac{1}{1-\sin(ix)}dx\right)-\frac{i\cosh(x)}{3(1-i\sinh(x))^2}$$
$$\Big\downarrow \text{3127}$$
$$\frac{1}{3}\left(3x+\frac{5i\cosh(x)}{1-i\sinh(x)}\right)-\frac{i\cosh(x)}{3(1-i\sinh(x))^2}$$

input
```
Int[Sinh[x]^2/(I + Sinh[x])^2,x]
```

output
```
(3*x + ((5*I)*Cosh[x])/(1 - I*Sinh[x]))/3 - ((I/3)*Cosh[x])/(1 - I*Sinh[x]
)^2
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3127
```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> Simp[-Cos[c +
 d*x]/(d*(b + a*Sin[c + d*x])), x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b
^2, 0]
```

rule 3214
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])/((c_.) + (d_.)*sin[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[b*(x/d), x] - Simp[(b*c - a*d)/d   Int[1/(c + d
*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

rule 3237
```
Int[sin[(e_.) + (f_.)*(x_)]^2*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_),
x_Symbol] :> Simp[b*Cos[e + f*x]*((a + b*Sin[e + f*x])^m/(a*f*(2*m + 1))),
x] - Simp[1/(a^2*(2*m + 1))   Int[(a + b*Sin[e + f*x])^(m + 1)*(a*m - b*(2*
m + 1)*Sin[e + f*x]), x], x] /; FreeQ[{a, b, e, f}, x] && EqQ[a^2 - b^2, 0]
 && LtQ[m, -2^(-1)]
```

Maple [A] (verified)

Time = 0.43 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.81

method	result
risch	$x+\frac{2i\left(9i\,\mathrm{e}^{x}+6\,\mathrm{e}^{2x}-5\right)}{3\left(\mathrm{e}^{x}+i\right)^{3}}$
default	$\ln\left(\tanh\left(\frac{x}{2}\right)+1\right)-\ln\left(\tanh\left(\frac{x}{2}\right)-1\right)-\frac{2i}{\left(\tanh\left(\frac{x}{2}\right)+i\right)^{2}}-\frac{4}{3\left(\tanh\left(\frac{x}{2}\right)+i\right)^{3}}-\frac{2}{\tanh\left(\frac{x}{2}\right)+i}$
parallelrisch	$\frac{\left(-3i\sinh\left(\frac{3x}{2}\right)-9i\sinh\left(\frac{x}{2}\right)+9\cosh\left(\frac{x}{2}\right)-3\cosh\left(\frac{3x}{2}\right)\right)\ln\left(1-\tanh\left(\frac{x}{2}\right)\right)+\left(3i\sinh\left(\frac{3x}{2}\right)+9i\sinh\left(\frac{x}{2}\right)-9\cosh\left(\frac{x}{2}\right)+3\cosh\left(\frac{3x}{2}\right)\right)\ln}{3i\sinh\left(\frac{3x}{2}\right)+9i\sinh\left(\frac{x}{2}\right)-9\cosh\left(\frac{x}{2}\right)+3\cosh\left(\frac{3x}{2}\right)}$

input
```
int(sinh(x)^2/(I+sinh(x))^2,x,method=_RETURNVERBOSE)
```

output
```
x+2/3*I*(9*I*exp(x)+6*exp(2*x)-5)/(exp(x)+I)^3
```

Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 50 vs. 2(22) = 44.

Time = 0.08 (sec) , antiderivative size = 50, normalized size of antiderivative = 1.56

$$\int\frac{\sinh^2(x)}{(i+\sinh(x))^2}\,dx=\frac{3\,xe^{(3\,x)}-3\,(-3i\,x-4i)e^{(2\,x)}-9\,(x+2)e^{x}-3i\,x-10i}{3\,\left(e^{(3\,x)}+3i\,e^{(2\,x)}-3\,e^{x}-i\right)}$$

input
```
integrate(sinh(x)^2/(I+sinh(x))^2,x, algorithm="fricas")
```

output
```
1/3*(3*x*e^(3*x) - 3*(-3*I*x - 4*I)*e^(2*x) - 9*(x + 2)*e^x - 3*I*x - 10*I
)/(e^(3*x) + 3*I*e^(2*x) - 3*e^x - I)
```

Sympy [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 41, normalized size of antiderivative = 1.28

$$\int \frac{\sinh^2(x)}{(i+\sinh(x))^2}\,dx = x + \frac{12ie^{2x} - 18e^x - 10i}{3e^{3x} + 9ie^{2x} - 9e^x - 3i}$$

input
```
integrate(sinh(x)**2/(I+sinh(x))**2,x)
```

output
```
x + (12*I*exp(2*x) - 18*exp(x) - 10*I)/(3*exp(3*x) + 9*I*exp(2*x) - 9*exp(
x) - 3*I)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 40, normalized size of antiderivative = 1.25

$$\int \frac{\sinh^2(x)}{(i+\sinh(x))^2}\,dx = x - \frac{2\left(9\,e^{(-x)} + 6i\,e^{(-2\,x)} - 5i\right)}{3\left(3\,e^{(-x)} + 3i\,e^{(-2\,x)} - e^{(-3\,x)} - i\right)}$$

input
```
integrate(sinh(x)^2/(I+sinh(x))^2,x, algorithm="maxima")
```

output
```
x - 2/3*(9*e^(-x) + 6*I*e^(-2*x) - 5*I)/(3*e^(-x) + 3*I*e^(-2*x) - e^(-3*x
) - I)
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.69

$$\int \frac{\sinh^2(x)}{(i+\sinh(x))^2}\,dx = x - \frac{2\left(-6i\,e^{(2\,x)} + 9\,e^x + 5i\right)}{3\left(e^x + i\right)^3}$$

input
```
integrate(sinh(x)^2/(I+sinh(x))^2,x, algorithm="giac")
```

output
```
x - 2/3*(-6*I*e^(2*x) + 9*e^x + 5*I)/(e^x + I)^3
```

Mupad [B] (verification not implemented)

Time = 1.73 (sec) , antiderivative size = 71, normalized size of antiderivative = 2.22

$$\int \frac{\sinh^2(x)}{(i+\sinh(x))^2}\,dx = x + \frac{-\frac{2}{3} + \frac{e^x\,4i}{3}}{e^{2x} - 1 + e^x\,2i} - \frac{\frac{4\,e^x}{3} - \frac{e^{2x}\,4i}{3} + \frac{4}{3}i}{e^{2x}\,3i + e^{3x} - 3\,e^x - i} + \frac{4i}{3\,(e^x + 1i)}$$

input
```
int(sinh(x)^2/(sinh(x) + 1i)^2,x)
```

output
```
x + ((exp(x)*4i)/3 - 2/3)/(exp(2*x) + exp(x)*2i - 1) - ((4*exp(x))/3 - (ex
p(2*x)*4i)/3 + 4i/3)/(exp(2*x)*3i + exp(3*x) - 3*exp(x) - 1i) + 4i/(3*(exp
(x) + 1i))
```

Reduce [F]

$$\int \frac{\sinh^2(x)}{(i+\sinh(x))^2}\,dx = \int \frac{\sinh(x)^2}{\sinh(x)^2 + 2\sinh(x)\,i - 1}dx$$

input
```
int(sinh(x)^2/(I+sinh(x))^2,x)
```

output
```
int(sinh(x)**2/(sinh(x)**2 + 2*sinh(x)*i - 1),x)
```

3.51 $\int \frac{\sinh(x)}{(i+\sinh(x))^2}\,dx$

Optimal result

Integrand size = 11, antiderivative size = 31

$$\int \frac{\sinh(x)}{(i+\sinh(x))^2}\,dx = -\frac{\cosh(x)}{3(i+\sinh(x))^2} - \frac{2i\cosh(x)}{3(i+\sinh(x))}$$

output
```
-1/3*cosh(x)/(I+sinh(x))^2-2/3*I*cosh(x)/(I+sinh(x))
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.71

$$\int \frac{\sinh(x)}{(i+\sinh(x))^2}\,dx = \frac{\cosh(x)(1-2i\sinh(x))}{3(i+\sinh(x))^2}$$

input
```
Integrate[Sinh[x]/(I + Sinh[x])^2,x]
```

output
```
(Cosh[x]*(1 - (2*I)*Sinh[x]))/(3*(I + Sinh[x])^2)
```

Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 41, normalized size of antiderivative = 1.32, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.545, Rules used = {3042, 26, 25, 3229, 3042, 3127}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sinh(x)}{(\sinh(x)+i)^2}dx$$

$$\Big\downarrow \text{3042}$$

$$\int -\frac{i\sin(ix)}{(i-i\sin(ix))^2}dx$$

$$\Big\downarrow \text{26}$$

$$-i\int -\frac{\sin(ix)}{(1-\sin(ix))^2}dx$$

$$\Big\downarrow \text{25}$$

$$i\int \frac{\sin(ix)}{(1-\sin(ix))^2}dx$$

$$\Big\downarrow \text{3229}$$

$$i\left(-\frac{2}{3}\int \frac{1}{1-i\sinh(x)}dx-\frac{i\cosh(x)}{3(1-i\sinh(x))^2}\right)$$

$$\Big\downarrow \text{3042}$$

$$i\left(-\frac{2}{3}\int \frac{1}{1-\sin(ix)}dx-\frac{i\cosh(x)}{3(1-i\sinh(x))^2}\right)$$

$$\Big\downarrow \text{3127}$$

$$i\left(\frac{2i\cosh(x)}{3(1-i\sinh(x))}-\frac{i\cosh(x)}{3(1-i\sinh(x))^2}\right)$$

input
```
Int[Sinh[x]/(I + Sinh[x])^2,x]
```

output
```
I*(((-1/3*I)*Cosh[x])/(1 - I*Sinh[x])^2 + (((2*I)/3)*Cosh[x])/(1 - I*Sinh[
x]))
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3127
```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> Simp[-Cos[c +
 d*x]/(d*(b + a*Sin[c + d*x])), x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b
^2, 0]
```

rule 3229
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Simp[(b*c - a*d)*Cos[e + f*x]*((a + b*Sin[e + f*
x])^m/(a*f*(2*m + 1))), x] + Simp[(a*d*m + b*c*(m + 1))/(a*b*(2*m + 1))   I
nt[(a + b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{a, b, c, d, e, f}, x] &&
NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && LtQ[m, -2^(-1)]
```

Maple [A] (verified)

Time = 0.27 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.74

method	result	size
risch	$-\frac{2\left(3 i \,{\mathrm e}^{x}+3 \,{\mathrm e}^{2 x}-2\right)}{3 \left({\mathrm e}^{x}+i\right)^{3}}$	23
default	$\frac{2}{\left(\tanh \left(\frac{x}{2}\right)+i\right)^{2}}-\frac{4 i}{3 \left(\tanh \left(\frac{x}{2}\right)+i\right)^{3}}$	25
parallelrisch	$\frac{2 i +6 \tanh \left(\frac{x}{2}\right)}{-9 \tanh \left(\frac{x}{2}\right)+9 i \tanh \left(\frac{x}{2}\right)^{2}+3 \tanh \left(\frac{x}{2}\right)^{3}-3 i}$	39

input
```
int(sinh(x)/(I+sinh(x))^2,x,method=_RETURNVERBOSE)
```

output
```
-2/3*(3*I*exp(x)+3*exp(2*x)-2)/(exp(x)+I)^3
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.03

$$\int \frac{\sinh(x)}{(i+\sinh(x))^2}\,dx = -\frac{2\left(3\,e^{(2\,x)}+3i\,e^{x}-2\right)}{3\left(e^{(3\,x)}+3i\,e^{(2\,x)}-3\,e^{x}-i\right)}$$

input
```
integrate(sinh(x)/(I+sinh(x))^2,x, algorithm="fricas")
```

output
```
-2/3*(3*e^(2*x) + 3*I*e^x - 2)/(e^(3*x) + 3*I*e^(2*x) - 3*e^x - I)
```

Sympy [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.19

$$\int \frac{\sinh(x)}{(i+\sinh(x))^2}\,dx = \frac{-6e^{2x}-6ie^{x}+4}{3e^{3x}+9ie^{2x}-9e^{x}-3i}$$

input
```
integrate(sinh(x)/(I+sinh(x))**2,x)
```

output
```
(-6*exp(2*x) - 6*I*exp(x) + 4)/(3*exp(3*x) + 9*I*exp(2*x) - 9*exp(x) - 3*I
)
```

Maxima [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 81 vs. 2(21) = 42.

Time = 0.04 (sec) , antiderivative size = 81, normalized size of antiderivative = 2.61

$$\int \frac{\sinh(x)}{(i+\sinh(x))^2}\,dx = -\frac{2i\,e^{(-x)}}{3\,e^{(-x)}+3i\,e^{(-2\,x)}-e^{(-3\,x)}-i}+\frac{2\,e^{(-2\,x)}}{3\,e^{(-x)}+3i\,e^{(-2\,x)}-e^{(-3\,x)}-i} - \frac{4}{3\,(3\,e^{(-x)}+3i\,e^{(-2\,x)}-e^{(-3\,x)}-i)}$$

input
```
integrate(sinh(x)/(I+sinh(x))^2,x, algorithm="maxima")
```

output
```
-2*I*e^(-x)/(3*e^(-x) + 3*I*e^(-2*x) - e^(-3*x) - I) + 2*e^(-2*x)/(3*e^(-x
) + 3*I*e^(-2*x) - e^(-3*x) - I) - 4/3/(3*e^(-x) + 3*I*e^(-2*x) - e^(-3*x)
 - I)
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.65

$$\int \frac{\sinh(x)}{(i+\sinh(x))^2}\,dx = -\frac{2\,(3\,e^{(2\,x)}+3i\,e^{x}-2)}{3\,(e^{x}+i)^3}$$

input
```
integrate(sinh(x)/(I+sinh(x))^2,x, algorithm="giac")
```

output
```
-2/3*(3*e^(2*x) + 3*I*e^x - 2)/(e^x + I)^3
```

Mupad [B] (verification not implemented)

Time = 1.68 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.81

$$\int \frac{\sinh(x)}{(i+\sinh(x))^2}\,dx = -\frac{2\,\left(3\,{\mathrm{e}}^x-{\mathrm{e}}^{2\,x}\,3{}\mathrm{i}+2{}\mathrm{i}\right)}{3\,{\left(-1+{\mathrm{e}}^x\,1{}\mathrm{i}\right)}^3}$$

input
```
int(sinh(x)/(sinh(x) + 1i)^2,x)
```

output
```
-(2*(3*exp(x) - exp(2*x)*3i + 2i))/(3*(exp(x)*1i - 1)^3)
```

Reduce [F]

$$\int \frac{\sinh(x)}{(i+\sinh(x))^2}\,dx = \int \frac{\sinh\left(x\right)}{\sinh\left(x\right)^{2}+2\sinh\left(x\right)i-1}dx$$

input
```
int(sinh(x)/(I+sinh(x))^2,x)
```

output
```
int(sinh(x)/(sinh(x)**2 + 2*sinh(x)*i - 1),x)
```

3.52 $\int \frac{\operatorname{csch}(x)}{(i+\sinh(x))^2}\,dx$

Optimal result

Integrand size = 11, antiderivative size = 34

$$\int \frac{\operatorname{csch}(x)}{(i+\sinh(x))^2}\,dx = \operatorname{arctanh}(\cosh(x)) + \frac{\cosh(x)}{3(i+\sinh(x))^2} - \frac{4i\cosh(x)}{3(i+\sinh(x))}$$

output
```
arctanh(cosh(x))+1/3*cosh(x)/(I+sinh(x))^2-4/3*I*cosh(x)/(I+sinh(x))
```

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.76

$$\int \frac{\operatorname{csch}(x)}{(i+\sinh(x))^2}\,dx = \operatorname{arctanh}(\cosh(x)) + \frac{\cosh(x)(5-4i\sinh(x))}{3(i+\sinh(x))^2}$$

input
```
Integrate[Csch[x]/(I + Sinh[x])^2,x]
```

output
```
ArcTanh[Cosh[x]] + (Cosh[x]*(5 - (4*I)*Sinh[x]))/(3*(I + Sinh[x])^2)
```

Rubi [A] (verified)

Time = 0.40 (sec) , antiderivative size = 51, normalized size of antiderivative = 1.50, number of steps used = 12, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 1.091, Rules used = {3042, 26, 25, 3245, 26, 3042, 26, 3457, 27, 3042, 26, 4257}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\text{csch}(x)}{(\sinh(x)+i)^2}dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \frac{i}{(i-i\sin(ix))^2\sin(ix)}dx$$

$$\Big\downarrow \text{ 26}$$

$$i\int -\frac{1}{(1-\sin(ix))^2\sin(ix)}dx$$

$$\Big\downarrow \text{ 25}$$

$$-i\int \frac{1}{(1-\sin(ix))^2\sin(ix)}dx$$

$$\Big\downarrow \text{ 3245}$$

$$-i\left(\frac{1}{3}\int -\frac{i\text{csch}(x)(i\sinh(x)+3)}{1-i\sinh(x)}dx-\frac{i\cosh(x)}{3(1-i\sinh(x))^2}\right)$$

$$\Big\downarrow \text{ 26}$$

$$-i\left(-\frac{1}{3}i\int \frac{\text{csch}(x)(i\sinh(x)+3)}{1-i\sinh(x)}dx-\frac{i\cosh(x)}{3(1-i\sinh(x))^2}\right)$$

$$\Big\downarrow \text{ 3042}$$

$$-i\left(-\frac{1}{3}i\int \frac{i(\sin(ix)+3)}{(1-\sin(ix))\sin(ix)}dx-\frac{i\cosh(x)}{3(1-i\sinh(x))^2}\right)$$

$$\Big\downarrow \text{ 26}$$

$$-i\left(\frac{1}{3}\int \frac{\sin(ix)+3}{(1-\sin(ix))\sin(ix)}dx-\frac{i\cosh(x)}{3(1-i\sinh(x))^2}\right)$$

$$\Big\downarrow \text{ 3457}$$

$$-i\left(\frac{1}{3}\left(\int -3i\text{csch}(x)dx-\frac{4i\cosh(x)}{1-i\sinh(x)}\right)-\frac{i\cosh(x)}{3(1-i\sinh(x))^2}\right)$$

$$\Big\downarrow \text{27}$$

$$-i\left(\frac{1}{3}\left(-3i\int \text{csch}(x)dx-\frac{4i\cosh(x)}{1-i\sinh(x)}\right)-\frac{i\cosh(x)}{3(1-i\sinh(x))^2}\right)$$

$$\Big\downarrow \text{3042}$$

$$-i\left(\frac{1}{3}\left(-3i\int i\csc(ix)dx-\frac{4i\cosh(x)}{1-i\sinh(x)}\right)-\frac{i\cosh(x)}{3(1-i\sinh(x))^2}\right)$$

$$\Big\downarrow \text{26}$$

$$-i\left(\frac{1}{3}\left(3\int \csc(ix)dx-\frac{4i\cosh(x)}{1-i\sinh(x)}\right)-\frac{i\cosh(x)}{3(1-i\sinh(x))^2}\right)$$

$$\Big\downarrow \text{4257}$$

$$-i\left(\frac{1}{3}\left(3i\text{arctanh}(\cosh(x))-\frac{4i\cosh(x)}{1-i\sinh(x)}\right)-\frac{i\cosh(x)}{3(1-i\sinh(x))^2}\right)$$

input
```
Int[Csch[x]/(I + Sinh[x])^2,x]
```

output
```
(-I)*(((3*I)*ArcTanh[Cosh[x]] - ((4*I)*Cosh[x])/(1 - I*Sinh[x]))/3 - ((I/3
)*Cosh[x])/(1 - I*Sinh[x])^2)
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3245
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_), x_Symbol] :> Simp[b^2*Cos[e + f*x]*(a + b*Sin[e + f*x])^
m*((c + d*Sin[e + f*x])^(n + 1)/(a*f*(2*m + 1)*(b*c - a*d))), x] + Simp[1/(
a*(2*m + 1)*(b*c - a*d))   Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e +
f*x])^n*Simp[b*c*(m + 1) - a*d*(2*m + n + 2) + b*d*(m + n + 2)*Sin[e + f*x]
, x], x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && EqQ
[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -1] &&  !GtQ[n, 0] && (Intege
rsQ[2*m, 2*n] || (IntegerQ[m] && EqQ[c, 0]))
```

rule 3457
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[b*(A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*((c + d*Sin[e + f*x])^(
n + 1)/(a*f*(2*m + 1)*(b*c - a*d))), x] + Simp[1/(a*(2*m + 1)*(b*c - a*d))
  Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[B*(a*c*m + b
*d*(n + 1)) + A*(b*c*(m + 1) - a*d*(2*m + n + 2)) + d*(A*b - a*B)*(m + n +
2)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ
[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -2^(-1)]
 &&  !GtQ[n, 0] && IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])
```

rule 4257
```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Maple [A] (verified)

Time = 0.48 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.06

method	result	size
risch	$-\frac{2(9i\,\mathrm{e}^{x}+3\,\mathrm{e}^{2x}-4)}{3(\mathrm{e}^{x}+i)^{3}}-\ln\left(\mathrm{e}^{x}-1\right)+\ln\left(\mathrm{e}^{x}+1\right)$	36
default	$\frac{4i}{3\left(\tanh\left(\frac{x}{2}\right)+i\right)^{3}}-\frac{4i}{\tanh\left(\frac{x}{2}\right)+i}-\frac{2}{\left(\tanh\left(\frac{x}{2}\right)+i\right)^{2}}-\ln\left(\tanh\left(\frac{x}{2}\right)\right)$	44
parallelrisch	$\frac{\left(-3\tanh\left(\frac{x}{2}\right)^{3}+9\tanh\left(\frac{x}{2}\right)-9i\tanh\left(\frac{x}{2}\right)^{2}+3i\right)\ln\left(\tanh\left(\frac{x}{2}\right)\right)+6i\tanh\left(\frac{x}{2}\right)^{2}+6\tanh\left(\frac{x}{2}\right)^{3}+4i}{-9\tanh\left(\frac{x}{2}\right)+9i\tanh\left(\frac{x}{2}\right)^{2}+3\tanh\left(\frac{x}{2}\right)^{3}-3i}$	82

input
```
int(csch(x)/(I+sinh(x))^2,x,method=_RETURNVERBOSE)
```

output
```
-2/3*(9*I*exp(x)+3*exp(2*x)-4)/(exp(x)+I)^3-ln(exp(x)-1)+ln(exp(x)+1)
```

Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 78 vs. 2(24) = 48.

Time = 0.08 (sec) , antiderivative size = 78, normalized size of antiderivative = 2.29

$$\int \frac{\operatorname{csch}(x)}{(i+\sinh(x))^2}\,dx = \frac{3\left(e^{(3x)}+3i\,e^{(2x)}-3\,e^{x}-i\right)\log\left(e^{x}+1\right)-3\left(e^{(3x)}+3i\,e^{(2x)}-3\,e^{x}-i\right)\log\left(e^{x}-1\right)-6\,e^{(2x)}-18i\,e^{x}-}{3\left(e^{(3x)}+3i\,e^{(2x)}-3\,e^{x}-i\right)}$$

input
```
integrate(csch(x)/(I+sinh(x))^2,x, algorithm="fricas")
```

output
```
1/3*(3*(e^(3*x) + 3*I*e^(2*x) - 3*e^x - I)*log(e^x + 1) - 3*(e^(3*x) + 3*I
*e^(2*x) - 3*e^x - I)*log(e^x - 1) - 6*e^(2*x) - 18*I*e^x + 8)/(e^(3*x) +
3*I*e^(2*x) - 3*e^x - I)
```

Sympy [F]

$$\int \frac{\operatorname{csch}(x)}{(i+\sinh(x))^2}\,dx = \int \frac{\operatorname{csch}(x)}{(\sinh(x)+i)^2}\,dx$$

input
```
integrate(csch(x)/(I+sinh(x))**2,x)
```

output
```
Integral(csch(x)/(sinh(x) + I)**2, x)
```

Maxima [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 55 vs. 2(24) = 48.

Time = 0.04 (sec) , antiderivative size = 55, normalized size of antiderivative = 1.62

$$\int \frac{\operatorname{csch}(x)}{(i+\sinh(x))^2}\,dx = \frac{2\left(-9i\,e^{(-x)}+3\,e^{(-2\,x)}-4\right)}{3\left(3\,e^{(-x)}+3i\,e^{(-2\,x)}-e^{(-3\,x)}-i\right)}+\log\left(e^{(-x)}+1\right)-\log\left(e^{(-x)}-1\right)$$

input
```
integrate(csch(x)/(I+sinh(x))^2,x, algorithm="maxima")
```

output
```
2/3*(-9*I*e^(-x) + 3*e^(-2*x) - 4)/(3*e^(-x) + 3*I*e^(-2*x) - e^(-3*x) - I
) + log(e^(-x) + 1) - log(e^(-x) - 1)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.00

$$\int \frac{\operatorname{csch}(x)}{(i+\sinh(x))^2}\,dx = -\frac{2\left(3\,e^{(2\,x)}+9i\,e^{x}-4\right)}{3\,(e^{x}+i)^3}+\log\left(e^{x}+1\right)-\log\left(|e^{x}-1|\right)$$

input
```
integrate(csch(x)/(I+sinh(x))^2,x, algorithm="giac")
```

output
```
-2/3*(3*e^(2*x) + 9*I*e^x - 4)/(e^x + I)^3 + log(e^x + 1) - log(abs(e^x -
1))
```

Mupad [B] (verification not implemented)

Time = 0.33 (sec) , antiderivative size = 41, normalized size of antiderivative = 1.21

$$\int \frac{\operatorname{csch}(x)}{(i+\sinh(x))^2}\,dx = \ln\left(\mathrm{e}^{x}+1\right)-\ln\left(\mathrm{e}^{x}-1\right)-\frac{2}{\mathrm{e}^{x}+1\mathrm{i}}-\frac{2\mathrm{i}}{(\mathrm{e}^{x}+1\mathrm{i})^2}-\frac{4}{3\,(\mathrm{e}^{x}+1\mathrm{i})^3}$$

input
```
int(1/(sinh(x)*(sinh(x) + 1i)^2),x)
```

output
```
log(exp(x) + 1) - log(exp(x) - 1) - 2/(exp(x) + 1i) - 2i/(exp(x) + 1i)^2 -
 4/(3*(exp(x) + 1i)^3)
```

Reduce [F]

$$\int \frac{\operatorname{csch}(x)}{(i+\sinh(x))^2}\,dx = \int \frac{\operatorname{csch}(x)}{\sinh{(x)}^2+2\sinh{(x)}\,i-1}dx$$

input
```
int(csch(x)/(I+sinh(x))^2,x)
```

output
```
int(csch(x)/(sinh(x)**2 + 2*sinh(x)*i - 1),x)
```

3.53 $\int \frac{\operatorname{csch}^2(x)}{(i+\sinh(x))^2}\,dx$

Optimal result

Integrand size = 13, antiderivative size = 42

$$\int \frac{\operatorname{csch}^2(x)}{(i+\sinh(x))^2}\,dx = 2i\operatorname{arctanh}(\cosh(x)) + \frac{10\coth(x)}{3} + \frac{\coth(x)}{3(i+\sinh(x))^2} - \frac{2i\coth(x)}{i+\sinh(x)}$$

output
```
2*I*arctanh(cosh(x))+10/3*coth(x)+1/3*coth(x)/(I+sinh(x))^2-2*I*coth(x)/(I
+sinh(x))
```

Mathematica [B] (verified)

Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 88 vs. 2(42) = 84.

Time = 1.11 (sec) , antiderivative size = 88, normalized size of antiderivative = 2.10

$$\int \frac{\operatorname{csch}^2(x)}{(i+\sinh(x))^2}\,dx = \frac{1}{6}\left(3\coth\left(\frac{x}{2}\right) + 12i\log\left(\cosh\left(\frac{x}{2}\right)\right) - 12i\log\left(\sinh\left(\frac{x}{2}\right)\right) + \frac{2}{i+\sinh(x)} - \frac{4\sinh\left(\frac{x}{2}\right)(8i+7\sinh(x))}{\left(i\cosh\left(\frac{x}{2}\right)+\sinh\left(\frac{x}{2}\right)\right)^3} + 3\tanh\left(\frac{x}{2}\right)\right)$$

input
```
Integrate[Csch[x]^2/(I + Sinh[x])^2,x]
```

output
```
(3*Coth[x/2] + (12*I)*Log[Cosh[x/2]] - (12*I)*Log[Sinh[x/2]] + 2/(I + Sinh
[x]) - (4*Sinh[x/2]*(8*I + 7*Sinh[x]))/(I*Cosh[x/2] + Sinh[x/2])^3 + 3*Tan
h[x/2])/6
```

Rubi [A] (verified)

Time = 0.56 (sec) , antiderivative size = 47, normalized size of antiderivative = 1.12, number of steps used = 21, number of rules used = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 1.538, Rules used = {3042, 25, 25, 3245, 27, 3042, 25, 3457, 25, 3042, 25, 3227, 25, 26, 3042, 25, 26, 4254, 24, 4257}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\text{csch}^2(x)}{(\sinh(x)+i)^2}dx$$
$$\Big\downarrow \text{3042}$$
$$\int -\frac{1}{(i-i\sin(ix))^2\sin(ix)^2}dx$$
$$\Big\downarrow \text{25}$$
$$-\int -\frac{1}{(1-\sin(ix))^2\sin(ix)^2}dx$$
$$\Big\downarrow \text{25}$$
$$\int \frac{1}{(1-\sin(ix))^2\sin(ix)^2}dx$$
$$\Big\downarrow \text{3245}$$
$$\frac{1}{3}\int -\frac{2\text{csch}^2(x)(i\sinh(x)+2)}{1-i\sinh(x)}dx-\frac{\coth(x)}{3(1-i\sinh(x))^2}$$
$$\Big\downarrow \text{27}$$
$$-\frac{2}{3}\int \frac{\text{csch}^2(x)(i\sinh(x)+2)}{1-i\sinh(x)}dx-\frac{\coth(x)}{3(1-i\sinh(x))^2}$$
$$\Big\downarrow \text{3042}$$
$$-\frac{2}{3}\int -\frac{\sin(ix)+2}{(1-\sin(ix))\sin(ix)^2}dx-\frac{\coth(x)}{3(1-i\sinh(x))^2}$$

$$\Big\downarrow \text{25}$$

$$\frac{2}{3}\int \frac{\sin(ix)+2}{(1-\sin(ix))\sin(ix)^2}dx-\frac{\coth(x)}{3(1-i\sinh(x))^2}$$

$$\Big\downarrow \text{3457}$$

$$\frac{2}{3}\left(\int -\text{csch}^2(x)(3i\sinh(x)+5)dx-\frac{3\coth(x)}{1-i\sinh(x)}\right)-\frac{\coth(x)}{3(1-i\sinh(x))^2}$$

$$\Big\downarrow \text{25}$$

$$\frac{2}{3}\left(-\int \text{csch}^2(x)(3i\sinh(x)+5)dx-\frac{3\coth(x)}{1-i\sinh(x)}\right)-\frac{\coth(x)}{3(1-i\sinh(x))^2}$$

$$\Big\downarrow \text{3042}$$

$$\frac{2}{3}\left(-\int -\frac{3\sin(ix)+5}{\sin(ix)^2}dx-\frac{3\coth(x)}{1-i\sinh(x)}\right)-\frac{\coth(x)}{3(1-i\sinh(x))^2}$$

$$\Big\downarrow \text{25}$$

$$\frac{2}{3}\left(\int \frac{3\sin(ix)+5}{\sin(ix)^2}dx-\frac{3\coth(x)}{1-i\sinh(x)}\right)-\frac{\coth(x)}{3(1-i\sinh(x))^2}$$

$$\Big\downarrow \text{3227}$$

$$\frac{2}{3}\left(5\int -\text{csch}^2(x)dx+3\int -i\text{csch}(x)dx-\frac{3\coth(x)}{1-i\sinh(x)}\right)-\frac{\coth(x)}{3(1-i\sinh(x))^2}$$

$$\Big\downarrow \text{25}$$

$$\frac{2}{3}\left(-5\int \text{csch}^2(x)dx+3\int -i\text{csch}(x)dx-\frac{3\coth(x)}{1-i\sinh(x)}\right)-\frac{\coth(x)}{3(1-i\sinh(x))^2}$$

$$\Big\downarrow \text{26}$$

$$\frac{2}{3}\left(-5\int \text{csch}^2(x)dx-3i\int \text{csch}(x)dx-\frac{3\coth(x)}{1-i\sinh(x)}\right)-\frac{\coth(x)}{3(1-i\sinh(x))^2}$$

$$\Big\downarrow \text{3042}$$

$$\frac{2}{3}\left(-3i\int i\csc(ix)dx-5\int -\csc(ix)^2dx-\frac{3\coth(x)}{1-i\sinh(x)}\right)-\frac{\coth(x)}{3(1-i\sinh(x))^2}$$

$$\Big\downarrow \text{25}$$

$$\frac{2}{3}\left(-3i\int i\csc(ix)dx+5\int \csc(ix)^2dx-\frac{3\coth(x)}{1-i\sinh(x)}\right)-\frac{\coth(x)}{3(1-i\sinh(x))^2}$$

$$\Big\downarrow \text{26}$$

$$\frac{2}{3}\left(3\int \csc(ix)dx+5\int \csc(ix)^2dx-\frac{3\coth(x)}{1-i\sinh(x)}\right)-\frac{\coth(x)}{3(1-i\sinh(x))^2}$$

$$\Big\downarrow \text{4254}$$

$$\frac{2}{3}\left(3\int \csc(ix)dx+5i\int 1d(-i\coth(x))-\frac{3\coth(x)}{1-i\sinh(x)}\right)-\frac{\coth(x)}{3(1-i\sinh(x))^2}$$

$$\Big\downarrow \text{24}$$

$$\frac{2}{3}\left(3\int \csc(ix)dx+5\coth(x)-\frac{3\coth(x)}{1-i\sinh(x)}\right)-\frac{\coth(x)}{3(1-i\sinh(x))^2}$$

$$\Big\downarrow \text{4257}$$

$$\frac{2}{3}\left(3i\text{arctanh}(\cosh(x))+5\coth(x)-\frac{3\coth(x)}{1-i\sinh(x)}\right)-\frac{\coth(x)}{3(1-i\sinh(x))^2}$$

input
```
Int[Csch[x]^2/(I + Sinh[x])^2,x]
```

output
```
(2*((3*I)*ArcTanh[Cosh[x]] + 5*Coth[x] - (3*Coth[x])/(1 - I*Sinh[x])))/3 -
 Coth[x]/(3*(1 - I*Sinh[x])^2)
```

Defintions of rubi rules used

rule 24
```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3227
```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Simp[c   Int[(b*Sin[e + f*x])^m, x], x] + Simp[d/b   Int
[(b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

rule 3245
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_), x_Symbol] :> Simp[b^2*Cos[e + f*x]*(a + b*Sin[e + f*x])^
m*((c + d*Sin[e + f*x])^(n + 1)/(a*f*(2*m + 1)*(b*c - a*d))), x] + Simp[1/(
a*(2*m + 1)*(b*c - a*d))   Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e +
f*x])^n*Simp[b*c*(m + 1) - a*d*(2*m + n + 2) + b*d*(m + n + 2)*Sin[e + f*x]
, x], x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && EqQ
[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -1] &&  !GtQ[n, 0] && (Intege
rsQ[2*m, 2*n] || (IntegerQ[m] && EqQ[c, 0]))
```

rule 3457
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[b*(A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*((c + d*Sin[e + f*x])^(
n + 1)/(a*f*(2*m + 1)*(b*c - a*d))), x] + Simp[1/(a*(2*m + 1)*(b*c - a*d))
  Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[B*(a*c*m + b
*d*(n + 1)) + A*(b*c*(m + 1) - a*d*(2*m + n + 2)) + d*(A*b - a*B)*(m + n +
2)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ
[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -2^(-1)]
 &&  !GtQ[n, 0] && IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])
```

rule 4254
```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Simp[-d^(-1)   Subst[Int[Exp
andIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
 d}, x] && IGtQ[n/2, 0]
```

rule 4257
```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Maple [A] (verified)

Time = 0.68 (sec) , antiderivative size = 58, normalized size of antiderivative = 1.38

method	result
default	$\frac{\tanh\left(\frac{x}{2}\right)}{2}-2i\ln\left(\tanh\left(\frac{x}{2}\right)\right)+\frac{1}{2\tanh\left(\frac{x}{2}\right)}-\frac{2i}{\left(\tanh\left(\frac{x}{2}\right)+i\right)^{2}}-\frac{4}{3\left(\tanh\left(\frac{x}{2}\right)+i\right)^{3}}+\frac{6}{\tanh\left(\frac{x}{2}\right)+i}$
risch	$-\frac{4i\left(9i\mathrm{e}^{3x}+3\,\mathrm{e}^{4x}-12i\mathrm{e}^{x}-11\,\mathrm{e}^{2x}+5\right)}{3\left(\mathrm{e}^{2x}-1\right)\left(\mathrm{e}^{x}+i\right)^{3}}+2i\ln\left(\mathrm{e}^{x}+1\right)-2i\ln\left(\mathrm{e}^{x}-1\right)$
parallelrisch	$\frac{\left(36i\tanh\left(\frac{x}{2}\right)^{2}+12\tanh\left(\frac{x}{2}\right)^{3}-12i-36\tanh\left(\frac{x}{2}\right)\right)\ln\left(\tanh\left(\frac{x}{2}\right)\right)+3i\tanh\left(\frac{x}{2}\right)^{4}-19\tanh\left(\frac{x}{2}\right)^{3}-31i+3\coth\left(\frac{x}{2}\right)-36\tanh\left(\frac{x}{2}\right)}{-18\tanh\left(\frac{x}{2}\right)^{2}+6i\tanh\left(\frac{x}{2}\right)^{3}+6-18i\tanh\left(\frac{x}{2}\right)}$

input
```
int(csch(x)^2/(I+sinh(x))^2,x,method=_RETURNVERBOSE)
```

output
```
1/2*tanh(1/2*x)-2*I*ln(tanh(1/2*x))+1/2/tanh(1/2*x)-2*I/(tanh(1/2*x)+I)^2-
4/3/(tanh(1/2*x)+I)^3+6/(tanh(1/2*x)+I)
```

Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 130 vs. $2(30) = 60$.

Time = 0.10 (sec) , antiderivative size = 130, normalized size of antiderivative = 3.10

$$\int \frac{\operatorname{csch}^2(x)}{(i+\sinh(x))^2}\,dx = -\frac{2\left(3\left(-i\,e^{(5\,x)}+3\,e^{(4\,x)}+4i\,e^{(3\,x)}-4\,e^{(2\,x)}-3i\,e^{x}+1\right)\log\left(e^{x}+1\right)+3\left(i\,e^{(5\,x)}-3\,e^{(4\,x)}-4i\,e^{(3\,x)}+4\,\ldots\right.}{3\left(e^{(5\,x)}+3i\,e^{(4\,x)}-4\,e^{(3\,x)}-4i\,e^{(2\,x)}\ldots\right.}$$

input
```
integrate(csch(x)^2/(I+sinh(x))^2,x, algorithm="fricas")
```

output
```
-2/3*(3*(-I*e^(5*x) + 3*e^(4*x) + 4*I*e^(3*x) - 4*e^(2*x) - 3*I*e^x + 1)*l
og(e^x + 1) + 3*(I*e^(5*x) - 3*e^(4*x) - 4*I*e^(3*x) + 4*e^(2*x) + 3*I*e^x
 - 1)*log(e^x - 1) + 6*I*e^(4*x) - 18*e^(3*x) - 22*I*e^(2*x) + 24*e^x + 10
*I)/(e^(5*x) + 3*I*e^(4*x) - 4*e^(3*x) - 4*I*e^(2*x) + 3*e^x + I)
```

Sympy [F]

$$\int \frac{\operatorname{csch}^2(x)}{(i+\sinh(x))^2}\,dx = \int \frac{\operatorname{csch}^2(x)}{(\sinh(x)+i)^2}\,dx$$

input
```
integrate(csch(x)**2/(I+sinh(x))**2,x)
```

output
```
Integral(csch(x)**2/(sinh(x) + I)**2, x)
```

Maxima [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 79 vs. 2(30) = 60.

Time = 0.03 (sec) , antiderivative size = 79, normalized size of antiderivative = 1.88

$$\int \frac{\operatorname{csch}^2(x)}{(i+\sinh(x))^2}\,dx = \frac{4\left(12\,e^{(-x)}+11i\,e^{(-2\,x)}-9\,e^{(-3\,x)}-3i\,e^{(-4\,x)}-5i\right)}{3\left(3\,e^{(-x)}+4i\,e^{(-2\,x)}-4\,e^{(-3\,x)}-3i\,e^{(-4\,x)}+e^{(-5\,x)}-i\right)} + 2i\,\log\left(e^{(-x)}+1\right)-2i\,\log\left(e^{(-x)}-1\right)$$

input
```
integrate(csch(x)^2/(I+sinh(x))^2,x, algorithm="maxima")
```

output
```
4/3*(12*e^(-x) + 11*I*e^(-2*x) - 9*e^(-3*x) - 3*I*e^(-4*x) - 5*I)/(3*e^(-x
) + 4*I*e^(-2*x) - 4*e^(-3*x) - 3*I*e^(-4*x) + e^(-5*x) - I) + 2*I*log(e^(
-x) + 1) - 2*I*log(e^(-x) - 1)
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.10

$$\int \frac{\operatorname{csch}^2(x)}{(i+\sinh(x))^2}\,dx = \frac{2}{e^{(2\,x)}-1} - \frac{2\left(6i\,e^{(2\,x)}-15\,e^{x}-7i\right)}{3\,(e^{x}+i)^3} + 2i\,\log\left(e^{x}+1\right)-2i\,\log\left(|e^{x}-1|\right)$$

input
```
integrate(csch(x)^2/(I+sinh(x))^2,x, algorithm="giac")
```

output
```
2/(e^(2*x) - 1) - 2/3*(6*I*e^(2*x) - 15*e^x - 7*I)/(e^x + I)^3 + 2*I*log(e
^x + 1) - 2*I*log(abs(e^x - 1))
```

Mupad [B] (verification not implemented)

Time = 1.95 (sec) , antiderivative size = 85, normalized size of antiderivative = 2.02

$$\int \frac{\mathrm{csch}^2(x)}{(i+\sinh(x))^2}\,dx = \frac{2}{e^{2x}-1+e^{x}\,2i}+\frac{2}{e^{2x}-1}-\ln\left(e^{x}\,4i-4i\right)\,2i+\ln\left(e^{x}\,4i+4i\right)\,2i-\frac{4i}{e^{x}+1i}-\frac{4i}{3\,\left(e^{2x}\,3i+e^{3x}-3\,e^{x}-i\right)}$$

input
```
int(1/(sinh(x)^2*(sinh(x) + 1i)^2),x)
```

output
```
log(exp(x)*4i + 4i)*2i - log(exp(x)*4i - 4i)*2i + 2/(exp(2*x) + exp(x)*2i
- 1) - 4i/(exp(x) + 1i) - 4i/(3*(exp(2*x)*3i + exp(3*x) - 3*exp(x) - 1i))
+ 2/(exp(2*x) - 1)
```

Reduce [F]

$$\int \frac{\mathrm{csch}^2(x)}{(i+\sinh(x))^2}\,dx = \int \frac{\mathrm{csch}(x)^2}{\sinh(x)^2+2\sinh(x)\,i-1}dx$$

input
```
int(csch(x)^2/(I+sinh(x))^2,x)
```

output
```
int(csch(x)**2/(sinh(x)**2 + 2*sinh(x)*i - 1),x)
```

3.54 $\int \frac{\operatorname{csch}^3(x)}{(i+\sinh(x))^2} \, dx$

Optimal result

Integrand size = 13, antiderivative size = 58

$$\int \frac{\operatorname{csch}^3(x)}{(i+\sinh(x))^2} \, dx = -\frac{7}{2}\operatorname{arctanh}(\cosh(x)) + \frac{16}{3}i\coth(x) + \frac{7}{2}\coth(x)\operatorname{csch}(x) + \frac{\coth(x)\operatorname{csch}(x)}{3(i+\sinh(x))^2} - \frac{8i\coth(x)\operatorname{csch}(x)}{3(i+\sinh(x))}$$

output

```
-7/2*arctanh(cosh(x))+16/3*I*coth(x)+7/2*coth(x)*csch(x)+1/3*coth(x)*csch(
x)/(I+sinh(x))^2-8/3*I*coth(x)*csch(x)/(I+sinh(x))
```

Mathematica [B] (verified)

Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 140 vs. 2(58) = 116.

Time = 1.02 (sec) , antiderivative size = 140, normalized size of antiderivative = 2.41

$$\int \frac{\operatorname{csch}^3(x)}{(i+\sinh(x))^2}\,dx = \frac{1}{24}\left(24i\coth\left(\frac{x}{2}\right)+3\operatorname{csch}^2\left(\frac{x}{2}\right)-84\log\left(\cosh\left(\frac{x}{2}\right)\right)+84\log\left(\sinh\left(\frac{x}{2}\right)\right)+3\operatorname{sech}^2\left(\frac{x}{2}\right)+\frac{8}{\left(\cosh\left(\frac{x}{2}\right)-i\sinh\left(\frac{x}{2}\right)\right)^2}+\frac{160i\sinh\left(\frac{x}{2}\right)}{\cosh\left(\frac{x}{2}\right)-i\sinh\left(\frac{x}{2}\right)}+\frac{16\sinh\left(\frac{x}{2}\right)}{\left(i\cosh\left(\frac{x}{2}\right)+\sinh\left(\frac{x}{2}\right)\right)^3}+24i\tanh\left(\frac{x}{2}\right)\right)$$

input
```
Integrate[Csch[x]^3/(I + Sinh[x])^2,x]
```

output
```
((24*I)*Coth[x/2] + 3*Csch[x/2]^2 - 84*Log[Cosh[x/2]] + 84*Log[Sinh[x/2]]
+ 3*Sech[x/2]^2 + 8/(Cosh[x/2] - I*Sinh[x/2])^2 + ((160*I)*Sinh[x/2])/(Cos
h[x/2] - I*Sinh[x/2]) + (16*Sinh[x/2])/(I*Cosh[x/2] + Sinh[x/2])^3 + (24*I
)*Tanh[x/2])/24
```

Rubi [A] (verified)

Time = 0.67 (sec) , antiderivative size = 74, normalized size of antiderivative = 1.28, number of steps used = 25, number of rules used = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 1.846, Rules used = {3042, 26, 25, 3245, 26, 3042, 26, 3457, 26, 3042, 26, 3227, 25, 26, 3042, 25, 26, 4254, 24, 4255, 26, 3042, 26, 4257}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\operatorname{csch}^3(x)}{(\sinh(x)+i)^2}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int -\frac{i}{(i-i\sin(ix))^2\sin(ix)^3}dx$$

$$\Big\downarrow \; 26$$

$$-i\int -\frac{1}{(1-\sin(ix))^2\sin(ix)^3}dx$$

$$\Big\downarrow \; 25$$

$$i\int \frac{1}{(1-\sin(ix))^2\sin(ix)^3}dx$$

$$\Big\downarrow \; 3245$$

$$i\left(\frac{1}{3}\int \frac{i\text{csch}^3(x)(3i\sinh(x)+5)}{1-i\sinh(x)}dx+\frac{i\coth(x)\text{csch}(x)}{3(1-i\sinh(x))^2}\right)$$

$$\Big\downarrow \; 26$$

$$i\left(\frac{1}{3}i\int \frac{\text{csch}^3(x)(3i\sinh(x)+5)}{1-i\sinh(x)}dx+\frac{i\coth(x)\text{csch}(x)}{3(1-i\sinh(x))^2}\right)$$

$$\Big\downarrow \; 3042$$

$$i\left(\frac{1}{3}i\int -\frac{i(3\sin(ix)+5)}{(1-\sin(ix))\sin(ix)^3}dx+\frac{i\coth(x)\text{csch}(x)}{3(1-i\sinh(x))^2}\right)$$

$$\Big\downarrow \; 26$$

$$i\left(\frac{1}{3}\int \frac{3\sin(ix)+5}{(1-\sin(ix))\sin(ix)^3}dx+\frac{i\coth(x)\text{csch}(x)}{3(1-i\sinh(x))^2}\right)$$

$$\Big\downarrow \; 3457$$

$$i\left(\frac{1}{3}\left(\int i\text{csch}^3(x)(16i\sinh(x)+21)dx+\frac{8i\coth(x)\text{csch}(x)}{1-i\sinh(x)}\right)+\frac{i\coth(x)\text{csch}(x)}{3(1-i\sinh(x))^2}\right)$$

$$\Big\downarrow \; 26$$

$$i\left(\frac{1}{3}\left(i\int \text{csch}^3(x)(16i\sinh(x)+21)dx+\frac{8i\coth(x)\text{csch}(x)}{1-i\sinh(x)}\right)+\frac{i\coth(x)\text{csch}(x)}{3(1-i\sinh(x))^2}\right)$$

$$\Big\downarrow \; 3042$$

$$i\left(\frac{1}{3}\left(i\int -\frac{i(16\sin(ix)+21)}{\sin(ix)^3}dx+\frac{8i\coth(x)\text{csch}(x)}{1-i\sinh(x)}\right)+\frac{i\coth(x)\text{csch}(x)}{3(1-i\sinh(x))^2}\right)$$

$$\Big\downarrow \; 26$$

$$i\left(\frac{1}{3}\left(\int \frac{16\sin(ix)+21}{\sin(ix)^3}dx+\frac{8i\coth(x)\text{csch}(x)}{1-i\sinh(x)}\right)+\frac{i\coth(x)\text{csch}(x)}{3(1-i\sinh(x))^2}\right)$$

$$\Big\downarrow \text{3227}$$

$$i\left(\frac{1}{3}\left(21\int i\text{csch}^3(x)dx+16\int -\text{csch}^2(x)dx+\frac{8i\coth(x)\text{csch}(x)}{1-i\sinh(x)}\right)+\frac{i\coth(x)\text{csch}(x)}{3(1-i\sinh(x))^2}\right)$$

$$\Big\downarrow \text{25}$$

$$i\left(\frac{1}{3}\left(21\int i\text{csch}^3(x)dx-16\int \text{csch}^2(x)dx+\frac{8i\coth(x)\text{csch}(x)}{1-i\sinh(x)}\right)+\frac{i\coth(x)\text{csch}(x)}{3(1-i\sinh(x))^2}\right)$$

$$\Big\downarrow \text{26}$$

$$i\left(\frac{1}{3}\left(21i\int \text{csch}^3(x)dx-16\int \text{csch}^2(x)dx+\frac{8i\coth(x)\text{csch}(x)}{1-i\sinh(x)}\right)+\frac{i\coth(x)\text{csch}(x)}{3(1-i\sinh(x))^2}\right)$$

$$\Big\downarrow \text{3042}$$

$$i\left(\frac{1}{3}\left(-16\int -\csc(ix)^2dx+21i\int -i\csc(ix)^3dx+\frac{8i\coth(x)\text{csch}(x)}{1-i\sinh(x)}\right)+\frac{i\coth(x)\text{csch}(x)}{3(1-i\sinh(x))^2}\right)$$

$$\Big\downarrow \text{25}$$

$$i\left(\frac{1}{3}\left(16\int \csc(ix)^2dx+21i\int -i\csc(ix)^3dx+\frac{8i\coth(x)\text{csch}(x)}{1-i\sinh(x)}\right)+\frac{i\coth(x)\text{csch}(x)}{3(1-i\sinh(x))^2}\right)$$

$$\Big\downarrow \text{26}$$

$$i\left(\frac{1}{3}\left(16\int \csc(ix)^2dx+21\int \csc(ix)^3dx+\frac{8i\coth(x)\text{csch}(x)}{1-i\sinh(x)}\right)+\frac{i\coth(x)\text{csch}(x)}{3(1-i\sinh(x))^2}\right)$$

$$\Big\downarrow \text{4254}$$

$$i\left(\frac{1}{3}\left(21\int \csc(ix)^3dx+16i\int 1d(-i\coth(x))+\frac{8i\coth(x)\text{csch}(x)}{1-i\sinh(x)}\right)+\frac{i\coth(x)\text{csch}(x)}{3(1-i\sinh(x))^2}\right)$$

$$\Big\downarrow \text{24}$$

$$i\left(\frac{1}{3}\left(21\int \csc(ix)^3dx+16\coth(x)+\frac{8i\coth(x)\text{csch}(x)}{1-i\sinh(x)}\right)+\frac{i\coth(x)\text{csch}(x)}{3(1-i\sinh(x))^2}\right)$$

$$\Big\downarrow \text{4255}$$

$$i\left(\frac{1}{3}\left(21\left(\frac{1}{2}\int -i\text{csch}(x)dx-\frac{1}{2}i\coth(x)\text{csch}(x)\right)+16\coth(x)+\frac{8i\coth(x)\text{csch}(x)}{1-i\sinh(x)}\right)+\frac{i\coth(x)\text{csch}(x)}{3(1-i\sinh(x))^2}\right)$$

$$\Big\downarrow \text{26}$$

$$i\left(\frac{1}{3}\left(21\left(-\frac{1}{2}i\int \text{csch}(x)dx-\frac{1}{2}i\coth(x)\text{csch}(x)\right)+16\coth(x)+\frac{8i\coth(x)\text{csch}(x)}{1-i\sinh(x)}\right)+\frac{i\coth(x)\text{csch}(x)}{3(1-i\sinh(x))^2}\right)$$

$$\Big\downarrow \text{3042}$$

$$i\left(\frac{1}{3}\left(21\left(-\frac{1}{2}i\int i\csc(ix)dx-\frac{1}{2}i\coth(x)\text{csch}(x)\right)+16\coth(x)+\frac{8i\coth(x)\text{csch}(x)}{1-i\sinh(x)}\right)+\frac{i\coth(x)\text{csch}(x)}{3(1-i\sinh(x))^2}\right)$$

$$\Big\downarrow \text{26}$$

$$i\left(\frac{1}{3}\left(21\left(\frac{1}{2}\int\csc(ix)dx-\frac{1}{2}i\coth(x)\text{csch}(x)\right)+16\coth(x)+\frac{8i\coth(x)\text{csch}(x)}{1-i\sinh(x)}\right)+\frac{i\coth(x)\text{csch}(x)}{3(1-i\sinh(x))^2}\right)$$

$$\Big\downarrow \text{4257}$$

$$i\left(\frac{1}{3}\left(21\left(\frac{1}{2}i\text{arctanh}(\cosh(x))-\frac{1}{2}i\coth(x)\text{csch}(x)\right)+16\coth(x)+\frac{8i\coth(x)\text{csch}(x)}{1-i\sinh(x)}\right)+\frac{i\coth(x)\text{csch}(x)}{3(1-i\sinh(x))^2}\right)$$

input
```
Int[Csch[x]^3/(I + Sinh[x])^2,x]
```

output
```
I*((16*Coth[x] + 21*((I/2)*ArcTanh[Cosh[x]] - (I/2)*Coth[x]*Csch[x]) + ((8
*I)*Coth[x]*Csch[x])/(1 - I*Sinh[x]))/3 + ((I/3)*Coth[x]*Csch[x])/(1 - I*S
inh[x])^2)
```

Defintions of rubi rules used

rule 24
```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3227
```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Simp[c   Int[(b*Sin[e + f*x])^m, x], x] + Simp[d/b   Int
[(b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

rule 3245
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_), x_Symbol] :> Simp[b^2*Cos[e + f*x]*(a + b*Sin[e + f*x])^
m*((c + d*Sin[e + f*x])^(n + 1)/(a*f*(2*m + 1)*(b*c - a*d))), x] + Simp[1/(
a*(2*m + 1)*(b*c - a*d))   Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e +
f*x])^n*Simp[b*c*(m + 1) - a*d*(2*m + n + 2) + b*d*(m + n + 2)*Sin[e + f*x]
, x], x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && EqQ
[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -1] &&  !GtQ[n, 0] && (Intege
rsQ[2*m, 2*n] || (IntegerQ[m] && EqQ[c, 0]))
```

rule 3457
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[b*(A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*((c + d*Sin[e + f*x])^(
n + 1)/(a*f*(2*m + 1)*(b*c - a*d))), x] + Simp[1/(a*(2*m + 1)*(b*c - a*d))
  Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[B*(a*c*m + b
*d*(n + 1)) + A*(b*c*(m + 1) - a*d*(2*m + n + 2)) + d*(A*b - a*B)*(m + n +
2)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ
[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -2^(-1)]
 &&  !GtQ[n, 0] && IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])
```

rule 4254
```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Simp[-d^(-1)   Subst[Int[Exp
andIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
 d}, x] && IGtQ[n/2, 0]
```

rule 4255
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Csc[c + d*x])^(n - 1)/(d*(n - 1))), x] + Simp[b^2*((n - 2)/(n - 1))
  Int[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
&& IntegerQ[2*n]
```

rule 4257
```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Maple [A] (verified)

Time = 0.96 (sec) , antiderivative size = 72, normalized size of antiderivative = 1.24

method	result
risch	$\frac{-98\,e^{4x}+63i e^{5x}+97\,e^{2x}-126i e^{3x}+21\,e^{6x}-32+75i e^{x}}{3(e^{2x}-1)^{2}(e^{x}+i)^{3}}+\frac{7\ln(e^{x}-1)}{2}-\frac{7\ln(e^{x}+1)}{2}$
default	$i\tanh\left(\frac{x}{2}\right)-\frac{\tanh(\frac{x}{2})^{2}}{8}+\frac{8i}{\tanh(\frac{x}{2})+i}-\frac{4i}{3(\tanh(\frac{x}{2})+i)^{3}}+\frac{2}{(\tanh(\frac{x}{2})+i)^{2}}+\frac{i}{\tanh(\frac{x}{2})}+\frac{1}{8\tanh(\frac{x}{2})^{2}}+\frac{7\ln(\mathrm{ta}$
parallelrisch	$\frac{\left(252i\tanh(\frac{x}{2})^{2}+84\tanh(\frac{x}{2})^{3}-84i-252\tanh(\frac{x}{2})\right)\ln(\tanh(\frac{x}{2}))+15i\tanh(\frac{x}{2})^{4}-3\tanh(\frac{x}{2})^{5}-3i\coth(\frac{x}{2})^{2}-112\tanh(\frac{x}{2})^{3}-}{-72\tanh(\frac{x}{2})+72i\tanh(\frac{x}{2})^{2}+24\tanh(\frac{x}{2})^{3}-24i}$

input
```
int(csch(x)^3/(I+sinh(x))^2,x,method=_RETURNVERBOSE)
```

output
```
1/3*(-98*exp(x)^4+63*I*exp(x)^5+97*exp(x)^2-126*I*exp(x)^3+21*exp(x)^6-32+
75*I*exp(x))/(exp(x)^2-1)^2/(exp(x)+I)^3+7/2*ln(exp(x)-1)-7/2*ln(exp(x)+1)
```

Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 174 vs. 2(40) = 80.

Time = 0.09 (sec) , antiderivative size = 174, normalized size of antiderivative = 3.00

$$\int \frac{\operatorname{csch}^3(x)}{(i+\sinh(x))^2}\,dx = -\frac{21\left(e^{(7x)}+3i\,e^{(6x)}-5\,e^{(5x)}-7i\,e^{(4x)}+7\,e^{(3x)}+5i\,e^{(2x)}-3\,e^{x}-i\right)\log\left(e^{x}+1\right)-21\left(e^{(7x)}+3i\,e^{(6x)}\right.}{6\left(e^{(7x)}+3i\,e^{(6x}\right.}$$

input
```
integrate(csch(x)^3/(I+sinh(x))^2,x, algorithm="fricas")
```

output
```
-1/6*(21*(e^(7*x) + 3*I*e^(6*x) - 5*e^(5*x) - 7*I*e^(4*x) + 7*e^(3*x) + 5*
I*e^(2*x) - 3*e^x - I)*log(e^x + 1) - 21*(e^(7*x) + 3*I*e^(6*x) - 5*e^(5*x
) - 7*I*e^(4*x) + 7*e^(3*x) + 5*I*e^(2*x) - 3*e^x - I)*log(e^x - 1) - 42*e
^(6*x) - 126*I*e^(5*x) + 196*e^(4*x) + 252*I*e^(3*x) - 194*e^(2*x) - 150*I
*e^x + 64)/(e^(7*x) + 3*I*e^(6*x) - 5*e^(5*x) - 7*I*e^(4*x) + 7*e^(3*x) +
5*I*e^(2*x) - 3*e^x - I)
```

Sympy [F]

$$\int \frac{\operatorname{csch}^3(x)}{(i+\sinh(x))^2}\,dx = \int \frac{\operatorname{csch}^3(x)}{(\sinh(x)+i)^2}\,dx$$

input
```
integrate(csch(x)**3/(I+sinh(x))**2,x)
```

output
```
Integral(csch(x)**3/(sinh(x) + I)**2, x)
```

Maxima [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 105 vs. $2(40) = 80$.

Time = 0.05 (sec) , antiderivative size = 105, normalized size of antiderivative = 1.81

$$\begin{aligned}\int \frac{\operatorname{csch}^3(x)}{(i+\sinh(x))^2}\,dx &= -\frac{-75i\,e^{(-x)} + 97\,e^{(-2\,x)} + 126i\,e^{(-3\,x)} - 98\,e^{(-4\,x)} - 63i\,e^{(-5\,x)} + 21\,e^{(-6\,x)} - 32}{3\left(3\,e^{(-x)} + 5i\,e^{(-2\,x)} - 7\,e^{(-3\,x)} - 7i\,e^{(-4\,x)} + 5\,e^{(-5\,x)} + 3i\,e^{(-6\,x)} - e^{(-7\,x)} - i\right)} \\ &\quad - \frac{7}{2}\log\left(e^{(-x)}+1\right) + \frac{7}{2}\log\left(e^{(-x)}-1\right)\end{aligned}$$

input
```
integrate(csch(x)^3/(I+sinh(x))^2,x, algorithm="maxima")
```

output
```
-1/3*(-75*I*e^(-x) + 97*e^(-2*x) + 126*I*e^(-3*x) - 98*e^(-4*x) - 63*I*e^(
-5*x) + 21*e^(-6*x) - 32)/(3*e^(-x) + 5*I*e^(-2*x) - 7*e^(-3*x) - 7*I*e^(-
4*x) + 5*e^(-5*x) + 3*I*e^(-6*x) - e^(-7*x) - I) - 7/2*log(e^(-x) + 1) + 7
/2*log(e^(-x) - 1)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 59, normalized size of antiderivative = 1.02

$$\int \frac{\operatorname{csch}^3(x)}{(i+\sinh(x))^2}\,dx = \frac{e^{(3x)} + 4i\,e^{(2x)} + e^x - 4i}{\left(e^{(2x)} - 1\right)^2} + \frac{2\left(9\,e^{(2x)} + 21i\,e^x - 10\right)}{3\left(e^x + i\right)^3} - \frac{7}{2}\log\left(e^x + 1\right) + \frac{7}{2}\log\left(|e^x - 1|\right)$$

input
```
integrate(csch(x)^3/(I+sinh(x))^2,x, algorithm="giac")
```

output
```
(e^(3*x) + 4*I*e^(2*x) + e^x - 4*I)/(e^(2*x) - 1)^2 + 2/3*(9*e^(2*x) + 21*
I*e^x - 10)/(e^x + I)^3 - 7/2*log(e^x + 1) + 7/2*log(abs(e^x - 1))
```

Mupad [B] (verification not implemented)

Time = 2.04 (sec) , antiderivative size = 79, normalized size of antiderivative = 1.36

$$\int \frac{\operatorname{csch}^3(x)}{(i+\sinh(x))^2}\,dx = \frac{e^x}{e^{2x} - 1} - \frac{7\ln\left(e^x + 1\right)}{2} - \frac{7\ln\left(\frac{1}{e^x - 1}\right)}{2} + \frac{2\,e^x}{\left(e^{2x} - 1\right)^2} + \frac{6}{e^x + 1i} + \frac{2i}{\left(e^x + 1i\right)^2} + \frac{4}{3\left(e^x + 1i\right)^3} + \frac{4i}{e^{2x} - 1}$$

input
```
int(1/(sinh(x)^3*(sinh(x) + 1i)^2),x)
```

output
```
exp(x)/(exp(2*x) - 1) - (7*log(exp(x) + 1))/2 - (7*log(1/(exp(x) - 1)))/2
+ (2*exp(x))/(exp(2*x) - 1)^2 + 6/(exp(x) + 1i) + 2i/(exp(x) + 1i)^2 + 4/(
3*(exp(x) + 1i)^3) + 4i/(exp(2*x) - 1)
```

Reduce [F]

$$\int \frac{\operatorname{csch}^3(x)}{(i+\sinh(x))^2}\,dx = \int \frac{\operatorname{csch}(x)^3}{\sinh(x)^2 + 2\sinh(x)\,i - 1}dx$$

input
```
int(csch(x)^3/(I+sinh(x))^2,x)
```

output
```
int(csch(x)**3/(sinh(x)**2 + 2*sinh(x)*i - 1),x)
```

3.55 $\int \frac{\operatorname{csch}^4(x)}{(i+\sinh(x))^2}\,dx$

Optimal result

Integrand size = 13, antiderivative size = 64

$$\int \frac{\operatorname{csch}^4(x)}{(i+\sinh(x))^2}\,dx = -5i\operatorname{arctanh}(\cosh(x)) - 12\coth(x) + 4\coth^3(x) + 5i\coth(x)\operatorname{csch}(x) + \frac{\coth(x)\operatorname{csch}^2(x)}{3(i+\sinh(x))^2} - \frac{10i\coth(x)\operatorname{csch}^2(x)}{3(i+\sinh(x))}$$

output
```
-5*I*arctanh(cosh(x))-12*coth(x)+4*coth(x)^3+5*I*coth(x)*csch(x)+1/3*coth(
x)*csch(x)^2/(I+sinh(x))^2-10/3*I*coth(x)*csch(x)^2/(I+sinh(x))
```

Mathematica [B] (verified)

Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 143 vs. $2(64) = 128$.

Time = 1.98 (sec) , antiderivative size = 143, normalized size of antiderivative = 2.23

$$\int \frac{\operatorname{csch}^4(x)}{(i+\sinh(x))^2}\,dx = \frac{1}{24}\Bigg(-44\coth\left(\frac{x}{2}\right) + 6i\operatorname{csch}^2\left(\frac{x}{2}\right) + \frac{1}{2}\operatorname{csch}^4\left(\frac{x}{2}\right)\sinh(x) + 2\Bigg(-60i\log\left(\cosh\left(\frac{x}{2}\right)\right) + 60i\log\left(\sinh\left(\frac{x}{2}\right)\right) + 3i\operatorname{sech}^2\left(\frac{x}{2}\right) - 4\operatorname{csch}^3(x)\sinh^4\left(\frac{x}{2}\right) - \frac{4}{i+\sinh(x)} + \frac{8\sinh\left(\frac{x}{2}\right)(14i+13\sinh(x))}{\left(i\cosh\left(\frac{x}{2}\right)+\sinh\left(\frac{x}{2}\right)\right)^3} - 22\tanh\left(\frac{x}{2}\right)\Bigg)\Bigg)$$

input
```
Integrate[Csch[x]^4/(I + Sinh[x])^2,x]
```

output
```
(-44*Coth[x/2] + (6*I)*Csch[x/2]^2 + (Csch[x/2]^4*Sinh[x])/2 + 2*((-60*I)*
Log[Cosh[x/2]] + (60*I)*Log[Sinh[x/2]] + (3*I)*Sech[x/2]^2 - 4*Csch[x]^3*S
inh[x/2]^4 - 4/(I + Sinh[x]) + (8*Sinh[x/2]*(14*I + 13*Sinh[x]))/(I*Cosh[x
/2] + Sinh[x/2])^3 - 22*Tanh[x/2]))/24
```

Rubi [A] (verified)

Time = 0.70 (sec) , antiderivative size = 88, normalized size of antiderivative = 1.38, number of steps used = 19, number of rules used = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 1.385$, Rules used = {3042, 3245, 27, 3042, 3457, 27, 3042, 3227, 26, 3042, 26, 4254, 2009, 4255, 26, 3042, 26, 4257}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\operatorname{csch}^4(x)}{(\sinh(x)+i)^2}\,dx$$

$$\Big\downarrow \; 3042$$

$$\int \frac{1}{(i-i\sin(ix))^2\sin(ix)^4}dx$$

$$\Big\downarrow \text{3245}$$

$$\frac{\coth(x)\text{csch}^2(x)}{3(\sinh(x)+i)^2}-\frac{1}{3}\int \frac{2\text{csch}^4(x)(3i-2\sinh(x))}{\sinh(x)+i}dx$$

$$\Big\downarrow \text{27}$$

$$\frac{\coth(x)\text{csch}^2(x)}{3(\sinh(x)+i)^2}-\frac{2}{3}\int \frac{\text{csch}^4(x)(3i-2\sinh(x))}{\sinh(x)+i}dx$$

$$\Big\downarrow \text{3042}$$

$$\frac{\coth(x)\text{csch}^2(x)}{3(\sinh(x)+i)^2}-\frac{2}{3}\int \frac{2i\sin(ix)+3i}{(i-i\sin(ix))\sin(ix)^4}dx$$

$$\Big\downarrow \text{3457}$$

$$\frac{\coth(x)\text{csch}^2(x)}{3(\sinh(x)+i)^2}-\frac{2}{3}\left(\frac{5i\coth(x)\text{csch}^2(x)}{\sinh(x)+i}-\int -3\text{csch}^4(x)(5i\sinh(x)+6)dx\right)$$

$$\Big\downarrow \text{27}$$

$$\frac{\coth(x)\text{csch}^2(x)}{3(\sinh(x)+i)^2}-\frac{2}{3}\left(3\int \text{csch}^4(x)(5i\sinh(x)+6)dx+\frac{5i\coth(x)\text{csch}^2(x)}{\sinh(x)+i}\right)$$

$$\Big\downarrow \text{3042}$$

$$\frac{\coth(x)\text{csch}^2(x)}{3(\sinh(x)+i)^2}-\frac{2}{3}\left(3\int \frac{5\sin(ix)+6}{\sin(ix)^4}dx+\frac{5i\coth(x)\text{csch}^2(x)}{\sinh(x)+i}\right)$$

$$\Big\downarrow \text{3227}$$

$$\frac{\coth(x)\text{csch}^2(x)}{3(\sinh(x)+i)^2}-\frac{2}{3}\left(3\left(6\int \text{csch}^4(x)dx+5\int i\text{csch}^3(x)dx\right)+\frac{5i\coth(x)\text{csch}^2(x)}{\sinh(x)+i}\right)$$

$$\Big\downarrow \text{26}$$

$$\frac{\coth(x)\text{csch}^2(x)}{3(\sinh(x)+i)^2}-\frac{2}{3}\left(3\left(6\int \text{csch}^4(x)dx+5i\int \text{csch}^3(x)dx\right)+\frac{5i\coth(x)\text{csch}^2(x)}{\sinh(x)+i}\right)$$

$$\Big\downarrow \text{3042}$$

$$\frac{\coth(x)\text{csch}^2(x)}{3(\sinh(x)+i)^2}-\frac{2}{3}\left(3\left(5i\int -i\csc(ix)^3dx+6\int \csc(ix)^4dx\right)+\frac{5i\coth(x)\text{csch}^2(x)}{\sinh(x)+i}\right)$$

$$\Big\downarrow \text{26}$$

$$\frac{\coth(x)\text{csch}^2(x)}{3(\sinh(x)+i)^2}-\frac{2}{3}\left(3\left(5\int \csc(ix)^3dx+6\int \csc(ix)^4dx\right)+\frac{5i\coth(x)\text{csch}^2(x)}{\sinh(x)+i}\right)$$

$$\Big\downarrow \; 4254$$

$$\frac{\coth(x)\text{csch}^2(x)}{3(\sinh(x)+i)^2}-\frac{2}{3}\left(3\left(5\int \csc(ix)^3dx+6i\int \left(1-\coth^2(x)\right)d(-i\coth(x))\right)+\frac{5i\coth(x)\text{csch}^2(x)}{\sinh(x)+i}\right)$$

$$\Big\downarrow \; 2009$$

$$\frac{\coth(x)\text{csch}^2(x)}{3(\sinh(x)+i)^2}-\frac{2}{3}\left(3\left(5\int \csc(ix)^3dx+6i\left(\frac{1}{3}i\coth^3(x)-i\coth(x)\right)\right)+\frac{5i\coth(x)\text{csch}^2(x)}{\sinh(x)+i}\right)$$

$$\Big\downarrow \; 4255$$

$$\frac{\coth(x)\text{csch}^2(x)}{3(\sinh(x)+i)^2}-\frac{2}{3}\left(3\left(5\left(\frac{1}{2}\int -i\text{csch}(x)dx-\frac{1}{2}i\coth(x)\text{csch}(x)\right)+6i\left(\frac{1}{3}i\coth^3(x)-i\coth(x)\right)\right)+\frac{5i\coth(x)\text{csch}^2(x)}{\sinh(x)+i}\right)$$

$$\Big\downarrow \; 26$$

$$\frac{\coth(x)\text{csch}^2(x)}{3(\sinh(x)+i)^2}-\frac{2}{3}\left(3\left(5\left(-\frac{1}{2}i\int \text{csch}(x)dx-\frac{1}{2}i\coth(x)\text{csch}(x)\right)+6i\left(\frac{1}{3}i\coth^3(x)-i\coth(x)\right)\right)+\frac{5i\coth(x)\text{csch}^2(x)}{\sinh(x)+i}\right)$$

$$\Big\downarrow \; 3042$$

$$\frac{\coth(x)\text{csch}^2(x)}{3(\sinh(x)+i)^2}-\frac{2}{3}\left(3\left(5\left(-\frac{1}{2}i\int i\csc(ix)dx-\frac{1}{2}i\coth(x)\text{csch}(x)\right)+6i\left(\frac{1}{3}i\coth^3(x)-i\coth(x)\right)\right)+\frac{5i\coth(x)\text{csch}^2(x)}{\sinh(x)+i}\right)$$

$$\Big\downarrow \; 26$$

$$\frac{\coth(x)\text{csch}^2(x)}{3(\sinh(x)+i)^2}-\frac{2}{3}\left(3\left(5\left(\frac{1}{2}\int \csc(ix)dx-\frac{1}{2}i\coth(x)\text{csch}(x)\right)+6i\left(\frac{1}{3}i\coth^3(x)-i\coth(x)\right)\right)+\frac{5i\coth(x)\text{csch}^2(x)}{\sinh(x)+i}\right)$$

$$\Big\downarrow \; 4257$$

$$\frac{\coth(x)\text{csch}^2(x)}{3(\sinh(x)+i)^2}-\frac{2}{3}\left(3\left(5\left(\frac{1}{2}i\text{arctanh}(\cosh(x))-\frac{1}{2}i\coth(x)\text{csch}(x)\right)+6i\left(\frac{1}{3}i\coth^3(x)-i\coth(x)\right)\right)+\frac{5i\coth(x)\text{csch}^2(x)}{\sinh(x)+i}\right)$$

input
```
Int[Csch[x]^4/(I + Sinh[x])^2,x]
```

output
```
(Coth[x]*Csch[x]^2)/(3*(I + Sinh[x])^2) - (2*(3*((6*I)*((-I)*Coth[x] + (I/
3)*Coth[x]^3) + 5*((I/2)*ArcTanh[Cosh[x]] - (I/2)*Coth[x]*Csch[x])) + ((5*
I)*Coth[x]*Csch[x]^2)/(I + Sinh[x])))/3
```

Defintions of rubi rules used

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3227
```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Simp[c   Int[(b*Sin[e + f*x])^m, x], x] + Simp[d/b   Int
[(b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

rule 3245
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_), x_Symbol] :> Simp[b^2*Cos[e + f*x]*(a + b*Sin[e + f*x])^
m*((c + d*Sin[e + f*x])^(n + 1)/(a*f*(2*m + 1)*(b*c - a*d))), x] + Simp[1/(
a*(2*m + 1)*(b*c - a*d))   Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e +
f*x])^n*Simp[b*c*(m + 1) - a*d*(2*m + n + 2) + b*d*(m + n + 2)*Sin[e + f*x]
, x], x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && EqQ
[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -1] &&  !GtQ[n, 0] && (Intege
rsQ[2*m, 2*n] || (IntegerQ[m] && EqQ[c, 0]))
```

rule 3457
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[b*(A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*((c + d*Sin[e + f*x])^(
n + 1)/(a*f*(2*m + 1)*(b*c - a*d))), x] + Simp[1/(a*(2*m + 1)*(b*c - a*d))
  Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[B*(a*c*m + b
*d*(n + 1)) + A*(b*c*(m + 1) - a*d*(2*m + n + 2)) + d*(A*b - a*B)*(m + n +
2)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ
[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -2^(-1)]
 &&  !GtQ[n, 0] && IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])
```

rule 4254
```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Simp[-d^(-1)   Subst[Int[Exp
andIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
 d}, x] && IGtQ[n/2, 0]
```

rule 4255
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Csc[c + d*x])^(n - 1)/(d*(n - 1))), x] + Simp[b^2*((n - 2)/(n - 1))
  Int[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
&& IntegerQ[2*n]
```

rule 4257
```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Maple [A] (verified)

Time = 1.30 (sec) , antiderivative size = 88, normalized size of antiderivative = 1.38

method	result
risch	$\frac{2i\left(45i\mathrm{e}^{7x}+15\,\mathrm{e}^{8x}-135i\mathrm{e}^{5x}-85\,\mathrm{e}^{6x}+155i\mathrm{e}^{3x}+153\,\mathrm{e}^{4x}-57i\mathrm{e}^{x}-99\,\mathrm{e}^{2x}+24\right)}{3\left(\mathrm{e}^{2x}-1\right)^{3}\left(\mathrm{e}^{x}+i\right)^{3}}+5i\ln\left(\mathrm{e}^{x}-1\right)-5i\ln\left(\mathrm{e}^{x}+1\right)$
default	$-\frac{15\tanh\left(\frac{x}{2}\right)}{8}+\frac{\tanh\left(\frac{x}{2}\right)^{3}}{24}-\frac{i\tanh\left(\frac{x}{2}\right)^{2}}{4}+\frac{2i}{\left(\tanh\left(\frac{x}{2}\right)+i\right)^{2}}+\frac{4}{3\left(\tanh\left(\frac{x}{2}\right)+i\right)^{3}}-\frac{10}{\tanh\left(\frac{x}{2}\right)+i}+\frac{i}{4\tanh\left(\frac{x}{2}\right)^{2}}+5i$
parallelrisch	$\frac{\left(-360i\tanh\left(\frac{x}{2}\right)^{2}-120\tanh\left(\frac{x}{2}\right)^{3}+120i+360\tanh\left(\frac{x}{2}\right)\right)\ln\left(\tanh\left(\frac{x}{2}\right)\right)+i\tanh\left(\frac{x}{2}\right)^{6}-30i\tanh\left(\frac{x}{2}\right)^{4}+3\tanh\left(\frac{x}{2}\right)^{5}+3i\coth\left(\frac{x}{2}\right)^{2}}{-72\tanh\left(\frac{x}{2}\right)^{2}+24i\tanh\left(\frac{x}{2}\right)^{3}+24-72i\tanh\left(\frac{x}{2}\right)}$

input
```
int(csch(x)^4/(I+sinh(x))^2,x,method=_RETURNVERBOSE)
```

output
```
2/3*I*(45*I*exp(x)^7+15*exp(x)^8-135*I*exp(x)^5-85*exp(x)^6+155*I*exp(x)^3
+153*exp(x)^4-57*I*exp(x)-99*exp(x)^2+24)/(exp(x)^2-1)^3/(exp(x)+I)^3+5*I*
ln(exp(x)-1)-5*I*ln(exp(x)+1)
```

Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 226 vs. $2(50) = 100$.

Time = 0.09 (sec) , antiderivative size = 226, normalized size of antiderivative = 3.53

$$\int \frac{\operatorname{csch}^4(x)}{(i+\sinh(x))^2}\,dx =$$
$$-\frac{15\left(i\,e^{(9\,x)}-3\,e^{(8\,x)}-6i\,e^{(7\,x)}+10\,e^{(6\,x)}+12i\,e^{(5\,x)}-12\,e^{(4\,x)}-10i\,e^{(3\,x)}+6\,e^{(2\,x)}+3i\,e^{x}-1\right)\log\left(e^{x}\right.}{}$$

input
```
integrate(csch(x)^4/(I+sinh(x))^2,x, algorithm="fricas")
```

output
```
-1/3*(15*(I*e^(9*x) - 3*e^(8*x) - 6*I*e^(7*x) + 10*e^(6*x) + 12*I*e^(5*x)
- 12*e^(4*x) - 10*I*e^(3*x) + 6*e^(2*x) + 3*I*e^x - 1)*log(e^x + 1) + 15*(
-I*e^(9*x) + 3*e^(8*x) + 6*I*e^(7*x) - 10*e^(6*x) - 12*I*e^(5*x) + 12*e^(4
*x) + 10*I*e^(3*x) - 6*e^(2*x) - 3*I*e^x + 1)*log(e^x - 1) - 30*I*e^(8*x)
+ 90*e^(7*x) + 170*I*e^(6*x) - 270*e^(5*x) - 306*I*e^(4*x) + 310*e^(3*x) +
 198*I*e^(2*x) - 114*e^x - 48*I)/(e^(9*x) + 3*I*e^(8*x) - 6*e^(7*x) - 10*I
*e^(6*x) + 12*e^(5*x) + 12*I*e^(4*x) - 10*e^(3*x) - 6*I*e^(2*x) + 3*e^x +
I)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\operatorname{csch}^4(x)}{(i+\sinh(x))^2}\,dx = \text{Timed out}$$

input
```
integrate(csch(x)**4/(I+sinh(x))**2,x)
```

output
```
Timed out
```

Maxima [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 127 vs. 2(50) = 100.

Time = 0.04 (sec) , antiderivative size = 127, normalized size of antiderivative = 1.98

$$\int \frac{\operatorname{csch}^4(x)}{(i+\sinh(x))^2}\,dx = -\frac{2\left(57\,e^{(-x)}+99i\,e^{(-2\,x)}-155\,e^{(-3\,x)}-153i\,e^{(-4\,x)}+135\,e^{(-5\,x)}+85i\,e^{(-6\,x)}-45\,e^{(-7\,x)}-15i\,e^{(-8\,x)}-\right.}{3\left(3\,e^{(-x)}+6i\,e^{(-2\,x)}-10\,e^{(-3\,x)}-12i\,e^{(-4\,x)}+12\,e^{(-5\,x)}+10i\,e^{(-6\,x)}-6\,e^{(-7\,x)}-3i\,e^{(-8\,x)}+e^{(-9\,x)}\right.} - 5i\,\log\left(e^{(-x)}+1\right)+5i\,\log\left(e^{(-x)}-1\right)$$

input
```
integrate(csch(x)^4/(I+sinh(x))^2,x, algorithm="maxima")
```

output
```
-2/3*(57*e^(-x) + 99*I*e^(-2*x) - 155*e^(-3*x) - 153*I*e^(-4*x) + 135*e^(-
5*x) + 85*I*e^(-6*x) - 45*e^(-7*x) - 15*I*e^(-8*x) - 24*I)/(3*e^(-x) + 6*I
*e^(-2*x) - 10*e^(-3*x) - 12*I*e^(-4*x) + 12*e^(-5*x) + 10*I*e^(-6*x) - 6*
e^(-7*x) - 3*I*e^(-8*x) + e^(-9*x) - I) - 5*I*log(e^(-x) + 1) + 5*I*log(e^
(-x) - 1)
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 84, normalized size of antiderivative = 1.31

$$\int \frac{\operatorname{csch}^4(x)}{(i+\sinh(x))^2}\,dx = -\frac{2\left(-15i\,e^{(8\,x)}+45\,e^{(7\,x)}+85i\,e^{(6\,x)}-135\,e^{(5\,x)}-153i\,e^{(4\,x)}+155\,e^{(3\,x)}+99i\,e^{(2\,x)}-57\,e^{x}-24i\right)}{3\left(e^{(3\,x)}+i\,e^{(2\,x)}-e^{x}-i\right)^3} - 5i\,\log\left(e^{x}+1\right)+5i\,\log\left(|e^{x}-1|\right)$$

input
```
integrate(csch(x)^4/(I+sinh(x))^2,x, algorithm="giac")
```

output
```
-2/3*(-15*I*e^(8*x) + 45*e^(7*x) + 85*I*e^(6*x) - 135*e^(5*x) - 153*I*e^(4
*x) + 155*e^(3*x) + 99*I*e^(2*x) - 57*e^x - 24*I)/(e^(3*x) + I*e^(2*x) - e
^x - I)^3 - 5*I*log(e^x + 1) + 5*I*log(abs(e^x - 1))
```

Mupad [B] (verification not implemented)

Time = 2.39 (sec) , antiderivative size = 189, normalized size of antiderivative = 2.95

$$\int \frac{\operatorname{csch}^4(x)}{(i+\sinh(x))^2}\,dx = -\ln\left(-e^x\,10i-10i\right)\,5i+\ln\left(-e^x\,10i+10i\right)\,5i - \frac{\frac{16\,e^x}{3}-\frac{e^{2\,x}\,32i}{3}+\frac{16}{3}i}{12\,e^{5\,x}-10\,e^{3\,x}+e^{4\,x}\,12i-e^{2\,x}\,6i-e^{6\,x}\,10i-6\,e^{7\,x}+e^{8\,x}\,3i+e^{9\,x}+3\,e^x+1i} + \frac{\frac{20\,e^{2\,x}}{3}-\frac{44}{3}+\frac{e^x\,16i}{3}}{3\,e^{2\,x}-3\,e^{4\,x}+e^{6\,x}-1-e^{3\,x}\,4i+e^{5\,x}\,2i+e^x\,2i} - \frac{10\,e^x-e^{2\,x}\,10i+\frac{20}{3}i}{e^{2\,x}\,1i+e^{3\,x}-e^x-i}$$

input
```
int(1/(sinh(x)^4*(sinh(x) + 1i)^2),x)
```

output
```
log(10i - exp(x)*10i)*5i - log(- exp(x)*10i - 10i)*5i - ((16*exp(x))/3 - (
exp(2*x)*32i)/3 + 16i/3)/(exp(4*x)*12i - 10*exp(3*x) - exp(2*x)*6i + 12*ex
p(5*x) - exp(6*x)*10i - 6*exp(7*x) + exp(8*x)*3i + exp(9*x) + 3*exp(x) + 1
i) + ((20*exp(2*x))/3 + (exp(x)*16i)/3 - 44/3)/(3*exp(2*x) - exp(3*x)*4i -
 3*exp(4*x) + exp(5*x)*2i + exp(6*x) + exp(x)*2i - 1) - (10*exp(x) - exp(2
*x)*10i + 20i/3)/(exp(2*x)*1i + exp(3*x) - exp(x) - 1i)
```

Reduce [F]

$$\int \frac{\operatorname{csch}^4(x)}{(i+\sinh(x))^2}\,dx = \int \frac{\operatorname{csch}(x)^4}{\sinh(x)^2+2\sinh(x)\,i-1}dx$$

input
```
int(csch(x)^4/(I+sinh(x))^2,x)
```

output
```
int(csch(x)**4/(sinh(x)**2 + 2*sinh(x)*i - 1),x)
```

3.56 $\int \frac{1}{1+i\sinh(c+dx)}\,dx$

Optimal result

Integrand size = 14, antiderivative size = 27

$$\int \frac{1}{1+i\sinh(c+dx)}\,dx = \frac{i\cosh(c+dx)}{d(1+i\sinh(c+dx))}$$

output
```
I*cosh(d*x+c)/d/(1+I*sinh(d*x+c))
```

Mathematica [A] (verified)

Time = 0.16 (sec) , antiderivative size = 42, normalized size of antiderivative = 1.56

$$\int \frac{1}{1+i\sinh(c+dx)}\,dx = \frac{2\sinh\left(\frac{1}{2}(c+dx)\right)}{d\left(\cosh\left(\frac{1}{2}(c+dx)\right)+i\sinh\left(\frac{1}{2}(c+dx)\right)\right)}$$

input
```
Integrate[(1 + I*Sinh[c + d*x])^(-1),x]
```

output
```
(2*Sinh[(c + d*x)/2])/(d*(Cosh[(c + d*x)/2] + I*Sinh[(c + d*x)/2]))
```

Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {3042, 3127}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{1+i\sinh(c+dx)}\,dx$$
$$\Big\downarrow \text{ 3042}$$
$$\int \frac{1}{1+\sin(ic+idx)}dx$$
$$\Big\downarrow \text{ 3127}$$
$$\frac{i\cosh(c+dx)}{d(1+i\sinh(c+dx))}$$

input
```
Int[(1 + I*Sinh[c + d*x])^(-1),x]
```

output
```
(I*Cosh[c + d*x])/(d*(1 + I*Sinh[c + d*x]))
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3127
```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> Simp[-Cos[c +
 d*x]/(d*(b + a*Sin[c + d*x])), x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b
^2, 0]
```

Maple [A] (verified)

Time = 0.14 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.67

method	result	size
risch	$\frac{2i}{d(e^{dx+c}-i)}$	18
derivativedivides	$\frac{2}{d\left(-i+\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}$	20
default	$\frac{2}{d\left(-i+\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}$	20
parallelrisch	$-\frac{2}{d\left(i-\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}$	22

input
```
int(1/(1+I*sinh(d*x+c)),x,method=_RETURNVERBOSE)
```

output
```
2*I/d/(exp(d*x+c)-I)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.59

$$\int \frac{1}{1+i\sinh(c+dx)}\,dx = \frac{2i}{de^{(dx+c)}-i\,d}$$

input
```
integrate(1/(1+I*sinh(d*x+c)),x, algorithm="fricas")
```

output
```
2*I/(d*e^(d*x + c) - I*d)
```

Sympy [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.56

$$\int \frac{1}{1+i\sinh(c+dx)}\,dx = \frac{2i}{de^c e^{dx} - id}$$

input
```
integrate(1/(1+I*sinh(d*x+c)),x)
```

output
```
2*I/(d*exp(c)*exp(d*x) - I*d)
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.74

$$\int \frac{1}{1+i\sinh(c+dx)}\,dx = -\frac{2}{d(i\,e^{(-dx-c)} - 1)}$$

input
```
integrate(1/(1+I*sinh(d*x+c)),x, algorithm="maxima")
```

output
```
-2/(d*(I*e^(-d*x - c) - 1))
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.56

$$\int \frac{1}{1+i\sinh(c+dx)}\,dx = \frac{2i}{d(e^{(dx+c)} - i)}$$

input
```
integrate(1/(1+I*sinh(d*x+c)),x, algorithm="giac")
```

output
```
2*I/(d*(e^(d*x + c) - I))
```

Mupad [B] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.63

$$\int \frac{1}{1+i\sinh(c+dx)}\,dx = \frac{2\mathrm{i}}{d\,\left(\mathrm{e}^{c+d\,x}-\mathrm{i}\right)}$$

input
```
int(1/(sinh(c + d*x)*1i + 1),x)
```

output
```
2i/(d*(exp(c + d*x) - 1i))
```

Reduce [F]

$$\int \frac{1}{1+i\sinh(c+dx)}\,dx = \int \frac{1}{\sinh\left(dx+c\right)i+1}dx$$

input
```
int(1/(1+I*sinh(d*x+c)),x)
```

output
```
int(1/(sinh(c + d*x)*i + 1),x)
```

3.57 $\int \frac{1}{(1+i\sinh(c+dx))^2}\,dx$

Optimal result

Integrand size = 14, antiderivative size = 59

$$\int \frac{1}{(1+i\sinh(c+dx))^2}\,dx = \frac{i\cosh(c+dx)}{3d(1+i\sinh(c+dx))^2} + \frac{i\cosh(c+dx)}{3d(1+i\sinh(c+dx))}$$

output
```
1/3*I*cosh(d*x+c)/d/(1+I*sinh(d*x+c))^2+1/3*I*cosh(d*x+c)/d/(1+I*sinh(d*x+
c))
```

Mathematica [A] (verified)

Time = 0.25 (sec) , antiderivative size = 61, normalized size of antiderivative = 1.03

$$\int \frac{1}{(1+i\sinh(c+dx))^2}\,dx = \frac{3i-4i\cosh(c+dx)-i\cosh(2(c+dx))-4\sinh(c+dx)+\sinh(2(c+dx))}{6d(-i+\sinh(c+dx))^2}$$

input
```
Integrate[(1 + I*Sinh[c + d*x])^(-2),x]
```

output
```
(3*I - (4*I)*Cosh[c + d*x] - I*Cosh[2*(c + d*x)] - 4*Sinh[c + d*x] + Sinh[
2*(c + d*x)])/(6*d*(-I + Sinh[c + d*x])^2)
```

Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 59, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$, Rules used = {3042, 3129, 3042, 3127}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(1+i\sinh(c+dx))^2}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{(1+\sin(ic+idx))^2}dx$$

$$\Big\downarrow \text{3129}$$

$$\frac{1}{3}\int \frac{1}{i\sinh(c+dx)+1}dx+\frac{i\cosh(c+dx)}{3d(1+i\sinh(c+dx))^2}$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{3}\int \frac{1}{\sin(ic+idx)+1}dx+\frac{i\cosh(c+dx)}{3d(1+i\sinh(c+dx))^2}$$

$$\Big\downarrow \text{3127}$$

$$\frac{i\cosh(c+dx)}{3d(1+i\sinh(c+dx))}+\frac{i\cosh(c+dx)}{3d(1+i\sinh(c+dx))^2}$$

input
```
Int[(1 + I*Sinh[c + d*x])^(-2),x]
```

output
```
((I/3)*Cosh[c + d*x])/(d*(1 + I*Sinh[c + d*x])^2) + ((I/3)*Cosh[c + d*x])/
(d*(1 + I*Sinh[c + d*x]))
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3127
```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> Simp[-Cos[c +
 d*x]/(d*(b + a*Sin[c + d*x])), x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b
^2, 0]
```

rule 3129
```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[b*Cos[c
+ d*x]*((a + b*Sin[c + d*x])^n/(a*d*(2*n + 1))), x] + Simp[(n + 1)/(a*(2*n
+ 1))   Int[(a + b*Sin[c + d*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d}, x]
&& EqQ[a^2 - b^2, 0] && LtQ[n, -1] && IntegerQ[2*n]
```

Maple [A] (verified)

Time = 0.25 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.47

method	result	size
risch	$\frac{-\frac{2i}{3}+2\,\mathrm{e}^{dx+c}}{\left(\mathrm{e}^{dx+c}-i\right)^{3} d}$	28
derivativedivides	$\frac{\frac{2}{-i+\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)}+\frac{2i}{\left(-i+\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^{2}}-\frac{4}{3\left(-i+\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^{3}}}{d}$	55
default	$\frac{\frac{2}{-i+\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)}+\frac{2i}{\left(-i+\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^{2}}-\frac{4}{3\left(-i+\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^{3}}}{d}$	55
parallelrisch	$\frac{4+6i\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)-6\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}}{3d\left(-\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^{3}+3\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)+3i\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}-i\right)}$	76

input
```
int(1/(1+I*sinh(d*x+c))^2,x,method=_RETURNVERBOSE)
```

output
```
2/3*(-I+3*exp(d*x+c))/(exp(d*x+c)-I)^3/d
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 50, normalized size of antiderivative = 0.85

$$\int \frac{1}{(1+i\sinh(c+dx))^2}\,dx = \frac{2\left(3\,e^{(dx+c)}-i\right)}{3\left(de^{(3\,dx+3\,c)}-3i\,de^{(2\,dx+2\,c)}-3\,de^{(dx+c)}+i\,d\right)}$$

input
```
integrate(1/(1+I*sinh(d*x+c))^2,x, algorithm="fricas")
```

output
```
2/3*(3*e^(d*x + c) - I)/(d*e^(3*d*x + 3*c) - 3*I*d*e^(2*d*x + 2*c) - 3*d*e
^(d*x + c) + I*d)
```

Sympy [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 61, normalized size of antiderivative = 1.03

$$\int \frac{1}{(1+i\sinh(c+dx))^2}\,dx = \frac{6e^ce^{dx}-2i}{3de^{3c}e^{3dx}-9ide^{2c}e^{2dx}-9de^ce^{dx}+3id}$$

input
```
integrate(1/(1+I*sinh(d*x+c))**2,x)
```

output
```
(6*exp(c)*exp(d*x) - 2*I)/(3*d*exp(3*c)*exp(3*d*x) - 9*I*d*exp(2*c)*exp(2*
d*x) - 9*d*exp(c)*exp(d*x) + 3*I*d)
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 94, normalized size of antiderivative = 1.59

$$\int \frac{1}{(1+i\sinh(c+dx))^2}\,dx = \frac{2\,e^{(-dx-c)}}{d(3\,e^{(-dx-c)}-3i\,e^{(-2\,dx-2\,c)}-e^{(-3\,dx-3\,c)}+i)} + \frac{2i}{3\,d(3\,e^{(-dx-c)}-3i\,e^{(-2\,dx-2\,c)}-e^{(-3\,dx-3\,c)}+i)}$$

input
```
integrate(1/(1+I*sinh(d*x+c))^2,x, algorithm="maxima")
```

output
```
2*e^(-d*x - c)/(d*(3*e^(-d*x - c) - 3*I*e^(-2*d*x - 2*c) - e^(-3*d*x - 3*c
) + I)) + 2/3*I/(d*(3*e^(-d*x - c) - 3*I*e^(-2*d*x - 2*c) - e^(-3*d*x - 3*
c) + I))
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.42

$$\int \frac{1}{(1+i\sinh(c+dx))^2}\,dx = \frac{2\left(3\,e^{(dx+c)}-i\right)}{3\,d\left(e^{(dx+c)}-i\right)^3}$$

input
```
integrate(1/(1+I*sinh(d*x+c))^2,x, algorithm="giac")
```

output
```
2/3*(3*e^(d*x + c) - I)/(d*(e^(d*x + c) - I)^3)
```

Mupad [B] (verification not implemented)

Time = 1.72 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.49

$$\int \frac{1}{(1+i\sinh(c+dx))^2}\,dx = -\frac{\frac{2}{3}+\mathrm{e}^{c+d\,x}\,2\mathrm{i}}{d\,{\left(1+\mathrm{e}^{c+d\,x}\,1\mathrm{i}\right)}^3}$$

input
```
int(1/(sinh(c + d*x)*1i + 1)^2,x)
```

output
```
-(exp(c + d*x)*2i + 2/3)/(d*(exp(c + d*x)*1i + 1)^3)
```

Reduce [F]

$$\int \frac{1}{(1+i\sinh(c+dx))^2}\,dx = -\left(\int \frac{1}{\sinh\left(dx+c\right)^2-2\sinh\left(dx+c\right)i-1}dx\right)$$

input
```
int(1/(1+I*sinh(d*x+c))^2,x)
```

output
```
- int(1/(sinh(c + d*x)**2 - 2*sinh(c + d*x)*i - 1),x)
```

3.58 $\int \frac{1}{(1+i\sinh(c+dx))^3}\,dx$

Optimal result

Integrand size = 14, antiderivative size = 88

$$\int \frac{1}{(1+i\sinh(c+dx))^3}\,dx=\frac{i\cosh(c+dx)}{5d(1+i\sinh(c+dx))^3}+\frac{2i\cosh(c+dx)}{15d(1+i\sinh(c+dx))^2}+\frac{2i\cosh(c+dx)}{15d(1+i\sinh(c+dx))}$$

output
```
1/5*I*cosh(d*x+c)/d/(1+I*sinh(d*x+c))^3+2/15*I*cosh(d*x+c)/d/(1+I*sinh(d*x
+c))^2+2/15*I*cosh(d*x+c)/d/(1+I*sinh(d*x+c))
```

Mathematica [A] (verified)

Time = 0.25 (sec) , antiderivative size = 81, normalized size of antiderivative = 0.92

$$\int \frac{1}{(1+i\sinh(c+dx))^3}\,dx=\frac{10-15\cosh(c+dx)-6\cosh(2(c+dx))+\cosh(3(c+dx))+15i\sinh(c+dx)-6i\sinh(2(c+dx))-i}{30d(-i+\sinh(c+dx))^3}$$

input
```
Integrate[(1 + I*Sinh[c + d*x])^(-3),x]
```

output
```
(10 - 15*Cosh[c + d*x] - 6*Cosh[2*(c + d*x)] + Cosh[3*(c + d*x)] + (15*I)*
Sinh[c + d*x] - (6*I)*Sinh[2*(c + d*x)] - I*Sinh[3*(c + d*x)])/(30*d*(-I +
 Sinh[c + d*x])^3)
```

Rubi [A] (verified)

Time = 0.34 (sec) , antiderivative size = 93, normalized size of antiderivative = 1.06, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.429$, Rules used = {3042, 3129, 3042, 3129, 3042, 3127}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(1+i\sinh(c+dx))^3}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{(1+\sin(ic+idx))^3}dx$$

$$\Big\downarrow \text{3129}$$

$$\frac{2}{5}\int \frac{1}{(i\sinh(c+dx)+1)^2}dx+\frac{i\cosh(c+dx)}{5d(1+i\sinh(c+dx))^3}$$

$$\Big\downarrow \text{3042}$$

$$\frac{2}{5}\int \frac{1}{(\sin(ic+idx)+1)^2}dx+\frac{i\cosh(c+dx)}{5d(1+i\sinh(c+dx))^3}$$

$$\Big\downarrow \text{3129}$$

$$\frac{2}{5}\left(\frac{1}{3}\int \frac{1}{i\sinh(c+dx)+1}dx+\frac{i\cosh(c+dx)}{3d(1+i\sinh(c+dx))^2}\right)+\frac{i\cosh(c+dx)}{5d(1+i\sinh(c+dx))^3}$$

$$\Big\downarrow \text{3042}$$

$$\frac{2}{5}\left(\frac{1}{3}\int \frac{1}{\sin(ic+idx)+1}dx+\frac{i\cosh(c+dx)}{3d(1+i\sinh(c+dx))^2}\right)+\frac{i\cosh(c+dx)}{5d(1+i\sinh(c+dx))^3}$$

$$\Big\downarrow \text{3127}$$

$$\frac{i\cosh(c+dx)}{5d(1+i\sinh(c+dx))^3}+\frac{2}{5}\left(\frac{i\cosh(c+dx)}{3d(1+i\sinh(c+dx))}+\frac{i\cosh(c+dx)}{3d(1+i\sinh(c+dx))^2}\right)$$

input
```
Int[(1 + I*Sinh[c + d*x])^(-3),x]
```

output
```
(2*(((I/3)*Cosh[c + d*x])/(d*(1 + I*Sinh[c + d*x])^2) + ((I/3)*Cosh[c + d*
x])/(d*(1 + I*Sinh[c + d*x]))))/5 + ((I/5)*Cosh[c + d*x])/(d*(1 + I*Sinh[c
 + d*x])^3)
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3127
```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> Simp[-Cos[c +
 d*x]/(d*(b + a*Sin[c + d*x])), x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b
^2, 0]
```

rule 3129
```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[b*Cos[c
+ d*x]*((a + b*Sin[c + d*x])^n/(a*d*(2*n + 1))), x] + Simp[(n + 1)/(a*(2*n
+ 1))   Int[(a + b*Sin[c + d*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d}, x]
&& EqQ[a^2 - b^2, 0] && LtQ[n, -1] && IntegerQ[2*n]
```

Maple [A] (verified)

Time = 0.30 (sec) , antiderivative size = 40, normalized size of antiderivative = 0.45

method	result	siz
risch	$-\frac{4i\left(-5i\,e^{dx+c}+10\,e^{2dx+2c}-1\right)}{15d\left(e^{dx+c}-i\right)^{5}}$	40
derivativedivides	$\frac{-\frac{4i}{\left(-i+\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^{4}}+\frac{8}{5\left(-i+\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^{5}}+\frac{2}{-i+\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)}-\frac{16}{3\left(-i+\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^{3}}+\frac{4i}{\left(-i+\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^{2}}}{d}$	88
default	$\frac{-\frac{4i}{\left(-i+\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^{4}}+\frac{8}{5\left(-i+\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^{5}}+\frac{2}{-i+\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)}-\frac{16}{3\left(-i+\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^{3}}+\frac{4i}{\left(-i+\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^{2}}}{d}$	88
parallelrisch	$\frac{\frac{14}{15}-\frac{16\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}}{3}-4i\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^{3}+\frac{8i\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)}{3}+2\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^{4}}{d\left(-10\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^{3}-5i\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^{4}+\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^{5}+5\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)+10i\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}-i\right)}$	12

input
```
int(1/(1+I*sinh(d*x+c))^3,x,method=_RETURNVERBOSE)
```

output
```
-4/15*I*(-5*I*exp(d*x+c)+10*exp(2*d*x+2*c)-1)/d/(exp(d*x+c)-I)^5
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 85, normalized size of antiderivative = 0.97

$$\int \frac{1}{(1+i\sinh(c+dx))^3}\,dx = -\frac{4\left(10i\,e^{(2\,dx+2\,c)}+5\,e^{(dx+c)}-i\right)}{15\left(de^{(5\,dx+5\,c)}-5i\,de^{(4\,dx+4\,c)}-10\,de^{(3\,dx+3\,c)}+10i\,de^{(2\,dx+2\,c)}+5\,de^{(dx+c)}-i\,d\right)}$$

input
```
integrate(1/(1+I*sinh(d*x+c))^3,x, algorithm="fricas")
```

output
```
-4/15*(10*I*e^(2*d*x + 2*c) + 5*e^(d*x + c) - I)/(d*e^(5*d*x + 5*c) - 5*I*
d*e^(4*d*x + 4*c) - 10*d*e^(3*d*x + 3*c) + 10*I*d*e^(2*d*x + 2*c) + 5*d*e^
(d*x + c) - I*d)
```

Sympy [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 109, normalized size of antiderivative = 1.24

$$\int \frac{1}{(1+i\sinh(c+dx))^3}\,dx = \frac{-40ie^{2c}e^{2dx}-20e^{c}e^{dx}+4i}{15de^{5c}e^{5dx}-75ide^{4c}e^{4dx}-150de^{3c}e^{3dx}+150ide^{2c}e^{2dx}+75de^{c}e^{dx}-15id}$$

input
```
integrate(1/(1+I*sinh(d*x+c))**3,x)
```

output
```
(-40*I*exp(2*c)*exp(2*d*x) - 20*exp(c)*exp(d*x) + 4*I)/(15*d*exp(5*c)*exp(
5*d*x) - 75*I*d*exp(4*c)*exp(4*d*x) - 150*d*exp(3*c)*exp(3*d*x) + 150*I*d*
exp(2*c)*exp(2*d*x) + 75*d*exp(c)*exp(d*x) - 15*I*d)
```

Maxima [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 211 vs. $2(70) = 140$.

Time = 0.04 (sec) , antiderivative size = 211, normalized size of antiderivative = 2.40

$$\int \frac{1}{(1+i\sinh(c+dx))^3}\,dx$$
$$= \frac{20i\,e^{(-dx-c)}}{-15\,d(-5i\,e^{(-dx-c)}-10\,e^{(-2\,dx-2\,c)}+10i\,e^{(-3\,dx-3\,c)}+5\,e^{(-4\,dx-4\,c)}-i\,e^{(-5\,dx-5\,c)}+1)}$$
$$+\frac{40\,e^{(-2\,dx-2\,c)}}{-15\,d(-5i\,e^{(-dx-c)}-10\,e^{(-2\,dx-2\,c)}+10i\,e^{(-3\,dx-3\,c)}+5\,e^{(-4\,dx-4\,c)}-i\,e^{(-5\,dx-5\,c)}+1)}$$
$$-\frac{4}{-15\,d(-5i\,e^{(-dx-c)}-10\,e^{(-2\,dx-2\,c)}+10i\,e^{(-3\,dx-3\,c)}+5\,e^{(-4\,dx-4\,c)}-i\,e^{(-5\,dx-5\,c)}+1)}$$

input
```
integrate(1/(1+I*sinh(d*x+c))^3,x, algorithm="maxima")
```

output
```
20*I*e^(-d*x - c)/(d*(75*I*e^(-d*x - c) + 150*e^(-2*d*x - 2*c) - 150*I*e^(
-3*d*x - 3*c) - 75*e^(-4*d*x - 4*c) + 15*I*e^(-5*d*x - 5*c) - 15)) + 40*e^
(-2*d*x - 2*c)/(d*(75*I*e^(-d*x - c) + 150*e^(-2*d*x - 2*c) - 150*I*e^(-3*
d*x - 3*c) - 75*e^(-4*d*x - 4*c) + 15*I*e^(-5*d*x - 5*c) - 15)) - 4/(d*(75
*I*e^(-d*x - c) + 150*e^(-2*d*x - 2*c) - 150*I*e^(-3*d*x - 3*c) - 75*e^(-4
*d*x - 4*c) + 15*I*e^(-5*d*x - 5*c) - 15))
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.41

$$\int \frac{1}{(1+i\sinh(c+dx))^3}\,dx = -\frac{4i\left(10\,e^{(2\,dx+2\,c)}-5i\,e^{(dx+c)}-1\right)}{15\,d(e^{(dx+c)}-i)^5}$$

input
```
integrate(1/(1+I*sinh(d*x+c))^3,x, algorithm="giac")
```

output
```
-4/15*I*(10*e^(2*d*x + 2*c) - 5*I*e^(d*x + c) - 1)/(d*(e^(d*x + c) - I)^5)
```

Mupad [B] (verification not implemented)

Time = 2.02 (sec) , antiderivative size = 40, normalized size of antiderivative = 0.45

$$\int \frac{1}{(1+i\sinh(c+dx))^3}\,dx = -\frac{\frac{4}{15}-\frac{8\,\mathrm{e}^{2\,c+2\,d\,x}}{3}+\frac{\mathrm{e}^{c+d\,x}\,4\mathrm{i}}{3}}{d\,\left(1+\mathrm{e}^{c+d\,x}\,1\mathrm{i}\right)^5}$$

input
```
int(1/(sinh(c + d*x)*1i + 1)^3,x)
```

output
```
-((exp(c + d*x)*4i)/3 - (8*exp(2*c + 2*d*x))/3 + 4/15)/(d*(exp(c + d*x)*1i
 + 1)^5)
```

Reduce [F]

$$\int \frac{1}{(1+i\sinh(c+dx))^3}\,dx = -\left(\int \frac{1}{\sinh\left(dx+c\right)^3 i+3\sinh\left(dx+c\right)^2-3\sinh\left(dx+c\right)i-1}dx\right)$$

input
```
int(1/(1+I*sinh(d*x+c))^3,x)
```

output
```
- int(1/(sinh(c + d*x)**3*i + 3*sinh(c + d*x)**2 - 3*sinh(c + d*x)*i - 1)
,x)
```

3.59 $\int \frac{1}{(1+i\sinh(c+dx))^4}\,dx$

Optimal result

Integrand size = 14, antiderivative size = 117

$$\int \frac{1}{(1+i\sinh(c+dx))^4}\,dx = \frac{i\cosh(c+dx)}{7d(1+i\sinh(c+dx))^4}+\frac{3i\cosh(c+dx)}{35d(1+i\sinh(c+dx))^3}+\frac{2i\cosh(c+dx)}{35d(1+i\sinh(c+dx))^2}+\frac{2i\cosh(c+dx)}{35d(1+i\sinh(c+dx))}$$

output
```
1/7*I*cosh(d*x+c)/d/(1+I*sinh(d*x+c))^4+3/35*I*cosh(d*x+c)/d/(1+I*sinh(d*x
+c))^3+2/35*I*cosh(d*x+c)/d/(1+I*sinh(d*x+c))^2+2/35*I*cosh(d*x+c)/d/(1+I*
sinh(d*x+c))
```

Mathematica [A] (verified)

Time = 0.17 (sec) , antiderivative size = 87, normalized size of antiderivative = 0.74

$$\int \frac{1}{(1+i\sinh(c+dx))^4}\,dx = \frac{21i\cosh\left(\frac{3}{2}(c+dx)\right)-i\cosh\left(\frac{7}{2}(c+dx)\right)+35\sinh\left(\frac{1}{2}(c+dx)\right)-7\sinh\left(\frac{5}{2}(c+dx)\right)}{70d\left(\cosh\left(\frac{1}{2}(c+dx)\right)+i\sinh\left(\frac{1}{2}(c+dx)\right)\right)^7}$$

input
```
Integrate[(1 + I*Sinh[c + d*x])^(-4),x]
```

output
```
((21*I)*Cosh[(3*(c + d*x))/2] - I*Cosh[(7*(c + d*x))/2] + 35*Sinh[(c + d*x
)/2] - 7*Sinh[(5*(c + d*x))/2])/(70*d*(Cosh[(c + d*x)/2] + I*Sinh[(c + d*x
)/2])^7)
```

Rubi [A] (verified)

Time = 0.45 (sec) , antiderivative size = 127, normalized size of antiderivative = 1.09, number of steps used = 8, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.571$, Rules used = {3042, 3129, 3042, 3129, 3042, 3129, 3042, 3127}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(1+i\sinh(c+dx))^4}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{(1+\sin(ic+idx))^4}dx$$

$$\Big\downarrow \text{3129}$$

$$\frac{3}{7}\int \frac{1}{(i\sinh(c+dx)+1)^3}dx+\frac{i\cosh(c+dx)}{7d(1+i\sinh(c+dx))^4}$$

$$\Big\downarrow \text{3042}$$

$$\frac{3}{7}\int \frac{1}{(\sin(ic+idx)+1)^3}dx+\frac{i\cosh(c+dx)}{7d(1+i\sinh(c+dx))^4}$$

$$\Big\downarrow \text{3129}$$

$$\frac{3}{7}\left(\frac{2}{5}\int \frac{1}{(i\sinh(c+dx)+1)^2}dx+\frac{i\cosh(c+dx)}{5d(1+i\sinh(c+dx))^3}\right)+\frac{i\cosh(c+dx)}{7d(1+i\sinh(c+dx))^4}$$

$$\Big\downarrow \text{3042}$$

$$\frac{3}{7}\left(\frac{2}{5}\int \frac{1}{(\sin(ic+idx)+1)^2}dx+\frac{i\cosh(c+dx)}{5d(1+i\sinh(c+dx))^3}\right)+\frac{i\cosh(c+dx)}{7d(1+i\sinh(c+dx))^4}$$

$$\Big\downarrow \text{3129}$$

$$\frac{3}{7}\left(\frac{2}{5}\left(\frac{1}{3}\int \frac{1}{i\sinh(c+dx)+1}dx+\frac{i\cosh(c+dx)}{3d(1+i\sinh(c+dx))^2}\right)+\frac{i\cosh(c+dx)}{5d(1+i\sinh(c+dx))^3}\right)+\frac{i\cosh(c+dx)}{7d(1+i\sinh(c+dx))^4}$$

$$\Big\downarrow \ 3042$$

$$\frac{3}{7}\left(\frac{2}{5}\left(\frac{1}{3}\int \frac{1}{\sin(ic+idx)+1}dx+\frac{i\cosh(c+dx)}{3d(1+i\sinh(c+dx))^2}\right)+\frac{i\cosh(c+dx)}{5d(1+i\sinh(c+dx))^3}\right)+\frac{i\cosh(c+dx)}{7d(1+i\sinh(c+dx))^4}$$

$$\Big\downarrow \ 3127$$

$$\frac{i\cosh(c+dx)}{7d(1+i\sinh(c+dx))^4}+\frac{3}{7}\left(\frac{i\cosh(c+dx)}{5d(1+i\sinh(c+dx))^3}+\frac{2}{5}\left(\frac{i\cosh(c+dx)}{3d(1+i\sinh(c+dx))}+\frac{i\cosh(c+dx)}{3d(1+i\sinh(c+dx))^2}\right)\right)$$

input
```
Int[(1 + I*Sinh[c + d*x])^(-4),x]
```

output
```
(3*((2*(((I/3)*Cosh[c + d*x])/(d*(1 + I*Sinh[c + d*x])^2) + ((I/3)*Cosh[c
+ d*x])/(d*(1 + I*Sinh[c + d*x]))))/5 + ((I/5)*Cosh[c + d*x])/(d*(1 + I*Si
nh[c + d*x])^3)))/7 + ((I/7)*Cosh[c + d*x])/(d*(1 + I*Sinh[c + d*x])^4)
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3127
```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> Simp[-Cos[c +
 d*x]/(d*(b + a*Sin[c + d*x])), x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b
^2, 0]
```

rule 3129
```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[b*Cos[c
+ d*x]*((a + b*Sin[c + d*x])^n/(a*d*(2*n + 1))), x] + Simp[(n + 1)/(a*(2*n
+ 1))   Int[(a + b*Sin[c + d*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d}, x]
&& EqQ[a^2 - b^2, 0] && LtQ[n, -1] && IntegerQ[2*n]
```

Maple [A] (verified)

Time = 0.36 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.44

method	result
risch	$-\frac{4\left(-7\,\mathrm{e}^{dx+c}-21i\mathrm{e}^{2dx+2c}+35\,\mathrm{e}^{3dx+3c}+i\right)}{35\left(\mathrm{e}^{dx+c}-i\right)^{7}d}$
derivativedivides	$\frac{\frac{6i}{\left(-i+\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^{2}}+\frac{72}{5\left(-i+\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^{5}}-\frac{12}{\left(-i+\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^{3}}+\frac{2}{-i+\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)}-\frac{16}{7\left(-i+\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^{7}}+ \frac{}{(-i+\mathrm{ta}}}{d}$
default	$\frac{\frac{6i}{\left(-i+\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^{2}}+\frac{72}{5\left(-i+\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^{5}}-\frac{12}{\left(-i+\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^{3}}+\frac{2}{-i+\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)}-\frac{16}{7\left(-i+\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^{7}}+ \frac{}{(-i+\mathrm{ta}}}{d}$
parallelrisch	$\frac{12-42\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}+14\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^{6}+12i\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^{7}-42i\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^{5}+14i\tanh\left(\frac{dx}{2}+\frac{c}{2}\right.}{245\left(i\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^{6}-\frac{\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^{7}}{7}-5i\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^{4}+3\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^{5}+3i\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}-5\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^{3}-\frac{i}{7}\right.}$

input
```
int(1/(1+I*sinh(d*x+c))^4,x,method=_RETURNVERBOSE)
```

output
```
-4/35*(-7*exp(d*x+c)-21*I*exp(2*d*x+2*c)+35*exp(3*d*x+3*c)+I)/(exp(d*x+c)-
I)^7/d
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 120, normalized size of antiderivative = 1.03

$$\int \frac{1}{(1+i\sinh(c+dx))^4}\,dx = -\frac{4\left(35\,e^{(3\,dx+3\,c)}-21i\,e^{(2\,dx+2\,c)}-7\,e^{(dx+c)}+i\right)}{35\left(de^{(7\,dx+7\,c)}-7i\,de^{(6\,dx+6\,c)}-21\,de^{(5\,dx+5\,c)}+35i\,de^{(4\,dx+4\,c)}+35\,de^{(3\,dx+3\,c)}-21i\,de^{(2\,dx+2\,c)}-7\,de^{(d}\right.}$$

input
```
integrate(1/(1+I*sinh(d*x+c))^4,x, algorithm="fricas")
```

output
```
-4/35*(35*e^(3*d*x + 3*c) - 21*I*e^(2*d*x + 2*c) - 7*e^(d*x + c) + I)/(d*e
^(7*d*x + 7*c) - 7*I*d*e^(6*d*x + 6*c) - 21*d*e^(5*d*x + 5*c) + 35*I*d*e^(
4*d*x + 4*c) + 35*d*e^(3*d*x + 3*c) - 21*I*d*e^(2*d*x + 2*c) - 7*d*e^(d*x
+ c) + I*d)
```

Sympy [A] (verification not implemented)

Time = 0.34 (sec) , antiderivative size = 155, normalized size of antiderivative = 1.32

$$\int \frac{1}{(1+i\sinh(c+dx))^4}\,dx = \frac{-140e^{3c}e^{3dx}+84ie^{2c}e^{2dx}+28e^{c}e^{dx}-4i}{35de^{7c}e^{7dx}-245ide^{6c}e^{6dx}-735de^{5c}e^{5dx}+1225ide^{4c}e^{4dx}+1225de^{3c}e^{3dx}-735ide^{2c}e^{2dx}-245de^{c}e^{dx}+3}$$

input
```
integrate(1/(1+I*sinh(d*x+c))**4,x)
```

output
```
(-140*exp(3*c)*exp(3*d*x) + 84*I*exp(2*c)*exp(2*d*x) + 28*exp(c)*exp(d*x)
- 4*I)/(35*d*exp(7*c)*exp(7*d*x) - 245*I*d*exp(6*c)*exp(6*d*x) - 735*d*exp
(5*c)*exp(5*d*x) + 1225*I*d*exp(4*c)*exp(4*d*x) + 1225*d*exp(3*c)*exp(3*d*
x) - 735*I*d*exp(2*c)*exp(2*d*x) - 245*d*exp(c)*exp(d*x) + 35*I*d)
```

Maxima [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 372 vs. 2(93) = 186.

Time = 0.05 (sec) , antiderivative size = 372, normalized size of antiderivative = 3.18

$$\int \frac{1}{(1+i\sinh(c+dx))^4}\,dx = \frac{4\,e^{(-dx-c)}}{5\,d(7\,e^{(-dx-c)}-21i\,e^{(-2\,dx-2\,c)}-35\,e^{(-3\,dx-3\,c)}+35i\,e^{(-4\,dx-4\,c)}+21\,e^{(-5\,dx-5\,c)}-7i\,e^{(-6\,dx-6\,c)}-e^{(-7\,dx-}}$$
$$-\frac{12i\,e^{(-2\,dx-2\,c)}}{5\,d(7\,e^{(-dx-c)}-21i\,e^{(-2\,dx-2\,c)}-35\,e^{(-3\,dx-3\,c)}+35i\,e^{(-4\,dx-4\,c)}+21\,e^{(-5\,dx-5\,c)}-7i\,e^{(-6\,dx-6\,c)}-e^{(-7\,d}}$$
$$-\frac{4\,e^{(-3\,dx-3\,c)}}{d(7\,e^{(-dx-c)}-21i\,e^{(-2\,dx-2\,c)}-35\,e^{(-3\,dx-3\,c)}+35i\,e^{(-4\,dx-4\,c)}+21\,e^{(-5\,dx-5\,c)}-7i\,e^{(-6\,dx-6\,c)}-e^{(-7\,dx-}}$$
$$+\frac{4i}{35\,d(7\,e^{(-dx-c)}-21i\,e^{(-2\,dx-2\,c)}-35\,e^{(-3\,dx-3\,c)}+35i\,e^{(-4\,dx-4\,c)}+21\,e^{(-5\,dx-5\,c)}-7i\,e^{(-6\,dx-6\,c)}-e^{(-7}}$$

input
```
integrate(1/(1+I*sinh(d*x+c))^4,x, algorithm="maxima")
```

output
```
4/5*e^(-d*x - c)/(d*(7*e^(-d*x - c) - 21*I*e^(-2*d*x - 2*c) - 35*e^(-3*d*x
 - 3*c) + 35*I*e^(-4*d*x - 4*c) + 21*e^(-5*d*x - 5*c) - 7*I*e^(-6*d*x - 6*
c) - e^(-7*d*x - 7*c) + I)) - 12/5*I*e^(-2*d*x - 2*c)/(d*(7*e^(-d*x - c) -
 21*I*e^(-2*d*x - 2*c) - 35*e^(-3*d*x - 3*c) + 35*I*e^(-4*d*x - 4*c) + 21*
e^(-5*d*x - 5*c) - 7*I*e^(-6*d*x - 6*c) - e^(-7*d*x - 7*c) + I)) - 4*e^(-3
*d*x - 3*c)/(d*(7*e^(-d*x - c) - 21*I*e^(-2*d*x - 2*c) - 35*e^(-3*d*x - 3*
c) + 35*I*e^(-4*d*x - 4*c) + 21*e^(-5*d*x - 5*c) - 7*I*e^(-6*d*x - 6*c) -
e^(-7*d*x - 7*c) + I)) + 4/35*I/(d*(7*e^(-d*x - c) - 21*I*e^(-2*d*x - 2*c)
 - 35*e^(-3*d*x - 3*c) + 35*I*e^(-4*d*x - 4*c) + 21*e^(-5*d*x - 5*c) - 7*I
*e^(-6*d*x - 6*c) - e^(-7*d*x - 7*c) + I))
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.40

$$\int \frac{1}{(1+i\sinh(c+dx))^4}\,dx = -\frac{4\left(35\,e^{(3\,dx+3\,c)}-21i\,e^{(2\,dx+2\,c)}-7\,e^{(dx+c)}+i\right)}{35\,d\left(e^{(dx+c)}-i\right)^7}$$

input
```
integrate(1/(1+I*sinh(d*x+c))^4,x, algorithm="giac")
```

output
```
-4/35*(35*e^(3*d*x + 3*c) - 21*I*e^(2*d*x + 2*c) - 7*e^(d*x + c) + I)/(d*(
e^(d*x + c) - I)^7)
```

Mupad [B] (verification not implemented)

Time = 2.27 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.45

$$\int \frac{1}{(1+i\sinh(c+dx))^4}\,dx = -\frac{\left(7\,e^{c+d\,x}+e^{2\,c+2\,d\,x}\,21\mathrm{i}-35\,e^{3\,c+3\,d\,x}-\mathrm{i}\right)\,4\mathrm{i}}{35\,d\,\left(1+e^{c+d\,x}\,1\mathrm{i}\right)^7}$$

input
```
int(1/(sinh(c + d*x)*1i + 1)^4,x)
```

output
```
-((7*exp(c + d*x) + exp(2*c + 2*d*x)*21i - 35*exp(3*c + 3*d*x) - 1i)*4i)/(
35*d*(exp(c + d*x)*1i + 1)^7)
```

Reduce [F]

$$
\begin{aligned}
&\int \frac{1}{(1+i\sinh(c+dx))^4}\,dx \\
&= \int \frac{1}{\sinh{(dx+c)}^4-4\sinh{(dx+c)}^3 i-6\sinh{(dx+c)}^2+4\sinh{(dx+c)}\, i+1}dx
\end{aligned}
$$

input
```
int(1/(1+I*sinh(d*x+c))^4,x)
```

output
```
int(1/(sinh(c + d*x)**4 - 4*sinh(c + d*x)**3*i - 6*sinh(c + d*x)**2 + 4*si
nh(c + d*x)*i + 1),x)
```

3.60 $\int \frac{1}{1-i \sinh(c+dx)}\,dx$

Optimal result

Integrand size = 14, antiderivative size = 27

$$\int \frac{1}{1-i\sinh(c+dx)}\,dx=-\frac{i\cosh(c+dx)}{d(1-i\sinh(c+dx))}$$

output
```
-I*cosh(d*x+c)/d/(1-I*sinh(d*x+c))
```

Mathematica [A] (verified)

Time = 0.17 (sec) , antiderivative size = 42, normalized size of antiderivative = 1.56

$$\int \frac{1}{1-i\sinh(c+dx)}\,dx=\frac{2\sinh\left(\frac{1}{2}(c+dx)\right)}{d\left(\cosh\left(\frac{1}{2}(c+dx)\right)-i\sinh\left(\frac{1}{2}(c+dx)\right)\right)}$$

input
```
Integrate[(1 - I*Sinh[c + d*x])^(-1),x]
```

output
```
(2*Sinh[(c + d*x)/2])/(d*(Cosh[(c + d*x)/2] - I*Sinh[(c + d*x)/2]))
```

Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {3042, 3127}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{1-i\sinh(c+dx)}\,dx$$
$$\Big\downarrow \text{3042}$$
$$\int \frac{1}{1-\sin(ic+idx)}dx$$
$$\Big\downarrow \text{3127}$$
$$-\frac{i\cosh(c+dx)}{d(1-i\sinh(c+dx))}$$

input
```
Int[(1 - I*Sinh[c + d*x])^(-1),x]
```

output
```
((-I)*Cosh[c + d*x])/(d*(1 - I*Sinh[c + d*x]))
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3127
```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> Simp[-Cos[c +
 d*x]/(d*(b + a*Sin[c + d*x])), x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b
^2, 0]
```

Maple [A] (verified)

Time = 0.16 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.67

method	result	size
risch	$-\frac{2i}{d(e^{dx+c}+i)}$	18
derivativedivides	$\frac{2}{d\left(i+\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}$	20
default	$\frac{2}{d\left(i+\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}$	20
parallelrisch	$\frac{2}{d\left(i+\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}$	20

input
```
int(1/(-I*sinh(d*x+c)+1),x,method=_RETURNVERBOSE)
```

output
```
-2*I/d/(exp(d*x+c)+I)
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.59

$$\int \frac{1}{1-i\sinh(c+dx)}\,dx = -\frac{2i}{de^{(dx+c)}+i\,d}$$

input
```
integrate(1/(1-I*sinh(d*x+c)),x, algorithm="fricas")
```

output
```
-2*I/(d*e^(d*x + c) + I*d)
```

Sympy [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.63

$$\int \frac{1}{1-i\sinh(c+dx)}\,dx = -\frac{2i}{de^{c}e^{dx}+id}$$

input
```
integrate(1/(1-I*sinh(d*x+c)),x)
```

output
```
-2*I/(d*exp(c)*exp(d*x) + I*d)
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.74

$$\int \frac{1}{1-i\sinh(c+dx)}\,dx = \frac{2}{d(i\,e^{(-dx-c)}+1)}$$

input
```
integrate(1/(1-I*sinh(d*x+c)),x, algorithm="maxima")
```

output
```
2/(d*(I*e^(-d*x - c) + 1))
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.56

$$\int \frac{1}{1-i\sinh(c+dx)}\,dx = -\frac{2i}{d(e^{(dx+c)}+i)}$$

input
```
integrate(1/(1-I*sinh(d*x+c)),x, algorithm="giac")
```

output
```
-2*I/(d*(e^(d*x + c) + I))
```

Mupad [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.63

$$\int \frac{1}{1 - i\sinh(c + dx)}\,dx = -\frac{2\mathrm{i}}{d\,\left(\mathrm{e}^{c+d\,x} + 1\mathrm{i}\right)}$$

input
```
int(-1/(sinh(c + d*x)*1i - 1),x)
```

output
```
-2i/(d*(exp(c + d*x) + 1i))
```

Reduce [F]

$$\int \frac{1}{1 - i\sinh(c + dx)}\,dx = -\left(\int \frac{1}{\sinh\left(dx + c\right)i - 1}dx\right)$$

input
```
int(1/(1-I*sinh(d*x+c)),x)
```

output
```
- int(1/(sinh(c + d*x)*i - 1),x)
```

3.61 $\int \frac{1}{(1-i\sinh(c+dx))^2}\,dx$

Optimal result

Integrand size = 14, antiderivative size = 59

$$\int \frac{1}{(1-i\sinh(c+dx))^2}\,dx = -\frac{i\cosh(c+dx)}{3d(1-i\sinh(c+dx))^2} - \frac{i\cosh(c+dx)}{3d(1-i\sinh(c+dx))}$$

output
```
-1/3*I*cosh(d*x+c)/d/(1-I*sinh(d*x+c))^2-1/3*I*cosh(d*x+c)/d/(1-I*sinh(d*x
+c))
```

Mathematica [A] (verified)

Time = 0.16 (sec) , antiderivative size = 59, normalized size of antiderivative = 1.00

$$\int \frac{1}{(1-i\sinh(c+dx))^2}\,dx = -\frac{\cosh\left(\frac{3}{2}(c+dx)\right)+3i\sinh\left(\frac{1}{2}(c+dx)\right)}{3d\left(i\cosh\left(\frac{1}{2}(c+dx)\right)+\sinh\left(\frac{1}{2}(c+dx)\right)\right)^3}$$

input
```
Integrate[(1 - I*Sinh[c + d*x])^(-2),x]
```

output
```
-1/3*(Cosh[(3*(c + d*x))/2] + (3*I)*Sinh[(c + d*x)/2])/(d*(I*Cosh[(c + d*x
)/2] + Sinh[(c + d*x)/2])^3)
```

Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 59, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$, Rules used = {3042, 3129, 3042, 3127}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(1-i\sinh(c+dx))^2}dx$$
$$\Big\downarrow \text{3042}$$
$$\int \frac{1}{(1-\sin(ic+idx))^2}dx$$
$$\Big\downarrow \text{3129}$$
$$\frac{1}{3}\int \frac{1}{1-i\sinh(c+dx)}dx-\frac{i\cosh(c+dx)}{3d(1-i\sinh(c+dx))^2}$$
$$\Big\downarrow \text{3042}$$
$$\frac{1}{3}\int \frac{1}{1-\sin(ic+idx)}dx-\frac{i\cosh(c+dx)}{3d(1-i\sinh(c+dx))^2}$$
$$\Big\downarrow \text{3127}$$
$$-\frac{i\cosh(c+dx)}{3d(1-i\sinh(c+dx))}-\frac{i\cosh(c+dx)}{3d(1-i\sinh(c+dx))^2}$$

input
```
Int[(1 - I*Sinh[c + d*x])^(-2),x]
```

output
```
((-1/3*I)*Cosh[c + d*x])/(d*(1 - I*Sinh[c + d*x])^2) - ((I/3)*Cosh[c + d*x
])/(d*(1 - I*Sinh[c + d*x]))
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3127
```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> Simp[-Cos[c +
 d*x]/(d*(b + a*Sin[c + d*x])), x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b
^2, 0]
```

rule 3129
```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[b*Cos[c
+ d*x]*((a + b*Sin[c + d*x])^n/(a*d*(2*n + 1))), x] + Simp[(n + 1)/(a*(2*n
+ 1))   Int[(a + b*Sin[c + d*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d}, x]
&& EqQ[a^2 - b^2, 0] && LtQ[n, -1] && IntegerQ[2*n]
```

Maple [A] (verified)

Time = 0.24 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.47

method	result	size
risch	$\frac{\frac{2i}{3}+2\,{\mathrm e}^{dx+c}}{d\left({\mathrm e}^{dx+c}+i\right)^{3}}$	28
derivativedivides	$\frac{\frac{2}{i+\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)}-\frac{2i}{\left(i+\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^{2}}-\frac{4}{3\left(i+\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^{3}}}{d}$	55
default	$\frac{\frac{2}{i+\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)}-\frac{2i}{\left(i+\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^{2}}-\frac{4}{3\left(i+\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^{3}}}{d}$	55
parallelrisch	$\frac{6i\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)+6\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}-4}{3d\left(-3\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)+3i\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}+\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^{3}-i\right)}$	74

input
```
int(1/(-I*sinh(d*x+c)+1)^2,x,method=_RETURNVERBOSE)
```

output
```
2/3*(I+3*exp(d*x+c))/d/(exp(d*x+c)+I)^3
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 50, normalized size of antiderivative = 0.85

$$\int \frac{1}{(1-i\sinh(c+dx))^2}\,dx = \frac{2\left(3\,e^{(dx+c)}+i\right)}{3\left(de^{(3\,dx+3\,c)}+3i\,de^{(2\,dx+2\,c)}-3\,de^{(dx+c)}-i\,d\right)}$$

input
```
integrate(1/(1-I*sinh(d*x+c))^2,x, algorithm="fricas")
```

output
```
2/3*(3*e^(d*x + c) + I)/(d*e^(3*d*x + 3*c) + 3*I*d*e^(2*d*x + 2*c) - 3*d*e
^(d*x + c) - I*d)
```

Sympy [A] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 61, normalized size of antiderivative = 1.03

$$\int \frac{1}{(1-i\sinh(c+dx))^2}\,dx = \frac{6e^{c}e^{dx}+2i}{3de^{3c}e^{3dx}+9ide^{2c}e^{2dx}-9de^{c}e^{dx}-3id}$$

input
```
integrate(1/(1-I*sinh(d*x+c))**2,x)
```

output
```
(6*exp(c)*exp(d*x) + 2*I)/(3*d*exp(3*c)*exp(3*d*x) + 9*I*d*exp(2*c)*exp(2*
d*x) - 9*d*exp(c)*exp(d*x) - 3*I*d)
```

Maxima [A] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 94, normalized size of antiderivative = 1.59

$$\int \frac{1}{(1-i\sinh(c+dx))^2}\,dx = \frac{2\,e^{(-dx-c)}}{d(3\,e^{(-dx-c)}+3i\,e^{(-2\,dx-2\,c)}-e^{(-3\,dx-3\,c)}-i)} - \frac{2i}{3\,d(3\,e^{(-dx-c)}+3i\,e^{(-2\,dx-2\,c)}-e^{(-3\,dx-3\,c)}-i)}$$

input
```
integrate(1/(1-I*sinh(d*x+c))^2,x, algorithm="maxima")
```

output
```
2*e^(-d*x - c)/(d*(3*e^(-d*x - c) + 3*I*e^(-2*d*x - 2*c) - e^(-3*d*x - 3*c
) - I)) - 2/3*I/(d*(3*e^(-d*x - c) + 3*I*e^(-2*d*x - 2*c) - e^(-3*d*x - 3*
c) - I))
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.42

$$\int \frac{1}{(1-i\sinh(c+dx))^2}\,dx = \frac{2\left(3\,e^{(dx+c)}+i\right)}{3\,d{\left(e^{(dx+c)}+i\right)}^3}$$

input
```
integrate(1/(1-I*sinh(d*x+c))^2,x, algorithm="giac")
```

output
```
2/3*(3*e^(d*x + c) + I)/(d*(e^(d*x + c) + I)^3)
```

Mupad [B] (verification not implemented)

Time = 1.72 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.49

$$\int \frac{1}{(1-i\sinh(c+dx))^2}\,dx = -\frac{2\left(-1+\mathrm{e}^{c+d\,x}\,3\mathrm{i}\right)}{3\,d\,{\left(-1+\mathrm{e}^{c+d\,x}\,1\mathrm{i}\right)}^3}$$

input
```
int(1/(sinh(c + d*x)*1i - 1)^2,x)
```

output
```
-(2*(exp(c + d*x)*3i - 1))/(3*d*(exp(c + d*x)*1i - 1)^3)
```

Reduce [F]

$$\int \frac{1}{(1-i\sinh(c+dx))^2}\,dx = -\left(\int \frac{1}{\sinh(dx+c)^2+2\sinh(dx+c)\,i-1}dx\right)$$

input
```
int(1/(1-I*sinh(d*x+c))^2,x)
```

output
```
- int(1/(sinh(c + d*x)**2 + 2*sinh(c + d*x)*i - 1),x)
```

3.62 $\int \frac{1}{(1-i\sinh(c+dx))^3}\,dx$

Optimal result

Integrand size = 14, antiderivative size = 88

$$\int \frac{1}{(1-i\sinh(c+dx))^3}\,dx = -\frac{i\cosh(c+dx)}{5d(1-i\sinh(c+dx))^3} - \frac{2i\cosh(c+dx)}{15d(1-i\sinh(c+dx))^2} - \frac{2i\cosh(c+dx)}{15d(1-i\sinh(c+dx))}$$

output
```
-1/5*I*cosh(d*x+c)/d/(1-I*sinh(d*x+c))^3-2/15*I*cosh(d*x+c)/d/(1-I*sinh(d*
x+c))^2-2/15*I*cosh(d*x+c)/d/(1-I*sinh(d*x+c))
```

Mathematica [A] (verified)

Time = 0.29 (sec) , antiderivative size = 81, normalized size of antiderivative = 0.92

$$\int \frac{1}{(1-i\sinh(c+dx))^3}\,dx = \frac{10-15\cosh(c+dx)-6\cosh(2(c+dx))+\cosh(3(c+dx))-15i\sinh(c+dx)+6i\sinh(2(c+dx))+i}{30d(i+\sinh(c+dx))^3}$$

input
```
Integrate[(1 - I*Sinh[c + d*x])^(-3),x]
```

output
```
(10 - 15*Cosh[c + d*x] - 6*Cosh[2*(c + d*x)] + Cosh[3*(c + d*x)] - (15*I)*
Sinh[c + d*x] + (6*I)*Sinh[2*(c + d*x)] + I*Sinh[3*(c + d*x)])/(30*d*(I +
Sinh[c + d*x])^3)
```

Rubi [A] (verified)

Time = 0.36 (sec) , antiderivative size = 93, normalized size of antiderivative = 1.06, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.429, Rules used = {3042, 3129, 3042, 3129, 3042, 3127}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(1-i\sinh(c+dx))^3}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{(1-\sin(ic+idx))^3}dx$$

$$\Big\downarrow \text{3129}$$

$$\frac{2}{5}\int \frac{1}{(1-i\sinh(c+dx))^2}dx-\frac{i\cosh(c+dx)}{5d(1-i\sinh(c+dx))^3}$$

$$\Big\downarrow \text{3042}$$

$$\frac{2}{5}\int \frac{1}{(1-\sin(ic+idx))^2}dx-\frac{i\cosh(c+dx)}{5d(1-i\sinh(c+dx))^3}$$

$$\Big\downarrow \text{3129}$$

$$\frac{2}{5}\left(\frac{1}{3}\int \frac{1}{1-i\sinh(c+dx)}dx-\frac{i\cosh(c+dx)}{3d(1-i\sinh(c+dx))^2}\right)-\frac{i\cosh(c+dx)}{5d(1-i\sinh(c+dx))^3}$$

$$\Big\downarrow \text{3042}$$

$$\frac{2}{5}\left(\frac{1}{3}\int \frac{1}{1-\sin(ic+idx)}dx-\frac{i\cosh(c+dx)}{3d(1-i\sinh(c+dx))^2}\right)-\frac{i\cosh(c+dx)}{5d(1-i\sinh(c+dx))^3}$$

$$\Big\downarrow \text{3127}$$

$$\frac{2}{5}\left(-\frac{i\cosh(c+dx)}{3d(1-i\sinh(c+dx))}-\frac{i\cosh(c+dx)}{3d(1-i\sinh(c+dx))^2}\right)-\frac{i\cosh(c+dx)}{5d(1-i\sinh(c+dx))^3}$$

input
```
Int[(1 - I*Sinh[c + d*x])^(-3),x]
```

output
```
(2*(((-1/3*I)*Cosh[c + d*x])/(d*(1 - I*Sinh[c + d*x])^2) - ((I/3)*Cosh[c +
 d*x])/(d*(1 - I*Sinh[c + d*x]))))/5 - ((I/5)*Cosh[c + d*x])/(d*(1 - I*Sin
h[c + d*x])^3)
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3127
```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> Simp[-Cos[c +
 d*x]/(d*(b + a*Sin[c + d*x])), x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b
^2, 0]
```

rule 3129
```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[b*Cos[c
+ d*x]*((a + b*Sin[c + d*x])^n/(a*d*(2*n + 1))), x] + Simp[(n + 1)/(a*(2*n
+ 1))   Int[(a + b*Sin[c + d*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d}, x]
&& EqQ[a^2 - b^2, 0] && LtQ[n, -1] && IntegerQ[2*n]
```

Maple [A] (verified)

Time = 0.29 (sec) , antiderivative size = 40, normalized size of antiderivative = 0.45

method	result	size
risch	$\frac{4i\left(5i\,\mathrm{e}^{dx+c}+10\,\mathrm{e}^{2dx+2c}-1\right)}{15d\left(\mathrm{e}^{dx+c}+i\right)^{5}}$	40
derivativedivides	$\frac{-\frac{16}{3\left(i+\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^{3}}+\frac{2}{i+\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)}-\frac{4i}{\left(i+\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^{2}}+\frac{4i}{\left(i+\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^{4}}+\frac{8}{5\left(i+\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^{5}}}{d}$	88
default	$\frac{-\frac{16}{3\left(i+\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^{3}}+\frac{2}{i+\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)}-\frac{4i}{\left(i+\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^{2}}+\frac{4i}{\left(i+\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^{4}}+\frac{8}{5\left(i+\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^{5}}}{d}$	88
parallelrisch	$\frac{\frac{14}{15}+4i\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^{3}+2\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^{4}-\frac{8i\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)}{3}-\frac{16\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}}{3}}{d\left(-10\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^{3}+5i\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^{4}+\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^{5}+5\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)-10i\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}+i\right)}$	128

input
```
int(1/(-I*sinh(d*x+c)+1)^3,x,method=_RETURNVERBOSE)
```

output
```
4/15*I*(5*I*exp(d*x+c)+10*exp(2*d*x+2*c)-1)/d/(exp(d*x+c)+I)^5
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 85, normalized size of antiderivative = 0.97

$$\int \frac{1}{(1-i\sinh(c+dx))^3}\,dx = -\frac{4\left(-10i\,e^{(2\,dx+2\,c)}+5\,e^{(dx+c)}+i\right)}{15\left(de^{(5\,dx+5\,c)}+5i\,de^{(4\,dx+4\,c)}-10\,de^{(3\,dx+3\,c)}-10i\,de^{(2\,dx+2\,c)}+5\,de^{(dx+c)}+i\,d\right)}$$

input
```
integrate(1/(1-I*sinh(d*x+c))^3,x, algorithm="fricas")
```

output
```
-4/15*(-10*I*e^(2*d*x + 2*c) + 5*e^(d*x + c) + I)/(d*e^(5*d*x + 5*c) + 5*I
*d*e^(4*d*x + 4*c) - 10*d*e^(3*d*x + 3*c) - 10*I*d*e^(2*d*x + 2*c) + 5*d*e
^(d*x + c) + I*d)
```

Sympy [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 109, normalized size of antiderivative = 1.24

$$\int \frac{1}{(1-i\sinh(c+dx))^3}\,dx = \frac{40ie^{2c}e^{2dx}-20e^{c}e^{dx}-4i}{15de^{5c}e^{5dx}+75ide^{4c}e^{4dx}-150de^{3c}e^{3dx}-150ide^{2c}e^{2dx}+75de^{c}e^{dx}+15id}$$

input
```
integrate(1/(1-I*sinh(d*x+c))**3,x)
```

output
```
(40*I*exp(2*c)*exp(2*d*x) - 20*exp(c)*exp(d*x) - 4*I)/(15*d*exp(5*c)*exp(5
*d*x) + 75*I*d*exp(4*c)*exp(4*d*x) - 150*d*exp(3*c)*exp(3*d*x) - 150*I*d*e
xp(2*c)*exp(2*d*x) + 75*d*exp(c)*exp(d*x) + 15*I*d)
```

Maxima [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 211 vs. $2(70) = 140$.

Time = 0.03 (sec) , antiderivative size = 211, normalized size of antiderivative = 2.40

$$\int \frac{1}{(1-i\sinh(c+dx))^3}\,dx$$
$$= \frac{20i\,e^{(-dx-c)}}{-15\,d(-5i\,e^{(-dx-c)}+10\,e^{(-2\,dx-2\,c)}+10i\,e^{(-3\,dx-3\,c)}-5\,e^{(-4\,dx-4\,c)}-i\,e^{(-5\,dx-5\,c)}-1)}$$
$$-\frac{40\,e^{(-2\,dx-2\,c)}}{-15\,d(-5i\,e^{(-dx-c)}+10\,e^{(-2\,dx-2\,c)}+10i\,e^{(-3\,dx-3\,c)}-5\,e^{(-4\,dx-4\,c)}-i\,e^{(-5\,dx-5\,c)}-1)}$$
$$+\frac{4}{-15\,d(-5i\,e^{(-dx-c)}+10\,e^{(-2\,dx-2\,c)}+10i\,e^{(-3\,dx-3\,c)}-5\,e^{(-4\,dx-4\,c)}-i\,e^{(-5\,dx-5\,c)}-1)}$$

input
```
integrate(1/(1-I*sinh(d*x+c))^3,x, algorithm="maxima")
```

output
```
20*I*e^(-d*x - c)/(d*(75*I*e^(-d*x - c) - 150*e^(-2*d*x - 2*c) - 150*I*e^(
-3*d*x - 3*c) + 75*e^(-4*d*x - 4*c) + 15*I*e^(-5*d*x - 5*c) + 15)) - 40*e^
(-2*d*x - 2*c)/(d*(75*I*e^(-d*x - c) - 150*e^(-2*d*x - 2*c) - 150*I*e^(-3*
d*x - 3*c) + 75*e^(-4*d*x - 4*c) + 15*I*e^(-5*d*x - 5*c) + 15)) + 4/(d*(75
*I*e^(-d*x - c) - 150*e^(-2*d*x - 2*c) - 150*I*e^(-3*d*x - 3*c) + 75*e^(-4
*d*x - 4*c) + 15*I*e^(-5*d*x - 5*c) + 15))
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.41

$$\int \frac{1}{(1-i\sinh(c+dx))^3}\,dx = \frac{4i\left(10\,e^{(2\,dx+2\,c)}+5i\,e^{(dx+c)}-1\right)}{15\,d\left(e^{(dx+c)}+i\right)^5}$$

input
```
integrate(1/(1-I*sinh(d*x+c))^3,x, algorithm="giac")
```

output
```
4/15*I*(10*e^(2*d*x + 2*c) + 5*I*e^(d*x + c) - 1)/(d*(e^(d*x + c) + I)^5)
```

Mupad [B] (verification not implemented)

Time = 1.91 (sec) , antiderivative size = 40, normalized size of antiderivative = 0.45

$$\int \frac{1}{(1-i\sinh(c+dx))^3}\,dx = -\frac{4\left(10\,\mathrm{e}^{2\,c+2\,d\,x}-1+\mathrm{e}^{c+d\,x}\,5\mathrm{i}\right)}{15\,d\left(-1+\mathrm{e}^{c+d\,x}\,1\mathrm{i}\right)^5}$$

input
```
int(-1/(sinh(c + d*x)*1i - 1)^3,x)
```

output
```
-(4*(exp(c + d*x)*5i + 10*exp(2*c + 2*d*x) - 1))/(15*d*(exp(c + d*x)*1i -
1)^5)
```

Reduce [F]

$$\int \frac{1}{(1-i\sinh(c+dx))^3}\,dx = \int \frac{1}{\sinh(dx+c)^3 i-3\sinh(dx+c)^2-3\sinh(dx+c)\,i+1}dx$$

input
```
int(1/(1-I*sinh(d*x+c))^3,x)
```

output
```
int(1/(sinh(c + d*x)**3*i - 3*sinh(c + d*x)**2 - 3*sinh(c + d*x)*i + 1),x)
```

3.63 $\int \frac{1}{(1-i\sinh(c+dx))^4}\,dx$

Optimal result

Integrand size = 14, antiderivative size = 117

$$\int \frac{1}{(1-i\sinh(c+dx))^4}\,dx = -\frac{i\cosh(c+dx)}{7d(1-i\sinh(c+dx))^4} - \frac{3i\cosh(c+dx)}{35d(1-i\sinh(c+dx))^3} - \frac{2i\cosh(c+dx)}{35d(1-i\sinh(c+dx))^2} - \frac{2i\cosh(c+dx)}{35d(1-i\sinh(c+dx))}$$

output
```
-1/7*I*cosh(d*x+c)/d/(1-I*sinh(d*x+c))^4-3/35*I*cosh(d*x+c)/d/(1-I*sinh(d*
x+c))^3-2/35*I*cosh(d*x+c)/d/(1-I*sinh(d*x+c))^2-2/35*I*cosh(d*x+c)/d/(1-I
*sinh(d*x+c))
```

Mathematica [A] (verified)

Time = 0.17 (sec) , antiderivative size = 87, normalized size of antiderivative = 0.74

$$\int \frac{1}{(1-i\sinh(c+dx))^4}\,dx = \frac{-21i\cosh\left(\frac{3}{2}(c+dx)\right)+i\cosh\left(\frac{7}{2}(c+dx)\right)+35\sinh\left(\frac{1}{2}(c+dx)\right)-7\sinh\left(\frac{5}{2}(c+dx)\right)}{70d\left(\cosh\left(\frac{1}{2}(c+dx)\right)-i\sinh\left(\frac{1}{2}(c+dx)\right)\right)^7}$$

input
```
Integrate[(1 - I*Sinh[c + d*x])^(-4),x]
```

output
```
((-21*I)*Cosh[(3*(c + d*x))/2] + I*Cosh[(7*(c + d*x))/2] + 35*Sinh[(c + d*
x)/2] - 7*Sinh[(5*(c + d*x))/2])/(70*d*(Cosh[(c + d*x)/2] - I*Sinh[(c + d*
x)/2])^7)
```

Rubi [A] (verified)

Time = 0.45 (sec) , antiderivative size = 127, normalized size of antiderivative = 1.09, number of steps used = 8, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.571$, Rules used = {3042, 3129, 3042, 3129, 3042, 3129, 3042, 3127}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(1-i\sinh(c+dx))^4}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{(1-\sin(ic+idx))^4}dx$$

$$\Big\downarrow \text{3129}$$

$$\frac{3}{7}\int \frac{1}{(1-i\sinh(c+dx))^3}dx-\frac{i\cosh(c+dx)}{7d(1-i\sinh(c+dx))^4}$$

$$\Big\downarrow \text{3042}$$

$$\frac{3}{7}\int \frac{1}{(1-\sin(ic+idx))^3}dx-\frac{i\cosh(c+dx)}{7d(1-i\sinh(c+dx))^4}$$

$$\Big\downarrow \text{3129}$$

$$\frac{3}{7}\left(\frac{2}{5}\int \frac{1}{(1-i\sinh(c+dx))^2}dx-\frac{i\cosh(c+dx)}{5d(1-i\sinh(c+dx))^3}\right)-\frac{i\cosh(c+dx)}{7d(1-i\sinh(c+dx))^4}$$

$$\Big\downarrow \text{3042}$$

$$\frac{3}{7}\left(\frac{2}{5}\int \frac{1}{(1-\sin(ic+idx))^2}dx-\frac{i\cosh(c+dx)}{5d(1-i\sinh(c+dx))^3}\right)-\frac{i\cosh(c+dx)}{7d(1-i\sinh(c+dx))^4}$$

$$\Big\downarrow \text{3129}$$

$$\frac{3}{7}\left(\frac{2}{5}\left(\frac{1}{3}\int \frac{1}{1-i\sinh(c+dx)}dx-\frac{i\cosh(c+dx)}{3d(1-i\sinh(c+dx))^2}\right)-\frac{i\cosh(c+dx)}{5d(1-i\sinh(c+dx))^3}\right)-\frac{i\cosh(c+dx)}{7d(1-i\sinh(c+dx))^4}$$

$\downarrow$ 3042

$$\frac{3}{7}\left(\frac{2}{5}\left(\frac{1}{3}\int \frac{1}{1-\sin(ic+idx)}dx-\frac{i\cosh(c+dx)}{3d(1-i\sinh(c+dx))^2}\right)-\frac{i\cosh(c+dx)}{5d(1-i\sinh(c+dx))^3}\right)-\frac{i\cosh(c+dx)}{7d(1-i\sinh(c+dx))^4}$$

$\downarrow$ 3127

$$\frac{3}{7}\left(\frac{2}{5}\left(-\frac{i\cosh(c+dx)}{3d(1-i\sinh(c+dx))}-\frac{i\cosh(c+dx)}{3d(1-i\sinh(c+dx))^2}\right)-\frac{i\cosh(c+dx)}{5d(1-i\sinh(c+dx))^3}\right)-\frac{i\cosh(c+dx)}{7d(1-i\sinh(c+dx))^4}$$

input
```
Int[(1 - I*Sinh[c + d*x])^(-4),x]
```

output
```
(3*((2*(((-1/3*I)*Cosh[c + d*x])/(d*(1 - I*Sinh[c + d*x])^2) - ((I/3)*Cosh
[c + d*x])/(d*(1 - I*Sinh[c + d*x]))))/5 - ((I/5)*Cosh[c + d*x])/(d*(1 - I
*Sinh[c + d*x])^3)))/7 - ((I/7)*Cosh[c + d*x])/(d*(1 - I*Sinh[c + d*x])^4)
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3127
```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> Simp[-Cos[c +
 d*x]/(d*(b + a*Sin[c + d*x])), x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b
^2, 0]
```

rule 3129
```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[b*Cos[c
+ d*x]*((a + b*Sin[c + d*x])^n/(a*d*(2*n + 1))), x] + Simp[(n + 1)/(a*(2*n
+ 1))   Int[(a + b*Sin[c + d*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d}, x]
&& EqQ[a^2 - b^2, 0] && LtQ[n, -1] && IntegerQ[2*n]
```

Maple [A] (verified)

Time = 0.36 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.44

method	result
risch	$-\frac{4\left(-7\,\mathrm{e}^{dx+c}+21 i\,\mathrm{e}^{2dx+2c}+35\,\mathrm{e}^{3dx+3c}-i\right)}{35d\left(\mathrm{e}^{dx+c}+i\right)^{7}}$
derivativedivides	$\frac{-\frac{6i}{\left(i+\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^{2}}+\frac{2}{i+\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)}-\frac{12}{\left(i+\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^{3}}-\frac{16}{7\left(i+\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^{7}}+\frac{72}{5\left(i+\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^{5}}+\frac{16i}{\left(i+\tanh\left(\frac{dx}{2}+\right.\right.}}{d}$
default	$\frac{-\frac{6i}{\left(i+\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^{2}}+\frac{2}{i+\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)}-\frac{12}{\left(i+\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^{3}}-\frac{16}{7\left(i+\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^{7}}+\frac{72}{5\left(i+\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^{5}}+\frac{16i}{\left(i+\tanh\left(\frac{dx}{2}+\right.\right.}}{d}$
parallelrisch	$\frac{-\frac{12}{35}-\frac{2\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^{6}}{5}+\frac{12i\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^{7}}{35}-\frac{6i\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^{5}}{5}+\frac{6\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}}{5}+\frac{2i\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)}{5}}{d\left(7i\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^{6}+\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^{7}-35i\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^{4}-21\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^{5}+21i\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}+35\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^{3}\right.}$

input
```
int(1/(-I*sinh(d*x+c)+1)^4,x,method=_RETURNVERBOSE)
```

output
```
-4/35*(-7*exp(d*x+c)+21*I*exp(2*d*x+2*c)+35*exp(3*d*x+3*c)-I)/d/(exp(d*x+c
)+I)^7
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 120, normalized size of antiderivative = 1.03

$$\int \frac{1}{(1-i\sinh(c+dx))^4}\,dx = -\frac{4\left(35\,e^{(3\,dx+3\,c)}+21i\,e^{(2\,dx+2\,c)}-7\,e^{(dx+c)}-i\right)}{35\left(de^{(7\,dx+7\,c)}+7i\,de^{(6\,dx+6\,c)}-21\,de^{(5\,dx+5\,c)}-35i\,de^{(4\,dx+4\,c)}+35\,de^{(3\,dx+3\,c)}+21i\,de^{(2\,dx+2\,c)}-7\,de^{(d\right.}}$$

input
```
integrate(1/(1-I*sinh(d*x+c))^4,x, algorithm="fricas")
```

output
```
-4/35*(35*e^(3*d*x + 3*c) + 21*I*e^(2*d*x + 2*c) - 7*e^(d*x + c) - I)/(d*e
^(7*d*x + 7*c) + 7*I*d*e^(6*d*x + 6*c) - 21*d*e^(5*d*x + 5*c) - 35*I*d*e^(
4*d*x + 4*c) + 35*d*e^(3*d*x + 3*c) + 21*I*d*e^(2*d*x + 2*c) - 7*d*e^(d*x
+ c) - I*d)
```

Sympy [A] (verification not implemented)

Time = 0.42 (sec) , antiderivative size = 155, normalized size of antiderivative = 1.32

$$\int \frac{1}{(1-i\sinh(c+dx))^4}\,dx$$
$$= \frac{-140e^{3c}e^{3dx} - 84ie^{2c}e^{2dx} + 28e^{c}e^{dx} + 4i}{35de^{7c}e^{7dx} + 245ide^{6c}e^{6dx} - 735de^{5c}e^{5dx} - 1225ide^{4c}e^{4dx} + 1225de^{3c}e^{3dx} + 735ide^{2c}e^{2dx} - 245de^{c}e^{dx} - 3}$$

input
```
integrate(1/(1-I*sinh(d*x+c))**4,x)
```

output
```
(-140*exp(3*c)*exp(3*d*x) - 84*I*exp(2*c)*exp(2*d*x) + 28*exp(c)*exp(d*x)
+ 4*I)/(35*d*exp(7*c)*exp(7*d*x) + 245*I*d*exp(6*c)*exp(6*d*x) - 735*d*exp
(5*c)*exp(5*d*x) - 1225*I*d*exp(4*c)*exp(4*d*x) + 1225*d*exp(3*c)*exp(3*d*
x) + 735*I*d*exp(2*c)*exp(2*d*x) - 245*d*exp(c)*exp(d*x) - 35*I*d)
```

Maxima [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 372 vs. $2(93) = 186$.

Time = 0.05 (sec) , antiderivative size = 372, normalized size of antiderivative = 3.18

$$\int \frac{1}{(1-i\sinh(c+dx))^4}\,dx$$
$$= \frac{4\,e^{(-dx-c)}}{5\,d(7\,e^{(-dx-c)} + 21i\,e^{(-2\,dx-2\,c)} - 35\,e^{(-3\,dx-3\,c)} - 35i\,e^{(-4\,dx-4\,c)} + 21\,e^{(-5\,dx-5\,c)} + 7i\,e^{(-6\,dx-6\,c)} - e^{(-7\,dx-}}$$
$$+ \frac{12i\,e^{(-2\,dx-2\,c)}}{5\,d(7\,e^{(-dx-c)} + 21i\,e^{(-2\,dx-2\,c)} - 35\,e^{(-3\,dx-3\,c)} - 35i\,e^{(-4\,dx-4\,c)} + 21\,e^{(-5\,dx-5\,c)} + 7i\,e^{(-6\,dx-6\,c)} - e^{(-7\,d}}$$
$$- \frac{4\,e^{(-3\,dx-3\,c)}}{d(7\,e^{(-dx-c)} + 21i\,e^{(-2\,dx-2\,c)} - 35\,e^{(-3\,dx-3\,c)} - 35i\,e^{(-4\,dx-4\,c)} + 21\,e^{(-5\,dx-5\,c)} + 7i\,e^{(-6\,dx-6\,c)} - e^{(-7\,dx-}}$$
$$- \frac{4i}{35\,d(7\,e^{(-dx-c)} + 21i\,e^{(-2\,dx-2\,c)} - 35\,e^{(-3\,dx-3\,c)} - 35i\,e^{(-4\,dx-4\,c)} + 21\,e^{(-5\,dx-5\,c)} + 7i\,e^{(-6\,dx-6\,c)} - e^{(-7}}$$

input
```
integrate(1/(1-I*sinh(d*x+c))^4,x, algorithm="maxima")
```

output
```
4/5*e^(-d*x - c)/(d*(7*e^(-d*x - c) + 21*I*e^(-2*d*x - 2*c) - 35*e^(-3*d*x
 - 3*c) - 35*I*e^(-4*d*x - 4*c) + 21*e^(-5*d*x - 5*c) + 7*I*e^(-6*d*x - 6*
c) - e^(-7*d*x - 7*c) - I)) + 12/5*I*e^(-2*d*x - 2*c)/(d*(7*e^(-d*x - c) +
 21*I*e^(-2*d*x - 2*c) - 35*e^(-3*d*x - 3*c) - 35*I*e^(-4*d*x - 4*c) + 21*
e^(-5*d*x - 5*c) + 7*I*e^(-6*d*x - 6*c) - e^(-7*d*x - 7*c) - I)) - 4*e^(-3
*d*x - 3*c)/(d*(7*e^(-d*x - c) + 21*I*e^(-2*d*x - 2*c) - 35*e^(-3*d*x - 3*
c) - 35*I*e^(-4*d*x - 4*c) + 21*e^(-5*d*x - 5*c) + 7*I*e^(-6*d*x - 6*c) -
e^(-7*d*x - 7*c) - I)) - 4/35*I/(d*(7*e^(-d*x - c) + 21*I*e^(-2*d*x - 2*c)
 - 35*e^(-3*d*x - 3*c) - 35*I*e^(-4*d*x - 4*c) + 21*e^(-5*d*x - 5*c) + 7*I
*e^(-6*d*x - 6*c) - e^(-7*d*x - 7*c) - I))
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.40

$$\int \frac{1}{(1-i\sinh(c+dx))^4}\,dx = -\frac{4\left(35\,e^{(3\,dx+3\,c)}+21i\,e^{(2\,dx+2\,c)}-7\,e^{(dx+c)}-i\right)}{35\,d\left(e^{(dx+c)}+i\right)^7}$$

input
```
integrate(1/(1-I*sinh(d*x+c))^4,x, algorithm="giac")
```

output
```
-4/35*(35*e^(3*d*x + 3*c) + 21*I*e^(2*d*x + 2*c) - 7*e^(d*x + c) - I)/(d*(
e^(d*x + c) + I)^7)
```

Mupad [B] (verification not implemented)

Time = 2.23 (sec) , antiderivative size = 52, normalized size of antiderivative = 0.44

$$\int \frac{1}{(1-i\sinh(c+dx))^4}\,dx = -\frac{4\left(21\,\mathrm{e}^{2\,c+2\,d\,x}-1+\mathrm{e}^{c+d\,x}\,7\mathrm{i}-\mathrm{e}^{3\,c+3\,d\,x}\,35\mathrm{i}\right)}{35\,d\,\left(-1+\mathrm{e}^{c+d\,x}\,1\mathrm{i}\right)^7}$$

input
```
int(1/(sinh(c + d*x)*1i - 1)^4,x)
```

output
```
-(4*(exp(c + d*x)*7i + 21*exp(2*c + 2*d*x) - exp(3*c + 3*d*x)*35i - 1))/(3
5*d*(exp(c + d*x)*1i - 1)^7)
```

Reduce [F]

$$\int \frac{1}{(1-i\sinh(c+dx))^4}\,dx$$
$$= \int \frac{1}{\sinh(dx+c)^4+4\sinh(dx+c)^3 i-6\sinh(dx+c)^2-4\sinh(dx+c) i+1}dx$$

input
```
int(1/(1-I*sinh(d*x+c))^4,x)
```

output
```
int(1/(sinh(c + d*x)**4 + 4*sinh(c + d*x)**3*i - 6*sinh(c + d*x)**2 - 4*si
nh(c + d*x)*i + 1),x)
```

3.64 $\int \frac{\sinh(x)}{\sqrt{a+ia\sinh(x)}}\,dx$

Optimal result

Integrand size = 16, antiderivative size = 57

$$\int \frac{\sinh(x)}{\sqrt{a+ia\sinh(x)}}\,dx=-\frac{\sqrt{2}\text{arctanh}\left(\frac{\sqrt{a}\cosh(x)}{\sqrt{2}\sqrt{a+ia\sinh(x)}}\right)}{\sqrt{a}}+\frac{2\cosh(x)}{\sqrt{a+ia\sinh(x)}}$$

output
```
-2^(1/2)*arctanh(1/2*a^(1/2)*cosh(x)*2^(1/2)/(a+I*a*sinh(x))^(1/2))/a^(1/2
)+2*cosh(x)/(a+I*a*sinh(x))^(1/2)
```

Mathematica [A] (verified)

Time = 0.23 (sec) , antiderivative size = 75, normalized size of antiderivative = 1.32

$$\int \frac{\sinh(x)}{\sqrt{a+ia\sinh(x)}}\,dx = \frac{2\left((1+i)\sqrt[4]{-1}\arctan\left(\frac{i+\tanh\left(\frac{x}{4}\right)}{\sqrt{2}}\right)+\cosh\left(\frac{x}{2}\right)-i\sinh\left(\frac{x}{2}\right)\right)\left(\cosh\left(\frac{x}{2}\right)+i\sinh\left(\frac{x}{2}\right)\right)}{\sqrt{a+ia\sinh(x)}}$$

input
```
Integrate[Sinh[x]/Sqrt[a + I*a*Sinh[x]],x]
```

output
```
(2*((1 + I)*(-1)^(1/4)*ArcTan[(I + Tanh[x/4])/Sqrt[2]] + Cosh[x/2] - I*Sin
h[x/2])*(Cosh[x/2] + I*Sinh[x/2]))/Sqrt[a + I*a*Sinh[x]]
```

Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 65, normalized size of antiderivative = 1.14, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.375, Rules used = {3042, 26, 3230, 3042, 3128, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sinh(x)}{\sqrt{a+ia\sinh(x)}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int -\frac{i\sin(ix)}{\sqrt{a+a\sin(ix)}}dx$$

$$\Big\downarrow \text{26}$$

$$-i\int \frac{\sin(ix)}{\sqrt{\sin(ix)a+a}}dx$$

$$\Big\downarrow \text{3230}$$

$$-i\left(\frac{2i\cosh(x)}{\sqrt{a+ia\sinh(x)}}-\int \frac{1}{\sqrt{i\sinh(x)a+a}}dx\right)$$

$$\Big\downarrow \text{3042}$$

$$-i\left(\frac{2i\cosh(x)}{\sqrt{a+ia\sinh(x)}}-\int \frac{1}{\sqrt{\sin(ix)a+a}}dx\right)$$

$$\Big\downarrow \text{3128}$$

$$-i\left(\frac{2i\cosh(x)}{\sqrt{a+ia\sinh(x)}}-2i\int \frac{1}{2a-\frac{a^2\cosh^2(x)}{i\sinh(x)a+a}}d\frac{a\cosh(x)}{\sqrt{i\sinh(x)a+a}}\right)$$

$$\Big\downarrow \text{219}$$

$$-i\left(\frac{2i\cosh(x)}{\sqrt{a+ia\sinh(x)}}-\frac{i\sqrt{2}\text{arctanh}\left(\frac{\sqrt{a}\cosh(x)}{\sqrt{2}\sqrt{a+ia\sinh(x)}}\right)}{\sqrt{a}}\right)$$

input
```
Int[Sinh[x]/Sqrt[a + I*a*Sinh[x]],x]
```

output
```
(-I)*(((-I)*Sqrt[2]*ArcTanh[(Sqrt[a]*Cosh[x])/(Sqrt[2]*Sqrt[a + I*a*Sinh[x
]])])/Sqrt[a] + ((2*I)*Cosh[x])/Sqrt[a + I*a*Sinh[x]])
```

Defintions of rubi rules used

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3128
```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[-2/d
Subst[Int[1/(2*a - x^2), x], x, b*(Cos[c + d*x]/Sqrt[a + b*Sin[c + d*x]])],
 x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]
```

rule 3230
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Simp[(-d)*Cos[e + f*x]*((a + b*Sin[e + f*x])^m/(
f*(m + 1))), x] + Simp[(a*d*m + b*c*(m + 1))/(b*(m + 1))   Int[(a + b*Sin[e
 + f*x])^m, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b*c - a*d, 0]
&& EqQ[a^2 - b^2, 0] &&  !LtQ[m, -2^(-1)]
```

Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 107 vs. 2(44) = 88.

Time = 0.70 (sec) , antiderivative size = 108, normalized size of antiderivative = 1.89

method	result	size
risch	$\frac{(e^x-i)^2\sqrt{2}\,e^{-x}}{\sqrt{a(ie^{2x}+2\,e^x-i)e^{-x}}}-\frac{2i(-e^x+i)\left(a^{\frac{3}{2}}+\arctan\left(\frac{\sqrt{ia\,e^x}}{\sqrt{a}}\right)a\sqrt{ia\,e^x}\right)\sqrt{2}\,e^{-x}}{a^{\frac{3}{2}}\sqrt{a(ie^{2x}+2\,e^x-i)e^{-x}}}$	108

input
```
int(sinh(x)/(a+I*a*sinh(x))^(1/2),x,method=_RETURNVERBOSE)
```

output
```
(exp(x)-I)^2*2^(1/2)/(a*(I*exp(x)^2+2*exp(x)-I)/exp(x))^(1/2)/exp(x)-2*I*(
-exp(x)+I)*(a^(3/2)+arctan((I*a*exp(x))^(1/2)/a^(1/2))*a*(I*a*exp(x))^(1/2
))/a^(3/2)*2^(1/2)/(a*(I*exp(x)^2+2*exp(x)-I)/exp(x))^(1/2)/exp(x)
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 76, normalized size of antiderivative = 1.33

$$\int \frac{\sinh(x)}{\sqrt{a+ia\sinh(x)}}\,dx = -\frac{\sqrt{2}\sqrt{a}\log\left(\frac{1}{2}\sqrt{2}\sqrt{a}+\sqrt{\frac{1}{2}i\,ae^{(-x)}}\right)-\sqrt{2}\sqrt{a}\log\left(-\frac{1}{2}\sqrt{2}\sqrt{a}+\sqrt{\frac{1}{2}i\,ae^{(-x)}}\right)+2\sqrt{\frac{1}{2}i\,ae^{(-x)}}(i\,e^x-1)}{a}$$

input
```
integrate(sinh(x)/(a+I*a*sinh(x))^(1/2),x, algorithm="fricas")
```

output
```
-(sqrt(2)*sqrt(a)*log(1/2*sqrt(2)*sqrt(a) + sqrt(1/2*I*a*e^(-x))) - sqrt(2
)*sqrt(a)*log(-1/2*sqrt(2)*sqrt(a) + sqrt(1/2*I*a*e^(-x))) + 2*sqrt(1/2*I*
a*e^(-x))*(I*e^x - 1))/a
```

Sympy [F]

$$\int \frac{\sinh(x)}{\sqrt{a + ia\sinh(x)}}\, dx = \int \frac{\sinh{(x)}}{\sqrt{ia\left(\sinh{(x)} - i\right)}}\, dx$$

input
```
integrate(sinh(x)/(a+I*a*sinh(x))**(1/2),x)
```

output
```
Integral(sinh(x)/sqrt(I*a*(sinh(x) - I)), x)
```

Maxima [F]

$$\int \frac{\sinh(x)}{\sqrt{a + ia\sinh(x)}}\, dx = \int \frac{\sinh{(x)}}{\sqrt{i\,a\sinh{(x)} + a}}\, dx$$

input
```
integrate(sinh(x)/(a+I*a*sinh(x))^(1/2),x, algorithm="maxima")
```

output
```
integrate(sinh(x)/sqrt(I*a*sinh(x) + a), x)
```

Giac [F]

$$\int \frac{\sinh(x)}{\sqrt{a + ia\sinh(x)}}\, dx = \int \frac{\sinh{(x)}}{\sqrt{i\,a\sinh{(x)} + a}}\, dx$$

input
```
integrate(sinh(x)/(a+I*a*sinh(x))^(1/2),x, algorithm="giac")
```

output
```
integrate(sinh(x)/sqrt(I*a*sinh(x) + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sinh(x)}{\sqrt{a+ia\sinh(x)}}\,dx = \int \frac{\sinh(x)}{\sqrt{a+a\sinh\left(x\right)\,1{}\mathrm{i}}}\,dx$$

input
```
int(sinh(x)/(a + a*sinh(x)*1i)^(1/2),x)
```

output
```
int(sinh(x)/(a + a*sinh(x)*1i)^(1/2), x)
```

Reduce [F]

$$\int \frac{\sinh(x)}{\sqrt{a+ia\sinh(x)}}\,dx = \frac{\sqrt{a}\left(-\left(\int \frac{\sqrt{\sinh(x)i+1}\,\sinh(x)^{2}}{\sinh(x)^{2}+1}dx\right)i+\int \frac{\sqrt{\sinh(x)i+1}\,\sinh(x)}{\sinh(x)^{2}+1}dx\right)}{a}$$

input
```
int(sinh(x)/(a+I*a*sinh(x))^(1/2),x)
```

output
```
(sqrt(a)*( - int((sqrt(sinh(x)*i + 1)*sinh(x)**2)/(sinh(x)**2 + 1),x)*i +
int((sqrt(sinh(x)*i + 1)*sinh(x))/(sinh(x)**2 + 1),x)))/a
```

3.65 $\int \frac{\sinh(x)}{\sqrt{a-ia\sinh(x)}}\,dx$

Optimal result

Integrand size = 16, antiderivative size = 57

$$\int \frac{\sinh(x)}{\sqrt{a-ia\sinh(x)}}\,dx=-\frac{\sqrt{2}\text{arctanh}\left(\frac{\sqrt{a}\cosh(x)}{\sqrt{2}\sqrt{a-ia\sinh(x)}}\right)}{\sqrt{a}}+\frac{2\cosh(x)}{\sqrt{a-ia\sinh(x)}}$$

output
```
-2^(1/2)*arctanh(1/2*a^(1/2)*cosh(x)*2^(1/2)/(a-I*a*sinh(x))^(1/2))/a^(1/2
)+2*cosh(x)/(a-I*a*sinh(x))^(1/2)
```

Mathematica [A] (verified)

Time = 0.24 (sec) , antiderivative size = 76, normalized size of antiderivative = 1.33

$$\int \frac{\sinh(x)}{\sqrt{a-ia\sinh(x)}}\,dx =\frac{2\left(\cosh\left(\frac{x}{2}\right)-i\sinh\left(\frac{x}{2}\right)\right)\left(\cosh\left(\frac{x}{2}\right)+i\left((1+i)(-1)^{3/4}\arctan\left(\frac{-i+\tanh\left(\frac{x}{4}\right)}{\sqrt{2}}\right)+\sinh\left(\frac{x}{2}\right)\right)\right)}{\sqrt{a-ia\sinh(x)}}$$

input
```
Integrate[Sinh[x]/Sqrt[a - I*a*Sinh[x]],x]
```

output
```
(2*(Cosh[x/2] - I*Sinh[x/2])*(Cosh[x/2] + I*((1 + I)*(-1)^(3/4)*ArcTan[(-I
 + Tanh[x/4])/Sqrt[2]] + Sinh[x/2])))/Sqrt[a - I*a*Sinh[x]]
```

Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 65, normalized size of antiderivative = 1.14, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.375$, Rules used = {3042, 26, 3230, 3042, 3128, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sinh(x)}{\sqrt{a-ia\sinh(x)}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int -\frac{i\sin(ix)}{\sqrt{a-a\sin(ix)}}dx$$

$$\Big\downarrow \text{26}$$

$$-i\int \frac{\sin(ix)}{\sqrt{a-a\sin(ix)}}dx$$

$$\Big\downarrow \text{3230}$$

$$-i\left(\int \frac{1}{\sqrt{a-ia\sinh(x)}}dx+\frac{2i\cosh(x)}{\sqrt{a-ia\sinh(x)}}\right)$$

$$\Big\downarrow \text{3042}$$

$$-i\left(\int \frac{1}{\sqrt{a-a\sin(ix)}}dx+\frac{2i\cosh(x)}{\sqrt{a-ia\sinh(x)}}\right)$$

$$\Big\downarrow \text{3128}$$

$$-i\left(2i\int \frac{1}{2a-\frac{a^2\cosh^2(x)}{a-ia\sinh(x)}}d\left(-\frac{a\cosh(x)}{\sqrt{a-ia\sinh(x)}}\right)+\frac{2i\cosh(x)}{\sqrt{a-ia\sinh(x)}}\right)$$

$$\Big\downarrow \text{219}$$

$$-i\left(\frac{2i\cosh(x)}{\sqrt{a-ia\sinh(x)}}-\frac{i\sqrt{2}\text{arctanh}\left(\frac{\sqrt{a}\cosh(x)}{\sqrt{2}\sqrt{a-ia\sinh(x)}}\right)}{\sqrt{a}}\right)$$

input
```
Int[Sinh[x]/Sqrt[a - I*a*Sinh[x]],x]
```

output
```
(-I)*(((-I)*Sqrt[2]*ArcTanh[(Sqrt[a]*Cosh[x])/(Sqrt[2]*Sqrt[a - I*a*Sinh[x
]])])/Sqrt[a] + ((2*I)*Cosh[x])/Sqrt[a - I*a*Sinh[x]])
```

Defintions of rubi rules used

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3128
```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[-2/d
Subst[Int[1/(2*a - x^2), x], x, b*(Cos[c + d*x]/Sqrt[a + b*Sin[c + d*x]])],
 x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]
```

rule 3230
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Simp[(-d)*Cos[e + f*x]*((a + b*Sin[e + f*x])^m/(
f*(m + 1))), x] + Simp[(a*d*m + b*c*(m + 1))/(b*(m + 1))   Int[(a + b*Sin[e
 + f*x])^m, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b*c - a*d, 0]
&& EqQ[a^2 - b^2, 0] &&  !LtQ[m, -2^(-1)]
```

Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 107 vs. $2(44) = 88$.

Time = 0.78 (sec) , antiderivative size = 108, normalized size of antiderivative = 1.89

method	result	size
risch	$\frac{(e^x+i)^2\sqrt{2}\,e^{-x}}{\sqrt{-a(i e^{2x}-i-2\,e^x)e^{-x}}} - \frac{2i(e^x+i)\left(\arctan\left(\frac{\sqrt{-ia\,e^x}}{\sqrt{a}}\right)a\sqrt{-ia\,e^x}+a^{\frac{3}{2}}\right)\sqrt{2}\,e^{-x}}{a^{\frac{3}{2}}\sqrt{-a(i e^{2x}-i-2\,e^x)e^{-x}}}$	108

input
```
int(sinh(x)/(a-I*a*sinh(x))^(1/2),x,method=_RETURNVERBOSE)
```

output
```
(exp(x)+I)^2*2^(1/2)/(-a*(I*exp(x)^2-I-2*exp(x))/exp(x))^(1/2)/exp(x)-2*I*
(exp(x)+I)*(arctan((-I*a*exp(x))^(1/2)/a^(1/2))*a*(-I*a*exp(x))^(1/2)+a^(3
/2))/a^(3/2)*2^(1/2)/(-a*(I*exp(x)^2-I-2*exp(x))/exp(x))^(1/2)/exp(x)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 76, normalized size of antiderivative = 1.33

$$\int \frac{\sinh(x)}{\sqrt{a-ia\sinh(x)}}\,dx = -\frac{\sqrt{2}\sqrt{a}\log\left(\frac{1}{2}\sqrt{2}\sqrt{a}+\sqrt{-\frac{1}{2}i\,ae^{(-x)}}\right)-\sqrt{2}\sqrt{a}\log\left(-\frac{1}{2}\sqrt{2}\sqrt{a}+\sqrt{-\frac{1}{2}i\,ae^{(-x)}}\right)+2\sqrt{-\frac{1}{2}i\,ae^{(-x)}}(-i\,e}{a}$$

input
```
integrate(sinh(x)/(a-I*a*sinh(x))^(1/2),x, algorithm="fricas")
```

output
```
-(sqrt(2)*sqrt(a)*log(1/2*sqrt(2)*sqrt(a) + sqrt(-1/2*I*a*e^(-x))) - sqrt(
2)*sqrt(a)*log(-1/2*sqrt(2)*sqrt(a) + sqrt(-1/2*I*a*e^(-x))) + 2*sqrt(-1/2
*I*a*e^(-x))*(-I*e^x - 1))/a
```

Sympy [F]

$$\int \frac{\sinh(x)}{\sqrt{a - ia\sinh(x)}}\,dx = \int \frac{\sinh(x)}{\sqrt{-ia(\sinh(x) + i)}}\,dx$$

input
```
integrate(sinh(x)/(a-I*a*sinh(x))**(1/2),x)
```

output
```
Integral(sinh(x)/sqrt(-I*a*(sinh(x) + I)), x)
```

Maxima [F]

$$\int \frac{\sinh(x)}{\sqrt{a - ia\sinh(x)}}\,dx = \int \frac{\sinh(x)}{\sqrt{-i\,a\sinh(x) + a}}\,dx$$

input
```
integrate(sinh(x)/(a-I*a*sinh(x))^(1/2),x, algorithm="maxima")
```

output
```
integrate(sinh(x)/sqrt(-I*a*sinh(x) + a), x)
```

Giac [F]

$$\int \frac{\sinh(x)}{\sqrt{a - ia\sinh(x)}}\,dx = \int \frac{\sinh(x)}{\sqrt{-i\,a\sinh(x) + a}}\,dx$$

input
```
integrate(sinh(x)/(a-I*a*sinh(x))^(1/2),x, algorithm="giac")
```

output
```
integrate(sinh(x)/sqrt(-I*a*sinh(x) + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sinh(x)}{\sqrt{a - ia\sinh(x)}}\,dx = \int \frac{\sinh(x)}{\sqrt{a - a\sinh\left(x\right)\,1\mathrm{i}}}\,dx$$

input
```
int(sinh(x)/(a - a*sinh(x)*1i)^(1/2),x)
```

output
```
int(sinh(x)/(a - a*sinh(x)*1i)^(1/2), x)
```

Reduce [F]

$$\int \frac{\sinh(x)}{\sqrt{a - ia\sinh(x)}}\,dx = \frac{\sqrt{a}\left(\left(\int \frac{\sqrt{-\sinh(x)i+1}\,\sinh(x)^2}{\sinh(x)^2+1}dx\right)i + \int \frac{\sqrt{-\sinh(x)i+1}\,\sinh(x)}{\sinh(x)^2+1}dx\right)}{a}$$

input
```
int(sinh(x)/(a-I*a*sinh(x))^(1/2),x)
```

output
```
(sqrt(a)*(int((sqrt( - sinh(x)*i + 1)*sinh(x)**2)/(sinh(x)**2 + 1),x)*i +
int((sqrt( - sinh(x)*i + 1)*sinh(x))/(sinh(x)**2 + 1),x)))/a
```

3.66 $\int (a + ia\sinh(c+dx))^{5/2}\,dx$

Optimal result

Integrand size = 17, antiderivative size = 104

$$\int (a+ia\sinh(c+dx))^{5/2}\,dx = \frac{64ia^3\cosh(c+dx)}{15d\sqrt{a+ia\sinh(c+dx)}} + \frac{16ia^2\cosh(c+dx)\sqrt{a+ia\sinh(c+dx)}}{15d} + \frac{2ia\cosh(c+dx)(a+ia\sinh(c+dx))^{3/2}}{5d}$$

output
```
64/15*I*a^3*cosh(d*x+c)/d/(a+I*a*sinh(d*x+c))^(1/2)+16/15*I*a^2*cosh(d*x+c
)*(a+I*a*sinh(d*x+c))^(1/2)/d+2/5*I*a*cosh(d*x+c)*(a+I*a*sinh(d*x+c))^(3/2
)/d
```

Mathematica [A] (verified)

Time = 5.96 (sec) , antiderivative size = 145, normalized size of antiderivative = 1.39

$$\int (a+ia\sinh(c+dx))^{5/2}\,dx = \frac{a^2(-i+\sinh(c+dx))^2\sqrt{a+ia\sinh(c+dx)}\left(-150i\cosh\left(\frac{1}{2}(c+dx)\right) - 25i\cosh\left(\frac{3}{2}(c+dx)\right)\right.}{30d\left(\cosh\left(\frac{1}{2}(c+dx)\right)\right.}$$

input
```
Integrate[(a + I*a*Sinh[c + d*x])^(5/2),x]
```

output
```
(a^2*(-I + Sinh[c + d*x])^2*Sqrt[a + I*a*Sinh[c + d*x]]*((-150*I)*Cosh[(c
+ d*x)/2] - (25*I)*Cosh[(3*(c + d*x))/2] + (3*I)*Cosh[(5*(c + d*x))/2] - 1
50*Sinh[(c + d*x)/2] + 25*Sinh[(3*(c + d*x))/2] + 3*Sinh[(5*(c + d*x))/2])
)/(30*d*(Cosh[(c + d*x)/2] + I*Sinh[(c + d*x)/2])^5)
```

Rubi [A] (verified)

Time = 0.37 (sec) , antiderivative size = 108, normalized size of antiderivative = 1.04, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.353$, Rules used = {3042, 3126, 3042, 3126, 3042, 3125}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a+ia\sinh(c+dx))^{5/2}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int (a+a\sin(ic+idx))^{5/2}dx$$

$$\Big\downarrow \text{3126}$$

$$\frac{8}{5}a\int (i\sinh(c+dx)a+a)^{3/2}dx+\frac{2ia\cosh(c+dx)(a+ia\sinh(c+dx))^{3/2}}{5d}$$

$$\Big\downarrow \text{3042}$$

$$\frac{8}{5}a\int (\sin(ic+idx)a+a)^{3/2}dx+\frac{2ia\cosh(c+dx)(a+ia\sinh(c+dx))^{3/2}}{5d}$$

$$\Big\downarrow \text{3126}$$

$$\frac{8}{5}a\left(\frac{4}{3}a\int \sqrt{i\sinh(c+dx)a+a}dx+\frac{2ia\cosh(c+dx)\sqrt{a+ia\sinh(c+dx)}}{3d}\right)+\frac{2ia\cosh(c+dx)(a+ia\sinh(c+dx))^{3/2}}{5d}$$

$$\Big\downarrow \text{3042}$$

$$\frac{8}{5}a\left(\frac{4}{3}a\int \sqrt{\sin(ic+idx)a+a}dx+\frac{2ia\cosh(c+dx)\sqrt{a+ia\sinh(c+dx)}}{3d}\right)+\frac{2ia\cosh(c+dx)(a+ia\sinh(c+dx))^{3/2}}{5d}$$

$$\Big\downarrow \text{3125}$$

$$\frac{8}{5}a\left(\frac{8ia^2\cosh(c+dx)}{3d\sqrt{a+ia\sinh(c+dx)}}+\frac{2ia\cosh(c+dx)\sqrt{a+ia\sinh(c+dx)}}{3d}\right)+\frac{2ia\cosh(c+dx)(a+ia\sinh(c+dx))^{3/2}}{5d}$$

input
```
Int[(a + I*a*Sinh[c + d*x])^(5/2),x]
```

output
```
(((2*I)/5)*a*Cosh[c + d*x]*(a + I*a*Sinh[c + d*x])^(3/2))/d + (8*a*((((8*I
)/3)*a^2*Cosh[c + d*x])/(d*Sqrt[a + I*a*Sinh[c + d*x]]) + (((2*I)/3)*a*Cos
h[c + d*x]*Sqrt[a + I*a*Sinh[c + d*x]])/d))/5
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3125
```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[-2*b*(Cos
[c + d*x]/(d*Sqrt[a + b*Sin[c + d*x]])), x] /; FreeQ[{a, b, c, d}, x] && Eq
Q[a^2 - b^2, 0]
```

rule 3126
```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos
[c + d*x]*((a + b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Simp[a*((2*n - 1)/n)
 Int[(a + b*Sin[c + d*x])^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] && EqQ[
a^2 - b^2, 0] && IGtQ[n - 1/2, 0]
```

Maple [A] (verified)

Time = 0.34 (sec) , antiderivative size = 93, normalized size of antiderivative = 0.89

method	result	size
default	$\frac{8a^3\cosh\left(\frac{c}{2}+\frac{i\pi}{4}+\frac{dx}{2}\right)\sinh\left(\frac{c}{2}+\frac{i\pi}{4}+\frac{dx}{2}\right)\left(3\cosh\left(\frac{c}{2}+\frac{i\pi}{4}+\frac{dx}{2}\right)^4+4\cosh\left(\frac{c}{2}+\frac{i\pi}{4}+\frac{dx}{2}\right)^2+8\right)\sqrt{2}}{15\sqrt{a\cosh\left(\frac{c}{2}+\frac{i\pi}{4}+\frac{dx}{2}\right)^2}\,d}$	93

input
```
int((a+I*a*sinh(d*x+c))^(5/2),x,method=_RETURNVERBOSE)
```

output
```
8/15*a^3*cosh(1/2*c+1/4*I*Pi+1/2*d*x)*sinh(1/2*c+1/4*I*Pi+1/2*d*x)*(3*cosh
(1/2*c+1/4*I*Pi+1/2*d*x)^4+4*cosh(1/2*c+1/4*I*Pi+1/2*d*x)^2+8)*2^(1/2)/(a*
cosh(1/2*c+1/4*I*Pi+1/2*d*x)^2)^(1/2)/d
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 101, normalized size of antiderivative = 0.97

$$\int (a+ia\sinh(c+dx))^{5/2}\,dx =$$
$$-\frac{\left(3\,a^2e^{(5\,dx+5\,c)}-25i\,a^2e^{(4\,dx+4\,c)}-150\,a^2e^{(3\,dx+3\,c)}-150i\,a^2e^{(2\,dx+2\,c)}-25\,a^2e^{(dx+c)}+3i\,a^2\right)\sqrt{\frac{1}{2}i\,ae^{(-dx-c)}}}{30\,d}$$

input
```
integrate((a+I*a*sinh(d*x+c))^(5/2),x, algorithm="fricas")
```

output
```
-1/30*(3*a^2*e^(5*d*x + 5*c) - 25*I*a^2*e^(4*d*x + 4*c) - 150*a^2*e^(3*d*x
 + 3*c) - 150*I*a^2*e^(2*d*x + 2*c) - 25*a^2*e^(d*x + c) + 3*I*a^2)*sqrt(1
/2*I*a*e^(-d*x - c))*e^(-2*d*x - 2*c)/d
```

Sympy [F(-1)]

Timed out.

$$\int (a + ia \sinh(c + dx))^{5/2} \, dx = \text{Timed out}$$

input
```
integrate((a+I*a*sinh(d*x+c))**(5/2),x)
```

output
```
Timed out
```

Maxima [F]

$$\int (a + ia \sinh(c + dx))^{5/2} \, dx = \int (i \, a \sinh (dx + c) + a)^{\frac{5}{2}} \, dx$$

input
```
integrate((a+I*a*sinh(d*x+c))^(5/2),x, algorithm="maxima")
```

output
```
integrate((I*a*sinh(d*x + c) + a)^(5/2), x)
```

Giac [F]

$$\int (a + ia \sinh(c + dx))^{5/2} \, dx = \int (i \, a \sinh (dx + c) + a)^{\frac{5}{2}} \, dx$$

input
```
integrate((a+I*a*sinh(d*x+c))^(5/2),x, algorithm="giac")
```

output
```
integrate((I*a*sinh(d*x + c) + a)^(5/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int (a+ia\sinh(c+dx))^{5/2}\,dx = \int (a+a\sinh(c+d\,x)\ 1\mathrm{i})^{5/2}\,dx$$

input
```
int((a + a*sinh(c + d*x)*1i)^(5/2),x)
```

output
```
int((a + a*sinh(c + d*x)*1i)^(5/2), x)
```

Reduce [F]

$$\int (a+ia\sinh(c+dx))^{5/2}\,dx = \sqrt{a}\,a^2\left(\int \sqrt{\sinh(dx+c)\,i+1}dx - \left(\int \sqrt{\sinh(dx+c)\,i+1}\,\sinh(dx+c)^2\,dx\right) + 2\left(\int \sqrt{\sinh(dx+c)\,i+1}\,\sinh(dx+c)\,dx\right)i\right)$$

input
```
int((a+I*a*sinh(d*x+c))^(5/2),x)
```

output
```
sqrt(a)*a**2*(int(sqrt(sinh(c + d*x)*i + 1),x) - int(sqrt(sinh(c + d*x)*i
+ 1)*sinh(c + d*x)**2,x) + 2*int(sqrt(sinh(c + d*x)*i + 1)*sinh(c + d*x),x
)*i)
```

3.67 $\int (a + ia\sinh(c + dx))^{3/2} dx$

Optimal result

Integrand size = 17, antiderivative size = 69

$$\int (a + ia\sinh(c + dx))^{3/2} dx = \frac{8ia^2\cosh(c + dx)}{3d\sqrt{a + ia\sinh(c + dx)}} + \frac{2ia\cosh(c + dx)\sqrt{a + ia\sinh(c + dx)}}{3d}$$

output
```
8/3*I*a^2*cosh(d*x+c)/d/(a+I*a*sinh(d*x+c))^(1/2)+2/3*I*a*cosh(d*x+c)*(a+I
*a*sinh(d*x+c))^(1/2)/d
```

Mathematica [A] (verified)

Time = 5.31 (sec) , antiderivative size = 113, normalized size of antiderivative = 1.64

$$\int (a + ia\sinh(c + dx))^{3/2} dx = -\frac{a(-i + \sinh(c + dx))\sqrt{a + ia\sinh(c + dx)}\left(9\cosh\left(\frac{1}{2}(c + dx)\right) + \cosh\left(\frac{3}{2}(c + dx)\right) - 9i\sinh\left(\frac{1}{2}(c + dx)\right)\right.}{3d\left(\cosh\left(\frac{1}{2}(c + dx)\right) + i\sinh\left(\frac{1}{2}(c + dx)\right)\right)^3}$$

input
```
Integrate[(a + I*a*Sinh[c + d*x])^(3/2),x]
```

output
```
-1/3*(a*(-I + Sinh[c + d*x])*Sqrt[a + I*a*Sinh[c + d*x]]*(9*Cosh[(c + d*x)
/2] + Cosh[(3*(c + d*x))/2] - (9*I)*Sinh[(c + d*x)/2] + I*Sinh[(3*(c + d*x
))/2]))/(d*(Cosh[(c + d*x)/2] + I*Sinh[(c + d*x)/2])^3)
```

Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 69, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.235$, Rules used = {3042, 3126, 3042, 3125}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a+ia\sinh(c+dx))^{3/2}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int (a+a\sin(ic+idx))^{3/2}dx$$

$$\Big\downarrow \text{3126}$$

$$\frac{4}{3}a\int \sqrt{i\sinh(c+dx)a+a}dx+\frac{2ia\cosh(c+dx)\sqrt{a+ia\sinh(c+dx)}}{3d}$$

$$\Big\downarrow \text{3042}$$

$$\frac{4}{3}a\int \sqrt{\sin(ic+idx)a+a}dx+\frac{2ia\cosh(c+dx)\sqrt{a+ia\sinh(c+dx)}}{3d}$$

$$\Big\downarrow \text{3125}$$

$$\frac{8ia^2\cosh(c+dx)}{3d\sqrt{a+ia\sinh(c+dx)}}+\frac{2ia\cosh(c+dx)\sqrt{a+ia\sinh(c+dx)}}{3d}$$

input
```
Int[(a + I*a*Sinh[c + d*x])^(3/2),x]
```

output
```
(((8*I)/3)*a^2*Cosh[c + d*x])/(d*Sqrt[a + I*a*Sinh[c + d*x]]) + (((2*I)/3)
*a*Cosh[c + d*x]*Sqrt[a + I*a*Sinh[c + d*x]])/d
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3125
```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[-2*b*(Cos
[c + d*x]/(d*Sqrt[a + b*Sin[c + d*x]])), x] /; FreeQ[{a, b, c, d}, x] && Eq
Q[a^2 - b^2, 0]
```

rule 3126
```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos
[c + d*x]*((a + b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Simp[a*((2*n - 1)/n)
 Int[(a + b*Sin[c + d*x])^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] && EqQ[
a^2 - b^2, 0] && IGtQ[n - 1/2, 0]
```

Maple [A] (verified)

Time = 0.27 (sec) , antiderivative size = 74, normalized size of antiderivative = 1.07

method	result	size
default	$\frac{4a^{2}\cosh\left(\frac{c}{2}+\frac{i\pi}{4}+\frac{dx}{2}\right)\sinh\left(\frac{c}{2}+\frac{i\pi}{4}+\frac{dx}{2}\right)\left(\cosh\left(\frac{c}{2}+\frac{i\pi}{4}+\frac{dx}{2}\right)^{2}+2\right)\sqrt{2}}{3\sqrt{a\cosh\left(\frac{c}{2}+\frac{i\pi}{4}+\frac{dx}{2}\right)^{2}}\,d}$	74

input
```
int((a+I*a*sinh(d*x+c))^(3/2),x,method=_RETURNVERBOSE)
```

output
```
4/3*a^2*cosh(1/2*c+1/4*I*Pi+1/2*d*x)*sinh(1/2*c+1/4*I*Pi+1/2*d*x)*(cosh(1/
2*c+1/4*I*Pi+1/2*d*x)^2+2)*2^(1/2)/(a*cosh(1/2*c+1/4*I*Pi+1/2*d*x)^2)^(1/2
)/d
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 63, normalized size of antiderivative = 0.91

$$\int (a+ia\sinh(c+dx))^{3/2}\,dx=\frac{\left(i\,ae^{(3\,dx+3\,c)}+9\,ae^{(2\,dx+2\,c)}+9i\,ae^{(dx+c)}+a\right)\sqrt{\frac{1}{2}i\,ae^{(-dx-c)}}e^{(-dx-c)}}{3\,d}$$

input
```
integrate((a+I*a*sinh(d*x+c))^(3/2),x, algorithm="fricas")
```

output
```
1/3*(I*a*e^(3*d*x + 3*c) + 9*a*e^(2*d*x + 2*c) + 9*I*a*e^(d*x + c) + a)*sq
rt(1/2*I*a*e^(-d*x - c))*e^(-d*x - c)/d
```

Sympy [F]

$$\int (a+ia\sinh(c+dx))^{3/2}\,dx=\int \left(ia\sinh\left(c+dx\right)+a\right)^{\frac{3}{2}}\,dx$$

input
```
integrate((a+I*a*sinh(d*x+c))**(3/2),x)
```

output
```
Integral((I*a*sinh(c + d*x) + a)**(3/2), x)
```

Maxima [F]

$$\int (a+ia\sinh(c+dx))^{3/2}\,dx=\int \left(i\,a\sinh\left(dx+c\right)+a\right)^{\frac{3}{2}}\,dx$$

input
```
integrate((a+I*a*sinh(d*x+c))^(3/2),x, algorithm="maxima")
```

output
```
integrate((I*a*sinh(d*x + c) + a)^(3/2), x)
```

Giac [A] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 59, normalized size of antiderivative = 0.86

$$\int (a+ia\sinh(c+dx))^{3/2}\,dx = -\frac{\left(-(i-1)\,a^{\frac{3}{2}}e^{(3\,dx+3\,c)}-(9i+9)\,a^{\frac{3}{2}}e^{(2\,dx+2\,c)}-(9i-9)\,a^{\frac{3}{2}}e^{(dx+c)}-(i+1)\,a^{\frac{3}{2}}\right)e^{\left(-\frac{3}{2}\,dx-\frac{3}{2}\,c\right)}}{6\,d}$$

input
```
integrate((a+I*a*sinh(d*x+c))^(3/2),x, algorithm="giac")
```

output
```
-1/6*(-(I - 1)*a^(3/2)*e^(3*d*x + 3*c) - (9*I + 9)*a^(3/2)*e^(2*d*x + 2*c)
 - (9*I - 9)*a^(3/2)*e^(d*x + c) - (I + 1)*a^(3/2))*e^(-3/2*d*x - 3/2*c)/d
```

Mupad [F(-1)]

Timed out.

$$\int (a+ia\sinh(c+dx))^{3/2}\,dx = \int \left(a+a\sinh(c+d\,x)\,1\mathrm{i}\right)^{3/2}\,dx$$

input
```
int((a + a*sinh(c + d*x)*1i)^(3/2),x)
```

output
```
int((a + a*sinh(c + d*x)*1i)^(3/2), x)
```

Reduce [F]

$$\int (a+ia\sinh(c+dx))^{3/2}\,dx = \sqrt{a}\,a\left(\int \sqrt{\sinh(dx+c)\,i+1}\,dx + \left(\int \sqrt{\sinh(dx+c)\,i+1}\,\sinh(dx+c)\,dx\right)i\right)$$

input
```
int((a+I*a*sinh(d*x+c))^(3/2),x)
```

output
```
sqrt(a)*a*(int(sqrt(sinh(c + d*x)*i + 1),x) + int(sqrt(sinh(c + d*x)*i + 1
)*sinh(c + d*x),x)*i)
```

3.68 $\int \sqrt{a+ia\sinh(c+dx)}\,dx$

Optimal result

Integrand size = 17, antiderivative size = 31

$$\int \sqrt{a+ia\sinh(c+dx)}\,dx = \frac{2ia\cosh(c+dx)}{d\sqrt{a+ia\sinh(c+dx)}}$$

output
```
2*I*a*cosh(d*x+c)/d/(a+I*a*sinh(d*x+c))^(1/2)
```

Mathematica [B] (verified)

Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 74 vs. 2(31) = 62.

Time = 0.03 (sec) , antiderivative size = 74, normalized size of antiderivative = 2.39

$$\int \sqrt{a+ia\sinh(c+dx)}\,dx = \frac{2\left(i\cosh\left(\frac{1}{2}(c+dx)\right)+\sinh\left(\frac{1}{2}(c+dx)\right)\right)\sqrt{a+ia\sinh(c+dx)}}{d\left(\cosh\left(\frac{1}{2}(c+dx)\right)+i\sinh\left(\frac{1}{2}(c+dx)\right)\right)}$$

input
```
Integrate[Sqrt[a + I*a*Sinh[c + d*x]],x]
```

output
```
(2*(I*Cosh[(c + d*x)/2] + Sinh[(c + d*x)/2])*Sqrt[a + I*a*Sinh[c + d*x]])/
(d*(Cosh[(c + d*x)/2] + I*Sinh[(c + d*x)/2]))
```

Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.118$, Rules used = {3042, 3125}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{a + ia\sinh(c + dx)}\, dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \sqrt{a + a\sin(ic + idx)}dx$$

$$\Big\downarrow \text{ 3125}$$

$$\frac{2ia\cosh(c + dx)}{d\sqrt{a + ia\sinh(c + dx)}}$$

input
```
Int[Sqrt[a + I*a*Sinh[c + d*x]],x]
```

output
```
((2*I)*a*Cosh[c + d*x])/(d*Sqrt[a + I*a*Sinh[c + d*x]])
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3125
```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[-2*b*(Cos
[c + d*x]/(d*Sqrt[a + b*Sin[c + d*x]])), x] /; FreeQ[{a, b, c, d}, x] && Eq
Q[a^2 - b^2, 0]
```

Maple [A] (verified)

Time = 0.33 (sec) , antiderivative size = 55, normalized size of antiderivative = 1.77

method	result	size
default	$\frac{2\sqrt{2}\,a\cosh\left(\frac{c}{2}+\frac{i\pi}{4}+\frac{dx}{2}\right)\sinh\left(\frac{c}{2}+\frac{i\pi}{4}+\frac{dx}{2}\right)}{\sqrt{a\cosh\left(\frac{c}{2}+\frac{i\pi}{4}+\frac{dx}{2}\right)^{2}}\,d}$	55
risch	$\frac{i\sqrt{2}\,\sqrt{a\left(i\mathrm{e}^{2dx+2c}-i+2\,\mathrm{e}^{dx+c}\right)\mathrm{e}^{-dx-c}}\,\left(\mathrm{e}^{dx+c}+i\right)\left(\mathrm{e}^{dx+c}-i\right)}{\left(i\mathrm{e}^{2dx+2c}-i+2\,\mathrm{e}^{dx+c}\right)d}$	89

input
```
int((a+I*a*sinh(d*x+c))^(1/2),x,method=_RETURNVERBOSE)
```

output
```
2*2^(1/2)*a*cosh(1/2*c+1/4*I*Pi+1/2*d*x)*sinh(1/2*c+1/4*I*Pi+1/2*d*x)/(a*c
osh(1/2*c+1/4*I*Pi+1/2*d*x)^2)^(1/2)/d
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.87

$$\int\sqrt{a+ia\sinh(c+dx)}\,dx=\frac{2\sqrt{\frac{1}{2}i\,ae^{(-dx-c)}}\left(e^{(dx+c)}+i\right)}{d}$$

input
```
integrate((a+I*a*sinh(d*x+c))^(1/2),x, algorithm="fricas")
```

output
```
2*sqrt(1/2*I*a*e^(-d*x - c))*(e^(d*x + c) + I)/d
```

Sympy [F]

$$\int\sqrt{a+ia\sinh(c+dx)}\,dx=\int\sqrt{ia\sinh{(c+dx)}+a}\,dx$$

input
```
integrate((a+I*a*sinh(d*x+c))**(1/2),x)
```

output
```
Integral(sqrt(I*a*sinh(c + d*x) + a), x)
```

Maxima [F]

$$\int \sqrt{a+ia\sinh(c+dx)}\,dx=\int \sqrt{i\,a\sinh\left(dx+c\right)+a}\,dx$$

input
```
integrate((a+I*a*sinh(d*x+c))^(1/2),x, algorithm="maxima")
```

output
```
integrate(sqrt(I*a*sinh(d*x + c) + a), x)
```

Giac [F]

$$\int \sqrt{a+ia\sinh(c+dx)}\,dx=\int \sqrt{i\,a\sinh\left(dx+c\right)+a}\,dx$$

input
```
integrate((a+I*a*sinh(d*x+c))^(1/2),x, algorithm="giac")
```

output
```
integrate(sqrt(I*a*sinh(d*x + c) + a), x)
```

Mupad [B] (verification not implemented)

Time = 2.11 (sec) , antiderivative size = 53, normalized size of antiderivative = 1.71

$$\int \sqrt{a+ia\sinh(c+dx)}\,dx=\frac{\sqrt{2}\,\left(\mathrm{e}^{c+d\,x}+1\mathrm{i}\right)\,\sqrt{a\,\mathrm{e}^{-c-d\,x}\,{\left(\mathrm{e}^{c+d\,x}-\mathrm{i}\right)}^2\,1\mathrm{i}}}{d\,\left(\mathrm{e}^{c+d\,x}-\mathrm{i}\right)}$$

input
```
int((a + a*sinh(c + d*x)*1i)^(1/2),x)
```

output
```
(2^(1/2)*(exp(c + d*x) + 1i)*(a*exp(- c - d*x)*(exp(c + d*x) - 1i)^2*1i)^(
1/2))/(d*(exp(c + d*x) - 1i))
```

Reduce [F]

$$\int \sqrt{a+ia\sinh(c+dx)}\,dx = \sqrt{a}\left(\int \sqrt{\sinh(dx+c)\,i+1}dx\right)$$

input
```
int((a+I*a*sinh(d*x+c))^(1/2),x)
```

output
```
sqrt(a)*int(sqrt(sinh(c + d*x)*i + 1),x)
```

3.69 $\int \frac{1}{\sqrt{a+ia\sinh(c+dx)}}\,dx$

Optimal result

Integrand size = 17, antiderivative size = 52

$$\int \frac{1}{\sqrt{a+ia\sinh(c+dx)}}\,dx = \frac{i\sqrt{2}\text{arctanh}\left(\frac{\sqrt{a}\cosh(c+dx)}{\sqrt{2}\sqrt{a+ia\sinh(c+dx)}}\right)}{\sqrt{a}d}$$

output
```
I*2^(1/2)*arctanh(1/2*a^(1/2)*cosh(d*x+c)*2^(1/2)/(a+I*a*sinh(d*x+c))^(1/2
))/a^(1/2)/d
```

Mathematica [A] (verified)

Time = 0.15 (sec) , antiderivative size = 84, normalized size of antiderivative = 1.62

$$\int \frac{1}{\sqrt{a+ia\sinh(c+dx)}}\,dx = \frac{(2+2i)\sqrt[4]{-1}\arctan\left(\left(\frac{1}{2}+\frac{i}{2}\right)\sqrt[4]{-1}\left(1-i\tanh\left(\frac{1}{4}(c+dx)\right)\right)\right)\left(-i\cosh\left(\frac{1}{2}(c+dx)\right)+\sinh\left(\frac{1}{2}(c+dx)\right)\right)}{d\sqrt{a+ia\sinh(c+dx)}}$$

input
```
Integrate[1/Sqrt[a + I*a*Sinh[c + d*x]],x]
```

output
```
((2 + 2*I)*(-1)^(1/4)*ArcTan[(1/2 + I/2)*(-1)^(1/4)*(1 - I*Tanh[(c + d*x)/
4])]*((-I)*Cosh[(c + d*x)/2] + Sinh[(c + d*x)/2]))/(d*Sqrt[a + I*a*Sinh[c
+ d*x]])
```

Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 52, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.176$, Rules used = {3042, 3128, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt{a+ia\sinh(c+dx)}}dx$$
$$\Big\downarrow \text{3042}$$
$$\int \frac{1}{\sqrt{a+a\sin(ic+idx)}}dx$$
$$\Big\downarrow \text{3128}$$
$$\frac{2i\int \frac{1}{2a-\frac{a^2\cosh^2(c+dx)}{i\sinh(c+dx)a+a}}d\frac{a\cosh(c+dx)}{\sqrt{i\sinh(c+dx)a+a}}}{d}$$
$$\Big\downarrow \text{219}$$
$$\frac{i\sqrt{2}\text{arctanh}\left(\frac{\sqrt{a}\cosh(c+dx)}{\sqrt{2}\sqrt{a+ia\sinh(c+dx)}}\right)}{\sqrt{a}d}$$

input
```
Int[1/Sqrt[a + I*a*Sinh[c + d*x]],x]
```

output
```
(I*Sqrt[2]*ArcTanh[(Sqrt[a]*Cosh[c + d*x])/(Sqrt[2]*Sqrt[a + I*a*Sinh[c +
d*x]])])/(Sqrt[a]*d)
```

Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3128
```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[-2/d
Subst[Int[1/(2*a - x^2), x], x, b*(Cos[c + d*x]/Sqrt[a + b*Sin[c + d*x]])],
 x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]
```

Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 126 vs. $2(41) = 82$.

Time = 0.79 (sec) , antiderivative size = 127, normalized size of antiderivative = 2.44

method	result	size
default	$-\frac{\cosh\left(\frac{c}{2}+\frac{i\pi}{4}+\frac{dx}{2}\right)\sqrt{\sinh\left(\frac{c}{2}+\frac{i\pi}{4}+\frac{dx}{2}\right)^{2}a}\,\ln\left(\frac{2\sqrt{-a}\,\sqrt{\sinh\left(\frac{c}{2}+\frac{i\pi}{4}+\frac{dx}{2}\right)^{2}a}-2a}{\cosh\left(\frac{c}{2}+\frac{i\pi}{4}+\frac{dx}{2}\right)}\right)\sqrt{2}}{\sqrt{-a}\,\sinh\left(\frac{c}{2}+\frac{i\pi}{4}+\frac{dx}{2}\right)\sqrt{a\cosh\left(\frac{c}{2}+\frac{i\pi}{4}+\frac{dx}{2}\right)^{2}}\,d}$	127
risch	$-\frac{2\left(\mathrm{e}^{dx+c}-i\right)\sqrt{2}\,\mathrm{e}^{-dx-c}}{d\sqrt{a\left(i\mathrm{e}^{2dx+2c}-i+2\,\mathrm{e}^{dx+c}\right)\mathrm{e}^{-dx-c}}}-\frac{2\left(-\mathrm{e}^{dx+c}+i\right)\left(a^{\frac{3}{2}}+\arctan\left(\frac{\sqrt{i\mathrm{e}^{dx+c}a}}{\sqrt{a}}\right)a\sqrt{i\mathrm{e}^{dx+c}a}\right)\sqrt{2}\,\mathrm{e}^{-dx-c}}{d\,a^{\frac{3}{2}}\sqrt{a\left(i\mathrm{e}^{2dx+2c}-i+2\,\mathrm{e}^{dx+c}\right)\mathrm{e}^{-dx-c}}}$	160

input
```
int(1/(a+I*a*sinh(d*x+c))^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-cosh(1/2*c+1/4*I*Pi+1/2*d*x)*(sinh(1/2*c+1/4*I*Pi+1/2*d*x)^2*a)^(1/2)/(-a
)^(1/2)*ln(2*((-a)^(1/2)*(sinh(1/2*c+1/4*I*Pi+1/2*d*x)^2*a)^(1/2)-a)/cosh(
1/2*c+1/4*I*Pi+1/2*d*x))/sinh(1/2*c+1/4*I*Pi+1/2*d*x)*2^(1/2)/(a*cosh(1/2*
c+1/4*I*Pi+1/2*d*x)^2)^(1/2)/d
```

Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 93 vs. $2(39) = 78$.

Time = 0.09 (sec) , antiderivative size = 93, normalized size of antiderivative = 1.79

$$\int \frac{1}{\sqrt{a+ia\sinh(c+dx)}}\,dx = i\sqrt{2}\sqrt{\frac{1}{ad^2}}\log\left(\frac{1}{2}\sqrt{2}ad\sqrt{\frac{1}{ad^2}}+\sqrt{\frac{1}{2}i\,ae^{(-dx-c)}}\right) - i\sqrt{2}\sqrt{\frac{1}{ad^2}}\log\left(-\frac{1}{2}\sqrt{2}ad\sqrt{\frac{1}{ad^2}}+\sqrt{\frac{1}{2}i\,ae^{(-dx-c)}}\right)$$

input
```
integrate(1/(a+I*a*sinh(d*x+c))^(1/2),x, algorithm="fricas")
```

output
```
I*sqrt(2)*sqrt(1/(a*d^2))*log(1/2*sqrt(2)*a*d*sqrt(1/(a*d^2)) + sqrt(1/2*I
*a*e^(-d*x - c))) - I*sqrt(2)*sqrt(1/(a*d^2))*log(-1/2*sqrt(2)*a*d*sqrt(1/
(a*d^2)) + sqrt(1/2*I*a*e^(-d*x - c)))
```

Sympy [F]

$$\int \frac{1}{\sqrt{a+ia\sinh(c+dx)}}\,dx = \int \frac{1}{\sqrt{ia\sinh(c+dx)+a}}\,dx$$

input
```
integrate(1/(a+I*a*sinh(d*x+c))**(1/2),x)
```

output
```
Integral(1/sqrt(I*a*sinh(c + d*x) + a), x)
```

Maxima [F]

$$\int \frac{1}{\sqrt{a + ia\sinh(c + dx)}}\,dx = \int \frac{1}{\sqrt{i\,a\sinh\left(dx + c\right) + a}}\,dx$$

input
```
integrate(1/(a+I*a*sinh(d*x+c))^(1/2),x, algorithm="maxima")
```

output
```
integrate(1/sqrt(I*a*sinh(d*x + c) + a), x)
```

Giac [F]

$$\int \frac{1}{\sqrt{a + ia\sinh(c + dx)}}\,dx = \int \frac{1}{\sqrt{i\,a\sinh\left(dx + c\right) + a}}\,dx$$

input
```
integrate(1/(a+I*a*sinh(d*x+c))^(1/2),x, algorithm="giac")
```

output
```
integrate(1/sqrt(I*a*sinh(d*x + c) + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt{a + ia\sinh(c + dx)}}\,dx = \int \frac{1}{\sqrt{a + a\sinh\left(c + d\,x\right)\,1\mathrm{i}}}\,dx$$

input
```
int(1/(a + a*sinh(c + d*x)*1i)^(1/2),x)
```

output
```
int(1/(a + a*sinh(c + d*x)*1i)^(1/2), x)
```

Reduce [F]

$$\int \frac{1}{\sqrt{a+ia\sinh(c+dx)}}\,dx = \frac{\sqrt{a}\left(\int \frac{\sqrt{\sinh(dx+c)i+1}}{\sinh(dx+c)^2+1}dx - \left(\int \frac{\sqrt{\sinh(dx+c)i+1}\,\sinh(dx+c)}{\sinh(dx+c)^2+1}dx\right)i\right)}{a}$$

input
```
int(1/(a+I*a*sinh(d*x+c))^(1/2),x)
```

output
```
(sqrt(a)*(int(sqrt(sinh(c + d*x)*i + 1)/(sinh(c + d*x)**2 + 1),x) - int((s
qrt(sinh(c + d*x)*i + 1)*sinh(c + d*x))/(sinh(c + d*x)**2 + 1),x)*i))/a
```

3.70 $\int \frac{1}{(a+ia\sinh(c+dx))^{3/2}}\,dx$

Optimal result

Integrand size = 17, antiderivative size = 87

$$\int \frac{1}{(a+ia\sinh(c+dx))^{3/2}}\,dx=\frac{i\text{arctanh}\left(\frac{\sqrt{a}\cosh(c+dx)}{\sqrt{2}\sqrt{a+ia\sinh(c+dx)}}\right)}{2\sqrt{2}a^{3/2}d}+\frac{i\cosh(c+dx)}{2d(a+ia\sinh(c+dx))^{3/2}}$$

output
```
1/4*I*arctanh(1/2*a^(1/2)*cosh(d*x+c)*2^(1/2)/(a+I*a*sinh(d*x+c))^(1/2))*2
^(1/2)/a^(3/2)/d+1/2*I*cosh(d*x+c)/d/(a+I*a*sinh(d*x+c))^(3/2)
```

Mathematica [A] (verified)

Time = 0.26 (sec) , antiderivative size = 156, normalized size of antiderivative = 1.79

$$\int \frac{1}{(a+ia\sinh(c+dx))^{3/2}}\,dx=\frac{\left(\cosh\left(\frac{1}{2}(c+dx)\right)+i\sinh\left(\frac{1}{2}(c+dx)\right)\right)\left(\cosh\left(\frac{1}{2}(c+dx)\right)-i\left((1-i)\sqrt[4]{-\cdots}\right.\right.}{2a\cdots}$$

input
```
Integrate[(a + I*a*Sinh[c + d*x])^(-3/2),x]
```

output
```
((Cosh[(c + d*x)/2] + I*Sinh[(c + d*x)/2])*(Cosh[(c + d*x)/2] - I*((1 - I)
*(-1)^(1/4)*ArcTan[(1/2 + I/2)*(-1)^(1/4)*(1 - I*Tanh[(c + d*x)/4])]*(Cosh
[(c + d*x)/2] + I*Sinh[(c + d*x)/2])^2 + Sinh[(c + d*x)/2])))/(2*a*d*(-I +
 Sinh[c + d*x])*Sqrt[a + I*a*Sinh[c + d*x]])
```

Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 87, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.294$, Rules used = {3042, 3129, 3042, 3128, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(a+ia\sinh(c+dx))^{3/2}}dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \frac{1}{(a+a\sin(ic+idx))^{3/2}}dx$$

$$\Big\downarrow \text{ 3129}$$

$$\frac{\int \frac{1}{\sqrt{i\sinh(c+dx)a+a}}dx}{4a}+\frac{i\cosh(c+dx)}{2d(a+ia\sinh(c+dx))^{3/2}}$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{\int \frac{1}{\sqrt{\sin(ic+idx)a+a}}dx}{4a}+\frac{i\cosh(c+dx)}{2d(a+ia\sinh(c+dx))^{3/2}}$$

$$\Big\downarrow \text{ 3128}$$

$$\frac{i\int \frac{1}{2a-\frac{a^2\cosh^2(c+dx)}{i\sinh(c+dx)a+a}}d\frac{a\cosh(c+dx)}{\sqrt{i\sinh(c+dx)a+a}}}{2ad}+\frac{i\cosh(c+dx)}{2d(a+ia\sinh(c+dx))^{3/2}}$$

$$\Big\downarrow \text{ 219}$$

$$\frac{i\text{arctanh}\left(\frac{\sqrt{a}\cosh(c+dx)}{\sqrt{2}\sqrt{a+ia\sinh(c+dx)}}\right)}{2\sqrt{2}a^{3/2}d}+\frac{i\cosh(c+dx)}{2d(a+ia\sinh(c+dx))^{3/2}}$$

input
```
Int[(a + I*a*Sinh[c + d*x])^(-3/2),x]
```

output
```
((I/2)*ArcTanh[(Sqrt[a]*Cosh[c + d*x])/(Sqrt[2]*Sqrt[a + I*a*Sinh[c + d*x]
])])/(Sqrt[2]*a^(3/2)*d) + ((I/2)*Cosh[c + d*x])/(d*(a + I*a*Sinh[c + d*x]
)^(3/2))
```

Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3128
```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[-2/d
Subst[Int[1/(2*a - x^2), x], x, b*(Cos[c + d*x]/Sqrt[a + b*Sin[c + d*x]])],
 x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]
```

rule 3129
```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[b*Cos[c
+ d*x]*((a + b*Sin[c + d*x])^n/(a*d*(2*n + 1))), x] + Simp[(n + 1)/(a*(2*n
+ 1))   Int[(a + b*Sin[c + d*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d}, x]
&& EqQ[a^2 - b^2, 0] && LtQ[n, -1] && IntegerQ[2*n]
```

Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 175 vs. $2(68) = 136$.

Time = 0.26 (sec) , antiderivative size = 176, normalized size of antiderivative = 2.02

method	result	size
default	$-\frac{\sqrt{\sinh\left(\frac{c}{2}+\frac{i\pi}{4}+\frac{dx}{2}\right)^{2}a}\,\left(\ln\left(\frac{2\sqrt{-a}\,\sqrt{\sinh\left(\frac{c}{2}+\frac{i\pi}{4}+\frac{dx}{2}\right)^{2}a}-2a}{\cosh\left(\frac{c}{2}+\frac{i\pi}{4}+\frac{dx}{2}\right)}\right)a\cosh\left(\frac{c}{2}+\frac{i\pi}{4}+\frac{dx}{2}\right)^{2}-\sqrt{-a}\,\sqrt{\sinh\left(\frac{c}{2}+\frac{i\pi}{4}+\frac{dx}{2}\right)^{2}a}\right)\sqrt{2}}{4a^{2}\cosh\left(\frac{c}{2}+\frac{i\pi}{4}+\frac{dx}{2}\right)\sqrt{-a}\,\sinh\left(\frac{c}{2}+\frac{i\pi}{4}+\frac{dx}{2}\right)\sqrt{a\cosh\left(\frac{c}{2}+\frac{i\pi}{4}+\frac{dx}{2}\right)^{2}}\,d}$	176

input
```
int(1/(a+I*a*sinh(d*x+c))^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-1/4*(sinh(1/2*c+1/4*I*Pi+1/2*d*x)^2*a)^(1/2)*(ln(2*((-a)^(1/2)*(sinh(1/2*
c+1/4*I*Pi+1/2*d*x)^2*a)^(1/2)-a)/cosh(1/2*c+1/4*I*Pi+1/2*d*x))*a*cosh(1/2
*c+1/4*I*Pi+1/2*d*x)^2-(-a)^(1/2)*(sinh(1/2*c+1/4*I*Pi+1/2*d*x)^2*a)^(1/2)
)/a^2/cosh(1/2*c+1/4*I*Pi+1/2*d*x)/(-a)^(1/2)/sinh(1/2*c+1/4*I*Pi+1/2*d*x)
*2^(1/2)/(a*cosh(1/2*c+1/4*I*Pi+1/2*d*x)^2)^(1/2)/d
```

Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 235 vs. $2(64) = 128$.

Time = 0.09 (sec) , antiderivative size = 235, normalized size of antiderivative = 2.70

$$\int \frac{1}{(a+ia\sinh(c+dx))^{3/2}}\,dx = \frac{\sqrt{\frac{1}{2}}\left(i\,a^{2}de^{(2\,dx+2\,c)}+2\,a^{2}de^{(dx+c)}-i\,a^{2}d\right)\sqrt{\frac{1}{a^{3}d^{2}}}\log\left(\sqrt{\frac{1}{2}}a^{2}d\sqrt{\frac{1}{a^{3}d^{2}}}+\sqrt{\frac{1}{2}\cdots}\right.}{}$$

input
```
integrate(1/(a+I*a*sinh(d*x+c))^(3/2),x, algorithm="fricas")
```

output
```
1/2*(sqrt(1/2)*(I*a^2*d*e^(2*d*x + 2*c) + 2*a^2*d*e^(d*x + c) - I*a^2*d)*s
qrt(1/(a^3*d^2))*log(sqrt(1/2)*a^2*d*sqrt(1/(a^3*d^2)) + sqrt(1/2*I*a*e^(-
d*x - c))) + sqrt(1/2)*(-I*a^2*d*e^(2*d*x + 2*c) - 2*a^2*d*e^(d*x + c) + I
*a^2*d)*sqrt(1/(a^3*d^2))*log(-sqrt(1/2)*a^2*d*sqrt(1/(a^3*d^2)) + sqrt(1/
2*I*a*e^(-d*x - c))) - 2*sqrt(1/2*I*a*e^(-d*x - c))*(I*e^(2*d*x + 2*c) - e
^(d*x + c)))/(a^2*d*e^(2*d*x + 2*c) - 2*I*a^2*d*e^(d*x + c) - a^2*d)
```

Sympy [F]

$$\int \frac{1}{(a+ia\sinh(c+dx))^{3/2}}\,dx = \int \frac{1}{(ia\sinh{(c+dx)}+a)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/(a+I*a*sinh(d*x+c))**(3/2),x)
```

output
```
Integral((I*a*sinh(c + d*x) + a)**(-3/2), x)
```

Maxima [F]

$$\int \frac{1}{(a+ia\sinh(c+dx))^{3/2}}\,dx = \int \frac{1}{(i\,a\sinh{(dx+c)}+a)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/(a+I*a*sinh(d*x+c))^(3/2),x, algorithm="maxima")
```

output
```
integrate((I*a*sinh(d*x + c) + a)^(-3/2), x)
```

Giac [F]

$$\int \frac{1}{(a+ia\sinh(c+dx))^{3/2}}\,dx = \int \frac{1}{(i\,a\sinh{(dx+c)}+a)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/(a+I*a*sinh(d*x+c))^(3/2),x, algorithm="giac")
```

output
```
integrate((I*a*sinh(d*x + c) + a)^(-3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(a+ia\sinh(c+dx))^{3/2}}\,dx = \int \frac{1}{(a+a\sinh\left(c+d\,x\right)\,1\mathrm{i})^{3/2}}\,dx$$

input
```
int(1/(a + a*sinh(c + d*x)*1i)^(3/2),x)
```

output
```
int(1/(a + a*sinh(c + d*x)*1i)^(3/2), x)
```

Reduce [F]

$$\int \frac{1}{(a+ia\sinh(c+dx))^{3/2}}\,dx = \frac{\sqrt{a}\left(\int \frac{\sqrt{\sinh(dx+c)i+1}}{\sinh(dx+c)^3i+\sinh(dx+c)^2+\sinh(dx+c)i+1}dx - \left(\int \frac{\sqrt{\sinh(dx+c)i+1}\,\sinh(dx}{\sinh(dx+c)^3i+\sinh(dx+c)^2+\sinh}\right.\right.}{a^2}$$

input
```
int(1/(a+I*a*sinh(d*x+c))^(3/2),x)
```

output
```
(sqrt(a)*(int(sqrt(sinh(c + d*x)*i + 1)/(sinh(c + d*x)**3*i + sinh(c + d*x
)**2 + sinh(c + d*x)*i + 1),x) - int((sqrt(sinh(c + d*x)*i + 1)*sinh(c + d
*x))/(sinh(c + d*x)**3*i + sinh(c + d*x)**2 + sinh(c + d*x)*i + 1),x)*i))/
a**2
```

3.71 $\int \frac{1}{(a+ia\sinh(c+dx))^{5/2}}\,dx$

Optimal result

Integrand size = 17, antiderivative size = 122

$$\int \frac{1}{(a+ia\sinh(c+dx))^{5/2}}\,dx = \frac{3i\operatorname{arctanh}\left(\frac{\sqrt{a}\cosh(c+dx)}{\sqrt{2}\sqrt{a+ia\sinh(c+dx)}}\right)}{16\sqrt{2}a^{5/2}d} + \frac{i\cosh(c+dx)}{4d(a+ia\sinh(c+dx))^{5/2}} + \frac{3i\cosh(c+dx)}{16ad(a+ia\sinh(c+dx))^{3/2}}$$

output
```
3/32*I*arctanh(1/2*a^(1/2)*cosh(d*x+c)*2^(1/2)/(a+I*a*sinh(d*x+c))^(1/2))*
2^(1/2)/a^(5/2)/d+1/4*I*cosh(d*x+c)/d/(a+I*a*sinh(d*x+c))^(5/2)+3/16*I*cos
h(d*x+c)/a/d/(a+I*a*sinh(d*x+c))^(3/2)
```

Mathematica [A] (verified)

Time = 0.24 (sec) , antiderivative size = 210, normalized size of antiderivative = 1.72

$$\int \frac{1}{(a+ia\sinh(c+dx))^{5/2}}\,dx = \frac{\left(\cosh\left(\frac{1}{2}(c+dx)\right)+i\sinh\left(\frac{1}{2}(c+dx)\right)\right)\left(4i\cosh\left(\frac{1}{2}(c+dx)\right)+(3-3i)\sqrt[4]{\cdots}\right.}{\cdots}$$

input
```
Integrate[(a + I*a*Sinh[c + d*x])^(-5/2),x]
```

output
```
((Cosh[(c + d*x)/2] + I*Sinh[(c + d*x)/2])*((4*I)*Cosh[(c + d*x)/2] + (3 -
 3*I)*(-1)^(1/4)*ArcTan[(1/2 + I/2)*(-1)^(1/4)*(1 - I*Tanh[(c + d*x)/4])]*
(Cosh[(c + d*x)/2] + I*Sinh[(c + d*x)/2])^4 + 4*Sinh[(c + d*x)/2] + 6*(Cos
h[(c + d*x)/2] + I*Sinh[(c + d*x)/2])^2*Sinh[(c + d*x)/2] + 3*((-I)*Cosh[(
c + d*x)/2] + Sinh[(c + d*x)/2])^3))/(16*d*(a + I*a*Sinh[c + d*x])^(5/2))
```

Rubi [A] (verified)

Time = 0.39 (sec) , antiderivative size = 127, normalized size of antiderivative = 1.04, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.412$, Rules used = {3042, 3129, 3042, 3129, 3042, 3128, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(a+ia\sinh(c+dx))^{5/2}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{(a+a\sin(ic+idx))^{5/2}}dx$$

$$\Big\downarrow \text{3129}$$

$$\frac{3\int \frac{1}{(i\sinh(c+dx)a+a)^{3/2}}dx}{8a}+\frac{i\cosh(c+dx)}{4d(a+ia\sinh(c+dx))^{5/2}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{3\int \frac{1}{(\sin(ic+idx)a+a)^{3/2}}dx}{8a}+\frac{i\cosh(c+dx)}{4d(a+ia\sinh(c+dx))^{5/2}}$$

$$\Big\downarrow \text{3129}$$

$$\frac{3\left(\frac{\int \frac{1}{\sqrt{i\sinh(c+dx)a+a}}dx}{4a}+\frac{i\cosh(c+dx)}{2d(a+ia\sinh(c+dx))^{3/2}}\right)}{8a}+\frac{i\cosh(c+dx)}{4d(a+ia\sinh(c+dx))^{5/2}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{3\left(\frac{\int \frac{1}{\sqrt{\sin(ic+idx)a+a}}dx}{4a}+\frac{i\cosh(c+dx)}{2d(a+ia\sinh(c+dx))^{3/2}}\right)}{8a}+\frac{i\cosh(c+dx)}{4d(a+ia\sinh(c+dx))^{5/2}}$$

$$\Big\downarrow \text{3128}$$

$$\frac{3\left(\frac{i\int \frac{1}{2a-\frac{a^2\cosh^2(c+dx)}{i\sinh(c+dx)a+a}}d\frac{a\cosh(c+dx)}{\sqrt{i\sinh(c+dx)a+a}}}{2ad}+\frac{i\cosh(c+dx)}{2d(a+ia\sinh(c+dx))^{3/2}}\right)}{8a}+\frac{i\cosh(c+dx)}{4d(a+ia\sinh(c+dx))^{5/2}}$$

$$\Big\downarrow \text{219}$$

$$\frac{3\left(\frac{i\text{arctanh}\left(\frac{\sqrt{a}\cosh(c+dx)}{\sqrt{2}\sqrt{a+ia\sinh(c+dx)}}\right)}{2\sqrt{2}a^{3/2}d}+\frac{i\cosh(c+dx)}{2d(a+ia\sinh(c+dx))^{3/2}}\right)}{8a}+\frac{i\cosh(c+dx)}{4d(a+ia\sinh(c+dx))^{5/2}}$$

input
```
Int[(a + I*a*Sinh[c + d*x])^(-5/2),x]
```

output
```
((I/4)*Cosh[c + d*x])/(d*(a + I*a*Sinh[c + d*x])^(5/2)) + (3*(((I/2)*ArcTa
nh[(Sqrt[a]*Cosh[c + d*x])/(Sqrt[2]*Sqrt[a + I*a*Sinh[c + d*x]])])/(Sqrt[2
]*a^(3/2)*d) + ((I/2)*Cosh[c + d*x])/(d*(a + I*a*Sinh[c + d*x])^(3/2))))/(
8*a)
```

Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3128
```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[-2/d
Subst[Int[1/(2*a - x^2), x], x, b*(Cos[c + d*x]/Sqrt[a + b*Sin[c + d*x]])],
 x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]
```

rule 3129
```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[b*Cos[c
+ d*x]*((a + b*Sin[c + d*x])^n/(a*d*(2*n + 1))), x] + Simp[(n + 1)/(a*(2*n
+ 1))   Int[(a + b*Sin[c + d*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d}, x]
&& EqQ[a^2 - b^2, 0] && LtQ[n, -1] && IntegerQ[2*n]
```

Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 217 vs. 2(97) = 194.

Time = 0.27 (sec) , antiderivative size = 218, normalized size of antiderivative = 1.79

method	result
default	$-\frac{\sqrt{\sinh\left(\frac{c}{2}+\frac{i\pi}{4}+\frac{dx}{2}\right)^{2} a}\left(3\ln\left(\frac{2\sqrt{-a}\sqrt{\sinh\left(\frac{c}{2}+\frac{i\pi}{4}+\frac{dx}{2}\right)^{2} a}-2a}{\cosh\left(\frac{c}{2}+\frac{i\pi}{4}+\frac{dx}{2}\right)}\right) a\cosh\left(\frac{c}{2}+\frac{i\pi}{4}+\frac{dx}{2}\right)^{4}-3\sqrt{\sinh\left(\frac{c}{2}+\frac{i\pi}{4}+\frac{dx}{2}\right)^{2} a}\, \cosh\left(\frac{c}{2}+\frac{i\pi}{4}+\frac{dx}{2}\right.}{32a^{3}\cosh\left(\frac{c}{2}+\frac{i\pi}{4}+\frac{dx}{2}\right)^{3}\sqrt{-a}\,\sinh\left(\frac{c}{2}+\frac{i\pi}{4}+\frac{dx}{2}\right)\sqrt{a\cosh\left(\frac{c}{2}+\frac{i\pi}{4}+\frac{dx}{2}\right)^{2}}\, d}$

input
```
int(1/(a+I*a*sinh(d*x+c))^(5/2),x,method=_RETURNVERBOSE)
```

output
```
-1/32*(sinh(1/2*c+1/4*I*Pi+1/2*d*x)^2*a)^(1/2)*(3*ln(2*((-a)^(1/2)*(sinh(1
/2*c+1/4*I*Pi+1/2*d*x)^2*a)^(1/2)-a)/cosh(1/2*c+1/4*I*Pi+1/2*d*x))*a*cosh(
1/2*c+1/4*I*Pi+1/2*d*x)^4-3*(sinh(1/2*c+1/4*I*Pi+1/2*d*x)^2*a)^(1/2)*cosh(
1/2*c+1/4*I*Pi+1/2*d*x)^2*(-a)^(1/2)-2*(-a)^(1/2)*(sinh(1/2*c+1/4*I*Pi+1/2
*d*x)^2*a)^(1/2))/a^3/cosh(1/2*c+1/4*I*Pi+1/2*d*x)^3/(-a)^(1/2)/sinh(1/2*c
+1/4*I*Pi+1/2*d*x)*2^(1/2)/(a*cosh(1/2*c+1/4*I*Pi+1/2*d*x)^2)^(1/2)/d
```

Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 348 vs. 2(91) = 182.

Time = 0.10 (sec) , antiderivative size = 348, normalized size of antiderivative = 2.85

$$\int \frac{1}{(a+ia\sinh(c+dx))^{5/2}}\,dx = -\frac{3\sqrt{\frac{1}{2}}\left(-i\,a^{3}de^{(4\,dx+4\,c)}-4\,a^{3}de^{(3\,dx+3\,c)}+6i\,a^{3}de^{(2\,dx+2\,c)}+4\,a^{3}de^{(dx+c)}-i\,a^{3}d\right)\sqrt{\frac{1}{a^{5}d^{2}}}\log\left(\sqrt{\frac{1}{2}}a^{3}d\sqrt{\frac{1}{a^{5}d^{2}}}\right.}{}$$

input
```
integrate(1/(a+I*a*sinh(d*x+c))^(5/2),x, algorithm="fricas")
```

output
```
-1/16*(3*sqrt(1/2)*(-I*a^3*d*e^(4*d*x + 4*c) - 4*a^3*d*e^(3*d*x + 3*c) + 6
*I*a^3*d*e^(2*d*x + 2*c) + 4*a^3*d*e^(d*x + c) - I*a^3*d)*sqrt(1/(a^5*d^2)
)*log(sqrt(1/2)*a^3*d*sqrt(1/(a^5*d^2)) + sqrt(1/2*I*a*e^(-d*x - c))) + 3*
sqrt(1/2)*(I*a^3*d*e^(4*d*x + 4*c) + 4*a^3*d*e^(3*d*x + 3*c) - 6*I*a^3*d*e
^(2*d*x + 2*c) - 4*a^3*d*e^(d*x + c) + I*a^3*d)*sqrt(1/(a^5*d^2))*log(-sqr
t(1/2)*a^3*d*sqrt(1/(a^5*d^2)) + sqrt(1/2*I*a*e^(-d*x - c))) - 2*sqrt(1/2*
I*a*e^(-d*x - c))*(-3*I*e^(4*d*x + 4*c) - 11*e^(3*d*x + 3*c) - 11*I*e^(2*d
*x + 2*c) - 3*e^(d*x + c)))/(a^3*d*e^(4*d*x + 4*c) - 4*I*a^3*d*e^(3*d*x +
3*c) - 6*a^3*d*e^(2*d*x + 2*c) + 4*I*a^3*d*e^(d*x + c) + a^3*d)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{1}{(a+ia\sinh(c+dx))^{5/2}}\,dx = \text{Timed out}$$

input
```
integrate(1/(a+I*a*sinh(d*x+c))**(5/2),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{1}{(a+ia\sinh(c+dx))^{5/2}}\,dx = \int \frac{1}{(i\,a\sinh(dx+c)+a)^{\frac{5}{2}}}\,dx$$

input
```
integrate(1/(a+I*a*sinh(d*x+c))^(5/2),x, algorithm="maxima")
```

output
```
integrate((I*a*sinh(d*x + c) + a)^(-5/2), x)
```

Giac [F]

$$\int \frac{1}{(a+ia\sinh(c+dx))^{5/2}}\,dx = \int \frac{1}{(i\,a\sinh\left(dx+c\right)+a)^{\frac{5}{2}}}\,dx$$

input
```
integrate(1/(a+I*a*sinh(d*x+c))^(5/2),x, algorithm="giac")
```

output
```
integrate((I*a*sinh(d*x + c) + a)^(-5/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(a+ia\sinh(c+dx))^{5/2}}\,dx = \int \frac{1}{{\left(a+a\,\mathrm{sinh}\left(c+d\,x\right)\,1{}\mathrm{i}\right)}^{5/2}}\,dx$$

input
```
int(1/(a + a*sinh(c + d*x)*1i)^(5/2),x)
```

output
```
int(1/(a + a*sinh(c + d*x)*1i)^(5/2), x)
```

Reduce [F]

$$\int \frac{1}{(a+ia\sinh(c+dx))^{5/2}}\,dx = -\frac{\int \frac{1}{\sqrt{\sinh(dx+c)i+1}\,\sinh(dx+c)^{2}-2\sqrt{\sinh(dx+c)i+1}\,\sinh(dx+c)i-\sqrt{\sinh(dx+c)i+1}}dx}{\sqrt{a}\,a^{2}}$$

input
```
int(1/(a+I*a*sinh(d*x+c))^(5/2),x)
```

output
```
( - int(1/(sqrt(sinh(c + d*x)*i + 1)*sinh(c + d*x)**2 - 2*sqrt(sinh(c + d*
x)*i + 1)*sinh(c + d*x)*i - sqrt(sinh(c + d*x)*i + 1)),x))/(sqrt(a)*a**2)
```

3.72 $\int \frac{\sinh^4(x)}{a+b\sinh(x)}\,dx$

Optimal result

Integrand size = 13, antiderivative size = 108

$$\int \frac{\sinh^4(x)}{a+b\sinh(x)}\,dx = -\frac{a(2a^2-b^2)\,x}{2b^4} - \frac{2a^4\text{arctanh}\left(\frac{b-a\tanh\left(\frac{x}{2}\right)}{\sqrt{a^2+b^2}}\right)}{b^4\sqrt{a^2+b^2}} - \frac{\left(2-\frac{3a^2}{b^2}\right)\cosh(x)}{3b} - \frac{a\cosh(x)\sinh(x)}{2b^2} + \frac{\cosh(x)\sinh^2(x)}{3b}$$

output
```
-1/2*a*(2*a^2-b^2)*x/b^4-2*a^4*arctanh((b-a*tanh(1/2*x))/(a^2+b^2)^(1/2))/
b^4/(a^2+b^2)^(1/2)-1/3*(2-3*a^2/b^2)*cosh(x)/b-1/2*a*cosh(x)*sinh(x)/b^2+
1/3*cosh(x)*sinh(x)^2/b
```

Mathematica [A] (verified)

Time = 1.54 (sec) , antiderivative size = 105, normalized size of antiderivative = 0.97

$$\int \frac{\sinh^4(x)}{a+b\sinh(x)}\,dx = \frac{3b(4a^2-3b^2)\cosh(x)+b^3\cosh(3x)+3a\left(-4a^2x+2b^2x+\frac{8a^3\arctan\left(\frac{b-a\tanh\left(\frac{x}{2}\right)}{\sqrt{-a^2-b^2}}\right)}{\sqrt{-a^2-b^2}}-b^2\sinh(2x)\right)}{12b^4}$$

input
```
Integrate[Sinh[x]^4/(a + b*Sinh[x]),x]
```

output
```
(3*b*(4*a^2 - 3*b^2)*Cosh[x] + b^3*Cosh[3*x] + 3*a*(-4*a^2*x + 2*b^2*x + (
8*a^3*ArcTan[(b - a*Tanh[x/2])/Sqrt[-a^2 - b^2]])/Sqrt[-a^2 - b^2] - b^2*S
inh[2*x]))/(12*b^4)
```

Rubi [C] (verified)

Result contains complex when optimal does not.

Time = 0.81 (sec) , antiderivative size = 138, normalized size of antiderivative = 1.28, number of steps used = 17, number of rules used = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 1.231$, Rules used = {3042, 3272, 26, 3042, 26, 3528, 25, 3042, 3502, 27, 3042, 3214, 3042, 3139, 1083, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sinh^4(x)}{a+b\sinh(x)}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\sin(ix)^4}{a-ib\sin(ix)}dx$$

$$\Big\downarrow \text{3272}$$

$$\frac{\sinh^2(x)\cosh(x)}{3b}+\frac{i\int \frac{i\sinh(x)\left(3a\sinh^2(x)+2b\sinh(x)+2a\right)}{a+b\sinh(x)}dx}{3b}$$

$$\Big\downarrow \text{26}$$

$$\frac{\sinh^2(x)\cosh(x)}{3b}-\frac{\int \frac{\sinh(x)\left(3a\sinh^2(x)+2b\sinh(x)+2a\right)}{a+b\sinh(x)}dx}{3b}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\sinh^2(x)\cosh(x)}{3b}-\frac{\int -\frac{i\sin(ix)\left(-3a\sin(ix)^2-2ib\sin(ix)+2a\right)}{a-ib\sin(ix)}dx}{3b}$$

$$\Big\downarrow \text{26}$$

$$\frac{\sinh^2(x)\cosh(x)}{3b}+\frac{i\int \frac{\sin(ix)\left(-3a\sin(ix)^2-2ib\sin(ix)+2a\right)}{a-ib\sin(ix)}dx}{3b}$$

$\Big\downarrow$ 3528

$$\frac{\sinh^2(x)\cosh(x)}{3b}+\frac{i\left(\frac{i\int -\frac{3a^2-b\sinh(x)a+2\left(3a^2-2b^2\right)\sinh^2(x)}{a+b\sinh(x)}dx}{2b}+\frac{3ia\sinh(x)\cosh(x)}{2b}\right)}{3b}$$

$\Big\downarrow$ 25

$$\frac{\sinh^2(x)\cosh(x)}{3b}+\frac{i\left(\frac{3ia\sinh(x)\cosh(x)}{2b}-\frac{i\int \frac{3a^2-b\sinh(x)a+2\left(3a^2-2b^2\right)\sinh^2(x)}{a+b\sinh(x)}dx}{2b}\right)}{3b}$$

$\Big\downarrow$ 3042

$$\frac{\sinh^2(x)\cosh(x)}{3b}+\frac{i\left(\frac{3ia\sinh(x)\cosh(x)}{2b}-\frac{i\int \frac{3a^2+ib\sin(ix)a-2\left(3a^2-2b^2\right)\sin(ix)^2}{a-ib\sin(ix)}dx}{2b}\right)}{3b}$$

$\Big\downarrow$ 3502

$$\frac{\sinh^2(x)\cosh(x)}{3b}+\frac{i\left(\frac{3ia\sinh(x)\cosh(x)}{2b}-\frac{i\left(\frac{2\left(3a^2-2b^2\right)\cosh(x)}{b}+\frac{i\int -\frac{3i\left(a^2b-a\left(2a^2-b^2\right)\sinh(x)\right)}{a+b\sinh(x)}dx}{b}\right)}{2b}\right)}{3b}$$

$\Big\downarrow$ 27

$$\frac{\sinh^2(x)\cosh(x)}{3b}+\frac{i\left(\frac{3ia\sinh(x)\cosh(x)}{2b}-\frac{i\left(\frac{3\int \frac{a^2b-a\left(2a^2-b^2\right)\sinh(x)}{a+b\sinh(x)}dx}{b}+\frac{2\left(3a^2-2b^2\right)\cosh(x)}{b}\right)}{2b}\right)}{3b}$$

$\Big\downarrow$ 3042

$$\frac{\sinh^2(x)\cosh(x)}{3b}+\frac{i\left(\frac{3ia\sinh(x)\cosh(x)}{2b}-\frac{i\left(\frac{2\left(3a^2-2b^2\right)\cosh(x)}{b}+\frac{3\int\frac{ba^2+i\left(2a^2-b^2\right)\sin(ix)a}{a-ib\sin(ix)}dx}{b}\right)}{2b}\right)}{3b}$$

$\Big\downarrow$ 3214

$$\frac{\sinh^2(x)\cosh(x)}{3b}+\frac{i\left(\frac{3ia\sinh(x)\cosh(x)}{2b}-\frac{i\left(\frac{3\left(\frac{2a^4\int\frac{1}{a+b\sinh(x)}dx}{b}-\frac{ax\left(2a^2-b^2\right)}{b}\right)}{b}+\frac{2\left(3a^2-2b^2\right)\cosh(x)}{b}\right)}{2b}\right)}{3b}$$

$\Big\downarrow$ 3042

$$\frac{\sinh^2(x)\cosh(x)}{3b}+\frac{i\left(\frac{3ia\sinh(x)\cosh(x)}{2b}-\frac{i\left(\frac{2\left(3a^2-2b^2\right)\cosh(x)}{b}+\frac{3\left(-\frac{ax\left(2a^2-b^2\right)}{b}+\frac{2a^4\int\frac{1}{a-ib\sin(ix)}dx}{b}\right)}{b}\right)}{2b}\right)}{3b}$$

$\Big\downarrow$ 3139

$$\frac{\sinh^2(x)\cosh(x)}{3b}+\frac{i\left(\frac{3ia\sinh(x)\cosh(x)}{2b}-\frac{i\left(\frac{3\left(\frac{4a^4\int\frac{1}{-a\tanh^2\left(\frac{x}{2}\right)+2b\tanh\left(\frac{x}{2}\right)+a}d\tanh\left(\frac{x}{2}\right)}{b}-\frac{ax\left(2a^2-b^2\right)}{b}\right)}{b}+\frac{2\left(3a^2-2b^2\right)\cosh(x)}{b}\right)}{2b}\right)}{3b}$$

$\Big\downarrow$ 1083

$$\frac{\sinh^2(x)\cosh(x)}{3b}+\frac{i\left(\frac{3ia\sinh(x)\cosh(x)}{2b}-\frac{i\left(\frac{3\left(-\frac{8a^4\int\frac{1}{4(a^2+b^2)-\left(2b-2a\tanh\left(\frac{x}{2}\right)\right)^2}d\left(2b-2a\tanh\left(\frac{x}{2}\right)\right)}{b}-\frac{ax(2a^2-b^2)}{b}\right)}{b}+\frac{2(3a^2-2b^2)\cosh(x)}{b}\right)}{2b}\right)}{3b}$$

$\downarrow$ 219

$$\frac{\sinh^2(x)\cosh(x)}{3b}+\frac{i\left(\frac{3ia\sinh(x)\cosh(x)}{2b}-\frac{i\left(\frac{2(3a^2-2b^2)\cosh(x)}{b}+\frac{3\left(-\frac{ax(2a^2-b^2)}{b}-\frac{4a^4\text{arctanh}\left(\frac{2b-2a\tanh\left(\frac{x}{2}\right)}{2\sqrt{a^2+b^2}}\right)}{b\sqrt{a^2+b^2}}\right)}{b}\right)}{2b}\right)}{3b}$$

input
```
Int[Sinh[x]^4/(a + b*Sinh[x]),x]
```

output
```
(Cosh[x]*Sinh[x]^2)/(3*b) + ((I/3)*(((-1/2*I)*((3*(-((a*(2*a^2 - b^2)*x)/b
) - (4*a^4*ArcTanh[(2*b - 2*a*Tanh[x/2])/(2*Sqrt[a^2 + b^2])])/(b*Sqrt[a^2
 + b^2])))/b + (2*(3*a^2 - 2*b^2)*Cosh[x])/b))/b + (((3*I)/2)*a*Cosh[x]*Si
nh[x])/b))/b
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1083
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Simp[-2   Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3139
```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Simp[2*(e/d)   Subst[Int[1/(a + 2*b*e*x + a
*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[a^2 - b^2, 0]
```

rule 3214
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])/((c_.) + (d_.)*sin[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[b*(x/d), x] - Simp[(b*c - a*d)/d   Int[1/(c + d
*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

rule 3272
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b^2)*Cos[e + f*x]*(a + b*Sin[e + f*
x])^(m - 2)*((c + d*Sin[e + f*x])^(n + 1)/(d*f*(m + n))), x] + Simp[1/(d*(m
 + n))   Int[(a + b*Sin[e + f*x])^(m - 3)*(c + d*Sin[e + f*x])^n*Simp[a^3*d
*(m + n) + b^2*(b*c*(m - 2) + a*d*(n + 1)) - b*(a*b*c - b^2*d*(m + n - 1) -
 3*a^2*d*(m + n))*Sin[e + f*x] - b^2*(b*c*(m - 1) - a*d*(3*m + 2*n - 2))*Si
n[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*
d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 2] && (IntegerQ[m
] || IntegersQ[2*m, 2*n]) &&  !(IGtQ[n, 2] && ( !IntegerQ[m] || (EqQ[a, 0]
&& NeQ[c, 0])))
```

rule 3502
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-C)*Co
s[e + f*x]*((a + b*Sin[e + f*x])^(m + 1)/(b*f*(m + 2))), x] + Simp[1/(b*(m
+ 2))   Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m
 + 2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x]
 &&  !LtQ[m, -1]
```

rule 3528
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*sin[(e_.)
+ (f_.)*(x_)])^(n_.)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_
.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-C)*Cos[e + f*x]*(a + b*Sin[e + f*x
])^m*((c + d*Sin[e + f*x])^(n + 1)/(d*f*(m + n + 2))), x] + Simp[1/(d*(m +
n + 2))   Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^n*Simp[a*A*
d*(m + n + 2) + C*(b*c*m + a*d*(n + 1)) + (d*(A*b + a*B)*(m + n + 2) - C*(a
*c - b*d*(m + n + 1)))*Sin[e + f*x] + (C*(a*d*m - b*c*(m + 1)) + b*B*d*(m +
 n + 2))*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, n}
, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[
m, 0] &&  !(IGtQ[n, 0] && ( !IntegerQ[m] || (EqQ[a, 0] && NeQ[c, 0])))
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 200 vs. 2(94) = 188.

Time = 0.40 (sec) , antiderivative size = 201, normalized size of antiderivative = 1.86

method	result
risch	$-\frac{a^{3}x}{b^{4}}+\frac{ax}{2b^{2}}+\frac{e^{3x}}{24b}-\frac{a\,e^{2x}}{8b^{2}}+\frac{e^{x}a^{2}}{2b^{3}}-\frac{3\,e^{x}}{8b}+\frac{e^{-x}a^{2}}{2b^{3}}-\frac{3\,e^{-x}}{8b}+\frac{a\,e^{-2x}}{8b^{2}}+\frac{e^{-3x}}{24b}+\frac{a^{4}\ln\left(e^{x}+\frac{a\sqrt{a^{2}+b^{2}}-a^{2}-b^{2}}{\sqrt{a^{2}+b^{2}}\,b}\right)}{\sqrt{a^{2}+b^{2}}\,b^{4}}-$
default	$-\frac{1}{3b\left(\tanh\left(\frac{x}{2}\right)-1\right)^{3}}-\frac{a+b}{2b^{2}\left(\tanh\left(\frac{x}{2}\right)-1\right)^{2}}-\frac{2a^{2}+ab-b^{2}}{2b^{3}\left(\tanh\left(\frac{x}{2}\right)-1\right)}+\frac{a\left(2a^{2}-b^{2}\right)\ln\left(\tanh\left(\frac{x}{2}\right)-1\right)}{2b^{4}}+\frac{2a^{4}\operatorname{arctanh}\left(\frac{2a\tanh\left(\frac{x}{2}\right)-2b}{2\sqrt{a^{2}+b^{2}}}\right)}{b^{4}\sqrt{a^{2}+b^{2}}}$

input
```
int(sinh(x)^4/(a+b*sinh(x)),x,method=_RETURNVERBOSE)
```

output
```
-a^3*x/b^4+1/2*a*x/b^2+1/24/b*exp(x)^3-1/8*a/b^2*exp(x)^2+1/2/b^3*exp(x)*a
^2-3/8/b*exp(x)+1/2/b^3/exp(x)*a^2-3/8/b/exp(x)+1/8*a/b^2/exp(x)^2+1/24/b/
exp(x)^3+1/(a^2+b^2)^(1/2)*a^4/b^4*ln(exp(x)+(a*(a^2+b^2)^(1/2)-a^2-b^2)/(
a^2+b^2)^(1/2)/b)-1/(a^2+b^2)^(1/2)*a^4/b^4*ln(exp(x)+(a*(a^2+b^2)^(1/2)+a
^2+b^2)/(a^2+b^2)^(1/2)/b)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 799 vs. 2(96) = 192.

Time = 0.10 (sec) , antiderivative size = 799, normalized size of antiderivative = 7.40

$$\int \frac{\sinh^4(x)}{a+b\sinh(x)}\,dx = \text{Too large to display}$$

input
```
integrate(sinh(x)^4/(a+b*sinh(x)),x, algorithm="fricas")
```

output
```
1/24*((a^2*b^3 + b^5)*cosh(x)^6 + (a^2*b^3 + b^5)*sinh(x)^6 - 3*(a^3*b^2 +
 a*b^4)*cosh(x)^5 - 3*(a^3*b^2 + a*b^4 - 2*(a^2*b^3 + b^5)*cosh(x))*sinh(x
)^5 + a^2*b^3 + b^5 - 12*(2*a^5 + a^3*b^2 - a*b^4)*x*cosh(x)^3 + 3*(4*a^4*
b + a^2*b^3 - 3*b^5)*cosh(x)^4 + 3*(4*a^4*b + a^2*b^3 - 3*b^5 + 5*(a^2*b^3
 + b^5)*cosh(x)^2 - 5*(a^3*b^2 + a*b^4)*cosh(x))*sinh(x)^4 + 2*(10*(a^2*b^
3 + b^5)*cosh(x)^3 - 15*(a^3*b^2 + a*b^4)*cosh(x)^2 - 6*(2*a^5 + a^3*b^2 -
 a*b^4)*x + 6*(4*a^4*b + a^2*b^3 - 3*b^5)*cosh(x))*sinh(x)^3 + 3*(4*a^4*b
+ a^2*b^3 - 3*b^5)*cosh(x)^2 + 3*(4*a^4*b + a^2*b^3 - 3*b^5 + 5*(a^2*b^3 +
 b^5)*cosh(x)^4 - 10*(a^3*b^2 + a*b^4)*cosh(x)^3 - 12*(2*a^5 + a^3*b^2 - a
*b^4)*x*cosh(x) + 6*(4*a^4*b + a^2*b^3 - 3*b^5)*cosh(x)^2)*sinh(x)^2 + 24*
(a^4*cosh(x)^3 + 3*a^4*cosh(x)^2*sinh(x) + 3*a^4*cosh(x)*sinh(x)^2 + a^4*s
inh(x)^3)*sqrt(a^2 + b^2)*log((b^2*cosh(x)^2 + b^2*sinh(x)^2 + 2*a*b*cosh(
x) + 2*a^2 + b^2 + 2*(b^2*cosh(x) + a*b)*sinh(x) - 2*sqrt(a^2 + b^2)*(b*co
sh(x) + b*sinh(x) + a))/(b*cosh(x)^2 + b*sinh(x)^2 + 2*a*cosh(x) + 2*(b*co
sh(x) + a)*sinh(x) - b)) + 3*(a^3*b^2 + a*b^4)*cosh(x) + 3*(2*(a^2*b^3 + b
^5)*cosh(x)^5 + a^3*b^2 + a*b^4 - 5*(a^3*b^2 + a*b^4)*cosh(x)^4 - 12*(2*a^
5 + a^3*b^2 - a*b^4)*x*cosh(x)^2 + 4*(4*a^4*b + a^2*b^3 - 3*b^5)*cosh(x)^3
 + 2*(4*a^4*b + a^2*b^3 - 3*b^5)*cosh(x))*sinh(x))/((a^2*b^4 + b^6)*cosh(x
)^3 + 3*(a^2*b^4 + b^6)*cosh(x)^2*sinh(x) + 3*(a^2*b^4 + b^6)*cosh(x)*sinh
(x)^2 + (a^2*b^4 + b^6)*sinh(x)^3)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\sinh^4(x)}{a + b\sinh(x)}\,dx = \text{Timed out}$$

input
```
integrate(sinh(x)**4/(a+b*sinh(x)),x)
```

output
```
Timed out
```

Maxima [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 158, normalized size of antiderivative = 1.46

$$\int \frac{\sinh^4(x)}{a+b\sinh(x)}\,dx = \frac{a^4\log\left(\frac{be^{(-x)}-a-\sqrt{a^2+b^2}}{be^{(-x)}-a+\sqrt{a^2+b^2}}\right)}{\sqrt{a^2+b^2}b^4} - \frac{\left(3\,abe^{(-x)}-b^2-3\,(4\,a^2-3\,b^2)e^{(-2\,x)}\right)e^{(3\,x)}}{24\,b^3} + \frac{3\,abe^{(-2\,x)}+b^2e^{(-3\,x)}+3\,(4\,a^2-3\,b^2)e^{(-x)}}{24\,b^3} - \frac{(2\,a^3-ab^2)x}{2\,b^4}$$

input
```
integrate(sinh(x)^4/(a+b*sinh(x)),x, algorithm="maxima")
```

output
```
a^4*log((b*e^(-x) - a - sqrt(a^2 + b^2))/(b*e^(-x) - a + sqrt(a^2 + b^2)))
/(sqrt(a^2 + b^2)*b^4) - 1/24*(3*a*b*e^(-x) - b^2 - 3*(4*a^2 - 3*b^2)*e^(-
2*x))*e^(3*x)/b^3 + 1/24*(3*a*b*e^(-2*x) + b^2*e^(-3*x) + 3*(4*a^2 - 3*b^2
)*e^(-x))/b^3 - 1/2*(2*a^3 - a*b^2)*x/b^4
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 156, normalized size of antiderivative = 1.44

$$\int \frac{\sinh^4(x)}{a+b\sinh(x)}\,dx = \frac{a^4\log\left(\frac{\left|2\,be^x+2\,a-2\,\sqrt{a^2+b^2}\right|}{\left|2\,be^x+2\,a+2\,\sqrt{a^2+b^2}\right|}\right)}{\sqrt{a^2+b^2}b^4} + \frac{b^2e^{(3\,x)}-3\,abe^{(2\,x)}+12\,a^2e^x-9\,b^2e^x}{24\,b^3} - \frac{(2\,a^3-ab^2)x}{2\,b^4} + \frac{\left(3\,ab^2e^x+b^3+3\,(4\,a^2b-3\,b^3)e^{(2\,x)}\right)e^{(-3\,x)}}{24\,b^4}$$

input
```
integrate(sinh(x)^4/(a+b*sinh(x)),x, algorithm="giac")
```

output
```
a^4*log(abs(2*b*e^x + 2*a - 2*sqrt(a^2 + b^2))/abs(2*b*e^x + 2*a + 2*sqrt(
a^2 + b^2)))/(sqrt(a^2 + b^2)*b^4) + 1/24*(b^2*e^(3*x) - 3*a*b*e^(2*x) + 1
2*a^2*e^x - 9*b^2*e^x)/b^3 - 1/2*(2*a^3 - a*b^2)*x/b^4 + 1/24*(3*a*b^2*e^x
 + b^3 + 3*(4*a^2*b - 3*b^3)*e^(2*x))*e^(-3*x)/b^4
```

Mupad [B] (verification not implemented)

Time = 2.11 (sec) , antiderivative size = 199, normalized size of antiderivative = 1.84

$$\int \frac{\sinh^4(x)}{a+b\sinh(x)}\,dx = \frac{e^{-3x}}{24b}+\frac{e^{3x}}{24b}+\frac{x\,(a\,b^2-2\,a^3)}{2\,b^4}+\frac{e^{x}\,(4\,a^2-3\,b^2)}{8\,b^3}+\frac{a\,e^{-2x}}{8\,b^2}-\frac{a\,e^{2x}}{8\,b^2}+\frac{e^{-x}\,(4\,a^2-3\,b^2)}{8\,b^3}-\frac{a^4\,\ln\left(-\frac{2\,a^4\,e^{x}}{b^5}-\frac{2\,a^4\,(b-a\,e^{x})}{b^5\,\sqrt{a^2+b^2}}\right)}{b^4\,\sqrt{a^2+b^2}}+\frac{a^4\,\ln\left(\frac{2\,a^4\,(b-a\,e^{x})}{b^5\,\sqrt{a^2+b^2}}-\frac{2\,a^4\,e^{x}}{b^5}\right)}{b^4\,\sqrt{a^2+b^2}}$$

input
```
int(sinh(x)^4/(a + b*sinh(x)),x)
```

output
```
exp(-3*x)/(24*b) + exp(3*x)/(24*b) + (x*(a*b^2 - 2*a^3))/(2*b^4) + (exp(x)
*(4*a^2 - 3*b^2))/(8*b^3) + (a*exp(-2*x))/(8*b^2) - (a*exp(2*x))/(8*b^2) +
 (exp(-x)*(4*a^2 - 3*b^2))/(8*b^3) - (a^4*log(- (2*a^4*exp(x))/b^5 - (2*a^
4*(b - a*exp(x)))/(b^5*(a^2 + b^2)^(1/2))))/(b^4*(a^2 + b^2)^(1/2)) + (a^4
*log((2*a^4*(b - a*exp(x)))/(b^5*(a^2 + b^2)^(1/2)) - (2*a^4*exp(x))/b^5))
/(b^4*(a^2 + b^2)^(1/2))
```

Reduce [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 243, normalized size of antiderivative = 2.25

$$\int \frac{\sinh^4(x)}{a+b\sinh(x)}\,dx = \frac{48e^{3x}\sqrt{a^2+b^2}\,atan\left(\frac{e^{x}bi+ai}{\sqrt{a^2+b^2}}\right)a^4 i+e^{6x}a^2b^3+e^{6x}b^5-3e^{5x}a^3b^2-3e^{5x}a\,b^4+12e^{4x}a^4b+3e^{4x}a^2b^3-9e^{4x}b^5}{24e^{3x}b^4 (}$$

input
```
int(sinh(x)^4/(a+b*sinh(x)),x)
```

output
```
(48*e**(3*x)*sqrt(a**2 + b**2)*atan((e**x*b*i + a*i)/sqrt(a**2 + b**2))*a*
*4*i + e**(6*x)*a**2*b**3 + e**(6*x)*b**5 - 3*e**(5*x)*a**3*b**2 - 3*e**(5
*x)*a*b**4 + 12*e**(4*x)*a**4*b + 3*e**(4*x)*a**2*b**3 - 9*e**(4*x)*b**5 -
 24*e**(3*x)*a**5*x - 12*e**(3*x)*a**3*b**2*x + 12*e**(3*x)*a*b**4*x + 12*
e**(2*x)*a**4*b + 3*e**(2*x)*a**2*b**3 - 9*e**(2*x)*b**5 + 3*e**x*a**3*b**
2 + 3*e**x*a*b**4 + a**2*b**3 + b**5)/(24*e**(3*x)*b**4*(a**2 + b**2))
```

3.73 $\int \frac{\sinh^3(x)}{a+b\sinh(x)}\,dx$

Optimal result

Integrand size = 13, antiderivative size = 82

$$\int \frac{\sinh^3(x)}{a+b\sinh(x)}\,dx = \frac{(2a^2-b^2)\,x}{2b^3} + \frac{2a^3\text{arctanh}\left(\frac{b-a\tanh\left(\frac{x}{2}\right)}{\sqrt{a^2+b^2}}\right)}{b^3\sqrt{a^2+b^2}} - \frac{a\cosh(x)}{b^2} + \frac{\cosh(x)\sinh(x)}{2b}$$

output
```
1/2*(2*a^2-b^2)*x/b^3+2*a^3*arctanh((b-a*tanh(1/2*x))/(a^2+b^2)^(1/2))/b^3
/(a^2+b^2)^(1/2)-a*cosh(x)/b^2+1/2*cosh(x)*sinh(x)/b
```

Mathematica [A] (verified)

Time = 0.86 (sec) , antiderivative size = 82, normalized size of antiderivative = 1.00

$$\int \frac{\sinh^3(x)}{a+b\sinh(x)}\,dx = \frac{4a^2x-2b^2x-\frac{8a^3\arctan\left(\frac{b-a\tanh\left(\frac{x}{2}\right)}{\sqrt{-a^2-b^2}}\right)}{\sqrt{-a^2-b^2}}-4ab\cosh(x)+b^2\sinh(2x)}{4b^3}$$

input
```
Integrate[Sinh[x]^3/(a + b*Sinh[x]),x]
```

output
```
(4*a^2*x - 2*b^2*x - (8*a^3*ArcTan[(b - a*Tanh[x/2])/Sqrt[-a^2 - b^2]])/Sq
rt[-a^2 - b^2] - 4*a*b*Cosh[x] + b^2*Sinh[2*x])/(4*b^3)
```

Rubi [C] (verified)

Result contains complex when optimal does not.

Time = 0.61 (sec) , antiderivative size = 106, normalized size of antiderivative = 1.29, number of steps used = 13, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.923$, Rules used = {3042, 26, 3272, 3042, 3502, 26, 3042, 3214, 3042, 3139, 1083, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sinh^3(x)}{a+b\sinh(x)}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{i\sin(ix)^3}{a-ib\sin(ix)}dx$$

$$\Big\downarrow \text{26}$$

$$i\int \frac{\sin(ix)^3}{a-ib\sin(ix)}dx$$

$$\Big\downarrow \text{3272}$$

$$i\left(\frac{i\int \frac{2a\sinh^2(x)+b\sinh(x)+a}{a+b\sinh(x)}dx}{2b}-\frac{i\sinh(x)\cosh(x)}{2b}\right)$$

$$\Big\downarrow \text{3042}$$

$$i\left(\frac{i\int \frac{-2a\sin(ix)^2-ib\sin(ix)+a}{a-ib\sin(ix)}dx}{2b}-\frac{i\sinh(x)\cosh(x)}{2b}\right)$$

$$\Big\downarrow \text{3502}$$

$$i\left(\frac{i\left(\frac{2a\cosh(x)}{b}+\frac{i\int -\frac{i\left(ab-\left(2a^2-b^2\right)\sinh(x)\right)}{a+b\sinh(x)}dx}{b}\right)}{2b}-\frac{i\sinh(x)\cosh(x)}{2b}\right)$$

$$\Big\downarrow \; 26$$

$$i\left(\frac{i\left(\frac{\int \frac{ab-\left(2a^2-b^2\right)\sinh(x)}{a+b\sinh(x)}dx}{b}+\frac{2a\cosh(x)}{b}\right)}{2b}-\frac{i\sinh(x)\cosh(x)}{2b}\right)$$

$$\Big\downarrow \; 3042$$

$$i\left(\frac{i\left(\frac{2a\cosh(x)}{b}+\frac{\int \frac{ab+i\left(2a^2-b^2\right)\sin(ix)}{a-ib\sin(ix)}dx}{b}\right)}{2b}-\frac{i\sinh(x)\cosh(x)}{2b}\right)$$

$$\Big\downarrow \; 3214$$

$$i\left(\frac{i\left(\frac{\frac{2a^3\int \frac{1}{a+b\sinh(x)}dx}{b}-\frac{x\left(2a^2-b^2\right)}{b}}{b}+\frac{2a\cosh(x)}{b}\right)}{2b}-\frac{i\sinh(x)\cosh(x)}{2b}\right)$$

$$\Big\downarrow \; 3042$$

$$i\left(\frac{i\left(\frac{2a\cosh(x)}{b}+\frac{-\frac{x\left(2a^2-b^2\right)}{b}+\frac{2a^3\int \frac{1}{a-ib\sin(ix)}dx}{b}}{b}\right)}{2b}-\frac{i\sinh(x)\cosh(x)}{2b}\right)$$

$$\Big\downarrow \; 3139$$

$$i\left(\frac{i\left(\frac{\frac{4a^3\int \frac{1}{-a\tanh^2\left(\frac{x}{2}\right)+2b\tanh\left(\frac{x}{2}\right)+a}d\tanh\left(\frac{x}{2}\right)}{b}-\frac{x(2a^2-b^2)}{b}}{b}+\frac{2a\cosh(x)}{b}\right)}{2b}-\frac{i\sinh(x)\cosh(x)}{2b}\right)$$

$\Big\downarrow$ 1083

$$i\left(\frac{i\left(\frac{-\frac{8a^3\int \frac{1}{4(a^2+b^2)-\left(2b-2a\tanh\left(\frac{x}{2}\right)\right)^2}d\left(2b-2a\tanh\left(\frac{x}{2}\right)\right)}{b}-\frac{x(2a^2-b^2)}{b}}{b}+\frac{2a\cosh(x)}{b}\right)}{2b}-\frac{i\sinh(x)\cosh(x)}{2b}\right)$$

$\Big\downarrow$ 219

$$i\left(\frac{i\left(\frac{-\frac{x(2a^2-b^2)}{b}-\frac{4a^3\text{arctanh}\left(\frac{2b-2a\tanh\left(\frac{x}{2}\right)}{2\sqrt{a^2+b^2}}\right)}{b\sqrt{a^2+b^2}}}{b}+\frac{2a\cosh(x)}{b}\right)}{2b}-\frac{i\sinh(x)\cosh(x)}{2b}\right)$$

input
```
Int[Sinh[x]^3/(a + b*Sinh[x]),x]
```

output
```
I*(((I/2)*((-(((2*a^2 - b^2)*x)/b) - (4*a^3*ArcTanh[(2*b - 2*a*Tanh[x/2])/
(2*Sqrt[a^2 + b^2])])/(b*Sqrt[a^2 + b^2]))/b + (2*a*Cosh[x])/b))/b - ((I/2
)*Cosh[x]*Sinh[x])/b)
```

Defintions of rubi rules used

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1083
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Simp[-2   Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3139
```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Simp[2*(e/d)   Subst[Int[1/(a + 2*b*e*x + a
*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[a^2 - b^2, 0]
```

rule 3214
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])/((c_.) + (d_.)*sin[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[b*(x/d), x] - Simp[(b*c - a*d)/d   Int[1/(c + d
*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

rule 3272
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b^2)*Cos[e + f*x]*(a + b*Sin[e + f*
x])^(m - 2)*((c + d*Sin[e + f*x])^(n + 1)/(d*f*(m + n))), x] + Simp[1/(d*(m
 + n))   Int[(a + b*Sin[e + f*x])^(m - 3)*(c + d*Sin[e + f*x])^n*Simp[a^3*d
*(m + n) + b^2*(b*c*(m - 2) + a*d*(n + 1)) - b*(a*b*c - b^2*d*(m + n - 1) -
 3*a^2*d*(m + n))*Sin[e + f*x] - b^2*(b*c*(m - 1) - a*d*(3*m + 2*n - 2))*Si
n[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*
d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 2] && (IntegerQ[m
] || IntegersQ[2*m, 2*n]) &&  !(IGtQ[n, 2] && ( !IntegerQ[m] || (EqQ[a, 0]
&& NeQ[c, 0])))
```

rule 3502
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-C)*Co
s[e + f*x]*((a + b*Sin[e + f*x])^(m + 1)/(b*f*(m + 2))), x] + Simp[1/(b*(m
+ 2))   Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m
 + 2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x]
 &&  !LtQ[m, -1]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 151 vs. $2(72) = 144$.

Time = 0.27 (sec) , antiderivative size = 152, normalized size of antiderivative = 1.85

method	result
default	$-\frac{2a^3 \operatorname{arctanh}\left(\frac{2a\tanh\left(\frac{x}{2}\right)-2b}{2\sqrt{a^2+b^2}}\right)}{b^3\sqrt{a^2+b^2}} + \frac{1}{2b\left(\tanh\left(\frac{x}{2}\right)-1\right)^2} - \frac{-b-2a}{2b^2\left(\tanh\left(\frac{x}{2}\right)-1\right)} + \frac{\left(-2a^2+b^2\right)\ln\left(\tanh\left(\frac{x}{2}\right)-1\right)}{2b^3} - \frac{1}{2b\left(\tanh\left(\frac{x}{2}\right)+1\right)}$
risch	$\frac{x a^2}{b^3} - \frac{x}{2b} + \frac{e^{2x}}{8b} - \frac{a e^{x}}{2b^2} - \frac{a e^{-x}}{2b^2} - \frac{e^{-2x}}{8b} + \frac{a^3 \ln\left(e^x + \frac{a\sqrt{a^2+b^2}+a^2+b^2}{\sqrt{a^2+b^2}\, b}\right)}{\sqrt{a^2+b^2}\, b^3} - \frac{a^3 \ln\left(e^x + \frac{a\sqrt{a^2+b^2}-a^2-b^2}{\sqrt{a^2+b^2}\, b}\right)}{\sqrt{a^2+b^2}\, b^3}$

input
```
int(sinh(x)^3/(a+b*sinh(x)),x,method=_RETURNVERBOSE)
```

output
```
-2*a^3/b^3/(a^2+b^2)^(1/2)*arctanh(1/2*(2*a*tanh(1/2*x)-2*b)/(a^2+b^2)^(1/
2))+1/2/b/(tanh(1/2*x)-1)^2-1/2*(-b-2*a)/b^2/(tanh(1/2*x)-1)+1/2/b^3*(-2*a
^2+b^2)*ln(tanh(1/2*x)-1)-1/2/b/(tanh(1/2*x)+1)^2-1/2*(-b+2*a)/b^2/(tanh(1
/2*x)+1)+1/2*(2*a^2-b^2)/b^3*ln(tanh(1/2*x)+1)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 459 vs. $2(74) = 148$.

Time = 0.10 (sec) , antiderivative size = 459, normalized size of antiderivative = 5.60

$$\int \frac{\sinh^3(x)}{a+b\sinh(x)}\,dx$$
$$= \frac{(a^2b^2+b^4)\cosh(x)^4+(a^2b^2+b^4)\sinh(x)^4-a^2b^2-b^4+4\,(2\,a^4+a^2b^2-b^4)x\cosh(x)^2-4\,(a^3b+ab^3)}{}$$

input
```
integrate(sinh(x)^3/(a+b*sinh(x)),x, algorithm="fricas")
```

output
```
1/8*((a^2*b^2 + b^4)*cosh(x)^4 + (a^2*b^2 + b^4)*sinh(x)^4 - a^2*b^2 - b^4
 + 4*(2*a^4 + a^2*b^2 - b^4)*x*cosh(x)^2 - 4*(a^3*b + a*b^3)*cosh(x)^3 - 4
*(a^3*b + a*b^3 - (a^2*b^2 + b^4)*cosh(x))*sinh(x)^3 + 2*(3*(a^2*b^2 + b^4
)*cosh(x)^2 + 2*(2*a^4 + a^2*b^2 - b^4)*x - 6*(a^3*b + a*b^3)*cosh(x))*sin
h(x)^2 + 8*(a^3*cosh(x)^2 + 2*a^3*cosh(x)*sinh(x) + a^3*sinh(x)^2)*sqrt(a^
2 + b^2)*log((b^2*cosh(x)^2 + b^2*sinh(x)^2 + 2*a*b*cosh(x) + 2*a^2 + b^2
+ 2*(b^2*cosh(x) + a*b)*sinh(x) + 2*sqrt(a^2 + b^2)*(b*cosh(x) + b*sinh(x)
 + a))/(b*cosh(x)^2 + b*sinh(x)^2 + 2*a*cosh(x) + 2*(b*cosh(x) + a)*sinh(x
) - b)) - 4*(a^3*b + a*b^3)*cosh(x) - 4*(a^3*b + a*b^3 - (a^2*b^2 + b^4)*c
osh(x)^3 - 2*(2*a^4 + a^2*b^2 - b^4)*x*cosh(x) + 3*(a^3*b + a*b^3)*cosh(x)
^2)*sinh(x))/((a^2*b^3 + b^5)*cosh(x)^2 + 2*(a^2*b^3 + b^5)*cosh(x)*sinh(x
) + (a^2*b^3 + b^5)*sinh(x)^2)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\sinh^3(x)}{a+b\sinh(x)}\,dx = \text{Timed out}$$

input
```
integrate(sinh(x)**3/(a+b*sinh(x)),x)
```

output
```
Timed out
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 118, normalized size of antiderivative = 1.44

$$\int \frac{\sinh^3(x)}{a+b\sinh(x)}\,dx = -\frac{a^3\log\left(\frac{be^{(-x)}-a-\sqrt{a^2+b^2}}{be^{(-x)}-a+\sqrt{a^2+b^2}}\right)}{\sqrt{a^2+b^2}b^3} - \frac{\left(4\,ae^{(-x)}-b\right)e^{(2\,x)}}{8\,b^2} - \frac{4\,ae^{(-x)}+be^{(-2\,x)}}{8\,b^2} + \frac{(2\,a^2-b^2)x}{2\,b^3}$$

input
```
integrate(sinh(x)^3/(a+b*sinh(x)),x, algorithm="maxima")
```

output
```
-a^3*log((b*e^(-x) - a - sqrt(a^2 + b^2))/(b*e^(-x) - a + sqrt(a^2 + b^2))
)/(sqrt(a^2 + b^2)*b^3) - 1/8*(4*a*e^(-x) - b)*e^(2*x)/b^2 - 1/8*(4*a*e^(-
x) + b*e^(-2*x))/b^2 + 1/2*(2*a^2 - b^2)*x/b^3
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 117, normalized size of antiderivative = 1.43

$$\int \frac{\sinh^3(x)}{a+b\sinh(x)}\,dx = -\frac{a^3\log\left(\frac{\left|2\,be^x+2\,a-2\,\sqrt{a^2+b^2}\right|}{\left|2\,be^x+2\,a+2\,\sqrt{a^2+b^2}\right|}\right)}{\sqrt{a^2+b^2}b^3} + \frac{be^{(2\,x)}-4\,ae^x}{8\,b^2} + \frac{(2\,a^2-b^2)x}{2\,b^3} - \frac{(4\,abe^x+b^2)e^{(-2\,x)}}{8\,b^3}$$

input
```
integrate(sinh(x)^3/(a+b*sinh(x)),x, algorithm="giac")
```

output
```
-a^3*log(abs(2*b*e^x + 2*a - 2*sqrt(a^2 + b^2))/abs(2*b*e^x + 2*a + 2*sqrt
(a^2 + b^2)))/(sqrt(a^2 + b^2)*b^3) + 1/8*(b*e^(2*x) - 4*a*e^x)/b^2 + 1/2*
(2*a^2 - b^2)*x/b^3 - 1/8*(4*a*b*e^x + b^2)*e^(-2*x)/b^3
```

Mupad [B] (verification not implemented)

Time = 1.92 (sec) , antiderivative size = 159, normalized size of antiderivative = 1.94

$$\int \frac{\sinh^3(x)}{a+b\sinh(x)}\,dx = \frac{e^{2x}}{8b} - \frac{e^{-2x}}{8b} + \frac{x\,(2a^2-b^2)}{2b^3} - \frac{a\,e^{x}}{2b^2} - \frac{a\,e^{-x}}{2b^2} - \frac{a^3\ln\left(\frac{2a^3e^x}{b^4} - \frac{2a^3(b-a\,e^x)}{b^4\sqrt{a^2+b^2}}\right)}{b^3\sqrt{a^2+b^2}} + \frac{a^3\ln\left(\frac{2a^3e^x}{b^4} + \frac{2a^3(b-a\,e^x)}{b^4\sqrt{a^2+b^2}}\right)}{b^3\sqrt{a^2+b^2}}$$

input
```
int(sinh(x)^3/(a + b*sinh(x)),x)
```

output
```
exp(2*x)/(8*b) - exp(-2*x)/(8*b) + (x*(2*a^2 - b^2))/(2*b^3) - (a*exp(x))/
(2*b^2) - (a*exp(-x))/(2*b^2) - (a^3*log((2*a^3*exp(x))/b^4 - (2*a^3*(b -
a*exp(x)))/(b^4*(a^2 + b^2)^(1/2))))/(b^3*(a^2 + b^2)^(1/2)) + (a^3*log((2
*a^3*exp(x))/b^4 + (2*a^3*(b - a*exp(x)))/(b^4*(a^2 + b^2)^(1/2))))/(b^3*(
a^2 + b^2)^(1/2))
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 173, normalized size of antiderivative = 2.11

$$\int \frac{\sinh^3(x)}{a+b\sinh(x)}\,dx = \frac{-16e^{2x}\sqrt{a^2+b^2}\,atan\left(\frac{e^{x}bi+ai}{\sqrt{a^2+b^2}}\right)a^3i + e^{4x}a^2b^2 + e^{4x}b^4 - 4e^{3x}a^3b - 4e^{3x}a\,b^3 + 8e^{2x}a^4x + 4e^{2x}a^2b^2x - 4e^{2x}}{8e^{2x}b^3\,(a^2+b^2)}$$

input
```
int(sinh(x)^3/(a+b*sinh(x)),x)
```

output
```
( - 16*e**(2*x)*sqrt(a**2 + b**2)*atan((e**x*b*i + a*i)/sqrt(a**2 + b**2))
*a**3*i + e**(4*x)*a**2*b**2 + e**(4*x)*b**4 - 4*e**(3*x)*a**3*b - 4*e**(3
*x)*a*b**3 + 8*e**(2*x)*a**4*x + 4*e**(2*x)*a**2*b**2*x - 4*e**(2*x)*b**4*
x - 4*e**x*a**3*b - 4*e**x*a*b**3 - a**2*b**2 - b**4)/(8*e**(2*x)*b**3*(a*
*2 + b**2))
```

3.74 $\int \frac{\sinh^2(x)}{a+b\sinh(x)}\,dx$

Optimal result

Integrand size = 13, antiderivative size = 57

$$\int \frac{\sinh^2(x)}{a+b\sinh(x)}\,dx = -\frac{ax}{b^2}-\frac{2a^2\operatorname{arctanh}\left(\frac{b-a\tanh\left(\frac{x}{2}\right)}{\sqrt{a^2+b^2}}\right)}{b^2\sqrt{a^2+b^2}}+\frac{\cosh(x)}{b}$$

output
```
-a*x/b^2-2*a^2*arctanh((b-a*tanh(1/2*x))/(a^2+b^2)^(1/2))/b^2/(a^2+b^2)^(1
/2)+cosh(x)/b
```

Mathematica [A] (verified)

Time = 0.23 (sec) , antiderivative size = 61, normalized size of antiderivative = 1.07

$$\int \frac{\sinh^2(x)}{a+b\sinh(x)}\,dx = \frac{a\left(-x+\frac{2a\arctan\left(\frac{b-a\tanh\left(\frac{x}{2}\right)}{\sqrt{-a^2-b^2}}\right)}{\sqrt{-a^2-b^2}}\right)+b\cosh(x)}{b^2}$$

input
```
Integrate[Sinh[x]^2/(a + b*Sinh[x]),x]
```

output
```
(a*(-x + (2*a*ArcTan[(b - a*Tanh[x/2])/Sqrt[-a^2 - b^2]])/Sqrt[-a^2 - b^2]
) + b*Cosh[x])/b^2
```

Rubi [C] (verified)

Result contains complex when optimal does not.

Time = 0.46 (sec) , antiderivative size = 72, normalized size of antiderivative = 1.26, number of steps used = 13, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.923$, Rules used = {3042, 25, 3225, 26, 27, 3042, 26, 3214, 3042, 3139, 1083, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sinh^2(x)}{a+b\sinh(x)}dx$$

$$\Big\downarrow \text{3042}$$

$$\int -\frac{\sin(ix)^2}{a-ib\sin(ix)}dx$$

$$\Big\downarrow \text{25}$$

$$-\int \frac{\sin(ix)^2}{a-ib\sin(ix)}dx$$

$$\Big\downarrow \text{3225}$$

$$\frac{\cosh(x)}{b}-\frac{i\int -\frac{ia\sinh(x)}{a+b\sinh(x)}dx}{b}$$

$$\Big\downarrow \text{26}$$

$$\frac{\cosh(x)}{b}-\frac{\int \frac{a\sinh(x)}{a+b\sinh(x)}dx}{b}$$

$$\Big\downarrow \text{27}$$

$$\frac{\cosh(x)}{b}-\frac{a\int \frac{\sinh(x)}{a+b\sinh(x)}dx}{b}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\cosh(x)}{b}-\frac{a\int -\frac{i\sin(ix)}{a-ib\sin(ix)}dx}{b}$$

$$\Big\downarrow \textbf{26}$$

$$\frac{\cosh(x)}{b}+\frac{ia\int \frac{\sin(ix)}{a-ib\sin(ix)}dx}{b}$$

$$\Big\downarrow \textbf{3214}$$

$$\frac{\cosh(x)}{b}+\frac{ia\left(\frac{ix}{b}-\frac{ia\int \frac{1}{a+b\sinh(x)}dx}{b}\right)}{b}$$

$$\Big\downarrow \textbf{3042}$$

$$\frac{\cosh(x)}{b}+\frac{ia\left(\frac{ix}{b}-\frac{ia\int \frac{1}{a-ib\sin(ix)}dx}{b}\right)}{b}$$

$$\Big\downarrow \textbf{3139}$$

$$\frac{\cosh(x)}{b}+\frac{ia\left(\frac{ix}{b}-\frac{2ia\int \frac{1}{-a\tanh^2\left(\frac{x}{2}\right)+2b\tanh\left(\frac{x}{2}\right)+a}d\tanh\left(\frac{x}{2}\right)}{b}\right)}{b}$$

$$\Big\downarrow \textbf{1083}$$

$$\frac{\cosh(x)}{b}+\frac{ia\left(\frac{4ia\int \frac{1}{4\left(a^2+b^2\right)-\left(2b-2a\tanh\left(\frac{x}{2}\right)\right)^2}d\left(2b-2a\tanh\left(\frac{x}{2}\right)\right)}{b}+\frac{ix}{b}\right)}{b}$$

$$\Big\downarrow \textbf{219}$$

$$\frac{\cosh(x)}{b}+\frac{ia\left(\frac{2ia\text{arctanh}\left(\frac{2b-2a\tanh\left(\frac{x}{2}\right)}{2\sqrt{a^2+b^2}}\right)}{b\sqrt{a^2+b^2}}+\frac{ix}{b}\right)}{b}$$

input
```
Int[Sinh[x]^2/(a + b*Sinh[x]),x]
```

output
```
(I*a*((I*x)/b + ((2*I)*a*ArcTanh[(2*b - 2*a*Tanh[x/2])/(2*Sqrt[a^2 + b^2])
])/(b*Sqrt[a^2 + b^2])))/b + Cosh[x]/b
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1083
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Simp[-2   Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3139
```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Simp[2*(e/d)   Subst[Int[1/(a + 2*b*e*x + a
*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[a^2 - b^2, 0]
```

rule 3214
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])/((c_.) + (d_.)*sin[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[b*(x/d), x] - Simp[(b*c - a*d)/d   Int[1/(c + d
*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

rule 3225
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^2/((c_.) + (d_.)*sin[(e_.) + (f
_.)*(x_)]), x_Symbol] :> Simp[(-b^2)*(Cos[e + f*x]/(d*f)), x] + Simp[1/d
Int[Simp[a^2*d - b*(b*c - 2*a*d)*Sin[e + f*x], x]/(c + d*Sin[e + f*x]), x],
 x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Maple [A] (verified)

Time = 0.16 (sec) , antiderivative size = 92, normalized size of antiderivative = 1.61

method	result	size
default	$-\frac{1}{b(\tanh(\frac{x}{2})-1)}+\frac{a\ln(\tanh(\frac{x}{2})-1)}{b^2}+\frac{1}{b(\tanh(\frac{x}{2})+1)}-\frac{a\ln(\tanh(\frac{x}{2})+1)}{b^2}+\frac{2a^2\,\text{arctanh}\left(\frac{2a\tanh(\frac{x}{2})-2b}{2\sqrt{a^2+b^2}}\right)}{b^2\sqrt{a^2+b^2}}$	92
risch	$-\frac{ax}{b^2}+\frac{e^x}{2b}+\frac{e^{-x}}{2b}+\frac{a^2\ln\left(e^x+\frac{a\sqrt{a^2+b^2}-a^2-b^2}{\sqrt{a^2+b^2}\,b}\right)}{\sqrt{a^2+b^2}\,b^2}-\frac{a^2\ln\left(e^x+\frac{a\sqrt{a^2+b^2}+a^2+b^2}{\sqrt{a^2+b^2}\,b}\right)}{\sqrt{a^2+b^2}\,b^2}$	132

input
```
int(sinh(x)^2/(a+b*sinh(x)),x,method=_RETURNVERBOSE)
```

output
```
-1/b/(tanh(1/2*x)-1)+a/b^2*ln(tanh(1/2*x)-1)+1/b/(tanh(1/2*x)+1)-a/b^2*ln(
tanh(1/2*x)+1)+2*a^2/b^2/(a^2+b^2)^(1/2)*arctanh(1/2*(2*a*tanh(1/2*x)-2*b)
/(a^2+b^2)^(1/2))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 238 vs. 2(53) = 106.

Time = 0.10 (sec) , antiderivative size = 238, normalized size of antiderivative = 4.18

$$\int \frac{\sinh^2(x)}{a+b\sinh(x)}\,dx$$
$$= \frac{a^2b+b^3-2\left(a^3+ab^2\right)x\cosh(x)+\left(a^2b+b^3\right)\cosh(x)^2+\left(a^2b+b^3\right)\sinh(x)^2+2\left(a^2\cosh(x)+a^2\sinh(\ldots\right.}{\ldots}$$

input
```
integrate(sinh(x)^2/(a+b*sinh(x)),x, algorithm="fricas")
```

output
```
1/2*(a^2*b + b^3 - 2*(a^3 + a*b^2)*x*cosh(x) + (a^2*b + b^3)*cosh(x)^2 + (
a^2*b + b^3)*sinh(x)^2 + 2*(a^2*cosh(x) + a^2*sinh(x))*sqrt(a^2 + b^2)*log
((b^2*cosh(x)^2 + b^2*sinh(x)^2 + 2*a*b*cosh(x) + 2*a^2 + b^2 + 2*(b^2*cos
h(x) + a*b)*sinh(x) - 2*sqrt(a^2 + b^2)*(b*cosh(x) + b*sinh(x) + a))/(b*co
sh(x)^2 + b*sinh(x)^2 + 2*a*cosh(x) + 2*(b*cosh(x) + a)*sinh(x) - b)) - 2*
((a^3 + a*b^2)*x - (a^2*b + b^3)*cosh(x))*sinh(x))/((a^2*b^2 + b^4)*cosh(x
) + (a^2*b^2 + b^4)*sinh(x))
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\sinh^2(x)}{a + b\sinh(x)}\, dx = \text{Timed out}$$

input
```
integrate(sinh(x)**2/(a+b*sinh(x)),x)
```

output
```
Timed out
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 84, normalized size of antiderivative = 1.47

$$\int \frac{\sinh^2(x)}{a + b\sinh(x)}\, dx = \frac{a^2 \log\left(\frac{be^{(-x)} - a - \sqrt{a^2+b^2}}{be^{(-x)} - a + \sqrt{a^2+b^2}}\right)}{\sqrt{a^2+b^2}b^2} - \frac{ax}{b^2} + \frac{e^{(-x)}}{2\,b} + \frac{e^x}{2\,b}$$

input
```
integrate(sinh(x)^2/(a+b*sinh(x)),x, algorithm="maxima")
```

output
```
a^2*log((b*e^(-x) - a - sqrt(a^2 + b^2))/(b*e^(-x) - a + sqrt(a^2 + b^2)))
/(sqrt(a^2 + b^2)*b^2) - a*x/b^2 + 1/2*e^(-x)/b + 1/2*e^x/b
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 86, normalized size of antiderivative = 1.51

$$\int \frac{\sinh^2(x)}{a+b\sinh(x)}\,dx = \frac{a^2\log\left(\frac{\left|2\,be^x+2\,a-2\,\sqrt{a^2+b^2}\right|}{\left|2\,be^x+2\,a+2\,\sqrt{a^2+b^2}\right|}\right)}{\sqrt{a^2+b^2}b^2} - \frac{ax}{b^2} + \frac{e^{(-x)}}{2\,b} + \frac{e^x}{2\,b}$$

input
```
integrate(sinh(x)^2/(a+b*sinh(x)),x, algorithm="giac")
```

output
```
a^2*log(abs(2*b*e^x + 2*a - 2*sqrt(a^2 + b^2))/abs(2*b*e^x + 2*a + 2*sqrt(
a^2 + b^2)))/(sqrt(a^2 + b^2)*b^2) - a*x/b^2 + 1/2*e^(-x)/b + 1/2*e^x/b
```

Mupad [B] (verification not implemented)

Time = 1.83 (sec) , antiderivative size = 129, normalized size of antiderivative = 2.26

$$\int \frac{\sinh^2(x)}{a+b\sinh(x)}\,dx = \frac{e^{-x}}{2\,b} + \frac{e^x}{2\,b} - \frac{a\,x}{b^2} - \frac{a^2\,\ln\left(-\frac{2\,a^2\,e^x}{b^3} - \frac{2\,a^2\,(b-a\,e^x)}{b^3\,\sqrt{a^2+b^2}}\right)}{b^2\,\sqrt{a^2+b^2}} + \frac{a^2\,\ln\left(\frac{2\,a^2\,(b-a\,e^x)}{b^3\,\sqrt{a^2+b^2}} - \frac{2\,a^2\,e^x}{b^3}\right)}{b^2\,\sqrt{a^2+b^2}}$$

input
```
int(sinh(x)^2/(a + b*sinh(x)),x)
```

output
```
exp(-x)/(2*b) + exp(x)/(2*b) - (a*x)/b^2 - (a^2*log(- (2*a^2*exp(x))/b^3 -
 (2*a^2*(b - a*exp(x)))/(b^3*(a^2 + b^2)^(1/2))))/(b^2*(a^2 + b^2)^(1/2))
+ (a^2*log((2*a^2*(b - a*exp(x)))/(b^3*(a^2 + b^2)^(1/2)) - (2*a^2*exp(x))
/b^3))/(b^2*(a^2 + b^2)^(1/2))
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 76, normalized size of antiderivative = 1.33

$$\int \frac{\sinh^2(x)}{a+b\sinh(x)}\,dx = \frac{2\sqrt{a^2+b^2}\,atan\left(\frac{e^x bi+ai}{\sqrt{a^2+b^2}}\right)a^2 i+\cosh(x)\,a^2 b+\cosh(x)\,b^3-a^3 x-a\,b^2 x}{b^2\,(a^2+b^2)}$$

input
```
int(sinh(x)^2/(a+b*sinh(x)),x)
```

output
```
(2*sqrt(a**2 + b**2)*atan((e**x*b*i + a*i)/sqrt(a**2 + b**2))*a**2*i + cos
h(x)*a**2*b + cosh(x)*b**3 - a**3*x - a*b**2*x)/(b**2*(a**2 + b**2))
```

3.75 $\int \frac{\sinh(x)}{a+b\sinh(x)}\,dx$

Optimal result

Integrand size = 11, antiderivative size = 47

$$\int \frac{\sinh(x)}{a+b\sinh(x)}\,dx = \frac{x}{b} + \frac{2a\text{arctanh}\left(\frac{b-a\tanh\left(\frac{x}{2}\right)}{\sqrt{a^2+b^2}}\right)}{b\sqrt{a^2+b^2}}$$

output
```
x/b+2*a*arctanh((b-a*tanh(1/2*x))/(a^2+b^2)^(1/2))/b/(a^2+b^2)^(1/2)
```

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 52, normalized size of antiderivative = 1.11

$$\int \frac{\sinh(x)}{a+b\sinh(x)}\,dx = \frac{x - \frac{2a\arctan\left(\frac{b-a\tanh\left(\frac{x}{2}\right)}{\sqrt{-a^2-b^2}}\right)}{\sqrt{-a^2-b^2}}}{b}$$

input
```
Integrate[Sinh[x]/(a + b*Sinh[x]),x]
```

output
```
(x - (2*a*ArcTan[(b - a*Tanh[x/2])/Sqrt[-a^2 - b^2]])/Sqrt[-a^2 - b^2])/b
```

Rubi [C] (verified)

Result contains complex when optimal does not.

Time = 0.32 (sec) , antiderivative size = 61, normalized size of antiderivative = 1.30, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.636, Rules used = {3042, 26, 3214, 3042, 3139, 1083, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sinh(x)}{a+b\sinh(x)}dx$$

$$\Big\downarrow \text{3042}$$

$$\int -\frac{i\sin(ix)}{a-ib\sin(ix)}dx$$

$$\Big\downarrow \text{26}$$

$$-i\int \frac{\sin(ix)}{a-ib\sin(ix)}dx$$

$$\Big\downarrow \text{3214}$$

$$-i\left(\frac{ix}{b}-\frac{ia\int \frac{1}{a+b\sinh(x)}dx}{b}\right)$$

$$\Big\downarrow \text{3042}$$

$$-i\left(\frac{ix}{b}-\frac{ia\int \frac{1}{a-ib\sin(ix)}dx}{b}\right)$$

$$\Big\downarrow \text{3139}$$

$$-i\left(\frac{ix}{b}-\frac{2ia\int \frac{1}{-a\tanh^2\left(\frac{x}{2}\right)+2b\tanh\left(\frac{x}{2}\right)+a}d\tanh\left(\frac{x}{2}\right)}{b}\right)$$

$$\Big\downarrow \text{1083}$$

$$-i\left(\frac{4ia\int \frac{1}{4(a^2+b^2)-\left(2b-2a\tanh\left(\frac{x}{2}\right)\right)^2}d\left(2b-2a\tanh\left(\frac{x}{2}\right)\right)}{b}+\frac{ix}{b}\right)$$

$$\Big\downarrow \text{219}$$

$$-i\left(\frac{2ia\,\text{arctanh}\left(\frac{2b-2a\tanh\left(\frac{x}{2}\right)}{2\sqrt{a^2+b^2}}\right)}{b\sqrt{a^2+b^2}}+\frac{ix}{b}\right)$$

input
```
Int[Sinh[x]/(a + b*Sinh[x]),x]
```

output
```
(-I)*((I*x)/b + ((2*I)*a*ArcTanh[(2*b - 2*a*Tanh[x/2])/(2*Sqrt[a^2 + b^2])
])/(b*Sqrt[a^2 + b^2]))
```

Defintions of rubi rules used

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1083
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Simp[-2   Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3139
```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Simp[2*(e/d)   Subst[Int[1/(a + 2*b*e*x + a
*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[a^2 - b^2, 0]
```

rule 3214
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])/((c_.) + (d_.)*sin[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[b*(x/d), x] - Simp[(b*c - a*d)/d   Int[1/(c + d
*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Maple [A] (verified)

Time = 0.09 (sec) , antiderivative size = 63, normalized size of antiderivative = 1.34

method	result	size
default	$-\frac{2a \,\operatorname{arctanh}\left(\frac{2a \tanh\left(\frac{x}{2}\right)-2b}{2\sqrt{a^{2}+b^{2}}}\right)}{b\sqrt{a^{2}+b^{2}}}-\frac{\ln\left(\tanh\left(\frac{x}{2}\right)-1\right)}{b}+\frac{\ln\left(\tanh\left(\frac{x}{2}\right)+1\right)}{b}$	63
risch	$\frac{x}{b}+\frac{a \ln\left(\mathrm{e}^{x}+\frac{a\sqrt{a^{2}+b^{2}}+a^{2}+b^{2}}{\sqrt{a^{2}+b^{2}}\, b}\right)}{\sqrt{a^{2}+b^{2}}\, b}-\frac{a \ln\left(\mathrm{e}^{x}+\frac{a\sqrt{a^{2}+b^{2}}-a^{2}-b^{2}}{\sqrt{a^{2}+b^{2}}\, b}\right)}{\sqrt{a^{2}+b^{2}}\, b}$	110

input
```
int(sinh(x)/(a+b*sinh(x)),x,method=_RETURNVERBOSE)
```

output
```
-2/b*a/(a^2+b^2)^(1/2)*arctanh(1/2*(2*a*tanh(1/2*x)-2*b)/(a^2+b^2)^(1/2))-
1/b*ln(tanh(1/2*x)-1)+1/b*ln(tanh(1/2*x)+1)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 134 vs. 2(43) = 86.

Time = 0.11 (sec) , antiderivative size = 134, normalized size of antiderivative = 2.85

$$\int \frac{\sinh(x)}{a+b\sinh(x)}\,dx$$
$$=\frac{\sqrt{a^2+b^2}a\log\left(\frac{b^2\cosh(x)^2+b^2\sinh(x)^2+2\,ab\cosh(x)+2\,a^2+b^2+2\,(b^2\cosh(x)+ab)\sinh(x)+2\,\sqrt{a^2+b^2}(b\cosh(x)+b\sinh(x)+a)}{b\cosh(x)^2+b\sinh(x)^2+2\,a\cosh(x)+2\,(b\cosh(x)+a)\sinh(x)-b}\right)+(a}{a^2b+b^3}$$

input
```
integrate(sinh(x)/(a+b*sinh(x)),x, algorithm="fricas")
```

output
```
(sqrt(a^2 + b^2)*a*log((b^2*cosh(x)^2 + b^2*sinh(x)^2 + 2*a*b*cosh(x) + 2*
a^2 + b^2 + 2*(b^2*cosh(x) + a*b)*sinh(x) + 2*sqrt(a^2 + b^2)*(b*cosh(x) +
 b*sinh(x) + a))/(b*cosh(x)^2 + b*sinh(x)^2 + 2*a*cosh(x) + 2*(b*cosh(x) +
 a)*sinh(x) - b)) + (a^2 + b^2)*x)/(a^2*b + b^3)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 18.52 (sec) , antiderivative size = 170, normalized size of antiderivative = 3.62

$$\int \frac{\sinh(x)}{a+b\sinh(x)}\,dx = \begin{cases} \tilde{\infty}x & \text{for } a=0 \wedge b=0 \\ \frac{x}{b} & \text{for } a=0 \\ \frac{\cosh(x)}{a} & \text{for } b=0 \\ \frac{x\tanh\left(\frac{x}{2}\right)}{b\tanh\left(\frac{x}{2}\right)-ib} - \frac{ix}{b\tanh\left(\frac{x}{2}\right)-ib} - \frac{2}{b\tanh\left(\frac{x}{2}\right)-ib} & \text{for } a=-ib \\ \frac{x\tanh\left(\frac{x}{2}\right)}{b\tanh\left(\frac{x}{2}\right)+ib} + \frac{ix}{b\tanh\left(\frac{x}{2}\right)+ib} - \frac{2}{b\tanh\left(\frac{x}{2}\right)+ib} & \text{for } a=ib \\ \frac{a\log\left(\tanh\left(\frac{x}{2}\right)-\frac{b}{a}-\frac{\sqrt{a^2+b^2}}{a}\right)}{b\sqrt{a^2+b^2}} - \frac{a\log\left(\tanh\left(\frac{x}{2}\right)-\frac{b}{a}+\frac{\sqrt{a^2+b^2}}{a}\right)}{b\sqrt{a^2+b^2}} + \frac{x}{b} & \text{otherwise} \end{cases}$$

input
```
integrate(sinh(x)/(a+b*sinh(x)),x)
```

output
```
Piecewise((zoo*x, Eq(a, 0) & Eq(b, 0)), (x/b, Eq(a, 0)), (cosh(x)/a, Eq(b,
 0)), (x*tanh(x/2)/(b*tanh(x/2) - I*b) - I*x/(b*tanh(x/2) - I*b) - 2/(b*ta
nh(x/2) - I*b), Eq(a, -I*b)), (x*tanh(x/2)/(b*tanh(x/2) + I*b) + I*x/(b*ta
nh(x/2) + I*b) - 2/(b*tanh(x/2) + I*b), Eq(a, I*b)), (a*log(tanh(x/2) - b/
a - sqrt(a**2 + b**2)/a)/(b*sqrt(a**2 + b**2)) - a*log(tanh(x/2) - b/a + s
qrt(a**2 + b**2)/a)/(b*sqrt(a**2 + b**2)) + x/b, True))
```

Maxima [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 65, normalized size of antiderivative = 1.38

$$\int \frac{\sinh(x)}{a+b\sinh(x)}\,dx = -\frac{a\log\left(\frac{be^{(-x)}-a-\sqrt{a^2+b^2}}{be^{(-x)}-a+\sqrt{a^2+b^2}}\right)}{\sqrt{a^2+b^2}b}+\frac{x}{b}$$

input
```
integrate(sinh(x)/(a+b*sinh(x)),x, algorithm="maxima")
```

output
```
-a*log((b*e^(-x) - a - sqrt(a^2 + b^2))/(b*e^(-x) - a + sqrt(a^2 + b^2)))/
(sqrt(a^2 + b^2)*b) + x/b
```

Giac [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 67, normalized size of antiderivative = 1.43

$$\int \frac{\sinh(x)}{a+b\sinh(x)}\,dx = -\frac{a\log\left(\frac{|2\,be^x+2\,a-2\,\sqrt{a^2+b^2}|}{|2\,be^x+2\,a+2\,\sqrt{a^2+b^2}|}\right)}{\sqrt{a^2+b^2}b}+\frac{x}{b}$$

input
```
integrate(sinh(x)/(a+b*sinh(x)),x, algorithm="giac")
```

output
```
-a*log(abs(2*b*e^x + 2*a - 2*sqrt(a^2 + b^2))/abs(2*b*e^x + 2*a + 2*sqrt(a
^2 + b^2)))/(sqrt(a^2 + b^2)*b) + x/b
```

Mupad [B] (verification not implemented)

Time = 1.84 (sec) , antiderivative size = 99, normalized size of antiderivative = 2.11

$$\int \frac{\sinh(x)}{a+b\sinh(x)}\,dx = \frac{x}{b}-\frac{a\,\ln\left(\frac{2\,a\,e^x}{b^2}-\frac{2\,a\,(b-a\,e^x)}{b^2\,\sqrt{a^2+b^2}}\right)}{b\,\sqrt{a^2+b^2}}+\frac{a\,\ln\left(\frac{2\,a\,e^x}{b^2}+\frac{2\,a\,(b-a\,e^x)}{b^2\,\sqrt{a^2+b^2}}\right)}{b\,\sqrt{a^2+b^2}}$$

input
```
int(sinh(x)/(a + b*sinh(x)),x)
```

output
```
x/b - (a*log((2*a*exp(x))/b^2 - (2*a*(b - a*exp(x)))/(b^2*(a^2 + b^2)^(1/2
))))/(b*(a^2 + b^2)^(1/2)) + (a*log((2*a*exp(x))/b^2 + (2*a*(b - a*exp(x))
)/(b^2*(a^2 + b^2)^(1/2))))/(b*(a^2 + b^2)^(1/2))
```

Reduce [B] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 58, normalized size of antiderivative = 1.23

$$\int \frac{\sinh(x)}{a + b\sinh(x)}\,dx = \frac{-2\sqrt{a^2+b^2}\,atan\left(\frac{e^x bi+ai}{\sqrt{a^2+b^2}}\right)ai + a^2x + b^2x}{b\,(a^2+b^2)}$$

input
```
int(sinh(x)/(a+b*sinh(x)),x)
```

output
```
( - 2*sqrt(a**2 + b**2)*atan((e**x*b*i + a*i)/sqrt(a**2 + b**2))*a*i + a**
2*x + b**2*x)/(b*(a**2 + b**2))
```

3.76 $\int \frac{\operatorname{csch}(x)}{a+b\sinh(x)}\,dx$

Optimal result

Integrand size = 11, antiderivative size = 50

$$\int \frac{\operatorname{csch}(x)}{a+b\sinh(x)}\,dx=-\frac{\operatorname{arctanh}(\cosh(x))}{a}+\frac{2b\operatorname{arctanh}\left(\frac{b-a\tanh\left(\frac{x}{2}\right)}{\sqrt{a^2+b^2}}\right)}{a\sqrt{a^2+b^2}}$$

output
```
-arctanh(cosh(x))/a+2*b*arctanh((b-a*tanh(1/2*x))/(a^2+b^2)^(1/2))/a/(a^2+
b^2)^(1/2)
```

Mathematica [A] (verified)

Time = 0.28 (sec) , antiderivative size = 67, normalized size of antiderivative = 1.34

$$\int \frac{\operatorname{csch}(x)}{a+b\sinh(x)}\,dx=\frac{-\frac{2b\arctan\left(\frac{b-a\tanh\left(\frac{x}{2}\right)}{\sqrt{-a^2-b^2}}\right)}{\sqrt{-a^2-b^2}}-\log\left(\cosh\left(\frac{x}{2}\right)\right)+\log\left(\sinh\left(\frac{x}{2}\right)\right)}{a}$$

input
```
Integrate[Csch[x]/(a + b*Sinh[x]),x]
```

output
```
((-2*b*ArcTan[(b - a*Tanh[x/2])/Sqrt[-a^2 - b^2]])/Sqrt[-a^2 - b^2] - Log[
Cosh[x/2]] + Log[Sinh[x/2]])/a
```

Rubi [C] (verified)

Result contains complex when optimal does not.

Time = 0.38 (sec) , antiderivative size = 63, normalized size of antiderivative = 1.26, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.909, Rules used = {3042, 26, 3226, 26, 3042, 26, 3139, 1083, 219, 4257}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\operatorname{csch}(x)}{a+b\sinh(x)}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{i}{\sin(ix)(a-ib\sin(ix))}dx$$

$$\Big\downarrow \text{26}$$

$$i\int \frac{1}{\sin(ix)(a-ib\sin(ix))}dx$$

$$\Big\downarrow \text{3226}$$

$$i\left(\frac{ib\int \frac{1}{a+b\sinh(x)}dx}{a}+\frac{\int -i\operatorname{csch}(x)dx}{a}\right)$$

$$\Big\downarrow \text{26}$$

$$i\left(\frac{ib\int \frac{1}{a+b\sinh(x)}dx}{a}-\frac{i\int \operatorname{csch}(x)dx}{a}\right)$$

$$\Big\downarrow \text{3042}$$

$$i\left(\frac{ib\int \frac{1}{a-ib\sin(ix)}dx}{a}-\frac{i\int i\csc(ix)dx}{a}\right)$$

$$\Big\downarrow \text{26}$$

$$i\left(\frac{ib\int \frac{1}{a-ib\sin(ix)}dx}{a}+\frac{\int \csc(ix)dx}{a}\right)$$

$$\Big\downarrow \text{3139}$$

$$i\left(\frac{2ib\int \frac{1}{-a\tanh^2\left(\frac{x}{2}\right)+2b\tanh\left(\frac{x}{2}\right)+a}d\tanh\left(\frac{x}{2}\right)}{a}+\frac{\int \csc(ix)dx}{a}\right)$$

$$\Big\downarrow \text{1083}$$

$$i\left(\frac{\int \csc(ix)dx}{a}-\frac{4ib\int \frac{1}{4(a^2+b^2)-\left(2b-2a\tanh\left(\frac{x}{2}\right)\right)^2}d\left(2b-2a\tanh\left(\frac{x}{2}\right)\right)}{a}\right)$$

$$\Big\downarrow \text{219}$$

$$i\left(\frac{\int \csc(ix)dx}{a}-\frac{2ib\text{arctanh}\left(\frac{2b-2a\tanh\left(\frac{x}{2}\right)}{2\sqrt{a^2+b^2}}\right)}{a\sqrt{a^2+b^2}}\right)$$

$$\Big\downarrow \text{4257}$$

$$i\left(\frac{i\text{arctanh}(\cosh(x))}{a}-\frac{2ib\text{arctanh}\left(\frac{2b-2a\tanh\left(\frac{x}{2}\right)}{2\sqrt{a^2+b^2}}\right)}{a\sqrt{a^2+b^2}}\right)$$

input
```
Int[Csch[x]/(a + b*Sinh[x]),x]
```

output
```
I*((I*ArcTanh[Cosh[x]])/a - ((2*I)*b*ArcTanh[(2*b - 2*a*Tanh[x/2])/(2*Sqrt
[a^2 + b^2])])/(a*Sqrt[a^2 + b^2]))
```

Defintions of rubi rules used

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1083
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Simp[-2   Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3139
```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Simp[2*(e/d)   Subst[Int[1/(a + 2*b*e*x + a
*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[a^2 - b^2, 0]
```

rule 3226
```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (
f_.)*(x_)])), x_Symbol] :> Simp[b/(b*c - a*d)   Int[1/(a + b*Sin[e + f*x]),
 x], x] - Simp[d/(b*c - a*d)   Int[1/(c + d*Sin[e + f*x]), x], x] /; FreeQ[
{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

rule 4257
```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Maple [A] (verified)

Time = 0.16 (sec) , antiderivative size = 49, normalized size of antiderivative = 0.98

method	result	size
default	$-\frac{2b\,\operatorname{arctanh}\left(\frac{2a\tanh\left(\frac{x}{2}\right)-2b}{2\sqrt{a^2+b^2}}\right)}{a\sqrt{a^2+b^2}}+\frac{\ln\left(\tanh\left(\frac{x}{2}\right)\right)}{a}$	49
risch	$\frac{b\ln\left(\mathrm{e}^{x}+\frac{a\sqrt{a^2+b^2}+a^2+b^2}{\sqrt{a^2+b^2}\,b}\right)}{\sqrt{a^2+b^2}\,a}-\frac{b\ln\left(\mathrm{e}^{x}+\frac{a\sqrt{a^2+b^2}-a^2-b^2}{\sqrt{a^2+b^2}\,b}\right)}{\sqrt{a^2+b^2}\,a}+\frac{\ln(\mathrm{e}^{x}-1)}{a}-\frac{\ln(\mathrm{e}^{x}+1)}{a}$	124

input
```
int(csch(x)/(a+b*sinh(x)),x,method=_RETURNVERBOSE)
```

output
```
-2*b/a/(a^2+b^2)^(1/2)*arctanh(1/2*(2*a*tanh(1/2*x)-2*b)/(a^2+b^2)^(1/2))+
1/a*ln(tanh(1/2*x))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 156 vs. $2(46) = 92$.

Time = 0.11 (sec) , antiderivative size = 156, normalized size of antiderivative = 3.12

$$\int \frac{\operatorname{csch}(x)}{a+b\sinh(x)}\,dx = \frac{\sqrt{a^2+b^2}b\log\left(\frac{b^2\cosh(x)^2+b^2\sinh(x)^2+2\,ab\cosh(x)+2\,a^2+b^2+2\,(b^2\cosh(x)+ab)\sinh(x)+2\,\sqrt{a^2+b^2}(b\cosh(x)+b\sinh(x)+a)}{b\cosh(x)^2+b\sinh(x)^2+2\,a\cosh(x)+2\,(b\cosh(x)+a)\sinh(x)-b}\right)-(a^2}{a^3+ab^2}$$

input
```
integrate(csch(x)/(a+b*sinh(x)),x, algorithm="fricas")
```

output
```
(sqrt(a^2 + b^2)*b*log((b^2*cosh(x)^2 + b^2*sinh(x)^2 + 2*a*b*cosh(x) + 2*
a^2 + b^2 + 2*(b^2*cosh(x) + a*b)*sinh(x) + 2*sqrt(a^2 + b^2)*(b*cosh(x) +
 b*sinh(x) + a))/(b*cosh(x)^2 + b*sinh(x)^2 + 2*a*cosh(x) + 2*(b*cosh(x) +
 a)*sinh(x) - b)) - (a^2 + b^2)*log(cosh(x) + sinh(x) + 1) + (a^2 + b^2)*l
og(cosh(x) + sinh(x) - 1))/(a^3 + a*b^2)
```

Sympy [F]

$$\int \frac{\operatorname{csch}(x)}{a+b\sinh(x)}\,dx = \int \frac{\operatorname{csch}(x)}{a+b\sinh(x)}\,dx$$

input
```
integrate(csch(x)/(a+b*sinh(x)),x)
```

output
```
Integral(csch(x)/(a + b*sinh(x)), x)
```

Maxima [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 83, normalized size of antiderivative = 1.66

$$\int \frac{\operatorname{csch}(x)}{a+b\sinh(x)}\,dx = -\frac{b\log\left(\frac{be^{(-x)}-a-\sqrt{a^2+b^2}}{be^{(-x)}-a+\sqrt{a^2+b^2}}\right)}{\sqrt{a^2+b^2}a} - \frac{\log\left(e^{(-x)}+1\right)}{a} + \frac{\log\left(e^{(-x)}-1\right)}{a}$$

input
```
integrate(csch(x)/(a+b*sinh(x)),x, algorithm="maxima")
```

output
```
-b*log((b*e^(-x) - a - sqrt(a^2 + b^2))/(b*e^(-x) - a + sqrt(a^2 + b^2)))/
(sqrt(a^2 + b^2)*a) - log(e^(-x) + 1)/a + log(e^(-x) - 1)/a
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 82, normalized size of antiderivative = 1.64

$$\int \frac{\operatorname{csch}(x)}{a+b\sinh(x)}\,dx = -\frac{b\log\left(\frac{\left|2\,be^x+2\,a-2\,\sqrt{a^2+b^2}\right|}{\left|2\,be^x+2\,a+2\,\sqrt{a^2+b^2}\right|}\right)}{\sqrt{a^2+b^2}a} - \frac{\log\left(e^x+1\right)}{a} + \frac{\log\left(\left|e^x-1\right|\right)}{a}$$

input
```
integrate(csch(x)/(a+b*sinh(x)),x, algorithm="giac")
```

output
```
-b*log(abs(2*b*e^x + 2*a - 2*sqrt(a^2 + b^2))/abs(2*b*e^x + 2*a + 2*sqrt(a
^2 + b^2)))/(sqrt(a^2 + b^2)*a) - log(e^x + 1)/a + log(abs(e^x - 1))/a
```

Mupad [B] (verification not implemented)

Time = 2.01 (sec) , antiderivative size = 287, normalized size of antiderivative = 5.74

$$\int \frac{\operatorname{csch}(x)}{a+b\sinh(x)}\,dx = \frac{\ln\left(32\,a-32\,a\,\mathrm{e}^x\right)}{a} - \frac{\ln\left(32\,a+32\,a\,\mathrm{e}^x\right)}{a} + \frac{b\,\ln\left(128\,a^5\,\mathrm{e}^x-64\,a^2\,b^3-64\,a^4\,b-128\,a^4\,\mathrm{e}^x\,\sqrt{a^2+b^2}+32\,a\,b^4\,\mathrm{e}^x+160\,a^3\,b^2\,\mathrm{e}^x+32\,a\,b^3\,\sqrt{a^2+b^2}+\right.}{a^3+a\,b^2} - \frac{b\,\ln\left(64\,a^4\,b+64\,a^2\,b^3-128\,a^5\,\mathrm{e}^x-128\,a^4\,\mathrm{e}^x\,\sqrt{a^2+b^2}-32\,a\,b^4\,\mathrm{e}^x-160\,a^3\,b^2\,\mathrm{e}^x+32\,a\,b^3\,\sqrt{a^2+b^2}+\right.}{a^3+a\,b^2}$$

input
```
int(1/(sinh(x)*(a + b*sinh(x))),x)
```

output
```
log(32*a - 32*a*exp(x))/a - log(32*a + 32*a*exp(x))/a + (b*log(128*a^5*exp
(x) - 64*a^2*b^3 - 64*a^4*b - 128*a^4*exp(x)*(a^2 + b^2)^(1/2) + 32*a*b^4*
exp(x) + 160*a^3*b^2*exp(x) + 32*a*b^3*(a^2 + b^2)^(1/2) + 64*a^3*b*(a^2 +
 b^2)^(1/2) - 96*a^2*b^2*exp(x)*(a^2 + b^2)^(1/2))*(a^2 + b^2)^(1/2))/(a*b
^2 + a^3) - (b*log(64*a^4*b + 64*a^2*b^3 - 128*a^5*exp(x) - 128*a^4*exp(x)
*(a^2 + b^2)^(1/2) - 32*a*b^4*exp(x) - 160*a^3*b^2*exp(x) + 32*a*b^3*(a^2
+ b^2)^(1/2) + 64*a^3*b*(a^2 + b^2)^(1/2) - 96*a^2*b^2*exp(x)*(a^2 + b^2)^
(1/2))*(a^2 + b^2)^(1/2))/(a*b^2 + a^3)
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 90, normalized size of antiderivative = 1.80

$$\int \frac{\operatorname{csch}(x)}{a+b\sinh(x)}\,dx = \frac{-2\sqrt{a^2+b^2}\,atan\left(\frac{e^x bi+ai}{\sqrt{a^2+b^2}}\right)bi+\log(e^x-1)\,a^2+\log(e^x-1)\,b^2-\log(e^x+1)\,a^2-\log(e^x+1)\,b^2}{a\,(a^2+b^2)}$$

input
```
int(csch(x)/(a+b*sinh(x)),x)
```

output
```
( - 2*sqrt(a**2 + b**2)*atan((e**x*b*i + a*i)/sqrt(a**2 + b**2))*b*i + log
(e**x - 1)*a**2 + log(e**x - 1)*b**2 - log(e**x + 1)*a**2 - log(e**x + 1)*
b**2)/(a*(a**2 + b**2))
```

3.77 $\int \frac{\operatorname{csch}^2(x)}{a+b\sinh(x)}\,dx$

Optimal result

Integrand size = 13, antiderivative size = 59

$$\int \frac{\operatorname{csch}^2(x)}{a+b\sinh(x)}\,dx=\frac{b\operatorname{arctanh}(\cosh(x))}{a^2}-\frac{2b^2\arctan\left(\frac{b-a\tanh\left(\frac{x}{2}\right)}{\sqrt{a^2+b^2}}\right)}{a^2\sqrt{a^2+b^2}}-\frac{\coth(x)}{a}$$

output
```
b*arctanh(cosh(x))/a^2-2*b^2*arctanh((b-a*tanh(1/2*x))/(a^2+b^2)^(1/2))/a^
2/(a^2+b^2)^(1/2)-coth(x)/a
```

Mathematica [A] (verified)

Time = 0.67 (sec) , antiderivative size = 90, normalized size of antiderivative = 1.53

$$\int \frac{\operatorname{csch}^2(x)}{a+b\sinh(x)}\,dx=-\frac{a\coth\left(\frac{x}{2}\right)+2b\left(-\frac{2b\arctan\left(\frac{b-a\tanh\left(\frac{x}{2}\right)}{\sqrt{-a^2-b^2}}\right)}{\sqrt{-a^2-b^2}}-\log\left(\cosh\left(\frac{x}{2}\right)\right)+\log\left(\sinh\left(\frac{x}{2}\right)\right)\right)+a\tanh\left(\frac{x}{2}\right)}{2a^2}$$

input
```
Integrate[Csch[x]^2/(a + b*Sinh[x]),x]
```

output
```
-1/2*(a*Coth[x/2] + 2*b*((-2*b*ArcTan[(b - a*Tanh[x/2])/Sqrt[-a^2 - b^2]])
/Sqrt[-a^2 - b^2] - Log[Cosh[x/2]] + Log[Sinh[x/2]]) + a*Tanh[x/2])/a^2
```

Rubi [C] (verified)

Result contains complex when optimal does not.

Time = 0.49 (sec) , antiderivative size = 75, normalized size of antiderivative = 1.27, number of steps used = 15, number of rules used = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 1.077$, Rules used = {3042, 25, 3281, 27, 3042, 26, 3226, 26, 3042, 26, 3139, 1083, 219, 4257}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\text{csch}^2(x)}{a+b\sinh(x)}dx$$

$$\Big\downarrow \text{3042}$$

$$\int -\frac{1}{\sin(ix)^2(a-ib\sin(ix))}dx$$

$$\Big\downarrow \text{25}$$

$$-\int \frac{1}{\sin(ix)^2(a-ib\sin(ix))}dx$$

$$\Big\downarrow \text{3281}$$

$$-\frac{\int \frac{b\text{csch}(x)}{a+b\sinh(x)}dx}{a}-\frac{\coth(x)}{a}$$

$$\Big\downarrow \text{27}$$

$$-\frac{b\int \frac{\text{csch}(x)}{a+b\sinh(x)}dx}{a}-\frac{\coth(x)}{a}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{\coth(x)}{a}-\frac{b\int \frac{i}{\sin(ix)(a-ib\sin(ix))}dx}{a}$$

$$\Big\downarrow \text{26}$$

$$-\frac{\coth(x)}{a}-\frac{ib\int \frac{1}{\sin(ix)(a-ib\sin(ix))}dx}{a}$$

$$\Big\downarrow \ 3226$$

$$-\frac{\coth(x)}{a}-\frac{ib\left(\frac{ib\int \frac{1}{a+b\sinh(x)}dx}{a}+\frac{\int -i\text{csch}(x)dx}{a}\right)}{a}$$

$$\Big\downarrow \ 26$$

$$-\frac{\coth(x)}{a}-\frac{ib\left(\frac{ib\int \frac{1}{a+b\sinh(x)}dx}{a}-\frac{i\int \text{csch}(x)dx}{a}\right)}{a}$$

$$\Big\downarrow \ 3042$$

$$-\frac{\coth(x)}{a}-\frac{ib\left(\frac{ib\int \frac{1}{a-ib\sin(ix)}dx}{a}-\frac{i\int i\csc(ix)dx}{a}\right)}{a}$$

$$\Big\downarrow \ 26$$

$$-\frac{\coth(x)}{a}-\frac{ib\left(\frac{ib\int \frac{1}{a-ib\sin(ix)}dx}{a}+\frac{\int \csc(ix)dx}{a}\right)}{a}$$

$$\Big\downarrow \ 3139$$

$$-\frac{\coth(x)}{a}-\frac{ib\left(\frac{2ib\int \frac{1}{-a\tanh^2\left(\frac{x}{2}\right)+2b\tanh\left(\frac{x}{2}\right)+a}d\tanh\left(\frac{x}{2}\right)}{a}+\frac{\int \csc(ix)dx}{a}\right)}{a}$$

$$\Big\downarrow \ 1083$$

$$-\frac{\coth(x)}{a}-\frac{ib\left(\frac{\int \csc(ix)dx}{a}-\frac{4ib\int \frac{1}{4\left(a^2+b^2\right)-\left(2b-2a\tanh\left(\frac{x}{2}\right)\right)^2}d\left(2b-2a\tanh\left(\frac{x}{2}\right)\right)}{a}\right)}{a}$$

$$\Big\downarrow \ 219$$

$$-\frac{\coth(x)}{a}-\frac{ib\left(\frac{\int \csc(ix)dx}{a}-\frac{2ib\text{arctanh}\left(\frac{2b-2a\tanh\left(\frac{x}{2}\right)}{2\sqrt{a^2+b^2}}\right)}{a\sqrt{a^2+b^2}}\right)}{a}$$

$$\Big\downarrow \ 4257$$

$$-\frac{\coth(x)}{a}-\frac{ib\left(\frac{i\text{arctanh}(\cosh(x))}{a}-\frac{2ib\text{arctanh}\left(\frac{2b-2a\tanh\left(\frac{x}{2}\right)}{2\sqrt{a^2+b^2}}\right)}{a\sqrt{a^2+b^2}}\right)}{a}$$

input
```
Int[Csch[x]^2/(a + b*Sinh[x]),x]
```

output
```
((-I)*b*((I*ArcTanh[Cosh[x]])/a - ((2*I)*b*ArcTanh[(2*b - 2*a*Tanh[x/2])/(
2*Sqrt[a^2 + b^2])])/(a*Sqrt[a^2 + b^2])))/a - Coth[x]/a
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1083
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Simp[-2   Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3139
```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Simp[2*(e/d)   Subst[Int[1/(a + 2*b*e*x + a
*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[a^2 - b^2, 0]
```

rule 3226
```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (
f_.)*(x_)])), x_Symbol] :> Simp[b/(b*c - a*d)   Int[1/(a + b*Sin[e + f*x]),
 x], x] - Simp[d/(b*c - a*d)   Int[1/(c + d*Sin[e + f*x]), x], x] /; FreeQ[
{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

rule 3281
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b^2)*Cos[e + f*x]*(a + b*Sin[e + f*
x])^(m + 1)*((c + d*Sin[e + f*x])^(n + 1)/(f*(m + 1)*(b*c - a*d)*(a^2 - b^2
))), x] + Simp[1/((m + 1)*(b*c - a*d)*(a^2 - b^2))   Int[(a + b*Sin[e + f*x
])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[a*(b*c - a*d)*(m + 1) + b^2*d*(m + n
 + 2) - (b^2*c + b*(b*c - a*d)*(m + 1))*Sin[e + f*x] - b^2*d*(m + n + 3)*Si
n[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*
d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -1] && IntegersQ[
2*m, 2*n] && ((EqQ[a, 0] && IntegerQ[m] &&  !IntegerQ[n]) ||  !(IntegerQ[2*
n] && LtQ[n, -1] && ((IntegerQ[n] &&  !IntegerQ[m]) || EqQ[a, 0])))
```

rule 4257
```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Maple [A] (verified)

Time = 0.19 (sec) , antiderivative size = 73, normalized size of antiderivative = 1.24

method	result	size
default	$-\frac{\tanh\left(\frac{x}{2}\right)}{2a}-\frac{1}{2a\tanh\left(\frac{x}{2}\right)}-\frac{b\ln\left(\tanh\left(\frac{x}{2}\right)\right)}{a^{2}}+\frac{2b^{2}\operatorname{arctanh}\left(\frac{2a\tanh\left(\frac{x}{2}\right)-2b}{2\sqrt{a^{2}+b^{2}}}\right)}{a^{2}\sqrt{a^{2}+b^{2}}}$	73
risch	$-\frac{2}{a\left(\mathrm{e}^{2x}-1\right)}+\frac{b\ln\left(\mathrm{e}^{x}+1\right)}{a^{2}}+\frac{b^{2}\ln\left(\mathrm{e}^{x}+\frac{a\sqrt{a^{2}+b^{2}}-a^{2}-b^{2}}{\sqrt{a^{2}+b^{2}}\,b}\right)}{\sqrt{a^{2}+b^{2}}\,a^{2}}-\frac{b^{2}\ln\left(\mathrm{e}^{x}+\frac{a\sqrt{a^{2}+b^{2}}+a^{2}+b^{2}}{\sqrt{a^{2}+b^{2}}\,b}\right)}{\sqrt{a^{2}+b^{2}}\,a^{2}}-\frac{b\ln\left(\mathrm{e}^{x}-1\right)}{a^{2}}$	143

input
```
int(csch(x)^2/(a+b*sinh(x)),x,method=_RETURNVERBOSE)
```

output
```
-1/2/a*tanh(1/2*x)-1/2/a/tanh(1/2*x)-1/a^2*b*ln(tanh(1/2*x))+2*b^2/a^2/(a^
2+b^2)^(1/2)*arctanh(1/2*(2*a*tanh(1/2*x)-2*b)/(a^2+b^2)^(1/2))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 345 vs. 2(55) = 110.

Time = 0.11 (sec) , antiderivative size = 345, normalized size of antiderivative = 5.85

$$\int \frac{\operatorname{csch}^2(x)}{a+b\sinh(x)}\,dx = \frac{2\,a^3+2\,ab^2-\left(b^2\cosh\left(x\right)^2+2\,b^2\cosh\left(x\right)\sinh\left(x\right)+b^2\sinh\left(x\right)^2-b^2\right)\sqrt{a^2+b^2}\log\left(\frac{b^2\cosh(x)^2+b^2\sinh(x)^2+\cdots}{b\cdots}\right)\cdots}{\cdots}$$

input
```
integrate(csch(x)^2/(a+b*sinh(x)),x, algorithm="fricas")
```

output
```
(2*a^3 + 2*a*b^2 - (b^2*cosh(x)^2 + 2*b^2*cosh(x)*sinh(x) + b^2*sinh(x)^2
- b^2)*sqrt(a^2 + b^2)*log((b^2*cosh(x)^2 + b^2*sinh(x)^2 + 2*a*b*cosh(x)
+ 2*a^2 + b^2 + 2*(b^2*cosh(x) + a*b)*sinh(x) - 2*sqrt(a^2 + b^2)*(b*cosh(
x) + b*sinh(x) + a))/(b*cosh(x)^2 + b*sinh(x)^2 + 2*a*cosh(x) + 2*(b*cosh(
x) + a)*sinh(x) - b)) + (a^2*b + b^3 - (a^2*b + b^3)*cosh(x)^2 - 2*(a^2*b
+ b^3)*cosh(x)*sinh(x) - (a^2*b + b^3)*sinh(x)^2)*log(cosh(x) + sinh(x) +
1) - (a^2*b + b^3 - (a^2*b + b^3)*cosh(x)^2 - 2*(a^2*b + b^3)*cosh(x)*sinh
(x) - (a^2*b + b^3)*sinh(x)^2)*log(cosh(x) + sinh(x) - 1))/(a^4 + a^2*b^2
- (a^4 + a^2*b^2)*cosh(x)^2 - 2*(a^4 + a^2*b^2)*cosh(x)*sinh(x) - (a^4 + a
^2*b^2)*sinh(x)^2)
```

Sympy [F]

$$\int \frac{\operatorname{csch}^2(x)}{a+b\sinh(x)}\,dx = \int \frac{\operatorname{csch}^2\left(x\right)}{a+b\sinh\left(x\right)}\,dx$$

input
```
integrate(csch(x)**2/(a+b*sinh(x)),x)
```

output
```
Integral(csch(x)**2/(a + b*sinh(x)), x)
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 100, normalized size of antiderivative = 1.69

$$\int \frac{\operatorname{csch}^2(x)}{a+b\sinh(x)}\,dx = \frac{b^2\log\left(\frac{be^{(-x)}-a-\sqrt{a^2+b^2}}{be^{(-x)}-a+\sqrt{a^2+b^2}}\right)}{\sqrt{a^2+b^2}a^2} + \frac{b\log\left(e^{(-x)}+1\right)}{a^2} - \frac{b\log\left(e^{(-x)}-1\right)}{a^2} + \frac{2}{ae^{(-2x)}-a}$$

input
```
integrate(csch(x)^2/(a+b*sinh(x)),x, algorithm="maxima")
```

output
```
b^2*log((b*e^(-x) - a - sqrt(a^2 + b^2))/(b*e^(-x) - a + sqrt(a^2 + b^2)))
/(sqrt(a^2 + b^2)*a^2) + b*log(e^(-x) + 1)/a^2 - b*log(e^(-x) - 1)/a^2 + 2
/(a*e^(-2*x) - a)
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 98, normalized size of antiderivative = 1.66

$$\int \frac{\operatorname{csch}^2(x)}{a+b\sinh(x)}\,dx = \frac{b^2\log\left(\frac{\left|2\,be^x+2\,a-2\,\sqrt{a^2+b^2}\right|}{\left|2\,be^x+2\,a+2\,\sqrt{a^2+b^2}\right|}\right)}{\sqrt{a^2+b^2}a^2} + \frac{b\log\left(e^x+1\right)}{a^2} - \frac{b\log\left(\left|e^x-1\right|\right)}{a^2} - \frac{2}{a(e^{(2x)}-1)}$$

input
```
integrate(csch(x)^2/(a+b*sinh(x)),x, algorithm="giac")
```

output
```
b^2*log(abs(2*b*e^x + 2*a - 2*sqrt(a^2 + b^2))/abs(2*b*e^x + 2*a + 2*sqrt(
a^2 + b^2)))/(sqrt(a^2 + b^2)*a^2) + b*log(e^x + 1)/a^2 - b*log(abs(e^x -
1))/a^2 - 2/(a*(e^(2*x) - 1))
```

Mupad [B] (verification not implemented)

Time = 2.00 (sec) , antiderivative size = 292, normalized size of antiderivative = 4.95

$$\int \frac{\operatorname{csch}^2(x)}{a+b\sinh(x)}\,dx = \frac{2}{a-a\,e^{2\,x}} - \frac{b\,\ln\left(32\,e^x-32\right)}{a^2} + \frac{b\,\ln\left(32\,e^x+32\right)}{a^2} + \frac{b^2\,\ln\left(128\,a^4\,e^x-64\,a\,b^3-64\,a^3\,b-32\,b^3\,\sqrt{a^2+b^2}+32\,b^4\,e^x+128\,a^3\,e^x\,\sqrt{a^2+b^2}+160\,a^2\,b^2\,e^x-64}{a^4+a^2\,b^2} - \frac{b^2\,\ln\left(32\,b^3\,\sqrt{a^2+b^2}-64\,a\,b^3-64\,a^3\,b+128\,a^4\,e^x+32\,b^4\,e^x-128\,a^3\,e^x\,\sqrt{a^2+b^2}+160\,a^2\,b^2\,e^x+64}{a^4+a^2\,b^2}$$

input
```
int(1/(sinh(x)^2*(a + b*sinh(x))),x)
```

output
```
2/(a - a*exp(2*x)) - (b*log(32*exp(x) - 32))/a^2 + (b*log(32*exp(x) + 32))
/a^2 + (b^2*log(128*a^4*exp(x) - 64*a*b^3 - 64*a^3*b - 32*b^3*(a^2 + b^2)^
(1/2) + 32*b^4*exp(x) + 128*a^3*exp(x)*(a^2 + b^2)^(1/2) + 160*a^2*b^2*exp
(x) - 64*a^2*b*(a^2 + b^2)^(1/2) + 96*a*b^2*exp(x)*(a^2 + b^2)^(1/2))*(a^2
 + b^2)^(1/2))/(a^4 + a^2*b^2) - (b^2*log(32*b^3*(a^2 + b^2)^(1/2) - 64*a*
b^3 - 64*a^3*b + 128*a^4*exp(x) + 32*b^4*exp(x) - 128*a^3*exp(x)*(a^2 + b^
2)^(1/2) + 160*a^2*b^2*exp(x) + 64*a^2*b*(a^2 + b^2)^(1/2) - 96*a*b^2*exp(
x)*(a^2 + b^2)^(1/2))*(a^2 + b^2)^(1/2))/(a^4 + a^2*b^2)
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 242, normalized size of antiderivative = 4.10

$$\int \frac{\operatorname{csch}^2(x)}{a+b\sinh(x)}\,dx = \frac{2e^{2x}\sqrt{a^2+b^2}\,atan\left(\frac{e^x bi+ai}{\sqrt{a^2+b^2}}\right)b^2i - 2\sqrt{a^2+b^2}\,atan\left(\frac{e^x bi+ai}{\sqrt{a^2+b^2}}\right)b^2i - e^{2x}\log(e^x-1)\,a^2b - e^{2x}\log(e^x-1)\,b^3+}{a^2}$$

input
```
int(csch(x)^2/(a+b*sinh(x)),x)
```

output
```
(2*e**(2*x)*sqrt(a**2 + b**2)*atan((e**x*b*i + a*i)/sqrt(a**2 + b**2))*b**
2*i - 2*sqrt(a**2 + b**2)*atan((e**x*b*i + a*i)/sqrt(a**2 + b**2))*b**2*i
- e**(2*x)*log(e**x - 1)*a**2*b - e**(2*x)*log(e**x - 1)*b**3 + e**(2*x)*l
og(e**x + 1)*a**2*b + e**(2*x)*log(e**x + 1)*b**3 - 2*e**(2*x)*a**3 - 2*e*
*(2*x)*a*b**2 + log(e**x - 1)*a**2*b + log(e**x - 1)*b**3 - log(e**x + 1)*
a**2*b - log(e**x + 1)*b**3)/(a**2*(e**(2*x)*a**2 + e**(2*x)*b**2 - a**2 -
 b**2))
```

3.78 $\int \frac{\operatorname{csch}^3(x)}{a+b\sinh(x)}\,dx$

Optimal result

Integrand size = 13, antiderivative size = 81

$$\int \frac{\operatorname{csch}^3(x)}{a+b\sinh(x)}\,dx=\frac{(a^2-2b^2)\operatorname{arctanh}(\cosh(x))}{2a^3}+\frac{2b^3\operatorname{arctanh}\left(\frac{b-a\tanh\left(\frac{x}{2}\right)}{\sqrt{a^2+b^2}}\right)}{a^3\sqrt{a^2+b^2}}+\frac{b\coth(x)}{a^2}-\frac{\coth(x)\operatorname{csch}(x)}{2a}$$

output
```
1/2*(a^2-2*b^2)*arctanh(cosh(x))/a^3+2*b^3*arctanh((b-a*tanh(1/2*x))/(a^2+
b^2)^(1/2))/a^3/(a^2+b^2)^(1/2)+b*coth(x)/a^2-1/2*coth(x)*csch(x)/a
```

Mathematica [A] (verified)

Time = 0.62 (sec) , antiderivative size = 136, normalized size of antiderivative = 1.68

$$\int \frac{\operatorname{csch}^3(x)}{a+b\sinh(x)}\,dx = -\frac{\frac{16b^3\arctan\left(\frac{b-a\tanh\left(\frac{x}{2}\right)}{\sqrt{-a^2-b^2}}\right)}{\sqrt{-a^2-b^2}}-4ab\coth\left(\frac{x}{2}\right)+a^2\operatorname{csch}^2\left(\frac{x}{2}\right)-4(a^2-2b^2)\log\left(\cosh\left(\frac{x}{2}\right)\right)+4(a^2-2b^2)\log\left(\sin\right.}{8a^3}$$

input
```
Integrate[Csch[x]^3/(a + b*Sinh[x]),x]
```

output
```
-1/8*((16*b^3*ArcTan[(b - a*Tanh[x/2])/Sqrt[-a^2 - b^2]])/Sqrt[-a^2 - b^2]
 - 4*a*b*Coth[x/2] + a^2*Csch[x/2]^2 - 4*(a^2 - 2*b^2)*Log[Cosh[x/2]] + 4*
(a^2 - 2*b^2)*Log[Sinh[x/2]] + a^2*Sech[x/2]^2 - 4*a*b*Tanh[x/2])/a^3
```

Rubi [C] (verified)

Result contains complex when optimal does not.

Time = 0.84 (sec) , antiderivative size = 113, normalized size of antiderivative = 1.40, number of steps used = 19, number of rules used = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 1.385$, Rules used = {3042, 26, 3281, 26, 3042, 25, 3534, 25, 3042, 26, 3480, 26, 3042, 26, 3139, 1083, 219, 4257}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\text{csch}^3(x)}{a+b\sinh(x)}dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int -\frac{i}{\sin(ix)^3(a-ib\sin(ix))}dx$$

$$\Big\downarrow \text{ 26}$$

$$-i\int \frac{1}{\sin(ix)^3(a-ib\sin(ix))}dx$$

$$\Big\downarrow \text{ 3281}$$

$$-i\left(\frac{\int -\frac{i\text{csch}^2(x)(b\sinh^2(x)+a\sinh(x)+2b)}{a+b\sinh(x)}dx}{2a}-\frac{i\coth(x)\text{csch}(x)}{2a}\right)$$

$$\Big\downarrow \text{ 26}$$

$$-i\left(-\frac{i\int \frac{\text{csch}^2(x)(b\sinh^2(x)+a\sinh(x)+2b)}{a+b\sinh(x)}dx}{2a}-\frac{i\coth(x)\text{csch}(x)}{2a}\right)$$

$$\Big\downarrow \text{ 3042}$$

$$-i\left(-\frac{i\int -\frac{-b\sin(ix)^2-ia\sin(ix)+2b}{\sin(ix)^2(a-ib\sin(ix))}dx}{2a}-\frac{i\coth(x)\text{csch}(x)}{2a}\right)$$

$$\Big\downarrow \text{ 25}$$

$$-i\left(\frac{i\int \frac{-b\sin(ix)^2-ia\sin(ix)+2b}{\sin(ix)^2(a-ib\sin(ix))}dx}{2a}-\frac{i\coth(x)\text{csch}(x)}{2a}\right)$$

$$\Big\downarrow \text{ 3534}$$

$$-i\left(\frac{i\left(\frac{\int -\frac{\text{csch}(x)\left(a^2+b\sinh(x)a-2b^2\right)}{a+b\sinh(x)}dx}{a}+\frac{2b\coth(x)}{a}\right)}{2a}-\frac{i\coth(x)\text{csch}(x)}{2a}\right)$$

$$\Big\downarrow \text{ 25}$$

$$-i\left(\frac{i\left(\frac{2b\coth(x)}{a}-\frac{\int \frac{\text{csch}(x)\left(a^2+b\sinh(x)a-2b^2\right)}{a+b\sinh(x)}dx}{a}\right)}{2a}-\frac{i\coth(x)\text{csch}(x)}{2a}\right)$$

$$\Big\downarrow \text{ 3042}$$

$$-i\left(\frac{i\left(\frac{2b\coth(x)}{a}-\frac{\int \frac{i\left(a^2-ib\sin(ix)a-2b^2\right)}{\sin(ix)(a-ib\sin(ix))}dx}{a}\right)}{2a}-\frac{i\coth(x)\text{csch}(x)}{2a}\right)$$

$$\Big\downarrow \text{ 26}$$

$$-i\left(\frac{i\left(\frac{2b\coth(x)}{a}-\frac{i\int \frac{a^2-ib\sin(ix)a-2b^2}{\sin(ix)(a-ib\sin(ix))}dx}{a}\right)}{2a}-\frac{i\coth(x)\text{csch}(x)}{2a}\right)$$

$$\Big\downarrow \text{ 3480}$$

$$-i\left(\frac{i\left(\frac{2b\coth(x)}{a}-\frac{i\left(\frac{\left(a^2-2b^2\right)\int -i\text{csch}(x)dx}{a}-\frac{2ib^3\int \frac{1}{a+b\sinh(x)}dx}{a}\right)}{a}\right)}{2a}-\frac{i\coth(x)\text{csch}(x)}{2a}\right)$$

$$\Big\downarrow \; 26$$

$$-i\left(\frac{i\left(\frac{2b\coth(x)}{a}-\frac{i\left(-\frac{i\left(a^2-2b^2\right)\int \text{csch}(x)dx}{a}-\frac{2ib^3\int \frac{1}{a+b\sinh(x)}dx}{a}\right)}{a}\right)}{2a}-\frac{i\coth(x)\text{csch}(x)}{2a}\right)$$

$$\Big\downarrow \; 3042$$

$$-i\left(\frac{i\left(\frac{2b\coth(x)}{a}-\frac{i\left(-\frac{i\left(a^2-2b^2\right)\int i\csc(ix)dx}{a}-\frac{2ib^3\int \frac{1}{a-ib\sin(ix)}dx}{a}\right)}{a}\right)}{2a}-\frac{i\coth(x)\text{csch}(x)}{2a}\right)$$

$$\Big\downarrow \; 26$$

$$-i\left(\frac{i\left(\frac{2b\coth(x)}{a}-\frac{i\left(\frac{\left(a^2-2b^2\right)\int \csc(ix)dx}{a}-\frac{2ib^3\int \frac{1}{a-ib\sin(ix)}dx}{a}\right)}{a}\right)}{2a}-\frac{i\coth(x)\text{csch}(x)}{2a}\right)$$

$$\Big\downarrow \; 3139$$

$$-i\left(\frac{i\left(\frac{2b\coth(x)}{a}-\frac{i\left(\frac{(a^2-2b^2)\int \csc(ix)dx}{a}-\frac{4ib^3\int \frac{1}{-a\tanh^2\left(\frac{x}{2}\right)+2b\tanh\left(\frac{x}{2}\right)+a}d\tanh\left(\frac{x}{2}\right)}{a}\right)}{a}\right)}{2a}-\frac{i\coth(x)\text{csch}(x)}{2a}\right)$$

$$\Big\downarrow \text{1083}$$

$$-i\left(\frac{i\left(\frac{2b\coth(x)}{a}-\frac{i\left(\frac{(a^2-2b^2)\int \csc(ix)dx}{a}+\frac{8ib^3\int \frac{1}{4(a^2+b^2)-\left(2b-2a\tanh\left(\frac{x}{2}\right)\right)^2}d\left(2b-2a\tanh\left(\frac{x}{2}\right)\right)}{a}\right)}{a}\right)}{2a}-\frac{i\coth(x)\text{csch}(x)}{2a}\right)$$

$$\Big\downarrow \text{219}$$

$$-i\left(\frac{i\left(\frac{2b\coth(x)}{a}-\frac{i\left(\frac{(a^2-2b^2)\int \csc(ix)dx}{a}+\frac{4ib^3\text{arctanh}\left(\frac{2b-2a\tanh\left(\frac{x}{2}\right)}{2\sqrt{a^2+b^2}}\right)}{a\sqrt{a^2+b^2}}\right)}{a}\right)}{2a}-\frac{i\coth(x)\text{csch}(x)}{2a}\right)$$

$$\Big\downarrow \text{4257}$$

$$-i\left(\frac{i\left(\frac{2b\coth(x)}{a}-\frac{i\left(\frac{i(a^2-2b^2)\operatorname{arctanh}(\cosh(x))}{a}+\frac{4ib^3\operatorname{arctanh}\left(\frac{2b-2a\tanh\left(\frac{x}{2}\right)}{2\sqrt{a^2+b^2}}\right)}{a\sqrt{a^2+b^2}}\right)}{a}\right)}{2a}-\frac{i\coth(x)\operatorname{csch}(x)}{2a}\right)$$

input
```
Int[Csch[x]^3/(a + b*Sinh[x]),x]
```

output
```
(-I)*(((I/2)*(((-I)*((I*(a^2 - 2*b^2)*ArcTanh[Cosh[x]])/a + ((4*I)*b^3*Arc
Tanh[(2*b - 2*a*Tanh[x/2])/(2*Sqrt[a^2 + b^2])])/(a*Sqrt[a^2 + b^2])))/a +
 (2*b*Coth[x])/a))/a - ((I/2)*Coth[x]*Csch[x])/a)
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1083
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Simp[-2   Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3139
```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Simp[2*(e/d)   Subst[Int[1/(a + 2*b*e*x + a
*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[a^2 - b^2, 0]
```

rule 3281
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b^2)*Cos[e + f*x]*(a + b*Sin[e + f*
x])^(m + 1)*((c + d*Sin[e + f*x])^(n + 1)/(f*(m + 1)*(b*c - a*d)*(a^2 - b^2
))), x] + Simp[1/((m + 1)*(b*c - a*d)*(a^2 - b^2))   Int[(a + b*Sin[e + f*x
])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[a*(b*c - a*d)*(m + 1) + b^2*d*(m + n
 + 2) - (b^2*c + b*(b*c - a*d)*(m + 1))*Sin[e + f*x] - b^2*d*(m + n + 3)*Si
n[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*
d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -1] && IntegersQ[
2*m, 2*n] && ((EqQ[a, 0] && IntegerQ[m] &&  !IntegerQ[n]) ||  !(IntegerQ[2*
n] && LtQ[n, -1] && ((IntegerQ[n] &&  !IntegerQ[m]) || EqQ[a, 0])))
```

rule 3480
```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((a_.) + (b_.)*sin[(e_.) + (f_
.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])), x_Symbol] :> Simp[(A*b
- a*B)/(b*c - a*d)   Int[1/(a + b*Sin[e + f*x]), x], x] + Simp[(B*c - A*d)/
(b*c - a*d)   Int[1/(c + d*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f
, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

rule 3534
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-(A*b^2 - a*b*B + a^2*C))*Cos[e + f*x
]*(a + b*Sin[e + f*x])^(m + 1)*((c + d*Sin[e + f*x])^(n + 1)/(f*(m + 1)*(b*
c - a*d)*(a^2 - b^2))), x] + Simp[1/((m + 1)*(b*c - a*d)*(a^2 - b^2))   Int
[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[(m + 1)*(b*c - a*
d)*(a*A - b*B + a*C) + d*(A*b^2 - a*b*B + a^2*C)*(m + n + 2) - (c*(A*b^2 -
a*b*B + a^2*C) + (m + 1)*(b*c - a*d)*(A*b - a*B + b*C))*Sin[e + f*x] - d*(A
*b^2 - a*b*B + a^2*C)*(m + n + 3)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b
, c, d, e, f, A, B, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] &&
NeQ[c^2 - d^2, 0] && LtQ[m, -1] && ((EqQ[a, 0] && IntegerQ[m] &&  !IntegerQ
[n]) ||  !(IntegerQ[2*n] && LtQ[n, -1] && ((IntegerQ[n] &&  !IntegerQ[m]) |
| EqQ[a, 0])))
```

rule 4257
```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Maple [A] (verified)

Time = 0.25 (sec) , antiderivative size = 108, normalized size of antiderivative = 1.33

method	result
default	$\frac{\frac{\tanh\left(\frac{x}{2}\right)^{2} a}{2}+2 b \tanh\left(\frac{x}{2}\right)}{4 a^{2}}-\frac{1}{8 a \tanh\left(\frac{x}{2}\right)^{2}}+\frac{\left(-2 a^{2}+4 b^{2}\right) \ln\left(\tanh\left(\frac{x}{2}\right)\right)}{4 a^{3}}+\frac{b}{2 a^{2} \tanh\left(\frac{x}{2}\right)}-\frac{2 b^{3} \operatorname{arctanh}\left(\frac{2 a \tanh\left(\frac{x}{2}\right)-2 b}{2 \sqrt{a^{2}+b^{2}}}\right)}{a^{3} \sqrt{a^{2}+b^{2}}}$
risch	$-\frac{a\,\mathrm{e}^{3 x}-2 b\,\mathrm{e}^{2 x}+\mathrm{e}^{x} a+2 b}{\left(\mathrm{e}^{2 x}-1\right)^{2} a^{2}}-\frac{\ln\left(\mathrm{e}^{x}-1\right)}{2 a}+\frac{\ln\left(\mathrm{e}^{x}-1\right) b^{2}}{a^{3}}+\frac{\ln\left(\mathrm{e}^{x}+1\right)}{2 a}-\frac{\ln\left(\mathrm{e}^{x}+1\right) b^{2}}{a^{3}}+\frac{b^{3} \ln\left(\mathrm{e}^{x}+\frac{a \sqrt{a^{2}+b^{2}}+a^{2}+b^{2}}{\sqrt{a^{2}+b^{2}}\, b}\right)}{\sqrt{a^{2}+b^{2}}\, a^{3}}-\frac{b^{3} \ln\left(\mathrm{e}\right.}{}$

input
```
int(csch(x)^3/(a+b*sinh(x)),x,method=_RETURNVERBOSE)
```

output
```
1/4/a^2*(1/2*tanh(1/2*x)^2*a+2*b*tanh(1/2*x))-1/8/a/tanh(1/2*x)^2+1/4/a^3*
(-2*a^2+4*b^2)*ln(tanh(1/2*x))+1/2/a^2*b/tanh(1/2*x)-2*b^3/a^3/(a^2+b^2)^(
1/2)*arctanh(1/2*(2*a*tanh(1/2*x)-2*b)/(a^2+b^2)^(1/2))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 929 vs. $2(73) = 146$.

Time = 0.19 (sec) , antiderivative size = 929, normalized size of antiderivative = 11.47

$$\int \frac{\operatorname{csch}^3(x)}{a + b\sinh(x)}\,dx = \text{Too large to display}$$

input
```
integrate(csch(x)^3/(a+b*sinh(x)),x, algorithm="fricas")
```

output
```
-1/2*(4*a^3*b + 4*a*b^3 + 2*(a^4 + a^2*b^2)*cosh(x)^3 + 2*(a^4 + a^2*b^2)*
sinh(x)^3 - 4*(a^3*b + a*b^3)*cosh(x)^2 - 2*(2*a^3*b + 2*a*b^3 - 3*(a^4 +
a^2*b^2)*cosh(x))*sinh(x)^2 - 2*(b^3*cosh(x)^4 + 4*b^3*cosh(x)*sinh(x)^3 +
 b^3*sinh(x)^4 - 2*b^3*cosh(x)^2 + b^3 + 2*(3*b^3*cosh(x)^2 - b^3)*sinh(x)
^2 + 4*(b^3*cosh(x)^3 - b^3*cosh(x))*sinh(x))*sqrt(a^2 + b^2)*log((b^2*cos
h(x)^2 + b^2*sinh(x)^2 + 2*a*b*cosh(x) + 2*a^2 + b^2 + 2*(b^2*cosh(x) + a*
b)*sinh(x) + 2*sqrt(a^2 + b^2)*(b*cosh(x) + b*sinh(x) + a))/(b*cosh(x)^2 +
 b*sinh(x)^2 + 2*a*cosh(x) + 2*(b*cosh(x) + a)*sinh(x) - b)) + 2*(a^4 + a^
2*b^2)*cosh(x) - ((a^4 - a^2*b^2 - 2*b^4)*cosh(x)^4 + 4*(a^4 - a^2*b^2 - 2
*b^4)*cosh(x)*sinh(x)^3 + (a^4 - a^2*b^2 - 2*b^4)*sinh(x)^4 + a^4 - a^2*b^
2 - 2*b^4 - 2*(a^4 - a^2*b^2 - 2*b^4)*cosh(x)^2 - 2*(a^4 - a^2*b^2 - 2*b^4
 - 3*(a^4 - a^2*b^2 - 2*b^4)*cosh(x)^2)*sinh(x)^2 + 4*((a^4 - a^2*b^2 - 2*
b^4)*cosh(x)^3 - (a^4 - a^2*b^2 - 2*b^4)*cosh(x))*sinh(x))*log(cosh(x) + s
inh(x) + 1) + ((a^4 - a^2*b^2 - 2*b^4)*cosh(x)^4 + 4*(a^4 - a^2*b^2 - 2*b^
4)*cosh(x)*sinh(x)^3 + (a^4 - a^2*b^2 - 2*b^4)*sinh(x)^4 + a^4 - a^2*b^2 -
 2*b^4 - 2*(a^4 - a^2*b^2 - 2*b^4)*cosh(x)^2 - 2*(a^4 - a^2*b^2 - 2*b^4 -
3*(a^4 - a^2*b^2 - 2*b^4)*cosh(x)^2)*sinh(x)^2 + 4*((a^4 - a^2*b^2 - 2*b^4
)*cosh(x)^3 - (a^4 - a^2*b^2 - 2*b^4)*cosh(x))*sinh(x))*log(cosh(x) + sinh
(x) - 1) + 2*(a^4 + a^2*b^2 + 3*(a^4 + a^2*b^2)*cosh(x)^2 - 4*(a^3*b + a*b
^3)*cosh(x))*sinh(x))/(a^5 + a^3*b^2 + (a^5 + a^3*b^2)*cosh(x)^4 + 4*(a...
```

Sympy [F]

$$\int \frac{\operatorname{csch}^3(x)}{a+b\sinh(x)}\,dx=\int \frac{\operatorname{csch}^3(x)}{a+b\sinh(x)}\,dx$$

input
```
integrate(csch(x)**3/(a+b*sinh(x)),x)
```

output
```
Integral(csch(x)**3/(a + b*sinh(x)), x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 154 vs. 2(73) = 146.

Time = 0.12 (sec) , antiderivative size = 154, normalized size of antiderivative = 1.90

$$\int \frac{\operatorname{csch}^3(x)}{a+b\sinh(x)}\,dx=-\frac{b^3\log\left(\frac{be^{(-x)}-a-\sqrt{a^2+b^2}}{be^{(-x)}-a+\sqrt{a^2+b^2}}\right)}{\sqrt{a^2+b^2}a^3}+\frac{ae^{(-x)}+2\,be^{(-2\,x)}+ae^{(-3\,x)}-2\,b}{2\,a^2e^{(-2\,x)}-a^2e^{(-4\,x)}-a^2}+\frac{(a^2-2\,b^2)\log\left(e^{(-x)}+1\right)}{2\,a^3}-\frac{(a^2-2\,b^2)\log\left(e^{(-x)}-1\right)}{2\,a^3}$$

input
```
integrate(csch(x)^3/(a+b*sinh(x)),x, algorithm="maxima")
```

output
```
-b^3*log((b*e^(-x) - a - sqrt(a^2 + b^2))/(b*e^(-x) - a + sqrt(a^2 + b^2))
)/(sqrt(a^2 + b^2)*a^3) + (a*e^(-x) + 2*b*e^(-2*x) + a*e^(-3*x) - 2*b)/(2*
a^2*e^(-2*x) - a^2*e^(-4*x) - a^2) + 1/2*(a^2 - 2*b^2)*log(e^(-x) + 1)/a^3
 - 1/2*(a^2 - 2*b^2)*log(e^(-x) - 1)/a^3
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 137, normalized size of antiderivative = 1.69

$$\int \frac{\operatorname{csch}^3(x)}{a+b\sinh(x)}\,dx = -\frac{b^3\log\left(\frac{|2\,be^x+2\,a-2\,\sqrt{a^2+b^2}|}{|2\,be^x+2\,a+2\,\sqrt{a^2+b^2}|}\right)}{\sqrt{a^2+b^2}a^3} + \frac{(a^2-2\,b^2)\log\left(e^x+1\right)}{2\,a^3} - \frac{(a^2-2\,b^2)\log\left(|e^x-1|\right)}{2\,a^3} - \frac{ae^{(3\,x)}-2\,be^{(2\,x)}+ae^x+2\,b}{a^2\left(e^{(2\,x)}-1\right)^2}$$

input
```
integrate(csch(x)^3/(a+b*sinh(x)),x, algorithm="giac")
```

output
```
-b^3*log(abs(2*b*e^x + 2*a - 2*sqrt(a^2 + b^2))/abs(2*b*e^x + 2*a + 2*sqrt
(a^2 + b^2)))/(sqrt(a^2 + b^2)*a^3) + 1/2*(a^2 - 2*b^2)*log(e^x + 1)/a^3 -
 1/2*(a^2 - 2*b^2)*log(abs(e^x - 1))/a^3 - (a*e^(3*x) - 2*b*e^(2*x) + a*e^
x + 2*b)/(a^2*(e^(2*x) - 1)^2)
```

Mupad [B] (verification not implemented)

Time = 2.34 (sec) , antiderivative size = 617, normalized size of antiderivative = 7.62

$$\int \frac{\operatorname{csch}^3(x)}{a+b\sinh(x)}\,dx = \frac{\mathrm{e}^x}{a-a\,\mathrm{e}^{2\,x}} - \frac{2\,\mathrm{e}^x}{a-2\,a\,\mathrm{e}^{2\,x}+a\,\mathrm{e}^{4\,x}} - \frac{\ln\left(4\,a^4+24\,b^4-20\,a^2\,b^2-4\,a^4\,\mathrm{e}^x-24\,b^4\,\mathrm{e}^x+20\,a^2\,b^2\,\mathrm{e}^x\right)}{2\,a} + \frac{\ln\left(4\,a^4+24\,b^4-20\,a^2\,b^2+4\,a^4\,\mathrm{e}^x+24\,b^4\,\mathrm{e}^x-20\,a^2\,b^2\,\mathrm{e}^x\right)}{2\,a} + \frac{2\,b}{a^2\,\mathrm{e}^{2\,x}-a^2} + \frac{b^2\,\ln\left(4\,a^4+24\,b^4-20\,a^2\,b^2-4\,a^4\,\mathrm{e}^x-24\,b^4\,\mathrm{e}^x+20\,a^2\,b^2\,\mathrm{e}^x\right)}{a^3} - \frac{b^2\,\ln\left(4\,a^4+24\,b^4-20\,a^2\,b^2+4\,a^4\,\mathrm{e}^x+24\,b^4\,\mathrm{e}^x-20\,a^2\,b^2\,\mathrm{e}^x\right)}{a^3} - \frac{b^3\,\ln\left(16\,a^5\,b-48\,a\,b^5-24\,b^5\,\sqrt{a^2+b^2}-32\,a^3\,b^3-32\,a^6\,\mathrm{e}^x+24\,b^6\,\mathrm{e}^x-40\,a^2\,b^3\,\sqrt{a^2+b^2}-32\,a^5\,\mathrm{e}^x\,\sqrt{}\right.}{a} + \frac{b^3\,\ln\left(24\,b^5\,\sqrt{a^2+b^2}-48\,a\,b^5+16\,a^5\,b-32\,a^3\,b^3-32\,a^6\,\mathrm{e}^x+24\,b^6\,\mathrm{e}^x+40\,a^2\,b^3\,\sqrt{a^2+b^2}+32\,a^5\,\mathrm{e}^x\,\sqrt{}\right.}{a}$$

input
```
int(1/(sinh(x)^3*(a + b*sinh(x))),x)
```

output
```
exp(x)/(a - a*exp(2*x)) - (2*exp(x))/(a - 2*a*exp(2*x) + a*exp(4*x)) - log
(4*a^4 + 24*b^4 - 20*a^2*b^2 - 4*a^4*exp(x) - 24*b^4*exp(x) + 20*a^2*b^2*e
xp(x))/(2*a) + log(4*a^4 + 24*b^4 - 20*a^2*b^2 + 4*a^4*exp(x) + 24*b^4*exp
(x) - 20*a^2*b^2*exp(x))/(2*a) + (2*b)/(a^2*exp(2*x) - a^2) + (b^2*log(4*a
^4 + 24*b^4 - 20*a^2*b^2 - 4*a^4*exp(x) - 24*b^4*exp(x) + 20*a^2*b^2*exp(x
)))/a^3 - (b^2*log(4*a^4 + 24*b^4 - 20*a^2*b^2 + 4*a^4*exp(x) + 24*b^4*exp
(x) - 20*a^2*b^2*exp(x)))/a^3 - (b^3*log(16*a^5*b - 48*a*b^5 - 24*b^5*(a^2
 + b^2)^(1/2) - 32*a^3*b^3 - 32*a^6*exp(x) + 24*b^6*exp(x) - 40*a^2*b^3*(a
^2 + b^2)^(1/2) - 32*a^5*exp(x)*(a^2 + b^2)^(1/2) + 112*a^2*b^4*exp(x) + 5
6*a^4*b^2*exp(x) + 16*a^4*b*(a^2 + b^2)^(1/2) + 72*a*b^4*exp(x)*(a^2 + b^2
)^(1/2) + 72*a^3*b^2*exp(x)*(a^2 + b^2)^(1/2))*(a^2 + b^2)^(1/2))/(a^5 + a
^3*b^2) + (b^3*log(24*b^5*(a^2 + b^2)^(1/2) - 48*a*b^5 + 16*a^5*b - 32*a^3
*b^3 - 32*a^6*exp(x) + 24*b^6*exp(x) + 40*a^2*b^3*(a^2 + b^2)^(1/2) + 32*a
^5*exp(x)*(a^2 + b^2)^(1/2) + 112*a^2*b^4*exp(x) + 56*a^4*b^2*exp(x) - 16*
a^4*b*(a^2 + b^2)^(1/2) - 72*a*b^4*exp(x)*(a^2 + b^2)^(1/2) - 72*a^3*b^2*e
xp(x)*(a^2 + b^2)^(1/2))*(a^2 + b^2)^(1/2))/(a^5 + a^3*b^2)
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 519, normalized size of antiderivative = 6.41

$$\int \frac{\operatorname{csch}^3(x)}{a+b\sinh(x)}\,dx = \frac{-\log(e^x-1)\,a^4+\log(e^x+1)\,a^4-2e^{3x}a^4-2e^xa^4+2\log(e^x-1)\,b^4-2\log(e^x+1)\,b^4-4e^{4x}\sqrt{a^2+b^2}\,a}{}$$

input
```
int(csch(x)^3/(a+b*sinh(x)),x)
```

output
```
( - 4*e**(4*x)*sqrt(a**2 + b**2)*atan((e**x*b*i + a*i)/sqrt(a**2 + b**2))*
b**3*i + 8*e**(2*x)*sqrt(a**2 + b**2)*atan((e**x*b*i + a*i)/sqrt(a**2 + b*
*2))*b**3*i - 4*sqrt(a**2 + b**2)*atan((e**x*b*i + a*i)/sqrt(a**2 + b**2))
*b**3*i - e**(4*x)*log(e**x - 1)*a**4 + e**(4*x)*log(e**x - 1)*a**2*b**2 +
 2*e**(4*x)*log(e**x - 1)*b**4 + e**(4*x)*log(e**x + 1)*a**4 - e**(4*x)*lo
g(e**x + 1)*a**2*b**2 - 2*e**(4*x)*log(e**x + 1)*b**4 + 2*e**(4*x)*a**3*b
+ 2*e**(4*x)*a*b**3 - 2*e**(3*x)*a**4 - 2*e**(3*x)*a**2*b**2 + 2*e**(2*x)*
log(e**x - 1)*a**4 - 2*e**(2*x)*log(e**x - 1)*a**2*b**2 - 4*e**(2*x)*log(e
**x - 1)*b**4 - 2*e**(2*x)*log(e**x + 1)*a**4 + 2*e**(2*x)*log(e**x + 1)*a
**2*b**2 + 4*e**(2*x)*log(e**x + 1)*b**4 - 2*e**x*a**4 - 2*e**x*a**2*b**2
- log(e**x - 1)*a**4 + log(e**x - 1)*a**2*b**2 + 2*log(e**x - 1)*b**4 + lo
g(e**x + 1)*a**4 - log(e**x + 1)*a**2*b**2 - 2*log(e**x + 1)*b**4 - 2*a**3
*b - 2*a*b**3)/(2*a**3*(e**(4*x)*a**2 + e**(4*x)*b**2 - 2*e**(2*x)*a**2 -
2*e**(2*x)*b**2 + a**2 + b**2))
```

3.79 $\int \frac{\operatorname{csch}^4(x)}{a+b\sinh(x)}\,dx$

Optimal result

Integrand size = 13, antiderivative size = 109

$$\int \frac{\operatorname{csch}^4(x)}{a+b\sinh(x)}\,dx = -\frac{b(a^2-2b^2)\operatorname{arctanh}(\cosh(x))}{2a^4} - \frac{2b^4\operatorname{arctanh}\left(\frac{b-a\tanh\left(\frac{x}{2}\right)}{\sqrt{a^2+b^2}}\right)}{a^4\sqrt{a^2+b^2}} + \frac{(2a^2-3b^2)\coth(x)}{3a^3} + \frac{b\coth(x)\operatorname{csch}(x)}{2a^2} - \frac{\coth(x)\operatorname{csch}^2(x)}{3a}$$

output
```
-1/2*b*(a^2-2*b^2)*arctanh(cosh(x))/a^4-2*b^4*arctanh((b-a*tanh(1/2*x))/(a
^2+b^2)^(1/2))/a^4/(a^2+b^2)^(1/2)+1/3*(2*a^2-3*b^2)*coth(x)/a^3+1/2*b*cot
h(x)*csch(x)/a^2-1/3*coth(x)*csch(x)^2/a
```

Mathematica [A] (verified)

Time = 0.93 (sec) , antiderivative size = 211, normalized size of antiderivative = 1.94

$$\int \frac{\operatorname{csch}^4(x)}{a+b\sinh(x)}\,dx$$
$$= \frac{\frac{48b^4\arctan\left(\frac{b-a\tanh\left(\frac{x}{2}\right)}{\sqrt{-a^2-b^2}}\right)}{\sqrt{-a^2-b^2}} + 4a(2a^2-3b^2)\coth\left(\frac{x}{2}\right) + 3a^2b\operatorname{csch}^2\left(\frac{x}{2}\right) - 12a^2b\log\left(\cosh\left(\frac{x}{2}\right)\right) + 24b^3\log\left(\cosh\left(\right.\right.}{}$$

input
```
Integrate[Csch[x]^4/(a + b*Sinh[x]),x]
```

output
```
((48*b^4*ArcTan[(b - a*Tanh[x/2])/Sqrt[-a^2 - b^2]])/Sqrt[-a^2 - b^2] + 4*
a*(2*a^2 - 3*b^2)*Coth[x/2] + 3*a^2*b*Csch[x/2]^2 - 12*a^2*b*Log[Cosh[x/2]
] + 24*b^3*Log[Cosh[x/2]] + 12*a^2*b*Log[Sinh[x/2]] - 24*b^3*Log[Sinh[x/2]
] + 3*a^2*b*Sech[x/2]^2 + 8*a^3*Csch[x]^3*Sinh[x/2]^4 - (a^3*Csch[x/2]^4*S
inh[x])/2 + 8*a^3*Tanh[x/2] - 12*a*b^2*Tanh[x/2])/(24*a^4)
```

Rubi [C] (verified)

Result contains complex when optimal does not.

Time = 1.02 (sec) , antiderivative size = 142, normalized size of antiderivative = 1.30, number of steps used = 21, number of rules used = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 1.538$, Rules used = {3042, 3281, 25, 3042, 26, 3534, 3042, 25, 3534, 27, 3042, 26, 3480, 26, 3042, 26, 3139, 1083, 219, 4257}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\operatorname{csch}^4(x)}{a+b\sinh(x)}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{\sin(ix)^4(a-ib\sin(ix))}dx$$

$$\Big\downarrow \text{3281}$$

$$\frac{\int -\frac{\operatorname{csch}^3(x)\left(2b\sinh^2(x)+2a\sinh(x)+3b\right)}{a+b\sinh(x)}dx}{3a}-\frac{\coth(x)\operatorname{csch}^2(x)}{3a}$$

$$\Big\downarrow \text{25}$$

$$-\frac{\int \frac{\operatorname{csch}^3(x)\left(2b\sinh^2(x)+2a\sinh(x)+3b\right)}{a+b\sinh(x)}dx}{3a}-\frac{\coth(x)\operatorname{csch}^2(x)}{3a}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{\coth(x)\operatorname{csch}^2(x)}{3a}-\frac{\int -\frac{i\left(-2b\sin(ix)^2-2ia\sin(ix)+3b\right)}{\sin(ix)^3(a-ib\sin(ix))}dx}{3a}$$

$$\Big\downarrow \color{blue}{26}$$

$$-\frac{\coth(x)\text{csch}^2(x)}{3a}+\frac{i\int \frac{-2b\sin(ix)^2-2ia\sin(ix)+3b}{\sin(ix)^3(a-ib\sin(ix))}dx}{3a}$$

$$\Big\downarrow \color{blue}{3534}$$

$$-\frac{\coth(x)\text{csch}^2(x)}{3a}+\frac{i\left(\frac{\int \frac{\text{csch}^2(x)\left(-3ib^2\sinh^2(x)+iab\sinh(x)+2\left(2ia^2-3ib^2\right)\right)}{a+b\sinh(x)}dx}{2a}-\frac{3ib\coth(x)\text{csch}(x)}{2a}\right)}{3a}$$

$$\Big\downarrow \color{blue}{3042}$$

$$-\frac{\coth(x)\text{csch}^2(x)}{3a}+\frac{i\left(\frac{\int -\frac{3ib^2\sin(ix)^2+ab\sin(ix)+2\left(2ia^2-3ib^2\right)}{\sin(ix)^2(a-ib\sin(ix))}dx}{2a}-\frac{3ib\coth(x)\text{csch}(x)}{2a}\right)}{3a}$$

$$\Big\downarrow \color{blue}{25}$$

$$-\frac{\coth(x)\text{csch}^2(x)}{3a}+\frac{i\left(-\frac{\int \frac{3ib^2\sin(ix)^2+ab\sin(ix)+2i\left(2a^2-3b^2\right)}{\sin(ix)^2(a-ib\sin(ix))}dx}{2a}-\frac{3ib\coth(x)\text{csch}(x)}{2a}\right)}{3a}$$

$$\Big\downarrow \color{blue}{3534}$$

$$-\frac{\coth(x)\text{csch}^2(x)}{3a}+\frac{i\left(-\frac{\frac{\int \frac{3i\text{csch}(x)\left(a\sinh(x)b^2+\left(a^2-2b^2\right)b\right)}{a+b\sinh(x)}dx}{a}+\frac{2i\left(2a^2-3b^2\right)\coth(x)}{a}}{2a}-\frac{3ib\coth(x)\text{csch}(x)}{2a}\right)}{3a}$$

$$\Big\downarrow \color{blue}{27}$$

$$-\frac{\coth(x)\text{csch}^2(x)}{3a}+\frac{i\left(-\frac{\frac{3i\int \frac{\text{csch}(x)\left(a\sinh(x)b^2+\left(a^2-2b^2\right)b\right)}{a+b\sinh(x)}dx}{a}+\frac{2i\left(2a^2-3b^2\right)\coth(x)}{a}}{2a}-\frac{3ib\coth(x)\text{csch}(x)}{2a}\right)}{3a}$$

$$\Big\downarrow \color{blue}{3042}$$

$$-\frac{\coth(x)\text{csch}^2(x)}{3a}+\frac{i\left(-\frac{\frac{3i\int \frac{i\left(b\left(a^2-2b^2\right)-iab^2\sin(ix)\right)}{\sin(ix)(a-ib\sin(ix))}dx}{a}+\frac{2i\left(2a^2-3b^2\right)\coth(x)}{a}}{2a}-\frac{3ib\coth(x)\text{csch}(x)}{2a}\right)}{3a}$$

$$\Big\downarrow \color{blue}{26}$$

$$-\frac{\coth(x)\text{csch}^2(x)}{3a}+\frac{i\left(-\frac{\frac{2i\left(2a^2-3b^2\right)\coth(x)}{a}-\frac{3\int\frac{b\left(a^2-2b^2\right)-iab^2\sin(ix)}{\sin(ix)(a-ib\sin(ix))}dx}{a}}{2a}-\frac{3ib\coth(x)\text{csch}(x)}{2a}\right)}{3a}$$

$\Big\downarrow$ 3480

$$-\frac{\coth(x)\text{csch}^2(x)}{3a}+\frac{i\left(-\frac{\frac{2i\left(2a^2-3b^2\right)\coth(x)}{a}-\frac{3\left(\frac{b\left(a^2-2b^2\right)\int-i\text{csch}(x)dx}{a}-\frac{2ib^4\int\frac{1}{a+b\sinh(x)}dx}{a}\right)}{a}}{2a}-\frac{3ib\coth(x)\text{csch}(x)}{2a}\right)}{3a}$$

$\Big\downarrow$ 26

$$-\frac{\coth(x)\text{csch}^2(x)}{3a}+\frac{i\left(-\frac{\frac{2i\left(2a^2-3b^2\right)\coth(x)}{a}-\frac{3\left(-\frac{ib\left(a^2-2b^2\right)\int\text{csch}(x)dx}{a}-\frac{2ib^4\int\frac{1}{a+b\sinh(x)}dx}{a}\right)}{a}}{2a}-\frac{3ib\coth(x)\text{csch}(x)}{2a}\right)}{3a}$$

$\Big\downarrow$ 3042

$$-\frac{\coth(x)\text{csch}^2(x)}{3a}+\frac{i\left(-\frac{\frac{2i\left(2a^2-3b^2\right)\coth(x)}{a}-\frac{3\left(-\frac{ib\left(a^2-2b^2\right)\int i\csc(ix)dx}{a}-\frac{2ib^4\int\frac{1}{a-ib\sin(ix)}dx}{a}\right)}{a}}{2a}-\frac{3ib\coth(x)\text{csch}(x)}{2a}\right)}{3a}$$

$\Big\downarrow$ 26

$$-\frac{\coth(x)\text{csch}^2(x)}{3a}+\frac{i\left(-\frac{\frac{2i\left(2a^2-3b^2\right)\coth(x)}{a}-\frac{3\left(\frac{b\left(a^2-2b^2\right)\int\csc(ix)dx}{a}-\frac{2ib^4\int\frac{1}{a-ib\sin(ix)}dx}{a}\right)}{a}}{2a}-\frac{3ib\coth(x)\text{csch}(x)}{2a}\right)}{3a}$$

$\Big\downarrow$ 3139

$$-\frac{\coth(x)\operatorname{csch}^2(x)}{3a}+\frac{i\left(-\frac{\frac{2i(2a^2-3b^2)\coth(x)}{a}-\frac{3\left(\frac{b(a^2-2b^2)\int\csc(ix)dx}{a}-\frac{4ib^4\int\frac{1}{-a\tanh^2\left(\frac{x}{2}\right)+2b\tanh\left(\frac{x}{2}\right)+a}d\tanh\left(\frac{x}{2}\right)}{a}\right)}{a}}{2a}-\frac{3ib\coth(x)\operatorname{csch}(x)}{2a}\right)}{3a}$$

$\Big\downarrow$ 1083

$$-\frac{\coth(x)\operatorname{csch}^2(x)}{3a}+\frac{i\left(-\frac{\frac{2i(2a^2-3b^2)\coth(x)}{a}-\frac{3\left(\frac{b(a^2-2b^2)\int\csc(ix)dx}{a}+\frac{8ib^4\int\frac{1}{4(a^2+b^2)-\left(2b-2a\tanh\left(\frac{x}{2}\right)\right)^2}d\left(2b-2a\tanh\left(\frac{x}{2}\right)\right)}{a}\right)}{a}}{2a}-\frac{3ib\coth(x)\operatorname{csch}(x)}{2a}\right)}{3a}$$

$\Big\downarrow$ 219

$$-\frac{\coth(x)\operatorname{csch}^2(x)}{3a}+\frac{i\left(-\frac{\frac{2i(2a^2-3b^2)\coth(x)}{a}-\frac{3\left(\frac{b(a^2-2b^2)\int\csc(ix)dx}{a}+\frac{4ib^4\operatorname{arctanh}\left(\frac{2b-2a\tanh\left(\frac{x}{2}\right)}{2\sqrt{a^2+b^2}}\right)}{a\sqrt{a^2+b^2}}\right)}{a}}{2a}-\frac{3ib\coth(x)\operatorname{csch}(x)}{2a}\right)}{3a}$$

$\Big\downarrow$ 4257

$$-\frac{\coth(x)\operatorname{csch}^2(x)}{3a}+\frac{i\left(-\frac{\frac{2i(2a^2-3b^2)\coth(x)}{a}-\frac{3\left(\frac{ib(a^2-2b^2)\operatorname{arctanh}(\cosh(x))}{a}+\frac{4ib^4\operatorname{arctanh}\left(\frac{2b-2a\tanh\left(\frac{x}{2}\right)}{2\sqrt{a^2+b^2}}\right)}{a\sqrt{a^2+b^2}}\right)}{a}}{2a}-\frac{3ib\coth(x)\operatorname{csch}(x)}{2a}\right)}{3a}$$

input
```
Int[Csch[x]^4/(a + b*Sinh[x]),x]
```

output
```
-1/3*(Coth[x]*Csch[x]^2)/a + ((I/3)*(-1/2*((-3*((I*b*(a^2 - 2*b^2)*ArcTanh
[Cosh[x]])/a + ((4*I)*b^4*ArcTanh[(2*b - 2*a*Tanh[x/2])/(2*Sqrt[a^2 + b^2]
)])/(a*Sqrt[a^2 + b^2])))/a + ((2*I)*(2*a^2 - 3*b^2)*Coth[x])/a)/a - (((3*
I)/2)*b*Coth[x]*Csch[x])/a))/a
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1083
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Simp[-2   Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3139
```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Simp[2*(e/d)   Subst[Int[1/(a + 2*b*e*x + a
*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[a^2 - b^2, 0]
```

rule 3281
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b^2)*Cos[e + f*x]*(a + b*Sin[e + f*
x])^(m + 1)*((c + d*Sin[e + f*x])^(n + 1)/(f*(m + 1)*(b*c - a*d)*(a^2 - b^2
))), x] + Simp[1/((m + 1)*(b*c - a*d)*(a^2 - b^2))   Int[(a + b*Sin[e + f*x
])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[a*(b*c - a*d)*(m + 1) + b^2*d*(m + n
 + 2) - (b^2*c + b*(b*c - a*d)*(m + 1))*Sin[e + f*x] - b^2*d*(m + n + 3)*Si
n[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*
d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -1] && IntegersQ[
2*m, 2*n] && ((EqQ[a, 0] && IntegerQ[m] &&  !IntegerQ[n]) ||  !(IntegerQ[2*
n] && LtQ[n, -1] && ((IntegerQ[n] &&  !IntegerQ[m]) || EqQ[a, 0])))
```

rule 3480
```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((a_.) + (b_.)*sin[(e_.) + (f_
.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])), x_Symbol] :> Simp[(A*b
- a*B)/(b*c - a*d)   Int[1/(a + b*Sin[e + f*x]), x], x] + Simp[(B*c - A*d)/
(b*c - a*d)   Int[1/(c + d*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f
, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

rule 3534
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-(A*b^2 - a*b*B + a^2*C))*Cos[e + f*x
]*(a + b*Sin[e + f*x])^(m + 1)*((c + d*Sin[e + f*x])^(n + 1)/(f*(m + 1)*(b*
c - a*d)*(a^2 - b^2))), x] + Simp[1/((m + 1)*(b*c - a*d)*(a^2 - b^2))   Int
[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[(m + 1)*(b*c - a*
d)*(a*A - b*B + a*C) + d*(A*b^2 - a*b*B + a^2*C)*(m + n + 2) - (c*(A*b^2 -
a*b*B + a^2*C) + (m + 1)*(b*c - a*d)*(A*b - a*B + b*C))*Sin[e + f*x] - d*(A
*b^2 - a*b*B + a^2*C)*(m + n + 3)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b
, c, d, e, f, A, B, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] &&
NeQ[c^2 - d^2, 0] && LtQ[m, -1] && ((EqQ[a, 0] && IntegerQ[m] &&  !IntegerQ
[n]) ||  !(IntegerQ[2*n] && LtQ[n, -1] && ((IntegerQ[n] &&  !IntegerQ[m]) |
| EqQ[a, 0])))
```

rule 4257
```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Maple [A] (verified)

Time = 0.34 (sec) , antiderivative size = 151, normalized size of antiderivative = 1.39

method	result
default	$-\frac{\frac{\tanh\left(\frac{x}{2}\right)^{3} a^{2}}{3}+\tanh\left(\frac{x}{2}\right)^{2} a b-3\tanh\left(\frac{x}{2}\right) a^{2}+4 b^{2}\tanh\left(\frac{x}{2}\right)}{8 a^{3}}-\frac{1}{24 a\tanh\left(\frac{x}{2}\right)^{3}}-\frac{-3 a^{2}+4 b^{2}}{8 a^{3}\tanh\left(\frac{x}{2}\right)}+\frac{b}{8 a^{2}\tanh\left(\frac{x}{2}\right)^{2}}+\frac{b\left(a^{2}-2 b^{2}\right)\ln}{2 a^{4}}$
risch	$-\frac{-3 a b\,\mathrm{e}^{5 x}+6 b^{2}\mathrm{e}^{4 x}+12 a^{2}\mathrm{e}^{2 x}-12 b^{2}\mathrm{e}^{2 x}+3 b\,\mathrm{e}^{x} a-4 a^{2}+6 b^{2}}{3 a^{3}\left(\mathrm{e}^{2 x}-1\right)^{3}}-\frac{b\ln\left(\mathrm{e}^{x}+1\right)}{2 a^{2}}+\frac{b^{3}\ln\left(\mathrm{e}^{x}+1\right)}{a^{4}}+\frac{b^{4}\ln\left(\mathrm{e}^{x}+\frac{a\sqrt{a^{2}+b^{2}}-a^{2}-b^{2}}{\sqrt{a^{2}+b^{2}}\, b}\right)}{\sqrt{a^{2}+b^{2}}\, a^{4}}-$

input
```
int(csch(x)^4/(a+b*sinh(x)),x,method=_RETURNVERBOSE)
```

output
```
-1/8/a^3*(1/3*tanh(1/2*x)^3*a^2+tanh(1/2*x)^2*a*b-3*tanh(1/2*x)*a^2+4*b^2*
tanh(1/2*x))-1/24/a/tanh(1/2*x)^3-1/8/a^3*(-3*a^2+4*b^2)/tanh(1/2*x)+1/8/a
^2*b/tanh(1/2*x)^2+1/2/a^4*b*(a^2-2*b^2)*ln(tanh(1/2*x))+2*b^4/a^4/(a^2+b^
2)^(1/2)*arctanh(1/2*(2*a*tanh(1/2*x)-2*b)/(a^2+b^2)^(1/2))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1676 vs. $2(97) = 194$.

Time = 0.18 (sec) , antiderivative size = 1676, normalized size of antiderivative = 15.38

$$\int \frac{\operatorname{csch}^4(x)}{a + b\sinh(x)}\, dx = \text{Too large to display}$$

input
```
integrate(csch(x)^4/(a+b*sinh(x)),x, algorithm="fricas")
```

output
```
1/6*(6*(a^4*b + a^2*b^3)*cosh(x)^5 + 6*(a^4*b + a^2*b^3)*sinh(x)^5 + 8*a^5
 - 4*a^3*b^2 - 12*a*b^4 - 12*(a^3*b^2 + a*b^4)*cosh(x)^4 - 6*(2*a^3*b^2 +
2*a*b^4 - 5*(a^4*b + a^2*b^3)*cosh(x))*sinh(x)^4 + 12*(5*(a^4*b + a^2*b^3)
*cosh(x)^2 - 4*(a^3*b^2 + a*b^4)*cosh(x))*sinh(x)^3 - 24*(a^5 - a*b^4)*cos
h(x)^2 - 12*(2*a^5 - 2*a*b^4 - 5*(a^4*b + a^2*b^3)*cosh(x)^3 + 6*(a^3*b^2
+ a*b^4)*cosh(x)^2)*sinh(x)^2 + 6*(b^4*cosh(x)^6 + 6*b^4*cosh(x)*sinh(x)^5
 + b^4*sinh(x)^6 - 3*b^4*cosh(x)^4 + 3*b^4*cosh(x)^2 + 3*(5*b^4*cosh(x)^2
- b^4)*sinh(x)^4 - b^4 + 4*(5*b^4*cosh(x)^3 - 3*b^4*cosh(x))*sinh(x)^3 + 3
*(5*b^4*cosh(x)^4 - 6*b^4*cosh(x)^2 + b^4)*sinh(x)^2 + 6*(b^4*cosh(x)^5 -
2*b^4*cosh(x)^3 + b^4*cosh(x))*sinh(x))*sqrt(a^2 + b^2)*log((b^2*cosh(x)^2
 + b^2*sinh(x)^2 + 2*a*b*cosh(x) + 2*a^2 + b^2 + 2*(b^2*cosh(x) + a*b)*sin
h(x) - 2*sqrt(a^2 + b^2)*(b*cosh(x) + b*sinh(x) + a))/(b*cosh(x)^2 + b*sin
h(x)^2 + 2*a*cosh(x) + 2*(b*cosh(x) + a)*sinh(x) - b)) - 6*(a^4*b + a^2*b^
3)*cosh(x) - 3*((a^4*b - a^2*b^3 - 2*b^5)*cosh(x)^6 + 6*(a^4*b - a^2*b^3 -
 2*b^5)*cosh(x)*sinh(x)^5 + (a^4*b - a^2*b^3 - 2*b^5)*sinh(x)^6 - a^4*b +
a^2*b^3 + 2*b^5 - 3*(a^4*b - a^2*b^3 - 2*b^5)*cosh(x)^4 - 3*(a^4*b - a^2*b
^3 - 2*b^5 - 5*(a^4*b - a^2*b^3 - 2*b^5)*cosh(x)^2)*sinh(x)^4 + 4*(5*(a^4*
b - a^2*b^3 - 2*b^5)*cosh(x)^3 - 3*(a^4*b - a^2*b^3 - 2*b^5)*cosh(x))*sinh
(x)^3 + 3*(a^4*b - a^2*b^3 - 2*b^5)*cosh(x)^2 + 3*(a^4*b - a^2*b^3 - 2*b^5
 + 5*(a^4*b - a^2*b^3 - 2*b^5)*cosh(x)^4 - 6*(a^4*b - a^2*b^3 - 2*b^5)*...
```

Sympy [F]

$$\int \frac{\operatorname{csch}^4(x)}{a+b\sinh(x)}\,dx=\int \frac{\operatorname{csch}^4(x)}{a+b\sinh(x)}\,dx$$

input
```
integrate(csch(x)**4/(a+b*sinh(x)),x)
```

output
```
Integral(csch(x)**4/(a + b*sinh(x)), x)
```

Maxima [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 194, normalized size of antiderivative = 1.78

$$\int \frac{\operatorname{csch}^4(x)}{a+b\sinh(x)}\,dx = \frac{b^4\log\left(\frac{be^{(-x)}-a-\sqrt{a^2+b^2}}{be^{(-x)}-a+\sqrt{a^2+b^2}}\right)}{\sqrt{a^2+b^2}a^4} - \frac{3\,abe^{(-x)}-6\,b^2e^{(-4\,x)}-3\,abe^{(-5\,x)}+4\,a^2-6\,b^2-12\,(a^2-b^2)e^{(-2\,x)}}{3\,(3\,a^3e^{(-2\,x)}-3\,a^3e^{(-4\,x)}+a^3e^{(-6\,x)}-a^3)} - \frac{(a^2b-2\,b^3)\log\left(e^{(-x)}+1\right)}{2\,a^4} + \frac{(a^2b-2\,b^3)\log\left(e^{(-x)}-1\right)}{2\,a^4}$$

input
```
integrate(csch(x)^4/(a+b*sinh(x)),x, algorithm="maxima")
```

output
```
b^4*log((b*e^(-x) - a - sqrt(a^2 + b^2))/(b*e^(-x) - a + sqrt(a^2 + b^2)))
/(sqrt(a^2 + b^2)*a^4) - 1/3*(3*a*b*e^(-x) - 6*b^2*e^(-4*x) - 3*a*b*e^(-5*
x) + 4*a^2 - 6*b^2 - 12*(a^2 - b^2)*e^(-2*x))/(3*a^3*e^(-2*x) - 3*a^3*e^(-
4*x) + a^3*e^(-6*x) - a^3) - 1/2*(a^2*b - 2*b^3)*log(e^(-x) + 1)/a^4 + 1/2
*(a^2*b - 2*b^3)*log(e^(-x) - 1)/a^4
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 171, normalized size of antiderivative = 1.57

$$\int \frac{\operatorname{csch}^4(x)}{a+b\sinh(x)}\,dx = \frac{b^4\log\left(\frac{\left|2\,be^x+2\,a-2\,\sqrt{a^2+b^2}\right|}{\left|2\,be^x+2\,a+2\,\sqrt{a^2+b^2}\right|}\right)}{\sqrt{a^2+b^2}a^4} - \frac{(a^2b-2\,b^3)\log\left(e^x+1\right)}{2\,a^4} + \frac{(a^2b-2\,b^3)\log\left(|e^x-1|\right)}{2\,a^4} + \frac{3\,abe^{(5\,x)}-6\,b^2e^{(4\,x)}-12\,a^2e^{(2\,x)}+12\,b^2e^{(2\,x)}-3\,abe^x+4\,a^2-6\,b^2}{3\,a^3\left(e^{(2\,x)}-1\right)^3}$$

input
```
integrate(csch(x)^4/(a+b*sinh(x)),x, algorithm="giac")
```

output
```
b^4*log(abs(2*b*e^x + 2*a - 2*sqrt(a^2 + b^2))/abs(2*b*e^x + 2*a + 2*sqrt(
a^2 + b^2)))/(sqrt(a^2 + b^2)*a^4) - 1/2*(a^2*b - 2*b^3)*log(e^x + 1)/a^4
+ 1/2*(a^2*b - 2*b^3)*log(abs(e^x - 1))/a^4 + 1/3*(3*a*b*e^(5*x) - 6*b^2*e
^(4*x) - 12*a^2*e^(2*x) + 12*b^2*e^(2*x) - 3*a*b*e^x + 4*a^2 - 6*b^2)/(a^3
*(e^(2*x) - 1)^3)
```

Mupad [B] (verification not implemented)

Time = 2.13 (sec) , antiderivative size = 694, normalized size of antiderivative = 6.37

$$\int \frac{\operatorname{csch}^4(x)}{a+b\sinh(x)}\,dx = \frac{8}{3\,\left(a-3\,a\,e^{2\,x}+3\,a\,e^{4\,x}-a\,e^{6\,x}\right)}-\frac{4}{a-2\,a\,e^{2\,x}+a\,e^{4\,x}}$$
$$-\frac{2\,b^2}{a^3\,e^{2\,x}-a^3}+\frac{b\,\ln\left(4\,a^4+24\,b^4-20\,a^2\,b^2-4\,a^4\,e^x-24\,b^4\,e^x+20\,a^2\,b^2\,e^x\right)}{2\,a^2}$$
$$-\frac{b\,\ln\left(4\,a^4+24\,b^4-20\,a^2\,b^2+4\,a^4\,e^x+24\,b^4\,e^x-20\,a^2\,b^2\,e^x\right)}{2\,a^2}$$
$$-\frac{b^3\,\ln\left(4\,a^4+24\,b^4-20\,a^2\,b^2-4\,a^4\,e^x-24\,b^4\,e^x+20\,a^2\,b^2\,e^x\right)}{a^4}$$
$$+\frac{b^3\,\ln\left(4\,a^4+24\,b^4-20\,a^2\,b^2+4\,a^4\,e^x+24\,b^4\,e^x-20\,a^2\,b^2\,e^x\right)}{a^4}$$
$$+\frac{2\,b\,e^x}{a^2\,e^{4\,x}-2\,a^2\,e^{2\,x}+a^2}+\frac{b\,e^x}{a^2\,e^{2\,x}-a^2}$$
$$+\frac{b^4\,\ln\left(16\,a^5\,b^2-48\,a\,b^6-32\,a^3\,b^4-24\,b^6\,\sqrt{a^2+b^2}+24\,b^7\,e^x-40\,a^2\,b^4\,\sqrt{a^2+b^2}+16\,a^4\,b^2\,\sqrt{a^2+b^2}-\right.}{}$$
$$-\frac{b^4\,\ln\left(24\,b^6\,\sqrt{a^2+b^2}-48\,a\,b^6-32\,a^3\,b^4+16\,a^5\,b^2+24\,b^7\,e^x+40\,a^2\,b^4\,\sqrt{a^2+b^2}-16\,a^4\,b^2\,\sqrt{a^2+b^2}-\right.}{}$$

input
```
int(1/(sinh(x)^4*(a + b*sinh(x))),x)
```

output
```
8/(3*(a - 3*a*exp(2*x) + 3*a*exp(4*x) - a*exp(6*x))) - 4/(a - 2*a*exp(2*x)
 + a*exp(4*x)) - (2*b^2)/(a^3*exp(2*x) - a^3) + (b*log(4*a^4 + 24*b^4 - 20
*a^2*b^2 - 4*a^4*exp(x) - 24*b^4*exp(x) + 20*a^2*b^2*exp(x)))/(2*a^2) - (b
*log(4*a^4 + 24*b^4 - 20*a^2*b^2 + 4*a^4*exp(x) + 24*b^4*exp(x) - 20*a^2*b
^2*exp(x)))/(2*a^2) - (b^3*log(4*a^4 + 24*b^4 - 20*a^2*b^2 - 4*a^4*exp(x)
- 24*b^4*exp(x) + 20*a^2*b^2*exp(x)))/a^4 + (b^3*log(4*a^4 + 24*b^4 - 20*a
^2*b^2 + 4*a^4*exp(x) + 24*b^4*exp(x) - 20*a^2*b^2*exp(x)))/a^4 + (2*b*exp
(x))/(a^2*exp(4*x) - 2*a^2*exp(2*x) + a^2) + (b*exp(x))/(a^2*exp(2*x) - a^
2) + (b^4*log(16*a^5*b^2 - 48*a*b^6 - 32*a^3*b^4 - 24*b^6*(a^2 + b^2)^(1/2
) + 24*b^7*exp(x) - 40*a^2*b^4*(a^2 + b^2)^(1/2) + 16*a^4*b^2*(a^2 + b^2)^
(1/2) - 32*a^6*b*exp(x) + 112*a^2*b^5*exp(x) + 56*a^4*b^3*exp(x) + 72*a*b^
5*exp(x)*(a^2 + b^2)^(1/2) - 32*a^5*b*exp(x)*(a^2 + b^2)^(1/2) + 72*a^3*b^
3*exp(x)*(a^2 + b^2)^(1/2))*(a^2 + b^2)^(1/2))/(a^6 + a^4*b^2) - (b^4*log(
24*b^6*(a^2 + b^2)^(1/2) - 48*a*b^6 - 32*a^3*b^4 + 16*a^5*b^2 + 24*b^7*exp
(x) + 40*a^2*b^4*(a^2 + b^2)^(1/2) - 16*a^4*b^2*(a^2 + b^2)^(1/2) - 32*a^6
*b*exp(x) + 112*a^2*b^5*exp(x) + 56*a^4*b^3*exp(x) - 72*a*b^5*exp(x)*(a^2
+ b^2)^(1/2) + 32*a^5*b*exp(x)*(a^2 + b^2)^(1/2) - 72*a^3*b^3*exp(x)*(a^2
+ b^2)^(1/2))*(a^2 + b^2)^(1/2))/(a^6 + a^4*b^2)
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 735, normalized size of antiderivative = 6.74

$$\int \frac{\operatorname{csch}^4(x)}{a+b\sinh(x)}\,dx$$

$$= \frac{-24e^{2x}a^5+6\log(e^x-1)\,b^5-6\log(e^x+1)\,b^5-8a\,b^4-6e^{6x}\log(e^x-1)\,b^5+6e^{6x}\log(e^x+1)\,b^5-4e^{6x}a^3}{}$$

input
```
int(csch(x)^4/(a+b*sinh(x)),x)
```

output

```
(12*e**(6*x)*sqrt(a**2 + b**2)*atan((e**x*b*i + a*i)/sqrt(a**2 + b**2))*b*
*4*i - 36*e**(4*x)*sqrt(a**2 + b**2)*atan((e**x*b*i + a*i)/sqrt(a**2 + b**
2))*b**4*i + 36*e**(2*x)*sqrt(a**2 + b**2)*atan((e**x*b*i + a*i)/sqrt(a**2
 + b**2))*b**4*i - 12*sqrt(a**2 + b**2)*atan((e**x*b*i + a*i)/sqrt(a**2 +
b**2))*b**4*i + 3*e**(6*x)*log(e**x - 1)*a**4*b - 3*e**(6*x)*log(e**x - 1)
*a**2*b**3 - 6*e**(6*x)*log(e**x - 1)*b**5 - 3*e**(6*x)*log(e**x + 1)*a**4
*b + 3*e**(6*x)*log(e**x + 1)*a**2*b**3 + 6*e**(6*x)*log(e**x + 1)*b**5 -
4*e**(6*x)*a**3*b**2 - 4*e**(6*x)*a*b**4 + 6*e**(5*x)*a**4*b + 6*e**(5*x)*
a**2*b**3 - 9*e**(4*x)*log(e**x - 1)*a**4*b + 9*e**(4*x)*log(e**x - 1)*a**
2*b**3 + 18*e**(4*x)*log(e**x - 1)*b**5 + 9*e**(4*x)*log(e**x + 1)*a**4*b
- 9*e**(4*x)*log(e**x + 1)*a**2*b**3 - 18*e**(4*x)*log(e**x + 1)*b**5 + 9*
e**(2*x)*log(e**x - 1)*a**4*b - 9*e**(2*x)*log(e**x - 1)*a**2*b**3 - 18*e*
*(2*x)*log(e**x - 1)*b**5 - 9*e**(2*x)*log(e**x + 1)*a**4*b + 9*e**(2*x)*l
og(e**x + 1)*a**2*b**3 + 18*e**(2*x)*log(e**x + 1)*b**5 - 24*e**(2*x)*a**5
 - 12*e**(2*x)*a**3*b**2 + 12*e**(2*x)*a*b**4 - 6*e**x*a**4*b - 6*e**x*a**
2*b**3 - 3*log(e**x - 1)*a**4*b + 3*log(e**x - 1)*a**2*b**3 + 6*log(e**x -
 1)*b**5 + 3*log(e**x + 1)*a**4*b - 3*log(e**x + 1)*a**2*b**3 - 6*log(e**x
 + 1)*b**5 + 8*a**5 - 8*a*b**4)/(6*a**4*(e**(6*x)*a**2 + e**(6*x)*b**2 - 3
*e**(4*x)*a**2 - 3*e**(4*x)*b**2 + 3*e**(2*x)*a**2 + 3*e**(2*x)*b**2 - a**
2 - b**2))
```

3.80 $\int \frac{\sinh^4(x)}{(a+b\sinh(x))^2}\,dx$

Optimal result

Integrand size = 13, antiderivative size = 162

$$\int \frac{\sinh^4(x)}{(a+b\sinh(x))^2}\,dx = \frac{(6a^2-b^2)\,x}{2b^4} + \frac{2a^3(3a^2+4b^2)\operatorname{arctanh}\left(\frac{b-a\tanh\left(\frac{x}{2}\right)}{\sqrt{a^2+b^2}}\right)}{b^4\,(a^2+b^2)^{3/2}} - \frac{a(3a^2+2b^2)\cosh(x)}{b^3\,(a^2+b^2)} + \frac{(3a^2+b^2)\cosh(x)\sinh(x)}{2b^2\,(a^2+b^2)} - \frac{a^2\cosh(x)\sinh^2(x)}{b\,(a^2+b^2)\,(a+b\sinh(x))}$$

output
```
1/2*(6*a^2-b^2)*x/b^4+2*a^3*(3*a^2+4*b^2)*arctanh((b-a*tanh(1/2*x))/(a^2+b
^2)^(1/2))/b^4/(a^2+b^2)^(3/2)-a*(3*a^2+2*b^2)*cosh(x)/b^3/(a^2+b^2)+1/2*(
3*a^2+b^2)*cosh(x)*sinh(x)/b^2/(a^2+b^2)-a^2*cosh(x)*sinh(x)^2/b/(a^2+b^2)
/(a+b*sinh(x))
```

Mathematica [A] (verified)

Time = 0.94 (sec) , antiderivative size = 118, normalized size of antiderivative = 0.73

$$\int \frac{\sinh^4(x)}{(a+b\sinh(x))^2}\,dx = \frac{-2(-6a^2+b^2)\,x+\frac{8a^3(3a^2+4b^2)\arctan\left(\frac{b-a\tanh\left(\frac{x}{2}\right)}{\sqrt{-a^2-b^2}}\right)}{(-a^2-b^2)^{3/2}}-8ab\cosh(x)-\frac{4a^4b\cosh(x)}{(a^2+b^2)(a+b\sinh(x))}+b^2\sinh(2x)}{4b^4}$$

input
```
Integrate[Sinh[x]^4/(a + b*Sinh[x])^2,x]
```

output
```
(-2*(-6*a^2 + b^2)*x + (8*a^3*(3*a^2 + 4*b^2)*ArcTan[(b - a*Tanh[x/2])/Sqr
t[-a^2 - b^2]])/(-a^2 - b^2)^(3/2) - 8*a*b*Cosh[x] - (4*a^4*b*Cosh[x])/((a
^2 + b^2)*(a + b*Sinh[x])) + b^2*Sinh[2*x])/(4*b^4)
```

Rubi [C] (verified)

Result contains complex when optimal does not.

Time = 1.00 (sec) , antiderivative size = 188, normalized size of antiderivative = 1.16, number of steps used = 17, number of rules used = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 1.231$, Rules used = {3042, 3271, 26, 3042, 26, 3528, 25, 3042, 3502, 26, 3042, 3214, 3042, 3139, 1083, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sinh^4(x)}{(a+b\sinh(x))^2}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\sin(ix)^4}{(a-ib\sin(ix))^2}dx$$

$$\Big\downarrow \text{3271}$$

$$-\frac{a^2\sinh^2(x)\cosh(x)}{b(a^2+b^2)(a+b\sinh(x))}-\frac{i\int\frac{i\sinh(x)(2a^2-b\sinh(x)a+(3a^2+b^2)\sinh^2(x))}{a+b\sinh(x)}dx}{b(a^2+b^2)}$$

$\Big\downarrow$ 26

$$\frac{\int\frac{\sinh(x)(2a^2-b\sinh(x)a+(3a^2+b^2)\sinh^2(x))}{a+b\sinh(x)}dx}{b(a^2+b^2)}-\frac{a^2\sinh^2(x)\cosh(x)}{b(a^2+b^2)(a+b\sinh(x))}$$

$\Big\downarrow$ 3042

$$-\frac{a^2\sinh^2(x)\cosh(x)}{b(a^2+b^2)(a+b\sinh(x))}+\frac{\int-\frac{i\sin(ix)(2a^2+ib\sin(ix)a-(3a^2+b^2)\sin(ix)^2)}{a-ib\sin(ix)}dx}{b(a^2+b^2)}$$

$\Big\downarrow$ 26

$$-\frac{a^2\sinh^2(x)\cosh(x)}{b(a^2+b^2)(a+b\sinh(x))}-\frac{i\int\frac{\sin(ix)(2a^2+ib\sin(ix)a-(3a^2+b^2)\sin(ix)^2)}{a-ib\sin(ix)}dx}{b(a^2+b^2)}$$

$\Big\downarrow$ 3528

$$-\frac{a^2\sinh^2(x)\cosh(x)}{b(a^2+b^2)(a+b\sinh(x))}-\frac{i\left(\frac{i\int-\frac{2a\left(3a^2+2b^2\right)\sinh^2(x)-b\left(a^2-b^2\right)\sinh(x)+a\left(3a^2+b^2\right)}{a+b\sinh(x)}dx}{2b}+\frac{i(3a^2+b^2)\sinh(x)\cosh(x)}{2b}\right)}{b(a^2+b^2)}$$

$\Big\downarrow$ 25

$$-\frac{a^2\sinh^2(x)\cosh(x)}{b(a^2+b^2)(a+b\sinh(x))}-\frac{i\left(\frac{i(3a^2+b^2)\sinh(x)\cosh(x)}{2b}-\frac{i\int\frac{2a\left(3a^2+2b^2\right)\sinh^2(x)-b\left(a^2-b^2\right)\sinh(x)+a\left(3a^2+b^2\right)}{a+b\sinh(x)}dx}{2b}\right)}{b(a^2+b^2)}$$

$\Big\downarrow$ 3042

$$-\frac{a^2\sinh^2(x)\cosh(x)}{b(a^2+b^2)(a+b\sinh(x))}-\frac{i\left(\frac{i(3a^2+b^2)\sinh(x)\cosh(x)}{2b}-\frac{i\int\frac{-2a\left(3a^2+2b^2\right)\sin(ix)^2+ib\left(a^2-b^2\right)\sin(ix)+a\left(3a^2+b^2\right)}{a-ib\sin(ix)}dx}{2b}\right)}{b(a^2+b^2)}$$

$\Big\downarrow$ 3502

$$-\frac{a^2\sinh^2(x)\cosh(x)}{b(a^2+b^2)(a+b\sinh(x))}-\frac{i\left(\frac{i(3a^2+b^2)\sinh(x)\cosh(x)}{2b}-\frac{i\left(\frac{2a\left(3a^2+2b^2\right)\cosh(x)}{b}+\frac{i\int -\frac{i\left(ab\left(3a^2+b^2\right)-\left(6a^2-b^2\right)\left(a^2+b^2\right)\sinh(x)\right)}{a+b\sinh(x)}dx}{b}\right)}{2b}\right)}{b(a^2+b^2)}$$

$\Big\downarrow$ 26

$$-\frac{a^2\sinh^2(x)\cosh(x)}{b(a^2+b^2)(a+b\sinh(x))}-\frac{i\left(\frac{i(3a^2+b^2)\sinh(x)\cosh(x)}{2b}-\frac{i\left(\frac{\int \frac{ab\left(3a^2+b^2\right)-\left(6a^2-b^2\right)\left(a^2+b^2\right)\sinh(x)}{a+b\sinh(x)}dx}{b}+\frac{2a\left(3a^2+2b^2\right)\cosh(x)}{b}\right)}{2b}\right)}{b(a^2+b^2)}$$

$\Big\downarrow$ 3042

$$-\frac{a^2\sinh^2(x)\cosh(x)}{b(a^2+b^2)(a+b\sinh(x))}-\frac{i\left(\frac{i(3a^2+b^2)\sinh(x)\cosh(x)}{2b}-\frac{i\left(\frac{2a\left(3a^2+2b^2\right)\cosh(x)}{b}+\frac{\int \frac{ab\left(3a^2+b^2\right)+i\left(6a^2-b^2\right)\left(a^2+b^2\right)\sin(ix)}{a-ib\sin(ix)}dx}{b}\right)}{2b}\right)}{b(a^2+b^2)}$$

$\Big\downarrow$ 3214

$$-\frac{a^2\sinh^2(x)\cosh(x)}{b(a^2+b^2)(a+b\sinh(x))}-\frac{i\left(\frac{i(3a^2+b^2)\sinh(x)\cosh(x)}{2b}-\frac{i\left(\frac{\frac{2a^3\left(3a^2+4b^2\right)\int \frac{1}{a+b\sinh(x)}dx}{b}-\frac{x\left(6a^2-b^2\right)\left(a^2+b^2\right)}{b}}{b}+\frac{2a\left(3a^2+2b^2\right)\cosh(x)}{b}\right)}{2b}\right)}{b(a^2+b^2)}$$

$\Big\downarrow$ 3042

$$\frac{-\frac{a^2\sinh^2(x)\cosh(x)}{b(a^2+b^2)(a+b\sinh(x))}-\frac{i\left(\frac{i(3a^2+b^2)\sinh(x)\cosh(x)}{2b}-\frac{i\left(\frac{2a\left(3a^2+2b^2\right)\cosh(x)}{b}+\frac{-\frac{x\left(6a^2-b^2\right)\left(a^2+b^2\right)}{b}+\frac{2a^3\left(3a^2+4b^2\right)\int\frac{1}{a-ib\sin(ix)}dx}{b}}{b}\right)}{2b}\right)}{b(a^2+b^2)}}{}$$

$\Big\downarrow$ 3139

$$-\frac{a^2\sinh^2(x)\cosh(x)}{b(a^2+b^2)(a+b\sinh(x))}-\frac{i\left(\frac{i(3a^2+b^2)\sinh(x)\cosh(x)}{2b}-\frac{i\left(\frac{\frac{4a^3\left(3a^2+4b^2\right)\int\frac{1}{-a\tanh^2\left(\frac{x}{2}\right)+2b\tanh\left(\frac{x}{2}\right)+a}d\tanh\left(\frac{x}{2}\right)}{b}-\frac{x\left(6a^2-b^2\right)\left(a^2+b^2\right)}{b}}{b}+\frac{2a\left(3a^2+2b^2\right)\cosh(x)}{b}\right)}{2b}\right)}{b(a^2+b^2)}$$

$\Big\downarrow$ 1083

$$-\frac{a^2\sinh^2(x)\cosh(x)}{b(a^2+b^2)(a+b\sinh(x))}-\frac{i\left(\frac{i(3a^2+b^2)\sinh(x)\cosh(x)}{2b}-\frac{i\left(\frac{-\frac{8a^3\left(3a^2+4b^2\right)\int\frac{1}{4\left(a^2+b^2\right)-\left(2b-2a\tanh\left(\frac{x}{2}\right)\right)^2}d\left(2b-2a\tanh\left(\frac{x}{2}\right)\right)}{b}-\frac{x\left(6a^2-b^2\right)\left(a^2+b^2\right)}{b}}{b}+\frac{2a\left(3a^2+2b^2\right)\cosh(x)}{b}\right)}{2b}\right)}{b(a^2+b^2)}$$

$\Big\downarrow$ 219

$$-\frac{a^2\sinh^2(x)\cosh(x)}{b\,(a^2+b^2)\,(a+b\sinh(x))}-\frac{i\left(\frac{i(3a^2+b^2)\sinh(x)\cosh(x)}{2b}-\frac{i\left(\frac{2a\left(3a^2+2b^2\right)\cosh(x)}{b}+\frac{-\frac{x\left(6a^2-b^2\right)\left(a^2+b^2\right)}{b}-\frac{4a^3\left(3a^2+4b^2\right)\operatorname{arctanh}\left(\frac{2b-2a\tanh\left(\frac{x}{2}\right)}{2\sqrt{a^2+b^2}}\right)}{b\sqrt{a^2+b^2}}}{b}\right)}{2b}\right)}{b\,(a^2+b^2)}$$

input
```
Int[Sinh[x]^4/(a + b*Sinh[x])^2,x]
```

output
```
-((a^2*Cosh[x]*Sinh[x]^2)/(b*(a^2 + b^2)*(a + b*Sinh[x]))) - (I*(((-1/2*I)
*((-(((6*a^2 - b^2)*(a^2 + b^2)*x)/b) - (4*a^3*(3*a^2 + 4*b^2)*ArcTanh[(2*
b - 2*a*Tanh[x/2])/(2*Sqrt[a^2 + b^2])])/(b*Sqrt[a^2 + b^2]))/b + (2*a*(3*
a^2 + 2*b^2)*Cosh[x])/b))/b + ((I/2)*(3*a^2 + b^2)*Cosh[x]*Sinh[x])/b))/(b
*(a^2 + b^2))
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1083
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Simp[-2   Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3139
```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Simp[2*(e/d)   Subst[Int[1/(a + 2*b*e*x + a
*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[a^2 - b^2, 0]
```

rule 3214
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])/((c_.) + (d_.)*sin[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[b*(x/d), x] - Simp[(b*c - a*d)/d   Int[1/(c + d
*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

rule 3271
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_), x_Symbol] :> Simp[(-(b^2*c^2 - 2*a*b*c*d + a^2*d^2))*Co
s[e + f*x]*(a + b*Sin[e + f*x])^(m - 2)*((c + d*Sin[e + f*x])^(n + 1)/(d*f*
(n + 1)*(c^2 - d^2))), x] + Simp[1/(d*(n + 1)*(c^2 - d^2))   Int[(a + b*Sin
[e + f*x])^(m - 3)*(c + d*Sin[e + f*x])^(n + 1)*Simp[b*(m - 2)*(b*c - a*d)^
2 + a*d*(n + 1)*(c*(a^2 + b^2) - 2*a*b*d) + (b*(n + 1)*(a*b*c^2 + c*d*(a^2
+ b^2) - 3*a*b*d^2) - a*(n + 2)*(b*c - a*d)^2)*Sin[e + f*x] + b*(b^2*(c^2 -
 d^2) - m*(b*c - a*d)^2 + d*n*(2*a*b*c - d*(a^2 + b^2)))*Sin[e + f*x]^2, x]
, x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 -
b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 2] && LtQ[n, -1] && (IntegerQ[m] ||
IntegersQ[2*m, 2*n])
```

rule 3502
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-C)*Co
s[e + f*x]*((a + b*Sin[e + f*x])^(m + 1)/(b*f*(m + 2))), x] + Simp[1/(b*(m
+ 2))   Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m
 + 2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x]
 &&  !LtQ[m, -1]
```

rule 3528
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*sin[(e_.)
+ (f_.)*(x_)])^(n_.)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_
.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-C)*Cos[e + f*x]*(a + b*Sin[e + f*x
])^m*((c + d*Sin[e + f*x])^(n + 1)/(d*f*(m + n + 2))), x] + Simp[1/(d*(m +
n + 2))   Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^n*Simp[a*A*
d*(m + n + 2) + C*(b*c*m + a*d*(n + 1)) + (d*(A*b + a*B)*(m + n + 2) - C*(a
*c - b*d*(m + n + 1)))*Sin[e + f*x] + (C*(a*d*m - b*c*(m + 1)) + b*B*d*(m +
 n + 2))*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, n}
, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[
m, 0] &&  !(IGtQ[n, 0] && ( !IntegerQ[m] || (EqQ[a, 0] && NeQ[c, 0])))
```

Maple [A] (verified)

Time = 0.50 (sec) , antiderivative size = 218, normalized size of antiderivative = 1.35

method	result
default	$-\frac{1}{2b^2\left(\tanh\left(\frac{x}{2}\right)+1\right)^2}-\frac{-b+4a}{2b^3\left(\tanh\left(\frac{x}{2}\right)+1\right)}+\frac{(6a^2-b^2)\ln\left(\tanh\left(\frac{x}{2}\right)+1\right)}{2b^4}+\frac{2a^3\left(\frac{\frac{b^2\tanh\left(\frac{x}{2}\right)}{a^2+b^2}+\frac{ab}{a^2+b^2}}{\tanh\left(\frac{x}{2}\right)^2 a-2b\tanh\left(\frac{x}{2}\right)-a}-\frac{\left(3a^2+4b^2\right)\operatorname{arctanh}\left(\ldots\right.}{\left(a^2+\ldots\right.}\right.}{b^4}$
risch	$\frac{3x\,a^2}{b^4}-\frac{x}{2b^2}+\frac{e^{2x}}{8b^2}-\frac{a\,e^{x}}{b^3}-\frac{a\,e^{-x}}{b^3}-\frac{e^{-2x}}{8b^2}+\frac{2a^4(e^{x}a-b)}{b^4(a^2+b^2)(b\,e^{2x}+2\,e^{x}a-b)}+\frac{3a^5\ln\left(e^{x}+\frac{a\left(a^2+b^2\right)^{\frac{3}{2}}+a^4+2a^2b^2+b^4}{b\left(a^2+b^2\right)^{\frac{3}{2}}}\right)}{(a^2+b^2)^{\frac{3}{2}}b^4}+\ldots$

input
```
int(sinh(x)^4/(a+b*sinh(x))^2,x,method=_RETURNVERBOSE)
```

output
```
-1/2/b^2/(tanh(1/2*x)+1)^2-1/2*(-b+4*a)/b^3/(tanh(1/2*x)+1)+1/2*(6*a^2-b^2
)/b^4*ln(tanh(1/2*x)+1)+2*a^3/b^4*((b^2/(a^2+b^2)*tanh(1/2*x)+a*b/(a^2+b^2
))/(tanh(1/2*x)^2*a-2*b*tanh(1/2*x)-a)-(3*a^2+4*b^2)/(a^2+b^2)^(3/2)*arcta
nh(1/2*(2*a*tanh(1/2*x)-2*b)/(a^2+b^2)^(1/2)))+1/2/b^2/(tanh(1/2*x)-1)^2-1
/2*(-b-4*a)/b^3/(tanh(1/2*x)-1)+1/2/b^4*(-6*a^2+b^2)*ln(tanh(1/2*x)-1)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1769 vs. $2(154) = 308$.

Time = 0.11 (sec) , antiderivative size = 1769, normalized size of antiderivative = 10.92

$$\int \frac{\sinh^4(x)}{(a+b\sinh(x))^2}\,dx = \text{Too large to display}$$

input
```
integrate(sinh(x)^4/(a+b*sinh(x))^2,x, algorithm="fricas")
```

output
```
1/8*(a^4*b^3 + 2*a^2*b^5 + b^7 + (a^4*b^3 + 2*a^2*b^5 + b^7)*cosh(x)^6 + (
a^4*b^3 + 2*a^2*b^5 + b^7)*sinh(x)^6 - 6*(a^5*b^2 + 2*a^3*b^4 + a*b^6)*cos
h(x)^5 - 6*(a^5*b^2 + 2*a^3*b^4 + a*b^6 - (a^4*b^3 + 2*a^2*b^5 + b^7)*cosh
(x))*sinh(x)^5 - (16*a^6*b + 33*a^4*b^3 + 18*a^2*b^5 + b^7 - 4*(6*a^6*b +
11*a^4*b^3 + 4*a^2*b^5 - b^7)*x)*cosh(x)^4 - (16*a^6*b + 33*a^4*b^3 + 18*a
^2*b^5 + b^7 - 15*(a^4*b^3 + 2*a^2*b^5 + b^7)*cosh(x)^2 - 4*(6*a^6*b + 11*
a^4*b^3 + 4*a^2*b^5 - b^7)*x + 30*(a^5*b^2 + 2*a^3*b^4 + a*b^6)*cosh(x))*s
inh(x)^4 + 8*(2*a^7 + 2*a^5*b^2 + (6*a^7 + 11*a^5*b^2 + 4*a^3*b^4 - a*b^6)
*x)*cosh(x)^3 + 4*(4*a^7 + 4*a^5*b^2 + 5*(a^4*b^3 + 2*a^2*b^5 + b^7)*cosh(
x)^3 - 15*(a^5*b^2 + 2*a^3*b^4 + a*b^6)*cosh(x)^2 + 2*(6*a^7 + 11*a^5*b^2
+ 4*a^3*b^4 - a*b^6)*x - (16*a^6*b + 33*a^4*b^3 + 18*a^2*b^5 + b^7 - 4*(6*
a^6*b + 11*a^4*b^3 + 4*a^2*b^5 - b^7)*x)*cosh(x))*sinh(x)^3 - (32*a^6*b +
49*a^4*b^3 + 18*a^2*b^5 + b^7 + 4*(6*a^6*b + 11*a^4*b^3 + 4*a^2*b^5 - b^7)
*x)*cosh(x)^2 - (32*a^6*b + 49*a^4*b^3 + 18*a^2*b^5 + b^7 - 15*(a^4*b^3 +
2*a^2*b^5 + b^7)*cosh(x)^4 + 60*(a^5*b^2 + 2*a^3*b^4 + a*b^6)*cosh(x)^3 +
6*(16*a^6*b + 33*a^4*b^3 + 18*a^2*b^5 + b^7 - 4*(6*a^6*b + 11*a^4*b^3 + 4*
a^2*b^5 - b^7)*x)*cosh(x)^2 + 4*(6*a^6*b + 11*a^4*b^3 + 4*a^2*b^5 - b^7)*x
 - 24*(2*a^7 + 2*a^5*b^2 + (6*a^7 + 11*a^5*b^2 + 4*a^3*b^4 - a*b^6)*x)*cos
h(x))*sinh(x)^2 + 8*((3*a^5*b + 4*a^3*b^3)*cosh(x)^4 + (3*a^5*b + 4*a^3*b^
3)*sinh(x)^4 + 2*(3*a^6 + 4*a^4*b^2)*cosh(x)^3 + 2*(3*a^6 + 4*a^4*b^2 +...
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\sinh^4(x)}{(a+b\sinh(x))^2}\,dx = \text{Timed out}$$

input
```
integrate(sinh(x)**4/(a+b*sinh(x))**2,x)
```

output
```
Timed out
```

Maxima [A] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 256, normalized size of antiderivative = 1.58

$$\int \frac{\sinh^4(x)}{(a+b\sinh(x))^2}\,dx = -\frac{(3\,a^2+4\,b^2)a^3\log\left(\frac{be^{(-x)}-a-\sqrt{a^2+b^2}}{be^{(-x)}-a+\sqrt{a^2+b^2}}\right)}{(a^2b^4+b^6)\sqrt{a^2+b^2}} + \frac{a^2b^3+b^5-6\,(a^3b^2+ab^4)e^{(-x)}-(32\,a^4b+17\,a^2b^3+b^5)e^{(-2\,x)}-8\,(2\,a^5-a^3b^2-ab^4)e^{(-3\,x)}}{8\,((a^2b^5+b^7)e^{(-2\,x)}+2\,(a^3b^4+ab^6)e^{(-3\,x)}-(a^2b^5+b^7)e^{(-4\,x)})} - \frac{8\,ae^{(-x)}+be^{(-2\,x)}}{8\,b^3} + \frac{(6\,a^2-b^2)x}{2\,b^4}$$

input
```
integrate(sinh(x)^4/(a+b*sinh(x))^2,x, algorithm="maxima")
```

output
```
-(3*a^2 + 4*b^2)*a^3*log((b*e^(-x) - a - sqrt(a^2 + b^2))/(b*e^(-x) - a +
sqrt(a^2 + b^2)))/((a^2*b^4 + b^6)*sqrt(a^2 + b^2)) + 1/8*(a^2*b^3 + b^5 -
 6*(a^3*b^2 + a*b^4)*e^(-x) - (32*a^4*b + 17*a^2*b^3 + b^5)*e^(-2*x) - 8*(
2*a^5 - a^3*b^2 - a*b^4)*e^(-3*x))/((a^2*b^5 + b^7)*e^(-2*x) + 2*(a^3*b^4
+ a*b^6)*e^(-3*x) - (a^2*b^5 + b^7)*e^(-4*x)) - 1/8*(8*a*e^(-x) + b*e^(-2*
x))/b^3 + 1/2*(6*a^2 - b^2)*x/b^4
```

Giac [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 235, normalized size of antiderivative = 1.45

$$\int \frac{\sinh^4(x)}{(a+b\sinh(x))^2}\,dx$$
$$= -\frac{(3a^5+4a^3b^2)\log\left(\frac{\left|2be^x+2a-2\sqrt{a^2+b^2}\right|}{\left|2be^x+2a+2\sqrt{a^2+b^2}\right|}\right)}{(a^2b^4+b^6)\sqrt{a^2+b^2}} + \frac{(6a^2-b^2)x}{2b^4} + \frac{b^2e^{(2x)}-8abe^x}{8b^4}$$
$$+ \frac{(a^2b^3+b^5+8(2a^5-a^3b^2-ab^4)e^{(3x)}-(32a^4b+17a^2b^3+b^5)e^{(2x)}+6(a^3b^2+ab^4)e^x)e^{(-2x)}}{8(a^2+b^2)(be^{(2x)}+2ae^x-b)b^4}$$

input
```
integrate(sinh(x)^4/(a+b*sinh(x))^2,x, algorithm="giac")
```

output
```
-(3*a^5 + 4*a^3*b^2)*log(abs(2*b*e^x + 2*a - 2*sqrt(a^2 + b^2))/abs(2*b*e^
x + 2*a + 2*sqrt(a^2 + b^2)))/((a^2*b^4 + b^6)*sqrt(a^2 + b^2)) + 1/2*(6*a
^2 - b^2)*x/b^4 + 1/8*(b^2*e^(2*x) - 8*a*b*e^x)/b^4 + 1/8*(a^2*b^3 + b^5 +
 8*(2*a^5 - a^3*b^2 - a*b^4)*e^(3*x) - (32*a^4*b + 17*a^2*b^3 + b^5)*e^(2*
x) + 6*(a^3*b^2 + a*b^4)*e^x)*e^(-2*x)/((a^2 + b^2)*(b*e^(2*x) + 2*a*e^x -
 b)*b^4)
```

Mupad [B] (verification not implemented)

Time = 2.10 (sec) , antiderivative size = 305, normalized size of antiderivative = 1.88

$$\int \frac{\sinh^4(x)}{(a+b\sinh(x))^2}\,dx = \frac{e^{2x}}{8b^2} - \frac{e^{-2x}}{8b^2} - \frac{\frac{2a^4}{b^2(a^2b+b^3)} - \frac{2a^5e^x}{b^3(a^2b+b^3)}}{2ae^x-b+be^{2x}} + \frac{x(6a^2-b^2)}{2b^4} - \frac{ae^x}{b^3}$$
$$- \frac{ae^{-x}}{b^3} - \frac{a^3\ln\left(\frac{2e^x(3a^5+4a^3b^2)}{a^2b^5+b^7} - \frac{2a^3(b-ae^x)(3a^2+4b^2)}{b^5(a^2+b^2)^{3/2}}\right)(3a^2+4b^2)}{b^4(a^2+b^2)^{3/2}}$$
$$+ \frac{a^3\ln\left(\frac{2e^x(3a^5+4a^3b^2)}{a^2b^5+b^7} + \frac{2a^3(b-ae^x)(3a^2+4b^2)}{b^5(a^2+b^2)^{3/2}}\right)(3a^2+4b^2)}{b^4(a^2+b^2)^{3/2}}$$

input
```
int(sinh(x)^4/(a + b*sinh(x))^2,x)
```

output
```
exp(2*x)/(8*b^2) - exp(-2*x)/(8*b^2) - ((2*a^4)/(b^2*(a^2*b + b^3)) - (2*a
^5*exp(x))/(b^3*(a^2*b + b^3)))/(2*a*exp(x) - b + b*exp(2*x)) + (x*(6*a^2
- b^2))/(2*b^4) - (a*exp(x))/b^3 - (a*exp(-x))/b^3 - (a^3*log((2*exp(x)*(3
*a^5 + 4*a^3*b^2))/(b^7 + a^2*b^5) - (2*a^3*(b - a*exp(x))*(3*a^2 + 4*b^2)
)/(b^5*(a^2 + b^2)^(3/2)))*(3*a^2 + 4*b^2))/(b^4*(a^2 + b^2)^(3/2)) + (a^3
*log((2*exp(x)*(3*a^5 + 4*a^3*b^2))/(b^7 + a^2*b^5) + (2*a^3*(b - a*exp(x)
)*(3*a^2 + 4*b^2))/(b^5*(a^2 + b^2)^(3/2)))*(3*a^2 + 4*b^2))/(b^4*(a^2 + b
^2)^(3/2))
```

Reduce [B] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 719, normalized size of antiderivative = 4.44

$$\int \frac{\sinh^4(x)}{(a+b\sinh(x))^2}\,dx$$
$$= \frac{24e^{4x}a^6bx + 44e^{4x}a^4b^3x + 16e^{4x}a^2b^5x + 88e^{3x}a^5b^2x + 32e^{3x}a^3b^4x - 8e^{3x}a\,b^6x - 24e^{2x}a^6bx - 44e^{2x}a^4b^3x}{}$$

input
```
int(sinh(x)^4/(a+b*sinh(x))^2,x)
```

output

```
( - 48*e**(4*x)*sqrt(a**2 + b**2)*atan((e**x*b*i + a*i)/sqrt(a**2 + b**2))
*a**5*b*i - 64*e**(4*x)*sqrt(a**2 + b**2)*atan((e**x*b*i + a*i)/sqrt(a**2
+ b**2))*a**3*b**3*i - 96*e**(3*x)*sqrt(a**2 + b**2)*atan((e**x*b*i + a*i)
/sqrt(a**2 + b**2))*a**6*i - 128*e**(3*x)*sqrt(a**2 + b**2)*atan((e**x*b*i
 + a*i)/sqrt(a**2 + b**2))*a**4*b**2*i + 48*e**(2*x)*sqrt(a**2 + b**2)*ata
n((e**x*b*i + a*i)/sqrt(a**2 + b**2))*a**5*b*i + 64*e**(2*x)*sqrt(a**2 + b
**2)*atan((e**x*b*i + a*i)/sqrt(a**2 + b**2))*a**3*b**3*i + e**(6*x)*a**4*
b**3 + 2*e**(6*x)*a**2*b**5 + e**(6*x)*b**7 - 6*e**(5*x)*a**5*b**2 - 12*e*
*(5*x)*a**3*b**4 - 6*e**(5*x)*a*b**6 + 24*e**(4*x)*a**6*b*x - 24*e**(4*x)*
a**6*b + 44*e**(4*x)*a**4*b**3*x - 41*e**(4*x)*a**4*b**3 + 16*e**(4*x)*a**
2*b**5*x - 18*e**(4*x)*a**2*b**5 - 4*e**(4*x)*b**7*x - e**(4*x)*b**7 + 48*
e**(3*x)*a**7*x + 88*e**(3*x)*a**5*b**2*x + 32*e**(3*x)*a**3*b**4*x - 8*e*
*(3*x)*a*b**6*x - 24*e**(2*x)*a**6*b*x - 24*e**(2*x)*a**6*b - 44*e**(2*x)*
a**4*b**3*x - 41*e**(2*x)*a**4*b**3 - 16*e**(2*x)*a**2*b**5*x - 18*e**(2*x
)*a**2*b**5 + 4*e**(2*x)*b**7*x - e**(2*x)*b**7 + 6*e**x*a**5*b**2 + 12*e*
*x*a**3*b**4 + 6*e**x*a*b**6 + a**4*b**3 + 2*a**2*b**5 + b**7)/(8*e**(2*x)
*b**4*(e**(2*x)*a**4*b + 2*e**(2*x)*a**2*b**3 + e**(2*x)*b**5 + 2*e**x*a**
5 + 4*e**x*a**3*b**2 + 2*e**x*a*b**4 - a**4*b - 2*a**2*b**3 - b**5))
```

3.81 $\int \frac{\sinh^3(x)}{(a+b\sinh(x))^2}\,dx$

Optimal result

Integrand size = 13, antiderivative size = 115

$$\int \frac{\sinh^3(x)}{(a+b\sinh(x))^2}\,dx = -\frac{2ax}{b^3} - \frac{2a^2(2a^2+3b^2)\,\text{arctanh}\left(\frac{b-a\tanh\left(\frac{x}{2}\right)}{\sqrt{a^2+b^2}}\right)}{b^3\,(a^2+b^2)^{3/2}} + \frac{(2a^2+b^2)\cosh(x)}{b^2\,(a^2+b^2)} - \frac{a^2\cosh(x)\sinh(x)}{b\,(a^2+b^2)\,(a+b\sinh(x))}$$

output
```
-2*a*x/b^3-2*a^2*(2*a^2+3*b^2)*arctanh((b-a*tanh(1/2*x))/(a^2+b^2)^(1/2))/
b^3/(a^2+b^2)^(3/2)+(2*a^2+b^2)*cosh(x)/b^2/(a^2+b^2)-a^2*cosh(x)*sinh(x)/
b/(a^2+b^2)/(a+b*sinh(x))
```

Mathematica [A] (verified)

Time = 0.40 (sec) , antiderivative size = 95, normalized size of antiderivative = 0.83

$$\int \frac{\sinh^3(x)}{(a+b\sinh(x))^2}\,dx = \frac{-2ax - \frac{2a^2(2a^2+3b^2)\arctan\left(\frac{b-a\tanh\left(\frac{x}{2}\right)}{\sqrt{-a^2-b^2}}\right)}{(-a^2-b^2)^{3/2}} + \cosh(x)\left(b + \frac{a^3b}{(a^2+b^2)(a+b\sinh(x))}\right)}{b^3}$$

input
```
Integrate[Sinh[x]^3/(a + b*Sinh[x])^2,x]
```

output
```
(-2*a*x - (2*a^2*(2*a^2 + 3*b^2)*ArcTan[(b - a*Tanh[x/2])/Sqrt[-a^2 - b^2]
])/(-a^2 - b^2)^(3/2) + Cosh[x]*(b + (a^3*b)/((a^2 + b^2)*(a + b*Sinh[x]))
))/b^3
```

Rubi [C] (verified)

Result contains complex when optimal does not.

Time = 0.69 (sec) , antiderivative size = 146, normalized size of antiderivative = 1.27, number of steps used = 13, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.923$, Rules used = {3042, 26, 3271, 3042, 3502, 26, 3042, 3214, 3042, 3139, 1083, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sinh^3(x)}{(a+b\sinh(x))^2}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{i\sin(ix)^3}{(a-ib\sin(ix))^2}dx$$

$$\Big\downarrow \text{26}$$

$$i\int \frac{\sin(ix)^3}{(a-ib\sin(ix))^2}dx$$

$$\Big\downarrow \text{3271}$$

$$i\left(\frac{ia^2\sinh(x)\cosh(x)}{b(a^2+b^2)(a+b\sinh(x))}-\frac{i\int \frac{a^2-b\sinh(x)a+(2a^2+b^2)\sinh^2(x)}{a+b\sinh(x)}dx}{b(a^2+b^2)}\right)$$

$$\Big\downarrow \text{3042}$$

$$i\left(\frac{ia^2\sinh(x)\cosh(x)}{b(a^2+b^2)(a+b\sinh(x))}-\frac{i\int \frac{a^2+ib\sin(ix)a-(2a^2+b^2)\sin(ix)^2}{a-ib\sin(ix)}dx}{b(a^2+b^2)}\right)$$

$$\Big\downarrow \text{3502}$$

$$i\left(\frac{ia^2\sinh(x)\cosh(x)}{b(a^2+b^2)(a+b\sinh(x))}-\frac{i\left(\frac{(2a^2+b^2)\cosh(x)}{b}+\frac{i\int -\frac{i\left(a^2b-2a\left(a^2+b^2\right)\sinh(x)\right)}{a+b\sinh(x)}dx}{b}\right)}{b(a^2+b^2)}\right)$$

$$\Big\downarrow \; 26$$

$$i\left(\frac{ia^2\sinh(x)\cosh(x)}{b(a^2+b^2)(a+b\sinh(x))}-\frac{i\left(\frac{\int\frac{a^2b-2a\left(a^2+b^2\right)\sinh(x)}{a+b\sinh(x)}dx}{b}+\frac{(2a^2+b^2)\cosh(x)}{b}\right)}{b(a^2+b^2)}\right)$$

$$\Big\downarrow \; 3042$$

$$i\left(\frac{ia^2\sinh(x)\cosh(x)}{b(a^2+b^2)(a+b\sinh(x))}-\frac{i\left(\frac{(2a^2+b^2)\cosh(x)}{b}+\frac{\int\frac{ba^2+2i\left(a^2+b^2\right)\sin(ix)a}{a-ib\sin(ix)}dx}{b}\right)}{b(a^2+b^2)}\right)$$

$$\Big\downarrow \; 3214$$

$$i\left(\frac{ia^2\sinh(x)\cosh(x)}{b(a^2+b^2)(a+b\sinh(x))}-\frac{i\left(\frac{\frac{a^2\left(2a^2+3b^2\right)\int\frac{1}{a+b\sinh(x)}dx}{b}-\frac{2ax\left(a^2+b^2\right)}{b}}{b}+\frac{(2a^2+b^2)\cosh(x)}{b}\right)}{b(a^2+b^2)}\right)$$

$$\Big\downarrow \; 3042$$

$$i\left(\frac{ia^2\sinh(x)\cosh(x)}{b(a^2+b^2)(a+b\sinh(x))}-\frac{i\left(\frac{(2a^2+b^2)\cosh(x)}{b}+\frac{-\frac{2ax\left(a^2+b^2\right)}{b}+\frac{a^2\left(2a^2+3b^2\right)\int\frac{1}{a-ib\sin(ix)}dx}{b}}{b}\right)}{b(a^2+b^2)}\right)$$

$$\Big\downarrow \; 3139$$

$$i\left(\frac{ia^2\sinh(x)\cosh(x)}{b\,(a^2+b^2)\,(a+b\sinh(x))}-\frac{i\left(\frac{\frac{2a^2\left(2a^2+3b^2\right)\int\frac{1}{-a\tanh^2\left(\frac{x}{2}\right)+2b\tanh\left(\frac{x}{2}\right)+a}d\tanh\left(\frac{x}{2}\right)}{b}-\frac{2ax\left(a^2+b^2\right)}{b}}{b}+\frac{(2a^2+b^2)\cosh(x)}{b}\right)}{b\,(a^2+b^2)}\right)$$

$$\Big\downarrow \text{1083}$$

$$i\left(\frac{ia^2\sinh(x)\cosh(x)}{b\,(a^2+b^2)\,(a+b\sinh(x))}-\frac{i\left(\frac{-\frac{4a^2\left(2a^2+3b^2\right)\int\frac{1}{4\left(a^2+b^2\right)-\left(2b-2a\tanh\left(\frac{x}{2}\right)\right)^2}d\left(2b-2a\tanh\left(\frac{x}{2}\right)\right)}{b}-\frac{2ax\left(a^2+b^2\right)}{b}}{b}+\frac{(2a^2+b^2)\cosh(x)}{b}\right)}{b\,(a^2+b^2)}\right)$$

$$\Big\downarrow \text{219}$$

$$i\left(\frac{ia^2\sinh(x)\cosh(x)}{b\,(a^2+b^2)\,(a+b\sinh(x))}-\frac{i\left(\frac{-\frac{2a^2\left(2a^2+3b^2\right)\text{arctanh}\left(\frac{2b-2a\tanh\left(\frac{x}{2}\right)}{2\sqrt{a^2+b^2}}\right)}{b\sqrt{a^2+b^2}}-\frac{2ax\left(a^2+b^2\right)}{b}}{b}+\frac{(2a^2+b^2)\cosh(x)}{b}\right)}{b\,(a^2+b^2)}\right)$$

input
```
Int[Sinh[x]^3/(a + b*Sinh[x])^2,x]
```

output
```
I*(((-I)*(((-2*a*(a^2 + b^2)*x)/b - (2*a^2*(2*a^2 + 3*b^2)*ArcTanh[(2*b -
2*a*Tanh[x/2])/(2*Sqrt[a^2 + b^2])])/(b*Sqrt[a^2 + b^2]))/b + ((2*a^2 + b^
2)*Cosh[x])/b))/(b*(a^2 + b^2)) + (I*a^2*Cosh[x]*Sinh[x])/(b*(a^2 + b^2)*(
a + b*Sinh[x])))
```

Defintions of rubi rules used

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1083
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Simp[-2   Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3139
```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Simp[2*(e/d)   Subst[Int[1/(a + 2*b*e*x + a
*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[a^2 - b^2, 0]
```

rule 3214
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])/((c_.) + (d_.)*sin[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[b*(x/d), x] - Simp[(b*c - a*d)/d   Int[1/(c + d
*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

rule 3271
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_), x_Symbol] :> Simp[(-(b^2*c^2 - 2*a*b*c*d + a^2*d^2))*Co
s[e + f*x]*(a + b*Sin[e + f*x])^(m - 2)*((c + d*Sin[e + f*x])^(n + 1)/(d*f*
(n + 1)*(c^2 - d^2))), x] + Simp[1/(d*(n + 1)*(c^2 - d^2))   Int[(a + b*Sin
[e + f*x])^(m - 3)*(c + d*Sin[e + f*x])^(n + 1)*Simp[b*(m - 2)*(b*c - a*d)^
2 + a*d*(n + 1)*(c*(a^2 + b^2) - 2*a*b*d) + (b*(n + 1)*(a*b*c^2 + c*d*(a^2
+ b^2) - 3*a*b*d^2) - a*(n + 2)*(b*c - a*d)^2)*Sin[e + f*x] + b*(b^2*(c^2 -
 d^2) - m*(b*c - a*d)^2 + d*n*(2*a*b*c - d*(a^2 + b^2)))*Sin[e + f*x]^2, x]
, x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 -
b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 2] && LtQ[n, -1] && (IntegerQ[m] ||
IntegersQ[2*m, 2*n])
```

rule 3502
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-C)*Co
s[e + f*x]*((a + b*Sin[e + f*x])^(m + 1)/(b*f*(m + 2))), x] + Simp[1/(b*(m
+ 2))   Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m
 + 2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x]
 &&  !LtQ[m, -1]
```

Maple [A] (verified)

Time = 0.31 (sec) , antiderivative size = 161, normalized size of antiderivative = 1.40

method	result
default	$-\frac{4a^2\left(\frac{\frac{b^2\tanh\left(\frac{x}{2}\right)}{2a^2+2b^2}+\frac{ab}{2a^2+2b^2}}{\tanh\left(\frac{x}{2}\right)^2 a-2b\tanh\left(\frac{x}{2}\right)-a}-\frac{\left(2a^2+3b^2\right)\operatorname{arctanh}\left(\frac{2a\tanh\left(\frac{x}{2}\right)-2b}{2\sqrt{a^2+b^2}}\right)}{2\left(a^2+b^2\right)^{\frac{3}{2}}}\right)}{b^3}-\frac{1}{b^2\left(\tanh\left(\frac{x}{2}\right)-1\right)}+\frac{2a\ln\left(\tanh\left(\frac{x}{2}\right)-1\right)}{b^3}+$
risch	$-\frac{2ax}{b^3}+\frac{e^x}{2b^2}+\frac{e^{-x}}{2b^2}-\frac{2a^3\left(e^x a-b\right)}{b^3\left(a^2+b^2\right)\left(b\,e^{2x}+2\,e^x a-b\right)}+\frac{2a^4\ln\left(e^x+\frac{a\left(a^2+b^2\right)^{\frac{3}{2}}-a^4-2a^2b^2-b^4}{b\left(a^2+b^2\right)^{\frac{3}{2}}}\right)}{\left(a^2+b^2\right)^{\frac{3}{2}}b^3}+$

input
```
int(sinh(x)^3/(a+b*sinh(x))^2,x,method=_RETURNVERBOSE)
```

output
```
-4/b^3*a^2*((1/2*b^2/(a^2+b^2)*tanh(1/2*x)+1/2*a*b/(a^2+b^2))/(tanh(1/2*x)
^2*a-2*b*tanh(1/2*x)-a)-1/2*(2*a^2+3*b^2)/(a^2+b^2)^(3/2)*arctanh(1/2*(2*a
*tanh(1/2*x)-2*b)/(a^2+b^2)^(1/2)))-1/b^2/(tanh(1/2*x)-1)+2/b^3*a*ln(tanh(
1/2*x)-1)+1/b^2/(tanh(1/2*x)+1)-2/b^3*a*ln(tanh(1/2*x)+1)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1053 vs. $2(111) = 222$.

Time = 0.11 (sec) , antiderivative size = 1053, normalized size of antiderivative = 9.16

$$\int \frac{\sinh^3(x)}{(a + b\sinh(x))^2}\,dx = \text{Too large to display}$$

input
```
integrate(sinh(x)^3/(a+b*sinh(x))^2,x, algorithm="fricas")
```

output
```
-1/2*(a^4*b^2 + 2*a^2*b^4 + b^6 - (a^4*b^2 + 2*a^2*b^4 + b^6)*cosh(x)^4 -
(a^4*b^2 + 2*a^2*b^4 + b^6)*sinh(x)^4 - 2*(a^5*b + 2*a^3*b^3 + a*b^5 - 2*(
a^5*b + 2*a^3*b^3 + a*b^5)*x)*cosh(x)^3 - 2*(a^5*b + 2*a^3*b^3 + a*b^5 - 2
*(a^5*b + 2*a^3*b^3 + a*b^5)*x + 2*(a^4*b^2 + 2*a^2*b^4 + b^6)*cosh(x))*si
nh(x)^3 + 4*(a^6 + a^4*b^2 + 2*(a^6 + 2*a^4*b^2 + a^2*b^4)*x)*cosh(x)^2 +
2*(2*a^6 + 2*a^4*b^2 - 3*(a^4*b^2 + 2*a^2*b^4 + b^6)*cosh(x)^2 + 4*(a^6 +
2*a^4*b^2 + a^2*b^4)*x - 3*(a^5*b + 2*a^3*b^3 + a*b^5 - 2*(a^5*b + 2*a^3*b
^3 + a*b^5)*x)*cosh(x))*sinh(x)^2 - 2*((2*a^4*b + 3*a^2*b^3)*cosh(x)^3 + (
2*a^4*b + 3*a^2*b^3)*sinh(x)^3 + 2*(2*a^5 + 3*a^3*b^2)*cosh(x)^2 + (4*a^5
+ 6*a^3*b^2 + 3*(2*a^4*b + 3*a^2*b^3)*cosh(x))*sinh(x)^2 - (2*a^4*b + 3*a^
2*b^3)*cosh(x) - (2*a^4*b + 3*a^2*b^3 - 3*(2*a^4*b + 3*a^2*b^3)*cosh(x)^2
- 4*(2*a^5 + 3*a^3*b^2)*cosh(x))*sinh(x))*sqrt(a^2 + b^2)*log((b^2*cosh(x)
^2 + b^2*sinh(x)^2 + 2*a*b*cosh(x) + 2*a^2 + b^2 + 2*(b^2*cosh(x) + a*b)*s
inh(x) - 2*sqrt(a^2 + b^2)*(b*cosh(x) + b*sinh(x) + a))/(b*cosh(x)^2 + b*s
inh(x)^2 + 2*a*cosh(x) + 2*(b*cosh(x) + a)*sinh(x) - b)) - 2*(3*a^5*b + 4*
a^3*b^3 + a*b^5 + 2*(a^5*b + 2*a^3*b^3 + a*b^5)*x)*cosh(x) - 2*(3*a^5*b +
4*a^3*b^3 + a*b^5 + 2*(a^4*b^2 + 2*a^2*b^4 + b^6)*cosh(x)^3 + 3*(a^5*b + 2
*a^3*b^3 + a*b^5 - 2*(a^5*b + 2*a^3*b^3 + a*b^5)*x)*cosh(x)^2 + 2*(a^5*b +
 2*a^3*b^3 + a*b^5)*x - 4*(a^6 + a^4*b^2 + 2*(a^6 + 2*a^4*b^2 + a^2*b^4)*x
)*cosh(x))*sinh(x))/((a^4*b^4 + 2*a^2*b^6 + b^8)*cosh(x)^3 + (a^4*b^4 +...
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\sinh^3(x)}{(a+b\sinh(x))^2}\,dx = \text{Timed out}$$

input
```
integrate(sinh(x)**3/(a+b*sinh(x))**2,x)
```

output
```
Timed out
```

Maxima [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 208, normalized size of antiderivative = 1.81

$$\int \frac{\sinh^3(x)}{(a+b\sinh(x))^2}\,dx = \frac{(2\,a^2+3\,b^2)a^2\log\left(\frac{be^{(-x)}-a-\sqrt{a^2+b^2}}{be^{(-x)}-a+\sqrt{a^2+b^2}}\right)}{(a^2b^3+b^5)\sqrt{a^2+b^2}} + \frac{a^2b^2+b^4+2\,(3\,a^3b+ab^3)e^{(-x)}+(4\,a^4-a^2b^2-b^4)e^{(-2\,x)}}{2\,((a^2b^4+b^6)e^{(-x)}+2\,(a^3b^3+ab^5)e^{(-2\,x)}-(a^2b^4+b^6)e^{(-3\,x)})} - \frac{2\,ax}{b^3}+\frac{e^{(-x)}}{2\,b^2}$$

input
```
integrate(sinh(x)^3/(a+b*sinh(x))^2,x, algorithm="maxima")
```

output
```
(2*a^2 + 3*b^2)*a^2*log((b*e^(-x) - a - sqrt(a^2 + b^2))/(b*e^(-x) - a + s
qrt(a^2 + b^2)))/((a^2*b^3 + b^5)*sqrt(a^2 + b^2)) + 1/2*(a^2*b^2 + b^4 +
2*(3*a^3*b + a*b^3)*e^(-x) + (4*a^4 - a^2*b^2 - b^4)*e^(-2*x))/((a^2*b^4 +
 b^6)*e^(-x) + 2*(a^3*b^3 + a*b^5)*e^(-2*x) - (a^2*b^4 + b^6)*e^(-3*x)) -
2*a*x/b^3 + 1/2*e^(-x)/b^2
```

Giac [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 184, normalized size of antiderivative = 1.60

$$\int \frac{\sinh^3(x)}{(a+b\sinh(x))^2}\,dx = \frac{(2\,a^4+3\,a^2b^2)\log\left(\frac{\left|2\,be^x+2\,a-2\,\sqrt{a^2+b^2}\right|}{\left|2\,be^x+2\,a+2\,\sqrt{a^2+b^2}\right|}\right)}{(a^2b^3+b^5)\sqrt{a^2+b^2}} - \frac{2\,ax}{b^3} + \frac{e^x}{2\,b^2} - \frac{\left(a^2b^2+b^4+(4\,a^4-a^2b^2-b^4)e^{(2\,x)}-2\,(3\,a^3b+ab^3)e^x\right)e^{(-x)}}{2\,(a^2+b^2)(be^{(2\,x)}+2\,ae^x-b)b^3}$$

input
```
integrate(sinh(x)^3/(a+b*sinh(x))^2,x, algorithm="giac")
```

output
```
(2*a^4 + 3*a^2*b^2)*log(abs(2*b*e^x + 2*a - 2*sqrt(a^2 + b^2))/abs(2*b*e^x
 + 2*a + 2*sqrt(a^2 + b^2)))/((a^2*b^3 + b^5)*sqrt(a^2 + b^2)) - 2*a*x/b^3
 + 1/2*e^x/b^2 - 1/2*(a^2*b^2 + b^4 + (4*a^4 - a^2*b^2 - b^4)*e^(2*x) - 2*
(3*a^3*b + a*b^3)*e^x)*e^(-x)/((a^2 + b^2)*(b*e^(2*x) + 2*a*e^x - b)*b^3)
```

Mupad [B] (verification not implemented)

Time = 2.02 (sec) , antiderivative size = 274, normalized size of antiderivative = 2.38

$$\int \frac{\sinh^3(x)}{(a+b\sinh(x))^2}\,dx = \frac{\mathrm{e}^{-x}}{2\,b^2} + \frac{\frac{2\,a^3}{b\,(a^2\,b+b^3)} - \frac{2\,a^4\,\mathrm{e}^x}{b^2\,(a^2\,b+b^3)}}{2\,a\,\mathrm{e}^x-b+b\,\mathrm{e}^{2\,x}} + \frac{\mathrm{e}^x}{2\,b^2} - \frac{2\,a\,x}{b^3} - \frac{a^2\,\ln\left(-\frac{2\,\mathrm{e}^x\,(2\,a^4+3\,a^2\,b^2)}{a^2\,b^4+b^6} - \frac{2\,a^2\,(b-a\,\mathrm{e}^x)\,(2\,a^2+3\,b^2)}{b^4\,(a^2+b^2)^{3/2}}\right)\,(2\,a^2+3\,b^2)}{b^3\,(a^2+b^2)^{3/2}} + \frac{a^2\,\ln\left(\frac{2\,a^2\,(b-a\,\mathrm{e}^x)\,(2\,a^2+3\,b^2)}{b^4\,(a^2+b^2)^{3/2}} - \frac{2\,\mathrm{e}^x\,(2\,a^4+3\,a^2\,b^2)}{a^2\,b^4+b^6}\right)\,(2\,a^2+3\,b^2)}{b^3\,(a^2+b^2)^{3/2}}$$

input
```
int(sinh(x)^3/(a + b*sinh(x))^2,x)
```

output
```
exp(-x)/(2*b^2) + ((2*a^3)/(b*(a^2*b + b^3)) - (2*a^4*exp(x))/(b^2*(a^2*b
+ b^3)))/(2*a*exp(x) - b + b*exp(2*x)) + exp(x)/(2*b^2) - (2*a*x)/b^3 - (a
^2*log(- (2*exp(x)*(2*a^4 + 3*a^2*b^2))/(b^6 + a^2*b^4) - (2*a^2*(b - a*ex
p(x))*(2*a^2 + 3*b^2))/(b^4*(a^2 + b^2)^(3/2)))*(2*a^2 + 3*b^2))/(b^3*(a^2
 + b^2)^(3/2)) + (a^2*log((2*a^2*(b - a*exp(x))*(2*a^2 + 3*b^2))/(b^4*(a^2
 + b^2)^(3/2)) - (2*exp(x)*(2*a^4 + 3*a^2*b^2))/(b^6 + a^2*b^4))*(2*a^2 +
3*b^2))/(b^3*(a^2 + b^2)^(3/2))
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 574, normalized size of antiderivative = 4.99

$$\int \frac{\sinh^3(x)}{(a+b\sinh(x))^2}\,dx$$

$$= \frac{-4e^{3x}a^5bx - 8e^{3x}a^3b^3x - 4e^{3x}a\,b^5x - 16e^{2x}a^4b^2x - 8e^{2x}a^2b^4x + 4e^xa^5bx + 8e^xa^3b^3x + 4e^xa\,b^5x - b^6 + 8}{}$$

input
```
int(sinh(x)^3/(a+b*sinh(x))^2,x)
```

output
```
(8*e**(3*x)*sqrt(a**2 + b**2)*atan((e**x*b*i + a*i)/sqrt(a**2 + b**2))*a**
4*b*i + 12*e**(3*x)*sqrt(a**2 + b**2)*atan((e**x*b*i + a*i)/sqrt(a**2 + b*
*2))*a**2*b**3*i + 16*e**(2*x)*sqrt(a**2 + b**2)*atan((e**x*b*i + a*i)/sqr
t(a**2 + b**2))*a**5*i + 24*e**(2*x)*sqrt(a**2 + b**2)*atan((e**x*b*i + a*
i)/sqrt(a**2 + b**2))*a**3*b**2*i - 8*e**x*sqrt(a**2 + b**2)*atan((e**x*b*
i + a*i)/sqrt(a**2 + b**2))*a**4*b*i - 12*e**x*sqrt(a**2 + b**2)*atan((e**
x*b*i + a*i)/sqrt(a**2 + b**2))*a**2*b**3*i + e**(4*x)*a**4*b**2 + 2*e**(4
*x)*a**2*b**4 + e**(4*x)*b**6 - 4*e**(3*x)*a**5*b*x + 4*e**(3*x)*a**5*b -
8*e**(3*x)*a**3*b**3*x + 6*e**(3*x)*a**3*b**3 - 4*e**(3*x)*a*b**5*x + 2*e*
*(3*x)*a*b**5 - 8*e**(2*x)*a**6*x - 16*e**(2*x)*a**4*b**2*x - 8*e**(2*x)*a
**2*b**4*x + 4*e**x*a**5*b*x + 4*e**x*a**5*b + 8*e**x*a**3*b**3*x + 6*e**x
*a**3*b**3 + 4*e**x*a*b**5*x + 2*e**x*a*b**5 - a**4*b**2 - 2*a**2*b**4 - b
**6)/(2*e**x*b**3*(e**(2*x)*a**4*b + 2*e**(2*x)*a**2*b**3 + e**(2*x)*b**5
+ 2*e**x*a**5 + 4*e**x*a**3*b**2 + 2*e**x*a*b**4 - a**4*b - 2*a**2*b**3 -
b**5))
```

3.82 $\int \frac{\sinh^2(x)}{(a+b\sinh(x))^2}\,dx$

Optimal result

Integrand size = 13, antiderivative size = 83

$$\int \frac{\sinh^2(x)}{(a+b\sinh(x))^2}\,dx=\frac{x}{b^2}+\frac{2a(a^2+2b^2)\operatorname{arctanh}\left(\frac{b-a\tanh\left(\frac{x}{2}\right)}{\sqrt{a^2+b^2}}\right)}{b^2\left(a^2+b^2\right)^{3/2}}-\frac{a^2\cosh(x)}{b\left(a^2+b^2\right)(a+b\sinh(x))}$$

output
```
x/b^2+2*a*(a^2+2*b^2)*arctanh((b-a*tanh(1/2*x))/(a^2+b^2)^(1/2))/b^2/(a^2+
b^2)^(3/2)-a^2*cosh(x)/b/(a^2+b^2)/(a+b*sinh(x))
```

Mathematica [A] (verified)

Time = 0.19 (sec) , antiderivative size = 86, normalized size of antiderivative = 1.04

$$\int \frac{\sinh^2(x)}{(a+b\sinh(x))^2}\,dx=\frac{x+\frac{2a(a^2+2b^2)\arctan\left(\frac{b-a\tanh\left(\frac{x}{2}\right)}{\sqrt{-a^2-b^2}}\right)}{(-a^2-b^2)^{3/2}}-\frac{a^2b\cosh(x)}{(a^2+b^2)(a+b\sinh(x))}}{b^2}$$

input
```
Integrate[Sinh[x]^2/(a + b*Sinh[x])^2,x]
```

output
```
(x + (2*a*(a^2 + 2*b^2)*ArcTan[(b - a*Tanh[x/2])/Sqrt[-a^2 - b^2]])/(-a^2
- b^2)^(3/2) - (a^2*b*Cosh[x])/((a^2 + b^2)*(a + b*Sinh[x])))/b^2
```

Rubi [A] (verified)

Time = 0.50 (sec) , antiderivative size = 111, normalized size of antiderivative = 1.34, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.769, Rules used = {3042, 25, 3269, 26, 3042, 3214, 3042, 3139, 1083, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sinh^2(x)}{(a+b\sinh(x))^2}dx$$

$$\Big\downarrow \ 3042$$

$$\int -\frac{\sin(ix)^2}{(a-ib\sin(ix))^2}dx$$

$$\Big\downarrow \ 25$$

$$-\int \frac{\sin(ix)^2}{(a-ib\sin(ix))^2}dx$$

$$\Big\downarrow \ 3269$$

$$-\frac{a^2\cosh(x)}{b(a^2+b^2)(a+b\sinh(x))}-\frac{i\int -\frac{i(ab-(a^2+b^2)\sinh(x))}{a+b\sinh(x)}dx}{b(a^2+b^2)}$$

$$\Big\downarrow \ 26$$

$$-\frac{\int \frac{ab-(a^2+b^2)\sinh(x)}{a+b\sinh(x)}dx}{b(a^2+b^2)}-\frac{a^2\cosh(x)}{b(a^2+b^2)(a+b\sinh(x))}$$

$$\Big\downarrow \ 3042$$

$$-\frac{a^2\cosh(x)}{b(a^2+b^2)(a+b\sinh(x))}-\frac{\int \frac{ab+i(a^2+b^2)\sin(ix)}{a-ib\sin(ix)}dx}{b(a^2+b^2)}$$

$$\Big\downarrow \ 3214$$

$$-\frac{\frac{a(a^2+2b^2)\int \frac{1}{a+b\sinh(x)}dx}{b}-\frac{x(a^2+b^2)}{b}}{b(a^2+b^2)}-\frac{a^2\cosh(x)}{b(a^2+b^2)(a+b\sinh(x))}$$

$$\Big\downarrow \ 3042$$

$$-\frac{a^2\cosh(x)}{b\left(a^2+b^2\right)\left(a+b\sinh(x)\right)}-\frac{-\frac{x\left(a^2+b^2\right)}{b}+\frac{a\left(a^2+2b^2\right)\int\frac{1}{a-ib\sin(ix)}dx}{b}}{b\left(a^2+b^2\right)}$$

$$\Big\downarrow \ 3139$$

$$-\frac{\frac{2a\left(a^2+2b^2\right)\int\frac{1}{-a\tanh^2\left(\frac{x}{2}\right)+2b\tanh\left(\frac{x}{2}\right)+a}d\tanh\left(\frac{x}{2}\right)}{b}-\frac{x\left(a^2+b^2\right)}{b}}{b\left(a^2+b^2\right)}-\frac{a^2\cosh(x)}{b\left(a^2+b^2\right)\left(a+b\sinh(x)\right)}$$

$$\Big\downarrow \ 1083$$

$$-\frac{-\frac{4a\left(a^2+2b^2\right)\int\frac{1}{4\left(a^2+b^2\right)-\left(2b-2a\tanh\left(\frac{x}{2}\right)\right)^2}d\left(2b-2a\tanh\left(\frac{x}{2}\right)\right)}{b}-\frac{x\left(a^2+b^2\right)}{b}}{b\left(a^2+b^2\right)}-\frac{a^2\cosh(x)}{b\left(a^2+b^2\right)\left(a+b\sinh(x)\right)}$$

$$\Big\downarrow \ 219$$

$$-\frac{-\frac{2a\left(a^2+2b^2\right)\text{arctanh}\left(\frac{2b-2a\tanh\left(\frac{x}{2}\right)}{2\sqrt{a^2+b^2}}\right)}{b\sqrt{a^2+b^2}}-\frac{x\left(a^2+b^2\right)}{b}}{b\left(a^2+b^2\right)}-\frac{a^2\cosh(x)}{b\left(a^2+b^2\right)\left(a+b\sinh(x)\right)}$$

input
```
Int[Sinh[x]^2/(a + b*Sinh[x])^2,x]
```

output
```
-((-(((a^2 + b^2)*x)/b) - (2*a*(a^2 + 2*b^2)*ArcTanh[(2*b - 2*a*Tanh[x/2])
/(2*Sqrt[a^2 + b^2])])/(b*Sqrt[a^2 + b^2]))/(b*(a^2 + b^2))) - (a^2*Cosh[x
])/(b*(a^2 + b^2)*(a + b*Sinh[x]))
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1083
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Simp[-2   Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3139
```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Simp[2*(e/d)   Subst[Int[1/(a + 2*b*e*x + a
*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[a^2 - b^2, 0]
```

rule 3214
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])/((c_.) + (d_.)*sin[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[b*(x/d), x] - Simp[(b*c - a*d)/d   Int[1/(c + d
*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

rule 3269
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^2, x_Symbol] :> Simp[(-(b^2*c^2 - 2*a*b*c*d + a^2*d^2))*Cos[e
+ f*x]*((a + b*Sin[e + f*x])^(m + 1)/(b*f*(m + 1)*(a^2 - b^2))), x] - Simp[
1/(b*(m + 1)*(a^2 - b^2))   Int[(a + b*Sin[e + f*x])^(m + 1)*Simp[b*(m + 1)
*(2*b*c*d - a*(c^2 + d^2)) + (a^2*d^2 - 2*a*b*c*d*(m + 2) + b^2*(d^2*(m + 1
) + c^2*(m + 2)))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f}, x]
&& NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && LtQ[m, -1]
```

Maple [A] (verified)

Time = 0.18 (sec) , antiderivative size = 127, normalized size of antiderivative = 1.53

method	result
default	$-\frac{\ln\left(\tanh\left(\frac{x}{2}\right)-1\right)}{b^{2}}+\frac{\ln\left(\tanh\left(\frac{x}{2}\right)+1\right)}{b^{2}}+\frac{2a\left(\frac{\frac{b^{2}\tanh\left(\frac{x}{2}\right)}{a^{2}+b^{2}}+\frac{ab}{a^{2}+b^{2}}}{\tanh\left(\frac{x}{2}\right)^{2}a-2b\tanh\left(\frac{x}{2}\right)-a}-\frac{\left(a^{2}+2b^{2}\right)\operatorname{arctanh}\left(\frac{2a\tanh\left(\frac{x}{2}\right)-2b}{2\sqrt{a^{2}+b^{2}}}\right)}{\left(a^{2}+b^{2}\right)^{\frac{3}{2}}}\right)}{b^{2}}$
risch	$\frac{x}{b^{2}}+\frac{2a^{2}(e^{x}a-b)}{b^{2}(a^{2}+b^{2})(b\,e^{2x}+2\,e^{x}a-b)}+\frac{a^{3}\ln\left(e^{x}+\frac{a\left(a^{2}+b^{2}\right)^{\frac{3}{2}}+a^{4}+2a^{2}b^{2}+b^{4}}{b\left(a^{2}+b^{2}\right)^{\frac{3}{2}}}\right)}{(a^{2}+b^{2})^{\frac{3}{2}}b^{2}}+\frac{2a\ln\left(e^{x}+\frac{a\left(a^{2}+b^{2}\right)^{\frac{3}{2}}+a^{4}+2a^{2}b^{2}+b^{4}}{b\left(a^{2}+b^{2}\right)^{\frac{3}{2}}}\right)}{(a^{2}+b^{2})^{\frac{3}{2}}}-\frac{a^{3}\ln}{}$

input
```
int(sinh(x)^2/(a+b*sinh(x))^2,x,method=_RETURNVERBOSE)
```

output
```
-1/b^2*ln(tanh(1/2*x)-1)+1/b^2*ln(tanh(1/2*x)+1)+2*a/b^2*((b^2/(a^2+b^2)*t
anh(1/2*x)+a*b/(a^2+b^2))/(tanh(1/2*x)^2*a-2*b*tanh(1/2*x)-a)-(a^2+2*b^2)/
(a^2+b^2)^(3/2)*arctanh(1/2*(2*a*tanh(1/2*x)-2*b)/(a^2+b^2)^(1/2)))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 521 vs. 2(79) = 158.

Time = 0.09 (sec) , antiderivative size = 521, normalized size of antiderivative = 6.28

$$\int\frac{\sinh^{2}(x)}{(a+b\sinh(x))^{2}}\,dx$$
$$=\frac{2\,a^{4}b+2\,a^{2}b^{3}-(a^{4}b+2\,a^{2}b^{3}+b^{5})x\cosh\left(x\right)^{2}-(a^{4}b+2\,a^{2}b^{3}+b^{5})x\sinh\left(x\right)^{2}+\left(a^{3}b+2\,ab^{3}-(a^{3}b+2\right.}{}$$

input
```
integrate(sinh(x)^2/(a+b*sinh(x))^2,x, algorithm="fricas")
```

output
```
(2*a^4*b + 2*a^2*b^3 - (a^4*b + 2*a^2*b^3 + b^5)*x*cosh(x)^2 - (a^4*b + 2*
a^2*b^3 + b^5)*x*sinh(x)^2 + (a^3*b + 2*a*b^3 - (a^3*b + 2*a*b^3)*cosh(x)^
2 - (a^3*b + 2*a*b^3)*sinh(x)^2 - 2*(a^4 + 2*a^2*b^2)*cosh(x) - 2*(a^4 + 2
*a^2*b^2 + (a^3*b + 2*a*b^3)*cosh(x))*sinh(x))*sqrt(a^2 + b^2)*log((b^2*co
sh(x)^2 + b^2*sinh(x)^2 + 2*a*b*cosh(x) + 2*a^2 + b^2 + 2*(b^2*cosh(x) + a
*b)*sinh(x) + 2*sqrt(a^2 + b^2)*(b*cosh(x) + b*sinh(x) + a))/(b*cosh(x)^2
+ b*sinh(x)^2 + 2*a*cosh(x) + 2*(b*cosh(x) + a)*sinh(x) - b)) + (a^4*b + 2
*a^2*b^3 + b^5)*x - 2*(a^5 + a^3*b^2 + (a^5 + 2*a^3*b^2 + a*b^4)*x)*cosh(x
) - 2*(a^5 + a^3*b^2 + (a^4*b + 2*a^2*b^3 + b^5)*x*cosh(x) + (a^5 + 2*a^3*
b^2 + a*b^4)*x)*sinh(x))/(a^4*b^3 + 2*a^2*b^5 + b^7 - (a^4*b^3 + 2*a^2*b^5
 + b^7)*cosh(x)^2 - (a^4*b^3 + 2*a^2*b^5 + b^7)*sinh(x)^2 - 2*(a^5*b^2 + 2
*a^3*b^4 + a*b^6)*cosh(x) - 2*(a^5*b^2 + 2*a^3*b^4 + a*b^6 + (a^4*b^3 + 2*
a^2*b^5 + b^7)*cosh(x))*sinh(x))
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\sinh^2(x)}{(a+b\sinh(x))^2}\,dx = \text{Timed out}$$

input
```
integrate(sinh(x)**2/(a+b*sinh(x))**2,x)
```

output
```
Timed out
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 149, normalized size of antiderivative = 1.80

$$\int \frac{\sinh^2(x)}{(a+b\sinh(x))^2}\,dx = -\frac{(a^2+2\,b^2)a\log\left(\frac{be^{(-x)}-a-\sqrt{a^2+b^2}}{be^{(-x)}-a+\sqrt{a^2+b^2}}\right)}{(a^2b^2+b^4)\sqrt{a^2+b^2}} - \frac{2\left(a^3e^{(-x)}+a^2b\right)}{a^2b^3+b^5+2\,(a^3b^2+ab^4)e^{(-x)}-(a^2b^3+b^5)e^{(-2\,x)}} + \frac{x}{b^2}$$

input
```
integrate(sinh(x)^2/(a+b*sinh(x))^2,x, algorithm="maxima")
```

output
```
-(a^2 + 2*b^2)*a*log((b*e^(-x) - a - sqrt(a^2 + b^2))/(b*e^(-x) - a + sqrt
(a^2 + b^2)))/((a^2*b^2 + b^4)*sqrt(a^2 + b^2)) - 2*(a^3*e^(-x) + a^2*b)/(
a^2*b^3 + b^5 + 2*(a^3*b^2 + a*b^4)*e^(-x) - (a^2*b^3 + b^5)*e^(-2*x)) + x
/b^2
```

Giac [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 131, normalized size of antiderivative = 1.58

$$\int \frac{\sinh^2(x)}{(a+b\sinh(x))^2}\,dx = -\frac{(a^3+2\,ab^2)\log\left(\frac{\left|2\,be^x+2\,a-2\,\sqrt{a^2+b^2}\right|}{\left|2\,be^x+2\,a+2\,\sqrt{a^2+b^2}\right|}\right)}{(a^2b^2+b^4)\sqrt{a^2+b^2}} + \frac{2\,(a^3e^x-a^2b)}{(a^2b^2+b^4)(be^{(2\,x)}+2\,ae^x-b)} + \frac{x}{b^2}$$

input
```
integrate(sinh(x)^2/(a+b*sinh(x))^2,x, algorithm="giac")
```

output
```
-(a^3 + 2*a*b^2)*log(abs(2*b*e^x + 2*a - 2*sqrt(a^2 + b^2))/abs(2*b*e^x +
2*a + 2*sqrt(a^2 + b^2)))/((a^2*b^2 + b^4)*sqrt(a^2 + b^2)) + 2*(a^3*e^x -
 a^2*b)/((a^2*b^2 + b^4)*(b*e^(2*x) + 2*a*e^x - b)) + x/b^2
```

Mupad [B] (verification not implemented)

Time = 2.03 (sec) , antiderivative size = 228, normalized size of antiderivative = 2.75

$$\int \frac{\sinh^2(x)}{(a+b\sinh(x))^2}\,dx = \frac{x}{b^2} - \frac{\frac{2\,a^2}{a^2\,b+b^3} - \frac{2\,a^3\,\mathrm{e}^x}{b\,(a^2\,b+b^3)}}{2\,a\,\mathrm{e}^x-b+b\,\mathrm{e}^{2\,x}} - \frac{a\,\ln\left(\frac{2\,\mathrm{e}^x\,(a^3+2\,a\,b^2)}{b^3\,(a^2+b^2)} - \frac{2\,a\,(a^2+2\,b^2)\,(b-a\,\mathrm{e}^x)}{b^3\,{(a^2+b^2)}^{3/2}}\right)\,(a^2+2\,b^2)}{b^2\,{(a^2+b^2)}^{3/2}} + \frac{a\,\ln\left(\frac{2\,\mathrm{e}^x\,(a^3+2\,a\,b^2)}{b^3\,(a^2+b^2)} + \frac{2\,a\,(a^2+2\,b^2)\,(b-a\,\mathrm{e}^x)}{b^3\,{(a^2+b^2)}^{3/2}}\right)\,(a^2+2\,b^2)}{b^2\,{(a^2+b^2)}^{3/2}}$$

input
```
int(sinh(x)^2/(a + b*sinh(x))^2,x)
```

output
```
x/b^2 - ((2*a^2)/(a^2*b + b^3) - (2*a^3*exp(x))/(b*(a^2*b + b^3)))/(2*a*ex
p(x) - b + b*exp(2*x)) - (a*log((2*exp(x)*(2*a*b^2 + a^3))/(b^3*(a^2 + b^2
)) - (2*a*(a^2 + 2*b^2)*(b - a*exp(x)))/(b^3*(a^2 + b^2)^(3/2)))*(a^2 + 2*
b^2))/(b^2*(a^2 + b^2)^(3/2)) + (a*log((2*exp(x)*(2*a*b^2 + a^3))/(b^3*(a^
2 + b^2)) + (2*a*(a^2 + 2*b^2)*(b - a*exp(x)))/(b^3*(a^2 + b^2)^(3/2)))*(a
^2 + 2*b^2))/(b^2*(a^2 + b^2)^(3/2))
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 452, normalized size of antiderivative = 5.45

$$\int \frac{\sinh^2(x)}{(a+b\sinh(x))^2}\,dx = \frac{-2e^{2x}\sqrt{a^2+b^2}\,atan\left(\frac{e^x bi+ai}{\sqrt{a^2+b^2}}\right)a^3bi-4e^{2x}\sqrt{a^2+b^2}\,atan\left(\frac{e^x bi+ai}{\sqrt{a^2+b^2}}\right)a\,b^3i-4e^{x}\sqrt{a^2+b^2}\,atan\left(\frac{e^x bi+ai}{\sqrt{a^2+b^2}}\right)a^4i}{}$$

input
```
int(sinh(x)^2/(a+b*sinh(x))^2,x)
```

output
```
( - 2*e**(2*x)*sqrt(a**2 + b**2)*atan((e**x*b*i + a*i)/sqrt(a**2 + b**2))*
a**3*b*i - 4*e**(2*x)*sqrt(a**2 + b**2)*atan((e**x*b*i + a*i)/sqrt(a**2 +
b**2))*a*b**3*i - 4*e**x*sqrt(a**2 + b**2)*atan((e**x*b*i + a*i)/sqrt(a**2
 + b**2))*a**4*i - 8*e**x*sqrt(a**2 + b**2)*atan((e**x*b*i + a*i)/sqrt(a**
2 + b**2))*a**2*b**2*i + 2*sqrt(a**2 + b**2)*atan((e**x*b*i + a*i)/sqrt(a*
*2 + b**2))*a**3*b*i + 4*sqrt(a**2 + b**2)*atan((e**x*b*i + a*i)/sqrt(a**2
 + b**2))*a*b**3*i + e**(2*x)*a**4*b*x - e**(2*x)*a**4*b + 2*e**(2*x)*a**2
*b**3*x - e**(2*x)*a**2*b**3 + e**(2*x)*b**5*x + 2*e**x*a**5*x + 4*e**x*a*
*3*b**2*x + 2*e**x*a*b**4*x - a**4*b*x - a**4*b - 2*a**2*b**3*x - a**2*b**
3 - b**5*x)/(b**2*(e**(2*x)*a**4*b + 2*e**(2*x)*a**2*b**3 + e**(2*x)*b**5
+ 2*e**x*a**5 + 4*e**x*a**3*b**2 + 2*e**x*a*b**4 - a**4*b - 2*a**2*b**3 -
b**5))
```

3.83 $\int \frac{\sinh(x)}{(a+b\sinh(x))^2}\,dx$

Optimal result

Integrand size = 11, antiderivative size = 60

$$\int \frac{\sinh(x)}{(a+b\sinh(x))^2}\,dx=-\frac{2b\text{arctanh}\left(\frac{b-a\tanh\left(\frac{x}{2}\right)}{\sqrt{a^2+b^2}}\right)}{(a^2+b^2)^{3/2}}+\frac{a\cosh(x)}{(a^2+b^2)\,(a+b\sinh(x))}$$

output
```
-2*b*arctanh((b-a*tanh(1/2*x))/(a^2+b^2)^(1/2))/(a^2+b^2)^(3/2)+a*cosh(x)/
(a^2+b^2)/(a+b*sinh(x))
```

Mathematica [A] (verified)

Time = 0.12 (sec) , antiderivative size = 68, normalized size of antiderivative = 1.13

$$\int \frac{\sinh(x)}{(a+b\sinh(x))^2}\,dx=-\frac{2b\arctan\left(\frac{b-a\tanh\left(\frac{x}{2}\right)}{\sqrt{-a^2-b^2}}\right)}{(-a^2-b^2)^{3/2}}+\frac{a\cosh(x)}{(a^2+b^2)\,(a+b\sinh(x))}$$

input
```
Integrate[Sinh[x]/(a + b*Sinh[x])^2,x]
```

output
```
(-2*b*ArcTan[(b - a*Tanh[x/2])/Sqrt[-a^2 - b^2]])/(-a^2 - b^2)^(3/2) + (a*
Cosh[x])/((a^2 + b^2)*(a + b*Sinh[x]))
```

Rubi [C] (verified)

Result contains complex when optimal does not.

Time = 0.33 (sec) , antiderivative size = 74, normalized size of antiderivative = 1.23, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.818, Rules used = {3042, 26, 3233, 26, 27, 3042, 3139, 1083, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sinh(x)}{(a+b\sinh(x))^2}dx$$

$$\Big\downarrow \text{3042}$$

$$\int -\frac{i\sin(ix)}{(a-ib\sin(ix))^2}dx$$

$$\Big\downarrow \text{26}$$

$$-i\int \frac{\sin(ix)}{(a-ib\sin(ix))^2}dx$$

$$\Big\downarrow \text{3233}$$

$$-i\left(\frac{ia\cosh(x)}{(a^2+b^2)(a+b\sinh(x))}-\frac{\int -\frac{ib}{a+b\sinh(x)}dx}{a^2+b^2}\right)$$

$$\Big\downarrow \text{26}$$

$$-i\left(\frac{i\int \frac{b}{a+b\sinh(x)}dx}{a^2+b^2}+\frac{ia\cosh(x)}{(a^2+b^2)(a+b\sinh(x))}\right)$$

$$\Big\downarrow \text{27}$$

$$-i\left(\frac{ib\int \frac{1}{a+b\sinh(x)}dx}{a^2+b^2}+\frac{ia\cosh(x)}{(a^2+b^2)(a+b\sinh(x))}\right)$$

$$\Big\downarrow \text{3042}$$

$$-i\left(\frac{ib\int \frac{1}{a-ib\sin(ix)}dx}{a^2+b^2}+\frac{ia\cosh(x)}{(a^2+b^2)(a+b\sinh(x))}\right)$$

$$\Big\downarrow \text{3139}$$

$$-i\left(\frac{2ib\int \frac{1}{-a\tanh^2\left(\frac{x}{2}\right)+2b\tanh\left(\frac{x}{2}\right)+a}d\tanh\left(\frac{x}{2}\right)}{a^2+b^2}+\frac{ia\cosh(x)}{(a^2+b^2)\left(a+b\sinh(x)\right)}\right)$$

$$\Big\downarrow \text{ 1083}$$

$$-i\left(\frac{ia\cosh(x)}{(a^2+b^2)\left(a+b\sinh(x)\right)}-\frac{4ib\int \frac{1}{4(a^2+b^2)-\left(2b-2a\tanh\left(\frac{x}{2}\right)\right)^2}d\left(2b-2a\tanh\left(\frac{x}{2}\right)\right)}{a^2+b^2}\right)$$

$$\Big\downarrow \text{ 219}$$

$$-i\left(\frac{ia\cosh(x)}{(a^2+b^2)\left(a+b\sinh(x)\right)}-\frac{2ib\text{arctanh}\left(\frac{2b-2a\tanh\left(\frac{x}{2}\right)}{2\sqrt{a^2+b^2}}\right)}{(a^2+b^2)^{3/2}}\right)$$

input
```
Int[Sinh[x]/(a + b*Sinh[x])^2,x]
```

output
```
(-I)*(((-2*I)*b*ArcTanh[(2*b - 2*a*Tanh[x/2])/(2*Sqrt[a^2 + b^2])])/(a^2 +
 b^2)^(3/2) + (I*a*Cosh[x])/((a^2 + b^2)*(a + b*Sinh[x])))
```

Defintions of rubi rules used

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1083
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Simp[-2   Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3139
```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Simp[2*(e/d)   Subst[Int[1/(a + 2*b*e*x + a
*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[a^2 - b^2, 0]
```

rule 3233
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Simp[(-(b*c - a*d))*Cos[e + f*x]*((a + b*Sin[e +
 f*x])^(m + 1)/(f*(m + 1)*(a^2 - b^2))), x] + Simp[1/((m + 1)*(a^2 - b^2))
  Int[(a + b*Sin[e + f*x])^(m + 1)*Simp[(a*c - b*d)*(m + 1) - (b*c - a*d)*(
m + 2)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c
- a*d, 0] && NeQ[a^2 - b^2, 0] && LtQ[m, -1] && IntegerQ[2*m]
```

Maple [A] (verified)

Time = 0.15 (sec) , antiderivative size = 97, normalized size of antiderivative = 1.62

method	result	size
default	$\frac{8b\tanh\left(\frac{x}{2}\right)+8a}{\left(-4a^2-4b^2\right)\left(\tanh\left(\frac{x}{2}\right)^2 a-2b\tanh\left(\frac{x}{2}\right)-a\right)}-\frac{8b\,\operatorname{arctanh}\left(\frac{2a\tanh\left(\frac{x}{2}\right)-2b}{2\sqrt{a^2+b^2}}\right)}{\left(-4a^2-4b^2\right)\sqrt{a^2+b^2}}$	97
risch	$-\frac{2a\left(e^{x}a-b\right)}{b\left(a^2+b^2\right)\left(b\,e^{2x}+2\,e^{x}a-b\right)}+\frac{b\ln\left(e^{x}+\frac{a\left(a^2+b^2\right)^{\frac{3}{2}}-a^4-2a^2b^2-b^4}{b\left(a^2+b^2\right)^{\frac{3}{2}}}\right)}{\left(a^2+b^2\right)^{\frac{3}{2}}}-\frac{b\ln\left(e^{x}+\frac{a\left(a^2+b^2\right)^{\frac{3}{2}}+a^4+2a^2b^2+b^4}{b\left(a^2+b^2\right)^{\frac{3}{2}}}\right)}{\left(a^2+b^2\right)^{\frac{3}{2}}}$	155

input
```
int(sinh(x)/(a+b*sinh(x))^2,x,method=_RETURNVERBOSE)
```

output
```
4*(2*b*tanh(1/2*x)+2*a)/(-4*a^2-4*b^2)/(tanh(1/2*x)^2*a-2*b*tanh(1/2*x)-a)
-8*b/(-4*a^2-4*b^2)/(a^2+b^2)^(1/2)*arctanh(1/2*(2*a*tanh(1/2*x)-2*b)/(a^2
+b^2)^(1/2))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 341 vs. $2(56) = 112$.

Time = 0.08 (sec) , antiderivative size = 341, normalized size of antiderivative = 5.68

$$\int \frac{\sinh(x)}{(a+b\sinh(x))^2}\,dx =$$
$$-\frac{2\,a^3b + 2\,ab^3 + \left(b^3\cosh\left(x\right)^2 + b^3\sinh\left(x\right)^2 + 2\,ab^2\cosh\left(x\right) - b^3 + 2\left(b^3\cosh\left(x\right) + ab^2\right)\sinh\left(x\right)\right)\sqrt{a^2 +}}{a^4b^2 + 2\,a^2b^4 + b^6 - \left(a^4b^2 + 2\,a^2b^4 + b^6\right)\cosh\left(x\right)^2 - \left(a^4b^2 + 2\,a^2b^4 + b\right.}$$

input
```
integrate(sinh(x)/(a+b*sinh(x))^2,x, algorithm="fricas")
```

output
```
-(2*a^3*b + 2*a*b^3 + (b^3*cosh(x)^2 + b^3*sinh(x)^2 + 2*a*b^2*cosh(x) - b
^3 + 2*(b^3*cosh(x) + a*b^2)*sinh(x))*sqrt(a^2 + b^2)*log((b^2*cosh(x)^2 +
 b^2*sinh(x)^2 + 2*a*b*cosh(x) + 2*a^2 + b^2 + 2*(b^2*cosh(x) + a*b)*sinh(
x) - 2*sqrt(a^2 + b^2)*(b*cosh(x) + b*sinh(x) + a))/(b*cosh(x)^2 + b*sinh(
x)^2 + 2*a*cosh(x) + 2*(b*cosh(x) + a)*sinh(x) - b)) - 2*(a^4 + a^2*b^2)*c
osh(x) - 2*(a^4 + a^2*b^2)*sinh(x))/(a^4*b^2 + 2*a^2*b^4 + b^6 - (a^4*b^2
+ 2*a^2*b^4 + b^6)*cosh(x)^2 - (a^4*b^2 + 2*a^2*b^4 + b^6)*sinh(x)^2 - 2*(
a^5*b + 2*a^3*b^3 + a*b^5)*cosh(x) - 2*(a^5*b + 2*a^3*b^3 + a*b^5 + (a^4*b
^2 + 2*a^2*b^4 + b^6)*cosh(x))*sinh(x))
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\sinh(x)}{(a+b\sinh(x))^2}\,dx = \text{Timed out}$$

input
```
integrate(sinh(x)/(a+b*sinh(x))**2,x)
```

output
```
Timed out
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 117 vs. $2(56) = 112$.

Time = 0.11 (sec) , antiderivative size = 117, normalized size of antiderivative = 1.95

$$\int \frac{\sinh(x)}{(a+b\sinh(x))^2}\,dx = \frac{b\log\left(\frac{be^{(-x)}-a-\sqrt{a^2+b^2}}{be^{(-x)}-a+\sqrt{a^2+b^2}}\right)}{(a^2+b^2)^{\frac{3}{2}}} + \frac{2\left(a^2e^{(-x)}+ab\right)}{a^2b^2+b^4+2\,(a^3b+ab^3)e^{(-x)}-(a^2b^2+b^4)e^{(-2\,x)}}$$

input
```
integrate(sinh(x)/(a+b*sinh(x))^2,x, algorithm="maxima")
```

output
```
b*log((b*e^(-x) - a - sqrt(a^2 + b^2))/(b*e^(-x) - a + sqrt(a^2 + b^2)))/(
a^2 + b^2)^(3/2) + 2*(a^2*e^(-x) + a*b)/(a^2*b^2 + b^4 + 2*(a^3*b + a*b^3)
*e^(-x) - (a^2*b^2 + b^4)*e^(-2*x))
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 99, normalized size of antiderivative = 1.65

$$\int \frac{\sinh(x)}{(a+b\sinh(x))^2}\,dx = \frac{b\log\left(\frac{\left|2\,be^x+2\,a-2\,\sqrt{a^2+b^2}\right|}{\left|2\,be^x+2\,a+2\,\sqrt{a^2+b^2}\right|}\right)}{(a^2+b^2)^{\frac{3}{2}}} - \frac{2\left(a^2e^x-ab\right)}{(a^2b+b^3)(be^{(2\,x)}+2\,ae^x-b)}$$

input
```
integrate(sinh(x)/(a+b*sinh(x))^2,x, algorithm="giac")
```

output
```
b*log(abs(2*b*e^x + 2*a - 2*sqrt(a^2 + b^2))/abs(2*b*e^x + 2*a + 2*sqrt(a^
2 + b^2)))/(a^2 + b^2)^(3/2) - 2*(a^2*e^x - a*b)/((a^2*b + b^3)*(b*e^(2*x)
 + 2*a*e^x - b))
```

Mupad [B] (verification not implemented)

Time = 1.92 (sec) , antiderivative size = 142, normalized size of antiderivative = 2.37

$$\int \frac{\sinh(x)}{(a+b\sinh(x))^2}\,dx = \frac{\frac{2\,a\,b}{a^2\,b+b^3}-\frac{2\,a^2\,e^x}{a^2\,b+b^3}}{2\,a\,e^x-b+b\,e^{2\,x}}-\frac{b\,\ln\left(-\frac{2\,e^x}{a^2+b^2}-\frac{2\,(b-a\,e^x)}{(a^2+b^2)^{3/2}}\right)}{(a^2+b^2)^{3/2}}+\frac{b\,\ln\left(\frac{2\,(b-a\,e^x)}{(a^2+b^2)^{3/2}}-\frac{2\,e^x}{a^2+b^2}\right)}{(a^2+b^2)^{3/2}}$$

input
```
int(sinh(x)/(a + b*sinh(x))^2,x)
```

output
```
((2*a*b)/(a^2*b + b^3) - (2*a^2*exp(x))/(a^2*b + b^3))/(2*a*exp(x) - b + b
*exp(2*x)) - (b*log(- (2*exp(x))/(a^2 + b^2) - (2*(b - a*exp(x)))/(a^2 + b
^2)^(3/2)))/(a^2 + b^2)^(3/2) + (b*log((2*(b - a*exp(x)))/(a^2 + b^2)^(3/2
) - (2*exp(x))/(a^2 + b^2)))/(a^2 + b^2)^(3/2)
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 226, normalized size of antiderivative = 3.77

$$\int \frac{\sinh(x)}{(a+b\sinh(x))^2}\,dx = \frac{2e^{2x}\sqrt{a^2+b^2}\,atan\left(\frac{e^x bi+ai}{\sqrt{a^2+b^2}}\right)b^2i+4e^x\sqrt{a^2+b^2}\,atan\left(\frac{e^x bi+ai}{\sqrt{a^2+b^2}}\right)abi-2\sqrt{a^2+b^2}\,atan\left(\frac{e^x bi+ai}{\sqrt{a^2+b^2}}\right)b^2i+e^{2x}a^3}{e^{2x}a^4b+2e^{2x}a^2b^3+e^{2x}b^5+2e^xa^5+4e^xa^3b^2+2e^xa\,b^4-a^4b-2a^2b^3-b^5}$$

input
```
int(sinh(x)/(a+b*sinh(x))^2,x)
```

output
```
(2*e**(2*x)*sqrt(a**2 + b**2)*atan((e**x*b*i + a*i)/sqrt(a**2 + b**2))*b**
2*i + 4*e**x*sqrt(a**2 + b**2)*atan((e**x*b*i + a*i)/sqrt(a**2 + b**2))*a*
b*i - 2*sqrt(a**2 + b**2)*atan((e**x*b*i + a*i)/sqrt(a**2 + b**2))*b**2*i
+ e**(2*x)*a**3 + e**(2*x)*a*b**2 + a**3 + a*b**2)/(e**(2*x)*a**4*b + 2*e*
*(2*x)*a**2*b**3 + e**(2*x)*b**5 + 2*e**x*a**5 + 4*e**x*a**3*b**2 + 2*e**x
*a*b**4 - a**4*b - 2*a**2*b**3 - b**5)
```

3.84 $\int \frac{\operatorname{csch}(x)}{(a+b\sinh(x))^2}\,dx$

Optimal result

Integrand size = 11, antiderivative size = 85

$$\int \frac{\operatorname{csch}(x)}{(a+b\sinh(x))^2}\,dx=-\frac{\operatorname{arctanh}(\cosh(x))}{a^2}+\frac{2b(2a^2+b^2)\operatorname{arctanh}\left(\frac{b-a\tanh\left(\frac{x}{2}\right)}{\sqrt{a^2+b^2}}\right)}{a^2\left(a^2+b^2\right)^{3/2}}+\frac{b^2\cosh(x)}{a\left(a^2+b^2\right)(a+b\sinh(x))}$$

output
```
-arctanh(cosh(x))/a^2+2*b*(2*a^2+b^2)*arctanh((b-a*tanh(1/2*x))/(a^2+b^2)^
(1/2))/a^2/(a^2+b^2)^(3/2)+b^2*cosh(x)/a/(a^2+b^2)/(a+b*sinh(x))
```

Mathematica [A] (verified)

Time = 0.42 (sec) , antiderivative size = 100, normalized size of antiderivative = 1.18

$$\int \frac{\operatorname{csch}(x)}{(a+b\sinh(x))^2}\,dx=\frac{\frac{2b(2a^2+b^2)\arctan\left(\frac{b-a\tanh\left(\frac{x}{2}\right)}{\sqrt{-a^2-b^2}}\right)}{(-a^2-b^2)^{3/2}}-\log\left(\cosh\left(\frac{x}{2}\right)\right)+\log\left(\sinh\left(\frac{x}{2}\right)\right)+\frac{ab^2\cosh(x)}{(a^2+b^2)(a+b\sinh(x))}}{a^2}$$

input
```
Integrate[Csch[x]/(a + b*Sinh[x])^2,x]
```

output
```
((2*b*(2*a^2 + b^2)*ArcTan[(b - a*Tanh[x/2])/Sqrt[-a^2 - b^2]])/(-a^2 - b^
2)^(3/2) - Log[Cosh[x/2]] + Log[Sinh[x/2]] + (a*b^2*Cosh[x])/((a^2 + b^2)*
(a + b*Sinh[x])))/a^2
```

Rubi [C] (verified)

Result contains complex when optimal does not.

Time = 0.62 (sec) , antiderivative size = 122, normalized size of antiderivative = 1.44, number of steps used = 15, number of rules used = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 1.273$, Rules used = {3042, 26, 3281, 26, 3042, 26, 3480, 26, 3042, 26, 3139, 1083, 219, 4257}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\text{csch}(x)}{(a+b\sinh(x))^2}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{i}{\sin(ix)(a-ib\sin(ix))^2}dx$$

$$\Big\downarrow \text{26}$$

$$i\int \frac{1}{\sin(ix)(a-ib\sin(ix))^2}dx$$

$$\Big\downarrow \text{3281}$$

$$i\left(\frac{\int -\frac{i\text{csch}(x)(a^2-b\sinh(x)a+b^2)}{a+b\sinh(x)}dx}{a(a^2+b^2)}-\frac{ib^2\cosh(x)}{a(a^2+b^2)(a+b\sinh(x))}\right)$$

$$\Big\downarrow \text{26}$$

$$i\left(-\frac{i\int \frac{\text{csch}(x)(a^2-b\sinh(x)a+b^2)}{a+b\sinh(x)}dx}{a(a^2+b^2)}-\frac{ib^2\cosh(x)}{a(a^2+b^2)(a+b\sinh(x))}\right)$$

$$\Big\downarrow \text{3042}$$

$$i\left(-\frac{i\int \frac{i(a^2+ib\sin(ix)a+b^2)}{\sin(ix)(a-ib\sin(ix))}dx}{a(a^2+b^2)}-\frac{ib^2\cosh(x)}{a(a^2+b^2)(a+b\sinh(x))}\right)$$

$$\Big\downarrow \text{26}$$

$$i\left(\frac{\int \frac{a^2+ib\sin(ix)a+b^2}{\sin(ix)(a-ib\sin(ix))}dx}{a(a^2+b^2)}-\frac{ib^2\cosh(x)}{a(a^2+b^2)(a+b\sinh(x))}\right)$$

$$\Big\downarrow \text{3480}$$

$$i\left(\frac{\frac{ib(2a^2+b^2)\int \frac{1}{a+b\sinh(x)}dx}{a}+\frac{(a^2+b^2)\int -i\text{csch}(x)dx}{a}}{a(a^2+b^2)}-\frac{ib^2\cosh(x)}{a(a^2+b^2)(a+b\sinh(x))}\right)$$

$$\Big\downarrow \text{26}$$

$$i\left(\frac{\frac{ib(2a^2+b^2)\int \frac{1}{a+b\sinh(x)}dx}{a}-\frac{i(a^2+b^2)\int \text{csch}(x)dx}{a}}{a(a^2+b^2)}-\frac{ib^2\cosh(x)}{a(a^2+b^2)(a+b\sinh(x))}\right)$$

$$\Big\downarrow \text{3042}$$

$$i\left(\frac{\frac{ib(2a^2+b^2)\int \frac{1}{a-ib\sin(ix)}dx}{a}-\frac{i(a^2+b^2)\int i\csc(ix)dx}{a}}{a(a^2+b^2)}-\frac{ib^2\cosh(x)}{a(a^2+b^2)(a+b\sinh(x))}\right)$$

$$\Big\downarrow \text{26}$$

$$i\left(\frac{\frac{ib(2a^2+b^2)\int \frac{1}{a-ib\sin(ix)}dx}{a}+\frac{(a^2+b^2)\int \csc(ix)dx}{a}}{a(a^2+b^2)}-\frac{ib^2\cosh(x)}{a(a^2+b^2)(a+b\sinh(x))}\right)$$

$$\Big\downarrow \text{3139}$$

$$i\left(\frac{\frac{(a^2+b^2)\int \csc(ix)dx}{a}+\frac{2ib(2a^2+b^2)\int \frac{1}{-a\tanh^2\left(\frac{x}{2}\right)+2b\tanh\left(\frac{x}{2}\right)+a}d\tanh\left(\frac{x}{2}\right)}{a}}{a(a^2+b^2)}-\frac{ib^2\cosh(x)}{a(a^2+b^2)(a+b\sinh(x))}\right)$$

$$\Big\downarrow \text{1083}$$

$$i\left(\frac{\frac{(a^2+b^2)\int \csc(ix)dx}{a}-\frac{4ib(2a^2+b^2)\int \frac{1}{4(a^2+b^2)-\left(2b-2a\tanh\left(\frac{x}{2}\right)\right)^2}d\left(2b-2a\tanh\left(\frac{x}{2}\right)\right)}{a}}{a(a^2+b^2)}-\frac{ib^2\cosh(x)}{a(a^2+b^2)(a+b\sinh(x))}\right)$$

$$\Big\downarrow \text{219}$$

$$i\left(\frac{\frac{(a^2+b^2)\int \csc(ix)dx}{a}-\frac{2ib(2a^2+b^2)\text{arctanh}\left(\frac{2b-2a\tanh\left(\frac{x}{2}\right)}{2\sqrt{a^2+b^2}}\right)}{a\sqrt{a^2+b^2}}}{a\left(a^2+b^2\right)}-\frac{ib^2\cosh(x)}{a\left(a^2+b^2\right)(a+b\sinh(x))}\right)$$

$$\Big\downarrow \text{4257}$$

$$i\left(\frac{\frac{i(a^2+b^2)\text{arctanh}(\cosh(x))}{a}-\frac{2ib(2a^2+b^2)\text{arctanh}\left(\frac{2b-2a\tanh\left(\frac{x}{2}\right)}{2\sqrt{a^2+b^2}}\right)}{a\sqrt{a^2+b^2}}}{a\left(a^2+b^2\right)}-\frac{ib^2\cosh(x)}{a\left(a^2+b^2\right)(a+b\sinh(x))}\right)$$

input
```
Int[Csch[x]/(a + b*Sinh[x])^2,x]
```

output
```
I*(((I*(a^2 + b^2)*ArcTanh[Cosh[x]])/a - ((2*I)*b*(2*a^2 + b^2)*ArcTanh[(2
*b - 2*a*Tanh[x/2])/(2*Sqrt[a^2 + b^2])])/(a*Sqrt[a^2 + b^2]))/(a*(a^2 + b
^2)) - (I*b^2*Cosh[x])/(a*(a^2 + b^2)*(a + b*Sinh[x])))
```

Defintions of rubi rules used

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1083
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Simp[-2   Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3139
```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Simp[2*(e/d)   Subst[Int[1/(a + 2*b*e*x + a
*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[a^2 - b^2, 0]
```

rule 3281
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b^2)*Cos[e + f*x]*(a + b*Sin[e + f*
x])^(m + 1)*((c + d*Sin[e + f*x])^(n + 1)/(f*(m + 1)*(b*c - a*d)*(a^2 - b^2
))), x] + Simp[1/((m + 1)*(b*c - a*d)*(a^2 - b^2))   Int[(a + b*Sin[e + f*x
])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[a*(b*c - a*d)*(m + 1) + b^2*d*(m + n
 + 2) - (b^2*c + b*(b*c - a*d)*(m + 1))*Sin[e + f*x] - b^2*d*(m + n + 3)*Si
n[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*
d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -1] && IntegersQ[
2*m, 2*n] && ((EqQ[a, 0] && IntegerQ[m] &&  !IntegerQ[n]) ||  !(IntegerQ[2*
n] && LtQ[n, -1] && ((IntegerQ[n] &&  !IntegerQ[m]) || EqQ[a, 0])))
```

rule 3480
```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((a_.) + (b_.)*sin[(e_.) + (f_
.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])), x_Symbol] :> Simp[(A*b
- a*B)/(b*c - a*d)   Int[1/(a + b*Sin[e + f*x]), x], x] + Simp[(B*c - A*d)/
(b*c - a*d)   Int[1/(c + d*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f
, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

rule 4257
```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Maple [A] (verified)

Time = 0.26 (sec) , antiderivative size = 115, normalized size of antiderivative = 1.35

method	result
default	$\frac{4b\left(\frac{-\frac{b^2\tanh\left(\frac{x}{2}\right)}{2\left(a^2+b^2\right)}-\frac{ab}{2\left(a^2+b^2\right)}}{\tanh\left(\frac{x}{2}\right)^2 a-2b\tanh\left(\frac{x}{2}\right)-a}-\frac{\left(2a^2+b^2\right)\operatorname{arctanh}\left(\frac{2a\tanh\left(\frac{x}{2}\right)-2b}{2\sqrt{a^2+b^2}}\right)}{2\left(a^2+b^2\right)^{\frac{3}{2}}}\right)}{a^2}+\frac{\ln\left(\tanh\left(\frac{x}{2}\right)\right)}{a^2}$
risch	$-\frac{2b(e^x a-b)}{a(a^2+b^2)(b\,e^{2x}+2\,e^x a-b)}-\frac{\ln(e^x+1)}{a^2}+\frac{2b\ln\left(e^x+\frac{a\left(a^2+b^2\right)^{\frac{3}{2}}+a^4+2a^2b^2+b^4}{b\left(a^2+b^2\right)^{\frac{3}{2}}}\right)}{(a^2+b^2)^{\frac{3}{2}}}+\frac{b^3\ln\left(e^x+\frac{a\left(a^2+b^2\right)^{\frac{3}{2}}+a^4+2a^2b^2+b^4}{b\left(a^2+b^2\right)^{\frac{3}{2}}}\right)}{(a^2+b^2)^{\frac{3}{2}}a^2}$

input
```
int(csch(x)/(a+b*sinh(x))^2,x,method=_RETURNVERBOSE)
```

output
```
4/a^2*b*((-1/2*b^2/(a^2+b^2)*tanh(1/2*x)-1/2*a*b/(a^2+b^2))/(tanh(1/2*x)^2
*a-2*b*tanh(1/2*x)-a)-1/2*(2*a^2+b^2)/(a^2+b^2)^(3/2)*arctanh(1/2*(2*a*tan
h(1/2*x)-2*b)/(a^2+b^2)^(1/2)))+1/a^2*ln(tanh(1/2*x))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 672 vs. $2(81) = 162$.

Time = 0.19 (sec) , antiderivative size = 672, normalized size of antiderivative = 7.91

$$\int \frac{\operatorname{csch}(x)}{(a+b\sinh(x))^2}\,dx = \text{Too large to display}$$

input
```
integrate(csch(x)/(a+b*sinh(x))^2,x, algorithm="fricas")
```

output
```
-(2*a^3*b^2 + 2*a*b^4 - (2*a^2*b^2 + b^4 - (2*a^2*b^2 + b^4)*cosh(x)^2 - (
2*a^2*b^2 + b^4)*sinh(x)^2 - 2*(2*a^3*b + a*b^3)*cosh(x) - 2*(2*a^3*b + a*
b^3 + (2*a^2*b^2 + b^4)*cosh(x))*sinh(x))*sqrt(a^2 + b^2)*log((b^2*cosh(x)
^2 + b^2*sinh(x)^2 + 2*a*b*cosh(x) + 2*a^2 + b^2 + 2*(b^2*cosh(x) + a*b)*s
inh(x) + 2*sqrt(a^2 + b^2)*(b*cosh(x) + b*sinh(x) + a))/(b*cosh(x)^2 + b*s
inh(x)^2 + 2*a*cosh(x) + 2*(b*cosh(x) + a)*sinh(x) - b)) - 2*(a^4*b + a^2*
b^3)*cosh(x) + (a^4*b + 2*a^2*b^3 + b^5 - (a^4*b + 2*a^2*b^3 + b^5)*cosh(x
)^2 - (a^4*b + 2*a^2*b^3 + b^5)*sinh(x)^2 - 2*(a^5 + 2*a^3*b^2 + a*b^4)*co
sh(x) - 2*(a^5 + 2*a^3*b^2 + a*b^4 + (a^4*b + 2*a^2*b^3 + b^5)*cosh(x))*si
nh(x))*log(cosh(x) + sinh(x) + 1) - (a^4*b + 2*a^2*b^3 + b^5 - (a^4*b + 2*
a^2*b^3 + b^5)*cosh(x)^2 - (a^4*b + 2*a^2*b^3 + b^5)*sinh(x)^2 - 2*(a^5 +
2*a^3*b^2 + a*b^4)*cosh(x) - 2*(a^5 + 2*a^3*b^2 + a*b^4 + (a^4*b + 2*a^2*b
^3 + b^5)*cosh(x))*sinh(x))*log(cosh(x) + sinh(x) - 1) - 2*(a^4*b + a^2*b^
3)*sinh(x))/(a^6*b + 2*a^4*b^3 + a^2*b^5 - (a^6*b + 2*a^4*b^3 + a^2*b^5)*c
osh(x)^2 - (a^6*b + 2*a^4*b^3 + a^2*b^5)*sinh(x)^2 - 2*(a^7 + 2*a^5*b^2 +
a^3*b^4)*cosh(x) - 2*(a^7 + 2*a^5*b^2 + a^3*b^4 + (a^6*b + 2*a^4*b^3 + a^2
*b^5)*cosh(x))*sinh(x))
```

Sympy [F]

$$\int \frac{\text{csch}(x)}{(a+b\sinh(x))^2}\,dx=\int \frac{\text{csch}\,(x)}{(a+b\sinh\,(x))^2}\,dx$$

input
```
integrate(csch(x)/(a+b*sinh(x))**2,x)
```

output
```
Integral(csch(x)/(a + b*sinh(x))**2, x)
```

Maxima [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 162, normalized size of antiderivative = 1.91

$$\int \frac{\operatorname{csch}(x)}{(a+b\sinh(x))^2}\,dx = -\frac{(2\,a^2b+b^3)\log\left(\frac{be^{(-x)}-a-\sqrt{a^2+b^2}}{be^{(-x)}-a+\sqrt{a^2+b^2}}\right)}{(a^4+a^2b^2)\sqrt{a^2+b^2}} + \frac{2\left(abe^{(-x)}+b^2\right)}{a^3b+ab^3+2\,(a^4+a^2b^2)e^{(-x)}-(a^3b+ab^3)e^{(-2\,x)}} - \frac{\log\left(e^{(-x)}+1\right)}{a^2} + \frac{\log\left(e^{(-x)}-1\right)}{a^2}$$

input
```
integrate(csch(x)/(a+b*sinh(x))^2,x, algorithm="maxima")
```

output
```
-(2*a^2*b + b^3)*log((b*e^(-x) - a - sqrt(a^2 + b^2))/(b*e^(-x) - a + sqrt
(a^2 + b^2)))/((a^4 + a^2*b^2)*sqrt(a^2 + b^2)) + 2*(a*b*e^(-x) + b^2)/(a^
3*b + a*b^3 + 2*(a^4 + a^2*b^2)*e^(-x) - (a^3*b + a*b^3)*e^(-2*x)) - log(e
^(-x) + 1)/a^2 + log(e^(-x) - 1)/a^2
```

Giac [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 142, normalized size of antiderivative = 1.67

$$\int \frac{\operatorname{csch}(x)}{(a+b\sinh(x))^2}\,dx = -\frac{(2\,a^2b+b^3)\log\left(\frac{\left|2\,be^x+2\,a-2\,\sqrt{a^2+b^2}\right|}{\left|2\,be^x+2\,a+2\,\sqrt{a^2+b^2}\right|}\right)}{(a^4+a^2b^2)\sqrt{a^2+b^2}} - \frac{2\,(abe^x-b^2)}{(a^3+ab^2)(be^{(2\,x)}+2\,ae^x-b)} - \frac{\log\left(e^x+1\right)}{a^2} + \frac{\log\left(|e^x-1|\right)}{a^2}$$

input
```
integrate(csch(x)/(a+b*sinh(x))^2,x, algorithm="giac")
```

output
```
-(2*a^2*b + b^3)*log(abs(2*b*e^x + 2*a - 2*sqrt(a^2 + b^2))/abs(2*b*e^x +
2*a + 2*sqrt(a^2 + b^2)))/((a^4 + a^2*b^2)*sqrt(a^2 + b^2)) - 2*(a*b*e^x -
 b^2)/((a^3 + a*b^2)*(b*e^(2*x) + 2*a*e^x - b)) - log(e^x + 1)/a^2 + log(a
bs(e^x - 1))/a^2
```

Mupad [B] (verification not implemented)

Time = 4.28 (sec) , antiderivative size = 1001, normalized size of antiderivative = 11.78

$$\int \frac{\operatorname{csch}(x)}{(a+b\sinh(x))^2}\,dx = \text{Too large to display}$$

input
```
int(1/(sinh(x)*(a + b*sinh(x))^2),x)
```

output
```
((2*b^5)/(a*(b^5 + a^2*b^3)) - (2*b^4*exp(x))/(b^5 + a^2*b^3))/(2*a*exp(x)
 - b + b*exp(2*x)) + log(exp(x) - 1)/a^2 - log(exp(x) + 1)/a^2 - (b*log((3
2*(4*a^4*b + 2*b^5 + 6*a^2*b^3 - 8*a^5*exp(x) - 3*a*b^4*exp(x) - 10*a^3*b^
2*exp(x)))/(a*(a^2*b^5 + a^4*b^3)*(a^2*b + b^3)) + (b*((32*(2*a^6*b + 2*b^
7 + 8*a^2*b^5 + 9*a^4*b^3 - 4*a^7*exp(x) - 3*a*b^6*exp(x) - 10*a^3*b^4*exp
(x) - 11*a^5*b^2*exp(x)))/(a*b^5*(a*b^4 + a^5 + 2*a^3*b^2)) - (b*(2*a^2 +
b^2)*((a^2 + b^2)^3)^(1/2)*((32*(2*a*b^3 + 4*a^3*b - 7*a^4*exp(x) - 4*a^2*
b^2*exp(x)))/(b*(b^5 + a^2*b^3)) + (32*(2*a^2 + b^2)*((a^2 + b^2)^3)^(1/2)
*(3*a^4*b + 2*a^2*b^3 - 4*a^5*exp(x) - 3*a^3*b^2*exp(x)))/(b^4*(a^8 + a^2*
b^6 + 3*a^4*b^4 + 3*a^6*b^2))))/(a^8 + a^2*b^6 + 3*a^4*b^4 + 3*a^6*b^2))*(
2*a^2 + b^2)*((a^2 + b^2)^3)^(1/2))/(a^8 + a^2*b^6 + 3*a^4*b^4 + 3*a^6*b^2
))*(2*a^2 + b^2)*((a^2 + b^2)^3)^(1/2))/(a^8 + a^2*b^6 + 3*a^4*b^4 + 3*a^6
*b^2) + (b*log((32*(4*a^4*b + 2*b^5 + 6*a^2*b^3 - 8*a^5*exp(x) - 3*a*b^4*e
xp(x) - 10*a^3*b^2*exp(x)))/(a*(a^2*b^5 + a^4*b^3)*(a^2*b + b^3)) - (b*((3
2*(2*a^6*b + 2*b^7 + 8*a^2*b^5 + 9*a^4*b^3 - 4*a^7*exp(x) - 3*a*b^6*exp(x)
 - 10*a^3*b^4*exp(x) - 11*a^5*b^2*exp(x)))/(a*b^5*(a*b^4 + a^5 + 2*a^3*b^2
)) + (b*(2*a^2 + b^2)*((a^2 + b^2)^3)^(1/2)*((32*(2*a*b^3 + 4*a^3*b - 7*a^
4*exp(x) - 4*a^2*b^2*exp(x)))/(b*(b^5 + a^2*b^3)) - (32*(2*a^2 + b^2)*((a^
2 + b^2)^3)^(1/2)*(3*a^4*b + 2*a^2*b^3 - 4*a^5*exp(x) - 3*a^3*b^2*exp(x)))
/(b^4*(a^8 + a^2*b^6 + 3*a^4*b^4 + 3*a^6*b^2))))/(a^8 + a^2*b^6 + 3*a^4...
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 627, normalized size of antiderivative = 7.38

$$\int \frac{\operatorname{csch}(x)}{(a+b\sinh(x))^2}\,dx$$

$$= \frac{-\log(e^x-1)\,b^5+\log(e^x+1)\,b^5+a\,b^4+e^{2x}\log(e^x-1)\,b^5-e^{2x}\log(e^x+1)\,b^5+e^{2x}a^3b^2+e^{2x}a\,b^4-\log(\ldots}{\ldots}$$

input
```
int(csch(x)/(a+b*sinh(x))^2,x)
```

output
```
( - 4*e**(2*x)*sqrt(a**2 + b**2)*atan((e**x*b*i + a*i)/sqrt(a**2 + b**2))*
a**2*b**2*i - 2*e**(2*x)*sqrt(a**2 + b**2)*atan((e**x*b*i + a*i)/sqrt(a**2
 + b**2))*b**4*i - 8*e**x*sqrt(a**2 + b**2)*atan((e**x*b*i + a*i)/sqrt(a**
2 + b**2))*a**3*b*i - 4*e**x*sqrt(a**2 + b**2)*atan((e**x*b*i + a*i)/sqrt(
a**2 + b**2))*a*b**3*i + 4*sqrt(a**2 + b**2)*atan((e**x*b*i + a*i)/sqrt(a*
*2 + b**2))*a**2*b**2*i + 2*sqrt(a**2 + b**2)*atan((e**x*b*i + a*i)/sqrt(a
**2 + b**2))*b**4*i + e**(2*x)*log(e**x - 1)*a**4*b + 2*e**(2*x)*log(e**x
- 1)*a**2*b**3 + e**(2*x)*log(e**x - 1)*b**5 - e**(2*x)*log(e**x + 1)*a**4
*b - 2*e**(2*x)*log(e**x + 1)*a**2*b**3 - e**(2*x)*log(e**x + 1)*b**5 + e*
*(2*x)*a**3*b**2 + e**(2*x)*a*b**4 + 2*e**x*log(e**x - 1)*a**5 + 4*e**x*lo
g(e**x - 1)*a**3*b**2 + 2*e**x*log(e**x - 1)*a*b**4 - 2*e**x*log(e**x + 1)
*a**5 - 4*e**x*log(e**x + 1)*a**3*b**2 - 2*e**x*log(e**x + 1)*a*b**4 - log
(e**x - 1)*a**4*b - 2*log(e**x - 1)*a**2*b**3 - log(e**x - 1)*b**5 + log(e
**x + 1)*a**4*b + 2*log(e**x + 1)*a**2*b**3 + log(e**x + 1)*b**5 + a**3*b*
*2 + a*b**4)/(a**2*(e**(2*x)*a**4*b + 2*e**(2*x)*a**2*b**3 + e**(2*x)*b**5
 + 2*e**x*a**5 + 4*e**x*a**3*b**2 + 2*e**x*a*b**4 - a**4*b - 2*a**2*b**3 -
 b**5))
```

3.85 $\int \frac{\operatorname{csch}^2(x)}{(a+b\sinh(x))^2}\,dx$

Optimal result

Integrand size = 13, antiderivative size = 115

$$\int \frac{\operatorname{csch}^2(x)}{(a+b\sinh(x))^2}\,dx = \frac{2b\operatorname{arctanh}(\cosh(x))}{a^3} - \frac{2b^2(3a^2+2b^2)\operatorname{arctanh}\left(\frac{b-a\tanh\left(\frac{x}{2}\right)}{\sqrt{a^2+b^2}}\right)}{a^3(a^2+b^2)^{3/2}} - \frac{(a^2+2b^2)\coth(x)}{a^2(a^2+b^2)} + \frac{b^2\coth(x)}{a(a^2+b^2)(a+b\sinh(x))}$$

output
```
2*b*arctanh(cosh(x))/a^3-2*b^2*(3*a^2+2*b^2)*arctanh((b-a*tanh(1/2*x))/(a^
2+b^2)^(1/2))/a^3/(a^2+b^2)^(3/2)-(a^2+2*b^2)*coth(x)/a^2/(a^2+b^2)+b^2*co
th(x)/a/(a^2+b^2)/(a+b*sinh(x))
```

Mathematica [A] (verified)

Time = 0.74 (sec) , antiderivative size = 128, normalized size of antiderivative = 1.11

$$\int \frac{\operatorname{csch}^2(x)}{(a+b\sinh(x))^2}\,dx = -\frac{\frac{4b^2(3a^2+2b^2)\arctan\left(\frac{b-a\tanh\left(\frac{x}{2}\right)}{\sqrt{-a^2-b^2}}\right)}{(-a^2-b^2)^{3/2}} + a\coth\left(\frac{x}{2}\right) - 4b\log\left(\cosh\left(\frac{x}{2}\right)\right) + 4b\log\left(\sinh\left(\frac{x}{2}\right)\right) + \frac{2ab^3\cosh(x)}{(a^2+b^2)(a+b\sinh(x))} +}{2a^3}$$

input
```
Integrate[Csch[x]^2/(a + b*Sinh[x])^2,x]
```

output
```
-1/2*((4*b^2*(3*a^2 + 2*b^2)*ArcTan[(b - a*Tanh[x/2])/Sqrt[-a^2 - b^2]])/(
-a^2 - b^2)^(3/2) + a*Coth[x/2] - 4*b*Log[Cosh[x/2]] + 4*b*Log[Sinh[x/2]]
+ (2*a*b^3*Cosh[x])/((a^2 + b^2)*(a + b*Sinh[x])) + a*Tanh[x/2])/a^3
```

Rubi [C] (verified)

Result contains complex when optimal does not.

Time = 0.90 (sec) , antiderivative size = 144, normalized size of antiderivative = 1.25, number of steps used = 18, number of rules used = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 1.308$, Rules used = {3042, 25, 3281, 25, 3042, 25, 3534, 3042, 26, 3480, 26, 3042, 26, 3139, 1083, 219, 4257}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\text{csch}^2(x)}{(a+b\sinh(x))^2}dx$$

$$\Big\downarrow \text{3042}$$

$$\int -\frac{1}{\sin(ix)^2(a-ib\sin(ix))^2}dx$$

$$\Big\downarrow \text{25}$$

$$-\int \frac{1}{\sin(ix)^2(a-ib\sin(ix))^2}dx$$

$$\Big\downarrow \text{3281}$$

$$\frac{b^2\coth(x)}{a(a^2+b^2)(a+b\sinh(x))}-\frac{\int -\frac{\text{csch}^2(x)(a^2-b\sinh(x)a+2b^2+b^2\sinh^2(x))}{a+b\sinh(x)}dx}{a(a^2+b^2)}$$

$$\Big\downarrow \text{25}$$

$$\frac{\int \frac{\text{csch}^2(x)(a^2-b\sinh(x)a+2b^2+b^2\sinh^2(x))}{a+b\sinh(x)}dx}{a(a^2+b^2)}+\frac{b^2\coth(x)}{a(a^2+b^2)(a+b\sinh(x))}$$

$$\Big\downarrow \text{3042}$$

$$\frac{b^2\coth(x)}{a\left(a^2+b^2\right)(a+b\sinh(x))}+\frac{\int -\frac{a^2+ib\sin(ix)a+2b^2-b^2\sin(ix)^2}{\sin(ix)^2(a-ib\sin(ix))}dx}{a\left(a^2+b^2\right)}$$

$\downarrow$ 25

$$\frac{b^2\coth(x)}{a\left(a^2+b^2\right)(a+b\sinh(x))}-\frac{\int \frac{a^2+ib\sin(ix)a+2b^2-b^2\sin(ix)^2}{\sin(ix)^2(a-ib\sin(ix))}dx}{a\left(a^2+b^2\right)}$$

$\downarrow$ 3534

$$\frac{b^2\coth(x)}{a\left(a^2+b^2\right)(a+b\sinh(x))}-\frac{\frac{\int \frac{\operatorname{csch}(x)\left(2b\left(a^2+b^2\right)-ab^2\sinh(x)\right)}{a+b\sinh(x)}dx}{a}+\frac{\left(a^2+2b^2\right)\coth(x)}{a}}{a\left(a^2+b^2\right)}$$

$\downarrow$ 3042

$$\frac{b^2\coth(x)}{a\left(a^2+b^2\right)(a+b\sinh(x))}-\frac{\frac{\left(a^2+2b^2\right)\coth(x)}{a}+\frac{\int \frac{i\left(ia\sin(ix)b^2+2\left(a^2+b^2\right)b\right)}{\sin(ix)(a-ib\sin(ix))}dx}{a}}{a\left(a^2+b^2\right)}$$

$\downarrow$ 26

$$\frac{b^2\coth(x)}{a\left(a^2+b^2\right)(a+b\sinh(x))}-\frac{\frac{\left(a^2+2b^2\right)\coth(x)}{a}+\frac{i\int \frac{ia\sin(ix)b^2+2\left(a^2+b^2\right)b}{\sin(ix)(a-ib\sin(ix))}dx}{a}}{a\left(a^2+b^2\right)}$$

$\downarrow$ 3480

$$\frac{b^2\coth(x)}{a\left(a^2+b^2\right)(a+b\sinh(x))}-\frac{\frac{\left(a^2+2b^2\right)\coth(x)}{a}+\frac{i\left(\frac{ib^2\left(3a^2+2b^2\right)\int \frac{1}{a+b\sinh(x)}dx}{a}+\frac{2b\left(a^2+b^2\right)\int -i\operatorname{csch}(x)dx}{a}\right)}{a}}{a\left(a^2+b^2\right)}$$

$\downarrow$ 26

$$\frac{b^2\coth(x)}{a\left(a^2+b^2\right)(a+b\sinh(x))}-\frac{\frac{\left(a^2+2b^2\right)\coth(x)}{a}+\frac{i\left(\frac{ib^2\left(3a^2+2b^2\right)\int \frac{1}{a+b\sinh(x)}dx}{a}-\frac{2ib\left(a^2+b^2\right)\int \operatorname{csch}(x)dx}{a}\right)}{a}}{a\left(a^2+b^2\right)}$$

$\downarrow$ 3042

$$\frac{b^2\coth(x)}{a\left(a^2+b^2\right)(a+b\sinh(x))}-\frac{\frac{\left(a^2+2b^2\right)\coth(x)}{a}+\frac{i\left(\frac{ib^2\left(3a^2+2b^2\right)\int \frac{1}{a-ib\sin(ix)}dx}{a}-\frac{2ib\left(a^2+b^2\right)\int i\csc(ix)dx}{a}\right)}{a}}{a\left(a^2+b^2\right)}$$

$\downarrow$ 26

$$\frac{b^2\coth(x)}{a\left(a^2+b^2\right)(a+b\sinh(x))}-\frac{\frac{\left(a^2+2b^2\right)\coth(x)}{a}+\frac{i\left(\frac{ib^2\left(3a^2+2b^2\right)\int\frac{1}{a-ib\sin(ix)}dx}{a}+\frac{2b\left(a^2+b^2\right)\int\csc(ix)dx}{a}\right)}{a}}{a\left(a^2+b^2\right)}$$

$$\Big\downarrow \text{3139}$$

$$\frac{b^2\coth(x)}{a\left(a^2+b^2\right)(a+b\sinh(x))}-\frac{\frac{\left(a^2+2b^2\right)\coth(x)}{a}+\frac{i\left(\frac{2b\left(a^2+b^2\right)\int\csc(ix)dx}{a}+\frac{2ib^2\left(3a^2+2b^2\right)\int\frac{1}{-a\tanh^2\left(\frac{x}{2}\right)+2b\tanh\left(\frac{x}{2}\right)+a}d\tanh\left(\frac{x}{2}\right)}{a}\right)}{a}}{a\left(a^2+b^2\right)}$$

$$\Big\downarrow \text{1083}$$

$$\frac{b^2\coth(x)}{a\left(a^2+b^2\right)(a+b\sinh(x))}-\frac{\frac{\left(a^2+2b^2\right)\coth(x)}{a}+\frac{i\left(\frac{2b\left(a^2+b^2\right)\int\csc(ix)dx}{a}-\frac{4ib^2\left(3a^2+2b^2\right)\int\frac{1}{4\left(a^2+b^2\right)-\left(2b-2a\tanh\left(\frac{x}{2}\right)\right)^2}d\left(2b-2a\tanh\left(\frac{x}{2}\right)\right)}{a}\right)}{a}}{a\left(a^2+b^2\right)}$$

$$\Big\downarrow \text{219}$$

$$\frac{b^2\coth(x)}{a\left(a^2+b^2\right)(a+b\sinh(x))}-\frac{\frac{\left(a^2+2b^2\right)\coth(x)}{a}+\frac{i\left(\frac{2b\left(a^2+b^2\right)\int\csc(ix)dx}{a}-\frac{2ib^2\left(3a^2+2b^2\right)\text{arctanh}\left(\frac{2b-2a\tanh\left(\frac{x}{2}\right)}{2\sqrt{a^2+b^2}}\right)}{a\sqrt{a^2+b^2}}\right)}{a}}{a\left(a^2+b^2\right)}$$

$$\Big\downarrow \text{4257}$$

$$\frac{b^2\coth(x)}{a\left(a^2+b^2\right)(a+b\sinh(x))}-\frac{\frac{\left(a^2+2b^2\right)\coth(x)}{a}+\frac{i\left(\frac{2ib\left(a^2+b^2\right)\text{arctanh}(\cosh(x))}{a}-\frac{2ib^2\left(3a^2+2b^2\right)\text{arctanh}\left(\frac{2b-2a\tanh\left(\frac{x}{2}\right)}{2\sqrt{a^2+b^2}}\right)}{a\sqrt{a^2+b^2}}\right)}{a}}{a\left(a^2+b^2\right)}$$

input
```
Int[Csch[x]^2/(a + b*Sinh[x])^2,x]
```

output
```
-(((I*(((2*I)*b*(a^2 + b^2)*ArcTanh[Cosh[x]])/a - ((2*I)*b^2*(3*a^2 + 2*b^
2)*ArcTanh[(2*b - 2*a*Tanh[x/2])/(2*Sqrt[a^2 + b^2])])/(a*Sqrt[a^2 + b^2])
))/a + ((a^2 + 2*b^2)*Coth[x])/a)/(a*(a^2 + b^2))) + (b^2*Coth[x])/(a*(a^2
 + b^2)*(a + b*Sinh[x]))
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1083
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Simp[-2   Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3139
```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Simp[2*(e/d)   Subst[Int[1/(a + 2*b*e*x + a
*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[a^2 - b^2, 0]
```

rule 3281
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b^2)*Cos[e + f*x]*(a + b*Sin[e + f*
x])^(m + 1)*((c + d*Sin[e + f*x])^(n + 1)/(f*(m + 1)*(b*c - a*d)*(a^2 - b^2
))), x] + Simp[1/((m + 1)*(b*c - a*d)*(a^2 - b^2))   Int[(a + b*Sin[e + f*x
])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[a*(b*c - a*d)*(m + 1) + b^2*d*(m + n
 + 2) - (b^2*c + b*(b*c - a*d)*(m + 1))*Sin[e + f*x] - b^2*d*(m + n + 3)*Si
n[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*
d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -1] && IntegersQ[
2*m, 2*n] && ((EqQ[a, 0] && IntegerQ[m] &&  !IntegerQ[n]) ||  !(IntegerQ[2*
n] && LtQ[n, -1] && ((IntegerQ[n] &&  !IntegerQ[m]) || EqQ[a, 0])))
```

rule 3480
```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((a_.) + (b_.)*sin[(e_.) + (f_
.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])), x_Symbol] :> Simp[(A*b
- a*B)/(b*c - a*d)   Int[1/(a + b*Sin[e + f*x]), x], x] + Simp[(B*c - A*d)/
(b*c - a*d)   Int[1/(c + d*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f
, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

rule 3534
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-(A*b^2 - a*b*B + a^2*C))*Cos[e + f*x
]*(a + b*Sin[e + f*x])^(m + 1)*((c + d*Sin[e + f*x])^(n + 1)/(f*(m + 1)*(b*
c - a*d)*(a^2 - b^2))), x] + Simp[1/((m + 1)*(b*c - a*d)*(a^2 - b^2))   Int
[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[(m + 1)*(b*c - a*
d)*(a*A - b*B + a*C) + d*(A*b^2 - a*b*B + a^2*C)*(m + n + 2) - (c*(A*b^2 -
a*b*B + a^2*C) + (m + 1)*(b*c - a*d)*(A*b - a*B + b*C))*Sin[e + f*x] - d*(A
*b^2 - a*b*B + a^2*C)*(m + n + 3)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b
, c, d, e, f, A, B, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] &&
NeQ[c^2 - d^2, 0] && LtQ[m, -1] && ((EqQ[a, 0] && IntegerQ[m] &&  !IntegerQ
[n]) ||  !(IntegerQ[2*n] && LtQ[n, -1] && ((IntegerQ[n] &&  !IntegerQ[m]) |
| EqQ[a, 0])))
```

rule 4257
```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Maple [A] (verified)

Time = 0.32 (sec) , antiderivative size = 141, normalized size of antiderivative = 1.23

method	result
default	$-\frac{\tanh\left(\frac{x}{2}\right)}{2a^2}-\frac{2b^2\left(\frac{-\frac{b^2\tanh\left(\frac{x}{2}\right)}{a^2+b^2}-\frac{ab}{a^2+b^2}}{\tanh\left(\frac{x}{2}\right)^2 a-2b\tanh\left(\frac{x}{2}\right)-a}-\frac{(3a^2+2b^2)\,\mathrm{arctanh}\left(\frac{2a\tanh\left(\frac{x}{2}\right)-2b}{2\sqrt{a^2+b^2}}\right)}{(a^2+b^2)^{\frac{3}{2}}}\right)}{a^3}-\frac{1}{2a^2\tanh\left(\frac{x}{2}\right)}-\frac{2b\ln\left(\tanh\left(\frac{x}{2}\right)\right)}{a^3}$
risch	$-\frac{2(-a\,b^2 e^{3x}+a^2 b\,e^{2x}+2b^3 e^{2x}+2a^3 e^x+3a\,b^2 e^x-a^2 b-2b^3)}{a^2(e^{2x}-1)(a^2+b^2)(b\,e^{2x}+2\,e^x a-b)}-\frac{2b\ln(e^x-1)}{a^3}+\frac{2b\ln(e^x+1)}{a^3}+\frac{3b^2\ln\left(e^x+\frac{a(a^2+b^2)^{\frac{3}{2}}-a^4-2a^2}{b(a^2+b^2)^{\frac{3}{2}}}\right.}{(a^2+b^2)^{\frac{3}{2}}a}$

input
```
int(csch(x)^2/(a+b*sinh(x))^2,x,method=_RETURNVERBOSE)
```

output
```
-1/2/a^2*tanh(1/2*x)-2/a^3*b^2*((-b^2/(a^2+b^2)*tanh(1/2*x)-a*b/(a^2+b^2))
/(tanh(1/2*x)^2*a-2*b*tanh(1/2*x)-a)-(3*a^2+2*b^2)/(a^2+b^2)^(3/2)*arctanh
(1/2*(2*a*tanh(1/2*x)-2*b)/(a^2+b^2)^(1/2)))-1/2/a^2/tanh(1/2*x)-2/a^3*b*l
n(tanh(1/2*x))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1740 vs. 2(111) = 222.

Time = 0.20 (sec) , antiderivative size = 1740, normalized size of antiderivative = 15.13

$$\int \frac{\mathrm{csch}^2(x)}{(a+b\sinh(x))^2}\,dx = \text{Too large to display}$$

input
```
integrate(csch(x)^2/(a+b*sinh(x))^2,x, algorithm="fricas")
```

output
```
(2*a^5*b + 6*a^3*b^3 + 4*a*b^5 + 2*(a^4*b^2 + a^2*b^4)*cosh(x)^3 + 2*(a^4*
b^2 + a^2*b^4)*sinh(x)^3 - 2*(a^5*b + 3*a^3*b^3 + 2*a*b^5)*cosh(x)^2 - 2*(
a^5*b + 3*a^3*b^3 + 2*a*b^5 - 3*(a^4*b^2 + a^2*b^4)*cosh(x))*sinh(x)^2 + (
3*a^2*b^3 + 2*b^5 + (3*a^2*b^3 + 2*b^5)*cosh(x)^4 + (3*a^2*b^3 + 2*b^5)*si
nh(x)^4 + 2*(3*a^3*b^2 + 2*a*b^4)*cosh(x)^3 + 2*(3*a^3*b^2 + 2*a*b^4 + 2*(
3*a^2*b^3 + 2*b^5)*cosh(x))*sinh(x)^3 - 2*(3*a^2*b^3 + 2*b^5)*cosh(x)^2 -
2*(3*a^2*b^3 + 2*b^5 - 3*(3*a^2*b^3 + 2*b^5)*cosh(x)^2 - 3*(3*a^3*b^2 + 2*
a*b^4)*cosh(x))*sinh(x)^2 - 2*(3*a^3*b^2 + 2*a*b^4)*cosh(x) - 2*(3*a^3*b^2
 + 2*a*b^4 - 2*(3*a^2*b^3 + 2*b^5)*cosh(x)^3 - 3*(3*a^3*b^2 + 2*a*b^4)*cos
h(x)^2 + 2*(3*a^2*b^3 + 2*b^5)*cosh(x))*sinh(x))*sqrt(a^2 + b^2)*log((b^2*
cosh(x)^2 + b^2*sinh(x)^2 + 2*a*b*cosh(x) + 2*a^2 + b^2 + 2*(b^2*cosh(x) +
 a*b)*sinh(x) - 2*sqrt(a^2 + b^2)*(b*cosh(x) + b*sinh(x) + a))/(b*cosh(x)^
2 + b*sinh(x)^2 + 2*a*cosh(x) + 2*(b*cosh(x) + a)*sinh(x) - b)) - 2*(2*a^6
 + 5*a^4*b^2 + 3*a^2*b^4)*cosh(x) + 2*(a^4*b^2 + 2*a^2*b^4 + b^6 + (a^4*b^
2 + 2*a^2*b^4 + b^6)*cosh(x)^4 + (a^4*b^2 + 2*a^2*b^4 + b^6)*sinh(x)^4 + 2
*(a^5*b + 2*a^3*b^3 + a*b^5)*cosh(x)^3 + 2*(a^5*b + 2*a^3*b^3 + a*b^5 + 2*
(a^4*b^2 + 2*a^2*b^4 + b^6)*cosh(x))*sinh(x)^3 - 2*(a^4*b^2 + 2*a^2*b^4 +
b^6)*cosh(x)^2 - 2*(a^4*b^2 + 2*a^2*b^4 + b^6 - 3*(a^4*b^2 + 2*a^2*b^4 + b
^6)*cosh(x)^2 - 3*(a^5*b + 2*a^3*b^3 + a*b^5)*cosh(x))*sinh(x)^2 - 2*(a^5*
b + 2*a^3*b^3 + a*b^5)*cosh(x) - 2*(a^5*b + 2*a^3*b^3 + a*b^5 - 2*(a^4*...
```

Sympy [F]

$$\int \frac{\operatorname{csch}^2(x)}{(a+b\sinh(x))^2}\,dx=\int \frac{\operatorname{csch}^{2}\left(x\right)}{\left(a+b\sinh\left(x\right)\right)^{2}}\,dx$$

input
```
integrate(csch(x)**2/(a+b*sinh(x))**2,x)
```

output
```
Integral(csch(x)**2/(a + b*sinh(x))**2, x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 251 vs. $2(111) = 222$.

Time = 0.16 (sec) , antiderivative size = 251, normalized size of antiderivative = 2.18

$$\int \frac{\operatorname{csch}^2(x)}{(a+b\sinh(x))^2}\,dx = \frac{(3\,a^2b^2+2\,b^4)\log\left(\frac{be^{(-x)}-a-\sqrt{a^2+b^2}}{be^{(-x)}-a+\sqrt{a^2+b^2}}\right)}{(a^5+a^3b^2)\sqrt{a^2+b^2}} + \frac{2\left(ab^2e^{(-3\,x)}-a^2b-2\,b^3-(2\,a^3+3\,ab^2)e^{(-x)}+(a^2b+2\,b^3)e^{(-2\,x)}\right)}{a^4b+a^2b^3+2\,(a^5+a^3b^2)e^{(-x)}-2\,(a^4b+a^2b^3)e^{(-2\,x)}-2\,(a^5+a^3b^2)e^{(-3\,x)}+(a^4b+a^2b^3)e^{(-4\,x)}} + \frac{2\,b\log\left(e^{(-x)}+1\right)}{a^3} - \frac{2\,b\log\left(e^{(-x)}-1\right)}{a^3}$$

input
```
integrate(csch(x)^2/(a+b*sinh(x))^2,x, algorithm="maxima")
```

output
```
(3*a^2*b^2 + 2*b^4)*log((b*e^(-x) - a - sqrt(a^2 + b^2))/(b*e^(-x) - a + s
qrt(a^2 + b^2)))/((a^5 + a^3*b^2)*sqrt(a^2 + b^2)) + 2*(a*b^2*e^(-3*x) - a
^2*b - 2*b^3 - (2*a^3 + 3*a*b^2)*e^(-x) + (a^2*b + 2*b^3)*e^(-2*x))/(a^4*b
 + a^2*b^3 + 2*(a^5 + a^3*b^2)*e^(-x) - 2*(a^4*b + a^2*b^3)*e^(-2*x) - 2*(
a^5 + a^3*b^2)*e^(-3*x) + (a^4*b + a^2*b^3)*e^(-4*x)) + 2*b*log(e^(-x) + 1
)/a^3 - 2*b*log(e^(-x) - 1)/a^3
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 205, normalized size of antiderivative = 1.78

$$\int \frac{\operatorname{csch}^2(x)}{(a+b\sinh(x))^2}\,dx = \frac{(3\,a^2b^2+2\,b^4)\log\left(\frac{\left|2\,be^x+2\,a-2\,\sqrt{a^2+b^2}\right|}{\left|2\,be^x+2\,a+2\,\sqrt{a^2+b^2}\right|}\right)}{(a^5+a^3b^2)\sqrt{a^2+b^2}} + \frac{2\left(ab^2e^{(3\,x)}-a^2be^{(2\,x)}-2\,b^3e^{(2\,x)}-2\,a^3e^x-3\,ab^2e^x+a^2b+2\,b^3\right)}{(a^4+a^2b^2)(be^{(4\,x)}+2\,ae^{(3\,x)}-2\,be^{(2\,x)}-2\,ae^x+b)} + \frac{2\,b\log\left(e^x+1\right)}{a^3} - \frac{2\,b\log\left(|e^x-1|\right)}{a^3}$$

input
```
integrate(csch(x)^2/(a+b*sinh(x))^2,x, algorithm="giac")
```

output
```
(3*a^2*b^2 + 2*b^4)*log(abs(2*b*e^x + 2*a - 2*sqrt(a^2 + b^2))/abs(2*b*e^x
 + 2*a + 2*sqrt(a^2 + b^2)))/((a^5 + a^3*b^2)*sqrt(a^2 + b^2)) + 2*(a*b^2*
e^(3*x) - a^2*b*e^(2*x) - 2*b^3*e^(2*x) - 2*a^3*e^x - 3*a*b^2*e^x + a^2*b
+ 2*b^3)/((a^4 + a^2*b^2)*(b*e^(4*x) + 2*a*e^(3*x) - 2*b*e^(2*x) - 2*a*e^x
 + b)) + 2*b*log(e^x + 1)/a^3 - 2*b*log(abs(e^x - 1))/a^3
```

Mupad [B] (verification not implemented)

Time = 5.05 (sec) , antiderivative size = 1017, normalized size of antiderivative = 8.84

$$\int \frac{\operatorname{csch}^2(x)}{(a+b\sinh(x))^2}\,dx = \text{Too large to display}$$

input
```
int(1/(sinh(x)^2*(a + b*sinh(x))^2),x)
```

output
```
((2*(32*a^2*b^12 + 96*a^4*b^10 + 90*a^6*b^8 + 25*a^8*b^6))/(a^4*b^2*(16*b^
9 + 56*a^2*b^7 + 65*a^4*b^5 + 25*a^6*b^3)) - (2*exp(x)*(48*a^3*b^12 + 152*
a^5*b^10 + 155*a^7*b^8 + 50*a^9*b^6))/(a^4*b^3*(16*b^9 + 56*a^2*b^7 + 65*a
^4*b^5 + 25*a^6*b^3)) - (2*exp(2*x)*(32*a^2*b^12 + 96*a^4*b^10 + 90*a^6*b^
8 + 25*a^8*b^6))/(a^4*b^2*(16*b^9 + 56*a^2*b^7 + 65*a^4*b^5 + 25*a^6*b^3))
 + (2*exp(3*x)*(16*a^3*b^12 + 40*a^5*b^10 + 25*a^7*b^8))/(a^4*b^3*(16*b^9
+ 56*a^2*b^7 + 65*a^4*b^5 + 25*a^6*b^3)))/(b - 2*a*exp(x) + 2*a*exp(3*x) -
 2*b*exp(2*x) + b*exp(4*x)) - (2*b*log(exp(x) - 1))/a^3 + (2*b*log(exp(x)
+ 1))/a^3 + (b^2*log(- (64*(3*a^2 + 2*b^2)*(4*a^2*b + 4*b^3 - 8*a^3*exp(x)
 - 7*a*b^2*exp(x)))/(a^6*b*(a^2 + b^2)^2) - (32*(3*a^2 + 2*b^2)*(8*a^9*b -
 8*b^7*((a^2 + b^2)^3)^(1/2) + 3*a^3*b^7 + 13*a^5*b^5 + 18*a^7*b^3 - 16*a^
10*exp(x) - 24*a^2*b^5*((a^2 + b^2)^3)^(1/2) - 18*a^4*b^3*((a^2 + b^2)^3)^
(1/2) - 9*a^4*b^6*exp(x) - 33*a^6*b^4*exp(x) - 40*a^8*b^2*exp(x) + 41*a^3*
b^4*exp(x)*((a^2 + b^2)^3)^(1/2) + 30*a^5*b^2*exp(x)*((a^2 + b^2)^3)^(1/2)
 + 14*a*b^6*exp(x)*((a^2 + b^2)^3)^(1/2)))/(a^6*b*((a^2 + b^2)^3)^(1/2)*(a
^2 + b^2)^4))*((a^2 + b^2)^3)^(1/2)*(3*a^2 + 2*b^2))/(a^9 + a^3*b^6 + 3*a^
5*b^4 + 3*a^7*b^2) - (b^2*log((32*(3*a^2 + 2*b^2)*(8*a^9*b + 8*b^7*((a^2 +
 b^2)^3)^(1/2) + 3*a^3*b^7 + 13*a^5*b^5 + 18*a^7*b^3 - 16*a^10*exp(x) + 24
*a^2*b^5*((a^2 + b^2)^3)^(1/2) + 18*a^4*b^3*((a^2 + b^2)^3)^(1/2) - 9*a^4*
b^6*exp(x) - 33*a^6*b^4*exp(x) - 40*a^8*b^2*exp(x) - 41*a^3*b^4*exp(x)*...
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 1168, normalized size of antiderivative = 10.16

$$\int \frac{\operatorname{csch}^2(x)}{(a+b\sinh(x))^2}\,dx = \text{Too large to display}$$

input
```
int(csch(x)^2/(a+b*sinh(x))^2,x)
```

output
```
(6*e**(4*x)*sqrt(a**2 + b**2)*atan((e**x*b*i + a*i)/sqrt(a**2 + b**2))*a**
2*b**3*i + 4*e**(4*x)*sqrt(a**2 + b**2)*atan((e**x*b*i + a*i)/sqrt(a**2 +
b**2))*b**5*i + 12*e**(3*x)*sqrt(a**2 + b**2)*atan((e**x*b*i + a*i)/sqrt(a
**2 + b**2))*a**3*b**2*i + 8*e**(3*x)*sqrt(a**2 + b**2)*atan((e**x*b*i + a
*i)/sqrt(a**2 + b**2))*a*b**4*i - 12*e**(2*x)*sqrt(a**2 + b**2)*atan((e**x
*b*i + a*i)/sqrt(a**2 + b**2))*a**2*b**3*i - 8*e**(2*x)*sqrt(a**2 + b**2)*
atan((e**x*b*i + a*i)/sqrt(a**2 + b**2))*b**5*i - 12*e**x*sqrt(a**2 + b**2
)*atan((e**x*b*i + a*i)/sqrt(a**2 + b**2))*a**3*b**2*i - 8*e**x*sqrt(a**2
+ b**2)*atan((e**x*b*i + a*i)/sqrt(a**2 + b**2))*a*b**4*i + 6*sqrt(a**2 +
b**2)*atan((e**x*b*i + a*i)/sqrt(a**2 + b**2))*a**2*b**3*i + 4*sqrt(a**2 +
 b**2)*atan((e**x*b*i + a*i)/sqrt(a**2 + b**2))*b**5*i - 2*e**(4*x)*log(e*
*x - 1)*a**4*b**2 - 4*e**(4*x)*log(e**x - 1)*a**2*b**4 - 2*e**(4*x)*log(e*
*x - 1)*b**6 + 2*e**(4*x)*log(e**x + 1)*a**4*b**2 + 4*e**(4*x)*log(e**x +
1)*a**2*b**4 + 2*e**(4*x)*log(e**x + 1)*b**6 - e**(4*x)*a**3*b**3 - e**(4*
x)*a*b**5 - 4*e**(3*x)*log(e**x - 1)*a**5*b - 8*e**(3*x)*log(e**x - 1)*a**
3*b**3 - 4*e**(3*x)*log(e**x - 1)*a*b**5 + 4*e**(3*x)*log(e**x + 1)*a**5*b
 + 8*e**(3*x)*log(e**x + 1)*a**3*b**3 + 4*e**(3*x)*log(e**x + 1)*a*b**5 +
4*e**(2*x)*log(e**x - 1)*a**4*b**2 + 8*e**(2*x)*log(e**x - 1)*a**2*b**4 +
4*e**(2*x)*log(e**x - 1)*b**6 - 4*e**(2*x)*log(e**x + 1)*a**4*b**2 - 8*e**
(2*x)*log(e**x + 1)*a**2*b**4 - 4*e**(2*x)*log(e**x + 1)*b**6 - 2*e**(2...
```

3.86 $\int \frac{\operatorname{csch}^3(x)}{(a+b\sinh(x))^2}\,dx$

Optimal result

Integrand size = 13, antiderivative size = 158

$$\int \frac{\operatorname{csch}^3(x)}{(a+b\sinh(x))^2}\,dx = \frac{(a^2-6b^2)\operatorname{arctanh}(\cosh(x))}{2a^4} + \frac{2b^3(4a^2+3b^2)\operatorname{arctanh}\left(\frac{b-a\tanh\left(\frac{x}{2}\right)}{\sqrt{a^2+b^2}}\right)}{a^4(a^2+b^2)^{3/2}} + \frac{b(2a^2+3b^2)\coth(x)}{a^3(a^2+b^2)} - \frac{(a^2+3b^2)\coth(x)\operatorname{csch}(x)}{2a^2(a^2+b^2)} + \frac{b^2\coth(x)\operatorname{csch}(x)}{a(a^2+b^2)(a+b\sinh(x))}$$

output
```
1/2*(a^2-6*b^2)*arctanh(cosh(x))/a^4+2*b^3*(4*a^2+3*b^2)*arctanh((b-a*tanh
(1/2*x))/(a^2+b^2)^(1/2))/a^4/(a^2+b^2)^(3/2)+b*(2*a^2+3*b^2)*coth(x)/a^3/
(a^2+b^2)-1/2*(a^2+3*b^2)*coth(x)*csch(x)/a^2/(a^2+b^2)+b^2*coth(x)*csch(x
)/a/(a^2+b^2)/(a+b*sinh(x))
```

Mathematica [A] (verified)

Time = 0.91 (sec) , antiderivative size = 174, normalized size of antiderivative = 1.10

$$\int \frac{\operatorname{csch}^3(x)}{(a+b\sinh(x))^2}\,dx$$
$$=\frac{\frac{16b^3(4a^2+3b^2)\arctan\left(\frac{b-a\tanh\left(\frac{x}{2}\right)}{\sqrt{-a^2-b^2}}\right)}{(-a^2-b^2)^{3/2}}+8ab\coth\left(\frac{x}{2}\right)-a^2\operatorname{csch}^2\left(\frac{x}{2}\right)+4(a^2-6b^2)\log\left(\cosh\left(\frac{x}{2}\right)\right)-4(a^2-6b^2)\log}{8a^4}$$

input
```
Integrate[Csch[x]^3/(a + b*Sinh[x])^2,x]
```

output
```
((16*b^3*(4*a^2 + 3*b^2)*ArcTan[(b - a*Tanh[x/2])/Sqrt[-a^2 - b^2]])/(-a^2
 - b^2)^(3/2) + 8*a*b*Coth[x/2] - a^2*Csch[x/2]^2 + 4*(a^2 - 6*b^2)*Log[Co
sh[x/2]] - 4*(a^2 - 6*b^2)*Log[Sinh[x/2]] - a^2*Sech[x/2]^2 + (8*a*b^4*Cos
h[x])/((a^2 + b^2)*(a + b*Sinh[x])) + 8*a*b*Tanh[x/2])/(8*a^4)
```

Rubi [C] (verified)

Result contains complex when optimal does not.

Time = 1.47 (sec) , antiderivative size = 196, normalized size of antiderivative = 1.24, number of steps used = 23, number of rules used = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 1.692, Rules used = {3042, 26, 3281, 26, 3042, 26, 3534, 26, 3042, 25, 3534, 25, 3042, 26, 3480, 26, 3042, 26, 3139, 1083, 219, 4257}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\operatorname{csch}^3(x)}{(a+b\sinh(x))^2}\,dx$$
$$\Big\downarrow \; 3042$$
$$\int -\frac{i}{\sin(ix)^3(a-ib\sin(ix))^2}dx$$
$$\Big\downarrow \; 26$$

$$-i\int \frac{1}{\sin(ix)^3(a-ib\sin(ix))^2}dx$$

$$\Big\downarrow \ 3281$$

$$-i\left(\frac{\int \frac{i\text{csch}^3(x)\left(a^2-b\sinh(x)a+3b^2+2b^2\sinh^2(x)\right)}{a+b\sinh(x)}dx}{a\left(a^2+b^2\right)}+\frac{ib^2\coth(x)\text{csch}(x)}{a\left(a^2+b^2\right)\left(a+b\sinh(x)\right)}\right)$$

$$\Big\downarrow \ 26$$

$$-i\left(\frac{i\int \frac{\text{csch}^3(x)\left(a^2-b\sinh(x)a+3b^2+2b^2\sinh^2(x)\right)}{a+b\sinh(x)}dx}{a\left(a^2+b^2\right)}+\frac{ib^2\coth(x)\text{csch}(x)}{a\left(a^2+b^2\right)\left(a+b\sinh(x)\right)}\right)$$

$$\Big\downarrow \ 3042$$

$$-i\left(\frac{i\int -\frac{i\left(a^2+ib\sin(ix)a+3b^2-2b^2\sin(ix)^2\right)}{\sin(ix)^3(a-ib\sin(ix))}dx}{a\left(a^2+b^2\right)}+\frac{ib^2\coth(x)\text{csch}(x)}{a\left(a^2+b^2\right)\left(a+b\sinh(x)\right)}\right)$$

$$\Big\downarrow \ 26$$

$$-i\left(\frac{\int \frac{a^2+ib\sin(ix)a+3b^2-2b^2\sin(ix)^2}{\sin(ix)^3(a-ib\sin(ix))}dx}{a\left(a^2+b^2\right)}+\frac{ib^2\coth(x)\text{csch}(x)}{a\left(a^2+b^2\right)\left(a+b\sinh(x)\right)}\right)$$

$$\Big\downarrow \ 3534$$

$$-i\left(\frac{\frac{\int -\frac{i\text{csch}^2(x)\left(b\left(a^2+3b^2\right)\sinh^2(x)+a\left(a^2-b^2\right)\sinh(x)+2b\left(2a^2+3b^2\right)\right)}{a+b\sinh(x)}dx}{2a}-\frac{i\left(a^2+3b^2\right)\coth(x)\text{csch}(x)}{2a}}{a\left(a^2+b^2\right)}+\frac{ib^2\coth(x)\text{csch}(x)}{a\left(a^2+b^2\right)\left(a+b\sinh(x)\right)}\right)$$

$$\Big\downarrow \ 26$$

$$-i\left(\frac{-\frac{i\int \frac{\text{csch}^2(x)\left(b\left(a^2+3b^2\right)\sinh^2(x)+a\left(a^2-b^2\right)\sinh(x)+2b\left(2a^2+3b^2\right)\right)}{a+b\sinh(x)}dx}{2a}-\frac{i\left(a^2+3b^2\right)\coth(x)\text{csch}(x)}{2a}}{a\left(a^2+b^2\right)}+\frac{ib^2\coth(x)\text{csch}(x)}{a\left(a^2+b^2\right)\left(a+b\sinh(x)\right)}\right)$$

$$\Big\downarrow \ 3042$$

$$-i\left(\frac{-\frac{i\int -\frac{-b\left(a^2+3b^2\right)\sin(ix)^2-ia\left(a^2-b^2\right)\sin(ix)+2b\left(2a^2+3b^2\right)}{\sin(ix)^2(a-ib\sin(ix))}dx}{2a}-\frac{i\left(a^2+3b^2\right)\coth(x)\text{csch}(x)}{2a}}{a\left(a^2+b^2\right)}+\frac{ib^2\coth(x)\text{csch}(x)}{a\left(a^2+b^2\right)\left(a+b\sinh(x)\right)}\right)$$

$$\Big\downarrow 25$$

$$-i\left(\frac{\frac{i\int \frac{-b(a^2+3b^2)\sin(ix)^2-ia(a^2-b^2)\sin(ix)+2b(2a^2+3b^2)}{\sin(ix)^2(a-ib\sin(ix))}dx}{2a}-\frac{i(a^2+3b^2)\coth(x)\text{csch}(x)}{2a}}{a(a^2+b^2)}+\frac{ib^2\coth(x)\text{csch}(x)}{a(a^2+b^2)(a+b\sinh(x))}\right)$$

$$\Big\downarrow 3534$$

$$-i\left(\frac{\frac{i\left(\frac{\int -\frac{\text{csch}(x)\left((a^2-6b^2)(a^2+b^2)+ab(a^2+3b^2)\sinh(x)\right)}{a+b\sinh(x)}dx}{a}+\frac{2b(2a^2+3b^2)\coth(x)}{a}\right)}{2a}-\frac{i(a^2+3b^2)\coth(x)\text{csch}(x)}{2a}}{a(a^2+b^2)}+\frac{ib^2\coth(x)\text{csch}(x)}{a(a^2+b^2)(a+b\sinh(x))}\right)$$

$$\Big\downarrow 25$$

$$-i\left(\frac{\frac{i\left(\frac{2b(2a^2+3b^2)\coth(x)}{a}-\frac{\int \frac{\text{csch}(x)\left((a^2-6b^2)(a^2+b^2)+ab(a^2+3b^2)\sinh(x)\right)}{a+b\sinh(x)}dx}{a}\right)}{2a}-\frac{i(a^2+3b^2)\coth(x)\text{csch}(x)}{2a}}{a(a^2+b^2)}+\frac{ib^2\coth(x)\text{csch}(x)}{a(a^2+b^2)(a+b\sinh(x))}\right)$$

$$\Big\downarrow 3042$$

$$-i\left(\frac{\frac{i\left(\frac{2b(2a^2+3b^2)\coth(x)}{a}-\frac{\int \frac{i\left((a^2-6b^2)(a^2+b^2)-iab(a^2+3b^2)\sin(ix)\right)}{\sin(ix)(a-ib\sin(ix))}dx}{a}\right)}{2a}-\frac{i(a^2+3b^2)\coth(x)\text{csch}(x)}{2a}}{a(a^2+b^2)}+\frac{ib^2\coth(x)\text{csch}(x)}{a(a^2+b^2)(a+b\sinh(x))}\right)$$

$$\Big\downarrow 26$$

$$-i\left(\frac{\frac{i\left(\frac{2b\left(2a^2+3b^2\right)\coth(x)}{a}-\frac{i\int\frac{\left(a^2-6b^2\right)\left(a^2+b^2\right)-iab\left(a^2+3b^2\right)\sin(ix)}{\sin(ix)(a-ib\sin(ix))}dx}{a}\right)}{2a}-\frac{i(a^2+3b^2)\coth(x)\text{csch}(x)}{2a}}{a\left(a^2+b^2\right)}+\frac{ib^2\coth(x)\text{csch}(x)}{a\left(a^2+b^2\right)\left(a+b\sinh(x\right.}\right.$$

$\downarrow$ 3480

$$-i\left(\frac{\frac{i\left(\frac{2b\left(2a^2+3b^2\right)\coth(x)}{a}-\frac{i\left(\frac{\left(a^2-6b^2\right)\left(a^2+b^2\right)\int-i\text{csch}(x)dx}{a}-\frac{2ib^3\left(4a^2+3b^2\right)\int\frac{1}{a+b\sinh(x)}dx}{a}\right)}{a}\right)}{2a}-\frac{i(a^2+3b^2)\coth(x)\text{csch}(x)}{2a}}{a\left(a^2+b^2\right)}+\frac{i}{a\left(a^2\right.}\right.$$

$\downarrow$ 26

$$-i\left(\frac{\frac{i\left(\frac{2b\left(2a^2+3b^2\right)\coth(x)}{a}-\frac{i\left(-\frac{i\left(a^2-6b^2\right)\left(a^2+b^2\right)\int\text{csch}(x)dx}{a}-\frac{2ib^3\left(4a^2+3b^2\right)\int\frac{1}{a+b\sinh(x)}dx}{a}\right)}{a}\right)}{2a}-\frac{i(a^2+3b^2)\coth(x)\text{csch}(x)}{2a}}{a\left(a^2+b^2\right)}+\frac{i}{a\left(a^2\right.}\right.$$

$\downarrow$ 3042

$$-i\left(\frac{\frac{i\left(\frac{2b\left(2a^2+3b^2\right)\coth(x)}{a}-\frac{i\left(-\frac{i\left(a^2-6b^2\right)\left(a^2+b^2\right)\int i\csc(ix)dx}{a}-\frac{2ib^3\left(4a^2+3b^2\right)\int\frac{1}{a-ib\sin(ix)}dx}{a}\right)}{a}\right)}{2a}-\frac{i(a^2+3b^2)\coth(x)\text{csch}(x)}{2a}}{a\left(a^2+b^2\right)}+\frac{ib}{a\left(a^2\right.}\right.$$

$\downarrow$ 26

$$-i\left(\frac{\frac{i\left(\frac{2b\left(2a^2+3b^2\right)\coth(x)}{a}-\frac{i\left(\frac{\left(a^2-6b^2\right)\left(a^2+b^2\right)\int\csc(ix)dx}{a}-\frac{2ib^3\left(4a^2+3b^2\right)\int\frac{1}{a-ib\sin(ix)}dx}{a}\right)}{a}\right)}{2a}-\frac{i(a^2+3b^2)\coth(x)\text{csch}(x)}{2a}}{a\left(a^2+b^2\right)}+\frac{ib^2\cdots}{a\left(a^2+\cdots\right.}\right.$$

$\downarrow$ 3139

$$-i\left(\frac{\frac{i\left(\frac{2b\left(2a^2+3b^2\right)\coth(x)}{a}-\frac{i\left(\frac{\left(a^2-6b^2\right)\left(a^2+b^2\right)\int\csc(ix)dx}{a}-\frac{4ib^3\left(4a^2+3b^2\right)\int\frac{1}{-a\tanh^2\left(\frac{x}{2}\right)+2b\tanh\left(\frac{x}{2}\right)+a}d\tanh\left(\frac{x}{2}\right)}{a}\right)}{a}\right)}{2a}-\frac{i(a^2+3b^2)\coth(x)\cdots}{2a}}{a\left(a^2+b^2\right)}\right.$$

$\downarrow$ 1083

$$-i\left(\frac{\frac{i\left(\frac{2b\left(2a^2+3b^2\right)\coth(x)}{a}-\frac{i\left(\frac{\left(a^2-6b^2\right)\left(a^2+b^2\right)\int\csc(ix)dx}{a}+\frac{8ib^3\left(4a^2+3b^2\right)\int\frac{1}{4\left(a^2+b^2\right)-\left(2b-2a\tanh\left(\frac{x}{2}\right)\right)^2}d\left(2b-2a\tanh\left(\frac{x}{2}\right)\right)}{a}\right)}{a}\right)}{2a}-\frac{i(a^2+3b^2\cdots}{\cdots}}{a\left(a^2+b^2\right)}\right.$$

$\downarrow$ 219

$$-i\left(\frac{\frac{i\left(\frac{2b\left(2a^2+3b^2\right)\coth(x)}{a}-\frac{i\left(\frac{\left(a^2-6b^2\right)\left(a^2+b^2\right)\int \csc(ix)dx}{a}+\frac{4ib^3\left(4a^2+3b^2\right)\text{arctanh}\left(\frac{2b-2a\tanh\left(\frac{x}{2}\right)}{2\sqrt{a^2+b^2}}\right)}{a\sqrt{a^2+b^2}}\right)}{a}\right)}{2a}-\frac{i(a^2+3b^2)\coth(x)\text{csch}(x)}{2a}}{a\left(a^2+b^2\right)}\right.$$

$\downarrow$ 4257

$$-i\left(\frac{\frac{i\left(\frac{2b\left(2a^2+3b^2\right)\coth(x)}{a}-\frac{i\left(\frac{i\left(a^2-6b^2\right)\left(a^2+b^2\right)\text{arctanh}(\cosh(x))}{a}+\frac{4ib^3\left(4a^2+3b^2\right)\text{arctanh}\left(\frac{2b-2a\tanh\left(\frac{x}{2}\right)}{2\sqrt{a^2+b^2}}\right)}{a\sqrt{a^2+b^2}}\right)}{a}\right)}{2a}-\frac{i(a^2+3b^2)\coth(x)\text{cs}}{2a}}{a\left(a^2+b^2\right)}\right.$$

input

```
Int[Csch[x]^3/(a + b*Sinh[x])^2,x]
```

output

```
(-I)*((((I/2)*(((-I)*((I*(a^2 - 6*b^2)*(a^2 + b^2)*ArcTanh[Cosh[x]])/a + (
(4*I)*b^3*(4*a^2 + 3*b^2)*ArcTanh[(2*b - 2*a*Tanh[x/2])/(2*Sqrt[a^2 + b^2]
)])/(a*Sqrt[a^2 + b^2])))/a + (2*b*(2*a^2 + 3*b^2)*Coth[x])/a))/a - ((I/2)
*(a^2 + 3*b^2)*Coth[x]*Csch[x])/a)/(a*(a^2 + b^2)) + (I*b^2*Coth[x]*Csch[x
])/(a*(a^2 + b^2)*(a + b*Sinh[x])))
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1083
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Simp[-2   Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3139
```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Simp[2*(e/d)   Subst[Int[1/(a + 2*b*e*x + a
*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[a^2 - b^2, 0]
```

rule 3281
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b^2)*Cos[e + f*x]*(a + b*Sin[e + f*
x])^(m + 1)*((c + d*Sin[e + f*x])^(n + 1)/(f*(m + 1)*(b*c - a*d)*(a^2 - b^2
))), x] + Simp[1/((m + 1)*(b*c - a*d)*(a^2 - b^2))   Int[(a + b*Sin[e + f*x
])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[a*(b*c - a*d)*(m + 1) + b^2*d*(m + n
 + 2) - (b^2*c + b*(b*c - a*d)*(m + 1))*Sin[e + f*x] - b^2*d*(m + n + 3)*Si
n[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*
d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -1] && IntegersQ[
2*m, 2*n] && ((EqQ[a, 0] && IntegerQ[m] &&  !IntegerQ[n]) ||  !(IntegerQ[2*
n] && LtQ[n, -1] && ((IntegerQ[n] &&  !IntegerQ[m]) || EqQ[a, 0])))
```

rule 3480
```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((a_.) + (b_.)*sin[(e_.) + (f_
.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])), x_Symbol] :> Simp[(A*b
- a*B)/(b*c - a*d)   Int[1/(a + b*Sin[e + f*x]), x], x] + Simp[(B*c - A*d)/
(b*c - a*d)   Int[1/(c + d*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f
, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

rule 3534
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-(A*b^2 - a*b*B + a^2*C))*Cos[e + f*x
]*(a + b*Sin[e + f*x])^(m + 1)*((c + d*Sin[e + f*x])^(n + 1)/(f*(m + 1)*(b*
c - a*d)*(a^2 - b^2))), x] + Simp[1/((m + 1)*(b*c - a*d)*(a^2 - b^2))   Int
[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[(m + 1)*(b*c - a*
d)*(a*A - b*B + a*C) + d*(A*b^2 - a*b*B + a^2*C)*(m + n + 2) - (c*(A*b^2 -
a*b*B + a^2*C) + (m + 1)*(b*c - a*d)*(A*b - a*B + b*C))*Sin[e + f*x] - d*(A
*b^2 - a*b*B + a^2*C)*(m + n + 3)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b
, c, d, e, f, A, B, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] &&
NeQ[c^2 - d^2, 0] && LtQ[m, -1] && ((EqQ[a, 0] && IntegerQ[m] &&  !IntegerQ
[n]) ||  !(IntegerQ[2*n] && LtQ[n, -1] && ((IntegerQ[n] &&  !IntegerQ[m]) |
| EqQ[a, 0])))
```

rule 4257
```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Maple [A] (verified)

Time = 0.40 (sec) , antiderivative size = 175, normalized size of antiderivative = 1.11

method	result
default	$\frac{\frac{\tanh\left(\frac{x}{2}\right)^{2} a}{2}+4 b \tanh\left(\frac{x}{2}\right)}{4 a^{3}}-\frac{1}{8 a^{2} \tanh\left(\frac{x}{2}\right)^{2}}+\frac{\left(-2 a^{2}+12 b^{2}\right) \ln\left(\tanh\left(\frac{x}{2}\right)\right)}{4 a^{4}}+\frac{b}{a^{3} \tanh\left(\frac{x}{2}\right)}+\frac{4 b^{3} \left(\frac{-\frac{b^{2} \tanh\left(\frac{x}{2}\right)}{2\left(a^{2}+b^{2}\right)}-\frac{a b}{2\left(a^{2}+b^{2}\right)}}{\tanh\left(\frac{x}{2}\right)^{2} a-2 b \tanh\left(\frac{x}{2}\right)-a}-\right.}{}$
risch	$-\frac{a^{3} b\,\mathrm{e}^{5 x}+3 a\,b^{3} \mathrm{e}^{5 x}+2 a^{4} \mathrm{e}^{4 x}-2 a^{2} b^{2} \mathrm{e}^{4 x}-6 b^{4} \mathrm{e}^{4 x}-8 a^{3} b\,\mathrm{e}^{3 x}-12 a\,b^{3} \mathrm{e}^{3 x}+2 a^{4} \mathrm{e}^{2 x}+10 a^{2} b^{2} \mathrm{e}^{2 x}+12 b^{4} \mathrm{e}^{2 x}+7 a^{3} b\,\mathrm{e}^{x}+9 b^{3} \mathrm{e}^{x} a-4 a^{2} b^{2}-6 b}{\left(a^{2}+b^{2}\right) a^{3} \left(b\,\mathrm{e}^{2 x}+2\,\mathrm{e}^{x} a-b\right) \left(\mathrm{e}^{2 x}-1\right)^{2}}$

input
```
int(csch(x)^3/(a+b*sinh(x))^2,x,method=_RETURNVERBOSE)
```

output
```
1/4/a^3*(1/2*tanh(1/2*x)^2*a+4*b*tanh(1/2*x))-1/8/a^2/tanh(1/2*x)^2+1/4/a^
4*(-2*a^2+12*b^2)*ln(tanh(1/2*x))+1/a^3*b/tanh(1/2*x)+4/a^4*b^3*((-1/2*b^2
/(a^2+b^2)*tanh(1/2*x)-1/2*a*b/(a^2+b^2))/(tanh(1/2*x)^2*a-2*b*tanh(1/2*x)
-a)-1/2*(4*a^2+3*b^2)/(a^2+b^2)^(3/2)*arctanh(1/2*(2*a*tanh(1/2*x)-2*b)/(a
^2+b^2)^(1/2)))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 3754 vs. $2(150) = 300$.

Time = 0.33 (sec) , antiderivative size = 3754, normalized size of antiderivative = 23.76

$$\int \frac{\operatorname{csch}^3(x)}{(a+b\sinh(x))^2}\,dx = \text{Too large to display}$$

input
```
integrate(csch(x)^3/(a+b*sinh(x))^2,x, algorithm="fricas")
```

output
```
Too large to include
```

Sympy [F]

$$\int \frac{\operatorname{csch}^3(x)}{(a+b\sinh(x))^2}\,dx = \int \frac{\operatorname{csch}^3(x)}{(a+b\sinh(x))^2}\,dx$$

input
```
integrate(csch(x)**3/(a+b*sinh(x))**2,x)
```

output
```
Integral(csch(x)**3/(a + b*sinh(x))**2, x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 363 vs. $2(150) = 300$.

Time = 0.15 (sec) , antiderivative size = 363, normalized size of antiderivative = 2.30

$$\int \frac{\operatorname{csch}^3(x)}{(a+b\sinh(x))^2}\,dx = -\frac{(4\,a^2b^3+3\,b^5)\log\left(\frac{be^{(-x)}-a-\sqrt{a^2+b^2}}{be^{(-x)}-a+\sqrt{a^2+b^2}}\right)}{(a^6+a^4b^2)\sqrt{a^2+b^2}}$$
$$+\frac{4\,a^2b^2+6\,b^4+(7\,a^3b+9\,ab^3)e^{(-x)}-2\,(a^4+5\,a^2b^2+6\,b^4)e^{(-2\,x)}-4\,(2\,a^3b+3\,ab^3)e^{(-3\,x)}-2\,(a^4-a}{a^5b+a^3b^3+2\,(a^6+a^4b^2)e^{(-x)}-3\,(a^5b+a^3b^3)e^{(-2\,x)}-4\,(a^6+a^4b^2)e^{(-3\,x)}+3\,(a^5b+a^3b^3)e^{(-4\,x)}+2}$$
$$+\frac{(a^2-6\,b^2)\log\left(e^{(-x)}+1\right)}{2\,a^4}-\frac{(a^2-6\,b^2)\log\left(e^{(-x)}-1\right)}{2\,a^4}$$

input
```
integrate(csch(x)^3/(a+b*sinh(x))^2,x, algorithm="maxima")
```

output
```
-(4*a^2*b^3 + 3*b^5)*log((b*e^(-x) - a - sqrt(a^2 + b^2))/(b*e^(-x) - a +
sqrt(a^2 + b^2)))/((a^6 + a^4*b^2)*sqrt(a^2 + b^2)) + (4*a^2*b^2 + 6*b^4 +
 (7*a^3*b + 9*a*b^3)*e^(-x) - 2*(a^4 + 5*a^2*b^2 + 6*b^4)*e^(-2*x) - 4*(2*
a^3*b + 3*a*b^3)*e^(-3*x) - 2*(a^4 - a^2*b^2 - 3*b^4)*e^(-4*x) + (a^3*b +
3*a*b^3)*e^(-5*x))/(a^5*b + a^3*b^3 + 2*(a^6 + a^4*b^2)*e^(-x) - 3*(a^5*b
+ a^3*b^3)*e^(-2*x) - 4*(a^6 + a^4*b^2)*e^(-3*x) + 3*(a^5*b + a^3*b^3)*e^(
-4*x) + 2*(a^6 + a^4*b^2)*e^(-5*x) - (a^5*b + a^3*b^3)*e^(-6*x)) + 1/2*(a^
2 - 6*b^2)*log(e^(-x) + 1)/a^4 - 1/2*(a^2 - 6*b^2)*log(e^(-x) - 1)/a^4
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 203, normalized size of antiderivative = 1.28

$$\int \frac{\operatorname{csch}^3(x)}{(a+b\sinh(x))^2}\,dx = -\frac{(4\,a^2b^3+3\,b^5)\log\left(\frac{\left|2\,be^x+2\,a-2\,\sqrt{a^2+b^2}\right|}{\left|2\,be^x+2\,a+2\,\sqrt{a^2+b^2}\right|}\right)}{(a^6+a^4b^2)\sqrt{a^2+b^2}}$$
$$-\frac{2\,(ab^3e^x-b^4)}{(a^5+a^3b^2)(be^{(2\,x)}+2\,ae^x-b)}+\frac{(a^2-6\,b^2)\log\left(e^x+1\right)}{2\,a^4}$$
$$-\frac{(a^2-6\,b^2)\log\left(|e^x-1|\right)}{2\,a^4}-\frac{ae^{(3\,x)}-4\,be^{(2\,x)}+ae^x+4\,b}{a^3\left(e^{(2\,x)}-1\right)^2}$$

input
```
integrate(csch(x)^3/(a+b*sinh(x))^2,x, algorithm="giac")
```

output
```
-(4*a^2*b^3 + 3*b^5)*log(abs(2*b*e^x + 2*a - 2*sqrt(a^2 + b^2))/abs(2*b*e^
x + 2*a + 2*sqrt(a^2 + b^2)))/((a^6 + a^4*b^2)*sqrt(a^2 + b^2)) - 2*(a*b^3
*e^x - b^4)/((a^5 + a^3*b^2)*(b*e^(2*x) + 2*a*e^x - b)) + 1/2*(a^2 - 6*b^2
)*log(e^x + 1)/a^4 - 1/2*(a^2 - 6*b^2)*log(abs(e^x - 1))/a^4 - (a*e^(3*x)
- 4*b*e^(2*x) + a*e^x + 4*b)/(a^3*(e^(2*x) - 1)^2)
```

Mupad [B] (verification not implemented)

Time = 5.13 (sec) , antiderivative size = 977, normalized size of antiderivative = 6.18

$$\int \frac{\operatorname{csch}^3(x)}{(a+b\sinh(x))^2}\,dx = \text{Too large to display}$$

input
```
int(1/(sinh(x)^3*(a + b*sinh(x))^2),x)
```

output
```
((4*b)/a^3 - exp(x)/a^2)/(exp(2*x) - 1) + ((2*b^7)/(a^3*(b^5 + a^2*b^3)) -
 (2*b^6*exp(x))/(a^2*(b^5 + a^2*b^3)))/(2*a*exp(x) - b + b*exp(2*x)) - (lo
g(exp(x) - 1)*(a^2 - 6*b^2))/(2*a^4) + (log(exp(x) + 1)*(a^2 - 6*b^2))/(2*
a^4) - (2*exp(x))/(a^2*(exp(4*x) - 2*exp(2*x) + 1)) + (b^3*log((8*(4*a^2 +
 3*b^2)*(20*a^9*b^5 - 72*b^11*((a^2 + b^2)^3)^(1/2) - 9*a^3*b^11 - 30*a^5*
b^9 - 18*a^7*b^7 - 2*a^13*b + 15*a^11*b^3 + 4*a^14*exp(x) - 192*a^2*b^9*((
a^2 + b^2)^3)^(1/2) - 128*a^4*b^7*((a^2 + b^2)^3)^(1/2) + 27*a^4*b^10*exp(
x) + 72*a^6*b^8*exp(x) + 30*a^8*b^6*exp(x) - 48*a^10*b^4*exp(x) - 29*a^12*
b^2*exp(x) + 312*a^3*b^8*exp(x)*((a^2 + b^2)^3)^(1/2) + 206*a^5*b^6*exp(x)
*((a^2 + b^2)^3)^(1/2) + 8*a*b^4*exp(x)*((a^2 + b^2)^3)^(3/2) + 118*a*b^10
*exp(x)*((a^2 + b^2)^3)^(1/2)))/(a^9*b^2*((a^2 + b^2)^3)^(1/2)*(a^2 + b^2)
^4) - (8*(18*b^4 - 4*a^4 + 21*a^2*b^2)*(2*a^4*b - 12*b^5 - 10*a^2*b^3 - 4*
a^5*exp(x) + 21*a*b^4*exp(x) + 19*a^3*b^2*exp(x)))/(a^9*b^2*(a^2 + b^2)^2)
)*((a^2 + b^2)^3)^(1/2)*(4*a^2 + 3*b^2))/(a^10 + a^4*b^6 + 3*a^6*b^4 + 3*a
^8*b^2) - (b^3*log((8*(4*a^2 + 3*b^2)*(2*a^13*b - 72*b^11*((a^2 + b^2)^3)^
(1/2) + 9*a^3*b^11 + 30*a^5*b^9 + 18*a^7*b^7 - 20*a^9*b^5 - 15*a^11*b^3 -
4*a^14*exp(x) - 192*a^2*b^9*((a^2 + b^2)^3)^(1/2) - 128*a^4*b^7*((a^2 + b^
2)^3)^(1/2) - 27*a^4*b^10*exp(x) - 72*a^6*b^8*exp(x) - 30*a^8*b^6*exp(x) +
 48*a^10*b^4*exp(x) + 29*a^12*b^2*exp(x) + 312*a^3*b^8*exp(x)*((a^2 + b^2)
^3)^(1/2) + 206*a^5*b^6*exp(x)*((a^2 + b^2)^3)^(1/2) + 8*a*b^4*exp(x)*(...
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 1964, normalized size of antiderivative = 12.43

$$\int \frac{\operatorname{csch}^3(x)}{(a+b\sinh(x))^2}\,dx = \text{Too large to display}$$

input
```
int(csch(x)^3/(a+b*sinh(x))^2,x)
```

output
```
( - 16*e**(6*x)*sqrt(a**2 + b**2)*atan((e**x*b*i + a*i)/sqrt(a**2 + b**2))
*a**2*b**4*i - 12*e**(6*x)*sqrt(a**2 + b**2)*atan((e**x*b*i + a*i)/sqrt(a*
*2 + b**2))*b**6*i - 32*e**(5*x)*sqrt(a**2 + b**2)*atan((e**x*b*i + a*i)/s
qrt(a**2 + b**2))*a**3*b**3*i - 24*e**(5*x)*sqrt(a**2 + b**2)*atan((e**x*b
*i + a*i)/sqrt(a**2 + b**2))*a*b**5*i + 48*e**(4*x)*sqrt(a**2 + b**2)*atan
((e**x*b*i + a*i)/sqrt(a**2 + b**2))*a**2*b**4*i + 36*e**(4*x)*sqrt(a**2 +
 b**2)*atan((e**x*b*i + a*i)/sqrt(a**2 + b**2))*b**6*i + 64*e**(3*x)*sqrt(
a**2 + b**2)*atan((e**x*b*i + a*i)/sqrt(a**2 + b**2))*a**3*b**3*i + 48*e**
(3*x)*sqrt(a**2 + b**2)*atan((e**x*b*i + a*i)/sqrt(a**2 + b**2))*a*b**5*i
- 48*e**(2*x)*sqrt(a**2 + b**2)*atan((e**x*b*i + a*i)/sqrt(a**2 + b**2))*a
**2*b**4*i - 36*e**(2*x)*sqrt(a**2 + b**2)*atan((e**x*b*i + a*i)/sqrt(a**2
 + b**2))*b**6*i - 32*e**x*sqrt(a**2 + b**2)*atan((e**x*b*i + a*i)/sqrt(a*
*2 + b**2))*a**3*b**3*i - 24*e**x*sqrt(a**2 + b**2)*atan((e**x*b*i + a*i)/
sqrt(a**2 + b**2))*a*b**5*i + 16*sqrt(a**2 + b**2)*atan((e**x*b*i + a*i)/s
qrt(a**2 + b**2))*a**2*b**4*i + 12*sqrt(a**2 + b**2)*atan((e**x*b*i + a*i)
/sqrt(a**2 + b**2))*b**6*i - e**(6*x)*log(e**x - 1)*a**6*b + 4*e**(6*x)*lo
g(e**x - 1)*a**4*b**3 + 11*e**(6*x)*log(e**x - 1)*a**2*b**5 + 6*e**(6*x)*l
og(e**x - 1)*b**7 + e**(6*x)*log(e**x + 1)*a**6*b - 4*e**(6*x)*log(e**x +
1)*a**4*b**3 - 11*e**(6*x)*log(e**x + 1)*a**2*b**5 - 6*e**(6*x)*log(e**x +
 1)*b**7 + e**(6*x)*a**5*b**2 + 4*e**(6*x)*a**3*b**4 + 3*e**(6*x)*a*b**...
```

3.87 $\int \frac{\operatorname{csch}^4(x)}{(a+b\sinh(x))^2}\,dx$

Optimal result

Integrand size = 13, antiderivative size = 198

$$\begin{aligned}\int \frac{\operatorname{csch}^4(x)}{(a+b\sinh(x))^2}\,dx = &-\frac{b(a^2-4b^2)\operatorname{arctanh}(\cosh(x))}{a^5}\\ &-\frac{2b^4(5a^2+4b^2)\operatorname{arctanh}\left(\frac{b-a\tanh\left(\frac{x}{2}\right)}{\sqrt{a^2+b^2}}\right)}{a^5(a^2+b^2)^{3/2}}\\ &+\frac{(2a^4-7a^2b^2-12b^4)\coth(x)}{3a^4(a^2+b^2)}+\frac{b(a^2+2b^2)\coth(x)\operatorname{csch}(x)}{a^3(a^2+b^2)}\\ &-\frac{(a^2+4b^2)\coth(x)\operatorname{csch}^2(x)}{3a^2(a^2+b^2)}+\frac{b^2\coth(x)\operatorname{csch}^2(x)}{a(a^2+b^2)(a+b\sinh(x))}\end{aligned}$$

output
```
-b*(a^2-4*b^2)*arctanh(cosh(x))/a^5-2*b^4*(5*a^2+4*b^2)*arctanh((b-a*tanh(
1/2*x))/(a^2+b^2)^(1/2))/a^5/(a^2+b^2)^(3/2)+1/3*(2*a^4-7*a^2*b^2-12*b^4)*
coth(x)/a^4/(a^2+b^2)+b*(a^2+2*b^2)*coth(x)*csch(x)/a^3/(a^2+b^2)-1/3*(a^2
+4*b^2)*coth(x)*csch(x)^2/a^2/(a^2+b^2)+b^2*coth(x)*csch(x)^2/a/(a^2+b^2)/
(a+b*sinh(x))
```

Mathematica [A] (verified)

Time = 1.06 (sec) , antiderivative size = 232, normalized size of antiderivative = 1.17

$$\int \frac{\operatorname{csch}^4(x)}{(a+b\sinh(x))^2}\,dx$$
$$= \frac{-\frac{48b^4(5a^2+4b^2)\arctan\left(\frac{b-a\tanh\left(\frac{x}{2}\right)}{\sqrt{-a^2-b^2}}\right)}{(-a^2-b^2)^{3/2}} + 4a(2a^2-9b^2)\coth\left(\frac{x}{2}\right) + 6a^2b\operatorname{csch}^2\left(\frac{x}{2}\right) - 24b(a^2-4b^2)\log\left(\cosh\left(\frac{x}{2}\right)\right)}{}$$

input
```
Integrate[Csch[x]^4/(a + b*Sinh[x])^2,x]
```

output
```
((-48*b^4*(5*a^2 + 4*b^2)*ArcTan[(b - a*Tanh[x/2])/Sqrt[-a^2 - b^2]])/(-a^
2 - b^2)^(3/2) + 4*a*(2*a^2 - 9*b^2)*Coth[x/2] + 6*a^2*b*Csch[x/2]^2 - 24*
b*(a^2 - 4*b^2)*Log[Cosh[x/2]] + 24*b*(a^2 - 4*b^2)*Log[Sinh[x/2]] + 6*a^2
*b*Sech[x/2]^2 + 8*a^3*Csch[x]^3*Sinh[x/2]^4 - (a^3*Csch[x/2]^4*Sinh[x])/2
 - (24*a*b^5*Cosh[x])/((a^2 + b^2)*(a + b*Sinh[x])) + 4*a*(2*a^2 - 9*b^2)*
Tanh[x/2])/(24*a^5)
```

Rubi [C] (verified)

Result contains complex when optimal does not.

Time = 1.87 (sec) , antiderivative size = 228, normalized size of antiderivative = 1.15, number of steps used = 24, number of rules used = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 1.769$, Rules used = {3042, 3281, 3042, 3534, 25, 3042, 26, 3534, 27, 3042, 25, 3534, 27, 3042, 26, 3480, 26, 3042, 26, 3139, 1083, 219, 4257}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\operatorname{csch}^4(x)}{(a+b\sinh(x))^2}\,dx$$
$$\Big\downarrow \ 3042$$
$$\int \frac{1}{\sin(ix)^4(a-ib\sin(ix))^2}dx$$

$\Big\downarrow$ 3281

$$\frac{\int \frac{\operatorname{csch}^4(x)\left(a^2-b\sinh(x)a+4b^2+3b^2\sinh^2(x)\right)}{a+b\sinh(x)}dx}{a\left(a^2+b^2\right)}+\frac{b^2\coth(x)\operatorname{csch}^2(x)}{a\left(a^2+b^2\right)\left(a+b\sinh(x)\right)}$$

$\Big\downarrow$ 3042

$$\frac{b^2\coth(x)\operatorname{csch}^2(x)}{a\left(a^2+b^2\right)\left(a+b\sinh(x)\right)}+\frac{\int \frac{a^2+ib\sin(ix)a+4b^2-3b^2\sin(ix)^2}{\sin(ix)^4\left(a-ib\sin(ix)\right)}dx}{a\left(a^2+b^2\right)}$$

$\Big\downarrow$ 3534

$$\frac{\frac{\int -\frac{\operatorname{csch}^3(x)\left(2b\left(a^2+4b^2\right)\sinh^2(x)+a\left(2a^2-b^2\right)\sinh(x)+6b\left(a^2+2b^2\right)\right)}{a+b\sinh(x)}dx}{3a}-\frac{\left(a^2+4b^2\right)\coth(x)\operatorname{csch}^2(x)}{3a}}{a\left(a^2+b^2\right)}+\frac{b^2\coth(x)\operatorname{csch}^2(x)}{a\left(a^2+b^2\right)\left(a+b\sinh(x)\right)}$$

$\Big\downarrow$ 25

$$\frac{-\frac{\int \frac{\operatorname{csch}^3(x)\left(2b\left(a^2+4b^2\right)\sinh^2(x)+a\left(2a^2-b^2\right)\sinh(x)+6b\left(a^2+2b^2\right)\right)}{a+b\sinh(x)}dx}{3a}-\frac{\left(a^2+4b^2\right)\coth(x)\operatorname{csch}^2(x)}{3a}}{a\left(a^2+b^2\right)}+\frac{b^2\coth(x)\operatorname{csch}^2(x)}{a\left(a^2+b^2\right)\left(a+b\sinh(x)\right)}$$

$\Big\downarrow$ 3042

$$\frac{b^2\coth(x)\operatorname{csch}^2(x)}{a\left(a^2+b^2\right)\left(a+b\sinh(x)\right)}+\frac{-\frac{\left(a^2+4b^2\right)\coth(x)\operatorname{csch}^2(x)}{3a}-\frac{\int -\frac{i\left(-2b\left(a^2+4b^2\right)\sin(ix)^2-ia\left(2a^2-b^2\right)\sin(ix)+6b\left(a^2+2b^2\right)\right)}{\sin(ix)^3\left(a-ib\sin(ix)\right)}dx}{3a}}{a\left(a^2+b^2\right)}$$

$\Big\downarrow$ 26

$$\frac{b^2\coth(x)\operatorname{csch}^2(x)}{a\left(a^2+b^2\right)\left(a+b\sinh(x)\right)}+\frac{-\frac{\left(a^2+4b^2\right)\coth(x)\operatorname{csch}^2(x)}{3a}+\frac{i\int \frac{-2b\left(a^2+4b^2\right)\sin(ix)^2-ia\left(2a^2-b^2\right)\sin(ix)+6b\left(a^2+2b^2\right)}{\sin(ix)^3\left(a-ib\sin(ix)\right)}dx}{3a}}{a\left(a^2+b^2\right)}$$

$\Big\downarrow$ 3534

$$\frac{b^2\coth(x)\text{csch}^2(x)}{a\left(a^2+b^2\right)\left(a+b\sinh(x)\right)}+\frac{-\frac{\left(a^2+4b^2\right)\coth(x)\text{csch}^2(x)}{3a}+\frac{i\left(\frac{\int\frac{2i\text{csch}^2(x)\left(2a^4-7b^2a^2-b\left(a^2-2b^2\right)\sinh(x)a-12b^4-3b^2\left(a^2+2b^2\right)\sinh^2(x)\right)}{a+b\sinh(x)}dx}{2a}-\frac{3ib\left(a^2+2b^2\right)\coth(x)\text{csch}(x)}{a}\right)}{3a}}{a\left(a^2+b^2\right)}$$

$\Big\downarrow$ 27

$$\frac{b^2\coth(x)\text{csch}^2(x)}{a\left(a^2+b^2\right)\left(a+b\sinh(x)\right)}+\frac{-\frac{\left(a^2+4b^2\right)\coth(x)\text{csch}^2(x)}{3a}+\frac{i\left(\frac{i\int\frac{\text{csch}^2(x)\left(2a^4-7b^2a^2-b\left(a^2-2b^2\right)\sinh(x)a-12b^4-3b^2\left(a^2+2b^2\right)\sinh^2(x)\right)}{a+b\sinh(x)}dx}{a}-\frac{3ib\left(a^2+2b^2\right)\coth(x)\text{csch}(x)}{a}\right)}{3a}}{a\left(a^2+b^2\right)}$$

$\Big\downarrow$ 3042

$$\frac{b^2\coth(x)\text{csch}^2(x)}{a\left(a^2+b^2\right)\left(a+b\sinh(x)\right)}+\frac{-\frac{\left(a^2+4b^2\right)\coth(x)\text{csch}^2(x)}{3a}+\frac{i\left(\frac{i\int-\frac{2a^4-7b^2a^2+ib\left(a^2-2b^2\right)\sin(ix)a-12b^4+3b^2\left(a^2+2b^2\right)\sin(ix)^2}{\sin(ix)^2\left(a-ib\sin(ix)\right)}dx}{a}-\frac{3ib\left(a^2+2b^2\right)\coth(x)\text{csch}(x)}{a}\right)}{3a}}{a\left(a^2+b^2\right)}$$

$\Big\downarrow$ 25

$$\frac{b^2\coth(x)\text{csch}^2(x)}{a\left(a^2+b^2\right)\left(a+b\sinh(x)\right)}+\frac{-\frac{\left(a^2+4b^2\right)\coth(x)\text{csch}^2(x)}{3a}+\frac{i\left(-\frac{i\int\frac{2a^4-7b^2a^2+ib\left(a^2-2b^2\right)\sin(ix)a-12b^4+3b^2\left(a^2+2b^2\right)\sin(ix)^2}{\sin(ix)^2\left(a-ib\sin(ix)\right)}dx}{a}-\frac{3ib\left(a^2+2b^2\right)\coth(x)\text{csch}(x)}{a}\right)}{3a}}{a\left(a^2+b^2\right)}$$

$\Big\downarrow$ 3534

$$\frac{b^2\coth(x)\text{csch}^2(x)}{a\left(a^2+b^2\right)\left(a+b\sinh(x)\right)}+\frac{-\frac{\left(a^2+4b^2\right)\coth(x)\text{csch}^2(x)}{3a}+\frac{i\left(-\frac{i\left(\frac{\int\frac{3\text{csch}(x)\left(a\left(a^2+2b^2\right)\sinh(x)b^2+\left(a^4-3b^2a^2-4b^4\right)b\right)}{a+b\sinh(x)}dx}{a}+\frac{\left(2a^4-7a^2b^2-12b^4\right)\coth(x)}{a}\right)}{a}-\frac{3ib\left(a^2+2b^2\right)\coth}{a}\right)}{3a}}{a\left(a^2+b^2\right)}$$

$$\Big\downarrow \; 27$$

$$\frac{\frac{b^2\coth(x)\operatorname{csch}^2(x)}{a(a^2+b^2)(a+b\sinh(x))}+\frac{-\frac{(a^2+4b^2)\coth(x)\operatorname{csch}^2(x)}{3a}+\frac{i\left(-\frac{i\left(\frac{3\int\frac{\operatorname{csch}(x)\left(a\left(a^2+2b^2\right)\sinh(x)b^2+\left(a^4-3b^2a^2-4b^4\right)b\right)}{a+b\sinh(x)}dx}{a}+\frac{\left(2a^4-7a^2b^2-12b^4\right)\coth(x)}{a}\right)}{a}-\frac{3ib\left(a^2+2b^2\right)\coth}{a}}{3a}}{a(a^2+b^2)}$$

$$\Big\downarrow \; 3042$$

$$\frac{\frac{b^2\coth(x)\operatorname{csch}^2(x)}{a(a^2+b^2)(a+b\sinh(x))}+\frac{-\frac{(a^2+4b^2)\coth(x)\operatorname{csch}^2(x)}{3a}+\frac{i\left(-\frac{i\left(\frac{\left(2a^4-7a^2b^2-12b^4\right)\coth(x)}{a}+\frac{3\int\frac{i\left(b\left(a^4-3b^2a^2-4b^4\right)-iab^2\left(a^2+2b^2\right)\sin(ix)\right)}{\sin(ix)(a-ib\sin(ix))}dx}{a}\right)}{a}-\frac{3ib\left(a^2+2b^2\right)\coth(x)\operatorname{csc}}{a}}{3a}}{a(a^2+b^2)}$$

$$\Big\downarrow \; 26$$

$$\frac{\frac{b^2\coth(x)\operatorname{csch}^2(x)}{a(a^2+b^2)(a+b\sinh(x))}+\frac{-\frac{(a^2+4b^2)\coth(x)\operatorname{csch}^2(x)}{3a}+\frac{i\left(-\frac{i\left(\frac{\left(2a^4-7a^2b^2-12b^4\right)\coth(x)}{a}+\frac{3i\int\frac{b\left(a^4-3b^2a^2-4b^4\right)-iab^2\left(a^2+2b^2\right)\sin(ix)}{\sin(ix)(a-ib\sin(ix))}dx}{a}\right)}{a}-\frac{3ib\left(a^2+2b^2\right)\coth(x)\operatorname{csch}(}{a}}{3a}}{a(a^2+b^2)}$$

$$\Big\downarrow \; 3480$$

$$\frac{b^2\coth(x)\text{csch}^2(x)}{a\left(a^2+b^2\right)\left(a+b\sinh(x)\right)}+\frac{-\frac{\left(a^2+4b^2\right)\coth(x)\text{csch}^2(x)}{3a}+\frac{i\left(-\frac{i\left(\frac{\left(2a^4-7a^2b^2-12b^4\right)\coth(x)}{a}+\frac{3i\left(\frac{b\left(a^4-3a^2b^2-4b^4\right)\int -i\text{csch}(x)dx}{a}-\frac{ib^4\left(5a^2+4b^2\right)\int\frac{1}{a+b\sinh(x)}dx}{a}\right)}{a}\right)}{a}-\frac{3i\cdots}{}\right)}{3a}}{a\left(a^2+b^2\right)}$$

$\downarrow$ 26

$$\frac{b^2\coth(x)\text{csch}^2(x)}{a\left(a^2+b^2\right)\left(a+b\sinh(x)\right)}+\frac{-\frac{\left(a^2+4b^2\right)\coth(x)\text{csch}^2(x)}{3a}+\frac{i\left(-\frac{i\left(\frac{\left(2a^4-7a^2b^2-12b^4\right)\coth(x)}{a}+\frac{3i\left(-\frac{ib^4\left(5a^2+4b^2\right)\int\frac{1}{a+b\sinh(x)}dx}{a}-\frac{ib\left(a^4-3a^2b^2-4b^4\right)\int\text{csch}(x)dx}{a}\right)}{a}\right)}{a}-\frac{3i\cdots}{}\right)}{3a}}{a\left(a^2+b^2\right)}$$

$\downarrow$ 3042

$$\frac{b^2\coth(x)\text{csch}^2(x)}{a\left(a^2+b^2\right)\left(a+b\sinh(x)\right)}+\frac{-\frac{\left(a^2+4b^2\right)\coth(x)\text{csch}^2(x)}{3a}+\frac{i\left(-\frac{i\left(\frac{\left(2a^4-7a^2b^2-12b^4\right)\coth(x)}{a}+\frac{3i\left(-\frac{ib^4\left(5a^2+4b^2\right)\int\frac{1}{a-ib\sin(ix)}dx}{a}-\frac{ib\left(a^4-3a^2b^2-4b^4\right)\int i\csc(ix)dx}{a}\right)}{a}\right)}{a}-\frac{3ib\cdots}{}\right)}{3a}}{a\left(a^2+b^2\right)}$$

$\downarrow$ 26

$$\frac{\frac{b^2\coth(x)\text{csch}^2(x)}{a(a^2+b^2)(a+b\sinh(x))}+\frac{-\frac{(a^2+4b^2)\coth(x)\text{csch}^2(x)}{3a}+\frac{i\left(-\frac{i\left(\frac{(2a^4-7a^2b^2-12b^4)\coth(x)}{a}+\frac{3i\left(\frac{b(a^4-3a^2b^2-4b^4)\int\csc(ix)dx}{a}-\frac{ib^4(5a^2+4b^2)\int\frac{1}{a-ib\sin(ix)}dx}{a}\right)}{a}\right)}{a}\right)-\frac{3ib(a^2}{}}{3a}}{a(a^2+b^2)}$$

$\downarrow$ 3139

$$\frac{\frac{b^2\coth(x)\text{csch}^2(x)}{a(a^2+b^2)(a+b\sinh(x))}+\frac{-\frac{(a^2+4b^2)\coth(x)\text{csch}^2(x)}{3a}+\frac{i\left(-\frac{i\left(\frac{(2a^4-7a^2b^2-12b^4)\coth(x)}{a}+\frac{3i\left(\frac{b(a^4-3a^2b^2-4b^4)\int\csc(ix)dx}{a}-\frac{2ib^4(5a^2+4b^2)\int\frac{1}{-a\tanh^2\left(\frac{x}{2}\right)+2b\tanh\left(\frac{x}{2}\right)}}{a}\right)}{a}\right)}{a}}{3a}}{a(a^2+b^2)}$$

$\downarrow$ 1083

$$\frac{\frac{b^2\coth(x)\text{csch}^2(x)}{a(a^2+b^2)(a+b\sinh(x))}+\frac{-\frac{(a^2+4b^2)\coth(x)\text{csch}^2(x)}{3a}+\frac{i\left(-\frac{i\left(\frac{(2a^4-7a^2b^2-12b^4)\coth(x)}{a}+\frac{3i\left(\frac{4ib^4(5a^2+4b^2)\int\frac{1}{4(a^2+b^2)-\left(2b-2a\tanh\left(\frac{x}{2}\right)\right)^2}d\left(2b-2a\tanh\left(\frac{x}{2}\right)\right)}{a}+\frac{b(a^4-}{}\right)}{a}\right)}{a}}{3a}}{a(a^2+b^2)}$$

$\downarrow$ 219

$$\frac{b^2\coth(x)\operatorname{csch}^2(x)}{a\left(a^2+b^2\right)(a+b\sinh(x))}+\frac{-\frac{(a^2+4b^2)\coth(x)\operatorname{csch}^2(x)}{3a}+\frac{i\left(-\frac{i\left(\frac{(2a^4-7a^2b^2-12b^4)\coth(x)}{a}+\frac{3i\left(\frac{b(a^4-3a^2b^2-4b^4)\int\csc(ix)dx}{a}+\frac{2ib^4(5a^2+4b^2)\operatorname{arctanh}\left(\frac{2b-2a\tanh\left(\frac{x}{2}\right)}{2\sqrt{a^2+b^2}}\right.}{a\sqrt{a^2+b^2}}\right.}{a}\right.}{a}\right.}{3a}}{a\left(a^2+b^2\right)}$$

$$\Big\downarrow \text{ 4257}$$

$$\frac{b^2\coth(x)\operatorname{csch}^2(x)}{a\left(a^2+b^2\right)(a+b\sinh(x))}+\frac{-\frac{(a^2+4b^2)\coth(x)\operatorname{csch}^2(x)}{3a}+\frac{i\left(-\frac{3ib(a^2+2b^2)\coth(x)\operatorname{csch}(x)}{a}-\frac{i\left(\frac{(2a^4-7a^2b^2-12b^4)\coth(x)}{a}+\frac{3i\left(\frac{2ib^4(5a^2+4b^2)\operatorname{arctanh}\left(\frac{2b-2a\tanh\left(\frac{x}{2}\right)}{2\sqrt{a^2+b^2}}\right.}{a\sqrt{a^2+b^2}}\right.}{a}\right.}{a}\right.}{3a}}{a\left(a^2+b^2\right)}$$

input
```
Int[Csch[x]^4/(a + b*Sinh[x])^2,x]
```

output
```
(-1/3*((a^2 + 4*b^2)*Coth[x]*Csch[x]^2)/a + ((I/3)*(((-I)*(((3*I)*((I*b*(a
^4 - 3*a^2*b^2 - 4*b^4)*ArcTanh[Cosh[x]])/a + ((2*I)*b^4*(5*a^2 + 4*b^2)*A
rcTanh[(2*b - 2*a*Tanh[x/2])/(2*Sqrt[a^2 + b^2])])/(a*Sqrt[a^2 + b^2])))/a
 + ((2*a^4 - 7*a^2*b^2 - 12*b^4)*Coth[x])/a))/a - ((3*I)*b*(a^2 + 2*b^2)*C
oth[x]*Csch[x])/a))/a)/(a*(a^2 + b^2)) + (b^2*Coth[x]*Csch[x]^2)/(a*(a^2 +
 b^2)*(a + b*Sinh[x]))
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1083
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Simp[-2   Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3139
```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Simp[2*(e/d)   Subst[Int[1/(a + 2*b*e*x + a
*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[a^2 - b^2, 0]
```

rule 3281
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b^2)*Cos[e + f*x]*(a + b*Sin[e + f*
x])^(m + 1)*((c + d*Sin[e + f*x])^(n + 1)/(f*(m + 1)*(b*c - a*d)*(a^2 - b^2
))), x] + Simp[1/((m + 1)*(b*c - a*d)*(a^2 - b^2))   Int[(a + b*Sin[e + f*x
])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[a*(b*c - a*d)*(m + 1) + b^2*d*(m + n
 + 2) - (b^2*c + b*(b*c - a*d)*(m + 1))*Sin[e + f*x] - b^2*d*(m + n + 3)*Si
n[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*
d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -1] && IntegersQ[
2*m, 2*n] && ((EqQ[a, 0] && IntegerQ[m] &&  !IntegerQ[n]) ||  !(IntegerQ[2*
n] && LtQ[n, -1] && ((IntegerQ[n] &&  !IntegerQ[m]) || EqQ[a, 0])))
```

rule 3480
```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((a_.) + (b_.)*sin[(e_.) + (f_
.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])), x_Symbol] :> Simp[(A*b
- a*B)/(b*c - a*d)   Int[1/(a + b*Sin[e + f*x]), x], x] + Simp[(B*c - A*d)/
(b*c - a*d)   Int[1/(c + d*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f
, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

rule 3534
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-(A*b^2 - a*b*B + a^2*C))*Cos[e + f*x
]*(a + b*Sin[e + f*x])^(m + 1)*((c + d*Sin[e + f*x])^(n + 1)/(f*(m + 1)*(b*
c - a*d)*(a^2 - b^2))), x] + Simp[1/((m + 1)*(b*c - a*d)*(a^2 - b^2))   Int
[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[(m + 1)*(b*c - a*
d)*(a*A - b*B + a*C) + d*(A*b^2 - a*b*B + a^2*C)*(m + n + 2) - (c*(A*b^2 -
a*b*B + a^2*C) + (m + 1)*(b*c - a*d)*(A*b - a*B + b*C))*Sin[e + f*x] - d*(A
*b^2 - a*b*B + a^2*C)*(m + n + 3)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b
, c, d, e, f, A, B, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] &&
NeQ[c^2 - d^2, 0] && LtQ[m, -1] && ((EqQ[a, 0] && IntegerQ[m] &&  !IntegerQ
[n]) ||  !(IntegerQ[2*n] && LtQ[n, -1] && ((IntegerQ[n] &&  !IntegerQ[m]) |
| EqQ[a, 0])))
```

rule 4257
```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Maple [A] (verified)

Time = 0.50 (sec) , antiderivative size = 219, normalized size of antiderivative = 1.11

method	result
default	$-\frac{\frac{\tanh(\frac{x}{2})^3 a^2}{3}+2\tanh(\frac{x}{2})^2 ab-3\tanh(\frac{x}{2})a^2+12b^2\tanh(\frac{x}{2})}{8a^4}-\frac{1}{24a^2\tanh(\frac{x}{2})^3}-\frac{-3a^2+12b^2}{8a^4\tanh(\frac{x}{2})}+\frac{b}{4a^3\tanh(\frac{x}{2})^2}+\frac{b(a^2-4b^2}{}$
risch	$-\frac{2(-3a^3b^2e^{7x}-6a\,b^4e^{7x}-6a^4b\,e^{6x}+3a^2b^3e^{6x}+12b^5e^{6x}+21a^3b^2e^{5x}+30a\,b^4e^{5x}+6a^4b\,e^{4x}-21a^2b^3e^{4x}-36b^5e^{4x}+12a^5e^{3x}-21a^3b}{3a^4(e^{2x}-1)^3(a^2+b^2)(b\,e^{2x}+2\,e^x a}$

input
```
int(csch(x)^4/(a+b*sinh(x))^2,x,method=_RETURNVERBOSE)
```

output
```
-1/8/a^4*(1/3*tanh(1/2*x)^3*a^2+2*tanh(1/2*x)^2*a*b-3*tanh(1/2*x)*a^2+12*b
^2*tanh(1/2*x))-1/24/a^2/tanh(1/2*x)^3-1/8/a^4*(-3*a^2+12*b^2)/tanh(1/2*x)
+1/4/a^3*b/tanh(1/2*x)^2+1/a^5*b*(a^2-4*b^2)*ln(tanh(1/2*x))-2*b^4/a^5*((-
b^2/(a^2+b^2)*tanh(1/2*x)-a*b/(a^2+b^2))/(tanh(1/2*x)^2*a-2*b*tanh(1/2*x)-
a)-(5*a^2+4*b^2)/(a^2+b^2)^(3/2)*arctanh(1/2*(2*a*tanh(1/2*x)-2*b)/(a^2+b^
2)^(1/2)))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 6430 vs. 2(190) = 380.

Time = 0.34 (sec) , antiderivative size = 6430, normalized size of antiderivative = 32.47

$$\int \frac{\operatorname{csch}^4(x)}{(a+b\sinh(x))^2}\,dx = \text{Too large to display}$$

input
```
integrate(csch(x)^4/(a+b*sinh(x))^2,x, algorithm="fricas")
```

output
```
Too large to include
```

Sympy [F]

$$\int \frac{\operatorname{csch}^4(x)}{(a+b\sinh(x))^2}\,dx = \int \frac{\operatorname{csch}^4(x)}{(a+b\sinh(x))^2}\,dx$$

input
```
integrate(csch(x)**4/(a+b*sinh(x))**2,x)
```

output
```
Integral(csch(x)**4/(a + b*sinh(x))**2, x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 477 vs. $2(190) = 380$.

Time = 0.13 (sec) , antiderivative size = 477, normalized size of antiderivative = 2.41

$$\int \frac{\operatorname{csch}^4(x)}{(a+b\sinh(x))^2}\,dx = \frac{(5\,a^2b^4+4\,b^6)\log\left(\frac{be^{(-x)}-a-\sqrt{a^2+b^2}}{be^{(-x)}-a+\sqrt{a^2+b^2}}\right)}{(a^7+a^5b^2)\sqrt{a^2+b^2}} + \frac{2\,(2\,a^4b-7\,a^2b^3-12\,b^5+(4\,a^5-11\,a^3b^2-18\,ab^4)e^{(-x)}-(2\,a^4b-25\,a^2b^3-36\,b^5)e^{(-2\,x)}-3\,(4\,a^5-}{3\,(a^6b+a^4b^3+2\,(a^7+a^5b^2)e^{(-x)}-4\,(a^6b+a^4b^3)e^{(-2\,x)}-6\,(a^7+a^5b^2)e^{(-3}} - \frac{(a^2b-4\,b^3)\log\left(e^{(-x)}+1\right)}{a^5} + \frac{(a^2b-4\,b^3)\log\left(e^{(-x)}-1\right)}{a^5}$$

input
```
integrate(csch(x)^4/(a+b*sinh(x))^2,x, algorithm="maxima")
```

output
```
(5*a^2*b^4 + 4*b^6)*log((b*e^(-x) - a - sqrt(a^2 + b^2))/(b*e^(-x) - a + s
qrt(a^2 + b^2)))/((a^7 + a^5*b^2)*sqrt(a^2 + b^2)) + 2/3*(2*a^4*b - 7*a^2*
b^3 - 12*b^5 + (4*a^5 - 11*a^3*b^2 - 18*a*b^4)*e^(-x) - (2*a^4*b - 25*a^2*
b^3 - 36*b^5)*e^(-2*x) - 3*(4*a^5 - 7*a^3*b^2 - 14*a*b^4)*e^(-3*x) + 3*(2*
a^4*b - 7*a^2*b^3 - 12*b^5)*e^(-4*x) - 3*(7*a^3*b^2 + 10*a*b^4)*e^(-5*x) -
 3*(2*a^4*b - a^2*b^3 - 4*b^5)*e^(-6*x) + 3*(a^3*b^2 + 2*a*b^4)*e^(-7*x))/
(a^6*b + a^4*b^3 + 2*(a^7 + a^5*b^2)*e^(-x) - 4*(a^6*b + a^4*b^3)*e^(-2*x)
 - 6*(a^7 + a^5*b^2)*e^(-3*x) + 6*(a^6*b + a^4*b^3)*e^(-4*x) + 6*(a^7 + a^
5*b^2)*e^(-5*x) - 4*(a^6*b + a^4*b^3)*e^(-6*x) - 2*(a^7 + a^5*b^2)*e^(-7*x
) + (a^6*b + a^4*b^3)*e^(-8*x)) - (a^2*b - 4*b^3)*log(e^(-x) + 1)/a^5 + (a
^2*b - 4*b^3)*log(e^(-x) - 1)/a^5
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 236, normalized size of antiderivative = 1.19

$$\int \frac{\operatorname{csch}^4(x)}{(a+b\sinh(x))^2}\,dx = \frac{(5\,a^2b^4+4\,b^6)\log\left(\frac{\left|2\,be^x+2\,a-2\,\sqrt{a^2+b^2}\right|}{\left|2\,be^x+2\,a+2\,\sqrt{a^2+b^2}\right|}\right)}{(a^7+a^5b^2)\sqrt{a^2+b^2}} + \frac{2\,(ab^4e^x-b^5)}{(a^6+a^4b^2)(be^{(2\,x)}+2\,ae^x-b)} - \frac{(a^2b-4\,b^3)\log(e^x+1)}{a^5} + \frac{(a^2b-4\,b^3)\log(|e^x-1|)}{a^5} + \frac{2\left(3\,abe^{(5\,x)}-9\,b^2e^{(4\,x)}-6\,a^2e^{(2\,x)}+18\,b^2e^{(2\,x)}-3\,abe^x+2\,a^2-9\,b^2\right)}{3\,a^4(e^{(2\,x)}-1)^3}$$

input
```
integrate(csch(x)^4/(a+b*sinh(x))^2,x, algorithm="giac")
```

output
```
(5*a^2*b^4 + 4*b^6)*log(abs(2*b*e^x + 2*a - 2*sqrt(a^2 + b^2))/abs(2*b*e^x
 + 2*a + 2*sqrt(a^2 + b^2)))/((a^7 + a^5*b^2)*sqrt(a^2 + b^2)) + 2*(a*b^4*
e^x - b^5)/((a^6 + a^4*b^2)*(b*e^(2*x) + 2*a*e^x - b)) - (a^2*b - 4*b^3)*l
og(e^x + 1)/a^5 + (a^2*b - 4*b^3)*log(abs(e^x - 1))/a^5 + 2/3*(3*a*b*e^(5*
x) - 9*b^2*e^(4*x) - 6*a^2*e^(2*x) + 18*b^2*e^(2*x) - 3*a*b*e^x + 2*a^2 -
9*b^2)/(a^4*(e^(2*x) - 1)^3)
```

Mupad [B] (verification not implemented)

Time = 4.69 (sec) , antiderivative size = 975, normalized size of antiderivative = 4.92

$$\int \frac{\operatorname{csch}^4(x)}{(a+b\sinh(x))^2}\,dx = \text{Too large to display}$$

input
```
int(1/(sinh(x)^4*(a + b*sinh(x))^2),x)
```

output
```
(log(exp(x) - 1)*(a^2*b - 4*b^3))/a^5 - 8/(3*a^2*(3*exp(2*x) - 3*exp(4*x)
+ exp(6*x) - 1)) - (4/a^2 - (4*b*exp(x))/a^3)/(exp(4*x) - 2*exp(2*x) + 1)
- ((6*b^2)/a^4 - (2*b*exp(x))/a^3)/(exp(2*x) - 1) - ((2*b^8)/(a^4*(b^5 + a
^2*b^3)) - (2*b^7*exp(x))/(a^3*(b^5 + a^2*b^3)))/(2*a*exp(x) - b + b*exp(2
*x)) - (log(exp(x) + 1)*(a^2*b - 4*b^3))/a^5 - (b^4*log((32*b*(16*b^4 - 5*
a^4 + 16*a^2*b^2)*(2*a^4*b - 8*b^5 - 6*a^2*b^3 - 4*a^5*exp(x) + 14*a*b^4*e
xp(x) + 11*a^3*b^2*exp(x)))/(a^12*(a^2 + b^2)^2) - (32*b*(5*a^2 + 4*b^2)*(
5*a^5*b^9 - 32*b^11*((a^2 + b^2)^3)^(1/2) - 2*a^13*b + 20*a^7*b^7 + 24*a^9
*b^5 + 7*a^11*b^3 + 4*a^14*exp(x) - 80*a^2*b^9*((a^2 + b^2)^3)^(1/2) - 50*
a^4*b^7*((a^2 + b^2)^3)^(1/2) - 15*a^6*b^8*exp(x) - 50*a^8*b^6*exp(x) - 52
*a^10*b^4*exp(x) - 13*a^12*b^2*exp(x) + 127*a^3*b^8*exp(x)*((a^2 + b^2)^3)
^(1/2) + 79*a^5*b^6*exp(x)*((a^2 + b^2)^3)^(1/2) + 5*a*b^4*exp(x)*((a^2 +
b^2)^3)^(3/2) + 51*a*b^10*exp(x)*((a^2 + b^2)^3)^(1/2)))/(a^12*((a^2 + b^2
)^3)^(1/2)*(a^2 + b^2)^4))*((a^2 + b^2)^3)^(1/2)*(5*a^2 + 4*b^2))/(a^11 +
a^5*b^6 + 3*a^7*b^4 + 3*a^9*b^2) + (b^4*log((32*b*(16*b^4 - 5*a^4 + 16*a^2
*b^2)*(2*a^4*b - 8*b^5 - 6*a^2*b^3 - 4*a^5*exp(x) + 14*a*b^4*exp(x) + 11*a
^3*b^2*exp(x)))/(a^12*(a^2 + b^2)^2) - (32*b*(5*a^2 + 4*b^2)*(2*a^13*b - 3
2*b^11*((a^2 + b^2)^3)^(1/2) - 5*a^5*b^9 - 20*a^7*b^7 - 24*a^9*b^5 - 7*a^1
1*b^3 - 4*a^14*exp(x) - 80*a^2*b^9*((a^2 + b^2)^3)^(1/2) - 50*a^4*b^7*((a^
2 + b^2)^3)^(1/2) + 15*a^6*b^8*exp(x) + 50*a^8*b^6*exp(x) + 52*a^10*b^4...
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 2608, normalized size of antiderivative = 13.17

$$\int \frac{\operatorname{csch}^4(x)}{(a+b\sinh(x))^2}\,dx = \text{Too large to display}$$

input
```
int(csch(x)^4/(a+b*sinh(x))^2,x)
```

output
```
(30*e**(8*x)*sqrt(a**2 + b**2)*atan((e**x*b*i + a*i)/sqrt(a**2 + b**2))*a*
*2*b**5*i + 24*e**(8*x)*sqrt(a**2 + b**2)*atan((e**x*b*i + a*i)/sqrt(a**2
+ b**2))*b**7*i + 60*e**(7*x)*sqrt(a**2 + b**2)*atan((e**x*b*i + a*i)/sqrt
(a**2 + b**2))*a**3*b**4*i + 48*e**(7*x)*sqrt(a**2 + b**2)*atan((e**x*b*i
+ a*i)/sqrt(a**2 + b**2))*a*b**6*i - 120*e**(6*x)*sqrt(a**2 + b**2)*atan((
e**x*b*i + a*i)/sqrt(a**2 + b**2))*a**2*b**5*i - 96*e**(6*x)*sqrt(a**2 + b
**2)*atan((e**x*b*i + a*i)/sqrt(a**2 + b**2))*b**7*i - 180*e**(5*x)*sqrt(a
**2 + b**2)*atan((e**x*b*i + a*i)/sqrt(a**2 + b**2))*a**3*b**4*i - 144*e**
(5*x)*sqrt(a**2 + b**2)*atan((e**x*b*i + a*i)/sqrt(a**2 + b**2))*a*b**6*i
+ 180*e**(4*x)*sqrt(a**2 + b**2)*atan((e**x*b*i + a*i)/sqrt(a**2 + b**2))*
a**2*b**5*i + 144*e**(4*x)*sqrt(a**2 + b**2)*atan((e**x*b*i + a*i)/sqrt(a*
*2 + b**2))*b**7*i + 180*e**(3*x)*sqrt(a**2 + b**2)*atan((e**x*b*i + a*i)/
sqrt(a**2 + b**2))*a**3*b**4*i + 144*e**(3*x)*sqrt(a**2 + b**2)*atan((e**x
*b*i + a*i)/sqrt(a**2 + b**2))*a*b**6*i - 120*e**(2*x)*sqrt(a**2 + b**2)*a
tan((e**x*b*i + a*i)/sqrt(a**2 + b**2))*a**2*b**5*i - 96*e**(2*x)*sqrt(a**
2 + b**2)*atan((e**x*b*i + a*i)/sqrt(a**2 + b**2))*b**7*i - 60*e**x*sqrt(a
**2 + b**2)*atan((e**x*b*i + a*i)/sqrt(a**2 + b**2))*a**3*b**4*i - 48*e**x
*sqrt(a**2 + b**2)*atan((e**x*b*i + a*i)/sqrt(a**2 + b**2))*a*b**6*i + 30*
sqrt(a**2 + b**2)*atan((e**x*b*i + a*i)/sqrt(a**2 + b**2))*a**2*b**5*i + 2
4*sqrt(a**2 + b**2)*atan((e**x*b*i + a*i)/sqrt(a**2 + b**2))*b**7*i + 3...
```

3.88 $\int \frac{1}{3+5i\sinh(c+dx)}\,dx$

Optimal result

Integrand size = 14, antiderivative size = 53

$$\int \frac{1}{3+5i\sinh(c+dx)}\,dx = -\frac{i\log\left(i-3\tanh\left(\frac{1}{2}(c+dx)\right)\right)}{4d}+\frac{i\log\left(3i-\tanh\left(\frac{1}{2}(c+dx)\right)\right)}{4d}$$

output
```
-1/4*I*ln(I-3*tanh(1/2*d*x+1/2*c))/d+1/4*I*ln(3*I-tanh(1/2*d*x+1/2*c))/d
```

Mathematica [A] (verified)

Time = 0.12 (sec) , antiderivative size = 81, normalized size of antiderivative = 1.53

$$\int \frac{1}{3+5i\sinh(c+dx)}\,dx = \frac{\arctan\left(3\coth\left(\frac{1}{2}(c+dx)\right)\right)}{4d}+\frac{\arctan\left(3\tanh\left(\frac{1}{2}(c+dx)\right)\right)}{4d} - \frac{i\log(4-5\cosh(c+dx))}{8d}+\frac{i\log(4+5\cosh(c+dx))}{8d}$$

input
```
Integrate[(3 + (5*I)*Sinh[c + d*x])^(-1),x]
```

output
```
ArcTan[3*Coth[(c + d*x)/2]]/(4*d) + ArcTan[3*Tanh[(c + d*x)/2]]/(4*d) - ((
I/8)*Log[4 - 5*Cosh[c + d*x]])/d + ((I/8)*Log[4 + 5*Cosh[c + d*x]])/d
```

Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 50, normalized size of antiderivative = 0.94, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {3042, 3139, 1081, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{3+5i\sinh(c+dx)}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{3+5\sin(ic+idx)}dx$$

$$\Big\downarrow \text{3139}$$

$$-\frac{2i\int \frac{1}{-3\tanh^2\left(\frac{1}{2}(c+dx)\right)+10i\tanh\left(\frac{1}{2}(c+dx)\right)+3}d\left(i\tanh\left(\frac{1}{2}(c+dx)\right)\right)}{d}$$

$$\Big\downarrow \text{1081}$$

$$-\frac{6i\int \left(\frac{1}{8\left(3i\tanh\left(\frac{1}{2}(c+dx)\right)+1\right)}-\frac{1}{24\left(i\tanh\left(\frac{1}{2}(c+dx)\right)+3\right)}\right)d\left(i\tanh\left(\frac{1}{2}(c+dx)\right)\right)}{d}$$

$$\Big\downarrow \text{2009}$$

$$-\frac{6i\left(\frac{1}{24}\log\left(1+3i\tanh\left(\frac{1}{2}(c+dx)\right)\right)-\frac{1}{24}\log\left(3+i\tanh\left(\frac{1}{2}(c+dx)\right)\right)\right)}{d}$$

input
```
Int[(3 + (5*I)*Sinh[c + d*x])^(-1),x]
```

output
```
((-6*I)*(-1/24*Log[3 + I*Tanh[(c + d*x)/2]] + Log[1 + (3*I)*Tanh[(c + d*x)
/2]]/24))/d
```

Defintions of rubi rules used

rule 1081
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = Rt[b^2
- 4*a*c, 2]}, Simp[c   Int[ExpandIntegrand[1/((b/2 - q/2 + c*x)*(b/2 + q/2
+ c*x)), x], x], x]] /; FreeQ[{a, b, c}, x] && NiceSqrtQ[b^2 - 4*a*c]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3139
```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Simp[2*(e/d)   Subst[Int[1/(a + 2*b*e*x + a
*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[a^2 - b^2, 0]
```

Maple [A] (verified)

Time = 0.31 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.68

method	result	size
risch	$\frac{i\ln\left(e^{dx+c}+\frac{4}{5}-\frac{3i}{5}\right)}{4d}-\frac{i\ln\left(e^{dx+c}-\frac{4}{5}-\frac{3i}{5}\right)}{4d}$	36
derivativedivides	$\frac{-\frac{i\ln\left(3\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)-i\right)}{4}+\frac{i\ln\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)-3i\right)}{4}}{d}$	40
default	$\frac{-\frac{i\ln\left(3\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)-i\right)}{4}+\frac{i\ln\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)-3i\right)}{4}}{d}$	40
parallelrisch	$\frac{i\left(\ln\left(3\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)-9i\right)-\ln\left(3\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)-i\right)\right)}{4d}$	40

input
```
int(1/(3+5*I*sinh(d*x+c)),x,method=_RETURNVERBOSE)
```

output
```
1/4*I/d*ln(exp(d*x+c)+4/5-3/5*I)-1/4*I/d*ln(exp(d*x+c)-4/5-3/5*I)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.53

$$\int \frac{1}{3+5i\sinh(c+dx)}\,dx = \frac{i\,\log\left(e^{(dx+c)}-\frac{3}{5}i+\frac{4}{5}\right)-i\,\log\left(e^{(dx+c)}-\frac{3}{5}i-\frac{4}{5}\right)}{4\,d}$$

input
```
integrate(1/(3+5*I*sinh(d*x+c)),x, algorithm="fricas")
```

output
```
1/4*(I*log(e^(d*x + c) - 3/5*I + 4/5) - I*log(e^(d*x + c) - 3/5*I - 4/5))/
d
```

Sympy [A] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.58

$$\int \frac{1}{3+5i\sinh(c+dx)}\,dx = \frac{\operatorname{RootSum}\left(16z^2+1,\left(i\mapsto i\log\left(\frac{(-16ii-3i)e^{-c}}{5}+e^{dx}\right)\right)\right)}{d}$$

input
```
integrate(1/(3+5*I*sinh(d*x+c)),x)
```

output
```
RootSum(16*_z**2 + 1, Lambda(_i, _i*log((-16*_i*I - 3*I)*exp(-c)/5 + exp(d
*x))))/d
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.36

$$\int \frac{1}{3+5i\sinh(c+dx)}\,dx = \frac{\arctan\left(\frac{5}{4}i\,e^{(-dx-c)}-\frac{3}{4}\right)}{2\,d}$$

input
```
integrate(1/(3+5*I*sinh(d*x+c)),x, algorithm="maxima")
```

output
```
1/2*arctan(5/4*I*e^(-d*x - c) - 3/4)/d
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.60

$$\int \frac{1}{3+5i\sinh(c+dx)}\,dx = -\frac{-i\,\log\left(-(i-2)\,e^{(dx+c)}-2i+1\right)+i\,\log\left(-(2i-1)\,e^{(dx+c)}+i-2\right)}{4\,d}$$

input
```
integrate(1/(3+5*I*sinh(d*x+c)),x, algorithm="giac")
```

output
```
-1/4*(-I*log(-(I - 2)*e^(d*x + c) - 2*I + 1) + I*log(-(2*I - 1)*e^(d*x + c
) + I - 2))/d
```

Mupad [B] (verification not implemented)

Time = 0.34 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.74

$$\int \frac{1}{3+5i\sinh(c+dx)}\,dx = -\frac{\ln\left(-\frac{5}{2}+e^{d\,x}\,e^{c}\left(2-\frac{3}{2}\mathrm{i}\right)\right)\,1\mathrm{i}}{4\,d}+\frac{\ln\left(\frac{5}{2}+e^{d\,x}\,e^{c}\left(2+\frac{3}{2}\mathrm{i}\right)\right)\,1\mathrm{i}}{4\,d}$$

input
```
int(1/(sinh(c + d*x)*5i + 3),x)
```

output
```
(log(exp(d*x)*exp(c)*(2 + 3i/2) + 5/2)*1i)/(4*d) - (log(exp(d*x)*exp(c)*(2
 - 3i/2) - 5/2)*1i)/(4*d)
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.34

$$\int \frac{1}{3+5i\sinh(c+dx)}\,dx = \frac{atan\left(\frac{5e^{dx+c}i}{4}+\frac{3}{4}\right)}{2d}$$

input
```
int(1/(3+5*I*sinh(d*x+c)),x)
```

output
```
atan((5*e**(c + d*x)*i + 3)/4)/(2*d)
```

3.89 $\int \frac{1}{(3+5i\sinh(c+dx))^2}\,dx$

Optimal result

Integrand size = 14, antiderivative size = 82

$$\int \frac{1}{(3+5i\sinh(c+dx))^2}\,dx = \frac{3i\log\left(i-3\tanh\left(\frac{1}{2}(c+dx)\right)\right)}{64d} - \frac{3i\log\left(3i-\tanh\left(\frac{1}{2}(c+dx)\right)\right)}{64d} + \frac{5i\cosh(c+dx)}{16d(3+5i\sinh(c+dx))}$$

output
```
3/64*I*ln(I-3*tanh(1/2*d*x+1/2*c))/d-3/64*I*ln(3*I-tanh(1/2*d*x+1/2*c))/d+
5/16*I*cosh(d*x+c)/d/(3+5*I*sinh(d*x+c))
```

Mathematica [A] (verified)

Time = 0.36 (sec) , antiderivative size = 142, normalized size of antiderivative = 1.73

$$\int \frac{1}{(3+5i\sinh(c+dx))^2}\,dx = \frac{-9\left(2\arctan\left(3\coth\left(\frac{1}{2}(c+dx)\right)\right)+2\arctan\left(3\tanh\left(\frac{1}{2}(c+dx)\right)\right)-i\log(4-5\cosh(c+dx))+i\log(4-\right.}{384d}$$

input
```
Integrate[(3 + (5*I)*Sinh[c + d*x])^(-2),x]
```

output
```
(-9*(2*ArcTan[3*Coth[(c + d*x)/2]] + 2*ArcTan[3*Tanh[(c + d*x)/2]] - I*Log
[4 - 5*Cosh[c + d*x]] + I*Log[4 + 5*Cosh[c + d*x]]) + 40*((3*Cosh[(c + d*x
)/2] + I*Sinh[(c + d*x)/2])^(-1) + 3/(Cosh[(c + d*x)/2] + (3*I)*Sinh[(c +
d*x)/2]))*Sinh[(c + d*x)/2])/(384*d)
```

Rubi [A] (verified)

Time = 0.33 (sec) , antiderivative size = 82, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$, Rules used = {3042, 3143, 27, 3042, 3139, 1081, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(3+5i\sinh(c+dx))^2}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{(3+5\sin(ic+idx))^2}dx$$

$$\Big\downarrow \text{3143}$$

$$\frac{1}{16}\int -\frac{3}{5i\sinh(c+dx)+3}dx+\frac{5i\cosh(c+dx)}{16d(3+5i\sinh(c+dx))}$$

$$\Big\downarrow \text{27}$$

$$\frac{5i\cosh(c+dx)}{16d(3+5i\sinh(c+dx))}-\frac{3}{16}\int \frac{1}{5i\sinh(c+dx)+3}dx$$

$$\Big\downarrow \text{3042}$$

$$\frac{5i\cosh(c+dx)}{16d(3+5i\sinh(c+dx))}-\frac{3}{16}\int \frac{1}{5\sin(ic+idx)+3}dx$$

$$\Big\downarrow \text{3139}$$

$$\frac{3i\int \frac{1}{-3\tanh^2\left(\frac{1}{2}(c+dx)\right)+10i\tanh\left(\frac{1}{2}(c+dx)\right)+3}d\left(i\tanh\left(\frac{1}{2}(c+dx)\right)\right)}{8d}+\frac{5i\cosh(c+dx)}{16d(3+5i\sinh(c+dx))}$$

$$\Big\downarrow \text{1081}$$

$$\frac{9i\int\left(\frac{1}{8(3i\tanh(\frac{1}{2}(c+dx))+1)}-\frac{1}{24(i\tanh(\frac{1}{2}(c+dx))+3)}\right)d\left(i\tanh\left(\frac{1}{2}(c+dx)\right)\right)}{8d}+\frac{5i\cosh(c+dx)}{16d(3+5i\sinh(c+dx))}$$

$$\Big\downarrow \text{2009}$$

$$\frac{9i\left(\frac{1}{24}\log\left(1+3i\tanh\left(\frac{1}{2}(c+dx)\right)\right)-\frac{1}{24}\log\left(3+i\tanh\left(\frac{1}{2}(c+dx)\right)\right)\right)}{8d}+\frac{5i\cosh(c+dx)}{16d(3+5i\sinh(c+dx))}$$

input
```
Int[(3 + (5*I)*Sinh[c + d*x])^(-2),x]
```

output
```
(((9*I)/8)*(-1/24*Log[3 + I*Tanh[(c + d*x)/2]] + Log[1 + (3*I)*Tanh[(c + d
*x)/2]]/24))/d + (((5*I)/16)*Cosh[c + d*x])/(d*(3 + (5*I)*Sinh[c + d*x]))
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 1081
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = Rt[b^2
- 4*a*c, 2]}, Simp[c   Int[ExpandIntegrand[1/((b/2 - q/2 + c*x)*(b/2 + q/2
+ c*x)), x], x], x]] /; FreeQ[{a, b, c}, x] && NiceSqrtQ[b^2 - 4*a*c]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3139
```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Simp[2*(e/d)   Subst[Int[1/(a + 2*b*e*x + a
*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[a^2 - b^2, 0]
```

rule 3143
```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos
[c + d*x]*((a + b*Sin[c + d*x])^(n + 1)/(d*(n + 1)*(a^2 - b^2))), x] + Simp
[1/((n + 1)*(a^2 - b^2))   Int[(a + b*Sin[c + d*x])^(n + 1)*Simp[a*(n + 1)
- b*(n + 2)*Sin[c + d*x], x], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
 b^2, 0] && LtQ[n, -1] && IntegerQ[2*n]
```

Maple [A] (verified)

Time = 0.32 (sec) , antiderivative size = 74, normalized size of antiderivative = 0.90

method	result
derivativedivides	$\frac{\frac{3i\ln\left(3\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)-i\right)}{64}+\frac{5}{48\left(3\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)-i\right)}-\frac{3i\ln\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)-3i\right)}{64}+\frac{5}{16\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)-3i\right)}}{d}$
default	$\frac{\frac{3i\ln\left(3\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)-i\right)}{64}+\frac{5}{48\left(3\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)-i\right)}-\frac{3i\ln\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)-3i\right)}{64}+\frac{5}{16\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)-3i\right)}}{d}$
risch	$\frac{i\left(3\,\mathrm{e}^{dx+c}-5i\right)}{8d\left(5\,\mathrm{e}^{2dx+2c}-5-6i\mathrm{e}^{dx+c}\right)}-\frac{3i\ln\left(\mathrm{e}^{dx+c}+\frac{4}{5}-\frac{3i}{5}\right)}{64d}+\frac{3i\ln\left(\mathrm{e}^{dx+c}-\frac{4}{5}-\frac{3i}{5}\right)}{64d}$
parallelrisch	$\frac{(-9-15i\sinh(dx+c))\ln\left(3\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)-i\right)+15i\sinh(dx+c)\ln\left(3\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)-9i\right)+\ln(19683)-20\cosh(dx+c)+9}{64d(3i-5\sinh(dx+c))}$

input
```
int(1/(3+5*I*sinh(d*x+c))^2,x,method=_RETURNVERBOSE)
```

output
```
1/d*(3/64*I*ln(3*tanh(1/2*d*x+1/2*c)-I)+5/48/(3*tanh(1/2*d*x+1/2*c)-I)-3/6
4*I*ln(tanh(1/2*d*x+1/2*c)-3*I)+5/16/(tanh(1/2*d*x+1/2*c)-3*I))
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 103, normalized size of antiderivative = 1.26

$$\int \frac{1}{(3+5i\sinh(c+dx))^2}\,dx = -\frac{3\left(5i\,e^{(2\,dx+2\,c)}+6\,e^{(dx+c)}-5i\right)\log\left(e^{(dx+c)}-\frac{3}{5}i+\frac{4}{5}\right)+3\left(-5i\,e^{(2\,dx+2\,c)}-6\,e^{(dx+c)}+5i\right)\log\left(e^{(dx+c)}-\right.}{64\left(5\,de^{(2\,dx+2\,c)}-6i\,de^{(dx+c)}-5\,d\right)}$$

input
```
integrate(1/(3+5*I*sinh(d*x+c))^2,x, algorithm="fricas")
```

output
```
-1/64*(3*(5*I*e^(2*d*x + 2*c) + 6*e^(d*x + c) - 5*I)*log(e^(d*x + c) - 3/5
*I + 4/5) + 3*(-5*I*e^(2*d*x + 2*c) - 6*e^(d*x + c) + 5*I)*log(e^(d*x + c)
 - 3/5*I - 4/5) - 24*I*e^(d*x + c) - 40)/(5*d*e^(2*d*x + 2*c) - 6*I*d*e^(d
*x + c) - 5*d)
```

Sympy [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 75, normalized size of antiderivative = 0.91

$$\int \frac{1}{(3+5i\sinh(c+dx))^2}\,dx = \frac{3ie^ce^{dx}+5}{40de^{2c}e^{2dx}-48ide^ce^{dx}-40d} + \frac{\operatorname{RootSum}\left(4096z^2+9,\left(i\mapsto i\log\left(\frac{(256ii-9i)e^{-c}}{15}+e^{dx}\right)\right)\right)}{d}$$

input
```
integrate(1/(3+5*I*sinh(d*x+c))**2,x)
```

output
```
(3*I*exp(c)*exp(d*x) + 5)/(40*d*exp(2*c)*exp(2*d*x) - 48*I*d*exp(c)*exp(d*
x) - 40*d) + RootSum(4096*_z**2 + 9, Lambda(_i, _i*log((256*_i*I - 9*I)*ex
p(-c)/15 + exp(d*x))))/d
```

Maxima [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 79, normalized size of antiderivative = 0.96

$$\int \frac{1}{(3+5i\sinh(c+dx))^2}\,dx = \frac{3i\,\log\left(\frac{5\,e^{(-dx-c)}+3i-4}{5\,e^{(-dx-c)}+3i+4}\right)}{64\,d} + \frac{3i\,e^{(-dx-c)}-5}{-8\,d(-6i\,e^{(-dx-c)}-5\,e^{(-2\,dx-2\,c)}+5)}$$

input
```
integrate(1/(3+5*I*sinh(d*x+c))^2,x, algorithm="maxima")
```

output
```
3/64*I*log((5*e^(-d*x - c) + 3*I - 4)/(5*e^(-d*x - c) + 3*I + 4))/d + (3*I
*e^(-d*x - c) - 5)/(d*(48*I*e^(-d*x - c) + 40*e^(-2*d*x - 2*c) - 40))
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 67, normalized size of antiderivative = 0.82

$$\int \frac{1}{(3+5i\sinh(c+dx))^2}\,dx = -\frac{\frac{8\,(-3i\,e^{(dx+c)}-5)}{5\,e^{(2\,dx+2\,c)}-6i\,e^{(dx+c)}-5} + 3i\,\log\left(-(i-2)\,e^{(dx+c)}-2i+1\right) - 3i\,\log\left(-(2i-1)\,e^{(dx+c)}+i-2\right)}{64\,d}$$

input
```
integrate(1/(3+5*I*sinh(d*x+c))^2,x, algorithm="giac")
```

output
```
-1/64*(8*(-3*I*e^(d*x + c) - 5)/(5*e^(2*d*x + 2*c) - 6*I*e^(d*x + c) - 5)
+ 3*I*log(-(I - 2)*e^(d*x + c) - 2*I + 1) - 3*I*log(-(2*I - 1)*e^(d*x + c)
 + I - 2))/d
```

Mupad [B] (verification not implemented)

Time = 2.11 (sec) , antiderivative size = 106, normalized size of antiderivative = 1.29

$$\int \frac{1}{(3+5i\sinh(c+dx))^2}\,dx = -\frac{5}{8\,\left(5\,d-5\,d\,\mathrm{e}^{2\,c+2\,d\,x}+d\,\mathrm{e}^{c+d\,x}\,6\mathrm{i}\right)} - \frac{\ln\left(-\frac{15}{4}+\mathrm{e}^{d\,x}\,\mathrm{e}^{c}\,\left(-3-\frac{9}{4}\mathrm{i}\right)\right)\,3\mathrm{i}}{64\,d} + \frac{\ln\left(\frac{15}{4}+\mathrm{e}^{d\,x}\,\mathrm{e}^{c}\,\left(-3+\frac{9}{4}\mathrm{i}\right)\right)\,3\mathrm{i}}{64\,d} - \frac{\mathrm{e}^{c+d\,x}\,3\mathrm{i}}{8\,\left(5\,d-5\,d\,\mathrm{e}^{2\,c+2\,d\,x}+d\,\mathrm{e}^{c+d\,x}\,6\mathrm{i}\right)}$$

input
```
int(1/(sinh(c + d*x)*5i + 3)^2,x)
```

output
```
(log(15/4 - exp(d*x)*exp(c)*(3 - 9i/4))*3i)/(64*d) - (log(- exp(d*x)*exp(c
)*(3 + 9i/4) - 15/4)*3i)/(64*d) - 5/(8*(5*d + d*exp(c + d*x)*6i - 5*d*exp(
2*c + 2*d*x))) - (exp(c + d*x)*3i)/(8*(5*d + d*exp(c + d*x)*6i - 5*d*exp(2
*c + 2*d*x)))
```

Reduce [F]

$$\int \frac{1}{(3+5i\sinh(c+dx))^2}\,dx = -\left(\int \frac{1}{25\sinh(dx+c)^2-30\sinh(dx+c)\,i-9}dx\right)$$

input
```
int(1/(3+5*I*sinh(d*x+c))^2,x)
```

output
```
- int(1/(25*sinh(c + d*x)**2 - 30*sinh(c + d*x)*i - 9),x)
```

3.90 $\int \frac{1}{(3+5i\sinh(c+dx))^3}\,dx$

Optimal result

Integrand size = 14, antiderivative size = 111

$$\int \frac{1}{(3+5i\sinh(c+dx))^3}\,dx = -\frac{43i\log\left(i-3\tanh\left(\frac{1}{2}(c+dx)\right)\right)}{2048d} + \frac{43i\log\left(3i-\tanh\left(\frac{1}{2}(c+dx)\right)\right)}{2048d} + \frac{5i\cosh(c+dx)}{32d(3+5i\sinh(c+dx))^2} - \frac{45i\cosh(c+dx)}{512d(3+5i\sinh(c+dx))}$$

output
```
-43/2048*I*ln(I-3*tanh(1/2*d*x+1/2*c))/d+43/2048*I*ln(3*I-tanh(1/2*d*x+1/2
*c))/d+5/32*I*cosh(d*x+c)/d/(3+5*I*sinh(d*x+c))^2-45/512*I*cosh(d*x+c)/d/(
3+5*I*sinh(d*x+c))
```

Mathematica [A] (verified)

Time = 0.52 (sec) , antiderivative size = 204, normalized size of antiderivative = 1.84

$$\int \frac{1}{(3+5i\sinh(c+dx))^3}\,dx = \frac{86\arctan\left(3\coth\left(\frac{1}{2}(c+dx)\right)\right) + 86\arctan\left(3\tanh\left(\frac{1}{2}(c+dx)\right)\right) - 43i\log(4-5\cosh(c+dx)) + 43i\log(}{}$$

input
```
Integrate[(3 + (5*I)*Sinh[c + d*x])^(-3),x]
```

output
```
(86*ArcTan[3*Coth[(c + d*x)/2]] + 86*ArcTan[3*Tanh[(c + d*x)/2]] - (43*I)*
Log[4 - 5*Cosh[c + d*x]] + (43*I)*Log[4 + 5*Cosh[c + d*x]] - (80*I)/(3*Cos
h[(c + d*x)/2] + I*Sinh[(c + d*x)/2])^2 + (80*I)/(Cosh[(c + d*x)/2] + (3*I
)*Sinh[(c + d*x)/2])^2 + (-120/(3*Cosh[(c + d*x)/2] + I*Sinh[(c + d*x)/2])
 - 360/(Cosh[(c + d*x)/2] + (3*I)*Sinh[(c + d*x)/2]))*Sinh[(c + d*x)/2])/(
4096*d)
```

Rubi [A] (verified)

Time = 0.44 (sec) , antiderivative size = 116, normalized size of antiderivative = 1.05, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.714$, Rules used = {3042, 3143, 25, 3042, 3233, 27, 3042, 3139, 1081, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(3+5 i \sinh (c+d x))^3} \, dx$$

$$\Big\downarrow \ 3042$$

$$\int \frac{1}{(3+5 \sin (i c+i d x))^3}dx$$

$$\Big\downarrow \ 3143$$

$$\frac{1}{32} \int -\frac{6-5 i \sinh (c+d x)}{(5 i \sinh (c+d x)+3)^2}dx+\frac{5 i \cosh (c+d x)}{32 d (3+5 i \sinh (c+d x))^2}$$

$$\Big\downarrow \ 25$$

$$\frac{5 i \cosh (c+d x)}{32 d (3+5 i \sinh (c+d x))^2}-\frac{1}{32} \int \frac{6-5 i \sinh (c+d x)}{(5 i \sinh (c+d x)+3)^2}dx$$

$$\Big\downarrow \ 3042$$

$$\frac{5 i \cosh (c+d x)}{32 d (3+5 i \sinh (c+d x))^2}-\frac{1}{32} \int \frac{6-5 \sin (i c+i d x)}{(5 \sin (i c+i d x)+3)^2}dx$$

$$\Big\downarrow \ 3233$$

$$\frac{1}{32}\left(-\frac{1}{16}\int -\frac{43}{5i\sinh(c+dx)+3}dx-\frac{45i\cosh(c+dx)}{16d(3+5i\sinh(c+dx))}\right)+\frac{5i\cosh(c+dx)}{32d(3+5i\sinh(c+dx))^2}$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{32}\left(\frac{43}{16}\int \frac{1}{5i\sinh(c+dx)+3}dx-\frac{45i\cosh(c+dx)}{16d(3+5i\sinh(c+dx))}\right)+\frac{5i\cosh(c+dx)}{32d(3+5i\sinh(c+dx))^2}$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{32}\left(\frac{43}{16}\int \frac{1}{5\sin(ic+idx)+3}dx-\frac{45i\cosh(c+dx)}{16d(3+5i\sinh(c+dx))}\right)+\frac{5i\cosh(c+dx)}{32d(3+5i\sinh(c+dx))^2}$$

$$\Big\downarrow \text{3139}$$

$$\frac{1}{32}\left(-\frac{43i\int \frac{1}{-3\tanh^2\left(\frac{1}{2}(c+dx)\right)+10i\tanh\left(\frac{1}{2}(c+dx)\right)+3}d\left(i\tanh\left(\frac{1}{2}(c+dx)\right)\right)}{8d}-\frac{45i\cosh(c+dx)}{16d(3+5i\sinh(c+dx))}\right)+\frac{5i\cosh(c+dx)}{32d(3+5i\sinh(c+dx))^2}$$

$$\Big\downarrow \text{1081}$$

$$\frac{1}{32}\left(-\frac{129i\int \left(\frac{1}{8\left(3i\tanh\left(\frac{1}{2}(c+dx)\right)+1\right)}-\frac{1}{24\left(i\tanh\left(\frac{1}{2}(c+dx)\right)+3\right)}\right)d\left(i\tanh\left(\frac{1}{2}(c+dx)\right)\right)}{8d}-\frac{45i\cosh(c+dx)}{16d(3+5i\sinh(c+dx))}\right)+\frac{5i\cosh(c+dx)}{32d(3+5i\sinh(c+dx))^2}$$

$$\Big\downarrow \text{2009}$$

$$\frac{5i\cosh(c+dx)}{32d(3+5i\sinh(c+dx))^2}+\frac{1}{32}\left(-\frac{129i\left(\frac{1}{24}\log\left(1+3i\tanh\left(\frac{1}{2}(c+dx)\right)\right)-\frac{1}{24}\log\left(3+i\tanh\left(\frac{1}{2}(c+dx)\right)\right)\right)}{8d}-\frac{45i\cosh(c+dx)}{16d(3+5i\sinh(c+dx))}\right)$$

input
```
Int[(3 + (5*I)*Sinh[c + d*x])^(-3),x]
```

output
```
((((-129*I)/8)*(-1/24*Log[3 + I*Tanh[(c + d*x)/2]] + Log[1 + (3*I)*Tanh[(c
 + d*x)/2]]/24))/d - (((45*I)/16)*Cosh[c + d*x])/(d*(3 + (5*I)*Sinh[c + d*
x])))/32 + (((5*I)/32)*Cosh[c + d*x])/(d*(3 + (5*I)*Sinh[c + d*x])^2)
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 1081
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = Rt[b^2
- 4*a*c, 2]}, Simp[c   Int[ExpandIntegrand[1/((b/2 - q/2 + c*x)*(b/2 + q/2
+ c*x)), x], x], x]] /; FreeQ[{a, b, c}, x] && NiceSqrtQ[b^2 - 4*a*c]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3139
```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Simp[2*(e/d)   Subst[Int[1/(a + 2*b*e*x + a
*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[a^2 - b^2, 0]
```

rule 3143
```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos
[c + d*x]*((a + b*Sin[c + d*x])^(n + 1)/(d*(n + 1)*(a^2 - b^2))), x] + Simp
[1/((n + 1)*(a^2 - b^2))   Int[(a + b*Sin[c + d*x])^(n + 1)*Simp[a*(n + 1)
- b*(n + 2)*Sin[c + d*x], x], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
 b^2, 0] && LtQ[n, -1] && IntegerQ[2*n]
```

rule 3233
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Simp[(-(b*c - a*d))*Cos[e + f*x]*((a + b*Sin[e +
 f*x])^(m + 1)/(f*(m + 1)*(a^2 - b^2))), x] + Simp[1/((m + 1)*(a^2 - b^2))
  Int[(a + b*Sin[e + f*x])^(m + 1)*Simp[(a*c - b*d)*(m + 1) - (b*c - a*d)*(
m + 2)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c
- a*d, 0] && NeQ[a^2 - b^2, 0] && LtQ[m, -1] && IntegerQ[2*m]
```

Maple [A] (verified)

Time = 0.57 (sec) , antiderivative size = 100, normalized size of antiderivative = 0.90

method	result
risch	$-\frac{i\left(-387 i \mathrm{e}^{2 d x+2 c}+215 \,\mathrm{e}^{3 d x+3 c}+225 i-325 \,\mathrm{e}^{d x+c}\right)}{256 d \left(5 \,\mathrm{e}^{2 d x+2 c}-5-6 i \mathrm{e}^{d x+c}\right)^{2}}-\frac{43 i \ln\left(\mathrm{e}^{d x+c}-\frac{4}{5}-\frac{3 i}{5}\right)}{2048 d}+\frac{43 i \ln\left(\mathrm{e}^{d x+c}+\frac{4}{5}-\frac{3 i}{5}\right)}{2048 d}$
derivativedivides	$\frac{\frac{25 i}{128 \left(\tanh\left(\frac{d x}{2}+\frac{c}{2}\right)-3 i\right)^{2}}+\frac{43 i \ln\left(\tanh\left(\frac{d x}{2}+\frac{c}{2}\right)-3 i\right)}{2048}+\frac{15}{512 \left(\tanh\left(\frac{d x}{2}+\frac{c}{2}\right)-3 i\right)}-\frac{43 i \ln\left(3 \tanh\left(\frac{d x}{2}+\frac{c}{2}\right)-i\right)}{2048}-\frac{25 i}{1152 \left(3 \tanh\left(\frac{d x}{2}+\cdots\right.\right.}}{d}$
default	$\frac{\frac{25 i}{128 \left(\tanh\left(\frac{d x}{2}+\frac{c}{2}\right)-3 i\right)^{2}}+\frac{43 i \ln\left(\tanh\left(\frac{d x}{2}+\frac{c}{2}\right)-3 i\right)}{2048}+\frac{15}{512 \left(\tanh\left(\frac{d x}{2}+\frac{c}{2}\right)-3 i\right)}-\frac{43 i \ln\left(3 \tanh\left(\frac{d x}{2}+\frac{c}{2}\right)-i\right)}{2048}-\frac{25 i}{1152 \left(3 \tanh\left(\frac{d x}{2}+\cdots\right.\right.}}{d}$
parallelrisch	$9460-6966 \ln\left(3 \tanh\left(\frac{d x}{2}+\frac{c}{2}\right)-9 i\right)+6966 \ln\left(3 \tanh\left(\frac{d x}{2}+\frac{c}{2}\right)-i\right)+1935 \ln\left(\tanh\left(\frac{d x}{2}+\frac{c}{2}\right)-\frac{i}{3}\right)\left(5-5 \cosh(2 d x+2 c)+1\cdots\right.$

input
```
int(1/(3+5*I*sinh(d*x+c))^3,x,method=_RETURNVERBOSE)
```

output
```
-1/256*I*(-387*I*exp(2*d*x+2*c)+215*exp(3*d*x+3*c)+225*I-325*exp(d*x+c))/d
/(5*exp(2*d*x+2*c)-5-6*I*exp(d*x+c))^2-43/2048*I/d*ln(exp(d*x+c)-4/5-3/5*I
)+43/2048*I/d*ln(exp(d*x+c)+4/5-3/5*I)
```

Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 193 vs. 2(85) = 170.

Time = 0.11 (sec) , antiderivative size = 193, normalized size of antiderivative = 1.74

$$\int \frac{1}{(3+5i\sinh(c+dx))^3}\,dx =$$
$$-\frac{43\left(-25i\,e^{(4\,dx+4\,c)}-60\,e^{(3\,dx+3\,c)}+86i\,e^{(2\,dx+2\,c)}+60\,e^{(dx+c)}-25i\right)\log\left(e^{(dx+c)}-\frac{3}{5}i+\frac{4}{5}\right)+43\left(25i\,e^{(\cdots}\right.}{2048\left(25\,de^{(4\,dx+4\,c)}\cdots\right.}$$

input
```
integrate(1/(3+5*I*sinh(d*x+c))^3,x, algorithm="fricas")
```

output
```
-1/2048*(43*(-25*I*e^(4*d*x + 4*c) - 60*e^(3*d*x + 3*c) + 86*I*e^(2*d*x +
2*c) + 60*e^(d*x + c) - 25*I)*log(e^(d*x + c) - 3/5*I + 4/5) + 43*(25*I*e^
(4*d*x + 4*c) + 60*e^(3*d*x + 3*c) - 86*I*e^(2*d*x + 2*c) - 60*e^(d*x + c)
 + 25*I)*log(e^(d*x + c) - 3/5*I - 4/5) + 1720*I*e^(3*d*x + 3*c) + 3096*e^
(2*d*x + 2*c) - 2600*I*e^(d*x + c) - 1800)/(25*d*e^(4*d*x + 4*c) - 60*I*d*
e^(3*d*x + 3*c) - 86*d*e^(2*d*x + 2*c) + 60*I*d*e^(d*x + c) + 25*d)
```

Sympy [A] (verification not implemented)

Time = 0.35 (sec) , antiderivative size = 138, normalized size of antiderivative = 1.24

$$\int \frac{1}{(3+5i\sinh(c+dx))^3}\,dx = \frac{-215ie^{3c}e^{3dx}-387e^{2c}e^{2dx}+325ie^{c}e^{dx}+225}{6400de^{4c}e^{4dx}-15360ide^{3c}e^{3dx}-22016de^{2c}e^{2dx}+15360ide^{c}e^{dx}+6400d} + \frac{\operatorname{RootSum}\left(4194304z^2+1849,\left(i\mapsto i\log\left(\frac{(-8192ii-129i)e^{-c}}{215}+e^{dx}\right)\right)\right)}{d}$$

input
```
integrate(1/(3+5*I*sinh(d*x+c))**3,x)
```

output
```
(-215*I*exp(3*c)*exp(3*d*x) - 387*exp(2*c)*exp(2*d*x) + 325*I*exp(c)*exp(d
*x) + 225)/(6400*d*exp(4*c)*exp(4*d*x) - 15360*I*d*exp(3*c)*exp(3*d*x) - 2
2016*d*exp(2*c)*exp(2*d*x) + 15360*I*d*exp(c)*exp(d*x) + 6400*d) + RootSum
(4194304*_z**2 + 1849, Lambda(_i, _i*log((-8192*_i*I - 129*I)*exp(-c)/215
+ exp(d*x))))/d
```

Maxima [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 124, normalized size of antiderivative = 1.12

$$\int \frac{1}{(3+5i\sinh(c+dx))^3}\,dx = -\frac{43i\,\log\left(\frac{5\,e^{(-dx-c)}+3i-4}{5\,e^{(-dx-c)}+3i+4}\right)}{2048\,d} - \frac{-325i\,e^{(-dx-c)}-387\,e^{(-2\,dx-2\,c)}+215i\,e^{(-3\,dx-3\,c)}+225}{-256\,d(60i\,e^{(-dx-c)}+86\,e^{(-2\,dx-2\,c)}-60i\,e^{(-3\,dx-3\,c)}-25\,e^{(-4\,dx-4\,c)}-25)}$$

input
```
integrate(1/(3+5*I*sinh(d*x+c))^3,x, algorithm="maxima")
```

output
```
-43/2048*I*log((5*e^(-d*x - c) + 3*I - 4)/(5*e^(-d*x - c) + 3*I + 4))/d -
(-325*I*e^(-d*x - c) - 387*e^(-2*d*x - 2*c) + 215*I*e^(-3*d*x - 3*c) + 225
)/(d*(-15360*I*e^(-d*x - c) - 22016*e^(-2*d*x - 2*c) + 15360*I*e^(-3*d*x -
 3*c) + 6400*e^(-4*d*x - 4*c) + 6400))
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 89, normalized size of antiderivative = 0.80

$$\int \frac{1}{(3+5i\sinh(c+dx))^3}\,dx = -\frac{\frac{8\left(-215i\,e^{(3\,dx+3\,c)}-387\,e^{(2\,dx+2\,c)}+325i\,e^{(dx+c)}+225\right)}{\left(-5i\,e^{(2\,dx+2\,c)}-6\,e^{(dx+c)}+5i\right)^2} - 43i\,\log\left(-(i-2)\,e^{(dx+c)}-2i+1\right) + 43i\,\log\left(-(2i-1)\,e\right.}{2048\,d}$$

input
```
integrate(1/(3+5*I*sinh(d*x+c))^3,x, algorithm="giac")
```

output
```
-1/2048*(8*(-215*I*e^(3*d*x + 3*c) - 387*e^(2*d*x + 2*c) + 325*I*e^(d*x +
c) + 225)/(-5*I*e^(2*d*x + 2*c) - 6*e^(d*x + c) + 5*I)^2 - 43*I*log(-(I -
2)*e^(d*x + c) - 2*I + 1) + 43*I*log(-(2*I - 1)*e^(d*x + c) + I - 2))/d
```

Mupad [B] (verification not implemented)

Time = 2.17 (sec) , antiderivative size = 147, normalized size of antiderivative = 1.32

$$\int \frac{1}{(3+5i\sinh(c+dx))^3}\,dx = \frac{\frac{129}{6400\,d}+\frac{e^{c+d\,x}\,43\mathrm{i}}{1280\,d}}{1-e^{2\,c+2\,d\,x}+\frac{e^{c+d\,x}\,6\mathrm{i}}{5}} - \frac{\ln\left(-\frac{215}{4}+e^{c+d\,x}\left(43-\frac{129}{4}\mathrm{i}\right)\right)\,43\mathrm{i}}{2048\,d} + \frac{\ln\left(\frac{215}{4}+e^{c+d\,x}\left(43+\frac{129}{4}\mathrm{i}\right)\right)\,43\mathrm{i}}{2048\,d} - \frac{-\frac{3}{200\,d}+\frac{e^{c+d\,x}\,7\mathrm{i}}{1000\,d}}{e^{4\,c+4\,d\,x}-\frac{86\,e^{2\,c+2\,d\,x}}{25}+1+\frac{e^{c+d\,x}\,12\mathrm{i}}{5}-\frac{e^{3\,c+3\,d\,x}\,12\mathrm{i}}{5}}$$

input
```
int(1/(sinh(c + d*x)*5i + 3)^3,x)
```

output
```
((exp(c + d*x)*43i)/(1280*d) + 129/(6400*d))/((exp(c + d*x)*6i)/5 - exp(2*
c + 2*d*x) + 1) - (log(exp(c + d*x)*(43 - 129i/4) - 215/4)*43i)/(2048*d) +
 (log(exp(c + d*x)*(43 + 129i/4) + 215/4)*43i)/(2048*d) - ((exp(c + d*x)*7
i)/(1000*d) - 3/(200*d))/((exp(c + d*x)*12i)/5 - (86*exp(2*c + 2*d*x))/25
- (exp(3*c + 3*d*x)*12i)/5 + exp(4*c + 4*d*x) + 1)
```

Reduce [F]

$$\int \frac{1}{(3+5i\sinh(c+dx))^3}\,dx$$
$$= -\left(\int \frac{1}{125\sinh(dx+c)^3 i + 225\sinh(dx+c)^2 - 135\sinh(dx+c)\, i - 27}dx\right)$$

input
```
int(1/(3+5*I*sinh(d*x+c))^3,x)
```

output
```
 - int(1/(125*sinh(c + d*x)**3*i + 225*sinh(c + d*x)**2 - 135*sinh(c + d*x
)*i - 27),x)
```

3.91 $\int \frac{1}{(3+5i\sinh(c+dx))^4}\,dx$

Optimal result

Integrand size = 14, antiderivative size = 140

$$\int \frac{1}{(3+5i\sinh(c+dx))^4}\,dx = \frac{279i\log\left(i-3\tanh\left(\frac{1}{2}(c+dx)\right)\right)}{32768d} - \frac{279i\log\left(3i-\tanh\left(\frac{1}{2}(c+dx)\right)\right)}{32768d} + \frac{5i\cosh(c+dx)}{48d(3+5i\sinh(c+dx))^3} - \frac{25i\cosh(c+dx)}{512d(3+5i\sinh(c+dx))^2} + \frac{995i\cosh(c+dx)}{24576d(3+5i\sinh(c+dx))}$$

output
```
279/32768*I*ln(I-3*tanh(1/2*d*x+1/2*c))/d-279/32768*I*ln(3*I-tanh(1/2*d*x+
1/2*c))/d+5/48*I*cosh(d*x+c)/d/(3+5*I*sinh(d*x+c))^3-25/512*I*cosh(d*x+c)/
d/(3+5*I*sinh(d*x+c))^2+995/24576*I*cosh(d*x+c)/d/(3+5*I*sinh(d*x+c))
```

Mathematica [A] (verified)

Time = 0.53 (sec) , antiderivative size = 265, normalized size of antiderivative = 1.89

$$\int \frac{1}{(3+5i\sinh(c+dx))^4}\,dx$$
$$= \frac{-5022\arctan\left(3\coth\left(\frac{1}{2}(c+dx)\right)\right)-5022\arctan\left(3\tanh\left(\frac{1}{2}(c+dx)\right)\right)+2511i\log(4-5\cosh(c+dx))}{}$$

input
```
Integrate[(3 + (5*I)*Sinh[c + d*x])^(-4),x]
```

output
```
(-5022*ArcTan[3*Coth[(c + d*x)/2]] - 5022*ArcTan[3*Tanh[(c + d*x)/2]] + (2
511*I)*Log[4 - 5*Cosh[c + d*x]] - (2511*I)*Log[4 + 5*Cosh[c + d*x]] + (464
0*I)/(3*Cosh[(c + d*x)/2] + I*Sinh[(c + d*x)/2])^2 - (1440*I)/(Cosh[(c + d
*x)/2] + (3*I)*Sinh[(c + d*x)/2])^2 + 40*(80/(3*Cosh[(c + d*x)/2] + I*Sinh
[(c + d*x)/2])^3 + 199/(3*Cosh[(c + d*x)/2] + I*Sinh[(c + d*x)/2]) + 240/(
Cosh[(c + d*x)/2] + (3*I)*Sinh[(c + d*x)/2])^3 + 597/(Cosh[(c + d*x)/2] +
(3*I)*Sinh[(c + d*x)/2]))*Sinh[(c + d*x)/2])/(589824*d)
```

Rubi [A] (verified)

Time = 0.59 (sec) , antiderivative size = 150, normalized size of antiderivative = 1.07, number of steps used = 14, number of rules used = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.929$, Rules used = {3042, 3143, 25, 3042, 3233, 25, 3042, 3233, 27, 3042, 3139, 1081, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(3+5i\sinh(c+dx))^4}dx$$
$$\Big\downarrow \; 3042$$
$$\int \frac{1}{(3+5\sin(ic+idx))^4}dx$$
$$\Big\downarrow \; 3143$$

$$\frac{1}{48}\int -\frac{9-10i\sinh(c+dx)}{(5i\sinh(c+dx)+3)^3}dx+\frac{5i\cosh(c+dx)}{48d(3+5i\sinh(c+dx))^3}$$

$\downarrow$ 25

$$\frac{5i\cosh(c+dx)}{48d(3+5i\sinh(c+dx))^3}-\frac{1}{48}\int \frac{9-10i\sinh(c+dx)}{(5i\sinh(c+dx)+3)^3}dx$$

$\downarrow$ 3042

$$\frac{5i\cosh(c+dx)}{48d(3+5i\sinh(c+dx))^3}-\frac{1}{48}\int \frac{9-10\sin(ic+idx)}{(5\sin(ic+idx)+3)^3}dx$$

$\downarrow$ 3233

$$\frac{1}{48}\left(-\frac{1}{32}\int -\frac{154-75i\sinh(c+dx)}{(5i\sinh(c+dx)+3)^2}dx-\frac{75i\cosh(c+dx)}{32d(3+5i\sinh(c+dx))^2}\right)+\frac{5i\cosh(c+dx)}{48d(3+5i\sinh(c+dx))^3}$$

$\downarrow$ 25

$$\frac{1}{48}\left(\frac{1}{32}\int \frac{154-75i\sinh(c+dx)}{(5i\sinh(c+dx)+3)^2}dx-\frac{75i\cosh(c+dx)}{32d(3+5i\sinh(c+dx))^2}\right)+\frac{5i\cosh(c+dx)}{48d(3+5i\sinh(c+dx))^3}$$

$\downarrow$ 3042

$$\frac{1}{48}\left(\frac{1}{32}\int \frac{154-75\sin(ic+idx)}{(5\sin(ic+idx)+3)^2}dx-\frac{75i\cosh(c+dx)}{32d(3+5i\sinh(c+dx))^2}\right)+\frac{5i\cosh(c+dx)}{48d(3+5i\sinh(c+dx))^3}$$

$\downarrow$ 3233

$$\frac{1}{48}\left(\frac{1}{32}\left(\frac{1}{16}\int -\frac{837}{5i\sinh(c+dx)+3}dx+\frac{995i\cosh(c+dx)}{16d(3+5i\sinh(c+dx))}\right)-\frac{75i\cosh(c+dx)}{32d(3+5i\sinh(c+dx))^2}\right)+\frac{5i\cosh(c+dx)}{48d(3+5i\sinh(c+dx))^3}$$

$\downarrow$ 27

$$\frac{1}{48}\left(\frac{1}{32}\left(\frac{995i\cosh(c+dx)}{16d(3+5i\sinh(c+dx))}-\frac{837}{16}\int \frac{1}{5i\sinh(c+dx)+3}dx\right)-\frac{75i\cosh(c+dx)}{32d(3+5i\sinh(c+dx))^2}\right)+\frac{5i\cosh(c+dx)}{48d(3+5i\sinh(c+dx))^3}$$

$\downarrow$ 3042

$$\frac{1}{48}\left(\frac{1}{32}\left(\frac{995i\cosh(c+dx)}{16d(3+5i\sinh(c+dx))}-\frac{837}{16}\int\frac{1}{5\sin(ic+idx)+3}dx\right)-\frac{75i\cosh(c+dx)}{32d(3+5i\sinh(c+dx))^2}\right)+\frac{5i\cosh(c+dx)}{48d(3+5i\sinh(c+dx))^3}$$

$$\Big\downarrow \text{3139}$$

$$\frac{1}{48}\left(\frac{1}{32}\left(\frac{837i\int\frac{1}{-3\tanh^2\left(\frac{1}{2}(c+dx)\right)+10i\tanh\left(\frac{1}{2}(c+dx)\right)+3}d\left(i\tanh\left(\frac{1}{2}(c+dx)\right)\right)}{8d}+\frac{995i\cosh(c+dx)}{16d(3+5i\sinh(c+dx))}\right)-\frac{7}{32d(3}\right.$$
$$\frac{5i\cosh(c+dx)}{48d(3+5i\sinh(c+dx))^3}$$

$$\Big\downarrow \text{1081}$$

$$\frac{1}{48}\left(\frac{1}{32}\left(\frac{2511i\int\left(\frac{1}{8\left(3i\tanh\left(\frac{1}{2}(c+dx)\right)+1\right)}-\frac{1}{24\left(i\tanh\left(\frac{1}{2}(c+dx)\right)+3\right)}\right)d\left(i\tanh\left(\frac{1}{2}(c+dx)\right)\right)}{8d}+\frac{995i\cosh(c+dx)}{16d(3+5i\sinh(c+dx))}\right.\right.$$
$$\frac{5i\cosh(c+dx)}{48d(3+5i\sinh(c+dx))^3}$$

$$\Big\downarrow \text{2009}$$

$$\frac{5i\cosh(c+dx)}{48d(3+5i\sinh(c+dx))^3}+$$
$$\frac{1}{48}\left(\frac{1}{32}\left(\frac{2511i\left(\frac{1}{24}\log\left(1+3i\tanh\left(\frac{1}{2}(c+dx)\right)\right)-\frac{1}{24}\log\left(3+i\tanh\left(\frac{1}{2}(c+dx)\right)\right)\right)}{8d}+\frac{995i\cosh(c+dx)}{16d(3+5i\sinh(c+dx))}\right)\right.$$

input
```
Int[(3 + (5*I)*Sinh[c + d*x])^(-4),x]
```

output
```
(((((2511*I)/8)*(-1/24*Log[3 + I*Tanh[(c + d*x)/2]] + Log[1 + (3*I)*Tanh[(
c + d*x)/2]]/24))/d + (((995*I)/16)*Cosh[c + d*x])/(d*(3 + (5*I)*Sinh[c +
d*x])))/32 - (((75*I)/32)*Cosh[c + d*x])/(d*(3 + (5*I)*Sinh[c + d*x])^2))/
48 + (((5*I)/48)*Cosh[c + d*x])/(d*(3 + (5*I)*Sinh[c + d*x])^3)
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 1081
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = Rt[b^2
- 4*a*c, 2]}, Simp[c   Int[ExpandIntegrand[1/((b/2 - q/2 + c*x)*(b/2 + q/2
+ c*x)), x], x], x]] /; FreeQ[{a, b, c}, x] && NiceSqrtQ[b^2 - 4*a*c]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3139
```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Simp[2*(e/d)   Subst[Int[1/(a + 2*b*e*x + a
*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[a^2 - b^2, 0]
```

rule 3143
```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos
[c + d*x]*((a + b*Sin[c + d*x])^(n + 1)/(d*(n + 1)*(a^2 - b^2))), x] + Simp
[1/((n + 1)*(a^2 - b^2))   Int[(a + b*Sin[c + d*x])^(n + 1)*Simp[a*(n + 1)
- b*(n + 2)*Sin[c + d*x], x], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
 b^2, 0] && LtQ[n, -1] && IntegerQ[2*n]
```

rule 3233
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Simp[(-(b*c - a*d))*Cos[e + f*x]*((a + b*Sin[e +
 f*x])^(m + 1)/(f*(m + 1)*(a^2 - b^2))), x] + Simp[1/((m + 1)*(a^2 - b^2))
  Int[(a + b*Sin[e + f*x])^(m + 1)*Simp[(a*c - b*d)*(m + 1) - (b*c - a*d)*(
m + 2)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c
- a*d, 0] && NeQ[a^2 - b^2, 0] && LtQ[m, -1] && IntegerQ[2*m]
```

Maple [A] (verified)

Time = 0.68 (sec) , antiderivative size = 123, normalized size of antiderivative = 0.88

method	result
risch	$\frac{i(-62775i e^{4dx+4c}+20925\,e^{5dx+5c}+119310i e^{2dx+2c}-111042\,e^{3dx+3c}-24875i+68625\,e^{dx+c})}{12288d(5\,e^{2dx+2c}-5-6i e^{dx+c})^3}-\frac{279i\ln(e^{dx+c}+\frac{4}{5}-}{32768d}$
derivativedivides	$\frac{-\frac{279i\ln(\tanh(\frac{dx}{2}+\frac{c}{2})-3i)}{32768}+\frac{75i}{1024(\tanh(\frac{dx}{2}+\frac{c}{2})-3i)^2}-\frac{125}{768(\tanh(\frac{dx}{2}+\frac{c}{2})-3i)^3}+\frac{345}{8192(\tanh(\frac{dx}{2}+\frac{c}{2})-3i)}+\frac{275}{27648(3\tanh(}}{d}$
default	$\frac{-\frac{279i\ln(\tanh(\frac{dx}{2}+\frac{c}{2})-3i)}{32768}+\frac{75i}{1024(\tanh(\frac{dx}{2}+\frac{c}{2})-3i)^2}-\frac{125}{768(\tanh(\frac{dx}{2}+\frac{c}{2})-3i)^3}+\frac{345}{8192(\tanh(\frac{dx}{2}+\frac{c}{2})-3i)}+\frac{275}{27648(3\tanh(}}{d}$
parallelrisch	$\frac{28968900i\sinh(dx+c)-2824875i\ln(3\tanh(\frac{dx}{2}+\frac{c}{2})-i)\sinh(3dx+3c)-3957500i\sinh(3dx+3c)+5151600i\sinh(2dx+}{}$

input
```
int(1/(3+5*I*sinh(d*x+c))^4,x,method=_RETURNVERBOSE)
```

output
```
1/12288*I*(-62775*I*exp(4*d*x+4*c)+20925*exp(5*d*x+5*c)+119310*I*exp(2*d*x
+2*c)-111042*exp(3*d*x+3*c)-24875*I+68625*exp(d*x+c))/d/(5*exp(2*d*x+2*c)-
5-6*I*exp(d*x+c))^3-279/32768*I/d*ln(exp(d*x+c)+4/5-3/5*I)+279/32768*I/d*l
n(exp(d*x+c)-4/5-3/5*I)
```

Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 283 vs. 2(108) = 216.

Time = 0.10 (sec) , antiderivative size = 283, normalized size of antiderivative = 2.02

$$\int \frac{1}{(3+5i\sinh(c+dx))^4}\,dx = -\frac{837\,(125i\,e^{(6\,dx+6\,c)}+450\,e^{(5\,dx+5\,c)}-915i\,e^{(4\,dx+4\,c)}-1116\,e^{(3\,dx+3\,c)}+915i\,e^{(2\,dx+2\,c)}+450\,e^{(dx+c)}-1}{}$$

input
```
integrate(1/(3+5*I*sinh(d*x+c))^4,x, algorithm="fricas")
```

output
```
-1/98304*(837*(125*I*e^(6*d*x + 6*c) + 450*e^(5*d*x + 5*c) - 915*I*e^(4*d*
x + 4*c) - 1116*e^(3*d*x + 3*c) + 915*I*e^(2*d*x + 2*c) + 450*e^(d*x + c)
- 125*I)*log(e^(d*x + c) - 3/5*I + 4/5) + 837*(-125*I*e^(6*d*x + 6*c) - 45
0*e^(5*d*x + 5*c) + 915*I*e^(4*d*x + 4*c) + 1116*e^(3*d*x + 3*c) - 915*I*e
^(2*d*x + 2*c) - 450*e^(d*x + c) + 125*I)*log(e^(d*x + c) - 3/5*I - 4/5) -
 167400*I*e^(5*d*x + 5*c) - 502200*e^(4*d*x + 4*c) + 888336*I*e^(3*d*x + 3
*c) + 954480*e^(2*d*x + 2*c) - 549000*I*e^(d*x + c) - 199000)/(125*d*e^(6*
d*x + 6*c) - 450*I*d*e^(5*d*x + 5*c) - 915*d*e^(4*d*x + 4*c) + 1116*I*d*e^
(3*d*x + 3*c) + 915*d*e^(2*d*x + 2*c) - 450*I*d*e^(d*x + c) - 125*d)
```

Sympy [A] (verification not implemented)

Time = 0.41 (sec) , antiderivative size = 197, normalized size of antiderivative = 1.41

$$\int \frac{1}{(3+5i\sinh(c+dx))^4}\,dx = \frac{20925ie^{5c}e^{5dx}+62775e^{4c}e^{4dx}-111042ie^{3c}e^{3dx}-119310e^{2c}e^{2dx}+68625ie^{c}e^{dx}+24}{1536000de^{6c}e^{6dx}-5529600ide^{5c}e^{5dx}-11243520de^{4c}e^{4dx}+13713408ide^{3c}e^{3dx}+11243520de^{2c}e^{2dx}-552} + \frac{\operatorname{RootSum}\left(1073741824z^2+77841, \left(i\mapsto i\log\left(\frac{(131072ii-837i)e^{-c}}{1395}+e^{dx}\right)\right)\right)}{d}$$

input
```
integrate(1/(3+5*I*sinh(d*x+c))**4,x)
```

output
```
(20925*I*exp(5*c)*exp(5*d*x) + 62775*exp(4*c)*exp(4*d*x) - 111042*I*exp(3*
c)*exp(3*d*x) - 119310*exp(2*c)*exp(2*d*x) + 68625*I*exp(c)*exp(d*x) + 248
75)/(1536000*d*exp(6*c)*exp(6*d*x) - 5529600*I*d*exp(5*c)*exp(5*d*x) - 112
43520*d*exp(4*c)*exp(4*d*x) + 13713408*I*d*exp(3*c)*exp(3*d*x) + 11243520*
d*exp(2*c)*exp(2*d*x) - 5529600*I*d*exp(c)*exp(d*x) - 1536000*d) + RootSum
(1073741824*_z**2 + 77841, Lambda(_i, _i*log((131072*_i*I - 837*I)*exp(-c)
/1395 + exp(d*x))))/d
```

Maxima [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 167, normalized size of antiderivative = 1.19

$$\int \frac{1}{(3+5i\sinh(c+dx))^4}\,dx = \frac{279i\,\log\left(\frac{5\,e^{(-dx-c)}+3i-4}{5\,e^{(-dx-c)}+3i+4}\right)}{32768\,d} + \frac{68625i\,e^{(-dx-c)}+119310\,e^{(-2\,dx-2\,c)}-111042i\,e^{(-3\,dx-3\,c)}-62775\,e^{(-4\,dx-4\,c)}+20925i\,e^{(-5\,dx-5}}{-12288\,d(-450i\,e^{(-dx-c)}-915\,e^{(-2\,dx-2\,c)}+1116i\,e^{(-3\,dx-3\,c)}+915\,e^{(-4\,dx-4\,c)}-450i\,e^{(-5\,dx-5\,c)}-125}$$

input
```
integrate(1/(3+5*I*sinh(d*x+c))^4,x, algorithm="maxima")
```

output
```
279/32768*I*log((5*e^(-d*x - c) + 3*I - 4)/(5*e^(-d*x - c) + 3*I + 4))/d +
 (68625*I*e^(-d*x - c) + 119310*e^(-2*d*x - 2*c) - 111042*I*e^(-3*d*x - 3*
c) - 62775*e^(-4*d*x - 4*c) + 20925*I*e^(-5*d*x - 5*c) - 24875)/(d*(552960
0*I*e^(-d*x - c) + 11243520*e^(-2*d*x - 2*c) - 13713408*I*e^(-3*d*x - 3*c)
 - 11243520*e^(-4*d*x - 4*c) + 5529600*I*e^(-5*d*x - 5*c) + 1536000*e^(-6*
d*x - 6*c) - 1536000))
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 111, normalized size of antiderivative = 0.79

$$\int \frac{1}{(3+5i\sinh(c+dx))^4}\,dx = \frac{\frac{8\,(20925i\,e^{(5\,dx+5\,c)}+62775\,e^{(4\,dx+4\,c)}-111042i\,e^{(3\,dx+3\,c)}-119310\,e^{(2\,dx+2\,c)}+68625i\,e^{(dx+c)}+24875)}{(5\,e^{(2\,dx+2\,c)}-6i\,e^{(dx+c)}-5)^3}-837i\,\log\left(-(i-2)\,e^{(dx+c}\right.}{98304\,d}$$

input
```
integrate(1/(3+5*I*sinh(d*x+c))^4,x, algorithm="giac")
```

output
```
1/98304*(8*(20925*I*e^(5*d*x + 5*c) + 62775*e^(4*d*x + 4*c) - 111042*I*e^(
3*d*x + 3*c) - 119310*e^(2*d*x + 2*c) + 68625*I*e^(d*x + c) + 24875)/(5*e^
(2*d*x + 2*c) - 6*I*e^(d*x + c) - 5)^3 - 837*I*log(-(I - 2)*e^(d*x + c) -
2*I + 1) + 837*I*log(-(2*I - 1)*e^(d*x + c) + I - 2))/d
```

Mupad [B] (verification not implemented)

Time = 2.47 (sec) , antiderivative size = 237, normalized size of antiderivative = 1.69

$$\int \frac{1}{(3+5i\sinh(c+dx))^4}\,dx$$
$$= -\frac{\frac{837}{102400\,d}+\frac{\mathrm{e}^{c+d\,x}\,279\mathrm{i}}{20480\,d}}{1-\mathrm{e}^{2\,c+2\,d\,x}+\frac{\mathrm{e}^{c+d\,x}\,6\mathrm{i}}{5}}$$
$$+\frac{\frac{7}{3750\,d}+\frac{\mathrm{e}^{c+d\,x}\,39\mathrm{i}}{6250\,d}}{\frac{183\,\mathrm{e}^{4\,c+4\,d\,x}}{25}-\frac{183\,\mathrm{e}^{2\,c+2\,d\,x}}{25}-\mathrm{e}^{6\,c+6\,d\,x}+1+\frac{\mathrm{e}^{c+d\,x}\,18\mathrm{i}}{5}-\frac{\mathrm{e}^{3\,c+3\,d\,x}\,1116\mathrm{i}}{125}+\frac{\mathrm{e}^{5\,c+5\,d\,x}\,18\mathrm{i}}{5}}$$
$$-\frac{\ln\left(-\frac{1395}{4}+\mathrm{e}^{c+d\,x}\left(-279-\frac{837}{4}\mathrm{i}\right)\right)\,279\mathrm{i}}{32768\,d}+\frac{\ln\left(\frac{1395}{4}+\mathrm{e}^{c+d\,x}\left(-279+\frac{837}{4}\mathrm{i}\right)\right)\,279\mathrm{i}}{32768\,d}$$
$$-\frac{\frac{791}{80000\,d}+\frac{\mathrm{e}^{c+d\,x}\,93\mathrm{i}}{16000\,d}}{\mathrm{e}^{4\,c+4\,d\,x}-\frac{86\,\mathrm{e}^{2\,c+2\,d\,x}}{25}+1+\frac{\mathrm{e}^{c+d\,x}\,12\mathrm{i}}{5}-\frac{\mathrm{e}^{3\,c+3\,d\,x}\,12\mathrm{i}}{5}}$$

input
```
int(1/(sinh(c + d*x)*5i + 3)^4,x)
```

output
```
((exp(c + d*x)*39i)/(6250*d) + 7/(3750*d))/((exp(c + d*x)*18i)/5 - (183*ex
p(2*c + 2*d*x))/25 - (exp(3*c + 3*d*x)*1116i)/125 + (183*exp(4*c + 4*d*x))
/25 + (exp(5*c + 5*d*x)*18i)/5 - exp(6*c + 6*d*x) + 1) - ((exp(c + d*x)*27
9i)/(20480*d) + 837/(102400*d))/((exp(c + d*x)*6i)/5 - exp(2*c + 2*d*x) +
1) - (log(- exp(c + d*x)*(279 + 837i/4) - 1395/4)*279i)/(32768*d) + (log(1
395/4 - exp(c + d*x)*(279 - 837i/4))*279i)/(32768*d) - ((exp(c + d*x)*93i)
/(16000*d) + 791/(80000*d))/((exp(c + d*x)*12i)/5 - (86*exp(2*c + 2*d*x))/
25 - (exp(3*c + 3*d*x)*12i)/5 + exp(4*c + 4*d*x) + 1)
```

Reduce [F]

$$\int \frac{1}{(3+5i\sinh(c+dx))^4}\,dx$$
$$= \int \frac{1}{625\sinh(dx+c)^4-1500\sinh(dx+c)^3 i-1350\sinh(dx+c)^2+540\sinh(dx+c)\,i+81}\,dx$$

input
```
int(1/(3+5*I*sinh(d*x+c))^4,x)
```

output
```
int(1/(625*sinh(c + d*x)**4 - 1500*sinh(c + d*x)**3*i - 1350*sinh(c + d*x)
**2 + 540*sinh(c + d*x)*i + 81),x)
```

3.92 $\int \frac{1}{5+3i\sinh(c+dx)}\,dx$

Optimal result

Integrand size = 14, antiderivative size = 37

$$\int \frac{1}{5+3i\sinh(c+dx)}\,dx = \frac{x}{4} - \frac{i\arctan\left(\frac{\cosh(c+dx)}{3+i\sinh(c+dx)}\right)}{2d}$$

output
```
1/4*x-1/2*I*arctan(cosh(d*x+c)/(3+I*sinh(d*x+c)))/d
```

Mathematica [B] (verified)

Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 171 vs. 2(37) = 74.

Time = 0.12 (sec) , antiderivative size = 171, normalized size of antiderivative = 4.62

$$\begin{aligned}\int \frac{1}{5+3i\sinh(c+dx)}\,dx &= -\frac{i\arctan\left(\frac{2\cosh\left(\frac{1}{2}(c+dx)\right)-\sinh\left(\frac{1}{2}(c+dx)\right)}{\cosh\left(\frac{1}{2}(c+dx)\right)-2\sinh\left(\frac{1}{2}(c+dx)\right)}\right)}{4d} \\ &+ \frac{i\arctan\left(\frac{\cosh\left(\frac{1}{2}(c+dx)\right)+2\sinh\left(\frac{1}{2}(c+dx)\right)}{2\cosh\left(\frac{1}{2}(c+dx)\right)+\sinh\left(\frac{1}{2}(c+dx)\right)}\right)}{4d} \\ &- \frac{\log(5\cosh(c+dx)-4\sinh(c+dx))}{8d} \\ &+ \frac{\log(5\cosh(c+dx)+4\sinh(c+dx))}{8d}\end{aligned}$$

input
```
Integrate[(5 + (3*I)*Sinh[c + d*x])^(-1),x]
```

output
```
((-1/4*I)*ArcTan[(2*Cosh[(c + d*x)/2] - Sinh[(c + d*x)/2])/(Cosh[(c + d*x)
/2] - 2*Sinh[(c + d*x)/2])])/d + ((I/4)*ArcTan[(Cosh[(c + d*x)/2] + 2*Sinh
[(c + d*x)/2])/(2*Cosh[(c + d*x)/2] + Sinh[(c + d*x)/2])])/d - Log[5*Cosh[
c + d*x] - 4*Sinh[c + d*x]]/(8*d) + Log[5*Cosh[c + d*x] + 4*Sinh[c + d*x]]
/(8*d)
```

Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {3042, 3136}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{5+3i\sinh(c+dx)}\,dx$$
$$\Big\downarrow \text{3042}$$
$$\int \frac{1}{5+3\sin(ic+idx)}dx$$
$$\Big\downarrow \text{3136}$$
$$\frac{x}{4}-\frac{i\arctan\left(\frac{\cosh(c+dx)}{3+i\sinh(c+dx)}\right)}{2d}$$

input
```
Int[(5 + (3*I)*Sinh[c + d*x])^(-1),x]
```

output
```
x/4 - ((I/2)*ArcTan[Cosh[c + d*x]/(3 + I*Sinh[c + d*x])])/d
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3136
```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{q = Rt[
a^2 - b^2, 2]}, Simp[x/q, x] + Simp[(2/(d*q))*ArcTan[b*(Cos[c + d*x]/(a + q
 + b*Sin[c + d*x]))], x]] /; FreeQ[{a, b, c, d}, x] && GtQ[a^2 - b^2, 0] &&
 PosQ[a]
```

Maple [A] (verified)

Time = 0.28 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.86

method	result	size
risch	$\frac{\ln\left(-\frac{i}{3}+{\rm e}^{d x +c}\right)}{4 d}-\frac{\ln\left({\rm e}^{d x +c}-3 i\right)}{4 d}$	32
parallelrisch	$\frac{\ln\left(5 \tanh\left(\frac{d x}{2}+\frac{c}{2}\right)+4-3 i\right)-\ln\left(5 \tanh\left(\frac{d x}{2}+\frac{c}{2}\right)-4-3 i\right)}{4 d}$	41
derivativedivides	$\frac{\frac{\ln\left(5 \tanh\left(\frac{d x}{2}+\frac{c}{2}\right)+4-3 i\right)}{4}-\frac{\ln\left(5 \tanh\left(\frac{d x}{2}+\frac{c}{2}\right)-4-3 i\right)}{4}}{d}$	42
default	$\frac{\frac{\ln\left(5 \tanh\left(\frac{d x}{2}+\frac{c}{2}\right)+4-3 i\right)}{4}-\frac{\ln\left(5 \tanh\left(\frac{d x}{2}+\frac{c}{2}\right)-4-3 i\right)}{4}}{d}$	42

input
```
int(1/(5+3*I*sinh(d*x+c)),x,method=_RETURNVERBOSE)
```

output
```
1/4/d*ln(-1/3*I+exp(d*x+c))-1/4/d*ln(exp(d*x+c)-3*I)
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.70

$$\int \frac{1}{5+3i\sinh(c+dx)}\,dx=\frac{\log\left(e^{(dx+c)}-\frac{1}{3}i\right)-\log\left(e^{(dx+c)}-3i\right)}{4\,d}$$

input
```
integrate(1/(5+3*I*sinh(d*x+c)),x, algorithm="fricas")
```

output
```
1/4*(log(e^(d*x + c) - 1/3*I) - log(e^(d*x + c) - 3*I))/d
```

Sympy [A] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.84

$$\int \frac{1}{5+3i\sinh(c+dx)}\,dx = \frac{-\frac{\log\left(e^{dx}-3ie^{-c}\right)}{4}+\frac{\log\left(e^{dx}-\frac{ie^{-c}}{3}\right)}{4}}{d}$$

input
```
integrate(1/(5+3*I*sinh(d*x+c)),x)
```

output
```
(-log(exp(d*x) - 3*I*exp(-c))/4 + log(exp(d*x) - I*exp(-c)/3)/4)/d
```

Maxima [A] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.97

$$\int \frac{1}{5+3i\sinh(c+dx)}\,dx = \frac{\log\left(-\frac{6\left(-i\,e^{(-dx-c)}+3\right)}{6i\,e^{(-dx-c)}-2}\right)}{4\,d}$$

input
```
integrate(1/(5+3*I*sinh(d*x+c)),x, algorithm="maxima")
```

output
```
1/4*log(-6*(-I*e^(-d*x - c) + 3)/(6*I*e^(-d*x - c) - 2))/d
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.76

$$\int \frac{1}{5+3i\sinh(c+dx)}\,dx = \frac{\log\left(3\,e^{(dx+c)}-i\right)-\log\left(e^{(dx+c)}-3i\right)}{4\,d}$$

input
```
integrate(1/(5+3*I*sinh(d*x+c)),x, algorithm="giac")
```

output
```
1/4*(log(3*e^(d*x + c) - I) - log(e^(d*x + c) - 3*I))/d
```

Mupad [B] (verification not implemented)

Time = 1.73 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.86

$$\int \frac{1}{5+3i\sinh(c+dx)}\,dx = -\frac{\ln\left(-\frac{e^{d\,x}\,e^{c}}{2}+\frac{3}{2}i\right)-\ln\left(\frac{9\,e^{d\,x}\,e^{c}}{2}-\frac{3}{2}i\right)}{4\,d}$$

input
```
int(1/(sinh(c + d*x)*3i + 5),x)
```

output
```
-(log(3i/2 - (exp(d*x)*exp(c))/2) - log((9*exp(d*x)*exp(c))/2 - 3i/2))/(4*
d)
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.89

$$\int \frac{1}{5+3i\sinh(c+dx)}\,dx = \frac{-\log\left(e^{dx+c}i+3\right)+\log\left(3e^{dx+c}i+1\right)}{4d}$$

input
```
int(1/(5+3*I*sinh(d*x+c)),x)
```

output
```
( - log(e**(c + d*x)*i + 3) + log(3*e**(c + d*x)*i + 1))/(4*d)
```

3.93 $\int \frac{1}{(5+3i\sinh(c+dx))^2}\,dx$

Optimal result

Integrand size = 14, antiderivative size = 66

$$\int \frac{1}{(5+3i\sinh(c+dx))^2}\,dx=\frac{5x}{64}-\frac{5i\arctan\left(\frac{\cosh(c+dx)}{3+i\sinh(c+dx)}\right)}{32d}-\frac{3i\cosh(c+dx)}{16d(5+3i\sinh(c+dx))}$$

output
```
5/64*x-5/32*I*arctan(cosh(d*x+c)/(3+I*sinh(d*x+c)))/d-3/16*I*cosh(d*x+c)/d
/(5+3*I*sinh(d*x+c))
```

Mathematica [B] (verified)

Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 183 vs. 2(66) = 132.

Time = 0.34 (sec) , antiderivative size = 183, normalized size of antiderivative = 2.77

$$\int \frac{1}{(5+3i\sinh(c+dx))^2}\,dx$$
$$=\frac{24i-50i\arctan\left(\frac{2\cosh\left(\frac{1}{2}(c+dx)\right)-\sinh\left(\frac{1}{2}(c+dx)\right)}{\cosh\left(\frac{1}{2}(c+dx)\right)-2\sinh\left(\frac{1}{2}(c+dx)\right)}\right)+50i\arctan\left(\frac{\cosh\left(\frac{1}{2}(c+dx)\right)+2\sinh\left(\frac{1}{2}(c+dx)\right)}{2\cosh\left(\frac{1}{2}(c+dx)\right)+\sinh\left(\frac{1}{2}(c+dx)\right)}\right)-25\log(5\cosh(c\ldots}{640d}$$

input
```
Integrate[(5 + (3*I)*Sinh[c + d*x])^(-2),x]
```

output
```
(24*I - (50*I)*ArcTan[(2*Cosh[(c + d*x)/2] - Sinh[(c + d*x)/2])/(Cosh[(c +
 d*x)/2] - 2*Sinh[(c + d*x)/2])] + (50*I)*ArcTan[(Cosh[(c + d*x)/2] + 2*Si
nh[(c + d*x)/2])/(2*Cosh[(c + d*x)/2] + Sinh[(c + d*x)/2])] - 25*Log[5*Cos
h[c + d*x] - 4*Sinh[c + d*x]] + 25*Log[5*Cosh[c + d*x] + 4*Sinh[c + d*x]]
- (120*Cosh[c + d*x])/(-5*I + 3*Sinh[c + d*x]))/(640*d)
```

Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 71, normalized size of antiderivative = 1.08, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.357$, Rules used = {3042, 3143, 27, 3042, 3136}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(5+3i\sinh(c+dx))^2}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{(5+3\sin(ic+idx))^2}dx$$

$$\Big\downarrow \text{3143}$$

$$-\frac{1}{16}\int -\frac{5}{3i\sinh(c+dx)+5}dx-\frac{3i\cosh(c+dx)}{16d(5+3i\sinh(c+dx))}$$

$$\Big\downarrow \text{27}$$

$$\frac{5}{16}\int \frac{1}{3i\sinh(c+dx)+5}dx-\frac{3i\cosh(c+dx)}{16d(5+3i\sinh(c+dx))}$$

$$\Big\downarrow \text{3042}$$

$$\frac{5}{16}\int \frac{1}{3\sin(ic+idx)+5}dx-\frac{3i\cosh(c+dx)}{16d(5+3i\sinh(c+dx))}$$

$$\Big\downarrow \text{3136}$$

$$\frac{5}{16}\left(\frac{x}{4}-\frac{i\arctan\left(\frac{\cosh(c+dx)}{3+i\sinh(c+dx)}\right)}{2d}\right)-\frac{3i\cosh(c+dx)}{16d(5+3i\sinh(c+dx))}$$

input
```
Int[(5 + (3*I)*Sinh[c + d*x])^(-2),x]
```

output
```
(5*(x/4 - ((I/2)*ArcTan[Cosh[c + d*x]/(3 + I*Sinh[c + d*x])])/d))/16 - (((
3*I)/16)*Cosh[c + d*x])/(d*(5 + (3*I)*Sinh[c + d*x]))
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3136
```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{q = Rt[
a^2 - b^2, 2]}, Simp[x/q, x] + Simp[(2/(d*q))*ArcTan[b*(Cos[c + d*x]/(a + q
 + b*Sin[c + d*x]))], x]] /; FreeQ[{a, b, c, d}, x] && GtQ[a^2 - b^2, 0] &&
 PosQ[a]
```

rule 3143
```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos
[c + d*x]*((a + b*Sin[c + d*x])^(n + 1)/(d*(n + 1)*(a^2 - b^2))), x] + Simp
[1/((n + 1)*(a^2 - b^2))   Int[(a + b*Sin[c + d*x])^(n + 1)*Simp[a*(n + 1)
- b*(n + 2)*Sin[c + d*x], x], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
 b^2, 0] && LtQ[n, -1] && IntegerQ[2*n]
```

Maple [A] (verified)

Time = 0.33 (sec) , antiderivative size = 73, normalized size of antiderivative = 1.11

method	result
risch	$-\frac{i(5\,\mathrm{e}^{dx+c}-3i)}{8d(3\,\mathrm{e}^{2dx+2c}-3-10i\mathrm{e}^{dx+c})}-\frac{5\ln(\mathrm{e}^{dx+c}-3i)}{64d}+\frac{5\ln(-\frac{i}{3}+\mathrm{e}^{dx+c})}{64d}$
derivativedivides	$\frac{\frac{-\frac{9}{80}-\frac{3i}{20}}{5\tanh(\frac{dx}{2}+\frac{c}{2})+4-3i}+\frac{5\ln(5\tanh(\frac{dx}{2}+\frac{c}{2})+4-3i)}{64}+\frac{-\frac{9}{80}+\frac{3i}{20}}{5\tanh(\frac{dx}{2}+\frac{c}{2})-4-3i}-\frac{5\ln(5\tanh(\frac{dx}{2}+\frac{c}{2})-4-3i)}{64}}{d}$
default	$\frac{\frac{-\frac{9}{80}-\frac{3i}{20}}{5\tanh(\frac{dx}{2}+\frac{c}{2})+4-3i}+\frac{5\ln(5\tanh(\frac{dx}{2}+\frac{c}{2})+4-3i)}{64}+\frac{-\frac{9}{80}+\frac{3i}{20}}{5\tanh(\frac{dx}{2}+\frac{c}{2})-4-3i}-\frac{5\ln(5\tanh(\frac{dx}{2}+\frac{c}{2})-4-3i)}{64}}{d}$
parallelrisch	$\frac{(-125i+75\sinh(dx+c))\ln(5\tanh(\frac{dx}{2}+\frac{c}{2})-4-3i)+(125i-75\sinh(dx+c))\ln(5\tanh(\frac{dx}{2}+\frac{c}{2})+4-3i)+36i\sinh(dx+}{320d(5i-3\sinh(dx+c))}$

input
```
int(1/(5+3*I*sinh(d*x+c))^2,x,method=_RETURNVERBOSE)
```

output
```
-1/8*I*(5*exp(d*x+c)-3*I)/d/(3*exp(2*d*x+2*c)-3-10*I*exp(d*x+c))-5/64/d*ln
(exp(d*x+c)-3*I)+5/64/d*ln(-1/3*I+exp(d*x+c))
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 103, normalized size of antiderivative = 1.56

$$\int \frac{1}{(5+3i\sinh(c+dx))^2}\,dx = \frac{5\left(3\,e^{(2\,dx+2\,c)}-10i\,e^{(dx+c)}-3\right)\log\left(e^{(dx+c)}-\frac{1}{3}i\right)-5\left(3\,e^{(2\,dx+2\,c)}-10i\,e^{(dx+c)}-3\right)\log\left(e^{(dx+c)}-3i\right)-}{64\left(3\,de^{(2\,dx+2\,c)}-10i\,de^{(dx+c)}-3\,d\right)}$$

input
```
integrate(1/(5+3*I*sinh(d*x+c))^2,x, algorithm="fricas")
```

output
```
1/64*(5*(3*e^(2*d*x + 2*c) - 10*I*e^(d*x + c) - 3)*log(e^(d*x + c) - 1/3*I
) - 5*(3*e^(2*d*x + 2*c) - 10*I*e^(d*x + c) - 3)*log(e^(d*x + c) - 3*I) -
40*I*e^(d*x + c) - 24)/(3*d*e^(2*d*x + 2*c) - 10*I*d*e^(d*x + c) - 3*d)
```

Sympy [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 82, normalized size of antiderivative = 1.24

$$\int \frac{1}{(5+3i\sinh(c+dx))^2}\,dx = \frac{-5ie^c e^{dx}-3}{24de^{2c}e^{2dx}-80ide^c e^{dx}-24d} + \frac{-\frac{5\log\left(e^{dx}-3ie^{-c}\right)}{64}+\frac{5\log\left(e^{dx}-\frac{ie^{-c}}{3}\right)}{64}}{d}$$

input
```
integrate(1/(5+3*I*sinh(d*x+c))**2,x)
```

output
```
(-5*I*exp(c)*exp(d*x) - 3)/(24*d*exp(2*c)*exp(2*d*x) - 80*I*d*exp(c)*exp(d
*x) - 24*d) + (-5*log(exp(d*x) - 3*I*exp(-c))/64 + 5*log(exp(d*x) - I*exp(
-c)/3)/64)/d
```

Maxima [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 64, normalized size of antiderivative = 0.97

$$\int \frac{1}{(5+3i\sinh(c+dx))^2}\,dx = -\frac{5i\arctan\left(\frac{3}{4}e^{(-dx-c)}+\frac{5}{4}i\right)}{32\,d} - \frac{5i\,e^{(-dx-c)}-3}{-8\,d(-10i\,e^{(-dx-c)}-3\,e^{(-2\,dx-2\,c)}+3)}$$

input
```
integrate(1/(5+3*I*sinh(d*x+c))^2,x, algorithm="maxima")
```

output
```
-5/32*I*arctan(3/4*e^(-d*x - c) + 5/4*I)/d - (5*I*e^(-d*x - c) - 3)/(d*(80
*I*e^(-d*x - c) + 24*e^(-2*d*x - 2*c) - 24))
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 65, normalized size of antiderivative = 0.98

$$\int \frac{1}{(5+3i\sinh(c+dx))^2}\,dx = -\frac{\frac{8\,(5i\,e^{(dx+c)}+3)}{3\,e^{(2\,dx+2\,c)}-10i\,e^{(dx+c)}-3} - 5\,\log\left(3\,e^{(dx+c)}-i\right) + 5\,\log\left(e^{(dx+c)}-3i\right)}{64\,d}$$

input
```
integrate(1/(5+3*I*sinh(d*x+c))^2,x, algorithm="giac")
```

output
```
-1/64*(8*(5*I*e^(d*x + c) + 3)/(3*e^(2*d*x + 2*c) - 10*I*e^(d*x + c) - 3)
- 5*log(3*e^(d*x + c) - I) + 5*log(e^(d*x + c) - 3*I))/d
```

Mupad [B] (verification not implemented)

Time = 2.10 (sec) , antiderivative size = 102, normalized size of antiderivative = 1.55

$$\int \frac{1}{(5+3i\sinh(c+dx))^2}\,dx = \frac{3}{8\,\left(3\,d-3\,d\,e^{2\,c+2\,d\,x}+d\,e^{c+d\,x}\,10i\right)} - \frac{5\,\ln\left(-\frac{5\,e^{d\,x}\,e^{c}}{4}+\frac{15}{4}i\right)}{64\,d} + \frac{5\,\ln\left(\frac{45\,e^{d\,x}\,e^{c}}{4}-\frac{15}{4}i\right)}{64\,d} + \frac{e^{c+d\,x}\,5i}{8\,\left(3\,d-3\,d\,e^{2\,c+2\,d\,x}+d\,e^{c+d\,x}\,10i\right)}$$

input
```
int(1/(sinh(c + d*x)*3i + 5)^2,x)
```

output
```
3/(8*(3*d + d*exp(c + d*x)*10i - 3*d*exp(2*c + 2*d*x))) - (5*log(15i/4 - (
5*exp(d*x)*exp(c))/4))/(64*d) + (5*log((45*exp(d*x)*exp(c))/4 - 15i/4))/(6
4*d) + (exp(c + d*x)*5i)/(8*(3*d + d*exp(c + d*x)*10i - 3*d*exp(2*c + 2*d*
x)))
```

Reduce [F]

$$\int \frac{1}{(5+3i\sinh(c+dx))^2}\,dx = -\left(\int \frac{1}{9\sinh{(dx+c)}^2-30\sinh{(dx+c)}\,i-25}dx\right)$$

input
```
int(1/(5+3*I*sinh(d*x+c))^2,x)
```

output
```
- int(1/(9*sinh(c + d*x)**2 - 30*sinh(c + d*x)*i - 25),x)
```

3.94 $\int \frac{1}{(5+3i\sinh(c+dx))^3}\,dx$

Optimal result

Integrand size = 14, antiderivative size = 95

$$\int \frac{1}{(5+3i\sinh(c+dx))^3}\,dx = \frac{59x}{2048} - \frac{59i\arctan\left(\frac{\cosh(c+dx)}{3+i\sinh(c+dx)}\right)}{1024d} - \frac{3i\cosh(c+dx)}{32d(5+3i\sinh(c+dx))^2} - \frac{45i\cosh(c+dx)}{512d(5+3i\sinh(c+dx))}$$

output
```
59/2048*x-59/1024*I*arctan(cosh(d*x+c)/(3+I*sinh(d*x+c)))/d-3/32*I*cosh(d*
x+c)/d/(5+3*I*sinh(d*x+c))^2-45/512*I*cosh(d*x+c)/d/(5+3*I*sinh(d*x+c))
```

Mathematica [B] (verified)

Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 277 vs. 2(95) = 190.

Time = 0.69 (sec) , antiderivative size = 277, normalized size of antiderivative = 2.92

$$\int \frac{1}{(5+3i\sinh(c+dx))^3}\,dx$$
$$= \frac{-118i\arctan\left(\frac{2\cosh\left(\frac{1}{2}(c+dx)\right)-\sinh\left(\frac{1}{2}(c+dx)\right)}{\cosh\left(\frac{1}{2}(c+dx)\right)-2\sinh\left(\frac{1}{2}(c+dx)\right)}\right) + 118i\arctan\left(\frac{\cosh\left(\frac{1}{2}(c+dx)\right)+2\sinh\left(\frac{1}{2}(c+dx)\right)}{2\cosh\left(\frac{1}{2}(c+dx)\right)+\sinh\left(\frac{1}{2}(c+dx)\right)}\right) - 59\log(5\cosh(c}{}$$

input
```
Integrate[(5 + (3*I)*Sinh[c + d*x])^(-3),x]
```

output
```
((-118*I)*ArcTan[(2*Cosh[(c + d*x)/2] - Sinh[(c + d*x)/2])/(Cosh[(c + d*x)
/2] - 2*Sinh[(c + d*x)/2])] + (118*I)*ArcTan[(Cosh[(c + d*x)/2] + 2*Sinh[(
c + d*x)/2])/(2*Cosh[(c + d*x)/2] + Sinh[(c + d*x)/2])] - 59*Log[5*Cosh[c
+ d*x] - 4*Sinh[c + d*x]] + 59*Log[5*Cosh[c + d*x] + 4*Sinh[c + d*x]] + 48
/((1 + 2*I)*Cosh[(c + d*x)/2] - (2 + I)*Sinh[(c + d*x)/2])^2 + 48/((2 + I)
*Cosh[(c + d*x)/2] + (1 + 2*I)*Sinh[(c + d*x)/2])^2 - (144*Sinh[(c + d*x)/
2]*((-3*I)*Cosh[(c + d*x)/2] + 5*Sinh[(c + d*x)/2]))/(-5*I + 3*Sinh[c + d*
x]))/(4096*d)
```

Rubi [A] (verified)

Time = 0.40 (sec) , antiderivative size = 105, normalized size of antiderivative = 1.11, number of steps used = 8, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.571$, Rules used = {3042, 3143, 25, 3042, 3233, 27, 3042, 3136}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(5+3i\sinh(c+dx))^3}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{(5+3\sin(ic+idx))^3}dx$$

$$\Big\downarrow \text{3143}$$

$$-\frac{1}{32}\int -\frac{10-3i\sinh(c+dx)}{(3i\sinh(c+dx)+5)^2}dx-\frac{3i\cosh(c+dx)}{32d(5+3i\sinh(c+dx))^2}$$

$$\Big\downarrow \text{25}$$

$$\frac{1}{32}\int \frac{10-3i\sinh(c+dx)}{(3i\sinh(c+dx)+5)^2}dx-\frac{3i\cosh(c+dx)}{32d(5+3i\sinh(c+dx))^2}$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{32}\int \frac{10-3\sin(ic+idx)}{(3\sin(ic+idx)+5)^2}dx-\frac{3i\cosh(c+dx)}{32d(5+3i\sinh(c+dx))^2}$$

$$\Big\downarrow \text{3233}$$

$$\frac{1}{32}\left(-\frac{1}{16}\int -\frac{59}{3i\sinh(c+dx)+5}dx-\frac{45i\cosh(c+dx)}{16d(5+3i\sinh(c+dx))}\right)-\frac{3i\cosh(c+dx)}{32d(5+3i\sinh(c+dx))^2}$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{32}\left(\frac{59}{16}\int \frac{1}{3i\sinh(c+dx)+5}dx-\frac{45i\cosh(c+dx)}{16d(5+3i\sinh(c+dx))}\right)-\frac{3i\cosh(c+dx)}{32d(5+3i\sinh(c+dx))^2}$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{32}\left(\frac{59}{16}\int \frac{1}{3\sin(ic+idx)+5}dx-\frac{45i\cosh(c+dx)}{16d(5+3i\sinh(c+dx))}\right)-\frac{3i\cosh(c+dx)}{32d(5+3i\sinh(c+dx))^2}$$

$$\Big\downarrow \text{3136}$$

$$\frac{1}{32}\left(\frac{59}{16}\left(\frac{x}{4}-\frac{i\arctan\left(\frac{\cosh(c+dx)}{3+i\sinh(c+dx)}\right)}{2d}\right)-\frac{45i\cosh(c+dx)}{16d(5+3i\sinh(c+dx))}\right)-\frac{3i\cosh(c+dx)}{32d(5+3i\sinh(c+dx))^2}$$

input
```
Int[(5 + (3*I)*Sinh[c + d*x])^(-3),x]
```

output
```
((59*(x/4 - ((I/2)*ArcTan[Cosh[c + d*x]/(3 + I*Sinh[c + d*x])])/d))/16 - (
((45*I)/16)*Cosh[c + d*x])/(d*(5 + (3*I)*Sinh[c + d*x])))/32 - (((3*I)/32)
*Cosh[c + d*x])/(d*(5 + (3*I)*Sinh[c + d*x])^2)
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3136
```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{q = Rt[
a^2 - b^2, 2]}, Simp[x/q, x] + Simp[(2/(d*q))*ArcTan[b*(Cos[c + d*x]/(a + q
 + b*Sin[c + d*x]))], x]] /; FreeQ[{a, b, c, d}, x] && GtQ[a^2 - b^2, 0] &&
 PosQ[a]
```

rule 3143
```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos
[c + d*x]*((a + b*Sin[c + d*x])^(n + 1)/(d*(n + 1)*(a^2 - b^2))), x] + Simp
[1/((n + 1)*(a^2 - b^2))   Int[(a + b*Sin[c + d*x])^(n + 1)*Simp[a*(n + 1)
- b*(n + 2)*Sin[c + d*x], x], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
 b^2, 0] && LtQ[n, -1] && IntegerQ[2*n]
```

rule 3233
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Simp[(-(b*c - a*d))*Cos[e + f*x]*((a + b*Sin[e +
 f*x])^(m + 1)/(f*(m + 1)*(a^2 - b^2))), x] + Simp[1/((m + 1)*(a^2 - b^2))
  Int[(a + b*Sin[e + f*x])^(m + 1)*Simp[(a*c - b*d)*(m + 1) - (b*c - a*d)*(
m + 2)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c
- a*d, 0] && NeQ[a^2 - b^2, 0] && LtQ[m, -1] && IntegerQ[2*m]
```

Maple [A] (verified)

Time = 0.46 (sec) , antiderivative size = 96, normalized size of antiderivative = 1.01

method	result
risch	$-\frac{3i\left(-295i\,\mathrm{e}^{2dx+2c}+59\,\mathrm{e}^{3dx+3c}+45i-241\,\mathrm{e}^{dx+c}\right)}{256d\left(3\,\mathrm{e}^{2dx+2c}-3-10i\,\mathrm{e}^{dx+c}\right)^{2}}-\frac{59\ln\left(\mathrm{e}^{dx+c}-3i\right)}{2048d}+\frac{59\ln\left(-\frac{i}{3}+\mathrm{e}^{dx+c}\right)}{2048d}$
derivativedivides	$\frac{\frac{\frac{63}{3200}-\frac{27i}{400}}{\left(5\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)+4-3i\right)^{2}}+\frac{-\frac{963}{12800}-\frac{123i}{1600}}{5\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)+4-3i}+\frac{59\ln\left(5\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)+4-3i\right)}{2048}+\frac{-\frac{63}{3200}-\frac{27i}{400}}{\left(5\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)-4-3i\right)^{2}}+\frac{-\frac{963}{12800}+\frac{12}{16}}{5\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)}}{d}$
default	$\frac{\frac{\frac{63}{3200}-\frac{27i}{400}}{\left(5\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)+4-3i\right)^{2}}+\frac{-\frac{963}{12800}-\frac{123i}{1600}}{5\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)+4-3i}+\frac{59\ln\left(5\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)+4-3i\right)}{2048}+\frac{-\frac{63}{3200}-\frac{27i}{400}}{\left(5\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)-4-3i\right)^{2}}+\frac{-\frac{963}{12800}+\frac{12}{16}}{5\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)}}{d}$
parallelrisch	$\frac{\left(-87025+13275\cosh\left(2dx+2c\right)-88500i\sinh\left(dx+c\right)\right)\ln\left(5\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)-4-3i\right)+\left(88500i\sinh\left(dx+c\right)-13275\cosh\left(2d\right.\right.}{51200d\left(59-9\right.}$

input
```
int(1/(5+3*I*sinh(d*x+c))^3,x,method=_RETURNVERBOSE)
```

output
```
-3/256*I*(-295*I*exp(2*d*x+2*c)+59*exp(3*d*x+3*c)+45*I-241*exp(d*x+c))/d/(
3*exp(2*d*x+2*c)-3-10*I*exp(d*x+c))^2-59/2048/d*ln(exp(d*x+c)-3*I)+59/2048
/d*ln(-1/3*I+exp(d*x+c))
```

Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 193 vs. $2(75) = 150$.

Time = 0.10 (sec) , antiderivative size = 193, normalized size of antiderivative = 2.03

$$\int \frac{1}{(5+3i\sinh(c+dx))^3}\,dx = \frac{59\left(9\,e^{(4\,dx+4\,c)}-60i\,e^{(3\,dx+3\,c)}-118\,e^{(2\,dx+2\,c)}+60i\,e^{(dx+c)}+9\right)\log\left(e^{(dx+c)}-\frac{1}{3}i\right)-59\left(9\,e^{(4\,dx+4\,c)}-60i\right.}{2048\left(9\,de^{(4\,dx+4\,c)}-60i\,de^{(3\,c}\right.}$$

input
```
integrate(1/(5+3*I*sinh(d*x+c))^3,x, algorithm="fricas")
```

output
```
1/2048*(59*(9*e^(4*d*x + 4*c) - 60*I*e^(3*d*x + 3*c) - 118*e^(2*d*x + 2*c)
 + 60*I*e^(d*x + c) + 9)*log(e^(d*x + c) - 1/3*I) - 59*(9*e^(4*d*x + 4*c)
- 60*I*e^(3*d*x + 3*c) - 118*e^(2*d*x + 2*c) + 60*I*e^(d*x + c) + 9)*log(e
^(d*x + c) - 3*I) - 1416*I*e^(3*d*x + 3*c) - 7080*e^(2*d*x + 2*c) + 5784*I
*e^(d*x + c) + 1080)/(9*d*e^(4*d*x + 4*c) - 60*I*d*e^(3*d*x + 3*c) - 118*d
*e^(2*d*x + 2*c) + 60*I*d*e^(d*x + c) + 9*d)
```

Sympy [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 141, normalized size of antiderivative = 1.48

$$\int \frac{1}{(5+3i\sinh(c+dx))^3}\,dx = \frac{-177ie^{3c}e^{3dx}-885e^{2c}e^{2dx}+723ie^{c}e^{dx}+135}{2304de^{4c}e^{4dx}-15360ide^{3c}e^{3dx}-30208de^{2c}e^{2dx}+15360ide^{c}e^{dx}+2304d} + \frac{-\frac{59\log\left(e^{dx}-3ie^{-c}\right)}{2048}+\frac{59\log\left(e^{dx}-\frac{ie^{-c}}{3}\right)}{2048}}{d}$$

input
```
integrate(1/(5+3*I*sinh(d*x+c))**3,x)
```

output
```
(-177*I*exp(3*c)*exp(3*d*x) - 885*exp(2*c)*exp(2*d*x) + 723*I*exp(c)*exp(d
*x) + 135)/(2304*d*exp(4*c)*exp(4*d*x) - 15360*I*d*exp(3*c)*exp(3*d*x) - 3
0208*d*exp(2*c)*exp(2*d*x) + 15360*I*d*exp(c)*exp(d*x) + 2304*d) + (-59*lo
g(exp(d*x) - 3*I*exp(-c))/2048 + 59*log(exp(d*x) - I*exp(-c)/3)/2048)/d
```

Maxima [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 108, normalized size of antiderivative = 1.14

$$\int \frac{1}{(5+3i\sinh(c+dx))^3}\,dx = -\frac{59i\,\arctan\left(\frac{3}{4}\,e^{(-dx-c)}+\frac{5}{4}i\right)}{1024\,d} + \frac{3\left(241i\,e^{(-dx-c)}+295\,e^{(-2\,dx-2\,c)}-59i\,e^{(-3\,dx-3\,c)}-45\right)}{-256\,d(60i\,e^{(-dx-c)}+118\,e^{(-2\,dx-2\,c)}-60i\,e^{(-3\,dx-3\,c)}-9\,e^{(-4\,dx-4\,c)}-9)}$$

input
```
integrate(1/(5+3*I*sinh(d*x+c))^3,x, algorithm="maxima")
```

output
```
-59/1024*I*arctan(3/4*e^(-d*x - c) + 5/4*I)/d + 3*(241*I*e^(-d*x - c) + 29
5*e^(-2*d*x - 2*c) - 59*I*e^(-3*d*x - 3*c) - 45)/(d*(-15360*I*e^(-d*x - c)
 - 30208*e^(-2*d*x - 2*c) + 15360*I*e^(-3*d*x - 3*c) + 2304*e^(-4*d*x - 4*
c) + 2304))
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 87, normalized size of antiderivative = 0.92

$$\int \frac{1}{(5+3i\sinh(c+dx))^3}\,dx = -\frac{\frac{24\left(-59i\,e^{(3\,dx+3\,c)}-295\,e^{(2\,dx+2\,c)}+241i\,e^{(dx+c)}+45\right)}{\left(-3i\,e^{(2\,dx+2\,c)}-10\,e^{(dx+c)}+3i\right)^2}-59\,\log\left(3\,e^{(dx+c)}-i\right)+59\,\log\left(e^{(dx+c)}-3i\right)}{2048\,d}$$

input
```
integrate(1/(5+3*I*sinh(d*x+c))^3,x, algorithm="giac")
```

output
```
-1/2048*(24*(-59*I*e^(3*d*x + 3*c) - 295*e^(2*d*x + 2*c) + 241*I*e^(d*x +
c) + 45)/(-3*I*e^(2*d*x + 2*c) - 10*e^(d*x + c) + 3*I)^2 - 59*log(3*e^(d*x
 + c) - I) + 59*log(e^(d*x + c) - 3*I))/d
```

Mupad [B] (verification not implemented)

Time = 2.62 (sec) , antiderivative size = 143, normalized size of antiderivative = 1.51

$$\int \frac{1}{(5+3i\sinh(c+dx))^3}\,dx = \frac{\frac{295}{2304\,d}+\frac{e^{c+d\,x}\,59\mathrm{i}}{768\,d}}{1-e^{2\,c+2\,d\,x}+\frac{e^{c+d\,x}\,10\mathrm{i}}{3}} - \frac{59\,\ln\left(-\frac{59\,e^{c+d\,x}}{4}+\frac{177}{4}\mathrm{i}\right)}{2048\,d} + \frac{59\,\ln\left(\frac{531\,e^{c+d\,x}}{4}-\frac{177}{4}\mathrm{i}\right)}{2048\,d} - \frac{\frac{5}{72\,d}+\frac{e^{c+d\,x}\,41\mathrm{i}}{216\,d}}{e^{4\,c+4\,d\,x}-\frac{118\,e^{2\,c+2\,d\,x}}{9}+1+\frac{e^{c+d\,x}\,20\mathrm{i}}{3}-\frac{e^{3\,c+3\,d\,x}\,20\mathrm{i}}{3}}$$

input
```
int(1/(sinh(c + d*x)*3i + 5)^3,x)
```

output
```
((exp(c + d*x)*59i)/(768*d) + 295/(2304*d))/((exp(c + d*x)*10i)/3 - exp(2*
c + 2*d*x) + 1) - (59*log(177i/4 - (59*exp(c + d*x))/4))/(2048*d) + (59*lo
g((531*exp(c + d*x))/4 - 177i/4))/(2048*d) - ((exp(c + d*x)*41i)/(216*d) +
 5/(72*d))/((exp(c + d*x)*20i)/3 - (118*exp(2*c + 2*d*x))/9 - (exp(3*c + 3
*d*x)*20i)/3 + exp(4*c + 4*d*x) + 1)
```

Reduce [F]

$$\int \frac{1}{(5+3i\sinh(c+dx))^3}\,dx = -\left(\int \frac{1}{27\sinh(dx+c)^3 i+135\sinh(dx+c)^2-225\sinh(dx+c)\,i-125}dx\right)$$

input
```
int(1/(5+3*I*sinh(d*x+c))^3,x)
```

output
```
- int(1/(27*sinh(c + d*x)**3*i + 135*sinh(c + d*x)**2 - 225*sinh(c + d*x)
*i - 125),x)
```

3.95 $\int \frac{1}{(5+3i\sinh(c+dx))^4}\,dx$

Optimal result

Integrand size = 14, antiderivative size = 124

$$\int \frac{1}{(5+3i\sinh(c+dx))^4}\,dx = \frac{385x}{32768} - \frac{385i\arctan\left(\frac{\cosh(c+dx)}{3+i\sinh(c+dx)}\right)}{16384d} - \frac{i\cosh(c+dx)}{16d(5+3i\sinh(c+dx))^3} - \frac{25i\cosh(c+dx)}{512d(5+3i\sinh(c+dx))^2} - \frac{311i\cosh(c+dx)}{8192d(5+3i\sinh(c+dx))}$$

output
```
385/32768*x-385/16384*I*arctan(cosh(d*x+c)/(3+I*sinh(d*x+c)))/d-1/16*I*cos
h(d*x+c)/d/(5+3*I*sinh(d*x+c))^3-25/512*I*cosh(d*x+c)/d/(5+3*I*sinh(d*x+c)
)^2-311/8192*I*cosh(d*x+c)/d/(5+3*I*sinh(d*x+c))
```

Mathematica [B] (verified)

Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 308 vs. 2(124) = 248.

Time = 1.38 (sec) , antiderivative size = 308, normalized size of antiderivative = 2.48

$$\int \frac{1}{(5+3i\sinh(c+dx))^4}\,dx = \frac{-3850i\arctan\left(\frac{2\cosh\left(\frac{1}{2}(c+dx)\right)-\sinh\left(\frac{1}{2}(c+dx)\right)}{\cosh\left(\frac{1}{2}(c+dx)\right)-2\sinh\left(\frac{1}{2}(c+dx)\right)}\right)+3850i\arctan\left(\frac{\cosh\left(\frac{1}{2}(c+dx)\right)+2\sinh\left(\frac{1}{2}(c+dx)\right)}{2\cosh\left(\frac{1}{2}(c+dx)\right)+\sinh\left(\frac{1}{2}(c+dx)\right)}\right)-1925\log(5\cos}{}$$

input
```
Integrate[(5 + (3*I)*Sinh[c + d*x])^(-4),x]
```

output
```
((-3850*I)*ArcTan[(2*Cosh[(c + d*x)/2] - Sinh[(c + d*x)/2])/(Cosh[(c + d*x
)/2] - 2*Sinh[(c + d*x)/2])] + (3850*I)*ArcTan[(Cosh[(c + d*x)/2] + 2*Sinh
[(c + d*x)/2])/(2*Cosh[(c + d*x)/2] + Sinh[(c + d*x)/2])] - 1925*Log[5*Cos
h[c + d*x] - 4*Sinh[c + d*x]] + 1925*Log[5*Cosh[c + d*x] + 4*Sinh[c + d*x]
] + (2656 - 192*I)/((1 + 2*I)*Cosh[(c + d*x)/2] - (2 + I)*Sinh[(c + d*x)/2
])^2 + (2656 + 192*I)/((2 + I)*Cosh[(c + d*x)/2] + (1 + 2*I)*Sinh[(c + d*x
)/2])^2 + (2*(-235150 + 166615*Cosh[c + d*x] + 82530*Cosh[2*(c + d*x)] - 1
3995*Cosh[3*(c + d*x)] - (298563*I)*Sinh[c + d*x] + (89364*I)*Sinh[2*(c +
d*x)] + (8397*I)*Sinh[3*(c + d*x)]))/(-5*I + 3*Sinh[c + d*x])^3)/(327680*d
)
```

Rubi [A] (verified)

Time = 0.55 (sec) , antiderivative size = 139, normalized size of antiderivative = 1.12, number of steps used = 11, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.786$, Rules used = {3042, 3143, 27, 3042, 3233, 25, 3042, 3233, 27, 3042, 3136}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(5+3i\sinh(c+dx))^4}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{(5+3\sin(ic+idx))^4}dx$$

$$\Big\downarrow \text{3143}$$

$$-\frac{1}{48}\int -\frac{3(5-2i\sinh(c+dx))}{(3i\sinh(c+dx)+5)^3}dx-\frac{i\cosh(c+dx)}{16d(5+3i\sinh(c+dx))^3}$$

$\Big\downarrow$ 27

$$\frac{1}{16}\int \frac{5-2i\sinh(c+dx)}{(3i\sinh(c+dx)+5)^3}dx-\frac{i\cosh(c+dx)}{16d(5+3i\sinh(c+dx))^3}$$

$\Big\downarrow$ 3042

$$\frac{1}{16}\int \frac{5-2\sin(ic+idx)}{(3\sin(ic+idx)+5)^3}dx-\frac{i\cosh(c+dx)}{16d(5+3i\sinh(c+dx))^3}$$

$\Big\downarrow$ 3233

$$\frac{1}{16}\left(-\frac{1}{32}\int -\frac{62-25i\sinh(c+dx)}{(3i\sinh(c+dx)+5)^2}dx-\frac{25i\cosh(c+dx)}{32d(5+3i\sinh(c+dx))^2}\right)-\frac{i\cosh(c+dx)}{16d(5+3i\sinh(c+dx))^3}$$

$\Big\downarrow$ 25

$$\frac{1}{16}\left(\frac{1}{32}\int \frac{62-25i\sinh(c+dx)}{(3i\sinh(c+dx)+5)^2}dx-\frac{25i\cosh(c+dx)}{32d(5+3i\sinh(c+dx))^2}\right)-\frac{i\cosh(c+dx)}{16d(5+3i\sinh(c+dx))^3}$$

$\Big\downarrow$ 3042

$$\frac{1}{16}\left(\frac{1}{32}\int \frac{62-25\sin(ic+idx)}{(3\sin(ic+idx)+5)^2}dx-\frac{25i\cosh(c+dx)}{32d(5+3i\sinh(c+dx))^2}\right)-\frac{i\cosh(c+dx)}{16d(5+3i\sinh(c+dx))^3}$$

$\Big\downarrow$ 3233

$$\frac{1}{16}\left(\frac{1}{32}\left(-\frac{1}{16}\int -\frac{385}{3i\sinh(c+dx)+5}dx-\frac{311i\cosh(c+dx)}{16d(5+3i\sinh(c+dx))}\right)-\frac{25i\cosh(c+dx)}{32d(5+3i\sinh(c+dx))^2}\right)-\frac{i\cosh(c+dx)}{16d(5+3i\sinh(c+dx))^3}$$

$\Big\downarrow$ 27

$$\frac{1}{16}\left(\frac{1}{32}\left(\frac{385}{16}\int \frac{1}{3i\sinh(c+dx)+5}dx-\frac{311i\cosh(c+dx)}{16d(5+3i\sinh(c+dx))}\right)-\frac{25i\cosh(c+dx)}{32d(5+3i\sinh(c+dx))^2}\right)-\frac{i\cosh(c+dx)}{16d(5+3i\sinh(c+dx))^3}$$

$\Big\downarrow$ 3042

$$\frac{1}{16}\left(\frac{1}{32}\left(\frac{385}{16}\int \frac{1}{3\sin(ic+idx)+5}dx-\frac{311i\cosh(c+dx)}{16d(5+3i\sinh(c+dx))}\right)-\frac{25i\cosh(c+dx)}{32d(5+3i\sinh(c+dx))^2}\right)-\frac{i\cosh(c+dx)}{16d(5+3i\sinh(c+dx))^3}$$

$$\Big\downarrow \text{3136}$$

$$\frac{1}{16}\left(\frac{1}{32}\left(\frac{385}{16}\left(\frac{x}{4}-\frac{i\arctan\left(\frac{\cosh(c+dx)}{3+i\sinh(c+dx)}\right)}{2d}\right)-\frac{311i\cosh(c+dx)}{16d(5+3i\sinh(c+dx))}\right)-\frac{25i\cosh(c+dx)}{32d(5+3i\sinh(c+dx))^2}\right)-\frac{i\cosh(c+dx)}{16d(5+3i\sinh(c+dx))^3}$$

input
```
Int[(5 + (3*I)*Sinh[c + d*x])^(-4),x]
```

output
```
(((385*(x/4 - ((I/2)*ArcTan[Cosh[c + d*x]/(3 + I*Sinh[c + d*x])])/d))/16 -
 (((311*I)/16)*Cosh[c + d*x])/(d*(5 + (3*I)*Sinh[c + d*x])))/32 - (((25*I)
/32)*Cosh[c + d*x])/(d*(5 + (3*I)*Sinh[c + d*x])^2))/16 - ((I/16)*Cosh[c +
 d*x])/(d*(5 + (3*I)*Sinh[c + d*x])^3)
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3136
```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{q = Rt[
a^2 - b^2, 2]}, Simp[x/q, x] + Simp[(2/(d*q))*ArcTan[b*(Cos[c + d*x]/(a + q
 + b*Sin[c + d*x]))], x]] /; FreeQ[{a, b, c, d}, x] && GtQ[a^2 - b^2, 0] &&
 PosQ[a]
```

rule 3143
```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos
[c + d*x]*((a + b*Sin[c + d*x])^(n + 1)/(d*(n + 1)*(a^2 - b^2))), x] + Simp
[1/((n + 1)*(a^2 - b^2))   Int[(a + b*Sin[c + d*x])^(n + 1)*Simp[a*(n + 1)
- b*(n + 2)*Sin[c + d*x], x], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
 b^2, 0] && LtQ[n, -1] && IntegerQ[2*n]
```

rule 3233
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Simp[(-(b*c - a*d))*Cos[e + f*x]*((a + b*Sin[e +
 f*x])^(m + 1)/(f*(m + 1)*(a^2 - b^2))), x] + Simp[1/((m + 1)*(a^2 - b^2))
  Int[(a + b*Sin[e + f*x])^(m + 1)*Simp[(a*c - b*d)*(m + 1) - (b*c - a*d)*(
m + 2)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c
- a*d, 0] && NeQ[a^2 - b^2, 0] && LtQ[m, -1] && IntegerQ[2*m]
```

Maple [A] (verified)

Time = 0.58 (sec) , antiderivative size = 119, normalized size of antiderivative = 0.96

method	result
risch	$-\frac{i\left(-86625 i \mathrm{e}^{4 d x+4 c}+10395\, \mathrm{e}^{5 d x+5 c}+218466 i \mathrm{e}^{2 d x+2 c}-239470\, \mathrm{e}^{3 d x+3 c}-8397 i+73575\, \mathrm{e}^{d x+c}\right)}{12288 d \left(3\, \mathrm{e}^{2 d x+2 c}-3-10 i \mathrm{e}^{d x+c}\right)^{3}}-\frac{385 \ln\left(\mathrm{e}^{d x+c}-3 i\right)}{32768 d}$
derivativedivides	$\frac{\frac{\frac{1053}{32000}-\frac{99 i}{8000}}{\left(5 \tanh\left(\frac{d x}{2}+\frac{c}{2}\right)+4-3 i\right)^{3}}+\frac{\frac{783}{128000}-\frac{3753 i}{64000}}{\left(5 \tanh\left(\frac{d x}{2}+\frac{c}{2}\right)+4-3 i\right)^{2}}+\frac{-\frac{39933}{1024000}-\frac{8361 i}{256000}}{5 \tanh\left(\frac{d x}{2}+\frac{c}{2}\right)+4-3 i}+\frac{385 \ln\left(5 \tanh\left(\frac{d x}{2}+\frac{c}{2}\right)+4-3 i\right)}{32768}+\frac{\frac{1053}{32000}+}{\left(5 \tanh\left(\frac{d x}{2}+\frac{c}{2}\right.\right.}}{d}$
default	$\frac{\frac{\frac{1053}{32000}-\frac{99 i}{8000}}{\left(5 \tanh\left(\frac{d x}{2}+\frac{c}{2}\right)+4-3 i\right)^{3}}+\frac{\frac{783}{128000}-\frac{3753 i}{64000}}{\left(5 \tanh\left(\frac{d x}{2}+\frac{c}{2}\right)+4-3 i\right)^{2}}+\frac{-\frac{39933}{1024000}-\frac{8361 i}{256000}}{5 \tanh\left(\frac{d x}{2}+\frac{c}{2}\right)+4-3 i}+\frac{385 \ln\left(5 \tanh\left(\frac{d x}{2}+\frac{c}{2}\right)+4-3 i\right)}{32768}+\frac{\frac{1053}{32000}+}{\left(5 \tanh\left(\frac{d x}{2}+\frac{c}{2}\right.\right.}}{d}$
parallelrisch	$\left(-47210625 i \sinh\left(d x+c\right)+1299375 i \sinh\left(3 d x+3 c\right)+12993750 \cosh\left(2 d x+2 c\right)-37056250\right) \ln\left(5 \tanh\left(\frac{d x}{2}+\frac{c}{2}\right)-4-3 i\right)+$

input
```
int(1/(5+3*I*sinh(d*x+c))^4,x,method=_RETURNVERBOSE)
```

output
```
-1/12288*I*(-86625*I*exp(4*d*x+4*c)+10395*exp(5*d*x+5*c)+218466*I*exp(2*d*
x+2*c)-239470*exp(3*d*x+3*c)-8397*I+73575*exp(d*x+c))/d/(3*exp(2*d*x+2*c)-
3-10*I*exp(d*x+c))^3-385/32768/d*ln(exp(d*x+c)-3*I)+385/32768/d*ln(-1/3*I+
exp(d*x+c))
```

Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 283 vs. $2(98) = 196$.

Time = 0.10 (sec) , antiderivative size = 283, normalized size of antiderivative = 2.28

$$\int \frac{1}{(5+3i\sinh(c+dx))^4}\,dx = \frac{1155\left(27\,e^{(6\,dx+6\,c)} - 270i\,e^{(5\,dx+5\,c)} - 981\,e^{(4\,dx+4\,c)} + 1540i\,e^{(3\,dx+3\,c)} + 981\,e^{(2\,dx+2\,c)} - 270i\,e^{(dx+c)} - 27\right)}{}$$

input
```
integrate(1/(5+3*I*sinh(d*x+c))^4,x, algorithm="fricas")
```

output
```
1/98304*(1155*(27*e^(6*d*x + 6*c) - 270*I*e^(5*d*x + 5*c) - 981*e^(4*d*x +
 4*c) + 1540*I*e^(3*d*x + 3*c) + 981*e^(2*d*x + 2*c) - 270*I*e^(d*x + c) -
 27)*log(e^(d*x + c) - 1/3*I) - 1155*(27*e^(6*d*x + 6*c) - 270*I*e^(5*d*x
+ 5*c) - 981*e^(4*d*x + 4*c) + 1540*I*e^(3*d*x + 3*c) + 981*e^(2*d*x + 2*c
) - 270*I*e^(d*x + c) - 27)*log(e^(d*x + c) - 3*I) - 83160*I*e^(5*d*x + 5*
c) - 693000*e^(4*d*x + 4*c) + 1915760*I*e^(3*d*x + 3*c) + 1747728*e^(2*d*x
 + 2*c) - 588600*I*e^(d*x + c) - 67176)/(27*d*e^(6*d*x + 6*c) - 270*I*d*e^
(5*d*x + 5*c) - 981*d*e^(4*d*x + 4*c) + 1540*I*d*e^(3*d*x + 3*c) + 981*d*e
^(2*d*x + 2*c) - 270*I*d*e^(d*x + c) - 27*d)
```

Sympy [A] (verification not implemented)

Time = 0.35 (sec) , antiderivative size = 202, normalized size of antiderivative = 1.63

$$\int \frac{1}{(5+3i\sinh(c+dx))^4}\,dx = \frac{-10395ie^{5c}e^{5dx} - 86625e^{4c}e^{4dx} + 239470ie^{3c}e^{3dx} + 218466e^{2c}e^{2dx} - 73575ie^{c}e^{dx} - 83}{331776de^{6c}e^{6dx} - 3317760ide^{5c}e^{5dx} - 12054528de^{4c}e^{4dx} + 18923520ide^{3c}e^{3dx} + 12054528de^{2c}e^{2dx} - 3317} + \frac{-\frac{385\log\left(e^{dx}-3ie^{-c}\right)}{32768} + \frac{385\log\left(e^{dx}-\frac{ie^{-c}}{3}\right)}{32768}}{d}$$

input
```
integrate(1/(5+3*I*sinh(d*x+c))**4,x)
```

output
```
(-10395*I*exp(5*c)*exp(5*d*x) - 86625*exp(4*c)*exp(4*d*x) + 239470*I*exp(3
*c)*exp(3*d*x) + 218466*exp(2*c)*exp(2*d*x) - 73575*I*exp(c)*exp(d*x) - 83
97)/(331776*d*exp(6*c)*exp(6*d*x) - 3317760*I*d*exp(5*c)*exp(5*d*x) - 1205
4528*d*exp(4*c)*exp(4*d*x) + 18923520*I*d*exp(3*c)*exp(3*d*x) + 12054528*d
*exp(2*c)*exp(2*d*x) - 3317760*I*d*exp(c)*exp(d*x) - 331776*d) + (-385*log
(exp(d*x) - 3*I*exp(-c))/32768 + 385*log(exp(d*x) - I*exp(-c)/3)/32768)/d
```

Maxima [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 152, normalized size of antiderivative = 1.23

$$\int \frac{1}{(5+3i\sinh(c+dx))^4}\,dx = -\frac{385i\arctan\left(\frac{3}{4}e^{(-dx-c)}+\frac{5}{4}i\right)}{16384\,d} - \frac{73575i\,e^{(-dx-c)}+218466\,e^{(-2\,dx-2\,c)}-239470i\,e^{(-3\,dx-3\,c)}-86625\,e^{(-4\,dx-4\,c)}+10395i\,e^{(-5\,dx-5}}{-12288\,d(-270i\,e^{(-dx-c)}-981\,e^{(-2\,dx-2\,c)}+1540i\,e^{(-3\,dx-3\,c)}+981\,e^{(-4\,dx-4\,c)}-270i\,e^{(-5\,dx-5\,c)}-27}$$

input
```
integrate(1/(5+3*I*sinh(d*x+c))^4,x, algorithm="maxima")
```

output
```
-385/16384*I*arctan(3/4*e^(-d*x - c) + 5/4*I)/d - (73575*I*e^(-d*x - c) +
218466*e^(-2*d*x - 2*c) - 239470*I*e^(-3*d*x - 3*c) - 86625*e^(-4*d*x - 4*
c) + 10395*I*e^(-5*d*x - 5*c) - 8397)/(d*(3317760*I*e^(-d*x - c) + 1205452
8*e^(-2*d*x - 2*c) - 18923520*I*e^(-3*d*x - 3*c) - 12054528*e^(-4*d*x - 4*
c) + 3317760*I*e^(-5*d*x - 5*c) + 331776*e^(-6*d*x - 6*c) - 331776))
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 109, normalized size of antiderivative = 0.88

$$\int \frac{1}{(5+3i\sinh(c+dx))^4}\,dx = -\frac{\frac{8\left(10395i\,e^{(5\,dx+5\,c)}+86625\,e^{(4\,dx+4\,c)}-239470i\,e^{(3\,dx+3\,c)}-218466\,e^{(2\,dx+2\,c)}+73575i\,e^{(dx+c)}+8397\right)}{\left(3\,e^{(2\,dx+2\,c)}-10i\,e^{(dx+c)}-3\right)^3}-1155\log\left(3\,e^{(dx+c)}-i\right)}{98304\,d}$$

input
```
integrate(1/(5+3*I*sinh(d*x+c))^4,x, algorithm="giac")
```

output
```
-1/98304*(8*(10395*I*e^(5*d*x + 5*c) + 86625*e^(4*d*x + 4*c) - 239470*I*e^
(3*d*x + 3*c) - 218466*e^(2*d*x + 2*c) + 73575*I*e^(d*x + c) + 8397)/(3*e^
(2*d*x + 2*c) - 10*I*e^(d*x + c) - 3)^3 - 1155*log(3*e^(d*x + c) - I) + 11
55*log(e^(d*x + c) - 3*I))/d
```

Mupad [B] (verification not implemented)

Time = 3.20 (sec) , antiderivative size = 232, normalized size of antiderivative = 1.87

$$\int \frac{1}{(5+3i\sinh(c+dx))^4}\,dx = \frac{\frac{1925}{36864\,d}+\frac{e^{c+d\,x}\,385i}{12288\,d}}{1-e^{2\,c+2\,d\,x}+\frac{e^{c+d\,x}\,10i}{3}} + \frac{\frac{41}{486\,d}+\frac{e^{c+d\,x}\,365i}{1458\,d}}{\frac{109\,e^{4\,c+4\,d\,x}}{3}-\frac{109\,e^{2\,c+2\,d\,x}}{3}-e^{6\,c+6\,d\,x}+1+e^{c+d\,x}\,10i-\frac{e^{3\,c+3\,d\,x}\,1540i}{27}+e^{5\,c+5\,d\,x}\,10i} - \frac{385\,\ln\left(-\frac{385\,e^{c+d\,x}}{4}+\frac{1155}{4}i\right)}{32768\,d} + \frac{385\,\ln\left(\frac{3465\,e^{c+d\,x}}{4}-\frac{1155}{4}i\right)}{32768\,d} - \frac{\frac{3461}{31104\,d}+\frac{e^{c+d\,x}\,385i}{10368\,d}}{e^{4\,c+4\,d\,x}-\frac{118\,e^{2\,c+2\,d\,x}}{9}+1+\frac{e^{c+d\,x}\,20i}{3}-\frac{e^{3\,c+3\,d\,x}\,20i}{3}}$$

input
```
int(1/(sinh(c + d*x)*3i + 5)^4,x)
```

output
```
((exp(c + d*x)*385i)/(12288*d) + 1925/(36864*d))/((exp(c + d*x)*10i)/3 - e
xp(2*c + 2*d*x) + 1) + ((exp(c + d*x)*365i)/(1458*d) + 41/(486*d))/(exp(c
+ d*x)*10i - (109*exp(2*c + 2*d*x))/3 - (exp(3*c + 3*d*x)*1540i)/27 + (109
*exp(4*c + 4*d*x))/3 + exp(5*c + 5*d*x)*10i - exp(6*c + 6*d*x) + 1) - (385
*log(1155i/4 - (385*exp(c + d*x))/4))/(32768*d) + (385*log((3465*exp(c + d
*x))/4 - 1155i/4))/(32768*d) - ((exp(c + d*x)*385i)/(10368*d) + 3461/(3110
4*d))/((exp(c + d*x)*20i)/3 - (118*exp(2*c + 2*d*x))/9 - (exp(3*c + 3*d*x)
*20i)/3 + exp(4*c + 4*d*x) + 1)
```

Reduce [F]

$$\int \frac{1}{(5+3i\sinh(c+dx))^4}\,dx$$
$$= \int \frac{1}{81\sinh(dx+c)^4 - 540\sinh(dx+c)^3 i - 1350\sinh(dx+c)^2 + 1500\sinh(dx+c)\,i + 625}dx$$

input
```
int(1/(5+3*I*sinh(d*x+c))^4,x)
```

output
```
int(1/(81*sinh(c + d*x)**4 - 540*sinh(c + d*x)**3*i - 1350*sinh(c + d*x)**
2 + 1500*sinh(c + d*x)*i + 625),x)
```

3.96 $\int (a+b\sinh(c+dx))^5\,dx$

Optimal result

Integrand size = 12, antiderivative size = 183

$$\begin{aligned}\int (a+b\sinh(c+dx))^5\,dx &= \frac{1}{8}a\left(8a^4-40a^2b^2+15b^4\right)x \\ &+ \frac{b(107a^4-192a^2b^2+16b^4)\cosh(c+dx)}{30d} \\ &+ \frac{7ab^2(22a^2-23b^2)\cosh(c+dx)\sinh(c+dx)}{120d} \\ &+ \frac{b(47a^2-16b^2)\cosh(c+dx)(a+b\sinh(c+dx))^2}{60d} \\ &+ \frac{9ab\cosh(c+dx)(a+b\sinh(c+dx))^3}{20d} \\ &+ \frac{b\cosh(c+dx)(a+b\sinh(c+dx))^4}{5d}\end{aligned}$$

output
```
1/8*a*(8*a^4-40*a^2*b^2+15*b^4)*x+1/30*b*(107*a^4-192*a^2*b^2+16*b^4)*cosh
(d*x+c)/d+7/120*a*b^2*(22*a^2-23*b^2)*cosh(d*x+c)*sinh(d*x+c)/d+1/60*b*(47
*a^2-16*b^2)*cosh(d*x+c)*(a+b*sinh(d*x+c))^2/d+9/20*a*b*cosh(d*x+c)*(a+b*s
inh(d*x+c))^3/d+1/5*b*cosh(d*x+c)*(a+b*sinh(d*x+c))^4/d
```

Mathematica [A] (verified)

Time = 0.90 (sec) , antiderivative size = 138, normalized size of antiderivative = 0.75

$$\int (a+b\sinh(c+dx))^5\,dx$$
$$=\frac{300b(8a^4-12a^2b^2+b^4)\cosh(c+dx)+50(8a^2b^3-b^5)\cosh(3(c+dx))+6b^5\cosh(5(c+dx))+15a(4(8\cdots}{480d}$$

input
```
Integrate[(a + b*Sinh[c + d*x])^5,x]
```

output
```
(300*b*(8*a^4 - 12*a^2*b^2 + b^4)*Cosh[c + d*x] + 50*(8*a^2*b^3 - b^5)*Cos
h[3*(c + d*x)] + 6*b^5*Cosh[5*(c + d*x)] + 15*a*(4*(8*a^4 - 40*a^2*b^2 + 1
5*b^4)*(c + d*x) + 40*(2*a^2*b^2 - b^4)*Sinh[2*(c + d*x)] + 5*b^4*Sinh[4*(
c + d*x)]))/(480*d)
```

Rubi [A] (verified)

Time = 0.74 (sec) , antiderivative size = 196, normalized size of antiderivative = 1.07, number of steps used = 8, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.667, Rules used = {3042, 3135, 3042, 3232, 3042, 3232, 3042, 3213}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a+b\sinh(c+dx))^5\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int (a-ib\sin(ic+idx))^5dx$$

$$\Big\downarrow \text{3135}$$

$$\frac{1}{5}\int (a+b\sinh(c+dx))^3\left(5a^2+9b\sinh(c+dx)a-4b^2\right)dx+\frac{b\cosh(c+dx)(a+b\sinh(c+dx))^4}{5d}$$

$$\Big\downarrow \text{3042}$$

$$\frac{b\cosh(c+dx)(a+b\sinh(c+dx))^4}{5d}+\frac{1}{5}\int(a-ib\sin(ic+idx))^3\left(5a^2-9ib\sin(ic+idx)a-4b^2\right)dx$$

$\big\downarrow$ 3232

$$\frac{1}{5}\left(\frac{1}{4}\int(a+b\sinh(c+dx))^2\left(a\left(20a^2-43b^2\right)+b\left(47a^2-16b^2\right)\sinh(c+dx)\right)dx+\frac{9ab\cosh(c+dx)(a+b\sinh(c+dx))^3}{4d}\right)+\frac{b\cosh(c+dx)(a+b\sinh(c+dx))^4}{5d}$$

$\big\downarrow$ 3042

$$\frac{b\cosh(c+dx)(a+b\sinh(c+dx))^4}{5d}+\frac{1}{5}\left(\frac{9ab\cosh(c+dx)(a+b\sinh(c+dx))^3}{4d}+\frac{1}{4}\int(a-ib\sin(ic+idx))^2\left(a\left(20a^2-43b^2\right)-ib\left(47a^2-16b^2\right)\sin(ic\right.\right.$$

$\big\downarrow$ 3232

$$\frac{1}{5}\left(\frac{1}{4}\left(\frac{1}{3}\int(a+b\sinh(c+dx))\left(60a^4-223b^2a^2+7b\left(22a^2-23b^2\right)\sinh(c+dx)a+32b^4\right)dx+\frac{b\left(47a^2-16b^2\right)c}{}\right.\right.$$
$$\frac{b\cosh(c+dx)(a+b\sinh(c+dx))^4}{5d}$$

$\big\downarrow$ 3042

$$\frac{b\cosh(c+dx)(a+b\sinh(c+dx))^4}{5d}+\frac{1}{5}\left(\frac{9ab\cosh(c+dx)(a+b\sinh(c+dx))^3}{4d}+\frac{1}{4}\left(\frac{b\left(47a^2-16b^2\right)\cosh(c+dx)(a+b\sinh(c+dx))^2}{3d}+\frac{1}{3}\int(a-ib\right.\right.$$

$\big\downarrow$ 3213

$$\frac{1}{5}\left(\frac{1}{4}\left(\frac{b\left(47a^2-16b^2\right)\cosh(c+dx)(a+b\sinh(c+dx))^2}{3d}+\frac{1}{3}\left(\frac{7ab^2\left(22a^2-23b^2\right)\sinh(c+dx)\cosh(c+dx)}{2d}+\frac{2}{}\right.\right.\right.$$
$$\frac{b\cosh(c+dx)(a+b\sinh(c+dx))^4}{5d}$$

input
```
Int[(a + b*Sinh[c + d*x])^5,x]
```

output
```
(b*Cosh[c + d*x]*(a + b*Sinh[c + d*x])^4)/(5*d) + ((9*a*b*Cosh[c + d*x]*(a
 + b*Sinh[c + d*x])^3)/(4*d) + ((b*(47*a^2 - 16*b^2)*Cosh[c + d*x]*(a + b*
Sinh[c + d*x])^2)/(3*d) + ((15*a*(8*a^4 - 40*a^2*b^2 + 15*b^4)*x)/2 + (2*b
*(107*a^4 - 192*a^2*b^2 + 16*b^4)*Cosh[c + d*x])/d + (7*a*b^2*(22*a^2 - 23
*b^2)*Cosh[c + d*x]*Sinh[c + d*x])/(2*d))/3)/4)/5
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3135
```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos
[c + d*x]*((a + b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Simp[1/n   Int[(a + b*
Sin[c + d*x])^(n - 2)*Simp[a^2*n + b^2*(n - 1) + a*b*(2*n - 1)*Sin[c + d*x]
, x], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[n, 1] &&
 IntegerQ[2*n]
```

rule 3213
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)
*(x_)]), x_Symbol] :> Simp[(2*a*c + b*d)*(x/2), x] + (-Simp[(b*c + a*d)*(Co
s[e + f*x]/f), x] - Simp[b*d*Cos[e + f*x]*(Sin[e + f*x]/(2*f)), x]) /; Free
Q[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

rule 3232
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Simp[(-d)*Cos[e + f*x]*((a + b*Sin[e + f*x])^m/(
f*(m + 1))), x] + Simp[1/(m + 1)   Int[(a + b*Sin[e + f*x])^(m - 1)*Simp[b*
d*m + a*c*(m + 1) + (a*d*m + b*c*(m + 1))*Sin[e + f*x], x], x], x] /; FreeQ
[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && GtQ[m,
 0] && IntegerQ[2*m]
```

Maple [A] (verified)

Time = 77.63 (sec) , antiderivative size = 155, normalized size of antiderivative = 0.85

method	result
derivativedivides	$\frac{b^5\left(\frac{8}{15}+\frac{\sinh(dx+c)^4}{5}-\frac{4\sinh(dx+c)^2}{15}\right)\cosh(dx+c)+5a\,b^4\left(\left(\frac{\sinh(dx+c)^3}{4}-\frac{3\sinh(dx+c)}{8}\right)\cosh(dx+c)+\frac{3dx}{8}+\frac{3c}{8}\right)+10a^2}{d}$
default	$\frac{b^5\left(\frac{8}{15}+\frac{\sinh(dx+c)^4}{5}-\frac{4\sinh(dx+c)^2}{15}\right)\cosh(dx+c)+5a\,b^4\left(\left(\frac{\sinh(dx+c)^3}{4}-\frac{3\sinh(dx+c)}{8}\right)\cosh(dx+c)+\frac{3dx}{8}+\frac{3c}{8}\right)+10a^2}{d}$
parallelrisch	$\frac{(400a^2b^3-50b^5)\cosh(3dx+3c)+(1200a^3b^2-600a\,b^4)\sinh(2dx+2c)+6b^5\cosh(5dx+5c)+75a\,b^4\sinh(4dx+4c)+(2400}{480d}$
parts	$a^5x+\frac{b^5\left(\frac{8}{15}+\frac{\sinh(dx+c)^4}{5}-\frac{4\sinh(dx+c)^2}{15}\right)\cosh(dx+c)}{d}+\frac{5a^4b\cosh(dx+c)}{d}+\frac{10a^3b^2\left(\frac{\sinh(dx+c)\cosh(dx+c)}{2}-\frac{d}{2}\right.}{d}$
risch	$a^5x-5a^3b^2x+\frac{15a\,b^4x}{8}+\frac{b^5e^{5dx+5c}}{160d}+\frac{5a\,b^4e^{4dx+4c}}{64d}+\frac{5b^3e^{3dx+3c}a^2}{12d}-\frac{5b^5e^{3dx+3c}}{96d}+\frac{5a^3b^2e^{2dx+2c}}{4d}-$
orering	Expression too large to display

input
```
int((a+b*sinh(d*x+c))^5,x,method=_RETURNVERBOSE)
```

output
```
1/d*(b^5*(8/15+1/5*sinh(d*x+c)^4-4/15*sinh(d*x+c)^2)*cosh(d*x+c)+5*a*b^4*(
(1/4*sinh(d*x+c)^3-3/8*sinh(d*x+c))*cosh(d*x+c)+3/8*d*x+3/8*c)+10*a^2*b^3*
(-2/3+1/3*sinh(d*x+c)^2)*cosh(d*x+c)+10*a^3*b^2*(1/2*sinh(d*x+c)*cosh(d*x+
c)-1/2*d*x-1/2*c)+5*a^4*b*cosh(d*x+c)+a^5*(d*x+c))
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 223, normalized size of antiderivative = 1.22

$$\int (a+b\sinh(c+dx))^5\,dx$$
$$=\frac{3\,b^5\cosh(dx+c)^5+15\,b^5\cosh(dx+c)\sinh(dx+c)^4+150\,ab^4\cosh(dx+c)\sinh(dx+c)^3+25\,(8\,a^2b^3}{}$$

input
```
integrate((a+b*sinh(d*x+c))^5,x, algorithm="fricas")
```

output
```
1/240*(3*b^5*cosh(d*x + c)^5 + 15*b^5*cosh(d*x + c)*sinh(d*x + c)^4 + 150*
a*b^4*cosh(d*x + c)*sinh(d*x + c)^3 + 25*(8*a^2*b^3 - b^5)*cosh(d*x + c)^3
 + 30*(8*a^5 - 40*a^3*b^2 + 15*a*b^4)*d*x + 15*(2*b^5*cosh(d*x + c)^3 + 5*
(8*a^2*b^3 - b^5)*cosh(d*x + c))*sinh(d*x + c)^2 + 150*(8*a^4*b - 12*a^2*b
^3 + b^5)*cosh(d*x + c) + 150*(a*b^4*cosh(d*x + c)^3 + 4*(2*a^3*b^2 - a*b^
4)*cosh(d*x + c))*sinh(d*x + c))/d
```

Sympy [A] (verification not implemented)

Time = 0.35 (sec) , antiderivative size = 314, normalized size of antiderivative = 1.72

$$\int (a + b\sinh(c + dx))^5 \, dx$$
$$= \begin{cases} a^5x + \frac{5a^4b\cosh(c+dx)}{d} + 5a^3b^2x\sinh^2(c+dx) - 5a^3b^2x\cosh^2(c+dx) + \frac{5a^3b^2\sinh(c+dx)\cosh(c+dx)}{d} + \frac{10a^2b^3\,\mathrm{s}}{} \\ x(a + b\sinh(c))^5 \end{cases}$$

input
```
integrate((a+b*sinh(d*x+c))**5,x)
```

output
```
Piecewise((a**5*x + 5*a**4*b*cosh(c + d*x)/d + 5*a**3*b**2*x*sinh(c + d*x)
**2 - 5*a**3*b**2*x*cosh(c + d*x)**2 + 5*a**3*b**2*sinh(c + d*x)*cosh(c +
d*x)/d + 10*a**2*b**3*sinh(c + d*x)**2*cosh(c + d*x)/d - 20*a**2*b**3*cosh
(c + d*x)**3/(3*d) + 15*a*b**4*x*sinh(c + d*x)**4/8 - 15*a*b**4*x*sinh(c +
 d*x)**2*cosh(c + d*x)**2/4 + 15*a*b**4*x*cosh(c + d*x)**4/8 + 25*a*b**4*s
inh(c + d*x)**3*cosh(c + d*x)/(8*d) - 15*a*b**4*sinh(c + d*x)*cosh(c + d*x
)**3/(8*d) + b**5*sinh(c + d*x)**4*cosh(c + d*x)/d - 4*b**5*sinh(c + d*x)*
*2*cosh(c + d*x)**3/(3*d) + 8*b**5*cosh(c + d*x)**5/(15*d), Ne(d, 0)), (x*
(a + b*sinh(c))**5, True))
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 272, normalized size of antiderivative = 1.49

$$\int (a+b\sinh(c+dx))^5\,dx$$
$$= \frac{5}{64}ab^4\left(24x+\frac{e^{(4dx+4c)}}{d}-\frac{8e^{(2dx+2c)}}{d}+\frac{8e^{(-2dx-2c)}}{d}-\frac{e^{(-4dx-4c)}}{d}\right) - \frac{5}{4}a^3b^2\left(4x-\frac{e^{(2dx+2c)}}{d}+\frac{e^{(-2dx-2c)}}{d}\right)+a^5x + \frac{1}{480}b^5\left(\frac{3e^{(5dx+5c)}}{d}-\frac{25e^{(3dx+3c)}}{d}+\frac{150e^{(dx+c)}}{d}+\frac{150e^{(-dx-c)}}{d}-\frac{25e^{(-3dx-3c)}}{d}+\frac{3e^{(-5dx-5c)}}{d}\right) + \frac{5}{12}a^2b^3\left(\frac{e^{(3dx+3c)}}{d}-\frac{9e^{(dx+c)}}{d}-\frac{9e^{(-dx-c)}}{d}+\frac{e^{(-3dx-3c)}}{d}\right)+\frac{5a^4b\cosh(dx+c)}{d}$$

input
```
integrate((a+b*sinh(d*x+c))^5,x, algorithm="maxima")
```

output
```
5/64*a*b^4*(24*x + e^(4*d*x + 4*c)/d - 8*e^(2*d*x + 2*c)/d + 8*e^(-2*d*x -
 2*c)/d - e^(-4*d*x - 4*c)/d) - 5/4*a^3*b^2*(4*x - e^(2*d*x + 2*c)/d + e^(
-2*d*x - 2*c)/d) + a^5*x + 1/480*b^5*(3*e^(5*d*x + 5*c)/d - 25*e^(3*d*x +
3*c)/d + 150*e^(d*x + c)/d + 150*e^(-d*x - c)/d - 25*e^(-3*d*x - 3*c)/d +
3*e^(-5*d*x - 5*c)/d) + 5/12*a^2*b^3*(e^(3*d*x + 3*c)/d - 9*e^(d*x + c)/d
- 9*e^(-d*x - c)/d + e^(-3*d*x - 3*c)/d) + 5*a^4*b*cosh(d*x + c)/d
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 269, normalized size of antiderivative = 1.47

$$\int (a+b\sinh(c+dx))^5\,dx = \frac{b^5e^{(5dx+5c)}}{160d}+\frac{5ab^4e^{(4dx+4c)}}{64d}-\frac{5ab^4e^{(-4dx-4c)}}{64d}+\frac{b^5e^{(-5dx-5c)}}{160d} + \frac{1}{8}(8a^5-40a^3b^2+15ab^4)x+\frac{5(8a^2b^3-b^5)e^{(3dx+3c)}}{96d} + \frac{5(2a^3b^2-ab^4)e^{(2dx+2c)}}{8d}+\frac{5(8a^4b-12a^2b^3+b^5)e^{(dx+c)}}{16d} + \frac{5(8a^4b-12a^2b^3+b^5)e^{(-dx-c)}}{16d} - \frac{5(2a^3b^2-ab^4)e^{(-2dx-2c)}}{8d}+\frac{5(8a^2b^3-b^5)e^{(-3dx-3c)}}{96d}$$

input
```
integrate((a+b*sinh(d*x+c))^5,x, algorithm="giac")
```

output
```
1/160*b^5*e^(5*d*x + 5*c)/d + 5/64*a*b^4*e^(4*d*x + 4*c)/d - 5/64*a*b^4*e^
(-4*d*x - 4*c)/d + 1/160*b^5*e^(-5*d*x - 5*c)/d + 1/8*(8*a^5 - 40*a^3*b^2
+ 15*a*b^4)*x + 5/96*(8*a^2*b^3 - b^5)*e^(3*d*x + 3*c)/d + 5/8*(2*a^3*b^2
- a*b^4)*e^(2*d*x + 2*c)/d + 5/16*(8*a^4*b - 12*a^2*b^3 + b^5)*e^(d*x + c)
/d + 5/16*(8*a^4*b - 12*a^2*b^3 + b^5)*e^(-d*x - c)/d - 5/8*(2*a^3*b^2 - a
*b^4)*e^(-2*d*x - 2*c)/d + 5/96*(8*a^2*b^3 - b^5)*e^(-3*d*x - 3*c)/d
```

Mupad [B] (verification not implemented)

Time = 0.64 (sec) , antiderivative size = 160, normalized size of antiderivative = 0.87

$$\int (a+b\sinh(c+dx))^5\,dx$$
$$= \frac{75\,b^5\cosh(c+d\,x)-\frac{25\,b^5\cosh(3\,c+3\,d\,x)}{2}+\frac{3\,b^5\cosh(5\,c+5\,d\,x)}{2}-900\,a^2\,b^3\cosh(c+d\,x)-150\,a\,b^4\sinh(2\,c+2\,d$$

input
```
int((a + b*sinh(c + d*x))^5,x)
```

output
```
(75*b^5*cosh(c + d*x) - (25*b^5*cosh(3*c + 3*d*x))/2 + (3*b^5*cosh(5*c + 5
*d*x))/2 - 900*a^2*b^3*cosh(c + d*x) - 150*a*b^4*sinh(2*c + 2*d*x) + (75*a
*b^4*sinh(4*c + 4*d*x))/4 + 100*a^2*b^3*cosh(3*c + 3*d*x) + 300*a^3*b^2*si
nh(2*c + 2*d*x) + 600*a^4*b*cosh(c + d*x) + 120*a^5*d*x + 225*a*b^4*d*x -
600*a^3*b^2*d*x)/(120*d)
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 354, normalized size of antiderivative = 1.93

$$\int (a+b\sinh(c+dx))^5\,dx$$
$$= \frac{6e^{10dx+10c}b^5+75e^{9dx+9c}a\,b^4+400e^{8dx+8c}a^2b^3-50e^{8dx+8c}b^5+1200e^{7dx+7c}a^3b^2-600e^{7dx+7c}a\,b^4+2400e^{6}$$

input
```
int((a+b*sinh(d*x+c))^5,x)
```

output

```
(6*e**(10*c + 10*d*x)*b**5 + 75*e**(9*c + 9*d*x)*a*b**4 + 400*e**(8*c + 8*
d*x)*a**2*b**3 - 50*e**(8*c + 8*d*x)*b**5 + 1200*e**(7*c + 7*d*x)*a**3*b**
2 - 600*e**(7*c + 7*d*x)*a*b**4 + 2400*e**(6*c + 6*d*x)*a**4*b - 3600*e**(
6*c + 6*d*x)*a**2*b**3 + 300*e**(6*c + 6*d*x)*b**5 + 960*e**(5*c + 5*d*x)*
a**5*d*x - 4800*e**(5*c + 5*d*x)*a**3*b**2*d*x + 1800*e**(5*c + 5*d*x)*a*b
**4*d*x + 2400*e**(4*c + 4*d*x)*a**4*b - 3600*e**(4*c + 4*d*x)*a**2*b**3 +
 300*e**(4*c + 4*d*x)*b**5 - 1200*e**(3*c + 3*d*x)*a**3*b**2 + 600*e**(3*c
 + 3*d*x)*a*b**4 + 400*e**(2*c + 2*d*x)*a**2*b**3 - 50*e**(2*c + 2*d*x)*b*
*5 - 75*e**(c + d*x)*a*b**4 + 6*b**5)/(960*e**(5*c + 5*d*x)*d)
```

3.97 $\int (a+b\sinh(c+dx))^4\,dx$

Optimal result

Integrand size = 12, antiderivative size = 137

$$\begin{aligned}\int (a+b\sinh(c+dx))^4\,dx &= \frac{1}{8}\left(8a^4-24a^2b^2+3b^4\right)x+\frac{ab(19a^2-16b^2)\cosh(c+dx)}{6d}\\ &+\frac{b^2(26a^2-9b^2)\cosh(c+dx)\sinh(c+dx)}{24d}\\ &+\frac{7ab\cosh(c+dx)(a+b\sinh(c+dx))^2}{12d}\\ &+\frac{b\cosh(c+dx)(a+b\sinh(c+dx))^3}{4d}\end{aligned}$$

output
```
1/8*(8*a^4-24*a^2*b^2+3*b^4)*x+1/6*a*b*(19*a^2-16*b^2)*cosh(d*x+c)/d+1/24*
b^2*(26*a^2-9*b^2)*cosh(d*x+c)*sinh(d*x+c)/d+7/12*a*b*cosh(d*x+c)*(a+b*sin
h(d*x+c))^2/d+1/4*b*cosh(d*x+c)*(a+b*sinh(d*x+c))^3/d
```

Mathematica [A] (verified)

Time = 0.46 (sec) , antiderivative size = 108, normalized size of antiderivative = 0.79

$$\int (a+b\sinh(c+dx))^4\,dx$$
$$= \frac{96ab(4a^2-3b^2)\cosh(c+dx)+32ab^3\cosh(3(c+dx))+3(4(8a^4-24a^2b^2+3b^4)(c+dx)+8(6a^2b^2-b^4)\sinh(2(c+dx))+b^4\sinh(4(c+dx)))}{96d}$$

input
```
Integrate[(a + b*Sinh[c + d*x])^4,x]
```

output
```
(96*a*b*(4*a^2 - 3*b^2)*Cosh[c + d*x] + 32*a*b^3*Cosh[3*(c + d*x)] + 3*(4*
(8*a^4 - 24*a^2*b^2 + 3*b^4)*(c + d*x) + 8*(6*a^2*b^2 - b^4)*Sinh[2*(c + d
*x)] + b^4*Sinh[4*(c + d*x)]))/(96*d)
```

Rubi [A] (verified)

Time = 0.51 (sec) , antiderivative size = 145, normalized size of antiderivative = 1.06, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$, Rules used = {3042, 3135, 3042, 3232, 3042, 3213}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a+b\sinh(c+dx))^4\,dx$$
$$\Big\downarrow \text{3042}$$
$$\int (a-ib\sin(ic+idx))^4dx$$
$$\Big\downarrow \text{3135}$$
$$\frac{1}{4}\int (a+b\sinh(c+dx))^2\left(4a^2+7b\sinh(c+dx)a-3b^2\right)dx+\frac{b\cosh(c+dx)(a+b\sinh(c+dx))^3}{4d}$$
$$\Big\downarrow \text{3042}$$

$$\frac{b\cosh(c+dx)(a+b\sinh(c+dx))^3}{4d}+\frac{1}{4}\int(a-ib\sin(ic+ idx))^2\left(4a^2-7ib\sin(ic+idx)a-3b^2\right)dx$$

$\Big\downarrow$ 3232

$$\frac{1}{4}\left(\frac{1}{3}\int(a+b\sinh(c+dx))\left(a\left(12a^2-23b^2\right)+b\left(26a^2-9b^2\right)\sinh(c+dx)\right)dx+\frac{7ab\cosh(c+dx)(a+b\sinh(c+\right.$$
$$\frac{b\cosh(c+dx)(a+b\sinh(c+dx))^3}{4d}$$

$\Big\downarrow$ 3042

$$\frac{b\cosh(c+dx)(a+b\sinh(c+dx))^3}{4d}+$$
$$\frac{1}{4}\left(\frac{7ab\cosh(c+dx)(a+b\sinh(c+dx))^2}{3d}+\frac{1}{3}\int(a-ib\sin(ic+idx))\left(a\left(12a^2-23b^2\right)-ib\left(26a^2-9b^2\right)\sin(ic+\right.\right.$$

$\Big\downarrow$ 3213

$$\frac{1}{4}\left(\frac{1}{3}\left(\frac{2ab\left(19a^2-16b^2\right)\cosh(c+dx)}{d}+\frac{b^2\left(26a^2-9b^2\right)\sinh(c+dx)\cosh(c+dx)}{2d}+\frac{3}{2}x\left(8a^4-24a^2b^2+3b^4\right)\right)\right.$$
$$\frac{b\cosh(c+dx)(a+b\sinh(c+dx))^3}{4d}$$

input
```
Int[(a + b*Sinh[c + d*x])^4,x]
```

output
```
(b*Cosh[c + d*x]*(a + b*Sinh[c + d*x])^3)/(4*d) + ((7*a*b*Cosh[c + d*x]*(a
 + b*Sinh[c + d*x])^2)/(3*d) + ((3*(8*a^4 - 24*a^2*b^2 + 3*b^4)*x)/2 + (2*
a*b*(19*a^2 - 16*b^2)*Cosh[c + d*x])/d + (b^2*(26*a^2 - 9*b^2)*Cosh[c + d*
x]*Sinh[c + d*x])/(2*d))/3)/4
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3135
```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos
[c + d*x]*((a + b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Simp[1/n   Int[(a + b*
Sin[c + d*x])^(n - 2)*Simp[a^2*n + b^2*(n - 1) + a*b*(2*n - 1)*Sin[c + d*x]
, x], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[n, 1] &&
 IntegerQ[2*n]
```

rule 3213
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)
*(x_)]), x_Symbol] :> Simp[(2*a*c + b*d)*(x/2), x] + (-Simp[(b*c + a*d)*(Co
s[e + f*x]/f), x] - Simp[b*d*Cos[e + f*x]*(Sin[e + f*x]/(2*f)), x]) /; Free
Q[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

rule 3232
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Simp[(-d)*Cos[e + f*x]*((a + b*Sin[e + f*x])^m/(
f*(m + 1))), x] + Simp[1/(m + 1)   Int[(a + b*Sin[e + f*x])^(m - 1)*Simp[b*
d*m + a*c*(m + 1) + (a*d*m + b*c*(m + 1))*Sin[e + f*x], x], x], x] /; FreeQ
[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && GtQ[m,
 0] && IntegerQ[2*m]
```

Maple [A] (verified)

Time = 10.64 (sec) , antiderivative size = 119, normalized size of antiderivative = 0.87

method	result
derivativedivides	$\frac{b^4\left(\left(\frac{\sinh(dx+c)^3}{4}-\frac{3\sinh(dx+c)}{8}\right)\cosh(dx+c)+\frac{3dx}{8}+\frac{3c}{8}\right)+4a\,b^3\left(-\frac{2}{3}+\frac{\sinh(dx+c)^2}{3}\right)\cosh(dx+c)+6a^2b^2\left(\frac{\sinh(dx+c)\,c\ldots}{2}\right.}{d}$
default	$\frac{b^4\left(\left(\frac{\sinh(dx+c)^3}{4}-\frac{3\sinh(dx+c)}{8}\right)\cosh(dx+c)+\frac{3dx}{8}+\frac{3c}{8}\right)+4a\,b^3\left(-\frac{2}{3}+\frac{\sinh(dx+c)^2}{3}\right)\cosh(dx+c)+6a^2b^2\left(\frac{\sinh(dx+c)\,c\ldots}{2}\right.}{d}$
parts	$x\,a^4+\frac{b^4\left(\left(\frac{\sinh(dx+c)^3}{4}-\frac{3\sinh(dx+c)}{8}\right)\cosh(dx+c)+\frac{3dx}{8}+\frac{3c}{8}\right)}{d}+\frac{4a^3b\cosh(dx+c)}{d}+\frac{6a^2b^2\left(\frac{\sinh(dx+c)\cosh(dx\ldots}{2}\right.}{d}$
parallelrisch	$\frac{96a^4dx-288a^2b^2dx+36b^4dx+384a^3b\cosh(dx+c)-288a\,b^3\cosh(dx+c)+32a\,b^3\cosh(3dx+3c)+3b^4\sinh(4dx+4c)+14\ldots}{96d}$
risch	$x\,a^4-3x\,a^2b^2+\frac{3x\,b^4}{8}+\frac{b^4\mathrm{e}^{4dx+4c}}{64d}+\frac{a\,b^3\mathrm{e}^{3dx+3c}}{6d}+\frac{3b^2\mathrm{e}^{2dx+2c}a^2}{4d}-\frac{b^4\mathrm{e}^{2dx+2c}}{8d}+\frac{2a^3b\,\mathrm{e}^{dx+c}}{d}-\frac{3a\,b\ldots}{}$

input
```
int((a+b*sinh(d*x+c))^4,x,method=_RETURNVERBOSE)
```

output
```
1/d*(b^4*((1/4*sinh(d*x+c)^3-3/8*sinh(d*x+c))*cosh(d*x+c)+3/8*d*x+3/8*c)+4
*a*b^3*(-2/3+1/3*sinh(d*x+c)^2)*cosh(d*x+c)+6*a^2*b^2*(1/2*sinh(d*x+c)*cos
h(d*x+c)-1/2*d*x-1/2*c)+4*a^3*b*cosh(d*x+c)+a^4*(d*x+c))
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 146, normalized size of antiderivative = 1.07

$$\int (a+b\sinh(c+dx))^4\,dx$$
$$=\frac{3\,b^4\cosh\left(dx+c\right)\sinh\left(dx+c\right)^3+8\,ab^3\cosh\left(dx+c\right)^3+24\,ab^3\cosh\left(dx+c\right)\sinh\left(dx+c\right)^2+3\,(8\,a^4-2}{}$$

input
```
integrate((a+b*sinh(d*x+c))^4,x, algorithm="fricas")
```

output
```
1/24*(3*b^4*cosh(d*x + c)*sinh(d*x + c)^3 + 8*a*b^3*cosh(d*x + c)^3 + 24*a
*b^3*cosh(d*x + c)*sinh(d*x + c)^2 + 3*(8*a^4 - 24*a^2*b^2 + 3*b^4)*d*x +
24*(4*a^3*b - 3*a*b^3)*cosh(d*x + c) + 3*(b^4*cosh(d*x + c)^3 + 4*(6*a^2*b
^2 - b^4)*cosh(d*x + c))*sinh(d*x + c))/d
```

Sympy [A] (verification not implemented)

Time = 0.33 (sec) , antiderivative size = 240, normalized size of antiderivative = 1.75

$$\int (a+b\sinh(c+dx))^4\,dx$$
$$=\begin{cases} a^4x+\frac{4a^3b\cosh\left(c+dx\right)}{d}+3a^2b^2x\sinh^2\left(c+dx\right)-3a^2b^2x\cosh^2\left(c+dx\right)+\frac{3a^2b^2\sinh\left(c+dx\right)\cosh\left(c+dx\right)}{d}+\frac{4ab^3\sin}{} \\ x(a+b\sinh\left(c\right))^4 \end{cases}$$

input
```
integrate((a+b*sinh(d*x+c))**4,x)
```

output
```
Piecewise((a**4*x + 4*a**3*b*cosh(c + d*x)/d + 3*a**2*b**2*x*sinh(c + d*x)
**2 - 3*a**2*b**2*x*cosh(c + d*x)**2 + 3*a**2*b**2*sinh(c + d*x)*cosh(c +
d*x)/d + 4*a*b**3*sinh(c + d*x)**2*cosh(c + d*x)/d - 8*a*b**3*cosh(c + d*x
)**3/(3*d) + 3*b**4*x*sinh(c + d*x)**4/8 - 3*b**4*x*sinh(c + d*x)**2*cosh(
c + d*x)**2/4 + 3*b**4*x*cosh(c + d*x)**4/8 + 5*b**4*sinh(c + d*x)**3*cosh
(c + d*x)/(8*d) - 3*b**4*sinh(c + d*x)*cosh(c + d*x)**3/(8*d), Ne(d, 0)),
(x*(a + b*sinh(c))**4, True))
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 182, normalized size of antiderivative = 1.33

$$\int (a + b\sinh(c + dx))^4\,dx = \frac{1}{64}b^4\left(24\,x + \frac{e^{(4\,dx+4\,c)}}{d} - \frac{8\,e^{(2\,dx+2\,c)}}{d} + \frac{8\,e^{(-2\,dx-2\,c)}}{d} - \frac{e^{(-4\,dx-4\,c)}}{d}\right) - \frac{3}{4}a^2b^2\left(4\,x - \frac{e^{(2\,dx+2\,c)}}{d} + \frac{e^{(-2\,dx-2\,c)}}{d}\right) + a^4x + \frac{1}{6}ab^3\left(\frac{e^{(3\,dx+3\,c)}}{d} - \frac{9\,e^{(dx+c)}}{d} - \frac{9\,e^{(-dx-c)}}{d} + \frac{e^{(-3\,dx-3\,c)}}{d}\right) + \frac{4\,a^3b\cosh\left(dx + c\right)}{d}$$

input
```
integrate((a+b*sinh(d*x+c))^4,x, algorithm="maxima")
```

output
```
1/64*b^4*(24*x + e^(4*d*x + 4*c)/d - 8*e^(2*d*x + 2*c)/d + 8*e^(-2*d*x - 2
*c)/d - e^(-4*d*x - 4*c)/d) - 3/4*a^2*b^2*(4*x - e^(2*d*x + 2*c)/d + e^(-2
*d*x - 2*c)/d) + a^4*x + 1/6*a*b^3*(e^(3*d*x + 3*c)/d - 9*e^(d*x + c)/d -
9*e^(-d*x - c)/d + e^(-3*d*x - 3*c)/d) + 4*a^3*b*cosh(d*x + c)/d
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 200, normalized size of antiderivative = 1.46

$$\int (a+b\sinh(c+dx))^4\,dx = \frac{b^4e^{(4\,dx+4\,c)}}{64\,d} + \frac{ab^3e^{(3\,dx+3\,c)}}{6\,d} + \frac{ab^3e^{(-3\,dx-3\,c)}}{6\,d} - \frac{b^4e^{(-4\,dx-4\,c)}}{64\,d} + \frac{1}{8}\left(8\,a^4 - 24\,a^2b^2 + 3\,b^4\right)x + \frac{(6\,a^2b^2 - b^4)e^{(2\,dx+2\,c)}}{8\,d} + \frac{(4\,a^3b - 3\,ab^3)e^{(dx+c)}}{2\,d} + \frac{(4\,a^3b - 3\,ab^3)e^{(-dx-c)}}{2\,d} - \frac{(6\,a^2b^2 - b^4)e^{(-2\,dx-2\,c)}}{8\,d}$$

input
```
integrate((a+b*sinh(d*x+c))^4,x, algorithm="giac")
```

output
```
1/64*b^4*e^(4*d*x + 4*c)/d + 1/6*a*b^3*e^(3*d*x + 3*c)/d + 1/6*a*b^3*e^(-3
*d*x - 3*c)/d - 1/64*b^4*e^(-4*d*x - 4*c)/d + 1/8*(8*a^4 - 24*a^2*b^2 + 3*
b^4)*x + 1/8*(6*a^2*b^2 - b^4)*e^(2*d*x + 2*c)/d + 1/2*(4*a^3*b - 3*a*b^3)
*e^(d*x + c)/d + 1/2*(4*a^3*b - 3*a*b^3)*e^(-d*x - c)/d - 1/8*(6*a^2*b^2 -
 b^4)*e^(-2*d*x - 2*c)/d
```

Mupad [B] (verification not implemented)

Time = 0.37 (sec) , antiderivative size = 114, normalized size of antiderivative = 0.83

$$\int (a+b\sinh(c+dx))^4\,dx = \frac{\frac{3\,b^4\sinh(4\,c+4\,d\,x)}{4} - 6\,b^4\sinh(2\,c+2\,d\,x) + 8\,a\,b^3\cosh(3\,c+3\,d\,x) + 36\,a^2\,b^2\sinh(2\,c+2\,d\,x) - 72\,a\,b^3\cosh(}{24\,d}$$

input
```
int((a + b*sinh(c + d*x))^4,x)
```

output
```
((3*b^4*sinh(4*c + 4*d*x))/4 - 6*b^4*sinh(2*c + 2*d*x) + 8*a*b^3*cosh(3*c
+ 3*d*x) + 36*a^2*b^2*sinh(2*c + 2*d*x) - 72*a*b^3*cosh(c + d*x) + 96*a^3*
b*cosh(c + d*x) + 24*a^4*d*x + 9*b^4*d*x - 72*a^2*b^2*d*x)/(24*d)
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 251, normalized size of antiderivative = 1.83

$$\int (a + b\sinh(c + dx))^4\,dx$$
$$= \frac{3e^{8dx+8c}b^4 + 32e^{7dx+7c}a\,b^3 + 144e^{6dx+6c}a^2b^2 - 24e^{6dx+6c}b^4 + 384e^{5dx+5c}a^3b - 288e^{5dx+5c}a\,b^3 + 192e^{4dx+4c}}{}$$

input
```
int((a+b*sinh(d*x+c))^4,x)
```

output
```
(3*e**(8*c + 8*d*x)*b**4 + 32*e**(7*c + 7*d*x)*a*b**3 + 144*e**(6*c + 6*d*
x)*a**2*b**2 - 24*e**(6*c + 6*d*x)*b**4 + 384*e**(5*c + 5*d*x)*a**3*b - 28
8*e**(5*c + 5*d*x)*a*b**3 + 192*e**(4*c + 4*d*x)*a**4*d*x - 576*e**(4*c +
4*d*x)*a**2*b**2*d*x + 72*e**(4*c + 4*d*x)*b**4*d*x + 384*e**(3*c + 3*d*x)
*a**3*b - 288*e**(3*c + 3*d*x)*a*b**3 - 144*e**(2*c + 2*d*x)*a**2*b**2 + 2
4*e**(2*c + 2*d*x)*b**4 + 32*e**(c + d*x)*a*b**3 - 3*b**4)/(192*e**(4*c +
4*d*x)*d)
```

3.98 $\int (a+b\sinh(c+dx))^3\,dx$

Optimal result

Integrand size = 12, antiderivative size = 92

$$\int (a+b\sinh(c+dx))^3\,dx = \frac{1}{2}a\left(2a^2-3b^2\right)x+\frac{2b(4a^2-b^2)\cosh(c+dx)}{3d} + \frac{5ab^2\cosh(c+dx)\sinh(c+dx)}{6d} + \frac{b\cosh(c+dx)(a+b\sinh(c+dx))^2}{3d}$$

output
```
1/2*a*(2*a^2-3*b^2)*x+2/3*b*(4*a^2-b^2)*cosh(d*x+c)/d+5/6*a*b^2*cosh(d*x+c
)*sinh(d*x+c)/d+1/3*b*cosh(d*x+c)*(a+b*sinh(d*x+c))^2/d
```

Mathematica [A] (verified)

Time = 0.24 (sec) , antiderivative size = 71, normalized size of antiderivative = 0.77

$$\int (a+b\sinh(c+dx))^3\,dx = \frac{6a(2a^2-3b^2)(c+dx)-9b(-4a^2+b^2)\cosh(c+dx)+b^3\cosh(3(c+dx))+9ab^2\sinh(2(c+dx))}{12d}$$

input
```
Integrate[(a + b*Sinh[c + d*x])^3,x]
```

output
```
(6*a*(2*a^2 - 3*b^2)*(c + d*x) - 9*b*(-4*a^2 + b^2)*Cosh[c + d*x] + b^3*Co
sh[3*(c + d*x)] + 9*a*b^2*Sinh[2*(c + d*x)])/(12*d)
```

Rubi [A] (verified)

Time = 0.34 (sec) , antiderivative size = 95, normalized size of antiderivative = 1.03, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {3042, 3135, 3042, 3213}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a+b\sinh(c+dx))^3\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int (a-ib\sin(ic+idx))^3dx$$

$$\Big\downarrow \text{3135}$$

$$\frac{1}{3}\int (a+b\sinh(c+dx))\left(3a^2+5b\sinh(c+dx)a-2b^2\right)dx+\frac{b\cosh(c+dx)(a+b\sinh(c+dx))^2}{3d}$$

$$\Big\downarrow \text{3042}$$

$$\frac{b\cosh(c+dx)(a+b\sinh(c+dx))^2}{3d}+\frac{1}{3}\int (a-ib\sin(ic+idx))\left(3a^2-5ib\sin(ic+idx)a-2b^2\right)dx$$

$$\Big\downarrow \text{3213}$$

$$\frac{1}{3}\left(\frac{2b(4a^2-b^2)\cosh(c+dx)}{d}+\frac{3}{2}ax(2a^2-3b^2)+\frac{5ab^2\sinh(c+dx)\cosh(c+dx)}{2d}\right)+\frac{b\cosh(c+dx)(a+b\sinh(c+dx))^2}{3d}$$

input
```
Int[(a + b*Sinh[c + d*x])^3,x]
```

output
```
(b*Cosh[c + d*x]*(a + b*Sinh[c + d*x])^2)/(3*d) + ((3*a*(2*a^2 - 3*b^2)*x)
/2 + (2*b*(4*a^2 - b^2)*Cosh[c + d*x])/d + (5*a*b^2*Cosh[c + d*x]*Sinh[c +
 d*x])/(2*d))/3
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3135
```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos
[c + d*x]*((a + b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Simp[1/n   Int[(a + b*
Sin[c + d*x])^(n - 2)*Simp[a^2*n + b^2*(n - 1) + a*b*(2*n - 1)*Sin[c + d*x]
, x], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[n, 1] &&
 IntegerQ[2*n]
```

rule 3213
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)
*(x_)]), x_Symbol] :> Simp[(2*a*c + b*d)*(x/2), x] + (-Simp[(b*c + a*d)*(Co
s[e + f*x]/f), x] - Simp[b*d*Cos[e + f*x]*(Sin[e + f*x]/(2*f)), x]) /; Free
Q[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Maple [A] (verified)

Time = 159.82 (sec) , antiderivative size = 77, normalized size of antiderivative = 0.84

method	result
derivativedivides	$\frac{b^{3}\left(-\frac{2}{3}+\frac{\sinh\left(dx+c\right)^{2}}{3}\right)\cosh\left(dx+c\right)+3a\,b^{2}\left(\frac{\sinh\left(dx+c\right)\cosh\left(dx+c\right)}{2}-\frac{dx}{2}-\frac{c}{2}\right)+3a^{2}b\cosh\left(dx+c\right)+a^{3}\left(dx+c\right)}{d}$
default	$\frac{b^{3}\left(-\frac{2}{3}+\frac{\sinh\left(dx+c\right)^{2}}{3}\right)\cosh\left(dx+c\right)+3a\,b^{2}\left(\frac{\sinh\left(dx+c\right)\cosh\left(dx+c\right)}{2}-\frac{dx}{2}-\frac{c}{2}\right)+3a^{2}b\cosh\left(dx+c\right)+a^{3}\left(dx+c\right)}{d}$
parts	$a^{3}x+\frac{b^{3}\left(-\frac{2}{3}+\frac{\sinh\left(dx+c\right)^{2}}{3}\right)\cosh\left(dx+c\right)}{d}+\frac{3a\,b^{2}\left(\frac{\sinh\left(dx+c\right)\cosh\left(dx+c\right)}{2}-\frac{dx}{2}-\frac{c}{2}\right)}{d}+\frac{3a^{2}b\cosh\left(dx+c\right)}{d}$
parallelrisch	$\frac{b^{3}\cosh\left(3dx+3c\right)+9a\,b^{2}\sinh\left(2dx+2c\right)+\left(36a^{2}b-9b^{3}\right)\cosh\left(dx+c\right)+12a^{3}dx-18a\,b^{2}dx+36a^{2}b-8b^{3}}{12d}$
risch	$a^{3}x-\frac{3a\,b^{2}x}{2}+\frac{b^{3}{\mathrm e}^{3dx+3c}}{24d}+\frac{3a\,b^{2}{\mathrm e}^{2dx+2c}}{8d}+\frac{3b\,{\mathrm e}^{dx+c}a^{2}}{2d}-\frac{3b^{3}{\mathrm e}^{dx+c}}{8d}+\frac{3b\,{\mathrm e}^{-dx-c}a^{2}}{2d}-\frac{3b^{3}{\mathrm e}^{-dx-c}}{8d}-3a$

input
```
int((a+b*sinh(d*x+c))^3,x,method=_RETURNVERBOSE)
```

output
```
1/d*(b^3*(-2/3+1/3*sinh(d*x+c)^2)*cosh(d*x+c)+3*a*b^2*(1/2*sinh(d*x+c)*cos
h(d*x+c)-1/2*d*x-1/2*c)+3*a^2*b*cosh(d*x+c)+a^3*(d*x+c))
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 91, normalized size of antiderivative = 0.99

$$\int (a+b\sinh(c+dx))^3\,dx = \frac{b^3\cosh(dx+c)^3+3\,b^3\cosh(dx+c)\sinh(dx+c)^2+18\,ab^2\cosh(dx+c)\sinh(dx+c)+6\,(2\,a^3-3\,ab^2}{12\,d}$$

input
```
integrate((a+b*sinh(d*x+c))^3,x, algorithm="fricas")
```

output
```
1/12*(b^3*cosh(d*x + c)^3 + 3*b^3*cosh(d*x + c)*sinh(d*x + c)^2 + 18*a*b^2
*cosh(d*x + c)*sinh(d*x + c) + 6*(2*a^3 - 3*a*b^2)*d*x + 9*(4*a^2*b - b^3)
*cosh(d*x + c))/d
```

Sympy [A] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 128, normalized size of antiderivative = 1.39

$$\int (a+b\sinh(c+dx))^3\,dx = \begin{cases} a^3x+\frac{3a^2b\cosh(c+dx)}{d}+\frac{3ab^2x\sinh^2(c+dx)}{2}-\frac{3ab^2x\cosh^2(c+dx)}{2}+\frac{3ab^2\sinh(c+dx)\cosh(c+dx)}{2d}+\frac{b^3\sinh^2(c+dx)\cosh(c+dx}{d} \\ x(a+b\sinh(c))^3 \end{cases}$$

input
```
integrate((a+b*sinh(d*x+c))**3,x)
```

output
```
Piecewise((a**3*x + 3*a**2*b*cosh(c + d*x)/d + 3*a*b**2*x*sinh(c + d*x)**2
/2 - 3*a*b**2*x*cosh(c + d*x)**2/2 + 3*a*b**2*sinh(c + d*x)*cosh(c + d*x)/
(2*d) + b**3*sinh(c + d*x)**2*cosh(c + d*x)/d - 2*b**3*cosh(c + d*x)**3/(3
*d), Ne(d, 0)), (x*(a + b*sinh(c))**3, True))
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 115, normalized size of antiderivative = 1.25

$$\int (a+b\sinh(c+dx))^3\,dx = -\frac{3}{8}\,ab^2\left(4\,x-\frac{e^{(2\,dx+2\,c)}}{d}+\frac{e^{(-2\,dx-2\,c)}}{d}\right)+a^3x+\frac{1}{24}\,b^3\left(\frac{e^{(3\,dx+3\,c)}}{d}-\frac{9\,e^{(dx+c)}}{d}-\frac{9\,e^{(-dx-c)}}{d}+\frac{e^{(-3\,dx-3\,c)}}{d}\right)+\frac{3\,a^2b\cosh\left(dx+c\right)}{d}$$

input
```
integrate((a+b*sinh(d*x+c))^3,x, algorithm="maxima")
```

output
```
-3/8*a*b^2*(4*x - e^(2*d*x + 2*c)/d + e^(-2*d*x - 2*c)/d) + a^3*x + 1/24*b
^3*(e^(3*d*x + 3*c)/d - 9*e^(d*x + c)/d - 9*e^(-d*x - c)/d + e^(-3*d*x - 3
*c)/d) + 3*a^2*b*cosh(d*x + c)/d
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 135, normalized size of antiderivative = 1.47

$$\int (a+b\sinh(c+dx))^3\,dx = \frac{b^3e^{(3\,dx+3\,c)}}{24\,d}+\frac{3\,ab^2e^{(2\,dx+2\,c)}}{8\,d}-\frac{3\,ab^2e^{(-2\,dx-2\,c)}}{8\,d}+\frac{b^3e^{(-3\,dx-3\,c)}}{24\,d}+\frac{1}{2}\left(2\,a^3-3\,ab^2\right)x+\frac{3\left(4\,a^2b-b^3\right)e^{(dx+c)}}{8\,d}+\frac{3\left(4\,a^2b-b^3\right)e^{(-dx-c)}}{8\,d}$$

input
```
integrate((a+b*sinh(d*x+c))^3,x, algorithm="giac")
```

output
```
1/24*b^3*e^(3*d*x + 3*c)/d + 3/8*a*b^2*e^(2*d*x + 2*c)/d - 3/8*a*b^2*e^(-2
*d*x - 2*c)/d + 1/24*b^3*e^(-3*d*x - 3*c)/d + 1/2*(2*a^3 - 3*a*b^2)*x + 3/
8*(4*a^2*b - b^3)*e^(d*x + c)/d + 3/8*(4*a^2*b - b^3)*e^(-d*x - c)/d
```

Mupad [B] (verification not implemented)

Time = 1.56 (sec) , antiderivative size = 75, normalized size of antiderivative = 0.82

$$\int (a + b\sinh(c + dx))^3\,dx$$
$$= \frac{6\,d\,x\,a^3 + 18\,a^2\,b\cosh(c+d\,x) + 9\sinh(c+d\,x)\;a\,b^2\cosh(c+d\,x) - 9\,d\,x\,a\,b^2 + 2\,b^3\cosh(c+d\,x)^3 - 6\,b}{6\,d}$$

input
```
int((a + b*sinh(c + d*x))^3,x)
```

output
```
(2*b^3*cosh(c + d*x)^3 - 6*b^3*cosh(c + d*x) + 18*a^2*b*cosh(c + d*x) + 6*
a^3*d*x + 9*a*b^2*cosh(c + d*x)*sinh(c + d*x) - 9*a*b^2*d*x)/(6*d)
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 161, normalized size of antiderivative = 1.75

$$\int (a + b\sinh(c + dx))^3\,dx$$
$$= \frac{e^{6dx+6c}b^3 + 9e^{5dx+5c}a\,b^2 + 36e^{4dx+4c}a^2b - 9e^{4dx+4c}b^3 + 24e^{3dx+3c}a^3dx - 36e^{3dx+3c}a\,b^2dx + 36e^{2dx+2c}a^2b -}{24e^{3dx+3c}d}$$

input
```
int((a+b*sinh(d*x+c))^3,x)
```

output
```
(e**(6*c + 6*d*x)*b**3 + 9*e**(5*c + 5*d*x)*a*b**2 + 36*e**(4*c + 4*d*x)*a
**2*b - 9*e**(4*c + 4*d*x)*b**3 + 24*e**(3*c + 3*d*x)*a**3*d*x - 36*e**(3*
c + 3*d*x)*a*b**2*d*x + 36*e**(2*c + 2*d*x)*a**2*b - 9*e**(2*c + 2*d*x)*b*
*3 - 9*e**(c + d*x)*a*b**2 + b**3)/(24*e**(3*c + 3*d*x)*d)
```

3.99 $\int (a+b\sinh(c+dx))^2\,dx$

Optimal result

Integrand size = 12, antiderivative size = 52

$$\int (a+b\sinh(c+dx))^2\,dx=\frac{1}{2}\left(2a^2-b^2\right)x+\frac{2ab\cosh(c+dx)}{d}+\frac{b^2\cosh(c+dx)\sinh(c+dx)}{2d}$$

output
```
1/2*(2*a^2-b^2)*x+2*a*b*cosh(d*x+c)/d+1/2*b^2*cosh(d*x+c)*sinh(d*x+c)/d
```

Mathematica [A] (verified)

Time = 0.13 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.92

$$\int (a+b\sinh(c+dx))^2\,dx=\frac{2\left(2a^2-b^2\right)(c+dx)+8ab\cosh(c+dx)+b^2\sinh(2(c+dx))}{4d}$$

input
```
Integrate[(a + b*Sinh[c + d*x])^2,x]
```

output
```
(2*(2*a^2 - b^2)*(c + d*x) + 8*a*b*Cosh[c + d*x] + b^2*Sinh[2*(c + d*x)])/
(4*d)
```

Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 52, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {3042, 3123}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a+b\sinh(c+dx))^2\,dx$$

$$\Big\downarrow \; 3042$$

$$\int (a-ib\sin(ic+idx))^2dx$$

$$\Big\downarrow \; 3123$$

$$\frac{1}{2}x\left(2a^2-b^2\right)+\frac{2ab\cosh(c+dx)}{d}+\frac{b^2\sinh(c+dx)\cosh(c+dx)}{2d}$$

input
```
Int[(a + b*Sinh[c + d*x])^2,x]
```

output
```
((2*a^2 - b^2)*x)/2 + (2*a*b*Cosh[c + d*x])/d + (b^2*Cosh[c + d*x]*Sinh[c
+ d*x])/(2*d)
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3123
```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^2, x_Symbol] :> Simp[(2*a^2 + b^
2)*(x/2), x] + (-Simp[2*a*b*(Cos[c + d*x]/d), x] - Simp[b^2*Cos[c + d*x]*(S
in[c + d*x]/(2*d)), x]) /; FreeQ[{a, b, c, d}, x]
```

Maple [A] (verified)

Time = 0.50 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.92

method	result
parallelrisch	$\frac{4a^2dx-2b^2dx+8ab\cosh(dx+c)+b^2\sinh(2dx+2c)+8ab}{4d}$
parts	$a^2x+\frac{b^2\left(\frac{\sinh(dx+c)\cosh(dx+c)}{2}-\frac{dx}{2}-\frac{c}{2}\right)}{d}+\frac{2ab\cosh(dx+c)}{d}$
derivativedivides	$\frac{b^2\left(\frac{\sinh(dx+c)\cosh(dx+c)}{2}-\frac{dx}{2}-\frac{c}{2}\right)+2ab\cosh(dx+c)+a^2(dx+c)}{d}$
default	$\frac{b^2\left(\frac{\sinh(dx+c)\cosh(dx+c)}{2}-\frac{dx}{2}-\frac{c}{2}\right)+2ab\cosh(dx+c)+a^2(dx+c)}{d}$
risch	$a^2x-\frac{x\,b^2}{2}+\frac{b^2\mathrm{e}^{2dx+2c}}{8d}+\frac{ab\,\mathrm{e}^{dx+c}}{d}+\frac{ab\,\mathrm{e}^{-dx-c}}{d}-\frac{b^2\mathrm{e}^{-2dx-2c}}{8d}$
orering	$x(a+b\sinh(dx+c))^2+\frac{5(a+b\sinh(dx+c))b\cosh(dx+c)}{2d}-\frac{5x\left(2b^2d^2\cosh(dx+c)^2+2(a+b\sinh(dx+c))b\right.}{4d^2}$

input
```
int((a+b*sinh(d*x+c))^2,x,method=_RETURNVERBOSE)
```

output
```
1/4*(4*a^2*d*x-2*b^2*d*x+8*a*b*cosh(d*x+c)+b^2*sinh(2*d*x+2*c)+8*a*b)/d
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.88

$$\int (a+b\sinh(c+dx))^2\,dx = \frac{b^2\cosh(dx+c)\sinh(dx+c)+(2\,a^2-b^2)dx+4\,ab\cosh(dx+c)}{2\,d}$$

input
```
integrate((a+b*sinh(d*x+c))^2,x, algorithm="fricas")
```

output
```
1/2*(b^2*cosh(d*x + c)*sinh(d*x + c) + (2*a^2 - b^2)*d*x + 4*a*b*cosh(d*x
+ c))/d
```

Sympy [A] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 78, normalized size of antiderivative = 1.50

$$\int (a+b\sinh(c+dx))^2\,dx$$
$$=\begin{cases} a^2x+\frac{2ab\cosh(c+dx)}{d}+\frac{b^2x\sinh^2(c+dx)}{2}-\frac{b^2x\cosh^2(c+dx)}{2}+\frac{b^2\sinh(c+dx)\cosh(c+dx)}{2d} & \text{for } d\neq 0 \\ x(a+b\sinh(c))^2 & \text{otherwise} \end{cases}$$

input
```
integrate((a+b*sinh(d*x+c))**2,x)
```

output
```
Piecewise((a**2*x + 2*a*b*cosh(c + d*x)/d + b**2*x*sinh(c + d*x)**2/2 - b*
*2*x*cosh(c + d*x)**2/2 + b**2*sinh(c + d*x)*cosh(c + d*x)/(2*d), Ne(d, 0)
), (x*(a + b*sinh(c))**2, True))
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 55, normalized size of antiderivative = 1.06

$$\int (a+b\sinh(c+dx))^2\,dx=-\frac{1}{8}b^2\left(4x-\frac{e^{(2dx+2c)}}{d}+\frac{e^{(-2dx-2c)}}{d}\right)+a^2x+\frac{2ab\cosh(dx+c)}{d}$$

input
```
integrate((a+b*sinh(d*x+c))^2,x, algorithm="maxima")
```

output
```
-1/8*b^2*(4*x - e^(2*d*x + 2*c)/d + e^(-2*d*x - 2*c)/d) + a^2*x + 2*a*b*co
sh(d*x + c)/d
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 76, normalized size of antiderivative = 1.46

$$\int (a+b\sinh(c+dx))^2\,dx = \frac{1}{2}\left(2\,a^2-b^2\right)x + \frac{b^2e^{(2\,dx+2\,c)}}{8\,d} + \frac{abe^{(dx+c)}}{d} + \frac{abe^{(-dx-c)}}{d} - \frac{b^2e^{(-2\,dx-2\,c)}}{8\,d}$$

input
```
integrate((a+b*sinh(d*x+c))^2,x, algorithm="giac")
```

output
```
1/2*(2*a^2 - b^2)*x + 1/8*b^2*e^(2*d*x + 2*c)/d + a*b*e^(d*x + c)/d + a*b*
e^(-d*x - c)/d - 1/8*b^2*e^(-2*d*x - 2*c)/d
```

Mupad [B] (verification not implemented)

Time = 1.56 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.79

$$\int (a+b\sinh(c+dx))^2\,dx = a^2\,x - \frac{b^2\,x}{2} + \frac{\frac{\sinh(2\,c+2\,d\,x)\,b^2}{4} + 2\,a\cosh(c+d\,x)\,b}{d}$$

input
```
int((a + b*sinh(c + d*x))^2,x)
```

output
```
a^2*x - (b^2*x)/2 + ((b^2*sinh(2*c + 2*d*x))/4 + 2*a*b*cosh(c + d*x))/d
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 96, normalized size of antiderivative = 1.85

$$\int (a+b\sinh(c+dx))^2\,dx = \frac{e^{4dx+4c}b^2+8e^{3dx+3c}ab+8e^{2dx+2c}a^2dx-4e^{2dx+2c}b^2dx+8e^{dx+c}ab-b^2}{8e^{2dx+2c}d}$$

input
```
int((a+b*sinh(d*x+c))^2,x)
```

output
```
(e**(4*c + 4*d*x)*b**2 + 8*e**(3*c + 3*d*x)*a*b + 8*e**(2*c + 2*d*x)*a**2*
d*x - 4*e**(2*c + 2*d*x)*b**2*d*x + 8*e**(c + d*x)*a*b - b**2)/(8*e**(2*c
+ 2*d*x)*d)
```

3.100 $\int (a+b\sinh(c+dx))\,dx$

Optimal result

Integrand size = 10, antiderivative size = 15

$$\int (a+b\sinh(c+dx))\,dx = ax+\frac{b\cosh(c+dx)}{d}$$

output
```
a*x+b*cosh(d*x+c)/d
```

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.73

$$\int (a+b\sinh(c+dx))\,dx = ax+\frac{b\cosh(c)\cosh(dx)}{d}+\frac{b\sinh(c)\sinh(dx)}{d}$$

input
```
Integrate[a + b*Sinh[c + d*x],x]
```

output
```
a*x + (b*Cosh[c]*Cosh[d*x])/d + (b*Sinh[c]*Sinh[d*x])/d
```

Rubi [A] (verified)

Time = 0.17 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a+b\sinh(c+dx))\,dx$$

$$\Big\downarrow \text{2009}$$

$$ax+\frac{b\cosh(c+dx)}{d}$$

input
```
Int[a + b*Sinh[c + d*x],x]
```

output
```
a*x + (b*Cosh[c + d*x])/d
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.15 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.07

method	result	size
default	$ax+\frac{b\cosh(dx+c)}{d}$	16
parts	$ax+\frac{b\cosh(dx+c)}{d}$	16
parallelrisch	$\frac{b(1+\cosh(dx+c))}{d}+ax$	18
derivativedivides	$\frac{(dx+c)a+b\cosh(dx+c)}{d}$	21
risch	$ax+\frac{b\,\mathrm{e}^{dx+c}}{2d}+\frac{b\,\mathrm{e}^{-dx-c}}{2d}$	32
orering	$x(a+b\sinh(dx+c))+\frac{b\cosh(dx+c)}{d}-xb\sinh(dx+c)$	35

input
```
int(a+b*sinh(d*x+c),x,method=_RETURNVERBOSE)
```

output
```
a*x+b*cosh(d*x+c)/d
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.13

$$\int (a+b\sinh(c+dx))\,dx=\frac{adx+b\cosh(dx+c)}{d}$$

input
```
integrate(a+b*sinh(d*x+c),x, algorithm="fricas")
```

output
```
(a*d*x + b*cosh(d*x + c))/d
```

Sympy [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.13

$$\int (a+b\sinh(c+dx))\,dx = ax+b\left(\begin{cases}\frac{\cosh(c+dx)}{d} & \text{for } d\neq 0\\ x\sinh(c) & \text{otherwise}\end{cases}\right)$$

input
```
integrate(a+b*sinh(d*x+c),x)
```

output
```
a*x + b*Piecewise((cosh(c + d*x)/d, Ne(d, 0)), (x*sinh(c), True))
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.00

$$\int (a+b\sinh(c+dx))\,dx = ax+\frac{b\cosh(dx+c)}{d}$$

input
```
integrate(a+b*sinh(d*x+c),x, algorithm="maxima")
```

output
```
a*x + b*cosh(d*x + c)/d
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 31 vs. 2(15) = 30.

Time = 0.12 (sec) , antiderivative size = 31, normalized size of antiderivative = 2.07

$$\int (a+b\sinh(c+dx))\,dx = ax+\frac{1}{2}b\left(\frac{e^{(dx+c)}}{d}+\frac{e^{(-dx-c)}}{d}\right)$$

input
```
integrate(a+b*sinh(d*x+c),x, algorithm="giac")
```

output
```
a*x + 1/2*b*(e^(d*x + c)/d + e^(-d*x - c)/d)
```

Mupad [B] (verification not implemented)

Time = 1.50 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.00

$$\int (a + b\sinh(c + dx))\,dx = a\,x + \frac{b\cosh(c + d\,x)}{d}$$

input
```
int(a + b*sinh(c + d*x),x)
```

output
```
a*x + (b*cosh(c + d*x))/d
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.13

$$\int (a + b\sinh(c + dx))\,dx = \frac{\cosh(dx + c)\,b + adx}{d}$$

input
```
int(a+b*sinh(d*x+c),x)
```

output
```
(cosh(c + d*x)*b + a*d*x)/d
```

3.101 $\int \frac{1}{a+b\sinh(c+dx)}\,dx$

Optimal result

Integrand size = 12, antiderivative size = 44

$$\int \frac{1}{a+b\sinh(c+dx)}\,dx = -\frac{2\text{arctanh}\left(\frac{b-a\tanh\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2+b^2}}\right)}{\sqrt{a^2+b^2}d}$$

output
```
-2*arctanh((b-a*tanh(1/2*d*x+1/2*c))/(a^2+b^2)^(1/2))/(a^2+b^2)^(1/2)/d
```

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 52, normalized size of antiderivative = 1.18

$$\int \frac{1}{a+b\sinh(c+dx)}\,dx = \frac{2\arctan\left(\frac{b-a\tanh\left(\frac{1}{2}(c+dx)\right)}{\sqrt{-a^2-b^2}}\right)}{\sqrt{-a^2-b^2}d}$$

input
```
Integrate[(a + b*Sinh[c + d*x])^(-1),x]
```

output
```
(2*ArcTan[(b - a*Tanh[(c + d*x)/2])/Sqrt[-a^2 - b^2]])/(Sqrt[-a^2 - b^2]*d
)
```

Rubi [A] (warning: unable to verify)

Time = 0.25 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.95, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {3042, 3139, 1083, 217}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{a+b\sinh(c+dx)}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{a-ib\sin(ic+idx)}dx$$

$$\Big\downarrow \text{3139}$$

$$-\frac{2i\int \frac{1}{-a\tanh^2\left(\frac{1}{2}(c+dx)\right)+2b\tanh\left(\frac{1}{2}(c+dx)\right)+a}d\left(i\tanh\left(\frac{1}{2}(c+dx)\right)\right)}{d}$$

$$\Big\downarrow \text{1083}$$

$$\frac{4i\int \frac{1}{\tanh^2\left(\frac{1}{2}(c+dx)\right)-4(a^2+b^2)}d\left(2ia\tanh\left(\frac{1}{2}(c+dx)\right)-2ib\right)}{d}$$

$$\Big\downarrow \text{217}$$

$$\frac{2\text{arctanh}\left(\frac{\tanh\left(\frac{1}{2}(c+dx)\right)}{2\sqrt{a^2+b^2}}\right)}{d\sqrt{a^2+b^2}}$$

input
```
Int[(a + b*Sinh[c + d*x])^(-1),x]
```

output
```
(2*ArcTanh[Tanh[(c + d*x)/2]/(2*Sqrt[a^2 + b^2])])/(Sqrt[a^2 + b^2]*d)
```

Defintions of rubi rules used

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 1083
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Simp[-2   Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3139
```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Simp[2*(e/d)   Subst[Int[1/(a + 2*b*e*x + a
*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[a^2 - b^2, 0]
```

Maple [A] (verified)

Time = 0.13 (sec) , antiderivative size = 43, normalized size of antiderivative = 0.98

method	result	size
derivativedivides	$\frac{2 \operatorname{arctanh}\left(\frac{2 a \tanh\left(\frac{d x}{2}+\frac{c}{2}\right)-2 b}{2 \sqrt{a^{2}+b^{2}}}\right)}{d \sqrt{a^{2}+b^{2}}}$	43
default	$\frac{2 \operatorname{arctanh}\left(\frac{2 a \tanh\left(\frac{d x}{2}+\frac{c}{2}\right)-2 b}{2 \sqrt{a^{2}+b^{2}}}\right)}{d \sqrt{a^{2}+b^{2}}}$	43
risch	$\frac{\ln\left({\mathrm e}^{d x +c}+\frac{a \sqrt{a^{2}+b^{2}}-a^{2}-b^{2}}{\sqrt{a^{2}+b^{2}}\, b}\right)}{\sqrt{a^{2}+b^{2}}\, d}-\frac{\ln\left({\mathrm e}^{d x +c}+\frac{a \sqrt{a^{2}+b^{2}}+a^{2}+b^{2}}{\sqrt{a^{2}+b^{2}}\, b}\right)}{\sqrt{a^{2}+b^{2}}\, d}$	111

input
```
int(1/(a+b*sinh(d*x+c)),x,method=_RETURNVERBOSE)
```

output
```
2/d/(a^2+b^2)^(1/2)*arctanh(1/2*(2*a*tanh(1/2*d*x+1/2*c)-2*b)/(a^2+b^2)^(1
/2))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 162 vs. $2(41) = 82$.

Time = 0.08 (sec) , antiderivative size = 162, normalized size of antiderivative = 3.68

$$\int \frac{1}{a + b\sinh(c + dx)}\,dx$$
$$= \frac{\log\left(\frac{b^2\cosh(dx+c)^2 + b^2\sinh(dx+c)^2 + 2\,ab\cosh(dx+c) + 2\,a^2 + b^2 + 2\,(b^2\cosh(dx+c) + ab)\sinh(dx+c) - 2\,\sqrt{a^2+b^2}(b\cosh(dx+c) + b\sinh(dx+c)}{b\cosh(dx+c)^2 + b\sinh(dx+c)^2 + 2\,a\cosh(dx+c) + 2\,(b\cosh(dx+c) + a)\sinh(dx+c) - b}\right)}{\sqrt{a^2+b^2}d}$$

input
```
integrate(1/(a+b*sinh(d*x+c)),x, algorithm="fricas")
```

output
```
log((b^2*cosh(d*x + c)^2 + b^2*sinh(d*x + c)^2 + 2*a*b*cosh(d*x + c) + 2*a
^2 + b^2 + 2*(b^2*cosh(d*x + c) + a*b)*sinh(d*x + c) - 2*sqrt(a^2 + b^2)*(
b*cosh(d*x + c) + b*sinh(d*x + c) + a))/(b*cosh(d*x + c)^2 + b*sinh(d*x +
c)^2 + 2*a*cosh(d*x + c) + 2*(b*cosh(d*x + c) + a)*sinh(d*x + c) - b))/(sq
rt(a^2 + b^2)*d)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 3.87 (sec) , antiderivative size = 155, normalized size of antiderivative = 3.52

$$\int \frac{1}{a+b\sinh(c+dx)}\,dx$$
$$= \begin{cases} \frac{\tilde{\infty}x}{\sinh{(c)}} & \text{for } a=0 \wedge b=0 \wedge d=0 \\ \frac{\log\left(\tanh\left(\frac{c}{2}+\frac{dx}{2}\right)\right)}{bd} & \text{for } a=0 \\ \frac{x}{a+b\sinh{(c)}} & \text{for } d=0 \\ \frac{2i}{bd\tanh\left(\frac{c}{2}+\frac{dx}{2}\right)-ibd} & \text{for } a=-ib \\ -\frac{2i}{bd\tanh\left(\frac{c}{2}+\frac{dx}{2}\right)+ibd} & \text{for } a=ib \\ -\frac{\log\left(\tanh\left(\frac{c}{2}+\frac{dx}{2}\right)-\frac{b}{a}-\frac{\sqrt{a^2+b^2}}{a}\right)}{d\sqrt{a^2+b^2}}+\frac{\log\left(\tanh\left(\frac{c}{2}+\frac{dx}{2}\right)-\frac{b}{a}+\frac{\sqrt{a^2+b^2}}{a}\right)}{d\sqrt{a^2+b^2}} & \text{otherwise} \end{cases}$$

input
```
integrate(1/(a+b*sinh(d*x+c)),x)
```

output
```
Piecewise((zoo*x/sinh(c), Eq(a, 0) & Eq(b, 0) & Eq(d, 0)), (log(tanh(c/2 +
 d*x/2))/(b*d), Eq(a, 0)), (x/(a + b*sinh(c)), Eq(d, 0)), (2*I/(b*d*tanh(c
/2 + d*x/2) - I*b*d), Eq(a, -I*b)), (-2*I/(b*d*tanh(c/2 + d*x/2) + I*b*d),
 Eq(a, I*b)), (-log(tanh(c/2 + d*x/2) - b/a - sqrt(a**2 + b**2)/a)/(d*sqrt
(a**2 + b**2)) + log(tanh(c/2 + d*x/2) - b/a + sqrt(a**2 + b**2)/a)/(d*sqr
t(a**2 + b**2)), True))
```

Maxima [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 67, normalized size of antiderivative = 1.52

$$\int \frac{1}{a+b\sinh(c+dx)}\,dx = \frac{\log\left(\frac{be^{(-dx-c)}-a-\sqrt{a^2+b^2}}{be^{(-dx-c)}-a+\sqrt{a^2+b^2}}\right)}{\sqrt{a^2+b^2}d}$$

input
```
integrate(1/(a+b*sinh(d*x+c)),x, algorithm="maxima")
```

output
```
log((b*e^(-d*x - c) - a - sqrt(a^2 + b^2))/(b*e^(-d*x - c) - a + sqrt(a^2
+ b^2)))/(sqrt(a^2 + b^2)*d)
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 67, normalized size of antiderivative = 1.52

$$\int \frac{1}{a+b\sinh(c+dx)}\,dx = \frac{\log\left(\frac{\left|2\,be^{(dx+c)}+2\,a-2\,\sqrt{a^2+b^2}\right|}{\left|2\,be^{(dx+c)}+2\,a+2\,\sqrt{a^2+b^2}\right|}\right)}{\sqrt{a^2+b^2}d}$$

input
```
integrate(1/(a+b*sinh(d*x+c)),x, algorithm="giac")
```

output
```
log(abs(2*b*e^(d*x + c) + 2*a - 2*sqrt(a^2 + b^2))/abs(2*b*e^(d*x + c) + 2
*a + 2*sqrt(a^2 + b^2)))/(sqrt(a^2 + b^2)*d)
```

Mupad [B] (verification not implemented)

Time = 1.81 (sec) , antiderivative size = 55, normalized size of antiderivative = 1.25

$$\int \frac{1}{a+b\sinh(c+dx)}\,dx = \frac{2\,\mathrm{atan}\left(\frac{a\,d+b\,d\,\mathrm{e}^{d\,x}\,\mathrm{e}^{c}}{\sqrt{-a^2\,d^2-b^2\,d^2}}\right)}{\sqrt{-a^2\,d^2-b^2\,d^2}}$$

input
```
int(1/(a + b*sinh(c + d*x)),x)
```

output
```
(2*atan((a*d + b*d*exp(d*x)*exp(c))/(- a^2*d^2 - b^2*d^2)^(1/2)))/(- a^2*d
^2 - b^2*d^2)^(1/2)
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 49, normalized size of antiderivative = 1.11

$$\int \frac{1}{a+b\sinh(c+dx)}\,dx=\frac{2\sqrt{a^2+b^2}\,atan\left(\frac{e^{dx+c}bi+ai}{\sqrt{a^2+b^2}}\right)i}{d\,(a^2+b^2)}$$

input
```
int(1/(a+b*sinh(d*x+c)),x)
```

output
```
(2*sqrt(a**2 + b**2)*atan((e**(c + d*x)*b*i + a*i)/sqrt(a**2 + b**2))*i)/(
d*(a**2 + b**2))
```

3.102 $\int \frac{1}{(a+b\sinh(c+dx))^2}\,dx$

Optimal result

Integrand size = 12, antiderivative size = 79

$$\int \frac{1}{(a+b\sinh(c+dx))^2}\,dx = -\frac{2a\text{arctanh}\left(\frac{b-a\tanh\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2+b^2}}\right)}{\left(a^2+b^2\right)^{3/2}d} - \frac{b\cosh(c+dx)}{(a^2+b^2)\,d(a+b\sinh(c+dx))}$$

output
```
-2*a*arctanh((b-a*tanh(1/2*d*x+1/2*c))/(a^2+b^2)^(1/2))/(a^2+b^2)^(3/2)/d-
b*cosh(d*x+c)/(a^2+b^2)/d/(a+b*sinh(d*x+c))
```

Mathematica [A] (verified)

Time = 0.22 (sec) , antiderivative size = 85, normalized size of antiderivative = 1.08

$$\int \frac{1}{(a+b\sinh(c+dx))^2}\,dx = -\frac{\frac{2a\arctan\left(\frac{b-a\tanh\left(\frac{1}{2}(c+dx)\right)}{\sqrt{-a^2-b^2}}\right)}{\left(-a^2-b^2\right)^{3/2}} + \frac{b\cosh(c+dx)}{(a^2+b^2)(a+b\sinh(c+dx))}}{d}$$

input
```
Integrate[(a + b*Sinh[c + d*x])^(-2),x]
```

output
```
-(((2*a*ArcTan[(b - a*Tanh[(c + d*x)/2])/Sqrt[-a^2 - b^2]])/(-a^2 - b^2)^(
3/2) + (b*Cosh[c + d*x])/((a^2 + b^2)*(a + b*Sinh[c + d*x])))/d)
```

Rubi [A] (warning: unable to verify)

Time = 0.35 (sec) , antiderivative size = 77, normalized size of antiderivative = 0.97, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.667, Rules used = {3042, 3143, 25, 27, 3042, 3139, 1083, 217}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(a+b\sinh(c+dx))^2}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{(a-ib\sin(ic+idx))^2}dx$$

$$\Big\downarrow \text{3143}$$

$$-\frac{\int -\frac{a}{a+b\sinh(c+dx)}dx}{a^2+b^2}-\frac{b\cosh(c+dx)}{d\left(a^2+b^2\right)(a+b\sinh(c+dx))}$$

$$\Big\downarrow \text{25}$$

$$\frac{\int \frac{a}{a+b\sinh(c+dx)}dx}{a^2+b^2}-\frac{b\cosh(c+dx)}{d\left(a^2+b^2\right)(a+b\sinh(c+dx))}$$

$$\Big\downarrow \text{27}$$

$$\frac{a\int \frac{1}{a+b\sinh(c+dx)}dx}{a^2+b^2}-\frac{b\cosh(c+dx)}{d\left(a^2+b^2\right)(a+b\sinh(c+dx))}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{b\cosh(c+dx)}{d\left(a^2+b^2\right)(a+b\sinh(c+dx))}+\frac{a\int \frac{1}{a-ib\sin(ic+idx)}dx}{a^2+b^2}$$

$$\Big\downarrow \text{3139}$$

$$-\frac{b\cosh(c+dx)}{d(a^2+b^2)(a+b\sinh(c+dx))}-\frac{2ia\int\frac{1}{-a\tanh^2\left(\frac{1}{2}(c+dx)\right)+2b\tanh\left(\frac{1}{2}(c+dx)\right)+a}d\left(i\tanh\left(\frac{1}{2}(c+dx)\right)\right)}{d(a^2+b^2)}$$

$$\Big\downarrow \text{1083}$$

$$-\frac{b\cosh(c+dx)}{d(a^2+b^2)(a+b\sinh(c+dx))}+\frac{4ia\int\frac{1}{\tanh^2\left(\frac{1}{2}(c+dx)\right)-4(a^2+b^2)}d\left(2ia\tanh\left(\frac{1}{2}(c+dx)\right)-2ib\right)}{d(a^2+b^2)}$$

$$\Big\downarrow \text{217}$$

$$\frac{2a\text{arctanh}\left(\frac{\tanh\left(\frac{1}{2}(c+dx)\right)}{2\sqrt{a^2+b^2}}\right)}{d(a^2+b^2)^{3/2}}-\frac{b\cosh(c+dx)}{d(a^2+b^2)(a+b\sinh(c+dx))}$$

input
```
Int[(a + b*Sinh[c + d*x])^(-2),x]
```

output
```
(2*a*ArcTanh[Tanh[(c + d*x)/2]/(2*Sqrt[a^2 + b^2])])/((a^2 + b^2)^(3/2)*d)
 - (b*Cosh[c + d*x])/((a^2 + b^2)*d*(a + b*Sinh[c + d*x]))
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 1083
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Simp[-2   Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3139
```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Simp[2*(e/d)   Subst[Int[1/(a + 2*b*e*x + a
*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[a^2 - b^2, 0]
```

rule 3143
```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos
[c + d*x]*((a + b*Sin[c + d*x])^(n + 1)/(d*(n + 1)*(a^2 - b^2))), x] + Simp
[1/((n + 1)*(a^2 - b^2))   Int[(a + b*Sin[c + d*x])^(n + 1)*Simp[a*(n + 1)
- b*(n + 2)*Sin[c + d*x], x], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
 b^2, 0] && LtQ[n, -1] && IntegerQ[2*n]
```

Maple [A] (verified)

Time = 0.20 (sec) , antiderivative size = 118, normalized size of antiderivative = 1.49

method	result
derivativedivides	$\frac{-\frac{2\left(-\frac{b^{2}\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)}{a\left(a^{2}+b^{2}\right)}-\frac{b}{a^{2}+b^{2}}\right)}{\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}a-2b\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)-a}+\frac{2a\,\operatorname{arctanh}\left(\frac{2a\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)-2b}{2\sqrt{a^{2}+b^{2}}}\right)}{\left(a^{2}+b^{2}\right)^{\frac{3}{2}}}}{d}$
default	$\frac{-\frac{2\left(-\frac{b^{2}\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)}{a\left(a^{2}+b^{2}\right)}-\frac{b}{a^{2}+b^{2}}\right)}{\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}a-2b\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)-a}+\frac{2a\,\operatorname{arctanh}\left(\frac{2a\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)-2b}{2\sqrt{a^{2}+b^{2}}}\right)}{\left(a^{2}+b^{2}\right)^{\frac{3}{2}}}}{d}$
risch	$\frac{2a\,e^{dx+c}-2b}{d(a^{2}+b^{2})\left(b\,e^{2dx+2c}+2a\,e^{dx+c}-b\right)}+\frac{a\ln\left(e^{dx+c}+\frac{a\left(a^{2}+b^{2}\right)^{\frac{3}{2}}-a^{4}-2a^{2}b^{2}-b^{4}}{b\left(a^{2}+b^{2}\right)^{\frac{3}{2}}}\right)}{(a^{2}+b^{2})^{\frac{3}{2}}d}-\frac{a\ln\left(e^{dx+c}+\frac{a\left(a^{2}+b^{2}\right)^{\frac{3}{2}}+a^{4}+2\ldots}{b\left(a^{2}+b^{2}\right)^{\frac{3}{2}}}\ldots\right)}{(a^{2}+b^{2})^{\frac{3}{2}}d}$

input
```
int(1/(a+b*sinh(d*x+c))^2,x,method=_RETURNVERBOSE)
```

output
```
1/d*(-2*(-b^2/a/(a^2+b^2)*tanh(1/2*d*x+1/2*c)-b/(a^2+b^2))/(tanh(1/2*d*x+1
/2*c)^2*a-2*b*tanh(1/2*d*x+1/2*c)-a)+2*a/(a^2+b^2)^(3/2)*arctanh(1/2*(2*a*
tanh(1/2*d*x+1/2*c)-2*b)/(a^2+b^2)^(1/2))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 423 vs. 2(76) = 152.

Time = 0.09 (sec) , antiderivative size = 423, normalized size of antiderivative = 5.35

$$\int \frac{1}{(a+b\sinh(c+dx))^2}\,dx = -\frac{2\,a^2b+2\,b^3-\left(ab\cosh\left(dx+c\right)^2+ab\sinh\left(dx+c\right)^2+2\,a^2\cosh\left(dx+c\right)-ab+2\left(ab\cosh\left(dx+c\right)+\right.\right.}{(a^4b+2\,a^2b^3+b^5)d\cosh\left(dx+c\right)^2+(a^4b+2\,a^2b^3+b^5)d\,\mathrm{si}}$$

input
```
integrate(1/(a+b*sinh(d*x+c))^2,x, algorithm="fricas")
```

output
```
-(2*a^2*b + 2*b^3 - (a*b*cosh(d*x + c)^2 + a*b*sinh(d*x + c)^2 + 2*a^2*cos
h(d*x + c) - a*b + 2*(a*b*cosh(d*x + c) + a^2)*sinh(d*x + c))*sqrt(a^2 + b
^2)*log((b^2*cosh(d*x + c)^2 + b^2*sinh(d*x + c)^2 + 2*a*b*cosh(d*x + c) +
 2*a^2 + b^2 + 2*(b^2*cosh(d*x + c) + a*b)*sinh(d*x + c) - 2*sqrt(a^2 + b^
2)*(b*cosh(d*x + c) + b*sinh(d*x + c) + a))/(b*cosh(d*x + c)^2 + b*sinh(d*
x + c)^2 + 2*a*cosh(d*x + c) + 2*(b*cosh(d*x + c) + a)*sinh(d*x + c) - b))
 - 2*(a^3 + a*b^2)*cosh(d*x + c) - 2*(a^3 + a*b^2)*sinh(d*x + c))/((a^4*b
+ 2*a^2*b^3 + b^5)*d*cosh(d*x + c)^2 + (a^4*b + 2*a^2*b^3 + b^5)*d*sinh(d*
x + c)^2 + 2*(a^5 + 2*a^3*b^2 + a*b^4)*d*cosh(d*x + c) - (a^4*b + 2*a^2*b^
3 + b^5)*d + 2*((a^4*b + 2*a^2*b^3 + b^5)*d*cosh(d*x + c) + (a^5 + 2*a^3*b
^2 + a*b^4)*d)*sinh(d*x + c))
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2082 vs. 2(68) = 136.

Time = 81.16 (sec) , antiderivative size = 2082, normalized size of antiderivative = 26.35

$$\int \frac{1}{(a+b\sinh(c+dx))^2}\,dx = \text{Too large to display}$$

input
```
integrate(1/(a+b*sinh(d*x+c))**2,x)
```

output
```
Piecewise((zoo*x/sinh(c)**2, Eq(a, 0) & Eq(b, 0) & Eq(d, 0)), ((-tanh(c/2
+ d*x/2)/(2*d) - 1/(2*d*tanh(c/2 + d*x/2)))/b**2, Eq(a, 0)), (-6*b*tanh(c/
2 + d*x/2)**2/(3*b**3*d*tanh(c/2 + d*x/2)**3 - 9*b**3*d*tanh(c/2 + d*x/2)
- 9*b**2*d*sqrt(-b**2)*tanh(c/2 + d*x/2)**2 + 3*b**2*d*sqrt(-b**2)) + 4*b/
(3*b**3*d*tanh(c/2 + d*x/2)**3 - 9*b**3*d*tanh(c/2 + d*x/2) - 9*b**2*d*sqr
t(-b**2)*tanh(c/2 + d*x/2)**2 + 3*b**2*d*sqrt(-b**2)) + 6*sqrt(-b**2)*tanh
(c/2 + d*x/2)/(3*b**3*d*tanh(c/2 + d*x/2)**3 - 9*b**3*d*tanh(c/2 + d*x/2)
- 9*b**2*d*sqrt(-b**2)*tanh(c/2 + d*x/2)**2 + 3*b**2*d*sqrt(-b**2)), Eq(a,
 -sqrt(-b**2))), (-6*b*tanh(c/2 + d*x/2)**2/(3*b**3*d*tanh(c/2 + d*x/2)**3
 - 9*b**3*d*tanh(c/2 + d*x/2) + 9*b**2*d*sqrt(-b**2)*tanh(c/2 + d*x/2)**2
- 3*b**2*d*sqrt(-b**2)) + 4*b/(3*b**3*d*tanh(c/2 + d*x/2)**3 - 9*b**3*d*ta
nh(c/2 + d*x/2) + 9*b**2*d*sqrt(-b**2)*tanh(c/2 + d*x/2)**2 - 3*b**2*d*sqr
t(-b**2)) - 6*sqrt(-b**2)*tanh(c/2 + d*x/2)/(3*b**3*d*tanh(c/2 + d*x/2)**3
 - 9*b**3*d*tanh(c/2 + d*x/2) + 9*b**2*d*sqrt(-b**2)*tanh(c/2 + d*x/2)**2
- 3*b**2*d*sqrt(-b**2)), Eq(a, sqrt(-b**2))), (x/(a + b*sinh(c))**2, Eq(d,
 0)), (-a**3*log(tanh(c/2 + d*x/2) - b/a - sqrt(a**2 + b**2)/a)*tanh(c/2 +
 d*x/2)**2/(a**4*d*sqrt(a**2 + b**2)*tanh(c/2 + d*x/2)**2 - a**4*d*sqrt(a*
*2 + b**2) - 2*a**3*b*d*sqrt(a**2 + b**2)*tanh(c/2 + d*x/2) + a**2*b**2*d*
sqrt(a**2 + b**2)*tanh(c/2 + d*x/2)**2 - a**2*b**2*d*sqrt(a**2 + b**2) - 2
*a*b**3*d*sqrt(a**2 + b**2)*tanh(c/2 + d*x/2)) + a**3*log(tanh(c/2 + d*...
```

Maxima [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 138, normalized size of antiderivative = 1.75

$$\int \frac{1}{(a+b\sinh(c+dx))^2}\,dx = \frac{a\log\left(\frac{be^{(-dx-c)}-a-\sqrt{a^2+b^2}}{be^{(-dx-c)}-a+\sqrt{a^2+b^2}}\right)}{(a^2+b^2)^{\frac{3}{2}}d} - \frac{2\left(ae^{(-dx-c)}+b\right)}{(a^2b+b^3+2\,(a^3+ab^2)e^{(-dx-c)}-(a^2b+b^3)e^{(-2\,dx-2\,c)})d}$$

input
```
integrate(1/(a+b*sinh(d*x+c))^2,x, algorithm="maxima")
```

output
```
a*log((b*e^(-d*x - c) - a - sqrt(a^2 + b^2))/(b*e^(-d*x - c) - a + sqrt(a^
2 + b^2)))/((a^2 + b^2)^(3/2)*d) - 2*(a*e^(-d*x - c) + b)/((a^2*b + b^3 +
2*(a^3 + a*b^2)*e^(-d*x - c) - (a^2*b + b^3)*e^(-2*d*x - 2*c))*d)
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 119, normalized size of antiderivative = 1.51

$$\int \frac{1}{(a+b\sinh(c+dx))^2}\,dx = \frac{\frac{a\log\left(\frac{\left|2\,be^{(dx+c)}+2\,a-2\,\sqrt{a^2+b^2}\right|}{\left|2\,be^{(dx+c)}+2\,a+2\,\sqrt{a^2+b^2}\right|}\right)}{(a^2+b^2)^{\frac{3}{2}}}+\frac{2\,(ae^{(dx+c)}-b)}{(a^2+b^2)(be^{(2\,dx+2\,c)}+2\,ae^{(dx+c)}-b)}}{d}$$

input
```
integrate(1/(a+b*sinh(d*x+c))^2,x, algorithm="giac")
```

output
```
(a*log(abs(2*b*e^(d*x + c) + 2*a - 2*sqrt(a^2 + b^2))/abs(2*b*e^(d*x + c)
+ 2*a + 2*sqrt(a^2 + b^2)))/(a^2 + b^2)^(3/2) + 2*(a*e^(d*x + c) - b)/((a^
2 + b^2)*(b*e^(2*d*x + 2*c) + 2*a*e^(d*x + c) - b)))/d
```

Mupad [B] (verification not implemented)

Time = 1.97 (sec) , antiderivative size = 200, normalized size of antiderivative = 2.53

$$\int \frac{1}{(a+b\sinh(c+dx))^2}\,dx = \frac{a\,\ln\left(\frac{2\,a\,\left(b-a\,e^{c+d\,x}\right)}{b\,{\left(a^2+b^2\right)}^{3/2}}-\frac{2\,a\,e^{c+d\,x}}{b\,\left(a^2+b^2\right)}\right)}{d\,{\left(a^2+b^2\right)}^{3/2}} - \frac{a\,\ln\left(-\frac{2\,a\,e^{c+d\,x}}{b\,\left(a^2+b^2\right)}-\frac{2\,a\,\left(b-a\,e^{c+d\,x}\right)}{b\,{\left(a^2+b^2\right)}^{3/2}}\right)}{d\,{\left(a^2+b^2\right)}^{3/2}} - \frac{\frac{2\,b^2}{d\,\left(a^2\,b+b^3\right)}-\frac{2\,a\,b\,e^{c+d\,x}}{d\,\left(a^2\,b+b^3\right)}}{2\,a\,e^{c+d\,x}-b+b\,e^{2\,c+2\,d\,x}}$$

input
```
int(1/(a + b*sinh(c + d*x))^2,x)
```

output
```
(a*log((2*a*(b - a*exp(c + d*x)))/(b*(a^2 + b^2)^(3/2)) - (2*a*exp(c + d*x
))/(b*(a^2 + b^2))))/(d*(a^2 + b^2)^(3/2)) - (a*log(- (2*a*exp(c + d*x))/(
b*(a^2 + b^2)) - (2*a*(b - a*exp(c + d*x)))/(b*(a^2 + b^2)^(3/2))))/(d*(a^
2 + b^2)^(3/2)) - ((2*b^2)/(d*(a^2*b + b^3)) - (2*a*b*exp(c + d*x))/(d*(a^
2*b + b^3)))/(2*a*exp(c + d*x) - b + b*exp(2*c + 2*d*x))
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 291, normalized size of antiderivative = 3.68

$$\int \frac{1}{(a+b\sinh(c+dx))^2}\,dx$$
$$= \frac{2e^{2dx+2c}\sqrt{a^2+b^2}\,atan\left(\frac{e^{dx+c}bi+ai}{\sqrt{a^2+b^2}}\right)abi+4e^{dx+c}\sqrt{a^2+b^2}\,atan\left(\frac{e^{dx+c}bi+ai}{\sqrt{a^2+b^2}}\right)a^2i-2\sqrt{a^2+b^2}\,atan\left(\frac{e^{dx+c}bi+a}{\sqrt{a^2+b^2}}\right.}{d\left(e^{2dx+2c}a^4b+2e^{2dx+2c}a^2b^3+e^{2dx+2c}b^5+2e^{dx+c}a^5+4e^{dx+c}a^3b^2+2e^{dx+c}a\,b^4-\right.}$$

input
```
int(1/(a+b*sinh(d*x+c))^2,x)
```

output
```
(2*e**(2*c + 2*d*x)*sqrt(a**2 + b**2)*atan((e**(c + d*x)*b*i + a*i)/sqrt(a
**2 + b**2))*a*b*i + 4*e**(c + d*x)*sqrt(a**2 + b**2)*atan((e**(c + d*x)*b
*i + a*i)/sqrt(a**2 + b**2))*a**2*i - 2*sqrt(a**2 + b**2)*atan((e**(c + d*
x)*b*i + a*i)/sqrt(a**2 + b**2))*a*b*i - e**(2*c + 2*d*x)*a**2*b - e**(2*c
 + 2*d*x)*b**3 - a**2*b - b**3)/(d*(e**(2*c + 2*d*x)*a**4*b + 2*e**(2*c +
2*d*x)*a**2*b**3 + e**(2*c + 2*d*x)*b**5 + 2*e**(c + d*x)*a**5 + 4*e**(c +
 d*x)*a**3*b**2 + 2*e**(c + d*x)*a*b**4 - a**4*b - 2*a**2*b**3 - b**5))
```

3.103 $\int \frac{1}{(a+b\sinh(c+dx))^3}\,dx$

Optimal result

Integrand size = 12, antiderivative size = 127

$$\int \frac{1}{(a+b\sinh(c+dx))^3}\,dx = -\frac{(2a^2-b^2)\operatorname{arctanh}\left(\frac{b-a\tanh\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2+b^2}}\right)}{\left(a^2+b^2\right)^{5/2}d} - \frac{b\cosh(c+dx)}{2\left(a^2+b^2\right)d(a+b\sinh(c+dx))^2} - \frac{3ab\cosh(c+dx)}{2\left(a^2+b^2\right)^2d(a+b\sinh(c+dx))}$$

output
```
-(2*a^2-b^2)*arctanh((b-a*tanh(1/2*d*x+1/2*c))/(a^2+b^2)^(1/2))/(a^2+b^2)^
(5/2)/d-1/2*b*cosh(d*x+c)/(a^2+b^2)/d/(a+b*sinh(d*x+c))^2-3/2*a*b*cosh(d*x
+c)/(a^2+b^2)^2/d/(a+b*sinh(d*x+c))
```

Mathematica [A] (verified)

Time = 0.23 (sec) , antiderivative size = 117, normalized size of antiderivative = 0.92

$$\int \frac{1}{(a+b\sinh(c+dx))^3}\,dx = \frac{\frac{2(2a^2-b^2)\arctan\left(\frac{b-a\tanh\left(\frac{1}{2}(c+dx)\right)}{\sqrt{-a^2-b^2}}\right)}{\sqrt{-a^2-b^2}} - \frac{b\cosh(c+dx)(4a^2+b^2+3ab\sinh(c+dx))}{(a+b\sinh(c+dx))^2}}{2\left(a^2+b^2\right)^2d}$$

input
```
Integrate[(a + b*Sinh[c + d*x])^(-3),x]
```

output
```
((2*(2*a^2 - b^2)*ArcTan[(b - a*Tanh[(c + d*x)/2])/Sqrt[-a^2 - b^2]])/Sqrt
[-a^2 - b^2] - (b*Cosh[c + d*x]*(4*a^2 + b^2 + 3*a*b*Sinh[c + d*x]))/(a +
b*Sinh[c + d*x])^2)/(2*(a^2 + b^2)^2*d)
```

Rubi [A] (warning: unable to verify)

Time = 0.58 (sec) , antiderivative size = 137, normalized size of antiderivative = 1.08, number of steps used = 12, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.917$, Rules used = {3042, 3143, 25, 3042, 3233, 25, 27, 3042, 3139, 1083, 217}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(a+b\sinh(c+dx))^3}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{(a-ib\sin(ic+idx))^3}dx$$

$$\Big\downarrow \text{3143}$$

$$-\frac{\int -\frac{2a-b\sinh(c+dx)}{(a+b\sinh(c+dx))^2}dx}{2\left(a^2+b^2\right)}-\frac{b\cosh(c+dx)}{2d\left(a^2+b^2\right)(a+b\sinh(c+dx))^2}$$

$$\Big\downarrow \text{25}$$

$$\frac{\int \frac{2a-b\sinh(c+dx)}{(a+b\sinh(c+dx))^2}dx}{2\left(a^2+b^2\right)}-\frac{b\cosh(c+dx)}{2d\left(a^2+b^2\right)(a+b\sinh(c+dx))^2}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{b\cosh(c+dx)}{2d\left(a^2+b^2\right)(a+b\sinh(c+dx))^2}+\frac{\int \frac{2a+ib\sin(ic+idx)}{(a-ib\sin(ic+idx))^2}dx}{2\left(a^2+b^2\right)}$$

$$\Big\downarrow \text{3233}$$

$$\frac{-\frac{\int -\frac{2a^2-b^2}{a+b\sinh(c+dx)}dx}{a^2+b^2}-\frac{3ab\cosh(c+dx)}{d(a^2+b^2)(a+b\sinh(c+dx))}}{2\left(a^2+b^2\right)}-\frac{b\cosh(c+dx)}{2d\left(a^2+b^2\right)(a+b\sinh(c+dx))^2}$$

$$\Big\downarrow \; 25$$

$$\frac{\frac{\int \frac{2a^2-b^2}{a+b\sinh(c+dx)}dx}{a^2+b^2}-\frac{3ab\cosh(c+dx)}{d(a^2+b^2)(a+b\sinh(c+dx))}}{2\left(a^2+b^2\right)}-\frac{b\cosh(c+dx)}{2d\left(a^2+b^2\right)(a+b\sinh(c+dx))^2}$$

$$\Big\downarrow \; 27$$

$$\frac{\frac{\left(2a^2-b^2\right)\int \frac{1}{a+b\sinh(c+dx)}dx}{a^2+b^2}-\frac{3ab\cosh(c+dx)}{d(a^2+b^2)(a+b\sinh(c+dx))}}{2\left(a^2+b^2\right)}-\frac{b\cosh(c+dx)}{2d\left(a^2+b^2\right)(a+b\sinh(c+dx))^2}$$

$$\Big\downarrow \; 3042$$

$$-\frac{b\cosh(c+dx)}{2d\left(a^2+b^2\right)(a+b\sinh(c+dx))^2}+\frac{-\frac{3ab\cosh(c+dx)}{d(a^2+b^2)(a+b\sinh(c+dx))}+\frac{\left(2a^2-b^2\right)\int \frac{1}{a-ib\sin(ic+idx)}dx}{a^2+b^2}}{2\left(a^2+b^2\right)}$$

$$\Big\downarrow \; 3139$$

$$-\frac{b\cosh(c+dx)}{2d\left(a^2+b^2\right)(a+b\sinh(c+dx))^2}+\frac{-\frac{3ab\cosh(c+dx)}{d(a^2+b^2)(a+b\sinh(c+dx))}-\frac{2i\left(2a^2-b^2\right)\int \frac{1}{-a\tanh^2\left(\frac{1}{2}(c+dx)\right)+2b\tanh\left(\frac{1}{2}(c+dx)\right)+a}d\left(i\tanh\left(\frac{1}{2}(c+dx)\right)\right)}{d(a^2+b^2)}}{2\left(a^2+b^2\right)}$$

$$\Big\downarrow \; 1083$$

$$-\frac{b\cosh(c+dx)}{2d\left(a^2+b^2\right)(a+b\sinh(c+dx))^2}+\frac{-\frac{3ab\cosh(c+dx)}{d(a^2+b^2)(a+b\sinh(c+dx))}+\frac{4i\left(2a^2-b^2\right)\int \frac{1}{\tanh^2\left(\frac{1}{2}(c+dx)\right)-4\left(a^2+b^2\right)}d\left(2ia\tanh\left(\frac{1}{2}(c+dx)\right)-2ib\right)}{d(a^2+b^2)}}{2\left(a^2+b^2\right)}$$

$$\Big\downarrow \; 217$$

$$\frac{\frac{2(2a^2-b^2)\operatorname{arctanh}\left(\frac{\tanh\left(\frac{1}{2}(c+dx)\right)}{2\sqrt{a^2+b^2}}\right)}{d(a^2+b^2)^{3/2}}-\frac{3ab\cosh(c+dx)}{d(a^2+b^2)(a+b\sinh(c+dx))}}{2\left(a^2+b^2\right)}-\frac{b\cosh(c+dx)}{2d\left(a^2+b^2\right)(a+b\sinh(c+dx))^2}$$

input
```
Int[(a + b*Sinh[c + d*x])^(-3),x]
```

output
```
-1/2*(b*Cosh[c + d*x])/((a^2 + b^2)*d*(a + b*Sinh[c + d*x])^2) + ((2*(2*a^
2 - b^2)*ArcTanh[Tanh[(c + d*x)/2]/(2*Sqrt[a^2 + b^2])])/((a^2 + b^2)^(3/2
)*d) - (3*a*b*Cosh[c + d*x])/((a^2 + b^2)*d*(a + b*Sinh[c + d*x])))/(2*(a^
2 + b^2))
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 1083
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Simp[-2   Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3139
```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Simp[2*(e/d)   Subst[Int[1/(a + 2*b*e*x + a
*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[a^2 - b^2, 0]
```

rule 3143
```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos
[c + d*x]*((a + b*Sin[c + d*x])^(n + 1)/(d*(n + 1)*(a^2 - b^2))), x] + Simp
[1/((n + 1)*(a^2 - b^2))   Int[(a + b*Sin[c + d*x])^(n + 1)*Simp[a*(n + 1)
- b*(n + 2)*Sin[c + d*x], x], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
 b^2, 0] && LtQ[n, -1] && IntegerQ[2*n]
```

rule 3233
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Simp[(-(b*c - a*d))*Cos[e + f*x]*((a + b*Sin[e +
 f*x])^(m + 1)/(f*(m + 1)*(a^2 - b^2))), x] + Simp[1/((m + 1)*(a^2 - b^2))
  Int[(a + b*Sin[e + f*x])^(m + 1)*Simp[(a*c - b*d)*(m + 1) - (b*c - a*d)*(
m + 2)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c
- a*d, 0] && NeQ[a^2 - b^2, 0] && LtQ[m, -1] && IntegerQ[2*m]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 279 vs. $2(118) = 236$.

Time = 0.36 (sec) , antiderivative size = 280, normalized size of antiderivative = 2.20

method	result
derivativedivides	$\frac{-\frac{2\left(-\frac{b^2\left(5a^2+2b^2\right)\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^3}{2a\left(a^4+2a^2b^2+b^4\right)}-\frac{b\left(4a^4-7a^2b^2-2b^4\right)\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^2}{2\left(a^4+2a^2b^2+b^4\right)a^2}+\frac{b^2\left(11a^2+2b^2\right)\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)}{2\left(a^4+2a^2b^2+b^4\right)a}+\frac{b\left(4a^2+b^2\right)}{2a^4+4a^2b^2+2b^4}\right)}{\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^2a-2b\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)-a\right)^2}+}{d}$
default	$\frac{-\frac{2\left(-\frac{b^2\left(5a^2+2b^2\right)\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^3}{2a\left(a^4+2a^2b^2+b^4\right)}-\frac{b\left(4a^4-7a^2b^2-2b^4\right)\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^2}{2\left(a^4+2a^2b^2+b^4\right)a^2}+\frac{b^2\left(11a^2+2b^2\right)\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)}{2\left(a^4+2a^2b^2+b^4\right)a}+\frac{b\left(4a^2+b^2\right)}{2a^4+4a^2b^2+2b^4}\right)}{\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^2a-2b\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)-a\right)^2}+}{d}$
risch	$\frac{2a^2b\,e^{3dx+3c}-b^3e^{3dx+3c}+6a^3e^{2dx+2c}-3a\,b^2e^{2dx+2c}-10a^2b\,e^{dx+c}-e^{dx+c}b^3+3a\,b^2}{d(a^2+b^2)^2\left(b\,e^{2dx+2c}+2a\,e^{dx+c}-b\right)^2}+\frac{\ln\left(e^{dx+c}+\frac{\left(a^2+b^2\right)^{\frac{5}{2}}a-a^6-}{b\left(a^2+\right.}\right.}{(a^2+b^2)^{\frac{5}{2}}}$

input
```
int(1/(a+b*sinh(d*x+c))^3,x,method=_RETURNVERBOSE)
```

output
```
1/d*(-2*(-1/2*b^2*(5*a^2+2*b^2)/a/(a^4+2*a^2*b^2+b^4)*tanh(1/2*d*x+1/2*c)^
3-1/2*b*(4*a^4-7*a^2*b^2-2*b^4)/(a^4+2*a^2*b^2+b^4)/a^2*tanh(1/2*d*x+1/2*c
)^2+1/2*b^2*(11*a^2+2*b^2)/(a^4+2*a^2*b^2+b^4)/a*tanh(1/2*d*x+1/2*c)+1/2*b
*(4*a^2+b^2)/(a^4+2*a^2*b^2+b^4))/(tanh(1/2*d*x+1/2*c)^2*a-2*b*tanh(1/2*d*
x+1/2*c)-a)^2+(2*a^2-b^2)/(a^4+2*a^2*b^2+b^4)/(a^2+b^2)^(1/2)*arctanh(1/2*
(2*a*tanh(1/2*d*x+1/2*c)-2*b)/(a^2+b^2)^(1/2)))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1347 vs. $2(120) = 240$.

Time = 0.11 (sec) , antiderivative size = 1347, normalized size of antiderivative = 10.61

$$\int \frac{1}{(a+b\sinh(c+dx))^3}\,dx = \text{Too large to display}$$

input
```
integrate(1/(a+b*sinh(d*x+c))^3,x, algorithm="fricas")
```

output
```
1/2*(6*a^3*b^2 + 6*a*b^4 + 2*(2*a^4*b + a^2*b^3 - b^5)*cosh(d*x + c)^3 + 2
*(2*a^4*b + a^2*b^3 - b^5)*sinh(d*x + c)^3 + 6*(2*a^5 + a^3*b^2 - a*b^4)*c
osh(d*x + c)^2 + 6*(2*a^5 + a^3*b^2 - a*b^4 + (2*a^4*b + a^2*b^3 - b^5)*co
sh(d*x + c))*sinh(d*x + c)^2 - ((2*a^2*b^2 - b^4)*cosh(d*x + c)^4 + (2*a^2
*b^2 - b^4)*sinh(d*x + c)^4 + 2*a^2*b^2 - b^4 + 4*(2*a^3*b - a*b^3)*cosh(d
*x + c)^3 + 4*(2*a^3*b - a*b^3 + (2*a^2*b^2 - b^4)*cosh(d*x + c))*sinh(d*x
 + c)^3 + 2*(4*a^4 - 4*a^2*b^2 + b^4)*cosh(d*x + c)^2 + 2*(4*a^4 - 4*a^2*b
^2 + b^4 + 3*(2*a^2*b^2 - b^4)*cosh(d*x + c)^2 + 6*(2*a^3*b - a*b^3)*cosh(
d*x + c))*sinh(d*x + c)^2 - 4*(2*a^3*b - a*b^3)*cosh(d*x + c) - 4*(2*a^3*b
 - a*b^3 - (2*a^2*b^2 - b^4)*cosh(d*x + c)^3 - 3*(2*a^3*b - a*b^3)*cosh(d*
x + c)^2 - (4*a^4 - 4*a^2*b^2 + b^4)*cosh(d*x + c))*sinh(d*x + c))*sqrt(a^
2 + b^2)*log((b^2*cosh(d*x + c)^2 + b^2*sinh(d*x + c)^2 + 2*a*b*cosh(d*x +
 c) + 2*a^2 + b^2 + 2*(b^2*cosh(d*x + c) + a*b)*sinh(d*x + c) + 2*sqrt(a^2
 + b^2)*(b*cosh(d*x + c) + b*sinh(d*x + c) + a))/(b*cosh(d*x + c)^2 + b*si
nh(d*x + c)^2 + 2*a*cosh(d*x + c) + 2*(b*cosh(d*x + c) + a)*sinh(d*x + c)
- b)) - 2*(10*a^4*b + 11*a^2*b^3 + b^5)*cosh(d*x + c) - 2*(10*a^4*b + 11*a
^2*b^3 + b^5 - 3*(2*a^4*b + a^2*b^3 - b^5)*cosh(d*x + c)^2 - 6*(2*a^5 + a^
3*b^2 - a*b^4)*cosh(d*x + c))*sinh(d*x + c))/((a^6*b^2 + 3*a^4*b^4 + 3*a^2
*b^6 + b^8)*d*cosh(d*x + c)^4 + (a^6*b^2 + 3*a^4*b^4 + 3*a^2*b^6 + b^8)*d*
sinh(d*x + c)^4 + 4*(a^7*b + 3*a^5*b^3 + 3*a^3*b^5 + a*b^7)*d*cosh(d*x ...
```

Sympy [F(-1)]

Timed out.

$$\int \frac{1}{(a + b\sinh(c + dx))^3}\,dx = \text{Timed out}$$

input
```
integrate(1/(a+b*sinh(d*x+c))**3,x)
```

output
```
Timed out
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 315 vs. 2(120) = 240.

Time = 0.15 (sec) , antiderivative size = 315, normalized size of antiderivative = 2.48

$$\int \frac{1}{(a + b\sinh(c + dx))^3}\,dx = \frac{(2\,a^2 - b^2)\log\left(\frac{be^{(-dx-c)} - a - \sqrt{a^2+b^2}}{be^{(-dx-c)} - a + \sqrt{a^2+b^2}}\right)}{2\,(a^4 + 2\,a^2b^2 + b^4)\sqrt{a^2+b^2}d} - \frac{3\,ab^2 + (10\,a^2b + b^3)e^{(-dx-c)} + 3\,(2\,a^3 - ab^2)e^{(-2\,dx-2\,c)} - (2\,a^2b}{(a^4b^2 + 2\,a^2b^4 + b^6 + 4\,(a^5b + 2\,a^3b^3 + ab^5)e^{(-dx-c)} + 2\,(2\,a^6 + 3\,a^4b^2 - b^6)e^{(-2\,dx-2\,c)} - 4\,(a^5b + 2\,a^3b^3}$$

input
```
integrate(1/(a+b*sinh(d*x+c))^3,x, algorithm="maxima")
```

output
```
1/2*(2*a^2 - b^2)*log((b*e^(-d*x - c) - a - sqrt(a^2 + b^2))/(b*e^(-d*x -
c) - a + sqrt(a^2 + b^2)))/((a^4 + 2*a^2*b^2 + b^4)*sqrt(a^2 + b^2)*d) - (
3*a*b^2 + (10*a^2*b + b^3)*e^(-d*x - c) + 3*(2*a^3 - a*b^2)*e^(-2*d*x - 2*
c) - (2*a^2*b - b^3)*e^(-3*d*x - 3*c))/((a^4*b^2 + 2*a^2*b^4 + b^6 + 4*(a^
5*b + 2*a^3*b^3 + a*b^5)*e^(-d*x - c) + 2*(2*a^6 + 3*a^4*b^2 - b^6)*e^(-2*
d*x - 2*c) - 4*(a^5*b + 2*a^3*b^3 + a*b^5)*e^(-3*d*x - 3*c) + (a^4*b^2 + 2
*a^2*b^4 + b^6)*e^(-4*d*x - 4*c))*d)
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 231, normalized size of antiderivative = 1.82

$$\int \frac{1}{(a+b\sinh(c+dx))^3}\,dx = \frac{\frac{(2a^2-b^2)\log\left(\frac{|2be^{(dx+c)}+2a-2\sqrt{a^2+b^2}|}{|2be^{(dx+c)}+2a+2\sqrt{a^2+b^2}|}\right)}{(a^4+2a^2b^2+b^4)\sqrt{a^2+b^2}} + \frac{2\left(2a^2be^{(3dx+3c)}-b^3e^{(3dx+3c)}+6a^3e^{(2dx+2c)}-3ab^2e^{(2dx+2c)}-10a^2be^{(dx+c)}-b^3e^{(dx+c)}+\right.}{(a^4+2a^2b^2+b^4)\left(be^{(2dx+2c)}+2ae^{(dx+c)}-b\right)^2}}{2d}$$

input
```
integrate(1/(a+b*sinh(d*x+c))^3,x, algorithm="giac")
```

output
```
1/2*((2*a^2 - b^2)*log(abs(2*b*e^(d*x + c) + 2*a - 2*sqrt(a^2 + b^2))/abs(
2*b*e^(d*x + c) + 2*a + 2*sqrt(a^2 + b^2)))/((a^4 + 2*a^2*b^2 + b^4)*sqrt(
a^2 + b^2)) + 2*(2*a^2*b*e^(3*d*x + 3*c) - b^3*e^(3*d*x + 3*c) + 6*a^3*e^(
2*d*x + 2*c) - 3*a*b^2*e^(2*d*x + 2*c) - 10*a^2*b*e^(d*x + c) - b^3*e^(d*x
 + c) + 3*a*b^2)/((a^4 + 2*a^2*b^2 + b^4)*(b*e^(2*d*x + 2*c) + 2*a*e^(d*x
+ c) - b)^2))/d
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(a+b\sinh(c+dx))^3}\,dx = \int \frac{1}{\left(a+b\sinh\left(c+d\,x\right)\right)^3}\,dx$$

input
```
int(1/(a + b*sinh(c + d*x))^3,x)
```

output
```
int(1/(a + b*sinh(c + d*x))^3, x)
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 1033, normalized size of antiderivative = 8.13

$$\int \frac{1}{(a+b\sinh(c+dx))^3}\,dx = \text{Too large to display}$$

input
```
int(1/(a+b*sinh(d*x+c))^3,x)
```

output
```
(8*e**(4*c + 4*d*x)*sqrt(a**2 + b**2)*atan((e**(c + d*x)*b*i + a*i)/sqrt(a
**2 + b**2))*a**3*b**2*i - 4*e**(4*c + 4*d*x)*sqrt(a**2 + b**2)*atan((e**(
c + d*x)*b*i + a*i)/sqrt(a**2 + b**2))*a*b**4*i + 32*e**(3*c + 3*d*x)*sqrt
(a**2 + b**2)*atan((e**(c + d*x)*b*i + a*i)/sqrt(a**2 + b**2))*a**4*b*i -
16*e**(3*c + 3*d*x)*sqrt(a**2 + b**2)*atan((e**(c + d*x)*b*i + a*i)/sqrt(a
**2 + b**2))*a**2*b**3*i + 32*e**(2*c + 2*d*x)*sqrt(a**2 + b**2)*atan((e**
(c + d*x)*b*i + a*i)/sqrt(a**2 + b**2))*a**5*i - 32*e**(2*c + 2*d*x)*sqrt(
a**2 + b**2)*atan((e**(c + d*x)*b*i + a*i)/sqrt(a**2 + b**2))*a**3*b**2*i
+ 8*e**(2*c + 2*d*x)*sqrt(a**2 + b**2)*atan((e**(c + d*x)*b*i + a*i)/sqrt(
a**2 + b**2))*a*b**4*i - 32*e**(c + d*x)*sqrt(a**2 + b**2)*atan((e**(c + d
*x)*b*i + a*i)/sqrt(a**2 + b**2))*a**4*b*i + 16*e**(c + d*x)*sqrt(a**2 + b
**2)*atan((e**(c + d*x)*b*i + a*i)/sqrt(a**2 + b**2))*a**2*b**3*i + 8*sqrt
(a**2 + b**2)*atan((e**(c + d*x)*b*i + a*i)/sqrt(a**2 + b**2))*a**3*b**2*i
 - 4*sqrt(a**2 + b**2)*atan((e**(c + d*x)*b*i + a*i)/sqrt(a**2 + b**2))*a*
b**4*i - 2*e**(4*c + 4*d*x)*a**4*b**2 - e**(4*c + 4*d*x)*a**2*b**4 + e**(4
*c + 4*d*x)*b**6 + 16*e**(2*c + 2*d*x)*a**6 + 12*e**(2*c + 2*d*x)*a**4*b**
2 - 6*e**(2*c + 2*d*x)*a**2*b**4 - 2*e**(2*c + 2*d*x)*b**6 - 32*e**(c + d*
x)*a**5*b - 40*e**(c + d*x)*a**3*b**3 - 8*e**(c + d*x)*a*b**5 + 10*a**4*b*
*2 + 11*a**2*b**4 + b**6)/(4*a*d*(e**(4*c + 4*d*x)*a**6*b**2 + 3*e**(4*c +
 4*d*x)*a**4*b**4 + 3*e**(4*c + 4*d*x)*a**2*b**6 + e**(4*c + 4*d*x)*b**...
```

3.104 $\int \frac{1}{(a+b\sinh(c+dx))^4}\,dx$

Optimal result

Integrand size = 12, antiderivative size = 174

$$\int \frac{1}{(a+b\sinh(c+dx))^4}\,dx = -\frac{a(2a^2-3b^2)\,\text{arctanh}\left(\frac{b-a\tanh\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2+b^2}}\right)}{\left(a^2+b^2\right)^{7/2}d} - \frac{b\cosh(c+dx)}{3\left(a^2+b^2\right)d(a+b\sinh(c+dx))^3} - \frac{5ab\cosh(c+dx)}{6\left(a^2+b^2\right)^2d(a+b\sinh(c+dx))^2} - \frac{b(11a^2-4b^2)\cosh(c+dx)}{6\left(a^2+b^2\right)^3d(a+b\sinh(c+dx))}$$

output
```
-a*(2*a^2-3*b^2)*arctanh((b-a*tanh(1/2*d*x+1/2*c))/(a^2+b^2)^(1/2))/(a^2+b
^2)^(7/2)/d-1/3*b*cosh(d*x+c)/(a^2+b^2)/d/(a+b*sinh(d*x+c))^3-5/6*a*b*cosh
(d*x+c)/(a^2+b^2)^2/d/(a+b*sinh(d*x+c))^2-1/6*b*(11*a^2-4*b^2)*cosh(d*x+c)
/(a^2+b^2)^3/d/(a+b*sinh(d*x+c))
```

Mathematica [A] (verified)

Time = 0.50 (sec) , antiderivative size = 159, normalized size of antiderivative = 0.91

$$\int \frac{1}{(a+b\sinh(c+dx))^4}\,dx = \frac{\frac{6a(2a^2-3b^2)\arctan\left(\frac{b-a\tanh\left(\frac{1}{2}(c+dx)\right)}{\sqrt{-a^2-b^2}}\right)}{\sqrt{-a^2-b^2}}+\frac{b\cosh(c+dx)\left(-18a^4-5a^2b^2-2b^4+3ab(-9a^2+b^2)\sinh(c+dx)+(-11a^2b^2+4b^4)\sinh^2(c+dx)\right)}{(a+b\sinh(c+dx))^3}}{6(a^2+b^2)^3d}$$

input
```
Integrate[(a + b*Sinh[c + d*x])^(-4),x]
```

output
```
((6*a*(2*a^2 - 3*b^2)*ArcTan[(b - a*Tanh[(c + d*x)/2])/Sqrt[-a^2 - b^2]])/
Sqrt[-a^2 - b^2] + (b*Cosh[c + d*x]*(-18*a^4 - 5*a^2*b^2 - 2*b^4 + 3*a*b*(
-9*a^2 + b^2)*Sinh[c + d*x] + (-11*a^2*b^2 + 4*b^4)*Sinh[c + d*x]^2))/(a +
 b*Sinh[c + d*x])^3)/(6*(a^2 + b^2)^3*d)
```

Rubi [A] (warning: unable to verify)

Time = 0.87 (sec) , antiderivative size = 198, normalized size of antiderivative = 1.14, number of steps used = 14, number of rules used = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 1.083$, Rules used = {3042, 3143, 25, 3042, 3233, 25, 3042, 3233, 27, 3042, 3139, 1083, 217}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(a+b\sinh(c+dx))^4}\,dx$$

$$\Big\downarrow \; 3042$$

$$\int \frac{1}{(a-ib\sin(ic+idx))^4}dx$$

$$\Big\downarrow \; 3143$$

$$-\frac{\int -\frac{3a-2b\sinh(c+dx)}{(a+b\sinh(c+dx))^3}dx}{3(a^2+b^2)}-\frac{b\cosh(c+dx)}{3d(a^2+b^2)(a+b\sinh(c+dx))^3}$$

$$\Big\downarrow \; 25$$

$$\frac{\int \frac{3a-2b\sinh(c+dx)}{(a+b\sinh(c+dx))^3}dx}{3(a^2+b^2)}-\frac{b\cosh(c+dx)}{3d(a^2+b^2)(a+b\sinh(c+dx))^3}$$

$\Big\downarrow$ 3042

$$-\frac{b\cosh(c+dx)}{3d(a^2+b^2)(a+b\sinh(c+dx))^3}+\frac{\int \frac{3a+2ib\sin(ic+idx)}{(a-ib\sin(ic+idx))^3}dx}{3(a^2+b^2)}$$

$\Big\downarrow$ 3233

$$\frac{-\frac{\int -\frac{2\left(3a^2-2b^2\right)-5ab\sinh(c+dx)}{(a+b\sinh(c+dx))^2}dx}{2(a^2+b^2)}-\frac{5ab\cosh(c+dx)}{2d(a^2+b^2)(a+b\sinh(c+dx))^2}}{3(a^2+b^2)}-\frac{b\cosh(c+dx)}{3d(a^2+b^2)(a+b\sinh(c+dx))^3}$$

$\Big\downarrow$ 25

$$\frac{\frac{\int \frac{2\left(3a^2-2b^2\right)-5ab\sinh(c+dx)}{(a+b\sinh(c+dx))^2}dx}{2(a^2+b^2)}-\frac{5ab\cosh(c+dx)}{2d(a^2+b^2)(a+b\sinh(c+dx))^2}}{3(a^2+b^2)}-\frac{b\cosh(c+dx)}{3d(a^2+b^2)(a+b\sinh(c+dx))^3}$$

$\Big\downarrow$ 3042

$$-\frac{b\cosh(c+dx)}{3d(a^2+b^2)(a+b\sinh(c+dx))^3}+\frac{-\frac{5ab\cosh(c+dx)}{2d(a^2+b^2)(a+b\sinh(c+dx))^2}+\frac{\int \frac{2\left(3a^2-2b^2\right)+5iab\sin(ic+idx)}{(a-ib\sin(ic+idx))^2}dx}{2(a^2+b^2)}}{3(a^2+b^2)}$$

$\Big\downarrow$ 3233

$$\frac{\frac{-\frac{\int -\frac{3a\left(2a^2-3b^2\right)}{a+b\sinh(c+dx)}dx}{a^2+b^2}-\frac{b\left(11a^2-4b^2\right)\cosh(c+dx)}{d\left(a^2+b^2\right)(a+b\sinh(c+dx))}}{2(a^2+b^2)}-\frac{5ab\cosh(c+dx)}{2d(a^2+b^2)(a+b\sinh(c+dx))^2}}{3(a^2+b^2)}-\frac{b\cosh(c+dx)}{3d(a^2+b^2)(a+b\sinh(c+dx))^3}$$

$\Big\downarrow$ 27

$$\frac{\frac{\frac{3a\left(2a^2-3b^2\right)\int \frac{1}{a+b\sinh(c+dx)}dx}{a^2+b^2}-\frac{b\left(11a^2-4b^2\right)\cosh(c+dx)}{d\left(a^2+b^2\right)(a+b\sinh(c+dx))}}{2(a^2+b^2)}-\frac{5ab\cosh(c+dx)}{2d(a^2+b^2)(a+b\sinh(c+dx))^2}}{3(a^2+b^2)}-\frac{b\cosh(c+dx)}{3d(a^2+b^2)(a+b\sinh(c+dx))^3}$$

$\Big\downarrow$ 3042

$$-\frac{b\cosh(c+dx)}{3d\left(a^2+b^2\right)\left(a+b\sinh(c+dx)\right)^3}+\frac{-\frac{5ab\cosh(c+dx)}{2d(a^2+b^2)(a+b\sinh(c+dx))^2}+\frac{-\frac{b\left(11a^2-4b^2\right)\cosh(c+dx)}{d\left(a^2+b^2\right)(a+b\sinh(c+dx))}+\frac{3a\left(2a^2-3b^2\right)\int\frac{1}{a-ib\sin(ic+idx)}dx}{a^2+b^2}}{2(a^2+b^2)}}{3\left(a^2+b^2\right)}$$

$\Big\downarrow$ 3139

$$-\frac{b\cosh(c+dx)}{3d\left(a^2+b^2\right)\left(a+b\sinh(c+dx)\right)^3}+\frac{-\frac{5ab\cosh(c+dx)}{2d(a^2+b^2)(a+b\sinh(c+dx))^2}+\frac{-\frac{b\left(11a^2-4b^2\right)\cosh(c+dx)}{d\left(a^2+b^2\right)(a+b\sinh(c+dx))}-\frac{6ia\left(2a^2-3b^2\right)\int\frac{1}{-a\tanh^2\left(\frac{1}{2}(c+dx)\right)+2b\tanh\left(\frac{1}{2}(c+dx)\right)+a}d\left(i\tanh\left(\frac{1}{2}(c+dx)\right)\right)}{d\left(a^2+b^2\right)}}{2(a^2+b^2)}}{3\left(a^2+b^2\right)}$$

$\Big\downarrow$ 1083

$$-\frac{b\cosh(c+dx)}{3d\left(a^2+b^2\right)\left(a+b\sinh(c+dx)\right)^3}+\frac{-\frac{5ab\cosh(c+dx)}{2d(a^2+b^2)(a+b\sinh(c+dx))^2}+\frac{-\frac{b\left(11a^2-4b^2\right)\cosh(c+dx)}{d\left(a^2+b^2\right)(a+b\sinh(c+dx))}+\frac{12ia\left(2a^2-3b^2\right)\int\frac{1}{\tanh^2\left(\frac{1}{2}(c+dx)\right)-4\left(a^2+b^2\right)}d\left(2ia\tanh\left(\frac{1}{2}(c+dx)\right)-2ib\right)}{d\left(a^2+b^2\right)}}{2(a^2+b^2)}}{3\left(a^2+b^2\right)}$$

$\Big\downarrow$ 217

$$\frac{\frac{\frac{6a\left(2a^2-3b^2\right)\text{arctanh}\left(\frac{\tanh\left(\frac{1}{2}(c+dx)\right)}{2\sqrt{a^2+b^2}}\right)}{d\left(a^2+b^2\right)^{3/2}}-\frac{b\left(11a^2-4b^2\right)\cosh(c+dx)}{d\left(a^2+b^2\right)(a+b\sinh(c+dx))}}{2(a^2+b^2)}-\frac{5ab\cosh(c+dx)}{2d(a^2+b^2)(a+b\sinh(c+dx))^2}}{3\left(a^2+b^2\right)}-\frac{b\cosh(c+dx)}{3d\left(a^2+b^2\right)\left(a+b\sinh(c+dx)\right)^3}$$

input
```
Int[(a + b*Sinh[c + d*x])^(-4),x]
```

output
```
-1/3*(b*Cosh[c + d*x])/((a^2 + b^2)*d*(a + b*Sinh[c + d*x])^3) + ((-5*a*b*
Cosh[c + d*x])/(2*(a^2 + b^2)*d*(a + b*Sinh[c + d*x])^2) + ((6*a*(2*a^2 -
3*b^2)*ArcTanh[Tanh[(c + d*x)/2]/(2*Sqrt[a^2 + b^2])])/((a^2 + b^2)^(3/2)*
d) - (b*(11*a^2 - 4*b^2)*Cosh[c + d*x])/((a^2 + b^2)*d*(a + b*Sinh[c + d*x
])))/(2*(a^2 + b^2)))/(3*(a^2 + b^2))
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 1083
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Simp[-2   Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3139
```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Simp[2*(e/d)   Subst[Int[1/(a + 2*b*e*x + a
*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[a^2 - b^2, 0]
```

rule 3143
```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos
[c + d*x]*((a + b*Sin[c + d*x])^(n + 1)/(d*(n + 1)*(a^2 - b^2))), x] + Simp
[1/((n + 1)*(a^2 - b^2))   Int[(a + b*Sin[c + d*x])^(n + 1)*Simp[a*(n + 1)
- b*(n + 2)*Sin[c + d*x], x], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
 b^2, 0] && LtQ[n, -1] && IntegerQ[2*n]
```

rule 3233
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Simp[(-(b*c - a*d))*Cos[e + f*x]*((a + b*Sin[e +
 f*x])^(m + 1)/(f*(m + 1)*(a^2 - b^2))), x] + Simp[1/((m + 1)*(a^2 - b^2))
  Int[(a + b*Sin[e + f*x])^(m + 1)*Simp[(a*c - b*d)*(m + 1) - (b*c - a*d)*(
m + 2)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c
- a*d, 0] && NeQ[a^2 - b^2, 0] && LtQ[m, -1] && IntegerQ[2*m]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 493 vs. $2(163) = 326$.

Time = 0.77 (sec) , antiderivative size = 494, normalized size of antiderivative = 2.84

method	result
derivativedivides	$-\frac{2\left(-\frac{b^2\left(9a^4+6a^2b^2+2b^4\right)\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^5}{2a\left(a^6+3a^4b^2+3a^2b^4+b^6\right)}-\frac{b\left(6a^6-27a^4b^2-12a^2b^4-4b^6\right)\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^4}{2a^2\left(a^6+3a^4b^2+3a^2b^4+b^6\right)}+\frac{b^2\left(54a^6-21a^4b^2-4a^2b^4-4b^6\right)\tan}{3a^3\left(a^6+3a^4b^2+3a^2b^4+b\right.}}{\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^2a-2\right.}$
default	$-\frac{2\left(-\frac{b^2\left(9a^4+6a^2b^2+2b^4\right)\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^5}{2a\left(a^6+3a^4b^2+3a^2b^4+b^6\right)}-\frac{b\left(6a^6-27a^4b^2-12a^2b^4-4b^6\right)\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^4}{2a^2\left(a^6+3a^4b^2+3a^2b^4+b^6\right)}+\frac{b^2\left(54a^6-21a^4b^2-4a^2b^4-4b^6\right)\tan}{3a^3\left(a^6+3a^4b^2+3a^2b^4+b\right.}}{\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^2a-2\right.}$
risch	$\frac{6a^3b^2e^{5dx+5c}-9a\,b^4e^{5dx+5c}+30a^4b\,e^{4dx+4c}-45a^2b^3e^{4dx+4c}+44a^5e^{3dx+3c}-82a^3b^2e^{3dx+3c}+24a\,b^4e^{3dx+3c}-102a^4b}{3d\left(a^2+b^2\right)^3\left(b\,e^{2dx+2c}+2a\,e^{dx+c}-b\right)}$

input
```
int(1/(a+b*sinh(d*x+c))^4,x,method=_RETURNVERBOSE)
```

output
```
1/d*(-2*(-1/2*b^2*(9*a^4+6*a^2*b^2+2*b^4)/a/(a^6+3*a^4*b^2+3*a^2*b^4+b^6)*
tanh(1/2*d*x+1/2*c)^5-1/2*b*(6*a^6-27*a^4*b^2-12*a^2*b^4-4*b^6)/a^2/(a^6+3
*a^4*b^2+3*a^2*b^4+b^6)*tanh(1/2*d*x+1/2*c)^4+1/3/a^3*b^2*(54*a^6-21*a^4*b
^2-4*a^2*b^4-4*b^6)/(a^6+3*a^4*b^2+3*a^2*b^4+b^6)*tanh(1/2*d*x+1/2*c)^3+1/
a^2*b*(6*a^6-20*a^4*b^2-3*a^2*b^4-2*b^6)/(a^6+3*a^4*b^2+3*a^2*b^4+b^6)*tan
h(1/2*d*x+1/2*c)^2-1/2/a*b^2*(27*a^4+4*a^2*b^2+2*b^4)/(a^6+3*a^4*b^2+3*a^2
*b^4+b^6)*tanh(1/2*d*x+1/2*c)-1/6*b*(18*a^4+5*a^2*b^2+2*b^4)/(a^6+3*a^4*b^
2+3*a^2*b^4+b^6))/(tanh(1/2*d*x+1/2*c)^2*a-2*b*tanh(1/2*d*x+1/2*c)-a)^3+a*
(2*a^2-3*b^2)/(a^6+3*a^4*b^2+3*a^2*b^4+b^6)/(a^2+b^2)^(1/2)*arctanh(1/2*(2
*a*tanh(1/2*d*x+1/2*c)-2*b)/(a^2+b^2)^(1/2)))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2934 vs. $2(165) = 330$.

Time = 0.14 (sec) , antiderivative size = 2934, normalized size of antiderivative = 16.86

$$\int \frac{1}{(a + b\sinh(c + dx))^4}\,dx = \text{Too large to display}$$

input
```
integrate(1/(a+b*sinh(d*x+c))^4,x, algorithm="fricas")
```

output
```
-1/6*(22*a^4*b^3 + 14*a^2*b^5 - 8*b^7 - 6*(2*a^5*b^2 - a^3*b^4 - 3*a*b^6)*
cosh(d*x + c)^5 - 6*(2*a^5*b^2 - a^3*b^4 - 3*a*b^6)*sinh(d*x + c)^5 - 30*(
2*a^6*b - a^4*b^3 - 3*a^2*b^5)*cosh(d*x + c)^4 - 30*(2*a^6*b - a^4*b^3 - 3
*a^2*b^5 + (2*a^5*b^2 - a^3*b^4 - 3*a*b^6)*cosh(d*x + c))*sinh(d*x + c)^4
- 4*(22*a^7 - 19*a^5*b^2 - 29*a^3*b^4 + 12*a*b^6)*cosh(d*x + c)^3 - 4*(22*
a^7 - 19*a^5*b^2 - 29*a^3*b^4 + 12*a*b^6 + 15*(2*a^5*b^2 - a^3*b^4 - 3*a*b
^6)*cosh(d*x + c)^2 + 30*(2*a^6*b - a^4*b^3 - 3*a^2*b^5)*cosh(d*x + c))*si
nh(d*x + c)^3 + 12*(17*a^6*b + 11*a^4*b^3 - 4*a^2*b^5 + 2*b^7)*cosh(d*x +
c)^2 + 12*(17*a^6*b + 11*a^4*b^3 - 4*a^2*b^5 + 2*b^7 - 5*(2*a^5*b^2 - a^3*
b^4 - 3*a*b^6)*cosh(d*x + c)^3 - 15*(2*a^6*b - a^4*b^3 - 3*a^2*b^5)*cosh(d
*x + c)^2 - (22*a^7 - 19*a^5*b^2 - 29*a^3*b^4 + 12*a*b^6)*cosh(d*x + c))*s
inh(d*x + c)^2 + 3*((2*a^3*b^3 - 3*a*b^5)*cosh(d*x + c)^6 + (2*a^3*b^3 - 3
*a*b^5)*sinh(d*x + c)^6 - 2*a^3*b^3 + 3*a*b^5 + 6*(2*a^4*b^2 - 3*a^2*b^4)*
cosh(d*x + c)^5 + 6*(2*a^4*b^2 - 3*a^2*b^4 + (2*a^3*b^3 - 3*a*b^5)*cosh(d*
x + c))*sinh(d*x + c)^5 + 3*(8*a^5*b - 14*a^3*b^3 + 3*a*b^5)*cosh(d*x + c)
^4 + 3*(8*a^5*b - 14*a^3*b^3 + 3*a*b^5 + 5*(2*a^3*b^3 - 3*a*b^5)*cosh(d*x
+ c)^2 + 10*(2*a^4*b^2 - 3*a^2*b^4)*cosh(d*x + c))*sinh(d*x + c)^4 + 4*(4*
a^6 - 12*a^4*b^2 + 9*a^2*b^4)*cosh(d*x + c)^3 + 4*(4*a^6 - 12*a^4*b^2 + 9*
a^2*b^4 + 5*(2*a^3*b^3 - 3*a*b^5)*cosh(d*x + c)^3 + 15*(2*a^4*b^2 - 3*a^2*
b^4)*cosh(d*x + c)^2 + 3*(8*a^5*b - 14*a^3*b^3 + 3*a*b^5)*cosh(d*x + c)...
```

Sympy [F(-1)]

Timed out.

$$\int \frac{1}{(a+b\sinh(c+dx))^4}\,dx = \text{Timed out}$$

input
```
integrate(1/(a+b*sinh(d*x+c))**4,x)
```

output
```
Timed out
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 551 vs. $2(165) = 330$.

Time = 0.15 (sec) , antiderivative size = 551, normalized size of antiderivative = 3.17

$$\int \frac{1}{(a+b\sinh(c+dx))^4}\,dx = \frac{(2\,a^2-3\,b^2)a\log\left(\frac{be^{(-dx-c)}-a-\sqrt{a^2+b^2}}{be^{(-dx-c)}-a+\sqrt{a^2+b^2}}\right)}{2\,(a^6+3\,a^4b^2+3\,a^2b^4+b^6)\sqrt{a^2+b^2}d} - \frac{11\,a^2b^3-4\,b^5+15\,(4\,a^3b^2-ab^4)e}{3\,(a^6b^3+3\,a^4b^5+3\,a^2b^7+b^9+6\,(a^7b^2+3\,a^5b^4+3\,a^3b^6+ab^8)e^{(-dx-c)}+3\,(4\,a^8b+11\,a^6b^3+9\,a^4b^5+}$$

input
```
integrate(1/(a+b*sinh(d*x+c))^4,x, algorithm="maxima")
```

output
```
1/2*(2*a^2 - 3*b^2)*a*log((b*e^(-d*x - c) - a - sqrt(a^2 + b^2))/(b*e^(-d*
x - c) - a + sqrt(a^2 + b^2)))/((a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6)*sqrt(a
^2 + b^2)*d) - 1/3*(11*a^2*b^3 - 4*b^5 + 15*(4*a^3*b^2 - a*b^4)*e^(-d*x -
c) + 6*(17*a^4*b - 6*a^2*b^3 + 2*b^5)*e^(-2*d*x - 2*c) + 2*(22*a^5 - 41*a^
3*b^2 + 12*a*b^4)*e^(-3*d*x - 3*c) - 15*(2*a^4*b - 3*a^2*b^3)*e^(-4*d*x -
4*c) + 3*(2*a^3*b^2 - 3*a*b^4)*e^(-5*d*x - 5*c))/((a^6*b^3 + 3*a^4*b^5 + 3
*a^2*b^7 + b^9 + 6*(a^7*b^2 + 3*a^5*b^4 + 3*a^3*b^6 + a*b^8)*e^(-d*x - c)
+ 3*(4*a^8*b + 11*a^6*b^3 + 9*a^4*b^5 + a^2*b^7 - b^9)*e^(-2*d*x - 2*c) +
4*(2*a^9 + 3*a^7*b^2 - 3*a^5*b^4 - 7*a^3*b^6 - 3*a*b^8)*e^(-3*d*x - 3*c) -
 3*(4*a^8*b + 11*a^6*b^3 + 9*a^4*b^5 + a^2*b^7 - b^9)*e^(-4*d*x - 4*c) + 6
*(a^7*b^2 + 3*a^5*b^4 + 3*a^3*b^6 + a*b^8)*e^(-5*d*x - 5*c) - (a^6*b^3 + 3
*a^4*b^5 + 3*a^2*b^7 + b^9)*e^(-6*d*x - 6*c))*d)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 357 vs. $2(165) = 330$.

Time = 0.14 (sec) , antiderivative size = 357, normalized size of antiderivative = 2.05

$$\int \frac{1}{(a+b\sinh(c+dx))^4}\,dx = \frac{\frac{3\,(2\,a^3-3\,ab^2)\log\left(\frac{\left|2\,be^{(dx+c)}+2\,a-2\,\sqrt{a^2+b^2}\right|}{\left|2\,be^{(dx+c)}+2\,a+2\,\sqrt{a^2+b^2}\right|}\right)}{(a^6+3\,a^4b^2+3\,a^2b^4+b^6)\sqrt{a^2+b^2}} + \frac{2\,(6\,a^3b^2e^{(5\,dx+5\,c)}-9\,ab^4e^{(5\,dx+5\,c)}+30\,a^4be^{(4\,dx+4\,c)}-45\,a^2b^3e^{(4\,dx+4\,c)}+44\,a^5e^{(3\,dx+3\,c)}}{}}{6}$$

input
```
integrate(1/(a+b*sinh(d*x+c))^4,x, algorithm="giac")
```

output
```
1/6*(3*(2*a^3 - 3*a*b^2)*log(abs(2*b*e^(d*x + c) + 2*a - 2*sqrt(a^2 + b^2)
)/abs(2*b*e^(d*x + c) + 2*a + 2*sqrt(a^2 + b^2)))/((a^6 + 3*a^4*b^2 + 3*a^
2*b^4 + b^6)*sqrt(a^2 + b^2)) + 2*(6*a^3*b^2*e^(5*d*x + 5*c) - 9*a*b^4*e^(
5*d*x + 5*c) + 30*a^4*b*e^(4*d*x + 4*c) - 45*a^2*b^3*e^(4*d*x + 4*c) + 44*
a^5*e^(3*d*x + 3*c) - 82*a^3*b^2*e^(3*d*x + 3*c) + 24*a*b^4*e^(3*d*x + 3*c
) - 102*a^4*b*e^(2*d*x + 2*c) + 36*a^2*b^3*e^(2*d*x + 2*c) - 12*b^5*e^(2*d
*x + 2*c) + 60*a^3*b^2*e^(d*x + c) - 15*a*b^4*e^(d*x + c) - 11*a^2*b^3 + 4
*b^5)/((a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6)*(b*e^(2*d*x + 2*c) + 2*a*e^(d*x
 + c) - b)^3))/d
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(a+b\sinh(c+dx))^4}\,dx = \int \frac{1}{{\left(a+b\,\mathrm{sinh}\left(c+d\,x\right)\right)}^4}\,dx$$

input
```
int(1/(a + b*sinh(c + d*x))^4,x)
```

output
```
int(1/(a + b*sinh(c + d*x))^4, x)
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 1722, normalized size of antiderivative = 9.90

$$\int \frac{1}{(a + b\sinh(c + dx))^4}\,dx = \text{Too large to display}$$

input
```
int(1/(a+b*sinh(d*x+c))^4,x)
```

output
```
(12*e**(6*c + 6*d*x)*sqrt(a**2 + b**2)*atan((e**(c + d*x)*b*i + a*i)/sqrt(
a**2 + b**2))*a**3*b**3*i - 18*e**(6*c + 6*d*x)*sqrt(a**2 + b**2)*atan((e*
*(c + d*x)*b*i + a*i)/sqrt(a**2 + b**2))*a*b**5*i + 72*e**(5*c + 5*d*x)*sq
rt(a**2 + b**2)*atan((e**(c + d*x)*b*i + a*i)/sqrt(a**2 + b**2))*a**4*b**2
*i - 108*e**(5*c + 5*d*x)*sqrt(a**2 + b**2)*atan((e**(c + d*x)*b*i + a*i)/
sqrt(a**2 + b**2))*a**2*b**4*i + 144*e**(4*c + 4*d*x)*sqrt(a**2 + b**2)*at
an((e**(c + d*x)*b*i + a*i)/sqrt(a**2 + b**2))*a**5*b*i - 252*e**(4*c + 4*
d*x)*sqrt(a**2 + b**2)*atan((e**(c + d*x)*b*i + a*i)/sqrt(a**2 + b**2))*a*
*3*b**3*i + 54*e**(4*c + 4*d*x)*sqrt(a**2 + b**2)*atan((e**(c + d*x)*b*i +
 a*i)/sqrt(a**2 + b**2))*a*b**5*i + 96*e**(3*c + 3*d*x)*sqrt(a**2 + b**2)*
atan((e**(c + d*x)*b*i + a*i)/sqrt(a**2 + b**2))*a**6*i - 288*e**(3*c + 3*
d*x)*sqrt(a**2 + b**2)*atan((e**(c + d*x)*b*i + a*i)/sqrt(a**2 + b**2))*a*
*4*b**2*i + 216*e**(3*c + 3*d*x)*sqrt(a**2 + b**2)*atan((e**(c + d*x)*b*i
+ a*i)/sqrt(a**2 + b**2))*a**2*b**4*i - 144*e**(2*c + 2*d*x)*sqrt(a**2 + b
**2)*atan((e**(c + d*x)*b*i + a*i)/sqrt(a**2 + b**2))*a**5*b*i + 252*e**(2
*c + 2*d*x)*sqrt(a**2 + b**2)*atan((e**(c + d*x)*b*i + a*i)/sqrt(a**2 + b*
*2))*a**3*b**3*i - 54*e**(2*c + 2*d*x)*sqrt(a**2 + b**2)*atan((e**(c + d*x
)*b*i + a*i)/sqrt(a**2 + b**2))*a*b**5*i + 72*e**(c + d*x)*sqrt(a**2 + b**
2)*atan((e**(c + d*x)*b*i + a*i)/sqrt(a**2 + b**2))*a**4*b**2*i - 108*e**(
c + d*x)*sqrt(a**2 + b**2)*atan((e**(c + d*x)*b*i + a*i)/sqrt(a**2 + b*...
```

3.105 $\int (a+b\sinh(x))^{5/2}\,dx$

Optimal result

Integrand size = 10, antiderivative size = 179

$$\int (a+b\sinh(x))^{5/2}\,dx = \frac{16}{15}ab\cosh(x)\sqrt{a+b\sinh(x)} + \frac{2}{5}b\cosh(x)(a+b\sinh(x))^{3/2} + \frac{2i(23a^2-9b^2)\,E\left(\frac{\pi}{4}-\frac{ix}{2}\middle|\frac{2b}{ia+b}\right)\sqrt{a+b\sinh(x)}}{15\sqrt{\frac{a+b\sinh(x)}{a-ib}}} - \frac{16ia(a^2+b^2)\,\text{EllipticF}\left(\frac{\pi}{4}-\frac{ix}{2},\frac{2b}{ia+b}\right)\sqrt{\frac{a+b\sinh(x)}{a-ib}}}{15\sqrt{a+b\sinh(x)}}$$

output
```
16/15*a*b*cosh(x)*(a+b*sinh(x))^(1/2)+2/5*b*cosh(x)*(a+b*sinh(x))^(3/2)+2/
15*I*(23*a^2-9*b^2)*EllipticE(cos(1/4*Pi+1/2*I*x),2^(1/2)*(b/(I*a+b))^(1/2
))*(a+b*sinh(x))^(1/2)/((a+b*sinh(x))/(a-I*b))^(1/2)+16/15*I*a*(a^2+b^2)*I
nverseJacobiAM(-1/4*Pi+1/2*I*x,2^(1/2)*(b/(I*a+b))^(1/2))*((a+b*sinh(x))/(
a-I*b))^(1/2)/(a+b*sinh(x))^(1/2)
```

Mathematica [A] (verified)

Time = 0.31 (sec) , antiderivative size = 178, normalized size of antiderivative = 0.99

$$\int (a+b\sinh(x))^{5/2}\,dx = \frac{2(23ia^3+23a^2b-9iab^2-9b^3)\,E\left(\frac{1}{4}(\pi-2ix)|-\frac{2ib}{a-ib}\right)\sqrt{\frac{a+b\sinh(x)}{a-ib}}-16ia(a^2+b^2)\,\text{Ellip}}{15\sqrt{a}}$$

input
```
Integrate[(a + b*Sinh[x])^(5/2),x]
```

output
```
(2*((23*I)*a^3 + 23*a^2*b - (9*I)*a*b^2 - 9*b^3)*EllipticE[(Pi - (2*I)*x)/
4, ((-2*I)*b)/(a - I*b)]*Sqrt[(a + b*Sinh[x])/(a - I*b)] - (16*I)*a*(a^2 +
 b^2)*EllipticF[(Pi - (2*I)*x)/4, ((-2*I)*b)/(a - I*b)]*Sqrt[(a + b*Sinh[x
])/(a - I*b)] + b*Cosh[x]*(22*a^2 - 3*b^2 + 3*b^2*Cosh[2*x] + 28*a*b*Sinh[
x]))/(15*Sqrt[a + b*Sinh[x]])
```

Rubi [A] (verified)

Time = 0.95 (sec) , antiderivative size = 185, normalized size of antiderivative = 1.03, number of steps used = 15, number of rules used = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 1.500$, Rules used = {3042, 3135, 27, 3042, 3232, 27, 3042, 3231, 3042, 3134, 3042, 3132, 3142, 3042, 3140}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a+b\sinh(x))^{5/2}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int (a-ib\sin(ix))^{5/2}dx$$

$$\Big\downarrow \text{3135}$$

$$\frac{2}{5}\int \frac{1}{2}\sqrt{a+b\sinh(x)}\left(5a^2+8b\sinh(x)a-3b^2\right)dx+\frac{2}{5}b\cosh(x)(a+b\sinh(x))^{3/2}$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{5}\int \sqrt{a+b\sinh(x)}\left(5a^2+8b\sinh(x)a-3b^2\right)dx+\frac{2}{5}b\cosh(x)(a+b\sinh(x))^{3/2}$$

$$\Big\downarrow \text{3042}$$

$$\frac{2}{5}b\cosh(x)(a+b\sinh(x))^{3/2}+\frac{1}{5}\int \sqrt{a-ib\sin(ix)}\left(5a^2-8ib\sin(ix)a-3b^2\right)dx$$

$$\Big\downarrow \text{3232}$$

$$\frac{1}{5}\left(\frac{2}{3}\int \frac{a\left(15a^2-17b^2\right)+b\left(23a^2-9b^2\right)\sinh(x)}{2\sqrt{a+b\sinh(x)}}dx+\frac{16}{3}ab\cosh(x)\sqrt{a+b\sinh(x)}\right)+\frac{2}{5}b\cosh(x)(a+b\sinh(x))^{3/2}$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{5}\left(\frac{1}{3}\int \frac{a\left(15a^2-17b^2\right)+b\left(23a^2-9b^2\right)\sinh(x)}{\sqrt{a+b\sinh(x)}}dx+\frac{16}{3}ab\cosh(x)\sqrt{a+b\sinh(x)}\right)+\frac{2}{5}b\cosh(x)(a+b\sinh(x))^{3/2}$$

$$\Big\downarrow \text{3042}$$

$$\frac{2}{5}b\cosh(x)(a+b\sinh(x))^{3/2}+\frac{1}{5}\left(\frac{16}{3}ab\cosh(x)\sqrt{a+b\sinh(x)}+\frac{1}{3}\int \frac{a\left(15a^2-17b^2\right)-ib\left(23a^2-9b^2\right)\sin(ix)}{\sqrt{a-ib\sin(ix)}}dx\right)$$

$$\Big\downarrow \text{3231}$$

$$\frac{1}{5}\left(\frac{1}{3}\left(\left(23a^2-9b^2\right)\int \sqrt{a+b\sinh(x)}dx-8a\left(a^2+b^2\right)\int \frac{1}{\sqrt{a+b\sinh(x)}}dx\right)+\frac{16}{3}ab\cosh(x)\sqrt{a+b\sinh(x)}\right)$$
$$\frac{2}{5}b\cosh(x)(a+b\sinh(x))^{3/2}$$

$$\Big\downarrow \text{3042}$$

$$\frac{2}{5}b\cosh(x)(a+b\sinh(x))^{3/2}+\frac{1}{5}\left(\frac{16}{3}ab\cosh(x)\sqrt{a+b\sinh(x)}+\frac{1}{3}\left(\left(23a^2-9b^2\right)\int \sqrt{a-ib\sin(ix)}dx-8a\left(a^2+b^2\right)\int \frac{1}{\sqrt{a-ib\sin(ix)}}dx\right)\right)$$

$$\Big\downarrow \text{3134}$$

$$\frac{2}{5}b\cosh(x)(a+b\sinh(x))^{3/2}+$$
$$\frac{1}{5}\left(\frac{16}{3}ab\cosh(x)\sqrt{a+b\sinh(x)}+\frac{1}{3}\left(\frac{\left(23a^2-9b^2\right)\sqrt{a+b\sinh(x)}\int\sqrt{\frac{a}{a-ib}+\frac{b\sinh(x)}{a-ib}}dx}{\sqrt{\frac{a+b\sinh(x)}{a-ib}}}-8a\left(a^2+b^2\right)\int\frac{}{\sqrt{a}}\right.\right.$$

$\downarrow$ 3042

$$\frac{2}{5}b\cosh(x)(a+b\sinh(x))^{3/2}+$$
$$\frac{1}{5}\left(\frac{16}{3}ab\cosh(x)\sqrt{a+b\sinh(x)}+\frac{1}{3}\left(\frac{\left(23a^2-9b^2\right)\sqrt{a+b\sinh(x)}\int\sqrt{\frac{a}{a-ib}-\frac{ib\sin(ix)}{a-ib}}dx}{\sqrt{\frac{a+b\sinh(x)}{a-ib}}}-8a\left(a^2+b^2\right)\int\frac{}{\sqrt{a}}\right.\right.$$

$\downarrow$ 3132

$$\frac{2}{5}b\cosh(x)(a+b\sinh(x))^{3/2}+$$
$$\frac{1}{5}\left(\frac{16}{3}ab\cosh(x)\sqrt{a+b\sinh(x)}+\frac{1}{3}\left(\frac{2i\left(23a^2-9b^2\right)\sqrt{a+b\sinh(x)}E\left(\frac{\pi}{4}-\frac{ix}{2}|\frac{2b}{ia+b}\right)}{\sqrt{\frac{a+b\sinh(x)}{a-ib}}}-8a\left(a^2+b^2\right)\int\frac{1}{\sqrt{a-ib}}\right.\right.$$

$\downarrow$ 3142

$$\frac{2}{5}b\cosh(x)(a+b\sinh(x))^{3/2}+$$
$$\frac{1}{5}\left(\frac{16}{3}ab\cosh(x)\sqrt{a+b\sinh(x)}+\frac{1}{3}\left(\frac{2i\left(23a^2-9b^2\right)\sqrt{a+b\sinh(x)}E\left(\frac{\pi}{4}-\frac{ix}{2}|\frac{2b}{ia+b}\right)}{\sqrt{\frac{a+b\sinh(x)}{a-ib}}}-\frac{8a\left(a^2+b^2\right)\sqrt{\frac{a+b\sinh(x)}{a-ib}}}{\sqrt{a+b\sinh}}\right.\right.$$

$\downarrow$ 3042

$$\frac{2}{5}b\cosh(x)(a+b\sinh(x))^{3/2}+$$
$$\frac{1}{5}\left(\frac{16}{3}ab\cosh(x)\sqrt{a+b\sinh(x)}+\frac{1}{3}\left(\frac{2i\left(23a^2-9b^2\right)\sqrt{a+b\sinh(x)}E\left(\frac{\pi}{4}-\frac{ix}{2}|\frac{2b}{ia+b}\right)}{\sqrt{\frac{a+b\sinh(x)}{a-ib}}}-\frac{8a\left(a^2+b^2\right)\sqrt{\frac{a+b\sinh(x)}{a-ib}}}{\sqrt{a+b\sinh}}\right.\right.$$

$\downarrow$ 3140

$$\frac{2}{5}b\cosh(x)(a+b\sinh(x))^{3/2}+$$
$$\frac{1}{5}\left(\frac{16}{3}ab\cosh(x)\sqrt{a+b\sinh(x)}+\frac{1}{3}\left(\frac{2i\left(23a^2-9b^2\right)\sqrt{a+b\sinh(x)}E\left(\frac{\pi}{4}-\frac{ix}{2}|\frac{2b}{ia+b}\right)}{\sqrt{\frac{a+b\sinh(x)}{a-ib}}}-\frac{16ia\left(a^2+b^2\right)\sqrt{\frac{a+b\sinh}{a-ib}}}{\sqrt{a+}}\right.\right.$$

input
```
Int[(a + b*Sinh[x])^(5/2),x]
```

output
```
(2*b*Cosh[x]*(a + b*Sinh[x])^(3/2))/5 + ((16*a*b*Cosh[x]*Sqrt[a + b*Sinh[x
]])/3 + (((2*I)*(23*a^2 - 9*b^2)*EllipticE[Pi/4 - (I/2)*x, (2*b)/(I*a + b)
]*Sqrt[a + b*Sinh[x]])/Sqrt[(a + b*Sinh[x])/(a - I*b)] - ((16*I)*a*(a^2 +
b^2)*EllipticF[Pi/4 - (I/2)*x, (2*b)/(I*a + b)]*Sqrt[(a + b*Sinh[x])/(a -
I*b)])/Sqrt[a + b*Sinh[x]])/3)/5
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3132
```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[2*(Sqrt[a
 + b]/d)*EllipticE[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

rule 3134
```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)]   Int[Sqrt[a/(a + b) + (
b/(a + b))*Sin[c + d*x]], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2
, 0] &&  !GtQ[a + b, 0]
```

rule 3135
```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos
[c + d*x]*((a + b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Simp[1/n   Int[(a + b*
Sin[c + d*x])^(n - 2)*Simp[a^2*n + b^2*(n - 1) + a*b*(2*n - 1)*Sin[c + d*x]
, x], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[n, 1] &&
 IntegerQ[2*n]
```

rule 3140
```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/(d*S
qrt[a + b]))*EllipticF[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

rule 3142
```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]]   Int[1/Sqrt[a/(a + b)
 + (b/(a + b))*Sin[c + d*x]], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
 b^2, 0] &&  !GtQ[a + b, 0]
```

rule 3231
```
Int[((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])/Sqrt[(a_) + (b_.)*sin[(e_.) + (
f_.)*(x_)]], x_Symbol] :> Simp[(b*c - a*d)/b   Int[1/Sqrt[a + b*Sin[e + f*x
]], x], x] + Simp[d/b   Int[Sqrt[a + b*Sin[e + f*x]], x], x] /; FreeQ[{a, b
, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
```

rule 3232
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Simp[(-d)*Cos[e + f*x]*((a + b*Sin[e + f*x])^m/(
f*(m + 1))), x] + Simp[1/(m + 1)   Int[(a + b*Sin[e + f*x])^(m - 1)*Simp[b*
d*m + a*c*(m + 1) + (a*d*m + b*c*(m + 1))*Sin[e + f*x], x], x], x] /; FreeQ
[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && GtQ[m,
 0] && IntegerQ[2*m]
```

Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 916 vs. $2(154) = 308$.

Time = 1.41 (sec) , antiderivative size = 917, normalized size of antiderivative = 5.12

method	result
default	$\frac{\frac{16i\sqrt{-\frac{a+b\sinh(x)}{ib-a}}\sqrt{\frac{(i-\sinh(x))b}{ib+a}}\sqrt{\frac{b(i+\sinh(x))}{ib-a}}\,\text{EllipticF}\left(\sqrt{-\frac{a+b\sinh(x)}{ib-a}},\sqrt{-\frac{ib-a}{ib+a}}\right)a^3b}{15}+\frac{16i\sqrt{-\frac{a+b\sinh(x)}{ib-a}}\sqrt{\frac{(i-\sinh(x))b}{ib+a}}\sqrt{\frac{b(i+\sinh(x}{ib-a}}}{15}}{}$

input
```
int((a+b*sinh(x))^(5/2),x,method=_RETURNVERBOSE)
```

output
```
2/15*(8*I*(-(a+b*sinh(x))/(I*b-a))^(1/2)*((I-sinh(x))*b/(I*b+a))^(1/2)*(b*
(I+sinh(x))/(I*b-a))^(1/2)*EllipticF((-(a+b*sinh(x))/(I*b-a))^(1/2),(-(I*b
-a)/(I*b+a))^(1/2))*a^3*b+8*I*(-(a+b*sinh(x))/(I*b-a))^(1/2)*((I-sinh(x))*
b/(I*b+a))^(1/2)*(b*(I+sinh(x))/(I*b-a))^(1/2)*EllipticF((-(a+b*sinh(x))/(
I*b-a))^(1/2),(-(I*b-a)/(I*b+a))^(1/2))*a*b^3+15*(-(a+b*sinh(x))/(I*b-a))^
(1/2)*((I-sinh(x))*b/(I*b+a))^(1/2)*(b*(I+sinh(x))/(I*b-a))^(1/2)*Elliptic
F((-(a+b*sinh(x))/(I*b-a))^(1/2),(-(I*b-a)/(I*b+a))^(1/2))*a^4+6*(-(a+b*si
nh(x))/(I*b-a))^(1/2)*((I-sinh(x))*b/(I*b+a))^(1/2)*(b*(I+sinh(x))/(I*b-a)
)^(1/2)*EllipticF((-(a+b*sinh(x))/(I*b-a))^(1/2),(-(I*b-a)/(I*b+a))^(1/2))
*a^2*b^2-9*(-(a+b*sinh(x))/(I*b-a))^(1/2)*((I-sinh(x))*b/(I*b+a))^(1/2)*(b
*(I+sinh(x))/(I*b-a))^(1/2)*EllipticF((-(a+b*sinh(x))/(I*b-a))^(1/2),(-(I*
b-a)/(I*b+a))^(1/2))*b^4-23*(-(a+b*sinh(x))/(I*b-a))^(1/2)*((I-sinh(x))*b/
(I*b+a))^(1/2)*(b*(I+sinh(x))/(I*b-a))^(1/2)*EllipticE((-(a+b*sinh(x))/(I*
b-a))^(1/2),(-(I*b-a)/(I*b+a))^(1/2))*a^4-14*(-(a+b*sinh(x))/(I*b-a))^(1/2
)*((I-sinh(x))*b/(I*b+a))^(1/2)*(b*(I+sinh(x))/(I*b-a))^(1/2)*EllipticE((-
(a+b*sinh(x))/(I*b-a))^(1/2),(-(I*b-a)/(I*b+a))^(1/2))*a^2*b^2+9*(-(a+b*si
nh(x))/(I*b-a))^(1/2)*((I-sinh(x))*b/(I*b+a))^(1/2)*(b*(I+sinh(x))/(I*b-a)
)^(1/2)*EllipticE((-(a+b*sinh(x))/(I*b-a))^(1/2),(-(I*b-a)/(I*b+a))^(1/2))
*b^4+3*b^4*sinh(x)^4+14*a*b^3*sinh(x)^3+11*a^2*b^2*sinh(x)^2+3*b^4*sinh(x)
^2+14*a*b^3*sinh(x)+11*a^2*b^2)/b/cosh(x)/(a+b*sinh(x))^(1/2)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 452 vs. $2(146) = 292$.

Time = 0.10 (sec) , antiderivative size = 452, normalized size of antiderivative = 2.53

$$\int (a + b\sinh(x))^{5/2}\,dx = \text{Too large to display}$$

input
```
integrate((a+b*sinh(x))^(5/2),x, algorithm="fricas")
```

output
```
-1/90*(8*sqrt(1/2)*((a^3 + 33*a*b^2)*cosh(x)^2 + 2*(a^3 + 33*a*b^2)*cosh(x
)*sinh(x) + (a^3 + 33*a*b^2)*sinh(x)^2)*sqrt(b)*weierstrassPInverse(4/3*(4
*a^2 + 3*b^2)/b^2, -8/27*(8*a^3 + 9*a*b^2)/b^3, 1/3*(3*b*cosh(x) + 3*b*sin
h(x) + 2*a)/b) + 24*sqrt(1/2)*((23*a^2*b - 9*b^3)*cosh(x)^2 + 2*(23*a^2*b
- 9*b^3)*cosh(x)*sinh(x) + (23*a^2*b - 9*b^3)*sinh(x)^2)*sqrt(b)*weierstra
ssZeta(4/3*(4*a^2 + 3*b^2)/b^2, -8/27*(8*a^3 + 9*a*b^2)/b^3, weierstrassPI
nverse(4/3*(4*a^2 + 3*b^2)/b^2, -8/27*(8*a^3 + 9*a*b^2)/b^3, 1/3*(3*b*cosh
(x) + 3*b*sinh(x) + 2*a)/b)) - 3*(3*b^3*cosh(x)^4 + 3*b^3*sinh(x)^4 + 22*a
*b^2*cosh(x)^3 + 22*a*b^2*cosh(x) + 2*(6*b^3*cosh(x) + 11*a*b^2)*sinh(x)^3
 - 3*b^3 - 4*(23*a^2*b - 9*b^3)*cosh(x)^2 + 2*(9*b^3*cosh(x)^2 + 33*a*b^2*
cosh(x) - 46*a^2*b + 18*b^3)*sinh(x)^2 + 2*(6*b^3*cosh(x)^3 + 33*a*b^2*cos
h(x)^2 + 11*a*b^2 - 4*(23*a^2*b - 9*b^3)*cosh(x))*sinh(x))*sqrt(b*sinh(x)
+ a))/(b*cosh(x)^2 + 2*b*cosh(x)*sinh(x) + b*sinh(x)^2)
```

Sympy [F(-1)]

Timed out.

$$\int (a + b\sinh(x))^{5/2} \, dx = \text{Timed out}$$

input
```
integrate((a+b*sinh(x))**(5/2),x)
```

output
```
Timed out
```

Maxima [F]

$$\int (a + b\sinh(x))^{5/2} \, dx = \int (b \sinh(x) + a)^{\frac{5}{2}} \, dx$$

input
```
integrate((a+b*sinh(x))^(5/2),x, algorithm="maxima")
```

output
```
integrate((b*sinh(x) + a)^(5/2), x)
```

Giac [F]

$$\int (a+b\sinh(x))^{5/2}\,dx = \int (b\sinh(x)+a)^{\frac{5}{2}}\,dx$$

input
```
integrate((a+b*sinh(x))^(5/2),x, algorithm="giac")
```

output
```
integrate((b*sinh(x) + a)^(5/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int (a+b\sinh(x))^{5/2}\,dx = \int (a+b\sinh(x))^{5/2}\,dx$$

input
```
int((a + b*sinh(x))^(5/2),x)
```

output
```
int((a + b*sinh(x))^(5/2), x)
```

Reduce [F]

$$\int (a+b\sinh(x))^{5/2}\,dx = \left(\int \sqrt{\sinh(x)b+a}dx\right)a^2 + \left(\int \sqrt{\sinh(x)b+a}\,\sinh(x)^2\,dx\right)b^2 + 2\left(\int \sqrt{\sinh(x)b+a}\,\sinh(x)\,dx\right)ab$$

input
```
int((a+b*sinh(x))^(5/2),x)
```

output
```
int(sqrt(sinh(x)*b + a),x)*a**2 + int(sqrt(sinh(x)*b + a)*sinh(x)**2,x)*b*
*2 + 2*int(sqrt(sinh(x)*b + a)*sinh(x),x)*a*b
```

3.106 $\int (a+b\sinh(x))^{3/2}\,dx$

Optimal result

Integrand size = 10, antiderivative size = 150

$$\int (a+b\sinh(x))^{3/2}\,dx=\frac{2}{3}b\cosh(x)\sqrt{a+b\sinh(x)}+\frac{8iaE\left(\frac{\pi}{4}-\frac{ix}{2}|\frac{2b}{ia+b}\right)\sqrt{a+b\sinh(x)}}{3\sqrt{\frac{a+b\sinh(x)}{a-ib}}}-\frac{2i(a^2+b^2)\,\text{EllipticF}\left(\frac{\pi}{4}-\frac{ix}{2},\frac{2b}{ia+b}\right)\sqrt{\frac{a+b\sinh(x)}{a-ib}}}{3\sqrt{a+b\sinh(x)}}$$

output
```
2/3*b*cosh(x)*(a+b*sinh(x))^(1/2)+8/3*I*a*EllipticE(cos(1/4*Pi+1/2*I*x),2^
(1/2)*(b/(I*a+b))^(1/2))*(a+b*sinh(x))^(1/2)/((a+b*sinh(x))/(a-I*b))^(1/2)
+2/3*I*(a^2+b^2)*InverseJacobiAM(-1/4*Pi+1/2*I*x,2^(1/2)*(b/(I*a+b))^(1/2)
)*((a+b*sinh(x))/(a-I*b))^(1/2)/(a+b*sinh(x))^(1/2)
```

Mathematica [A] (verified)

Time = 0.25 (sec) , antiderivative size = 139, normalized size of antiderivative = 0.93

$$\int (a+b\sinh(x))^{3/2}\,dx = \frac{2b\cosh(x)(a+b\sinh(x))+8a(ia+b)E\left(\frac{1}{4}(\pi-2ix)|-\frac{2ib}{a-ib}\right)\sqrt{\frac{a+b\sinh(x)}{a-ib}}-2i(a^2+b^2)\,\mathrm{I}}{3\sqrt{a+b\sinh(x)}}$$

input
```
Integrate[(a + b*Sinh[x])^(3/2),x]
```

output
```
(2*b*Cosh[x]*(a + b*Sinh[x]) + 8*a*(I*a + b)*EllipticE[(Pi - (2*I)*x)/4, (
(-2*I)*b)/(a - I*b)]*Sqrt[(a + b*Sinh[x])/(a - I*b)] - (2*I)*(a^2 + b^2)*E
llipticF[(Pi - (2*I)*x)/4, ((-2*I)*b)/(a - I*b)]*Sqrt[(a + b*Sinh[x])/(a -
 I*b)])/(3*Sqrt[a + b*Sinh[x]])
```

Rubi [A] (verified)

Time = 0.87 (sec) , antiderivative size = 151, normalized size of antiderivative = 1.01, number of steps used = 12, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 1.200, Rules used = {3042, 3135, 27, 3042, 3231, 3042, 3134, 3042, 3132, 3142, 3042, 3140}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a+b\sinh(x))^{3/2}\,dx$$

$$\Big\downarrow \; 3042$$

$$\int (a-ib\sin(ix))^{3/2}dx$$

$$\Big\downarrow \; 3135$$

$$\frac{2}{3}\int \frac{3a^2+4b\sinh(x)a-b^2}{2\sqrt{a+b\sinh(x)}}dx+\frac{2}{3}b\cosh(x)\sqrt{a+b\sinh(x)}$$

$$\Big\downarrow \; 27$$

$$\frac{1}{3}\int \frac{3a^2+4b\sinh(x)a-b^2}{\sqrt{a+b\sinh(x)}}dx+\frac{2}{3}b\cosh(x)\sqrt{a+b\sinh(x)}$$

$$\Big\downarrow \text{3042}$$

$$\frac{2}{3}b\cosh(x)\sqrt{a+b\sinh(x)}+\frac{1}{3}\int \frac{3a^2-4ib\sin(ix)a-b^2}{\sqrt{a-ib\sin(ix)}}dx$$

$$\Big\downarrow \text{3231}$$

$$\frac{1}{3}\left(4a\int \sqrt{a+b\sinh(x)}dx-\left(a^2+b^2\right)\int \frac{1}{\sqrt{a+b\sinh(x)}}dx\right)+\frac{2}{3}b\cosh(x)\sqrt{a+b\sinh(x)}$$

$$\Big\downarrow \text{3042}$$

$$\frac{2}{3}b\cosh(x)\sqrt{a+b\sinh(x)}+\frac{1}{3}\left(4a\int \sqrt{a-ib\sin(ix)}dx-\left(a^2+b^2\right)\int \frac{1}{\sqrt{a-ib\sin(ix)}}dx\right)$$

$$\Big\downarrow \text{3134}$$

$$\frac{2}{3}b\cosh(x)\sqrt{a+b\sinh(x)}+\frac{1}{3}\left(\frac{4a\sqrt{a+b\sinh(x)}\int \sqrt{\frac{a}{a-ib}+\frac{b\sinh(x)}{a-ib}}dx}{\sqrt{\frac{a+b\sinh(x)}{a-ib}}}-\left(a^2+b^2\right)\int \frac{1}{\sqrt{a-ib\sin(ix)}}dx\right)$$

$$\Big\downarrow \text{3042}$$

$$\frac{2}{3}b\cosh(x)\sqrt{a+b\sinh(x)}+\frac{1}{3}\left(\frac{4a\sqrt{a+b\sinh(x)}\int \sqrt{\frac{a}{a-ib}-\frac{ib\sin(ix)}{a-ib}}dx}{\sqrt{\frac{a+b\sinh(x)}{a-ib}}}-\left(a^2+b^2\right)\int \frac{1}{\sqrt{a-ib\sin(ix)}}dx\right)$$

$$\Big\downarrow \text{3132}$$

$$\frac{2}{3}b\cosh(x)\sqrt{a+b\sinh(x)}+\frac{1}{3}\left(\frac{8ia\sqrt{a+b\sinh(x)}E\left(\frac{\pi}{4}-\frac{ix}{2}|\frac{2b}{ia+b}\right)}{\sqrt{\frac{a+b\sinh(x)}{a-ib}}}-\left(a^2+b^2\right)\int \frac{1}{\sqrt{a-ib\sin(ix)}}dx\right)$$

$$\Big\downarrow \text{3142}$$

$$\frac{2}{3}b\cosh(x)\sqrt{a+b\sinh(x)}+\frac{1}{3}\left(\frac{8ia\sqrt{a+b\sinh(x)}E\left(\frac{\pi}{4}-\frac{ix}{2}|\frac{2b}{ia+b}\right)}{\sqrt{\frac{a+b\sinh(x)}{a-ib}}}-\frac{\left(a^2+b^2\right)\sqrt{\frac{a+b\sinh(x)}{a-ib}}\int \frac{1}{\sqrt{\frac{a}{a-ib}+\frac{b\sinh(x)}{a-ib}}}dx}{\sqrt{a+b\sinh(x)}}\right)$$

$\Big\downarrow$ 3042

$$\frac{2}{3}b\cosh(x)\sqrt{a+b\sinh(x)}+\frac{1}{3}\left(\frac{8ia\sqrt{a+b\sinh(x)}E\left(\frac{\pi}{4}-\frac{ix}{2}|\frac{2b}{ia+b}\right)}{\sqrt{\frac{a+b\sinh(x)}{a-ib}}}-\frac{\left(a^2+b^2\right)\sqrt{\frac{a+b\sinh(x)}{a-ib}}\int\frac{1}{\sqrt{\frac{a}{a-ib}-\frac{ib\sin(ix)}{a-ib}}}dx}{\sqrt{a+b\sinh(x)}}\right)$$

$\Big\downarrow$ 3140

$$\frac{2}{3}b\cosh(x)\sqrt{a+b\sinh(x)}+\frac{1}{3}\left(\frac{8ia\sqrt{a+b\sinh(x)}E\left(\frac{\pi}{4}-\frac{ix}{2}|\frac{2b}{ia+b}\right)}{\sqrt{\frac{a+b\sinh(x)}{a-ib}}}-\frac{2i\left(a^2+b^2\right)\sqrt{\frac{a+b\sinh(x)}{a-ib}}\operatorname{EllipticF}\left(\frac{\pi}{4}-\frac{ix}{2},\frac{2b}{ia+b}\right)}{\sqrt{a+b\sinh(x)}}\right)$$

input
```
Int[(a + b*Sinh[x])^(3/2),x]
```

output
```
(2*b*Cosh[x]*Sqrt[a + b*Sinh[x]])/3 + (((8*I)*a*EllipticE[Pi/4 - (I/2)*x,
(2*b)/(I*a + b)]*Sqrt[a + b*Sinh[x]])/Sqrt[(a + b*Sinh[x])/(a - I*b)] - ((
2*I)*(a^2 + b^2)*EllipticF[Pi/4 - (I/2)*x, (2*b)/(I*a + b)]*Sqrt[(a + b*Si
nh[x])/(a - I*b)])/Sqrt[a + b*Sinh[x]])/3
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3132
```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[2*(Sqrt[a
 + b]/d)*EllipticE[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

rule 3134
```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)]   Int[Sqrt[a/(a + b) + (
b/(a + b))*Sin[c + d*x]], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2
, 0] &&  !GtQ[a + b, 0]
```

rule 3135
```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos
[c + d*x]*((a + b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Simp[1/n   Int[(a + b*
Sin[c + d*x])^(n - 2)*Simp[a^2*n + b^2*(n - 1) + a*b*(2*n - 1)*Sin[c + d*x]
, x], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[n, 1] &&
 IntegerQ[2*n]
```

rule 3140
```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/(d*S
qrt[a + b]))*EllipticF[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

rule 3142
```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]]   Int[1/Sqrt[a/(a + b)
 + (b/(a + b))*Sin[c + d*x]], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
 b^2, 0] &&  !GtQ[a + b, 0]
```

rule 3231
```
Int[((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])/Sqrt[(a_) + (b_.)*sin[(e_.) + (
f_.)*(x_)]], x_Symbol] :> Simp[(b*c - a*d)/b   Int[1/Sqrt[a + b*Sin[e + f*x
]], x], x] + Simp[d/b   Int[Sqrt[a + b*Sin[e + f*x]], x], x] /; FreeQ[{a, b
, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
```

Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 675 vs. $2(129) = 258$.

Time = 0.89 (sec) , antiderivative size = 676, normalized size of antiderivative = 4.51

method	result
default	$\frac{2i\sqrt{-\frac{a+b\sinh(x)}{ib-a}}\sqrt{\frac{(i-\sinh(x))b}{ib+a}}\sqrt{\frac{b(i+\sinh(x))}{ib-a}}\,\text{EllipticF}\left(\sqrt{-\frac{a+b\sinh(x)}{ib-a}},\sqrt{-\frac{ib-a}{ib+a}}\right)a^2b}{3}+\frac{2i\sqrt{-\frac{a+b\sinh(x)}{ib-a}}\sqrt{\frac{(i-\sinh(x))b}{ib+a}}\sqrt{\frac{b(i+\sinh(x))}{ib-a}}}{3}$

input
```
int((a+b*sinh(x))^(3/2),x,method=_RETURNVERBOSE)
```

output
```
2/3*(I*(-(a+b*sinh(x))/(I*b-a))^(1/2)*((I-sinh(x))*b/(I*b+a))^(1/2)*(b*(I+
sinh(x))/(I*b-a))^(1/2)*EllipticF((-(a+b*sinh(x))/(I*b-a))^(1/2),(-(I*b-a)
/(I*b+a))^(1/2))*a^2*b+I*(-(a+b*sinh(x))/(I*b-a))^(1/2)*((I-sinh(x))*b/(I*
b+a))^(1/2)*(b*(I+sinh(x))/(I*b-a))^(1/2)*EllipticF((-(a+b*sinh(x))/(I*b-a
))^(1/2),(-(I*b-a)/(I*b+a))^(1/2))*b^3+3*(-(a+b*sinh(x))/(I*b-a))^(1/2)*((
I-sinh(x))*b/(I*b+a))^(1/2)*(b*(I+sinh(x))/(I*b-a))^(1/2)*EllipticF((-(a+b
*sinh(x))/(I*b-a))^(1/2),(-(I*b-a)/(I*b+a))^(1/2))*a^3+3*(-(a+b*sinh(x))/(
I*b-a))^(1/2)*((I-sinh(x))*b/(I*b+a))^(1/2)*(b*(I+sinh(x))/(I*b-a))^(1/2)*
EllipticF((-(a+b*sinh(x))/(I*b-a))^(1/2),(-(I*b-a)/(I*b+a))^(1/2))*a*b^2-4
*(-(a+b*sinh(x))/(I*b-a))^(1/2)*((I-sinh(x))*b/(I*b+a))^(1/2)*(b*(I+sinh(x
))/(I*b-a))^(1/2)*EllipticE((-(a+b*sinh(x))/(I*b-a))^(1/2),(-(I*b-a)/(I*b+
a))^(1/2))*a^3-4*(-(a+b*sinh(x))/(I*b-a))^(1/2)*((I-sinh(x))*b/(I*b+a))^(1
/2)*(b*(I+sinh(x))/(I*b-a))^(1/2)*EllipticE((-(a+b*sinh(x))/(I*b-a))^(1/2)
,(-(I*b-a)/(I*b+a))^(1/2))*a*b^2+b^3*sinh(x)^3+a*b^2*sinh(x)^2+sinh(x)*b^3
+a*b^2)/b/cosh(x)/(a+b*sinh(x))^(1/2)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 257 vs. $2(125) = 250$.

Time = 0.09 (sec) , antiderivative size = 257, normalized size of antiderivative = 1.71

$$\int (a+b\sinh(x))^{3/2}\,dx = \frac{4\sqrt{\frac{1}{2}}\left(\left(a^2-3\,b^2\right)\cosh\left(x\right)+\left(a^2-3\,b^2\right)\sinh\left(x\right)\right)\sqrt{b}\,\mathrm{weierstrassPInverse}\left(\frac{4\,\left(4\,a^2+3\,b^2\right)}{3\,b^2},-\frac{8}{}\right.}{}$$

input
```
integrate((a+b*sinh(x))^(3/2),x, algorithm="fricas")
```

output
```
1/9*(4*sqrt(1/2)*((a^2 - 3*b^2)*cosh(x) + (a^2 - 3*b^2)*sinh(x))*sqrt(b)*w
eierstrassPInverse(4/3*(4*a^2 + 3*b^2)/b^2, -8/27*(8*a^3 + 9*a*b^2)/b^3, 1
/3*(3*b*cosh(x) + 3*b*sinh(x) + 2*a)/b) - 48*sqrt(1/2)*(a*b*cosh(x) + a*b*
sinh(x))*sqrt(b)*weierstrassZeta(4/3*(4*a^2 + 3*b^2)/b^2, -8/27*(8*a^3 + 9
*a*b^2)/b^3, weierstrassPInverse(4/3*(4*a^2 + 3*b^2)/b^2, -8/27*(8*a^3 + 9
*a*b^2)/b^3, 1/3*(3*b*cosh(x) + 3*b*sinh(x) + 2*a)/b)) + 3*(b^2*cosh(x)^2
+ b^2*sinh(x)^2 - 8*a*b*cosh(x) + b^2 + 2*(b^2*cosh(x) - 4*a*b)*sinh(x))*s
qrt(b*sinh(x) + a))/(b*cosh(x) + b*sinh(x))
```

Sympy [F]

$$\int (a+b\sinh(x))^{3/2}\,dx=\int (a+b\sinh{(x)})^{\frac{3}{2}}\,dx$$

input
```
integrate((a+b*sinh(x))**(3/2),x)
```

output
```
Integral((a + b*sinh(x))**(3/2), x)
```

Maxima [F]

$$\int (a+b\sinh(x))^{3/2}\,dx=\int (b\sinh{(x)}+a)^{\frac{3}{2}}\,dx$$

input
```
integrate((a+b*sinh(x))^(3/2),x, algorithm="maxima")
```

output
```
integrate((b*sinh(x) + a)^(3/2), x)
```

Giac [F]

$$\int (a + b\sinh(x))^{3/2} \, dx = \int (b\sinh(x) + a)^{\frac{3}{2}} \, dx$$

input
```
integrate((a+b*sinh(x))^(3/2),x, algorithm="giac")
```

output
```
integrate((b*sinh(x) + a)^(3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int (a + b\sinh(x))^{3/2} \, dx = \int (a + b\sinh(x))^{3/2} \, dx$$

input
```
int((a + b*sinh(x))^(3/2),x)
```

output
```
int((a + b*sinh(x))^(3/2), x)
```

Reduce [F]

$$\int (a+b\sinh(x))^{3/2} \, dx = \left(\int \sqrt{\sinh(x)\,b + a}dx\right) a + \left(\int \sqrt{\sinh(x)\,b + a}\,\sinh(x)\,dx\right) b$$

input
```
int((a+b*sinh(x))^(3/2),x)
```

output
```
int(sqrt(sinh(x)*b + a),x)*a + int(sqrt(sinh(x)*b + a)*sinh(x),x)*b
```

3.107 $\int \sqrt{a+b\sinh(x)}\,dx$

Optimal result

Integrand size = 10, antiderivative size = 60

$$\int \sqrt{a+b\sinh(x)}\,dx=\frac{2iE\left(\frac{\pi}{4}-\frac{ix}{2}\middle|\frac{2b}{ia+b}\right)\sqrt{a+b\sinh(x)}}{\sqrt{\frac{a+b\sinh(x)}{a-ib}}}$$

output
```
2*I*EllipticE(cos(1/4*Pi+1/2*I*x),2^(1/2)*(b/(I*a+b))^(1/2))*(a+b*sinh(x))
^(1/2)/((a+b*sinh(x))/(a-I*b))^(1/2)
```

Mathematica [A] (verified)

Time = 0.13 (sec) , antiderivative size = 65, normalized size of antiderivative = 1.08

$$\int \sqrt{a+b\sinh(x)}\,dx=\frac{2(ia+b)E\left(\frac{1}{4}(\pi-2ix)\middle|-\frac{2ib}{a-ib}\right)\sqrt{\frac{a+b\sinh(x)}{a-ib}}}{\sqrt{a+b\sinh(x)}}$$

input
```
Integrate[Sqrt[a + b*Sinh[x]],x]
```

output
```
(2*(I*a + b)*EllipticE[(Pi - (2*I)*x)/4, ((-2*I)*b)/(a - I*b)]*Sqrt[(a + b
*Sinh[x])/(a - I*b)])/Sqrt[a + b*Sinh[x]]
```

Rubi [A] (verified)

Time = 0.39 (sec) , antiderivative size = 60, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.400$, Rules used = {3042, 3134, 3042, 3132}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{a+b\sinh(x)}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \sqrt{a-ib\sin(ix)}dx$$

$$\Big\downarrow \text{3134}$$

$$\frac{\sqrt{a+b\sinh(x)}\int \sqrt{\frac{a}{a-ib}+\frac{b\sinh(x)}{a-ib}}dx}{\sqrt{\frac{a+b\sinh(x)}{a-ib}}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\sqrt{a+b\sinh(x)}\int \sqrt{\frac{a}{a-ib}-\frac{ib\sin(ix)}{a-ib}}dx}{\sqrt{\frac{a+b\sinh(x)}{a-ib}}}$$

$$\Big\downarrow \text{3132}$$

$$\frac{2i\sqrt{a+b\sinh(x)}E\left(\frac{\pi}{4}-\frac{ix}{2}|\frac{2b}{ia+b}\right)}{\sqrt{\frac{a+b\sinh(x)}{a-ib}}}$$

input
```
Int[Sqrt[a + b*Sinh[x]],x]
```

output
```
((2*I)*EllipticE[Pi/4 - (I/2)*x, (2*b)/(I*a + b)]*Sqrt[a + b*Sinh[x]])/Sqr
t[(a + b*Sinh[x])/(a - I*b)]
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3132
```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[2*(Sqrt[a
 + b]/d)*EllipticE[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

rule 3134
```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)]   Int[Sqrt[a/(a + b) + (
b/(a + b))*Sin[c + d*x]], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2
, 0] &&  !GtQ[a + b, 0]
```

Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 261 vs. $2(54) = 108$.

Time = 0.93 (sec) , antiderivative size = 262, normalized size of antiderivative = 4.37

method	result
default	$-\frac{2(ib-a)\sqrt{-\frac{a+b\sinh(x)}{ib-a}}\,\sqrt{\frac{(i-\sinh(x))b}{ib+a}}\,\sqrt{\frac{b(i+\sinh(x))}{ib-a}}\,\left(i\,\text{EllipticF}\left(\sqrt{-\frac{a+b\sinh(x)}{ib-a}},\sqrt{-\frac{ib-a}{ib+a}}\right)b-i\,\text{EllipticE}\left(\sqrt{-\frac{a+b\sinh(x)}{ib-a}},\sqrt{-\right.\right.}{b\cosh(x)\sqrt{a+b\sinh(x)}}$
risch	$\sqrt{2}\,\sqrt{(b\,\mathrm{e}^{2x}+2\,\mathrm{e}^{x}a-b)\,\mathrm{e}^{-x}}+\frac{\left(\frac{4a\left(a+\sqrt{a^2+b^2}\right)\sqrt{\frac{\left(\mathrm{e}^{x}+\frac{a+\sqrt{a^2+b^2}}{b}\right)b}{a+\sqrt{a^2+b^2}}}\,\sqrt{\frac{\mathrm{e}^{x}-\frac{-a+\sqrt{a^2+b^2}}{b}}{-\frac{a+\sqrt{a^2+b^2}}{b}-\frac{-a+\sqrt{a^2+b^2}}{b}}}\,\sqrt{-\frac{\mathrm{e}^{x}b}{a+\sqrt{a^2+b^2}}}\,\text{Elli}}{b\sqrt{\mathrm{e}^{3x}b+2a\,\mathrm{e}^{2x}-\mathrm{e}^{x}b}}\right.}{}$

input
```
int((a+b*sinh(x))^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-2*(I*b-a)*(-(a+b*sinh(x))/(I*b-a))^(1/2)*((I-sinh(x))*b/(I*b+a))^(1/2)*(b
*(I+sinh(x))/(I*b-a))^(1/2)/b*(I*EllipticF((-(a+b*sinh(x))/(I*b-a))^(1/2),
(-(I*b-a)/(I*b+a))^(1/2))*b-I*EllipticE((-(a+b*sinh(x))/(I*b-a))^(1/2),(-(
I*b-a)/(I*b+a))^(1/2))*b+EllipticF((-(a+b*sinh(x))/(I*b-a))^(1/2),(-(I*b-a
)/(I*b+a))^(1/2))*a-EllipticE((-(a+b*sinh(x))/(I*b-a))^(1/2),(-(I*b-a)/(I*
b+a))^(1/2))*a)/cosh(x)/(a+b*sinh(x))^(1/2)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 174 vs. $2(50) = 100$.

Time = 0.09 (sec) , antiderivative size = 174, normalized size of antiderivative = 2.90

$$\int \sqrt{a+b\sinh(x)}\,dx$$
$$= \frac{2\left(2\sqrt{\frac{1}{2}}a\sqrt{b}\text{weierstrassPInverse}\left(\frac{4\,(4\,a^2+3\,b^2)}{3\,b^2},-\frac{8\,(8\,a^3+9\,ab^2)}{27\,b^3},\frac{3\,b\cosh(x)+3\,b\sinh(x)+2\,a}{3\,b}\right)-6\sqrt{\frac{1}{2}}b^{\frac{3}{2}}\text{weierstrassZ}\right.}{}$$

input
```
integrate((a+b*sinh(x))^(1/2),x, algorithm="fricas")
```

output
```
2/3*(2*sqrt(1/2)*a*sqrt(b)*weierstrassPInverse(4/3*(4*a^2 + 3*b^2)/b^2, -8
/27*(8*a^3 + 9*a*b^2)/b^3, 1/3*(3*b*cosh(x) + 3*b*sinh(x) + 2*a)/b) - 6*sq
rt(1/2)*b^(3/2)*weierstrassZeta(4/3*(4*a^2 + 3*b^2)/b^2, -8/27*(8*a^3 + 9*
a*b^2)/b^3, weierstrassPInverse(4/3*(4*a^2 + 3*b^2)/b^2, -8/27*(8*a^3 + 9*
a*b^2)/b^3, 1/3*(3*b*cosh(x) + 3*b*sinh(x) + 2*a)/b)) - 3*sqrt(b*sinh(x) +
 a)*b)/b
```

Sympy [F]

$$\int \sqrt{a+b\sinh(x)}\,dx = \int \sqrt{a+b\sinh\left(x\right)}\,dx$$

input
```
integrate((a+b*sinh(x))**(1/2),x)
```

output
```
Integral(sqrt(a + b*sinh(x)), x)
```

Maxima [F]

$$\int \sqrt{a+b\sinh(x)}\,dx=\int \sqrt{b\sinh\left(x\right)+a}\,dx$$

input
```
integrate((a+b*sinh(x))^(1/2),x, algorithm="maxima")
```

output
```
integrate(sqrt(b*sinh(x) + a), x)
```

Giac [F]

$$\int \sqrt{a+b\sinh(x)}\,dx=\int \sqrt{b\sinh\left(x\right)+a}\,dx$$

input
```
integrate((a+b*sinh(x))^(1/2),x, algorithm="giac")
```

output
```
integrate(sqrt(b*sinh(x) + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int \sqrt{a+b\sinh(x)}\,dx=\int \sqrt{a+b\sinh\left(x\right)}\,dx$$

input
```
int((a + b*sinh(x))^(1/2),x)
```

output
```
int((a + b*sinh(x))^(1/2), x)
```

Reduce [F]

$$\int \sqrt{a+b\sinh(x)}\,dx=\int \sqrt{\sinh\left(x\right)b+a}dx$$

input
```
int((a+b*sinh(x))^(1/2),x)
```

output
```
int(sqrt(sinh(x)*b + a),x)
```

3.108 $\int \frac{1}{\sqrt{a+b\sinh(x)}}\,dx$

Optimal result

Integrand size = 10, antiderivative size = 60

$$\int \frac{1}{\sqrt{a+b\sinh(x)}}\,dx=\frac{2i\,\text{EllipticF}\left(\frac{\pi}{4}-\frac{ix}{2},\frac{2b}{ia+b}\right)\sqrt{\frac{a+b\sinh(x)}{a-ib}}}{\sqrt{a+b\sinh(x)}}$$

output
```
-2*I*InverseJacobiAM(-1/4*Pi+1/2*I*x,2^(1/2)*(b/(I*a+b))^(1/2))*((a+b*sinh
(x))/(a-I*b))^(1/2)/(a+b*sinh(x))^(1/2)
```

Mathematica [A] (verified)

Time = 0.13 (sec) , antiderivative size = 60, normalized size of antiderivative = 1.00

$$\int \frac{1}{\sqrt{a+b\sinh(x)}}\,dx=\frac{2i\,\text{EllipticF}\left(\frac{1}{4}(\pi-2ix),-\frac{2ib}{a-ib}\right)\sqrt{\frac{a+b\sinh(x)}{a-ib}}}{\sqrt{a+b\sinh(x)}}$$

input
```
Integrate[1/Sqrt[a + b*Sinh[x]],x]
```

output
```
((2*I)*EllipticF[(Pi - (2*I)*x)/4, ((-2*I)*b)/(a - I*b)]*Sqrt[(a + b*Sinh[
x])/(a - I*b)])/Sqrt[a + b*Sinh[x]]
```

Rubi [A] (verified)

Time = 0.44 (sec) , antiderivative size = 60, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.400, Rules used = {3042, 3142, 3042, 3140}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt{a+b\sinh(x)}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{\sqrt{a-ib\sin(ix)}}dx$$

$$\Big\downarrow \text{3142}$$

$$\frac{\sqrt{\frac{a+b\sinh(x)}{a-ib}}\int \frac{1}{\sqrt{\frac{a}{a-ib}+\frac{b\sinh(x)}{a-ib}}}dx}{\sqrt{a+b\sinh(x)}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\sqrt{\frac{a+b\sinh(x)}{a-ib}}\int \frac{1}{\sqrt{\frac{a}{a-ib}-\frac{ib\sin(ix)}{a-ib}}}dx}{\sqrt{a+b\sinh(x)}}$$

$$\Big\downarrow \text{3140}$$

$$\frac{2i\sqrt{\frac{a+b\sinh(x)}{a-ib}}\text{EllipticF}\left(\frac{\pi}{4}-\frac{ix}{2},\frac{2b}{ia+b}\right)}{\sqrt{a+b\sinh(x)}}$$

input
```
Int[1/Sqrt[a + b*Sinh[x]],x]
```

output
```
((2*I)*EllipticF[Pi/4 - (I/2)*x, (2*b)/(I*a + b)]*Sqrt[(a + b*Sinh[x])/(a
- I*b)])/Sqrt[a + b*Sinh[x]]
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3140
```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/(d*S
qrt[a + b]))*EllipticF[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

rule 3142
```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]]   Int[1/Sqrt[a/(a + b)
 + (b/(a + b))*Sin[c + d*x]], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
 b^2, 0] &&  !GtQ[a + b, 0]
```

Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 124 vs. $2(53) = 106$.

Time = 0.25 (sec) , antiderivative size = 125, normalized size of antiderivative = 2.08

method	result	size
default	$-\frac{2(ib-a)\sqrt{-\frac{a+b\sinh(x)}{ib-a}}\,\sqrt{\frac{(i-\sinh(x))b}{ib+a}}\,\sqrt{\frac{b(i+\sinh(x))}{ib-a}}\,\operatorname{EllipticF}\left(\sqrt{-\frac{a+b\sinh(x)}{ib-a}},\sqrt{-\frac{ib-a}{ib+a}}\right)}{b\cosh(x)\sqrt{a+b\sinh(x)}}$	125

input
```
int(1/(a+b*sinh(x))^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-2*(I*b-a)*(-(a+b*sinh(x))/(I*b-a))^(1/2)*((I-sinh(x))*b/(I*b+a))^(1/2)*(b
*(I+sinh(x))/(I*b-a))^(1/2)*EllipticF((-(a+b*sinh(x))/(I*b-a))^(1/2),(-(I*
b-a)/(I*b+a))^(1/2))/b/cosh(x)/(a+b*sinh(x))^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 61, normalized size of antiderivative = 1.02

$$\int \frac{1}{\sqrt{a+b\sinh(x)}}\,dx = \frac{4\sqrt{\frac{1}{2}}\,\mathrm{weierstrassPInverse}\left(\frac{4\,(4\,a^2+3\,b^2)}{3\,b^2},-\frac{8\,(8\,a^3+9\,ab^2)}{27\,b^3},\frac{3\,b\cosh(x)+3\,b\sinh(x)+2\,a}{3\,b}\right)}{\sqrt{b}}$$

input
```
integrate(1/(a+b*sinh(x))^(1/2),x, algorithm="fricas")
```

output
```
4*sqrt(1/2)*weierstrassPInverse(4/3*(4*a^2 + 3*b^2)/b^2, -8/27*(8*a^3 + 9*
a*b^2)/b^3, 1/3*(3*b*cosh(x) + 3*b*sinh(x) + 2*a)/b)/sqrt(b)
```

Sympy [F]

$$\int \frac{1}{\sqrt{a+b\sinh(x)}}\,dx = \int \frac{1}{\sqrt{a+b\sinh{(x)}}}\,dx$$

input
```
integrate(1/(a+b*sinh(x))**(1/2),x)
```

output
```
Integral(1/sqrt(a + b*sinh(x)), x)
```

Maxima [F]

$$\int \frac{1}{\sqrt{a+b\sinh(x)}}\,dx = \int \frac{1}{\sqrt{b\sinh{(x)}+a}}\,dx$$

input
```
integrate(1/(a+b*sinh(x))^(1/2),x, algorithm="maxima")
```

output
```
integrate(1/sqrt(b*sinh(x) + a), x)
```

Giac [F]

$$\int \frac{1}{\sqrt{a+b\sinh(x)}}\,dx=\int \frac{1}{\sqrt{b\sinh{(x)}+a}}\,dx$$

input
```
integrate(1/(a+b*sinh(x))^(1/2),x, algorithm="giac")
```

output
```
integrate(1/sqrt(b*sinh(x) + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt{a+b\sinh(x)}}\,dx=\int \frac{1}{\sqrt{a+b\sinh{(x)}}}\,dx$$

input
```
int(1/(a + b*sinh(x))^(1/2),x)
```

output
```
int(1/(a + b*sinh(x))^(1/2), x)
```

Reduce [F]

$$\int \frac{1}{\sqrt{a+b\sinh(x)}}\,dx=\int \frac{\sqrt{\sinh{(x)}\,b+a}}{\sinh{(x)}\,b+a}dx$$

input
```
int(1/(a+b*sinh(x))^(1/2),x)
```

output
```
int(sqrt(sinh(x)*b + a)/(sinh(x)*b + a),x)
```

3.109 $\int \frac{1}{(a+b\sinh(x))^{3/2}}\,dx$

Optimal result

Integrand size = 10, antiderivative size = 94

$$\int \frac{1}{(a+b\sinh(x))^{3/2}}\,dx = -\frac{2b\cosh(x)}{(a^2+b^2)\sqrt{a+b\sinh(x)}} + \frac{2iE\left(\frac{\pi}{4}-\frac{ix}{2}\middle|\frac{2b}{ia+b}\right)\sqrt{a+b\sinh(x)}}{(a^2+b^2)\sqrt{\frac{a+b\sinh(x)}{a-ib}}}$$

output
```
-2*b*cosh(x)/(a^2+b^2)/(a+b*sinh(x))^(1/2)+2*I*EllipticE(cos(1/4*Pi+1/2*I*
x),2^(1/2)*(b/(I*a+b))^(1/2))*(a+b*sinh(x))^(1/2)/(a^2+b^2)/((a+b*sinh(x))
/(a-I*b))^(1/2)
```

Mathematica [A] (verified)

Time = 0.11 (sec) , antiderivative size = 81, normalized size of antiderivative = 0.86

$$\int \frac{1}{(a+b\sinh(x))^{3/2}}\,dx = \frac{-2b\cosh(x)+2(ia+b)E\left(\frac{1}{4}(\pi-2ix)\middle|-\frac{2ib}{a-ib}\right)\sqrt{\frac{a+b\sinh(x)}{a-ib}}}{(a^2+b^2)\sqrt{a+b\sinh(x)}}$$

input
```
Integrate[(a + b*Sinh[x])^(-3/2),x]
```

output
```
(-2*b*Cosh[x] + 2*(I*a + b)*EllipticE[(Pi - (2*I)*x)/4, ((-2*I)*b)/(a - I*
b)]*Sqrt[(a + b*Sinh[x])/(a - I*b)])/((a^2 + b^2)*Sqrt[a + b*Sinh[x]])
```

Rubi [A] (verified)

Time = 0.65 (sec) , antiderivative size = 94, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.700$, Rules used = {3042, 3143, 27, 3042, 3134, 3042, 3132}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(a+b\sinh(x))^{3/2}}dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \frac{1}{(a-ib\sin(ix))^{3/2}}dx$$

$$\Big\downarrow \text{ 3143}$$

$$-\frac{2\int -\frac{1}{2}\sqrt{a+b\sinh(x)}dx}{a^2+b^2}-\frac{2b\cosh(x)}{(a^2+b^2)\sqrt{a+b\sinh(x)}}$$

$$\Big\downarrow \text{ 27}$$

$$\frac{\int \sqrt{a+b\sinh(x)}dx}{a^2+b^2}-\frac{2b\cosh(x)}{(a^2+b^2)\sqrt{a+b\sinh(x)}}$$

$$\Big\downarrow \text{ 3042}$$

$$-\frac{2b\cosh(x)}{(a^2+b^2)\sqrt{a+b\sinh(x)}}+\frac{\int \sqrt{a-ib\sin(ix)}dx}{a^2+b^2}$$

$$\Big\downarrow \text{ 3134}$$

$$-\frac{2b\cosh(x)}{(a^2+b^2)\sqrt{a+b\sinh(x)}}+\frac{\sqrt{a+b\sinh(x)}\int \sqrt{\frac{a}{a-ib}+\frac{b\sinh(x)}{a-ib}}dx}{(a^2+b^2)\sqrt{\frac{a+b\sinh(x)}{a-ib}}}$$

$$\Big\downarrow \text{ 3042}$$

$$-\frac{2b\cosh(x)}{(a^2+b^2)\sqrt{a+b\sinh(x)}}+\frac{\sqrt{a+b\sinh(x)}\int\sqrt{\frac{a}{a-ib}-\frac{ib\sin(ix)}{a-ib}}dx}{(a^2+b^2)\sqrt{\frac{a+b\sinh(x)}{a-ib}}}$$

$$\Big\downarrow \text{3132}$$

$$-\frac{2b\cosh(x)}{(a^2+b^2)\sqrt{a+b\sinh(x)}}+\frac{2i\sqrt{a+b\sinh(x)}E\left(\frac{\pi}{4}-\frac{ix}{2}|\frac{2b}{ia+b}\right)}{(a^2+b^2)\sqrt{\frac{a+b\sinh(x)}{a-ib}}}$$

input
```
Int[(a + b*Sinh[x])^(-3/2),x]
```

output
```
(-2*b*Cosh[x])/((a^2 + b^2)*Sqrt[a + b*Sinh[x]]) + ((2*I)*EllipticE[Pi/4 -
 (I/2)*x, (2*b)/(I*a + b)]*Sqrt[a + b*Sinh[x]])/((a^2 + b^2)*Sqrt[(a + b*S
inh[x])/(a - I*b)])
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3132
```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[2*(Sqrt[a
 + b]/d)*EllipticE[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

rule 3134
```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)]   Int[Sqrt[a/(a + b) + (
b/(a + b))*Sin[c + d*x]], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2
, 0] &&  !GtQ[a + b, 0]
```

rule 3143
```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos
[c + d*x]*((a + b*Sin[c + d*x])^(n + 1)/(d*(n + 1)*(a^2 - b^2))), x] + Simp
[1/((n + 1)*(a^2 - b^2))   Int[(a + b*Sin[c + d*x])^(n + 1)*Simp[a*(n + 1)
- b*(n + 2)*Sin[c + d*x], x], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
 b^2, 0] && LtQ[n, -1] && IntegerQ[2*n]
```

Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 455 vs. $2(86) = 172$.

Time = 0.34 (sec) , antiderivative size = 456, normalized size of antiderivative = 4.85

method	result
default	$\frac{2\sqrt{-\frac{a+b\sinh(x)}{ib-a}}\,\sqrt{\frac{(i-\sinh(x))b}{ib+a}}\,\sqrt{\frac{b(i+\sinh(x))}{ib-a}}\,\mathrm{EllipticF}\left(\sqrt{-\frac{a+b\sinh(x)}{ib-a}},\sqrt{-\frac{ib-a}{ib+a}}\right)a^2+2\sqrt{-\frac{a+b\sinh(x)}{ib-a}}\,\sqrt{\frac{(i-\sinh(x))b}{ib+a}}\,\sqrt{\frac{b(i+\sinh}{ib-a}}}{}$

input
```
int(1/(a+b*sinh(x))^(3/2),x,method=_RETURNVERBOSE)
```

output
```
2*((-(a+b*sinh(x))/(I*b-a))^(1/2)*((I-sinh(x))*b/(I*b+a))^(1/2)*(b*(I+sinh
(x))/(I*b-a))^(1/2)*EllipticF((-(a+b*sinh(x))/(I*b-a))^(1/2),(-(I*b-a)/(I*
b+a))^(1/2))*a^2+(-(a+b*sinh(x))/(I*b-a))^(1/2)*((I-sinh(x))*b/(I*b+a))^(1
/2)*(b*(I+sinh(x))/(I*b-a))^(1/2)*EllipticF((-(a+b*sinh(x))/(I*b-a))^(1/2)
,(-(I*b-a)/(I*b+a))^(1/2))*b^2-(-(a+b*sinh(x))/(I*b-a))^(1/2)*((I-sinh(x))
*b/(I*b+a))^(1/2)*(b*(I+sinh(x))/(I*b-a))^(1/2)*EllipticE((-(a+b*sinh(x))/
(I*b-a))^(1/2),(-(I*b-a)/(I*b+a))^(1/2))*a^2-(-(a+b*sinh(x))/(I*b-a))^(1/2
)*((I-sinh(x))*b/(I*b+a))^(1/2)*(b*(I+sinh(x))/(I*b-a))^(1/2)*EllipticE((-
(a+b*sinh(x))/(I*b-a))^(1/2),(-(I*b-a)/(I*b+a))^(1/2))*b^2-b^2*sinh(x)^2-b
^2)/(a^2+b^2)/b/cosh(x)/(a+b*sinh(x))^(1/2)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 376 vs. $2(82) = 164$.

Time = 0.10 (sec) , antiderivative size = 376, normalized size of antiderivative = 4.00

$$\int \frac{1}{(a+b\sinh(x))^{3/2}}\,dx = -\frac{4\left(\sqrt{\frac{1}{2}}\left(ab\cosh(x)^2+ab\sinh(x)^2+2\,a^2\cosh(x)-ab+2\,(ab\cosh(x)+a^2)\sinh(x)\right)\sqrt{b}\text{weierstrassPInv}\right.}{}$$

input
```
integrate(1/(a+b*sinh(x))^(3/2),x, algorithm="fricas")
```

output
```
-4/3*(sqrt(1/2)*(a*b*cosh(x)^2 + a*b*sinh(x)^2 + 2*a^2*cosh(x) - a*b + 2*(
a*b*cosh(x) + a^2)*sinh(x))*sqrt(b)*weierstrassPInverse(4/3*(4*a^2 + 3*b^2
)/b^2, -8/27*(8*a^3 + 9*a*b^2)/b^3, 1/3*(3*b*cosh(x) + 3*b*sinh(x) + 2*a)/
b) - 3*sqrt(1/2)*(b^2*cosh(x)^2 + b^2*sinh(x)^2 + 2*a*b*cosh(x) - b^2 + 2*
(b^2*cosh(x) + a*b)*sinh(x))*sqrt(b)*weierstrassZeta(4/3*(4*a^2 + 3*b^2)/b
^2, -8/27*(8*a^3 + 9*a*b^2)/b^3, weierstrassPInverse(4/3*(4*a^2 + 3*b^2)/b
^2, -8/27*(8*a^3 + 9*a*b^2)/b^3, 1/3*(3*b*cosh(x) + 3*b*sinh(x) + 2*a)/b))
 - 3*(b^2*cosh(x)^2 + b^2*sinh(x)^2 + a*b*cosh(x) + (2*b^2*cosh(x) + a*b)*
sinh(x))*sqrt(b*sinh(x) + a))/(a^2*b^2 + b^4 - (a^2*b^2 + b^4)*cosh(x)^2 -
 (a^2*b^2 + b^4)*sinh(x)^2 - 2*(a^3*b + a*b^3)*cosh(x) - 2*(a^3*b + a*b^3
+ (a^2*b^2 + b^4)*cosh(x))*sinh(x))
```

Sympy [F]

$$\int \frac{1}{(a+b\sinh(x))^{3/2}}\,dx = \int \frac{1}{(a+b\sinh(x))^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/(a+b*sinh(x))**(3/2),x)
```

output
```
Integral((a + b*sinh(x))**(-3/2), x)
```

Maxima [F]

$$\int \frac{1}{(a+b\sinh(x))^{3/2}}\,dx = \int \frac{1}{(b\sinh(x)+a)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/(a+b*sinh(x))^(3/2),x, algorithm="maxima")
```

output
```
integrate((b*sinh(x) + a)^(-3/2), x)
```

Giac [F]

$$\int \frac{1}{(a+b\sinh(x))^{3/2}}\,dx = \int \frac{1}{(b\sinh(x)+a)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/(a+b*sinh(x))^(3/2),x, algorithm="giac")
```

output
```
integrate((b*sinh(x) + a)^(-3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(a+b\sinh(x))^{3/2}}\,dx = \int \frac{1}{(a+b\sinh(x))^{3/2}}\,dx$$

input
```
int(1/(a + b*sinh(x))^(3/2),x)
```

output
```
int(1/(a + b*sinh(x))^(3/2), x)
```

Reduce [F]

$$\int \frac{1}{(a+b\sinh(x))^{3/2}}\,dx=\int \frac{\sqrt{\sinh(x)\,b+a}}{\sinh(x)^{2}\,b^{2}+2\sinh(x)\,ab+a^{2}}dx$$

input
```
int(1/(a+b*sinh(x))^(3/2),x)
```

output
```
int(sqrt(sinh(x)*b + a)/(sinh(x)**2*b**2 + 2*sinh(x)*a*b + a**2),x)
```

3.110 $\int \frac{1}{(a+b\sinh(x))^{5/2}}\,dx$

Optimal result

Integrand size = 10, antiderivative size = 197

$$\int \frac{1}{(a+b\sinh(x))^{5/2}}\,dx = -\frac{2b\cosh(x)}{3(a^2+b^2)(a+b\sinh(x))^{3/2}} - \frac{8ab\cosh(x)}{3(a^2+b^2)^2\sqrt{a+b\sinh(x)}} + \frac{8iaE\left(\frac{\pi}{4}-\frac{ix}{2}\middle|\frac{2b}{ia+b}\right)\sqrt{a+b\sinh(x)}}{3(a^2+b^2)^2\sqrt{\frac{a+b\sinh(x)}{a-ib}}} - \frac{2i\,\text{EllipticF}\left(\frac{\pi}{4}-\frac{ix}{2},\frac{2b}{ia+b}\right)\sqrt{\frac{a+b\sinh(x)}{a-ib}}}{3(a^2+b^2)\sqrt{a+b\sinh(x)}}$$

output
```
-2/3*b*cosh(x)/(a^2+b^2)/(a+b*sinh(x))^(3/2)-8/3*a*b*cosh(x)/(a^2+b^2)^2/(
a+b*sinh(x))^(1/2)+8/3*I*a*EllipticE(cos(1/4*Pi+1/2*I*x),2^(1/2)*(b/(I*a+b
))^(1/2))*(a+b*sinh(x))^(1/2)/(a^2+b^2)^2/((a+b*sinh(x))/(a-I*b))^(1/2)+2/
3*I*InverseJacobiAM(-1/4*Pi+1/2*I*x,2^(1/2)*(b/(I*a+b))^(1/2))*((a+b*sinh(
x))/(a-I*b))^(1/2)/(a^2+b^2)/(a+b*sinh(x))^(1/2)
```

Mathematica [A] (verified)

Time = 0.46 (sec) , antiderivative size = 166, normalized size of antiderivative = 0.84

$$\int \frac{1}{(a+b\sinh(x))^{5/2}}\,dx = \frac{\frac{8iaE\left(\frac{1}{4}(\pi-2ix)|-\frac{2ib}{a-ib}\right)(a+b\sinh(x))^2}{\sqrt{\frac{a+b\sinh(x)}{a-ib}}} - 2i(a^2+b^2)\,\text{EllipticF}\left(\frac{1}{4}(\pi-2ix),-\frac{2ib}{a-ib}\right)(a+b \cdots}{3\left(a^2+b^2\right)^2(a+b\sinh(x \cdots}$$

input
```
Integrate[(a + b*Sinh[x])^(-5/2),x]
```

output
```
(((8*I)*a*EllipticE[(Pi - (2*I)*x)/4, ((-2*I)*b)/(a - I*b)]*(a + b*Sinh[x]
)^2)/Sqrt[(a + b*Sinh[x])/(a - I*b)] - (2*I)*(a^2 + b^2)*EllipticF[(Pi - (
2*I)*x)/4, ((-2*I)*b)/(a - I*b)]*(a + b*Sinh[x])*Sqrt[(a + b*Sinh[x])/(a -
 I*b)] - 2*b*Cosh[x]*(5*a^2 + b^2 + 4*a*b*Sinh[x]))/(3*(a^2 + b^2)^2*(a +
b*Sinh[x])^(3/2))
```

Rubi [A] (verified)

Time = 1.22 (sec) , antiderivative size = 205, normalized size of antiderivative = 1.04, number of steps used = 15, number of rules used = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 1.500$, Rules used = {3042, 3143, 27, 3042, 3233, 27, 3042, 3231, 3042, 3134, 3042, 3132, 3142, 3042, 3140}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(a+b\sinh(x))^{5/2}}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{(a-ib\sin(ix))^{5/2}}dx$$

$$\Big\downarrow \text{3143}$$

$$-\frac{2\int -\frac{3a-b\sinh(x)}{2(a+b\sinh(x))^{3/2}}dx}{3\left(a^2+b^2\right)}-\frac{2b\cosh(x)}{3\left(a^2+b^2\right)(a+b\sinh(x))^{3/2}}$$

$$\Big\downarrow \text{27}$$

$$\frac{\int \frac{3a-b\sinh(x)}{(a+b\sinh(x))^{3/2}}dx}{3(a^2+b^2)}-\frac{2b\cosh(x)}{3(a^2+b^2)(a+b\sinh(x))^{3/2}}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{2b\cosh(x)}{3(a^2+b^2)(a+b\sinh(x))^{3/2}}+\frac{\int \frac{3a+ib\sin(ix)}{(a-ib\sin(ix))^{3/2}}dx}{3(a^2+b^2)}$$

$$\Big\downarrow \text{3233}$$

$$\frac{-\frac{2\int -\frac{3a^2+4b\sinh(x)a-b^2}{2\sqrt{a+b\sinh(x)}}dx}{a^2+b^2}-\frac{8ab\cosh(x)}{(a^2+b^2)\sqrt{a+b\sinh(x)}}}{3(a^2+b^2)}-\frac{2b\cosh(x)}{3(a^2+b^2)(a+b\sinh(x))^{3/2}}$$

$$\Big\downarrow \text{27}$$

$$\frac{\frac{\int \frac{3a^2+4b\sinh(x)a-b^2}{\sqrt{a+b\sinh(x)}}dx}{a^2+b^2}-\frac{8ab\cosh(x)}{(a^2+b^2)\sqrt{a+b\sinh(x)}}}{3(a^2+b^2)}-\frac{2b\cosh(x)}{3(a^2+b^2)(a+b\sinh(x))^{3/2}}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{2b\cosh(x)}{3(a^2+b^2)(a+b\sinh(x))^{3/2}}+\frac{-\frac{8ab\cosh(x)}{(a^2+b^2)\sqrt{a+b\sinh(x)}}+\frac{\int \frac{3a^2-4ib\sin(ix)a-b^2}{\sqrt{a-ib\sin(ix)}}dx}{a^2+b^2}}{3(a^2+b^2)}$$

$$\Big\downarrow \text{3231}$$

$$\frac{\frac{4a\int \sqrt{a+b\sinh(x)}dx-(a^2+b^2)\int \frac{1}{\sqrt{a+b\sinh(x)}}dx}{a^2+b^2}-\frac{8ab\cosh(x)}{(a^2+b^2)\sqrt{a+b\sinh(x)}}}{3(a^2+b^2)}-\frac{2b\cosh(x)}{3(a^2+b^2)(a+b\sinh(x))^{3/2}}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{2b\cosh(x)}{3(a^2+b^2)(a+b\sinh(x))^{3/2}}+\frac{-\frac{8ab\cosh(x)}{(a^2+b^2)\sqrt{a+b\sinh(x)}}+\frac{4a\int \sqrt{a-ib\sin(ix)}dx-(a^2+b^2)\int \frac{1}{\sqrt{a-ib\sin(ix)}}dx}{a^2+b^2}}{3(a^2+b^2)}$$

$$\Big\downarrow \text{3134}$$

$$-\frac{2b\cosh(x)}{3(a^2+b^2)(a+b\sinh(x))^{3/2}}+\frac{-\frac{8ab\cosh(x)}{(a^2+b^2)\sqrt{a+b\sinh(x)}}+\frac{\frac{4a\sqrt{a+b\sinh(x)}\int \sqrt{\frac{a}{a-ib}+\frac{b\sinh(x)}{a-ib}}dx}{\sqrt{\frac{a+b\sinh(x)}{a-ib}}}-(a^2+b^2)\int \frac{1}{\sqrt{a-ib\sin(ix)}}dx}{a^2+b^2}}{3(a^2+b^2)}$$

$\Big\downarrow$ 3042

$$-\frac{2b\cosh(x)}{3\left(a^2+b^2\right)\left(a+b\sinh(x)\right)^{3/2}}+\frac{-\frac{8ab\cosh(x)}{\left(a^2+b^2\right)\sqrt{a+b\sinh(x)}}+\frac{\frac{4a\sqrt{a+b\sinh(x)}\int\sqrt{\frac{a}{a-ib}-\frac{ib\sin(ix)}{a-ib}}dx}{\sqrt{\frac{a+b\sinh(x)}{a-ib}}}-\left(a^2+b^2\right)\int\frac{1}{\sqrt{a-ib\sin(ix)}}dx}{a^2+b^2}}{3\left(a^2+b^2\right)}$$

$\Big\downarrow$ 3132

$$-\frac{2b\cosh(x)}{3\left(a^2+b^2\right)\left(a+b\sinh(x)\right)^{3/2}}+\frac{-\frac{8ab\cosh(x)}{\left(a^2+b^2\right)\sqrt{a+b\sinh(x)}}+\frac{\frac{8ia\sqrt{a+b\sinh(x)}E\left(\frac{\pi}{4}-\frac{ix}{2}|\frac{2b}{ia+b}\right)}{\sqrt{\frac{a+b\sinh(x)}{a-ib}}}-\left(a^2+b^2\right)\int\frac{1}{\sqrt{a-ib\sin(ix)}}dx}{a^2+b^2}}{3\left(a^2+b^2\right)}$$

$\Big\downarrow$ 3142

$$-\frac{2b\cosh(x)}{3\left(a^2+b^2\right)\left(a+b\sinh(x)\right)^{3/2}}+\frac{-\frac{8ab\cosh(x)}{\left(a^2+b^2\right)\sqrt{a+b\sinh(x)}}+\frac{\frac{8ia\sqrt{a+b\sinh(x)}E\left(\frac{\pi}{4}-\frac{ix}{2}|\frac{2b}{ia+b}\right)}{\sqrt{\frac{a+b\sinh(x)}{a-ib}}}-\frac{\left(a^2+b^2\right)\sqrt{\frac{a+b\sinh(x)}{a-ib}}\int\frac{1}{\sqrt{\frac{a}{a-ib}+\frac{b\sinh(x)}{a-ib}}}dx}{\sqrt{a+b\sinh(x)}}}{a^2+b^2}}{3\left(a^2+b^2\right)}$$

$\Big\downarrow$ 3042

$$-\frac{2b\cosh(x)}{3\left(a^2+b^2\right)\left(a+b\sinh(x)\right)^{3/2}}+\frac{-\frac{8ab\cosh(x)}{\left(a^2+b^2\right)\sqrt{a+b\sinh(x)}}+\frac{\frac{8ia\sqrt{a+b\sinh(x)}E\left(\frac{\pi}{4}-\frac{ix}{2}|\frac{2b}{ia+b}\right)}{\sqrt{\frac{a+b\sinh(x)}{a-ib}}}-\frac{\left(a^2+b^2\right)\sqrt{\frac{a+b\sinh(x)}{a-ib}}\int\frac{1}{\sqrt{\frac{a}{a-ib}-\frac{ib\sin(ix)}{a-ib}}}dx}{\sqrt{a+b\sinh(x)}}}{a^2+b^2}}{3\left(a^2+b^2\right)}$$

$\Big\downarrow$ 3140

$$-\frac{2b\cosh(x)}{3\left(a^2+b^2\right)\left(a+b\sinh(x)\right)^{3/2}}+\frac{-\frac{8ab\cosh(x)}{\left(a^2+b^2\right)\sqrt{a+b\sinh(x)}}+\frac{\frac{8ia\sqrt{a+b\sinh(x)}E\left(\frac{\pi}{4}-\frac{ix}{2}|\frac{2b}{ia+b}\right)}{\sqrt{\frac{a+b\sinh(x)}{a-ib}}}-\frac{2i\left(a^2+b^2\right)\sqrt{\frac{a+b\sinh(x)}{a-ib}}\,\text{EllipticF}\left(\frac{\pi}{4}-\frac{ix}{2},\frac{2b}{ia+b}\right)}{\sqrt{a+b\sinh(x)}}}{a^2+b^2}}{3\left(a^2+b^2\right)}$$

input
```
Int[(a + b*Sinh[x])^(-5/2),x]
```

output
```
(-2*b*Cosh[x])/(3*(a^2 + b^2)*(a + b*Sinh[x])^(3/2)) + ((-8*a*b*Cosh[x])/(
(a^2 + b^2)*Sqrt[a + b*Sinh[x]]) + (((8*I)*a*EllipticE[Pi/4 - (I/2)*x, (2*
b)/(I*a + b)]*Sqrt[a + b*Sinh[x]])/Sqrt[(a + b*Sinh[x])/(a - I*b)] - ((2*I
)*(a^2 + b^2)*EllipticF[Pi/4 - (I/2)*x, (2*b)/(I*a + b)]*Sqrt[(a + b*Sinh[
x])/(a - I*b)])/Sqrt[a + b*Sinh[x]])/(a^2 + b^2))/(3*(a^2 + b^2))
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3132
```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[2*(Sqrt[a
 + b]/d)*EllipticE[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

rule 3134
```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)]   Int[Sqrt[a/(a + b) + (
b/(a + b))*Sin[c + d*x]], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2
, 0] &&  !GtQ[a + b, 0]
```

rule 3140
```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/(d*S
qrt[a + b]))*EllipticF[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

rule 3142
```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]]   Int[1/Sqrt[a/(a + b)
 + (b/(a + b))*Sin[c + d*x]], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
 b^2, 0] &&  !GtQ[a + b, 0]
```

rule 3143
```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos
[c + d*x]*((a + b*Sin[c + d*x])^(n + 1)/(d*(n + 1)*(a^2 - b^2))), x] + Simp
[1/((n + 1)*(a^2 - b^2))   Int[(a + b*Sin[c + d*x])^(n + 1)*Simp[a*(n + 1)
- b*(n + 2)*Sin[c + d*x], x], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
 b^2, 0] && LtQ[n, -1] && IntegerQ[2*n]
```

rule 3231
```
Int[((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])/Sqrt[(a_) + (b_.)*sin[(e_.) + (
f_.)*(x_)]], x_Symbol] :> Simp[(b*c - a*d)/b   Int[1/Sqrt[a + b*Sin[e + f*x
]], x], x] + Simp[d/b   Int[Sqrt[a + b*Sin[e + f*x]], x], x] /; FreeQ[{a, b
, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
```

rule 3233
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Simp[(-(b*c - a*d))*Cos[e + f*x]*((a + b*Sin[e +
 f*x])^(m + 1)/(f*(m + 1)*(a^2 - b^2))), x] + Simp[1/((m + 1)*(a^2 - b^2))
  Int[(a + b*Sin[e + f*x])^(m + 1)*Simp[(a*c - b*d)*(m + 1) - (b*c - a*d)*(
m + 2)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c
- a*d, 0] && NeQ[a^2 - b^2, 0] && LtQ[m, -1] && IntegerQ[2*m]
```

Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 437 vs. $2(172) = 344$.

Time = 0.46 (sec) , antiderivative size = 438, normalized size of antiderivative = 2.22

method	result
default	$\frac{\sqrt{\cosh(x)^2(a+b\sinh(x))}\left(-\frac{2\sqrt{\cosh(x)^2(a+b\sinh(x))}}{3b\left(a^2+b^2\right)\left(\sinh(x)+\frac{a}{b}\right)^2}-\frac{8b\cosh(x)^2a}{3\left(a^2+b^2\right)^2\sqrt{\cosh(x)^2(a+b\sinh(x))}}+\frac{2\left(3a^2-b^2\right)\left(\frac{a}{b}-i\right)\sqrt{\frac{-a-b\sinh(x)}{ib-a}}\sqrt{\frac{(i-\sinh}{ib+}}}{\left(3a^4+6a^2b^2+\right.}\right.}{}$

input
```
int(1/(a+b*sinh(x))^(5/2),x,method=_RETURNVERBOSE)
```

output
```
(cosh(x)^2*(a+b*sinh(x)))^(1/2)*(-2/3/b/(a^2+b^2)*(cosh(x)^2*(a+b*sinh(x))
)^(1/2)/(sinh(x)+1/b*a)^2-8/3*b*cosh(x)^2/(a^2+b^2)^2*a/(cosh(x)^2*(a+b*si
nh(x)))^(1/2)+2*(3*a^2-b^2)/(3*a^4+6*a^2*b^2+3*b^4)*(1/b*a-I)*((-a-b*sinh(
x))/(I*b-a))^(1/2)*((I-sinh(x))*b/(I*b+a))^(1/2)*(b*(I+sinh(x))/(I*b-a))^(
1/2)/(cosh(x)^2*(a+b*sinh(x)))^(1/2)*EllipticF(((-a-b*sinh(x))/(I*b-a))^(1
/2),((a-I*b)/(I*b+a))^(1/2))+8/3*a*b/(a^2+b^2)^2*(1/b*a-I)*((-a-b*sinh(x))
/(I*b-a))^(1/2)*((I-sinh(x))*b/(I*b+a))^(1/2)*(b*(I+sinh(x))/(I*b-a))^(1/2
)/(cosh(x)^2*(a+b*sinh(x)))^(1/2)*((-1/b*a-I)*EllipticE(((-a-b*sinh(x))/(I
*b-a))^(1/2),((a-I*b)/(I*b+a))^(1/2))+I*EllipticF(((-a-b*sinh(x))/(I*b-a))
^(1/2),((a-I*b)/(I*b+a))^(1/2))))/cosh(x)/(a+b*sinh(x))^(1/2)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1198 vs. $2(164) = 328$.

Time = 0.12 (sec) , antiderivative size = 1198, normalized size of antiderivative = 6.08

$$\int \frac{1}{(a+b\sinh(x))^{5/2}}\,dx = \text{Too large to display}$$

input
```
integrate(1/(a+b*sinh(x))^(5/2),x, algorithm="fricas")
```

output

```
4/9*(sqrt(1/2)*((a^2*b^2 - 3*b^4)*cosh(x)^4 + (a^2*b^2 - 3*b^4)*sinh(x)^4
+ a^2*b^2 - 3*b^4 + 4*(a^3*b - 3*a*b^3)*cosh(x)^3 + 4*(a^3*b - 3*a*b^3 + (
a^2*b^2 - 3*b^4)*cosh(x))*sinh(x)^3 + 2*(2*a^4 - 7*a^2*b^2 + 3*b^4)*cosh(x
)^2 + 2*(2*a^4 - 7*a^2*b^2 + 3*b^4 + 3*(a^2*b^2 - 3*b^4)*cosh(x)^2 + 6*(a^
3*b - 3*a*b^3)*cosh(x))*sinh(x)^2 - 4*(a^3*b - 3*a*b^3)*cosh(x) - 4*(a^3*b
 - 3*a*b^3 - (a^2*b^2 - 3*b^4)*cosh(x)^3 - 3*(a^3*b - 3*a*b^3)*cosh(x)^2 -
 (2*a^4 - 7*a^2*b^2 + 3*b^4)*cosh(x))*sinh(x))*sqrt(b)*weierstrassPInverse
(4/3*(4*a^2 + 3*b^2)/b^2, -8/27*(8*a^3 + 9*a*b^2)/b^3, 1/3*(3*b*cosh(x) +
3*b*sinh(x) + 2*a)/b) - 12*sqrt(1/2)*(a*b^3*cosh(x)^4 + a*b^3*sinh(x)^4 +
4*a^2*b^2*cosh(x)^3 - 4*a^2*b^2*cosh(x) + a*b^3 + 4*(a*b^3*cosh(x) + a^2*b
^2)*sinh(x)^3 + 2*(2*a^3*b - a*b^3)*cosh(x)^2 + 2*(3*a*b^3*cosh(x)^2 + 6*a
^2*b^2*cosh(x) + 2*a^3*b - a*b^3)*sinh(x)^2 + 4*(a*b^3*cosh(x)^3 + 3*a^2*b
^2*cosh(x)^2 - a^2*b^2 + (2*a^3*b - a*b^3)*cosh(x))*sinh(x))*sqrt(b)*weier
strassZeta(4/3*(4*a^2 + 3*b^2)/b^2, -8/27*(8*a^3 + 9*a*b^2)/b^3, weierstra
ssPInverse(4/3*(4*a^2 + 3*b^2)/b^2, -8/27*(8*a^3 + 9*a*b^2)/b^3, 1/3*(3*b*
cosh(x) + 3*b*sinh(x) + 2*a)/b)) - 3*(4*a*b^3*cosh(x)^4 + 4*a*b^3*sinh(x)^
4 + (13*a^2*b^2 + b^4)*cosh(x)^3 + (16*a*b^3*cosh(x) + 13*a^2*b^2 + b^4)*s
inh(x)^3 + 4*(2*a^3*b - a*b^3)*cosh(x)^2 + (24*a*b^3*cosh(x)^2 + 8*a^3*b -
 4*a*b^3 + 3*(13*a^2*b^2 + b^4)*cosh(x))*sinh(x)^2 - (3*a^2*b^2 - b^4)*cos
h(x) + (16*a*b^3*cosh(x)^3 - 3*a^2*b^2 + b^4 + 3*(13*a^2*b^2 + b^4)*cos...
```

Sympy [F]

$$\int \frac{1}{(a+b\sinh(x))^{5/2}}\,dx = \int \frac{1}{(a+b\sinh(x))^{\frac{5}{2}}}\,dx$$

input

```
integrate(1/(a+b*sinh(x))**(5/2),x)
```

output

```
Integral((a + b*sinh(x))**(-5/2), x)
```

Maxima [F]

$$\int \frac{1}{(a+b\sinh(x))^{5/2}}\,dx = \int \frac{1}{(b\sinh(x)+a)^{\frac{5}{2}}}\,dx$$

input
```
integrate(1/(a+b*sinh(x))^(5/2),x, algorithm="maxima")
```

output
```
integrate((b*sinh(x) + a)^(-5/2), x)
```

Giac [F]

$$\int \frac{1}{(a+b\sinh(x))^{5/2}}\,dx = \int \frac{1}{(b\sinh(x)+a)^{\frac{5}{2}}}\,dx$$

input
```
integrate(1/(a+b*sinh(x))^(5/2),x, algorithm="giac")
```

output
```
integrate((b*sinh(x) + a)^(-5/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(a+b\sinh(x))^{5/2}}\,dx = \int \frac{1}{(a+b\sinh(x))^{5/2}}\,dx$$

input
```
int(1/(a + b*sinh(x))^(5/2),x)
```

output
```
int(1/(a + b*sinh(x))^(5/2), x)
```

Reduce [F]

$$\int \frac{1}{(a+b\sinh(x))^{5/2}}\,dx = \int \frac{\sqrt{\sinh(x)\,b+a}}{\sinh(x)^{3}\,b^{3}+3\sinh(x)^{2}\,a\,b^{2}+3\sinh(x)\,a^{2}b+a^{3}}dx$$

input
```
int(1/(a+b*sinh(x))^(5/2),x)
```

output
```
int(sqrt(sinh(x)*b + a)/(sinh(x)**3*b**3 + 3*sinh(x)**2*a*b**2 + 3*sinh(x)
*a**2*b + a**3),x)
```

3.111 $\int \frac{\sinh(x)}{\sqrt{a+b\sinh(x)}}\,dx$

Optimal result

Integrand size = 13, antiderivative size = 128

$$\int \frac{\sinh(x)}{\sqrt{a+b\sinh(x)}}\,dx = \frac{2iE\left(\frac{\pi}{4}-\frac{ix}{2}\middle|\frac{2b}{ia+b}\right)\sqrt{a+b\sinh(x)}}{b\sqrt{\frac{a+b\sinh(x)}{a-ib}}} - \frac{2ia\,\text{EllipticF}\left(\frac{\pi}{4}-\frac{ix}{2},\frac{2b}{ia+b}\right)\sqrt{\frac{a+b\sinh(x)}{a-ib}}}{b\sqrt{a+b\sinh(x)}}$$

output
```
2*I*EllipticE(cos(1/4*Pi+1/2*I*x),2^(1/2)*(b/(I*a+b))^(1/2))*(a+b*sinh(x))
^(1/2)/b/((a+b*sinh(x))/(a-I*b))^(1/2)+2*I*a*InverseJacobiAM(-1/4*Pi+1/2*I
*x,2^(1/2)*(b/(I*a+b))^(1/2))*((a+b*sinh(x))/(a-I*b))^(1/2)/b/(a+b*sinh(x)
)^(1/2)
```

Mathematica [A] (verified)

Time = 0.30 (sec) , antiderivative size = 101, normalized size of antiderivative = 0.79

$$\int \frac{\sinh(x)}{\sqrt{a+b\sinh(x)}}\,dx = \frac{2\left((ia+b)E\left(\frac{1}{4}(\pi-2ix)\middle|-\frac{2ib}{a-ib}\right)-ia\,\text{EllipticF}\left(\frac{1}{4}(\pi-2ix),-\frac{2ib}{a-ib}\right)\right)\sqrt{\frac{a+b\sinh(x)}{a-ib}}}{b\sqrt{a+b\sinh(x)}}$$

input
```
Integrate[Sinh[x]/Sqrt[a + b*Sinh[x]],x]
```

output
```
(2*((I*a + b)*EllipticE[(Pi - (2*I)*x)/4, ((-2*I)*b)/(a - I*b)] - I*a*Elli
pticF[(Pi - (2*I)*x)/4, ((-2*I)*b)/(a - I*b)])*Sqrt[(a + b*Sinh[x])/(a - I
*b)])/(b*Sqrt[a + b*Sinh[x]])
```

Rubi [A] (verified)

Time = 0.56 (sec) , antiderivative size = 128, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.769$, Rules used = {3042, 26, 3231, 3042, 3134, 3042, 3132, 3142, 3042, 3140}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sinh(x)}{\sqrt{a+b\sinh(x)}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int -\frac{i\sin(ix)}{\sqrt{a-ib\sin(ix)}}dx$$

$$\Big\downarrow \text{26}$$

$$-i\int \frac{\sin(ix)}{\sqrt{a-ib\sin(ix)}}dx$$

$$\Big\downarrow \text{3231}$$

$$-i\left(\frac{i\int \sqrt{a+b\sinh(x)}dx}{b}-\frac{ia\int \frac{1}{\sqrt{a+b\sinh(x)}}dx}{b}\right)$$

$$\Big\downarrow \text{3042}$$

$$-i\left(\frac{i\int \sqrt{a-ib\sin(ix)}dx}{b}-\frac{ia\int \frac{1}{\sqrt{a-ib\sin(ix)}}dx}{b}\right)$$

$$\Big\downarrow \text{3134}$$

$$-i\left(\frac{i\sqrt{a+b\sinh(x)}\int\sqrt{\frac{a}{a-ib}+\frac{b\sinh(x)}{a-ib}}dx}{b\sqrt{\frac{a+b\sinh(x)}{a-ib}}}-\frac{ia\int\frac{1}{\sqrt{a-ib\sin(ix)}}dx}{b}\right)$$

$$\Big\downarrow \text{3042}$$

$$-i\left(\frac{i\sqrt{a+b\sinh(x)}\int\sqrt{\frac{a}{a-ib}-\frac{ib\sin(ix)}{a-ib}}dx}{b\sqrt{\frac{a+b\sinh(x)}{a-ib}}}-\frac{ia\int\frac{1}{\sqrt{a-ib\sin(ix)}}dx}{b}\right)$$

$$\Big\downarrow \text{3132}$$

$$-i\left(-\frac{ia\int\frac{1}{\sqrt{a-ib\sin(ix)}}dx}{b}-\frac{2\sqrt{a+b\sinh(x)}E\left(\frac{\pi}{4}-\frac{ix}{2}|\frac{2b}{ia+b}\right)}{b\sqrt{\frac{a+b\sinh(x)}{a-ib}}}\right)$$

$$\Big\downarrow \text{3142}$$

$$-i\left(-\frac{ia\sqrt{\frac{a+b\sinh(x)}{a-ib}}\int\frac{1}{\sqrt{\frac{a}{a-ib}+\frac{b\sinh(x)}{a-ib}}}dx}{b\sqrt{a+b\sinh(x)}}-\frac{2\sqrt{a+b\sinh(x)}E\left(\frac{\pi}{4}-\frac{ix}{2}|\frac{2b}{ia+b}\right)}{b\sqrt{\frac{a+b\sinh(x)}{a-ib}}}\right)$$

$$\Big\downarrow \text{3042}$$

$$-i\left(-\frac{ia\sqrt{\frac{a+b\sinh(x)}{a-ib}}\int\frac{1}{\sqrt{\frac{a}{a-ib}-\frac{ib\sin(ix)}{a-ib}}}dx}{b\sqrt{a+b\sinh(x)}}-\frac{2\sqrt{a+b\sinh(x)}E\left(\frac{\pi}{4}-\frac{ix}{2}|\frac{2b}{ia+b}\right)}{b\sqrt{\frac{a+b\sinh(x)}{a-ib}}}\right)$$

$$\Big\downarrow \text{3140}$$

$$-i\left(\frac{2a\sqrt{\frac{a+b\sinh(x)}{a-ib}}\,\text{EllipticF}\left(\frac{\pi}{4}-\frac{ix}{2},\frac{2b}{ia+b}\right)}{b\sqrt{a+b\sinh(x)}}-\frac{2\sqrt{a+b\sinh(x)}E\left(\frac{\pi}{4}-\frac{ix}{2}|\frac{2b}{ia+b}\right)}{b\sqrt{\frac{a+b\sinh(x)}{a-ib}}}\right)$$

input

```
Int[Sinh[x]/Sqrt[a + b*Sinh[x]],x]
```

output

```
(-I)*((-2*EllipticE[Pi/4 - (I/2)*x, (2*b)/(I*a + b)]*Sqrt[a + b*Sinh[x]])/
(b*Sqrt[(a + b*Sinh[x])/(a - I*b)]) + (2*a*EllipticF[Pi/4 - (I/2)*x, (2*b)
/(I*a + b)]*Sqrt[(a + b*Sinh[x])/(a - I*b)])/(b*Sqrt[a + b*Sinh[x]]))
```

Defintions of rubi rules used

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3132
```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[2*(Sqrt[a
 + b]/d)*EllipticE[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

rule 3134
```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)]   Int[Sqrt[a/(a + b) + (
b/(a + b))*Sin[c + d*x]], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2
, 0] &&  !GtQ[a + b, 0]
```

rule 3140
```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/(d*S
qrt[a + b]))*EllipticF[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

rule 3142
```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]]   Int[1/Sqrt[a/(a + b)
 + (b/(a + b))*Sin[c + d*x]], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
 b^2, 0] &&  !GtQ[a + b, 0]
```

rule 3231
```
Int[((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])/Sqrt[(a_) + (b_.)*sin[(e_.) + (
f_.)*(x_)]], x_Symbol] :> Simp[(b*c - a*d)/b   Int[1/Sqrt[a + b*Sin[e + f*x
]], x], x] + Simp[d/b   Int[Sqrt[a + b*Sin[e + f*x]], x], x] /; FreeQ[{a, b
, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
```

Maple [A] (verified)

Time = 0.40 (sec) , antiderivative size = 218, normalized size of antiderivative = 1.70

method	result
default	$\frac{2(ib-a)\sqrt{-\frac{a+b\sinh(x)}{ib-a}}\sqrt{\frac{(i-\sinh(x))b}{ib+a}}\sqrt{\frac{b(i+\sinh(x))}{ib-a}}\left(i\,\mathrm{EllipticE}\left(\sqrt{-\frac{a+b\sinh(x)}{ib-a}},\sqrt{-\frac{ib-a}{ib+a}}\right)b-i\,\mathrm{EllipticF}\left(\sqrt{-\frac{a+b\sinh(x)}{ib-a}},\sqrt{-\frac{ib}{ib}}\right.\right.}{b^2\cosh(x)\sqrt{a+b\sinh(x)}}$
risch	$\frac{(b\,e^{2x}+2\,e^{x}a-b)\sqrt{2}\,e^{-x}}{b\sqrt{(b\,e^{2x}+2\,e^{x}a-b)e^{-x}}}+\frac{\left(-\frac{4(b\,e^{2x}+2\,e^{x}a-b)}{b\sqrt{(b\,e^{2x}+2\,e^{x}a-b)e^{x}}}+\frac{4\left(a+\sqrt{a^2+b^2}\right)\sqrt{\frac{\left(e^{x}+\frac{a+\sqrt{a^2+b^2}}{b}\right)b}{a+\sqrt{a^2+b^2}}}\sqrt{\frac{e^{x}-\frac{-a+\sqrt{a^2+b^2}}{b}}{-\frac{a+\sqrt{a^2+b^2}}{b}-\frac{-a+\sqrt{a^2+b^2}}{b}}}\sqrt{-\frac{e}{a+\sqrt{}}}}{}\right.}{}$

input
```
int(sinh(x)/(a+b*sinh(x))^(1/2),x,method=_RETURNVERBOSE)
```

output
```
2*(I*b-a)*(-(a+b*sinh(x))/(I*b-a))^(1/2)*((I-sinh(x))*b/(I*b+a))^(1/2)*(b*
(I+sinh(x))/(I*b-a))^(1/2)*(I*EllipticE((-(a+b*sinh(x))/(I*b-a))^(1/2),(-(
I*b-a)/(I*b+a))^(1/2))*b-I*EllipticF((-(a+b*sinh(x))/(I*b-a))^(1/2),(-(I*b
-a)/(I*b+a))^(1/2))*b+EllipticE((-(a+b*sinh(x))/(I*b-a))^(1/2),(-(I*b-a)/(
I*b+a))^(1/2))*a)/b^2/cosh(x)/(a+b*sinh(x))^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 174, normalized size of antiderivative = 1.36

$$\int \frac{\sinh(x)}{\sqrt{a+b\sinh(x)}}\,dx = -\frac{2\left(4\sqrt{\frac{1}{2}}a\sqrt{b}\,\mathrm{weierstrassPInverse}\left(\frac{4\,(4\,a^2+3\,b^2)}{3\,b^2},-\frac{8\,(8\,a^3+9\,ab^2)}{27\,b^3},\frac{3\,b\cosh(x)+3\,b\sinh(x)+2\,a}{3\,b}\right)+6\sqrt{\frac{1}{2}}b^{\frac{3}{2}}\mathrm{weierstras}\right.}{}$$

input
```
integrate(sinh(x)/(a+b*sinh(x))^(1/2),x, algorithm="fricas")
```

output
```
-2/3*(4*sqrt(1/2)*a*sqrt(b)*weierstrassPInverse(4/3*(4*a^2 + 3*b^2)/b^2, -
8/27*(8*a^3 + 9*a*b^2)/b^3, 1/3*(3*b*cosh(x) + 3*b*sinh(x) + 2*a)/b) + 6*s
qrt(1/2)*b^(3/2)*weierstrassZeta(4/3*(4*a^2 + 3*b^2)/b^2, -8/27*(8*a^3 + 9
*a*b^2)/b^3, weierstrassPInverse(4/3*(4*a^2 + 3*b^2)/b^2, -8/27*(8*a^3 + 9
*a*b^2)/b^3, 1/3*(3*b*cosh(x) + 3*b*sinh(x) + 2*a)/b)) + 3*sqrt(b*sinh(x)
+ a)*b)/b^2
```

Sympy [F]

$$\int \frac{\sinh(x)}{\sqrt{a+b\sinh(x)}}\,dx=\int \frac{\sinh{(x)}}{\sqrt{a+b\sinh{(x)}}}\,dx$$

input
```
integrate(sinh(x)/(a+b*sinh(x))**(1/2),x)
```

output
```
Integral(sinh(x)/sqrt(a + b*sinh(x)), x)
```

Maxima [F]

$$\int \frac{\sinh(x)}{\sqrt{a+b\sinh(x)}}\,dx=\int \frac{\sinh{(x)}}{\sqrt{b\sinh{(x)}+a}}\,dx$$

input
```
integrate(sinh(x)/(a+b*sinh(x))^(1/2),x, algorithm="maxima")
```

output
```
integrate(sinh(x)/sqrt(b*sinh(x) + a), x)
```

Giac [F]

$$\int \frac{\sinh(x)}{\sqrt{a+b\sinh(x)}}\,dx = \int \frac{\sinh(x)}{\sqrt{b\sinh(x)+a}}\,dx$$

input
```
integrate(sinh(x)/(a+b*sinh(x))^(1/2),x, algorithm="giac")
```

output
```
integrate(sinh(x)/sqrt(b*sinh(x) + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sinh(x)}{\sqrt{a+b\sinh(x)}}\,dx = \int \frac{\sinh(x)}{\sqrt{a+b\sinh(x)}}\,dx$$

input
```
int(sinh(x)/(a + b*sinh(x))^(1/2),x)
```

output
```
int(sinh(x)/(a + b*sinh(x))^(1/2), x)
```

Reduce [F]

$$\int \frac{\sinh(x)}{\sqrt{a+b\sinh(x)}}\,dx = \int \frac{\sqrt{\sinh(x)\,b+a}\,\sinh(x)}{\sinh(x)\,b+a}\,dx$$

input
```
int(sinh(x)/(a+b*sinh(x))^(1/2),x)
```

output
```
int((sqrt(sinh(x)*b + a)*sinh(x))/(sinh(x)*b + a),x)
```

3.112 $\int (a+ia\sinh(x))^{5/2}(A+B\sinh(x))\,dx$

Optimal result

Integrand size = 20, antiderivative size = 112

$$\int (a+ia\sinh(x))^{5/2}(A+B\sinh(x))\,dx = \frac{64a^3(7iA+5B)\cosh(x)}{105\sqrt{a+ia\sinh(x)}} + \frac{16}{105}a^2(7iA+5B)\cosh(x)\sqrt{a+ia\sinh(x)} + \frac{2}{35}a(7iA+5B)\cosh(x)(a+ia\sinh(x))^{3/2} + \frac{2}{7}B\cosh(x)(a+ia\sinh(x))^{5/2}$$

output
```
64/105*a^3*(7*I*A+5*B)*cosh(x)/(a+I*a*sinh(x))^(1/2)+16/105*a^2*(7*I*A+5*B
)*cosh(x)*(a+I*a*sinh(x))^(1/2)+2/35*a*(7*I*A+5*B)*cosh(x)*(a+I*a*sinh(x))
^(3/2)+2/7*B*cosh(x)*(a+I*a*sinh(x))^(5/2)
```

Mathematica [A] (verified)

Time = 2.68 (sec) , antiderivative size = 100, normalized size of antiderivative = 0.89

$$\int (a+ia\sinh(x))^{5/2}(A+B\sinh(x))\,dx = \frac{a^2\left(\cosh\left(\frac{x}{2}\right)-i\sinh\left(\frac{x}{2}\right)\right)\sqrt{a+ia\sinh(x)}(1246iA+1040B+(-42iA-120B)\cosh(2x}{210\left(\cosh\left(\frac{x}{2}\right)+i\sinh\left(\frac{x}{2}\right)\right)}$$

input
```
Integrate[(a + I*a*Sinh[x])^(5/2)*(A + B*Sinh[x]),x]
```

output
```
(a^2*(Cosh[x/2] - I*Sinh[x/2])*Sqrt[a + I*a*Sinh[x]]*((1246*I)*A + 1040*B
+ ((-42*I)*A - 120*B)*Cosh[2*x] + (-392*A + (505*I)*B)*Sinh[x] - (15*I)*B*
Sinh[3*x]))/(210*(Cosh[x/2] + I*Sinh[x/2]))
```

Rubi [A] (verified)

Time = 0.47 (sec) , antiderivative size = 109, normalized size of antiderivative = 0.97, number of steps used = 8, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.400$, Rules used = {3042, 3230, 3042, 3126, 3042, 3126, 3042, 3125}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a+ia\sinh(x))^{5/2}(A+B\sinh(x))\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int (a+a\sin(ix))^{5/2}(A-iB\sin(ix))dx$$

$$\Big\downarrow \text{3230}$$

$$\frac{1}{7}(7A-5iB)\int (i\sinh(x)a+a)^{5/2}dx+\frac{2}{7}B\cosh(x)(a+ia\sinh(x))^{5/2}$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{7}(7A-5iB)\int (\sin(ix)a+a)^{5/2}dx+\frac{2}{7}B\cosh(x)(a+ia\sinh(x))^{5/2}$$

$$\Big\downarrow \text{3126}$$

$$\frac{1}{7}(7A-5iB)\left(\frac{8}{5}a\int (i\sinh(x)a+a)^{3/2}dx+\frac{2}{5}ia\cosh(x)(a+ia\sinh(x))^{3/2}\right)+\frac{2}{7}B\cosh(x)(a+ia\sinh(x))^{5/2}$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{7}(7A-5iB)\left(\frac{8}{5}a\int (\sin(ix)a+a)^{3/2}dx+\frac{2}{5}ia\cosh(x)(a+ia\sinh(x))^{3/2}\right)+\frac{2}{7}B\cosh(x)(a+ia\sinh(x))^{5/2}$$

$\Big\downarrow$ 3126

$$\frac{1}{7}(7A-5iB)\left(\frac{8}{5}a\left(\frac{4}{3}a\int\sqrt{i\sinh(x)a+a}dx+\frac{2}{3}ia\cosh(x)\sqrt{a+ia\sinh(x)}\right)+\frac{2}{5}ia\cosh(x)(a+ia\sinh(x))^{3/2}\right)+\frac{2}{7}B\cosh(x)(a+ia\sinh(x))^{5/2}$$

$\Big\downarrow$ 3042

$$\frac{1}{7}(7A-5iB)\left(\frac{8}{5}a\left(\frac{4}{3}a\int\sqrt{\sin(ix)a+a}dx+\frac{2}{3}ia\cosh(x)\sqrt{a+ia\sinh(x)}\right)+\frac{2}{5}ia\cosh(x)(a+ia\sinh(x))^{3/2}\right)+\frac{2}{7}B\cosh(x)(a+ia\sinh(x))^{5/2}$$

$\Big\downarrow$ 3125

$$\frac{1}{7}(7A-5iB)\left(\frac{8}{5}a\left(\frac{8ia^2\cosh(x)}{3\sqrt{a+ia\sinh(x)}}+\frac{2}{3}ia\cosh(x)\sqrt{a+ia\sinh(x)}\right)+\frac{2}{5}ia\cosh(x)(a+ia\sinh(x))^{3/2}\right)+\frac{2}{7}B\cosh(x)(a+ia\sinh(x))^{5/2}$$

input
```
Int[(a + I*a*Sinh[x])^(5/2)*(A + B*Sinh[x]),x]
```

output
```
(2*B*Cosh[x]*(a + I*a*Sinh[x])^(5/2))/7 + ((7*A - (5*I)*B)*(((2*I)/5)*a*Co
sh[x]*(a + I*a*Sinh[x])^(3/2) + (8*a*((((8*I)/3)*a^2*Cosh[x])/Sqrt[a + I*a
*Sinh[x]] + ((2*I)/3)*a*Cosh[x]*Sqrt[a + I*a*Sinh[x]]))/5))/7
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3125
```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[-2*b*(Cos
[c + d*x]/(d*Sqrt[a + b*Sin[c + d*x]])), x] /; FreeQ[{a, b, c, d}, x] && Eq
Q[a^2 - b^2, 0]
```

rule 3126
```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos
[c + d*x]*((a + b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Simp[a*((2*n - 1)/n)
 Int[(a + b*Sin[c + d*x])^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] && EqQ[
a^2 - b^2, 0] && IGtQ[n - 1/2, 0]
```

rule 3230
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Simp[(-d)*Cos[e + f*x]*((a + b*Sin[e + f*x])^m/(
f*(m + 1))), x] + Simp[(a*d*m + b*c*(m + 1))/(b*(m + 1))   Int[(a + b*Sin[e
 + f*x])^m, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b*c - a*d, 0]
&& EqQ[a^2 - b^2, 0] &&  !LtQ[m, -2^(-1)]
```

Maple [F]

$$\int (a+ia\sinh(x))^{\frac{5}{2}} (A+B\sinh(x))\, dx$$

input
```
int((a+I*a*sinh(x))^(5/2)*(A+B*sinh(x)),x)
```

output
```
int((a+I*a*sinh(x))^(5/2)*(A+B*sinh(x)),x)
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 126, normalized size of antiderivative = 1.12

$$\int (a+ia\sinh(x))^{5/2}(A+B\sinh(x))\, dx =$$
$$-\frac{1}{420} \left(15\, Ba^2 e^{(7\,x)} + 21\,(2\,A-5i\,B)a^2 e^{(6\,x)} + 35\,(-10i\,A-11\,B)a^2 e^{(5\,x)} - 525\,(4\,A-3i\,B)a^2 e^{(4\,x)} + 525\right.$$

input
```
integrate((a+I*a*sinh(x))^(5/2)*(A+B*sinh(x)),x, algorithm="fricas")
```

output
```
-1/420*(15*B*a^2*e^(7*x) + 21*(2*A - 5*I*B)*a^2*e^(6*x) + 35*(-10*I*A - 11
*B)*a^2*e^(5*x) - 525*(4*A - 3*I*B)*a^2*e^(4*x) + 525*(-4*I*A - 3*B)*a^2*e
^(3*x) - 35*(10*A - 11*I*B)*a^2*e^(2*x) + 21*(2*I*A + 5*B)*a^2*e^x - 15*I*
B*a^2)*sqrt(1/2*I*a*e^(-x))*e^(-3*x)
```

Sympy [F(-1)]

Timed out.

$$\int (a+ia\sinh(x))^{5/2}(A+B\sinh(x))\,dx = \text{Timed out}$$

input
```
integrate((a+I*a*sinh(x))**(5/2)*(A+B*sinh(x)),x)
```

output
```
Timed out
```

Maxima [F]

$$\int (a+ia\sinh(x))^{5/2}(A+B\sinh(x))\,dx = \int (B\sinh(x)+A)(i\,a\sinh(x)+a)^{\frac{5}{2}}\,dx$$

input
```
integrate((a+I*a*sinh(x))^(5/2)*(A+B*sinh(x)),x, algorithm="maxima")
```

output
```
integrate((B*sinh(x) + A)*(I*a*sinh(x) + a)^(5/2), x)
```

Giac [F]

$$\int (a+ia\sinh(x))^{5/2}(A+B\sinh(x))\,dx = \int (B\sinh(x)+A)(i\,a\sinh(x)+a)^{\frac{5}{2}}\,dx$$

input
```
integrate((a+I*a*sinh(x))^(5/2)*(A+B*sinh(x)),x, algorithm="giac")
```

output
```
integrate((B*sinh(x) + A)*(I*a*sinh(x) + a)^(5/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int (a+ia\sinh(x))^{5/2}(A+B\sinh(x))\,dx=\int (A+B\sinh(x))\,\left(a+a\sinh(x)\,1\mathrm{i}\right)^{5/2}\,dx$$

input
```
int((A + B*sinh(x))*(a + a*sinh(x)*1i)^(5/2),x)
```

output
```
int((A + B*sinh(x))*(a + a*sinh(x)*1i)^(5/2), x)
```

Reduce [F]

$$\int (a+ia\sinh(x))^{5/2}(A+B\sinh(x))\,dx=\sqrt{a}\,a^2\Bigg(\left(\int\sqrt{\sinh(x)\,i+1}dx\right)a-\left(\int\sqrt{\sinh(x)\,i+1}\,\sinh(x)^3\,dx\right)b-\left(\int\sqrt{\sinh(x)\,i+1}\,\sinh(x)^2\,dx\right)a+2\left(\int\sqrt{\sinh(x)\,i+1}\,\sinh(x)^2\,dx\right)bi+2\left(\int\sqrt{\sinh(x)\,i+1}\,\sinh(x)\,dx\right)ai+\left(\int\sqrt{\sinh(x)\,i+1}\,\sinh(x)\,dx\right)b\Bigg)$$

input
```
int((a+I*a*sinh(x))^(5/2)*(A+B*sinh(x)),x)
```

output
```
sqrt(a)*a**2*(int(sqrt(sinh(x)*i + 1),x)*a - int(sqrt(sinh(x)*i + 1)*sinh(
x)**3,x)*b - int(sqrt(sinh(x)*i + 1)*sinh(x)**2,x)*a + 2*int(sqrt(sinh(x)*
i + 1)*sinh(x)**2,x)*b*i + 2*int(sqrt(sinh(x)*i + 1)*sinh(x),x)*a*i + int(
sqrt(sinh(x)*i + 1)*sinh(x),x)*b)
```

3.113 $\int (a+ia\sinh(x))^{3/2}(A+B\sinh(x))\,dx$

Optimal result

Integrand size = 20, antiderivative size = 81

$$\int (a+ia\sinh(x))^{3/2}(A+B\sinh(x))\,dx=\frac{8a^2(5iA+3B)\cosh(x)}{15\sqrt{a+ia\sinh(x)}}+\frac{2}{15}a(5iA+3B)\cosh(x)\sqrt{a+ia\sinh(x)}+\frac{2}{5}B\cosh(x)(a+ia\sinh(x))^{3/2}$$

output
```
8/15*a^2*(5*I*A+3*B)*cosh(x)/(a+I*a*sinh(x))^(1/2)+2/15*a*(5*I*A+3*B)*cosh
(x)*(a+I*a*sinh(x))^(1/2)+2/5*B*cosh(x)*(a+I*a*sinh(x))^(3/2)
```

Mathematica [A] (verified)

Time = 1.06 (sec) , antiderivative size = 83, normalized size of antiderivative = 1.02

$$\int (a+ia\sinh(x))^{3/2}(A+B\sinh(x))\,dx=-\frac{a\left(\cosh\left(\frac{x}{2}\right)-i\sinh\left(\frac{x}{2}\right)\right)\sqrt{a+ia\sinh(x)}(-50iA-39B+3B\cosh(2x)+2(5A-9iB)\sinh(x))}{15\left(\cosh\left(\frac{x}{2}\right)+i\sinh\left(\frac{x}{2}\right)\right)}$$

input
```
Integrate[(a + I*a*Sinh[x])^(3/2)*(A + B*Sinh[x]),x]
```

output
```
-1/15*(a*(Cosh[x/2] - I*Sinh[x/2])*Sqrt[a + I*a*Sinh[x]]*((-50*I)*A - 39*B
 + 3*B*Cosh[2*x] + 2*(5*A - (9*I)*B)*Sinh[x]))/(Cosh[x/2] + I*Sinh[x/2])
```

Rubi [A] (verified)

Time = 0.38 (sec) , antiderivative size = 81, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.300$, Rules used = {3042, 3230, 3042, 3126, 3042, 3125}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a+ia\sinh(x))^{3/2}(A+B\sinh(x))\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int (a+a\sin(ix))^{3/2}(A-iB\sin(ix))dx$$

$$\Big\downarrow \text{3230}$$

$$\frac{1}{5}(5A-3iB)\int (i\sinh(x)a+a)^{3/2}dx+\frac{2}{5}B\cosh(x)(a+ia\sinh(x))^{3/2}$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{5}(5A-3iB)\int (\sin(ix)a+a)^{3/2}dx+\frac{2}{5}B\cosh(x)(a+ia\sinh(x))^{3/2}$$

$$\Big\downarrow \text{3126}$$

$$\frac{1}{5}(5A-3iB)\left(\frac{4}{3}a\int \sqrt{i\sinh(x)a+a}dx+\frac{2}{3}ia\cosh(x)\sqrt{a+ia\sinh(x)}\right)+\frac{2}{5}B\cosh(x)(a+ia\sinh(x))^{3/2}$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{5}(5A-3iB)\left(\frac{4}{3}a\int \sqrt{\sin(ix)a+a}dx+\frac{2}{3}ia\cosh(x)\sqrt{a+ia\sinh(x)}\right)+\frac{2}{5}B\cosh(x)(a+ia\sinh(x))^{3/2}$$

$$\Big\downarrow \text{3125}$$

$$\frac{1}{5}(5A-3iB)\left(\frac{8ia^2\cosh(x)}{3\sqrt{a+ia\sinh(x)}}+\frac{2}{3}ia\cosh(x)\sqrt{a+ia\sinh(x)}\right)+\frac{2}{5}B\cosh(x)(a+ia\sinh(x))^{3/2}$$

input
```
Int[(a + I*a*Sinh[x])^(3/2)*(A + B*Sinh[x]),x]
```

output
```
(2*B*Cosh[x]*(a + I*a*Sinh[x])^(3/2))/5 + ((5*A - (3*I)*B)*((((8*I)/3)*a^2
*Cosh[x])/Sqrt[a + I*a*Sinh[x]] + ((2*I)/3)*a*Cosh[x]*Sqrt[a + I*a*Sinh[x]
]))/5
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3125
```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[-2*b*(Cos
[c + d*x]/(d*Sqrt[a + b*Sin[c + d*x]])), x] /; FreeQ[{a, b, c, d}, x] && Eq
Q[a^2 - b^2, 0]
```

rule 3126
```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos
[c + d*x]*((a + b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Simp[a*((2*n - 1)/n)
 Int[(a + b*Sin[c + d*x])^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] && EqQ[
a^2 - b^2, 0] && IGtQ[n - 1/2, 0]
```

rule 3230
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Simp[(-d)*Cos[e + f*x]*((a + b*Sin[e + f*x])^m/(
f*(m + 1))), x] + Simp[(a*d*m + b*c*(m + 1))/(b*(m + 1))   Int[(a + b*Sin[e
 + f*x])^m, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b*c - a*d, 0]
&& EqQ[a^2 - b^2, 0] &&  !LtQ[m, -2^(-1)]
```

Maple [F]

$$\int (a + ia\sinh(x))^{\frac{3}{2}} (A + B\sinh(x))\, dx$$

input
```
int((a+I*a*sinh(x))^(3/2)*(A+B*sinh(x)),x)
```

output
```
int((a+I*a*sinh(x))^(3/2)*(A+B*sinh(x)),x)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 82, normalized size of antiderivative = 1.01

$$\int (a + ia\sinh(x))^{3/2}(A + B\sinh(x))\, dx = \frac{1}{30}\left(3i\,Bae^{(5\,x)} - 5\,(-2i\,A - 3\,B)ae^{(4\,x)} + 30\,(3\,A - 2i\,B)ae^{(3\,x)} - 30\,(-3i\,A - 2\,B)ae^{(2\,x}\right.$$

input
```
integrate((a+I*a*sinh(x))^(3/2)*(A+B*sinh(x)),x, algorithm="fricas")
```

output
```
1/30*(3*I*B*a*e^(5*x) - 5*(-2*I*A - 3*B)*a*e^(4*x) + 30*(3*A - 2*I*B)*a*e^
(3*x) - 30*(-3*I*A - 2*B)*a*e^(2*x) + 5*(2*A - 3*I*B)*a*e^x - 3*B*a)*sqrt(
1/2*I*a*e^(-x))*e^(-2*x)
```

Sympy [F]

$$\int (a + ia\sinh(x))^{3/2}(A + B\sinh(x))\, dx = \int (ia(\sinh(x) - i))^{\frac{3}{2}} (A + B\sinh(x))\, dx$$

input
```
integrate((a+I*a*sinh(x))**(3/2)*(A+B*sinh(x)),x)
```

output
```
Integral((I*a*(sinh(x) - I))**(3/2)*(A + B*sinh(x)), x)
```

Maxima [F]

$$\int (a+ia\sinh(x))^{3/2}(A+B\sinh(x))\,dx = \int (B\sinh{(x)}+A)(i\,a\sinh{(x)}+a)^{\frac{3}{2}}\,dx$$

input
```
integrate((a+I*a*sinh(x))^(3/2)*(A+B*sinh(x)),x, algorithm="maxima")
```

output
```
integrate((B*sinh(x) + A)*(I*a*sinh(x) + a)^(3/2), x)
```

Giac [F]

$$\int (a+ia\sinh(x))^{3/2}(A+B\sinh(x))\,dx = \int (B\sinh{(x)}+A)(i\,a\sinh{(x)}+a)^{\frac{3}{2}}\,dx$$

input
```
integrate((a+I*a*sinh(x))^(3/2)*(A+B*sinh(x)),x, algorithm="giac")
```

output
```
integrate((B*sinh(x) + A)*(I*a*sinh(x) + a)^(3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int (a+ia\sinh(x))^{3/2}(A+B\sinh(x))\,dx = \int (A+B\sinh(x))\,\left(a+a\sinh(x)\,1\mathrm{i}\right)^{3/2}\,dx$$

input
```
int((A + B*sinh(x))*(a + a*sinh(x)*1i)^(3/2),x)
```

output
```
int((A + B*sinh(x))*(a + a*sinh(x)*1i)^(3/2), x)
```

Reduce [F]

$$\int (a+ia\sinh(x))^{3/2}(A+B\sinh(x))\,dx = \sqrt{a}\,a\left(\left(\int \sqrt{\sinh(x)\,i+1}dx\right)a + \left(\int \sqrt{\sinh(x)\,i+1}\,\sinh(x)^2\,dx\right)bi + \left(\int \sqrt{\sinh(x)\,i+1}\,\sinh(x)\,dx\right)ai + \left(\int \sqrt{\sinh(x)\,i+1}\,\sinh(x)\,dx\right)b\right)$$

input
```
int((a+I*a*sinh(x))^(3/2)*(A+B*sinh(x)),x)
```

output
```
sqrt(a)*a*(int(sqrt(sinh(x)*i + 1),x)*a + int(sqrt(sinh(x)*i + 1)*sinh(x)*
*2,x)*b*i + int(sqrt(sinh(x)*i + 1)*sinh(x),x)*a*i + int(sqrt(sinh(x)*i +
1)*sinh(x),x)*b)
```

3.114 $\int \sqrt{a+ia\sinh(x)}(A+B\sinh(x))\,dx$

Optimal result

Integrand size = 20, antiderivative size = 48

$$\int \sqrt{a+ia\sinh(x)}(A+B\sinh(x))\,dx = \frac{2a(3iA+B)\cosh(x)}{3\sqrt{a+ia\sinh(x)}} + \frac{2}{3}B\cosh(x)\sqrt{a+ia\sinh(x)}$$

output
```
2/3*a*(3*I*A+B)*cosh(x)/(a+I*a*sinh(x))^(1/2)+2/3*B*cosh(x)*(a+I*a*sinh(x)
)^(1/2)
```

Mathematica [A] (verified)

Time = 0.14 (sec) , antiderivative size = 66, normalized size of antiderivative = 1.38

$$\int \sqrt{a+ia\sinh(x)}(A+B\sinh(x))\,dx = \frac{2\left(i\cosh\left(\frac{x}{2}\right)+\sinh\left(\frac{x}{2}\right)\right)\sqrt{a+ia\sinh(x)}(3A-2iB+B\sinh(x))}{3\left(\cosh\left(\frac{x}{2}\right)+i\sinh\left(\frac{x}{2}\right)\right)}$$

input
```
Integrate[Sqrt[a + I*a*Sinh[x]]*(A + B*Sinh[x]),x]
```

output
```
(2*(I*Cosh[x/2] + Sinh[x/2])*Sqrt[a + I*a*Sinh[x]]*(3*A - (2*I)*B + B*Sinh
[x]))/(3*(Cosh[x/2] + I*Sinh[x/2]))
```

Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 52, normalized size of antiderivative = 1.08, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {3042, 3230, 3042, 3125}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{a+ia\sinh(x)}(A+B\sinh(x))\,dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \sqrt{a+a\sin(ix)}(A-iB\sin(ix))dx$$

$$\Big\downarrow \text{ 3230}$$

$$\frac{1}{3}(3A-iB)\int \sqrt{i\sinh(x)a+a}dx+\frac{2}{3}B\cosh(x)\sqrt{a+ia\sinh(x)}$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{1}{3}(3A-iB)\int \sqrt{\sin(ix)a+a}dx+\frac{2}{3}B\cosh(x)\sqrt{a+ia\sinh(x)}$$

$$\Big\downarrow \text{ 3125}$$

$$\frac{2ia(3A-iB)\cosh(x)}{3\sqrt{a+ia\sinh(x)}}+\frac{2}{3}B\cosh(x)\sqrt{a+ia\sinh(x)}$$

input
```
Int[Sqrt[a + I*a*Sinh[x]]*(A + B*Sinh[x]),x]
```

output
```
(((2*I)/3)*a*(3*A - I*B)*Cosh[x])/Sqrt[a + I*a*Sinh[x]] + (2*B*Cosh[x]*Sqr
t[a + I*a*Sinh[x]])/3
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3125
```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[-2*b*(Cos
[c + d*x]/(d*Sqrt[a + b*Sin[c + d*x]])), x] /; FreeQ[{a, b, c, d}, x] && Eq
Q[a^2 - b^2, 0]
```

rule 3230
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Simp[(-d)*Cos[e + f*x]*((a + b*Sin[e + f*x])^m/(
f*(m + 1))), x] + Simp[(a*d*m + b*c*(m + 1))/(b*(m + 1))   Int[(a + b*Sin[e
 + f*x])^m, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b*c - a*d, 0]
&& EqQ[a^2 - b^2, 0] &&  !LtQ[m, -2^(-1)]
```

Maple [F]

$$\int \sqrt{a + ia \sinh{(x)}}\,(A + B \sinh{(x)})\,dx$$

input
```
int((a+I*a*sinh(x))^(1/2)*(A+B*sinh(x)),x)
```

output
```
int((a+I*a*sinh(x))^(1/2)*(A+B*sinh(x)),x)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 49, normalized size of antiderivative = 1.02

$$\int \sqrt{a + ia \sinh(x)}(A + B \sinh(x))\,dx = \frac{1}{3}\left(Be^{(3x)} + 3(2A - iB)e^{(2x)} - 3(-2iA - B)e^{x} - iB\right)\sqrt{\frac{1}{2}i\,ae^{(-x)}}e^{(-x)}$$

input
```
integrate((a+I*a*sinh(x))^(1/2)*(A+B*sinh(x)),x, algorithm="fricas")
```

output
```
1/3*(B*e^(3*x) + 3*(2*A - I*B)*e^(2*x) - 3*(-2*I*A - B)*e^x - I*B)*sqrt(1/
2*I*a*e^(-x))*e^(-x)
```

Sympy [F]

$$\int \sqrt{a + ia\sinh(x)}(A + B\sinh(x))\,dx = \int \sqrt{ia\,(\sinh{(x)} - i)}(A + B\sinh{(x)})\,\,dx$$

input
```
integrate((a+I*a*sinh(x))**(1/2)*(A+B*sinh(x)),x)
```

output
```
Integral(sqrt(I*a*(sinh(x) - I))*(A + B*sinh(x)), x)
```

Maxima [F]

$$\int \sqrt{a + ia\sinh(x)}(A + B\sinh(x))\,dx = \int (B\sinh{(x)} + A)\sqrt{i\,a\sinh{(x)} + a}\,dx$$

input
```
integrate((a+I*a*sinh(x))^(1/2)*(A+B*sinh(x)),x, algorithm="maxima")
```

output
```
integrate((B*sinh(x) + A)*sqrt(I*a*sinh(x) + a), x)
```

Giac [F]

$$\int \sqrt{a + ia\sinh(x)}(A + B\sinh(x))\,dx = \int (B\sinh{(x)} + A)\sqrt{i\,a\sinh{(x)} + a}\,dx$$

input
```
integrate((a+I*a*sinh(x))^(1/2)*(A+B*sinh(x)),x, algorithm="giac")
```

output
```
integrate((B*sinh(x) + A)*sqrt(I*a*sinh(x) + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int \sqrt{a+ia\sinh(x)}(A+B\sinh(x))\,dx=\int (A+B\sinh(x))\ \sqrt{a+a\sinh(x)\ 1\mathrm{i}}\,dx$$

input
```
int((A + B*sinh(x))*(a + a*sinh(x)*1i)^(1/2),x)
```

output
```
int((A + B*sinh(x))*(a + a*sinh(x)*1i)^(1/2), x)
```

Reduce [F]

$$\int \sqrt{a+ia\sinh(x)}(A+B\sinh(x))\,dx=\sqrt{a}\left(\left(\int \sqrt{\sinh(x)\,i+1}dx\right)a+\left(\int \sqrt{\sinh(x)\,i+1}\ \sinh(x)\,dx\right)b\right)$$

input
```
int((a+I*a*sinh(x))^(1/2)*(A+B*sinh(x)),x)
```

output
```
sqrt(a)*(int(sqrt(sinh(x)*i + 1),x)*a + int(sqrt(sinh(x)*i + 1)*sinh(x),x)
*b)
```

3.115 $\int \frac{A+B\sinh(x)}{i+\sinh(x)}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 23

$$\int \frac{A+B\sinh(x)}{i+\sinh(x)}\,dx = Bx-\frac{(iA+B)\cosh(x)}{i+\sinh(x)}$$

output
```
B*x-(I*A+B)*cosh(x)/(I+sinh(x))
```

Mathematica [B] (verified)

Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 53 vs. 2(23) = 46.

Time = 0.18 (sec) , antiderivative size = 53, normalized size of antiderivative = 2.30

$$\int \frac{A+B\sinh(x)}{i+\sinh(x)}\,dx = \cosh(x)\left(\frac{2iB\arcsin\left(\frac{\sqrt{1-i\sinh(x)}}{\sqrt{2}}\right)}{\sqrt{\cosh^2(x)}}-\frac{iA+B}{i+\sinh(x)}\right)$$

input
```
Integrate[(A + B*Sinh[x])/(I + Sinh[x]),x]
```

output
```
Cosh[x]*(((2*I)*B*ArcSin[Sqrt[1 - I*Sinh[x]]/Sqrt[2]])/Sqrt[Cosh[x]^2] - (
I*A + B)/(I + Sinh[x]))
```

Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.09, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.267$, Rules used = {3042, 3214, 3042, 3127}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+B\sinh(x)}{\sinh(x)+i}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{A-iB\sin(ix)}{i-i\sin(ix)}dx$$

$$\Big\downarrow \text{3214}$$

$$Bx+(A-iB)\int \frac{1}{\sinh(x)+i}dx$$

$$\Big\downarrow \text{3042}$$

$$Bx+(A-iB)\int \frac{1}{i-i\sin(ix)}dx$$

$$\Big\downarrow \text{3127}$$

$$Bx-\frac{i(A-iB)\cosh(x)}{\sinh(x)+i}$$

input
```
Int[(A + B*Sinh[x])/(I + Sinh[x]),x]
```

output
```
B*x - (I*(A - I*B)*Cosh[x])/(I + Sinh[x])
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3127
```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> Simp[-Cos[c +
 d*x]/(d*(b + a*Sin[c + d*x])), x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b
^2, 0]
```

rule 3214
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])/((c_.) + (d_.)*sin[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[b*(x/d), x] - Simp[(b*c - a*d)/d   Int[1/(c + d
*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Maple [A] (verified)

Time = 0.22 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.13

method	result	size
risch	$Bx-\frac{2A}{{\mathrm e}^{x}+i}+\frac{2iB}{{\mathrm e}^{x}+i}$	26
parallelrisch	$\frac{iBx+\tanh\left(\frac{x}{2}\right)xB-2iA-2B}{\tanh\left(\frac{x}{2}\right)+i}$	31
default	$-B\ln\left(\tanh\left(\frac{x}{2}\right)-1\right)+B\ln\left(\tanh\left(\frac{x}{2}\right)+1\right)-\frac{2i(-iB+A)}{\tanh\left(\frac{x}{2}\right)+i}$	39

input
```
int((A+B*sinh(x))/(I+sinh(x)),x,method=_RETURNVERBOSE)
```

output
```
B*x-2/(exp(x)+I)*A+2*I/(exp(x)+I)*B
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00

$$\int \frac{A + B\sinh(x)}{i + \sinh(x)}\, dx = \frac{Bxe^x + i\,Bx - 2\,A + 2i\,B}{e^x + i}$$

input
```
integrate((A+B*sinh(x))/(I+sinh(x)),x, algorithm="fricas")
```

output
```
(B*x*e^x + I*B*x - 2*A + 2*I*B)/(e^x + I)
```

Sympy [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.65

$$\int \frac{A + B\sinh(x)}{i + \sinh(x)}\, dx = Bx + \frac{-2A + 2iB}{e^x + i}$$

input
```
integrate((A+B*sinh(x))/(I+sinh(x)),x)
```

output
```
B*x + (-2*A + 2*I*B)/(exp(x) + I)
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.13

$$\int \frac{A + B\sinh(x)}{i + \sinh(x)}\, dx = B\left(x + \frac{2i}{e^{(-x)} - i}\right) - \frac{2\,A}{e^{(-x)} - i}$$

input
```
integrate((A+B*sinh(x))/(I+sinh(x)),x, algorithm="maxima")
```

output
```
B*(x + 2*I/(e^(-x) - I)) - 2*A/(e^(-x) - I)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.74

$$\int \frac{A+B\sinh(x)}{i+\sinh(x)}\,dx = Bx - \frac{2\,(A-i\,B)}{e^x+i}$$

input
```
integrate((A+B*sinh(x))/(I+sinh(x)),x, algorithm="giac")
```

output
```
B*x - 2*(A - I*B)/(e^x + I)
```

Mupad [B] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.91

$$\int \frac{A+B\sinh(x)}{i+\sinh(x)}\,dx = B\,x - \frac{2\,A-B\,2\mathrm{i}}{\mathrm{e}^x+1\mathrm{i}}$$

input
```
int((A + B*sinh(x))/(sinh(x) + 1i),x)
```

output
```
B*x - (2*A - B*2i)/(exp(x) + 1i)
```

Reduce [F]

$$\int \frac{A+B\sinh(x)}{i+\sinh(x)}\,dx = \left(\int \frac{1}{\sinh(x)+i}dx\right)a - \left(\int \frac{1}{\sinh(x)+i}dx\right)bi + bx$$

input
```
int((A+B*sinh(x))/(I+sinh(x)),x)
```

output
```
int(1/(sinh(x) + i),x)*a - int(1/(sinh(x) + i),x)*b*i + b*x
```

3.116 $\int \frac{A+B\sinh(x)}{(i+\sinh(x))^2}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 43

$$\int \frac{A+B\sinh(x)}{(i+\sinh(x))^2}\,dx = -\frac{(iA+B)\cosh(x)}{3(i+\sinh(x))^2} - \frac{(A+2iB)\cosh(x)}{3(i+\sinh(x))}$$

output
```
-1/3*(I*A+B)*cosh(x)/(I+sinh(x))^2-(A+2*I*B)*cosh(x)/(3*I+3*sinh(x))
```

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.74

$$\int \frac{A+B\sinh(x)}{(i+\sinh(x))^2}\,dx = \frac{\cosh(x)(-2iA+B-(A+2iB)\sinh(x))}{3(i+\sinh(x))^2}$$

input
```
Integrate[(A + B*Sinh[x])/(I + Sinh[x])^2,x]
```

output
```
(Cosh[x]*((-2*I)*A + B - (A + (2*I)*B)*Sinh[x]))/(3*(I + Sinh[x])^2)
```

Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 47, normalized size of antiderivative = 1.09, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.267$, Rules used = {3042, 3229, 3042, 3127}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+B\sinh(x)}{(\sinh(x)+i)^2}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{A-iB\sin(ix)}{(i-i\sin(ix))^2}dx$$

$$\Big\downarrow \text{3229}$$

$$-\frac{1}{3}(-2B+iA)\int \frac{1}{\sinh(x)+i}dx-\frac{(B+iA)\cosh(x)}{3(\sinh(x)+i)^2}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{1}{3}(-2B+iA)\int \frac{1}{i-i\sin(ix)}dx-\frac{(B+iA)\cosh(x)}{3(\sinh(x)+i)^2}$$

$$\Big\downarrow \text{3127}$$

$$\frac{i(-2B+iA)\cosh(x)}{3(\sinh(x)+i)}-\frac{(B+iA)\cosh(x)}{3(\sinh(x)+i)^2}$$

input
```
Int[(A + B*Sinh[x])/(I + Sinh[x])^2,x]
```

output
```
-1/3*((I*A + B)*Cosh[x])/(I + Sinh[x])^2 + ((I/3)*(I*A - 2*B)*Cosh[x])/(I
+ Sinh[x])
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3127
```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> Simp[-Cos[c +
 d*x]/(d*(b + a*Sin[c + d*x])), x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b
^2, 0]
```

rule 3229
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Simp[(b*c - a*d)*Cos[e + f*x]*((a + b*Sin[e + f*
x])^m/(a*f*(2*m + 1))), x] + Simp[(a*d*m + b*c*(m + 1))/(a*b*(2*m + 1))   I
nt[(a + b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{a, b, c, d, e, f}, x] &&
NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && LtQ[m, -2^(-1)]
```

Maple [A] (verified)

Time = 0.31 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.84

method	result	size
risch	$-\frac{2\left(3A\,{\mathrm e}^{x}-2B+3B\,{\mathrm e}^{2x}+iA+3iB\,{\mathrm e}^{x}\right)}{3\left({\mathrm e}^{x}+i\right)^{3}}$	36
default	$-\frac{-2iA-2B}{\left(\tanh\left(\frac{x}{2}\right)+i\right)^{2}}-\frac{2A}{\tanh\left(\frac{x}{2}\right)+i}-\frac{2\left(2iB-2A\right)}{3\left(\tanh\left(\frac{x}{2}\right)+i\right)^{3}}$	52
parallelrisch	$\frac{-6A\tanh\left(\frac{x}{2}\right)^{2}+\left(-6iA+6B\right)\tanh\left(\frac{x}{2}\right)+2iB+4A}{-9\tanh\left(\frac{x}{2}\right)+9i\tanh\left(\frac{x}{2}\right)^{2}+3\tanh\left(\frac{x}{2}\right)^{3}-3i}$	60

input
```
int((A+B*sinh(x))/(I+sinh(x))^2,x,method=_RETURNVERBOSE)
```

output
```
-2/3*(3*A*exp(x)-2*B+3*B*exp(x)^2+I*A+3*I*B*exp(x))/(exp(x)+I)^3
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 43, normalized size of antiderivative = 1.00

$$\int \frac{A+B\sinh(x)}{(i+\sinh(x))^2}\,dx = -\frac{2\left(3\,Be^{(2\,x)}+3\,(A+i\,B)e^{x}+i\,A-2\,B\right)}{3\left(e^{(3\,x)}+3i\,e^{(2\,x)}-3\,e^{x}-i\right)}$$

input
```
integrate((A+B*sinh(x))/(I+sinh(x))^2,x, algorithm="fricas")
```

output
```
-2/3*(3*B*e^(2*x) + 3*(A + I*B)*e^x + I*A - 2*B)/(e^(3*x) + 3*I*e^(2*x) -
3*e^x - I)
```

Sympy [A] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 53, normalized size of antiderivative = 1.23

$$\int \frac{A+B\sinh(x)}{(i+\sinh(x))^2}\,dx = \frac{-2iA-6Be^{2x}+4B+(-6A-6iB)\,e^{x}}{3e^{3x}+9ie^{2x}-9e^{x}-3i}$$

input
```
integrate((A+B*sinh(x))/(I+sinh(x))**2,x)
```

output
```
(-2*I*A - 6*B*exp(2*x) + 4*B + (-6*A - 6*I*B)*exp(x))/(3*exp(3*x) + 9*I*ex
p(2*x) - 9*exp(x) - 3*I)
```

Maxima [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 141 vs. 2(31) = 62.

Time = 0.04 (sec) , antiderivative size = 141, normalized size of antiderivative = 3.28

$$\int \frac{A+B\sinh(x)}{(i+\sinh(x))^2}\,dx$$
$$= -\frac{2}{3}\,A\left(\frac{3\,e^{(-x)}}{3\,e^{(-x)}+3i\,e^{(-2\,x)}-e^{(-3\,x)}-i}-\frac{i}{3\,e^{(-x)}+3i\,e^{(-2\,x)}-e^{(-3\,x)}-i}\right)$$
$$-\frac{2}{3}\,B\left(\frac{3i\,e^{(-x)}}{3\,e^{(-x)}+3i\,e^{(-2\,x)}-e^{(-3\,x)}-i}-\frac{3\,e^{(-2\,x)}}{3\,e^{(-x)}+3i\,e^{(-2\,x)}-e^{(-3\,x)}-i}+\frac{2}{3\,e^{(-x)}+3i\,e^{(-2\,x)}-e^{(-3\,x)}-\right.$$

input
```
integrate((A+B*sinh(x))/(I+sinh(x))^2,x, algorithm="maxima")
```

output
```
-2/3*A*(3*e^(-x)/(3*e^(-x) + 3*I*e^(-2*x) - e^(-3*x) - I) - I/(3*e^(-x) +
3*I*e^(-2*x) - e^(-3*x) - I)) - 2/3*B*(3*I*e^(-x)/(3*e^(-x) + 3*I*e^(-2*x)
 - e^(-3*x) - I) - 3*e^(-2*x)/(3*e^(-x) + 3*I*e^(-2*x) - e^(-3*x) - I) + 2
/(3*e^(-x) + 3*I*e^(-2*x) - e^(-3*x) - I))
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.74

$$\int \frac{A+B\sinh(x)}{(i+\sinh(x))^2}\,dx = -\frac{2\left(3\,Be^{(2\,x)}+3\,Ae^{x}+3i\,Be^{x}+i\,A-2\,B\right)}{3\,(e^{x}+i)^{3}}$$

input
```
integrate((A+B*sinh(x))/(I+sinh(x))^2,x, algorithm="giac")
```

output
```
-2/3*(3*B*e^(2*x) + 3*A*e^x + 3*I*B*e^x + I*A - 2*B)/(e^x + I)^3
```

Mupad [B] (verification not implemented)

Time = 1.52 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.91

$$\int \frac{A+B\sinh(x)}{(i+\sinh(x))^2}\,dx = -\frac{\frac{2\,A}{3}+\frac{B\,4\mathrm{i}}{3}-\mathrm{e}^{x}\,(-2\,B+A\,2\mathrm{i})-B\,\mathrm{e}^{2\,x}\,2\mathrm{i}}{(-1+\mathrm{e}^{x}\,1\mathrm{i})^{3}}$$

input
```
int((A + B*sinh(x))/(sinh(x) + 1i)^2,x)
```

output
```
-((2*A)/3 + (B*4i)/3 - exp(x)*(A*2i - 2*B) - B*exp(2*x)*2i)/(exp(x)*1i - 1
)^3
```

Reduce [F]

$$\int \frac{A+B\sinh(x)}{(i+\sinh(x))^2}\,dx = \left(\int \frac{\sinh(x)}{\sinh(x)^2+2\sinh(x)\,i-1}dx\right)b + \left(\int \frac{1}{\sinh(x)^2+2\sinh(x)\,i-1}dx\right)a$$

input
```
int((A+B*sinh(x))/(I+sinh(x))^2,x)
```

output
```
int(sinh(x)/(sinh(x)**2 + 2*sinh(x)*i - 1),x)*b + int(1/(sinh(x)**2 + 2*si
nh(x)*i - 1),x)*a
```

3.117 $\int \frac{A+B\sinh(x)}{(i+\sinh(x))^3}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 68

$$\int \frac{A+B\sinh(x)}{(i+\sinh(x))^3}\,dx = -\frac{(iA+B)\cosh(x)}{5(i+\sinh(x))^3} - \frac{(2A+3iB)\cosh(x)}{15(i+\sinh(x))^2} + \frac{(2iA-3B)\cosh(x)}{15(i+\sinh(x))}$$

output
```
-1/5*(I*A+B)*cosh(x)/(I+sinh(x))^3-1/15*(2*A+3*I*B)*cosh(x)/(I+sinh(x))^2+
(2*I*A-3*B)*cosh(x)/(15*I+15*sinh(x))
```

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 50, normalized size of antiderivative = 0.74

$$\int \frac{A+B\sinh(x)}{(i+\sinh(x))^3}\,dx = \frac{\cosh(x)\left(-7iA+3B-3(2A+3iB)\sinh(x)+(2iA-3B)\sinh^2(x)\right)}{15(i+\sinh(x))^3}$$

input
```
Integrate[(A + B*Sinh[x])/(I + Sinh[x])^3,x]
```

output
```
(Cosh[x]*((-7*I)*A + 3*B - 3*(2*A + (3*I)*B)*Sinh[x] + ((2*I)*A - 3*B)*Sin
h[x]^2))/(15*(I + Sinh[x])^3)
```

Rubi [A] (verified)

Time = 0.37 (sec) , antiderivative size = 66, normalized size of antiderivative = 0.97, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.400$, Rules used = {3042, 3229, 3042, 3129, 3042, 3127}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+B\sinh(x)}{(\sinh(x)+i)^3}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{A-iB\sin(ix)}{(i-i\sin(ix))^3}dx$$

$$\Big\downarrow \text{3229}$$

$$-\frac{1}{5}(-3B+2iA)\int \frac{1}{(\sinh(x)+i)^2}dx-\frac{(B+iA)\cosh(x)}{5(\sinh(x)+i)^3}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{1}{5}(-3B+2iA)\int \frac{1}{(i-i\sin(ix))^2}dx-\frac{(B+iA)\cosh(x)}{5(\sinh(x)+i)^3}$$

$$\Big\downarrow \text{3129}$$

$$-\frac{1}{5}(-3B+2iA)\left(-\frac{1}{3}i\int \frac{1}{\sinh(x)+i}dx-\frac{i\cosh(x)}{3(\sinh(x)+i)^2}\right)-\frac{(B+iA)\cosh(x)}{5(\sinh(x)+i)^3}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{1}{5}(-3B+2iA)\left(-\frac{1}{3}i\int \frac{1}{i-i\sin(ix)}dx-\frac{i\cosh(x)}{3(\sinh(x)+i)^2}\right)-\frac{(B+iA)\cosh(x)}{5(\sinh(x)+i)^3}$$

$$\Big\downarrow \text{3127}$$

$$-\frac{(B+iA)\cosh(x)}{5(\sinh(x)+i)^3}-\frac{1}{5}(-3B+2iA)\left(-\frac{\cosh(x)}{3(\sinh(x)+i)}-\frac{i\cosh(x)}{3(\sinh(x)+i)^2}\right)$$

input
```
Int[(A + B*Sinh[x])/(I + Sinh[x])^3,x]
```

output
```
-1/5*((I*A + B)*Cosh[x])/(I + Sinh[x])^3 - (((2*I)*A - 3*B)*(((-1/3*I)*Cos
h[x])/(I + Sinh[x])^2 - Cosh[x]/(3*(I + Sinh[x]))))/5
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3127
```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> Simp[-Cos[c +
 d*x]/(d*(b + a*Sin[c + d*x])), x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b
^2, 0]
```

rule 3129
```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[b*Cos[c
+ d*x]*((a + b*Sin[c + d*x])^n/(a*d*(2*n + 1))), x] + Simp[(n + 1)/(a*(2*n
+ 1))   Int[(a + b*Sin[c + d*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d}, x]
&& EqQ[a^2 - b^2, 0] && LtQ[n, -1] && IntegerQ[2*n]
```

rule 3229
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Simp[(b*c - a*d)*Cos[e + f*x]*((a + b*Sin[e + f*
x])^m/(a*f*(2*m + 1))), x] + Simp[(a*d*m + b*c*(m + 1))/(a*b*(2*m + 1))   I
nt[(a + b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{a, b, c, d, e, f}, x] &&
NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && LtQ[m, -2^(-1)]
```

Maple [A] (verified)

Time = 0.44 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.75

method	result
risch	$-\frac{2\left(15B\,e^{3x}+20A\,e^{2x}-15B\,e^{x}+15iB\,e^{2x}-2A+10iA\,e^{x}-3iB\right)}{15\left(e^{x}+i\right)^{5}}$
default	$\frac{2iA}{\tanh\left(\frac{x}{2}\right)+i}-\frac{2\left(-4iA-4B\right)}{5\left(\tanh\left(\frac{x}{2}\right)+i\right)^{5}}-\frac{2\left(8iA+6B\right)}{3\left(\tanh\left(\frac{x}{2}\right)+i\right)^{3}}-\frac{-8iB+8A}{2\left(\tanh\left(\frac{x}{2}\right)+i\right)^{4}}-\frac{2iB-4A}{\left(\tanh\left(\frac{x}{2}\right)+i\right)^{2}}$
parallelrisch	$-\frac{2\tanh\left(\frac{x}{2}\right)\left(3iB\tanh\left(\frac{x}{2}\right)^{4}+20i\tanh\left(\frac{x}{2}\right)^{3}A+7\tanh\left(\frac{x}{2}\right)^{4}A-15iB\tanh\left(\frac{x}{2}\right)^{2}-15\tanh\left(\frac{x}{2}\right)^{3}B-30iA\tanh\left(\frac{x}{2}\right)-40A\tanh(\frac{x}{2}\right.}{15\left(-10\tanh\left(\frac{x}{2}\right)^{3}+5i\tanh\left(\frac{x}{2}\right)^{4}+\tanh\left(\frac{x}{2}\right)^{5}+5\tanh\left(\frac{x}{2}\right)-10i\tanh\left(\frac{x}{2}\right)^{2}+i\right)}$

input
```
int((A+B*sinh(x))/(I+sinh(x))^3,x,method=_RETURNVERBOSE)
```

output
```
-2/15*(15*B*exp(x)^3+20*A*exp(x)^2-15*B*exp(x)+15*I*B*exp(x)^2-2*A+10*I*A*
exp(x)-3*I*B)/(exp(x)+I)^5
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 70, normalized size of antiderivative = 1.03

$$\int \frac{A+B\sinh(x)}{(i+\sinh(x))^3}\,dx = -\frac{2\left(15\,Be^{(3\,x)}+5\,(4\,A+3i\,B)e^{(2\,x)}+5\,(2i\,A-3\,B)e^{x}-2\,A-3i\,B\right)}{15\left(e^{(5\,x)}+5i\,e^{(4\,x)}-10\,e^{(3\,x)}-10i\,e^{(2\,x)}+5\,e^{x}+i\right)}$$

input
```
integrate((A+B*sinh(x))/(I+sinh(x))^3,x, algorithm="fricas")
```

output
```
-2/15*(15*B*e^(3*x) + 5*(4*A + 3*I*B)*e^(2*x) + 5*(2*I*A - 3*B)*e^x - 2*A
- 3*I*B)/(e^(5*x) + 5*I*e^(4*x) - 10*e^(3*x) - 10*I*e^(2*x) + 5*e^x + I)
```

Sympy [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 82, normalized size of antiderivative = 1.21

$$\int \frac{A+B\sinh(x)}{(i+\sinh(x))^3}\,dx = \frac{4A-30Be^{3x}+6iB+(-40A-30iB)\,e^{2x}+(-20iA+30B)\,e^{x}}{15e^{5x}+75ie^{4x}-150e^{3x}-150ie^{2x}+75e^{x}+15i}$$

input
```
integrate((A+B*sinh(x))/(I+sinh(x))**3,x)
```

output
```
(4*A - 30*B*exp(3*x) + 6*I*B + (-40*A - 30*I*B)*exp(2*x) + (-20*I*A + 30*B
)*exp(x))/(15*exp(5*x) + 75*I*exp(4*x) - 150*exp(3*x) - 150*I*exp(2*x) + 7
5*exp(x) + 15*I)
```

Maxima [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 267 vs. $2(50) = 100$.

Time = 0.05 (sec) , antiderivative size = 267, normalized size of antiderivative = 3.93

$$\int \frac{A+B\sinh(x)}{(i+\sinh(x))^3}\,dx = -\frac{2}{5}\,B\left(\frac{5\,e^{(-x)}}{5\,e^{(-x)}+10i\,e^{(-2\,x)}-10\,e^{(-3\,x)}-5i\,e^{(-4\,x)}+e^{(-5\,x)}-i}+\frac{5i\,e^{(-2\,x)}}{5\,e^{(-x)}+10i\,e^{(-2\,x)}-10\,e^{(-3\,x)}-5i\,e^{(-4\,x)}+e^{(-5\,x)}-i}\right. - \frac{4}{15}\,A\left(-\frac{5i\,e^{(-x)}}{5\,e^{(-x)}+10i\,e^{(-2\,x)}-10\,e^{(-3\,x)}-5i\,e^{(-4\,x)}+e^{(-5\,x)}-i}+\frac{10\,e^{(-2\,x)}}{5\,e^{(-x)}+10i\,e^{(-2\,x)}-10\,e^{(-3\,x)}-5i\,e^{(-4\,x)}+e^{(-5\,x)}-i}\right.$$

input
```
integrate((A+B*sinh(x))/(I+sinh(x))^3,x, algorithm="maxima")
```

output
```
-2/5*B*(5*e^(-x)/(5*e^(-x) + 10*I*e^(-2*x) - 10*e^(-3*x) - 5*I*e^(-4*x) +
e^(-5*x) - I) + 5*I*e^(-2*x)/(5*e^(-x) + 10*I*e^(-2*x) - 10*e^(-3*x) - 5*I
*e^(-4*x) + e^(-5*x) - I) - 5*e^(-3*x)/(5*e^(-x) + 10*I*e^(-2*x) - 10*e^(-
3*x) - 5*I*e^(-4*x) + e^(-5*x) - I) - I/(5*e^(-x) + 10*I*e^(-2*x) - 10*e^(
-3*x) - 5*I*e^(-4*x) + e^(-5*x) - I)) - 4/15*A*(-5*I*e^(-x)/(5*e^(-x) + 10
*I*e^(-2*x) - 10*e^(-3*x) - 5*I*e^(-4*x) + e^(-5*x) - I) + 10*e^(-2*x)/(5*
e^(-x) + 10*I*e^(-2*x) - 10*e^(-3*x) - 5*I*e^(-4*x) + e^(-5*x) - I) - 1/(5
*e^(-x) + 10*I*e^(-2*x) - 10*e^(-3*x) - 5*I*e^(-4*x) + e^(-5*x) - I))
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.68

$$\int \frac{A+B\sinh(x)}{(i+\sinh(x))^3}\,dx = -\frac{2\left(15\,Be^{(3\,x)}+20\,Ae^{(2\,x)}+15i\,Be^{(2\,x)}+10i\,Ae^{x}-15\,Be^{x}-2\,A-3i\,B\right)}{15\left(e^{x}+i\right)^{5}}$$

input
```
integrate((A+B*sinh(x))/(I+sinh(x))^3,x, algorithm="giac")
```

output
```
-2/15*(15*B*e^(3*x) + 20*A*e^(2*x) + 15*I*B*e^(2*x) + 10*I*A*e^x - 15*B*e^
x - 2*A - 3*I*B)/(e^x + I)^5
```

Mupad [B] (verification not implemented)

Time = 1.74 (sec) , antiderivative size = 52, normalized size of antiderivative = 0.76

$$\int \frac{A+B\sinh(x)}{(i+\sinh(x))^3}\,dx = \frac{\frac{A\,4\mathrm{i}}{15}-\frac{2\,B}{5}-\frac{A\,\mathrm{e}^{2\,x}\,8\mathrm{i}}{3}+\mathrm{e}^{x}\,\left(\frac{4\,A}{3}+B\,2\mathrm{i}\right)+2\,B\,\mathrm{e}^{2\,x}-B\,\mathrm{e}^{3\,x}\,2\mathrm{i}}{{\left(-1+\mathrm{e}^{x}\,1\mathrm{i}\right)}^5}$$

input
```
int((A + B*sinh(x))/(sinh(x) + 1i)^3,x)
```

output
```
((A*4i)/15 - (2*B)/5 - (A*exp(2*x)*8i)/3 + exp(x)*((4*A)/3 + B*2i) + 2*B*e
xp(2*x) - B*exp(3*x)*2i)/(exp(x)*1i - 1)^5
```

Reduce [F]

$$\int \frac{A+B\sinh(x)}{(i+\sinh(x))^3}\,dx = \left(\int \frac{\sinh(x)}{\sinh(x)^{3}+3\sinh(x)^{2}i-3\sinh(x)-i}dx\right)b + \left(\int \frac{1}{\sinh(x)^{3}+3\sinh(x)^{2}i-3\sinh(x)-i}dx\right)a$$

input
```
int((A+B*sinh(x))/(I+sinh(x))^3,x)
```

output
```
int(sinh(x)/(sinh(x)**3 + 3*sinh(x)**2*i - 3*sinh(x) - i),x)*b + int(1/(si
nh(x)**3 + 3*sinh(x)**2*i - 3*sinh(x) - i),x)*a
```

3.118 $\int \frac{A+B\sinh(x)}{(i+\sinh(x))^4}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 91

$$\int \frac{A+B\sinh(x)}{(i+\sinh(x))^4}\,dx = -\frac{(iA+B)\cosh(x)}{7(i+\sinh(x))^4} - \frac{(3A+4iB)\cosh(x)}{35(i+\sinh(x))^3} + \frac{2(3iA-4B)\cosh(x)}{105(i+\sinh(x))^2} + \frac{2(3A+4iB)\cosh(x)}{105(i+\sinh(x))}$$

output
```
-1/7*(I*A+B)*cosh(x)/(I+sinh(x))^4-1/35*(3*A+4*I*B)*cosh(x)/(I+sinh(x))^3+
2/105*(3*I*A-4*B)*cosh(x)/(I+sinh(x))^2+2*(3*A+4*I*B)*cosh(x)/(105*I+105*s
inh(x))
```

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 67, normalized size of antiderivative = 0.74

$$\int \frac{A+B\sinh(x)}{(i+\sinh(x))^4}\,dx = \frac{\cosh(x)\left(-36iA+13B-13(3A+4iB)\sinh(x)+8i(3A+4iB)\sinh^2(x)+(6A+8iB)\sinh^3(x)\right)}{105(i+\sinh(x))^4}$$

input
```
Integrate[(A + B*Sinh[x])/(I + Sinh[x])^4,x]
```

output
```
(Cosh[x]*((-36*I)*A + 13*B - 13*(3*A + (4*I)*B)*Sinh[x] + (8*I)*(3*A + (4*
I)*B)*Sinh[x]^2 + (6*A + (8*I)*B)*Sinh[x]^3))/(105*(I + Sinh[x])^4)
```

Rubi [A] (verified)

Time = 0.67 (sec) , antiderivative size = 89, normalized size of antiderivative = 0.98, number of steps used = 8, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.533$, Rules used = {3042, 3229, 3042, 3129, 3042, 3129, 3042, 3127}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+B\sinh(x)}{(\sinh(x)+i)^4}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{A-iB\sin(ix)}{(i-i\sin(ix))^4}dx$$

$$\Big\downarrow \text{3229}$$

$$-\frac{1}{7}(-4B+3iA)\int \frac{1}{(\sinh(x)+i)^3}dx-\frac{(B+iA)\cosh(x)}{7(\sinh(x)+i)^4}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{1}{7}(-4B+3iA)\int \frac{1}{(i-i\sin(ix))^3}dx-\frac{(B+iA)\cosh(x)}{7(\sinh(x)+i)^4}$$

$$\Big\downarrow \text{3129}$$

$$-\frac{1}{7}(-4B+3iA)\left(-\frac{2}{5}i\int \frac{1}{(\sinh(x)+i)^2}dx-\frac{i\cosh(x)}{5(\sinh(x)+i)^3}\right)-\frac{(B+iA)\cosh(x)}{7(\sinh(x)+i)^4}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{1}{7}(-4B+3iA)\left(-\frac{2}{5}i\int \frac{1}{(i-i\sin(ix))^2}dx-\frac{i\cosh(x)}{5(\sinh(x)+i)^3}\right)-\frac{(B+iA)\cosh(x)}{7(\sinh(x)+i)^4}$$

$$\Big\downarrow \text{3129}$$

$$-\frac{1}{7}(-4B+3iA)\left(-\frac{2}{5}i\left(-\frac{1}{3}i\int \frac{1}{\sinh(x)+i}dx-\frac{i\cosh(x)}{3(\sinh(x)+i)^2}\right)-\frac{i\cosh(x)}{5(\sinh(x)+i)^3}\right)-\frac{(B+iA)\cosh(x)}{7(\sinh(x)+i)^4}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{1}{7}(-4B+3iA)\left(-\frac{2}{5}i\left(-\frac{1}{3}i\int\frac{1}{i-i\sin(ix)}dx-\frac{i\cosh(x)}{3(\sinh(x)+i)^2}\right)-\frac{i\cosh(x)}{5(\sinh(x)+i)^3}\right)-\frac{(B+iA)\cosh(x)}{7(\sinh(x)+i)^4}$$

$$\Big\downarrow \text{3127}$$

$$-\frac{(B+iA)\cosh(x)}{7(\sinh(x)+i)^4}-\frac{1}{7}(-4B+3iA)\left(-\frac{i\cosh(x)}{5(\sinh(x)+i)^3}-\frac{2}{5}i\left(-\frac{\cosh(x)}{3(\sinh(x)+i)}-\frac{i\cosh(x)}{3(\sinh(x)+i)^2}\right)\right)$$

input
```
Int[(A + B*Sinh[x])/(I + Sinh[x])^4,x]
```

output
```
-1/7*((I*A + B)*Cosh[x])/(I + Sinh[x])^4 - (((3*I)*A - 4*B)*(((-1/5*I)*Cos
h[x])/(I + Sinh[x])^3 - ((2*I)/5)*(((-1/3*I)*Cosh[x])/(I + Sinh[x])^2 - Co
sh[x]/(3*(I + Sinh[x])))))/7
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3127
```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> Simp[-Cos[c +
 d*x]/(d*(b + a*Sin[c + d*x])), x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b
^2, 0]
```

rule 3129
```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[b*Cos[c
+ d*x]*((a + b*Sin[c + d*x])^n/(a*d*(2*n + 1))), x] + Simp[(n + 1)/(a*(2*n
+ 1))   Int[(a + b*Sin[c + d*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d}, x]
&& EqQ[a^2 - b^2, 0] && LtQ[n, -1] && IntegerQ[2*n]
```

rule 3229
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Simp[(b*c - a*d)*Cos[e + f*x]*((a + b*Sin[e + f*
x])^m/(a*f*(2*m + 1))), x] + Simp[(a*d*m + b*c*(m + 1))/(a*b*(2*m + 1))   I
nt[(a + b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{a, b, c, d, e, f}, x] &&
NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && LtQ[m, -2^(-1)]
```

Maple [A] (verified)

Time = 0.65 (sec) , antiderivative size = 66, normalized size of antiderivative = 0.73

method	result
risch	$-\frac{4(4B-21A\,e^{x}-84B\,e^{2x}-3iA+70iB\,e^{3x}+63iA\,e^{2x}-28iB\,e^{x}+70B\,e^{4x}+105A\,e^{3x})}{105(e^{x}+i)^{7}}$
default	$-\frac{2(-8iB+8A)}{7(\tanh(\frac{x}{2})+i)^{7}}+\frac{2A}{\tanh(\frac{x}{2})+i}-\frac{24iA+24B}{3(\tanh(\frac{x}{2})+i)^{6}}-\frac{6iA+2B}{(\tanh(\frac{x}{2})+i)^{2}}-\frac{2(32iB-36A)}{5(\tanh(\frac{x}{2})+i)^{5}}-\frac{2(-10iB+18A)}{3(\tanh(\frac{x}{2})+i)^{3}}-$ ...
parallelrisch	$\frac{210A\tanh(\frac{x}{2})^{6}+(630iA-210B)\tanh(\frac{x}{2})^{5}+(-350iB-1260A)\tanh(\frac{x}{2})^{4}+(-1260iA+560B)\tanh(\frac{x}{2})^{3}+(336iB+882A)\tan \ldots}{-2205\tanh(\frac{x}{2})^{5}+735i\tanh(\frac{x}{2})^{6}+105\tanh(\frac{x}{2})^{7}+3675\tanh(\frac{x}{2})^{3}-3675i\tanh(\frac{x}{2})^{4}-735\tanh(\frac{x}{2})+\ldots}$

input
```
int((A+B*sinh(x))/(I+sinh(x))^4,x,method=_RETURNVERBOSE)
```

output
```
-4/105*(4*B-21*A*exp(x)-84*B*exp(x)^2-3*I*A+70*I*B*exp(x)^3+63*I*A*exp(x)^
2-28*I*B*exp(x)+70*B*exp(x)^4+105*A*exp(x)^3)/(exp(x)+I)^7
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 95, normalized size of antiderivative = 1.04

$$\int \frac{A+B\sinh(x)}{(i+\sinh(x))^4}\,dx = -\frac{4\left(70\,Be^{(4\,x)}+35\,(3\,A+2i\,B)e^{(3\,x)}+21\,(3i\,A-4\,B)e^{(2\,x)}-7\,(3\,A+4i\,B)e^{x}-3i\,A+4\,B\right)}{105\left(e^{(7\,x)}+7i\,e^{(6\,x)}-21\,e^{(5\,x)}-35i\,e^{(4\,x)}+35\,e^{(3\,x)}+21i\,e^{(2\,x)}-7\,e^{x}-i\right)}$$

input
```
integrate((A+B*sinh(x))/(I+sinh(x))^4,x, algorithm="fricas")
```

output
```
-4/105*(70*B*e^(4*x) + 35*(3*A + 2*I*B)*e^(3*x) + 21*(3*I*A - 4*B)*e^(2*x)
 - 7*(3*A + 4*I*B)*e^x - 3*I*A + 4*B)/(e^(7*x) + 7*I*e^(6*x) - 21*e^(5*x)
- 35*I*e^(4*x) + 35*e^(3*x) + 21*I*e^(2*x) - 7*e^x - I)
```

Sympy [A] (verification not implemented)

Time = 0.42 (sec) , antiderivative size = 110, normalized size of antiderivative = 1.21

$$\int \frac{A+B\sinh(x)}{(i+\sinh(x))^4}\,dx = \frac{12iA - 280Be^{4x} - 16B + (-420A - 280iB)\,e^{3x} + (84A + 112iB)\,e^{x} + (-252iA + 336B)\,e^{2x}}{105e^{7x} + 735ie^{6x} - 2205e^{5x} - 3675ie^{4x} + 3675e^{3x} + 2205ie^{2x} - 735e^{x} - 105i}$$

input
```
integrate((A+B*sinh(x))/(I+sinh(x))**4,x)
```

output
```
(12*I*A - 280*B*exp(4*x) - 16*B + (-420*A - 280*I*B)*exp(3*x) + (84*A + 11
2*I*B)*exp(x) + (-252*I*A + 336*B)*exp(2*x))/(105*exp(7*x) + 735*I*exp(6*x
) - 2205*exp(5*x) - 3675*I*exp(4*x) + 3675*exp(3*x) + 2205*I*exp(2*x) - 73
5*exp(x) - 105*I)
```

Maxima [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 469 vs. 2(67) = 134.

Time = 0.04 (sec) , antiderivative size = 469, normalized size of antiderivative = 5.15

$$\int \frac{A+B\sinh(x)}{(i+\sinh(x))^4}\,dx = \text{Too large to display}$$

input
```
integrate((A+B*sinh(x))/(I+sinh(x))^4,x, algorithm="maxima")
```

output
```
4/35*A*(7*e^(-x)/(7*e^(-x) + 21*I*e^(-2*x) - 35*e^(-3*x) - 35*I*e^(-4*x) +
 21*e^(-5*x) + 7*I*e^(-6*x) - e^(-7*x) - I) + 21*I*e^(-2*x)/(7*e^(-x) + 21
*I*e^(-2*x) - 35*e^(-3*x) - 35*I*e^(-4*x) + 21*e^(-5*x) + 7*I*e^(-6*x) - e
^(-7*x) - I) - 35*e^(-3*x)/(7*e^(-x) + 21*I*e^(-2*x) - 35*e^(-3*x) - 35*I*
e^(-4*x) + 21*e^(-5*x) + 7*I*e^(-6*x) - e^(-7*x) - I) - I/(7*e^(-x) + 21*I
*e^(-2*x) - 35*e^(-3*x) - 35*I*e^(-4*x) + 21*e^(-5*x) + 7*I*e^(-6*x) - e^(
-7*x) - I)) - 8/105*B*(-14*I*e^(-x)/(7*e^(-x) + 21*I*e^(-2*x) - 35*e^(-3*x
) - 35*I*e^(-4*x) + 21*e^(-5*x) + 7*I*e^(-6*x) - e^(-7*x) - I) + 42*e^(-2*
x)/(7*e^(-x) + 21*I*e^(-2*x) - 35*e^(-3*x) - 35*I*e^(-4*x) + 21*e^(-5*x) +
 7*I*e^(-6*x) - e^(-7*x) - I) + 35*I*e^(-3*x)/(7*e^(-x) + 21*I*e^(-2*x) -
35*e^(-3*x) - 35*I*e^(-4*x) + 21*e^(-5*x) + 7*I*e^(-6*x) - e^(-7*x) - I) -
 35*e^(-4*x)/(7*e^(-x) + 21*I*e^(-2*x) - 35*e^(-3*x) - 35*I*e^(-4*x) + 21*
e^(-5*x) + 7*I*e^(-6*x) - e^(-7*x) - I) - 2/(7*e^(-x) + 21*I*e^(-2*x) - 35
*e^(-3*x) - 35*I*e^(-4*x) + 21*e^(-5*x) + 7*I*e^(-6*x) - e^(-7*x) - I))
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 60, normalized size of antiderivative = 0.66

$$\int \frac{A + B\sinh(x)}{(i + \sinh(x))^4}\,dx = -\frac{4\left(70\,Be^{(4\,x)} + 105\,Ae^{(3\,x)} + 70i\,Be^{(3\,x)} + 63i\,Ae^{(2\,x)} - 84\,Be^{(2\,x)} - 21\,Ae^{x} - 28i\,Be^{x} - 3i\,A + 4\,B\right)}{105\,(e^{x} + i)^{7}}$$

input
```
integrate((A+B*sinh(x))/(I+sinh(x))^4,x, algorithm="giac")
```

output
```
-4/105*(70*B*e^(4*x) + 105*A*e^(3*x) + 70*I*B*e^(3*x) + 63*I*A*e^(2*x) - 8
4*B*e^(2*x) - 21*A*e^x - 28*I*B*e^x - 3*I*A + 4*B)/(e^x + I)^7
```

Mupad [B] (verification not implemented)

Time = 1.99 (sec) , antiderivative size = 66, normalized size of antiderivative = 0.73

$$\int \frac{A+B\sinh(x)}{(i+\sinh(x))^4}\,dx$$
$$= -\frac{\frac{16\,B}{105}+4\,A\,\mathrm{e}^{3\,x}-\mathrm{e}^{x}\,\left(\frac{4\,A}{5}+\frac{B\,16\mathrm{i}}{15}\right)-\frac{16\,B\,\mathrm{e}^{2\,x}}{5}+\frac{8\,B\,\mathrm{e}^{4\,x}}{3}-\frac{A\,4\mathrm{i}}{35}+\frac{A\,\mathrm{e}^{2\,x}\,12\mathrm{i}}{5}+\frac{B\,\mathrm{e}^{3\,x}\,8\mathrm{i}}{3}}{{\left(\mathrm{e}^{x}+1\mathrm{i}\right)}^7}$$

input
```
int((A + B*sinh(x))/(sinh(x) + 1i)^4,x)
```

output
```
-((16*B)/105 - (A*4i)/35 + (A*exp(2*x)*12i)/5 + 4*A*exp(3*x) - exp(x)*((4*
A)/5 + (B*16i)/15) - (16*B*exp(2*x))/5 + (B*exp(3*x)*8i)/3 + (8*B*exp(4*x)
)/3)/(exp(x) + 1i)^7
```

Reduce [F]

$$\int \frac{A+B\sinh(x)}{(i+\sinh(x))^4}\,dx$$
$$= \left(\int \frac{\sinh(x)}{\sinh(x)^4+4\sinh(x)^3 i-6\sinh(x)^2-4\sinh(x)\,i+1}dx\right)b$$
$$+\left(\int \frac{1}{\sinh(x)^4+4\sinh(x)^3 i-6\sinh(x)^2-4\sinh(x)\,i+1}dx\right)a$$

input
```
int((A+B*sinh(x))/(I+sinh(x))^4,x)
```

output
```
int(sinh(x)/(sinh(x)**4 + 4*sinh(x)**3*i - 6*sinh(x)**2 - 4*sinh(x)*i + 1)
,x)*b + int(1/(sinh(x)**4 + 4*sinh(x)**3*i - 6*sinh(x)**2 - 4*sinh(x)*i +
1),x)*a
```

3.119 $\int \frac{A+B\sinh(x)}{i-\sinh(x)}\,dx$

Optimal result

Integrand size = 17, antiderivative size = 27

$$\int \frac{A+B\sinh(x)}{i-\sinh(x)}\,dx = -Bx + \frac{(iA-B)\cosh(x)}{i-\sinh(x)}$$

output
```
-B*x+(I*A-B)*cosh(x)/(I-sinh(x))
```

Mathematica [A] (verified)

Time = 0.50 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.26

$$\int \frac{A+B\sinh(x)}{i-\sinh(x)}\,dx = \cosh(x)\left(-\frac{B\operatorname{arcsinh}(\sinh(x))}{\sqrt{\cosh^2(x)}} + \frac{-iA+B}{-i+\sinh(x)}\right)$$

input
```
Integrate[(A + B*Sinh[x])/(I - Sinh[x]),x]
```

output
```
Cosh[x]*(-((B*ArcSinh[Sinh[x]])/Sqrt[Cosh[x]^2]) + ((-I)*A + B)/(-I + Sinh
[x]))
```

Rubi [A] (verified)

Time = 0.43 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.04, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.235, Rules used = {3042, 3214, 3042, 3127}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+B\sinh(x)}{-\sinh(x)+i}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{A-iB\sin(ix)}{i\sin(ix)+i}dx$$

$$\Big\downarrow \text{3214}$$

$$-Bx+(A+iB)\int \frac{1}{i-\sinh(x)}dx$$

$$\Big\downarrow \text{3042}$$

$$-Bx+(A+iB)\int \frac{1}{i\sin(ix)+i}dx$$

$$\Big\downarrow \text{3127}$$

$$-Bx+\frac{i(A+iB)\cosh(x)}{-\sinh(x)+i}$$

input
```
Int[(A + B*Sinh[x])/(I - Sinh[x]),x]
```

output
```
-(B*x) + (I*(A + I*B)*Cosh[x])/(I - Sinh[x])
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3127
```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> Simp[-Cos[c +
 d*x]/(d*(b + a*Sin[c + d*x])), x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b
^2, 0]
```

rule 3214
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])/((c_.) + (d_.)*sin[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[b*(x/d), x] - Simp[(b*c - a*d)/d   Int[1/(c + d
*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Maple [A] (verified)

Time = 0.17 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.00

method	result	size
risch	$-Bx+\frac{2A}{{\mathrm e}^{x}-i}+\frac{2iB}{{\mathrm e}^{x}-i}$	27
parallelrisch	$\frac{-iBx+\tanh\left(\frac{x}{2}\right)xB+2iA-2B}{i-\tanh\left(\frac{x}{2}\right)}$	33
default	$B\ln\left(\tanh\left(\frac{x}{2}\right)-1\right)-\frac{2i(iB+A)}{\tanh\left(\frac{x}{2}\right)-i}-B\ln\left(\tanh\left(\frac{x}{2}\right)+1\right)$	39

input
```
int((A+B*sinh(x))/(I-sinh(x)),x,method=_RETURNVERBOSE)
```

output
```
-B*x+2/(exp(x)-I)*A+2*I/(exp(x)-I)*B
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.89

$$\int \frac{A+B\sinh(x)}{i-\sinh(x)}\,dx = -\frac{Bxe^{x}-i\,Bx-2\,A-2i\,B}{e^{x}-i}$$

input
```
integrate((A+B*sinh(x))/(I-sinh(x)),x, algorithm="fricas")
```

output
```
-(B*x*e^x - I*B*x - 2*A - 2*I*B)/(e^x - I)
```

Sympy [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.56

$$\int \frac{A+B\sinh(x)}{i-\sinh(x)}\,dx = -Bx+\frac{2A+2iB}{e^{x}-i}$$

input
```
integrate((A+B*sinh(x))/(I-sinh(x)),x)
```

output
```
-B*x + (2*A + 2*I*B)/(exp(x) - I)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.00

$$\int \frac{A+B\sinh(x)}{i-\sinh(x)}\,dx = -B\left(x-\frac{2i}{e^{(-x)}+i}\right)+\frac{2\,A}{e^{(-x)}+i}$$

input
```
integrate((A+B*sinh(x))/(I-sinh(x)),x, algorithm="maxima")
```

output
```
-B*(x - 2*I/(e^(-x) + I)) + 2*A/(e^(-x) + I)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.67

$$\int \frac{A + B\sinh(x)}{i - \sinh(x)}\,dx = -Bx + \frac{2\,(A + i\,B)}{e^x - i}$$

input
```
integrate((A+B*sinh(x))/(I-sinh(x)),x, algorithm="giac")
```

output
```
-B*x + 2*(A + I*B)/(e^x - I)
```

Mupad [B] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.78

$$\int \frac{A + B\sinh(x)}{i - \sinh(x)}\,dx = -B\,x + \frac{2\,A + B\,2\mathrm{i}}{\mathrm{e}^x - \mathrm{i}}$$

input
```
int(-(A + B*sinh(x))/(sinh(x) - 1i),x)
```

output
```
(2*A + B*2i)/(exp(x) - 1i) - B*x
```

Reduce [F]

$$\int \frac{A + B\sinh(x)}{i - \sinh(x)}\,dx = -\left(\int \frac{1}{\sinh(x) - i}dx\right)a - \left(\int \frac{1}{\sinh(x) - i}dx\right)bi - bx$$

input
```
int((A+B*sinh(x))/(I-sinh(x)),x)
```

output
```
- (int(1/(sinh(x) - i),x)*a + int(1/(sinh(x) - i),x)*b*i + b*x)
```

3.120 $\int \frac{A+B\sinh(x)}{(i-\sinh(x))^2}\,dx$

Optimal result

Integrand size = 17, antiderivative size = 49

$$\int \frac{A+B\sinh(x)}{(i-\sinh(x))^2}\,dx=\frac{(iA-B)\cosh(x)}{3(i-\sinh(x))^2}+\frac{(A-2iB)\cosh(x)}{3(i-\sinh(x))}$$

output
```
1/3*(I*A-B)*cosh(x)/(I-sinh(x))^2+(A-2*I*B)*cosh(x)/(3*I-3*sinh(x))
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.65

$$\int \frac{A+B\sinh(x)}{(i-\sinh(x))^2}\,dx=\frac{\cosh(x)(2iA+B-(A-2iB)\sinh(x))}{3(-i+\sinh(x))^2}$$

input
```
Integrate[(A + B*Sinh[x])/(I - Sinh[x])^2,x]
```

output
```
(Cosh[x]*((2*I)*A + B - (A - (2*I)*B)*Sinh[x]))/(3*(-I + Sinh[x])^2)
```

Rubi [A] (verified)

Time = 0.43 (sec) , antiderivative size = 53, normalized size of antiderivative = 1.08, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.235, Rules used = {3042, 3229, 3042, 3127}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+B\sinh(x)}{(-\sinh(x)+i)^2}\,dx$$

$$\Big\downarrow \; 3042$$

$$\int \frac{A-iB\sin(ix)}{(i\sin(ix)+i)^2}dx$$

$$\Big\downarrow \; 3229$$

$$\frac{(-B+iA)\cosh(x)}{3(-\sinh(x)+i)^2}-\frac{1}{3}(2B+iA)\int \frac{1}{i-\sinh(x)}dx$$

$$\Big\downarrow \; 3042$$

$$\frac{(-B+iA)\cosh(x)}{3(-\sinh(x)+i)^2}-\frac{1}{3}(2B+iA)\int \frac{1}{i\sin(ix)+i}dx$$

$$\Big\downarrow \; 3127$$

$$\frac{(-B+iA)\cosh(x)}{3(-\sinh(x)+i)^2}-\frac{i(2B+iA)\cosh(x)}{3(-\sinh(x)+i)}$$

input
```
Int[(A + B*Sinh[x])/(I - Sinh[x])^2,x]
```

output
```
((I*A - B)*Cosh[x])/(3*(I - Sinh[x])^2) - ((I/3)*(I*A + 2*B)*Cosh[x])/(I -
 Sinh[x])
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3127
```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> Simp[-Cos[c +
 d*x]/(d*(b + a*Sin[c + d*x])), x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b
^2, 0]
```

rule 3229
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Simp[(b*c - a*d)*Cos[e + f*x]*((a + b*Sin[e + f*
x])^m/(a*f*(2*m + 1))), x] + Simp[(a*d*m + b*c*(m + 1))/(a*b*(2*m + 1))   I
nt[(a + b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{a, b, c, d, e, f}, x] &&
NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && LtQ[m, -2^(-1)]
```

Maple [A] (verified)

Time = 0.30 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.73

method	result	size
risch	$-\frac{2\left(3A\,{\mathrm e}^{x}-2B+3B\,{\mathrm e}^{2x}-iA-3iB\,{\mathrm e}^{x}\right)}{3\left({\mathrm e}^{x}-i\right)^{3}}$	36
default	$-\frac{2A}{\tanh\left(\frac{x}{2}\right)-i}-\frac{2iA-2B}{\left(\tanh\left(\frac{x}{2}\right)-i\right)^{2}}-\frac{2\left(-2iB-2A\right)}{3\left(\tanh\left(\frac{x}{2}\right)-i\right)^{3}}$	52
parallelrisch	$\frac{6A\tanh\left(\frac{x}{2}\right)^{2}+\left(-6iA-6B\right)\tanh\left(\frac{x}{2}\right)+2iB-4A}{9i\tanh\left(\frac{x}{2}\right)^{2}-3\tanh\left(\frac{x}{2}\right)^{3}-3i+9\tanh\left(\frac{x}{2}\right)}$	60

input
```
int((A+B*sinh(x))/(I-sinh(x))^2,x,method=_RETURNVERBOSE)
```

output
```
-2/3*(3*A*exp(x)-2*B+3*B*exp(x)^2-I*A-3*I*B*exp(x))/(exp(x)-I)^3
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 43, normalized size of antiderivative = 0.88

$$\int \frac{A+B\sinh(x)}{(i-\sinh(x))^2}\,dx = -\frac{2\left(3\,Be^{(2\,x)}+3\,(A-i\,B)e^{x}-i\,A-2\,B\right)}{3\left(e^{(3\,x)}-3i\,e^{(2\,x)}-3\,e^{x}+i\right)}$$

input
```
integrate((A+B*sinh(x))/(I-sinh(x))^2,x, algorithm="fricas")
```

output
```
-2/3*(3*B*e^(2*x) + 3*(A - I*B)*e^x - I*A - 2*B)/(e^(3*x) - 3*I*e^(2*x) -
3*e^x + I)
```

Sympy [A] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 51, normalized size of antiderivative = 1.04

$$\int \frac{A+B\sinh(x)}{(i-\sinh(x))^2}\,dx = \frac{2iA-6Be^{2x}+4B+(-6A+6iB)\,e^{x}}{3e^{3x}-9ie^{2x}-9e^{x}+3i}$$

input
```
integrate((A+B*sinh(x))/(I-sinh(x))**2,x)
```

output
```
(2*I*A - 6*B*exp(2*x) + 4*B + (-6*A + 6*I*B)*exp(x))/(3*exp(3*x) - 9*I*exp
(2*x) - 9*exp(x) + 3*I)
```

Maxima [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 141 vs. 2(31) = 62.

Time = 0.05 (sec) , antiderivative size = 141, normalized size of antiderivative = 2.88

$$\begin{aligned}\int \frac{A+B\sinh(x)}{(i-\sinh(x))^2}\,dx &= -\frac{2}{3}\,A\left(\frac{3\,e^{(-x)}}{3\,e^{(-x)}-3i\,e^{(-2\,x)}-e^{(-3\,x)}+i}+\frac{i}{3\,e^{(-x)}-3i\,e^{(-2\,x)}-e^{(-3\,x)}+i}\right)\\ &\quad -\frac{2}{3}\,B\left(-\frac{3i\,e^{(-x)}}{3\,e^{(-x)}-3i\,e^{(-2\,x)}-e^{(-3\,x)}+i}-\frac{3\,e^{(-2\,x)}}{3\,e^{(-x)}-3i\,e^{(-2\,x)}-e^{(-3\,x)}+i}+\frac{2}{3\,e^{(-x)}-3i\,e^{(-2\,x)}-e^{(-3\,x)}}\right.\end{aligned}$$

input
```
integrate((A+B*sinh(x))/(I-sinh(x))^2,x, algorithm="maxima")
```

output
```
-2/3*A*(3*e^(-x)/(3*e^(-x) - 3*I*e^(-2*x) - e^(-3*x) + I) + I/(3*e^(-x) -
3*I*e^(-2*x) - e^(-3*x) + I)) - 2/3*B*(-3*I*e^(-x)/(3*e^(-x) - 3*I*e^(-2*x
) - e^(-3*x) + I) - 3*e^(-2*x)/(3*e^(-x) - 3*I*e^(-2*x) - e^(-3*x) + I) +
2/(3*e^(-x) - 3*I*e^(-2*x) - e^(-3*x) + I))
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.65

$$\int \frac{A+B\sinh(x)}{(i-\sinh(x))^2}\,dx = -\frac{2\left(3\,Be^{(2\,x)}+3\,Ae^{x}-3i\,Be^{x}-i\,A-2\,B\right)}{3\left(e^{x}-i\right)^{3}}$$

input
```
integrate((A+B*sinh(x))/(I-sinh(x))^2,x, algorithm="giac")
```

output
```
-2/3*(3*B*e^(2*x) + 3*A*e^x - 3*I*B*e^x - I*A - 2*B)/(e^x - I)^3
```

Mupad [B] (verification not implemented)

Time = 1.50 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.76

$$\int \frac{A+B\sinh(x)}{(i-\sinh(x))^2}\,dx = \frac{\frac{2\,A}{3}-\frac{B\,4\mathrm{i}}{3}+\mathrm{e}^{x}\,\left(2\,B+A\,2\mathrm{i}\right)+B\,\mathrm{e}^{2\,x}\,2\mathrm{i}}{{\left(1+\mathrm{e}^{x}\,1\mathrm{i}\right)}^{3}}$$

input
```
int((A + B*sinh(x))/(sinh(x) - 1i)^2,x)
```

output
```
((2*A)/3 - (B*4i)/3 + exp(x)*(A*2i + 2*B) + B*exp(2*x)*2i)/(exp(x)*1i + 1)
^3
```

Reduce [F]

$$\int \frac{A + B\sinh(x)}{(i - \sinh(x))^2}\,dx = \left(\int \frac{\sinh(x)}{\sinh(x)^2 - 2\sinh(x)\,i - 1}dx\right)b + \left(\int \frac{1}{\sinh(x)^2 - 2\sinh(x)\,i - 1}dx\right)a$$

input
```
int((A+B*sinh(x))/(I-sinh(x))^2,x)
```

output
```
int(sinh(x)/(sinh(x)**2 - 2*sinh(x)*i - 1),x)*b + int(1/(sinh(x)**2 - 2*si
nh(x)*i - 1),x)*a
```

3.121 $\int \frac{A+B\sinh(x)}{(i-\sinh(x))^3}\,dx$

Optimal result

Integrand size = 17, antiderivative size = 76

$$\int \frac{A+B\sinh(x)}{(i-\sinh(x))^3}\,dx=\frac{(iA-B)\cosh(x)}{5(i-\sinh(x))^3}+\frac{(2A-3iB)\cosh(x)}{15(i-\sinh(x))^2}-\frac{(2iA+3B)\cosh(x)}{15(i-\sinh(x))}$$

output
```
1/5*(I*A-B)*cosh(x)/(I-sinh(x))^3+1/15*(2*A-3*I*B)*cosh(x)/(I-sinh(x))^2-(
2*I*A+3*B)*cosh(x)/(15*I-15*sinh(x))
```

Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 49, normalized size of antiderivative = 0.64

$$\int \frac{A+B\sinh(x)}{(i-\sinh(x))^3}\,dx = \frac{\cosh(x)\left(-7iA-3B+(6A-9iB)\sinh(x)+(2iA+3B)\sinh^2(x)\right)}{15(-i+\sinh(x))^3}$$

input
```
Integrate[(A + B*Sinh[x])/(I - Sinh[x])^3,x]
```

output
```
(Cosh[x]*((-7*I)*A - 3*B + (6*A - (9*I)*B)*Sinh[x] + ((2*I)*A + 3*B)*Sinh[
x]^2))/(15*(-I + Sinh[x])^3)
```

Rubi [A] (verified)

Time = 0.62 (sec) , antiderivative size = 74, normalized size of antiderivative = 0.97, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.353, Rules used = {3042, 3229, 3042, 3129, 3042, 3127}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+B\sinh(x)}{(-\sinh(x)+i)^3}dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \frac{A-iB\sin(ix)}{(i\sin(ix)+i)^3}dx$$

$$\Big\downarrow \text{ 3229}$$

$$\frac{(-B+iA)\cosh(x)}{5(-\sinh(x)+i)^3}-\frac{1}{5}(3B+2iA)\int \frac{1}{(i-\sinh(x))^2}dx$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{(-B+iA)\cosh(x)}{5(-\sinh(x)+i)^3}-\frac{1}{5}(3B+2iA)\int \frac{1}{(i\sin(ix)+i)^2}dx$$

$$\Big\downarrow \text{ 3129}$$

$$\frac{(-B+iA)\cosh(x)}{5(-\sinh(x)+i)^3}-\frac{1}{5}(3B+2iA)\left(\frac{i\cosh(x)}{3(-\sinh(x)+i)^2}-\frac{1}{3}i\int \frac{1}{i-\sinh(x)}dx\right)$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{(-B+iA)\cosh(x)}{5(-\sinh(x)+i)^3}-\frac{1}{5}(3B+2iA)\left(\frac{i\cosh(x)}{3(-\sinh(x)+i)^2}-\frac{1}{3}i\int \frac{1}{i\sin(ix)+i}dx\right)$$

$$\Big\downarrow \text{ 3127}$$

$$\frac{(-B+iA)\cosh(x)}{5(-\sinh(x)+i)^3}-\frac{1}{5}(3B+2iA)\left(\frac{\cosh(x)}{3(-\sinh(x)+i)}+\frac{i\cosh(x)}{3(-\sinh(x)+i)^2}\right)$$

input
```
Int[(A + B*Sinh[x])/(I - Sinh[x])^3,x]
```

output
```
-1/5*(((2*I)*A + 3*B)*(((I/3)*Cosh[x])/(I - Sinh[x])^2 + Cosh[x]/(3*(I - S
inh[x])))) + ((I*A - B)*Cosh[x])/(5*(I - Sinh[x])^3)
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3127
```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> Simp[-Cos[c +
 d*x]/(d*(b + a*Sin[c + d*x])), x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b
^2, 0]
```

rule 3129
```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[b*Cos[c
+ d*x]*((a + b*Sin[c + d*x])^n/(a*d*(2*n + 1))), x] + Simp[(n + 1)/(a*(2*n
+ 1))   Int[(a + b*Sin[c + d*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d}, x]
&& EqQ[a^2 - b^2, 0] && LtQ[n, -1] && IntegerQ[2*n]
```

rule 3229
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Simp[(b*c - a*d)*Cos[e + f*x]*((a + b*Sin[e + f*
x])^m/(a*f*(2*m + 1))), x] + Simp[(a*d*m + b*c*(m + 1))/(a*b*(2*m + 1))   I
nt[(a + b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{a, b, c, d, e, f}, x] &&
NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && LtQ[m, -2^(-1)]
```

Maple [A] (verified)

Time = 0.44 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.67

method	result	size
risch	$\frac{2B\,{\mathrm e}^{3x}+\frac{8A\,{\mathrm e}^{2x}}{3}-2B\,{\mathrm e}^{x}-2iB\,{\mathrm e}^{2x}-\frac{4A}{15}-\frac{4iA\,{\mathrm e}^{x}}{3}+\frac{2iB}{5}}{\left({\mathrm e}^{x}-i\right)^{5}}$	51
default	$-\frac{2(-4iA+4B)}{5\left(\tanh\left(\frac{x}{2}\right)-i\right)^{5}}-\frac{2iB+4A}{\left(\tanh\left(\frac{x}{2}\right)-i\right)^{2}}-\frac{2(8iA-6B)}{3\left(\tanh\left(\frac{x}{2}\right)-i\right)^{3}}-\frac{-8iB-8A}{2\left(\tanh\left(\frac{x}{2}\right)-i\right)^{4}}+\frac{2iA}{\tanh\left(\frac{x}{2}\right)-i}$	91
parallelrisch	$\frac{(3iB-6A)\tanh\left(\frac{x}{2}\right)^{5}+15B\tanh\left(\frac{x}{2}\right)^{4}+20iA\tanh\left(\frac{x}{2}\right)^{2}+(-15iB+10A)\tanh\left(\frac{x}{2}\right)-8iA-3B}{75i\tanh\left(\frac{x}{2}\right)^{4}-15\tanh\left(\frac{x}{2}\right)^{5}-150i\tanh\left(\frac{x}{2}\right)^{2}+150\tanh\left(\frac{x}{2}\right)^{3}+15i-75\tanh\left(\frac{x}{2}\right)}$	102

input
```
int((A+B*sinh(x))/(I-sinh(x))^3,x,method=_RETURNVERBOSE)
```

output
```
2/15*(15*B*exp(x)^3+20*A*exp(x)^2-15*B*exp(x)-15*I*B*exp(x)^2-2*A-10*I*A*e
xp(x)+3*I*B)/(exp(x)-I)^5
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 70, normalized size of antiderivative = 0.92

$$\int \frac{A+B\sinh(x)}{(i-\sinh(x))^3}\,dx = \frac{2\left(15\,Be^{(3\,x)}+5\,(4\,A-3i\,B)e^{(2\,x)}-5\,(2i\,A+3\,B)e^{x}-2\,A+3i\,B\right)}{15\,\left(e^{(5\,x)}-5i\,e^{(4\,x)}-10\,e^{(3\,x)}+10i\,e^{(2\,x)}+5\,e^{x}-i\right)}$$

input
```
integrate((A+B*sinh(x))/(I-sinh(x))^3,x, algorithm="fricas")
```

output
```
2/15*(15*B*e^(3*x) + 5*(4*A - 3*I*B)*e^(2*x) - 5*(2*I*A + 3*B)*e^x - 2*A +
 3*I*B)/(e^(5*x) - 5*I*e^(4*x) - 10*e^(3*x) + 10*I*e^(2*x) + 5*e^x - I)
```

Sympy [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 82, normalized size of antiderivative = 1.08

$$\int \frac{A+B\sinh(x)}{(i-\sinh(x))^3}\,dx = \frac{-4A+30Be^{3x}+6iB+(40A-30iB)\,e^{2x}+(-20iA-30B)\,e^{x}}{15e^{5x}-75ie^{4x}-150e^{3x}+150ie^{2x}+75e^{x}-15i}$$

input
```
integrate((A+B*sinh(x))/(I-sinh(x))**3,x)
```

output
```
(-4*A + 30*B*exp(3*x) + 6*I*B + (40*A - 30*I*B)*exp(2*x) + (-20*I*A - 30*B
)*exp(x))/(15*exp(5*x) - 75*I*exp(4*x) - 150*exp(3*x) + 150*I*exp(2*x) + 7
5*exp(x) - 15*I)
```

Maxima [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 267 vs. $2(50) = 100$.

Time = 0.05 (sec) , antiderivative size = 267, normalized size of antiderivative = 3.51

$$\int \frac{A+B\sinh(x)}{(i-\sinh(x))^3}\,dx$$
$$= \frac{2}{5}B\left(\frac{5\,e^{(-x)}}{5\,e^{(-x)}-10i\,e^{(-2\,x)}-10\,e^{(-3\,x)}+5i\,e^{(-4\,x)}+e^{(-5\,x)}+i}-\frac{5i\,e^{(-2\,x)}}{5\,e^{(-x)}-10i\,e^{(-2\,x)}-10\,e^{(-3\,x)}+5i\,e^{(-4}\right.$$
$$+\frac{4}{15}A\left(\frac{5i\,e^{(-x)}}{5\,e^{(-x)}-10i\,e^{(-2\,x)}-10\,e^{(-3\,x)}+5i\,e^{(-4\,x)}+e^{(-5\,x)}+i}+\frac{10\,e^{(-2\,x)}}{5\,e^{(-x)}-10i\,e^{(-2\,x)}-10\,e^{(-3\,x)}+5i\,e}\right.$$

input
```
integrate((A+B*sinh(x))/(I-sinh(x))^3,x, algorithm="maxima")
```

output
```
2/5*B*(5*e^(-x)/(5*e^(-x) - 10*I*e^(-2*x) - 10*e^(-3*x) + 5*I*e^(-4*x) + e
^(-5*x) + I) - 5*I*e^(-2*x)/(5*e^(-x) - 10*I*e^(-2*x) - 10*e^(-3*x) + 5*I*
e^(-4*x) + e^(-5*x) + I) - 5*e^(-3*x)/(5*e^(-x) - 10*I*e^(-2*x) - 10*e^(-3
*x) + 5*I*e^(-4*x) + e^(-5*x) + I) + I/(5*e^(-x) - 10*I*e^(-2*x) - 10*e^(-
3*x) + 5*I*e^(-4*x) + e^(-5*x) + I)) + 4/15*A*(5*I*e^(-x)/(5*e^(-x) - 10*I
*e^(-2*x) - 10*e^(-3*x) + 5*I*e^(-4*x) + e^(-5*x) + I) + 10*e^(-2*x)/(5*e^
(-x) - 10*I*e^(-2*x) - 10*e^(-3*x) + 5*I*e^(-4*x) + e^(-5*x) + I) - 1/(5*e
^(-x) - 10*I*e^(-2*x) - 10*e^(-3*x) + 5*I*e^(-4*x) + e^(-5*x) + I))
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.61

$$\int \frac{A+B\sinh(x)}{(i-\sinh(x))^3}\,dx$$
$$= \frac{2\left(15\,Be^{(3\,x)}+20\,Ae^{(2\,x)}-15i\,Be^{(2\,x)}-10i\,Ae^{x}-15\,Be^{x}-2\,A+3i\,B\right)}{15\,(e^{x}-i)^{5}}$$

input
```
integrate((A+B*sinh(x))/(I-sinh(x))^3,x, algorithm="giac")
```

output
```
2/15*(15*B*e^(3*x) + 20*A*e^(2*x) - 15*I*B*e^(2*x) - 10*I*A*e^x - 15*B*e^x
 - 2*A + 3*I*B)/(e^x - I)^5
```

Mupad [B] (verification not implemented)

Time = 1.73 (sec) , antiderivative size = 52, normalized size of antiderivative = 0.68

$$\int \frac{A+B\sinh(x)}{(i-\sinh(x))^3}\,dx = \frac{2\,B\,\mathrm{e}^{2\,x}-\frac{2\,B}{5}+\frac{A\,\mathrm{e}^{2\,x}\,8\mathrm{i}}{3}+\mathrm{e}^{x}\,\left(\frac{4\,A}{3}-B\,2\mathrm{i}\right)-\frac{A\,4\mathrm{i}}{15}+B\,\mathrm{e}^{3\,x}\,2\mathrm{i}}{{\left(1+\mathrm{e}^{x}\,1\mathrm{i}\right)}^5}$$

input
```
int(-(A + B*sinh(x))/(sinh(x) - 1i)^3,x)
```

output
```
((A*exp(2*x)*8i)/3 - (2*B)/5 - (A*4i)/15 + exp(x)*((4*A)/3 - B*2i) + 2*B*e
xp(2*x) + B*exp(3*x)*2i)/(exp(x)*1i + 1)^5
```

Reduce [F]

$$\int \frac{A+B\sinh(x)}{(i-\sinh(x))^3}\,dx = -\left(\int \frac{\sinh(x)}{\sinh(x)^3-3\sinh(x)^2 i-3\sinh(x)+i}dx\right)b - \left(\int \frac{1}{\sinh(x)^3-3\sinh(x)^2 i-3\sinh(x)+i}dx\right)a$$

input
```
int((A+B*sinh(x))/(I-sinh(x))^3,x)
```

output
```
 - (int(sinh(x)/(sinh(x)**3 - 3*sinh(x)**2*i - 3*sinh(x) + i),x)*b + int(1
/(sinh(x)**3 - 3*sinh(x)**2*i - 3*sinh(x) + i),x)*a)
```

3.122 $\int \frac{A+B\sinh(x)}{(i-\sinh(x))^4}\,dx$

Optimal result

Integrand size = 17, antiderivative size = 101

$$\int \frac{A+B\sinh(x)}{(i-\sinh(x))^4}\,dx = \frac{(iA-B)\cosh(x)}{7(i-\sinh(x))^4}+\frac{(3A-4iB)\cosh(x)}{35(i-\sinh(x))^3} - \frac{2(3iA+4B)\cosh(x)}{105(i-\sinh(x))^2}-\frac{2(3A-4iB)\cosh(x)}{105(i-\sinh(x))}$$

output
```
1/7*(I*A-B)*cosh(x)/(I-sinh(x))^4+1/35*(3*A-4*I*B)*cosh(x)/(I-sinh(x))^3-2
/105*(3*I*A+4*B)*cosh(x)/(I-sinh(x))^2-2*(3*A-4*I*B)*cosh(x)/(105*I-105*si
nh(x))
```

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 63, normalized size of antiderivative = 0.62

$$\int \frac{A+B\sinh(x)}{(i-\sinh(x))^4}\,dx = \frac{\cosh(x)\left(36iA+13B+(-39A+52iB)\sinh(x)+(-24iA-32B)\sinh^2(x)+(6A-8iB)\sinh^3(x)\right)}{105(-i+\sinh(x))^4}$$

input
```
Integrate[(A + B*Sinh[x])/(I - Sinh[x])^4,x]
```

output
```
(Cosh[x]*((36*I)*A + 13*B + (-39*A + (52*I)*B)*Sinh[x] + ((-24*I)*A - 32*B
)*Sinh[x]^2 + (6*A - (8*I)*B)*Sinh[x]^3))/(105*(-I + Sinh[x])^4)
```

Rubi [A] (verified)

Time = 0.64 (sec) , antiderivative size = 99, normalized size of antiderivative = 0.98, number of steps used = 8, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.471$, Rules used = {3042, 3229, 3042, 3129, 3042, 3129, 3042, 3127}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+B\sinh(x)}{(-\sinh(x)+i)^4}dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \frac{A-iB\sin(ix)}{(i\sin(ix)+i)^4}dx$$

$$\Big\downarrow \text{ 3229}$$

$$\frac{(-B+iA)\cosh(x)}{7(-\sinh(x)+i)^4}-\frac{1}{7}(4B+3iA)\int \frac{1}{(i-\sinh(x))^3}dx$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{(-B+iA)\cosh(x)}{7(-\sinh(x)+i)^4}-\frac{1}{7}(4B+3iA)\int \frac{1}{(i\sin(ix)+i)^3}dx$$

$$\Big\downarrow \text{ 3129}$$

$$\frac{(-B+iA)\cosh(x)}{7(-\sinh(x)+i)^4}-\frac{1}{7}(4B+3iA)\left(\frac{i\cosh(x)}{5(-\sinh(x)+i)^3}-\frac{2}{5}i\int \frac{1}{(i-\sinh(x))^2}dx\right)$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{(-B+iA)\cosh(x)}{7(-\sinh(x)+i)^4}-\frac{1}{7}(4B+3iA)\left(\frac{i\cosh(x)}{5(-\sinh(x)+i)^3}-\frac{2}{5}i\int \frac{1}{(i\sin(ix)+i)^2}dx\right)$$

$$\Big\downarrow \text{ 3129}$$

$$\frac{(-B+iA)\cosh(x)}{7(-\sinh(x)+i)^4}-\frac{1}{7}(4B+3iA)\left(\frac{i\cosh(x)}{5(-\sinh(x)+i)^3}-\frac{2}{5}i\left(\frac{i\cosh(x)}{3(-\sinh(x)+i)^2}-\frac{1}{3}i\int \frac{1}{i-\sinh(x)}dx\right)\right)$$

$$\Big\downarrow \text{3042}$$

$$\frac{(-B+iA)\cosh(x)}{7(-\sinh(x)+i)^4}-\frac{1}{7}(4B+3iA)\left(\frac{i\cosh(x)}{5(-\sinh(x)+i)^3}-\frac{2}{5}i\left(\frac{i\cosh(x)}{3(-\sinh(x)+i)^2}-\frac{1}{3}i\int\frac{1}{i\sin(ix)+i}dx\right)\right)$$

$$\Big\downarrow \text{3127}$$

$$\frac{(-B+iA)\cosh(x)}{7(-\sinh(x)+i)^4}-\frac{1}{7}(4B+3iA)\left(\frac{i\cosh(x)}{5(-\sinh(x)+i)^3}-\frac{2}{5}i\left(\frac{\cosh(x)}{3(-\sinh(x)+i)}+\frac{i\cosh(x)}{3(-\sinh(x)+i)^2}\right)\right)$$

input
```
Int[(A + B*Sinh[x])/(I - Sinh[x])^4,x]
```

output
```
-1/7*(((3*I)*A + 4*B)*(((-2*I)/5)*(((I/3)*Cosh[x])/(I - Sinh[x])^2 + Cosh[
x]/(3*(I - Sinh[x]))) + ((I/5)*Cosh[x])/(I - Sinh[x])^3)) + ((I*A - B)*Cos
h[x])/(7*(I - Sinh[x])^4)
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3127
```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> Simp[-Cos[c +
 d*x]/(d*(b + a*Sin[c + d*x])), x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b
^2, 0]
```

rule 3129
```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[b*Cos[c
+ d*x]*((a + b*Sin[c + d*x])^n/(a*d*(2*n + 1))), x] + Simp[(n + 1)/(a*(2*n
+ 1))   Int[(a + b*Sin[c + d*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d}, x]
&& EqQ[a^2 - b^2, 0] && LtQ[n, -1] && IntegerQ[2*n]
```

rule 3229
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Simp[(b*c - a*d)*Cos[e + f*x]*((a + b*Sin[e + f*
x])^m/(a*f*(2*m + 1))), x] + Simp[(a*d*m + b*c*(m + 1))/(a*b*(2*m + 1))   I
nt[(a + b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{a, b, c, d, e, f}, x] &&
NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && LtQ[m, -2^(-1)]
```

Maple [A] (verified)

Time = 0.59 (sec) , antiderivative size = 66, normalized size of antiderivative = 0.65

method	result
risch	$-\frac{4\left(4B-84B\,{\mathrm e}^{2x}-21A\,{\mathrm e}^{x}-70iB\,{\mathrm e}^{3x}-63iA\,{\mathrm e}^{2x}+28iB\,{\mathrm e}^{x}+70B\,{\mathrm e}^{4x}+105A\,{\mathrm e}^{3x}+3iA\right)}{105\left({\mathrm e}^{x}-i\right)^{7}}$
default	$-\frac{-6iA+2B}{\left(\tanh\left(\frac{x}{2}\right)-i\right)^{2}}+\frac{2A}{\tanh\left(\frac{x}{2}\right)-i}-\frac{2(8iB+8A)}{7\left(\tanh\left(\frac{x}{2}\right)-i\right)^{7}}-\frac{2(10iB+18A)}{3\left(\tanh\left(\frac{x}{2}\right)-i\right)^{3}}-\frac{32iA-24B}{2\left(\tanh\left(\frac{x}{2}\right)-i\right)^{4}}-\frac{-24iA+24B}{3\left(\tanh\left(\frac{x}{2}\right)-i\right)^{6}}-\frac{2(-}{5(t}$
parallelrisch	$\frac{-210A\tanh\left(\frac{x}{2}\right)^{6}+(630iA+210B)\tanh\left(\frac{x}{2}\right)^{5}+(-350iB+1260A)\tanh\left(\frac{x}{2}\right)^{4}+(-1260iA-560B)\tanh\left(\frac{x}{2}\right)^{3}+(336iB-882A)\,t}{735i\tanh\left(\frac{x}{2}\right)^{6}-105\tanh\left(\frac{x}{2}\right)^{7}-3675i\tanh\left(\frac{x}{2}\right)^{4}+2205\tanh\left(\frac{x}{2}\right)^{5}+2205i\tanh\left(\frac{x}{2}\right)^{2}-3675\tanh\left(\frac{x}{2}\right)}$

input
```
int((A+B*sinh(x))/(I-sinh(x))^4,x,method=_RETURNVERBOSE)
```

output
```
-4/105*(4*B-84*B*exp(x)^2-21*A*exp(x)-70*I*B*exp(x)^3-63*I*A*exp(x)^2+28*I
*B*exp(x)+70*B*exp(x)^4+105*A*exp(x)^3+3*I*A)/(exp(x)-I)^7
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 95, normalized size of antiderivative = 0.94

$$\int \frac{A+B\sinh(x)}{(i-\sinh(x))^4}\,dx = -\frac{4\left(70\,Be^{(4\,x)}+35\,(3\,A-2i\,B)e^{(3\,x)}+21\,(-3i\,A-4\,B)e^{(2\,x)}-7\,(3\,A-4i\,B)e^{x}+3i\,A+4\,B\right)}{105\left(e^{(7\,x)}-7i\,e^{(6\,x)}-21\,e^{(5\,x)}+35i\,e^{(4\,x)}+35\,e^{(3\,x)}-21i\,e^{(2\,x)}-7\,e^{x}+i\right)}$$

input
```
integrate((A+B*sinh(x))/(I-sinh(x))^4,x, algorithm="fricas")
```

output
```
-4/105*(70*B*e^(4*x) + 35*(3*A - 2*I*B)*e^(3*x) + 21*(-3*I*A - 4*B)*e^(2*x
) - 7*(3*A - 4*I*B)*e^x + 3*I*A + 4*B)/(e^(7*x) - 7*I*e^(6*x) - 21*e^(5*x)
 + 35*I*e^(4*x) + 35*e^(3*x) - 21*I*e^(2*x) - 7*e^x + I)
```

Sympy [A] (verification not implemented)

Time = 0.44 (sec) , antiderivative size = 109, normalized size of antiderivative = 1.08

$$\int \frac{A+B\sinh(x)}{(i-\sinh(x))^4}\,dx = \frac{-12iA-280Be^{4x}-16B+(-420A+280iB)\,e^{3x}+(84A-112iB)\,e^{x}+(252iA+336B)\,e^{2x}}{105e^{7x}-735ie^{6x}-2205e^{5x}+3675ie^{4x}+3675e^{3x}-2205ie^{2x}-735e^{x}+105i}$$

input
```
integrate((A+B*sinh(x))/(I-sinh(x))**4,x)
```

output
```
(-12*I*A - 280*B*exp(4*x) - 16*B + (-420*A + 280*I*B)*exp(3*x) + (84*A - 1
12*I*B)*exp(x) + (252*I*A + 336*B)*exp(2*x))/(105*exp(7*x) - 735*I*exp(6*x
) - 2205*exp(5*x) + 3675*I*exp(4*x) + 3675*exp(3*x) - 2205*I*exp(2*x) - 73
5*exp(x) + 105*I)
```

Maxima [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 469 vs. 2(67) = 134.

Time = 0.06 (sec) , antiderivative size = 469, normalized size of antiderivative = 4.64

$$\int \frac{A+B\sinh(x)}{(i-\sinh(x))^4}\,dx = \text{Too large to display}$$

input
```
integrate((A+B*sinh(x))/(I-sinh(x))^4,x, algorithm="maxima")
```

output
```
4/35*A*(7*e^(-x)/(7*e^(-x) - 21*I*e^(-2*x) - 35*e^(-3*x) + 35*I*e^(-4*x) +
 21*e^(-5*x) - 7*I*e^(-6*x) - e^(-7*x) + I) - 21*I*e^(-2*x)/(7*e^(-x) - 21
*I*e^(-2*x) - 35*e^(-3*x) + 35*I*e^(-4*x) + 21*e^(-5*x) - 7*I*e^(-6*x) - e
^(-7*x) + I) - 35*e^(-3*x)/(7*e^(-x) - 21*I*e^(-2*x) - 35*e^(-3*x) + 35*I*
e^(-4*x) + 21*e^(-5*x) - 7*I*e^(-6*x) - e^(-7*x) + I) + I/(7*e^(-x) - 21*I
*e^(-2*x) - 35*e^(-3*x) + 35*I*e^(-4*x) + 21*e^(-5*x) - 7*I*e^(-6*x) - e^(
-7*x) + I)) - 8/105*B*(14*I*e^(-x)/(7*e^(-x) - 21*I*e^(-2*x) - 35*e^(-3*x)
 + 35*I*e^(-4*x) + 21*e^(-5*x) - 7*I*e^(-6*x) - e^(-7*x) + I) + 42*e^(-2*x
)/(7*e^(-x) - 21*I*e^(-2*x) - 35*e^(-3*x) + 35*I*e^(-4*x) + 21*e^(-5*x) -
7*I*e^(-6*x) - e^(-7*x) + I) - 35*I*e^(-3*x)/(7*e^(-x) - 21*I*e^(-2*x) - 3
5*e^(-3*x) + 35*I*e^(-4*x) + 21*e^(-5*x) - 7*I*e^(-6*x) - e^(-7*x) + I) -
35*e^(-4*x)/(7*e^(-x) - 21*I*e^(-2*x) - 35*e^(-3*x) + 35*I*e^(-4*x) + 21*e
^(-5*x) - 7*I*e^(-6*x) - e^(-7*x) + I) - 2/(7*e^(-x) - 21*I*e^(-2*x) - 35*
e^(-3*x) + 35*I*e^(-4*x) + 21*e^(-5*x) - 7*I*e^(-6*x) - e^(-7*x) + I))
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 60, normalized size of antiderivative = 0.59

$$\int \frac{A+B\sinh(x)}{(i-\sinh(x))^4}\,dx = -\frac{4\left(70\,Be^{(4\,x)}+105\,Ae^{(3\,x)}-70i\,Be^{(3\,x)}-63i\,Ae^{(2\,x)}-84\,Be^{(2\,x)}-21\,Ae^{x}+28i\,Be^{x}+3i\,A+4\,B\right)}{105\,(e^{x}-i)^{7}}$$

input
```
integrate((A+B*sinh(x))/(I-sinh(x))^4,x, algorithm="giac")
```

output
```
-4/105*(70*B*e^(4*x) + 105*A*e^(3*x) - 70*I*B*e^(3*x) - 63*I*A*e^(2*x) - 8
4*B*e^(2*x) - 21*A*e^x + 28*I*B*e^x + 3*I*A + 4*B)/(e^x - I)^7
```

Mupad [B] (verification not implemented)

Time = 1.96 (sec) , antiderivative size = 68, normalized size of antiderivative = 0.67

$$\int \frac{A+B\sinh(x)}{(i-\sinh(x))^4}\,dx = \frac{\frac{12\,A\,\mathrm{e}^{2\,x}}{5}+\frac{B\,16\mathrm{i}}{105}-\frac{4\,A}{35}+A\,\mathrm{e}^{3\,x}\,4\mathrm{i}-\mathrm{e}^{x}\,\left(\frac{16\,B}{15}+\frac{A\,4\mathrm{i}}{5}\right)-\frac{B\,\mathrm{e}^{2\,x}\,16\mathrm{i}}{5}+\frac{8\,B\,\mathrm{e}^{3\,x}}{3}+\frac{B\,\mathrm{e}^{4\,x}\,8\mathrm{i}}{3}}{{\left(1+\mathrm{e}^{x}\,1\mathrm{i}\right)}^7}$$

input
```
int((A + B*sinh(x))/(sinh(x) - 1i)^4,x)
```

output
```
((B*16i)/105 - (4*A)/35 + (12*A*exp(2*x))/5 + A*exp(3*x)*4i - exp(x)*((A*4
i)/5 + (16*B)/15) - (B*exp(2*x)*16i)/5 + (8*B*exp(3*x))/3 + (B*exp(4*x)*8i
)/3)/(exp(x)*1i + 1)^7
```

Reduce [F]

$$\int \frac{A+B\sinh(x)}{(i-\sinh(x))^4}\,dx = \left(\int \frac{\sinh(x)}{\sinh(x)^4-4\sinh(x)^3 i-6\sinh(x)^2+4\sinh(x)\,i+1}dx\right)b + \left(\int \frac{1}{\sinh(x)^4-4\sinh(x)^3 i-6\sinh(x)^2+4\sinh(x)\,i+1}dx\right)a$$

input
```
int((A+B*sinh(x))/(I-sinh(x))^4,x)
```

output
```
int(sinh(x)/(sinh(x)**4 - 4*sinh(x)**3*i - 6*sinh(x)**2 + 4*sinh(x)*i + 1)
,x)*b + int(1/(sinh(x)**4 - 4*sinh(x)**3*i - 6*sinh(x)**2 + 4*sinh(x)*i +
1),x)*a
```

3.123 $\int \frac{A+B\sinh(x)}{\sqrt{a+ia\sinh(x)}}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 66

$$\int \frac{A+B\sinh(x)}{\sqrt{a+ia\sinh(x)}}\,dx=\frac{\sqrt{2}(iA-B)\text{arctanh}\left(\frac{\sqrt{a}\cosh(x)}{\sqrt{2}\sqrt{a+ia\sinh(x)}}\right)}{\sqrt{a}}+\frac{2B\cosh(x)}{\sqrt{a+ia\sinh(x)}}$$

output
```
2^(1/2)*(I*A-B)*arctanh(1/2*a^(1/2)*cosh(x)*2^(1/2)/(a+I*a*sinh(x))^(1/2))
/a^(1/2)+2*B*cosh(x)/(a+I*a*sinh(x))^(1/2)
```

Mathematica [A] (verified)

Time = 0.21 (sec) , antiderivative size = 85, normalized size of antiderivative = 1.29

$$\int \frac{A+B\sinh(x)}{\sqrt{a+ia\sinh(x)}}\,dx$$
$$=\frac{2\left(\cosh\left(\frac{x}{2}\right)+i\sinh\left(\frac{x}{2}\right)\right)\left((1+i)\sqrt[4]{-1}(-iA+B)\arctan\left(\frac{i+\tanh\left(\frac{x}{4}\right)}{\sqrt{2}}\right)+B\cosh\left(\frac{x}{2}\right)-iB\sinh\left(\frac{x}{2}\right)\right)}{\sqrt{a+ia\sinh(x)}}$$

input
```
Integrate[(A + B*Sinh[x])/Sqrt[a + I*a*Sinh[x]],x]
```

output
```
(2*(Cosh[x/2] + I*Sinh[x/2])*((1 + I)*(-1)^(1/4)*((-I)*A + B)*ArcTan[(I +
Tanh[x/4])/Sqrt[2]] + B*Cosh[x/2] - I*B*Sinh[x/2]))/Sqrt[a + I*a*Sinh[x]]
```

Rubi [A] (verified)

Time = 0.33 (sec) , antiderivative size = 67, normalized size of antiderivative = 1.02, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {3042, 3230, 3042, 3128, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+B\sinh(x)}{\sqrt{a+ia\sinh(x)}}dx$$
$$\Big\downarrow \text{ 3042}$$
$$\int \frac{A-iB\sin(ix)}{\sqrt{a+a\sin(ix)}}dx$$
$$\Big\downarrow \text{ 3230}$$
$$(A+iB)\int \frac{1}{\sqrt{i\sinh(x)a+a}}dx+\frac{2B\cosh(x)}{\sqrt{a+ia\sinh(x)}}$$
$$\Big\downarrow \text{ 3042}$$
$$(A+iB)\int \frac{1}{\sqrt{\sin(ix)a+a}}dx+\frac{2B\cosh(x)}{\sqrt{a+ia\sinh(x)}}$$
$$\Big\downarrow \text{ 3128}$$
$$2i(A+iB)\int \frac{1}{2a-\frac{a^2\cosh^2(x)}{i\sinh(x)a+a}}d\frac{a\cosh(x)}{\sqrt{i\sinh(x)a+a}}+\frac{2B\cosh(x)}{\sqrt{a+ia\sinh(x)}}$$
$$\Big\downarrow \text{ 219}$$
$$\frac{i\sqrt{2}(A+iB)\text{arctanh}\left(\frac{\sqrt{a}\cosh(x)}{\sqrt{2}\sqrt{a+ia\sinh(x)}}\right)}{\sqrt{a}}+\frac{2B\cosh(x)}{\sqrt{a+ia\sinh(x)}}$$

input
```
Int[(A + B*Sinh[x])/Sqrt[a + I*a*Sinh[x]],x]
```

output
```
(I*Sqrt[2]*(A + I*B)*ArcTanh[(Sqrt[a]*Cosh[x])/(Sqrt[2]*Sqrt[a + I*a*Sinh[
x]])])/Sqrt[a] + (2*B*Cosh[x])/Sqrt[a + I*a*Sinh[x]]
```

Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3128
```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[-2/d
Subst[Int[1/(2*a - x^2), x], x, b*(Cos[c + d*x]/Sqrt[a + b*Sin[c + d*x]])],
 x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]
```

rule 3230
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Simp[(-d)*Cos[e + f*x]*((a + b*Sin[e + f*x])^m/(
f*(m + 1))), x] + Simp[(a*d*m + b*c*(m + 1))/(b*(m + 1))   Int[(a + b*Sin[e
 + f*x])^m, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b*c - a*d, 0]
&& EqQ[a^2 - b^2, 0] &&  !LtQ[m, -2^(-1)]
```

Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 125 vs. $2(52) = 104$.

Time = 1.22 (sec) , antiderivative size = 126, normalized size of antiderivative = 1.91

method	result	size
risch	$\frac{\left(-2A-iB+B\,\mathrm{e}^{x}\right)\left(\mathrm{e}^{x}-i\right)\sqrt{2}\,\mathrm{e}^{-x}}{\sqrt{a\left(i\mathrm{e}^{2x}+2\,\mathrm{e}^{x}-i\right)\mathrm{e}^{-x}}}+\frac{i\left(2iA-2B\right)\left(-\mathrm{e}^{x}+i\right)\left(a^{\frac{3}{2}}+\arctan\left(\frac{\sqrt{ia\,\mathrm{e}^{x}}}{\sqrt{a}}\right)a\sqrt{ia\,\mathrm{e}^{x}}\right)\sqrt{2}\,\mathrm{e}^{-x}}{a^{\frac{3}{2}}\sqrt{a\left(i\mathrm{e}^{2x}+2\,\mathrm{e}^{x}-i\right)\mathrm{e}^{-x}}}$	126

input
```
int((A+B*sinh(x))/(a+I*a*sinh(x))^(1/2),x,method=_RETURNVERBOSE)
```

output
```
(-2*A-I*B+B*exp(x))*(exp(x)-I)*2^(1/2)/(a*(I*exp(x)^2+2*exp(x)-I)/exp(x))^
(1/2)/exp(x)+I*(2*I*A-2*B)*(-exp(x)+I)*(a^(3/2)+arctan((I*a*exp(x))^(1/2)/
a^(1/2))*a*(I*a*exp(x))^(1/2))/a^(3/2)*2^(1/2)/(a*(I*exp(x)^2+2*exp(x)-I)/
exp(x))^(1/2)/exp(x)
```

Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 188 vs. $2(49) = 98$.

Time = 0.09 (sec) , antiderivative size = 188, normalized size of antiderivative = 2.85

$$\int \frac{A + B\sinh(x)}{\sqrt{a + ia\sinh(x)}}\,dx = \frac{\sqrt{2}a\sqrt{-\frac{A^2+2i\,AB-B^2}{a}}\log\left(-\frac{2\left(\sqrt{2}a\sqrt{-\frac{A^2+2i\,AB-B^2}{a}}+2\sqrt{\frac{1}{2}i\,ae^{(-x)}}(i\,A-B)\right)}{-4i\,A+4\,B}\right) - \sqrt{2}a\sqrt{-\frac{A^2+2i\,AB-B^2}{a}}\log\left(\frac{2\left(\sqrt{2}a\right.}{}\right.}{a}$$

input
```
integrate((A+B*sinh(x))/(a+I*a*sinh(x))^(1/2),x, algorithm="fricas")
```

output
```
(sqrt(2)*a*sqrt(-(A^2 + 2*I*A*B - B^2)/a)*log(-2*(sqrt(2)*a*sqrt(-(A^2 + 2
*I*A*B - B^2)/a) + 2*sqrt(1/2*I*a*e^(-x))*(I*A - B))/(-4*I*A + 4*B)) - sqr
t(2)*a*sqrt(-(A^2 + 2*I*A*B - B^2)/a)*log(2*(sqrt(2)*a*sqrt(-(A^2 + 2*I*A*
B - B^2)/a) - 2*sqrt(1/2*I*a*e^(-x))*(I*A - B))/(-4*I*A + 4*B)) - 2*sqrt(1
/2*I*a*e^(-x))*(I*B*e^x - B))/a
```

Sympy [F]

$$\int \frac{A + B\sinh(x)}{\sqrt{a + ia\sinh(x)}}\,dx = \int \frac{A + B\sinh{(x)}}{\sqrt{ia\left(\sinh{(x)} - i\right)}}\,dx$$

input
```
integrate((A+B*sinh(x))/(a+I*a*sinh(x))**(1/2),x)
```

output
```
Integral((A + B*sinh(x))/sqrt(I*a*(sinh(x) - I)), x)
```

Maxima [F]

$$\int \frac{A + B\sinh(x)}{\sqrt{a + ia\sinh(x)}}\,dx = \int \frac{B\sinh(x) + A}{\sqrt{i\,a\sinh(x) + a}}\,dx$$

input
```
integrate((A+B*sinh(x))/(a+I*a*sinh(x))^(1/2),x, algorithm="maxima")
```

output
```
integrate((B*sinh(x) + A)/sqrt(I*a*sinh(x) + a), x)
```

Giac [F]

$$\int \frac{A + B\sinh(x)}{\sqrt{a + ia\sinh(x)}}\,dx = \int \frac{B\sinh(x) + A}{\sqrt{i\,a\sinh(x) + a}}\,dx$$

input
```
integrate((A+B*sinh(x))/(a+I*a*sinh(x))^(1/2),x, algorithm="giac")
```

output
```
integrate((B*sinh(x) + A)/sqrt(I*a*sinh(x) + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{A + B\sinh(x)}{\sqrt{a + ia\sinh(x)}}\,dx = \int \frac{A + B\sinh(x)}{\sqrt{a + a\sinh(x)\;1\mathrm{i}}}\,dx$$

input
```
int((A + B*sinh(x))/(a + a*sinh(x)*1i)^(1/2),x)
```

output
```
int((A + B*sinh(x))/(a + a*sinh(x)*1i)^(1/2), x)
```

Reduce [F]

$$\int \frac{A+B\sinh(x)}{\sqrt{a+ia\sinh(x)}}\,dx$$
$$= \frac{\sqrt{a}\left(\left(\int \frac{\sqrt{\sinh(x)i+1}}{\sinh(x)^2+1}dx\right)a-\left(\int \frac{\sqrt{\sinh(x)i+1}\,\sinh(x)^2}{\sinh(x)^2+1}dx\right)bi-\left(\int \frac{\sqrt{\sinh(x)i+1}\,\sinh(x)}{\sinh(x)^2+1}dx\right)ai+\left(\int \frac{\sqrt{\sinh(x)i+1}\,\sinh(x)}{\sinh(x)^2+1}\right.\right.}{a}$$

input
```
int((A+B*sinh(x))/(a+I*a*sinh(x))^(1/2),x)
```

output
```
(sqrt(a)*(int(sqrt(sinh(x)*i + 1)/(sinh(x)**2 + 1),x)*a - int((sqrt(sinh(x
)*i + 1)*sinh(x)**2)/(sinh(x)**2 + 1),x)*b*i - int((sqrt(sinh(x)*i + 1)*si
nh(x))/(sinh(x)**2 + 1),x)*a*i + int((sqrt(sinh(x)*i + 1)*sinh(x))/(sinh(x
)**2 + 1),x)*b))/a
```

3.124 $\int \frac{A+B\sinh(x)}{(a+ia\sinh(x))^{3/2}}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 79

$$\int \frac{A+B\sinh(x)}{(a+ia\sinh(x))^{3/2}}\,dx = \frac{(iA+3B)\text{arctanh}\left(\frac{\sqrt{a}\cosh(x)}{\sqrt{2}\sqrt{a+ia\sinh(x)}}\right)}{2\sqrt{2}a^{3/2}} + \frac{(iA-B)\cosh(x)}{2(a+ia\sinh(x))^{3/2}}$$

output
```
1/4*(I*A+3*B)*arctanh(1/2*a^(1/2)*cosh(x)*2^(1/2)/(a+I*a*sinh(x))^(1/2))*2
^(1/2)/a^(3/2)+1/2*(I*A-B)*cosh(x)/(a+I*a*sinh(x))^(3/2)
```

Mathematica [A] (warning: unable to verify)

Time = 0.34 (sec) , antiderivative size = 105, normalized size of antiderivative = 1.33

$$\int \frac{A+B\sinh(x)}{(a+ia\sinh(x))^{3/2}}\,dx = \frac{\left(\cosh\left(\frac{x}{2}\right)+i\sinh\left(\frac{x}{2}\right)\right)\left(i(A+iB)\cosh\left(\frac{x}{2}\right)+(A+iB)\sinh\left(\frac{x}{2}\right)+(1+i)\sqrt[4]{-\right.}{2(a+ia\sinh(x))^{3/2}}$$

input
```
Integrate[(A + B*Sinh[x])/(a + I*a*Sinh[x])^(3/2),x]
```

output
```
((Cosh[x/2] + I*Sinh[x/2])*(I*(A + I*B)*Cosh[x/2] + (A + I*B)*Sinh[x/2] +
(1 + I)*(-1)^(1/4)*(A - (3*I)*B)*ArcTan[(I + Tanh[x/4])/Sqrt[2]]*(-I + Sin
h[x])))/(2*(a + I*a*Sinh[x])^(3/2))
```

Rubi [A] (verified)

Time = 0.32 (sec) , antiderivative size = 79, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {3042, 3229, 3042, 3128, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+B\sinh(x)}{(a+ia\sinh(x))^{3/2}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{A-iB\sin(ix)}{(a+a\sin(ix))^{3/2}}dx$$

$$\Big\downarrow \text{3229}$$

$$\frac{(A-3iB)\int \frac{1}{\sqrt{i\sinh(x)a+a}}dx}{4a}+\frac{(-B+iA)\cosh(x)}{2(a+ia\sinh(x))^{3/2}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{(A-3iB)\int \frac{1}{\sqrt{\sin(ix)a+a}}dx}{4a}+\frac{(-B+iA)\cosh(x)}{2(a+ia\sinh(x))^{3/2}}$$

$$\Big\downarrow \text{3128}$$

$$\frac{i(A-3iB)\int \frac{1}{2a-\frac{a^2\cosh^2(x)}{i\sinh(x)a+a}}d\frac{a\cosh(x)}{\sqrt{i\sinh(x)a+a}}}{2a}+\frac{(-B+iA)\cosh(x)}{2(a+ia\sinh(x))^{3/2}}$$

$$\Big\downarrow \text{219}$$

$$\frac{i(A-3iB)\text{arctanh}\left(\frac{\sqrt{a}\cosh(x)}{\sqrt{2}\sqrt{a+ia\sinh(x)}}\right)}{2\sqrt{2}a^{3/2}}+\frac{(-B+iA)\cosh(x)}{2(a+ia\sinh(x))^{3/2}}$$

input
```
Int[(A + B*Sinh[x])/(a + I*a*Sinh[x])^(3/2),x]
```

output
```
((I/2)*(A - (3*I)*B)*ArcTanh[(Sqrt[a]*Cosh[x])/(Sqrt[2]*Sqrt[a + I*a*Sinh[
x]])])/(Sqrt[2]*a^(3/2)) + ((I*A - B)*Cosh[x])/(2*(a + I*a*Sinh[x])^(3/2))
```

Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3128
```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[-2/d
Subst[Int[1/(2*a - x^2), x], x, b*(Cos[c + d*x]/Sqrt[a + b*Sin[c + d*x]])],
 x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]
```

rule 3229
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Simp[(b*c - a*d)*Cos[e + f*x]*((a + b*Sin[e + f*
x])^m/(a*f*(2*m + 1))), x] + Simp[(a*d*m + b*c*(m + 1))/(a*b*(2*m + 1))   I
nt[(a + b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{a, b, c, d, e, f}, x] &&
NegQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && LtQ[m, -2^(-1)]
```

Maple [F]

$$\int \frac{A + B\sinh(x)}{(a + ia\sinh(x))^{\frac{3}{2}}} dx$$

input
```
int((A+B*sinh(x))/(a+I*a*sinh(x))^(3/2),x)
```

output
```
int((A+B*sinh(x))/(a+I*a*sinh(x))^(3/2),x)
```

Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 264 vs. 2(54) = 108.

Time = 0.09 (sec) , antiderivative size = 264, normalized size of antiderivative = 3.34

$$\int \frac{A+B\sinh(x)}{(a+ia\sinh(x))^{3/2}}\,dx = \frac{\sqrt{\frac{1}{2}}\left(a^2e^{(2x)}-2i\,a^2e^x-a^2\right)\sqrt{-\frac{A^2-6i\,AB-9\,B^2}{a^3}}\log\left(\frac{\sqrt{\frac{1}{2}}a^2\sqrt{-\frac{A^2-6i\,AB-9\,B^2}{a^3}}+\sqrt{\frac{1}{2}i\,ae^{(-}}}{i\,A+3\,B}\right.}{}$$

input
```
integrate((A+B*sinh(x))/(a+I*a*sinh(x))^(3/2),x, algorithm="fricas")
```

output
```
1/2*(sqrt(1/2)*(a^2*e^(2*x) - 2*I*a^2*e^x - a^2)*sqrt(-(A^2 - 6*I*A*B - 9*
B^2)/a^3)*log((sqrt(1/2)*a^2*sqrt(-(A^2 - 6*I*A*B - 9*B^2)/a^3) + sqrt(1/2
*I*a*e^(-x))*(I*A + 3*B))/(I*A + 3*B)) - sqrt(1/2)*(a^2*e^(2*x) - 2*I*a^2*
e^x - a^2)*sqrt(-(A^2 - 6*I*A*B - 9*B^2)/a^3)*log(-(sqrt(1/2)*a^2*sqrt(-(A
^2 - 6*I*A*B - 9*B^2)/a^3) - sqrt(1/2*I*a*e^(-x))*(I*A + 3*B))/(I*A + 3*B)
) - 2*((I*A - B)*e^(2*x) - (A + I*B)*e^x)*sqrt(1/2*I*a*e^(-x)))/(a^2*e^(2*
x) - 2*I*a^2*e^x - a^2)
```

Sympy [F]

$$\int \frac{A+B\sinh(x)}{(a+ia\sinh(x))^{3/2}}\,dx = \int \frac{A+B\sinh{(x)}}{(ia\,(\sinh{(x)}-i))^{\frac{3}{2}}}\,dx$$

input
```
integrate((A+B*sinh(x))/(a+I*a*sinh(x))**(3/2),x)
```

output
```
Integral((A + B*sinh(x))/(I*a*(sinh(x) - I))**(3/2), x)
```

Maxima [F]

$$\int \frac{A+B\sinh(x)}{(a+ia\sinh(x))^{3/2}}\,dx = \int \frac{B\sinh\left(x\right)+A}{\left(i\,a\sinh\left(x\right)+a\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate((A+B*sinh(x))/(a+I*a*sinh(x))^(3/2),x, algorithm="maxima")
```

output
```
integrate((B*sinh(x) + A)/(I*a*sinh(x) + a)^(3/2), x)
```

Giac [F]

$$\int \frac{A+B\sinh(x)}{(a+ia\sinh(x))^{3/2}}\,dx = \int \frac{B\sinh\left(x\right)+A}{\left(i\,a\sinh\left(x\right)+a\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate((A+B*sinh(x))/(a+I*a*sinh(x))^(3/2),x, algorithm="giac")
```

output
```
integrate((B*sinh(x) + A)/(I*a*sinh(x) + a)^(3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{A+B\sinh(x)}{(a+ia\sinh(x))^{3/2}}\,dx = \int \frac{A+B\sinh(x)}{{\left(a+a\sinh\left(x\right)\,1{}\mathrm{i}\right)}^{3/2}}\,dx$$

input
```
int((A + B*sinh(x))/(a + a*sinh(x)*1i)^(3/2),x)
```

output
```
int((A + B*sinh(x))/(a + a*sinh(x)*1i)^(3/2), x)
```

Reduce [F]

$$\int \frac{A+B\sinh(x)}{(a+ia\sinh(x))^{3/2}}\,dx = \frac{\sqrt{a}\left(\left(\int \frac{\sqrt{\sinh(x)i+1}}{\sinh(x)^3i+\sinh(x)^2+\sinh(x)i+1}dx\right)a-\left(\int \frac{\sqrt{\sinh(x)i+1}\,\sinh(x)^2}{\sinh(x)^3i+\sinh(x)^2+\sinh(x)i+1}dx\right)bi-}{a^2}$$

input
```
int((A+B*sinh(x))/(a+I*a*sinh(x))^(3/2),x)
```

output
```
(sqrt(a)*(int(sqrt(sinh(x)*i + 1)/(sinh(x)**3*i + sinh(x)**2 + sinh(x)*i +
 1),x)*a - int((sqrt(sinh(x)*i + 1)*sinh(x)**2)/(sinh(x)**3*i + sinh(x)**2
 + sinh(x)*i + 1),x)*b*i - int((sqrt(sinh(x)*i + 1)*sinh(x))/(sinh(x)**3*i
 + sinh(x)**2 + sinh(x)*i + 1),x)*a*i + int((sqrt(sinh(x)*i + 1)*sinh(x))/
(sinh(x)**3*i + sinh(x)**2 + sinh(x)*i + 1),x)*b))/a**2
```

3.125 $\int \frac{A+B\sinh(x)}{(a+ia\sinh(x))^{5/2}}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 110

$$\int \frac{A+B\sinh(x)}{(a+ia\sinh(x))^{5/2}}\,dx=\frac{(3iA+5B)\text{arctanh}\left(\frac{\sqrt{a}\cosh(x)}{\sqrt{2}\sqrt{a+ia\sinh(x)}}\right)}{16\sqrt{2}a^{5/2}}+\frac{(iA-B)\cosh(x)}{4(a+ia\sinh(x))^{5/2}}+\frac{(3iA+5B)\cosh(x)}{16a(a+ia\sinh(x))^{3/2}}$$

output
```
1/32*(3*I*A+5*B)*arctanh(1/2*a^(1/2)*cosh(x)*2^(1/2)/(a+I*a*sinh(x))^(1/2)
)*2^(1/2)/a^(5/2)+1/4*(I*A-B)*cosh(x)/(a+I*a*sinh(x))^(5/2)+1/16*(3*I*A+5*
B)*cosh(x)/a/(a+I*a*sinh(x))^(3/2)
```

Mathematica [A] (warning: unable to verify)

Time = 0.31 (sec) , antiderivative size = 184, normalized size of antiderivative = 1.67

$$\int \frac{A+B\sinh(x)}{(a+ia\sinh(x))^{5/2}}\,dx=\frac{\left(\cosh\left(\frac{x}{2}\right)+i\sinh\left(\frac{x}{2}\right)\right)\left(4i(A+iB)\left(\cosh\left(\frac{x}{2}\right)+i\sinh\left(\frac{x}{2}\right)\right)+(3iA+5B)\left(\cos\right.\right.}{}$$

input
```
Integrate[(A + B*Sinh[x])/(a + I*a*Sinh[x])^(5/2),x]
```

output
```
((Cosh[x/2] + I*Sinh[x/2])*((4*I)*(A + I*B)*(Cosh[x/2] + I*Sinh[x/2]) + ((
3*I)*A + 5*B)*(Cosh[x/2] + I*Sinh[x/2])^3 + (1 - I)*(-1)^(1/4)*(3*A - (5*I
)*B)*ArcTan[(I + Tanh[x/4])/Sqrt[2]]*(Cosh[x/2] + I*Sinh[x/2])^4 + 8*(A +
I*B)*Sinh[x/2] + 2*((3*I)*A + 5*B)*Sinh[x/2]*(-I + Sinh[x])))/(16*(a + I*a
*Sinh[x])^(5/2))
```

Rubi [A] (verified)

Time = 0.43 (sec) , antiderivative size = 110, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.350$, Rules used = {3042, 3229, 3042, 3129, 3042, 3128, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+B\sinh(x)}{(a+ia\sinh(x))^{5/2}}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{A-iB\sin(ix)}{(a+a\sin(ix))^{5/2}}dx$$

$$\Big\downarrow \text{3229}$$

$$\frac{(3A-5iB)\int \frac{1}{(i\sinh(x)a+a)^{3/2}}dx}{8a}+\frac{(-B+iA)\cosh(x)}{4(a+ia\sinh(x))^{5/2}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{(3A-5iB)\int \frac{1}{(\sin(ix)a+a)^{3/2}}dx}{8a}+\frac{(-B+iA)\cosh(x)}{4(a+ia\sinh(x))^{5/2}}$$

$$\Big\downarrow \text{3129}$$

$$\frac{(3A-5iB)\left(\frac{\int \frac{1}{\sqrt{i\sinh(x)a+a}}dx}{4a}+\frac{i\cosh(x)}{2(a+ia\sinh(x))^{3/2}}\right)}{8a}+\frac{(-B+iA)\cosh(x)}{4(a+ia\sinh(x))^{5/2}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{(3A-5iB)\left(\frac{\int \frac{1}{\sqrt{\sin(ix)a+a}}dx}{4a}+\frac{i\cosh(x)}{2(a+ia\sinh(x))^{3/2}}\right)}{8a}+\frac{(-B+iA)\cosh(x)}{4(a+ia\sinh(x))^{5/2}}$$

$$\Big\downarrow \text{3128}$$

$$\frac{(3A-5iB)\left(\frac{i\int \frac{1}{2a-\frac{a^2\cosh^2(x)}{i\sinh(x)a+a}}d\frac{a\cosh(x)}{\sqrt{i\sinh(x)a+a}}}{2a}+\frac{i\cosh(x)}{2(a+ia\sinh(x))^{3/2}}\right)}{8a}+\frac{(-B+iA)\cosh(x)}{4(a+ia\sinh(x))^{5/2}}$$

$$\Big\downarrow \text{219}$$

$$\frac{(3A-5iB)\left(\frac{i\text{arctanh}\left(\frac{\sqrt{a}\cosh(x)}{\sqrt{2}\sqrt{a+ia\sinh(x)}}\right)}{2\sqrt{2}a^{3/2}}+\frac{i\cosh(x)}{2(a+ia\sinh(x))^{3/2}}\right)}{8a}+\frac{(-B+iA)\cosh(x)}{4(a+ia\sinh(x))^{5/2}}$$

input
```
Int[(A + B*Sinh[x])/(a + I*a*Sinh[x])^(5/2),x]
```

output
```
((I*A - B)*Cosh[x])/(4*(a + I*a*Sinh[x])^(5/2)) + ((3*A - (5*I)*B)*(((I/2)
*ArcTanh[(Sqrt[a]*Cosh[x])/(Sqrt[2]*Sqrt[a + I*a*Sinh[x]])])/(Sqrt[2]*a^(3
/2)) + ((I/2)*Cosh[x])/(a + I*a*Sinh[x])^(3/2)))/(8*a)
```

Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3128
```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[-2/d
Subst[Int[1/(2*a - x^2), x], x, b*(Cos[c + d*x]/Sqrt[a + b*Sin[c + d*x]])],
 x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]
```

rule 3129
```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[b*Cos[c
+ d*x]*((a + b*Sin[c + d*x])^n/(a*d*(2*n + 1))), x] + Simp[(n + 1)/(a*(2*n
+ 1))   Int[(a + b*Sin[c + d*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d}, x]
&& EqQ[a^2 - b^2, 0] && LtQ[n, -1] && IntegerQ[2*n]
```

rule 3229
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Simp[(b*c - a*d)*Cos[e + f*x]*((a + b*Sin[e + f*
x])^m/(a*f*(2*m + 1))), x] + Simp[(a*d*m + b*c*(m + 1))/(a*b*(2*m + 1))   I
nt[(a + b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{a, b, c, d, e, f}, x] &&
NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && LtQ[m, -2^(-1)]
```

Maple [F]

$$\int \frac{A + B\sinh(x)}{(a + ia\sinh(x))^{\frac{5}{2}}}dx$$

input
```
int((A+B*sinh(x))/(a+I*a*sinh(x))^(5/2),x)
```

output
```
int((A+B*sinh(x))/(a+I*a*sinh(x))^(5/2),x)
```

Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 347 vs. $2(77) = 154$.

Time = 0.10 (sec) , antiderivative size = 347, normalized size of antiderivative = 3.15

$$\int \frac{A + B\sinh(x)}{(a + ia\sinh(x))^{5/2}}\,dx = \frac{\sqrt{\frac{1}{2}}\left(a^3e^{(4x)} - 4i\,a^3e^{(3x)} - 6\,a^3e^{(2x)} + 4i\,a^3e^x + a^3\right)\sqrt{-\frac{9\,A^2 - 30i\,AB - 25\,B^2}{a^5}}\log\left(\sqrt{\right.}}{}$$

input
```
integrate((A+B*sinh(x))/(a+I*a*sinh(x))^(5/2),x, algorithm="fricas")
```

output
```
1/16*(sqrt(1/2)*(a^3*e^(4*x) - 4*I*a^3*e^(3*x) - 6*a^3*e^(2*x) + 4*I*a^3*e
^x + a^3)*sqrt(-(9*A^2 - 30*I*A*B - 25*B^2)/a^5)*log((sqrt(1/2)*a^3*sqrt(-
(9*A^2 - 30*I*A*B - 25*B^2)/a^5) + sqrt(1/2*I*a*e^(-x))*(3*I*A + 5*B))/(3*
I*A + 5*B)) - sqrt(1/2)*(a^3*e^(4*x) - 4*I*a^3*e^(3*x) - 6*a^3*e^(2*x) + 4
*I*a^3*e^x + a^3)*sqrt(-(9*A^2 - 30*I*A*B - 25*B^2)/a^5)*log(-(sqrt(1/2)*a
^3*sqrt(-(9*A^2 - 30*I*A*B - 25*B^2)/a^5) - sqrt(1/2*I*a*e^(-x))*(3*I*A +
5*B))/(3*I*A + 5*B)) + 2*((-3*I*A - 5*B)*e^(4*x) - (11*A + 3*I*B)*e^(3*x)
+ (-11*I*A + 3*B)*e^(2*x) - (3*A - 5*I*B)*e^x)*sqrt(1/2*I*a*e^(-x)))/(a^3*
e^(4*x) - 4*I*a^3*e^(3*x) - 6*a^3*e^(2*x) + 4*I*a^3*e^x + a^3)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{A+B\sinh(x)}{(a+ia\sinh(x))^{5/2}}\,dx = \text{Timed out}$$

input
```
integrate((A+B*sinh(x))/(a+I*a*sinh(x))**(5/2),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{A+B\sinh(x)}{(a+ia\sinh(x))^{5/2}}\,dx = \int \frac{B\sinh(x)+A}{(i\,a\sinh(x)+a)^{\frac{5}{2}}}\,dx$$

input
```
integrate((A+B*sinh(x))/(a+I*a*sinh(x))^(5/2),x, algorithm="maxima")
```

output
```
integrate((B*sinh(x) + A)/(I*a*sinh(x) + a)^(5/2), x)
```

Giac [F]

$$\int \frac{A+B\sinh(x)}{(a+ia\sinh(x))^{5/2}}\,dx=\int \frac{B\sinh{(x)}+A}{(i\,a\sinh{(x)}+a)^{\frac{5}{2}}}\,dx$$

input
```
integrate((A+B*sinh(x))/(a+I*a*sinh(x))^(5/2),x, algorithm="giac")
```

output
```
integrate((B*sinh(x) + A)/(I*a*sinh(x) + a)^(5/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{A+B\sinh(x)}{(a+ia\sinh(x))^{5/2}}\,dx=\int \frac{A+B\sinh(x)}{(a+a\sinh{(x)}\ 1\mathrm{i})^{5/2}}\,dx$$

input
```
int((A + B*sinh(x))/(a + a*sinh(x)*1i)^(5/2),x)
```

output
```
int((A + B*sinh(x))/(a + a*sinh(x)*1i)^(5/2), x)
```

Reduce [F]

$$\int \frac{A+B\sinh(x)}{(a+ia\sinh(x))^{5/2}}\,dx=\frac{-\left(\int \frac{\sinh(x)}{\sqrt{\sinh(x)i+1}\,\sinh(x)^2-2\sqrt{\sinh(x)i+1}\,\sinh(x)i-\sqrt{\sinh(x)i+1}}dx\right)b-\left(\int \frac{}{\sqrt{\sinh(x)i+1}\,\sinh(}\right.}{\sqrt{a}\,a^2}$$

input
```
int((A+B*sinh(x))/(a+I*a*sinh(x))^(5/2),x)
```

output
```
( - (int(sinh(x)/(sqrt(sinh(x)*i + 1)*sinh(x)**2 - 2*sqrt(sinh(x)*i + 1)*s
inh(x)*i - sqrt(sinh(x)*i + 1)),x)*b + int(1/(sqrt(sinh(x)*i + 1)*sinh(x)*
*2 - 2*sqrt(sinh(x)*i + 1)*sinh(x)*i - sqrt(sinh(x)*i + 1)),x)*a))/(sqrt(a
)*a**2)
```

3.126 $\int (a+b\sinh(x))^{5/2}(A+B\sinh(x))\,dx$

Optimal result

Integrand size = 17, antiderivative size = 259

$$\begin{aligned}\int (a+b\sinh(x))^{5/2}(A+B\sinh(x))\,dx &= \frac{2}{105}\big(56aAb \\ &+ 15a^2B-25b^2B\big)\cosh(x)\sqrt{a+b\sinh(x)} \\ &+ \frac{2}{35}(7Ab+5aB)\cosh(x)(a+b\sinh(x))^{3/2}+\frac{2}{7}B\cosh(x)(a+b\sinh(x))^{5/2} \\ &+ \frac{2i(161a^2Ab-63Ab^3+15a^3B-145ab^2B)\,E\left(\frac{\pi}{4}-\frac{ix}{2}\middle|\frac{2b}{ia+b}\right)\sqrt{a+b\sinh(x)}}{105b\sqrt{\frac{a+b\sinh(x)}{a-ib}}} \\ &- \frac{2i(a^2+b^2)\left(56aAb+15a^2B-25b^2B\right)\text{EllipticF}\left(\frac{\pi}{4}-\frac{ix}{2},\frac{2b}{ia+b}\right)\sqrt{\frac{a+b\sinh(x)}{a-ib}}}{105b\sqrt{a+b\sinh(x)}}\end{aligned}$$

output

```
2/105*(56*A*a*b+15*B*a^2-25*B*b^2)*cosh(x)*(a+b*sinh(x))^(1/2)+2/35*(7*A*b
+5*B*a)*cosh(x)*(a+b*sinh(x))^(3/2)+2/7*B*cosh(x)*(a+b*sinh(x))^(5/2)+2/10
5*I*(161*A*a^2*b-63*A*b^3+15*B*a^3-145*B*a*b^2)*EllipticE(cos(1/4*Pi+1/2*I
*x),2^(1/2)*(b/(I*a+b))^(1/2))*(a+b*sinh(x))^(1/2)/b/((a+b*sinh(x))/(a-I*b
))^(1/2)+2/105*I*(a^2+b^2)*(56*A*a*b+15*B*a^2-25*B*b^2)*InverseJacobiAM(-1
/4*Pi+1/2*I*x,2^(1/2)*(b/(I*a+b))^(1/2))*((a+b*sinh(x))/(a-I*b))^(1/2)/b/(
a+b*sinh(x))^(1/2)
```

Mathematica [A] (verified)

Time = 0.59 (sec) , antiderivative size = 241, normalized size of antiderivative = 0.93

$$\int (a+b\sinh(x))^{5/2}(A+B\sinh(x))\,dx = \frac{\frac{2i\left(b(105a^3A-119aAb^2-135a^2bB+25b^3B)\,\text{EllipticF}\left(\frac{1}{4}(\pi-2ix),-\frac{2ib}{a-ib}\right)+(161a^2Ab-63Ab^3+15a^3B-145ab^2B)\left((a-ib)\right.}{b}}{}$$

input
```
Integrate[(a + b*Sinh[x])^(5/2)*(A + B*Sinh[x]),x]
```

output
```
(((2*I)*(b*(105*a^3*A - 119*a*A*b^2 - 135*a^2*b*B + 25*b^3*B)*EllipticF[(P
i - (2*I)*x)/4, ((-2*I)*b)/(a - I*b)] + (161*a^2*A*b - 63*A*b^3 + 15*a^3*B
 - 145*a*b^2*B)*((a - I*b)*EllipticE[(Pi - (2*I)*x)/4, ((-2*I)*b)/(a - I*b
)] - a*EllipticF[(Pi - (2*I)*x)/4, ((-2*I)*b)/(a - I*b)]))*Sqrt[(a + b*Sin
h[x])/(a - I*b)])/b + Cosh[x]*(a + b*Sinh[x])*(154*a*A*b + 90*a^2*B - 65*b
^2*B + 15*b^2*B*Cosh[2*x] + 6*b*(7*A*b + 15*a*B)*Sinh[x]))/(105*Sqrt[a + b
*Sinh[x]])
```

Rubi [A] (verified)

Time = 1.52 (sec) , antiderivative size = 270, normalized size of antiderivative = 1.04, number of steps used = 18, number of rules used = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 1.059$, Rules used = {3042, 3232, 27, 3042, 3232, 27, 3042, 3232, 27, 3042, 3231, 3042, 3134, 3042, 3132, 3142, 3042, 3140}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a+b\sinh(x))^{5/2}(A+B\sinh(x))\,dx$$

$$\Big\downarrow \; 3042$$

$$\int (a-ib\sin(ix))^{5/2}(A-iB\sin(ix))dx$$

$$\Big\downarrow \; 3232$$

$$\frac{2}{7}\int \frac{1}{2}(a+b\sinh(x))^{3/2}(7aA-5bB+(7Ab+5aB)\sinh(x))dx+\frac{2}{7}B\cosh(x)(a+b\sinh(x))^{5/2}$$

$$\Big\downarrow \; 27$$

$$\frac{1}{7}\int (a+b\sinh(x))^{3/2}(7aA-5bB+(7Ab+5aB)\sinh(x))dx+\frac{2}{7}B\cosh(x)(a+b\sinh(x))^{5/2}$$

$$\Big\downarrow \; 3042$$

$$\frac{2}{7}B\cosh(x)(a+b\sinh(x))^{5/2}+\frac{1}{7}\int (a-ib\sin(ix))^{3/2}(7aA-5bB-i(7Ab+5aB)\sin(ix))dx$$

$$\Big\downarrow \; 3232$$

$$\frac{1}{7}\left(\frac{2}{5}\int \frac{1}{2}\sqrt{a+b\sinh(x)}\left(35Aa^2-40bBa-21Ab^2+\left(15Ba^2+56Aba-25b^2B\right)\sinh(x)\right)dx+\frac{2}{5}\cosh(x)(5aB\right.$$
$$\frac{2}{7}B\cosh(x)(a+b\sinh(x))^{5/2}$$

$$\Big\downarrow \; 27$$

$$\frac{1}{7}\left(\frac{1}{5}\int \sqrt{a+b\sinh(x)}\left(35Aa^2-40bBa-21Ab^2+\left(15Ba^2+56Aba-25b^2B\right)\sinh(x)\right)dx+\frac{2}{5}\cosh(x)(5aB+\right.$$
$$\frac{2}{7}B\cosh(x)(a+b\sinh(x))^{5/2}$$

$$\Big\downarrow \; 3042$$

$$\frac{2}{7}B\cosh(x)(a+b\sinh(x))^{5/2}+$$
$$\frac{1}{7}\left(\frac{2}{5}\cosh(x)(5aB+7Ab)(a+b\sinh(x))^{3/2}+\frac{1}{5}\int \sqrt{a-ib\sin(ix)}\left(35Aa^2-40bBa-21Ab^2-i\left(15Ba^2+56Ab\right.\right.\right.$$

$$\Big\downarrow \; 3232$$

$$\frac{1}{7}\left(\frac{1}{5}\left(\frac{2}{3}\int \frac{105Aa^3-135bBa^2-119Ab^2a+25b^3B+\left(15Ba^3+161Aba^2-145b^2Ba-63Ab^3\right)\sinh(x)}{2\sqrt{a+b\sinh(x)}}dx+\frac{2}{3}\right.\right.$$
$$\frac{2}{7}B\cosh(x)(a+b\sinh(x))^{5/2}$$

$$\Big\downarrow \; 27$$

$$\frac{1}{7}\left(\frac{1}{5}\left(\frac{1}{3}\int \frac{105Aa^3-135bBa^2-119Ab^2a+25b^3B+\left(15Ba^3+161Aba^2-145b^2Ba-63Ab^3\right)\sinh(x)}{\sqrt{a+b\sinh(x)}}dx+\frac{2}{3}\right.\right.$$
$$\frac{2}{7}B\cosh(x)(a+b\sinh(x))^{5/2}$$

$$\Big\downarrow \text{3042}$$

$$\frac{2}{7}B\cosh(x)(a+b\sinh(x))^{5/2}+$$

$$\frac{1}{7}\left(\frac{2}{5}\cosh(x)(5aB+7Ab)(a+b\sinh(x))^{3/2}+\frac{1}{5}\left(\frac{2}{3}\cosh(x)\left(15a^2B+56aAb-25b^2B\right)\sqrt{a+b\sinh(x)}+\frac{1}{3}\int \frac{1}{}\right.\right.$$

$$\Big\downarrow \text{3231}$$

$$\frac{1}{7}\left(\frac{1}{5}\left(\frac{1}{3}\left(\frac{\left(15a^3B+161a^2Ab-145ab^2B-63Ab^3\right)\int\sqrt{a+b\sinh(x)}dx}{b}-\frac{\left(a^2+b^2\right)\left(15a^2B+56aAb-25b^2B\right)}{b}\right.\right.\right.$$

$$\frac{2}{7}B\cosh(x)(a+b\sinh(x))^{5/2}$$

$$\Big\downarrow \text{3042}$$

$$\frac{2}{7}B\cosh(x)(a+b\sinh(x))^{5/2}+$$

$$\frac{1}{7}\left(\frac{2}{5}\cosh(x)(5aB+7Ab)(a+b\sinh(x))^{3/2}+\frac{1}{5}\left(\frac{2}{3}\cosh(x)\left(15a^2B+56aAb-25b^2B\right)\sqrt{a+b\sinh(x)}+\frac{1}{3}\left(\frac{(1}{}\right.\right.\right.$$

$$\Big\downarrow \text{3134}$$

$$\frac{2}{7}B\cosh(x)(a+b\sinh(x))^{5/2}+$$

$$\frac{1}{7}\left(\frac{2}{5}\cosh(x)(5aB+7Ab)(a+b\sinh(x))^{3/2}+\frac{1}{5}\left(\frac{2}{3}\cosh(x)\left(15a^2B+56aAb-25b^2B\right)\sqrt{a+b\sinh(x)}+\frac{1}{3}\left(\frac{(1}{}\right.\right.\right.$$

$$\Big\downarrow \text{3042}$$

$$\frac{2}{7}B\cosh(x)(a+b\sinh(x))^{5/2}+$$

$$\frac{1}{7}\left(\frac{2}{5}\cosh(x)(5aB+7Ab)(a+b\sinh(x))^{3/2}+\frac{1}{5}\left(\frac{2}{3}\cosh(x)\left(15a^2B+56aAb-25b^2B\right)\sqrt{a+b\sinh(x)}+\frac{1}{3}\left(\frac{(1}{}\right.\right.\right.$$

$$\Big\downarrow \text{3132}$$

$$\frac{2}{7}B\cosh(x)(a+b\sinh(x))^{5/2}+$$

$$\frac{1}{7}\left(\frac{2}{5}\cosh(x)(5aB+7Ab)(a+b\sinh(x))^{3/2}+\frac{1}{5}\left(\frac{2}{3}\cosh(x)\left(15a^2B+56aAb-25b^2B\right)\sqrt{a+b\sinh(x)}+\frac{1}{3}\left(\frac{2a}{}\right.\right.\right.$$

$$\Big\downarrow \text{3142}$$

$$\frac{2}{7}B\cosh(x)(a+b\sinh(x))^{5/2}+$$
$$\frac{1}{7}\left(\frac{2}{5}\cosh(x)(5aB+7Ab)(a+b\sinh(x))^{3/2}+\frac{1}{5}\left(\frac{2}{3}\cosh(x)\left(15a^2B+56aAb-25b^2B\right)\sqrt{a+b\sinh(x)}+\frac{1}{3}\left(\frac{2i}{\ldots}\right.\right.\right.$$

$\big\downarrow$ 3042

$$\frac{2}{7}B\cosh(x)(a+b\sinh(x))^{5/2}+$$
$$\frac{1}{7}\left(\frac{2}{5}\cosh(x)(5aB+7Ab)(a+b\sinh(x))^{3/2}+\frac{1}{5}\left(\frac{2}{3}\cosh(x)\left(15a^2B+56aAb-25b^2B\right)\sqrt{a+b\sinh(x)}+\frac{1}{3}\left(\frac{2i}{\ldots}\right.\right.\right.$$

$\big\downarrow$ 3140

$$\frac{2}{7}B\cosh(x)(a+b\sinh(x))^{5/2}+$$
$$\frac{1}{7}\left(\frac{2}{5}\cosh(x)(5aB+7Ab)(a+b\sinh(x))^{3/2}+\frac{1}{5}\left(\frac{2}{3}\cosh(x)\left(15a^2B+56aAb-25b^2B\right)\sqrt{a+b\sinh(x)}+\frac{1}{3}\left(\frac{2i}{\ldots}\right.\right.\right.$$

input
```
Int[(a + b*Sinh[x])^(5/2)*(A + B*Sinh[x]),x]
```

output
```
(2*B*Cosh[x]*(a + b*Sinh[x])^(5/2))/7 + ((2*(7*A*b + 5*a*B)*Cosh[x]*(a + b
*Sinh[x])^(3/2))/5 + ((2*(56*a*A*b + 15*a^2*B - 25*b^2*B)*Cosh[x]*Sqrt[a +
 b*Sinh[x]])/3 + (((2*I)*(161*a^2*A*b - 63*A*b^3 + 15*a^3*B - 145*a*b^2*B)
*EllipticE[Pi/4 - (I/2)*x, (2*b)/(I*a + b)]*Sqrt[a + b*Sinh[x]])/(b*Sqrt[(
a + b*Sinh[x])/(a - I*b)]) - ((2*I)*(a^2 + b^2)*(56*a*A*b + 15*a^2*B - 25*
b^2*B)*EllipticF[Pi/4 - (I/2)*x, (2*b)/(I*a + b)]*Sqrt[(a + b*Sinh[x])/(a
- I*b)])/(b*Sqrt[a + b*Sinh[x]]))/3)/5)/7
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3132
```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[2*(Sqrt[a
 + b]/d)*EllipticE[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

rule 3134
```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)]   Int[Sqrt[a/(a + b) + (
b/(a + b))*Sin[c + d*x]], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2
, 0] &&  !GtQ[a + b, 0]
```

rule 3140
```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/(d*S
qrt[a + b]))*EllipticF[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

rule 3142
```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]]   Int[1/Sqrt[a/(a + b)
 + (b/(a + b))*Sin[c + d*x]], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
 b^2, 0] &&  !GtQ[a + b, 0]
```

rule 3231
```
Int[((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])/Sqrt[(a_) + (b_.)*sin[(e_.) + (
f_.)*(x_)]], x_Symbol] :> Simp[(b*c - a*d)/b   Int[1/Sqrt[a + b*Sin[e + f*x
]], x], x] + Simp[d/b   Int[Sqrt[a + b*Sin[e + f*x]], x], x] /; FreeQ[{a, b
, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
```

rule 3232
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Simp[(-d)*Cos[e + f*x]*((a + b*Sin[e + f*x])^m/(
f*(m + 1))), x] + Simp[1/(m + 1)   Int[(a + b*Sin[e + f*x])^(m - 1)*Simp[b*
d*m + a*c*(m + 1) + (a*d*m + b*c*(m + 1))*Sin[e + f*x], x], x], x] /; FreeQ
[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && GtQ[m,
 0] && IntegerQ[2*m]
```

Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 1877 vs. $2(230) = 460$.

Time = 4.02 (sec) , antiderivative size = 1878, normalized size of antiderivative = 7.25

method	result	size
parts	Expression too large to display	1878
default	Expression too large to display	1893

input
```
int((a+b*sinh(x))^(5/2)*(A+B*sinh(x)),x,method=_RETURNVERBOSE)
```

output
```
2/15*A*(8*I*(-(a+b*sinh(x))/(I*b-a))^(1/2)*((I-sinh(x))*b/(I*b+a))^(1/2)*(
b*(I+sinh(x))/(I*b-a))^(1/2)*EllipticF((-(a+b*sinh(x))/(I*b-a))^(1/2),(-(I
*b-a)/(I*b+a))^(1/2))*a^3*b+8*I*(-(a+b*sinh(x))/(I*b-a))^(1/2)*((I-sinh(x)
)*b/(I*b+a))^(1/2)*(b*(I+sinh(x))/(I*b-a))^(1/2)*EllipticF((-(a+b*sinh(x))
/(I*b-a))^(1/2),(-(I*b-a)/(I*b+a))^(1/2))*a*b^3+15*(-(a+b*sinh(x))/(I*b-a)
)^(1/2)*((I-sinh(x))*b/(I*b+a))^(1/2)*(b*(I+sinh(x))/(I*b-a))^(1/2)*Ellipt
icF((-(a+b*sinh(x))/(I*b-a))^(1/2),(-(I*b-a)/(I*b+a))^(1/2))*a^4+6*(-(a+b*
sinh(x))/(I*b-a))^(1/2)*((I-sinh(x))*b/(I*b+a))^(1/2)*(b*(I+sinh(x))/(I*b-
a))^(1/2)*EllipticF((-(a+b*sinh(x))/(I*b-a))^(1/2),(-(I*b-a)/(I*b+a))^(1/2
))*a^2*b^2-9*(-(a+b*sinh(x))/(I*b-a))^(1/2)*((I-sinh(x))*b/(I*b+a))^(1/2)*
(b*(I+sinh(x))/(I*b-a))^(1/2)*EllipticF((-(a+b*sinh(x))/(I*b-a))^(1/2),(-(
I*b-a)/(I*b+a))^(1/2))*b^4-23*(-(a+b*sinh(x))/(I*b-a))^(1/2)*((I-sinh(x))*
b/(I*b+a))^(1/2)*(b*(I+sinh(x))/(I*b-a))^(1/2)*EllipticE((-(a+b*sinh(x))/(
I*b-a))^(1/2),(-(I*b-a)/(I*b+a))^(1/2))*a^4-14*(-(a+b*sinh(x))/(I*b-a))^(1
/2)*((I-sinh(x))*b/(I*b+a))^(1/2)*(b*(I+sinh(x))/(I*b-a))^(1/2)*EllipticE(
(-(a+b*sinh(x))/(I*b-a))^(1/2),(-(I*b-a)/(I*b+a))^(1/2))*a^2*b^2+9*(-(a+b*
sinh(x))/(I*b-a))^(1/2)*((I-sinh(x))*b/(I*b+a))^(1/2)*(b*(I+sinh(x))/(I*b-
a))^(1/2)*EllipticE((-(a+b*sinh(x))/(I*b-a))^(1/2),(-(I*b-a)/(I*b+a))^(1/2
))*b^4+3*b^4*sinh(x)^4+14*a*b^3*sinh(x)^3+11*a^2*b^2*sinh(x)^2+3*b^4*sinh(
x)^2+14*a*b^3*sinh(x)+11*a^2*b^2)/b/cosh(x)/(a+b*sinh(x))^(1/2)+2/21*B*...
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1121 vs. $2(226) = 452$.

Time = 0.13 (sec) , antiderivative size = 1121, normalized size of antiderivative = 4.33

$$\int (a + b\sinh(x))^{5/2}(A + B\sinh(x))\, dx = \text{Too large to display}$$

input
```
integrate((a+b*sinh(x))^(5/2)*(A+B*sinh(x)),x, algorithm="fricas")
```

output
```
-1/1260*(16*sqrt(1/2)*((30*B*a^4 + 7*A*a^3*b + 115*B*a^2*b^2 + 231*A*a*b^3
 - 75*B*b^4)*cosh(x)^3 + 3*(30*B*a^4 + 7*A*a^3*b + 115*B*a^2*b^2 + 231*A*a
*b^3 - 75*B*b^4)*cosh(x)^2*sinh(x) + 3*(30*B*a^4 + 7*A*a^3*b + 115*B*a^2*b
^2 + 231*A*a*b^3 - 75*B*b^4)*cosh(x)*sinh(x)^2 + (30*B*a^4 + 7*A*a^3*b + 1
15*B*a^2*b^2 + 231*A*a*b^3 - 75*B*b^4)*sinh(x)^3)*sqrt(b)*weierstrassPInve
rse(4/3*(4*a^2 + 3*b^2)/b^2, -8/27*(8*a^3 + 9*a*b^2)/b^3, 1/3*(3*b*cosh(x)
 + 3*b*sinh(x) + 2*a)/b) + 48*sqrt(1/2)*((15*B*a^3*b + 161*A*a^2*b^2 - 145
*B*a*b^3 - 63*A*b^4)*cosh(x)^3 + 3*(15*B*a^3*b + 161*A*a^2*b^2 - 145*B*a*b
^3 - 63*A*b^4)*cosh(x)^2*sinh(x) + 3*(15*B*a^3*b + 161*A*a^2*b^2 - 145*B*a
*b^3 - 63*A*b^4)*cosh(x)*sinh(x)^2 + (15*B*a^3*b + 161*A*a^2*b^2 - 145*B*a
*b^3 - 63*A*b^4)*sinh(x)^3)*sqrt(b)*weierstrassZeta(4/3*(4*a^2 + 3*b^2)/b^
2, -8/27*(8*a^3 + 9*a*b^2)/b^3, weierstrassPInverse(4/3*(4*a^2 + 3*b^2)/b^
2, -8/27*(8*a^3 + 9*a*b^2)/b^3, 1/3*(3*b*cosh(x) + 3*b*sinh(x) + 2*a)/b))
- 3*(15*B*b^4*cosh(x)^6 + 15*B*b^4*sinh(x)^6 + 6*(15*B*a*b^3 + 7*A*b^4)*co
sh(x)^5 + 6*(15*B*b^4*cosh(x) + 15*B*a*b^3 + 7*A*b^4)*sinh(x)^5 + 15*B*b^4
 + (180*B*a^2*b^2 + 308*A*a*b^3 - 115*B*b^4)*cosh(x)^4 + (225*B*b^4*cosh(x
)^2 + 180*B*a^2*b^2 + 308*A*a*b^3 - 115*B*b^4 + 30*(15*B*a*b^3 + 7*A*b^4)*
cosh(x))*sinh(x)^4 - 8*(15*B*a^3*b + 161*A*a^2*b^2 - 145*B*a*b^3 - 63*A*b^
4)*cosh(x)^3 + 4*(75*B*b^4*cosh(x)^3 - 30*B*a^3*b - 322*A*a^2*b^2 + 290*B*
a*b^3 + 126*A*b^4 + 15*(15*B*a*b^3 + 7*A*b^4)*cosh(x)^2 + (180*B*a^2*b^...
```

Sympy [F(-1)]

Timed out.

$$\int (a+b\sinh(x))^{5/2}(A+B\sinh(x))\,dx = \text{Timed out}$$

input
```
integrate((a+b*sinh(x))**(5/2)*(A+B*sinh(x)),x)
```

output
```
Timed out
```

Maxima [F]

$$\int (a+b\sinh(x))^{5/2}(A+B\sinh(x))\,dx=\int (B\sinh{(x)}+A)(b\sinh{(x)}+a)^{\frac{5}{2}}\,dx$$

input
```
integrate((a+b*sinh(x))^(5/2)*(A+B*sinh(x)),x, algorithm="maxima")
```

output
```
integrate((B*sinh(x) + A)*(b*sinh(x) + a)^(5/2), x)
```

Giac [F]

$$\int (a+b\sinh(x))^{5/2}(A+B\sinh(x))\,dx=\int (B\sinh{(x)}+A)(b\sinh{(x)}+a)^{\frac{5}{2}}\,dx$$

input
```
integrate((a+b*sinh(x))^(5/2)*(A+B*sinh(x)),x, algorithm="giac")
```

output
```
integrate((B*sinh(x) + A)*(b*sinh(x) + a)^(5/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int (a+b\sinh(x))^{5/2}(A+B\sinh(x))\,dx=\int (A+B\sinh(x))\ (a+b\sinh(x))^{5/2}\,dx$$

input
```
int((A + B*sinh(x))*(a + b*sinh(x))^(5/2),x)
```

output
```
int((A + B*sinh(x))*(a + b*sinh(x))^(5/2), x)
```

Reduce [F]

$$\int (a+b\sinh(x))^{5/2}(A+B\sinh(x))\,dx = \left(\int \sqrt{\sinh(x)\,b+a}dx\right)a^3 + \left(\int \sqrt{\sinh(x)\,b+a}\,\sinh(x)^3\,dx\right)b^3 + 3\left(\int \sqrt{\sinh(x)\,b+a}\,\sinh(x)^2\,dx\right)a\,b^2 + 3\left(\int \sqrt{\sinh(x)\,b+a}\,\sinh(x)\,dx\right)a^2b$$

input
```
int((a+b*sinh(x))^(5/2)*(A+B*sinh(x)),x)
```

output
```
int(sqrt(sinh(x)*b + a),x)*a**3 + int(sqrt(sinh(x)*b + a)*sinh(x)**3,x)*b*
*3 + 3*int(sqrt(sinh(x)*b + a)*sinh(x)**2,x)*a*b**2 + 3*int(sqrt(sinh(x)*b
 + a)*sinh(x),x)*a**2*b
```

3.127 $\int (a+b\sinh(x))^{3/2}(A+B\sinh(x))\,dx$

Optimal result

Integrand size = 17, antiderivative size = 207

$$\int (a+b\sinh(x))^{3/2}(A+B\sinh(x))\,dx = \frac{2}{15}(5Ab+3aB)\cosh(x)\sqrt{a+b\sinh(x)} + \frac{2}{5}B\cosh(x)(a+b\sinh(x))^{3/2} + \frac{2i(20aAb+3a^2B-9b^2B)\,E\left(\frac{\pi}{4}-\frac{ix}{2}\middle|\frac{2b}{ia+b}\right)\sqrt{a+b\sinh(x)}}{15b\sqrt{\frac{a+b\sinh(x)}{a-ib}}} - \frac{2i(a^2+b^2)(5Ab+3aB)\,\text{EllipticF}\left(\frac{\pi}{4}-\frac{ix}{2},\frac{2b}{ia+b}\right)\sqrt{\frac{a+b\sinh(x)}{a-ib}}}{15b\sqrt{a+b\sinh(x)}}$$

output
```
2/15*(5*A*b+3*B*a)*cosh(x)*(a+b*sinh(x))^(1/2)+2/5*B*cosh(x)*(a+b*sinh(x))
^(3/2)+2/15*I*(20*A*a*b+3*B*a^2-9*B*b^2)*EllipticE(cos(1/4*Pi+1/2*I*x),2^(
1/2)*(b/(I*a+b))^(1/2))*(a+b*sinh(x))^(1/2)/b/((a+b*sinh(x))/(a-I*b))^(1/2
)+2/15*I*(a^2+b^2)*(5*A*b+3*B*a)*InverseJacobiAM(-1/4*Pi+1/2*I*x,2^(1/2)*(
b/(I*a+b))^(1/2))*((a+b*sinh(x))/(a-I*b))^(1/2)/b/(a+b*sinh(x))^(1/2)
```

Mathematica [A] (verified)

Time = 0.41 (sec) , antiderivative size = 196, normalized size of antiderivative = 0.95

$$\int (a+b\sinh(x))^{3/2}(A+B\sinh(x))\,dx = \frac{2\left(\frac{i\left(b(15a^2A-5Ab^2-12abB)\operatorname{EllipticF}\left(\frac{1}{4}(\pi-2ix),-\frac{2ib}{a-ib}\right)+(20aAb+3a^2B-9b^2B)\left((a-ib)E\left(\frac{1}{4}(\pi-2ix)|-\frac{2ib}{a-ib}\right)-a\operatorname{E}\right.\right.}{b}\right.}{15\sqrt{a+}}$$

input
```
Integrate[(a + b*Sinh[x])^(3/2)*(A + B*Sinh[x]),x]
```

output
```
(2*((I*(b*(15*a^2*A - 5*A*b^2 - 12*a*b*B)*EllipticF[(Pi - (2*I)*x)/4, ((-2
*I)*b)/(a - I*b)] + (20*a*A*b + 3*a^2*B - 9*b^2*B)*((a - I*b)*EllipticE[(P
i - (2*I)*x)/4, ((-2*I)*b)/(a - I*b)] - a*EllipticF[(Pi - (2*I)*x)/4, ((-2
*I)*b)/(a - I*b)]))*Sqrt[(a + b*Sinh[x])/(a - I*b)])/b + Cosh[x]*(a + b*Si
nh[x])*(5*A*b + 6*a*B + 3*b*B*Sinh[x])))/(15*Sqrt[a + b*Sinh[x]])
```

Rubi [A] (verified)

Time = 1.10 (sec) , antiderivative size = 213, normalized size of antiderivative = 1.03, number of steps used = 15, number of rules used = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.882$, Rules used = {3042, 3232, 27, 3042, 3232, 27, 3042, 3231, 3042, 3134, 3042, 3132, 3142, 3042, 3140}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a+b\sinh(x))^{3/2}(A+B\sinh(x))\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int (a-ib\sin(ix))^{3/2}(A-iB\sin(ix))dx$$

$$\Big\downarrow \text{3232}$$

$$\frac{2}{5}\int \frac{1}{2}\sqrt{a+b\sinh(x)}(5aA-3bB+(5Ab+3aB)\sinh(x))dx+\frac{2}{5}B\cosh(x)(a+b\sinh(x))^{3/2}$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{5}\int \sqrt{a+b\sinh(x)}(5aA-3bB+(5Ab+3aB)\sinh(x))dx+\frac{2}{5}B\cosh(x)(a+b\sinh(x))^{3/2}$$

$$\Big\downarrow \text{3042}$$

$$\frac{2}{5}B\cosh(x)(a+b\sinh(x))^{3/2}+\frac{1}{5}\int \sqrt{a-ib\sin(ix)}(5aA-3bB-i(5Ab+3aB)\sin(ix))dx$$

$$\Big\downarrow \text{3232}$$

$$\frac{1}{5}\left(\frac{2}{3}\int \frac{15Aa^2-12bBa-5Ab^2+\left(3Ba^2+20Aba-9b^2B\right)\sinh(x)}{2\sqrt{a+b\sinh(x)}}dx+\frac{2}{3}\cosh(x)(3aB+5Ab)\sqrt{a+b\sinh(x)}\right)$$
$$\frac{2}{5}B\cosh(x)(a+b\sinh(x))^{3/2}$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{5}\left(\frac{1}{3}\int \frac{15Aa^2-12bBa-5Ab^2+\left(3Ba^2+20Aba-9b^2B\right)\sinh(x)}{\sqrt{a+b\sinh(x)}}dx+\frac{2}{3}\cosh(x)(3aB+5Ab)\sqrt{a+b\sinh(x)}\right)$$
$$\frac{2}{5}B\cosh(x)(a+b\sinh(x))^{3/2}$$

$$\Big\downarrow \text{3042}$$

$$\frac{2}{5}B\cosh(x)(a+b\sinh(x))^{3/2}+$$
$$\frac{1}{5}\left(\frac{2}{3}\cosh(x)(3aB+5Ab)\sqrt{a+b\sinh(x)}+\frac{1}{3}\int \frac{15Aa^2-12bBa-5Ab^2-i\left(3Ba^2+20Aba-9b^2B\right)\sin(ix)}{\sqrt{a-ib\sin(ix)}}dx\right)$$

$$\Big\downarrow \text{3231}$$

$$\frac{1}{5}\left(\frac{1}{3}\left(\frac{\left(3a^2B+20aAb-9b^2B\right)\int \sqrt{a+b\sinh(x)}dx}{b}-\frac{\left(a^2+b^2\right)(3aB+5Ab)\int \frac{1}{\sqrt{a+b\sinh(x)}}dx}{b}\right)+\frac{2}{3}\cosh(x)(3$$
$$\frac{2}{5}B\cosh(x)(a+b\sinh(x))^{3/2}$$

$$\Big\downarrow \text{3042}$$

$$\frac{2}{5}B\cosh(x)(a+b\sinh(x))^{3/2}+$$
$$\frac{1}{5}\left(\frac{2}{3}\cosh(x)(3aB+5Ab)\sqrt{a+b\sinh(x)}+\frac{1}{3}\left(\frac{\left(3a^2B+20aAb-9b^2B\right)\int \sqrt{a-ib\sin(ix)}dx}{b}-\frac{\left(a^2+b^2\right)(3aB}{}\right.\right.$$

$$\Big\downarrow \text{3134}$$

$$\frac{2}{5}B\cosh(x)(a+b\sinh(x))^{3/2}+$$
$$\frac{1}{5}\left(\frac{2}{3}\cosh(x)(3aB+5Ab)\sqrt{a+b\sinh(x)}+\frac{1}{3}\left(\frac{\left(3a^2B+20aAb-9b^2B\right)\sqrt{a+b\sinh(x)}\int\sqrt{\frac{a}{a-ib}+\frac{b\sinh(x)}{a-ib}}dx}{b\sqrt{\frac{a+b\sinh(x)}{a-ib}}}\right.\right.$$

$\downarrow$ 3042

$$\frac{2}{5}B\cosh(x)(a+b\sinh(x))^{3/2}+$$
$$\frac{1}{5}\left(\frac{2}{3}\cosh(x)(3aB+5Ab)\sqrt{a+b\sinh(x)}+\frac{1}{3}\left(\frac{\left(3a^2B+20aAb-9b^2B\right)\sqrt{a+b\sinh(x)}\int\sqrt{\frac{a}{a-ib}-\frac{ib\sin(ix)}{a-ib}}dx}{b\sqrt{\frac{a+b\sinh(x)}{a-ib}}}\right.\right.$$

$\downarrow$ 3132

$$\frac{2}{5}B\cosh(x)(a+b\sinh(x))^{3/2}+$$
$$\frac{1}{5}\left(\frac{2}{3}\cosh(x)(3aB+5Ab)\sqrt{a+b\sinh(x)}+\frac{1}{3}\left(\frac{2i\left(3a^2B+20aAb-9b^2B\right)\sqrt{a+b\sinh(x)}E\left(\frac{\pi}{4}-\frac{ix}{2}|\frac{2b}{ia+b}\right)}{b\sqrt{\frac{a+b\sinh(x)}{a-ib}}}-\right.\right.$$

$\downarrow$ 3142

$$\frac{2}{5}B\cosh(x)(a+b\sinh(x))^{3/2}+$$
$$\frac{1}{5}\left(\frac{2}{3}\cosh(x)(3aB+5Ab)\sqrt{a+b\sinh(x)}+\frac{1}{3}\left(\frac{2i\left(3a^2B+20aAb-9b^2B\right)\sqrt{a+b\sinh(x)}E\left(\frac{\pi}{4}-\frac{ix}{2}|\frac{2b}{ia+b}\right)}{b\sqrt{\frac{a+b\sinh(x)}{a-ib}}}-\right.\right.$$

$\downarrow$ 3042

$$\frac{2}{5}B\cosh(x)(a+b\sinh(x))^{3/2}+$$
$$\frac{1}{5}\left(\frac{2}{3}\cosh(x)(3aB+5Ab)\sqrt{a+b\sinh(x)}+\frac{1}{3}\left(\frac{2i\left(3a^2B+20aAb-9b^2B\right)\sqrt{a+b\sinh(x)}E\left(\frac{\pi}{4}-\frac{ix}{2}|\frac{2b}{ia+b}\right)}{b\sqrt{\frac{a+b\sinh(x)}{a-ib}}}-\right.\right.$$

$\downarrow$ 3140

$$\frac{2}{5}B\cosh(x)(a+b\sinh(x))^{3/2}+$$
$$\frac{1}{5}\left(\frac{2}{3}\cosh(x)(3aB+5Ab)\sqrt{a+b\sinh(x)}+\frac{1}{3}\left(\frac{2i\left(3a^2B+20aAb-9b^2B\right)\sqrt{a+b\sinh(x)}E\left(\frac{\pi}{4}-\frac{ix}{2}|\frac{2b}{ia+b}\right)}{b\sqrt{\frac{a+b\sinh(x)}{a-ib}}}-\right.\right.$$

input
```
Int[(a + b*Sinh[x])^(3/2)*(A + B*Sinh[x]),x]
```

output
```
(2*B*Cosh[x]*(a + b*Sinh[x])^(3/2))/5 + ((2*(5*A*b + 3*a*B)*Cosh[x]*Sqrt[a
 + b*Sinh[x]])/3 + (((2*I)*(20*a*A*b + 3*a^2*B - 9*b^2*B)*EllipticE[Pi/4 -
 (I/2)*x, (2*b)/(I*a + b)]*Sqrt[a + b*Sinh[x]])/(b*Sqrt[(a + b*Sinh[x])/(a
 - I*b)]) - ((2*I)*(a^2 + b^2)*(5*A*b + 3*a*B)*EllipticF[Pi/4 - (I/2)*x, (
2*b)/(I*a + b)]*Sqrt[(a + b*Sinh[x])/(a - I*b)])/(b*Sqrt[a + b*Sinh[x]]))/
3)/5
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3132
```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[2*(Sqrt[a
 + b]/d)*EllipticE[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

rule 3134
```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)]   Int[Sqrt[a/(a + b) + (
b/(a + b))*Sin[c + d*x]], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2
, 0] &&  !GtQ[a + b, 0]
```

rule 3140
```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/(d*S
qrt[a + b]))*EllipticF[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

rule 3142
```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]]   Int[1/Sqrt[a/(a + b)
 + (b/(a + b))*Sin[c + d*x]], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
 b^2, 0] &&  !GtQ[a + b, 0]
```

rule 3231
```
Int[((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])/Sqrt[(a_) + (b_.)*sin[(e_.) + (
f_.)*(x_)]], x_Symbol] :> Simp[(b*c - a*d)/b   Int[1/Sqrt[a + b*Sin[e + f*x
]], x], x] + Simp[d/b   Int[Sqrt[a + b*Sin[e + f*x]], x], x] /; FreeQ[{a, b
, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
```

rule 3232
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Simp[(-d)*Cos[e + f*x]*((a + b*Sin[e + f*x])^m/(
f*(m + 1))), x] + Simp[1/(m + 1)   Int[(a + b*Sin[e + f*x])^(m - 1)*Simp[b*
d*m + a*c*(m + 1) + (a*d*m + b*c*(m + 1))*Sin[e + f*x], x], x], x] /; FreeQ
[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && GtQ[m,
 0] && IntegerQ[2*m]
```

Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 1032 vs. $2(182) = 364$.

Time = 2.50 (sec) , antiderivative size = 1033, normalized size of antiderivative = 4.99

method	result	size
default	Expression too large to display	1033
parts	Expression too large to display	1489

input
```
int((a+b*sinh(x))^(3/2)*(A+B*sinh(x)),x,method=_RETURNVERBOSE)
```

output
```
(cosh(x)^2*(a+b*sinh(x)))^(1/2)*(2*a^2*A*(1/b*a-I)*((-a-b*sinh(x))/(I*b-a)
)^(1/2)*((I-sinh(x))*b/(I*b+a))^(1/2)*(b*(I+sinh(x))/(I*b-a))^(1/2)/(cosh(
x)^2*(a+b*sinh(x)))^(1/2)*EllipticF(((-a-b*sinh(x))/(I*b-a))^(1/2),((a-I*b
)/(I*b+a))^(1/2))+b*(A*b+2*B*a)*(2/3/b*(cosh(x)^2*(a+b*sinh(x)))^(1/2)-2/3
*(1/b*a-I)*((-a-b*sinh(x))/(I*b-a))^(1/2)*((I-sinh(x))*b/(I*b+a))^(1/2)*(b
*(I+sinh(x))/(I*b-a))^(1/2)/(cosh(x)^2*(a+b*sinh(x)))^(1/2)*EllipticF(((-a
-b*sinh(x))/(I*b-a))^(1/2),((a-I*b)/(I*b+a))^(1/2))-4/3/b*a*(1/b*a-I)*((-a
-b*sinh(x))/(I*b-a))^(1/2)*((I-sinh(x))*b/(I*b+a))^(1/2)*(b*(I+sinh(x))/(I
*b-a))^(1/2)/(cosh(x)^2*(a+b*sinh(x)))^(1/2)*((-1/b*a-I)*EllipticE(((-a-b*
sinh(x))/(I*b-a))^(1/2),((a-I*b)/(I*b+a))^(1/2))+I*EllipticF(((-a-b*sinh(x
))/(I*b-a))^(1/2),((a-I*b)/(I*b+a))^(1/2))))+2*a*(2*A*b+B*a)*(1/b*a-I)*((-
a-b*sinh(x))/(I*b-a))^(1/2)*((I-sinh(x))*b/(I*b+a))^(1/2)*(b*(I+sinh(x))/(
I*b-a))^(1/2)/(cosh(x)^2*(a+b*sinh(x)))^(1/2)*((-1/b*a-I)*EllipticE(((-a-b
*sinh(x))/(I*b-a))^(1/2),((a-I*b)/(I*b+a))^(1/2))+I*EllipticF(((-a-b*sinh(
x))/(I*b-a))^(1/2),((a-I*b)/(I*b+a))^(1/2)))+B*b^2*(2/5/b*sinh(x)*(cosh(x)
^2*(a+b*sinh(x)))^(1/2)-8/15*a/b^2*(cosh(x)^2*(a+b*sinh(x)))^(1/2)-4/15/b*
a*(1/b*a-I)*((-a-b*sinh(x))/(I*b-a))^(1/2)*((I-sinh(x))*b/(I*b+a))^(1/2)*(
b*(I+sinh(x))/(I*b-a))^(1/2)/(cosh(x)^2*(a+b*sinh(x)))^(1/2)*EllipticF(((-
a-b*sinh(x))/(I*b-a))^(1/2),((a-I*b)/(I*b+a))^(1/2))+2*(-3/5+8/15*a^2/b^2)
*(1/b*a-I)*((-a-b*sinh(x))/(I*b-a))^(1/2)*((I-sinh(x))*b/(I*b+a))^(1/2)...
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 623 vs. $2(178) = 356$.

Time = 0.12 (sec) , antiderivative size = 623, normalized size of antiderivative = 3.01

$$\int (a + b\sinh(x))^{3/2}(A + B\sinh(x))\,dx = \text{Too large to display}$$

input
```
integrate((a+b*sinh(x))^(3/2)*(A+B*sinh(x)),x, algorithm="fricas")
```

output
```
-1/90*(8*sqrt(1/2)*((6*B*a^3 - 5*A*a^2*b + 18*B*a*b^2 + 15*A*b^3)*cosh(x)^
2 + 2*(6*B*a^3 - 5*A*a^2*b + 18*B*a*b^2 + 15*A*b^3)*cosh(x)*sinh(x) + (6*B
*a^3 - 5*A*a^2*b + 18*B*a*b^2 + 15*A*b^3)*sinh(x)^2)*sqrt(b)*weierstrassPI
nverse(4/3*(4*a^2 + 3*b^2)/b^2, -8/27*(8*a^3 + 9*a*b^2)/b^3, 1/3*(3*b*cosh
(x) + 3*b*sinh(x) + 2*a)/b) + 24*sqrt(1/2)*((3*B*a^2*b + 20*A*a*b^2 - 9*B*
b^3)*cosh(x)^2 + 2*(3*B*a^2*b + 20*A*a*b^2 - 9*B*b^3)*cosh(x)*sinh(x) + (3
*B*a^2*b + 20*A*a*b^2 - 9*B*b^3)*sinh(x)^2)*sqrt(b)*weierstrassZeta(4/3*(4
*a^2 + 3*b^2)/b^2, -8/27*(8*a^3 + 9*a*b^2)/b^3, weierstrassPInverse(4/3*(4
*a^2 + 3*b^2)/b^2, -8/27*(8*a^3 + 9*a*b^2)/b^3, 1/3*(3*b*cosh(x) + 3*b*sin
h(x) + 2*a)/b)) - 3*(3*B*b^3*cosh(x)^4 + 3*B*b^3*sinh(x)^4 - 3*B*b^3 + 2*(
6*B*a*b^2 + 5*A*b^3)*cosh(x)^3 + 2*(6*B*b^3*cosh(x) + 6*B*a*b^2 + 5*A*b^3)
*sinh(x)^3 - 4*(3*B*a^2*b + 20*A*a*b^2 - 9*B*b^3)*cosh(x)^2 + 2*(9*B*b^3*c
osh(x)^2 - 6*B*a^2*b - 40*A*a*b^2 + 18*B*b^3 + 3*(6*B*a*b^2 + 5*A*b^3)*cos
h(x))*sinh(x)^2 + 2*(6*B*a*b^2 + 5*A*b^3)*cosh(x) + 2*(6*B*b^3*cosh(x)^3 +
 6*B*a*b^2 + 5*A*b^3 + 3*(6*B*a*b^2 + 5*A*b^3)*cosh(x)^2 - 4*(3*B*a^2*b +
20*A*a*b^2 - 9*B*b^3)*cosh(x))*sinh(x))*sqrt(b*sinh(x) + a))/(b^2*cosh(x)^
2 + 2*b^2*cosh(x)*sinh(x) + b^2*sinh(x)^2)
```

Sympy [F]

$$\int (a+b\sinh(x))^{3/2}(A+B\sinh(x))\,dx=\int (A+B\sinh{(x)})\,(a+b\sinh{(x)})^{\frac{3}{2}}\,dx$$

input
```
integrate((a+b*sinh(x))**(3/2)*(A+B*sinh(x)),x)
```

output
```
Integral((A + B*sinh(x))*(a + b*sinh(x))**(3/2), x)
```

Maxima [F]

$$\int (a+b\sinh(x))^{3/2}(A+B\sinh(x))\,dx=\int (B\sinh{(x)}+A)(b\sinh{(x)}+a)^{\frac{3}{2}}\,dx$$

input
```
integrate((a+b*sinh(x))^(3/2)*(A+B*sinh(x)),x, algorithm="maxima")
```

output
```
integrate((B*sinh(x) + A)*(b*sinh(x) + a)^(3/2), x)
```

Giac [F]

$$\int (a+b\sinh(x))^{3/2}(A+B\sinh(x))\,dx=\int (B\sinh{(x)}+A)(b\sinh{(x)}+a)^{\frac{3}{2}}\,dx$$

input
```
integrate((a+b*sinh(x))^(3/2)*(A+B*sinh(x)),x, algorithm="giac")
```

output
```
integrate((B*sinh(x) + A)*(b*sinh(x) + a)^(3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int (a+b\sinh(x))^{3/2}(A+B\sinh(x))\,dx=\int (A+B\sinh(x))\,\,{(a+b\sinh(x))}^{3/2}\,dx$$

input
```
int((A + B*sinh(x))*(a + b*sinh(x))^(3/2),x)
```

output
```
int((A + B*sinh(x))*(a + b*sinh(x))^(3/2), x)
```

Reduce [F]

$$\int (a+b\sinh(x))^{3/2}(A+B\sinh(x))\,dx=\left(\int \sqrt{\sinh{(x)}\,b+a}dx\right)a^{2}+\left(\int \sqrt{\sinh{(x)}\,b+a}\,\sinh{(x)}^{2}\,dx\right)b^{2}+2\left(\int \sqrt{\sinh{(x)}\,b+a}\,\sinh{(x)}\,dx\right)ab$$

input
```
int((a+b*sinh(x))^(3/2)*(A+B*sinh(x)),x)
```

output
```
int(sqrt(sinh(x)*b + a),x)*a**2 + int(sqrt(sinh(x)*b + a)*sinh(x)**2,x)*b*
*2 + 2*int(sqrt(sinh(x)*b + a)*sinh(x),x)*a*b
```

3.128 $\int \sqrt{a+b\sinh(x)}(A+B\sinh(x))\,dx$

Optimal result

Integrand size = 17, antiderivative size = 164

$$\int \sqrt{a+b\sinh(x)}(A+B\sinh(x))\,dx = \frac{2}{3}B\cosh(x)\sqrt{a+b\sinh(x)}+\frac{2i(3Ab+aB)E\left(\frac{\pi}{4}-\frac{ix}{2}\middle|\frac{2b}{ia+b}\right)\sqrt{a+b\sinh(x)}}{3b\sqrt{\frac{a+b\sinh(x)}{a-ib}}} - \frac{2i(a^2+b^2)\,B\,\text{EllipticF}\left(\frac{\pi}{4}-\frac{ix}{2},\frac{2b}{ia+b}\right)\sqrt{\frac{a+b\sinh(x)}{a-ib}}}{3b\sqrt{a+b\sinh(x)}}$$

output
```
2/3*B*cosh(x)*(a+b*sinh(x))^(1/2)+2/3*I*(3*A*b+B*a)*EllipticE(cos(1/4*Pi+1
/2*I*x),2^(1/2)*(b/(I*a+b))^(1/2))*(a+b*sinh(x))^(1/2)/b/((a+b*sinh(x))/(a
-I*b))^(1/2)+2/3*I*(a^2+b^2)*B*InverseJacobiAM(-1/4*Pi+1/2*I*x,2^(1/2)*(b/
(I*a+b))^(1/2))*((a+b*sinh(x))/(a-I*b))^(1/2)/b/(a+b*sinh(x))^(1/2)
```

Mathematica [A] (verified)

Time = 0.31 (sec) , antiderivative size = 151, normalized size of antiderivative = 0.92

$$\int \sqrt{a+b\sinh(x)}(A+B\sinh(x))\,dx$$

$$= \frac{2bB\cosh(x)(a+b\sinh(x))+2(ia+b)(3Ab+aB)E\left(\frac{1}{4}(\pi-2ix)\middle|-\frac{2ib}{a-ib}\right)\sqrt{\frac{a+b\sinh(x)}{a-ib}}-2i(a^2+b^2)\,B\,\mathrm{Ell}}{3b\sqrt{a+b\sinh(x)}}$$

input
```
Integrate[Sqrt[a + b*Sinh[x]]*(A + B*Sinh[x]),x]
```

output
```
(2*b*B*Cosh[x]*(a + b*Sinh[x]) + 2*(I*a + b)*(3*A*b + a*B)*EllipticE[(Pi -
 (2*I)*x)/4, ((-2*I)*b)/(a - I*b)]*Sqrt[(a + b*Sinh[x])/(a - I*b)] - (2*I)
*(a^2 + b^2)*B*EllipticF[(Pi - (2*I)*x)/4, ((-2*I)*b)/(a - I*b)]*Sqrt[(a +
 b*Sinh[x])/(a - I*b)])/(3*b*Sqrt[a + b*Sinh[x]])
```

Rubi [A] (verified)

Time = 1.20 (sec) , antiderivative size = 165, normalized size of antiderivative = 1.01, number of steps used = 12, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.706$, Rules used = {3042, 3232, 27, 3042, 3231, 3042, 3134, 3042, 3132, 3142, 3042, 3140}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{a+b\sinh(x)}(A+B\sinh(x))\,dx$$

$$\Big\downarrow \; 3042$$

$$\int \sqrt{a-ib\sin(ix)}(A-iB\sin(ix))dx$$

$$\Big\downarrow \; 3232$$

$$\frac{2}{3}\int \frac{3aA-bB+(3Ab+aB)\sinh(x)}{2\sqrt{a+b\sinh(x)}}dx+\frac{2}{3}B\cosh(x)\sqrt{a+b\sinh(x)}$$

$$\Big\downarrow \; 27$$

$$\frac{1}{3}\int \frac{3aA-bB+(3Ab+aB)\sinh(x)}{\sqrt{a+b\sinh(x)}}dx+\frac{2}{3}B\cosh(x)\sqrt{a+b\sinh(x)}$$

$$\Big\downarrow \text{3042}$$

$$\frac{2}{3}B\cosh(x)\sqrt{a+b\sinh(x)}+\frac{1}{3}\int \frac{3aA-bB-i(3Ab+aB)\sin(ix)}{\sqrt{a-ib\sin(ix)}}dx$$

$$\Big\downarrow \text{3231}$$

$$\frac{1}{3}\left(\frac{(aB+3Ab)\int \sqrt{a+b\sinh(x)}dx}{b}-\frac{B\left(a^2+b^2\right)\int \frac{1}{\sqrt{a+b\sinh(x)}}dx}{b}\right)+\frac{2}{3}B\cosh(x)\sqrt{a+b\sinh(x)}$$

$$\Big\downarrow \text{3042}$$

$$\frac{2}{3}B\cosh(x)\sqrt{a+b\sinh(x)}+\frac{1}{3}\left(\frac{(aB+3Ab)\int \sqrt{a-ib\sin(ix)}dx}{b}-\frac{B\left(a^2+b^2\right)\int \frac{1}{\sqrt{a-ib\sin(ix)}}dx}{b}\right)$$

$$\Big\downarrow \text{3134}$$

$$\frac{2}{3}B\cosh(x)\sqrt{a+b\sinh(x)}+\frac{1}{3}\left(\frac{(aB+3Ab)\sqrt{a+b\sinh(x)}\int \sqrt{\frac{a}{a-ib}+\frac{b\sinh(x)}{a-ib}}dx}{b\sqrt{\frac{a+b\sinh(x)}{a-ib}}}-\frac{B\left(a^2+b^2\right)\int \frac{1}{\sqrt{a-ib\sin(ix)}}dx}{b}\right)$$

$$\Big\downarrow \text{3042}$$

$$\frac{2}{3}B\cosh(x)\sqrt{a+b\sinh(x)}+\frac{1}{3}\left(\frac{(aB+3Ab)\sqrt{a+b\sinh(x)}\int \sqrt{\frac{a}{a-ib}-\frac{ib\sin(ix)}{a-ib}}dx}{b\sqrt{\frac{a+b\sinh(x)}{a-ib}}}-\frac{B\left(a^2+b^2\right)\int \frac{1}{\sqrt{a-ib\sin(ix)}}dx}{b}\right)$$

$$\Big\downarrow \text{3132}$$

$$\frac{2}{3}B\cosh(x)\sqrt{a+b\sinh(x)}+\frac{1}{3}\left(\frac{2i(aB+3Ab)\sqrt{a+b\sinh(x)}E\left(\frac{\pi}{4}-\frac{ix}{2}|\frac{2b}{ia+b}\right)}{b\sqrt{\frac{a+b\sinh(x)}{a-ib}}}-\frac{B\left(a^2+b^2\right)\int \frac{1}{\sqrt{a-ib\sin(ix)}}dx}{b}\right)$$

$$\Big\downarrow \text{3142}$$

$$\frac{2}{3}B\cosh(x)\sqrt{a+b\sinh(x)}+\frac{1}{3}\left(\frac{2i(aB+3Ab)\sqrt{a+b\sinh(x)}E\left(\frac{\pi}{4}-\frac{ix}{2}|\frac{2b}{ia+b}\right)}{b\sqrt{\frac{a+b\sinh(x)}{a-ib}}}-\frac{B(a^2+b^2)\sqrt{\frac{a+b\sinh(x)}{a-ib}}\int\frac{1}{\sqrt{\frac{a}{a-ib}+\frac{b\sinh(x)}{a-ib}}}dx}{b\sqrt{a+b\sinh(x)}}\right)$$

$\Big\downarrow$ 3042

$$\frac{2}{3}B\cosh(x)\sqrt{a+b\sinh(x)}+\frac{1}{3}\left(\frac{2i(aB+3Ab)\sqrt{a+b\sinh(x)}E\left(\frac{\pi}{4}-\frac{ix}{2}|\frac{2b}{ia+b}\right)}{b\sqrt{\frac{a+b\sinh(x)}{a-ib}}}-\frac{B(a^2+b^2)\sqrt{\frac{a+b\sinh(x)}{a-ib}}\int\frac{1}{\sqrt{\frac{a}{a-ib}-\frac{ib\sin(ix)}{a-ib}}}dx}{b\sqrt{a+b\sinh(x)}}\right)$$

$\Big\downarrow$ 3140

$$\frac{2}{3}B\cosh(x)\sqrt{a+b\sinh(x)}+\frac{1}{3}\left(\frac{2i(aB+3Ab)\sqrt{a+b\sinh(x)}E\left(\frac{\pi}{4}-\frac{ix}{2}|\frac{2b}{ia+b}\right)}{b\sqrt{\frac{a+b\sinh(x)}{a-ib}}}-\frac{2iB(a^2+b^2)\sqrt{\frac{a+b\sinh(x)}{a-ib}}\text{EllipticF}\left(\frac{\pi}{4}-\frac{ix}{2},\frac{2b}{ia+b}\right)}{b\sqrt{a+b\sinh(x)}}\right)$$

input
```
Int[Sqrt[a + b*Sinh[x]]*(A + B*Sinh[x]),x]
```

output
```
(2*B*Cosh[x]*Sqrt[a + b*Sinh[x]])/3 + (((2*I)*(3*A*b + a*B)*EllipticE[Pi/4
 - (I/2)*x, (2*b)/(I*a + b)]*Sqrt[a + b*Sinh[x]])/(b*Sqrt[(a + b*Sinh[x])/
(a - I*b)]) - ((2*I)*(a^2 + b^2)*B*EllipticF[Pi/4 - (I/2)*x, (2*b)/(I*a +
b)]*Sqrt[(a + b*Sinh[x])/(a - I*b)])/(b*Sqrt[a + b*Sinh[x]]))/3
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3132
```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[2*(Sqrt[a
 + b]/d)*EllipticE[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

rule 3134
```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)]   Int[Sqrt[a/(a + b) + (
b/(a + b))*Sin[c + d*x]], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2
, 0] &&  !GtQ[a + b, 0]
```

rule 3140
```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/(d*S
qrt[a + b]))*EllipticF[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

rule 3142
```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]]   Int[1/Sqrt[a/(a + b)
 + (b/(a + b))*Sin[c + d*x]], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
 b^2, 0] &&  !GtQ[a + b, 0]
```

rule 3231
```
Int[((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])/Sqrt[(a_) + (b_.)*sin[(e_.) + (
f_.)*(x_)]], x_Symbol] :> Simp[(b*c - a*d)/b   Int[1/Sqrt[a + b*Sin[e + f*x
]], x], x] + Simp[d/b   Int[Sqrt[a + b*Sin[e + f*x]], x], x] /; FreeQ[{a, b
, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
```

rule 3232
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Simp[(-d)*Cos[e + f*x]*((a + b*Sin[e + f*x])^m/(
f*(m + 1))), x] + Simp[1/(m + 1)   Int[(a + b*Sin[e + f*x])^(m - 1)*Simp[b*
d*m + a*c*(m + 1) + (a*d*m + b*c*(m + 1))*Sin[e + f*x], x], x], x] /; FreeQ
[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && GtQ[m,
 0] && IntegerQ[2*m]
```

Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 730 vs. $2(143) = 286$.

Time = 1.66 (sec) , antiderivative size = 731, normalized size of antiderivative = 4.46

method	result
parts	$-\frac{2A(ib-a)\sqrt{-\frac{a+b\sinh(x)}{ib-a}}\sqrt{\frac{(i-\sinh(x))b}{ib+a}}\sqrt{\frac{b(i+\sinh(x))}{ib-a}}\left(i\,\mathrm{EllipticF}\left(\sqrt{-\frac{a+b\sinh(x)}{ib-a}},\sqrt{-\frac{ib-a}{ib+a}}\right)b-i\,\mathrm{EllipticE}\left(\sqrt{-\frac{a+b\sinh(x)}{ib-a}},\sqrt{\right.\right.}{b\cosh(x)\sqrt{a+b\sinh(x)}}$
default	$\frac{\frac{2iB\sqrt{-\frac{a+b\sinh(x)}{ib-a}}\sqrt{\frac{(i-\sinh(x))b}{ib+a}}\sqrt{\frac{b(i+\sinh(x))}{ib-a}}\,\mathrm{EllipticF}\left(\sqrt{-\frac{a+b\sinh(x)}{ib-a}},\sqrt{-\frac{ib-a}{ib+a}}\right)a^2b}{3}+\frac{2iB\sqrt{-\frac{a+b\sinh(x)}{ib-a}}\sqrt{\frac{(i-\sinh(x))b}{ib+a}}\sqrt{\frac{b(i+\sinh(}{ib-a}}}{3}}{}$

input
```
int((a+b*sinh(x))^(1/2)*(A+B*sinh(x)),x,method=_RETURNVERBOSE)
```

output
```
-2*A*(I*b-a)*(-(a+b*sinh(x))/(I*b-a))^(1/2)*((I-sinh(x))*b/(I*b+a))^(1/2)*
(b*(I+sinh(x))/(I*b-a))^(1/2)/b*(I*EllipticF((-(a+b*sinh(x))/(I*b-a))^(1/2
),(-(I*b-a)/(I*b+a))^(1/2))*b-I*EllipticE((-(a+b*sinh(x))/(I*b-a))^(1/2),(
-(I*b-a)/(I*b+a))^(1/2))*b+EllipticF((-(a+b*sinh(x))/(I*b-a))^(1/2),(-(I*b
-a)/(I*b+a))^(1/2))*a-EllipticE((-(a+b*sinh(x))/(I*b-a))^(1/2),(-(I*b-a)/(
I*b+a))^(1/2))*a)/cosh(x)/(a+b*sinh(x))^(1/2)+2/3*B*(I*(-(a+b*sinh(x))/(I*
b-a))^(1/2)*((I-sinh(x))*b/(I*b+a))^(1/2)*(b*(I+sinh(x))/(I*b-a))^(1/2)*El
lipticF((-(a+b*sinh(x))/(I*b-a))^(1/2),(-(I*b-a)/(I*b+a))^(1/2))*a^2*b+I*(
-(a+b*sinh(x))/(I*b-a))^(1/2)*((I-sinh(x))*b/(I*b+a))^(1/2)*(b*(I+sinh(x))
/(I*b-a))^(1/2)*EllipticF((-(a+b*sinh(x))/(I*b-a))^(1/2),(-(I*b-a)/(I*b+a)
)^(1/2))*b^3-(-(a+b*sinh(x))/(I*b-a))^(1/2)*((I-sinh(x))*b/(I*b+a))^(1/2)*
(b*(I+sinh(x))/(I*b-a))^(1/2)*EllipticE((-(a+b*sinh(x))/(I*b-a))^(1/2),(-(
I*b-a)/(I*b+a))^(1/2))*a^3-(-(a+b*sinh(x))/(I*b-a))^(1/2)*((I-sinh(x))*b/(
I*b+a))^(1/2)*(b*(I+sinh(x))/(I*b-a))^(1/2)*EllipticE((-(a+b*sinh(x))/(I*b
-a))^(1/2),(-(I*b-a)/(I*b+a))^(1/2))*a*b^2+b^3*sinh(x)^3+a*b^2*sinh(x)^2+s
inh(x)*b^3+a*b^2)/b^2/cosh(x)/(a+b*sinh(x))^(1/2)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 318 vs. $2(140) = 280$.

Time = 0.10 (sec) , antiderivative size = 318, normalized size of antiderivative = 1.94

$$\int \sqrt{a+b\sinh(x)}(A+B\sinh(x))\,dx =$$
$$-\frac{4\sqrt{\frac{1}{2}}\left(\left(2\,Ba^2-3\,Aab+3\,Bb^2\right)\cosh\left(x\right)+\left(2\,Ba^2-3\,Aab+3\,Bb^2\right)\sinh\left(x\right)\right)\sqrt{b}\text{weierstrassPInverse}\left(\frac{4}{\right.}$$

input
```
integrate((a+b*sinh(x))^(1/2)*(A+B*sinh(x)),x, algorithm="fricas")
```

output
```
-1/9*(4*sqrt(1/2)*((2*B*a^2 - 3*A*a*b + 3*B*b^2)*cosh(x) + (2*B*a^2 - 3*A*
a*b + 3*B*b^2)*sinh(x))*sqrt(b)*weierstrassPInverse(4/3*(4*a^2 + 3*b^2)/b^
2, -8/27*(8*a^3 + 9*a*b^2)/b^3, 1/3*(3*b*cosh(x) + 3*b*sinh(x) + 2*a)/b) +
 12*sqrt(1/2)*((B*a*b + 3*A*b^2)*cosh(x) + (B*a*b + 3*A*b^2)*sinh(x))*sqrt
(b)*weierstrassZeta(4/3*(4*a^2 + 3*b^2)/b^2, -8/27*(8*a^3 + 9*a*b^2)/b^3,
weierstrassPInverse(4/3*(4*a^2 + 3*b^2)/b^2, -8/27*(8*a^3 + 9*a*b^2)/b^3,
1/3*(3*b*cosh(x) + 3*b*sinh(x) + 2*a)/b)) - 3*(B*b^2*cosh(x)^2 + B*b^2*sin
h(x)^2 + B*b^2 - 2*(B*a*b + 3*A*b^2)*cosh(x) + 2*(B*b^2*cosh(x) - B*a*b -
3*A*b^2)*sinh(x))*sqrt(b*sinh(x) + a))/(b^2*cosh(x) + b^2*sinh(x))
```

Sympy [F]

$$\int \sqrt{a+b\sinh(x)}(A+B\sinh(x))\,dx = \int \left(A+B\sinh\left(x\right)\right)\sqrt{a+b\sinh\left(x\right)}\,dx$$

input
```
integrate((a+b*sinh(x))**(1/2)*(A+B*sinh(x)),x)
```

output
```
Integral((A + B*sinh(x))*sqrt(a + b*sinh(x)), x)
```

Maxima [F]

$$\int \sqrt{a+b\sinh(x)}(A+B\sinh(x))\,dx=\int (B\sinh\left(x\right)+A)\sqrt{b\sinh\left(x\right)+a}\,dx$$

input
```
integrate((a+b*sinh(x))^(1/2)*(A+B*sinh(x)),x, algorithm="maxima")
```

output
```
integrate((B*sinh(x) + A)*sqrt(b*sinh(x) + a), x)
```

Giac [F]

$$\int \sqrt{a+b\sinh(x)}(A+B\sinh(x))\,dx=\int (B\sinh\left(x\right)+A)\sqrt{b\sinh\left(x\right)+a}\,dx$$

input
```
integrate((a+b*sinh(x))^(1/2)*(A+B*sinh(x)),x, algorithm="giac")
```

output
```
integrate((B*sinh(x) + A)*sqrt(b*sinh(x) + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int \sqrt{a+b\sinh(x)}(A+B\sinh(x))\,dx=\int (A+B\sinh(x))\ \sqrt{a+b\sinh\left(x\right)}\,dx$$

input
```
int((A + B*sinh(x))*(a + b*sinh(x))^(1/2),x)
```

output
```
int((A + B*sinh(x))*(a + b*sinh(x))^(1/2), x)
```

Reduce [F]

$$\int \sqrt{a+b\sinh(x)}(A+B\sinh(x))\,dx = \left(\int \sqrt{\sinh(x)\,b+a}dx\right)a + \left(\int \sqrt{\sinh(x)\,b+a}\,\sinh(x)\,dx\right)b$$

input
```
int((a+b*sinh(x))^(1/2)*(A+B*sinh(x)),x)
```

output
```
int(sqrt(sinh(x)*b + a),x)*a + int(sqrt(sinh(x)*b + a)*sinh(x),x)*b
```

3.129 $\int \frac{A+B\sinh(x)}{a+b\sinh(x)}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 55

$$\int \frac{A+B\sinh(x)}{a+b\sinh(x)}\,dx = \frac{Bx}{b} - \frac{2(Ab-aB)\text{arctanh}\left(\frac{b-a\tanh\left(\frac{x}{2}\right)}{\sqrt{a^2+b^2}}\right)}{b\sqrt{a^2+b^2}}$$

output
```
B*x/b-2*(A*b-B*a)*arctanh((b-a*tanh(1/2*x))/(a^2+b^2)^(1/2))/b/(a^2+b^2)^(
1/2)
```

Mathematica [A] (verified)

Time = 0.14 (sec) , antiderivative size = 61, normalized size of antiderivative = 1.11

$$\int \frac{A+B\sinh(x)}{a+b\sinh(x)}\,dx = \frac{Bx + \frac{2(Ab-aB)\arctan\left(\frac{b-a\tanh\left(\frac{x}{2}\right)}{\sqrt{-a^2-b^2}}\right)}{\sqrt{-a^2-b^2}}}{b}$$

input
```
Integrate[(A + B*Sinh[x])/(a + b*Sinh[x]),x]
```

output
```
(B*x + (2*(A*b - a*B)*ArcTan[(b - a*Tanh[x/2])/Sqrt[-a^2 - b^2]])/Sqrt[-a^
2 - b^2])/b
```

Rubi [A] (verified)

Time = 0.50 (sec) , antiderivative size = 60, normalized size of antiderivative = 1.09, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.400, Rules used = {3042, 3214, 3042, 3139, 1083, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+B\sinh(x)}{a+b\sinh(x)}dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \frac{A-iB\sin(ix)}{a-ib\sin(ix)}dx$$

$$\Big\downarrow \text{ 3214}$$

$$\frac{(Ab-aB)\int \frac{1}{a+b\sinh(x)}dx}{b}+\frac{Bx}{b}$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{Bx}{b}+\frac{(Ab-aB)\int \frac{1}{a-ib\sin(ix)}dx}{b}$$

$$\Big\downarrow \text{ 3139}$$

$$\frac{2(Ab-aB)\int \frac{1}{-a\tanh^2\left(\frac{x}{2}\right)+2b\tanh\left(\frac{x}{2}\right)+a}d\tanh\left(\frac{x}{2}\right)}{b}+\frac{Bx}{b}$$

$$\Big\downarrow \text{ 1083}$$

$$\frac{Bx}{b}-\frac{4(Ab-aB)\int \frac{1}{4(a^2+b^2)-\left(2b-2a\tanh\left(\frac{x}{2}\right)\right)^2}d\left(2b-2a\tanh\left(\frac{x}{2}\right)\right)}{b}$$

$$\Big\downarrow \text{ 219}$$

$$\frac{Bx}{b}-\frac{2(Ab-aB)\text{arctanh}\left(\frac{2b-2a\tanh\left(\frac{x}{2}\right)}{2\sqrt{a^2+b^2}}\right)}{b\sqrt{a^2+b^2}}$$

input
```
Int[(A + B*Sinh[x])/(a + b*Sinh[x]),x]
```

output
```
(B*x)/b - (2*(A*b - a*B)*ArcTanh[(2*b - 2*a*Tanh[x/2])/(2*Sqrt[a^2 + b^2])
])/(b*Sqrt[a^2 + b^2])
```

Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1083
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Simp[-2   Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3139
```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Simp[2*(e/d)   Subst[Int[1/(a + 2*b*e*x + a
*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[a^2 - b^2, 0]
```

rule 3214
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])/((c_.) + (d_.)*sin[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[b*(x/d), x] - Simp[(b*c - a*d)/d   Int[1/(c + d
*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Maple [A] (verified)

Time = 0.14 (sec) , antiderivative size = 72, normalized size of antiderivative = 1.31

method	result
default	$-\frac{2(-Ab+aB)\operatorname{arctanh}\left(\frac{2a\tanh\left(\frac{x}{2}\right)-2b}{2\sqrt{a^2+b^2}}\right)}{b\sqrt{a^2+b^2}}+\frac{B\ln\left(\tanh\left(\frac{x}{2}\right)+1\right)}{b}-\frac{B\ln\left(\tanh\left(\frac{x}{2}\right)-1\right)}{b}$
risch	$\frac{Bx}{b}+\frac{\ln\left(e^x+\frac{a\sqrt{a^2+b^2}-a^2-b^2}{\sqrt{a^2+b^2}\,b}\right)A}{\sqrt{a^2+b^2}}-\frac{\ln\left(e^x+\frac{a\sqrt{a^2+b^2}-a^2-b^2}{\sqrt{a^2+b^2}\,b}\right)aB}{\sqrt{a^2+b^2}\,b}-\frac{\ln\left(e^x+\frac{a\sqrt{a^2+b^2}+a^2+b^2}{\sqrt{a^2+b^2}\,b}\right)A}{\sqrt{a^2+b^2}}+\frac{\ln\left(e^x+\frac{a\sqrt{a^2+b^2}+a^2}{\sqrt{a^2+b^2}\,b}\right.}{\sqrt{a^2+b^2}\,b}$

input
```
int((A+B*sinh(x))/(a+b*sinh(x)),x,method=_RETURNVERBOSE)
```

output
```
-2*(-A*b+B*a)/b/(a^2+b^2)^(1/2)*arctanh(1/2*(2*a*tanh(1/2*x)-2*b)/(a^2+b^2
)^(1/2))+B/b*ln(tanh(1/2*x)+1)-B/b*ln(tanh(1/2*x)-1)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 147 vs. $2(51) = 102$.

Time = 0.10 (sec) , antiderivative size = 147, normalized size of antiderivative = 2.67

$$\int \frac{A+B\sinh(x)}{a+b\sinh(x)}\,dx = -\frac{(Ba-Ab)\sqrt{a^2+b^2}\log\left(\frac{b^2\cosh(x)^2+b^2\sinh(x)^2+2\,ab\cosh(x)+2\,a^2+b^2+2\,(b^2\cosh(x)+ab)\sinh(x)-2\,\sqrt{a^2+b^2}(b\cosh(x)+b\sinh}{b\cosh(x)^2+b\sinh(x)^2+2\,a\cosh(x)+2\,(b\cosh(x)+a)\sinh(x)-b}\right.}{a^2b+b^3}$$

input
```
integrate((A+B*sinh(x))/(a+b*sinh(x)),x, algorithm="fricas")
```

output
```
-((B*a - A*b)*sqrt(a^2 + b^2)*log((b^2*cosh(x)^2 + b^2*sinh(x)^2 + 2*a*b*c
osh(x) + 2*a^2 + b^2 + 2*(b^2*cosh(x) + a*b)*sinh(x) - 2*sqrt(a^2 + b^2)*(
b*cosh(x) + b*sinh(x) + a))/(b*cosh(x)^2 + b*sinh(x)^2 + 2*a*cosh(x) + 2*(
b*cosh(x) + a)*sinh(x) - b)) - (B*a^2 + B*b^2)*x)/(a^2*b + b^3)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 21.07 (sec) , antiderivative size = 309, normalized size of antiderivative = 5.62

$$\int \frac{A+B\sinh(x)}{a+b\sinh(x)}\,dx$$
$$= \begin{cases} \tilde{\infty}\left(A\log\left(\tanh\left(\frac{x}{2}\right)\right)+Bx\right) \\ \frac{A\log\left(\tanh\left(\frac{x}{2}\right)\right)+Bx}{b} \\ \frac{Ax+B\cosh(x)}{a} \\ \frac{2iA}{b\tanh\left(\frac{x}{2}\right)-ib}+\frac{Bx\tanh\left(\frac{x}{2}\right)}{b\tanh\left(\frac{x}{2}\right)-ib}-\frac{iBx}{b\tanh\left(\frac{x}{2}\right)-ib}-\frac{2B}{b\tanh\left(\frac{x}{2}\right)-ib} \\ -\frac{2iA}{b\tanh\left(\frac{x}{2}\right)+ib}+\frac{Bx\tanh\left(\frac{x}{2}\right)}{b\tanh\left(\frac{x}{2}\right)+ib}+\frac{iBx}{b\tanh\left(\frac{x}{2}\right)+ib}-\frac{2B}{b\tanh\left(\frac{x}{2}\right)+ib} \\ -\frac{A\log\left(\tanh\left(\frac{x}{2}\right)-\frac{b}{a}-\frac{\sqrt{a^2+b^2}}{a}\right)}{\sqrt{a^2+b^2}}+\frac{A\log\left(\tanh\left(\frac{x}{2}\right)-\frac{b}{a}+\frac{\sqrt{a^2+b^2}}{a}\right)}{\sqrt{a^2+b^2}}+\frac{Ba\log\left(\tanh\left(\frac{x}{2}\right)-\frac{b}{a}-\frac{\sqrt{a^2+b^2}}{a}\right)}{b\sqrt{a^2+b^2}}-\frac{Ba\log\left(\tanh\left(\frac{x}{2}\right)-\frac{b}{a}+\sqrt{\right.}}{b\sqrt{a^2+b^2}} \end{cases}$$

input
```
integrate((A+B*sinh(x))/(a+b*sinh(x)),x)
```

output
```
Piecewise((zoo*(A*log(tanh(x/2)) + B*x), Eq(a, 0) & Eq(b, 0)), ((A*log(tan
h(x/2)) + B*x)/b, Eq(a, 0)), ((A*x + B*cosh(x))/a, Eq(b, 0)), (2*I*A/(b*ta
nh(x/2) - I*b) + B*x*tanh(x/2)/(b*tanh(x/2) - I*b) - I*B*x/(b*tanh(x/2) -
I*b) - 2*B/(b*tanh(x/2) - I*b), Eq(a, -I*b)), (-2*I*A/(b*tanh(x/2) + I*b)
+ B*x*tanh(x/2)/(b*tanh(x/2) + I*b) + I*B*x/(b*tanh(x/2) + I*b) - 2*B/(b*t
anh(x/2) + I*b), Eq(a, I*b)), (-A*log(tanh(x/2) - b/a - sqrt(a**2 + b**2)/
a)/sqrt(a**2 + b**2) + A*log(tanh(x/2) - b/a + sqrt(a**2 + b**2)/a)/sqrt(a
**2 + b**2) + B*a*log(tanh(x/2) - b/a - sqrt(a**2 + b**2)/a)/(b*sqrt(a**2
+ b**2)) - B*a*log(tanh(x/2) - b/a + sqrt(a**2 + b**2)/a)/(b*sqrt(a**2 + b
**2)) + B*x/b, True))
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 124 vs. $2(51) = 102$.

Time = 0.13 (sec) , antiderivative size = 124, normalized size of antiderivative = 2.25

$$\int \frac{A + B\sinh(x)}{a + b\sinh(x)}\,dx = -B\left(\frac{a\log\left(\frac{be^{(-x)} - a - \sqrt{a^2+b^2}}{be^{(-x)} - a + \sqrt{a^2+b^2}}\right)}{\sqrt{a^2+b^2}b} - \frac{x}{b}\right) + \frac{A\log\left(\frac{be^{(-x)} - a - \sqrt{a^2+b^2}}{be^{(-x)} - a + \sqrt{a^2+b^2}}\right)}{\sqrt{a^2+b^2}}$$

input
```
integrate((A+B*sinh(x))/(a+b*sinh(x)),x, algorithm="maxima")
```

output
```
-B*(a*log((b*e^(-x) - a - sqrt(a^2 + b^2))/(b*e^(-x) - a + sqrt(a^2 + b^2)
))/(sqrt(a^2 + b^2)*b) - x/b) + A*log((b*e^(-x) - a - sqrt(a^2 + b^2))/(b*
e^(-x) - a + sqrt(a^2 + b^2)))/sqrt(a^2 + b^2)
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 75, normalized size of antiderivative = 1.36

$$\int \frac{A + B\sinh(x)}{a + b\sinh(x)}\,dx = \frac{Bx}{b} - \frac{(Ba - Ab)\log\left(\frac{|2\,be^x + 2\,a - 2\,\sqrt{a^2+b^2}|}{|2\,be^x + 2\,a + 2\,\sqrt{a^2+b^2}|}\right)}{\sqrt{a^2+b^2}b}$$

input
```
integrate((A+B*sinh(x))/(a+b*sinh(x)),x, algorithm="giac")
```

output
```
B*x/b - (B*a - A*b)*log(abs(2*b*e^x + 2*a - 2*sqrt(a^2 + b^2))/abs(2*b*e^x
 + 2*a + 2*sqrt(a^2 + b^2)))/(sqrt(a^2 + b^2)*b)
```

Mupad [B] (verification not implemented)

Time = 1.85 (sec) , antiderivative size = 269, normalized size of antiderivative = 4.89

$$\int \frac{A+B\sinh(x)}{a+b\sinh(x)}\,dx = \frac{B\,x}{b} - \frac{2\,\mathrm{atan}\left(\frac{b^2\,\mathrm{e}^x\,\sqrt{-a^2\,b^2-b^4}\,\left(\frac{2\,\left(A\,b\,\sqrt{-a^2\,b^2-b^4}-B\,a\,\sqrt{-a^2\,b^2-b^4}\right)}{b^4\,\sqrt{-a^2\,b^2-b^4}\,\sqrt{{\left(A\,b-B\,a\right)}^2}}+\frac{2\,a^2\,\sqrt{A^2\,b^2-2\,A\,B\,a\,b+B^2\,a^2}}{b^2\,\sqrt{-b^2\,\left(a^2+b^2\right)}\,\sqrt{-a^2\,b^2-b^4}\,\left(A\,b-B\,a\right)}\right)}{2}-\frac{a\,b\,\sqrt{A^2\,b^2-2\,A\,B\,a\,b+B^2\,a^2}}{\sqrt{-b^2\,\left(a^2+b^2\right)}\,\left(A\,b-B\,a\right)}\right)\ldots}{\sqrt{-a^2\,b^2-b^4}}$$

input
```
int((A + B*sinh(x))/(a + b*sinh(x)),x)
```

output
```
(B*x)/b - (2*atan((b^2*exp(x)*(- b^4 - a^2*b^2)^(1/2)*((2*(A*b*(- b^4 - a^
2*b^2)^(1/2) - B*a*(- b^4 - a^2*b^2)^(1/2)))/(b^4*(- b^4 - a^2*b^2)^(1/2)*
((A*b - B*a)^2)^(1/2)) + (2*a^2*(A^2*b^2 + B^2*a^2 - 2*A*B*a*b)^(1/2))/(b^
2*(-b^2*(a^2 + b^2))^(1/2)*(- b^4 - a^2*b^2)^(1/2)*(A*b - B*a))))/2 - (a*b
*(A^2*b^2 + B^2*a^2 - 2*A*B*a*b)^(1/2))/((-b^2*(a^2 + b^2))^(1/2)*(A*b - B
*a)))*(A^2*b^2 + B^2*a^2 - 2*A*B*a*b)^(1/2))/(- b^4 - a^2*b^2)^(1/2)
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 1, normalized size of antiderivative = 0.02

$$\int \frac{A+B\sinh(x)}{a+b\sinh(x)}\,dx = x$$

input
```
int((A+B*sinh(x))/(a+b*sinh(x)),x)
```

output
```
x
```

3.130 $\int \frac{A+B\sinh(x)}{(a+b\sinh(x))^2}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 74

$$\int \frac{A+B\sinh(x)}{(a+b\sinh(x))^2}\,dx = -\frac{2(aA+bB)\operatorname{arctanh}\left(\frac{b-a\tanh\left(\frac{x}{2}\right)}{\sqrt{a^2+b^2}}\right)}{(a^2+b^2)^{3/2}} - \frac{(Ab-aB)\cosh(x)}{(a^2+b^2)\,(a+b\sinh(x))}$$

output
```
-2*(A*a+B*b)*arctanh((b-a*tanh(1/2*x))/(a^2+b^2)^(1/2))/(a^2+b^2)^(3/2)-(A
*b-B*a)*cosh(x)/(a^2+b^2)/(a+b*sinh(x))
```

Mathematica [A] (verified)

Time = 0.14 (sec) , antiderivative size = 82, normalized size of antiderivative = 1.11

$$\int \frac{A+B\sinh(x)}{(a+b\sinh(x))^2}\,dx = \frac{\frac{2(aA+bB)\arctan\left(\frac{b-a\tanh\left(\frac{x}{2}\right)}{\sqrt{-a^2-b^2}}\right)}{\sqrt{-a^2-b^2}} + \frac{(-Ab+aB)\cosh(x)}{a+b\sinh(x)}}{a^2+b^2}$$

input
```
Integrate[(A + B*Sinh[x])/(a + b*Sinh[x])^2,x]
```

output
```
((2*(a*A + b*B)*ArcTan[(b - a*Tanh[x/2])/Sqrt[-a^2 - b^2]])/Sqrt[-a^2 - b^
2] + ((-(A*b) + a*B)*Cosh[x])/(a + b*Sinh[x]))/(a^2 + b^2)
```

Rubi [A] (verified)

Time = 0.37 (sec) , antiderivative size = 79, normalized size of antiderivative = 1.07, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.533, Rules used = {3042, 3233, 25, 27, 3042, 3139, 1083, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+B\sinh(x)}{(a+b\sinh(x))^2}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{A-iB\sin(ix)}{(a-ib\sin(ix))^2}dx$$

$$\Big\downarrow \text{3233}$$

$$-\frac{\int -\frac{aA+bB}{a+b\sinh(x)}dx}{a^2+b^2}-\frac{\cosh(x)(Ab-aB)}{(a^2+b^2)(a+b\sinh(x))}$$

$$\Big\downarrow \text{25}$$

$$\frac{\int \frac{aA+bB}{a+b\sinh(x)}dx}{a^2+b^2}-\frac{\cosh(x)(Ab-aB)}{(a^2+b^2)(a+b\sinh(x))}$$

$$\Big\downarrow \text{27}$$

$$\frac{(aA+bB)\int \frac{1}{a+b\sinh(x)}dx}{a^2+b^2}-\frac{\cosh(x)(Ab-aB)}{(a^2+b^2)(a+b\sinh(x))}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{\cosh(x)(Ab-aB)}{(a^2+b^2)(a+b\sinh(x))}+\frac{(aA+bB)\int \frac{1}{a-ib\sin(ix)}dx}{a^2+b^2}$$

$$\Big\downarrow \text{3139}$$

$$\frac{2(aA+bB)\int \frac{1}{-a\tanh^2\left(\frac{x}{2}\right)+2b\tanh\left(\frac{x}{2}\right)+a}d\tanh\left(\frac{x}{2}\right)}{a^2+b^2}-\frac{\cosh(x)(Ab-aB)}{(a^2+b^2)(a+b\sinh(x))}$$

$$\Big\downarrow \text{1083}$$

$$-\frac{4(aA+bB)\int \frac{1}{4(a^2+b^2)-\left(2b-2a\tanh\left(\frac{x}{2}\right)\right)^2}d\left(2b-2a\tanh\left(\frac{x}{2}\right)\right)}{a^2+b^2}-\frac{\cosh(x)(Ab-aB)}{(a^2+b^2)(a+b\sinh(x))}$$

$$\Big\downarrow \text{219}$$

$$-\frac{2(aA+bB)\text{arctanh}\left(\frac{2b-2a\tanh\left(\frac{x}{2}\right)}{2\sqrt{a^2+b^2}}\right)}{(a^2+b^2)^{3/2}}-\frac{\cosh(x)(Ab-aB)}{(a^2+b^2)\,(a+b\sinh(x))}$$

input
```
Int[(A + B*Sinh[x])/(a + b*Sinh[x])^2,x]
```

output
```
(-2*(a*A + b*B)*ArcTanh[(2*b - 2*a*Tanh[x/2])/(2*Sqrt[a^2 + b^2])])/(a^2 +
 b^2)^(3/2) - ((A*b - a*B)*Cosh[x])/((a^2 + b^2)*(a + b*Sinh[x]))
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1083
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Simp[-2   Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3139
```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Simp[2*(e/d)   Subst[Int[1/(a + 2*b*e*x + a
*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[a^2 - b^2, 0]
```

rule 3233
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Simp[(-(b*c - a*d))*Cos[e + f*x]*((a + b*Sin[e +
 f*x])^(m + 1)/(f*(m + 1)*(a^2 - b^2))), x] + Simp[1/((m + 1)*(a^2 - b^2))
  Int[(a + b*Sin[e + f*x])^(m + 1)*Simp[(a*c - b*d)*(m + 1) - (b*c - a*d)*(
m + 2)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c
- a*d, 0] && NeQ[a^2 - b^2, 0] && LtQ[m, -1] && IntegerQ[2*m]
```

Maple [A] (verified)

Time = 0.18 (sec) , antiderivative size = 113, normalized size of antiderivative = 1.53

method	result
default	$-\frac{2\left(-\frac{b(Ab-aB)\tanh\left(\frac{x}{2}\right)}{a\left(a^2+b^2\right)}-\frac{Ab-aB}{a^2+b^2}\right)}{\tanh\left(\frac{x}{2}\right)^2 a-2b\tanh\left(\frac{x}{2}\right)-a}+\frac{2(Aa+bB)\,\mathrm{arctanh}\left(\frac{2a\tanh\left(\frac{x}{2}\right)-2b}{2\sqrt{a^2+b^2}}\right)}{\left(a^2+b^2\right)^{\frac{3}{2}}}$
risch	$\frac{2(Ab-aB)(\mathrm{e}^x a-b)}{b(a^2+b^2)(b\,\mathrm{e}^{2x}+2\,\mathrm{e}^x a-b)}+\frac{\ln\left(\mathrm{e}^x+\frac{a\left(a^2+b^2\right)^{\frac{3}{2}}-a^4-2a^2b^2-b^4}{b\left(a^2+b^2\right)^{\frac{3}{2}}}\right)Aa}{\left(a^2+b^2\right)^{\frac{3}{2}}}+\frac{\ln\left(\mathrm{e}^x+\frac{a\left(a^2+b^2\right)^{\frac{3}{2}}-a^4-2a^2b^2-b^4}{b\left(a^2+b^2\right)^{\frac{3}{2}}}\right)bB}{\left(a^2+b^2\right)^{\frac{3}{2}}}-\frac{\ln\left(\mathrm{e}^x+\frac{a(\cdots}{\cdots}\right.}{\cdots}$

input
```
int((A+B*sinh(x))/(a+b*sinh(x))^2,x,method=_RETURNVERBOSE)
```

output
```
-2*(-b*(A*b-B*a)/a/(a^2+b^2)*tanh(1/2*x)-(A*b-B*a)/(a^2+b^2))/(tanh(1/2*x)
^2*a-2*b*tanh(1/2*x)-a)+2*(A*a+B*b)/(a^2+b^2)^(3/2)*arctanh(1/2*(2*a*tanh(
1/2*x)-2*b)/(a^2+b^2)^(1/2))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 444 vs. 2(69) = 138.

Time = 0.14 (sec) , antiderivative size = 444, normalized size of antiderivative = 6.00

$$\int \frac{A+B\sinh(x)}{(a+b\sinh(x))^2}\,dx =$$

$$-\frac{2\,Ba^3b-2\,Aa^2b^2+2\,Bab^3-2\,Ab^4-\left(Aab^2+Bb^3-(Aab^2+Bb^3)\cosh{(x)}^2-(Aab^2+Bb^3)\sinh{(x)}\cdots\right.}{a^4b^2+\cdots}$$

input
```
integrate((A+B*sinh(x))/(a+b*sinh(x))^2,x, algorithm="fricas")
```

output
```
-(2*B*a^3*b - 2*A*a^2*b^2 + 2*B*a*b^3 - 2*A*b^4 - (A*a*b^2 + B*b^3 - (A*a*
b^2 + B*b^3)*cosh(x)^2 - (A*a*b^2 + B*b^3)*sinh(x)^2 - 2*(A*a^2*b + B*a*b^
2)*cosh(x) - 2*(A*a^2*b + B*a*b^2 + (A*a*b^2 + B*b^3)*cosh(x))*sinh(x))*sq
rt(a^2 + b^2)*log((b^2*cosh(x)^2 + b^2*sinh(x)^2 + 2*a*b*cosh(x) + 2*a^2 +
 b^2 + 2*(b^2*cosh(x) + a*b)*sinh(x) - 2*sqrt(a^2 + b^2)*(b*cosh(x) + b*si
nh(x) + a))/(b*cosh(x)^2 + b*sinh(x)^2 + 2*a*cosh(x) + 2*(b*cosh(x) + a)*s
inh(x) - b)) - 2*(B*a^4 - A*a^3*b + B*a^2*b^2 - A*a*b^3)*cosh(x) - 2*(B*a^
4 - A*a^3*b + B*a^2*b^2 - A*a*b^3)*sinh(x))/(a^4*b^2 + 2*a^2*b^4 + b^6 - (
a^4*b^2 + 2*a^2*b^4 + b^6)*cosh(x)^2 - (a^4*b^2 + 2*a^2*b^4 + b^6)*sinh(x)
^2 - 2*(a^5*b + 2*a^3*b^3 + a*b^5)*cosh(x) - 2*(a^5*b + 2*a^3*b^3 + a*b^5
+ (a^4*b^2 + 2*a^2*b^4 + b^6)*cosh(x))*sinh(x))
```

Sympy [F(-1)]

Timed out.

$$\int \frac{A+B\sinh(x)}{(a+b\sinh(x))^2}\,dx = \text{Timed out}$$

input
```
integrate((A+B*sinh(x))/(a+b*sinh(x))**2,x)
```

output
```
Timed out
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 229 vs. $2(69) = 138$.

Time = 0.12 (sec) , antiderivative size = 229, normalized size of antiderivative = 3.09

$$\int \frac{A+B\sinh(x)}{(a+b\sinh(x))^2}\,dx = A\left(\frac{a\log\left(\frac{be^{(-x)}-a-\sqrt{a^2+b^2}}{be^{(-x)}-a+\sqrt{a^2+b^2}}\right)}{(a^2+b^2)^{\frac{3}{2}}} - \frac{2\left(ae^{(-x)}+b\right)}{a^2b+b^3+2\left(a^3+ab^2\right)e^{(-x)}-\left(a^2b+b^3\right)e^{(-2x)}}\right) + B\left(\frac{b\log\left(\frac{be^{(-x)}-a-\sqrt{a^2+b^2}}{be^{(-x)}-a+\sqrt{a^2+b^2}}\right)}{(a^2+b^2)^{\frac{3}{2}}} + \frac{2\left(a^2e^{(-x)}+ab\right)}{a^2b^2+b^4+2\left(a^3b+ab^3\right)e^{(-x)}-\left(a^2b^2+b^4\right)e^{(-2x)}}\right)$$

input
```
integrate((A+B*sinh(x))/(a+b*sinh(x))^2,x, algorithm="maxima")
```

output
```
A*(a*log((b*e^(-x) - a - sqrt(a^2 + b^2))/(b*e^(-x) - a + sqrt(a^2 + b^2))
)/(a^2 + b^2)^(3/2) - 2*(a*e^(-x) + b)/(a^2*b + b^3 + 2*(a^3 + a*b^2)*e^(-
x) - (a^2*b + b^3)*e^(-2*x))) + B*(b*log((b*e^(-x) - a - sqrt(a^2 + b^2))/
(b*e^(-x) - a + sqrt(a^2 + b^2)))/(a^2 + b^2)^(3/2) + 2*(a^2*e^(-x) + a*b)
/(a^2*b^2 + b^4 + 2*(a^3*b + a*b^3)*e^(-x) - (a^2*b^2 + b^4)*e^(-2*x)))
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 119, normalized size of antiderivative = 1.61

$$\int \frac{A+B\sinh(x)}{(a+b\sinh(x))^2}\,dx = \frac{(Aa+Bb)\log\left(\frac{\left|2\,be^x+2\,a-2\,\sqrt{a^2+b^2}\right|}{\left|2\,be^x+2\,a+2\,\sqrt{a^2+b^2}\right|}\right)}{(a^2+b^2)^{\frac{3}{2}}} - \frac{2\left(Ba^2e^x-Aabe^x-Bab+Ab^2\right)}{(a^2b+b^3)(be^{(2x)}+2\,ae^x-b)}$$

input
```
integrate((A+B*sinh(x))/(a+b*sinh(x))^2,x, algorithm="giac")
```

output
```
(A*a + B*b)*log(abs(2*b*e^x + 2*a - 2*sqrt(a^2 + b^2))/abs(2*b*e^x + 2*a +
 2*sqrt(a^2 + b^2)))/(a^2 + b^2)^(3/2) - 2*(B*a^2*e^x - A*a*b*e^x - B*a*b
+ A*b^2)/((a^2*b + b^3)*(b*e^(2*x) + 2*a*e^x - b))
```

Mupad [B] (verification not implemented)

Time = 1.79 (sec) , antiderivative size = 223, normalized size of antiderivative = 3.01

$$\int \frac{A+B\sinh(x)}{(a+b\sinh(x))^2}\,dx = \frac{\ln\left(\frac{2\,(b-a\,e^x)\,(A\,a+B\,b)}{b\,(a^2+b^2)^{3/2}}-\frac{2\,e^x\,(A\,a+B\,b)}{a^2\,b+b^3}\right)\,(A\,a+B\,b)}{(a^2+b^2)^{3/2}} - \frac{\ln\left(-\frac{2\,e^x\,(A\,a+B\,b)}{a^2\,b+b^3}-\frac{2\,(b-a\,e^x)\,(A\,a+B\,b)}{b\,(a^2+b^2)^{3/2}}\right)\,(A\,a+B\,b)}{(a^2+b^2)^{3/2}} - \frac{\frac{2\,(A\,b^3-B\,a\,b^2)}{b\,(a^2\,b+b^3)}+\frac{2\,e^x\,(B\,a^2\,b^2-A\,a\,b^3)}{b^2\,(a^2\,b+b^3)}}{2\,a\,e^x-b+b\,e^{2\,x}}$$

input
```
int((A + B*sinh(x))/(a + b*sinh(x))^2,x)
```

output
```
(log((2*(b - a*exp(x))*(A*a + B*b))/(b*(a^2 + b^2)^(3/2)) - (2*exp(x)*(A*a
 + B*b))/(a^2*b + b^3))*(A*a + B*b))/(a^2 + b^2)^(3/2) - (log(- (2*exp(x)*
(A*a + B*b))/(a^2*b + b^3) - (2*(b - a*exp(x))*(A*a + B*b))/(b*(a^2 + b^2)
^(3/2)))*(A*a + B*b))/(a^2 + b^2)^(3/2) - ((2*(A*b^3 - B*a*b^2))/(b*(a^2*b
 + b^3)) + (2*exp(x)*(B*a^2*b^2 - A*a*b^3))/(b^2*(a^2*b + b^3)))/(2*a*exp(
x) - b + b*exp(2*x))
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.57

$$\int \frac{A+B\sinh(x)}{(a+b\sinh(x))^2}\,dx = \frac{2\sqrt{a^2+b^2}\,atan\left(\frac{e^x bi+ai}{\sqrt{a^2+b^2}}\right)i}{a^2+b^2}$$

input
```
int((A+B*sinh(x))/(a+b*sinh(x))^2,x)
```

output
```
(2*sqrt(a**2 + b**2)*atan((e**x*b*i + a*i)/sqrt(a**2 + b**2))*i)/(a**2 + b
**2)
```

3.131 $\int \frac{A+B\sinh(x)}{(a+b\sinh(x))^3}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 128

$$\int \frac{A+B\sinh(x)}{(a+b\sinh(x))^3}\,dx = -\frac{(2a^2A-Ab^2+3abB)\,\text{arctanh}\left(\frac{b-a\tanh\left(\frac{x}{2}\right)}{\sqrt{a^2+b^2}}\right)}{(a^2+b^2)^{5/2}} - \frac{(Ab-aB)\cosh(x)}{2(a^2+b^2)(a+b\sinh(x))^2} - \frac{(3aAb-a^2B+2b^2B)\cosh(x)}{2(a^2+b^2)^2(a+b\sinh(x))}$$

output
```
-(2*A*a^2-A*b^2+3*B*a*b)*arctanh((b-a*tanh(1/2*x))/(a^2+b^2)^(1/2))/(a^2+b
^2)^(5/2)-1/2*(A*b-B*a)*cosh(x)/(a^2+b^2)/(a+b*sinh(x))^2-1/2*(3*A*a*b-B*a
^2+2*B*b^2)*cosh(x)/(a^2+b^2)^2/(a+b*sinh(x))
```

Mathematica [A] (verified)

Time = 0.21 (sec) , antiderivative size = 131, normalized size of antiderivative = 1.02

$$\int \frac{A+B\sinh(x)}{(a+b\sinh(x))^3}\,dx = \frac{\frac{2(2a^2A-Ab^2+3abB)\arctan\left(\frac{b-a\tanh\left(\frac{x}{2}\right)}{\sqrt{-a^2-b^2}}\right)}{\sqrt{-a^2-b^2}} + \frac{(a^2+b^2)(-Ab+aB)\cosh(x)}{(a+b\sinh(x))^2} + \frac{(-3aAb+a^2B-2b^2B)\cosh(x)}{a+b\sinh(x)}}{2(a^2+b^2)^2}$$

input
```
Integrate[(A + B*Sinh[x])/(a + b*Sinh[x])^3,x]
```

output
```
((2*(2*a^2*A - A*b^2 + 3*a*b*B)*ArcTan[(b - a*Tanh[x/2])/Sqrt[-a^2 - b^2]]
)/Sqrt[-a^2 - b^2] + ((a^2 + b^2)*(-(A*b) + a*B)*Cosh[x])/(a + b*Sinh[x])^
2 + ((-3*a*A*b + a^2*B - 2*b^2*B)*Cosh[x])/(a + b*Sinh[x]))/(2*(a^2 + b^2)
^2)
```

Rubi [A] (verified)

Time = 0.56 (sec) , antiderivative size = 145, normalized size of antiderivative = 1.13, number of steps used = 12, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.733$, Rules used = {3042, 3233, 25, 3042, 3233, 25, 27, 3042, 3139, 1083, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+B\sinh(x)}{(a+b\sinh(x))^3}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{A-iB\sin(ix)}{(a-ib\sin(ix))^3}dx$$

$$\Big\downarrow \text{3233}$$

$$-\frac{\int -\frac{2(aA+bB)-(Ab-aB)\sinh(x)}{(a+b\sinh(x))^2}dx}{2(a^2+b^2)}-\frac{\cosh(x)(Ab-aB)}{2(a^2+b^2)(a+b\sinh(x))^2}$$

$$\Big\downarrow \text{25}$$

$$\frac{\int \frac{2(aA+bB)-(Ab-aB)\sinh(x)}{(a+b\sinh(x))^2}dx}{2(a^2+b^2)}-\frac{\cosh(x)(Ab-aB)}{2(a^2+b^2)(a+b\sinh(x))^2}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{\cosh(x)(Ab-aB)}{2(a^2+b^2)(a+b\sinh(x))^2}+\frac{\int \frac{2(aA+bB)+i(Ab-aB)\sin(ix)}{(a-ib\sin(ix))^2}dx}{2(a^2+b^2)}$$

$$\Big\downarrow \text{3233}$$

$$\frac{-\frac{\int -\frac{2Aa^2+3bBa-Ab^2}{a+b\sinh(x)}dx}{a^2+b^2}-\frac{\cosh(x)(a^2(-B)+3aAb+2b^2B)}{(a^2+b^2)(a+b\sinh(x))}}{2(a^2+b^2)}-\frac{\cosh(x)(Ab-aB)}{2(a^2+b^2)(a+b\sinh(x))^2}$$

$$\Big\downarrow \text{25}$$

$$\frac{\frac{\int \frac{2Aa^2+3bBa-Ab^2}{a+b\sinh(x)}dx}{a^2+b^2}-\frac{\cosh(x)(a^2(-B)+3aAb+2b^2B)}{(a^2+b^2)(a+b\sinh(x))}}{2(a^2+b^2)}-\frac{\cosh(x)(Ab-aB)}{2(a^2+b^2)(a+b\sinh(x))^2}$$

$$\Big\downarrow \text{27}$$

$$\frac{\frac{(2a^2A+3abB-Ab^2)\int \frac{1}{a+b\sinh(x)}dx}{a^2+b^2}-\frac{\cosh(x)(a^2(-B)+3aAb+2b^2B)}{(a^2+b^2)(a+b\sinh(x))}}{2(a^2+b^2)}-\frac{\cosh(x)(Ab-aB)}{2(a^2+b^2)(a+b\sinh(x))^2}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{\cosh(x)(Ab-aB)}{2(a^2+b^2)(a+b\sinh(x))^2}+\frac{-\frac{\cosh(x)(a^2(-B)+3aAb+2b^2B)}{(a^2+b^2)(a+b\sinh(x))}+\frac{(2a^2A+3abB-Ab^2)\int \frac{1}{a-ib\sin(ix)}dx}{a^2+b^2}}{2(a^2+b^2)}$$

$$\Big\downarrow \text{3139}$$

$$\frac{\frac{2(2a^2A+3abB-Ab^2)\int \frac{1}{-a\tanh^2\left(\frac{x}{2}\right)+2b\tanh\left(\frac{x}{2}\right)+a}d\tanh\left(\frac{x}{2}\right)}{a^2+b^2}-\frac{\cosh(x)(a^2(-B)+3aAb+2b^2B)}{(a^2+b^2)(a+b\sinh(x))}}{2(a^2+b^2)}-\frac{\cosh(x)(Ab-aB)}{2(a^2+b^2)(a+b\sinh(x))^2}$$

$$\Big\downarrow \text{1083}$$

$$\frac{-\frac{4(2a^2A+3abB-Ab^2)\int \frac{1}{4(a^2+b^2)-\left(2b-2a\tanh\left(\frac{x}{2}\right)\right)^2}d\left(2b-2a\tanh\left(\frac{x}{2}\right)\right)}{a^2+b^2}-\frac{\cosh(x)(a^2(-B)+3aAb+2b^2B)}{(a^2+b^2)(a+b\sinh(x))}}{2(a^2+b^2)}-\frac{\cosh(x)(Ab-aB)}{2(a^2+b^2)(a+b\sinh(x))^2}$$

$$\Big\downarrow \text{219}$$

$$\frac{-\frac{2(2a^2A+3abB-Ab^2)\text{arctanh}\left(\frac{2b-2a\tanh\left(\frac{x}{2}\right)}{2\sqrt{a^2+b^2}}\right)}{(a^2+b^2)^{3/2}}-\frac{\cosh(x)(a^2(-B)+3aAb+2b^2B)}{(a^2+b^2)(a+b\sinh(x))}}{2(a^2+b^2)}-\frac{\cosh(x)(Ab-aB)}{2(a^2+b^2)(a+b\sinh(x))^2}$$

input

```
Int[(A + B*Sinh[x])/(a + b*Sinh[x])^3,x]
```

output
```
-1/2*((A*b - a*B)*Cosh[x])/((a^2 + b^2)*(a + b*Sinh[x])^2) + ((-2*(2*a^2*A
 - A*b^2 + 3*a*b*B)*ArcTanh[(2*b - 2*a*Tanh[x/2])/(2*Sqrt[a^2 + b^2])])/(a
^2 + b^2)^(3/2) - ((3*a*A*b - a^2*B + 2*b^2*B)*Cosh[x])/((a^2 + b^2)*(a +
b*Sinh[x])))/(2*(a^2 + b^2))
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1083
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Simp[-2   Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3139
```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Simp[2*(e/d)   Subst[Int[1/(a + 2*b*e*x + a
*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[a^2 - b^2, 0]
```

rule 3233
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Simp[(-(b*c - a*d))*Cos[e + f*x]*((a + b*Sin[e +
 f*x])^(m + 1)/(f*(m + 1)*(a^2 - b^2))), x] + Simp[1/((m + 1)*(a^2 - b^2))
  Int[(a + b*Sin[e + f*x])^(m + 1)*Simp[(a*c - b*d)*(m + 1) - (b*c - a*d)*(
m + 2)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c
- a*d, 0] && NeQ[a^2 - b^2, 0] && LtQ[m, -1] && IntegerQ[2*m]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 313 vs. $2(118) = 236$.

Time = 0.31 (sec) , antiderivative size = 314, normalized size of antiderivative = 2.45

method	result
default	$-\frac{2\left(-\frac{b\left(5A\,a^2b+2A\,b^3-3a^3B\right)\tanh\left(\frac{x}{2}\right)^3}{2a\left(a^4+2a^2b^2+b^4\right)}-\frac{\left(4A\,a^4b-7A\,b^3a^2-2A\,b^5-2B\,a^5+5B\,a^3b^2-2Ba\,b^4\right)\tanh\left(\frac{x}{2}\right)^2}{2\left(a^4+2a^2b^2+b^4\right)a^2}+\frac{b\left(11A\,a^2b+2A\,b^3-5a^3B+4Ba\right.}{2\left(a^4+2a^2b^2+b^4\right)a}\right.}{\left(\tanh\left(\frac{x}{2}\right)^2a-2b\tanh\left(\frac{x}{2}\right)-a\right)^2}$
risch	$\frac{2A\,a^2b^2e^{3x}-A\,b^4e^{3x}+3Ba\,b^3e^{3x}+6A\,a^3b\,e^{2x}-3Aa\,b^3e^{2x}-2B\,a^4e^{2x}+5B\,a^2b^2e^{2x}-2B\,b^4e^{2x}-10A\,a^2b^2e^{x}-A\,b^4e^{x}+4B\,a^3b\,e^{x}-5B}{b(a^2+b^2)^2(b\,e^{2x}+2\,e^{x}a-b)^2}$

input
```
int((A+B*sinh(x))/(a+b*sinh(x))^3,x,method=_RETURNVERBOSE)
```

output
```
-2*(-1/2*b*(5*A*a^2*b+2*A*b^3-3*B*a^3)/a/(a^4+2*a^2*b^2+b^4)*tanh(1/2*x)^3
-1/2*(4*A*a^4*b-7*A*a^2*b^3-2*A*b^5-2*B*a^5+5*B*a^3*b^2-2*B*a*b^4)/(a^4+2*
a^2*b^2+b^4)/a^2*tanh(1/2*x)^2+1/2*b*(11*A*a^2*b+2*A*b^3-5*B*a^3+4*B*a*b^2
)/(a^4+2*a^2*b^2+b^4)/a*tanh(1/2*x)+1/2*(4*A*a^2*b+A*b^3-2*B*a^3+B*a*b^2)/
(a^4+2*a^2*b^2+b^4))/(tanh(1/2*x)^2*a-2*b*tanh(1/2*x)-a)^2+(2*A*a^2-A*b^2+
3*B*a*b)/(a^4+2*a^2*b^2+b^4)/(a^2+b^2)^(1/2)*arctanh(1/2*(2*a*tanh(1/2*x)-
2*b)/(a^2+b^2)^(1/2))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1614 vs. $2(119) = 238$.

Time = 0.11 (sec) , antiderivative size = 1614, normalized size of antiderivative = 12.61

$$\int \frac{A + B\sinh(x)}{(a + b\sinh(x))^3}\,dx = \text{Too large to display}$$

input
```
integrate((A+B*sinh(x))/(a+b*sinh(x))^3,x, algorithm="fricas")
```

output
```
-1/2*(2*B*a^4*b^2 - 6*A*a^3*b^3 - 2*B*a^2*b^4 - 6*A*a*b^5 - 4*B*b^6 - 2*(2
*A*a^4*b^2 + 3*B*a^3*b^3 + A*a^2*b^4 + 3*B*a*b^5 - A*b^6)*cosh(x)^3 - 2*(2
*A*a^4*b^2 + 3*B*a^3*b^3 + A*a^2*b^4 + 3*B*a*b^5 - A*b^6)*sinh(x)^3 + 2*(2
*B*a^6 - 6*A*a^5*b - 3*B*a^4*b^2 - 3*A*a^3*b^3 - 3*B*a^2*b^4 + 3*A*a*b^5 +
 2*B*b^6)*cosh(x)^2 + 2*(2*B*a^6 - 6*A*a^5*b - 3*B*a^4*b^2 - 3*A*a^3*b^3 -
 3*B*a^2*b^4 + 3*A*a*b^5 + 2*B*b^6 - 3*(2*A*a^4*b^2 + 3*B*a^3*b^3 + A*a^2*
b^4 + 3*B*a*b^5 - A*b^6)*cosh(x))*sinh(x)^2 + (2*A*a^2*b^3 + 3*B*a*b^4 - A
*b^5 + (2*A*a^2*b^3 + 3*B*a*b^4 - A*b^5)*cosh(x)^4 + (2*A*a^2*b^3 + 3*B*a*
b^4 - A*b^5)*sinh(x)^4 + 4*(2*A*a^3*b^2 + 3*B*a^2*b^3 - A*a*b^4)*cosh(x)^3
 + 4*(2*A*a^3*b^2 + 3*B*a^2*b^3 - A*a*b^4 + (2*A*a^2*b^3 + 3*B*a*b^4 - A*b
^5)*cosh(x))*sinh(x)^3 + 2*(4*A*a^4*b + 6*B*a^3*b^2 - 4*A*a^2*b^3 - 3*B*a*
b^4 + A*b^5)*cosh(x)^2 + 2*(4*A*a^4*b + 6*B*a^3*b^2 - 4*A*a^2*b^3 - 3*B*a*
b^4 + A*b^5 + 3*(2*A*a^2*b^3 + 3*B*a*b^4 - A*b^5)*cosh(x)^2 + 6*(2*A*a^3*b
^2 + 3*B*a^2*b^3 - A*a*b^4)*cosh(x))*sinh(x)^2 - 4*(2*A*a^3*b^2 + 3*B*a^2*
b^3 - A*a*b^4)*cosh(x) - 4*(2*A*a^3*b^2 + 3*B*a^2*b^3 - A*a*b^4 - (2*A*a^2
*b^3 + 3*B*a*b^4 - A*b^5)*cosh(x)^3 - 3*(2*A*a^3*b^2 + 3*B*a^2*b^3 - A*a*b
^4)*cosh(x)^2 - (4*A*a^4*b + 6*B*a^3*b^2 - 4*A*a^2*b^3 - 3*B*a*b^4 + A*b^5
)*cosh(x))*sinh(x))*sqrt(a^2 + b^2)*log((b^2*cosh(x)^2 + b^2*sinh(x)^2 + 2
*a*b*cosh(x) + 2*a^2 + b^2 + 2*(b^2*cosh(x) + a*b)*sinh(x) + 2*sqrt(a^2 +
b^2)*(b*cosh(x) + b*sinh(x) + a))/(b*cosh(x)^2 + b*sinh(x)^2 + 2*a*cosh...
```

Sympy [F(-1)]

Timed out.

$$\int \frac{A+B\sinh(x)}{(a+b\sinh(x))^3}\,dx = \text{Timed out}$$

input
```
integrate((A+B*sinh(x))/(a+b*sinh(x))**3,x)
```

output
```
Timed out
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 537 vs. 2(119) = 238.

Time = 0.14 (sec) , antiderivative size = 537, normalized size of antiderivative = 4.20

$$\int \frac{A+B\sinh(x)}{(a+b\sinh(x))^3}\,dx$$
$$= \frac{1}{2}\left(\frac{3\,ab\log\left(\frac{be^{(-x)}-a-\sqrt{a^2+b^2}}{be^{(-x)}-a+\sqrt{a^2+b^2}}\right)}{(a^4+2\,a^2b^2+b^4)\sqrt{a^2+b^2}} + \frac{2\left(3\,ab^3e^{(-3\,x)}+a^2b^2-2\,b^4+(4\,a^3b-5\,\right.}{a^4b^3+2\,a^2b^5+b^7+4\,(a^5b^2+2\,a^3b^4+ab^6)e^{(-x)}+2\,(2\,a^6b+3\,a^4b^3-}\right.$$
$$+\frac{1}{2}\,A\left(\frac{(2\,a^2-b^2)\log\left(\frac{be^{(-x)}-a-\sqrt{a^2+b^2}}{be^{(-x)}-a+\sqrt{a^2+b^2}}\right)}{(a^4+2\,a^2b^2+b^4)\sqrt{a^2+b^2}} - \frac{2\left(3\,ab^2+(10\,a^2b+b^3)e^{(-x)}\right.}{a^4b^2+2\,a^2b^4+b^6+4\,(a^5b+2\,a^3b^3+ab^5)e^{(-x)}+2\,(2\,a^6+3}\right.$$

input
```
integrate((A+B*sinh(x))/(a+b*sinh(x))^3,x, algorithm="maxima")
```

output
```
1/2*(3*a*b*log((b*e^(-x) - a - sqrt(a^2 + b^2))/(b*e^(-x) - a + sqrt(a^2 +
 b^2)))/((a^4 + 2*a^2*b^2 + b^4)*sqrt(a^2 + b^2)) + 2*(3*a*b^3*e^(-3*x) +
a^2*b^2 - 2*b^4 + (4*a^3*b - 5*a*b^3)*e^(-x) + (2*a^4 - 5*a^2*b^2 + 2*b^4)
*e^(-2*x))/(a^4*b^3 + 2*a^2*b^5 + b^7 + 4*(a^5*b^2 + 2*a^3*b^4 + a*b^6)*e^
(-x) + 2*(2*a^6*b + 3*a^4*b^3 - b^7)*e^(-2*x) - 4*(a^5*b^2 + 2*a^3*b^4 + a
*b^6)*e^(-3*x) + (a^4*b^3 + 2*a^2*b^5 + b^7)*e^(-4*x)))*B + 1/2*A*((2*a^2
- b^2)*log((b*e^(-x) - a - sqrt(a^2 + b^2))/(b*e^(-x) - a + sqrt(a^2 + b^2
)))/((a^4 + 2*a^2*b^2 + b^4)*sqrt(a^2 + b^2)) - 2*(3*a*b^2 + (10*a^2*b + b
^3)*e^(-x) + 3*(2*a^3 - a*b^2)*e^(-2*x) - (2*a^2*b - b^3)*e^(-3*x))/(a^4*b
^2 + 2*a^2*b^4 + b^6 + 4*(a^5*b + 2*a^3*b^3 + a*b^5)*e^(-x) + 2*(2*a^6 + 3
*a^4*b^2 - b^6)*e^(-2*x) - 4*(a^5*b + 2*a^3*b^3 + a*b^5)*e^(-3*x) + (a^4*b
^2 + 2*a^2*b^4 + b^6)*e^(-4*x)))
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 279 vs. $2(119) = 238$.

Time = 0.14 (sec) , antiderivative size = 279, normalized size of antiderivative = 2.18

$$\int \frac{A+B\sinh(x)}{(a+b\sinh(x))^3}\,dx = -\frac{(2\,Aa^2+3\,Bab-Ab^2)\log\left(\frac{\left|-2\,be^x-2\,a-2\,\sqrt{a^2+b^2}\right|}{\left|-2\,be^x-2\,a+2\,\sqrt{a^2+b^2}\right|}\right)}{2\,(a^4+2\,a^2b^2+b^4)\sqrt{a^2+b^2}} + \frac{2\,Aa^2b^2e^{(3\,x)}+3\,Bab^3e^{(3\,x)}-Ab^4e^{(3\,x)}-2\,Ba^4e^{(2\,x)}+6\,Aa^3be^{(2\,x)}+5\,Ba^2b^2e^{(2\,x)}-3\,Aab^3e^{(2\,x)}-2\,Bb}{(a^4b+2\,a^2b^3+b^5)(be^{(2\,x)}+2\,a}$$

input
```
integrate((A+B*sinh(x))/(a+b*sinh(x))^3,x, algorithm="giac")
```

output
```
-1/2*(2*A*a^2 + 3*B*a*b - A*b^2)*log(abs(-2*b*e^x - 2*a - 2*sqrt(a^2 + b^2
))/abs(-2*b*e^x - 2*a + 2*sqrt(a^2 + b^2)))/((a^4 + 2*a^2*b^2 + b^4)*sqrt(
a^2 + b^2)) + (2*A*a^2*b^2*e^(3*x) + 3*B*a*b^3*e^(3*x) - A*b^4*e^(3*x) - 2
*B*a^4*e^(2*x) + 6*A*a^3*b*e^(2*x) + 5*B*a^2*b^2*e^(2*x) - 3*A*a*b^3*e^(2*
x) - 2*B*b^4*e^(2*x) + 4*B*a^3*b*e^x - 10*A*a^2*b^2*e^x - 5*B*a*b^3*e^x -
A*b^4*e^x - B*a^2*b^2 + 3*A*a*b^3 + 2*B*b^4)/((a^4*b + 2*a^2*b^3 + b^5)*(b
*e^(2*x) + 2*a*e^x - b)^2)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{A+B\sinh(x)}{(a+b\sinh(x))^3}\,dx=\int \frac{A+B\sinh(x)}{\left(a+b\sinh\left(x\right)\right)^3}\,dx$$

input
```
int((A + B*sinh(x))/(a + b*sinh(x))^3,x)
```

output
```
int((A + B*sinh(x))/(a + b*sinh(x))^3, x)
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 230, normalized size of antiderivative = 1.80

$$\int \frac{A+B\sinh(x)}{(a+b\sinh(x))^3}\,dx = \frac{2e^{2x}\sqrt{a^2+b^2}\,atan\left(\frac{e^x bi+ai}{\sqrt{a^2+b^2}}\right)abi+4e^x\sqrt{a^2+b^2}\,atan\left(\frac{e^x bi+ai}{\sqrt{a^2+b^2}}\right)a^2 i-2\sqrt{a^2+b^2}\,atan\left(\frac{e^x bi+ai}{\sqrt{a^2+b^2}}\right)abi-e^{2x}a^2}{e^{2x}a^4b+2e^{2x}a^2b^3+e^{2x}b^5+2e^xa^5+4e^xa^3b^2+2e^xa\,b^4-a^4b-2a^2b^3-b^5}$$

input
```
int((A+B*sinh(x))/(a+b*sinh(x))^3,x)
```

output
```
(2*e**(2*x)*sqrt(a**2 + b**2)*atan((e**x*b*i + a*i)/sqrt(a**2 + b**2))*a*b
*i + 4*e**x*sqrt(a**2 + b**2)*atan((e**x*b*i + a*i)/sqrt(a**2 + b**2))*a**
2*i - 2*sqrt(a**2 + b**2)*atan((e**x*b*i + a*i)/sqrt(a**2 + b**2))*a*b*i -
 e**(2*x)*a**2*b - e**(2*x)*b**3 - a**2*b - b**3)/(e**(2*x)*a**4*b + 2*e**
(2*x)*a**2*b**3 + e**(2*x)*b**5 + 2*e**x*a**5 + 4*e**x*a**3*b**2 + 2*e**x*
a*b**4 - a**4*b - 2*a**2*b**3 - b**5)
```

3.132 $\int \frac{A+B\sinh(x)}{(a+b\sinh(x))^4}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 187

$$\int \frac{A+B\sinh(x)}{(a+b\sinh(x))^4}\,dx = -\frac{(2a^3A-3aAb^2+4a^2bB-b^3B)\operatorname{arctanh}\left(\frac{b-a\tanh\left(\frac{x}{2}\right)}{\sqrt{a^2+b^2}}\right)}{(a^2+b^2)^{7/2}} - \frac{(Ab-aB)\cosh(x)}{3(a^2+b^2)(a+b\sinh(x))^3} - \frac{(5aAb-2a^2B+3b^2B)\cosh(x)}{6(a^2+b^2)^2(a+b\sinh(x))^2} - \frac{(11a^2Ab-4Ab^3-2a^3B+13ab^2B)\cosh(x)}{6(a^2+b^2)^3(a+b\sinh(x))}$$

output
```
-(2*A*a^3-3*A*a*b^2+4*B*a^2*b-B*b^3)*arctanh((b-a*tanh(1/2*x))/(a^2+b^2)^(
1/2))/(a^2+b^2)^(7/2)-1/3*(A*b-B*a)*cosh(x)/(a^2+b^2)/(a+b*sinh(x))^3-1/6*
(5*A*a*b-2*B*a^2+3*B*b^2)*cosh(x)/(a^2+b^2)^2/(a+b*sinh(x))^2-1/6*(11*A*a^
2*b-4*A*b^3-2*B*a^3+13*B*a*b^2)*cosh(x)/(a^2+b^2)^3/(a+b*sinh(x))
```

Mathematica [A] (verified)

Time = 0.31 (sec) , antiderivative size = 189, normalized size of antiderivative = 1.01

$$\int \frac{A+B\sinh(x)}{(a+b\sinh(x))^4}\,dx = \frac{\frac{6(2a^3A-3aAb^2+4a^2bB-b^3B)\arctan\left(\frac{b-a\tanh\left(\frac{x}{2}\right)}{\sqrt{-a^2-b^2}}\right)}{\sqrt{-a^2-b^2}} + \frac{2(a^2+b^2)^2(-Ab+aB)\cosh(x)}{(a+b\sinh(x))^3} + \frac{(a^2+b^2)(-5aAb+2a^2B-3b^2B)\cosh(x)}{(a+b\sinh(x))^2} + (-1}{6(a^2+b^2)^3}$$

input
```
Integrate[(A + B*Sinh[x])/(a + b*Sinh[x])^4,x]
```

output
```
((6*(2*a^3*A - 3*a*A*b^2 + 4*a^2*b*B - b^3*B)*ArcTan[(b - a*Tanh[x/2])/Sqr
t[-a^2 - b^2]])/Sqrt[-a^2 - b^2] + (2*(a^2 + b^2)^2*(-(A*b) + a*B)*Cosh[x]
)/(a + b*Sinh[x])^3 + ((a^2 + b^2)*(-5*a*A*b + 2*a^2*B - 3*b^2*B)*Cosh[x])
/(a + b*Sinh[x])^2 + ((-11*a^2*A*b + 4*A*b^3 + 2*a^3*B - 13*a*b^2*B)*Cosh[
x])/(a + b*Sinh[x]))/(6*(a^2 + b^2)^3)
```

Rubi [A] (verified)

Time = 0.86 (sec) , antiderivative size = 218, normalized size of antiderivative = 1.17, number of steps used = 14, number of rules used = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.867$, Rules used = {3042, 3233, 25, 3042, 3233, 25, 3042, 3233, 27, 3042, 3139, 1083, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+B\sinh(x)}{(a+b\sinh(x))^4}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{A-iB\sin(ix)}{(a-ib\sin(ix))^4}dx$$

$$\Big\downarrow \text{3233}$$

$$-\frac{\int -\frac{3(aA+bB)-2(Ab-aB)\sinh(x)}{(a+b\sinh(x))^3}dx}{3(a^2+b^2)} - \frac{\cosh(x)(Ab-aB)}{3(a^2+b^2)(a+b\sinh(x))^3}$$

$$\Big\downarrow \textbf{25}$$

$$\frac{\int \frac{3(aA+bB)-2(Ab-aB)\sinh(x)}{(a+b\sinh(x))^3}dx}{3(a^2+b^2)}-\frac{\cosh(x)(Ab-aB)}{3(a^2+b^2)(a+b\sinh(x))^3}$$

$$\Big\downarrow \textbf{3042}$$

$$-\frac{\cosh(x)(Ab-aB)}{3(a^2+b^2)(a+b\sinh(x))^3}+\frac{\int \frac{3(aA+bB)+2i(Ab-aB)\sin(ix)}{(a-ib\sin(ix))^3}dx}{3(a^2+b^2)}$$

$$\Big\downarrow \textbf{3233}$$

$$\frac{-\frac{\int -\frac{2\left(3Aa^2+5bBa-2Ab^2\right)-\left(-2Ba^2+5Aba+3b^2B\right)\sinh(x)}{(a+b\sinh(x))^2}dx}{2(a^2+b^2)}-\frac{\cosh(x)\left(-2a^2B+5aAb+3b^2B\right)}{2(a^2+b^2)(a+b\sinh(x))^2}}{3(a^2+b^2)}-\frac{\cosh(x)(Ab-aB)}{3(a^2+b^2)(a+b\sinh(x))^3}$$

$$\Big\downarrow \textbf{25}$$

$$\frac{\frac{\int \frac{2\left(3Aa^2+5bBa-2Ab^2\right)-\left(-2Ba^2+5Aba+3b^2B\right)\sinh(x)}{(a+b\sinh(x))^2}dx}{2(a^2+b^2)}-\frac{\cosh(x)\left(-2a^2B+5aAb+3b^2B\right)}{2(a^2+b^2)(a+b\sinh(x))^2}}{3(a^2+b^2)}-\frac{\cosh(x)(Ab-aB)}{3(a^2+b^2)(a+b\sinh(x))^3}$$

$$\Big\downarrow \textbf{3042}$$

$$-\frac{\cosh(x)(Ab-aB)}{3(a^2+b^2)(a+b\sinh(x))^3}+\frac{-\frac{\cosh(x)\left(-2a^2B+5aAb+3b^2B\right)}{2(a^2+b^2)(a+b\sinh(x))^2}+\frac{\int \frac{2\left(3Aa^2+5bBa-2Ab^2\right)+i\left(-2Ba^2+5Aba+3b^2B\right)\sin(ix)}{(a-ib\sin(ix))^2}dx}{2(a^2+b^2)}}{3(a^2+b^2)}$$

$$\Big\downarrow \textbf{3233}$$

$$\frac{\frac{-\frac{\int -\frac{3\left(2Aa^3+4bBa^2-3Ab^2a-b^3B\right)}{a+b\sinh(x)}dx}{a^2+b^2}-\frac{\cosh(x)\left(-2a^3B+11a^2Ab+13ab^2B-4Ab^3\right)}{\left(a^2+b^2\right)(a+b\sinh(x))}}{2(a^2+b^2)}-\frac{\cosh(x)\left(-2a^2B+5aAb+3b^2B\right)}{2(a^2+b^2)(a+b\sinh(x))^2}}{3(a^2+b^2)}-\frac{\cosh(x)(Ab-aB)}{3(a^2+b^2)(a+b\sinh(x))^3}$$

$$\Big\downarrow \textbf{27}$$

$$\frac{\frac{\frac{3\left(2a^3A+4a^2bB-3aAb^2-b^3B\right)\int\frac{1}{a+b\sinh(x)}dx}{a^2+b^2}-\frac{\cosh(x)\left(-2a^3B+11a^2Ab+13ab^2B-4Ab^3\right)}{\left(a^2+b^2\right)(a+b\sinh(x))}}{2(a^2+b^2)}-\frac{\cosh(x)(-2a^2B+5aAb+3b^2B)}{2(a^2+b^2)(a+b\sinh(x))^2}}{3\left(a^2+b^2\right)}-\frac{\cosh(x)(Ab-aB)}{3\left(a^2+b^2\right)(a+b\sinh(x))^3}$$

$$\Big\downarrow \text{ 3042}$$

$$-\frac{\cosh(x)(Ab-aB)}{3\left(a^2+b^2\right)(a+b\sinh(x))^3}+\frac{-\frac{\cosh(x)(-2a^2B+5aAb+3b^2B)}{2(a^2+b^2)(a+b\sinh(x))^2}+\frac{-\frac{\cosh(x)\left(-2a^3B+11a^2Ab+13ab^2B-4Ab^3\right)}{\left(a^2+b^2\right)(a+b\sinh(x))}+\frac{3\left(2a^3A+4a^2bB-3aAb^2-b^3B\right)\int\frac{1}{a-ib\sin(ix)}dx}{a^2+b^2}}{2(a^2+b^2)}}{3\left(a^2+b^2\right)}$$

$$\Big\downarrow \text{ 3139}$$

$$\frac{\frac{\frac{6\left(2a^3A+4a^2bB-3aAb^2-b^3B\right)\int\frac{1}{-a\tanh^2\left(\frac{x}{2}\right)+2b\tanh\left(\frac{x}{2}\right)+a}d\tanh\left(\frac{x}{2}\right)}{a^2+b^2}-\frac{\cosh(x)\left(-2a^3B+11a^2Ab+13ab^2B-4Ab^3\right)}{\left(a^2+b^2\right)(a+b\sinh(x))}}{2(a^2+b^2)}-\frac{\cosh(x)(-2a^2B+5aAb+3b^2B)}{2(a^2+b^2)(a+b\sinh(x))^2}}{3\left(a^2+b^2\right)}-\frac{\cosh(x)(Ab-aB)}{3\left(a^2+b^2\right)(a+b\sinh(x))^3}$$

$$\Big\downarrow \text{ 1083}$$

$$\frac{\frac{-\frac{12\left(2a^3A+4a^2bB-3aAb^2-b^3B\right)\int\frac{1}{4\left(a^2+b^2\right)-\left(2b-2a\tanh\left(\frac{x}{2}\right)\right)^2}d\left(2b-2a\tanh\left(\frac{x}{2}\right)\right)}{a^2+b^2}-\frac{\cosh(x)\left(-2a^3B+11a^2Ab+13ab^2B-4Ab^3\right)}{\left(a^2+b^2\right)(a+b\sinh(x))}}{2(a^2+b^2)}-\frac{\cosh(x)(-2a^2B+5aAb+3b^2B)}{2(a^2+b^2)(a+b\sinh(x))^2}}{3\left(a^2+b^2\right)}-\frac{\cosh(x)(Ab-aB)}{3\left(a^2+b^2\right)(a+b\sinh(x))^3}$$

$$\Big\downarrow \text{ 219}$$

$$\frac{\frac{-\frac{6\left(2a^3A+4a^2bB-3aAb^2-b^3B\right)\text{arctanh}\left(\frac{2b-2a\tanh\left(\frac{x}{2}\right)}{2\sqrt{a^2+b^2}}\right)}{\left(a^2+b^2\right)^{3/2}}-\frac{\cosh(x)\left(-2a^3B+11a^2Ab+13ab^2B-4Ab^3\right)}{\left(a^2+b^2\right)(a+b\sinh(x))}}{2(a^2+b^2)}-\frac{\cosh(x)(-2a^2B+5aAb+3b^2B)}{2(a^2+b^2)(a+b\sinh(x))^2}}{3\left(a^2+b^2\right)}-\frac{\cosh(x)(Ab-aB)}{3\left(a^2+b^2\right)(a+b\sinh(x))^3}$$

input
```
Int[(A + B*Sinh[x])/(a + b*Sinh[x])^4,x]
```

output
```
-1/3*((A*b - a*B)*Cosh[x])/((a^2 + b^2)*(a + b*Sinh[x])^3) + (-1/2*((5*a*A
*b - 2*a^2*B + 3*b^2*B)*Cosh[x])/((a^2 + b^2)*(a + b*Sinh[x])^2) + ((-6*(2
*a^3*A - 3*a*A*b^2 + 4*a^2*b*B - b^3*B)*ArcTanh[(2*b - 2*a*Tanh[x/2])/(2*S
qrt[a^2 + b^2])])/(a^2 + b^2)^(3/2) - ((11*a^2*A*b - 4*A*b^3 - 2*a^3*B + 1
3*a*b^2*B)*Cosh[x])/((a^2 + b^2)*(a + b*Sinh[x])))/(2*(a^2 + b^2)))/(3*(a^
2 + b^2))
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1083
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Simp[-2   Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3139
```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Simp[2*(e/d)   Subst[Int[1/(a + 2*b*e*x + a
*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[a^2 - b^2, 0]
```

rule 3233
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Simp[(-(b*c - a*d))*Cos[e + f*x]*((a + b*Sin[e +
 f*x])^(m + 1)/(f*(m + 1)*(a^2 - b^2))), x] + Simp[1/((m + 1)*(a^2 - b^2))
  Int[(a + b*Sin[e + f*x])^(m + 1)*Simp[(a*c - b*d)*(m + 1) - (b*c - a*d)*(
m + 2)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c
- a*d, 0] && NeQ[a^2 - b^2, 0] && LtQ[m, -1] && IntegerQ[2*m]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 632 vs. $2(175) = 350$.

Time = 0.52 (sec) , antiderivative size = 633, normalized size of antiderivative = 3.39

method	result
default	$-\frac{2\left(-\frac{b\left(9A\,a^4b+6A\,b^3a^2+2A\,b^5-4B\,a^5+B\,a^3b^2\right)\tanh\left(\frac{x}{2}\right)^5}{2a\left(a^6+3a^4b^2+3a^2b^4+b^6\right)}-\frac{\left(6A\,a^6b-27A\,a^4b^3-12A\,a^2b^5-4A\,b^7-2B\,a^7+14B\,a^5b^2-11B\,a^3b^4-2Ba\,b^6\right)\text{t}}{2\left(a^6+3a^4b^2+3a^2b^4+b^6\right)a^2}}{}$
risch	$\frac{-11A\,a^2b^4+2B\,a^3b^3+4A\,b^6+60B\,a^3b^3\mathrm{e}^{4x}-15Ba\,b^5\mathrm{e}^{4x}+44A\,a^5b\,\mathrm{e}^{3x}-82A\,a^3b^3\mathrm{e}^{3x}+24Aa\,b^5\mathrm{e}^{3x}+64B\,a^4b^2\mathrm{e}^{3x}-78B\,a^2b^4\mathrm{e}^{3x}-10}{}$

input
```
int((A+B*sinh(x))/(a+b*sinh(x))^4,x,method=_RETURNVERBOSE)
```

output
```
-2*(-1/2*b*(9*A*a^4*b+6*A*a^2*b^3+2*A*b^5-4*B*a^5+B*a^3*b^2)/a/(a^6+3*a^4*
b^2+3*a^2*b^4+b^6)*tanh(1/2*x)^5-1/2*(6*A*a^6*b-27*A*a^4*b^3-12*A*a^2*b^5-
4*A*b^7-2*B*a^7+14*B*a^5*b^2-11*B*a^3*b^4-2*B*a*b^6)/(a^6+3*a^4*b^2+3*a^2*
b^4+b^6)/a^2*tanh(1/2*x)^4+1/3/a^3*b*(54*A*a^6*b-21*A*a^4*b^3-4*A*a^2*b^5-
4*A*b^7-18*B*a^7+42*B*a^5*b^2-17*B*a^3*b^4-2*B*a*b^6)/(a^6+3*a^4*b^2+3*a^2
*b^4+b^6)*tanh(1/2*x)^3+1/a^2*(6*A*a^6*b-20*A*a^4*b^3-3*A*a^2*b^5-2*A*b^7-
2*B*a^7+10*B*a^5*b^2-14*B*a^3*b^4-B*a*b^6)/(a^6+3*a^4*b^2+3*a^2*b^4+b^6)*t
anh(1/2*x)^2-1/2/a*b*(27*A*a^4*b+4*A*a^2*b^3+2*A*b^5-8*B*a^5+19*B*a^3*b^2+
2*B*a*b^4)/(a^6+3*a^4*b^2+3*a^2*b^4+b^6)*tanh(1/2*x)-1/6*(18*A*a^4*b+5*A*a
^2*b^3+2*A*b^5-6*B*a^5+10*B*a^3*b^2+B*a*b^4)/(a^6+3*a^4*b^2+3*a^2*b^4+b^6)
)/(tanh(1/2*x)^2*a-2*b*tanh(1/2*x)-a)^3+(2*A*a^3-3*A*a*b^2+4*B*a^2*b-B*b^3
)/(a^6+3*a^4*b^2+3*a^2*b^4+b^6)/(a^2+b^2)^(1/2)*arctanh(1/2*(2*a*tanh(1/2*
x)-2*b)/(a^2+b^2)^(1/2))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 3870 vs. $2(177) = 354$.

Time = 0.24 (sec) , antiderivative size = 3870, normalized size of antiderivative = 20.70

$$\int \frac{A + B\sinh(x)}{(a + b\sinh(x))^4} \, dx = \text{Too large to display}$$

input
```
integrate((A+B*sinh(x))/(a+b*sinh(x))^4,x, algorithm="fricas")
```

output
```
Too large to include
```

Sympy [F(-1)]

Timed out.

$$\int \frac{A + B\sinh(x)}{(a + b\sinh(x))^4} \, dx = \text{Timed out}$$

input
```
integrate((A+B*sinh(x))/(a+b*sinh(x))**4,x)
```

output
```
Timed out
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 982 vs. $2(177) = 354$.

Time = 0.18 (sec) , antiderivative size = 982, normalized size of antiderivative = 5.25

$$\int \frac{A + B\sinh(x)}{(a + b\sinh(x))^4} \, dx = \text{Too large to display}$$

input
```
integrate((A+B*sinh(x))/(a+b*sinh(x))^4,x, algorithm="maxima")
```

output
```
1/6*(3*(2*a^2 - 3*b^2)*a*log((b*e^(-x) - a - sqrt(a^2 + b^2))/(b*e^(-x) -
a + sqrt(a^2 + b^2)))/((a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6)*sqrt(a^2 + b^2)
) - 2*(11*a^2*b^3 - 4*b^5 + 15*(4*a^3*b^2 - a*b^4)*e^(-x) + 6*(17*a^4*b -
6*a^2*b^3 + 2*b^5)*e^(-2*x) + 2*(22*a^5 - 41*a^3*b^2 + 12*a*b^4)*e^(-3*x)
- 15*(2*a^4*b - 3*a^2*b^3)*e^(-4*x) + 3*(2*a^3*b^2 - 3*a*b^4)*e^(-5*x))/(a
^6*b^3 + 3*a^4*b^5 + 3*a^2*b^7 + b^9 + 6*(a^7*b^2 + 3*a^5*b^4 + 3*a^3*b^6
+ a*b^8)*e^(-x) + 3*(4*a^8*b + 11*a^6*b^3 + 9*a^4*b^5 + a^2*b^7 - b^9)*e^(
-2*x) + 4*(2*a^9 + 3*a^7*b^2 - 3*a^5*b^4 - 7*a^3*b^6 - 3*a*b^8)*e^(-3*x) -
 3*(4*a^8*b + 11*a^6*b^3 + 9*a^4*b^5 + a^2*b^7 - b^9)*e^(-4*x) + 6*(a^7*b^
2 + 3*a^5*b^4 + 3*a^3*b^6 + a*b^8)*e^(-5*x) - (a^6*b^3 + 3*a^4*b^5 + 3*a^2
*b^7 + b^9)*e^(-6*x)))*A + 1/6*B*(3*(4*a^2*b - b^3)*log((b*e^(-x) - a - sq
rt(a^2 + b^2))/(b*e^(-x) - a + sqrt(a^2 + b^2)))/((a^6 + 3*a^4*b^2 + 3*a^2
*b^4 + b^6)*sqrt(a^2 + b^2)) + 2*(2*a^3*b^3 - 13*a*b^5 + 3*(4*a^4*b^2 - 22
*a^2*b^4 - b^6)*e^(-x) + 6*(4*a^5*b - 17*a^3*b^3 + 4*a*b^5)*e^(-2*x) + 2*(
4*a^6 - 32*a^4*b^2 + 39*a^2*b^4)*e^(-3*x) + 15*(4*a^3*b^3 - a*b^5)*e^(-4*x
) - 3*(4*a^2*b^4 - b^6)*e^(-5*x))/(a^6*b^4 + 3*a^4*b^6 + 3*a^2*b^8 + b^10
+ 6*(a^7*b^3 + 3*a^5*b^5 + 3*a^3*b^7 + a*b^9)*e^(-x) + 3*(4*a^8*b^2 + 11*a
^6*b^4 + 9*a^4*b^6 + a^2*b^8 - b^10)*e^(-2*x) + 4*(2*a^9*b + 3*a^7*b^3 - 3
*a^5*b^5 - 7*a^3*b^7 - 3*a*b^9)*e^(-3*x) - 3*(4*a^8*b^2 + 11*a^6*b^4 + 9*a
^4*b^6 + a^2*b^8 - b^10)*e^(-4*x) + 6*(a^7*b^3 + 3*a^5*b^5 + 3*a^3*b^7 ...
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 477 vs. $2(177) = 354$.

Time = 0.15 (sec) , antiderivative size = 477, normalized size of antiderivative = 2.55

$$\int \frac{A + B\sinh(x)}{(a + b\sinh(x))^4}\,dx = \frac{(2\,Aa^3 + 4\,Ba^2b - 3\,Aab^2 - Bb^3)\log\left(\frac{\left|2\,be^x + 2\,a - 2\,\sqrt{a^2+b^2}\right|}{\left|2\,be^x + 2\,a + 2\,\sqrt{a^2+b^2}\right|}\right)}{2\,(a^6 + 3\,a^4b^2 + 3\,a^2b^4 + b^6)\sqrt{a^2+b^2}} + \frac{6\,Aa^3b^3e^{(5\,x)} + 12\,Ba^2b^4e^{(5\,x)} - 9\,Aab^5e^{(5\,x)} - 3\,Bb^6e^{(5\,x)} + 30\,Aa^4b^2e^{(4\,x)} + 60\,Ba^3b^3e^{(4\,x)} - 45\,Aa^2b^4e^{(}}{}$$

input
```
integrate((A+B*sinh(x))/(a+b*sinh(x))^4,x, algorithm="giac")
```

output
```
1/2*(2*A*a^3 + 4*B*a^2*b - 3*A*a*b^2 - B*b^3)*log(abs(2*b*e^x + 2*a - 2*sq
rt(a^2 + b^2))/abs(2*b*e^x + 2*a + 2*sqrt(a^2 + b^2)))/((a^6 + 3*a^4*b^2 +
 3*a^2*b^4 + b^6)*sqrt(a^2 + b^2)) + 1/3*(6*A*a^3*b^3*e^(5*x) + 12*B*a^2*b
^4*e^(5*x) - 9*A*a*b^5*e^(5*x) - 3*B*b^6*e^(5*x) + 30*A*a^4*b^2*e^(4*x) +
60*B*a^3*b^3*e^(4*x) - 45*A*a^2*b^4*e^(4*x) - 15*B*a*b^5*e^(4*x) - 8*B*a^6
*e^(3*x) + 44*A*a^5*b*e^(3*x) + 64*B*a^4*b^2*e^(3*x) - 82*A*a^3*b^3*e^(3*x
) - 78*B*a^2*b^4*e^(3*x) + 24*A*a*b^5*e^(3*x) + 24*B*a^5*b*e^(2*x) - 102*A
*a^4*b^2*e^(2*x) - 102*B*a^3*b^3*e^(2*x) + 36*A*a^2*b^4*e^(2*x) + 24*B*a*b
^5*e^(2*x) - 12*A*b^6*e^(2*x) - 12*B*a^4*b^2*e^x + 60*A*a^3*b^3*e^x + 66*B
*a^2*b^4*e^x - 15*A*a*b^5*e^x + 3*B*b^6*e^x + 2*B*a^3*b^3 - 11*A*a^2*b^4 -
 13*B*a*b^5 + 4*A*b^6)/((a^6*b + 3*a^4*b^3 + 3*a^2*b^5 + b^7)*(b*e^(2*x) +
 2*a*e^x - b)^3)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{A+B\sinh(x)}{(a+b\sinh(x))^4}\,dx = \int \frac{A+B\sinh(x)}{(a+b\sinh(x))^4}\,dx$$

input
```
int((A + B*sinh(x))/(a + b*sinh(x))^4,x)
```

output
```
int((A + B*sinh(x))/(a + b*sinh(x))^4, x)
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 815, normalized size of antiderivative = 4.36

$$\int \frac{A+B\sinh(x)}{(a+b\sinh(x))^4}\,dx = \frac{8e^{4x}\sqrt{a^2+b^2}\,atan\left(\frac{e^x bi+ai}{\sqrt{a^2+b^2}}\right)a^3b^2i - 4e^{4x}\sqrt{a^2+b^2}\,atan\left(\frac{e^x bi+ai}{\sqrt{a^2+b^2}}\right)a\,b^4i + 32e^{3x}\sqrt{a^2+b^2}\,atan\left(\frac{e^x bi+ai}{\sqrt{a^2+b^2}}\right)a^4b}{}$$

input
```
int((A+B*sinh(x))/(a+b*sinh(x))^4,x)
```

output

```
(8*e**(4*x)*sqrt(a**2 + b**2)*atan((e**x*b*i + a*i)/sqrt(a**2 + b**2))*a**
3*b**2*i - 4*e**(4*x)*sqrt(a**2 + b**2)*atan((e**x*b*i + a*i)/sqrt(a**2 +
b**2))*a*b**4*i + 32*e**(3*x)*sqrt(a**2 + b**2)*atan((e**x*b*i + a*i)/sqrt
(a**2 + b**2))*a**4*b*i - 16*e**(3*x)*sqrt(a**2 + b**2)*atan((e**x*b*i + a
*i)/sqrt(a**2 + b**2))*a**2*b**3*i + 32*e**(2*x)*sqrt(a**2 + b**2)*atan((e
**x*b*i + a*i)/sqrt(a**2 + b**2))*a**5*i - 32*e**(2*x)*sqrt(a**2 + b**2)*a
tan((e**x*b*i + a*i)/sqrt(a**2 + b**2))*a**3*b**2*i + 8*e**(2*x)*sqrt(a**2
 + b**2)*atan((e**x*b*i + a*i)/sqrt(a**2 + b**2))*a*b**4*i - 32*e**x*sqrt(
a**2 + b**2)*atan((e**x*b*i + a*i)/sqrt(a**2 + b**2))*a**4*b*i + 16*e**x*s
qrt(a**2 + b**2)*atan((e**x*b*i + a*i)/sqrt(a**2 + b**2))*a**2*b**3*i + 8*
sqrt(a**2 + b**2)*atan((e**x*b*i + a*i)/sqrt(a**2 + b**2))*a**3*b**2*i - 4
*sqrt(a**2 + b**2)*atan((e**x*b*i + a*i)/sqrt(a**2 + b**2))*a*b**4*i - 2*e
**(4*x)*a**4*b**2 - e**(4*x)*a**2*b**4 + e**(4*x)*b**6 + 16*e**(2*x)*a**6
+ 12*e**(2*x)*a**4*b**2 - 6*e**(2*x)*a**2*b**4 - 2*e**(2*x)*b**6 - 32*e**x
*a**5*b - 40*e**x*a**3*b**3 - 8*e**x*a*b**5 + 10*a**4*b**2 + 11*a**2*b**4
+ b**6)/(4*a*(e**(4*x)*a**6*b**2 + 3*e**(4*x)*a**4*b**4 + 3*e**(4*x)*a**2*
b**6 + e**(4*x)*b**8 + 4*e**(3*x)*a**7*b + 12*e**(3*x)*a**5*b**3 + 12*e**(
3*x)*a**3*b**5 + 4*e**(3*x)*a*b**7 + 4*e**(2*x)*a**8 + 10*e**(2*x)*a**6*b*
*2 + 6*e**(2*x)*a**4*b**4 - 2*e**(2*x)*a**2*b**6 - 2*e**(2*x)*b**8 - 4*e**
x*a**7*b - 12*e**x*a**5*b**3 - 12*e**x*a**3*b**5 - 4*e**x*a*b**7 + a**6...
```

3.133 $\int \frac{\frac{bB}{a}+B\sinh(x)}{a+b\sinh(x)}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 60

$$\int \frac{\frac{bB}{a}+B\sinh(x)}{a+b\sinh(x)}\,dx = \frac{Bx}{b}+\frac{2(a^2-b^2)\,B\text{arctanh}\left(\frac{b-a\tanh\left(\frac{x}{2}\right)}{\sqrt{a^2+b^2}}\right)}{ab\sqrt{a^2+b^2}}$$

output
```
B*x/b+2*(a^2-b^2)*B*arctanh((b-a*tanh(1/2*x))/(a^2+b^2)^(1/2))/a/b/(a^2+b^
2)^(1/2)
```

Mathematica [A] (verified)

Time = 0.11 (sec) , antiderivative size = 66, normalized size of antiderivative = 1.10

$$\int \frac{\frac{bB}{a}+B\sinh(x)}{a+b\sinh(x)}\,dx = \frac{B\left(ax-\frac{2(a^2-b^2)\arctan\left(\frac{b-a\tanh\left(\frac{x}{2}\right)}{\sqrt{-a^2-b^2}}\right)}{\sqrt{-a^2-b^2}}\right)}{ab}$$

input
```
Integrate[((b*B)/a + B*Sinh[x])/(a + b*Sinh[x]),x]
```

output
```
(B*(a*x - (2*(a^2 - b^2)*ArcTan[(b - a*Tanh[x/2])/Sqrt[-a^2 - b^2]])/Sqrt[
-a^2 - b^2]))/(a*b)
```

Rubi [A] (verified)

Time = 0.33 (sec) , antiderivative size = 65, normalized size of antiderivative = 1.08, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.300$, Rules used = {3042, 3214, 3042, 3139, 1083, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\frac{bB}{a}+B\sinh(x)}{a+b\sinh(x)}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\frac{bB}{a}-iB\sin(ix)}{a-ib\sin(ix)}dx$$

$$\Big\downarrow \text{3214}$$

$$\frac{Bx}{b}-\frac{B(a^2-b^2)\int \frac{1}{a+b\sinh(x)}dx}{ab}$$

$$\Big\downarrow \text{3042}$$

$$\frac{Bx}{b}-\frac{B(a^2-b^2)\int \frac{1}{a-ib\sin(ix)}dx}{ab}$$

$$\Big\downarrow \text{3139}$$

$$\frac{Bx}{b}-\frac{2B(a^2-b^2)\int \frac{1}{-a\tanh^2\left(\frac{x}{2}\right)+2b\tanh\left(\frac{x}{2}\right)+a}d\tanh\left(\frac{x}{2}\right)}{ab}$$

$$\Big\downarrow \text{1083}$$

$$\frac{4B(a^2-b^2)\int \frac{1}{4(a^2+b^2)-\left(2b-2a\tanh\left(\frac{x}{2}\right)\right)^2}d\left(2b-2a\tanh\left(\frac{x}{2}\right)\right)}{ab}+\frac{Bx}{b}$$

$$\Big\downarrow \text{219}$$

$$\frac{2B(a^2-b^2)\,\text{arctanh}\left(\frac{2b-2a\tanh\left(\frac{x}{2}\right)}{2\sqrt{a^2+b^2}}\right)}{ab\sqrt{a^2+b^2}}+\frac{Bx}{b}$$

input
```
Int[((b*B)/a + B*Sinh[x])/(a + b*Sinh[x]),x]
```

output
```
(B*x)/b + (2*(a^2 - b^2)*B*ArcTanh[(2*b - 2*a*Tanh[x/2])/(2*Sqrt[a^2 + b^2
])])/(a*b*Sqrt[a^2 + b^2])
```

Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1083
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Simp[-2   Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3139
```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Simp[2*(e/d)   Subst[Int[1/(a + 2*b*e*x + a
*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[a^2 - b^2, 0]
```

rule 3214
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])/((c_.) + (d_.)*sin[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[b*(x/d), x] - Simp[(b*c - a*d)/d   Int[1/(c + d
*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Maple [A] (verified)

Time = 0.16 (sec) , antiderivative size = 80, normalized size of antiderivative = 1.33

method	result
default	$\frac{2B\left(-\frac{a\ln\left(\tanh\left(\frac{x}{2}\right)-1\right)}{2b}+\frac{a\ln\left(\tanh\left(\frac{x}{2}\right)+1\right)}{2b}-\frac{\left(a^{2}-b^{2}\right)\operatorname{arctanh}\left(\frac{2a\tanh\left(\frac{x}{2}\right)-2b}{2\sqrt{a^{2}+b^{2}}}\right)}{b\sqrt{a^{2}+b^{2}}}\right)}{a}$
risch	$\frac{Bx}{b}+\frac{Ba\ln\left(\mathrm{e}^{x}+\frac{a\sqrt{a^{2}+b^{2}}+a^{2}+b^{2}}{\sqrt{a^{2}+b^{2}}\,b}\right)}{\sqrt{a^{2}+b^{2}}\,b}-\frac{Bb\ln\left(\mathrm{e}^{x}+\frac{a\sqrt{a^{2}+b^{2}}+a^{2}+b^{2}}{\sqrt{a^{2}+b^{2}}\,b}\right)}{\sqrt{a^{2}+b^{2}}\,a}-\frac{\ln\left(\mathrm{e}^{x}+\frac{a\sqrt{a^{2}+b^{2}}-a^{2}-b^{2}}{\sqrt{a^{2}+b^{2}}\,b}\right)aB}{\sqrt{a^{2}+b^{2}}\,b}+\frac{Bb\ln\left(\mathrm{e}^{x}+\frac{a\sqrt{a^{2}+}}{\sqrt{a}}\right.}{\sqrt{a^{2}+b^{2}}}$

input
```
int((b*B/a+B*sinh(x))/(a+b*sinh(x)),x,method=_RETURNVERBOSE)
```

output
```
2*B/a*(-1/2/b*a*ln(tanh(1/2*x)-1)+1/2/b*a*ln(tanh(1/2*x)+1)-(a^2-b^2)/b/(a
^2+b^2)^(1/2)*arctanh(1/2*(2*a*tanh(1/2*x)-2*b)/(a^2+b^2)^(1/2)))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 154 vs. $2(56) = 112$.

Time = 0.10 (sec) , antiderivative size = 154, normalized size of antiderivative = 2.57

$$\int \frac{\frac{bB}{a}+B\sinh(x)}{a+b\sinh(x)}\,dx = -\frac{(Ba^{2}-Bb^{2})\sqrt{a^{2}+b^{2}}\log\left(\frac{b^{2}\cosh(x)^{2}+b^{2}\sinh(x)^{2}+2\,ab\cosh(x)+2\,a^{2}+b^{2}+2\,(b^{2}\cosh(x)+ab)\sinh(x)-2\,\sqrt{a^{2}+b^{2}}(b\cosh(x)+b\sinh}{b\cosh(x)^{2}+b\sinh(x)^{2}+2\,a\cosh(x)+2\,(b\cosh(x)+a)\sinh(x)-b}\right.}{a^{3}b+ab^{3}}$$

input
```
integrate((b*B/a+B*sinh(x))/(a+b*sinh(x)),x, algorithm="fricas")
```

output
```
-((B*a^2 - B*b^2)*sqrt(a^2 + b^2)*log((b^2*cosh(x)^2 + b^2*sinh(x)^2 + 2*a
*b*cosh(x) + 2*a^2 + b^2 + 2*(b^2*cosh(x) + a*b)*sinh(x) - 2*sqrt(a^2 + b^
2)*(b*cosh(x) + b*sinh(x) + a))/(b*cosh(x)^2 + b*sinh(x)^2 + 2*a*cosh(x) +
 2*(b*cosh(x) + a)*sinh(x) - b)) - (B*a^3 + B*a*b^2)*x)/(a^3*b + a*b^3)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 18.82 (sec) , antiderivative size = 258, normalized size of antiderivative = 4.30

$$\int \frac{\frac{bB}{a}+B\sinh(x)}{a+b\sinh(x)}\,dx = \begin{cases} \text{NaN} \\ \frac{B\cosh(x)}{a} \\ \frac{Bx\tanh\left(\frac{x}{2}\right)}{b\tanh\left(\frac{x}{2}\right)-ib} - \frac{iBx}{b\tanh\left(\frac{x}{2}\right)-ib} - \frac{4B}{b\tanh\left(\frac{x}{2}\right)-ib} \\ \frac{Bx\tanh\left(\frac{x}{2}\right)}{b\tanh\left(\frac{x}{2}\right)+ib} + \frac{iBx}{b\tanh\left(\frac{x}{2}\right)+ib} - \frac{4B}{b\tanh\left(\frac{x}{2}\right)+ib} \\ \frac{Ba\log\left(\tanh\left(\frac{x}{2}\right)-\frac{b}{a}-\frac{\sqrt{a^2+b^2}}{a}\right)}{b\sqrt{a^2+b^2}} - \frac{Ba\log\left(\tanh\left(\frac{x}{2}\right)-\frac{b}{a}+\frac{\sqrt{a^2+b^2}}{a}\right)}{b\sqrt{a^2+b^2}} + \frac{Bx}{b} - \frac{Bb\log\left(\tanh\left(\frac{x}{2}\right)-\frac{b}{a}-\frac{\sqrt{a^2+b^2}}{a}\right)}{a\sqrt{a^2+b^2}} + \frac{Bb\log\left(\tanh\left(\frac{x}{2}\right)-\frac{b}{a}+\frac{\sqrt{a^2+b^2}}{a}\right)}{a\sqrt{a^2+b^2}} \end{cases}$$

input

```
integrate((b*B/a+B*sinh(x))/(a+b*sinh(x)),x)
```

output

```
Piecewise((nan, Eq(a, 0) & Eq(b, 0)), (B*cosh(x)/a, Eq(b, 0)), (B*x*tanh(x
/2)/(b*tanh(x/2) - I*b) - I*B*x/(b*tanh(x/2) - I*b) - 4*B/(b*tanh(x/2) - I
*b), Eq(a, -I*b)), (B*x*tanh(x/2)/(b*tanh(x/2) + I*b) + I*B*x/(b*tanh(x/2)
 + I*b) - 4*B/(b*tanh(x/2) + I*b), Eq(a, I*b)), (B*a*log(tanh(x/2) - b/a -
 sqrt(a**2 + b**2)/a)/(b*sqrt(a**2 + b**2)) - B*a*log(tanh(x/2) - b/a + sq
rt(a**2 + b**2)/a)/(b*sqrt(a**2 + b**2)) + B*x/b - B*b*log(tanh(x/2) - b/a
 - sqrt(a**2 + b**2)/a)/(a*sqrt(a**2 + b**2)) + B*b*log(tanh(x/2) - b/a +
sqrt(a**2 + b**2)/a)/(a*sqrt(a**2 + b**2)), True))
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 128 vs. 2(56) = 112.

Time = 0.12 (sec) , antiderivative size = 128, normalized size of antiderivative = 2.13

$$\int \frac{\frac{bB}{a}+B\sinh(x)}{a+b\sinh(x)}\,dx = -B\left(\frac{a\log\left(\frac{be^{(-x)}-a-\sqrt{a^2+b^2}}{be^{(-x)}-a+\sqrt{a^2+b^2}}\right)}{\sqrt{a^2+b^2}b} - \frac{x}{b}\right) + \frac{Bb\log\left(\frac{be^{(-x)}-a-\sqrt{a^2+b^2}}{be^{(-x)}-a+\sqrt{a^2+b^2}}\right)}{\sqrt{a^2+b^2}a}$$

input
```
integrate((b*B/a+B*sinh(x))/(a+b*sinh(x)),x, algorithm="maxima")
```

output
```
-B*(a*log((b*e^(-x) - a - sqrt(a^2 + b^2))/(b*e^(-x) - a + sqrt(a^2 + b^2)
))/(sqrt(a^2 + b^2)*b) - x/b) + B*b*log((b*e^(-x) - a - sqrt(a^2 + b^2))/(
b*e^(-x) - a + sqrt(a^2 + b^2)))/(sqrt(a^2 + b^2)*a)
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 82, normalized size of antiderivative = 1.37

$$\int \frac{\frac{bB}{a} + B\sinh(x)}{a + b\sinh(x)}\,dx = \frac{Bx}{b} - \frac{(Ba^2 - Bb^2)\log\left(\frac{\left|2\,be^x + 2\,a - 2\,\sqrt{a^2+b^2}\right|}{\left|2\,be^x + 2\,a + 2\,\sqrt{a^2+b^2}\right|}\right)}{\sqrt{a^2+b^2}ab}$$

input
```
integrate((b*B/a+B*sinh(x))/(a+b*sinh(x)),x, algorithm="giac")
```

output
```
B*x/b - (B*a^2 - B*b^2)*log(abs(2*b*e^x + 2*a - 2*sqrt(a^2 + b^2))/abs(2*b
*e^x + 2*a + 2*sqrt(a^2 + b^2)))/(sqrt(a^2 + b^2)*a*b)
```

Mupad [B] (verification not implemented)

Time = 2.13 (sec) , antiderivative size = 331, normalized size of antiderivative = 5.52

$$\int \frac{\frac{bB}{a} + B\sinh(x)}{a + b\sinh(x)}\,dx$$
$$= \frac{2\,\mathrm{atan}\left(\frac{a\,b^2\,\mathrm{e}^x\,\sqrt{-a^4\,b^2-a^2\,b^4}\,\left(\frac{2\,\left(B\,a^2\,\sqrt{-a^4\,b^2-a^2\,b^4}-B\,b^2\,\sqrt{-a^4\,b^2-a^2\,b^4}\right)}{a^2\,b^4\,\sqrt{-a^4\,b^2-a^2\,b^4}\,\sqrt{B^2\,\left(a^2-b^2\right)^2}}+\frac{2\,a^2\,\sqrt{B^2\,a^4-2\,B^2\,a^2\,b^2+B^2\,b^4}}{B\,b^2\,\sqrt{-a^4\,b^2-a^2\,b^4}\,\left(a^2-b^2\right)\,\sqrt{-a^2\,b^2\,\left(a^2+b^2\right)}}\right)}{2} - \frac{a^2\,b\,\sqrt{B^2}}{B\,(a^2-}\right.}{\sqrt{-a^4\,b^2-a^2\,b^4}}$$
$$+ \frac{B\,x}{b}$$

input
```
int((B*sinh(x) + (B*b)/a)/(a + b*sinh(x)),x)
```

output
```
(2*atan((a*b^2*exp(x)*(- a^2*b^4 - a^4*b^2)^(1/2)*((2*(B*a^2*(- a^2*b^4 -
a^4*b^2)^(1/2) - B*b^2*(- a^2*b^4 - a^4*b^2)^(1/2)))/(a^2*b^4*(- a^2*b^4 -
 a^4*b^2)^(1/2)*(B^2*(a^2 - b^2)^2)^(1/2)) + (2*a^2*(B^2*a^4 + B^2*b^4 - 2
*B^2*a^2*b^2)^(1/2))/(B*b^2*(- a^2*b^4 - a^4*b^2)^(1/2)*(a^2 - b^2)*(-a^2*
b^2*(a^2 + b^2))^(1/2))))/2 - (a^2*b*(B^2*a^4 + B^2*b^4 - 2*B^2*a^2*b^2)^(
1/2))/(B*(a^2 - b^2)*(-a^2*b^2*(a^2 + b^2))^(1/2)))*(B^2*a^4 + B^2*b^4 - 2
*B^2*a^2*b^2)^(1/2))/(- a^2*b^4 - a^4*b^2)^(1/2) + (B*x)/b
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 97, normalized size of antiderivative = 1.62

$$\int \frac{\frac{bB}{a} + B\sinh(x)}{a + b\sinh(x)}\,dx = \frac{-2\sqrt{a^2+b^2}\,atan\left(\frac{e^x bi+ai}{\sqrt{a^2+b^2}}\right)a^2i + 2\sqrt{a^2+b^2}\,atan\left(\frac{e^x bi+ai}{\sqrt{a^2+b^2}}\right)b^2i + a^3x + a\,b^2x}{a\,(a^2+b^2)}$$

input
```
int((b*B/a+B*sinh(x))/(a+b*sinh(x)),x)
```

output
```
( - 2*sqrt(a**2 + b**2)*atan((e**x*b*i + a*i)/sqrt(a**2 + b**2))*a**2*i +
2*sqrt(a**2 + b**2)*atan((e**x*b*i + a*i)/sqrt(a**2 + b**2))*b**2*i + a**3
*x + a*b**2*x)/(a*(a**2 + b**2))
```

3.134 $\int \frac{\frac{aB}{b}+B\sinh(x)}{a+b\sinh(x)}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 6

$$\int \frac{\frac{aB}{b}+B\sinh(x)}{a+b\sinh(x)}\,dx=\frac{Bx}{b}$$

output
```
B*x/b
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 6, normalized size of antiderivative = 1.00

$$\int \frac{\frac{aB}{b}+B\sinh(x)}{a+b\sinh(x)}\,dx=\frac{Bx}{b}$$

input
```
Integrate[((a*B)/b + B*Sinh[x])/(a + b*Sinh[x]),x]
```

output
```
(B*x)/b
```

Rubi [A] (verified)

Time = 0.15 (sec) , antiderivative size = 6, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {2011, 24}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\frac{aB}{b}+B\sinh(x)}{a+b\sinh(x)}\,dx$$

$$\Big\downarrow \ 2011$$

$$\frac{B\int 1dx}{b}$$

$$\Big\downarrow \ 24$$

$$\frac{Bx}{b}$$

input
```
Int[((a*B)/b + B*Sinh[x])/(a + b*Sinh[x]),x]
```

output
```
(B*x)/b
```

Defintions of rubi rules used

rule 24
```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

rule 2011
```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_.)*((c_) + (d_.)*(v_))^(n_.), x_Symbol] :>
 Simp[(b/d)^m   Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x
] && EqQ[b*c - a*d, 0] && IntegerQ[m] && ( !IntegerQ[n] || SimplerQ[c + d*x
, a + b*x])
```

Maple [A] (verified)

Time = 0.05 (sec) , antiderivative size = 7, normalized size of antiderivative = 1.17

method	result	size
default	$\frac{Bx}{b}$	7
risch	$\frac{Bx}{b}$	7
orering	$\frac{x\left(\frac{aB}{b}+B\sinh(x)\right)}{a+b\sinh(x)}$	22

input
```
int((a*B/b+B*sinh(x))/(a+b*sinh(x)),x,method=_RETURNVERBOSE)
```

output
```
B*x/b
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 6, normalized size of antiderivative = 1.00

$$\int \frac{\frac{aB}{b}+B\sinh(x)}{a+b\sinh(x)}\,dx=\frac{Bx}{b}$$

input
```
integrate((a*B/b+B*sinh(x))/(a+b*sinh(x)),x, algorithm="fricas")
```

output
```
B*x/b
```

Sympy [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 3, normalized size of antiderivative = 0.50

$$\int \frac{\frac{aB}{b}+B\sinh(x)}{a+b\sinh(x)}\,dx=\frac{Bx}{b}$$

input
```
integrate((a*B/b+B*sinh(x))/(a+b*sinh(x)),x)
```

output
```
B*x/b
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 128 vs. 2(6) = 12.

Time = 0.14 (sec) , antiderivative size = 128, normalized size of antiderivative = 21.33

$$\int \frac{\frac{aB}{b} + B\sinh(x)}{a + b\sinh(x)}\,dx = -B\left(\frac{a\log\left(\frac{be^{(-x)}-a-\sqrt{a^2+b^2}}{be^{(-x)}-a+\sqrt{a^2+b^2}}\right)}{\sqrt{a^2+b^2}b} - \frac{x}{b}\right) + \frac{Ba\log\left(\frac{be^{(-x)}-a-\sqrt{a^2+b^2}}{be^{(-x)}-a+\sqrt{a^2+b^2}}\right)}{\sqrt{a^2+b^2}b}$$

input
```
integrate((a*B/b+B*sinh(x))/(a+b*sinh(x)),x, algorithm="maxima")
```

output
```
-B*(a*log((b*e^(-x) - a - sqrt(a^2 + b^2))/(b*e^(-x) - a + sqrt(a^2 + b^2)
))/(sqrt(a^2 + b^2)*b) - x/b) + B*a*log((b*e^(-x) - a - sqrt(a^2 + b^2))/(
b*e^(-x) - a + sqrt(a^2 + b^2)))/(sqrt(a^2 + b^2)*b)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 6, normalized size of antiderivative = 1.00

$$\int \frac{\frac{aB}{b} + B\sinh(x)}{a + b\sinh(x)}\,dx = \frac{Bx}{b}$$

input
```
integrate((a*B/b+B*sinh(x))/(a+b*sinh(x)),x, algorithm="giac")
```

output
```
B*x/b
```

Mupad [B] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 6, normalized size of antiderivative = 1.00

$$\int \frac{\frac{aB}{b} + B\sinh(x)}{a + b\sinh(x)}\,dx = \frac{B\,x}{b}$$

input
```
int((B*sinh(x) + (B*a)/b)/(a + b*sinh(x)),x)
```

output
```
(B*x)/b
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 1, normalized size of antiderivative = 0.17

$$\int \frac{\frac{aB}{b} + B\sinh(x)}{a + b\sinh(x)}\,dx = x$$

input
```
int((a*B/b+B*sinh(x))/(a+b*sinh(x)),x)
```

output
```
x
```

3.135 $\int \frac{a-b\sinh(x)}{(b+a\sinh(x))^2}\,dx$

Optimal result

Integrand size = 16, antiderivative size = 12

$$\int \frac{a-b\sinh(x)}{(b+a\sinh(x))^2}\,dx = -\frac{\cosh(x)}{b+a\sinh(x)}$$

output
```
-cosh(x)/(b+a*sinh(x))
```

Mathematica [A] (verified)

Time = 0.16 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.00

$$\int \frac{a-b\sinh(x)}{(b+a\sinh(x))^2}\,dx = -\frac{\cosh(x)}{b+a\sinh(x)}$$

input
```
Integrate[(a - b*Sinh[x])/(b + a*Sinh[x])^2,x]
```

output
```
-(Cosh[x]/(b + a*Sinh[x]))
```

Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.188$, Rules used = {3042, 3233, 24}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{a-b\sinh(x)}{(a\sinh(x)+b)^2}dx$$
$$\Big\downarrow \text{3042}$$
$$\int \frac{a+ib\sin(ix)}{(b-ia\sin(ix))^2}dx$$
$$\Big\downarrow \text{3233}$$
$$-\frac{\int 0dx}{a^2+b^2}-\frac{\cosh(x)}{a\sinh(x)+b}$$
$$\Big\downarrow \text{24}$$
$$-\frac{\cosh(x)}{a\sinh(x)+b}$$

input
```
Int[(a - b*Sinh[x])/(b + a*Sinh[x])^2,x]
```

output
```
-(Cosh[x]/(b + a*Sinh[x]))
```

Defintions of rubi rules used

rule 24
```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3233
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Simp[(-(b*c - a*d))*Cos[e + f*x]*((a + b*Sin[e +
 f*x])^(m + 1)/(f*(m + 1)*(a^2 - b^2))), x] + Simp[1/((m + 1)*(a^2 - b^2))
  Int[(a + b*Sin[e + f*x])^(m + 1)*Simp[(a*c - b*d)*(m + 1) - (b*c - a*d)*(
m + 2)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c
- a*d, 0] && NeQ[a^2 - b^2, 0] && LtQ[m, -1] && IntegerQ[2*m]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 25 vs. $2(12) = 24$.

Time = 0.18 (sec) , antiderivative size = 26, normalized size of antiderivative = 2.17

method	result	size
parallelrisch	$\frac{-a\sinh(x)-b(1+\cosh(x))}{b(b+a\sinh(x))}$	26
risch	$-\frac{2(-\mathrm{e}^{x}b+a)}{a(a\,\mathrm{e}^{2x}+2\,\mathrm{e}^{x}b-a)}$	30
default	$-\frac{2\left(\frac{a\tanh\left(\frac{x}{2}\right)}{2b}+\frac{1}{2}\right)}{-\frac{\tanh\left(\frac{x}{2}\right)^{2}b}{2}+a\tanh\left(\frac{x}{2}\right)+\frac{b}{2}}$	36

input
```
int((a-b*sinh(x))/(b+a*sinh(x))^2,x,method=_RETURNVERBOSE)
```

output
```
(-a*sinh(x)-b*(1+cosh(x)))/b/(b+a*sinh(x))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 58 vs. $2(12) = 24$.

Time = 0.07 (sec) , antiderivative size = 58, normalized size of antiderivative = 4.83

$$\int \frac{a-b\sinh(x)}{(b+a\sinh(x))^2}\,dx = \frac{2\,(b\cosh{(x)}+b\sinh{(x)}-a)}{a^2\cosh{(x)}^2+a^2\sinh{(x)}^2+2\,ab\cosh{(x)}-a^2+2\,(a^2\cosh{(x)}+ab)\sinh{(x)}}$$

input
```
integrate((a-b*sinh(x))/(b+a*sinh(x))^2,x, algorithm="fricas")
```

output
```
2*(b*cosh(x) + b*sinh(x) - a)/(a^2*cosh(x)^2 + a^2*sinh(x)^2 + 2*a*b*cosh(
x) - a^2 + 2*(a^2*cosh(x) + a*b)*sinh(x))
```

Sympy [F(-1)]

Timed out.

$$\int \frac{a - b\sinh(x)}{(b + a\sinh(x))^2}\,dx = \text{Timed out}$$

input
```
integrate((a-b*sinh(x))/(b+a*sinh(x))**2,x)
```

output
```
Timed out
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 230 vs. $2(12) = 24$.

Time = 0.15 (sec) , antiderivative size = 230, normalized size of antiderivative = 19.17

$$\begin{aligned}\int \frac{a - b\sinh(x)}{(b + a\sinh(x))^2}\,dx &= -b\left(\frac{a\log\left(\frac{ae^{(-x)}-b-\sqrt{a^2+b^2}}{ae^{(-x)}-b+\sqrt{a^2+b^2}}\right)}{(a^2+b^2)^{\frac{3}{2}}} + \frac{2\,(b^2e^{(-x)}+ab)}{a^4+a^2b^2+2\,(a^3b+ab^3)e^{(-x)}-(a^4+a^2b^2)e^{(-2\,x)}}\right) \\ &\quad + a\left(\frac{b\log\left(\frac{ae^{(-x)}-b-\sqrt{a^2+b^2}}{ae^{(-x)}-b+\sqrt{a^2+b^2}}\right)}{(a^2+b^2)^{\frac{3}{2}}} - \frac{2\,(be^{(-x)}+a)}{a^3+ab^2+2\,(a^2b+b^3)e^{(-x)}-(a^3+ab^2)e^{(-2\,x)}}\right)\end{aligned}$$

input
```
integrate((a-b*sinh(x))/(b+a*sinh(x))^2,x, algorithm="maxima")
```

output
```
-b*(a*log((a*e^(-x) - b - sqrt(a^2 + b^2))/(a*e^(-x) - b + sqrt(a^2 + b^2)
))/(a^2 + b^2)^(3/2) + 2*(b^2*e^(-x) + a*b)/(a^4 + a^2*b^2 + 2*(a^3*b + a*
b^3)*e^(-x) - (a^4 + a^2*b^2)*e^(-2*x))) + a*(b*log((a*e^(-x) - b - sqrt(a
^2 + b^2))/(a*e^(-x) - b + sqrt(a^2 + b^2)))/(a^2 + b^2)^(3/2) - 2*(b*e^(-
x) + a)/(a^3 + a*b^2 + 2*(a^2*b + b^3)*e^(-x) - (a^3 + a*b^2)*e^(-2*x)))
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 30 vs. $2(12) = 24$.

Time = 0.14 (sec) , antiderivative size = 30, normalized size of antiderivative = 2.50

$$\int \frac{a - b\sinh(x)}{(b + a\sinh(x))^2}\,dx = \frac{2\,(be^x - a)}{(ae^{(2\,x)} + 2\,be^x - a)a}$$

input
```
integrate((a-b*sinh(x))/(b+a*sinh(x))^2,x, algorithm="giac")
```

output
```
2*(b*e^x - a)/((a*e^(2*x) + 2*b*e^x - a)*a)
```

Mupad [B] (verification not implemented)

Time = 1.48 (sec) , antiderivative size = 49, normalized size of antiderivative = 4.08

$$\int \frac{a - b\sinh(x)}{(b + a\sinh(x))^2}\,dx = \frac{\frac{2\,e^x\,(a^3\,b + a\,b^3)}{a\,(a^3 + a\,b^2)} - 2}{2\,b\,e^x - a + a\,e^{2\,x}}$$

input
```
int((a - b*sinh(x))/(b + a*sinh(x))^2,x)
```

output
```
((2*exp(x)*(a*b^3 + a^3*b))/(a*(a*b^2 + a^3)) - 2)/(2*b*exp(x) - a + a*exp
(2*x))
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 29, normalized size of antiderivative = 2.42

$$\int \frac{a - b\sinh(x)}{(b + a\sinh(x))^2}\,dx = \frac{-e^{2x} - 1}{e^{2x}a + 2e^x b - a}$$

input
```
int((a-b*sinh(x))/(b+a*sinh(x))^2,x)
```

output
```
( - (e**(2*x) + 1))/(e**(2*x)*a + 2*e***x*b - a)
```

3.136 $\int \frac{2-\sinh(x)}{2+\sinh(x)}\,dx$

Optimal result

Integrand size = 13, antiderivative size = 34

$$\int \frac{2-\sinh(x)}{2+\sinh(x)}\,dx = -x+\frac{4x}{\sqrt{5}}-\frac{8\operatorname{arctanh}\left(\frac{\cosh(x)}{2+\sqrt{5}+\sinh(x)}\right)}{\sqrt{5}}$$

output
```
-x+4/5*x*5^(1/2)-8/5*arctanh(cosh(x)/(2+5^(1/2)+sinh(x)))*5^(1/2)
```

Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.82

$$\int \frac{2-\sinh(x)}{2+\sinh(x)}\,dx = -x-\frac{8\operatorname{arctanh}\left(\frac{1-2\tanh\left(\frac{x}{2}\right)}{\sqrt{5}}\right)}{\sqrt{5}}$$

input
```
Integrate[(2 - Sinh[x])/(2 + Sinh[x]),x]
```

output
```
-x - (8*ArcTanh[(1 - 2*Tanh[x/2])/Sqrt[5]])/Sqrt[5]
```

Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.06, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.308, Rules used = {3042, 3214, 3042, 3136}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{2-\sinh(x)}{\sinh(x)+2}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{2+i\sin(ix)}{2-i\sin(ix)}dx$$

$$\Big\downarrow \text{3214}$$

$$4\int \frac{1}{\sinh(x)+2}dx-x$$

$$\Big\downarrow \text{3042}$$

$$-x+4\int \frac{1}{2-i\sin(ix)}dx$$

$$\Big\downarrow \text{3136}$$

$$4\left(\frac{x}{\sqrt{5}}-\frac{2\text{arctanh}\left(\frac{\cosh(x)}{\sinh(x)+\sqrt{5}+2}\right)}{\sqrt{5}}\right)-x$$

input
```
Int[(2 - Sinh[x])/(2 + Sinh[x]),x]
```

output
```
-x + 4*(x/Sqrt[5] - (2*ArcTanh[Cosh[x]/(2 + Sqrt[5] + Sinh[x])])/Sqrt[5])
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3136
```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{q = Rt[
a^2 - b^2, 2]}, Simp[x/q, x] + Simp[(2/(d*q))*ArcTan[b*(Cos[c + d*x]/(a + q
 + b*Sin[c + d*x]))], x]] /; FreeQ[{a, b, c, d}, x] && GtQ[a^2 - b^2, 0] &&
 PosQ[a]
```

rule 3214
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])/((c_.) + (d_.)*sin[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[b*(x/d), x] - Simp[(b*c - a*d)/d   Int[1/(c + d
*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Maple [A] (verified)

Time = 0.11 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.97

method	result	size
risch	$-x+\frac{4\sqrt{5}\, \ln\left({\mathrm e}^{x}+2-\sqrt{5}\right)}{5}-\frac{4\sqrt{5}\, \ln\left({\mathrm e}^{x}+2+\sqrt{5}\right)}{5}$	33
default	$-\ln\left(\tanh\left(\frac{x}{2}\right)+1\right)+\frac{8\sqrt{5}\, \operatorname{arctanh}\left(\frac{\left(2\tanh\left(\frac{x}{2}\right)-1\right)\sqrt{5}}{5}\right)}{5}+\ln\left(\tanh\left(\frac{x}{2}\right)-1\right)$	37

input
```
int((2-sinh(x))/(2+sinh(x)),x,method=_RETURNVERBOSE)
```

output
```
-x+4/5*5^(1/2)*ln(exp(x)+2-5^(1/2))-4/5*5^(1/2)*ln(exp(x)+2+5^(1/2))
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 42, normalized size of antiderivative = 1.24

$$\int \frac{2-\sinh(x)}{2+\sinh(x)}\,dx = \frac{4}{5}\sqrt{5}\log\left(-\frac{\left(2\sqrt{5}-5\right)\cosh\left(x\right)-2\left(\sqrt{5}-2\right)\sinh\left(x\right)+\sqrt{5}-2}{\sinh\left(x\right)+2}\right) - x$$

input
```
integrate((2-sinh(x))/(2+sinh(x)),x, algorithm="fricas")
```

output
```
4/5*sqrt(5)*log(-((2*sqrt(5) - 5)*cosh(x) - 2*(sqrt(5) - 2)*sinh(x) + sqrt
(5) - 2)/(sinh(x) + 2)) - x
```

Sympy [A] (verification not implemented)

Time = 1.04 (sec) , antiderivative size = 51, normalized size of antiderivative = 1.50

$$\int \frac{2-\sinh(x)}{2+\sinh(x)}\,dx = -x + \frac{4\sqrt{5}\log\left(\tanh\left(\frac{x}{2}\right)-\frac{1}{2}+\frac{\sqrt{5}}{2}\right)}{5} - \frac{4\sqrt{5}\log\left(\tanh\left(\frac{x}{2}\right)-\frac{\sqrt{5}}{2}-\frac{1}{2}\right)}{5}$$

input
```
integrate((2-sinh(x))/(2+sinh(x)),x)
```

output
```
-x + 4*sqrt(5)*log(tanh(x/2) - 1/2 + sqrt(5)/2)/5 - 4*sqrt(5)*log(tanh(x/2
) - sqrt(5)/2 - 1/2)/5
```

Maxima [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.00

$$\int \frac{2-\sinh(x)}{2+\sinh(x)}\,dx = \frac{4}{5}\sqrt{5}\log\left(-\frac{\sqrt{5}-e^{(-x)}+2}{\sqrt{5}+e^{(-x)}-2}\right) - x$$

input
```
integrate((2-sinh(x))/(2+sinh(x)),x, algorithm="maxima")
```

output
```
4/5*sqrt(5)*log(-(sqrt(5) - e^(-x) + 2)/(sqrt(5) + e^(-x) - 2)) - x
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.97

$$\int \frac{2-\sinh(x)}{2+\sinh(x)}\,dx = \frac{4}{5}\sqrt{5}\log\left(\frac{|-2\sqrt{5}+2e^x+4|}{2\left(\sqrt{5}+e^x+2\right)}\right)-x$$

input
```
integrate((2-sinh(x))/(2+sinh(x)),x, algorithm="giac")
```

output
```
4/5*sqrt(5)*log(1/2*abs(-2*sqrt(5) + 2*e^x + 4)/(sqrt(5) + e^x + 2)) - x
```

Mupad [B] (verification not implemented)

Time = 1.54 (sec) , antiderivative size = 48, normalized size of antiderivative = 1.41

$$\int \frac{2-\sinh(x)}{2+\sinh(x)}\,dx = \frac{4\sqrt{5}\,\ln\left(-8\,e^x-\frac{4\sqrt{5}\,(4\,e^x-2)}{5}\right)}{5}-x-\frac{4\sqrt{5}\,\ln\left(\frac{4\sqrt{5}\,(4\,e^x-2)}{5}-8\,e^x\right)}{5}$$

input
```
int(-(sinh(x) - 2)/(sinh(x) + 2),x)
```

output
```
(4*5^(1/2)*log(- 8*exp(x) - (4*5^(1/2)*(4*exp(x) - 2))/5))/5 - x - (4*5^(1
/2)*log((4*5^(1/2)*(4*exp(x) - 2))/5 - 8*exp(x)))/5
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 30, normalized size of antiderivative = 0.88

$$\int \frac{2-\sinh(x)}{2+\sinh(x)}\,dx = \frac{4\sqrt{5}\log\left(e^{x}-\sqrt{5}+2\right)}{5} - \frac{4\sqrt{5}\log\left(e^{x}+\sqrt{5}+2\right)}{5} - x$$

input
```
int((2-sinh(x))/(2+sinh(x)),x)
```

output
```
(4*sqrt(5)*log(e**x - sqrt(5) + 2) - 4*sqrt(5)*log(e**x + sqrt(5) + 2) - 5
*x)/5
```

3.137 $\int \frac{A+B\sinh(x)}{\sqrt{a+b\sinh(x)}}\,dx$

Optimal result

Integrand size = 17, antiderivative size = 136

$$\int \frac{A+B\sinh(x)}{\sqrt{a+b\sinh(x)}}\,dx = \frac{2iBE\left(\frac{\pi}{4}-\frac{ix}{2}\middle|\frac{2b}{ia+b}\right)\sqrt{a+b\sinh(x)}}{b\sqrt{\frac{a+b\sinh(x)}{a-ib}}} + \frac{2i(Ab-aB)\operatorname{EllipticF}\left(\frac{\pi}{4}-\frac{ix}{2},\frac{2b}{ia+b}\right)\sqrt{\frac{a+b\sinh(x)}{a-ib}}}{b\sqrt{a+b\sinh(x)}}$$

output
```
2*I*B*EllipticE(cos(1/4*Pi+1/2*I*x),2^(1/2)*(b/(I*a+b))^(1/2))*(a+b*sinh(x
))^(1/2)/b/((a+b*sinh(x))/(a-I*b))^(1/2)-2*I*(A*b-B*a)*InverseJacobiAM(-1/
4*Pi+1/2*I*x,2^(1/2)*(b/(I*a+b))^(1/2))*((a+b*sinh(x))/(a-I*b))^(1/2)/b/(a
+b*sinh(x))^(1/2)
```

Mathematica [A] (verified)

Time = 0.38 (sec) , antiderivative size = 109, normalized size of antiderivative = 0.80

$$\int \frac{A+B\sinh(x)}{\sqrt{a+b\sinh(x)}}\,dx = \frac{2\left((ia+b)BE\left(\frac{1}{4}(\pi-2ix)\middle|-\frac{2ib}{a-ib}\right)+i(Ab-aB)\operatorname{EllipticF}\left(\frac{1}{4}(\pi-2ix),-\frac{2ib}{a-ib}\right)\right)\sqrt{\frac{a+b\sinh(x)}{a-ib}}}{b\sqrt{a+b\sinh(x)}}$$

input
```
Integrate[(A + B*Sinh[x])/Sqrt[a + b*Sinh[x]],x]
```

output
```
(2*((I*a + b)*B*EllipticE[(Pi - (2*I)*x)/4, ((-2*I)*b)/(a - I*b)] + I*(A*b
 - a*B)*EllipticF[(Pi - (2*I)*x)/4, ((-2*I)*b)/(a - I*b)])*Sqrt[(a + b*Sin
h[x])/(a - I*b)])/(b*Sqrt[a + b*Sinh[x]])
```

Rubi [A] (verified)

Time = 0.61 (sec) , antiderivative size = 136, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.529$, Rules used = {3042, 3231, 3042, 3134, 3042, 3132, 3142, 3042, 3140}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+B\sinh(x)}{\sqrt{a+b\sinh(x)}}dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \frac{A-iB\sin(ix)}{\sqrt{a-ib\sin(ix)}}dx$$

$$\Big\downarrow \text{ 3231}$$

$$\frac{(Ab-aB)\int \frac{1}{\sqrt{a+b\sinh(x)}}dx}{b}+\frac{B\int \sqrt{a+b\sinh(x)}dx}{b}$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{(Ab-aB)\int \frac{1}{\sqrt{a-ib\sin(ix)}}dx}{b}+\frac{B\int \sqrt{a-ib\sin(ix)}dx}{b}$$

$$\Big\downarrow \text{ 3134}$$

$$\frac{(Ab-aB)\int \frac{1}{\sqrt{a-ib\sin(ix)}}dx}{b}+\frac{B\sqrt{a+b\sinh(x)}\int \sqrt{\frac{a}{a-ib}+\frac{b\sinh(x)}{a-ib}}dx}{b\sqrt{\frac{a+b\sinh(x)}{a-ib}}}$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{(Ab-aB)\int \frac{1}{\sqrt{a-ib\sin(ix)}}dx}{b}+\frac{B\sqrt{a+b\sinh(x)}\int \sqrt{\frac{a}{a-ib}-\frac{ib\sin(ix)}{a-ib}}dx}{b\sqrt{\frac{a+b\sinh(x)}{a-ib}}}$$

$$\Big\downarrow \text{3132}$$

$$\frac{(Ab-aB)\int \frac{1}{\sqrt{a-ib\sin(ix)}}dx}{b}+\frac{2iB\sqrt{a+b\sinh(x)}E\left(\frac{\pi}{4}-\frac{ix}{2}|\frac{2b}{ia+b}\right)}{b\sqrt{\frac{a+b\sinh(x)}{a-ib}}}$$

$$\Big\downarrow \text{3142}$$

$$\frac{(Ab-aB)\sqrt{\frac{a+b\sinh(x)}{a-ib}}\int \frac{1}{\sqrt{\frac{a}{a-ib}+\frac{b\sinh(x)}{a-ib}}}dx}{b\sqrt{a+b\sinh(x)}}+\frac{2iB\sqrt{a+b\sinh(x)}E\left(\frac{\pi}{4}-\frac{ix}{2}|\frac{2b}{ia+b}\right)}{b\sqrt{\frac{a+b\sinh(x)}{a-ib}}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{(Ab-aB)\sqrt{\frac{a+b\sinh(x)}{a-ib}}\int \frac{1}{\sqrt{\frac{a}{a-ib}-\frac{ib\sin(ix)}{a-ib}}}dx}{b\sqrt{a+b\sinh(x)}}+\frac{2iB\sqrt{a+b\sinh(x)}E\left(\frac{\pi}{4}-\frac{ix}{2}|\frac{2b}{ia+b}\right)}{b\sqrt{\frac{a+b\sinh(x)}{a-ib}}}$$

$$\Big\downarrow \text{3140}$$

$$\frac{2i(Ab-aB)\sqrt{\frac{a+b\sinh(x)}{a-ib}}\,\text{EllipticF}\left(\frac{\pi}{4}-\frac{ix}{2},\frac{2b}{ia+b}\right)}{b\sqrt{a+b\sinh(x)}}+\frac{2iB\sqrt{a+b\sinh(x)}E\left(\frac{\pi}{4}-\frac{ix}{2}|\frac{2b}{ia+b}\right)}{b\sqrt{\frac{a+b\sinh(x)}{a-ib}}}$$

input
```
Int[(A + B*Sinh[x])/Sqrt[a + b*Sinh[x]],x]
```

output
```
((2*I)*B*EllipticE[Pi/4 - (I/2)*x, (2*b)/(I*a + b)]*Sqrt[a + b*Sinh[x]])/(
b*Sqrt[(a + b*Sinh[x])/(a - I*b)]) + ((2*I)*(A*b - a*B)*EllipticF[Pi/4 - (
I/2)*x, (2*b)/(I*a + b)]*Sqrt[(a + b*Sinh[x])/(a - I*b)])/(b*Sqrt[a + b*Si
nh[x]])
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3132
```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[2*(Sqrt[a
 + b]/d)*EllipticE[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

rule 3134
```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)]   Int[Sqrt[a/(a + b) + (
b/(a + b))*Sin[c + d*x]], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2
, 0] &&  !GtQ[a + b, 0]
```

rule 3140
```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/(d*S
qrt[a + b]))*EllipticF[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

rule 3142
```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]]   Int[1/Sqrt[a/(a + b)
 + (b/(a + b))*Sin[c + d*x]], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
 b^2, 0] &&  !GtQ[a + b, 0]
```

rule 3231
```
Int[((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])/Sqrt[(a_) + (b_.)*sin[(e_.) + (
f_.)*(x_)]], x_Symbol] :> Simp[(b*c - a*d)/b   Int[1/Sqrt[a + b*Sin[e + f*x
]], x], x] + Simp[d/b   Int[Sqrt[a + b*Sin[e + f*x]], x], x] /; FreeQ[{a, b
, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
```

Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 265 vs. $2(123) = 246$.

Time = 1.13 (sec) , antiderivative size = 266, normalized size of antiderivative = 1.96

method	result
default	$-\frac{2(ib-a)\sqrt{-\frac{a+b\sinh(x)}{ib-a}}\sqrt{\frac{(i-\sinh(x))b}{ib+a}}\sqrt{\frac{b(i+\sinh(x))}{ib-a}}\left(iB\,\text{EllipticF}\left(\sqrt{-\frac{a+b\sinh(x)}{ib-a}},\sqrt{-\frac{ib-a}{ib+a}}\right)b-iB\,\text{EllipticE}\left(\sqrt{-\frac{a+b\sinh(x)}{ib-a}}\right.\right.}{b^2\cosh(x)\sqrt{a+b\sinh(x)}}$
parts	$-\frac{2A(ib-a)\sqrt{-\frac{a+b\sinh(x)}{ib-a}}\sqrt{\frac{(i-\sinh(x))b}{ib+a}}\sqrt{\frac{b(i+\sinh(x))}{ib-a}}\,\text{EllipticF}\left(\sqrt{-\frac{a+b\sinh(x)}{ib-a}},\sqrt{-\frac{ib-a}{ib+a}}\right)}{b\cosh(x)\sqrt{a+b\sinh(x)}}+\frac{2B(ib-a)\sqrt{-\frac{a+b\sinh(x)}{ib-a}}\sqrt{\frac{(i}{}}}{}$
risch	$\frac{B\left(b\,e^{2x}+2\,e^{x}a-b\right)\sqrt{2}\,e^{-x}}{b\sqrt{\left(b\,e^{2x}+2\,e^{x}a-b\right)e^{-x}}}+\frac{\left(\frac{4A\left(a+\sqrt{a^2+b^2}\right)\sqrt{\frac{\left(e^{x}+\frac{a+\sqrt{a^2+b^2}}{b}\right)b}{a+\sqrt{a^2+b^2}}}\sqrt{\frac{e^{x}-\frac{-a+\sqrt{a^2+b^2}}{b}}{-\frac{a+\sqrt{a^2+b^2}}{b}-\frac{-a+\sqrt{a^2+b^2}}{b}}}\sqrt{-\frac{e^{x}b}{a+\sqrt{a^2+b^2}}}\,\text{EllipticF}\left(\sqrt{\frac{\left(e^{x}\right.}{}}\right.}{b\sqrt{e^{3x}b+2a\,e^{2x}-e^{x}b}}\right.}{}$

input
```
int((A+B*sinh(x))/(a+b*sinh(x))^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-2*(I*b-a)*(-(a+b*sinh(x))/(I*b-a))^(1/2)*((I-sinh(x))*b/(I*b+a))^(1/2)*(b
*(I+sinh(x))/(I*b-a))^(1/2)*(I*B*EllipticF((-(a+b*sinh(x))/(I*b-a))^(1/2),
(-(I*b-a)/(I*b+a))^(1/2))*b-I*B*EllipticE((-(a+b*sinh(x))/(I*b-a))^(1/2),(
-(I*b-a)/(I*b+a))^(1/2))*b+A*EllipticF((-(a+b*sinh(x))/(I*b-a))^(1/2),(-(I
*b-a)/(I*b+a))^(1/2))*b-B*EllipticE((-(a+b*sinh(x))/(I*b-a))^(1/2),(-(I*b-
a)/(I*b+a))^(1/2))*a)/b^2/cosh(x)/(a+b*sinh(x))^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 184, normalized size of antiderivative = 1.35

$$\int \frac{A+B\sinh(x)}{\sqrt{a+b\sinh(x)}}\,dx =$$
$$-\frac{2\left(6\sqrt{\frac{1}{2}}Bb^{\frac{3}{2}}\text{weierstrassZeta}\left(\frac{4\,(4\,a^2+3\,b^2)}{3\,b^2},-\frac{8\,(8\,a^3+9\,ab^2)}{27\,b^3},\text{weierstrassPInverse}\left(\frac{4\,(4\,a^2+3\,b^2)}{3\,b^2},-\frac{8\,(8\,a^3+9\,ab^2)}{27\,b^3},\right.\right.\right.}{}$$

input
```
integrate((A+B*sinh(x))/(a+b*sinh(x))^(1/2),x, algorithm="fricas")
```

output
```
-2/3*(6*sqrt(1/2)*B*b^(3/2)*weierstrassZeta(4/3*(4*a^2 + 3*b^2)/b^2, -8/27
*(8*a^3 + 9*a*b^2)/b^3, weierstrassPInverse(4/3*(4*a^2 + 3*b^2)/b^2, -8/27
*(8*a^3 + 9*a*b^2)/b^3, 1/3*(3*b*cosh(x) + 3*b*sinh(x) + 2*a)/b)) + 2*sqrt
(1/2)*(2*B*a - 3*A*b)*sqrt(b)*weierstrassPInverse(4/3*(4*a^2 + 3*b^2)/b^2,
 -8/27*(8*a^3 + 9*a*b^2)/b^3, 1/3*(3*b*cosh(x) + 3*b*sinh(x) + 2*a)/b) + 3
*sqrt(b*sinh(x) + a)*B*b)/b^2
```

Sympy [F]

$$\int \frac{A+B\sinh(x)}{\sqrt{a+b\sinh(x)}}\,dx = \int \frac{A+B\sinh{(x)}}{\sqrt{a+b\sinh{(x)}}}\,dx$$

input
```
integrate((A+B*sinh(x))/(a+b*sinh(x))**(1/2),x)
```

output
```
Integral((A + B*sinh(x))/sqrt(a + b*sinh(x)), x)
```

Maxima [F]

$$\int \frac{A+B\sinh(x)}{\sqrt{a+b\sinh(x)}}\,dx=\int \frac{B\sinh\left(x\right)+A}{\sqrt{b\sinh\left(x\right)+a}}\,dx$$

input
```
integrate((A+B*sinh(x))/(a+b*sinh(x))^(1/2),x, algorithm="maxima")
```

output
```
integrate((B*sinh(x) + A)/sqrt(b*sinh(x) + a), x)
```

Giac [F]

$$\int \frac{A+B\sinh(x)}{\sqrt{a+b\sinh(x)}}\,dx=\int \frac{B\sinh\left(x\right)+A}{\sqrt{b\sinh\left(x\right)+a}}\,dx$$

input
```
integrate((A+B*sinh(x))/(a+b*sinh(x))^(1/2),x, algorithm="giac")
```

output
```
integrate((B*sinh(x) + A)/sqrt(b*sinh(x) + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{A+B\sinh(x)}{\sqrt{a+b\sinh(x)}}\,dx=\int \frac{A+B\sinh(x)}{\sqrt{a+b\sinh\left(x\right)}}\,dx$$

input
```
int((A + B*sinh(x))/(a + b*sinh(x))^(1/2),x)
```

output
```
int((A + B*sinh(x))/(a + b*sinh(x))^(1/2), x)
```

Reduce [F]

$$\int \frac{A + B\sinh(x)}{\sqrt{a + b\sinh(x)}}\,dx = \int \sqrt{\sinh\left(x\right)b + a}dx$$

input
```
int((A+B*sinh(x))/(a+b*sinh(x))^(1/2),x)
```

output
```
int(sqrt(sinh(x)*b + a),x)
```

3.138 $\int \frac{A+B\sinh(x)}{(a+b\sinh(x))^{3/2}}\,dx$

Optimal result

Integrand size = 17, antiderivative size = 176

$$\int \frac{A+B\sinh(x)}{(a+b\sinh(x))^{3/2}}\,dx = -\frac{2(Ab-aB)\cosh(x)}{(a^2+b^2)\sqrt{a+b\sinh(x)}} + \frac{2i(Ab-aB)E\left(\frac{\pi}{4}-\frac{ix}{2}\middle|\frac{2b}{ia+b}\right)\sqrt{a+b\sinh(x)}}{b(a^2+b^2)\sqrt{\frac{a+b\sinh(x)}{a-ib}}} + \frac{2iB\,\text{EllipticF}\left(\frac{\pi}{4}-\frac{ix}{2},\frac{2b}{ia+b}\right)\sqrt{\frac{a+b\sinh(x)}{a-ib}}}{b\sqrt{a+b\sinh(x)}}$$

output
```
-2*(A*b-B*a)*cosh(x)/(a^2+b^2)/(a+b*sinh(x))^(1/2)+2*I*(A*b-B*a)*EllipticE
(cos(1/4*Pi+1/2*I*x),2^(1/2)*(b/(I*a+b))^(1/2))*(a+b*sinh(x))^(1/2)/b/(a^2
+b^2)/((a+b*sinh(x))/(a-I*b))^(1/2)-2*I*B*InverseJacobiAM(-1/4*Pi+1/2*I*x,
2^(1/2)*(b/(I*a+b))^(1/2))*((a+b*sinh(x))/(a-I*b))^(1/2)/b/(a+b*sinh(x))^(
1/2)
```

Mathematica [A] (verified)

Time = 0.49 (sec) , antiderivative size = 159, normalized size of antiderivative = 0.90

$$\int \frac{A+B\sinh(x)}{(a+b\sinh(x))^{3/2}}\,dx = \frac{2b(-Ab+aB)\cosh(x)+\frac{2i(Ab-aB)E\left(\frac{1}{4}(\pi-2ix)|-\frac{2ib}{a-ib}\right)(a+b\sinh(x))}{\sqrt{\frac{a+b\sinh(x)}{a-ib}}}+2i(a^2+b^2)\,B\,\mathrm{El}}{b\,(a^2+b^2)\,\sqrt{a+b\sinh(x)}}$$

input
```
Integrate[(A + B*Sinh[x])/(a + b*Sinh[x])^(3/2),x]
```

output
```
(2*b*(-(A*b) + a*B)*Cosh[x] + ((2*I)*(A*b - a*B)*EllipticE[(Pi - (2*I)*x)/
4, ((-2*I)*b)/(a - I*b)]*(a + b*Sinh[x]))/Sqrt[(a + b*Sinh[x])/(a - I*b)]
+ (2*I)*(a^2 + b^2)*B*EllipticF[(Pi - (2*I)*x)/4, ((-2*I)*b)/(a - I*b)]*Sq
rt[(a + b*Sinh[x])/(a - I*b)])/(b*(a^2 + b^2)*Sqrt[a + b*Sinh[x]])
```

Rubi [A] (verified)

Time = 0.84 (sec) , antiderivative size = 185, normalized size of antiderivative = 1.05, number of steps used = 12, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.706$, Rules used = {3042, 3233, 27, 3042, 3231, 3042, 3134, 3042, 3132, 3142, 3042, 3140}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+B\sinh(x)}{(a+b\sinh(x))^{3/2}}\,dx$$

$$\Big\downarrow \ 3042$$

$$\int \frac{A-iB\sin(ix)}{(a-ib\sin(ix))^{3/2}}dx$$

$$\Big\downarrow \ 3233$$

$$-\frac{2\int -\frac{aA+bB+(Ab-aB)\sinh(x)}{2\sqrt{a+b\sinh(x)}}dx}{a^2+b^2}-\frac{2\cosh(x)(Ab-aB)}{(a^2+b^2)\sqrt{a+b\sinh(x)}}$$

$$\Big\downarrow \ 27$$

$$\frac{\int \frac{aA+bB+(Ab-aB)\sinh(x)}{\sqrt{a+b\sinh(x)}}dx}{a^2+b^2}-\frac{2\cosh(x)(Ab-aB)}{(a^2+b^2)\sqrt{a+b\sinh(x)}}$$

$\Big\downarrow$ 3042

$$-\frac{2\cosh(x)(Ab-aB)}{(a^2+b^2)\sqrt{a+b\sinh(x)}}+\frac{\int \frac{aA+bB-i(Ab-aB)\sin(ix)}{\sqrt{a-ib\sin(ix)}}dx}{a^2+b^2}$$

$\Big\downarrow$ 3231

$$\frac{\frac{B(a^2+b^2)\int \frac{1}{\sqrt{a+b\sinh(x)}}dx}{b}+\frac{(Ab-aB)\int \sqrt{a+b\sinh(x)}dx}{b}}{a^2+b^2}-\frac{2\cosh(x)(Ab-aB)}{(a^2+b^2)\sqrt{a+b\sinh(x)}}$$

$\Big\downarrow$ 3042

$$-\frac{2\cosh(x)(Ab-aB)}{(a^2+b^2)\sqrt{a+b\sinh(x)}}+\frac{\frac{B(a^2+b^2)\int \frac{1}{\sqrt{a-ib\sin(ix)}}dx}{b}+\frac{(Ab-aB)\int \sqrt{a-ib\sin(ix)}dx}{b}}{a^2+b^2}$$

$\Big\downarrow$ 3134

$$-\frac{2\cosh(x)(Ab-aB)}{(a^2+b^2)\sqrt{a+b\sinh(x)}}+\frac{\frac{B(a^2+b^2)\int \frac{1}{\sqrt{a-ib\sin(ix)}}dx}{b}+\frac{(Ab-aB)\sqrt{a+b\sinh(x)}\int \sqrt{\frac{a}{a-ib}+\frac{b\sinh(x)}{a-ib}}dx}{b\sqrt{\frac{a+b\sinh(x)}{a-ib}}}}{a^2+b^2}$$

$\Big\downarrow$ 3042

$$-\frac{2\cosh(x)(Ab-aB)}{(a^2+b^2)\sqrt{a+b\sinh(x)}}+\frac{\frac{B(a^2+b^2)\int \frac{1}{\sqrt{a-ib\sin(ix)}}dx}{b}+\frac{(Ab-aB)\sqrt{a+b\sinh(x)}\int \sqrt{\frac{a}{a-ib}-\frac{ib\sin(ix)}{a-ib}}dx}{b\sqrt{\frac{a+b\sinh(x)}{a-ib}}}}{a^2+b^2}$$

$\Big\downarrow$ 3132

$$-\frac{2\cosh(x)(Ab-aB)}{(a^2+b^2)\sqrt{a+b\sinh(x)}}+\frac{\frac{B(a^2+b^2)\int \frac{1}{\sqrt{a-ib\sin(ix)}}dx}{b}+\frac{2i(Ab-aB)\sqrt{a+b\sinh(x)}E\left(\frac{\pi}{4}-\frac{ix}{2}|\frac{2b}{ia+b}\right)}{b\sqrt{\frac{a+b\sinh(x)}{a-ib}}}}{a^2+b^2}$$

$\Big\downarrow$ 3142

$$-\frac{2\cosh(x)(Ab-aB)}{(a^2+b^2)\sqrt{a+b\sinh(x)}}+$$
$$\frac{\frac{B(a^2+b^2)\sqrt{\frac{a+b\sinh(x)}{a-ib}}\int \frac{1}{\sqrt{\frac{a}{a-ib}+\frac{b\sinh(x)}{a-ib}}}dx}{b\sqrt{a+b\sinh(x)}}+\frac{2i(Ab-aB)\sqrt{a+b\sinh(x)}E\left(\frac{\pi}{4}-\frac{ix}{2}|\frac{2b}{ia+b}\right)}{b\sqrt{\frac{a+b\sinh(x)}{a-ib}}}}{a^2+b^2}$$

$\Big\downarrow$ 3042

$$-\frac{2\cosh(x)(Ab-aB)}{(a^2+b^2)\sqrt{a+b\sinh(x)}}+\frac{\frac{B(a^2+b^2)\sqrt{\frac{a+b\sinh(x)}{a-ib}}\int\frac{1}{\sqrt{\frac{a}{a-ib}-\frac{ib\sin(ix)}{a-ib}}}dx}{b\sqrt{a+b\sinh(x)}}+\frac{2i(Ab-aB)\sqrt{a+b\sinh(x)}E\left(\frac{\pi}{4}-\frac{ix}{2}|\frac{2b}{ia+b}\right)}{b\sqrt{\frac{a+b\sinh(x)}{a-ib}}}}{a^2+b^2}$$

$$\Big\downarrow \text{3140}$$

$$-\frac{2\cosh(x)(Ab-aB)}{(a^2+b^2)\sqrt{a+b\sinh(x)}}+\frac{\frac{2iB(a^2+b^2)\sqrt{\frac{a+b\sinh(x)}{a-ib}}\text{EllipticF}\left(\frac{\pi}{4}-\frac{ix}{2},\frac{2b}{ia+b}\right)}{b\sqrt{a+b\sinh(x)}}+\frac{2i(Ab-aB)\sqrt{a+b\sinh(x)}E\left(\frac{\pi}{4}-\frac{ix}{2}|\frac{2b}{ia+b}\right)}{b\sqrt{\frac{a+b\sinh(x)}{a-ib}}}}{a^2+b^2}$$

input
```
Int[(A + B*Sinh[x])/(a + b*Sinh[x])^(3/2),x]
```

output
```
(-2*(A*b - a*B)*Cosh[x])/((a^2 + b^2)*Sqrt[a + b*Sinh[x]]) + (((2*I)*(A*b
- a*B)*EllipticE[Pi/4 - (I/2)*x, (2*b)/(I*a + b)]*Sqrt[a + b*Sinh[x]])/(b*
Sqrt[(a + b*Sinh[x])/(a - I*b)]) + ((2*I)*(a^2 + b^2)*B*EllipticF[Pi/4 - (
I/2)*x, (2*b)/(I*a + b)]*Sqrt[(a + b*Sinh[x])/(a - I*b)])/(b*Sqrt[a + b*Si
nh[x]]))/(a^2 + b^2)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3132
```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[2*(Sqrt[a
 + b]/d)*EllipticE[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

rule 3134
```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)]   Int[Sqrt[a/(a + b) + (
b/(a + b))*Sin[c + d*x]], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2
, 0] &&  !GtQ[a + b, 0]
```

rule 3140
```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/(d*S
qrt[a + b]))*EllipticF[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

rule 3142
```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]]   Int[1/Sqrt[a/(a + b)
 + (b/(a + b))*Sin[c + d*x]], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
 b^2, 0] &&  !GtQ[a + b, 0]
```

rule 3231
```
Int[((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])/Sqrt[(a_) + (b_.)*sin[(e_.) + (
f_.)*(x_)]], x_Symbol] :> Simp[(b*c - a*d)/b   Int[1/Sqrt[a + b*Sin[e + f*x
]], x], x] + Simp[d/b   Int[Sqrt[a + b*Sin[e + f*x]], x], x] /; FreeQ[{a, b
, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
```

rule 3233
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Simp[(-(b*c - a*d))*Cos[e + f*x]*((a + b*Sin[e +
 f*x])^(m + 1)/(f*(m + 1)*(a^2 - b^2))), x] + Simp[1/((m + 1)*(a^2 - b^2))
  Int[(a + b*Sin[e + f*x])^(m + 1)*Simp[(a*c - b*d)*(m + 1) - (b*c - a*d)*(
m + 2)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c
- a*d, 0] && NeQ[a^2 - b^2, 0] && LtQ[m, -1] && IntegerQ[2*m]
```

Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 516 vs. $2(161) = 322$.

Time = 1.29 (sec) , antiderivative size = 517, normalized size of antiderivative = 2.94

method	result
default	$\frac{\sqrt{\cosh(x)^2(a+b\sinh(x))}\left(\frac{2B\left(\frac{a}{b}-i\right)\sqrt{\frac{-a-b\sinh(x)}{ib-a}}\sqrt{\frac{(i-\sinh(x))b}{ib+a}}\sqrt{\frac{b(i+\sinh(x))}{ib-a}}\,\text{EllipticF}\left(\sqrt{\frac{-a-b\sinh(x)}{ib-a}},\sqrt{\frac{-ib+a}{ib+a}}\right)}{b\sqrt{\cosh(x)^2(a+b\sinh(x))}}+\frac{(Ab-aB)\left(-\cdots\right)}{\cdots}\right)}{\cdots}$
parts	$\frac{2A\left(\sqrt{-\frac{a+b\sinh(x)}{ib-a}}\sqrt{\frac{(i-\sinh(x))b}{ib+a}}\sqrt{\frac{b(i+\sinh(x))}{ib-a}}\,\text{EllipticF}\left(\sqrt{-\frac{a+b\sinh(x)}{ib-a}},\sqrt{-\frac{ib-a}{ib+a}}\right)a^2+\sqrt{-\frac{a+b\sinh(x)}{ib-a}}\sqrt{\frac{(i-\sinh(x))b}{ib+a}}\sqrt{\frac{b(i+\cdots}{ib\cdots}}\cdots\right)}{\cdots}$

input
```
int((A+B*sinh(x))/(a+b*sinh(x))^(3/2),x,method=_RETURNVERBOSE)
```

output
```
(cosh(x)^2*(a+b*sinh(x)))^(1/2)*(2*B/b*(1/b*a-I)*((-a-b*sinh(x))/(I*b-a))^
(1/2)*((I-sinh(x))*b/(I*b+a))^(1/2)*(b*(I+sinh(x))/(I*b-a))^(1/2)/(cosh(x)
^2*(a+b*sinh(x)))^(1/2)*EllipticF(((-a-b*sinh(x))/(I*b-a))^(1/2),((a-I*b)/
(I*b+a))^(1/2))+(A*b-B*a)/b*(-2*b*cosh(x)^2/(a^2+b^2)/(cosh(x)^2*(a+b*sinh
(x)))^(1/2)+2*a/(a^2+b^2)*(1/b*a-I)*((-a-b*sinh(x))/(I*b-a))^(1/2)*((I-sin
h(x))*b/(I*b+a))^(1/2)*(b*(I+sinh(x))/(I*b-a))^(1/2)/(cosh(x)^2*(a+b*sinh(
x)))^(1/2)*EllipticF(((-a-b*sinh(x))/(I*b-a))^(1/2),((a-I*b)/(I*b+a))^(1/2
))+2*b/(a^2+b^2)*(1/b*a-I)*((-a-b*sinh(x))/(I*b-a))^(1/2)*((I-sinh(x))*b/(
I*b+a))^(1/2)*(b*(I+sinh(x))/(I*b-a))^(1/2)/(cosh(x)^2*(a+b*sinh(x)))^(1/2
)*((-1/b*a-I)*EllipticE(((-a-b*sinh(x))/(I*b-a))^(1/2),((a-I*b)/(I*b+a))^(
1/2))+I*EllipticF(((-a-b*sinh(x))/(I*b-a))^(1/2),((a-I*b)/(I*b+a))^(1/2)))
))/cosh(x)/(a+b*sinh(x))^(1/2)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 597 vs. 2(154) = 308.

Time = 0.11 (sec) , antiderivative size = 597, normalized size of antiderivative = 3.39

$$\int \frac{A+B\sinh(x)}{(a+b\sinh(x))^{3/2}}\,dx = \text{Too large to display}$$

input
```
integrate((A+B*sinh(x))/(a+b*sinh(x))^(3/2),x, algorithm="fricas")
```

output
```
4/3*(sqrt(1/2)*(2*B*a^2*b + A*a*b^2 + 3*B*b^3 - (2*B*a^2*b + A*a*b^2 + 3*B
*b^3)*cosh(x)^2 - (2*B*a^2*b + A*a*b^2 + 3*B*b^3)*sinh(x)^2 - 2*(2*B*a^3 +
 A*a^2*b + 3*B*a*b^2)*cosh(x) - 2*(2*B*a^3 + A*a^2*b + 3*B*a*b^2 + (2*B*a^
2*b + A*a*b^2 + 3*B*b^3)*cosh(x))*sinh(x))*sqrt(b)*weierstrassPInverse(4/3
*(4*a^2 + 3*b^2)/b^2, -8/27*(8*a^3 + 9*a*b^2)/b^3, 1/3*(3*b*cosh(x) + 3*b*
sinh(x) + 2*a)/b) + 3*sqrt(1/2)*(B*a*b^2 - A*b^3 - (B*a*b^2 - A*b^3)*cosh(
x)^2 - (B*a*b^2 - A*b^3)*sinh(x)^2 - 2*(B*a^2*b - A*a*b^2)*cosh(x) - 2*(B*
a^2*b - A*a*b^2 + (B*a*b^2 - A*b^3)*cosh(x))*sinh(x))*sqrt(b)*weierstrassZ
eta(4/3*(4*a^2 + 3*b^2)/b^2, -8/27*(8*a^3 + 9*a*b^2)/b^3, weierstrassPInve
rse(4/3*(4*a^2 + 3*b^2)/b^2, -8/27*(8*a^3 + 9*a*b^2)/b^3, 1/3*(3*b*cosh(x)
 + 3*b*sinh(x) + 2*a)/b)) - 3*((B*a*b^2 - A*b^3)*cosh(x)^2 + (B*a*b^2 - A*
b^3)*sinh(x)^2 + (B*a^2*b - A*a*b^2)*cosh(x) + (B*a^2*b - A*a*b^2 + 2*(B*a
*b^2 - A*b^3)*cosh(x))*sinh(x))*sqrt(b*sinh(x) + a))/(a^2*b^3 + b^5 - (a^2
*b^3 + b^5)*cosh(x)^2 - (a^2*b^3 + b^5)*sinh(x)^2 - 2*(a^3*b^2 + a*b^4)*co
sh(x) - 2*(a^3*b^2 + a*b^4 + (a^2*b^3 + b^5)*cosh(x))*sinh(x))
```

Sympy [F(-1)]

Timed out.

$$\int \frac{A + B\sinh(x)}{(a + b\sinh(x))^{3/2}}\, dx = \text{Timed out}$$

input
```
integrate((A+B*sinh(x))/(a+b*sinh(x))**(3/2),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{A + B\sinh(x)}{(a + b\sinh(x))^{3/2}}\, dx = \int \frac{B\sinh(x) + A}{(b\sinh(x) + a)^{\frac{3}{2}}}\, dx$$

input
```
integrate((A+B*sinh(x))/(a+b*sinh(x))^(3/2),x, algorithm="maxima")
```

output
```
integrate((B*sinh(x) + A)/(b*sinh(x) + a)^(3/2), x)
```

Giac [F]

$$\int \frac{A + B\sinh(x)}{(a + b\sinh(x))^{3/2}}\, dx = \int \frac{B\sinh(x) + A}{(b\sinh(x) + a)^{\frac{3}{2}}}\, dx$$

input
```
integrate((A+B*sinh(x))/(a+b*sinh(x))^(3/2),x, algorithm="giac")
```

output
```
integrate((B*sinh(x) + A)/(b*sinh(x) + a)^(3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{A + B\sinh(x)}{(a + b\sinh(x))^{3/2}}\, dx = \int \frac{A + B\sinh(x)}{(a + b\sinh(x))^{3/2}}\, dx$$

input
```
int((A + B*sinh(x))/(a + b*sinh(x))^(3/2),x)
```

output
```
int((A + B*sinh(x))/(a + b*sinh(x))^(3/2), x)
```

Reduce [F]

$$\int \frac{A + B\sinh(x)}{(a + b\sinh(x))^{3/2}}\, dx = \int \frac{\sqrt{\sinh(x)b + a}}{\sinh(x)b + a} dx$$

input
```
int((A+B*sinh(x))/(a+b*sinh(x))^(3/2),x)
```

output
```
int(sqrt(sinh(x)*b + a)/(sinh(x)*b + a),x)
```

3.139 $\int \frac{A+B\sinh(x)}{(a+b\sinh(x))^{5/2}}\,dx$

Optimal result

Integrand size = 17, antiderivative size = 251

$$\int \frac{A+B\sinh(x)}{(a+b\sinh(x))^{5/2}}\,dx = -\frac{2(Ab-aB)\cosh(x)}{3(a^2+b^2)(a+b\sinh(x))^{3/2}} - \frac{2(4aAb-a^2B+3b^2B)\cosh(x)}{3(a^2+b^2)^2\sqrt{a+b\sinh(x)}} + \frac{2i(4aAb-a^2B+3b^2B)E\left(\frac{\pi}{4}-\frac{ix}{2}\middle|\frac{2b}{ia+b}\right)\sqrt{a+b\sinh(x)}}{3b(a^2+b^2)^2\sqrt{\frac{a+b\sinh(x)}{a-ib}}} - \frac{2i(Ab-aB)\,\text{EllipticF}\left(\frac{\pi}{4}-\frac{ix}{2},\frac{2b}{ia+b}\right)\sqrt{\frac{a+b\sinh(x)}{a-ib}}}{3b(a^2+b^2)\sqrt{a+b\sinh(x)}}$$

output
```
-2/3*(A*b-B*a)*cosh(x)/(a^2+b^2)/(a+b*sinh(x))^(3/2)-2/3*(4*A*a*b-B*a^2+3*
B*b^2)*cosh(x)/(a^2+b^2)^2/(a+b*sinh(x))^(1/2)+2/3*I*(4*A*a*b-B*a^2+3*B*b^
2)*EllipticE(cos(1/4*Pi+1/2*I*x),2^(1/2)*(b/(I*a+b))^(1/2))*(a+b*sinh(x))^
(1/2)/b/(a^2+b^2)^2/((a+b*sinh(x))/(a-I*b))^(1/2)+2/3*I*(A*b-B*a)*InverseJ
acobiAM(-1/4*Pi+1/2*I*x,2^(1/2)*(b/(I*a+b))^(1/2))*((a+b*sinh(x))/(a-I*b))
^(1/2)/b/(a^2+b^2)/(a+b*sinh(x))^(1/2)
```

Mathematica [A] (verified)

Time = 0.60 (sec) , antiderivative size = 236, normalized size of antiderivative = 0.94

$$\int \frac{A+B\sinh(x)}{(a+b\sinh(x))^{5/2}}\,dx = \frac{2i\left(\left(b(3a^2A-Ab^2+4abB)\operatorname{EllipticF}\left(\frac{1}{4}(\pi-2ix),-\frac{2ib}{a-ib}\right)+(4aAb-a^2B+3b^2\right.\right.}{}$$

input
```
Integrate[(A + B*Sinh[x])/(a + b*Sinh[x])^(5/2),x]
```

output
```
(((2*I)/3)*((b*(3*a^2*A - A*b^2 + 4*a*b*B)*EllipticF[(Pi - (2*I)*x)/4, ((-
2*I)*b)/(a - I*b)] + (4*a*A*b - a^2*B + 3*b^2*B)*((a - I*b)*EllipticE[(Pi
- (2*I)*x)/4, ((-2*I)*b)/(a - I*b)] - a*EllipticF[(Pi - (2*I)*x)/4, ((-2*I
)*b)/(a - I*b)]))*(a + b*Sinh[x])*Sqrt[(a + b*Sinh[x])/(a - I*b)] + I*b*Co
sh[x]*(-((a^2 + b^2)*(-(A*b) + a*B)) - (-4*a*A*b + a^2*B - 3*b^2*B)*(a + b
*Sinh[x]))))/(b*(a^2 + b^2)^2*(a + b*Sinh[x])^(3/2))
```

Rubi [A] (verified)

Time = 1.38 (sec) , antiderivative size = 259, normalized size of antiderivative = 1.03, number of steps used = 15, number of rules used = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.882$, Rules used = {3042, 3233, 27, 3042, 3233, 27, 3042, 3231, 3042, 3134, 3042, 3132, 3142, 3042, 3140}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+B\sinh(x)}{(a+b\sinh(x))^{5/2}}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{A-iB\sin(ix)}{(a-ib\sin(ix))^{5/2}}dx$$

$$\Big\downarrow \text{3233}$$

$$-\frac{2\int -\frac{3(aA+bB)-(Ab-aB)\sinh(x)}{2(a+b\sinh(x))^{3/2}}dx}{3\left(a^2+b^2\right)}-\frac{2\cosh(x)(Ab-aB)}{3\left(a^2+b^2\right)(a+b\sinh(x))^{3/2}}$$

$$\Big\downarrow \; 27$$

$$\frac{\int \frac{3(aA+bB)-(Ab-aB)\sinh(x)}{(a+b\sinh(x))^{3/2}}dx}{3(a^2+b^2)}-\frac{2\cosh(x)(Ab-aB)}{3(a^2+b^2)(a+b\sinh(x))^{3/2}}$$

$$\Big\downarrow \; 3042$$

$$-\frac{2\cosh(x)(Ab-aB)}{3(a^2+b^2)(a+b\sinh(x))^{3/2}}+\frac{\int \frac{3(aA+bB)+i(Ab-aB)\sin(ix)}{(a-ib\sin(ix))^{3/2}}dx}{3(a^2+b^2)}$$

$$\Big\downarrow \; 3233$$

$$\frac{-\frac{2\int -\frac{3Aa^2+4bBa-Ab^2+\left(-Ba^2+4Aba+3b^2B\right)\sinh(x)}{2\sqrt{a+b\sinh(x)}}dx}{a^2+b^2}-\frac{2\cosh(x)\left(a^2(-B)+4aAb+3b^2B\right)}{(a^2+b^2)\sqrt{a+b\sinh(x)}}}{3(a^2+b^2)}-\frac{2\cosh(x)(Ab-aB)}{3(a^2+b^2)(a+b\sinh(x))^{3/2}}$$

$$\Big\downarrow \; 27$$

$$\frac{\frac{\int \frac{3Aa^2+4bBa-Ab^2+\left(-Ba^2+4Aba+3b^2B\right)\sinh(x)}{\sqrt{a+b\sinh(x)}}dx}{a^2+b^2}-\frac{2\cosh(x)\left(a^2(-B)+4aAb+3b^2B\right)}{(a^2+b^2)\sqrt{a+b\sinh(x)}}}{3(a^2+b^2)}-\frac{2\cosh(x)(Ab-aB)}{3(a^2+b^2)(a+b\sinh(x))^{3/2}}$$

$$\Big\downarrow \; 3042$$

$$-\frac{2\cosh(x)(Ab-aB)}{3(a^2+b^2)(a+b\sinh(x))^{3/2}}+\frac{-\frac{2\cosh(x)\left(a^2(-B)+4aAb+3b^2B\right)}{(a^2+b^2)\sqrt{a+b\sinh(x)}}+\frac{\int \frac{3Aa^2+4bBa-Ab^2-i\left(-Ba^2+4Aba+3b^2B\right)\sin(ix)}{\sqrt{a-ib\sin(ix)}}dx}{a^2+b^2}}{3(a^2+b^2)}$$

$$\Big\downarrow \; 3231$$

$$\frac{\frac{\frac{\left(a^2(-B)+4aAb+3b^2B\right)\int \sqrt{a+b\sinh(x)}dx}{b}-\frac{\left(a^2+b^2\right)(Ab-aB)\int \frac{1}{\sqrt{a+b\sinh(x)}}dx}{b}}{a^2+b^2}-\frac{2\cosh(x)\left(a^2(-B)+4aAb+3b^2B\right)}{(a^2+b^2)\sqrt{a+b\sinh(x)}}}{3(a^2+b^2)}-\frac{2\cosh(x)(Ab-aB)}{3(a^2+b^2)(a+b\sinh(x))^{3/2}}$$

$$\Big\downarrow \; 3042$$

$$-\frac{2\cosh(x)(Ab-aB)}{3\left(a^2+b^2\right)(a+b\sinh(x))^{3/2}}+\frac{-\frac{2\cosh(x)\left(a^2(-B)+4aAb+3b^2B\right)}{\left(a^2+b^2\right)\sqrt{a+b\sinh(x)}}+\frac{\frac{\left(a^2(-B)+4aAb+3b^2B\right)\int\sqrt{a-ib\sin(ix)}dx}{b}-\frac{\left(a^2+b^2\right)(Ab-aB)\int\frac{1}{\sqrt{a-ib\sin(ix)}}dx}{b}}{a^2+b^2}}{3\left(a^2+b^2\right)}$$

$\downarrow$ 3134

$$-\frac{2\cosh(x)(Ab-aB)}{3\left(a^2+b^2\right)(a+b\sinh(x))^{3/2}}+\frac{-\frac{2\cosh(x)\left(a^2(-B)+4aAb+3b^2B\right)}{\left(a^2+b^2\right)\sqrt{a+b\sinh(x)}}+\frac{\frac{\left(a^2(-B)+4aAb+3b^2B\right)\sqrt{a+b\sinh(x)}\int\sqrt{\frac{a}{a-ib}+\frac{b\sinh(x)}{a-ib}}dx}{b\sqrt{\frac{a+b\sinh(x)}{a-ib}}}-\frac{\left(a^2+b^2\right)(Ab-aB)\int\frac{1}{\sqrt{a-ib\sin(ix)}}dx}{b}}{a^2+b^2}}{3\left(a^2+b^2\right)}$$

$\downarrow$ 3042

$$-\frac{2\cosh(x)(Ab-aB)}{3\left(a^2+b^2\right)(a+b\sinh(x))^{3/2}}+\frac{-\frac{2\cosh(x)\left(a^2(-B)+4aAb+3b^2B\right)}{\left(a^2+b^2\right)\sqrt{a+b\sinh(x)}}+\frac{\frac{\left(a^2(-B)+4aAb+3b^2B\right)\sqrt{a+b\sinh(x)}\int\sqrt{\frac{a}{a-ib}-\frac{ib\sin(ix)}{a-ib}}dx}{b\sqrt{\frac{a+b\sinh(x)}{a-ib}}}-\frac{\left(a^2+b^2\right)(Ab-aB)\int\frac{1}{\sqrt{a-ib\sin(ix)}}dx}{b}}{a^2+b^2}}{3\left(a^2+b^2\right)}$$

$\downarrow$ 3132

$$-\frac{2\cosh(x)(Ab-aB)}{3\left(a^2+b^2\right)(a+b\sinh(x))^{3/2}}+\frac{-\frac{2\cosh(x)\left(a^2(-B)+4aAb+3b^2B\right)}{\left(a^2+b^2\right)\sqrt{a+b\sinh(x)}}+\frac{\frac{2i\left(a^2(-B)+4aAb+3b^2B\right)\sqrt{a+b\sinh(x)}E\left(\frac{\pi}{4}-\frac{ix}{2}|\frac{2b}{ia+b}\right)}{b\sqrt{\frac{a+b\sinh(x)}{a-ib}}}-\frac{\left(a^2+b^2\right)(Ab-aB)\int\frac{1}{\sqrt{a-ib\sin(ix)}}dx}{b}}{a^2+b^2}}{3\left(a^2+b^2\right)}$$

$\downarrow$ 3142

$$-\frac{2\cosh(x)(Ab-aB)}{3\left(a^2+b^2\right)(a+b\sinh(x))^{3/2}}+\frac{-\frac{2\cosh(x)\left(a^2(-B)+4aAb+3b^2B\right)}{\left(a^2+b^2\right)\sqrt{a+b\sinh(x)}}+\frac{\frac{2i\left(a^2(-B)+4aAb+3b^2B\right)\sqrt{a+b\sinh(x)}E\left(\frac{\pi}{4}-\frac{ix}{2}|\frac{2b}{ia+b}\right)}{b\sqrt{\frac{a+b\sinh(x)}{a-ib}}}-\frac{\left(a^2+b^2\right)(Ab-aB)\sqrt{\frac{a+b\sinh(x)}{a-ib}}\int\frac{1}{\sqrt{\frac{a}{a-ib}+\frac{b\sinh(x)}{a-ib}}}dx}{b\sqrt{a+b\sinh(x)}}}{a^2+b^2}}{3\left(a^2+b^2\right)}$$

$\downarrow$ 3042

$$-\frac{2\cosh(x)(Ab-aB)}{3(a^2+b^2)(a+b\sinh(x))^{3/2}}+\frac{-\frac{2\cosh(x)(a^2(-B)+4aAb+3b^2B)}{(a^2+b^2)\sqrt{a+b\sinh(x)}}+\frac{\frac{2i(a^2(-B)+4aAb+3b^2B)\sqrt{a+b\sinh(x)}E\left(\frac{\pi}{4}-\frac{ix}{2}|\frac{2b}{ia+b}\right)}{b\sqrt{\frac{a+b\sinh(x)}{a-ib}}}-\frac{(a^2+b^2)(Ab-aB)\sqrt{\frac{a+b\sinh(x)}{a-ib}}\int\frac{1}{\sqrt{\frac{a}{a-ib}-\frac{ib\sin(ix)}{a-ib}}}}{b\sqrt{a+b\sinh(x)}}}{a^2+b^2}}{3(a^2+b^2)}$$

$\Big\downarrow$ 3140

$$-\frac{2\cosh(x)(Ab-aB)}{3(a^2+b^2)(a+b\sinh(x))^{3/2}}+\frac{-\frac{2\cosh(x)(a^2(-B)+4aAb+3b^2B)}{(a^2+b^2)\sqrt{a+b\sinh(x)}}+\frac{\frac{2i(a^2(-B)+4aAb+3b^2B)\sqrt{a+b\sinh(x)}E\left(\frac{\pi}{4}-\frac{ix}{2}|\frac{2b}{ia+b}\right)}{b\sqrt{\frac{a+b\sinh(x)}{a-ib}}}-\frac{2i(a^2+b^2)(Ab-aB)\sqrt{\frac{a+b\sinh(x)}{a-ib}}\text{EllipticF}\left(\frac{\pi}{4}-\frac{ix}{2},\frac{2}{ia}\right.}{b\sqrt{a+b\sinh(x)}}}{a^2+b^2}}{3(a^2+b^2)}$$

input
```
Int[(A + B*Sinh[x])/(a + b*Sinh[x])^(5/2),x]
```

output
```
(-2*(A*b - a*B)*Cosh[x])/(3*(a^2 + b^2)*(a + b*Sinh[x])^(3/2)) + ((-2*(4*a
*A*b - a^2*B + 3*b^2*B)*Cosh[x])/((a^2 + b^2)*Sqrt[a + b*Sinh[x]]) + (((2*
I)*(4*a*A*b - a^2*B + 3*b^2*B)*EllipticE[Pi/4 - (I/2)*x, (2*b)/(I*a + b)]*
Sqrt[a + b*Sinh[x]])/(b*Sqrt[(a + b*Sinh[x])/(a - I*b)]) - ((2*I)*(a^2 + b
^2)*(A*b - a*B)*EllipticF[Pi/4 - (I/2)*x, (2*b)/(I*a + b)]*Sqrt[(a + b*Sin
h[x])/(a - I*b)])/(b*Sqrt[a + b*Sinh[x]]))/(a^2 + b^2))/(3*(a^2 + b^2))
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3132
```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[2*(Sqrt[a
 + b]/d)*EllipticE[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

rule 3134
```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)]   Int[Sqrt[a/(a + b) + (
b/(a + b))*Sin[c + d*x]], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2
, 0] &&  !GtQ[a + b, 0]
```

rule 3140
```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/(d*S
qrt[a + b]))*EllipticF[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

rule 3142
```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]]   Int[1/Sqrt[a/(a + b)
 + (b/(a + b))*Sin[c + d*x]], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
 b^2, 0] &&  !GtQ[a + b, 0]
```

rule 3231
```
Int[((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])/Sqrt[(a_) + (b_.)*sin[(e_.) + (
f_.)*(x_)]], x_Symbol] :> Simp[(b*c - a*d)/b   Int[1/Sqrt[a + b*Sin[e + f*x
]], x], x] + Simp[d/b   Int[Sqrt[a + b*Sin[e + f*x]], x], x] /; FreeQ[{a, b
, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
```

rule 3233
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Simp[(-(b*c - a*d))*Cos[e + f*x]*((a + b*Sin[e +
 f*x])^(m + 1)/(f*(m + 1)*(a^2 - b^2))), x] + Simp[1/((m + 1)*(a^2 - b^2))
  Int[(a + b*Sin[e + f*x])^(m + 1)*Simp[(a*c - b*d)*(m + 1) - (b*c - a*d)*(
m + 2)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c
- a*d, 0] && NeQ[a^2 - b^2, 0] && LtQ[m, -1] && IntegerQ[2*m]
```

Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 805 vs. $2(226) = 452$.

Time = 1.94 (sec) , antiderivative size = 806, normalized size of antiderivative = 3.21

method	result
default	$\dfrac{\sqrt{\cosh(x)^2(a+b\sinh(x))}\left(\dfrac{B\left(-\frac{2b\cosh(x)^2}{(a^2+b^2)\sqrt{\cosh(x)^2(a+b\sinh(x))}}+\frac{2a\left(\frac{a}{b}-i\right)\sqrt{\frac{-a-b\sinh(x)}{ib-a}}\sqrt{\frac{(i-\sinh(x))b}{ib+a}}\sqrt{\frac{b(i+\sinh(x))}{ib-a}}\,\mathrm{EllipticF}\left(\sqrt{-\right.}{(a^2+b^2)\sqrt{\cosh(x)^2(a+b\sinh(x))}}\right.}{}\right.}{}$
parts	Expression too large to display

input
```
int((A+B*sinh(x))/(a+b*sinh(x))^(5/2),x,method=_RETURNVERBOSE)
```

output
```
(cosh(x)^2*(a+b*sinh(x)))^(1/2)*(B/b*(-2*b*cosh(x)^2/(a^2+b^2)/(cosh(x)^2*
(a+b*sinh(x)))^(1/2)+2*a/(a^2+b^2)*(1/b*a-I)*((-a-b*sinh(x))/(I*b-a))^(1/2
)*((I-sinh(x))*b/(I*b+a))^(1/2)*(b*(I+sinh(x))/(I*b-a))^(1/2)/(cosh(x)^2*(
a+b*sinh(x)))^(1/2)*EllipticF(((-a-b*sinh(x))/(I*b-a))^(1/2),((a-I*b)/(I*b
+a))^(1/2))+2*b/(a^2+b^2)*(1/b*a-I)*((-a-b*sinh(x))/(I*b-a))^(1/2)*((I-sin
h(x))*b/(I*b+a))^(1/2)*(b*(I+sinh(x))/(I*b-a))^(1/2)/(cosh(x)^2*(a+b*sinh(
x)))^(1/2)*((-1/b*a-I)*EllipticE(((-a-b*sinh(x))/(I*b-a))^(1/2),((a-I*b)/(
I*b+a))^(1/2))+I*EllipticF(((-a-b*sinh(x))/(I*b-a))^(1/2),((a-I*b)/(I*b+a)
)^(1/2))))+(A*b-B*a)/b*(-2/3/b/(a^2+b^2)*(cosh(x)^2*(a+b*sinh(x)))^(1/2)/(
sinh(x)+1/b*a)^2-8/3*b*cosh(x)^2/(a^2+b^2)^2*a/(cosh(x)^2*(a+b*sinh(x)))^(
1/2)+2*(3*a^2-b^2)/(3*a^4+6*a^2*b^2+3*b^4)*(1/b*a-I)*((-a-b*sinh(x))/(I*b-
a))^(1/2)*((I-sinh(x))*b/(I*b+a))^(1/2)*(b*(I+sinh(x))/(I*b-a))^(1/2)/(cos
h(x)^2*(a+b*sinh(x)))^(1/2)*EllipticF(((-a-b*sinh(x))/(I*b-a))^(1/2),((a-I
*b)/(I*b+a))^(1/2))+8/3*a*b/(a^2+b^2)^2*(1/b*a-I)*((-a-b*sinh(x))/(I*b-a))
^(1/2)*((I-sinh(x))*b/(I*b+a))^(1/2)*(b*(I+sinh(x))/(I*b-a))^(1/2)/(cosh(x
)^2*(a+b*sinh(x)))^(1/2)*((-1/b*a-I)*EllipticE(((-a-b*sinh(x))/(I*b-a))^(1
/2),((a-I*b)/(I*b+a))^(1/2))+I*EllipticF(((-a-b*sinh(x))/(I*b-a))^(1/2),((
a-I*b)/(I*b+a))^(1/2)))))/cosh(x)/(a+b*sinh(x))^(1/2)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2069 vs. $2(218) = 436$.

Time = 0.16 (sec) , antiderivative size = 2069, normalized size of antiderivative = 8.24

$$\int \frac{A + B\sinh(x)}{(a + b\sinh(x))^{5/2}}\,dx = \text{Too large to display}$$

input
```
integrate((A+B*sinh(x))/(a+b*sinh(x))^(5/2),x, algorithm="fricas")
```

output
```
4/9*(sqrt(1/2)*(2*B*a^3*b^2 + A*a^2*b^3 + 6*B*a*b^4 - 3*A*b^5 + (2*B*a^3*b
^2 + A*a^2*b^3 + 6*B*a*b^4 - 3*A*b^5)*cosh(x)^4 + (2*B*a^3*b^2 + A*a^2*b^3
 + 6*B*a*b^4 - 3*A*b^5)*sinh(x)^4 + 4*(2*B*a^4*b + A*a^3*b^2 + 6*B*a^2*b^3
 - 3*A*a*b^4)*cosh(x)^3 + 4*(2*B*a^4*b + A*a^3*b^2 + 6*B*a^2*b^3 - 3*A*a*b
^4 + (2*B*a^3*b^2 + A*a^2*b^3 + 6*B*a*b^4 - 3*A*b^5)*cosh(x))*sinh(x)^3 +
2*(4*B*a^5 + 2*A*a^4*b + 10*B*a^3*b^2 - 7*A*a^2*b^3 - 6*B*a*b^4 + 3*A*b^5)
*cosh(x)^2 + 2*(4*B*a^5 + 2*A*a^4*b + 10*B*a^3*b^2 - 7*A*a^2*b^3 - 6*B*a*b
^4 + 3*A*b^5 + 3*(2*B*a^3*b^2 + A*a^2*b^3 + 6*B*a*b^4 - 3*A*b^5)*cosh(x)^2
 + 6*(2*B*a^4*b + A*a^3*b^2 + 6*B*a^2*b^3 - 3*A*a*b^4)*cosh(x))*sinh(x)^2
- 4*(2*B*a^4*b + A*a^3*b^2 + 6*B*a^2*b^3 - 3*A*a*b^4)*cosh(x) - 4*(2*B*a^4
*b + A*a^3*b^2 + 6*B*a^2*b^3 - 3*A*a*b^4 - (2*B*a^3*b^2 + A*a^2*b^3 + 6*B*
a*b^4 - 3*A*b^5)*cosh(x)^3 - 3*(2*B*a^4*b + A*a^3*b^2 + 6*B*a^2*b^3 - 3*A*
a*b^4)*cosh(x)^2 - (4*B*a^5 + 2*A*a^4*b + 10*B*a^3*b^2 - 7*A*a^2*b^3 - 6*B
*a*b^4 + 3*A*b^5)*cosh(x))*sinh(x))*sqrt(b)*weierstrassPInverse(4/3*(4*a^2
 + 3*b^2)/b^2, -8/27*(8*a^3 + 9*a*b^2)/b^3, 1/3*(3*b*cosh(x) + 3*b*sinh(x)
 + 2*a)/b) + 3*sqrt(1/2)*(B*a^2*b^3 - 4*A*a*b^4 - 3*B*b^5 + (B*a^2*b^3 - 4
*A*a*b^4 - 3*B*b^5)*cosh(x)^4 + (B*a^2*b^3 - 4*A*a*b^4 - 3*B*b^5)*sinh(x)^
4 + 4*(B*a^3*b^2 - 4*A*a^2*b^3 - 3*B*a*b^4)*cosh(x)^3 + 4*(B*a^3*b^2 - 4*A
*a^2*b^3 - 3*B*a*b^4 + (B*a^2*b^3 - 4*A*a*b^4 - 3*B*b^5)*cosh(x))*sinh(x)^
3 + 2*(2*B*a^4*b - 8*A*a^3*b^2 - 7*B*a^2*b^3 + 4*A*a*b^4 + 3*B*b^5)*cos...
```

Sympy [F(-1)]

Timed out.

$$\int \frac{A + B\sinh(x)}{(a + b\sinh(x))^{5/2}}\, dx = \text{Timed out}$$

input
```
integrate((A+B*sinh(x))/(a+b*sinh(x))**(5/2),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{A+B\sinh(x)}{(a+b\sinh(x))^{5/2}}\,dx = \int \frac{B\sinh\left(x\right)+A}{\left(b\sinh\left(x\right)+a\right)^{\frac{5}{2}}}\,dx$$

input
```
integrate((A+B*sinh(x))/(a+b*sinh(x))^(5/2),x, algorithm="maxima")
```

output
```
integrate((B*sinh(x) + A)/(b*sinh(x) + a)^(5/2), x)
```

Giac [F]

$$\int \frac{A+B\sinh(x)}{(a+b\sinh(x))^{5/2}}\,dx = \int \frac{B\sinh\left(x\right)+A}{\left(b\sinh\left(x\right)+a\right)^{\frac{5}{2}}}\,dx$$

input
```
integrate((A+B*sinh(x))/(a+b*sinh(x))^(5/2),x, algorithm="giac")
```

output
```
integrate((B*sinh(x) + A)/(b*sinh(x) + a)^(5/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{A+B\sinh(x)}{(a+b\sinh(x))^{5/2}}\,dx = \int \frac{A+B\sinh(x)}{\left(a+b\sinh\left(x\right)\right)^{5/2}}\,dx$$

input
```
int((A + B*sinh(x))/(a + b*sinh(x))^(5/2),x)
```

output
```
int((A + B*sinh(x))/(a + b*sinh(x))^(5/2), x)
```

Reduce [F]

$$\int \frac{A + B\sinh(x)}{(a + b\sinh(x))^{5/2}}\,dx = \int \frac{\sqrt{\sinh(x)\,b + a}}{\sinh(x)^2\,b^2 + 2\sinh(x)\,ab + a^2}dx$$

input
```
int((A+B*sinh(x))/(a+b*sinh(x))^(5/2),x)
```

output
```
int(sqrt(sinh(x)*b + a)/(sinh(x)**2*b**2 + 2*sinh(x)*a*b + a**2),x)
```

3.140 $\int \left(a \sinh^2(x)\right)^{5/2} dx$

Optimal result

Integrand size = 10, antiderivative size = 53

$$\int \left(a \sinh^2(x)\right)^{5/2} dx = \frac{8}{15}a^2\coth(x)\sqrt{a\sinh^2(x)} - \frac{4}{15}a\coth(x)\left(a\sinh^2(x)\right)^{3/2} + \frac{1}{5}\coth(x)\left(a\sinh^2(x)\right)^{5/2}$$

output
```
8/15*a^2*coth(x)*(a*sinh(x)^2)^(1/2)-4/15*a*coth(x)*(a*sinh(x)^2)^(3/2)+1/
5*coth(x)*(a*sinh(x)^2)^(5/2)
```

Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.68

$$\int \left(a \sinh^2(x)\right)^{5/2} dx = \frac{1}{240}a^2(150\cosh(x)-25\cosh(3x)+3\cosh(5x))\text{csch}(x)\sqrt{a\sinh^2(x)}$$

input
```
Integrate[(a*Sinh[x]^2)^(5/2),x]
```

output
```
(a^2*(150*Cosh[x] - 25*Cosh[3*x] + 3*Cosh[5*x])*Csch[x]*Sqrt[a*Sinh[x]^2])
/240
```

Rubi [A] (verified)

Time = 0.42 (sec) , antiderivative size = 56, normalized size of antiderivative = 1.06, number of steps used = 9, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.900$, Rules used = {3042, 3682, 3042, 3682, 3042, 3686, 3042, 26, 3118}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(a \sinh^2(x)\right)^{5/2} dx$$

$$\Big\downarrow \text{3042}$$

$$\int \left(-a \sin(ix)^2\right)^{5/2} dx$$

$$\Big\downarrow \text{3682}$$

$$\frac{1}{5} \coth(x) \left(a \sinh^2(x)\right)^{5/2} - \frac{4}{5} a \int \left(a \sinh^2(x)\right)^{3/2} dx$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{5} \coth(x) \left(a \sinh^2(x)\right)^{5/2} - \frac{4}{5} a \int \left(-a \sin(ix)^2\right)^{3/2} dx$$

$$\Big\downarrow \text{3682}$$

$$\frac{1}{5} \coth(x) \left(a \sinh^2(x)\right)^{5/2} - \frac{4}{5} a \left(\frac{1}{3} \coth(x) \left(a \sinh^2(x)\right)^{3/2} - \frac{2}{3} a \int \sqrt{a \sinh^2(x)} dx\right)$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{5} \coth(x) \left(a \sinh^2(x)\right)^{5/2} - \frac{4}{5} a \left(\frac{1}{3} \coth(x) \left(a \sinh^2(x)\right)^{3/2} - \frac{2}{3} a \int \sqrt{-a \sin(ix)^2} dx\right)$$

$$\Big\downarrow \text{3686}$$

$$\frac{1}{5} \coth(x) \left(a \sinh^2(x)\right)^{5/2} - \frac{4}{5} a \left(\frac{1}{3} \coth(x) \left(a \sinh^2(x)\right)^{3/2} - \frac{2}{3} a \operatorname{csch}(x) \sqrt{a \sinh^2(x)} \int \sinh(x) dx\right)$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{5} \coth(x) \left(a \sinh^2(x)\right)^{5/2} - \frac{4}{5} a \left(\frac{1}{3} \coth(x) \left(a \sinh^2(x)\right)^{3/2} - \frac{2}{3} a \operatorname{csch}(x) \sqrt{a \sinh^2(x)} \int -i \sin(ix) dx\right)$$

$$\Big\downarrow 26$$

$$\frac{1}{5}\coth(x)\left(a\sinh^2(x)\right)^{5/2}-\frac{4}{5}a\left(\frac{1}{3}\coth(x)\left(a\sinh^2(x)\right)^{3/2}+\frac{2}{3}ia\text{csch}(x)\sqrt{a\sinh^2(x)}\int\sin(ix)dx\right)$$

$$\Big\downarrow 3118$$

$$\frac{1}{5}\coth(x)\left(a\sinh^2(x)\right)^{5/2}-\frac{4}{5}a\left(\frac{1}{3}\coth(x)\left(a\sinh^2(x)\right)^{3/2}-\frac{2}{3}a\coth(x)\sqrt{a\sinh^2(x)}\right)$$

input

```
Int[(a*Sinh[x]^2)^(5/2),x]
```

output

```
(Coth[x]*(a*Sinh[x]^2)^(5/2))/5 - (4*a*((-2*a*Coth[x]*Sqrt[a*Sinh[x]^2])/3
 + (Coth[x]*(a*Sinh[x]^2)^(3/2))/3))/5
```

Defintions of rubi rules used

rule 26

```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 3042

```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3118

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

rule 3682

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> Simp[(-Cot[e + f*x
])*((b*Sin[e + f*x]^2)^p/(2*f*p)), x] + Simp[b*((2*p - 1)/(2*p))   Int[(b*S
in[e + f*x]^2)^(p - 1), x], x] /; FreeQ[{b, e, f}, x] &&  !IntegerQ[p] && G
tQ[p, 1]
```

rule 3686
```
Int[(u_.)*((b_.)*sin[(e_.) + (f_.)*(x_)]^(n_))^(p_), x_Symbol] :> With[{ff
= FreeFactors[Sin[e + f*x], x]}, Simp[(b*ff^n)^IntPart[p]*((b*Sin[e + f*x]^
n)^FracPart[p]/(Sin[e + f*x]/ff)^(n*FracPart[p]))   Int[ActivateTrig[u]*(Si
n[e + f*x]/ff)^(n*p), x], x]] /; FreeQ[{b, e, f, n, p}, x] &&  !IntegerQ[p]
 && IntegerQ[n] && (EqQ[u, 1] || MatchQ[u, ((d_.)*(trig_)[e + f*x])^(m_.) /
; FreeQ[{d, m}, x] && MemberQ[{sin, cos, tan, cot, sec, csc}, trig]])
```

Maple [A] (verified)

Time = 0.44 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.60

method	result
default	$\frac{a^3 \sinh(x) \cosh(x) \left(3 \sinh(x)^4 - 4 \sinh(x)^2 + 8\right)}{15 \sqrt{a \sinh(x)^2}}$
risch	$\frac{a^2 e^{6x} \sqrt{a(e^{2x}-1)^2 e^{-2x}}}{160 e^{2x} - 160} - \frac{5a^2 e^{4x} \sqrt{a(e^{2x}-1)^2 e^{-2x}}}{96(e^{2x}-1)} + \frac{5a^2 e^{2x} \sqrt{a(e^{2x}-1)^2 e^{-2x}}}{16(e^{2x}-1)} + \frac{5 \sqrt{a(e^{2x}-1)^2 e^{-2x}}\, a^2}{16(e^{2x}-1)} - \frac{5a^2 e^{-2x} \sqrt{a(e^{2x}}}{96(e^{2x}-}$

input
```
int((a*sinh(x)^2)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
1/15*a^3*sinh(x)*cosh(x)*(3*sinh(x)^4-4*sinh(x)^2+8)/(a*sinh(x)^2)^(1/2)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 511 vs. $2(41) = 82$.

Time = 0.12 (sec) , antiderivative size = 511, normalized size of antiderivative = 9.64

$$\int \left(a \sinh^2(x)\right)^{5/2} dx = \text{Too large to display}$$

input
```
integrate((a*sinh(x)^2)^(5/2),x, algorithm="fricas")
```

output
```
1/480*(30*a^2*cosh(x)*e^x*sinh(x)^9 + 3*a^2*e^x*sinh(x)^10 + 5*(27*a^2*cos
h(x)^2 - 5*a^2)*e^x*sinh(x)^8 + 40*(9*a^2*cosh(x)^3 - 5*a^2*cosh(x))*e^x*s
inh(x)^7 + 10*(63*a^2*cosh(x)^4 - 70*a^2*cosh(x)^2 + 15*a^2)*e^x*sinh(x)^6
 + 4*(189*a^2*cosh(x)^5 - 350*a^2*cosh(x)^3 + 225*a^2*cosh(x))*e^x*sinh(x)
^5 + 10*(63*a^2*cosh(x)^6 - 175*a^2*cosh(x)^4 + 225*a^2*cosh(x)^2 + 15*a^2
)*e^x*sinh(x)^4 + 40*(9*a^2*cosh(x)^7 - 35*a^2*cosh(x)^5 + 75*a^2*cosh(x)^
3 + 15*a^2*cosh(x))*e^x*sinh(x)^3 + 5*(27*a^2*cosh(x)^8 - 140*a^2*cosh(x)^
6 + 450*a^2*cosh(x)^4 + 180*a^2*cosh(x)^2 - 5*a^2)*e^x*sinh(x)^2 + 10*(3*a
^2*cosh(x)^9 - 20*a^2*cosh(x)^7 + 90*a^2*cosh(x)^5 + 60*a^2*cosh(x)^3 - 5*
a^2*cosh(x))*e^x*sinh(x) + (3*a^2*cosh(x)^10 - 25*a^2*cosh(x)^8 + 150*a^2*
cosh(x)^6 + 150*a^2*cosh(x)^4 - 25*a^2*cosh(x)^2 + 3*a^2)*e^x)*sqrt(a*e^(4
*x) - 2*a*e^(2*x) + a)*e^(-x)/(cosh(x)^5*e^(2*x) + (e^(2*x) - 1)*sinh(x)^5
 - cosh(x)^5 + 5*(cosh(x)*e^(2*x) - cosh(x))*sinh(x)^4 + 10*(cosh(x)^2*e^(
2*x) - cosh(x)^2)*sinh(x)^3 + 10*(cosh(x)^3*e^(2*x) - cosh(x)^3)*sinh(x)^2
 + 5*(cosh(x)^4*e^(2*x) - cosh(x)^4)*sinh(x))
```

Sympy [F]

$$\int \left(a\sinh^2(x)\right)^{5/2}\,dx=\int \left(a\sinh^{2}\left(x\right)\right)^{\frac{5}{2}}\,dx$$

input
```
integrate((a*sinh(x)**2)**(5/2),x)
```

output
```
Integral((a*sinh(x)**2)**(5/2), x)
```

Maxima [A] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 53, normalized size of antiderivative = 1.00

$$\int \left(a\sinh^2(x)\right)^{5/2}\,dx=-\frac{1}{160}\,a^{\frac{5}{2}}e^{(5\,x)}+\frac{5}{96}\,a^{\frac{5}{2}}e^{(3\,x)}-\frac{5}{16}\,a^{\frac{5}{2}}e^{(-x)}+\frac{5}{96}\,a^{\frac{5}{2}}e^{(-3\,x)}-\frac{1}{160}\,a^{\frac{5}{2}}e^{(-5\,x)}-\frac{5}{16}\,a^{\frac{5}{2}}e^{x}$$

input
```
integrate((a*sinh(x)^2)^(5/2),x, algorithm="maxima")
```

output
```
-1/160*a^(5/2)*e^(5*x) + 5/96*a^(5/2)*e^(3*x) - 5/16*a^(5/2)*e^(-x) + 5/96
*a^(5/2)*e^(-3*x) - 1/160*a^(5/2)*e^(-5*x) - 5/16*a^(5/2)*e^x
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 102 vs. $2(41) = 82$.

Time = 0.12 (sec) , antiderivative size = 102, normalized size of antiderivative = 1.92

$$\int \left(a\sinh^2(x)\right)^{5/2} dx = \frac{1}{480}\left(\left(150\,e^{(4\,x)}\text{sgn}\left(e^{(3\,x)} - e^{x}\right) - 25\,e^{(2\,x)}\text{sgn}\left(e^{(3\,x)} - e^{x}\right) + 3\,\text{sgn}\left(e^{(3\,x)} - e^{x}\right)\right)e^{(-5\,x}$$

input
```
integrate((a*sinh(x)^2)^(5/2),x, algorithm="giac")
```

output
```
1/480*((150*e^(4*x)*sgn(e^(3*x) - e^x) - 25*e^(2*x)*sgn(e^(3*x) - e^x) + 3
*sgn(e^(3*x) - e^x))*e^(-5*x) + 3*e^(5*x)*sgn(e^(3*x) - e^x) - 25*e^(3*x)*
sgn(e^(3*x) - e^x) + 150*e^x*sgn(e^(3*x) - e^x))*a^(5/2)
```

Mupad [F(-1)]

Timed out.

$$\int \left(a\sinh^2(x)\right)^{5/2} dx = \int \left(a\,\sinh(x)^2\right)^{5/2} dx$$

input
```
int((a*sinh(x)^2)^(5/2),x)
```

output
```
int((a*sinh(x)^2)^(5/2), x)
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.96

$$\int \left(a\sinh^2(x)\right)^{5/2}\,dx = \frac{\sqrt{a}\,a^2(3e^{10x}-25e^{8x}+150e^{6x}+150e^{4x}-25e^{2x}+3)}{480e^{5x}}$$

input
```
int((a*sinh(x)^2)^(5/2),x)
```

output
```
(sqrt(a)*a**2*(3*e**(10*x) - 25*e**(8*x) + 150*e**(6*x) + 150*e**(4*x) - 2
5*e**(2*x) + 3))/(480*e**(5*x))
```

3.141 $\int \left(a\sinh^2(x)\right)^{3/2}\,dx$

Optimal result

Integrand size = 10, antiderivative size = 34

$$\int \left(a\sinh^2(x)\right)^{3/2}\,dx=-\frac{2}{3}a\coth(x)\sqrt{a\sinh^2(x)}+\frac{1}{3}\coth(x)\left(a\sinh^2(x)\right)^{3/2}$$

output
```
-2/3*a*coth(x)*(a*sinh(x)^2)^(1/2)+1/3*coth(x)*(a*sinh(x)^2)^(3/2)
```

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.76

$$\int \left(a\sinh^2(x)\right)^{3/2}\,dx=\frac{1}{12}a(-9\cosh(x)+\cosh(3x))\operatorname{csch}(x)\sqrt{a\sinh^2(x)}$$

input
```
Integrate[(a*Sinh[x]^2)^(3/2),x]
```

output
```
(a*(-9*Cosh[x] + Cosh[3*x])*Csch[x]*Sqrt[a*Sinh[x]^2])/12
```

Rubi [A] (verified)

Time = 0.34 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.700, Rules used = {3042, 3682, 3042, 3686, 3042, 26, 3118}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(a\sinh^2(x)\right)^{3/2}dx$$
$$\Big\downarrow \text{3042}$$
$$\int \left(-a\sin(ix)^2\right)^{3/2}dx$$
$$\Big\downarrow \text{3682}$$
$$\frac{1}{3}\coth(x)\left(a\sinh^2(x)\right)^{3/2}-\frac{2}{3}a\int \sqrt{a\sinh^2(x)}dx$$
$$\Big\downarrow \text{3042}$$
$$\frac{1}{3}\coth(x)\left(a\sinh^2(x)\right)^{3/2}-\frac{2}{3}a\int \sqrt{-a\sin(ix)^2}dx$$
$$\Big\downarrow \text{3686}$$
$$\frac{1}{3}\coth(x)\left(a\sinh^2(x)\right)^{3/2}-\frac{2}{3}a\text{csch}(x)\sqrt{a\sinh^2(x)}\int \sinh(x)dx$$
$$\Big\downarrow \text{3042}$$
$$\frac{1}{3}\coth(x)\left(a\sinh^2(x)\right)^{3/2}-\frac{2}{3}a\text{csch}(x)\sqrt{a\sinh^2(x)}\int -i\sin(ix)dx$$
$$\Big\downarrow \text{26}$$
$$\frac{1}{3}\coth(x)\left(a\sinh^2(x)\right)^{3/2}+\frac{2}{3}ia\text{csch}(x)\sqrt{a\sinh^2(x)}\int \sin(ix)dx$$
$$\Big\downarrow \text{3118}$$
$$\frac{1}{3}\coth(x)\left(a\sinh^2(x)\right)^{3/2}-\frac{2}{3}a\coth(x)\sqrt{a\sinh^2(x)}$$

input
```
Int[(a*Sinh[x]^2)^(3/2),x]
```

output
```
(-2*a*Coth[x]*Sqrt[a*Sinh[x]^2])/3 + (Coth[x]*(a*Sinh[x]^2)^(3/2))/3
```

Defintions of rubi rules used

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3118
```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

rule 3682
```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> Simp[(-Cot[e + f*x
])*((b*Sin[e + f*x]^2)^p/(2*f*p)), x] + Simp[b*((2*p - 1)/(2*p))   Int[(b*S
in[e + f*x]^2)^(p - 1), x], x] /; FreeQ[{b, e, f}, x] &&  !IntegerQ[p] && G
tQ[p, 1]
```

rule 3686
```
Int[(u_.)*((b_.)*sin[(e_.) + (f_.)*(x_)]^(n_))^(p_), x_Symbol] :> With[{ff
= FreeFactors[Sin[e + f*x], x]}, Simp[(b*ff^n)^IntPart[p]*((b*Sin[e + f*x]^
n)^FracPart[p]/(Sin[e + f*x]/ff)^(n*FracPart[p]))   Int[ActivateTrig[u]*(Si
n[e + f*x]/ff)^(n*p), x], x]] /; FreeQ[{b, e, f, n, p}, x] &&  !IntegerQ[p]
 && IntegerQ[n] && (EqQ[u, 1] || MatchQ[u, ((d_.)*(trig_)[e + f*x])^(m_.) /
; FreeQ[{d, m}, x] && MemberQ[{sin, cos, tan, cot, sec, csc}, trig]])
```

Maple [A] (verified)

Time = 0.12 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.71

method	result	size
default	$\frac{a^{2} \sinh(x) \cosh(x) \left(\sinh(x)^{2}-2\right)}{3 \sqrt{a \sinh(x)^{2}}}$	24
risch	$\frac{a\, e^{4x} \sqrt{a(e^{2x}-1)^{2} e^{-2x}}}{24\, e^{2x}-24}-\frac{3a\, e^{2x} \sqrt{a(e^{2x}-1)^{2} e^{-2x}}}{8(e^{2x}-1)}-\frac{3 \sqrt{a(e^{2x}-1)^{2} e^{-2x}}\, a}{8(e^{2x}-1)}+\frac{a\, e^{-2x} \sqrt{a(e^{2x}-1)^{2} e^{-2x}}}{24\, e^{2x}-24}$	122

input
```
int((a*sinh(x)^2)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
1/3*a^2*sinh(x)*cosh(x)*(sinh(x)^2-2)/(a*sinh(x)^2)^(1/2)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 226 vs. 2(26) = 52.

Time = 0.09 (sec) , antiderivative size = 226, normalized size of antiderivative = 6.65

$$\int \left(a\sinh^2(x)\right)^{3/2}\,dx = \frac{\left(6\,a\cosh\left(x\right)e^{x}\sinh\left(x\right)^{5}+ae^{x}\sinh\left(x\right)^{6}+3\left(5\,a\cosh\left(x\right)^{2}-3\,a\right)e^{x}\sinh\left(x\right)^{4}+4\left(5\right.\right.}{}$$

input
```
integrate((a*sinh(x)^2)^(3/2),x, algorithm="fricas")
```

output
```
1/24*(6*a*cosh(x)*e^x*sinh(x)^5 + a*e^x*sinh(x)^6 + 3*(5*a*cosh(x)^2 - 3*a
)*e^x*sinh(x)^4 + 4*(5*a*cosh(x)^3 - 9*a*cosh(x))*e^x*sinh(x)^3 + 3*(5*a*c
osh(x)^4 - 18*a*cosh(x)^2 - 3*a)*e^x*sinh(x)^2 + 6*(a*cosh(x)^5 - 6*a*cosh
(x)^3 - 3*a*cosh(x))*e^x*sinh(x) + (a*cosh(x)^6 - 9*a*cosh(x)^4 - 9*a*cosh
(x)^2 + a)*e^x)*sqrt(a*e^(4*x) - 2*a*e^(2*x) + a)*e^(-x)/(cosh(x)^3*e^(2*x
) + (e^(2*x) - 1)*sinh(x)^3 - cosh(x)^3 + 3*(cosh(x)*e^(2*x) - cosh(x))*si
nh(x)^2 + 3*(cosh(x)^2*e^(2*x) - cosh(x)^2)*sinh(x))
```

Sympy [F]

$$\int \left(a\sinh^2(x)\right)^{3/2}\,dx=\int \left(a\sinh^2{(x)}\right)^{\frac{3}{2}}\,dx$$

input
```
integrate((a*sinh(x)**2)**(3/2),x)
```

output
```
Integral((a*sinh(x)**2)**(3/2), x)
```

Maxima [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 35, normalized size of antiderivative = 1.03

$$\int \left(a\sinh^2(x)\right)^{3/2}\,dx=-\frac{1}{24}\,a^{\frac{3}{2}}e^{(3\,x)}+\frac{3}{8}\,a^{\frac{3}{2}}e^{(-x)}-\frac{1}{24}\,a^{\frac{3}{2}}e^{(-3\,x)}+\frac{3}{8}\,a^{\frac{3}{2}}e^{x}$$

input
```
integrate((a*sinh(x)^2)^(3/2),x, algorithm="maxima")
```

output
```
-1/24*a^(3/2)*e^(3*x) + 3/8*a^(3/2)*e^(-x) - 1/24*a^(3/2)*e^(-3*x) + 3/8*a
^(3/2)*e^x
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 70 vs. $2(26) = 52$.

Time = 0.12 (sec) , antiderivative size = 70, normalized size of antiderivative = 2.06

$$\int \left(a\sinh^2(x)\right)^{3/2}\,dx= -\frac{1}{24}\left(\left(9\,e^{(2\,x)}\mathrm{sgn}\left(e^{(3\,x)}-e^{x}\right)-\mathrm{sgn}\left(e^{(3\,x)}-e^{x}\right)\right)e^{(-3\,x)}-e^{(3\,x)}\mathrm{sgn}\left(e^{(3\,x)}-e^{x}\right)+9\,e^{x}\mathrm{sgn}\left(e^{(3\,x)}-e^{x}\right)\right)a^{\frac{3}{2}}$$

input
```
integrate((a*sinh(x)^2)^(3/2),x, algorithm="giac")
```

output
```
-1/24*((9*e^(2*x)*sgn(e^(3*x) - e^x) - sgn(e^(3*x) - e^x))*e^(-3*x) - e^(3
*x)*sgn(e^(3*x) - e^x) + 9*e^x*sgn(e^(3*x) - e^x))*a^(3/2)
```

Mupad [F(-1)]

Timed out.

$$\int \left(a\sinh^2(x)\right)^{3/2}\,dx = \int \left(a\sinh(x)^2\right)^{3/2}\,dx$$

input
```
int((a*sinh(x)^2)^(3/2),x)
```

output
```
int((a*sinh(x)^2)^(3/2), x)
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.97

$$\int \left(a\sinh^2(x)\right)^{3/2}\,dx = \frac{\sqrt{a}\,a(e^{6x}-9e^{4x}-9e^{2x}+1)}{24e^{3x}}$$

input
```
int((a*sinh(x)^2)^(3/2),x)
```

output
```
(sqrt(a)*a*(e**(6*x) - 9*e**(4*x) - 9*e**(2*x) + 1))/(24*e**(3*x))
```

3.142 $\int \sqrt{a \sinh^2(x)}\, dx$

Optimal result

Integrand size = 10, antiderivative size = 13

$$\int \sqrt{a \sinh^2(x)}\, dx = \coth(x)\sqrt{a \sinh^2(x)}$$

output
```
coth(x)*(a*sinh(x)^2)^(1/2)
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 13, normalized size of antiderivative = 1.00

$$\int \sqrt{a \sinh^2(x)}\, dx = \coth(x)\sqrt{a \sinh^2(x)}$$

input
```
Integrate[Sqrt[a*Sinh[x]^2],x]
```

output
```
Coth[x]*Sqrt[a*Sinh[x]^2]
```

Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 13, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {3042, 3686, 3042, 26, 3118}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{a\sinh^2(x)}\,dx$$
$$\Big\downarrow \text{3042}$$
$$\int \sqrt{-a\sin(ix)^2}dx$$
$$\Big\downarrow \text{3686}$$
$$\text{csch}(x)\sqrt{a\sinh^2(x)}\int \sinh(x)dx$$
$$\Big\downarrow \text{3042}$$
$$\text{csch}(x)\sqrt{a\sinh^2(x)}\int -i\sin(ix)dx$$
$$\Big\downarrow \text{26}$$
$$-i\text{csch}(x)\sqrt{a\sinh^2(x)}\int \sin(ix)dx$$
$$\Big\downarrow \text{3118}$$
$$\coth(x)\sqrt{a\sinh^2(x)}$$

input
```
Int[Sqrt[a*Sinh[x]^2],x]
```

output
```
Coth[x]*Sqrt[a*Sinh[x]^2]
```

Defintions of rubi rules used

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3118
```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

rule 3686
```
Int[(u_.)*((b_.)*sin[(e_.) + (f_.)*(x_)]^(n_))^(p_), x_Symbol] :> With[{ff
= FreeFactors[Sin[e + f*x], x]}, Simp[(b*ff^n)^IntPart[p]*((b*Sin[e + f*x]^
n)^FracPart[p]/(Sin[e + f*x]/ff)^(n*FracPart[p]))   Int[ActivateTrig[u]*(Si
n[e + f*x]/ff)^(n*p), x], x]] /; FreeQ[{b, e, f, n, p}, x] &&  !IntegerQ[p]
 && IntegerQ[n] && (EqQ[u, 1] || MatchQ[u, ((d_.)*(trig_)[e + f*x])^(m_.) /
; FreeQ[{d, m}, x] && MemberQ[{sin, cos, tan, cot, sec, csc}, trig]])
```

Maple [A] (verified)

Time = 0.11 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.15

method	result	size
default	$\frac{a \sinh(x)\cosh(x)}{\sqrt{a \sinh(x)^{2}}}$	15
risch	$\frac{\sqrt{a(\mathrm{e}^{2x}-1)^{2}\mathrm{e}^{-2x}}\,\mathrm{e}^{2x}}{2\,\mathrm{e}^{2x}-2}+\frac{\sqrt{a(\mathrm{e}^{2x}-1)^{2}\mathrm{e}^{-2x}}}{2\,\mathrm{e}^{2x}-2}$	58

input
```
int((a*sinh(x)^2)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/(a*sinh(x)^2)^(1/2)*a*sinh(x)*cosh(x)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 71 vs. $2(11) = 22$.

Time = 0.11 (sec) , antiderivative size = 71, normalized size of antiderivative = 5.46

$$\int \sqrt{a \sinh^2(x)}\,dx = \frac{\left(2 \cosh(x)\, e^{x} \sinh(x) + e^{x} \sinh(x)^{2} + \left(\cosh(x)^{2} + 1\right) e^{x}\right) \sqrt{a e^{(4\,x)} - 2\,a e^{(2\,x)} + a}\, e^{(-x)}}{2 \left(\cosh(x)\, e^{(2\,x)} + \left(e^{(2\,x)} - 1\right) \sinh(x) - \cosh(x)\right)}$$

input
```
integrate((a*sinh(x)^2)^(1/2),x, algorithm="fricas")
```

output
```
1/2*(2*cosh(x)*e^x*sinh(x) + e^x*sinh(x)^2 + (cosh(x)^2 + 1)*e^x)*sqrt(a*e
^(4*x) - 2*a*e^(2*x) + a)*e^(-x)/(cosh(x)*e^(2*x) + (e^(2*x) - 1)*sinh(x)
- cosh(x))
```

Sympy [A] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.15

$$\int \sqrt{a \sinh^2(x)}\,dx = \frac{\sqrt{a \sinh^2(x)} \cosh(x)}{\sinh(x)}$$

input
```
integrate((a*sinh(x)**2)**(1/2),x)
```

output
```
sqrt(a*sinh(x)**2)*cosh(x)/sinh(x)
```

Maxima [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.31

$$\int \sqrt{a\sinh^2(x)}\,dx = -\frac{1}{2}\sqrt{a}e^{(-x)} - \frac{1}{2}\sqrt{a}e^{x}$$

input
```
integrate((a*sinh(x)^2)^(1/2),x, algorithm="maxima")
```

output
```
-1/2*sqrt(a)*e^(-x) - 1/2*sqrt(a)*e^x
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 34 vs. 2(11) = 22.

Time = 0.12 (sec) , antiderivative size = 34, normalized size of antiderivative = 2.62

$$\int \sqrt{a\sinh^2(x)}\,dx = \frac{1}{2}\left(e^{(-x)}\mathrm{sgn}\left(e^{(3x)} - e^{x}\right) + e^{x}\mathrm{sgn}\left(e^{(3x)} - e^{x}\right)\right)\sqrt{a}$$

input
```
integrate((a*sinh(x)^2)^(1/2),x, algorithm="giac")
```

output
```
1/2*(e^(-x)*sgn(e^(3*x) - e^x) + e^x*sgn(e^(3*x) - e^x))*sqrt(a)
```

Mupad [B] (verification not implemented)

Time = 1.40 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.62

$$\int \sqrt{a\sinh^2(x)}\,dx = \sqrt{a}\coth(x)\sqrt{\left(\frac{e^{-x}}{2} - \frac{e^{x}}{2}\right)^2}$$

input
```
int((a*sinh(x)^2)^(1/2),x)
```

output
```
a^(1/2)*coth(x)*((exp(-x)/2 - exp(x)/2)^2)^(1/2)
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 5, normalized size of antiderivative = 0.38

$$\int \sqrt{a \sinh^2(x)}\, dx = \sqrt{a} \cosh(x)$$

input
```
int((a*sinh(x)^2)^(1/2),x)
```

output
```
sqrt(a)*cosh(x)
```

3.143 $\int \frac{1}{\sqrt{a \sinh^2(x)}} dx$

Optimal result

Integrand size = 10, antiderivative size = 17

$$\int \frac{1}{\sqrt{a \sinh^2(x)}} dx = -\frac{\text{arctanh}(\cosh(x)) \sinh(x)}{\sqrt{a \sinh^2(x)}}$$

output
```
-arctanh(cosh(x))*sinh(x)/(a*sinh(x)^2)^(1/2)
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.00

$$\int \frac{1}{\sqrt{a \sinh^2(x)}} dx = -\frac{\text{arctanh}(\cosh(x)) \sinh(x)}{\sqrt{a \sinh^2(x)}}$$

input
```
Integrate[1/Sqrt[a*Sinh[x]^2],x]
```

output
```
-((ArcTanh[Cosh[x]]*Sinh[x])/Sqrt[a*Sinh[x]^2])
```

Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {3042, 3686, 3042, 26, 4257}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt{a \sinh^2(x)}} dx$$
$$\Big\downarrow \text{ 3042}$$
$$\int \frac{1}{\sqrt{-a \sin(ix)^2}} dx$$
$$\Big\downarrow \text{ 3686}$$
$$\frac{\sinh(x) \int \operatorname{csch}(x) dx}{\sqrt{a \sinh^2(x)}}$$
$$\Big\downarrow \text{ 3042}$$
$$\frac{\sinh(x) \int i \csc(ix) dx}{\sqrt{a \sinh^2(x)}}$$
$$\Big\downarrow \text{ 26}$$
$$\frac{i \sinh(x) \int \csc(ix) dx}{\sqrt{a \sinh^2(x)}}$$
$$\Big\downarrow \text{ 4257}$$
$$-\frac{\sinh(x) \operatorname{arctanh}(\cosh(x))}{\sqrt{a \sinh^2(x)}}$$

input
```
Int[1/Sqrt[a*Sinh[x]^2],x]
```

output
```
-((ArcTanh[Cosh[x]]*Sinh[x])/Sqrt[a*Sinh[x]^2])
```

Defintions of rubi rules used

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3686
```
Int[(u_.)*((b_.)*sin[(e_.) + (f_.)*(x_)]^(n_))^(p_), x_Symbol] :> With[{ff
= FreeFactors[Sin[e + f*x], x]}, Simp[(b*ff^n)^IntPart[p]*((b*Sin[e + f*x]^
n)^FracPart[p]/(Sin[e + f*x]/ff)^(n*FracPart[p]))   Int[ActivateTrig[u]*(Si
n[e + f*x]/ff)^(n*p), x], x]] /; FreeQ[{b, e, f, n, p}, x] &&  !IntegerQ[p]
 && IntegerQ[n] && (EqQ[u, 1] || MatchQ[u, ((d_.)*(trig_)[e + f*x])^(m_.) /
; FreeQ[{d, m}, x] && MemberQ[{sin, cos, tan, cot, sec, csc}, trig]])
```

rule 4257
```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 48 vs. $2(15) = 30$.

Time = 0.12 (sec) , antiderivative size = 49, normalized size of antiderivative = 2.88

method	result	size
default	$-\frac{\sinh(x)\sqrt{a\cosh(x)^{2}}\,\ln\left(\frac{2\sqrt{a}\,\sqrt{a\cosh(x)^{2}}+2a}{\sinh(x)}\right)}{\sqrt{a}\,\cosh(x)\sqrt{a\sinh(x)^{2}}}$	49
risch	$\frac{\mathrm{e}^{-x}\left(\mathrm{e}^{2x}-1\right)\ln\left(\mathrm{e}^{x}-1\right)}{\sqrt{a\left(\mathrm{e}^{2x}-1\right)^{2}\mathrm{e}^{-2x}}}-\frac{\mathrm{e}^{-x}\left(\mathrm{e}^{2x}-1\right)\ln\left(\mathrm{e}^{x}+1\right)}{\sqrt{a\left(\mathrm{e}^{2x}-1\right)^{2}\mathrm{e}^{-2x}}}$	67

input
```
int(1/(a*sinh(x)^2)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-sinh(x)*(a*cosh(x)^2)^(1/2)/a^(1/2)*ln(2*(a^(1/2)*(a*cosh(x)^2)^(1/2)+a)/
sinh(x))/cosh(x)/(a*sinh(x)^2)^(1/2)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 63 vs. $2(15) = 30$.

Time = 0.09 (sec) , antiderivative size = 110, normalized size of antiderivative = 6.47

$$\int \frac{1}{\sqrt{a\sinh^2(x)}}\,dx = \left[\frac{\sqrt{ae^{(4x)} - 2ae^{(2x)} + a}\log\left(\frac{\cosh(x)+\sinh(x)-1}{\cosh(x)+\sinh(x)+1}\right)}{ae^{(2x)} - a}, \frac{2\sqrt{-a}\arctan\left(\frac{\sqrt{ae^{(4x)}-2ae^{(2x)}+a}\sqrt{-a}}{a\cosh(x)e^{(2x)}-a\cosh(x)+(ae^{(2x)}-a)\sinh(x)}\right)}{a}\right]$$

input
```
integrate(1/(a*sinh(x)^2)^(1/2),x, algorithm="fricas")
```

output
```
[sqrt(a*e^(4*x) - 2*a*e^(2*x) + a)*log((cosh(x) + sinh(x) - 1)/(cosh(x) +
sinh(x) + 1))/(a*e^(2*x) - a), 2*sqrt(-a)*arctan(sqrt(a*e^(4*x) - 2*a*e^(2
*x) + a)*sqrt(-a)/(a*cosh(x)*e^(2*x) - a*cosh(x) + (a*e^(2*x) - a)*sinh(x)
))/a]
```

Sympy [F]

$$\int \frac{1}{\sqrt{a\sinh^2(x)}}\,dx = \int \frac{1}{\sqrt{a\sinh^2(x)}}\,dx$$

input
```
integrate(1/(a*sinh(x)**2)**(1/2),x)
```

output
```
Integral(1/sqrt(a*sinh(x)**2), x)
```

Maxima [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.41

$$\int \frac{1}{\sqrt{a\sinh^2(x)}}\,dx = \frac{\log\left(e^{(-x)}+1\right)}{\sqrt{a}} - \frac{\log\left(e^{(-x)}-1\right)}{\sqrt{a}}$$

input
```
integrate(1/(a*sinh(x)^2)^(1/2),x, algorithm="maxima")
```

output
```
log(e^(-x) + 1)/sqrt(a) - log(e^(-x) - 1)/sqrt(a)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 1, normalized size of antiderivative = 0.06

$$\int \frac{1}{\sqrt{a\sinh^2(x)}}\,dx = 0$$

input
```
integrate(1/(a*sinh(x)^2)^(1/2),x, algorithm="giac")
```

output
```
0
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt{a\sinh^2(x)}}\,dx = \int \frac{1}{\sqrt{a\sinh(x)^2}}\,dx$$

input
```
int(1/(a*sinh(x)^2)^(1/2),x)
```

output
```
int(1/(a*sinh(x)^2)^(1/2), x)
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.24

$$\int \frac{1}{\sqrt{a\sinh^2(x)}}\,dx = \frac{\sqrt{a}\,(\log(e^x-1)-\log(e^x+1))}{a}$$

input
```
int(1/(a*sinh(x)^2)^(1/2),x)
```

output
```
(sqrt(a)*(log(e**x - 1) - log(e**x + 1)))/a
```

3.144 $\int \frac{1}{\left(a\sinh^2(x)\right)^{3/2}}\,dx$

Optimal result

Integrand size = 10, antiderivative size = 42

$$\int \frac{1}{\left(a\sinh^2(x)\right)^{3/2}}\,dx=-\frac{\coth(x)}{2a\sqrt{a\sinh^2(x)}}+\frac{\operatorname{arctanh}(\cosh(x))\sinh(x)}{2a\sqrt{a\sinh^2(x)}}$$

output
```
-1/2*coth(x)/a/(a*sinh(x)^2)^(1/2)+1/2*arctanh(cosh(x))*sinh(x)/a/(a*sinh(
x)^2)^(1/2)
```

Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 53, normalized size of antiderivative = 1.26

$$\int \frac{1}{\left(a\sinh^2(x)\right)^{3/2}}\,dx = -\frac{\left(\operatorname{csch}^2\left(\frac{x}{2}\right)-4\log\left(\cosh\left(\frac{x}{2}\right)\right)+4\log\left(\sinh\left(\frac{x}{2}\right)\right)+\operatorname{sech}^2\left(\frac{x}{2}\right)\right)\sinh^3(x)}{8\left(a\sinh^2(x)\right)^{3/2}}$$

input
```
Integrate[(a*Sinh[x]^2)^(-3/2),x]
```

output
```
-1/8*((Csch[x/2]^2 - 4*Log[Cosh[x/2]] + 4*Log[Sinh[x/2]] + Sech[x/2]^2)*Si
nh[x]^3)/(a*Sinh[x]^2)^(3/2)
```

Rubi [A] (verified)

Time = 0.31 (sec) , antiderivative size = 42, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.700$, Rules used = {3042, 3683, 3042, 3686, 3042, 26, 4257}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\left(a\sinh^2(x)\right)^{3/2}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{\left(-a\sin(ix)^2\right)^{3/2}}dx$$

$$\Big\downarrow \text{3683}$$

$$-\frac{\int \frac{1}{\sqrt{a\sinh^2(x)}}dx}{2a}-\frac{\coth(x)}{2a\sqrt{a\sinh^2(x)}}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{\coth(x)}{2a\sqrt{a\sinh^2(x)}}-\frac{\int \frac{1}{\sqrt{-a\sin(ix)^2}}dx}{2a}$$

$$\Big\downarrow \text{3686}$$

$$-\frac{\sinh(x)\int \text{csch}(x)dx}{2a\sqrt{a\sinh^2(x)}}-\frac{\coth(x)}{2a\sqrt{a\sinh^2(x)}}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{\coth(x)}{2a\sqrt{a\sinh^2(x)}}-\frac{\sinh(x)\int i\csc(ix)dx}{2a\sqrt{a\sinh^2(x)}}$$

$$\Big\downarrow \text{26}$$

$$-\frac{\coth(x)}{2a\sqrt{a\sinh^2(x)}}-\frac{i\sinh(x)\int\csc(ix)dx}{2a\sqrt{a\sinh^2(x)}}$$

$$\Big\downarrow \text{4257}$$

$$\frac{\sinh(x)\text{arctanh}(\cosh(x))}{2a\sqrt{a\sinh^2(x)}}-\frac{\coth(x)}{2a\sqrt{a\sinh^2(x)}}$$

input
```
Int[(a*Sinh[x]^2)^(-3/2),x]
```

output
```
-1/2*Coth[x]/(a*Sqrt[a*Sinh[x]^2]) + (ArcTanh[Cosh[x]]*Sinh[x])/(2*a*Sqrt[
a*Sinh[x]^2])
```

Defintions of rubi rules used

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3683
```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> Simp[Cot[e + f*x]*
((b*Sin[e + f*x]^2)^(p + 1)/(b*f*(2*p + 1))), x] + Simp[2*((p + 1)/(b*(2*p
+ 1)))   Int[(b*Sin[e + f*x]^2)^(p + 1), x], x] /; FreeQ[{b, e, f}, x] &&
!IntegerQ[p] && LtQ[p, -1]
```

rule 3686
```
Int[(u_.)*((b_.)*sin[(e_.) + (f_.)*(x_)]^(n_))^(p_), x_Symbol] :> With[{ff
= FreeFactors[Sin[e + f*x], x]}, Simp[(b*ff^n)^IntPart[p]*((b*Sin[e + f*x]^
n)^FracPart[p]/(Sin[e + f*x]/ff)^(n*FracPart[p]))   Int[ActivateTrig[u]*(Si
n[e + f*x]/ff)^(n*p), x], x]] /; FreeQ[{b, e, f, n, p}, x] &&  !IntegerQ[p]
 && IntegerQ[n] && (EqQ[u, 1] || MatchQ[u, ((d_.)*(trig_)[e + f*x])^(m_.) /
; FreeQ[{d, m}, x] && MemberQ[{sin, cos, tan, cot, sec, csc}, trig]])
```

rule 4257
```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 70 vs. $2(34) = 68$.

Time = 0.15 (sec) , antiderivative size = 71, normalized size of antiderivative = 1.69

method	result	size
default	$-\frac{\sqrt{a \cosh(x)^{2}}\,\left(-\ln\left(\frac{2\sqrt{a}\,\sqrt{a \cosh(x)^{2}}+2a}{\sinh(x)}\right) a \sinh(x)^{2}+\sqrt{a}\,\sqrt{a \cosh(x)^{2}}\right)}{2a^{\frac{5}{2}} \sinh(x) \cosh(x) \sqrt{a \sinh(x)^{2}}}$	71
risch	$-\frac{\mathrm{e}^{2x}+1}{a(\mathrm{e}^{2x}-1)\sqrt{a(\mathrm{e}^{2x}-1)^{2}\mathrm{e}^{-2x}}}-\frac{(\mathrm{e}^{2x}-1)\mathrm{e}^{-x}\ln(\mathrm{e}^{x}-1)}{2a\sqrt{a(\mathrm{e}^{2x}-1)^{2}\mathrm{e}^{-2x}}}+\frac{(\mathrm{e}^{2x}-1)\mathrm{e}^{-x}\ln(\mathrm{e}^{x}+1)}{2a\sqrt{a(\mathrm{e}^{2x}-1)^{2}\mathrm{e}^{-2x}}}$	109

input
```
int(1/(a*sinh(x)^2)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-1/2/a^(5/2)/sinh(x)*(a*cosh(x)^2)^(1/2)*(-ln(2*(a^(1/2)*(a*cosh(x)^2)^(1/
2)+a)/sinh(x))*a*sinh(x)^2+a^(1/2)*(a*cosh(x)^2)^(1/2))/cosh(x)/(a*sinh(x)
^2)^(1/2)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 327 vs. $2(34) = 68$.

Time = 0.09 (sec) , antiderivative size = 327, normalized size of antiderivative = 7.79

$$\int \frac{1}{\left(a \sinh^2(x)\right)^{3/2}}\,dx = \frac{\left(6\,\cosh\left(x\right)e^{x}\sinh\left(x\right)^{2}+2\,e^{x}\sinh\left(x\right)^{3}+2\,\left(3\,\cosh\left(x\right)^{2}+1\right)e^{x}\sinh\left(x\right)+2\,\left(\cosh\right.\right.}{2\,\left(a^{2}\cosh\left(x\right)^{4}-\left(a^{2}e^{(2\,x)}-a^{2}\right)\sinh\left(x\right)^{4}-2\,a^{2}\cosh\left(x\right)^{2}-4\,\left(a^{2}\cosh\left(x\right)e^{(2\,x)}\right.\right.}$$

input
```
integrate(1/(a*sinh(x)^2)^(3/2),x, algorithm="fricas")
```

output
```
1/2*(6*cosh(x)*e^x*sinh(x)^2 + 2*e^x*sinh(x)^3 + 2*(3*cosh(x)^2 + 1)*e^x*s
inh(x) + 2*(cosh(x)^3 + cosh(x))*e^x - (4*cosh(x)*e^x*sinh(x)^3 + e^x*sinh
(x)^4 + 2*(3*cosh(x)^2 - 1)*e^x*sinh(x)^2 + 4*(cosh(x)^3 - cosh(x))*e^x*si
nh(x) + (cosh(x)^4 - 2*cosh(x)^2 + 1)*e^x)*log((cosh(x) + sinh(x) + 1)/(co
sh(x) + sinh(x) - 1)))*sqrt(a*e^(4*x) - 2*a*e^(2*x) + a)*e^(-x)/(a^2*cosh(
x)^4 - (a^2*e^(2*x) - a^2)*sinh(x)^4 - 2*a^2*cosh(x)^2 - 4*(a^2*cosh(x)*e^
(2*x) - a^2*cosh(x))*sinh(x)^3 + 2*(3*a^2*cosh(x)^2 - a^2 - (3*a^2*cosh(x)
^2 - a^2)*e^(2*x))*sinh(x)^2 + a^2 - (a^2*cosh(x)^4 - 2*a^2*cosh(x)^2 + a^
2)*e^(2*x) + 4*(a^2*cosh(x)^3 - a^2*cosh(x) - (a^2*cosh(x)^3 - a^2*cosh(x)
)*e^(2*x))*sinh(x))
```

Sympy [F]

$$\int \frac{1}{\left(a\sinh^2(x)\right)^{3/2}}\,dx = \int \frac{1}{\left(a\sinh^2\left(x\right)\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/(a*sinh(x)**2)**(3/2),x)
```

output
```
Integral((a*sinh(x)**2)**(-3/2), x)
```

Maxima [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 62, normalized size of antiderivative = 1.48

$$\int \frac{1}{\left(a\sinh^2(x)\right)^{3/2}}\,dx = -\frac{e^{(-x)}+e^{(-3x)}}{2a^{\frac{3}{2}}e^{(-2x)}-a^{\frac{3}{2}}e^{(-4x)}-a^{\frac{3}{2}}}-\frac{\log\left(e^{(-x)}+1\right)}{2a^{\frac{3}{2}}}+\frac{\log\left(e^{(-x)}-1\right)}{2a^{\frac{3}{2}}}$$

input
```
integrate(1/(a*sinh(x)^2)^(3/2),x, algorithm="maxima")
```

output
```
-(e^(-x) + e^(-3*x))/(2*a^(3/2)*e^(-2*x) - a^(3/2)*e^(-4*x) - a^(3/2)) - 1
/2*log(e^(-x) + 1)/a^(3/2) + 1/2*log(e^(-x) - 1)/a^(3/2)
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.88

$$\int \frac{1}{\left(a\sinh^2(x)\right)^{3/2}}\,dx = -\frac{e^{(-x)}+e^{x}}{\left(\left(e^{(-x)}+e^{x}\right)^2-4\right)a^{\frac{3}{2}}\operatorname{sgn}\left(e^{(3\,x)}-e^{x}\right)}$$

input
```
integrate(1/(a*sinh(x)^2)^(3/2),x, algorithm="giac")
```

output
```
-(e^(-x) + e^x)/(((e^(-x) + e^x)^2 - 4)*a^(3/2)*sgn(e^(3*x) - e^x))
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\left(a\sinh^2(x)\right)^{3/2}}\,dx = \int \frac{1}{\left(a\sinh(x)^2\right)^{3/2}}\,dx$$

input
```
int(1/(a*sinh(x)^2)^(3/2),x)
```

output
```
int(1/(a*sinh(x)^2)^(3/2), x)
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 101, normalized size of antiderivative = 2.40

$$\int \frac{1}{\left(a\sinh^2(x)\right)^{3/2}}\,dx = \frac{\sqrt{a}\left(-e^{4x}\log(e^{x}-1)+e^{4x}\log(e^{x}+1)-2e^{3x}+2e^{2x}\log(e^{x}-1)-2e^{2x}\log(e^{x}+1)\right.}{2a^2\left(e^{4x}-2e^{2x}+1\right)}$$

input
```
int(1/(a*sinh(x)^2)^(3/2),x)
```

output
```
(sqrt(a)*( - e**(4*x)*log(e**x - 1) + e**(4*x)*log(e**x + 1) - 2*e**(3*x)
+ 2*e**(2*x)*log(e**x - 1) - 2*e**(2*x)*log(e**x + 1) - 2*e**x - log(e**x
- 1) + log(e**x + 1)))/(2*a**2*(e**(4*x) - 2*e**(2*x) + 1))
```

3.145 $\int \frac{1}{\left(a\sinh^2(x)\right)^{5/2}}\,dx$

Optimal result

Integrand size = 10, antiderivative size = 61

$$\int \frac{1}{\left(a\sinh^2(x)\right)^{5/2}}\,dx = -\frac{\coth(x)}{4a\left(a\sinh^2(x)\right)^{3/2}} + \frac{3\coth(x)}{8a^2\sqrt{a\sinh^2(x)}} - \frac{3\text{arctanh}(\cosh(x))\sinh(x)}{8a^2\sqrt{a\sinh^2(x)}}$$

output
```
-1/4*coth(x)/a/(a*sinh(x)^2)^(3/2)+3/8*coth(x)/a^2/(a*sinh(x)^2)^(1/2)-3/8
*arctanh(cosh(x))*sinh(x)/a^2/(a*sinh(x)^2)^(1/2)
```

Mathematica [A] (verified)

Time = 0.16 (sec) , antiderivative size = 77, normalized size of antiderivative = 1.26

$$\int \frac{1}{\left(a\sinh^2(x)\right)^{5/2}}\,dx = -\frac{\text{csch}(x)\left(-6\text{csch}^2\left(\frac{x}{2}\right)+\text{csch}^4\left(\frac{x}{2}\right)+24\left(\log\left(\cosh\left(\frac{x}{2}\right)\right)-\log\left(\sinh\left(\frac{x}{2}\right)\right)\right)-6\text{sech}^2\left(\frac{x}{2}\right)-\text{sech}^4\left(\frac{x}{2}\right)\right)\sqrt{a\sin}}{64a^3}$$

input
```
Integrate[(a*Sinh[x]^2)^(-5/2),x]
```

output
```
-1/64*(Csch[x]*(-6*Csch[x/2]^2 + Csch[x/2]^4 + 24*(Log[Cosh[x/2]] - Log[Si
nh[x/2]]) - 6*Sech[x/2]^2 - Sech[x/2]^4)*Sqrt[a*Sinh[x]^2])/a^3
```

Rubi [A] (verified)

Time = 0.41 (sec) , antiderivative size = 69, normalized size of antiderivative = 1.13, number of steps used = 9, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.900$, Rules used = {3042, 3683, 3042, 3683, 3042, 3686, 3042, 26, 4257}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\left(a\sinh^2(x)\right)^{5/2}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{\left(-a\sin(ix)^2\right)^{5/2}}dx$$

$$\Big\downarrow \text{3683}$$

$$-\frac{3\int \frac{1}{\left(a\sinh^2(x)\right)^{3/2}}dx}{4a}-\frac{\coth(x)}{4a\left(a\sinh^2(x)\right)^{3/2}}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{\coth(x)}{4a\left(a\sinh^2(x)\right)^{3/2}}-\frac{3\int \frac{1}{\left(-a\sin(ix)^2\right)^{3/2}}dx}{4a}$$

$$\Big\downarrow \text{3683}$$

$$-\frac{3\left(-\frac{\int \frac{1}{\sqrt{a\sinh^2(x)}}dx}{2a}-\frac{\coth(x)}{2a\sqrt{a\sinh^2(x)}}\right)}{4a}-\frac{\coth(x)}{4a\left(a\sinh^2(x)\right)^{3/2}}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{\coth(x)}{4a\left(a\sinh^2(x)\right)^{3/2}}-\frac{3\left(-\frac{\coth(x)}{2a\sqrt{a\sinh^2(x)}}-\frac{\int \frac{1}{\sqrt{-a\sin(ix)^2}}dx}{2a}\right)}{4a}$$

$$\Big\downarrow \text{3686}$$

$$-\frac{3\left(-\frac{\sinh(x)\int \operatorname{csch}(x)dx}{2a\sqrt{a\sinh^2(x)}}-\frac{\coth(x)}{2a\sqrt{a\sinh^2(x)}}\right)}{4a}-\frac{\coth(x)}{4a\left(a\sinh^2(x)\right)^{3/2}}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{\coth(x)}{4a\left(a\sinh^2(x)\right)^{3/2}}-\frac{3\left(-\frac{\coth(x)}{2a\sqrt{a\sinh^2(x)}}-\frac{\sinh(x)\int i\csc(ix)dx}{2a\sqrt{a\sinh^2(x)}}\right)}{4a}$$

$$\Big\downarrow \text{26}$$

$$-\frac{\coth(x)}{4a\left(a\sinh^2(x)\right)^{3/2}}-\frac{3\left(-\frac{\coth(x)}{2a\sqrt{a\sinh^2(x)}}-\frac{i\sinh(x)\int \csc(ix)dx}{2a\sqrt{a\sinh^2(x)}}\right)}{4a}$$

$$\Big\downarrow \text{4257}$$

$$-\frac{3\left(\frac{\sinh(x)\text{arctanh}(\cosh(x))}{2a\sqrt{a\sinh^2(x)}}-\frac{\coth(x)}{2a\sqrt{a\sinh^2(x)}}\right)}{4a}-\frac{\coth(x)}{4a\left(a\sinh^2(x)\right)^{3/2}}$$

input
```
Int[(a*Sinh[x]^2)^(-5/2),x]
```

output
```
-1/4*Coth[x]/(a*(a*Sinh[x]^2)^(3/2)) - (3*(-1/2*Coth[x]/(a*Sqrt[a*Sinh[x]^
2]) + (ArcTanh[Cosh[x]]*Sinh[x])/(2*a*Sqrt[a*Sinh[x]^2])))/(4*a)
```

Defintions of rubi rules used

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3683
```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> Simp[Cot[e + f*x]*
((b*Sin[e + f*x]^2)^(p + 1)/(b*f*(2*p + 1))), x] + Simp[2*((p + 1)/(b*(2*p
+ 1)))   Int[(b*Sin[e + f*x]^2)^(p + 1), x], x] /; FreeQ[{b, e, f}, x] &&
!IntegerQ[p] && LtQ[p, -1]
```

rule 3686
```
Int[(u_.)*((b_.)*sin[(e_.) + (f_.)*(x_)]^(n_))^(p_), x_Symbol] :> With[{ff
= FreeFactors[Sin[e + f*x], x]}, Simp[(b*ff^n)^IntPart[p]*((b*Sin[e + f*x]^
n)^FracPart[p]/(Sin[e + f*x]/ff)^(n*FracPart[p]))   Int[ActivateTrig[u]*(Si
n[e + f*x]/ff)^(n*p), x], x]] /; FreeQ[{b, e, f, n, p}, x] &&  !IntegerQ[p]
 && IntegerQ[n] && (EqQ[u, 1] || MatchQ[u, ((d_.)*(trig_)[e + f*x])^(m_.) /
; FreeQ[{d, m}, x] && MemberQ[{sin, cos, tan, cot, sec, csc}, trig]])
```

rule 4257
```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Maple [A] (verified)

Time = 0.14 (sec) , antiderivative size = 89, normalized size of antiderivative = 1.46

method	result	size
default	$\frac{\sqrt{a\cosh(x)^2}\left(-3\ln\left(\frac{2\sqrt{a}\,\sqrt{a\cosh(x)^2}+2a}{\sinh(x)}\right)a\sinh(x)^4+3\sinh(x)^2\sqrt{a\cosh(x)^2}\,\sqrt{a}-2\sqrt{a}\,\sqrt{a\cosh(x)^2}\right)}{8a^{\frac{7}{2}}\sinh(x)^3\cosh(x)\sqrt{a\sinh(x)^2}}$	89
risch	$\frac{3\,\mathrm{e}^{6x}-11\,\mathrm{e}^{4x}-11\,\mathrm{e}^{2x}+3}{4a^2(\mathrm{e}^{2x}-1)^3\sqrt{a(\mathrm{e}^{2x}-1)^2\mathrm{e}^{-2x}}}-\frac{3(\mathrm{e}^{2x}-1)\mathrm{e}^{-x}\ln(\mathrm{e}^x+1)}{8a^2\sqrt{a(\mathrm{e}^{2x}-1)^2\mathrm{e}^{-2x}}}+\frac{3(\mathrm{e}^{2x}-1)\mathrm{e}^{-x}\ln(\mathrm{e}^x-1)}{8a^2\sqrt{a(\mathrm{e}^{2x}-1)^2\mathrm{e}^{-2x}}}$	123

input
```
int(1/(a*sinh(x)^2)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
1/8*(a*cosh(x)^2)^(1/2)*(-3*ln(2*(a^(1/2)*(a*cosh(x)^2)^(1/2)+a)/sinh(x))*
a*sinh(x)^4+3*sinh(x)^2*(a*cosh(x)^2)^(1/2)*a^(1/2)-2*a^(1/2)*(a*cosh(x)^2
)^(1/2))/a^(7/2)/sinh(x)^3/cosh(x)/(a*sinh(x)^2)^(1/2)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 875 vs. $2(49) = 98$.

Time = 0.10 (sec) , antiderivative size = 875, normalized size of antiderivative = 14.34

$$\int \frac{1}{\left(a\sinh^2(x)\right)^{5/2}}\,dx = \text{Too large to display}$$

input
```
integrate(1/(a*sinh(x)^2)^(5/2),x, algorithm="fricas")
```

output
```
-1/8*(42*cosh(x)*e^x*sinh(x)^6 + 6*e^x*sinh(x)^7 + 2*(63*cosh(x)^2 - 11)*e
^x*sinh(x)^5 + 10*(21*cosh(x)^3 - 11*cosh(x))*e^x*sinh(x)^4 + 2*(105*cosh(
x)^4 - 110*cosh(x)^2 - 11)*e^x*sinh(x)^3 + 2*(63*cosh(x)^5 - 110*cosh(x)^3
 - 33*cosh(x))*e^x*sinh(x)^2 + 2*(21*cosh(x)^6 - 55*cosh(x)^4 - 33*cosh(x)
^2 + 3)*e^x*sinh(x) + 2*(3*cosh(x)^7 - 11*cosh(x)^5 - 11*cosh(x)^3 + 3*cos
h(x))*e^x + 3*(8*cosh(x)*e^x*sinh(x)^7 + e^x*sinh(x)^8 + 4*(7*cosh(x)^2 -
1)*e^x*sinh(x)^6 + 8*(7*cosh(x)^3 - 3*cosh(x))*e^x*sinh(x)^5 + 2*(35*cosh(
x)^4 - 30*cosh(x)^2 + 3)*e^x*sinh(x)^4 + 8*(7*cosh(x)^5 - 10*cosh(x)^3 + 3
*cosh(x))*e^x*sinh(x)^3 + 4*(7*cosh(x)^6 - 15*cosh(x)^4 + 9*cosh(x)^2 - 1)
*e^x*sinh(x)^2 + 8*(cosh(x)^7 - 3*cosh(x)^5 + 3*cosh(x)^3 - cosh(x))*e^x*s
inh(x) + (cosh(x)^8 - 4*cosh(x)^6 + 6*cosh(x)^4 - 4*cosh(x)^2 + 1)*e^x)*lo
g((cosh(x) + sinh(x) - 1)/(cosh(x) + sinh(x) + 1)))*sqrt(a*e^(4*x) - 2*a*e
^(2*x) + a)*e^(-x)/(a^3*cosh(x)^8 - 4*a^3*cosh(x)^6 - (a^3*e^(2*x) - a^3)*
sinh(x)^8 - 8*(a^3*cosh(x)*e^(2*x) - a^3*cosh(x))*sinh(x)^7 + 6*a^3*cosh(x
)^4 + 4*(7*a^3*cosh(x)^2 - a^3 - (7*a^3*cosh(x)^2 - a^3)*e^(2*x))*sinh(x)^
6 + 8*(7*a^3*cosh(x)^3 - 3*a^3*cosh(x) - (7*a^3*cosh(x)^3 - 3*a^3*cosh(x))
*e^(2*x))*sinh(x)^5 - 4*a^3*cosh(x)^2 + 2*(35*a^3*cosh(x)^4 - 30*a^3*cosh(
x)^2 + 3*a^3 - (35*a^3*cosh(x)^4 - 30*a^3*cosh(x)^2 + 3*a^3)*e^(2*x))*sinh
(x)^4 + 8*(7*a^3*cosh(x)^5 - 10*a^3*cosh(x)^3 + 3*a^3*cosh(x) - (7*a^3*cos
h(x)^5 - 10*a^3*cosh(x)^3 + 3*a^3*cosh(x))*e^(2*x))*sinh(x)^3 + a^3 + 4...
```

Sympy [F]

$$\int \frac{1}{\left(a\sinh^2(x)\right)^{5/2}}\,dx = \int \frac{1}{\left(a\sinh^2{(x)}\right)^{\frac{5}{2}}}\,dx$$

input
```
integrate(1/(a*sinh(x)**2)**(5/2),x)
```

output
```
Integral((a*sinh(x)**2)**(-5/2), x)
```

Maxima [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 96, normalized size of antiderivative = 1.57

$$\int \frac{1}{\left(a\sinh^2(x)\right)^{5/2}}\,dx = \frac{3\,e^{(-x)} - 11\,e^{(-3\,x)} - 11\,e^{(-5\,x)} + 3\,e^{(-7\,x)}}{4\left(4\,a^{\frac{5}{2}}e^{(-2\,x)} - 6\,a^{\frac{5}{2}}e^{(-4\,x)} + 4\,a^{\frac{5}{2}}e^{(-6\,x)} - a^{\frac{5}{2}}e^{(-8\,x)} - a^{\frac{5}{2}}\right)} + \frac{3\,\log\left(e^{(-x)} + 1\right)}{8\,a^{\frac{5}{2}}} - \frac{3\,\log\left(e^{(-x)} - 1\right)}{8\,a^{\frac{5}{2}}}$$

input
```
integrate(1/(a*sinh(x)^2)^(5/2),x, algorithm="maxima")
```

output
```
1/4*(3*e^(-x) - 11*e^(-3*x) - 11*e^(-5*x) + 3*e^(-7*x))/(4*a^(5/2)*e^(-2*x
) - 6*a^(5/2)*e^(-4*x) + 4*a^(5/2)*e^(-6*x) - a^(5/2)*e^(-8*x) - a^(5/2))
+ 3/8*log(e^(-x) + 1)/a^(5/2) - 3/8*log(e^(-x) - 1)/a^(5/2)
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 52, normalized size of antiderivative = 0.85

$$\int \frac{1}{\left(a\sinh^2(x)\right)^{5/2}}\,dx = \frac{3\left(e^{(-x)} + e^{x}\right)^3 - 20\,e^{(-x)} - 20\,e^{x}}{4\left(\left(e^{(-x)} + e^{x}\right)^2 - 4\right)^2 a^{\frac{5}{2}}\operatorname{sgn}\left(e^{(3\,x)} - e^{x}\right)}$$

input
```
integrate(1/(a*sinh(x)^2)^(5/2),x, algorithm="giac")
```

output
```
1/4*(3*(e^(-x) + e^x)^3 - 20*e^(-x) - 20*e^x)/(((e^(-x) + e^x)^2 - 4)^2*a^
(5/2)*sgn(e^(3*x) - e^x))
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\left(a\sinh^2(x)\right)^{5/2}}\,dx = \int \frac{1}{\left(a\sinh(x)^2\right)^{5/2}}\,dx$$

input
```
int(1/(a*sinh(x)^2)^(5/2),x)
```

output
```
int(1/(a*sinh(x)^2)^(5/2), x)
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 184, normalized size of antiderivative = 3.02

$$\int \frac{1}{\left(a\sinh^2(x)\right)^{5/2}}\,dx = \frac{\sqrt{a}\,(3e^{8x}\log(e^x-1) - 3e^{8x}\log(e^x+1) + 6e^{7x} - 12e^{6x}\log(e^x-1) + 12e^{6x}\log(e^x+}{}$$

input
```
int(1/(a*sinh(x)^2)^(5/2),x)
```

output
```
(sqrt(a)*(3*e**(8*x)*log(e**x - 1) - 3*e**(8*x)*log(e**x + 1) + 6*e**(7*x)
 - 12*e**(6*x)*log(e**x - 1) + 12*e**(6*x)*log(e**x + 1) - 22*e**(5*x) + 1
8*e**(4*x)*log(e**x - 1) - 18*e**(4*x)*log(e**x + 1) - 22*e**(3*x) - 12*e*
*(2*x)*log(e**x - 1) + 12*e**(2*x)*log(e**x + 1) + 6*e**x + 3*log(e**x - 1
) - 3*log(e**x + 1)))/(8*a**3*(e**(8*x) - 4*e**(6*x) + 6*e**(4*x) - 4*e**(
2*x) + 1))
```

3.146 $\int \left(a\sinh^3(x)\right)^{5/2}\,dx$

Optimal result

Integrand size = 10, antiderivative size = 135

$$\begin{aligned}\int \left(a\sinh^3(x)\right)^{5/2}\,dx = &-\frac{26}{77}a^2\coth(x)\sqrt{a\sinh^3(x)}\\ &+\frac{26}{77}ia^2\operatorname{csch}^2(x)\operatorname{EllipticF}\left(\frac{\pi}{4}-\frac{ix}{2},2\right)\sqrt{i\sinh(x)}\sqrt{a\sinh^3(x)}\\ &+\frac{78}{385}a^2\cosh(x)\sinh(x)\sqrt{a\sinh^3(x)}-\frac{26}{165}a^2\cosh(x)\sinh^3(x)\sqrt{a\sinh^3(x)}\\ &+\frac{2}{15}a^2\cosh(x)\sinh^5(x)\sqrt{a\sinh^3(x)}\end{aligned}$$

output
```
-26/77*a^2*coth(x)*(a*sinh(x)^3)^(1/2)-26/77*I*a^2*csch(x)^2*InverseJacobi
AM(-1/4*Pi+1/2*I*x,2^(1/2))*(I*sinh(x))^(1/2)*(a*sinh(x)^3)^(1/2)+78/385*a
^2*cosh(x)*sinh(x)*(a*sinh(x)^3)^(1/2)-26/165*a^2*cosh(x)*sinh(x)^3*(a*sin
h(x)^3)^(1/2)+2/15*a^2*cosh(x)*sinh(x)^5*(a*sinh(x)^3)^(1/2)
```

Mathematica [A] (verified)

Time = 0.15 (sec) , antiderivative size = 67, normalized size of antiderivative = 0.50

$$\int \left(a\sinh^3(x)\right)^{5/2} dx = \frac{a^2\text{csch}(x)\left(-15465\cosh(x)+3657\cosh(3x)-749\cosh(5x)+77\cosh(7x)-\frac{12480}{}\right.}{36960}$$

input
```
Integrate[(a*Sinh[x]^3)^(5/2),x]
```

output
```
(a^2*Csch[x]*(-15465*Cosh[x] + 3657*Cosh[3*x] - 749*Cosh[5*x] + 77*Cosh[7*
x] - (12480*EllipticF[(Pi - (2*I)*x)/4, 2])/Sqrt[I*Sinh[x]])*Sqrt[a*Sinh[x
]^3])/36960
```

Rubi [A] (verified)

Time = 0.59 (sec) , antiderivative size = 121, normalized size of antiderivative = 0.90, number of steps used = 14, number of rules used = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 1.400$, Rules used = {3042, 3686, 3042, 3115, 3042, 3115, 3042, 3115, 3042, 3115, 3042, 3121, 3042, 3120}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(a\sinh^3(x)\right)^{5/2} dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \left(ia\sin(ix)^3\right)^{5/2} dx$$

$$\Big\downarrow \text{ 3686}$$

$$\frac{a^2\sqrt{a\sinh^3(x)}\int \sinh^{\frac{15}{2}}(x)dx}{\sinh^{\frac{3}{2}}(x)}$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{a^2\sqrt{a\sinh^3(x)}\int(-i\sin(ix))^{15/2}dx}{\sinh^{\frac{3}{2}}(x)}$$

$$\Big\downarrow \text{3115}$$

$$\frac{a^2\sqrt{a\sinh^3(x)}\left(\frac{2}{15}\sinh^{\frac{13}{2}}(x)\cosh(x)-\frac{13}{15}\int\sinh^{\frac{11}{2}}(x)dx\right)}{\sinh^{\frac{3}{2}}(x)}$$

$$\Big\downarrow \text{3042}$$

$$\frac{a^2\sqrt{a\sinh^3(x)}\left(\frac{2}{15}\sinh^{\frac{13}{2}}(x)\cosh(x)-\frac{13}{15}\int(-i\sin(ix))^{11/2}dx\right)}{\sinh^{\frac{3}{2}}(x)}$$

$$\Big\downarrow \text{3115}$$

$$\frac{a^2\sqrt{a\sinh^3(x)}\left(\frac{2}{15}\sinh^{\frac{13}{2}}(x)\cosh(x)-\frac{13}{15}\left(\frac{2}{11}\sinh^{\frac{9}{2}}(x)\cosh(x)-\frac{9}{11}\int\sinh^{\frac{7}{2}}(x)dx\right)\right)}{\sinh^{\frac{3}{2}}(x)}$$

$$\Big\downarrow \text{3042}$$

$$\frac{a^2\sqrt{a\sinh^3(x)}\left(\frac{2}{15}\sinh^{\frac{13}{2}}(x)\cosh(x)-\frac{13}{15}\left(\frac{2}{11}\sinh^{\frac{9}{2}}(x)\cosh(x)-\frac{9}{11}\int(-i\sin(ix))^{7/2}dx\right)\right)}{\sinh^{\frac{3}{2}}(x)}$$

$$\Big\downarrow \text{3115}$$

$$\frac{a^2\sqrt{a\sinh^3(x)}\left(\frac{2}{15}\sinh^{\frac{13}{2}}(x)\cosh(x)-\frac{13}{15}\left(\frac{2}{11}\sinh^{\frac{9}{2}}(x)\cosh(x)-\frac{9}{11}\left(\frac{2}{7}\sinh^{\frac{5}{2}}(x)\cosh(x)-\frac{5}{7}\int\sinh^{\frac{3}{2}}(x)dx\right)\right)\right)}{\sinh^{\frac{3}{2}}(x)}$$

$$\Big\downarrow \text{3042}$$

$$\frac{a^2\sqrt{a\sinh^3(x)}\left(\frac{2}{15}\sinh^{\frac{13}{2}}(x)\cosh(x)-\frac{13}{15}\left(\frac{2}{11}\sinh^{\frac{9}{2}}(x)\cosh(x)-\frac{9}{11}\left(\frac{2}{7}\sinh^{\frac{5}{2}}(x)\cosh(x)-\frac{5}{7}\int(-i\sin(ix))^{3/2}dx\right.\right.\right.}{\sinh^{\frac{3}{2}}(x)}$$

$$\Big\downarrow \text{3115}$$

$$\frac{a^2\sqrt{a\sinh^3(x)}\left(\frac{2}{15}\sinh^{\frac{13}{2}}(x)\cosh(x)-\frac{13}{15}\left(\frac{2}{11}\sinh^{\frac{9}{2}}(x)\cosh(x)-\frac{9}{11}\left(\frac{2}{7}\sinh^{\frac{5}{2}}(x)\cosh(x)-\frac{5}{7}\left(\frac{2}{3}\sqrt{\sinh(x)}\cosh(x\right.\right.\right.\right.}{\sinh^{\frac{3}{2}}(x)}$$

$$\Big\downarrow \text{3042}$$

$$\frac{a^2\sqrt{a\sinh^3(x)}\Big(\frac{2}{15}\sinh^{\frac{13}{2}}(x)\cosh(x)-\frac{13}{15}\Big(\frac{2}{11}\sinh^{\frac{9}{2}}(x)\cosh(x)-\frac{9}{11}\Big(\frac{2}{7}\sinh^{\frac{5}{2}}(x)\cosh(x)-\frac{5}{7}\Big(\frac{2}{3}\sqrt{\sinh(x)}\cosh(x}{\sinh^{\frac{3}{2}}(x)}$$

$$\Big\downarrow \; 3121$$

$$\frac{a^2\sqrt{a\sinh^3(x)}\Bigg(\frac{2}{15}\sinh^{\frac{13}{2}}(x)\cosh(x)-\frac{13}{15}\Bigg(\frac{2}{11}\sinh^{\frac{9}{2}}(x)\cosh(x)-\frac{9}{11}\Bigg(\frac{2}{7}\sinh^{\frac{5}{2}}(x)\cosh(x)-\frac{5}{7}\Bigg(\frac{2}{3}\sqrt{\sinh(x)}\cosh(}{\sinh^{\frac{3}{2}}(x)}$$

$$\Big\downarrow \; 3042$$

$$\frac{a^2\sqrt{a\sinh^3(x)}\Bigg(\frac{2}{15}\sinh^{\frac{13}{2}}(x)\cosh(x)-\frac{13}{15}\Bigg(\frac{2}{11}\sinh^{\frac{9}{2}}(x)\cosh(x)-\frac{9}{11}\Bigg(\frac{2}{7}\sinh^{\frac{5}{2}}(x)\cosh(x)-\frac{5}{7}\Bigg(\frac{2}{3}\sqrt{\sinh(x)}\cosh(}{\sinh^{\frac{3}{2}}(x)}$$

$$\Big\downarrow \; 3120$$

$$\frac{a^2\sqrt{a\sinh^3(x)}\Bigg(\frac{2}{15}\sinh^{\frac{13}{2}}(x)\cosh(x)-\frac{13}{15}\Bigg(\frac{2}{11}\sinh^{\frac{9}{2}}(x)\cosh(x)-\frac{9}{11}\Bigg(\frac{2}{7}\sinh^{\frac{5}{2}}(x)\cosh(x)-\frac{5}{7}\Bigg(\frac{2}{3}\sqrt{\sinh(x)}\cosh(}{\sinh^{\frac{3}{2}}(x)}$$

input
```
Int[(a*Sinh[x]^3)^(5/2),x]
```

output
```
(a^2*Sqrt[a*Sinh[x]^3]*((2*Cosh[x]*Sinh[x]^(13/2))/15 - (13*((2*Cosh[x]*Si
nh[x]^(9/2))/11 - (9*((-5*((((-2*I)/3)*EllipticF[Pi/4 - (I/2)*x, 2]*Sqrt[I
*Sinh[x]])/Sqrt[Sinh[x]] + (2*Cosh[x]*Sqrt[Sinh[x]])/3))/7 + (2*Cosh[x]*Si
nh[x]^(5/2))/7))/11))/15))/Sinh[x]^(3/2)
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3115
```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Simp[b^2*((n - 1)/n)   Int[(b*Sin
[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[
2*n]
```

rule 3120
```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3121
```
Int[((b_)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*Sin[c + d*x])
^n/Sin[c + d*x]^n   Int[Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] && Lt
Q[-1, n, 1] && IntegerQ[2*n]
```

rule 3686
```
Int[(u_.)*((b_.)*sin[(e_.) + (f_.)*(x_)]^(n_))^(p_), x_Symbol] :> With[{ff
= FreeFactors[Sin[e + f*x], x]}, Simp[(b*ff^n)^IntPart[p]*((b*Sin[e + f*x]^
n)^FracPart[p]/(Sin[e + f*x]/ff)^(n*FracPart[p]))   Int[ActivateTrig[u]*(Si
n[e + f*x]/ff)^(n*p), x], x]] /; FreeQ[{b, e, f, n, p}, x] &&  !IntegerQ[p]
 && IntegerQ[n] && (EqQ[u, 1] || MatchQ[u, ((d_.)*(trig_)[e + f*x])^(m_.) /
; FreeQ[{d, m}, x] && MemberQ[{sin, cos, tan, cot, sec, csc}, trig]])
```

Maple [F]

$$\int \left(a \sinh \left(x\right)^{3}\right)^{\frac{5}{2}}dx$$

input
```
int((a*sinh(x)^3)^(5/2),x)
```

output
```
int((a*sinh(x)^3)^(5/2),x)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 802 vs. $2(105) = 210$.

Time = 0.11 (sec) , antiderivative size = 802, normalized size of antiderivative = 5.94

$$\int \left(a\sinh^3(x)\right)^{5/2} dx = \text{Too large to display}$$

input
```
integrate((a*sinh(x)^3)^(5/2),x, algorithm="fricas")
```

output
```
1/73920*(49920*sqrt(1/2)*(a^2*cosh(x)^7 + 7*a^2*cosh(x)^6*sinh(x) + 21*a^2
*cosh(x)^5*sinh(x)^2 + 35*a^2*cosh(x)^4*sinh(x)^3 + 35*a^2*cosh(x)^3*sinh(
x)^4 + 21*a^2*cosh(x)^2*sinh(x)^5 + 7*a^2*cosh(x)*sinh(x)^6 + a^2*sinh(x)^
7)*sqrt(a)*weierstrassPInverse(4, 0, cosh(x) + sinh(x)) + (77*a^2*cosh(x)^
14 + 1078*a^2*cosh(x)*sinh(x)^13 + 77*a^2*sinh(x)^14 - 749*a^2*cosh(x)^12
+ 7*(1001*a^2*cosh(x)^2 - 107*a^2)*sinh(x)^12 + 3657*a^2*cosh(x)^10 + 28*(
1001*a^2*cosh(x)^3 - 321*a^2*cosh(x))*sinh(x)^11 + (77077*a^2*cosh(x)^4 -
49434*a^2*cosh(x)^2 + 3657*a^2)*sinh(x)^10 - 15465*a^2*cosh(x)^8 + 2*(7707
7*a^2*cosh(x)^5 - 82390*a^2*cosh(x)^3 + 18285*a^2*cosh(x))*sinh(x)^9 + 3*(
77077*a^2*cosh(x)^6 - 123585*a^2*cosh(x)^4 + 54855*a^2*cosh(x)^2 - 5155*a^
2)*sinh(x)^8 - 15465*a^2*cosh(x)^6 + 24*(11011*a^2*cosh(x)^7 - 24717*a^2*c
osh(x)^5 + 18285*a^2*cosh(x)^3 - 5155*a^2*cosh(x))*sinh(x)^7 + 3*(77077*a^
2*cosh(x)^8 - 230692*a^2*cosh(x)^6 + 255990*a^2*cosh(x)^4 - 144340*a^2*cos
h(x)^2 - 5155*a^2)*sinh(x)^6 + 3657*a^2*cosh(x)^4 + 2*(77077*a^2*cosh(x)^9
 - 296604*a^2*cosh(x)^7 + 460782*a^2*cosh(x)^5 - 433020*a^2*cosh(x)^3 - 46
395*a^2*cosh(x))*sinh(x)^5 + (77077*a^2*cosh(x)^10 - 370755*a^2*cosh(x)^8
+ 767970*a^2*cosh(x)^6 - 1082550*a^2*cosh(x)^4 - 231975*a^2*cosh(x)^2 + 36
57*a^2)*sinh(x)^4 - 749*a^2*cosh(x)^2 + 4*(7007*a^2*cosh(x)^11 - 41195*a^2
*cosh(x)^9 + 109710*a^2*cosh(x)^7 - 216510*a^2*cosh(x)^5 - 77325*a^2*cosh(
x)^3 + 3657*a^2*cosh(x))*sinh(x)^3 + (7007*a^2*cosh(x)^12 - 49434*a^2*c...
```

Sympy [F]

$$\int \left(a\sinh^3(x)\right)^{5/2} dx = \int \left(a\sinh^3{(x)}\right)^{\frac{5}{2}} dx$$

input
```
integrate((a*sinh(x)**3)**(5/2),x)
```

output
```
Integral((a*sinh(x)**3)**(5/2), x)
```

Maxima [F]

$$\int \left(a\sinh^3(x)\right)^{5/2} dx = \int \left(a\sinh{(x)}^3\right)^{\frac{5}{2}} dx$$

input
```
integrate((a*sinh(x)^3)^(5/2),x, algorithm="maxima")
```

output
```
integrate((a*sinh(x)^3)^(5/2), x)
```

Giac [F]

$$\int \left(a\sinh^3(x)\right)^{5/2} dx = \int \left(a\sinh{(x)}^3\right)^{\frac{5}{2}} dx$$

input
```
integrate((a*sinh(x)^3)^(5/2),x, algorithm="giac")
```

output
```
integrate((a*sinh(x)^3)^(5/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \left(a\sinh^3(x)\right)^{5/2}\,dx=\int \left(a\,\sinh(x)^3\right)^{5/2}\,dx$$

input
```
int((a*sinh(x)^3)^(5/2),x)
```

output
```
int((a*sinh(x)^3)^(5/2), x)
```

Reduce [F]

$$\int \left(a\sinh^3(x)\right)^{5/2}\,dx=\sqrt{a}\left(\int \sqrt{\sinh\left(x\right)}\,\sinh\left(x\right)^7dx\right)a^2$$

input
```
int((a*sinh(x)^3)^(5/2),x)
```

output
```
sqrt(a)*int(sqrt(sinh(x))*sinh(x)**7,x)*a**2
```

3.147 $\int \left(a\sinh^3(x)\right)^{3/2}\,dx$

Optimal result

Integrand size = 10, antiderivative size = 83

$$\int \left(a\sinh^3(x)\right)^{3/2}\,dx=-\frac{14}{45}a\cosh(x)\sqrt{a\sinh^3(x)}+\frac{14iacsch(x)E\left(\frac{\pi}{4}-\frac{ix}{2}\middle|2\right)\sqrt{a\sinh^3(x)}}{15\sqrt{i\sinh(x)}}+\frac{2}{9}a\cosh(x)\sinh^2(x)\sqrt{a\sinh^3(x)}$$

output
```
-14/45*a*cosh(x)*(a*sinh(x)^3)^(1/2)+14/15*I*a*csch(x)*EllipticE(cos(1/4*P
i+1/2*I*x),2^(1/2))*(a*sinh(x)^3)^(1/2)/(I*sinh(x))^(1/2)+2/9*a*cosh(x)*si
nh(x)^2*(a*sinh(x)^3)^(1/2)
```

Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.69

$$\int \left(a\sinh^3(x)\right)^{3/2}\,dx=\frac{1}{180}acsch(x)\sqrt{a\sinh^3(x)}\left(168csch(x)E\left(\frac{1}{4}(\pi-2ix)\middle|2\right)\sqrt{i\sinh(x)}-38\sinh(2x)+5\sinh(4x)\right)$$

input
```
Integrate[(a*Sinh[x]^3)^(3/2),x]
```

output
```
(a*Csch[x]*Sqrt[a*Sinh[x]^3]*(168*Csch[x]*EllipticE[(Pi - (2*I)*x)/4, 2]*S
qrt[I*Sinh[x]] - 38*Sinh[2*x] + 5*Sinh[4*x]))/180
```

Rubi [A] (verified)

Time = 0.42 (sec) , antiderivative size = 85, normalized size of antiderivative = 1.02, number of steps used = 10, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 1.000$, Rules used = {3042, 3686, 3042, 3115, 3042, 3115, 3042, 3121, 3042, 3119}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(a\sinh^3(x)\right)^{3/2}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \left(ia\sin(ix)^3\right)^{3/2}dx$$

$$\Big\downarrow \text{3686}$$

$$\frac{a\sqrt{a\sinh^3(x)}\int \sinh^{\frac{9}{2}}(x)dx}{\sinh^{\frac{3}{2}}(x)}$$

$$\Big\downarrow \text{3042}$$

$$\frac{a\sqrt{a\sinh^3(x)}\int (-i\sin(ix))^{9/2}dx}{\sinh^{\frac{3}{2}}(x)}$$

$$\Big\downarrow \text{3115}$$

$$\frac{a\sqrt{a\sinh^3(x)}\left(\frac{2}{9}\sinh^{\frac{7}{2}}(x)\cosh(x)-\frac{7}{9}\int \sinh^{\frac{5}{2}}(x)dx\right)}{\sinh^{\frac{3}{2}}(x)}$$

$$\Big\downarrow \text{3042}$$

$$\frac{a\sqrt{a\sinh^3(x)}\left(\frac{2}{9}\sinh^{\frac{7}{2}}(x)\cosh(x)-\frac{7}{9}\int (-i\sin(ix))^{5/2}dx\right)}{\sinh^{\frac{3}{2}}(x)}$$

$$\Big\downarrow \text{3115}$$

$$\frac{a\sqrt{a\sinh^3(x)}\left(\frac{2}{9}\sinh^{\frac{7}{2}}(x)\cosh(x)-\frac{7}{9}\left(\frac{2}{5}\sinh^{\frac{3}{2}}(x)\cosh(x)-\frac{3}{5}\int\sqrt{\sinh(x)}dx\right)\right)}{\sinh^{\frac{3}{2}}(x)}$$

$$\Big\downarrow \text{3042}$$

$$\frac{a\sqrt{a\sinh^3(x)}\left(\frac{2}{9}\sinh^{\frac{7}{2}}(x)\cosh(x)-\frac{7}{9}\left(\frac{2}{5}\sinh^{\frac{3}{2}}(x)\cosh(x)-\frac{3}{5}\int\sqrt{-i\sin(ix)}dx\right)\right)}{\sinh^{\frac{3}{2}}(x)}$$

$$\Big\downarrow \text{3121}$$

$$\frac{a\sqrt{a\sinh^3(x)}\left(\frac{2}{9}\sinh^{\frac{7}{2}}(x)\cosh(x)-\frac{7}{9}\left(\frac{2}{5}\sinh^{\frac{3}{2}}(x)\cosh(x)-\frac{3\sqrt{\sinh(x)}\int\sqrt{i\sinh(x)}dx}{5\sqrt{i\sinh(x)}}\right)\right)}{\sinh^{\frac{3}{2}}(x)}$$

$$\Big\downarrow \text{3042}$$

$$\frac{a\sqrt{a\sinh^3(x)}\left(\frac{2}{9}\sinh^{\frac{7}{2}}(x)\cosh(x)-\frac{7}{9}\left(\frac{2}{5}\sinh^{\frac{3}{2}}(x)\cosh(x)-\frac{3\sqrt{\sinh(x)}\int\sqrt{\sin(ix)}dx}{5\sqrt{i\sinh(x)}}\right)\right)}{\sinh^{\frac{3}{2}}(x)}$$

$$\Big\downarrow \text{3119}$$

$$\frac{a\sqrt{a\sinh^3(x)}\left(\frac{2}{9}\sinh^{\frac{7}{2}}(x)\cosh(x)-\frac{7}{9}\left(\frac{2}{5}\sinh^{\frac{3}{2}}(x)\cosh(x)-\frac{6i\sqrt{\sinh(x)}E\left(\frac{\pi}{4}-\frac{ix}{2}|2\right)}{5\sqrt{i\sinh(x)}}\right)\right)}{\sinh^{\frac{3}{2}}(x)}$$

input
```
Int[(a*Sinh[x]^3)^(3/2),x]
```

output
```
(a*Sqrt[a*Sinh[x]^3]*((2*Cosh[x]*Sinh[x]^(7/2))/9 - (7*((((-6*I)/5)*Ellipt
icE[Pi/4 - (I/2)*x, 2]*Sqrt[Sinh[x]])/Sqrt[I*Sinh[x]] + (2*Cosh[x]*Sinh[x]
^(3/2))/5))/9))/Sinh[x]^(3/2)
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3115
```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Simp[b^2*((n - 1)/n)   Int[(b*Sin
[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[
2*n]
```

rule 3119
```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3121
```
Int[((b_)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*Sin[c + d*x])
^n/Sin[c + d*x]^n   Int[Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] && Lt
Q[-1, n, 1] && IntegerQ[2*n]
```

rule 3686
```
Int[(u_.)*((b_.)*sin[(e_.) + (f_.)*(x_)]^(n_))^(p_), x_Symbol] :> With[{ff
= FreeFactors[Sin[e + f*x], x]}, Simp[(b*ff^n)^IntPart[p]*((b*Sin[e + f*x]^
n)^FracPart[p]/(Sin[e + f*x]/ff)^(n*FracPart[p]))   Int[ActivateTrig[u]*(Si
n[e + f*x]/ff)^(n*p), x], x]] /; FreeQ[{b, e, f, n, p}, x] &&  !IntegerQ[p]
 && IntegerQ[n] && (EqQ[u, 1] || MatchQ[u, ((d_.)*(trig_)[e + f*x])^(m_.) /
; FreeQ[{d, m}, x] && MemberQ[{sin, cos, tan, cot, sec, csc}, trig]])
```

Maple [F]

$$\int \left(a \sinh \left(x \right)^{3}\right)^{\frac{3}{2}}d x$$

input
```
int((a*sinh(x)^3)^(3/2),x)
```

output
```
int((a*sinh(x)^3)^(3/2),x)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 305 vs. $2(62) = 124$.

Time = 0.10 (sec) , antiderivative size = 305, normalized size of antiderivative = 3.67

$$\int \left(a\sinh^3(x)\right)^{3/2} dx =$$

$$-\frac{672\sqrt{\frac{1}{2}}\left(a\cosh(x)^4+4\,a\cosh(x)^3\sinh(x)+6\,a\cosh(x)^2\sinh(x)^2+4\,a\cosh(x)\sinh(x)^3+a\sinh(x)^4\right.}{}$$

input
```
integrate((a*sinh(x)^3)^(3/2),x, algorithm="fricas")
```

output
```
-1/360*(672*sqrt(1/2)*(a*cosh(x)^4 + 4*a*cosh(x)^3*sinh(x) + 6*a*cosh(x)^2
*sinh(x)^2 + 4*a*cosh(x)*sinh(x)^3 + a*sinh(x)^4)*sqrt(a)*weierstrassZeta(
4, 0, weierstrassPInverse(4, 0, cosh(x) + sinh(x))) - (5*a*cosh(x)^8 + 40*
a*cosh(x)*sinh(x)^7 + 5*a*sinh(x)^8 - 38*a*cosh(x)^6 + 2*(70*a*cosh(x)^2 -
 19*a)*sinh(x)^6 + 4*(70*a*cosh(x)^3 - 57*a*cosh(x))*sinh(x)^5 - 336*a*cos
h(x)^4 + 2*(175*a*cosh(x)^4 - 285*a*cosh(x)^2 - 168*a)*sinh(x)^4 + 8*(35*a
*cosh(x)^5 - 95*a*cosh(x)^3 - 168*a*cosh(x))*sinh(x)^3 + 38*a*cosh(x)^2 +
2*(70*a*cosh(x)^6 - 285*a*cosh(x)^4 - 1008*a*cosh(x)^2 + 19*a)*sinh(x)^2 +
 4*(10*a*cosh(x)^7 - 57*a*cosh(x)^5 - 336*a*cosh(x)^3 + 19*a*cosh(x))*sinh
(x) - 5*a)*sqrt(a*sinh(x)))/(cosh(x)^4 + 4*cosh(x)^3*sinh(x) + 6*cosh(x)^2
*sinh(x)^2 + 4*cosh(x)*sinh(x)^3 + sinh(x)^4)
```

Sympy [F]

$$\int \left(a\sinh^3(x)\right)^{3/2} dx = \int \left(a\sinh^3(x)\right)^{\frac{3}{2}} dx$$

input
```
integrate((a*sinh(x)**3)**(3/2),x)
```

output
```
Integral((a*sinh(x)**3)**(3/2), x)
```

Maxima [F]

$$\int \left(a\sinh^3(x)\right)^{3/2} dx = \int \left(a\sinh\left(x\right)^3\right)^{\frac{3}{2}} dx$$

input
```
integrate((a*sinh(x)^3)^(3/2),x, algorithm="maxima")
```

output
```
integrate((a*sinh(x)^3)^(3/2), x)
```

Giac [F]

$$\int \left(a\sinh^3(x)\right)^{3/2} dx = \int \left(a\sinh\left(x\right)^3\right)^{\frac{3}{2}} dx$$

input
```
integrate((a*sinh(x)^3)^(3/2),x, algorithm="giac")
```

output
```
integrate((a*sinh(x)^3)^(3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \left(a\sinh^3(x)\right)^{3/2} dx = \int \left(a\sinh(x)^3\right)^{3/2} dx$$

input
```
int((a*sinh(x)^3)^(3/2),x)
```

output
```
int((a*sinh(x)^3)^(3/2), x)
```

Reduce [F]

$$\int \left(a\sinh^3(x)\right)^{3/2} dx = \sqrt{a}\left(\int \sqrt{\sinh(x)}\,\sinh(x)^4\,dx\right)a$$

input
```
int((a*sinh(x)^3)^(3/2),x)
```

output
```
sqrt(a)*int(sqrt(sinh(x))*sinh(x)**4,x)*a
```

3.148 $\int \sqrt{a \sinh^3(x)}\,dx$

Optimal result

Integrand size = 10, antiderivative size = 62

$$\int \sqrt{a \sinh^3(x)}\,dx = \frac{2}{3}\coth(x)\sqrt{a \sinh^3(x)} - \frac{2}{3} i \operatorname{csch}^2(x)\,\text{EllipticF}\left(\frac{\pi}{4}-\frac{ix}{2},2\right)\sqrt{i\sinh(x)}\sqrt{a\sinh^3(x)}$$

output
```
2/3*coth(x)*(a*sinh(x)^3)^(1/2)+2/3*I*csch(x)^2*InverseJacobiAM(-1/4*Pi+1/
2*I*x,2^(1/2))*(I*sinh(x))^(1/2)*(a*sinh(x)^3)^(1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 0.08 (sec) , antiderivative size = 60, normalized size of antiderivative = 0.97

$$\int \sqrt{a \sinh^3(x)}\,dx = \frac{2}{3}\sqrt{a\sinh^3(x)}\Big(\coth(x) - \sqrt{2}\operatorname{csch}^2(x)\,\text{Hypergeometric2F1}\left(\frac{1}{4},\frac{1}{2},\frac{5}{4},\cosh(2x)+\sinh(2x)\right)\sqrt{-\sinh(x)(\cosh(x)+\sinh(x))}\Big)$$

input
```
Integrate[Sqrt[a*Sinh[x]^3],x]
```

output
```
(2*Sqrt[a*Sinh[x]^3]*(Coth[x] - Sqrt[2]*Csch[x]^2*Hypergeometric2F1[1/4, 1
/2, 5/4, Cosh[2*x] + Sinh[2*x]]*Sqrt[-(Sinh[x]*(Cosh[x] + Sinh[x]))]))/3
```

Rubi [A] (verified)

Time = 0.35 (sec) , antiderivative size = 67, normalized size of antiderivative = 1.08, number of steps used = 8, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.800$, Rules used = {3042, 3686, 3042, 3115, 3042, 3121, 3042, 3120}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{a \sinh^3(x)} \, dx$$

$$\Big\downarrow \text{3042}$$

$$\int \sqrt{ia \sin(ix)^3}dx$$

$$\Big\downarrow \text{3686}$$

$$\frac{\sqrt{a \sinh^3(x)} \int \sinh^{\frac{3}{2}}(x)dx}{\sinh^{\frac{3}{2}}(x)}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\sqrt{a \sinh^3(x)} \int (-i \sin(ix))^{3/2}dx}{\sinh^{\frac{3}{2}}(x)}$$

$$\Big\downarrow \text{3115}$$

$$\frac{\sqrt{a \sinh^3(x)} \left(\frac{2}{3}\sqrt{\sinh(x)}\cosh(x) - \frac{1}{3}\int \frac{1}{\sqrt{\sinh(x)}}dx\right)}{\sinh^{\frac{3}{2}}(x)}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\sqrt{a \sinh^3(x)} \left(\frac{2}{3}\sqrt{\sinh(x)}\cosh(x) - \frac{1}{3}\int \frac{1}{\sqrt{-i \sin(ix)}}dx\right)}{\sinh^{\frac{3}{2}}(x)}$$

$$\Big\downarrow \text{3121}$$

$$\frac{\sqrt{a\sinh^3(x)}\left(\frac{2}{3}\sqrt{\sinh(x)}\cosh(x)-\frac{\sqrt{i\sinh(x)}\int\frac{1}{\sqrt{i\sinh(x)}}dx}{3\sqrt{\sinh(x)}}\right)}{\sinh^{\frac{3}{2}}(x)}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\sqrt{a\sinh^3(x)}\left(\frac{2}{3}\sqrt{\sinh(x)}\cosh(x)-\frac{\sqrt{i\sinh(x)}\int\frac{1}{\sqrt{\sin(ix)}}dx}{3\sqrt{\sinh(x)}}\right)}{\sinh^{\frac{3}{2}}(x)}$$

$$\Big\downarrow \text{3120}$$

$$\frac{\sqrt{a\sinh^3(x)}\left(\frac{2}{3}\sqrt{\sinh(x)}\cosh(x)-\frac{2i\sqrt{i\sinh(x)}\,\text{EllipticF}\left(\frac{\pi}{4}-\frac{ix}{2},2\right)}{3\sqrt{\sinh(x)}}\right)}{\sinh^{\frac{3}{2}}(x)}$$

input

```
Int[Sqrt[a*Sinh[x]^3],x]
```

output

```
(((((-2*I)/3)*EllipticF[Pi/4 - (I/2)*x, 2]*Sqrt[I*Sinh[x]])/Sqrt[Sinh[x]]
+ (2*Cosh[x]*Sqrt[Sinh[x]])/3)*Sqrt[a*Sinh[x]^3])/Sinh[x]^(3/2)
```

Defintions of rubi rules used

rule 3042

```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3115

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Simp[b^2*((n - 1)/n)   Int[(b*Sin
[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[
2*n]
```

rule 3120

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3121
```
Int[((b_)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*Sin[c + d*x])
^n/Sin[c + d*x]^n   Int[Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] && Lt
Q[-1, n, 1] && IntegerQ[2*n]
```

rule 3686
```
Int[(u_.)*((b_.)*sin[(e_.) + (f_.)*(x_)]^(n_))^(p_), x_Symbol] :> With[{ff
= FreeFactors[Sin[e + f*x], x]}, Simp[(b*ff^n)^IntPart[p]*((b*Sin[e + f*x]^
n)^FracPart[p]/(Sin[e + f*x]/ff)^(n*FracPart[p]))   Int[ActivateTrig[u]*(Si
n[e + f*x]/ff)^(n*p), x], x]] /; FreeQ[{b, e, f, n, p}, x] &&  !IntegerQ[p]
 && IntegerQ[n] && (EqQ[u, 1] || MatchQ[u, ((d_.)*(trig_)[e + f*x])^(m_.) /
; FreeQ[{d, m}, x] && MemberQ[{sin, cos, tan, cot, sec, csc}, trig]])
```

Maple [F]

$$\int \sqrt{a \sinh(x)^3}dx$$

input
```
int((a*sinh(x)^3)^(1/2),x)
```

output
```
int((a*sinh(x)^3)^(1/2),x)
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 55, normalized size of antiderivative = 0.89

$$\int \sqrt{a \sinh^3(x)}\,dx = -\frac{4\sqrt{\frac{1}{2}}\sqrt{a}(\cosh(x)+\sinh(x))\text{weierstrassPInverse}(4,0,\cosh(x)+\sinh(x)) - (\cosh(x)^2 + 2\cosh(x)\,\mathrm{s}}{3(\cosh(x)+\sinh(x))}$$

input
```
integrate((a*sinh(x)^3)^(1/2),x, algorithm="fricas")
```

output
```
-1/3*(4*sqrt(1/2)*sqrt(a)*(cosh(x) + sinh(x))*weierstrassPInverse(4, 0, co
sh(x) + sinh(x)) - (cosh(x)^2 + 2*cosh(x)*sinh(x) + sinh(x)^2 + 1)*sqrt(a*
sinh(x)))/(cosh(x) + sinh(x))
```

Sympy [F]

$$\int \sqrt{a \sinh^3(x)}\, dx = \int \sqrt{a \sinh^3{(x)}}\, dx$$

input
```
integrate((a*sinh(x)**3)**(1/2),x)
```

output
```
Integral(sqrt(a*sinh(x)**3), x)
```

Maxima [F]

$$\int \sqrt{a \sinh^3(x)}\, dx = \int \sqrt{a \sinh{(x)}^3}\, dx$$

input
```
integrate((a*sinh(x)^3)^(1/2),x, algorithm="maxima")
```

output
```
integrate(sqrt(a*sinh(x)^3), x)
```

Giac [F]

$$\int \sqrt{a \sinh^3(x)}\, dx = \int \sqrt{a \sinh{(x)}^3}\, dx$$

input
```
integrate((a*sinh(x)^3)^(1/2),x, algorithm="giac")
```

output
```
integrate(sqrt(a*sinh(x)^3), x)
```

Mupad [F(-1)]

Timed out.

$$\int \sqrt{a\sinh^3(x)}\,dx = \int \sqrt{a\sinh(x)^3}\,dx$$

input
```
int((a*sinh(x)^3)^(1/2),x)
```

output
```
int((a*sinh(x)^3)^(1/2), x)
```

Reduce [F]

$$\int \sqrt{a\sinh^3(x)}\,dx = \sqrt{a}\left(\int \sqrt{\sinh(x)}\,\sinh(x)\,dx\right)$$

input
```
int((a*sinh(x)^3)^(1/2),x)
```

output
```
sqrt(a)*int(sqrt(sinh(x))*sinh(x),x)
```

3.149 $\int \frac{1}{\sqrt{a \sinh^3(x)}} \, dx$

Optimal result

Integrand size = 10, antiderivative size = 60

$$\int \frac{1}{\sqrt{a \sinh^3(x)}} \, dx = -\frac{2\cosh(x)\sinh(x)}{\sqrt{a\sinh^3(x)}} + \frac{2iE\left(\frac{\pi}{4}-\frac{ix}{2}\middle|2\right)\sinh^2(x)}{\sqrt{i\sinh(x)}\sqrt{a\sinh^3(x)}}$$

output
```
-2*cosh(x)*sinh(x)/(a*sinh(x)^3)^(1/2)+2*I*EllipticE(cos(1/4*Pi+1/2*I*x),2
^(1/2))*sinh(x)^2/(I*sinh(x))^(1/2)/(a*sinh(x)^3)^(1/2)
```

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.70

$$\int \frac{1}{\sqrt{a \sinh^3(x)}} \, dx = -\frac{2\left(\cosh(x) - E\left(\frac{1}{4}(\pi-2ix)\middle|2\right)\sqrt{i\sinh(x)}\right)\sinh(x)}{\sqrt{a\sinh^3(x)}}$$

input
```
Integrate[1/Sqrt[a*Sinh[x]^3],x]
```

output
```
(-2*(Cosh[x] - EllipticE[(Pi - (2*I)*x)/4, 2]*Sqrt[I*Sinh[x]])*Sinh[x])/Sq
rt[a*Sinh[x]^3]
```

Rubi [A] (verified)

Time = 0.35 (sec) , antiderivative size = 63, normalized size of antiderivative = 1.05, number of steps used = 8, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.800, Rules used = {3042, 3686, 3042, 3116, 3042, 3121, 3042, 3119}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt{a\sinh^3(x)}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{\sqrt{ia\sin(ix)^3}}dx$$

$$\Big\downarrow \text{3686}$$

$$\frac{\sinh^{\frac{3}{2}}(x)\int \frac{1}{\sinh^{\frac{3}{2}}(x)}dx}{\sqrt{a\sinh^3(x)}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\sinh^{\frac{3}{2}}(x)\int \frac{1}{(-i\sin(ix))^{3/2}}dx}{\sqrt{a\sinh^3(x)}}$$

$$\Big\downarrow \text{3116}$$

$$\frac{\sinh^{\frac{3}{2}}(x)\left(\int \sqrt{\sinh(x)}dx-\frac{2\cosh(x)}{\sqrt{\sinh(x)}}\right)}{\sqrt{a\sinh^3(x)}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\sinh^{\frac{3}{2}}(x)\left(-\frac{2\cosh(x)}{\sqrt{\sinh(x)}}+\int \sqrt{-i\sin(ix)}dx\right)}{\sqrt{a\sinh^3(x)}}$$

$$\Big\downarrow \text{3121}$$

$$\frac{\sinh^{\frac{3}{2}}(x)\left(-\frac{2\cosh(x)}{\sqrt{\sinh(x)}}+\frac{\sqrt{\sinh(x)}\int \sqrt{i\sinh(x)}dx}{\sqrt{i\sinh(x)}}\right)}{\sqrt{a\sinh^3(x)}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\sinh^{\frac{3}{2}}(x)\left(-\frac{2\cosh(x)}{\sqrt{\sinh(x)}}+\frac{\sqrt{\sinh(x)}\int\sqrt{\sin(ix)}dx}{\sqrt{i\sinh(x)}}\right)}{\sqrt{a\sinh^3(x)}}$$

$$\Big\downarrow \text{3119}$$

$$\frac{\sinh^{\frac{3}{2}}(x)\left(-\frac{2\cosh(x)}{\sqrt{\sinh(x)}}+\frac{2i\sqrt{\sinh(x)}E\left(\frac{\pi}{4}-\frac{ix}{2}|2\right)}{\sqrt{i\sinh(x)}}\right)}{\sqrt{a\sinh^3(x)}}$$

input
```
Int[1/Sqrt[a*Sinh[x]^3],x]
```

output
```
(((-2*Cosh[x])/Sqrt[Sinh[x]] + ((2*I)*EllipticE[Pi/4 - (I/2)*x, 2]*Sqrt[Si
nh[x]])/Sqrt[I*Sinh[x]])*Sinh[x]^(3/2))/Sqrt[a*Sinh[x]^3]
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3116
```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1))), x] + Simp[(n + 2)/(b^2*(n + 1))   I
nt[(b*Sin[c + d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] &&
 IntegerQ[2*n]
```

rule 3119
```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3121
```
Int[((b_)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*Sin[c + d*x])
^n/Sin[c + d*x]^n   Int[Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] && Lt
Q[-1, n, 1] && IntegerQ[2*n]
```

rule 3686
```
Int[(u_.)*((b_.)*sin[(e_.) + (f_.)*(x_)]^(n_))^(p_), x_Symbol] :> With[{ff
= FreeFactors[Sin[e + f*x], x]}, Simp[(b*ff^n)^IntPart[p]*((b*Sin[e + f*x]^
n)^FracPart[p]/(Sin[e + f*x]/ff)^(n*FracPart[p]))   Int[ActivateTrig[u]*(Si
n[e + f*x]/ff)^(n*p), x], x]] /; FreeQ[{b, e, f, n, p}, x] &&  !IntegerQ[p]
 && IntegerQ[n] && (EqQ[u, 1] || MatchQ[u, ((d_.)*(trig_)[e + f*x])^(m_.) /
; FreeQ[{d, m}, x] && MemberQ[{sin, cos, tan, cot, sec, csc}, trig]])
```

Maple [F]

$$\int \frac{1}{\sqrt{a \sinh(x)^3}} dx$$

input
```
int(1/(a*sinh(x)^3)^(1/2),x)
```

output
```
int(1/(a*sinh(x)^3)^(1/2),x)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 84, normalized size of antiderivative = 1.40

$$\int \frac{1}{\sqrt{a \sinh^3(x)}} dx = -\frac{4 \left(\sqrt{\frac{1}{2}} (\cosh(x)^2 + 2 \cosh(x) \sinh(x) + \sinh(x)^2 - 1) \sqrt{a} \text{weierstrassZeta}(4, 0, \text{weierstrassPInverse}(4\right.}{a \cosh(x)^2 + 2 a \cosh(x) \sinh(x)}$$

input
```
integrate(1/(a*sinh(x)^3)^(1/2),x, algorithm="fricas")
```

output
```
-4*(sqrt(1/2)*(cosh(x)^2 + 2*cosh(x)*sinh(x) + sinh(x)^2 - 1)*sqrt(a)*weie
rstrassZeta(4, 0, weierstrassPInverse(4, 0, cosh(x) + sinh(x))) + (cosh(x)
^2 + 2*cosh(x)*sinh(x) + sinh(x)^2)*sqrt(a*sinh(x)))/(a*cosh(x)^2 + 2*a*co
sh(x)*sinh(x) + a*sinh(x)^2 - a)
```

Sympy [F]

$$\int \frac{1}{\sqrt{a \sinh^3(x)}} \, dx = \int \frac{1}{\sqrt{a \sinh^3(x)}} \, dx$$

input
```
integrate(1/(a*sinh(x)**3)**(1/2),x)
```

output
```
Integral(1/sqrt(a*sinh(x)**3), x)
```

Maxima [F]

$$\int \frac{1}{\sqrt{a \sinh^3(x)}} \, dx = \int \frac{1}{\sqrt{a \sinh(x)^3}} \, dx$$

input
```
integrate(1/(a*sinh(x)^3)^(1/2),x, algorithm="maxima")
```

output
```
integrate(1/sqrt(a*sinh(x)^3), x)
```

Giac [F]

$$\int \frac{1}{\sqrt{a \sinh^3(x)}} \, dx = \int \frac{1}{\sqrt{a \sinh(x)^3}} \, dx$$

input
```
integrate(1/(a*sinh(x)^3)^(1/2),x, algorithm="giac")
```

output
```
integrate(1/sqrt(a*sinh(x)^3), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt{a\sinh^3(x)}}\,dx = \int \frac{1}{\sqrt{a\sinh(x)^3}}\,dx$$

input
```
int(1/(a*sinh(x)^3)^(1/2),x)
```

output
```
int(1/(a*sinh(x)^3)^(1/2), x)
```

Reduce [F]

$$\int \frac{1}{\sqrt{a\sinh^3(x)}}\,dx = \frac{\sqrt{a}\left(\int \frac{\sqrt{\sinh(x)}}{\sinh(x)^2}dx\right)}{a}$$

input
```
int(1/(a*sinh(x)^3)^(1/2),x)
```

output
```
(sqrt(a)*int(sqrt(sinh(x))/sinh(x)**2,x))/a
```

3.150 $\int \frac{1}{\left(a\sinh^3(x)\right)^{3/2}}\,dx$

Optimal result

Integrand size = 10, antiderivative size = 87

$$\int \frac{1}{\left(a\sinh^3(x)\right)^{3/2}}\,dx=\frac{10\cosh(x)}{21a\sqrt{a\sinh^3(x)}}-\frac{2\coth(x)\operatorname{csch}(x)}{7a\sqrt{a\sinh^3(x)}}+\frac{10i\,\text{EllipticF}\left(\frac{\pi}{4}-\frac{ix}{2},2\right)\sqrt{i\sinh(x)}\sinh(x)}{21a\sqrt{a\sinh^3(x)}}$$

output
```
10/21*cosh(x)/a/(a*sinh(x)^3)^(1/2)-2/7*coth(x)*csch(x)/a/(a*sinh(x)^3)^(1
/2)-10/21*I*InverseJacobiAM(-1/4*Pi+1/2*I*x,2^(1/2))*(I*sinh(x))^(1/2)*sin
h(x)/a/(a*sinh(x)^3)^(1/2)
```

Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.61

$$\int \frac{1}{\left(a\sinh^3(x)\right)^{3/2}}\,dx=\frac{2\left(5\cosh(x)-3\coth(x)\operatorname{csch}(x)+5\,\text{EllipticF}\left(\frac{1}{4}(\pi-2ix),2\right)(i\sinh(x))^{3/2}\right)}{21a\sqrt{a\sinh^3(x)}}$$

input
```
Integrate[(a*Sinh[x]^3)^(-3/2),x]
```

output
```
(2*(5*Cosh[x] - 3*Coth[x]*Csch[x] + 5*EllipticF[(Pi - (2*I)*x)/4, 2]*(I*Si
nh[x])^(3/2)))/(21*a*Sqrt[a*Sinh[x]^3])
```

Rubi [A] (verified)

Time = 0.42 (sec) , antiderivative size = 87, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 1.000, Rules used = {3042, 3686, 3042, 3116, 3042, 3116, 3042, 3121, 3042, 3120}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\left(a\sinh^3(x)\right)^{3/2}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{(ia\sin(ix)^3)^{3/2}}dx$$

$$\Big\downarrow \text{3686}$$

$$\frac{\sinh^{\frac{3}{2}}(x)\int \frac{1}{\sinh^{\frac{9}{2}}(x)}dx}{a\sqrt{a\sinh^3(x)}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\sinh^{\frac{3}{2}}(x)\int \frac{1}{(-i\sin(ix))^{9/2}}dx}{a\sqrt{a\sinh^3(x)}}$$

$$\Big\downarrow \text{3116}$$

$$\frac{\sinh^{\frac{3}{2}}(x)\left(-\frac{5}{7}\int \frac{1}{\sinh^{\frac{5}{2}}(x)}dx-\frac{2\cosh(x)}{7\sinh^{\frac{7}{2}}(x)}\right)}{a\sqrt{a\sinh^3(x)}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\sinh^{\frac{3}{2}}(x)\left(-\frac{2\cosh(x)}{7\sinh^{\frac{7}{2}}(x)}-\frac{5}{7}\int \frac{1}{(-i\sin(ix))^{5/2}}dx\right)}{a\sqrt{a\sinh^3(x)}}$$

$$\Big\downarrow \text{3116}$$

$$\frac{\sinh^{\frac{3}{2}}(x)\left(-\frac{5}{7}\left(-\frac{1}{3}\int\frac{1}{\sqrt{\sinh(x)}}dx-\frac{2\cosh(x)}{3\sinh^{\frac{3}{2}}(x)}\right)-\frac{2\cosh(x)}{7\sinh^{\frac{7}{2}}(x)}\right)}{a\sqrt{a\sinh^3(x)}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\sinh^{\frac{3}{2}}(x)\left(-\frac{2\cosh(x)}{7\sinh^{\frac{7}{2}}(x)}-\frac{5}{7}\left(-\frac{2\cosh(x)}{3\sinh^{\frac{3}{2}}(x)}-\frac{1}{3}\int\frac{1}{\sqrt{-i\sin(ix)}}dx\right)\right)}{a\sqrt{a\sinh^3(x)}}$$

$$\Big\downarrow \text{3121}$$

$$\frac{\sinh^{\frac{3}{2}}(x)\left(-\frac{2\cosh(x)}{7\sinh^{\frac{7}{2}}(x)}-\frac{5}{7}\left(-\frac{2\cosh(x)}{3\sinh^{\frac{3}{2}}(x)}-\frac{\sqrt{i\sinh(x)}\int\frac{1}{\sqrt{i\sinh(x)}}dx}{3\sqrt{\sinh(x)}}\right)\right)}{a\sqrt{a\sinh^3(x)}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\sinh^{\frac{3}{2}}(x)\left(-\frac{2\cosh(x)}{7\sinh^{\frac{7}{2}}(x)}-\frac{5}{7}\left(-\frac{2\cosh(x)}{3\sinh^{\frac{3}{2}}(x)}-\frac{\sqrt{i\sinh(x)}\int\frac{1}{\sqrt{\sin(ix)}}dx}{3\sqrt{\sinh(x)}}\right)\right)}{a\sqrt{a\sinh^3(x)}}$$

$$\Big\downarrow \text{3120}$$

$$\frac{\sinh^{\frac{3}{2}}(x)\left(-\frac{2\cosh(x)}{7\sinh^{\frac{7}{2}}(x)}-\frac{5}{7}\left(-\frac{2\cosh(x)}{3\sinh^{\frac{3}{2}}(x)}-\frac{2i\sqrt{i\sinh(x)}\,\text{EllipticF}\left(\frac{\pi}{4}-\frac{ix}{2},2\right)}{3\sqrt{\sinh(x)}}\right)\right)}{a\sqrt{a\sinh^3(x)}}$$

input

```
Int[(a*Sinh[x]^3)^(-3/2),x]
```

output

```
(((-5*((-2*Cosh[x])/(3*Sinh[x]^(3/2)) - (((2*I)/3)*EllipticF[Pi/4 - (I/2)*
x, 2]*Sqrt[I*Sinh[x]])/Sqrt[Sinh[x]]))/7 - (2*Cosh[x])/(7*Sinh[x]^(7/2)))*
Sinh[x]^(3/2))/(a*Sqrt[a*Sinh[x]^3])
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3116
```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1))), x] + Simp[(n + 2)/(b^2*(n + 1))   I
nt[(b*Sin[c + d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] &&
 IntegerQ[2*n]
```

rule 3120
```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3121
```
Int[((b_)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*Sin[c + d*x])
^n/Sin[c + d*x]^n   Int[Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] && Lt
Q[-1, n, 1] && IntegerQ[2*n]
```

rule 3686
```
Int[(u_.)*((b_.)*sin[(e_.) + (f_.)*(x_)]^(n_))^(p_), x_Symbol] :> With[{ff
= FreeFactors[Sin[e + f*x], x]}, Simp[(b*ff^n)^IntPart[p]*((b*Sin[e + f*x]^
n)^FracPart[p]/(Sin[e + f*x]/ff)^(n*FracPart[p]))   Int[ActivateTrig[u]*(Si
n[e + f*x]/ff)^(n*p), x], x]] /; FreeQ[{b, e, f, n, p}, x] &&  !IntegerQ[p]
 && IntegerQ[n] && (EqQ[u, 1] || MatchQ[u, ((d_.)*(trig_)[e + f*x])^(m_.) /
; FreeQ[{d, m}, x] && MemberQ[{sin, cos, tan, cot, sec, csc}, trig]])
```

Maple [F]

$$\int \frac{1}{\left(a \sinh \left(x\right)^{3}\right)^{\frac{3}{2}}}dx$$

input
```
int(1/(a*sinh(x)^3)^(3/2),x)
```

output
```
int(1/(a*sinh(x)^3)^(3/2),x)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 561 vs. $2(65) = 130$.

Time = 0.10 (sec) , antiderivative size = 561, normalized size of antiderivative = 6.45

$$\int \frac{1}{\left(a\sinh^3(x)\right)^{3/2}}\,dx = \text{Too large to display}$$

input
```
integrate(1/(a*sinh(x)^3)^(3/2),x, algorithm="fricas")
```

output
```
4/21*(5*sqrt(1/2)*(cosh(x)^8 + 8*cosh(x)*sinh(x)^7 + sinh(x)^8 + 4*(7*cosh
(x)^2 - 1)*sinh(x)^6 - 4*cosh(x)^6 + 8*(7*cosh(x)^3 - 3*cosh(x))*sinh(x)^5
 + 2*(35*cosh(x)^4 - 30*cosh(x)^2 + 3)*sinh(x)^4 + 6*cosh(x)^4 + 8*(7*cosh
(x)^5 - 10*cosh(x)^3 + 3*cosh(x))*sinh(x)^3 + 4*(7*cosh(x)^6 - 15*cosh(x)^
4 + 9*cosh(x)^2 - 1)*sinh(x)^2 - 4*cosh(x)^2 + 8*(cosh(x)^7 - 3*cosh(x)^5
+ 3*cosh(x)^3 - cosh(x))*sinh(x) + 1)*sqrt(a)*weierstrassPInverse(4, 0, co
sh(x) + sinh(x)) + (5*cosh(x)^7 + 35*cosh(x)*sinh(x)^6 + 5*sinh(x)^7 + (10
5*cosh(x)^2 - 17)*sinh(x)^5 - 17*cosh(x)^5 + 5*(35*cosh(x)^3 - 17*cosh(x))
*sinh(x)^4 + (175*cosh(x)^4 - 170*cosh(x)^2 - 17)*sinh(x)^3 - 17*cosh(x)^3
 + (105*cosh(x)^5 - 170*cosh(x)^3 - 51*cosh(x))*sinh(x)^2 + (35*cosh(x)^6
- 85*cosh(x)^4 - 51*cosh(x)^2 + 5)*sinh(x) + 5*cosh(x))*sqrt(a*sinh(x)))/(
a^2*cosh(x)^8 + 8*a^2*cosh(x)*sinh(x)^7 + a^2*sinh(x)^8 - 4*a^2*cosh(x)^6
+ 4*(7*a^2*cosh(x)^2 - a^2)*sinh(x)^6 + 6*a^2*cosh(x)^4 + 8*(7*a^2*cosh(x)
^3 - 3*a^2*cosh(x))*sinh(x)^5 + 2*(35*a^2*cosh(x)^4 - 30*a^2*cosh(x)^2 + 3
*a^2)*sinh(x)^4 - 4*a^2*cosh(x)^2 + 8*(7*a^2*cosh(x)^5 - 10*a^2*cosh(x)^3
+ 3*a^2*cosh(x))*sinh(x)^3 + 4*(7*a^2*cosh(x)^6 - 15*a^2*cosh(x)^4 + 9*a^2
*cosh(x)^2 - a^2)*sinh(x)^2 + a^2 + 8*(a^2*cosh(x)^7 - 3*a^2*cosh(x)^5 + 3
*a^2*cosh(x)^3 - a^2*cosh(x))*sinh(x))
```

Sympy [F]

$$\int \frac{1}{\left(a\sinh^3(x)\right)^{3/2}}\,dx = \int \frac{1}{\left(a\sinh^3{(x)}\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/(a*sinh(x)**3)**(3/2),x)
```

output
```
Integral((a*sinh(x)**3)**(-3/2), x)
```

Maxima [F]

$$\int \frac{1}{\left(a\sinh^3(x)\right)^{3/2}}\,dx = \int \frac{1}{\left(a\sinh\left(x\right)^3\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/(a*sinh(x)^3)^(3/2),x, algorithm="maxima")
```

output
```
integrate((a*sinh(x)^3)^(-3/2), x)
```

Giac [F]

$$\int \frac{1}{\left(a\sinh^3(x)\right)^{3/2}}\,dx = \int \frac{1}{\left(a\sinh\left(x\right)^3\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/(a*sinh(x)^3)^(3/2),x, algorithm="giac")
```

output
```
integrate((a*sinh(x)^3)^(-3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\left(a\sinh^3(x)\right)^{3/2}}\,dx = \int \frac{1}{\left(a\sinh\left(x\right)^3\right)^{3/2}}\,dx$$

input
```
int(1/(a*sinh(x)^3)^(3/2),x)
```

output
```
int(1/(a*sinh(x)^3)^(3/2), x)
```

Reduce [F]

$$\int \frac{1}{\left(a\sinh^3(x)\right)^{3/2}}\,dx = \frac{\sqrt{a}\left(\int \frac{\sqrt{\sinh(x)}}{\sinh(x)^5}dx\right)}{a^2}$$

input
```
int(1/(a*sinh(x)^3)^(3/2),x)
```

output
```
(sqrt(a)*int(sqrt(sinh(x))/sinh(x)**5,x))/a**2
```

3.151 $\int \frac{1}{\left(a\sinh^3(x)\right)^{5/2}}\,dx$

Optimal result

Integrand size = 10, antiderivative size = 135

$$\int \frac{1}{\left(a\sinh^3(x)\right)^{5/2}}\,dx = -\frac{154\coth(x)}{585a^2\sqrt{a\sinh^3(x)}}+\frac{22\coth(x)\text{csch}^2(x)}{117a^2\sqrt{a\sinh^3(x)}} - \frac{2\coth(x)\text{csch}^4(x)}{13a^2\sqrt{a\sinh^3(x)}}+\frac{154\cosh(x)\sinh(x)}{195a^2\sqrt{a\sinh^3(x)}}-\frac{154iE\left(\frac{\pi}{4}-\frac{ix}{2}\middle|2\right)\sinh^2(x)}{195a^2\sqrt{i\sinh(x)}\sqrt{a\sinh^3(x)}}$$

output
```
-154/585*coth(x)/a^2/(a*sinh(x)^3)^(1/2)+22/117*coth(x)*csch(x)^2/a^2/(a*s
inh(x)^3)^(1/2)-2/13*coth(x)*csch(x)^4/a^2/(a*sinh(x)^3)^(1/2)+154/195*cos
h(x)*sinh(x)/a^2/(a*sinh(x)^3)^(1/2)-154/195*I*EllipticE(cos(1/4*Pi+1/2*I*
x),2^(1/2))*sinh(x)^2/a^2/(I*sinh(x))^(1/2)/(a*sinh(x)^3)^(1/2)
```

Mathematica [A] (verified)

Time = 0.16 (sec) , antiderivative size = 69, normalized size of antiderivative = 0.51

$$\int \frac{1}{\left(a\sinh^3(x)\right)^{5/2}}\,dx = \frac{-2\coth(x)\left(77-55\text{csch}^2(x)+45\text{csch}^4(x)\right)+462iE\left(\frac{1}{4}(\pi-2ix)\middle|2\right)(i\sinh(x))^{3/2}}{585a^2\sqrt{a\sinh^3(x)}}$$

input
```
Integrate[(a*Sinh[x]^3)^(-5/2),x]
```

output
```
(-2*Coth[x]*(77 - 55*Csch[x]^2 + 45*Csch[x]^4) + (462*I)*EllipticE[(Pi - (
2*I)*x)/4, 2]*(I*Sinh[x])^(3/2) + 462*Cosh[x]*Sinh[x])/(585*a^2*Sqrt[a*Sin
h[x]^3])
```

Rubi [A] (verified)

Time = 0.59 (sec) , antiderivative size = 117, normalized size of antiderivative = 0.87, number of steps used = 14, number of rules used = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 1.400$, Rules used = {3042, 3686, 3042, 3116, 3042, 3116, 3042, 3116, 3042, 3116, 3042, 3121, 3042, 3119}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\left(a\sinh^3(x)\right)^{5/2}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{(ia\sin(ix)^3)^{5/2}}dx$$

$$\Big\downarrow \text{3686}$$

$$\frac{\sinh^{\frac{3}{2}}(x)\int \frac{1}{\sinh^{\frac{15}{2}}(x)}dx}{a^2\sqrt{a\sinh^3(x)}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\sinh^{\frac{3}{2}}(x)\int \frac{1}{(-i\sin(ix))^{15/2}}dx}{a^2\sqrt{a\sinh^3(x)}}$$

$$\Big\downarrow \text{3116}$$

$$\frac{\sinh^{\frac{3}{2}}(x)\left(-\frac{11}{13}\int \frac{1}{\sinh^{\frac{11}{2}}(x)}dx-\frac{2\cosh(x)}{13\sinh^{\frac{13}{2}}(x)}\right)}{a^2\sqrt{a\sinh^3(x)}}$$

$$\Big\downarrow \; 3042$$

$$\frac{\sinh^{\frac{3}{2}}(x)\left(-\frac{2\cosh(x)}{13\sinh^{\frac{13}{2}}(x)}-\frac{11}{13}\int\frac{1}{(-i\sin(ix))^{11/2}}dx\right)}{a^2\sqrt{a\sinh^3(x)}}$$

$$\Big\downarrow \; 3116$$

$$\frac{\sinh^{\frac{3}{2}}(x)\left(-\frac{11}{13}\left(-\frac{7}{9}\int\frac{1}{\sinh^{\frac{7}{2}}(x)}dx-\frac{2\cosh(x)}{9\sinh^{\frac{9}{2}}(x)}\right)-\frac{2\cosh(x)}{13\sinh^{\frac{13}{2}}(x)}\right)}{a^2\sqrt{a\sinh^3(x)}}$$

$$\Big\downarrow \; 3042$$

$$\frac{\sinh^{\frac{3}{2}}(x)\left(-\frac{2\cosh(x)}{13\sinh^{\frac{13}{2}}(x)}-\frac{11}{13}\left(-\frac{2\cosh(x)}{9\sinh^{\frac{9}{2}}(x)}-\frac{7}{9}\int\frac{1}{(-i\sin(ix))^{7/2}}dx\right)\right)}{a^2\sqrt{a\sinh^3(x)}}$$

$$\Big\downarrow \; 3116$$

$$\frac{\sinh^{\frac{3}{2}}(x)\left(-\frac{11}{13}\left(-\frac{7}{9}\left(-\frac{3}{5}\int\frac{1}{\sinh^{\frac{3}{2}}(x)}dx-\frac{2\cosh(x)}{5\sinh^{\frac{5}{2}}(x)}\right)-\frac{2\cosh(x)}{9\sinh^{\frac{9}{2}}(x)}\right)-\frac{2\cosh(x)}{13\sinh^{\frac{13}{2}}(x)}\right)}{a^2\sqrt{a\sinh^3(x)}}$$

$$\Big\downarrow \; 3042$$

$$\frac{\sinh^{\frac{3}{2}}(x)\left(-\frac{2\cosh(x)}{13\sinh^{\frac{13}{2}}(x)}-\frac{11}{13}\left(-\frac{2\cosh(x)}{9\sinh^{\frac{9}{2}}(x)}-\frac{7}{9}\left(-\frac{2\cosh(x)}{5\sinh^{\frac{5}{2}}(x)}-\frac{3}{5}\int\frac{1}{(-i\sin(ix))^{3/2}}dx\right)\right)\right)}{a^2\sqrt{a\sinh^3(x)}}$$

$$\Big\downarrow \; 3116$$

$$\frac{\sinh^{\frac{3}{2}}(x)\left(-\frac{11}{13}\left(-\frac{7}{9}\left(-\frac{3}{5}\left(\int\sqrt{\sinh(x)}dx-\frac{2\cosh(x)}{\sqrt{\sinh(x)}}\right)-\frac{2\cosh(x)}{5\sinh^{\frac{5}{2}}(x)}\right)-\frac{2\cosh(x)}{9\sinh^{\frac{9}{2}}(x)}\right)-\frac{2\cosh(x)}{13\sinh^{\frac{13}{2}}(x)}\right)}{a^2\sqrt{a\sinh^3(x)}}$$

$$\Big\downarrow \; 3042$$

$$\frac{\sinh^{\frac{3}{2}}(x)\left(-\frac{2\cosh(x)}{13\sinh^{\frac{13}{2}}(x)}-\frac{11}{13}\left(-\frac{2\cosh(x)}{9\sinh^{\frac{9}{2}}(x)}-\frac{7}{9}\left(-\frac{2\cosh(x)}{5\sinh^{\frac{5}{2}}(x)}-\frac{3}{5}\left(-\frac{2\cosh(x)}{\sqrt{\sinh(x)}}+\int\sqrt{-i\sin(ix)}dx\right)\right)\right)\right)}{a^2\sqrt{a\sinh^3(x)}}$$

$$\Big\downarrow \; 3121$$

$$\frac{\sinh^{\frac{3}{2}}(x)\left(-\frac{2\cosh(x)}{13\sinh^{\frac{13}{2}}(x)}-\frac{11}{13}\left(-\frac{2\cosh(x)}{9\sinh^{\frac{9}{2}}(x)}-\frac{7}{9}\left(-\frac{2\cosh(x)}{5\sinh^{\frac{5}{2}}(x)}-\frac{3}{5}\left(-\frac{2\cosh(x)}{\sqrt{\sinh(x)}}+\frac{\sqrt{\sinh(x)}\int\sqrt{i\sinh(x)}dx}{\sqrt{i\sinh(x)}}\right)\right)\right)\right)}{a^2\sqrt{a\sinh^3(x)}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\sinh^{\frac{3}{2}}(x)\left(-\frac{2\cosh(x)}{13\sinh^{\frac{13}{2}}(x)}-\frac{11}{13}\left(-\frac{2\cosh(x)}{9\sinh^{\frac{9}{2}}(x)}-\frac{7}{9}\left(-\frac{2\cosh(x)}{5\sinh^{\frac{5}{2}}(x)}-\frac{3}{5}\left(-\frac{2\cosh(x)}{\sqrt{\sinh(x)}}+\frac{\sqrt{\sinh(x)}\int\sqrt{\sin(ix)}dx}{\sqrt{i\sinh(x)}}\right)\right)\right)\right)}{a^2\sqrt{a\sinh^3(x)}}$$

$$\Big\downarrow \text{3119}$$

$$\frac{\sinh^{\frac{3}{2}}(x)\left(-\frac{2\cosh(x)}{13\sinh^{\frac{13}{2}}(x)}-\frac{11}{13}\left(-\frac{2\cosh(x)}{9\sinh^{\frac{9}{2}}(x)}-\frac{7}{9}\left(-\frac{2\cosh(x)}{5\sinh^{\frac{5}{2}}(x)}-\frac{3}{5}\left(-\frac{2\cosh(x)}{\sqrt{\sinh(x)}}+\frac{2i\sqrt{\sinh(x)}E\left(\frac{\pi}{4}-\frac{ix}{2}|2\right)}{\sqrt{i\sinh(x)}}\right)\right)\right)\right)}{a^2\sqrt{a\sinh^3(x)}}$$

input
```
Int[(a*Sinh[x]^3)^(-5/2),x]
```

output
```
(((-11*((-7*((-3*((-2*Cosh[x])/Sqrt[Sinh[x]] + ((2*I)*EllipticE[Pi/4 - (I/
2)*x, 2]*Sqrt[Sinh[x]])/Sqrt[I*Sinh[x]]))/5 - (2*Cosh[x])/(5*Sinh[x]^(5/2)
)))/9 - (2*Cosh[x])/(9*Sinh[x]^(9/2))))/13 - (2*Cosh[x])/(13*Sinh[x]^(13/2
)))*Sinh[x]^(3/2))/(a^2*Sqrt[a*Sinh[x]^3])
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3116
```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1))), x] + Simp[(n + 2)/(b^2*(n + 1))   I
nt[(b*Sin[c + d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] &&
 IntegerQ[2*n]
```

rule 3119
```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3121
```
Int[((b_)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*Sin[c + d*x])
^n/Sin[c + d*x]^n   Int[Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] && Lt
Q[-1, n, 1] && IntegerQ[2*n]
```

rule 3686
```
Int[(u_.)*((b_.)*sin[(e_.) + (f_.)*(x_)]^(n_))^(p_), x_Symbol] :> With[{ff
= FreeFactors[Sin[e + f*x], x]}, Simp[(b*ff^n)^IntPart[p]*((b*Sin[e + f*x]^
n)^FracPart[p]/(Sin[e + f*x]/ff)^(n*FracPart[p]))   Int[ActivateTrig[u]*(Si
n[e + f*x]/ff)^(n*p), x], x]] /; FreeQ[{b, e, f, n, p}, x] &&  !IntegerQ[p]
 && IntegerQ[n] && (EqQ[u, 1] || MatchQ[u, ((d_.)*(trig_)[e + f*x])^(m_.) /
; FreeQ[{d, m}, x] && MemberQ[{sin, cos, tan, cot, sec, csc}, trig]])
```

Maple [F]

$$\int \frac{1}{\left(a \sinh \left(x \right)^{3}\right)^{\frac{5}{2}}}dx$$

input
```
int(1/(a*sinh(x)^3)^(5/2),x)
```

output
```
int(1/(a*sinh(x)^3)^(5/2),x)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1477 vs. $2(106) = 212$.

Time = 0.13 (sec) , antiderivative size = 1477, normalized size of antiderivative = 10.94

$$\int \frac{1}{\left(a \sinh^3(x)\right)^{5/2}}\, dx = \text{Too large to display}$$

input
```
integrate(1/(a*sinh(x)^3)^(5/2),x, algorithm="fricas")
```

output
```
4/585*(231*sqrt(1/2)*(cosh(x)^14 + 14*cosh(x)*sinh(x)^13 + sinh(x)^14 + 7*
(13*cosh(x)^2 - 1)*sinh(x)^12 - 7*cosh(x)^12 + 28*(13*cosh(x)^3 - 3*cosh(x
))*sinh(x)^11 + 7*(143*cosh(x)^4 - 66*cosh(x)^2 + 3)*sinh(x)^10 + 21*cosh(
x)^10 + 14*(143*cosh(x)^5 - 110*cosh(x)^3 + 15*cosh(x))*sinh(x)^9 + 7*(429
*cosh(x)^6 - 495*cosh(x)^4 + 135*cosh(x)^2 - 5)*sinh(x)^8 - 35*cosh(x)^8 +
 8*(429*cosh(x)^7 - 693*cosh(x)^5 + 315*cosh(x)^3 - 35*cosh(x))*sinh(x)^7
+ 7*(429*cosh(x)^8 - 924*cosh(x)^6 + 630*cosh(x)^4 - 140*cosh(x)^2 + 5)*si
nh(x)^6 + 35*cosh(x)^6 + 14*(143*cosh(x)^9 - 396*cosh(x)^7 + 378*cosh(x)^5
 - 140*cosh(x)^3 + 15*cosh(x))*sinh(x)^5 + 7*(143*cosh(x)^10 - 495*cosh(x)
^8 + 630*cosh(x)^6 - 350*cosh(x)^4 + 75*cosh(x)^2 - 3)*sinh(x)^4 - 21*cosh
(x)^4 + 28*(13*cosh(x)^11 - 55*cosh(x)^9 + 90*cosh(x)^7 - 70*cosh(x)^5 + 2
5*cosh(x)^3 - 3*cosh(x))*sinh(x)^3 + 7*(13*cosh(x)^12 - 66*cosh(x)^10 + 13
5*cosh(x)^8 - 140*cosh(x)^6 + 75*cosh(x)^4 - 18*cosh(x)^2 + 1)*sinh(x)^2 +
 7*cosh(x)^2 + 14*(cosh(x)^13 - 6*cosh(x)^11 + 15*cosh(x)^9 - 20*cosh(x)^7
 + 15*cosh(x)^5 - 6*cosh(x)^3 + cosh(x))*sinh(x) - 1)*sqrt(a)*weierstrassZ
eta(4, 0, weierstrassPInverse(4, 0, cosh(x) + sinh(x))) + (231*cosh(x)^14
+ 3234*cosh(x)*sinh(x)^13 + 231*sinh(x)^14 + 77*(273*cosh(x)^2 - 20)*sinh(
x)^12 - 1540*cosh(x)^12 + 924*(91*cosh(x)^3 - 20*cosh(x))*sinh(x)^11 + 11*
(21021*cosh(x)^4 - 9240*cosh(x)^2 + 397)*sinh(x)^10 + 4367*cosh(x)^10 + 22
*(21021*cosh(x)^5 - 15400*cosh(x)^3 + 1985*cosh(x))*sinh(x)^9 + (693693...
```

Sympy [F]

$$\int \frac{1}{\left(a\sinh^3(x)\right)^{5/2}}\,dx=\int \frac{1}{\left(a\sinh^3\left(x\right)\right)^{\frac{5}{2}}}\,dx$$

input
```
integrate(1/(a*sinh(x)**3)**(5/2),x)
```

output
```
Integral((a*sinh(x)**3)**(-5/2), x)
```

Maxima [F]

$$\int \frac{1}{\left(a\sinh^3(x)\right)^{5/2}}\,dx = \int \frac{1}{\left(a\sinh\left(x\right)^3\right)^{\frac{5}{2}}}\,dx$$

input
```
integrate(1/(a*sinh(x)^3)^(5/2),x, algorithm="maxima")
```

output
```
integrate((a*sinh(x)^3)^(-5/2), x)
```

Giac [F]

$$\int \frac{1}{\left(a\sinh^3(x)\right)^{5/2}}\,dx = \int \frac{1}{\left(a\sinh\left(x\right)^3\right)^{\frac{5}{2}}}\,dx$$

input
```
integrate(1/(a*sinh(x)^3)^(5/2),x, algorithm="giac")
```

output
```
integrate((a*sinh(x)^3)^(-5/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\left(a\sinh^3(x)\right)^{5/2}}\,dx = \int \frac{1}{\left(a\sinh\left(x\right)^3\right)^{5/2}}\,dx$$

input
```
int(1/(a*sinh(x)^3)^(5/2),x)
```

output
```
int(1/(a*sinh(x)^3)^(5/2), x)
```

Reduce [F]

$$\int \frac{1}{\left(a \sinh^3(x)\right)^{5/2}}\,dx = \frac{\sqrt{a}\left(\int \frac{\sqrt{\sinh(x)}}{\sinh(x)^8}dx\right)}{a^3}$$

input
```
int(1/(a*sinh(x)^3)^(5/2),x)
```

output
```
(sqrt(a)*int(sqrt(sinh(x))/sinh(x)**8,x))/a**3
```

3.152 $\int \left(a\sinh^4(x)\right)^{5/2}\,dx$

Optimal result

Integrand size = 10, antiderivative size = 132

$$\int \left(a\sinh^4(x)\right)^{5/2}\,dx = \frac{63}{256}a^2\coth(x)\sqrt{a\sinh^4(x)} - \frac{63}{256}a^2x\operatorname{csch}^2(x)\sqrt{a\sinh^4(x)} - \frac{21}{128}a^2\cosh(x)\sinh(x)\sqrt{a\sinh^4(x)} + \frac{21}{160}a^2\cosh(x)\sinh^3(x)\sqrt{a\sinh^4(x)} - \frac{9}{80}a^2\cosh(x)\sinh^5(x)\sqrt{a\sinh^4(x)} + \frac{1}{10}a^2\cosh(x)\sinh^7(x)\sqrt{a\sinh^4(x)}$$

output
```
63/256*a^2*coth(x)*(a*sinh(x)^4)^(1/2)-63/256*a^2*x*csch(x)^2*(a*sinh(x)^4
)^(1/2)-21/128*a^2*cosh(x)*sinh(x)*(a*sinh(x)^4)^(1/2)+21/160*a^2*cosh(x)*
sinh(x)^3*(a*sinh(x)^4)^(1/2)-9/80*a^2*cosh(x)*sinh(x)^5*(a*sinh(x)^4)^(1/
2)+1/10*a^2*cosh(x)*sinh(x)^7*(a*sinh(x)^4)^(1/2)
```

Mathematica [A] (verified)

Time = 0.13 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.40

$$\int \left(a\sinh^4(x)\right)^{5/2}\,dx = \frac{a\operatorname{csch}^6(x)\left(a\sinh^4(x)\right)^{3/2}\left(-2520x+2100\sinh(2x)-600\sinh(4x)+150\sinh(6x)\right.}{10240}$$

input
```
Integrate[(a*Sinh[x]^4)^(5/2),x]
```

output
```
(a*Csch[x]^6*(a*Sinh[x]^4)^(3/2)*(-2520*x + 2100*Sinh[2*x] - 600*Sinh[4*x]
 + 150*Sinh[6*x] - 25*Sinh[8*x] + 2*Sinh[10*x]))/10240
```

Rubi [A] (verified)

Time = 0.49 (sec) , antiderivative size = 93, normalized size of antiderivative = 0.70, number of steps used = 18, number of rules used = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 1.800, Rules used = {3042, 3686, 3042, 25, 3115, 3042, 3115, 25, 3042, 25, 3115, 3042, 3115, 25, 3042, 25, 3115, 24}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(a\sinh^4(x)\right)^{5/2}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \left(a\sin(ix)^4\right)^{5/2}dx$$

$$\Big\downarrow \text{3686}$$

$$a^2\text{csch}^2(x)\sqrt{a\sinh^4(x)}\int \sinh^{10}(x)dx$$

$$\Big\downarrow \text{3042}$$

$$a^2\text{csch}^2(x)\sqrt{a\sinh^4(x)}\int -\sin(ix)^{10}dx$$

$$\Big\downarrow \text{25}$$

$$-a^2\text{csch}^2(x)\sqrt{a\sinh^4(x)}\int \sin(ix)^{10}dx$$

$$\Big\downarrow \text{3115}$$

$$-a^2\text{csch}^2(x)\sqrt{a\sinh^4(x)}\left(\frac{9}{10}\int \sinh^8(x)dx-\frac{1}{10}\sinh^9(x)\cosh(x)\right)$$

$$\Big\downarrow \text{3042}$$

$$-a^2\text{csch}^2(x)\sqrt{a\sinh^4(x)}\left(-\frac{1}{10}\sinh^9(x)\cosh(x)+\frac{9}{10}\int \sin(ix)^8dx\right)$$

$$\Big\downarrow \text{3115}$$

$$-a^2\text{csch}^2(x)\sqrt{a\sinh^4(x)}\left(\frac{9}{10}\left(\frac{7}{8}\int -\sinh^6(x)dx+\frac{1}{8}\sinh^7(x)\cosh(x)\right)-\frac{1}{10}\sinh^9(x)\cosh(x)\right)$$

$\downarrow$ 25

$$-a^2\text{csch}^2(x)\sqrt{a\sinh^4(x)}\left(\frac{9}{10}\left(\frac{1}{8}\sinh^7(x)\cosh(x)-\frac{7}{8}\int \sinh^6(x)dx\right)-\frac{1}{10}\sinh^9(x)\cosh(x)\right)$$

$\downarrow$ 3042

$$-a^2\text{csch}^2(x)\sqrt{a\sinh^4(x)}\left(-\frac{1}{10}\sinh^9(x)\cosh(x)+\frac{9}{10}\left(\frac{1}{8}\sinh^7(x)\cosh(x)-\frac{7}{8}\int -\sin(ix)^6dx\right)\right)$$

$\downarrow$ 25

$$-a^2\text{csch}^2(x)\sqrt{a\sinh^4(x)}\left(-\frac{1}{10}\sinh^9(x)\cosh(x)+\frac{9}{10}\left(\frac{1}{8}\sinh^7(x)\cosh(x)+\frac{7}{8}\int \sin(ix)^6dx\right)\right)$$

$\downarrow$ 3115

$$-a^2\text{csch}^2(x)\sqrt{a\sinh^4(x)}\left(\frac{9}{10}\left(\frac{7}{8}\left(\frac{5}{6}\int \sinh^4(x)dx-\frac{1}{6}\sinh^5(x)\cosh(x)\right)+\frac{1}{8}\sinh^7(x)\cosh(x)\right)-\frac{1}{10}\sinh^9(x)\,\text{c}$$

$\downarrow$ 3042

$$-a^2\text{csch}^2(x)\sqrt{a\sinh^4(x)}\left(-\frac{1}{10}\sinh^9(x)\cosh(x)+\frac{9}{10}\left(\frac{1}{8}\sinh^7(x)\cosh(x)+\frac{7}{8}\left(-\frac{1}{6}\sinh^5(x)\cosh(x)+\frac{5}{6}\int \sin(\right.\right.\right.$$

$\downarrow$ 3115

$$-a^2\text{csch}^2(x)\sqrt{a\sinh^4(x)}\left(\frac{9}{10}\left(\frac{7}{8}\left(\frac{5}{6}\left(\frac{3}{4}\int -\sinh^2(x)dx+\frac{1}{4}\sinh^3(x)\cosh(x)\right)-\frac{1}{6}\sinh^5(x)\cosh(x)\right)+\frac{1}{8}\sinh^7\right.\right.$$

$\downarrow$ 25

$$-a^2\text{csch}^2(x)\sqrt{a\sinh^4(x)}\left(\frac{9}{10}\left(\frac{7}{8}\left(\frac{5}{6}\left(\frac{1}{4}\sinh^3(x)\cosh(x)-\frac{3}{4}\int \sinh^2(x)dx\right)-\frac{1}{6}\sinh^5(x)\cosh(x)\right)+\frac{1}{8}\sinh^7(x\right.\right.$$

$\downarrow$ 3042

$$-a^2\text{csch}^2(x)\sqrt{a\sinh^4(x)}\left(-\frac{1}{10}\sinh^9(x)\cosh(x)+\frac{9}{10}\left(\frac{1}{8}\sinh^7(x)\cosh(x)+\frac{7}{8}\left(-\frac{1}{6}\sinh^5(x)\cosh(x)+\frac{5}{6}\left(\frac{1}{4}\sin\right.\right.\right.\right.$$

$\downarrow$ 25

$$-a^2\text{csch}^2(x)\sqrt{a\sinh^4(x)}\left(-\frac{1}{10}\sinh^9(x)\cosh(x)+\frac{9}{10}\left(\frac{1}{8}\sinh^7(x)\cosh(x)+\frac{7}{8}\left(-\frac{1}{6}\sinh^5(x)\cosh(x)+\frac{5}{6}\left(\frac{1}{4}\sin\right.\right.\right.\right.$$

$$\Big\downarrow \text{ 3115}$$

$$-a^2\text{csch}^2(x)\sqrt{a\sinh^4(x)}\left(\frac{9}{10}\left(\frac{7}{8}\left(\frac{5}{6}\left(\frac{3}{4}\left(\frac{\int 1dx}{2}-\frac{1}{2}\sinh(x)\cosh(x)\right)+\frac{1}{4}\sinh^3(x)\cosh(x)\right)-\frac{1}{6}\sinh^5(x)\cosh(\right.\right.\right.\right.$$

$$\Big\downarrow \text{ 24}$$

$$-a^2\text{csch}^2(x)\sqrt{a\sinh^4(x)}\left(\frac{9}{10}\left(\frac{1}{8}\sinh^7(x)\cosh(x)+\frac{7}{8}\left(\frac{5}{6}\left(\frac{1}{4}\sinh^3(x)\cosh(x)+\frac{3}{4}\left(\frac{x}{2}-\frac{1}{2}\sinh(x)\cosh(x)\right)\right)-\right.\right.\right.$$

input
```
Int[(a*Sinh[x]^4)^(5/2),x]
```

output
```
-(a^2*Csch[x]^2*Sqrt[a*Sinh[x]^4]*(-1/10*(Cosh[x]*Sinh[x]^9) + (9*((Cosh[x
]*Sinh[x]^7)/8 + (7*(-1/6*(Cosh[x]*Sinh[x]^5) + (5*((Cosh[x]*Sinh[x]^3)/4
+ (3*(x/2 - (Cosh[x]*Sinh[x])/2))/4))/6))/8))/10))
```

Defintions of rubi rules used

rule 24
```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3115
```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Simp[b^2*((n - 1)/n)   Int[(b*Sin
[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[
2*n]
```

rule 3686
```
Int[(u_.)*((b_.)*sin[(e_.) + (f_.)*(x_)]^(n_))^(p_), x_Symbol] :> With[{ff
= FreeFactors[Sin[e + f*x], x]}, Simp[(b*ff^n)^IntPart[p]*((b*Sin[e + f*x]^
n)^FracPart[p]/(Sin[e + f*x]/ff)^(n*FracPart[p]))   Int[ActivateTrig[u]*(Si
n[e + f*x]/ff)^(n*p), x], x]] /; FreeQ[{b, e, f, n, p}, x] &&  !IntegerQ[p]
 && IntegerQ[n] && (EqQ[u, 1] || MatchQ[u, ((d_.)*(trig_)[e + f*x])^(m_.) /
; FreeQ[{d, m}, x] && MemberQ[{sin, cos, tan, cot, sec, csc}, trig]])
```

Maple [A] (verified)

Time = 7.42 (sec) , antiderivative size = 171, normalized size of antiderivative = 1.30

method	result
default	$\frac{a^{\frac{3}{2}}(-1+\cosh(2x))\sqrt{a(-1+\cosh(2x))(1+\cosh(2x))}\left(8\sqrt{a\sinh(2x)^2}\sqrt{a}\sinh(2x)^4-50\sqrt{a\sinh(2x)^2}\sqrt{a}\cosh(2x)\sinh(2x)^2+160\sqrt{\cdots}\right.}{2560\sinh(2x)\sqrt{(-1\cdots}}$
risch	$-\frac{63a^2e^{2x}\sqrt{a(e^{2x}-1)^4e^{-4x}}\,x}{256(e^{2x}-1)^2}+\frac{a^2e^{12x}\sqrt{a(e^{2x}-1)^4e^{-4x}}}{10240(e^{2x}-1)^2}-\frac{5a^2e^{10x}\sqrt{a(e^{2x}-1)^4e^{-4x}}}{4096(e^{2x}-1)^2}+\frac{15a^2e^{8x}\sqrt{a(e^{2x}-1)^4e^{-4x}}}{2048(e^{2x}-1)^2}-\frac{15a^2\cdots}{\cdots}$

input
```
int((a*sinh(x)^4)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
1/2560*a^(3/2)*(-1+cosh(2*x))*(a*(-1+cosh(2*x))*(1+cosh(2*x)))^(1/2)*(8*(a
*sinh(2*x)^2)^(1/2)*a^(1/2)*sinh(2*x)^4-50*(a*sinh(2*x)^2)^(1/2)*a^(1/2)*c
osh(2*x)*sinh(2*x)^2+160*(a*sinh(2*x)^2)^(1/2)*a^(1/2)*sinh(2*x)^2-325*cos
h(2*x)*(a*sinh(2*x)^2)^(1/2)*a^(1/2)+640*(a*sinh(2*x)^2)^(1/2)*a^(1/2)-315
*ln(a^(1/2)*cosh(2*x)+(a*sinh(2*x)^2)^(1/2))*a)/sinh(2*x)/((-1+cosh(2*x))^
2*a)^(1/2)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1597 vs. 2(108) = 216.

Time = 0.17 (sec) , antiderivative size = 1597, normalized size of antiderivative = 12.10

$$\int \left(a\sinh^4(x)\right)^{5/2}\,dx = \text{Too large to display}$$

input
```
integrate((a*sinh(x)^4)^(5/2),x, algorithm="fricas")
```

output
```
1/20480*(40*a^2*cosh(x)*e^(2*x)*sinh(x)^19 + 2*a^2*e^(2*x)*sinh(x)^20 + 5*
(76*a^2*cosh(x)^2 - 5*a^2)*e^(2*x)*sinh(x)^18 + 30*(76*a^2*cosh(x)^3 - 15*
a^2*cosh(x))*e^(2*x)*sinh(x)^17 + 15*(646*a^2*cosh(x)^4 - 255*a^2*cosh(x)^
2 + 10*a^2)*e^(2*x)*sinh(x)^16 + 48*(646*a^2*cosh(x)^5 - 425*a^2*cosh(x)^3
 + 50*a^2*cosh(x))*e^(2*x)*sinh(x)^15 + 60*(1292*a^2*cosh(x)^6 - 1275*a^2*
cosh(x)^4 + 300*a^2*cosh(x)^2 - 10*a^2)*e^(2*x)*sinh(x)^14 + 120*(1292*a^2
*cosh(x)^7 - 1785*a^2*cosh(x)^5 + 700*a^2*cosh(x)^3 - 70*a^2*cosh(x))*e^(2
*x)*sinh(x)^13 + 60*(4199*a^2*cosh(x)^8 - 7735*a^2*cosh(x)^6 + 4550*a^2*co
sh(x)^4 - 910*a^2*cosh(x)^2 + 35*a^2)*e^(2*x)*sinh(x)^12 + 80*(4199*a^2*co
sh(x)^9 - 9945*a^2*cosh(x)^7 + 8190*a^2*cosh(x)^5 - 2730*a^2*cosh(x)^3 + 3
15*a^2*cosh(x))*e^(2*x)*sinh(x)^11 + 2*(184756*a^2*cosh(x)^10 - 546975*a^2
*cosh(x)^8 + 600600*a^2*cosh(x)^6 - 300300*a^2*cosh(x)^4 + 69300*a^2*cosh(
x)^2 - 2520*a^2*x)*e^(2*x)*sinh(x)^10 + 20*(16796*a^2*cosh(x)^11 - 60775*a
^2*cosh(x)^9 + 85800*a^2*cosh(x)^7 - 60060*a^2*cosh(x)^5 + 23100*a^2*cosh(
x)^3 - 2520*a^2*x*cosh(x))*e^(2*x)*sinh(x)^9 + 30*(8398*a^2*cosh(x)^12 - 3
6465*a^2*cosh(x)^10 + 64350*a^2*cosh(x)^8 - 60060*a^2*cosh(x)^6 + 34650*a^
2*cosh(x)^4 - 7560*a^2*x*cosh(x)^2 - 70*a^2)*e^(2*x)*sinh(x)^8 + 240*(646*
a^2*cosh(x)^13 - 3315*a^2*cosh(x)^11 + 7150*a^2*cosh(x)^9 - 8580*a^2*cosh(
x)^7 + 6930*a^2*cosh(x)^5 - 2520*a^2*x*cosh(x)^3 - 70*a^2*cosh(x))*e^(2*x)
*sinh(x)^7 + 60*(1292*a^2*cosh(x)^14 - 7735*a^2*cosh(x)^12 + 20020*a^2*...
```

Sympy [F]

$$\int \left(a\sinh^4(x)\right)^{5/2}\,dx=\int \left(a\sinh^4{(x)}\right)^{\frac{5}{2}}\,dx$$

input
```
integrate((a*sinh(x)**4)**(5/2),x)
```

output
```
Integral((a*sinh(x)**4)**(5/2), x)
```

Maxima [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 100, normalized size of antiderivative = 0.76

$$\int \left(a \sinh^4(x)\right)^{5/2} \, dx = -\frac{63}{256} a^{\frac{5}{2}} x - \frac{1}{20480} \left(25 a^{\frac{5}{2}} e^{(-2x)} - 150 a^{\frac{5}{2}} e^{(-4x)} + 600 a^{\frac{5}{2}} e^{(-6x)} - 2100 a^{\frac{5}{2}} e^{(-8x)} + 2100 a^{\frac{5}{2}} e^{(-12x)} - 600 a^{\frac{5}{2}} e^{(-14x)} + 1\right.$$

input
```
integrate((a*sinh(x)^4)^(5/2),x, algorithm="maxima")
```

output
```
-63/256*a^(5/2)*x - 1/20480*(25*a^(5/2)*e^(-2*x) - 150*a^(5/2)*e^(-4*x) +
600*a^(5/2)*e^(-6*x) - 2100*a^(5/2)*e^(-8*x) + 2100*a^(5/2)*e^(-12*x) - 60
0*a^(5/2)*e^(-14*x) + 150*a^(5/2)*e^(-16*x) - 25*a^(5/2)*e^(-18*x) + 2*a^(
5/2)*e^(-20*x) - 2*a^(5/2))*e^(10*x)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 76, normalized size of antiderivative = 0.58

$$\int \left(a \sinh^4(x)\right)^{5/2} \, dx = \frac{1}{20480} \left(\left(5754 e^{(10x)} - 2100 e^{(8x)} + 600 e^{(6x)} - 150 e^{(4x)} + 25 e^{(2x)} - 2\right) e^{(-10x)} - 50\right.$$

input
```
integrate((a*sinh(x)^4)^(5/2),x, algorithm="giac")
```

output
```
1/20480*((5754*e^(10*x) - 2100*e^(8*x) + 600*e^(6*x) - 150*e^(4*x) + 25*e^
(2*x) - 2)*e^(-10*x) - 5040*x + 2*e^(10*x) - 25*e^(8*x) + 150*e^(6*x) - 60
0*e^(4*x) + 2100*e^(2*x))*a^(5/2)
```

Mupad [F(-1)]

Timed out.

$$\int \left(a\sinh^4(x)\right)^{5/2}\,dx=\int \left(a\,\sinh(x)^4\right)^{5/2}\,dx$$

input
```
int((a*sinh(x)^4)^(5/2),x)
```

output
```
int((a*sinh(x)^4)^(5/2), x)
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 87, normalized size of antiderivative = 0.66

$$\int \left(a\sinh^4(x)\right)^{5/2}\,dx=\frac{\sqrt{a}\,a^2(2e^{20x}-25e^{18x}+150e^{16x}-600e^{14x}+2100e^{12x}-5040e^{10x}x-2100e^{8x}+60}{20480e^{10x}}$$

input
```
int((a*sinh(x)^4)^(5/2),x)
```

output
```
(sqrt(a)*a**2*(2*e**(20*x) - 25*e**(18*x) + 150*e**(16*x) - 600*e**(14*x)
+ 2100*e**(12*x) - 5040*e**(10*x)*x - 2100*e**(8*x) + 600*e**(6*x) - 150*e
**(4*x) + 25*e**(2*x) - 2))/(20480*e**(10*x))
```

3.153 $\int \left(a\sinh^4(x)\right)^{3/2}\,dx$

Optimal result

Integrand size = 10, antiderivative size = 78

$$\int \left(a\sinh^4(x)\right)^{3/2}\,dx=\frac{5}{16}a\coth(x)\sqrt{a\sinh^4(x)}-\frac{5}{16}ax\text{csch}^2(x)\sqrt{a\sinh^4(x)}-\frac{5}{24}a\cosh(x)\sinh(x)\sqrt{a\sinh^4(x)}+\frac{1}{6}a\cosh(x)\sinh^3(x)\sqrt{a\sinh^4(x)}$$

output
```
5/16*a*coth(x)*(a*sinh(x)^4)^(1/2)-5/16*a*x*csch(x)^2*(a*sinh(x)^4)^(1/2)-
5/24*a*cosh(x)*sinh(x)*(a*sinh(x)^4)^(1/2)+1/6*a*cosh(x)*sinh(x)^3*(a*sinh
(x)^4)^(1/2)
```

Mathematica [A] (verified)

Time = 0.09 (sec) , antiderivative size = 38, normalized size of antiderivative = 0.49

$$\int \left(a\sinh^4(x)\right)^{3/2}\,dx=\frac{1}{192}\text{csch}^6(x)\left(a\sinh^4(x)\right)^{3/2}(-60x+45\sinh(2x)-9\sinh(4x)+\sinh(6x))$$

input
```
Integrate[(a*Sinh[x]^4)^(3/2),x]
```

output
```
(Csch[x]^6*(a*Sinh[x]^4)^(3/2)*(-60*x + 45*Sinh[2*x] - 9*Sinh[4*x] + Sinh[
6*x]))/192
```

Rubi [A] (verified)

Time = 0.36 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.78, number of steps used = 12, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 1.200, Rules used = {3042, 3686, 3042, 25, 3115, 3042, 3115, 25, 3042, 25, 3115, 24}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(a \sinh^4(x)\right)^{3/2} dx$$

$$\Big\downarrow \text{3042}$$

$$\int \left(a \sin(ix)^4\right)^{3/2} dx$$

$$\Big\downarrow \text{3686}$$

$$a\text{csch}^2(x)\sqrt{a \sinh^4(x)}\int \sinh^6(x)dx$$

$$\Big\downarrow \text{3042}$$

$$a\text{csch}^2(x)\sqrt{a \sinh^4(x)}\int -\sin(ix)^6dx$$

$$\Big\downarrow \text{25}$$

$$-a\text{csch}^2(x)\sqrt{a \sinh^4(x)}\int \sin(ix)^6dx$$

$$\Big\downarrow \text{3115}$$

$$-a\text{csch}^2(x)\sqrt{a \sinh^4(x)}\left(\frac{5}{6}\int \sinh^4(x)dx-\frac{1}{6}\sinh^5(x)\cosh(x)\right)$$

$$\Big\downarrow \text{3042}$$

$$-a\text{csch}^2(x)\sqrt{a \sinh^4(x)}\left(-\frac{1}{6}\sinh^5(x)\cosh(x)+\frac{5}{6}\int \sin(ix)^4dx\right)$$

$$\Big\downarrow \text{3115}$$

$$-a\text{csch}^2(x)\sqrt{a \sinh^4(x)}\left(\frac{5}{6}\left(\frac{3}{4}\int -\sinh^2(x)dx+\frac{1}{4}\sinh^3(x)\cosh(x)\right)-\frac{1}{6}\sinh^5(x)\cosh(x)\right)$$

$$\Big\downarrow \text{25}$$

$$-a\text{csch}^2(x)\sqrt{a \sinh^4(x)}\left(\frac{5}{6}\left(\frac{1}{4}\sinh^3(x)\cosh(x)-\frac{3}{4}\int \sinh^2(x)dx\right)-\frac{1}{6}\sinh^5(x)\cosh(x)\right)$$

$$\Big\downarrow \text{3042}$$

$$-a\text{csch}^2(x)\sqrt{a\sinh^4(x)}\left(-\frac{1}{6}\sinh^5(x)\cosh(x)+\frac{5}{6}\left(\frac{1}{4}\sinh^3(x)\cosh(x)-\frac{3}{4}\int-\sin(ix)^2dx\right)\right)$$

$$\Big\downarrow \text{25}$$

$$-a\text{csch}^2(x)\sqrt{a\sinh^4(x)}\left(-\frac{1}{6}\sinh^5(x)\cosh(x)+\frac{5}{6}\left(\frac{1}{4}\sinh^3(x)\cosh(x)+\frac{3}{4}\int\sin(ix)^2dx\right)\right)$$

$$\Big\downarrow \text{3115}$$

$$-a\text{csch}^2(x)\sqrt{a\sinh^4(x)}\left(\frac{5}{6}\left(\frac{3}{4}\left(\frac{\int 1dx}{2}-\frac{1}{2}\sinh(x)\cosh(x)\right)+\frac{1}{4}\sinh^3(x)\cosh(x)\right)-\frac{1}{6}\sinh^5(x)\cosh(x)\right)$$

$$\Big\downarrow \text{24}$$

$$-a\text{csch}^2(x)\sqrt{a\sinh^4(x)}\left(\frac{5}{6}\left(\frac{1}{4}\sinh^3(x)\cosh(x)+\frac{3}{4}\left(\frac{x}{2}-\frac{1}{2}\sinh(x)\cosh(x)\right)\right)-\frac{1}{6}\sinh^5(x)\cosh(x)\right)$$

input
```
Int[(a*Sinh[x]^4)^(3/2),x]
```

output
```
-(a*Csch[x]^2*Sqrt[a*Sinh[x]^4]*(-1/6*(Cosh[x]*Sinh[x]^5) + (5*((Cosh[x]*S
inh[x]^3)/4 + (3*(x/2 - (Cosh[x]*Sinh[x])/2))/4))/6))
```

Defintions of rubi rules used

rule 24
```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3115
```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Simp[b^2*((n - 1)/n)   Int[(b*Sin
[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[
2*n]
```

rule 3686
```
Int[(u_.)*((b_.)*sin[(e_.) + (f_.)*(x_)]^(n_))^(p_), x_Symbol] :> With[{ff
= FreeFactors[Sin[e + f*x], x]}, Simp[(b*ff^n)^IntPart[p]*((b*Sin[e + f*x]^
n)^FracPart[p]/(Sin[e + f*x]/ff)^(n*FracPart[p]))   Int[ActivateTrig[u]*(Si
n[e + f*x]/ff)^(n*p), x], x]] /; FreeQ[{b, e, f, n, p}, x] &&  !IntegerQ[p]
 && IntegerQ[n] && (EqQ[u, 1] || MatchQ[u, ((d_.)*(trig_)[e + f*x])^(m_.) /
; FreeQ[{d, m}, x] && MemberQ[{sin, cos, tan, cot, sec, csc}, trig]])
```

Maple [A] (verified)

Time = 0.22 (sec) , antiderivative size = 125, normalized size of antiderivative = 1.60

method	result
default	$\frac{\sqrt{a}\,(-1+\cosh(2x))\sqrt{a(-1+\cosh(2x))(1+\cosh(2x))}\left(2\sqrt{a\sinh(2x)^2}\,\sqrt{a}\,\sinh(2x)^2-9\cosh(2x)\sqrt{a\sinh(2x)^2}\,\sqrt{a}+24\sqrt{a\sinh(2x)^2}\right.}{96\sinh(2x)\sqrt{(-1+\cosh(2x))^2 a}}$
risch	$-\frac{5a\,e^{2x}\sqrt{a(e^{2x}-1)^4 e^{-4x}}\,x}{16(e^{2x}-1)^2}+\frac{a\,e^{8x}\sqrt{a(e^{2x}-1)^4 e^{-4x}}}{384(e^{2x}-1)^2}-\frac{3a\,e^{6x}\sqrt{a(e^{2x}-1)^4 e^{-4x}}}{128(e^{2x}-1)^2}+\frac{15a\,e^{4x}\sqrt{a(e^{2x}-1)^4 e^{-4x}}}{128(e^{2x}-1)^2}-\frac{15\sqrt{a(e^{2x}}{128(e}$

input
```
int((a*sinh(x)^4)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
1/96*a^(1/2)*(-1+cosh(2*x))*(a*(-1+cosh(2*x))*(1+cosh(2*x)))^(1/2)*(2*(a*s
inh(2*x)^2)^(1/2)*a^(1/2)*sinh(2*x)^2-9*cosh(2*x)*(a*sinh(2*x)^2)^(1/2)*a^
(1/2)+24*(a*sinh(2*x)^2)^(1/2)*a^(1/2)-15*ln(a^(1/2)*cosh(2*x)+(a*sinh(2*x
)^2)^(1/2))*a)/sinh(2*x)/((-1+cosh(2*x))^2*a)^(1/2)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 659 vs. $2(62) = 124$.

Time = 0.11 (sec) , antiderivative size = 659, normalized size of antiderivative = 8.45

$$\int \left(a\sinh^4(x)\right)^{3/2} dx = \text{Too large to display}$$

input
```
integrate((a*sinh(x)^4)^(3/2),x, algorithm="fricas")
```

output
```
1/384*(12*a*cosh(x)*e^(2*x)*sinh(x)^11 + a*e^(2*x)*sinh(x)^12 + 3*(22*a*co
sh(x)^2 - 3*a)*e^(2*x)*sinh(x)^10 + 10*(22*a*cosh(x)^3 - 9*a*cosh(x))*e^(2
*x)*sinh(x)^9 + 45*(11*a*cosh(x)^4 - 9*a*cosh(x)^2 + a)*e^(2*x)*sinh(x)^8
+ 72*(11*a*cosh(x)^5 - 15*a*cosh(x)^3 + 5*a*cosh(x))*e^(2*x)*sinh(x)^7 + 6
*(154*a*cosh(x)^6 - 315*a*cosh(x)^4 + 210*a*cosh(x)^2 - 20*a*x)*e^(2*x)*si
nh(x)^6 + 36*(22*a*cosh(x)^7 - 63*a*cosh(x)^5 + 70*a*cosh(x)^3 - 20*a*x*co
sh(x))*e^(2*x)*sinh(x)^5 + 45*(11*a*cosh(x)^8 - 42*a*cosh(x)^6 + 70*a*cosh
(x)^4 - 40*a*x*cosh(x)^2 - a)*e^(2*x)*sinh(x)^4 + 20*(11*a*cosh(x)^9 - 54*
a*cosh(x)^7 + 126*a*cosh(x)^5 - 120*a*x*cosh(x)^3 - 9*a*cosh(x))*e^(2*x)*s
inh(x)^3 + 3*(22*a*cosh(x)^10 - 135*a*cosh(x)^8 + 420*a*cosh(x)^6 - 600*a*
x*cosh(x)^4 - 90*a*cosh(x)^2 + 3*a)*e^(2*x)*sinh(x)^2 + 6*(2*a*cosh(x)^11
- 15*a*cosh(x)^9 + 60*a*cosh(x)^7 - 120*a*x*cosh(x)^5 - 30*a*cosh(x)^3 + 3
*a*cosh(x))*e^(2*x)*sinh(x) + (a*cosh(x)^12 - 9*a*cosh(x)^10 + 45*a*cosh(x
)^8 - 120*a*x*cosh(x)^6 - 45*a*cosh(x)^4 + 9*a*cosh(x)^2 - a)*e^(2*x))*sqr
t(a*e^(8*x) - 4*a*e^(6*x) + 6*a*e^(4*x) - 4*a*e^(2*x) + a)*e^(-2*x)/(cosh(
x)^6*e^(4*x) - 2*cosh(x)^6*e^(2*x) + (e^(4*x) - 2*e^(2*x) + 1)*sinh(x)^6 +
 cosh(x)^6 + 6*(cosh(x)*e^(4*x) - 2*cosh(x)*e^(2*x) + cosh(x))*sinh(x)^5 +
 15*(cosh(x)^2*e^(4*x) - 2*cosh(x)^2*e^(2*x) + cosh(x)^2)*sinh(x)^4 + 20*(
cosh(x)^3*e^(4*x) - 2*cosh(x)^3*e^(2*x) + cosh(x)^3)*sinh(x)^3 + 15*(cosh(
x)^4*e^(4*x) - 2*cosh(x)^4*e^(2*x) + cosh(x)^4)*sinh(x)^2 + 6*(cosh(x)^...
```

Sympy [F]

$$\int \left(a\sinh^4(x)\right)^{3/2}\,dx = \int \left(a\sinh^4\left(x\right)\right)^{\frac{3}{2}}\,dx$$

input
```
integrate((a*sinh(x)**4)**(3/2),x)
```

output
```
Integral((a*sinh(x)**4)**(3/2), x)
```

Maxima [A] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 63, normalized size of antiderivative = 0.81

$$\int \left(a\sinh^4(x)\right)^{3/2}\,dx = -\frac{5}{16}\,a^{\frac{3}{2}}x - \frac{1}{384}\left(9\,a^{\frac{3}{2}}e^{(-2\,x)} - 45\,a^{\frac{3}{2}}e^{(-4\,x)} + 45\,a^{\frac{3}{2}}e^{(-8\,x)} - 9\,a^{\frac{3}{2}}e^{(-10\,x)} + a^{\frac{3}{2}}e^{(-12\,x)} - a^{\frac{3}{2}}\right)e^{(6\,x)}$$

input
```
integrate((a*sinh(x)^4)^(3/2),x, algorithm="maxima")
```

output
```
-5/16*a^(3/2)*x - 1/384*(9*a^(3/2)*e^(-2*x) - 45*a^(3/2)*e^(-4*x) + 45*a^(
3/2)*e^(-8*x) - 9*a^(3/2)*e^(-10*x) + a^(3/2)*e^(-12*x) - a^(3/2))*e^(6*x)
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 50, normalized size of antiderivative = 0.64

$$\int \left(a\sinh^4(x)\right)^{3/2}\,dx = \frac{1}{384}\left(\left(110\,e^{(6\,x)} - 45\,e^{(4\,x)} + 9\,e^{(2\,x)} - 1\right)e^{(-6\,x)} - 120\,x + e^{(6\,x)} - 9\,e^{(4\,x)} + 45\,e^{(2\,x)}\right)$$

input
```
integrate((a*sinh(x)^4)^(3/2),x, algorithm="giac")
```

output
```
1/384*((110*e^(6*x) - 45*e^(4*x) + 9*e^(2*x) - 1)*e^(-6*x) - 120*x + e^(6*
x) - 9*e^(4*x) + 45*e^(2*x))*a^(3/2)
```

Mupad [F(-1)]

Timed out.

$$\int \left(a\sinh^4(x)\right)^{3/2} dx = \int \left(a\sinh(x)^4\right)^{3/2} dx$$

input
```
int((a*sinh(x)^4)^(3/2),x)
```

output
```
int((a*sinh(x)^4)^(3/2), x)
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 55, normalized size of antiderivative = 0.71

$$\int \left(a\sinh^4(x)\right)^{3/2} dx = \frac{\sqrt{a}\,a(e^{12x} - 9e^{10x} + 45e^{8x} - 120e^{6x}x - 45e^{4x} + 9e^{2x} - 1)}{384e^{6x}}$$

input
```
int((a*sinh(x)^4)^(3/2),x)
```

output
```
(sqrt(a)*a*(e**(12*x) - 9*e**(10*x) + 45*e**(8*x) - 120*e**(6*x)*x - 45*e*
*(4*x) + 9*e**(2*x) - 1))/(384*e**(6*x))
```

3.154 $\int \sqrt{a \sinh^4(x)}\,dx$

Optimal result

Integrand size = 10, antiderivative size = 36

$$\int \sqrt{a \sinh^4(x)}\,dx = \frac{1}{2}\coth(x)\sqrt{a \sinh^4(x)} - \frac{1}{2}x\operatorname{csch}^2(x)\sqrt{a \sinh^4(x)}$$

output
```
1/2*coth(x)*(a*sinh(x)^4)^(1/2)-1/2*x*csch(x)^2*(a*sinh(x)^4)^(1/2)
```

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.67

$$\int \sqrt{a \sinh^4(x)}\,dx = \frac{1}{2}\left(\coth(x) - x\operatorname{csch}^2(x)\right)\sqrt{a \sinh^4(x)}$$

input
```
Integrate[Sqrt[a*Sinh[x]^4],x]
```

output
```
((Coth[x] - x*Csch[x]^2)*Sqrt[a*Sinh[x]^4])/2
```

Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 30, normalized size of antiderivative = 0.83, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.600$, Rules used = {3042, 3686, 3042, 25, 3115, 24}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{a \sinh^4(x)}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \sqrt{a \sin(ix)^4}dx$$

$$\Big\downarrow \text{3686}$$

$$\operatorname{csch}^2(x)\sqrt{a \sinh^4(x)}\int \sinh^2(x)dx$$

$$\Big\downarrow \text{3042}$$

$$\operatorname{csch}^2(x)\sqrt{a \sinh^4(x)}\int -\sin(ix)^2dx$$

$$\Big\downarrow \text{25}$$

$$-\operatorname{csch}^2(x)\sqrt{a \sinh^4(x)}\int \sin(ix)^2dx$$

$$\Big\downarrow \text{3115}$$

$$-\operatorname{csch}^2(x)\sqrt{a \sinh^4(x)}\left(\frac{\int 1dx}{2}-\frac{1}{2}\sinh(x)\cosh(x)\right)$$

$$\Big\downarrow \text{24}$$

$$-\operatorname{csch}^2(x)\sqrt{a \sinh^4(x)}\left(\frac{x}{2}-\frac{1}{2}\sinh(x)\cosh(x)\right)$$

input
```
Int[Sqrt[a*Sinh[x]^4],x]
```

output
```
-(Csch[x]^2*Sqrt[a*Sinh[x]^4]*(x/2 - (Cosh[x]*Sinh[x])/2))
```

Defintions of rubi rules used

rule 24
```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3115
```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Simp[b^2*((n - 1)/n)   Int[(b*Sin
[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[
2*n]
```

rule 3686
```
Int[(u_.)*((b_.)*sin[(e_.) + (f_.)*(x_)]^(n_))^(p_), x_Symbol] :> With[{ff
= FreeFactors[Sin[e + f*x], x]}, Simp[(b*ff^n)^IntPart[p]*((b*Sin[e + f*x]^
n)^FracPart[p]/(Sin[e + f*x]/ff)^(n*FracPart[p]))   Int[ActivateTrig[u]*(Si
n[e + f*x]/ff)^(n*p), x], x]] /; FreeQ[{b, e, f, n, p}, x] &&  !IntegerQ[p]
 && IntegerQ[n] && (EqQ[u, 1] || MatchQ[u, ((d_.)*(trig_)[e + f*x])^(m_.) /
; FreeQ[{d, m}, x] && MemberQ[{sin, cos, tan, cot, sec, csc}, trig]])
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 83 vs. $2(28) = 56$.

Time = 0.24 (sec) , antiderivative size = 84, normalized size of antiderivative = 2.33

method	result	size
default	$\frac{(-1+\cosh(2x))\sqrt{a(-1+\cosh(2x))(1+\cosh(2x))}\left(\sqrt{a\sinh(2x)^2}\,\sqrt{a}-\ln\left(\sqrt{a}\,\cosh(2x)+\sqrt{a\sinh(2x)^2}\right)a\right)}{4\sqrt{a}\,\sinh(2x)\sqrt{(-1+\cosh(2x))^2 a}}$	84
risch	$-\frac{\sqrt{a(e^{2x}-1)^4 e^{-4x}}\,e^{2x}x}{2(e^{2x}-1)^2}+\frac{\sqrt{a(e^{2x}-1)^4 e^{-4x}}\,e^{4x}}{8(e^{2x}-1)^2}-\frac{\sqrt{a(e^{2x}-1)^4 e^{-4x}}}{8(e^{2x}-1)^2}$	89

input
```
int((a*sinh(x)^4)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/4*(-1+cosh(2*x))*(a*(-1+cosh(2*x))*(1+cosh(2*x)))^(1/2)*((a*sinh(2*x)^2)
^(1/2)*a^(1/2)-ln(a^(1/2)*cosh(2*x)+(a*sinh(2*x)^2)^(1/2))*a)/a^(1/2)/sinh
(2*x)/((-1+cosh(2*x))^2*a)^(1/2)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 180 vs. 2(28) = 56.

Time = 0.08 (sec) , antiderivative size = 180, normalized size of antiderivative = 5.00

$$\int \sqrt{a\sinh^4(x)}\,dx = \frac{\left(4\cosh(x)\,e^{(2x)}\sinh(x)^3 + e^{(2x)}\sinh(x)^4 + 2\left(3\cosh(x)^2 - 2x\right)e^{(2x)}\sinh(x)^2 + 4\left(\cosh(x)^3 - 2x\cos\right.\right.}{8\left(\cosh(x)^2 e^{(4x)} - 2\cosh(x)^2 e^{(2x)} + \left(e^{(4x)} - 2e^{(2x)} + 1\right)\sinh(x)\right.}$$

input
```
integrate((a*sinh(x)^4)^(1/2),x, algorithm="fricas")
```

output
```
1/8*(4*cosh(x)*e^(2*x)*sinh(x)^3 + e^(2*x)*sinh(x)^4 + 2*(3*cosh(x)^2 - 2*
x)*e^(2*x)*sinh(x)^2 + 4*(cosh(x)^3 - 2*x*cosh(x))*e^(2*x)*sinh(x) + (cosh
(x)^4 - 4*x*cosh(x)^2 - 1)*e^(2*x))*sqrt(a*e^(8*x) - 4*a*e^(6*x) + 6*a*e^(
4*x) - 4*a*e^(2*x) + a)*e^(-2*x)/(cosh(x)^2*e^(4*x) - 2*cosh(x)^2*e^(2*x)
+ (e^(4*x) - 2*e^(2*x) + 1)*sinh(x)^2 + cosh(x)^2 + 2*(cosh(x)*e^(4*x) - 2
*cosh(x)*e^(2*x) + cosh(x))*sinh(x))
```

Sympy [F]

$$\int \sqrt{a\sinh^4(x)}\,dx = \int \sqrt{a\sinh^4(x)}\,dx$$

input
```
integrate((a*sinh(x)**4)**(1/2),x)
```

output
```
Integral(sqrt(a*sinh(x)**4), x)
```

Maxima [A] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.75

$$\int \sqrt{a \sinh^4(x)}\, dx = -\frac{1}{8} \left(\sqrt{a}e^{(-4\,x)} - \sqrt{a}\right)e^{(2\,x)} - \frac{1}{2}\,\sqrt{a}x$$

input
```
integrate((a*sinh(x)^4)^(1/2),x, algorithm="maxima")
```

output
```
-1/8*(sqrt(a)*e^(-4*x) - sqrt(a))*e^(2*x) - 1/2*sqrt(a)*x
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.72

$$\int \sqrt{a \sinh^4(x)}\, dx = \frac{1}{8} \left(\left(2\,e^{(2\,x)} - 1\right)e^{(-2\,x)} - 4\,x + e^{(2\,x)}\right)\sqrt{a}$$

input
```
integrate((a*sinh(x)^4)^(1/2),x, algorithm="giac")
```

output
```
1/8*((2*e^(2*x) - 1)*e^(-2*x) - 4*x + e^(2*x))*sqrt(a)
```

Mupad [F(-1)]

Timed out.

$$\int \sqrt{a \sinh^4(x)}\, dx = \int \sqrt{a\,{\sinh\left(x\right)}^4}\,dx$$

input
```
int((a*sinh(x)^4)^(1/2),x)
```

output
```
int((a*sinh(x)^4)^(1/2), x)
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.72

$$\int \sqrt{a\sinh^4(x)}\,dx = \frac{\sqrt{a}\,(e^{4x} - 4e^{2x}x - 1)}{8e^{2x}}$$

input
```
int((a*sinh(x)^4)^(1/2),x)
```

output
```
(sqrt(a)*(e**(4*x) - 4*e**(2*x)*x - 1))/(8*e**(2*x))
```

3.155 $\int \frac{1}{\sqrt{a \sinh^4(x)}} dx$

Optimal result

Integrand size = 10, antiderivative size = 16

$$\int \frac{1}{\sqrt{a \sinh^4(x)}} dx = -\frac{\cosh(x)\sinh(x)}{\sqrt{a \sinh^4(x)}}$$

output
```
-cosh(x)*sinh(x)/(a*sinh(x)^4)^(1/2)
```

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00

$$\int \frac{1}{\sqrt{a \sinh^4(x)}} dx = -\frac{\cosh(x)\sinh(x)}{\sqrt{a \sinh^4(x)}}$$

input
```
Integrate[1/Sqrt[a*Sinh[x]^4],x]
```

output
```
-((Cosh[x]*Sinh[x])/Sqrt[a*Sinh[x]^4])
```

Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.600$, Rules used = {3042, 3686, 3042, 25, 4254, 24}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt{a \sinh^4(x)}} \, dx$$

$$\Big\downarrow \; 3042$$

$$\int \frac{1}{\sqrt{a \sin(ix)^4}} dx$$

$$\Big\downarrow \; 3686$$

$$\frac{\sinh^2(x) \int \operatorname{csch}^2(x) dx}{\sqrt{a \sinh^4(x)}}$$

$$\Big\downarrow \; 3042$$

$$\frac{\sinh^2(x) \int -\csc(ix)^2 dx}{\sqrt{a \sinh^4(x)}}$$

$$\Big\downarrow \; 25$$

$$-\frac{\sinh^2(x) \int \csc(ix)^2 dx}{\sqrt{a \sinh^4(x)}}$$

$$\Big\downarrow \; 4254$$

$$-\frac{i \sinh^2(x) \int 1 d(-i \coth(x))}{\sqrt{a \sinh^4(x)}}$$

$$\Big\downarrow \; 24$$

$$-\frac{\sinh(x) \cosh(x)}{\sqrt{a \sinh^4(x)}}$$

input
```
Int[1/Sqrt[a*Sinh[x]^4],x]
```

output
```
-((Cosh[x]*Sinh[x])/Sqrt[a*Sinh[x]^4])
```

Defintions of rubi rules used

rule 24
```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3686
```
Int[(u_.)*((b_.)*sin[(e_.) + (f_.)*(x_)]^(n_))^(p_), x_Symbol] :> With[{ff
= FreeFactors[Sin[e + f*x], x]}, Simp[(b*ff^n)^IntPart[p]*((b*Sin[e + f*x]^
n)^FracPart[p]/(Sin[e + f*x]/ff)^(n*FracPart[p]))   Int[ActivateTrig[u]*(Si
n[e + f*x]/ff)^(n*p), x], x]] /; FreeQ[{b, e, f, n, p}, x] &&  !IntegerQ[p]
 && IntegerQ[n] && (EqQ[u, 1] || MatchQ[u, ((d_.)*(trig_)[e + f*x])^(m_.) /
; FreeQ[{d, m}, x] && MemberQ[{sin, cos, tan, cot, sec, csc}, trig]])
```

rule 4254
```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Simp[-d^(-1)   Subst[Int[Exp
andIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
 d}, x] && IGtQ[n/2, 0]
```

Maple [A] (verified)

Time = 0.23 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.81

method	result	size
risch	$-\frac{2\,\mathrm{e}^{-2x}\left(\mathrm{e}^{2x}-1\right)}{\sqrt{a\left(\mathrm{e}^{2x}-1\right)^{4}\mathrm{e}^{-4x}}}$	29
default	$-\frac{\sqrt{a\left(-1+\cosh\left(2x\right)\right)\left(1+\cosh\left(2x\right)\right)}\,\sqrt{a\sinh\left(2x\right)^{2}}}{a\sinh\left(2x\right)\sqrt{\left(-1+\cosh\left(2x\right)\right)^{2}a}}$	50

input
```
int(1/(a*sinh(x)^4)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-2/(a*(exp(2*x)-1)^4*exp(-4*x))^(1/2)*exp(-2*x)*(exp(2*x)-1)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 122 vs. $2(14) = 28$.

Time = 0.08 (sec) , antiderivative size = 122, normalized size of antiderivative = 7.62

$$\int \frac{1}{\sqrt{a \sinh^4(x)}}\,dx =$$

$$-\frac{2\sqrt{ae^{(8\,x)} - 4\,ae^{(6\,x)} + 6\,ae^{(4\,x)} - 4\,ae^{(2\,x)} + \cdots}}{a\cosh\left(x\right)^{2} + \left(ae^{(4\,x)} - 2\,ae^{(2\,x)} + a\right)\sinh\left(x\right)^{2} + \left(a\cosh\left(x\right)^{2} - a\right)e^{(4\,x)} - 2\left(a\cosh\left(x\right)^{2} - a\right)e^{(2\,x)} + 2\,(\cdots}$$

input
```
integrate(1/(a*sinh(x)^4)^(1/2),x, algorithm="fricas")
```

output
```
-2*sqrt(a*e^(8*x) - 4*a*e^(6*x) + 6*a*e^(4*x) - 4*a*e^(2*x) + a)/(a*cosh(x
)^2 + (a*e^(4*x) - 2*a*e^(2*x) + a)*sinh(x)^2 + (a*cosh(x)^2 - a)*e^(4*x)
- 2*(a*cosh(x)^2 - a)*e^(2*x) + 2*(a*cosh(x)*e^(4*x) - 2*a*cosh(x)*e^(2*x)
 + a*cosh(x))*sinh(x) - a)
```

Sympy [F]

$$\int \frac{1}{\sqrt{a \sinh^4(x)}}\,dx = \int \frac{1}{\sqrt{a \sinh^4\left(x\right)}}\,dx$$

input
```
integrate(1/(a*sinh(x)**4)**(1/2),x)
```

output
```
Integral(1/sqrt(a*sinh(x)**4), x)
```

Maxima [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.12

$$\int \frac{1}{\sqrt{a\sinh^4(x)}}\,dx = \frac{2}{\sqrt{a}e^{(-2\,x)} - \sqrt{a}}$$

input
```
integrate(1/(a*sinh(x)^4)^(1/2),x, algorithm="maxima")
```

output
```
2/(sqrt(a)*e^(-2*x) - sqrt(a))
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.81

$$\int \frac{1}{\sqrt{a\sinh^4(x)}}\,dx = -\frac{2}{\sqrt{a}(e^{(2\,x)} - 1)}$$

input
```
integrate(1/(a*sinh(x)^4)^(1/2),x, algorithm="giac")
```

output
```
-2/(sqrt(a)*(e^(2*x) - 1))
```

Mupad [B] (verification not implemented)

Time = 1.33 (sec) , antiderivative size = 38, normalized size of antiderivative = 2.38

$$\int \frac{1}{\sqrt{a\sinh^4(x)}}\,dx = \frac{e^{-x}\sqrt{a\left(\frac{e^{-x}}{2} - \frac{e^{x}}{2}\right)^4}}{a\left(\frac{e^{-x}}{2} - \frac{e^{x}}{2}\right)^3}$$

input
```
int(1/(a*sinh(x)^4)^(1/2),x)
```

output
```
(exp(-x)*(a*(exp(-x)/2 - exp(x)/2)^4)^(1/2))/(a*(exp(-x)/2 - exp(x)/2)^3)
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.31

$$\int \frac{1}{\sqrt{a\sinh^4(x)}}\,dx = -\frac{2e^{2x}\sqrt{a}}{a\left(e^{2x}-1\right)}$$

input
```
int(1/(a*sinh(x)^4)^(1/2),x)
```

output
```
( - 2*e**(2*x)*sqrt(a))/(a*(e**(2*x) - 1))
```

3.156 $\int \frac{1}{\left(a\sinh^4(x)\right)^{3/2}}\,dx$

Optimal result

Integrand size = 10, antiderivative size = 68

$$\int \frac{1}{\left(a\sinh^4(x)\right)^{3/2}}\,dx=\frac{2\cosh^2(x)\coth(x)}{3a\sqrt{a\sinh^4(x)}}-\frac{\cosh^2(x)\coth^3(x)}{5a\sqrt{a\sinh^4(x)}}-\frac{\cosh(x)\sinh(x)}{a\sqrt{a\sinh^4(x)}}$$

output
```
2/3*cosh(x)^2*coth(x)/a/(a*sinh(x)^4)^(1/2)-1/5*cosh(x)^2*coth(x)^3/a/(a*s
inh(x)^4)^(1/2)-cosh(x)*sinh(x)/a/(a*sinh(x)^4)^(1/2)
```

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.50

$$\int \frac{1}{\left(a\sinh^4(x)\right)^{3/2}}\,dx=-\frac{\cosh(x)\left(8-4\text{csch}^2(x)+3\text{csch}^4(x)\right)\sinh^5(x)}{15\left(a\sinh^4(x)\right)^{3/2}}$$

input
```
Integrate[(a*Sinh[x]^4)^(-3/2),x]
```

output
```
-1/15*(Cosh[x]*(8 - 4*Csch[x]^2 + 3*Csch[x]^4)*Sinh[x]^5)/(a*Sinh[x]^4)^(3
/2)
```

Rubi [C] (verified)

Result contains complex when optimal does not.

Time = 0.26 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.71, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.600$, Rules used = {3042, 3686, 3042, 25, 4254, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\left(a \sinh^4(x)\right)^{3/2}} \, dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{\left(a \sin(ix)^4\right)^{3/2}}dx$$

$$\Big\downarrow \text{3686}$$

$$\frac{\sinh^2(x) \int \operatorname{csch}^6(x)dx}{a\sqrt{a \sinh^4(x)}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\sinh^2(x) \int -\csc(ix)^6dx}{a\sqrt{a \sinh^4(x)}}$$

$$\Big\downarrow \text{25}$$

$$-\frac{\sinh^2(x) \int \csc(ix)^6dx}{a\sqrt{a \sinh^4(x)}}$$

$$\Big\downarrow \text{4254}$$

$$-\frac{i \sinh^2(x) \int \left(\coth^4(x)-2 \coth^2(x)+1\right)d(-i \coth(x))}{a\sqrt{a \sinh^4(x)}}$$

$$\Big\downarrow \text{2009}$$

$$-\frac{i \sinh^2(x) \left(-\frac{1}{5} i \coth^5(x)+\frac{2}{3} i \coth^3(x)-i \coth(x)\right)}{a\sqrt{a \sinh^4(x)}}$$

input
```
Int[(a*Sinh[x]^4)^(-3/2),x]
```

output
```
((-I)*((-I)*Coth[x] + ((2*I)/3)*Coth[x]^3 - (I/5)*Coth[x]^5)*Sinh[x]^2)/(a
*Sqrt[a*Sinh[x]^4])
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3686
```
Int[(u_.)*((b_.)*sin[(e_.) + (f_.)*(x_)]^(n_))^(p_), x_Symbol] :> With[{ff
= FreeFactors[Sin[e + f*x], x]}, Simp[(b*ff^n)^IntPart[p]*((b*Sin[e + f*x]^
n)^FracPart[p]/(Sin[e + f*x]/ff)^(n*FracPart[p]))   Int[ActivateTrig[u]*(Si
n[e + f*x]/ff)^(n*p), x], x]] /; FreeQ[{b, e, f, n, p}, x] &&  !IntegerQ[p]
 && IntegerQ[n] && (EqQ[u, 1] || MatchQ[u, ((d_.)*(trig_)[e + f*x])^(m_.) /
; FreeQ[{d, m}, x] && MemberQ[{sin, cos, tan, cot, sec, csc}, trig]])
```

rule 4254
```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Simp[-d^(-1)   Subst[Int[Exp
andIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
 d}, x] && IGtQ[n/2, 0]
```

Maple [A] (verified)

Time = 0.26 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.71

method	result	size
risch	$-\frac{16\,{\mathrm e}^{-2 x} \left(10\,{\mathrm e}^{4 x}-5\,{\mathrm e}^{2 x}+1\right)}{15 a \left({\mathrm e}^{2 x}-1\right)^{3} \sqrt{a \left({\mathrm e}^{2 x}-1\right)^{4} {\mathrm e}^{-4 x}}}$	48
default	$-\frac{4 \left(2 \cosh \left(2 x\right)^{2}-6 \cosh \left(2 x\right)+7\right) \sqrt{a \sinh \left(2 x\right)^{2}}\, \sqrt{a \left(-1+\cosh \left(2 x\right)\right) \left(1+\cosh \left(2 x\right)\right)}}{15 a^{2} \left(-1+\cosh \left(2 x\right)\right)^{2} \sinh \left(2 x\right) \sqrt{\left(-1+\cosh \left(2 x\right)\right)^{2} a}}$	74

input
```
int(1/(a*sinh(x)^4)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-16/15/a/(exp(2*x)-1)^3*exp(-2*x)/(a*(exp(2*x)-1)^4*exp(-4*x))^(1/2)*(10*e
xp(4*x)-5*exp(2*x)+1)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1163 vs. $2(58) = 116$.

Time = 0.11 (sec) , antiderivative size = 1163, normalized size of antiderivative = 17.10

$$\int \frac{1}{\left(a\sinh^4(x)\right)^{3/2}}\,dx = \text{Too large to display}$$

input
```
integrate(1/(a*sinh(x)^4)^(3/2),x, algorithm="fricas")
```

output
```
-16/15*(40*cosh(x)*e^(2*x)*sinh(x)^3 + 10*e^(2*x)*sinh(x)^4 + 5*(12*cosh(x
)^2 - 1)*e^(2*x)*sinh(x)^2 + 10*(4*cosh(x)^3 - cosh(x))*e^(2*x)*sinh(x) +
(10*cosh(x)^4 - 5*cosh(x)^2 + 1)*e^(2*x))*sqrt(a*e^(8*x) - 4*a*e^(6*x) + 6
*a*e^(4*x) - 4*a*e^(2*x) + a)*e^(-2*x)/(a^2*cosh(x)^10 + (a^2*e^(4*x) - 2*
a^2*e^(2*x) + a^2)*sinh(x)^10 - 5*a^2*cosh(x)^8 + 10*(a^2*cosh(x)*e^(4*x)
- 2*a^2*cosh(x)*e^(2*x) + a^2*cosh(x))*sinh(x)^9 + 5*(9*a^2*cosh(x)^2 - a^
2 + (9*a^2*cosh(x)^2 - a^2)*e^(4*x) - 2*(9*a^2*cosh(x)^2 - a^2)*e^(2*x))*s
inh(x)^8 + 10*a^2*cosh(x)^6 + 40*(3*a^2*cosh(x)^3 - a^2*cosh(x) + (3*a^2*c
osh(x)^3 - a^2*cosh(x))*e^(4*x) - 2*(3*a^2*cosh(x)^3 - a^2*cosh(x))*e^(2*x
))*sinh(x)^7 + 10*(21*a^2*cosh(x)^4 - 14*a^2*cosh(x)^2 + a^2 + (21*a^2*cos
h(x)^4 - 14*a^2*cosh(x)^2 + a^2)*e^(4*x) - 2*(21*a^2*cosh(x)^4 - 14*a^2*co
sh(x)^2 + a^2)*e^(2*x))*sinh(x)^6 - 10*a^2*cosh(x)^4 + 4*(63*a^2*cosh(x)^5
 - 70*a^2*cosh(x)^3 + 15*a^2*cosh(x) + (63*a^2*cosh(x)^5 - 70*a^2*cosh(x)^
3 + 15*a^2*cosh(x))*e^(4*x) - 2*(63*a^2*cosh(x)^5 - 70*a^2*cosh(x)^3 + 15*
a^2*cosh(x))*e^(2*x))*sinh(x)^5 + 10*(21*a^2*cosh(x)^6 - 35*a^2*cosh(x)^4
+ 15*a^2*cosh(x)^2 - a^2 + (21*a^2*cosh(x)^6 - 35*a^2*cosh(x)^4 + 15*a^2*c
osh(x)^2 - a^2)*e^(4*x) - 2*(21*a^2*cosh(x)^6 - 35*a^2*cosh(x)^4 + 15*a^2*
cosh(x)^2 - a^2)*e^(2*x))*sinh(x)^4 + 5*a^2*cosh(x)^2 + 40*(3*a^2*cosh(x)^
7 - 7*a^2*cosh(x)^5 + 5*a^2*cosh(x)^3 - a^2*cosh(x) + (3*a^2*cosh(x)^7 - 7
*a^2*cosh(x)^5 + 5*a^2*cosh(x)^3 - a^2*cosh(x))*e^(4*x) - 2*(3*a^2*cosh...
```

Sympy [F]

$$\int \frac{1}{\left(a\sinh^4(x)\right)^{3/2}}\,dx = \int \frac{1}{\left(a\sinh^4\left(x\right)\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/(a*sinh(x)**4)**(3/2),x)
```

output
```
Integral((a*sinh(x)**4)**(-3/2), x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 171 vs. $2(58) = 116$.

Time = 0.14 (sec) , antiderivative size = 171, normalized size of antiderivative = 2.51

$$\int \frac{1}{\left(a\sinh^4(x)\right)^{3/2}}\,dx = -\frac{16\,e^{(-2\,x)}}{3\left(5\,a^{\frac{3}{2}}e^{(-2\,x)} - 10\,a^{\frac{3}{2}}e^{(-4\,x)} + 10\,a^{\frac{3}{2}}e^{(-6\,x)} - 5\,a^{\frac{3}{2}}e^{(-8\,x)} + a^{\frac{3}{2}}e^{(-10\,x)} - a^{\frac{3}{2}}\right)} + \frac{32\,e^{(-4\,x)}}{3\left(5\,a^{\frac{3}{2}}e^{(-2\,x)} - 10\,a^{\frac{3}{2}}e^{(-4\,x)} + 10\,a^{\frac{3}{2}}e^{(-6\,x)} - 5\,a^{\frac{3}{2}}e^{(-8\,x)} + a^{\frac{3}{2}}e^{(-10\,x)} - a^{\frac{3}{2}}\right)} + \frac{16}{15\left(5\,a^{\frac{3}{2}}e^{(-2\,x)} - 10\,a^{\frac{3}{2}}e^{(-4\,x)} + 10\,a^{\frac{3}{2}}e^{(-6\,x)} - 5\,a^{\frac{3}{2}}e^{(-8\,x)} + a^{\frac{3}{2}}e^{(-10\,x)} - a^{\frac{3}{2}}\right)}$$

input
```
integrate(1/(a*sinh(x)^4)^(3/2),x, algorithm="maxima")
```

output
```
-16/3*e^(-2*x)/(5*a^(3/2)*e^(-2*x) - 10*a^(3/2)*e^(-4*x) + 10*a^(3/2)*e^(-
6*x) - 5*a^(3/2)*e^(-8*x) + a^(3/2)*e^(-10*x) - a^(3/2)) + 32/3*e^(-4*x)/(
5*a^(3/2)*e^(-2*x) - 10*a^(3/2)*e^(-4*x) + 10*a^(3/2)*e^(-6*x) - 5*a^(3/2)
*e^(-8*x) + a^(3/2)*e^(-10*x) - a^(3/2)) + 16/15/(5*a^(3/2)*e^(-2*x) - 10*
a^(3/2)*e^(-4*x) + 10*a^(3/2)*e^(-6*x) - 5*a^(3/2)*e^(-8*x) + a^(3/2)*e^(-
10*x) - a^(3/2))
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.40

$$\int \frac{1}{\left(a\sinh^4(x)\right)^{3/2}}\,dx = -\frac{16\left(10\,e^{(4\,x)} - 5\,e^{(2\,x)} + 1\right)}{15\,a^{\frac{3}{2}}\left(e^{(2\,x)} - 1\right)^5}$$

input
```
integrate(1/(a*sinh(x)^4)^(3/2),x, algorithm="giac")
```

output
```
-16/15*(10*e^(4*x) - 5*e^(2*x) + 1)/(a^(3/2)*(e^(2*x) - 1)^5)
```

Mupad [B] (verification not implemented)

Time = 1.47 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.71

$$\int \frac{1}{\left(a\sinh^4(x)\right)^{3/2}}\,dx = -\frac{64\,\mathrm{e}^{2\,x}\,\sqrt{a\,{\left(\frac{\mathrm{e}^{-x}}{2}-\frac{\mathrm{e}^{x}}{2}\right)}^4}\,\left(10\,\mathrm{e}^{4\,x}-5\,\mathrm{e}^{2\,x}+1\right)}{15\,a^2\,{\left(\mathrm{e}^{2\,x}-1\right)}^7}$$

input
```
int(1/(a*sinh(x)^4)^(3/2),x)
```

output
```
-(64*exp(2*x)*(a*(exp(-x)/2 - exp(x)/2)^4)^(1/2)*(10*exp(4*x) - 5*exp(2*x)
 + 1))/(15*a^2*(exp(2*x) - 1)^7)
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 60, normalized size of antiderivative = 0.88

$$\int \frac{1}{\left(a\sinh^4(x)\right)^{3/2}}\,dx = \frac{16\sqrt{a}\,(-10e^{4x}+5e^{2x}-1)}{15a^2\,(e^{10x}-5e^{8x}+10e^{6x}-10e^{4x}+5e^{2x}-1)}$$

input
```
int(1/(a*sinh(x)^4)^(3/2),x)
```

output
```
(16*sqrt(a)*( - 10*e**(4*x) + 5*e**(2*x) - 1))/(15*a**2*(e**(10*x) - 5*e**
(8*x) + 10*e**(6*x) - 10*e**(4*x) + 5*e**(2*x) - 1))
```

3.157 $\int \frac{1}{(a\sinh^4(x))^{5/2}}\,dx$

Optimal result

Integrand size = 10, antiderivative size = 118

$$\int \frac{1}{(a\sinh^4(x))^{5/2}}\,dx = \frac{4\cosh^2(x)\coth(x)}{3a^2\sqrt{a\sinh^4(x)}} - \frac{6\cosh^2(x)\coth^3(x)}{5a^2\sqrt{a\sinh^4(x)}} + \frac{4\cosh^2(x)\coth^5(x)}{7a^2\sqrt{a\sinh^4(x)}} - \frac{\cosh^2(x)\coth^7(x)}{9a^2\sqrt{a\sinh^4(x)}} - \frac{\cosh(x)\sinh(x)}{a^2\sqrt{a\sinh^4(x)}}$$

output
```
4/3*cosh(x)^2*coth(x)/a^2/(a*sinh(x)^4)^(1/2)-6/5*cosh(x)^2*coth(x)^3/a^2/
(a*sinh(x)^4)^(1/2)+4/7*cosh(x)^2*coth(x)^5/a^2/(a*sinh(x)^4)^(1/2)-1/9*co
sh(x)^2*coth(x)^7/a^2/(a*sinh(x)^4)^(1/2)-cosh(x)*sinh(x)/a^2/(a*sinh(x)^4
)^(1/2)
```

Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.40

$$\int \frac{1}{(a\sinh^4(x))^{5/2}}\,dx = -\frac{\cosh(x)\left(128 - 64\text{csch}^2(x) + 48\text{csch}^4(x) - 40\text{csch}^6(x) + 35\text{csch}^8(x)\right)\sinh(x)}{315a^2\sqrt{a\sinh^4(x)}}$$

input
```
Integrate[(a*Sinh[x]^4)^(-5/2),x]
```

output
```
-1/315*(Cosh[x]*(128 - 64*Csch[x]^2 + 48*Csch[x]^4 - 40*Csch[x]^6 + 35*Csc
h[x]^8)*Sinh[x])/(a^2*Sqrt[a*Sinh[x]^4])
```

Rubi [C] (verified)

Result contains complex when optimal does not.

Time = 0.27 (sec) , antiderivative size = 68, normalized size of antiderivative = 0.58, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.600, Rules used = {3042, 3686, 3042, 25, 4254, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\left(a\sinh^4(x)\right)^{5/2}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{\left(a\sin(ix)^4\right)^{5/2}}dx$$

$$\Big\downarrow \text{3686}$$

$$\frac{\sinh^2(x)\int \text{csch}^{10}(x)dx}{a^2\sqrt{a\sinh^4(x)}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\sinh^2(x)\int -\csc(ix)^{10}dx}{a^2\sqrt{a\sinh^4(x)}}$$

$$\Big\downarrow \text{25}$$

$$-\frac{\sinh^2(x)\int \csc(ix)^{10}dx}{a^2\sqrt{a\sinh^4(x)}}$$

$$\Big\downarrow \text{4254}$$

$$-\frac{i\sinh^2(x)\int\left(\coth^8(x)-4\coth^6(x)+6\coth^4(x)-4\coth^2(x)+1\right)d(-i\coth(x))}{a^2\sqrt{a\sinh^4(x)}}$$

$$\Big\downarrow \text{2009}$$

$$-\frac{i\sinh^2(x)\left(-\frac{1}{9}i\coth^9(x)+\frac{4}{7}i\coth^7(x)-\frac{6}{5}i\coth^5(x)+\frac{4}{3}i\coth^3(x)-i\coth(x)\right)}{a^2\sqrt{a\sinh^4(x)}}$$

input
```
Int[(a*Sinh[x]^4)^(-5/2),x]
```

output
```
((-I)*((-I)*Coth[x] + ((4*I)/3)*Coth[x]^3 - ((6*I)/5)*Coth[x]^5 + ((4*I)/7
)*Coth[x]^7 - (I/9)*Coth[x]^9)*Sinh[x]^2)/(a^2*Sqrt[a*Sinh[x]^4])
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3686
```
Int[(u_.)*((b_.)*sin[(e_.) + (f_.)*(x_)]^(n_))^(p_), x_Symbol] :> With[{ff
= FreeFactors[Sin[e + f*x], x]}, Simp[(b*ff^n)^IntPart[p]*((b*Sin[e + f*x]^
n)^FracPart[p]/(Sin[e + f*x]/ff)^(n*FracPart[p]))   Int[ActivateTrig[u]*(Si
n[e + f*x]/ff)^(n*p), x], x]] /; FreeQ[{b, e, f, n, p}, x] &&  !IntegerQ[p]
 && IntegerQ[n] && (EqQ[u, 1] || MatchQ[u, ((d_.)*(trig_)[e + f*x])^(m_.) /
; FreeQ[{d, m}, x] && MemberQ[{sin, cos, tan, cot, sec, csc}, trig]])
```

rule 4254
```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Simp[-d^(-1)   Subst[Int[Exp
andIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
 d}, x] && IGtQ[n/2, 0]
```

Maple [A] (verified)

Time = 0.25 (sec) , antiderivative size = 60, normalized size of antiderivative = 0.51

method	result	size
risch	$-\frac{256\,\mathrm{e}^{-2x}\left(126\,\mathrm{e}^{8x}-84\,\mathrm{e}^{6x}+36\,\mathrm{e}^{4x}-9\,\mathrm{e}^{2x}+1\right)}{315a^{2}\left(\mathrm{e}^{2x}-1\right)^{7}\sqrt{a\left(\mathrm{e}^{2x}-1\right)^{4}\mathrm{e}^{-4x}}}$	60
default	$-\frac{16\left(8\cosh(2x)^{4}-40\cosh(2x)^{3}+84\cosh(2x)^{2}-100\cosh(2x)+83\right)\sqrt{a\sinh(2x)^{2}}\,\sqrt{a(-1+\cosh(2x))(1+\cosh(2x))}}{315a^{3}(-1+\cosh(2x))^{4}\sinh(2x)\sqrt{(-1+\cosh(2x))^{2}a}}$	90

input
```
int(1/(a*sinh(x)^4)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
-256/315/a^2/(exp(2*x)-1)^7*exp(-2*x)/(a*(exp(2*x)-1)^4*exp(-4*x))^(1/2)*(
126*exp(8*x)-84*exp(6*x)+36*exp(4*x)-9*exp(2*x)+1)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 3093 vs. 2(100) = 200.

Time = 0.17 (sec) , antiderivative size = 3093, normalized size of antiderivative = 26.21

$$\int \frac{1}{\left(a\sinh^4(x)\right)^{5/2}}\,dx = \text{Too large to display}$$

input
```
integrate(1/(a*sinh(x)^4)^(5/2),x, algorithm="fricas")
```

output
```
Too large to include
```

Sympy [F]

$$\int \frac{1}{\left(a\sinh^4(x)\right)^{5/2}}\,dx = \int \frac{1}{\left(a\sinh^4\left(x\right)\right)^{\frac{5}{2}}}\,dx$$

input
```
integrate(1/(a*sinh(x)**4)**(5/2),x)
```

output
```
Integral((a*sinh(x)**4)**(-5/2), x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 467 vs. $2(100) = 200$.

Time = 0.13 (sec) , antiderivative size = 467, normalized size of antiderivative = 3.96

$$\int \frac{1}{\left(a\sinh^4(x)\right)^{5/2}}\,dx = \text{Too large to display}$$

input
```
integrate(1/(a*sinh(x)^4)^(5/2),x, algorithm="maxima")
```

output
```
-256/35*e^(-2*x)/(9*a^(5/2)*e^(-2*x) - 36*a^(5/2)*e^(-4*x) + 84*a^(5/2)*e^
(-6*x) - 126*a^(5/2)*e^(-8*x) + 126*a^(5/2)*e^(-10*x) - 84*a^(5/2)*e^(-12*
x) + 36*a^(5/2)*e^(-14*x) - 9*a^(5/2)*e^(-16*x) + a^(5/2)*e^(-18*x) - a^(5
/2)) + 1024/35*e^(-4*x)/(9*a^(5/2)*e^(-2*x) - 36*a^(5/2)*e^(-4*x) + 84*a^(
5/2)*e^(-6*x) - 126*a^(5/2)*e^(-8*x) + 126*a^(5/2)*e^(-10*x) - 84*a^(5/2)*
e^(-12*x) + 36*a^(5/2)*e^(-14*x) - 9*a^(5/2)*e^(-16*x) + a^(5/2)*e^(-18*x)
 - a^(5/2)) - 1024/15*e^(-6*x)/(9*a^(5/2)*e^(-2*x) - 36*a^(5/2)*e^(-4*x) +
 84*a^(5/2)*e^(-6*x) - 126*a^(5/2)*e^(-8*x) + 126*a^(5/2)*e^(-10*x) - 84*a
^(5/2)*e^(-12*x) + 36*a^(5/2)*e^(-14*x) - 9*a^(5/2)*e^(-16*x) + a^(5/2)*e^
(-18*x) - a^(5/2)) + 512/5*e^(-8*x)/(9*a^(5/2)*e^(-2*x) - 36*a^(5/2)*e^(-4
*x) + 84*a^(5/2)*e^(-6*x) - 126*a^(5/2)*e^(-8*x) + 126*a^(5/2)*e^(-10*x) -
 84*a^(5/2)*e^(-12*x) + 36*a^(5/2)*e^(-14*x) - 9*a^(5/2)*e^(-16*x) + a^(5/
2)*e^(-18*x) - a^(5/2)) + 256/315/(9*a^(5/2)*e^(-2*x) - 36*a^(5/2)*e^(-4*x
) + 84*a^(5/2)*e^(-6*x) - 126*a^(5/2)*e^(-8*x) + 126*a^(5/2)*e^(-10*x) - 8
4*a^(5/2)*e^(-12*x) + 36*a^(5/2)*e^(-14*x) - 9*a^(5/2)*e^(-16*x) + a^(5/2)
*e^(-18*x) - a^(5/2))
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.33

$$\int \frac{1}{\left(a\sinh^4(x)\right)^{5/2}}\,dx = -\frac{256\left(126\,e^{(8\,x)} - 84\,e^{(6\,x)} + 36\,e^{(4\,x)} - 9\,e^{(2\,x)} + 1\right)}{315\,a^{\frac{5}{2}}\left(e^{(2\,x)} - 1\right)^{9}}$$

input
```
integrate(1/(a*sinh(x)^4)^(5/2),x, algorithm="giac")
```

output
```
-256/315*(126*e^(8*x) - 84*e^(6*x) + 36*e^(4*x) - 9*e^(2*x) + 1)/(a^(5/2)*
(e^(2*x) - 1)^9)
```

Mupad [B] (verification not implemented)

Time = 1.42 (sec) , antiderivative size = 256, normalized size of antiderivative = 2.17

$$\begin{aligned}\int \frac{1}{\left(a\sinh^4(x)\right)^{5/2}}\,dx = &-\frac{2048\,e^{4\,x}\sqrt{a\left(\frac{e^{-x}}{2}-\frac{e^{x}}{2}\right)^4}}{5\,a^3\left(e^{2\,x}-1\right)^5\left(e^{2\,x}-2\,e^{4\,x}+e^{6\,x}\right)} \\ &-\frac{4096\,e^{4\,x}\sqrt{a\left(\frac{e^{-x}}{2}-\frac{e^{x}}{2}\right)^4}}{3\,a^3\left(e^{2\,x}-1\right)^6\left(e^{2\,x}-2\,e^{4\,x}+e^{6\,x}\right)}-\frac{12288\,e^{4\,x}\sqrt{a\left(\frac{e^{-x}}{2}-\frac{e^{x}}{2}\right)^4}}{7\,a^3\left(e^{2\,x}-1\right)^7\left(e^{2\,x}-2\,e^{4\,x}+e^{6\,x}\right)} \\ &-\frac{1024\,e^{4\,x}\sqrt{a\left(\frac{e^{-x}}{2}-\frac{e^{x}}{2}\right)^4}}{a^3\left(e^{2\,x}-1\right)^8\left(e^{2\,x}-2\,e^{4\,x}+e^{6\,x}\right)}-\frac{2048\,e^{4\,x}\sqrt{a\left(\frac{e^{-x}}{2}-\frac{e^{x}}{2}\right)^4}}{9\,a^3\left(e^{2\,x}-1\right)^9\left(e^{2\,x}-2\,e^{4\,x}+e^{6\,x}\right)}\end{aligned}$$

input
```
int(1/(a*sinh(x)^4)^(5/2),x)
```

output
```
- (2048*exp(4*x)*(a*(exp(-x)/2 - exp(x)/2)^4)^(1/2))/(5*a^3*(exp(2*x) - 1)
^5*(exp(2*x) - 2*exp(4*x) + exp(6*x))) - (4096*exp(4*x)*(a*(exp(-x)/2 - ex
p(x)/2)^4)^(1/2))/(3*a^3*(exp(2*x) - 1)^6*(exp(2*x) - 2*exp(4*x) + exp(6*x
))) - (12288*exp(4*x)*(a*(exp(-x)/2 - exp(x)/2)^4)^(1/2))/(7*a^3*(exp(2*x)
 - 1)^7*(exp(2*x) - 2*exp(4*x) + exp(6*x))) - (1024*exp(4*x)*(a*(exp(-x)/2
 - exp(x)/2)^4)^(1/2))/(a^3*(exp(2*x) - 1)^8*(exp(2*x) - 2*exp(4*x) + exp(
6*x))) - (2048*exp(4*x)*(a*(exp(-x)/2 - exp(x)/2)^4)^(1/2))/(9*a^3*(exp(2*
x) - 1)^9*(exp(2*x) - 2*exp(4*x) + exp(6*x)))
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 102, normalized size of antiderivative = 0.86

$$\int \frac{1}{\left(a\sinh^4(x)\right)^{5/2}}\,dx = \frac{256\sqrt{a}\left(-126e^{8x}+84e^{6x}-36e^{4x}+9e^{2x}-1\right)}{315a^3\left(e^{18x}-9e^{16x}+36e^{14x}-84e^{12x}+126e^{10x}-126e^{8x}+84e^{6x}-36e^{4x}+9e^{2x}-\right.}$$

input
```
int(1/(a*sinh(x)^4)^(5/2),x)
```

output
```
(256*sqrt(a)*( - 126*e**(8*x) + 84*e**(6*x) - 36*e**(4*x) + 9*e**(2*x) - 1
))/(315*a**3*(e**(18*x) - 9*e**(16*x) + 36*e**(14*x) - 84*e**(12*x) + 126*
e**(10*x) - 126*e**(8*x) + 84*e**(6*x) - 36*e**(4*x) + 9*e**(2*x) - 1))
```

3.158 $\int \frac{\cosh^8(x)}{i+\sinh(x)}\,dx$

Optimal result

Integrand size = 13, antiderivative size = 50

$$\int \frac{\cosh^8(x)}{i+\sinh(x)}\,dx = -\frac{5ix}{16}+\frac{\cosh^7(x)}{7}-\frac{5}{16}i\cosh(x)\sinh(x) -\frac{5}{24}i\cosh^3(x)\sinh(x)-\frac{1}{6}i\cosh^5(x)\sinh(x)$$

output
```
-5/16*I*x+1/7*cosh(x)^7-5/16*I*cosh(x)*sinh(x)-5/24*I*cosh(x)^3*sinh(x)-1/
6*I*cosh(x)^5*sinh(x)
```

Mathematica [B] (verified)

Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 219 vs. 2(50) = 100.

Time = 0.12 (sec) , antiderivative size = 219, normalized size of antiderivative = 4.38

$$\int \frac{\cosh^8(x)}{i+\sinh(x)}\,dx = \frac{\cosh^9(x)\left(6i\left(35\arcsin\left(\frac{\sqrt{1-i\sinh(x)}}{\sqrt{2}}\right)\sqrt{1-i\sinh(x)}+8\sqrt{1+i\sinh(x)}\right)+279\sqrt{1+i\sinh(x)}\sinh(x)-\cdots}{\cdots}$$

input
```
Integrate[Cosh[x]^8/(I + Sinh[x]),x]
```

output
```
(Cosh[x]^9*((6*I)*(35*ArcSin[Sqrt[1 - I*Sinh[x]]/Sqrt[2]]*Sqrt[1 - I*Sinh[
x]] + 8*Sqrt[1 + I*Sinh[x]]) + 279*Sqrt[1 + I*Sinh[x]]*Sinh[x] - (87*I)*Sq
rt[1 + I*Sinh[x]]*Sinh[x]^2 + 326*Sqrt[1 + I*Sinh[x]]*Sinh[x]^3 - (38*I)*S
qrt[1 + I*Sinh[x]]*Sinh[x]^4 + 200*Sqrt[1 + I*Sinh[x]]*Sinh[x]^5 - (8*I)*S
qrt[1 + I*Sinh[x]]*Sinh[x]^6 + 48*Sqrt[1 + I*Sinh[x]]*Sinh[x]^7))/(336*Sqr
t[1 + I*Sinh[x]]*(-I + Sinh[x])^4*(I + Sinh[x])^5)
```

Rubi [A] (verified)

Time = 0.36 (sec) , antiderivative size = 57, normalized size of antiderivative = 1.14, number of steps used = 9, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.692$, Rules used = {3042, 3161, 3042, 3115, 3042, 3115, 3042, 3115, 24}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cosh^8(x)}{\sinh(x)+i}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\cos(ix)^8}{i-i\sin(ix)}dx$$

$$\Big\downarrow \text{3161}$$

$$\frac{\cosh^7(x)}{7}-i\int \cosh^6(x)dx$$

$$\Big\downarrow \text{3042}$$

$$\frac{\cosh^7(x)}{7}-i\int \sin\left(ix+\frac{\pi}{2}\right)^6dx$$

$$\Big\downarrow \text{3115}$$

$$\frac{\cosh^7(x)}{7}-i\left(\frac{5}{6}\int \cosh^4(x)dx+\frac{1}{6}\sinh(x)\cosh^5(x)\right)$$

$$\Big\downarrow \text{3042}$$

$$\frac{\cosh^7(x)}{7} - i\left(\frac{1}{6}\sinh(x)\cosh^5(x) + \frac{5}{6}\int \sin\left(ix + \frac{\pi}{2}\right)^4 dx\right)$$

$$\Big\downarrow \text{3115}$$

$$\frac{\cosh^7(x)}{7} - i\left(\frac{5}{6}\left(\frac{3}{4}\int \cosh^2(x)dx + \frac{1}{4}\sinh(x)\cosh^3(x)\right) + \frac{1}{6}\sinh(x)\cosh^5(x)\right)$$

$$\Big\downarrow \text{3042}$$

$$\frac{\cosh^7(x)}{7} - i\left(\frac{1}{6}\sinh(x)\cosh^5(x) + \frac{5}{6}\left(\frac{1}{4}\sinh(x)\cosh^3(x) + \frac{3}{4}\int \sin\left(ix + \frac{\pi}{2}\right)^2 dx\right)\right)$$

$$\Big\downarrow \text{3115}$$

$$\frac{\cosh^7(x)}{7} - i\left(\frac{5}{6}\left(\frac{3}{4}\left(\frac{\int 1dx}{2} + \frac{1}{2}\sinh(x)\cosh(x)\right) + \frac{1}{4}\sinh(x)\cosh^3(x)\right) + \frac{1}{6}\sinh(x)\cosh^5(x)\right)$$

$$\Big\downarrow \text{24}$$

$$\frac{\cosh^7(x)}{7} - i\left(\frac{1}{6}\sinh(x)\cosh^5(x) + \frac{5}{6}\left(\frac{1}{4}\sinh(x)\cosh^3(x) + \frac{3}{4}\left(\frac{x}{2} + \frac{1}{2}\sinh(x)\cosh(x)\right)\right)\right)$$

input
```
Int[Cosh[x]^8/(I + Sinh[x]),x]
```

output
```
Cosh[x]^7/7 - I*((Cosh[x]^5*Sinh[x])/6 + (5*((Cosh[x]^3*Sinh[x])/4 + (3*(x
/2 + (Cosh[x]*Sinh[x])/2))/4))/6)
```

Defintions of rubi rules used

rule 24
```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3115
```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Simp[b^2*((n - 1)/n)   Int[(b*Sin
[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[
2*n]
```

rule 3161
```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)/((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Simp[g*((g*Cos[e + f*x])^(p - 1)/(b*f*(p - 1))), x] + Si
mp[g^2/a   Int[(g*Cos[e + f*x])^(p - 2), x], x] /; FreeQ[{a, b, e, f, g}, x
] && EqQ[a^2 - b^2, 0] && GtQ[p, 1] && IntegerQ[2*p]
```

Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 185 vs. $2(36) = 72$.

Time = 0.09 (sec) , antiderivative size = 186, normalized size of antiderivative = 3.72

$$\frac{\frac{5}{16}-\frac{11i}{16}}{\tanh\left(\frac{x}{2}\right)+1}+\frac{-\frac{5}{4}+i}{\left(\tanh\left(\frac{x}{2}\right)+1\right)^{4}}+\frac{-\frac{11}{16}-\frac{19i}{16}}{\left(\tanh\left(\frac{x}{2}\right)-1\right)^{2}}+\frac{1-\frac{i}{2}}{\left(\tanh\left(\frac{x}{2}\right)+1\right)^{5}}+\frac{-\frac{9}{8}-\frac{7i}{6}}{\left(\tanh\left(\frac{x}{2}\right)-1\right)^{3}}+\frac{-\frac{5}{16}-\frac{11i}{16}}{\tanh\left(\frac{x}{2}\right)-1}+\frac{}{(t}$$

input
```
int(cosh(x)^8/(I+sinh(x)),x)
```

output
```
(5/16-11/16*I)/(tanh(1/2*x)+1)+(-5/4+I)/(tanh(1/2*x)+1)^4-(11/16+19/16*I)/
(tanh(1/2*x)-1)^2+(1-1/2*I)/(tanh(1/2*x)+1)^5-(9/8+7/6*I)/(tanh(1/2*x)-1)^
3-(5/16+11/16*I)/(tanh(1/2*x)-1)-(1/2+1/6*I)/(tanh(1/2*x)-1)^6+1/7/(tanh(1
/2*x)+1)^7-(1+1/2*I)/(tanh(1/2*x)-1)^5+(9/8-7/6*I)/(tanh(1/2*x)+1)^3+(-1/2
+1/6*I)/(tanh(1/2*x)+1)^6-(5/4+I)/(tanh(1/2*x)-1)^4+(-11/16+19/16*I)/(tanh
(1/2*x)+1)^2+5/16*I*ln(tanh(1/2*x)-1)-5/16*I*ln(tanh(1/2*x)+1)-1/7/(tanh(1
/2*x)-1)^7
```

Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 91 vs. $2(32) = 64$.

Time = 0.09 (sec) , antiderivative size = 91, normalized size of antiderivative = 1.82

$$\int \frac{\cosh^8(x)}{i+\sinh(x)}\,dx$$
$$= \frac{1}{2688}\left(-840i\,xe^{(7\,x)} + 3\,e^{(14\,x)} - 7i\,e^{(13\,x)} + 21\,e^{(12\,x)} - 63i\,e^{(11\,x)} + 63\,e^{(10\,x)} - 315i\,e^{(9\,x)} + 105\,e^{(8\,x)} + 105\right.$$

input
```
integrate(cosh(x)^8/(I+sinh(x)),x, algorithm="fricas")
```

output
```
1/2688*(-840*I*x*e^(7*x) + 3*e^(14*x) - 7*I*e^(13*x) + 21*e^(12*x) - 63*I*
e^(11*x) + 63*e^(10*x) - 315*I*e^(9*x) + 105*e^(8*x) + 105*e^(6*x) + 315*I
*e^(5*x) + 63*e^(4*x) + 63*I*e^(3*x) + 21*e^(2*x) + 7*I*e^x + 3)*e^(-7*x)
```

Sympy [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 124 vs. $2(49) = 98$.

Time = 0.21 (sec) , antiderivative size = 124, normalized size of antiderivative = 2.48

$$\int \frac{\cosh^8(x)}{i+\sinh(x)}\,dx = -\frac{5ix}{16} + \frac{e^{7x}}{896} - \frac{ie^{6x}}{384} + \frac{e^{5x}}{128} - \frac{3ie^{4x}}{128} + \frac{3e^{3x}}{128} - \frac{15ie^{2x}}{128} + \frac{5e^{x}}{128} + \frac{5e^{-x}}{128} + \frac{15ie^{-2x}}{128} + \frac{3e^{-3x}}{128} + \frac{3ie^{-4x}}{128} + \frac{e^{-5x}}{128} + \frac{ie^{-6x}}{384} + \frac{e^{-7x}}{896}$$

input
```
integrate(cosh(x)**8/(I+sinh(x)),x)
```

output
```
-5*I*x/16 + exp(7*x)/896 - I*exp(6*x)/384 + exp(5*x)/128 - 3*I*exp(4*x)/12
8 + 3*exp(3*x)/128 - 15*I*exp(2*x)/128 + 5*exp(x)/128 + 5*exp(-x)/128 + 15
*I*exp(-2*x)/128 + 3*exp(-3*x)/128 + 3*I*exp(-4*x)/128 + exp(-5*x)/128 + I
*exp(-6*x)/384 + exp(-7*x)/896
```

Maxima [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 90 vs. $2(32) = 64$.

Time = 0.03 (sec) , antiderivative size = 90, normalized size of antiderivative = 1.80

$$\int \frac{\cosh^8(x)}{i+\sinh(x)}\,dx = -\frac{1}{2688}\left(7i\,e^{(-x)} - 21\,e^{(-2\,x)} + 63i\,e^{(-3\,x)} - 63\,e^{(-4\,x)} + 315i\,e^{(-5\,x)} - 105\,e^{(-6\,x)} - 3\right)e^{(7\,x)} - \frac{5}{16}i\,x + \frac{5}{128}\,e^{(-x)} + \frac{15}{128}i\,e^{(-2\,x)} + \frac{3}{128}\,e^{(-3\,x)} + \frac{3}{128}i\,e^{(-4\,x)} + \frac{1}{128}\,e^{(-5\,x)} + \frac{1}{384}i\,e^{(-6\,x)} + \frac{1}{896}\,e^{(-7\,x)}$$

input
```
integrate(cosh(x)^8/(I+sinh(x)),x, algorithm="maxima")
```

output
```
-1/2688*(7*I*e^(-x) - 21*e^(-2*x) + 63*I*e^(-3*x) - 63*e^(-4*x) + 315*I*e^
(-5*x) - 105*e^(-6*x) - 3)*e^(7*x) - 5/16*I*x + 5/128*e^(-x) + 15/128*I*e^
(-2*x) + 3/128*e^(-3*x) + 3/128*I*e^(-4*x) + 1/128*e^(-5*x) + 1/384*I*e^(-
6*x) + 1/896*e^(-7*x)
```

Giac [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 86 vs. $2(32) = 64$.

Time = 0.13 (sec) , antiderivative size = 86, normalized size of antiderivative = 1.72

$$\int \frac{\cosh^8(x)}{i+\sinh(x)}\,dx = \frac{1}{2688}\left(105\,e^{(6\,x)} + 315i\,e^{(5\,x)} + 63\,e^{(4\,x)} + 63i\,e^{(3\,x)} + 21\,e^{(2\,x)} + 7i\,e^{x} + 3\right)e^{(-7\,x)} - \frac{5}{16}i\,x + \frac{1}{896}\,e^{(7\,x)} - \frac{1}{384}i\,e^{(6\,x)} + \frac{1}{128}\,e^{(5\,x)} - \frac{3}{128}i\,e^{(4\,x)} + \frac{3}{128}\,e^{(3\,x)} - \frac{15}{128}i\,e^{(2\,x)} + \frac{5}{128}\,e^{x}$$

input
```
integrate(cosh(x)^8/(I+sinh(x)),x, algorithm="giac")
```

output
```
1/2688*(105*e^(6*x) + 315*I*e^(5*x) + 63*e^(4*x) + 63*I*e^(3*x) + 21*e^(2*
x) + 7*I*e^x + 3)*e^(-7*x) - 5/16*I*x + 1/896*e^(7*x) - 1/384*I*e^(6*x) +
1/128*e^(5*x) - 3/128*I*e^(4*x) + 3/128*e^(3*x) - 15/128*I*e^(2*x) + 5/128
*e^x
```

Mupad [B] (verification not implemented)

Time = 1.65 (sec) , antiderivative size = 93, normalized size of antiderivative = 1.86

$$\int \frac{\cosh^8(x)}{i+\sinh(x)}\,dx = \frac{5\,\mathrm{e}^{-x}}{128}+\frac{3\,\mathrm{e}^{-3\,x}}{128}+\frac{3\,\mathrm{e}^{3\,x}}{128}+\frac{\mathrm{e}^{-5\,x}}{128}+\frac{\mathrm{e}^{5\,x}}{128}+\frac{\mathrm{e}^{-7\,x}}{896}+\frac{\mathrm{e}^{7\,x}}{896}+\frac{5\,\mathrm{e}^{x}}{128} -\frac{x\,5\mathrm{i}}{16}+\frac{\mathrm{e}^{-2\,x}\,15\mathrm{i}}{128}-\frac{\mathrm{e}^{2\,x}\,15\mathrm{i}}{128}+\frac{\mathrm{e}^{-4\,x}\,3\mathrm{i}}{128}-\frac{\mathrm{e}^{4\,x}\,3\mathrm{i}}{128}+\frac{\mathrm{e}^{-6\,x}\,1\mathrm{i}}{384}-\frac{\mathrm{e}^{6\,x}\,1\mathrm{i}}{384}$$

input
```
int(cosh(x)^8/(sinh(x) + 1i),x)
```

output
```
(5*exp(-x))/128 - (x*5i)/16 + (exp(-2*x)*15i)/128 - (exp(2*x)*15i)/128 + (
3*exp(-3*x))/128 + (3*exp(3*x))/128 + (exp(-4*x)*3i)/128 - (exp(4*x)*3i)/1
28 + exp(-5*x)/128 + exp(5*x)/128 + (exp(-6*x)*1i)/384 - (exp(6*x)*1i)/384
 + exp(-7*x)/896 + exp(7*x)/896 + (5*exp(x))/128
```

Reduce [F]

$$\int \frac{\cosh^8(x)}{i+\sinh(x)}\,dx = \int \frac{\cosh(x)^8}{\sinh(x)+i}dx$$

input
```
int(cosh(x)^8/(I+sinh(x)),x)
```

output
```
int(cosh(x)**8/(sinh(x) + i),x)
```

3.159 $\int \frac{\cosh^7(x)}{i+\sinh(x)}\,dx$

Optimal result

Integrand size = 13, antiderivative size = 43

$$\int \frac{\cosh^7(x)}{i+\sinh(x)}\,dx = -(i-\sinh(x))^4 - \frac{4}{5}i(i-\sinh(x))^5 + \frac{1}{6}(i-\sinh(x))^6$$

output
```
-(I-sinh(x))^4-4/5*I*(I-sinh(x))^5+1/6*(I-sinh(x))^6
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.98

$$\int \frac{\cosh^7(x)}{i+\sinh(x)}\,dx = \frac{1}{30}\sinh(x)\left(-30i+15\sinh(x)-20i\sinh^2(x)+15\sinh^3(x)-6i\sinh^4(x)+5\sinh^5(x)\right)$$

input
```
Integrate[Cosh[x]^7/(I + Sinh[x]),x]
```

output
```
(Sinh[x]*(-30*I + 15*Sinh[x] - (20*I)*Sinh[x]^2 + 15*Sinh[x]^3 - (6*I)*Sin
h[x]^4 + 5*Sinh[x]^5))/30
```

Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 43, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.308, Rules used = {3042, 3146, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cosh^7(x)}{\sinh(x)+i}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\cos(ix)^7}{i-i\sin(ix)}dx$$

$$\Big\downarrow \text{3146}$$

$$-\int (i-\sinh(x))^3(\sinh(x)+i)^2d\sinh(x)$$

$$\Big\downarrow \text{49}$$

$$-\int \left((i-\sinh(x))^5-4i(i-\sinh(x))^4-4(i-\sinh(x))^3\right)d\sinh(x)$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{6}(-\sinh(x)+i)^6-\frac{4}{5}i(-\sinh(x)+i)^5-(-\sinh(x)+i)^4$$

input
```
Int[Cosh[x]^7/(I + Sinh[x]),x]
```

output
```
-(I - Sinh[x])^4 - ((4*I)/5)*(I - Sinh[x])^5 + (I - Sinh[x])^6/6
```

Defintions of rubi rules used

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3146
```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Simp[1/(b^p*f)   Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x
)^((p - 1)/2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && I
ntegerQ[(p - 1)/2] && EqQ[a^2 - b^2, 0] && (GeQ[p, -1] ||  !IntegerQ[m + 1/
2])
```

Maple [A] (verified)

Time = 0.08 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.91

$$-i\sinh(x)+\frac{\sinh(x)^6}{6}-\frac{i\sinh(x)^5}{5}+\frac{\sinh(x)^4}{2}-\frac{2i\sinh(x)^3}{3}+\frac{\sinh(x)^2}{2}$$

input
```
int(cosh(x)^7/(I+sinh(x)),x)
```

output
```
-I*sinh(x)+1/6*sinh(x)^6-1/5*I*sinh(x)^5+1/2*sinh(x)^4-2/3*I*sinh(x)^3+1/2
*sinh(x)^2
```

Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 72 vs. $2(25) = 50$.

Time = 0.08 (sec) , antiderivative size = 72, normalized size of antiderivative = 1.67

$$\int \frac{\cosh^7(x)}{i+\sinh(x)}\,dx$$
$$= \frac{1}{1920}\left(5\,e^{(12\,x)} - 12i\,e^{(11\,x)} + 30\,e^{(10\,x)} - 100i\,e^{(9\,x)} + 75\,e^{(8\,x)} - 600i\,e^{(7\,x)} + 600i\,e^{(5\,x)} + 75\,e^{(4\,x)} + 100i\,e^{(\right.$$

input
```
integrate(cosh(x)^7/(I+sinh(x)),x, algorithm="fricas")
```

output
```
1/1920*(5*e^(12*x) - 12*I*e^(11*x) + 30*e^(10*x) - 100*I*e^(9*x) + 75*e^(8
*x) - 600*I*e^(7*x) + 600*I*e^(5*x) + 75*e^(4*x) + 100*I*e^(3*x) + 30*e^(2
*x) + 12*I*e^x + 5)*e^(-6*x)
```

Sympy [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 100 vs. $2(26) = 52$.

Time = 0.18 (sec) , antiderivative size = 100, normalized size of antiderivative = 2.33

$$\int \frac{\cosh^7(x)}{i+\sinh(x)}\,dx = \frac{e^{6x}}{384} - \frac{ie^{5x}}{160} + \frac{e^{4x}}{64} - \frac{5ie^{3x}}{96} + \frac{5e^{2x}}{128} - \frac{5ie^{x}}{16} + \frac{5ie^{-x}}{16} + \frac{5e^{-2x}}{128} + \frac{5ie^{-3x}}{96} + \frac{e^{-4x}}{64} + \frac{ie^{-5x}}{160} + \frac{e^{-6x}}{384}$$

input
```
integrate(cosh(x)**7/(I+sinh(x)),x)
```

output
```
exp(6*x)/384 - I*exp(5*x)/160 + exp(4*x)/64 - 5*I*exp(3*x)/96 + 5*exp(2*x)
/128 - 5*I*exp(x)/16 + 5*I*exp(-x)/16 + 5*exp(-2*x)/128 + 5*I*exp(-3*x)/96
 + exp(-4*x)/64 + I*exp(-5*x)/160 + exp(-6*x)/384
```

Maxima [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 75 vs. 2(25) = 50.

Time = 0.05 (sec) , antiderivative size = 75, normalized size of antiderivative = 1.74

$$\int \frac{\cosh^7(x)}{i+\sinh(x)}\,dx = -\frac{1}{1920}\left(12i\,e^{(-x)} - 30\,e^{(-2\,x)} + 100i\,e^{(-3\,x)} - 75\,e^{(-4\,x)} + 600i\,e^{(-5\,x)} - 5\right)e^{(6\,x)} + \frac{5}{16}i\,e^{(-x)} + \frac{5}{128}\,e^{(-2\,x)} + \frac{5}{96}i\,e^{(-3\,x)} + \frac{1}{64}\,e^{(-4\,x)} + \frac{1}{160}i\,e^{(-5\,x)} + \frac{1}{384}\,e^{(-6\,x)}$$

input
```
integrate(cosh(x)^7/(I+sinh(x)),x, algorithm="maxima")
```

output
```
-1/1920*(12*I*e^(-x) - 30*e^(-2*x) + 100*I*e^(-3*x) - 75*e^(-4*x) + 600*I*
e^(-5*x) - 5)*e^(6*x) + 5/16*I*e^(-x) + 5/128*e^(-2*x) + 5/96*I*e^(-3*x) +
 1/64*e^(-4*x) + 1/160*I*e^(-5*x) + 1/384*e^(-6*x)
```

Giac [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 71 vs. 2(25) = 50.

Time = 0.12 (sec) , antiderivative size = 71, normalized size of antiderivative = 1.65

$$\int \frac{\cosh^7(x)}{i+\sinh(x)}\,dx = -\frac{1}{1920}\left(-600i\,e^{(5\,x)} - 75\,e^{(4\,x)} - 100i\,e^{(3\,x)} - 30\,e^{(2\,x)} - 12i\,e^{x} - 5\right)e^{(-6\,x)} + \frac{1}{384}\,e^{(6\,x)} - \frac{1}{160}i\,e^{(5\,x)} + \frac{1}{64}\,e^{(4\,x)} - \frac{5}{96}i\,e^{(3\,x)} + \frac{5}{128}\,e^{(2\,x)} - \frac{5}{16}i\,e^{x}$$

input
```
integrate(cosh(x)^7/(I+sinh(x)),x, algorithm="giac")
```

output
```
-1/1920*(-600*I*e^(5*x) - 75*e^(4*x) - 100*I*e^(3*x) - 30*e^(2*x) - 12*I*e
^x - 5)*e^(-6*x) + 1/384*e^(6*x) - 1/160*I*e^(5*x) + 1/64*e^(4*x) - 5/96*I
*e^(3*x) + 5/128*e^(2*x) - 5/16*I*e^x
```

Mupad [B] (verification not implemented)

Time = 1.53 (sec) , antiderivative size = 77, normalized size of antiderivative = 1.79

$$\int \frac{\cosh^7(x)}{i+\sinh(x)}\,dx = \frac{e^{-x}\,5i}{16} + \frac{5\,e^{-2\,x}}{128} + \frac{5\,e^{2\,x}}{128} + \frac{e^{-3\,x}\,5i}{96} - \frac{e^{3\,x}\,5i}{96} + \frac{e^{-4\,x}}{64} + \frac{e^{4\,x}}{64} + \frac{e^{-5\,x}\,1i}{160} - \frac{e^{5\,x}\,1i}{160} + \frac{e^{-6\,x}}{384} + \frac{e^{6\,x}}{384} - \frac{e^{x}\,5i}{16}$$

input
```
int(cosh(x)^7/(sinh(x) + 1i),x)
```

output
```
(exp(-x)*5i)/16 + (5*exp(-2*x))/128 + (5*exp(2*x))/128 + (exp(-3*x)*5i)/96
 - (exp(3*x)*5i)/96 + exp(-4*x)/64 + exp(4*x)/64 + (exp(-5*x)*1i)/160 - (e
xp(5*x)*1i)/160 + exp(-6*x)/384 + exp(6*x)/384 - (exp(x)*5i)/16
```

Reduce [F]

$$\int \frac{\cosh^7(x)}{i+\sinh(x)}\,dx = \int \frac{\cosh(x)^7}{\sinh(x)+i}dx$$

input
```
int(cosh(x)^7/(I+sinh(x)),x)
```

output
```
int(cosh(x)**7/(sinh(x) + i),x)
```

3.160 $\int \frac{\cosh^6(x)}{i+\sinh(x)}\,dx$

Optimal result

Integrand size = 13, antiderivative size = 38

$$\int \frac{\cosh^6(x)}{i+\sinh(x)}\,dx=-\frac{3ix}{8}+\frac{\cosh^5(x)}{5}-\frac{3}{8}i\cosh(x)\sinh(x)-\frac{1}{4}i\cosh^3(x)\sinh(x)$$

output
```
-3/8*I*x+1/5*cosh(x)^5-3/8*I*cosh(x)*sinh(x)-1/4*I*cosh(x)^3*sinh(x)
```

Mathematica [B] (verified)

Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 131 vs. 2(38) = 76.

Time = 0.16 (sec) , antiderivative size = 131, normalized size of antiderivative = 3.45

$$\int \frac{\cosh^6(x)}{i+\sinh(x)}\,dx = -\frac{i\cosh^7(x)\left(8i+\frac{30i\arcsin\left(\frac{\sqrt{1-i\sinh(x)}}{\sqrt{2}}\right)\sqrt{1-i\sinh(x)}}{\sqrt{1+i\sinh(x)}}+33\sinh(x)-9i\sinh^2(x)+26\sinh^3(x)-2i\sinh^4(x)\right.}{40\left(\cosh\left(\frac{x}{2}\right)-i\sinh\left(\frac{x}{2}\right)\right)^8\left(\cosh\left(\frac{x}{2}\right)+i\sinh\left(\frac{x}{2}\right)\right)^6}$$

input
```
Integrate[Cosh[x]^6/(I + Sinh[x]),x]
```

output
```
((-1/40*I)*Cosh[x]^7*(8*I + ((30*I)*ArcSin[Sqrt[1 - I*Sinh[x]]/Sqrt[2]]*Sq
rt[1 - I*Sinh[x]])/Sqrt[1 + I*Sinh[x]] + 33*Sinh[x] - (9*I)*Sinh[x]^2 + 26
*Sinh[x]^3 - (2*I)*Sinh[x]^4 + 8*Sinh[x]^5))/((Cosh[x/2] - I*Sinh[x/2])^8*
(Cosh[x/2] + I*Sinh[x/2])^6)
```

Rubi [A] (verified)

Time = 0.32 (sec) , antiderivative size = 42, normalized size of antiderivative = 1.11, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.538$, Rules used = {3042, 3161, 3042, 3115, 3042, 3115, 24}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cosh^6(x)}{\sinh(x)+i}dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \frac{\cos(ix)^6}{i-i\sin(ix)}dx$$

$$\Big\downarrow \text{ 3161}$$

$$\frac{\cosh^5(x)}{5}-i\int \cosh^4(x)dx$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{\cosh^5(x)}{5}-i\int \sin\left(ix+\frac{\pi}{2}\right)^4dx$$

$$\Big\downarrow \text{ 3115}$$

$$\frac{\cosh^5(x)}{5}-i\left(\frac{3}{4}\int \cosh^2(x)dx+\frac{1}{4}\sinh(x)\cosh^3(x)\right)$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{\cosh^5(x)}{5}-i\left(\frac{1}{4}\sinh(x)\cosh^3(x)+\frac{3}{4}\int \sin\left(ix+\frac{\pi}{2}\right)^2dx\right)$$

$$\Big\downarrow \text{ 3115}$$

$$\frac{\cosh^5(x)}{5}-i\left(\frac{3}{4}\left(\frac{\int 1dx}{2}+\frac{1}{2}\sinh(x)\cosh(x)\right)+\frac{1}{4}\sinh(x)\cosh^3(x)\right)$$

$$\Big\downarrow \text{24}$$

$$\frac{\cosh^5(x)}{5} - i\left(\frac{1}{4}\sinh(x)\cosh^3(x) + \frac{3}{4}\left(\frac{x}{2} + \frac{1}{2}\sinh(x)\cosh(x)\right)\right)$$

input
```
Int[Cosh[x]^6/(I + Sinh[x]),x]
```

output
```
Cosh[x]^5/5 - I*((Cosh[x]^3*Sinh[x])/4 + (3*(x/2 + (Cosh[x]*Sinh[x])/2))/4
)
```

Defintions of rubi rules used

rule 24
```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3115
```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Simp[b^2*((n - 1)/n)   Int[(b*Sin
[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[
2*n]
```

rule 3161
```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)/((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Simp[g*((g*Cos[e + f*x])^(p - 1)/(b*f*(p - 1))), x] + Si
mp[g^2/a   Int[(g*Cos[e + f*x])^(p - 2), x], x] /; FreeQ[{a, b, e, f, g}, x
] && EqQ[a^2 - b^2, 0] && GtQ[p, 1] && IntegerQ[2*p]
```

Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 137 vs. $2(27) = 54$.

Time = 0.08 (sec) , antiderivative size = 138, normalized size of antiderivative = 3.63

$$-\frac{3i\ln\left(\tanh\left(\frac{x}{2}\right)+1\right)}{8}+\frac{-\frac{1}{2}+\frac{i}{4}}{\left(\tanh\left(\frac{x}{2}\right)+1\right)^{4}}+\frac{\frac{3}{4}-\frac{i}{2}}{\left(\tanh\left(\frac{x}{2}\right)+1\right)^{3}}+\frac{\frac{3}{8}-\frac{5i}{8}}{\tanh\left(\frac{x}{2}\right)+1}+\frac{-\frac{5}{8}+\frac{7i}{8}}{\left(\tanh\left(\frac{x}{2}\right)+1\right)^{2}}+\frac{1}{5\left(\tanh\left(\frac{x}{2}\right)\right.}$$

input
```
int(cosh(x)^6/(I+sinh(x)),x)
```

output
```
-3/8*I*ln(tanh(1/2*x)+1)+(-1/2+1/4*I)/(tanh(1/2*x)+1)^4+(3/4-1/2*I)/(tanh(
1/2*x)+1)^3+(3/8-5/8*I)/(tanh(1/2*x)+1)+(-5/8+7/8*I)/(tanh(1/2*x)+1)^2+1/5
/(tanh(1/2*x)+1)^5+3/8*I*ln(tanh(1/2*x)-1)-(1/2+1/4*I)/(tanh(1/2*x)-1)^4-(
3/8+5/8*I)/(tanh(1/2*x)-1)-(5/8+7/8*I)/(tanh(1/2*x)-1)^2-(3/4+1/2*I)/(tanh
(1/2*x)-1)^3-1/5/(tanh(1/2*x)-1)^5
```

Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 67 vs. $2(24) = 48$.

Time = 0.09 (sec) , antiderivative size = 67, normalized size of antiderivative = 1.76

$$\int \frac{\cosh^6(x)}{i+\sinh(x)}\,dx$$
$$=\frac{1}{320}\left(-120i\,xe^{(5\,x)}+2\,e^{(10\,x)}-5i\,e^{(9\,x)}+10\,e^{(8\,x)}-40i\,e^{(7\,x)}+20\,e^{(6\,x)}+20\,e^{(4\,x)}+40i\,e^{(3\,x)}+10\,e^{(2\,x)}+\right.$$

input
```
integrate(cosh(x)^6/(I+sinh(x)),x, algorithm="fricas")
```

output
```
1/320*(-120*I*x*e^(5*x) + 2*e^(10*x) - 5*I*e^(9*x) + 10*e^(8*x) - 40*I*e^(
7*x) + 20*e^(6*x) + 20*e^(4*x) + 40*I*e^(3*x) + 10*e^(2*x) + 5*I*e^x + 2)*
e^(-5*x)
```

Sympy [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 82 vs. 2(36) = 72.

Time = 0.15 (sec) , antiderivative size = 82, normalized size of antiderivative = 2.16

$$\int \frac{\cosh^6(x)}{i+\sinh(x)}\,dx = -\frac{3ix}{8} + \frac{e^{5x}}{160} - \frac{ie^{4x}}{64} + \frac{e^{3x}}{32} - \frac{ie^{2x}}{8} + \frac{e^{x}}{16} + \frac{e^{-x}}{16} + \frac{ie^{-2x}}{8} + \frac{e^{-3x}}{32} + \frac{ie^{-4x}}{64} + \frac{e^{-5x}}{160}$$

input
```
integrate(cosh(x)**6/(I+sinh(x)),x)
```

output
```
-3*I*x/8 + exp(5*x)/160 - I*exp(4*x)/64 + exp(3*x)/32 - I*exp(2*x)/8 + exp
(x)/16 + exp(-x)/16 + I*exp(-2*x)/8 + exp(-3*x)/32 + I*exp(-4*x)/64 + exp(
-5*x)/160
```

Maxima [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 66 vs. 2(24) = 48.

Time = 0.03 (sec) , antiderivative size = 66, normalized size of antiderivative = 1.74

$$\int \frac{\cosh^6(x)}{i+\sinh(x)}\,dx = -\frac{1}{320}\left(5i\,e^{(-x)} - 10\,e^{(-2\,x)} + 40i\,e^{(-3\,x)} - 20\,e^{(-4\,x)} - 2\right)e^{(5\,x)} - \frac{3}{8}i\,x + \frac{1}{16}\,e^{(-x)} + \frac{1}{8}i\,e^{(-2\,x)} + \frac{1}{32}\,e^{(-3\,x)} + \frac{1}{64}i\,e^{(-4\,x)} + \frac{1}{160}\,e^{(-5\,x)}$$

input
```
integrate(cosh(x)^6/(I+sinh(x)),x, algorithm="maxima")
```

output
```
-1/320*(5*I*e^(-x) - 10*e^(-2*x) + 40*I*e^(-3*x) - 20*e^(-4*x) - 2)*e^(5*x
) - 3/8*I*x + 1/16*e^(-x) + 1/8*I*e^(-2*x) + 1/32*e^(-3*x) + 1/64*I*e^(-4*
x) + 1/160*e^(-5*x)
```

Giac [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 62 vs. $2(24) = 48$.

Time = 0.12 (sec) , antiderivative size = 62, normalized size of antiderivative = 1.63

$$\int \frac{\cosh^6(x)}{i+\sinh(x)}\,dx = \frac{1}{320}\left(20\,e^{(4\,x)} + 40i\,e^{(3\,x)} + 10\,e^{(2\,x)} + 5i\,e^{x} + 2\right)e^{(-5\,x)} - \frac{3}{8}i\,x + \frac{1}{160}\,e^{(5\,x)} - \frac{1}{64}i\,e^{(4\,x)} + \frac{1}{32}\,e^{(3\,x)} - \frac{1}{8}i\,e^{(2\,x)} + \frac{1}{16}\,e^{x}$$

input
```
integrate(cosh(x)^6/(I+sinh(x)),x, algorithm="giac")
```

output
```
1/320*(20*e^(4*x) + 40*I*e^(3*x) + 10*e^(2*x) + 5*I*e^x + 2)*e^(-5*x) - 3/
8*I*x + 1/160*e^(5*x) - 1/64*I*e^(4*x) + 1/32*e^(3*x) - 1/8*I*e^(2*x) + 1/
16*e^x
```

Mupad [B] (verification not implemented)

Time = 1.48 (sec) , antiderivative size = 67, normalized size of antiderivative = 1.76

$$\int \frac{\cosh^6(x)}{i+\sinh(x)}\,dx = \frac{e^{-x}}{16} + \frac{e^{-3\,x}}{32} + \frac{e^{3\,x}}{32} + \frac{e^{-5\,x}}{160} + \frac{e^{5\,x}}{160} + \frac{e^{x}}{16} - \frac{x\,3\mathrm{i}}{8} + \frac{e^{-2\,x}\,1\mathrm{i}}{8} - \frac{e^{2\,x}\,1\mathrm{i}}{8} + \frac{e^{-4\,x}\,1\mathrm{i}}{64} - \frac{e^{4\,x}\,1\mathrm{i}}{64}$$

input
```
int(cosh(x)^6/(sinh(x) + 1i),x)
```

output
```
exp(-x)/16 - (x*3i)/8 + (exp(-2*x)*1i)/8 - (exp(2*x)*1i)/8 + exp(-3*x)/32
+ exp(3*x)/32 + (exp(-4*x)*1i)/64 - (exp(4*x)*1i)/64 + exp(-5*x)/160 + exp
(5*x)/160 + exp(x)/16
```

Reduce [F]

$$\int \frac{\cosh^6(x)}{i + \sinh(x)}\, dx = \int \frac{\cosh(x)^6}{\sinh(x) + i} dx$$

input

```
int(cosh(x)^6/(I+sinh(x)),x)
```

output

```
int(cosh(x)**6/(sinh(x) + i),x)
```

3.161 $\int \frac{\cosh^5(x)}{i+\sinh(x)}\,dx$

Optimal result

Integrand size = 13, antiderivative size = 33

$$\int \frac{\cosh^5(x)}{i+\sinh(x)}\,dx = -i\sinh(x)+\frac{\sinh^2(x)}{2}-\frac{1}{3}i\sinh^3(x)+\frac{\sinh^4(x)}{4}$$

output
```
-I*sinh(x)+1/2*sinh(x)^2-1/3*I*sinh(x)^3+1/4*sinh(x)^4
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.85

$$\int \frac{\cosh^5(x)}{i+\sinh(x)}\,dx = \frac{1}{12}\sinh(x)\left(-12i+6\sinh(x)-4i\sinh^2(x)+3\sinh^3(x)\right)$$

input
```
Integrate[Cosh[x]^5/(I + Sinh[x]),x]
```

output
```
(Sinh[x]*(-12*I + 6*Sinh[x] - (4*I)*Sinh[x]^2 + 3*Sinh[x]^3))/12
```

Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 33, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.308$, Rules used = {3042, 3146, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cosh^5(x)}{\sinh(x)+i}dx$$

$$\Big\downarrow \; 3042$$

$$\int \frac{\cos(ix)^5}{i-i\sin(ix)}dx$$

$$\Big\downarrow \; 3146$$

$$\int (-\sinh(x)+i)^2(\sinh(x)+i)d\sinh(x)$$

$$\Big\downarrow \; 49$$

$$\int \left(\sinh^3(x)-i\sinh^2(x)+\sinh(x)-i\right)d\sinh(x)$$

$$\Big\downarrow \; 2009$$

$$\frac{\sinh^4(x)}{4}-\frac{1}{3}i\sinh^3(x)+\frac{\sinh^2(x)}{2}-i\sinh(x)$$

input
```
Int[Cosh[x]^5/(I + Sinh[x]),x]
```

output
```
(-I)*Sinh[x] + Sinh[x]^2/2 - (I/3)*Sinh[x]^3 + Sinh[x]^4/4
```

Defintions of rubi rules used

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3146
```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Simp[1/(b^p*f)   Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x
)^((p - 1)/2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && I
ntegerQ[(p - 1)/2] && EqQ[a^2 - b^2, 0] && (GeQ[p, -1] ||  !IntegerQ[m + 1/
2])
```

Maple [A] (verified)

Time = 91.65 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.79

method	result	size
derivativedivides	$-i \sinh \left(x \right)+\frac{\sinh \left(x \right)^{2}}{2}-\frac{i \sinh \left(x \right)^{3}}{3}+\frac{\sinh \left(x \right)^{4}}{4}$	26
default	$-i \sinh \left(x \right)+\frac{\sinh \left(x \right)^{2}}{2}-\frac{i \sinh \left(x \right)^{3}}{3}+\frac{\sinh \left(x \right)^{4}}{4}$	26
risch	$\frac{{\mathrm e}^{4 x}}{64}-\frac{i {\mathrm e}^{3 x}}{24}+\frac{{\mathrm e}^{2 x}}{16}-\frac{3 i {\mathrm e}^{x}}{8}+\frac{3 i {\mathrm e}^{-x}}{8}+\frac{{\mathrm e}^{-2 x}}{16}+\frac{i {\mathrm e}^{-3 x}}{24}+\frac{{\mathrm e}^{-4 x}}{64}$	52

input
```
int(cosh(x)^5/(I+sinh(x)),x,method=_RETURNVERBOSE)
```

output
```
-I*sinh(x)+1/2*sinh(x)^2-1/3*I*sinh(x)^3+1/4*sinh(x)^4
```

Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 48 vs. 2(23) = 46.

Time = 0.10 (sec) , antiderivative size = 48, normalized size of antiderivative = 1.45

$$\int \frac{\cosh^5(x)}{i+\sinh(x)}\,dx = \frac{1}{192}\left(3\,e^{(8\,x)} - 8i\,e^{(7\,x)} + 12\,e^{(6\,x)} - 72i\,e^{(5\,x)} + 72i\,e^{(3\,x)} + 12\,e^{(2\,x)} + 8i\,e^{x} + 3\right)e^{(-4\,x)}$$

input
```
integrate(cosh(x)^5/(I+sinh(x)),x, algorithm="fricas")
```

output
```
1/192*(3*e^(8*x) - 8*I*e^(7*x) + 12*e^(6*x) - 72*I*e^(5*x) + 72*I*e^(3*x)
+ 12*e^(2*x) + 8*I*e^x + 3)*e^(-4*x)
```

Sympy [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 63 vs. 2(26) = 52.

Time = 0.14 (sec) , antiderivative size = 63, normalized size of antiderivative = 1.91

$$\int \frac{\cosh^5(x)}{i+\sinh(x)}\,dx = \frac{e^{4x}}{64} - \frac{ie^{3x}}{24} + \frac{e^{2x}}{16} - \frac{3ie^{x}}{8} + \frac{3ie^{-x}}{8} + \frac{e^{-2x}}{16} + \frac{ie^{-3x}}{24} + \frac{e^{-4x}}{64}$$

input
```
integrate(cosh(x)**5/(I+sinh(x)),x)
```

output
```
exp(4*x)/64 - I*exp(3*x)/24 + exp(2*x)/16 - 3*I*exp(x)/8 + 3*I*exp(-x)/8 +
 exp(-2*x)/16 + I*exp(-3*x)/24 + exp(-4*x)/64
```

Maxima [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 51 vs. 2(23) = 46.

Time = 0.04 (sec) , antiderivative size = 51, normalized size of antiderivative = 1.55

$$\int \frac{\cosh^5(x)}{i+\sinh(x)}\,dx = -\frac{1}{192}\left(8i\,e^{(-x)} - 12\,e^{(-2\,x)} + 72i\,e^{(-3\,x)} - 3\right)e^{(4\,x)} + \frac{3}{8}i\,e^{(-x)} + \frac{1}{16}\,e^{(-2\,x)} + \frac{1}{24}i\,e^{(-3\,x)} + \frac{1}{64}\,e^{(-4\,x)}$$

input
```
integrate(cosh(x)^5/(I+sinh(x)),x, algorithm="maxima")
```

output
```
-1/192*(8*I*e^(-x) - 12*e^(-2*x) + 72*I*e^(-3*x) - 3)*e^(4*x) + 3/8*I*e^(-
x) + 1/16*e^(-2*x) + 1/24*I*e^(-3*x) + 1/64*e^(-4*x)
```

Giac [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 47 vs. 2(23) = 46.

Time = 0.12 (sec) , antiderivative size = 47, normalized size of antiderivative = 1.42

$$\int \frac{\cosh^5(x)}{i+\sinh(x)}\,dx = -\frac{1}{192}\left(-72i\,e^{(3\,x)} - 12\,e^{(2\,x)} - 8i\,e^{x} - 3\right)e^{(-4\,x)} + \frac{1}{64}\,e^{(4\,x)} - \frac{1}{24}i\,e^{(3\,x)} + \frac{1}{16}\,e^{(2\,x)} - \frac{3}{8}i\,e^{x}$$

input
```
integrate(cosh(x)^5/(I+sinh(x)),x, algorithm="giac")
```

output
```
-1/192*(-72*I*e^(3*x) - 12*e^(2*x) - 8*I*e^x - 3)*e^(-4*x) + 1/64*e^(4*x)
- 1/24*I*e^(3*x) + 1/16*e^(2*x) - 3/8*I*e^x
```

Mupad [B] (verification not implemented)

Time = 1.44 (sec) , antiderivative size = 51, normalized size of antiderivative = 1.55

$$\int \frac{\cosh^5(x)}{i+\sinh(x)}\,dx = \frac{e^{-x}\,3i}{8} + \frac{e^{-2x}}{16} + \frac{e^{2x}}{16} + \frac{e^{-3x}\,1i}{24} - \frac{e^{3x}\,1i}{24} + \frac{e^{-4x}}{64} + \frac{e^{4x}}{64} - \frac{e^{x}\,3i}{8}$$

input
```
int(cosh(x)^5/(sinh(x) + 1i),x)
```

output
```
(exp(-x)*3i)/8 + exp(-2*x)/16 + exp(2*x)/16 + (exp(-3*x)*1i)/24 - (exp(3*x
)*1i)/24 + exp(-4*x)/64 + exp(4*x)/64 - (exp(x)*3i)/8
```

Reduce [F]

$$\int \frac{\cosh^5(x)}{i+\sinh(x)}\,dx = \int \frac{\cosh(x)^5}{\sinh(x)+i}dx$$

input
```
int(cosh(x)^5/(I+sinh(x)),x)
```

output
```
int(cosh(x)**5/(sinh(x) + i),x)
```

3.162 $\int \frac{\cosh^4(x)}{i+\sinh(x)}\,dx$

Optimal result

Integrand size = 13, antiderivative size = 26

$$\int \frac{\cosh^4(x)}{i+\sinh(x)}\,dx=-\frac{ix}{2}+\frac{\cosh^3(x)}{3}-\frac{1}{2}i\cosh(x)\sinh(x)$$

output
```
-1/2*I*x+1/3*cosh(x)^3-1/2*I*cosh(x)*sinh(x)
```

Mathematica [B] (verified)

Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 93 vs. 2(26) = 52.

Time = 0.12 (sec) , antiderivative size = 93, normalized size of antiderivative = 3.58

$$\int \frac{\cosh^4(x)}{i+\sinh(x)}\,dx =\frac{\cosh^5(x)\left(2i+\frac{6i\arcsin\left(\frac{\sqrt{1-i\sinh(x)}}{\sqrt{2}}\right)\sqrt{1-i\sinh(x)}}{\sqrt{1+i\sinh(x)}}+5\sinh(x)-i\sinh^2(x)+2\sinh^3(x)\right)}{6(-i+\sinh(x))^2(i+\sinh(x))^3}$$

input
```
Integrate[Cosh[x]^4/(I + Sinh[x]),x]
```

output
```
(Cosh[x]^5*(2*I + ((6*I)*ArcSin[Sqrt[1 - I*Sinh[x]]/Sqrt[2]]*Sqrt[1 - I*Si
nh[x]])/Sqrt[1 + I*Sinh[x]] + 5*Sinh[x] - I*Sinh[x]^2 + 2*Sinh[x]^3))/(6*(
-I + Sinh[x])^2*(I + Sinh[x])^3)
```

Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.04, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.385$, Rules used = {3042, 3161, 3042, 3115, 24}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cosh^4(x)}{\sinh(x)+i}dx$$
$$\Big\downarrow \text{3042}$$
$$\int \frac{\cos(ix)^4}{i-i\sin(ix)}dx$$
$$\Big\downarrow \text{3161}$$
$$\frac{\cosh^3(x)}{3}-i\int \cosh^2(x)dx$$
$$\Big\downarrow \text{3042}$$
$$\frac{\cosh^3(x)}{3}-i\int \sin\left(ix+\frac{\pi}{2}\right)^2dx$$
$$\Big\downarrow \text{3115}$$
$$\frac{\cosh^3(x)}{3}-i\left(\frac{\int 1dx}{2}+\frac{1}{2}\sinh(x)\cosh(x)\right)$$
$$\Big\downarrow \text{24}$$
$$\frac{\cosh^3(x)}{3}-i\left(\frac{x}{2}+\frac{1}{2}\sinh(x)\cosh(x)\right)$$

input
```
Int[Cosh[x]^4/(I + Sinh[x]),x]
```

output
```
Cosh[x]^3/3 - I*(x/2 + (Cosh[x]*Sinh[x])/2)
```

Defintions of rubi rules used

rule 24
```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3115
```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Simp[b^2*((n - 1)/n)   Int[(b*Sin
[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[
2*n]
```

rule 3161
```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)/((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Simp[g*((g*Cos[e + f*x])^(p - 1)/(b*f*(p - 1))), x] + Si
mp[g^2/a   Int[(g*Cos[e + f*x])^(p - 2), x], x] /; FreeQ[{a, b, e, f, g}, x
] && EqQ[a^2 - b^2, 0] && GtQ[p, 1] && IntegerQ[2*p]
```

Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 41 vs. $2(18) = 36$.

Time = 12.16 (sec) , antiderivative size = 42, normalized size of antiderivative = 1.62

method	result
risch	$-\frac{ix}{2}+\frac{{\mathrm e}^{3x}}{24}-\frac{i{\mathrm e}^{2x}}{8}+\frac{{\mathrm e}^{x}}{8}+\frac{{\mathrm e}^{-x}}{8}+\frac{i{\mathrm e}^{-2x}}{8}+\frac{{\mathrm e}^{-3x}}{24}$
default	$-\frac{i\ln\left(\tanh\left(\frac{x}{2}\right)+1\right)}{2}+\frac{\frac{1}{2}-\frac{i}{2}}{\tanh\left(\frac{x}{2}\right)+1}+\frac{-\frac{1}{2}+\frac{i}{2}}{\left(\tanh\left(\frac{x}{2}\right)+1\right)^{2}}+\frac{1}{3\left(\tanh\left(\frac{x}{2}\right)+1\right)^{3}}+\frac{i\ln\left(\tanh\left(\frac{x}{2}\right)-1\right)}{2}+\frac{-\frac{1}{2}-\frac{i}{2}}{\left(\tanh\left(\frac{x}{2}\right)-1\right)^{2}}+\frac{-\frac{1}{2}-}{\tanh(}$

input
```
int(cosh(x)^4/(I+sinh(x)),x,method=_RETURNVERBOSE)
```

output
```
-1/2*I*x+1/24*exp(x)^3-1/8*I*exp(x)^2+1/8*exp(x)+1/8/exp(x)+1/8*I/exp(x)^2
+1/24/exp(x)^3
```

Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 41 vs. $2(16) = 32$.

Time = 0.10 (sec) , antiderivative size = 41, normalized size of antiderivative = 1.58

$$\int \frac{\cosh^4(x)}{i+\sinh(x)}\,dx = \frac{1}{24}\left(-12i\,xe^{(3\,x)} + e^{(6\,x)} - 3i\,e^{(5\,x)} + 3\,e^{(4\,x)} + 3\,e^{(2\,x)} + 3i\,e^{x} + 1\right)e^{(-3\,x)}$$

input
```
integrate(cosh(x)^4/(I+sinh(x)),x, algorithm="fricas")
```

output
```
1/24*(-12*I*x*e^(3*x) + e^(6*x) - 3*I*e^(5*x) + 3*e^(4*x) + 3*e^(2*x) + 3*
I*e^x + 1)*e^(-3*x)
```

Sympy [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 48 vs. $2(20) = 40$.

Time = 0.10 (sec) , antiderivative size = 48, normalized size of antiderivative = 1.85

$$\int \frac{\cosh^4(x)}{i+\sinh(x)}\,dx = -\frac{ix}{2} + \frac{e^{3x}}{24} - \frac{ie^{2x}}{8} + \frac{e^{x}}{8} + \frac{e^{-x}}{8} + \frac{ie^{-2x}}{8} + \frac{e^{-3x}}{24}$$

input
```
integrate(cosh(x)**4/(I+sinh(x)),x)
```

output
```
-I*x/2 + exp(3*x)/24 - I*exp(2*x)/8 + exp(x)/8 + exp(-x)/8 + I*exp(-2*x)/8
 + exp(-3*x)/24
```

Maxima [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 42 vs. 2(16) = 32.

Time = 0.04 (sec) , antiderivative size = 42, normalized size of antiderivative = 1.62

$$\int \frac{\cosh^4(x)}{i+\sinh(x)}\,dx = -\frac{1}{24}\left(3i\,e^{(-x)} - 3\,e^{(-2\,x)} - 1\right)e^{(3\,x)} - \frac{1}{2}i\,x + \frac{1}{8}\,e^{(-x)} + \frac{1}{8}i\,e^{(-2\,x)} + \frac{1}{24}\,e^{(-3\,x)}$$

input
```
integrate(cosh(x)^4/(I+sinh(x)),x, algorithm="maxima")
```

output
```
-1/24*(3*I*e^(-x) - 3*e^(-2*x) - 1)*e^(3*x) - 1/2*I*x + 1/8*e^(-x) + 1/8*I
*e^(-2*x) + 1/24*e^(-3*x)
```

Giac [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 38 vs. 2(16) = 32.

Time = 0.12 (sec) , antiderivative size = 38, normalized size of antiderivative = 1.46

$$\int \frac{\cosh^4(x)}{i+\sinh(x)}\,dx = \frac{1}{24}\left(3\,e^{(2\,x)} + 3i\,e^{x} + 1\right)e^{(-3\,x)} - \frac{1}{2}i\,x + \frac{1}{24}\,e^{(3\,x)} - \frac{1}{8}i\,e^{(2\,x)} + \frac{1}{8}\,e^{x}$$

input
```
integrate(cosh(x)^4/(I+sinh(x)),x, algorithm="giac")
```

output
```
1/24*(3*e^(2*x) + 3*I*e^x + 1)*e^(-3*x) - 1/2*I*x + 1/24*e^(3*x) - 1/8*I*e
^(2*x) + 1/8*e^x
```

Mupad [B] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 41, normalized size of antiderivative = 1.58

$$\int \frac{\cosh^4(x)}{i+\sinh(x)}\,dx = \frac{e^{-x}}{8} + \frac{e^{-3\,x}}{24} + \frac{e^{3\,x}}{24} + \frac{e^{x}}{8} - \frac{x\,1i}{2} + \frac{e^{-2\,x}\,1i}{8} - \frac{e^{2\,x}\,1i}{8}$$

input
```
int(cosh(x)^4/(sinh(x) + 1i),x)
```

output
```
exp(-x)/8 - (x*1i)/2 + (exp(-2*x)*1i)/8 - (exp(2*x)*1i)/8 + exp(-3*x)/24 +
 exp(3*x)/24 + exp(x)/8
```

Reduce [F]

$$\int \frac{\cosh^4(x)}{i+\sinh(x)}\,dx = \int \frac{\cosh(x)^4}{\sinh(x)+i}dx$$

input
```
int(cosh(x)^4/(I+sinh(x)),x)
```

output
```
int(cosh(x)**4/(sinh(x) + i),x)
```

3.163 $\int \frac{\cosh^3(x)}{i+\sinh(x)}\,dx$

Optimal result

Integrand size = 13, antiderivative size = 14

$$\int \frac{\cosh^3(x)}{i+\sinh(x)}\,dx = \frac{1}{2}(i-\sinh(x))^2$$

output
```
1/2*(I-sinh(x))^2
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.86

$$\int \frac{\cosh^3(x)}{i+\sinh(x)}\,dx = \frac{1}{2}\sinh(x)(-2i+\sinh(x))$$

input
```
Integrate[Cosh[x]^3/(I + Sinh[x]),x]
```

output
```
(Sinh[x]*(-2*I + Sinh[x]))/2
```

Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.231$, Rules used = {3042, 3146, 17}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cosh^3(x)}{\sinh(x)+i}dx$$
$$\Big\downarrow \text{3042}$$
$$\int \frac{\cos(ix)^3}{i-i\sin(ix)}dx$$
$$\Big\downarrow \text{3146}$$
$$-\int (i-\sinh(x))d\sinh(x)$$
$$\Big\downarrow \text{17}$$
$$\frac{1}{2}(-\sinh(x)+i)^2$$

input
```
Int[Cosh[x]^3/(I + Sinh[x]),x]
```

output
```
(I - Sinh[x])^2/2
```

Defintions of rubi rules used

rule 17
```
Int[(c_.)*((a_.) + (b_.)*(x_))^(m_.), x_Symbol] :> Simp[c*((a + b*x)^(m + 1
)/(b*(m + 1))), x] /; FreeQ[{a, b, c, m}, x] && NeQ[m, -1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3146
```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Simp[1/(b^p*f)   Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x
)^((p - 1)/2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && I
ntegerQ[(p - 1)/2] && EqQ[a^2 - b^2, 0] && (GeQ[p, -1] ||  !IntegerQ[m + 1/
2])
```

Maple [A] (verified)

Time = 3.79 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.93

method	result	size
derivativedivides	$\frac{\sinh(x)^2}{2} - i\sinh\left(x\right)$	13
default	$\frac{\sinh(x)^2}{2} - i\sinh\left(x\right)$	13
risch	$\frac{e^{2x}}{8} - \frac{i e^{x}}{2} + \frac{i e^{-x}}{2} + \frac{e^{-2x}}{8}$	26

input
```
int(cosh(x)^3/(I+sinh(x)),x,method=_RETURNVERBOSE)
```

output
```
1/2*sinh(x)^2-I*sinh(x)
```

Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 22 vs. 2(8) = 16.

Time = 0.08 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.57

$$\int \frac{\cosh^3(x)}{i+\sinh(x)}\,dx = \frac{1}{8}\left(e^{(4\,x)} - 4i\,e^{(3\,x)} + 4i\,e^{x} + 1\right)e^{(-2\,x)}$$

input
```
integrate(cosh(x)^3/(I+sinh(x)),x, algorithm="fricas")
```

output
```
1/8*(e^(4*x) - 4*I*e^(3*x) + 4*I*e^x + 1)*e^(-2*x)
```

Sympy [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 27 vs. 2(7) = 14.

Time = 0.09 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.93

$$\int \frac{\cosh^3(x)}{i+\sinh(x)}\,dx = \frac{e^{2x}}{8} - \frac{ie^{x}}{2} + \frac{ie^{-x}}{2} + \frac{e^{-2x}}{8}$$

input
```
integrate(cosh(x)**3/(I+sinh(x)),x)
```

output
```
exp(2*x)/8 - I*exp(x)/2 + I*exp(-x)/2 + exp(-2*x)/8
```

Maxima [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 27 vs. 2(8) = 16.

Time = 0.03 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.93

$$\int \frac{\cosh^3(x)}{i+\sinh(x)}\,dx = \frac{1}{8}\left(-4i\,e^{(-x)}+1\right)e^{(2\,x)} + \frac{1}{2}i\,e^{(-x)} + \frac{1}{8}\,e^{(-2\,x)}$$

input
```
integrate(cosh(x)^3/(I+sinh(x)),x, algorithm="maxima")
```

output
```
1/8*(-4*I*e^(-x) + 1)*e^(2*x) + 1/2*I*e^(-x) + 1/8*e^(-2*x)
```

Giac [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 23 vs. $2(8) = 16$.

Time = 0.12 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.64

$$\int \frac{\cosh^3(x)}{i+\sinh(x)}\,dx = -\frac{1}{8}\,(-4i\,e^x-1)e^{(-2\,x)}+\frac{1}{8}\,e^{(2\,x)}-\frac{1}{2}i\,e^x$$

input
```
integrate(cosh(x)^3/(I+sinh(x)),x, algorithm="giac")
```

output
```
-1/8*(-4*I*e^x - 1)*e^(-2*x) + 1/8*e^(2*x) - 1/2*I*e^x
```

Mupad [B] (verification not implemented)

Time = 1.43 (sec) , antiderivative size = 31, normalized size of antiderivative = 2.21

$$\int \frac{\cosh^3(x)}{i+\sinh(x)}\,dx = \frac{e^{-2\,x}\,(e^{4\,x}+1)}{8}-\frac{e^{-2\,x}\,(4\,e^{3\,x}-4\,e^{x})\,1i}{8}$$

input
```
int(cosh(x)^3/(sinh(x) + 1i),x)
```

output
```
(exp(-2*x)*(exp(4*x) + 1))/8 - (exp(-2*x)*(4*exp(3*x) - 4*exp(x))*1i)/8
```

Reduce [F]

$$\int \frac{\cosh^3(x)}{i+\sinh(x)}\,dx = \int \frac{\cosh(x)^3}{\sinh(x)+i}dx$$

input
```
int(cosh(x)^3/(I+sinh(x)),x)
```

output
```
int(cosh(x)**3/(sinh(x) + i),x)
```

3.164 $\int \frac{\cosh^2(x)}{i+\sinh(x)} \, dx$

Optimal result

Integrand size = 13, antiderivative size = 8

$$\int \frac{\cosh^2(x)}{i+\sinh(x)} \, dx = -ix + \cosh(x)$$

output
```
-I*x+cosh(x)
```

Mathematica [B] (verified)

Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 34 vs. 2(8) = 16.

Time = 0.03 (sec) , antiderivative size = 34, normalized size of antiderivative = 4.25

$$\int \frac{\cosh^2(x)}{i+\sinh(x)} \, dx = \cosh(x) + 2 \arcsin\left(\frac{\sqrt{1 - i\sinh(x)}}{\sqrt{2}}\right) \sqrt{\cosh^2(x)} \operatorname{sech}(x)$$

input
```
Integrate[Cosh[x]^2/(I + Sinh[x]),x]
```

output
```
Cosh[x] + 2*ArcSin[Sqrt[1 - I*Sinh[x]]/Sqrt[2]]*Sqrt[Cosh[x]^2]*Sech[x]
```

Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 8, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used = {3042, 3161, 24}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cosh^2(x)}{\sinh(x)+i}dx$$
$$\Big\downarrow \text{3042}$$
$$\int \frac{\cos(ix)^2}{i-i\sin(ix)}dx$$
$$\Big\downarrow \text{3161}$$
$$\cosh(x)-i\int 1dx$$
$$\Big\downarrow \text{24}$$
$$\cosh(x)-ix$$

input
```
Int[Cosh[x]^2/(I + Sinh[x]),x]
```

output
```
(-I)*x + Cosh[x]
```

Defintions of rubi rules used

rule 24
```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3161
```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)/((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Simp[g*((g*Cos[e + f*x])^(p - 1)/(b*f*(p - 1))), x] + Si
mp[g^2/a   Int[(g*Cos[e + f*x])^(p - 2), x], x] /; FreeQ[{a, b, e, f, g}, x
] && EqQ[a^2 - b^2, 0] && GtQ[p, 1] && IntegerQ[2*p]
```

Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 15 vs. 2(7) = 14.

Time = 1.27 (sec) , antiderivative size = 16, normalized size of antiderivative = 2.00

method	result	size
risch	$-ix+\frac{\mathrm{e}^{x}}{2}+\frac{\mathrm{e}^{-x}}{2}$	16
default	$i\ln\left(\tanh\left(\frac{x}{2}\right)-1\right)-\frac{1}{\tanh\left(\frac{x}{2}\right)-1}-i\ln\left(\tanh\left(\frac{x}{2}\right)+1\right)+\frac{1}{\tanh\left(\frac{x}{2}\right)+1}$	40

input
```
int(cosh(x)^2/(I+sinh(x)),x,method=_RETURNVERBOSE)
```

output
```
-I*x+1/2*exp(x)+1/2*exp(-x)
```

Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 17 vs. 2(6) = 12.

Time = 0.09 (sec) , antiderivative size = 17, normalized size of antiderivative = 2.12

$$\int\frac{\cosh^2(x)}{i+\sinh(x)}\,dx=\frac{1}{2}\left(-2i\,xe^{x}+e^{(2\,x)}+1\right)e^{(-x)}$$

input
```
integrate(cosh(x)^2/(I+sinh(x)),x, algorithm="fricas")
```

output
```
1/2*(-2*I*x*e^x + e^(2*x) + 1)*e^(-x)
```

Sympy [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 14 vs. 2(5) = 10.

Time = 0.08 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.75

$$\int \frac{\cosh^2(x)}{i+\sinh(x)}\,dx = -ix + \frac{e^x}{2} + \frac{e^{-x}}{2}$$

input
```
integrate(cosh(x)**2/(I+sinh(x)),x)
```

output
```
-I*x + exp(x)/2 + exp(-x)/2
```

Maxima [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 14 vs. 2(6) = 12.

Time = 0.04 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.75

$$\int \frac{\cosh^2(x)}{i+\sinh(x)}\,dx = -i\,x + \frac{1}{2}\,e^{(-x)} + \frac{1}{2}\,e^x$$

input
```
integrate(cosh(x)^2/(I+sinh(x)),x, algorithm="maxima")
```

output
```
-I*x + 1/2*e^(-x) + 1/2*e^x
```

Giac [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 14 vs. 2(6) = 12.

Time = 0.12 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.75

$$\int \frac{\cosh^2(x)}{i+\sinh(x)}\,dx = -i\,x + \frac{1}{2}\,e^{(-x)} + \frac{1}{2}\,e^{x}$$

input
```
integrate(cosh(x)^2/(I+sinh(x)),x, algorithm="giac")
```

output
```
-I*x + 1/2*e^(-x) + 1/2*e^x
```

Mupad [B] (verification not implemented)

Time = 1.43 (sec) , antiderivative size = 7, normalized size of antiderivative = 0.88

$$\int \frac{\cosh^2(x)}{i+\sinh(x)}\,dx = \cosh(x) - x\,1\mathrm{i}$$

input
```
int(cosh(x)^2/(sinh(x) + 1i),x)
```

output
```
cosh(x) - x*1i
```

Reduce [F]

$$\int \frac{\cosh^2(x)}{i+\sinh(x)}\,dx = \int \frac{\cosh(x)^2}{\sinh(x)+i}dx$$

input
```
int(cosh(x)^2/(I+sinh(x)),x)
```

output
```
int(cosh(x)**2/(sinh(x) + i),x)
```

3.165 $\int \frac{\cosh(x)}{i+\sinh(x)}\,dx$

Optimal result

Integrand size = 11, antiderivative size = 7

$$\int \frac{\cosh(x)}{i+\sinh(x)}\,dx=\log(i+\sinh(x))$$

output
```
ln(I+sinh(x))
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 7, normalized size of antiderivative = 1.00

$$\int \frac{\cosh(x)}{i+\sinh(x)}\,dx=\log(i+\sinh(x))$$

input
```
Integrate[Cosh[x]/(I + Sinh[x]),x]
```

output
```
Log[I + Sinh[x]]
```

Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 7, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.273$, Rules used = {3042, 3146, 16}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cosh(x)}{\sinh(x)+i}dx$$
$$\Big\downarrow \text{ 3042}$$
$$\int \frac{\cos(ix)}{i-i\sin(ix)}dx$$
$$\Big\downarrow \text{ 3146}$$
$$\int \frac{1}{\sinh(x)+i}d\sinh(x)$$
$$\Big\downarrow \text{ 16}$$
$$\log(\sinh(x)+i)$$

input
```
Int[Cosh[x]/(I + Sinh[x]),x]
```

output
```
Log[I + Sinh[x]]
```

Defintions of rubi rules used

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3146
```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Simp[1/(b^p*f)   Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x
)^((p - 1)/2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && I
ntegerQ[(p - 1)/2] && EqQ[a^2 - b^2, 0] && (GeQ[p, -1] ||  !IntegerQ[m + 1/
2])
```

Maple [A] (verified)

Time = 0.38 (sec) , antiderivative size = 7, normalized size of antiderivative = 1.00

method	result	size
derivativedivides	$\ln(i+\sinh(x))$	7
default	$\ln(i+\sinh(x))$	7
risch	$-x+2\ln(e^{x}+i)$	13

input
```
int(cosh(x)/(I+sinh(x)),x,method=_RETURNVERBOSE)
```

output
```
ln(I+sinh(x))
```

Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 11 vs. 2(5) = 10.

Time = 0.09 (sec) , antiderivative size = 11, normalized size of antiderivative = 1.57

$$\int \frac{\cosh(x)}{i+\sinh(x)}\,dx = -x + 2\,\log\left(e^{x}+i\right)$$

input
```
integrate(cosh(x)/(I+sinh(x)),x, algorithm="fricas")
```

output
```
-x + 2*log(e^x + I)
```

Sympy [A] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 8, normalized size of antiderivative = 1.14

$$\int \frac{\cosh(x)}{i+\sinh(x)}\,dx = -x+2\log\left(e^x+i\right)$$

input
```
integrate(cosh(x)/(I+sinh(x)),x)
```

output
```
-x + 2*log(exp(x) + I)
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 5, normalized size of antiderivative = 0.71

$$\int \frac{\cosh(x)}{i+\sinh(x)}\,dx = \log\left(\sinh\left(x\right)+i\right)$$

input
```
integrate(cosh(x)/(I+sinh(x)),x, algorithm="maxima")
```

output
```
log(sinh(x) + I)
```

Giac [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 11 vs. 2(5) = 10.

Time = 0.12 (sec) , antiderivative size = 11, normalized size of antiderivative = 1.57

$$\int \frac{\cosh(x)}{i+\sinh(x)}\,dx = -x+2\,\log\left(e^x+i\right)$$

input
```
integrate(cosh(x)/(I+sinh(x)),x, algorithm="giac")
```

output
```
-x + 2*log(e^x + I)
```

Mupad [B] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 10, normalized size of antiderivative = 1.43

$$\int \frac{\cosh(x)}{i+\sinh(x)} \, dx = \ln\left(\cosh(x)\right) - \text{atan}(\sinh(x)) \text{ 1i}$$

input
```
int(cosh(x)/(sinh(x) + 1i),x)
```

output
```
log(cosh(x)) - atan(sinh(x))*1i
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 5, normalized size of antiderivative = 0.71

$$\int \frac{\cosh(x)}{i+\sinh(x)} \, dx = \log(\sinh(x) + i)$$

input
```
int(cosh(x)/(I+sinh(x)),x)
```

output
```
log(sinh(x) + i)
```

3.166 $\int \frac{\operatorname{sech}(x)}{i+\sinh(x)}\,dx$

Optimal result

Integrand size = 11, antiderivative size = 24

$$\int \frac{\operatorname{sech}(x)}{i+\sinh(x)}\,dx=-\frac{1}{2}i\arctan(\sinh(x))-\frac{i}{2(i+\sinh(x))}$$

output
```
-1/2*I*arctan(sinh(x))-1/2*I/(I+sinh(x))
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.75

$$\int \frac{\operatorname{sech}(x)}{i+\sinh(x)}\,dx=-\frac{1}{2}i\left(\arctan(\sinh(x))+\frac{1}{i+\sinh(x)}\right)$$

input
```
Integrate[Sech[x]/(I + Sinh[x]),x]
```

output
```
(-1/2*I)*(ArcTan[Sinh[x]] + (I + Sinh[x])^(-1))
```

Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.364, Rules used = {3042, 3146, 54, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\text{sech}(x)}{\sinh(x)+i}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{(i-i\sin(ix))\cos(ix)}dx$$

$$\Big\downarrow \text{3146}$$

$$-\int \frac{1}{(i-\sinh(x))(\sinh(x)+i)^2}d\sinh(x)$$

$$\Big\downarrow \text{54}$$

$$-\int \left(\frac{i}{2\left(\sinh^2(x)+1\right)}-\frac{i}{2(\sinh(x)+i)^2}\right)d\sinh(x)$$

$$\Big\downarrow \text{2009}$$

$$-\frac{1}{2}i\arctan(\sinh(x))-\frac{i}{2(\sinh(x)+i)}$$

input
```
Int[Sech[x]/(I + Sinh[x]),x]
```

output
```
(-1/2*I)*ArcTan[Sinh[x]] - (I/2)/(I + Sinh[x])
```

Defintions of rubi rules used

rule 54
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m + n + 2, 0])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3146
```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Simp[1/(b^p*f)   Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x
)^((p - 1)/2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && I
ntegerQ[(p - 1)/2] && EqQ[a^2 - b^2, 0] && (GeQ[p, -1] ||  !IntegerQ[m + 1/
2])
```

Maple [A] (verified)

Time = 1.51 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.25

method	result	size
risch	$-\frac{i e^{x}}{\left(e^{x}+i\right)^{2}}+\frac{\ln\left(e^{x}+i\right)}{2}-\frac{\ln\left(e^{x}-i\right)}{2}$	30
default	$-\frac{\ln\left(\tanh\left(\frac{x}{2}\right)-i\right)}{2}+\frac{i}{\tanh\left(\frac{x}{2}\right)+i}+\frac{1}{\left(\tanh\left(\frac{x}{2}\right)+i\right)^{2}}+\frac{\ln\left(\tanh\left(\frac{x}{2}\right)+i\right)}{2}$	43

input
```
int(sech(x)/(I+sinh(x)),x,method=_RETURNVERBOSE)
```

output
```
-I*exp(x)/(exp(x)+I)^2+1/2*ln(exp(x)+I)-1/2*ln(exp(x)-I)
```

Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 52 vs. 2(14) = 28.

Time = 0.10 (sec) , antiderivative size = 52, normalized size of antiderivative = 2.17

$$\int \frac{\operatorname{sech}(x)}{i+\sinh(x)}\,dx = \frac{\left(e^{(2x)}+2i\,e^{x}-1\right)\log\left(e^{x}+i\right)-\left(e^{(2x)}+2i\,e^{x}-1\right)\log\left(e^{x}-i\right)-2i\,e^{x}}{2\left(e^{(2x)}+2i\,e^{x}-1\right)}$$

input
```
integrate(sech(x)/(I+sinh(x)),x, algorithm="fricas")
```

output
```
1/2*((e^(2*x) + 2*I*e^x - 1)*log(e^x + I) - (e^(2*x) + 2*I*e^x - 1)*log(e^
x - I) - 2*I*e^x)/(e^(2*x) + 2*I*e^x - 1)
```

Sympy [F]

$$\int \frac{\operatorname{sech}(x)}{i+\sinh(x)}\,dx = \int \frac{\operatorname{sech}(x)}{\sinh(x)+i}\,dx$$

input
```
integrate(sech(x)/(I+sinh(x)),x)
```

output
```
Integral(sech(x)/(sinh(x) + I), x)
```

Maxima [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 41 vs. 2(14) = 28.

Time = 0.04 (sec) , antiderivative size = 41, normalized size of antiderivative = 1.71

$$\int \frac{\operatorname{sech}(x)}{i+\sinh(x)}\,dx = \frac{2i\,e^{(-x)}}{-4i\,e^{(-x)}+2\,e^{(-2x)}-2}-\frac{1}{2}\log\left(e^{(-x)}+i\right)+\frac{1}{2}\log\left(e^{(-x)}-i\right)$$

input
```
integrate(sech(x)/(I+sinh(x)),x, algorithm="maxima")
```

output
```
2*I*e^(-x)/(-4*I*e^(-x) + 2*e^(-2*x) - 2) - 1/2*log(e^(-x) + I) + 1/2*log(
e^(-x) - I)
```

Giac [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 51 vs. 2(14) = 28.

Time = 0.12 (sec) , antiderivative size = 51, normalized size of antiderivative = 2.12

$$\int \frac{\operatorname{sech}(x)}{i+\sinh(x)}\,dx = -\frac{e^{(-x)}-e^x-6i}{4\left(e^{(-x)}-e^x-2i\right)}+\frac{1}{4}\log\left(-e^{(-x)}+e^x+2i\right) - \frac{1}{4}\log\left(-e^{(-x)}+e^x-2i\right)$$

input
```
integrate(sech(x)/(I+sinh(x)),x, algorithm="giac")
```

output
```
-1/4*(e^(-x) - e^x - 6*I)/(e^(-x) - e^x - 2*I) + 1/4*log(-e^(-x) + e^x + 2
*I) - 1/4*log(-e^(-x) + e^x - 2*I)
```

Mupad [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.92

$$\int \frac{\operatorname{sech}(x)}{i+\sinh(x)}\,dx = \frac{\ln\left(-1+\mathrm{e}^x\,1\mathrm{i}\right)}{2}-\frac{\ln\left(1+\mathrm{e}^x\,1\mathrm{i}\right)}{2}-\frac{1}{\mathrm{e}^{2\,x}-1+\mathrm{e}^x\,2\mathrm{i}}-\frac{1\mathrm{i}}{\mathrm{e}^x+1\mathrm{i}}$$

input
```
int(1/(cosh(x)*(sinh(x) + 1i)),x)
```

output
```
log(exp(x)*1i - 1)/2 - log(exp(x)*1i + 1)/2 - 1/(exp(2*x) + exp(x)*2i - 1)
 - 1i/(exp(x) + 1i)
```

Reduce [F]

$$\int \frac{\operatorname{sech}(x)}{i+\sinh(x)}\,dx = \int \frac{\operatorname{sech}(x)}{\sinh(x)+i}dx$$

input
```
int(sech(x)/(I+sinh(x)),x)
```

output
```
int(sech(x)/(sinh(x) + i),x)
```

3.167 $\int \frac{\text{sech}^2(x)}{i+\sinh(x)} dx$

Optimal result

Integrand size = 13, antiderivative size = 25

$$\int \frac{\text{sech}^2(x)}{i+\sinh(x)} dx = -\frac{i\text{sech}(x)}{3(i+\sinh(x))} - \frac{2}{3}i\tanh(x)$$

output
```
-1/3*I*sech(x)/(I+sinh(x))-2/3*I*tanh(x)
```

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.88

$$\int \frac{\text{sech}^2(x)}{i+\sinh(x)} dx = -\frac{1}{3}i\left(\frac{\text{sech}(x)}{i+\sinh(x)} + 2\tanh(x)\right)$$

input
```
Integrate[Sech[x]^2/(I + Sinh[x]),x]
```

output
```
(-1/3*I)*(Sech[x]/(I + Sinh[x]) + 2*Tanh[x])
```

Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.385$, Rules used = {3042, 3151, 3042, 4254, 24}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\text{sech}^2(x)}{\sinh(x)+i}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{(i-i\sin(ix))\cos(ix)^2}dx$$

$$\Big\downarrow \text{3151}$$

$$-\frac{2}{3}i\int \text{sech}^2(x)dx-\frac{i\text{sech}(x)}{3(\sinh(x)+i)}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{2}{3}i\int \csc\left(ix+\frac{\pi}{2}\right)^2 dx-\frac{i\text{sech}(x)}{3(\sinh(x)+i)}$$

$$\Big\downarrow \text{4254}$$

$$\frac{2}{3}\int 1d(-i\tanh(x))-\frac{i\text{sech}(x)}{3(\sinh(x)+i)}$$

$$\Big\downarrow \text{24}$$

$$-\frac{2}{3}i\tanh(x)-\frac{i\text{sech}(x)}{3(\sinh(x)+i)}$$

input
```
Int[Sech[x]^2/(I + Sinh[x]),x]
```

output
```
((-1/3*I)*Sech[x])/(I + Sinh[x]) - ((2*I)/3)*Tanh[x]
```

Defintions of rubi rules used

rule 24
```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3151
```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[b*(g*Cos[e + f*x])^(p + 1)*((a + b*Sin[e + f*x
])^m/(a*f*g*Simplify[2*m + p + 1])), x] + Simp[Simplify[m + p + 1]/(a*Simpl
ify[2*m + p + 1])   Int[(g*Cos[e + f*x])^p*(a + b*Sin[e + f*x])^(m + 1), x]
, x] /; FreeQ[{a, b, e, f, g, m, p}, x] && EqQ[a^2 - b^2, 0] && ILtQ[Simpli
fy[m + p + 1], 0] && NeQ[2*m + p + 1, 0] &&  !IGtQ[m, 0]
```

rule 4254
```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Simp[-d^(-1)   Subst[Int[Exp
andIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
 d}, x] && IGtQ[n/2, 0]
```

Maple [A] (verified)

Time = 5.17 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.96

method	result	size
risch	$-\frac{4(2\,\mathrm{e}^{x}+i)}{3(\mathrm{e}^{x}+i)^{3}(\mathrm{e}^{x}-i)}$	24
default	$-\frac{i}{2(\tanh(\frac{x}{2})-i)}-\frac{1}{(\tanh(\frac{x}{2})+i)^{2}}+\frac{2i}{3(\tanh(\frac{x}{2})+i)^{3}}-\frac{3i}{2(\tanh(\frac{x}{2})+i)}$	49

input
```
int(sech(x)^2/(I+sinh(x)),x,method=_RETURNVERBOSE)
```

output
```
-4/3*(2*exp(x)+I)/(exp(x)+I)^3/(exp(x)-I)
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.04

$$\int \frac{\operatorname{sech}^2(x)}{i+\sinh(x)}\,dx = -\frac{4\,(2\,e^x+i)}{3\,(e^{(4\,x)}+2i\,e^{(3\,x)}+2i\,e^x-1)}$$

input
```
integrate(sech(x)^2/(I+sinh(x)),x, algorithm="fricas")
```

output
```
-4/3*(2*e^x + I)/(e^(4*x) + 2*I*e^(3*x) + 2*I*e^x - 1)
```

Sympy [F]

$$\int \frac{\operatorname{sech}^2(x)}{i+\sinh(x)}\,dx = \int \frac{\operatorname{sech}^2(x)}{\sinh(x)+i}\,dx$$

input
```
integrate(sech(x)**2/(I+sinh(x)),x)
```

output
```
Integral(sech(x)**2/(sinh(x) + I), x)
```

Maxima [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 53 vs. 2(15) = 30.

Time = 0.03 (sec) , antiderivative size = 53, normalized size of antiderivative = 2.12

$$\int \frac{\operatorname{sech}^2(x)}{i+\sinh(x)}\,dx = -\frac{8\,e^{(-x)}}{-6i\,e^{(-x)}-6i\,e^{(-3\,x)}+3\,e^{(-4\,x)}-3} + \frac{4i}{-6i\,e^{(-x)}-6i\,e^{(-3\,x)}+3\,e^{(-4\,x)}-3}$$

input
```
integrate(sech(x)^2/(I+sinh(x)),x, algorithm="maxima")
```

output
```
-8*e^(-x)/(-6*I*e^(-x) - 6*I*e^(-3*x) + 3*e^(-4*x) - 3) + 4*I/(-6*I*e^(-x)
 - 6*I*e^(-3*x) + 3*e^(-4*x) - 3)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.16

$$\int \frac{\operatorname{sech}^2(x)}{i+\sinh(x)}\,dx = \frac{1}{2\,(e^x-i)} - \frac{3\,e^{(2\,x)}+12i\,e^x-5}{6\,(e^x+i)^3}$$

input
```
integrate(sech(x)^2/(I+sinh(x)),x, algorithm="giac")
```

output
```
1/2/(e^x - I) - 1/6*(3*e^(2*x) + 12*I*e^x - 5)/(e^x + I)^3
```

Mupad [B] (verification not implemented)

Time = 1.52 (sec) , antiderivative size = 63, normalized size of antiderivative = 2.52

$$\int \frac{\operatorname{sech}^2(x)}{i+\sinh(x)}\,dx = -\frac{8\,e^x}{3\,(e^{2\,x}+1)^3} - \frac{8\,e^x\,(e^{2\,x}-1)}{3\,(e^{2\,x}+1)^3} + \frac{e^{2\,x}\,16i}{3\,(e^{2\,x}+1)^3} - \frac{(e^{2\,x}-1)\,4i}{3\,(e^{2\,x}+1)^3}$$

input
```
int(1/(cosh(x)^2*(sinh(x) + 1i)),x)
```

output
```
(exp(2*x)*16i)/(3*(exp(2*x) + 1)^3) - (8*exp(x))/(3*(exp(2*x) + 1)^3) - ((
exp(2*x) - 1)*4i)/(3*(exp(2*x) + 1)^3) - (8*exp(x)*(exp(2*x) - 1))/(3*(exp
(2*x) + 1)^3)
```

Reduce [F]

$$\int \frac{\operatorname{sech}^2(x)}{i+\sinh(x)}\,dx = \int \frac{\operatorname{sech}(x)^2}{\sinh(x)+i}dx$$

input
```
int(sech(x)^2/(I+sinh(x)),x)
```

output
```
int(sech(x)**2/(sinh(x) + i),x)
```

3.168 $\int \frac{\operatorname{sech}^3(x)}{i+\sinh(x)} \, dx$

Optimal result

Integrand size = 13, antiderivative size = 52

$$\int \frac{\operatorname{sech}^3(x)}{i+\sinh(x)} \, dx = -\frac{3}{8} i \arctan(\sinh(x)) + \frac{i}{8(i-\sinh(x))} + \frac{1}{8(i+\sinh(x))^2} - \frac{i}{4(i+\sinh(x))}$$

output
```
-3/8*I*arctan(sinh(x))+1/8*I/(I-sinh(x))+1/8/(I+sinh(x))^2-1/4*I/(I+sinh(x
))
```

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 61, normalized size of antiderivative = 1.17

$$\int \frac{\operatorname{sech}^3(x)}{i+\sinh(x)} \, dx = -\frac{i \operatorname{sech}^2(x) \left(2 + 3i \arctan(\sinh(x)) + 3(i + \arctan(\sinh(x))) \sinh(x) + (3 + 3i \arctan(\sinh(x))) \sinh^2(x\right.}{8(i+\sinh(x))}$$

input
```
Integrate[Sech[x]^3/(I + Sinh[x]),x]
```

output
```
((-1/8*I)*Sech[x]^2*(2 + (3*I)*ArcTan[Sinh[x]] + 3*(I + ArcTan[Sinh[x]])*S
inh[x] + (3 + (3*I)*ArcTan[Sinh[x]])*Sinh[x]^2 + 3*ArcTan[Sinh[x]]*Sinh[x]
^3))/(I + Sinh[x])
```

Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 52, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.308$, Rules used = {3042, 3146, 54, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\operatorname{sech}^3(x)}{\sinh(x)+i}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{(i-i\sin(ix))\cos(ix)^3}dx$$

$$\Big\downarrow \text{3146}$$

$$\int \frac{1}{(-\sinh(x)+i)^2(\sinh(x)+i)^3}d\sinh(x)$$

$$\Big\downarrow \text{54}$$

$$\int \left(-\frac{3i}{8\left(\sinh^2(x)+1\right)}+\frac{i}{8(\sinh(x)-i)^2}+\frac{i}{4(\sinh(x)+i)^2}-\frac{1}{4(\sinh(x)+i)^3}\right)d\sinh(x)$$

$$\Big\downarrow \text{2009}$$

$$-\frac{3}{8}i\arctan(\sinh(x))+\frac{i}{8(-\sinh(x)+i)}-\frac{i}{4(\sinh(x)+i)}+\frac{1}{8(\sinh(x)+i)^2}$$

input
```
Int[Sech[x]^3/(I + Sinh[x]),x]
```

output
```
((-3*I)/8)*ArcTan[Sinh[x]] + (I/8)/(I - Sinh[x]) + 1/(8*(I + Sinh[x])^2) -
 (I/4)/(I + Sinh[x])
```

Defintions of rubi rules used

rule 54
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m + n + 2, 0])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3146
```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Simp[1/(b^p*f)   Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x
)^((p - 1)/2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && I
ntegerQ[(p - 1)/2] && EqQ[a^2 - b^2, 0] && (GeQ[p, -1] ||  !IntegerQ[m + 1/
2])
```

Maple [A] (verified)

Time = 19.83 (sec) , antiderivative size = 63, normalized size of antiderivative = 1.21

method	result
risch	$-\frac{i \mathrm{e}^{x} \left(6 i \mathrm{e}^{3 x}+3 \,\mathrm{e}^{4 x}-6 i \mathrm{e}^{x}+2 \,\mathrm{e}^{2 x}+3\right)}{4 \left(\mathrm{e}^{x}+i\right)^{4} \left(\mathrm{e}^{x}-i\right)^{2}}-\frac{3 \ln \left(\mathrm{e}^{x}-i\right)}{8}+\frac{3 \ln \left(\mathrm{e}^{x}+i\right)}{8}$
default	$-\frac{1}{2 \left(\tanh \left(\frac{x}{2}\right)+i\right)^{4}}+\frac{i}{\tanh \left(\frac{x}{2}\right)+i}-\frac{i}{\left(\tanh \left(\frac{x}{2}\right)+i\right)^{3}}+\frac{3}{2 \left(\tanh \left(\frac{x}{2}\right)+i\right)^{2}}+\frac{3 \ln \left(\tanh \left(\frac{x}{2}\right)+i\right)}{8}+\frac{i}{4 \tanh \left(\frac{x}{2}\right)-4 i}-\frac{1}{4 (\tanh (\frac{x}{2})}$

input
```
int(sech(x)^3/(I+sinh(x)),x,method=_RETURNVERBOSE)
```

output
```
-1/4*I*exp(x)*(6*I*exp(x)^3+3*exp(x)^4-6*I*exp(x)+2*exp(x)^2+3)/(exp(x)+I)
^4/(exp(x)-I)^2-3/8*ln(exp(x)-I)+3/8*ln(exp(x)+I)
```

Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 143 vs. $2(30) = 60$.

Time = 0.11 (sec) , antiderivative size = 143, normalized size of antiderivative = 2.75

$$\int \frac{\operatorname{sech}^3(x)}{i+\sinh(x)}\,dx$$
$$= \frac{3\left(e^{(6x)}+2i\,e^{(5x)}+e^{(4x)}+4i\,e^{(3x)}-e^{(2x)}+2i\,e^{x}-1\right)\log\left(e^{x}+i\right)-3\left(e^{(6x)}+2i\,e^{(5x)}+e^{(4x)}+4i\,e^{(3x)}\right.}{8\left(e^{(6x)}+2i\,e^{(5x)}+e^{(4x)}+4i\,e^{(3x)}-e\right.}$$

input
```
integrate(sech(x)^3/(I+sinh(x)),x, algorithm="fricas")
```

output
```
1/8*(3*(e^(6*x) + 2*I*e^(5*x) + e^(4*x) + 4*I*e^(3*x) - e^(2*x) + 2*I*e^x
- 1)*log(e^x + I) - 3*(e^(6*x) + 2*I*e^(5*x) + e^(4*x) + 4*I*e^(3*x) - e^(
2*x) + 2*I*e^x - 1)*log(e^x - I) - 6*I*e^(5*x) + 12*e^(4*x) - 4*I*e^(3*x)
- 12*e^(2*x) - 6*I*e^x)/(e^(6*x) + 2*I*e^(5*x) + e^(4*x) + 4*I*e^(3*x) - e
^(2*x) + 2*I*e^x - 1)
```

Sympy [F]

$$\int \frac{\operatorname{sech}^3(x)}{i+\sinh(x)}\,dx = \int \frac{\operatorname{sech}^3(x)}{\sinh(x)+i}\,dx$$

input
```
integrate(sech(x)**3/(I+sinh(x)),x)
```

output
```
Integral(sech(x)**3/(sinh(x) + I), x)
```

Maxima [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 92 vs. 2(30) = 60.

Time = 0.04 (sec) , antiderivative size = 92, normalized size of antiderivative = 1.77

$$\int \frac{\operatorname{sech}^3(x)}{i+\sinh(x)}\,dx = \frac{8\left(3i\,e^{(-x)}-6\,e^{(-2\,x)}+2i\,e^{(-3\,x)}+6\,e^{(-4\,x)}+3i\,e^{(-5\,x)}\right)}{-64i\,e^{(-x)}-32\,e^{(-2\,x)}-128i\,e^{(-3\,x)}+32\,e^{(-4\,x)}-64i\,e^{(-5\,x)}+32\,e^{(-6\,x)}-32} - \frac{3}{8}\log\left(e^{(-x)}+i\right)+\frac{3}{8}\log\left(e^{(-x)}-i\right)$$

input
```
integrate(sech(x)^3/(I+sinh(x)),x, algorithm="maxima")
```

output
```
8*(3*I*e^(-x) - 6*e^(-2*x) + 2*I*e^(-3*x) + 6*e^(-4*x) + 3*I*e^(-5*x))/(-6
4*I*e^(-x) - 32*e^(-2*x) - 128*I*e^(-3*x) + 32*e^(-4*x) - 64*I*e^(-5*x) +
32*e^(-6*x) - 32) - 3/8*log(e^(-x) + I) + 3/8*log(e^(-x) - I)
```

Giac [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 92 vs. 2(30) = 60.

Time = 0.12 (sec) , antiderivative size = 92, normalized size of antiderivative = 1.77

$$\int \frac{\operatorname{sech}^3(x)}{i+\sinh(x)}\,dx = \frac{3\,e^{(-x)}-3\,e^{x}+10i}{16\left(e^{(-x)}-e^{x}+2i\right)} - \frac{9\left(e^{(-x)}-e^{x}\right)^2-52i\,e^{(-x)}+52i\,e^{x}-84}{32\left(e^{(-x)}-e^{x}-2i\right)^2} + \frac{3}{16}\log\left(-e^{(-x)}+e^{x}+2i\right)-\frac{3}{16}\log\left(-e^{(-x)}+e^{x}-2i\right)$$

input
```
integrate(sech(x)^3/(I+sinh(x)),x, algorithm="giac")
```

output
```
1/16*(3*e^(-x) - 3*e^x + 10*I)/(e^(-x) - e^x + 2*I) - 1/32*(9*(e^(-x) - e^
x)^2 - 52*I*e^(-x) + 52*I*e^x - 84)/(e^(-x) - e^x - 2*I)^2 + 3/16*log(-e^(
-x) + e^x + 2*I) - 3/16*log(-e^(-x) + e^x - 2*I)
```

Mupad [B] (verification not implemented)

Time = 1.85 (sec) , antiderivative size = 115, normalized size of antiderivative = 2.21

$$\int \frac{\operatorname{sech}^3(x)}{i+\sinh(x)}\,dx = \frac{3\,\ln\left(-\frac{3}{4}+\frac{e^x\,3i}{4}\right)}{8}-\frac{3\,\ln\left(\frac{3}{4}+\frac{e^x\,3i}{4}\right)}{8} - \frac{1}{2\,\left(e^{4\,x}-6\,e^{2\,x}+1+e^{3\,x}\,4i-e^x\,4i\right)}-\frac{1}{4\,\left(1-e^{2\,x}+e^x\,2i\right)} - \frac{1i}{4\,\left(e^x-i\right)}-\frac{1i}{2\,\left(e^x+1i\right)}-\frac{1i}{e^{2\,x}\,3i+e^{3\,x}-3\,e^x-i}$$

input
```
int(1/(cosh(x)^3*(sinh(x) + 1i)),x)
```

output
```
(3*log((exp(x)*3i)/4 - 3/4))/8 - (3*log((exp(x)*3i)/4 + 3/4))/8 - 1/(2*(ex
p(3*x)*4i - 6*exp(2*x) + exp(4*x) - exp(x)*4i + 1)) - 1/(4*(exp(x)*2i - ex
p(2*x) + 1)) - 1i/(4*(exp(x) - 1i)) - 1i/(2*(exp(x) + 1i)) - 1i/(exp(2*x)*
3i + exp(3*x) - 3*exp(x) - 1i)
```

Reduce [F]

$$\int \frac{\operatorname{sech}^3(x)}{i+\sinh(x)}\,dx = \int \frac{\operatorname{sech}(x)^3}{\sinh(x)+i}dx$$

input
```
int(sech(x)^3/(I+sinh(x)),x)
```

output
```
int(sech(x)**3/(sinh(x) + i),x)
```

3.169 $\int \frac{\operatorname{sech}^4(x)}{i+\sinh(x)}\,dx$

Optimal result

Integrand size = 13, antiderivative size = 37

$$\int \frac{\operatorname{sech}^4(x)}{i+\sinh(x)}\,dx = -\frac{i\operatorname{sech}^3(x)}{5(i+\sinh(x))} - \frac{4}{5}i\tanh(x) + \frac{4}{15}i\tanh^3(x)$$

output
```
-1/5*I*sech(x)^3/(I+sinh(x))-4/5*I*tanh(x)+4/15*I*tanh(x)^3
```

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.95

$$\int \frac{\operatorname{sech}^4(x)}{i+\sinh(x)}\,dx = -\frac{1}{15}i\left(\frac{3\operatorname{sech}^3(x)}{i+\sinh(x)} + 12\operatorname{sech}^2(x)\tanh(x) + 8\tanh^3(x)\right)$$

input
```
Integrate[Sech[x]^4/(I + Sinh[x]),x]
```

output
```
(-1/15*I)*((3*Sech[x]^3)/(I + Sinh[x]) + 12*Sech[x]^2*Tanh[x] + 8*Tanh[x]^
3)
```

Rubi [A] (verified)

Time = 0.31 (sec) , antiderivative size = 40, normalized size of antiderivative = 1.08, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.385$, Rules used = {3042, 3151, 3042, 4254, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\operatorname{sech}^4(x)}{\sinh(x)+i}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{(i-i\sin(ix))\cos(ix)^4}dx$$

$$\Big\downarrow \text{3151}$$

$$-\frac{4}{5}i\int \operatorname{sech}^4(x)dx-\frac{i\operatorname{sech}^3(x)}{5(\sinh(x)+i)}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{4}{5}i\int \csc\left(ix+\frac{\pi}{2}\right)^4dx-\frac{i\operatorname{sech}^3(x)}{5(\sinh(x)+i)}$$

$$\Big\downarrow \text{4254}$$

$$\frac{4}{5}\int \left(1-\tanh^2(x)\right)d(-i\tanh(x))-\frac{i\operatorname{sech}^3(x)}{5(\sinh(x)+i)}$$

$$\Big\downarrow \text{2009}$$

$$\frac{4}{5}\left(\frac{1}{3}i\tanh^3(x)-i\tanh(x)\right)-\frac{i\operatorname{sech}^3(x)}{5(\sinh(x)+i)}$$

input
```
Int[Sech[x]^4/(I + Sinh[x]),x]
```

output
```
((-1/5*I)*Sech[x]^3)/(I + Sinh[x]) + (4*((-I)*Tanh[x] + (I/3)*Tanh[x]^3))/
5
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3151
```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[b*(g*Cos[e + f*x])^(p + 1)*((a + b*Sin[e + f*x
])^m/(a*f*g*Simplify[2*m + p + 1])), x] + Simp[Simplify[m + p + 1]/(a*Simpl
ify[2*m + p + 1])   Int[(g*Cos[e + f*x])^p*(a + b*Sin[e + f*x])^(m + 1), x]
, x] /; FreeQ[{a, b, e, f, g, m, p}, x] && EqQ[a^2 - b^2, 0] && ILtQ[Simpli
fy[m + p + 1], 0] && NeQ[2*m + p + 1, 0] &&  !IGtQ[m, 0]
```

rule 4254
```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Simp[-d^(-1)   Subst[Int[Exp
andIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
 d}, x] && IGtQ[n/2, 0]
```

Maple [A] (verified)

Time = 143.93 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.00

method	result
risch	$-\frac{16\left(6\,\mathrm{e}^{3x}+2i\mathrm{e}^{2x}+2\,\mathrm{e}^{x}+i\right)}{15\left(\mathrm{e}^{x}+i\right)^{5}\left(\mathrm{e}^{x}-i\right)^{3}}$
default	$\frac{i}{6\left(\tanh\left(\frac{x}{2}\right)-i\right)^{3}}-\frac{5i}{8\left(\tanh\left(\frac{x}{2}\right)-i\right)}+\frac{1}{4\left(\tanh\left(\frac{x}{2}\right)-i\right)^{2}}-\frac{2i}{5\left(\tanh\left(\frac{x}{2}\right)+i\right)^{5}}+\frac{5i}{3\left(\tanh\left(\frac{x}{2}\right)+i\right)^{3}}-\frac{11i}{8\left(\tanh\left(\frac{x}{2}\right)+i\right)}+\frac{1}{\left(\tanh\left(\frac{x}{2}\right.\right.}$

input
```
int(sech(x)^4/(I+sinh(x)),x,method=_RETURNVERBOSE)
```

output
```
-16/15*(6*exp(x)^3+2*I*exp(x)^2+2*exp(x)+I)/(exp(x)+I)^5/(exp(x)-I)^3
```

Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 62 vs. 2(23) = 46.

Time = 0.07 (sec) , antiderivative size = 62, normalized size of antiderivative = 1.68

$$\int \frac{\operatorname{sech}^4(x)}{i+\sinh(x)}\,dx = -\frac{16\left(6\,e^{(3\,x)}+2i\,e^{(2\,x)}+2\,e^{x}+i\right)}{15\left(e^{(8\,x)}+2i\,e^{(7\,x)}+2\,e^{(6\,x)}+6i\,e^{(5\,x)}+6i\,e^{(3\,x)}-2\,e^{(2\,x)}+2i\,e^{x}-1\right)}$$

input
```
integrate(sech(x)^4/(I+sinh(x)),x, algorithm="fricas")
```

output
```
-16/15*(6*e^(3*x) + 2*I*e^(2*x) + 2*e^x + I)/(e^(8*x) + 2*I*e^(7*x) + 2*e^
(6*x) + 6*I*e^(5*x) + 6*I*e^(3*x) - 2*e^(2*x) + 2*I*e^x - 1)
```

Sympy [F]

$$\int \frac{\operatorname{sech}^4(x)}{i+\sinh(x)}\,dx = \int \frac{\operatorname{sech}^4(x)}{\sinh(x)+i}\,dx$$

input
```
integrate(sech(x)**4/(I+sinh(x)),x)
```

output
```
Integral(sech(x)**4/(sinh(x) + I), x)
```

Maxima [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 205 vs. 2(23) = 46.

Time = 0.04 (sec) , antiderivative size = 205, normalized size of antiderivative = 5.54

$$\int \frac{\operatorname{sech}^4(x)}{i+\sinh(x)}\,dx = -\frac{32\,e^{(-x)}}{-30i\,e^{(-x)}-30\,e^{(-2\,x)}-90i\,e^{(-3\,x)}-90i\,e^{(-5\,x)}+30\,e^{(-6\,x)}-30i\,e^{(-7\,x)}+15\,e^{(-8\,x)}-15} + \frac{32i\,e^{(-2\,x)}}{-30i\,e^{(-x)}-30\,e^{(-2\,x)}-90i\,e^{(-3\,x)}-90i\,e^{(-5\,x)}+30\,e^{(-6\,x)}-30i\,e^{(-7\,x)}+15\,e^{(-8\,x)}-15} - \frac{96\,e^{(-3\,x)}}{-30i\,e^{(-x)}-30\,e^{(-2\,x)}-90i\,e^{(-3\,x)}-90i\,e^{(-5\,x)}+30\,e^{(-6\,x)}-30i\,e^{(-7\,x)}+15\,e^{(-8\,x)}-15} + \frac{16i}{-30i\,e^{(-x)}-30\,e^{(-2\,x)}-90i\,e^{(-3\,x)}-90i\,e^{(-5\,x)}+30\,e^{(-6\,x)}-30i\,e^{(-7\,x)}+15\,e^{(-8\,x)}-15}$$

input
```
integrate(sech(x)^4/(I+sinh(x)),x, algorithm="maxima")
```

output
```
-32*e^(-x)/(-30*I*e^(-x) - 30*e^(-2*x) - 90*I*e^(-3*x) - 90*I*e^(-5*x) + 3
0*e^(-6*x) - 30*I*e^(-7*x) + 15*e^(-8*x) - 15) + 32*I*e^(-2*x)/(-30*I*e^(-
x) - 30*e^(-2*x) - 90*I*e^(-3*x) - 90*I*e^(-5*x) + 30*e^(-6*x) - 30*I*e^(-
7*x) + 15*e^(-8*x) - 15) - 96*e^(-3*x)/(-30*I*e^(-x) - 30*e^(-2*x) - 90*I*
e^(-3*x) - 90*I*e^(-5*x) + 30*e^(-6*x) - 30*I*e^(-7*x) + 15*e^(-8*x) - 15)
 + 16*I/(-30*I*e^(-x) - 30*e^(-2*x) - 90*I*e^(-3*x) - 90*I*e^(-5*x) + 30*e
^(-6*x) - 30*I*e^(-7*x) + 15*e^(-8*x) - 15)
```

Giac [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 53 vs. 2(23) = 46.

Time = 0.13 (sec) , antiderivative size = 53, normalized size of antiderivative = 1.43

$$\int \frac{\operatorname{sech}^4(x)}{i+\sinh(x)}\,dx = \frac{9\,e^{(2\,x)}-24i\,e^{x}-11}{24\,(e^{x}-i)^3} - \frac{45\,e^{(4\,x)}+240i\,e^{(3\,x)}-490\,e^{(2\,x)}-320i\,e^{x}+73}{120\,(e^{x}+i)^5}$$

input
```
integrate(sech(x)^4/(I+sinh(x)),x, algorithm="giac")
```

output
```
1/24*(9*e^(2*x) - 24*I*e^x - 11)/(e^x - I)^3 - 1/120*(45*e^(4*x) + 240*I*e
^(3*x) - 490*e^(2*x) - 320*I*e^x + 73)/(e^x + I)^5
```

Mupad [B] (verification not implemented)

Time = 1.95 (sec) , antiderivative size = 231, normalized size of antiderivative = 6.24

$$\int \frac{\operatorname{sech}^4(x)}{i+\sinh(x)}\,dx = -\frac{1}{6\,\left(e^{2\,x}\,3\mathrm{i}-e^{3\,x}+3\,e^{x}-\mathrm{i}\right)}-\frac{\frac{3\,e^{x}}{40}+\frac{1}{8}\mathrm{i}}{e^{2\,x}-1+e^{x}\,2\mathrm{i}} -\frac{\frac{3\,e^{2\,x}}{40}-\frac{5}{24}+\frac{e^{x}\,1\mathrm{i}}{4}}{e^{2\,x}\,3\mathrm{i}+e^{3\,x}-3\,e^{x}-\mathrm{i}}+\frac{1\mathrm{i}}{4\,\left(1-e^{2\,x}+e^{x}\,2\mathrm{i}\right)}+\frac{3}{8\,\left(e^{x}-\mathrm{i}\right)} -\frac{3}{40\,\left(e^{x}+1\mathrm{i}\right)}-\frac{\frac{e^{2\,x}\,3\mathrm{i}}{8}+\frac{3\,e^{3\,x}}{40}-\frac{5\,e^{x}}{8}-\frac{1}{8}\mathrm{i}}{e^{4\,x}-6\,e^{2\,x}+1+e^{3\,x}\,4\mathrm{i}-e^{x}\,4\mathrm{i}} -\frac{\frac{3\,e^{4\,x}}{40}-\frac{5\,e^{2\,x}}{4}+\frac{3}{40}+\frac{e^{3\,x}\,1\mathrm{i}}{2}-\frac{e^{x}\,1\mathrm{i}}{2}}{e^{5\,x}-10\,e^{3\,x}+e^{4\,x}\,5\mathrm{i}-e^{2\,x}\,10\mathrm{i}+5\,e^{x}+1\mathrm{i}}$$

input
```
int(1/(cosh(x)^4*(sinh(x) + 1i)),x)
```

output
```
1i/(4*(exp(x)*2i - exp(2*x) + 1)) - ((3*exp(x))/40 + 1i/8)/(exp(2*x) + exp
(x)*2i - 1) - ((3*exp(2*x))/40 + (exp(x)*1i)/4 - 5/24)/(exp(2*x)*3i + exp(
3*x) - 3*exp(x) - 1i) - 1/(6*(exp(2*x)*3i - exp(3*x) + 3*exp(x) - 1i)) + 3
/(8*(exp(x) - 1i)) - 3/(40*(exp(x) + 1i)) - ((exp(2*x)*3i)/8 + (3*exp(3*x)
)/40 - (5*exp(x))/8 - 1i/8)/(exp(3*x)*4i - 6*exp(2*x) + exp(4*x) - exp(x)*
4i + 1) - ((exp(3*x)*1i)/2 - (5*exp(2*x))/4 + (3*exp(4*x))/40 - (exp(x)*1i
)/2 + 3/40)/(exp(4*x)*5i - 10*exp(3*x) - exp(2*x)*10i + exp(5*x) + 5*exp(x
) + 1i)
```

Reduce [F]

$$\int \frac{\operatorname{sech}^4(x)}{i+\sinh(x)}\,dx = \int \frac{\operatorname{sech}(x)^4}{\sinh(x)+i}dx$$

input
```
int(sech(x)^4/(I+sinh(x)),x)
```

output
```
int(sech(x)**4/(sinh(x) + i),x)
```

3.170 $\int \frac{\text{sech}^5(x)}{i+\sinh(x)}\,dx$

Optimal result

Integrand size = 13, antiderivative size = 80

$$\int \frac{\text{sech}^5(x)}{i+\sinh(x)}\,dx = -\frac{5}{16}i\arctan(\sinh(x)) - \frac{1}{32(i-\sinh(x))^2} + \frac{i}{8(i-\sinh(x))} + \frac{i}{24(i+\sinh(x))^3} + \frac{3}{32(i+\sinh(x))^2} - \frac{3i}{16(i+\sinh(x))}$$

output
```
-5/16*I*arctan(sinh(x))-1/32/(I-sinh(x))^2+1/8*I/(I-sinh(x))+1/24*I/(I+sin
h(x))^3+3/32/(I+sinh(x))^2-3/16*I/(I+sinh(x))
```

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 94, normalized size of antiderivative = 1.18

$$\int \frac{\text{sech}^5(x)}{i+\sinh(x)}\,dx = -\frac{i\text{sech}^4(x)\left(8+15i\arctan(\sinh(x))+5(5i+3\arctan(\sinh(x)))\sinh(x)+5(5+6i\arctan(\sinh(x)))\sin\right.}{48}$$

input
```
Integrate[Sech[x]^5/(I + Sinh[x]),x]
```

output
```
((-1/48*I)*Sech[x]^4*(8 + (15*I)*ArcTan[Sinh[x]] + 5*(5*I + 3*ArcTan[Sinh[
x]])*Sinh[x] + 5*(5 + (6*I)*ArcTan[Sinh[x]])*Sinh[x]^2 + 15*(I + 2*ArcTan[
Sinh[x]])*Sinh[x]^3 + 15*(1 + I*ArcTan[Sinh[x]])*Sinh[x]^4 + 15*ArcTan[Sin
h[x]]*Sinh[x]^5))/(I + Sinh[x])
```

Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 80, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.308$, Rules used = {3042, 3146, 54, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\operatorname{sech}^5(x)}{\sinh(x)+i}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{(i-i\sin(ix))\cos(ix)^5}dx$$

$$\Big\downarrow \text{3146}$$

$$-\int \frac{1}{(i-\sinh(x))^3(\sinh(x)+i)^4}d\sinh(x)$$

$$\Big\downarrow \text{54}$$

$$-\int \left(-\frac{i}{8(\sinh(x)-i)^2}-\frac{3i}{16(\sinh(x)+i)^2}-\frac{1}{16(\sinh(x)-i)^3}+\frac{3}{16(\sinh(x)+i)^3}+\frac{i}{8(\sinh(x)+i)^4}+\frac{}{16(\sinh$$

$$\Big\downarrow \text{2009}$$

$$-\frac{5}{16}i\arctan(\sinh(x))+\frac{i}{8(-\sinh(x)+i)}-\frac{3i}{16(\sinh(x)+i)}-\frac{1}{32(-\sinh(x)+i)^2}+\frac{3}{32(\sinh(x)+i)^2}+\frac{i}{24(\sinh(x)+i)^3}$$

input
```
Int[Sech[x]^5/(I + Sinh[x]),x]
```

output
```
((-5*I)/16)*ArcTan[Sinh[x]] - 1/(32*(I - Sinh[x])^2) + (I/8)/(I - Sinh[x])
 + (I/24)/(I + Sinh[x])^3 + 3/(32*(I + Sinh[x])^2) - ((3*I)/16)/(I + Sinh[
x])
```

Defintions of rubi rules used

rule 54
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m + n + 2, 0])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3146
```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Simp[1/(b^p*f)   Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x
)^((p - 1)/2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && I
ntegerQ[(p - 1)/2] && EqQ[a^2 - b^2, 0] && (GeQ[p, -1] ||  !IntegerQ[m + 1/
2])
```

Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 136 vs. 2(59) = 118.

Time = 0.08 (sec) , antiderivative size = 137, normalized size of antiderivative = 1.71

$$\frac{3i}{8\left(\tanh\left(\frac{x}{2}\right)-i\right)}-\frac{i}{4\left(\tanh\left(\frac{x}{2}\right)-i\right)^{3}}+\frac{1}{8\left(\tanh\left(\frac{x}{2}\right)-i\right)^{4}}-\frac{1}{2\left(\tanh\left(\frac{x}{2}\right)-i\right)^{2}}-\frac{5\ln\left(\tanh\left(\frac{x}{2}\right)-i\right)}{16}+\frac{}{\left(\tanh\left(\right.\right.}$$

input
```
int(sech(x)^5/(I+sinh(x)),x)
```

output
```
3/8*I/(tanh(1/2*x)-I)-1/4*I/(tanh(1/2*x)-I)^3+1/8/(tanh(1/2*x)-I)^4-1/2/(t
anh(1/2*x)-I)^2-5/16*ln(tanh(1/2*x)-I)+I/(tanh(1/2*x)+I)^5+I/(tanh(1/2*x)+
I)-25/12*I/(tanh(1/2*x)+I)^3+1/3/(tanh(1/2*x)+I)^6-15/8/(tanh(1/2*x)+I)^4+
15/8/(tanh(1/2*x)+I)^2+5/16*ln(tanh(1/2*x)+I)
```

Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 245 vs. $2(46) = 92$.

Time = 0.11 (sec) , antiderivative size = 245, normalized size of antiderivative = 3.06

$$\int \frac{\operatorname{sech}^5(x)}{i+\sinh(x)}\,dx = \frac{15\left(e^{(10\,x)} + 2i\,e^{(9\,x)} + 3\,e^{(8\,x)} + 8i\,e^{(7\,x)} + 2\,e^{(6\,x)} + 12i\,e^{(5\,x)} - 2\,e^{(4\,x)} + 8i\,e^{(3\,x)} - 3\,e^{(2\,x)} + 2i\,e^{x} - 1\right)\log(\ldots}{\ldots}$$

input
```
integrate(sech(x)^5/(I+sinh(x)),x, algorithm="fricas")
```

output
```
1/48*(15*(e^(10*x) + 2*I*e^(9*x) + 3*e^(8*x) + 8*I*e^(7*x) + 2*e^(6*x) + 1
2*I*e^(5*x) - 2*e^(4*x) + 8*I*e^(3*x) - 3*e^(2*x) + 2*I*e^x - 1)*log(e^x +
 I) - 15*(e^(10*x) + 2*I*e^(9*x) + 3*e^(8*x) + 8*I*e^(7*x) + 2*e^(6*x) + 1
2*I*e^(5*x) - 2*e^(4*x) + 8*I*e^(3*x) - 3*e^(2*x) + 2*I*e^x - 1)*log(e^x -
 I) - 30*I*e^(9*x) + 60*e^(8*x) - 80*I*e^(7*x) + 220*e^(6*x) - 36*I*e^(5*x
) - 220*e^(4*x) - 80*I*e^(3*x) - 60*e^(2*x) - 30*I*e^x)/(e^(10*x) + 2*I*e^
(9*x) + 3*e^(8*x) + 8*I*e^(7*x) + 2*e^(6*x) + 12*I*e^(5*x) - 2*e^(4*x) + 8
*I*e^(3*x) - 3*e^(2*x) + 2*I*e^x - 1)
```

Sympy [F]

$$\int \frac{\operatorname{sech}^5(x)}{i+\sinh(x)}\,dx = \int \frac{\operatorname{sech}^5(x)}{\sinh(x)+i}\,dx$$

input
```
integrate(sech(x)**5/(I+sinh(x)),x)
```

output
```
Integral(sech(x)**5/(sinh(x) + I), x)
```

Maxima [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 140 vs. $2(46) = 92$.

Time = 0.04 (sec) , antiderivative size = 140, normalized size of antiderivative = 1.75

$$\int \frac{\operatorname{sech}^5(x)}{i + \sinh(x)}\, dx$$
$$= \frac{32\left(15i\, e^{(-x)} - 30\, e^{(-2\, x)} + 40i\, e^{(-3\, x)} - 110\, e^{(-4\, x)} + 18i\, e^{(-5\, x)} + 110\, e^{(-6\, x)} + 40i\, e^{(-7\,}\right.}{-1536i\, e^{(-x)} - 2304\, e^{(-2\, x)} - 6144i\, e^{(-3\, x)} - 1536\, e^{(-4\, x)} - 9216i\, e^{(-5\, x)} + 1536\, e^{(-6\, x)} - 6144i\, e^{(-7\, x)} + 23}$$
$$- \frac{5}{16} \log\left(e^{(-x)} + i\right) + \frac{5}{16} \log\left(e^{(-x)} - i\right)$$

input
```
integrate(sech(x)^5/(I+sinh(x)),x, algorithm="maxima")
```

output
```
32*(15*I*e^(-x) - 30*e^(-2*x) + 40*I*e^(-3*x) - 110*e^(-4*x) + 18*I*e^(-5*
x) + 110*e^(-6*x) + 40*I*e^(-7*x) + 30*e^(-8*x) + 15*I*e^(-9*x))/(-1536*I*
e^(-x) - 2304*e^(-2*x) - 6144*I*e^(-3*x) - 1536*e^(-4*x) - 9216*I*e^(-5*x)
 + 1536*e^(-6*x) - 6144*I*e^(-7*x) + 2304*e^(-8*x) - 1536*I*e^(-9*x) + 768
*e^(-10*x) - 768) - 5/16*log(e^(-x) + I) + 5/16*log(e^(-x) - I)
```

Giac [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 118 vs. $2(46) = 92$.

Time = 0.13 (sec) , antiderivative size = 118, normalized size of antiderivative = 1.48

$$\int \frac{\operatorname{sech}^5(x)}{i+\sinh(x)}\,dx = \frac{15\left(e^{(-x)}-e^{x}\right)^2+76i\,e^{(-x)}-76i\,e^{x}-100}{64\left(e^{(-x)}-e^{x}+2i\right)^2} - \frac{55\left(e^{(-x)}-e^{x}\right)^3-402i\left(e^{(-x)}-e^{x}\right)^2-1020\,e^{(-x)}+1020\,e^{x}+936i}{192\left(e^{(-x)}-e^{x}-2i\right)^3} + \frac{5}{32}\log\left(-e^{(-x)}+e^{x}+2i\right) - \frac{5}{32}\log\left(-e^{(-x)}+e^{x}-2i\right)$$

input
```
integrate(sech(x)^5/(I+sinh(x)),x, algorithm="giac")
```

output
```
1/64*(15*(e^(-x) - e^x)^2 + 76*I*e^(-x) - 76*I*e^x - 100)/(e^(-x) - e^x +
2*I)^2 - 1/192*(55*(e^(-x) - e^x)^3 - 402*I*(e^(-x) - e^x)^2 - 1020*e^(-x)
 + 1020*e^x + 936*I)/(e^(-x) - e^x - 2*I)^3 + 5/32*log(-e^(-x) + e^x + 2*I
) - 5/32*log(-e^(-x) + e^x - 2*I)
```

Mupad [B] (verification not implemented)

Time = 2.73 (sec) , antiderivative size = 249, normalized size of antiderivative = 3.11

$$\int \frac{\operatorname{sech}^5(x)}{i+\sinh(x)}\,dx = \frac{5\ln\left(-\frac{5}{8}+\frac{e^x\,5i}{8}\right)}{16} - \frac{5\ln\left(\frac{5}{8}+\frac{e^x\,5i}{8}\right)}{16} - \frac{1i}{e^{5x}-10\,e^{3x}+e^{4x}\,5i-e^{2x}\,10i+5\,e^x+1i} + \frac{1i}{4\,\left(e^{2x}\,3i-e^{3x}+3\,e^x-i\right)} + \frac{1}{8\,\left(e^{4x}-6\,e^{2x}+1-e^{3x}\,4i+e^x\,4i\right)} + \frac{5}{8\,\left(e^{4x}-6\,e^{2x}+1+e^{3x}\,4i-e^x\,4i\right)} - \frac{1}{8\,\left(1-e^{2x}+e^x\,2i\right)} - \frac{1i}{4\,\left(e^x-i\right)} - \frac{3i}{8\,\left(e^x+1i\right)} - \frac{1}{3\,\left(15\,e^{2x}-15\,e^{4x}+e^{6x}-1-e^{3x}\,20i+e^{5x}\,6i+e^x\,6i\right)} - \frac{5i}{12\,\left(e^{2x}\,3i+e^{3x}-3\,e^x-i\right)}$$

input
```
int(1/(cosh(x)^5*(sinh(x) + 1i)),x)
```

output
```
(5*log((exp(x)*5i)/8 - 5/8))/16 - (5*log((exp(x)*5i)/8 + 5/8))/16 - 1i/(ex
p(4*x)*5i - 10*exp(3*x) - exp(2*x)*10i + exp(5*x) + 5*exp(x) + 1i) + 1i/(4
*(exp(2*x)*3i - exp(3*x) + 3*exp(x) - 1i)) + 1/(8*(exp(4*x) - exp(3*x)*4i
- 6*exp(2*x) + exp(x)*4i + 1)) + 5/(8*(exp(3*x)*4i - 6*exp(2*x) + exp(4*x)
 - exp(x)*4i + 1)) - 1/(8*(exp(x)*2i - exp(2*x) + 1)) - 1i/(4*(exp(x) - 1i
)) - 3i/(8*(exp(x) + 1i)) - 1/(3*(15*exp(2*x) - exp(3*x)*20i - 15*exp(4*x)
 + exp(5*x)*6i + exp(6*x) + exp(x)*6i - 1)) - 5i/(12*(exp(2*x)*3i + exp(3*
x) - 3*exp(x) - 1i))
```

Reduce [F]

$$\int \frac{\operatorname{sech}^5(x)}{i+\sinh(x)}\,dx = \int \frac{\operatorname{sech}(x)^5}{\sinh(x)+i}dx$$

input
```
int(sech(x)^5/(I+sinh(x)),x)
```

output
```
int(sech(x)**5/(sinh(x) + i),x)
```

3.171 $\int \frac{\cosh^6(x)}{(i+\sinh(x))^2} \, dx$

Optimal result

Integrand size = 13, antiderivative size = 40

$$\int \frac{\cosh^6(x)}{(i+\sinh(x))^2} \, dx = -\frac{5x}{8} - \frac{5}{12} i \cosh^3(x) - \frac{5}{8} \cosh(x) \sinh(x) + \frac{\cosh^5(x)}{4(i+\sinh(x))}$$

output
```
-5/8*x-5/12*I*cosh(x)^3-5/8*cosh(x)*sinh(x)+cosh(x)^5/(4*I+4*sinh(x))
```

Mathematica [B] (verified)

Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 121 vs. 2(40) = 80.

Time = 0.14 (sec) , antiderivative size = 121, normalized size of antiderivative = 3.02

$$\int \frac{\cosh^6(x)}{(i+\sinh(x))^2} \, dx = -\frac{i \cosh^7(x) \left(16 + \frac{30 \arcsin\left(\frac{\sqrt{1-i\sinh(x)}}{\sqrt{2}}\right) \sqrt{1-i\sinh(x)}}{\sqrt{1+i\sinh(x)}} - 25 i \sinh(x) + 7\sinh^2(x) - 10 i \sinh^3(x) + 6\sinh^4(x)\right)}{24 \left(\cosh\left(\frac{x}{2}\right) - i \sinh\left(\frac{x}{2}\right)\right)^8 \left(\cosh\left(\frac{x}{2}\right) + i \sinh\left(\frac{x}{2}\right)\right)^6}$$

input
```
Integrate[Cosh[x]^6/(I + Sinh[x])^2,x]
```

output
```
((-1/24*I)*Cosh[x]^7*(16 + (30*ArcSin[Sqrt[1 - I*Sinh[x]]/Sqrt[2]]*Sqrt[1
- I*Sinh[x]])/Sqrt[1 + I*Sinh[x]] - (25*I)*Sinh[x] + 7*Sinh[x]^2 - (10*I)*
Sinh[x]^3 + 6*Sinh[x]^4))/((Cosh[x/2] - I*Sinh[x/2])^8*(Cosh[x/2] + I*Sinh
[x/2])^6)
```

Rubi [A] (verified)

Time = 0.40 (sec) , antiderivative size = 50, normalized size of antiderivative = 1.25, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.538$, Rules used = {3042, 3158, 3042, 3161, 3042, 3115, 24}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cosh^6(x)}{(\sinh(x)+i)^2} \, dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\cos(ix)^6}{(i-i\sin(ix))^2}dx$$

$$\Big\downarrow \text{3158}$$

$$\frac{\cosh^5(x)}{4(\sinh(x)+i)}-\frac{5}{4}i\int \frac{\cosh^4(x)}{\sinh(x)+i}dx$$

$$\Big\downarrow \text{3042}$$

$$\frac{\cosh^5(x)}{4(\sinh(x)+i)}-\frac{5}{4}i\int \frac{\cos(ix)^4}{i-i\sin(ix)}dx$$

$$\Big\downarrow \text{3161}$$

$$\frac{\cosh^5(x)}{4(\sinh(x)+i)}-\frac{5}{4}i\left(\frac{\cosh^3(x)}{3}-i\int \cosh^2(x)dx\right)$$

$$\Big\downarrow \text{3042}$$

$$\frac{\cosh^5(x)}{4(\sinh(x)+i)}-\frac{5}{4}i\left(\frac{\cosh^3(x)}{3}-i\int \sin\left(ix+\frac{\pi}{2}\right)^2dx\right)$$

$$\Big\downarrow \text{3115}$$

$$\frac{\cosh^5(x)}{4(\sinh(x)+i)}-\frac{5}{4}i\left(\frac{\cosh^3(x)}{3}-i\left(\frac{\int 1dx}{2}+\frac{1}{2}\sinh(x)\cosh(x)\right)\right)$$

$$\Big\downarrow \text{24}$$

$$\frac{\cosh^5(x)}{4(\sinh(x)+i)}-\frac{5}{4}i\left(\frac{\cosh^3(x)}{3}-i\left(\frac{x}{2}+\frac{1}{2}\sinh(x)\cosh(x)\right)\right)$$

input
```
Int[Cosh[x]^6/(I + Sinh[x])^2,x]
```

output
```
Cosh[x]^5/(4*(I + Sinh[x])) - ((5*I)/4)*(Cosh[x]^3/3 - I*(x/2 + (Cosh[x]*S
inh[x])/2))
```

Defintions of rubi rules used

rule 24
```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3115
```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Simp[b^2*((n - 1)/n)   Int[(b*Sin
[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[
2*n]
```

rule 3158
```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[g*(g*Cos[e + f*x])^(p - 1)*((a + b*Sin[e + f*x
])^(m + 1)/(b*f*(m + p))), x] + Simp[g^2*((p - 1)/(a*(m + p)))   Int[(g*Cos
[e + f*x])^(p - 2)*(a + b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{a, b, e,
f, g}, x] && EqQ[a^2 - b^2, 0] && LtQ[m, -1] && GtQ[p, 1] && (GtQ[m, -2] ||
 EqQ[2*m + p + 1, 0] || (EqQ[m, -2] && IntegerQ[p])) && NeQ[m + p, 0] && In
tegersQ[2*m, 2*p]
```

rule 3161
```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)/((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Simp[g*((g*Cos[e + f*x])^(p - 1)/(b*f*(p - 1))), x] + Si
mp[g^2/a   Int[(g*Cos[e + f*x])^(p - 2), x], x] /; FreeQ[{a, b, e, f, g}, x
] && EqQ[a^2 - b^2, 0] && GtQ[p, 1] && IntegerQ[2*p]
```

Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 111 vs. 2(31) = 62.

Time = 0.09 (sec) , antiderivative size = 112, normalized size of antiderivative = 2.80

$$\frac{\frac{1}{2}+\frac{2i}{3}}{\left(\tanh\left(\frac{x}{2}\right)-1\right)^{3}}+\frac{-\frac{1}{8}+i}{\left(\tanh\left(\frac{x}{2}\right)-1\right)^{2}}+\frac{-\frac{3}{8}+i}{\tanh\left(\frac{x}{2}\right)-1}+\frac{1}{4\left(\tanh\left(\frac{x}{2}\right)-1\right)^{4}}+\frac{5\ln\left(\tanh\left(\frac{x}{2}\right)-1\right)}{8}+\frac{\frac{1}{2}-\frac{2i}{3}}{\left(\tanh\left(\frac{x}{2}\right)+\right.}$$

input
```
int(cosh(x)^6/(I+sinh(x))^2,x)
```

output
```
(1/2+2/3*I)/(tanh(1/2*x)-1)^3+(-1/8+I)/(tanh(1/2*x)-1)^2+(-3/8+I)/(tanh(1/
2*x)-1)+1/4/(tanh(1/2*x)-1)^4+5/8*ln(tanh(1/2*x)-1)+(1/2-2/3*I)/(tanh(1/2*
x)+1)^3+(1/8+I)/(tanh(1/2*x)+1)^2-(3/8+I)/(tanh(1/2*x)+1)-1/4/(tanh(1/2*x)
+1)^4-5/8*ln(tanh(1/2*x)+1)
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 55, normalized size of antiderivative = 1.38

$$\int\frac{\cosh^{6}(x)}{(i+\sinh(x))^{2}}\,dx = -\frac{1}{192}\left(120\,xe^{(4\,x)}-3\,e^{(8\,x)}+16i\,e^{(7\,x)}+24\,e^{(6\,x)}+48i\,e^{(5\,x)}+48i\,e^{(3\,x)}-24\,e^{(2\,x)}+16i\,e^{x}+3\right)e^{(-4\,x)}$$

input
```
integrate(cosh(x)^6/(I+sinh(x))^2,x, algorithm="fricas")
```

output
```
-1/192*(120*x*e^(4*x) - 3*e^(8*x) + 16*I*e^(7*x) + 24*e^(6*x) + 48*I*e^(5*
x) + 48*I*e^(3*x) - 24*e^(2*x) + 16*I*e^x + 3)*e^(-4*x)
```

Sympy [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 65, normalized size of antiderivative = 1.62

$$\int \frac{\cosh^6(x)}{(i+\sinh(x))^2}\,dx = -\frac{5x}{8} + \frac{e^{4x}}{64} - \frac{ie^{3x}}{12} - \frac{e^{2x}}{8} - \frac{ie^{x}}{4} - \frac{ie^{-x}}{4} + \frac{e^{-2x}}{8} - \frac{ie^{-3x}}{12} - \frac{e^{-4x}}{64}$$

input
```
integrate(cosh(x)**6/(I+sinh(x))**2,x)
```

output
```
-5*x/8 + exp(4*x)/64 - I*exp(3*x)/12 - exp(2*x)/8 - I*exp(x)/4 - I*exp(-x)
/4 + exp(-2*x)/8 - I*exp(-3*x)/12 - exp(-4*x)/64
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 54, normalized size of antiderivative = 1.35

$$\int \frac{\cosh^6(x)}{(i+\sinh(x))^2}\,dx = -\frac{1}{192}\left(16i\,e^{(-x)} + 24\,e^{(-2\,x)} + 48i\,e^{(-3\,x)} - 3\right)e^{(4\,x)} - \frac{5}{8}\,x - \frac{1}{4}i\,e^{(-x)} + \frac{1}{8}\,e^{(-2\,x)} - \frac{1}{12}i\,e^{(-3\,x)} - \frac{1}{64}\,e^{(-4\,x)}$$

input
```
integrate(cosh(x)^6/(I+sinh(x))^2,x, algorithm="maxima")
```

output
```
-1/192*(16*I*e^(-x) + 24*e^(-2*x) + 48*I*e^(-3*x) - 3)*e^(4*x) - 5/8*x - 1
/4*I*e^(-x) + 1/8*e^(-2*x) - 1/12*I*e^(-3*x) - 1/64*e^(-4*x)
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 50, normalized size of antiderivative = 1.25

$$\int \frac{\cosh^6(x)}{(i+\sinh(x))^2}\,dx = -\frac{1}{192}\left(48i\,e^{(3\,x)} - 24\,e^{(2\,x)} + 16i\,e^{x} + 3\right)e^{(-4\,x)} - \frac{5}{8}\,x + \frac{1}{64}\,e^{(4\,x)} - \frac{1}{12}i\,e^{(3\,x)} - \frac{1}{8}\,e^{(2\,x)} - \frac{1}{4}i\,e^{x}$$

input
```
integrate(cosh(x)^6/(I+sinh(x))^2,x, algorithm="giac")
```

output
```
-1/192*(48*I*e^(3*x) - 24*e^(2*x) + 16*I*e^x + 3)*e^(-4*x) - 5/8*x + 1/64*
e^(4*x) - 1/12*I*e^(3*x) - 1/8*e^(2*x) - 1/4*I*e^x
```

Mupad [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 54, normalized size of antiderivative = 1.35

$$\int \frac{\cosh^6(x)}{(i+\sinh(x))^2}\,dx = \frac{e^{-2x}}{8} - \frac{e^{-x}\,1i}{4} - \frac{5x}{8} - \frac{e^{2x}}{8} - \frac{e^{-3x}\,1i}{12} - \frac{e^{3x}\,1i}{12} - \frac{e^{-4x}}{64} + \frac{e^{4x}}{64} - \frac{e^{x}\,1i}{4}$$

input
```
int(cosh(x)^6/(sinh(x) + 1i)^2,x)
```

output
```
exp(-2*x)/8 - (exp(-x)*1i)/4 - (5*x)/8 - exp(2*x)/8 - (exp(-3*x)*1i)/12 -
(exp(3*x)*1i)/12 - exp(-4*x)/64 + exp(4*x)/64 - (exp(x)*1i)/4
```

Reduce [F]

$$\int \frac{\cosh^6(x)}{(i+\sinh(x))^2}\,dx = \int \frac{\cosh(x)^6}{\sinh(x)^2 + 2\sinh(x)\,i - 1}dx$$

input
```
int(cosh(x)^6/(I+sinh(x))^2,x)
```

output
```
int(cosh(x)**6/(sinh(x)**2 + 2*sinh(x)*i - 1),x)
```

3.172 $\int \frac{\cosh^5(x)}{(i+\sinh(x))^2} dx$

Optimal result

Integrand size = 13, antiderivative size = 14

$$\int \frac{\cosh^5(x)}{(i+\sinh(x))^2} dx = -\frac{1}{3}(i-\sinh(x))^3$$

output
```
-1/3*(I-sinh(x))^3
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.29

$$\int \frac{\cosh^5(x)}{(i+\sinh(x))^2} dx = \frac{1}{6}(-7+\cosh(2x)-6i\sinh(x))\sinh(x)$$

input
```
Integrate[Cosh[x]^5/(I + Sinh[x])^2,x]
```

output
```
((-7 + Cosh[2*x] - (6*I)*Sinh[x])*Sinh[x])/6
```

Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.231$, Rules used = {3042, 3146, 17}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cosh^5(x)}{(\sinh(x)+i)^2}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\cos(ix)^5}{(i-i\sin(ix))^2}dx$$

$$\Big\downarrow \text{3146}$$

$$\int (-\sinh(x)+i)^2 d\sinh(x)$$

$$\Big\downarrow \text{17}$$

$$-\frac{1}{3}(-\sinh(x)+i)^3$$

input
```
Int[Cosh[x]^5/(I + Sinh[x])^2,x]
```

output
```
-1/3*(I - Sinh[x])^3
```

Defintions of rubi rules used

rule 17
```
Int[(c_.)*((a_.) + (b_.)*(x_))^(m_.), x_Symbol] :> Simp[c*((a + b*x)^(m + 1
)/(b*(m + 1))), x] /; FreeQ[{a, b, c, m}, x] && NeQ[m, -1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3146
```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Simp[1/(b^p*f)   Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x
)^((p - 1)/2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && I
ntegerQ[(p - 1)/2] && EqQ[a^2 - b^2, 0] && (GeQ[p, -1] ||  !IntegerQ[m + 1/
2])
```

Maple [A] (verified)

Time = 160.73 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.86

method	result	size
derivativedivides	$-\frac{(i-\sinh(x))^3}{3}$	12
default	$-\frac{(i-\sinh(x))^3}{3}$	12
risch	$\frac{e^{3x}}{24}-\frac{ie^{2x}}{4}-\frac{5\,e^{x}}{8}+\frac{5\,e^{-x}}{8}-\frac{ie^{-2x}}{4}-\frac{e^{-3x}}{24}$	38

input
```
int(cosh(x)^5/(I+sinh(x))^2,x,method=_RETURNVERBOSE)
```

output
```
-1/3*(I-sinh(x))^3
```

Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 34 vs. 2(8) = 16.

Time = 0.09 (sec) , antiderivative size = 34, normalized size of antiderivative = 2.43

$$\int \frac{\cosh^5(x)}{(i+\sinh(x))^2}\,dx = \frac{1}{24}\left(e^{(6\,x)}-6i\,e^{(5\,x)}-15\,e^{(4\,x)}+15\,e^{(2\,x)}-6i\,e^{x}-1\right)e^{(-3\,x)}$$

input
```
integrate(cosh(x)^5/(I+sinh(x))^2,x, algorithm="fricas")
```

output
```
1/24*(e^(6*x) - 6*I*e^(5*x) - 15*e^(4*x) + 15*e^(2*x) - 6*I*e^x - 1)*e^(-3
*x)
```

Sympy [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 44 vs. $2(8) = 16$.

Time = 0.13 (sec) , antiderivative size = 44, normalized size of antiderivative = 3.14

$$\int \frac{\cosh^5(x)}{(i+\sinh(x))^2}\,dx = \frac{e^{3x}}{24} - \frac{ie^{2x}}{4} - \frac{5e^{x}}{8} + \frac{5e^{-x}}{8} - \frac{ie^{-2x}}{4} - \frac{e^{-3x}}{24}$$

input
```
integrate(cosh(x)**5/(I+sinh(x))**2,x)
```

output
```
exp(3*x)/24 - I*exp(2*x)/4 - 5*exp(x)/8 + 5*exp(-x)/8 - I*exp(-2*x)/4 - ex
p(-3*x)/24
```

Maxima [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 39 vs. $2(8) = 16$.

Time = 0.03 (sec) , antiderivative size = 39, normalized size of antiderivative = 2.79

$$\int \frac{\cosh^5(x)}{(i+\sinh(x))^2}\,dx = -\frac{1}{24}\left(6i\,e^{(-x)} + 15\,e^{(-2\,x)} - 1\right)e^{(3\,x)} + \frac{5}{8}\,e^{(-x)} - \frac{1}{4}i\,e^{(-2\,x)} - \frac{1}{24}\,e^{(-3\,x)}$$

input
```
integrate(cosh(x)^5/(I+sinh(x))^2,x, algorithm="maxima")
```

output
```
-1/24*(6*I*e^(-x) + 15*e^(-2*x) - 1)*e^(3*x) + 5/8*e^(-x) - 1/4*I*e^(-2*x)
 - 1/24*e^(-3*x)
```

Giac [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 35 vs. 2(8) = 16.

Time = 0.12 (sec) , antiderivative size = 35, normalized size of antiderivative = 2.50

$$\int \frac{\cosh^5(x)}{(i+\sinh(x))^2}\,dx = \frac{1}{24}\left(15\,e^{(2\,x)} - 6i\,e^x - 1\right)e^{(-3\,x)} + \frac{1}{24}\,e^{(3\,x)} - \frac{1}{4}i\,e^{(2\,x)} - \frac{5}{8}\,e^x$$

input
```
integrate(cosh(x)^5/(I+sinh(x))^2,x, algorithm="giac")
```

output
```
1/24*(15*e^(2*x) - 6*I*e^x - 1)*e^(-3*x) + 1/24*e^(3*x) - 1/4*I*e^(2*x) -
5/8*e^x
```

Mupad [B] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 37, normalized size of antiderivative = 2.64

$$\int \frac{\cosh^5(x)}{(i+\sinh(x))^2}\,dx = \frac{5\,\mathrm{e}^{-x}}{8} - \frac{\mathrm{e}^{-3\,x}}{24} + \frac{\mathrm{e}^{3\,x}}{24} - \frac{5\,\mathrm{e}^{x}}{8} - \frac{\mathrm{e}^{-2\,x}\,1\mathrm{i}}{4} - \frac{\mathrm{e}^{2\,x}\,1\mathrm{i}}{4}$$

input
```
int(cosh(x)^5/(sinh(x) + 1i)^2,x)
```

output
```
(5*exp(-x))/8 - (exp(-2*x)*1i)/4 - (exp(2*x)*1i)/4 - exp(-3*x)/24 + exp(3*
x)/24 - (5*exp(x))/8
```

Reduce [F]

$$\int \frac{\cosh^5(x)}{(i+\sinh(x))^2}\,dx = \int \frac{\cosh(x)^5}{\sinh(x)^2 + 2\sinh(x)\,i - 1}dx$$

input
```
int(cosh(x)^5/(I+sinh(x))^2,x)
```

output
```
int(cosh(x)**5/(sinh(x)**2 + 2*sinh(x)*i - 1),x)
```

3.173 $\int \frac{\cosh^4(x)}{(i+\sinh(x))^2}\,dx$

Optimal result

Integrand size = 13, antiderivative size = 30

$$\int \frac{\cosh^4(x)}{(i+\sinh(x))^2}\,dx = -\frac{3x}{2} - \frac{3}{2} i\cosh(x) + \frac{\cosh^3(x)}{2(i+\sinh(x))}$$

output
```
-3/2*x-3/2*I*cosh(x)+cosh(x)^3/(2*I+2*sinh(x))
```

Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.53

$$\int \frac{\cosh^4(x)}{(i+\sinh(x))^2}\,dx = -3i\arcsin\left(\frac{\sqrt{1-i\sinh(x)}}{\sqrt{2}}\right)\sqrt{\cosh^2(x)}\operatorname{sech}(x) + \frac{1}{2}\cosh(x)(-4i+\sinh(x))$$

input
```
Integrate[Cosh[x]^4/(I + Sinh[x])^2,x]
```

output
```
(-3*I)*ArcSin[Sqrt[1 - I*Sinh[x]]/Sqrt[2]]*Sqrt[Cosh[x]^2]*Sech[x] + (Cosh
[x]*(-4*I + Sinh[x]))/2
```

Rubi [A] (verified)

Time = 0.31 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.03, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.385, Rules used = {3042, 3158, 3042, 3161, 24}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cosh^4(x)}{(\sinh(x)+i)^2}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\cos(ix)^4}{(i-i\sin(ix))^2}dx$$

$$\Big\downarrow \text{3158}$$

$$\frac{\cosh^3(x)}{2(\sinh(x)+i)}-\frac{3}{2}i\int \frac{\cosh^2(x)}{\sinh(x)+i}dx$$

$$\Big\downarrow \text{3042}$$

$$\frac{\cosh^3(x)}{2(\sinh(x)+i)}-\frac{3}{2}i\int \frac{\cos(ix)^2}{i-i\sin(ix)}dx$$

$$\Big\downarrow \text{3161}$$

$$\frac{\cosh^3(x)}{2(\sinh(x)+i)}-\frac{3}{2}i\left(\cosh(x)-i\int 1dx\right)$$

$$\Big\downarrow \text{24}$$

$$\frac{\cosh^3(x)}{2(\sinh(x)+i)}-\frac{3}{2}i(\cosh(x)-ix)$$

input
```
Int[Cosh[x]^4/(I + Sinh[x])^2,x]
```

output
```
((-3*I)/2)*((-I)*x + Cosh[x]) + Cosh[x]^3/(2*(I + Sinh[x]))
```

Defintions of rubi rules used

rule 24
```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3158
```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[g*(g*Cos[e + f*x])^(p - 1)*((a + b*Sin[e + f*x
])^(m + 1)/(b*f*(m + p))), x] + Simp[g^2*((p - 1)/(a*(m + p)))   Int[(g*Cos
[e + f*x])^(p - 2)*(a + b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{a, b, e,
f, g}, x] && EqQ[a^2 - b^2, 0] && LtQ[m, -1] && GtQ[p, 1] && (GtQ[m, -2] ||
 EqQ[2*m + p + 1, 0] || (EqQ[m, -2] && IntegerQ[p])) && NeQ[m + p, 0] && In
tegersQ[2*m, 2*p]
```

rule 3161
```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)/((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Simp[g*((g*Cos[e + f*x])^(p - 1)/(b*f*(p - 1))), x] + Si
mp[g^2/a   Int[(g*Cos[e + f*x])^(p - 2), x], x] /; FreeQ[{a, b, e, f, g}, x
] && EqQ[a^2 - b^2, 0] && GtQ[p, 1] && IntegerQ[2*p]
```

Maple [A] (verified)

Time = 17.56 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.97

method	result	size
risch	$-\frac{3x}{2}+\frac{{\rm e}^{2x}}{8}-i{\rm e}^{x}-i{\rm e}^{-x}-\frac{{\rm e}^{-2x}}{8}$	29
default	$\frac{\frac{1}{2}-2i}{\tanh\left(\frac{x}{2}\right)+1}-\frac{1}{2\left(\tanh\left(\frac{x}{2}\right)+1\right)^{2}}-\frac{3\ln\left(\tanh\left(\frac{x}{2}\right)+1\right)}{2}+\frac{\frac{1}{2}+2i}{\tanh\left(\frac{x}{2}\right)-1}+\frac{1}{2\left(\tanh\left(\frac{x}{2}\right)-1\right)^{2}}+\frac{3\ln\left(\tanh\left(\frac{x}{2}\right)-1\right)}{2}$	64

input
```
int(cosh(x)^4/(I+sinh(x))^2,x,method=_RETURNVERBOSE)
```

output
```
-3/2*x+1/8*exp(x)^2-I*exp(x)-I/exp(x)-1/8/exp(x)^2
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.03

$$\int \frac{\cosh^4(x)}{(i+\sinh(x))^2}\,dx = -\frac{1}{8}\left(12\,xe^{(2\,x)} - e^{(4\,x)} + 8i\,e^{(3\,x)} + 8i\,e^{x} + 1\right)e^{(-2\,x)}$$

input
```
integrate(cosh(x)^4/(I+sinh(x))^2,x, algorithm="fricas")
```

output
```
-1/8*(12*x*e^(2*x) - e^(4*x) + 8*I*e^(3*x) + 8*I*e^x + 1)*e^(-2*x)
```

Sympy [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.97

$$\int \frac{\cosh^4(x)}{(i+\sinh(x))^2}\,dx = -\frac{3x}{2} + \frac{e^{2x}}{8} - ie^{x} - ie^{-x} - \frac{e^{-2x}}{8}$$

input
```
integrate(cosh(x)**4/(I+sinh(x))**2,x)
```

output
```
-3*x/2 + exp(2*x)/8 - I*exp(x) - I*exp(-x) - exp(-2*x)/8
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.00

$$\int \frac{\cosh^4(x)}{(i+\sinh(x))^2}\,dx = -\frac{1}{8}\left(8i\,e^{(-x)} - 1\right)e^{(2\,x)} - \frac{3}{2}\,x - i\,e^{(-x)} - \frac{1}{8}\,e^{(-2\,x)}$$

input
```
integrate(cosh(x)^4/(I+sinh(x))^2,x, algorithm="maxima")
```

output
```
-1/8*(8*I*e^(-x) - 1)*e^(2*x) - 3/2*x - I*e^(-x) - 1/8*e^(-2*x)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.87

$$\int \frac{\cosh^4(x)}{(i+\sinh(x))^2}\,dx = -\frac{1}{8}\left(8i\,e^x+1\right)e^{(-2\,x)} - \frac{3}{2}\,x + \frac{1}{8}\,e^{(2\,x)} - i\,e^x$$

input
```
integrate(cosh(x)^4/(I+sinh(x))^2,x, algorithm="giac")
```

output
```
-1/8*(8*I*e^x + 1)*e^(-2*x) - 3/2*x + 1/8*e^(2*x) - I*e^x
```

Mupad [B] (verification not implemented)

Time = 1.32 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.93

$$\int \frac{\cosh^4(x)}{(i+\sinh(x))^2}\,dx = \frac{e^{2\,x}}{8} - e^{-x}\,1i - \frac{e^{-2\,x}}{8} - \frac{3\,x}{2} - e^{x}\,1i$$

input
```
int(cosh(x)^4/(sinh(x) + 1i)^2,x)
```

output
```
exp(2*x)/8 - exp(-x)*1i - exp(-2*x)/8 - (3*x)/2 - exp(x)*1i
```

Reduce [F]

$$\int \frac{\cosh^4(x)}{(i+\sinh(x))^2}\,dx = \int \frac{\cosh\left(x\right)^4}{\sinh\left(x\right)^2+2\sinh\left(x\right)i-1}dx$$

input
```
int(cosh(x)^4/(I+sinh(x))^2,x)
```

output
```
int(cosh(x)**4/(sinh(x)**2 + 2*sinh(x)*i - 1),x)
```

3.174 $\int \frac{\cosh^3(x)}{(i+\sinh(x))^2} \, dx$

Optimal result

Integrand size = 13, antiderivative size = 14

$$\int \frac{\cosh^3(x)}{(i+\sinh(x))^2} \, dx = -2i\log(i+\sinh(x))+\sinh(x)$$

output
```
-2*I*ln(I+sinh(x))+sinh(x)
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.00

$$\int \frac{\cosh^3(x)}{(i+\sinh(x))^2} \, dx = -2i\log(i+\sinh(x))+\sinh(x)$$

input
```
Integrate[Cosh[x]^3/(I + Sinh[x])^2,x]
```

output
```
(-2*I)*Log[I + Sinh[x]] + Sinh[x]
```

Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.308$, Rules used = {3042, 3146, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cosh^3(x)}{(\sinh(x)+i)^2}\,dx$$
$$\Big\downarrow \text{3042}$$
$$\int \frac{\cos(ix)^3}{(i-i\sin(ix))^2}dx$$
$$\Big\downarrow \text{3146}$$
$$-\int \frac{i-\sinh(x)}{\sinh(x)+i}d\sinh(x)$$
$$\Big\downarrow \text{49}$$
$$-\int \left(\frac{2i}{\sinh(x)+i}-1\right)d\sinh(x)$$
$$\Big\downarrow \text{2009}$$
$$\sinh(x)-2i\log(\sinh(x)+i)$$

input
```
Int[Cosh[x]^3/(I + Sinh[x])^2,x]
```

output
```
(-2*I)*Log[I + Sinh[x]] + Sinh[x]
```

Defintions of rubi rules used

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3146
```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Simp[1/(b^p*f)   Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x
)^((p - 1)/2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && I
ntegerQ[(p - 1)/2] && EqQ[a^2 - b^2, 0] && (GeQ[p, -1] ||  !IntegerQ[m + 1/
2])
```

Maple [A] (verified)

Time = 6.56 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.79

method	result	size
risch	$2ix+\frac{{\mathrm e}^{x}}{2}-\frac{{\mathrm e}^{-x}}{2}-4 i \ln \left({\mathrm e}^{x}+i\right)$	25
default	$2 i \ln \left(\tanh \left(\frac{x}{2}\right)+1\right)-\frac{1}{\tanh \left(\frac{x}{2}\right)+1}-4 i \ln \left(\tanh \left(\frac{x}{2}\right)+i\right)+2 i \ln \left(\tanh \left(\frac{x}{2}\right)-1\right)-\frac{1}{\tanh \left(\frac{x}{2}\right)-1}$	53

input
```
int(cosh(x)^3/(I+sinh(x))^2,x,method=_RETURNVERBOSE)
```

output
```
2*I*x+1/2*exp(x)-1/2*exp(-x)-4*I*ln(exp(x)+I)
```

Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 26 vs. 2(10) = 20.

Time = 0.09 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.86

$$\int \frac{\cosh^3(x)}{(i+\sinh(x))^2}\,dx = \frac{1}{2}\left(4i\,xe^{x} - 8i\,e^{x}\log\left(e^{x}+i\right) + e^{(2\,x)} - 1\right)e^{(-x)}$$

input
```
integrate(cosh(x)^3/(I+sinh(x))^2,x, algorithm="fricas")
```

output
```
1/2*(4*I*x*e^x - 8*I*e^x*log(e^x + I) + e^(2*x) - 1)*e^(-x)
```

Sympy [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 26 vs. 2(12) = 24.

Time = 0.13 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.86

$$\int \frac{\cosh^3(x)}{(i+\sinh(x))^2}\,dx = 2ix + \frac{e^{x}}{2} - 4i\log\left(e^{x}+i\right) - \frac{e^{-x}}{2}$$

input
```
integrate(cosh(x)**3/(I+sinh(x))**2,x)
```

output
```
2*I*x + exp(x)/2 - 4*I*log(exp(x) + I) - exp(-x)/2
```

Maxima [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 23 vs. 2(10) = 20.

Time = 0.04 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.64

$$\int \frac{\cosh^3(x)}{(i+\sinh(x))^2}\,dx = -2i\,x - \frac{1}{2}\,e^{(-x)} + \frac{1}{2}\,e^{x} - 4i\,\log\left(e^{(-x)} - i\right)$$

input
```
integrate(cosh(x)^3/(I+sinh(x))^2,x, algorithm="maxima")
```

output
```
-2*I*x - 1/2*e^(-x) + 1/2*e^x - 4*I*log(e^(-x) - I)
```

Giac [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 21 vs. 2(10) = 20.

Time = 0.12 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.50

$$\int \frac{\cosh^3(x)}{(i+\sinh(x))^2}\,dx = 2i\,x - \frac{1}{2}\,e^{(-x)} + \frac{1}{2}\,e^{x} - 4i\,\log\left(e^{x} + i\right)$$

input
```
integrate(cosh(x)^3/(I+sinh(x))^2,x, algorithm="giac")
```

output
```
2*I*x - 1/2*e^(-x) + 1/2*e^x - 4*I*log(e^x + I)
```

Mupad [B] (verification not implemented)

Time = 1.39 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.71

$$\int \frac{\cosh^3(x)}{(i+\sinh(x))^2}\,dx = \frac{e^x}{2} - \frac{e^{-x}}{2} + x\,2i - \ln\left(e^x + 1i\right)\,4i$$

input
```
int(cosh(x)^3/(sinh(x) + 1i)^2,x)
```

output
```
x*2i - exp(-x)/2 - log(exp(x) + 1i)*4i + exp(x)/2
```

Reduce [F]

$$\int \frac{\cosh^3(x)}{(i+\sinh(x))^2}\,dx = \int \frac{\cosh(x)^3}{\sinh(x)^2 + 2\sinh(x)\,i - 1}dx$$

input
```
int(cosh(x)^3/(I+sinh(x))^2,x)
```

output
```
int(cosh(x)**3/(sinh(x)**2 + 2*sinh(x)*i - 1),x)
```

3.175 $\int \frac{\cosh^2(x)}{(i+\sinh(x))^2} \, dx$

Optimal result

Integrand size = 13, antiderivative size = 14

$$\int \frac{\cosh^2(x)}{(i+\sinh(x))^2} \, dx = x - \frac{2\cosh(x)}{i+\sinh(x)}$$

output
```
x-2*cosh(x)/(I+sinh(x))
```

Mathematica [B] (verified)

Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 69 vs. 2(14) = 28.

Time = 0.05 (sec) , antiderivative size = 69, normalized size of antiderivative = 4.93

$$\int \frac{\cosh^2(x)}{(i+\sinh(x))^2} \, dx = \frac{2\cosh^3(x)\left(-1 - \frac{\arcsin\left(\frac{\sqrt{1-i\sinh(x)}}{\sqrt{2}}\right)\sqrt{1-i\sinh(x)}}{\sqrt{1+i\sinh(x)}}\right)}{(-i+\sinh(x))(i+\sinh(x))^2}$$

input
```
Integrate[Cosh[x]^2/(I + Sinh[x])^2,x]
```

output
```
(2*Cosh[x]^3*(-1 - (ArcSin[Sqrt[1 - I*Sinh[x]]/Sqrt[2]]*Sqrt[1 - I*Sinh[x]
])/Sqrt[1 + I*Sinh[x]]))/((-I + Sinh[x])*(I + Sinh[x])^2)
```

Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.231$, Rules used = {3042, 3159, 24}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cosh^2(x)}{(\sinh(x)+i)^2} \, dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\cos(ix)^2}{(i-i\sin(ix))^2}dx$$

$$\Big\downarrow \text{3159}$$

$$\int 1dx - \frac{2\cosh(x)}{\sinh(x)+i}$$

$$\Big\downarrow \text{24}$$

$$x - \frac{2\cosh(x)}{\sinh(x)+i}$$

input
```
Int[Cosh[x]^2/(I + Sinh[x])^2,x]
```

output
```
x - (2*Cosh[x])/(I + Sinh[x])
```

Defintions of rubi rules used

rule 24
```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3159
```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[2*g*(g*Cos[e + f*x])^(p - 1)*((a + b*Sin[e + f
*x])^(m + 1)/(b*f*(2*m + p + 1))), x] + Simp[g^2*((p - 1)/(b^2*(2*m + p + 1
)))   Int[(g*Cos[e + f*x])^(p - 2)*(a + b*Sin[e + f*x])^(m + 2), x], x] /;
FreeQ[{a, b, e, f, g}, x] && EqQ[a^2 - b^2, 0] && LeQ[m, -2] && GtQ[p, 1] &
& NeQ[2*m + p + 1, 0] &&  !ILtQ[m + p + 1, 0] && IntegersQ[2*m, 2*p]
```

Maple [A] (verified)

Time = 2.23 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.93

method	result	size
risch	$x+\frac{4i}{{\mathrm e}^{x}+i}$	13
default	$-\frac{4}{\tanh\left(\frac{x}{2}\right)+i}+\ln\left(\tanh\left(\frac{x}{2}\right)+1\right)-\ln\left(\tanh\left(\frac{x}{2}\right)-1\right)$	29

input
```
int(cosh(x)^2/(I+sinh(x))^2,x,method=_RETURNVERBOSE)
```

output
```
x+4*I/(exp(x)+I)
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.14

$$\int \frac{\cosh^2(x)}{(i+\sinh(x))^2}\,dx = \frac{xe^x + i\,x + 4i}{e^x + i}$$

input
```
integrate(cosh(x)^2/(I+sinh(x))^2,x, algorithm="fricas")
```

output
```
(x*e^x + I*x + 4*I)/(e^x + I)
```

Sympy [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 8, normalized size of antiderivative = 0.57

$$\int \frac{\cosh^2(x)}{(i+\sinh(x))^2}\,dx = x + \frac{4i}{e^x + i}$$

input
```
integrate(cosh(x)**2/(I+sinh(x))**2,x)
```

output
```
x + 4*I/(exp(x) + I)
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.86

$$\int \frac{\cosh^2(x)}{(i+\sinh(x))^2}\,dx = x + \frac{4i}{e^{(-x)} - i}$$

input
```
integrate(cosh(x)^2/(I+sinh(x))^2,x, algorithm="maxima")
```

output
```
x + 4*I/(e^(-x) - I)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.71

$$\int \frac{\cosh^2(x)}{(i+\sinh(x))^2}\,dx = x + \frac{4i}{e^x + i}$$

input
```
integrate(cosh(x)^2/(I+sinh(x))^2,x, algorithm="giac")
```

output
```
x + 4*I/(e^x + I)
```

Mupad [B] (verification not implemented)

Time = 1.40 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.86

$$\int \frac{\cosh^2(x)}{(i+\sinh(x))^2}\,dx = x + \frac{4\mathrm{i}}{\mathrm{e}^x + 1\mathrm{i}}$$

input
```
int(cosh(x)^2/(sinh(x) + 1i)^2,x)
```

output
```
x + 4i/(exp(x) + 1i)
```

Reduce [F]

$$\int \frac{\cosh^2(x)}{(i+\sinh(x))^2}\,dx = \int \frac{\cosh(x)^2}{\sinh(x)^2 + 2\sinh(x)\,i - 1}dx$$

input
```
int(cosh(x)^2/(I+sinh(x))^2,x)
```

output
```
int(cosh(x)**2/(sinh(x)**2 + 2*sinh(x)*i - 1),x)
```

3.176 $\int \frac{\cosh(x)}{(i+\sinh(x))^2}\,dx$

Optimal result

Integrand size = 11, antiderivative size = 10

$$\int \frac{\cosh(x)}{(i+\sinh(x))^2}\,dx = -\frac{1}{i+\sinh(x)}$$

output
```
-1/(I+sinh(x))
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 10, normalized size of antiderivative = 1.00

$$\int \frac{\cosh(x)}{(i+\sinh(x))^2}\,dx = -\frac{1}{i+\sinh(x)}$$

input
```
Integrate[Cosh[x]/(I + Sinh[x])^2,x]
```

output
```
-(I + Sinh[x])^(-1)
```

Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 10, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.273$, Rules used = {3042, 3146, 17}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cosh(x)}{(\sinh(x)+i)^2}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\cos(ix)}{(i-i\sin(ix))^2}dx$$

$$\Big\downarrow \text{3146}$$

$$\int \frac{1}{(\sinh(x)+i)^2}d\sinh(x)$$

$$\Big\downarrow \text{17}$$

$$-\frac{1}{\sinh(x)+i}$$

input
```
Int[Cosh[x]/(I + Sinh[x])^2,x]
```

output
```
-(I + Sinh[x])^(-1)
```

Defintions of rubi rules used

rule 17
```
Int[(c_.)*((a_.) + (b_.)*(x_))^(m_.), x_Symbol] :> Simp[c*((a + b*x)^(m + 1
)/(b*(m + 1))), x] /; FreeQ[{a, b, c, m}, x] && NeQ[m, -1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3146
```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Simp[1/(b^p*f)   Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x
)^((p - 1)/2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && I
ntegerQ[(p - 1)/2] && EqQ[a^2 - b^2, 0] && (GeQ[p, -1] ||  !IntegerQ[m + 1/
2])
```

Maple [A] (verified)

Time = 1.89 (sec) , antiderivative size = 10, normalized size of antiderivative = 1.00

method	result	size
derivativedivides	$-\frac{1}{i+\sinh(x)}$	10
default	$-\frac{1}{i+\sinh(x)}$	10
risch	$-\frac{2\,\mathrm{e}^{x}}{(\mathrm{e}^{x}+i)^{2}}$	12

input
```
int(cosh(x)/(I+sinh(x))^2,x,method=_RETURNVERBOSE)
```

output
```
-1/(I+sinh(x))
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.60

$$\int \frac{\cosh(x)}{(i+\sinh(x))^2}\,dx = -\frac{2\,e^{x}}{e^{(2\,x)} + 2i\,e^{x} - 1}$$

input
```
integrate(cosh(x)/(I+sinh(x))^2,x, algorithm="fricas")
```

output
```
-2*e^x/(e^(2*x) + 2*I*e^x - 1)
```

Sympy [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 19 vs. 2(7) = 14.

Time = 0.07 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.90

$$\int \frac{\cosh(x)}{(i+\sinh(x))^2}\,dx = -\frac{2e^x}{e^{2x}+2ie^x-1}$$

input
```
integrate(cosh(x)/(I+sinh(x))**2,x)
```

output
```
-2*exp(x)/(exp(2*x) + 2*I*exp(x) - 1)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 8, normalized size of antiderivative = 0.80

$$\int \frac{\cosh(x)}{(i+\sinh(x))^2}\,dx = -\frac{1}{\sinh{(x)}+i}$$

input
```
integrate(cosh(x)/(I+sinh(x))^2,x, algorithm="maxima")
```

output
```
-1/(sinh(x) + I)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 10, normalized size of antiderivative = 1.00

$$\int \frac{\cosh(x)}{(i+\sinh(x))^2}\,dx = -\frac{2\,e^x}{\left(e^x+i\right)^2}$$

input
```
integrate(cosh(x)/(I+sinh(x))^2,x, algorithm="giac")
```

output
```
-2*e^x/(e^x + I)^2
```

Mupad [B] (verification not implemented)

Time = 1.41 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.20

$$\int \frac{\cosh(x)}{(i+\sinh(x))^2}\,dx = -\frac{1\mathrm{i}}{-1+\sinh(x)\ 1\mathrm{i}}$$

input
```
int(cosh(x)/(sinh(x) + 1i)^2,x)
```

output
```
-1i/(sinh(x)*1i - 1)
```

Reduce [F]

$$\int \frac{\cosh(x)}{(i+\sinh(x))^2}\,dx = \int \frac{\cosh(x)}{\sinh(x)^2+2\sinh(x)\,i-1}dx$$

input
```
int(cosh(x)/(I+sinh(x))^2,x)
```

output
```
int(cosh(x)/(sinh(x)**2 + 2*sinh(x)*i - 1),x)
```

3.177 $\int \frac{\operatorname{sech}(x)}{(i+\sinh(x))^2}\,dx$

Optimal result

Integrand size = 11, antiderivative size = 34

$$\int \frac{\operatorname{sech}(x)}{(i+\sinh(x))^2}\,dx = -\frac{1}{4}\arctan(\sinh(x)) - \frac{i}{4(i+\sinh(x))^2} - \frac{1}{4(i+\sinh(x))}$$

output
```
-1/4*arctan(sinh(x))-1/4*I/(I+sinh(x))^2-1/(4*I+4*sinh(x))
```

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.76

$$\int \frac{\operatorname{sech}(x)}{(i+\sinh(x))^2}\,dx = \frac{1}{4}\left(-\arctan(\sinh(x)) - \frac{2i+\sinh(x)}{(i+\sinh(x))^2}\right)$$

input
```
Integrate[Sech[x]/(I + Sinh[x])^2,x]
```

output
```
(-ArcTan[Sinh[x]] - (2*I + Sinh[x])/(I + Sinh[x])^2)/4
```

Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.364$, Rules used = {3042, 3146, 54, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\text{sech}(x)}{(\sinh(x)+i)^2} \, dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{(i-i\sin(ix))^2\cos(ix)}dx$$

$$\Big\downarrow \text{3146}$$

$$-\int \frac{1}{(i-\sinh(x))(\sinh(x)+i)^3}d\sinh(x)$$

$$\Big\downarrow \text{54}$$

$$-\int \left(-\frac{1}{4(\sinh(x)+i)^2}-\frac{i}{2(\sinh(x)+i)^3}+\frac{1}{4\left(\sinh^2(x)+1\right)}\right)d\sinh(x)$$

$$\Big\downarrow \text{2009}$$

$$-\frac{1}{4}\arctan(\sinh(x))-\frac{1}{4(\sinh(x)+i)}-\frac{i}{4(\sinh(x)+i)^2}$$

input
```
Int[Sech[x]/(I + Sinh[x])^2,x]
```

output
```
-1/4*ArcTan[Sinh[x]] - (I/4)/(I + Sinh[x])^2 - 1/(4*(I + Sinh[x]))
```

Defintions of rubi rules used

rule 54
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m + n + 2, 0])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3146
```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Simp[1/(b^p*f)   Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x
)^((p - 1)/2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && I
ntegerQ[(p - 1)/2] && EqQ[a^2 - b^2, 0] && (GeQ[p, -1] ||  !IntegerQ[m + 1/
2])
```

Maple [A] (verified)

Time = 8.44 (sec) , antiderivative size = 45, normalized size of antiderivative = 1.32

method	result	size
risch	$-\frac{-{\mathrm e}^{x}+4 i {\mathrm e}^{2 x}+{\mathrm e}^{3 x}}{2 \left({\mathrm e}^{x}+i\right)^{4}}+\frac{i \ln \left({\mathrm e}^{x}-i\right)}{4}-\frac{i \ln \left({\mathrm e}^{x}+i\right)}{4}$	45
default	$\frac{i}{\left(\tanh \left(\frac{x}{2}\right)+i\right)^{4}}-\frac{i \ln \left(\tanh \left(\frac{x}{2}\right)+i\right)}{4}-\frac{5 i}{2 \left(\tanh \left(\frac{x}{2}\right)+i\right)^{2}}-\frac{2}{\left(\tanh \left(\frac{x}{2}\right)+i\right)^{3}}+\frac{3}{2 \left(\tanh \left(\frac{x}{2}\right)+i\right)}+\frac{i \ln \left(\tanh \left(\frac{x}{2}\right)-i\right)}{4}$	70

input
```
int(sech(x)/(I+sinh(x))^2,x,method=_RETURNVERBOSE)
```

output
```
-1/2*(-exp(x)+4*I*exp(x)^2+exp(x)^3)/(exp(x)+I)^4+1/4*I*ln(exp(x)-I)-1/4*I
*ln(exp(x)+I)
```

Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 103 vs. $2(22) = 44$.

Time = 0.09 (sec) , antiderivative size = 103, normalized size of antiderivative = 3.03

$$\int \frac{\operatorname{sech}(x)}{(i+\sinh(x))^2}\,dx = \frac{\left(-i\,e^{(4\,x)}+4\,e^{(3\,x)}+6i\,e^{(2\,x)}-4\,e^{x}-i\right)\log\left(e^{x}+i\right)+\left(i\,e^{(4\,x)}-4\,e^{(3\,x)}-6i\,e^{(2\,x)}+4\,e^{x}+i\right)\log\left(e^{x}-i\right)}{4\left(e^{(4\,x)}+4i\,e^{(3\,x)}-6\,e^{(2\,x)}-4i\,e^{x}+1\right)}$$

input
```
integrate(sech(x)/(I+sinh(x))^2,x, algorithm="fricas")
```

output
```
1/4*((-I*e^(4*x) + 4*e^(3*x) + 6*I*e^(2*x) - 4*e^x - I)*log(e^x + I) + (I*
e^(4*x) - 4*e^(3*x) - 6*I*e^(2*x) + 4*e^x + I)*log(e^x - I) - 2*e^(3*x) -
8*I*e^(2*x) + 2*e^x)/(e^(4*x) + 4*I*e^(3*x) - 6*e^(2*x) - 4*I*e^x + 1)
```

Sympy [F]

$$\int \frac{\operatorname{sech}(x)}{(i+\sinh(x))^2}\,dx = \int \frac{\operatorname{sech}(x)}{(\sinh(x)+i)^2}\,dx$$

input
```
integrate(sech(x)/(I+sinh(x))**2,x)
```

output
```
Integral(sech(x)/(sinh(x) + I)**2, x)
```

Maxima [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 70 vs. $2(22) = 44$.

Time = 0.04 (sec) , antiderivative size = 70, normalized size of antiderivative = 2.06

$$\int \frac{\operatorname{sech}(x)}{(i+\sinh(x))^2}\,dx = -\frac{2\left(e^{(-x)} + 4i\,e^{(-2\,x)} - e^{(-3\,x)}\right)}{16i\,e^{(-x)} - 24\,e^{(-2\,x)} - 16i\,e^{(-3\,x)} + 4\,e^{(-4\,x)} + 4} - \frac{1}{4}i\,\log\left(i\,e^{(-x)} + 1\right) + \frac{1}{4}i\,\log\left(i\,e^{(-x)} - 1\right)$$

input
```
integrate(sech(x)/(I+sinh(x))^2,x, algorithm="maxima")
```

output
```
-2*(e^(-x) + 4*I*e^(-2*x) - e^(-3*x))/(16*I*e^(-x) - 24*e^(-2*x) - 16*I*e^
(-3*x) + 4*e^(-4*x) + 4) - 1/4*I*log(I*e^(-x) + 1) + 1/4*I*log(I*e^(-x) -
1)
```

Giac [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 66 vs. $2(22) = 44$.

Time = 0.13 (sec) , antiderivative size = 66, normalized size of antiderivative = 1.94

$$\int \frac{\operatorname{sech}(x)}{(i+\sinh(x))^2}\,dx = \frac{3i\left(e^{(-x)} - e^{x}\right)^2 + 20\,e^{(-x)} - 20\,e^{x} - 44i}{16\left(e^{(-x)} - e^{x} - 2i\right)^2} - \frac{1}{8}i\,\log\left(-e^{(-x)} + e^{x} + 2i\right) + \frac{1}{8}i\,\log\left(-e^{(-x)} + e^{x} - 2i\right)$$

input
```
integrate(sech(x)/(I+sinh(x))^2,x, algorithm="giac")
```

output
```
1/16*(3*I*(e^(-x) - e^x)^2 + 20*e^(-x) - 20*e^x - 44*I)/(e^(-x) - e^x - 2*
I)^2 - 1/8*I*log(-e^(-x) + e^x + 2*I) + 1/8*I*log(-e^(-x) + e^x - 2*I)
```

Mupad [B] (verification not implemented)

Time = 1.67 (sec) , antiderivative size = 86, normalized size of antiderivative = 2.53

$$\int \frac{\operatorname{sech}(x)}{(i+\sinh(x))^2}\,dx = -\frac{\operatorname{atan}(e^x)}{2} - \frac{1i}{2\,(e^{2\,x}-1+e^x\,2i)} + \frac{1i}{e^{4\,x}-6\,e^{2\,x}+1+e^{3\,x}\,4i-e^x\,4i} - \frac{1}{2\,(e^x+1i)} - \frac{2}{e^{2\,x}\,3i+e^{3\,x}-3\,e^x-i}$$

input
```
int(1/(cosh(x)*(sinh(x) + 1i)^2),x)
```

output
```
1i/(exp(3*x)*4i - 6*exp(2*x) + exp(4*x) - exp(x)*4i + 1) - 1i/(2*(exp(2*x)
 + exp(x)*2i - 1)) - atan(exp(x))/2 - 1/(2*(exp(x) + 1i)) - 2/(exp(2*x)*3i
 + exp(3*x) - 3*exp(x) - 1i)
```

Reduce [F]

$$\int \frac{\operatorname{sech}(x)}{(i+\sinh(x))^2}\,dx = \int \frac{\operatorname{sech}(x)}{\sinh(x)^2+2\sinh(x)\,i-1}dx$$

input
```
int(sech(x)/(I+sinh(x))^2,x)
```

output
```
int(sech(x)/(sinh(x)**2 + 2*sinh(x)*i - 1),x)
```

3.178 $\int \frac{\operatorname{sech}^2(x)}{(i+\sinh(x))^2}\,dx$

Optimal result

Integrand size = 13, antiderivative size = 37

$$\int \frac{\operatorname{sech}^2(x)}{(i+\sinh(x))^2}\,dx = -\frac{i\operatorname{sech}(x)}{5(i+\sinh(x))^2}-\frac{\operatorname{sech}(x)}{5(i+\sinh(x))}-\frac{2\tanh(x)}{5}$$

output
```
-1/5*I*sech(x)/(I+sinh(x))^2-sech(x)/(5*I+5*sinh(x))-2/5*tanh(x)
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.84

$$\int \frac{\operatorname{sech}^2(x)}{(i+\sinh(x))^2}\,dx = -\frac{\operatorname{sech}(x)(4i\cosh(2x)-5\sinh(x)+\sinh(3x))}{10(i+\sinh(x))^2}$$

input
```
Integrate[Sech[x]^2/(I + Sinh[x])^2,x]
```

output
```
-1/10*(Sech[x]*((4*I)*Cosh[2*x] - 5*Sinh[x] + Sinh[3*x]))/(I + Sinh[x])^2
```

Rubi [A] (verified)

Time = 0.35 (sec) , antiderivative size = 48, normalized size of antiderivative = 1.30, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.538, Rules used = {3042, 3151, 3042, 3151, 3042, 4254, 24}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\text{sech}^2(x)}{(\sinh(x)+i)^2}dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \frac{1}{(i-i\sin(ix))^2\cos(ix)^2}dx$$

$$\Big\downarrow \text{ 3151}$$

$$-\frac{3}{5}i\int \frac{\text{sech}^2(x)}{\sinh(x)+i}dx-\frac{i\text{sech}(x)}{5(\sinh(x)+i)^2}$$

$$\Big\downarrow \text{ 3042}$$

$$-\frac{3}{5}i\int \frac{1}{\cos(ix)^2(i-i\sin(ix))}dx-\frac{i\text{sech}(x)}{5(\sinh(x)+i)^2}$$

$$\Big\downarrow \text{ 3151}$$

$$-\frac{3}{5}i\left(-\frac{2}{3}i\int \text{sech}^2(x)dx-\frac{i\text{sech}(x)}{3(\sinh(x)+i)}\right)-\frac{i\text{sech}(x)}{5(\sinh(x)+i)^2}$$

$$\Big\downarrow \text{ 3042}$$

$$-\frac{3}{5}i\left(-\frac{2}{3}i\int \csc\left(ix+\frac{\pi}{2}\right)^2dx-\frac{i\text{sech}(x)}{3(\sinh(x)+i)}\right)-\frac{i\text{sech}(x)}{5(\sinh(x)+i)^2}$$

$$\Big\downarrow \text{ 4254}$$

$$-\frac{3}{5}i\left(\frac{2}{3}\int 1d(-i\tanh(x))-\frac{i\text{sech}(x)}{3(\sinh(x)+i)}\right)-\frac{i\text{sech}(x)}{5(\sinh(x)+i)^2}$$

$$\Big\downarrow \text{ 24}$$

$$-\frac{i\text{sech}(x)}{5(\sinh(x)+i)^2}-\frac{3}{5}i\left(-\frac{2}{3}i\tanh(x)-\frac{i\text{sech}(x)}{3(\sinh(x)+i)}\right)$$

input
```
Int[Sech[x]^2/(I + Sinh[x])^2,x]
```

output
```
((-1/5*I)*Sech[x])/(I + Sinh[x])^2 - ((3*I)/5)*(((-1/3*I)*Sech[x])/(I + Si
nh[x]) - ((2*I)/3)*Tanh[x])
```

Defintions of rubi rules used

rule 24
```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3151
```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[b*(g*Cos[e + f*x])^(p + 1)*((a + b*Sin[e + f*x
])^m/(a*f*g*Simplify[2*m + p + 1])), x] + Simp[Simplify[m + p + 1]/(a*Simpl
ify[2*m + p + 1])   Int[(g*Cos[e + f*x])^p*(a + b*Sin[e + f*x])^(m + 1), x]
, x] /; FreeQ[{a, b, e, f, g, m, p}, x] && EqQ[a^2 - b^2, 0] && ILtQ[Simpli
fy[m + p + 1], 0] && NeQ[2*m + p + 1, 0] &&  !IGtQ[m, 0]
```

rule 4254
```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Simp[-d^(-1)   Subst[Int[Exp
andIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
 d}, x] && IGtQ[n/2, 0]
```

Maple [A] (verified)

Time = 24.71 (sec) , antiderivative size = 30, normalized size of antiderivative = 0.81

method	result	size
risch	$-\frac{4\left(5\,{\mathrm e}^{2 x}+4 i {\mathrm e}^{x}-1\right)}{5\left({\mathrm e}^{x}-i\right)\left({\mathrm e}^{x}+i\right)^{5}}$	30
default	$-\frac{2 i}{\left(\tanh\left(\frac{x}{2}\right)+i\right)^{4}}+\frac{5 i}{2\left(\tanh\left(\frac{x}{2}\right)+i\right)^{2}}-\frac{4}{5\left(\tanh\left(\frac{x}{2}\right)+i\right)^{5}}+\frac{3}{\left(\tanh\left(\frac{x}{2}\right)+i\right)^{3}}-\frac{7}{4\left(\tanh\left(\frac{x}{2}\right)+i\right)}-\frac{1}{4\left(\tanh\left(\frac{x}{2}\right)-i\right)}$	70

input
```
int(sech(x)^2/(I+sinh(x))^2,x,method=_RETURNVERBOSE)
```

output
```
-4/5*(5*exp(2*x)+4*I*exp(x)-1)/(exp(x)-I)/(exp(x)+I)^5
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 44, normalized size of antiderivative = 1.19

$$\int \frac{\operatorname{sech}^2(x)}{(i+\sinh(x))^2}\,dx = -\frac{4\left(5\,e^{(2\,x)}+4i\,e^{x}-1\right)}{5\left(e^{(6\,x)}+4i\,e^{(5\,x)}-5\,e^{(4\,x)}-5\,e^{(2\,x)}-4i\,e^{x}+1\right)}$$

input
```
integrate(sech(x)^2/(I+sinh(x))^2,x, algorithm="fricas")
```

output
```
-4/5*(5*e^(2*x) + 4*I*e^x - 1)/(e^(6*x) + 4*I*e^(5*x) - 5*e^(4*x) - 5*e^(2
*x) - 4*I*e^x + 1)
```

Sympy [F]

$$\int \frac{\operatorname{sech}^2(x)}{(i+\sinh(x))^2}\,dx = \int \frac{\operatorname{sech}^2(x)}{(\sinh(x)+i)^2}\,dx$$

input
```
integrate(sech(x)**2/(I+sinh(x))**2,x)
```

output
```
Integral(sech(x)**2/(sinh(x) + I)**2, x)
```

Maxima [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 117 vs. 2(25) = 50.

Time = 0.04 (sec) , antiderivative size = 117, normalized size of antiderivative = 3.16

$$\int \frac{\operatorname{sech}^2(x)}{(i+\sinh(x))^2}\,dx = -\frac{16i\,e^{(-x)}}{20i\,e^{(-x)}-25\,e^{(-2\,x)}-25\,e^{(-4\,x)}-20i\,e^{(-5\,x)}+5\,e^{(-6\,x)}+5} + \frac{20\,e^{(-2\,x)}}{20i\,e^{(-x)}-25\,e^{(-2\,x)}-25\,e^{(-4\,x)}-20i\,e^{(-5\,x)}+5\,e^{(-6\,x)}+5} - \frac{4}{20i\,e^{(-x)}-25\,e^{(-2\,x)}-25\,e^{(-4\,x)}-20i\,e^{(-5\,x)}+5\,e^{(-6\,x)}+5}$$

input
```
integrate(sech(x)^2/(I+sinh(x))^2,x, algorithm="maxima")
```

output
```
-16*I*e^(-x)/(20*I*e^(-x) - 25*e^(-2*x) - 25*e^(-4*x) - 20*I*e^(-5*x) + 5*
e^(-6*x) + 5) + 20*e^(-2*x)/(20*I*e^(-x) - 25*e^(-2*x) - 25*e^(-4*x) - 20*
I*e^(-5*x) + 5*e^(-6*x) + 5) - 4/(20*I*e^(-x) - 25*e^(-2*x) - 25*e^(-4*x)
- 20*I*e^(-5*x) + 5*e^(-6*x) + 5)
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 41, normalized size of antiderivative = 1.11

$$\int \frac{\operatorname{sech}^2(x)}{(i+\sinh(x))^2}\,dx = -\frac{i}{4\,(e^x-i)} - \frac{-5i\,e^{(4\,x)}+30\,e^{(3\,x)}+80i\,e^{(2\,x)}-50\,e^x-11i}{20\,(e^x+i)^5}$$

input
```
integrate(sech(x)^2/(I+sinh(x))^2,x, algorithm="giac")
```

output
```
-1/4*I/(e^x - I) - 1/20*(-5*I*e^(4*x) + 30*e^(3*x) + 80*I*e^(2*x) - 50*e^x
 - 11*I)/(e^x + I)^5
```

Mupad [B] (verification not implemented)

Time = 1.58 (sec) , antiderivative size = 109, normalized size of antiderivative = 2.95

$$\int \frac{\operatorname{sech}^2(x)}{(i+\sinh(x))^2}\,dx = -\frac{16\,e^x\,(4\,e^{3\,x}-4\,e^x)}{5\,(e^{2\,x}+1)^5}-\frac{\left(4\,e^{2\,x}-\frac{4}{5}\right)\,(e^{4\,x}-6\,e^{2\,x}+1)}{(e^{2\,x}+1)^5} -\frac{e^x\,(e^{4\,x}-6\,e^{2\,x}+1)\,16\mathrm{i}}{5\,(e^{2\,x}+1)^5}+\frac{(4\,e^{3\,x}-4\,e^x)\,\left(4\,e^{2\,x}-\frac{4}{5}\right)\,1\mathrm{i}}{(e^{2\,x}+1)^5}$$

input
```
int(1/(cosh(x)^2*(sinh(x) + 1i)^2),x)
```

output
```
((4*exp(3*x) - 4*exp(x))*(4*exp(2*x) - 4/5)*1i)/(exp(2*x) + 1)^5 - (exp(x)
*(exp(4*x) - 6*exp(2*x) + 1)*16i)/(5*(exp(2*x) + 1)^5) - (16*exp(x)*(4*exp
(3*x) - 4*exp(x)))/(5*(exp(2*x) + 1)^5) - ((4*exp(2*x) - 4/5)*(exp(4*x) -
6*exp(2*x) + 1))/(exp(2*x) + 1)^5
```

Reduce [F]

$$\int \frac{\operatorname{sech}^2(x)}{(i+\sinh(x))^2}\,dx = \int \frac{\operatorname{sech}(x)^2}{\sinh(x)^2+2\sinh(x)\,i-1}dx$$

input
```
int(sech(x)^2/(I+sinh(x))^2,x)
```

output
```
int(sech(x)**2/(sinh(x)**2 + 2*sinh(x)*i - 1),x)
```

3.179 $\int \frac{\mathrm{sech}^3(x)}{(i+\sinh(x))^2} \, dx$

Optimal result

Integrand size = 13, antiderivative size = 60

$$\int \frac{\mathrm{sech}^3(x)}{(i+\sinh(x))^2} \, dx = -\frac{1}{4}\arctan(\sinh(x)) + \frac{1}{16(i-\sinh(x))} + \frac{1}{12(i+\sinh(x))^3} - \frac{i}{8(i+\sinh(x))^2} - \frac{3}{16(i+\sinh(x))}$$

output
```
-1/4*arctan(sinh(x))+1/(16*I-16*sinh(x))+1/12/(I+sinh(x))^3-1/8*I/(I+sinh(
x))^2-3/(16*I+16*sinh(x))
```

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 68, normalized size of antiderivative = 1.13

$$\int \frac{\mathrm{sech}^3(x)}{(i+\sinh(x))^2} \, dx = -\frac{\mathrm{sech}^2(x)\,(4i - 3\arctan(\sinh(x)) + (-1+6i\arctan(\sinh(x)))\sinh(x) + 6i\sinh^2(x) + (3+6i\arctan(\mathrm{s}}{12(i+\sinh(x))^2}$$

input
```
Integrate[Sech[x]^3/(I + Sinh[x])^2,x]
```

output
```
-1/12*(Sech[x]^2*(4*I - 3*ArcTan[Sinh[x]] + (-1 + (6*I)*ArcTan[Sinh[x]])*S
inh[x] + (6*I)*Sinh[x]^2 + (3 + (6*I)*ArcTan[Sinh[x]])*Sinh[x]^3 + 3*ArcTa
n[Sinh[x]]*Sinh[x]^4))/(I + Sinh[x])^2
```

Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 60, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.308$, Rules used = {3042, 3146, 54, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\operatorname{sech}^3(x)}{(\sinh(x)+i)^2}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{(i-i\sin(ix))^2\cos(ix)^3}dx$$

$$\Big\downarrow \text{3146}$$

$$\int \frac{1}{(-\sinh(x)+i)^2(\sinh(x)+i)^4}d\sinh(x)$$

$$\Big\downarrow \text{54}$$

$$\int \left(-\frac{1}{4\left(\sinh^2(x)+1\right)}+\frac{1}{16(\sinh(x)-i)^2}+\frac{3}{16(\sinh(x)+i)^2}+\frac{i}{4(\sinh(x)+i)^3}-\frac{1}{4(\sinh(x)+i)^4}\right)d\sinh(x)$$

$$\Big\downarrow \text{2009}$$

$$-\frac{1}{4}\arctan(\sinh(x))+\frac{1}{16(-\sinh(x)+i)}-\frac{3}{16(\sinh(x)+i)}-\frac{i}{8(\sinh(x)+i)^2}+\frac{1}{12(\sinh(x)+i)^3}$$

input
```
Int[Sech[x]^3/(I + Sinh[x])^2,x]
```

output
```
-1/4*ArcTan[Sinh[x]] + 1/(16*(I - Sinh[x])) + 1/(12*(I + Sinh[x])^3) - (I/
8)/(I + Sinh[x])^2 - 3/(16*(I + Sinh[x]))
```

Defintions of rubi rules used

rule 54
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m + n + 2, 0])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3146
```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Simp[1/(b^p*f)   Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x
)^((p - 1)/2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && I
ntegerQ[(p - 1)/2] && EqQ[a^2 - b^2, 0] && (GeQ[p, -1] ||  !IntegerQ[m + 1/
2])
```

Maple [A] (verified)

Time = 169.53 (sec) , antiderivative size = 80, normalized size of antiderivative = 1.33

method	result
risch	$-\frac{-13\,\mathrm{e}^{5x}+8i\mathrm{e}^{4x}+13\,\mathrm{e}^{3x}+12i\mathrm{e}^{2x}+12i\mathrm{e}^{6x}+3\,\mathrm{e}^{7x}-3\,\mathrm{e}^{x}}{6(\mathrm{e}^{x}-i)^{2}(\mathrm{e}^{x}+i)^{6}}-\frac{i\ln(\mathrm{e}^{x}+i)}{4}+\frac{i\ln(\mathrm{e}^{x}-i)}{4}$
default	$\frac{7i}{2(\tanh(\frac{x}{2})+i)^{4}}-\frac{2i}{3(\tanh(\frac{x}{2})+i)^{6}}-\frac{i\ln(\tanh(\frac{x}{2})+i)}{4}-\frac{23i}{8(\tanh(\frac{x}{2})+i)^{2}}+\frac{2}{(\tanh(\frac{x}{2})+i)^{5}}-\frac{11}{3(\tanh(\frac{x}{2})+i)^{3}}+\frac{1}{8(\tanh}$

input
```
int(sech(x)^3/(I+sinh(x))^2,x,method=_RETURNVERBOSE)
```

output
```
-1/6*(-13*exp(x)^5+8*I*exp(x)^4+13*exp(x)^3+12*I*exp(x)^2+12*I*exp(x)^6+3*
exp(x)^7-3*exp(x))/(exp(x)-I)^2/(exp(x)+I)^6-1/4*I*ln(exp(x)+I)+1/4*I*ln(e
xp(x)-I)
```

Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 201 vs. $2(38) = 76$.

Time = 0.09 (sec) , antiderivative size = 201, normalized size of antiderivative = 3.35

$$\int \frac{\operatorname{sech}^3(x)}{(i+\sinh(x))^2}\,dx =$$
$$-\frac{3\left(i\,e^{(8\,x)} - 4\,e^{(7\,x)} - 4i\,e^{(6\,x)} - 4\,e^{(5\,x)} - 10i\,e^{(4\,x)} + 4\,e^{(3\,x)} - 4i\,e^{(2\,x)} + 4\,e^{x} + i\right)\log\left(e^{x}+i\right) + 3\left(-i\,e^{(8}\right.}{12\left(e^{(8\,x)} + 4i\right.}$$

input
```
integrate(sech(x)^3/(I+sinh(x))^2,x, algorithm="fricas")
```

output
```
-1/12*(3*(I*e^(8*x) - 4*e^(7*x) - 4*I*e^(6*x) - 4*e^(5*x) - 10*I*e^(4*x) +
 4*e^(3*x) - 4*I*e^(2*x) + 4*e^x + I)*log(e^x + I) + 3*(-I*e^(8*x) + 4*e^(
7*x) + 4*I*e^(6*x) + 4*e^(5*x) + 10*I*e^(4*x) - 4*e^(3*x) + 4*I*e^(2*x) -
4*e^x - I)*log(e^x - I) + 6*e^(7*x) + 24*I*e^(6*x) - 26*e^(5*x) + 16*I*e^(
4*x) + 26*e^(3*x) + 24*I*e^(2*x) - 6*e^x)/(e^(8*x) + 4*I*e^(7*x) - 4*e^(6*
x) + 4*I*e^(5*x) - 10*e^(4*x) - 4*I*e^(3*x) - 4*e^(2*x) - 4*I*e^x + 1)
```

Sympy [F]

$$\int \frac{\operatorname{sech}^3(x)}{(i+\sinh(x))^2}\,dx = \int \frac{\operatorname{sech}^3(x)}{(\sinh(x)+i)^2}\,dx$$

input
```
integrate(sech(x)**3/(I+sinh(x))**2,x)
```

output
```
Integral(sech(x)**3/(sinh(x) + I)**2, x)
```

Maxima [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 120 vs. $2(38) = 76$.

Time = 0.05 (sec) , antiderivative size = 120, normalized size of antiderivative = 2.00

$$\int \frac{\operatorname{sech}^3(x)}{(i+\sinh(x))^2}\,dx = -\frac{8\left(3\,e^{(-x)} + 12i\,e^{(-2\,x)} - 13\,e^{(-3\,x)} + 8i\,e^{(-4\,x)} + 13\,e^{(-5\,x)} + 12i\,e^{(-6\,x)} - 3\,e^{(-7\,x)}\right)}{192i\,e^{(-x)} - 192\,e^{(-2\,x)} + 192i\,e^{(-3\,x)} - 480\,e^{(-4\,x)} - 192i\,e^{(-5\,x)} - 192\,e^{(-6\,x)} - 192i\,e^{(-7\,x)} + 48\,e^{(-8\,x)}} - \frac{1}{4}i\,\log\left(i\,e^{(-x)} + 1\right) + \frac{1}{4}i\,\log\left(i\,e^{(-x)} - 1\right)$$

input
```
integrate(sech(x)^3/(I+sinh(x))^2,x, algorithm="maxima")
```

output
```
-8*(3*e^(-x) + 12*I*e^(-2*x) - 13*e^(-3*x) + 8*I*e^(-4*x) + 13*e^(-5*x) +
12*I*e^(-6*x) - 3*e^(-7*x))/(192*I*e^(-x) - 192*e^(-2*x) + 192*I*e^(-3*x)
- 480*e^(-4*x) - 192*I*e^(-5*x) - 192*e^(-6*x) - 192*I*e^(-7*x) + 48*e^(-8
*x) + 48) - 1/4*I*log(I*e^(-x) + 1) + 1/4*I*log(I*e^(-x) - 1)
```

Giac [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 105 vs. $2(38) = 76$.

Time = 0.13 (sec) , antiderivative size = 105, normalized size of antiderivative = 1.75

$$\begin{aligned}\int \frac{\operatorname{sech}^3(x)}{(i+\sinh(x))^2}\,dx &= \frac{-i\,e^{(-x)} + i\,e^{x} + 3}{8\left(e^{(-x)} - e^{x} + 2i\right)} \\ &\quad + \frac{11i\left(e^{(-x)} - e^{x}\right)^3 + 84\left(e^{(-x)} - e^{x}\right)^2 - 228i\,e^{(-x)} + 228i\,e^{x} - 240}{48\left(e^{(-x)} - e^{x} - 2i\right)^3} \\ &\quad - \frac{1}{8}i\,\log\left(-e^{(-x)} + e^{x} + 2i\right) + \frac{1}{8}i\,\log\left(-e^{(-x)} + e^{x} - 2i\right)\end{aligned}$$

input
```
integrate(sech(x)^3/(I+sinh(x))^2,x, algorithm="giac")
```

output
```
1/8*(-I*e^(-x) + I*e^x + 3)/(e^(-x) - e^x + 2*I) + 1/48*(11*I*(e^(-x) - e^
x)^3 + 84*(e^(-x) - e^x)^2 - 228*I*e^(-x) + 228*I*e^x - 240)/(e^(-x) - e^x
 - 2*I)^3 - 1/8*I*log(-e^(-x) + e^x + 2*I) + 1/8*I*log(-e^(-x) + e^x - 2*I
)
```

Mupad [B] (verification not implemented)

Time = 2.06 (sec) , antiderivative size = 198, normalized size of antiderivative = 3.30

$$\int \frac{\operatorname{sech}^3(x)}{(i+\sinh(x))^2}\,dx = -\frac{\operatorname{atan}(e^x)}{2} - \frac{2}{e^{5x} - 10\,e^{3x} + e^{4x}\,5i - e^{2x}\,10i + 5\,e^x + 1i} - \frac{1i}{8\,(e^{2x} - 1 + e^x\,2i)} - \frac{3i}{2\,(e^{4x} - 6\,e^{2x} + 1 + e^{3x}\,4i - e^x\,4i)} + \frac{1i}{8\,(1 - e^{2x} + e^x\,2i)} - \frac{1}{8\,(e^x - i)} - \frac{3}{8\,(e^x + 1i)} + \frac{2i}{3\,(15\,e^{2x} - 15\,e^{4x} + e^{6x} - 1 - e^{3x}\,20i + e^{5x}\,6i + e^x\,6i)} - \frac{1}{3\,(e^{2x}\,3i + e^{3x} - 3\,e^x - i)}$$

input
```
int(1/(cosh(x)^3*(sinh(x) + 1i)^2),x)
```

output
```
1i/(8*(exp(x)*2i - exp(2*x) + 1)) - 2/(exp(4*x)*5i - 10*exp(3*x) - exp(2*x
)*10i + exp(5*x) + 5*exp(x) + 1i) - 1i/(8*(exp(2*x) + exp(x)*2i - 1)) - 3i
/(2*(exp(3*x)*4i - 6*exp(2*x) + exp(4*x) - exp(x)*4i + 1)) - atan(exp(x))/
2 - 1/(8*(exp(x) - 1i)) - 3/(8*(exp(x) + 1i)) + 2i/(3*(15*exp(2*x) - exp(3
*x)*20i - 15*exp(4*x) + exp(5*x)*6i + exp(6*x) + exp(x)*6i - 1)) - 1/(3*(e
xp(2*x)*3i + exp(3*x) - 3*exp(x) - 1i))
```

Reduce [F]

$$\int \frac{\operatorname{sech}^3(x)}{(i+\sinh(x))^2}\,dx = \int \frac{\operatorname{sech}(x)^3}{\sinh(x)^2+2\sinh(x)\,i-1}dx$$

input
```
int(sech(x)^3/(I+sinh(x))^2,x)
```

output
```
int(sech(x)**3/(sinh(x)**2 + 2*sinh(x)*i - 1),x)
```

3.180 $\int \frac{\operatorname{sech}^4(x)}{(i+\sinh(x))^2} \, dx$

Optimal result

Integrand size = 13, antiderivative size = 49

$$\int \frac{\operatorname{sech}^4(x)}{(i+\sinh(x))^2} \, dx = -\frac{i\operatorname{sech}^3(x)}{7(i+\sinh(x))^2}-\frac{\operatorname{sech}^3(x)}{7(i+\sinh(x))}-\frac{4\tanh(x)}{7}+\frac{4\tanh^3(x)}{21}$$

output
```
-1/7*I*sech(x)^3/(I+sinh(x))^2-sech(x)^3/(7*I+7*sinh(x))-4/7*tanh(x)+4/21*
tanh(x)^3
```

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.96

$$\int \frac{\operatorname{sech}^4(x)}{(i+\sinh(x))^2} \, dx = -\frac{\operatorname{sech}^3(x)(8i\cosh(2x)+4i\cosh(4x)-14\sinh(x)-3\sinh(3x)+\sinh(5x))}{42(i+\sinh(x))^2}$$

input
```
Integrate[Sech[x]^4/(I + Sinh[x])^2,x]
```

output
```
-1/42*(Sech[x]^3*((8*I)*Cosh[2*x] + (4*I)*Cosh[4*x] - 14*Sinh[x] - 3*Sinh[
3*x] + Sinh[5*x]))/(I + Sinh[x])^2
```

Rubi [A] (verified)

Time = 0.38 (sec) , antiderivative size = 65, normalized size of antiderivative = 1.33, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.538, Rules used = {3042, 3151, 3042, 3151, 3042, 4254, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\text{sech}^4(x)}{(\sinh(x)+i)^2}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{(i-i\sin(ix))^2\cos(ix)^4}dx$$

$$\Big\downarrow \text{3151}$$

$$-\frac{5}{7}i\int \frac{\text{sech}^4(x)}{\sinh(x)+i}dx-\frac{i\text{sech}^3(x)}{7(\sinh(x)+i)^2}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{5}{7}i\int \frac{1}{\cos(ix)^4(i-i\sin(ix))}dx-\frac{i\text{sech}^3(x)}{7(\sinh(x)+i)^2}$$

$$\Big\downarrow \text{3151}$$

$$-\frac{5}{7}i\left(-\frac{4}{5}i\int \text{sech}^4(x)dx-\frac{i\text{sech}^3(x)}{5(\sinh(x)+i)}\right)-\frac{i\text{sech}^3(x)}{7(\sinh(x)+i)^2}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{5}{7}i\left(-\frac{4}{5}i\int \csc\left(ix+\frac{\pi}{2}\right)^4dx-\frac{i\text{sech}^3(x)}{5(\sinh(x)+i)}\right)-\frac{i\text{sech}^3(x)}{7(\sinh(x)+i)^2}$$

$$\Big\downarrow \text{4254}$$

$$-\frac{5}{7}i\left(\frac{4}{5}\int \left(1-\tanh^2(x)\right)d(-i\tanh(x))-\frac{i\text{sech}^3(x)}{5(\sinh(x)+i)}\right)-\frac{i\text{sech}^3(x)}{7(\sinh(x)+i)^2}$$

$$\Big\downarrow \text{2009}$$

$$-\frac{i\text{sech}^3(x)}{7(\sinh(x)+i)^2}-\frac{5}{7}i\left(\frac{4}{5}\left(\frac{1}{3}i\tanh^3(x)-i\tanh(x)\right)-\frac{i\text{sech}^3(x)}{5(\sinh(x)+i)}\right)$$

input
```
Int[Sech[x]^4/(I + Sinh[x])^2,x]
```

output
```
((-1/7*I)*Sech[x]^3)/(I + Sinh[x])^2 - ((5*I)/7)*(((-1/5*I)*Sech[x]^3)/(I
+ Sinh[x]) + (4*((-I)*Tanh[x] + (I/3)*Tanh[x]^3))/5)
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3151
```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[b*(g*Cos[e + f*x])^(p + 1)*((a + b*Sin[e + f*x
])^m/(a*f*g*Simplify[2*m + p + 1])), x] + Simp[Simplify[m + p + 1]/(a*Simpl
ify[2*m + p + 1])   Int[(g*Cos[e + f*x])^p*(a + b*Sin[e + f*x])^(m + 1), x]
, x] /; FreeQ[{a, b, e, f, g, m, p}, x] && EqQ[a^2 - b^2, 0] && ILtQ[Simpli
fy[m + p + 1], 0] && NeQ[2*m + p + 1, 0] &&  !IGtQ[m, 0]
```

rule 4254
```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Simp[-d^(-1)   Subst[Int[Exp
andIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
 d}, x] && IGtQ[n/2, 0]
```

Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 115 vs. 2(40) = 80.

Time = 0.08 (sec) , antiderivative size = 116, normalized size of antiderivative = 2.37

$$\frac{2i}{\left(\tanh\left(\frac{x}{2}\right)+i\right)^{6}}-\frac{5i}{\left(\tanh\left(\frac{x}{2}\right)+i\right)^{4}}+\frac{23i}{8\left(\tanh\left(\frac{x}{2}\right)+i\right)^{2}}+\frac{4}{7\left(\tanh\left(\frac{x}{2}\right)+i\right)^{7}}-\frac{4}{\left(\tanh\left(\frac{x}{2}\right)+i\right)^{5}}+\frac{55}{12\left(\tanh\left(\frac{x}{2}\right)\right.}$$

input
```
int(sech(x)^4/(I+sinh(x))^2,x)
```

output
```
2*I/(tanh(1/2*x)+I)^6-5*I/(tanh(1/2*x)+I)^4+23/8*I/(tanh(1/2*x)+I)^2+4/7/(
tanh(1/2*x)+I)^7-4/(tanh(1/2*x)+I)^5+55/12/(tanh(1/2*x)+I)^3-13/8/(tanh(1/
2*x)+I)-1/8*I/(tanh(1/2*x)-I)^2+1/12/(tanh(1/2*x)-I)^3-3/8/(tanh(1/2*x)-I)
```

Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 80 vs. 2(35) = 70.

Time = 0.09 (sec) , antiderivative size = 80, normalized size of antiderivative = 1.63

$$\int \frac{\operatorname{sech}^4(x)}{(i+\sinh(x))^2}\,dx = -\frac{16\left(14\,e^{(4\,x)}+8i\,e^{(3\,x)}+3\,e^{(2\,x)}+4i\,e^{x}-1\right)}{21\left(e^{(10\,x)}+4i\,e^{(9\,x)}-3\,e^{(8\,x)}+8i\,e^{(7\,x)}-14\,e^{(6\,x)}-14\,e^{(4\,x)}-8i\,e^{(3\,x)}-3\,e^{(2\,x)}-4i\,e^{x}+1\right)}$$

input
```
integrate(sech(x)^4/(I+sinh(x))^2,x, algorithm="fricas")
```

output
```
-16/21*(14*e^(4*x) + 8*I*e^(3*x) + 3*e^(2*x) + 4*I*e^x - 1)/(e^(10*x) + 4*
I*e^(9*x) - 3*e^(8*x) + 8*I*e^(7*x) - 14*e^(6*x) - 14*e^(4*x) - 8*I*e^(3*x
) - 3*e^(2*x) - 4*I*e^x + 1)
```

Sympy [F]

$$\int \frac{\operatorname{sech}^4(x)}{(i+\sinh(x))^2}\,dx = \int \frac{\operatorname{sech}^4(x)}{(\sinh(x)+i)^2}\,dx$$

input
```
integrate(sech(x)**4/(I+sinh(x))**2,x)
```

output
```
Integral(sech(x)**4/(sinh(x) + I)**2, x)
```

Maxima [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 317 vs. 2(35) = 70.

Time = 0.04 (sec) , antiderivative size = 317, normalized size of antiderivative = 6.47

$$\int \frac{\operatorname{sech}^4(x)}{(i+\sinh(x))^2}\,dx =$$

$$-\frac{64i\,e^{(-x)}}{84i\,e^{(-x)}-63\,e^{(-2\,x)}+168i\,e^{(-3\,x)}-294\,e^{(-4\,x)}-294\,e^{(-6\,x)}-168i\,e^{(-7\,x)}-63\,e^{(-8\,x)}-84i\,e^{(-9\,x)}+2}$$
$$+\frac{48\,e^{(-2\,x)}}{84i\,e^{(-x)}-63\,e^{(-2\,x)}+168i\,e^{(-3\,x)}-294\,e^{(-4\,x)}-294\,e^{(-6\,x)}-168i\,e^{(-7\,x)}-63\,e^{(-8\,x)}-84i\,e^{(-9\,x)}+2}$$
$$-\frac{128i\,e^{(-3\,x)}}{84i\,e^{(-x)}-63\,e^{(-2\,x)}+168i\,e^{(-3\,x)}-294\,e^{(-4\,x)}-294\,e^{(-6\,x)}-168i\,e^{(-7\,x)}-63\,e^{(-8\,x)}-84i\,e^{(-9\,x)}+2}$$
$$+\frac{224\,e^{(-4\,x)}}{84i\,e^{(-x)}-63\,e^{(-2\,x)}+168i\,e^{(-3\,x)}-294\,e^{(-4\,x)}-294\,e^{(-6\,x)}-168i\,e^{(-7\,x)}-63\,e^{(-8\,x)}-84i\,e^{(-9\,x)}+2}$$
$$-\frac{16}{84i\,e^{(-x)}-63\,e^{(-2\,x)}+168i\,e^{(-3\,x)}-294\,e^{(-4\,x)}-294\,e^{(-6\,x)}-168i\,e^{(-7\,x)}-63\,e^{(-8\,x)}-84i\,e^{(-9\,x)}+2}$$

input
```
integrate(sech(x)^4/(I+sinh(x))^2,x, algorithm="maxima")
```

output
```
-64*I*e^(-x)/(84*I*e^(-x) - 63*e^(-2*x) + 168*I*e^(-3*x) - 294*e^(-4*x) -
294*e^(-6*x) - 168*I*e^(-7*x) - 63*e^(-8*x) - 84*I*e^(-9*x) + 21*e^(-10*x)
 + 21) + 48*e^(-2*x)/(84*I*e^(-x) - 63*e^(-2*x) + 168*I*e^(-3*x) - 294*e^(
-4*x) - 294*e^(-6*x) - 168*I*e^(-7*x) - 63*e^(-8*x) - 84*I*e^(-9*x) + 21*e
^(-10*x) + 21) - 128*I*e^(-3*x)/(84*I*e^(-x) - 63*e^(-2*x) + 168*I*e^(-3*x
) - 294*e^(-4*x) - 294*e^(-6*x) - 168*I*e^(-7*x) - 63*e^(-8*x) - 84*I*e^(-
9*x) + 21*e^(-10*x) + 21) + 224*e^(-4*x)/(84*I*e^(-x) - 63*e^(-2*x) + 168*
I*e^(-3*x) - 294*e^(-4*x) - 294*e^(-6*x) - 168*I*e^(-7*x) - 63*e^(-8*x) -
84*I*e^(-9*x) + 21*e^(-10*x) + 21) - 16/(84*I*e^(-x) - 63*e^(-2*x) + 168*I
*e^(-3*x) - 294*e^(-4*x) - 294*e^(-6*x) - 168*I*e^(-7*x) - 63*e^(-8*x) - 8
4*I*e^(-9*x) + 21*e^(-10*x) + 21)
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 65, normalized size of antiderivative = 1.33

$$\int \frac{\operatorname{sech}^4(x)}{(i+\sinh(x))^2}\,dx = -\frac{6i\,e^{(2\,x)}+15\,e^{x}-7i}{24\,(e^{x}-i)^3} - \frac{-42i\,e^{(6\,x)}+315\,e^{(5\,x)}+1015i\,e^{(4\,x)}-1750\,e^{(3\,x)}-1344i\,e^{(2\,x)}+511\,e^{x}+79i}{168\,(e^{x}+i)^7}$$

input
```
integrate(sech(x)^4/(I+sinh(x))^2,x, algorithm="giac")
```

output
```
-1/24*(6*I*e^(2*x) + 15*e^x - 7*I)/(e^x - I)^3 - 1/168*(-42*I*e^(6*x) + 31
5*e^(5*x) + 1015*I*e^(4*x) - 1750*e^(3*x) - 1344*I*e^(2*x) + 511*e^x + 79*
I)/(e^x + I)^7
```

Mupad [B] (verification not implemented)

Time = 0.53 (sec) , antiderivative size = 139, normalized size of antiderivative = 2.84

$$\int \frac{\operatorname{sech}^4(x)}{(i+\sinh(x))^2}\,dx = \frac{(4\,e^{3\,x}-4\,e^{x})\,\left(\frac{16\,e^{2\,x}}{7}+\frac{32\,e^{4\,x}}{3}-\frac{16}{21}\right)\,1i}{{\left(e^{2\,x}+1\right)}^7} - \frac{(e^{4\,x}-6\,e^{2\,x}+1)\,\left(\frac{16\,e^{2\,x}}{7}+\frac{32\,e^{4\,x}}{3}-\frac{16}{21}\right)}{{\left(e^{2\,x}+1\right)}^7} - \frac{(4\,e^{3\,x}-4\,e^{x})\,\left(\frac{128\,e^{3\,x}}{21}+\frac{64\,e^{x}}{21}\right)}{{\left(e^{2\,x}+1\right)}^7} - \frac{\left(\frac{128\,e^{3\,x}}{21}+\frac{64\,e^{x}}{21}\right)\,(e^{4\,x}-6\,e^{2\,x}+1)\,1i}{{\left(e^{2\,x}+1\right)}^7}$$

input
```
int(1/(cosh(x)^4*(sinh(x) + 1i)^2),x)
```

output
```
((4*exp(3*x) - 4*exp(x))*((16*exp(2*x))/7 + (32*exp(4*x))/3 - 16/21)*1i)/(
exp(2*x) + 1)^7 - ((exp(4*x) - 6*exp(2*x) + 1)*((16*exp(2*x))/7 + (32*exp(
4*x))/3 - 16/21))/(exp(2*x) + 1)^7 - ((4*exp(3*x) - 4*exp(x))*((128*exp(3*
x))/21 + (64*exp(x))/21))/(exp(2*x) + 1)^7 - (((128*exp(3*x))/21 + (64*exp
(x))/21)*(exp(4*x) - 6*exp(2*x) + 1)*1i)/(exp(2*x) + 1)^7
```

Reduce [F]

$$\int \frac{\operatorname{sech}^4(x)}{(i+\sinh(x))^2}\,dx = \int \frac{\operatorname{sech}(x)^4}{\sinh(x)^2+2\sinh(x)\,i-1}dx$$

input
```
int(sech(x)^4/(I+sinh(x))^2,x)
```

output
```
int(sech(x)**4/(sinh(x)**2 + 2*sinh(x)*i - 1),x)
```

3.181 $\int \frac{\cosh^3(x)}{(1+i\sinh(x))^3}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 28

$$\int \frac{\cosh^3(x)}{(1+i\sinh(x))^3}\,dx = i\log(i-\sinh(x)) + \frac{2i}{1+i\sinh(x)}$$

output
```
I*ln(I-sinh(x))+2*I/(1+I*sinh(x))
```

Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 47, normalized size of antiderivative = 1.68

$$\int \frac{\cosh^3(x)}{(1+i\sinh(x))^3}\,dx = \frac{\cosh^4(x)(2+\log(i-\sinh(x))+i\log(i-\sinh(x))\sinh(x))}{(-i+\sinh(x))^3(i+\sinh(x))^2}$$

input
```
Integrate[Cosh[x]^3/(1 + I*Sinh[x])^3,x]
```

output
```
(Cosh[x]^4*(2 + Log[I - Sinh[x]] + I*Log[I - Sinh[x]]*Sinh[x]))/((-I + Sin
h[x])^3*(I + Sinh[x])^2)
```

Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.267, Rules used = {3042, 3146, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cosh^3(x)}{(1+i\sinh(x))^3}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\cos(ix)^3}{(1+\sin(ix))^3}dx$$

$$\Big\downarrow \text{3146}$$

$$-i\int \frac{1-i\sinh(x)}{(i\sinh(x)+1)^2}d(i\sinh(x))$$

$$\Big\downarrow \text{49}$$

$$-i\int \left(\frac{2}{(i\sinh(x)+1)^2}+\frac{1}{-i\sinh(x)-1}\right)d(i\sinh(x))$$

$$\Big\downarrow \text{2009}$$

$$-i\left(-\frac{2}{1+i\sinh(x)}-\log(1+i\sinh(x))\right)$$

input
```
Int[Cosh[x]^3/(1 + I*Sinh[x])^3,x]
```

output
```
(-I)*(-Log[1 + I*Sinh[x]] - 2/(1 + I*Sinh[x]))
```

Defintions of rubi rules used

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3146
```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Simp[1/(b^p*f)   Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x
)^((p - 1)/2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && I
ntegerQ[(p - 1)/2] && EqQ[a^2 - b^2, 0] && (GeQ[p, -1] ||  !IntegerQ[m + 1/
2])
```

Maple [A] (verified)

Time = 35.28 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.93

method	result	size
derivativedivides	$\frac{i\ln\left(1+\sinh(x)^{2}\right)}{2}-\arctan\left(\sinh\left(x\right)\right)+\frac{2}{\sinh(x)-i}$	26
default	$\frac{i\ln\left(1+\sinh(x)^{2}\right)}{2}-\arctan\left(\sinh\left(x\right)\right)+\frac{2}{\sinh(x)-i}$	26
risch	$-ix+\frac{4\,\mathrm{e}^{x}}{\left(\mathrm{e}^{x}-i\right)^{2}}+2i\ln\left(\mathrm{e}^{x}-i\right)$	26

input
```
int(cosh(x)^3/(1+I*sinh(x))^3,x,method=_RETURNVERBOSE)
```

output
```
1/2*I*ln(1+sinh(x)^2)-arctan(sinh(x))+2/(sinh(x)-I)
```

Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 50 vs. 2(20) = 40.

Time = 0.11 (sec) , antiderivative size = 50, normalized size of antiderivative = 1.79

$$\int \frac{\cosh^3(x)}{(1+i\sinh(x))^3}\,dx = \frac{-i\,xe^{(2\,x)}-2\,(x-2)e^x-2\left(-i\,e^{(2\,x)}-2\,e^x+i\right)\log\left(e^x-i\right)+i\,x}{e^{(2\,x)}-2i\,e^x-1}$$

input
```
integrate(cosh(x)^3/(1+I*sinh(x))^3,x, algorithm="fricas")
```

output
```
(-I*x*e^(2*x) - 2*(x - 2)*e^x - 2*(-I*e^(2*x) - 2*e^x + I)*log(e^x - I) +
I*x)/(e^(2*x) - 2*I*e^x - 1)
```

Sympy [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.11

$$\int \frac{\cosh^3(x)}{(1+i\sinh(x))^3}\,dx = -ix+2i\log\left(e^x-i\right)+\frac{4e^x}{e^{2x}-2ie^x-1}$$

input
```
integrate(cosh(x)**3/(1+I*sinh(x))**3,x)
```

output
```
-I*x + 2*I*log(exp(x) - I) + 4*exp(x)/(exp(2*x) - 2*I*exp(x) - 1)
```

Maxima [A] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 33, normalized size of antiderivative = 1.18

$$\int \frac{\cosh^3(x)}{(1+i\sinh(x))^3}\,dx = i\,x - \frac{4\,e^{(-x)}}{2i\,e^{(-x)} + e^{(-2\,x)} - 1} + 2i\,\log\left(e^{(-x)} + i\right)$$

input
```
integrate(cosh(x)^3/(1+I*sinh(x))^3,x, algorithm="maxima")
```

output
```
I*x - 4*e^(-x)/(2*I*e^(-x) + e^(-2*x) - 1) + 2*I*log(e^(-x) + I)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.75

$$\int \frac{\cosh^3(x)}{(1+i\sinh(x))^3}\,dx = -i\,x + \frac{4\,e^x}{(e^x - i)^2} + 2i\,\log\left(e^x - i\right)$$

input
```
integrate(cosh(x)^3/(1+I*sinh(x))^3,x, algorithm="giac")
```

output
```
-I*x + 4*e^x/(e^x - I)^2 + 2*I*log(e^x - I)
```

Mupad [B] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 41, normalized size of antiderivative = 1.46

$$\int \frac{\cosh^3(x)}{(1+i\sinh(x))^3}\,dx = -x\,1\mathrm{i} + \ln\left(\mathrm{e}^x - \mathrm{i}\right)\,2\mathrm{i} - \frac{4\mathrm{i}}{1 - \mathrm{e}^{2\,x} + \mathrm{e}^x\,2\mathrm{i}} + \frac{4}{\mathrm{e}^x - \mathrm{i}}$$

input
```
int(cosh(x)^3/(sinh(x)*1i + 1)^3,x)
```

output
```
log(exp(x) - 1i)*2i - x*1i - 4i/(exp(x)*2i - exp(2*x) + 1) + 4/(exp(x) - 1
i)
```

Reduce [F]

$$\int \frac{\cosh^3(x)}{(1+i\sinh(x))^3}\,dx = -12\left(\int \frac{e^{4x}}{e^{6x}-6e^{5x}i-15e^{4x}+20e^{3x}i+15e^{2x}-6e^{x}i-1}dx\right)i - 40\left(\int \frac{e^{3x}}{e^{6x}-6e^{5x}i-15e^{4x}+20e^{3x}i+15e^{2x}-6e^{x}i-1}dx\right) + 48\left(\int \frac{e^{2x}}{e^{6x}-6e^{5x}i-15e^{4x}+20e^{3x}i+15e^{2x}-6e^{x}i-1}dx\right)i + 24\left(\int \frac{e^{x}}{e^{6x}-6e^{5x}i-15e^{4x}+20e^{3x}i+15e^{2x}-6e^{x}i-1}dx\right) - 4\left(\int \frac{1}{e^{6x}-6e^{5x}i-15e^{4x}+20e^{3x}i+15e^{2x}-6e^{x}i-1}dx\right)i + \log\left(e^{6x}i+6e^{5x}-15e^{4x}i-20e^{3x}+15e^{2x}i+6e^{x}-i\right)i-5ix$$

input
```
int(cosh(x)^3/(1+I*sinh(x))^3,x)
```

output
```
- 12*int(e**(4*x)/(e**(6*x) - 6*e**(5*x)*i - 15*e**(4*x) + 20*e**(3*x)*i
+ 15*e**(2*x) - 6*e**x*i - 1),x)*i - 40*int(e**(3*x)/(e**(6*x) - 6*e**(5*x
)*i - 15*e**(4*x) + 20*e**(3*x)*i + 15*e**(2*x) - 6*e**x*i - 1),x) + 48*in
t(e**(2*x)/(e**(6*x) - 6*e**(5*x)*i - 15*e**(4*x) + 20*e**(3*x)*i + 15*e**
(2*x) - 6*e**x*i - 1),x)*i + 24*int(e**x/(e**(6*x) - 6*e**(5*x)*i - 15*e**
(4*x) + 20*e**(3*x)*i + 15*e**(2*x) - 6*e**x*i - 1),x) - 4*int(1/(e**(6*x)
 - 6*e**(5*x)*i - 15*e**(4*x) + 20*e**(3*x)*i + 15*e**(2*x) - 6*e**x*i - 1
),x)*i + log(e**(6*x)*i + 6*e**(5*x) - 15*e**(4*x)*i - 20*e**(3*x) + 15*e*
*(2*x)*i + 6*e**x - i)*i - 5*i*x
```

3.182 $\int \frac{\cosh^2(x)}{(1+i\sinh(x))^3}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 20

$$\int \frac{\cosh^2(x)}{(1+i\sinh(x))^3}\,dx = \frac{i\cosh^3(x)}{3(1+i\sinh(x))^3}$$

output
```
1/3*I*cosh(x)^3/(1+I*sinh(x))^3
```

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.80

$$\int \frac{\cosh^2(x)}{(1+i\sinh(x))^3}\,dx = -\frac{\cosh^3(x)}{3(-i+\sinh(x))^3}$$

input
```
Integrate[Cosh[x]^2/(1 + I*Sinh[x])^3,x]
```

output
```
-1/3*Cosh[x]^3/(-I + Sinh[x])^3
```

Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$, Rules used = {3042, 3150}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cosh^2(x)}{(1+i\sinh(x))^3}\,dx$$
$$\Big\downarrow \text{ 3042}$$
$$\int \frac{\cos(ix)^2}{(1+\sin(ix))^3}dx$$
$$\Big\downarrow \text{ 3150}$$
$$\frac{i\cosh^3(x)}{3(1+i\sinh(x))^3}$$

input
```
Int[Cosh[x]^2/(1 + I*Sinh[x])^3,x]
```

output
```
((I/3)*Cosh[x]^3)/(1 + I*Sinh[x])^3
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3150
```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[b*(g*Cos[e + f*x])^(p + 1)*((a + b*Sin[e + f*x
])^m/(a*f*g*m)), x] /; FreeQ[{a, b, e, f, g, m, p}, x] && EqQ[a^2 - b^2, 0]
 && EqQ[Simplify[m + p + 1], 0] && !ILtQ[p, 0]
```

Maple [A] (verified)

Time = 33.90 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.95

method	result	size
risch	$-\frac{2i(3\,\mathrm{e}^{2x}-1)}{3(\mathrm{e}^{x}-i)^{3}}$	19
default	$\frac{2}{\tanh\left(\frac{x}{2}\right)-i}-\frac{8}{3\left(\tanh\left(\frac{x}{2}\right)-i\right)^{3}}+\frac{4i}{\left(\tanh\left(\frac{x}{2}\right)-i\right)^{2}}$	36

input
```
int(cosh(x)^2/(1+I*sinh(x))^3,x,method=_RETURNVERBOSE)
```

output
```
-2/3*I*(3*exp(2*x)-1)/(exp(x)-I)^3
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.40

$$\int \frac{\cosh^2(x)}{(1+i\sinh(x))^3}\,dx = -\frac{2\left(3i\,e^{(2\,x)}-i\right)}{3\left(e^{(3\,x)}-3i\,e^{(2\,x)}-3\,e^{x}+i\right)}$$

input
```
integrate(cosh(x)^2/(1+I*sinh(x))^3,x, algorithm="fricas")
```

output
```
-2/3*(3*I*e^(2*x) - I)/(e^(3*x) - 3*I*e^(2*x) - 3*e^x + I)
```

Sympy [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 34 vs. 2(15) = 30.

Time = 0.11 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.70

$$\int \frac{\cosh^2(x)}{(1+i\sinh(x))^3}\,dx = \frac{-6ie^{2x}+2i}{3e^{3x}-9ie^{2x}-9e^{x}+3i}$$

input
```
integrate(cosh(x)**2/(1+I*sinh(x))**3,x)
```

output
```
(-6*I*exp(2*x) + 2*I)/(3*exp(3*x) - 9*I*exp(2*x) - 9*exp(x) + 3*I)
```

Maxima [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 53 vs. 2(14) = 28.

Time = 0.04 (sec) , antiderivative size = 53, normalized size of antiderivative = 2.65

$$\int \frac{\cosh^2(x)}{(1+i\sinh(x))^3}\,dx = \frac{6\,e^{(-2\,x)}}{-9i\,e^{(-x)}-9\,e^{(-2\,x)}+3i\,e^{(-3\,x)}+3} - \frac{2}{-9i\,e^{(-x)}-9\,e^{(-2\,x)}+3i\,e^{(-3\,x)}+3}$$

input
```
integrate(cosh(x)^2/(1+I*sinh(x))^3,x, algorithm="maxima")
```

output
```
6*e^(-2*x)/(-9*I*e^(-x) - 9*e^(-2*x) + 3*I*e^(-3*x) + 3) - 2/(-9*I*e^(-x)
- 9*e^(-2*x) + 3*I*e^(-3*x) + 3)
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.80

$$\int \frac{\cosh^2(x)}{(1+i\sinh(x))^3}\,dx = -\frac{2\left(3i\,e^{(2\,x)}-i\right)}{3\,(e^x-i)^3}$$

input
```
integrate(cosh(x)^2/(1+I*sinh(x))^3,x, algorithm="giac")
```

output
```
-2/3*(3*I*e^(2*x) - I)/(e^x - I)^3
```

Mupad [B] (verification not implemented)

Time = 1.60 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.95

$$\int \frac{\cosh^2(x)}{(1+i\sinh(x))^3}\,dx = -\frac{2\,\mathrm{e}^{2\,x}-\frac{2}{3}}{\left(1+\mathrm{e}^{x}\,1\mathrm{i}\right)^3}$$

input
```
int(cosh(x)^2/(sinh(x)*1i + 1)^3,x)
```

output
```
-(2*exp(2*x) - 2/3)/(exp(x)*1i + 1)^3
```

Reduce [F]

$$\begin{aligned}\int \frac{\cosh^2(x)}{(1+i\sinh(x))^3}\,dx = &-10\left(\int \frac{e^{4x}}{e^{6x}i+6e^{5x}-15e^{4x}i-20e^{3x}+15e^{2x}i+6e^{x}-i}dx\right)i\\ &-24\left(\int \frac{e^{3x}}{e^{6x}i+6e^{5x}-15e^{4x}i-20e^{3x}+15e^{2x}i+6e^{x}-i}dx\right)\\ &+20\left(\int \frac{e^{2x}}{e^{6x}i+6e^{5x}-15e^{4x}i-20e^{3x}+15e^{2x}i+6e^{x}-i}dx\right)i\\ &+8\left(\int \frac{e^{x}}{e^{6x}i+6e^{5x}-15e^{4x}i-20e^{3x}+15e^{2x}i+6e^{x}-i}dx\right)\\ &-2\left(\int \frac{1}{e^{6x}i+6e^{5x}-15e^{4x}i-20e^{3x}+15e^{2x}i+6e^{x}-i}dx\right)i\\ &+\frac{\log(e^{6x}i+6e^{5x}-15e^{4x}i-20e^{3x}+15e^{2x}i+6e^{x}-i)}{3}-2x\end{aligned}$$

input
```
int(cosh(x)^2/(1+I*sinh(x))^3,x)
```

output

```
( - 30*int(e**(4*x)/(e**(6*x)*i + 6*e**(5*x) - 15*e**(4*x)*i - 20*e**(3*x)
 + 15*e**(2*x)*i + 6*e**x - i),x)*i - 72*int(e**(3*x)/(e**(6*x)*i + 6*e**(
5*x) - 15*e**(4*x)*i - 20*e**(3*x) + 15*e**(2*x)*i + 6*e**x - i),x) + 60*i
nt(e**(2*x)/(e**(6*x)*i + 6*e**(5*x) - 15*e**(4*x)*i - 20*e**(3*x) + 15*e*
*(2*x)*i + 6*e**x - i),x)*i + 24*int(e**x/(e**(6*x)*i + 6*e**(5*x) - 15*e*
*(4*x)*i - 20*e**(3*x) + 15*e**(2*x)*i + 6*e**x - i),x) - 6*int(1/(e**(6*x
)*i + 6*e**(5*x) - 15*e**(4*x)*i - 20*e**(3*x) + 15*e**(2*x)*i + 6*e**x -
i),x)*i + log(e**(6*x)*i + 6*e**(5*x) - 15*e**(4*x)*i - 20*e**(3*x) + 15*e
**(2*x)*i + 6*e**x - i) - 6*x)/3
```

3.183 $\int \frac{\cosh(x)}{(1+i\sinh(x))^3} dx$

Optimal result

Integrand size = 13, antiderivative size = 16

$$\int \frac{\cosh(x)}{(1+i\sinh(x))^3} dx = \frac{i}{2(1+i\sinh(x))^2}$$

output
```
1/2*I/(1+I*sinh(x))^2
```

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.88

$$\int \frac{\cosh(x)}{(1+i\sinh(x))^3} dx = -\frac{i}{2(-i+\sinh(x))^2}$$

input
```
Integrate[Cosh[x]/(1 + I*Sinh[x])^3,x]
```

output
```
(-1/2*I)/(-I + Sinh[x])^2
```

Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used = {3042, 3146, 17}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cosh(x)}{(1+i\sinh(x))^3}\,dx$$
$$\Big\downarrow \text{ 3042}$$
$$\int \frac{\cos(ix)}{(1+\sin(ix))^3}dx$$
$$\Big\downarrow \text{ 3146}$$
$$-i\int \frac{1}{(i\sinh(x)+1)^3}d(i\sinh(x))$$
$$\Big\downarrow \text{ 17}$$
$$\frac{i}{2(1+i\sinh(x))^2}$$

input
```
Int[Cosh[x]/(1 + I*Sinh[x])^3,x]
```

output
```
(I/2)/(1 + I*Sinh[x])^2
```

Defintions of rubi rules used

rule 17
```
Int[(c_.)*((a_.) + (b_.)*(x_))^(m_.), x_Symbol] :> Simp[c*((a + b*x)^(m + 1
)/(b*(m + 1))), x] /; FreeQ[{a, b, c, m}, x] && NeQ[m, -1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3146
```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Simp[1/(b^p*f)   Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x
)^((p - 1)/2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && I
ntegerQ[(p - 1)/2] && EqQ[a^2 - b^2, 0] && (GeQ[p, -1] ||  !IntegerQ[m + 1/
2])
```

Maple [A] (verified)

Time = 33.94 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.81

method	result	size
derivativedivides	$\frac{i}{2(1+i\sinh(x))^2}$	13
default	$\frac{i}{2(1+i\sinh(x))^2}$	13
risch	$-\frac{2i\mathrm{e}^{2x}}{(\mathrm{e}^{x}-i)^4}$	15

input
```
int(cosh(x)/(1+I*sinh(x))^3,x,method=_RETURNVERBOSE)
```

output
```
1/2*I/(1+I*sinh(x))^2
```

Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 30 vs. 2(10) = 20.

Time = 0.08 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.88

$$\int \frac{\cosh(x)}{(1+i\sinh(x))^3}\,dx = -\frac{2i\,e^{(2\,x)}}{e^{(4\,x)}-4i\,e^{(3\,x)}-6\,e^{(2\,x)}+4i\,e^{x}+1}$$

input
```
integrate(cosh(x)/(1+I*sinh(x))^3,x, algorithm="fricas")
```

output
```
-2*I*e^(2*x)/(e^(4*x) - 4*I*e^(3*x) - 6*e^(2*x) + 4*I*e^x + 1)
```

Sympy [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 37 vs. 2(10) = 20.

Time = 0.10 (sec) , antiderivative size = 37, normalized size of antiderivative = 2.31

$$\int \frac{\cosh(x)}{(1+i\sinh(x))^3}\,dx = -\frac{2ie^{2x}}{e^{4x}-4ie^{3x}-6e^{2x}+4ie^{x}+1}$$

input
```
integrate(cosh(x)/(1+I*sinh(x))**3,x)
```

output
```
-2*I*exp(2*x)/(exp(4*x) - 4*I*exp(3*x) - 6*exp(2*x) + 4*I*exp(x) + 1)
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.62

$$\int \frac{\cosh(x)}{(1+i\sinh(x))^3}\,dx = \frac{i}{2\,(i\,\sinh(x)+1)^2}$$

input
```
integrate(cosh(x)/(1+I*sinh(x))^3,x, algorithm="maxima")
```

output
```
1/2*I/(I*sinh(x) + 1)^2
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.75

$$\int \frac{\cosh(x)}{(1+i\sinh(x))^3}\,dx = -\frac{2i\,e^{(2\,x)}}{(e^{x}-i)^4}$$

input
```
integrate(cosh(x)/(1+I*sinh(x))^3,x, algorithm="giac")
```

output
```
-2*I*e^(2*x)/(e^x - I)^4
```

Mupad [B] (verification not implemented)

Time = 1.56 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00

$$\int \frac{\cosh(x)}{(1+i\sinh(x))^3}\,dx = -\frac{e^{2x}\,2i}{(1+e^{x}\,1i)^4}$$

input
```
int(cosh(x)/(sinh(x)*1i + 1)^3,x)
```

output
```
-(exp(2*x)*2i)/(exp(x)*1i + 1)^4
```

Reduce [F]

$$\int \frac{\cosh(x)}{(1+i\sinh(x))^3}\,dx = -\left(\int \frac{\cosh(x)}{\sinh(x)^3 i + 3\sinh(x)^2 - 3\sinh(x) i - 1}dx\right)$$

input
```
int(cosh(x)/(1+I*sinh(x))^3,x)
```

output
```
- int(cosh(x)/(sinh(x)**3*i + 3*sinh(x)**2 - 3*sinh(x)*i - 1),x)
```

3.184 $\int \frac{\cosh^3(x)}{(1-i\sinh(x))^3}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 26

$$\int \frac{\cosh^3(x)}{(1-i\sinh(x))^3}\,dx = -i\log(i+\sinh(x)) - \frac{2i}{1-i\sinh(x)}$$

output
```
-I*ln(I+sinh(x))-2*I/(1-I*sinh(x))
```

Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 43, normalized size of antiderivative = 1.65

$$\int \frac{\cosh^3(x)}{(1-i\sinh(x))^3}\,dx = \frac{\cosh^4(x)(2+\log(i+\sinh(x)) - i\log(i+\sinh(x))\sinh(x))}{(-i+\sinh(x))^2(i+\sinh(x))^3}$$

input
```
Integrate[Cosh[x]^3/(1 - I*Sinh[x])^3,x]
```

output
```
(Cosh[x]^4*(2 + Log[I + Sinh[x]] - I*Log[I + Sinh[x]]*Sinh[x]))/((-I + Sin
h[x])^2*(I + Sinh[x])^3)
```

Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.08, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.267$, Rules used = {3042, 3146, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cosh^3(x)}{(1-i\sinh(x))^3}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\cos(ix)^3}{(1-\sin(ix))^3}dx$$

$$\Big\downarrow \text{3146}$$

$$i\int \frac{i\sinh(x)+1}{(1-i\sinh(x))^2}d(-i\sinh(x))$$

$$\Big\downarrow \text{49}$$

$$i\int \left(\frac{2}{(1-i\sinh(x))^2}+\frac{1}{i\sinh(x)-1}\right)d(-i\sinh(x))$$

$$\Big\downarrow \text{2009}$$

$$i\left(-\frac{2}{1-i\sinh(x)}-\log(1-i\sinh(x))\right)$$

input
```
Int[Cosh[x]^3/(1 - I*Sinh[x])^3,x]
```

output
```
I*(-Log[1 - I*Sinh[x]] - 2/(1 - I*Sinh[x]))
```

Defintions of rubi rules used

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3146
```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Simp[1/(b^p*f)   Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x
)^((p - 1)/2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && I
ntegerQ[(p - 1)/2] && EqQ[a^2 - b^2, 0] && (GeQ[p, -1] ||  !IntegerQ[m + 1/
2])
```

Maple [A] (verified)

Time = 36.04 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.00

method	result	size
derivativedivides	$-\frac{i \ln\left(1+\sinh(x)^2\right)}{2}-\arctan\left(\sinh\left(x\right)\right)+\frac{2}{i+\sinh(x)}$	26
default	$-\frac{i \ln\left(1+\sinh(x)^2\right)}{2}-\arctan\left(\sinh\left(x\right)\right)+\frac{2}{i+\sinh(x)}$	26
risch	$ix+\frac{4\,\mathrm{e}^{x}}{(\mathrm{e}^{x}+i)^2}-2i\ln\left(\mathrm{e}^{x}+i\right)$	26

input
```
int(cosh(x)^3/(1-I*sinh(x))^3,x,method=_RETURNVERBOSE)
```

output
```
-1/2*I*ln(1+sinh(x)^2)-arctan(sinh(x))+2/(I+sinh(x))
```

Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 50 vs. 2(18) = 36.

Time = 0.08 (sec) , antiderivative size = 50, normalized size of antiderivative = 1.92

$$\int \frac{\cosh^3(x)}{(1-i\sinh(x))^3}\,dx = \frac{i\,xe^{(2\,x)}-2\,(x-2)e^x-2\,\left(i\,e^{(2\,x)}-2\,e^x-i\right)\log\left(e^x+i\right)-i\,x}{e^{(2\,x)}+2i\,e^x-1}$$

input
```
integrate(cosh(x)^3/(1-I*sinh(x))^3,x, algorithm="fricas")
```

output
```
(I*x*e^(2*x) - 2*(x - 2)*e^x - 2*(I*e^(2*x) - 2*e^x - I)*log(e^x + I) - I*
x)/(e^(2*x) + 2*I*e^x - 1)
```

Sympy [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.19

$$\int \frac{\cosh^3(x)}{(1-i\sinh(x))^3}\,dx = ix-2i\log\left(e^x+i\right)+\frac{4e^x}{e^{2x}+2ie^x-1}$$

input
```
integrate(cosh(x)**3/(1-I*sinh(x))**3,x)
```

output
```
I*x - 2*I*log(exp(x) + I) + 4*exp(x)/(exp(2*x) + 2*I*exp(x) - 1)
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 33, normalized size of antiderivative = 1.27

$$\int \frac{\cosh^3(x)}{(1-i\sinh(x))^3}\,dx = -i\,x-\frac{4\,e^{(-x)}}{-2i\,e^{(-x)}+e^{(-2\,x)}-1}-2i\,\log\left(e^{(-x)}-i\right)$$

input
```
integrate(cosh(x)^3/(1-I*sinh(x))^3,x, algorithm="maxima")
```

output
```
-I*x - 4*e^(-x)/(-2*I*e^(-x) + e^(-2*x) - 1) - 2*I*log(e^(-x) - I)
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.81

$$\int \frac{\cosh^3(x)}{(1-i\sinh(x))^3}\,dx = i\,x + \frac{4\,e^x}{(e^x+i)^2} - 2i\,\log\left(e^x+i\right)$$

input
```
integrate(cosh(x)^3/(1-I*sinh(x))^3,x, algorithm="giac")
```

output
```
I*x + 4*e^x/(e^x + I)^2 - 2*I*log(e^x + I)
```

Mupad [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 39, normalized size of antiderivative = 1.50

$$\int \frac{\cosh^3(x)}{(1-i\sinh(x))^3}\,dx = x\,1\mathrm{i} - \ln\left(\mathrm{e}^x+1\mathrm{i}\right)\,2\mathrm{i} - \frac{4\mathrm{i}}{\mathrm{e}^{2\,x}-1+\mathrm{e}^x\,2\mathrm{i}} + \frac{4}{\mathrm{e}^x+1\mathrm{i}}$$

input
```
int(-cosh(x)^3/(sinh(x)*1i - 1)^3,x)
```

output
```
x*1i - log(exp(x) + 1i)*2i - 4i/(exp(2*x) + exp(x)*2i - 1) + 4/(exp(x) + 1
i)
```

Reduce [F]

$$\begin{aligned}\int \frac{\cosh^3(x)}{(1-i\sinh(x))^3}\,dx &= 12\left(\int \frac{e^{4x}}{e^{6x}+6e^{5x}i-15e^{4x}-20e^{3x}i+15e^{2x}+6e^{x}i-1}dx\right)i\\&\quad-40\left(\int \frac{e^{3x}}{e^{6x}+6e^{5x}i-15e^{4x}-20e^{3x}i+15e^{2x}+6e^{x}i-1}dx\right)\\&\quad-48\left(\int \frac{e^{2x}}{e^{6x}+6e^{5x}i-15e^{4x}-20e^{3x}i+15e^{2x}+6e^{x}i-1}dx\right)i\\&\quad+24\left(\int \frac{e^{x}}{e^{6x}+6e^{5x}i-15e^{4x}-20e^{3x}i+15e^{2x}+6e^{x}i-1}dx\right)\\&\quad+4\left(\int \frac{1}{e^{6x}+6e^{5x}i-15e^{4x}-20e^{3x}i+15e^{2x}+6e^{x}i-1}dx\right)i\\&\quad-\log\left(e^{6x}i-6e^{5x}-15e^{4x}i+20e^{3x}+15e^{2x}i-6e^{x}-i\right)i+5ix\end{aligned}$$

input
```
int(cosh(x)^3/(1-I*sinh(x))^3,x)
```

output
```
12*int(e**(4*x)/(e**(6*x) + 6*e**(5*x)*i - 15*e**(4*x) - 20*e**(3*x)*i + 1
5*e**(2*x) + 6*e**x*i - 1),x)*i - 40*int(e**(3*x)/(e**(6*x) + 6*e**(5*x)*i
 - 15*e**(4*x) - 20*e**(3*x)*i + 15*e**(2*x) + 6*e**x*i - 1),x) - 48*int(e
**(2*x)/(e**(6*x) + 6*e**(5*x)*i - 15*e**(4*x) - 20*e**(3*x)*i + 15*e**(2*
x) + 6*e**x*i - 1),x)*i + 24*int(e**x/(e**(6*x) + 6*e**(5*x)*i - 15*e**(4*
x) - 20*e**(3*x)*i + 15*e**(2*x) + 6*e**x*i - 1),x) + 4*int(1/(e**(6*x) +
6*e**(5*x)*i - 15*e**(4*x) - 20*e**(3*x)*i + 15*e**(2*x) + 6*e**x*i - 1),x
)*i - log(e**(6*x)*i - 6*e**(5*x) - 15*e**(4*x)*i + 20*e**(3*x) + 15*e**(2
*x)*i - 6*e**x - i)*i + 5*i*x
```

3.185 $\int \frac{\cosh^2(x)}{(1-i\sinh(x))^3}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 20

$$\int \frac{\cosh^2(x)}{(1-i\sinh(x))^3}\,dx = -\frac{i\cosh^3(x)}{3(1-i\sinh(x))^3}$$

output
```
-1/3*I*cosh(x)^3/(1-I*sinh(x))^3
```

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.80

$$\int \frac{\cosh^2(x)}{(1-i\sinh(x))^3}\,dx = -\frac{\cosh^3(x)}{3(i+\sinh(x))^3}$$

input
```
Integrate[Cosh[x]^2/(1 - I*Sinh[x])^3,x]
```

output
```
-1/3*Cosh[x]^3/(I + Sinh[x])^3
```

Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {3042, 3150}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cosh^2(x)}{(1-i\sinh(x))^3}\,dx$$
$$\Big\downarrow \text{ 3042}$$
$$\int \frac{\cos(ix)^2}{(1-\sin(ix))^3}dx$$
$$\Big\downarrow \text{ 3150}$$
$$-\frac{i\cosh^3(x)}{3(1-i\sinh(x))^3}$$

input
```
Int[Cosh[x]^2/(1 - I*Sinh[x])^3,x]
```

output
```
((-1/3*I)*Cosh[x]^3)/(1 - I*Sinh[x])^3
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3150
```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[b*(g*Cos[e + f*x])^(p + 1)*((a + b*Sin[e + f*x
])^m/(a*f*g*m)), x] /; FreeQ[{a, b, e, f, g, m, p}, x] && EqQ[a^2 - b^2, 0]
 && EqQ[Simplify[m + p + 1], 0] && !ILtQ[p, 0]
```

Maple [A] (verified)

Time = 34.63 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.95

method	result	size
risch	$\frac{2i(3\,\mathrm{e}^{2x}-1)}{3(\mathrm{e}^{x}+i)^{3}}$	19
default	$\frac{2}{\tanh(\frac{x}{2})+i}-\frac{4i}{(\tanh(\frac{x}{2})+i)^{2}}-\frac{8}{3(\tanh(\frac{x}{2})+i)^{3}}$	36

input
```
int(cosh(x)^2/(1-I*sinh(x))^3,x,method=_RETURNVERBOSE)
```

output
```
2/3*I*(3*exp(2*x)-1)/(exp(x)+I)^3
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.40

$$\int \frac{\cosh^2(x)}{(1-i\sinh(x))^3}\,dx = -\frac{2\left(-3i\,e^{(2\,x)}+i\right)}{3\left(e^{(3\,x)}+3i\,e^{(2\,x)}-3\,e^{x}-i\right)}$$

input
```
integrate(cosh(x)^2/(1-I*sinh(x))^3,x, algorithm="fricas")
```

output
```
-2/3*(-3*I*e^(2*x) + I)/(e^(3*x) + 3*I*e^(2*x) - 3*e^x - I)
```

Sympy [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.70

$$\int \frac{\cosh^2(x)}{(1-i\sinh(x))^3}\,dx = \frac{6ie^{2x}-2i}{3e^{3x}+9ie^{2x}-9e^{x}-3i}$$

input
```
integrate(cosh(x)**2/(1-I*sinh(x))**3,x)
```

output
```
(6*I*exp(2*x) - 2*I)/(3*exp(3*x) + 9*I*exp(2*x) - 9*exp(x) - 3*I)
```

Maxima [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 53 vs. 2(14) = 28.

Time = 0.04 (sec) , antiderivative size = 53, normalized size of antiderivative = 2.65

$$\int \frac{\cosh^2(x)}{(1 - i\sinh(x))^3}\,dx = -\frac{6\,e^{(-2\,x)}}{-9i\,e^{(-x)} + 9\,e^{(-2\,x)} + 3i\,e^{(-3\,x)} - 3} + \frac{2}{-9i\,e^{(-x)} + 9\,e^{(-2\,x)} + 3i\,e^{(-3\,x)} - 3}$$

input
```
integrate(cosh(x)^2/(1-I*sinh(x))^3,x, algorithm="maxima")
```

output
```
-6*e^(-2*x)/(-9*I*e^(-x) + 9*e^(-2*x) + 3*I*e^(-3*x) - 3) + 2/(-9*I*e^(-x)
 + 9*e^(-2*x) + 3*I*e^(-3*x) - 3)
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.80

$$\int \frac{\cosh^2(x)}{(1 - i\sinh(x))^3}\,dx = -\frac{2\left(-3i\,e^{(2\,x)} + i\right)}{3\left(e^x + i\right)^3}$$

input
```
integrate(cosh(x)^2/(1-I*sinh(x))^3,x, algorithm="giac")
```

output
```
-2/3*(-3*I*e^(2*x) + I)/(e^x + I)^3
```

Mupad [B] (verification not implemented)

Time = 1.55 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.95

$$\int \frac{\cosh^2(x)}{(1-i\sinh(x))^3}\,dx = \frac{2\,(3\,\mathrm{e}^{2\,x}-1)}{3\,{(-1+\mathrm{e}^{x}\,1\mathrm{i})}^3}$$

input
```
int(-cosh(x)^2/(sinh(x)*1i - 1)^3,x)
```

output
```
(2*(3*exp(2*x) - 1))/(3*(exp(x)*1i - 1)^3)
```

Reduce [F]

$$\begin{aligned}\int \frac{\cosh^2(x)}{(1-i\sinh(x))^3}\,dx = &-10\left(\int \frac{e^{4x}}{e^{6x}i-6e^{5x}-15e^{4x}i+20e^{3x}+15e^{2x}i-6e^{x}-i}dx\right)i\\ &+24\left(\int \frac{e^{3x}}{e^{6x}i-6e^{5x}-15e^{4x}i+20e^{3x}+15e^{2x}i-6e^{x}-i}dx\right)\\ &+20\left(\int \frac{e^{2x}}{e^{6x}i-6e^{5x}-15e^{4x}i+20e^{3x}+15e^{2x}i-6e^{x}-i}dx\right)i\\ &-8\left(\int \frac{e^{x}}{e^{6x}i-6e^{5x}-15e^{4x}i+20e^{3x}+15e^{2x}i-6e^{x}-i}dx\right)\\ &-2\left(\int \frac{1}{e^{6x}i-6e^{5x}-15e^{4x}i+20e^{3x}+15e^{2x}i-6e^{x}-i}dx\right)i\\ &+\frac{\log(e^{6x}i-6e^{5x}-15e^{4x}i+20e^{3x}+15e^{2x}i-6e^{x}-i)}{3}-2x\end{aligned}$$

input
```
int(cosh(x)^2/(1-I*sinh(x))^3,x)
```

output

```
( - 30*int(e**(4*x)/(e**(6*x)*i - 6*e**(5*x) - 15*e**(4*x)*i + 20*e**(3*x)
 + 15*e**(2*x)*i - 6*e**x - i),x)*i + 72*int(e**(3*x)/(e**(6*x)*i - 6*e**(
5*x) - 15*e**(4*x)*i + 20*e**(3*x) + 15*e**(2*x)*i - 6*e**x - i),x) + 60*i
nt(e**(2*x)/(e**(6*x)*i - 6*e**(5*x) - 15*e**(4*x)*i + 20*e**(3*x) + 15*e*
*(2*x)*i - 6*e**x - i),x)*i - 24*int(e**x/(e**(6*x)*i - 6*e**(5*x) - 15*e*
*(4*x)*i + 20*e**(3*x) + 15*e**(2*x)*i - 6*e**x - i),x) - 6*int(1/(e**(6*x
)*i - 6*e**(5*x) - 15*e**(4*x)*i + 20*e**(3*x) + 15*e**(2*x)*i - 6*e**x -
i),x)*i + log(e**(6*x)*i - 6*e**(5*x) - 15*e**(4*x)*i + 20*e**(3*x) + 15*e
**(2*x)*i - 6*e**x - i) - 6*x)/3
```

3.186 $\int \frac{\cosh(x)}{(1-i\sinh(x))^3}\,dx$

Optimal result

Integrand size = 13, antiderivative size = 16

$$\int \frac{\cosh(x)}{(1-i\sinh(x))^3}\,dx = -\frac{i}{2(1-i\sinh(x))^2}$$

output
```
-1/2*I/(1-I*sinh(x))^2
```

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.88

$$\int \frac{\cosh(x)}{(1-i\sinh(x))^3}\,dx = \frac{i}{2(i+\sinh(x))^2}$$

input
```
Integrate[Cosh[x]/(1 - I*Sinh[x])^3,x]
```

output
```
(I/2)/(I + Sinh[x])^2
```

Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used = {3042, 3146, 17}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cosh(x)}{(1-i\sinh(x))^3}\,dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \frac{\cos(ix)}{(1-\sin(ix))^3}dx$$

$$\Big\downarrow \text{ 3146}$$

$$i\int \frac{1}{(1-i\sinh(x))^3}d(-i\sinh(x))$$

$$\Big\downarrow \text{ 17}$$

$$-\frac{i}{2(1-i\sinh(x))^2}$$

input
```
Int[Cosh[x]/(1 - I*Sinh[x])^3,x]
```

output
```
(-1/2*I)/(1 - I*Sinh[x])^2
```

Defintions of rubi rules used

rule 17
```
Int[(c_.)*((a_.) + (b_.)*(x_))^(m_.), x_Symbol] :> Simp[c*((a + b*x)^(m + 1
)/(b*(m + 1))), x] /; FreeQ[{a, b, c, m}, x] && NeQ[m, -1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3146
```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Simp[1/(b^p*f)   Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x
)^((p - 1)/2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && I
ntegerQ[(p - 1)/2] && EqQ[a^2 - b^2, 0] && (GeQ[p, -1] ||  !IntegerQ[m + 1/
2])
```

Maple [A] (verified)

Time = 34.52 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.81

method	result	size
derivativedivides	$-\frac{i}{2(1-i\sinh(x))^2}$	13
default	$-\frac{i}{2(1-i\sinh(x))^2}$	13
risch	$\frac{2ie^{2x}}{(e^x+i)^4}$	15

input
```
int(cosh(x)/(1-I*sinh(x))^3,x,method=_RETURNVERBOSE)
```

output
```
-1/2*I/(1-I*sinh(x))^2
```

Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 30 vs. 2(10) = 20.

Time = 0.07 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.88

$$\int \frac{\cosh(x)}{(1-i\sinh(x))^3}\,dx = \frac{2i\,e^{(2\,x)}}{e^{(4\,x)}+4i\,e^{(3\,x)}-6\,e^{(2\,x)}-4i\,e^{x}+1}$$

input
```
integrate(cosh(x)/(1-I*sinh(x))^3,x, algorithm="fricas")
```

output
```
2*I*e^(2*x)/(e^(4*x) + 4*I*e^(3*x) - 6*e^(2*x) - 4*I*e^x + 1)
```

Sympy [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 36 vs. 2(12) = 24.

Time = 0.09 (sec) , antiderivative size = 36, normalized size of antiderivative = 2.25

$$\int \frac{\cosh(x)}{(1-i\sinh(x))^3}\,dx = \frac{2ie^{2x}}{e^{4x}+4ie^{3x}-6e^{2x}-4ie^{x}+1}$$

input
```
integrate(cosh(x)/(1-I*sinh(x))**3,x)
```

output
```
2*I*exp(2*x)/(exp(4*x) + 4*I*exp(3*x) - 6*exp(2*x) - 4*I*exp(x) + 1)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.62

$$\int \frac{\cosh(x)}{(1-i\sinh(x))^3}\,dx = -\frac{i}{2\left(-i\,\sinh\left(x\right)+1\right)^2}$$

input
```
integrate(cosh(x)/(1-I*sinh(x))^3,x, algorithm="maxima")
```

output
```
-1/2*I/(-I*sinh(x) + 1)^2
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.75

$$\int \frac{\cosh(x)}{(1-i\sinh(x))^3}\,dx = \frac{2i\,e^{(2\,x)}}{\left(e^{x}+i\right)^4}$$

input
```
integrate(cosh(x)/(1-I*sinh(x))^3,x, algorithm="giac")
```

output
```
2*I*e^(2*x)/(e^x + I)^4
```

Mupad [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00

$$\int \frac{\cosh(x)}{(1 - i\sinh(x))^3}\,dx = \frac{e^{2x}\,2i}{(-1 + e^x\,1i)^4}$$

input
```
int(-cosh(x)/(sinh(x)*1i - 1)^3,x)
```

output
```
(exp(2*x)*2i)/(exp(x)*1i - 1)^4
```

Reduce [F]

$$\int \frac{\cosh(x)}{(1 - i\sinh(x))^3}\,dx = \int \frac{\cosh(x)}{\sinh(x)^3 i - 3\sinh(x)^2 - 3\sinh(x) i + 1}\,dx$$

input
```
int(cosh(x)/(1-I*sinh(x))^3,x)
```

output
```
int(cosh(x)/(sinh(x)**3*i - 3*sinh(x)**2 - 3*sinh(x)*i + 1),x)
```

3.187 $\int \frac{\cosh^7(x)}{a+b\sinh(x)}\,dx$

Optimal result

Integrand size = 13, antiderivative size = 138

$$\int \frac{\cosh^7(x)}{a+b\sinh(x)}\,dx = \frac{(a^2+b^2)^3\log(a+b\sinh(x))}{b^7} - \frac{a(a^4+3a^2b^2+3b^4)\sinh(x)}{b^6} + \frac{(a^4+3a^2b^2+3b^4)\sinh^2(x)}{2b^5} - \frac{a(a^2+3b^2)\sinh^3(x)}{3b^4} + \frac{(a^2+3b^2)\sinh^4(x)}{4b^3} - \frac{a\sinh^5(x)}{5b^2} + \frac{\sinh^6(x)}{6b}$$

output

```
(a^2+b^2)^3*ln(a+b*sinh(x))/b^7-a*(a^4+3*a^2*b^2+3*b^4)*sinh(x)/b^6+1/2*(a
^4+3*a^2*b^2+3*b^4)*sinh(x)^2/b^5-1/3*a*(a^2+3*b^2)*sinh(x)^3/b^4+1/4*(a^2
+3*b^2)*sinh(x)^4/b^3-1/5*a*sinh(x)^5/b^2+1/6*sinh(x)^6/b
```

Mathematica [A] (verified)

Time = 0.11 (sec) , antiderivative size = 121, normalized size of antiderivative = 0.88

$$\int \frac{\cosh^7(x)}{a+b\sinh(x)}\,dx = \frac{15b^4(a^2+b^2)\cosh^4(x)+10b^6\cosh^6(x)+60(a^2+b^2)^3\log(a+b\sinh(x))-60ab(a^4+3a^2b^2+3b^4)\sinh(}{60b^7}$$

input
```
Integrate[Cosh[x]^7/(a + b*Sinh[x]),x]
```

output
```
(15*b^4*(a^2 + b^2)*Cosh[x]^4 + 10*b^6*Cosh[x]^6 + 60*(a^2 + b^2)^3*Log[a
+ b*Sinh[x]] - 60*a*b*(a^4 + 3*a^2*b^2 + 3*b^4)*Sinh[x] + 30*b^2*(a^2 + b^
2)^2*Sinh[x]^2 - 20*a*b^3*(a^2 + 3*b^2)*Sinh[x]^3 - 12*a*b^5*Sinh[x]^5)/(6
0*b^7)
```

Rubi [A] (verified)

Time = 0.35 (sec) , antiderivative size = 138, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.385$, Rules used = {3042, 3147, 25, 476, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cosh^7(x)}{a+b\sinh(x)}dx$$

$$\Big\downarrow \ 3042$$

$$\int \frac{\cos(ix)^7}{a-ib\sin(ix)}dx$$

$$\Big\downarrow \ 3147$$

$$-\frac{\int -\frac{\left(\sinh^2(x)b^2+b^2\right)^3}{a+b\sinh(x)}d(b\sinh(x))}{b^7}$$

$$\Big\downarrow \ 25$$

$$\frac{\int \frac{\left(\sinh^2(x)b^2+b^2\right)^3}{a+b\sinh(x)}d(b\sinh(x))}{b^7}$$

$$\Big\downarrow \ 476$$

$$\frac{\int \Big(b^5\sinh^5(x)-ab^4\sinh^4(x)+b^3\left(a^2+3b^2\right)\sinh^3(x)-ab^2\left(a^2+3b^2\right)\sinh^2(x)+b\left(a^4+3b^2a^2+3b^4\right)\sinh(x)-}{b^7}$$

$$\Big\downarrow \ 2009$$

$$-\frac{-\left(a^2+b^2\right)^3 \log(a+b\sinh(x))-\frac{1}{4}b^4\left(a^2+3b^2\right)\sinh^4(x)+\frac{1}{3}ab^3\left(a^2+3b^2\right)\sinh^3(x)-\frac{1}{2}b^2\left(a^4+3a^2b^2+3b^4\right)\text{si}}{b^7}$$

input
```
Int[Cosh[x]^7/(a + b*Sinh[x]),x]
```

output
```
-((-((a^2 + b^2)^3*Log[a + b*Sinh[x]]) + a*b*(a^4 + 3*a^2*b^2 + 3*b^4)*Sin
h[x] - (b^2*(a^4 + 3*a^2*b^2 + 3*b^4)*Sinh[x]^2)/2 + (a*b^3*(a^2 + 3*b^2)*
Sinh[x]^3)/3 - (b^4*(a^2 + 3*b^2)*Sinh[x]^4)/4 + (a*b^5*Sinh[x]^5)/5 - (b^
6*Sinh[x]^6)/6)/b^7)
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 476
```
Int[((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[
ExpandIntegrand[(c + d*x)^n*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d, n},
 x] && IGtQ[p, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3147
```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Simp[1/(b^p*f)   Subst[Int[(a + x)^m*(b^2 - x^2)^((p - 1)
/2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && IntegerQ[(p
 - 1)/2] && NeQ[a^2 - b^2, 0]
```

Maple [A] (verified)

Time = 122.62 (sec) , antiderivative size = 146, normalized size of antiderivative = 1.06

method	result
derivativedivides	$-\frac{-\frac{\sinh(x)^6 b^5}{6}+\frac{a \sinh(x)^5 b^4}{5}-\frac{b\left(a^2b^2+3b^4\right)\sinh(x)^4}{4}+\frac{a\left(a^2b^2+3b^4\right)\sinh(x)^3}{3}-\frac{\left(a^4+3a^2b^2+3b^4\right)\sinh(x)^2 b}{2}+a\left(a^4+3a^2b^2+\right.}{b^6}$
default	$-\frac{-\frac{\sinh(x)^6 b^5}{6}+\frac{a \sinh(x)^5 b^4}{5}-\frac{b\left(a^2b^2+3b^4\right)\sinh(x)^4}{4}+\frac{a\left(a^2b^2+3b^4\right)\sinh(x)^3}{3}-\frac{\left(a^4+3a^2b^2+3b^4\right)\sinh(x)^2 b}{2}+a\left(a^4+3a^2b^2+\right.}{b^6}$
risch	$-\frac{3a^4x}{b^5}-\frac{x}{b}-\frac{3x\,a^2}{b^3}-\frac{19a\,e^{x}}{16b^2}+\frac{e^{4x}}{32b}+\frac{29\,e^{2x}}{128b}+\frac{29\,e^{-2x}}{128b}+\frac{e^{-4x}}{32b}+\frac{\ln\left(e^{2x}+\frac{2a\,e^{x}}{b}-1\right)}{b}+\frac{e^{-6x}}{384b}+\frac{e^{6x}}{384b}$

input
```
int(cosh(x)^7/(a+b*sinh(x)),x,method=_RETURNVERBOSE)
```

output
```
-1/b^6*(-1/6*sinh(x)^6*b^5+1/5*a*sinh(x)^5*b^4-1/4*b*(a^2*b^2+3*b^4)*sinh(
x)^4+1/3*a*(a^2*b^2+3*b^4)*sinh(x)^3-1/2*(a^4+3*a^2*b^2+3*b^4)*sinh(x)^2*b
+a*(a^4+3*a^2*b^2+3*b^4)*sinh(x))+(a^6+3*a^4*b^2+3*a^2*b^4+b^6)/b^7*ln(a+b
*sinh(x))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2105 vs. $2(128) = 256$.

Time = 0.12 (sec) , antiderivative size = 2105, normalized size of antiderivative = 15.25

$$\int \frac{\cosh^7(x)}{a+b\sinh(x)}\,dx = \text{Too large to display}$$

input
```
integrate(cosh(x)^7/(a+b*sinh(x)),x, algorithm="fricas")
```

output
```
1/1920*(5*b^6*cosh(x)^12 + 5*b^6*sinh(x)^12 - 12*a*b^5*cosh(x)^11 + 12*(5*
b^6*cosh(x) - a*b^5)*sinh(x)^11 + 30*(a^2*b^4 + 2*b^6)*cosh(x)^10 + 6*(55*
b^6*cosh(x)^2 - 22*a*b^5*cosh(x) + 5*a^2*b^4 + 10*b^6)*sinh(x)^10 - 20*(4*
a^3*b^3 + 9*a*b^5)*cosh(x)^9 + 20*(55*b^6*cosh(x)^3 - 33*a*b^5*cosh(x)^2 -
 4*a^3*b^3 - 9*a*b^5 + 15*(a^2*b^4 + 2*b^6)*cosh(x))*sinh(x)^9 + 15*(16*a^
4*b^2 + 40*a^2*b^4 + 29*b^6)*cosh(x)^8 + 15*(165*b^6*cosh(x)^4 - 132*a*b^5
*cosh(x)^3 + 16*a^4*b^2 + 40*a^2*b^4 + 29*b^6 + 90*(a^2*b^4 + 2*b^6)*cosh(
x)^2 - 12*(4*a^3*b^3 + 9*a*b^5)*cosh(x))*sinh(x)^8 - 1920*(a^6 + 3*a^4*b^2
 + 3*a^2*b^4 + b^6)*x*cosh(x)^6 - 120*(8*a^5*b + 22*a^3*b^3 + 19*a*b^5)*co
sh(x)^7 + 120*(33*b^6*cosh(x)^5 - 33*a*b^5*cosh(x)^4 - 8*a^5*b - 22*a^3*b^
3 - 19*a*b^5 + 30*(a^2*b^4 + 2*b^6)*cosh(x)^3 - 6*(4*a^3*b^3 + 9*a*b^5)*co
sh(x)^2 + (16*a^4*b^2 + 40*a^2*b^4 + 29*b^6)*cosh(x))*sinh(x)^7 + 12*a*b^5
*cosh(x) + 12*(385*b^6*cosh(x)^6 - 462*a*b^5*cosh(x)^5 + 525*(a^2*b^4 + 2*
b^6)*cosh(x)^4 - 140*(4*a^3*b^3 + 9*a*b^5)*cosh(x)^3 + 35*(16*a^4*b^2 + 40
*a^2*b^4 + 29*b^6)*cosh(x)^2 - 160*(a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6)*x -
 70*(8*a^5*b + 22*a^3*b^3 + 19*a*b^5)*cosh(x))*sinh(x)^6 + 5*b^6 + 120*(8*
a^5*b + 22*a^3*b^3 + 19*a*b^5)*cosh(x)^5 + 24*(165*b^6*cosh(x)^7 - 231*a*b
^5*cosh(x)^6 + 40*a^5*b + 110*a^3*b^3 + 95*a*b^5 + 315*(a^2*b^4 + 2*b^6)*c
osh(x)^5 - 105*(4*a^3*b^3 + 9*a*b^5)*cosh(x)^4 + 35*(16*a^4*b^2 + 40*a^2*b
^4 + 29*b^6)*cosh(x)^3 - 480*(a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6)*x*cosh...
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\cosh^7(x)}{a+b\sinh(x)}\,dx = \text{Timed out}$$

input
```
integrate(cosh(x)**7/(a+b*sinh(x)),x)
```

output
```
Timed out
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 308 vs. $2(128) = 256$.

Time = 0.04 (sec) , antiderivative size = 308, normalized size of antiderivative = 2.23

$$\int \frac{\cosh^7(x)}{a+b\sinh(x)}\,dx = -\frac{\left(12\,ab^4e^{(-x)} - 5\,b^5 - 30\,(a^2b^3+2\,b^5)e^{(-2\,x)} + 20\,(4\,a^3b^2+9\,ab^4)e^{(-3\,x)} - 15\,(16\,a^4b+40\,a^2b^3+29\,b^5)e^{(-4\,x)} + 120\,(8\,a^5+22\,a^3b^2+19\,ab^4)e^{(-5\,x)}\right)e^{(6\,x)}}{1920\,b^6} + \frac{12\,ab^4e^{(-5\,x)} + 5\,b^5e^{(-6\,x)} + 120\,(8\,a^5+22\,a^3b^2+19\,ab^4)e^{(-x)} + 15\,(16\,a^4b+40\,a^2b^3+29\,b^5)e^{(-2\,x)} + 20\,(4\,a^3b^2+9\,ab^4)e^{(-3\,x)} + 30\,(a^2b^3+2\,b^5)e^{(-4\,x)}}{1920\,b^6} + \frac{(a^6+3\,a^4b^2+3\,a^2b^4+b^6)x}{b^7} + \frac{(a^6+3\,a^4b^2+3\,a^2b^4+b^6)\log\left(-2\,ae^{(-x)}+be^{(-2\,x)}-b\right)}{b^7}$$

input
```
integrate(cosh(x)^7/(a+b*sinh(x)),x, algorithm="maxima")
```

output
```
-1/1920*(12*a*b^4*e^(-x) - 5*b^5 - 30*(a^2*b^3 + 2*b^5)*e^(-2*x) + 20*(4*a
^3*b^2 + 9*a*b^4)*e^(-3*x) - 15*(16*a^4*b + 40*a^2*b^3 + 29*b^5)*e^(-4*x)
+ 120*(8*a^5 + 22*a^3*b^2 + 19*a*b^4)*e^(-5*x))*e^(6*x)/b^6 + 1/1920*(12*a
*b^4*e^(-5*x) + 5*b^5*e^(-6*x) + 120*(8*a^5 + 22*a^3*b^2 + 19*a*b^4)*e^(-x
) + 15*(16*a^4*b + 40*a^2*b^3 + 29*b^5)*e^(-2*x) + 20*(4*a^3*b^2 + 9*a*b^4
)*e^(-3*x) + 30*(a^2*b^3 + 2*b^5)*e^(-4*x))/b^6 + (a^6 + 3*a^4*b^2 + 3*a^2
*b^4 + b^6)*x/b^7 + (a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6)*log(-2*a*e^(-x) +
b*e^(-2*x) - b)/b^7
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 254, normalized size of antiderivative = 1.84

$$\int \frac{\cosh^7(x)}{a+b\sinh(x)}\,dx = \frac{5\,b^5\left(e^{(-x)}-e^x\right)^6 + 12\,ab^4\left(e^{(-x)}-e^x\right)^5 + 30\,a^2b^3\left(e^{(-x)}-e^x\right)^4 + 90\,b^5\left(e^{(-x)}-e^x\right)^4 + 80\,a^3b^2\left(e^{(-x)}-e^x\right)^3 \cdots}{\cdots} + \frac{(a^6+3\,a^4b^2+3\,a^2b^4+b^6)\log\left(\left|-b\left(e^{(-x)}-e^x\right)+2\,a\right|\right)}{b^7}$$

input
```
integrate(cosh(x)^7/(a+b*sinh(x)),x, algorithm="giac")
```

output
```
1/1920*(5*b^5*(e^(-x) - e^x)^6 + 12*a*b^4*(e^(-x) - e^x)^5 + 30*a^2*b^3*(e
^(-x) - e^x)^4 + 90*b^5*(e^(-x) - e^x)^4 + 80*a^3*b^2*(e^(-x) - e^x)^3 + 2
40*a*b^4*(e^(-x) - e^x)^3 + 240*a^4*b*(e^(-x) - e^x)^2 + 720*a^2*b^3*(e^(-
x) - e^x)^2 + 720*b^5*(e^(-x) - e^x)^2 + 960*a^5*(e^(-x) - e^x) + 2880*a^3
*b^2*(e^(-x) - e^x) + 2880*a*b^4*(e^(-x) - e^x))/b^6 + (a^6 + 3*a^4*b^2 +
3*a^2*b^4 + b^6)*log(abs(-b*(e^(-x) - e^x) + 2*a))/b^7
```

Mupad [B] (verification not implemented)

Time = 2.14 (sec) , antiderivative size = 287, normalized size of antiderivative = 2.08

$$\int \frac{\cosh^7(x)}{a+b\sinh(x)}\,dx = \frac{e^{-6x}}{384\,b}+\frac{e^{6x}}{384\,b}+\frac{e^{-x}\,(8\,a^5+22\,a^3\,b^2+19\,a\,b^4)}{16\,b^6}+\frac{e^{-3x}\,(4\,a^3+9\,a\,b^2)}{96\,b^4} - \frac{e^{3x}\,(4\,a^3+9\,a\,b^2)}{96\,b^4}+\frac{e^{-4x}\,(a^2+2\,b^2)}{64\,b^3}+\frac{e^{4x}\,(a^2+2\,b^2)}{64\,b^3}+\frac{a\,e^{-5x}}{160\,b^2} - \frac{a\,e^{5x}}{160\,b^2}-\frac{x\,(a^2+b^2)^3}{b^7}+\frac{e^{-2x}\,(16\,a^4+40\,a^2\,b^2+29\,b^4)}{128\,b^5} + \frac{e^{2x}\,(16\,a^4+40\,a^2\,b^2+29\,b^4)}{128\,b^5}-\frac{e^{x}\,(8\,a^5+22\,a^3\,b^2+19\,a\,b^4)}{16\,b^6} + \frac{\ln\left(2\,a\,e^{x}-b+b\,e^{2x}\right)\,(a^6+3\,a^4\,b^2+3\,a^2\,b^4+b^6)}{b^7}$$

input
```
int(cosh(x)^7/(a + b*sinh(x)),x)
```

output
```
exp(-6*x)/(384*b) + exp(6*x)/(384*b) + (exp(-x)*(19*a*b^4 + 8*a^5 + 22*a^3
*b^2))/(16*b^6) + (exp(-3*x)*(9*a*b^2 + 4*a^3))/(96*b^4) - (exp(3*x)*(9*a*
b^2 + 4*a^3))/(96*b^4) + (exp(-4*x)*(a^2 + 2*b^2))/(64*b^3) + (exp(4*x)*(a
^2 + 2*b^2))/(64*b^3) + (a*exp(-5*x))/(160*b^2) - (a*exp(5*x))/(160*b^2) -
 (x*(a^2 + b^2)^3)/b^7 + (exp(-2*x)*(16*a^4 + 29*b^4 + 40*a^2*b^2))/(128*b
^5) + (exp(2*x)*(16*a^4 + 29*b^4 + 40*a^2*b^2))/(128*b^5) - (exp(x)*(19*a*
b^4 + 8*a^5 + 22*a^3*b^2))/(16*b^6) + (log(2*a*exp(x) - b + b*exp(2*x))*(a
^6 + b^6 + 3*a^2*b^4 + 3*a^4*b^2))/b^7
```

Reduce [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 452, normalized size of antiderivative = 3.28

$$\int \frac{\cosh^7(x)}{a+b\sinh(x)}\,dx$$
$$= \frac{5b^6 + 5760e^{6x}\log(e^{2x}b + 2e^xa - b)\,a^4b^2 + 5760e^{6x}\log(e^{2x}b + 2e^xa - b)\,a^2b^4 - 5760e^{6x}a^4b^2x - 5760e^{6x}a^2}{}$$

input
```
int(cosh(x)^7/(a+b*sinh(x)),x)
```

output
```
(5*e**(12*x)*b**6 - 12*e**(11*x)*a*b**5 + 30*e**(10*x)*a**2*b**4 + 60*e**(
10*x)*b**6 - 80*e**(9*x)*a**3*b**3 - 180*e**(9*x)*a*b**5 + 240*e**(8*x)*a*
*4*b**2 + 600*e**(8*x)*a**2*b**4 + 435*e**(8*x)*b**6 - 960*e**(7*x)*a**5*b
 - 2640*e**(7*x)*a**3*b**3 - 2280*e**(7*x)*a*b**5 + 1920*e**(6*x)*log(e**(
2*x)*b + 2*e**x*a - b)*a**6 + 5760*e**(6*x)*log(e**(2*x)*b + 2*e**x*a - b)
*a**4*b**2 + 5760*e**(6*x)*log(e**(2*x)*b + 2*e**x*a - b)*a**2*b**4 + 1920
*e**(6*x)*log(e**(2*x)*b + 2*e**x*a - b)*b**6 - 1920*e**(6*x)*a**6*x - 576
0*e**(6*x)*a**4*b**2*x - 5760*e**(6*x)*a**2*b**4*x - 1920*e**(6*x)*b**6*x
 + 960*e**(5*x)*a**5*b + 2640*e**(5*x)*a**3*b**3 + 2280*e**(5*x)*a*b**5 + 2
40*e**(4*x)*a**4*b**2 + 600*e**(4*x)*a**2*b**4 + 435*e**(4*x)*b**6 + 80*e*
*(3*x)*a**3*b**3 + 180*e**(3*x)*a*b**5 + 30*e**(2*x)*a**2*b**4 + 60*e**(2*
x)*b**6 + 12*e**x*a*b**5 + 5*b**6)/(1920*e**(6*x)*b**7)
```

3.188 $\int \frac{\cosh^6(x)}{a+b\sinh(x)}\,dx$

Optimal result

Integrand size = 13, antiderivative size = 145

$$\int \frac{\cosh^6(x)}{a+b\sinh(x)}\,dx = -\frac{a(8a^4+20a^2b^2+15b^4)\,x}{8b^6} - \frac{2(a^2+b^2)^{5/2}\,\text{arctanh}\left(\frac{b-a\tanh\left(\frac{x}{2}\right)}{\sqrt{a^2+b^2}}\right)}{b^6} + \frac{\cosh^5(x)}{5b} + \frac{\cosh^3(x)\,(4(a^2+b^2)-3ab\sinh(x))}{12b^3} + \frac{\cosh(x)\left(8(a^2+b^2)^2-ab(4a^2+7b^2)\sinh(x)\right)}{8b^5}$$

output
```
-1/8*a*(8*a^4+20*a^2*b^2+15*b^4)*x/b^6-2*(a^2+b^2)^(5/2)*arctanh((b-a*tanh
(1/2*x))/(a^2+b^2)^(1/2))/b^6+1/5*cosh(x)^5/b+1/12*cosh(x)^3*(4*a^2+4*b^2-
3*a*b*sinh(x))/b^3+1/8*cosh(x)*(8*(a^2+b^2)^2-a*b*(4*a^2+7*b^2)*sinh(x))/b
^5
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 4.41 (sec) , antiderivative size = 463, normalized size of antiderivative = 3.19

$$\int \frac{\cosh^6(x)}{a+b\sinh(x)}\,dx$$

$$= \frac{\cosh(x)\left(8(15a^4+35a^2b^2+23b^4)-15ab(4a^2+9b^2)\sinh(x)+8b^2(5a^2+11b^2)\sinh^2(x)-30ab^3\sinh^3(\ldots\right.}{}$$

input
```
Integrate[Cosh[x]^6/(a + b*Sinh[x]),x]
```

output
```
(Cosh[x]*(8*(15*a^4 + 35*a^2*b^2 + 23*b^4) - 15*a*b*(4*a^2 + 9*b^2)*Sinh[x
] + 8*b^2*(5*a^2 + 11*b^2)*Sinh[x]^2 - 30*a*b^3*Sinh[x]^3 + 24*b^4*Sinh[x]
^4 - (30*(-1)^(3/4)*Sqrt[b]*(8*a^4 - (4*I)*a^3*b + 16*a^2*b^2 - (7*I)*a*b^
3 + 8*b^4)*ArcSin[((1/2 + I/2)*Sqrt[a - I*b]*Sqrt[-((b*(I + Sinh[x]))/(a -
 I*b))])/Sqrt[b]])/(Sqrt[a - I*b]*Sqrt[1 + I*Sinh[x]]*Sqrt[-((b*(I + Sinh[
x]))/(a - I*b))]) - (240*(a^2 + b^2)^2*ArcTanh[Sqrt[-((b*(I + Sinh[x]))/(a
 - I*b))]/Sqrt[-((b*(-I + Sinh[x]))/(a + I*b))]])/(Sqrt[-((b*(-I + Sinh[x]
))/(a + I*b))]*Sqrt[-((b*(I + Sinh[x]))/(a - I*b))]) + (240*(a - I*b)^(5/2
)*(a + I*b)^(3/2)*ArcTanh[(Sqrt[a - I*b]*Sqrt[-((b*(I + Sinh[x]))/(a - I*b
))])/(Sqrt[a + I*b]*Sqrt[-((b*(-I + Sinh[x]))/(a + I*b))])])/(Sqrt[-((b*(-
I + Sinh[x]))/(a + I*b))]*Sqrt[-((b*(I + Sinh[x]))/(a - I*b))])))/(120*b^5
)
```

Rubi [A] (verified)

Time = 0.98 (sec) , antiderivative size = 169, normalized size of antiderivative = 1.17, number of steps used = 16, number of rules used = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 1.154$, Rules used = {3042, 3174, 26, 3042, 3344, 25, 3042, 3344, 25, 3042, 3214, 3042, 3139, 1083, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cosh^6(x)}{a+b\sinh(x)}\,dx$$

$\Big\downarrow$ 3042

$$\int \frac{\cos(ix)^6}{a-ib\sin(ix)}dx$$

$\Big\downarrow$ 3174

$$\frac{\cosh^5(x)}{5b}+\frac{i\int -\frac{i\cosh^4(x)(b-a\sinh(x))}{a+b\sinh(x)}dx}{b}$$

$\Big\downarrow$ 26

$$\frac{\int \frac{\cosh^4(x)(b-a\sinh(x))}{a+b\sinh(x)}dx}{b}+\frac{\cosh^5(x)}{5b}$$

$\Big\downarrow$ 3042

$$\frac{\cosh^5(x)}{5b}+\frac{\int \frac{\cos(ix)^4(b+ia\sin(ix))}{a-ib\sin(ix)}dx}{b}$$

$\Big\downarrow$ 3344

$$\frac{\frac{\cosh^3(x)(4(a^2+b^2)-3ab\sinh(x))}{12b^2}-\frac{\int -\frac{\cosh^2(x)\left(b\left(a^2+4b^2\right)-a\left(4a^2+7b^2\right)\sinh(x)\right)}{a+b\sinh(x)}dx}{4b^2}}{b}+\frac{\cosh^5(x)}{5b}$$

$\Big\downarrow$ 25

$$\frac{\frac{\int \frac{\cosh^2(x)\left(b\left(a^2+4b^2\right)-a\left(4a^2+7b^2\right)\sinh(x)\right)}{a+b\sinh(x)}dx}{4b^2}+\frac{\cosh^3(x)(4(a^2+b^2)-3ab\sinh(x))}{12b^2}}{b}+\frac{\cosh^5(x)}{5b}$$

$\Big\downarrow$ 3042

$$\frac{\cosh^5(x)}{5b}+\frac{\frac{\cosh^3(x)(4(a^2+b^2)-3ab\sinh(x))}{12b^2}+\frac{\int \frac{\cos(ix)^2\left(b\left(a^2+4b^2\right)+ia\left(4a^2+7b^2\right)\sin(ix)\right)}{a-ib\sin(ix)}dx}{4b^2}}{b}$$

$\Big\downarrow$ 3344

$$\frac{\frac{\frac{\cosh(x)\left(8\left(a^2+b^2\right)^2-ab\left(4a^2+7b^2\right)\sinh(x)\right)}{2b^2}-\frac{\int -\frac{b\left(4a^4+9b^2a^2+8b^4\right)-a\left(8a^4+20b^2a^2+15b^4\right)\sinh(x)}{a+b\sinh(x)}dx}{2b^2}}{4b^2}+\frac{\cosh^3(x)(4(a^2+b^2)-3ab\sinh(x))}{12b^2}}{b}+\frac{\cosh^5(x)}{5b}$$

$\Big\downarrow$ 25

$$\frac{\frac{\frac{\int \frac{b\left(4a^4+9b^2a^2+8b^4\right)-a\left(8a^4+20b^2a^2+15b^4\right)\sinh(x)}{a+b\sinh(x)}dx}{2b^2}+\frac{\cosh(x)\left(8\left(a^2+b^2\right)^2-ab\left(4a^2+7b^2\right)\sinh(x)\right)}{2b^2}}{4b^2}+\frac{\cosh^3(x)\left(4\left(a^2+b^2\right)-3ab\sinh(x)\right)}{12b^2}}{b}+\frac{\cosh^5(x)}{5b}$$

$\Big\downarrow$ 3042

$$\frac{\cosh^5(x)}{5b}+\frac{\frac{\cosh^3(x)\left(4\left(a^2+b^2\right)-3ab\sinh(x)\right)}{12b^2}+\frac{\frac{\cosh(x)\left(8\left(a^2+b^2\right)^2-ab\left(4a^2+7b^2\right)\sinh(x)\right)}{2b^2}+\frac{\int \frac{b\left(4a^4+9b^2a^2+8b^4\right)+ia\left(8a^4+20b^2a^2+15b^4\right)\sin(ix)}{a-ib\sin(ix)}dx}{2b^2}}{4b^2}}{b}$$

$\Big\downarrow$ 3214

$$\frac{\frac{\frac{\frac{8\left(a^2+b^2\right)^3\int \frac{1}{a+b\sinh(x)}dx}{b}-\frac{ax\left(8a^4+20a^2b^2+15b^4\right)}{b}}{2b^2}+\frac{\cosh(x)\left(8\left(a^2+b^2\right)^2-ab\left(4a^2+7b^2\right)\sinh(x)\right)}{2b^2}}{4b^2}+\frac{\cosh^3(x)\left(4\left(a^2+b^2\right)-3ab\sinh(x)\right)}{12b^2}}{b}+\frac{\cosh^5(x)}{5b}$$

$\Big\downarrow$ 3042

$$\frac{\cosh^5(x)}{5b}+\frac{\frac{\cosh^3(x)\left(4\left(a^2+b^2\right)-3ab\sinh(x)\right)}{12b^2}+\frac{\frac{\cosh(x)\left(8\left(a^2+b^2\right)^2-ab\left(4a^2+7b^2\right)\sinh(x)\right)}{2b^2}+\frac{-\frac{ax\left(8a^4+20a^2b^2+15b^4\right)}{b}+\frac{8\left(a^2+b^2\right)^3\int \frac{1}{a-ib\sin(ix)}dx}{b}}{2b^2}}{4b^2}}{b}$$

$\Big\downarrow$ 3139

$$\frac{\frac{\frac{\frac{16\left(a^2+b^2\right)^3\int \frac{1}{-a\tanh^2\left(\frac{x}{2}\right)+2b\tanh\left(\frac{x}{2}\right)+a}d\tanh\left(\frac{x}{2}\right)}{b}-\frac{ax\left(8a^4+20a^2b^2+15b^4\right)}{b}}{2b^2}+\frac{\cosh(x)\left(8\left(a^2+b^2\right)^2-ab\left(4a^2+7b^2\right)\sinh(x)\right)}{2b^2}}{4b^2}+\frac{\cosh^3(x)\left(4\left(a^2+b^2\right)-\right.}{12b^2}}{b}$$

$$\frac{\cosh^5(x)}{5b}$$

$\Big\downarrow$ 1083

$$\frac{\frac{-\frac{32\left(a^2+b^2\right)^3\int \frac{1}{4\left(a^2+b^2\right)-\left(2b-2a\tanh\left(\frac{x}{2}\right)\right)^2}d\left(2b-2a\tanh\left(\frac{x}{2}\right)\right)}{b}-\frac{ax\left(8a^4+20a^2b^2+15b^4\right)}{b}}{2b^2}+\frac{\cosh(x)\left(8\left(a^2+b^2\right)^2-ab\left(4a^2+7b^2\right)\sinh(x)\right)}{2b^2}}{4b^2}+\frac{\cosh^3(x)(\ldots}{b}$$

$$\frac{\cosh^5(x)}{5b}$$

$\Big\downarrow$ 219

$$\frac{\frac{\cosh^3(x)\left(4\left(a^2+b^2\right)-3ab\sinh(x)\right)}{12b^2}+\frac{\frac{\cosh(x)\left(8\left(a^2+b^2\right)^2-ab\left(4a^2+7b^2\right)\sinh(x)\right)}{2b^2}+\frac{-\frac{16\left(a^2+b^2\right)^{5/2}\text{arctanh}\left(\frac{2b-2a\tanh\left(\frac{x}{2}\right)}{2\sqrt{a^2+b^2}}\right)}{b}-\frac{ax\left(8a^4+20a^2b^2+\ldots}{b}}{2b^2}}{4b^2}}{b}$$

$$\frac{\cosh^5(x)}{5b}$$

input
```
Int[Cosh[x]^6/(a + b*Sinh[x]),x]
```

output
```
Cosh[x]^5/(5*b) + ((Cosh[x]^3*(4*(a^2 + b^2) - 3*a*b*Sinh[x]))/(12*b^2) +
((-((a*(8*a^4 + 20*a^2*b^2 + 15*b^4)*x)/b) - (16*(a^2 + b^2)^(5/2)*ArcTanh
[(2*b - 2*a*Tanh[x/2])/(2*Sqrt[a^2 + b^2])])/b)/(2*b^2) + (Cosh[x]*(8*(a^2
 + b^2)^2 - a*b*(4*a^2 + 7*b^2)*Sinh[x]))/(2*b^2))/(4*b^2))/b
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1083
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Simp[-2   Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3139
```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Simp[2*(e/d)   Subst[Int[1/(a + 2*b*e*x + a
*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[a^2 - b^2, 0]
```

rule 3174
```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[g*(g*Cos[e + f*x])^(p - 1)*((a + b*Sin[e + f*x
])^(m + 1)/(b*f*(m + p))), x] + Simp[g^2*((p - 1)/(b*(m + p)))   Int[(g*Cos
[e + f*x])^(p - 2)*(a + b*Sin[e + f*x])^m*(b + a*Sin[e + f*x]), x], x] /; F
reeQ[{a, b, e, f, g, m}, x] && NeQ[a^2 - b^2, 0] && GtQ[p, 1] && NeQ[m + p,
 0] && IntegersQ[2*m, 2*p]
```

rule 3214
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])/((c_.) + (d_.)*sin[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[b*(x/d), x] - Simp[(b*c - a*d)/d   Int[1/(c + d
*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

rule 3344
```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[g*(g*
Cos[e + f*x])^(p - 1)*(a + b*Sin[e + f*x])^(m + 1)*((b*c*(m + p + 1) - a*d*
p + b*d*(m + p)*Sin[e + f*x])/(b^2*f*(m + p)*(m + p + 1))), x] + Simp[g^2*(
(p - 1)/(b^2*(m + p)*(m + p + 1)))   Int[(g*Cos[e + f*x])^(p - 2)*(a + b*Si
n[e + f*x])^m*Simp[b*(a*d*m + b*c*(m + p + 1)) + (a*b*c*(m + p + 1) - d*(a^
2*p - b^2*(m + p)))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, g,
 m}, x] && NeQ[a^2 - b^2, 0] && GtQ[p, 1] && NeQ[m + p, 0] && NeQ[m + p + 1
, 0] && IntegerQ[2*m]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 343 vs. 2(132) = 264.

Time = 55.25 (sec) , antiderivative size = 344, normalized size of antiderivative = 2.37

method	result
risch	$-\frac{a^5x}{b^6}-\frac{5a^3x}{2b^4}-\frac{15ax}{8b^2}+\frac{e^{5x}}{160b}-\frac{a\,e^{4x}}{64b^2}+\frac{e^{3x}a^2}{24b^3}+\frac{7\,e^{3x}}{96b}-\frac{a^3e^{2x}}{8b^4}-\frac{a\,e^{2x}}{4b^2}+\frac{e^{x}a^4}{2b^5}+\frac{9\,e^{x}a^2}{8b^3}+\frac{11\,e^{x}}{16b}+\frac{e^{-x}a^4}{2b^5}+$
default	$\frac{1}{5b\left(\tanh\left(\frac{x}{2}\right)+1\right)^5}-\frac{2b-a}{4b^2\left(\tanh\left(\frac{x}{2}\right)+1\right)^4}-\frac{-4a^2+6ab-13b^2}{12b^3\left(\tanh\left(\frac{x}{2}\right)+1\right)^3}-\frac{-4a^3+4a^2b-11a\,b^2+9b^3}{8b^4\left(\tanh\left(\frac{x}{2}\right)+1\right)^2}-\frac{-8a^4+4a^3b-20a^2b^2+9a\,b^3-15}{8b^5\left(\tanh\left(\frac{x}{2}\right)+1\right)}$

input
```
int(cosh(x)^6/(a+b*sinh(x)),x,method=_RETURNVERBOSE)
```

output
```
-a^5*x/b^6-5/2*a^3*x/b^4-15/8*a*x/b^2+1/160/b*exp(x)^5-1/64*a/b^2*exp(x)^4
+1/24/b^3*exp(x)^3*a^2+7/96/b*exp(x)^3-1/8*a^3/b^4*exp(x)^2-1/4*a/b^2*exp(
x)^2+1/2/b^5*exp(x)*a^4+9/8/b^3*exp(x)*a^2+11/16/b*exp(x)+1/2/b^5/exp(x)*a
^4+9/8/b^3/exp(x)*a^2+11/16/b/exp(x)+1/8*a^3/b^4/exp(x)^2+1/4*a/b^2/exp(x)
^2+1/24/b^3/exp(x)^3*a^2+7/96/b/exp(x)^3+1/64*a/b^2/exp(x)^4+1/160/b/exp(x
)^5+(a^2+b^2)^(5/2)/b^6*ln(exp(x)-((a^2+b^2)^(5/2)-a^5-2*a^3*b^2-a*b^4)/b/
(a^4+2*a^2*b^2+b^4))-(a^2+b^2)^(5/2)/b^6*ln(exp(x)+((a^2+b^2)^(5/2)+a^5+2*
a^3*b^2+a*b^4)/b/(a^4+2*a^2*b^2+b^4))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1486 vs. 2(133) = 266.

Time = 0.11 (sec) , antiderivative size = 1486, normalized size of antiderivative = 10.25

$$\int \frac{\cosh^6(x)}{a+b\sinh(x)}\,dx = \text{Too large to display}$$

input
```
integrate(cosh(x)^6/(a+b*sinh(x)),x, algorithm="fricas")
```

output
```
1/960*(6*b^5*cosh(x)^10 + 6*b^5*sinh(x)^10 - 15*a*b^4*cosh(x)^9 + 15*(4*b^
5*cosh(x) - a*b^4)*sinh(x)^9 + 10*(4*a^2*b^3 + 7*b^5)*cosh(x)^8 + 5*(54*b^
5*cosh(x)^2 - 27*a*b^4*cosh(x) + 8*a^2*b^3 + 14*b^5)*sinh(x)^8 - 120*(a^3*
b^2 + 2*a*b^4)*cosh(x)^7 + 20*(36*b^5*cosh(x)^3 - 27*a*b^4*cosh(x)^2 - 6*a
^3*b^2 - 12*a*b^4 + 4*(4*a^2*b^3 + 7*b^5)*cosh(x))*sinh(x)^7 - 120*(8*a^5
+ 20*a^3*b^2 + 15*a*b^4)*x*cosh(x)^5 + 60*(8*a^4*b + 18*a^2*b^3 + 11*b^5)*
cosh(x)^6 + 20*(63*b^5*cosh(x)^4 - 63*a*b^4*cosh(x)^3 + 24*a^4*b + 54*a^2*
b^3 + 33*b^5 + 14*(4*a^2*b^3 + 7*b^5)*cosh(x)^2 - 42*(a^3*b^2 + 2*a*b^4)*c
osh(x))*sinh(x)^6 + 15*a*b^4*cosh(x) + 2*(756*b^5*cosh(x)^5 - 945*a*b^4*co
sh(x)^4 + 280*(4*a^2*b^3 + 7*b^5)*cosh(x)^3 - 1260*(a^3*b^2 + 2*a*b^4)*cos
h(x)^2 - 60*(8*a^5 + 20*a^3*b^2 + 15*a*b^4)*x + 180*(8*a^4*b + 18*a^2*b^3
+ 11*b^5)*cosh(x))*sinh(x)^5 + 6*b^5 + 60*(8*a^4*b + 18*a^2*b^3 + 11*b^5)*
cosh(x)^4 + 10*(126*b^5*cosh(x)^6 - 189*a*b^4*cosh(x)^5 + 48*a^4*b + 108*a
^2*b^3 + 66*b^5 + 70*(4*a^2*b^3 + 7*b^5)*cosh(x)^4 - 420*(a^3*b^2 + 2*a*b^
4)*cosh(x)^3 - 60*(8*a^5 + 20*a^3*b^2 + 15*a*b^4)*x*cosh(x) + 90*(8*a^4*b
+ 18*a^2*b^3 + 11*b^5)*cosh(x)^2)*sinh(x)^4 + 120*(a^3*b^2 + 2*a*b^4)*cosh
(x)^3 + 20*(36*b^5*cosh(x)^7 - 63*a*b^4*cosh(x)^6 + 28*(4*a^2*b^3 + 7*b^5)
*cosh(x)^5 + 6*a^3*b^2 + 12*a*b^4 - 210*(a^3*b^2 + 2*a*b^4)*cosh(x)^4 - 60
*(8*a^5 + 20*a^3*b^2 + 15*a*b^4)*x*cosh(x)^2 + 60*(8*a^4*b + 18*a^2*b^3 +
11*b^5)*cosh(x)^3 + 12*(8*a^4*b + 18*a^2*b^3 + 11*b^5)*cosh(x))*sinh(x)...
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\cosh^6(x)}{a+b\sinh(x)}\,dx=\text{Timed out}$$

input
```
integrate(cosh(x)**6/(a+b*sinh(x)),x)
```

output
```
Timed out
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 283 vs. $2(133) = 266$.

Time = 0.12 (sec) , antiderivative size = 283, normalized size of antiderivative = 1.95

$$\int \frac{\cosh^6(x)}{a+b\sinh(x)}\,dx = -\frac{\left(15\,ab^3e^{(-x)} - 6\,b^4 - 10\,(4\,a^2b^2 + 7\,b^4)e^{(-2\,x)} + 120\,(a^3b + 2\,ab^3)e^{(-3\,x)} - 60\,(8\,a^4 + 18\,a^2b^2 + 11\,b^4)e^{(-4\,x)}\right)e^{(5\,x)}}{960\,b^5} + \frac{15\,ab^3e^{(-4\,x)} + 6\,b^4e^{(-5\,x)} + 60\,(8\,a^4 + 18\,a^2b^2 + 11\,b^4)e^{(-x)} + 120\,(a^3b + 2\,ab^3)e^{(-2\,x)} + 10\,(4\,a^2b^2 + 7\,b^4)e^{(-3\,x)}}{960\,b^5} - \frac{(8\,a^5 + 20\,a^3b^2 + 15\,ab^4)x}{8\,b^6} + \frac{(a^6 + 3\,a^4b^2 + 3\,a^2b^4 + b^6)\log\left(\frac{be^{(-x)} - a - \sqrt{a^2+b^2}}{be^{(-x)} - a + \sqrt{a^2+b^2}}\right)}{\sqrt{a^2+b^2}\,b^6}$$

input

```
integrate(cosh(x)^6/(a+b*sinh(x)),x, algorithm="maxima")
```

output

```
-1/960*(15*a*b^3*e^(-x) - 6*b^4 - 10*(4*a^2*b^2 + 7*b^4)*e^(-2*x) + 120*(a
^3*b + 2*a*b^3)*e^(-3*x) - 60*(8*a^4 + 18*a^2*b^2 + 11*b^4)*e^(-4*x))*e^(5
*x)/b^5 + 1/960*(15*a*b^3*e^(-4*x) + 6*b^4*e^(-5*x) + 60*(8*a^4 + 18*a^2*b
^2 + 11*b^4)*e^(-x) + 120*(a^3*b + 2*a*b^3)*e^(-2*x) + 10*(4*a^2*b^2 + 7*b
^4)*e^(-3*x))/b^5 - 1/8*(8*a^5 + 20*a^3*b^2 + 15*a*b^4)*x/b^6 + (a^6 + 3*a
^4*b^2 + 3*a^2*b^4 + b^6)*log((b*e^(-x) - a - sqrt(a^2 + b^2))/(b*e^(-x) -
 a + sqrt(a^2 + b^2)))/(sqrt(a^2 + b^2)*b^6)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 288 vs. $2(133) = 266$.

Time = 0.15 (sec) , antiderivative size = 288, normalized size of antiderivative = 1.99

$$\int \frac{\cosh^6(x)}{a + b\sinh(x)}\,dx$$
$$= \frac{6\,b^4e^{(5\,x)} - 15\,ab^3e^{(4\,x)} + 40\,a^2b^2e^{(3\,x)} + 70\,b^4e^{(3\,x)} - 120\,a^3be^{(2\,x)} - 240\,ab^3e^{(2\,x)} + 480\,a^4e^x + 1080\,a^2b^2e^x}{960\,b^5}$$
$$- \frac{(8\,a^5 + 20\,a^3b^2 + 15\,ab^4)x}{8\,b^6}$$
$$+ \frac{(15\,ab^4e^x + 6\,b^5 + 60\,(8\,a^4b + 18\,a^2b^3 + 11\,b^5)e^{(4\,x)} + 120\,(a^3b^2 + 2\,ab^4)e^{(3\,x)} + 10\,(4\,a^2b^3 + 7\,b^5)e^{(2\,x)})e}{960\,b^6}$$
$$+ \frac{(a^6 + 3\,a^4b^2 + 3\,a^2b^4 + b^6)\log\left(\frac{\left|2\,be^x + 2\,a - 2\,\sqrt{a^2 + b^2}\right|}{\left|2\,be^x + 2\,a + 2\,\sqrt{a^2 + b^2}\right|}\right)}{\sqrt{a^2 + b^2}b^6}$$

input
```
integrate(cosh(x)^6/(a+b*sinh(x)),x, algorithm="giac")
```

output
```
1/960*(6*b^4*e^(5*x) - 15*a*b^3*e^(4*x) + 40*a^2*b^2*e^(3*x) + 70*b^4*e^(3
*x) - 120*a^3*b*e^(2*x) - 240*a*b^3*e^(2*x) + 480*a^4*e^x + 1080*a^2*b^2*e
^x + 660*b^4*e^x)/b^5 - 1/8*(8*a^5 + 20*a^3*b^2 + 15*a*b^4)*x/b^6 + 1/960*
(15*a*b^4*e^x + 6*b^5 + 60*(8*a^4*b + 18*a^2*b^3 + 11*b^5)*e^(4*x) + 120*(
a^3*b^2 + 2*a*b^4)*e^(3*x) + 10*(4*a^2*b^3 + 7*b^5)*e^(2*x))*e^(-5*x)/b^6
+ (a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6)*log(abs(2*b*e^x + 2*a - 2*sqrt(a^2 +
 b^2))/abs(2*b*e^x + 2*a + 2*sqrt(a^2 + b^2)))/(sqrt(a^2 + b^2)*b^6)
```

Mupad [B] (verification not implemented)

Time = 2.08 (sec) , antiderivative size = 302, normalized size of antiderivative = 2.08

$$\int \frac{\cosh^6(x)}{a+b\sinh(x)}\,dx = \frac{e^{-5x}}{160\,b}+\frac{e^{5x}}{160\,b}-\frac{\ln\left(-\frac{2\,e^x\,(a^2+b^2)^3}{b^7}-\frac{2\,(b-a\,e^x)\,(a^2+b^2)^{5/2}}{b^7}\right)\,(a^2+b^2)^{5/2}}{b^6}+\frac{\ln\left(\frac{2\,(b-a\,e^x)\,(a^2+b^2)^{5/2}}{b^7}-\frac{2\,e^x\,(a^2+b^2)^3}{b^7}\right)\,(a^2+b^2)^{5/2}}{b^6}-\frac{x\,(8\,a^5+20\,a^3\,b^2+15\,a\,b^4)}{8\,b^6}+\frac{e^x\,(8\,a^4+18\,a^2\,b^2+11\,b^4)}{16\,b^5}+\frac{a\,e^{-4x}}{64\,b^2}-\frac{a\,e^{4x}}{64\,b^2}+\frac{e^{-x}\,(8\,a^4+18\,a^2\,b^2+11\,b^4)}{16\,b^5}+\frac{e^{-3x}\,(4\,a^2+7\,b^2)}{96\,b^3}+\frac{e^{3x}\,(4\,a^2+7\,b^2)}{96\,b^3}+\frac{e^{-2x}\,(a^3+2\,a\,b^2)}{8\,b^4}-\frac{e^{2x}\,(a^3+2\,a\,b^2)}{8\,b^4}$$

input
```
int(cosh(x)^6/(a + b*sinh(x)),x)
```

output
```
exp(-5*x)/(160*b) + exp(5*x)/(160*b) - (log(- (2*exp(x)*(a^2 + b^2)^3)/b^7
 - (2*(b - a*exp(x))*(a^2 + b^2)^(5/2))/b^7)*(a^2 + b^2)^(5/2))/b^6 + (log
((2*(b - a*exp(x))*(a^2 + b^2)^(5/2))/b^7 - (2*exp(x)*(a^2 + b^2)^3)/b^7)*
(a^2 + b^2)^(5/2))/b^6 - (x*(15*a*b^4 + 8*a^5 + 20*a^3*b^2))/(8*b^6) + (ex
p(x)*(8*a^4 + 11*b^4 + 18*a^2*b^2))/(16*b^5) + (a*exp(-4*x))/(64*b^2) - (a
*exp(4*x))/(64*b^2) + (exp(-x)*(8*a^4 + 11*b^4 + 18*a^2*b^2))/(16*b^5) + (
exp(-3*x)*(4*a^2 + 7*b^2))/(96*b^3) + (exp(3*x)*(4*a^2 + 7*b^2))/(96*b^3)
+ (exp(-2*x)*(2*a*b^2 + a^3))/(8*b^4) - (exp(2*x)*(2*a*b^2 + a^3))/(8*b^4)
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 373, normalized size of antiderivative = 2.57

$$\int \frac{\cosh^6(x)}{a+b\sinh(x)}\,dx = \frac{1920e^{5x}\sqrt{a^2+b^2}\,atan\left(\frac{e^x bi+ai}{\sqrt{a^2+b^2}}\right)a^4i+3840e^{5x}\sqrt{a^2+b^2}\,atan\left(\frac{e^x bi+ai}{\sqrt{a^2+b^2}}\right)a^2b^2i+1920e^{5x}\sqrt{a^2+b^2}\,atan\left(\frac{e^x bi}{\sqrt{a^2}}\right.}{}$$

input
```
int(cosh(x)^6/(a+b*sinh(x)),x)
```

output
```
(1920*e**(5*x)*sqrt(a**2 + b**2)*atan((e**x*b*i + a*i)/sqrt(a**2 + b**2))*
a**4*i + 3840*e**(5*x)*sqrt(a**2 + b**2)*atan((e**x*b*i + a*i)/sqrt(a**2 +
 b**2))*a**2*b**2*i + 1920*e**(5*x)*sqrt(a**2 + b**2)*atan((e**x*b*i + a*i
)/sqrt(a**2 + b**2))*b**4*i + 6*e**(10*x)*b**5 - 15*e**(9*x)*a*b**4 + 40*e
**(8*x)*a**2*b**3 + 70*e**(8*x)*b**5 - 120*e**(7*x)*a**3*b**2 - 240*e**(7*
x)*a*b**4 + 480*e**(6*x)*a**4*b + 1080*e**(6*x)*a**2*b**3 + 660*e**(6*x)*b
**5 - 960*e**(5*x)*a**5*x - 2400*e**(5*x)*a**3*b**2*x - 1800*e**(5*x)*a*b*
*4*x + 480*e**(4*x)*a**4*b + 1080*e**(4*x)*a**2*b**3 + 660*e**(4*x)*b**5 +
 120*e**(3*x)*a**3*b**2 + 240*e**(3*x)*a*b**4 + 40*e**(2*x)*a**2*b**3 + 70
*e**(2*x)*b**5 + 15*e**x*a*b**4 + 6*b**5)/(960*e**(5*x)*b**6)
```

3.189 $\int \frac{\cosh^5(x)}{a+b\sinh(x)}\,dx$

Optimal result

Integrand size = 13, antiderivative size = 81

$$\int \frac{\cosh^5(x)}{a+b\sinh(x)}\,dx = \frac{(a^2+b^2)^2\log(a+b\sinh(x))}{b^5} - \frac{a(a^2+2b^2)\sinh(x)}{b^4} + \frac{(a^2+2b^2)\sinh^2(x)}{2b^3} - \frac{a\sinh^3(x)}{3b^2} + \frac{\sinh^4(x)}{4b}$$

output
```
(a^2+b^2)^2*ln(a+b*sinh(x))/b^5-a*(a^2+2*b^2)*sinh(x)/b^4+1/2*(a^2+2*b^2)*
sinh(x)^2/b^3-1/3*a*sinh(x)^3/b^2+1/4*sinh(x)^4/b
```

Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 76, normalized size of antiderivative = 0.94

$$\int \frac{\cosh^5(x)}{a+b\sinh(x)}\,dx = \frac{3b^4\cosh^4(x) + 12(a^2+b^2)^2\log(a+b\sinh(x)) - 12ab(a^2+2b^2)\sinh(x) + 6b^2(a^2+b^2)\sinh^2(x) - 4ab^3\,\mathrm{si}}{12b^5}$$

input
```
Integrate[Cosh[x]^5/(a + b*Sinh[x]),x]
```

output
```
(3*b^4*Cosh[x]^4 + 12*(a^2 + b^2)^2*Log[a + b*Sinh[x]] - 12*a*b*(a^2 + 2*b
^2)*Sinh[x] + 6*b^2*(a^2 + b^2)*Sinh[x]^2 - 4*a*b^3*Sinh[x]^3)/(12*b^5)
```

Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 80, normalized size of antiderivative = 0.99, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.308$, Rules used = {3042, 3147, 476, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cosh^5(x)}{a+b\sinh(x)}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\cos(ix)^5}{a-ib\sin(ix)}dx$$

$$\Big\downarrow \text{3147}$$

$$\frac{\int \frac{\left(\sinh^2(x)b^2+b^2\right)^2}{a+b\sinh(x)}d(b\sinh(x))}{b^5}$$

$$\Big\downarrow \text{476}$$

$$\frac{\int \left(b^3\sinh^3(x)-ab^2\sinh^2(x)+b\left(a^2+2b^2\right)\sinh(x)-a\left(a^2+2b^2\right)+\frac{\left(a^2+b^2\right)^2}{a+b\sinh(x)}\right)d(b\sinh(x))}{b^5}$$

$$\Big\downarrow \text{2009}$$

$$\frac{\frac{1}{2}b^2\left(a^2+2b^2\right)\sinh^2(x)-ab\left(a^2+2b^2\right)\sinh(x)+\left(a^2+b^2\right)^2\log(a+b\sinh(x))-\frac{1}{3}ab^3\sinh^3(x)+\frac{1}{4}b^4\sinh^4(x)}{b^5}$$

input
```
Int[Cosh[x]^5/(a + b*Sinh[x]),x]
```

output
```
((a^2 + b^2)^2*Log[a + b*Sinh[x]] - a*b*(a^2 + 2*b^2)*Sinh[x] + (b^2*(a^2
+ 2*b^2)*Sinh[x]^2)/2 - (a*b^3*Sinh[x]^3)/3 + (b^4*Sinh[x]^4)/4)/b^5
```

Defintions of rubi rules used

rule 476
```
Int[((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[
ExpandIntegrand[(c + d*x)^n*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d, n},
 x] && IGtQ[p, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3147
```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Simp[1/(b^p*f)   Subst[Int[(a + x)^m*(b^2 - x^2)^((p - 1)
/2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && IntegerQ[(p
 - 1)/2] && NeQ[a^2 - b^2, 0]
```

Maple [A] (verified)

Time = 22.77 (sec) , antiderivative size = 82, normalized size of antiderivative = 1.01

method	result
derivativedivides	$-\frac{-\frac{\sinh(x)^4 b^3}{4}+\frac{a\,b^2\sinh(x)^3}{3}-\frac{(a^2+2b^2)\sinh(x)^2 b}{2}+a(a^2+2b^2)\sinh(x)}{b^4}+\frac{(a^4+2a^2b^2+b^4)\ln(a+b\sinh(x))}{b^5}$
default	$-\frac{-\frac{\sinh(x)^4 b^3}{4}+\frac{a\,b^2\sinh(x)^3}{3}-\frac{(a^2+2b^2)\sinh(x)^2 b}{2}+a(a^2+2b^2)\sinh(x)}{b^4}+\frac{(a^4+2a^2b^2+b^4)\ln(a+b\sinh(x))}{b^5}$
risch	$-\frac{a^4x}{b^5}-\frac{2xa^2}{b^3}-\frac{x}{b}+\frac{e^{4x}}{64b}-\frac{a\,e^{3x}}{24b^2}+\frac{e^{2x}a^2}{8b^3}+\frac{3\,e^{2x}}{16b}-\frac{a^3e^x}{2b^4}-\frac{7a\,e^x}{8b^2}+\frac{a^3e^{-x}}{2b^4}+\frac{7a\,e^{-x}}{8b^2}+\frac{e^{-2x}a^2}{8b^3}+$

input
```
int(cosh(x)^5/(a+b*sinh(x)),x,method=_RETURNVERBOSE)
```

output
```
-1/b^4*(-1/4*sinh(x)^4*b^3+1/3*a*b^2*sinh(x)^3-1/2*(a^2+2*b^2)*sinh(x)^2*b
+a*(a^2+2*b^2)*sinh(x))+(a^4+2*a^2*b^2+b^4)/b^5*ln(a+b*sinh(x))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 865 vs. $2(75) = 150$.

Time = 0.12 (sec) , antiderivative size = 865, normalized size of antiderivative = 10.68

$$\int \frac{\cosh^5(x)}{a + b\sinh(x)}\,dx = \text{Too large to display}$$

input
```
integrate(cosh(x)^5/(a+b*sinh(x)),x, algorithm="fricas")
```

output
```
1/192*(3*b^4*cosh(x)^8 + 3*b^4*sinh(x)^8 - 8*a*b^3*cosh(x)^7 + 8*(3*b^4*co
sh(x) - a*b^3)*sinh(x)^7 + 12*(2*a^2*b^2 + 3*b^4)*cosh(x)^6 + 4*(21*b^4*co
sh(x)^2 - 14*a*b^3*cosh(x) + 6*a^2*b^2 + 9*b^4)*sinh(x)^6 - 192*(a^4 + 2*a
^2*b^2 + b^4)*x*cosh(x)^4 - 24*(4*a^3*b + 7*a*b^3)*cosh(x)^5 + 24*(7*b^4*c
osh(x)^3 - 7*a*b^3*cosh(x)^2 - 4*a^3*b - 7*a*b^3 + 3*(2*a^2*b^2 + 3*b^4)*c
osh(x))*sinh(x)^5 + 8*a*b^3*cosh(x) + 2*(105*b^4*cosh(x)^4 - 140*a*b^3*cos
h(x)^3 + 90*(2*a^2*b^2 + 3*b^4)*cosh(x)^2 - 96*(a^4 + 2*a^2*b^2 + b^4)*x -
 60*(4*a^3*b + 7*a*b^3)*cosh(x))*sinh(x)^4 + 3*b^4 + 24*(4*a^3*b + 7*a*b^3
)*cosh(x)^3 + 8*(21*b^4*cosh(x)^5 - 35*a*b^3*cosh(x)^4 + 12*a^3*b + 21*a*b
^3 + 30*(2*a^2*b^2 + 3*b^4)*cosh(x)^3 - 96*(a^4 + 2*a^2*b^2 + b^4)*x*cosh(
x) - 30*(4*a^3*b + 7*a*b^3)*cosh(x)^2)*sinh(x)^3 + 12*(2*a^2*b^2 + 3*b^4)*
cosh(x)^2 + 12*(7*b^4*cosh(x)^6 - 14*a*b^3*cosh(x)^5 + 15*(2*a^2*b^2 + 3*b
^4)*cosh(x)^4 + 2*a^2*b^2 + 3*b^4 - 96*(a^4 + 2*a^2*b^2 + b^4)*x*cosh(x)^2
 - 20*(4*a^3*b + 7*a*b^3)*cosh(x)^3 + 6*(4*a^3*b + 7*a*b^3)*cosh(x))*sinh(
x)^2 + 192*((a^4 + 2*a^2*b^2 + b^4)*cosh(x)^4 + 4*(a^4 + 2*a^2*b^2 + b^4)*
cosh(x)^3*sinh(x) + 6*(a^4 + 2*a^2*b^2 + b^4)*cosh(x)^2*sinh(x)^2 + 4*(a^4
 + 2*a^2*b^2 + b^4)*cosh(x)*sinh(x)^3 + (a^4 + 2*a^2*b^2 + b^4)*sinh(x)^4)
*log(2*(b*sinh(x) + a)/(cosh(x) - sinh(x))) + 8*(3*b^4*cosh(x)^7 - 7*a*b^3
*cosh(x)^6 + 9*(2*a^2*b^2 + 3*b^4)*cosh(x)^5 - 96*(a^4 + 2*a^2*b^2 + b^4)*
x*cosh(x)^3 - 15*(4*a^3*b + 7*a*b^3)*cosh(x)^4 + a*b^3 + 9*(4*a^3*b + 7...
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\cosh^5(x)}{a+b\sinh(x)}\,dx = \text{Timed out}$$

input
```
integrate(cosh(x)**5/(a+b*sinh(x)),x)
```

output
```
Timed out
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 180 vs. $2(75) = 150$.

Time = 0.04 (sec) , antiderivative size = 180, normalized size of antiderivative = 2.22

$$\begin{aligned}\int \frac{\cosh^5(x)}{a+b\sinh(x)}\,dx &= -\frac{\left(8\,ab^2e^{(-x)}-3\,b^3-12\,(2\,a^2b+3\,b^3)e^{(-2\,x)}+24\,(4\,a^3+7\,ab^2)e^{(-3\,x)}\right)e^{(4\,x)}}{192\,b^4}\\&\quad+\frac{8\,ab^2e^{(-3\,x)}+3\,b^3e^{(-4\,x)}+24\,(4\,a^3+7\,ab^2)e^{(-x)}+12\,(2\,a^2b+3\,b^3)e^{(-2\,x)}}{192\,b^4}\\&\quad+\frac{(a^4+2\,a^2b^2+b^4)x}{b^5}+\frac{(a^4+2\,a^2b^2+b^4)\log\left(-2\,ae^{(-x)}+be^{(-2\,x)}-b\right)}{b^5}\end{aligned}$$

input
```
integrate(cosh(x)^5/(a+b*sinh(x)),x, algorithm="maxima")
```

output
```
-1/192*(8*a*b^2*e^(-x) - 3*b^3 - 12*(2*a^2*b + 3*b^3)*e^(-2*x) + 24*(4*a^3
 + 7*a*b^2)*e^(-3*x))*e^(4*x)/b^4 + 1/192*(8*a*b^2*e^(-3*x) + 3*b^3*e^(-4*
x) + 24*(4*a^3 + 7*a*b^2)*e^(-x) + 12*(2*a^2*b + 3*b^3)*e^(-2*x))/b^4 + (a
^4 + 2*a^2*b^2 + b^4)*x/b^5 + (a^4 + 2*a^2*b^2 + b^4)*log(-2*a*e^(-x) + b*
e^(-2*x) - b)/b^5
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 139, normalized size of antiderivative = 1.72

$$\int \frac{\cosh^5(x)}{a+b\sinh(x)}\,dx$$
$$= \frac{3\,b^3\left(e^{(-x)}-e^x\right)^4+8\,ab^2\left(e^{(-x)}-e^x\right)^3+24\,a^2b\left(e^{(-x)}-e^x\right)^2+48\,b^3\left(e^{(-x)}-e^x\right)^2+96\,a^3\left(e^{(-x)}-e^x\right)+1}{192\,b^4}$$
$$+\frac{(a^4+2\,a^2b^2+b^4)\log\left(\left|-b\left(e^{(-x)}-e^x\right)+2\,a\right|\right)}{b^5}$$

input
```
integrate(cosh(x)^5/(a+b*sinh(x)),x, algorithm="giac")
```

output
```
1/192*(3*b^3*(e^(-x) - e^x)^4 + 8*a*b^2*(e^(-x) - e^x)^3 + 24*a^2*b*(e^(-x
) - e^x)^2 + 48*b^3*(e^(-x) - e^x)^2 + 96*a^3*(e^(-x) - e^x) + 192*a*b^2*(
e^(-x) - e^x))/b^4 + (a^4 + 2*a^2*b^2 + b^4)*log(abs(-b*(e^(-x) - e^x) + 2
*a))/b^5
```

Mupad [B] (verification not implemented)

Time = 1.72 (sec) , antiderivative size = 169, normalized size of antiderivative = 2.09

$$\int \frac{\cosh^5(x)}{a+b\sinh(x)}\,dx = \frac{e^{-4\,x}}{64\,b}+\frac{e^{4\,x}}{64\,b}+\frac{\ln\left(2\,a\,e^x-b+b\,e^{2\,x}\right)\,\left(a^4+2\,a^2\,b^2+b^4\right)}{b^5}$$
$$+\frac{e^{-x}\,\left(4\,a^3+7\,a\,b^2\right)}{8\,b^4}+\frac{a\,e^{-3\,x}}{24\,b^2}-\frac{a\,e^{3\,x}}{24\,b^2}-\frac{x\,\left(a^2+b^2\right)^2}{b^5}$$
$$+\frac{e^{-2\,x}\,\left(2\,a^2+3\,b^2\right)}{16\,b^3}+\frac{e^{2\,x}\,\left(2\,a^2+3\,b^2\right)}{16\,b^3}-\frac{e^x\,\left(4\,a^3+7\,a\,b^2\right)}{8\,b^4}$$

input
```
int(cosh(x)^5/(a + b*sinh(x)),x)
```

output
```
exp(-4*x)/(64*b) + exp(4*x)/(64*b) + (log(2*a*exp(x) - b + b*exp(2*x))*(a^
4 + b^4 + 2*a^2*b^2))/b^5 + (exp(-x)*(7*a*b^2 + 4*a^3))/(8*b^4) + (a*exp(-
3*x))/(24*b^2) - (a*exp(3*x))/(24*b^2) - (x*(a^2 + b^2)^2)/b^5 + (exp(-2*x
)*(2*a^2 + 3*b^2))/(16*b^3) + (exp(2*x)*(2*a^2 + 3*b^2))/(16*b^3) - (exp(x
)*(7*a*b^2 + 4*a^3))/(8*b^4)
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 261, normalized size of antiderivative = 3.22

$$\int \frac{\cosh^5(x)}{a+b\sinh(x)}\,dx$$
$$= \frac{3e^{8x}b^4 - 8e^{7x}a\,b^3 + 24e^{6x}a^2b^2 + 36e^{6x}b^4 - 96e^{5x}a^3b - 168e^{5x}a\,b^3 + 192e^{4x}\log(e^{2x}b + 2e^x a - b)\,a^4 + 384e}{}$$

input
```
int(cosh(x)^5/(a+b*sinh(x)),x)
```

output
```
(3*e**(8*x)*b**4 - 8*e**(7*x)*a*b**3 + 24*e**(6*x)*a**2*b**2 + 36*e**(6*x)
*b**4 - 96*e**(5*x)*a**3*b - 168*e**(5*x)*a*b**3 + 192*e**(4*x)*log(e**(2*
x)*b + 2*e**x*a - b)*a**4 + 384*e**(4*x)*log(e**(2*x)*b + 2*e**x*a - b)*a*
*2*b**2 + 192*e**(4*x)*log(e**(2*x)*b + 2*e**x*a - b)*b**4 - 192*e**(4*x)*
a**4*x - 384*e**(4*x)*a**2*b**2*x - 192*e**(4*x)*b**4*x + 96*e**(3*x)*a**3
*b + 168*e**(3*x)*a*b**3 + 24*e**(2*x)*a**2*b**2 + 36*e**(2*x)*b**4 + 8*e*
*x*a*b**3 + 3*b**4)/(192*e**(4*x)*b**5)
```

3.190 $\int \frac{\cosh^4(x)}{a+b\sinh(x)}\,dx$

Optimal result

Integrand size = 13, antiderivative size = 97

$$\int \frac{\cosh^4(x)}{a+b\sinh(x)}\,dx = -\frac{a(2a^2+3b^2)\,x}{2b^4} - \frac{2(a^2+b^2)^{3/2}\operatorname{arctanh}\left(\frac{b-a\tanh\left(\frac{x}{2}\right)}{\sqrt{a^2+b^2}}\right)}{b^4} + \frac{\cosh^3(x)}{3b} + \frac{\cosh(x)\left(2(a^2+b^2)-ab\sinh(x)\right)}{2b^3}$$

output
```
-1/2*a*(2*a^2+3*b^2)*x/b^4-2*(a^2+b^2)^(3/2)*arctanh((b-a*tanh(1/2*x))/(a^
2+b^2)^(1/2))/b^4+1/3*cosh(x)^3/b+1/2*cosh(x)*(2*a^2+2*b^2-a*b*sinh(x))/b^
3
```

Mathematica [C] (warning: unable to verify)

Result contains complex when optimal does not.

Time = 1.66 (sec) , antiderivative size = 553, normalized size of antiderivative = 5.70

$$\int \frac{\cosh^4(x)}{a+b\sinh(x)}\,dx$$
$$= \frac{\cosh^3(x)\left(-12\sqrt{a-ib}\sqrt{a+ib}(a^2+b^2)\operatorname{arctanh}\left(\frac{\sqrt{-\frac{b(i+\sinh(x))}{a-ib}}}{\sqrt{-\frac{b(-i+\sinh(x))}{a+ib}}}\right)\sqrt{1+i\sinh(x)}+12(a-ib)^2(a+ib)a\right.}{}$$

input
```
Integrate[Cosh[x]^4/(a + b*Sinh[x]),x]
```

output
```
(Cosh[x]^3*(-12*Sqrt[a - I*b]*Sqrt[a + I*b]*(a^2 + b^2)*ArcTanh[Sqrt[-((b*
(I + Sinh[x]))/(a - I*b))]/Sqrt[-((b*(-I + Sinh[x]))/(a + I*b))]]*Sqrt[1 +
 I*Sinh[x]] + 12*(a - I*b)^2*(a + I*b)*ArcTanh[(Sqrt[a - I*b]*Sqrt[-((b*(I
 + Sinh[x]))/(a - I*b))])/(Sqrt[a + I*b]*Sqrt[-((b*(-I + Sinh[x]))/(a + I*
b))])]*Sqrt[1 + I*Sinh[x]] + Sqrt[a + I*b]*Sqrt[-((b*(-I + Sinh[x]))/(a +
I*b))]*((3 - 3*I)*Sqrt[2]*Sqrt[b]*(2*a^2 - I*a*b + 2*b^2)*ArcSin[((1/2 + I
/2)*Sqrt[a - I*b]*Sqrt[-((b*(I + Sinh[x]))/(a - I*b))])/Sqrt[b]] + 2*Sqrt[
a - I*b]*(3*a^2 + 4*b^2)*Sqrt[1 + I*Sinh[x]]*Sqrt[-((b*(I + Sinh[x]))/(a -
 I*b))] - 3*a*Sqrt[a - I*b]*b*Sqrt[1 + I*Sinh[x]]*Sinh[x]*Sqrt[-((b*(I + S
inh[x]))/(a - I*b))] + 2*Sqrt[a - I*b]*b^2*Sqrt[1 + I*Sinh[x]]*Sinh[x]^2*S
qrt[-((b*(I + Sinh[x]))/(a - I*b))])))/(6*(a - I*b)^(3/2)*(a + I*b)^(3/2)*
b*Sqrt[1 + I*Sinh[x]]*(-((b*(-I + Sinh[x]))/(a + I*b)))^(3/2)*(-((b*(I + S
inh[x]))/(a - I*b)))^(3/2))
```

Rubi [A] (verified)

Time = 0.65 (sec) , antiderivative size = 113, normalized size of antiderivative = 1.16, number of steps used = 13, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.923$, Rules used = {3042, 3174, 26, 3042, 3344, 25, 3042, 3214, 3042, 3139, 1083, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cosh^4(x)}{a+b\sinh(x)}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\cos(ix)^4}{a-ib\sin(ix)}dx$$

$$\Big\downarrow \text{3174}$$

$$\frac{\cosh^3(x)}{3b}+\frac{i\int -\frac{i\cosh^2(x)(b-a\sinh(x))}{a+b\sinh(x)}dx}{b}$$

$$\Big\downarrow \text{26}$$

$$\frac{\int \frac{\cosh^2(x)(b-a\sinh(x))}{a+b\sinh(x)}dx}{b}+\frac{\cosh^3(x)}{3b}$$

$\big\downarrow$ 3042

$$\frac{\cosh^3(x)}{3b}+\frac{\int \frac{\cos(ix)^2(b+ia\sin(ix))}{a-ib\sin(ix)}dx}{b}$$

$\big\downarrow$ 3344

$$\frac{\frac{\cosh(x)(2(a^2+b^2)-ab\sinh(x))}{2b^2}-\frac{\int -\frac{b(a^2+2b^2)-a(2a^2+3b^2)\sinh(x)}{a+b\sinh(x)}dx}{2b^2}}{b}+\frac{\cosh^3(x)}{3b}$$

$\big\downarrow$ 25

$$\frac{\frac{\int \frac{b(a^2+2b^2)-a(2a^2+3b^2)\sinh(x)}{a+b\sinh(x)}dx}{2b^2}+\frac{\cosh(x)(2(a^2+b^2)-ab\sinh(x))}{2b^2}}{b}+\frac{\cosh^3(x)}{3b}$$

$\big\downarrow$ 3042

$$\frac{\cosh^3(x)}{3b}+\frac{\frac{\cosh(x)(2(a^2+b^2)-ab\sinh(x))}{2b^2}+\frac{\int \frac{b(a^2+2b^2)+ia(2a^2+3b^2)\sin(ix)}{a-ib\sin(ix)}dx}{2b^2}}{b}$$

$\big\downarrow$ 3214

$$\frac{\frac{\frac{2(a^2+b^2)^2\int \frac{1}{a+b\sinh(x)}dx}{b}-\frac{ax(2a^2+3b^2)}{b}}{2b^2}+\frac{\cosh(x)(2(a^2+b^2)-ab\sinh(x))}{2b^2}}{b}+\frac{\cosh^3(x)}{3b}$$

$\big\downarrow$ 3042

$$\frac{\cosh^3(x)}{3b}+\frac{\frac{\cosh(x)(2(a^2+b^2)-ab\sinh(x))}{2b^2}+\frac{-\frac{ax(2a^2+3b^2)}{b}+\frac{2(a^2+b^2)^2\int \frac{1}{a-ib\sin(ix)}dx}{b}}{2b^2}}{b}$$

$\big\downarrow$ 3139

$$\frac{\frac{\frac{4(a^2+b^2)^2\int \frac{1}{-a\tanh^2\left(\frac{x}{2}\right)+2b\tanh\left(\frac{x}{2}\right)+a}d\tanh\left(\frac{x}{2}\right)}{b}-\frac{ax(2a^2+3b^2)}{b}}{2b^2}+\frac{\cosh(x)(2(a^2+b^2)-ab\sinh(x))}{2b^2}}{b}+\frac{\cosh^3(x)}{3b}$$

$\big\downarrow$ 1083

$$\frac{\frac{-\frac{8(a^2+b^2)^2\int \frac{1}{4(a^2+b^2)-\left(2b-2a\tanh\left(\frac{x}{2}\right)\right)^2}d\left(2b-2a\tanh\left(\frac{x}{2}\right)\right)}{b}-\frac{ax(2a^2+3b^2)}{b}}{2b^2}+\frac{\cosh(x)(2(a^2+b^2)-ab\sinh(x))}{2b^2}}{b}+\frac{\cosh^3(x)}{3b}$$

$$\Big\downarrow \text{219}$$

$$\frac{\frac{-\frac{4(a^2+b^2)^{3/2}\text{arctanh}\left(\frac{2b-2a\tanh\left(\frac{x}{2}\right)}{2\sqrt{a^2+b^2}}\right)}{b}-\frac{ax(2a^2+3b^2)}{b}}{2b^2}+\frac{\cosh(x)(2(a^2+b^2)-ab\sinh(x))}{2b^2}}{b}+\frac{\cosh^3(x)}{3b}$$

input
```
Int[Cosh[x]^4/(a + b*Sinh[x]),x]
```

output
```
Cosh[x]^3/(3*b) + ((-((a*(2*a^2 + 3*b^2)*x)/b) - (4*(a^2 + b^2)^(3/2)*ArcT
anh[(2*b - 2*a*Tanh[x/2])/(2*Sqrt[a^2 + b^2])])/b)/(2*b^2) + (Cosh[x]*(2*(
a^2 + b^2) - a*b*Sinh[x]))/(2*b^2))/b
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1083
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Simp[-2   Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3139
```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Simp[2*(e/d)   Subst[Int[1/(a + 2*b*e*x + a
*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[a^2 - b^2, 0]
```

rule 3174
```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[g*(g*Cos[e + f*x])^(p - 1)*((a + b*Sin[e + f*x
])^(m + 1)/(b*f*(m + p))), x] + Simp[g^2*((p - 1)/(b*(m + p)))   Int[(g*Cos
[e + f*x])^(p - 2)*(a + b*Sin[e + f*x])^m*(b + a*Sin[e + f*x]), x], x] /; F
reeQ[{a, b, e, f, g, m}, x] && NeQ[a^2 - b^2, 0] && GtQ[p, 1] && NeQ[m + p,
 0] && IntegersQ[2*m, 2*p]
```

rule 3214
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])/((c_.) + (d_.)*sin[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[b*(x/d), x] - Simp[(b*c - a*d)/d   Int[1/(c + d
*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

rule 3344
```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[g*(g*
Cos[e + f*x])^(p - 1)*(a + b*Sin[e + f*x])^(m + 1)*((b*c*(m + p + 1) - a*d*
p + b*d*(m + p)*Sin[e + f*x])/(b^2*f*(m + p)*(m + p + 1))), x] + Simp[g^2*(
(p - 1)/(b^2*(m + p)*(m + p + 1)))   Int[(g*Cos[e + f*x])^(p - 2)*(a + b*Si
n[e + f*x])^m*Simp[b*(a*d*m + b*c*(m + p + 1)) + (a*b*c*(m + p + 1) - d*(a^
2*p - b^2*(m + p)))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, g,
 m}, x] && NeQ[a^2 - b^2, 0] && GtQ[p, 1] && NeQ[m + p, 0] && NeQ[m + p + 1
, 0] && IntegerQ[2*m]
```

Maple [A] (verified)

Time = 8.22 (sec) , antiderivative size = 162, normalized size of antiderivative = 1.67

method	result
risch	$-\frac{a^3 x}{b^4}-\frac{3 a x}{2 b^2}+\frac{{\mathrm e}^{3 x}}{24 b}-\frac{a\,{\mathrm e}^{2 x}}{8 b^2}+\frac{{\mathrm e}^{x} a^2}{2 b^3}+\frac{5\,{\mathrm e}^{x}}{8 b}+\frac{{\mathrm e}^{-x} a^2}{2 b^3}+\frac{5\,{\mathrm e}^{-x}}{8 b}+\frac{a\,{\mathrm e}^{-2 x}}{8 b^2}+\frac{{\mathrm e}^{-3 x}}{24 b}+\frac{\left(a^2+b^2\right)^{\frac{3}{2}} \ln\left({\mathrm e}^{x}-\frac{-a+\sqrt{a^2+b^2}}{b}\right)}{b^4}$
default	$\frac{1}{3 b \left(\tanh\left(\frac{x}{2}\right)+1\right)^3}-\frac{-a+b}{2 b^2 \left(\tanh\left(\frac{x}{2}\right)+1\right)^2}-\frac{-2 a^2+a b-3 b^2}{2 b^3 \left(\tanh\left(\frac{x}{2}\right)+1\right)}-\frac{a \left(2 a^2+3 b^2\right) \ln\left(\tanh\left(\frac{x}{2}\right)+1\right)}{2 b^4}-\frac{2\left(-a^4-2 a^2 b^2-b^4\right) \operatorname{arctanh}\left(\right.}{b^4 \sqrt{a^2+b^2}}$

input
```
int(cosh(x)^4/(a+b*sinh(x)),x,method=_RETURNVERBOSE)
```

output
```
-a^3*x/b^4-3/2*a*x/b^2+1/24/b*exp(x)^3-1/8*a/b^2*exp(x)^2+1/2/b^3*exp(x)*a
^2+5/8/b*exp(x)+1/2/b^3/exp(x)*a^2+5/8/b/exp(x)+1/8*a/b^2/exp(x)^2+1/24/b/
exp(x)^3+(a^2+b^2)^(3/2)/b^4*ln(exp(x)-(-a+(a^2+b^2)^(1/2))/b)-(a^2+b^2)^(
3/2)/b^4*ln(exp(x)+(a+(a^2+b^2)^(1/2))/b)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 569 vs. $2(87) = 174$.

Time = 0.10 (sec) , antiderivative size = 569, normalized size of antiderivative = 5.87

$$\int \frac{\cosh^4(x)}{a + b\sinh(x)}\,dx = \text{Too large to display}$$

input
```
integrate(cosh(x)^4/(a+b*sinh(x)),x, algorithm="fricas")
```

output
```
1/24*(b^3*cosh(x)^6 + b^3*sinh(x)^6 - 3*a*b^2*cosh(x)^5 + 3*(2*b^3*cosh(x)
 - a*b^2)*sinh(x)^5 - 12*(2*a^3 + 3*a*b^2)*x*cosh(x)^3 + 3*(4*a^2*b + 5*b^
3)*cosh(x)^4 + 3*(5*b^3*cosh(x)^2 - 5*a*b^2*cosh(x) + 4*a^2*b + 5*b^3)*sin
h(x)^4 + 3*a*b^2*cosh(x) + 2*(10*b^3*cosh(x)^3 - 15*a*b^2*cosh(x)^2 - 6*(2
*a^3 + 3*a*b^2)*x + 6*(4*a^2*b + 5*b^3)*cosh(x))*sinh(x)^3 + b^3 + 3*(4*a^
2*b + 5*b^3)*cosh(x)^2 + 3*(5*b^3*cosh(x)^4 - 10*a*b^2*cosh(x)^3 + 4*a^2*b
 + 5*b^3 - 12*(2*a^3 + 3*a*b^2)*x*cosh(x) + 6*(4*a^2*b + 5*b^3)*cosh(x)^2)
*sinh(x)^2 + 24*((a^2 + b^2)*cosh(x)^3 + 3*(a^2 + b^2)*cosh(x)^2*sinh(x) +
 3*(a^2 + b^2)*cosh(x)*sinh(x)^2 + (a^2 + b^2)*sinh(x)^3)*sqrt(a^2 + b^2)*
log((b^2*cosh(x)^2 + b^2*sinh(x)^2 + 2*a*b*cosh(x) + 2*a^2 + b^2 + 2*(b^2*
cosh(x) + a*b)*sinh(x) - 2*sqrt(a^2 + b^2)*(b*cosh(x) + b*sinh(x) + a))/(b
*cosh(x)^2 + b*sinh(x)^2 + 2*a*cosh(x) + 2*(b*cosh(x) + a)*sinh(x) - b)) +
 3*(2*b^3*cosh(x)^5 - 5*a*b^2*cosh(x)^4 - 12*(2*a^3 + 3*a*b^2)*x*cosh(x)^2
 + 4*(4*a^2*b + 5*b^3)*cosh(x)^3 + a*b^2 + 2*(4*a^2*b + 5*b^3)*cosh(x))*si
nh(x))/(b^4*cosh(x)^3 + 3*b^4*cosh(x)^2*sinh(x) + 3*b^4*cosh(x)*sinh(x)^2
+ b^4*sinh(x)^3)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\cosh^4(x)}{a+b\sinh(x)}\,dx = \text{Timed out}$$

input
```
integrate(cosh(x)**4/(a+b*sinh(x)),x)
```

output
```
Timed out
```

Maxima [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 170, normalized size of antiderivative = 1.75

$$\int \frac{\cosh^4(x)}{a+b\sinh(x)}\,dx = -\frac{\left(3\,abe^{(-x)}-b^2-3\,(4\,a^2+5\,b^2)e^{(-2\,x)}\right)e^{(3\,x)}}{24\,b^3} + \frac{3\,abe^{(-2\,x)}+b^2e^{(-3\,x)}+3\,(4\,a^2+5\,b^2)e^{(-x)}}{24\,b^3} - \frac{(2\,a^3+3\,ab^2)x}{2\,b^4} + \frac{(a^4+2\,a^2b^2+b^4)\log\left(\frac{be^{(-x)}-a-\sqrt{a^2+b^2}}{be^{(-x)}-a+\sqrt{a^2+b^2}}\right)}{\sqrt{a^2+b^2}b^4}$$

input
```
integrate(cosh(x)^4/(a+b*sinh(x)),x, algorithm="maxima")
```

output
```
-1/24*(3*a*b*e^(-x) - b^2 - 3*(4*a^2 + 5*b^2)*e^(-2*x))*e^(3*x)/b^3 + 1/24
*(3*a*b*e^(-2*x) + b^2*e^(-3*x) + 3*(4*a^2 + 5*b^2)*e^(-x))/b^3 - 1/2*(2*a
^3 + 3*a*b^2)*x/b^4 + (a^4 + 2*a^2*b^2 + b^4)*log((b*e^(-x) - a - sqrt(a^2
 + b^2))/(b*e^(-x) - a + sqrt(a^2 + b^2)))/(sqrt(a^2 + b^2)*b^4)
```

Giac [A] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 168, normalized size of antiderivative = 1.73

$$\int \frac{\cosh^4(x)}{a+b\sinh(x)}\,dx = \frac{b^2e^{(3x)} - 3\,abe^{(2x)} + 12\,a^2e^x + 15\,b^2e^x}{24\,b^3} - \frac{(2\,a^3+3\,ab^2)x}{2\,b^4} + \frac{\left(3\,ab^2e^x + b^3 + 3\,(4\,a^2b+5\,b^3)e^{(2x)}\right)e^{(-3x)}}{24\,b^4} + \frac{(a^4+2\,a^2b^2+b^4)\log\left(\frac{\left|2\,be^x+2\,a-2\,\sqrt{a^2+b^2}\right|}{\left|2\,be^x+2\,a+2\,\sqrt{a^2+b^2}\right|}\right)}{\sqrt{a^2+b^2}b^4}$$

input

```
integrate(cosh(x)^4/(a+b*sinh(x)),x, algorithm="giac")
```

output

```
1/24*(b^2*e^(3*x) - 3*a*b*e^(2*x) + 12*a^2*e^x + 15*b^2*e^x)/b^3 - 1/2*(2*
a^3 + 3*a*b^2)*x/b^4 + 1/24*(3*a*b^2*e^x + b^3 + 3*(4*a^2*b + 5*b^3)*e^(2*
x))*e^(-3*x)/b^4 + (a^4 + 2*a^2*b^2 + b^4)*log(abs(2*b*e^x + 2*a - 2*sqrt(
a^2 + b^2))/abs(2*b*e^x + 2*a + 2*sqrt(a^2 + b^2)))/(sqrt(a^2 + b^2)*b^4)
```

Mupad [B] (verification not implemented)

Time = 1.71 (sec) , antiderivative size = 200, normalized size of antiderivative = 2.06

$$\int \frac{\cosh^4(x)}{a+b\sinh(x)}\,dx = \frac{e^{-3x}}{24\,b} + \frac{e^{3x}}{24\,b} - \frac{\ln\left(-\frac{2\,e^x\,(a^2+b^2)^2}{b^5} - \frac{2\,(b-a\,e^x)\,(a^2+b^2)^{3/2}}{b^5}\right)\,(a^2+b^2)^{3/2}}{b^4} + \frac{\ln\left(\frac{2\,(b-a\,e^x)\,(a^2+b^2)^{3/2}}{b^5} - \frac{2\,e^x\,(a^2+b^2)^2}{b^5}\right)\,(a^2+b^2)^{3/2}}{b^4} - \frac{x\,(2\,a^3+3\,a\,b^2)}{2\,b^4} + \frac{e^x\,(4\,a^2+5\,b^2)}{8\,b^3} + \frac{a\,e^{-2x}}{8\,b^2} - \frac{a\,e^{2x}}{8\,b^2} + \frac{e^{-x}\,(4\,a^2+5\,b^2)}{8\,b^3}$$

input

```
int(cosh(x)^4/(a + b*sinh(x)),x)
```

output
```
exp(-3*x)/(24*b) + exp(3*x)/(24*b) - (log(- (2*exp(x)*(a^2 + b^2)^2)/b^5 -
 (2*(b - a*exp(x))*(a^2 + b^2)^(3/2))/b^5)*(a^2 + b^2)^(3/2))/b^4 + (log((
2*(b - a*exp(x))*(a^2 + b^2)^(3/2))/b^5 - (2*exp(x)*(a^2 + b^2)^2)/b^5)*(a
^2 + b^2)^(3/2))/b^4 - (x*(3*a*b^2 + 2*a^3))/(2*b^4) + (exp(x)*(4*a^2 + 5*
b^2))/(8*b^3) + (a*exp(-2*x))/(8*b^2) - (a*exp(2*x))/(8*b^2) + (exp(-x)*(4
*a^2 + 5*b^2))/(8*b^3)
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 192, normalized size of antiderivative = 1.98

$$\int \frac{\cosh^4(x)}{a+b\sinh(x)}\,dx$$
$$= \frac{48e^{3x}\sqrt{a^2+b^2}\,atan\left(\frac{e^x bi+ai}{\sqrt{a^2+b^2}}\right)a^2 i+48e^{3x}\sqrt{a^2+b^2}\,atan\left(\frac{e^x bi+ai}{\sqrt{a^2+b^2}}\right)b^2 i+e^{6x}b^3-3e^{5x}a\,b^2+12e^{4x}a^2b+15e}{24e^{3x}b^4}$$

input
```
int(cosh(x)^4/(a+b*sinh(x)),x)
```

output
```
(48*e**(3*x)*sqrt(a**2 + b**2)*atan((e**x*b*i + a*i)/sqrt(a**2 + b**2))*a*
*2*i + 48*e**(3*x)*sqrt(a**2 + b**2)*atan((e**x*b*i + a*i)/sqrt(a**2 + b**
2))*b**2*i + e**(6*x)*b**3 - 3*e**(5*x)*a*b**2 + 12*e**(4*x)*a**2*b + 15*e
**(4*x)*b**3 - 24*e**(3*x)*a**3*x - 36*e**(3*x)*a*b**2*x + 12*e**(2*x)*a**
2*b + 15*e**(2*x)*b**3 + 3*e**x*a*b**2 + b**3)/(24*e**(3*x)*b**4)
```

3.191 $\int \frac{\cosh^3(x)}{a+b\sinh(x)}\,dx$

Optimal result

Integrand size = 13, antiderivative size = 38

$$\int \frac{\cosh^3(x)}{a+b\sinh(x)}\,dx = \frac{(a^2+b^2)\log(a+b\sinh(x))}{b^3} - \frac{a\sinh(x)}{b^2} + \frac{\sinh^2(x)}{2b}$$

output
```
(a^2+b^2)*ln(a+b*sinh(x))/b^3-a*sinh(x)/b^2+1/2*sinh(x)^2/b
```

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 38, normalized size of antiderivative = 1.00

$$\int \frac{\cosh^3(x)}{a+b\sinh(x)}\,dx = -\frac{-((a^2+b^2)\log(a+b\sinh(x))) + ab\sinh(x) - \frac{1}{2}b^2\sinh^2(x)}{b^3}$$

input
```
Integrate[Cosh[x]^3/(a + b*Sinh[x]),x]
```

output
```
-((-((a^2 + b^2)*Log[a + b*Sinh[x]]) + a*b*Sinh[x] - (b^2*Sinh[x]^2)/2)/b^
3)
```

Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 38, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.385, Rules used = {3042, 3147, 25, 476, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cosh^3(x)}{a+b\sinh(x)}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\cos(ix)^3}{a-ib\sin(ix)}dx$$

$$\Big\downarrow \text{3147}$$

$$-\frac{\int -\frac{\sinh^2(x)b^2+b^2}{a+b\sinh(x)}d(b\sinh(x))}{b^3}$$

$$\Big\downarrow \text{25}$$

$$\frac{\int \frac{\sinh^2(x)b^2+b^2}{a+b\sinh(x)}d(b\sinh(x))}{b^3}$$

$$\Big\downarrow \text{476}$$

$$\frac{\int \left(-a+b\sinh(x)+\frac{a^2+b^2}{a+b\sinh(x)}\right)d(b\sinh(x))}{b^3}$$

$$\Big\downarrow \text{2009}$$

$$-\frac{-(a^2+b^2)\log(a+b\sinh(x))+ab\sinh(x)-\frac{1}{2}b^2\sinh^2(x)}{b^3}$$

input
```
Int[Cosh[x]^3/(a + b*Sinh[x]),x]
```

output
```
-((-((a^2 + b^2)*Log[a + b*Sinh[x]]) + a*b*Sinh[x] - (b^2*Sinh[x]^2)/2)/b^
3)
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 476
```
Int[((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[
ExpandIntegrand[(c + d*x)^n*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d, n},
 x] && IGtQ[p, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3147
```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Simp[1/(b^p*f)   Subst[Int[(a + x)^m*(b^2 - x^2)^((p - 1)
/2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && IntegerQ[(p
 - 1)/2] && NeQ[a^2 - b^2, 0]
```

Maple [A] (verified)

Time = 2.89 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.97

method	result	size
derivativedivides	$-\frac{-\frac{b\sinh(x)^2}{2}+a\sinh(x)}{b^2}+\frac{(a^2+b^2)\ln(a+b\sinh(x))}{b^3}$	37
default	$-\frac{-\frac{b\sinh(x)^2}{2}+a\sinh(x)}{b^2}+\frac{(a^2+b^2)\ln(a+b\sinh(x))}{b^3}$	37
risch	$-\frac{x\,a^2}{b^3}-\frac{x}{b}+\frac{e^{2x}}{8b}-\frac{a\,e^{x}}{2b^2}+\frac{a\,e^{-x}}{2b^2}+\frac{e^{-2x}}{8b}+\frac{\ln\left(e^{2x}+\frac{2a\,e^{x}}{b}-1\right)a^2}{b^3}+\frac{\ln\left(e^{2x}+\frac{2a\,e^{x}}{b}-1\right)}{b}$	94

input
```
int(cosh(x)^3/(a+b*sinh(x)),x,method=_RETURNVERBOSE)
```

output
```
-1/b^2*(-1/2*b*sinh(x)^2+a*sinh(x))+(a^2+b^2)*ln(a+b*sinh(x))/b^3
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 221 vs. $2(36) = 72$.

Time = 0.09 (sec) , antiderivative size = 221, normalized size of antiderivative = 5.82

$$\int \frac{\cosh^3(x)}{a + b\sinh(x)}\,dx = \frac{b^2\cosh{(x)}^4 + b^2\sinh{(x)}^4 - 4\,ab\cosh{(x)}^3 - 8\,(a^2 + b^2)x\cosh{(x)}^2 + 4\,(b^2\cosh{(x)} - ab)\sinh{(x)}^3 + 4\,ab}{}$$

input
```
integrate(cosh(x)^3/(a+b*sinh(x)),x, algorithm="fricas")
```

output
```
1/8*(b^2*cosh(x)^4 + b^2*sinh(x)^4 - 4*a*b*cosh(x)^3 - 8*(a^2 + b^2)*x*cos
h(x)^2 + 4*(b^2*cosh(x) - a*b)*sinh(x)^3 + 4*a*b*cosh(x) + 2*(3*b^2*cosh(x
)^2 - 6*a*b*cosh(x) - 4*(a^2 + b^2)*x)*sinh(x)^2 + b^2 + 8*((a^2 + b^2)*co
sh(x)^2 + 2*(a^2 + b^2)*cosh(x)*sinh(x) + (a^2 + b^2)*sinh(x)^2)*log(2*(b*
sinh(x) + a)/(cosh(x) - sinh(x))) + 4*(b^2*cosh(x)^3 - 3*a*b*cosh(x)^2 - 4
*(a^2 + b^2)*x*cosh(x) + a*b)*sinh(x))/(b^3*cosh(x)^2 + 2*b^3*cosh(x)*sinh
(x) + b^3*sinh(x)^2)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\cosh^3(x)}{a + b\sinh(x)}\,dx = \text{Timed out}$$

input
```
integrate(cosh(x)**3/(a+b*sinh(x)),x)
```

output
```
Timed out
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 81 vs. $2(36) = 72$.

Time = 0.04 (sec) , antiderivative size = 81, normalized size of antiderivative = 2.13

$$\int \frac{\cosh^3(x)}{a + b\sinh(x)}\,dx = -\frac{\left(4\,ae^{(-x)} - b\right)e^{(2\,x)}}{8\,b^2} + \frac{4\,ae^{(-x)} + be^{(-2\,x)}}{8\,b^2} + \frac{(a^2 + b^2)x}{b^3} + \frac{(a^2 + b^2)\log\left(-2\,ae^{(-x)} + be^{(-2\,x)} - b\right)}{b^3}$$

input
```
integrate(cosh(x)^3/(a+b*sinh(x)),x, algorithm="maxima")
```

output
```
-1/8*(4*a*e^(-x) - b)*e^(2*x)/b^2 + 1/8*(4*a*e^(-x) + b*e^(-2*x))/b^2 + (a
^2 + b^2)*x/b^3 + (a^2 + b^2)*log(-2*a*e^(-x) + b*e^(-2*x) - b)/b^3
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 61, normalized size of antiderivative = 1.61

$$\int \frac{\cosh^3(x)}{a + b\sinh(x)}\,dx = \frac{b\left(e^{(-x)} - e^x\right)^2 + 4\,a\left(e^{(-x)} - e^x\right)}{8\,b^2} + \frac{(a^2 + b^2)\log\left(\left|-b\left(e^{(-x)} - e^x\right) + 2\,a\right|\right)}{b^3}$$

input
```
integrate(cosh(x)^3/(a+b*sinh(x)),x, algorithm="giac")
```

output
```
1/8*(b*(e^(-x) - e^x)^2 + 4*a*(e^(-x) - e^x))/b^2 + (a^2 + b^2)*log(abs(-b
*(e^(-x) - e^x) + 2*a))/b^3
```

Mupad [B] (verification not implemented)

Time = 1.48 (sec) , antiderivative size = 77, normalized size of antiderivative = 2.03

$$\int \frac{\cosh^3(x)}{a + b\sinh(x)}\,dx = \frac{e^{-2x}}{8b} + \frac{e^{2x}}{8b} + \frac{\ln\left(2\,a\,e^{x} - b + b\,e^{2x}\right)\,\left(a^2 + b^2\right)}{b^3} - \frac{a\,e^{x}}{2\,b^2} - \frac{x\,\left(a^2 + b^2\right)}{b^3} + \frac{a\,e^{-x}}{2\,b^2}$$

input
```
int(cosh(x)^3/(a + b*sinh(x)),x)
```

output
```
exp(-2*x)/(8*b) + exp(2*x)/(8*b) + (log(2*a*exp(x) - b + b*exp(2*x))*(a^2
+ b^2))/b^3 - (a*exp(x))/(2*b^2) - (x*(a^2 + b^2))/b^3 + (a*exp(-x))/(2*b^
2)
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 119, normalized size of antiderivative = 3.13

$$\int \frac{\cosh^3(x)}{a + b\sinh(x)}\,dx = \frac{e^{4x}b^2 - 4e^{3x}ab + 8e^{2x}\log(e^{2x}b + 2e^{x}a - b)\,a^2 + 8e^{2x}\log(e^{2x}b + 2e^{x}a - b)\,b^2 - 8e^{2x}a^2x - 8e^{2x}b^2x + 4e^{x}ab}{8e^{2x}b^3}$$

input
```
int(cosh(x)^3/(a+b*sinh(x)),x)
```

output
```
(e**(4*x)*b**2 - 4*e**(3*x)*a*b + 8*e**(2*x)*log(e**(2*x)*b + 2*e**x*a - b
)*a**2 + 8*e**(2*x)*log(e**(2*x)*b + 2*e**x*a - b)*b**2 - 8*e**(2*x)*a**2*
x - 8*e**(2*x)*b**2*x + 4*e**x*a*b + b**2)/(8*e**(2*x)*b**3)
```

3.192 $\int \frac{\cosh^2(x)}{a+b\sinh(x)}\,dx$

Optimal result

Integrand size = 13, antiderivative size = 54

$$\int \frac{\cosh^2(x)}{a+b\sinh(x)}\,dx = -\frac{ax}{b^2} - \frac{2\sqrt{a^2+b^2}\operatorname{arctanh}\left(\frac{b-a\tanh\left(\frac{x}{2}\right)}{\sqrt{a^2+b^2}}\right)}{b^2} + \frac{\cosh(x)}{b}$$

output
```
-a*x/b^2-2*(a^2+b^2)^(1/2)*arctanh((b-a*tanh(1/2*x))/(a^2+b^2)^(1/2))/b^2+
cosh(x)/b
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.67 (sec) , antiderivative size = 396, normalized size of antiderivative = 7.33

$$\int \frac{\cosh^2(x)}{a+b\sinh(x)}\,dx$$
$$= \frac{\cosh(x)\left(-2\sqrt{a-ib}\sqrt{a+ib}\operatorname{arctanh}\left(\frac{\sqrt{-\frac{b(i+\sinh(x))}{a-ib}}}{\sqrt{-\frac{b(-i+\sinh(x))}{a+ib}}}\right)\sqrt{1+i\sinh(x)} + 2(a-ib)\operatorname{arctanh}\left(\frac{\sqrt{a-ib}\sqrt{-\frac{b(i+\sinh}{a-}}}{\sqrt{a+ib}\sqrt{-\frac{b(-i+\sinh}{a-}}}\right.\right.}{\sqrt{a-ib}\sqrt{a+}}$$

input
```
Integrate[Cosh[x]^2/(a + b*Sinh[x]),x]
```

output
```
(Cosh[x]*(-2*Sqrt[a - I*b]*Sqrt[a + I*b]*ArcTanh[Sqrt[-((b*(I + Sinh[x]))/
(a - I*b))]/Sqrt[-((b*(-I + Sinh[x]))/(a + I*b))]]*Sqrt[1 + I*Sinh[x]] + 2
*(a - I*b)*ArcTanh[(Sqrt[a - I*b]*Sqrt[-((b*(I + Sinh[x]))/(a - I*b))])/(S
qrt[a + I*b]*Sqrt[-((b*(-I + Sinh[x]))/(a + I*b))])]*Sqrt[1 + I*Sinh[x]] +
 Sqrt[a + I*b]*Sqrt[-((b*(-I + Sinh[x]))/(a + I*b))]*(-2*(-1)^(3/4)*Sqrt[b
]*ArcSin[((1/2 + I/2)*Sqrt[a - I*b]*Sqrt[-((b*(I + Sinh[x]))/(a - I*b))])/
Sqrt[b]] + Sqrt[a - I*b]*Sqrt[1 + I*Sinh[x]]*Sqrt[-((b*(I + Sinh[x]))/(a -
 I*b))])))/(Sqrt[a - I*b]*Sqrt[a + I*b]*b*Sqrt[1 + I*Sinh[x]]*Sqrt[-((b*(-
I + Sinh[x]))/(a + I*b))]*Sqrt[-((b*(I + Sinh[x]))/(a - I*b))])
```

Rubi [A] (verified)

Time = 0.42 (sec) , antiderivative size = 64, normalized size of antiderivative = 1.19, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.692$, Rules used = {3042, 3174, 26, 3042, 3214, 3042, 3139, 1083, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cosh^2(x)}{a+b\sinh(x)}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\cos(ix)^2}{a-ib\sin(ix)}dx$$

$$\Big\downarrow \text{3174}$$

$$\frac{\cosh(x)}{b}+\frac{i\int -\frac{i(b-a\sinh(x))}{a+b\sinh(x)}dx}{b}$$

$$\Big\downarrow \text{26}$$

$$\frac{\int \frac{b-a\sinh(x)}{a+b\sinh(x)}dx}{b}+\frac{\cosh(x)}{b}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\cosh(x)}{b}+\frac{\int \frac{b+ia\sin(ix)}{a-ib\sin(ix)}dx}{b}$$

$$\Big\downarrow \text{3214}$$

$$\frac{\frac{(a^2+b^2)\int \frac{1}{a+b\sinh(x)}dx}{b}-\frac{ax}{b}}{b}+\frac{\cosh(x)}{b}$$

$\Big\downarrow$ 3042

$$\frac{\cosh(x)}{b}+\frac{-\frac{ax}{b}+\frac{(a^2+b^2)\int \frac{1}{a-ib\sin(ix)}dx}{b}}{b}$$

$\Big\downarrow$ 3139

$$\frac{\frac{2(a^2+b^2)\int \frac{1}{-a\tanh^2\left(\frac{x}{2}\right)+2b\tanh\left(\frac{x}{2}\right)+a}d\tanh\left(\frac{x}{2}\right)}{b}-\frac{ax}{b}}{b}+\frac{\cosh(x)}{b}$$

$\Big\downarrow$ 1083

$$\frac{-\frac{4(a^2+b^2)\int \frac{1}{4(a^2+b^2)-\left(2b-2a\tanh\left(\frac{x}{2}\right)\right)^2}d\left(2b-2a\tanh\left(\frac{x}{2}\right)\right)}{b}-\frac{ax}{b}}{b}+\frac{\cosh(x)}{b}$$

$\Big\downarrow$ 219

$$\frac{-\frac{2\sqrt{a^2+b^2}\text{arctanh}\left(\frac{2b-2a\tanh\left(\frac{x}{2}\right)}{2\sqrt{a^2+b^2}}\right)}{b}-\frac{ax}{b}}{b}+\frac{\cosh(x)}{b}$$

input
```
Int[Cosh[x]^2/(a + b*Sinh[x]),x]
```

output
```
(-((a*x)/b) - (2*Sqrt[a^2 + b^2]*ArcTanh[(2*b - 2*a*Tanh[x/2])/(2*Sqrt[a^2
 + b^2])])/b)/b + Cosh[x]/b
```

Defintions of rubi rules used

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1083
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Simp[-2   Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3139
```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Simp[2*(e/d)   Subst[Int[1/(a + 2*b*e*x + a
*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[a^2 - b^2, 0]
```

rule 3174
```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[g*(g*Cos[e + f*x])^(p - 1)*((a + b*Sin[e + f*x
])^(m + 1)/(b*f*(m + p))), x] + Simp[g^2*((p - 1)/(b*(m + p)))   Int[(g*Cos
[e + f*x])^(p - 2)*(a + b*Sin[e + f*x])^m*(b + a*Sin[e + f*x]), x], x] /; F
reeQ[{a, b, e, f, g, m}, x] && NeQ[a^2 - b^2, 0] && GtQ[p, 1] && NeQ[m + p,
 0] && IntegersQ[2*m, 2*p]
```

rule 3214
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])/((c_.) + (d_.)*sin[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[b*(x/d), x] - Simp[(b*c - a*d)/d   Int[1/(c + d
*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Maple [A] (verified)

Time = 0.93 (sec) , antiderivative size = 93, normalized size of antiderivative = 1.72

method	result	size
risch	$-\frac{ax}{b^{2}}+\frac{{\mathrm e}^{x}}{2b}+\frac{{\mathrm e}^{-x}}{2b}+\frac{\sqrt{a^{2}+b^{2}}\,\ln\left({\mathrm e}^{x}-\frac{-a+\sqrt{a^{2}+b^{2}}}{b}\right)}{b^{2}}-\frac{\sqrt{a^{2}+b^{2}}\,\ln\left({\mathrm e}^{x}+\frac{a+\sqrt{a^{2}+b^{2}}}{b}\right)}{b^{2}}$	93
default	$\frac{1}{b\left(\tanh\left(\frac{x}{2}\right)+1\right)}-\frac{a\ln\left(\tanh\left(\frac{x}{2}\right)+1\right)}{b^{2}}-\frac{2\left(-a^{2}-b^{2}\right)\operatorname{arctanh}\left(\frac{2a\tanh\left(\frac{x}{2}\right)-2b}{2\sqrt{a^{2}+b^{2}}}\right)}{b^{2}\sqrt{a^{2}+b^{2}}}-\frac{1}{b\left(\tanh\left(\frac{x}{2}\right)-1\right)}+\frac{a\ln\left(\tanh\left(\frac{x}{2}\right)-1\right)}{b^{2}}$	100

input
```
int(cosh(x)^2/(a+b*sinh(x)),x,method=_RETURNVERBOSE)
```

output
```
-a*x/b^2+1/2/b*exp(x)+1/2/b/exp(x)+(a^2+b^2)^(1/2)/b^2*ln(exp(x)-(-a+(a^2+
b^2)^(1/2))/b)-(a^2+b^2)^(1/2)/b^2*ln(exp(x)+(a+(a^2+b^2)^(1/2))/b)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 171 vs. $2(50) = 100$.

Time = 0.09 (sec) , antiderivative size = 171, normalized size of antiderivative = 3.17

$$\int \frac{\cosh^2(x)}{a+b\sinh(x)}\,dx = -\frac{2\,ax\cosh(x) - b\cosh(x)^2 - b\sinh(x)^2 - 2\sqrt{a^2+b^2}(\cosh(x)+\sinh(x))\log\left(\frac{b^2\cosh(x)^2+b^2\sinh(x)^2+2\,ab\,\cdots}{b\cosh\cdots}\right.}{2\left(b^2\cosh(x)+b^2\cdots\right.}$$

input
```
integrate(cosh(x)^2/(a+b*sinh(x)),x, algorithm="fricas")
```

output
```
-1/2*(2*a*x*cosh(x) - b*cosh(x)^2 - b*sinh(x)^2 - 2*sqrt(a^2 + b^2)*(cosh(
x) + sinh(x))*log((b^2*cosh(x)^2 + b^2*sinh(x)^2 + 2*a*b*cosh(x) + 2*a^2 +
 b^2 + 2*(b^2*cosh(x) + a*b)*sinh(x) - 2*sqrt(a^2 + b^2)*(b*cosh(x) + b*si
nh(x) + a))/(b*cosh(x)^2 + b*sinh(x)^2 + 2*a*cosh(x) + 2*(b*cosh(x) + a)*s
inh(x) - b)) + 2*(a*x - b*cosh(x))*sinh(x) - b)/(b^2*cosh(x) + b^2*sinh(x)
)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 377 vs. $2(46) = 92$.

Time = 79.56 (sec) , antiderivative size = 377, normalized size of antiderivative = 6.98

$$\int \frac{\cosh^2(x)}{a+b\sinh(x)}\,dx = \begin{cases} \tilde{\infty}\left(\frac{\log\left(\tanh\left(\frac{x}{2}\right)\right)\tanh^2\left(\frac{x}{2}\right)}{\tanh^2\left(\frac{x}{2}\right)-1} - \frac{\log\left(\tanh\left(\frac{x}{2}\right)\right)}{\tanh^2\left(\frac{x}{2}\right)-1} - \frac{2}{\tanh^2\left(\frac{x}{2}\right)-1}\right) \\ \frac{\frac{\log\left(\tanh\left(\frac{x}{2}\right)\right)\tanh^2\left(\frac{x}{2}\right)}{\tanh^2\left(\frac{x}{2}\right)-1} - \frac{\log\left(\tanh\left(\frac{x}{2}\right)\right)}{\tanh^2\left(\frac{x}{2}\right)-1} - \frac{2}{\tanh^2\left(\frac{x}{2}\right)-1}}{b} \\ \frac{-\frac{x\sinh^2(x)}{2} + \frac{x\cosh^2(x)}{2} + \frac{\sinh(x)\cosh(x)}{2}}{a} \\ -\frac{ax\tanh^2\left(\frac{x}{2}\right)}{b^2\tanh^2\left(\frac{x}{2}\right)-b^2} + \frac{ax}{b^2\tanh^2\left(\frac{x}{2}\right)-b^2} - \frac{2b}{b^2\tanh^2\left(\frac{x}{2}\right)-b^2} - \frac{\sqrt{a^2+b^2}\log\left(\tanh\left(\frac{x}{2}\right)-\frac{b}{a}-\frac{\sqrt{a^2+b^2}}{a}\right)\tanh^2\left(\frac{x}{2}\right)}{b^2\tanh^2\left(\frac{x}{2}\right)-b^2} + \frac{\sqrt{a^2+b^2}\log\left(\tanh\right.}{b^2\tanh^2} \end{cases}$$

input
```
integrate(cosh(x)**2/(a+b*sinh(x)),x)
```

output
```
Piecewise((zoo*(log(tanh(x/2))*tanh(x/2)**2/(tanh(x/2)**2 - 1) - log(tanh(
x/2))/(tanh(x/2)**2 - 1) - 2/(tanh(x/2)**2 - 1)), Eq(a, 0) & Eq(b, 0)), ((
log(tanh(x/2))*tanh(x/2)**2/(tanh(x/2)**2 - 1) - log(tanh(x/2))/(tanh(x/2)
**2 - 1) - 2/(tanh(x/2)**2 - 1))/b, Eq(a, 0)), ((-x*sinh(x)**2/2 + x*cosh(
x)**2/2 + sinh(x)*cosh(x)/2)/a, Eq(b, 0)), (-a*x*tanh(x/2)**2/(b**2*tanh(x
/2)**2 - b**2) + a*x/(b**2*tanh(x/2)**2 - b**2) - 2*b/(b**2*tanh(x/2)**2 -
 b**2) - sqrt(a**2 + b**2)*log(tanh(x/2) - b/a - sqrt(a**2 + b**2)/a)*tanh
(x/2)**2/(b**2*tanh(x/2)**2 - b**2) + sqrt(a**2 + b**2)*log(tanh(x/2) - b/
a - sqrt(a**2 + b**2)/a)/(b**2*tanh(x/2)**2 - b**2) + sqrt(a**2 + b**2)*lo
g(tanh(x/2) - b/a + sqrt(a**2 + b**2)/a)*tanh(x/2)**2/(b**2*tanh(x/2)**2 -
 b**2) - sqrt(a**2 + b**2)*log(tanh(x/2) - b/a + sqrt(a**2 + b**2)/a)/(b**
2*tanh(x/2)**2 - b**2), True))
```

Maxima [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 81, normalized size of antiderivative = 1.50

$$\int \frac{\cosh^2(x)}{a+b\sinh(x)}\,dx = -\frac{ax}{b^2} + \frac{e^{(-x)}}{2\,b} + \frac{e^x}{2\,b} + \frac{\sqrt{a^2+b^2}\log\left(\frac{be^{(-x)}-a-\sqrt{a^2+b^2}}{be^{(-x)}-a+\sqrt{a^2+b^2}}\right)}{b^2}$$

input
```
integrate(cosh(x)^2/(a+b*sinh(x)),x, algorithm="maxima")
```

output
```
-a*x/b^2 + 1/2*e^(-x)/b + 1/2*e^x/b + sqrt(a^2 + b^2)*log((b*e^(-x) - a -
sqrt(a^2 + b^2))/(b*e^(-x) - a + sqrt(a^2 + b^2)))/b^2
```

Giac [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 83, normalized size of antiderivative = 1.54

$$\int \frac{\cosh^2(x)}{a+b\sinh(x)}\,dx = -\frac{ax}{b^2} + \frac{e^{(-x)}}{2\,b} + \frac{e^x}{2\,b} + \frac{\sqrt{a^2+b^2}\log\left(\frac{\left|2\,be^x+2\,a-2\,\sqrt{a^2+b^2}\right|}{\left|2\,be^x+2\,a+2\,\sqrt{a^2+b^2}\right|}\right)}{b^2}$$

input
```
integrate(cosh(x)^2/(a+b*sinh(x)),x, algorithm="giac")
```

output
```
-a*x/b^2 + 1/2*e^(-x)/b + 1/2*e^x/b + sqrt(a^2 + b^2)*log(abs(2*b*e^x + 2*
a - 2*sqrt(a^2 + b^2))/abs(2*b*e^x + 2*a + 2*sqrt(a^2 + b^2)))/b^2
```

Mupad [B] (verification not implemented)

Time = 1.49 (sec) , antiderivative size = 87, normalized size of antiderivative = 1.61

$$\int \frac{\cosh^2(x)}{a+b\sinh(x)}\,dx = \frac{e^{-x}}{2\,b} + \frac{e^x}{2\,b} - \frac{2\,\mathrm{atan}\left(\frac{a\,\sqrt{-b^4}}{b^2\,\sqrt{a^2+b^2}} + \frac{e^x\,\sqrt{-b^4}}{b\,\sqrt{a^2+b^2}}\right)\,\sqrt{a^2+b^2}}{\sqrt{-b^4}} - \frac{a\,x}{b^2}$$

input
```
int(cosh(x)^2/(a + b*sinh(x)),x)
```

output
```
exp(-x)/(2*b) + exp(x)/(2*b) - (2*atan((a*(-b^4)^(1/2))/(b^2*(a^2 + b^2)^(
1/2)) + (exp(x)*(-b^4)^(1/2))/(b*(a^2 + b^2)^(1/2)))*(a^2 + b^2)^(1/2))/(-
b^4)^(1/2) - (a*x)/b^2
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 62, normalized size of antiderivative = 1.15

$$\int \frac{\cosh^2(x)}{a+b\sinh(x)}\,dx = \frac{4e^x\sqrt{a^2+b^2}\,atan\left(\frac{e^x bi+ai}{\sqrt{a^2+b^2}}\right)i+e^{2x}b-2e^x ax+b}{2e^x b^2}$$

input
```
int(cosh(x)^2/(a+b*sinh(x)),x)
```

output
```
(4*e**x*sqrt(a**2 + b**2)*atan((e**x*b*i + a*i)/sqrt(a**2 + b**2))*i + e**
(2*x)*b - 2*e**x*a*x + b)/(2*e**x*b**2)
```

3.193 $\int \frac{\cosh(x)}{a+b\sinh(x)}\,dx$

Optimal result

Integrand size = 11, antiderivative size = 11

$$\int \frac{\cosh(x)}{a+b\sinh(x)}\,dx = \frac{\log(a+b\sinh(x))}{b}$$

output
```
ln(a+b*sinh(x))/b
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 11, normalized size of antiderivative = 1.00

$$\int \frac{\cosh(x)}{a+b\sinh(x)}\,dx = \frac{\log(a+b\sinh(x))}{b}$$

input
```
Integrate[Cosh[x]/(a + b*Sinh[x]),x]
```

output
```
Log[a + b*Sinh[x]]/b
```

Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 11, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.273$, Rules used = {3042, 3147, 16}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cosh(x)}{a+b\sinh(x)}dx$$

$$\Big\downarrow \; 3042$$

$$\int \frac{\cos(ix)}{a-ib\sin(ix)}dx$$

$$\Big\downarrow \; 3147$$

$$\frac{\int \frac{1}{a+b\sinh(x)}d(b\sinh(x))}{b}$$

$$\Big\downarrow \; 16$$

$$\frac{\log(a+b\sinh(x))}{b}$$

input
```
Int[Cosh[x]/(a + b*Sinh[x]),x]
```

output
```
Log[a + b*Sinh[x]]/b
```

Defintions of rubi rules used

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3147
```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Simp[1/(b^p*f)   Subst[Int[(a + x)^m*(b^2 - x^2)^((p - 1)
/2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && IntegerQ[(p
 - 1)/2] && NeQ[a^2 - b^2, 0]
```

Maple [A] (verified)

Time = 0.24 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.09

method	result	size
derivativedivides	$\frac{\ln(a+b\sinh(x))}{b}$	12
default	$\frac{\ln(a+b\sinh(x))}{b}$	12
risch	$-\frac{x}{b}+\frac{\ln\left(e^{2x}+\frac{2a\,e^{x}}{b}-1\right)}{b}$	27

input
```
int(cosh(x)/(a+b*sinh(x)),x,method=_RETURNVERBOSE)
```

output
```
ln(a+b*sinh(x))/b
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 27 vs. 2(11) = 22.

Time = 0.09 (sec) , antiderivative size = 27, normalized size of antiderivative = 2.45

$$\int \frac{\cosh(x)}{a+b\sinh(x)}\,dx = -\frac{x-\log\left(\frac{2\,(b\sinh(x)+a)}{\cosh(x)-\sinh(x)}\right)}{b}$$

input
```
integrate(cosh(x)/(a+b*sinh(x)),x, algorithm="fricas")
```

output
```
-(x - log(2*(b*sinh(x) + a)/(cosh(x) - sinh(x))))/b
```

Sympy [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.27

$$\int \frac{\cosh(x)}{a+b\sinh(x)}\,dx=\begin{cases}\frac{\log\left(\frac{a}{b}+\sinh(x)\right)}{b} & \text{for } b\neq 0\\ \frac{\sinh(x)}{a} & \text{otherwise}\end{cases}$$

input
```
integrate(cosh(x)/(a+b*sinh(x)),x)
```

output
```
Piecewise((log(a/b + sinh(x))/b, Ne(b, 0)), (sinh(x)/a, True))
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 11, normalized size of antiderivative = 1.00

$$\int \frac{\cosh(x)}{a+b\sinh(x)}\,dx=\frac{\log\left(b\sinh(x)+a\right)}{b}$$

input
```
integrate(cosh(x)/(a+b*sinh(x)),x, algorithm="maxima")
```

output
```
log(b*sinh(x) + a)/b
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 22, normalized size of antiderivative = 2.00

$$\int \frac{\cosh(x)}{a+b\sinh(x)}\,dx=\frac{\log\left(\left|-b\left(e^{(-x)}-e^{x}\right)+2\,a\right|\right)}{b}$$

input
```
integrate(cosh(x)/(a+b*sinh(x)),x, algorithm="giac")
```

output
```
log(abs(-b*(e^(-x) - e^x) + 2*a))/b
```

Mupad [B] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 11, normalized size of antiderivative = 1.00

$$\int \frac{\cosh(x)}{a+b\sinh(x)}\,dx=\frac{\ln\left(a+b\sinh(x)\right)}{b}$$

input
```
int(cosh(x)/(a + b*sinh(x)),x)
```

output
```
log(a + b*sinh(x))/b
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 11, normalized size of antiderivative = 1.00

$$\int \frac{\cosh(x)}{a+b\sinh(x)}\,dx=\frac{\log(\sinh\left(x\right)b+a)}{b}$$

input
```
int(cosh(x)/(a+b*sinh(x)),x)
```

output
```
log(sinh(x)*b + a)/b
```

3.194 $\int \frac{\text{sech}(x)}{a+b\sinh(x)}\,dx$

Optimal result

Integrand size = 11, antiderivative size = 48

$$\int \frac{\text{sech}(x)}{a+b\sinh(x)}\,dx=\frac{a\arctan(\sinh(x))}{a^2+b^2}-\frac{b\log(\cosh(x))}{a^2+b^2}+\frac{b\log(a+b\sinh(x))}{a^2+b^2}$$

output
```
a*arctan(sinh(x))/(a^2+b^2)-b*ln(cosh(x))/(a^2+b^2)+b*ln(a+b*sinh(x))/(a^2
+b^2)
```

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 99 vs. 2(48) = 96.

Time = 0.07 (sec) , antiderivative size = 99, normalized size of antiderivative = 2.06

$$\int \frac{\text{sech}(x)}{a+b\sinh(x)}\,dx = -\frac{b\left(\left(-a+\sqrt{-b^2}\right)\log\left(\sqrt{-b^2}-b\sinh(x)\right)-2\sqrt{-b^2}\log(a+b\sinh(x))+\left(a+\sqrt{-b^2}\right)\log\left(\sqrt{-b^2}+b\sin\right.\right.}{2\sqrt{-b^2}\,(a^2+b^2)}$$

input
```
Integrate[Sech[x]/(a + b*Sinh[x]),x]
```

output
```
-1/2*(b*((-a + Sqrt[-b^2])*Log[Sqrt[-b^2] - b*Sinh[x]] - 2*Sqrt[-b^2]*Log[
a + b*Sinh[x]] + (a + Sqrt[-b^2])*Log[Sqrt[-b^2] + b*Sinh[x]]))/(Sqrt[-b^2
]*(a^2 + b^2))
```

Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 59, normalized size of antiderivative = 1.23, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.636$, Rules used = {3042, 3147, 25, 479, 452, 216, 240}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\operatorname{sech}(x)}{a+b\sinh(x)}dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \frac{1}{\cos(ix)(a-ib\sin(ix))}dx$$

$$\Big\downarrow \text{ 3147}$$

$$-b\int -\frac{1}{(a+b\sinh(x))\left(\sinh^2(x)b^2+b^2\right)}d(b\sinh(x))$$

$$\Big\downarrow \text{ 25}$$

$$b\int \frac{1}{(a+b\sinh(x))\left(\sinh^2(x)b^2+b^2\right)}d(b\sinh(x))$$

$$\Big\downarrow \text{ 479}$$

$$-b\left(-\frac{\int \frac{a-b\sinh(x)}{\sinh^2(x)b^2+b^2}d(b\sinh(x))}{a^2+b^2}-\frac{\log(a+b\sinh(x))}{a^2+b^2}\right)$$

$$\Big\downarrow \text{ 452}$$

$$-b\left(-\frac{a\int \frac{1}{\sinh^2(x)b^2+b^2}d(b\sinh(x))-\int \frac{b\sinh(x)}{\sinh^2(x)b^2+b^2}d(b\sinh(x))}{a^2+b^2}-\frac{\log(a+b\sinh(x))}{a^2+b^2}\right)$$

$$\Big\downarrow \text{ 216}$$

$$-b\left(-\frac{\frac{a\arctan(\sinh(x))}{b}-\int \frac{b\sinh(x)}{\sinh^2(x)b^2+b^2}d(b\sinh(x))}{a^2+b^2}-\frac{\log(a+b\sinh(x))}{a^2+b^2}\right)$$

$$\Big\downarrow \text{240}$$

$$-b\left(-\frac{\frac{a\arctan(\sinh(x))}{b}-\frac{1}{2}\log\left(b^2\sinh^2(x)+b^2\right)}{a^2+b^2}-\frac{\log(a+b\sinh(x))}{a^2+b^2}\right)$$

input
```
Int[Sech[x]/(a + b*Sinh[x]),x]
```

output
```
-(b*(-(Log[a + b*Sinh[x]]/(a^2 + b^2)) - ((a*ArcTan[Sinh[x]])/b - Log[b^2
+ b^2*Sinh[x]^2]/2)/(a^2 + b^2)))
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 240
```
Int[(x_)/((a_) + (b_.)*(x_)^2), x_Symbol] :> Simp[Log[RemoveContent[a + b*x
^2, x]]/(2*b), x] /; FreeQ[{a, b}, x]
```

rule 452
```
Int[((c_) + (d_.)*(x_))/((a_) + (b_.)*(x_)^2), x_Symbol] :> Simp[c   Int[1/
(a + b*x^2), x], x] + Simp[d   Int[x/(a + b*x^2), x], x] /; FreeQ[{a, b, c,
 d}, x] && NeQ[b*c^2 + a*d^2, 0]
```

rule 479
```
Int[1/(((c_) + (d_.)*(x_))*((a_) + (b_.)*(x_)^2)), x_Symbol] :> Simp[d*(Log
[RemoveContent[c + d*x, x]]/(b*c^2 + a*d^2)), x] + Simp[b/(b*c^2 + a*d^2)
 Int[(c - d*x)/(a + b*x^2), x], x] /; FreeQ[{a, b, c, d}, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3147
```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Simp[1/(b^p*f)   Subst[Int[(a + x)^m*(b^2 - x^2)^((p - 1)
/2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && IntegerQ[(p
 - 1)/2] && NeQ[a^2 - b^2, 0]
```

Maple [A] (verified)

Time = 1.96 (sec) , antiderivative size = 64, normalized size of antiderivative = 1.33

method	result	size
default	$\frac{b\ln\left(\tanh\left(\frac{x}{2}\right)^{2}a-2b\tanh\left(\frac{x}{2}\right)-a\right)}{a^{2}+b^{2}}+\frac{-b\ln\left(1+\tanh\left(\frac{x}{2}\right)^{2}\right)+2a\arctan\left(\tanh\left(\frac{x}{2}\right)\right)}{a^{2}+b^{2}}$	64
risch	$\frac{i\ln(e^{x}+i)a}{a^{2}+b^{2}}-\frac{\ln(e^{x}+i)b}{a^{2}+b^{2}}-\frac{i\ln(e^{x}-i)a}{a^{2}+b^{2}}-\frac{\ln(e^{x}-i)b}{a^{2}+b^{2}}+\frac{b\ln\left(e^{2x}+\frac{2a\,e^{x}}{b}-1\right)}{a^{2}+b^{2}}$	102

input
```
int(sech(x)/(a+b*sinh(x)),x,method=_RETURNVERBOSE)
```

output
```
b/(a^2+b^2)*ln(tanh(1/2*x)^2*a-2*b*tanh(1/2*x)-a)+2/(a^2+b^2)*(-1/2*b*ln(1
+tanh(1/2*x)^2)+a*arctan(tanh(1/2*x)))
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 57, normalized size of antiderivative = 1.19

$$\int\frac{\operatorname{sech}(x)}{a+b\sinh(x)}\,dx=\frac{2\,a\arctan\left(\cosh\left(x\right)+\sinh\left(x\right)\right)+b\log\left(\frac{2\,(b\sinh(x)+a)}{\cosh(x)-\sinh(x)}\right)-b\log\left(\frac{2\,\cosh(x)}{\cosh(x)-\sinh(x)}\right)}{a^{2}+b^{2}}$$

input
```
integrate(sech(x)/(a+b*sinh(x)),x, algorithm="fricas")
```

output
```
(2*a*arctan(cosh(x) + sinh(x)) + b*log(2*(b*sinh(x) + a)/(cosh(x) - sinh(x
))) - b*log(2*cosh(x)/(cosh(x) - sinh(x))))/(a^2 + b^2)
```

Sympy [F]

$$\int \frac{\operatorname{sech}(x)}{a+b\sinh(x)}\,dx=\int \frac{\operatorname{sech}(x)}{a+b\sinh(x)}\,dx$$

input
```
integrate(sech(x)/(a+b*sinh(x)),x)
```

output
```
Integral(sech(x)/(a + b*sinh(x)), x)
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 66, normalized size of antiderivative = 1.38

$$\int \frac{\operatorname{sech}(x)}{a+b\sinh(x)}\,dx=-\frac{2\,a\arctan\left(e^{(-x)}\right)}{a^2+b^2}+\frac{b\log\left(-2\,ae^{(-x)}+be^{(-2\,x)}-b\right)}{a^2+b^2}-\frac{b\log\left(e^{(-2\,x)}+1\right)}{a^2+b^2}$$

input
```
integrate(sech(x)/(a+b*sinh(x)),x, algorithm="maxima")
```

output
```
-2*a*arctan(e^(-x))/(a^2 + b^2) + b*log(-2*a*e^(-x) + b*e^(-2*x) - b)/(a^2
 + b^2) - b*log(e^(-2*x) + 1)/(a^2 + b^2)
```

Giac [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 89, normalized size of antiderivative = 1.85

$$\int \frac{\operatorname{sech}(x)}{a+b\sinh(x)}\,dx = \frac{b^2\log\left(\left|-b\left(e^{(-x)}-e^x\right)+2\,a\right|\right)}{a^2b+b^3} + \frac{\left(\pi+2\,\arctan\left(\frac{1}{2}\left(e^{(2\,x)}-1\right)e^{(-x)}\right)\right)a}{2\left(a^2+b^2\right)} - \frac{b\log\left(\left(e^{(-x)}-e^x\right)^2+4\right)}{2\left(a^2+b^2\right)}$$

input
```
integrate(sech(x)/(a+b*sinh(x)),x, algorithm="giac")
```

output
```
b^2*log(abs(-b*(e^(-x) - e^x) + 2*a))/(a^2*b + b^3) + 1/2*(pi + 2*arctan(1
/2*(e^(2*x) - 1)*e^(-x)))*a/(a^2 + b^2) - 1/2*b*log((e^(-x) - e^x)^2 + 4)/
(a^2 + b^2)
```

Mupad [B] (verification not implemented)

Time = 2.24 (sec) , antiderivative size = 93, normalized size of antiderivative = 1.94

$$\int \frac{\operatorname{sech}(x)}{a+b\sinh(x)}\,dx = \frac{b\,\ln\left(4\,b^3\,e^{2\,x}-a^2\,b-4\,b^3+2\,a^3\,e^x+8\,a\,b^2\,e^x+a^2\,b\,e^{2\,x}\right)}{a^2+b^2} - \frac{\ln\left(e^x+1i\right)}{b+a\,1i} - \frac{\ln\left(1+e^x\,1i\right)\,1i}{a+b\,1i}$$

input
```
int(1/(cosh(x)*(a + b*sinh(x))),x)
```

output
```
(b*log(4*b^3*exp(2*x) - a^2*b - 4*b^3 + 2*a^3*exp(x) + 8*a*b^2*exp(x) + a^
2*b*exp(2*x)))/(a^2 + b^2) - log(exp(x) + 1i)/(a*1i + b) - (log(exp(x)*1i
+ 1)*1i)/(a + b*1i)
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 49, normalized size of antiderivative = 1.02

$$\int \frac{\operatorname{sech}(x)}{a+b\sinh(x)}\,dx = \frac{2atan(e^{x})\,a-\log(e^{2x}+1)\,b+\log(e^{2x}b+2e^{x}a-b)\,b}{a^{2}+b^{2}}$$

input
```
int(sech(x)/(a+b*sinh(x)),x)
```

output
```
(2*atan(e**x)*a - log(e**(2*x) + 1)*b + log(e**(2*x)*b + 2*e**x*a - b)*b)/
(a**2 + b**2)
```

3.195 $\int \frac{\operatorname{sech}^2(x)}{a+b\sinh(x)}\,dx$

Optimal result

Integrand size = 13, antiderivative size = 59

$$\int \frac{\operatorname{sech}^2(x)}{a+b\sinh(x)}\,dx = -\frac{2b^2\operatorname{arctanh}\left(\frac{b-a\tanh\left(\frac{x}{2}\right)}{\sqrt{a^2+b^2}}\right)}{(a^2+b^2)^{3/2}} + \frac{\operatorname{sech}(x)(b+a\sinh(x))}{a^2+b^2}$$

output
```
-2*b^2*arctanh((b-a*tanh(1/2*x))/(a^2+b^2)^(1/2))/(a^2+b^2)^(3/2)+sech(x)*
(b+a*sinh(x))/(a^2+b^2)
```

Mathematica [A] (verified)

Time = 0.14 (sec) , antiderivative size = 67, normalized size of antiderivative = 1.14

$$\int \frac{\operatorname{sech}^2(x)}{a+b\sinh(x)}\,dx = \frac{\frac{2b^2\arctan\left(\frac{b-a\tanh\left(\frac{x}{2}\right)}{\sqrt{-a^2-b^2}}\right)}{\sqrt{-a^2-b^2}} + b\operatorname{sech}(x) + a\tanh(x)}{a^2+b^2}$$

input
```
Integrate[Sech[x]^2/(a + b*Sinh[x]),x]
```

output
```
((2*b^2*ArcTan[(b - a*Tanh[x/2])/Sqrt[-a^2 - b^2]])/Sqrt[-a^2 - b^2] + b*S
ech[x] + a*Tanh[x])/(a^2 + b^2)
```

Rubi [A] (verified)

Time = 0.39 (sec) , antiderivative size = 64, normalized size of antiderivative = 1.08, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.615, Rules used = {3042, 3175, 25, 27, 3042, 3139, 1083, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\text{sech}^2(x)}{a+b\sinh(x)}dx$$

$$\Big\downarrow \ 3042$$

$$\int \frac{1}{\cos(ix)^2(a-ib\sin(ix))}dx$$

$$\Big\downarrow \ 3175$$

$$\frac{\text{sech}(x)(a\sinh(x)+b)}{a^2+b^2}-\frac{\int -\frac{b^2}{a+b\sinh(x)}dx}{a^2+b^2}$$

$$\Big\downarrow \ 25$$

$$\frac{\int \frac{b^2}{a+b\sinh(x)}dx}{a^2+b^2}+\frac{\text{sech}(x)(a\sinh(x)+b)}{a^2+b^2}$$

$$\Big\downarrow \ 27$$

$$\frac{b^2\int \frac{1}{a+b\sinh(x)}dx}{a^2+b^2}+\frac{\text{sech}(x)(a\sinh(x)+b)}{a^2+b^2}$$

$$\Big\downarrow \ 3042$$

$$\frac{\text{sech}(x)(a\sinh(x)+b)}{a^2+b^2}+\frac{b^2\int \frac{1}{a-ib\sin(ix)}dx}{a^2+b^2}$$

$$\Big\downarrow \ 3139$$

$$\frac{2b^2\int \frac{1}{-a\tanh^2\left(\frac{x}{2}\right)+2b\tanh\left(\frac{x}{2}\right)+a}d\tanh\left(\frac{x}{2}\right)}{a^2+b^2}+\frac{\text{sech}(x)(a\sinh(x)+b)}{a^2+b^2}$$

$$\Big\downarrow \ 1083$$

$$\frac{\text{sech}(x)(a\sinh(x)+b)}{a^2+b^2}-\frac{4b^2\int \frac{1}{4(a^2+b^2)-\left(2b-2a\tanh\left(\frac{x}{2}\right)\right)^2}d\left(2b-2a\tanh\left(\frac{x}{2}\right)\right)}{a^2+b^2}$$

$$\Big\downarrow \ 219$$

$$\frac{\operatorname{sech}(x)(a\sinh(x)+b)}{a^2+b^2}-\frac{2b^2\operatorname{arctanh}\left(\frac{2b-2a\tanh\left(\frac{x}{2}\right)}{2\sqrt{a^2+b^2}}\right)}{(a^2+b^2)^{3/2}}$$

input
```
Int[Sech[x]^2/(a + b*Sinh[x]),x]
```

output
```
(-2*b^2*ArcTanh[(2*b - 2*a*Tanh[x/2])/(2*Sqrt[a^2 + b^2])])/(a^2 + b^2)^(3
/2) + (Sech[x]*(b + a*Sinh[x]))/(a^2 + b^2)
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1083
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Simp[-2   Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3139
```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Simp[2*(e/d)   Subst[Int[1/(a + 2*b*e*x + a
*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[a^2 - b^2, 0]
```

rule 3175
```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x])^
(m + 1)*((b - a*Sin[e + f*x])/(f*g*(a^2 - b^2)*(p + 1))), x] + Simp[1/(g^2*
(a^2 - b^2)*(p + 1))   Int[(g*Cos[e + f*x])^(p + 2)*(a + b*Sin[e + f*x])^m*
(a^2*(p + 2) - b^2*(m + p + 2) + a*b*(m + p + 3)*Sin[e + f*x]), x], x] /; F
reeQ[{a, b, e, f, g, m}, x] && NeQ[a^2 - b^2, 0] && LtQ[p, -1] && IntegersQ
[2*m, 2*p]
```

Maple [A] (verified)

Time = 6.45 (sec) , antiderivative size = 71, normalized size of antiderivative = 1.20

method	result	size
default	$\frac{2b^2\,\mathrm{arctanh}\left(\frac{2a\tanh\left(\frac{x}{2}\right)-2b}{2\sqrt{a^2+b^2}}\right)}{\left(a^2+b^2\right)^{\frac{3}{2}}}-\frac{2\left(-a\tanh\left(\frac{x}{2}\right)-b\right)}{\left(a^2+b^2\right)\left(1+\tanh\left(\frac{x}{2}\right)^2\right)}$	71
risch	$-\frac{2\left(-e^x b+a\right)}{\left(e^{2x}+1\right)\left(a^2+b^2\right)}+\frac{b^2\ln\left(e^x+\frac{a\left(a^2+b^2\right)^{\frac{3}{2}}-a^4-2a^2b^2-b^4}{b\left(a^2+b^2\right)^{\frac{3}{2}}}\right)}{\left(a^2+b^2\right)^{\frac{3}{2}}}-\frac{b^2\ln\left(e^x+\frac{a\left(a^2+b^2\right)^{\frac{3}{2}}+a^4+2a^2b^2+b^4}{b\left(a^2+b^2\right)^{\frac{3}{2}}}\right)}{\left(a^2+b^2\right)^{\frac{3}{2}}}$	145

input
```
int(sech(x)^2/(a+b*sinh(x)),x,method=_RETURNVERBOSE)
```

output
```
2*b^2/(a^2+b^2)^(3/2)*arctanh(1/2*(2*a*tanh(1/2*x)-2*b)/(a^2+b^2)^(1/2))-2
/(a^2+b^2)*(-a*tanh(1/2*x)-b)/(1+tanh(1/2*x)^2)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 259 vs. $2(55) = 110$.

Time = 0.09 (sec) , antiderivative size = 259, normalized size of antiderivative = 4.39

$$\int \frac{\operatorname{sech}^2(x)}{a+b\sinh(x)}\,dx =$$

$$-\frac{2\,a^3+2\,ab^2-\left(b^2\cosh\left(x\right)^2+2\,b^2\cosh\left(x\right)\sinh\left(x\right)+b^2\sinh\left(x\right)^2+b^2\right)\sqrt{a^2+b^2}\log\left(\frac{b^2\cosh(x)^2+b^2\sinh(x)}{}\right.}{a^4+2\,a^2b^2+b^4+\left(a^4+2\,a^2b^2+b^4\right)\cosh\left(x\right)^2+2\,(a}$$

input
```
integrate(sech(x)^2/(a+b*sinh(x)),x, algorithm="fricas")
```

output
```
-(2*a^3 + 2*a*b^2 - (b^2*cosh(x)^2 + 2*b^2*cosh(x)*sinh(x) + b^2*sinh(x)^2
 + b^2)*sqrt(a^2 + b^2)*log((b^2*cosh(x)^2 + b^2*sinh(x)^2 + 2*a*b*cosh(x)
 + 2*a^2 + b^2 + 2*(b^2*cosh(x) + a*b)*sinh(x) - 2*sqrt(a^2 + b^2)*(b*cosh
(x) + b*sinh(x) + a))/(b*cosh(x)^2 + b*sinh(x)^2 + 2*a*cosh(x) + 2*(b*cosh
(x) + a)*sinh(x) - b)) - 2*(a^2*b + b^3)*cosh(x) - 2*(a^2*b + b^3)*sinh(x)
)/(a^4 + 2*a^2*b^2 + b^4 + (a^4 + 2*a^2*b^2 + b^4)*cosh(x)^2 + 2*(a^4 + 2*
a^2*b^2 + b^4)*cosh(x)*sinh(x) + (a^4 + 2*a^2*b^2 + b^4)*sinh(x)^2)
```

Sympy [F]

$$\int \frac{\operatorname{sech}^2(x)}{a+b\sinh(x)}\,dx=\int \frac{\operatorname{sech}^2{(x)}}{a+b\sinh{(x)}}\,dx$$

input
```
integrate(sech(x)**2/(a+b*sinh(x)),x)
```

output
```
Integral(sech(x)**2/(a + b*sinh(x)), x)
```

Maxima [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 89, normalized size of antiderivative = 1.51

$$\int \frac{\operatorname{sech}^2(x)}{a+b\sinh(x)}\,dx=\frac{b^2\log\left(\frac{be^{(-x)}-a-\sqrt{a^2+b^2}}{be^{(-x)}-a+\sqrt{a^2+b^2}}\right)}{(a^2+b^2)^{\frac{3}{2}}}+\frac{2\left(be^{(-x)}+a\right)}{a^2+b^2+(a^2+b^2)e^{(-2x)}}$$

input
```
integrate(sech(x)^2/(a+b*sinh(x)),x, algorithm="maxima")
```

output
```
b^2*log((b*e^(-x) - a - sqrt(a^2 + b^2))/(b*e^(-x) - a + sqrt(a^2 + b^2)))
/(a^2 + b^2)^(3/2) + 2*(b*e^(-x) + a)/(a^2 + b^2 + (a^2 + b^2)*e^(-2*x))
```

Giac [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 87, normalized size of antiderivative = 1.47

$$\int \frac{\operatorname{sech}^2(x)}{a+b\sinh(x)}\,dx = \frac{b^2\log\left(\frac{\left|2\,be^x+2\,a-2\,\sqrt{a^2+b^2}\right|}{\left|2\,be^x+2\,a+2\,\sqrt{a^2+b^2}\right|}\right)}{(a^2+b^2)^{\frac{3}{2}}} + \frac{2\,(be^x-a)}{(a^2+b^2)(e^{(2\,x)}+1)}$$

input
```
integrate(sech(x)^2/(a+b*sinh(x)),x, algorithm="giac")
```

output
```
b^2*log(abs(2*b*e^x + 2*a - 2*sqrt(a^2 + b^2))/abs(2*b*e^x + 2*a + 2*sqrt(
a^2 + b^2)))/(a^2 + b^2)^(3/2) + 2*(b*e^x - a)/((a^2 + b^2)*(e^(2*x) + 1))
```

Mupad [B] (verification not implemented)

Time = 1.94 (sec) , antiderivative size = 321, normalized size of antiderivative = 5.44

$$\int \frac{\operatorname{sech}^2(x)}{a+b\sinh(x)}\,dx = -\frac{\frac{2\,a}{a^2+b^2}-\frac{2\,b\,\mathrm{e}^x}{a^2+b^2}}{\mathrm{e}^{2\,x}+1} - \frac{2\,\mathrm{atan}\left(\left(\mathrm{e}^x\left(\frac{2}{\sqrt{b^4}\,(a^2+b^2)^2}+\frac{2\,a\left(a^3\,\sqrt{b^4}+a\,b^2\,\sqrt{b^4}\right)}{b^4\,\sqrt{-(a^2+b^2)^3}\,(a^2+b^2)\,\sqrt{-a^6-3\,a^4\,b^2-3\,a^2\,b^4-b^6}}\right)-\frac{2\,a\left(b^3\,\sqrt{b^4}+a^2\,b\,\sqrt{b^4}\right)}{b^4\,\sqrt{-(a^2+b^2)^3}\,(a^2+b^2)\,\sqrt{-a^6-3\,a^4\,b^2-3}}\right.\right.}{\sqrt{-a^6-3\,a^4\,b^2-3\,a^2\,b^4-b^6}}$$

input
```
int(1/(cosh(x)^2*(a + b*sinh(x))),x)
```

output
```
- ((2*a)/(a^2 + b^2) - (2*b*exp(x))/(a^2 + b^2))/(exp(2*x) + 1) - (2*atan(
(exp(x)*(2/((b^4)^(1/2)*(a^2 + b^2)^2) + (2*a*(a^3*(b^4)^(1/2) + a*b^2*(b^
4)^(1/2)))/(b^4*(-(a^2 + b^2)^3)^(1/2)*(a^2 + b^2)*(- a^6 - b^6 - 3*a^2*b^
4 - 3*a^4*b^2)^(1/2))) - (2*a*(b^3*(b^4)^(1/2) + a^2*b*(b^4)^(1/2)))/(b^4*
(-(a^2 + b^2)^3)^(1/2)*(a^2 + b^2)*(- a^6 - b^6 - 3*a^2*b^4 - 3*a^4*b^2)^(
1/2)))*((b^3*(- a^6 - b^6 - 3*a^2*b^4 - 3*a^4*b^2)^(1/2))/2 + (a^2*b*(- a^
6 - b^6 - 3*a^2*b^4 - 3*a^4*b^2)^(1/2))/2))*(b^4)^(1/2))/(- a^6 - b^6 - 3*
a^2*b^4 - 3*a^4*b^2)^(1/2)
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 160, normalized size of antiderivative = 2.71

$$\int \frac{\operatorname{sech}^2(x)}{a + b\sinh(x)}\,dx$$
$$= \frac{2e^{2x}\sqrt{a^2+b^2}\,atan\left(\frac{e^x bi+ai}{\sqrt{a^2+b^2}}\right)b^2 i + 2\sqrt{a^2+b^2}\,atan\left(\frac{e^x bi+ai}{\sqrt{a^2+b^2}}\right)b^2 i + 2e^{2x}a^3 + 2e^{2x}a\,b^2 + 2e^x a^2 b + 2e^x b^3}{e^{2x}a^4 + 2e^{2x}a^2b^2 + e^{2x}b^4 + a^4 + 2a^2b^2 + b^4}$$

input
```
int(sech(x)^2/(a+b*sinh(x)),x)
```

output
```
(2*(e**(2*x)*sqrt(a**2 + b**2)*atan((e**x*b*i + a*i)/sqrt(a**2 + b**2))*b*
*2*i + sqrt(a**2 + b**2)*atan((e**x*b*i + a*i)/sqrt(a**2 + b**2))*b**2*i +
 e**(2*x)*a**3 + e**(2*x)*a*b**2 + e**x*a**2*b + e**x*b**3))/(e**(2*x)*a**
4 + 2*e**(2*x)*a**2*b**2 + e**(2*x)*b**4 + a**4 + 2*a**2*b**2 + b**4)
```

3.196 $\int \frac{\operatorname{sech}^3(x)}{a+b\sinh(x)}\,dx$

Optimal result

Integrand size = 13, antiderivative size = 87

$$\int \frac{\operatorname{sech}^3(x)}{a+b\sinh(x)}\,dx=\frac{a(a^2+3b^2)\arctan(\sinh(x))}{2(a^2+b^2)^2}-\frac{b^3\log(\cosh(x))}{(a^2+b^2)^2}+\frac{b^3\log(a+b\sinh(x))}{(a^2+b^2)^2}+\frac{\operatorname{sech}^2(x)(b+a\sinh(x))}{2(a^2+b^2)}$$

output
```
1/2*a*(a^2+3*b^2)*arctan(sinh(x))/(a^2+b^2)^2-b^3*ln(cosh(x))/(a^2+b^2)^2+
b^3*ln(a+b*sinh(x))/(a^2+b^2)^2+sech(x)^2*(b+a*sinh(x))/(2*a^2+2*b^2)
```

Mathematica [A] (verified)

Time = 0.13 (sec) , antiderivative size = 77, normalized size of antiderivative = 0.89

$$\int \frac{\operatorname{sech}^3(x)}{a+b\sinh(x)}\,dx$$
$$=\frac{2a(a^2+3b^2)\arctan\left(\tanh\left(\frac{x}{2}\right)\right)+2b^3(-\log(\cosh(x))+\log(a+b\sinh(x)))+b(a^2+b^2)\operatorname{sech}^2(x)+a(a^2}{2(a^2+b^2)^2}$$

input
```
Integrate[Sech[x]^3/(a + b*Sinh[x]),x]
```

output
```
(2*a*(a^2 + 3*b^2)*ArcTan[Tanh[x/2]] + 2*b^3*(-Log[Cosh[x]] + Log[a + b*Si
nh[x]]) + b*(a^2 + b^2)*Sech[x]^2 + a*(a^2 + b^2)*Sech[x]*Tanh[x])/(2*(a^2
 + b^2)^2)
```

Rubi [A] (verified)

Time = 0.40 (sec) , antiderivative size = 135, normalized size of antiderivative = 1.55, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.462, Rules used = {3042, 3147, 496, 25, 657, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\text{sech}^3(x)}{a+b\sinh(x)}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{\cos(ix)^3(a-ib\sin(ix))}dx$$

$$\Big\downarrow \text{3147}$$

$$b^3\int \frac{1}{(a+b\sinh(x))\left(\sinh^2(x)b^2+b^2\right)^2}d(b\sinh(x))$$

$$\Big\downarrow \text{496}$$

$$b^3\left(\frac{ab\sinh(x)+b^2}{2b^2\left(a^2+b^2\right)\left(b^2\sinh^2(x)+b^2\right)}-\frac{\int -\frac{a^2+b\sinh(x)a+2b^2}{(a+b\sinh(x))\left(\sinh^2(x)b^2+b^2\right)}d(b\sinh(x))}{2b^2\left(a^2+b^2\right)}\right)$$

$$\Big\downarrow \text{25}$$

$$b^3\left(\frac{\int \frac{a^2+b\sinh(x)a+2b^2}{(a+b\sinh(x))\left(\sinh^2(x)b^2+b^2\right)}d(b\sinh(x))}{2b^2\left(a^2+b^2\right)}+\frac{ab\sinh(x)+b^2}{2b^2\left(a^2+b^2\right)\left(b^2\sinh^2(x)+b^2\right)}\right)$$

$$\Big\downarrow \text{657}$$

$$b^3\left(\frac{\int \left(\frac{2b^2}{\left(a^2+b^2\right)(a+b\sinh(x))}+\frac{a^3+3b^2a-2b^3\sinh(x)}{\left(a^2+b^2\right)\left(\sinh^2(x)b^2+b^2\right)}\right)d(b\sinh(x))}{2b^2\left(a^2+b^2\right)}+\frac{ab\sinh(x)+b^2}{2b^2\left(a^2+b^2\right)\left(b^2\sinh^2(x)+b^2\right)}\right)$$

$$\Big\downarrow \text{2009}$$

$$b^3\left(\frac{\frac{a(a^2+3b^2)\arctan(\sinh(x))}{b(a^2+b^2)}-\frac{b^2\log(b^2\sinh^2(x)+b^2)}{a^2+b^2}+\frac{2b^2\log(a+b\sinh(x))}{a^2+b^2}}{2b^2(a^2+b^2)}+\frac{ab\sinh(x)+b^2}{2b^2(a^2+b^2)(b^2\sinh^2(x)+b^2)}\right)$$

input
```
Int[Sech[x]^3/(a + b*Sinh[x]),x]
```

output
```
b^3*(((a*(a^2 + 3*b^2)*ArcTan[Sinh[x]])/(b*(a^2 + b^2)) + (2*b^2*Log[a + b
*Sinh[x]])/(a^2 + b^2) - (b^2*Log[b^2 + b^2*Sinh[x]^2])/(a^2 + b^2))/(2*b^
2*(a^2 + b^2)) + (b^2 + a*b*Sinh[x])/(2*b^2*(a^2 + b^2)*(b^2 + b^2*Sinh[x]
^2)))
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 496
```
Int[((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(-(a*d + b*c*x))*(c + d*x)^(n + 1)*((a + b*x^2)^(p + 1)/(2*a*(p + 1)*(b*c^2
 + a*d^2))), x] + Simp[1/(2*a*(p + 1)*(b*c^2 + a*d^2))   Int[(c + d*x)^n*(a
 + b*x^2)^(p + 1)*Simp[b*c^2*(2*p + 3) + a*d^2*(n + 2*p + 3) + b*c*d*(n + 2
*p + 4)*x, x], x], x] /; FreeQ[{a, b, c, d, n}, x] && LtQ[p, -1] && IntQuad
raticQ[a, 0, b, c, d, n, p, x]
```

rule 657
```
Int[(((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))^(n_.))/((a_) + (c_.)*(
x_)^2), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*((f + g*x)^n/(a + c*x^
2)), x], x] /; FreeQ[{a, c, d, e, f, g, m}, x] && IntegersQ[n]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3147
```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Simp[1/(b^p*f)   Subst[Int[(a + x)^m*(b^2 - x^2)^((p - 1)
/2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && IntegerQ[(p
 - 1)/2] && NeQ[a^2 - b^2, 0]
```

Maple [A] (verified)

Time = 16.36 (sec) , antiderivative size = 161, normalized size of antiderivative = 1.85

method	result
default	$\frac{b^3\ln\left(\tanh\left(\frac{x}{2}\right)^2 a-2b\tanh\left(\frac{x}{2}\right)-a\right)}{a^4+2a^2b^2+b^4}+\frac{\frac{2\left(\left(-\frac{1}{2}a^3-\frac{1}{2}a\,b^2\right)\tanh\left(\frac{x}{2}\right)^3+\left(-a^2b-b^3\right)\tanh\left(\frac{x}{2}\right)^2+\left(\frac{1}{2}a^3+\frac{1}{2}a\,b^2\right)\tanh\left(\frac{x}{2}\right)\right)}{\left(1+\tanh\left(\frac{x}{2}\right)^2\right)^2}-b^3\ln\left(1+\tanh\left(\right.\right.}{a^4+2a^2b^2+b^4}$
risch	$\frac{e^x\left(a\,e^{2x}+2\,e^x b-a\right)}{\left(e^{2x}+1\right)^2\left(a^2+b^2\right)}+\frac{i\ln\left(e^x+i\right)a^3}{2a^4+4a^2b^2+2b^4}+\frac{3i\ln\left(e^x+i\right)a\,b^2}{2\left(a^4+2a^2b^2+b^4\right)}-\frac{\ln\left(e^x+i\right)b^3}{a^4+2a^2b^2+b^4}-\frac{i\ln\left(e^x-i\right)a^3}{2\left(a^4+2a^2b^2+b^4\right)}-\frac{3i\ln\left(e^x-i\right)a\,b^2}{2\left(a^4+2a^2b^2+b^4\right)}-$

input
```
int(sech(x)^3/(a+b*sinh(x)),x,method=_RETURNVERBOSE)
```

output
```
b^3/(a^4+2*a^2*b^2+b^4)*ln(tanh(1/2*x)^2*a-2*b*tanh(1/2*x)-a)+2/(a^4+2*a^2
*b^2+b^4)*(((-1/2*a^3-1/2*a*b^2)*tanh(1/2*x)^3+(-a^2*b-b^3)*tanh(1/2*x)^2+
(1/2*a^3+1/2*a*b^2)*tanh(1/2*x))/(1+tanh(1/2*x)^2)^2-1/2*b^3*ln(1+tanh(1/2
*x)^2)+1/2*(a^3+3*a*b^2)*arctan(tanh(1/2*x)))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 652 vs. 2(83) = 166.

Time = 0.11 (sec) , antiderivative size = 652, normalized size of antiderivative = 7.49

$$\int \frac{\operatorname{sech}^3(x)}{a+b\sinh(x)}\,dx = \text{Too large to display}$$

input
```
integrate(sech(x)^3/(a+b*sinh(x)),x, algorithm="fricas")
```

output
```
((a^3 + a*b^2)*cosh(x)^3 + (a^3 + a*b^2)*sinh(x)^3 + 2*(a^2*b + b^3)*cosh(
x)^2 + (2*a^2*b + 2*b^3 + 3*(a^3 + a*b^2)*cosh(x))*sinh(x)^2 + ((a^3 + 3*a
*b^2)*cosh(x)^4 + 4*(a^3 + 3*a*b^2)*cosh(x)*sinh(x)^3 + (a^3 + 3*a*b^2)*si
nh(x)^4 + a^3 + 3*a*b^2 + 2*(a^3 + 3*a*b^2)*cosh(x)^2 + 2*(a^3 + 3*a*b^2 +
 3*(a^3 + 3*a*b^2)*cosh(x)^2)*sinh(x)^2 + 4*((a^3 + 3*a*b^2)*cosh(x)^3 + (
a^3 + 3*a*b^2)*cosh(x))*sinh(x))*arctan(cosh(x) + sinh(x)) - (a^3 + a*b^2)
*cosh(x) + (b^3*cosh(x)^4 + 4*b^3*cosh(x)*sinh(x)^3 + b^3*sinh(x)^4 + 2*b^
3*cosh(x)^2 + b^3 + 2*(3*b^3*cosh(x)^2 + b^3)*sinh(x)^2 + 4*(b^3*cosh(x)^3
 + b^3*cosh(x))*sinh(x))*log(2*(b*sinh(x) + a)/(cosh(x) - sinh(x))) - (b^3
*cosh(x)^4 + 4*b^3*cosh(x)*sinh(x)^3 + b^3*sinh(x)^4 + 2*b^3*cosh(x)^2 + b
^3 + 2*(3*b^3*cosh(x)^2 + b^3)*sinh(x)^2 + 4*(b^3*cosh(x)^3 + b^3*cosh(x))
*sinh(x))*log(2*cosh(x)/(cosh(x) - sinh(x))) - (a^3 + a*b^2 - 3*(a^3 + a*b
^2)*cosh(x)^2 - 4*(a^2*b + b^3)*cosh(x))*sinh(x))/((a^4 + 2*a^2*b^2 + b^4)
*cosh(x)^4 + 4*(a^4 + 2*a^2*b^2 + b^4)*cosh(x)*sinh(x)^3 + (a^4 + 2*a^2*b^
2 + b^4)*sinh(x)^4 + a^4 + 2*a^2*b^2 + b^4 + 2*(a^4 + 2*a^2*b^2 + b^4)*cos
h(x)^2 + 2*(a^4 + 2*a^2*b^2 + b^4 + 3*(a^4 + 2*a^2*b^2 + b^4)*cosh(x)^2)*s
inh(x)^2 + 4*((a^4 + 2*a^2*b^2 + b^4)*cosh(x)^3 + (a^4 + 2*a^2*b^2 + b^4)*
cosh(x))*sinh(x))
```

Sympy [F]

$$\int \frac{\operatorname{sech}^3(x)}{a+b\sinh(x)}\,dx = \int \frac{\operatorname{sech}^3{(x)}}{a+b\sinh{(x)}}\,dx$$

input
```
integrate(sech(x)**3/(a+b*sinh(x)),x)
```

output
```
Integral(sech(x)**3/(a + b*sinh(x)), x)
```

Maxima [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 159, normalized size of antiderivative = 1.83

$$\int \frac{\operatorname{sech}^3(x)}{a+b\sinh(x)}\,dx = \frac{b^3\log\left(-2\,ae^{(-x)}+be^{(-2\,x)}-b\right)}{a^4+2\,a^2b^2+b^4} - \frac{b^3\log\left(e^{(-2\,x)}+1\right)}{a^4+2\,a^2b^2+b^4} - \frac{(a^3+3\,ab^2)\arctan\left(e^{(-x)}\right)}{a^4+2\,a^2b^2+b^4} + \frac{ae^{(-x)}+2\,be^{(-2\,x)}-ae^{(-3\,x)}}{a^2+b^2+2\,(a^2+b^2)e^{(-2\,x)}+(a^2+b^2)e^{(-4\,x)}}$$

input
```
integrate(sech(x)^3/(a+b*sinh(x)),x, algorithm="maxima")
```

output
```
b^3*log(-2*a*e^(-x) + b*e^(-2*x) - b)/(a^4 + 2*a^2*b^2 + b^4) - b^3*log(e^
(-2*x) + 1)/(a^4 + 2*a^2*b^2 + b^4) - (a^3 + 3*a*b^2)*arctan(e^(-x))/(a^4
+ 2*a^2*b^2 + b^4) + (a*e^(-x) + 2*b*e^(-2*x) - a*e^(-3*x))/(a^2 + b^2 + 2
*(a^2 + b^2)*e^(-2*x) + (a^2 + b^2)*e^(-4*x))
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 214 vs. 2(83) = 166.

Time = 0.14 (sec) , antiderivative size = 214, normalized size of antiderivative = 2.46

$$\int \frac{\operatorname{sech}^3(x)}{a+b\sinh(x)}\,dx = \frac{b^4\log\left(\left|-b\left(e^{(-x)}-e^x\right)+2\,a\right|\right)}{a^4b+2\,a^2b^3+b^5} - \frac{b^3\log\left(\left(e^{(-x)}-e^x\right)^2+4\right)}{2\,(a^4+2\,a^2b^2+b^4)} + \frac{\left(\pi+2\,\arctan\left(\frac{1}{2}\left(e^{(2\,x)}-1\right)e^{(-x)}\right)\right)(a^3+3\,ab^2)}{4\,(a^4+2\,a^2b^2+b^4)} + \frac{b^3\left(e^{(-x)}-e^x\right)^2-2\,a^3\left(e^{(-x)}-e^x\right)-2\,ab^2\left(e^{(-x)}-e^x\right)+4\,a^2b+8\,b^3}{2\,(a^4+2\,a^2b^2+b^4)\left(\left(e^{(-x)}-e^x\right)^2+4\right)}$$

input
```
integrate(sech(x)^3/(a+b*sinh(x)),x, algorithm="giac")
```

output
```
b^4*log(abs(-b*(e^(-x) - e^x) + 2*a))/(a^4*b + 2*a^2*b^3 + b^5) - 1/2*b^3*
log((e^(-x) - e^x)^2 + 4)/(a^4 + 2*a^2*b^2 + b^4) + 1/4*(pi + 2*arctan(1/2
*(e^(2*x) - 1)*e^(-x)))*(a^3 + 3*a*b^2)/(a^4 + 2*a^2*b^2 + b^4) + 1/2*(b^3
*(e^(-x) - e^x)^2 - 2*a^3*(e^(-x) - e^x) - 2*a*b^2*(e^(-x) - e^x) + 4*a^2*
b + 8*b^3)/((a^4 + 2*a^2*b^2 + b^4)*((e^(-x) - e^x)^2 + 4))
```

Mupad [B] (verification not implemented)

Time = 3.18 (sec) , antiderivative size = 291, normalized size of antiderivative = 3.34

$$\int \frac{\operatorname{sech}^3(x)}{a+b\sinh(x)}\,dx = \frac{\frac{2\,(a^2\,b+b^3)}{(a^2+b^2)^2}+\frac{e^x\,(a^3+a\,b^2)}{(a^2+b^2)^2}}{e^{2\,x}+1}-\frac{\frac{2\,b}{a^2+b^2}+\frac{2\,a\,e^x}{a^2+b^2}}{2\,e^{2\,x}+e^{4\,x}+1}-\frac{\ln\left(1+e^x\,1i\right)\,\left(a+b\,2i\right)}{2\,\left(-a^2\,1i+2\,a\,b+b^2\,1i\right)} + \frac{b^3\,\ln\left(16\,b^7\,e^{2\,x}-a^6\,b-16\,b^7-9\,a^2\,b^5-6\,a^4\,b^3+2\,a^7\,e^x+9\,a^2\,b^5\,e^{2\,x}+6\,a^4\,b^3\,e^{2\,x}+32\,a\,b^6\,e^x+a^6\,b\,e\right.}{a^4+2\,a^2\,b^2+b^4} - \frac{\ln\left(e^x+1i\right)\,\left(2\,b+a\,1i\right)}{2\,\left(-a^2+a\,b\,2i+b^2\right)}$$

input
```
int(1/(cosh(x)^3*(a + b*sinh(x))),x)
```

output
```
((2*(a^2*b + b^3))/(a^2 + b^2)^2 + (exp(x)*(a*b^2 + a^3))/(a^2 + b^2)^2)/(
exp(2*x) + 1) - ((2*b)/(a^2 + b^2) + (2*a*exp(x))/(a^2 + b^2))/(2*exp(2*x)
 + exp(4*x) + 1) - (log(exp(x)*1i + 1)*(a + b*2i))/(2*(2*a*b - a^2*1i + b^
2*1i)) + (b^3*log(16*b^7*exp(2*x) - a^6*b - 16*b^7 - 9*a^2*b^5 - 6*a^4*b^3
 + 2*a^7*exp(x) + 9*a^2*b^5*exp(2*x) + 6*a^4*b^3*exp(2*x) + 32*a*b^6*exp(x
) + a^6*b*exp(2*x) + 18*a^3*b^4*exp(x) + 12*a^5*b^2*exp(x)))/(a^4 + b^4 +
2*a^2*b^2) - (log(exp(x) + 1i)*(a*1i + 2*b))/(2*(a*b*2i - a^2 + b^2))
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 352, normalized size of antiderivative = 4.05

$$\int \frac{\operatorname{sech}^3(x)}{a+b\sinh(x)}\,dx$$
$$= \frac{e^{4x}atan(e^x)\,a^3+3e^{4x}atan(e^x)\,a\,b^2+2e^{2x}atan(e^x)\,a^3+6e^{2x}atan(e^x)\,a\,b^2+atan(e^x)\,a^3+3atan(e^x)\,a\,b^2}{}$$

input
```
int(sech(x)^3/(a+b*sinh(x)),x)
```

output
```
(e**(4*x)*atan(e**x)*a**3 + 3*e**(4*x)*atan(e**x)*a*b**2 + 2*e**(2*x)*atan
(e**x)*a**3 + 6*e**(2*x)*atan(e**x)*a*b**2 + atan(e**x)*a**3 + 3*atan(e**x
)*a*b**2 - e**(4*x)*log(e**(2*x) + 1)*b**3 + e**(4*x)*log(e**(2*x)*b + 2*e
**x*a - b)*b**3 - e**(4*x)*a**2*b - e**(4*x)*b**3 + e**(3*x)*a**3 + e**(3*
x)*a*b**2 - 2*e**(2*x)*log(e**(2*x) + 1)*b**3 + 2*e**(2*x)*log(e**(2*x)*b
+ 2*e**x*a - b)*b**3 - e**x*a**3 - e**x*a*b**2 - log(e**(2*x) + 1)*b**3 +
log(e**(2*x)*b + 2*e**x*a - b)*b**3 - a**2*b - b**3)/(e**(4*x)*a**4 + 2*e*
*(4*x)*a**2*b**2 + e**(4*x)*b**4 + 2*e**(2*x)*a**4 + 4*e**(2*x)*a**2*b**2
+ 2*e**(2*x)*b**4 + a**4 + 2*a**2*b**2 + b**4)
```

3.197 $\int \frac{\operatorname{sech}^4(x)}{a+b\sinh(x)}\,dx$

Optimal result

Integrand size = 13, antiderivative size = 100

$$\int \frac{\operatorname{sech}^4(x)}{a+b\sinh(x)}\,dx=-\frac{2b^4\operatorname{arctanh}\left(\frac{b-a\tanh\left(\frac{x}{2}\right)}{\sqrt{a^2+b^2}}\right)}{\left(a^2+b^2\right)^{5/2}}+\frac{\operatorname{sech}^3(x)(b+a\sinh(x))}{3\left(a^2+b^2\right)}+\frac{\operatorname{sech}(x)\left(3b^3+a\left(2a^2+5b^2\right)\sinh(x)\right)}{3\left(a^2+b^2\right)^2}$$

output
```
-2*b^4*arctanh((b-a*tanh(1/2*x))/(a^2+b^2)^(1/2))/(a^2+b^2)^(5/2)+sech(x)^
3*(b+a*sinh(x))/(3*a^2+3*b^2)+1/3*sech(x)*(3*b^3+a*(2*a^2+5*b^2)*sinh(x))/
(a^2+b^2)^2
```

Mathematica [A] (verified)

Time = 0.26 (sec) , antiderivative size = 102, normalized size of antiderivative = 1.02

$$\int \frac{\operatorname{sech}^4(x)}{a+b\sinh(x)}\,dx=\frac{\frac{6b^4\arctan\left(\frac{b-a\tanh\left(\frac{x}{2}\right)}{\sqrt{-a^2-b^2}}\right)}{\sqrt{-a^2-b^2}}+3b^3\operatorname{sech}(x)+\left(a^2+b^2\right)\operatorname{sech}^3(x)(b+a\sinh(x))+a\left(2a^2+5b^2\right)\tanh(x)}{3\left(a^2+b^2\right)^2}$$

input
```
Integrate[Sech[x]^4/(a + b*Sinh[x]),x]
```

output
```
((6*b^4*ArcTan[(b - a*Tanh[x/2])/Sqrt[-a^2 - b^2]])/Sqrt[-a^2 - b^2] + 3*b
^3*Sech[x] + (a^2 + b^2)*Sech[x]^3*(b + a*Sinh[x]) + a*(2*a^2 + 5*b^2)*Tan
h[x])/(3*(a^2 + b^2)^2)
```

Rubi [A] (verified)

Time = 0.69 (sec) , antiderivative size = 116, normalized size of antiderivative = 1.16, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.769$, Rules used = {3042, 3175, 25, 3042, 3345, 27, 3042, 3139, 1083, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\text{sech}^4(x)}{a+b\sinh(x)}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{\cos(ix)^4(a-ib\sin(ix))}dx$$

$$\Big\downarrow \text{3175}$$

$$\frac{\text{sech}^3(x)(a\sinh(x)+b)}{3(a^2+b^2)}-\frac{\int -\frac{\text{sech}^2(x)\left(2a^2+2b\sinh(x)a+3b^2\right)}{a+b\sinh(x)}dx}{3(a^2+b^2)}$$

$$\Big\downarrow \text{25}$$

$$\frac{\int \frac{\text{sech}^2(x)\left(2a^2+2b\sinh(x)a+3b^2\right)}{a+b\sinh(x)}dx}{3(a^2+b^2)}+\frac{\text{sech}^3(x)(a\sinh(x)+b)}{3(a^2+b^2)}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\text{sech}^3(x)(a\sinh(x)+b)}{3(a^2+b^2)}+\frac{\int \frac{2a^2-2ib\sin(ix)a+3b^2}{\cos(ix)^2(a-ib\sin(ix))}dx}{3(a^2+b^2)}$$

$$\Big\downarrow \text{3345}$$

$$\frac{\frac{\text{sech}(x)\left(a\left(2a^2+5b^2\right)\sinh(x)+3b^3\right)}{a^2+b^2}-\frac{\int -\frac{3b^4}{a+b\sinh(x)}dx}{a^2+b^2}}{3\left(a^2+b^2\right)}+\frac{\text{sech}^3(x)(a\sinh(x)+b)}{3\left(a^2+b^2\right)}$$

$$\Big\downarrow \text{ 27}$$

$$\frac{\frac{3b^4\int \frac{1}{a+b\sinh(x)}dx}{a^2+b^2}+\frac{\text{sech}(x)\left(a\left(2a^2+5b^2\right)\sinh(x)+3b^3\right)}{a^2+b^2}}{3\left(a^2+b^2\right)}+\frac{\text{sech}^3(x)(a\sinh(x)+b)}{3\left(a^2+b^2\right)}$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{\text{sech}^3(x)(a\sinh(x)+b)}{3\left(a^2+b^2\right)}+\frac{\frac{\text{sech}(x)\left(a\left(2a^2+5b^2\right)\sinh(x)+3b^3\right)}{a^2+b^2}+\frac{3b^4\int \frac{1}{a-ib\sin(ix)}dx}{a^2+b^2}}{3\left(a^2+b^2\right)}$$

$$\Big\downarrow \text{ 3139}$$

$$\frac{\frac{6b^4\int \frac{1}{-a\tanh^2\left(\frac{x}{2}\right)+2b\tanh\left(\frac{x}{2}\right)+a}d\tanh\left(\frac{x}{2}\right)}{a^2+b^2}+\frac{\text{sech}(x)\left(a\left(2a^2+5b^2\right)\sinh(x)+3b^3\right)}{a^2+b^2}}{3\left(a^2+b^2\right)}+\frac{\text{sech}^3(x)(a\sinh(x)+b)}{3\left(a^2+b^2\right)}$$

$$\Big\downarrow \text{ 1083}$$

$$\frac{\frac{\text{sech}(x)\left(a\left(2a^2+5b^2\right)\sinh(x)+3b^3\right)}{a^2+b^2}-\frac{12b^4\int \frac{1}{4\left(a^2+b^2\right)-\left(2b-2a\tanh\left(\frac{x}{2}\right)\right)^2}d\left(2b-2a\tanh\left(\frac{x}{2}\right)\right)}{a^2+b^2}}{3\left(a^2+b^2\right)}+\frac{\text{sech}^3(x)(a\sinh(x)+b)}{3\left(a^2+b^2\right)}$$

$$\Big\downarrow \text{ 219}$$

$$\frac{\frac{\text{sech}(x)\left(a\left(2a^2+5b^2\right)\sinh(x)+3b^3\right)}{a^2+b^2}-\frac{6b^4\text{arctanh}\left(\frac{2b-2a\tanh\left(\frac{x}{2}\right)}{2\sqrt{a^2+b^2}}\right)}{\left(a^2+b^2\right)^{3/2}}}{3\left(a^2+b^2\right)}+\frac{\text{sech}^3(x)(a\sinh(x)+b)}{3\left(a^2+b^2\right)}$$

input
```
Int[Sech[x]^4/(a + b*Sinh[x]),x]
```

output
```
(Sech[x]^3*(b + a*Sinh[x]))/(3*(a^2 + b^2)) + ((-6*b^4*ArcTanh[(2*b - 2*a*
Tanh[x/2])/(2*Sqrt[a^2 + b^2])])/(a^2 + b^2)^(3/2) + (Sech[x]*(3*b^3 + a*(
2*a^2 + 5*b^2)*Sinh[x]))/(a^2 + b^2))/(3*(a^2 + b^2))
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1083
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Simp[-2   Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3139
```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Simp[2*(e/d)   Subst[Int[1/(a + 2*b*e*x + a
*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[a^2 - b^2, 0]
```

rule 3175
```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x])^
(m + 1)*((b - a*Sin[e + f*x])/(f*g*(a^2 - b^2)*(p + 1))), x] + Simp[1/(g^2*
(a^2 - b^2)*(p + 1))   Int[(g*Cos[e + f*x])^(p + 2)*(a + b*Sin[e + f*x])^m*
(a^2*(p + 2) - b^2*(m + p + 2) + a*b*(m + p + 3)*Sin[e + f*x]), x], x] /; F
reeQ[{a, b, e, f, g, m}, x] && NeQ[a^2 - b^2, 0] && LtQ[p, -1] && IntegersQ
[2*m, 2*p]
```

rule 3345
```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[(g*Co
s[e + f*x])^(p + 1)*(a + b*Sin[e + f*x])^(m + 1)*((b*c - a*d - (a*c - b*d)*
Sin[e + f*x])/(f*g*(a^2 - b^2)*(p + 1))), x] + Simp[1/(g^2*(a^2 - b^2)*(p +
 1))   Int[(g*Cos[e + f*x])^(p + 2)*(a + b*Sin[e + f*x])^m*Simp[c*(a^2*(p +
 2) - b^2*(m + p + 2)) + a*b*d*m + b*(a*c - b*d)*(m + p + 3)*Sin[e + f*x],
x], x], x] /; FreeQ[{a, b, c, d, e, f, g, m}, x] && NeQ[a^2 - b^2, 0] && Lt
Q[p, -1] && IntegerQ[2*m]
```

Maple [A] (verified)

Time = 39.14 (sec) , antiderivative size = 182, normalized size of antiderivative = 1.82

method	result
default	$\frac{2b^4 \operatorname{arctanh}\left(\frac{2a\tanh\left(\frac{x}{2}\right)-2b}{2\sqrt{a^2+b^2}}\right)}{(a^4+2a^2b^2+b^4)\sqrt{a^2+b^2}}-\frac{2\left(\left(-a^3-2a\,b^2\right)\tanh\left(\frac{x}{2}\right)^5+\left(-a^2b-2b^3\right)\tanh\left(\frac{x}{2}\right)^4+\left(-\frac{2}{3}a^3-\frac{8}{3}a\,b^2\right)\tanh\left(\frac{x}{2}\right)^3-2\tanh\left(\frac{x}{2}\right)^2b^3+\right.}{(a^4+2a^2b^2+b^4)\left(1+\tanh\left(\frac{x}{2}\right)^2\right)^3}$
risch	$-\frac{2\left(-3b^3e^{5x}+3\,e^{4x}a\,b^2-4a^2b\,e^{3x}-10\,e^{3x}b^3+6a^3e^{2x}+12a\,e^{2x}b^2-3b^3e^x+2a^3+5a\,b^2\right)}{3(a^2+b^2)^2(e^{2x}+1)^3}+\frac{b^4\ln\left(e^x+\frac{\left(a^2+b^2\right)^{\frac{5}{2}}a-a^6-3a^4b^2-3a^2b^4}{b\left(a^2+b^2\right)^{\frac{5}{2}}}\right.}{(a^2+b^2)^{\frac{5}{2}}}$

input
```
int(sech(x)^4/(a+b*sinh(x)),x,method=_RETURNVERBOSE)
```

output
```
2*b^4/(a^4+2*a^2*b^2+b^4)/(a^2+b^2)^(1/2)*arctanh(1/2*(2*a*tanh(1/2*x)-2*b
)/(a^2+b^2)^(1/2))-2/(a^4+2*a^2*b^2+b^4)*((-a^3-2*a*b^2)*tanh(1/2*x)^5+(-a
^2*b-2*b^3)*tanh(1/2*x)^4+(-2/3*a^3-8/3*a*b^2)*tanh(1/2*x)^3-2*tanh(1/2*x)
^2*b^3+(-a^3-2*a*b^2)*tanh(1/2*x)-1/3*a^2*b-4/3*b^3)/(1+tanh(1/2*x)^2)^3
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1142 vs. 2(92) = 184.

Time = 0.12 (sec) , antiderivative size = 1142, normalized size of antiderivative = 11.42

$$\int \frac{\operatorname{sech}^4(x)}{a+b\sinh(x)}\,dx = \text{Too large to display}$$

input
```
integrate(sech(x)^4/(a+b*sinh(x)),x, algorithm="fricas")
```

output
```
1/3*(6*(a^2*b^3 + b^5)*cosh(x)^5 + 6*(a^2*b^3 + b^5)*sinh(x)^5 - 4*a^5 - 1
4*a^3*b^2 - 10*a*b^4 - 6*(a^3*b^2 + a*b^4)*cosh(x)^4 - 6*(a^3*b^2 + a*b^4
- 5*(a^2*b^3 + b^5)*cosh(x))*sinh(x)^4 + 4*(2*a^4*b + 7*a^2*b^3 + 5*b^5)*c
osh(x)^3 + 4*(2*a^4*b + 7*a^2*b^3 + 5*b^5 + 15*(a^2*b^3 + b^5)*cosh(x)^2 -
 6*(a^3*b^2 + a*b^4)*cosh(x))*sinh(x)^3 - 12*(a^5 + 3*a^3*b^2 + 2*a*b^4)*c
osh(x)^2 - 12*(a^5 + 3*a^3*b^2 + 2*a*b^4 - 5*(a^2*b^3 + b^5)*cosh(x)^3 + 3
*(a^3*b^2 + a*b^4)*cosh(x)^2 - (2*a^4*b + 7*a^2*b^3 + 5*b^5)*cosh(x))*sinh
(x)^2 + 3*(b^4*cosh(x)^6 + 6*b^4*cosh(x)*sinh(x)^5 + b^4*sinh(x)^6 + 3*b^4
*cosh(x)^4 + 3*b^4*cosh(x)^2 + 3*(5*b^4*cosh(x)^2 + b^4)*sinh(x)^4 + b^4 +
 4*(5*b^4*cosh(x)^3 + 3*b^4*cosh(x))*sinh(x)^3 + 3*(5*b^4*cosh(x)^4 + 6*b^
4*cosh(x)^2 + b^4)*sinh(x)^2 + 6*(b^4*cosh(x)^5 + 2*b^4*cosh(x)^3 + b^4*co
sh(x))*sinh(x))*sqrt(a^2 + b^2)*log((b^2*cosh(x)^2 + b^2*sinh(x)^2 + 2*a*b
*cosh(x) + 2*a^2 + b^2 + 2*(b^2*cosh(x) + a*b)*sinh(x) - 2*sqrt(a^2 + b^2)
*(b*cosh(x) + b*sinh(x) + a))/(b*cosh(x)^2 + b*sinh(x)^2 + 2*a*cosh(x) + 2
*(b*cosh(x) + a)*sinh(x) - b)) + 6*(a^2*b^3 + b^5)*cosh(x) + 6*(a^2*b^3 +
b^5 + 5*(a^2*b^3 + b^5)*cosh(x)^4 - 4*(a^3*b^2 + a*b^4)*cosh(x)^3 + 2*(2*a
^4*b + 7*a^2*b^3 + 5*b^5)*cosh(x)^2 - 4*(a^5 + 3*a^3*b^2 + 2*a*b^4)*cosh(x
))*sinh(x))/((a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6)*cosh(x)^6 + 6*(a^6 + 3*a^
4*b^2 + 3*a^2*b^4 + b^6)*cosh(x)*sinh(x)^5 + (a^6 + 3*a^4*b^2 + 3*a^2*b^4
+ b^6)*sinh(x)^6 + a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6 + 3*(a^6 + 3*a^4*b...
```

Sympy [F]

$$\int \frac{\operatorname{sech}^4(x)}{a+b\sinh(x)}\,dx=\int \frac{\operatorname{sech}^4{(x)}}{a+b\sinh{(x)}}\,dx$$

input
```
integrate(sech(x)**4/(a+b*sinh(x)),x)
```

output
```
Integral(sech(x)**4/(a + b*sinh(x)), x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 230 vs. $2(92) = 184$.

Time = 0.15 (sec) , antiderivative size = 230, normalized size of antiderivative = 2.30

$$\int \frac{\operatorname{sech}^4(x)}{a+b\sinh(x)}\,dx = \frac{b^4\log\left(\frac{be^{(-x)}-a-\sqrt{a^2+b^2}}{be^{(-x)}-a+\sqrt{a^2+b^2}}\right)}{(a^4+2\,a^2b^2+b^4)\sqrt{a^2+b^2}} + \frac{2\left(3\,b^3e^{(-x)}+3\,ab^2e^{(-4\,x)}+3\,b^3e^{(-5\,x)}+2\,a^3+5\,ab^2+6\,(a^3+2\,ab^2)e^{(-2\,x)}+2\,(2\,a^2b+5\,b^3)e^{(-3\,x)}\right)}{3\,(a^4+2\,a^2b^2+b^4+3\,(a^4+2\,a^2b^2+b^4)e^{(-2\,x)}+3\,(a^4+2\,a^2b^2+b^4)e^{(-4\,x)}+(a^4+2\,a^2b^2+b^4)e^{(-6\,x)})}$$

input
```
integrate(sech(x)^4/(a+b*sinh(x)),x, algorithm="maxima")
```

output
```
b^4*log((b*e^(-x) - a - sqrt(a^2 + b^2))/(b*e^(-x) - a + sqrt(a^2 + b^2)))
/((a^4 + 2*a^2*b^2 + b^4)*sqrt(a^2 + b^2)) + 2/3*(3*b^3*e^(-x) + 3*a*b^2*e
^(-4*x) + 3*b^3*e^(-5*x) + 2*a^3 + 5*a*b^2 + 6*(a^3 + 2*a*b^2)*e^(-2*x) +
2*(2*a^2*b + 5*b^3)*e^(-3*x))/(a^4 + 2*a^2*b^2 + b^4 + 3*(a^4 + 2*a^2*b^2
+ b^4)*e^(-2*x) + 3*(a^4 + 2*a^2*b^2 + b^4)*e^(-4*x) + (a^4 + 2*a^2*b^2 +
b^4)*e^(-6*x))
```

Giac [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 180, normalized size of antiderivative = 1.80

$$\int \frac{\operatorname{sech}^4(x)}{a+b\sinh(x)}\,dx = \frac{b^4\log\left(\frac{\left|2\,be^x+2\,a-2\,\sqrt{a^2+b^2}\right|}{\left|2\,be^x+2\,a+2\,\sqrt{a^2+b^2}\right|}\right)}{(a^4+2\,a^2b^2+b^4)\sqrt{a^2+b^2}} + \frac{2\left(3\,b^3e^{(5\,x)}-3\,ab^2e^{(4\,x)}+4\,a^2be^{(3\,x)}+10\,b^3e^{(3\,x)}-6\,a^3e^{(2\,x)}-12\,ab^2e^{(2\,x)}+3\,b^3e^x-2\,a^3-5\,ab^2\right)}{3\,(a^4+2\,a^2b^2+b^4)(e^{(2\,x)}+1)^3}$$

input
```
integrate(sech(x)^4/(a+b*sinh(x)),x, algorithm="giac")
```

output
```
b^4*log(abs(2*b*e^x + 2*a - 2*sqrt(a^2 + b^2))/abs(2*b*e^x + 2*a + 2*sqrt(
a^2 + b^2)))/((a^4 + 2*a^2*b^2 + b^4)*sqrt(a^2 + b^2)) + 2/3*(3*b^3*e^(5*x
) - 3*a*b^2*e^(4*x) + 4*a^2*b*e^(3*x) + 10*b^3*e^(3*x) - 6*a^3*e^(2*x) - 1
2*a*b^2*e^(2*x) + 3*b^3*e^x - 2*a^3 - 5*a*b^2)/((a^4 + 2*a^2*b^2 + b^4)*(e
^(2*x) + 1)^3)
```

Mupad [B] (verification not implemented)

Time = 2.54 (sec) , antiderivative size = 634, normalized size of antiderivative = 6.34

$$\int \frac{\text{sech}^4(x)}{a+b\sinh(x)}\,dx = \frac{\frac{2\,b^3\,e^x}{(a^2+b^2)^2}-\frac{2\,a\,b^2}{(a^2+b^2)^2}}{e^{2\,x}+1}-\frac{\frac{4\,(a^3+a\,b^2)}{(a^2+b^2)^2}-\frac{8\,e^x\,(a^2\,b+b^3)}{3\,(a^2+b^2)^2}}{2\,e^{2\,x}+e^{4\,x}+1}+\frac{\frac{8\,a}{3\,(a^2+b^2)}-\frac{8\,b\,e^x}{3\,(a^2+b^2)}}{3\,e^{2\,x}+3\,e^{4\,x}+e^{6\,x}+1} - \frac{2\,\text{atan}\left(\left(e^x\left(\frac{2\,b^2}{\sqrt{b^8}\,(a^2+b^2)^2\,(a^4+2\,a^2\,b^2+b^4)}+\frac{2\,a\,\left(a^5\,\sqrt{b^8}+2\,a^3\,b^2\,\sqrt{b^8}+a\,b^4\,\sqrt{b^8}\right)}{b^6\,\sqrt{-(a^2+b^2)^5}\,(a^4+2\,a^2\,b^2+b^4)\,\sqrt{-a^{10}-5\,a^8\,b^2-10\,a^6\,b^4-10\,a^4\,b^6-5\,a^2\,b^8-b^{10}}}\right)\right.\right.}{}$$

input
```
int(1/(cosh(x)^4*(a + b*sinh(x))),x)
```

output
```
((2*b^3*exp(x))/(a^2 + b^2)^2 - (2*a*b^2)/(a^2 + b^2)^2)/(exp(2*x) + 1) -
((4*(a*b^2 + a^3))/(a^2 + b^2)^2 - (8*exp(x)*(a^2*b + b^3))/(3*(a^2 + b^2)
^2))/(2*exp(2*x) + exp(4*x) + 1) + ((8*a)/(3*(a^2 + b^2)) - (8*b*exp(x))/(
3*(a^2 + b^2)))/(3*exp(2*x) + 3*exp(4*x) + exp(6*x) + 1) - (2*atan((exp(x)
*((2*b^2)/((b^8)^(1/2)*(a^2 + b^2)^2*(a^4 + b^4 + 2*a^2*b^2)) + (2*a*(a^5*
(b^8)^(1/2) + 2*a^3*b^2*(b^8)^(1/2) + a*b^4*(b^8)^(1/2)))/(b^6*(-(a^2 + b^
2)^5)^(1/2)*(a^4 + b^4 + 2*a^2*b^2)*(- a^10 - b^10 - 5*a^2*b^8 - 10*a^4*b^
6 - 10*a^6*b^4 - 5*a^8*b^2)^(1/2))) - (2*a*(b^5*(b^8)^(1/2) + 2*a^2*b^3*(b
^8)^(1/2) + a^4*b*(b^8)^(1/2)))/(b^6*(-(a^2 + b^2)^5)^(1/2)*(a^4 + b^4 + 2
*a^2*b^2)*(- a^10 - b^10 - 5*a^2*b^8 - 10*a^4*b^6 - 10*a^6*b^4 - 5*a^8*b^2
)^(1/2)))*((b^5*(- a^10 - b^10 - 5*a^2*b^8 - 10*a^4*b^6 - 10*a^6*b^4 - 5*a
^8*b^2)^(1/2))/2 + (a^4*b*(- a^10 - b^10 - 5*a^2*b^8 - 10*a^4*b^6 - 10*a^6
*b^4 - 5*a^8*b^2)^(1/2))/2 + a^2*b^3*(- a^10 - b^10 - 5*a^2*b^8 - 10*a^4*b
^6 - 10*a^6*b^4 - 5*a^8*b^2)^(1/2)))*(b^8)^(1/2))/(- a^10 - b^10 - 5*a^2*b
^8 - 10*a^4*b^6 - 10*a^6*b^4 - 5*a^8*b^2)^(1/2)
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 480, normalized size of antiderivative = 4.80

$$\int \frac{\operatorname{sech}^4(x)}{a+b\sinh(x)}\,dx = \frac{6e^{6x}\sqrt{a^2+b^2}\, atan\left(\frac{e^x bi+ai}{\sqrt{a^2+b^2}}\right) b^4 i + 18e^{4x}\sqrt{a^2+b^2}\, atan\left(\frac{e^x bi+ai}{\sqrt{a^2+b^2}}\right) b^4 i + 18e^{2x}\sqrt{a^2+b^2}\, atan\left(\frac{e^x bi+ai}{\sqrt{a^2+b^2}}\right) b^4 i +}{3e^{6x}a^6+9e^{6x}a^4b^2+9e^{6x}a^2b^4+3e^{6x}b^6}$$

input

```
int(sech(x)^4/(a+b*sinh(x)),x)
```

output

```
(2*(3*e**(6*x)*sqrt(a**2 + b**2)*atan((e**x*b*i + a*i)/sqrt(a**2 + b**2))*
b**4*i + 9*e**(4*x)*sqrt(a**2 + b**2)*atan((e**x*b*i + a*i)/sqrt(a**2 + b*
*2))*b**4*i + 9*e**(2*x)*sqrt(a**2 + b**2)*atan((e**x*b*i + a*i)/sqrt(a**2
 + b**2))*b**4*i + 3*sqrt(a**2 + b**2)*atan((e**x*b*i + a*i)/sqrt(a**2 + b
**2))*b**4*i + e**(6*x)*a**3*b**2 + e**(6*x)*a*b**4 + 3*e**(5*x)*a**2*b**3
 + 3*e**(5*x)*b**5 + 4*e**(3*x)*a**4*b + 14*e**(3*x)*a**2*b**3 + 10*e**(3*
x)*b**5 - 6*e**(2*x)*a**5 - 15*e**(2*x)*a**3*b**2 - 9*e**(2*x)*a*b**4 + 3*
e**x*a**2*b**3 + 3*e**x*b**5 - 2*a**5 - 6*a**3*b**2 - 4*a*b**4))/(3*(e**(6
*x)*a**6 + 3*e**(6*x)*a**4*b**2 + 3*e**(6*x)*a**2*b**4 + e**(6*x)*b**6 + 3
*e**(4*x)*a**6 + 9*e**(4*x)*a**4*b**2 + 9*e**(4*x)*a**2*b**4 + 3*e**(4*x)*
b**6 + 3*e**(2*x)*a**6 + 9*e**(2*x)*a**4*b**2 + 9*e**(2*x)*a**2*b**4 + 3*e
**(2*x)*b**6 + a**6 + 3*a**4*b**2 + 3*a**2*b**4 + b**6))
```

3.198 $\int \frac{\operatorname{sech}^5(x)}{a+b\sinh(x)}\,dx$

Optimal result

Integrand size = 13, antiderivative size = 135

$$\int \frac{\operatorname{sech}^5(x)}{a+b\sinh(x)}\,dx = \frac{a(3a^4+10a^2b^2+15b^4)\arctan(\sinh(x))}{8\,(a^2+b^2)^3} - \frac{b^5\log(\cosh(x))}{(a^2+b^2)^3} + \frac{b^5\log(a+b\sinh(x))}{(a^2+b^2)^3} + \frac{\operatorname{sech}^4(x)(b+a\sinh(x))}{4\,(a^2+b^2)} + \frac{\operatorname{sech}^2(x)\,(4b^3+a(3a^2+7b^2)\sinh(x))}{8\,(a^2+b^2)^2}$$

output
```
1/8*a*(3*a^4+10*a^2*b^2+15*b^4)*arctan(sinh(x))/(a^2+b^2)^3-b^5*ln(cosh(x)
)/(a^2+b^2)^3+b^5*ln(a+b*sinh(x))/(a^2+b^2)^3+sech(x)^4*(b+a*sinh(x))/(4*a
^2+4*b^2)+1/8*sech(x)^2*(4*b^3+a*(3*a^2+7*b^2)*sinh(x))/(a^2+b^2)^2
```

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 284 vs. $2(135) = 270$.

Time = 0.31 (sec) , antiderivative size = 284, normalized size of antiderivative = 2.10

$$\int \frac{\operatorname{sech}^5(x)}{a+b\sinh(x)}\,dx = \frac{-\left(\left(8b^6+3a^5\sqrt{-b^2}+15ab^4\sqrt{-b^2}-10a^3(-b^2)^{3/2}\right)\log\left(\sqrt{-b^2}-b\sinh(x)\right)\right)+16b^6\log(a+b\sinh(x))-}{}$$

input
```
Integrate[Sech[x]^5/(a + b*Sinh[x]),x]
```

output
```
(-((8*b^6 + 3*a^5*Sqrt[-b^2] + 15*a*b^4*Sqrt[-b^2] - 10*a^3*(-b^2)^(3/2))*
Log[Sqrt[-b^2] - b*Sinh[x]]) + 16*b^6*Log[a + b*Sinh[x]] - 8*b^6*Log[Sqrt[
-b^2] + b*Sinh[x]] + 3*a^5*Sqrt[-b^2]*Log[Sqrt[-b^2] + b*Sinh[x]] + 15*a*b
^4*Sqrt[-b^2]*Log[Sqrt[-b^2] + b*Sinh[x]] - 10*a^3*(-b^2)^(3/2)*Log[Sqrt[-
b^2] + b*Sinh[x]] + 8*b^4*(a^2 + b^2)*Sech[x]^2 + 4*b^2*(a^2 + b^2)^2*Sech
[x]^4 + 2*a*b*(3*a^4 + 10*a^2*b^2 + 7*b^4)*Sech[x]*Tanh[x] + 4*a*b*(a^2 +
b^2)^2*Sech[x]^3*Tanh[x])/(16*b*(a^2 + b^2)^3)
```

Rubi [A] (verified)

Time = 0.55 (sec) , antiderivative size = 215, normalized size of antiderivative = 1.59, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.692$, Rules used = {3042, 3147, 25, 496, 25, 686, 25, 657, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\operatorname{sech}^5(x)}{a+b\sinh(x)}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{\cos(ix)^5(a-ib\sin(ix))}dx$$

$$\Big\downarrow \text{3147}$$

$$-b^5 \int -\frac{1}{(a+b\sinh(x))\left(\sinh^2(x)b^2+b^2\right)^3}d(b\sinh(x))$$

$\Big\downarrow$ 25

$$b^5 \int \frac{1}{(a+b\sinh(x))\left(\sinh^2(x)b^2+b^2\right)^3}d(b\sinh(x))$$

$\Big\downarrow$ 496

$$-b^5 \left(\frac{\int -\frac{3a^2+3b\sinh(x)a+4b^2}{(a+b\sinh(x))\left(\sinh^2(x)b^2+b^2\right)^2}d(b\sinh(x))}{4b^2(a^2+b^2)}-\frac{ab\sinh(x)+b^2}{4b^2(a^2+b^2)\left(b^2\sinh^2(x)+b^2\right)^2}\right)$$

$\Big\downarrow$ 25

$$-b^5 \left(-\frac{\int \frac{3a^2+3b\sinh(x)a+4b^2}{(a+b\sinh(x))\left(\sinh^2(x)b^2+b^2\right)^2}d(b\sinh(x))}{4b^2(a^2+b^2)}-\frac{ab\sinh(x)+b^2}{4b^2(a^2+b^2)\left(b^2\sinh^2(x)+b^2\right)^2}\right)$$

$\Big\downarrow$ 686

$$-b^5 \left(-\frac{\frac{ab(3a^2+7b^2)\sinh(x)+4b^4}{2b^2(a^2+b^2)(b^2\sinh^2(x)+b^2)}-\frac{\int -\frac{3a^4+7b^2a^2+b\left(3a^2+7b^2\right)\sinh(x)a+8b^4}{(a+b\sinh(x))\left(\sinh^2(x)b^2+b^2\right)}d(b\sinh(x))}{2b^2(a^2+b^2)}}{4b^2(a^2+b^2)}-\frac{ab\sinh(x)+b^2}{4b^2(a^2+b^2)\left(b^2\sinh^2(x)+b^2\right)^2}\right)$$

$\Big\downarrow$ 25

$$-b^5 \left(-\frac{\frac{\int \frac{3a^4+7b^2a^2+b\left(3a^2+7b^2\right)\sinh(x)a+8b^4}{(a+b\sinh(x))\left(\sinh^2(x)b^2+b^2\right)}d(b\sinh(x))}{2b^2(a^2+b^2)}+\frac{ab(3a^2+7b^2)\sinh(x)+4b^4}{2b^2(a^2+b^2)(b^2\sinh^2(x)+b^2)}}{4b^2(a^2+b^2)}-\frac{ab\sinh(x)+b^2}{4b^2(a^2+b^2)\left(b^2\sinh^2(x)+b^2\right)^2}\right)$$

$\Big\downarrow$ 657

$$-b^5 \left(-\frac{\frac{\int \left(\frac{8b^4}{\left(a^2+b^2\right)(a+b\sinh(x))}+\frac{3a^5+10b^2a^3+15b^4a-8b^5\sinh(x)}{\left(a^2+b^2\right)\left(\sinh^2(x)b^2+b^2\right)}\right)d(b\sinh(x))}{2b^2(a^2+b^2)}+\frac{ab(3a^2+7b^2)\sinh(x)+4b^4}{2b^2(a^2+b^2)(b^2\sinh^2(x)+b^2)}}{4b^2(a^2+b^2)}-\frac{ab\sinh(x)+}{4b^2(a^2+b^2)\left(b^2\sinh\right.}\right.$$

$\Big\downarrow$ 2009

$$-b^5\left(-\frac{ab\sinh(x)+b^2}{4b^2\left(a^2+b^2\right)\left(b^2\sinh^2(x)+b^2\right)^2}-\frac{\frac{ab(3a^2+7b^2)\sinh(x)+4b^4}{2b^2(a^2+b^2)(b^2\sinh^2(x)+b^2)}+\frac{-\frac{4b^4\log\left(b^2\sinh^2(x)+b^2\right)}{a^2+b^2}+\frac{8b^4\log(a+b\sinh(x))}{a^2+b^2}+\frac{a\left(3a^4+1\right.}{}}{2b^2(a^2+b^2)}}{4b^2\left(a^2+b^2\right)}\right.$$

input
```
Int[Sech[x]^5/(a + b*Sinh[x]),x]
```

output
```
-(b^5*(-1/4*(b^2 + a*b*Sinh[x])/(b^2*(a^2 + b^2)*(b^2 + b^2*Sinh[x]^2)^2)
- (((a*(3*a^4 + 10*a^2*b^2 + 15*b^4)*ArcTan[Sinh[x]])/(b*(a^2 + b^2)) + (8
*b^4*Log[a + b*Sinh[x]])/(a^2 + b^2) - (4*b^4*Log[b^2 + b^2*Sinh[x]^2])/(a
^2 + b^2))/(2*b^2*(a^2 + b^2)) + (4*b^4 + a*b*(3*a^2 + 7*b^2)*Sinh[x])/(2*
b^2*(a^2 + b^2)*(b^2 + b^2*Sinh[x]^2)))/(4*b^2*(a^2 + b^2))))
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 496
```
Int[((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(-(a*d + b*c*x))*(c + d*x)^(n + 1)*((a + b*x^2)^(p + 1)/(2*a*(p + 1)*(b*c^2
 + a*d^2))), x] + Simp[1/(2*a*(p + 1)*(b*c^2 + a*d^2))   Int[(c + d*x)^n*(a
 + b*x^2)^(p + 1)*Simp[b*c^2*(2*p + 3) + a*d^2*(n + 2*p + 3) + b*c*d*(n + 2
*p + 4)*x, x], x], x] /; FreeQ[{a, b, c, d, n}, x] && LtQ[p, -1] && IntQuad
raticQ[a, 0, b, c, d, n, p, x]
```

rule 657
```
Int[(((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))^(n_.))/((a_) + (c_.)*(
x_)^2), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*((f + g*x)^n/(a + c*x^
2)), x], x] /; FreeQ[{a, c, d, e, f, g, m}, x] && IntegersQ[n]
```

rule 686
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_), x_Symbol] :> Simp[(-(d + e*x)^(m + 1))*(f*a*c*e - a*g*c*d + c*(c*d*f +
a*e*g)*x)*((a + c*x^2)^(p + 1)/(2*a*c*(p + 1)*(c*d^2 + a*e^2))), x] + Simp[
1/(2*a*c*(p + 1)*(c*d^2 + a*e^2))   Int[(d + e*x)^m*(a + c*x^2)^(p + 1)*Sim
p[f*(c^2*d^2*(2*p + 3) + a*c*e^2*(m + 2*p + 3)) - a*c*d*e*g*m + c*e*(c*d*f
+ a*e*g)*(m + 2*p + 4)*x, x], x], x] /; FreeQ[{a, c, d, e, f, g}, x] && LtQ
[p, -1] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3147
```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Simp[1/(b^p*f)   Subst[Int[(a + x)^m*(b^2 - x^2)^((p - 1)
/2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && IntegerQ[(p
 - 1)/2] && NeQ[a^2 - b^2, 0]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 312 vs. $2(132) = 264$.

Time = 78.74 (sec) , antiderivative size = 313, normalized size of antiderivative = 2.32

method	result
default	$\frac{b^{5} \ln\left(\tanh\left(\frac{x}{2}\right)^{2} a-2 b \tanh\left(\frac{x}{2}\right)-a\right)}{a^{6}+3 a^{4} b^{2}+3 a^{2} b^{4}+b^{6}}+\frac{2\left(\left(-\frac{5}{8} a^{5}-\frac{7}{4} a^{3} b^{2}-\frac{9}{8} a\,b^{4}\right) \tanh\left(\frac{x}{2}\right)^{7}+\left(-a^{4} b-3 a^{2} b^{3}-2 b^{5}\right) \tanh\left(\frac{x}{2}\right)^{6}+\left(\frac{3}{8} a^{5}+\frac{1}{4} a^{3} b^{2}-\frac{1}{8} a\,b^{4}\right) \tanh}{}$
risch	$\frac{\left(3\,{\mathrm e}^{6 x} a^{3}+7\,{\mathrm e}^{6 x} a\,b^{2}+8 b^{3} {\mathrm e}^{5 x}+11\,{\mathrm e}^{4 x} a^{3}+15\,{\mathrm e}^{4 x} a\,b^{2}+16 a^{2} b\,{\mathrm e}^{3 x}+32\,{\mathrm e}^{3 x} b^{3}-11 a^{3} {\mathrm e}^{2 x}-15 a\,{\mathrm e}^{2 x} b^{2}+8 b^{3} {\mathrm e}^{x}-3 a^{3}-7 a\,b^{2}\right) {\mathrm e}^{x}}{4\left(a^{4}+2 a^{2} b^{2}+b^{4}\right) \left({\mathrm e}^{2 x}+1\right)^{4}}+\frac{3 i \ln}{8(a^{6}+3 a^{4}}$

input
```
int(sech(x)^5/(a+b*sinh(x)),x,method=_RETURNVERBOSE)
```

output
```
b^5/(a^6+3*a^4*b^2+3*a^2*b^4+b^6)*ln(tanh(1/2*x)^2*a-2*b*tanh(1/2*x)-a)+2/
(a^6+3*a^4*b^2+3*a^2*b^4+b^6)*(((-5/8*a^5-7/4*a^3*b^2-9/8*a*b^4)*tanh(1/2*
x)^7+(-a^4*b-3*a^2*b^3-2*b^5)*tanh(1/2*x)^6+(3/8*a^5+1/4*a^3*b^2-1/8*a*b^4
)*tanh(1/2*x)^5+(-2*a^2*b^3-2*b^5)*tanh(1/2*x)^4+(-3/8*a^5-1/4*a^3*b^2+1/8
*a*b^4)*tanh(1/2*x)^3+(-a^4*b-3*a^2*b^3-2*b^5)*tanh(1/2*x)^2+(5/8*a^5+7/4*
a^3*b^2+9/8*a*b^4)*tanh(1/2*x))/(1+tanh(1/2*x)^2)^4-1/2*b^5*ln(1+tanh(1/2*
x)^2)+1/8*(3*a^5+10*a^3*b^2+15*a*b^4)*arctan(tanh(1/2*x)))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2707 vs. $2(129) = 258$.

Time = 0.14 (sec) , antiderivative size = 2707, normalized size of antiderivative = 20.05

$$\int \frac{\operatorname{sech}^5(x)}{a+b\sinh(x)}\,dx = \text{Too large to display}$$

input
```
integrate(sech(x)^5/(a+b*sinh(x)),x, algorithm="fricas")
```

output
```
1/4*((3*a^5 + 10*a^3*b^2 + 7*a*b^4)*cosh(x)^7 + (3*a^5 + 10*a^3*b^2 + 7*a*
b^4)*sinh(x)^7 + 8*(a^2*b^3 + b^5)*cosh(x)^6 + (8*a^2*b^3 + 8*b^5 + 7*(3*a
^5 + 10*a^3*b^2 + 7*a*b^4)*cosh(x))*sinh(x)^6 + (11*a^5 + 26*a^3*b^2 + 15*
a*b^4)*cosh(x)^5 + (11*a^5 + 26*a^3*b^2 + 15*a*b^4 + 21*(3*a^5 + 10*a^3*b^
2 + 7*a*b^4)*cosh(x)^2 + 48*(a^2*b^3 + b^5)*cosh(x))*sinh(x)^5 + 16*(a^4*b
 + 3*a^2*b^3 + 2*b^5)*cosh(x)^4 + (16*a^4*b + 48*a^2*b^3 + 32*b^5 + 35*(3*
a^5 + 10*a^3*b^2 + 7*a*b^4)*cosh(x)^3 + 120*(a^2*b^3 + b^5)*cosh(x)^2 + 5*
(11*a^5 + 26*a^3*b^2 + 15*a*b^4)*cosh(x))*sinh(x)^4 - (11*a^5 + 26*a^3*b^2
 + 15*a*b^4)*cosh(x)^3 - (11*a^5 + 26*a^3*b^2 + 15*a*b^4 - 35*(3*a^5 + 10*
a^3*b^2 + 7*a*b^4)*cosh(x)^4 - 160*(a^2*b^3 + b^5)*cosh(x)^3 - 10*(11*a^5
+ 26*a^3*b^2 + 15*a*b^4)*cosh(x)^2 - 64*(a^4*b + 3*a^2*b^3 + 2*b^5)*cosh(x
))*sinh(x)^3 + 8*(a^2*b^3 + b^5)*cosh(x)^2 + (21*(3*a^5 + 10*a^3*b^2 + 7*a
*b^4)*cosh(x)^5 + 8*a^2*b^3 + 8*b^5 + 120*(a^2*b^3 + b^5)*cosh(x)^4 + 10*(
11*a^5 + 26*a^3*b^2 + 15*a*b^4)*cosh(x)^3 + 96*(a^4*b + 3*a^2*b^3 + 2*b^5)
*cosh(x)^2 - 3*(11*a^5 + 26*a^3*b^2 + 15*a*b^4)*cosh(x))*sinh(x)^2 + ((3*a
^5 + 10*a^3*b^2 + 15*a*b^4)*cosh(x)^8 + 8*(3*a^5 + 10*a^3*b^2 + 15*a*b^4)*
cosh(x)*sinh(x)^7 + (3*a^5 + 10*a^3*b^2 + 15*a*b^4)*sinh(x)^8 + 4*(3*a^5 +
 10*a^3*b^2 + 15*a*b^4)*cosh(x)^6 + 4*(3*a^5 + 10*a^3*b^2 + 15*a*b^4 + 7*(
3*a^5 + 10*a^3*b^2 + 15*a*b^4)*cosh(x)^2)*sinh(x)^6 + 8*(7*(3*a^5 + 10*a^3
*b^2 + 15*a*b^4)*cosh(x)^3 + 3*(3*a^5 + 10*a^3*b^2 + 15*a*b^4)*cosh(x))...
```

Sympy [F]

$$\int \frac{\operatorname{sech}^5(x)}{a+b\sinh(x)}\,dx=\int \frac{\operatorname{sech}^5{(x)}}{a+b\sinh{(x)}}\,dx$$

input
```
integrate(sech(x)**5/(a+b*sinh(x)),x)
```

output
```
Integral(sech(x)**5/(a + b*sinh(x)), x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 345 vs. 2(129) = 258.

Time = 0.13 (sec) , antiderivative size = 345, normalized size of antiderivative = 2.56

$$\int \frac{\operatorname{sech}^5(x)}{a+b\sinh(x)}\,dx = \frac{b^5\log\left(-2\,ae^{(-x)}+be^{(-2\,x)}-b\right)}{a^6+3\,a^4b^2+3\,a^2b^4+b^6} - \frac{b^5\log\left(e^{(-2\,x)}+1\right)}{a^6+3\,a^4b^2+3\,a^2b^4+b^6} - \frac{(3\,a^5+10\,a^3b^2+15\,ab^4)\arctan\left(e^{(-x)}\right)}{4\,(a^6+3\,a^4b^2+3\,a^2b^4+b^6)} + \frac{8\,b^3e^{(-2\,x)}+8\,b^3e^{(-6\,x)}+(3\,a^3+7\,ab^2)e^{(-x)}+(11\,a^3+15\,ab^2)e^{(-3\,x)}+16\,(a^2b+2\,b^3)e^{(-4\,x)}-(11\,a^3+}{4\,(a^4+2\,a^2b^2+b^4+4\,(a^4+2\,a^2b^2+b^4)e^{(-2\,x)}+6\,(a^4+2\,a^2b^2+b^4)e^{(-4\,x)}+4\,(a^4+2\,a^2b^2+b^4)e}$$

input
```
integrate(sech(x)^5/(a+b*sinh(x)),x, algorithm="maxima")
```

output
```
b^5*log(-2*a*e^(-x) + b*e^(-2*x) - b)/(a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6)
- b^5*log(e^(-2*x) + 1)/(a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6) - 1/4*(3*a^5 +
 10*a^3*b^2 + 15*a*b^4)*arctan(e^(-x))/(a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6)
 + 1/4*(8*b^3*e^(-2*x) + 8*b^3*e^(-6*x) + (3*a^3 + 7*a*b^2)*e^(-x) + (11*a
^3 + 15*a*b^2)*e^(-3*x) + 16*(a^2*b + 2*b^3)*e^(-4*x) - (11*a^3 + 15*a*b^2
)*e^(-5*x) - (3*a^3 + 7*a*b^2)*e^(-7*x))/(a^4 + 2*a^2*b^2 + b^4 + 4*(a^4 +
 2*a^2*b^2 + b^4)*e^(-2*x) + 6*(a^4 + 2*a^2*b^2 + b^4)*e^(-4*x) + 4*(a^4 +
 2*a^2*b^2 + b^4)*e^(-6*x) + (a^4 + 2*a^2*b^2 + b^4)*e^(-8*x))
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 369 vs. 2(129) = 258.

Time = 0.11 (sec) , antiderivative size = 369, normalized size of antiderivative = 2.73

$$\int \frac{\operatorname{sech}^5(x)}{a+b\sinh(x)}\,dx = \frac{b^6\log\left(\left|-b\left(e^{(-x)}-e^x\right)+2\,a\right|\right)}{a^6b+3\,a^4b^3+3\,a^2b^5+b^7} - \frac{b^5\log\left(\left(e^{(-x)}-e^x\right)^2+4\right)}{2\,(a^6+3\,a^4b^2+3\,a^2b^4+b^6)} + \frac{\left(\pi+2\,\arctan\left(\frac{1}{2}\left(e^{(2\,x)}-1\right)e^{(-x)}\right)\right)(3\,a^5+10\,a^3b^2+15\,ab^4)}{16\,(a^6+3\,a^4b^2+3\,a^2b^4+b^6)} + \frac{3\,b^5\left(e^{(-x)}-e^x\right)^4-3\,a^5\left(e^{(-x)}-e^x\right)^3-10\,a^3b^2\left(e^{(-x)}-e^x\right)^3-7\,ab^4\left(e^{(-x)}-e^x\right)^3+8\,a^2b^3\left(e^{(-x)}-e^x\right)^2}{4\,(a^6+3\,a^4b^2+3\,a^2}$$

input
```
integrate(sech(x)^5/(a+b*sinh(x)),x, algorithm="giac")
```

output
```
b^6*log(abs(-b*(e^(-x) - e^x) + 2*a))/(a^6*b + 3*a^4*b^3 + 3*a^2*b^5 + b^7
) - 1/2*b^5*log((e^(-x) - e^x)^2 + 4)/(a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6)
+ 1/16*(pi + 2*arctan(1/2*(e^(2*x) - 1)*e^(-x)))*(3*a^5 + 10*a^3*b^2 + 15*
a*b^4)/(a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6) + 1/4*(3*b^5*(e^(-x) - e^x)^4 -
 3*a^5*(e^(-x) - e^x)^3 - 10*a^3*b^2*(e^(-x) - e^x)^3 - 7*a*b^4*(e^(-x) -
e^x)^3 + 8*a^2*b^3*(e^(-x) - e^x)^2 + 32*b^5*(e^(-x) - e^x)^2 - 20*a^5*(e^
(-x) - e^x) - 56*a^3*b^2*(e^(-x) - e^x) - 36*a*b^4*(e^(-x) - e^x) + 16*a^4
*b + 64*a^2*b^3 + 96*b^5)/((a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6)*((e^(-x) -
e^x)^2 + 4)^2)
```

Mupad [B] (verification not implemented)

Time = 5.85 (sec) , antiderivative size = 548, normalized size of antiderivative = 4.06

$$\int \frac{\mathrm{sech}^5(x)}{a+b\sinh(x)}\,dx = \frac{\frac{2\,(2\,a^2\,b+b^3)}{(a^2+b^2)^2}-\frac{e^x\,(3\,a\,b^2-a^3)}{2\,(a^2+b^2)^2}}{2\,e^{2\,x}+e^{4\,x}+1}-\frac{\frac{8\,(a^2\,b+b^3)}{(a^2+b^2)^2}+\frac{6\,e^x\,(a^3+a\,b^2)}{(a^2+b^2)^2}}{3\,e^{2\,x}+3\,e^{4\,x}+e^{6\,x}+1}+\frac{\frac{4\,b}{a^2+b^2}+\frac{4\,a\,e^x}{a^2+b^2}}{4\,e^{2\,x}+6\,e^{4\,x}+4\,e^{6\,x}+e^{8\,x}+1}+\frac{\frac{2\,(a^2\,b^3+b^5)}{(a^2+b^2)^3}+\frac{e^x\,(3\,a^5+10\,a^3\,b^2+7\,a\,b^4)}{4\,(a^2+b^2)^3}}{e^{2\,x}+1}+\frac{b^5\,\ln\left(256\,b^{11}\,e^{2\,x}-9\,a^{10}\,b-256\,b^{11}-225\,a^2\,b^9-300\,a^4\,b^7-190\,a^6\,b^5-60\,a^8\,b^3+18\,a^{11}\,e^x+225\,a^2\,b\right.}{}-\frac{\ln\left(1+e^x\,1i\right)\,\left(-a^2\,3i+9\,a\,b+b^2\,8i\right)}{8\,\left(-a^3-a^2\,b\,3i+3\,a\,b^2+b^3\,1i\right)}-\frac{\ln\left(e^x+1i\right)\,\left(-3\,a^2+a\,b\,9i+8\,b^2\right)}{8\,\left(-a^3\,1i-3\,a^2\,b+a\,b^2\,3i+b^3\right)}$$

input
```
int(1/(cosh(x)^5*(a + b*sinh(x))),x)
```

output
```
((2*(2*a^2*b + b^3))/(a^2 + b^2)^2 - (exp(x)*(3*a*b^2 - a^3))/(2*(a^2 + b^
2)^2))/(2*exp(2*x) + exp(4*x) + 1) - ((8*(a^2*b + b^3))/(a^2 + b^2)^2 + (6
*exp(x)*(a*b^2 + a^3))/(a^2 + b^2)^2)/(3*exp(2*x) + 3*exp(4*x) + exp(6*x)
+ 1) + ((4*b)/(a^2 + b^2) + (4*a*exp(x))/(a^2 + b^2))/(4*exp(2*x) + 6*exp(
4*x) + 4*exp(6*x) + exp(8*x) + 1) + ((2*(b^5 + a^2*b^3))/(a^2 + b^2)^3 + (
exp(x)*(7*a*b^4 + 3*a^5 + 10*a^3*b^2))/(4*(a^2 + b^2)^3))/(exp(2*x) + 1) +
 (b^5*log(256*b^11*exp(2*x) - 9*a^10*b - 256*b^11 - 225*a^2*b^9 - 300*a^4*
b^7 - 190*a^6*b^5 - 60*a^8*b^3 + 18*a^11*exp(x) + 225*a^2*b^9*exp(2*x) + 3
00*a^4*b^7*exp(2*x) + 190*a^6*b^5*exp(2*x) + 60*a^8*b^3*exp(2*x) + 512*a*b
^10*exp(x) + 9*a^10*b*exp(2*x) + 450*a^3*b^8*exp(x) + 600*a^5*b^6*exp(x) +
 380*a^7*b^4*exp(x) + 120*a^9*b^2*exp(x)))/(a^6 + b^6 + 3*a^2*b^4 + 3*a^4*
b^2) - (log(exp(x)*1i + 1)*(9*a*b - a^2*3i + b^2*8i))/(8*(3*a*b^2 - a^2*b*
3i - a^3 + b^3*1i)) - (log(exp(x) + 1i)*(a*b*9i - 3*a^2 + 8*b^2))/(8*(a*b^
2*3i - 3*a^2*b - a^3*1i + b^3))
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 850, normalized size of antiderivative = 6.30

$$\int \frac{\operatorname{sech}^5(x)}{a+b\sinh(x)}\,dx = \text{Too large to display}$$

input
```
int(sech(x)^5/(a+b*sinh(x)),x)
```

output

```
(3*e**(8*x)*atan(e**x)*a**5 + 10*e**(8*x)*atan(e**x)*a**3*b**2 + 15*e**(8*
x)*atan(e**x)*a*b**4 + 12*e**(6*x)*atan(e**x)*a**5 + 40*e**(6*x)*atan(e**x
)*a**3*b**2 + 60*e**(6*x)*atan(e**x)*a*b**4 + 18*e**(4*x)*atan(e**x)*a**5
+ 60*e**(4*x)*atan(e**x)*a**3*b**2 + 90*e**(4*x)*atan(e**x)*a*b**4 + 12*e*
*(2*x)*atan(e**x)*a**5 + 40*e**(2*x)*atan(e**x)*a**3*b**2 + 60*e**(2*x)*at
an(e**x)*a*b**4 + 3*atan(e**x)*a**5 + 10*atan(e**x)*a**3*b**2 + 15*atan(e*
*x)*a*b**4 - 4*e**(8*x)*log(e**(2*x) + 1)*b**5 + 4*e**(8*x)*log(e**(2*x)*b
 + 2*e**x*a - b)*b**5 - 2*e**(8*x)*a**2*b**3 - 2*e**(8*x)*b**5 + 3*e**(7*x
)*a**5 + 10*e**(7*x)*a**3*b**2 + 7*e**(7*x)*a*b**4 - 16*e**(6*x)*log(e**(2
*x) + 1)*b**5 + 16*e**(6*x)*log(e**(2*x)*b + 2*e**x*a - b)*b**5 + 11*e**(5
*x)*a**5 + 26*e**(5*x)*a**3*b**2 + 15*e**(5*x)*a*b**4 - 24*e**(4*x)*log(e*
*(2*x) + 1)*b**5 + 24*e**(4*x)*log(e**(2*x)*b + 2*e**x*a - b)*b**5 + 16*e*
*(4*x)*a**4*b + 36*e**(4*x)*a**2*b**3 + 20*e**(4*x)*b**5 - 11*e**(3*x)*a**
5 - 26*e**(3*x)*a**3*b**2 - 15*e**(3*x)*a*b**4 - 16*e**(2*x)*log(e**(2*x)
+ 1)*b**5 + 16*e**(2*x)*log(e**(2*x)*b + 2*e**x*a - b)*b**5 - 3*e**x*a**5
- 10*e**x*a**3*b**2 - 7*e**x*a*b**4 - 4*log(e**(2*x) + 1)*b**5 + 4*log(e**
(2*x)*b + 2*e**x*a - b)*b**5 - 2*a**2*b**3 - 2*b**5)/(4*(e**(8*x)*a**6 + 3
*e**(8*x)*a**4*b**2 + 3*e**(8*x)*a**2*b**4 + e**(8*x)*b**6 + 4*e**(6*x)*a*
*6 + 12*e**(6*x)*a**4*b**2 + 12*e**(6*x)*a**2*b**4 + 4*e**(6*x)*b**6 + 6*e
**(4*x)*a**6 + 18*e**(4*x)*a**4*b**2 + 18*e**(4*x)*a**2*b**4 + 6*e**(4*...
```

3.199 $\int \frac{\operatorname{sech}^6(x)}{a+b\sinh(x)}\,dx$

Optimal result

Integrand size = 13, antiderivative size = 146

$$\int \frac{\operatorname{sech}^6(x)}{a+b\sinh(x)}\,dx = -\frac{2b^6\operatorname{arctanh}\left(\frac{b-a\tanh\left(\frac{x}{2}\right)}{\sqrt{a^2+b^2}}\right)}{(a^2+b^2)^{7/2}} + \frac{\operatorname{sech}^5(x)(b+a\sinh(x))}{5\,(a^2+b^2)} + \frac{\operatorname{sech}^3(x)\,(5b^3+a(4a^2+9b^2)\sinh(x))}{15\,(a^2+b^2)^2} + \frac{\operatorname{sech}(x)\,(15b^5+a(8a^4+26a^2b^2+33b^4)\sinh(x))}{15\,(a^2+b^2)^3}$$

output
```
-2*b^6*arctanh((b-a*tanh(1/2*x))/(a^2+b^2)^(1/2))/(a^2+b^2)^(7/2)+sech(x)^
5*(b+a*sinh(x))/(5*a^2+5*b^2)+1/15*sech(x)^3*(5*b^3+a*(4*a^2+9*b^2)*sinh(x
))/(a^2+b^2)^2+1/15*sech(x)*(15*b^5+a*(8*a^4+26*a^2*b^2+33*b^4)*sinh(x))/(
a^2+b^2)^3
```

Mathematica [A] (verified)

Time = 0.35 (sec) , antiderivative size = 146, normalized size of antiderivative = 1.00

$$\int \frac{\operatorname{sech}^6(x)}{a+b\sinh(x)}\,dx$$
$$= \frac{\frac{30b^6\arctan\left(\frac{b-a\tanh\left(\frac{x}{2}\right)}{\sqrt{-a^2-b^2}}\right)}{\sqrt{-a^2-b^2}}+15b^5\operatorname{sech}(x)+3(a^2+b^2)^2\operatorname{sech}^5(x)(b+a\sinh(x))+(a^2+b^2)\operatorname{sech}^3(x)\left(5b^3+a(4\right.}{15\left(a^2+b^2\right)^3}$$

input
```
Integrate[Sech[x]^6/(a + b*Sinh[x]),x]
```

output
```
((30*b^6*ArcTan[(b - a*Tanh[x/2])/Sqrt[-a^2 - b^2]])/Sqrt[-a^2 - b^2] + 15
*b^5*Sech[x] + 3*(a^2 + b^2)^2*Sech[x]^5*(b + a*Sinh[x]) + (a^2 + b^2)*Sec
h[x]^3*(5*b^3 + a*(4*a^2 + 9*b^2)*Sinh[x]) + a*(8*a^4 + 26*a^2*b^2 + 33*b^
4)*Tanh[x])/(15*(a^2 + b^2)^3)
```

Rubi [A] (verified)

Time = 1.01 (sec) , antiderivative size = 176, normalized size of antiderivative = 1.21, number of steps used = 14, number of rules used = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 1.000, Rules used = {3042, 3175, 25, 3042, 3345, 25, 3042, 3345, 27, 3042, 3139, 1083, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\operatorname{sech}^6(x)}{a+b\sinh(x)}\,dx$$
$$\Big\downarrow \text{3042}$$
$$\int \frac{1}{\cos(ix)^6(a-ib\sin(ix))}dx$$
$$\Big\downarrow \text{3175}$$
$$\frac{\operatorname{sech}^5(x)(a\sinh(x)+b)}{5\left(a^2+b^2\right)}-\frac{\int -\frac{\operatorname{sech}^4(x)\left(4a^2+4b\sinh(x)a+5b^2\right)}{a+b\sinh(x)}dx}{5\left(a^2+b^2\right)}$$

$$\Big\downarrow \text{25}$$

$$\frac{\int \frac{\text{sech}^4(x)\left(4a^2+4b\sinh(x)a+5b^2\right)}{a+b\sinh(x)}dx}{5\left(a^2+b^2\right)}+\frac{\text{sech}^5(x)(a\sinh(x)+b)}{5\left(a^2+b^2\right)}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\text{sech}^5(x)(a\sinh(x)+b)}{5\left(a^2+b^2\right)}+\frac{\int \frac{4a^2-4ib\sin(ix)a+5b^2}{\cos(ix)^4(a-ib\sin(ix))}dx}{5\left(a^2+b^2\right)}$$

$$\Big\downarrow \text{3345}$$

$$\frac{\frac{\text{sech}^3(x)\left(a\left(4a^2+9b^2\right)\sinh(x)+5b^3\right)}{3(a^2+b^2)}-\frac{\int -\frac{\text{sech}^2(x)\left(8a^4+18b^2a^2+2b\left(4a^2+9b^2\right)\sinh(x)a+15b^4\right)}{a+b\sinh(x)}dx}{3(a^2+b^2)}}{5\left(a^2+b^2\right)}+\frac{\text{sech}^5(x)(a\sinh(x)+b)}{5\left(a^2+b^2\right)}$$

$$\Big\downarrow \text{25}$$

$$\frac{\frac{\int \frac{\text{sech}^2(x)\left(8a^4+18b^2a^2+2b\left(4a^2+9b^2\right)\sinh(x)a+15b^4\right)}{a+b\sinh(x)}dx}{3(a^2+b^2)}+\frac{\text{sech}^3(x)\left(a\left(4a^2+9b^2\right)\sinh(x)+5b^3\right)}{3(a^2+b^2)}}{5\left(a^2+b^2\right)}+\frac{\text{sech}^5(x)(a\sinh(x)+b)}{5\left(a^2+b^2\right)}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\text{sech}^5(x)(a\sinh(x)+b)}{5\left(a^2+b^2\right)}+\frac{\frac{\text{sech}^3(x)\left(a\left(4a^2+9b^2\right)\sinh(x)+5b^3\right)}{3(a^2+b^2)}+\frac{\int \frac{8a^4+18b^2a^2-2ib\left(4a^2+9b^2\right)\sin(ix)a+15b^4}{\cos(ix)^2(a-ib\sin(ix))}dx}{3(a^2+b^2)}}{5\left(a^2+b^2\right)}$$

$$\Big\downarrow \text{3345}$$

$$\frac{\frac{\frac{\text{sech}(x)\left(a\left(8a^4+26a^2b^2+33b^4\right)\sinh(x)+15b^5\right)}{a^2+b^2}-\frac{\int -\frac{15b^6}{a+b\sinh(x)}dx}{a^2+b^2}}{3(a^2+b^2)}+\frac{\text{sech}^3(x)\left(a\left(4a^2+9b^2\right)\sinh(x)+5b^3\right)}{3(a^2+b^2)}}{5\left(a^2+b^2\right)}+\frac{\text{sech}^5(x)(a\sinh(x)+b)}{5\left(a^2+b^2\right)}$$

$$\Big\downarrow \text{27}$$

$$\frac{\frac{\frac{15b^6\int \frac{1}{a+b\sinh(x)}dx}{a^2+b^2}+\frac{\text{sech}(x)\left(a\left(8a^4+26a^2b^2+33b^4\right)\sinh(x)+15b^5\right)}{a^2+b^2}}{3(a^2+b^2)}+\frac{\text{sech}^3(x)\left(a\left(4a^2+9b^2\right)\sinh(x)+5b^3\right)}{3(a^2+b^2)}}{5\left(a^2+b^2\right)}+\frac{\text{sech}^5(x)(a\sinh(x)+b)}{5\left(a^2+b^2\right)}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\operatorname{sech}^5(x)(a\sinh(x)+b)}{5(a^2+b^2)}+\frac{\frac{\operatorname{sech}^3(x)\left(a\left(4a^2+9b^2\right)\sinh(x)+5b^3\right)}{3(a^2+b^2)}+\frac{\frac{\operatorname{sech}(x)\left(a\left(8a^4+26a^2b^2+33b^4\right)\sinh(x)+15b^5\right)}{a^2+b^2}+\frac{15b^6\int\frac{1}{a-ib\sin(ix)}dx}{a^2+b^2}}{3(a^2+b^2)}}{5(a^2+b^2)}$$

$$\Big\downarrow \text{3139}$$

$$\frac{\frac{\frac{30b^6\int\frac{1}{-a\tanh^2\left(\frac{x}{2}\right)+2b\tanh\left(\frac{x}{2}\right)+a}d\tanh\left(\frac{x}{2}\right)}{a^2+b^2}+\frac{\operatorname{sech}(x)\left(a\left(8a^4+26a^2b^2+33b^4\right)\sinh(x)+15b^5\right)}{a^2+b^2}}{3(a^2+b^2)}+\frac{\operatorname{sech}^3(x)\left(a\left(4a^2+9b^2\right)\sinh(x)+5b^3\right)}{3(a^2+b^2)}}{5(a^2+b^2)}+\frac{\operatorname{sech}^5(x)(a\sinh(x)+b)}{5(a^2+b^2)}$$

$$\Big\downarrow \text{1083}$$

$$\frac{\frac{\frac{\operatorname{sech}(x)\left(a\left(8a^4+26a^2b^2+33b^4\right)\sinh(x)+15b^5\right)}{a^2+b^2}-\frac{60b^6\int\frac{1}{4\left(a^2+b^2\right)-\left(2b-2a\tanh\left(\frac{x}{2}\right)\right)^2}d\left(2b-2a\tanh\left(\frac{x}{2}\right)\right)}{a^2+b^2}}{3(a^2+b^2)}+\frac{\operatorname{sech}^3(x)\left(a\left(4a^2+9b^2\right)\sinh(x)+5b^3\right)}{3(a^2+b^2)}}{5(a^2+b^2)}+\frac{\operatorname{sech}^5(x)(a\sinh(x)+b)}{5(a^2+b^2)}$$

$$\Big\downarrow \text{219}$$

$$\frac{\operatorname{sech}^5(x)(a\sinh(x)+b)}{5(a^2+b^2)}+\frac{\frac{\operatorname{sech}^3(x)\left(a\left(4a^2+9b^2\right)\sinh(x)+5b^3\right)}{3(a^2+b^2)}+\frac{\frac{\operatorname{sech}(x)\left(a\left(8a^4+26a^2b^2+33b^4\right)\sinh(x)+15b^5\right)}{a^2+b^2}-\frac{30b^6\operatorname{arctanh}\left(\frac{2b-2a\tanh\left(\frac{x}{2}\right)}{2\sqrt{a^2+b^2}}\right)}{\left(a^2+b^2\right)^{3/2}}}{3(a^2+b^2)}}{5(a^2+b^2)}$$

input
```
Int[Sech[x]^6/(a + b*Sinh[x]),x]
```

output
```
(Sech[x]^5*(b + a*Sinh[x]))/(5*(a^2 + b^2)) + ((Sech[x]^3*(5*b^3 + a*(4*a^
2 + 9*b^2)*Sinh[x]))/(3*(a^2 + b^2)) + ((-30*b^6*ArcTanh[(2*b - 2*a*Tanh[x
/2])/(2*Sqrt[a^2 + b^2])])/(a^2 + b^2)^(3/2) + (Sech[x]*(15*b^5 + a*(8*a^4
 + 26*a^2*b^2 + 33*b^4)*Sinh[x]))/(a^2 + b^2))/(3*(a^2 + b^2)))/(5*(a^2 +
b^2))
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1083
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Simp[-2   Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3139
```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Simp[2*(e/d)   Subst[Int[1/(a + 2*b*e*x + a
*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[a^2 - b^2, 0]
```

rule 3175
```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x])^
(m + 1)*((b - a*Sin[e + f*x])/(f*g*(a^2 - b^2)*(p + 1))), x] + Simp[1/(g^2*
(a^2 - b^2)*(p + 1))   Int[(g*Cos[e + f*x])^(p + 2)*(a + b*Sin[e + f*x])^m*
(a^2*(p + 2) - b^2*(m + p + 2) + a*b*(m + p + 3)*Sin[e + f*x]), x], x] /; F
reeQ[{a, b, e, f, g, m}, x] && NeQ[a^2 - b^2, 0] && LtQ[p, -1] && IntegersQ
[2*m, 2*p]
```

rule 3345
```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[(g*Co
s[e + f*x])^(p + 1)*(a + b*Sin[e + f*x])^(m + 1)*((b*c - a*d - (a*c - b*d)*
Sin[e + f*x])/(f*g*(a^2 - b^2)*(p + 1))), x] + Simp[1/(g^2*(a^2 - b^2)*(p +
 1))   Int[(g*Cos[e + f*x])^(p + 2)*(a + b*Sin[e + f*x])^m*Simp[c*(a^2*(p +
 2) - b^2*(m + p + 2)) + a*b*d*m + b*(a*c - b*d)*(m + p + 3)*Sin[e + f*x],
x], x], x] /; FreeQ[{a, b, c, d, e, f, g, m}, x] && NeQ[a^2 - b^2, 0] && Lt
Q[p, -1] && IntegerQ[2*m]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 349 vs. $2(137) = 274$.

Time = 150.02 (sec) , antiderivative size = 350, normalized size of antiderivative = 2.40

method	result
default	$\frac{2b^6 \operatorname{arctanh}\left(\frac{2a\tanh\left(\frac{x}{2}\right)-2b}{2\sqrt{a^2+b^2}}\right)}{(a^6+3a^4b^2+3a^2b^4+b^6)\sqrt{a^2+b^2}} - \frac{2\left(\left(-a^5-3a^3b^2-3ab^4\right)\tanh\left(\frac{x}{2}\right)^9+\left(-a^4b-3a^2b^3-3b^5\right)\tanh\left(\frac{x}{2}\right)^8+\left(-\frac{4}{3}a^5-\frac{16}{3}a^3b^2-8ab^4\right) \mathrm{ta}}{}$
risch	$-\frac{2\left(-15b^5e^{9x}+15ab^4e^{8x}-20a^2b^3e^{7x}-80b^5e^{7x}+30a^3b^2e^{6x}+90ab^4e^{6x}-48a^4be^{5x}-136a^2b^3e^{5x}-178e^{5x}b^5+80a^5e^{4x}+230a^3b^2e^{4x}\right.}{15(a^4+2a^2b^2+b^4)(e^{2x}+1)^5(a}$

input
```
int(sech(x)^6/(a+b*sinh(x)),x,method=_RETURNVERBOSE)
```

output
```
2*b^6/(a^6+3*a^4*b^2+3*a^2*b^4+b^6)/(a^2+b^2)^(1/2)*arctanh(1/2*(2*a*tanh(
1/2*x)-2*b)/(a^2+b^2)^(1/2))-2/(a^6+3*a^4*b^2+3*a^2*b^4+b^6)*((-a^5-3*a^3*
b^2-3*a*b^4)*tanh(1/2*x)^9+(-a^4*b-3*a^2*b^3-3*b^5)*tanh(1/2*x)^8+(-4/3*a^
5-16/3*a^3*b^2-8*a*b^4)*tanh(1/2*x)^7+(-2*a^2*b^3-6*b^5)*tanh(1/2*x)^6+(-5
8/15*a^5-166/15*a^3*b^2-66/5*a*b^4)*tanh(1/2*x)^5+(-2*a^4*b-16/3*a^2*b^3-2
8/3*b^5)*tanh(1/2*x)^4+(-4/3*a^5-16/3*a^3*b^2-8*a*b^4)*tanh(1/2*x)^3+(-2/3
*a^2*b^3-14/3*b^5)*tanh(1/2*x)^2+(-a^5-3*a^3*b^2-3*a*b^4)*tanh(1/2*x)-1/5*
a^4*b-11/15*a^2*b^3-23/15*b^5)/(1+tanh(1/2*x)^2)^5
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 3175 vs. 2(136) = 272.

Time = 0.11 (sec) , antiderivative size = 3175, normalized size of antiderivative = 21.75

$$\int \frac{\operatorname{sech}^6(x)}{a+b\sinh(x)}\,dx = \text{Too large to display}$$

input
```
integrate(sech(x)^6/(a+b*sinh(x)),x, algorithm="fricas")
```

output
```
Too large to include
```

Sympy [F]

$$\int \frac{\operatorname{sech}^6(x)}{a+b\sinh(x)}\,dx = \int \frac{\operatorname{sech}^6(x)}{a+b\sinh(x)}\,dx$$

input
```
integrate(sech(x)**6/(a+b*sinh(x)),x)
```

output
```
Integral(sech(x)**6/(a + b*sinh(x)), x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 438 vs. 2(136) = 272.

Time = 0.13 (sec) , antiderivative size = 438, normalized size of antiderivative = 3.00

$$\int \frac{\operatorname{sech}^6(x)}{a+b\sinh(x)}\,dx = \frac{b^6\log\left(\frac{be^{(-x)}-a-\sqrt{a^2+b^2}}{be^{(-x)}-a+\sqrt{a^2+b^2}}\right)}{(a^6+3a^4b^2+3a^2b^4+b^6)\sqrt{a^2+b^2}} + \frac{2\left(15b^5e^{(-x)}+15ab^4e^{(-8x)}+15b^5e^{(-9x)}+8a^5+26a^3b^2+33ab^4+10(4a^5+13a^3b^2+15ab^4)e^{(-2x)}\right.}{15\left(a^6+3a^4b^2+3a^2b^4+b^6+5(a^6+3a^4b^2+3a^2b^4+b^6)e^{(-2x)}+10(a^6+3a^4b^2+\right.}$$

input
```
integrate(sech(x)^6/(a+b*sinh(x)),x, algorithm="maxima")
```

output
```
b^6*log((b*e^(-x) - a - sqrt(a^2 + b^2))/(b*e^(-x) - a + sqrt(a^2 + b^2)))
/((a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6)*sqrt(a^2 + b^2)) + 2/15*(15*b^5*e^(-
x) + 15*a*b^4*e^(-8*x) + 15*b^5*e^(-9*x) + 8*a^5 + 26*a^3*b^2 + 33*a*b^4 +
 10*(4*a^5 + 13*a^3*b^2 + 15*a*b^4)*e^(-2*x) + 20*(a^2*b^3 + 4*b^5)*e^(-3*
x) + 10*(8*a^5 + 23*a^3*b^2 + 24*a*b^4)*e^(-4*x) + 2*(24*a^4*b + 68*a^2*b^
3 + 89*b^5)*e^(-5*x) + 30*(a^3*b^2 + 3*a*b^4)*e^(-6*x) + 20*(a^2*b^3 + 4*b
^5)*e^(-7*x))/(a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6 + 5*(a^6 + 3*a^4*b^2 + 3*
a^2*b^4 + b^6)*e^(-2*x) + 10*(a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6)*e^(-4*x)
+ 10*(a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6)*e^(-6*x) + 5*(a^6 + 3*a^4*b^2 + 3
*a^2*b^4 + b^6)*e^(-8*x) + (a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6)*e^(-10*x))
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 323 vs. $2(136) = 272$.

Time = 0.13 (sec) , antiderivative size = 323, normalized size of antiderivative = 2.21

$$\int \frac{\operatorname{sech}^6(x)}{a+b\sinh(x)}\,dx = \frac{b^6\log\left(\frac{\left|2\,be^x+2\,a-2\,\sqrt{a^2+b^2}\right|}{\left|2\,be^x+2\,a+2\,\sqrt{a^2+b^2}\right|}\right)}{(a^6+3\,a^4b^2+3\,a^2b^4+b^6)\sqrt{a^2+b^2}} + \frac{2\left(15\,b^5e^{(9\,x)}-15\,ab^4e^{(8\,x)}+20\,a^2b^3e^{(7\,x)}+80\,b^5e^{(7\,x)}-30\,a^3b^2e^{(6\,x)}-90\,ab^4e^{(6\,x)}+48\,a^4be^{(5\,x)}+136\,a\right.}{}$$

input
```
integrate(sech(x)^6/(a+b*sinh(x)),x, algorithm="giac")
```

output
```
b^6*log(abs(2*b*e^x + 2*a - 2*sqrt(a^2 + b^2))/abs(2*b*e^x + 2*a + 2*sqrt(
a^2 + b^2)))/((a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6)*sqrt(a^2 + b^2)) + 2/15*
(15*b^5*e^(9*x) - 15*a*b^4*e^(8*x) + 20*a^2*b^3*e^(7*x) + 80*b^5*e^(7*x) -
 30*a^3*b^2*e^(6*x) - 90*a*b^4*e^(6*x) + 48*a^4*b*e^(5*x) + 136*a^2*b^3*e^
(5*x) + 178*b^5*e^(5*x) - 80*a^5*e^(4*x) - 230*a^3*b^2*e^(4*x) - 240*a*b^4
*e^(4*x) + 20*a^2*b^3*e^(3*x) + 80*b^5*e^(3*x) - 40*a^5*e^(2*x) - 130*a^3*
b^2*e^(2*x) - 150*a*b^4*e^(2*x) + 15*b^5*e^x - 8*a^5 - 26*a^3*b^2 - 33*a*b
^4)/((a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6)*(e^(2*x) + 1)^5)
```

Mupad [B] (verification not implemented)

Time = 3.06 (sec) , antiderivative size = 1010, normalized size of antiderivative = 6.92

$$\int \frac{\operatorname{sech}^6(x)}{a+b\sinh(x)}\,dx = \text{Too large to display}$$

input

```
int(1/(cosh(x)^6*(a + b*sinh(x))),x)
```

output

```
((2*b^5*exp(x))/(a^2 + b^2)^3 - (2*a*b^4)/(a^2 + b^2)^3)/(exp(2*x) + 1) -
((8*(3*a*b^2 + 4*a^3))/(3*(a^2 + b^2)^2) - (8*exp(x)*(12*a^2*b + 7*b^3))/(
15*(a^2 + b^2)^2))/(3*exp(2*x) + 3*exp(4*x) + exp(6*x) + 1) - ((4*(a*b^4 +
 a^3*b^2))/(a^2 + b^2)^3 - (8*exp(x)*(b^5 + a^2*b^3))/(3*(a^2 + b^2)^3))/(
2*exp(2*x) + exp(4*x) + 1) - ((32*a)/(5*(a^2 + b^2)) - (32*b*exp(x))/(5*(a
^2 + b^2)))/(5*exp(2*x) + 10*exp(4*x) + 10*exp(6*x) + 5*exp(8*x) + exp(10*
x) + 1) + ((16*(a*b^2 + a^3))/(a^2 + b^2)^2 - (64*exp(x)*(a^2*b + b^3))/(5
*(a^2 + b^2)^2))/(4*exp(2*x) + 6*exp(4*x) + 4*exp(6*x) + exp(8*x) + 1) - (
2*atan((exp(x)*((2*b^4)/((b^12)^(1/2)*(a^2 + b^2)^3*(a^6 + b^6 + 3*a^2*b^4
 + 3*a^4*b^2)) + (2*a*(a^7*(b^12)^(1/2) + 3*a^3*b^4*(b^12)^(1/2) + 3*a^5*b
^2*(b^12)^(1/2) + a*b^6*(b^12)^(1/2)))/(b^8*(-(a^2 + b^2)^7)^(1/2)*(a^6 +
b^6 + 3*a^2*b^4 + 3*a^4*b^2)*(- a^14 - b^14 - 7*a^2*b^12 - 21*a^4*b^10 - 3
5*a^6*b^8 - 35*a^8*b^6 - 21*a^10*b^4 - 7*a^12*b^2)^(1/2))) - (2*a*(b^7*(b^
12)^(1/2) + 3*a^2*b^5*(b^12)^(1/2) + 3*a^4*b^3*(b^12)^(1/2) + a^6*b*(b^12)
^(1/2)))/(b^8*(-(a^2 + b^2)^7)^(1/2)*(a^6 + b^6 + 3*a^2*b^4 + 3*a^4*b^2)*(
- a^14 - b^14 - 7*a^2*b^12 - 21*a^4*b^10 - 35*a^6*b^8 - 35*a^8*b^6 - 21*a^
10*b^4 - 7*a^12*b^2)^(1/2)))*((b^7*(- a^14 - b^14 - 7*a^2*b^12 - 21*a^4*b^
10 - 35*a^6*b^8 - 35*a^8*b^6 - 21*a^10*b^4 - 7*a^12*b^2)^(1/2))/2 + (3*a^2
*b^5*(- a^14 - b^14 - 7*a^2*b^12 - 21*a^4*b^10 - 35*a^6*b^8 - 35*a^8*b^6 -
 21*a^10*b^4 - 7*a^12*b^2)^(1/2))/2 + (3*a^4*b^3*(- a^14 - b^14 - 7*a^2...
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 919, normalized size of antiderivative = 6.29

$$\int \frac{\operatorname{sech}^6(x)}{a+b\sinh(x)}\,dx$$

$$= \frac{6e^{10x}a^3b^4 + 6e^{10x}a\,b^6 + 30e^{9x}a^2b^5 + 40e^{7x}a^4b^3 + 200e^{7x}a^2b^5 - 60e^{6x}a^5b^2 - 180e^{6x}a^3b^4 - 120e^{6x}a\,b^6 + 96}{}$$

input
```
int(sech(x)^6/(a+b*sinh(x)),x)
```

output
```
(2*(15*e**(10*x)*sqrt(a**2 + b**2)*atan((e**x*b*i + a*i)/sqrt(a**2 + b**2)
)*b**6*i + 75*e**(8*x)*sqrt(a**2 + b**2)*atan((e**x*b*i + a*i)/sqrt(a**2 +
 b**2))*b**6*i + 150*e**(6*x)*sqrt(a**2 + b**2)*atan((e**x*b*i + a*i)/sqrt
(a**2 + b**2))*b**6*i + 150*e**(4*x)*sqrt(a**2 + b**2)*atan((e**x*b*i + a*
i)/sqrt(a**2 + b**2))*b**6*i + 75*e**(2*x)*sqrt(a**2 + b**2)*atan((e**x*b*
i + a*i)/sqrt(a**2 + b**2))*b**6*i + 15*sqrt(a**2 + b**2)*atan((e**x*b*i +
 a*i)/sqrt(a**2 + b**2))*b**6*i + 3*e**(10*x)*a**3*b**4 + 3*e**(10*x)*a*b*
*6 + 15*e**(9*x)*a**2*b**5 + 15*e**(9*x)*b**7 + 20*e**(7*x)*a**4*b**3 + 10
0*e**(7*x)*a**2*b**5 + 80*e**(7*x)*b**7 - 30*e**(6*x)*a**5*b**2 - 90*e**(6
*x)*a**3*b**4 - 60*e**(6*x)*a*b**6 + 48*e**(5*x)*a**6*b + 184*e**(5*x)*a**
4*b**3 + 314*e**(5*x)*a**2*b**5 + 178*e**(5*x)*b**7 - 80*e**(4*x)*a**7 - 3
10*e**(4*x)*a**5*b**2 - 440*e**(4*x)*a**3*b**4 - 210*e**(4*x)*a*b**6 + 20*
e**(3*x)*a**4*b**3 + 100*e**(3*x)*a**2*b**5 + 80*e**(3*x)*b**7 - 40*e**(2*
x)*a**7 - 170*e**(2*x)*a**5*b**2 - 265*e**(2*x)*a**3*b**4 - 135*e**(2*x)*a
*b**6 + 15*e**x*a**2*b**5 + 15*e**x*b**7 - 8*a**7 - 34*a**5*b**2 - 56*a**3
*b**4 - 30*a*b**6))/(15*(e**(10*x)*a**8 + 4*e**(10*x)*a**6*b**2 + 6*e**(10
*x)*a**4*b**4 + 4*e**(10*x)*a**2*b**6 + e**(10*x)*b**8 + 5*e**(8*x)*a**8 +
 20*e**(8*x)*a**6*b**2 + 30*e**(8*x)*a**4*b**4 + 20*e**(8*x)*a**2*b**6 + 5
*e**(8*x)*b**8 + 10*e**(6*x)*a**8 + 40*e**(6*x)*a**6*b**2 + 60*e**(6*x)*a*
*4*b**4 + 40*e**(6*x)*a**2*b**6 + 10*e**(6*x)*b**8 + 10*e**(4*x)*a**8 +...
```

3.200 $\int \frac{\cosh^4(x)}{(a+b\sinh(x))^2}\,dx$

Optimal result

Integrand size = 13, antiderivative size = 94

$$\int \frac{\cosh^4(x)}{(a+b\sinh(x))^2}\,dx = \frac{3(2a^2+b^2)\,x}{2b^4} + \frac{6a\sqrt{a^2+b^2}\operatorname{arctanh}\left(\frac{b-a\tanh\left(\frac{x}{2}\right)}{\sqrt{a^2+b^2}}\right)}{b^4} - \frac{3\cosh(x)(2a-b\sinh(x))}{2b^3} - \frac{\cosh^3(x)}{b(a+b\sinh(x))}$$

output
```
3/2*(2*a^2+b^2)*x/b^4+6*a*(a^2+b^2)^(1/2)*arctanh((b-a*tanh(1/2*x))/(a^2+b
^2)^(1/2))/b^4-3/2*cosh(x)*(2*a-b*sinh(x))/b^3-cosh(x)^3/b/(a+b*sinh(x))
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 3.37 (sec) , antiderivative size = 660, normalized size of antiderivative = 7.02

$$\int \frac{\cosh^4(x)}{(a+b\sinh(x))^2}\,dx = \frac{\cosh^3(x)\left(12a\sqrt{a-ib}(a+ib)^{3/2}\operatorname{arctanh}\left(\frac{\sqrt{-\frac{b(i+\sinh(x))}{a-ib}}}{\sqrt{-\frac{b(-i+\sinh(x))}{a+ib}}}\right)\sqrt{1+i\sinh(x)}(a+b\sinh(x))-12a(a^2+b^2)\right.}{}$$

input
```
Integrate[Cosh[x]^4/(a + b*Sinh[x])^2,x]
```

output
```
(Cosh[x]^3*(12*a*Sqrt[a - I*b]*(a + I*b)^(3/2)*ArcTanh[Sqrt[-((b*(I + Sinh
[x]))/(a - I*b))]/Sqrt[-((b*(-I + Sinh[x]))/(a + I*b))]]*Sqrt[1 + I*Sinh[x
]]*(a + b*Sinh[x]) - 12*a*(a^2 + b^2)*ArcTanh[(Sqrt[a - I*b]*Sqrt[-((b*(I
+ Sinh[x]))/(a - I*b))])/(Sqrt[a + I*b]*Sqrt[-((b*(-I + Sinh[x]))/(a + I*b
))])]*Sqrt[1 + I*Sinh[x]]*(a + b*Sinh[x]) + Sqrt[a + I*b]*Sqrt[-((b*(-I +
Sinh[x]))/(a + I*b))]*(6*(-1)^(3/4)*a*Sqrt[b]*(2*a^2 + I*a*b + b^2)*ArcSin
[((1/2 + I/2)*Sqrt[a - I*b]*Sqrt[-((b*(I + Sinh[x]))/(a - I*b))])/Sqrt[b]]
 + 6*(-1)^(3/4)*b^(3/2)*(2*a^2 + I*a*b + b^2)*ArcSin[((1/2 + I/2)*Sqrt[a -
 I*b]*Sqrt[-((b*(I + Sinh[x]))/(a - I*b))])/Sqrt[b]]*Sinh[x] - 2*Sqrt[a -
I*b]*(3*a^3 + (3*I)*a^2*b + a*b^2 + I*b^3)*Sqrt[1 + I*Sinh[x]]*Sqrt[-((b*(
I + Sinh[x]))/(a - I*b))] - 3*a*Sqrt[a - I*b]*(a + I*b)*b*Sqrt[1 + I*Sinh[
x]]*Sinh[x]*Sqrt[-((b*(I + Sinh[x]))/(a - I*b))] + Sqrt[a - I*b]*(a + I*b)
*b^2*Sqrt[1 + I*Sinh[x]]*Sinh[x]^2*Sqrt[-((b*(I + Sinh[x]))/(a - I*b))])))
/(2*(a - I*b)^(3/2)*(a + I*b)^(5/2)*b*Sqrt[1 + I*Sinh[x]]*(-((b*(-I + Sinh
[x]))/(a + I*b)))^(3/2)*(-((b*(I + Sinh[x]))/(a - I*b)))^(3/2)*(a + b*Sinh
[x]))
```

Rubi [C] (verified)

Result contains complex when optimal does not.

Time = 0.60 (sec) , antiderivative size = 117, normalized size of antiderivative = 1.24, number of steps used = 14, number of rules used = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 1.000$, Rules used = {3042, 3172, 26, 3042, 26, 3344, 26, 3042, 3214, 3042, 3139, 1083, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cosh^4(x)}{(a+b\sinh(x))^2}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\cos(ix)^4}{(a-ib\sin(ix))^2}dx$$

$$\Big\downarrow \text{3172}$$

$$-\frac{\cosh^3(x)}{b(a+b\sinh(x))}-\frac{3i\int\frac{i\cosh^2(x)\sinh(x)}{a+b\sinh(x)}dx}{b}$$

$\downarrow$ 26

$$\frac{3\int\frac{\cosh^2(x)\sinh(x)}{a+b\sinh(x)}dx}{b}-\frac{\cosh^3(x)}{b(a+b\sinh(x))}$$

$\downarrow$ 3042

$$-\frac{\cosh^3(x)}{b(a+b\sinh(x))}+\frac{3\int-\frac{i\cos(ix)^2\sin(ix)}{a-ib\sin(ix)}dx}{b}$$

$\downarrow$ 26

$$-\frac{\cosh^3(x)}{b(a+b\sinh(x))}-\frac{3i\int\frac{\cos(ix)^2\sin(ix)}{a-ib\sin(ix)}dx}{b}$$

$\downarrow$ 3344

$$-\frac{\cosh^3(x)}{b(a+b\sinh(x))}-\frac{3i\left(-\frac{\int\frac{i\left(ab-\left(2a^2+b^2\right)\sinh(x)\right)}{a+b\sinh(x)}dx}{2b^2}-\frac{i\cosh(x)(2a-b\sinh(x))}{2b^2}\right)}{b}$$

$\downarrow$ 26

$$-\frac{\cosh^3(x)}{b(a+b\sinh(x))}-\frac{3i\left(-\frac{i\int\frac{ab-\left(2a^2+b^2\right)\sinh(x)}{a+b\sinh(x)}dx}{2b^2}-\frac{i\cosh(x)(2a-b\sinh(x))}{2b^2}\right)}{b}$$

$\downarrow$ 3042

$$-\frac{\cosh^3(x)}{b(a+b\sinh(x))}-\frac{3i\left(-\frac{i\int\frac{ab+i\left(2a^2+b^2\right)\sin(ix)}{a-ib\sin(ix)}dx}{2b^2}-\frac{i\cosh(x)(2a-b\sinh(x))}{2b^2}\right)}{b}$$

$\downarrow$ 3214

$$-\frac{\cosh^3(x)}{b(a+b\sinh(x))}-\frac{3i\left(-\frac{i\left(\frac{2a\left(a^2+b^2\right)\int\frac{1}{a+b\sinh(x)}dx}{b}-\frac{x\left(2a^2+b^2\right)}{b}\right)}{2b^2}-\frac{i\cosh(x)(2a-b\sinh(x))}{2b^2}\right)}{b}$$

$\downarrow$ 3042

$$-\frac{\cosh^3(x)}{b(a+b\sinh(x))}-\frac{3i\left(-\frac{i\left(-\frac{x(2a^2+b^2)}{b}+\frac{2a(a^2+b^2)\int\frac{1}{a-ib\sin(ix)}dx}{b}\right)}{2b^2}-\frac{i\cosh(x)(2a-b\sinh(x))}{2b^2}\right)}{b}$$

$\downarrow$ 3139

$$-\frac{\cosh^3(x)}{b(a+b\sinh(x))}-\frac{3i\left(-\frac{i\left(\frac{4a(a^2+b^2)\int\frac{1}{-a\tanh^2\left(\frac{x}{2}\right)+2b\tanh\left(\frac{x}{2}\right)+a}d\tanh\left(\frac{x}{2}\right)}{b}-\frac{x(2a^2+b^2)}{b}\right)}{2b^2}-\frac{i\cosh(x)(2a-b\sinh(x))}{2b^2}\right)}{b}$$

$\downarrow$ 1083

$$-\frac{\cosh^3(x)}{b(a+b\sinh(x))}-\frac{3i\left(-\frac{i\left(-\frac{8a(a^2+b^2)\int\frac{1}{4(a^2+b^2)-\left(2b-2a\tanh\left(\frac{x}{2}\right)\right)^2}d\left(2b-2a\tanh\left(\frac{x}{2}\right)\right)}{b}-\frac{x(2a^2+b^2)}{b}\right)}{2b^2}-\frac{i\cosh(x)(2a-b\sinh(x))}{2b^2}\right)}{b}$$

$\downarrow$ 219

$$-\frac{\cosh^3(x)}{b(a+b\sinh(x))}-\frac{3i\left(-\frac{i\left(-\frac{4a\sqrt{a^2+b^2}\,\text{arctanh}\left(\frac{2b-2a\tanh\left(\frac{x}{2}\right)}{2\sqrt{a^2+b^2}}\right)}{b}-\frac{x(2a^2+b^2)}{b}\right)}{2b^2}-\frac{i\cosh(x)(2a-b\sinh(x))}{2b^2}\right)}{b}$$

input
```
Int[Cosh[x]^4/(a + b*Sinh[x])^2,x]
```

output
```
-(Cosh[x]^3/(b*(a + b*Sinh[x]))) - ((3*I)*(((-1/2*I)*(-(((2*a^2 + b^2)*x)/
b) - (4*a*Sqrt[a^2 + b^2]*ArcTanh[(2*b - 2*a*Tanh[x/2])/(2*Sqrt[a^2 + b^2]
)])/b))/b^2 - ((I/2)*Cosh[x]*(2*a - b*Sinh[x]))/b^2))/b
```

Defintions of rubi rules used

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1083
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Simp[-2   Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3139
```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Simp[2*(e/d)   Subst[Int[1/(a + 2*b*e*x + a
*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[a^2 - b^2, 0]
```

rule 3172
```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[g*(g*Cos[e + f*x])^(p - 1)*((a + b*Sin[e + f*x
])^(m + 1)/(b*f*(m + 1))), x] + Simp[g^2*((p - 1)/(b*(m + 1)))   Int[(g*Cos
[e + f*x])^(p - 2)*(a + b*Sin[e + f*x])^(m + 1)*Sin[e + f*x], x], x] /; Fre
eQ[{a, b, e, f, g}, x] && NeQ[a^2 - b^2, 0] && LtQ[m, -1] && GtQ[p, 1] && I
ntegersQ[2*m, 2*p]
```

rule 3214
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])/((c_.) + (d_.)*sin[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[b*(x/d), x] - Simp[(b*c - a*d)/d   Int[1/(c + d
*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

rule 3344
```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[g*(g*
Cos[e + f*x])^(p - 1)*(a + b*Sin[e + f*x])^(m + 1)*((b*c*(m + p + 1) - a*d*
p + b*d*(m + p)*Sin[e + f*x])/(b^2*f*(m + p)*(m + p + 1))), x] + Simp[g^2*(
(p - 1)/(b^2*(m + p)*(m + p + 1)))   Int[(g*Cos[e + f*x])^(p - 2)*(a + b*Si
n[e + f*x])^m*Simp[b*(a*d*m + b*c*(m + p + 1)) + (a*b*c*(m + p + 1) - d*(a^
2*p - b^2*(m + p)))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, g,
 m}, x] && NeQ[a^2 - b^2, 0] && GtQ[p, 1] && NeQ[m + p, 0] && NeQ[m + p + 1
, 0] && IntegerQ[2*m]
```

Maple [A] (verified)

Time = 16.70 (sec) , antiderivative size = 161, normalized size of antiderivative = 1.71

method	result
risch	$\frac{3x\,a^2}{b^4}+\frac{3x}{2b^2}+\frac{e^{2x}}{8b^2}-\frac{a\,e^{x}}{b^3}-\frac{a\,e^{-x}}{b^3}-\frac{e^{-2x}}{8b^2}+\frac{2(a^2+b^2)(e^{x}a-b)}{b^4(b\,e^{2x}+2\,e^{x}a-b)}+\frac{3\sqrt{a^2+b^2}\,a\ln\left(e^{x}+\frac{a+\sqrt{a^2+b^2}}{b}\right)}{b^4}-\frac{3\sqrt{a^2+b^2}\,a\ln\left(\right.}{}$
default	$-\frac{1}{2(\tanh(\frac{x}{2})+1)^2b^2}-\frac{4a-b}{2b^3(\tanh(\frac{x}{2})+1)}+\frac{(6a^2+3b^2)\ln(\tanh(\frac{x}{2})+1)}{2b^4}+\frac{\frac{2\left(\frac{b^2(a^2+b^2)\tanh(\frac{x}{2})}{a}+b(a^2+b^2)\right)}{\tanh(\frac{x}{2})^2a-2b\tanh(\frac{x}{2})-a}-6a\sqrt{a^2+b^2}\ \text{ar}}{b^4}$

input
```
int(cosh(x)^4/(a+b*sinh(x))^2,x,method=_RETURNVERBOSE)
```

output
```
3*x/b^4*a^2+3/2*x/b^2+1/8/b^2*exp(x)^2-1/b^3*a*exp(x)-1/b^3*a/exp(x)-1/8/b
^2/exp(x)^2+2*(a^2+b^2)*(exp(x)*a-b)/b^4/(b*exp(x)^2+2*exp(x)*a-b)+3*(a^2+
b^2)^(1/2)*a/b^4*ln(exp(x)+(a+(a^2+b^2)^(1/2))/b)-3*(a^2+b^2)^(1/2)*a/b^4*
ln(exp(x)-(-a+(a^2+b^2)^(1/2))/b)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 833 vs. $2(85) = 170$.

Time = 0.10 (sec) , antiderivative size = 833, normalized size of antiderivative = 8.86

$$\int \frac{\cosh^4(x)}{(a + b\sinh(x))^2}\,dx = \text{Too large to display}$$

input
```
integrate(cosh(x)^4/(a+b*sinh(x))^2,x, algorithm="fricas")
```

output
```
1/8*(b^3*cosh(x)^6 + b^3*sinh(x)^6 - 6*a*b^2*cosh(x)^5 + 6*(b^3*cosh(x) -
a*b^2)*sinh(x)^5 - (16*a^2*b + b^3 - 12*(2*a^2*b + b^3)*x)*cosh(x)^4 + (15
*b^3*cosh(x)^2 - 30*a*b^2*cosh(x) - 16*a^2*b - b^3 + 12*(2*a^2*b + b^3)*x)
*sinh(x)^4 + 6*a*b^2*cosh(x) + 8*(2*a^3 + 2*a*b^2 + 3*(2*a^3 + a*b^2)*x)*c
osh(x)^3 + 4*(5*b^3*cosh(x)^3 - 15*a*b^2*cosh(x)^2 + 4*a^3 + 4*a*b^2 + 6*(
2*a^3 + a*b^2)*x - (16*a^2*b + b^3 - 12*(2*a^2*b + b^3)*x)*cosh(x))*sinh(x
)^3 + b^3 - (32*a^2*b + 17*b^3 + 12*(2*a^2*b + b^3)*x)*cosh(x)^2 + (15*b^3
*cosh(x)^4 - 60*a*b^2*cosh(x)^3 - 32*a^2*b - 17*b^3 - 6*(16*a^2*b + b^3 -
12*(2*a^2*b + b^3)*x)*cosh(x)^2 - 12*(2*a^2*b + b^3)*x + 24*(2*a^3 + 2*a*b
^2 + 3*(2*a^3 + a*b^2)*x)*cosh(x))*sinh(x)^2 + 24*(a*b*cosh(x)^4 + a*b*sin
h(x)^4 + 2*a^2*cosh(x)^3 - a*b*cosh(x)^2 + 2*(2*a*b*cosh(x) + a^2)*sinh(x)
^3 + (6*a*b*cosh(x)^2 + 6*a^2*cosh(x) - a*b)*sinh(x)^2 + 2*(2*a*b*cosh(x)^
3 + 3*a^2*cosh(x)^2 - a*b*cosh(x))*sinh(x))*sqrt(a^2 + b^2)*log((b^2*cosh(
x)^2 + b^2*sinh(x)^2 + 2*a*b*cosh(x) + 2*a^2 + b^2 + 2*(b^2*cosh(x) + a*b)
*sinh(x) + 2*sqrt(a^2 + b^2)*(b*cosh(x) + b*sinh(x) + a))/(b*cosh(x)^2 + b
*sinh(x)^2 + 2*a*cosh(x) + 2*(b*cosh(x) + a)*sinh(x) - b)) + 2*(3*b^3*cosh
(x)^5 - 15*a*b^2*cosh(x)^4 - 2*(16*a^2*b + b^3 - 12*(2*a^2*b + b^3)*x)*cos
h(x)^3 + 3*a*b^2 + 12*(2*a^3 + 2*a*b^2 + 3*(2*a^3 + a*b^2)*x)*cosh(x)^2 -
(32*a^2*b + 17*b^3 + 12*(2*a^2*b + b^3)*x)*cosh(x))*sinh(x))/(b^5*cosh(x)^
4 + b^5*sinh(x)^4 + 2*a*b^4*cosh(x)^3 - b^5*cosh(x)^2 + 2*(2*b^5*cosh(x...
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\cosh^4(x)}{(a+b\sinh(x))^2}\,dx = \text{Timed out}$$

input
```
integrate(cosh(x)**4/(a+b*sinh(x))**2,x)
```

output
```
Timed out
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 176 vs. 2(85) = 170.

Time = 0.12 (sec) , antiderivative size = 176, normalized size of antiderivative = 1.87

$$\begin{aligned}\int \frac{\cosh^4(x)}{(a+b\sinh(x))^2}\,dx &= -\frac{6\,ab^2e^{(-x)} - b^3 + (32\,a^2b + 17\,b^3)e^{(-2\,x)} + 8\,(2\,a^3 + ab^2)e^{(-3\,x)}}{8\,(b^5e^{(-2\,x)} + 2\,ab^4e^{(-3\,x)} - b^5e^{(-4\,x)})} \\ &\quad - \frac{3\,\sqrt{a^2+b^2}a\log\left(\frac{be^{(-x)}-a-\sqrt{a^2+b^2}}{be^{(-x)}-a+\sqrt{a^2+b^2}}\right)}{b^4} \\ &\quad - \frac{8\,ae^{(-x)} + be^{(-2\,x)}}{8\,b^3} + \frac{3\,(2\,a^2+b^2)x}{2\,b^4}\end{aligned}$$

input
```
integrate(cosh(x)^4/(a+b*sinh(x))^2,x, algorithm="maxima")
```

output
```
-1/8*(6*a*b^2*e^(-x) - b^3 + (32*a^2*b + 17*b^3)*e^(-2*x) + 8*(2*a^3 + a*b
^2)*e^(-3*x))/(b^5*e^(-2*x) + 2*a*b^4*e^(-3*x) - b^5*e^(-4*x)) - 3*sqrt(a^
2 + b^2)*a*log((b*e^(-x) - a - sqrt(a^2 + b^2))/(b*e^(-x) - a + sqrt(a^2 +
 b^2)))/b^4 - 1/8*(8*a*e^(-x) + b*e^(-2*x))/b^3 + 3/2*(2*a^2 + b^2)*x/b^4
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 178 vs. $2(85) = 170$.

Time = 0.14 (sec) , antiderivative size = 178, normalized size of antiderivative = 1.89

$$\int \frac{\cosh^4(x)}{(a+b\sinh(x))^2}\,dx = \frac{3\,(2\,a^2+b^2)x}{2\,b^4} - \frac{3\,(a^3+ab^2)\log\left(\frac{\left|2\,be^x+2\,a-2\,\sqrt{a^2+b^2}\right|}{\left|2\,be^x+2\,a+2\,\sqrt{a^2+b^2}\right|}\right)}{\sqrt{a^2+b^2}b^4} + \frac{b^2e^{(2\,x)}-8\,abe^x}{8\,b^4} + \frac{\left(6\,ab^2e^x+b^3+8\,(2\,a^3+ab^2)e^{(3\,x)}-(32\,a^2b+17\,b^3)e^{(2\,x)}\right)e^{(-2\,x)}}{8\,(be^{(2\,x)}+2\,ae^x-b)b^4}$$

input
```
integrate(cosh(x)^4/(a+b*sinh(x))^2,x, algorithm="giac")
```

output
```
3/2*(2*a^2 + b^2)*x/b^4 - 3*(a^3 + a*b^2)*log(abs(2*b*e^x + 2*a - 2*sqrt(a
^2 + b^2))/abs(2*b*e^x + 2*a + 2*sqrt(a^2 + b^2)))/(sqrt(a^2 + b^2)*b^4) +
 1/8*(b^2*e^(2*x) - 8*a*b*e^x)/b^4 + 1/8*(6*a*b^2*e^x + b^3 + 8*(2*a^3 + a
*b^2)*e^(3*x) - (32*a^2*b + 17*b^3)*e^(2*x))*e^(-2*x)/((b*e^(2*x) + 2*a*e^
x - b)*b^4)
```

Mupad [B] (verification not implemented)

Time = 1.74 (sec) , antiderivative size = 256, normalized size of antiderivative = 2.72

$$\int \frac{\cosh^4(x)}{(a+b\sinh(x))^2}\,dx = \frac{e^{2\,x}}{8\,b^2} - \frac{e^{-2\,x}}{8\,b^2} - \frac{\frac{2\,(a^4\,b^2+2\,a^2\,b^4+b^6)}{b^4\,(a^2\,b+b^3)} - \frac{2\,e^x\,(a^5\,b^2+2\,a^3\,b^4+a\,b^6)}{b^5\,(a^2\,b+b^3)}}{2\,a\,e^x-b+b\,e^{2\,x}} + \frac{x\,(6\,a^2+3\,b^2)}{2\,b^4} - \frac{a\,e^x}{b^3} - \frac{a\,e^{-x}}{b^3} - \frac{3\,a\,\ln\left(\frac{6\,a\,e^x\,(a^2+b^2)}{b^5} - \frac{6\,a\,(b-a\,e^x)\,\sqrt{a^2+b^2}}{b^5}\right)\sqrt{a^2+b^2}}{b^4} + \frac{3\,a\,\ln\left(\frac{6\,a\,(b-a\,e^x)\,\sqrt{a^2+b^2}}{b^5} + \frac{6\,a\,e^x\,(a^2+b^2)}{b^5}\right)\sqrt{a^2+b^2}}{b^4}$$

input
```
int(cosh(x)^4/(a + b*sinh(x))^2,x)
```

output
```
exp(2*x)/(8*b^2) - exp(-2*x)/(8*b^2) - ((2*(b^6 + 2*a^2*b^4 + a^4*b^2))/(b
^4*(a^2*b + b^3)) - (2*exp(x)*(a*b^6 + 2*a^3*b^4 + a^5*b^2))/(b^5*(a^2*b +
 b^3)))/(2*a*exp(x) - b + b*exp(2*x)) + (x*(6*a^2 + 3*b^2))/(2*b^4) - (a*e
xp(x))/b^3 - (a*exp(-x))/b^3 - (3*a*log((6*a*exp(x)*(a^2 + b^2))/b^5 - (6*
a*(b - a*exp(x))*(a^2 + b^2)^(1/2))/b^5)*(a^2 + b^2)^(1/2))/b^4 + (3*a*log
((6*a*(b - a*exp(x))*(a^2 + b^2)^(1/2))/b^5 + (6*a*exp(x)*(a^2 + b^2))/b^5
)*(a^2 + b^2)^(1/2))/b^4
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 296, normalized size of antiderivative = 3.15

$$\int \frac{\cosh^4(x)}{(a+b\sinh(x))^2}\,dx = \frac{-48e^{4x}\sqrt{a^2+b^2}\,atan\left(\frac{e^x bi+ai}{\sqrt{a^2+b^2}}\right)abi - 96e^{3x}\sqrt{a^2+b^2}\,atan\left(\frac{e^x bi+ai}{\sqrt{a^2+b^2}}\right)a^2i + 48e^{2x}\sqrt{a^2+b^2}\,atan\left(\frac{e^x bi+ai}{\sqrt{a^2+b^2}}\right)ab}{}$$

input
```
int(cosh(x)^4/(a+b*sinh(x))^2,x)
```

output
```
( - 48*e**(4*x)*sqrt(a**2 + b**2)*atan((e**x*b*i + a*i)/sqrt(a**2 + b**2))
*a*b*i - 96*e**(3*x)*sqrt(a**2 + b**2)*atan((e**x*b*i + a*i)/sqrt(a**2 + b
**2))*a**2*i + 48*e**(2*x)*sqrt(a**2 + b**2)*atan((e**x*b*i + a*i)/sqrt(a*
*2 + b**2))*a*b*i + e**(6*x)*b**3 - 6*e**(5*x)*a*b**2 + 24*e**(4*x)*a**2*b
*x - 24*e**(4*x)*a**2*b + 12*e**(4*x)*b**3*x - 9*e**(4*x)*b**3 + 48*e**(3*
x)*a**3*x + 24*e**(3*x)*a*b**2*x - 24*e**(2*x)*a**2*b*x - 24*e**(2*x)*a**2
*b - 12*e**(2*x)*b**3*x - 9*e**(2*x)*b**3 + 6*e**x*a*b**2 + b**3)/(8*e**(2
*x)*b**4*(e**(2*x)*b + 2*e**x*a - b))
```

3.201 $\int \frac{\cosh^3(x)}{(a+b\sinh(x))^2}\,dx$

Optimal result

Integrand size = 13, antiderivative size = 40

$$\int \frac{\cosh^3(x)}{(a+b\sinh(x))^2}\,dx = -\frac{2a\log(a+b\sinh(x))}{b^3} + \frac{\sinh(x)}{b^2} - \frac{a^2+b^2}{b^3(a+b\sinh(x))}$$

output
```
-2*a*ln(a+b*sinh(x))/b^3+sinh(x)/b^2-(a^2+b^2)/b^3/(a+b*sinh(x))
```

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.92

$$\int \frac{\cosh^3(x)}{(a+b\sinh(x))^2}\,dx = -\frac{2a\log(a+b\sinh(x)) - b\sinh(x) + \frac{a^2+b^2}{a+b\sinh(x)}}{b^3}$$

input
```
Integrate[Cosh[x]^3/(a + b*Sinh[x])^2,x]
```

output
```
-((2*a*Log[a + b*Sinh[x]] - b*Sinh[x] + (a^2 + b^2)/(a + b*Sinh[x]))/b^3)
```

Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.92, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.385$, Rules used = {3042, 3147, 25, 476, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cosh^3(x)}{(a+b\sinh(x))^2}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\cos(ix)^3}{(a-ib\sin(ix))^2}dx$$

$$\Big\downarrow \text{3147}$$

$$-\frac{\int -\frac{\sinh^2(x)b^2+b^2}{(a+b\sinh(x))^2}d(b\sinh(x))}{b^3}$$

$$\Big\downarrow \text{25}$$

$$\frac{\int \frac{\sinh^2(x)b^2+b^2}{(a+b\sinh(x))^2}d(b\sinh(x))}{b^3}$$

$$\Big\downarrow \text{476}$$

$$\frac{\int \left(-\frac{2a}{a+b\sinh(x)}+\frac{a^2+b^2}{(a+b\sinh(x))^2}+1\right)d(b\sinh(x))}{b^3}$$

$$\Big\downarrow \text{2009}$$

$$-\frac{\frac{a^2+b^2}{a+b\sinh(x)}+2a\log(a+b\sinh(x))-b\sinh(x)}{b^3}$$

input
```
Int[Cosh[x]^3/(a + b*Sinh[x])^2,x]
```

output
```
-((2*a*Log[a + b*Sinh[x]] - b*Sinh[x] + (a^2 + b^2)/(a + b*Sinh[x]))/b^3)
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 476
```
Int[((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[
ExpandIntegrand[(c + d*x)^n*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d, n},
 x] && IGtQ[p, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3147
```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Simp[1/(b^p*f)   Subst[Int[(a + x)^m*(b^2 - x^2)^((p - 1)
/2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && IntegerQ[(p
 - 1)/2] && NeQ[a^2 - b^2, 0]
```

Maple [A] (verified)

Time = 5.97 (sec) , antiderivative size = 41, normalized size of antiderivative = 1.02

method	result	size
derivativedivides	$-\frac{2a\ln(a+b\sinh(x))}{b^3}+\frac{\sinh(x)}{b^2}-\frac{a^2+b^2}{b^3(a+b\sinh(x))}$	41
default	$-\frac{2a\ln(a+b\sinh(x))}{b^3}+\frac{\sinh(x)}{b^2}-\frac{a^2+b^2}{b^3(a+b\sinh(x))}$	41
risch	$\frac{2ax}{b^3}+\frac{e^x}{2b^2}-\frac{e^{-x}}{2b^2}-\frac{2(a^2+b^2)e^x}{b^3(b\,e^{2x}+2\,e^x a-b)}-\frac{2a\ln\left(e^{2x}+\frac{2a\,e^x}{b}-1\right)}{b^3}$	77

input
```
int(cosh(x)^3/(a+b*sinh(x))^2,x,method=_RETURNVERBOSE)
```

output
```
-2*a*ln(a+b*sinh(x))/b^3+sinh(x)/b^2-(a^2+b^2)/b^3/(a+b*sinh(x))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 370 vs. $2(40) = 80$.

Time = 0.12 (sec) , antiderivative size = 370, normalized size of antiderivative = 9.25

$$\int \frac{\cosh^3(x)}{(a+b\sinh(x))^2}\,dx$$
$$= \frac{b^2\cosh(x)^4 + b^2\sinh(x)^4 + 2(2abx+ab)\cosh(x)^3 + 2(2abx+2b^2\cosh(x)+ab)\sinh(x)^3 + 2(4a^2x \cdots}{}$$

input
```
integrate(cosh(x)^3/(a+b*sinh(x))^2,x, algorithm="fricas")
```

output
```
1/2*(b^2*cosh(x)^4 + b^2*sinh(x)^4 + 2*(2*a*b*x + a*b)*cosh(x)^3 + 2*(2*a*
b*x + 2*b^2*cosh(x) + a*b)*sinh(x)^3 + 2*(4*a^2*x - 2*a^2 - 3*b^2)*cosh(x)
^2 + 2*(3*b^2*cosh(x)^2 + 4*a^2*x - 2*a^2 - 3*b^2 + 3*(2*a*b*x + a*b)*cosh
(x))*sinh(x)^2 + b^2 - 2*(2*a*b*x + a*b)*cosh(x) - 4*(a*b*cosh(x)^3 + a*b*
sinh(x)^3 + 2*a^2*cosh(x)^2 - a*b*cosh(x) + (3*a*b*cosh(x) + 2*a^2)*sinh(x
)^2 + (3*a*b*cosh(x)^2 + 4*a^2*cosh(x) - a*b)*sinh(x))*log(2*(b*sinh(x) +
a)/(cosh(x) - sinh(x))) + 2*(2*b^2*cosh(x)^3 - 2*a*b*x + 3*(2*a*b*x + a*b)
*cosh(x)^2 - a*b + 2*(4*a^2*x - 2*a^2 - 3*b^2)*cosh(x))*sinh(x))/(b^4*cosh
(x)^3 + b^4*sinh(x)^3 + 2*a*b^3*cosh(x)^2 - b^4*cosh(x) + (3*b^4*cosh(x) +
 2*a*b^3)*sinh(x)^2 + (3*b^4*cosh(x)^2 + 4*a*b^3*cosh(x) - b^4)*sinh(x))
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 133 vs. $2(37) = 74$.

Time = 0.37 (sec) , antiderivative size = 133, normalized size of antiderivative = 3.32

$$\int \frac{\cosh^3(x)}{(a+b\sinh(x))^2}\,dx$$
$$= \begin{cases} -\frac{2a^2\log\left(\frac{a}{b}+\sinh(x)\right)}{ab^3+b^4\sinh(x)} - \frac{2a^2}{ab^3+b^4\sinh(x)} - \frac{2ab\log\left(\frac{a}{b}+\sinh(x)\right)\sinh(x)}{ab^3+b^4\sinh(x)} + \frac{2b^2\sinh^2(x)}{ab^3+b^4\sinh(x)} - \frac{b^2\cosh^2(x)}{ab^3+b^4\sinh(x)} & \text{for } b \neq 0 \\ \frac{-\frac{2\sinh^3(x)}{3}+\sinh(x)\cosh^2(x)}{a^2} & \text{otherwise} \end{cases}$$

input
```
integrate(cosh(x)**3/(a+b*sinh(x))**2,x)
```

output
```
Piecewise((-2*a**2*log(a/b + sinh(x))/(a*b**3 + b**4*sinh(x)) - 2*a**2/(a*
b**3 + b**4*sinh(x)) - 2*a*b*log(a/b + sinh(x))*sinh(x)/(a*b**3 + b**4*sin
h(x)) + 2*b**2*sinh(x)**2/(a*b**3 + b**4*sinh(x)) - b**2*cosh(x)**2/(a*b**
3 + b**4*sinh(x)), Ne(b, 0)), ((-2*sinh(x)**3/3 + sinh(x)*cosh(x)**2)/a**2
, True))
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 102 vs. 2(40) = 80.

Time = 0.05 (sec) , antiderivative size = 102, normalized size of antiderivative = 2.55

$$\int \frac{\cosh^3(x)}{(a+b\sinh(x))^2}\,dx = \frac{2\,abe^{(-x)}+b^2-(4\,a^2+5\,b^2)e^{(-2\,x)}}{2\,(b^4e^{(-x)}+2\,ab^3e^{(-2\,x)}-b^4e^{(-3\,x)})} - \frac{2\,ax}{b^3} - \frac{e^{(-x)}}{2\,b^2} - \frac{2\,a\log\left(-2\,ae^{(-x)}+be^{(-2\,x)}-b\right)}{b^3}$$

input
```
integrate(cosh(x)^3/(a+b*sinh(x))^2,x, algorithm="maxima")
```

output
```
1/2*(2*a*b*e^(-x) + b^2 - (4*a^2 + 5*b^2)*e^(-2*x))/(b^4*e^(-x) + 2*a*b^3*
e^(-2*x) - b^4*e^(-3*x)) - 2*a*x/b^3 - 1/2*e^(-x)/b^2 - 2*a*log(-2*a*e^(-x
) + b*e^(-2*x) - b)/b^3
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 82 vs. 2(40) = 80.

Time = 0.13 (sec) , antiderivative size = 82, normalized size of antiderivative = 2.05

$$\int \frac{\cosh^3(x)}{(a+b\sinh(x))^2}\,dx = -\frac{e^{(-x)}-e^x}{2\,b^2} - \frac{2\,a\log\left(\left|-b\left(e^{(-x)}-e^x\right)+2\,a\right|\right)}{b^3} + \frac{2\left(ab\left(e^{(-x)}-e^x\right)-a^2+b^2\right)}{(b(e^{(-x)}-e^x)-2\,a)b^3}$$

input
```
integrate(cosh(x)^3/(a+b*sinh(x))^2,x, algorithm="giac")
```

output
```
-1/2*(e^(-x) - e^x)/b^2 - 2*a*log(abs(-b*(e^(-x) - e^x) + 2*a))/b^3 + 2*(a
*b*(e^(-x) - e^x) - a^2 + b^2)/((b*(e^(-x) - e^x) - 2*a)*b^3)
```

Mupad [B] (verification not implemented)

Time = 1.57 (sec) , antiderivative size = 60, normalized size of antiderivative = 1.50

$$\int \frac{\cosh^3(x)}{(a+b\sinh(x))^2}\,dx = \frac{\frac{\cosh(x)^2}{b}-\frac{2\sinh(x)^3}{a}+\frac{2\cosh(x)^2\sinh(x)}{a}+\frac{2\,a\sinh(x)}{b^2}}{a+b\sinh(x)} - \frac{2\,a\,\ln(a+b\sinh(x))}{b^3}$$

input
```
int(cosh(x)^3/(a + b*sinh(x))^2,x)
```

output
```
(cosh(x)^2/b - (2*sinh(x)^3)/a + (2*cosh(x)^2*sinh(x))/a + (2*a*sinh(x))/b
^2)/(a + b*sinh(x)) - (2*a*log(a + b*sinh(x)))/b^3
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 62, normalized size of antiderivative = 1.55

$$\int \frac{\cosh^3(x)}{(a+b\sinh(x))^2}\,dx = \frac{-\cosh(x)^2 b^2 - 2\log(\sinh(x)\,b+a)\sinh(x)\,ab - 2\log(\sinh(x)\,b+a)\,a^2 + 2\sinh(x)^2 b^2 + 2\sinh(x)\,ab}{b^3(\sinh(x)\,b+a)}$$

input
```
int(cosh(x)^3/(a+b*sinh(x))^2,x)
```

output
```
( - cosh(x)**2*b**2 - 2*log(sinh(x)*b + a)*sinh(x)*a*b - 2*log(sinh(x)*b +
 a)*a**2 + 2*sinh(x)**2*b**2 + 2*sinh(x)*a*b)/(b**3*(sinh(x)*b + a))
```

3.202 $\int \frac{\cosh^2(x)}{(a+b\sinh(x))^2}\,dx$

Optimal result

Integrand size = 13, antiderivative size = 62

$$\int \frac{\cosh^2(x)}{(a+b\sinh(x))^2}\,dx=\frac{x}{b^2}+\frac{2a\text{arctanh}\left(\frac{b-a\tanh\left(\frac{x}{2}\right)}{\sqrt{a^2+b^2}}\right)}{b^2\sqrt{a^2+b^2}}-\frac{\cosh(x)}{b(a+b\sinh(x))}$$

output
```
x/b^2+2*a*arctanh((b-a*tanh(1/2*x))/(a^2+b^2)^(1/2))/b^2/(a^2+b^2)^(1/2)-c
osh(x)/b/(a+b*sinh(x))
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 1.29 (sec) , antiderivative size = 502, normalized size of antiderivative = 8.10

$$\int \frac{\cosh^2(x)}{(a+b\sinh(x))^2}\,dx$$
$$=\frac{\cosh(x)\left(2a\sqrt{a-ib}\sqrt{a+ib}\text{arctanh}\left(\frac{\sqrt{-\frac{b(i+\sinh(x))}{a-ib}}}{\sqrt{-\frac{b(-i+\sinh(x))}{a+ib}}}\right)\sqrt{1+i\sinh(x)}(a+b\sinh(x))-2a(a-ib)\text{arctanh}\right.}{}$$

input
```
Integrate[Cosh[x]^2/(a + b*Sinh[x])^2,x]
```

output
```
(Cosh[x]*(2*a*Sqrt[a - I*b]*Sqrt[a + I*b]*ArcTanh[Sqrt[-((b*(I + Sinh[x]))
/(a - I*b))]/Sqrt[-((b*(-I + Sinh[x]))/(a + I*b))]]*Sqrt[1 + I*Sinh[x]]*(a
 + b*Sinh[x]) - 2*a*(a - I*b)*ArcTanh[(Sqrt[a - I*b]*Sqrt[-((b*(I + Sinh[x
]))/(a - I*b))])/(Sqrt[a + I*b]*Sqrt[-((b*(-I + Sinh[x]))/(a + I*b))])]*Sq
rt[1 + I*Sinh[x]]*(a + b*Sinh[x]) + Sqrt[a + I*b]*Sqrt[-((b*(-I + Sinh[x])
)/(a + I*b))]*(2*(-1)^(1/4)*a*Sqrt[b]*(I*a + b)*ArcSin[((1/2 + I/2)*Sqrt[a
 - I*b]*Sqrt[-((b*(I + Sinh[x]))/(a - I*b))])/Sqrt[b]] + 2*(-1)^(1/4)*b^(3
/2)*(I*a + b)*ArcSin[((1/2 + I/2)*Sqrt[a - I*b]*Sqrt[-((b*(I + Sinh[x]))/(
a - I*b))])/Sqrt[b]]*Sinh[x] - Sqrt[a - I*b]*(a^2 + b^2)*Sqrt[1 + I*Sinh[x
]]*Sqrt[-((b*(I + Sinh[x]))/(a - I*b))])))/((a - I*b)^(3/2)*(a + I*b)^(3/2
)*b*Sqrt[1 + I*Sinh[x]]*Sqrt[-((b*(-I + Sinh[x]))/(a + I*b))]*Sqrt[-((b*(I
 + Sinh[x]))/(a - I*b))]*(a + b*Sinh[x]))
```

Rubi [C] (verified)

Result contains complex when optimal does not.

Time = 0.40 (sec) , antiderivative size = 80, normalized size of antiderivative = 1.29, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.769, Rules used = {3042, 3172, 26, 3042, 26, 3214, 3042, 3139, 1083, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cosh^2(x)}{(a+b\sinh(x))^2}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\cos(ix)^2}{(a-ib\sin(ix))^2}dx$$

$$\Big\downarrow \text{3172}$$

$$-\frac{\cosh(x)}{b(a+b\sinh(x))}-\frac{i\int \frac{i\sinh(x)}{a+b\sinh(x)}dx}{b}$$

$$\Big\downarrow \text{26}$$

$$\frac{\int \frac{\sinh(x)}{a+b\sinh(x)}dx}{b}-\frac{\cosh(x)}{b(a+b\sinh(x))}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{\cosh(x)}{b(a+b\sinh(x))}+\frac{\int -\frac{i\sin(ix)}{a-ib\sin(ix)}dx}{b}$$

$$\Big\downarrow \text{26}$$

$$-\frac{\cosh(x)}{b(a+b\sinh(x))}-\frac{i\int \frac{\sin(ix)}{a-ib\sin(ix)}dx}{b}$$

$$\Big\downarrow \text{3214}$$

$$-\frac{\cosh(x)}{b(a+b\sinh(x))}-\frac{i\left(\frac{ix}{b}-\frac{ia\int \frac{1}{a+b\sinh(x)}dx}{b}\right)}{b}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{\cosh(x)}{b(a+b\sinh(x))}-\frac{i\left(\frac{ix}{b}-\frac{ia\int \frac{1}{a-ib\sin(ix)}dx}{b}\right)}{b}$$

$$\Big\downarrow \text{3139}$$

$$-\frac{\cosh(x)}{b(a+b\sinh(x))}-\frac{i\left(\frac{ix}{b}-\frac{2ia\int \frac{1}{-a\tanh^2\left(\frac{x}{2}\right)+2b\tanh\left(\frac{x}{2}\right)+a}d\tanh\left(\frac{x}{2}\right)}{b}\right)}{b}$$

$$\Big\downarrow \text{1083}$$

$$-\frac{\cosh(x)}{b(a+b\sinh(x))}-\frac{i\left(\frac{4ia\int \frac{1}{4\left(a^2+b^2\right)-\left(2b-2a\tanh\left(\frac{x}{2}\right)\right)^2}d\left(2b-2a\tanh\left(\frac{x}{2}\right)\right)}{b}+\frac{ix}{b}\right)}{b}$$

$$\Big\downarrow \text{219}$$

$$-\frac{\cosh(x)}{b(a+b\sinh(x))}-\frac{i\left(\frac{2ia\text{arctanh}\left(\frac{2b-2a\tanh\left(\frac{x}{2}\right)}{2\sqrt{a^2+b^2}}\right)}{b\sqrt{a^2+b^2}}+\frac{ix}{b}\right)}{b}$$

input
```
Int[Cosh[x]^2/(a + b*Sinh[x])^2,x]
```

output
```
((-I)*((I*x)/b + ((2*I)*a*ArcTanh[(2*b - 2*a*Tanh[x/2])/(2*Sqrt[a^2 + b^2]
)])/(b*Sqrt[a^2 + b^2])))/b - Cosh[x]/(b*(a + b*Sinh[x]))
```

Defintions of rubi rules used

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1083
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Simp[-2   Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3139
```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Simp[2*(e/d)   Subst[Int[1/(a + 2*b*e*x + a
*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[a^2 - b^2, 0]
```

rule 3172
```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[g*(g*Cos[e + f*x])^(p - 1)*((a + b*Sin[e + f*x
])^(m + 1)/(b*f*(m + 1))), x] + Simp[g^2*((p - 1)/(b*(m + 1)))   Int[(g*Cos
[e + f*x])^(p - 2)*(a + b*Sin[e + f*x])^(m + 1)*Sin[e + f*x], x], x] /; Fre
eQ[{a, b, e, f, g}, x] && NeQ[a^2 - b^2, 0] && LtQ[m, -1] && GtQ[p, 1] && I
ntegersQ[2*m, 2*p]
```

rule 3214
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])/((c_.) + (d_.)*sin[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[b*(x/d), x] - Simp[(b*c - a*d)/d   Int[1/(c + d
*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Maple [A] (verified)

Time = 2.21 (sec) , antiderivative size = 101, normalized size of antiderivative = 1.63

method	result	size
default	$\frac{\ln\left(\tanh\left(\frac{x}{2}\right)+1\right)}{b^2}+\frac{\frac{2\left(\frac{b^2\tanh\left(\frac{x}{2}\right)}{a}+b\right)}{\tanh\left(\frac{x}{2}\right)^2 a-2b\tanh\left(\frac{x}{2}\right)-a}-\frac{2a\,\mathrm{arctanh}\left(\frac{2a\tanh\left(\frac{x}{2}\right)-2b}{2\sqrt{a^2+b^2}}\right)}{\sqrt{a^2+b^2}}}{b^2}-\frac{\ln\left(\tanh\left(\frac{x}{2}\right)-1\right)}{b^2}$	101
risch	$\frac{x}{b^2}+\frac{2\,\mathrm{e}^x a-2b}{b^2\left(b\,\mathrm{e}^{2x}+2\,\mathrm{e}^x a-b\right)}+\frac{a\ln\left(\mathrm{e}^x+\frac{a\sqrt{a^2+b^2}+a^2+b^2}{\sqrt{a^2+b^2}\,b}\right)}{\sqrt{a^2+b^2}\,b^2}-\frac{a\ln\left(\mathrm{e}^x+\frac{a\sqrt{a^2+b^2}-a^2-b^2}{\sqrt{a^2+b^2}\,b}\right)}{\sqrt{a^2+b^2}\,b^2}$	140

input
```
int(cosh(x)^2/(a+b*sinh(x))^2,x,method=_RETURNVERBOSE)
```

output
```
1/b^2*ln(tanh(1/2*x)+1)+2/b^2*((1/a*b^2*tanh(1/2*x)+b)/(tanh(1/2*x)^2*a-2*
b*tanh(1/2*x)-a)-a/(a^2+b^2)^(1/2)*arctanh(1/2*(2*a*tanh(1/2*x)-2*b)/(a^2+
b^2)^(1/2)))-1/b^2*ln(tanh(1/2*x)-1)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 362 vs. $2(58) = 116$.

Time = 0.11 (sec) , antiderivative size = 362, normalized size of antiderivative = 5.84

$$\int \frac{\cosh^2(x)}{(a+b\sinh(x))^2}\,dx =$$

$$-\frac{(a^2b+b^3)x\cosh(x)^2+(a^2b+b^3)x\sinh(x)^2-2\,a^2b-2\,b^3+\left(ab\cosh(x)^2+ab\sinh(x)^2+2\,a^2\cosh(\right.}{}$$

input
```
integrate(cosh(x)^2/(a+b*sinh(x))^2,x, algorithm="fricas")
```

output
```
-((a^2*b + b^3)*x*cosh(x)^2 + (a^2*b + b^3)*x*sinh(x)^2 - 2*a^2*b - 2*b^3
+ (a*b*cosh(x)^2 + a*b*sinh(x)^2 + 2*a^2*cosh(x) - a*b + 2*(a*b*cosh(x) +
a^2)*sinh(x))*sqrt(a^2 + b^2)*log((b^2*cosh(x)^2 + b^2*sinh(x)^2 + 2*a*b*c
osh(x) + 2*a^2 + b^2 + 2*(b^2*cosh(x) + a*b)*sinh(x) + 2*sqrt(a^2 + b^2)*(
b*cosh(x) + b*sinh(x) + a))/(b*cosh(x)^2 + b*sinh(x)^2 + 2*a*cosh(x) + 2*(
b*cosh(x) + a)*sinh(x) - b)) - (a^2*b + b^3)*x + 2*(a^3 + a*b^2 + (a^3 + a
*b^2)*x)*cosh(x) + 2*(a^3 + a*b^2 + (a^2*b + b^3)*x*cosh(x) + (a^3 + a*b^2
)*x)*sinh(x))/(a^2*b^3 + b^5 - (a^2*b^3 + b^5)*cosh(x)^2 - (a^2*b^3 + b^5)
*sinh(x)^2 - 2*(a^3*b^2 + a*b^4)*cosh(x) - 2*(a^3*b^2 + a*b^4 + (a^2*b^3 +
 b^5)*cosh(x))*sinh(x))
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\cosh^2(x)}{(a+b\sinh(x))^2}\,dx = \text{Timed out}$$

input
```
integrate(cosh(x)**2/(a+b*sinh(x))**2,x)
```

output
```
Timed out
```

Maxima [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 100, normalized size of antiderivative = 1.61

$$\int \frac{\cosh^2(x)}{(a+b\sinh(x))^2}\,dx = -\frac{2\left(ae^{(-x)}+b\right)}{2\,ab^2e^{(-x)}-b^3e^{(-2\,x)}+b^3} - \frac{a\log\left(\frac{be^{(-x)}-a-\sqrt{a^2+b^2}}{be^{(-x)}-a+\sqrt{a^2+b^2}}\right)}{\sqrt{a^2+b^2}b^2} + \frac{x}{b^2}$$

input
```
integrate(cosh(x)^2/(a+b*sinh(x))^2,x, algorithm="maxima")
```

output
```
-2*(a*e^(-x) + b)/(2*a*b^2*e^(-x) - b^3*e^(-2*x) + b^3) - a*log((b*e^(-x)
- a - sqrt(a^2 + b^2))/(b*e^(-x) - a + sqrt(a^2 + b^2)))/(sqrt(a^2 + b^2)*
b^2) + x/b^2
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 97, normalized size of antiderivative = 1.56

$$\int \frac{\cosh^2(x)}{(a+b\sinh(x))^2}\,dx = -\frac{a\log\left(\frac{\left|2\,be^x+2\,a-2\,\sqrt{a^2+b^2}\right|}{\left|2\,be^x+2\,a+2\,\sqrt{a^2+b^2}\right|}\right)}{\sqrt{a^2+b^2}b^2}+\frac{x}{b^2}+\frac{2\,(ae^x-b)}{(be^{(2\,x)}+2\,ae^x-b)b^2}$$

input
```
integrate(cosh(x)^2/(a+b*sinh(x))^2,x, algorithm="giac")
```

output
```
-a*log(abs(2*b*e^x + 2*a - 2*sqrt(a^2 + b^2))/abs(2*b*e^x + 2*a + 2*sqrt(a
^2 + b^2)))/(sqrt(a^2 + b^2)*b^2) + x/b^2 + 2*(a*e^x - b)/((b*e^(2*x) + 2*
a*e^x - b)*b^2)
```

Mupad [B] (verification not implemented)

Time = 1.54 (sec) , antiderivative size = 132, normalized size of antiderivative = 2.13

$$\int \frac{\cosh^2(x)}{(a+b\sinh(x))^2}\,dx = \frac{x}{b^2}-\frac{\frac{2}{b}-\frac{2\,a\,e^x}{b^2}}{2\,a\,e^x-b+b\,e^{2\,x}}-\frac{a\,\ln\left(\frac{2\,a\,e^x}{b^3}-\frac{2\,a\,(b-a\,e^x)}{b^3\,\sqrt{a^2+b^2}}\right)}{b^2\,\sqrt{a^2+b^2}}+\frac{a\,\ln\left(\frac{2\,a\,e^x}{b^3}+\frac{2\,a\,(b-a\,e^x)}{b^3\,\sqrt{a^2+b^2}}\right)}{b^2\,\sqrt{a^2+b^2}}$$

input
```
int(cosh(x)^2/(a + b*sinh(x))^2,x)
```

output
```
x/b^2 - (2/b - (2*a*exp(x))/b^2)/(2*a*exp(x) - b + b*exp(2*x)) - (a*log((2
*a*exp(x))/b^3 - (2*a*(b - a*exp(x)))/(b^3*(a^2 + b^2)^(1/2))))/(b^2*(a^2
+ b^2)^(1/2)) + (a*log((2*a*exp(x))/b^3 + (2*a*(b - a*exp(x)))/(b^3*(a^2 +
 b^2)^(1/2))))/(b^2*(a^2 + b^2)^(1/2))
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 143, normalized size of antiderivative = 2.31

$$\int \frac{\cosh^2(x)}{(a+b\sinh(x))^2}\,dx$$
$$= \frac{-2\sqrt{a^2+b^2}\,atan\left(\frac{e^x bi+ai}{\sqrt{a^2+b^2}}\right)\sinh(x)\,abi-2\sqrt{a^2+b^2}\,atan\left(\frac{e^x bi+ai}{\sqrt{a^2+b^2}}\right)a^2i-\cosh(x)\,a^2b-\cosh(x)\,b^3+\sinh}{b^2\left(\sinh(x)\,a^2b+\sinh(x)\,b^3+a^3+a\,b^2\right)}$$

input
```
int(cosh(x)^2/(a+b*sinh(x))^2,x)
```

output
```
( - 2*sqrt(a**2 + b**2)*atan((e**x*b*i + a*i)/sqrt(a**2 + b**2))*sinh(x)*a
*b*i - 2*sqrt(a**2 + b**2)*atan((e**x*b*i + a*i)/sqrt(a**2 + b**2))*a**2*i
 - cosh(x)*a**2*b - cosh(x)*b**3 + sinh(x)*a**2*b*x + sinh(x)*b**3*x + a**
3*x + a*b**2*x)/(b**2*(sinh(x)*a**2*b + sinh(x)*b**3 + a**3 + a*b**2))
```

3.203 $\int \frac{\cosh(x)}{(a+b\sinh(x))^2}\,dx$

Optimal result

Integrand size = 11, antiderivative size = 13

$$\int \frac{\cosh(x)}{(a+b\sinh(x))^2}\,dx = -\frac{1}{b(a+b\sinh(x))}$$

output
```
-1/b/(a+b*sinh(x))
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 13, normalized size of antiderivative = 1.00

$$\int \frac{\cosh(x)}{(a+b\sinh(x))^2}\,dx = -\frac{1}{b(a+b\sinh(x))}$$

input
```
Integrate[Cosh[x]/(a + b*Sinh[x])^2,x]
```

output
```
-(1/(b*(a + b*Sinh[x])))
```

Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 13, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.273, Rules used = {3042, 3147, 17}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cosh(x)}{(a+b\sinh(x))^2}dx$$
$$\Big\downarrow \text{3042}$$
$$\int \frac{\cos(ix)}{(a-ib\sin(ix))^2}dx$$
$$\Big\downarrow \text{3147}$$
$$\frac{\int \frac{1}{(a+b\sinh(x))^2}d(b\sinh(x))}{b}$$
$$\Big\downarrow \text{17}$$
$$-\frac{1}{b(a+b\sinh(x))}$$

input
```
Int[Cosh[x]/(a + b*Sinh[x])^2,x]
```

output
```
-(1/(b*(a + b*Sinh[x])))
```

Defintions of rubi rules used

rule 17
```
Int[(c_.)*((a_.) + (b_.)*(x_))^(m_.), x_Symbol] :> Simp[c*((a + b*x)^(m + 1
)/(b*(m + 1))), x] /; FreeQ[{a, b, c, m}, x] && NeQ[m, -1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3147
```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Simp[1/(b^p*f)   Subst[Int[(a + x)^m*(b^2 - x^2)^((p - 1)
/2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && IntegerQ[(p
 - 1)/2] && NeQ[a^2 - b^2, 0]
```

Maple [A] (verified)

Time = 1.39 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.08

method	result	size
derivativedivides	$-\frac{1}{b(a+b\sinh(x))}$	14
default	$-\frac{1}{b(a+b\sinh(x))}$	14
risch	$-\frac{2\,{\rm e}^{x}}{b(b\,{\rm e}^{2x}+2\,{\rm e}^{x}a-b)}$	25

input
```
int(cosh(x)/(a+b*sinh(x))^2,x,method=_RETURNVERBOSE)
```

output
```
-1/b/(a+b*sinh(x))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 51 vs. 2(13) = 26.

Time = 0.08 (sec) , antiderivative size = 51, normalized size of antiderivative = 3.92

$$\int \frac{\cosh(x)}{(a+b\sinh(x))^2}\,dx = -\frac{2\,(\cosh\left(x\right)+\sinh\left(x\right))}{b^2\cosh\left(x\right)^2+b^2\sinh\left(x\right)^2+2\,ab\cosh\left(x\right)-b^2+2\,(b^2\cosh\left(x\right)+ab)\sinh\left(x\right)}$$

input
```
integrate(cosh(x)/(a+b*sinh(x))^2,x, algorithm="fricas")
```

output
```
-2*(cosh(x) + sinh(x))/(b^2*cosh(x)^2 + b^2*sinh(x)^2 + 2*a*b*cosh(x) - b^
2 + 2*(b^2*cosh(x) + a*b)*sinh(x))
```

Sympy [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.46

$$\int \frac{\cosh(x)}{(a+b\sinh(x))^2}\,dx = \begin{cases} -\frac{1}{ab+b^2\sinh{(x)}} & \text{for } b\neq 0 \\ \frac{\sinh{(x)}}{a^2} & \text{otherwise} \end{cases}$$

input
```
integrate(cosh(x)/(a+b*sinh(x))**2,x)
```

output
```
Piecewise((-1/(a*b + b**2*sinh(x)), Ne(b, 0)), (sinh(x)/a**2, True))
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 13, normalized size of antiderivative = 1.00

$$\int \frac{\cosh(x)}{(a+b\sinh(x))^2}\,dx = -\frac{1}{(b\sinh{(x)}+a)b}$$

input
```
integrate(cosh(x)/(a+b*sinh(x))^2,x, algorithm="maxima")
```

output
```
-1/((b*sinh(x) + a)*b)
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.69

$$\int \frac{\cosh(x)}{(a+b\sinh(x))^2}\,dx = \frac{2}{(b(e^{(-x)}-e^x)-2\,a)b}$$

input
```
integrate(cosh(x)/(a+b*sinh(x))^2,x, algorithm="giac")
```

output
```
2/((b*(e^(-x) - e^x) - 2*a)*b)
```

Mupad [B] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.08

$$\int \frac{\cosh(x)}{(a+b\sinh(x))^2}\,dx = \frac{\sinh(x)}{a\ (a+b\sinh{(x)})}$$

input
```
int(cosh(x)/(a + b*sinh(x))^2,x)
```

output
```
sinh(x)/(a*(a + b*sinh(x)))
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.08

$$\int \frac{\cosh(x)}{(a+b\sinh(x))^2}\,dx = \frac{\sinh{(x)}}{a\,(\sinh{(x)}\,b+a)}$$

input
```
int(cosh(x)/(a+b*sinh(x))^2,x)
```

output
```
sinh(x)/(a*(sinh(x)*b + a))
```

3.204 $\int \frac{\text{sech}(x)}{(a+b\sinh(x))^2}\,dx$

Optimal result

Integrand size = 11, antiderivative size = 79

$$\int \frac{\text{sech}(x)}{(a+b\sinh(x))^2}\,dx = \frac{(a^2-b^2)\arctan(\sinh(x))}{(a^2+b^2)^2} - \frac{2ab\log(\cosh(x))}{(a^2+b^2)^2} + \frac{2ab\log(a+b\sinh(x))}{(a^2+b^2)^2} - \frac{b}{(a^2+b^2)(a+b\sinh(x))}$$

output
```
(a^2-b^2)*arctan(sinh(x))/(a^2+b^2)^2-2*a*b*ln(cosh(x))/(a^2+b^2)^2+2*a*b*
ln(a+b*sinh(x))/(a^2+b^2)^2-b/(a^2+b^2)/(a+b*sinh(x))
```

Mathematica [A] (verified)

Time = 0.29 (sec) , antiderivative size = 121, normalized size of antiderivative = 1.53

$$\int \frac{\text{sech}(x)}{(a+b\sinh(x))^2}\,dx = -\frac{b\left(\left(2a+\frac{-a^2+b^2}{\sqrt{-b^2}}\right)\log\left(\sqrt{-b^2}-b\sinh(x)\right)-4a\log(a+b\sinh(x))+\left(2a+\frac{a^2-b^2}{\sqrt{-b^2}}\right)\log\left(\sqrt{-b^2}+b\sinh(\right.\right.}{2\left(a^2+b^2\right)^2}$$

input
```
Integrate[Sech[x]/(a + b*Sinh[x])^2,x]
```

output
```
-1/2*(b*((2*a + (-a^2 + b^2)/Sqrt[-b^2])*Log[Sqrt[-b^2] - b*Sinh[x]] - 4*a
*Log[a + b*Sinh[x]] + (2*a + (a^2 - b^2)/Sqrt[-b^2])*Log[Sqrt[-b^2] + b*Si
nh[x]] + (2*(a^2 + b^2))/(a + b*Sinh[x])))/(a^2 + b^2)^2
```

Rubi [A] (verified)

Time = 0.32 (sec) , antiderivative size = 103, normalized size of antiderivative = 1.30, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.545$, Rules used = {3042, 3147, 25, 480, 657, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\text{sech}(x)}{(a+b\sinh(x))^2}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{\cos(ix)(a-ib\sin(ix))^2}dx$$

$$\Big\downarrow \text{3147}$$

$$-b\int -\frac{1}{(a+b\sinh(x))^2\left(\sinh^2(x)b^2+b^2\right)}d(b\sinh(x))$$

$$\Big\downarrow \text{25}$$

$$b\int \frac{1}{(a+b\sinh(x))^2\left(\sinh^2(x)b^2+b^2\right)}d(b\sinh(x))$$

$$\Big\downarrow \text{480}$$

$$-b\left(\frac{1}{(a^2+b^2)(a+b\sinh(x))}-\frac{\int \frac{a-b\sinh(x)}{(a+b\sinh(x))\left(\sinh^2(x)b^2+b^2\right)}d(b\sinh(x))}{a^2+b^2}\right)$$

$$\Big\downarrow \text{657}$$

$$-b\left(\frac{1}{(a^2+b^2)(a+b\sinh(x))}-\frac{\int \left(\frac{2a}{(a^2+b^2)(a+b\sinh(x))}+\frac{a^2-2b\sinh(x)a-b^2}{(a^2+b^2)\left(\sinh^2(x)b^2+b^2\right)}\right)d(b\sinh(x))}{a^2+b^2}\right)$$

$$\Big\downarrow \text{2009}$$

$$-b\left(\frac{1}{(a^2+b^2)(a+b\sinh(x))}-\frac{\frac{(a^2-b^2)\arctan(\sinh(x))}{b(a^2+b^2)}-\frac{a\log(b^2\sinh^2(x)+b^2)}{a^2+b^2}+\frac{2a\log(a+b\sinh(x))}{a^2+b^2}}{a^2+b^2}\right)$$

input
```
Int[Sech[x]/(a + b*Sinh[x])^2,x]
```

output
```
-(b*(-((((a^2 - b^2)*ArcTan[Sinh[x]])/(b*(a^2 + b^2)) + (2*a*Log[a + b*Sin
h[x]])/(a^2 + b^2) - (a*Log[b^2 + b^2*Sinh[x]^2])/(a^2 + b^2))/(a^2 + b^2)
) + 1/((a^2 + b^2)*(a + b*Sinh[x]))))
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 480
```
Int[((c_) + (d_.)*(x_))^(n_)/((a_) + (b_.)*(x_)^2), x_Symbol] :> Simp[d*((c
 + d*x)^(n + 1)/((n + 1)*(b*c^2 + a*d^2))), x] + Simp[b/(b*c^2 + a*d^2)   I
nt[(c + d*x)^(n + 1)*((c - d*x)/(a + b*x^2)), x], x] /; FreeQ[{a, b, c, d},
 x] && ILtQ[n, -1]
```

rule 657
```
Int[(((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))^(n_.))/((a_) + (c_.)*(
x_)^2), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*((f + g*x)^n/(a + c*x^
2)), x], x] /; FreeQ[{a, c, d, e, f, g, m}, x] && IntegersQ[n]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3147
```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Simp[1/(b^p*f)   Subst[Int[(a + x)^m*(b^2 - x^2)^((p - 1)
/2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && IntegerQ[(p
 - 1)/2] && NeQ[a^2 - b^2, 0]
```

Maple [A] (verified)

Time = 15.28 (sec) , antiderivative size = 123, normalized size of antiderivative = 1.56

method	result
default	$\frac{2b\left(-\frac{b\left(a^2+b^2\right)\tanh\left(\frac{x}{2}\right)}{a\left(\tanh\left(\frac{x}{2}\right)^2 a-2b\tanh\left(\frac{x}{2}\right)-a\right)}+a\ln\left(\tanh\left(\frac{x}{2}\right)^2 a-2b\tanh\left(\frac{x}{2}\right)-a\right)\right)}{\left(a^2+b^2\right)^2}+\frac{-2ab\ln\left(1+\tanh\left(\frac{x}{2}\right)^2\right)+2\left(a^2-b^2\right)\arctan\left(\tanh\left(\frac{x}{2}\right)\right.}{a^4+2a^2b^2+b^4}$
risch	$-\frac{2b\,e^x}{(a^2+b^2)(b\,e^{2x}+2\,e^x a-b)}-\frac{i\ln(e^x-i)a^2}{a^4+2a^2b^2+b^4}+\frac{i\ln(e^x-i)b^2}{a^4+2a^2b^2+b^4}-\frac{2\ln(e^x-i)ab}{a^4+2a^2b^2+b^4}+\frac{i\ln(e^x+i)a^2}{a^4+2a^2b^2+b^4}-\frac{i\ln(e^x+i)b^2}{a^4+2a^2b^2+b^4}-\frac{2\ln}{a^4+}$

input
```
int(sech(x)/(a+b*sinh(x))^2,x,method=_RETURNVERBOSE)
```

output
```
2*b/(a^2+b^2)^2*(-b*(a^2+b^2)/a*tanh(1/2*x)/(tanh(1/2*x)^2*a-2*b*tanh(1/2*
x)-a)+a*ln(tanh(1/2*x)^2*a-2*b*tanh(1/2*x)-a))+2/(a^4+2*a^2*b^2+b^4)*(-a*b
*ln(1+tanh(1/2*x)^2)+(a^2-b^2)*arctan(tanh(1/2*x)))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 383 vs. $2(79) = 158$.

Time = 0.11 (sec) , antiderivative size = 383, normalized size of antiderivative = 4.85

$$\int \frac{\operatorname{sech}(x)}{(a+b\sinh(x))^2}\,dx$$
$$= \frac{2\left(\left(a^2b-b^3-(a^2b-b^3)\cosh(x)^2-(a^2b-b^3)\sinh(x)^2-2(a^3-ab^2)\cosh(x)-2(a^3-ab^2+(a^2b-\right.\right.}{}$$

input
```
integrate(sech(x)/(a+b*sinh(x))^2,x, algorithm="fricas")
```

output
```
2*((a^2*b - b^3 - (a^2*b - b^3)*cosh(x)^2 - (a^2*b - b^3)*sinh(x)^2 - 2*(a
^3 - a*b^2)*cosh(x) - 2*(a^3 - a*b^2 + (a^2*b - b^3)*cosh(x))*sinh(x))*arc
tan(cosh(x) + sinh(x)) + (a^2*b + b^3)*cosh(x) - (a*b^2*cosh(x)^2 + a*b^2*
sinh(x)^2 + 2*a^2*b*cosh(x) - a*b^2 + 2*(a*b^2*cosh(x) + a^2*b)*sinh(x))*l
og(2*(b*sinh(x) + a)/(cosh(x) - sinh(x))) + (a*b^2*cosh(x)^2 + a*b^2*sinh(
x)^2 + 2*a^2*b*cosh(x) - a*b^2 + 2*(a*b^2*cosh(x) + a^2*b)*sinh(x))*log(2*
cosh(x)/(cosh(x) - sinh(x))) + (a^2*b + b^3)*sinh(x))/(a^4*b + 2*a^2*b^3 +
 b^5 - (a^4*b + 2*a^2*b^3 + b^5)*cosh(x)^2 - (a^4*b + 2*a^2*b^3 + b^5)*sin
h(x)^2 - 2*(a^5 + 2*a^3*b^2 + a*b^4)*cosh(x) - 2*(a^5 + 2*a^3*b^2 + a*b^4
+ (a^4*b + 2*a^2*b^3 + b^5)*cosh(x))*sinh(x))
```

Sympy [F]

$$\int \frac{\operatorname{sech}(x)}{(a+b\sinh(x))^2}\,dx=\int \frac{\operatorname{sech}(x)}{(a+b\sinh(x))^2}\,dx$$

input
```
integrate(sech(x)/(a+b*sinh(x))**2,x)
```

output
```
Integral(sech(x)/(a + b*sinh(x))**2, x)
```

Maxima [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 149, normalized size of antiderivative = 1.89

$$\int \frac{\operatorname{sech}(x)}{(a+b\sinh(x))^2}\,dx=\frac{2\,ab\log\left(-2\,ae^{(-x)}+be^{(-2\,x)}-b\right)}{a^4+2\,a^2b^2+b^4}-\frac{2\,ab\log\left(e^{(-2\,x)}+1\right)}{a^4+2\,a^2b^2+b^4}-\frac{2\,(a^2-b^2)\arctan\left(e^{(-x)}\right)}{a^4+2\,a^2b^2+b^4}-\frac{2\,be^{(-x)}}{a^2b+b^3+2\,(a^3+ab^2)e^{(-x)}-(a^2b+b^3)e^{(-2\,x)}}$$

input
```
integrate(sech(x)/(a+b*sinh(x))^2,x, algorithm="maxima")
```

output
```
2*a*b*log(-2*a*e^(-x) + b*e^(-2*x) - b)/(a^4 + 2*a^2*b^2 + b^4) - 2*a*b*lo
g(e^(-2*x) + 1)/(a^4 + 2*a^2*b^2 + b^4) - 2*(a^2 - b^2)*arctan(e^(-x))/(a^
4 + 2*a^2*b^2 + b^4) - 2*b*e^(-x)/(a^2*b + b^3 + 2*(a^3 + a*b^2)*e^(-x) -
(a^2*b + b^3)*e^(-2*x))
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 186 vs. $2(79) = 158$.

Time = 0.13 (sec) , antiderivative size = 186, normalized size of antiderivative = 2.35

$$\int \frac{\operatorname{sech}(x)}{(a+b\sinh(x))^2}\,dx = \frac{2\,ab^2\log\left(\left|-b\left(e^{(-x)}-e^x\right)+2\,a\right|\right)}{a^4b+2\,a^2b^3+b^5} - \frac{ab\log\left(\left(e^{(-x)}-e^x\right)^2+4\right)}{a^4+2\,a^2b^2+b^4} + \frac{\left(\pi+2\,\arctan\left(\frac{1}{2}\left(e^{(2\,x)}-1\right)e^{(-x)}\right)\right)(a^2-b^2)}{2\,(a^4+2\,a^2b^2+b^4)} - \frac{2\left(ab^2\left(e^{(-x)}-e^x\right)-3\,a^2b-b^3\right)}{(a^4+2\,a^2b^2+b^4)\left(b\left(e^{(-x)}-e^x\right)-2\,a\right)}$$

input
```
integrate(sech(x)/(a+b*sinh(x))^2,x, algorithm="giac")
```

output
```
2*a*b^2*log(abs(-b*(e^(-x) - e^x) + 2*a))/(a^4*b + 2*a^2*b^3 + b^5) - a*b*
log((e^(-x) - e^x)^2 + 4)/(a^4 + 2*a^2*b^2 + b^4) + 1/2*(pi + 2*arctan(1/2
*(e^(2*x) - 1)*e^(-x)))*(a^2 - b^2)/(a^4 + 2*a^2*b^2 + b^4) - 2*(a*b^2*(e^
(-x) - e^x) - 3*a^2*b - b^3)/((a^4 + 2*a^2*b^2 + b^4)*(b*(e^(-x) - e^x) -
2*a))
```

Mupad [B] (verification not implemented)

Time = 2.64 (sec) , antiderivative size = 186, normalized size of antiderivative = 2.35

$$\int \frac{\operatorname{sech}(x)}{(a+b\sinh(x))^2}\,dx = \frac{2\,a\,b\,\ln\left(b^5\,e^{2\,x}-a^4\,b-b^5-14\,a^2\,b^3+2\,a^5\,e^x+14\,a^2\,b^3\,e^{2\,x}+2\,a\,b^4\,e^x+a^4\,b\,e^{2\,x}+28\,a^3\,b^2\,e^x\right)}{a^4+2\,a^2\,b^2+b^4} - \frac{2\,b^2\,e^x}{\left(a^2\,b+b^3\right)\,\left(2\,a\,e^x-b+b\,e^{2\,x}\right)} - \frac{\ln\left(1+e^x\,1i\right)}{-a^2\,1i+2\,a\,b+b^2\,1i} - \frac{\ln\left(e^x+1i\right)\,1i}{-a^2+a\,b\,2i+b^2}$$

input
```
int(1/(cosh(x)*(a + b*sinh(x))^2),x)
```

output
```
(2*a*b*log(b^5*exp(2*x) - a^4*b - b^5 - 14*a^2*b^3 + 2*a^5*exp(x) + 14*a^2
*b^3*exp(2*x) + 2*a*b^4*exp(x) + a^4*b*exp(2*x) + 28*a^3*b^2*exp(x)))/(a^4
 + b^4 + 2*a^2*b^2) - (log(exp(x) + 1i)*1i)/(a*b*2i - a^2 + b^2) - (2*b^2*
exp(x))/((a^2*b + b^3)*(2*a*exp(x) - b + b*exp(2*x))) - log(exp(x)*1i + 1)
/(2*a*b - a^2*1i + b^2*1i)
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 336, normalized size of antiderivative = 4.25

$$\int \frac{\operatorname{sech}(x)}{(a+b\sinh(x))^2}\,dx = \frac{2e^{2x}\,atan(e^x)\,a^3b-2e^{2x}\,atan(e^x)\,a\,b^3+4e^x\,atan(e^x)\,a^4-4e^x\,atan(e^x)\,a^2b^2-2\,atan(e^x)\,a^3b+2\,atan(e^x)\,a}{}$$

input
```
int(sech(x)/(a+b*sinh(x))^2,x)
```

output
```
(2*e**(2*x)*atan(e**x)*a**3*b - 2*e**(2*x)*atan(e**x)*a*b**3 + 4*e**x*atan
(e**x)*a**4 - 4*e**x*atan(e**x)*a**2*b**2 - 2*atan(e**x)*a**3*b + 2*atan(e
**x)*a*b**3 - 2*e**(2*x)*log(e**(2*x) + 1)*a**2*b**2 + 2*e**(2*x)*log(e**(
2*x)*b + 2*e**x*a - b)*a**2*b**2 + e**(2*x)*a**2*b**2 + e**(2*x)*b**4 - 4*
e**x*log(e**(2*x) + 1)*a**3*b + 4*e**x*log(e**(2*x)*b + 2*e**x*a - b)*a**3
*b + 2*log(e**(2*x) + 1)*a**2*b**2 - 2*log(e**(2*x)*b + 2*e**x*a - b)*a**2
*b**2 - a**2*b**2 - b**4)/(a*(e**(2*x)*a**4*b + 2*e**(2*x)*a**2*b**3 + e**
(2*x)*b**5 + 2*e**x*a**5 + 4*e**x*a**3*b**2 + 2*e**x*a*b**4 - a**4*b - 2*a
**2*b**3 - b**5))
```

3.205 $\int \frac{\operatorname{sech}^2(x)}{(a+b\sinh(x))^2}\,dx$

Optimal result

Integrand size = 13, antiderivative size = 93

$$\int \frac{\operatorname{sech}^2(x)}{(a+b\sinh(x))^2}\,dx = -\frac{6ab^2\operatorname{arctanh}\left(\frac{b-a\tanh\left(\frac{x}{2}\right)}{\sqrt{a^2+b^2}}\right)}{(a^2+b^2)^{5/2}} - \frac{b\operatorname{sech}(x)}{(a^2+b^2)\,(a+b\sinh(x))} + \frac{\operatorname{sech}(x)\,(3ab+(a^2-2b^2)\sinh(x))}{(a^2+b^2)^2}$$

output
```
-6*a*b^2*arctanh((b-a*tanh(1/2*x))/(a^2+b^2)^(1/2))/(a^2+b^2)^(5/2)-b*sech
(x)/(a^2+b^2)/(a+b*sinh(x))+sech(x)*(3*a*b+(a^2-2*b^2)*sinh(x))/(a^2+b^2)^
2
```

Mathematica [A] (verified)

Time = 0.18 (sec) , antiderivative size = 94, normalized size of antiderivative = 1.01

$$\int \frac{\operatorname{sech}^2(x)}{(a+b\sinh(x))^2}\,dx = \frac{\frac{6ab^2\arctan\left(\frac{b-a\tanh\left(\frac{x}{2}\right)}{\sqrt{-a^2-b^2}}\right)}{\sqrt{-a^2-b^2}} + 2ab\operatorname{sech}(x) - \frac{b^3\cosh(x)}{a+b\sinh(x)} + a^2\tanh(x) - b^2\tanh(x)}{(a^2+b^2)^2}$$

input
```
Integrate[Sech[x]^2/(a + b*Sinh[x])^2,x]
```

output
```
((6*a*b^2*ArcTan[(b - a*Tanh[x/2])/Sqrt[-a^2 - b^2]])/Sqrt[-a^2 - b^2] + 2
*a*b*Sech[x] - (b^3*Cosh[x])/(a + b*Sinh[x]) + a^2*Tanh[x] - b^2*Tanh[x])/
(a^2 + b^2)^2
```

Rubi [A] (verified)

Time = 0.51 (sec) , antiderivative size = 109, normalized size of antiderivative = 1.17, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.769$, Rules used = {3042, 3173, 25, 3042, 3345, 27, 3042, 3139, 1083, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\text{sech}^2(x)}{(a+b\sinh(x))^2}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{\cos(ix)^2(a-ib\sin(ix))^2}dx$$

$$\Big\downarrow \text{3173}$$

$$-\frac{\int -\frac{\text{sech}^2(x)(a-2b\sinh(x))}{a+b\sinh(x)}dx}{a^2+b^2}-\frac{b\text{sech}(x)}{(a^2+b^2)(a+b\sinh(x))}$$

$$\Big\downarrow \text{25}$$

$$\frac{\int \frac{\text{sech}^2(x)(a-2b\sinh(x))}{a+b\sinh(x)}dx}{a^2+b^2}-\frac{b\text{sech}(x)}{(a^2+b^2)(a+b\sinh(x))}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{b\text{sech}(x)}{(a^2+b^2)(a+b\sinh(x))}+\frac{\int \frac{a+2ib\sin(ix)}{\cos(ix)^2(a-ib\sin(ix))}dx}{a^2+b^2}$$

$$\Big\downarrow \text{3345}$$

$$\frac{\frac{\text{sech}(x)((a^2-2b^2)\sinh(x)+3ab)}{a^2+b^2}-\frac{\int -\frac{3ab^2}{a+b\sinh(x)}dx}{a^2+b^2}}{a^2+b^2}-\frac{b\text{sech}(x)}{(a^2+b^2)(a+b\sinh(x))}$$

$$\Big\downarrow \text{27}$$

$$\frac{\frac{3ab^2\int \frac{1}{a+b\sinh(x)}dx}{a^2+b^2}+\frac{\text{sech}(x)\left(\left(a^2-2b^2\right)\sinh(x)+3ab\right)}{a^2+b^2}}{a^2+b^2}-\frac{b\text{sech}(x)}{\left(a^2+b^2\right)\left(a+b\sinh(x)\right)}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{b\text{sech}(x)}{\left(a^2+b^2\right)\left(a+b\sinh(x)\right)}+\frac{\frac{\text{sech}(x)\left(\left(a^2-2b^2\right)\sinh(x)+3ab\right)}{a^2+b^2}+\frac{3ab^2\int \frac{1}{a-ib\sin(ix)}dx}{a^2+b^2}}{a^2+b^2}$$

$$\Big\downarrow \text{3139}$$

$$\frac{\frac{6ab^2\int \frac{1}{-a\tanh^2\left(\frac{x}{2}\right)+2b\tanh\left(\frac{x}{2}\right)+a}d\tanh\left(\frac{x}{2}\right)}{a^2+b^2}+\frac{\text{sech}(x)\left(\left(a^2-2b^2\right)\sinh(x)+3ab\right)}{a^2+b^2}}{a^2+b^2}-\frac{b\text{sech}(x)}{\left(a^2+b^2\right)\left(a+b\sinh(x)\right)}$$

$$\Big\downarrow \text{1083}$$

$$\frac{\frac{\text{sech}(x)\left(\left(a^2-2b^2\right)\sinh(x)+3ab\right)}{a^2+b^2}-\frac{12ab^2\int \frac{1}{4\left(a^2+b^2\right)-\left(2b-2a\tanh\left(\frac{x}{2}\right)\right)^2}d\left(2b-2a\tanh\left(\frac{x}{2}\right)\right)}{a^2+b^2}}{a^2+b^2}-\frac{b\text{sech}(x)}{\left(a^2+b^2\right)\left(a+b\sinh(x)\right)}$$

$$\Big\downarrow \text{219}$$

$$\frac{\frac{\text{sech}(x)\left(\left(a^2-2b^2\right)\sinh(x)+3ab\right)}{a^2+b^2}-\frac{6ab^2\text{arctanh}\left(\frac{2b-2a\tanh\left(\frac{x}{2}\right)}{2\sqrt{a^2+b^2}}\right)}{\left(a^2+b^2\right)^{3/2}}}{a^2+b^2}-\frac{b\text{sech}(x)}{\left(a^2+b^2\right)\left(a+b\sinh(x)\right)}$$

input
```
Int[Sech[x]^2/(a + b*Sinh[x])^2,x]
```

output
```
-((b*Sech[x])/((a^2 + b^2)*(a + b*Sinh[x]))) + ((-6*a*b^2*ArcTanh[(2*b - 2
*a*Tanh[x/2])/(2*Sqrt[a^2 + b^2])])/(a^2 + b^2)^(3/2) + (Sech[x]*(3*a*b +
(a^2 - 2*b^2)*Sinh[x]))/(a^2 + b^2))/(a^2 + b^2)
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1083
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Simp[-2   Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3139
```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Simp[2*(e/d)   Subst[Int[1/(a + 2*b*e*x + a
*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[a^2 - b^2, 0]
```

rule 3173
```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(-b)*(g*Cos[e + f*x])^(p + 1)*((a + b*Sin[e +
f*x])^(m + 1)/(f*g*(a^2 - b^2)*(m + 1))), x] + Simp[1/((a^2 - b^2)*(m + 1))
   Int[(g*Cos[e + f*x])^p*(a + b*Sin[e + f*x])^(m + 1)*(a*(m + 1) - b*(m +
p + 2)*Sin[e + f*x]), x], x] /; FreeQ[{a, b, e, f, g, p}, x] && NeQ[a^2 - b
^2, 0] && LtQ[m, -1] && IntegersQ[2*m, 2*p]
```

rule 3345
```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[(g*Co
s[e + f*x])^(p + 1)*(a + b*Sin[e + f*x])^(m + 1)*((b*c - a*d - (a*c - b*d)*
Sin[e + f*x])/(f*g*(a^2 - b^2)*(p + 1))), x] + Simp[1/(g^2*(a^2 - b^2)*(p +
 1))   Int[(g*Cos[e + f*x])^(p + 2)*(a + b*Sin[e + f*x])^m*Simp[c*(a^2*(p +
 2) - b^2*(m + p + 2)) + a*b*d*m + b*(a*c - b*d)*(m + p + 3)*Sin[e + f*x],
x], x], x] /; FreeQ[{a, b, c, d, e, f, g, m}, x] && NeQ[a^2 - b^2, 0] && Lt
Q[p, -1] && IntegerQ[2*m]
```

Maple [A] (verified)

Time = 42.93 (sec) , antiderivative size = 138, normalized size of antiderivative = 1.48

method	result	
default	$-\frac{2b^2\left(\frac{-\frac{b^2\tanh\left(\frac{x}{2}\right)}{a}-b}{\tanh\left(\frac{x}{2}\right)^2a-2b\tanh\left(\frac{x}{2}\right)-a}-\frac{3a\,\operatorname{arctanh}\left(\frac{2a\tanh\left(\frac{x}{2}\right)-2b}{2\sqrt{a^2+b^2}}\right)}{\sqrt{a^2+b^2}}\right)}{\left(a^2+b^2\right)^2}-\frac{2\left(\left(-a^2+b^2\right)\tanh\left(\frac{x}{2}\right)-2ab\right)}{\left(a^4+2a^2b^2+b^4\right)\left(1+\tanh\left(\frac{x}{2}\right)^2\right)}$	
risch	$-\frac{2\left(-3a\,b^2e^{3x}-3a^2b\,e^{2x}+2a^3e^x-a\,b^2e^x-a^2b+2b^3\right)}{\left(a^2+b^2\right)^2\left(e^{2x}+1\right)\left(b\,e^{2x}+2\,e^xa-b\right)}+\frac{3b^2a\ln\left(e^x+\frac{\left(a^2+b^2\right)^{\frac{5}{2}}a-a^6-3a^4b^2-3a^2b^4-b^6}{b\left(a^2+b^2\right)^{\frac{5}{2}}}\right)}{\left(a^2+b^2\right)^{\frac{5}{2}}}-\frac{3b^2a\ln\left(e^x+\frac{(a^2+b\ldots\right.}{\ldots}$	

input
```
int(sech(x)^2/(a+b*sinh(x))^2,x,method=_RETURNVERBOSE)
```

output
```
-2/(a^2+b^2)^2*b^2*((-1/a*b^2*tanh(1/2*x)-b)/(tanh(1/2*x)^2*a-2*b*tanh(1/2
*x)-a)-3*a/(a^2+b^2)^(1/2)*arctanh(1/2*(2*a*tanh(1/2*x)-2*b)/(a^2+b^2)^(1/
2)))-2/(a^4+2*a^2*b^2+b^4)*((-a^2+b^2)*tanh(1/2*x)-2*a*b)/(1+tanh(1/2*x)^2
)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 802 vs. $2(89) = 178$.

Time = 0.10 (sec) , antiderivative size = 802, normalized size of antiderivative = 8.62

$$\int \frac{\operatorname{sech}^2(x)}{(a+b\sinh(x))^2}\,dx = \text{Too large to display}$$

input

```
integrate(sech(x)^2/(a+b*sinh(x))^2,x, algorithm="fricas")
```

output

```
-(2*a^4*b - 2*a^2*b^3 - 4*b^5 + 6*(a^3*b^2 + a*b^4)*cosh(x)^3 + 6*(a^3*b^2
 + a*b^4)*sinh(x)^3 + 6*(a^4*b + a^2*b^3)*cosh(x)^2 + 6*(a^4*b + a^2*b^3 +
 3*(a^3*b^2 + a*b^4)*cosh(x))*sinh(x)^2 + 3*(a*b^3*cosh(x)^4 + a*b^3*sinh(
x)^4 + 2*a^2*b^2*cosh(x)^3 + 2*a^2*b^2*cosh(x) - a*b^3 + 2*(2*a*b^3*cosh(x
) + a^2*b^2)*sinh(x)^3 + 6*(a*b^3*cosh(x)^2 + a^2*b^2*cosh(x))*sinh(x)^2 +
 2*(2*a*b^3*cosh(x)^3 + 3*a^2*b^2*cosh(x)^2 + a^2*b^2)*sinh(x))*sqrt(a^2 +
 b^2)*log((b^2*cosh(x)^2 + b^2*sinh(x)^2 + 2*a*b*cosh(x) + 2*a^2 + b^2 + 2
*(b^2*cosh(x) + a*b)*sinh(x) - 2*sqrt(a^2 + b^2)*(b*cosh(x) + b*sinh(x) +
a))/(b*cosh(x)^2 + b*sinh(x)^2 + 2*a*cosh(x) + 2*(b*cosh(x) + a)*sinh(x) -
 b)) - 2*(2*a^5 + a^3*b^2 - a*b^4)*cosh(x) - 2*(2*a^5 + a^3*b^2 - a*b^4 -
9*(a^3*b^2 + a*b^4)*cosh(x)^2 - 6*(a^4*b + a^2*b^3)*cosh(x))*sinh(x))/(a^6
*b + 3*a^4*b^3 + 3*a^2*b^5 + b^7 - (a^6*b + 3*a^4*b^3 + 3*a^2*b^5 + b^7)*c
osh(x)^4 - (a^6*b + 3*a^4*b^3 + 3*a^2*b^5 + b^7)*sinh(x)^4 - 2*(a^7 + 3*a^
5*b^2 + 3*a^3*b^4 + a*b^6)*cosh(x)^3 - 2*(a^7 + 3*a^5*b^2 + 3*a^3*b^4 + a*
b^6 + 2*(a^6*b + 3*a^4*b^3 + 3*a^2*b^5 + b^7)*cosh(x))*sinh(x)^3 - 6*((a^6
*b + 3*a^4*b^3 + 3*a^2*b^5 + b^7)*cosh(x)^2 + (a^7 + 3*a^5*b^2 + 3*a^3*b^4
 + a*b^6)*cosh(x))*sinh(x)^2 - 2*(a^7 + 3*a^5*b^2 + 3*a^3*b^4 + a*b^6)*cos
h(x) - 2*(a^7 + 3*a^5*b^2 + 3*a^3*b^4 + a*b^6 + 2*(a^6*b + 3*a^4*b^3 + 3*a
^2*b^5 + b^7)*cosh(x)^3 + 3*(a^7 + 3*a^5*b^2 + 3*a^3*b^4 + a*b^6)*cosh(x)^
2)*sinh(x))
```

Sympy [F]

$$\int \frac{\operatorname{sech}^2(x)}{(a+b\sinh(x))^2}\,dx = \int \frac{\operatorname{sech}^2(x)}{(a+b\sinh(x))^2}\,dx$$

input
```
integrate(sech(x)**2/(a+b*sinh(x))**2,x)
```

output
```
Integral(sech(x)**2/(a + b*sinh(x))**2, x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 215 vs. 2(89) = 178.

Time = 0.13 (sec) , antiderivative size = 215, normalized size of antiderivative = 2.31

$$\int \frac{\operatorname{sech}^2(x)}{(a+b\sinh(x))^2}\,dx = \frac{3\,ab^2\log\left(\frac{be^{(-x)}-a-\sqrt{a^2+b^2}}{be^{(-x)}-a+\sqrt{a^2+b^2}}\right)}{(a^4+2\,a^2b^2+b^4)\sqrt{a^2+b^2}} + \frac{2\left(3\,a^2be^{(-2\,x)}-3\,ab^2e^{(-3\,x)}+a^2b-2\,b^3+(2\,a^3-ab^2)e^{(-x)}\right)}{a^4b+2\,a^2b^3+b^5+2\,(a^5+2\,a^3b^2+ab^4)e^{(-x)}+2\,(a^5+2\,a^3b^2+ab^4)e^{(-3\,x)}-(a^4b+2\,a^2b^3+b^5)e^{(-4\,x)}}$$

input
```
integrate(sech(x)^2/(a+b*sinh(x))^2,x, algorithm="maxima")
```

output
```
3*a*b^2*log((b*e^(-x) - a - sqrt(a^2 + b^2))/(b*e^(-x) - a + sqrt(a^2 + b^
2)))/((a^4 + 2*a^2*b^2 + b^4)*sqrt(a^2 + b^2)) + 2*(3*a^2*b*e^(-2*x) - 3*a
*b^2*e^(-3*x) + a^2*b - 2*b^3 + (2*a^3 - a*b^2)*e^(-x))/(a^4*b + 2*a^2*b^3
 + b^5 + 2*(a^5 + 2*a^3*b^2 + a*b^4)*e^(-x) + 2*(a^5 + 2*a^3*b^2 + a*b^4)*
e^(-3*x) - (a^4*b + 2*a^2*b^3 + b^5)*e^(-4*x))
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 167, normalized size of antiderivative = 1.80

$$\int \frac{\operatorname{sech}^2(x)}{(a+b\sinh(x))^2}\,dx = \frac{3\,ab^2\log\left(\frac{\left|2\,be^x+2\,a-2\,\sqrt{a^2+b^2}\right|}{\left|2\,be^x+2\,a+2\,\sqrt{a^2+b^2}\right|}\right)}{(a^4+2\,a^2b^2+b^4)\sqrt{a^2+b^2}} + \frac{2\left(3\,ab^2e^{(3\,x)}+3\,a^2be^{(2\,x)}-2\,a^3e^x+ab^2e^x+a^2b-2\,b^3\right)}{(a^4+2\,a^2b^2+b^4)(be^{(4\,x)}+2\,ae^{(3\,x)}+2\,ae^x-b)}$$

input
```
integrate(sech(x)^2/(a+b*sinh(x))^2,x, algorithm="giac")
```

output
```
3*a*b^2*log(abs(2*b*e^x + 2*a - 2*sqrt(a^2 + b^2))/abs(2*b*e^x + 2*a + 2*s
qrt(a^2 + b^2)))/((a^4 + 2*a^2*b^2 + b^4)*sqrt(a^2 + b^2)) + 2*(3*a*b^2*e^
(3*x) + 3*a^2*b*e^(2*x) - 2*a^3*e^x + a*b^2*e^x + a^2*b - 2*b^3)/((a^4 + 2
*a^2*b^2 + b^4)*(b*e^(4*x) + 2*a*e^(3*x) + 2*a*e^x - b))
```

Mupad [B] (verification not implemented)

Time = 1.83 (sec) , antiderivative size = 302, normalized size of antiderivative = 3.25

$$\int \frac{\operatorname{sech}^2(x)}{(a+b\sinh(x))^2}\,dx = \frac{\frac{6\,a^4\,b^4\,\mathrm{e}^{2\,x}}{(a^3+a\,b^2)\,(a^3\,b^3+a\,b^5)}-\frac{2\,(2\,a^2\,b^6-a^4\,b^4)}{(a^3+a\,b^2)\,(a^3\,b^3+a\,b^5)}+\frac{6\,a^3\,b^5\,\mathrm{e}^{3\,x}}{(a^3+a\,b^2)\,(a^3\,b^3+a\,b^5)}+\frac{2\,a\,\mathrm{e}^x\,(a^2\,b^6-2\,a^4\,b^4)}{b\,(a^3+a\,b^2)\,(a^3\,b^3+a\,b^5)}}{2\,a\,\mathrm{e}^x-b+2\,a\,\mathrm{e}^{3\,x}+b\,\mathrm{e}^{4\,x}} - \frac{3\,a\,b^2\,\ln\left(-\frac{6\,a\,b\,\mathrm{e}^x}{(a^2+b^2)^2}-\frac{6\,a\,b\,(b-a\,\mathrm{e}^x)}{(a^2+b^2)^{5/2}}\right)}{(a^2+b^2)^{5/2}} + \frac{3\,a\,b^2\,\ln\left(\frac{6\,a\,b\,(b-a\,\mathrm{e}^x)}{(a^2+b^2)^{5/2}}-\frac{6\,a\,b\,\mathrm{e}^x}{(a^2+b^2)^2}\right)}{(a^2+b^2)^{5/2}}$$

input
```
int(1/(cosh(x)^2*(a + b*sinh(x))^2),x)
```

output
```
((6*a^4*b^4*exp(2*x))/((a*b^2 + a^3)*(a*b^5 + a^3*b^3)) - (2*(2*a^2*b^6 -
a^4*b^4))/((a*b^2 + a^3)*(a*b^5 + a^3*b^3)) + (6*a^3*b^5*exp(3*x))/((a*b^2
 + a^3)*(a*b^5 + a^3*b^3)) + (2*a*exp(x)*(a^2*b^6 - 2*a^4*b^4))/(b*(a*b^2
+ a^3)*(a*b^5 + a^3*b^3)))/(2*a*exp(x) - b + 2*a*exp(3*x) + b*exp(4*x)) -
(3*a*b^2*log(- (6*a*b*exp(x))/(a^2 + b^2)^2 - (6*a*b*(b - a*exp(x)))/(a^2
+ b^2)^(5/2)))/(a^2 + b^2)^(5/2) + (3*a*b^2*log((6*a*b*(b - a*exp(x)))/(a^
2 + b^2)^(5/2) - (6*a*b*exp(x))/(a^2 + b^2)^2))/(a^2 + b^2)^(5/2)
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 421, normalized size of antiderivative = 4.53

$$\int \frac{\operatorname{sech}^2(x)}{(a+b\sinh(x))^2}\,dx = \frac{6e^{4x}\sqrt{a^2+b^2}\,atan\left(\frac{e^x bi+ai}{\sqrt{a^2+b^2}}\right)a\,b^3 i+12e^{3x}\sqrt{a^2+b^2}\,atan\left(\frac{e^x bi+ai}{\sqrt{a^2+b^2}}\right)a^2b^2 i+12e^x\sqrt{a^2+b^2}\,atan\left(\frac{e^x bi+ai}{\sqrt{a^2+b^2}}\right)a^2 b}{e^{4x}a^6b+3e^{4x}a^4b^3+3e^{4x}a^2b^5+e^{4x}b^7+2e^{3x}a^7+6e^{3x}a^5b^2+6e}$$

input
```
int(sech(x)^2/(a+b*sinh(x))^2,x)
```

output
```
(6*e**(4*x)*sqrt(a**2 + b**2)*atan((e**x*b*i + a*i)/sqrt(a**2 + b**2))*a*b
**3*i + 12*e**(3*x)*sqrt(a**2 + b**2)*atan((e**x*b*i + a*i)/sqrt(a**2 + b*
*2))*a**2*b**2*i + 12*e**x*sqrt(a**2 + b**2)*atan((e**x*b*i + a*i)/sqrt(a*
*2 + b**2))*a**2*b**2*i - 6*sqrt(a**2 + b**2)*atan((e**x*b*i + a*i)/sqrt(a
**2 + b**2))*a*b**3*i - 3*e**(4*x)*a**2*b**3 - 3*e**(4*x)*b**5 + 6*e**(2*x
)*a**4*b + 6*e**(2*x)*a**2*b**3 - 4*e**x*a**5 - 8*e**x*a**3*b**2 - 4*e**x*
a*b**4 + 2*a**4*b + a**2*b**3 - b**5)/(e**(4*x)*a**6*b + 3*e**(4*x)*a**4*b
**3 + 3*e**(4*x)*a**2*b**5 + e**(4*x)*b**7 + 2*e**(3*x)*a**7 + 6*e**(3*x)*
a**5*b**2 + 6*e**(3*x)*a**3*b**4 + 2*e**(3*x)*a*b**6 + 2*e**x*a**7 + 6*e**
x*a**5*b**2 + 6*e**x*a**3*b**4 + 2*e**x*a*b**6 - a**6*b - 3*a**4*b**3 - 3*
a**2*b**5 - b**7)
```

3.206 $\int \frac{\operatorname{sech}^3(x)}{(a+b\sinh(x))^2}\,dx$

Optimal result

Integrand size = 13, antiderivative size = 136

$$\int \frac{\operatorname{sech}^3(x)}{(a+b\sinh(x))^2}\,dx = \frac{(a^4+6a^2b^2-3b^4)\arctan(\sinh(x))}{2(a^2+b^2)^3} - \frac{4ab^3\log(\cosh(x))}{(a^2+b^2)^3} + \frac{4ab^3\log(a+b\sinh(x))}{(a^2+b^2)^3} + \frac{b(a^2-3b^2)}{2(a^2+b^2)^2(a+b\sinh(x))} + \frac{\operatorname{sech}^2(x)(b+a\sinh(x))}{2(a^2+b^2)(a+b\sinh(x))}$$

output
```
1/2*(a^4+6*a^2*b^2-3*b^4)*arctan(sinh(x))/(a^2+b^2)^3-4*a*b^3*ln(cosh(x))/
(a^2+b^2)^3+4*a*b^3*ln(a+b*sinh(x))/(a^2+b^2)^3+1/2*b*(a^2-3*b^2)/(a^2+b^2
)^2/(a+b*sinh(x))+1/2*sech(x)^2*(b+a*sinh(x))/(a^2+b^2)/(a+b*sinh(x))
```

Mathematica [A] (verified)

Time = 1.44 (sec) , antiderivative size = 260, normalized size of antiderivative = 1.91

$$\int \frac{\operatorname{sech}^3(x)}{(a+b\sinh(x))^2}\,dx = -\frac{-\frac{2\operatorname{sech}^2(x)(b+a\sinh(x))}{a+b\sinh(x)}+\frac{b\left(\frac{2a\left(a^2+b^2\right)\left(\left(-a+\sqrt{-b^2}\right)\log\left(\sqrt{-b^2}-b\sinh(x)\right)-2\sqrt{-b^2}\log(a+b\sinh(x))+\left(a+\sqrt{-b^2}\right)\log\left(\sqrt{-b^2}+b\sinh(x)\right)\right)}{\sqrt{-b^2}}+(\cdots\right.}{\cdots}}{4\left(a^2+b^2\cdots\right.}$$

input
```
Integrate[Sech[x]^3/(a + b*Sinh[x])^2,x]
```

output
```
-1/4*((-2*Sech[x]^2*(b + a*Sinh[x]))/(a + b*Sinh[x]) + (b*((2*a*(a^2 + b^2
)*((-a + Sqrt[-b^2])*Log[Sqrt[-b^2] - b*Sinh[x]] - 2*Sqrt[-b^2]*Log[a + b*
Sinh[x]] + (a + Sqrt[-b^2])*Log[Sqrt[-b^2] + b*Sinh[x]]))/Sqrt[-b^2] + (-a
^2 + 3*b^2)*((2*a + (-a^2 + b^2)/Sqrt[-b^2])*Log[Sqrt[-b^2] - b*Sinh[x]] -
 4*a*Log[a + b*Sinh[x]] + (2*a + (a^2 - b^2)/Sqrt[-b^2])*Log[Sqrt[-b^2] +
b*Sinh[x]] + (2*(a^2 + b^2))/(a + b*Sinh[x]))))/(a^2 + b^2)^2)/(a^2 + b^2)
```

Rubi [A] (verified)

Time = 0.42 (sec) , antiderivative size = 179, normalized size of antiderivative = 1.32, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.462$, Rules used = {3042, 3147, 496, 25, 657, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\operatorname{sech}^3(x)}{(a+b\sinh(x))^2}dx$$

$$\Big\downarrow \ 3042$$

$$\int \frac{1}{\cos(ix)^3(a-ib\sin(ix))^2}dx$$

$$\Big\downarrow \ 3147$$

$$b^3 \int \frac{1}{(a+b\sinh(x))^2 \left(\sinh^2(x)b^2+b^2\right)^2}d(b\sinh(x))$$

$$\Big\downarrow \textbf{496}$$

$$b^3 \left(\frac{ab\sinh(x)+b^2}{2b^2(a^2+b^2)\left(b^2\sinh^2(x)+b^2\right)(a+b\sinh(x))}-\frac{\int -\frac{a^2+2b\sinh(x)a+3b^2}{(a+b\sinh(x))^2\left(\sinh^2(x)b^2+b^2\right)}d(b\sinh(x))}{2b^2(a^2+b^2)}\right)$$

$$\Big\downarrow \textbf{25}$$

$$b^3 \left(\frac{\int \frac{a^2+2b\sinh(x)a+3b^2}{(a+b\sinh(x))^2\left(\sinh^2(x)b^2+b^2\right)}d(b\sinh(x))}{2b^2(a^2+b^2)}+\frac{ab\sinh(x)+b^2}{2b^2(a^2+b^2)\left(b^2\sinh^2(x)+b^2\right)(a+b\sinh(x))}\right)$$

$$\Big\downarrow \textbf{657}$$

$$b^3 \left(\frac{\int \left(\frac{8ab^2}{(a^2+b^2)^2(a+b\sinh(x))}+\frac{a^4+6b^2a^2-8b^3\sinh(x)a-3b^4}{(a^2+b^2)^2\left(\sinh^2(x)b^2+b^2\right)}+\frac{3b^2-a^2}{(a^2+b^2)(a+b\sinh(x))^2}\right)d(b\sinh(x))}{2b^2(a^2+b^2)}+\frac{ab\sin}{2b^2(a^2+b^2)\left(b^2\sinh\right.}\right.$$

$$\Big\downarrow \textbf{2009}$$

$$b^3 \left(\frac{ab\sinh(x)+b^2}{2b^2(a^2+b^2)\left(b^2\sinh^2(x)+b^2\right)(a+b\sinh(x))}+\frac{\frac{a^2-3b^2}{(a^2+b^2)(a+b\sinh(x))}-\frac{4ab^2\log\left(b^2\sinh^2(x)+b^2\right)}{(a^2+b^2)^2}+\frac{8ab^2\log(a+b\sinh(}{(a^2+b^2)^2}}{2b^2(a^2+b^2)}\right.$$

input
```
Int[Sech[x]^3/(a + b*Sinh[x])^2,x]
```

output
```
b^3*((b^2 + a*b*Sinh[x])/(2*b^2*(a^2 + b^2)*(a + b*Sinh[x])*(b^2 + b^2*Sin
h[x]^2)) + (((a^4 + 6*a^2*b^2 - 3*b^4)*ArcTan[Sinh[x]])/(b*(a^2 + b^2)^2)
+ (8*a*b^2*Log[a + b*Sinh[x]])/(a^2 + b^2)^2 - (4*a*b^2*Log[b^2 + b^2*Sinh
[x]^2])/(a^2 + b^2)^2 + (a^2 - 3*b^2)/((a^2 + b^2)*(a + b*Sinh[x])))/(2*b^
2*(a^2 + b^2)))
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 496
```
Int[((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(-(a*d + b*c*x))*(c + d*x)^(n + 1)*((a + b*x^2)^(p + 1)/(2*a*(p + 1)*(b*c^2
 + a*d^2))), x] + Simp[1/(2*a*(p + 1)*(b*c^2 + a*d^2))   Int[(c + d*x)^n*(a
 + b*x^2)^(p + 1)*Simp[b*c^2*(2*p + 3) + a*d^2*(n + 2*p + 3) + b*c*d*(n + 2
*p + 4)*x, x], x], x] /; FreeQ[{a, b, c, d, n}, x] && LtQ[p, -1] && IntQuad
raticQ[a, 0, b, c, d, n, p, x]
```

rule 657
```
Int[(((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))^(n_.))/((a_) + (c_.)*(
x_)^2), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*((f + g*x)^n/(a + c*x^
2)), x], x] /; FreeQ[{a, c, d, e, f, g, m}, x] && IntegersQ[n]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3147
```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Simp[1/(b^p*f)   Subst[Int[(a + x)^m*(b^2 - x^2)^((p - 1)
/2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && IntegerQ[(p
 - 1)/2] && NeQ[a^2 - b^2, 0]
```

Maple [A] (verified)

Time = 89.44 (sec) , antiderivative size = 211, normalized size of antiderivative = 1.55

method	result
default	$\frac{2b^3\left(-\frac{b\left(a^2+b^2\right)\tanh\left(\frac{x}{2}\right)}{a\left(\tanh\left(\frac{x}{2}\right)^2a-2b\tanh\left(\frac{x}{2}\right)-a\right)}+2a\ln\left(\tanh\left(\frac{x}{2}\right)^2a-2b\tanh\left(\frac{x}{2}\right)-a\right)\right)}{\left(a^2+b^2\right)^3}+\frac{2\left(\left(-\frac{a^4}{2}+\frac{b^4}{2}\right)\tanh\left(\frac{x}{2}\right)^3+\left(-2a^3b-2a\,b^3\right)\tanh\left(\frac{x}{2}\right)^2}{\left(1+\tanh\left(\frac{x}{2}\right)^2\right)^2}$
risch	$\frac{\left(a^2b\,e^{4x}-3\,e^{4x}b^3+2a^3e^{3x}+2a\,b^2e^{3x}+6a^2b\,e^{2x}-2b^3e^{2x}-2a^3e^x-2a\,b^2e^x+a^2b-3b^3\right)e^x}{\left(a^2+b^2\right)^2\left(e^{2x}+1\right)^2\left(b\,e^{2x}+2\,e^xa-b\right)}-\frac{i\ln\left(e^x-i\right)a^4}{2\left(a^6+3a^4b^2+3a^2b^4+b^6\right)}-\frac{3i\ln(e^x}{a^6+3a^4b^2}$

input
```
int(sech(x)^3/(a+b*sinh(x))^2,x,method=_RETURNVERBOSE)
```

output
```
2*b^3/(a^2+b^2)^3*(-b*(a^2+b^2)/a*tanh(1/2*x)/(tanh(1/2*x)^2*a-2*b*tanh(1/
2*x)-a)+2*a*ln(tanh(1/2*x)^2*a-2*b*tanh(1/2*x)-a))+2/(a^6+3*a^4*b^2+3*a^2*
b^4+b^6)*(((-1/2*a^4+1/2*b^4)*tanh(1/2*x)^3+(-2*a^3*b-2*a*b^3)*tanh(1/2*x)
^2+(1/2*a^4-1/2*b^4)*tanh(1/2*x))/(1+tanh(1/2*x)^2)^2-2*a*b^3*ln(1+tanh(1/
2*x)^2)+1/2*(a^4+6*a^2*b^2-3*b^4)*arctan(tanh(1/2*x)))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2615 vs. $2(130) = 260$.

Time = 0.15 (sec) , antiderivative size = 2615, normalized size of antiderivative = 19.23

$$\int \frac{\operatorname{sech}^3(x)}{(a+b\sinh(x))^2}\,dx = \text{Too large to display}$$

input
```
integrate(sech(x)^3/(a+b*sinh(x))^2,x, algorithm="fricas")
```

output
```
-((a^4*b - 2*a^2*b^3 - 3*b^5)*cosh(x)^5 + (a^4*b - 2*a^2*b^3 - 3*b^5)*sinh
(x)^5 + 2*(a^5 + 2*a^3*b^2 + a*b^4)*cosh(x)^4 + (2*a^5 + 4*a^3*b^2 + 2*a*b
^4 + 5*(a^4*b - 2*a^2*b^3 - 3*b^5)*cosh(x))*sinh(x)^4 + 2*(3*a^4*b + 2*a^2
*b^3 - b^5)*cosh(x)^3 + 2*(3*a^4*b + 2*a^2*b^3 - b^5 + 5*(a^4*b - 2*a^2*b^
3 - 3*b^5)*cosh(x)^2 + 4*(a^5 + 2*a^3*b^2 + a*b^4)*cosh(x))*sinh(x)^3 - 2*
(a^5 + 2*a^3*b^2 + a*b^4)*cosh(x)^2 - 2*(a^5 + 2*a^3*b^2 + a*b^4 - 5*(a^4*
b - 2*a^2*b^3 - 3*b^5)*cosh(x)^3 - 6*(a^5 + 2*a^3*b^2 + a*b^4)*cosh(x)^2 -
 3*(3*a^4*b + 2*a^2*b^3 - b^5)*cosh(x))*sinh(x)^2 + ((a^4*b + 6*a^2*b^3 -
3*b^5)*cosh(x)^6 + (a^4*b + 6*a^2*b^3 - 3*b^5)*sinh(x)^6 + 2*(a^5 + 6*a^3*
b^2 - 3*a*b^4)*cosh(x)^5 + 2*(a^5 + 6*a^3*b^2 - 3*a*b^4 + 3*(a^4*b + 6*a^2
*b^3 - 3*b^5)*cosh(x))*sinh(x)^5 - a^4*b - 6*a^2*b^3 + 3*b^5 + (a^4*b + 6*
a^2*b^3 - 3*b^5)*cosh(x)^4 + (a^4*b + 6*a^2*b^3 - 3*b^5 + 15*(a^4*b + 6*a^
2*b^3 - 3*b^5)*cosh(x)^2 + 10*(a^5 + 6*a^3*b^2 - 3*a*b^4)*cosh(x))*sinh(x)
^4 + 4*(a^5 + 6*a^3*b^2 - 3*a*b^4)*cosh(x)^3 + 4*(a^5 + 6*a^3*b^2 - 3*a*b^
4 + 5*(a^4*b + 6*a^2*b^3 - 3*b^5)*cosh(x)^3 + 5*(a^5 + 6*a^3*b^2 - 3*a*b^4
)*cosh(x)^2 + (a^4*b + 6*a^2*b^3 - 3*b^5)*cosh(x))*sinh(x)^3 - (a^4*b + 6*
a^2*b^3 - 3*b^5)*cosh(x)^2 - (a^4*b + 6*a^2*b^3 - 3*b^5 - 15*(a^4*b + 6*a^
2*b^3 - 3*b^5)*cosh(x)^4 - 20*(a^5 + 6*a^3*b^2 - 3*a*b^4)*cosh(x)^3 - 6*(a
^4*b + 6*a^2*b^3 - 3*b^5)*cosh(x)^2 - 12*(a^5 + 6*a^3*b^2 - 3*a*b^4)*cosh(
x))*sinh(x)^2 + 2*(a^5 + 6*a^3*b^2 - 3*a*b^4)*cosh(x) + 2*(3*(a^4*b + 6...
```

Sympy [F]

$$\int \frac{\operatorname{sech}^3(x)}{(a+b\sinh(x))^2}\,dx = \int \frac{\operatorname{sech}^3(x)}{(a+b\sinh(x))^2}\,dx$$

input
```
integrate(sech(x)**3/(a+b*sinh(x))**2,x)
```

output
```
Integral(sech(x)**3/(a + b*sinh(x))**2, x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 375 vs. $2(130) = 260$.

Time = 0.13 (sec) , antiderivative size = 375, normalized size of antiderivative = 2.76

$$\int \frac{\operatorname{sech}^3(x)}{(a+b\sinh(x))^2}\,dx = \frac{4\,ab^3\log\left(-2\,ae^{(-x)}+be^{(-2\,x)}-b\right)}{a^6+3\,a^4b^2+3\,a^2b^4+b^6} - \frac{4\,ab^3\log\left(e^{(-2\,x)}+1\right)}{a^6+3\,a^4b^2+3\,a^2b^4+b^6} - \frac{(a^4+6\,a^2b^2-3\,b^4)\arctan\left(e^{(-x)}\right)}{a^6+3\,a^4b^2+3\,a^2b^4+b^6} + \frac{(a^2b-3\,b^3)e^{(-x)}+2\,(a^3+ab^2)e^{(-2\,x)}+2\,(3\,a^2b-b^3)e^{(-3\,x)}}{a^4b+2\,a^2b^3+b^5+2\,(a^5+2\,a^3b^2+ab^4)e^{(-x)}+(a^4b+2\,a^2b^3+b^5)e^{(-2\,x)}+4\,(a^5+2\,a^3b^2+ab^4)e^{(-3\,x)}}$$

input
```
integrate(sech(x)^3/(a+b*sinh(x))^2,x, algorithm="maxima")
```

output
```
4*a*b^3*log(-2*a*e^(-x) + b*e^(-2*x) - b)/(a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b
^6) - 4*a*b^3*log(e^(-2*x) + 1)/(a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6) - (a^4
 + 6*a^2*b^2 - 3*b^4)*arctan(e^(-x))/(a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6) +
 ((a^2*b - 3*b^3)*e^(-x) + 2*(a^3 + a*b^2)*e^(-2*x) + 2*(3*a^2*b - b^3)*e^
(-3*x) - 2*(a^3 + a*b^2)*e^(-4*x) + (a^2*b - 3*b^3)*e^(-5*x))/(a^4*b + 2*a
^2*b^3 + b^5 + 2*(a^5 + 2*a^3*b^2 + a*b^4)*e^(-x) + (a^4*b + 2*a^2*b^3 + b
^5)*e^(-2*x) + 4*(a^5 + 2*a^3*b^2 + a*b^4)*e^(-3*x) - (a^4*b + 2*a^2*b^3 +
 b^5)*e^(-4*x) + 2*(a^5 + 2*a^3*b^2 + a*b^4)*e^(-5*x) - (a^4*b + 2*a^2*b^3
 + b^5)*e^(-6*x))
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 295 vs. $2(130) = 260$.

Time = 0.14 (sec) , antiderivative size = 295, normalized size of antiderivative = 2.17

$$\int \frac{\operatorname{sech}^3(x)}{(a+b\sinh(x))^2}\,dx = \frac{4\,ab^4\log\left(\left|-b\left(e^{(-x)}-e^x\right)+2\,a\right|\right)}{a^6b+3\,a^4b^3+3\,a^2b^5+b^7} - \frac{2\,ab^3\log\left(\left(e^{(-x)}-e^x\right)^2+4\right)}{a^6+3\,a^4b^2+3\,a^2b^4+b^6} + \frac{\left(\pi+2\,\arctan\left(\frac{1}{2}\left(e^{(2\,x)}-1\right)e^{(-x)}\right)\right)(a^4+6\,a^2b^2-3\,b^4)}{4\,(a^6+3\,a^4b^2+3\,a^2b^4+b^6)} - \frac{a^2b\left(e^{(-x)}-e^x\right)^2-3\,b^3\left(e^{(-x)}-e^x\right)^2-2\,a^3\left(e^{(-x)}-e^x\right)-2\,ab^2\left(e^{(-x)}-e^x\right)+8\,a^2b-8\,b^3}{(a^4+2\,a^2b^2+b^4)\left(b\left(e^{(-x)}-e^x\right)^3-2\,a\left(e^{(-x)}-e^x\right)^2+4\,b\left(e^{(-x)}-e^x\right)-8\,a\right)}$$

input
```
integrate(sech(x)^3/(a+b*sinh(x))^2,x, algorithm="giac")
```

output
```
4*a*b^4*log(abs(-b*(e^(-x) - e^x) + 2*a))/(a^6*b + 3*a^4*b^3 + 3*a^2*b^5 +
 b^7) - 2*a*b^3*log((e^(-x) - e^x)^2 + 4)/(a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b
^6) + 1/4*(pi + 2*arctan(1/2*(e^(2*x) - 1)*e^(-x)))*(a^4 + 6*a^2*b^2 - 3*b
^4)/(a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6) - (a^2*b*(e^(-x) - e^x)^2 - 3*b^3*
(e^(-x) - e^x)^2 - 2*a^3*(e^(-x) - e^x) - 2*a*b^2*(e^(-x) - e^x) + 8*a^2*b
 - 8*b^3)/((a^4 + 2*a^2*b^2 + b^4)*(b*(e^(-x) - e^x)^3 - 2*a*(e^(-x) - e^x
)^2 + 4*b*(e^(-x) - e^x) - 8*a))
```

Mupad [B] (verification not implemented)

Time = 6.03 (sec) , antiderivative size = 519, normalized size of antiderivative = 3.82

$$\int \frac{\operatorname{sech}^3(x)}{(a+b\sinh(x))^2}\,dx = \frac{\frac{4\,\left(a^7\,b+3\,a^5\,b^3+3\,a^3\,b^5+a\,b^7\right)}{\left(a^2+b^2\right)\,{\left(a^4+2\,a^2\,b^2+b^4\right)}^2}+\frac{{\mathrm{e}}^x\,\left(a^8+2\,a^6\,b^2-2\,a^2\,b^6-b^8\right)}{\left(a^2+b^2\right)\,{\left(a^4+2\,a^2\,b^2+b^4\right)}^2}}{{\mathrm{e}}^{2\,x}+1}-\frac{\frac{4\,a\,b}{a^4+2\,a^2\,b^2+b^4}+\frac{2\,{\mathrm{e}}^x\,\left(a^2-b^2\right)}{a^4+2\,a^2\,b^2+b^4}}{2\,{\mathrm{e}}^{2\,x}+{\mathrm{e}}^{4\,x}+1} + \frac{\ln\left({\mathrm{e}}^x+1{}\mathrm{i}\right)\,\left(a-b\,3{}\mathrm{i}\right)}{2\,\left(-a^3\,1{}\mathrm{i}-3\,a^2\,b+a\,b^2\,3{}\mathrm{i}+b^3\right)}+\frac{\ln\left(1+{\mathrm{e}}^x\,1{}\mathrm{i}\right)\,\left(-3\,b+a\,1{}\mathrm{i}\right)}{2\,\left(-a^3-a^2\,b\,3{}\mathrm{i}+3\,a\,b^2+b^3\,1{}\mathrm{i}\right)} + \frac{4\,a\,b^3\,\ln\left(9\,b^9\,{\mathrm{e}}^{2\,x}-a^8\,b-9\,b^9-220\,a^2\,b^7-30\,a^4\,b^5-12\,a^6\,b^3+2\,a^9\,{\mathrm{e}}^x+220\,a^2\,b^7\,{\mathrm{e}}^{2\,x}+30\,a^4\,b^5\,{\mathrm{e}}^{2\,x}+\right.}{a^6+3\,a^4\,b^2+3\,a^2\,b^4+b^6} - \frac{2\,{\mathrm{e}}^x\,\left(a^4\,b^6+2\,a^2\,b^8+b^{10}\right)}{b^2\,\left(a^2\,b+b^3\right)\,\left(a^2+b^2\right)\,\left(2\,a\,{\mathrm{e}}^x-b+b\,{\mathrm{e}}^{2\,x}\right)\,\left(a^4+2\,a^2\,b^2+b^4\right)}$$

input
```
int(1/(cosh(x)^3*(a + b*sinh(x))^2),x)
```

output
```
((4*(a*b^7 + a^7*b + 3*a^3*b^5 + 3*a^5*b^3))/((a^2 + b^2)*(a^4 + b^4 + 2*a
^2*b^2)^2) + (exp(x)*(a^8 - b^8 - 2*a^2*b^6 + 2*a^6*b^2))/((a^2 + b^2)*(a^
4 + b^4 + 2*a^2*b^2)^2))/(exp(2*x) + 1) - ((4*a*b)/(a^4 + b^4 + 2*a^2*b^2)
 + (2*exp(x)*(a^2 - b^2))/(a^4 + b^4 + 2*a^2*b^2))/(2*exp(2*x) + exp(4*x)
+ 1) + (log(exp(x) + 1i)*(a - b*3i))/(2*(a*b^2*3i - 3*a^2*b - a^3*1i + b^3
)) + (log(exp(x)*1i + 1)*(a*1i - 3*b))/(2*(3*a*b^2 - a^2*b*3i - a^3 + b^3*
1i)) + (4*a*b^3*log(9*b^9*exp(2*x) - a^8*b - 9*b^9 - 220*a^2*b^7 - 30*a^4*
b^5 - 12*a^6*b^3 + 2*a^9*exp(x) + 220*a^2*b^7*exp(2*x) + 30*a^4*b^5*exp(2*
x) + 12*a^6*b^3*exp(2*x) + 18*a*b^8*exp(x) + a^8*b*exp(2*x) + 440*a^3*b^6*
exp(x) + 60*a^5*b^4*exp(x) + 24*a^7*b^2*exp(x)))/(a^6 + b^6 + 3*a^2*b^4 +
3*a^4*b^2) - (2*exp(x)*(b^10 + 2*a^2*b^8 + a^4*b^6))/(b^2*(a^2*b + b^3)*(a
^2 + b^2)*(2*a*exp(x) - b + b*exp(2*x))*(a^4 + b^4 + 2*a^2*b^2))
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 1150, normalized size of antiderivative = 8.46

$$\int \frac{\operatorname{sech}^3(x)}{(a+b\sinh(x))^2}\,dx = \text{Too large to display}$$

input
```
int(sech(x)^3/(a+b*sinh(x))^2,x)
```

output
```
(2*e**(6*x)*atan(e**x)*a**5*b + 12*e**(6*x)*atan(e**x)*a**3*b**3 - 6*e**(6
*x)*atan(e**x)*a*b**5 + 4*e**(5*x)*atan(e**x)*a**6 + 24*e**(5*x)*atan(e**x
)*a**4*b**2 - 12*e**(5*x)*atan(e**x)*a**2*b**4 + 2*e**(4*x)*atan(e**x)*a**
5*b + 12*e**(4*x)*atan(e**x)*a**3*b**3 - 6*e**(4*x)*atan(e**x)*a*b**5 + 8*
e**(3*x)*atan(e**x)*a**6 + 48*e**(3*x)*atan(e**x)*a**4*b**2 - 24*e**(3*x)*
atan(e**x)*a**2*b**4 - 2*e**(2*x)*atan(e**x)*a**5*b - 12*e**(2*x)*atan(e**
x)*a**3*b**3 + 6*e**(2*x)*atan(e**x)*a*b**5 + 4*e**x*atan(e**x)*a**6 + 24*
e**x*atan(e**x)*a**4*b**2 - 12*e**x*atan(e**x)*a**2*b**4 - 2*atan(e**x)*a*
*5*b - 12*atan(e**x)*a**3*b**3 + 6*atan(e**x)*a*b**5 - 8*e**(6*x)*log(e**(
2*x) + 1)*a**2*b**4 + 8*e**(6*x)*log(e**(2*x)*b + 2*e**x*a - b)*a**2*b**4
- e**(6*x)*a**4*b**2 + 2*e**(6*x)*a**2*b**4 + 3*e**(6*x)*b**6 - 16*e**(5*x
)*log(e**(2*x) + 1)*a**3*b**3 + 16*e**(5*x)*log(e**(2*x)*b + 2*e**x*a - b)
*a**3*b**3 - 8*e**(4*x)*log(e**(2*x) + 1)*a**2*b**4 + 8*e**(4*x)*log(e**(2
*x)*b + 2*e**x*a - b)*a**2*b**4 + 4*e**(4*x)*a**6 + 7*e**(4*x)*a**4*b**2 +
 6*e**(4*x)*a**2*b**4 + 3*e**(4*x)*b**6 - 32*e**(3*x)*log(e**(2*x) + 1)*a*
*3*b**3 + 32*e**(3*x)*log(e**(2*x)*b + 2*e**x*a - b)*a**3*b**3 + 8*e**(3*x
)*a**5*b + 16*e**(3*x)*a**3*b**3 + 8*e**(3*x)*a*b**5 + 8*e**(2*x)*log(e**(
2*x) + 1)*a**2*b**4 - 8*e**(2*x)*log(e**(2*x)*b + 2*e**x*a - b)*a**2*b**4
- 4*e**(2*x)*a**6 - 7*e**(2*x)*a**4*b**2 - 6*e**(2*x)*a**2*b**4 - 3*e**(2*
x)*b**6 - 16*e**x*log(e**(2*x) + 1)*a**3*b**3 + 16*e**x*log(e**(2*x)*b ...
```

3.207 $\int \frac{\operatorname{sech}^4(x)}{(a+b\sinh(x))^2}\,dx$

Optimal result

Integrand size = 13, antiderivative size = 144

$$\int \frac{\operatorname{sech}^4(x)}{(a+b\sinh(x))^2}\,dx = -\frac{10ab^4\operatorname{arctanh}\left(\frac{b-a\tanh\left(\frac{x}{2}\right)}{\sqrt{a^2+b^2}}\right)}{(a^2+b^2)^{7/2}} - \frac{b\operatorname{sech}^3(x)}{(a^2+b^2)(a+b\sinh(x))} + \frac{\operatorname{sech}^3(x)\left(5ab+(a^2-4b^2)\sinh(x)\right)}{3(a^2+b^2)^2} + \frac{\operatorname{sech}(x)\left(15ab^3+(2a^4+9a^2b^2-8b^4)\sinh(x)\right)}{3(a^2+b^2)^3}$$

output
```
-10*a*b^4*arctanh((b-a*tanh(1/2*x))/(a^2+b^2)^(1/2))/(a^2+b^2)^(7/2)-b*sec
h(x)^3/(a^2+b^2)/(a+b*sinh(x))+1/3*sech(x)^3*(5*a*b+(a^2-4*b^2)*sinh(x))/(
a^2+b^2)^2+1/3*sech(x)*(15*a*b^3+(2*a^4+9*a^2*b^2-8*b^4)*sinh(x))/(a^2+b^2
)^3
```

Mathematica [A] (verified)

Time = 0.32 (sec) , antiderivative size = 137, normalized size of antiderivative = 0.95

$$\int \frac{\operatorname{sech}^4(x)}{(a+b\sinh(x))^2}\,dx$$
$$=\frac{\frac{30ab^4\arctan\left(\frac{b-a\tanh\left(\frac{x}{2}\right)}{\sqrt{-a^2-b^2}}\right)}{\sqrt{-a^2-b^2}}+12ab^3\operatorname{sech}(x)-\frac{3b^5\cosh(x)}{a+b\sinh(x)}+(a^2+b^2)\operatorname{sech}^3(x)\left(2ab+(a^2-b^2)\sinh(x)\right)+(2a^4}{3\left(a^2+b^2\right)^3}$$

input

```
Integrate[Sech[x]^4/(a + b*Sinh[x])^2,x]
```

output

```
((30*a*b^4*ArcTan[(b - a*Tanh[x/2])/Sqrt[-a^2 - b^2]])/Sqrt[-a^2 - b^2] +
12*a*b^3*Sech[x] - (3*b^5*Cosh[x])/(a + b*Sinh[x]) + (a^2 + b^2)*Sech[x]^3
*(2*a*b + (a^2 - b^2)*Sinh[x]) + (2*a^4 + 9*a^2*b^2 - 5*b^4)*Tanh[x])/(3*(
a^2 + b^2)^3)
```

Rubi [A] (verified)

Time = 0.82 (sec) , antiderivative size = 171, normalized size of antiderivative = 1.19, number of steps used = 14, number of rules used = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 1.000$, Rules used = {3042, 3173, 25, 3042, 3345, 25, 3042, 3345, 27, 3042, 3139, 1083, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\operatorname{sech}^4(x)}{(a+b\sinh(x))^2}\,dx$$
$$\Big\downarrow \ 3042$$
$$\int \frac{1}{\cos(ix)^4(a-ib\sin(ix))^2}dx$$
$$\Big\downarrow \ 3173$$
$$-\frac{\int -\frac{\operatorname{sech}^4(x)(a-4b\sinh(x))}{a+b\sinh(x)}dx}{a^2+b^2}-\frac{b\operatorname{sech}^3(x)}{(a^2+b^2)(a+b\sinh(x))}$$

$$\Big\downarrow \; 25$$

$$\frac{\int \frac{\text{sech}^4(x)(a-4b\sinh(x))}{a+b\sinh(x)}dx}{a^2+b^2}-\frac{b\text{sech}^3(x)}{(a^2+b^2)\,(a+b\sinh(x))}$$

$$\Big\downarrow \; 3042$$

$$-\frac{b\text{sech}^3(x)}{(a^2+b^2)\,(a+b\sinh(x))}+\frac{\int \frac{a+4ib\sin(ix)}{\cos(ix)^4(a-ib\sin(ix))}dx}{a^2+b^2}$$

$$\Big\downarrow \; 3345$$

$$\frac{\frac{\text{sech}^3(x)((a^2-4b^2)\sinh(x)+5ab)}{3(a^2+b^2)}-\frac{\int -\frac{\text{sech}^2(x)\left(a\left(2a^2+7b^2\right)+2b\left(a^2-4b^2\right)\sinh(x)\right)}{a+b\sinh(x)}dx}{3(a^2+b^2)}}{a^2+b^2}-\frac{b\text{sech}^3(x)}{(a^2+b^2)\,(a+b\sinh(x))}$$

$$\Big\downarrow \; 25$$

$$\frac{\frac{\int \frac{\text{sech}^2(x)\left(a\left(2a^2+7b^2\right)+2b\left(a^2-4b^2\right)\sinh(x)\right)}{a+b\sinh(x)}dx}{3(a^2+b^2)}+\frac{\text{sech}^3(x)((a^2-4b^2)\sinh(x)+5ab)}{3(a^2+b^2)}}{a^2+b^2}-\frac{b\text{sech}^3(x)}{(a^2+b^2)\,(a+b\sinh(x))}$$

$$\Big\downarrow \; 3042$$

$$-\frac{b\text{sech}^3(x)}{(a^2+b^2)\,(a+b\sinh(x))}+\frac{\frac{\text{sech}^3(x)((a^2-4b^2)\sinh(x)+5ab)}{3(a^2+b^2)}+\frac{\int \frac{a\left(2a^2+7b^2\right)-2ib\left(a^2-4b^2\right)\sin(ix)}{\cos(ix)^2(a-ib\sin(ix))}dx}{3(a^2+b^2)}}{a^2+b^2}$$

$$\Big\downarrow \; 3345$$

$$\frac{\frac{\frac{\text{sech}(x)\left(\left(2a^4+9a^2b^2-8b^4\right)\sinh(x)+15ab^3\right)}{a^2+b^2}-\frac{\int -\frac{15ab^4}{a+b\sinh(x)}dx}{a^2+b^2}}{3(a^2+b^2)}+\frac{\text{sech}^3(x)((a^2-4b^2)\sinh(x)+5ab)}{3(a^2+b^2)}}{a^2+b^2}-\frac{b\text{sech}^3(x)}{(a^2+b^2)\,(a+b\sinh(x))}$$

$$\Big\downarrow \; 27$$

$$\frac{\frac{\frac{15ab^4\int \frac{1}{a+b\sinh(x)}dx}{a^2+b^2}+\frac{\text{sech}(x)\left(\left(2a^4+9a^2b^2-8b^4\right)\sinh(x)+15ab^3\right)}{a^2+b^2}}{3(a^2+b^2)}+\frac{\text{sech}^3(x)((a^2-4b^2)\sinh(x)+5ab)}{3(a^2+b^2)}}{a^2+b^2}-\frac{b\text{sech}^3(x)}{(a^2+b^2)\,(a+b\sinh(x))}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{b\operatorname{sech}^3(x)}{(a^2+b^2)(a+b\sinh(x))}+\frac{\frac{\operatorname{sech}^3(x)((a^2-4b^2)\sinh(x)+5ab)}{3(a^2+b^2)}+\frac{\frac{\operatorname{sech}(x)\left(\left(2a^4+9a^2b^2-8b^4\right)\sinh(x)+15ab^3\right)}{a^2+b^2}+\frac{15ab^4\int\frac{1}{a-ib\sin(ix)}dx}{a^2+b^2}}{3(a^2+b^2)}}{a^2+b^2}$$

$$\Big\downarrow \text{3139}$$

$$\frac{\frac{\frac{30ab^4\int\frac{1}{-a\tanh^2\left(\frac{x}{2}\right)+2b\tanh\left(\frac{x}{2}\right)+a}d\tanh\left(\frac{x}{2}\right)}{a^2+b^2}+\frac{\operatorname{sech}(x)\left(\left(2a^4+9a^2b^2-8b^4\right)\sinh(x)+15ab^3\right)}{a^2+b^2}}{3(a^2+b^2)}+\frac{\operatorname{sech}^3(x)((a^2-4b^2)\sinh(x)+5ab)}{3(a^2+b^2)}}{a^2+b^2}-\frac{b\operatorname{sech}^3(x)}{(a^2+b^2)(a+b\sinh(x))}$$

$$\Big\downarrow \text{1083}$$

$$\frac{\frac{\frac{\operatorname{sech}(x)\left(\left(2a^4+9a^2b^2-8b^4\right)\sinh(x)+15ab^3\right)}{a^2+b^2}-\frac{60ab^4\int\frac{1}{4\left(a^2+b^2\right)-\left(2b-2a\tanh\left(\frac{x}{2}\right)\right)^2}d\left(2b-2a\tanh\left(\frac{x}{2}\right)\right)}{a^2+b^2}}{3(a^2+b^2)}+\frac{\operatorname{sech}^3(x)((a^2-4b^2)\sinh(x)+5ab)}{3(a^2+b^2)}}{a^2+b^2}-\frac{b\operatorname{sech}^3(x)}{(a^2+b^2)(a+b\sinh(x))}$$

$$\Big\downarrow \text{219}$$

$$\frac{\frac{\operatorname{sech}^3(x)((a^2-4b^2)\sinh(x)+5ab)}{3(a^2+b^2)}+\frac{\frac{\operatorname{sech}(x)\left(\left(2a^4+9a^2b^2-8b^4\right)\sinh(x)+15ab^3\right)}{a^2+b^2}-\frac{30ab^4\operatorname{arctanh}\left(\frac{2b-2a\tanh\left(\frac{x}{2}\right)}{2\sqrt{a^2+b^2}}\right)}{\left(a^2+b^2\right)^{3/2}}}{3(a^2+b^2)}}{a^2+b^2}-\frac{b\operatorname{sech}^3(x)}{(a^2+b^2)(a+b\sinh(x))}$$

input
```
Int[Sech[x]^4/(a + b*Sinh[x])^2,x]
```

output
```
-((b*Sech[x]^3)/((a^2 + b^2)*(a + b*Sinh[x]))) + ((Sech[x]^3*(5*a*b + (a^2
 - 4*b^2)*Sinh[x]))/(3*(a^2 + b^2)) + ((-30*a*b^4*ArcTanh[(2*b - 2*a*Tanh[
x/2])/(2*Sqrt[a^2 + b^2])])/(a^2 + b^2)^(3/2) + (Sech[x]*(15*a*b^3 + (2*a^
4 + 9*a^2*b^2 - 8*b^4)*Sinh[x]))/(a^2 + b^2))/(3*(a^2 + b^2)))/(a^2 + b^2)
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1083
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Simp[-2   Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3139
```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Simp[2*(e/d)   Subst[Int[1/(a + 2*b*e*x + a
*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[a^2 - b^2, 0]
```

rule 3173
```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(-b)*(g*Cos[e + f*x])^(p + 1)*((a + b*Sin[e +
f*x])^(m + 1)/(f*g*(a^2 - b^2)*(m + 1))), x] + Simp[1/((a^2 - b^2)*(m + 1))
   Int[(g*Cos[e + f*x])^p*(a + b*Sin[e + f*x])^(m + 1)*(a*(m + 1) - b*(m +
p + 2)*Sin[e + f*x]), x], x] /; FreeQ[{a, b, e, f, g, p}, x] && NeQ[a^2 - b
^2, 0] && LtQ[m, -1] && IntegersQ[2*m, 2*p]
```

rule 3345
```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[(g*Co
s[e + f*x])^(p + 1)*(a + b*Sin[e + f*x])^(m + 1)*((b*c - a*d - (a*c - b*d)*
Sin[e + f*x])/(f*g*(a^2 - b^2)*(p + 1))), x] + Simp[1/(g^2*(a^2 - b^2)*(p +
 1))   Int[(g*Cos[e + f*x])^(p + 2)*(a + b*Sin[e + f*x])^m*Simp[c*(a^2*(p +
 2) - b^2*(m + p + 2)) + a*b*d*m + b*(a*c - b*d)*(m + p + 3)*Sin[e + f*x],
x], x], x] /; FreeQ[{a, b, c, d, e, f, g, m}, x] && NeQ[a^2 - b^2, 0] && Lt
Q[p, -1] && IntegerQ[2*m]
```

Maple [A] (verified)

Time = 223.30 (sec) , antiderivative size = 266, normalized size of antiderivative = 1.85

method	result
default	$-\frac{2b^4\left(\frac{-\frac{b^2\tanh\left(\frac{x}{2}\right)}{a}-b}{\tanh\left(\frac{x}{2}\right)^2 a-2b\tanh\left(\frac{x}{2}\right)-a}-\frac{5a\,\operatorname{arctanh}\left(\frac{2a\tanh\left(\frac{x}{2}\right)-2b}{2\sqrt{a^2+b^2}}\right)}{\sqrt{a^2+b^2}}\right)}{(a^4+2a^2b^2+b^4)(a^2+b^2)}-\frac{2\left(\left(-a^4-3a^2b^2+2b^4\right)\tanh\left(\frac{x}{2}\right)^5+\left(-2a^3b-6a\,b^3\right)\tanh\left(\frac{x}{2}\right)^4 \cdots\right.}{(\cdots}$ (cut off)
risch	$-\frac{2(-15a\,b^4e^{7x}-15a^2b^3e^{6x}+10a^3b^2e^{5x}-35a\,b^4e^{5x}-10a^4b\,e^{4x}-55a^2b^3e^{4x}+12a^5e^{3x}+44a^3b^2e^{3x}-13a\,b^4e^{3x}-4a^4b\,e^{2x}-33a^2b^3e^{2x}\cdots}{3(a^2+b^2)(a^4+2a^2b^2+b^4)(e^{2x}+1)^3(b\,e^{2x}+2\,e^x a-b)}$ (cut off)

input
```
int(sech(x)^4/(a+b*sinh(x))^2,x,method=_RETURNVERBOSE)
```

output
```
-2*b^4/(a^4+2*a^2*b^2+b^4)/(a^2+b^2)*((-1/a*b^2*tanh(1/2*x)-b)/(tanh(1/2*x
)^2*a-2*b*tanh(1/2*x)-a)-5*a/(a^2+b^2)^(1/2)*arctanh(1/2*(2*a*tanh(1/2*x)-
2*b)/(a^2+b^2)^(1/2)))-2/(a^4+2*a^2*b^2+b^4)/(a^2+b^2)*((-a^4-3*a^2*b^2+2*
b^4)*tanh(1/2*x)^5+(-2*a^3*b-6*a*b^3)*tanh(1/2*x)^4+(-2/3*a^4-6*a^2*b^2+8/
3*b^4)*tanh(1/2*x)^3-8*a*b^3*tanh(1/2*x)^2+(-a^4-3*a^2*b^2+2*b^4)*tanh(1/2
*x)-2/3*a^3*b-14/3*a*b^3)/(1+tanh(1/2*x)^2)^3
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 3044 vs. $2(136) = 272$.

Time = 0.14 (sec) , antiderivative size = 3044, normalized size of antiderivative = 21.14

$$\int \frac{\operatorname{sech}^4(x)}{(a+b\sinh(x))^2}\,dx = \text{Too large to display}$$

input
```
integrate(sech(x)^4/(a+b*sinh(x))^2,x, algorithm="fricas")
```

output
```
Too large to include
```

Sympy [F]

$$\int \frac{\operatorname{sech}^4(x)}{(a+b\sinh(x))^2}\,dx = \int \frac{\operatorname{sech}^4(x)}{(a+b\sinh(x))^2}\,dx$$

input
```
integrate(sech(x)**4/(a+b*sinh(x))**2,x)
```

output
```
Integral(sech(x)**4/(a + b*sinh(x))**2, x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 490 vs. $2(136) = 272$.

Time = 0.16 (sec) , antiderivative size = 490, normalized size of antiderivative = 3.40

$$\int \frac{\operatorname{sech}^4(x)}{(a+b\sinh(x))^2}\,dx = \frac{5\,ab^4\log\left(\frac{be^{(-x)}-a-\sqrt{a^2+b^2}}{be^{(-x)}-a+\sqrt{a^2+b^2}}\right)}{(a^6+3\,a^4b^2+3\,a^2b^4+b^6)\sqrt{a^2+b^2}} + \frac{2\left(15\,a^2b^3e^{(-6\,x)}-15\,ab^4e^{(-7\,x)}+2\,a^4b+9\,a^2b^3-8\,b^5+(4\right.}{3\left(a^6b+3\,a^4b^3+3\,a^2b^5+b^7+2\,(a^7+3\,a^5b^2+3\,a^3b^4+ab^6)e^{(-x)}+2\,(a^6b+3\,a^4b^3+3\,a^2b^5+b^7)e^{(-2\,x}\right.}$$

input
```
integrate(sech(x)^4/(a+b*sinh(x))^2,x, algorithm="maxima")
```

output
```
5*a*b^4*log((b*e^(-x) - a - sqrt(a^2 + b^2))/(b*e^(-x) - a + sqrt(a^2 + b^
2)))/((a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6)*sqrt(a^2 + b^2)) + 2/3*(15*a^2*b
^3*e^(-6*x) - 15*a*b^4*e^(-7*x) + 2*a^4*b + 9*a^2*b^3 - 8*b^5 + (4*a^5 + 1
8*a^3*b^2 - a*b^4)*e^(-x) + (4*a^4*b + 33*a^2*b^3 - 16*b^5)*e^(-2*x) + (12
*a^5 + 44*a^3*b^2 - 13*a*b^4)*e^(-3*x) + 5*(2*a^4*b + 11*a^2*b^3)*e^(-4*x)
 + 5*(2*a^3*b^2 - 7*a*b^4)*e^(-5*x))/(a^6*b + 3*a^4*b^3 + 3*a^2*b^5 + b^7
+ 2*(a^7 + 3*a^5*b^2 + 3*a^3*b^4 + a*b^6)*e^(-x) + 2*(a^6*b + 3*a^4*b^3 +
3*a^2*b^5 + b^7)*e^(-2*x) + 6*(a^7 + 3*a^5*b^2 + 3*a^3*b^4 + a*b^6)*e^(-3*
x) + 6*(a^7 + 3*a^5*b^2 + 3*a^3*b^4 + a*b^6)*e^(-5*x) - 2*(a^6*b + 3*a^4*b
^3 + 3*a^2*b^5 + b^7)*e^(-6*x) + 2*(a^7 + 3*a^5*b^2 + 3*a^3*b^4 + a*b^6)*e
^(-7*x) - (a^6*b + 3*a^4*b^3 + 3*a^2*b^5 + b^7)*e^(-8*x))
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 287 vs. $2(136) = 272$.

Time = 0.14 (sec) , antiderivative size = 287, normalized size of antiderivative = 1.99

$$\int \frac{\operatorname{sech}^4(x)}{(a+b\sinh(x))^2}\,dx$$
$$= \frac{5\,ab^4\log\left(\frac{\left|2\,be^x+2\,a-2\,\sqrt{a^2+b^2}\right|}{\left|2\,be^x+2\,a+2\,\sqrt{a^2+b^2}\right|}\right)}{(a^6+3\,a^4b^2+3\,a^2b^4+b^6)\sqrt{a^2+b^2}} + \frac{2\,(ab^4e^x-b^5)}{(a^6+3\,a^4b^2+3\,a^2b^4+b^6)(be^{(2\,x)}+2\,ae^x-b)}$$
$$+ \frac{2\,\left(12\,ab^3e^{(5\,x)}-9\,a^2b^2e^{(4\,x)}+3\,b^4e^{(4\,x)}+8\,a^3be^{(3\,x)}+32\,ab^3e^{(3\,x)}-6\,a^4e^{(2\,x)}-18\,a^2b^2e^{(2\,x)}+12\,b^4e^{(2\,x)}\right.}{3\,(a^6+3\,a^4b^2+3\,a^2b^4+b^6)(e^{(2\,x)}+1)^3}$$

input
```
integrate(sech(x)^4/(a+b*sinh(x))^2,x, algorithm="giac")
```

output
```
5*a*b^4*log(abs(2*b*e^x + 2*a - 2*sqrt(a^2 + b^2))/abs(2*b*e^x + 2*a + 2*s
qrt(a^2 + b^2)))/((a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6)*sqrt(a^2 + b^2)) + 2
*(a*b^4*e^x - b^5)/((a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6)*(b*e^(2*x) + 2*a*e
^x - b)) + 2/3*(12*a*b^3*e^(5*x) - 9*a^2*b^2*e^(4*x) + 3*b^4*e^(4*x) + 8*a
^3*b*e^(3*x) + 32*a*b^3*e^(3*x) - 6*a^4*e^(2*x) - 18*a^2*b^2*e^(2*x) + 12*
b^4*e^(2*x) + 12*a*b^3*e^x - 2*a^4 - 9*a^2*b^2 + 5*b^4)/((a^6 + 3*a^4*b^2
+ 3*a^2*b^4 + b^6)*(e^(2*x) + 1)^3)
```

Mupad [B] (verification not implemented)

Time = 1.89 (sec) , antiderivative size = 476, normalized size of antiderivative = 3.31

$$\int \frac{\operatorname{sech}^4(x)}{(a+b\sinh(x))^2}\,dx = \frac{\frac{8\,(a^2-b^2)}{3\,(a^4+2\,a^2\,b^2+b^4)}-\frac{16\,a\,b\,e^x}{3\,(a^4+2\,a^2\,b^2+b^4)}}{3\,e^{2\,x}+3\,e^{4\,x}+e^{6\,x}+1} - \frac{\frac{4\,(a^6+a^4\,b^2-a^2\,b^4-b^6)}{(a^4+2\,a^2\,b^2+b^4)^2}-\frac{16\,e^x\,(a^5\,b+2\,a^3\,b^3+a\,b^5)}{3\,(a^4+2\,a^2\,b^2+b^4)^2}}{2\,e^{2\,x}+e^{4\,x}+1} - \frac{\frac{2\,(3\,a^4\,b^2+2\,a^2\,b^4-b^6)}{(a^4+2\,a^2\,b^2+b^4)^2}-\frac{8\,e^x\,(a^3\,b^3+a\,b^5)}{(a^4+2\,a^2\,b^2+b^4)^2}}{e^{2\,x}+1} - \frac{\frac{2\,(a^2\,b^9+b^{11})}{b^3\,(a^2\,b+b^3)\,(a^2+b^2)^3}-\frac{2\,e^x\,(a^3\,b^9+a\,b^{11})}{b^4\,(a^2\,b+b^3)\,(a^2+b^2)^3}}{2\,a\,e^x-b+b\,e^{2\,x}} - \frac{5\,a\,b^4\,\ln\left(-\frac{10\,a\,b^3\,(b-a\,e^x)}{(a^2+b^2)^{7/2}}-\frac{10\,a\,b^3\,e^x}{(a^2+b^2)^3}\right)}{(a^2+b^2)^{7/2}} + \frac{5\,a\,b^4\,\ln\left(\frac{10\,a\,b^3\,(b-a\,e^x)}{(a^2+b^2)^{7/2}}-\frac{10\,a\,b^3\,e^x}{(a^2+b^2)^3}\right)}{(a^2+b^2)^{7/2}}$$

input
```
int(1/(cosh(x)^4*(a + b*sinh(x))^2),x)
```

output
```
((8*(a^2 - b^2))/(3*(a^4 + b^4 + 2*a^2*b^2)) - (16*a*b*exp(x))/(3*(a^4 + b
^4 + 2*a^2*b^2)))/(3*exp(2*x) + 3*exp(4*x) + exp(6*x) + 1) - ((4*(a^6 - b^
6 - a^2*b^4 + a^4*b^2))/(a^4 + b^4 + 2*a^2*b^2)^2 - (16*exp(x)*(a*b^5 + a^
5*b + 2*a^3*b^3))/(3*(a^4 + b^4 + 2*a^2*b^2)^2))/(2*exp(2*x) + exp(4*x) +
1) - ((2*(2*a^2*b^4 - b^6 + 3*a^4*b^2))/(a^4 + b^4 + 2*a^2*b^2)^2 - (8*exp
(x)*(a*b^5 + a^3*b^3))/(a^4 + b^4 + 2*a^2*b^2)^2)/(exp(2*x) + 1) - ((2*(b^
11 + a^2*b^9))/(b^3*(a^2*b + b^3)*(a^2 + b^2)^3) - (2*exp(x)*(a*b^11 + a^3
*b^9))/(b^4*(a^2*b + b^3)*(a^2 + b^2)^3))/(2*a*exp(x) - b + b*exp(2*x)) -
(5*a*b^4*log(- (10*a*b^3*(b - a*exp(x)))/(a^2 + b^2)^(7/2) - (10*a*b^3*exp
(x))/(a^2 + b^2)^3))/(a^2 + b^2)^(7/2) + (5*a*b^4*log((10*a*b^3*(b - a*exp
(x)))/(a^2 + b^2)^(7/2) - (10*a*b^3*exp(x))/(a^2 + b^2)^3))/(a^2 + b^2)^(7
/2)
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 1067, normalized size of antiderivative = 7.41

$$\int \frac{\operatorname{sech}^4(x)}{(a+b\sinh(x))^2}\,dx = \text{Too large to display}$$

input
```
int(sech(x)^4/(a+b*sinh(x))^2,x)
```

output
```
(30*e**(8*x)*sqrt(a**2 + b**2)*atan((e**x*b*i + a*i)/sqrt(a**2 + b**2))*a*
b**5*i + 60*e**(7*x)*sqrt(a**2 + b**2)*atan((e**x*b*i + a*i)/sqrt(a**2 + b
**2))*a**2*b**4*i + 60*e**(6*x)*sqrt(a**2 + b**2)*atan((e**x*b*i + a*i)/sq
rt(a**2 + b**2))*a*b**5*i + 180*e**(5*x)*sqrt(a**2 + b**2)*atan((e**x*b*i
+ a*i)/sqrt(a**2 + b**2))*a**2*b**4*i + 180*e**(3*x)*sqrt(a**2 + b**2)*ata
n((e**x*b*i + a*i)/sqrt(a**2 + b**2))*a**2*b**4*i - 60*e**(2*x)*sqrt(a**2
+ b**2)*atan((e**x*b*i + a*i)/sqrt(a**2 + b**2))*a*b**5*i + 60*e**x*sqrt(a
**2 + b**2)*atan((e**x*b*i + a*i)/sqrt(a**2 + b**2))*a**2*b**4*i - 30*sqrt
(a**2 + b**2)*atan((e**x*b*i + a*i)/sqrt(a**2 + b**2))*a*b**5*i - 15*e**(8
*x)*a**2*b**5 - 15*e**(8*x)*b**7 + 30*e**(6*x)*a**4*b**3 - 30*e**(6*x)*b**
7 - 20*e**(5*x)*a**5*b**2 - 40*e**(5*x)*a**3*b**4 - 20*e**(5*x)*a*b**6 + 2
0*e**(4*x)*a**6*b + 130*e**(4*x)*a**4*b**3 + 110*e**(4*x)*a**2*b**5 - 24*e
**(3*x)*a**7 - 112*e**(3*x)*a**5*b**2 - 152*e**(3*x)*a**3*b**4 - 64*e**(3*
x)*a*b**6 + 8*e**(2*x)*a**6*b + 74*e**(2*x)*a**4*b**3 + 64*e**(2*x)*a**2*b
**5 - 2*e**(2*x)*b**7 - 8*e**x*a**7 - 44*e**x*a**5*b**2 - 64*e**x*a**3*b**
4 - 28*e**x*a*b**6 + 4*a**6*b + 22*a**4*b**3 + 17*a**2*b**5 - b**7)/(3*(e*
*(8*x)*a**8*b + 4*e**(8*x)*a**6*b**3 + 6*e**(8*x)*a**4*b**5 + 4*e**(8*x)*a
**2*b**7 + e**(8*x)*b**9 + 2*e**(7*x)*a**9 + 8*e**(7*x)*a**7*b**2 + 12*e**
(7*x)*a**5*b**4 + 8*e**(7*x)*a**3*b**6 + 2*e**(7*x)*a*b**8 + 2*e**(6*x)*a*
*8*b + 8*e**(6*x)*a**6*b**3 + 12*e**(6*x)*a**4*b**5 + 8*e**(6*x)*a**2*b...
```

3.208 $\int \frac{\tanh^4(x)}{i+\sinh(x)} \, dx$

Optimal result

Integrand size = 13, antiderivative size = 31

$$\int \frac{\tanh^4(x)}{i+\sinh(x)} \, dx = -\operatorname{sech}(x)+\frac{2\operatorname{sech}^3(x)}{3}-\frac{\operatorname{sech}^5(x)}{5}-\frac{1}{5}i\tanh^5(x)$$

output
```
-sech(x)+2/3*sech(x)^3-1/5*sech(x)^5-1/5*I*tanh(x)^5
```

Mathematica [B] (verified)

Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 96 vs. 2(31) = 62.

Time = 0.13 (sec) , antiderivative size = 96, normalized size of antiderivative = 3.10

$$\int \frac{\tanh^4(x)}{i+\sinh(x)} \, dx = -\frac{200-534\cosh(x)+288\cosh(2x)-178\cosh(3x)+24\cosh(4x)+64i\sinh(x)+178i\sinh(2x)-192i}{960\left(\cosh\left(\frac{x}{2}\right)-i\sinh\left(\frac{x}{2}\right)\right)^5\left(\cosh\left(\frac{x}{2}\right)+i\sinh\left(\frac{x}{2}\right)\right)^3}$$

input
```
Integrate[Tanh[x]^4/(I + Sinh[x]),x]
```

output
```
-1/960*(200 - 534*Cosh[x] + 288*Cosh[2*x] - 178*Cosh[3*x] + 24*Cosh[4*x] +
 (64*I)*Sinh[x] + (178*I)*Sinh[2*x] - (192*I)*Sinh[3*x] + (89*I)*Sinh[4*x]
)/((Cosh[x/2] - I*Sinh[x/2])^5*(Cosh[x/2] + I*Sinh[x/2])^3)
```

Rubi [A] (verified)

Time = 0.33 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.769$, Rules used = {3042, 3185, 26, 3042, 26, 3086, 210, 2009, 3087, 15}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\tanh^4(x)}{\sinh(x)+i}dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \frac{\tan(ix)^4}{i-i\sin(ix)}dx$$

$$\Big\downarrow \text{ 3185}$$

$$-i\int \text{sech}^2(x)\tanh^4(x)dx-i\int i\text{sech}(x)\tanh^5(x)dx$$

$$\Big\downarrow \text{ 26}$$

$$\int \text{sech}(x)\tanh^5(x)dx-i\int \text{sech}^2(x)\tanh^4(x)dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int -i\sec(ix)\tan(ix)^5dx-i\int \sec(ix)^2\tan(ix)^4dx$$

$$\Big\downarrow \text{ 26}$$

$$-i\int \sec(ix)^2\tan(ix)^4dx-i\int \sec(ix)\tan(ix)^5dx$$

$$\Big\downarrow \text{ 3086}$$

$$-\int \left(\text{sech}^2(x)-1\right)^2d\text{sech}(x)-i\int \sec(ix)^2\tan(ix)^4dx$$

$$\Big\downarrow \text{ 210}$$

$$-\int \left(\text{sech}^4(x)-2\text{sech}^2(x)+1\right)d\text{sech}(x)-i\int \sec(ix)^2\tan(ix)^4dx$$

$$\Big\downarrow \text{2009}$$

$$-i\int \sec(ix)^2\tan(ix)^4dx-\frac{1}{5}\text{sech}^5(x)+\frac{2\text{sech}^3(x)}{3}-\text{sech}(x)$$

$$\Big\downarrow \text{3087}$$

$$-\int \tanh^4(x)d(i\tanh(x))-\frac{1}{5}\text{sech}^5(x)+\frac{2\text{sech}^3(x)}{3}-\text{sech}(x)$$

$$\Big\downarrow \text{15}$$

$$-\frac{1}{5}i\tanh^5(x)-\frac{1}{5}\text{sech}^5(x)+\frac{2\text{sech}^3(x)}{3}-\text{sech}(x)$$

input
```
Int[Tanh[x]^4/(I + Sinh[x]),x]
```

output
```
-Sech[x] + (2*Sech[x]^3)/3 - Sech[x]^5/5 - (I/5)*Tanh[x]^5
```

Defintions of rubi rules used

rule 15
```
Int[(a_.)*(x_)^(m_.), x_Symbol] :> Simp[a*(x^(m + 1)/(m + 1)), x] /; FreeQ[
{a, m}, x] && NeQ[m, -1]
```

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 210
```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Int[ExpandIntegrand[(a + b*x^2
)^p, x], x] /; FreeQ[{a, b}, x] && IGtQ[p, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3086
```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Simp[a/f   Subst[Int[(a*x)^(m - 1)*(-1 + x^2)^((n - 1)/2
), x], x, Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2
] &&  !(IntegerQ[m/2] && LtQ[0, m, n + 1])
```

rule 3087
```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> Simp[1/f   Subst[Int[(b*x)^n*(1 + x^2)^(m/2 - 1), x], x, Tan[e +
f*x]], x] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2] &&  !(IntegerQ[(n - 1)
/2] && LtQ[0, n, m - 1])
```

rule 3185
```
Int[((g_.)*tan[(e_.) + (f_.)*(x_)])^(p_.)/((a_) + (b_.)*sin[(e_.) + (f_.)*(
x_)]), x_Symbol] :> Simp[1/a   Int[Sec[e + f*x]^2*(g*Tan[e + f*x])^p, x], x
] - Simp[1/(b*g)   Int[Sec[e + f*x]*(g*Tan[e + f*x])^(p + 1), x], x] /; Fre
eQ[{a, b, e, f, g, p}, x] && EqQ[a^2 - b^2, 0] && NeQ[p, -1]
```

Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 62 vs. $2(24) = 48$.

Time = 4.17 (sec) , antiderivative size = 63, normalized size of antiderivative = 2.03

method	result
risch	$-\frac{2(25i e^{4x}+5\,e^{5x}+21i e^{2x}+13\,e^{3x}+15i e^{6x}+15\,e^{7x}-9\,e^{x}+3i)}{15(e^{x}+i)^5(e^{x}-i)^3}$
default	$\frac{i}{3(\tanh(\frac{x}{2})+i)^3}-\frac{2i}{5(\tanh(\frac{x}{2})+i)^5}-\frac{3i}{8(\tanh(\frac{x}{2})+i)}+\frac{1}{(\tanh(\frac{x}{2})+i)^4}+\frac{1}{2(\tanh(\frac{x}{2})+i)^2}+\frac{3i}{8(\tanh(\frac{x}{2})-i)}+\frac{i}{6(\tanh(\frac{x}{2})}$

input
```
int(tanh(x)^4/(I+sinh(x)),x,method=_RETURNVERBOSE)
```

output
```
-2/15*(25*I*exp(x)^4+5*exp(x)^5+21*I*exp(x)^2+13*exp(x)^3+15*I*exp(x)^6+15
*exp(x)^7-9*exp(x)+3*I)/(exp(x)+I)^5/(exp(x)-I)^3
```

Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 86 vs. 2(23) = 46.

Time = 0.07 (sec) , antiderivative size = 86, normalized size of antiderivative = 2.77

$$\int \frac{\tanh^4(x)}{i+\sinh(x)}\,dx = -\frac{2\left(15\,e^{(7\,x)}+15i\,e^{(6\,x)}+5\,e^{(5\,x)}+25i\,e^{(4\,x)}+13\,e^{(3\,x)}+21i\,e^{(2\,x)}-9\,e^{x}+3i\right)}{15\left(e^{(8\,x)}+2i\,e^{(7\,x)}+2\,e^{(6\,x)}+6i\,e^{(5\,x)}+6i\,e^{(3\,x)}-2\,e^{(2\,x)}+2i\,e^{x}-1\right)}$$

input
```
integrate(tanh(x)^4/(I+sinh(x)),x, algorithm="fricas")
```

output
```
-2/15*(15*e^(7*x) + 15*I*e^(6*x) + 5*e^(5*x) + 25*I*e^(4*x) + 13*e^(3*x) +
 21*I*e^(2*x) - 9*e^x + 3*I)/(e^(8*x) + 2*I*e^(7*x) + 2*e^(6*x) + 6*I*e^(5
*x) + 6*I*e^(3*x) - 2*e^(2*x) + 2*I*e^x - 1)
```

Sympy [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 107 vs. 2(26) = 52.

Time = 0.12 (sec) , antiderivative size = 107, normalized size of antiderivative = 3.45

$$\int \frac{\tanh^4(x)}{i+\sinh(x)}\,dx = \frac{-30e^{7x}-30ie^{6x}-10e^{5x}-50ie^{4x}-26e^{3x}-42ie^{2x}+18e^{x}-6i}{15e^{8x}+30ie^{7x}+30e^{6x}+90ie^{5x}+90ie^{3x}-30e^{2x}+30ie^{x}-15}$$

input
```
integrate(tanh(x)**4/(I+sinh(x)),x)
```

output
```
(-30*exp(7*x) - 30*I*exp(6*x) - 10*exp(5*x) - 50*I*exp(4*x) - 26*exp(3*x)
- 42*I*exp(2*x) + 18*exp(x) - 6*I)/(15*exp(8*x) + 30*I*exp(7*x) + 30*exp(6
*x) + 90*I*exp(5*x) + 90*I*exp(3*x) - 30*exp(2*x) + 30*I*exp(x) - 15)
```

Maxima [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 413 vs. $2(23) = 46$.

Time = 0.04 (sec) , antiderivative size = 413, normalized size of antiderivative = 13.32

$$\int \frac{\tanh^4(x)}{i + \sinh(x)}\, dx = \text{Too large to display}$$

input
```
integrate(tanh(x)^4/(I+sinh(x)),x, algorithm="maxima")
```

output
```
18*e^(-x)/(-30*I*e^(-x) - 30*e^(-2*x) - 90*I*e^(-3*x) - 90*I*e^(-5*x) + 30
*e^(-6*x) - 30*I*e^(-7*x) + 15*e^(-8*x) - 15) + 42*I*e^(-2*x)/(-30*I*e^(-x
) - 30*e^(-2*x) - 90*I*e^(-3*x) - 90*I*e^(-5*x) + 30*e^(-6*x) - 30*I*e^(-7
*x) + 15*e^(-8*x) - 15) - 26*e^(-3*x)/(-30*I*e^(-x) - 30*e^(-2*x) - 90*I*e
^(-3*x) - 90*I*e^(-5*x) + 30*e^(-6*x) - 30*I*e^(-7*x) + 15*e^(-8*x) - 15)
+ 50*I*e^(-4*x)/(-30*I*e^(-x) - 30*e^(-2*x) - 90*I*e^(-3*x) - 90*I*e^(-5*x
) + 30*e^(-6*x) - 30*I*e^(-7*x) + 15*e^(-8*x) - 15) - 10*e^(-5*x)/(-30*I*e
^(-x) - 30*e^(-2*x) - 90*I*e^(-3*x) - 90*I*e^(-5*x) + 30*e^(-6*x) - 30*I*e
^(-7*x) + 15*e^(-8*x) - 15) + 30*I*e^(-6*x)/(-30*I*e^(-x) - 30*e^(-2*x) -
90*I*e^(-3*x) - 90*I*e^(-5*x) + 30*e^(-6*x) - 30*I*e^(-7*x) + 15*e^(-8*x)
- 15) - 30*e^(-7*x)/(-30*I*e^(-x) - 30*e^(-2*x) - 90*I*e^(-3*x) - 90*I*e^(
-5*x) + 30*e^(-6*x) - 30*I*e^(-7*x) + 15*e^(-8*x) - 15) + 6*I/(-30*I*e^(-x
) - 30*e^(-2*x) - 90*I*e^(-3*x) - 90*I*e^(-5*x) + 30*e^(-6*x) - 30*I*e^(-7
*x) + 15*e^(-8*x) - 15)
```

Giac [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 53 vs. $2(23) = 46$.

Time = 0.12 (sec) , antiderivative size = 53, normalized size of antiderivative = 1.71

$$\int \frac{\tanh^4(x)}{i + \sinh(x)}\, dx = -\frac{15\, e^{(2\, x)} - 24i\, e^{x} - 13}{24\, (e^{x} - i)^{3}} - \frac{165\, e^{(4\, x)} + 480i\, e^{(3\, x)} - 650\, e^{(2\, x)} - 400i\, e^{x} + 113}{120\, (e^{x} + i)^{5}}$$

input
```
integrate(tanh(x)^4/(I+sinh(x)),x, algorithm="giac")
```

output
```
-1/24*(15*e^(2*x) - 24*I*e^x - 13)/(e^x - I)^3 - 1/120*(165*e^(4*x) + 480*
I*e^(3*x) - 650*e^(2*x) - 400*I*e^x + 113)/(e^x + I)^5
```

Mupad [B] (verification not implemented)

Time = 2.02 (sec) , antiderivative size = 231, normalized size of antiderivative = 7.45

$$\int \frac{\tanh^4(x)}{i+\sinh(x)}\,dx = -\frac{1}{6\,\left(\mathrm{e}^{2\,x}\,3\mathrm{i}-\mathrm{e}^{3\,x}+3\,\mathrm{e}^{x}-\mathrm{i}\right)} - \frac{\frac{11\,\mathrm{e}^{x}}{40}+\frac{1}{8}\mathrm{i}}{\mathrm{e}^{2\,x}-1+\mathrm{e}^{x}\,2\mathrm{i}} - \frac{\frac{11\,\mathrm{e}^{2\,x}}{40}-\frac{17}{120}+\frac{\mathrm{e}^{x}\,1\mathrm{i}}{4}}{\mathrm{e}^{2\,x}\,3\mathrm{i}+\mathrm{e}^{3\,x}-3\,\mathrm{e}^{x}-\mathrm{i}} + \frac{1\mathrm{i}}{4\,\left(1-\mathrm{e}^{2\,x}+\mathrm{e}^{x}\,2\mathrm{i}\right)} - \frac{5}{8\,\left(\mathrm{e}^{x}-\mathrm{i}\right)} - \frac{11}{40\,\left(\mathrm{e}^{x}+1\mathrm{i}\right)} - \frac{\frac{\mathrm{e}^{2\,x}\,3\mathrm{i}}{8}+\frac{11\,\mathrm{e}^{3\,x}}{40}-\frac{17\,\mathrm{e}^{x}}{40}-\frac{1}{8}\mathrm{i}}{\mathrm{e}^{4\,x}-6\,\mathrm{e}^{2\,x}+1+\mathrm{e}^{3\,x}\,4\mathrm{i}-\mathrm{e}^{x}\,4\mathrm{i}} - \frac{\frac{11\,\mathrm{e}^{4\,x}}{40}-\frac{17\,\mathrm{e}^{2\,x}}{20}+\frac{11}{40}+\frac{\mathrm{e}^{3\,x}\,1\mathrm{i}}{2}-\frac{\mathrm{e}^{x}\,1\mathrm{i}}{2}}{\mathrm{e}^{5\,x}-10\,\mathrm{e}^{3\,x}+\mathrm{e}^{4\,x}\,5\mathrm{i}-\mathrm{e}^{2\,x}\,10\mathrm{i}+5\,\mathrm{e}^{x}+1\mathrm{i}}$$

input
```
int(tanh(x)^4/(sinh(x) + 1i),x)
```

output
```
1i/(4*(exp(x)*2i - exp(2*x) + 1)) - ((11*exp(x))/40 + 1i/8)/(exp(2*x) + ex
p(x)*2i - 1) - ((11*exp(2*x))/40 + (exp(x)*1i)/4 - 17/120)/(exp(2*x)*3i +
exp(3*x) - 3*exp(x) - 1i) - 1/(6*(exp(2*x)*3i - exp(3*x) + 3*exp(x) - 1i))
 - 5/(8*(exp(x) - 1i)) - 11/(40*(exp(x) + 1i)) - ((exp(2*x)*3i)/8 + (11*ex
p(3*x))/40 - (17*exp(x))/40 - 1i/8)/(exp(3*x)*4i - 6*exp(2*x) + exp(4*x) -
 exp(x)*4i + 1) - ((exp(3*x)*1i)/2 - (17*exp(2*x))/20 + (11*exp(4*x))/40 -
 (exp(x)*1i)/2 + 11/40)/(exp(4*x)*5i - 10*exp(3*x) - exp(2*x)*10i + exp(5*
x) + 5*exp(x) + 1i)
```

Reduce [F]

$$\int \frac{\tanh^4(x)}{i+\sinh(x)}\,dx = \int \frac{\tanh(x)^4}{\sinh(x)+i}dx$$

input
```
int(tanh(x)^4/(I+sinh(x)),x)
```

output
```
int(tanh(x)**4/(sinh(x) + i),x)
```

3.209 $\int \frac{\tanh^3(x)}{i+\sinh(x)} dx$

Optimal result

Integrand size = 13, antiderivative size = 36

$$\int \frac{\tanh^3(x)}{i+\sinh(x)} dx = \frac{3}{8}\arctan(\sinh(x)) - \frac{3}{8}\text{sech}(x)\tanh(x) - \frac{1}{4}\text{sech}(x)\tanh^3(x) - \frac{1}{4}i\tanh^4(x)$$

output
```
3/8*arctan(sinh(x))-3/8*sech(x)*tanh(x)-1/4*sech(x)*tanh(x)^3-1/4*I*tanh(x
)^4
```

Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 42, normalized size of antiderivative = 1.17

$$\int \frac{\tanh^3(x)}{i+\sinh(x)} dx = \frac{1}{8}\left(3\arctan(\sinh(x)) - \frac{2+i\sinh(x)+5\sinh^2(x)}{(-i+\sinh(x))(i+\sinh(x))^2}\right)$$

input
```
Integrate[Tanh[x]^3/(I + Sinh[x]),x]
```

output
```
(3*ArcTan[Sinh[x]] - (2 + I*Sinh[x] + 5*Sinh[x]^2)/((-I + Sinh[x])*(I + Si
nh[x])^2))/8
```

Rubi [A] (verified)

Time = 0.44 (sec) , antiderivative size = 41, normalized size of antiderivative = 1.14, number of steps used = 17, number of rules used = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 1.231, Rules used = {3042, 26, 26, 3185, 26, 3042, 26, 3087, 15, 3091, 25, 3042, 25, 3091, 3042, 4257}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\tanh^3(x)}{\sinh(x)+i}dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \frac{i\tan(ix)^3}{i-i\sin(ix)}dx$$

$$\Big\downarrow \text{ 26}$$

$$i\int -\frac{i\tan(ix)^3}{1-\sin(ix)}dx$$

$$\Big\downarrow \text{ 26}$$

$$\int \frac{\tan(ix)^3}{1-\sin(ix)}dx$$

$$\Big\downarrow \text{ 3185}$$

$$\int \text{sech}(x)\tanh^4(x)dx+\int -i\text{sech}^2(x)\tanh^3(x)dx$$

$$\Big\downarrow \text{ 26}$$

$$\int \text{sech}(x)\tanh^4(x)dx-i\int \text{sech}^2(x)\tanh^3(x)dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \sec(ix)\tan(ix)^4dx-i\int i\sec(ix)^2\tan(ix)^3dx$$

$$\Big\downarrow \text{ 26}$$

$$\int \sec(ix)^2\tan(ix)^3dx+\int \sec(ix)\tan(ix)^4dx$$

$$\Big\downarrow \text{ 3087}$$

$$\int \sec(ix)\tan(ix)^4dx - i\int -i\tanh^3(x)d(i\tanh(x))$$

$$\Big\downarrow \text{15}$$

$$\int \sec(ix)\tan(ix)^4dx - \frac{1}{4}i\tanh^4(x)$$

$$\Big\downarrow \text{3091}$$

$$-\frac{3}{4}\int -\text{sech}(x)\tanh^2(x)dx - \frac{1}{4}i\tanh^4(x) - \frac{1}{4}\tanh^3(x)\text{sech}(x)$$

$$\Big\downarrow \text{25}$$

$$\frac{3}{4}\int \text{sech}(x)\tanh^2(x)dx - \frac{1}{4}i\tanh^4(x) - \frac{1}{4}\tanh^3(x)\text{sech}(x)$$

$$\Big\downarrow \text{3042}$$

$$\frac{3}{4}\int -\sec(ix)\tan(ix)^2dx - \frac{1}{4}i\tanh^4(x) - \frac{1}{4}\tanh^3(x)\text{sech}(x)$$

$$\Big\downarrow \text{25}$$

$$-\frac{3}{4}\int \sec(ix)\tan(ix)^2dx - \frac{1}{4}i\tanh^4(x) - \frac{1}{4}\tanh^3(x)\text{sech}(x)$$

$$\Big\downarrow \text{3091}$$

$$-\frac{3}{4}\left(\frac{1}{2}\tanh(x)\text{sech}(x) - \frac{\int \text{sech}(x)dx}{2}\right) - \frac{1}{4}i\tanh^4(x) - \frac{1}{4}\tanh^3(x)\text{sech}(x)$$

$$\Big\downarrow \text{3042}$$

$$-\frac{3}{4}\left(\frac{1}{2}\tanh(x)\text{sech}(x) - \frac{1}{2}\int \csc\left(ix+\frac{\pi}{2}\right)dx\right) - \frac{1}{4}i\tanh^4(x) - \frac{1}{4}\tanh^3(x)\text{sech}(x)$$

$$\Big\downarrow \text{4257}$$

$$-\frac{3}{4}\left(\frac{1}{2}\tanh(x)\text{sech}(x) - \frac{1}{2}\arctan(\sinh(x))\right) - \frac{1}{4}i\tanh^4(x) - \frac{1}{4}\tanh^3(x)\text{sech}(x)$$

input
```
Int[Tanh[x]^3/(I + Sinh[x]),x]
```

output
```
-1/4*(Sech[x]*Tanh[x]^3) - (I/4)*Tanh[x]^4 - (3*(-1/2*ArcTan[Sinh[x]] + (S
ech[x]*Tanh[x])/2))/4
```

Defintions of rubi rules used

rule 15
```
Int[(a_.)*(x_)^(m_.), x_Symbol] :> Simp[a*(x^(m + 1)/(m + 1)), x] /; FreeQ[
{a, m}, x] && NeQ[m, -1]
```

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3087
```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> Simp[1/f   Subst[Int[(b*x)^n*(1 + x^2)^(m/2 - 1), x], x, Tan[e +
f*x]], x] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2] &&  !(IntegerQ[(n - 1)
/2] && LtQ[0, n, m - 1])
```

rule 3091
```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[b*(a*Sec[e + f*x])^m*((b*Tan[e + f*x])^(n - 1)/(f*(m
 + n - 1))), x] - Simp[b^2*((n - 1)/(m + n - 1))   Int[(a*Sec[e + f*x])^m*(
b*Tan[e + f*x])^(n - 2), x], x] /; FreeQ[{a, b, e, f, m}, x] && GtQ[n, 1] &
& NeQ[m + n - 1, 0] && IntegersQ[2*m, 2*n]
```

rule 3185
```
Int[((g_.)*tan[(e_.) + (f_.)*(x_)])^(p_.)/((a_) + (b_.)*sin[(e_.) + (f_.)*(
x_)]), x_Symbol] :> Simp[1/a   Int[Sec[e + f*x]^2*(g*Tan[e + f*x])^p, x], x
] - Simp[1/(b*g)   Int[Sec[e + f*x]*(g*Tan[e + f*x])^(p + 1), x], x] /; Fre
eQ[{a, b, e, f, g, p}, x] && EqQ[a^2 - b^2, 0] && NeQ[p, -1]
```

rule 4257
```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 66 vs. 2(27) = 54.

Time = 2.55 (sec) , antiderivative size = 67, normalized size of antiderivative = 1.86

method	result
risch	$-\frac{2i e^{4x}-2\,e^{3x}-2i e^{2x}+5\,e^{5x}+5\,e^{x}}{4(e^{x}+i)^{4}(e^{x}-i)^{2}}-\frac{3i\ln(e^{x}-i)}{8}+\frac{3i\ln(e^{x}+i)}{8}$
default	$-\frac{i}{2(\tanh(\frac{x}{2})+i)^{4}}+\frac{3i\ln(\tanh(\frac{x}{2})+i)}{8}+\frac{1}{(\tanh(\frac{x}{2})+i)^{3}}+\frac{1}{2\tanh(\frac{x}{2})+2i}-\frac{3i\ln(\tanh(\frac{x}{2})-i)}{8}+\frac{i}{4(\tanh(\frac{x}{2})-i)^{2}}+\frac{}{4\,\text{ta}}$

input
```
int(tanh(x)^3/(I+sinh(x)),x,method=_RETURNVERBOSE)
```

output
```
-1/4*(2*I*exp(x)^4-2*exp(x)^3-2*I*exp(x)^2+5*exp(x)^5+5*exp(x))/(exp(x)+I)
^4/(exp(x)-I)^2-3/8*I*ln(exp(x)-I)+3/8*I*ln(exp(x)+I)
```

Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 151 vs. 2(26) = 52.

Time = 0.09 (sec) , antiderivative size = 151, normalized size of antiderivative = 4.19

$$\int \frac{\tanh^3(x)}{i+\sinh(x)}\,dx = -\frac{3\left(-i\,e^{(6\,x)}+2\,e^{(5\,x)}-i\,e^{(4\,x)}+4\,e^{(3\,x)}+i\,e^{(2\,x)}+2\,e^{x}+i\right)\log\left(e^{x}+i\right)+3\left(i\,e^{(6\,x)}-2\,e^{(5\,x)}+i\,e^{(4\,x)}-\right.}{8\left(e^{(6\,x)}+2i\,e^{(5\,x)}+e^{(4\,x)}+4i\,e^{(3\,x)}\right.}$$

input
```
integrate(tanh(x)^3/(I+sinh(x)),x, algorithm="fricas")
```

output
```
-1/8*(3*(-I*e^(6*x) + 2*e^(5*x) - I*e^(4*x) + 4*e^(3*x) + I*e^(2*x) + 2*e^
x + I)*log(e^x + I) + 3*(I*e^(6*x) - 2*e^(5*x) + I*e^(4*x) - 4*e^(3*x) - I
*e^(2*x) - 2*e^x - I)*log(e^x - I) + 10*e^(5*x) + 4*I*e^(4*x) - 4*e^(3*x)
- 4*I*e^(2*x) + 10*e^x)/(e^(6*x) + 2*I*e^(5*x) + e^(4*x) + 4*I*e^(3*x) - e
^(2*x) + 2*I*e^x - 1)
```

Sympy [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 99 vs. $2(36) = 72$.

Time = 0.16 (sec) , antiderivative size = 99, normalized size of antiderivative = 2.75

$$\int \frac{\tanh^3(x)}{i+\sinh(x)}\,dx = \frac{-5e^{5x}-2ie^{4x}+2e^{3x}+2ie^{2x}-5e^{x}}{4e^{6x}+8ie^{5x}+4e^{4x}+16ie^{3x}-4e^{2x}+8ie^{x}-4} + \mathrm{RootSum}\left(64z^2+9,\left(i\mapsto i\log\left(\frac{8i}{3}+e^{x}\right)\right)\right)$$

input
```
integrate(tanh(x)**3/(I+sinh(x)),x)
```

output
```
(-5*exp(5*x) - 2*I*exp(4*x) + 2*exp(3*x) + 2*I*exp(2*x) - 5*exp(x))/(4*exp
(6*x) + 8*I*exp(5*x) + 4*exp(4*x) + 16*I*exp(3*x) - 4*exp(2*x) + 8*I*exp(x
) - 4) + RootSum(64*_z**2 + 9, Lambda(_i, _i*log(8*_i/3 + exp(x))))
```

Maxima [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 95 vs. $2(26) = 52$.

Time = 0.03 (sec) , antiderivative size = 95, normalized size of antiderivative = 2.64

$$\int \frac{\tanh^3(x)}{i+\sinh(x)}\,dx = \frac{5\,e^{(-x)}+2i\,e^{(-2\,x)}-2\,e^{(-3\,x)}-2i\,e^{(-4\,x)}+5\,e^{(-5\,x)}}{-8i\,e^{(-x)}-4\,e^{(-2\,x)}-16i\,e^{(-3\,x)}+4\,e^{(-4\,x)}-8i\,e^{(-5\,x)}+4\,e^{(-6\,x)}-4} + \frac{3}{8}i\,\log\left(i\,e^{(-x)}+1\right)-\frac{3}{8}i\,\log\left(i\,e^{(-x)}-1\right)$$

input
```
integrate(tanh(x)^3/(I+sinh(x)),x, algorithm="maxima")
```

output
```
(5*e^(-x) + 2*I*e^(-2*x) - 2*e^(-3*x) - 2*I*e^(-4*x) + 5*e^(-5*x))/(-8*I*e
^(-x) - 4*e^(-2*x) - 16*I*e^(-3*x) + 4*e^(-4*x) - 8*I*e^(-5*x) + 4*e^(-6*x
) - 4) + 3/8*I*log(I*e^(-x) + 1) - 3/8*I*log(I*e^(-x) - 1)
```

Giac [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 92 vs. $2(26) = 52$.

Time = 0.12 (sec) , antiderivative size = 92, normalized size of antiderivative = 2.56

$$\int \frac{\tanh^3(x)}{i+\sinh(x)}\,dx = \frac{3i\,e^{(-x)} - 3i\,e^{x} - 2}{16\,(e^{(-x)} - e^{x} + 2i)} - \frac{9i\,\left(e^{(-x)} - e^{x}\right)^2 + 4\,e^{(-x)} - 4\,e^{x} + 12i}{32\,(e^{(-x)} - e^{x} - 2i)^2} + \frac{3}{16}i\,\log\left(-e^{(-x)} + e^{x} + 2i\right) - \frac{3}{16}i\,\log\left(-e^{(-x)} + e^{x} - 2i\right)$$

input
```
integrate(tanh(x)^3/(I+sinh(x)),x, algorithm="giac")
```

output
```
1/16*(3*I*e^(-x) - 3*I*e^x - 2)/(e^(-x) - e^x + 2*I) - 1/32*(9*I*(e^(-x) -
 e^x)^2 + 4*e^(-x) - 4*e^x + 12*I)/(e^(-x) - e^x - 2*I)^2 + 3/16*I*log(-e^
(-x) + e^x + 2*I) - 3/16*I*log(-e^(-x) + e^x - 2*I)
```

Mupad [B] (verification not implemented)

Time = 0.45 (sec) , antiderivative size = 113, normalized size of antiderivative = 3.14

$$\int \frac{\tanh^3(x)}{i+\sinh(x)}\,dx = \frac{3\,\mathrm{atan}(e^{x})}{4} + \frac{3i}{2\,\left(e^{2\,x} - 1 + e^{x}\,2i\right)} - \frac{1i}{2\,\left(e^{4\,x} - 6\,e^{2\,x} + 1 + e^{3\,x}\,4i - e^{x}\,4i\right)} + \frac{1i}{4\,\left(1 - e^{2\,x} + e^{x}\,2i\right)} - \frac{1}{4\,\left(e^{x} - i\right)} - \frac{1}{e^{x} + 1i} + \frac{1}{e^{2\,x}\,3i + e^{3\,x} - 3\,e^{x} - i}$$

input
```
int(tanh(x)^3/(sinh(x) + 1i),x)
```

output
```
(3*atan(exp(x)))/4 + 3i/(2*(exp(2*x) + exp(x)*2i - 1)) - 1i/(2*(exp(3*x)*4
i - 6*exp(2*x) + exp(4*x) - exp(x)*4i + 1)) + 1i/(4*(exp(x)*2i - exp(2*x)
+ 1)) - 1/(4*(exp(x) - 1i)) - 1/(exp(x) + 1i) + 1/(exp(2*x)*3i + exp(3*x)
- 3*exp(x) - 1i)
```

Reduce [F]

$$\int \frac{\tanh^3(x)}{i+\sinh(x)}\,dx = \int \frac{\tanh(x)^3}{\sinh(x)+i}dx$$

input
```
int(tanh(x)^3/(I+sinh(x)),x)
```

output
```
int(tanh(x)**3/(sinh(x) + i),x)
```

3.210 $\int \frac{\tanh^2(x)}{i+\sinh(x)} dx$

Optimal result

Integrand size = 13, antiderivative size = 23

$$\int \frac{\tanh^2(x)}{i+\sinh(x)} dx = -\operatorname{sech}(x) + \frac{\operatorname{sech}^3(x)}{3} - \frac{1}{3} i \tanh^3(x)$$

output
```
-sech(x)+1/3*sech(x)^3-1/3*I*tanh(x)^3
```

Mathematica [B] (verified)

Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 67 vs. 2(23) = 46.

Time = 0.09 (sec) , antiderivative size = 67, normalized size of antiderivative = 2.91

$$\int \frac{\tanh^2(x)}{i+\sinh(x)} dx = \frac{-3-\cosh(2x)+\cosh(x)(5-5i\sinh(x))+4i\sinh(x)}{6\left(\cosh\left(\frac{x}{2}\right)-i\sinh\left(\frac{x}{2}\right)\right)^3\left(\cosh\left(\frac{x}{2}\right)+i\sinh\left(\frac{x}{2}\right)\right)}$$

input
```
Integrate[Tanh[x]^2/(I + Sinh[x]),x]
```

output
```
(-3 - Cosh[2*x] + Cosh[x]*(5 - (5*I)*Sinh[x]) + (4*I)*Sinh[x])/(6*(Cosh[x/
2] - I*Sinh[x/2])^3*(Cosh[x/2] + I*Sinh[x/2]))
```

Rubi [A] (verified)

Time = 0.33 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.30, number of steps used = 14, number of rules used = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 1.000, Rules used = {3042, 25, 26, 3185, 25, 26, 3042, 25, 26, 3086, 2009, 3087, 15}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\tanh^2(x)}{\sinh(x)+i}dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int -\frac{\tan(ix)^2}{i-i\sin(ix)}dx$$

$$\Big\downarrow \text{ 25}$$

$$-\int -\frac{i\tan(ix)^2}{1-\sin(ix)}dx$$

$$\Big\downarrow \text{ 26}$$

$$i\int \frac{\tan(ix)^2}{1-\sin(ix)}dx$$

$$\Big\downarrow \text{ 3185}$$

$$i\left(\int -\text{sech}^2(x)\tanh^2(x)dx+\int -i\text{sech}(x)\tanh^3(x)dx\right)$$

$$\Big\downarrow \text{ 25}$$

$$i\left(-\int \text{sech}^2(x)\tanh^2(x)dx+\int -i\text{sech}(x)\tanh^3(x)dx\right)$$

$$\Big\downarrow \text{ 26}$$

$$i\left(-\int \text{sech}^2(x)\tanh^2(x)dx-i\int \text{sech}(x)\tanh^3(x)dx\right)$$

$$\Big\downarrow \text{ 3042}$$

$$i\left(-\int -\sec(ix)^2\tan(ix)^2dx-i\int i\sec(ix)\tan(ix)^3dx\right)$$

$$\Big\downarrow \text{ 25}$$

$$i\left(\int \sec(ix)^2\tan(ix)^2dx - i\int i\sec(ix)\tan(ix)^3dx\right)$$

$\big\downarrow$ 26

$$i\left(\int \sec(ix)^2\tan(ix)^2dx + \int \sec(ix)\tan(ix)^3dx\right)$$

$\big\downarrow$ 3086

$$i\left(\int \sec(ix)^2\tan(ix)^2dx - i\int \left(\text{sech}^2(x)-1\right)d\text{sech}(x)\right)$$

$\big\downarrow$ 2009

$$i\left(\int \sec(ix)^2\tan(ix)^2dx - i\left(\frac{\text{sech}^3(x)}{3}-\text{sech}(x)\right)\right)$$

$\big\downarrow$ 3087

$$i\left(-i\int -\tanh^2(x)d(i\tanh(x)) - i\left(\frac{\text{sech}^3(x)}{3}-\text{sech}(x)\right)\right)$$

$\big\downarrow$ 15

$$i\left(-\frac{1}{3}\tanh^3(x) - i\left(\frac{\text{sech}^3(x)}{3}-\text{sech}(x)\right)\right)$$

input
```
Int[Tanh[x]^2/(I + Sinh[x]),x]
```

output
```
I*((-I)*(-Sech[x] + Sech[x]^3/3) - Tanh[x]^3/3)
```

Defintions of rubi rules used

rule 15
```
Int[(a_.)*(x_)^(m_.), x_Symbol] :> Simp[a*(x^(m + 1)/(m + 1)), x] /; FreeQ[
{a, m}, x] && NeQ[m, -1]
```

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3086
```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Simp[a/f   Subst[Int[(a*x)^(m - 1)*(-1 + x^2)^((n - 1)/2
), x], x, Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2
] &&  !(IntegerQ[m/2] && LtQ[0, m, n + 1])
```

rule 3087
```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> Simp[1/f   Subst[Int[(b*x)^n*(1 + x^2)^(m/2 - 1), x], x, Tan[e +
f*x]], x] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2] &&  !(IntegerQ[(n - 1)
/2] && LtQ[0, n, m - 1])
```

rule 3185
```
Int[((g_.)*tan[(e_.) + (f_.)*(x_)])^(p_.)/((a_) + (b_.)*sin[(e_.) + (f_.)*(
x_)]), x_Symbol] :> Simp[1/a   Int[Sec[e + f*x]^2*(g*Tan[e + f*x])^p, x], x
] - Simp[1/(b*g)   Int[Sec[e + f*x]*(g*Tan[e + f*x])^(p + 1), x], x] /; Fre
eQ[{a, b, e, f, g, p}, x] && EqQ[a^2 - b^2, 0] && NeQ[p, -1]
```

Maple [A] (verified)

Time = 1.54 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.61

method	result	size
risch	$-\frac{2\left(3 i \mathrm{e}^{2 x}+3 \,\mathrm{e}^{3 x}+i-\mathrm{e}^{x}\right)}{3\left(\mathrm{e}^{x}+i\right)^{3} \left(\mathrm{e}^{x}-i\right)}$	37
default	$-\frac{2 i}{3 \left(\tanh \left(\frac{x}{2}\right)+i\right)^{3}}-\frac{i}{2 \left(\tanh \left(\frac{x}{2}\right)+i\right)}+\frac{1}{\left(\tanh \left(\frac{x}{2}\right)+i\right)^{2}}+\frac{i}{2 \tanh \left(\frac{x}{2}\right)-2 i}$	47

input
```
int(tanh(x)^2/(I+sinh(x)),x,method=_RETURNVERBOSE)
```

output
```
-2/3*(3*I*exp(x)^2+3*exp(x)^3+I-exp(x))/(exp(x)+I)^3/(exp(x)-I)
```

Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 38 vs. 2(17) = 34.

Time = 0.08 (sec) , antiderivative size = 38, normalized size of antiderivative = 1.65

$$\int \frac{\tanh^2(x)}{i+\sinh(x)}\,dx = -\frac{2\left(3\,e^{(3\,x)}+3i\,e^{(2\,x)}-e^{x}+i\right)}{3\left(e^{(4\,x)}+2i\,e^{(3\,x)}+2i\,e^{x}-1\right)}$$

input
```
integrate(tanh(x)^2/(I+sinh(x)),x, algorithm="fricas")
```

output
```
-2/3*(3*e^(3*x) + 3*I*e^(2*x) - e^x + I)/(e^(4*x) + 2*I*e^(3*x) + 2*I*e^x
- 1)
```

Sympy [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 46 vs. 2(17) = 34.

Time = 0.08 (sec) , antiderivative size = 46, normalized size of antiderivative = 2.00

$$\int \frac{\tanh^2(x)}{i+\sinh(x)}\,dx = \frac{-6e^{3x}-6ie^{2x}+2e^{x}-2i}{3e^{4x}+6ie^{3x}+6ie^{x}-3}$$

input
```
integrate(tanh(x)**2/(I+sinh(x)),x)
```

output
```
(-6*exp(3*x) - 6*I*exp(2*x) + 2*exp(x) - 2*I)/(3*exp(4*x) + 6*I*exp(3*x) +
 6*I*exp(x) - 3)
```

Maxima [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 109 vs. $2(17) = 34$.

Time = 0.04 (sec) , antiderivative size = 109, normalized size of antiderivative = 4.74

$$\int \frac{\tanh^2(x)}{i+\sinh(x)}\,dx = \frac{2\,e^{(-x)}}{-6i\,e^{(-x)} - 6i\,e^{(-3\,x)} + 3\,e^{(-4\,x)} - 3} + \frac{6i\,e^{(-2\,x)}}{-6i\,e^{(-x)} - 6i\,e^{(-3\,x)} + 3\,e^{(-4\,x)} - 3} - \frac{6\,e^{(-3\,x)}}{-6i\,e^{(-x)} - 6i\,e^{(-3\,x)} + 3\,e^{(-4\,x)} - 3} + \frac{2i}{-6i\,e^{(-x)} - 6i\,e^{(-3\,x)} + 3\,e^{(-4\,x)} - 3}$$

input
```
integrate(tanh(x)^2/(I+sinh(x)),x, algorithm="maxima")
```

output
```
2*e^(-x)/(-6*I*e^(-x) - 6*I*e^(-3*x) + 3*e^(-4*x) - 3) + 6*I*e^(-2*x)/(-6*
I*e^(-x) - 6*I*e^(-3*x) + 3*e^(-4*x) - 3) - 6*e^(-3*x)/(-6*I*e^(-x) - 6*I*
e^(-3*x) + 3*e^(-4*x) - 3) + 2*I/(-6*I*e^(-x) - 6*I*e^(-3*x) + 3*e^(-4*x)
- 3)
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.26

$$\int \frac{\tanh^2(x)}{i+\sinh(x)}\,dx = -\frac{1}{2\,(e^x - i)} - \frac{9\,e^{(2\,x)} + 12i\,e^x - 7}{6\,(e^x + i)^3}$$

input
```
integrate(tanh(x)^2/(I+sinh(x)),x, algorithm="giac")
```

output
```
-1/2/(e^x - I) - 1/6*(9*e^(2*x) + 12*I*e^x - 7)/(e^x + I)^3
```

Mupad [B] (verification not implemented)

Time = 1.51 (sec) , antiderivative size = 80, normalized size of antiderivative = 3.48

$$\int \frac{\tanh^2(x)}{i+\sinh(x)}\,dx = -\frac{\frac{e^x}{2}+\frac{1}{6}\mathrm{i}}{e^{2x}-1+e^x\,2\mathrm{i}} - \frac{\frac{e^{2x}}{2}-\frac{1}{2}+\frac{e^x\,1\mathrm{i}}{3}}{e^{2x}\,3\mathrm{i}+e^{3x}-3\,e^x-\mathrm{i}} - \frac{1}{2\,(e^x-\mathrm{i})} - \frac{1}{2\,(e^x+1\mathrm{i})}$$

input
```
int(tanh(x)^2/(sinh(x) + 1i),x)
```

output
```
- (exp(x)/2 + 1i/6)/(exp(2*x) + exp(x)*2i - 1) - (exp(2*x)/2 + (exp(x)*1i)
/3 - 1/2)/(exp(2*x)*3i + exp(3*x) - 3*exp(x) - 1i) - 1/(2*(exp(x) - 1i)) -
 1/(2*(exp(x) + 1i))
```

Reduce [F]

$$\int \frac{\tanh^2(x)}{i+\sinh(x)}\,dx = \int \frac{\tanh(x)^2}{\sinh(x)+i}dx$$

input
```
int(tanh(x)^2/(I+sinh(x)),x)
```

output
```
int(tanh(x)**2/(sinh(x) + i),x)
```

3.211 $\int \frac{\tanh(x)}{i+\sinh(x)}\,dx$

Optimal result

Integrand size = 11, antiderivative size = 26

$$\int \frac{\tanh(x)}{i+\sinh(x)}\,dx = \frac{1}{2}\arctan(\sinh(x)) + \frac{1}{2}i\operatorname{sech}^2(x) - \frac{1}{2}\operatorname{sech}(x)\tanh(x)$$

output
```
1/2*arctan(sinh(x))+1/2*I*sech(x)^2-1/2*sech(x)*tanh(x)
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.77

$$\int \frac{\tanh(x)}{i+\sinh(x)}\,dx = \frac{1}{2}\arctan(\sinh(x)) - \frac{1}{2(i+\sinh(x))}$$

input
```
Integrate[Tanh[x]/(I + Sinh[x]),x]
```

output
```
ArcTan[Sinh[x]]/2 - 1/(2*(I + Sinh[x]))
```

Rubi [A] (verified)

Time = 0.35 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.00, number of steps used = 15, number of rules used = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 1.273, Rules used = {3042, 26, 26, 3185, 25, 26, 3042, 25, 26, 3086, 15, 3091, 3042, 4257}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\tanh(x)}{\sinh(x)+i}dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int -\frac{i\tan(ix)}{i-i\sin(ix)}dx$$

$$\Big\downarrow \text{ 26}$$

$$-i\int -\frac{i\tan(ix)}{1-\sin(ix)}dx$$

$$\Big\downarrow \text{ 26}$$

$$-\int \frac{\tan(ix)}{1-\sin(ix)}dx$$

$$\Big\downarrow \text{ 3185}$$

$$-\int -\text{sech}(x)\tanh^2(x)dx-\int i\text{sech}^2(x)\tanh(x)dx$$

$$\Big\downarrow \text{ 25}$$

$$\int \text{sech}(x)\tanh^2(x)dx-\int i\text{sech}^2(x)\tanh(x)dx$$

$$\Big\downarrow \text{ 26}$$

$$\int \text{sech}(x)\tanh^2(x)dx-i\int \text{sech}^2(x)\tanh(x)dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int -\sec(ix)\tan(ix)^2dx-i\int -i\sec(ix)^2\tan(ix)dx$$

$$\Big\downarrow \text{ 25}$$

$$-i\int -i\sec(ix)^2\tan(ix)dx-\int \sec(ix)\tan(ix)^2dx$$

$$\Big\downarrow \text{26}$$

$$-\int \sec(ix)^2\tan(ix)dx-\int \sec(ix)\tan(ix)^2dx$$

$$\Big\downarrow \text{3086}$$

$$i\int \text{sech}(x)d\text{sech}(x)-\int \sec(ix)\tan(ix)^2dx$$

$$\Big\downarrow \text{15}$$

$$\frac{1}{2}i\text{sech}^2(x)-\int \sec(ix)\tan(ix)^2dx$$

$$\Big\downarrow \text{3091}$$

$$\frac{\int \text{sech}(x)dx}{2}+\frac{1}{2}i\text{sech}^2(x)-\frac{1}{2}\tanh(x)\text{sech}(x)$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{2}\int \csc\left(ix+\frac{\pi}{2}\right)dx+\frac{1}{2}i\text{sech}^2(x)-\frac{1}{2}\tanh(x)\text{sech}(x)$$

$$\Big\downarrow \text{4257}$$

$$\frac{1}{2}\arctan(\sinh(x))+\frac{1}{2}i\text{sech}^2(x)-\frac{1}{2}\tanh(x)\text{sech}(x)$$

input
```
Int[Tanh[x]/(I + Sinh[x]),x]
```

output
```
ArcTan[Sinh[x]]/2 + (I/2)*Sech[x]^2 - (Sech[x]*Tanh[x])/2
```

Defintions of rubi rules used

rule 15
```
Int[(a_.)*(x_)^(m_.), x_Symbol] :> Simp[a*(x^(m + 1)/(m + 1)), x] /; FreeQ[
{a, m}, x] && NeQ[m, -1]
```

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3086
```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Simp[a/f   Subst[Int[(a*x)^(m - 1)*(-1 + x^2)^((n - 1)/2
), x], x, Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2
] &&  !(IntegerQ[m/2] && LtQ[0, m, n + 1])
```

rule 3091
```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[b*(a*Sec[e + f*x])^m*((b*Tan[e + f*x])^(n - 1)/(f*(m
 + n - 1))), x] - Simp[b^2*((n - 1)/(m + n - 1))   Int[(a*Sec[e + f*x])^m*(
b*Tan[e + f*x])^(n - 2), x], x] /; FreeQ[{a, b, e, f, m}, x] && GtQ[n, 1] &
& NeQ[m + n - 1, 0] && IntegersQ[2*m, 2*n]
```

rule 3185
```
Int[((g_.)*tan[(e_.) + (f_.)*(x_)])^(p_.)/((a_) + (b_.)*sin[(e_.) + (f_.)*(
x_)]), x_Symbol] :> Simp[1/a   Int[Sec[e + f*x]^2*(g*Tan[e + f*x])^p, x], x
] - Simp[1/(b*g)   Int[Sec[e + f*x]*(g*Tan[e + f*x])^(p + 1), x], x] /; Fre
eQ[{a, b, e, f, g, p}, x] && EqQ[a^2 - b^2, 0] && NeQ[p, -1]
```

rule 4257
```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Maple [A] (verified)

Time = 0.97 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.19

method	result	size
risch	$-\frac{{\mathrm e}^{x}}{\left({\mathrm e}^{x}+i\right)^{2}}+\frac{i \ln \left({\mathrm e}^{x}+i\right)}{2}-\frac{i \ln \left({\mathrm e}^{x}-i\right)}{2}$	31
default	$-\frac{i}{\left(\tanh \left(\frac{x}{2}\right)+i\right)^{2}}+\frac{i \ln \left(\tanh \left(\frac{x}{2}\right)+i\right)}{2}+\frac{1}{\tanh \left(\frac{x}{2}\right)+i}-\frac{i \ln \left(\tanh \left(\frac{x}{2}\right)-i\right)}{2}$	45

input
```
int(tanh(x)/(I+sinh(x)),x,method=_RETURNVERBOSE)
```

output
```
-1/(exp(x)+I)^2*exp(x)+1/2*I*ln(exp(x)+I)-1/2*I*ln(exp(x)-I)
```

Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 55 vs. $2(18) = 36$.

Time = 0.10 (sec) , antiderivative size = 55, normalized size of antiderivative = 2.12

$$\int \frac{\tanh(x)}{i+\sinh(x)}\,dx = \frac{\left(i\,e^{(2\,x)}-2\,e^{x}-i\right)\log\left(e^{x}+i\right)+\left(-i\,e^{(2\,x)}+2\,e^{x}+i\right)\log\left(e^{x}-i\right)-2\,e^{x}}{2\left(e^{(2\,x)}+2i\,e^{x}-1\right)}$$

input
```
integrate(tanh(x)/(I+sinh(x)),x, algorithm="fricas")
```

output
```
1/2*((I*e^(2*x) - 2*e^x - I)*log(e^x + I) + (-I*e^(2*x) + 2*e^x + I)*log(e
^x - I) - 2*e^x)/(e^(2*x) + 2*I*e^x - 1)
```

Sympy [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.23

$$\int \frac{\tanh(x)}{i+\sinh(x)}\,dx = \operatorname{RootSum}\left(4z^2+1,\left(i\mapsto i\log\left(2i+e^{x}\right)\right)\right)-\frac{e^{x}}{e^{2x}+2ie^{x}-1}$$

input
```
integrate(tanh(x)/(I+sinh(x)),x)
```

output
```
RootSum(4*_z**2 + 1, Lambda(_i, _i*log(2*_i + exp(x)))) - exp(x)/(exp(2*x)
 + 2*I*exp(x) - 1)
```

Maxima [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 42 vs. 2(18) = 36.

Time = 0.03 (sec) , antiderivative size = 42, normalized size of antiderivative = 1.62

$$\int \frac{\tanh(x)}{i+\sinh(x)}\,dx = \frac{e^{(-x)}}{-2i\,e^{(-x)}+e^{(-2\,x)}-1}+\frac{1}{2}i\,\log\left(i\,e^{(-x)}+1\right)-\frac{1}{2}i\,\log\left(i\,e^{(-x)}-1\right)$$

input
```
integrate(tanh(x)/(I+sinh(x)),x, algorithm="maxima")
```

output
```
e^(-x)/(-2*I*e^(-x) + e^(-2*x) - 1) + 1/2*I*log(I*e^(-x) + 1) - 1/2*I*log(
I*e^(-x) - 1)
```

Giac [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 53 vs. 2(18) = 36.

Time = 0.12 (sec) , antiderivative size = 53, normalized size of antiderivative = 2.04

$$\int \frac{\tanh(x)}{i+\sinh(x)}\,dx = \frac{-i\,e^{(-x)}+i\,e^{x}+2}{4\left(e^{(-x)}-e^{x}-2i\right)}+\frac{1}{4}i\,\log\left(-e^{(-x)}+e^{x}+2i\right) - \frac{1}{4}i\,\log\left(-e^{(-x)}+e^{x}-2i\right)$$

input
```
integrate(tanh(x)/(I+sinh(x)),x, algorithm="giac")
```

output
```
1/4*(-I*e^(-x) + I*e^x + 2)/(e^(-x) - e^x - 2*I) + 1/4*I*log(-e^(-x) + e^x
 + 2*I) - 1/4*I*log(-e^(-x) + e^x - 2*I)
```

Mupad [B] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.12

$$\int \frac{\tanh(x)}{i+\sinh(x)}\,dx = \mathrm{atan}(\mathrm{e}^x) + \frac{1\mathrm{i}}{\mathrm{e}^{2\,x} - 1 + \mathrm{e}^x\,2\mathrm{i}} - \frac{1}{\mathrm{e}^x + 1\mathrm{i}}$$

input
```
int(tanh(x)/(sinh(x) + 1i),x)
```

output
```
atan(exp(x)) + 1i/(exp(2*x) + exp(x)*2i - 1) - 1/(exp(x) + 1i)
```

Reduce [F]

$$\int \frac{\tanh(x)}{i+\sinh(x)}\,dx = \int \frac{\tanh(x)}{\sinh(x)+i}dx$$

input
```
int(tanh(x)/(I+sinh(x)),x)
```

output
```
int(tanh(x)/(sinh(x) + i),x)
```

3.212 $\int \frac{\coth(x)}{i+\sinh(x)}\,dx$

Optimal result

Integrand size = 11, antiderivative size = 19

$$\int \frac{\coth(x)}{i+\sinh(x)}\,dx = -i\log(\sinh(x)) + i\log(i+\sinh(x))$$

output
```
-I*ln(sinh(x))+I*ln(I+sinh(x))
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.89

$$\int \frac{\coth(x)}{i+\sinh(x)}\,dx = i(-\log(\sinh(x)) + \log(i+\sinh(x)))$$

input
```
Integrate[Coth[x]/(I + Sinh[x]),x]
```

output
```
I*(-Log[Sinh[x]] + Log[I + Sinh[x]])
```

Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.21, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.636$, Rules used = {3042, 26, 26, 3186, 47, 14, 16}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\coth(x)}{\sinh(x)+i}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{i}{(i-i\sin(ix))\tan(ix)}dx$$

$$\Big\downarrow \text{26}$$

$$i\int -\frac{i}{(1-\sin(ix))\tan(ix)}dx$$

$$\Big\downarrow \text{26}$$

$$\int \frac{1}{(1-\sin(ix))\tan(ix)}dx$$

$$\Big\downarrow \text{3186}$$

$$-i\int \frac{i\text{csch}(x)}{1-i\sinh(x)}d(-i\sinh(x))$$

$$\Big\downarrow \text{47}$$

$$-i\left(\int i\text{csch}(x)d(-i\sinh(x))-\int \frac{1}{1-i\sinh(x)}d(-i\sinh(x))\right)$$

$$\Big\downarrow \text{14}$$

$$-i\left(\log(-i\sinh(x))-\int \frac{1}{1-i\sinh(x)}d(-i\sinh(x))\right)$$

$$\Big\downarrow \text{16}$$

$$-i(\log(-i\sinh(x))-\log(1-i\sinh(x)))$$

input
```
Int[Coth[x]/(I + Sinh[x]),x]
```

output
```
(-I)*(-Log[1 - I*Sinh[x]] + Log[(-I)*Sinh[x]])
```

Defintions of rubi rules used

rule 14
```
Int[(a_.)/(x_), x_Symbol] :> Simp[a*Log[x], x] /; FreeQ[a, x]
```

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 47
```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] :> Simp[b/(b*c
 - a*d)   Int[1/(a + b*x), x], x] - Simp[d/(b*c - a*d)   Int[1/(c + d*x), x
], x] /; FreeQ[{a, b, c, d}, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3186
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*tan[(e_.) + (f_.)*(x_)]^(p
_.), x_Symbol] :> Simp[1/f   Subst[Int[x^p*((a + x)^(m - (p + 1)/2)/(a - x)
^((p + 1)/2)), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && E
qQ[a^2 - b^2, 0] && IntegerQ[(p + 1)/2]
```

Maple [A] (verified)

Time = 0.91 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.11

method	result	size
default	$-i \ln \left(\tanh \left(\frac{x}{2}\right)\right)+2 i \ln \left(\tanh \left(\frac{x}{2}\right)+i\right)$	21
risch	$2 i \ln \left({\mathrm e}^{x}+i\right)-i \ln \left({\mathrm e}^{2 x}-1\right)$	21

input
```
int(coth(x)/(I+sinh(x)),x,method=_RETURNVERBOSE)
```

output
```
-I*ln(tanh(1/2*x))+2*I*ln(tanh(1/2*x)+I)
```

Fricas [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.89

$$\int \frac{\coth(x)}{i+\sinh(x)}\,dx = -i\,\log\left(e^{(2\,x)}-1\right)+2i\,\log\left(e^{x}+i\right)$$

input
```
integrate(coth(x)/(I+sinh(x)),x, algorithm="fricas")
```

output
```
-I*log(e^(2*x) - 1) + 2*I*log(e^x + I)
```

Sympy [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.00

$$\int \frac{\coth(x)}{i+\sinh(x)}\,dx = 2i\log\left(e^{x}+i\right)-i\log\left(e^{2x}-1\right)$$

input
```
integrate(coth(x)/(I+sinh(x)),x)
```

output
```
2*I*log(exp(x) + I) - I*log(exp(2*x) - 1)
```

Maxima [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 28 vs. 2(13) = 26.

Time = 0.05 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.47

$$\int \frac{\coth(x)}{i+\sinh(x)}\,dx = -i\log\left(e^{(-x)}+1\right)+2i\log\left(e^{(-x)}-i\right)-i\log\left(e^{(-x)}-1\right)$$

input
```
integrate(coth(x)/(I+sinh(x)),x, algorithm="maxima")
```

output
```
-I*log(e^(-x) + 1) + 2*I*log(e^(-x) - I) - I*log(e^(-x) - 1)
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.21

$$\int \frac{\coth(x)}{i+\sinh(x)}\,dx = -i\log\left(e^{x}+1\right)+2i\log\left(e^{x}+i\right)-i\log\left(|e^{x}-1|\right)$$

input
```
integrate(coth(x)/(I+sinh(x)),x, algorithm="giac")
```

output
```
-I*log(e^x + 1) + 2*I*log(e^x + I) - I*log(abs(e^x - 1))
```

Mupad [B] (verification not implemented)

Time = 1.48 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.26

$$\int \frac{\coth(x)}{i+\sinh(x)}\,dx = \ln\left(-36\,\mathrm{e}^{x}-36\mathrm{i}\right)\,2\mathrm{i}-\ln\left(3-3\,\mathrm{e}^{2\,x}\right)\,1\mathrm{i}$$

input
```
int(coth(x)/(sinh(x) + 1i),x)
```

output
```
log(- 36*exp(x) - 36i)*2i - log(3 - 3*exp(2*x))*1i
```

Reduce [F]

$$\int \frac{\coth(x)}{i + \sinh(x)}\, dx = \int \frac{\coth (x)}{\sinh (x) + i} dx$$

input
```
int(coth(x)/(I+sinh(x)),x)
```

output
```
int(coth(x)/(sinh(x) + i),x)
```

3.213 $\int \frac{\coth^2(x)}{i+\sinh(x)}\,dx$

Optimal result

Integrand size = 13, antiderivative size = 12

$$\int \frac{\coth^2(x)}{i+\sinh(x)}\,dx = -\operatorname{arctanh}(\cosh(x)) + i\coth(x)$$

output
```
-arctanh(cosh(x))+I*coth(x)
```

Mathematica [B] (verified)

Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 41 vs. 2(12) = 24.

Time = 0.07 (sec) , antiderivative size = 41, normalized size of antiderivative = 3.42

$$\int \frac{\coth^2(x)}{i+\sinh(x)}\,dx = \frac{1}{2}i\coth\left(\frac{x}{2}\right) - \log\left(\cosh\left(\frac{x}{2}\right)\right) + \log\left(\sinh\left(\frac{x}{2}\right)\right) + \frac{1}{2}i\tanh\left(\frac{x}{2}\right)$$

input
```
Integrate[Coth[x]^2/(I + Sinh[x]),x]
```

output
```
(I/2)*Coth[x/2] - Log[Cosh[x/2]] + Log[Sinh[x/2]] + (I/2)*Tanh[x/2]
```

Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.17, number of steps used = 13, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.923, Rules used = {3042, 25, 26, 3185, 25, 26, 3042, 25, 26, 4254, 24, 4257}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\coth^2(x)}{\sinh(x)+i}dx$$

$$\Big\downarrow \text{3042}$$

$$\int -\frac{1}{(i-i\sin(ix))\tan(ix)^2}dx$$

$$\Big\downarrow \text{25}$$

$$-\int -\frac{i}{(1-\sin(ix))\tan(ix)^2}dx$$

$$\Big\downarrow \text{26}$$

$$i\int \frac{1}{(1-\sin(ix))\tan(ix)^2}dx$$

$$\Big\downarrow \text{3185}$$

$$i\left(\int -\text{csch}^2(x)dx+\int -i\text{csch}(x)dx\right)$$

$$\Big\downarrow \text{25}$$

$$i\left(-\int \text{csch}^2(x)dx+\int -i\text{csch}(x)dx\right)$$

$$\Big\downarrow \text{26}$$

$$i\left(-\int \text{csch}^2(x)dx-i\int \text{csch}(x)dx\right)$$

$$\Big\downarrow \text{3042}$$

$$i\left(-i\int i\csc(ix)dx-\int -\csc(ix)^2dx\right)$$

$$\Big\downarrow \text{25}$$

$$i\left(\int \csc(ix)^2dx - i\int i\csc(ix)dx\right)$$

$$\Big\downarrow \ 26$$

$$i\left(\int \csc(ix)dx + \int \csc(ix)^2dx\right)$$

$$\Big\downarrow \ 4254$$

$$i(\int \csc(ix)dx + i\int 1d(-i\coth(x)))$$

$$\Big\downarrow \ 24$$

$$i(\coth(x) + \int \csc(ix)dx)$$

$$\Big\downarrow \ 4257$$

$$i(\coth(x) + i\text{arctanh}(\cosh(x)))$$

input
```
Int[Coth[x]^2/(I + Sinh[x]),x]
```

output
```
I*(I*ArcTanh[Cosh[x]] + Coth[x])
```

Defintions of rubi rules used

rule 24
```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3185
```
Int[((g_.)*tan[(e_.) + (f_.)*(x_)])^(p_.)/((a_) + (b_.)*sin[(e_.) + (f_.)*(
x_)]), x_Symbol] :> Simp[1/a   Int[Sec[e + f*x]^2*(g*Tan[e + f*x])^p, x], x
] - Simp[1/(b*g)   Int[Sec[e + f*x]*(g*Tan[e + f*x])^(p + 1), x], x] /; Fre
eQ[{a, b, e, f, g, p}, x] && EqQ[a^2 - b^2, 0] && NeQ[p, -1]
```

rule 4254
```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Simp[-d^(-1)   Subst[Int[Exp
andIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
 d}, x] && IGtQ[n/2, 0]
```

rule 4257
```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Maple [A] (verified)

Time = 1.56 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.92

method	result	size
default	$\frac{i\tanh\left(\frac{x}{2}\right)}{2}+\ln\left(\tanh\left(\frac{x}{2}\right)\right)+\frac{i}{2\tanh\left(\frac{x}{2}\right)}$	23
risch	$\frac{2i}{e^{2x}-1}+\ln\left(e^{x}-1\right)-\ln\left(e^{x}+1\right)$	25

input
```
int(coth(x)^2/(I+sinh(x)),x,method=_RETURNVERBOSE)
```

output
```
1/2*I*tanh(1/2*x)+ln(tanh(1/2*x))+1/2*I/tanh(1/2*x)
```

Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 37 vs. 2(10) = 20.

Time = 0.10 (sec) , antiderivative size = 37, normalized size of antiderivative = 3.08

$$\int \frac{\coth^2(x)}{i+\sinh(x)}\,dx = -\frac{\left(e^{(2\,x)}-1\right)\log\left(e^{x}+1\right)-\left(e^{(2\,x)}-1\right)\log\left(e^{x}-1\right)-2i}{e^{(2\,x)}-1}$$

input
```
integrate(coth(x)^2/(I+sinh(x)),x, algorithm="fricas")
```

output
```
-((e^(2*x) - 1)*log(e^x + 1) - (e^(2*x) - 1)*log(e^x - 1) - 2*I)/(e^(2*x)
- 1)
```

Sympy [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 22 vs. 2(8) = 16.

Time = 0.08 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.83

$$\int \frac{\coth^2(x)}{i + \sinh(x)}\, dx = \log\left(e^x - 1\right) - \log\left(e^x + 1\right) + \frac{2i}{e^{2x} - 1}$$

input
```
integrate(coth(x)**2/(I+sinh(x)),x)
```

output
```
log(exp(x) - 1) - log(exp(x) + 1) + 2*I/(exp(2*x) - 1)
```

Maxima [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 27 vs. 2(10) = 20.

Time = 0.04 (sec) , antiderivative size = 27, normalized size of antiderivative = 2.25

$$\int \frac{\coth^2(x)}{i + \sinh(x)}\, dx = -\frac{2i}{e^{(-2x)} - 1} - \log\left(e^{(-x)} + 1\right) + \log\left(e^{(-x)} - 1\right)$$

input
```
integrate(coth(x)^2/(I+sinh(x)),x, algorithm="maxima")
```

output
```
-2*I/(e^(-2*x) - 1) - log(e^(-x) + 1) + log(e^(-x) - 1)
```

Giac [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 24 vs. 2(10) = 20.

Time = 0.12 (sec) , antiderivative size = 24, normalized size of antiderivative = 2.00

$$\int \frac{\coth^2(x)}{i+\sinh(x)}\,dx = \frac{2i}{e^{(2x)}-1} - \log\left(e^x+1\right) + \log\left(|e^x-1|\right)$$

input
```
integrate(coth(x)^2/(I+sinh(x)),x, algorithm="giac")
```

output
```
2*I/(e^(2*x) - 1) - log(e^x + 1) + log(abs(e^x - 1))
```

Mupad [B] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 28, normalized size of antiderivative = 2.33

$$\int \frac{\coth^2(x)}{i+\sinh(x)}\,dx = \ln\left(2-2\,\mathrm{e}^x\right) - \ln\left(-2\,\mathrm{e}^x-2\right) + \frac{2\mathrm{i}}{\mathrm{e}^{2x}-1}$$

input
```
int(coth(x)^2/(sinh(x) + 1i),x)
```

output
```
log(2 - 2*exp(x)) - log(- 2*exp(x) - 2) + 2i/(exp(2*x) - 1)
```

Reduce [F]

$$\int \frac{\coth^2(x)}{i+\sinh(x)}\,dx = \int \frac{\coth(x)^2}{\sinh(x)+i}dx$$

input
```
int(coth(x)^2/(I+sinh(x)),x)
```

output
```
int(coth(x)**2/(sinh(x) + i),x)
```

3.214 $\int \frac{\coth^3(x)}{i+\sinh(x)}\,dx$

Optimal result

Integrand size = 13, antiderivative size = 15

$$\int \frac{\coth^3(x)}{i+\sinh(x)}\,dx = -\operatorname{csch}(x)+\frac{1}{2}i\operatorname{csch}^2(x)$$

output
```
-csch(x)+1/2*I*csch(x)^2
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.00

$$\int \frac{\coth^3(x)}{i+\sinh(x)}\,dx = -\operatorname{csch}(x)+\frac{1}{2}i\operatorname{csch}^2(x)$$

input
```
Integrate[Coth[x]^3/(I + Sinh[x]),x]
```

output
```
-Csch[x] + (I/2)*Csch[x]^2
```

Rubi [A] (verified)

Time = 0.32 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.846$, Rules used = {3042, 26, 26, 3185, 25, 26, 3042, 26, 3086, 15, 24}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\coth^3(x)}{\sinh(x)+i}dx$$

$$\Big\downarrow \text{3042}$$

$$\int -\frac{i}{(i-i\sin(ix))\tan(ix)^3}dx$$

$$\Big\downarrow \text{26}$$

$$-i\int -\frac{i}{(1-\sin(ix))\tan(ix)^3}dx$$

$$\Big\downarrow \text{26}$$

$$-\int \frac{1}{(1-\sin(ix))\tan(ix)^3}dx$$

$$\Big\downarrow \text{3185}$$

$$-\int -\coth(x)\text{csch}(x)dx-\int i\coth(x)\text{csch}^2(x)dx$$

$$\Big\downarrow \text{25}$$

$$\int \coth(x)\text{csch}(x)dx-\int i\coth(x)\text{csch}^2(x)dx$$

$$\Big\downarrow \text{26}$$

$$\int \coth(x)\text{csch}(x)dx-i\int \coth(x)\text{csch}^2(x)dx$$

$$\Big\downarrow \text{3042}$$

$$\int \sec\left(ix-\frac{\pi}{2}\right)\tan\left(ix-\frac{\pi}{2}\right)dx-i\int i\sec\left(ix-\frac{\pi}{2}\right)^2\tan\left(ix-\frac{\pi}{2}\right)dx$$

$$\Big\downarrow \text{26}$$

$$\int \sec\left(ix-\frac{\pi}{2}\right)\tan\left(ix-\frac{\pi}{2}\right)dx+\int \sec\left(ix-\frac{\pi}{2}\right)^2\tan\left(ix-\frac{\pi}{2}\right)dx$$

$$\Big\downarrow \ 3086$$

$$-i\int 1d(-i\text{csch}(x))-i\int -i\text{csch}(x)d(-i\text{csch}(x))$$

$$\Big\downarrow \ 15$$

$$\frac{1}{2}i\text{csch}^2(x)-i\int 1d(-i\text{csch}(x))$$

$$\Big\downarrow \ 24$$

$$-\text{csch}(x)+\frac{1}{2}i\text{csch}^2(x)$$

input
```
Int[Coth[x]^3/(I + Sinh[x]),x]
```

output
```
-Csch[x] + (I/2)*Csch[x]^2
```

Defintions of rubi rules used

rule 15
```
Int[(a_.)*(x_)^(m_.), x_Symbol] :> Simp[a*(x^(m + 1)/(m + 1)), x] /; FreeQ[
{a, m}, x] && NeQ[m, -1]
```

rule 24
```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3086
```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Simp[a/f   Subst[Int[(a*x)^(m - 1)*(-1 + x^2)^((n - 1)/2
), x], x, Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2
] &&  !(IntegerQ[m/2] && LtQ[0, m, n + 1])
```

rule 3185
```
Int[((g_.)*tan[(e_.) + (f_.)*(x_)])^(p_.)/((a_) + (b_.)*sin[(e_.) + (f_.)*(
x_)]), x_Symbol] :> Simp[1/a   Int[Sec[e + f*x]^2*(g*Tan[e + f*x])^p, x], x
] - Simp[1/(b*g)   Int[Sec[e + f*x]*(g*Tan[e + f*x])^(p + 1), x], x] /; Fre
eQ[{a, b, e, f, g, p}, x] && EqQ[a^2 - b^2, 0] && NeQ[p, -1]
```

Maple [A] (verified)

Time = 2.80 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.60

method	result	size
risch	$-\frac{2\,{\mathrm e}^{x} \left({\mathrm e}^{2 x}-i {\mathrm e}^{x}-1\right)}{\left({\mathrm e}^{2 x}-1\right)^{2}}$	24
default	$\frac{\tanh\left(\frac{x}{2}\right)}{2}+\frac{i \tanh\left(\frac{x}{2}\right)^{2}}{8}+\frac{i}{8 \tanh\left(\frac{x}{2}\right)^{2}}-\frac{1}{2 \tanh\left(\frac{x}{2}\right)}$	34

input
```
int(coth(x)^3/(I+sinh(x)),x,method=_RETURNVERBOSE)
```

output
```
-2*exp(x)*(exp(2*x)-I*exp(x)-1)/(exp(2*x)-1)^2
```

Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 31 vs. 2(11) = 22.

Time = 0.10 (sec) , antiderivative size = 31, normalized size of antiderivative = 2.07

$$\int \frac{\coth^3(x)}{i+\sinh(x)}\,dx=-\frac{2\left(e^{(3\,x)}-i\,e^{(2\,x)}-e^{x}\right)}{e^{(4\,x)}-2\,e^{(2\,x)}+1}$$

input
```
integrate(coth(x)^3/(I+sinh(x)),x, algorithm="fricas")
```

output
```
-2*(e^(3*x) - I*e^(2*x) - e^x)/(e^(4*x) - 2*e^(2*x) + 1)
```

Sympy [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 32 vs. $2(10) = 20$.

Time = 0.08 (sec) , antiderivative size = 32, normalized size of antiderivative = 2.13

$$\int \frac{\coth^3(x)}{i + \sinh(x)}\, dx = \frac{-2e^{3x} + 2ie^{2x} + 2e^x}{e^{4x} - 2e^{2x} + 1}$$

input
```
integrate(coth(x)**3/(I+sinh(x)),x)
```

output
```
(-2*exp(3*x) + 2*I*exp(2*x) + 2*exp(x))/(exp(4*x) - 2*exp(2*x) + 1)
```

Maxima [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 67 vs. $2(11) = 22$.

Time = 0.03 (sec) , antiderivative size = 67, normalized size of antiderivative = 4.47

$$\int \frac{\coth^3(x)}{i + \sinh(x)}\, dx = \frac{2\, e^{(-x)}}{2\, e^{(-2\,x)} - e^{(-4\,x)} - 1} - \frac{2i\, e^{(-2\,x)}}{2\, e^{(-2\,x)} - e^{(-4\,x)} - 1} - \frac{2\, e^{(-3\,x)}}{2\, e^{(-2\,x)} - e^{(-4\,x)} - 1}$$

input
```
integrate(coth(x)^3/(I+sinh(x)),x, algorithm="maxima")
```

output
```
2*e^(-x)/(2*e^(-2*x) - e^(-4*x) - 1) - 2*I*e^(-2*x)/(2*e^(-2*x) - e^(-4*x)
 - 1) - 2*e^(-3*x)/(2*e^(-2*x) - e^(-4*x) - 1)
```

Giac [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 23 vs. 2(11) = 22.

Time = 0.12 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.53

$$\int \frac{\coth^3(x)}{i+\sinh(x)}\,dx = \frac{2\left(e^{(-x)}-e^x+i\right)}{\left(e^{(-x)}-e^x\right)^2}$$

input
```
integrate(coth(x)^3/(I+sinh(x)),x, algorithm="giac")
```

output
```
2*(e^(-x) - e^x + I)/(e^(-x) - e^x)^2
```

Mupad [B] (verification not implemented)

Time = 1.44 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.67

$$\int \frac{\coth^3(x)}{i+\sinh(x)}\,dx = \frac{2\,\mathrm{e}^x\,\left(1-\mathrm{e}^{2\,x}+\mathrm{e}^x\,1\mathrm{i}\right)}{\left(\mathrm{e}^{2\,x}-1\right)^2}$$

input
```
int(coth(x)^3/(sinh(x) + 1i),x)
```

output
```
(2*exp(x)*(exp(x)*1i - exp(2*x) + 1))/(exp(2*x) - 1)^2
```

Reduce [F]

$$\int \frac{\coth^3(x)}{i+\sinh(x)}\,dx = \int \frac{\coth(x)^3}{\sinh(x)+i}dx$$

input
```
int(coth(x)^3/(I+sinh(x)),x)
```

output
```
int(coth(x)**3/(sinh(x) + i),x)
```

3.215 $\int \frac{\coth^4(x)}{i+\sinh(x)} \, dx$

Optimal result

Integrand size = 13, antiderivative size = 26

$$\int \frac{\coth^4(x)}{i+\sinh(x)} \, dx = -\frac{1}{2}\text{arctanh}(\cosh(x)) + \frac{1}{3}i\coth^3(x) - \frac{1}{2}\coth(x)\text{csch}(x)$$

output
```
-1/2*arctanh(cosh(x))+1/3*I*coth(x)^3-1/2*coth(x)*csch(x)
```

Mathematica [B] (verified)

Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 111 vs. 2(26) = 52.

Time = 0.05 (sec) , antiderivative size = 111, normalized size of antiderivative = 4.27

$$\begin{aligned}\int \frac{\coth^4(x)}{i+\sinh(x)} \, dx = &\frac{1}{6}i\coth\left(\frac{x}{2}\right) - \frac{1}{8}\text{csch}^2\left(\frac{x}{2}\right) + \frac{1}{24}i\coth\left(\frac{x}{2}\right)\text{csch}^2\left(\frac{x}{2}\right) \\ &- \frac{1}{2}\log\left(\cosh\left(\frac{x}{2}\right)\right) + \frac{1}{2}\log\left(\sinh\left(\frac{x}{2}\right)\right) - \frac{1}{8}\text{sech}^2\left(\frac{x}{2}\right) \\ &+ \frac{1}{6}i\tanh\left(\frac{x}{2}\right) - \frac{1}{24}i\text{sech}^2\left(\frac{x}{2}\right)\tanh\left(\frac{x}{2}\right)\end{aligned}$$

input
```
Integrate[Coth[x]^4/(I + Sinh[x]),x]
```

output
```
(I/6)*Coth[x/2] - Csch[x/2]^2/8 + (I/24)*Coth[x/2]*Csch[x/2]^2 - Log[Cosh[
x/2]]/2 + Log[Sinh[x/2]]/2 - Sech[x/2]^2/8 + (I/6)*Tanh[x/2] - (I/24)*Sech
[x/2]^2*Tanh[x/2]
```

Rubi [A] (verified)

Time = 0.42 (sec) , antiderivative size = 35, normalized size of antiderivative = 1.35, number of steps used = 13, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.923$, Rules used = {3042, 3185, 26, 3042, 26, 3087, 15, 3091, 26, 3042, 26, 4257}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\coth^4(x)}{\sinh(x)+i}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{(i-i\sin(ix))\tan(ix)^4}dx$$

$$\Big\downarrow \text{3185}$$

$$-i\int \coth^2(x)\text{csch}^2(x)dx-i\int i\coth^2(x)\text{csch}(x)dx$$

$$\Big\downarrow \text{26}$$

$$\int \coth^2(x)\text{csch}(x)dx-i\int \coth^2(x)\text{csch}^2(x)dx$$

$$\Big\downarrow \text{3042}$$

$$\int -i\sec\left(ix-\frac{\pi}{2}\right)\tan\left(ix-\frac{\pi}{2}\right)^2dx-i\int \sec\left(ix-\frac{\pi}{2}\right)^2\tan\left(ix-\frac{\pi}{2}\right)^2dx$$

$$\Big\downarrow \text{26}$$

$$-i\int \sec\left(ix-\frac{\pi}{2}\right)\tan\left(ix-\frac{\pi}{2}\right)^2dx-i\int \sec\left(ix-\frac{\pi}{2}\right)^2\tan\left(ix-\frac{\pi}{2}\right)^2dx$$

$$\Big\downarrow \text{3087}$$

$$-\int -\coth^2(x)d(i\coth(x))-i\int \sec\left(ix-\frac{\pi}{2}\right)\tan\left(ix-\frac{\pi}{2}\right)^2dx$$

$$\Big\downarrow \text{15}$$

$$\frac{1}{3}i\coth^3(x)-i\int \sec\left(ix-\frac{\pi}{2}\right)\tan\left(ix-\frac{\pi}{2}\right)^2dx$$

$$\Big\downarrow \text{3091}$$

$$\frac{1}{3}i\coth^3(x)-i\left(-\frac{1}{2}\int -i\text{csch}(x)dx-\frac{1}{2}i\coth(x)\text{csch}(x)\right)$$

$$\Big\downarrow \text{26}$$

$$\frac{1}{3}i\coth^3(x)-i\left(\frac{1}{2}i\int \text{csch}(x)dx-\frac{1}{2}i\coth(x)\text{csch}(x)\right)$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{3}i\coth^3(x)-i\left(\frac{1}{2}i\int i\csc(ix)dx-\frac{1}{2}i\coth(x)\text{csch}(x)\right)$$

$$\Big\downarrow \text{26}$$

$$\frac{1}{3}i\coth^3(x)-i\left(-\frac{1}{2}\int \csc(ix)dx-\frac{1}{2}i\coth(x)\text{csch}(x)\right)$$

$$\Big\downarrow \text{4257}$$

$$\frac{1}{3}i\coth^3(x)-i\left(-\frac{1}{2}i\text{arctanh}(\cosh(x))-\frac{1}{2}i\coth(x)\text{csch}(x)\right)$$

input
```
Int[Coth[x]^4/(I + Sinh[x]),x]
```

output
```
(I/3)*Coth[x]^3 - I*((-1/2*I)*ArcTanh[Cosh[x]] - (I/2)*Coth[x]*Csch[x])
```

Defintions of rubi rules used

rule 15
```
Int[(a_.)*(x_)^(m_.), x_Symbol] :> Simp[a*(x^(m + 1)/(m + 1)), x] /; FreeQ[
{a, m}, x] && NeQ[m, -1]
```

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3087
```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> Simp[1/f   Subst[Int[(b*x)^n*(1 + x^2)^(m/2 - 1), x], x, Tan[e +
f*x]], x] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2] &&  !(IntegerQ[(n - 1)
/2] && LtQ[0, n, m - 1])
```

rule 3091
```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[b*(a*Sec[e + f*x])^m*((b*Tan[e + f*x])^(n - 1)/(f*(m
 + n - 1))), x] - Simp[b^2*((n - 1)/(m + n - 1))   Int[(a*Sec[e + f*x])^m*(
b*Tan[e + f*x])^(n - 2), x], x] /; FreeQ[{a, b, e, f, m}, x] && GtQ[n, 1] &
& NeQ[m + n - 1, 0] && IntegersQ[2*m, 2*n]
```

rule 3185
```
Int[((g_.)*tan[(e_.) + (f_.)*(x_)])^(p_.)/((a_) + (b_.)*sin[(e_.) + (f_.)*(
x_)]), x_Symbol] :> Simp[1/a   Int[Sec[e + f*x]^2*(g*Tan[e + f*x])^p, x], x
] - Simp[1/(b*g)   Int[Sec[e + f*x]*(g*Tan[e + f*x])^(p + 1), x], x] /; Fre
eQ[{a, b, e, f, g, p}, x] && EqQ[a^2 - b^2, 0] && NeQ[p, -1]
```

rule 4257
```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 45 vs. $2(19) = 38$.

Time = 4.92 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.77

method	result	size
risch	$-\frac{-6i\,{\mathrm e}^{4x}+3\,{\mathrm e}^{5x}-2i-3\,{\mathrm e}^{x}}{3\left({\mathrm e}^{2x}-1\right)^{3}}+\frac{\ln\left({\mathrm e}^{x}-1\right)}{2}-\frac{\ln\left({\mathrm e}^{x}+1\right)}{2}$	46
default	$\frac{i\tanh\left(\frac{x}{2}\right)}{8}+\frac{i\tanh\left(\frac{x}{2}\right)^{3}}{24}+\frac{\tanh\left(\frac{x}{2}\right)^{2}}{8}+\frac{\ln\left(\tanh\left(\frac{x}{2}\right)\right)}{2}-\frac{1}{8\tanh\left(\frac{x}{2}\right)^{2}}+\frac{i}{24\tanh\left(\frac{x}{2}\right)^{3}}+\frac{i}{8\tanh\left(\frac{x}{2}\right)}$	59

input
```
int(coth(x)^4/(I+sinh(x)),x,method=_RETURNVERBOSE)
```

output
```
-1/3*(-6*I*exp(x)^4+3*exp(x)^5-2*I-3*exp(x))/(exp(x)^2-1)^3+1/2*ln(exp(x)-
1)-1/2*ln(exp(x)+1)
```

Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 90 vs. $2(18) = 36$.

Time = 0.10 (sec) , antiderivative size = 90, normalized size of antiderivative = 3.46

$$\int \frac{\coth^4(x)}{i+\sinh(x)}\,dx = -\frac{3\left(e^{(6x)} - 3e^{(4x)} + 3e^{(2x)} - 1\right)\log\left(e^x+1\right) - 3\left(e^{(6x)} - 3e^{(4x)} + 3e^{(2x)} - 1\right)\log\left(e^x-1\right) + 6e^{(5x)} - 1}{6\left(e^{(6x)} - 3e^{(4x)} + 3e^{(2x)} - 1\right)}$$

input
```
integrate(coth(x)^4/(I+sinh(x)),x, algorithm="fricas")
```

output
```
-1/6*(3*(e^(6*x) - 3*e^(4*x) + 3*e^(2*x) - 1)*log(e^x + 1) - 3*(e^(6*x) -
3*e^(4*x) + 3*e^(2*x) - 1)*log(e^x - 1) + 6*e^(5*x) - 12*I*e^(4*x) - 6*e^x
 - 4*I)/(e^(6*x) - 3*e^(4*x) + 3*e^(2*x) - 1)
```

Sympy [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 61 vs. $2(22) = 44$.

Time = 0.12 (sec) , antiderivative size = 61, normalized size of antiderivative = 2.35

$$\int \frac{\coth^4(x)}{i+\sinh(x)}\,dx = \frac{-3e^{5x} + 6ie^{4x} + 3e^x + 2i}{3e^{6x} - 9e^{4x} + 9e^{2x} - 3} + \frac{\log\left(e^x-1\right)}{2} - \frac{\log\left(e^x+1\right)}{2}$$

input
```
integrate(coth(x)**4/(I+sinh(x)),x)
```

output
```
(-3*exp(5*x) + 6*I*exp(4*x) + 3*exp(x) + 2*I)/(3*exp(6*x) - 9*exp(4*x) + 9
*exp(2*x) - 3) + log(exp(x) - 1)/2 - log(exp(x) + 1)/2
```

Maxima [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 61 vs. 2(18) = 36.

Time = 0.04 (sec) , antiderivative size = 61, normalized size of antiderivative = 2.35

$$\int \frac{\coth^4(x)}{i+\sinh(x)}\,dx = \frac{3\,e^{(-x)} - 6i\,e^{(-4\,x)} - 3\,e^{(-5\,x)} - 2i}{3\,(3\,e^{(-2\,x)} - 3\,e^{(-4\,x)} + e^{(-6\,x)} - 1)} - \frac{1}{2}\,\log\left(e^{(-x)} + 1\right) + \frac{1}{2}\,\log\left(e^{(-x)} - 1\right)$$

input
```
integrate(coth(x)^4/(I+sinh(x)),x, algorithm="maxima")
```

output
```
1/3*(3*e^(-x) - 6*I*e^(-4*x) - 3*e^(-5*x) - 2*I)/(3*e^(-2*x) - 3*e^(-4*x)
+ e^(-6*x) - 1) - 1/2*log(e^(-x) + 1) + 1/2*log(e^(-x) - 1)
```

Giac [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 44 vs. 2(18) = 36.

Time = 0.12 (sec) , antiderivative size = 44, normalized size of antiderivative = 1.69

$$\int \frac{\coth^4(x)}{i+\sinh(x)}\,dx = -\frac{3\,e^{(5\,x)} - 6i\,e^{(4\,x)} - 3\,e^{x} - 2i}{3\,\left(e^{(2\,x)} - 1\right)^3} - \frac{1}{2}\,\log\left(e^{x} + 1\right) + \frac{1}{2}\,\log\left(|e^{x} - 1|\right)$$

input
```
integrate(coth(x)^4/(I+sinh(x)),x, algorithm="giac")
```

output
```
-1/3*(3*e^(5*x) - 6*I*e^(4*x) - 3*e^x - 2*I)/(e^(2*x) - 1)^3 - 1/2*log(e^x
 + 1) + 1/2*log(abs(e^x - 1))
```

Mupad [B] (verification not implemented)

Time = 0.32 (sec) , antiderivative size = 74, normalized size of antiderivative = 2.85

$$\int \frac{\coth^4(x)}{i+\sinh(x)}\,dx = \frac{\ln(1-e^x)}{2} - \frac{\ln(e^x+1)}{2} - \frac{e^x}{e^{2x}-1} - \frac{2\,e^x}{(e^{2x}-1)^2} + \frac{2i}{e^{2x}-1} + \frac{4i}{(e^{2x}-1)^2} + \frac{8i}{3\,(e^{2x}-1)^3}$$

input
```
int(coth(x)^4/(sinh(x) + 1i),x)
```

output
```
log(1 - exp(x))/2 - log(exp(x) + 1)/2 - exp(x)/(exp(2*x) - 1) - (2*exp(x))
/(exp(2*x) - 1)^2 + 2i/(exp(2*x) - 1) + 4i/(exp(2*x) - 1)^2 + 8i/(3*(exp(2
*x) - 1)^3)
```

Reduce [F]

$$\int \frac{\coth^4(x)}{i+\sinh(x)}\,dx = \int \frac{\coth(x)^4}{\sinh(x)+i}dx$$

input
```
int(coth(x)^4/(I+sinh(x)),x)
```

output
```
int(coth(x)**4/(sinh(x) + i),x)
```

3.216 $\int \frac{\coth^5(x)}{i+\sinh(x)}\,dx$

Optimal result

Integrand size = 13, antiderivative size = 23

$$\int \frac{\coth^5(x)}{i+\sinh(x)}\,dx=\frac{1}{4}i\coth^4(x)-\operatorname{csch}(x)-\frac{\operatorname{csch}^3(x)}{3}$$

output
```
1/4*I*coth(x)^4-csch(x)-1/3*csch(x)^3
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 33, normalized size of antiderivative = 1.43

$$\int \frac{\coth^5(x)}{i+\sinh(x)}\,dx=-\operatorname{csch}(x)+\frac{1}{2}i\operatorname{csch}^2(x)-\frac{\operatorname{csch}^3(x)}{3}+\frac{1}{4}i\operatorname{csch}^4(x)$$

input
```
Integrate[Coth[x]^5/(I + Sinh[x]),x]
```

output
```
-Csch[x] + (I/2)*Csch[x]^2 - Csch[x]^3/3 + (I/4)*Csch[x]^4
```

Rubi [A] (verified)

Time = 0.39 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.39, number of steps used = 13, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.923$, Rules used = {3042, 26, 26, 3185, 26, 3042, 25, 26, 3086, 2009, 3087, 15}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\coth^5(x)}{\sinh(x)+i}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{i}{(i-i\sin(ix))\tan(ix)^5}dx$$

$$\Big\downarrow \text{26}$$

$$i\int -\frac{i}{(1-\sin(ix))\tan(ix)^5}dx$$

$$\Big\downarrow \text{26}$$

$$\int \frac{1}{(1-\sin(ix))\tan(ix)^5}dx$$

$$\Big\downarrow \text{3185}$$

$$\int \coth^3(x)\text{csch}(x)dx+\int -i\coth^3(x)\text{csch}^2(x)dx$$

$$\Big\downarrow \text{26}$$

$$\int \coth^3(x)\text{csch}(x)dx-i\int \coth^3(x)\text{csch}^2(x)dx$$

$$\Big\downarrow \text{3042}$$

$$\int -\sec\left(ix-\frac{\pi}{2}\right)\tan\left(ix-\frac{\pi}{2}\right)^3dx-i\int -i\sec\left(ix-\frac{\pi}{2}\right)^2\tan\left(ix-\frac{\pi}{2}\right)^3dx$$

$$\Big\downarrow \text{25}$$

$$-\int \sec\left(ix-\frac{\pi}{2}\right)\tan\left(ix-\frac{\pi}{2}\right)^3dx-i\int -i\sec\left(ix-\frac{\pi}{2}\right)^2\tan\left(ix-\frac{\pi}{2}\right)^3dx$$

$$\Big\downarrow \text{26}$$

$$-\int \sec\left(ix-\frac{\pi}{2}\right)\tan\left(ix-\frac{\pi}{2}\right)^3dx-\int \sec\left(ix-\frac{\pi}{2}\right)^2\tan\left(ix-\frac{\pi}{2}\right)^3dx$$

$$\Big\downarrow \; 3086$$

$$i\int \left(-\text{csch}^2(x)-1\right)d(-i\text{csch}(x))-\int \sec\left(ix-\frac{\pi}{2}\right)^2\tan\left(ix-\frac{\pi}{2}\right)^3dx$$

$$\Big\downarrow \; 2009$$

$$i\left(\frac{1}{3}i\text{csch}^3(x)+i\text{csch}(x)\right)-\int \sec\left(ix-\frac{\pi}{2}\right)^2\tan\left(ix-\frac{\pi}{2}\right)^3dx$$

$$\Big\downarrow \; 3087$$

$$i\int -i\coth^3(x)d(i\coth(x))+i\left(\frac{1}{3}i\text{csch}^3(x)+i\text{csch}(x)\right)$$

$$\Big\downarrow \; 15$$

$$\frac{1}{4}i\coth^4(x)+i\left(\frac{1}{3}i\text{csch}^3(x)+i\text{csch}(x)\right)$$

input
```
Int[Coth[x]^5/(I + Sinh[x]),x]
```

output
```
(I/4)*Coth[x]^4 + I*(I*Csch[x] + (I/3)*Csch[x]^3)
```

Defintions of rubi rules used

rule 15
```
Int[(a_.)*(x_)^(m_.), x_Symbol] :> Simp[a*(x^(m + 1)/(m + 1)), x] /; FreeQ[
{a, m}, x] && NeQ[m, -1]
```

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3086
```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Simp[a/f   Subst[Int[(a*x)^(m - 1)*(-1 + x^2)^((n - 1)/2
), x], x, Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2
] &&  !(IntegerQ[m/2] && LtQ[0, m, n + 1])
```

rule 3087
```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> Simp[1/f   Subst[Int[(b*x)^n*(1 + x^2)^(m/2 - 1), x], x, Tan[e +
f*x]], x] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2] &&  !(IntegerQ[(n - 1)
/2] && LtQ[0, n, m - 1])
```

rule 3185
```
Int[((g_.)*tan[(e_.) + (f_.)*(x_)])^(p_.)/((a_) + (b_.)*sin[(e_.) + (f_.)*(
x_)]), x_Symbol] :> Simp[1/a   Int[Sec[e + f*x]^2*(g*Tan[e + f*x])^p, x], x
] - Simp[1/(b*g)   Int[Sec[e + f*x]*(g*Tan[e + f*x])^(p + 1), x], x] /; Fre
eQ[{a, b, e, f, g, p}, x] && EqQ[a^2 - b^2, 0] && NeQ[p, -1]
```

Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 44 vs. $2(18) = 36$.

Time = 7.82 (sec) , antiderivative size = 45, normalized size of antiderivative = 1.96

method	result	siz
risch	$-\frac{2\,e^{x}\left(-3 i\,e^{5x}+3\,e^{6x}-5\,e^{4x}-3 i\,e^{x}+5\,e^{2x}-3\right)}{3\left(e^{2x}-1\right)^{4}}$	45
default	$\frac{3\tanh\left(\frac{x}{2}\right)}{8}+\frac{i\tanh\left(\frac{x}{2}\right)^{4}}{64}+\frac{\tanh\left(\frac{x}{2}\right)^{3}}{24}+\frac{i\tanh\left(\frac{x}{2}\right)^{2}}{16}+\frac{i}{16\tanh\left(\frac{x}{2}\right)^{2}}-\frac{1}{24\tanh\left(\frac{x}{2}\right)^{3}}+\frac{i}{64\tanh\left(\frac{x}{2}\right)^{4}}-\frac{3}{8\tanh\left(\frac{x}{2}\right)}$	68

input
```
int(coth(x)^5/(I+sinh(x)),x,method=_RETURNVERBOSE)
```

output
```
-2/3*exp(x)*(-3*I*exp(x)^5+3*exp(x)^6-5*exp(x)^4-3*I*exp(x)+5*exp(x)^2-3)/
(exp(x)^2-1)^4
```

Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 63 vs. 2(17) = 34.

Time = 0.10 (sec) , antiderivative size = 63, normalized size of antiderivative = 2.74

$$\int \frac{\coth^5(x)}{i+\sinh(x)}\,dx = -\frac{2\left(3\,e^{(7\,x)} - 3i\,e^{(6\,x)} - 5\,e^{(5\,x)} + 5\,e^{(3\,x)} - 3i\,e^{(2\,x)} - 3\,e^{x}\right)}{3\left(e^{(8\,x)} - 4\,e^{(6\,x)} + 6\,e^{(4\,x)} - 4\,e^{(2\,x)} + 1\right)}$$

input
```
integrate(coth(x)^5/(I+sinh(x)),x, algorithm="fricas")
```

output
```
-2/3*(3*e^(7*x) - 3*I*e^(6*x) - 5*e^(5*x) + 5*e^(3*x) - 3*I*e^(2*x) - 3*e^
x)/(e^(8*x) - 4*e^(6*x) + 6*e^(4*x) - 4*e^(2*x) + 1)
```

Sympy [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 70 vs. 2(17) = 34.

Time = 0.11 (sec) , antiderivative size = 70, normalized size of antiderivative = 3.04

$$\int \frac{\coth^5(x)}{i+\sinh(x)}\,dx = \frac{-6e^{7x} + 6ie^{6x} + 10e^{5x} - 10e^{3x} + 6ie^{2x} + 6e^{x}}{3e^{8x} - 12e^{6x} + 18e^{4x} - 12e^{2x} + 3}$$

input
```
integrate(coth(x)**5/(I+sinh(x)),x)
```

output
```
(-6*exp(7*x) + 6*I*exp(6*x) + 10*exp(5*x) - 10*exp(3*x) + 6*I*exp(2*x) + 6
*exp(x))/(3*exp(8*x) - 12*exp(6*x) + 18*exp(4*x) - 12*exp(2*x) + 3)
```

Maxima [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 205 vs. $2(17) = 34$.

Time = 0.04 (sec) , antiderivative size = 205, normalized size of antiderivative = 8.91

$$\int \frac{\coth^5(x)}{i+\sinh(x)}\,dx = \frac{2\,e^{(-x)}}{4\,e^{(-2\,x)}-6\,e^{(-4\,x)}+4\,e^{(-6\,x)}-e^{(-8\,x)}-1} - \frac{2i\,e^{(-2\,x)}}{4\,e^{(-2\,x)}-6\,e^{(-4\,x)}+4\,e^{(-6\,x)}-e^{(-8\,x)}-1} - \frac{10\,e^{(-3\,x)}}{3\,(4\,e^{(-2\,x)}-6\,e^{(-4\,x)}+4\,e^{(-6\,x)}-e^{(-8\,x)}-1)} + \frac{10\,e^{(-5\,x)}}{3\,(4\,e^{(-2\,x)}-6\,e^{(-4\,x)}+4\,e^{(-6\,x)}-e^{(-8\,x)}-1)} - \frac{2i\,e^{(-6\,x)}}{4\,e^{(-2\,x)}-6\,e^{(-4\,x)}+4\,e^{(-6\,x)}-e^{(-8\,x)}-1} - \frac{2\,e^{(-7\,x)}}{4\,e^{(-2\,x)}-6\,e^{(-4\,x)}+4\,e^{(-6\,x)}-e^{(-8\,x)}-1}$$

input
```
integrate(coth(x)^5/(I+sinh(x)),x, algorithm="maxima")
```

output
```
2*e^(-x)/(4*e^(-2*x) - 6*e^(-4*x) + 4*e^(-6*x) - e^(-8*x) - 1) - 2*I*e^(-2
*x)/(4*e^(-2*x) - 6*e^(-4*x) + 4*e^(-6*x) - e^(-8*x) - 1) - 10/3*e^(-3*x)/
(4*e^(-2*x) - 6*e^(-4*x) + 4*e^(-6*x) - e^(-8*x) - 1) + 10/3*e^(-5*x)/(4*e
^(-2*x) - 6*e^(-4*x) + 4*e^(-6*x) - e^(-8*x) - 1) - 2*I*e^(-6*x)/(4*e^(-2*
x) - 6*e^(-4*x) + 4*e^(-6*x) - e^(-8*x) - 1) - 2*e^(-7*x)/(4*e^(-2*x) - 6*
e^(-4*x) + 4*e^(-6*x) - e^(-8*x) - 1)
```

Giac [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 51 vs. $2(17) = 34$.

Time = 0.12 (sec) , antiderivative size = 51, normalized size of antiderivative = 2.22

$$\int \frac{\coth^5(x)}{i+\sinh(x)}\,dx = \frac{2\left(3\left(e^{(-x)}-e^{x}\right)^3+3i\left(e^{(-x)}-e^{x}\right)^2+4\,e^{(-x)}-4\,e^{x}+6i\right)}{3\left(e^{(-x)}-e^{x}\right)^4}$$

input
```
integrate(coth(x)^5/(I+sinh(x)),x, algorithm="giac")
```

output
```
2/3*(3*(e^(-x) - e^x)^3 + 3*I*(e^(-x) - e^x)^2 + 4*e^(-x) - 4*e^x + 6*I)/(
e^(-x) - e^x)^4
```

Mupad [B] (verification not implemented)

Time = 1.48 (sec) , antiderivative size = 44, normalized size of antiderivative = 1.91

$$\int \frac{\coth^5(x)}{i+\sinh(x)}\,dx = \frac{2\,\mathrm{e}^{x}\,\left(5\,\mathrm{e}^{4\,x}-5\,\mathrm{e}^{2\,x}-3\,\mathrm{e}^{6\,x}+3+\mathrm{e}^{5\,x}\,3\mathrm{i}+\mathrm{e}^{x}\,3\mathrm{i}\right)}{3\,{\left(\mathrm{e}^{2\,x}-1\right)}^4}$$

input
```
int(coth(x)^5/(sinh(x) + 1i),x)
```

output
```
(2*exp(x)*(5*exp(4*x) - 5*exp(2*x) + exp(5*x)*3i - 3*exp(6*x) + exp(x)*3i
+ 3))/(3*(exp(2*x) - 1)^4)
```

Reduce [F]

$$\int \frac{\coth^5(x)}{i+\sinh(x)}\,dx = \int \frac{\coth\left(x\right)^5}{\sinh\left(x\right)+i}dx$$

input
```
int(coth(x)^5/(I+sinh(x)),x)
```

output
```
int(coth(x)**5/(sinh(x) + i),x)
```

3.217 $\int \frac{\coth^6(x)}{i+\sinh(x)}\,dx$

Optimal result

Integrand size = 13, antiderivative size = 36

$$\int \frac{\coth^6(x)}{i+\sinh(x)}\,dx = -\frac{3}{8}\operatorname{arctanh}(\cosh(x)) + \frac{1}{5}i\coth^5(x) - \frac{3}{8}\coth(x)\operatorname{csch}(x) - \frac{1}{4}\coth^3(x)\operatorname{csch}(x)$$

output
```
-3/8*arctanh(cosh(x))+1/5*I*coth(x)^5-3/8*coth(x)*csch(x)-1/4*coth(x)^3*cs
ch(x)
```

Mathematica [B] (verified)

Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 175 vs. 2(36) = 72.

Time = 0.07 (sec) , antiderivative size = 175, normalized size of antiderivative = 4.86

$$\int \frac{\coth^6(x)}{i+\sinh(x)}\,dx = \frac{1}{10}i\coth\left(\frac{x}{2}\right) - \frac{5}{32}\operatorname{csch}^2\left(\frac{x}{2}\right) + \frac{7}{160}i\coth\left(\frac{x}{2}\right)\operatorname{csch}^2\left(\frac{x}{2}\right) - \frac{1}{64}\operatorname{csch}^4\left(\frac{x}{2}\right) + \frac{1}{160}i\coth\left(\frac{x}{2}\right)\operatorname{csch}^4\left(\frac{x}{2}\right) - \frac{3}{8}\log\left(\cosh\left(\frac{x}{2}\right)\right) + \frac{3}{8}\log\left(\sinh\left(\frac{x}{2}\right)\right) - \frac{5}{32}\operatorname{sech}^2\left(\frac{x}{2}\right) + \frac{1}{64}\operatorname{sech}^4\left(\frac{x}{2}\right) + \frac{1}{10}i\tanh\left(\frac{x}{2}\right) - \frac{7}{160}i\operatorname{sech}^2\left(\frac{x}{2}\right)\tanh\left(\frac{x}{2}\right) + \frac{1}{160}i\operatorname{sech}^4\left(\frac{x}{2}\right)\tanh\left(\frac{x}{2}\right)$$

input
```
Integrate[Coth[x]^6/(I + Sinh[x]),x]
```

output
```
(I/10)*Coth[x/2] - (5*Csch[x/2]^2)/32 + ((7*I)/160)*Coth[x/2]*Csch[x/2]^2
- Csch[x/2]^4/64 + (I/160)*Coth[x/2]*Csch[x/2]^4 - (3*Log[Cosh[x/2]])/8 +
(3*Log[Sinh[x/2]])/8 - (5*Sech[x/2]^2)/32 + Sech[x/2]^4/64 + (I/10)*Tanh[x
/2] - ((7*I)/160)*Sech[x/2]^2*Tanh[x/2] + (I/160)*Sech[x/2]^4*Tanh[x/2]
```

Rubi [A] (verified)

Time = 0.50 (sec) , antiderivative size = 49, normalized size of antiderivative = 1.36, number of steps used = 21, number of rules used = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 1.538$, Rules used = {3042, 25, 26, 3185, 25, 26, 3042, 25, 26, 3087, 15, 3091, 26, 3042, 26, 3091, 26, 3042, 26, 4257}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\coth^6(x)}{\sinh(x)+i}dx$$

$$\Big\downarrow \text{3042}$$

$$\int -\frac{1}{(i-i\sin(ix))\tan(ix)^6}dx$$

$$\Big\downarrow \text{25}$$

$$-\int -\frac{i}{(1-\sin(ix))\tan(ix)^6}dx$$

$$\Big\downarrow \text{26}$$

$$i\int \frac{1}{(1-\sin(ix))\tan(ix)^6}dx$$

$$\Big\downarrow \text{3185}$$

$$i\left(\int -\coth^4(x)\text{csch}^2(x)dx+\int -i\coth^4(x)\text{csch}(x)dx\right)$$

$$\Big\downarrow \text{25}$$

$$i\left(-\int \coth^4(x)\text{csch}^2(x)dx+\int -i\coth^4(x)\text{csch}(x)dx\right)$$

$$\Big\downarrow \text{26}$$

$$i\left(-\int \coth^4(x)\text{csch}^2(x)dx-i\int \coth^4(x)\text{csch}(x)dx\right)$$

$$\Big\downarrow \text{3042}$$

$$i\left(-i\int i\sec\left(ix-\frac{\pi}{2}\right)\tan\left(ix-\frac{\pi}{2}\right)^4dx-\int -\sec\left(ix-\frac{\pi}{2}\right)^2\tan\left(ix-\frac{\pi}{2}\right)^4dx\right)$$

$$\Big\downarrow \text{25}$$

$$i\left(\int \sec\left(ix-\frac{\pi}{2}\right)^2\tan\left(ix-\frac{\pi}{2}\right)^4dx-i\int i\sec\left(ix-\frac{\pi}{2}\right)\tan\left(ix-\frac{\pi}{2}\right)^4dx\right)$$

$$\Big\downarrow \text{26}$$

$$i\left(\int \sec\left(ix-\frac{\pi}{2}\right)\tan\left(ix-\frac{\pi}{2}\right)^4dx+\int \sec\left(ix-\frac{\pi}{2}\right)^2\tan\left(ix-\frac{\pi}{2}\right)^4dx\right)$$

$$\Big\downarrow \text{3087}$$

$$i\left(\int \sec\left(ix-\frac{\pi}{2}\right)\tan\left(ix-\frac{\pi}{2}\right)^4dx-i\int \coth^4(x)d(i\coth(x))\right)$$

$$\Big\downarrow \text{15}$$

$$i\left(\frac{\coth^5(x)}{5}+\int \sec\left(ix-\frac{\pi}{2}\right)\tan\left(ix-\frac{\pi}{2}\right)^4dx\right)$$

$$\Big\downarrow \text{3091}$$

$$i\left(-\frac{3}{4}\int i\coth^2(x)\text{csch}(x)dx+\frac{\coth^5(x)}{5}+\frac{1}{4}i\coth^3(x)\text{csch}(x)\right)$$

$$\Big\downarrow \text{26}$$

$$i\left(-\frac{3}{4}i\int \coth^2(x)\text{csch}(x)dx+\frac{\coth^5(x)}{5}+\frac{1}{4}i\coth^3(x)\text{csch}(x)\right)$$

$$\Big\downarrow \text{3042}$$

$$i\left(-\frac{3}{4}i\int -i\sec\left(ix-\frac{\pi}{2}\right)\tan\left(ix-\frac{\pi}{2}\right)^2dx+\frac{\coth^5(x)}{5}+\frac{1}{4}i\coth^3(x)\text{csch}(x)\right)$$

$$\Big\downarrow \text{26}$$

$$i\left(-\frac{3}{4}\int \sec\left(ix-\frac{\pi}{2}\right)\tan\left(ix-\frac{\pi}{2}\right)^2dx+\frac{\coth^5(x)}{5}+\frac{1}{4}i\coth^3(x)\text{csch}(x)\right)$$

$$\Big\downarrow \text{3091}$$

$$i\left(-\frac{3}{4}\left(-\frac{1}{2}\int -i\text{csch}(x)dx-\frac{1}{2}i\coth(x)\text{csch}(x)\right)+\frac{\coth^5(x)}{5}+\frac{1}{4}i\coth^3(x)\text{csch}(x)\right)$$

$$\Big\downarrow \text{26}$$

$$i\left(-\frac{3}{4}\left(\frac{1}{2}i\int \text{csch}(x)dx-\frac{1}{2}i\coth(x)\text{csch}(x)\right)+\frac{\coth^5(x)}{5}+\frac{1}{4}i\coth^3(x)\text{csch}(x)\right)$$

$$\Big\downarrow \text{3042}$$

$$i\left(-\frac{3}{4}\left(\frac{1}{2}i\int i\csc(ix)dx-\frac{1}{2}i\coth(x)\text{csch}(x)\right)+\frac{\coth^5(x)}{5}+\frac{1}{4}i\coth^3(x)\text{csch}(x)\right)$$

$$\Big\downarrow \text{26}$$

$$i\left(-\frac{3}{4}\left(-\frac{1}{2}\int \csc(ix)dx-\frac{1}{2}i\coth(x)\text{csch}(x)\right)+\frac{\coth^5(x)}{5}+\frac{1}{4}i\coth^3(x)\text{csch}(x)\right)$$

$$\Big\downarrow \text{4257}$$

$$i\left(-\frac{3}{4}\left(-\frac{1}{2}i\text{arctanh}(\cosh(x))-\frac{1}{2}i\coth(x)\text{csch}(x)\right)+\frac{\coth^5(x)}{5}+\frac{1}{4}i\coth^3(x)\text{csch}(x)\right)$$

input
```
Int[Coth[x]^6/(I + Sinh[x]),x]
```

output
```
I*(Coth[x]^5/5 + (I/4)*Coth[x]^3*Csch[x] - (3*((-1/2*I)*ArcTanh[Cosh[x]] -
 (I/2)*Coth[x]*Csch[x]))/4)
```

Defintions of rubi rules used

rule 15
```
Int[(a_.)*(x_)^(m_.), x_Symbol] :> Simp[a*(x^(m + 1)/(m + 1)), x] /; FreeQ[
{a, m}, x] && NeQ[m, -1]
```

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3087
```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> Simp[1/f   Subst[Int[(b*x)^n*(1 + x^2)^(m/2 - 1), x], x, Tan[e +
f*x]], x] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2] &&  !(IntegerQ[(n - 1)
/2] && LtQ[0, n, m - 1])
```

rule 3091
```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[b*(a*Sec[e + f*x])^m*((b*Tan[e + f*x])^(n - 1)/(f*(m
 + n - 1))), x] - Simp[b^2*((n - 1)/(m + n - 1))   Int[(a*Sec[e + f*x])^m*(
b*Tan[e + f*x])^(n - 2), x], x] /; FreeQ[{a, b, e, f, m}, x] && GtQ[n, 1] &
& NeQ[m + n - 1, 0] && IntegersQ[2*m, 2*n]
```

rule 3185
```
Int[((g_.)*tan[(e_.) + (f_.)*(x_)])^(p_.)/((a_) + (b_.)*sin[(e_.) + (f_.)*(
x_)]), x_Symbol] :> Simp[1/a   Int[Sec[e + f*x]^2*(g*Tan[e + f*x])^p, x], x
] - Simp[1/(b*g)   Int[Sec[e + f*x]*(g*Tan[e + f*x])^(p + 1), x], x] /; Fre
eQ[{a, b, e, f, g, p}, x] && EqQ[a^2 - b^2, 0] && NeQ[p, -1]
```

rule 4257
```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 64 vs. $2(27) = 54$.

Time = 12.51 (sec) , antiderivative size = 65, normalized size of antiderivative = 1.81

method	result
risch	$-\frac{-40 i \,{\mathrm e}^{8 x}+25 \,{\mathrm e}^{9 x}-10 \,{\mathrm e}^{7 x}-80 i \,{\mathrm e}^{4 x}+10 \,{\mathrm e}^{3 x}-8 i -25 \,{\mathrm e}^{x}}{20 \left({\mathrm e}^{2 x}-1\right)^{5}}-\frac{3 \ln \left({\mathrm e}^{x}+1\right)}{8}+\frac{3 \ln \left({\mathrm e}^{x}-1\right)}{8}$
default	$\frac{i \tanh \left(\frac{x}{2}\right)}{16}+\frac{i \tanh \left(\frac{x}{2}\right)^{5}}{160}+\frac{\tanh \left(\frac{x}{2}\right)^{4}}{64}+\frac{i \tanh \left(\frac{x}{2}\right)^{3}}{32}+\frac{\tanh \left(\frac{x}{2}\right)^{2}}{8}+\frac{3 \ln \left(\tanh \left(\frac{x}{2}\right)\right)}{8}-\frac{1}{8 \tanh \left(\frac{x}{2}\right)^{2}}+\frac{i}{32 \tanh \left(\frac{x}{2}\right)^{3}}+$ 16

input
```
int(coth(x)^6/(I+sinh(x)),x,method=_RETURNVERBOSE)
```

output
```
-1/20*(-40*I*exp(x)^8+25*exp(x)^9-10*exp(x)^7-80*I*exp(x)^4+10*exp(x)^3-8*
I-25*exp(x))/(exp(x)^2-1)^5-3/8*ln(exp(x)+1)+3/8*ln(exp(x)-1)
```

Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 144 vs. $2(26) = 52$.

Time = 0.11 (sec) , antiderivative size = 144, normalized size of antiderivative = 4.00

$$\int \frac{\coth^6(x)}{i + \sinh(x)}\,dx = -\frac{15\left(e^{(10\,x)} - 5\,e^{(8\,x)} + 10\,e^{(6\,x)} - 10\,e^{(4\,x)} + 5\,e^{(2\,x)} - 1\right)\log\left(e^{x} + 1\right) - 15\left(e^{(10\,x)} - 5\,e^{(8\,x)} + 10\,e^{(6\,x)} - 1}{40\left(e^{(10\,x)} - 5\,e^{(8\,x)} + 10\,e^{(6}}$$

input
```
integrate(coth(x)^6/(I+sinh(x)),x, algorithm="fricas")
```

output
```
-1/40*(15*(e^(10*x) - 5*e^(8*x) + 10*e^(6*x) - 10*e^(4*x) + 5*e^(2*x) - 1)
*log(e^x + 1) - 15*(e^(10*x) - 5*e^(8*x) + 10*e^(6*x) - 10*e^(4*x) + 5*e^(
2*x) - 1)*log(e^x - 1) + 50*e^(9*x) - 80*I*e^(8*x) - 20*e^(7*x) - 160*I*e^
(4*x) + 20*e^(3*x) - 50*e^x - 16*I)/(e^(10*x) - 5*e^(8*x) + 10*e^(6*x) - 1
0*e^(4*x) + 5*e^(2*x) - 1)
```

Sympy [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 100 vs. $2(36) = 72$.

Time = 0.14 (sec) , antiderivative size = 100, normalized size of antiderivative = 2.78

$$\int \frac{\coth^6(x)}{i + \sinh(x)}\,dx = \frac{3\log\left(e^{x} - 1\right)}{8} - \frac{3\log\left(e^{x} + 1\right)}{8} + \frac{-25e^{9x} + 40ie^{8x} + 10e^{7x} + 80ie^{4x} - 10e^{3x} + 25e^{x} + 8i}{20e^{10x} - 100e^{8x} + 200e^{6x} - 200e^{4x} + 100e^{2x} - 20}$$

input
```
integrate(coth(x)**6/(I+sinh(x)),x)
```

output
```
3*log(exp(x) - 1)/8 - 3*log(exp(x) + 1)/8 + (-25*exp(9*x) + 40*I*exp(8*x)
+ 10*exp(7*x) + 80*I*exp(4*x) - 10*exp(3*x) + 25*exp(x) + 8*I)/(20*exp(10*
x) - 100*exp(8*x) + 200*exp(6*x) - 200*exp(4*x) + 100*exp(2*x) - 20)
```

Maxima [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 91 vs. 2(26) = 52.

Time = 0.05 (sec) , antiderivative size = 91, normalized size of antiderivative = 2.53

$$\int \frac{\coth^6(x)}{i+\sinh(x)}\,dx = \frac{25\,e^{(-x)}-10\,e^{(-3\,x)}-80i\,e^{(-4\,x)}+10\,e^{(-7\,x)}-40i\,e^{(-8\,x)}-25\,e^{(-9\,x)}-8i}{20\,\left(5\,e^{(-2\,x)}-10\,e^{(-4\,x)}+10\,e^{(-6\,x)}-5\,e^{(-8\,x)}+e^{(-10\,x)}-1\right)} - \frac{3}{8}\,\log\left(e^{(-x)}+1\right)+\frac{3}{8}\,\log\left(e^{(-x)}-1\right)$$

input
```
integrate(coth(x)^6/(I+sinh(x)),x, algorithm="maxima")
```

output
```
1/20*(25*e^(-x) - 10*e^(-3*x) - 80*I*e^(-4*x) + 10*e^(-7*x) - 40*I*e^(-8*x
) - 25*e^(-9*x) - 8*I)/(5*e^(-2*x) - 10*e^(-4*x) + 10*e^(-6*x) - 5*e^(-8*x
) + e^(-10*x) - 1) - 3/8*log(e^(-x) + 1) + 3/8*log(e^(-x) - 1)
```

Giac [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 62 vs. 2(26) = 52.

Time = 0.12 (sec) , antiderivative size = 62, normalized size of antiderivative = 1.72

$$\int \frac{\coth^6(x)}{i+\sinh(x)}\,dx = -\frac{25\,e^{(9\,x)}-40i\,e^{(8\,x)}-10\,e^{(7\,x)}-80i\,e^{(4\,x)}+10\,e^{(3\,x)}-25\,e^{x}-8i}{20\,\left(e^{(2\,x)}-1\right)^{5}} - \frac{3}{8}\,\log\left(e^{x}+1\right)+\frac{3}{8}\,\log\left(\left|e^{x}-1\right|\right)$$

input
```
integrate(coth(x)^6/(I+sinh(x)),x, algorithm="giac")
```

output
```
-1/20*(25*e^(9*x) - 40*I*e^(8*x) - 10*e^(7*x) - 80*I*e^(4*x) + 10*e^(3*x)
- 25*e^x - 8*I)/(e^(2*x) - 1)^5 - 3/8*log(e^x + 1) + 3/8*log(abs(e^x - 1))
```

Mupad [B] (verification not implemented)

Time = 1.80 (sec) , antiderivative size = 124, normalized size of antiderivative = 3.44

$$\int \frac{\coth^6(x)}{i+\sinh(x)}\,dx = \frac{3\ \ln\left(\frac{3}{4}-\frac{3\,\mathrm{e}^x}{4}\right)}{8} - \frac{3\ \ln\left(\frac{3\,\mathrm{e}^x}{4}+\frac{3}{4}\right)}{8} - \frac{5\,\mathrm{e}^x}{4\,\left(\mathrm{e}^{2\,x}-1\right)} - \frac{9\,\mathrm{e}^x}{2\,{\left(\mathrm{e}^{2\,x}-1\right)}^2} - \frac{6\,\mathrm{e}^x}{{\left(\mathrm{e}^{2\,x}-1\right)}^3} - \frac{4\,\mathrm{e}^x}{{\left(\mathrm{e}^{2\,x}-1\right)}^4} + \frac{2\mathrm{i}}{\mathrm{e}^{2\,x}-1} + \frac{8\mathrm{i}}{{\left(\mathrm{e}^{2\,x}-1\right)}^2} + \frac{16\mathrm{i}}{{\left(\mathrm{e}^{2\,x}-1\right)}^3} + \frac{16\mathrm{i}}{{\left(\mathrm{e}^{2\,x}-1\right)}^4} + \frac{32\mathrm{i}}{5\,{\left(\mathrm{e}^{2\,x}-1\right)}^5}$$

input
```
int(coth(x)^6/(sinh(x) + 1i),x)
```

output
```
(3*log(3/4 - (3*exp(x))/4))/8 - (3*log((3*exp(x))/4 + 3/4))/8 - (5*exp(x))
/(4*(exp(2*x) - 1)) - (9*exp(x))/(2*(exp(2*x) - 1)^2) - (6*exp(x))/(exp(2*
x) - 1)^3 - (4*exp(x))/(exp(2*x) - 1)^4 + 2i/(exp(2*x) - 1) + 8i/(exp(2*x)
 - 1)^2 + 16i/(exp(2*x) - 1)^3 + 16i/(exp(2*x) - 1)^4 + 32i/(5*(exp(2*x) -
 1)^5)
```

Reduce [F]

$$\int \frac{\coth^6(x)}{i+\sinh(x)}\,dx = \int \frac{\coth(x)^6}{\sinh(x)+i}dx$$

input
```
int(coth(x)^6/(I+sinh(x)),x)
```

output
```
int(coth(x)**6/(sinh(x) + i),x)
```

3.218 $\int \frac{\tanh^4(x)}{(i+\sinh(x))^2} \, dx$

Optimal result

Integrand size = 13, antiderivative size = 47

$$\int \frac{\tanh^4(x)}{(i+\sinh(x))^2} \, dx = \frac{2}{3} i \text{sech}^3(x) - \frac{4}{5} i \text{sech}^5(x) + \frac{2}{7} i \text{sech}^7(x) - \frac{\tanh^5(x)}{5} + \frac{2 \tanh^7(x)}{7}$$

output
```
2/3*I*sech(x)^3-4/5*I*sech(x)^5+2/7*I*sech(x)^7-1/5*tanh(x)^5+2/7*tanh(x)^
7
```

Mathematica [B] (verified)

Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 112 vs. 2(47) = 94.

Time = 0.15 (sec) , antiderivative size = 112, normalized size of antiderivative = 2.38

$$\int \frac{\tanh^4(x)}{(i+\sinh(x))^2} \, dx = -\frac{-672i + 1442i \cosh(x) - 1664i \cosh(2x) + 309i \cosh(3x) + 288i \cosh(4x) - 103i \cosh(5x) + 1232 \sinh}{13440 \left(\cosh\left(\frac{x}{2}\right) - i \sinh\left(\frac{x}{2}\right)\right)^7 \left(\cosh\left(\frac{x}{2}\right) + i \sinh\right.}$$

input
```
Integrate[Tanh[x]^4/(I + Sinh[x])^2,x]
```

output
```
-1/13440*(-672*I + (1442*I)*Cosh[x] - (1664*I)*Cosh[2*x] + (309*I)*Cosh[3*
x] + (288*I)*Cosh[4*x] - (103*I)*Cosh[5*x] + 1232*Sinh[x] + 824*Sinh[2*x]
- 1896*Sinh[3*x] + 412*Sinh[4*x] + 72*Sinh[5*x])/((Cosh[x/2] - I*Sinh[x/2]
)^7*(Cosh[x/2] + I*Sinh[x/2])^3)
```

Rubi [A] (verified)

Time = 0.34 (sec) , antiderivative size = 47, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.231$, Rules used = {3042, 3190, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\tanh^4(x)}{(\sinh(x)+i)^2} dx$$
$$\Big\downarrow \text{3042}$$
$$\int \frac{\tan(ix)^4}{(i-i\sin(ix))^2}dx$$
$$\Big\downarrow \text{3190}$$
$$\int \left(-\tanh^4(x)\text{sech}^4(x)-2i\tanh^5(x)\text{sech}^3(x)+\tanh^6(x)\text{sech}^2(x)\right)dx$$
$$\Big\downarrow \text{2009}$$
$$\frac{2\tanh^7(x)}{7}-\frac{\tanh^5(x)}{5}+\frac{2}{7}i\text{sech}^7(x)-\frac{4}{5}i\text{sech}^5(x)+\frac{2}{3}i\text{sech}^3(x)$$

input
```
Int[Tanh[x]^4/(I + Sinh[x])^2,x]
```

output
```
((2*I)/3)*Sech[x]^3 - ((4*I)/5)*Sech[x]^5 + ((2*I)/7)*Sech[x]^7 - Tanh[x]^
5/5 + (2*Tanh[x]^7)/7
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3190
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((g_.)*tan[(e_.) + (f_.)*(x
_)])^(p_.), x_Symbol] :> Simp[a^(2*m)   Int[ExpandIntegrand[(g*Tan[e + f*x]
)^p/Sec[e + f*x]^m, (a*Sec[e + f*x] - b*Tan[e + f*x])^(-m), x], x], x] /; F
reeQ[{a, b, e, f, g, p}, x] && EqQ[a^2 - b^2, 0] && ILtQ[m, 0]
```

Maple [A] (verified)

Time = 18.53 (sec) , antiderivative size = 69, normalized size of antiderivative = 1.47

method	result
risch	$-\frac{2\left(68i\,{\mathrm e}^{3x}-132\,{\mathrm e}^{2x}-36i\,{\mathrm e}^{x}+14\,{\mathrm e}^{4x}+9+84i\,{\mathrm e}^{5x}-140\,{\mathrm e}^{6x}+140i\,{\mathrm e}^{7x}+105\,{\mathrm e}^{8x}\right)}{105\left({\mathrm e}^{x}+i\right)^{7}\left({\mathrm e}^{x}-i\right)^{3}}$
default	$\frac{2i}{\left(\tanh\left(\frac{x}{2}\right)+i\right)^{6}}-\frac{i}{\left(\tanh\left(\frac{x}{2}\right)+i\right)^{4}}-\frac{i}{8\left(\tanh\left(\frac{x}{2}\right)+i\right)^{2}}+\frac{4}{7\left(\tanh\left(\frac{x}{2}\right)+i\right)^{7}}-\frac{12}{5\left(\tanh\left(\frac{x}{2}\right)+i\right)^{5}}-\frac{1}{12\left(\tanh\left(\frac{x}{2}\right)+i\right)^{3}}-\frac{1}{8(\tanh($

input
```
int(tanh(x)^4/(I+sinh(x))^2,x,method=_RETURNVERBOSE)
```

output
```
-2/105*(68*I*exp(x)^3-132*exp(x)^2-36*I*exp(x)+14*exp(x)^4+9+84*I*exp(x)^5
-140*exp(x)^6+140*I*exp(x)^7+105*exp(x)^8)/(exp(x)+I)^7/(exp(x)-I)^3
```

Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 104 vs. 2(31) = 62.

Time = 0.08 (sec) , antiderivative size = 104, normalized size of antiderivative = 2.21

$$\int \frac{\tanh^4(x)}{(i+\sinh(x))^2}\,dx = -\frac{2\left(105\,e^{(8\,x)}+140i\,e^{(7\,x)}-140\,e^{(6\,x)}+84i\,e^{(5\,x)}+14\,e^{(4\,x)}+68i\,e^{(3\,x)}-132\,e^{(2\,x)}-36i\,e^{x}+9\right)}{105\left(e^{(10\,x)}+4i\,e^{(9\,x)}-3\,e^{(8\,x)}+8i\,e^{(7\,x)}-14\,e^{(6\,x)}-14\,e^{(4\,x)}-8i\,e^{(3\,x)}-3\,e^{(2\,x)}-4i\,e^{x}+1\right)}$$

input
```
integrate(tanh(x)^4/(I+sinh(x))^2,x, algorithm="fricas")
```

output
```
-2/105*(105*e^(8*x) + 140*I*e^(7*x) - 140*e^(6*x) + 84*I*e^(5*x) + 14*e^(4
*x) + 68*I*e^(3*x) - 132*e^(2*x) - 36*I*e^x + 9)/(e^(10*x) + 4*I*e^(9*x) -
 3*e^(8*x) + 8*I*e^(7*x) - 14*e^(6*x) - 14*e^(4*x) - 8*I*e^(3*x) - 3*e^(2*
x) - 4*I*e^x + 1)
```

Sympy [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 128 vs. $2(44) = 88$.

Time = 0.13 (sec) , antiderivative size = 128, normalized size of antiderivative = 2.72

$$\int \frac{\tanh^4(x)}{(i+\sinh(x))^2}\,dx = \frac{-210e^{8x}-280ie^{7x}+280e^{6x}-168ie^{5x}-28e^{4x}-136ie^{3x}+264e^{2x}+72ie^{x}-18}{105e^{10x}+420ie^{9x}-315e^{8x}+840ie^{7x}-1470e^{6x}-1470e^{4x}-840ie^{3x}-315e^{2x}-420ie^{x}+105}$$

input
```
integrate(tanh(x)**4/(I+sinh(x))**2,x)
```

output
```
(-210*exp(8*x) - 280*I*exp(7*x) + 280*exp(6*x) - 168*I*exp(5*x) - 28*exp(4
*x) - 136*I*exp(3*x) + 264*exp(2*x) + 72*I*exp(x) - 18)/(105*exp(10*x) + 4
20*I*exp(9*x) - 315*exp(8*x) + 840*I*exp(7*x) - 1470*exp(6*x) - 1470*exp(4
*x) - 840*I*exp(3*x) - 315*exp(2*x) - 420*I*exp(x) + 105)
```

Maxima [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 573 vs. $2(31) = 62$.

Time = 0.04 (sec) , antiderivative size = 573, normalized size of antiderivative = 12.19

$$\int \frac{\tanh^4(x)}{(i + \sinh(x))^2}\, dx = \text{Too large to display}$$

input
```
integrate(tanh(x)^4/(I+sinh(x))^2,x, algorithm="maxima")
```

output
```
72*I*e^(-x)/(420*I*e^(-x) - 315*e^(-2*x) + 840*I*e^(-3*x) - 1470*e^(-4*x)
- 1470*e^(-6*x) - 840*I*e^(-7*x) - 315*e^(-8*x) - 420*I*e^(-9*x) + 105*e^(
-10*x) + 105) - 264*e^(-2*x)/(420*I*e^(-x) - 315*e^(-2*x) + 840*I*e^(-3*x)
 - 1470*e^(-4*x) - 1470*e^(-6*x) - 840*I*e^(-7*x) - 315*e^(-8*x) - 420*I*e
^(-9*x) + 105*e^(-10*x) + 105) - 136*I*e^(-3*x)/(420*I*e^(-x) - 315*e^(-2*
x) + 840*I*e^(-3*x) - 1470*e^(-4*x) - 1470*e^(-6*x) - 840*I*e^(-7*x) - 315
*e^(-8*x) - 420*I*e^(-9*x) + 105*e^(-10*x) + 105) + 28*e^(-4*x)/(420*I*e^(
-x) - 315*e^(-2*x) + 840*I*e^(-3*x) - 1470*e^(-4*x) - 1470*e^(-6*x) - 840*
I*e^(-7*x) - 315*e^(-8*x) - 420*I*e^(-9*x) + 105*e^(-10*x) + 105) - 168*I*
e^(-5*x)/(420*I*e^(-x) - 315*e^(-2*x) + 840*I*e^(-3*x) - 1470*e^(-4*x) - 1
470*e^(-6*x) - 840*I*e^(-7*x) - 315*e^(-8*x) - 420*I*e^(-9*x) + 105*e^(-10
*x) + 105) - 280*e^(-6*x)/(420*I*e^(-x) - 315*e^(-2*x) + 840*I*e^(-3*x) -
1470*e^(-4*x) - 1470*e^(-6*x) - 840*I*e^(-7*x) - 315*e^(-8*x) - 420*I*e^(-
9*x) + 105*e^(-10*x) + 105) - 280*I*e^(-7*x)/(420*I*e^(-x) - 315*e^(-2*x)
+ 840*I*e^(-3*x) - 1470*e^(-4*x) - 1470*e^(-6*x) - 840*I*e^(-7*x) - 315*e^
(-8*x) - 420*I*e^(-9*x) + 105*e^(-10*x) + 105) + 210*e^(-8*x)/(420*I*e^(-x
) - 315*e^(-2*x) + 840*I*e^(-3*x) - 1470*e^(-4*x) - 1470*e^(-6*x) - 840*I*
e^(-7*x) - 315*e^(-8*x) - 420*I*e^(-9*x) + 105*e^(-10*x) + 105) + 18/(420*
I*e^(-x) - 315*e^(-2*x) + 840*I*e^(-3*x) - 1470*e^(-4*x) - 1470*e^(-6*x) -
 840*I*e^(-7*x) - 315*e^(-8*x) - 420*I*e^(-9*x) + 105*e^(-10*x) + 105)
```

Giac [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 65 vs. $2(31) = 62$.

Time = 0.12 (sec) , antiderivative size = 65, normalized size of antiderivative = 1.38

$$\int \frac{\tanh^4(x)}{(i+\sinh(x))^2}\,dx = -\frac{-6i\,e^{(2\,x)} - 9\,e^x + 5i}{24\,(e^x - i)^3} - \frac{210i\,e^{(6\,x)} - 105\,e^{(5\,x)} + 175i\,e^{(4\,x)} - 910\,e^{(3\,x)} - 756i\,e^{(2\,x)} + 427\,e^x + 31i}{840\,(e^x + i)^7}$$

input
```
integrate(tanh(x)^4/(I+sinh(x))^2,x, algorithm="giac")
```

output
```
-1/24*(-6*I*e^(2*x) - 9*e^x + 5*I)/(e^x - I)^3 - 1/840*(210*I*e^(6*x) - 10
5*e^(5*x) + 175*I*e^(4*x) - 910*e^(3*x) - 756*I*e^(2*x) + 427*e^x + 31*I)/
(e^x + I)^7
```

Mupad [B] (verification not implemented)

Time = 5.06 (sec) , antiderivative size = 395, normalized size of antiderivative = 8.40

$$\int \frac{\tanh^4(x)}{(i+\sinh(x))^2}\,dx = \text{Too large to display}$$

input
```
int(tanh(x)^4/(sinh(x) + 1i)^2,x)
```

output
```
1i/(12*(exp(2*x)*3i - exp(3*x) + 3*exp(x) - 1i)) - ((exp(3*x)*5i)/42 - exp
(2*x)/4 + (25*exp(4*x))/168 + (exp(5*x)*1i)/28 - (exp(x)*5i)/84 + 5/168)/(
15*exp(2*x) - exp(3*x)*20i - 15*exp(4*x) + exp(5*x)*6i + exp(6*x) + exp(x)
*6i - 1) - ((exp(x)*1i)/28 + 5/168)/(exp(2*x) + exp(x)*2i - 1) - ((exp(4*x
)*5i)/28 - exp(3*x)/2 - (exp(2*x)*5i)/28 + (5*exp(5*x))/28 + (exp(6*x)*1i)
/28 + (5*exp(x))/28 - 1i/28)/(exp(2*x)*21i + 35*exp(3*x) - exp(4*x)*35i -
21*exp(5*x) + exp(6*x)*7i + exp(7*x) - 7*exp(x) - 1i) - ((exp(2*x)*1i)/28
+ (5*exp(x))/84 + 1i/84)/(exp(2*x)*3i + exp(3*x) - 3*exp(x) - 1i) + 1/(8*(
exp(x)*2i - exp(2*x) + 1)) + 1i/(4*(exp(x) - 1i)) - 1i/(28*(exp(x) + 1i))
- ((5*exp(2*x))/56 + (exp(3*x)*1i)/28 + (exp(x)*1i)/28 - 1/40)/(exp(3*x)*4
i - 6*exp(2*x) + exp(4*x) - exp(x)*4i + 1) - ((exp(2*x)*1i)/14 + (5*exp(3*
x))/42 + (exp(4*x)*1i)/28 - exp(x)/10 - 1i/84)/(exp(4*x)*5i - 10*exp(3*x)
- exp(2*x)*10i + exp(5*x) + 5*exp(x) + 1i)
```

Reduce [F]

$$\int \frac{\tanh^4(x)}{(i+\sinh(x))^2}\,dx = \int \frac{\tanh(x)^4}{\sinh(x)^2+2\sinh(x)\,i-1}dx$$

input
```
int(tanh(x)^4/(I+sinh(x))^2,x)
```

output
```
int(tanh(x)**4/(sinh(x)**2 + 2*sinh(x)*i - 1),x)
```

3.219 $\int \frac{\tanh^3(x)}{(i+\sinh(x))^2} \, dx$

Optimal result

Integrand size = 13, antiderivative size = 66

$$\int \frac{\tanh^3(x)}{(i+\sinh(x))^2} \, dx = -\frac{1}{8} i \arctan(\sinh(x)) - \frac{i}{16(i-\sinh(x))} + \frac{i}{12(i+\sinh(x))^3} - \frac{1}{4(i+\sinh(x))^2} - \frac{3i}{16(i+\sinh(x))}$$

output
```
-1/8*I*arctan(sinh(x))-1/16*I/(I-sinh(x))+1/12*I/(I+sinh(x))^3-1/4/(I+sinh
(x))^2-3/16*I/(I+sinh(x))
```

Mathematica [A] (verified)

Time = 0.09 (sec) , antiderivative size = 52, normalized size of antiderivative = 0.79

$$\int \frac{\tanh^3(x)}{(i+\sinh(x))^2} \, dx = \frac{1}{48} i \left(-6 \arctan(\sinh(x)) - \frac{2(2i + 7\sinh(x) - 6i\sinh^2(x) + 3\sinh^3(x))}{(-i+\sinh(x))(i+\sinh(x))^3} \right)$$

input
```
Integrate[Tanh[x]^3/(I + Sinh[x])^2,x]
```

output
```
(I/48)*(-6*ArcTan[Sinh[x]] - (2*(2*I + 7*Sinh[x] - (6*I)*Sinh[x]^2 + 3*Sin
h[x]^3))/((-I + Sinh[x])*(I + Sinh[x])^3))
```

Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 66, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.462$, Rules used = {3042, 26, 25, 3186, 99, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\tanh^3(x)}{(\sinh(x)+i)^2} \, dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \frac{i\tan(ix)^3}{(i-i\sin(ix))^2}dx$$

$$\Big\downarrow \text{ 26}$$

$$i\int -\frac{\tan(ix)^3}{(1-\sin(ix))^2}dx$$

$$\Big\downarrow \text{ 25}$$

$$-i\int \frac{\tan(ix)^3}{(1-\sin(ix))^2}dx$$

$$\Big\downarrow \text{ 3186}$$

$$-\int \frac{i\sinh^3(x)}{(1-i\sinh(x))^4(i\sinh(x)+1)^2}d(-i\sinh(x))$$

$$\Big\downarrow \text{ 99}$$

$$-\int \left(\frac{1}{16(-i\sinh(x)-1)^2}-\frac{3}{16(1-i\sinh(x))^2}+\frac{1}{2(1-i\sinh(x))^3}-\frac{1}{4(1-i\sinh(x))^4}+\frac{1}{8\left(-\sinh^2(x)-1\right)}\right)$$

$$\Big\downarrow \text{ 2009}$$

$$-\frac{1}{8}i\arctan(\sinh(x))-\frac{3}{16(1-i\sinh(x))}-\frac{1}{16(1+i\sinh(x))}+\frac{1}{4(1-i\sinh(x))^2}-\frac{1}{12(1-i\sinh(x))^3}$$

input
```
Int[Tanh[x]^3/(I + Sinh[x])^2,x]
```

output
```
(-1/8*I)*ArcTan[Sinh[x]] - 1/(12*(1 - I*Sinh[x])^3) + 1/(4*(1 - I*Sinh[x])
^2) - 3/(16*(1 - I*Sinh[x])) - 1/(16*(1 + I*Sinh[x]))
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 99
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (IntegerQ[p] |
| (GtQ[m, 0] && GeQ[n, -1]))
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3186
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*tan[(e_.) + (f_.)*(x_)]^(p
_.), x_Symbol] :> Simp[1/f   Subst[Int[x^p*((a + x)^(m - (p + 1)/2)/(a - x)
^((p + 1)/2)), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && E
qQ[a^2 - b^2, 0] && IntegerQ[(p + 1)/2]
```

Maple [A] (verified)

Time = 10.87 (sec) , antiderivative size = 76, normalized size of antiderivative = 1.15

method	result
risch	$-\frac{i e^{x}\left(-12 i e^{5 x}+3 e^{6 x}+40 i e^{3 x}+19 e^{4 x}-12 i e^{x}-19 e^{2 x}-3\right)}{12\left(e^{x}-i\right)^{2}\left(e^{x}+i\right)^{6}}-\frac{\ln\left(e^{x}-i\right)}{8}+\frac{\ln\left(e^{x}+i\right)}{8}$
default	$\frac{2i}{\left(\tanh\left(\frac{x}{2}\right)+i\right)^{5}}-\frac{2i}{3\left(\tanh\left(\frac{x}{2}\right)+i\right)^{3}}-\frac{i}{8\left(\tanh\left(\frac{x}{2}\right)+i\right)}+\frac{2}{3\left(\tanh\left(\frac{x}{2}\right)+i\right)^{6}}-\frac{2}{\left(\tanh\left(\frac{x}{2}\right)+i\right)^{4}}-\frac{1}{8\left(\tanh\left(\frac{x}{2}\right)+i\right)^{2}}+\frac{\ln\left(\tanh\left(\frac{x}{2}\right.\right.}{8}$

input
```
int(tanh(x)^3/(I+sinh(x))^2,x,method=_RETURNVERBOSE)
```

output
```
-1/12*I*exp(x)*(-12*I*exp(x)^5+3*exp(x)^6+40*I*exp(x)^3+19*exp(x)^4-12*I*e
xp(x)-19*exp(x)^2-3)/(exp(x)-I)^2/(exp(x)+I)^6-1/8*ln(exp(x)-I)+1/8*ln(exp
(x)+I)
```

Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 197 vs. 2(38) = 76.

Time = 0.09 (sec) , antiderivative size = 197, normalized size of antiderivative = 2.98

$$\int \frac{\tanh^{3}(x)}{(i+\sinh(x))^{2}}\,dx = \frac{3\left(e^{(8x)}+4i\,e^{(7x)}-4\,e^{(6x)}+4i\,e^{(5x)}-10\,e^{(4x)}-4i\,e^{(3x)}-4\,e^{(2x)}-4i\,e^{x}+1\right)\log\left(e^{x}+i\right)-3\left(e^{(8x)}+\cdots\right.}{24\left(e^{(8x)}+4i\,e^{(7\cdots}\right.}$$

input
```
integrate(tanh(x)^3/(I+sinh(x))^2,x, algorithm="fricas")
```

output
```
1/24*(3*(e^(8*x) + 4*I*e^(7*x) - 4*e^(6*x) + 4*I*e^(5*x) - 10*e^(4*x) - 4*
I*e^(3*x) - 4*e^(2*x) - 4*I*e^x + 1)*log(e^x + I) - 3*(e^(8*x) + 4*I*e^(7*
x) - 4*e^(6*x) + 4*I*e^(5*x) - 10*e^(4*x) - 4*I*e^(3*x) - 4*e^(2*x) - 4*I*
e^x + 1)*log(e^x - I) - 6*I*e^(7*x) - 24*e^(6*x) - 38*I*e^(5*x) + 80*e^(4*
x) + 38*I*e^(3*x) - 24*e^(2*x) + 6*I*e^x)/(e^(8*x) + 4*I*e^(7*x) - 4*e^(6*
x) + 4*I*e^(5*x) - 10*e^(4*x) - 4*I*e^(3*x) - 4*e^(2*x) - 4*I*e^x + 1)
```

Sympy [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 129 vs. $2(49) = 98$.

Time = 0.14 (sec) , antiderivative size = 129, normalized size of antiderivative = 1.95

$$\int \frac{\tanh^3(x)}{(i+\sinh(x))^2}\,dx = \frac{-3ie^{7x} - 12e^{6x} - 19ie^{5x} + 40e^{4x} + 19ie^{3x} - 12e^{2x} + 3ie^{x}}{12e^{8x} + 48ie^{7x} - 48e^{6x} + 48ie^{5x} - 120e^{4x} - 48ie^{3x} - 48e^{2x} - 48ie^{x} + 12} - \frac{\log(e^x - i)}{8} + \frac{\log(e^x + i)}{8}$$

input
```
integrate(tanh(x)**3/(I+sinh(x))**2,x)
```

output
```
(-3*I*exp(7*x) - 12*exp(6*x) - 19*I*exp(5*x) + 40*exp(4*x) + 19*I*exp(3*x)
 - 12*exp(2*x) + 3*I*exp(x))/(12*exp(8*x) + 48*I*exp(7*x) - 48*exp(6*x) +
48*I*exp(5*x) - 120*exp(4*x) - 48*I*exp(3*x) - 48*exp(2*x) - 48*I*exp(x) +
 12) - log(exp(x) - I)/8 + log(exp(x) + I)/8
```

Maxima [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 115 vs. $2(38) = 76$.

Time = 0.05 (sec) , antiderivative size = 115, normalized size of antiderivative = 1.74

$$\int \frac{\tanh^3(x)}{(i+\sinh(x))^2}\,dx = \frac{-3i\,e^{(-x)} - 12\,e^{(-2\,x)} - 19i\,e^{(-3\,x)} + 40\,e^{(-4\,x)} + 19i\,e^{(-5\,x)} - 12\,e^{(-6\,x)} + 3i\,e^{(-7\,x)}}{48i\,e^{(-x)} - 48\,e^{(-2\,x)} + 48i\,e^{(-3\,x)} - 120\,e^{(-4\,x)} - 48i\,e^{(-5\,x)} - 48\,e^{(-6\,x)} - 48i\,e^{(-7\,x)} + 12\,e^{(-8\,x)} + 12} - \frac{1}{8}\log\left(e^{(-x)} + i\right) + \frac{1}{8}\log\left(e^{(-x)} - i\right)$$

input
```
integrate(tanh(x)^3/(I+sinh(x))^2,x, algorithm="maxima")
```

output
```
(-3*I*e^(-x) - 12*e^(-2*x) - 19*I*e^(-3*x) + 40*e^(-4*x) + 19*I*e^(-5*x) -
 12*e^(-6*x) + 3*I*e^(-7*x))/(48*I*e^(-x) - 48*e^(-2*x) + 48*I*e^(-3*x) -
120*e^(-4*x) - 48*I*e^(-5*x) - 48*e^(-6*x) - 48*I*e^(-7*x) + 12*e^(-8*x) +
 12) - 1/8*log(e^(-x) + I) + 1/8*log(e^(-x) - I)
```

Giac [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 102 vs. $2(38) = 76$.

Time = 0.12 (sec) , antiderivative size = 102, normalized size of antiderivative = 1.55

$$\int \frac{\tanh^3(x)}{(i+\sinh(x))^2}\,dx = \frac{e^{(-x)} - e^x}{16\,(e^{(-x)} - e^x + 2i)} - \frac{11\,\left(e^{(-x)} - e^x\right)^3 - 102i\,\left(e^{(-x)} - e^x\right)^2 - 180\,e^{(-x)} + 180\,e^x + 104i}{96\,(e^{(-x)} - e^x - 2i)^3} + \frac{1}{16}\,\log\left(-e^{(-x)} + e^x + 2i\right) - \frac{1}{16}\,\log\left(-e^{(-x)} + e^x - 2i\right)$$

input
```
integrate(tanh(x)^3/(I+sinh(x))^2,x, algorithm="giac")
```

output
```
1/16*(e^(-x) - e^x)/(e^(-x) - e^x + 2*I) - 1/96*(11*(e^(-x) - e^x)^3 - 102
*I*(e^(-x) - e^x)^2 - 180*e^(-x) + 180*e^x + 104*I)/(e^(-x) - e^x - 2*I)^3
 + 1/16*log(-e^(-x) + e^x + 2*I) - 1/16*log(-e^(-x) + e^x - 2*I)
```

Mupad [B] (verification not implemented)

Time = 2.42 (sec) , antiderivative size = 209, normalized size of antiderivative = 3.17

$$\int \frac{\tanh^3(x)}{(i+\sinh(x))^2}\,dx = \frac{\ln\left(-\frac{1}{4}+\frac{e^x\,1i}{4}\right)}{8} - \frac{\ln\left(\frac{1}{4}+\frac{e^x\,1i}{4}\right)}{8} - \frac{2i}{e^{5x}-10\,e^{3x}+e^{4x}\,5i-e^{2x}\,10i+5\,e^x+1i} - \frac{11}{8\,\left(e^{2x}-1+e^x\,2i\right)} + \frac{3}{e^{4x}-6\,e^{2x}+1+e^{3x}\,4i-e^x\,4i} + \frac{1}{8\,\left(1-e^{2x}+e^x\,2i\right)} + \frac{1i}{8\,\left(e^x-i\right)} - \frac{3i}{8\,\left(e^x+1i\right)} - \frac{2}{3\,\left(15\,e^{2x}-15\,e^{4x}+e^{6x}-1-e^{3x}\,20i+e^{5x}\,6i+e^x\,6i\right)} + \frac{8i}{3\,\left(e^{2x}\,3i+e^{3x}-3\,e^x-i\right)}$$

input
```
int(tanh(x)^3/(sinh(x) + 1i)^2,x)
```

output
```
log((exp(x)*1i)/4 - 1/4)/8 - log((exp(x)*1i)/4 + 1/4)/8 - 2i/(exp(4*x)*5i
 - 10*exp(3*x) - exp(2*x)*10i + exp(5*x) + 5*exp(x) + 1i) - 11/(8*(exp(2*x)
 + exp(x)*2i - 1)) + 3/(exp(3*x)*4i - 6*exp(2*x) + exp(4*x) - exp(x)*4i +
1) + 1/(8*(exp(x)*2i - exp(2*x) + 1)) + 1i/(8*(exp(x) - 1i)) - 3i/(8*(exp(
x) + 1i)) - 2/(3*(15*exp(2*x) - exp(3*x)*20i - 15*exp(4*x) + exp(5*x)*6i +
 exp(6*x) + exp(x)*6i - 1)) + 8i/(3*(exp(2*x)*3i + exp(3*x) - 3*exp(x) - 1
i))
```

Reduce [F]

$$\int \frac{\tanh^3(x)}{(i+\sinh(x))^2}\,dx = \int \frac{\tanh(x)^3}{\sinh(x)^2+2\sinh(x)\,i-1}\,dx$$

input
```
int(tanh(x)^3/(I+sinh(x))^2,x)
```

output
```
int(tanh(x)**3/(sinh(x)**2 + 2*sinh(x)*i - 1),x)
```

3.220 $\int \frac{\tanh^2(x)}{(i+\sinh(x))^2}\,dx$

Optimal result

Integrand size = 13, antiderivative size = 37

$$\int \frac{\tanh^2(x)}{(i+\sinh(x))^2}\,dx = \frac{2}{3}i\mathrm{sech}^3(x) - \frac{2}{5}i\mathrm{sech}^5(x) - \frac{\tanh^3(x)}{3} + \frac{2\tanh^5(x)}{5}$$

output
```
2/3*I*sech(x)^3-2/5*I*sech(x)^5-1/3*tanh(x)^3+2/5*tanh(x)^5
```

Mathematica [B] (verified)

Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 84 vs. 2(37) = 74.

Time = 0.12 (sec) , antiderivative size = 84, normalized size of antiderivative = 2.27

$$\int \frac{\tanh^2(x)}{(i+\sinh(x))^2}\,dx = \frac{80i-55i\cosh(x)-16i\cosh(2x)+11i\cosh(3x)+140\sinh(x)-44\sinh(2x)-4\sinh(3x)}{240\left(\cosh\left(\frac{x}{2}\right)-i\sinh\left(\frac{x}{2}\right)\right)^5\left(\cosh\left(\frac{x}{2}\right)+i\sinh\left(\frac{x}{2}\right)\right)}$$

input
```
Integrate[Tanh[x]^2/(I + Sinh[x])^2,x]
```

output
```
(80*I - (55*I)*Cosh[x] - (16*I)*Cosh[2*x] + (11*I)*Cosh[3*x] + 140*Sinh[x]
 - 44*Sinh[2*x] - 4*Sinh[3*x])/(240*(Cosh[x/2] - I*Sinh[x/2])^5*(Cosh[x/2]
 + I*Sinh[x/2]))
```

Rubi [A] (verified)

Time = 0.35 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.385$, Rules used = {3042, 25, 25, 3190, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\tanh^2(x)}{(\sinh(x)+i)^2} \, dx$$

$$\Big\downarrow \text{3042}$$

$$\int -\frac{\tan(ix)^2}{(i-i\sin(ix))^2}dx$$

$$\Big\downarrow \text{25}$$

$$-\int -\frac{\tan(ix)^2}{(1-\sin(ix))^2}dx$$

$$\Big\downarrow \text{25}$$

$$\int \frac{\tan(ix)^2}{(1-\sin(ix))^2}dx$$

$$\Big\downarrow \text{3190}$$

$$\int \left(-\tanh^2(x)\text{sech}^4(x)-2i\tanh^3(x)\text{sech}^3(x)+\tanh^4(x)\text{sech}^2(x)\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{2\tanh^5(x)}{5}-\frac{\tanh^3(x)}{3}-\frac{2}{5}i\text{sech}^5(x)+\frac{2}{3}i\text{sech}^3(x)$$

input
```
Int[Tanh[x]^2/(I + Sinh[x])^2,x]
```

output
```
((2*I)/3)*Sech[x]^3 - ((2*I)/5)*Sech[x]^5 - Tanh[x]^3/3 + (2*Tanh[x]^5)/5
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3190
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((g_.)*tan[(e_.) + (f_.)*(x
_)])^(p_.), x_Symbol] :> Simp[a^(2*m)   Int[ExpandIntegrand[(g*Tan[e + f*x]
)^p/Sec[e + f*x]^m, (a*Sec[e + f*x] - b*Tan[e + f*x])^(-m), x], x], x] /; F
reeQ[{a, b, e, f, g, p}, x] && EqQ[a^2 - b^2, 0] && ILtQ[m, 0]
```

Maple [A] (verified)

Time = 6.24 (sec) , antiderivative size = 43, normalized size of antiderivative = 1.16

method	result	size
risch	$-\frac{2\left(-20\,{\mathrm e}^{2 x}-4 i {\mathrm e}^{x}+1+20 i {\mathrm e}^{3 x}+15\,{\mathrm e}^{4 x}\right)}{15\left({\mathrm e}^{x}-i\right) \left({\mathrm e}^{x}+i\right)^{5}}$	43
default	$\frac{2 i}{\left(\tanh \left(\frac{x}{2}\right)+i\right)^{4}}-\frac{i}{2 \left(\tanh \left(\frac{x}{2}\right)+i\right)^{2}}+\frac{4}{5 \left(\tanh \left(\frac{x}{2}\right)+i\right)^{5}}-\frac{5}{3 \left(\tanh \left(\frac{x}{2}\right)+i\right)^{3}}-\frac{1}{4 \left(\tanh \left(\frac{x}{2}\right)+i\right)}+\frac{1}{4 \tanh \left(\frac{x}{2}\right)-4 i}$	70

input
```
int(tanh(x)^2/(I+sinh(x))^2,x,method=_RETURNVERBOSE)
```

output
```
-2/15*(-20*exp(x)^2-4*I*exp(x)+1+20*I*exp(x)^3+15*exp(x)^4)/(exp(x)-I)/(ex
p(x)+I)^5
```

Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 56 vs. 2(25) = 50.

Time = 0.08 (sec) , antiderivative size = 56, normalized size of antiderivative = 1.51

$$\int \frac{\tanh^2(x)}{(i+\sinh(x))^2}\,dx = -\frac{2\left(15\,e^{(4\,x)}+20i\,e^{(3\,x)}-20\,e^{(2\,x)}-4i\,e^{x}+1\right)}{15\left(e^{(6\,x)}+4i\,e^{(5\,x)}-5\,e^{(4\,x)}-5\,e^{(2\,x)}-4i\,e^{x}+1\right)}$$

input
```
integrate(tanh(x)^2/(I+sinh(x))^2,x, algorithm="fricas")
```

output
```
-2/15*(15*e^(4*x) + 20*I*e^(3*x) - 20*e^(2*x) - 4*I*e^x + 1)/(e^(6*x) + 4*
I*e^(5*x) - 5*e^(4*x) - 5*e^(2*x) - 4*I*e^x + 1)
```

Sympy [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 66, normalized size of antiderivative = 1.78

$$\int \frac{\tanh^2(x)}{(i+\sinh(x))^2}\,dx = \frac{-30e^{4x}-40ie^{3x}+40e^{2x}+8ie^{x}-2}{15e^{6x}+60ie^{5x}-75e^{4x}-75e^{2x}-60ie^{x}+15}$$

input
```
integrate(tanh(x)**2/(I+sinh(x))**2,x)
```

output
```
(-30*exp(4*x) - 40*I*exp(3*x) + 40*exp(2*x) + 8*I*exp(x) - 2)/(15*exp(6*x)
 + 60*I*exp(5*x) - 75*exp(4*x) - 75*exp(2*x) - 60*I*exp(x) + 15)
```

Maxima [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 197 vs. 2(25) = 50.

Time = 0.05 (sec) , antiderivative size = 197, normalized size of antiderivative = 5.32

$$\int \frac{\tanh^2(x)}{(i+\sinh(x))^2}\,dx = \frac{8i\,e^{(-x)}}{60i\,e^{(-x)}-75\,e^{(-2\,x)}-75\,e^{(-4\,x)}-60i\,e^{(-5\,x)}+15\,e^{(-6\,x)}+15} - \frac{40\,e^{(-2\,x)}}{60i\,e^{(-x)}-75\,e^{(-2\,x)}-75\,e^{(-4\,x)}-60i\,e^{(-5\,x)}+15\,e^{(-6\,x)}+15} - \frac{40i\,e^{(-3\,x)}}{60i\,e^{(-x)}-75\,e^{(-2\,x)}-75\,e^{(-4\,x)}-60i\,e^{(-5\,x)}+15\,e^{(-6\,x)}+15} + \frac{30\,e^{(-4\,x)}}{60i\,e^{(-x)}-75\,e^{(-2\,x)}-75\,e^{(-4\,x)}-60i\,e^{(-5\,x)}+15\,e^{(-6\,x)}+15} + \frac{2}{60i\,e^{(-x)}-75\,e^{(-2\,x)}-75\,e^{(-4\,x)}-60i\,e^{(-5\,x)}+15\,e^{(-6\,x)}+15}$$

input
```
integrate(tanh(x)^2/(I+sinh(x))^2,x, algorithm="maxima")
```

output
```
8*I*e^(-x)/(60*I*e^(-x) - 75*e^(-2*x) - 75*e^(-4*x) - 60*I*e^(-5*x) + 15*e
^(-6*x) + 15) - 40*e^(-2*x)/(60*I*e^(-x) - 75*e^(-2*x) - 75*e^(-4*x) - 60*
I*e^(-5*x) + 15*e^(-6*x) + 15) - 40*I*e^(-3*x)/(60*I*e^(-x) - 75*e^(-2*x)
- 75*e^(-4*x) - 60*I*e^(-5*x) + 15*e^(-6*x) + 15) + 30*e^(-4*x)/(60*I*e^(-
x) - 75*e^(-2*x) - 75*e^(-4*x) - 60*I*e^(-5*x) + 15*e^(-6*x) + 15) + 2/(60
*I*e^(-x) - 75*e^(-2*x) - 75*e^(-4*x) - 60*I*e^(-5*x) + 15*e^(-6*x) + 15)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 41, normalized size of antiderivative = 1.11

$$\int \frac{\tanh^2(x)}{(i+\sinh(x))^2}\,dx = \frac{i}{4\,(e^x-i)} - \frac{15i\,e^{(4\,x)}+30\,e^{(3\,x)}+40i\,e^{(2\,x)}-50\,e^x-7i}{60\,(e^x+i)^5}$$

input
```
integrate(tanh(x)^2/(I+sinh(x))^2,x, algorithm="giac")
```

output
```
1/4*I/(e^x - I) - 1/60*(15*I*e^(4*x) + 30*e^(3*x) + 40*I*e^(2*x) - 50*e^x
- 7*I)/(e^x + I)^5
```

Mupad [B] (verification not implemented)

Time = 0.32 (sec) , antiderivative size = 139, normalized size of antiderivative = 3.76

$$\int \frac{\tanh^2(x)}{(i+\sinh(x))^2}\,dx = \frac{(4\,e^{3\,x}-4\,e^{x})\,\left(2\,e^{4\,x}-\frac{8\,e^{2\,x}}{3}+\frac{2}{15}\right)\,1\mathrm{i}}{{\left(e^{2\,x}+1\right)}^5} - \frac{(e^{4\,x}-6\,e^{2\,x}+1)\,\left(2\,e^{4\,x}-\frac{8\,e^{2\,x}}{3}+\frac{2}{15}\right)}{{\left(e^{2\,x}+1\right)}^5} - \frac{(4\,e^{3\,x}-4\,e^{x})\,\left(\frac{8\,e^{3\,x}}{3}-\frac{8\,e^{x}}{15}\right)}{{\left(e^{2\,x}+1\right)}^5} - \frac{\left(\frac{8\,e^{3\,x}}{3}-\frac{8\,e^{x}}{15}\right)\,(e^{4\,x}-6\,e^{2\,x}+1)\,1\mathrm{i}}{{\left(e^{2\,x}+1\right)}^5}$$

input
```
int(tanh(x)^2/(sinh(x) + 1i)^2,x)
```

output
```
((4*exp(3*x) - 4*exp(x))*(2*exp(4*x) - (8*exp(2*x))/3 + 2/15)*1i)/(exp(2*x
) + 1)^5 - ((exp(4*x) - 6*exp(2*x) + 1)*(2*exp(4*x) - (8*exp(2*x))/3 + 2/1
5))/(exp(2*x) + 1)^5 - ((4*exp(3*x) - 4*exp(x))*((8*exp(3*x))/3 - (8*exp(x
))/15))/(exp(2*x) + 1)^5 - (((8*exp(3*x))/3 - (8*exp(x))/15)*(exp(4*x) - 6
*exp(2*x) + 1)*1i)/(exp(2*x) + 1)^5
```

Reduce [F]

$$\int \frac{\tanh^2(x)}{(i+\sinh(x))^2}\,dx = \int \frac{\tanh\left(x\right)^2}{\sinh\left(x\right)^2+2\sinh\left(x\right)i-1}dx$$

input
```
int(tanh(x)^2/(I+sinh(x))^2,x)
```

output
```
int(tanh(x)**2/(sinh(x)**2 + 2*sinh(x)*i - 1),x)
```

3.221 $\int \frac{\tanh(x)}{(i+\sinh(x))^2}\,dx$

Optimal result

Integrand size = 11, antiderivative size = 36

$$\int \frac{\tanh(x)}{(i+\sinh(x))^2}\,dx = -\frac{1}{4}i\arctan(\sinh(x)) - \frac{1}{4(i+\sinh(x))^2} - \frac{i}{4(i+\sinh(x))}$$

output
```
-1/4*I*arctan(sinh(x))-1/4/(I+sinh(x))^2-1/4*I/(I+sinh(x))
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.81

$$\int \frac{\tanh(x)}{(i+\sinh(x))^2}\,dx = -\frac{i(\sinh(x)+\arctan(\sinh(x))(i+\sinh(x))^2)}{4(i+\sinh(x))^2}$$

input
```
Integrate[Tanh[x]/(I + Sinh[x])^2,x]
```

output
```
((-1/4*I)*(Sinh[x] + ArcTan[Sinh[x]]*(I + Sinh[x])^2))/(I + Sinh[x])^2
```

Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 38, normalized size of antiderivative = 1.06, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.545, Rules used = {3042, 26, 25, 3186, 86, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\tanh(x)}{(\sinh(x)+i)^2}dx$$
$$\Big\downarrow \text{3042}$$
$$\int -\frac{i\tan(ix)}{(i-i\sin(ix))^2}dx$$
$$\Big\downarrow \text{26}$$
$$-i\int -\frac{\tan(ix)}{(1-\sin(ix))^2}dx$$
$$\Big\downarrow \text{25}$$
$$i\int \frac{\tan(ix)}{(1-\sin(ix))^2}dx$$
$$\Big\downarrow \text{3186}$$
$$\int -\frac{i\sinh(x)}{(1-i\sinh(x))^3(1+i\sinh(x))}d(-i\sinh(x))$$
$$\Big\downarrow \text{86}$$
$$\int \left(-\frac{1}{4\left(-\sinh^2(x)-1\right)}+\frac{1}{4(1-i\sinh(x))^2}-\frac{1}{2(1-i\sinh(x))^3}\right)d(-i\sinh(x))$$
$$\Big\downarrow \text{2009}$$
$$-\frac{1}{4}i\arctan(\sinh(x))-\frac{1}{4(1-i\sinh(x))}+\frac{1}{4(1-i\sinh(x))^2}$$

input
```
Int[Tanh[x]/(I + Sinh[x])^2,x]
```

output
```
(-1/4*I)*ArcTan[Sinh[x]] + 1/(4*(1 - I*Sinh[x])^2) - 1/(4*(1 - I*Sinh[x]))
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 86
```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x], x] /;
 FreeQ[{a, b, c, d, e, f, n}, x] && ((ILtQ[n, 0] && ILtQ[p, 0]) || EqQ[p, 1
] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p + 5*(n + 2), 0] || GeQ[n + p
+ 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b, c, d, e, f]))))
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3186
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*tan[(e_.) + (f_.)*(x_)]^(p
_.), x_Symbol] :> Simp[1/f   Subst[Int[x^p*((a + x)^(m - (p + 1)/2)/(a - x)
^((p + 1)/2)), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && E
qQ[a^2 - b^2, 0] && IntegerQ[(p + 1)/2]
```

Maple [A] (verified)

Time = 3.61 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.00

method	result	size
risch	$-\frac{i\left({\mathrm e}^{2 x}-1\right) {\mathrm e}^{x}}{2 \left({\mathrm e}^{x}+i\right)^{4}}+\frac{\ln \left({\mathrm e}^{x}+i\right)}{4}-\frac{\ln \left({\mathrm e}^{x}-i\right)}{4}$	36
default	$\frac{2 i}{\left(\tanh \left(\frac{x}{2}\right)+i\right)^{3}}-\frac{i}{2 \left(\tanh \left(\frac{x}{2}\right)+i\right)}+\frac{1}{\left(\tanh \left(\frac{x}{2}\right)+i\right)^{4}}-\frac{3}{2 \left(\tanh \left(\frac{x}{2}\right)+i\right)^{2}}+\frac{\ln \left(\tanh \left(\frac{x}{2}\right)+i\right)}{4}-\frac{\ln \left(\tanh \left(\frac{x}{2}\right)-i\right)}{4}$	66

input
```
int(tanh(x)/(I+sinh(x))^2,x,method=_RETURNVERBOSE)
```

output
```
-1/2*I*(exp(2*x)-1)*exp(x)/(exp(x)+I)^4+1/4*ln(exp(x)+I)-1/4*ln(exp(x)-I)
```

Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 94 vs. 2(22) = 44.

Time = 0.08 (sec) , antiderivative size = 94, normalized size of antiderivative = 2.61

$$\int \frac{\tanh(x)}{(i+\sinh(x))^2}\,dx = \frac{\left(e^{(4\,x)}+4i\,e^{(3\,x)}-6\,e^{(2\,x)}-4i\,e^{x}+1\right)\log\left(e^{x}+i\right)-\left(e^{(4\,x)}+4i\,e^{(3\,x)}-6\,e^{(2\,x)}-4i\,e^{x}+1\right)\log\left(e^{x}-i\right)-}{4\left(e^{(4\,x)}+4i\,e^{(3\,x)}-6\,e^{(2\,x)}-4i\,e^{x}+1\right)}$$

input
```
integrate(tanh(x)/(I+sinh(x))^2,x, algorithm="fricas")
```

output
```
1/4*((e^(4*x) + 4*I*e^(3*x) - 6*e^(2*x) - 4*I*e^x + 1)*log(e^x + I) - (e^(
4*x) + 4*I*e^(3*x) - 6*e^(2*x) - 4*I*e^x + 1)*log(e^x - I) - 2*I*e^(3*x) +
 2*I*e^x)/(e^(4*x) + 4*I*e^(3*x) - 6*e^(2*x) - 4*I*e^x + 1)
```

Sympy [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 58, normalized size of antiderivative = 1.61

$$\int \frac{\tanh(x)}{(i+\sinh(x))^2}\,dx = \frac{-ie^{3x}+ie^{x}}{2e^{4x}+8ie^{3x}-12e^{2x}-8ie^{x}+2} - \frac{\log\left(e^{x}-i\right)}{4} + \frac{\log\left(e^{x}+i\right)}{4}$$

input
```
integrate(tanh(x)/(I+sinh(x))**2,x)
```

output
```
(-I*exp(3*x) + I*exp(x))/(2*exp(4*x) + 8*I*exp(3*x) - 12*exp(2*x) - 8*I*ex
p(x) + 2) - log(exp(x) - I)/4 + log(exp(x) + I)/4
```

Maxima [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 61 vs. 2(22) = 44.

Time = 0.04 (sec) , antiderivative size = 61, normalized size of antiderivative = 1.69

$$\int \frac{\tanh(x)}{(i+\sinh(x))^2}\,dx = \frac{-i\,e^{(-x)}+i\,e^{(-3\,x)}}{8i\,e^{(-x)}-12\,e^{(-2\,x)}-8i\,e^{(-3\,x)}+2\,e^{(-4\,x)}+2} - \frac{1}{4}\,\log\left(e^{(-x)}+i\right)+\frac{1}{4}\,\log\left(e^{(-x)}-i\right)$$

input
```
integrate(tanh(x)/(I+sinh(x))^2,x, algorithm="maxima")
```

output
```
(-I*e^(-x) + I*e^(-3*x))/(8*I*e^(-x) - 12*e^(-2*x) - 8*I*e^(-3*x) + 2*e^(-
4*x) + 2) - 1/4*log(e^(-x) + I) + 1/4*log(e^(-x) - I)
```

Giac [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 66 vs. 2(22) = 44.

Time = 0.12 (sec) , antiderivative size = 66, normalized size of antiderivative = 1.83

$$\int \frac{\tanh(x)}{(i+\sinh(x))^2}\,dx = -\frac{3\left(e^{(-x)}-e^{x}\right)^2-20i\,e^{(-x)}+20i\,e^{x}-12}{16\left(e^{(-x)}-e^{x}-2i\right)^2} + \frac{1}{8}\,\log\left(-e^{(-x)}+e^{x}+2i\right)-\frac{1}{8}\,\log\left(-e^{(-x)}+e^{x}-2i\right)$$

input
```
integrate(tanh(x)/(I+sinh(x))^2,x, algorithm="giac")
```

output
```
-1/16*(3*(e^(-x) - e^x)^2 - 20*I*e^(-x) + 20*I*e^x - 12)/(e^(-x) - e^x - 2
*I)^2 + 1/8*log(-e^(-x) + e^x + 2*I) - 1/8*log(-e^(-x) + e^x - 2*I)
```

Mupad [B] (verification not implemented)

Time = 1.79 (sec) , antiderivative size = 99, normalized size of antiderivative = 2.75

$$\int \frac{\tanh(x)}{(i+\sinh(x))^2}\,dx = \frac{\ln\left(-\frac{1}{2}+\frac{e^x\,1i}{2}\right)}{4} - \frac{\ln\left(\frac{1}{2}+\frac{e^x\,1i}{2}\right)}{4} - \frac{3}{2\,\left(e^{2\,x}-1+e^x\,2i\right)} + \frac{1}{e^{4\,x}-6\,e^{2\,x}+1+e^{3\,x}\,4i-e^x\,4i} - \frac{1i}{2\,\left(e^x+1i\right)} + \frac{2i}{e^{2\,x}\,3i+e^{3\,x}-3\,e^x-i}$$

input
```
int(tanh(x)/(sinh(x) + 1i)^2,x)
```

output
```
log((exp(x)*1i)/2 - 1/2)/4 - log((exp(x)*1i)/2 + 1/2)/4 - 3/(2*(exp(2*x) +
 exp(x)*2i - 1)) + 1/(exp(3*x)*4i - 6*exp(2*x) + exp(4*x) - exp(x)*4i + 1)
 - 1i/(2*(exp(x) + 1i)) + 2i/(exp(2*x)*3i + exp(3*x) - 3*exp(x) - 1i)
```

Reduce [F]

$$\int \frac{\tanh(x)}{(i+\sinh(x))^2}\,dx = \int \frac{\tanh(x)}{\sinh(x)^2+2\sinh(x)\,i-1}dx$$

input
```
int(tanh(x)/(I+sinh(x))^2,x)
```

output
```
int(tanh(x)/(sinh(x)**2 + 2*sinh(x)*i - 1),x)
```

3.222 $\int \frac{\coth(x)}{(i+\sinh(x))^2} dx$

Optimal result

Integrand size = 11, antiderivative size = 25

$$\int \frac{\coth(x)}{(i+\sinh(x))^2} dx = -\log(\sinh(x)) + \log(i+\sinh(x)) - \frac{i}{i+\sinh(x)}$$

output
```
-ln(sinh(x))+ln(I+sinh(x))-I/(I+sinh(x))
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00

$$\int \frac{\coth(x)}{(i+\sinh(x))^2} dx = -\log(\sinh(x)) + \log(i+\sinh(x)) - \frac{i}{i+\sinh(x)}$$

input
```
Integrate[Coth[x]/(I + Sinh[x])^2,x]
```

output
```
-Log[Sinh[x]] + Log[I + Sinh[x]] - I/(I + Sinh[x])
```

Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.24, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.545$, Rules used = {3042, 26, 25, 3186, 54, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\coth(x)}{(\sinh(x)+i)^2}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{i}{(i-i\sin(ix))^2\tan(ix)}dx$$

$$\Big\downarrow \text{26}$$

$$i\int -\frac{1}{(1-\sin(ix))^2\tan(ix)}dx$$

$$\Big\downarrow \text{25}$$

$$-i\int \frac{1}{(1-\sin(ix))^2\tan(ix)}dx$$

$$\Big\downarrow \text{3186}$$

$$-\int \frac{i\text{csch}(x)}{(1-i\sinh(x))^2}d(-i\sinh(x))$$

$$\Big\downarrow \text{54}$$

$$-\int \left(i\text{csch}(x)+\frac{1}{i\sinh(x)-1}-\frac{1}{(1-i\sinh(x))^2}\right)d(-i\sinh(x))$$

$$\Big\downarrow \text{2009}$$

$$-\frac{1}{1-i\sinh(x)}+\log(1-i\sinh(x))-\log(-i\sinh(x))$$

input
```
Int[Coth[x]/(I + Sinh[x])^2,x]
```

output
```
Log[1 - I*Sinh[x]] - Log[(-I)*Sinh[x]] - (1 - I*Sinh[x])^(-1)
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 54
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m + n + 2, 0])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3186
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*tan[(e_.) + (f_.)*(x_)]^(p
_.), x_Symbol] :> Simp[1/f   Subst[Int[x^p*((a + x)^(m - (p + 1)/2)/(a - x)
^((p + 1)/2)), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && E
qQ[a^2 - b^2, 0] && IntegerQ[(p + 1)/2]
```

Maple [A] (verified)

Time = 2.24 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.24

method	result	size
risch	$-\frac{2i\,\mathrm{e}^{x}}{\left(\mathrm{e}^{x}+i\right)^{2}}-\ln\left(\mathrm{e}^{2x}-1\right)+2\ln\left(\mathrm{e}^{x}+i\right)$	31
default	$-\ln\left(\tanh\left(\frac{x}{2}\right)\right)+\frac{2i}{\tanh\left(\frac{x}{2}\right)+i}+\frac{2}{\left(\tanh\left(\frac{x}{2}\right)+i\right)^{2}}+2\ln\left(\tanh\left(\frac{x}{2}\right)+i\right)$	42

input
```
int(coth(x)/(I+sinh(x))^2,x,method=_RETURNVERBOSE)
```

output
```
-2*I*exp(x)/(exp(x)+I)^2-ln(exp(2*x)-1)+2*ln(exp(x)+I)
```

Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 54 vs. $2(19) = 38$.

Time = 0.09 (sec) , antiderivative size = 54, normalized size of antiderivative = 2.16

$$\int \frac{\coth(x)}{(i+\sinh(x))^2}\,dx = -\frac{\left(e^{(2\,x)}+2i\,e^x-1\right)\log\left(e^{(2\,x)}-1\right)-2\left(e^{(2\,x)}+2i\,e^x-1\right)\log\left(e^x+i\right)+2i\,e^x}{e^{(2\,x)}+2i\,e^x-1}$$

input
```
integrate(coth(x)/(I+sinh(x))^2,x, algorithm="fricas")
```

output
```
-((e^(2*x) + 2*I*e^x - 1)*log(e^(2*x) - 1) - 2*(e^(2*x) + 2*I*e^x - 1)*log
(e^x + I) + 2*I*e^x)/(e^(2*x) + 2*I*e^x - 1)
```

Sympy [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 36 vs. $2(17) = 34$.

Time = 0.10 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.44

$$\int \frac{\coth(x)}{(i+\sinh(x))^2}\,dx = 2\log\left(e^x+i\right)-\log\left(e^{2x}-1\right)-\frac{2ie^x}{e^{2x}+2ie^x-1}$$

input
```
integrate(coth(x)/(I+sinh(x))**2,x)
```

output
```
2*log(exp(x) + I) - log(exp(2*x) - 1) - 2*I*exp(x)/(exp(2*x) + 2*I*exp(x)
- 1)
```

Maxima [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 48 vs. $2(19) = 38$.

Time = 0.04 (sec) , antiderivative size = 48, normalized size of antiderivative = 1.92

$$\int \frac{\coth(x)}{(i+\sinh(x))^2}\,dx = \frac{2i\,e^{(-x)}}{-2i\,e^{(-x)}+e^{(-2\,x)}-1} - \log\left(e^{(-x)}+1\right) + 2\,\log\left(e^{(-x)}-i\right) - \log\left(e^{(-x)}-1\right)$$

input
```
integrate(coth(x)/(I+sinh(x))^2,x, algorithm="maxima")
```

output
```
2*I*e^(-x)/(-2*I*e^(-x) + e^(-2*x) - 1) - log(e^(-x) + 1) + 2*log(e^(-x) -
 I) - log(e^(-x) - 1)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 33, normalized size of antiderivative = 1.32

$$\int \frac{\coth(x)}{(i+\sinh(x))^2}\,dx = -\frac{2i\,e^{x}}{(e^{x}+i)^2} - \log\left(e^{x}+1\right) + 2\,\log\left(e^{x}+i\right) - \log\left(|e^{x}-1|\right)$$

input
```
integrate(coth(x)/(I+sinh(x))^2,x, algorithm="giac")
```

output
```
-2*I*e^x/(e^x + I)^2 - log(e^x + 1) + 2*log(e^x + I) - log(abs(e^x - 1))
```

Mupad [B] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 49, normalized size of antiderivative = 1.96

$$\int \frac{\coth(x)}{(i+\sinh(x))^2}\,dx = 2\,\ln\left(36\,\mathrm{e}^{x}+36\mathrm{i}\right) - \ln\left(\mathrm{e}^{2\,x}\,3\mathrm{i}-3\mathrm{i}\right) - \frac{2}{\mathrm{e}^{2\,x}-1+\mathrm{e}^{x}\,2\mathrm{i}} - \frac{2\mathrm{i}}{\mathrm{e}^{x}+1\mathrm{i}}$$

input
```
int(coth(x)/(sinh(x) + 1i)^2,x)
```

output
```
2*log(36*exp(x) + 36i) - log(exp(2*x)*3i - 3i) - 2/(exp(2*x) + exp(x)*2i -
 1) - 2i/(exp(x) + 1i)
```

Reduce [F]

$$\int \frac{\coth(x)}{(i+\sinh(x))^2}\,dx = \int \frac{\coth(x)}{\sinh(x)^2 + 2\sinh(x)\,i - 1}dx$$

input
```
int(coth(x)/(I+sinh(x))^2,x)
```

output
```
int(coth(x)/(sinh(x)**2 + 2*sinh(x)*i - 1),x)
```

3.223 $\int \frac{\coth^2(x)}{(i+\sinh(x))^2}\,dx$

Optimal result

Integrand size = 13, antiderivative size = 26

$$\int \frac{\coth^2(x)}{(i+\sinh(x))^2}\,dx = 2i\text{arctanh}(\cosh(x)) + \coth(x) + \frac{2i\coth(x)}{i-\text{csch}(x)}$$

output
```
2*I*arctanh(cosh(x))+coth(x)+2*I*coth(x)/(I-csch(x))
```

Mathematica [B] (verified)

Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 66 vs. 2(26) = 52.

Time = 0.15 (sec) , antiderivative size = 66, normalized size of antiderivative = 2.54

$$\int \frac{\coth^2(x)}{(i+\sinh(x))^2}\,dx = \frac{1}{2}\left(\coth\left(\frac{x}{2}\right) + 4i\log\left(\cosh\left(\frac{x}{2}\right)\right) - 4i\log\left(\sinh\left(\frac{x}{2}\right)\right) + \frac{8\sinh\left(\frac{x}{2}\right)}{\cosh\left(\frac{x}{2}\right) - i\sinh\left(\frac{x}{2}\right)} + \tanh\left(\frac{x}{2}\right)\right)$$

input
```
Integrate[Coth[x]^2/(I + Sinh[x])^2,x]
```

output
```
(Coth[x/2] + (4*I)*Log[Cosh[x/2]] - (4*I)*Log[Sinh[x/2]] + (8*Sinh[x/2])/(
Cosh[x/2] - I*Sinh[x/2]) + Tanh[x/2])/2
```

Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.385, Rules used = {3042, 25, 25, 3188, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\coth^2(x)}{(\sinh(x)+i)^2}dx$$
$$\Big\downarrow \text{3042}$$
$$\int -\frac{1}{(i-i\sin(ix))^2\tan(ix)^2}dx$$
$$\Big\downarrow \text{25}$$
$$-\int -\frac{1}{(1-\sin(ix))^2\tan(ix)^2}dx$$
$$\Big\downarrow \text{25}$$
$$\int \frac{1}{(1-\sin(ix))^2\tan(ix)^2}dx$$
$$\Big\downarrow \text{3188}$$
$$\int \left(-\text{csch}^2(x)-2i\text{csch}(x)-\frac{2i}{-\text{csch}(x)+i}+2\right)dx$$
$$\Big\downarrow \text{2009}$$
$$2i\text{arctanh}(\cosh(x))+\coth(x)+\frac{2i\coth(x)}{-\text{csch}(x)+i}$$

input
```
Int[Coth[x]^2/(I + Sinh[x])^2,x]
```

output
```
(2*I)*ArcTanh[Cosh[x]] + Coth[x] + ((2*I)*Coth[x])/(I - Csch[x])
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3188
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*tan[(e_.) + (f_.)*(x_)]^(p_
), x_Symbol] :> Simp[a^p   Int[ExpandIntegrand[Sin[e + f*x]^p*((a + b*Sin[e
 + f*x])^(m - p/2)/(a - b*Sin[e + f*x])^(p/2)), x], x], x] /; FreeQ[{a, b,
e, f}, x] && EqQ[a^2 - b^2, 0] && IntegersQ[m, p/2] && (LtQ[p, 0] || GtQ[m
- p/2, 0])
```

Maple [A] (verified)

Time = 6.35 (sec) , antiderivative size = 35, normalized size of antiderivative = 1.35

method	result	size
default	$\frac{\tanh\left(\frac{x}{2}\right)}{2}-2i\ln\left(\tanh\left(\frac{x}{2}\right)\right)+\frac{1}{2\tanh\left(\frac{x}{2}\right)}+\frac{4}{\tanh\left(\frac{x}{2}\right)+i}$	35
risch	$-\frac{2i\left(i\,\mathrm{e}^{x}+2\,\mathrm{e}^{2x}-3\right)}{\left(\mathrm{e}^{2x}-1\right)\left(\mathrm{e}^{x}+i\right)}-2i\ln\left(\mathrm{e}^{x}-1\right)+2i\ln\left(\mathrm{e}^{x}+1\right)$	49

input
```
int(coth(x)^2/(I+sinh(x))^2,x,method=_RETURNVERBOSE)
```

output
```
1/2*tanh(1/2*x)-2*I*ln(tanh(1/2*x))+1/2/tanh(1/2*x)+4/(tanh(1/2*x)+I)
```

Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 78 vs. 2(18) = 36.

Time = 0.08 (sec) , antiderivative size = 78, normalized size of antiderivative = 3.00

$$\int \frac{\coth^2(x)}{(i+\sinh(x))^2}\,dx = -\frac{2\left(\left(-i\,e^{(3\,x)}+e^{(2\,x)}+i\,e^{x}-1\right)\log\left(e^{x}+1\right)+\left(i\,e^{(3\,x)}-e^{(2\,x)}-i\,e^{x}+1\right)\log\left(e^{x}-1\right)+2i\,e^{(2\,x)}-e^{x}-\right.}{e^{(3\,x)}+i\,e^{(2\,x)}-e^{x}-i}$$

input
```
integrate(coth(x)^2/(I+sinh(x))^2,x, algorithm="fricas")
```

output
```
-2*((-I*e^(3*x) + e^(2*x) + I*e^x - 1)*log(e^x + 1) + (I*e^(3*x) - e^(2*x)
 - I*e^x + 1)*log(e^x - 1) + 2*I*e^(2*x) - e^x - 3*I)/(e^(3*x) + I*e^(2*x)
 - e^x - I)
```

Sympy [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 49 vs. 2(22) = 44.

Time = 0.12 (sec) , antiderivative size = 49, normalized size of antiderivative = 1.88

$$\int \frac{\coth^2(x)}{(i+\sinh(x))^2}\,dx = \frac{-4ie^{2x}+2e^{x}+6i}{e^{3x}+ie^{2x}-e^{x}-i}+2\,\mathrm{RootSum}\left(z^2+1,\left(i\mapsto i\log\left(-ii+e^{x}\right)\right)\right)$$

input
```
integrate(coth(x)**2/(I+sinh(x))**2,x)
```

output
```
(-4*I*exp(2*x) + 2*exp(x) + 6*I)/(exp(3*x) + I*exp(2*x) - exp(x) - I) + 2*
RootSum(_z**2 + 1, Lambda(_i, _i*log(-_i*I + exp(x))))
```

Maxima [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 53 vs. 2(18) = 36.

Time = 0.04 (sec) , antiderivative size = 53, normalized size of antiderivative = 2.04

$$\int \frac{\coth^2(x)}{(i+\sinh(x))^2}\,dx = \frac{2\left(e^{(-x)}+2i\,e^{(-2\,x)}-3i\right)}{e^{(-x)}+i\,e^{(-2\,x)}-e^{(-3\,x)}-i}+2i\,\log\left(e^{(-x)}+1\right)-2i\,\log\left(e^{(-x)}-1\right)$$

input
```
integrate(coth(x)^2/(I+sinh(x))^2,x, algorithm="maxima")
```

output
```
2*(e^(-x) + 2*I*e^(-2*x) - 3*I)/(e^(-x) + I*e^(-2*x) - e^(-3*x) - I) + 2*I
*log(e^(-x) + 1) - 2*I*log(e^(-x) - 1)
```

Giac [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 48 vs. 2(18) = 36.

Time = 0.13 (sec) , antiderivative size = 48, normalized size of antiderivative = 1.85

$$\int \frac{\coth^2(x)}{(i+\sinh(x))^2}\,dx = -\frac{2\left(2i\,e^{(2\,x)}-e^{x}-3i\right)}{e^{(3\,x)}+i\,e^{(2\,x)}-e^{x}-i}+2i\,\log\left(e^{x}+1\right)-2i\,\log\left(|e^{x}-1|\right)$$

input
```
integrate(coth(x)^2/(I+sinh(x))^2,x, algorithm="giac")
```

output
```
-2*(2*I*e^(2*x) - e^x - 3*I)/(e^(3*x) + I*e^(2*x) - e^x - I) + 2*I*log(e^x
 + 1) - 2*I*log(abs(e^x - 1))
```

Mupad [B] (verification not implemented)

Time = 1.63 (sec) , antiderivative size = 60, normalized size of antiderivative = 2.31

$$\int \frac{\coth^2(x)}{(i+\sinh(x))^2}\,dx = -\ln\left(e^x\,4\mathrm{i}-4\mathrm{i}\right)\,2\mathrm{i}+\ln\left(e^x\,4\mathrm{i}+4\mathrm{i}\right)\,2\mathrm{i}+\frac{2\,e^x-e^{2\,x}\,4\mathrm{i}+6\mathrm{i}}{e^{2\,x}\,1\mathrm{i}+e^{3\,x}-e^x-\mathrm{i}}$$

input
```
int(coth(x)^2/(sinh(x) + 1i)^2,x)
```

output
```
log(exp(x)*4i + 4i)*2i - log(exp(x)*4i - 4i)*2i + (2*exp(x) - exp(2*x)*4i
+ 6i)/(exp(2*x)*1i + exp(3*x) - exp(x) - 1i)
```

Reduce [F]

$$\int \frac{\coth^2(x)}{(i+\sinh(x))^2}\,dx = \int \frac{\coth(x)^2}{\sinh(x)^2+2\sinh(x)\,i-1}dx$$

input
```
int(coth(x)^2/(I+sinh(x))^2,x)
```

output
```
int(coth(x)**2/(sinh(x)**2 + 2*sinh(x)*i - 1),x)
```

3.224 $\int \frac{\coth^3(x)}{(i+\sinh(x))^2} dx$

Optimal result

Integrand size = 13, antiderivative size = 29

$$\int \frac{\coth^3(x)}{(i+\sinh(x))^2} dx = 2i\text{csch}(x) + \frac{\text{csch}^2(x)}{2} + 2\log(\sinh(x)) - 2\log(i+\sinh(x))$$

output
```
2*I*csch(x)+1/2*csch(x)^2+2*ln(sinh(x))-2*ln(I+sinh(x))
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.00

$$\int \frac{\coth^3(x)}{(i+\sinh(x))^2} dx = 2i\text{csch}(x) + \frac{\text{csch}^2(x)}{2} + 2\log(\sinh(x)) - 2\log(i+\sinh(x))$$

input
```
Integrate[Coth[x]^3/(I + Sinh[x])^2,x]
```

output
```
(2*I)*Csch[x] + Csch[x]^2/2 + 2*Log[Sinh[x]] - 2*Log[I + Sinh[x]]
```

Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 35, normalized size of antiderivative = 1.21, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.462, Rules used = {3042, 26, 25, 3186, 86, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\coth^3(x)}{(\sinh(x)+i)^2}dx$$

$$\Big\downarrow \text{3042}$$

$$\int -\frac{i}{(i-i\sin(ix))^2\tan(ix)^3}dx$$

$$\Big\downarrow \text{26}$$

$$-i\int -\frac{1}{(1-\sin(ix))^2\tan(ix)^3}dx$$

$$\Big\downarrow \text{25}$$

$$i\int \frac{1}{(1-\sin(ix))^2\tan(ix)^3}dx$$

$$\Big\downarrow \text{3186}$$

$$\int -\frac{i(1+i\sinh(x))\text{csch}^3(x)}{1-i\sinh(x)}d(-i\sinh(x))$$

$$\Big\downarrow \text{86}$$

$$\int \left(-\frac{2}{1-i\sinh(x)}-i\text{csch}^3(x)+2\text{csch}^2(x)+2i\text{csch}(x)\right)d(-i\sinh(x))$$

$$\Big\downarrow \text{2009}$$

$$\frac{\text{csch}^2(x)}{2}+2i\text{csch}(x)-2\log(1-i\sinh(x))+2\log(-i\sinh(x))$$

input
```
Int[Coth[x]^3/(I + Sinh[x])^2,x]
```

output
```
(2*I)*Csch[x] + Csch[x]^2/2 - 2*Log[1 - I*Sinh[x]] + 2*Log[(-I)*Sinh[x]]
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 86
```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x], x] /;
 FreeQ[{a, b, c, d, e, f, n}, x] && ((ILtQ[n, 0] && ILtQ[p, 0]) || EqQ[p, 1
] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p + 5*(n + 2), 0] || GeQ[n + p
+ 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b, c, d, e, f]))))
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3186
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*tan[(e_.) + (f_.)*(x_)]^(p
_.), x_Symbol] :> Simp[1/f   Subst[Int[x^p*((a + x)^(m - (p + 1)/2)/(a - x)
^((p + 1)/2)), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && E
qQ[a^2 - b^2, 0] && IntegerQ[(p + 1)/2]
```

Maple [A] (verified)

Time = 12.18 (sec) , antiderivative size = 45, normalized size of antiderivative = 1.55

method	result	size
risch	$\frac{2 i \,{\mathrm e}^{x} \left(2 \,{\mathrm e}^{2 x}-2-i {\mathrm e}^{x}\right)}{\left({\mathrm e}^{2 x}-1\right)^{2}}+2 \ln \left({\mathrm e}^{2 x}-1\right)-4 \ln \left({\mathrm e}^{x}+i\right)$	45
default	$-i \tanh \left(\frac{x}{2}\right)+\frac{\tanh \left(\frac{x}{2}\right)^{2}}{8}+\frac{i}{\tanh \left(\frac{x}{2}\right)}+\frac{1}{8 \tanh \left(\frac{x}{2}\right)^{2}}+2 \ln \left(\tanh \left(\frac{x}{2}\right)\right)-4 \ln \left(\tanh \left(\frac{x}{2}\right)+i\right)$	51

input
```
int(coth(x)^3/(I+sinh(x))^2,x,method=_RETURNVERBOSE)
```

output
```
2*I*exp(x)*(2*exp(2*x)-2-I*exp(x))/(exp(2*x)-1)^2+2*ln(exp(2*x)-1)-4*ln(ex
p(x)+I)
```

Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 70 vs. $2(23) = 46$.

Time = 0.10 (sec) , antiderivative size = 70, normalized size of antiderivative = 2.41

$$\int \frac{\coth^3(x)}{(i+\sinh(x))^2}\,dx = \frac{2\left(\left(e^{(4x)}-2e^{(2x)}+1\right)\log\left(e^{(2x)}-1\right)-2\left(e^{(4x)}-2e^{(2x)}+1\right)\log\left(e^{x}+i\right)+2i\,e^{(3x)}+e^{(2x)}-2i\,e^{x}\right)}{e^{(4x)}-2e^{(2x)}+1}$$

input
```
integrate(coth(x)^3/(I+sinh(x))^2,x, algorithm="fricas")
```

output
```
2*((e^(4*x) - 2*e^(2*x) + 1)*log(e^(2*x) - 1) - 2*(e^(4*x) - 2*e^(2*x) + 1
)*log(e^x + I) + 2*I*e^(3*x) + e^(2*x) - 2*I*e^x)/(e^(4*x) - 2*e^(2*x) + 1
)
```

Sympy [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 53, normalized size of antiderivative = 1.83

$$\int \frac{\coth^3(x)}{(i+\sinh(x))^2}\,dx = \frac{4ie^{3x}+2e^{2x}-4ie^{x}}{e^{4x}-2e^{2x}+1}-4\log\left(e^{x}+i\right)+2\log\left(e^{2x}-1\right)$$

input
```
integrate(coth(x)**3/(I+sinh(x))**2,x)
```

output
```
(4*I*exp(3*x) + 2*exp(2*x) - 4*I*exp(x))/(exp(4*x) - 2*exp(2*x) + 1) - 4*l
og(exp(x) + I) + 2*log(exp(2*x) - 1)
```

Maxima [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 63 vs. 2(23) = 46.

Time = 0.04 (sec) , antiderivative size = 63, normalized size of antiderivative = 2.17

$$\int \frac{\coth^3(x)}{(i+\sinh(x))^2}\,dx = -\frac{2\left(2i\,e^{(-x)}+e^{(-2\,x)}-2i\,e^{(-3\,x)}\right)}{2\,e^{(-2\,x)}-e^{(-4\,x)}-1}+2\,\log\left(e^{(-x)}+1\right) - 4\,\log\left(e^{(-x)}-i\right)+2\,\log\left(e^{(-x)}-1\right)$$

input
```
integrate(coth(x)^3/(I+sinh(x))^2,x, algorithm="maxima")
```

output
```
-2*(2*I*e^(-x) + e^(-2*x) - 2*I*e^(-3*x))/(2*e^(-2*x) - e^(-4*x) - 1) + 2*
log(e^(-x) + 1) - 4*log(e^(-x) - I) + 2*log(e^(-x) - 1)
```

Giac [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 54 vs. 2(23) = 46.

Time = 0.12 (sec) , antiderivative size = 54, normalized size of antiderivative = 1.86

$$\int \frac{\coth^3(x)}{(i+\sinh(x))^2}\,dx = -\frac{2\left(-2i\,e^{(3\,x)}-e^{(2\,x)}+2i\,e^{x}\right)}{(e^x+1)^2(e^x-1)^2}+2\,\log\left(e^x+1\right) - 4\,\log\left(e^x+i\right)+2\,\log\left(|e^x-1|\right)$$

input
```
integrate(coth(x)^3/(I+sinh(x))^2,x, algorithm="giac")
```

output
```
-2*(-2*I*e^(3*x) - e^(2*x) + 2*I*e^x)/((e^x + 1)^2*(e^x - 1)^2) + 2*log(e^
x + 1) - 4*log(e^x + I) + 2*log(abs(e^x - 1))
```

Mupad [B] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 56, normalized size of antiderivative = 1.93

$$\int \frac{\coth^3(x)}{(i+\sinh(x))^2}\,dx = \frac{2}{e^{4x}-2e^{2x}+1} + 2\ln\left(-e^{2x}\,6i+6i\right) - 4\ln\left(144\,e^{x}+144i\right) + \frac{2+e^{x}\,4i}{e^{2x}-1}$$

input
```
int(coth(x)^3/(sinh(x) + 1i)^2,x)
```

output
```
2*log(6i - exp(2*x)*6i) - 4*log(144*exp(x) + 144i) + 2/(exp(4*x) - 2*exp(2
*x) + 1) + (exp(x)*4i + 2)/(exp(2*x) - 1)
```

Reduce [F]

$$\int \frac{\coth^3(x)}{(i+\sinh(x))^2}\,dx = \int \frac{\coth(x)^3}{\sinh(x)^2+2\sinh(x)\,i-1}dx$$

input
```
int(coth(x)^3/(I+sinh(x))^2,x)
```

output
```
int(coth(x)**3/(sinh(x)**2 + 2*sinh(x)*i - 1),x)
```

3.225 $\int \frac{\coth^4(x)}{(i+\sinh(x))^2} \, dx$

Optimal result

Integrand size = 13, antiderivative size = 28

$$\int \frac{\coth^4(x)}{(i+\sinh(x))^2} \, dx = -i\text{arctanh}(\cosh(x)) - 2\coth(x) + \frac{\coth^3(x)}{3} + i\coth(x)\text{csch}(x)$$

output
```
-I*arctanh(cosh(x))-2*coth(x)+1/3*coth(x)^3+I*coth(x)*csch(x)
```

Mathematica [B] (verified)

Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 107 vs. 2(28) = 56.

Time = 0.08 (sec) , antiderivative size = 107, normalized size of antiderivative = 3.82

$$\begin{aligned}\int \frac{\coth^4(x)}{(i+\sinh(x))^2} \, dx = & -\frac{5}{6}\coth\left(\frac{x}{2}\right) + \frac{1}{4}i\text{csch}^2\left(\frac{x}{2}\right) + \frac{1}{24}\coth\left(\frac{x}{2}\right)\text{csch}^2\left(\frac{x}{2}\right) \\ & - i\log\left(\cosh\left(\frac{x}{2}\right)\right) + i\log\left(\sinh\left(\frac{x}{2}\right)\right) + \frac{1}{4}i\text{sech}^2\left(\frac{x}{2}\right) \\ & - \frac{5}{6}\tanh\left(\frac{x}{2}\right) - \frac{1}{24}\text{sech}^2\left(\frac{x}{2}\right)\tanh\left(\frac{x}{2}\right)\end{aligned}$$

input
```
Integrate[Coth[x]^4/(I + Sinh[x])^2,x]
```

output
```
(-5*Coth[x/2])/6 + (I/4)*Csch[x/2]^2 + (Coth[x/2]*Csch[x/2]^2)/24 - I*Log[
Cosh[x/2]] + I*Log[Sinh[x/2]] + (I/4)*Sech[x/2]^2 - (5*Tanh[x/2])/6 - (Sec
h[x/2]^2*Tanh[x/2])/24
```

Rubi [A] (verified)

Time = 0.31 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.385$, Rules used = {3042, 3187, 3042, 3236, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\coth^4(x)}{(\sinh(x)+i)^2}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{(i-i\sin(ix))^2\tan(ix)^4}dx$$

$$\Big\downarrow \text{3187}$$

$$\int (-\sinh(x)+i)^2\text{csch}^4(x)dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{(i\sin(ix)+i)^2}{\sin(ix)^4}dx$$

$$\Big\downarrow \text{3236}$$

$$\int \left(-\text{csch}^4(x)-2i\text{csch}^3(x)+\text{csch}^2(x)\right)dx$$

$$\Big\downarrow \text{2009}$$

$$-i\text{arctanh}(\cosh(x))+\frac{\coth^3(x)}{3}-2\coth(x)+i\coth(x)\text{csch}(x)$$

input
```
Int[Coth[x]^4/(I + Sinh[x])^2,x]
```

output
```
(-I)*ArcTanh[Cosh[x]] - 2*Coth[x] + Coth[x]^3/3 + I*Coth[x]*Csch[x]
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3187
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*tan[(e_.) + (f_.)*(x_)]^(p
_), x_Symbol] :> Simp[a^p   Int[Sin[e + f*x]^p/(a - b*Sin[e + f*x])^m, x],
x] /; FreeQ[{a, b, e, f}, x] && EqQ[a^2 - b^2, 0] && IntegersQ[m, p] && EqQ
[p, 2*m]
```

rule 3236
```
Int[((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(
x_)])^(m_.), x_Symbol] :> Int[ExpandTrig[(a + b*sin[e + f*x])^m*(d*sin[e +
f*x])^n, x], x] /; FreeQ[{a, b, d, e, f, n}, x] && EqQ[a^2 - b^2, 0] && IGt
Q[m, 0] && RationalQ[n]
```

Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 55 vs. $2(24) = 48$.

Time = 19.58 (sec) , antiderivative size = 56, normalized size of antiderivative = 2.00

method	result	size
risch	$\frac{2i(3i\,e^{4x}+3\,e^{5x}-12i\,e^{2x}+5i-3\,e^{x})}{3(e^{2x}-1)^3}+i\ln\left(e^{x}-1\right)-i\ln\left(e^{x}+1\right)$	56
default	$-\frac{7\tanh\left(\frac{x}{2}\right)}{8}+\frac{\tanh\left(\frac{x}{2}\right)^3}{24}-\frac{i\tanh\left(\frac{x}{2}\right)^2}{4}+i\ln\left(\tanh\left(\frac{x}{2}\right)\right)+\frac{i}{4\tanh\left(\frac{x}{2}\right)^2}+\frac{1}{24\tanh\left(\frac{x}{2}\right)^3}-\frac{7}{8\tanh\left(\frac{x}{2}\right)}$	58

input
```
int(coth(x)^4/(I+sinh(x))^2,x,method=_RETURNVERBOSE)
```

output
```
2/3*I*(3*I*exp(x)^4+3*exp(x)^5-12*I*exp(x)^2+5*I-3*exp(x))/(exp(x)^2-1)^3+
I*ln(exp(x)-1)-I*ln(exp(x)+1)
```

Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 100 vs. 2(22) = 44.

Time = 0.11 (sec) , antiderivative size = 100, normalized size of antiderivative = 3.57

$$\int \frac{\coth^4(x)}{(i+\sinh(x))^2}\,dx = -\frac{3\left(i\,e^{(6\,x)}-3i\,e^{(4\,x)}+3i\,e^{(2\,x)}-i\right)\log\left(e^x+1\right)+3\left(-i\,e^{(6\,x)}+3i\,e^{(4\,x)}-3i\,e^{(2\,x)}+i\right)\log\left(e^x-1\right)-6i\,e}{3\left(e^{(6\,x)}-3\,e^{(4\,x)}+3\,e^{(2\,x)}-1\right)}$$

input
```
integrate(coth(x)^4/(I+sinh(x))^2,x, algorithm="fricas")
```

output
```
-1/3*(3*(I*e^(6*x) - 3*I*e^(4*x) + 3*I*e^(2*x) - I)*log(e^x + 1) + 3*(-I*e
^(6*x) + 3*I*e^(4*x) - 3*I*e^(2*x) + I)*log(e^x - 1) - 6*I*e^(5*x) + 6*e^(
4*x) - 24*e^(2*x) + 6*I*e^x + 10)/(e^(6*x) - 3*e^(4*x) + 3*e^(2*x) - 1)
```

Sympy [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 66 vs. 2(26) = 52.

Time = 0.13 (sec) , antiderivative size = 66, normalized size of antiderivative = 2.36

$$\int \frac{\coth^4(x)}{(i+\sinh(x))^2}\,dx = \operatorname{RootSum}\left(z^2+1,\left(i\mapsto i\log\left(ii+e^x\right)\right)\right) + \frac{6ie^{5x}-6e^{4x}+24e^{2x}-6ie^x-10}{3e^{6x}-9e^{4x}+9e^{2x}-3}$$

input
```
integrate(coth(x)**4/(I+sinh(x))**2,x)
```

output
```
RootSum(_z**2 + 1, Lambda(_i, _i*log(_i*I + exp(x)))) + (6*I*exp(5*x) - 6*
exp(4*x) + 24*exp(2*x) - 6*I*exp(x) - 10)/(3*exp(6*x) - 9*exp(4*x) + 9*exp
(2*x) - 3)
```

Maxima [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 67 vs. $2(22) = 44$.

Time = 0.05 (sec) , antiderivative size = 67, normalized size of antiderivative = 2.39

$$\int \frac{\coth^4(x)}{(i+\sinh(x))^2}\,dx = -\frac{2\left(3i\,e^{(-x)} + 12\,e^{(-2\,x)} - 3\,e^{(-4\,x)} - 3i\,e^{(-5\,x)} - 5\right)}{3\left(3\,e^{(-2\,x)} - 3\,e^{(-4\,x)} + e^{(-6\,x)} - 1\right)} - i\,\log\left(e^{(-x)} + 1\right) + i\,\log\left(e^{(-x)} - 1\right)$$

input
```
integrate(coth(x)^4/(I+sinh(x))^2,x, algorithm="maxima")
```

output
```
-2/3*(3*I*e^(-x) + 12*e^(-2*x) - 3*e^(-4*x) - 3*I*e^(-5*x) - 5)/(3*e^(-2*x
) - 3*e^(-4*x) + e^(-6*x) - 1) - I*log(e^(-x) + 1) + I*log(e^(-x) - 1)
```

Giac [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 50 vs. $2(22) = 44$.

Time = 0.12 (sec) , antiderivative size = 50, normalized size of antiderivative = 1.79

$$\int \frac{\coth^4(x)}{(i+\sinh(x))^2}\,dx = -\frac{2\left(-3i\,e^{(5\,x)} + 3\,e^{(4\,x)} - 12\,e^{(2\,x)} + 3i\,e^{x} + 5\right)}{3\left(e^{(2\,x)} - 1\right)^3} - i\,\log\left(e^{x} + 1\right) + i\,\log\left(|e^{x} - 1|\right)$$

input
```
integrate(coth(x)^4/(I+sinh(x))^2,x, algorithm="giac")
```

output
```
-2/3*(-3*I*e^(5*x) + 3*e^(4*x) - 12*e^(2*x) + 3*I*e^x + 5)/(e^(2*x) - 1)^3
 - I*log(e^x + 1) + I*log(abs(e^x - 1))
```

Mupad [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 111, normalized size of antiderivative = 3.96

$$\int \frac{\coth^4(x)}{(i+\sinh(x))^2}\,dx = -\ln\left(-e^x\,2i-2i\right)\,1i+\ln\left(-e^x\,2i+2i\right)\,1i - \frac{\frac{2\,e^{4\,x}}{3}-4\,e^{2\,x}+\frac{2}{3}-\frac{e^{3\,x}\,8i}{3}+\frac{e^x\,8i}{3}}{3\,e^{2\,x}-3\,e^{4\,x}+e^{6\,x}-1}+\frac{\frac{4}{3}+\frac{e^x\,4i}{3}}{e^{4\,x}-2\,e^{2\,x}+1}+\frac{-\frac{4}{3}+e^x\,2i}{e^{2\,x}-1}$$

input
```
int(coth(x)^4/(sinh(x) + 1i)^2,x)
```

output
```
log(2i - exp(x)*2i)*1i - log(- exp(x)*2i - 2i)*1i - ((2*exp(4*x))/3 - (exp
(3*x)*8i)/3 - 4*exp(2*x) + (exp(x)*8i)/3 + 2/3)/(3*exp(2*x) - 3*exp(4*x) +
 exp(6*x) - 1) + ((exp(x)*4i)/3 + 4/3)/(exp(4*x) - 2*exp(2*x) + 1) + (exp(
x)*2i - 4/3)/(exp(2*x) - 1)
```

Reduce [F]

$$\int \frac{\coth^4(x)}{(i+\sinh(x))^2}\,dx = \int \frac{\coth(x)^4}{\sinh(x)^2+2\sinh(x)\,i-1}dx$$

input
```
int(coth(x)^4/(I+sinh(x))^2,x)
```

output
```
int(coth(x)**4/(sinh(x)**2 + 2*sinh(x)*i - 1),x)
```

3.226 $\int \frac{\coth^5(x)}{(i+\sinh(x))^2}\,dx$

Optimal result

Integrand size = 13, antiderivative size = 27

$$\int \frac{\coth^5(x)}{(i+\sinh(x))^2}\,dx = -\frac{1}{2}\text{csch}^2(x) + \frac{2}{3}i\text{csch}^3(x) + \frac{\text{csch}^4(x)}{4}$$

output
```
-1/2*csch(x)^2+2/3*I*csch(x)^3+1/4*csch(x)^4
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.00

$$\int \frac{\coth^5(x)}{(i+\sinh(x))^2}\,dx = -\frac{1}{2}\text{csch}^2(x) + \frac{2}{3}i\text{csch}^3(x) + \frac{\text{csch}^4(x)}{4}$$

input
```
Integrate[Coth[x]^5/(I + Sinh[x])^2,x]
```

output
```
-1/2*Csch[x]^2 + ((2*I)/3)*Csch[x]^3 + Csch[x]^4/4
```

Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.462, Rules used = {3042, 26, 25, 3186, 53, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\coth^5(x)}{(\sinh(x)+i)^2}dx$$
$$\Big\downarrow \text{ 3042}$$
$$\int \frac{i}{(i-i\sin(ix))^2\tan(ix)^5}dx$$
$$\Big\downarrow \text{ 26}$$
$$i\int -\frac{1}{(1-\sin(ix))^2\tan(ix)^5}dx$$
$$\Big\downarrow \text{ 25}$$
$$-i\int \frac{1}{(1-\sin(ix))^2\tan(ix)^5}dx$$
$$\Big\downarrow \text{ 3186}$$
$$-\int i\text{csch}^5(x)(i\sinh(x)+1)^2d(-i\sinh(x))$$
$$\Big\downarrow \text{ 53}$$
$$-\int \left(i\text{csch}^5(x)-2\text{csch}^4(x)-i\text{csch}^3(x)\right)d(-i\sinh(x))$$
$$\Big\downarrow \text{ 2009}$$
$$\frac{\text{csch}^4(x)}{4}+\frac{2}{3}i\text{csch}^3(x)-\frac{\text{csch}^2(x)}{2}$$

input
```
Int[Coth[x]^5/(I + Sinh[x])^2,x]
```

output
```
-1/2*Csch[x]^2 + ((2*I)/3)*Csch[x]^3 + Csch[x]^4/4
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 53
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && LeQ[7*m + 4*n + 4, 0])
|| LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3186
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*tan[(e_.) + (f_.)*(x_)]^(p
_.), x_Symbol] :> Simp[1/f   Subst[Int[x^p*((a + x)^(m - (p + 1)/2)/(a - x)
^((p + 1)/2)), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && E
qQ[a^2 - b^2, 0] && IntegerQ[(p + 1)/2]
```

Maple [A] (verified)

Time = 31.44 (sec) , antiderivative size = 41, normalized size of antiderivative = 1.52

method	result	siz
risch	$-\frac{2\,e^{2x}\left(-8 i\,e^{3x}+3\,e^{4x}+8 i\,e^{x}-12\,e^{2x}+3\right)}{3\left(e^{2x}-1\right)^{4}}$	41
default	$\frac{i\tanh\left(\frac{x}{2}\right)}{4}+\frac{\tanh\left(\frac{x}{2}\right)^{4}}{64}-\frac{i\tanh\left(\frac{x}{2}\right)^{3}}{12}-\frac{3\tanh\left(\frac{x}{2}\right)^{2}}{16}-\frac{3}{16\tanh\left(\frac{x}{2}\right)^{2}}+\frac{i}{12\tanh\left(\frac{x}{2}\right)^{3}}+\frac{1}{64\tanh\left(\frac{x}{2}\right)^{4}}-\frac{i}{4\tanh\left(\frac{x}{2}\right)}$	68

input
```
int(coth(x)^5/(I+sinh(x))^2,x,method=_RETURNVERBOSE)
```

output
```
-2/3*exp(x)^2*(-8*I*exp(x)^3+3*exp(x)^4+8*I*exp(x)-12*exp(x)^2+3)/(exp(x)^
2-1)^4
```

Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 59 vs. $2(19) = 38$.

Time = 0.07 (sec) , antiderivative size = 59, normalized size of antiderivative = 2.19

$$\int \frac{\coth^5(x)}{(i+\sinh(x))^2}\,dx = -\frac{2\left(3\,e^{(6\,x)} - 8i\,e^{(5\,x)} - 12\,e^{(4\,x)} + 8i\,e^{(3\,x)} + 3\,e^{(2\,x)}\right)}{3\left(e^{(8\,x)} - 4\,e^{(6\,x)} + 6\,e^{(4\,x)} - 4\,e^{(2\,x)} + 1\right)}$$

input
```
integrate(coth(x)^5/(I+sinh(x))^2,x, algorithm="fricas")
```

output
```
-2/3*(3*e^(6*x) - 8*I*e^(5*x) - 12*e^(4*x) + 8*I*e^(3*x) + 3*e^(2*x))/(e^(
8*x) - 4*e^(6*x) + 6*e^(4*x) - 4*e^(2*x) + 1)
```

Sympy [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 65 vs. $2(22) = 44$.

Time = 0.11 (sec) , antiderivative size = 65, normalized size of antiderivative = 2.41

$$\int \frac{\coth^5(x)}{(i+\sinh(x))^2}\,dx = \frac{-6e^{6x} + 16ie^{5x} + 24e^{4x} - 16ie^{3x} - 6e^{2x}}{3e^{8x} - 12e^{6x} + 18e^{4x} - 12e^{2x} + 3}$$

input
```
integrate(coth(x)**5/(I+sinh(x))**2,x)
```

output
```
(-6*exp(6*x) + 16*I*exp(5*x) + 24*exp(4*x) - 16*I*exp(3*x) - 6*exp(2*x))/(
3*exp(8*x) - 12*exp(6*x) + 18*exp(4*x) - 12*exp(2*x) + 3)
```

Maxima [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 171 vs. $2(19) = 38$.

Time = 0.04 (sec) , antiderivative size = 171, normalized size of antiderivative = 6.33

$$\int \frac{\coth^5(x)}{(i+\sinh(x))^2}\,dx = \frac{2\,e^{(-2\,x)}}{4\,e^{(-2\,x)} - 6\,e^{(-4\,x)} + 4\,e^{(-6\,x)} - e^{(-8\,x)} - 1} - \frac{16i\,e^{(-3\,x)}}{3\,(4\,e^{(-2\,x)} - 6\,e^{(-4\,x)} + 4\,e^{(-6\,x)} - e^{(-8\,x)} - 1)} - \frac{8\,e^{(-4\,x)}}{4\,e^{(-2\,x)} - 6\,e^{(-4\,x)} + 4\,e^{(-6\,x)} - e^{(-8\,x)} - 1} + \frac{16i\,e^{(-5\,x)}}{3\,(4\,e^{(-2\,x)} - 6\,e^{(-4\,x)} + 4\,e^{(-6\,x)} - e^{(-8\,x)} - 1)} + \frac{2\,e^{(-6\,x)}}{4\,e^{(-2\,x)} - 6\,e^{(-4\,x)} + 4\,e^{(-6\,x)} - e^{(-8\,x)} - 1}$$

input
```
integrate(coth(x)^5/(I+sinh(x))^2,x, algorithm="maxima")
```

output
```
2*e^(-2*x)/(4*e^(-2*x) - 6*e^(-4*x) + 4*e^(-6*x) - e^(-8*x) - 1) - 16/3*I*
e^(-3*x)/(4*e^(-2*x) - 6*e^(-4*x) + 4*e^(-6*x) - e^(-8*x) - 1) - 8*e^(-4*x
)/(4*e^(-2*x) - 6*e^(-4*x) + 4*e^(-6*x) - e^(-8*x) - 1) + 16/3*I*e^(-5*x)/
(4*e^(-2*x) - 6*e^(-4*x) + 4*e^(-6*x) - e^(-8*x) - 1) + 2*e^(-6*x)/(4*e^(-
2*x) - 6*e^(-4*x) + 4*e^(-6*x) - e^(-8*x) - 1)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 38, normalized size of antiderivative = 1.41

$$\int \frac{\coth^5(x)}{(i+\sinh(x))^2}\,dx = -\frac{2\left(3\left(e^{(-x)} - e^{x}\right)^2 + 8i\,e^{(-x)} - 8i\,e^{x} - 6\right)}{3\left(e^{(-x)} - e^{x}\right)^4}$$

input
```
integrate(coth(x)^5/(I+sinh(x))^2,x, algorithm="giac")
```

output
```
-2/3*(3*(e^(-x) - e^x)^2 + 8*I*e^(-x) - 8*I*e^x - 6)/(e^(-x) - e^x)^4
```

Mupad [B] (verification not implemented)

Time = 1.50 (sec) , antiderivative size = 40, normalized size of antiderivative = 1.48

$$\int \frac{\coth^5(x)}{(i+\sinh(x))^2}\,dx = -\frac{2\,\mathrm{e}^{2\,x}\,\left(3\,\mathrm{e}^{4\,x}-12\,\mathrm{e}^{2\,x}+3-\mathrm{e}^{3\,x}\,8\mathrm{i}+\mathrm{e}^{x}\,8\mathrm{i}\right)}{3\,{\left(\mathrm{e}^{2\,x}-1\right)}^4}$$

input
```
int(coth(x)^5/(sinh(x) + 1i)^2,x)
```

output
```
-(2*exp(2*x)*(3*exp(4*x) - exp(3*x)*8i - 12*exp(2*x) + exp(x)*8i + 3))/(3*
(exp(2*x) - 1)^4)
```

Reduce [F]

$$\int \frac{\coth^5(x)}{(i+\sinh(x))^2}\,dx = \int \frac{\coth\left(x\right)^5}{\sinh\left(x\right)^2+2\sinh\left(x\right)i-1}dx$$

input
```
int(coth(x)^5/(I+sinh(x))^2,x)
```

output
```
int(coth(x)**5/(sinh(x)**2 + 2*sinh(x)*i - 1),x)
```

3.227 $\int \frac{\coth^6(x)}{(i+\sinh(x))^2} dx$

Optimal result

Integrand size = 13, antiderivative size = 48

$$\int \frac{\coth^6(x)}{(i+\sinh(x))^2} dx = -\frac{1}{4}i\text{arctanh}(\cosh(x)) - \frac{2\coth^3(x)}{3} + \frac{\coth^5(x)}{5} + \frac{1}{4}i\coth(x)\text{csch}(x) + \frac{1}{2}i\coth(x)\text{csch}^3(x)$$

output
```
-1/4*I*arctanh(cosh(x))-2/3*coth(x)^3+1/5*coth(x)^5+1/4*I*coth(x)*csch(x)+
1/2*I*coth(x)*csch(x)^3
```

Mathematica [B] (verified)

Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 175 vs. $2(48) = 96$.

Time = 0.10 (sec) , antiderivative size = 175, normalized size of antiderivative = 3.65

$$\int \frac{\coth^6(x)}{(i+\sinh(x))^2}\,dx = -\frac{7}{30}\coth\left(\frac{x}{2}\right)+\frac{1}{16}i\operatorname{csch}^2\left(\frac{x}{2}\right)-\frac{19}{480}\coth\left(\frac{x}{2}\right)\operatorname{csch}^2\left(\frac{x}{2}\right)+\frac{1}{32}i\operatorname{csch}^4\left(\frac{x}{2}\right)+\frac{1}{160}\coth\left(\frac{x}{2}\right)\operatorname{csch}^4\left(\frac{x}{2}\right)-\frac{1}{4}i\log\left(\cosh\left(\frac{x}{2}\right)\right)+\frac{1}{4}i\log\left(\sinh\left(\frac{x}{2}\right)\right)+\frac{1}{16}i\operatorname{sech}^2\left(\frac{x}{2}\right)-\frac{1}{32}i\operatorname{sech}^4\left(\frac{x}{2}\right)-\frac{7}{30}\tanh\left(\frac{x}{2}\right)+\frac{19}{480}\operatorname{sech}^2\left(\frac{x}{2}\right)\tanh\left(\frac{x}{2}\right)+\frac{1}{160}\operatorname{sech}^4\left(\frac{x}{2}\right)\tanh\left(\frac{x}{2}\right)$$

input
```
Integrate[Coth[x]^6/(I + Sinh[x])^2,x]
```

output
```
(-7*Coth[x/2])/30 + (I/16)*Csch[x/2]^2 - (19*Coth[x/2]*Csch[x/2]^2)/480 +
(I/32)*Csch[x/2]^4 + (Coth[x/2]*Csch[x/2]^4)/160 - (I/4)*Log[Cosh[x/2]] +
(I/4)*Log[Sinh[x/2]] + (I/16)*Sech[x/2]^2 - (I/32)*Sech[x/2]^4 - (7*Tanh[x
/2])/30 + (19*Sech[x/2]^2*Tanh[x/2])/480 + (Sech[x/2]^4*Tanh[x/2])/160
```

Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 48, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.385$, Rules used = {3042, 25, 25, 3188, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\coth^6(x)}{(\sinh(x)+i)^2}\,dx$$

$$\Big\downarrow \; 3042$$

$$\int -\frac{1}{(i-i\sin(ix))^2\tan(ix)^6}dx$$
$$\Big\downarrow \text{ 25}$$
$$-\int -\frac{1}{(1-\sin(ix))^2\tan(ix)^6}dx$$
$$\Big\downarrow \text{ 25}$$
$$\int \frac{1}{(1-\sin(ix))^2\tan(ix)^6}dx$$
$$\Big\downarrow \text{ 3188}$$
$$\int \left(-\text{csch}^6(x)-2i\text{csch}^5(x)-2i\text{csch}^3(x)+\text{csch}^2(x)\right)dx$$
$$\Big\downarrow \text{ 2009}$$
$$-\frac{1}{4}i\text{arctanh}(\cosh(x))+\frac{\coth^5(x)}{5}-\frac{2\coth^3(x)}{3}+\frac{1}{2}i\coth(x)\text{csch}^3(x)+\frac{1}{4}i\coth(x)\text{csch}(x)$$

input
```
Int[Coth[x]^6/(I + Sinh[x])^2,x]
```

output
```
(-1/4*I)*ArcTanh[Cosh[x]] - (2*Coth[x]^3)/3 + Coth[x]^5/5 + (I/4)*Coth[x]*
Csch[x] + (I/2)*Coth[x]*Csch[x]^3
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3188
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*tan[(e_.) + (f_.)*(x_)]^(p_
), x_Symbol] :> Simp[a^p   Int[ExpandIntegrand[Sin[e + f*x]^p*((a + b*Sin[e
 + f*x])^(m - p/2)/(a - b*Sin[e + f*x])^(p/2)), x], x], x] /; FreeQ[{a, b,
e, f}, x] && EqQ[a^2 - b^2, 0] && IntegersQ[m, p/2] && (LtQ[p, 0] || GtQ[m
- p/2, 0])
```

Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 73 vs. 2(35) = 70.

Time = 59.10 (sec) , antiderivative size = 74, normalized size of antiderivative = 1.54

method	result
default	$-\frac{3\tanh\left(\frac{x}{2}\right)}{16}+\frac{\tanh\left(\frac{x}{2}\right)^{5}}{160}-\frac{i\tanh\left(\frac{x}{2}\right)^{4}}{32}-\frac{5\tanh\left(\frac{x}{2}\right)^{3}}{96}+\frac{i\ln\left(\tanh\left(\frac{x}{2}\right)\right)}{4}-\frac{5}{96\tanh\left(\frac{x}{2}\right)^{3}}+\frac{1}{160\tanh\left(\frac{x}{2}\right)^{5}}+\frac{i}{32\tanh\left(\frac{x}{2}\right)}$
risch	$\frac{i\left(60i\,e^{8x}+15\,e^{9x}-240i\,e^{6x}+90\,e^{7x}+40i\,e^{4x}-80i\,e^{2x}-90\,e^{3x}+28i-15\,e^{x}\right)}{30\left(e^{2x}-1\right)^{5}}+\frac{i\ln\left(e^{x}-1\right)}{4}-\frac{i\ln\left(e^{x}+1\right)}{4}$

input
```
int(coth(x)^6/(I+sinh(x))^2,x,method=_RETURNVERBOSE)
```

output
```
-3/16*tanh(1/2*x)+1/160*tanh(1/2*x)^5-1/32*I*tanh(1/2*x)^4-5/96*tanh(1/2*x
)^3+1/4*I*ln(tanh(1/2*x))-5/96/tanh(1/2*x)^3+1/160/tanh(1/2*x)^5+1/32*I/ta
nh(1/2*x)^4-3/16/tanh(1/2*x)
```

Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 160 vs. 2(32) = 64.

Time = 0.10 (sec) , antiderivative size = 160, normalized size of antiderivative = 3.33

$$\int \frac{\coth^6(x)}{(i+\sinh(x))^2}\,dx = -\frac{15\left(i\,e^{(10\,x)}-5i\,e^{(8\,x)}+10i\,e^{(6\,x)}-10i\,e^{(4\,x)}+5i\,e^{(2\,x)}-i\right)\log\left(e^{x}+1\right)+15\left(-i\,e^{(10\,x)}+5i\,e^{(8\,x)}-10i}{60\left(e^{(10\,x)}}$$

input
```
integrate(coth(x)^6/(I+sinh(x))^2,x, algorithm="fricas")
```

output
```
-1/60*(15*(I*e^(10*x) - 5*I*e^(8*x) + 10*I*e^(6*x) - 10*I*e^(4*x) + 5*I*e^
(2*x) - I)*log(e^x + 1) + 15*(-I*e^(10*x) + 5*I*e^(8*x) - 10*I*e^(6*x) + 1
0*I*e^(4*x) - 5*I*e^(2*x) + I)*log(e^x - 1) - 30*I*e^(9*x) + 120*e^(8*x) -
 180*I*e^(7*x) - 480*e^(6*x) + 80*e^(4*x) + 180*I*e^(3*x) - 160*e^(2*x) +
30*I*e^x + 56)/(e^(10*x) - 5*e^(8*x) + 10*e^(6*x) - 10*e^(4*x) + 5*e^(2*x)
 - 1)
```

Sympy [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 114 vs. $2(44) = 88$.

Time = 0.18 (sec) , antiderivative size = 114, normalized size of antiderivative = 2.38

$$\int \frac{\coth^6(x)}{(i+\sinh(x))^2} dx$$
$$= \mathrm{RootSum}\left(16z^2+1, \left(i \mapsto i\log\left(4ii+e^x\right)\right)\right) + \frac{15ie^{9x}-60e^{8x}+90ie^{7x}+240e^{6x}-40e^{4x}-90ie^{3x}+80e^{2x}-15ie^x-28}{30e^{10x}-150e^{8x}+300e^{6x}-300e^{4x}+150e^{2x}-30}$$

input
```
integrate(coth(x)**6/(I+sinh(x))**2,x)
```

output
```
RootSum(16*_z**2 + 1, Lambda(_i, _i*log(4*_i*I + exp(x)))) + (15*I*exp(9*x
) - 60*exp(8*x) + 90*I*exp(7*x) + 240*exp(6*x) - 40*exp(4*x) - 90*I*exp(3*
x) + 80*exp(2*x) - 15*I*exp(x) - 28)/(30*exp(10*x) - 150*exp(8*x) + 300*ex
p(6*x) - 300*exp(4*x) + 150*exp(2*x) - 30)
```

Maxima [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 103 vs. $2(32) = 64$.

Time = 0.04 (sec) , antiderivative size = 103, normalized size of antiderivative = 2.15

$$\int \frac{\coth^6(x)}{(i+\sinh(x))^2}\,dx = \frac{-15i\,e^{(-x)} - 80\,e^{(-2\,x)} - 90i\,e^{(-3\,x)} + 40\,e^{(-4\,x)} - 240\,e^{(-6\,x)} + 90i\,e^{(-7\,x)} + 60\,e^{(-8\,x)} + 15i\,e^{(-9\,x)} + 28}{30\left(5\,e^{(-2\,x)} - 10\,e^{(-4\,x)} + 10\,e^{(-6\,x)} - 5\,e^{(-8\,x)} + e^{(-10\,x)} - 1\right)} - \frac{1}{4}i\,\log\left(e^{(-x)} + 1\right) + \frac{1}{4}i\,\log\left(e^{(-x)} - 1\right)$$

input
```
integrate(coth(x)^6/(I+sinh(x))^2,x, algorithm="maxima")
```

output
```
1/30*(-15*I*e^(-x) - 80*e^(-2*x) - 90*I*e^(-3*x) + 40*e^(-4*x) - 240*e^(-6
*x) + 90*I*e^(-7*x) + 60*e^(-8*x) + 15*I*e^(-9*x) + 28)/(5*e^(-2*x) - 10*e
^(-4*x) + 10*e^(-6*x) - 5*e^(-8*x) + e^(-10*x) - 1) - 1/4*I*log(e^(-x) + 1
) + 1/4*I*log(e^(-x) - 1)
```

Giac [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 74 vs. $2(32) = 64$.

Time = 0.13 (sec) , antiderivative size = 74, normalized size of antiderivative = 1.54

$$\int \frac{\coth^6(x)}{(i+\sinh(x))^2}\,dx = -\frac{-15i\,e^{(9\,x)} + 60\,e^{(8\,x)} - 90i\,e^{(7\,x)} - 240\,e^{(6\,x)} + 40\,e^{(4\,x)} + 90i\,e^{(3\,x)} - 80\,e^{(2\,x)} + 15i\,e^{x} + 28}{30\left(e^{(2\,x)} - 1\right)^5} - \frac{1}{4}i\,\log\left(e^{x} + 1\right) + \frac{1}{4}i\,\log\left(|e^{x} - 1|\right)$$

input
```
integrate(coth(x)^6/(I+sinh(x))^2,x, algorithm="giac")
```

output
```
-1/30*(-15*I*e^(9*x) + 60*e^(8*x) - 90*I*e^(7*x) - 240*e^(6*x) + 40*e^(4*x
) + 90*I*e^(3*x) - 80*e^(2*x) + 15*I*e^x + 28)/(e^(2*x) - 1)^5 - 1/4*I*log
(e^x + 1) + 1/4*I*log(abs(e^x - 1))
```

Mupad [B] (verification not implemented)

Time = 1.70 (sec) , antiderivative size = 246, normalized size of antiderivative = 5.12

$$\int \frac{\coth^6(x)}{(i+\sinh(x))^2}\,dx = -\frac{80\,e^{4\,x}-160\,e^{2\,x}-480\,e^{6\,x}+120\,e^{8\,x}+56-\ln\left(-\frac{e^x\,1i}{2}-\frac{1}{2}i\right)\,15i+\ln\left(-\frac{e^x\,1i}{2}+\frac{1}{2}i\right)\,15i+e^{3\,x}\,180i-e^{7}$$

input
```
int(coth(x)^6/(sinh(x) + 1i)^2,x)
```

output
```
-(log(1i/2 - (exp(x)*1i)/2)*15i - log(- (exp(x)*1i)/2 - 1i/2)*15i - 160*ex
p(2*x) + exp(3*x)*180i + 80*exp(4*x) - 480*exp(6*x) - exp(7*x)*180i + 120*
exp(8*x) - exp(9*x)*30i + exp(x)*30i + log(- (exp(x)*1i)/2 - 1i/2)*exp(2*x
)*75i - log(1i/2 - (exp(x)*1i)/2)*exp(2*x)*75i - log(- (exp(x)*1i)/2 - 1i/
2)*exp(4*x)*150i + log(1i/2 - (exp(x)*1i)/2)*exp(4*x)*150i + log(- (exp(x)
*1i)/2 - 1i/2)*exp(6*x)*150i - log(1i/2 - (exp(x)*1i)/2)*exp(6*x)*150i - l
og(- (exp(x)*1i)/2 - 1i/2)*exp(8*x)*75i + log(1i/2 - (exp(x)*1i)/2)*exp(8*
x)*75i + log(- (exp(x)*1i)/2 - 1i/2)*exp(10*x)*15i - log(1i/2 - (exp(x)*1i
)/2)*exp(10*x)*15i + 56)/(60*(exp(2*x) - 1)^5)
```

Reduce [F]

$$\int \frac{\coth^6(x)}{(i+\sinh(x))^2}\,dx = \int \frac{\coth(x)^6}{\sinh(x)^2+2\sinh(x)\,i-1}dx$$

input
```
int(coth(x)^6/(I+sinh(x))^2,x)
```

output
```
int(coth(x)**6/(sinh(x)**2 + 2*sinh(x)*i - 1),x)
```

3.228 $\int \frac{\tanh^4(x)}{a+b\sinh(x)}\,dx$

Optimal result

Integrand size = 13, antiderivative size = 124

$$\int \frac{\tanh^4(x)}{a+b\sinh(x)}\,dx = -\frac{2a^4\operatorname{arctanh}\left(\frac{b-a\tanh\left(\frac{x}{2}\right)}{\sqrt{a^2+b^2}}\right)}{(a^2+b^2)^{5/2}} - \frac{a^2b\operatorname{sech}(x)}{(a^2+b^2)^2} - \frac{b\operatorname{sech}(x)}{a^2+b^2} + \frac{b\operatorname{sech}^3(x)}{3(a^2+b^2)} - \frac{a^3\tanh(x)}{(a^2+b^2)^2} - \frac{a\tanh^3(x)}{3(a^2+b^2)}$$

output

```
-2*a^4*arctanh((b-a*tanh(1/2*x))/(a^2+b^2)^(1/2))/(a^2+b^2)^(5/2)-a^2*b*se
ch(x)/(a^2+b^2)^2-b*sech(x)/(a^2+b^2)+b*sech(x)^3/(3*a^2+3*b^2)-a^3*tanh(x
)/(a^2+b^2)^2-a*tanh(x)^3/(3*a^2+3*b^2)
```

Mathematica [A] (verified)

Time = 0.28 (sec) , antiderivative size = 108, normalized size of antiderivative = 0.87

$$\int \frac{\tanh^4(x)}{a+b\sinh(x)}\,dx = \frac{\frac{6a^4\arctan\left(\frac{b-a\tanh\left(\frac{x}{2}\right)}{\sqrt{-a^2-b^2}}\right)}{\sqrt{-a^2-b^2}} - 3b(2a^2+b^2)\operatorname{sech}(x) + (a^2+b^2)\operatorname{sech}^3(x)(b+a\sinh(x)) - a(4a^2+b^2)\tanh(x)}{3(a^2+b^2)^2}$$

input
```
Integrate[Tanh[x]^4/(a + b*Sinh[x]),x]
```

output
```
((6*a^4*ArcTan[(b - a*Tanh[x/2])/Sqrt[-a^2 - b^2]])/Sqrt[-a^2 - b^2] - 3*b
*(2*a^2 + b^2)*Sech[x] + (a^2 + b^2)*Sech[x]^3*(b + a*Sinh[x]) - a*(4*a^2
+ b^2)*Tanh[x])/(3*(a^2 + b^2)^2)
```

Rubi [A] (verified)

Time = 0.83 (sec) , antiderivative size = 129, normalized size of antiderivative = 1.04, number of steps used = 23, number of rules used = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 1.692$, Rules used = {3042, 3206, 25, 26, 3042, 25, 26, 3086, 2009, 3087, 15, 3206, 26, 3042, 26, 3086, 24, 3139, 1083, 219, 4254, 24}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\tanh^4(x)}{a+b\sinh(x)}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\tan(ix)^4}{a-ib\sin(ix)}dx$$

$$\Big\downarrow \text{3206}$$

$$-\frac{a^2\int -\frac{\tanh^2(x)}{a+b\sinh(x)}dx}{a^2+b^2}+\frac{a\int -\text{sech}^2(x)\tanh^2(x)dx}{a^2+b^2}+\frac{ib\int -i\text{sech}(x)\tanh^3(x)dx}{a^2+b^2}$$

$$\Big\downarrow \text{25}$$

$$\frac{a^2\int \frac{\tanh^2(x)}{a+b\sinh(x)}dx}{a^2+b^2}-\frac{a\int \text{sech}^2(x)\tanh^2(x)dx}{a^2+b^2}+\frac{ib\int -i\text{sech}(x)\tanh^3(x)dx}{a^2+b^2}$$

$$\Big\downarrow \text{26}$$

$$\frac{a^2\int \frac{\tanh^2(x)}{a+b\sinh(x)}dx}{a^2+b^2}-\frac{a\int \text{sech}^2(x)\tanh^2(x)dx}{a^2+b^2}+\frac{b\int \text{sech}(x)\tanh^3(x)dx}{a^2+b^2}$$

$$\Big\downarrow \text{3042}$$

$$\frac{a^2\int -\frac{\tan(ix)^2}{a-ib\sin(ix)}dx}{a^2+b^2}-\frac{a\int -\sec(ix)^2\tan(ix)^2dx}{a^2+b^2}+\frac{b\int i\sec(ix)\tan(ix)^3dx}{a^2+b^2}$$

$$\Big\downarrow \; 25$$

$$-\frac{a^2\int \frac{\tan(ix)^2}{a-ib\sin(ix)}dx}{a^2+b^2}+\frac{a\int \sec(ix)^2\tan(ix)^2dx}{a^2+b^2}+\frac{b\int i\sec(ix)\tan(ix)^3dx}{a^2+b^2}$$

$$\Big\downarrow \; 26$$

$$-\frac{a^2\int \frac{\tan(ix)^2}{a-ib\sin(ix)}dx}{a^2+b^2}+\frac{a\int \sec(ix)^2\tan(ix)^2dx}{a^2+b^2}+\frac{ib\int \sec(ix)\tan(ix)^3dx}{a^2+b^2}$$

$$\Big\downarrow \; 3086$$

$$\frac{b\int \left(\text{sech}^2(x)-1\right)d\text{sech}(x)}{a^2+b^2}-\frac{a^2\int \frac{\tan(ix)^2}{a-ib\sin(ix)}dx}{a^2+b^2}+\frac{a\int \sec(ix)^2\tan(ix)^2dx}{a^2+b^2}$$

$$\Big\downarrow \; 2009$$

$$-\frac{a^2\int \frac{\tan(ix)^2}{a-ib\sin(ix)}dx}{a^2+b^2}+\frac{a\int \sec(ix)^2\tan(ix)^2dx}{a^2+b^2}+\frac{b\left(\frac{\text{sech}^3(x)}{3}-\text{sech}(x)\right)}{a^2+b^2}$$

$$\Big\downarrow \; 3087$$

$$-\frac{ia\int -\tanh^2(x)d(i\tanh(x))}{a^2+b^2}-\frac{a^2\int \frac{\tan(ix)^2}{a-ib\sin(ix)}dx}{a^2+b^2}+\frac{b\left(\frac{\text{sech}^3(x)}{3}-\text{sech}(x)\right)}{a^2+b^2}$$

$$\Big\downarrow \; 15$$

$$-\frac{a^2\int \frac{\tan(ix)^2}{a-ib\sin(ix)}dx}{a^2+b^2}-\frac{a\tanh^3(x)}{3(a^2+b^2)}+\frac{b\left(\frac{\text{sech}^3(x)}{3}-\text{sech}(x)\right)}{a^2+b^2}$$

$$\Big\downarrow \; 3206$$

$$-\frac{a^2\left(-\frac{a^2\int \frac{1}{a+b\sinh(x)}dx}{a^2+b^2}+\frac{a\int \text{sech}^2(x)dx}{a^2+b^2}+\frac{ib\int i\text{sech}(x)\tanh(x)dx}{a^2+b^2}\right)}{a^2+b^2}-\frac{a\tanh^3(x)}{3(a^2+b^2)}+\frac{b\left(\frac{\text{sech}^3(x)}{3}-\text{sech}(x)\right)}{a^2+b^2}$$

$$\Big\downarrow \; 26$$

$$-\frac{a^2\left(-\frac{a^2\int \frac{1}{a+b\sinh(x)}dx}{a^2+b^2}+\frac{a\int \operatorname{sech}^2(x)dx}{a^2+b^2}-\frac{b\int \operatorname{sech}(x)\tanh(x)dx}{a^2+b^2}\right)}{a^2+b^2}-\frac{a\tanh^3(x)}{3(a^2+b^2)}+\frac{b\left(\frac{\operatorname{sech}^3(x)}{3}-\operatorname{sech}(x)\right)}{a^2+b^2}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{a^2\left(-\frac{a^2\int \frac{1}{a-ib\sin(ix)}dx}{a^2+b^2}+\frac{a\int \csc\left(ix+\frac{\pi}{2}\right)^2dx}{a^2+b^2}-\frac{b\int -i\sec(ix)\tan(ix)dx}{a^2+b^2}\right)}{a^2+b^2}-\frac{a\tanh^3(x)}{3(a^2+b^2)}+\frac{b\left(\frac{\operatorname{sech}^3(x)}{3}-\operatorname{sech}(x)\right)}{a^2+b^2}$$

$$\Big\downarrow \text{26}$$

$$-\frac{a^2\left(-\frac{a^2\int \frac{1}{a-ib\sin(ix)}dx}{a^2+b^2}+\frac{a\int \csc\left(ix+\frac{\pi}{2}\right)^2dx}{a^2+b^2}+\frac{ib\int \sec(ix)\tan(ix)dx}{a^2+b^2}\right)}{a^2+b^2}-\frac{a\tanh^3(x)}{3(a^2+b^2)}+\frac{b\left(\frac{\operatorname{sech}^3(x)}{3}-\operatorname{sech}(x)\right)}{a^2+b^2}$$

$$\Big\downarrow \text{3086}$$

$$-\frac{a^2\left(-\frac{a^2\int \frac{1}{a-ib\sin(ix)}dx}{a^2+b^2}+\frac{a\int \csc\left(ix+\frac{\pi}{2}\right)^2dx}{a^2+b^2}+\frac{b\int 1d\operatorname{sech}(x)}{a^2+b^2}\right)}{a^2+b^2}-\frac{a\tanh^3(x)}{3(a^2+b^2)}+\frac{b\left(\frac{\operatorname{sech}^3(x)}{3}-\operatorname{sech}(x)\right)}{a^2+b^2}$$

$$\Big\downarrow \text{24}$$

$$-\frac{a^2\left(-\frac{a^2\int \frac{1}{a-ib\sin(ix)}dx}{a^2+b^2}+\frac{a\int \csc\left(ix+\frac{\pi}{2}\right)^2dx}{a^2+b^2}+\frac{b\operatorname{sech}(x)}{a^2+b^2}\right)}{a^2+b^2}-\frac{a\tanh^3(x)}{3(a^2+b^2)}+\frac{b\left(\frac{\operatorname{sech}^3(x)}{3}-\operatorname{sech}(x)\right)}{a^2+b^2}$$

$$\Big\downarrow \text{3139}$$

$$-\frac{a^2\left(\frac{a\int \csc\left(ix+\frac{\pi}{2}\right)^2dx}{a^2+b^2}-\frac{2a^2\int \frac{1}{-a\tanh^2\left(\frac{x}{2}\right)+2b\tanh\left(\frac{x}{2}\right)+a}d\tanh\left(\frac{x}{2}\right)}{a^2+b^2}+\frac{b\operatorname{sech}(x)}{a^2+b^2}\right)}{a^2+b^2}-\frac{a\tanh^3(x)}{3(a^2+b^2)}+\frac{b\left(\frac{\operatorname{sech}^3(x)}{3}-\operatorname{sech}(x)\right)}{a^2+b^2}$$

$$\Big\downarrow \; 1083$$

$$-\frac{a^2\left(\frac{a\int \csc\left(ix+\frac{\pi}{2}\right)^2dx}{a^2+b^2}+\frac{4a^2\int \frac{1}{4(a^2+b^2)-\left(2b-2a\tanh\left(\frac{x}{2}\right)\right)^2}d\left(2b-2a\tanh\left(\frac{x}{2}\right)\right)}{a^2+b^2}+\frac{b\text{sech}(x)}{a^2+b^2}\right)}{a^2+b^2}-\frac{a\tanh^3(x)}{3(a^2+b^2)}+\frac{b\left(\frac{\text{sech}^3(x)}{3}-\text{sech}(x)\right)}{a^2+b^2}$$

$$\Big\downarrow \; 219$$

$$-\frac{a^2\left(\frac{a\int \csc\left(ix+\frac{\pi}{2}\right)^2dx}{a^2+b^2}+\frac{2a^2\text{arctanh}\left(\frac{2b-2a\tanh\left(\frac{x}{2}\right)}{2\sqrt{a^2+b^2}}\right)}{(a^2+b^2)^{3/2}}+\frac{b\text{sech}(x)}{a^2+b^2}\right)}{a^2+b^2}-\frac{a\tanh^3(x)}{3(a^2+b^2)}+\frac{b\left(\frac{\text{sech}^3(x)}{3}-\text{sech}(x)\right)}{a^2+b^2}$$

$$\Big\downarrow \; 4254$$

$$-\frac{a^2\left(\frac{ia\int 1d(-i\tanh(x))}{a^2+b^2}+\frac{2a^2\text{arctanh}\left(\frac{2b-2a\tanh\left(\frac{x}{2}\right)}{2\sqrt{a^2+b^2}}\right)}{(a^2+b^2)^{3/2}}+\frac{b\text{sech}(x)}{a^2+b^2}\right)}{a^2+b^2}-\frac{a\tanh^3(x)}{3(a^2+b^2)}+\frac{b\left(\frac{\text{sech}^3(x)}{3}-\text{sech}(x)\right)}{a^2+b^2}$$

$$\Big\downarrow \; 24$$

$$-\frac{a^2\left(\frac{2a^2\text{arctanh}\left(\frac{2b-2a\tanh\left(\frac{x}{2}\right)}{2\sqrt{a^2+b^2}}\right)}{(a^2+b^2)^{3/2}}+\frac{a\tanh(x)}{a^2+b^2}+\frac{b\text{sech}(x)}{a^2+b^2}\right)}{a^2+b^2}-\frac{a\tanh^3(x)}{3(a^2+b^2)}+\frac{b\left(\frac{\text{sech}^3(x)}{3}-\text{sech}(x)\right)}{a^2+b^2}$$

input
```
Int[Tanh[x]^4/(a + b*Sinh[x]),x]
```

output
```
(b*(-Sech[x] + Sech[x]^3/3))/(a^2 + b^2) - (a*Tanh[x]^3)/(3*(a^2 + b^2)) -
 (a^2*((2*a^2*ArcTanh[(2*b - 2*a*Tanh[x/2])/(2*Sqrt[a^2 + b^2])])/(a^2 + b
^2)^(3/2) + (b*Sech[x])/(a^2 + b^2) + (a*Tanh[x])/(a^2 + b^2)))/(a^2 + b^2
)
```

Defintions of rubi rules used

rule 15
```
Int[(a_.)*(x_)^(m_.), x_Symbol] :> Simp[a*(x^(m + 1)/(m + 1)), x] /; FreeQ[
{a, m}, x] && NeQ[m, -1]
```

rule 24
```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1083
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Simp[-2   Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3086
```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Simp[a/f   Subst[Int[(a*x)^(m - 1)*(-1 + x^2)^((n - 1)/2
), x], x, Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2
] &&  !(IntegerQ[m/2] && LtQ[0, m, n + 1])
```

rule 3087
```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> Simp[1/f   Subst[Int[(b*x)^n*(1 + x^2)^(m/2 - 1), x], x, Tan[e +
f*x]], x] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2] &&  !(IntegerQ[(n - 1)
/2] && LtQ[0, n, m - 1])
```

rule 3139
```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Simp[2*(e/d)   Subst[Int[1/(a + 2*b*e*x + a
*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[a^2 - b^2, 0]
```

rule 3206
```
Int[((g_.)*tan[(e_.) + (f_.)*(x_)])^(p_)/((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Simp[a/(a^2 - b^2)   Int[(g*Tan[e + f*x])^p/Sin[e + f*x]
^2, x], x] + (-Simp[b*(g/(a^2 - b^2))   Int[(g*Tan[e + f*x])^(p - 1)/Cos[e
+ f*x], x], x] - Simp[a^2*(g^2/(a^2 - b^2))   Int[(g*Tan[e + f*x])^(p - 2)/
(a + b*Sin[e + f*x]), x], x]) /; FreeQ[{a, b, e, f, g}, x] && NeQ[a^2 - b^2
, 0] && IntegersQ[2*p] && GtQ[p, 1]
```

rule 4254
```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Simp[-d^(-1)   Subst[Int[Exp
andIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
 d}, x] && IGtQ[n/2, 0]
```

Maple [A] (verified)

Time = 1.26 (sec) , antiderivative size = 171, normalized size of antiderivative = 1.38

method	result
default	$\frac{32a^4\operatorname{arctanh}\left(\frac{2a\tanh\left(\frac{x}{2}\right)-2b}{2\sqrt{a^2+b^2}}\right)}{(16a^4+32a^2b^2+16b^4)\sqrt{a^2+b^2}}+\frac{-2a^3\tanh\left(\frac{x}{2}\right)^5-2a^2b\tanh\left(\frac{x}{2}\right)^4+2\left(-\frac{10}{3}a^3-\frac{4}{3}a\,b^2\right)\tanh\left(\frac{x}{2}\right)^3+2\left(-4a^2b-2b^3\right)\tanh\left(\frac{x}{2}\right)^2-2a^3}{(a^4+2a^2b^2+b^4)\left(1+\tanh\left(\frac{x}{2}\right)^2\right)^3}$
risch	$\frac{-4a^2b\,e^{5x}-2b^3e^{5x}+4\,e^{4x}a^3+2\,e^{4x}a\,b^2-\frac{16a^2b\,e^{3x}}{3}-\frac{4\,e^{3x}b^3}{3}+4a^3e^{2x}-4\,e^x a^2b-2b^3e^x+\frac{8a^3}{3}+\frac{2a\,b^2}{3}}{(a^2+b^2)^2(e^{2x}+1)^3}+\frac{a^4\ln\left(e^x+\frac{\left(a^2+b^2\right)^{\frac{5}{2}}a-a^6-}{b\left(a^2+\right.}\right.}{(a^2+b^2)^{\frac{5}{2}}}$

input
```
int(tanh(x)^4/(a+b*sinh(x)),x,method=_RETURNVERBOSE)
```

output
```
32*a^4/(16*a^4+32*a^2*b^2+16*b^4)/(a^2+b^2)^(1/2)*arctanh(1/2*(2*a*tanh(1/
2*x)-2*b)/(a^2+b^2)^(1/2))+2/(a^4+2*a^2*b^2+b^4)*(-a^3*tanh(1/2*x)^5-a^2*b
*tanh(1/2*x)^4+(-10/3*a^3-4/3*a*b^2)*tanh(1/2*x)^3+(-4*a^2*b-2*b^3)*tanh(1
/2*x)^2-a^3*tanh(1/2*x)-5/3*a^2*b-2/3*b^3)/(1+tanh(1/2*x)^2)^3
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1199 vs. 2(116) = 232.

Time = 0.11 (sec) , antiderivative size = 1199, normalized size of antiderivative = 9.67

$$\int \frac{\tanh^4(x)}{a+b\sinh(x)}\,dx = \text{Too large to display}$$

input
```
integrate(tanh(x)^4/(a+b*sinh(x)),x, algorithm="fricas")
```

output
```
-1/3*(6*(2*a^4*b + 3*a^2*b^3 + b^5)*cosh(x)^5 + 6*(2*a^4*b + 3*a^2*b^3 + b
^5)*sinh(x)^5 - 8*a^5 - 10*a^3*b^2 - 2*a*b^4 - 6*(2*a^5 + 3*a^3*b^2 + a*b^
4)*cosh(x)^4 - 6*(2*a^5 + 3*a^3*b^2 + a*b^4 - 5*(2*a^4*b + 3*a^2*b^3 + b^5
)*cosh(x))*sinh(x)^4 + 4*(4*a^4*b + 5*a^2*b^3 + b^5)*cosh(x)^3 + 4*(4*a^4*
b + 5*a^2*b^3 + b^5 + 15*(2*a^4*b + 3*a^2*b^3 + b^5)*cosh(x)^2 - 6*(2*a^5
+ 3*a^3*b^2 + a*b^4)*cosh(x))*sinh(x)^3 - 12*(a^5 + a^3*b^2)*cosh(x)^2 - 1
2*(a^5 + a^3*b^2 - 5*(2*a^4*b + 3*a^2*b^3 + b^5)*cosh(x)^3 + 3*(2*a^5 + 3*
a^3*b^2 + a*b^4)*cosh(x)^2 - (4*a^4*b + 5*a^2*b^3 + b^5)*cosh(x))*sinh(x)^
2 - 3*(a^4*cosh(x)^6 + 6*a^4*cosh(x)*sinh(x)^5 + a^4*sinh(x)^6 + 3*a^4*cos
h(x)^4 + 3*a^4*cosh(x)^2 + 3*(5*a^4*cosh(x)^2 + a^4)*sinh(x)^4 + a^4 + 4*(
5*a^4*cosh(x)^3 + 3*a^4*cosh(x))*sinh(x)^3 + 3*(5*a^4*cosh(x)^4 + 6*a^4*co
sh(x)^2 + a^4)*sinh(x)^2 + 6*(a^4*cosh(x)^5 + 2*a^4*cosh(x)^3 + a^4*cosh(x
))*sinh(x))*sqrt(a^2 + b^2)*log((b^2*cosh(x)^2 + b^2*sinh(x)^2 + 2*a*b*cos
h(x) + 2*a^2 + b^2 + 2*(b^2*cosh(x) + a*b)*sinh(x) - 2*sqrt(a^2 + b^2)*(b*
cosh(x) + b*sinh(x) + a))/(b*cosh(x)^2 + b*sinh(x)^2 + 2*a*cosh(x) + 2*(b*
cosh(x) + a)*sinh(x) - b)) + 6*(2*a^4*b + 3*a^2*b^3 + b^5)*cosh(x) + 6*(2*
a^4*b + 3*a^2*b^3 + b^5 + 5*(2*a^4*b + 3*a^2*b^3 + b^5)*cosh(x)^4 - 4*(2*a
^5 + 3*a^3*b^2 + a*b^4)*cosh(x)^3 + 2*(4*a^4*b + 5*a^2*b^3 + b^5)*cosh(x)^
2 - 4*(a^5 + a^3*b^2)*cosh(x))*sinh(x))/((a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^
6)*cosh(x)^6 + 6*(a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6)*cosh(x)*sinh(x)^5 ...
```

Sympy [F]

$$\int \frac{\tanh^4(x)}{a+b\sinh(x)}\,dx = \int \frac{\tanh^4{(x)}}{a+b\sinh{(x)}}\,dx$$

input
```
integrate(tanh(x)**4/(a+b*sinh(x)),x)
```

output
```
Integral(tanh(x)**4/(a + b*sinh(x)), x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 241 vs. $2(116) = 232$.

Time = 0.12 (sec) , antiderivative size = 241, normalized size of antiderivative = 1.94

$$\int \frac{\tanh^4(x)}{a + b\sinh(x)}\,dx = \frac{a^4 \log\left(\frac{be^{(-x)} - a - \sqrt{a^2+b^2}}{be^{(-x)} - a + \sqrt{a^2+b^2}}\right)}{(a^4 + 2\,a^2b^2 + b^4)\sqrt{a^2+b^2}} - \frac{2\left(6\,a^3e^{(-2\,x)} + 4\,a^3 + ab^2 + 3\,(2\,a^2b + b^3)e^{(-x)} + 2\,(4\,a^2b + b^3)e^{(-3\,x)} + 3\,(2\,a^3 + ab^2)e^{(-4\,x)} + 3\,(2\,a^2b \cdots\right.}{3\,(a^4 + 2\,a^2b^2 + b^4 + 3\,(a^4 + 2\,a^2b^2 + b^4)e^{(-2\,x)} + 3\,(a^4 + 2\,a^2b^2 + b^4)e^{(-4\,x)} + (a^4 + 2\,a^2b^2 + b^4)e \cdots}$$

input
```
integrate(tanh(x)^4/(a+b*sinh(x)),x, algorithm="maxima")
```

output
```
a^4*log((b*e^(-x) - a - sqrt(a^2 + b^2))/(b*e^(-x) - a + sqrt(a^2 + b^2)))
/((a^4 + 2*a^2*b^2 + b^4)*sqrt(a^2 + b^2)) - 2/3*(6*a^3*e^(-2*x) + 4*a^3 +
 a*b^2 + 3*(2*a^2*b + b^3)*e^(-x) + 2*(4*a^2*b + b^3)*e^(-3*x) + 3*(2*a^3
+ a*b^2)*e^(-4*x) + 3*(2*a^2*b + b^3)*e^(-5*x))/(a^4 + 2*a^2*b^2 + b^4 + 3
*(a^4 + 2*a^2*b^2 + b^4)*e^(-2*x) + 3*(a^4 + 2*a^2*b^2 + b^4)*e^(-4*x) + (
a^4 + 2*a^2*b^2 + b^4)*e^(-6*x))
```

Giac [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 197, normalized size of antiderivative = 1.59

$$\int \frac{\tanh^4(x)}{a + b\sinh(x)}\,dx = \frac{a^4 \log\left(\frac{\left|2\,be^x + 2\,a - 2\,\sqrt{a^2+b^2}\right|}{\left|2\,be^x + 2\,a + 2\,\sqrt{a^2+b^2}\right|}\right)}{(a^4 + 2\,a^2b^2 + b^4)\sqrt{a^2+b^2}} - \frac{2\left(6\,a^2be^{(5\,x)} + 3\,b^3e^{(5\,x)} - 6\,a^3e^{(4\,x)} - 3\,ab^2e^{(4\,x)} + 8\,a^2be^{(3\,x)} + 2\,b^3e^{(3\,x)} - 6\,a^3e^{(2\,x)} + 6\,a^2be^x + 3\,b^3e^x \cdots\right.}{3\,(a^4 + 2\,a^2b^2 + b^4)\left(e^{(2\,x)} + 1\right)^3}$$

input
```
integrate(tanh(x)^4/(a+b*sinh(x)),x, algorithm="giac")
```

output
```
a^4*log(abs(2*b*e^x + 2*a - 2*sqrt(a^2 + b^2))/abs(2*b*e^x + 2*a + 2*sqrt(
a^2 + b^2)))/((a^4 + 2*a^2*b^2 + b^4)*sqrt(a^2 + b^2)) - 2/3*(6*a^2*b*e^(5
*x) + 3*b^3*e^(5*x) - 6*a^3*e^(4*x) - 3*a*b^2*e^(4*x) + 8*a^2*b*e^(3*x) +
2*b^3*e^(3*x) - 6*a^3*e^(2*x) + 6*a^2*b*e^x + 3*b^3*e^x - 4*a^3 - a*b^2)/(
(a^4 + 2*a^2*b^2 + b^4)*(e^(2*x) + 1)^3)
```

Mupad [B] (verification not implemented)

Time = 2.42 (sec) , antiderivative size = 654, normalized size of antiderivative = 5.27

$$\int \frac{\tanh^4(x)}{a+b\sinh(x)}\,dx$$
$$= \frac{\frac{2a(2a^2+b^2)}{(a^2+b^2)^2}-\frac{2b\,e^x(2a^2+b^2)}{(a^2+b^2)^2}}{e^{2x}+1}-\frac{\frac{4(a^3+ab^2)}{(a^2+b^2)^2}-\frac{8e^x(a^2b+b^3)}{3(a^2+b^2)^2}}{2e^{2x}+e^{4x}+1}+\frac{\frac{8a}{3(a^2+b^2)}-\frac{8b\,e^x}{3(a^2+b^2)}}{3e^{2x}+3e^{4x}+e^{6x}+1}$$
$$-\frac{2\,\mathrm{atan}\left(\left(\left(e^x\left(\frac{2a^4}{b^2\sqrt{a^8}(a^2+b^2)^2(a^4+2a^2b^2+b^4)}+\frac{2\left(a^5\sqrt{a^8}+2a^3b^2\sqrt{a^8}+ab^4\sqrt{a^8}\right)}{a^3b^2\sqrt{-(a^2+b^2)^5}(a^4+2a^2b^2+b^4)\sqrt{-a^{10}-5a^8b^2-10a^6b^4-10a^4b^6-5a^2b^8-\cdots}}\right.\right.\right.\right.}{}$$

input
```
int(tanh(x)^4/(a + b*sinh(x)),x)
```

output
```
((2*a*(2*a^2 + b^2))/(a^2 + b^2)^2 - (2*b*exp(x)*(2*a^2 + b^2))/(a^2 + b^2
)^2)/(exp(2*x) + 1) - ((4*(a*b^2 + a^3))/(a^2 + b^2)^2 - (8*exp(x)*(a^2*b
+ b^3))/(3*(a^2 + b^2)^2))/(2*exp(2*x) + exp(4*x) + 1) + ((8*a)/(3*(a^2 +
b^2)) - (8*b*exp(x))/(3*(a^2 + b^2)))/(3*exp(2*x) + 3*exp(4*x) + exp(6*x)
+ 1) - (2*atan((exp(x)*((2*a^4)/(b^2*(a^8)^(1/2)*(a^2 + b^2)^2*(a^4 + b^4
+ 2*a^2*b^2)) + (2*(a^5*(a^8)^(1/2) + 2*a^3*b^2*(a^8)^(1/2) + a*b^4*(a^8)^
(1/2)))/(a^3*b^2*(-(a^2 + b^2)^5)^(1/2)*(a^4 + b^4 + 2*a^2*b^2)*(- a^10 -
b^10 - 5*a^2*b^8 - 10*a^4*b^6 - 10*a^6*b^4 - 5*a^8*b^2)^(1/2))) - (2*(b^5*
(a^8)^(1/2) + 2*a^2*b^3*(a^8)^(1/2) + a^4*b*(a^8)^(1/2)))/(a^3*b^2*(-(a^2
+ b^2)^5)^(1/2)*(a^4 + b^4 + 2*a^2*b^2)*(- a^10 - b^10 - 5*a^2*b^8 - 10*a^
4*b^6 - 10*a^6*b^4 - 5*a^8*b^2)^(1/2)))*((b^5*(- a^10 - b^10 - 5*a^2*b^8 -
 10*a^4*b^6 - 10*a^6*b^4 - 5*a^8*b^2)^(1/2))/2 + (a^4*b*(- a^10 - b^10 - 5
*a^2*b^8 - 10*a^4*b^6 - 10*a^6*b^4 - 5*a^8*b^2)^(1/2))/2 + a^2*b^3*(- a^10
 - b^10 - 5*a^2*b^8 - 10*a^4*b^6 - 10*a^6*b^4 - 5*a^8*b^2)^(1/2)))*(a^8)^(
1/2))/(- a^10 - b^10 - 5*a^2*b^8 - 10*a^4*b^6 - 10*a^6*b^4 - 5*a^8*b^2)^(1
/2)
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 496, normalized size of antiderivative = 4.00

$$\int \frac{\tanh^4(x)}{a+b\sinh(x)}\,dx = \frac{6e^{6x}\sqrt{a^2+b^2}\,atan\left(\frac{e^x bi+ai}{\sqrt{a^2+b^2}}\right)a^4 i+18e^{4x}\sqrt{a^2+b^2}\,atan\left(\frac{e^x bi+ai}{\sqrt{a^2+b^2}}\right)a^4 i+18e^{2x}\sqrt{a^2+b^2}\,atan\left(\frac{e^x bi+ai}{\sqrt{a^2+b^2}}\right)a^4 i+}{3e^{6x}a^6+9e^{6x}a^4b^2+9e^{6x}a^2b^4+}$$

input

```
int(tanh(x)^4/(a+b*sinh(x)),x)
```

output

```
(2*(3*e**(6*x)*sqrt(a**2 + b**2)*atan((e**x*b*i + a*i)/sqrt(a**2 + b**2))*
a**4*i + 9*e**(4*x)*sqrt(a**2 + b**2)*atan((e**x*b*i + a*i)/sqrt(a**2 + b*
*2))*a**4*i + 9*e**(2*x)*sqrt(a**2 + b**2)*atan((e**x*b*i + a*i)/sqrt(a**2
 + b**2))*a**4*i + 3*sqrt(a**2 + b**2)*atan((e**x*b*i + a*i)/sqrt(a**2 + b
**2))*a**4*i - 2*e**(6*x)*a**5 - 3*e**(6*x)*a**3*b**2 - e**(6*x)*a*b**4 -
6*e**(5*x)*a**4*b - 9*e**(5*x)*a**2*b**3 - 3*e**(5*x)*b**5 - 8*e**(3*x)*a*
*4*b - 10*e**(3*x)*a**2*b**3 - 2*e**(3*x)*b**5 - 3*e**(2*x)*a**3*b**2 - 3*
e**(2*x)*a*b**4 - 6*e**x*a**4*b - 9*e**x*a**2*b**3 - 3*e**x*b**5 + 2*a**5
+ 2*a**3*b**2))/(3*(e**(6*x)*a**6 + 3*e**(6*x)*a**4*b**2 + 3*e**(6*x)*a**2
*b**4 + e**(6*x)*b**6 + 3*e**(4*x)*a**6 + 9*e**(4*x)*a**4*b**2 + 9*e**(4*x
)*a**2*b**4 + 3*e**(4*x)*b**6 + 3*e**(2*x)*a**6 + 9*e**(2*x)*a**4*b**2 + 9
*e**(2*x)*a**2*b**4 + 3*e**(2*x)*b**6 + a**6 + 3*a**4*b**2 + 3*a**2*b**4 +
 b**6))
```

3.229 $\int \frac{\tanh^3(x)}{a+b\sinh(x)}\,dx$

Optimal result

Integrand size = 13, antiderivative size = 88

$$\int \frac{\tanh^3(x)}{a+b\sinh(x)}\,dx = \frac{b(3a^2+b^2)\arctan(\sinh(x))}{2\left(a^2+b^2\right)^2}+\frac{a^3\log(\cosh(x))}{\left(a^2+b^2\right)^2} - \frac{a^3\log(a+b\sinh(x))}{\left(a^2+b^2\right)^2}+\frac{\operatorname{sech}^2(x)(a-b\sinh(x))}{2\left(a^2+b^2\right)}$$

output
```
1/2*b*(3*a^2+b^2)*arctan(sinh(x))/(a^2+b^2)^2+a^3*ln(cosh(x))/(a^2+b^2)^2-
a^3*ln(a+b*sinh(x))/(a^2+b^2)^2+sech(x)^2*(a-b*sinh(x))/(2*a^2+2*b^2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.13 (sec) , antiderivative size = 153, normalized size of antiderivative = 1.74

$$\int \frac{\tanh^3(x)}{a+b\sinh(x)}\,dx = -\frac{b\arctan(\sinh(x))}{2\left(a^2+b^2\right)}+\frac{\left(a^3-i\left(2a^2b+b^3\right)\right)\log(i-\sinh(x))}{2\left(a^2+b^2\right)^2} + \frac{\left(a^3+i\left(2a^2b+b^3\right)\right)\log(i+\sinh(x))}{2\left(a^2+b^2\right)^2} - \frac{a^3\log(a+b\sinh(x))}{\left(a^2+b^2\right)^2}+\frac{a\operatorname{sech}^2(x)}{2\left(a^2+b^2\right)}-\frac{b\operatorname{sech}(x)\tanh(x)}{2\left(a^2+b^2\right)}$$

input
```
Integrate[Tanh[x]^3/(a + b*Sinh[x]),x]
```

output
```
-1/2*(b*ArcTan[Sinh[x]])/(a^2 + b^2) + ((a^3 - I*(2*a^2*b + b^3))*Log[I -
Sinh[x]])/(2*(a^2 + b^2)^2) + ((a^3 + I*(2*a^2*b + b^3))*Log[I + Sinh[x]])
/(2*(a^2 + b^2)^2) - (a^3*Log[a + b*Sinh[x]])/(a^2 + b^2)^2 + (a*Sech[x]^2
)/(2*(a^2 + b^2)) - (b*Sech[x]*Tanh[x])/(2*(a^2 + b^2))
```

Rubi [A] (verified)

Time = 0.37 (sec) , antiderivative size = 122, normalized size of antiderivative = 1.39, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.615$, Rules used = {3042, 26, 3200, 601, 25, 27, 657, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\tanh^3(x)}{a+b\sinh(x)}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{i\tan(ix)^3}{a-ib\sin(ix)}dx$$

$$\Big\downarrow \text{26}$$

$$i\int \frac{\tan(ix)^3}{a-ib\sin(ix)}dx$$

$$\Big\downarrow \text{3200}$$

$$\int \frac{b^3\sinh^3(x)}{\left(b^2\sinh^2(x)+b^2\right)^2(a+b\sinh(x))}d(b\sinh(x))$$

$$\Big\downarrow \text{601}$$

$$\frac{b^2(a-b\sinh(x))}{2\left(a^2+b^2\right)\left(b^2\sinh^2(x)+b^2\right)}-\frac{\int -\frac{b^2\left(ab^2+\left(2a^2+b^2\right)\sinh(x)b\right)}{\left(a^2+b^2\right)(a+b\sinh(x))\left(\sinh^2(x)b^2+b^2\right)}d(b\sinh(x))}{2b^2}$$

$$\Big\downarrow \text{25}$$

$$\frac{\int \frac{b^2\left(ab^2+\left(2a^2+b^2\right)\sinh(x)b\right)}{\left(a^2+b^2\right)(a+b\sinh(x))\left(\sinh^2(x)b^2+b^2\right)}d(b\sinh(x))}{2b^2}+\frac{b^2(a-b\sinh(x))}{2\left(a^2+b^2\right)\left(b^2\sinh^2(x)+b^2\right)}$$

$$\Big\downarrow \; 27$$

$$\frac{\int \frac{ab^2+\left(2a^2+b^2\right)\sinh(x)b}{(a+b\sinh(x))\left(\sinh^2(x)b^2+b^2\right)}d(b\sinh(x))}{2\left(a^2+b^2\right)}+\frac{b^2(a-b\sinh(x))}{2\left(a^2+b^2\right)\left(b^2\sinh^2(x)+b^2\right)}$$

$$\Big\downarrow \; 657$$

$$\frac{\int \left(\frac{b^4+3a^2b^2+2a^3\sinh(x)b}{\left(a^2+b^2\right)\left(\sinh^2(x)b^2+b^2\right)}-\frac{2a^3}{\left(a^2+b^2\right)(a+b\sinh(x))}\right)d(b\sinh(x))}{2\left(a^2+b^2\right)}+\frac{b^2(a-b\sinh(x))}{2\left(a^2+b^2\right)\left(b^2\sinh^2(x)+b^2\right)}$$

$$\Big\downarrow \; 2009$$

$$\frac{b^2(a-b\sinh(x))}{2\left(a^2+b^2\right)\left(b^2\sinh^2(x)+b^2\right)}+\frac{\frac{b\left(3a^2+b^2\right)\arctan(\sinh(x))}{a^2+b^2}+\frac{a^3\log\left(b^2\sinh^2(x)+b^2\right)}{a^2+b^2}-\frac{2a^3\log(a+b\sinh(x))}{a^2+b^2}}{2\left(a^2+b^2\right)}$$

input
```
Int[Tanh[x]^3/(a + b*Sinh[x]),x]
```

output
```
((b*(3*a^2 + b^2)*ArcTan[Sinh[x]])/(a^2 + b^2) - (2*a^3*Log[a + b*Sinh[x]]
)/(a^2 + b^2) + (a^3*Log[b^2 + b^2*Sinh[x]^2])/(a^2 + b^2))/(2*(a^2 + b^2)
) + (b^2*(a - b*Sinh[x]))/(2*(a^2 + b^2)*(b^2 + b^2*Sinh[x]^2))
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 601
```
Int[(x_)^(m_)*((c_) + (d_.)*(x_))^(n_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbo
l] :> With[{Qx = PolynomialQuotient[x^m*(c + d*x)^n, a + b*x^2, x], e = Coe
ff[PolynomialRemainder[x^m*(c + d*x)^n, a + b*x^2, x], x, 0], f = Coeff[Pol
ynomialRemainder[x^m*(c + d*x)^n, a + b*x^2, x], x, 1]}, Simp[(a*f - b*e*x)
*((a + b*x^2)^(p + 1)/(2*a*b*(p + 1))), x] + Simp[1/(2*a*(p + 1))   Int[(c
+ d*x)^n*(a + b*x^2)^(p + 1)*ExpandToSum[(2*a*(p + 1)*Qx)/(c + d*x)^n + (e*
(2*p + 3))/(c + d*x)^n, x], x], x]] /; FreeQ[{a, b, c, d}, x] && IGtQ[m, 1]
 && LtQ[p, -1] && ILtQ[n, 0] && NeQ[b*c^2 + a*d^2, 0]
```

rule 657
```
Int[(((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))^(n_.))/((a_) + (c_.)*(
x_)^2), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*((f + g*x)^n/(a + c*x^
2)), x], x] /; FreeQ[{a, c, d, e, f, g, m}, x] && IntegersQ[n]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3200
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*tan[(e_.) + (f_.)*(x_)]^(p
_.), x_Symbol] :> Simp[1/f   Subst[Int[(x^p*(a + x)^m)/(b^2 - x^2)^((p + 1)
/2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && NeQ[a^2 - b
^2, 0] && IntegerQ[(p + 1)/2]
```

Maple [A] (verified)

Time = 0.83 (sec) , antiderivative size = 166, normalized size of antiderivative = 1.89

method	result
default	$-\frac{8a^3 \ln\left(\tanh\left(\frac{x}{2}\right)^2 a-2b\tanh\left(\frac{x}{2}\right)-a\right)}{8a^4+16a^2b^2+8b^4}+\frac{\frac{2\left(\left(\frac{1}{2}a^2b+\frac{1}{2}b^3\right)\tanh\left(\frac{x}{2}\right)^3+\left(-a^3-a\,b^2\right)\tanh\left(\frac{x}{2}\right)^2+\left(-\frac{1}{2}a^2b-\frac{1}{2}b^3\right)\tanh\left(\frac{x}{2}\right)\right)}{\left(1+\tanh\left(\frac{x}{2}\right)^2\right)^2}+a^3\ln\left(1+\mathrm{ta}\right.}{a^4+2a^2b^2+b^4}$
risch	$\frac{\mathrm{e}^{x}\left(-b\,\mathrm{e}^{2x}+2\,\mathrm{e}^{x}a+b\right)}{\left(\mathrm{e}^{2x}+1\right)^2\left(a^2+b^2\right)}+\frac{3i\ln(\mathrm{e}^{x}+i)a^2b}{2(a^4+2a^2b^2+b^4)}+\frac{i\ln(\mathrm{e}^{x}+i)b^3}{2a^4+4a^2b^2+2b^4}+\frac{\ln(\mathrm{e}^{x}+i)a^3}{a^4+2a^2b^2+b^4}-\frac{3i\ln(\mathrm{e}^{x}-i)a^2b}{2(a^4+2a^2b^2+b^4)}-\frac{i\ln(\mathrm{e}^{x}-i)b^3}{2(a^4+2a^2b^2+b^4)}+$

input
```
int(tanh(x)^3/(a+b*sinh(x)),x,method=_RETURNVERBOSE)
```

output
```
-8*a^3/(8*a^4+16*a^2*b^2+8*b^4)*ln(tanh(1/2*x)^2*a-2*b*tanh(1/2*x)-a)+2/(a
^4+2*a^2*b^2+b^4)*(((1/2*a^2*b+1/2*b^3)*tanh(1/2*x)^3+(-a^3-a*b^2)*tanh(1/
2*x)^2+(-1/2*a^2*b-1/2*b^3)*tanh(1/2*x))/(1+tanh(1/2*x)^2)^2+1/2*a^3*ln(1+
tanh(1/2*x)^2)+1/2*(3*a^2*b+b^3)*arctan(tanh(1/2*x)))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 655 vs. $2(85) = 170$.

Time = 0.11 (sec) , antiderivative size = 655, normalized size of antiderivative = 7.44

$$\int \frac{\tanh^3(x)}{a + b\sinh(x)}\,dx = \text{Too large to display}$$

input
```
integrate(tanh(x)^3/(a+b*sinh(x)),x, algorithm="fricas")
```

output
```
-((a^2*b + b^3)*cosh(x)^3 + (a^2*b + b^3)*sinh(x)^3 - 2*(a^3 + a*b^2)*cosh
(x)^2 - (2*a^3 + 2*a*b^2 - 3*(a^2*b + b^3)*cosh(x))*sinh(x)^2 - ((3*a^2*b
+ b^3)*cosh(x)^4 + 4*(3*a^2*b + b^3)*cosh(x)*sinh(x)^3 + (3*a^2*b + b^3)*s
inh(x)^4 + 3*a^2*b + b^3 + 2*(3*a^2*b + b^3)*cosh(x)^2 + 2*(3*a^2*b + b^3
+ 3*(3*a^2*b + b^3)*cosh(x)^2)*sinh(x)^2 + 4*((3*a^2*b + b^3)*cosh(x)^3 +
(3*a^2*b + b^3)*cosh(x))*sinh(x))*arctan(cosh(x) + sinh(x)) - (a^2*b + b^3
)*cosh(x) + (a^3*cosh(x)^4 + 4*a^3*cosh(x)*sinh(x)^3 + a^3*sinh(x)^4 + 2*a
^3*cosh(x)^2 + a^3 + 2*(3*a^3*cosh(x)^2 + a^3)*sinh(x)^2 + 4*(a^3*cosh(x)^
3 + a^3*cosh(x))*sinh(x))*log(2*(b*sinh(x) + a)/(cosh(x) - sinh(x))) - (a^
3*cosh(x)^4 + 4*a^3*cosh(x)*sinh(x)^3 + a^3*sinh(x)^4 + 2*a^3*cosh(x)^2 +
a^3 + 2*(3*a^3*cosh(x)^2 + a^3)*sinh(x)^2 + 4*(a^3*cosh(x)^3 + a^3*cosh(x)
)*sinh(x))*log(2*cosh(x)/(cosh(x) - sinh(x))) - (a^2*b + b^3 - 3*(a^2*b +
b^3)*cosh(x)^2 + 4*(a^3 + a*b^2)*cosh(x))*sinh(x))/((a^4 + 2*a^2*b^2 + b^4
)*cosh(x)^4 + 4*(a^4 + 2*a^2*b^2 + b^4)*cosh(x)*sinh(x)^3 + (a^4 + 2*a^2*b
^2 + b^4)*sinh(x)^4 + a^4 + 2*a^2*b^2 + b^4 + 2*(a^4 + 2*a^2*b^2 + b^4)*co
sh(x)^2 + 2*(a^4 + 2*a^2*b^2 + b^4 + 3*(a^4 + 2*a^2*b^2 + b^4)*cosh(x)^2)*
sinh(x)^2 + 4*((a^4 + 2*a^2*b^2 + b^4)*cosh(x)^3 + (a^4 + 2*a^2*b^2 + b^4)
*cosh(x))*sinh(x))
```

Sympy [F]

$$\int \frac{\tanh^3(x)}{a + b\sinh(x)}\,dx = \int \frac{\tanh^3{(x)}}{a + b\sinh{(x)}}\,dx$$

input
```
integrate(tanh(x)**3/(a+b*sinh(x)),x)
```

output
```
Integral(tanh(x)**3/(a + b*sinh(x)), x)
```

Maxima [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 160, normalized size of antiderivative = 1.82

$$\int \frac{\tanh^3(x)}{a + b\sinh(x)}\,dx = -\frac{a^3\log\left(-2\,ae^{(-x)} + be^{(-2\,x)} - b\right)}{a^4 + 2\,a^2b^2 + b^4} + \frac{a^3\log\left(e^{(-2\,x)} + 1\right)}{a^4 + 2\,a^2b^2 + b^4} - \frac{(3\,a^2b + b^3)\arctan\left(e^{(-x)}\right)}{a^4 + 2\,a^2b^2 + b^4} - \frac{be^{(-x)} - 2\,ae^{(-2\,x)} - be^{(-3\,x)}}{a^2 + b^2 + 2\,(a^2 + b^2)e^{(-2\,x)} + (a^2 + b^2)e^{(-4\,x)}}$$

input
```
integrate(tanh(x)^3/(a+b*sinh(x)),x, algorithm="maxima")
```

output
```
-a^3*log(-2*a*e^(-x) + b*e^(-2*x) - b)/(a^4 + 2*a^2*b^2 + b^4) + a^3*log(e
^(-2*x) + 1)/(a^4 + 2*a^2*b^2 + b^4) - (3*a^2*b + b^3)*arctan(e^(-x))/(a^4
 + 2*a^2*b^2 + b^4) - (b*e^(-x) - 2*a*e^(-2*x) - b*e^(-3*x))/(a^2 + b^2 +
2*(a^2 + b^2)*e^(-2*x) + (a^2 + b^2)*e^(-4*x))
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 211 vs. $2(85) = 170$.

Time = 0.14 (sec) , antiderivative size = 211, normalized size of antiderivative = 2.40

$$\int \frac{\tanh^3(x)}{a+b\sinh(x)}\,dx = -\frac{a^3 b\log\left(\left|-b\left(e^{(-x)}-e^{x}\right)+2\,a\right|\right)}{a^4 b+2\,a^2 b^3+b^5}+\frac{a^3\log\left(\left(e^{(-x)}-e^{x}\right)^2+4\right)}{2\,(a^4+2\,a^2b^2+b^4)} + \frac{\left(\pi+2\,\arctan\left(\frac{1}{2}\left(e^{(2\,x)}-1\right)e^{(-x)}\right)\right)(3\,a^2 b+b^3)}{4\,(a^4+2\,a^2b^2+b^4)} - \frac{a^3\left(e^{(-x)}-e^{x}\right)^2-2\,a^2 b\left(e^{(-x)}-e^{x}\right)-2\,b^3\left(e^{(-x)}-e^{x}\right)-4\,ab^2}{2\,(a^4+2\,a^2b^2+b^4)\left(\left(e^{(-x)}-e^{x}\right)^2+4\right)}$$

input
```
integrate(tanh(x)^3/(a+b*sinh(x)),x, algorithm="giac")
```

output
```
-a^3*b*log(abs(-b*(e^(-x) - e^x) + 2*a))/(a^4*b + 2*a^2*b^3 + b^5) + 1/2*a
^3*log((e^(-x) - e^x)^2 + 4)/(a^4 + 2*a^2*b^2 + b^4) + 1/4*(pi + 2*arctan(
1/2*(e^(2*x) - 1)*e^(-x)))*(3*a^2*b + b^3)/(a^4 + 2*a^2*b^2 + b^4) - 1/2*(
a^3*(e^(-x) - e^x)^2 - 2*a^2*b*(e^(-x) - e^x) - 2*b^3*(e^(-x) - e^x) - 4*a
*b^2)/((a^4 + 2*a^2*b^2 + b^4)*((e^(-x) - e^x)^2 + 4))
```

Mupad [B] (verification not implemented)

Time = 3.24 (sec) , antiderivative size = 291, normalized size of antiderivative = 3.31

$$\int \frac{\tanh^3(x)}{a+b\sinh(x)}\,dx = \frac{\frac{2\,(a^3+a\,b^2)}{(a^2+b^2)^2}-\frac{e^x\,(a^2\,b+b^3)}{(a^2+b^2)^2}}{e^{2\,x}+1}-\frac{\frac{2\,a}{a^2+b^2}-\frac{2\,b\,e^x}{a^2+b^2}}{2\,e^{2\,x}+e^{4\,x}+1}+\frac{\ln\left(1+e^x\,1i\right)\,\left(2\,a+b\,1i\right)}{2\,\left(a^2+a\,b\,2i-b^2\right)} - \frac{a^3\,\ln\left(b^7\,e^{2\,x}-16\,a^6\,b-b^7-6\,a^2\,b^5-9\,a^4\,b^3+32\,a^7\,e^x+6\,a^2\,b^5\,e^{2\,x}+9\,a^4\,b^3\,e^{2\,x}+2\,a\,b^6\,e^x+16\,a^6\,b\,e\right.}{a^4+2\,a^2\,b^2+b^4} + \frac{\ln\left(e^x+1i\right)\,\left(b+a\,2i\right)}{2\,\left(a^2\,1i+2\,a\,b-b^2\,1i\right)}$$

input
```
int(tanh(x)^3/(a + b*sinh(x)),x)
```

output
```
((2*(a*b^2 + a^3))/(a^2 + b^2)^2 - (exp(x)*(a^2*b + b^3))/(a^2 + b^2)^2)/(
exp(2*x) + 1) - ((2*a)/(a^2 + b^2) - (2*b*exp(x))/(a^2 + b^2))/(2*exp(2*x)
 + exp(4*x) + 1) + (log(exp(x)*1i + 1)*(2*a + b*1i))/(2*(a*b*2i + a^2 - b^
2)) - (a^3*log(b^7*exp(2*x) - 16*a^6*b - b^7 - 6*a^2*b^5 - 9*a^4*b^3 + 32*
a^7*exp(x) + 6*a^2*b^5*exp(2*x) + 9*a^4*b^3*exp(2*x) + 2*a*b^6*exp(x) + 16
*a^6*b*exp(2*x) + 12*a^3*b^4*exp(x) + 18*a^5*b^2*exp(x)))/(a^4 + b^4 + 2*a
^2*b^2) + (log(exp(x) + 1i)*(a*2i + b))/(2*(2*a*b + a^2*1i - b^2*1i))
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 352, normalized size of antiderivative = 4.00

$$\int \frac{\tanh^3(x)}{a+b\sinh(x)}\,dx = \frac{3e^{4x}atan(e^x)\,a^2b+e^{4x}atan(e^x)\,b^3+6e^{2x}atan(e^x)\,a^2b+2e^{2x}atan(e^x)\,b^3+3atan(e^x)\,a^2b+atan(e^x)\,b^3+}{}$$

input
```
int(tanh(x)^3/(a+b*sinh(x)),x)
```

output
```
(3*e**(4*x)*atan(e**x)*a**2*b + e**(4*x)*atan(e**x)*b**3 + 6*e**(2*x)*atan
(e**x)*a**2*b + 2*e**(2*x)*atan(e**x)*b**3 + 3*atan(e**x)*a**2*b + atan(e*
*x)*b**3 + e**(4*x)*log(e**(2*x) + 1)*a**3 - e**(4*x)*log(e**(2*x)*b + 2*e
**x*a - b)*a**3 - e**(4*x)*a**3 - e**(4*x)*a*b**2 - e**(3*x)*a**2*b - e**(
3*x)*b**3 + 2*e**(2*x)*log(e**(2*x) + 1)*a**3 - 2*e**(2*x)*log(e**(2*x)*b
+ 2*e**x*a - b)*a**3 + e**x*a**2*b + e**x*b**3 + log(e**(2*x) + 1)*a**3 -
log(e**(2*x)*b + 2*e**x*a - b)*a**3 - a**3 - a*b**2)/(e**(4*x)*a**4 + 2*e*
*(4*x)*a**2*b**2 + e**(4*x)*b**4 + 2*e**(2*x)*a**4 + 4*e**(2*x)*a**2*b**2
+ 2*e**(2*x)*b**4 + a**4 + 2*a**2*b**2 + b**4)
```

3.230 $\int \frac{\tanh^2(x)}{a+b\sinh(x)}\,dx$

Optimal result

Integrand size = 13, antiderivative size = 69

$$\int \frac{\tanh^2(x)}{a+b\sinh(x)}\,dx=-\frac{2a^2\operatorname{arctanh}\left(\frac{b-a\tanh\left(\frac{x}{2}\right)}{\sqrt{a^2+b^2}}\right)}{\left(a^2+b^2\right)^{3/2}}-\frac{b\operatorname{sech}(x)}{a^2+b^2}-\frac{a\tanh(x)}{a^2+b^2}$$

output
```
-2*a^2*arctanh((b-a*tanh(1/2*x))/(a^2+b^2)^(1/2))/(a^2+b^2)^(3/2)-b*sech(x
)/(a^2+b^2)-a*tanh(x)/(a^2+b^2)
```

Mathematica [A] (verified)

Time = 0.17 (sec) , antiderivative size = 69, normalized size of antiderivative = 1.00

$$\int \frac{\tanh^2(x)}{a+b\sinh(x)}\,dx=\frac{-b\operatorname{sech}(x)+a\left(\frac{2a\arctan\left(\frac{b-a\tanh\left(\frac{x}{2}\right)}{\sqrt{-a^2-b^2}}\right)}{\sqrt{-a^2-b^2}}-\tanh(x)\right)}{a^2+b^2}$$

input
```
Integrate[Tanh[x]^2/(a + b*Sinh[x]),x]
```

output
```
(-(b*Sech[x]) + a*((2*a*ArcTan[(b - a*Tanh[x/2])/Sqrt[-a^2 - b^2]])/Sqrt[-
a^2 - b^2] - Tanh[x]))/(a^2 + b^2)
```

Rubi [A] (verified)

Time = 0.47 (sec) , antiderivative size = 74, normalized size of antiderivative = 1.07, number of steps used = 14, number of rules used = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 1.000$, Rules used = {3042, 25, 3206, 26, 3042, 26, 3086, 24, 3139, 1083, 219, 4254, 24}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\tanh^2(x)}{a+b\sinh(x)}dx$$

$$\Big\downarrow \text{3042}$$

$$\int -\frac{\tan(ix)^2}{a-ib\sin(ix)}dx$$

$$\Big\downarrow \text{25}$$

$$-\int \frac{\tan(ix)^2}{a-ib\sin(ix)}dx$$

$$\Big\downarrow \text{3206}$$

$$\frac{a^2\int \frac{1}{a+b\sinh(x)}dx}{a^2+b^2}-\frac{a\int \operatorname{sech}^2(x)dx}{a^2+b^2}-\frac{ib\int i\operatorname{sech}(x)\tanh(x)dx}{a^2+b^2}$$

$$\Big\downarrow \text{26}$$

$$\frac{a^2\int \frac{1}{a+b\sinh(x)}dx}{a^2+b^2}-\frac{a\int \operatorname{sech}^2(x)dx}{a^2+b^2}+\frac{b\int \operatorname{sech}(x)\tanh(x)dx}{a^2+b^2}$$

$$\Big\downarrow \text{3042}$$

$$\frac{a^2\int \frac{1}{a-ib\sin(ix)}dx}{a^2+b^2}-\frac{a\int \csc\left(ix+\frac{\pi}{2}\right)^2dx}{a^2+b^2}+\frac{b\int -i\sec(ix)\tan(ix)dx}{a^2+b^2}$$

$$\Big\downarrow \text{26}$$

$$\frac{a^2\int \frac{1}{a-ib\sin(ix)}dx}{a^2+b^2}-\frac{a\int \csc\left(ix+\frac{\pi}{2}\right)^2dx}{a^2+b^2}-\frac{ib\int \sec(ix)\tan(ix)dx}{a^2+b^2}$$

$$\Big\downarrow \text{3086}$$

$$\frac{a^2\int \frac{1}{a-ib\sin(ix)}dx}{a^2+b^2}-\frac{a\int \csc\left(ix+\frac{\pi}{2}\right)^2dx}{a^2+b^2}-\frac{b\int 1d\text{sech}(x)}{a^2+b^2}$$

$$\Big\downarrow \ \text{24}$$

$$\frac{a^2\int \frac{1}{a-ib\sin(ix)}dx}{a^2+b^2}-\frac{a\int \csc\left(ix+\frac{\pi}{2}\right)^2dx}{a^2+b^2}-\frac{b\text{sech}(x)}{a^2+b^2}$$

$$\Big\downarrow \ \text{3139}$$

$$-\frac{a\int \csc\left(ix+\frac{\pi}{2}\right)^2dx}{a^2+b^2}+\frac{2a^2\int \frac{1}{-a\tanh^2\left(\frac{x}{2}\right)+2b\tanh\left(\frac{x}{2}\right)+a}d\tanh\left(\frac{x}{2}\right)}{a^2+b^2}-\frac{b\text{sech}(x)}{a^2+b^2}$$

$$\Big\downarrow \ \text{1083}$$

$$-\frac{a\int \csc\left(ix+\frac{\pi}{2}\right)^2dx}{a^2+b^2}-\frac{4a^2\int \frac{1}{4(a^2+b^2)-\left(2b-2a\tanh\left(\frac{x}{2}\right)\right)^2}d\left(2b-2a\tanh\left(\frac{x}{2}\right)\right)}{a^2+b^2}-\frac{b\text{sech}(x)}{a^2+b^2}$$

$$\Big\downarrow \ \text{219}$$

$$-\frac{a\int \csc\left(ix+\frac{\pi}{2}\right)^2dx}{a^2+b^2}-\frac{2a^2\text{arctanh}\left(\frac{2b-2a\tanh\left(\frac{x}{2}\right)}{2\sqrt{a^2+b^2}}\right)}{(a^2+b^2)^{3/2}}-\frac{b\text{sech}(x)}{a^2+b^2}$$

$$\Big\downarrow \ \text{4254}$$

$$-\frac{ia\int 1d(-i\tanh(x))}{a^2+b^2}-\frac{2a^2\text{arctanh}\left(\frac{2b-2a\tanh\left(\frac{x}{2}\right)}{2\sqrt{a^2+b^2}}\right)}{(a^2+b^2)^{3/2}}-\frac{b\text{sech}(x)}{a^2+b^2}$$

$$\Big\downarrow \ \text{24}$$

$$-\frac{2a^2\text{arctanh}\left(\frac{2b-2a\tanh\left(\frac{x}{2}\right)}{2\sqrt{a^2+b^2}}\right)}{(a^2+b^2)^{3/2}}-\frac{a\tanh(x)}{a^2+b^2}-\frac{b\text{sech}(x)}{a^2+b^2}$$

input
```
Int[Tanh[x]^2/(a + b*Sinh[x]),x]
```

output
```
(-2*a^2*ArcTanh[(2*b - 2*a*Tanh[x/2])/(2*Sqrt[a^2 + b^2])])/(a^2 + b^2)^(3
/2) - (b*Sech[x])/(a^2 + b^2) - (a*Tanh[x])/(a^2 + b^2)
```

Defintions of rubi rules used

rule 24
```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1083
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Simp[-2   Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3086
```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Simp[a/f   Subst[Int[(a*x)^(m - 1)*(-1 + x^2)^((n - 1)/2
), x], x, Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2
] &&  !(IntegerQ[m/2] && LtQ[0, m, n + 1])
```

rule 3139
```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Simp[2*(e/d)   Subst[Int[1/(a + 2*b*e*x + a
*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[a^2 - b^2, 0]
```

rule 3206
```
Int[((g_.)*tan[(e_.) + (f_.)*(x_)])^(p_)/((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Simp[a/(a^2 - b^2)   Int[(g*Tan[e + f*x])^p/Sin[e + f*x]
^2, x], x] + (-Simp[b*(g/(a^2 - b^2))   Int[(g*Tan[e + f*x])^(p - 1)/Cos[e
+ f*x], x], x] - Simp[a^2*(g^2/(a^2 - b^2))   Int[(g*Tan[e + f*x])^(p - 2)/
(a + b*Sin[e + f*x]), x], x]) /; FreeQ[{a, b, e, f, g}, x] && NeQ[a^2 - b^2
, 0] && IntegersQ[2*p] && GtQ[p, 1]
```

rule 4254
```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Simp[-d^(-1)   Subst[Int[Exp
andIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
 d}, x] && IGtQ[n/2, 0]
```

Maple [A] (verified)

Time = 0.48 (sec) , antiderivative size = 84, normalized size of antiderivative = 1.22

method	result	size
default	$\frac{8a^{2}\operatorname{arctanh}\left(\frac{2a\tanh\left(\frac{x}{2}\right)-2b}{2\sqrt{a^{2}+b^{2}}}\right)}{\left(4a^{2}+4b^{2}\right)\sqrt{a^{2}+b^{2}}}+\frac{-2a\tanh\left(\frac{x}{2}\right)-2b}{\left(a^{2}+b^{2}\right)\left(1+\tanh\left(\frac{x}{2}\right)^{2}\right)}$	84
risch	$\frac{-2\,e^{x}b+2a}{\left(e^{2x}+1\right)\left(a^{2}+b^{2}\right)}+\frac{a^{2}\ln\left(e^{x}+\frac{a\left(a^{2}+b^{2}\right)^{\frac{3}{2}}-a^{4}-2a^{2}b^{2}-b^{4}}{b\left(a^{2}+b^{2}\right)^{\frac{3}{2}}}\right)}{\left(a^{2}+b^{2}\right)^{\frac{3}{2}}}-\frac{a^{2}\ln\left(e^{x}+\frac{a\left(a^{2}+b^{2}\right)^{\frac{3}{2}}+a^{4}+2a^{2}b^{2}+b^{4}}{b\left(a^{2}+b^{2}\right)^{\frac{3}{2}}}\right)}{\left(a^{2}+b^{2}\right)^{\frac{3}{2}}}$	145

input
```
int(tanh(x)^2/(a+b*sinh(x)),x,method=_RETURNVERBOSE)
```

output
```
8*a^2/(4*a^2+4*b^2)/(a^2+b^2)^(1/2)*arctanh(1/2*(2*a*tanh(1/2*x)-2*b)/(a^2
+b^2)^(1/2))+2/(a^2+b^2)*(-a*tanh(1/2*x)-b)/(1+tanh(1/2*x)^2)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 257 vs. $2(65) = 130$.

Time = 0.09 (sec) , antiderivative size = 257, normalized size of antiderivative = 3.72

$$\int \frac{\tanh^2(x)}{a + b\sinh(x)}\,dx$$
$$= \frac{2\,a^3 + 2\,ab^2 + \left(a^2\cosh\left(x\right)^2 + 2\,a^2\cosh\left(x\right)\sinh\left(x\right) + a^2\sinh\left(x\right)^2 + a^2\right)\sqrt{a^2+b^2}\log\left(\frac{b^2\cosh(x)^2+b^2\sinh(x)^2}{}\right.}{a^4 + 2\,a^2b^2 + b^4 + \left(a^4 + 2\,a^2b^2 + b^4\right)\cosh\left(x\right)^2 + 2\left(a^4\right.}$$

input
```
integrate(tanh(x)^2/(a+b*sinh(x)),x, algorithm="fricas")
```

output
```
(2*a^3 + 2*a*b^2 + (a^2*cosh(x)^2 + 2*a^2*cosh(x)*sinh(x) + a^2*sinh(x)^2
+ a^2)*sqrt(a^2 + b^2)*log((b^2*cosh(x)^2 + b^2*sinh(x)^2 + 2*a*b*cosh(x)
+ 2*a^2 + b^2 + 2*(b^2*cosh(x) + a*b)*sinh(x) - 2*sqrt(a^2 + b^2)*(b*cosh(
x) + b*sinh(x) + a))/(b*cosh(x)^2 + b*sinh(x)^2 + 2*a*cosh(x) + 2*(b*cosh(
x) + a)*sinh(x) - b)) - 2*(a^2*b + b^3)*cosh(x) - 2*(a^2*b + b^3)*sinh(x))
/(a^4 + 2*a^2*b^2 + b^4 + (a^4 + 2*a^2*b^2 + b^4)*cosh(x)^2 + 2*(a^4 + 2*a
^2*b^2 + b^4)*cosh(x)*sinh(x) + (a^4 + 2*a^2*b^2 + b^4)*sinh(x)^2)
```

Sympy [F]

$$\int \frac{\tanh^2(x)}{a + b\sinh(x)}\,dx = \int \frac{\tanh^2\left(x\right)}{a + b\sinh\left(x\right)}\,dx$$

input
```
integrate(tanh(x)**2/(a+b*sinh(x)),x)
```

output
```
Integral(tanh(x)**2/(a + b*sinh(x)), x)
```

Maxima [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 89, normalized size of antiderivative = 1.29

$$\int \frac{\tanh^2(x)}{a+b\sinh(x)}\,dx = \frac{a^2\log\left(\frac{be^{(-x)}-a-\sqrt{a^2+b^2}}{be^{(-x)}-a+\sqrt{a^2+b^2}}\right)}{(a^2+b^2)^{\frac{3}{2}}} - \frac{2\left(be^{(-x)}+a\right)}{a^2+b^2+(a^2+b^2)e^{(-2x)}}$$

input
```
integrate(tanh(x)^2/(a+b*sinh(x)),x, algorithm="maxima")
```

output
```
a^2*log((b*e^(-x) - a - sqrt(a^2 + b^2))/(b*e^(-x) - a + sqrt(a^2 + b^2)))
/(a^2 + b^2)^(3/2) - 2*(b*e^(-x) + a)/(a^2 + b^2 + (a^2 + b^2)*e^(-2*x))
```

Giac [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 87, normalized size of antiderivative = 1.26

$$\int \frac{\tanh^2(x)}{a+b\sinh(x)}\,dx = \frac{a^2\log\left(\frac{\left|2\,be^x+2\,a-2\,\sqrt{a^2+b^2}\right|}{\left|2\,be^x+2\,a+2\,\sqrt{a^2+b^2}\right|}\right)}{(a^2+b^2)^{\frac{3}{2}}} - \frac{2\,(be^x-a)}{(a^2+b^2)(e^{(2x)}+1)}$$

input
```
integrate(tanh(x)^2/(a+b*sinh(x)),x, algorithm="giac")
```

output
```
a^2*log(abs(2*b*e^x + 2*a - 2*sqrt(a^2 + b^2))/abs(2*b*e^x + 2*a + 2*sqrt(
a^2 + b^2)))/(a^2 + b^2)^(3/2) - 2*(b*e^x - a)/((a^2 + b^2)*(e^(2*x) + 1))
```

Mupad [B] (verification not implemented)

Time = 1.86 (sec) , antiderivative size = 330, normalized size of antiderivative = 4.78

$$\int \frac{\tanh^2(x)}{a+b\sinh(x)}\,dx = \frac{\frac{2\,a}{a^2+b^2}-\frac{2\,b\,\mathrm{e}^x}{a^2+b^2}}{\mathrm{e}^{2\,x}+1} - \frac{2\,\mathrm{atan}\left(\left(\frac{b^3\,\sqrt{-a^6-3\,a^4\,b^2-3\,a^2\,b^4-b^6}}{2}+\frac{a^2\,b\,\sqrt{-a^6-3\,a^4\,b^2-3\,a^2\,b^4-b^6}}{2}\right)\left(\mathrm{e}^x\left(\frac{2\,a^2}{b^2\,\sqrt{a^4}\,(a^2+b^2)^2}+\frac{2\left(a^3\,\sqrt{a^4}+\right.}{a\,b^2\,\sqrt{-(a^2+b^2)^3}\,(a^2+b^2)\,\sqrt{}}\right.\right.\right.}{\sqrt{-a^6-3\,a^4\,b^2-3\,a^2\,b^4-b^6}}$$

input
```
int(tanh(x)^2/(a + b*sinh(x)),x)
```

output
```
((2*a)/(a^2 + b^2) - (2*b*exp(x))/(a^2 + b^2))/(exp(2*x) + 1) - (2*atan(((
b^3*(- a^6 - b^6 - 3*a^2*b^4 - 3*a^4*b^2)^(1/2))/2 + (a^2*b*(- a^6 - b^6 -
 3*a^2*b^4 - 3*a^4*b^2)^(1/2))/2)*(exp(x)*((2*a^2)/(b^2*(a^4)^(1/2)*(a^2 +
 b^2)^2) + (2*(a^3*(a^4)^(1/2) + a*b^2*(a^4)^(1/2)))/(a*b^2*(-(a^2 + b^2)^
3)^(1/2)*(a^2 + b^2)*(- a^6 - b^6 - 3*a^2*b^4 - 3*a^4*b^2)^(1/2))) - (2*(b
^3*(a^4)^(1/2) + a^2*b*(a^4)^(1/2)))/(a*b^2*(-(a^2 + b^2)^3)^(1/2)*(a^2 +
b^2)*(- a^6 - b^6 - 3*a^2*b^4 - 3*a^4*b^2)^(1/2))))*(a^4)^(1/2))/(- a^6 -
b^6 - 3*a^2*b^4 - 3*a^4*b^2)^(1/2)
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 164, normalized size of antiderivative = 2.38

$$\int \frac{\tanh^2(x)}{a+b\sinh(x)}\,dx = \frac{2e^{2x}\sqrt{a^2+b^2}\,atan\left(\frac{e^x bi+ai}{\sqrt{a^2+b^2}}\right)a^2 i+2\sqrt{a^2+b^2}\,atan\left(\frac{e^x bi+ai}{\sqrt{a^2+b^2}}\right)a^2 i-2e^{2x}a^3-2e^{2x}a\,b^2-2e^x a^2 b-2e^x b^3}{e^{2x}a^4+2e^{2x}a^2b^2+e^{2x}b^4+a^4+2a^2b^2+b^4}$$

input
```
int(tanh(x)^2/(a+b*sinh(x)),x)
```

output
```
(2*(e**(2*x)*sqrt(a**2 + b**2)*atan((e**x*b*i + a*i)/sqrt(a**2 + b**2))*a*
*2*i + sqrt(a**2 + b**2)*atan((e**x*b*i + a*i)/sqrt(a**2 + b**2))*a**2*i -
 e**(2*x)*a**3 - e**(2*x)*a*b**2 - e**x*a**2*b - e**x*b**3))/(e**(2*x)*a**
4 + 2*e**(2*x)*a**2*b**2 + e**(2*x)*b**4 + a**4 + 2*a**2*b**2 + b**4)
```

3.231 $\int \frac{\tanh(x)}{a+b\sinh(x)}\,dx$

Optimal result

Integrand size = 11, antiderivative size = 48

$$\int \frac{\tanh(x)}{a+b\sinh(x)}\,dx = \frac{b\arctan(\sinh(x))}{a^2+b^2} + \frac{a\log(\cosh(x))}{a^2+b^2} - \frac{a\log(a+b\sinh(x))}{a^2+b^2}$$

output
```
b*arctan(sinh(x))/(a^2+b^2)+a*ln(cosh(x))/(a^2+b^2)-a*ln(a+b*sinh(x))/(a^2
+b^2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.04 (sec) , antiderivative size = 56, normalized size of antiderivative = 1.17

$$\int \frac{\tanh(x)}{a+b\sinh(x)}\,dx = \frac{(a-ib)\log(i-\sinh(x))+(a+ib)\log(i+\sinh(x))-2a\log(a+b\sinh(x))}{2(a^2+b^2)}$$

input
```
Integrate[Tanh[x]/(a + b*Sinh[x]),x]
```

output
```
((a - I*b)*Log[I - Sinh[x]] + (a + I*b)*Log[I + Sinh[x]] - 2*a*Log[a + b*S
inh[x]])/(2*(a^2 + b^2))
```

Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 54, normalized size of antiderivative = 1.12, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.818$, Rules used = {3042, 26, 3200, 25, 587, 16, 452, 216, 240}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\tanh(x)}{a+b\sinh(x)}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int -\frac{i\tan(ix)}{a-ib\sin(ix)}dx$$

$$\Big\downarrow \text{26}$$

$$-i\int \frac{\tan(ix)}{a-ib\sin(ix)}dx$$

$$\Big\downarrow \text{3200}$$

$$-\int -\frac{b\sinh(x)}{(a+b\sinh(x))\left(\sinh^2(x)b^2+b^2\right)}d(b\sinh(x))$$

$$\Big\downarrow \text{25}$$

$$\int \frac{b\sinh(x)}{\left(b^2\sinh^2(x)+b^2\right)(a+b\sinh(x))}d(b\sinh(x))$$

$$\Big\downarrow \text{587}$$

$$\frac{\int \frac{b^2+a\sinh(x)b}{\sinh^2(x)b^2+b^2}d(b\sinh(x))}{a^2+b^2}-\frac{a\int \frac{1}{a+b\sinh(x)}d(b\sinh(x))}{a^2+b^2}$$

$$\Big\downarrow \text{16}$$

$$\frac{\int \frac{b^2+a\sinh(x)b}{\sinh^2(x)b^2+b^2}d(b\sinh(x))}{a^2+b^2}-\frac{a\log(a+b\sinh(x))}{a^2+b^2}$$

$$\Big\downarrow \text{452}$$

$$\frac{a\int \frac{b\sinh(x)}{\sinh^2(x)b^2+b^2}d(b\sinh(x))+b^2\int \frac{1}{\sinh^2(x)b^2+b^2}d(b\sinh(x))}{a^2+b^2}-\frac{a\log(a+b\sinh(x))}{a^2+b^2}$$

$$\Big\downarrow \text{216}$$

$$\frac{a\int \frac{b\sinh(x)}{\sinh^2(x)b^2+b^2}d(b\sinh(x))+b\arctan(\sinh(x))}{a^2+b^2}-\frac{a\log(a+b\sinh(x))}{a^2+b^2}$$

$$\Big\downarrow \text{240}$$

$$\frac{\frac{1}{2}a\log\left(b^2\sinh^2(x)+b^2\right)+b\arctan(\sinh(x))}{a^2+b^2}-\frac{a\log(a+b\sinh(x))}{a^2+b^2}$$

input
```
Int[Tanh[x]/(a + b*Sinh[x]),x]
```

output
```
-((a*Log[a + b*Sinh[x]])/(a^2 + b^2)) + (b*ArcTan[Sinh[x]] + (a*Log[b^2 +
b^2*Sinh[x]^2])/2)/(a^2 + b^2)
```

Defintions of rubi rules used

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 240
```
Int[(x_)/((a_) + (b_.)*(x_)^2), x_Symbol] :> Simp[Log[RemoveContent[a + b*x
^2, x]]/(2*b), x] /; FreeQ[{a, b}, x]
```

rule 452
```
Int[((c_) + (d_.)*(x_))/((a_) + (b_.)*(x_)^2), x_Symbol] :> Simp[c   Int[1/
(a + b*x^2), x], x] + Simp[d   Int[x/(a + b*x^2), x], x] /; FreeQ[{a, b, c,
 d}, x] && NeQ[b*c^2 + a*d^2, 0]
```

rule 587
```
Int[(x_.)/(((c_) + (d_.)*(x_))*((a_) + (b_.)*(x_)^2)), x_Symbol] :> Simp[(-
c)*(d/(b*c^2 + a*d^2))   Int[1/(c + d*x), x], x] + Simp[1/(b*c^2 + a*d^2)
 Int[(a*d + b*c*x)/(a + b*x^2), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c
^2 + a*d^2, 0]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3200
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*tan[(e_.) + (f_.)*(x_)]^(p
_.), x_Symbol] :> Simp[1/f   Subst[Int[(x^p*(a + x)^m)/(b^2 - x^2)^((p + 1)
/2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && NeQ[a^2 - b
^2, 0] && IntegerQ[(p + 1)/2]
```

Maple [A] (verified)

Time = 0.33 (sec) , antiderivative size = 73, normalized size of antiderivative = 1.52

method	result	size
default	$-\frac{2a\ln\left(\tanh\left(\frac{x}{2}\right)^{2}a-2b\tanh\left(\frac{x}{2}\right)-a\right)}{2a^{2}+2b^{2}}+\frac{2a\ln\left(1+\tanh\left(\frac{x}{2}\right)^{2}\right)+4b\arctan\left(\tanh\left(\frac{x}{2}\right)\right)}{2a^{2}+2b^{2}}$	73
risch	$\frac{i\ln\left(\mathrm{e}^{x}+i\right)b}{a^{2}+b^{2}}+\frac{\ln\left(\mathrm{e}^{x}+i\right)a}{a^{2}+b^{2}}-\frac{i\ln\left(\mathrm{e}^{x}-i\right)b}{a^{2}+b^{2}}+\frac{\ln\left(\mathrm{e}^{x}-i\right)a}{a^{2}+b^{2}}-\frac{a\ln\left(\mathrm{e}^{2x}+\frac{2a\,\mathrm{e}^{x}}{b}-1\right)}{a^{2}+b^{2}}$	101

input
```
int(tanh(x)/(a+b*sinh(x)),x,method=_RETURNVERBOSE)
```

output
```
-2*a/(2*a^2+2*b^2)*ln(tanh(1/2*x)^2*a-2*b*tanh(1/2*x)-a)+4/(2*a^2+2*b^2)*(
1/2*a*ln(1+tanh(1/2*x)^2)+b*arctan(tanh(1/2*x)))
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 57, normalized size of antiderivative = 1.19

$$\int \frac{\tanh(x)}{a+b\sinh(x)}\,dx = \frac{2\,b\arctan\left(\cosh\left(x\right)+\sinh\left(x\right)\right)-a\log\left(\frac{2\,(b\sinh(x)+a)}{\cosh(x)-\sinh(x)}\right)+a\log\left(\frac{2\,\cosh(x)}{\cosh(x)-\sinh(x)}\right)}{a^2+b^2}$$

input
```
integrate(tanh(x)/(a+b*sinh(x)),x, algorithm="fricas")
```

output
```
(2*b*arctan(cosh(x) + sinh(x)) - a*log(2*(b*sinh(x) + a)/(cosh(x) - sinh(x
))) + a*log(2*cosh(x)/(cosh(x) - sinh(x))))/(a^2 + b^2)
```

Sympy [F]

$$\int \frac{\tanh(x)}{a+b\sinh(x)}\,dx = \int \frac{\tanh\left(x\right)}{a+b\sinh\left(x\right)}\,dx$$

input
```
integrate(tanh(x)/(a+b*sinh(x)),x)
```

output
```
Integral(tanh(x)/(a + b*sinh(x)), x)
```

Maxima [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 66, normalized size of antiderivative = 1.38

$$\int \frac{\tanh(x)}{a+b\sinh(x)}\,dx = -\frac{2\,b\arctan\left(e^{(-x)}\right)}{a^2+b^2} - \frac{a\log\left(-2\,ae^{(-x)}+be^{(-2\,x)}-b\right)}{a^2+b^2} + \frac{a\log\left(e^{(-2\,x)}+1\right)}{a^2+b^2}$$

input

```
integrate(tanh(x)/(a+b*sinh(x)),x, algorithm="maxima")
```

output

```
-2*b*arctan(e^(-x))/(a^2 + b^2) - a*log(-2*a*e^(-x) + b*e^(-2*x) - b)/(a^2
 + b^2) + a*log(e^(-2*x) + 1)/(a^2 + b^2)
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 89, normalized size of antiderivative = 1.85

$$\int \frac{\tanh(x)}{a+b\sinh(x)}\,dx = -\frac{ab\log\left(\left|-b\left(e^{(-x)}-e^x\right)+2\,a\right|\right)}{a^2b+b^3} + \frac{\left(\pi+2\,\arctan\left(\frac{1}{2}\left(e^{(2\,x)}-1\right)e^{(-x)}\right)\right)b}{2\,(a^2+b^2)} + \frac{a\log\left(\left(e^{(-x)}-e^x\right)^2+4\right)}{2\,(a^2+b^2)}$$

input

```
integrate(tanh(x)/(a+b*sinh(x)),x, algorithm="giac")
```

output

```
-a*b*log(abs(-b*(e^(-x) - e^x) + 2*a))/(a^2*b + b^3) + 1/2*(pi + 2*arctan(
1/2*(e^(2*x) - 1)*e^(-x)))*b/(a^2 + b^2) + 1/2*a*log((e^(-x) - e^x)^2 + 4)
/(a^2 + b^2)
```

Mupad [B] (verification not implemented)

Time = 2.34 (sec) , antiderivative size = 95, normalized size of antiderivative = 1.98

$$\int \frac{\tanh(x)}{a+b\sinh(x)}\,dx = \frac{\ln\left(e^x+1i\right)}{a-b\,1i} - \frac{a\,\ln\left(b^3\,e^{2\,x}-4\,a^2\,b-b^3+8\,a^3\,e^x+2\,a\,b^2\,e^x+4\,a^2\,b\,e^{2\,x}\right)}{a^2+b^2} + \frac{\ln\left(1+e^x\,1i\right)\,1i}{-b+a\,1i}$$

input
```
int(tanh(x)/(a + b*sinh(x)),x)
```

output
```
(log(exp(x)*1i + 1)*1i)/(a*1i - b) + log(exp(x) + 1i)/(a - b*1i) - (a*log(
b^3*exp(2*x) - 4*a^2*b - b^3 + 8*a^3*exp(x) + 2*a*b^2*exp(x) + 4*a^2*b*exp
(2*x)))/(a^2 + b^2)
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 49, normalized size of antiderivative = 1.02

$$\int \frac{\tanh(x)}{a+b\sinh(x)}\,dx = \frac{2atan(e^x)\,b+\log(e^{2x}+1)\,a-\log(e^{2x}b+2e^xa-b)\,a}{a^2+b^2}$$

input
```
int(tanh(x)/(a+b*sinh(x)),x)
```

output
```
(2*atan(e**x)*b + log(e**(2*x) + 1)*a - log(e**(2*x)*b + 2*e**x*a - b)*a)/
(a**2 + b**2)
```

3.232 $\int \frac{\coth(x)}{a+b\sinh(x)} dx$

Optimal result

Integrand size = 11, antiderivative size = 20

$$\int \frac{\coth(x)}{a+b\sinh(x)} dx = \frac{\log(\sinh(x))}{a} - \frac{\log(a+b\sinh(x))}{a}$$

output
```
ln(sinh(x))/a-ln(a+b*sinh(x))/a
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00

$$\int \frac{\coth(x)}{a+b\sinh(x)} dx = \frac{\log(\sinh(x))}{a} - \frac{\log(a+b\sinh(x))}{a}$$

input
```
Integrate[Coth[x]/(a + b*Sinh[x]),x]
```

output
```
Log[Sinh[x]]/a - Log[a + b*Sinh[x]]/a
```

Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.10, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.545$, Rules used = {3042, 26, 3200, 47, 14, 16}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\coth(x)}{a+b\sinh(x)}dx$$
$$\Big\downarrow \text{ 3042}$$
$$\int \frac{i}{\tan(ix)(a-ib\sin(ix))}dx$$
$$\Big\downarrow \text{ 26}$$
$$i\int \frac{1}{(a-ib\sin(ix))\tan(ix)}dx$$
$$\Big\downarrow \text{ 3200}$$
$$\int \frac{\operatorname{csch}(x)}{b(a+b\sinh(x))}d(b\sinh(x))$$
$$\Big\downarrow \text{ 47}$$
$$\frac{\int \frac{\operatorname{csch}(x)}{b}d(b\sinh(x))}{a}-\frac{\int \frac{1}{a+b\sinh(x)}d(b\sinh(x))}{a}$$
$$\Big\downarrow \text{ 14}$$
$$\frac{\log(b\sinh(x))}{a}-\frac{\int \frac{1}{a+b\sinh(x)}d(b\sinh(x))}{a}$$
$$\Big\downarrow \text{ 16}$$
$$\frac{\log(b\sinh(x))}{a}-\frac{\log(a+b\sinh(x))}{a}$$

input
```
Int[Coth[x]/(a + b*Sinh[x]),x]
```

output
```
Log[b*Sinh[x]]/a - Log[a + b*Sinh[x]]/a
```

Defintions of rubi rules used

rule 14
```
Int[(a_.)/(x_), x_Symbol] :> Simp[a*Log[x], x] /; FreeQ[a, x]
```

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a + 
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 47
```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] :> Simp[b/(b*c
 - a*d)   Int[1/(a + b*x), x], x] - Simp[d/(b*c - a*d)   Int[1/(c + d*x), x
], x] /; FreeQ[{a, b, c, d}, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3200
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*tan[(e_.) + (f_.)*(x_)]^(p
_.), x_Symbol] :> Simp[1/f   Subst[Int[(x^p*(a + x)^m)/(b^2 - x^2)^((p + 1)
/2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && NeQ[a^2 - b
^2, 0] && IntegerQ[(p + 1)/2]
```

Maple [A] (verified)

Time = 0.43 (sec) , antiderivative size = 33, normalized size of antiderivative = 1.65

method	result	size
risch	$\frac{\ln(e^{2x}-1)}{a}-\frac{\ln\left(e^{2x}+\frac{2a\,e^{x}}{b}-1\right)}{a}$	33
default	$\frac{\ln\left(\tanh\left(\frac{x}{2}\right)\right)}{a}-\frac{\ln\left(\tanh\left(\frac{x}{2}\right)^{2}a-2b\tanh\left(\frac{x}{2}\right)-a\right)}{a}$	36

input
```
int(coth(x)/(a+b*sinh(x)),x,method=_RETURNVERBOSE)
```

output
```
1/a*ln(exp(2*x)-1)-1/a*ln(exp(2*x)+2/b*a*exp(x)-1)
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 40, normalized size of antiderivative = 2.00

$$\int \frac{\coth(x)}{a+b\sinh(x)}\,dx = -\frac{\log\left(\frac{2\,(b\sinh(x)+a)}{\cosh(x)-\sinh(x)}\right)-\log\left(\frac{2\,\sinh(x)}{\cosh(x)-\sinh(x)}\right)}{a}$$

input
```
integrate(coth(x)/(a+b*sinh(x)),x, algorithm="fricas")
```

output
```
-(log(2*(b*sinh(x) + a)/(cosh(x) - sinh(x))) - log(2*sinh(x)/(cosh(x) - si
nh(x))))/a
```

Sympy [F]

$$\int \frac{\coth(x)}{a+b\sinh(x)}\,dx = \int \frac{\coth(x)}{a+b\sinh(x)}\,dx$$

input
```
integrate(coth(x)/(a+b*sinh(x)),x)
```

output
```
Integral(coth(x)/(a + b*sinh(x)), x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 46 vs. $2(20) = 40$.

Time = 0.04 (sec) , antiderivative size = 46, normalized size of antiderivative = 2.30

$$\int \frac{\coth(x)}{a+b\sinh(x)}\,dx = -\frac{\log\left(-2\,ae^{(-x)}+be^{(-2\,x)}-b\right)}{a}+\frac{\log\left(e^{(-x)}+1\right)}{a}+\frac{\log\left(e^{(-x)}-1\right)}{a}$$

input
```
integrate(coth(x)/(a+b*sinh(x)),x, algorithm="maxima")
```

output
```
-log(-2*a*e^(-x) + b*e^(-2*x) - b)/a + log(e^(-x) + 1)/a + log(e^(-x) - 1)
/a
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 39, normalized size of antiderivative = 1.95

$$\int \frac{\coth(x)}{a+b\sinh(x)}\,dx = -\frac{\log\left(\left|-b\left(e^{(-x)}-e^{x}\right)+2\,a\right|\right)}{a}+\frac{\log\left(\left|-e^{(-x)}+e^{x}\right|\right)}{a}$$

input
```
integrate(coth(x)/(a+b*sinh(x)),x, algorithm="giac")
```

output
```
-log(abs(-b*(e^(-x) - e^x) + 2*a))/a + log(abs(-e^(-x) + e^x))/a
```

Mupad [B] (verification not implemented)

Time = 0.43 (sec) , antiderivative size = 195, normalized size of antiderivative = 9.75

$$\int \frac{\coth(x)}{a+b\sinh(x)}\,dx = \frac{2\,\mathrm{atan}\left(\frac{a\sqrt{-a^2}+b\,e^{x}\sqrt{-a^2}-2\,a\,e^{2\,x}\sqrt{-a^2}-b\,e^{3\,x}\sqrt{-a^2}}{a^2}\right)}{\sqrt{-a^2}} - \frac{2\,\mathrm{atan}\left(\left(4\,a^4\,b\sqrt{-a^2}+4\,a^2\,b^3\sqrt{-a^2}\right)\left(\frac{1}{8\,a\,b\,(a^2+b^2)^2}-e^{x}\left(\frac{1}{16\,b^2\,(a^2+b^2)^2}-\frac{(a^2+2\,b^2)^2}{16\,a^4\,b^2\,(a^2+b^2)^2}\right)+\frac{a^2+2\,b^2}{8\,a^3\,b\,(a^2+b^2)^2}\right)\right.}{\sqrt{-a^2}}$$

input
```
int(coth(x)/(a + b*sinh(x)),x)
```

output
```
(2*atan((a*(-a^2)^(1/2) + b*exp(x)*(-a^2)^(1/2) - 2*a*exp(2*x)*(-a^2)^(1/2
) - b*exp(3*x)*(-a^2)^(1/2))/a^2))/(-a^2)^(1/2) - (2*atan((4*a^4*b*(-a^2)^
(1/2) + 4*a^2*b^3*(-a^2)^(1/2))*(1/(8*a*b*(a^2 + b^2)^2) - exp(x)*(1/(16*b
^2*(a^2 + b^2)^2) - (a^2 + 2*b^2)^2/(16*a^4*b^2*(a^2 + b^2)^2)) + (a^2 + 2
*b^2)/(8*a^3*b*(a^2 + b^2)^2))))/(-a^2)^(1/2)
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.85

$$\int \frac{\coth(x)}{a + b\sinh(x)}\,dx = \frac{\log(e^x - 1) + \log(e^x + 1) - \log(e^{2x}b + 2e^x a - b)}{a}$$

input
```
int(coth(x)/(a+b*sinh(x)),x)
```

output
```
(log(e**x - 1) + log(e**x + 1) - log(e**(2*x)*b + 2*e**x*a - b))/a
```

3.233 $\int \frac{\coth^2(x)}{a+b\sinh(x)}\,dx$

Optimal result

Integrand size = 13, antiderivative size = 56

$$\int \frac{\coth^2(x)}{a+b\sinh(x)}\,dx=\frac{b\text{arctanh}(\cosh(x))}{a^2}-\frac{2\sqrt{a^2+b^2}\text{arctanh}\left(\frac{b-a\tanh\left(\frac{x}{2}\right)}{\sqrt{a^2+b^2}}\right)}{a^2}-\frac{\coth(x)}{a}$$

output
```
b*arctanh(cosh(x))/a^2-2*(a^2+b^2)^(1/2)*arctanh((b-a*tanh(1/2*x))/(a^2+b^
2)^(1/2))/a^2-coth(x)/a
```

Mathematica [A] (verified)

Time = 0.22 (sec) , antiderivative size = 89, normalized size of antiderivative = 1.59

$$\int \frac{\coth^2(x)}{a+b\sinh(x)}\,dx = -\frac{4\sqrt{-a^2-b^2}\arctan\left(\frac{b-a\tanh\left(\frac{x}{2}\right)}{\sqrt{-a^2-b^2}}\right)+a\coth\left(\frac{x}{2}\right)-2b\log\left(\cosh\left(\frac{x}{2}\right)\right)+2b\log\left(\sinh\left(\frac{x}{2}\right)\right)+a\tanh\left(\frac{x}{2}\right)}{2a^2}$$

input
```
Integrate[Coth[x]^2/(a + b*Sinh[x]),x]
```

output
```
-1/2*(4*Sqrt[-a^2 - b^2]*ArcTan[(b - a*Tanh[x/2])/Sqrt[-a^2 - b^2]] + a*Co
th[x/2] - 2*b*Log[Cosh[x/2]] + 2*b*Log[Sinh[x/2]] + a*Tanh[x/2])/a^2
```

Rubi [C] (verified)

Result contains complex when optimal does not.

Time = 0.63 (sec) , antiderivative size = 74, normalized size of antiderivative = 1.32, number of steps used = 18, number of rules used = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 1.308, Rules used = {3042, 25, 3202, 25, 3042, 25, 3535, 3042, 26, 3480, 26, 3042, 26, 3139, 1083, 219, 4257}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\coth^2(x)}{a+b\sinh(x)}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int -\frac{1}{\tan(ix)^2(a-ib\sin(ix))}dx$$

$$\Big\downarrow \text{25}$$

$$-\int \frac{1}{(a-ib\sin(ix))\tan(ix)^2}dx$$

$$\Big\downarrow \text{3202}$$

$$-\int -\frac{\text{csch}^2(x)\left(\sinh^2(x)+1\right)}{a+b\sinh(x)}dx$$

$$\Big\downarrow \text{25}$$

$$\int \frac{\left(\sinh^2(x)+1\right)\text{csch}^2(x)}{a+b\sinh(x)}dx$$

$$\Big\downarrow \text{3042}$$

$$\int -\frac{1-\sin(ix)^2}{\sin(ix)^2(a-ib\sin(ix))}dx$$

$$\Big\downarrow \text{25}$$

$$-\int \frac{1-\sin(ix)^2}{\sin(ix)^2(a-ib\sin(ix))}dx$$

$$\Big\downarrow \text{3535}$$

$$-\frac{\int \frac{\text{csch}(x)(b-a\sinh(x))}{a+b\sinh(x)}dx}{a}-\frac{\coth(x)}{a}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{\coth(x)}{a}-\frac{\int \frac{i(b+ia\sin(ix))}{\sin(ix)(a-ib\sin(ix))}dx}{a}$$

$$\Big\downarrow \text{26}$$

$$-\frac{\coth(x)}{a}-\frac{i\int \frac{b+ia\sin(ix)}{\sin(ix)(a-ib\sin(ix))}dx}{a}$$

$$\Big\downarrow \text{3480}$$

$$-\frac{\coth(x)}{a}-\frac{i\left(\frac{i(a^2+b^2)\int \frac{1}{a+b\sinh(x)}dx}{a}+\frac{b\int -i\text{csch}(x)dx}{a}\right)}{a}$$

$$\Big\downarrow \text{26}$$

$$-\frac{\coth(x)}{a}-\frac{i\left(\frac{i(a^2+b^2)\int \frac{1}{a+b\sinh(x)}dx}{a}-\frac{ib\int \text{csch}(x)dx}{a}\right)}{a}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{\coth(x)}{a}-\frac{i\left(\frac{i(a^2+b^2)\int \frac{1}{a-ib\sin(ix)}dx}{a}-\frac{ib\int i\csc(ix)dx}{a}\right)}{a}$$

$$\Big\downarrow \text{26}$$

$$-\frac{\coth(x)}{a}-\frac{i\left(\frac{i(a^2+b^2)\int \frac{1}{a-ib\sin(ix)}dx}{a}+\frac{b\int \csc(ix)dx}{a}\right)}{a}$$

$$\Big\downarrow \text{3139}$$

$$-\frac{\coth(x)}{a}-\frac{i\left(\frac{2i(a^2+b^2)\int \frac{1}{-a\tanh^2\left(\frac{x}{2}\right)+2b\tanh\left(\frac{x}{2}\right)+a}d\tanh\left(\frac{x}{2}\right)}{a}+\frac{b\int \csc(ix)dx}{a}\right)}{a}$$

$$\Big\downarrow \text{1083}$$

$$-\frac{\coth(x)}{a}-\frac{i\left(\frac{b\int \csc(ix)dx}{a}-\frac{4i(a^2+b^2)\int \frac{1}{4(a^2+b^2)-\left(2b-2a\tanh\left(\frac{x}{2}\right)\right)^2}d\left(2b-2a\tanh\left(\frac{x}{2}\right)\right)}{a}\right)}{a}$$

$$\Big\downarrow \text{219}$$

$$-\frac{\coth(x)}{a}-\frac{i\left(\frac{b\int \csc(ix)dx}{a}-\frac{2i\sqrt{a^2+b^2}\text{arctanh}\left(\frac{2b-2a\tanh\left(\frac{x}{2}\right)}{2\sqrt{a^2+b^2}}\right)}{a}\right)}{a}$$

$$\Big\downarrow \text{4257}$$

$$-\frac{\coth(x)}{a}-\frac{i\left(\frac{ib\text{arctanh}(\cosh(x))}{a}-\frac{2i\sqrt{a^2+b^2}\text{arctanh}\left(\frac{2b-2a\tanh\left(\frac{x}{2}\right)}{2\sqrt{a^2+b^2}}\right)}{a}\right)}{a}$$

input
```
Int[Coth[x]^2/(a + b*Sinh[x]),x]
```

output
```
((-I)*((I*b*ArcTanh[Cosh[x]])/a - ((2*I)*Sqrt[a^2 + b^2]*ArcTanh[(2*b - 2*
a*Tanh[x/2])/(2*Sqrt[a^2 + b^2])])/a))/a - Coth[x]/a
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1083
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Simp[-2   Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3139
```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Simp[2*(e/d)   Subst[Int[1/(a + 2*b*e*x + a
*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[a^2 - b^2, 0]
```

rule 3202
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)/tan[(e_.) + (f_.)*(x_)]^2,
x_Symbol] :> Int[(a + b*Sin[e + f*x])^m*((1 - Sin[e + f*x]^2)/Sin[e + f*x]^
2), x] /; FreeQ[{a, b, e, f, m}, x] && NeQ[a^2 - b^2, 0]
```

rule 3480
```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((a_.) + (b_.)*sin[(e_.) + (f_
.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])), x_Symbol] :> Simp[(A*b
- a*B)/(b*c - a*d)   Int[1/(a + b*Sin[e + f*x]), x], x] + Simp[(B*c - A*d)/
(b*c - a*d)   Int[1/(c + d*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f
, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

rule 3535
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :>
Simp[(-(A*b^2 + a^2*C))*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m + 1)*((c + d*S
in[e + f*x])^(n + 1)/(f*(m + 1)*(b*c - a*d)*(a^2 - b^2))), x] + Simp[1/((m
+ 1)*(b*c - a*d)*(a^2 - b^2))   Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin
[e + f*x])^n*Simp[a*(m + 1)*(b*c - a*d)*(A + C) + d*(A*b^2 + a^2*C)*(m + n
+ 2) - (c*(A*b^2 + a^2*C) + b*(m + 1)*(b*c - a*d)*(A + C))*Sin[e + f*x] - d
*(A*b^2 + a^2*C)*(m + n + 3)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c,
d, e, f, A, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2
- d^2, 0] && LtQ[m, -1] && ((EqQ[a, 0] && IntegerQ[m] &&  !IntegerQ[n]) ||
 !(IntegerQ[2*n] && LtQ[n, -1] && ((IntegerQ[n] &&  !IntegerQ[m]) || EqQ[a,
 0])))
```

rule 4257
```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Maple [A] (verified)

Time = 0.82 (sec) , antiderivative size = 81, normalized size of antiderivative = 1.45

method	result	size
default	$-\frac{\tanh\left(\frac{x}{2}\right)}{2a}-\frac{1}{2a\tanh\left(\frac{x}{2}\right)}-\frac{b\ln\left(\tanh\left(\frac{x}{2}\right)\right)}{a^2}-\frac{\left(-4a^2-4b^2\right)\operatorname{arctanh}\left(\frac{2a\tanh\left(\frac{x}{2}\right)-2b}{2\sqrt{a^2+b^2}}\right)}{2a^2\sqrt{a^2+b^2}}$	81
risch	$-\frac{2}{a\left(e^{2x}-1\right)}+\frac{b\ln\left(e^x+1\right)}{a^2}-\frac{b\ln\left(e^x-1\right)}{a^2}+\frac{\sqrt{a^2+b^2}\,\ln\left(e^x-\frac{-a+\sqrt{a^2+b^2}}{b}\right)}{a^2}-\frac{\sqrt{a^2+b^2}\,\ln\left(e^x+\frac{a+\sqrt{a^2+b^2}}{b}\right)}{a^2}$	104

input
```
int(coth(x)^2/(a+b*sinh(x)),x,method=_RETURNVERBOSE)
```

output
```
-1/2/a*tanh(1/2*x)-1/2/a/tanh(1/2*x)-1/a^2*b*ln(tanh(1/2*x))-1/2/a^2*(-4*a
^2-4*b^2)/(a^2+b^2)^(1/2)*arctanh(1/2*(2*a*tanh(1/2*x)-2*b)/(a^2+b^2)^(1/2
))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 228 vs. $2(52) = 104$.

Time = 0.10 (sec) , antiderivative size = 228, normalized size of antiderivative = 4.07

$$\int \frac{\coth^2(x)}{a+b\sinh(x)}\,dx$$
$$= \frac{\sqrt{a^2+b^2}\left(\cosh(x)^2+2\cosh(x)\sinh(x)+\sinh(x)^2-1\right)\log\left(\frac{b^2\cosh(x)^2+b^2\sinh(x)^2+2ab\cosh(x)+2a^2+b^2+2\left(b^2\,\text{co}\right.}{b\cosh(x)^2+b\sinh(x)^2+2a\,\text{cosh}}\right.}{}$$

input
```
integrate(coth(x)^2/(a+b*sinh(x)),x, algorithm="fricas")
```

output
```
(sqrt(a^2 + b^2)*(cosh(x)^2 + 2*cosh(x)*sinh(x) + sinh(x)^2 - 1)*log((b^2*
cosh(x)^2 + b^2*sinh(x)^2 + 2*a*b*cosh(x) + 2*a^2 + b^2 + 2*(b^2*cosh(x) +
 a*b)*sinh(x) - 2*sqrt(a^2 + b^2)*(b*cosh(x) + b*sinh(x) + a))/(b*cosh(x)^
2 + b*sinh(x)^2 + 2*a*cosh(x) + 2*(b*cosh(x) + a)*sinh(x) - b)) + (b*cosh(
x)^2 + 2*b*cosh(x)*sinh(x) + b*sinh(x)^2 - b)*log(cosh(x) + sinh(x) + 1) -
 (b*cosh(x)^2 + 2*b*cosh(x)*sinh(x) + b*sinh(x)^2 - b)*log(cosh(x) + sinh(
x) - 1) - 2*a)/(a^2*cosh(x)^2 + 2*a^2*cosh(x)*sinh(x) + a^2*sinh(x)^2 - a^
2)
```

Sympy [F]

$$\int \frac{\coth^2(x)}{a+b\sinh(x)}\,dx=\int \frac{\coth^2{(x)}}{a+b\sinh{(x)}}\,dx$$

input
```
integrate(coth(x)**2/(a+b*sinh(x)),x)
```

output
```
Integral(coth(x)**2/(a + b*sinh(x)), x)
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 97, normalized size of antiderivative = 1.73

$$\int \frac{\coth^2(x)}{a+b\sinh(x)}\,dx=\frac{b\log\left(e^{(-x)}+1\right)}{a^2}-\frac{b\log\left(e^{(-x)}-1\right)}{a^2}+\frac{\sqrt{a^2+b^2}\log\left(\frac{be^{(-x)}-a-\sqrt{a^2+b^2}}{be^{(-x)}-a+\sqrt{a^2+b^2}}\right)}{a^2}+\frac{2}{ae^{(-2x)}-a}$$

input
```
integrate(coth(x)^2/(a+b*sinh(x)),x, algorithm="maxima")
```

output
```
b*log(e^(-x) + 1)/a^2 - b*log(e^(-x) - 1)/a^2 + sqrt(a^2 + b^2)*log((b*e^(
-x) - a - sqrt(a^2 + b^2))/(b*e^(-x) - a + sqrt(a^2 + b^2)))/a^2 + 2/(a*e^
(-2*x) - a)
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 95, normalized size of antiderivative = 1.70

$$\int \frac{\coth^2(x)}{a+b\sinh(x)}\,dx = \frac{b\log(e^x+1)}{a^2} - \frac{b\log(|e^x-1|)}{a^2} + \frac{\sqrt{a^2+b^2}\log\left(\frac{\left|2\,be^x+2\,a-2\,\sqrt{a^2+b^2}\right|}{\left|2\,be^x+2\,a+2\,\sqrt{a^2+b^2}\right|}\right)}{a^2} - \frac{2}{a(e^{(2\,x)}-1)}$$

input
```
integrate(coth(x)^2/(a+b*sinh(x)),x, algorithm="giac")
```

output
```
b*log(e^x + 1)/a^2 - b*log(abs(e^x - 1))/a^2 + sqrt(a^2 + b^2)*log(abs(2*b
*e^x + 2*a - 2*sqrt(a^2 + b^2))/abs(2*b*e^x + 2*a + 2*sqrt(a^2 + b^2)))/a^
2 - 2/(a*(e^(2*x) - 1))
```

Mupad [B] (verification not implemented)

Time = 1.72 (sec) , antiderivative size = 304, normalized size of antiderivative = 5.43

$$\int \frac{\coth^2(x)}{a+b\sinh(x)}\,dx = \frac{2}{a-a\,e^{2\,x}} - \frac{b\,\ln\left(32\,a^2+32\,b^2-32\,a^2\,e^x-32\,b^2\,e^x\right)}{a^2} + \frac{b\,\ln\left(32\,a^2+32\,b^2+32\,a^2\,e^x+32\,b^2\,e^x\right)}{a^2} + \frac{\ln\left(128\,a^4\,e^x-64\,a\,b^3-64\,a^3\,b-32\,b^3\,\sqrt{a^2+b^2}+32\,b^4\,e^x+128\,a^3\,e^x\,\sqrt{a^2+b^2}+160\,a^2\,b^2\,e^x-64\,a^2\right.}{a^2} - \frac{\ln\left(32\,b^3\,\sqrt{a^2+b^2}-64\,a\,b^3-64\,a^3\,b+128\,a^4\,e^x+32\,b^4\,e^x-128\,a^3\,e^x\,\sqrt{a^2+b^2}+160\,a^2\,b^2\,e^x+64\,a^2\right.}{a^2}$$

input
```
int(coth(x)^2/(a + b*sinh(x)),x)
```

output
```
2/(a - a*exp(2*x)) - (b*log(32*a^2 + 32*b^2 - 32*a^2*exp(x) - 32*b^2*exp(x
)))/a^2 + (b*log(32*a^2 + 32*b^2 + 32*a^2*exp(x) + 32*b^2*exp(x)))/a^2 + (
log(128*a^4*exp(x) - 64*a*b^3 - 64*a^3*b - 32*b^3*(a^2 + b^2)^(1/2) + 32*b
^4*exp(x) + 128*a^3*exp(x)*(a^2 + b^2)^(1/2) + 160*a^2*b^2*exp(x) - 64*a^2
*b*(a^2 + b^2)^(1/2) + 96*a*b^2*exp(x)*(a^2 + b^2)^(1/2))*(a^2 + b^2)^(1/2
))/a^2 - (log(32*b^3*(a^2 + b^2)^(1/2) - 64*a*b^3 - 64*a^3*b + 128*a^4*exp
(x) + 32*b^4*exp(x) - 128*a^3*exp(x)*(a^2 + b^2)^(1/2) + 160*a^2*b^2*exp(x
) + 64*a^2*b*(a^2 + b^2)^(1/2) - 96*a*b^2*exp(x)*(a^2 + b^2)^(1/2))*(a^2 +
 b^2)^(1/2))/a^2
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 137, normalized size of antiderivative = 2.45

$$\int \frac{\coth^2(x)}{a+b\sinh(x)}\,dx = \frac{2e^{2x}\sqrt{a^2+b^2}\,atan\left(\frac{e^x bi+ai}{\sqrt{a^2+b^2}}\right)i-2\sqrt{a^2+b^2}\,atan\left(\frac{e^x bi+ai}{\sqrt{a^2+b^2}}\right)i-e^{2x}\log(e^x-1)\,b+e^{2x}\log(e^x+1)\,b-2e^{2x}a}{a^2\left(e^{2x}-1\right)}$$

input
```
int(coth(x)^2/(a+b*sinh(x)),x)
```

output
```
(2*e**(2*x)*sqrt(a**2 + b**2)*atan((e**x*b*i + a*i)/sqrt(a**2 + b**2))*i -
 2*sqrt(a**2 + b**2)*atan((e**x*b*i + a*i)/sqrt(a**2 + b**2))*i - e**(2*x)
*log(e**x - 1)*b + e**(2*x)*log(e**x + 1)*b - 2*e**(2*x)*a + log(e**x - 1)
*b - log(e**x + 1)*b)/(a**2*(e**(2*x) - 1))
```

3.234 $\int \frac{\coth^3(x)}{a+b\sinh(x)}\,dx$

Optimal result

Integrand size = 13, antiderivative size = 52

$$\int \frac{\coth^3(x)}{a+b\sinh(x)}\,dx = \frac{b\operatorname{csch}(x)}{a^2} - \frac{\operatorname{csch}^2(x)}{2a} + \frac{(a^2+b^2)\log(\sinh(x))}{a^3} - \frac{(a^2+b^2)\log(a+b\sinh(x))}{a^3}$$

output
```
b*csch(x)/a^2-1/2*csch(x)^2/a+(a^2+b^2)*ln(sinh(x))/a^3-(a^2+b^2)*ln(a+b*s
inh(x))/a^3
```

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 45, normalized size of antiderivative = 0.87

$$\int \frac{\coth^3(x)}{a+b\sinh(x)}\,dx = \frac{2ab\operatorname{csch}(x) - a^2\operatorname{csch}^2(x) + 2(a^2+b^2)\left(\log(\sinh(x)) - \log(a+b\sinh(x))\right)}{2a^3}$$

input
```
Integrate[Coth[x]^3/(a + b*Sinh[x]),x]
```

output
```
(2*a*b*Csch[x] - a^2*Csch[x]^2 + 2*(a^2 + b^2)*(Log[Sinh[x]] - Log[a + b*S
inh[x]]))/(2*a^3)
```

Rubi [A] (verified)

Time = 0.31 (sec) , antiderivative size = 54, normalized size of antiderivative = 1.04, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.462, Rules used = {3042, 26, 3200, 25, 522, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\coth^3(x)}{a+b\sinh(x)}dx$$

$$\Big\downarrow \text{3042}$$

$$\int -\frac{i}{\tan(ix)^3(a-ib\sin(ix))}dx$$

$$\Big\downarrow \text{26}$$

$$-i\int \frac{1}{(a-ib\sin(ix))\tan(ix)^3}dx$$

$$\Big\downarrow \text{3200}$$

$$-\int -\frac{\operatorname{csch}^3(x)\left(\sinh^2(x)b^2+b^2\right)}{b^3(a+b\sinh(x))}d(b\sinh(x))$$

$$\Big\downarrow \text{25}$$

$$\int \frac{\operatorname{csch}^3(x)\left(b^2\sinh^2(x)+b^2\right)}{b^3(a+b\sinh(x))}d(b\sinh(x))$$

$$\Big\downarrow \text{522}$$

$$\int \left(-\frac{\operatorname{csch}^2(x)}{a^2}+\frac{-a^2-b^2}{a^3(a+b\sinh(x))}+\frac{\left(a^2+b^2\right)\operatorname{csch}(x)}{a^3b}+\frac{\operatorname{csch}^3(x)}{ab}\right)d(b\sinh(x))$$

$$\Big\downarrow \text{2009}$$

$$\frac{b\operatorname{csch}(x)}{a^2}+\frac{\left(a^2+b^2\right)\log(b\sinh(x))}{a^3}-\frac{\left(a^2+b^2\right)\log(a+b\sinh(x))}{a^3}-\frac{\operatorname{csch}^2(x)}{2a}$$

input
```
Int[Coth[x]^3/(a + b*Sinh[x]),x]
```

output
```
(b*Csch[x])/a^2 - Csch[x]^2/(2*a) + ((a^2 + b^2)*Log[b*Sinh[x]])/a^3 - ((a
^2 + b^2)*Log[a + b*Sinh[x]])/a^3
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 522
```
Int[((e_.)*(x_))^(m_.)*((c_) + (d_.)*(x_))^(n_.)*((a_) + (b_.)*(x_)^2)^(p_.
), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(c + d*x)^n*(a + b*x^2)^p, x],
x] /; FreeQ[{a, b, c, d, e, m, n}, x] && IGtQ[p, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3200
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*tan[(e_.) + (f_.)*(x_)]^(p
_.), x_Symbol] :> Simp[1/f   Subst[Int[(x^p*(a + x)^m)/(b^2 - x^2)^((p + 1)
/2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && NeQ[a^2 - b
^2, 0] && IntegerQ[(p + 1)/2]
```

Maple [A] (verified)

Time = 1.28 (sec) , antiderivative size = 98, normalized size of antiderivative = 1.88

method	result
risch	$-\frac{2\,{\rm e}^{x}\left(-b\,{\rm e}^{2x}+{\rm e}^{x}a+b\right)}{\left({\rm e}^{2x}-1\right)^{2}a^{2}}-\frac{\ln\left({\rm e}^{2x}+\frac{2a\,{\rm e}^{x}}{b}-1\right)}{a}-\frac{\ln\left({\rm e}^{2x}+\frac{2a\,{\rm e}^{x}}{b}-1\right)b^{2}}{a^{3}}+\frac{\ln\left({\rm e}^{2x}-1\right)}{a}+\frac{\ln\left({\rm e}^{2x}-1\right)b^{2}}{a^{3}}$
default	$-\frac{\frac{\tanh\left(\frac{x}{2}\right)^{2}a}{2}+2b\tanh\left(\frac{x}{2}\right)}{4a^{2}}-\frac{1}{8a\tanh\left(\frac{x}{2}\right)^{2}}+\frac{\left(4a^{2}+4b^{2}\right)\ln\left(\tanh\left(\frac{x}{2}\right)\right)}{4a^{3}}+\frac{b}{2a^{2}\tanh\left(\frac{x}{2}\right)}+\frac{\left(-4a^{2}-4b^{2}\right)\ln\left(\tanh\left(\frac{x}{2}\right)^{2}a-2b\tan\right.}{4a^{3}}$

input
```
int(coth(x)^3/(a+b*sinh(x)),x,method=_RETURNVERBOSE)
```

output
```
-2*exp(x)*(-b*exp(2*x)+exp(x)*a+b)/(exp(2*x)-1)^2/a^2-1/a*ln(exp(2*x)+2/b*
a*exp(x)-1)-1/a^3*ln(exp(2*x)+2/b*a*exp(x)-1)*b^2+1/a*ln(exp(2*x)-1)+1/a^3
*ln(exp(2*x)-1)*b^2
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 427 vs. 2(50) = 100.

Time = 0.10 (sec) , antiderivative size = 427, normalized size of antiderivative = 8.21

$$\int \frac{\coth^3(x)}{a+b\sinh(x)}\,dx$$
$$=\frac{2\,ab\cosh\left(x\right)^{3}+2\,ab\sinh\left(x\right)^{3}-2\,a^{2}\cosh\left(x\right)^{2}-2\,ab\cosh\left(x\right)+2\left(3\,ab\cosh\left(x\right)-a^{2}\right)\sinh\left(x\right)^{2}-\left(\left(a^{2}+\right.\right.}{}$$

input
```
integrate(coth(x)^3/(a+b*sinh(x)),x, algorithm="fricas")
```

output
```
(2*a*b*cosh(x)^3 + 2*a*b*sinh(x)^3 - 2*a^2*cosh(x)^2 - 2*a*b*cosh(x) + 2*(
3*a*b*cosh(x) - a^2)*sinh(x)^2 - ((a^2 + b^2)*cosh(x)^4 + 4*(a^2 + b^2)*co
sh(x)*sinh(x)^3 + (a^2 + b^2)*sinh(x)^4 - 2*(a^2 + b^2)*cosh(x)^2 + 2*(3*(
a^2 + b^2)*cosh(x)^2 - a^2 - b^2)*sinh(x)^2 + a^2 + b^2 + 4*((a^2 + b^2)*c
osh(x)^3 - (a^2 + b^2)*cosh(x))*sinh(x))*log(2*(b*sinh(x) + a)/(cosh(x) -
sinh(x))) + ((a^2 + b^2)*cosh(x)^4 + 4*(a^2 + b^2)*cosh(x)*sinh(x)^3 + (a^
2 + b^2)*sinh(x)^4 - 2*(a^2 + b^2)*cosh(x)^2 + 2*(3*(a^2 + b^2)*cosh(x)^2
- a^2 - b^2)*sinh(x)^2 + a^2 + b^2 + 4*((a^2 + b^2)*cosh(x)^3 - (a^2 + b^2
)*cosh(x))*sinh(x))*log(2*sinh(x)/(cosh(x) - sinh(x))) + 2*(3*a*b*cosh(x)^
2 - 2*a^2*cosh(x) - a*b)*sinh(x))/(a^3*cosh(x)^4 + 4*a^3*cosh(x)*sinh(x)^3
 + a^3*sinh(x)^4 - 2*a^3*cosh(x)^2 + a^3 + 2*(3*a^3*cosh(x)^2 - a^3)*sinh(
x)^2 + 4*(a^3*cosh(x)^3 - a^3*cosh(x))*sinh(x))
```

Sympy [F]

$$\int \frac{\coth^3(x)}{a+b\sinh(x)}\,dx=\int \frac{\coth^3{(x)}}{a+b\sinh{(x)}}\,dx$$

input
```
integrate(coth(x)**3/(a+b*sinh(x)),x)
```

output
```
Integral(coth(x)**3/(a + b*sinh(x)), x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 116 vs. $2(50) = 100$.

Time = 0.05 (sec) , antiderivative size = 116, normalized size of antiderivative = 2.23

$$\int \frac{\coth^3(x)}{a+b\sinh(x)}\,dx=-\frac{2\left(be^{(-x)}-ae^{(-2x)}-be^{(-3x)}\right)}{2a^2e^{(-2x)}-a^2e^{(-4x)}-a^2}-\frac{(a^2+b^2)\log\left(-2ae^{(-x)}+be^{(-2x)}-b\right)}{a^3}+\frac{(a^2+b^2)\log\left(e^{(-x)}+1\right)}{a^3}+\frac{(a^2+b^2)\log\left(e^{(-x)}-1\right)}{a^3}$$

input
```
integrate(coth(x)^3/(a+b*sinh(x)),x, algorithm="maxima")
```

output
```
-2*(b*e^(-x) - a*e^(-2*x) - b*e^(-3*x))/(2*a^2*e^(-2*x) - a^2*e^(-4*x) - a
^2) - (a^2 + b^2)*log(-2*a*e^(-x) + b*e^(-2*x) - b)/a^3 + (a^2 + b^2)*log(
e^(-x) + 1)/a^3 + (a^2 + b^2)*log(e^(-x) - 1)/a^3
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 125 vs. $2(50) = 100$.

Time = 0.14 (sec) , antiderivative size = 125, normalized size of antiderivative = 2.40

$$\int \frac{\coth^3(x)}{a+b\sinh(x)}\,dx = \frac{(a^2+b^2)\log\left(\left|-e^{(-x)}+e^x\right|\right)}{a^3} - \frac{(a^2b+b^3)\log\left(\left|-b\left(e^{(-x)}-e^x\right)+2\,a\right|\right)}{a^3b} - \frac{3\,a^2\left(e^{(-x)}-e^x\right)^2+3\,b^2\left(e^{(-x)}-e^x\right)^2+4\,ab\left(e^{(-x)}-e^x\right)+4\,a^2}{2\,a^3\left(e^{(-x)}-e^x\right)^2}$$

input
```
integrate(coth(x)^3/(a+b*sinh(x)),x, algorithm="giac")
```

output
```
(a^2 + b^2)*log(abs(-e^(-x) + e^x))/a^3 - (a^2*b + b^3)*log(abs(-b*(e^(-x)
 - e^x) + 2*a))/(a^3*b) - 1/2*(3*a^2*(e^(-x) - e^x)^2 + 3*b^2*(e^(-x) - e^
x)^2 + 4*a*b*(e^(-x) - e^x) + 4*a^2)/(a^3*(e^(-x) - e^x)^2)
```

Mupad [B] (verification not implemented)

Time = 2.13 (sec) , antiderivative size = 1163, normalized size of antiderivative = 22.37

$$\int \frac{\coth^3(x)}{a+b\sinh(x)}\,dx = \text{Too large to display}$$

input
```
int(coth(x)^3/(a + b*sinh(x)),x)
```

output
```
((2*atan((a^2*(-a^6)^(1/2)*(a^4 + b^4 + 2*a^2*b^2)^(1/2) + 2*b^2*(-a^6)^(1
/2)*(a^4 + b^4 + 2*a^2*b^2)^(1/2))/(2*a^3*(a^2 + b^2)^2) + ((a^7 + a^5*b^2
)*(-a^6)^(1/2))/(2*a^6*((a^2 + b^2)^2)^(1/2)*(a^2 + b^2)) - (a^6*b^2*exp(x
)*(-a^6)^(1/2)*((8*(a^4 + b^4 + 2*a^2*b^2))/(a^8*b*(a^2 + b^2)^2) - (4*(2*
a^6*b + 2*a^4*b^3)*(a^4 + b^4 + 2*a^2*b^2)^(1/2))/(a^12*b^2*((a^2 + b^2)^2
)^(1/2)*(a^2 + b^2)) + (2*(a^7 + a^5*b^2)*(a^4 + b^4 + 2*a^2*b^2)^(1/2))/(
a^11*b^3*((a^2 + b^2)^2)^(1/2)*(a^2 + b^2)) - (2*(a^2 + 2*b^2)*(a^2*(-a^6)
^(1/2)*(a^4 + b^4 + 2*a^2*b^2)^(1/2) + 2*b^2*(-a^6)^(1/2)*(a^4 + b^4 + 2*a
^2*b^2)^(1/2))*(a^4 + b^4 + 2*a^2*b^2)^(1/2))/(a^10*b^3*(-a^6)^(1/2)*(a^2
+ b^2)^2)))/(8*(a^4 + b^4 + 2*a^2*b^2)^(1/2)) - (a^6*b^2*exp(2*x)*(-a^6)^(
1/2)*((4*(a^2 + 2*b^2)*(a^4 + b^4 + 2*a^2*b^2))/(a^9*b^2*(a^2 + b^2)^2) +
(4*(a^2*(-a^6)^(1/2)*(a^4 + b^4 + 2*a^2*b^2)^(1/2) + 2*b^2*(-a^6)^(1/2)*(a
^4 + b^4 + 2*a^2*b^2)^(1/2))*(a^4 + b^4 + 2*a^2*b^2)^(1/2))/(a^9*b^2*(-a^6
)^(1/2)*(a^2 + b^2)^2) + (2*(2*a^6*b + 2*a^4*b^3)*(a^4 + b^4 + 2*a^2*b^2)^
(1/2))/(a^11*b^3*((a^2 + b^2)^2)^(1/2)*(a^2 + b^2)) + (4*(a^7 + a^5*b^2)*(
a^4 + b^4 + 2*a^2*b^2)^(1/2))/(a^12*b^2*((a^2 + b^2)^2)^(1/2)*(a^2 + b^2))
))/(8*(a^4 + b^4 + 2*a^2*b^2)^(1/2)) + (a^6*b^2*exp(3*x)*((2*(a^7 + a^5*b^
2)*(a^4 + b^4 + 2*a^2*b^2)^(1/2))/(a^11*b^3*((a^2 + b^2)^2)^(1/2)*(a^2 + b
^2)) - (2*(a^2 + 2*b^2)*(a^2*(-a^6)^(1/2)*(a^4 + b^4 + 2*a^2*b^2)^(1/2) +
2*b^2*(-a^6)^(1/2)*(a^4 + b^4 + 2*a^2*b^2)^(1/2))*(a^4 + b^4 + 2*a^2*b^...
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 374, normalized size of antiderivative = 7.19

$$\int \frac{\coth^3(x)}{a+b\sinh(x)}\,dx = \frac{e^{4x}\log(e^x-1)\,a^2+e^{4x}\log(e^x-1)\,b^2+e^{4x}\log(e^x+1)\,a^2+e^{4x}\log(e^x+1)\,b^2-e^{4x}\log(e^{2x}b+2e^xa-b)\,a^2}{}$$

input
```
int(coth(x)^3/(a+b*sinh(x)),x)
```

output
```
(e**(4*x)*log(e**x - 1)*a**2 + e**(4*x)*log(e**x - 1)*b**2 + e**(4*x)*log(
e**x + 1)*a**2 + e**(4*x)*log(e**x + 1)*b**2 - e**(4*x)*log(e**(2*x)*b + 2
*e**x*a - b)*a**2 - e**(4*x)*log(e**(2*x)*b + 2*e**x*a - b)*b**2 - e**(4*x
)*a**2 + 2*e**(3*x)*a*b - 2*e**(2*x)*log(e**x - 1)*a**2 - 2*e**(2*x)*log(e
**x - 1)*b**2 - 2*e**(2*x)*log(e**x + 1)*a**2 - 2*e**(2*x)*log(e**x + 1)*b
**2 + 2*e**(2*x)*log(e**(2*x)*b + 2*e**x*a - b)*a**2 + 2*e**(2*x)*log(e**(
2*x)*b + 2*e**x*a - b)*b**2 - 2*e**x*a*b + log(e**x - 1)*a**2 + log(e**x -
 1)*b**2 + log(e**x + 1)*a**2 + log(e**x + 1)*b**2 - log(e**(2*x)*b + 2*e*
*x*a - b)*a**2 - log(e**(2*x)*b + 2*e**x*a - b)*b**2 - a**2)/(a**3*(e**(4*
x) - 2*e**(2*x) + 1))
```

3.235 $\int \frac{\coth^4(x)}{a+b\sinh(x)}\,dx$

Optimal result

Integrand size = 13, antiderivative size = 108

$$\int \frac{\coth^4(x)}{a+b\sinh(x)}\,dx = \frac{b(3a^2+2b^2)\operatorname{arctanh}(\cosh(x))}{2a^4} - \frac{2(a^2+b^2)^{3/2}\operatorname{arctanh}\left(\frac{b-a\tanh\left(\frac{x}{2}\right)}{\sqrt{a^2+b^2}}\right)}{a^4} - \frac{(4a^2+3b^2)\coth(x)}{3a^3} + \frac{b\coth(x)\operatorname{csch}(x)}{2a^2} - \frac{\coth(x)\operatorname{csch}^2(x)}{3a}$$

output
```
1/2*b*(3*a^2+2*b^2)*arctanh(cosh(x))/a^4-2*(a^2+b^2)^(3/2)*arctanh((b-a*ta
nh(1/2*x))/(a^2+b^2)^(1/2))/a^4-1/3*(4*a^2+3*b^2)*coth(x)/a^3+1/2*b*coth(x
)*csch(x)/a^2-1/3*coth(x)*csch(x)^2/a
```

Mathematica [A] (verified)

Time = 0.35 (sec) , antiderivative size = 197, normalized size of antiderivative = 1.82

$$\int \frac{\coth^4(x)}{a+b\sinh(x)}\,dx = \frac{48(-a^2-b^2)^{3/2}\arctan\left(\frac{b-a\tanh\left(\frac{x}{2}\right)}{\sqrt{-a^2-b^2}}\right) - 4a(4a^2+3b^2)\coth\left(\frac{x}{2}\right) + 3a^2b\operatorname{csch}^2\left(\frac{x}{2}\right) + 12b(3a^2+2b^2)\log\left(\cos\right.}{}$$

input
```
Integrate[Coth[x]^4/(a + b*Sinh[x]),x]
```

output
```
(48*(-a^2 - b^2)^(3/2)*ArcTan[(b - a*Tanh[x/2])/Sqrt[-a^2 - b^2]] - 4*a*(4
*a^2 + 3*b^2)*Coth[x/2] + 3*a^2*b*Csch[x/2]^2 + 12*b*(3*a^2 + 2*b^2)*Log[C
osh[x/2]] - 12*b*(3*a^2 + 2*b^2)*Log[Sinh[x/2]] + 3*a^2*b*Sech[x/2]^2 + 8*
a^3*Csch[x]^3*Sinh[x/2]^4 - (a^3*Csch[x/2]^4*Sinh[x])/2 - 4*a*(4*a^2 + 3*b
^2)*Tanh[x/2])/(24*a^4)
```

Rubi [C] (verified)

Result contains complex when optimal does not.

Time = 0.92 (sec) , antiderivative size = 129, normalized size of antiderivative = 1.19, number of steps used = 18, number of rules used = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 1.308$, Rules used = {3042, 3204, 25, 3042, 25, 3534, 27, 3042, 26, 3480, 26, 3042, 26, 3139, 1083, 219, 4257}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\coth^4(x)}{a+b\sinh(x)}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{\tan(ix)^4(a-ib\sin(ix))}dx$$

$$\Big\downarrow \text{3204}$$

$$-\frac{\int -\frac{\text{csch}^2(x)\left(3\left(2a^2+b^2\right)\sinh^2(x)-ab\sinh(x)+2\left(4a^2+3b^2\right)\right)}{a+b\sinh(x)}dx}{6a^2}+\frac{b\coth(x)\text{csch}(x)}{2a^2}-\frac{\coth(x)\text{csch}^2(x)}{3a}$$

$$\Big\downarrow \text{25}$$

$$\frac{\int \frac{\text{csch}^2(x)\left(3\left(2a^2+b^2\right)\sinh^2(x)-ab\sinh(x)+2\left(4a^2+3b^2\right)\right)}{a+b\sinh(x)}dx}{6a^2}+\frac{b\coth(x)\text{csch}(x)}{2a^2}-\frac{\coth(x)\text{csch}^2(x)}{3a}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\int -\frac{-3\left(2a^2+b^2\right)\sin(ix)^2+iab\sin(ix)+2\left(4a^2+3b^2\right)}{\sin(ix)^2(a-ib\sin(ix))}dx}{6a^2}+\frac{b\coth(x)\text{csch}(x)}{2a^2}-\frac{\coth(x)\text{csch}^2(x)}{3a}$$

$$\Big\downarrow \text{25}$$

$$-\frac{\int \frac{-3(2a^2+b^2)\sin(ix)^2+iab\sin(ix)+2(4a^2+3b^2)}{\sin(ix)^2(a-ib\sin(ix))}dx}{6a^2}+\frac{b\coth(x)\text{csch}(x)}{2a^2}-\frac{\coth(x)\text{csch}^2(x)}{3a}$$

$\downarrow$ 3534

$$-\frac{\frac{\int \frac{3\text{csch}(x)\left(b\left(3a^2+2b^2\right)-a\left(2a^2+b^2\right)\sinh(x)\right)}{a+b\sinh(x)}dx}{a}+\frac{2(4a^2+3b^2)\coth(x)}{a}}{6a^2}+\frac{b\coth(x)\text{csch}(x)}{2a^2}-\frac{\coth(x)\text{csch}^2(x)}{3a}$$

$\downarrow$ 27

$$-\frac{\frac{3\int \frac{\text{csch}(x)\left(b\left(3a^2+2b^2\right)-a\left(2a^2+b^2\right)\sinh(x)\right)}{a+b\sinh(x)}dx}{a}+\frac{2(4a^2+3b^2)\coth(x)}{a}}{6a^2}+\frac{b\coth(x)\text{csch}(x)}{2a^2}-\frac{\coth(x)\text{csch}^2(x)}{3a}$$

$\downarrow$ 3042

$$-\frac{\frac{2(4a^2+3b^2)\coth(x)}{a}+\frac{3\int \frac{i\left(b\left(3a^2+2b^2\right)+ia\left(2a^2+b^2\right)\sin(ix)\right)}{\sin(ix)(a-ib\sin(ix))}dx}{a}}{6a^2}+\frac{b\coth(x)\text{csch}(x)}{2a^2}-\frac{\coth(x)\text{csch}^2(x)}{3a}$$

$\downarrow$ 26

$$-\frac{\frac{2(4a^2+3b^2)\coth(x)}{a}+\frac{3i\int \frac{b\left(3a^2+2b^2\right)+ia\left(2a^2+b^2\right)\sin(ix)}{\sin(ix)(a-ib\sin(ix))}dx}{a}}{6a^2}+\frac{b\coth(x)\text{csch}(x)}{2a^2}-\frac{\coth(x)\text{csch}^2(x)}{3a}$$

$\downarrow$ 3480

$$-\frac{\frac{2(4a^2+3b^2)\coth(x)}{a}+\frac{3i\left(\frac{2i\left(a^2+b^2\right)^2\int \frac{1}{a+b\sinh(x)}dx}{a}+\frac{b\left(3a^2+2b^2\right)\int -i\text{csch}(x)dx}{a}\right)}{a}}{6a^2}+\frac{b\coth(x)\text{csch}(x)}{2a^2}-\frac{\coth(x)\text{csch}^2(x)}{3a}$$

$\downarrow$ 26

$$-\frac{\frac{2(4a^2+3b^2)\coth(x)}{a}+\frac{3i\left(\frac{2i\left(a^2+b^2\right)^2\int \frac{1}{a+b\sinh(x)}dx}{a}-\frac{ib\left(3a^2+2b^2\right)\int \text{csch}(x)dx}{a}\right)}{a}}{6a^2}+\frac{b\coth(x)\text{csch}(x)}{2a^2}-\frac{\coth(x)\text{csch}^2(x)}{3a}$$

$\downarrow$ 3042

$$-\frac{-\frac{2(4a^2+3b^2)\coth(x)}{a}+\frac{3i\left(\frac{2i\left(a^2+b^2\right)^2\int\frac{1}{a-ib\sin(ix)}dx}{a}-\frac{ib\left(3a^2+2b^2\right)\int i\csc(ix)dx}{a}\right)}{a}}{6a^2}+\frac{b\coth(x)\text{csch}(x)}{2a^2}-\frac{\coth(x)\text{csch}^2(x)}{3a}$$

$$\Big\downarrow \; 26$$

$$-\frac{-\frac{2(4a^2+3b^2)\coth(x)}{a}+\frac{3i\left(\frac{2i\left(a^2+b^2\right)^2\int\frac{1}{a-ib\sin(ix)}dx}{a}+\frac{b\left(3a^2+2b^2\right)\int\csc(ix)dx}{a}\right)}{a}}{6a^2}+\frac{b\coth(x)\text{csch}(x)}{2a^2}-\frac{\coth(x)\text{csch}^2(x)}{3a}$$

$$\Big\downarrow \; 3139$$

$$-\frac{-\frac{2(4a^2+3b^2)\coth(x)}{a}+\frac{3i\left(\frac{b\left(3a^2+2b^2\right)\int\csc(ix)dx}{a}+\frac{4i\left(a^2+b^2\right)^2\int\frac{1}{-a\tanh^2\left(\frac{x}{2}\right)+2b\tanh\left(\frac{x}{2}\right)+a}d\tanh\left(\frac{x}{2}\right)}{a}\right)}{a}}{6a^2}+\frac{b\coth(x)\text{csch}(x)}{2a^2}-\frac{\coth(x)\text{csch}^2(x)}{3a}$$

$$\Big\downarrow \; 1083$$

$$-\frac{-\frac{2(4a^2+3b^2)\coth(x)}{a}+\frac{3i\left(\frac{b\left(3a^2+2b^2\right)\int\csc(ix)dx}{a}-\frac{8i\left(a^2+b^2\right)^2\int\frac{1}{4\left(a^2+b^2\right)-\left(2b-2a\tanh\left(\frac{x}{2}\right)\right)^2}d\left(2b-2a\tanh\left(\frac{x}{2}\right)\right)}{a}\right)}{a}}{6a^2}+\frac{b\coth(x)\text{csch}(x)}{2a^2}-\frac{\coth(x)\text{csch}^2(x)}{3a}$$

$$\Big\downarrow \; 219$$

$$-\frac{-\frac{2(4a^2+3b^2)\coth(x)}{a}+\frac{3i\left(\frac{b\left(3a^2+2b^2\right)\int\csc(ix)dx}{a}-\frac{4i\left(a^2+b^2\right)^{3/2}\text{arctanh}\left(\frac{2b-2a\tanh\left(\frac{x}{2}\right)}{2\sqrt{a^2+b^2}}\right)}{a}\right)}{a}}{6a^2}+\frac{b\coth(x)\text{csch}(x)}{2a^2}-\frac{\coth(x)\text{csch}^2(x)}{3a}$$

$$\Big\downarrow \; 4257$$

$$-\frac{-\frac{2(4a^2+3b^2)\coth(x)}{a}+\frac{3i\left(\frac{ib(3a^2+2b^2)\text{arctanh}(\cosh(x))}{a}-\frac{4i(a^2+b^2)^{3/2}\text{arctanh}\left(\frac{2b-2a\tanh\left(\frac{x}{2}\right)}{2\sqrt{a^2+b^2}}\right)}{a}\right)}{a}}{6a^2}+\frac{b\coth(x)\text{csch}(x)}{2a^2}-\frac{\coth(x)\text{csch}^2(x)}{3a}$$

input
```
Int[Coth[x]^4/(a + b*Sinh[x]),x]
```

output
```
-1/6*(((3*I)*((I*b*(3*a^2 + 2*b^2)*ArcTanh[Cosh[x]])/a - ((4*I)*(a^2 + b^2
)^(3/2)*ArcTanh[(2*b - 2*a*Tanh[x/2])/(2*Sqrt[a^2 + b^2])])/a))/a + (2*(4*
a^2 + 3*b^2)*Coth[x])/a)/a^2 + (b*Coth[x]*Csch[x])/(2*a^2) - (Coth[x]*Csch
[x]^2)/(3*a)
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1083
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Simp[-2   Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3139
```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Simp[2*(e/d)   Subst[Int[1/(a + 2*b*e*x + a
*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[a^2 - b^2, 0]
```

rule 3204
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)/tan[(e_.) + (f_.)*(x_)]^4,
x_Symbol] :> Simp[(-Cos[e + f*x])*((a + b*Sin[e + f*x])^(m + 1)/(3*a*f*Sin[
e + f*x]^3)), x] + (-Simp[b*(m - 2)*Cos[e + f*x]*((a + b*Sin[e + f*x])^(m +
 1)/(6*a^2*f*Sin[e + f*x]^2)), x] - Simp[1/(6*a^2)   Int[((a + b*Sin[e + f*
x])^m/Sin[e + f*x]^2)*Simp[8*a^2 - b^2*(m - 1)*(m - 2) + a*b*m*Sin[e + f*x]
 - (6*a^2 - b^2*m*(m - 2))*Sin[e + f*x]^2, x], x], x]) /; FreeQ[{a, b, e, f
, m}, x] && NeQ[a^2 - b^2, 0] &&  !LtQ[m, -1] && IntegerQ[2*m]
```

rule 3480
```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((a_.) + (b_.)*sin[(e_.) + (f_
.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])), x_Symbol] :> Simp[(A*b
- a*B)/(b*c - a*d)   Int[1/(a + b*Sin[e + f*x]), x], x] + Simp[(B*c - A*d)/
(b*c - a*d)   Int[1/(c + d*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f
, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

rule 3534
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-(A*b^2 - a*b*B + a^2*C))*Cos[e + f*x
]*(a + b*Sin[e + f*x])^(m + 1)*((c + d*Sin[e + f*x])^(n + 1)/(f*(m + 1)*(b*
c - a*d)*(a^2 - b^2))), x] + Simp[1/((m + 1)*(b*c - a*d)*(a^2 - b^2))   Int
[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[(m + 1)*(b*c - a*
d)*(a*A - b*B + a*C) + d*(A*b^2 - a*b*B + a^2*C)*(m + n + 2) - (c*(A*b^2 -
a*b*B + a^2*C) + (m + 1)*(b*c - a*d)*(A*b - a*B + b*C))*Sin[e + f*x] - d*(A
*b^2 - a*b*B + a^2*C)*(m + n + 3)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b
, c, d, e, f, A, B, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] &&
NeQ[c^2 - d^2, 0] && LtQ[m, -1] && ((EqQ[a, 0] && IntegerQ[m] &&  !IntegerQ
[n]) ||  !(IntegerQ[2*n] && LtQ[n, -1] && ((IntegerQ[n] &&  !IntegerQ[m]) |
| EqQ[a, 0])))
```

rule 4257
```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Maple [A] (verified)

Time = 2.20 (sec) , antiderivative size = 169, normalized size of antiderivative = 1.56

method	result
default	$-\frac{\frac{\tanh\left(\frac{x}{2}\right)^{3} a^{2}}{3}+\tanh\left(\frac{x}{2}\right)^{2} a b+5\tanh\left(\frac{x}{2}\right) a^{2}+4b^{2}\tanh\left(\frac{x}{2}\right)}{8a^{3}}-\frac{1}{24a\tanh\left(\frac{x}{2}\right)^{3}}-\frac{5a^{2}+4b^{2}}{8a^{3}\tanh\left(\frac{x}{2}\right)}+\frac{b}{8a^{2}\tanh\left(\frac{x}{2}\right)^{2}}-\frac{b(3a^{2}+2b^{2})\,\mathrm{l}}{2a}$
risch	$-\frac{-3ab\,e^{5x}+12\,e^{4x}a^{2}+6b^{2}e^{4x}-12\,e^{2x}a^{2}-12\,e^{2x}b^{2}+3b\,e^{x}a+8a^{2}+6b^{2}}{3(e^{2x}-1)^{3}a^{3}}+\frac{3b\ln(e^{x}+1)}{2a^{2}}+\frac{b^{3}\ln(e^{x}+1)}{a^{4}}-\frac{3b\ln(e^{x}-1)}{2a^{2}}-\frac{b^{3}\ln(e}{a^{4}}$

input
```
int(coth(x)^4/(a+b*sinh(x)),x,method=_RETURNVERBOSE)
```

output
```
-1/8/a^3*(1/3*tanh(1/2*x)^3*a^2+tanh(1/2*x)^2*a*b+5*tanh(1/2*x)*a^2+4*b^2*
tanh(1/2*x))-1/24/a/tanh(1/2*x)^3-1/8*(5*a^2+4*b^2)/a^3/tanh(1/2*x)+1/8/a^
2*b/tanh(1/2*x)^2-1/2/a^4*b*(3*a^2+2*b^2)*ln(tanh(1/2*x))-1/8/a^4*(-16*a^4
-32*a^2*b^2-16*b^4)/(a^2+b^2)^(1/2)*arctanh(1/2*(2*a*tanh(1/2*x)-2*b)/(a^2
+b^2)^(1/2))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1303 vs. $2(96) = 192$.

Time = 0.16 (sec) , antiderivative size = 1303, normalized size of antiderivative = 12.06

$$\int \frac{\coth^4(x)}{a+b\sinh(x)}\,dx = \text{Too large to display}$$

input
```
integrate(coth(x)^4/(a+b*sinh(x)),x, algorithm="fricas")
```

output
```
1/6*(6*a^2*b*cosh(x)^5 + 6*a^2*b*sinh(x)^5 - 12*(2*a^3 + a*b^2)*cosh(x)^4
+ 6*(5*a^2*b*cosh(x) - 4*a^3 - 2*a*b^2)*sinh(x)^4 - 6*a^2*b*cosh(x) + 12*(
5*a^2*b*cosh(x)^2 - 4*(2*a^3 + a*b^2)*cosh(x))*sinh(x)^3 - 16*a^3 - 12*a*b
^2 + 24*(a^3 + a*b^2)*cosh(x)^2 + 12*(5*a^2*b*cosh(x)^3 + 2*a^3 + 2*a*b^2
- 6*(2*a^3 + a*b^2)*cosh(x)^2)*sinh(x)^2 + 6*((a^2 + b^2)*cosh(x)^6 + 6*(a
^2 + b^2)*cosh(x)*sinh(x)^5 + (a^2 + b^2)*sinh(x)^6 - 3*(a^2 + b^2)*cosh(x
)^4 + 3*(5*(a^2 + b^2)*cosh(x)^2 - a^2 - b^2)*sinh(x)^4 + 4*(5*(a^2 + b^2)
*cosh(x)^3 - 3*(a^2 + b^2)*cosh(x))*sinh(x)^3 + 3*(a^2 + b^2)*cosh(x)^2 +
3*(5*(a^2 + b^2)*cosh(x)^4 - 6*(a^2 + b^2)*cosh(x)^2 + a^2 + b^2)*sinh(x)^
2 - a^2 - b^2 + 6*((a^2 + b^2)*cosh(x)^5 - 2*(a^2 + b^2)*cosh(x)^3 + (a^2
+ b^2)*cosh(x))*sinh(x))*sqrt(a^2 + b^2)*log((b^2*cosh(x)^2 + b^2*sinh(x)^
2 + 2*a*b*cosh(x) + 2*a^2 + b^2 + 2*(b^2*cosh(x) + a*b)*sinh(x) - 2*sqrt(a
^2 + b^2)*(b*cosh(x) + b*sinh(x) + a))/(b*cosh(x)^2 + b*sinh(x)^2 + 2*a*co
sh(x) + 2*(b*cosh(x) + a)*sinh(x) - b)) + 3*((3*a^2*b + 2*b^3)*cosh(x)^6 +
 6*(3*a^2*b + 2*b^3)*cosh(x)*sinh(x)^5 + (3*a^2*b + 2*b^3)*sinh(x)^6 - 3*(
3*a^2*b + 2*b^3)*cosh(x)^4 - 3*(3*a^2*b + 2*b^3 - 5*(3*a^2*b + 2*b^3)*cosh
(x)^2)*sinh(x)^4 + 4*(5*(3*a^2*b + 2*b^3)*cosh(x)^3 - 3*(3*a^2*b + 2*b^3)*
cosh(x))*sinh(x)^3 - 3*a^2*b - 2*b^3 + 3*(3*a^2*b + 2*b^3)*cosh(x)^2 + 3*(
5*(3*a^2*b + 2*b^3)*cosh(x)^4 + 3*a^2*b + 2*b^3 - 6*(3*a^2*b + 2*b^3)*cosh
(x)^2)*sinh(x)^2 + 6*((3*a^2*b + 2*b^3)*cosh(x)^5 - 2*(3*a^2*b + 2*b^3)...
```

Sympy [F]

$$\int \frac{\coth^4(x)}{a+b\sinh(x)}\,dx=\int \frac{\coth^4{(x)}}{a+b\sinh{(x)}}\,dx$$

input
```
integrate(coth(x)**4/(a+b*sinh(x)),x)
```

output
```
Integral(coth(x)**4/(a + b*sinh(x)), x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 212 vs. 2(96) = 192.

Time = 0.13 (sec) , antiderivative size = 212, normalized size of antiderivative = 1.96

$$\int \frac{\coth^4(x)}{a+b\sinh(x)}\,dx = -\frac{3\,abe^{(-x)} - 3\,abe^{(-5\,x)} - 8\,a^2 - 6\,b^2 + 12\,(a^2+b^2)e^{(-2\,x)} - 6\,(2\,a^2+b^2)e^{(-4\,x)}}{3\,(3\,a^3e^{(-2\,x)} - 3\,a^3e^{(-4\,x)} + a^3e^{(-6\,x)} - a^3)} + \frac{(3\,a^2b+2\,b^3)\log\left(e^{(-x)}+1\right)}{2\,a^4} - \frac{(3\,a^2b+2\,b^3)\log\left(e^{(-x)}-1\right)}{2\,a^4} + \frac{(a^4+2\,a^2b^2+b^4)\log\left(\frac{be^{(-x)}-a-\sqrt{a^2+b^2}}{be^{(-x)}-a+\sqrt{a^2+b^2}}\right)}{\sqrt{a^2+b^2}a^4}$$

input
```
integrate(coth(x)^4/(a+b*sinh(x)),x, algorithm="maxima")
```

output
```
-1/3*(3*a*b*e^(-x) - 3*a*b*e^(-5*x) - 8*a^2 - 6*b^2 + 12*(a^2 + b^2)*e^(-2
*x) - 6*(2*a^2 + b^2)*e^(-4*x))/(3*a^3*e^(-2*x) - 3*a^3*e^(-4*x) + a^3*e^(
-6*x) - a^3) + 1/2*(3*a^2*b + 2*b^3)*log(e^(-x) + 1)/a^4 - 1/2*(3*a^2*b +
2*b^3)*log(e^(-x) - 1)/a^4 + (a^4 + 2*a^2*b^2 + b^4)*log((b*e^(-x) - a - s
qrt(a^2 + b^2))/(b*e^(-x) - a + sqrt(a^2 + b^2)))/(sqrt(a^2 + b^2)*a^4)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 194 vs. 2(96) = 192.

Time = 0.14 (sec) , antiderivative size = 194, normalized size of antiderivative = 1.80

$$\int \frac{\coth^4(x)}{a+b\sinh(x)}\,dx = \frac{(3\,a^2b+2\,b^3)\log\left(e^x+1\right)}{2\,a^4} - \frac{(3\,a^2b+2\,b^3)\log\left(|e^x-1|\right)}{2\,a^4} + \frac{(a^4+2\,a^2b^2+b^4)\log\left(\frac{\left|2\,be^x+2\,a-2\,\sqrt{a^2+b^2}\right|}{\left|2\,be^x+2\,a+2\,\sqrt{a^2+b^2}\right|}\right)}{\sqrt{a^2+b^2}a^4} + \frac{3\,abe^{(5\,x)} - 12\,a^2e^{(4\,x)} - 6\,b^2e^{(4\,x)} + 12\,a^2e^{(2\,x)} + 12\,b^2e^{(2\,x)} - 3\,abe^x - 8\,a^2 - 6\,b^2}{3\,a^3\left(e^{(2\,x)}-1\right)^3}$$

input
```
integrate(coth(x)^4/(a+b*sinh(x)),x, algorithm="giac")
```

output
```
1/2*(3*a^2*b + 2*b^3)*log(e^x + 1)/a^4 - 1/2*(3*a^2*b + 2*b^3)*log(abs(e^x
 - 1))/a^4 + (a^4 + 2*a^2*b^2 + b^4)*log(abs(2*b*e^x + 2*a - 2*sqrt(a^2 +
b^2))/abs(2*b*e^x + 2*a + 2*sqrt(a^2 + b^2)))/(sqrt(a^2 + b^2)*a^4) + 1/3*
(3*a*b*e^(5*x) - 12*a^2*e^(4*x) - 6*b^2*e^(4*x) + 12*a^2*e^(2*x) + 12*b^2*
e^(2*x) - 3*a*b*e^x - 8*a^2 - 6*b^2)/(a^3*(e^(2*x) - 1)^3)
```

Mupad [B] (verification not implemented)

Time = 2.31 (sec) , antiderivative size = 778, normalized size of antiderivative = 7.20

$$\int \frac{\coth^4(x)}{a+b\sinh(x)}\,dx = \text{Too large to display}$$

input
```
int(coth(x)^4/(a + b*sinh(x)),x)
```

output
```
(log(- (8*(18*a^8*b + 8*b^9 + 40*a^2*b^7 + 74*a^4*b^5 + 60*a^6*b^3 - 30*a^
9*exp(x) - 14*a*b^8*exp(x) - 69*a^3*b^6*exp(x) - 126*a^5*b^4*exp(x) - 101*
a^7*b^2*exp(x)))/(a^9*b^3) - (((a^2 + b^2)^3)^(1/2)*((8*(4*a^8 + 8*b^8 + 3
6*a^2*b^6 + 57*a^4*b^4 + 34*a^6*b^2 - 12*a*b^7*exp(x) - 36*a^7*b*exp(x) -
52*a^3*b^5*exp(x) - 75*a^5*b^3*exp(x)))/(a^6*b^4) - (((16*(4*a^4*b + 4*b^5
 + 8*a^2*b^3 - 8*a^5*exp(x) - 7*a*b^4*exp(x) - 15*a^3*b^2*exp(x)))/(a*b^5)
 + (32*((a^2 + b^2)^3)^(1/2)*(3*a^4*b + 2*a^2*b^3 - 4*a^5*exp(x) - 3*a^3*b
^2*exp(x)))/(a^4*b^5))*((a^2 + b^2)^3)^(1/2))/a^4))/a^4)*((a^2 + b^2)^3)^(
1/2))/a^4 - ((2*(2*a^2 + b^2))/a^3 - (b*exp(x))/a^2)/(exp(2*x) - 1) - (4/a
 - (2*b*exp(x))/a^2)/(exp(4*x) - 2*exp(2*x) + 1) - (log((((a^2 + b^2)^3)^(
1/2)*((8*(4*a^8 + 8*b^8 + 36*a^2*b^6 + 57*a^4*b^4 + 34*a^6*b^2 - 12*a*b^7*
exp(x) - 36*a^7*b*exp(x) - 52*a^3*b^5*exp(x) - 75*a^5*b^3*exp(x)))/(a^6*b^
4) + (((16*(4*a^4*b + 4*b^5 + 8*a^2*b^3 - 8*a^5*exp(x) - 7*a*b^4*exp(x) -
15*a^3*b^2*exp(x)))/(a*b^5) - (32*((a^2 + b^2)^3)^(1/2)*(3*a^4*b + 2*a^2*b
^3 - 4*a^5*exp(x) - 3*a^3*b^2*exp(x)))/(a^4*b^5))*((a^2 + b^2)^3)^(1/2))/a
^4))/a^4 - (8*(18*a^8*b + 8*b^9 + 40*a^2*b^7 + 74*a^4*b^5 + 60*a^6*b^3 - 3
0*a^9*exp(x) - 14*a*b^8*exp(x) - 69*a^3*b^6*exp(x) - 126*a^5*b^4*exp(x) -
101*a^7*b^2*exp(x)))/(a^9*b^3))*((a^2 + b^2)^3)^(1/2))/a^4 - 8/(3*a*(3*exp
(2*x) - 3*exp(4*x) + exp(6*x) - 1)) - (log(exp(x) - 1)*(3*a^2*b + 2*b^3))/
(2*a^4) + (log(exp(x) + 1)*(3*a^2*b + 2*b^3))/(2*a^4)
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 654, normalized size of antiderivative = 6.06

$$\int \frac{\coth^4(x)}{a+b\sinh(x)}\,dx$$
$$= \frac{-6e^{6x}\log(e^x-1)\,b^3+6e^{6x}\log(e^x+1)\,b^3-4e^{6x}a\,b^2+6e^{5x}a^2b+18e^{4x}\log(e^x-1)\,b^3-18e^{4x}\log(e^x+1)\,b}{}$$

input
```
int(coth(x)^4/(a+b*sinh(x)),x)
```

output
```
(12*e**(6*x)*sqrt(a**2 + b**2)*atan((e**x*b*i + a*i)/sqrt(a**2 + b**2))*a*
*2*i + 12*e**(6*x)*sqrt(a**2 + b**2)*atan((e**x*b*i + a*i)/sqrt(a**2 + b**
2))*b**2*i - 36*e**(4*x)*sqrt(a**2 + b**2)*atan((e**x*b*i + a*i)/sqrt(a**2
 + b**2))*a**2*i - 36*e**(4*x)*sqrt(a**2 + b**2)*atan((e**x*b*i + a*i)/sqr
t(a**2 + b**2))*b**2*i + 36*e**(2*x)*sqrt(a**2 + b**2)*atan((e**x*b*i + a*
i)/sqrt(a**2 + b**2))*a**2*i + 36*e**(2*x)*sqrt(a**2 + b**2)*atan((e**x*b*
i + a*i)/sqrt(a**2 + b**2))*b**2*i - 12*sqrt(a**2 + b**2)*atan((e**x*b*i +
 a*i)/sqrt(a**2 + b**2))*a**2*i - 12*sqrt(a**2 + b**2)*atan((e**x*b*i + a*
i)/sqrt(a**2 + b**2))*b**2*i - 9*e**(6*x)*log(e**x - 1)*a**2*b - 6*e**(6*x
)*log(e**x - 1)*b**3 + 9*e**(6*x)*log(e**x + 1)*a**2*b + 6*e**(6*x)*log(e*
*x + 1)*b**3 - 8*e**(6*x)*a**3 - 4*e**(6*x)*a*b**2 + 6*e**(5*x)*a**2*b + 2
7*e**(4*x)*log(e**x - 1)*a**2*b + 18*e**(4*x)*log(e**x - 1)*b**3 - 27*e**(
4*x)*log(e**x + 1)*a**2*b - 18*e**(4*x)*log(e**x + 1)*b**3 - 27*e**(2*x)*l
og(e**x - 1)*a**2*b - 18*e**(2*x)*log(e**x - 1)*b**3 + 27*e**(2*x)*log(e**
x + 1)*a**2*b + 18*e**(2*x)*log(e**x + 1)*b**3 + 12*e**(2*x)*a*b**2 - 6*e*
*x*a**2*b + 9*log(e**x - 1)*a**2*b + 6*log(e**x - 1)*b**3 - 9*log(e**x + 1
)*a**2*b - 6*log(e**x + 1)*b**3 - 8*a**3 - 8*a*b**2)/(6*a**4*(e**(6*x) - 3
*e**(4*x) + 3*e**(2*x) - 1))
```

3.236 $\int \frac{\tanh^4(x)}{(a+b\sinh(x))^2}\,dx$

Optimal result

Integrand size = 13, antiderivative size = 224

$$\int \frac{\tanh^4(x)}{(a+b\sinh(x))^2}\,dx = -\frac{2a^5\text{arctanh}\left(\frac{b-a\tanh\left(\frac{x}{2}\right)}{\sqrt{a^2+b^2}}\right)}{(a^2+b^2)^{7/2}}+\frac{8a^3b^2\text{arctanh}\left(\frac{b-a\tanh\left(\frac{x}{2}\right)}{\sqrt{a^2+b^2}}\right)}{(a^2+b^2)^{7/2}} - \frac{4a^3b\text{sech}(x)}{(a^2+b^2)^3}+\frac{2ab\text{sech}^3(x)}{3(a^2+b^2)^2} - \frac{a^4b\cosh(x)}{(a^2+b^2)^3(a+b\sinh(x))}+\frac{(a^2-b^2)\tanh(x)}{(a^2+b^2)^2} - \frac{(2a^4-3a^2b^2-b^4)\tanh(x)}{(a^2+b^2)^3}-\frac{(a^2-b^2)\tanh^3(x)}{3(a^2+b^2)^2}$$

output
```
-2*a^5*arctanh((b-a*tanh(1/2*x))/(a^2+b^2)^(1/2))/(a^2+b^2)^(7/2)+8*a^3*b^
2*arctanh((b-a*tanh(1/2*x))/(a^2+b^2)^(1/2))/(a^2+b^2)^(7/2)-4*a^3*b*sech(
x)/(a^2+b^2)^3+2/3*a*b*sech(x)^3/(a^2+b^2)^2-a^4*b*cosh(x)/(a^2+b^2)^3/(a+
b*sinh(x))+(a^2-b^2)*tanh(x)/(a^2+b^2)^2-(2*a^4-3*a^2*b^2-b^4)*tanh(x)/(a^
2+b^2)^3-1/3*(a^2-b^2)*tanh(x)^3/(a^2+b^2)^2
```

Mathematica [A] (verified)

Time = 0.29 (sec) , antiderivative size = 144, normalized size of antiderivative = 0.64

$$\int \frac{\tanh^4(x)}{(a+b\sinh(x))^2}\,dx$$
$$= \frac{\frac{6a^3(a^2-4b^2)\arctan\left(\frac{b-a\tanh\left(\frac{x}{2}\right)}{\sqrt{-a^2-b^2}}\right)}{\sqrt{-a^2-b^2}} - 12a^3b\text{sech}(x) - \frac{3a^4b\cosh(x)}{a+b\sinh(x)} + (a^2+b^2)\,\text{sech}^3(x)\,(2ab+(a^2-b^2)\sinh(x)) +}{3\,(a^2+b^2)^3}$$

input
```
Integrate[Tanh[x]^4/(a + b*Sinh[x])^2,x]
```

output
```
((6*a^3*(a^2 - 4*b^2)*ArcTan[(b - a*Tanh[x/2])/Sqrt[-a^2 - b^2]])/Sqrt[-a^
2 - b^2] - 12*a^3*b*Sech[x] - (3*a^4*b*Cosh[x])/(a + b*Sinh[x]) + (a^2 + b
^2)*Sech[x]^3*(2*a*b + (a^2 - b^2)*Sinh[x]) + (-4*a^4 + 9*a^2*b^2 + b^4)*T
anh[x])/(3*(a^2 + b^2)^3)
```

Rubi [A] (verified)

Time = 0.73 (sec) , antiderivative size = 224, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.231$, Rules used = {3042, 3210, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\tanh^4(x)}{(a+b\sinh(x))^2}\,dx$$
$$\Big\downarrow \text{3042}$$
$$\int \frac{\tan(ix)^4}{(a-ib\sin(ix))^2}dx$$
$$\Big\downarrow \text{3210}$$
$$\int \left(\frac{\text{sech}^4(x)\left(a^2\left(1-\frac{b^2}{a^2}\right)-2ab\sinh(x)\right)}{(a^2+b^2)^2} + \frac{a^4}{(a^2+b^2)^2\,(a+b\sinh(x))^2} - \frac{4a^3b^2}{(a^2+b^2)^3\,(a+b\sinh(x))} + \frac{\text{sech}^2(x)}{}\right.$$

$$\Big\downarrow \text{2009}$$

$$-\frac{(a^2-b^2)\tanh^3(x)}{3(a^2+b^2)^2}+\frac{(a^2-b^2)\tanh(x)}{(a^2+b^2)^2}+\frac{2ab\text{sech}^3(x)}{3(a^2+b^2)^2}-\frac{2a^5\text{arctanh}\left(\frac{b-a\tanh\left(\frac{x}{2}\right)}{\sqrt{a^2+b^2}}\right)}{(a^2+b^2)^{7/2}}-\frac{a^4b\cosh(x)}{(a^2+b^2)^3(a+b\sinh(x))}-\frac{(2a^4-3a^2b^2-b^4)\tanh(x)}{(a^2+b^2)^3}+\frac{8a^3b^2\text{arctanh}\left(\frac{b-a\tanh\left(\frac{x}{2}\right)}{\sqrt{a^2+b^2}}\right)}{(a^2+b^2)^{7/2}}-\frac{4a^3b\text{sech}(x)}{(a^2+b^2)^3}$$

input
```
Int[Tanh[x]^4/(a + b*Sinh[x])^2,x]
```

output
```
(-2*a^5*ArcTanh[(b - a*Tanh[x/2])/Sqrt[a^2 + b^2]])/(a^2 + b^2)^(7/2) + (8
*a^3*b^2*ArcTanh[(b - a*Tanh[x/2])/Sqrt[a^2 + b^2]])/(a^2 + b^2)^(7/2) - (
4*a^3*b*Sech[x])/(a^2 + b^2)^3 + (2*a*b*Sech[x]^3)/(3*(a^2 + b^2)^2) - (a^
4*b*Cosh[x])/((a^2 + b^2)^3*(a + b*Sinh[x])) + ((a^2 - b^2)*Tanh[x])/(a^2
+ b^2)^2 - ((2*a^4 - 3*a^2*b^2 - b^4)*Tanh[x])/(a^2 + b^2)^3 - ((a^2 - b^2
)*Tanh[x]^3)/(3*(a^2 + b^2)^2)
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3210
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*tan[(e_.) + (f_.)*(x_)]^(p_
), x_Symbol] :> Int[ExpandIntegrand[Sin[e + f*x]^p*((a + b*Sin[e + f*x])^m/
(1 - Sin[e + f*x]^2)^(p/2)), x], x] /; FreeQ[{a, b, e, f}, x] && NeQ[a^2 -
b^2, 0] && IntegersQ[m, p/2]
```

Maple [A] (verified)

Time = 3.23 (sec) , antiderivative size = 245, normalized size of antiderivative = 1.09

method	result
default	$-\frac{2a^3\left(\frac{-b^2\tanh\left(\frac{x}{2}\right)-ab}{\tanh\left(\frac{x}{2}\right)^2 a-2b\tanh\left(\frac{x}{2}\right)-a}-\frac{\left(a^2-4b^2\right)\operatorname{arctanh}\left(\frac{2a\tanh\left(\frac{x}{2}\right)-2b}{2\sqrt{a^2+b^2}}\right)}{\sqrt{a^2+b^2}}\right)}{\left(a^4+2a^2b^2+b^4\right)\left(a^2+b^2\right)}+\frac{2\left(-a^4+3a^2b^2\right)\tanh\left(\frac{x}{2}\right)^5+2\left(-2a^3b+2a\,b^3\right)\tanh\left(\frac{x}{2}\right.}{}$
risch	$\frac{2a^5\mathrm{e}^{7x}-8a^3b^2\mathrm{e}^{7x}-14a^4b\,\mathrm{e}^{6x}-6a^2b^3\mathrm{e}^{6x}-2b^5\mathrm{e}^{6x}+14a^5\mathrm{e}^{5x}-\frac{44a^3b^2\mathrm{e}^{5x}}{3}+\frac{4a\,b^4\mathrm{e}^{5x}}{3}-\frac{82a^4b\,\mathrm{e}^{4x}}{3}+\frac{14a^2b^3\mathrm{e}^{4x}}{3}+2b^5\mathrm{e}^{4x}+14a^5\mathrm{e}^{3x}-\frac{64a^3b}{3}}{\left(a^2+b^2\right)\left(a^4+2a^2b^2+b^4\right)\left(\mathrm{e}^{2x}+1\right)^3\left(b\,\mathrm{e}^{2x}+2\right.}$

input
```
int(tanh(x)^4/(a+b*sinh(x))^2,x,method=_RETURNVERBOSE)
```

output
```
-2*a^3/(a^4+2*a^2*b^2+b^4)/(a^2+b^2)*((-b^2*tanh(1/2*x)-a*b)/(tanh(1/2*x)^
2*a-2*b*tanh(1/2*x)-a)-(a^2-4*b^2)/(a^2+b^2)^(1/2)*arctanh(1/2*(2*a*tanh(1
/2*x)-2*b)/(a^2+b^2)^(1/2)))+2/(a^2+b^2)^3*((-a^4+3*a^2*b^2)*tanh(1/2*x)^5
+(-2*a^3*b+2*a*b^3)*tanh(1/2*x)^4+(-10/3*a^4+6*a^2*b^2+4/3*b^4)*tanh(1/2*x
)^3-8*a^3*b*tanh(1/2*x)^2+(-a^4+3*a^2*b^2)*tanh(1/2*x)-10/3*a^3*b+2/3*a*b^
3)/(1+tanh(1/2*x)^2)^3
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 3534 vs. 2(212) = 424.

Time = 0.16 (sec) , antiderivative size = 3534, normalized size of antiderivative = 15.78

$$\int \frac{\tanh^4(x)}{(a+b\sinh(x))^2}\,dx = \text{Too large to display}$$

input
```
integrate(tanh(x)^4/(a+b*sinh(x))^2,x, algorithm="fricas")
```

output
```
Too large to include
```

Sympy [F]

$$\int \frac{\tanh^4(x)}{(a+b\sinh(x))^2}\,dx = \int \frac{\tanh^4(x)}{(a+b\sinh(x))^2}\,dx$$

input
```
integrate(tanh(x)**4/(a+b*sinh(x))**2,x)
```

output
```
Integral(tanh(x)**4/(a + b*sinh(x))**2, x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 523 vs. 2(212) = 424.

Time = 0.14 (sec) , antiderivative size = 523, normalized size of antiderivative = 2.33

$$\int \frac{\tanh^4(x)}{(a+b\sinh(x))^2}\,dx = \frac{(a^2-4\,b^2)a^3\log\left(\frac{be^{(-x)}-a-\sqrt{a^2+b^2}}{be^{(-x)}-a+\sqrt{a^2+b^2}}\right)}{(a^6+3\,a^4b^2+3\,a^2b^4+b^6)\sqrt{a^2+b^2}} - \frac{2\,(7\,a^4b-9\,a^2b^3-b^5+(11\,a^5-6\,a^3b^2-2\,ab^4)e^{(-x)}+(35\,a^4b-9\,a^2b^3+}{3\,(a^6b+3\,a^4b^3+3\,a^2b^5+b^7+2\,(a^7+3\,a^5b^2+3\,a^3b^4+ab^6)e^{(-x)}+2\,(a^6b+3\,a^4b^3+3\,a^2b^5+b^7)e^{(-2\,x}}$$

input
```
integrate(tanh(x)^4/(a+b*sinh(x))^2,x, algorithm="maxima")
```

output
```
(a^2 - 4*b^2)*a^3*log((b*e^(-x) - a - sqrt(a^2 + b^2))/(b*e^(-x) - a + sqr
t(a^2 + b^2)))/((a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6)*sqrt(a^2 + b^2)) - 2/3
*(7*a^4*b - 9*a^2*b^3 - b^5 + (11*a^5 - 6*a^3*b^2 - 2*a*b^4)*e^(-x) + (35*
a^4*b - 9*a^2*b^3 + b^5)*e^(-2*x) + (21*a^5 - 32*a^3*b^2 - 8*a*b^4)*e^(-3*
x) + (41*a^4*b - 7*a^2*b^3 - 3*b^5)*e^(-4*x) + (21*a^5 - 22*a^3*b^2 + 2*a*
b^4)*e^(-5*x) + 3*(7*a^4*b + 3*a^2*b^3 + b^5)*e^(-6*x) + 3*(a^5 - 4*a^3*b^
2)*e^(-7*x))/(a^6*b + 3*a^4*b^3 + 3*a^2*b^5 + b^7 + 2*(a^7 + 3*a^5*b^2 + 3
*a^3*b^4 + a*b^6)*e^(-x) + 2*(a^6*b + 3*a^4*b^3 + 3*a^2*b^5 + b^7)*e^(-2*x
) + 6*(a^7 + 3*a^5*b^2 + 3*a^3*b^4 + a*b^6)*e^(-3*x) + 6*(a^7 + 3*a^5*b^2
+ 3*a^3*b^4 + a*b^6)*e^(-5*x) - 2*(a^6*b + 3*a^4*b^3 + 3*a^2*b^5 + b^7)*e^
(-6*x) + 2*(a^7 + 3*a^5*b^2 + 3*a^3*b^4 + a*b^6)*e^(-7*x) - (a^6*b + 3*a^4
*b^3 + 3*a^2*b^5 + b^7)*e^(-8*x))
```

Giac [A] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 292, normalized size of antiderivative = 1.30

$$\int \frac{\tanh^4(x)}{(a+b\sinh(x))^2}\,dx$$
$$= \frac{(a^5-4a^3b^2)\log\left(\frac{|2be^x+2a-2\sqrt{a^2+b^2}|}{|2be^x+2a+2\sqrt{a^2+b^2}|}\right)}{(a^6+3a^4b^2+3a^2b^4+b^6)\sqrt{a^2+b^2}} + \frac{2(a^5e^x-a^4b)}{(a^6+3a^4b^2+3a^2b^4+b^6)(be^{(2x)}+2ae^x-b)}$$
$$- \frac{2\left(12a^3be^{(5x)}-6a^4e^{(4x)}+9a^2b^2e^{(4x)}+3b^4e^{(4x)}+16a^3be^{(3x)}-8ab^3e^{(3x)}-6a^4e^{(2x)}+18a^2b^2e^{(2x)}\right.}{3(a^6+3a^4b^2+3a^2b^4+b^6)(e^{(2x)}+1)^3}$$

input
```
integrate(tanh(x)^4/(a+b*sinh(x))^2,x, algorithm="giac")
```

output
```
(a^5 - 4*a^3*b^2)*log(abs(2*b*e^x + 2*a - 2*sqrt(a^2 + b^2))/abs(2*b*e^x +
 2*a + 2*sqrt(a^2 + b^2)))/((a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6)*sqrt(a^2 +
 b^2)) + 2*(a^5*e^x - a^4*b)/((a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6)*(b*e^(2*
x) + 2*a*e^x - b)) - 2/3*(12*a^3*b*e^(5*x) - 6*a^4*e^(4*x) + 9*a^2*b^2*e^(
4*x) + 3*b^4*e^(4*x) + 16*a^3*b*e^(3*x) - 8*a*b^3*e^(3*x) - 6*a^4*e^(2*x)
+ 18*a^2*b^2*e^(2*x) + 12*a^3*b*e^x - 4*a^4 + 9*a^2*b^2 + b^4)/((a^6 + 3*a
^4*b^2 + 3*a^2*b^4 + b^6)*(e^(2*x) + 1)^3)
```

Mupad [B] (verification not implemented)

Time = 2.14 (sec) , antiderivative size = 543, normalized size of antiderivative = 2.42

$$\int \frac{\tanh^4(x)}{(a+b\sinh(x))^2}\,dx = \frac{\frac{8(a^2-b^2)}{3(a^4+2a^2b^2+b^4)} - \frac{16\,a\,b\,e^x}{3(a^4+2a^2b^2+b^4)}}{3e^{2x}+3e^{4x}+e^{6x}+1} - \frac{\frac{4(a^6+a^4b^2-a^2b^4-b^6)}{(a^4+2a^2b^2+b^4)^2} - \frac{16e^x(a^5b+2a^3b^3+ab^5)}{3(a^4+2a^2b^2+b^4)^2}}{2e^{2x}+e^{4x}+1} - \frac{\frac{2(a^6b^5+a^4b^7)}{b^3(a^2b+b^3)(a^2+b^2)^3} - \frac{2e^x(a^7b^5+a^5b^7)}{b^4(a^2b+b^3)(a^2+b^2)^3}}{2ae^x-b+be^{2x}} - \frac{\frac{2(-2a^6+a^4b^2+4a^2b^4+b^6)}{(a^4+2a^2b^2+b^4)^2} + \frac{8e^x(a^5b+a^3b^3)}{(a^4+2a^2b^2+b^4)^2}}{e^{2x}+1} - \frac{\ln\left(-\frac{2e^x(a^5-4a^3b^2)}{b(a^2+b^2)^3} - \frac{2(a^5-4a^3b^2)(b-ae^x)}{b(a^2+b^2)^{7/2}}\right)(a^5-4a^3b^2)}{(a^2+b^2)^{7/2}} + \frac{\ln\left(\frac{2(a^5-4a^3b^2)(b-ae^x)}{b(a^2+b^2)^{7/2}} - \frac{2e^x(a^5-4a^3b^2)}{b(a^2+b^2)^3}\right)(a^5-4a^3b^2)}{(a^2+b^2)^{7/2}}$$

input

```
int(tanh(x)^4/(a + b*sinh(x))^2,x)
```

output

```
((8*(a^2 - b^2))/(3*(a^4 + b^4 + 2*a^2*b^2)) - (16*a*b*exp(x))/(3*(a^4 + b
^4 + 2*a^2*b^2)))/(3*exp(2*x) + 3*exp(4*x) + exp(6*x) + 1) - ((4*(a^6 - b^
6 - a^2*b^4 + a^4*b^2))/(a^4 + b^4 + 2*a^2*b^2)^2 - (16*exp(x)*(a*b^5 + a^
5*b + 2*a^3*b^3))/(3*(a^4 + b^4 + 2*a^2*b^2)^2))/(2*exp(2*x) + exp(4*x) +
1) - ((2*(a^4*b^7 + a^6*b^5))/(b^3*(a^2*b + b^3)*(a^2 + b^2)^3) - (2*exp(x
)*(a^5*b^7 + a^7*b^5))/(b^4*(a^2*b + b^3)*(a^2 + b^2)^3))/(2*a*exp(x) - b
+ b*exp(2*x)) - ((2*(b^6 - 2*a^6 + 4*a^2*b^4 + a^4*b^2))/(a^4 + b^4 + 2*a^
2*b^2)^2 + (8*exp(x)*(a^5*b + a^3*b^3))/(a^4 + b^4 + 2*a^2*b^2)^2)/(exp(2*
x) + 1) - (log(- (2*exp(x)*(a^5 - 4*a^3*b^2))/(b*(a^2 + b^2)^3) - (2*(a^5
- 4*a^3*b^2)*(b - a*exp(x)))/(b*(a^2 + b^2)^(7/2)))*(a^5 - 4*a^3*b^2))/(a^
2 + b^2)^(7/2) + (log((2*(a^5 - 4*a^3*b^2)*(b - a*exp(x)))/(b*(a^2 + b^2)^
(7/2)) - (2*exp(x)*(a^5 - 4*a^3*b^2))/(b*(a^2 + b^2)^3))*(a^5 - 4*a^3*b^2)
)/(a^2 + b^2)^(7/2)
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 1445, normalized size of antiderivative = 6.45

$$\int \frac{\tanh^4(x)}{(a+b\sinh(x))^2}\,dx = \text{Too large to display}$$

input

```
int(tanh(x)^4/(a+b*sinh(x))^2,x)
```

output

```
(6*e**(8*x)*sqrt(a**2 + b**2)*atan((e**x*b*i + a*i)/sqrt(a**2 + b**2))*a**
5*b*i - 24*e**(8*x)*sqrt(a**2 + b**2)*atan((e**x*b*i + a*i)/sqrt(a**2 + b*
*2))*a**3*b**3*i + 12*e**(7*x)*sqrt(a**2 + b**2)*atan((e**x*b*i + a*i)/sqr
t(a**2 + b**2))*a**6*i - 48*e**(7*x)*sqrt(a**2 + b**2)*atan((e**x*b*i + a*
i)/sqrt(a**2 + b**2))*a**4*b**2*i + 12*e**(6*x)*sqrt(a**2 + b**2)*atan((e*
*x*b*i + a*i)/sqrt(a**2 + b**2))*a**5*b*i - 48*e**(6*x)*sqrt(a**2 + b**2)*
atan((e**x*b*i + a*i)/sqrt(a**2 + b**2))*a**3*b**3*i + 36*e**(5*x)*sqrt(a*
*2 + b**2)*atan((e**x*b*i + a*i)/sqrt(a**2 + b**2))*a**6*i - 144*e**(5*x)*
sqrt(a**2 + b**2)*atan((e**x*b*i + a*i)/sqrt(a**2 + b**2))*a**4*b**2*i + 3
6*e**(3*x)*sqrt(a**2 + b**2)*atan((e**x*b*i + a*i)/sqrt(a**2 + b**2))*a**6
*i - 144*e**(3*x)*sqrt(a**2 + b**2)*atan((e**x*b*i + a*i)/sqrt(a**2 + b**2
))*a**4*b**2*i - 12*e**(2*x)*sqrt(a**2 + b**2)*atan((e**x*b*i + a*i)/sqrt(
a**2 + b**2))*a**5*b*i + 48*e**(2*x)*sqrt(a**2 + b**2)*atan((e**x*b*i + a*
i)/sqrt(a**2 + b**2))*a**3*b**3*i + 12*e**x*sqrt(a**2 + b**2)*atan((e**x*b
*i + a*i)/sqrt(a**2 + b**2))*a**6*i - 48*e**x*sqrt(a**2 + b**2)*atan((e**x
*b*i + a*i)/sqrt(a**2 + b**2))*a**4*b**2*i - 6*sqrt(a**2 + b**2)*atan((e**
x*b*i + a*i)/sqrt(a**2 + b**2))*a**5*b*i + 24*sqrt(a**2 + b**2)*atan((e**x
*b*i + a*i)/sqrt(a**2 + b**2))*a**3*b**3*i - 3*e**(8*x)*a**6*b + 9*e**(8*x
)*a**4*b**3 + 12*e**(8*x)*a**2*b**5 - 48*e**(6*x)*a**6*b - 42*e**(6*x)*a**
4*b**3 - 6*e**(6*x)*b**7 + 24*e**(5*x)*a**7 + 52*e**(5*x)*a**5*b**2 + 3...
```

3.237 $\int \frac{\tanh^3(x)}{(a+b\sinh(x))^2}\,dx$

Optimal result

Integrand size = 13, antiderivative size = 135

$$\begin{aligned}\int \frac{\tanh^3(x)}{(a+b\sinh(x))^2}\,dx &= \frac{ab(3a^2-b^2)\arctan(\sinh(x))}{(a^2+b^2)^3}+\frac{a^2(a^2-3b^2)\log(\cosh(x))}{(a^2+b^2)^3}\\&\quad-\frac{a^2(a^2-3b^2)\log(a+b\sinh(x))}{(a^2+b^2)^3}+\frac{a^3}{(a^2+b^2)^2(a+b\sinh(x))}\\&\quad+\frac{\operatorname{sech}^2(x)\,(a^2-b^2-2ab\sinh(x))}{2\,(a^2+b^2)^2}\end{aligned}$$

output
```
a*b*(3*a^2-b^2)*arctan(sinh(x))/(a^2+b^2)^3+a^2*(a^2-3*b^2)*ln(cosh(x))/(a
^2+b^2)^3-a^2*(a^2-3*b^2)*ln(a+b*sinh(x))/(a^2+b^2)^3+a^3/(a^2+b^2)^2/(a+b
*sinh(x))+1/2*sech(x)^2*(a^2-b^2-2*a*b*sinh(x))/(a^2+b^2)^2
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.47 (sec) , antiderivative size = 150, normalized size of antiderivative = 1.11

$$\int \frac{\tanh^3(x)}{(a+b\sinh(x))^2}\,dx$$

$$= \frac{-2ab(a^2+b^2)\arctan(\sinh(x))+a^2(a-ib)(a-3ib)\log(i-\sinh(x))+a^2(a+ib)(a+3ib)\log(i+\sinh(}{2\,(a^2+}$$

input
```
Integrate[Tanh[x]^3/(a + b*Sinh[x])^2,x]
```

output
```
(-2*a*b*(a^2 + b^2)*ArcTan[Sinh[x]] + a^2*(a - I*b)*(a - (3*I)*b)*Log[I -
Sinh[x]] + a^2*(a + I*b)*(a + (3*I)*b)*Log[I + Sinh[x]] - 2*a^2*(a^2 - 3*b
^2)*Log[a + b*Sinh[x]] + (a^4 - b^4)*Sech[x]^2 + (2*a^3*(a^2 + b^2))/(a +
b*Sinh[x]) - 2*a*b*(a^2 + b^2)*Sech[x]*Tanh[x])/(2*(a^2 + b^2)^3)
```

Rubi [A] (verified)

Time = 0.67 (sec) , antiderivative size = 177, normalized size of antiderivative = 1.31, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.538$, Rules used = {3042, 26, 3200, 601, 27, 2160, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\tanh^3(x)}{(a+b\sinh(x))^2}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{i\tan(ix)^3}{(a-ib\sin(ix))^2}dx$$

$$\Big\downarrow \text{26}$$

$$i\int \frac{\tan(ix)^3}{(a-ib\sin(ix))^2}dx$$

$$\Big\downarrow \text{3200}$$

$$\int \frac{b^3\sinh^3(x)}{\left(b^2\sinh^2(x)+b^2\right)^2(a+b\sinh(x))^2}d(b\sinh(x))$$

$$\Big\downarrow \text{601}$$

$$\frac{b^2\left(a^2-2ab\sinh(x)-b^2\right)}{2\left(a^2+b^2\right)^2\left(b^2\sinh^2(x)+b^2\right)}-\frac{\int -\frac{2\left(-\frac{a\sinh^2(x)b^6}{\left(a^2+b^2\right)^2}+\frac{a^3b^4}{\left(a^2+b^2\right)^2}+\frac{a^2\sinh(x)b^3}{a^2+b^2}\right)}{(a+b\sinh(x))^2\left(\sinh^2(x)b^2+b^2\right)}d(b\sinh(x))}{2b^2}$$

$$\Big\downarrow \text{27}$$

$$\frac{\int \frac{-\frac{a\sinh^2(x)b^6}{(a^2+b^2)^2}+\frac{a^3b^4}{(a^2+b^2)^2}+\frac{a^2\sinh(x)b^3}{a^2+b^2}}{(a+b\sinh(x))^2(\sinh^2(x)b^2+b^2)}d(b\sinh(x))}{b^2}+\frac{b^2(a^2-2ab\sinh(x)-b^2)}{2(a^2+b^2)^2(b^2\sinh^2(x)+b^2)}$$

$$\Big\downarrow \text{2160}$$

$$\frac{\int \left(-\frac{b^2a^3}{(a^2+b^2)^2(a+b\sinh(x))^2}+\frac{b^2((3a^2-b^2)b^2+a(a^2-3b^2)\sinh(x)b)a}{(a^2+b^2)^3(\sinh^2(x)b^2+b^2)}+\frac{3a^2b^4-a^4b^2}{(a^2+b^2)^3(a+b\sinh(x))}\right)d(b\sinh(x))}{b^2}+\frac{b^2(a^2-2ab\sinh(x)-b^2)}{2(a^2+b^2)^2(b^2\sinh^2(x)+b^2)}$$

$$\Big\downarrow \text{2009}$$

$$\frac{b^2(a^2-2ab\sinh(x)-b^2)}{2(a^2+b^2)^2(b^2\sinh^2(x)+b^2)}+\frac{\frac{ab^3(3a^2-b^2)\arctan(\sinh(x))}{(a^2+b^2)^3}+\frac{a^2b^2(a^2-3b^2)\log(b^2\sinh^2(x)+b^2)}{2(a^2+b^2)^3}-\frac{a^2b^2(a^2-3b^2)\log(a+b\sinh(x))}{(a^2+b^2)^3}+\frac{a^3b^2}{(a^2+b^2)^2(a+b\sinh(x))}}{b^2}$$

input

```
Int[Tanh[x]^3/(a + b*Sinh[x])^2,x]
```

output

```
(b^2*(a^2 - b^2 - 2*a*b*Sinh[x]))/(2*(a^2 + b^2)^2*(b^2 + b^2*Sinh[x]^2))
+ ((a*b^3*(3*a^2 - b^2)*ArcTan[Sinh[x]])/(a^2 + b^2)^3 - (a^2*b^2*(a^2 - 3
*b^2)*Log[a + b*Sinh[x]])/(a^2 + b^2)^3 + (a^2*b^2*(a^2 - 3*b^2)*Log[b^2 +
 b^2*Sinh[x]^2])/(2*(a^2 + b^2)^3) + (a^3*b^2)/((a^2 + b^2)^2*(a + b*Sinh[
x])))/b^2
```

Defintions of rubi rules used

rule 26

```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 27

```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 601
```
Int[(x_)^(m_)*((c_) + (d_.)*(x_))^(n_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbo
l] :> With[{Qx = PolynomialQuotient[x^m*(c + d*x)^n, a + b*x^2, x], e = Coe
ff[PolynomialRemainder[x^m*(c + d*x)^n, a + b*x^2, x], x, 0], f = Coeff[Pol
ynomialRemainder[x^m*(c + d*x)^n, a + b*x^2, x], x, 1]}, Simp[(a*f - b*e*x)
*((a + b*x^2)^(p + 1)/(2*a*b*(p + 1))), x] + Simp[1/(2*a*(p + 1))   Int[(c
+ d*x)^n*(a + b*x^2)^(p + 1)*ExpandToSum[(2*a*(p + 1)*Qx)/(c + d*x)^n + (e*
(2*p + 3))/(c + d*x)^n, x], x], x]] /; FreeQ[{a, b, c, d}, x] && IGtQ[m, 1]
 && LtQ[p, -1] && ILtQ[n, 0] && NeQ[b*c^2 + a*d^2, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2160
```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol]
:> Int[ExpandIntegrand[(d + e*x)^m*Pq*(a + b*x^2)^p, x], x] /; FreeQ[{a, b,
 d, e, m}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3200
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*tan[(e_.) + (f_.)*(x_)]^(p
_.), x_Symbol] :> Simp[1/f   Subst[Int[(x^p*(a + x)^m)/(b^2 - x^2)^((p + 1)
/2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && NeQ[a^2 - b
^2, 0] && IntegerQ[(p + 1)/2]
```

Maple [A] (verified)

Time = 2.18 (sec) , antiderivative size = 237, normalized size of antiderivative = 1.76

method	result
default	$-\frac{2a^2\left(\frac{\left(-a^2b-b^3\right)\tanh\left(\frac{x}{2}\right)}{\tanh\left(\frac{x}{2}\right)^2a-2b\tanh\left(\frac{x}{2}\right)-a}+\frac{\left(a^2-3b^2\right)\ln\left(\tanh\left(\frac{x}{2}\right)^2a-2b\tanh\left(\frac{x}{2}\right)-a\right)}{2}\right)}{(a^4+2a^2b^2+b^4)(a^2+b^2)}+\frac{\frac{2\left(\left(a^3b+a\,b^3\right)\tanh\left(\frac{x}{2}\right)^3+\left(-a^4+b^4\right)\tanh\left(\frac{x}{2}\right)^2+\left(\right.}{\left(1+\tanh\left(\frac{x}{2}\right)^2\right)^2}}{}$
risch	$\frac{2\,\mathrm{e}^x\left(\mathrm{e}^{4x}a^3-\mathrm{e}^{4x}a\,b^2-a^2b\,\mathrm{e}^{3x}-\mathrm{e}^{3x}b^3+4a^3\mathrm{e}^{2x}+\mathrm{e}^xa^2b+b^3\mathrm{e}^x+a^3-a\,b^2\right)}{(\mathrm{e}^{2x}+1)^2(a^4+2a^2b^2+b^4)(b\,\mathrm{e}^{2x}+2\,\mathrm{e}^xa-b)}-\frac{3ia^3\ln(\mathrm{e}^x-i)b}{a^6+3a^4b^2+3a^2b^4+b^6}+\frac{ia\ln(\mathrm{e}^x-i)b^3}{a^6+3a^4b^2+3a^2b^4+b^6}+\frac{}{a^6}$

input
```
int(tanh(x)^3/(a+b*sinh(x))^2,x,method=_RETURNVERBOSE)
```

output
```
-2*a^2/(a^4+2*a^2*b^2+b^4)/(a^2+b^2)*((-a^2*b-b^3)*tanh(1/2*x)/(tanh(1/2*x
)^2*a-2*b*tanh(1/2*x)-a)+1/2*(a^2-3*b^2)*ln(tanh(1/2*x)^2*a-2*b*tanh(1/2*x
)-a))+2/(a^6+3*a^4*b^2+3*a^2*b^4+b^6)*(((a^3*b+a*b^3)*tanh(1/2*x)^3+(-a^4+
b^4)*tanh(1/2*x)^2+(-a^3*b-a*b^3)*tanh(1/2*x))/(1+tanh(1/2*x)^2)^2+a*(1/2*
(a^3-3*a*b^2)*ln(1+tanh(1/2*x)^2)+(3*a^2*b-b^3)*arctan(tanh(1/2*x))))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2850 vs. $2(133) = 266$.

Time = 0.13 (sec) , antiderivative size = 2850, normalized size of antiderivative = 21.11

$$\int \frac{\tanh^3(x)}{(a+b\sinh(x))^2}\,dx = \text{Too large to display}$$

input
```
integrate(tanh(x)^3/(a+b*sinh(x))^2,x, algorithm="fricas")
```

output
```
-(2*(a^5 - a*b^4)*cosh(x)^5 + 2*(a^5 - a*b^4)*sinh(x)^5 - 2*(a^4*b + 2*a^2
*b^3 + b^5)*cosh(x)^4 - 2*(a^4*b + 2*a^2*b^3 + b^5 - 5*(a^5 - a*b^4)*cosh(
x))*sinh(x)^4 + 8*(a^5 + a^3*b^2)*cosh(x)^3 + 4*(2*a^5 + 2*a^3*b^2 + 5*(a^
5 - a*b^4)*cosh(x)^2 - 2*(a^4*b + 2*a^2*b^3 + b^5)*cosh(x))*sinh(x)^3 + 2*
(a^4*b + 2*a^2*b^3 + b^5)*cosh(x)^2 + 2*(a^4*b + 2*a^2*b^3 + b^5 + 10*(a^5
 - a*b^4)*cosh(x)^3 - 6*(a^4*b + 2*a^2*b^3 + b^5)*cosh(x)^2 + 12*(a^5 + a^
3*b^2)*cosh(x))*sinh(x)^2 + 2*((3*a^3*b^2 - a*b^4)*cosh(x)^6 + (3*a^3*b^2
- a*b^4)*sinh(x)^6 + 2*(3*a^4*b - a^2*b^3)*cosh(x)^5 + 2*(3*a^4*b - a^2*b^
3 + 3*(3*a^3*b^2 - a*b^4)*cosh(x))*sinh(x)^5 - 3*a^3*b^2 + a*b^4 + (3*a^3*
b^2 - a*b^4)*cosh(x)^4 + (3*a^3*b^2 - a*b^4 + 15*(3*a^3*b^2 - a*b^4)*cosh(
x)^2 + 10*(3*a^4*b - a^2*b^3)*cosh(x))*sinh(x)^4 + 4*(3*a^4*b - a^2*b^3)*c
osh(x)^3 + 4*(3*a^4*b - a^2*b^3 + 5*(3*a^3*b^2 - a*b^4)*cosh(x)^3 + 5*(3*a
^4*b - a^2*b^3)*cosh(x)^2 + (3*a^3*b^2 - a*b^4)*cosh(x))*sinh(x)^3 - (3*a^
3*b^2 - a*b^4)*cosh(x)^2 - (3*a^3*b^2 - a*b^4 - 15*(3*a^3*b^2 - a*b^4)*cos
h(x)^4 - 20*(3*a^4*b - a^2*b^3)*cosh(x)^3 - 6*(3*a^3*b^2 - a*b^4)*cosh(x)^
2 - 12*(3*a^4*b - a^2*b^3)*cosh(x))*sinh(x)^2 + 2*(3*a^4*b - a^2*b^3)*cosh
(x) + 2*(3*(3*a^3*b^2 - a*b^4)*cosh(x)^5 + 3*a^4*b - a^2*b^3 + 5*(3*a^4*b
- a^2*b^3)*cosh(x)^4 + 2*(3*a^3*b^2 - a*b^4)*cosh(x)^3 + 6*(3*a^4*b - a^2*
b^3)*cosh(x)^2 - (3*a^3*b^2 - a*b^4)*cosh(x))*sinh(x))*arctan(cosh(x) + si
nh(x)) + 2*(a^5 - a*b^4)*cosh(x) - ((a^4*b - 3*a^2*b^3)*cosh(x)^6 + (a^...
```

Sympy [F]

$$\int \frac{\tanh^3(x)}{(a+b\sinh(x))^2}\,dx = \int \frac{\tanh^3{(x)}}{(a+b\sinh{(x)})^2}\,dx$$

input
```
integrate(tanh(x)**3/(a+b*sinh(x))**2,x)
```

output
```
Integral(tanh(x)**3/(a + b*sinh(x))**2, x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 375 vs. $2(133) = 266$.

Time = 0.15 (sec) , antiderivative size = 375, normalized size of antiderivative = 2.78

$$\int \frac{\tanh^3(x)}{(a+b\sinh(x))^2}\,dx = -\frac{2\,(3\,a^3b-ab^3)\arctan\left(e^{(-x)}\right)}{a^6+3\,a^4b^2+3\,a^2b^4+b^6} - \frac{(a^4-3\,a^2b^2)\log\left(-2\,ae^{(-x)}+be^{(-2\,x)}-b\right)}{a^6+3\,a^4b^2+3\,a^2b^4+b^6} + \frac{(a^4-3\,a^2b^2)\log\left(e^{(-2\,x)}+1\right)}{a^6+3\,a^4b^2+3\,a^2b^4+b^6} + \frac{2\left(4\,a^3e^{(-3\,x)}+(a^3-ab^2)e^{(-x)}-(a^2b+b^3)e^{(-2\,x)}+(\right.}{a^4b+2\,a^2b^3+b^5+2\,(a^5+2\,a^3b^2+ab^4)e^{(-x)}+(a^4b+2\,a^2b^3+b^5)e^{(-2\,x)}+4\,(a^5+2\,a^3b^2+ab^4)e^{(-3\,x)}}$$

input

```
integrate(tanh(x)^3/(a+b*sinh(x))^2,x, algorithm="maxima")
```

output

```
-2*(3*a^3*b - a*b^3)*arctan(e^(-x))/(a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6) -
(a^4 - 3*a^2*b^2)*log(-2*a*e^(-x) + b*e^(-2*x) - b)/(a^6 + 3*a^4*b^2 + 3*a
^2*b^4 + b^6) + (a^4 - 3*a^2*b^2)*log(e^(-2*x) + 1)/(a^6 + 3*a^4*b^2 + 3*a
^2*b^4 + b^6) + 2*(4*a^3*e^(-3*x) + (a^3 - a*b^2)*e^(-x) - (a^2*b + b^3)*e
^(-2*x) + (a^2*b + b^3)*e^(-4*x) + (a^3 - a*b^2)*e^(-5*x))/(a^4*b + 2*a^2*
b^3 + b^5 + 2*(a^5 + 2*a^3*b^2 + a*b^4)*e^(-x) + (a^4*b + 2*a^2*b^3 + b^5)
*e^(-2*x) + 4*(a^5 + 2*a^3*b^2 + a*b^4)*e^(-3*x) - (a^4*b + 2*a^2*b^3 + b^
5)*e^(-4*x) + 2*(a^5 + 2*a^3*b^2 + a*b^4)*e^(-5*x) - (a^4*b + 2*a^2*b^3 +
b^5)*e^(-6*x))
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 307 vs. $2(133) = 266$.

Time = 0.14 (sec) , antiderivative size = 307, normalized size of antiderivative = 2.27

$$\int \frac{\tanh^3(x)}{(a+b\sinh(x))^2}\,dx = \frac{\left(\pi+2\,\arctan\left(\frac{1}{2}\left(e^{(2\,x)}-1\right)e^{(-x)}\right)\right)(3\,a^3b-ab^3)}{2\,(a^6+3\,a^4b^2+3\,a^2b^4+b^6)} + \frac{(a^4-3\,a^2b^2)\log\left(\left(e^{(-x)}-e^x\right)^2+4\right)}{2\,(a^6+3\,a^4b^2+3\,a^2b^4+b^6)} - \frac{(a^4b-3\,a^2b^3)\log\left(\left|-b\left(e^{(-x)}-e^x\right)+2\,a\right|\right)}{a^6b+3\,a^4b^3+3\,a^2b^5+b^7} - \frac{2\left(a^3\left(e^{(-x)}-e^x\right)^2-ab^2\left(e^{(-x)}-e^x\right)^2+a^2b\left(e^{(-x)}-e^x\right)+b^3\left(e^{(-x)}-e^x\right)+6\,a^3-2\,ab^2\right)}{(a^4+2\,a^2b^2+b^4)\left(b\left(e^{(-x)}-e^x\right)^3-2\,a\left(e^{(-x)}-e^x\right)^2+4\,b\left(e^{(-x)}-e^x\right)-8\,a\right)}$$

input
```
integrate(tanh(x)^3/(a+b*sinh(x))^2,x, algorithm="giac")
```

output
```
1/2*(pi + 2*arctan(1/2*(e^(2*x) - 1)*e^(-x)))*(3*a^3*b - a*b^3)/(a^6 + 3*a
^4*b^2 + 3*a^2*b^4 + b^6) + 1/2*(a^4 - 3*a^2*b^2)*log((e^(-x) - e^x)^2 + 4
)/(a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6) - (a^4*b - 3*a^2*b^3)*log(abs(-b*(e^
(-x) - e^x) + 2*a))/(a^6*b + 3*a^4*b^3 + 3*a^2*b^5 + b^7) - 2*(a^3*(e^(-x)
 - e^x)^2 - a*b^2*(e^(-x) - e^x)^2 + a^2*b*(e^(-x) - e^x) + b^3*(e^(-x) -
e^x) + 6*a^3 - 2*a*b^2)/((a^4 + 2*a^2*b^2 + b^4)*(b*(e^(-x) - e^x)^3 - 2*a
*(e^(-x) - e^x)^2 + 4*b*(e^(-x) - e^x) - 8*a))
```

Mupad [B] (verification not implemented)

Time = 4.90 (sec) , antiderivative size = 501, normalized size of antiderivative = 3.71

$$\int \frac{\tanh^3(x)}{(a+b\sinh(x))^2}\,dx = \frac{\frac{2\,\left(a^8+2\,a^6\,b^2-2\,a^2\,b^6-b^8\right)}{\left(a^2+b^2\right)\,{\left(a^4+2\,a^2\,b^2+b^4\right)}^2}-\frac{2\,{\mathrm{e}}^x\,\left(a^7\,b+3\,a^5\,b^3+3\,a^3\,b^5+a\,b^7\right)}{\left(a^2+b^2\right)\,{\left(a^4+2\,a^2\,b^2+b^4\right)}^2}}{{\mathrm{e}}^{2\,x}+1}$$
$$-\frac{\frac{2\,\left(a^2-b^2\right)}{a^4+2\,a^2\,b^2+b^4}-\frac{4\,a\,b\,{\mathrm{e}}^x}{a^4+2\,a^2\,b^2+b^4}}{2\,{\mathrm{e}}^{2\,x}+{\mathrm{e}}^{4\,x}+1}-\frac{a\,\ln\left({\mathrm{e}}^x+1{}\mathrm{i}\right)}{-a^3+a^2\,b\,3{}\mathrm{i}+3\,a\,b^2-b^3\,1{}\mathrm{i}}$$
$$-\frac{\ln\left(15\,a^6\,b^3-a^2\,b^7-30\,a^4\,b^5-4\,a^8\,b+8\,a^9\,{\mathrm{e}}^x+a^2\,b^7\,{\mathrm{e}}^{2\,x}+30\,a^4\,b^5\,{\mathrm{e}}^{2\,x}-15\,a^6\,b^3\,{\mathrm{e}}^{2\,x}+4\,a^8\,b\,{\mathrm{e}}^{2\,x}+2\,a\right.}{a^6+3\,a^4\,b^2+3\,a^2\,b^4+b^6}$$
$$+\frac{2\,{\mathrm{e}}^x\,\left(a^7\,b^2+2\,a^5\,b^4+a^3\,b^6\right)}{b\,\left(a^2\,b+b^3\right)\,\left(a^2+b^2\right)\,\left(2\,a\,{\mathrm{e}}^x-b+b\,{\mathrm{e}}^{2\,x}\right)\,\left(a^4+2\,a^2\,b^2+b^4\right)}$$
$$-\frac{a\,\ln\left(1+{\mathrm{e}}^x\,1{}\mathrm{i}\right)\,1{}\mathrm{i}}{-a^3\,1{}\mathrm{i}+3\,a^2\,b+a\,b^2\,3{}\mathrm{i}-b^3}$$

input
```
int(tanh(x)^3/(a + b*sinh(x))^2,x)
```

output
```
((2*(a^8 - b^8 - 2*a^2*b^6 + 2*a^6*b^2))/((a^2 + b^2)*(a^4 + b^4 + 2*a^2*b
^2)^2) - (2*exp(x)*(a*b^7 + a^7*b + 3*a^3*b^5 + 3*a^5*b^3))/((a^2 + b^2)*(
a^4 + b^4 + 2*a^2*b^2)^2))/(exp(2*x) + 1) - ((2*(a^2 - b^2))/(a^4 + b^4 +
2*a^2*b^2) - (4*a*b*exp(x))/(a^4 + b^4 + 2*a^2*b^2))/(2*exp(2*x) + exp(4*x
) + 1) - (a*log(exp(x) + 1i))/(3*a*b^2 + a^2*b*3i - a^3 - b^3*1i) - (log(1
5*a^6*b^3 - a^2*b^7 - 30*a^4*b^5 - 4*a^8*b + 8*a^9*exp(x) + a^2*b^7*exp(2*
x) + 30*a^4*b^5*exp(2*x) - 15*a^6*b^3*exp(2*x) + 4*a^8*b*exp(2*x) + 2*a^3*
b^6*exp(x) + 60*a^5*b^4*exp(x) - 30*a^7*b^2*exp(x))*(a^4 - 3*a^2*b^2))/(a^
6 + b^6 + 3*a^2*b^4 + 3*a^4*b^2) - (a*log(exp(x)*1i + 1)*1i)/(a*b^2*3i + 3
*a^2*b - a^3*1i - b^3) + (2*exp(x)*(a^3*b^6 + 2*a^5*b^4 + a^7*b^2))/(b*(a^
2*b + b^3)*(a^2 + b^2)*(2*a*exp(x) - b + b*exp(2*x))*(a^4 + b^4 + 2*a^2*b^
2))
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 1305, normalized size of antiderivative = 9.67

$$\int \frac{\tanh^3(x)}{(a + b\sinh(x))^2} dx = \text{Too large to display}$$

input
```
int(tanh(x)^3/(a+b*sinh(x))^2,x)
```

output
```
(6*e**(6*x)*atan(e**x)*a**3*b**2 - 2*e**(6*x)*atan(e**x)*a*b**4 + 12*e**(5
*x)*atan(e**x)*a**4*b - 4*e**(5*x)*atan(e**x)*a**2*b**3 + 6*e**(4*x)*atan(
e**x)*a**3*b**2 - 2*e**(4*x)*atan(e**x)*a*b**4 + 24*e**(3*x)*atan(e**x)*a*
*4*b - 8*e**(3*x)*atan(e**x)*a**2*b**3 - 6*e**(2*x)*atan(e**x)*a**3*b**2 +
 2*e**(2*x)*atan(e**x)*a*b**4 + 12*e**x*atan(e**x)*a**4*b - 4*e**x*atan(e*
*x)*a**2*b**3 - 6*atan(e**x)*a**3*b**2 + 2*atan(e**x)*a*b**4 + e**(6*x)*lo
g(e**(2*x) + 1)*a**4*b - 3*e**(6*x)*log(e**(2*x) + 1)*a**2*b**3 - e**(6*x)
*log(e**(2*x)*b + 2*e**x*a - b)*a**4*b + 3*e**(6*x)*log(e**(2*x)*b + 2*e**
x*a - b)*a**2*b**3 - e**(6*x)*a**4*b + e**(6*x)*b**5 + 2*e**(5*x)*log(e**(
2*x) + 1)*a**5 - 6*e**(5*x)*log(e**(2*x) + 1)*a**3*b**2 - 2*e**(5*x)*log(e
**(2*x)*b + 2*e**x*a - b)*a**5 + 6*e**(5*x)*log(e**(2*x)*b + 2*e**x*a - b)
*a**3*b**2 + e**(4*x)*log(e**(2*x) + 1)*a**4*b - 3*e**(4*x)*log(e**(2*x) +
 1)*a**2*b**3 - e**(4*x)*log(e**(2*x)*b + 2*e**x*a - b)*a**4*b + 3*e**(4*x
)*log(e**(2*x)*b + 2*e**x*a - b)*a**2*b**3 - 3*e**(4*x)*a**4*b - 4*e**(4*x
)*a**2*b**3 - e**(4*x)*b**5 + 4*e**(3*x)*log(e**(2*x) + 1)*a**5 - 12*e**(3
*x)*log(e**(2*x) + 1)*a**3*b**2 - 4*e**(3*x)*log(e**(2*x)*b + 2*e**x*a - b
)*a**5 + 12*e**(3*x)*log(e**(2*x)*b + 2*e**x*a - b)*a**3*b**2 + 4*e**(3*x)
*a**5 + 8*e**(3*x)*a**3*b**2 + 4*e**(3*x)*a*b**4 - e**(2*x)*log(e**(2*x) +
 1)*a**4*b + 3*e**(2*x)*log(e**(2*x) + 1)*a**2*b**3 + e**(2*x)*log(e**(2*x
)*b + 2*e**x*a - b)*a**4*b - 3*e**(2*x)*log(e**(2*x)*b + 2*e**x*a - b)*...
```

3.238 $\int \frac{\tanh^2(x)}{(a+b\sinh(x))^2}\,dx$

Optimal result

Integrand size = 13, antiderivative size = 144

$$\int \frac{\tanh^2(x)}{(a+b\sinh(x))^2}\,dx = -\frac{2a^3\operatorname{arctanh}\left(\frac{b-a\tanh\left(\frac{x}{2}\right)}{\sqrt{a^2+b^2}}\right)}{(a^2+b^2)^{5/2}}+\frac{4ab^2\operatorname{arctanh}\left(\frac{b-a\tanh\left(\frac{x}{2}\right)}{\sqrt{a^2+b^2}}\right)}{(a^2+b^2)^{5/2}} - \frac{2ab\operatorname{sech}(x)}{(a^2+b^2)^2}-\frac{a^2b\cosh(x)}{(a^2+b^2)^2(a+b\sinh(x))}-\frac{(a^2-b^2)\tanh(x)}{(a^2+b^2)^2}$$

output
```
-2*a^3*arctanh((b-a*tanh(1/2*x))/(a^2+b^2)^(1/2))/(a^2+b^2)^(5/2)+4*a*b^2*
arctanh((b-a*tanh(1/2*x))/(a^2+b^2)^(1/2))/(a^2+b^2)^(5/2)-2*a*b*sech(x)/(
a^2+b^2)^2-a^2*b*cosh(x)/(a^2+b^2)^2/(a+b*sinh(x))-(a^2-b^2)*tanh(x)/(a^2+
b^2)^2
```

Mathematica [A] (verified)

Time = 0.21 (sec) , antiderivative size = 100, normalized size of antiderivative = 0.69

$$\int \frac{\tanh^2(x)}{(a+b\sinh(x))^2}\,dx = \frac{\frac{2a(a^2-2b^2)\arctan\left(\frac{b-a\tanh\left(\frac{x}{2}\right)}{\sqrt{-a^2-b^2}}\right)}{\sqrt{-a^2-b^2}}-2ab\operatorname{sech}(x)-\frac{a^2b\cosh(x)}{a+b\sinh(x)}+(-a^2+b^2)\tanh(x)}{(a^2+b^2)^2}$$

input
```
Integrate[Tanh[x]^2/(a + b*Sinh[x])^2,x]
```

output
```
((2*a*(a^2 - 2*b^2)*ArcTan[(b - a*Tanh[x/2])/Sqrt[-a^2 - b^2]])/Sqrt[-a^2
- b^2] - 2*a*b*Sech[x] - (a^2*b*Cosh[x])/(a + b*Sinh[x]) + (-a^2 + b^2)*Ta
nh[x])/(a^2 + b^2)^2
```

Rubi [A] (verified)

Time = 0.59 (sec) , antiderivative size = 144, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.308, Rules used = {3042, 25, 3210, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\tanh^2(x)}{(a+b\sinh(x))^2}dx$$

$$\Big\downarrow \text{3042}$$

$$\int -\frac{\tan(ix)^2}{(a-ib\sin(ix))^2}dx$$

$$\Big\downarrow \text{25}$$

$$-\int \frac{\tan(ix)^2}{(a-ib\sin(ix))^2}dx$$

$$\Big\downarrow \text{3210}$$

$$-\int \left(-\frac{a^2}{(a^2+b^2)(a+b\sinh(x))^2}+\frac{2b^2a}{(a^2+b^2)^2(a+b\sinh(x))}+\frac{\text{sech}^2(x)\left(a^2\left(1-\frac{b^2}{a^2}\right)-2ab\sinh(x)\right)}{(a^2+b^2)^2}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{4ab^2\text{arctanh}\left(\frac{b-a\tanh\left(\frac{x}{2}\right)}{\sqrt{a^2+b^2}}\right)}{(a^2+b^2)^{5/2}}-\frac{(a^2-b^2)\tanh(x)}{(a^2+b^2)^2}-\frac{2ab\text{sech}(x)}{(a^2+b^2)^2}-\frac{a^2b\cosh(x)}{(a^2+b^2)^2(a+b\sinh(x))}-\frac{2a^3\text{arctanh}\left(\frac{b-a\tanh\left(\frac{x}{2}\right)}{\sqrt{a^2+b^2}}\right)}{(a^2+b^2)^{5/2}}$$

input
```
Int[Tanh[x]^2/(a + b*Sinh[x])^2,x]
```

output
```
(-2*a^3*ArcTanh[(b - a*Tanh[x/2])/Sqrt[a^2 + b^2]])/(a^2 + b^2)^(5/2) + (4
*a*b^2*ArcTanh[(b - a*Tanh[x/2])/Sqrt[a^2 + b^2]])/(a^2 + b^2)^(5/2) - (2*
a*b*Sech[x])/(a^2 + b^2)^2 - (a^2*b*Cosh[x])/((a^2 + b^2)^2*(a + b*Sinh[x]
)) - ((a^2 - b^2)*Tanh[x])/(a^2 + b^2)^2
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3210
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*tan[(e_.) + (f_.)*(x_)]^(p_
), x_Symbol] :> Int[ExpandIntegrand[Sin[e + f*x]^p*((a + b*Sin[e + f*x])^m/
(1 - Sin[e + f*x]^2)^(p/2)), x], x] /; FreeQ[{a, b, e, f}, x] && NeQ[a^2 -
b^2, 0] && IntegersQ[m, p/2]
```

Maple [A] (verified)

Time = 1.23 (sec) , antiderivative size = 142, normalized size of antiderivative = 0.99

method	result
default	$-\frac{2a\left(\frac{-b^{2}\tanh\left(\frac{x}{2}\right)-ab}{\tanh\left(\frac{x}{2}\right)^{2}a-2b\tanh\left(\frac{x}{2}\right)-a}-\frac{\left(a^{2}-2b^{2}\right)\operatorname{arctanh}\left(\frac{2a\tanh\left(\frac{x}{2}\right)-2b}{2\sqrt{a^{2}+b^{2}}}\right)}{\sqrt{a^{2}+b^{2}}}\right)}{\left(a^{2}+b^{2}\right)^{2}}+\frac{2\left(-a^{2}+b^{2}\right)\tanh\left(\frac{x}{2}\right)-4ab}{\left(a^{4}+2a^{2}b^{2}+b^{4}\right)\left(1+\tanh\left(\frac{x}{2}\right)^{2}\right)}$
risch	$\frac{2a^{3}e^{3x}-4a\,b^{2}e^{3x}-8a^{2}b\,e^{2x}-2b^{3}e^{2x}+6a^{3}e^{x}-4a^{2}b+2b^{3}}{\left(a^{2}+b^{2}\right)^{2}\left(e^{2x}+1\right)\left(b\,e^{2x}+2\,e^{x}a-b\right)}+\frac{a^{3}\ln\left(e^{x}+\frac{\left(a^{2}+b^{2}\right)^{\frac{5}{2}}a-a^{6}-3a^{4}b^{2}-3a^{2}b^{4}-b^{6}}{b\left(a^{2}+b^{2}\right)^{\frac{5}{2}}}\right)}{\left(a^{2}+b^{2}\right)^{\frac{5}{2}}}-2b^{2}a\ln\left(e^{x}+(a^{2}+b\right.$

input
```
int(tanh(x)^2/(a+b*sinh(x))^2,x,method=_RETURNVERBOSE)
```

output
```
-2*a/(a^2+b^2)^2*((-b^2*tanh(1/2*x)-a*b)/(tanh(1/2*x)^2*a-2*b*tanh(1/2*x)-
a)-(a^2-2*b^2)/(a^2+b^2)^(1/2)*arctanh(1/2*(2*a*tanh(1/2*x)-2*b)/(a^2+b^2)
^(1/2)))+2/(a^4+2*a^2*b^2+b^4)*((-a^2+b^2)*tanh(1/2*x)-2*a*b)/(1+tanh(1/2*
x)^2)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 900 vs. $2(136) = 272$.

Time = 0.09 (sec) , antiderivative size = 900, normalized size of antiderivative = 6.25

$$\int \frac{\tanh^2(x)}{(a + b\sinh(x))^2}\,dx = \text{Too large to display}$$

input
```
integrate(tanh(x)^2/(a+b*sinh(x))^2,x, algorithm="fricas")
```

output
```
(4*a^4*b + 2*a^2*b^3 - 2*b^5 - 2*(a^5 - a^3*b^2 - 2*a*b^4)*cosh(x)^3 - 2*(
a^5 - a^3*b^2 - 2*a*b^4)*sinh(x)^3 + 2*(4*a^4*b + 5*a^2*b^3 + b^5)*cosh(x)
^2 + 2*(4*a^4*b + 5*a^2*b^3 + b^5 - 3*(a^5 - a^3*b^2 - 2*a*b^4)*cosh(x))*s
inh(x)^2 + ((a^3*b - 2*a*b^3)*cosh(x)^4 + (a^3*b - 2*a*b^3)*sinh(x)^4 - a^
3*b + 2*a*b^3 + 2*(a^4 - 2*a^2*b^2)*cosh(x)^3 + 2*(a^4 - 2*a^2*b^2 + 2*(a^
3*b - 2*a*b^3)*cosh(x))*sinh(x)^3 + 6*((a^3*b - 2*a*b^3)*cosh(x)^2 + (a^4
- 2*a^2*b^2)*cosh(x))*sinh(x)^2 + 2*(a^4 - 2*a^2*b^2)*cosh(x) + 2*(a^4 - 2
*a^2*b^2 + 2*(a^3*b - 2*a*b^3)*cosh(x)^3 + 3*(a^4 - 2*a^2*b^2)*cosh(x)^2)*
sinh(x))*sqrt(a^2 + b^2)*log((b^2*cosh(x)^2 + b^2*sinh(x)^2 + 2*a*b*cosh(x
) + 2*a^2 + b^2 + 2*(b^2*cosh(x) + a*b)*sinh(x) + 2*sqrt(a^2 + b^2)*(b*cos
h(x) + b*sinh(x) + a))/(b*cosh(x)^2 + b*sinh(x)^2 + 2*a*cosh(x) + 2*(b*cos
h(x) + a)*sinh(x) - b)) - 6*(a^5 + a^3*b^2)*cosh(x) - 2*(3*a^5 + 3*a^3*b^2
 + 3*(a^5 - a^3*b^2 - 2*a*b^4)*cosh(x)^2 - 2*(4*a^4*b + 5*a^2*b^3 + b^5)*c
osh(x))*sinh(x))/(a^6*b + 3*a^4*b^3 + 3*a^2*b^5 + b^7 - (a^6*b + 3*a^4*b^3
 + 3*a^2*b^5 + b^7)*cosh(x)^4 - (a^6*b + 3*a^4*b^3 + 3*a^2*b^5 + b^7)*sinh
(x)^4 - 2*(a^7 + 3*a^5*b^2 + 3*a^3*b^4 + a*b^6)*cosh(x)^3 - 2*(a^7 + 3*a^5
*b^2 + 3*a^3*b^4 + a*b^6 + 2*(a^6*b + 3*a^4*b^3 + 3*a^2*b^5 + b^7)*cosh(x)
)*sinh(x)^3 - 6*((a^6*b + 3*a^4*b^3 + 3*a^2*b^5 + b^7)*cosh(x)^2 + (a^7 +
3*a^5*b^2 + 3*a^3*b^4 + a*b^6)*cosh(x))*sinh(x)^2 - 2*(a^7 + 3*a^5*b^2 + 3
*a^3*b^4 + a*b^6)*cosh(x) - 2*(a^7 + 3*a^5*b^2 + 3*a^3*b^4 + a*b^6 + 2*...
```

Sympy [F]

$$\int \frac{\tanh^2(x)}{(a+b\sinh(x))^2}\,dx=\int \frac{\tanh^2{(x)}}{(a+b\sinh{(x)})^2}\,dx$$

input
```
integrate(tanh(x)**2/(a+b*sinh(x))**2,x)
```

output
```
Integral(tanh(x)**2/(a + b*sinh(x))**2, x)
```

Maxima [A] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 223, normalized size of antiderivative = 1.55

$$\int \frac{\tanh^2(x)}{(a+b\sinh(x))^2}\,dx = \frac{(a^2-2\,b^2)a\log\left(\frac{be^{(-x)}-a-\sqrt{a^2+b^2}}{be^{(-x)}-a+\sqrt{a^2+b^2}}\right)}{(a^4+2\,a^2b^2+b^4)\sqrt{a^2+b^2}} - \frac{2\left(3\,a^3e^{(-x)}+2\,a^2b-b^3+(4\,a^2b+b^3)e^{(-2\,x)}+(a^3-2\,ab^2)e^{(-3\,x)}\right)}{a^4b+2\,a^2b^3+b^5+2\,(a^5+2\,a^3b^2+ab^4)e^{(-x)}+2\,(a^5+2\,a^3b^2+ab^4)e^{(-3\,x)}-(a^4b+2\,a^2b^3+b^5)e^{(-4\,x)}}$$

input
```
integrate(tanh(x)^2/(a+b*sinh(x))^2,x, algorithm="maxima")
```

output
```
(a^2 - 2*b^2)*a*log((b*e^(-x) - a - sqrt(a^2 + b^2))/(b*e^(-x) - a + sqrt(
a^2 + b^2)))/((a^4 + 2*a^2*b^2 + b^4)*sqrt(a^2 + b^2)) - 2*(3*a^3*e^(-x) +
 2*a^2*b - b^3 + (4*a^2*b + b^3)*e^(-2*x) + (a^3 - 2*a*b^2)*e^(-3*x))/(a^4
*b + 2*a^2*b^3 + b^5 + 2*(a^5 + 2*a^3*b^2 + a*b^4)*e^(-x) + 2*(a^5 + 2*a^3
*b^2 + a*b^4)*e^(-3*x) - (a^4*b + 2*a^2*b^3 + b^5)*e^(-4*x))
```

Giac [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 181, normalized size of antiderivative = 1.26

$$\int \frac{\tanh^2(x)}{(a+b\sinh(x))^2}\,dx = \frac{(a^3-2\,ab^2)\log\left(\frac{\left|2\,be^x+2\,a-2\,\sqrt{a^2+b^2}\right|}{\left|2\,be^x+2\,a+2\,\sqrt{a^2+b^2}\right|}\right)}{(a^4+2\,a^2b^2+b^4)\sqrt{a^2+b^2}} + \frac{2\left(a^3e^{(3\,x)}-2\,ab^2e^{(3\,x)}-4\,a^2be^{(2\,x)}-b^3e^{(2\,x)}+3\,a^3e^x-2\,a^2b+b^3\right)}{(a^4+2\,a^2b^2+b^4)(be^{(4\,x)}+2\,ae^{(3\,x)}+2\,ae^x-b)}$$

input
```
integrate(tanh(x)^2/(a+b*sinh(x))^2,x, algorithm="giac")
```

output
```
(a^3 - 2*a*b^2)*log(abs(2*b*e^x + 2*a - 2*sqrt(a^2 + b^2))/abs(2*b*e^x + 2
*a + 2*sqrt(a^2 + b^2)))/((a^4 + 2*a^2*b^2 + b^4)*sqrt(a^2 + b^2)) + 2*(a^
3*e^(3*x) - 2*a*b^2*e^(3*x) - 4*a^2*b*e^(2*x) - b^3*e^(2*x) + 3*a^3*e^x -
2*a^2*b + b^3)/((a^4 + 2*a^2*b^2 + b^4)*(b*e^(4*x) + 2*a*e^(3*x) + 2*a*e^x
 - b))
```

Mupad [B] (verification not implemented)

Time = 1.99 (sec) , antiderivative size = 377, normalized size of antiderivative = 2.62

$$\int \frac{\tanh^2(x)}{(a+b\sinh(x))^2}\,dx = \frac{\frac{2\,(a^2\,b^9-2\,a^4\,b^7)}{b^3\,(a^3+a\,b^2)\,(a^3\,b^3+a\,b^5)}-\frac{2\,e^{2\,x}\,(4\,a^4\,b^7+a^2\,b^9)}{b^3\,(a^3+a\,b^2)\,(a^3\,b^3+a\,b^5)}+\frac{6\,a^5\,b^3\,e^x}{(a^3+a\,b^2)\,(a^3\,b^3+a\,b^5)}-\frac{2\,a\,e^{3\,x}\,(2\,a^2\,b^9-a^4\,b^7)}{b^4\,(a^3+a\,b^2)\,(a^3\,b^3+a\,b^5)}}{2\,a\,e^x-b+2\,a\,e^{3\,x}+b\,e^{4\,x}} - \frac{a\,\ln\left(\frac{2\,e^x\,(2\,a\,b^2-a^3)}{b\,(a^2+b^2)^2}-\frac{2\,a\,(a^2-2\,b^2)\,(b-a\,e^x)}{b\,(a^2+b^2)^{5/2}}\right)\,(a^2-2\,b^2)}{(a^2+b^2)^{5/2}} + \frac{a\,\ln\left(\frac{2\,e^x\,(2\,a\,b^2-a^3)}{b\,(a^2+b^2)^2}+\frac{2\,a\,(a^2-2\,b^2)\,(b-a\,e^x)}{b\,(a^2+b^2)^{5/2}}\right)\,(a^2-2\,b^2)}{(a^2+b^2)^{5/2}}$$

input
```
int(tanh(x)^2/(a + b*sinh(x))^2,x)
```

output
```
((2*(a^2*b^9 - 2*a^4*b^7))/(b^3*(a*b^2 + a^3)*(a*b^5 + a^3*b^3)) - (2*exp(
2*x)*(a^2*b^9 + 4*a^4*b^7))/(b^3*(a*b^2 + a^3)*(a*b^5 + a^3*b^3)) + (6*a^5
*b^3*exp(x))/((a*b^2 + a^3)*(a*b^5 + a^3*b^3)) - (2*a*exp(3*x)*(2*a^2*b^9
- a^4*b^7))/(b^4*(a*b^2 + a^3)*(a*b^5 + a^3*b^3)))/(2*a*exp(x) - b + 2*a*e
xp(3*x) + b*exp(4*x)) - (a*log((2*exp(x)*(2*a*b^2 - a^3))/(b*(a^2 + b^2)^2
) - (2*a*(a^2 - 2*b^2)*(b - a*exp(x)))/(b*(a^2 + b^2)^(5/2)))*(a^2 - 2*b^2
))/(a^2 + b^2)^(5/2) + (a*log((2*exp(x)*(2*a*b^2 - a^3))/(b*(a^2 + b^2)^2)
 + (2*a*(a^2 - 2*b^2)*(b - a*exp(x)))/(b*(a^2 + b^2)^(5/2)))*(a^2 - 2*b^2)
)/(a^2 + b^2)^(5/2)
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 596, normalized size of antiderivative = 4.14

$$\int \frac{\tanh^2(x)}{(a+b\sinh(x))^2}\,dx = \frac{2e^{4x}\sqrt{a^2+b^2}\,atan\left(\frac{e^x bi+ai}{\sqrt{a^2+b^2}}\right)a^3bi-4e^{4x}\sqrt{a^2+b^2}\,atan\left(\frac{e^x bi+ai}{\sqrt{a^2+b^2}}\right)a\,b^3i+4e^{3x}\sqrt{a^2+b^2}\,atan\left(\frac{e^x bi+ai}{\sqrt{a^2+b^2}}\right)a^4i-}{}$$

input
```
int(tanh(x)^2/(a+b*sinh(x))^2,x)
```

output

```
(2*e**(4*x)*sqrt(a**2 + b**2)*atan((e**x*b*i + a*i)/sqrt(a**2 + b**2))*a**
3*b*i - 4*e**(4*x)*sqrt(a**2 + b**2)*atan((e**x*b*i + a*i)/sqrt(a**2 + b**
2))*a*b**3*i + 4*e**(3*x)*sqrt(a**2 + b**2)*atan((e**x*b*i + a*i)/sqrt(a**
2 + b**2))*a**4*i - 8*e**(3*x)*sqrt(a**2 + b**2)*atan((e**x*b*i + a*i)/sqr
t(a**2 + b**2))*a**2*b**2*i + 4*e**x*sqrt(a**2 + b**2)*atan((e**x*b*i + a*
i)/sqrt(a**2 + b**2))*a**4*i - 8*e**x*sqrt(a**2 + b**2)*atan((e**x*b*i + a
*i)/sqrt(a**2 + b**2))*a**2*b**2*i - 2*sqrt(a**2 + b**2)*atan((e**x*b*i +
a*i)/sqrt(a**2 + b**2))*a**3*b*i + 4*sqrt(a**2 + b**2)*atan((e**x*b*i + a*
i)/sqrt(a**2 + b**2))*a*b**3*i - e**(4*x)*a**4*b + e**(4*x)*a**2*b**3 + 2*
e**(4*x)*b**5 - 8*e**(2*x)*a**4*b - 10*e**(2*x)*a**2*b**3 - 2*e**(2*x)*b**
5 + 4*e**x*a**5 + 8*e**x*a**3*b**2 + 4*e**x*a*b**4 - 3*a**4*b - 3*a**2*b**
3)/(e**(4*x)*a**6*b + 3*e**(4*x)*a**4*b**3 + 3*e**(4*x)*a**2*b**5 + e**(4*
x)*b**7 + 2*e**(3*x)*a**7 + 6*e**(3*x)*a**5*b**2 + 6*e**(3*x)*a**3*b**4 +
2*e**(3*x)*a*b**6 + 2*e**x*a**7 + 6*e**x*a**5*b**2 + 6*e**x*a**3*b**4 + 2*
e**x*a*b**6 - a**6*b - 3*a**4*b**3 - 3*a**2*b**5 - b**7)
```

3.239 $\int \frac{\tanh(x)}{(a+b\sinh(x))^2} \, dx$

Optimal result

Integrand size = 11, antiderivative size = 85

$$\int \frac{\tanh(x)}{(a+b\sinh(x))^2} \, dx = \frac{2ab\arctan(\sinh(x))}{(a^2+b^2)^2} + \frac{(a^2-b^2)\log(\cosh(x))}{(a^2+b^2)^2} - \frac{(a^2-b^2)\log(a+b\sinh(x))}{(a^2+b^2)^2} + \frac{a}{(a^2+b^2)(a+b\sinh(x))}$$

output
```
2*a*b*arctan(sinh(x))/(a^2+b^2)^2+(a^2-b^2)*ln(cosh(x))/(a^2+b^2)^2-(a^2-b
^2)*ln(a+b*sinh(x))/(a^2+b^2)^2+a/(a^2+b^2)/(a+b*sinh(x))
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.17 (sec) , antiderivative size = 146, normalized size of antiderivative = 1.72

$$\int \frac{\tanh(x)}{(a+b\sinh(x))^2} \, dx = \frac{a((a-ib)^2\log(i-\sinh(x)) + (a+ib)^2\log(i+\sinh(x)) + 2(a^2+b^2+(-a^2+b^2)\log(a+b\sinh(x)))) +}{2(a^2+b^2)^2(a+}$$

input
```
Integrate[Tanh[x]/(a + b*Sinh[x])^2,x]
```

output
```
(a*((a - I*b)^2*Log[I - Sinh[x]] + (a + I*b)^2*Log[I + Sinh[x]] + 2*(a^2 +
 b^2 + (-a^2 + b^2)*Log[a + b*Sinh[x]])) + b*((a - I*b)^2*Log[I - Sinh[x]]
 + (a + I*b)^2*Log[I + Sinh[x]] + 2*(-a^2 + b^2)*Log[a + b*Sinh[x]])*Sinh[
x])/(2*(a^2 + b^2)^2*(a + b*Sinh[x]))
```

Rubi [A] (verified)

Time = 0.38 (sec) , antiderivative size = 109, normalized size of antiderivative = 1.28, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.727$, Rules used = {3042, 26, 3200, 25, 594, 25, 657, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\tanh(x)}{(a+b\sinh(x))^2}dx$$

$$\Big\downarrow \text{3042}$$

$$\int -\frac{i\tan(ix)}{(a-ib\sin(ix))^2}dx$$

$$\Big\downarrow \text{26}$$

$$-i\int \frac{\tan(ix)}{(a-ib\sin(ix))^2}dx$$

$$\Big\downarrow \text{3200}$$

$$-\int -\frac{b\sinh(x)}{(a+b\sinh(x))^2\left(\sinh^2(x)b^2+b^2\right)}d(b\sinh(x))$$

$$\Big\downarrow \text{25}$$

$$\int \frac{b\sinh(x)}{\left(b^2\sinh^2(x)+b^2\right)(a+b\sinh(x))^2}d(b\sinh(x))$$

$$\Big\downarrow \text{594}$$

$$\frac{a}{(a^2+b^2)(a+b\sinh(x))}-\frac{\int -\frac{b^2+a\sinh(x)b}{(a+b\sinh(x))\left(\sinh^2(x)b^2+b^2\right)}d(b\sinh(x))}{a^2+b^2}$$

$$\Big\downarrow \text{25}$$

$$\frac{\int \frac{b^2+a\sinh(x)b}{(a+b\sinh(x))(\sinh^2(x)b^2+b^2)}d(b\sinh(x))}{a^2+b^2}+\frac{a}{(a^2+b^2)(a+b\sinh(x))}$$

$$\Big\downarrow \text{657}$$

$$\frac{\int \left(\frac{b^2-a^2}{(a^2+b^2)(a+b\sinh(x))}+\frac{2ab^2+(a^2-b^2)\sinh(x)b}{(a^2+b^2)(\sinh^2(x)b^2+b^2)}\right)d(b\sinh(x))}{a^2+b^2}+\frac{a}{(a^2+b^2)(a+b\sinh(x))}$$

$$\Big\downarrow \text{2009}$$

$$\frac{\frac{2ab\arctan(\sinh(x))}{a^2+b^2}+\frac{(a^2-b^2)\log(b^2\sinh^2(x)+b^2)}{2(a^2+b^2)}-\frac{(a^2-b^2)\log(a+b\sinh(x))}{a^2+b^2}}{a^2+b^2}+\frac{a}{(a^2+b^2)(a+b\sinh(x))}$$

input
```
Int[Tanh[x]/(a + b*Sinh[x])^2,x]
```

output
```
((2*a*b*ArcTan[Sinh[x]])/(a^2 + b^2) - ((a^2 - b^2)*Log[a + b*Sinh[x]])/(a
^2 + b^2) + ((a^2 - b^2)*Log[b^2 + b^2*Sinh[x]^2])/(2*(a^2 + b^2)))/(a^2 +
 b^2) + a/((a^2 + b^2)*(a + b*Sinh[x]))
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 594
```
Int[(x_)*((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :>
Simp[(-c)*(c + d*x)^(n + 1)*((a + b*x^2)^(p + 1)/((n + 1)*(b*c^2 + a*d^2)))
, x] + Simp[1/((n + 1)*(b*c^2 + a*d^2))   Int[(c + d*x)^(n + 1)*(a + b*x^2)
^p*(a*d*(n + 1) + b*c*(n + 2*p + 3)*x), x], x] /; FreeQ[{a, b, c, d, p}, x]
 && LtQ[n, -1] && NeQ[b*c^2 + a*d^2, 0]
```

rule 657
```
Int[(((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))^(n_.))/((a_) + (c_.)*(
x_)^2), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*((f + g*x)^n/(a + c*x^
2)), x], x] /; FreeQ[{a, c, d, e, f, g, m}, x] && IntegersQ[n]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3200
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*tan[(e_.) + (f_.)*(x_)]^(p
_.), x_Symbol] :> Simp[1/f   Subst[Int[(x^p*(a + x)^m)/(b^2 - x^2)^((p + 1)
/2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && NeQ[a^2 - b
^2, 0] && IntegerQ[(p + 1)/2]
```

Maple [A] (verified)

Time = 0.88 (sec) , antiderivative size = 136, normalized size of antiderivative = 1.60

method	result
default	$-\frac{2\left(\frac{\left(-a^{2} b-b^{3}\right) \tanh\left(\frac{x}{2}\right)}{\tanh\left(\frac{x}{2}\right)^{2} a-2 b \tanh\left(\frac{x}{2}\right)-a}+\frac{\left(a^{2}-b^{2}\right) \ln\left(\tanh\left(\frac{x}{2}\right)^{2} a-2 b \tanh\left(\frac{x}{2}\right)-a\right)}{2}\right)}{\left(a^{2}+b^{2}\right)^{2}}+\frac{2\left(a^{2}-b^{2}\right) \ln\left(1+\tanh\left(\frac{x}{2}\right)^{2}\right)+8 a b \arctan\left(\tanh\left(\frac{x}{2}\right)\right)}{2 a^{4}+4 a^{2} b^{2}+2 b^{4}}$
risch	$\frac{2 a\, e^{x}}{\left(a^{2}+b^{2}\right)\left(b\, e^{2 x}+2\, e^{x} a-b\right)}-\frac{\ln\left(e^{2 x}+\frac{2 a\, e^{x}}{b}-1\right) a^{2}}{a^{4}+2 a^{2} b^{2}+b^{4}}+\frac{\ln\left(e^{2 x}+\frac{2 a\, e^{x}}{b}-1\right) b^{2}}{a^{4}+2 a^{2} b^{2}+b^{4}}+\frac{2 i \ln\left(e^{x}+i\right) a b}{a^{4}+2 a^{2} b^{2}+b^{4}}+\frac{\ln\left(e^{x}+i\right) a^{2}}{a^{4}+2 a^{2} b^{2}+b^{4}}-\frac{\ln\left(e^{x}+i\right) b^{2}}{a^{4}+2 a^{2} b^{2}+b^{4}}$

input
```
int(tanh(x)/(a+b*sinh(x))^2,x,method=_RETURNVERBOSE)
```

output
```
-2/(a^2+b^2)^2*((-a^2*b-b^3)*tanh(1/2*x)/(tanh(1/2*x)^2*a-2*b*tanh(1/2*x)-
a)+1/2*(a^2-b^2)*ln(tanh(1/2*x)^2*a-2*b*tanh(1/2*x)-a))+4/(2*a^4+4*a^2*b^2
+2*b^4)*(1/2*(a^2-b^2)*ln(1+tanh(1/2*x)^2)+2*a*b*arctan(tanh(1/2*x)))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 423 vs. $2(85) = 170$.

Time = 0.09 (sec) , antiderivative size = 423, normalized size of antiderivative = 4.98

$$\int \frac{\tanh(x)}{(a+b\sinh(x))^2}\,dx =$$
$$-\frac{4\left(ab^2\cosh(x)^2+ab^2\sinh(x)^2+2\,a^2b\cosh(x)-ab^2+2\left(ab^2\cosh(x)+a^2b\right)\sinh(x)\right)\arctan\left(\cosh(x\right.}{}$$

input
```
integrate(tanh(x)/(a+b*sinh(x))^2,x, algorithm="fricas")
```

output
```
-(4*(a*b^2*cosh(x)^2 + a*b^2*sinh(x)^2 + 2*a^2*b*cosh(x) - a*b^2 + 2*(a*b^
2*cosh(x) + a^2*b)*sinh(x))*arctan(cosh(x) + sinh(x)) + 2*(a^3 + a*b^2)*co
sh(x) + (a^2*b - b^3 - (a^2*b - b^3)*cosh(x)^2 - (a^2*b - b^3)*sinh(x)^2 -
 2*(a^3 - a*b^2)*cosh(x) - 2*(a^3 - a*b^2 + (a^2*b - b^3)*cosh(x))*sinh(x)
)*log(2*(b*sinh(x) + a)/(cosh(x) - sinh(x))) - (a^2*b - b^3 - (a^2*b - b^3
)*cosh(x)^2 - (a^2*b - b^3)*sinh(x)^2 - 2*(a^3 - a*b^2)*cosh(x) - 2*(a^3 -
 a*b^2 + (a^2*b - b^3)*cosh(x))*sinh(x))*log(2*cosh(x)/(cosh(x) - sinh(x))
) + 2*(a^3 + a*b^2)*sinh(x))/(a^4*b + 2*a^2*b^3 + b^5 - (a^4*b + 2*a^2*b^3
 + b^5)*cosh(x)^2 - (a^4*b + 2*a^2*b^3 + b^5)*sinh(x)^2 - 2*(a^5 + 2*a^3*b
^2 + a*b^4)*cosh(x) - 2*(a^5 + 2*a^3*b^2 + a*b^4 + (a^4*b + 2*a^2*b^3 + b^
5)*cosh(x))*sinh(x))
```

Sympy [F]

$$\int \frac{\tanh(x)}{(a+b\sinh(x))^2}\,dx = \int \frac{\tanh(x)}{(a+b\sinh(x))^2}\,dx$$

input
```
integrate(tanh(x)/(a+b*sinh(x))**2,x)
```

output
```
Integral(tanh(x)/(a + b*sinh(x))**2, x)
```

Maxima [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 155, normalized size of antiderivative = 1.82

$$\int \frac{\tanh(x)}{(a+b\sinh(x))^2}\,dx = -\frac{4\,ab\arctan\left(e^{(-x)}\right)}{a^4+2\,a^2b^2+b^4} + \frac{2\,ae^{(-x)}}{a^2b+b^3+2\,(a^3+ab^2)e^{(-x)}-(a^2b+b^3)e^{(-2\,x)}} - \frac{(a^2-b^2)\log\left(-2\,ae^{(-x)}+be^{(-2\,x)}-b\right)}{a^4+2\,a^2b^2+b^4} + \frac{(a^2-b^2)\log\left(e^{(-2\,x)}+1\right)}{a^4+2\,a^2b^2+b^4}$$

input
```
integrate(tanh(x)/(a+b*sinh(x))^2,x, algorithm="maxima")
```

output
```
-4*a*b*arctan(e^(-x))/(a^4 + 2*a^2*b^2 + b^4) + 2*a*e^(-x)/(a^2*b + b^3 +
2*(a^3 + a*b^2)*e^(-x) - (a^2*b + b^3)*e^(-2*x)) - (a^2 - b^2)*log(-2*a*e^
(-x) + b*e^(-2*x) - b)/(a^4 + 2*a^2*b^2 + b^4) + (a^2 - b^2)*log(e^(-2*x)
+ 1)/(a^4 + 2*a^2*b^2 + b^4)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 199 vs. 2(85) = 170.

Time = 0.13 (sec) , antiderivative size = 199, normalized size of antiderivative = 2.34

$$\int \frac{\tanh(x)}{(a+b\sinh(x))^2}\,dx = \frac{\left(\pi+2\,\arctan\left(\frac{1}{2}\left(e^{(2\,x)}-1\right)e^{(-x)}\right)\right)ab}{a^4+2\,a^2b^2+b^4} + \frac{(a^2-b^2)\log\left(\left(e^{(-x)}-e^x\right)^2+4\right)}{2\,(a^4+2\,a^2b^2+b^4)} - \frac{(a^2b-b^3)\log\left(\left|-b\left(e^{(-x)}-e^x\right)+2\,a\right|\right)}{a^4b+2\,a^2b^3+b^5} + \frac{a^2b\left(e^{(-x)}-e^x\right)-b^3\left(e^{(-x)}-e^x\right)-4\,a^3}{(a^4+2\,a^2b^2+b^4)(b\left(e^{(-x)}-e^x\right)-2\,a)}$$

input
```
integrate(tanh(x)/(a+b*sinh(x))^2,x, algorithm="giac")
```

output
```
(pi + 2*arctan(1/2*(e^(2*x) - 1)*e^(-x)))*a*b/(a^4 + 2*a^2*b^2 + b^4) + 1/
2*(a^2 - b^2)*log((e^(-x) - e^x)^2 + 4)/(a^4 + 2*a^2*b^2 + b^4) - (a^2*b -
 b^3)*log(abs(-b*(e^(-x) - e^x) + 2*a))/(a^4*b + 2*a^2*b^3 + b^5) + (a^2*b
*(e^(-x) - e^x) - b^3*(e^(-x) - e^x) - 4*a^3)/((a^4 + 2*a^2*b^2 + b^4)*(b*
(e^(-x) - e^x) - 2*a))
```

Mupad [B] (verification not implemented)

Time = 2.63 (sec) , antiderivative size = 190, normalized size of antiderivative = 2.24

$$\int \frac{\tanh(x)}{(a+b\sinh(x))^2}\,dx = \frac{\ln\left(1+\mathrm{e}^x\,1\mathrm{i}\right)}{a^2+a\,b\,2\mathrm{i}-b^2} - \frac{\ln\left(b^5\,\mathrm{e}^{2\,x}-a^4\,b-b^5+a^2\,b^3+2\,a^5\,\mathrm{e}^x-a^2\,b^3\,\mathrm{e}^{2\,x}+2\,a\,b^4\,\mathrm{e}^x+a^4\,b\,\mathrm{e}^{2\,x}-2\,a^3\,b^2\,\mathrm{e}^x\right)\left(a^2-b^2\right)}{a^4+2\,a^2\,b^2+b^4} + \frac{2\,a\,b\,\mathrm{e}^x}{\left(a^2\,b+b^3\right)\left(2\,a\,\mathrm{e}^x-b+b\,\mathrm{e}^{2\,x}\right)} + \frac{\ln\left(\mathrm{e}^x+1\mathrm{i}\right)\,1\mathrm{i}}{a^2\,1\mathrm{i}+2\,a\,b-b^2\,1\mathrm{i}}$$

input
```
int(tanh(x)/(a + b*sinh(x))^2,x)
```

output
```
log(exp(x)*1i + 1)/(a*b*2i + a^2 - b^2) + (log(exp(x) + 1i)*1i)/(2*a*b + a
^2*1i - b^2*1i) - (log(b^5*exp(2*x) - a^4*b - b^5 + a^2*b^3 + 2*a^5*exp(x)
 - a^2*b^3*exp(2*x) + 2*a*b^4*exp(x) + a^4*b*exp(2*x) - 2*a^3*b^2*exp(x))*
(a^2 - b^2))/(a^4 + b^4 + 2*a^2*b^2) + (2*a*b*exp(x))/((a^2*b + b^3)*(2*a*
exp(x) - b + b*exp(2*x)))
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 401, normalized size of antiderivative = 4.72

$$\int \frac{\tanh(x)}{(a+b\sinh(x))^2}\,dx = \frac{4e^{2x}atan(e^x)\,a\,b^2+8e^x atan(e^x)\,a^2b-4atan(e^x)\,a\,b^2+e^{2x}\log(e^{2x}+1)\,a^2b-e^{2x}\log(e^{2x}+1)\,b^3-e^{2x}\log(}{}$$

input
```
int(tanh(x)/(a+b*sinh(x))^2,x)
```

output

```
(4*e**(2*x)*atan(e**x)*a*b**2 + 8*e**x*atan(e**x)*a**2*b - 4*atan(e**x)*a*
b**2 + e**(2*x)*log(e**(2*x) + 1)*a**2*b - e**(2*x)*log(e**(2*x) + 1)*b**3
 - e**(2*x)*log(e**(2*x)*b + 2*e**x*a - b)*a**2*b + e**(2*x)*log(e**(2*x)*
b + 2*e**x*a - b)*b**3 - e**(2*x)*a**2*b - e**(2*x)*b**3 + 2*e**x*log(e**(
2*x) + 1)*a**3 - 2*e**x*log(e**(2*x) + 1)*a*b**2 - 2*e**x*log(e**(2*x)*b +
 2*e**x*a - b)*a**3 + 2*e**x*log(e**(2*x)*b + 2*e**x*a - b)*a*b**2 - log(e
**(2*x) + 1)*a**2*b + log(e**(2*x) + 1)*b**3 + log(e**(2*x)*b + 2*e**x*a -
 b)*a**2*b - log(e**(2*x)*b + 2*e**x*a - b)*b**3 + a**2*b + b**3)/(e**(2*x
)*a**4*b + 2*e**(2*x)*a**2*b**3 + e**(2*x)*b**5 + 2*e**x*a**5 + 4*e**x*a**
3*b**2 + 2*e**x*a*b**4 - a**4*b - 2*a**2*b**3 - b**5)
```

3.240 $\int \frac{\coth(x)}{(a+b\sinh(x))^2}\,dx$

Optimal result

Integrand size = 11, antiderivative size = 32

$$\int \frac{\coth(x)}{(a+b\sinh(x))^2}\,dx = \frac{\log(\sinh(x))}{a^2} - \frac{\log(a+b\sinh(x))}{a^2} + \frac{1}{a(a+b\sinh(x))}$$

output
```
ln(sinh(x))/a^2-ln(a+b*sinh(x))/a^2+1/a/(a+b*sinh(x))
```

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.84

$$\int \frac{\coth(x)}{(a+b\sinh(x))^2}\,dx = \frac{\log(\sinh(x)) - \log(a+b\sinh(x)) + \frac{a}{a+b\sinh(x)}}{a^2}$$

input
```
Integrate[Coth[x]/(a + b*Sinh[x])^2,x]
```

output
```
(Log[Sinh[x]] - Log[a + b*Sinh[x]] + a/(a + b*Sinh[x]))/a^2
```

Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.06, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.455, Rules used = {3042, 26, 3200, 54, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\coth(x)}{(a+b\sinh(x))^2}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{i}{\tan(ix)(a-ib\sin(ix))^2}dx$$

$$\Big\downarrow \text{26}$$

$$i\int \frac{1}{(a-ib\sin(ix))^2\tan(ix)}dx$$

$$\Big\downarrow \text{3200}$$

$$\int \frac{\text{csch}(x)}{b(a+b\sinh(x))^2}d(b\sinh(x))$$

$$\Big\downarrow \text{54}$$

$$\int \left(-\frac{1}{a^2(a+b\sinh(x))}+\frac{\text{csch}(x)}{a^2b}-\frac{1}{a(a+b\sinh(x))^2}\right)d(b\sinh(x))$$

$$\Big\downarrow \text{2009}$$

$$\frac{\log(b\sinh(x))}{a^2}-\frac{\log(a+b\sinh(x))}{a^2}+\frac{1}{a(a+b\sinh(x))}$$

input
```
Int[Coth[x]/(a + b*Sinh[x])^2,x]
```

output
```
Log[b*Sinh[x]]/a^2 - Log[a + b*Sinh[x]]/a^2 + 1/(a*(a + b*Sinh[x]))
```

Defintions of rubi rules used

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 54
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m + n + 2, 0])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3200
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*tan[(e_.) + (f_.)*(x_)]^(p
_.), x_Symbol] :> Simp[1/f   Subst[Int[(x^p*(a + x)^m)/(b^2 - x^2)^((p + 1)
/2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && NeQ[a^2 - b
^2, 0] && IntegerQ[(p + 1)/2]
```

Maple [A] (verified)

Time = 1.44 (sec) , antiderivative size = 57, normalized size of antiderivative = 1.78

method	result	size
risch	$\frac{2\,\mathrm{e}^{x}}{a\left(b\,\mathrm{e}^{2x}+2\,\mathrm{e}^{x} a-b\right)}+\frac{\ln\left(\mathrm{e}^{2x}-1\right)}{a^{2}}-\frac{\ln\left(\mathrm{e}^{2x}+\frac{2a\,\mathrm{e}^{x}}{b}-1\right)}{a^{2}}$	57
default	$\frac{\ln\left(\tanh\left(\frac{x}{2}\right)\right)}{a^{2}}-\frac{2\left(-\frac{b\tanh\left(\frac{x}{2}\right)}{\tanh\left(\frac{x}{2}\right)^{2} a-2b\tanh\left(\frac{x}{2}\right)-a}+\frac{\ln\left(\tanh\left(\frac{x}{2}\right)^{2} a-2b\tanh\left(\frac{x}{2}\right)-a\right)}{2}\right)}{a^{2}}$	67

input
```
int(coth(x)/(a+b*sinh(x))^2,x,method=_RETURNVERBOSE)
```

output
```
2/a*exp(x)/(b*exp(2*x)+2*exp(x)*a-b)+1/a^2*ln(exp(2*x)-1)-1/a^2*ln(exp(2*x
)+2/b*a*exp(x)-1)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 158 vs. $2(32) = 64$.

Time = 0.09 (sec) , antiderivative size = 158, normalized size of antiderivative = 4.94

$$\int \frac{\coth(x)}{(a + b\sinh(x))^2}\,dx$$
$$= \frac{2\,a\cosh\left(x\right) - \left(b\cosh\left(x\right)^2 + b\sinh\left(x\right)^2 + 2\,a\cosh\left(x\right) + 2\left(b\cosh\left(x\right) + a\right)\sinh\left(x\right) - b\right)\log\left(\frac{2\,(b\sinh(x)+a)}{\cosh(x)-\sinh(x)}\right. \cdots}{a^2b\cosh\left(x\right)^2 + a^2b\sinh\left(x\right)^2 + 2\,a^3\,\mathrm{co}\cdots}$$

input
```
integrate(coth(x)/(a+b*sinh(x))^2,x, algorithm="fricas")
```

output
```
(2*a*cosh(x) - (b*cosh(x)^2 + b*sinh(x)^2 + 2*a*cosh(x) + 2*(b*cosh(x) + a
)*sinh(x) - b)*log(2*(b*sinh(x) + a)/(cosh(x) - sinh(x))) + (b*cosh(x)^2 +
 b*sinh(x)^2 + 2*a*cosh(x) + 2*(b*cosh(x) + a)*sinh(x) - b)*log(2*sinh(x)/
(cosh(x) - sinh(x))) + 2*a*sinh(x))/(a^2*b*cosh(x)^2 + a^2*b*sinh(x)^2 + 2
*a^3*cosh(x) - a^2*b + 2*(a^2*b*cosh(x) + a^3)*sinh(x))
```

Sympy [F]

$$\int \frac{\coth(x)}{(a + b\sinh(x))^2}\,dx = \int \frac{\coth\left(x\right)}{\left(a + b\sinh\left(x\right)\right)^2}\,dx$$

input
```
integrate(coth(x)/(a+b*sinh(x))**2,x)
```

output
```
Integral(coth(x)/(a + b*sinh(x))**2, x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 75 vs. $2(32) = 64$.

Time = 0.04 (sec) , antiderivative size = 75, normalized size of antiderivative = 2.34

$$\int \frac{\coth(x)}{(a+b\sinh(x))^2}\,dx = \frac{2\,e^{(-x)}}{2\,a^2e^{(-x)} - abe^{(-2\,x)} + ab} - \frac{\log\left(-2\,ae^{(-x)} + be^{(-2\,x)} - b\right)}{a^2} + \frac{\log\left(e^{(-x)} + 1\right)}{a^2} + \frac{\log\left(e^{(-x)} - 1\right)}{a^2}$$

input
```
integrate(coth(x)/(a+b*sinh(x))^2,x, algorithm="maxima")
```

output
```
2*e^(-x)/(2*a^2*e^(-x) - a*b*e^(-2*x) + a*b) - log(-2*a*e^(-x) + b*e^(-2*x
) - b)/a^2 + log(e^(-x) + 1)/a^2 + log(e^(-x) - 1)/a^2
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 75 vs. $2(32) = 64$.

Time = 0.13 (sec) , antiderivative size = 75, normalized size of antiderivative = 2.34

$$\int \frac{\coth(x)}{(a+b\sinh(x))^2}\,dx = -\frac{\log\left(\left|-b\left(e^{(-x)} - e^x\right) + 2\,a\right|\right)}{a^2} + \frac{\log\left(\left|-e^{(-x)} + e^x\right|\right)}{a^2} + \frac{b\left(e^{(-x)} - e^x\right) - 4\,a}{\left(b\left(e^{(-x)} - e^x\right) - 2\,a\right)a^2}$$

input
```
integrate(coth(x)/(a+b*sinh(x))^2,x, algorithm="giac")
```

output
```
-log(abs(-b*(e^(-x) - e^x) + 2*a))/a^2 + log(abs(-e^(-x) + e^x))/a^2 + (b*
(e^(-x) - e^x) - 4*a)/((b*(e^(-x) - e^x) - 2*a)*a^2)
```

Mupad [B] (verification not implemented)

Time = 1.80 (sec) , antiderivative size = 240, normalized size of antiderivative = 7.50

$$\int \frac{\coth(x)}{(a+b\sinh(x))^2}\,dx = \frac{2\,\mathrm{atan}\left(\frac{a\sqrt{-a^4}+b\,\mathrm{e}^x\sqrt{-a^4}-2\,a\,\mathrm{e}^{2\,x}\sqrt{-a^4}-b\,\mathrm{e}^{3\,x}\sqrt{-a^4}}{a^3}\right)-2\,\mathrm{atan}\left(\left(4\,a^5\,b\sqrt{-a^4}+4\,a^3\,b^3\sqrt{-a^4}\right)\left(\frac{1}{8\,a^3\,b\,(a^2+b^2)^2}-\mathrm{e}^x\right.\right.}{\sqrt{-a^4}} + \frac{2\,b^3\,\mathrm{e}^x\,(a^2+b^2)}{a\,(a^2\,b^3+b^5)\,(2\,a\,\mathrm{e}^x-b+b\,\mathrm{e}^{2\,x})}$$

input
```
int(coth(x)/(a + b*sinh(x))^2,x)
```

output
```
(2*atan((a*(-a^4)^(1/2) + b*exp(x)*(-a^4)^(1/2) - 2*a*exp(2*x)*(-a^4)^(1/2
) - b*exp(3*x)*(-a^4)^(1/2))/a^3) - 2*atan((4*a^5*b*(-a^4)^(1/2) + 4*a^3*b
^3*(-a^4)^(1/2))*(1/(8*a^3*b*(a^2 + b^2)^2) - exp(x)*(1/(16*a^2*b^2*(a^2 +
 b^2)^2) - (a^2 + 2*b^2)^2/(16*a^6*b^2*(a^2 + b^2)^2)) + (a^2 + 2*b^2)/(8*
a^5*b*(a^2 + b^2)^2))))/(-a^4)^(1/2) + (2*b^3*exp(x)*(a^2 + b^2))/(a*(b^5
+ a^2*b^3)*(2*a*exp(x) - b + b*exp(2*x)))
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 171, normalized size of antiderivative = 5.34

$$\int \frac{\coth(x)}{(a+b\sinh(x))^2}\,dx = \frac{e^{2x}\log(e^x-1)\,b+e^{2x}\log(e^x+1)\,b-e^{2x}\log(e^{2x}b+2e^xa-b)\,b-e^{2x}b+2e^x\log(e^x-1)\,a+2e^x\log(e^x+1}{a^2\,(e^{2x}b+2e^xa-}$$

input
```
int(coth(x)/(a+b*sinh(x))^2,x)
```

output
```
(e**(2*x)*log(e**x - 1)*b + e**(2*x)*log(e**x + 1)*b - e**(2*x)*log(e**(2*
x)*b + 2*e**x*a - b)*b - e**(2*x)*b + 2*e**x*log(e**x - 1)*a + 2*e**x*log(
e**x + 1)*a - 2*e**x*log(e**(2*x)*b + 2*e**x*a - b)*a - log(e**x - 1)*b -
log(e**x + 1)*b + log(e**(2*x)*b + 2*e**x*a - b)*b + b)/(a**2*(e**(2*x)*b
+ 2*e**x*a - b))
```

3.241 $\int \frac{\coth^2(x)}{(a+b\sinh(x))^2}\,dx$

Optimal result

Integrand size = 13, antiderivative size = 80

$$\int \frac{\coth^2(x)}{(a+b\sinh(x))^2}\,dx = \frac{2b\text{arctanh}(\cosh(x))}{a^3} - \frac{2(a^2+2b^2)\,\text{arctanh}\left(\frac{b-a\tanh\left(\frac{x}{2}\right)}{\sqrt{a^2+b^2}}\right)}{a^3\sqrt{a^2+b^2}} - \frac{2\coth(x)}{a^2} + \frac{\coth(x)}{a(a+b\sinh(x))}$$

output
```
2*b*arctanh(cosh(x))/a^3-2*(a^2+2*b^2)*arctanh((b-a*tanh(1/2*x))/(a^2+b^2)
^(1/2))/a^3/(a^2+b^2)^(1/2)-2*coth(x)/a^2+coth(x)/a/(a+b*sinh(x))
```

Mathematica [A] (verified)

Time = 0.41 (sec) , antiderivative size = 112, normalized size of antiderivative = 1.40

$$\int \frac{\coth^2(x)}{(a+b\sinh(x))^2}\,dx = -\frac{-\frac{4(a^2+2b^2)\arctan\left(\frac{b-a\tanh\left(\frac{x}{2}\right)}{\sqrt{-a^2-b^2}}\right)}{\sqrt{-a^2-b^2}} + a\coth\left(\frac{x}{2}\right) - 4b\log\left(\cosh\left(\frac{x}{2}\right)\right) + 4b\log\left(\sinh\left(\frac{x}{2}\right)\right) + \frac{2ab\cosh(x)}{a+b\sinh(x)} + a\tanh}{2a^3}$$

input
```
Integrate[Coth[x]^2/(a + b*Sinh[x])^2,x]
```

output
```
-1/2*((-4*(a^2 + 2*b^2)*ArcTan[(b - a*Tanh[x/2])/Sqrt[-a^2 - b^2]])/Sqrt[-
a^2 - b^2] + a*Coth[x/2] - 4*b*Log[Cosh[x/2]] + 4*b*Log[Sinh[x/2]] + (2*a*
b*Cosh[x])/(a + b*Sinh[x]) + a*Tanh[x/2])/a^3
```

Rubi [C] (verified)

Result contains complex when optimal does not.

Time = 1.12 (sec) , antiderivative size = 126, normalized size of antiderivative = 1.58, number of steps used = 22, number of rules used = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 1.615$, Rules used = {3042, 25, 3202, 25, 3042, 25, 3535, 25, 3042, 25, 3535, 3042, 26, 3480, 26, 3042, 26, 3139, 1083, 219, 4257}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\coth^2(x)}{(a+b\sinh(x))^2}dx$$

$$\Big\downarrow \text{3042}$$

$$\int -\frac{1}{\tan(ix)^2(a-ib\sin(ix))^2}dx$$

$$\Big\downarrow \text{25}$$

$$-\int \frac{1}{(a-ib\sin(ix))^2\tan(ix)^2}dx$$

$$\Big\downarrow \text{3202}$$

$$-\int -\frac{\text{csch}^2(x)\left(\sinh^2(x)+1\right)}{(a+b\sinh(x))^2}dx$$

$$\Big\downarrow \text{25}$$

$$\int \frac{\left(\sinh^2(x)+1\right)\text{csch}^2(x)}{(a+b\sinh(x))^2}dx$$

$$\Big\downarrow \text{3042}$$

$$\int -\frac{1-\sin(ix)^2}{\sin(ix)^2(a-ib\sin(ix))^2}dx$$

$$\Big\downarrow \text{25}$$

$$-\int \frac{1-\sin(ix)^2}{\sin(ix)^2(a-ib\sin(ix))^2}dx$$

$$\Big\downarrow \text{3535}$$

$$\frac{\coth(x)}{a(a+b\sinh(x))}-\frac{\int -\frac{\text{csch}^2(x)\left(\left(a^2+b^2\right)\sinh^2(x)+2\left(a^2+b^2\right)\right)}{a+b\sinh(x)}dx}{a\left(a^2+b^2\right)}$$

$$\Big\downarrow \text{25}$$

$$\frac{\int \frac{\text{csch}^2(x)\left(\left(a^2+b^2\right)\sinh^2(x)+2\left(a^2+b^2\right)\right)}{a+b\sinh(x)}dx}{a\left(a^2+b^2\right)}+\frac{\coth(x)}{a(a+b\sinh(x))}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\coth(x)}{a(a+b\sinh(x))}+\frac{\int -\frac{2\left(a^2+b^2\right)-\left(a^2+b^2\right)\sin(ix)^2}{\sin(ix)^2(a-ib\sin(ix))}dx}{a\left(a^2+b^2\right)}$$

$$\Big\downarrow \text{25}$$

$$\frac{\coth(x)}{a(a+b\sinh(x))}-\frac{\int \frac{2\left(a^2+b^2\right)-\left(a^2+b^2\right)\sin(ix)^2}{\sin(ix)^2(a-ib\sin(ix))}dx}{a\left(a^2+b^2\right)}$$

$$\Big\downarrow \text{3535}$$

$$\frac{\coth(x)}{a(a+b\sinh(x))}-\frac{\frac{\int \frac{\text{csch}(x)\left(2b\left(a^2+b^2\right)-a\left(a^2+b^2\right)\sinh(x)\right)}{a+b\sinh(x)}dx}{a}+\frac{2\left(a^2+b^2\right)\coth(x)}{a}}{a\left(a^2+b^2\right)}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\coth(x)}{a(a+b\sinh(x))}-\frac{\frac{2\left(a^2+b^2\right)\coth(x)}{a}+\frac{\int \frac{i\left(2b\left(a^2+b^2\right)+ia\sin(ix)\left(a^2+b^2\right)\right)}{\sin(ix)(a-ib\sin(ix))}dx}{a}}{a\left(a^2+b^2\right)}$$

$$\Big\downarrow \text{26}$$

$$\frac{\coth(x)}{a(a+b\sinh(x))}-\frac{\frac{2\left(a^2+b^2\right)\coth(x)}{a}+\frac{i\int \frac{2b\left(a^2+b^2\right)+ia\sin(ix)\left(a^2+b^2\right)}{\sin(ix)(a-ib\sin(ix))}dx}{a}}{a\left(a^2+b^2\right)}$$

$$\Big\downarrow \text{3480}$$

$$\frac{\coth(x)}{a(a+b\sinh(x))}-\frac{\frac{2\left(a^2+b^2\right)\coth(x)}{a}+\frac{i\left(\frac{i\left(a^2+b^2\right)\left(a^2+2b^2\right)\int \frac{1}{a+b\sinh(x)}dx}{a}+\frac{2b\left(a^2+b^2\right)\int -i\text{csch}(x)dx}{a}\right)}{a}}{a\left(a^2+b^2\right)}$$

$$\Big\downarrow \text{26}$$

$$\frac{\coth(x)}{a(a+b\sinh(x))}-\frac{\frac{2(a^2+b^2)\coth(x)}{a}+\frac{i\left(\frac{i(a^2+b^2)(a^2+2b^2)\int\frac{1}{a+b\sinh(x)}dx}{a}-\frac{2ib(a^2+b^2)\int\text{csch}(x)dx}{a}\right)}{a}}{a\left(a^2+b^2\right)}$$

$\big\downarrow$ 3042

$$\frac{\coth(x)}{a(a+b\sinh(x))}-\frac{\frac{2(a^2+b^2)\coth(x)}{a}+\frac{i\left(\frac{i(a^2+b^2)(a^2+2b^2)\int\frac{1}{a-ib\sin(ix)}dx}{a}-\frac{2ib(a^2+b^2)\int i\csc(ix)dx}{a}\right)}{a}}{a\left(a^2+b^2\right)}$$

$\big\downarrow$ 26

$$\frac{\coth(x)}{a(a+b\sinh(x))}-\frac{\frac{2(a^2+b^2)\coth(x)}{a}+\frac{i\left(\frac{i(a^2+b^2)(a^2+2b^2)\int\frac{1}{a-ib\sin(ix)}dx}{a}+\frac{2b(a^2+b^2)\int\csc(ix)dx}{a}\right)}{a}}{a\left(a^2+b^2\right)}$$

$\big\downarrow$ 3139

$$\frac{\coth(x)}{a(a+b\sinh(x))}-\frac{\frac{2(a^2+b^2)\coth(x)}{a}+\frac{i\left(\frac{2b(a^2+b^2)\int\csc(ix)dx}{a}+\frac{2i(a^2+b^2)(a^2+2b^2)\int\frac{1}{-a\tanh^2\left(\frac{x}{2}\right)+2b\tanh\left(\frac{x}{2}\right)+a}d\tanh\left(\frac{x}{2}\right)}{a}\right)}{a}}{a\left(a^2+b^2\right)}$$

$\big\downarrow$ 1083

$$\frac{\coth(x)}{a(a+b\sinh(x))}-\frac{\frac{2(a^2+b^2)\coth(x)}{a}+\frac{i\left(\frac{2b(a^2+b^2)\int\csc(ix)dx}{a}-\frac{4i(a^2+b^2)(a^2+2b^2)\int\frac{1}{4(a^2+b^2)-\left(2b-2a\tanh\left(\frac{x}{2}\right)\right)^2}d\left(2b-2a\tanh\left(\frac{x}{2}\right)\right)}{a}\right)}{a}}{a\left(a^2+b^2\right)}$$

$\big\downarrow$ 219

$$\frac{\coth(x)}{a(a+b\sinh(x))}-\frac{\frac{2(a^2+b^2)\coth(x)}{a}+\frac{i\left(\frac{2b(a^2+b^2)\int\csc(ix)dx}{a}-\frac{2i\sqrt{a^2+b^2}(a^2+2b^2)\text{arctanh}\left(\frac{2b-2a\tanh\left(\frac{x}{2}\right)}{2\sqrt{a^2+b^2}}\right)}{a}\right)}{a}}{a\left(a^2+b^2\right)}$$

$\big\downarrow$ 4257

$$\frac{\coth(x)}{a(a+b\sinh(x))}-\frac{\frac{2(a^2+b^2)\coth(x)}{a}+\frac{i\left(\frac{2ib(a^2+b^2)\operatorname{arctanh}(\cosh(x))}{a}-\frac{2i\sqrt{a^2+b^2}(a^2+2b^2)\operatorname{arctanh}\left(\frac{2b-2a\tanh\left(\frac{x}{2}\right)}{2\sqrt{a^2+b^2}}\right)}{a}\right)}{a}}{a(a^2+b^2)}$$

input
```
Int[Coth[x]^2/(a + b*Sinh[x])^2,x]
```

output
```
-(((I*(((2*I)*b*(a^2 + b^2)*ArcTanh[Cosh[x]])/a - ((2*I)*Sqrt[a^2 + b^2]*(
a^2 + 2*b^2)*ArcTanh[(2*b - 2*a*Tanh[x/2])/(2*Sqrt[a^2 + b^2])])/a))/a + (
2*(a^2 + b^2)*Coth[x])/a)/(a*(a^2 + b^2))) + Coth[x]/(a*(a + b*Sinh[x]))
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1083
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Simp[-2   Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3139
```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Simp[2*(e/d)   Subst[Int[1/(a + 2*b*e*x + a
*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[a^2 - b^2, 0]
```

rule 3202
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)/tan[(e_.) + (f_.)*(x_)]^2,
x_Symbol] :> Int[(a + b*Sin[e + f*x])^m*((1 - Sin[e + f*x]^2)/Sin[e + f*x]^
2), x] /; FreeQ[{a, b, e, f, m}, x] && NeQ[a^2 - b^2, 0]
```

rule 3480
```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((a_.) + (b_.)*sin[(e_.) + (f_
.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])), x_Symbol] :> Simp[(A*b
- a*B)/(b*c - a*d)   Int[1/(a + b*Sin[e + f*x]), x], x] + Simp[(B*c - A*d)/
(b*c - a*d)   Int[1/(c + d*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f
, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

rule 3535
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :>
Simp[(-(A*b^2 + a^2*C))*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m + 1)*((c + d*S
in[e + f*x])^(n + 1)/(f*(m + 1)*(b*c - a*d)*(a^2 - b^2))), x] + Simp[1/((m
+ 1)*(b*c - a*d)*(a^2 - b^2))   Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin
[e + f*x])^n*Simp[a*(m + 1)*(b*c - a*d)*(A + C) + d*(A*b^2 + a^2*C)*(m + n
+ 2) - (c*(A*b^2 + a^2*C) + b*(m + 1)*(b*c - a*d)*(A + C))*Sin[e + f*x] - d
*(A*b^2 + a^2*C)*(m + n + 3)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c,
d, e, f, A, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2
- d^2, 0] && LtQ[m, -1] && ((EqQ[a, 0] && IntegerQ[m] &&  !IntegerQ[n]) ||
 !(IntegerQ[2*n] && LtQ[n, -1] && ((IntegerQ[n] &&  !IntegerQ[m]) || EqQ[a,
 0])))
```

rule 4257
```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Maple [A] (verified)

Time = 3.51 (sec) , antiderivative size = 118, normalized size of antiderivative = 1.48

method	result
default	$-\frac{\tanh\left(\frac{x}{2}\right)}{2a^2}-\frac{1}{2a^2\tanh\left(\frac{x}{2}\right)}-\frac{2b\ln\left(\tanh\left(\frac{x}{2}\right)\right)}{a^3}-\frac{2\left(\frac{-b^2\tanh\left(\frac{x}{2}\right)-ab}{\tanh\left(\frac{x}{2}\right)^2 a-2b\tanh\left(\frac{x}{2}\right)-a}-\frac{\left(a^2+2b^2\right)\operatorname{arctanh}\left(\frac{2a\tanh\left(\frac{x}{2}\right)-2b}{2\sqrt{a^2+b^2}}\right)}{\sqrt{a^2+b^2}}\right)}{a^3}$
risch	$\frac{2a\,e^{3x}-4b\,e^{2x}-6\,e^{x}a+4b}{\left(b\,e^{2x}+2\,e^{x}a-b\right)a^2\left(e^{2x}-1\right)}+\frac{\ln\left(e^{x}+\frac{a\sqrt{a^2+b^2}-a^2-b^2}{\sqrt{a^2+b^2}\,b}\right)}{\sqrt{a^2+b^2}\,a}+\frac{2\ln\left(e^{x}+\frac{a\sqrt{a^2+b^2}-a^2-b^2}{\sqrt{a^2+b^2}\,b}\right)b^2}{\sqrt{a^2+b^2}\,a^3}-\frac{\ln\left(e^{x}+\frac{a\sqrt{a^2+b^2}+a^2+b^2}{\sqrt{a^2+b^2}\,b}\right)}{\sqrt{a^2+b^2}\,a}-$

input
```
int(coth(x)^2/(a+b*sinh(x))^2,x,method=_RETURNVERBOSE)
```

output
```
-1/2/a^2*tanh(1/2*x)-1/2/a^2/tanh(1/2*x)-2/a^3*b*ln(tanh(1/2*x))-2/a^3*((-
b^2*tanh(1/2*x)-a*b)/(tanh(1/2*x)^2*a-2*b*tanh(1/2*x)-a)-(a^2+2*b^2)/(a^2+
b^2)^(1/2)*arctanh(1/2*(2*a*tanh(1/2*x)-2*b)/(a^2+b^2)^(1/2)))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1257 vs. $2(76) = 152$.

Time = 0.13 (sec) , antiderivative size = 1257, normalized size of antiderivative = 15.71

$$\int \frac{\coth^2(x)}{(a+b\sinh(x))^2}\,dx = \text{Too large to display}$$

input
```
integrate(coth(x)^2/(a+b*sinh(x))^2,x, algorithm="fricas")
```

output
```
(4*a^3*b + 4*a*b^3 + 2*(a^4 + a^2*b^2)*cosh(x)^3 + 2*(a^4 + a^2*b^2)*sinh(
x)^3 - 4*(a^3*b + a*b^3)*cosh(x)^2 - 2*(2*a^3*b + 2*a*b^3 - 3*(a^4 + a^2*b
^2)*cosh(x))*sinh(x)^2 + ((a^2*b + 2*b^3)*cosh(x)^4 + (a^2*b + 2*b^3)*sinh
(x)^4 + 2*(a^3 + 2*a*b^2)*cosh(x)^3 + 2*(a^3 + 2*a*b^2 + 2*(a^2*b + 2*b^3)
*cosh(x))*sinh(x)^3 + a^2*b + 2*b^3 - 2*(a^2*b + 2*b^3)*cosh(x)^2 - 2*(a^2
*b + 2*b^3 - 3*(a^2*b + 2*b^3)*cosh(x)^2 - 3*(a^3 + 2*a*b^2)*cosh(x))*sinh
(x)^2 - 2*(a^3 + 2*a*b^2)*cosh(x) + 2*(2*(a^2*b + 2*b^3)*cosh(x)^3 - a^3 -
 2*a*b^2 + 3*(a^3 + 2*a*b^2)*cosh(x)^2 - 2*(a^2*b + 2*b^3)*cosh(x))*sinh(x
))*sqrt(a^2 + b^2)*log((b^2*cosh(x)^2 + b^2*sinh(x)^2 + 2*a*b*cosh(x) + 2*
a^2 + b^2 + 2*(b^2*cosh(x) + a*b)*sinh(x) - 2*sqrt(a^2 + b^2)*(b*cosh(x) +
 b*sinh(x) + a))/(b*cosh(x)^2 + b*sinh(x)^2 + 2*a*cosh(x) + 2*(b*cosh(x) +
 a)*sinh(x) - b)) - 6*(a^4 + a^2*b^2)*cosh(x) + 2*((a^2*b^2 + b^4)*cosh(x)
^4 + (a^2*b^2 + b^4)*sinh(x)^4 + a^2*b^2 + b^4 + 2*(a^3*b + a*b^3)*cosh(x)
^3 + 2*(a^3*b + a*b^3 + 2*(a^2*b^2 + b^4)*cosh(x))*sinh(x)^3 - 2*(a^2*b^2
+ b^4)*cosh(x)^2 - 2*(a^2*b^2 + b^4 - 3*(a^2*b^2 + b^4)*cosh(x)^2 - 3*(a^3
*b + a*b^3)*cosh(x))*sinh(x)^2 - 2*(a^3*b + a*b^3)*cosh(x) - 2*(a^3*b + a*
b^3 - 2*(a^2*b^2 + b^4)*cosh(x)^3 - 3*(a^3*b + a*b^3)*cosh(x)^2 + 2*(a^2*b
^2 + b^4)*cosh(x))*sinh(x))*log(cosh(x) + sinh(x) + 1) - 2*((a^2*b^2 + b^4
)*cosh(x)^4 + (a^2*b^2 + b^4)*sinh(x)^4 + a^2*b^2 + b^4 + 2*(a^3*b + a*b^3
)*cosh(x)^3 + 2*(a^3*b + a*b^3 + 2*(a^2*b^2 + b^4)*cosh(x))*sinh(x)^3 -...
```

Sympy [F]

$$\int \frac{\coth^2(x)}{(a+b\sinh(x))^2}\,dx = \int \frac{\coth^2{(x)}}{(a+b\sinh{(x)})^2}\,dx$$

input
```
integrate(coth(x)**2/(a+b*sinh(x))**2,x)
```

output
```
Integral(coth(x)**2/(a + b*sinh(x))**2, x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 165 vs. $2(76) = 152$.

Time = 0.12 (sec) , antiderivative size = 165, normalized size of antiderivative = 2.06

$$\int \frac{\coth^2(x)}{(a+b\sinh(x))^2}\,dx = -\frac{2\left(3\,ae^{(-x)} - 2\,be^{(-2\,x)} - ae^{(-3\,x)} + 2\,b\right)}{2\,a^3e^{(-x)} - 2\,a^2be^{(-2\,x)} - 2\,a^3e^{(-3\,x)} + a^2be^{(-4\,x)} + a^2b} + \frac{2\,b\log\left(e^{(-x)} + 1\right)}{a^3} - \frac{2\,b\log\left(e^{(-x)} - 1\right)}{a^3} + \frac{(a^2 + 2\,b^2)\log\left(\frac{be^{(-x)} - a - \sqrt{a^2+b^2}}{be^{(-x)} - a + \sqrt{a^2+b^2}}\right)}{\sqrt{a^2+b^2}a^3}$$

input
```
integrate(coth(x)^2/(a+b*sinh(x))^2,x, algorithm="maxima")
```

output
```
-2*(3*a*e^(-x) - 2*b*e^(-2*x) - a*e^(-3*x) + 2*b)/(2*a^3*e^(-x) - 2*a^2*b*
e^(-2*x) - 2*a^3*e^(-3*x) + a^2*b*e^(-4*x) + a^2*b) + 2*b*log(e^(-x) + 1)/
a^3 - 2*b*log(e^(-x) - 1)/a^3 + (a^2 + 2*b^2)*log((b*e^(-x) - a - sqrt(a^2
 + b^2))/(b*e^(-x) - a + sqrt(a^2 + b^2)))/(sqrt(a^2 + b^2)*a^3)
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 148, normalized size of antiderivative = 1.85

$$\int \frac{\coth^2(x)}{(a+b\sinh(x))^2}\,dx = \frac{2\,b\log\left(e^x + 1\right)}{a^3} - \frac{2\,b\log\left(|e^x - 1|\right)}{a^3} + \frac{(a^2 + 2\,b^2)\log\left(\frac{\left|2\,be^x + 2\,a - 2\,\sqrt{a^2+b^2}\right|}{\left|2\,be^x + 2\,a + 2\,\sqrt{a^2+b^2}\right|}\right)}{\sqrt{a^2+b^2}a^3} + \frac{2\left(ae^{(3\,x)} - 2\,be^{(2\,x)} - 3\,ae^x + 2\,b\right)}{(be^{(4\,x)} + 2\,ae^{(3\,x)} - 2\,be^{(2\,x)} - 2\,ae^x + b)a^2}$$

input
```
integrate(coth(x)^2/(a+b*sinh(x))^2,x, algorithm="giac")
```

output
```
2*b*log(e^x + 1)/a^3 - 2*b*log(abs(e^x - 1))/a^3 + (a^2 + 2*b^2)*log(abs(2
*b*e^x + 2*a - 2*sqrt(a^2 + b^2))/abs(2*b*e^x + 2*a + 2*sqrt(a^2 + b^2)))/
(sqrt(a^2 + b^2)*a^3) + 2*(a*e^(3*x) - 2*b*e^(2*x) - 3*a*e^x + 2*b)/((b*e^
(4*x) + 2*a*e^(3*x) - 2*b*e^(2*x) - 2*a*e^x + b)*a^2)
```

Mupad [B] (verification not implemented)

Time = 2.61 (sec) , antiderivative size = 897, normalized size of antiderivative = 11.21

$$\int \frac{\coth^2(x)}{(a+b\sinh(x))^2}\,dx = \text{Too large to display}$$

input
```
int(coth(x)^2/(a + b*sinh(x))^2,x)
```

output
```
((4*(16*a^2*b^14 + 56*a^4*b^12 + 65*a^6*b^10 + 25*a^8*b^8))/(a^4*b^4*(16*b
^9 + 56*a^2*b^7 + 65*a^4*b^5 + 25*a^6*b^3)) - (6*exp(x)*(16*a^3*b^14 + 56*
a^5*b^12 + 65*a^7*b^10 + 25*a^9*b^8))/(a^4*b^5*(16*b^9 + 56*a^2*b^7 + 65*a
^4*b^5 + 25*a^6*b^3)) - (4*exp(2*x)*(16*a^2*b^14 + 56*a^4*b^12 + 65*a^6*b^
10 + 25*a^8*b^8))/(a^4*b^4*(16*b^9 + 56*a^2*b^7 + 65*a^4*b^5 + 25*a^6*b^3)
) + (2*exp(3*x)*(16*a^3*b^14 + 56*a^5*b^12 + 65*a^7*b^10 + 25*a^9*b^8))/(a
^4*b^5*(16*b^9 + 56*a^2*b^7 + 65*a^4*b^5 + 25*a^6*b^3)))/(b - 2*a*exp(x) +
 2*a*exp(3*x) - 2*b*exp(2*x) + b*exp(4*x)) - (2*b*log(64*exp(x) - 64))/a^3
 + (2*b*log(64*exp(x) + 64))/a^3 - (log(((a^2 + 2*b^2)*((32*(a^4 + 8*b^4 +
 12*a^2*b^2 - 12*a*b^3*exp(x) - 16*a^3*b*exp(x)))/(a^4*b^4) + ((a^2 + 2*b^
2)*((32*(2*a^2*b + 4*b^3 - 4*a^3*exp(x) - 7*a*b^2*exp(x)))/b^5 - (32*(a^2
+ 2*b^2)*(a^2 + b^2)^(1/2)*(3*a^4*b + 2*a^2*b^3 - 4*a^5*exp(x) - 3*a^3*b^2
*exp(x)))/(b^5*(a^5 + a^3*b^2)))*(a^2 + b^2)^(1/2))/(a^5 + a^3*b^2))*(a^2
+ b^2)^(1/2))/(a^5 + a^3*b^2) - (64*(a^2 + 2*b^2)*(4*b - 7*a*exp(x)))/(a^6
*b^3))*(a^2 + 2*b^2)*(a^2 + b^2)^(1/2))/(a^5 + a^3*b^2) + (log(- ((a^2 + 2
*b^2)*((32*(a^4 + 8*b^4 + 12*a^2*b^2 - 12*a*b^3*exp(x) - 16*a^3*b*exp(x)))
/(a^4*b^4) - ((a^2 + 2*b^2)*((32*(2*a^2*b + 4*b^3 - 4*a^3*exp(x) - 7*a*b^2
*exp(x)))/b^5 + (32*(a^2 + 2*b^2)*(a^2 + b^2)^(1/2)*(3*a^4*b + 2*a^2*b^3 -
 4*a^5*exp(x) - 3*a^3*b^2*exp(x)))/(b^5*(a^5 + a^3*b^2)))*(a^2 + b^2)^(1/2
))/(a^5 + a^3*b^2))*(a^2 + b^2)^(1/2))/(a^5 + a^3*b^2) - (64*(a^2 + 2*b...
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 888, normalized size of antiderivative = 11.10

$$\int \frac{\coth^2(x)}{(a+b\sinh(x))^2}\,dx = \text{Too large to display}$$

input
```
int(coth(x)^2/(a+b*sinh(x))^2,x)
```

output
```
(2*e**(4*x)*sqrt(a**2 + b**2)*atan((e**x*b*i + a*i)/sqrt(a**2 + b**2))*a**
2*b*i + 4*e**(4*x)*sqrt(a**2 + b**2)*atan((e**x*b*i + a*i)/sqrt(a**2 + b**
2))*b**3*i + 4*e**(3*x)*sqrt(a**2 + b**2)*atan((e**x*b*i + a*i)/sqrt(a**2
+ b**2))*a**3*i + 8*e**(3*x)*sqrt(a**2 + b**2)*atan((e**x*b*i + a*i)/sqrt(
a**2 + b**2))*a*b**2*i - 4*e**(2*x)*sqrt(a**2 + b**2)*atan((e**x*b*i + a*i
)/sqrt(a**2 + b**2))*a**2*b*i - 8*e**(2*x)*sqrt(a**2 + b**2)*atan((e**x*b*
i + a*i)/sqrt(a**2 + b**2))*b**3*i - 4*e**x*sqrt(a**2 + b**2)*atan((e**x*b
*i + a*i)/sqrt(a**2 + b**2))*a**3*i - 8*e**x*sqrt(a**2 + b**2)*atan((e**x*
b*i + a*i)/sqrt(a**2 + b**2))*a*b**2*i + 2*sqrt(a**2 + b**2)*atan((e**x*b*
i + a*i)/sqrt(a**2 + b**2))*a**2*b*i + 4*sqrt(a**2 + b**2)*atan((e**x*b*i
+ a*i)/sqrt(a**2 + b**2))*b**3*i - 2*e**(4*x)*log(e**x - 1)*a**2*b**2 - 2*
e**(4*x)*log(e**x - 1)*b**4 + 2*e**(4*x)*log(e**x + 1)*a**2*b**2 + 2*e**(4
*x)*log(e**x + 1)*b**4 - e**(4*x)*a**3*b - e**(4*x)*a*b**3 - 4*e**(3*x)*lo
g(e**x - 1)*a**3*b - 4*e**(3*x)*log(e**x - 1)*a*b**3 + 4*e**(3*x)*log(e**x
 + 1)*a**3*b + 4*e**(3*x)*log(e**x + 1)*a*b**3 + 4*e**(2*x)*log(e**x - 1)*
a**2*b**2 + 4*e**(2*x)*log(e**x - 1)*b**4 - 4*e**(2*x)*log(e**x + 1)*a**2*
b**2 - 4*e**(2*x)*log(e**x + 1)*b**4 - 2*e**(2*x)*a**3*b - 2*e**(2*x)*a*b*
*3 + 4*e**x*log(e**x - 1)*a**3*b + 4*e**x*log(e**x - 1)*a*b**3 - 4*e**x*lo
g(e**x + 1)*a**3*b - 4*e**x*log(e**x + 1)*a*b**3 - 4*e**x*a**4 - 4*e**x*a*
*2*b**2 - 2*log(e**x - 1)*a**2*b**2 - 2*log(e**x - 1)*b**4 + 2*log(e**x...
```

3.242 $\int \frac{\coth^3(x)}{(a+b\sinh(x))^2}\,dx$

Optimal result

Integrand size = 13, antiderivative size = 76

$$\int \frac{\coth^3(x)}{(a+b\sinh(x))^2}\,dx = \frac{2b\text{csch}(x)}{a^3} - \frac{\text{csch}^2(x)}{2a^2} + \frac{(a^2+3b^2)\log(\sinh(x))}{a^4} - \frac{(a^2+3b^2)\log(a+b\sinh(x))}{a^4} + \frac{a^2+b^2}{a^3(a+b\sinh(x))}$$

output
```
2*b*csch(x)/a^3-1/2*csch(x)^2/a^2+(a^2+3*b^2)*ln(sinh(x))/a^4-(a^2+3*b^2)*
ln(a+b*sinh(x))/a^4+(a^2+b^2)/a^3/(a+b*sinh(x))
```

Mathematica [A] (verified)

Time = 0.13 (sec) , antiderivative size = 73, normalized size of antiderivative = 0.96

$$\int \frac{\coth^3(x)}{(a+b\sinh(x))^2}\,dx = \frac{4ab\text{csch}(x) - a^2\text{csch}^2(x) + 2(a^2+3b^2)\log(\sinh(x)) - 2(a^2+3b^2)\log(a+b\sinh(x)) + \frac{2a(a^2+b^2)}{a+b\sinh(x)}}{2a^4}$$

input
```
Integrate[Coth[x]^3/(a + b*Sinh[x])^2,x]
```

output
```
(4*a*b*Csch[x] - a^2*Csch[x]^2 + 2*(a^2 + 3*b^2)*Log[Sinh[x]] - 2*(a^2 + 3
*b^2)*Log[a + b*Sinh[x]] + (2*a*(a^2 + b^2))/(a + b*Sinh[x]))/(2*a^4)
```

Rubi [A] (verified)

Time = 0.36 (sec) , antiderivative size = 78, normalized size of antiderivative = 1.03, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.462$, Rules used = {3042, 26, 3200, 25, 522, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\coth^3(x)}{(a+b\sinh(x))^2}dx$$

$$\Big\downarrow \text{3042}$$

$$\int -\frac{i}{\tan(ix)^3(a-ib\sin(ix))^2}dx$$

$$\Big\downarrow \text{26}$$

$$-i\int \frac{1}{(a-ib\sin(ix))^2\tan(ix)^3}dx$$

$$\Big\downarrow \text{3200}$$

$$-\int -\frac{\operatorname{csch}^3(x)\left(\sinh^2(x)b^2+b^2\right)}{b^3(a+b\sinh(x))^2}d(b\sinh(x))$$

$$\Big\downarrow \text{25}$$

$$\int \frac{\operatorname{csch}^3(x)\left(b^2\sinh^2(x)+b^2\right)}{b^3(a+b\sinh(x))^2}d(b\sinh(x))$$

$$\Big\downarrow \text{522}$$

$$\int \left(-\frac{2\operatorname{csch}^2(x)}{a^3}+\frac{\operatorname{csch}^3(x)}{a^2b}+\frac{-a^2-3b^2}{a^4(a+b\sinh(x))}+\frac{\left(a^2+3b^2\right)\operatorname{csch}(x)}{a^4b}+\frac{-a^2-b^2}{a^3(a+b\sinh(x))^2}\right)d(b\sinh(x))$$

$$\Big\downarrow \text{2009}$$

$$\frac{2b\text{csch}(x)}{a^3}-\frac{\text{csch}^2(x)}{2a^2}+\frac{(a^2+3b^2)\log(b\sinh(x))}{a^4}-\frac{(a^2+3b^2)\log(a+b\sinh(x))}{a^4}+\frac{a^2+b^2}{a^3(a+b\sinh(x))}$$

input
```
Int[Coth[x]^3/(a + b*Sinh[x])^2,x]
```

output
```
(2*b*Csch[x])/a^3 - Csch[x]^2/(2*a^2) + ((a^2 + 3*b^2)*Log[b*Sinh[x]])/a^4
 - ((a^2 + 3*b^2)*Log[a + b*Sinh[x]])/a^4 + (a^2 + b^2)/(a^3*(a + b*Sinh[x
]))
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 522
```
Int[((e_.)*(x_))^(m_.)*((c_) + (d_.)*(x_))^(n_.)*((a_) + (b_.)*(x_)^2)^(p_.
), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(c + d*x)^n*(a + b*x^2)^p, x],
x] /; FreeQ[{a, b, c, d, e, m, n}, x] && IGtQ[p, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3200
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*tan[(e_.) + (f_.)*(x_)]^(p
_.), x_Symbol] :> Simp[1/f   Subst[Int[(x^p*(a + x)^m)/(b^2 - x^2)^((p + 1)
/2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && NeQ[a^2 - b
^2, 0] && IntegerQ[(p + 1)/2]
```

Maple [A] (verified)

Time = 5.52 (sec) , antiderivative size = 142, normalized size of antiderivative = 1.87

method	result
default	$-\frac{\frac{\tanh\left(\frac{x}{2}\right)^{2} a}{2}+4b\tanh\left(\frac{x}{2}\right)}{4a^{3}}-\frac{1}{8a^{2}\tanh\left(\frac{x}{2}\right)^{2}}+\frac{\left(4a^{2}+12b^{2}\right)\ln\left(\tanh\left(\frac{x}{2}\right)\right)}{4a^{4}}+\frac{b}{\tanh\left(\frac{x}{2}\right)a^{3}}-\frac{2\left(\frac{\left(-a^{2}b-b^{3}\right)\tanh\left(\frac{x}{2}\right)}{\tanh\left(\frac{x}{2}\right)^{2}a-2b\tanh\left(\frac{x}{2}\right)-a}+\frac{(a^{2}}{}\right.}{}$
risch	$\frac{2\,\mathrm{e}^{x}\left(\mathrm{e}^{4x}a^{2}+3b^{2}\mathrm{e}^{4x}+3ab\,\mathrm{e}^{3x}-4\,\mathrm{e}^{2x}a^{2}-6\,\mathrm{e}^{2x}b^{2}-3b\,\mathrm{e}^{x}a+a^{2}+3b^{2}\right)}{\left(\mathrm{e}^{2x}-1\right)^{2}a^{3}\left(b\,\mathrm{e}^{2x}+2\,\mathrm{e}^{x}a-b\right)}+\frac{\ln\left(\mathrm{e}^{2x}-1\right)}{a^{2}}+\frac{3\ln\left(\mathrm{e}^{2x}-1\right)b^{2}}{a^{4}}-\frac{\ln\left(\mathrm{e}^{2x}+\frac{2a\,\mathrm{e}^{x}}{b}-1\right)}{a^{2}}-\frac{3\ln}{}$

input
```
int(coth(x)^3/(a+b*sinh(x))^2,x,method=_RETURNVERBOSE)
```

output
```
-1/4/a^3*(1/2*tanh(1/2*x)^2*a+4*b*tanh(1/2*x))-1/8/a^2/tanh(1/2*x)^2+1/4/a
^4*(4*a^2+12*b^2)*ln(tanh(1/2*x))+b/tanh(1/2*x)/a^3-2/a^4*((-a^2*b-b^3)*ta
nh(1/2*x)/(tanh(1/2*x)^2*a-2*b*tanh(1/2*x)-a)+1/2*(a^2+3*b^2)*ln(tanh(1/2*
x)^2*a-2*b*tanh(1/2*x)-a))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1463 vs. 2(74) = 148.

Time = 0.10 (sec) , antiderivative size = 1463, normalized size of antiderivative = 19.25

$$\int \frac{\coth^3(x)}{(a+b\sinh(x))^2}\,dx = \text{Too large to display}$$

input
```
integrate(coth(x)^3/(a+b*sinh(x))^2,x, algorithm="fricas")
```

output
```
(6*a^2*b*cosh(x)^4 + 2*(a^3 + 3*a*b^2)*cosh(x)^5 + 2*(a^3 + 3*a*b^2)*sinh(
x)^5 - 6*a^2*b*cosh(x)^2 + 2*(3*a^2*b + 5*(a^3 + 3*a*b^2)*cosh(x))*sinh(x)
^4 - 4*(2*a^3 + 3*a*b^2)*cosh(x)^3 + 4*(6*a^2*b*cosh(x) - 2*a^3 - 3*a*b^2
+ 5*(a^3 + 3*a*b^2)*cosh(x)^2)*sinh(x)^3 + 2*(18*a^2*b*cosh(x)^2 + 10*(a^3
 + 3*a*b^2)*cosh(x)^3 - 3*a^2*b - 6*(2*a^3 + 3*a*b^2)*cosh(x))*sinh(x)^2 +
 2*(a^3 + 3*a*b^2)*cosh(x) - ((a^2*b + 3*b^3)*cosh(x)^6 + (a^2*b + 3*b^3)*
sinh(x)^6 + 2*(a^3 + 3*a*b^2)*cosh(x)^5 + 2*(a^3 + 3*a*b^2 + 3*(a^2*b + 3*
b^3)*cosh(x))*sinh(x)^5 - 3*(a^2*b + 3*b^3)*cosh(x)^4 - (3*a^2*b + 9*b^3 -
 15*(a^2*b + 3*b^3)*cosh(x)^2 - 10*(a^3 + 3*a*b^2)*cosh(x))*sinh(x)^4 - 4*
(a^3 + 3*a*b^2)*cosh(x)^3 + 4*(5*(a^2*b + 3*b^3)*cosh(x)^3 - a^3 - 3*a*b^2
 + 5*(a^3 + 3*a*b^2)*cosh(x)^2 - 3*(a^2*b + 3*b^3)*cosh(x))*sinh(x)^3 - a^
2*b - 3*b^3 + 3*(a^2*b + 3*b^3)*cosh(x)^2 + (15*(a^2*b + 3*b^3)*cosh(x)^4
+ 20*(a^3 + 3*a*b^2)*cosh(x)^3 + 3*a^2*b + 9*b^3 - 18*(a^2*b + 3*b^3)*cosh
(x)^2 - 12*(a^3 + 3*a*b^2)*cosh(x))*sinh(x)^2 + 2*(a^3 + 3*a*b^2)*cosh(x)
+ 2*(3*(a^2*b + 3*b^3)*cosh(x)^5 + 5*(a^3 + 3*a*b^2)*cosh(x)^4 - 6*(a^2*b
+ 3*b^3)*cosh(x)^3 + a^3 + 3*a*b^2 - 6*(a^3 + 3*a*b^2)*cosh(x)^2 + 3*(a^2*
b + 3*b^3)*cosh(x))*sinh(x))*log(2*(b*sinh(x) + a)/(cosh(x) - sinh(x))) +
((a^2*b + 3*b^3)*cosh(x)^6 + (a^2*b + 3*b^3)*sinh(x)^6 + 2*(a^3 + 3*a*b^2)
*cosh(x)^5 + 2*(a^3 + 3*a*b^2 + 3*(a^2*b + 3*b^3)*cosh(x))*sinh(x)^5 - 3*(
a^2*b + 3*b^3)*cosh(x)^4 - (3*a^2*b + 9*b^3 - 15*(a^2*b + 3*b^3)*cosh(x...
```

Sympy [F]

$$\int \frac{\coth^3(x)}{(a+b\sinh(x))^2}\,dx=\int \frac{\coth^3{(x)}}{(a+b\sinh{(x)})^2}\,dx$$

input
```
integrate(coth(x)**3/(a+b*sinh(x))**2,x)
```

output
```
Integral(coth(x)**3/(a + b*sinh(x))**2, x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 202 vs. $2(74) = 148$.

Time = 0.05 (sec) , antiderivative size = 202, normalized size of antiderivative = 2.66

$$\int \frac{\coth^3(x)}{(a+b\sinh(x))^2}\,dx = \frac{2\left(3\,abe^{(-2\,x)} - 3\,abe^{(-4\,x)} + (a^2+3\,b^2)e^{(-x)} - 2\,(2\,a^2+3\,b^2)e^{(-3\,x)} + (a^2+3\,b^2)e^{(-5\,x)}\right)}{2\,a^4e^{(-x)} - 3\,a^3be^{(-2\,x)} - 4\,a^4e^{(-3\,x)} + 3\,a^3be^{(-4\,x)} + 2\,a^4e^{(-5\,x)} - a^3be^{(-6\,x)} + a^3b} - \frac{(a^2+3\,b^2)\log\left(-2\,ae^{(-x)} + be^{(-2\,x)} - b\right)}{a^4} + \frac{(a^2+3\,b^2)\log\left(e^{(-x)}+1\right)}{a^4} + \frac{(a^2+3\,b^2)\log\left(e^{(-x)}-1\right)}{a^4}$$

input
```
integrate(coth(x)^3/(a+b*sinh(x))^2,x, algorithm="maxima")
```

output
```
2*(3*a*b*e^(-2*x) - 3*a*b*e^(-4*x) + (a^2 + 3*b^2)*e^(-x) - 2*(2*a^2 + 3*b
^2)*e^(-3*x) + (a^2 + 3*b^2)*e^(-5*x))/(2*a^4*e^(-x) - 3*a^3*b*e^(-2*x) -
4*a^4*e^(-3*x) + 3*a^3*b*e^(-4*x) + 2*a^4*e^(-5*x) - a^3*b*e^(-6*x) + a^3*
b) - (a^2 + 3*b^2)*log(-2*a*e^(-x) + b*e^(-2*x) - b)/a^4 + (a^2 + 3*b^2)*l
og(e^(-x) + 1)/a^4 + (a^2 + 3*b^2)*log(e^(-x) - 1)/a^4
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 190 vs. $2(74) = 148$.

Time = 0.13 (sec) , antiderivative size = 190, normalized size of antiderivative = 2.50

$$\int \frac{\coth^3(x)}{(a+b\sinh(x))^2}\,dx = \frac{(a^2+3\,b^2)\log\left(\left|-e^{(-x)}+e^x\right|\right)}{a^4} - \frac{(a^2b+3\,b^3)\log\left(\left|-b\left(e^{(-x)}-e^x\right)+2\,a\right|\right)}{a^4b} + \frac{a^2b\left(e^{(-x)}-e^x\right) + 3\,b^3\left(e^{(-x)}-e^x\right) - 4\,a^3 - 8\,ab^2}{\left(b\left(e^{(-x)}-e^x\right)-2\,a\right)a^4} - \frac{3\,a^2\left(e^{(-x)}-e^x\right)^2 + 9\,b^2\left(e^{(-x)}-e^x\right)^2 + 8\,ab\left(e^{(-x)}-e^x\right) + 4\,a^2}{2\,a^4\left(e^{(-x)}-e^x\right)^2}$$

input
```
integrate(coth(x)^3/(a+b*sinh(x))^2,x, algorithm="giac")
```

output
```
(a^2 + 3*b^2)*log(abs(-e^(-x) + e^x))/a^4 - (a^2*b + 3*b^3)*log(abs(-b*(e^
(-x) - e^x) + 2*a))/(a^4*b) + (a^2*b*(e^(-x) - e^x) + 3*b^3*(e^(-x) - e^x)
 - 4*a^3 - 8*a*b^2)/((b*(e^(-x) - e^x) - 2*a)*a^4) - 1/2*(3*a^2*(e^(-x) -
e^x)^2 + 9*b^2*(e^(-x) - e^x)^2 + 8*a*b*(e^(-x) - e^x) + 4*a^2)/(a^4*(e^(-
x) - e^x)^2)
```

Mupad [B] (verification not implemented)

Time = 2.47 (sec) , antiderivative size = 1375, normalized size of antiderivative = 18.09

$$\int \frac{\coth^3(x)}{(a+b\sinh(x))^2}\,dx = \text{Too large to display}$$

input
```
int(coth(x)^3/(a + b*sinh(x))^2,x)
```

output
```
(2*exp(x)*(a*b^7 + 2*a^3*b^5 + a^5*b^3))/(a^4*(b^5 + a^2*b^3)*(2*a*exp(x)
- b + b*exp(2*x))) - 2/(a^2*(exp(4*x) - 2*exp(2*x) + 1)) - ((2*atan((4*a^9
*b*((a^2 + 3*b^2)^2)^(1/2)*(-a^8)^(1/2) + 12*a^5*b^5*((a^2 + 3*b^2)^2)^(1/
2)*(-a^8)^(1/2) + 16*a^7*b^3*((a^2 + 3*b^2)^2)^(1/2)*(-a^8)^(1/2))*(exp(x)
*((a^2 + 2*b^2)^2/(16*a^10*b^2*(a^4 + 3*b^4 + 4*a^2*b^2)^2) - 1/(16*a^6*b^
2*(a^2 + 3*b^2)^2*(a^2 + b^2)^2)) + (a^2 + 2*b^2)/(8*a^9*b*(a^4 + 3*b^4 +
4*a^2*b^2)^2) + 1/(8*a^7*b*(a^2 + 3*b^2)^2*(a^2 + b^2)^2))) - 2*atan((a^2*
(-a^8)^(1/2)*(a^4 + 9*b^4 + 6*a^2*b^2)^(1/2) + 2*b^2*(-a^8)^(1/2)*(a^4 + 9
*b^4 + 6*a^2*b^2)^(1/2))/(2*a^4*(a^4 + 3*b^4 + 4*a^2*b^2)) + ((a^8 + 3*a^6
*b^2)*(-a^8)^(1/2))/(2*a^8*((a^2 + 3*b^2)^2)^(1/2)*(a^2 + b^2)) - (a^8*b^2
*exp(2*x)*(-a^8)^(1/2)*((4*(a^2 + 2*b^2)*(a^4 + 9*b^4 + 6*a^2*b^2))/(a^12*
b^2*(a^4 + 3*b^4 + 4*a^2*b^2)) + (4*(a^2*(-a^8)^(1/2)*(a^4 + 9*b^4 + 6*a^2
*b^2)^(1/2) + 2*b^2*(-a^8)^(1/2)*(a^4 + 9*b^4 + 6*a^2*b^2)^(1/2))*(a^4 + 9
*b^4 + 6*a^2*b^2)^(1/2))/(a^12*b^2*(-a^8)^(1/2)*(a^4 + 3*b^4 + 4*a^2*b^2))
 + (2*(2*a^7*b + 6*a^5*b^3)*(a^4 + 9*b^4 + 6*a^2*b^2)^(1/2))/(a^15*b^3*((a
^2 + 3*b^2)^2)^(1/2)*(a^2 + b^2)) + (4*(a^8 + 3*a^6*b^2)*(a^4 + 9*b^4 + 6*
a^2*b^2)^(1/2))/(a^16*b^2*((a^2 + 3*b^2)^2)^(1/2)*(a^2 + b^2))))/(8*(a^4 +
 9*b^4 + 6*a^2*b^2)^(1/2)) + (a^8*b^2*exp(3*x)*((2*(a^8 + 3*a^6*b^2)*(a^4
+ 9*b^4 + 6*a^2*b^2)^(1/2))/(a^15*b^3*((a^2 + 3*b^2)^2)^(1/2)*(a^2 + b^2))
 - (2*(a^2 + 2*b^2)*(a^2*(-a^8)^(1/2)*(a^4 + 9*b^4 + 6*a^2*b^2)^(1/2) +...
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 955, normalized size of antiderivative = 12.57

$$\int \frac{\coth^3(x)}{(a+b\sinh(x))^2}\,dx = \text{Too large to display}$$

input
```
int(coth(x)^3/(a+b*sinh(x))^2,x)
```

output
```
(e**(6*x)*log(e**x - 1)*a**2*b + 3*e**(6*x)*log(e**x - 1)*b**3 + e**(6*x)*
log(e**x + 1)*a**2*b + 3*e**(6*x)*log(e**x + 1)*b**3 - e**(6*x)*log(e**(2*
x)*b + 2*e**x*a - b)*a**2*b - 3*e**(6*x)*log(e**(2*x)*b + 2*e**x*a - b)*b*
*3 - e**(6*x)*a**2*b - 3*e**(6*x)*b**3 + 2*e**(5*x)*log(e**x - 1)*a**3 + 6
*e**(5*x)*log(e**x - 1)*a*b**2 + 2*e**(5*x)*log(e**x + 1)*a**3 + 6*e**(5*x
)*log(e**x + 1)*a*b**2 - 2*e**(5*x)*log(e**(2*x)*b + 2*e**x*a - b)*a**3 -
6*e**(5*x)*log(e**(2*x)*b + 2*e**x*a - b)*a*b**2 - 3*e**(4*x)*log(e**x - 1
)*a**2*b - 9*e**(4*x)*log(e**x - 1)*b**3 - 3*e**(4*x)*log(e**x + 1)*a**2*b
 - 9*e**(4*x)*log(e**x + 1)*b**3 + 3*e**(4*x)*log(e**(2*x)*b + 2*e**x*a -
b)*a**2*b + 9*e**(4*x)*log(e**(2*x)*b + 2*e**x*a - b)*b**3 + 9*e**(4*x)*a*
*2*b + 9*e**(4*x)*b**3 - 4*e**(3*x)*log(e**x - 1)*a**3 - 12*e**(3*x)*log(e
**x - 1)*a*b**2 - 4*e**(3*x)*log(e**x + 1)*a**3 - 12*e**(3*x)*log(e**x + 1
)*a*b**2 + 4*e**(3*x)*log(e**(2*x)*b + 2*e**x*a - b)*a**3 + 12*e**(3*x)*lo
g(e**(2*x)*b + 2*e**x*a - b)*a*b**2 - 4*e**(3*x)*a**3 + 3*e**(2*x)*log(e**
x - 1)*a**2*b + 9*e**(2*x)*log(e**x - 1)*b**3 + 3*e**(2*x)*log(e**x + 1)*a
**2*b + 9*e**(2*x)*log(e**x + 1)*b**3 - 3*e**(2*x)*log(e**(2*x)*b + 2*e**x
*a - b)*a**2*b - 9*e**(2*x)*log(e**(2*x)*b + 2*e**x*a - b)*b**3 - 9*e**(2*
x)*a**2*b - 9*e**(2*x)*b**3 + 2*e**x*log(e**x - 1)*a**3 + 6*e**x*log(e**x
- 1)*a*b**2 + 2*e**x*log(e**x + 1)*a**3 + 6*e**x*log(e**x + 1)*a*b**2 - 2*
e**x*log(e**(2*x)*b + 2*e**x*a - b)*a**3 - 6*e**x*log(e**(2*x)*b + 2*e*...
```

3.243 $\int \frac{\coth^4(x)}{(a+b\sinh(x))^2}\,dx$

Optimal result

Integrand size = 13, antiderivative size = 159

$$\int \frac{\coth^4(x)}{(a+b\sinh(x))^2}\,dx = \frac{b(3a^2+4b^2)\operatorname{arctanh}(\cosh(x))}{a^5} - \frac{2\sqrt{a^2+b^2}(a^2+4b^2)\arctan\left(\frac{b-a\tanh\left(\frac{x}{2}\right)}{\sqrt{a^2+b^2}}\right)}{a^5} - \frac{(7a^2+12b^2)\coth(x)}{3a^4} + \frac{(a^2+2b^2)\coth(x)\operatorname{csch}(x)}{a^3b} - \frac{\left(3+\frac{4b^2}{a^2}\right)\coth(x)\operatorname{csch}(x)}{3b(a+b\sinh(x))} - \frac{\coth(x)\operatorname{csch}^2(x)}{3a(a+b\sinh(x))}$$

output
```
b*(3*a^2+4*b^2)*arctanh(cosh(x))/a^5-2*(a^2+b^2)^(1/2)*(a^2+4*b^2)*arctanh
((b-a*tanh(1/2*x))/(a^2+b^2)^(1/2))/a^5-1/3*(7*a^2+12*b^2)*coth(x)/a^4+(a^
2+2*b^2)*coth(x)*csch(x)/a^3/b-1/3*(3+4*b^2/a^2)*coth(x)*csch(x)/b/(a+b*si
nh(x))-1/3*coth(x)*csch(x)^2/a/(a+b*sinh(x))
```

Mathematica [A] (verified)

Time = 0.59 (sec) , antiderivative size = 235, normalized size of antiderivative = 1.48

$$\int \frac{\coth^4(x)}{(a+b\sinh(x))^2}\,dx$$
$$= \frac{\frac{48(a^4+5a^2b^2+4b^4)\arctan\left(\frac{b-a\tanh\left(\frac{x}{2}\right)}{\sqrt{-a^2-b^2}}\right)}{\sqrt{-a^2-b^2}} - 4a(4a^2+9b^2)\coth\left(\frac{x}{2}\right) + 6a^2b\operatorname{csch}^2\left(\frac{x}{2}\right) + 24b(3a^2+4b^2)\log\left(\cosh\left(\frac{x}{2}\right.\right.}{}$$

input
```
Integrate[Coth[x]^4/(a + b*Sinh[x])^2,x]
```

output
```
((48*(a^4 + 5*a^2*b^2 + 4*b^4)*ArcTan[(b - a*Tanh[x/2])/Sqrt[-a^2 - b^2]])
/Sqrt[-a^2 - b^2] - 4*a*(4*a^2 + 9*b^2)*Coth[x/2] + 6*a^2*b*Csch[x/2]^2 +
24*b*(3*a^2 + 4*b^2)*Log[Cosh[x/2]] - 24*b*(3*a^2 + 4*b^2)*Log[Sinh[x/2]]
+ 6*a^2*b*Sech[x/2]^2 + 8*a^3*Csch[x]^3*Sinh[x/2]^4 - (a^3*Csch[x/2]^4*Sin
h[x])/2 - (24*a*b*(a^2 + b^2)*Cosh[x])/(a + b*Sinh[x]) - 4*a*(4*a^2 + 9*b^
2)*Tanh[x/2])/(24*a^5)
```

Rubi [C] (verified)

Result contains complex when optimal does not.

Time = 1.48 (sec) , antiderivative size = 199, normalized size of antiderivative = 1.25, number of steps used = 22, number of rules used = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 1.615$, Rules used = {3042, 3203, 26, 3042, 26, 3534, 27, 3042, 25, 3534, 27, 3042, 26, 3480, 26, 3042, 26, 3139, 1083, 219, 4257}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\coth^4(x)}{(a+b\sinh(x))^2}\,dx$$
$$\Big\downarrow \text{ 3042}$$
$$\int \frac{1}{\tan(ix)^4(a-ib\sin(ix))^2}dx$$

$$\Big\downarrow \text{3203}$$

$$\frac{i\int \frac{i\text{csch}^3(x)\left(\left(3a^2+8b^2\right)\sinh^2(x)-ab\sinh(x)+6\left(a^2+2b^2\right)\right)}{a+b\sinh(x)}dx}{3a^2b}-\frac{\left(\frac{4b^2}{a^2}+3\right)\coth(x)\text{csch}(x)}{3b(a+b\sinh(x))}-\frac{\coth(x)\text{csch}^2(x)}{3a(a+b\sinh(x))}$$

$$\Big\downarrow \text{26}$$

$$-\frac{\int \frac{\text{csch}^3(x)\left(\left(3a^2+8b^2\right)\sinh^2(x)-ab\sinh(x)+6\left(a^2+2b^2\right)\right)}{a+b\sinh(x)}dx}{3a^2b}-\frac{\left(\frac{4b^2}{a^2}+3\right)\coth(x)\text{csch}(x)}{3b(a+b\sinh(x))}-\frac{\coth(x)\text{csch}^2(x)}{3a(a+b\sinh(x))}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{\int -\frac{i\left(-\left(\left(3a^2+8b^2\right)\sin(ix)^2\right)+iab\sin(ix)+6\left(a^2+2b^2\right)\right)}{\sin(ix)^3(a-ib\sin(ix))}dx}{3a^2b}-\frac{\left(\frac{4b^2}{a^2}+3\right)\coth(x)\text{csch}(x)}{3b(a+b\sinh(x))}-\frac{\coth(x)\text{csch}^2(x)}{3a(a+b\sinh(x))}$$

$$\Big\downarrow \text{26}$$

$$\frac{i\int \frac{-\left(\left(3a^2+8b^2\right)\sin(ix)^2\right)+iab\sin(ix)+6\left(a^2+2b^2\right)}{\sin(ix)^3(a-ib\sin(ix))}dx}{3a^2b}-\frac{\left(\frac{4b^2}{a^2}+3\right)\coth(x)\text{csch}(x)}{3b(a+b\sinh(x))}-\frac{\coth(x)\text{csch}^2(x)}{3a(a+b\sinh(x))}$$

$$\Big\downarrow \text{3534}$$

$$\frac{i\left(\frac{\int -\frac{2i\text{csch}^2(x)\left(-2a\sinh(x)b^2+3\left(a^2+2b^2\right)\sinh^2(x)b+\left(7a^2+12b^2\right)b\right)}{a+b\sinh(x)}dx}{2a}-\frac{3i\left(a^2+2b^2\right)\coth(x)\text{csch}(x)}{a}\right)}{3a^2b}-\frac{\left(\frac{4b^2}{a^2}+3\right)\coth(x)\text{csch}(x)}{3b(a+b\sinh(x))}-\frac{\coth(x)\text{csch}^2(x)}{3a(a+b\sinh(x))}$$

$$\Big\downarrow \text{27}$$

$$\frac{i\left(-\frac{i\int \frac{\text{csch}^2(x)\left(-2a\sinh(x)b^2+3\left(a^2+2b^2\right)\sinh^2(x)b+\left(7a^2+12b^2\right)b\right)}{a+b\sinh(x)}dx}{a}-\frac{3i\left(a^2+2b^2\right)\coth(x)\text{csch}(x)}{a}\right)}{3a^2b}-\frac{\left(\frac{4b^2}{a^2}+3\right)\coth(x)\text{csch}(x)}{3b(a+b\sinh(x))}-\frac{\coth(x)\text{csch}^2(x)}{3a(a+b\sinh(x))}$$

$$\Big\downarrow \text{3042}$$

$$\frac{i\left(-\frac{i\int -\frac{2ia\sin(ix)b^2-3\left(a^2+2b^2\right)\sin(ix)^2b+\left(7a^2+12b^2\right)b}{\sin(ix)^2(a-ib\sin(ix))}dx}{a}-\frac{3i(a^2+2b^2)\coth(x)\operatorname{csch}(x)}{a}\right)}{3a^2b}-\frac{\left(\frac{4b^2}{a^2}+3\right)\coth(x)\operatorname{csch}(x)}{3b(a+b\sinh(x))}-\frac{\coth(x)\operatorname{csch}^2(x)}{3a(a+b\sinh(x))}$$

$\downarrow$ 25

$$\frac{i\left(\frac{i\int \frac{2ia\sin(ix)b^2-3\left(a^2+2b^2\right)\sin(ix)^2b+\left(7a^2+12b^2\right)b}{\sin(ix)^2(a-ib\sin(ix))}dx}{a}-\frac{3i(a^2+2b^2)\coth(x)\operatorname{csch}(x)}{a}\right)}{3a^2b}-\frac{\left(\frac{4b^2}{a^2}+3\right)\coth(x)\operatorname{csch}(x)}{3b(a+b\sinh(x))}-\frac{\coth(x)\operatorname{csch}^2(x)}{3a(a+b\sinh(x))}$$

$\downarrow$ 3534

$$\frac{i\left(\frac{i\left(\frac{\int \frac{3\operatorname{csch}(x)\left(b^2\left(3a^2+4b^2\right)-ab\left(a^2+2b^2\right)\sinh(x)\right)}{a+b\sinh(x)}dx}{a}+\frac{b\left(7a^2+12b^2\right)\coth(x)}{a}\right)}{a}-\frac{3i(a^2+2b^2)\coth(x)\operatorname{csch}(x)}{a}\right)}{3a^2b}-\frac{\left(\frac{4b^2}{a^2}+3\right)\coth(x)\operatorname{csch}(x)}{3b(a+b\sinh(x))}-\frac{\coth(x)\operatorname{csch}^2(x)}{3a(a+b\sinh(x))}$$

$\downarrow$ 27

$$\frac{i\left(\frac{i\left(\frac{3\int \frac{\operatorname{csch}(x)\left(b^2\left(3a^2+4b^2\right)-ab\left(a^2+2b^2\right)\sinh(x)\right)}{a+b\sinh(x)}dx}{a}+\frac{b\left(7a^2+12b^2\right)\coth(x)}{a}\right)}{a}-\frac{3i(a^2+2b^2)\coth(x)\operatorname{csch}(x)}{a}\right)}{3a^2b}-\frac{\left(\frac{4b^2}{a^2}+3\right)\coth(x)\operatorname{csch}(x)}{3b(a+b\sinh(x))}-\frac{\coth(x)\operatorname{csch}^2(x)}{3a(a+b\sinh(x))}$$

$\downarrow$ 3042

$$\frac{i\left(\frac{i\left(\frac{b\left(7a^2+12b^2\right)\coth(x)}{a}+\frac{3\int\frac{i\left(\left(3a^2+4b^2\right)b^2+ia\left(a^2+2b^2\right)\sin(ix)b\right)}{\sin(ix)(a-ib\sin(ix))}dx}{a}\right)}{a}-\frac{3i(a^2+2b^2)\coth(x)\text{csch}(x)}{a}\right)}{3a^2b}-\frac{\left(\frac{4b^2}{a^2}+3\right)\coth(x)\text{csch}(x)}{3b(a+b\sinh(x))}-\frac{\coth(x)\text{csch}^2(x)}{3a(a+b\sinh(x))}$$

$\big\downarrow$ 26

$$\frac{i\left(\frac{i\left(\frac{b\left(7a^2+12b^2\right)\coth(x)}{a}+\frac{3i\int\frac{\left(3a^2+4b^2\right)b^2+ia\left(a^2+2b^2\right)\sin(ix)b}{\sin(ix)(a-ib\sin(ix))}dx}{a}\right)}{a}-\frac{3i(a^2+2b^2)\coth(x)\text{csch}(x)}{a}\right)}{3a^2b}-\frac{\left(\frac{4b^2}{a^2}+3\right)\coth(x)\text{csch}(x)}{3b(a+b\sinh(x))}-\frac{\coth(x)\text{csch}^2(x)}{3a(a+b\sinh(x))}$$

$\big\downarrow$ 3480

$$\frac{i\left(\frac{i\left(\frac{b\left(7a^2+12b^2\right)\coth(x)}{a}+\frac{3i\left(\frac{ib\left(a^2+b^2\right)\left(a^2+4b^2\right)\int\frac{1}{a+b\sinh(x)}dx}{a}+\frac{b^2\left(3a^2+4b^2\right)\int-i\text{csch}(x)dx}{a}\right)}{a}\right)}{a}-\frac{3i(a^2+2b^2)\coth(x)\text{csch}(x)}{a}\right)}{3a^2b}-\frac{\left(\frac{4b^2}{a^2}+3\right)\coth(x)\text{csch}(x)}{3b(a+b\sinh(x))}-\frac{\coth(x)\text{csch}^2(x)}{3a(a+b\sinh(x))}$$

$\big\downarrow$ 26

$$\frac{i\left(\frac{i\left(\frac{b(7a^2+12b^2)\coth(x)}{a}+\frac{3i\left(\frac{ib(a^2+b^2)(a^2+4b^2)\int\frac{1}{a+b\sinh(x)}dx}{a}-\frac{ib^2(3a^2+4b^2)\int\operatorname{csch}(x)dx}{a}\right)}{a}\right)}{a}-\frac{3i(a^2+2b^2)\coth(x)\operatorname{csch}(x)}{a}\right)}{3a^2b}-\frac{\left(\frac{4b^2}{a^2}+3\right)\coth(x)\operatorname{csch}(x)}{3b(a+b\sinh(x))}-\frac{\coth(x)\operatorname{csch}^2(x)}{3a(a+b\sinh(x))}$$

$\Big\downarrow$ 3042

$$\frac{i\left(\frac{i\left(\frac{b(7a^2+12b^2)\coth(x)}{a}+\frac{3i\left(\frac{ib(a^2+b^2)(a^2+4b^2)\int\frac{1}{a-ib\sin(ix)}dx}{a}-\frac{ib^2(3a^2+4b^2)\int i\csc(ix)dx}{a}\right)}{a}\right)}{a}-\frac{3i(a^2+2b^2)\coth(x)\operatorname{csch}(x)}{a}\right)}{3a^2b}-\frac{\left(\frac{4b^2}{a^2}+3\right)\coth(x)\operatorname{csch}(x)}{3b(a+b\sinh(x))}-\frac{\coth(x)\operatorname{csch}^2(x)}{3a(a+b\sinh(x))}$$

$\Big\downarrow$ 26

$$\frac{i\left(\frac{i\left(\frac{b(7a^2+12b^2)\coth(x)}{a}+\frac{3i\left(\frac{ib(a^2+b^2)(a^2+4b^2)\int\frac{1}{a-ib\sin(ix)}dx}{a}+\frac{b^2(3a^2+4b^2)\int\csc(ix)dx}{a}\right)}{a}\right)}{a}-\frac{3i(a^2+2b^2)\coth(x)\operatorname{csch}(x)}{a}\right)}{3a^2b}-\frac{\left(\frac{4b^2}{a^2}+3\right)\coth(x)\operatorname{csch}(x)}{3b(a+b\sinh(x))}-\frac{\coth(x)\operatorname{csch}^2(x)}{3a(a+b\sinh(x))}$$

$\Big\downarrow$ 3139

$$\frac{i\left(\frac{i\left(\frac{b\left(7a^2+12b^2\right)\coth(x)}{a}+\frac{3i\left(\frac{b^2\left(3a^2+4b^2\right)\int \csc(ix)dx}{a}+\frac{2ib\left(a^2+b^2\right)\left(a^2+4b^2\right)\int \frac{1}{-a\tanh^2\left(\frac{x}{2}\right)+2b\tanh\left(\frac{x}{2}\right)+a}d\tanh\left(\frac{x}{2}\right)}{a}\right)}{a}\right)}{a}-\frac{3i\left(a^2+2b^2\right)\coth(x}{a}\right.}{3a^2b}$$

$$\frac{\left(\frac{4b^2}{a^2}+3\right)\coth(x)\text{csch}(x)}{3b(a+b\sinh(x))}-\frac{\coth(x)\text{csch}^2(x)}{3a(a+b\sinh(x))}$$

$$\Big\downarrow \text{1083}$$

$$\frac{i\left(\frac{i\left(\frac{b\left(7a^2+12b^2\right)\coth(x)}{a}+\frac{3i\left(\frac{b^2\left(3a^2+4b^2\right)\int \csc(ix)dx}{a}-\frac{4ib\left(a^2+b^2\right)\left(a^2+4b^2\right)\int \frac{1}{4\left(a^2+b^2\right)-\left(2b-2a\tanh\left(\frac{x}{2}\right)\right)^2}d\left(2b-2a\tanh\left(\frac{x}{2}\right)\right)}{a}\right)}{a}\right)}{a}-\frac{3i\left(a^2+2b\right.}{}\right.}{3a^2b}$$

$$\frac{\left(\frac{4b^2}{a^2}+3\right)\coth(x)\text{csch}(x)}{3b(a+b\sinh(x))}-\frac{\coth(x)\text{csch}^2(x)}{3a(a+b\sinh(x))}$$

$$\Big\downarrow \text{219}$$

$$\frac{i\left(\frac{i\left(\frac{b(7a^2+12b^2)\coth(x)}{a}+\frac{3i\left(\frac{b^2(3a^2+4b^2)\int\csc(ix)dx}{a}-\frac{2ib\sqrt{a^2+b^2}(a^2+4b^2)\text{arctanh}\left(\frac{2b-2a\tanh\left(\frac{x}{2}\right)}{2\sqrt{a^2+b^2}}\right)}{a}\right)}{a}\right)}{a}-\frac{3i(a^2+2b^2)\coth(x)\text{csch}(x)}{a}\right)}{3a^2b}$$

$$\frac{\left(\frac{4b^2}{a^2}+3\right)\coth(x)\text{csch}(x)}{3b(a+b\sinh(x))}-\frac{\coth(x)\text{csch}^2(x)}{3a(a+b\sinh(x))}$$

$$\Big\downarrow \; 4257$$

$$\frac{i\left(\frac{i\left(\frac{b(7a^2+12b^2)\coth(x)}{a}+\frac{3i\left(\frac{ib^2(3a^2+4b^2)\text{arctanh}(\cosh(x))}{a}-\frac{2ib\sqrt{a^2+b^2}(a^2+4b^2)\text{arctanh}\left(\frac{2b-2a\tanh\left(\frac{x}{2}\right)}{2\sqrt{a^2+b^2}}\right)}{a}\right)}{a}\right)}{a}-\frac{3i(a^2+2b^2)\coth(x)\text{csch}(x)}{a}\right)}{3a^2b}$$

$$\frac{\left(\frac{4b^2}{a^2}+3\right)\coth(x)\text{csch}(x)}{3b(a+b\sinh(x))}-\frac{\coth(x)\text{csch}^2(x)}{3a(a+b\sinh(x))}$$

input
```
Int[Coth[x]^4/(a + b*Sinh[x])^2,x]
```

output
```
((I/3)*((I*(((3*I)*((I*b^2*(3*a^2 + 4*b^2)*ArcTanh[Cosh[x]])/a - ((2*I)*b*
Sqrt[a^2 + b^2]*(a^2 + 4*b^2)*ArcTanh[(2*b - 2*a*Tanh[x/2])/(2*Sqrt[a^2 +
b^2])])/a))/a + (b*(7*a^2 + 12*b^2)*Coth[x])/a))/a - ((3*I)*(a^2 + 2*b^2)*
Coth[x]*Csch[x])/a))/(a^2*b) - ((3 + (4*b^2)/a^2)*Coth[x]*Csch[x])/(3*b*(a
 + b*Sinh[x])) - (Coth[x]*Csch[x]^2)/(3*a*(a + b*Sinh[x]))
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1083
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Simp[-2   Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3139
```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Simp[2*(e/d)   Subst[Int[1/(a + 2*b*e*x + a
*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[a^2 - b^2, 0]
```

rule 3203
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)/tan[(e_.) + (f_.)*(x_)]^4,
x_Symbol] :> Simp[(-Cos[e + f*x])*((a + b*Sin[e + f*x])^(m + 1)/(3*a*f*Sin[
e + f*x]^3)), x] + (-Simp[(3*a^2 + b^2*(m - 2))*Cos[e + f*x]*((a + b*Sin[e
+ f*x])^(m + 1)/(3*a^2*b*f*(m + 1)*Sin[e + f*x]^2)), x] - Simp[1/(3*a^2*b*(
m + 1))   Int[((a + b*Sin[e + f*x])^(m + 1)/Sin[e + f*x]^3)*Simp[6*a^2 - b^
2*(m - 1)*(m - 2) + a*b*(m + 1)*Sin[e + f*x] - (3*a^2 - b^2*m*(m - 2))*Sin[
e + f*x]^2, x], x], x]) /; FreeQ[{a, b, e, f}, x] && NeQ[a^2 - b^2, 0] && L
tQ[m, -1] && IntegerQ[2*m]
```

rule 3480
```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((a_.) + (b_.)*sin[(e_.) + (f_
.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])), x_Symbol] :> Simp[(A*b
- a*B)/(b*c - a*d)   Int[1/(a + b*Sin[e + f*x]), x], x] + Simp[(B*c - A*d)/
(b*c - a*d)   Int[1/(c + d*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f
, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

rule 3534
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-(A*b^2 - a*b*B + a^2*C))*Cos[e + f*x
]*(a + b*Sin[e + f*x])^(m + 1)*((c + d*Sin[e + f*x])^(n + 1)/(f*(m + 1)*(b*
c - a*d)*(a^2 - b^2))), x] + Simp[1/((m + 1)*(b*c - a*d)*(a^2 - b^2))   Int
[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[(m + 1)*(b*c - a*
d)*(a*A - b*B + a*C) + d*(A*b^2 - a*b*B + a^2*C)*(m + n + 2) - (c*(A*b^2 -
a*b*B + a^2*C) + (m + 1)*(b*c - a*d)*(A*b - a*B + b*C))*Sin[e + f*x] - d*(A
*b^2 - a*b*B + a^2*C)*(m + n + 3)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b
, c, d, e, f, A, B, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] &&
NeQ[c^2 - d^2, 0] && LtQ[m, -1] && ((EqQ[a, 0] && IntegerQ[m] &&  !IntegerQ
[n]) ||  !(IntegerQ[2*n] && LtQ[n, -1] && ((IntegerQ[n] &&  !IntegerQ[m]) |
| EqQ[a, 0])))
```

rule 4257
```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Maple [A] (verified)

Time = 8.51 (sec) , antiderivative size = 221, normalized size of antiderivative = 1.39

method	result
default	$-\frac{\frac{\tanh\left(\frac{x}{2}\right)^{3}a^{2}}{3}+2\tanh\left(\frac{x}{2}\right)^{2}ab+5\tanh\left(\frac{x}{2}\right)a^{2}+12b^{2}\tanh\left(\frac{x}{2}\right)}{8a^{4}}-\frac{1}{24a^{2}\tanh\left(\frac{x}{2}\right)^{3}}-\frac{5a^{2}+12b^{2}}{8a^{4}\tanh\left(\frac{x}{2}\right)}+\frac{b}{4a^{3}\tanh\left(\frac{x}{2}\right)^{2}}-\frac{b(3a^{2}+4b`
risch	$\frac{2a^{3}e^{7x}+4a\,b^{2}e^{7x}-2a^{2}b\,e^{6x}-8b^{3}e^{6x}-14a^{3}e^{5x}-20a\,b^{2}e^{5x}+14a^{2}b\,e^{4x}+24\,e^{4x}b^{3}+14a^{3}e^{3x}+28a\,b^{2}e^{3x}-\frac{50a^{2}b\,e^{2x}}{3}-24b^{3}e^{2x}-\frac{22a^{3}e^{x}}{3}}{(e^{2x}-1)^{3}a^{4}(b\,e^{2x}+2\,e^{x}a-b)}$

input
```
int(coth(x)^4/(a+b*sinh(x))^2,x,method=_RETURNVERBOSE)
```

output
```
-1/8/a^4*(1/3*tanh(1/2*x)^3*a^2+2*tanh(1/2*x)^2*a*b+5*tanh(1/2*x)*a^2+12*b
^2*tanh(1/2*x))-1/24/a^2/tanh(1/2*x)^3-1/8*(5*a^2+12*b^2)/a^4/tanh(1/2*x)+
1/4/a^3*b/tanh(1/2*x)^2-1/a^5*b*(3*a^2+4*b^2)*ln(tanh(1/2*x))-2/a^5*((-b^2
*(a^2+b^2)*tanh(1/2*x)-(a^2+b^2)*a*b)/(tanh(1/2*x)^2*a-2*b*tanh(1/2*x)-a)-
(a^4+5*a^2*b^2+4*b^4)/(a^2+b^2)^(1/2)*arctanh(1/2*(2*a*tanh(1/2*x)-2*b)/(a
^2+b^2)^(1/2)))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 3648 vs. 2(149) = 298.

Time = 0.16 (sec) , antiderivative size = 3648, normalized size of antiderivative = 22.94

$$\int \frac{\coth^4(x)}{(a+b\sinh(x))^2}\,dx = \text{Too large to display}$$

input
```
integrate(coth(x)^4/(a+b*sinh(x))^2,x, algorithm="fricas")
```

output
```
Too large to include
```

Sympy [F]

$$\int \frac{\coth^4(x)}{(a+b\sinh(x))^2}\,dx=\int \frac{\coth^4{(x)}}{(a+b\sinh{(x)})^2}\,dx$$

input
```
integrate(coth(x)**4/(a+b*sinh(x))**2,x)
```

output
```
Integral(coth(x)**4/(a + b*sinh(x))**2, x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 339 vs. 2(149) = 298.

Time = 0.13 (sec) , antiderivative size = 339, normalized size of antiderivative = 2.13

$$\int \frac{\coth^4(x)}{(a+b\sinh(x))^2}\,dx = -\frac{2\left(7\,a^2b+12\,b^3+(11\,a^3+18\,ab^2)e^{(-x)}-(25\,a^2b+36\,b^3)e^{(-2\,x)}-21\,(a^3+2\,ab^2)e^{(-3\,x)}+3\,(7\,a^2b+12\right.}{3\left(2\,a^5e^{(-x)}-4\,a^4be^{(-2\,x)}-6\,a^5e^{(-3\,x)}+6\,a^4be^{(-4\,x)}+6\,a^5e^{(-5\,x)}-\right.} + \frac{(3\,a^2b+4\,b^3)\log\left(e^{(-x)}+1\right)}{a^5} - \frac{(3\,a^2b+4\,b^3)\log\left(e^{(-x)}-1\right)}{a^5} + \frac{(a^4+5\,a^2b^2+4\,b^4)\log\left(\frac{be^{(-x)}-a-\sqrt{a^2+b^2}}{be^{(-x)}-a+\sqrt{a^2+b^2}}\right)}{\sqrt{a^2+b^2}a^5}$$

input
```
integrate(coth(x)^4/(a+b*sinh(x))^2,x, algorithm="maxima")
```

output
```
-2/3*(7*a^2*b + 12*b^3 + (11*a^3 + 18*a*b^2)*e^(-x) - (25*a^2*b + 36*b^3)*
e^(-2*x) - 21*(a^3 + 2*a*b^2)*e^(-3*x) + 3*(7*a^2*b + 12*b^3)*e^(-4*x) + 3
*(7*a^3 + 10*a*b^2)*e^(-5*x) - 3*(a^2*b + 4*b^3)*e^(-6*x) - 3*(a^3 + 2*a*b
^2)*e^(-7*x))/(2*a^5*e^(-x) - 4*a^4*b*e^(-2*x) - 6*a^5*e^(-3*x) + 6*a^4*b*
e^(-4*x) + 6*a^5*e^(-5*x) - 4*a^4*b*e^(-6*x) - 2*a^5*e^(-7*x) + a^4*b*e^(-
8*x) + a^4*b) + (3*a^2*b + 4*b^3)*log(e^(-x) + 1)/a^5 - (3*a^2*b + 4*b^3)*
log(e^(-x) - 1)/a^5 + (a^4 + 5*a^2*b^2 + 4*b^4)*log((b*e^(-x) - a - sqrt(a
^2 + b^2))/(b*e^(-x) - a + sqrt(a^2 + b^2)))/(sqrt(a^2 + b^2)*a^5)
```

Giac [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 242, normalized size of antiderivative = 1.52

$$\int \frac{\coth^4(x)}{(a+b\sinh(x))^2}\,dx = \frac{(3\,a^2b+4\,b^3)\log(e^x+1)}{a^5} - \frac{(3\,a^2b+4\,b^3)\log(|e^x-1|)}{a^5} + \frac{(a^4+5\,a^2b^2+4\,b^4)\log\left(\frac{\left|2\,be^x+2\,a-2\,\sqrt{a^2+b^2}\right|}{\left|2\,be^x+2\,a+2\,\sqrt{a^2+b^2}\right|}\right)}{\sqrt{a^2+b^2}a^5} + \frac{2\,(a^3e^x+ab^2e^x-a^2b-b^3)}{(be^{(2\,x)}+2\,ae^x-b)a^4} + \frac{2\left(3\,abe^{(5\,x)}-6\,a^2e^{(4\,x)}-9\,b^2e^{(4\,x)}+6\,a^2e^{(2\,x)}+18\,b^2e^{(2\,x)}-3\,abe^x-4\,a^2-9\,b^2\right)}{3\,a^4(e^{(2\,x)}-1)^3}$$

input
```
integrate(coth(x)^4/(a+b*sinh(x))^2,x, algorithm="giac")
```

output
```
(3*a^2*b + 4*b^3)*log(e^x + 1)/a^5 - (3*a^2*b + 4*b^3)*log(abs(e^x - 1))/a
^5 + (a^4 + 5*a^2*b^2 + 4*b^4)*log(abs(2*b*e^x + 2*a - 2*sqrt(a^2 + b^2))/
abs(2*b*e^x + 2*a + 2*sqrt(a^2 + b^2)))/(sqrt(a^2 + b^2)*a^5) + 2*(a^3*e^x
 + a*b^2*e^x - a^2*b - b^3)/((b*e^(2*x) + 2*a*e^x - b)*a^4) + 2/3*(3*a*b*e
^(5*x) - 6*a^2*e^(4*x) - 9*b^2*e^(4*x) + 6*a^2*e^(2*x) + 18*b^2*e^(2*x) -
3*a*b*e^x - 4*a^2 - 9*b^2)/(a^4*(e^(2*x) - 1)^3)
```

Mupad [B] (verification not implemented)

Time = 2.34 (sec) , antiderivative size = 1450, normalized size of antiderivative = 9.12

$$\int \frac{\coth^4(x)}{(a+b\sinh(x))^2}\,dx = \text{Too large to display}$$

input
```
int(coth(x)^4/(a + b*sinh(x))^2,x)
```

output
```
(3*b*log(96*a^4 + 128*b^4 + 224*a^2*b^2 + 96*a^4*exp(x) + 128*b^4*exp(x) +
 224*a^2*b^2*exp(x)))/a^3 - 4/(a^2*exp(2*x) - a^2) - (6*b^2)/(a^4*exp(2*x)
 - a^4) - 8/(3*(3*a^2*exp(2*x) - 3*a^2*exp(4*x) + a^2*exp(6*x) - a^2)) - (
4*a^3*b^7)/(a^5*b^7*exp(2*x) - a^7*b^5 - a^5*b^7 + a^7*b^5*exp(2*x) + 2*a^
6*b^6*exp(x) + 2*a^8*b^4*exp(x)) - (2*a^5*b^5)/(a^5*b^7*exp(2*x) - a^7*b^5
 - a^5*b^7 + a^7*b^5*exp(2*x) + 2*a^6*b^6*exp(x) + 2*a^8*b^4*exp(x)) - (3*
b*log(96*a^4 + 128*b^4 + 224*a^2*b^2 - 96*a^4*exp(x) - 128*b^4*exp(x) - 22
4*a^2*b^2*exp(x)))/a^3 - 4/(a^2*exp(4*x) - 2*a^2*exp(2*x) + a^2) - (4*b^3*
log(96*a^4 + 128*b^4 + 224*a^2*b^2 - 96*a^4*exp(x) - 128*b^4*exp(x) - 224*
a^2*b^2*exp(x)))/a^5 + (4*b^3*log(96*a^4 + 128*b^4 + 224*a^2*b^2 + 96*a^4*
exp(x) + 128*b^4*exp(x) + 224*a^2*b^2*exp(x)))/a^5 + (log(128*a^6*exp(x) -
 256*a*b^5 - 64*a^5*b - 320*a^3*b^3 - 128*b^5*(a^2 + b^2)^(1/2) + 128*b^6*
exp(x) - 288*a^2*b^3*(a^2 + b^2)^(1/2) + 128*a^5*exp(x)*(a^2 + b^2)^(1/2)
+ 672*a^2*b^4*exp(x) + 672*a^4*b^2*exp(x) - 64*a^4*b*(a^2 + b^2)^(1/2) + 3
84*a*b^4*exp(x)*(a^2 + b^2)^(1/2) + 608*a^3*b^2*exp(x)*(a^2 + b^2)^(1/2))*
(a^2 + b^2)^(1/2))/a^3 - (log(128*b^5*(a^2 + b^2)^(1/2) - 256*a*b^5 - 64*a
^5*b - 320*a^3*b^3 + 128*a^6*exp(x) + 128*b^6*exp(x) + 288*a^2*b^3*(a^2 +
b^2)^(1/2) - 128*a^5*exp(x)*(a^2 + b^2)^(1/2) + 672*a^2*b^4*exp(x) + 672*a
^4*b^2*exp(x) + 64*a^4*b*(a^2 + b^2)^(1/2) - 384*a*b^4*exp(x)*(a^2 + b^2)^
(1/2) - 608*a^3*b^2*exp(x)*(a^2 + b^2)^(1/2))*(a^2 + b^2)^(1/2))/a^3 - ...
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 1552, normalized size of antiderivative = 9.76

$$\int \frac{\coth^4(x)}{(a+b\sinh(x))^2}\,dx = \text{Too large to display}$$

input
```
int(coth(x)^4/(a+b*sinh(x))^2,x)
```

output

```
(6*e**(8*x)*sqrt(a**2 + b**2)*atan((e**x*b*i + a*i)/sqrt(a**2 + b**2))*a**
2*b*i + 24*e**(8*x)*sqrt(a**2 + b**2)*atan((e**x*b*i + a*i)/sqrt(a**2 + b*
*2))*b**3*i + 12*e**(7*x)*sqrt(a**2 + b**2)*atan((e**x*b*i + a*i)/sqrt(a**
2 + b**2))*a**3*i + 48*e**(7*x)*sqrt(a**2 + b**2)*atan((e**x*b*i + a*i)/sq
rt(a**2 + b**2))*a*b**2*i - 24*e**(6*x)*sqrt(a**2 + b**2)*atan((e**x*b*i +
 a*i)/sqrt(a**2 + b**2))*a**2*b*i - 96*e**(6*x)*sqrt(a**2 + b**2)*atan((e*
*x*b*i + a*i)/sqrt(a**2 + b**2))*b**3*i - 36*e**(5*x)*sqrt(a**2 + b**2)*at
an((e**x*b*i + a*i)/sqrt(a**2 + b**2))*a**3*i - 144*e**(5*x)*sqrt(a**2 + b
**2)*atan((e**x*b*i + a*i)/sqrt(a**2 + b**2))*a*b**2*i + 36*e**(4*x)*sqrt(
a**2 + b**2)*atan((e**x*b*i + a*i)/sqrt(a**2 + b**2))*a**2*b*i + 144*e**(4
*x)*sqrt(a**2 + b**2)*atan((e**x*b*i + a*i)/sqrt(a**2 + b**2))*b**3*i + 36
*e**(3*x)*sqrt(a**2 + b**2)*atan((e**x*b*i + a*i)/sqrt(a**2 + b**2))*a**3*
i + 144*e**(3*x)*sqrt(a**2 + b**2)*atan((e**x*b*i + a*i)/sqrt(a**2 + b**2)
)*a*b**2*i - 24*e**(2*x)*sqrt(a**2 + b**2)*atan((e**x*b*i + a*i)/sqrt(a**2
 + b**2))*a**2*b*i - 96*e**(2*x)*sqrt(a**2 + b**2)*atan((e**x*b*i + a*i)/s
qrt(a**2 + b**2))*b**3*i - 12*e**x*sqrt(a**2 + b**2)*atan((e**x*b*i + a*i)
/sqrt(a**2 + b**2))*a**3*i - 48*e**x*sqrt(a**2 + b**2)*atan((e**x*b*i + a*
i)/sqrt(a**2 + b**2))*a*b**2*i + 6*sqrt(a**2 + b**2)*atan((e**x*b*i + a*i)
/sqrt(a**2 + b**2))*a**2*b*i + 24*sqrt(a**2 + b**2)*atan((e**x*b*i + a*i)/
sqrt(a**2 + b**2))*b**3*i - 9*e**(8*x)*log(e**x - 1)*a**2*b**2 - 12*e**...
```

3.244 $\int \coth(x)\sqrt{a+b\sinh(x)}\,dx$

Optimal result

Integrand size = 13, antiderivative size = 37

$$\int \coth(x)\sqrt{a+b\sinh(x)}\,dx = -2\sqrt{a}\operatorname{arctanh}\left(\frac{\sqrt{a+b\sinh(x)}}{\sqrt{a}}\right)+2\sqrt{a+b\sinh(x)}$$

output
```
-2*a^(1/2)*arctanh((a+b*sinh(x))^(1/2)/a^(1/2))+2*(a+b*sinh(x))^(1/2)
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.00

$$\int \coth(x)\sqrt{a+b\sinh(x)}\,dx = -2\sqrt{a}\operatorname{arctanh}\left(\frac{\sqrt{a+b\sinh(x)}}{\sqrt{a}}\right)+2\sqrt{a+b\sinh(x)}$$

input
```
Integrate[Coth[x]*Sqrt[a + b*Sinh[x]],x]
```

output
```
-2*Sqrt[a]*ArcTanh[Sqrt[a + b*Sinh[x]]/Sqrt[a]] + 2*Sqrt[a + b*Sinh[x]]
```

Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.462$, Rules used = {3042, 26, 3200, 60, 73, 220}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \coth(x)\sqrt{a+b\sinh(x)}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{i\sqrt{a-ib\sin(ix)}}{\tan(ix)}dx$$

$$\Big\downarrow \text{26}$$

$$i\int \frac{\sqrt{a-ib\sin(ix)}}{\tan(ix)}dx$$

$$\Big\downarrow \text{3200}$$

$$\int \frac{\text{csch}(x)\sqrt{a+b\sinh(x)}}{b}d(b\sinh(x))$$

$$\Big\downarrow \text{60}$$

$$a\int \frac{\text{csch}(x)}{b\sqrt{a+b\sinh(x)}}d(b\sinh(x))+2\sqrt{a+b\sinh(x)}$$

$$\Big\downarrow \text{73}$$

$$2a\int \frac{1}{b^2\sinh^2(x)-a}d\sqrt{a+b\sinh(x)}+2\sqrt{a+b\sinh(x)}$$

$$\Big\downarrow \text{220}$$

$$2\sqrt{a+b\sinh(x)}-2\sqrt{a}\text{arctanh}\left(\frac{\sqrt{a+b\sinh(x)}}{\sqrt{a}}\right)$$

input
```
Int[Coth[x]*Sqrt[a + b*Sinh[x]],x]
```

output
```
-2*Sqrt[a]*ArcTanh[Sqrt[a + b*Sinh[x]]/Sqrt[a]] + 2*Sqrt[a + b*Sinh[x]]
```

Defintions of rubi rules used

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 60
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Simp[n*((b*c - a*d)/(
b*(m + n + 1)))   Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
 c, d}, x] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ[m, 0] && ( !Integer
Q[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n + 2, 0] && IntLinear
Q[a, b, c, d, m, n, x]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 220
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[b, 2])^(-
1))*ArcTanh[Rt[b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] &&
 (LtQ[a, 0] || GtQ[b, 0])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3200
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*tan[(e_.) + (f_.)*(x_)]^(p
_.), x_Symbol] :> Simp[1/f   Subst[Int[(x^p*(a + x)^m)/(b^2 - x^2)^((p + 1)
/2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && NeQ[a^2 - b
^2, 0] && IntegerQ[(p + 1)/2]
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.29 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.57

method	result	size
default	$`\mathrm{int/indef0}`\left(\frac{\frac{a}{\sinh(x)}+b}{\sqrt{a+b\sinh(x)}}, \sinh\left(x\right)\right)$	21

input
```
int(coth(x)*(a+b*sinh(x))^(1/2),x,method=_RETURNVERBOSE)
```

output
```
`int/indef0`((1/sinh(x)*a+b)/(a+b*sinh(x))^(1/2),sinh(x))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 107 vs. 2(29) = 58.

Time = 0.31 (sec) , antiderivative size = 390, normalized size of antiderivative = 10.54

$$\int \coth(x)\sqrt{a+b\sinh(x)}\,dx$$
$$= \left[\frac{1}{2}\sqrt{a}\log\left(-\frac{b^2\cosh\left(x\right)^4+b^2\sinh\left(x\right)^4+16\,ab\cosh\left(x\right)^3+4\left(b^2\cosh\left(x\right)+4\,ab\right)\sinh\left(x\right)^3-16\,ab\cosh(\right.\right.\right.$$
$$+2\sqrt{b\sinh\left(x\right)+a},\ \sqrt{-a}\arctan\left(\frac{\left(b\cosh\left(x\right)^2+b\sinh\left(x\right)^2+4\,a\cosh\left(x\right)+2\left(b\cosh\left(x\right)+2\,a\right)\sinh\left(x\right)\right.}{2\left(ab\cosh\left(x\right)^2+ab\sinh\left(x\right)^2+2\,a^2\cosh\left(x\right)-ab+2\left(ab\cosh\right.\right.}\right.$$
$$\left.+2\sqrt{b\sinh\left(x\right)+a}\right]$$

input
```
integrate(coth(x)*(a+b*sinh(x))^(1/2),x, algorithm="fricas")
```

output
```
[1/2*sqrt(a)*log(-(b^2*cosh(x)^4 + b^2*sinh(x)^4 + 16*a*b*cosh(x)^3 + 4*(b
^2*cosh(x) + 4*a*b)*sinh(x)^3 - 16*a*b*cosh(x) + 2*(16*a^2 - b^2)*cosh(x)^
2 + 2*(3*b^2*cosh(x)^2 + 24*a*b*cosh(x) + 16*a^2 - b^2)*sinh(x)^2 - 8*(b*c
osh(x)^3 + b*sinh(x)^3 + 4*a*cosh(x)^2 + (3*b*cosh(x) + 4*a)*sinh(x)^2 - b
*cosh(x) + (3*b*cosh(x)^2 + 8*a*cosh(x) - b)*sinh(x))*sqrt(b*sinh(x) + a)*
sqrt(a) + b^2 + 4*(b^2*cosh(x)^3 + 12*a*b*cosh(x)^2 - 4*a*b + (16*a^2 - b^
2)*cosh(x))*sinh(x))/(cosh(x)^4 + 4*cosh(x)*sinh(x)^3 + sinh(x)^4 + 2*(3*c
osh(x)^2 - 1)*sinh(x)^2 - 2*cosh(x)^2 + 4*(cosh(x)^3 - cosh(x))*sinh(x) +
1)) + 2*sqrt(b*sinh(x) + a), sqrt(-a)*arctan(1/2*(b*cosh(x)^2 + b*sinh(x)^
2 + 4*a*cosh(x) + 2*(b*cosh(x) + 2*a)*sinh(x) - b)*sqrt(b*sinh(x) + a)*sqr
t(-a)/(a*b*cosh(x)^2 + a*b*sinh(x)^2 + 2*a^2*cosh(x) - a*b + 2*(a*b*cosh(x
) + a^2)*sinh(x))) + 2*sqrt(b*sinh(x) + a)]
```

Sympy [F]

$$\int \coth(x)\sqrt{a+b\sinh(x)}\,dx = \int \sqrt{a+b\sinh(x)}\coth(x)\,dx$$

input
```
integrate(coth(x)*(a+b*sinh(x))**(1/2),x)
```

output
```
Integral(sqrt(a + b*sinh(x))*coth(x), x)
```

Maxima [F]

$$\int \coth(x)\sqrt{a+b\sinh(x)}\,dx = \int \sqrt{b\sinh(x)+a}\coth(x)\,dx$$

input
```
integrate(coth(x)*(a+b*sinh(x))^(1/2),x, algorithm="maxima")
```

output
```
integrate(sqrt(b*sinh(x) + a)*coth(x), x)
```

Giac [F]

$$\int \coth(x)\sqrt{a+b\sinh(x)}\,dx = \int \sqrt{b\sinh(x)+a}\coth(x)\,dx$$

input
```
integrate(coth(x)*(a+b*sinh(x))^(1/2),x, algorithm="giac")
```

output
```
integrate(sqrt(b*sinh(x) + a)*coth(x), x)
```

Mupad [F(-1)]

Timed out.

$$\int \coth(x)\sqrt{a+b\sinh(x)}\,dx = \int \coth(x)\,\sqrt{a+b\sinh(x)}\,dx$$

input
```
int(coth(x)*(a + b*sinh(x))^(1/2),x)
```

output
```
int(coth(x)*(a + b*sinh(x))^(1/2), x)
```

Reduce [F]

$$\int \coth(x)\sqrt{a+b\sinh(x)}\,dx = \int \sqrt{\sinh(x)\,b+a}\,\coth(x)\,dx$$

input
```
int(coth(x)*(a+b*sinh(x))^(1/2),x)
```

output
```
int(sqrt(sinh(x)*b + a)*coth(x),x)
```

3.245 $\int \frac{\coth(x)}{\sqrt{a+b\sinh(x)}}\,dx$

Optimal result

Integrand size = 13, antiderivative size = 24

$$\int \frac{\coth(x)}{\sqrt{a+b\sinh(x)}}\,dx = -\frac{2\text{arctanh}\left(\frac{\sqrt{a+b\sinh(x)}}{\sqrt{a}}\right)}{\sqrt{a}}$$

output
```
-2*arctanh((a+b*sinh(x))^(1/2)/a^(1/2))/a^(1/2)
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00

$$\int \frac{\coth(x)}{\sqrt{a+b\sinh(x)}}\,dx = -\frac{2\text{arctanh}\left(\frac{\sqrt{a+b\sinh(x)}}{\sqrt{a}}\right)}{\sqrt{a}}$$

input
```
Integrate[Coth[x]/Sqrt[a + b*Sinh[x]],x]
```

output
```
(-2*ArcTanh[Sqrt[a + b*Sinh[x]]/Sqrt[a]])/Sqrt[a]
```

Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.385, Rules used = {3042, 26, 3200, 73, 220}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\coth(x)}{\sqrt{a+b\sinh(x)}}dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \frac{i}{\tan(ix)\sqrt{a-ib\sin(ix)}}dx$$

$$\Big\downarrow \text{ 26}$$

$$i\int \frac{1}{\sqrt{a-ib\sin(ix)}\tan(ix)}dx$$

$$\Big\downarrow \text{ 3200}$$

$$\int \frac{\text{csch}(x)}{b\sqrt{a+b\sinh(x)}}d(b\sinh(x))$$

$$\Big\downarrow \text{ 73}$$

$$2\int \frac{1}{b^2\sinh^2(x)-a}d\sqrt{a+b\sinh(x)}$$

$$\Big\downarrow \text{ 220}$$

$$-\frac{2\text{arctanh}\left(\frac{\sqrt{a+b\sinh(x)}}{\sqrt{a}}\right)}{\sqrt{a}}$$

input
```
Int[Coth[x]/Sqrt[a + b*Sinh[x]],x]
```

output
```
(-2*ArcTanh[Sqrt[a + b*Sinh[x]]/Sqrt[a]])/Sqrt[a]
```

Defintions of rubi rules used

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 220
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[b, 2])^(-
1))*ArcTanh[Rt[b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] &&
 (LtQ[a, 0] || GtQ[b, 0])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3200
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*tan[(e_.) + (f_.)*(x_)]^(p
_.), x_Symbol] :> Simp[1/f   Subst[Int[(x^p*(a + x)^m)/(b^2 - x^2)^((p + 1)
/2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && NeQ[a^2 - b
^2, 0] && IntegerQ[(p + 1)/2]
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.20 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.71

method	result	size
default	‘int/indef0‘ $\left(\frac{1}{\sinh(x)\sqrt{a+b\sinh(x)}}, \sinh(x)\right)$	17

input
```
int(coth(x)/(a+b*sinh(x))^(1/2),x,method=_RETURNVERBOSE)
```

output
```
`int/indef0`(1/sinh(x)/(a+b*sinh(x))^(1/2),sinh(x))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 99 vs. $2(18) = 36$.

Time = 0.12 (sec) , antiderivative size = 370, normalized size of antiderivative = 15.42

$$\int \frac{\coth(x)}{\sqrt{a+b\sinh(x)}}\,dx = \left[\frac{\log\left(\frac{b^2\cosh(x)^4+b^2\sinh(x)^4+16\,ab\cosh(x)^3+4\,(b^2\cosh(x)+4\,ab)\sinh(x)^3-16\,ab\cosh(x)+2\,(16\,a^2-b^2)\cosh(x)^2+2\,\left(3\,b^2\cosh(x)^2+24\,a\right.}{\cosh}\right.}{}\right.$$

input
```
integrate(coth(x)/(a+b*sinh(x))^(1/2),x, algorithm="fricas")
```

output
```
[1/2*log((b^2*cosh(x)^4 + b^2*sinh(x)^4 + 16*a*b*cosh(x)^3 + 4*(b^2*cosh(x
) + 4*a*b)*sinh(x)^3 - 16*a*b*cosh(x) + 2*(16*a^2 - b^2)*cosh(x)^2 + 2*(3*
b^2*cosh(x)^2 + 24*a*b*cosh(x) + 16*a^2 - b^2)*sinh(x)^2 - 8*(b*cosh(x)^3
+ b*sinh(x)^3 + 4*a*cosh(x)^2 + (3*b*cosh(x) + 4*a)*sinh(x)^2 - b*cosh(x)
+ (3*b*cosh(x)^2 + 8*a*cosh(x) - b)*sinh(x))*sqrt(b*sinh(x) + a)*sqrt(a) +
 b^2 + 4*(b^2*cosh(x)^3 + 12*a*b*cosh(x)^2 - 4*a*b + (16*a^2 - b^2)*cosh(x
))*sinh(x))/(cosh(x)^4 + 4*cosh(x)*sinh(x)^3 + sinh(x)^4 + 2*(3*cosh(x)^2
- 1)*sinh(x)^2 - 2*cosh(x)^2 + 4*(cosh(x)^3 - cosh(x))*sinh(x) + 1))/sqrt(
a), sqrt(-a)*arctan(1/2*(b*cosh(x)^2 + b*sinh(x)^2 + 4*a*cosh(x) + 2*(b*co
sh(x) + 2*a)*sinh(x) - b)*sqrt(b*sinh(x) + a)*sqrt(-a)/(a*b*cosh(x)^2 + a*
b*sinh(x)^2 + 2*a^2*cosh(x) - a*b + 2*(a*b*cosh(x) + a^2)*sinh(x)))/a]
```

Sympy [F]

$$\int \frac{\coth(x)}{\sqrt{a + b\sinh(x)}}\,dx = \int \frac{\coth(x)}{\sqrt{a + b\sinh(x)}}\,dx$$

input
```
integrate(coth(x)/(a+b*sinh(x))**(1/2),x)
```

output
```
Integral(coth(x)/sqrt(a + b*sinh(x)), x)
```

Maxima [F]

$$\int \frac{\coth(x)}{\sqrt{a + b\sinh(x)}}\,dx = \int \frac{\coth(x)}{\sqrt{b\sinh(x) + a}}\,dx$$

input
```
integrate(coth(x)/(a+b*sinh(x))^(1/2),x, algorithm="maxima")
```

output
```
integrate(coth(x)/sqrt(b*sinh(x) + a), x)
```

Giac [F]

$$\int \frac{\coth(x)}{\sqrt{a + b\sinh(x)}}\,dx = \int \frac{\coth(x)}{\sqrt{b\sinh(x) + a}}\,dx$$

input
```
integrate(coth(x)/(a+b*sinh(x))^(1/2),x, algorithm="giac")
```

output
```
integrate(coth(x)/sqrt(b*sinh(x) + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\coth(x)}{\sqrt{a+b\sinh(x)}}\,dx = \int \frac{\coth(x)}{\sqrt{a+b\sinh\left(x\right)}}\,dx$$

input
```
int(coth(x)/(a + b*sinh(x))^(1/2),x)
```

output
```
int(coth(x)/(a + b*sinh(x))^(1/2), x)
```

Reduce [F]

$$\int \frac{\coth(x)}{\sqrt{a+b\sinh(x)}}\,dx = \int \frac{\sqrt{\sinh\left(x\right)b+a}\,\coth\left(x\right)}{\sinh\left(x\right)b+a}dx$$

input
```
int(coth(x)/(a+b*sinh(x))^(1/2),x)
```

output
```
int((sqrt(sinh(x)*b + a)*coth(x))/(sinh(x)*b + a),x)
```

3.246 $\int \frac{A+B\cosh(x)}{a+b\sinh(x)}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 51

$$\int \frac{A+B\cosh(x)}{a+b\sinh(x)}\,dx=-\frac{2A\text{arctanh}\left(\frac{b-a\tanh\left(\frac{x}{2}\right)}{\sqrt{a^2+b^2}}\right)}{\sqrt{a^2+b^2}}+\frac{B\log(a+b\sinh(x))}{b}$$

output
```
-2*A*arctanh((b-a*tanh(1/2*x))/(a^2+b^2)^(1/2))/(a^2+b^2)^(1/2)+B*ln(a+b*s
inh(x))/b
```

Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 59, normalized size of antiderivative = 1.16

$$\int \frac{A+B\cosh(x)}{a+b\sinh(x)}\,dx=\frac{2A\arctan\left(\frac{b-a\tanh\left(\frac{x}{2}\right)}{\sqrt{-a^2-b^2}}\right)}{\sqrt{-a^2-b^2}}+\frac{B\log(a+b\sinh(x))}{b}$$

input
```
Integrate[(A + B*Cosh[x])/(a + b*Sinh[x]),x]
```

output
```
(2*A*ArcTan[(b - a*Tanh[x/2])/Sqrt[-a^2 - b^2]])/Sqrt[-a^2 - b^2] + (B*Log
[a + b*Sinh[x]])/b
```

Rubi [A] (verified)

Time = 0.36 (sec) , antiderivative size = 51, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {3042, 4901, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+B\cosh(x)}{a+b\sinh(x)}\,dx$$

$$\Big\downarrow \; 3042$$

$$\int \frac{A+B\cos(ix)}{a-ib\sin(ix)}dx$$

$$\Big\downarrow \; 4901$$

$$\int \left(\frac{A}{a+b\sinh(x)}+\frac{B\cosh(x)}{a+b\sinh(x)}\right)dx$$

$$\Big\downarrow \; 2009$$

$$\frac{B\log(a+b\sinh(x))}{b}-\frac{2A\text{arctanh}\left(\frac{b-a\tanh\left(\frac{x}{2}\right)}{\sqrt{a^2+b^2}}\right)}{\sqrt{a^2+b^2}}$$

input
```
Int[(A + B*Cosh[x])/(a + b*Sinh[x]),x]
```

output
```
(-2*A*ArcTanh[(b - a*Tanh[x/2])/Sqrt[a^2 + b^2]])/Sqrt[a^2 + b^2] + (B*Log
[a + b*Sinh[x]])/b
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4901
```
Int[u_, x_Symbol] :> With[{v = ExpandTrig[u, x]}, Int[v, x] /; SumQ[v]] /;
 !InertTrigFreeQ[u]
```

Maple [A] (verified)

Time = 0.64 (sec) , antiderivative size = 49, normalized size of antiderivative = 0.96

method	result
parts	$\frac{2A \operatorname{arctanh}\left(\frac{2a\tanh\left(\frac{x}{2}\right)-2b}{2\sqrt{a^2+b^2}}\right)}{\sqrt{a^2+b^2}}+\frac{B\ln(a+b\sinh(x))}{b}$
default	$-\frac{B\ln\left(\tanh\left(\frac{x}{2}\right)+1\right)}{b}+\frac{B\ln\left(\tanh\left(\frac{x}{2}\right)^2 a-2b\tanh\left(\frac{x}{2}\right)-a\right)+\frac{2Ab \operatorname{arctanh}\left(\frac{2a\tanh\left(\frac{x}{2}\right)-2b}{2\sqrt{a^2+b^2}}\right)}{\sqrt{a^2+b^2}}}{b}-\frac{B\ln\left(\tanh\left(\frac{x}{2}\right)-1\right)}{b}$
risch	$\frac{Bx}{b}-\frac{2xB\,a^2b}{a^2b^2+b^4}-\frac{2xB\,b^3}{a^2b^2+b^4}+\frac{\ln\left(\mathrm{e}^x+\frac{Aab-\sqrt{A^2a^2b^2+A^2b^4}}{A\,b^2}\right)B\,a^2}{(a^2+b^2)b}+\frac{b\ln\left(\mathrm{e}^x+\frac{Aab-\sqrt{A^2a^2b^2+A^2b^4}}{A\,b^2}\right)B}{a^2+b^2}+\ln\left(\mathrm{e}^x+\frac{Aab-\sqrt{A}}{}\right.$

input
```
int((A+B*cosh(x))/(a+b*sinh(x)),x,method=_RETURNVERBOSE)
```

output
```
2*A/(a^2+b^2)^(1/2)*arctanh(1/2*(2*a*tanh(1/2*x)-2*b)/(a^2+b^2)^(1/2))+B*l
n(a+b*sinh(x))/b
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 170 vs. $2(47) = 94$.

Time = 0.09 (sec) , antiderivative size = 170, normalized size of antiderivative = 3.33

$$\int \frac{A+B\cosh(x)}{a+b\sinh(x)}\,dx = \frac{\sqrt{a^2+b^2}Ab\log\left(\frac{b^2\cosh(x)^2+b^2\sinh(x)^2+2\,ab\cosh(x)+2\,a^2+b^2+2\,(b^2\cosh(x)+ab)\sinh(x)-2\,\sqrt{a^2+b^2}(b\cosh(x)+b\sinh(x)+a)}{b\cosh(x)^2+b\sinh(x)^2+2\,a\cosh(x)+2\,(b\cosh(x)+a)\sinh(x)-b}\right)-(}{a^2b+b^3}$$

input
```
integrate((A+B*cosh(x))/(a+b*sinh(x)),x, algorithm="fricas")
```

output
```
(sqrt(a^2 + b^2)*A*b*log((b^2*cosh(x)^2 + b^2*sinh(x)^2 + 2*a*b*cosh(x) +
2*a^2 + b^2 + 2*(b^2*cosh(x) + a*b)*sinh(x) - 2*sqrt(a^2 + b^2)*(b*cosh(x)
 + b*sinh(x) + a))/(b*cosh(x)^2 + b*sinh(x)^2 + 2*a*cosh(x) + 2*(b*cosh(x)
 + a)*sinh(x) - b)) - (B*a^2 + B*b^2)*x + (B*a^2 + B*b^2)*log(2*(b*sinh(x)
 + a)/(cosh(x) - sinh(x))))/(a^2*b + b^3)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 21.54 (sec) , antiderivative size = 517, normalized size of antiderivative = 10.14

$$\int \frac{A + B\cosh(x)}{a + b\sinh(x)}\,dx = \text{Too large to display}$$

input
```
integrate((A+B*cosh(x))/(a+b*sinh(x)),x)
```

output
```
Piecewise((zoo*(A*log(tanh(x/2)) + B*x - 2*B*log(tanh(x/2) + 1) + B*log(ta
nh(x/2))), Eq(a, 0) & Eq(b, 0)), ((A*log(tanh(x/2)) + B*x - 2*B*log(tanh(x
/2) + 1) + B*log(tanh(x/2)))/b, Eq(a, 0)), ((A*x + B*sinh(x))/a, Eq(b, 0))
, (2*I*A/(b*tanh(x/2) - I*b) + B*x*tanh(x/2)/(b*tanh(x/2) - I*b) - I*B*x/(
b*tanh(x/2) - I*b) - 2*B*log(tanh(x/2) + 1)*tanh(x/2)/(b*tanh(x/2) - I*b)
+ 2*I*B*log(tanh(x/2) + 1)/(b*tanh(x/2) - I*b) + 2*B*log(tanh(x/2) - I)*ta
nh(x/2)/(b*tanh(x/2) - I*b) - 2*I*B*log(tanh(x/2) - I)/(b*tanh(x/2) - I*b)
, Eq(a, -I*b)), (-2*I*A/(b*tanh(x/2) + I*b) + B*x*tanh(x/2)/(b*tanh(x/2) +
 I*b) + I*B*x/(b*tanh(x/2) + I*b) - 2*B*log(tanh(x/2) + 1)*tanh(x/2)/(b*ta
nh(x/2) + I*b) - 2*I*B*log(tanh(x/2) + 1)/(b*tanh(x/2) + I*b) + 2*B*log(ta
nh(x/2) + I)*tanh(x/2)/(b*tanh(x/2) + I*b) + 2*I*B*log(tanh(x/2) + I)/(b*t
anh(x/2) + I*b), Eq(a, I*b)), (-A*log(tanh(x/2) - b/a - sqrt(a**2 + b**2)/
a)/sqrt(a**2 + b**2) + A*log(tanh(x/2) - b/a + sqrt(a**2 + b**2)/a)/sqrt(a
**2 + b**2) + B*x/b - 2*B*log(tanh(x/2) + 1)/b + B*log(tanh(x/2) - b/a - s
qrt(a**2 + b**2)/a)/b + B*log(tanh(x/2) - b/a + sqrt(a**2 + b**2)/a)/b, Tr
ue))
```

Maxima [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 68, normalized size of antiderivative = 1.33

$$\int \frac{A + B\cosh(x)}{a + b\sinh(x)}\,dx = \frac{A\log\left(\frac{be^{(-x)} - a - \sqrt{a^2+b^2}}{be^{(-x)} - a + \sqrt{a^2+b^2}}\right)}{\sqrt{a^2+b^2}} + \frac{B\log\left(b\sinh\left(x\right) + a\right)}{b}$$

input
```
integrate((A+B*cosh(x))/(a+b*sinh(x)),x, algorithm="maxima")
```

output
```
A*log((b*e^(-x) - a - sqrt(a^2 + b^2))/(b*e^(-x) - a + sqrt(a^2 + b^2)))/s
qrt(a^2 + b^2) + B*log(b*sinh(x) + a)/b
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 87, normalized size of antiderivative = 1.71

$$\int \frac{A + B\cosh(x)}{a + b\sinh(x)}\,dx = \frac{A\log\left(\frac{\left|2\,be^x + 2\,a - 2\,\sqrt{a^2+b^2}\right|}{\left|2\,be^x + 2\,a + 2\,\sqrt{a^2+b^2}\right|}\right)}{\sqrt{a^2+b^2}} - \frac{Bx}{b} + \frac{B\log\left(\left|be^{(2\,x)} + 2\,ae^x - b\right|\right)}{b}$$

input
```
integrate((A+B*cosh(x))/(a+b*sinh(x)),x, algorithm="giac")
```

output
```
A*log(abs(2*b*e^x + 2*a - 2*sqrt(a^2 + b^2))/abs(2*b*e^x + 2*a + 2*sqrt(a^
2 + b^2)))/sqrt(a^2 + b^2) - B*x/b + B*log(abs(b*e^(2*x) + 2*a*e^x - b))/b
```

Mupad [B] (verification not implemented)

Time = 3.48 (sec) , antiderivative size = 198, normalized size of antiderivative = 3.88

$$\int \frac{A + B\cosh(x)}{a + b\sinh(x)}\,dx = \frac{B\,b^3\,\ln\left(8\,A^2\,a\,\mathrm{e}^x - 4\,A^2\,b + 4\,A^2\,b\,\mathrm{e}^{2\,x}\right)}{a^2\,b^2 + b^4} - \frac{B\,x}{b} - \frac{2\,\mathrm{atan}\left(\frac{A^2\,b^2\,\mathrm{e}^x\,\sqrt{-a^2-b^2}}{\left(A\,a^2\,b + A\,b^3\right)\sqrt{A^2}} + \frac{A^2\,a\,b\,\sqrt{-a^2-b^2}}{\left(A\,a^2\,b + A\,b^3\right)\sqrt{A^2}}\right)\sqrt{A^2}}{\sqrt{-a^2-b^2}} + \frac{B\,a^2\,b\,\ln\left(8\,A^2\,a\,\mathrm{e}^x - 4\,A^2\,b + 4\,A^2\,b\,\mathrm{e}^{2\,x}\right)}{a^2\,b^2 + b^4}$$

input
```
int((A + B*cosh(x))/(a + b*sinh(x)),x)
```

output
```
(B*b^3*log(8*A^2*a*exp(x) - 4*A^2*b + 4*A^2*b*exp(2*x)))/(b^4 + a^2*b^2) -
 (B*x)/b - (2*atan((A^2*b^2*exp(x)*(- a^2 - b^2)^(1/2))/((A*b^3 + A*a^2*b)
*(A^2)^(1/2)) + (A^2*a*b*(- a^2 - b^2)^(1/2))/((A*b^3 + A*a^2*b)*(A^2)^(1/
2)))*(A^2)^(1/2))/(- a^2 - b^2)^(1/2) + (B*a^2*b*log(8*A^2*a*exp(x) - 4*A^
2*b + 4*A^2*b*exp(2*x)))/(b^4 + a^2*b^2)
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 101, normalized size of antiderivative = 1.98

$$\int \frac{A + B\cosh(x)}{a + b\sinh(x)}\,dx = \frac{2\sqrt{a^2+b^2}\,atan\left(\frac{e^x bi+ai}{\sqrt{a^2+b^2}}\right)ai + \log(e^{2x}b + 2e^x a - b)\,a^2 + \log(e^{2x}b + 2e^x a - b)\,b^2 - a^2x - b^2x}{a^2+b^2}$$

input
```
int((A+B*cosh(x))/(a+b*sinh(x)),x)
```

output
```
(2*sqrt(a**2 + b**2)*atan((e**x*b*i + a*i)/sqrt(a**2 + b**2))*a*i + log(e*
*(2*x)*b + 2*e**x*a - b)*a**2 + log(e**(2*x)*b + 2*e**x*a - b)*b**2 - a**2
*x - b**2*x)/(a**2 + b**2)
```

3.247 $\int \frac{A+B\cosh(x)}{i+\sinh(x)}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 25

$$\int \frac{A+B\cosh(x)}{i+\sinh(x)}\,dx = B\log(i+\sinh(x)) - \frac{A\cosh(x)}{1-i\sinh(x)}$$

output
```
B*ln(I+sinh(x))-A*cosh(x)/(1-I*sinh(x))
```

Mathematica [A] (verified)

Time = 0.12 (sec) , antiderivative size = 48, normalized size of antiderivative = 1.92

$$\int \frac{A+B\cosh(x)}{i+\sinh(x)}\,dx = -2iB\arctan\left(\tanh\left(\frac{x}{2}\right)\right) + B\log(\cosh(x)) - \frac{2iA\sinh\left(\frac{x}{2}\right)}{\cosh\left(\frac{x}{2}\right) - i\sinh\left(\frac{x}{2}\right)}$$

input
```
Integrate[(A + B*Cosh[x])/(I + Sinh[x]),x]
```

output
```
(-2*I)*B*ArcTan[Tanh[x/2]] + B*Log[Cosh[x]] - ((2*I)*A*Sinh[x/2])/(Cosh[x/
2] - I*Sinh[x/2])
```

Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {3042, 4901, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+B\cosh(x)}{\sinh(x)+i}dx$$
$$\Big\downarrow \text{3042}$$
$$\int \frac{A+B\cos(ix)}{i-i\sin(ix)}dx$$
$$\Big\downarrow \text{4901}$$
$$\int \left(\frac{iA}{-1+i\sinh(x)}+\frac{iB\cosh(x)}{-1+i\sinh(x)}\right)dx$$
$$\Big\downarrow \text{2009}$$
$$B\log(\sinh(x)+i)-\frac{A\cosh(x)}{1-i\sinh(x)}$$

input
```
Int[(A + B*Cosh[x])/(I + Sinh[x]),x]
```

output
```
B*Log[I + Sinh[x]] - (A*Cosh[x])/(1 - I*Sinh[x])
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4901
```
Int[u_, x_Symbol] :> With[{v = ExpandTrig[u, x]}, Int[v, x] /; SumQ[v]] /;
 !InertTrigFreeQ[u]
```

Maple [A] (verified)

Time = 0.58 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.92

method	result	size
parts	$-\frac{2iA}{\tanh\left(\frac{x}{2}\right)+i}+B\ln\left(i+\sinh\left(x\right)\right)$	23
risch	$-Bx-\frac{2A}{{\mathrm e}^{x}+i}+2B\ln\left({\mathrm e}^{x}+i\right)$	25
default	$-B\ln\left(\tanh\left(\frac{x}{2}\right)+1\right)+2B\ln\left(\tanh\left(\frac{x}{2}\right)+i\right)-\frac{2iA}{\tanh\left(\frac{x}{2}\right)+i}-B\ln\left(\tanh\left(\frac{x}{2}\right)-1\right)$	46

input
```
int((A+B*cosh(x))/(I+sinh(x)),x,method=_RETURNVERBOSE)
```

output
```
-2*I*A/(tanh(1/2*x)+I)+B*ln(I+sinh(x))
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.44

$$\int \frac{A+B\cosh(x)}{i+\sinh(x)}\,dx=-\frac{Bxe^{x}+i\,Bx-2\,(Be^{x}+i\,B)\log\left(e^{x}+i\right)+2\,A}{e^{x}+i}$$

input
```
integrate((A+B*cosh(x))/(I+sinh(x)),x, algorithm="fricas")
```

output
```
-(B*x*e^x + I*B*x - 2*(B*e^x + I*B)*log(e^x + I) + 2*A)/(e^x + I)
```

Sympy [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.80

$$\int \frac{A+B\cosh(x)}{i+\sinh(x)}\,dx = -\frac{2A}{e^x+i} - Bx + 2B\log\left(e^x+i\right)$$

input
```
integrate((A+B*cosh(x))/(I+sinh(x)),x)
```

output
```
-2*A/(exp(x) + I) - B*x + 2*B*log(exp(x) + I)
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.76

$$\int \frac{A+B\cosh(x)}{i+\sinh(x)}\,dx = B\log\left(\sinh\left(x\right)+i\right) - \frac{2\,A}{e^{(-x)}-i}$$

input
```
integrate((A+B*cosh(x))/(I+sinh(x)),x, algorithm="maxima")
```

output
```
B*log(sinh(x) + I) - 2*A/(e^(-x) - I)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.88

$$\int \frac{A+B\cosh(x)}{i+\sinh(x)}\,dx = -Bx + 2\,B\log\left(e^x+i\right) - \frac{2\,A}{e^x+i}$$

input
```
integrate((A+B*cosh(x))/(I+sinh(x)),x, algorithm="giac")
```

output
```
-B*x + 2*B*log(e^x + I) - 2*A/(e^x + I)
```

Mupad [B] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.96

$$\int \frac{A + B\cosh(x)}{i + \sinh(x)}\,dx = -B\,x - \frac{2\,A}{e^x + 1i} + 2\,B\,\ln\left(e^x + 1i\right)$$

input
```
int((A + B*cosh(x))/(sinh(x) + 1i),x)
```

output
```
2*B*log(exp(x) + 1i) - (2*A)/(exp(x) + 1i) - B*x
```

Reduce [F]

$$\int \frac{A + B\cosh(x)}{i + \sinh(x)}\,dx = \left(\int \frac{1}{\sinh(x) + i}dx\right)a + \log(\sinh(x) + i)\,b$$

input
```
int((A+B*cosh(x))/(I+sinh(x)),x)
```

output
```
int(1/(sinh(x) + i),x)*a + log(sinh(x) + i)*b
```

3.248 $\int \frac{A+B\cosh(x)}{i-\sinh(x)}\,dx$

Optimal result

Integrand size = 17, antiderivative size = 27

$$\int \frac{A+B\cosh(x)}{i-\sinh(x)}\,dx = -B\log(i-\sinh(x)) + \frac{A\cosh(x)}{1+i\sinh(x)}$$

output
```
-B*ln(I-sinh(x))+A*cosh(x)/(1+I*sinh(x))
```

Mathematica [B] (verified)

Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 81 vs. 2(27) = 54.

Time = 0.35 (sec) , antiderivative size = 81, normalized size of antiderivative = 3.00

$$\int \frac{A+B\cosh(x)}{i-\sinh(x)}\,dx = -\frac{\left(\cosh\left(\frac{x}{2}\right)+i\sinh\left(\frac{x}{2}\right)\right)\left(B\cosh\left(\frac{x}{2}\right)\left(2\arctan\left(\tanh\left(\frac{x}{2}\right)\right)-i\log(\cosh(x))\right)+\left(2A+2iB\arctan\left(\tanh\right.\right.\right.}{-i+\sinh(x)}$$

input
```
Integrate[(A + B*Cosh[x])/(I - Sinh[x]),x]
```

output
```
-(((Cosh[x/2] + I*Sinh[x/2])*(B*Cosh[x/2]*(2*ArcTan[Tanh[x/2]] - I*Log[Cos
h[x]]) + (2*A + (2*I)*B*ArcTan[Tanh[x/2]] + B*Log[Cosh[x]])*Sinh[x/2]))/(-
I + Sinh[x]))
```

Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.176$, Rules used = {3042, 4901, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+B\cosh(x)}{-\sinh(x)+i}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{A+B\cos(ix)}{i\sin(ix)+i}dx$$

$$\Big\downarrow \text{4901}$$

$$\int \left(-\frac{iA}{1+i\sinh(x)}-\frac{iB\cosh(x)}{1+i\sinh(x)}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{A\cosh(x)}{1+i\sinh(x)}-B\log(-\sinh(x)+i)$$

input
```
Int[(A + B*Cosh[x])/(I - Sinh[x]),x]
```

output
```
-(B*Log[I - Sinh[x]]) + (A*Cosh[x])/(1 + I*Sinh[x])
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4901
```
Int[u_, x_Symbol] :> With[{v = ExpandTrig[u, x]}, Int[v, x] /; SumQ[v]] /;
 !InertTrigFreeQ[u]
```

Maple [A] (verified)

Time = 0.56 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.89

method	result	size
risch	$Bx+\frac{2A}{e^{x}-i}-2B\ln\left(e^{x}-i\right)$	24
parts	$-\frac{2iA}{\tanh\left(\frac{x}{2}\right)-i}+B\left(-\frac{\ln\left(1+\sinh\left(x\right)^{2}\right)}{2}-i\arctan\left(\sinh\left(x\right)\right)\right)$	33
default	$B\ln\left(\tanh\left(\frac{x}{2}\right)+1\right)-\frac{2iA}{\tanh\left(\frac{x}{2}\right)-i}-2B\ln\left(\tanh\left(\frac{x}{2}\right)-i\right)+B\ln\left(\tanh\left(\frac{x}{2}\right)-1\right)$	44

input
```
int((A+B*cosh(x))/(I-sinh(x)),x,method=_RETURNVERBOSE)
```

output
```
B*x+2*A/(exp(x)-I)-2*B*ln(exp(x)-I)
```

Fricas [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 35, normalized size of antiderivative = 1.30

$$\int\frac{A+B\cosh(x)}{i-\sinh(x)}\,dx=\frac{Bxe^{x}-i\,Bx-2\left(Be^{x}-i\,B\right)\log\left(e^{x}-i\right)+2\,A}{e^{x}-i}$$

input
```
integrate((A+B*cosh(x))/(I-sinh(x)),x, algorithm="fricas")
```

output
```
(B*x*e^x - I*B*x - 2*(B*e^x - I*B)*log(e^x - I) + 2*A)/(e^x - I)
```

Sympy [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.74

$$\int \frac{A + B\cosh(x)}{i - \sinh(x)}\, dx = \frac{2A}{e^x - i} + Bx - 2B\log\left(e^x - i\right)$$

input
```
integrate((A+B*cosh(x))/(I-sinh(x)),x)
```

output
```
2*A/(exp(x) - I) + B*x - 2*B*log(exp(x) - I)
```

Maxima [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.74

$$\int \frac{A + B\cosh(x)}{i - \sinh(x)}\, dx = -B\log\left(\sinh\left(x\right) - i\right) + \frac{2\,A}{e^{(-x)} + i}$$

input
```
integrate((A+B*cosh(x))/(I-sinh(x)),x, algorithm="maxima")
```

output
```
-B*log(sinh(x) - I) + 2*A/(e^(-x) + I)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.78

$$\int \frac{A + B\cosh(x)}{i - \sinh(x)}\, dx = Bx - 2\,B\log\left(e^x - i\right) + \frac{2\,A}{e^x - i}$$

input
```
integrate((A+B*cosh(x))/(I-sinh(x)),x, algorithm="giac")
```

output
```
B*x - 2*B*log(e^x - I) + 2*A/(e^x - I)
```

Mupad [B] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.85

$$\int \frac{A+B\cosh(x)}{i-\sinh(x)}\,dx = B\,x+\frac{2\,A}{\mathrm{e}^x-\mathrm{i}}-2\,B\,\ln\left(\mathrm{e}^x-\mathrm{i}\right)$$

input
```
int(-(A + B*cosh(x))/(sinh(x) - 1i),x)
```

output
```
B*x + (2*A)/(exp(x) - 1i) - 2*B*log(exp(x) - 1i)
```

Reduce [F]

$$\int \frac{A+B\cosh(x)}{i-\sinh(x)}\,dx = -\left(\int \frac{1}{\sinh\left(x\right)-i}dx\right)a-\log(\sinh\left(x\right)-i)\,b$$

input
```
int((A+B*cosh(x))/(I-sinh(x)),x)
```

output
```
- (int(1/(sinh(x) - i),x)*a + log(sinh(x) - i)*b)
```

3.249 $\int \frac{A+B\tanh(x)}{a+b\sinh(x)}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 89

$$\int \frac{A+B\tanh(x)}{a+b\sinh(x)}\,dx = \frac{bB\arctan(\sinh(x))}{a^2+b^2} - \frac{2A\text{arctanh}\left(\frac{b-a\tanh\left(\frac{x}{2}\right)}{\sqrt{a^2+b^2}}\right)}{\sqrt{a^2+b^2}} + \frac{aB\log(\cosh(x))}{a^2+b^2} - \frac{aB\log(a+b\sinh(x))}{a^2+b^2}$$

output
```
b*B*arctan(sinh(x))/(a^2+b^2)-2*A*arctanh((b-a*tanh(1/2*x))/(a^2+b^2)^(1/2
))/(a^2+b^2)^(1/2)+a*B*ln(cosh(x))/(a^2+b^2)-a*B*ln(a+b*sinh(x))/(a^2+b^2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.58 (sec) , antiderivative size = 149, normalized size of antiderivative = 1.67

$$\int \frac{A+B\tanh(x)}{a+b\sinh(x)}\,dx = \frac{\cosh(x)\left(2b\sqrt{-a^2-b^2}B\arctan\left(\tanh\left(\frac{x}{2}\right)\right)+2A(a^2+b^2)\arctan\left(\frac{b-a\tanh\left(\frac{x}{2}\right)}{\sqrt{-a^2-b^2}}\right)+a\sqrt{-a^2-b^2}B(\log(\cosh}{(a-ib)(a+ib)\sqrt{-a^2-b^2}(A\cosh(x)+B\sinh(x))}$$

input
```
Integrate[(A + B*Tanh[x])/(a + b*Sinh[x]),x]
```

output
```
(Cosh[x]*(2*b*Sqrt[-a^2 - b^2]*B*ArcTan[Tanh[x/2]] + 2*A*(a^2 + b^2)*ArcTa
n[(b - a*Tanh[x/2])/Sqrt[-a^2 - b^2]] + a*Sqrt[-a^2 - b^2]*B*(Log[Cosh[x]]
 - Log[a + b*Sinh[x]]))*(A + B*Tanh[x]))/((a - I*b)*(a + I*b)*Sqrt[-a^2 -
b^2]*(A*Cosh[x] + B*Sinh[x]))
```

Rubi [A] (verified)

Time = 0.42 (sec) , antiderivative size = 89, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {3042, 4901, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+B\tanh(x)}{a+b\sinh(x)}dx$$
$$\Big\downarrow \text{3042}$$
$$\int \frac{A-iB\tan(ix)}{a-ib\sin(ix)}dx$$
$$\Big\downarrow \text{4901}$$
$$\int \left(\frac{A}{a+b\sinh(x)}+\frac{B\tanh(x)}{a+b\sinh(x)}\right)dx$$
$$\Big\downarrow \text{2009}$$
$$-\frac{2A\text{arctanh}\left(\frac{b-a\tanh\left(\frac{x}{2}\right)}{\sqrt{a^2+b^2}}\right)}{\sqrt{a^2+b^2}}+\frac{bB\arctan(\sinh(x))}{a^2+b^2}-\frac{aB\log(a+b\sinh(x))}{a^2+b^2}+\frac{aB\log(\cosh(x))}{a^2+b^2}$$

input
```
Int[(A + B*Tanh[x])/(a + b*Sinh[x]),x]
```

output
```
(b*B*ArcTan[Sinh[x]])/(a^2 + b^2) - (2*A*ArcTanh[(b - a*Tanh[x/2])/Sqrt[a^
2 + b^2]])/Sqrt[a^2 + b^2] + (a*B*Log[Cosh[x]])/(a^2 + b^2) - (a*B*Log[a +
 b*Sinh[x]])/(a^2 + b^2)
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4901
```
Int[u_, x_Symbol] :> With[{v = ExpandTrig[u, x]}, Int[v, x] /; SumQ[v]] /;
 !InertTrigFreeQ[u]
```

Maple [A] (verified)

Time = 0.64 (sec) , antiderivative size = 117, normalized size of antiderivative = 1.31

method	result
default	$\frac{-Ba\ln\left(\tanh\left(\frac{x}{2}\right)^2a-2b\tanh\left(\frac{x}{2}\right)-a\right)-\frac{2\left(-a^2A-A\,b^2\right)\operatorname{arctanh}\left(\frac{2a\tanh\left(\frac{x}{2}\right)-2b}{2\sqrt{a^2+b^2}}\right)}{\sqrt{a^2+b^2}}}{a^2+b^2}+\frac{2B\left(\frac{a\ln\left(1+\tanh\left(\frac{x}{2}\right)^2\right)}{2}+b\arctan\left(\tanh\left(\frac{x}{2}\right)\right)\right)}{a^2+b^2}$
risch	$-\frac{2xaB}{a^2+b^2}-\frac{2x\,a^3B}{-a^4-2a^2b^2-b^4}-\frac{2xBa\,b^2}{-a^4-2a^2b^2-b^4}+\frac{iB\ln(\mathrm{e}^x+i)b}{a^2+b^2}+\frac{B\ln(\mathrm{e}^x+i)a}{a^2+b^2}-\frac{iB\ln(\mathrm{e}^x-i)b}{a^2+b^2}+\frac{B\ln(\mathrm{e}^x-i)a}{a^2+b^2}-\frac{\ln\left(\mathrm{e}^x\right.}{}$

input
```
int((A+B*tanh(x))/(a+b*sinh(x)),x,method=_RETURNVERBOSE)
```

output
```
2/(a^2+b^2)*(-1/2*B*a*ln(tanh(1/2*x)^2*a-2*b*tanh(1/2*x)-a)-(-A*a^2-A*b^2)
/(a^2+b^2)^(1/2)*arctanh(1/2*(2*a*tanh(1/2*x)-2*b)/(a^2+b^2)^(1/2)))+2*B/(
a^2+b^2)*(1/2*a*ln(1+tanh(1/2*x)^2)+b*arctan(tanh(1/2*x)))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 172 vs. $2(85) = 170$.

Time = 1.05 (sec) , antiderivative size = 172, normalized size of antiderivative = 1.93

$$\int \frac{A + B\tanh(x)}{a + b\sinh(x)}\,dx$$
$$= \frac{2\,Bb\arctan\left(\cosh\left(x\right) + \sinh\left(x\right)\right) - Ba\log\left(\frac{2\,(b\sinh(x)+a)}{\cosh(x)-\sinh(x)}\right) + Ba\log\left(\frac{2\,\cosh(x)}{\cosh(x)-\sinh(x)}\right) + \sqrt{a^2+b^2}A\log\left(\frac{b\ldots}{\ldots}\right.}{a^2+b^2}$$

input
```
integrate((A+B*tanh(x))/(a+b*sinh(x)),x, algorithm="fricas")
```

output
```
(2*B*b*arctan(cosh(x) + sinh(x)) - B*a*log(2*(b*sinh(x) + a)/(cosh(x) - si
nh(x))) + B*a*log(2*cosh(x)/(cosh(x) - sinh(x))) + sqrt(a^2 + b^2)*A*log((
b^2*cosh(x)^2 + b^2*sinh(x)^2 + 2*a*b*cosh(x) + 2*a^2 + b^2 + 2*(b^2*cosh(
x) + a*b)*sinh(x) - 2*sqrt(a^2 + b^2)*(b*cosh(x) + b*sinh(x) + a))/(b*cosh
(x)^2 + b*sinh(x)^2 + 2*a*cosh(x) + 2*(b*cosh(x) + a)*sinh(x) - b)))/(a^2
+ b^2)
```

Sympy [F]

$$\int \frac{A + B\tanh(x)}{a + b\sinh(x)}\,dx = \int \frac{A + B\tanh\left(x\right)}{a + b\sinh\left(x\right)}\,dx$$

input
```
integrate((A+B*tanh(x))/(a+b*sinh(x)),x)
```

output
```
Integral((A + B*tanh(x))/(a + b*sinh(x)), x)
```

Maxima [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 125, normalized size of antiderivative = 1.40

$$\int \frac{A+B\tanh(x)}{a+b\sinh(x)}\,dx = -B\left(\frac{2\,b\arctan\left(e^{(-x)}\right)}{a^2+b^2}+\frac{a\log\left(-2\,ae^{(-x)}+be^{(-2\,x)}-b\right)}{a^2+b^2}-\frac{a\log\left(e^{(-2\,x)}+1\right)}{a^2+b^2}\right)+\frac{A\log\left(\frac{be^{(-x)}-a-\sqrt{a^2+b^2}}{be^{(-x)}-a+\sqrt{a^2+b^2}}\right)}{\sqrt{a^2+b^2}}$$

input
```
integrate((A+B*tanh(x))/(a+b*sinh(x)),x, algorithm="maxima")
```

output
```
-B*(2*b*arctan(e^(-x))/(a^2 + b^2) + a*log(-2*a*e^(-x) + b*e^(-2*x) - b)/(
a^2 + b^2) - a*log(e^(-2*x) + 1)/(a^2 + b^2)) + A*log((b*e^(-x) - a - sqrt
(a^2 + b^2))/(b*e^(-x) - a + sqrt(a^2 + b^2)))/sqrt(a^2 + b^2)
```

Giac [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 123, normalized size of antiderivative = 1.38

$$\int \frac{A+B\tanh(x)}{a+b\sinh(x)}\,dx = \frac{2\,Bb\arctan\left(e^{x}\right)}{a^2+b^2}+\frac{Ba\log\left(e^{(2\,x)}+1\right)}{a^2+b^2}-\frac{Ba\log\left(\left|be^{(2\,x)}+2\,ae^{x}-b\right|\right)}{a^2+b^2}+\frac{A\log\left(\frac{\left|2\,be^{x}+2\,a-2\,\sqrt{a^2+b^2}\right|}{\left|2\,be^{x}+2\,a+2\,\sqrt{a^2+b^2}\right|}\right)}{\sqrt{a^2+b^2}}$$

input
```
integrate((A+B*tanh(x))/(a+b*sinh(x)),x, algorithm="giac")
```

output
```
2*B*b*arctan(e^x)/(a^2 + b^2) + B*a*log(e^(2*x) + 1)/(a^2 + b^2) - B*a*log
(abs(b*e^(2*x) + 2*a*e^x - b))/(a^2 + b^2) + A*log(abs(2*b*e^x + 2*a - 2*s
qrt(a^2 + b^2))/abs(2*b*e^x + 2*a + 2*sqrt(a^2 + b^2)))/sqrt(a^2 + b^2)
```

Mupad [B] (verification not implemented)

Time = 9.48 (sec) , antiderivative size = 914, normalized size of antiderivative = 10.27

$$\int \frac{A+B\tanh(x)}{a+b\sinh(x)}\,dx = \text{Too large to display}$$

input

```
int((A + B*tanh(x))/(a + b*sinh(x)),x)
```

output

```
(B*log(exp(x) + 1i))/(a - b*1i) - (log((32*B*(A^2*b^2*exp(x) - 4*B^2*a^2*e
xp(x) + A^2*a*b + B^2*a*b - 4*A*B*a^2*exp(x) - A*B*b^2*exp(x) + 2*A*B*a*b)
)/b^5 - (((32*(A^2*b^3 + B^2*b^3 - A^2*a^2*b - 3*B^2*a^2*b + 4*B^2*a^3*exp
(x) - 5*B^2*a*b^2*exp(x) - 4*A*B*a^2*b + 8*A*B*a^3*exp(x) - 2*A^2*a*b^2*ex
p(x) + 2*A*B*a*b^2*exp(x)))/b^5 - ((B*a^3 - A*((a^2 + b^2)^3)^(1/2) + B*a*
b^2)*(a*b^5*(64*A - 128*B) + a^5*b*(64*A - 128*B) + 96*b^6*exp(x)*(A - 3*B
) + a^3*b^3*(128*A - 256*B) - 128*exp(x)*(A - 2*B)*(a^2 + b^2)^3 + 192*a^2
*b^4*exp(x)*(A - 3*B) + 96*a^4*b^2*exp(x)*(A - 3*B) + 96*A*a^2*b*((a^2 + b
^2)^3)^(1/2) - 128*A*a^3*exp(x)*((a^2 + b^2)^3)^(1/2) - 32*A*a*b^2*exp(x)*
((a^2 + b^2)^3)^(1/2)))/(b^5*(a^2 + b^2)^3))*(B*a^3 - A*((a^2 + b^2)^3)^(1
/2) + B*a*b^2))/(a^2 + b^2)^2)*(B*a^3 - A*((a^2 + b^2)^3)^(1/2) + B*a*b^2)
)/(a^4 + b^4 + 2*a^2*b^2) - (log((32*B*(A^2*b^2*exp(x) - 4*B^2*a^2*exp(x)
+ A^2*a*b + B^2*a*b - 4*A*B*a^2*exp(x) - A*B*b^2*exp(x) + 2*A*B*a*b))/b^5
- (((32*(A^2*b^3 + B^2*b^3 - A^2*a^2*b - 3*B^2*a^2*b + 4*B^2*a^3*exp(x) -
5*B^2*a*b^2*exp(x) - 4*A*B*a^2*b + 8*A*B*a^3*exp(x) - 2*A^2*a*b^2*exp(x) +
 2*A*B*a*b^2*exp(x)))/b^5 - ((A*((a^2 + b^2)^3)^(1/2) + B*a^3 + B*a*b^2)*(
a*b^5*(64*A - 128*B) + a^5*b*(64*A - 128*B) + 96*b^6*exp(x)*(A - 3*B) + a^
3*b^3*(128*A - 256*B) - 128*exp(x)*(A - 2*B)*(a^2 + b^2)^3 + 192*a^2*b^4*e
xp(x)*(A - 3*B) + 96*a^4*b^2*exp(x)*(A - 3*B) - 96*A*a^2*b*((a^2 + b^2)^3)
^(1/2) + 128*A*a^3*exp(x)*((a^2 + b^2)^3)^(1/2) + 32*A*a*b^2*exp(x)*((a...
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 87, normalized size of antiderivative = 0.98

$$\int \frac{A+B\tanh(x)}{a+b\sinh(x)}\,dx = \frac{2atan(e^x)\,b^2+2\sqrt{a^2+b^2}\,atan\left(\frac{e^x bi+ai}{\sqrt{a^2+b^2}}\right)ai+\log(e^{2x}+1)\,ab-\log(e^{2x}b+2e^x a-b)\,ab}{a^2+b^2}$$

input
```
int((A+B*tanh(x))/(a+b*sinh(x)),x)
```

output
```
(2*atan(e**x)*b**2 + 2*sqrt(a**2 + b**2)*atan((e**x*b*i + a*i)/sqrt(a**2 +
 b**2))*a*i + log(e**(2*x) + 1)*a*b - log(e**(2*x)*b + 2*e**x*a - b)*a*b)/
(a**2 + b**2)
```

3.250 $\int \frac{A+B\coth(x)}{a+b\sinh(x)}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 60

$$\int \frac{A+B\coth(x)}{a+b\sinh(x)}\,dx = -\frac{2A\operatorname{arctanh}\left(\frac{b-a\tanh\left(\frac{x}{2}\right)}{\sqrt{a^2+b^2}}\right)}{\sqrt{a^2+b^2}}+\frac{B\log(\sinh(x))}{a}-\frac{B\log(a+b\sinh(x))}{a}$$

output
```
-2*A*arctanh((b-a*tanh(1/2*x))/(a^2+b^2)^(1/2))/(a^2+b^2)^(1/2)+B*ln(sinh(
x))/a-B*ln(a+b*sinh(x))/a
```

Mathematica [A] (verified)

Time = 1.25 (sec) , antiderivative size = 65, normalized size of antiderivative = 1.08

$$\int \frac{A+B\coth(x)}{a+b\sinh(x)}\,dx = \frac{2A\arctan\left(\frac{b-a\tanh\left(\frac{x}{2}\right)}{\sqrt{-a^2-b^2}}\right)}{\sqrt{-a^2-b^2}}+\frac{B(\log(\sinh(x))-\log(a+b\sinh(x)))}{a}$$

input
```
Integrate[(A + B*Coth[x])/(a + b*Sinh[x]),x]
```

output
```
(2*A*ArcTan[(b - a*Tanh[x/2])/Sqrt[-a^2 - b^2]])/Sqrt[-a^2 - b^2] + (B*(Lo
g[Sinh[x]] - Log[a + b*Sinh[x]]))/a
```

Rubi [A] (verified)

Time = 0.38 (sec) , antiderivative size = 60, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {3042, 4901, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+B\coth(x)}{a+b\sinh(x)}dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \frac{A+iB\cot(ix)}{a-ib\sin(ix)}dx$$

$$\Big\downarrow \text{ 4901}$$

$$\int \left(\frac{A}{a+b\sinh(x)}+\frac{B\coth(x)}{a+b\sinh(x)}\right)dx$$

$$\Big\downarrow \text{ 2009}$$

$$-\frac{2A\text{arctanh}\left(\frac{b-a\tanh\left(\frac{x}{2}\right)}{\sqrt{a^2+b^2}}\right)}{\sqrt{a^2+b^2}}-\frac{B\log(a+b\sinh(x))}{a}+\frac{B\log(\sinh(x))}{a}$$

input
```
Int[(A + B*Coth[x])/(a + b*Sinh[x]),x]
```

output
```
(-2*A*ArcTanh[(b - a*Tanh[x/2])/Sqrt[a^2 + b^2]])/Sqrt[a^2 + b^2] + (B*Log
[Sinh[x]])/a - (B*Log[a + b*Sinh[x]])/a
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4901
```
Int[u_, x_Symbol] :> With[{v = ExpandTrig[u, x]}, Int[v, x] /; SumQ[v]] /;
 !InertTrigFreeQ[u]
```

Maple [A] (verified)

Time = 0.65 (sec) , antiderivative size = 73, normalized size of antiderivative = 1.22

method	result
parts	$\frac{2A\,\mathrm{arctanh}\left(\frac{2a\tanh\left(\frac{x}{2}\right)-2b}{2\sqrt{a^2+b^2}}\right)}{\sqrt{a^2+b^2}}-\frac{B\ln\left(\tanh\left(\frac{x}{2}\right)^2a-2b\tanh\left(\frac{x}{2}\right)-a\right)}{a}+\frac{B\ln\left(\tanh\left(\frac{x}{2}\right)\right)}{a}$
default	$\frac{B\ln\left(\tanh\left(\frac{x}{2}\right)\right)}{a}+\frac{-B\ln\left(\tanh\left(\frac{x}{2}\right)^2a-2b\tanh\left(\frac{x}{2}\right)-a\right)+\frac{2Aa\,\mathrm{arctanh}\left(\frac{2a\tanh\left(\frac{x}{2}\right)-2b}{2\sqrt{a^2+b^2}}\right)}{\sqrt{a^2+b^2}}}{a}$
risch	$-\frac{2xB}{a}-\frac{2x\,a^3B}{-a^4-a^2b^2}-\frac{2xBa\,b^2}{-a^4-a^2b^2}+\frac{B\ln\left(\mathrm{e}^{2x}-1\right)}{a}-\frac{a\ln\left(\mathrm{e}^x+\frac{a^2A-\sqrt{A^2a^4+A^2a^2b^2}}{Aab}\right)B}{a^2+b^2}-\frac{\ln\left(\mathrm{e}^x+\frac{a^2A-\sqrt{A^2a^4+A^2a^2b^2}}{Aab}\right)}{(a^2+b^2)a}$

input
```
int((A+B*coth(x))/(a+b*sinh(x)),x,method=_RETURNVERBOSE)
```

output
```
2*A/(a^2+b^2)^(1/2)*arctanh(1/2*(2*a*tanh(1/2*x)-2*b)/(a^2+b^2)^(1/2))-B/a
*ln(tanh(1/2*x)^2*a-2*b*tanh(1/2*x)-a)+B/a*ln(tanh(1/2*x))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 183 vs. 2(56) = 112.

Time = 0.11 (sec) , antiderivative size = 183, normalized size of antiderivative = 3.05

$$\int \frac{A+B\coth(x)}{a+b\sinh(x)}\,dx = \frac{\sqrt{a^2+b^2}Aa\log\left(\frac{b^2\cosh(x)^2+b^2\sinh(x)^2+2\,ab\cosh(x)+2\,a^2+b^2+2\,(b^2\cosh(x)+ab)\sinh(x)-2\,\sqrt{a^2+b^2}(b\cosh(x)+b\sinh(x)+a)}{b\cosh(x)^2+b\sinh(x)^2+2\,a\cosh(x)+2\,(b\cosh(x)+a)\sinh(x)-b}\right)-(}{a^3+ab^2}$$

input
```
integrate((A+B*coth(x))/(a+b*sinh(x)),x, algorithm="fricas")
```

output
```
(sqrt(a^2 + b^2)*A*a*log((b^2*cosh(x)^2 + b^2*sinh(x)^2 + 2*a*b*cosh(x) +
2*a^2 + b^2 + 2*(b^2*cosh(x) + a*b)*sinh(x) - 2*sqrt(a^2 + b^2)*(b*cosh(x)
 + b*sinh(x) + a))/(b*cosh(x)^2 + b*sinh(x)^2 + 2*a*cosh(x) + 2*(b*cosh(x)
 + a)*sinh(x) - b)) - (B*a^2 + B*b^2)*log(2*(b*sinh(x) + a)/(cosh(x) - sin
h(x))) + (B*a^2 + B*b^2)*log(2*sinh(x)/(cosh(x) - sinh(x))))/(a^3 + a*b^2)
```

Sympy [F]

$$\int \frac{A+B\coth(x)}{a+b\sinh(x)}\,dx=\int \frac{A+B\coth{(x)}}{a+b\sinh{(x)}}\,dx$$

input
```
integrate((A+B*coth(x))/(a+b*sinh(x)),x)
```

output
```
Integral((A + B*coth(x))/(a + b*sinh(x)), x)
```

Maxima [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 106, normalized size of antiderivative = 1.77

$$\int \frac{A+B\coth(x)}{a+b\sinh(x)}\,dx = -B\left(\frac{\log\left(-2\,ae^{(-x)}+be^{(-2\,x)}-b\right)}{a}-\frac{\log\left(e^{(-x)}+1\right)}{a}-\frac{\log\left(e^{(-x)}-1\right)}{a}\right) + \frac{A\log\left(\frac{be^{(-x)}-a-\sqrt{a^2+b^2}}{be^{(-x)}-a+\sqrt{a^2+b^2}}\right)}{\sqrt{a^2+b^2}}$$

input
```
integrate((A+B*coth(x))/(a+b*sinh(x)),x, algorithm="maxima")
```

output
```
-B*(log(-2*a*e^(-x) + b*e^(-2*x) - b)/a - log(e^(-x) + 1)/a - log(e^(-x) -
 1)/a) + A*log((b*e^(-x) - a - sqrt(a^2 + b^2))/(b*e^(-x) - a + sqrt(a^2 +
 b^2)))/sqrt(a^2 + b^2)
```

Giac [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 102, normalized size of antiderivative = 1.70

$$\int \frac{A+B\coth(x)}{a+b\sinh(x)}\,dx = \frac{A\log\left(\frac{\left|2\,be^{x}+2\,a-2\,\sqrt{a^2+b^2}\right|}{\left|2\,be^{x}+2\,a+2\,\sqrt{a^2+b^2}\right|}\right)}{\sqrt{a^2+b^2}} + \frac{B\log\left(e^{x}+1\right)}{a} - \frac{B\log\left(\left|be^{(2\,x)}+2\,ae^{x}-b\right|\right)}{a} + \frac{B\log\left(\left|e^{x}-1\right|\right)}{a}$$

input
```
integrate((A+B*coth(x))/(a+b*sinh(x)),x, algorithm="giac")
```

output
```
A*log(abs(2*b*e^x + 2*a - 2*sqrt(a^2 + b^2))/abs(2*b*e^x + 2*a + 2*sqrt(a^
2 + b^2)))/sqrt(a^2 + b^2) + B*log(e^x + 1)/a - B*log(abs(b*e^(2*x) + 2*a*
e^x - b))/a + B*log(abs(e^x - 1))/a
```

Mupad [B] (verification not implemented)

Time = 11.95 (sec) , antiderivative size = 164, normalized size of antiderivative = 2.73

$$\int \frac{A+B\coth(x)}{a+b\sinh(x)}\,dx = \frac{B\,\ln\left(16\,B^2\,a^2+16\,B^2\,b^2-16\,B^2\,a^2\,e^{2\,x}-16\,B^2\,b^2\,e^{2\,x}\right)}{a} - \frac{2\,\mathrm{atan}\left(\frac{A^2\,b^2\,e^{x}\,\sqrt{-a^2-b^2}+A^2\,a\,b\,\sqrt{-a^2-b^2}}{A\,b\,\sqrt{A^2}\,\left(a^2+b^2\right)}\right)\,\sqrt{A^2}}{\sqrt{-a^2-b^2}} - \frac{B\,\ln\left(32\,B^2\,a\,e^{x}-16\,B^2\,b+16\,B^2\,b\,e^{2\,x}\right)}{a}$$

input
```
int((A + B*coth(x))/(a + b*sinh(x)),x)
```

output
```
(B*log(16*B^2*a^2 + 16*B^2*b^2 - 16*B^2*a^2*exp(2*x) - 16*B^2*b^2*exp(2*x)
))/a - (2*atan((A^2*b^2*exp(x)*(- a^2 - b^2)^(1/2) + A^2*a*b*(- a^2 - b^2)
^(1/2))/(A*b*(A^2)^(1/2)*(a^2 + b^2)))*(A^2)^(1/2))/(- a^2 - b^2)^(1/2) -
(B*log(32*B^2*a*exp(x) - 16*B^2*b + 16*B^2*b*exp(2*x)))/a
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 139, normalized size of antiderivative = 2.32

$$\int \frac{A + B\coth(x)}{a + b\sinh(x)}\,dx = \frac{2\sqrt{a^2+b^2}\,atan\left(\frac{e^x bi + ai}{\sqrt{a^2+b^2}}\right)a^2 i + \log(e^x-1)\,a^2 b + \log(e^x-1)\,b^3 + \log(e^x+1)\,a^2 b + \log(e^x+1)\,b^3 - \log(e}{a\,(a^2+b^2)}$$

input
```
int((A+B*coth(x))/(a+b*sinh(x)),x)
```

output
```
(2*sqrt(a**2 + b**2)*atan((e**x*b*i + a*i)/sqrt(a**2 + b**2))*a**2*i + log
(e**x - 1)*a**2*b + log(e**x - 1)*b**3 + log(e**x + 1)*a**2*b + log(e**x +
 1)*b**3 - log(e**(2*x)*b + 2*e**x*a - b)*a**2*b - log(e**(2*x)*b + 2*e**x
*a - b)*b**3)/(a*(a**2 + b**2))
```

3.251 $\int \frac{A+B\text{sech}(x)}{a+b\sinh(x)}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 89

$$\int \frac{A+B\text{sech}(x)}{a+b\sinh(x)}\,dx=\frac{aB\arctan(\sinh(x))}{a^2+b^2}-\frac{2A\text{arctanh}\left(\frac{b-a\tanh\left(\frac{x}{2}\right)}{\sqrt{a^2+b^2}}\right)}{\sqrt{a^2+b^2}}-\frac{bB\log(\cosh(x))}{a^2+b^2}+\frac{bB\log(a+b\sinh(x))}{a^2+b^2}$$

output
```
a*B*arctan(sinh(x))/(a^2+b^2)-2*A*arctanh((b-a*tanh(1/2*x))/(a^2+b^2)^(1/2
))/(a^2+b^2)^(1/2)-b*B*ln(cosh(x))/(a^2+b^2)+b*B*ln(a+b*sinh(x))/(a^2+b^2)
```

Mathematica [A] (verified)

Time = 0.25 (sec) , antiderivative size = 93, normalized size of antiderivative = 1.04

$$\int \frac{A+B\text{sech}(x)}{a+b\sinh(x)}\,dx=\frac{2aB\arctan\left(\tanh\left(\frac{x}{2}\right)\right)}{a^2+b^2}+\frac{2A\arctan\left(\frac{b-a\tanh\left(\frac{x}{2}\right)}{\sqrt{-a^2-b^2}}\right)}{\sqrt{-a^2-b^2}}-\frac{bB(\log(\cosh(x))-\log(a+b\sinh(x)))}{a^2+b^2}$$

input
```
Integrate[(A + B*Sech[x])/(a + b*Sinh[x]),x]
```

output
```
(2*a*B*ArcTan[Tanh[x/2]])/(a^2 + b^2) + (2*A*ArcTan[(b - a*Tanh[x/2])/Sqrt
[-a^2 - b^2]])/Sqrt[-a^2 - b^2] - (b*B*(Log[Cosh[x]] - Log[a + b*Sinh[x]])
)/(a^2 + b^2)
```

Rubi [A] (verified)

Time = 0.52 (sec) , antiderivative size = 89, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {3042, 4714, 3042, 4901, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+B\text{sech}(x)}{a+b\sinh(x)}dx$$
$$\Big\downarrow \text{3042}$$
$$\int \frac{A+B\sec(ix)}{a-ib\sin(ix)}dx$$
$$\Big\downarrow \text{4714}$$
$$\int \frac{\text{sech}(x)(A\cosh(x)+B)}{a+b\sinh(x)}dx$$
$$\Big\downarrow \text{3042}$$
$$\int \frac{B+A\cos(ix)}{\cos(ix)(a-ib\sin(ix))}dx$$
$$\Big\downarrow \text{4901}$$
$$\int \left(\frac{A}{a+b\sinh(x)}+\frac{B\text{sech}(x)}{a+b\sinh(x)}\right)dx$$
$$\Big\downarrow \text{2009}$$
$$-\frac{2A\text{arctanh}\left(\frac{b-a\tanh\left(\frac{x}{2}\right)}{\sqrt{a^2+b^2}}\right)}{\sqrt{a^2+b^2}}+\frac{aB\arctan(\sinh(x))}{a^2+b^2}+\frac{bB\log(a+b\sinh(x))}{a^2+b^2}-\frac{bB\log(\cosh(x))}{a^2+b^2}$$

input
```
Int[(A + B*Sech[x])/(a + b*Sinh[x]),x]
```

output
```
(a*B*ArcTan[Sinh[x]])/(a^2 + b^2) - (2*A*ArcTanh[(b - a*Tanh[x/2])/Sqrt[a^
2 + b^2]])/Sqrt[a^2 + b^2] - (b*B*Log[Cosh[x]])/(a^2 + b^2) + (b*B*Log[a +
 b*Sinh[x]])/(a^2 + b^2)
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4714
```
Int[(u_)*((A_) + (B_.)*sec[(a_.) + (b_.)*(x_)]), x_Symbol] :> Int[ActivateT
rig[u]*((B + A*Cos[a + b*x])/Cos[a + b*x]), x] /; FreeQ[{a, b, A, B}, x] &&
 KnownSineIntegrandQ[u, x]
```

rule 4901
```
Int[u_, x_Symbol] :> With[{v = ExpandTrig[u, x]}, Int[v, x] /; SumQ[v]] /;
 !InertTrigFreeQ[u]
```

Maple [A] (verified)

Time = 2.19 (sec) , antiderivative size = 102, normalized size of antiderivative = 1.15

method	result
parts	$\frac{2A\,\operatorname{arctanh}\left(\frac{2a\tanh\left(\frac{x}{2}\right)-2b}{2\sqrt{a^2+b^2}}\right)}{\sqrt{a^2+b^2}}+B\left(\frac{b\ln\left(\tanh\left(\frac{x}{2}\right)^2a-2b\tanh\left(\frac{x}{2}\right)-a\right)}{a^2+b^2}+\frac{-b\ln\left(1+\tanh\left(\frac{x}{2}\right)^2\right)+2a\arctan\left(\tanh\left(\frac{x}{2}\right)\right)}{a^2+b^2}\right)$
default	$\frac{Bb\ln\left(\tanh\left(\frac{x}{2}\right)^2a-2b\tanh\left(\frac{x}{2}\right)-a\right)-\frac{2\left(-a^2A-A\,b^2\right)\operatorname{arctanh}\left(\frac{2a\tanh\left(\frac{x}{2}\right)-2b}{2\sqrt{a^2+b^2}}\right)}{\sqrt{a^2+b^2}}}{a^2+b^2}+\frac{2B\left(-\frac{b\ln\left(1+\tanh\left(\frac{x}{2}\right)^2\right)}{2}+a\arctan\left(\tanh\left(\frac{x}{2}\right)\right)\right)}{a^2+b^2}$
risch	$\frac{2xBb}{a^2+b^2}+\frac{2xB\,a^2b}{-a^4-2a^2b^2-b^4}+\frac{2xB\,b^3}{-a^4-2a^2b^2-b^4}+\frac{iB\ln(\mathrm{e}^x+i)a}{a^2+b^2}-\frac{B\ln(\mathrm{e}^x+i)b}{a^2+b^2}-\frac{iB\ln(\mathrm{e}^x-i)a}{a^2+b^2}-\frac{B\ln(\mathrm{e}^x-i)b}{a^2+b^2}+\frac{\ln\left(\mathrm{e}^x+\right.}{}$

input
```
int((A+B*sech(x))/(a+b*sinh(x)),x,method=_RETURNVERBOSE)
```

output
```
2*A/(a^2+b^2)^(1/2)*arctanh(1/2*(2*a*tanh(1/2*x)-2*b)/(a^2+b^2)^(1/2))+B*(
b/(a^2+b^2)*ln(tanh(1/2*x)^2*a-2*b*tanh(1/2*x)-a)+2/(a^2+b^2)*(-1/2*b*ln(1
+tanh(1/2*x)^2)+a*arctan(tanh(1/2*x))))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 172 vs. 2(85) = 170.

Time = 1.76 (sec) , antiderivative size = 172, normalized size of antiderivative = 1.93

$$\int \frac{A+B\mathrm{sech}(x)}{a+b\sinh(x)}\,dx = \frac{2\,Ba\arctan\left(\cosh\left(x\right)+\sinh\left(x\right)\right)+Bb\log\left(\frac{2\,(b\sinh(x)+a)}{\cosh(x)-\sinh(x)}\right)-Bb\log\left(\frac{2\,\cosh(x)}{\cosh(x)-\sinh(x)}\right)+\sqrt{a^2+b^2}A\log\left(\frac{b^2 \cdots}{\cdots}\right.}{a^2+b^2}$$

input
```
integrate((A+B*sech(x))/(a+b*sinh(x)),x, algorithm="fricas")
```

output
```
(2*B*a*arctan(cosh(x) + sinh(x)) + B*b*log(2*(b*sinh(x) + a)/(cosh(x) - si
nh(x))) - B*b*log(2*cosh(x)/(cosh(x) - sinh(x))) + sqrt(a^2 + b^2)*A*log((
b^2*cosh(x)^2 + b^2*sinh(x)^2 + 2*a*b*cosh(x) + 2*a^2 + b^2 + 2*(b^2*cosh(
x) + a*b)*sinh(x) - 2*sqrt(a^2 + b^2)*(b*cosh(x) + b*sinh(x) + a))/(b*cosh
(x)^2 + b*sinh(x)^2 + 2*a*cosh(x) + 2*(b*cosh(x) + a)*sinh(x) - b)))/(a^2
+ b^2)
```

Sympy [F]

$$\int \frac{A+B\mathrm{sech}(x)}{a+b\sinh(x)}\,dx = \int \frac{A+B\operatorname{sech}\left(x\right)}{a+b\sinh\left(x\right)}\,dx$$

input
```
integrate((A+B*sech(x))/(a+b*sinh(x)),x)
```

output
```
Integral((A + B*sech(x))/(a + b*sinh(x)), x)
```

Maxima [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 125, normalized size of antiderivative = 1.40

$$\int \frac{A+B\operatorname{sech}(x)}{a+b\sinh(x)}\,dx = -B\left(\frac{2\,a\arctan\left(e^{(-x)}\right)}{a^2+b^2} - \frac{b\log\left(-2\,ae^{(-x)}+be^{(-2\,x)}-b\right)}{a^2+b^2} + \frac{b\log\left(e^{(-2\,x)}+1\right)}{a^2+b^2}\right) + \frac{A\log\left(\frac{be^{(-x)}-a-\sqrt{a^2+b^2}}{be^{(-x)}-a+\sqrt{a^2+b^2}}\right)}{\sqrt{a^2+b^2}}$$

input
```
integrate((A+B*sech(x))/(a+b*sinh(x)),x, algorithm="maxima")
```

output
```
-B*(2*a*arctan(e^(-x))/(a^2 + b^2) - b*log(-2*a*e^(-x) + b*e^(-2*x) - b)/(
a^2 + b^2) + b*log(e^(-2*x) + 1)/(a^2 + b^2)) + A*log((b*e^(-x) - a - sqrt
(a^2 + b^2))/(b*e^(-x) - a + sqrt(a^2 + b^2)))/sqrt(a^2 + b^2)
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 123, normalized size of antiderivative = 1.38

$$\int \frac{A+B\operatorname{sech}(x)}{a+b\sinh(x)}\,dx = \frac{2\,Ba\arctan\left(e^{x}\right)}{a^2+b^2} - \frac{Bb\log\left(e^{(2\,x)}+1\right)}{a^2+b^2} + \frac{Bb\log\left(\left|be^{(2\,x)}+2\,ae^{x}-b\right|\right)}{a^2+b^2} + \frac{A\log\left(\frac{\left|2\,be^{x}+2\,a-2\,\sqrt{a^2+b^2}\right|}{\left|2\,be^{x}+2\,a+2\,\sqrt{a^2+b^2}\right|}\right)}{\sqrt{a^2+b^2}}$$

input
```
integrate((A+B*sech(x))/(a+b*sinh(x)),x, algorithm="giac")
```

output
```
2*B*a*arctan(e^x)/(a^2 + b^2) - B*b*log(e^(2*x) + 1)/(a^2 + b^2) + B*b*log
(abs(b*e^(2*x) + 2*a*e^x - b))/(a^2 + b^2) + A*log(abs(2*b*e^x + 2*a - 2*s
qrt(a^2 + b^2))/abs(2*b*e^x + 2*a + 2*sqrt(a^2 + b^2)))/sqrt(a^2 + b^2)
```

Mupad [B] (verification not implemented)

Time = 11.60 (sec) , antiderivative size = 864, normalized size of antiderivative = 9.71

$$\int \frac{A + B\mathrm{sech}(x)}{a + b\sinh(x)}\, dx = \text{Too large to display}$$

input
```
int((A + B/cosh(x))/(a + b*sinh(x)),x)
```

output
```
(log(((A*((a^2 + b^2)^3)^(1/2) + B*b^3 + B*a^2*b)*(b^3*(32*A^2 + 64*B^2 -
96*A*B*exp(x)) - 128*A^2*exp(x)*((a^2 + b^2)^3)^(1/2) - a*b^2*(96*A^2*exp(
x) + 128*B^2*exp(x) - 192*A*B) - 128*a^3*exp(x)*(A^2 + B^2) + a^2*b*(64*A^
2 + 64*B^2 - 384*A*B*exp(x)) + (32*A*b^6*(2*B + 3*A*exp(x)))/((a^2 + b^2)^
3)^(1/2) + (32*A*a^4*b^2*(5*B + 3*A*exp(x)))/((a^2 + b^2)^3)^(1/2) + (32*A
*a^2*b^4*(7*B + 6*A*exp(x)))/((a^2 + b^2)^3)^(1/2) + (32*A*a^3*b^3*(4*A -
19*B*exp(x)))/((a^2 + b^2)^3)^(1/2) + (64*A*a*b^5*(A - 4*B*exp(x)))/((a^2
+ b^2)^3)^(1/2) + (32*A*a^5*b*(2*A - 11*B*exp(x)))/((a^2 + b^2)^3)^(1/2)))
/(b^5*(a^2 + b^2)^2) - (32*B*(2*B^2*b^2 - A^2*b^2 + 4*A*B*a^2*exp(x) + A*B
*b^2*exp(x) + A^2*a*b*exp(x) - 4*B^2*a*b*exp(x) - 2*A*B*a*b))/b^5)*(A*((a^
2 + b^2)^3)^(1/2) + B*b^3 + B*a^2*b))/(a^4 + b^4 + 2*a^2*b^2) - (B*log(exp
(x) + 1i))/(a*1i + b) - (B*log(exp(x) - 1i)*1i)/(a + b*1i) + (log(- (32*B*
(2*B^2*b^2 - A^2*b^2 + 4*A*B*a^2*exp(x) + A*B*b^2*exp(x) + A^2*a*b*exp(x)
- 4*B^2*a*b*exp(x) - 2*A*B*a*b))/b^5 - ((B*b^3 - A*((a^2 + b^2)^3)^(1/2) +
 B*a^2*b)*(a*b^2*(96*A^2*exp(x) + 128*B^2*exp(x) - 192*A*B) - 128*A^2*exp(
x)*((a^2 + b^2)^3)^(1/2) - b^3*(32*A^2 + 64*B^2 - 96*A*B*exp(x)) + 128*a^3
*exp(x)*(A^2 + B^2) - a^2*b*(64*A^2 + 64*B^2 - 384*A*B*exp(x)) + (32*A*b^6
*(2*B + 3*A*exp(x)))/((a^2 + b^2)^3)^(1/2) + (32*A*a^4*b^2*(5*B + 3*A*exp(
x)))/((a^2 + b^2)^3)^(1/2) + (32*A*a^2*b^4*(7*B + 6*A*exp(x)))/((a^2 + b^2
)^3)^(1/2) + (32*A*a^3*b^3*(4*A - 19*B*exp(x)))/((a^2 + b^2)^3)^(1/2) +...
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 88, normalized size of antiderivative = 0.99

$$\int \frac{A + B\mathrm{sech}(x)}{a + b\sinh(x)}\, dx$$
$$= \frac{2atan(e^x)\, ab + 2\sqrt{a^2+b^2}\, atan\left(\frac{e^x bi+ai}{\sqrt{a^2+b^2}}\right) ai - \log(e^{2x}+1)\, b^2 + \log(e^{2x}b + 2e^x a - b)\, b^2}{a^2+b^2}$$

input
```
int((A+B*sech(x))/(a+b*sinh(x)),x)
```

output
```
(2*atan(e**x)*a*b + 2*sqrt(a**2 + b**2)*atan((e**x*b*i + a*i)/sqrt(a**2 +
b**2))*a*i - log(e**(2*x) + 1)*b**2 + log(e**(2*x)*b + 2*e**x*a - b)*b**2)
/(a**2 + b**2)
```

3.252 $\int \frac{A+B\operatorname{csch}(x)}{a+b\sinh(x)}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 58

$$\int \frac{A+B\operatorname{csch}(x)}{a+b\sinh(x)}\,dx=-\frac{B\operatorname{arctanh}(\cosh(x))}{a}-\frac{2(aA-bB)\operatorname{arctanh}\left(\frac{b-a\tanh\left(\frac{x}{2}\right)}{\sqrt{a^2+b^2}}\right)}{a\sqrt{a^2+b^2}}$$

output
```
-B*arctanh(cosh(x))/a-2*(A*a-B*b)*arctanh((b-a*tanh(1/2*x))/(a^2+b^2)^(1/2
))/a/(a^2+b^2)^(1/2)
```

Mathematica [A] (verified)

Time = 1.05 (sec) , antiderivative size = 77, normalized size of antiderivative = 1.33

$$\int \frac{A+B\operatorname{csch}(x)}{a+b\sinh(x)}\,dx=\frac{\frac{2(aA-bB)\arctan\left(\frac{b-a\tanh\left(\frac{x}{2}\right)}{\sqrt{-a^2-b^2}}\right)}{\sqrt{-a^2-b^2}}+B\left(-\log\left(\cosh\left(\frac{x}{2}\right)\right)+\log\left(\sinh\left(\frac{x}{2}\right)\right)\right)}{a}$$

input
```
Integrate[(A + B*Csch[x])/(a + b*Sinh[x]),x]
```

output
```
((2*(a*A - b*B)*ArcTan[(b - a*Tanh[x/2])/Sqrt[-a^2 - b^2]])/Sqrt[-a^2 - b^
2] + B*(-Log[Cosh[x/2]] + Log[Sinh[x/2]]))/a
```

Rubi [C] (verified)

Result contains complex when optimal does not.

Time = 0.48 (sec) , antiderivative size = 71, normalized size of antiderivative = 1.22, number of steps used = 15, number of rules used = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.933$, Rules used = {3042, 3307, 26, 26, 3042, 26, 3480, 26, 3042, 26, 3139, 1083, 219, 4257}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+B\text{csch}(x)}{a+b\sinh(x)}dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \frac{A+iB\csc(ix)}{a-ib\sin(ix)}dx$$

$$\Big\downarrow \text{ 3307}$$

$$\int -\frac{i\text{csch}(x)(iA\sinh(x)+iB)}{a+b\sinh(x)}dx$$

$$\Big\downarrow \text{ 26}$$

$$-i\int \frac{i\text{csch}(x)(B+A\sinh(x))}{a+b\sinh(x)}dx$$

$$\Big\downarrow \text{ 26}$$

$$\int \frac{\text{csch}(x)(A\sinh(x)+B)}{a+b\sinh(x)}dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \frac{i(B-iA\sin(ix))}{\sin(ix)(a-ib\sin(ix))}dx$$

$$\Big\downarrow \text{ 26}$$

$$i\int \frac{B-iA\sin(ix)}{\sin(ix)(a-ib\sin(ix))}dx$$

$$\Big\downarrow \text{ 3480}$$

$$i\left(\frac{B\int -i\text{csch}(x)dx}{a}-\frac{i(aA-bB)\int \frac{1}{a+b\sinh(x)}dx}{a}\right)$$

$$\downarrow \text{26}$$

$$i\left(-\frac{i(aA-bB)\int \frac{1}{a+b\sinh(x)}dx}{a}-\frac{iB\int \operatorname{csch}(x)dx}{a}\right)$$

$$\downarrow \text{3042}$$

$$i\left(-\frac{i(aA-bB)\int \frac{1}{a-ib\sin(ix)}dx}{a}-\frac{iB\int i\csc(ix)dx}{a}\right)$$

$$\downarrow \text{26}$$

$$i\left(\frac{B\int \csc(ix)dx}{a}-\frac{i(aA-bB)\int \frac{1}{a-ib\sin(ix)}dx}{a}\right)$$

$$\downarrow \text{3139}$$

$$i\left(\frac{B\int \csc(ix)dx}{a}-\frac{2i(aA-bB)\int \frac{1}{-a\tanh^2\left(\frac{x}{2}\right)+2b\tanh\left(\frac{x}{2}\right)+a}d\tanh\left(\frac{x}{2}\right)}{a}\right)$$

$$\downarrow \text{1083}$$

$$i\left(\frac{4i(aA-bB)\int \frac{1}{4(a^2+b^2)-\left(2b-2a\tanh\left(\frac{x}{2}\right)\right)^2}d\left(2b-2a\tanh\left(\frac{x}{2}\right)\right)}{a}+\frac{B\int \csc(ix)dx}{a}\right)$$

$$\downarrow \text{219}$$

$$i\left(\frac{B\int \csc(ix)dx}{a}+\frac{2i(aA-bB)\text{arctanh}\left(\frac{2b-2a\tanh\left(\frac{x}{2}\right)}{2\sqrt{a^2+b^2}}\right)}{a\sqrt{a^2+b^2}}\right)$$

$$\downarrow \text{4257}$$

$$i\left(\frac{2i(aA-bB)\text{arctanh}\left(\frac{2b-2a\tanh\left(\frac{x}{2}\right)}{2\sqrt{a^2+b^2}}\right)}{a\sqrt{a^2+b^2}}+\frac{iB\text{arctanh}(\cosh(x))}{a}\right)$$

input
```
Int[(A + B*Csch[x])/(a + b*Sinh[x]),x]
```

output
```
I*((I*B*ArcTanh[Cosh[x]])/a + ((2*I)*(a*A - b*B)*ArcTanh[(2*b - 2*a*Tanh[x
/2])/(2*Sqrt[a^2 + b^2])])/(a*Sqrt[a^2 + b^2]))
```

Defintions of rubi rules used

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1083
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Simp[-2   Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3139
```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Simp[2*(e/d)   Subst[Int[1/(a + 2*b*e*x + a
*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[a^2 - b^2, 0]
```

rule 3307
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))^(n_.)*((a_) + (b_.)*sin[(e_.) +
(f_.)*(x_)])^(m_.), x_Symbol] :> Int[(a + b*Sin[e + f*x])^m*((d + c*Sin[e +
 f*x])^n/Sin[e + f*x]^n), x] /; FreeQ[{a, b, c, d, e, f, m}, x] && IntegerQ
[n]
```

rule 3480
```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((a_.) + (b_.)*sin[(e_.) + (f_
.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])), x_Symbol] :> Simp[(A*b
- a*B)/(b*c - a*d)   Int[1/(a + b*Sin[e + f*x]), x], x] + Simp[(B*c - A*d)/
(b*c - a*d)   Int[1/(c + d*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f
, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

rule 4257
```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Maple [A] (verified)

Time = 0.54 (sec) , antiderivative size = 58, normalized size of antiderivative = 1.00

method	result
default	$\frac{B\ln\left(\tanh\left(\frac{x}{2}\right)\right)}{a}-\frac{(-2Aa+2bB)\,\mathrm{arctanh}\left(\frac{2a\tanh\left(\frac{x}{2}\right)-2b}{2\sqrt{a^2+b^2}}\right)}{a\sqrt{a^2+b^2}}$
parts	$\frac{2A\,\mathrm{arctanh}\left(\frac{2a\tanh\left(\frac{x}{2}\right)-2b}{2\sqrt{a^2+b^2}}\right)}{\sqrt{a^2+b^2}}-\frac{2Bb\,\mathrm{arctanh}\left(\frac{2a\tanh\left(\frac{x}{2}\right)-2b}{2\sqrt{a^2+b^2}}\right)}{a\sqrt{a^2+b^2}}+\frac{B\ln\left(\tanh\left(\frac{x}{2}\right)\right)}{a}$
risch	$\frac{B\ln(\mathrm{e}^x-1)}{a}-\frac{B\ln(\mathrm{e}^x+1)}{a}+\frac{\ln\left(\mathrm{e}^x+\frac{a\sqrt{a^2+b^2}-a^2-b^2}{\sqrt{a^2+b^2}\,b}\right)A}{\sqrt{a^2+b^2}}-\frac{Bb\ln\left(\mathrm{e}^x+\frac{a\sqrt{a^2+b^2}-a^2-b^2}{\sqrt{a^2+b^2}\,b}\right)}{\sqrt{a^2+b^2}\,a}-\frac{\ln\left(\mathrm{e}^x+\frac{a\sqrt{a^2+b^2}+a^2+b^2}{\sqrt{a^2+b^2}\,b}\right)A}{\sqrt{a^2+b^2}}$

input
```
int((A+B*csch(x))/(a+b*sinh(x)),x,method=_RETURNVERBOSE)
```

output
```
B/a*ln(tanh(1/2*x))-(-2*A*a+2*B*b)/a/(a^2+b^2)^(1/2)*arctanh(1/2*(2*a*tanh
(1/2*x)-2*b)/(a^2+b^2)^(1/2))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 172 vs. $2(54) = 108$.

Time = 0.33 (sec) , antiderivative size = 172, normalized size of antiderivative = 2.97

$$\int \frac{A+B\operatorname{csch}(x)}{a+b\sinh(x)}\,dx =$$
$$-\frac{(Aa-Bb)\sqrt{a^2+b^2}\log\left(\frac{b^2\cosh(x)^2+b^2\sinh(x)^2+2\,ab\cosh(x)+2\,a^2+b^2+2\,(b^2\cosh(x)+ab)\sinh(x)+2\,\sqrt{a^2+b^2}(b\cosh(x)+b\sinh}{b\cosh(x)^2+b\sinh(x)^2+2\,a\cosh(x)+2\,(b\cosh(x)+a)\sinh(x)-b}\right.}{a^3+ab}$$

input
```
integrate((A+B*csch(x))/(a+b*sinh(x)),x, algorithm="fricas")
```

output
```
-((A*a - B*b)*sqrt(a^2 + b^2)*log((b^2*cosh(x)^2 + b^2*sinh(x)^2 + 2*a*b*c
osh(x) + 2*a^2 + b^2 + 2*(b^2*cosh(x) + a*b)*sinh(x) + 2*sqrt(a^2 + b^2)*(
b*cosh(x) + b*sinh(x) + a))/(b*cosh(x)^2 + b*sinh(x)^2 + 2*a*cosh(x) + 2*(
b*cosh(x) + a)*sinh(x) - b)) + (B*a^2 + B*b^2)*log(cosh(x) + sinh(x) + 1)
- (B*a^2 + B*b^2)*log(cosh(x) + sinh(x) - 1))/(a^3 + a*b^2)
```

Sympy [F]

$$\int \frac{A+B\text{csch}(x)}{a+b\sinh(x)}\,dx=\int \frac{A+B\operatorname{csch}(x)}{a+b\sinh(x)}\,dx$$

input
```
integrate((A+B*csch(x))/(a+b*sinh(x)),x)
```

output
```
Integral((A + B*csch(x))/(a + b*sinh(x)), x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 141 vs. 2(54) = 108.

Time = 0.12 (sec) , antiderivative size = 141, normalized size of antiderivative = 2.43

$$\int \frac{A+B\text{csch}(x)}{a+b\sinh(x)}\,dx = -B\left(\frac{b\log\left(\frac{be^{(-x)}-a-\sqrt{a^2+b^2}}{be^{(-x)}-a+\sqrt{a^2+b^2}}\right)}{\sqrt{a^2+b^2}a}+\frac{\log\left(e^{(-x)}+1\right)}{a}-\frac{\log\left(e^{(-x)}-1\right)}{a}\right)+\frac{A\log\left(\frac{be^{(-x)}-a-\sqrt{a^2+b^2}}{be^{(-x)}-a+\sqrt{a^2+b^2}}\right)}{\sqrt{a^2+b^2}}$$

input
```
integrate((A+B*csch(x))/(a+b*sinh(x)),x, algorithm="maxima")
```

output
```
-B*(b*log((b*e^(-x) - a - sqrt(a^2 + b^2))/(b*e^(-x) - a + sqrt(a^2 + b^2)
))/(sqrt(a^2 + b^2)*a) + log(e^(-x) + 1)/a - log(e^(-x) - 1)/a) + A*log((b
*e^(-x) - a - sqrt(a^2 + b^2))/(b*e^(-x) - a + sqrt(a^2 + b^2)))/sqrt(a^2
+ b^2)
```

Giac [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 90, normalized size of antiderivative = 1.55

$$\int \frac{A+B\text{csch}(x)}{a+b\sinh(x)}\,dx = -\frac{B\log\left(e^x+1\right)}{a}+\frac{B\log\left(|e^x-1|\right)}{a}+\frac{(Aa-Bb)\log\left(\frac{\left|2\,be^x+2\,a-2\,\sqrt{a^2+b^2}\right|}{\left|2\,be^x+2\,a+2\,\sqrt{a^2+b^2}\right|}\right)}{\sqrt{a^2+b^2}a}$$

input
```
integrate((A+B*csch(x))/(a+b*sinh(x)),x, algorithm="giac")
```

output
```
-B*log(e^x + 1)/a + B*log(abs(e^x - 1))/a + (A*a - B*b)*log(abs(2*b*e^x +
2*a - 2*sqrt(a^2 + b^2))/abs(2*b*e^x + 2*a + 2*sqrt(a^2 + b^2)))/(sqrt(a^2
 + b^2)*a)
```

Mupad [B] (verification not implemented)

Time = 2.16 (sec) , antiderivative size = 539, normalized size of antiderivative = 9.29

$$\int \frac{A+B\operatorname{csch}(x)}{a+b\sinh(x)}\,dx = \frac{B\,\ln\left(e^x-1\right)}{a} - \frac{B\,\ln\left(e^x+1\right)}{a}$$

$$-\frac{\ln\left(\frac{(A\,a-B\,b)\left(\frac{32\left(A^2\,a^2\,b-2\,A\,B\,a\,b^2-4\,e^x\,B^2\,a^3+2\,B^2\,a^2\,b-3\,e^x\,B^2\,a\,b^2+2\,B^2\,b^3\right)}{b^5}-\frac{(A\,a-B\,b)\left(\frac{32\,a^2\left(2\,B\,b^2+4\,A\,a^2\,e^x+A\,b^2\,e^x-2\,A\,a\,b-3\,B\,a\,b\cdots}{b^5}\cdots\right.}{a\,\sqrt{a^2\cdots}}\right.}{a\,\sqrt{a^2+b^2}}\right.}{a^3+a\cdots}$$

$$+\frac{\ln\left(\frac{32\,B\,(A\,a-B\,b)\,(A\,b\,e^x-2\,B\,b+4\,B\,a\,e^x)}{b^5}-\frac{(A\,a-B\,b)\left(\frac{32\left(A^2\,a^2\,b-2\,A\,B\,a\,b^2-4\,e^x\,B^2\,a^3+2\,B^2\,a^2\,b-3\,e^x\,B^2\,a\,b^2+2\,B^2\,b^3\right)}{b^5}+\frac{(A\,a-B\,b)\cdots}{\cdots}\right.}{\cdots}\right.}{a^3+a\cdots}$$

input
```
int((A + B/sinh(x))/(a + b*sinh(x)),x)
```

output
```
(B*log(exp(x) - 1))/a - (B*log(exp(x) + 1))/a - (log(((A*a - B*b)*((32*(2*
B^2*b^3 + A^2*a^2*b + 2*B^2*a^2*b - 4*B^2*a^3*exp(x) - 3*B^2*a*b^2*exp(x)
- 2*A*B*a*b^2))/b^5 - ((A*a - B*b)*((32*a^2*(2*B*b^2 + 4*A*a^2*exp(x) + A*
b^2*exp(x) - 2*A*a*b - 3*B*a*b*exp(x)))/b^5 + (32*a*(A*a - B*b)*(3*a^2*b +
 2*b^3 - 4*a^3*exp(x) - 3*a*b^2*exp(x)))/(b^5*(a^2 + b^2)^(1/2))))/(a*(a^2
 + b^2)^(1/2))))/(a*(a^2 + b^2)^(1/2)) + (32*B*(A*a - B*b)*(A*b*exp(x) - 2
*B*b + 4*B*a*exp(x)))/b^5)*(A*a - B*b)*(a^2 + b^2)^(1/2))/(a*b^2 + a^3) +
(log((32*B*(A*a - B*b)*(A*b*exp(x) - 2*B*b + 4*B*a*exp(x)))/b^5 - ((A*a -
B*b)*((32*(2*B^2*b^3 + A^2*a^2*b + 2*B^2*a^2*b - 4*B^2*a^3*exp(x) - 3*B^2*
a*b^2*exp(x) - 2*A*B*a*b^2))/b^5 + ((A*a - B*b)*((32*a^2*(2*B*b^2 + 4*A*a^
2*exp(x) + A*b^2*exp(x) - 2*A*a*b - 3*B*a*b*exp(x)))/b^5 - (32*a*(A*a - B*
b)*(3*a^2*b + 2*b^3 - 4*a^3*exp(x) - 3*a*b^2*exp(x)))/(b^5*(a^2 + b^2)^(1/
2))))/(a*(a^2 + b^2)^(1/2))))/(a*(a^2 + b^2)^(1/2)))*(A*a - B*b)*(a^2 + b^
2)^(1/2))/(a*b^2 + a^3)
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 130, normalized size of antiderivative = 2.24

$$\int \frac{A + B\text{csch}(x)}{a + b\sinh(x)}\,dx$$
$$= \frac{2\sqrt{a^2+b^2}\,atan\left(\frac{e^x bi+ai}{\sqrt{a^2+b^2}}\right)a^2 i - 2\sqrt{a^2+b^2}\,atan\left(\frac{e^x bi+ai}{\sqrt{a^2+b^2}}\right)b^2 i + \log(e^x-1)\,a^2 b + \log(e^x-1)\,b^3 - \log(e^x +}{a\,(a^2+b^2)}$$

input
```
int((A+B*csch(x))/(a+b*sinh(x)),x)
```

output
```
(2*sqrt(a**2 + b**2)*atan((e**x*b*i + a*i)/sqrt(a**2 + b**2))*a**2*i - 2*s
qrt(a**2 + b**2)*atan((e**x*b*i + a*i)/sqrt(a**2 + b**2))*b**2*i + log(e**
x - 1)*a**2*b + log(e**x - 1)*b**3 - log(e**x + 1)*a**2*b - log(e**x + 1)*
b**3)/(a*(a**2 + b**2))
```

3.253 $\int \frac{A+B\cosh(d+ex)+C\sinh(d+ex)}{a+c\sinh(d+ex)}\,dx$

Optimal result

Integrand size = 31, antiderivative size = 81

$$\int \frac{A+B\cosh(d+ex)+C\sinh(d+ex)}{a+c\sinh(d+ex)}\,dx = \frac{Cx}{c}-\frac{2(Ac-aC)\text{arctanh}\left(\frac{c-a\tanh\left(\frac{1}{2}(d+ex)\right)}{\sqrt{a^2+c^2}}\right)}{c\sqrt{a^2+c^2}e}+\frac{B\log(a+c\sinh(d+ex))}{ce}$$

output
```
C*x/c-2*(A*c-C*a)*arctanh((c-a*tanh(1/2*e*x+1/2*d))/(a^2+c^2)^(1/2))/c/(a^
2+c^2)^(1/2)/e+B*ln(a+c*sinh(e*x+d))/c/e
```

Mathematica [A] (verified)

Time = 2.13 (sec) , antiderivative size = 85, normalized size of antiderivative = 1.05

$$\int \frac{A+B\cosh(d+ex)+C\sinh(d+ex)}{a+c\sinh(d+ex)}\,dx = \frac{C(d+ex)+\frac{2(Ac-aC)\arctan\left(\frac{c-a\tanh\left(\frac{1}{2}(d+ex)\right)}{\sqrt{-a^2-c^2}}\right)}{\sqrt{-a^2-c^2}}+B\log(a+c\sinh(d+ex))}{ce}$$

input
```
Integrate[(A + B*Cosh[d + e*x] + C*Sinh[d + e*x])/(a + c*Sinh[d + e*x]),x]
```

output
```
(C*(d + e*x) + (2*(A*c - a*C)*ArcTan[(c - a*Tanh[(d + e*x)/2])/Sqrt[-a^2 -
 c^2]])/Sqrt[-a^2 - c^2] + B*Log[a + c*Sinh[d + e*x]])/(c*e)
```

Rubi [A] (warning: unable to verify)

Time = 0.55 (sec) , antiderivative size = 79, normalized size of antiderivative = 0.98, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.323$, Rules used = {3042, 4876, 3042, 3147, 16, 3214, 3042, 3139, 1083, 217}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+B\cosh(d+ex)+C\sinh(d+ex)}{a+c\sinh(d+ex)}dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \frac{A+B\cos(id+iex)-iC\sin(id+iex)}{a-ic\sin(id+iex)}dx$$

$$\Big\downarrow \text{ 4876}$$

$$\int \frac{A+C\sinh(d+ex)}{a+c\sinh(d+ex)}dx+B\int \frac{\cosh(d+ex)}{a+c\sinh(d+ex)}dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \frac{A-iC\sin(id+iex)}{a-ic\sin(id+iex)}dx+B\int \frac{\cos(id+iex)}{a-ic\sin(id+iex)}dx$$

$$\Big\downarrow \text{ 3147}$$

$$\frac{B\int \frac{1}{a+c\sinh(d+ex)}d(c\sinh(d+ex))}{ce}+\int \frac{A-iC\sin(id+iex)}{a-ic\sin(id+iex)}dx$$

$$\Big\downarrow \text{ 16}$$

$$\frac{B\log(a+c\sinh(d+ex))}{ce}+\int \frac{A-iC\sin(id+iex)}{a-ic\sin(id+iex)}dx$$

$$\Big\downarrow \text{ 3214}$$

$$\frac{(Ac-aC)\int \frac{1}{a+c\sinh(d+ex)}dx}{c}+\frac{B\log(a+c\sinh(d+ex))}{ce}+\frac{Cx}{c}$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{(Ac-aC)\int \frac{1}{a-ic\sin(id+iex)}dx}{c}+\frac{B\log(a+c\sinh(d+ex))}{ce}+\frac{Cx}{c}$$

$$\Big\downarrow \text{ 3139}$$

$$-\frac{2i(Ac-aC)\int \frac{1}{-a\tanh^2\left(\frac{1}{2}(d+ex)\right)+2c\tanh\left(\frac{1}{2}(d+ex)\right)+a}d\left(i\tanh\left(\frac{1}{2}(d+ex)\right)\right)}{ce}+\frac{B\log(a+c\sinh(d+ex))}{ce}+\frac{Cx}{c}$$

$$\Big\downarrow \text{ 1083}$$

$$\frac{4i(Ac-aC)\int \frac{1}{\tanh^2\left(\frac{1}{2}(d+ex)\right)-4(a^2+c^2)}d\left(2ia\tanh\left(\frac{1}{2}(d+ex)\right)-2ic\right)}{ce}+\frac{B\log(a+c\sinh(d+ex))}{ce}+\frac{Cx}{c}$$

$$\Big\downarrow \text{ 217}$$

$$\frac{2(Ac-aC)\text{arctanh}\left(\frac{\tanh\left(\frac{1}{2}(d+ex)\right)}{2\sqrt{a^2+c^2}}\right)}{ce\sqrt{a^2+c^2}}+\frac{B\log(a+c\sinh(d+ex))}{ce}+\frac{Cx}{c}$$

input
```
Int[(A + B*Cosh[d + e*x] + C*Sinh[d + e*x])/(a + c*Sinh[d + e*x]),x]
```

output
```
(C*x)/c + (2*(A*c - a*C)*ArcTanh[Tanh[(d + e*x)/2]/(2*Sqrt[a^2 + c^2])])/(
c*Sqrt[a^2 + c^2]*e) + (B*Log[a + c*Sinh[d + e*x]])/(c*e)
```

Defintions of rubi rules used

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 1083
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Simp[-2   Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3139
```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Simp[2*(e/d)   Subst[Int[1/(a + 2*b*e*x + a
*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[a^2 - b^2, 0]
```

rule 3147
```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Simp[1/(b^p*f)   Subst[Int[(a + x)^m*(b^2 - x^2)^((p - 1)
/2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && IntegerQ[(p
 - 1)/2] && NeQ[a^2 - b^2, 0]
```

rule 3214
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])/((c_.) + (d_.)*sin[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[b*(x/d), x] - Simp[(b*c - a*d)/d   Int[1/(c + d
*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

rule 4876
```
Int[(u_)*((v_) + (d_.)*(F_)[(c_.)*((a_.) + (b_.)*(x_))]^(n_.)), x_Symbol] :
> With[{e = FreeFactors[Sin[c*(a + b*x)], x]}, Int[ActivateTrig[u*v], x] +
Simp[d   Int[ActivateTrig[u]*Cos[c*(a + b*x)]^n, x], x] /; FunctionOfQ[Sin[
c*(a + b*x)]/e, u, x]] /; FreeQ[{a, b, c, d}, x] && !FreeQ[v, x] && Intege
rQ[(n - 1)/2] && NonsumQ[u] && (EqQ[F, Cos] || EqQ[F, cos])
```

Maple [A] (verified)

Time = 1.37 (sec) , antiderivative size = 111, normalized size of antiderivative = 1.37

method	result
parts	$\frac{\frac{C\ln\left(\tanh\left(\frac{ex}{2}+\frac{d}{2}\right)+1\right)}{c}-\frac{2(-Ac+Ca)\operatorname{arctanh}\left(\frac{2a\tanh\left(\frac{ex}{2}+\frac{d}{2}\right)-2c}{2\sqrt{a^2+c^2}}\right)}{c\sqrt{a^2+c^2}}-\frac{C\ln\left(\tanh\left(\frac{ex}{2}+\frac{d}{2}\right)-1\right)}{c}}{e}+\frac{B\ln(a+c\sinh(ex+d))}{ce}$
derivativedivides	$\frac{\frac{(-B-C)\ln\left(\tanh\left(\frac{ex}{2}+\frac{d}{2}\right)-1\right)}{c}+\frac{B\ln\left(a\tanh\left(\frac{ex}{2}+\frac{d}{2}\right)^2-2c\tanh\left(\frac{ex}{2}+\frac{d}{2}\right)-a\right)-\frac{2(-Ac+Ca)\operatorname{arctanh}\left(\frac{2a\tanh\left(\frac{ex}{2}+\frac{d}{2}\right)-2c}{2\sqrt{a^2+c^2}}\right)}{\sqrt{a^2+c^2}}}{c}+}{e}$
default	$\frac{\frac{(-B-C)\ln\left(\tanh\left(\frac{ex}{2}+\frac{d}{2}\right)-1\right)}{c}+\frac{B\ln\left(a\tanh\left(\frac{ex}{2}+\frac{d}{2}\right)^2-2c\tanh\left(\frac{ex}{2}+\frac{d}{2}\right)-a\right)-\frac{2(-Ac+Ca)\operatorname{arctanh}\left(\frac{2a\tanh\left(\frac{ex}{2}+\frac{d}{2}\right)-2c}{2\sqrt{a^2+c^2}}\right)}{\sqrt{a^2+c^2}}}{c}+}{e}$
risch	$\frac{xB}{c}+\frac{Cx}{c}-\frac{2Ba^2ce^2x}{a^2c^2e^2+c^4e^2}-\frac{2Bc^3e^2x}{a^2c^2e^2+c^4e^2}-\frac{2Ba^2cde}{a^2c^2e^2+c^4e^2}-\frac{2Bc^3de}{a^2c^2e^2+c^4e^2}+\frac{\ln\left(e^{ex+d}+\frac{Aac-a^2C-\sqrt{A^2a^2c}}{}\right.}{}$

input
```
int((A+B*cosh(e*x+d)+C*sinh(e*x+d))/(a+c*sinh(e*x+d)),x,method=_RETURNVERB
OSE)
```

output
```
1/e*(C/c*ln(tanh(1/2*e*x+1/2*d)+1)-2*(-A*c+C*a)/c/(a^2+c^2)^(1/2)*arctanh(
1/2*(2*a*tanh(1/2*e*x+1/2*d)-2*c)/(a^2+c^2)^(1/2))-C/c*ln(tanh(1/2*e*x+1/2
*d)-1))+B*ln(a+c*sinh(e*x+d))/c/e
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 249 vs. $2(78) = 156$.

Time = 0.10 (sec) , antiderivative size = 249, normalized size of antiderivative = 3.07

$$\int \frac{A+B\cosh(d+ex)+C\sinh(d+ex)}{a+c\sinh(d+ex)}\,dx = -\frac{((B-C)a^2+(B-C)c^2)ex+(Ca-Ac)\sqrt{a^2+c^2}\log\left(\frac{c^2\cosh(ex+d)^2+c^2\sinh(ex+d)^2+2ac\cosh(ex+d)+2a^2+c^2+}{c\cosh(ex+d)^2+c\sinh(ex+d)^2+2}\right.}{(a^2c+}$$

input
```
integrate((A+B*cosh(e*x+d)+C*sinh(e*x+d))/(a+c*sinh(e*x+d)),x, algorithm="
fricas")
```

output
```
-(((B - C)*a^2 + (B - C)*c^2)*e*x + (C*a - A*c)*sqrt(a^2 + c^2)*log((c^2*c
osh(e*x + d)^2 + c^2*sinh(e*x + d)^2 + 2*a*c*cosh(e*x + d) + 2*a^2 + c^2 +
 2*(c^2*cosh(e*x + d) + a*c)*sinh(e*x + d) - 2*sqrt(a^2 + c^2)*(c*cosh(e*x
 + d) + c*sinh(e*x + d) + a))/(c*cosh(e*x + d)^2 + c*sinh(e*x + d)^2 + 2*a
*cosh(e*x + d) + 2*(c*cosh(e*x + d) + a)*sinh(e*x + d) - c)) - (B*a^2 + B*
c^2)*log(2*(c*sinh(e*x + d) + a)/(cosh(e*x + d) - sinh(e*x + d))))/((a^2*c
 + c^3)*e)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 21.03 (sec) , antiderivative size = 1318, normalized size of antiderivative = 16.27

$$\int \frac{A + B\cosh(d + ex) + C\sinh(d + ex)}{a + c\sinh(d + ex)}\, dx = \text{Too large to display}$$

input
```
integrate((A+B*cosh(e*x+d)+C*sinh(e*x+d))/(a+c*sinh(e*x+d)),x)
```

output
```
Piecewise((zoo*x*(A + B*cosh(d) + C*sinh(d))/sinh(d), Eq(a, 0) & Eq(c, 0)
& Eq(e, 0)), ((A*log(tanh(d/2 + e*x/2))/e + B*x - 2*B*log(tanh(d/2 + e*x/2
) + 1)/e + B*log(tanh(d/2 + e*x/2))/e + C*x)/c, Eq(a, 0)), (2*I*A/(c*e*tan
h(d/2 + e*x/2) - I*c*e) + B*e*x*tanh(d/2 + e*x/2)/(c*e*tanh(d/2 + e*x/2) -
 I*c*e) - I*B*e*x/(c*e*tanh(d/2 + e*x/2) - I*c*e) - 2*B*log(tanh(d/2 + e*x
/2) + 1)*tanh(d/2 + e*x/2)/(c*e*tanh(d/2 + e*x/2) - I*c*e) + 2*I*B*log(tan
h(d/2 + e*x/2) + 1)/(c*e*tanh(d/2 + e*x/2) - I*c*e) + 2*B*log(tanh(d/2 + e
*x/2) - I)*tanh(d/2 + e*x/2)/(c*e*tanh(d/2 + e*x/2) - I*c*e) - 2*I*B*log(t
anh(d/2 + e*x/2) - I)/(c*e*tanh(d/2 + e*x/2) - I*c*e) + C*e*x*tanh(d/2 + e
*x/2)/(c*e*tanh(d/2 + e*x/2) - I*c*e) - I*C*e*x/(c*e*tanh(d/2 + e*x/2) - I
*c*e) - 2*C/(c*e*tanh(d/2 + e*x/2) - I*c*e), Eq(a, -I*c)), (-2*I*A/(c*e*ta
nh(d/2 + e*x/2) + I*c*e) + B*e*x*tanh(d/2 + e*x/2)/(c*e*tanh(d/2 + e*x/2)
+ I*c*e) + I*B*e*x/(c*e*tanh(d/2 + e*x/2) + I*c*e) - 2*B*log(tanh(d/2 + e*
x/2) + 1)*tanh(d/2 + e*x/2)/(c*e*tanh(d/2 + e*x/2) + I*c*e) - 2*I*B*log(ta
nh(d/2 + e*x/2) + 1)/(c*e*tanh(d/2 + e*x/2) + I*c*e) + 2*B*log(tanh(d/2 +
e*x/2) + I)*tanh(d/2 + e*x/2)/(c*e*tanh(d/2 + e*x/2) + I*c*e) + 2*I*B*log(
tanh(d/2 + e*x/2) + I)/(c*e*tanh(d/2 + e*x/2) + I*c*e) + C*e*x*tanh(d/2 +
e*x/2)/(c*e*tanh(d/2 + e*x/2) + I*c*e) + I*C*e*x/(c*e*tanh(d/2 + e*x/2) +
I*c*e) - 2*C/(c*e*tanh(d/2 + e*x/2) + I*c*e), Eq(a, I*c)), ((A*x + B*sinh(
d + e*x)/e + C*cosh(d + e*x)/e)/a, Eq(c, 0)), (x*(A + B*cosh(d) + C*sin...
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 176 vs. $2(78) = 156$.

Time = 0.12 (sec) , antiderivative size = 176, normalized size of antiderivative = 2.17

$$\begin{aligned}
&\int \frac{A + B\cosh(d+ex) + C\sinh(d+ex)}{a + c\sinh(d+ex)}\,dx \\
&= -C\left(\frac{a\log\left(\frac{ce^{(-ex-d)}-a-\sqrt{a^2+c^2}}{ce^{(-ex-d)}-a+\sqrt{a^2+c^2}}\right)}{\sqrt{a^2+c^2}ce} - \frac{ex+d}{ce}\right) \\
&\quad + \frac{A\log\left(\frac{ce^{(-ex-d)}-a-\sqrt{a^2+c^2}}{ce^{(-ex-d)}-a+\sqrt{a^2+c^2}}\right)}{\sqrt{a^2+c^2}e} + \frac{B\log\left(c\sinh\left(ex+d\right)+a\right)}{ce}
\end{aligned}$$

input
```
integrate((A+B*cosh(e*x+d)+C*sinh(e*x+d))/(a+c*sinh(e*x+d)),x, algorithm="
maxima")
```

output
```
-C*(a*log((c*e^(-e*x - d) - a - sqrt(a^2 + c^2))/(c*e^(-e*x - d) - a + sqr
t(a^2 + c^2)))/(sqrt(a^2 + c^2)*c*e) - (e*x + d)/(c*e)) + A*log((c*e^(-e*x
 - d) - a - sqrt(a^2 + c^2))/(c*e^(-e*x - d) - a + sqrt(a^2 + c^2)))/(sqrt
(a^2 + c^2)*e) + B*log(c*sinh(e*x + d) + a)/(c*e)
```

Giac [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 127, normalized size of antiderivative = 1.57

$$\int \frac{A+B\cosh(d+ex)+C\sinh(d+ex)}{a+c\sinh(d+ex)}\,dx = -\frac{\frac{(ex+d)(B-C)}{c} - \frac{B\log\left(\left|ce^{(2\,ex+2\,d)}+2\,ae^{(ex+d)}-c\right|\right)}{c} + \frac{(Ca-Ac)\log\left(\frac{\left|2\,ce^{(ex+d)}+2\,a-2\,\sqrt{a^2+c^2}\right|}{\left|2\,ce^{(ex+d)}+2\,a+2\,\sqrt{a^2+c^2}\right|}\right)}{\sqrt{a^2+c^2}c}}{e}$$

input
```
integrate((A+B*cosh(e*x+d)+C*sinh(e*x+d))/(a+c*sinh(e*x+d)),x, algorithm="
giac")
```

output
```
-((e*x + d)*(B - C)/c - B*log(abs(c*e^(2*e*x + 2*d) + 2*a*e^(e*x + d) - c)
)/c + (C*a - A*c)*log(abs(2*c*e^(e*x + d) + 2*a - 2*sqrt(a^2 + c^2))/abs(2
*c*e^(e*x + d) + 2*a + 2*sqrt(a^2 + c^2)))/(sqrt(a^2 + c^2)*c))/e
```

Mupad [B] (verification not implemented)

Time = 2.70 (sec) , antiderivative size = 656, normalized size of antiderivative = 8.10

$$\int \frac{A+B\cosh(d+ex)+C\sinh(d+ex)}{a+c\sinh(d+ex)}\,dx = \frac{C\,x}{c} - \frac{B\,x}{c}$$
$$-\frac{2\,\mathrm{atan}\left(\frac{a\,\sqrt{-a^2\,c^2\,e^2-c^4\,e^2}\,\sqrt{A^2\,c^2-2\,A\,C\,a\,c+C^2\,a^2}}{-C\,e\,a^3\,c+A\,e\,a^2\,c^2-C\,e\,a\,c^3+A\,e\,c^4} - \frac{a^2\,c^2\,\mathrm{e}^{e\,x}\,\mathrm{e}^d\,\sqrt{-a^2\,c^2\,e^2-c^4\,e^2}\,\sqrt{A^2\,c^2-2\,A\,C\,a\,c+C^2\,a^2}}{-C\,e\,a^3\,c^4+A\,e\,a^2\,c^5-C\,e\,a\,c^6+A\,e\,c^7} + \frac{A\,\mathrm{e}^{e\,x}\,\mathrm{e}^d\,\sqrt{-a^2\,c^2}}{c\,e\,\sqrt{A^2\,c^2-2\,A\,C}}\right.}{\sqrt{-a^2\,c^2\,e^2-c^4\,e^2}}$$
$$+\frac{B\,c^3\,e\,\ln\left(8\,A\,C\,a\,c^2-4\,C^2\,a^2\,c-4\,A^2\,c^3+8\,C^2\,a^3\,\mathrm{e}^{e\,x}\,\mathrm{e}^d+4\,A^2\,c^3\,\mathrm{e}^{2\,d}\,\mathrm{e}^{2\,e\,x}+8\,A^2\,a\,c^2\,\mathrm{e}^{e\,x}\,\mathrm{e}^d+4\,C^2\,a^2\right.}{a^2\,c^2\,e^2+c^4\,e^2}$$
$$+\frac{B\,a^2\,c\,e\,\ln\left(8\,A\,C\,a\,c^2-4\,C^2\,a^2\,c-4\,A^2\,c^3+8\,C^2\,a^3\,\mathrm{e}^{e\,x}\,\mathrm{e}^d+4\,A^2\,c^3\,\mathrm{e}^{2\,d}\,\mathrm{e}^{2\,e\,x}+8\,A^2\,a\,c^2\,\mathrm{e}^{e\,x}\,\mathrm{e}^d+4\,C^2\right.}{a^2\,c^2\,e^2+c^4\,e^2}$$

input
```
int((A + B*cosh(d + e*x) + C*sinh(d + e*x))/(a + c*sinh(d + e*x)),x)
```

output
```
(C*x)/c - (B*x)/c - (2*atan((a*(- c^4*e^2 - a^2*c^2*e^2)^(1/2)*(A^2*c^2 +
C^2*a^2 - 2*A*C*a*c)^(1/2))/(A*c^4*e - C*a*c^3*e - C*a^3*c*e + A*a^2*c^2*e
) - (a^2*c^2*exp(e*x)*exp(d)*(- c^4*e^2 - a^2*c^2*e^2)^(1/2)*(A^2*c^2 + C^
2*a^2 - 2*A*C*a*c)^(1/2))/(A*c^7*e - C*a*c^6*e + A*a^2*c^5*e - C*a^3*c^4*e
) + (A*exp(e*x)*exp(d)*(- c^4*e^2 - a^2*c^2*e^2)^(1/2))/(c*e*(A^2*c^2 + C^
2*a^2 - 2*A*C*a*c)^(1/2)) - (C*a*exp(e*x)*exp(d)*(- c^4*e^2 - a^2*c^2*e^2)
^(1/2))/(c^2*e*(A^2*c^2 + C^2*a^2 - 2*A*C*a*c)^(1/2)))*(A^2*c^2 + C^2*a^2
- 2*A*C*a*c)^(1/2))/(- c^4*e^2 - a^2*c^2*e^2)^(1/2) + (B*c^3*e*log(8*A*C*a
*c^2 - 4*C^2*a^2*c - 4*A^2*c^3 + 8*C^2*a^3*exp(e*x)*exp(d) + 4*A^2*c^3*exp
(2*d)*exp(2*e*x) + 8*A^2*a*c^2*exp(e*x)*exp(d) + 4*C^2*a^2*c*exp(2*d)*exp(
2*e*x) - 16*A*C*a^2*c*exp(e*x)*exp(d) - 8*A*C*a*c^2*exp(2*d)*exp(2*e*x)))/
(c^4*e^2 + a^2*c^2*e^2) + (B*a^2*c*e*log(8*A*C*a*c^2 - 4*C^2*a^2*c - 4*A^2
*c^3 + 8*C^2*a^3*exp(e*x)*exp(d) + 4*A^2*c^3*exp(2*d)*exp(2*e*x) + 8*A^2*a
*c^2*exp(e*x)*exp(d) + 4*C^2*a^2*c*exp(2*d)*exp(2*e*x) - 16*A*C*a^2*c*exp(
e*x)*exp(d) - 8*A*C*a*c^2*exp(2*d)*exp(2*e*x)))/(c^4*e^2 + a^2*c^2*e^2)
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.31

$$\int \frac{A + B\cosh(d + ex) + C\sinh(d + ex)}{a + c\sinh(d + ex)}\, dx = \frac{\log(\sinh\left(ex + d\right)c + a\right)b + cex}{ce}$$

input
```
int((A+B*cosh(e*x+d)+C*sinh(e*x+d))/(a+c*sinh(e*x+d)),x)
```

output
```
(log(sinh(d + e*x)*c + a)*b + c*e*x)/(c*e)
```

3.254 $\int \frac{A+B\cosh(d+ex)+C\sinh(d+ex)}{(a+c\sinh(d+ex))^2}\,dx$

Optimal result

Integrand size = 31, antiderivative size = 113

$$\int \frac{A+B\cosh(d+ex)+C\sinh(d+ex)}{(a+c\sinh(d+ex))^2}\,dx = -\frac{2(aA+cC)\operatorname{arctanh}\left(\frac{c-a\tanh\left(\frac{1}{2}(d+ex)\right)}{\sqrt{a^2+c^2}}\right)}{\left(a^2+c^2\right)^{3/2}e} - \frac{B}{ce(a+c\sinh(d+ex))} - \frac{(Ac-aC)\cosh(d+ex)}{(a^2+c^2)\,e(a+c\sinh(d+ex))}$$

output
```
-2*(A*a+C*c)*arctanh((c-a*tanh(1/2*e*x+1/2*d))/(a^2+c^2)^(1/2))/(a^2+c^2)^
(3/2)/e-B/c/e/(a+c*sinh(e*x+d))-(A*c-C*a)*cosh(e*x+d)/(a^2+c^2)/e/(a+c*sin
h(e*x+d))
```

Mathematica [A] (verified)

Time = 1.22 (sec) , antiderivative size = 113, normalized size of antiderivative = 1.00

$$\int \frac{A+B\cosh(d+ex)+C\sinh(d+ex)}{(a+c\sinh(d+ex))^2}\,dx = \frac{\frac{2(aA+cC)\arctan\left(\frac{c-a\tanh\left(\frac{1}{2}(d+ex)\right)}{\sqrt{-a^2-c^2}}\right)}{\sqrt{-a^2-c^2}} - \frac{B(a^2+c^2)+c(Ac-aC)\cosh(d+ex)}{c(a+c\sinh(d+ex))}}{(a^2+c^2)\,e}$$

input
```
Integrate[(A + B*Cosh[d + e*x] + C*Sinh[d + e*x])/(a + c*Sinh[d + e*x])^2,
x]
```

output
```
((2*(a*A + c*C)*ArcTan[(c - a*Tanh[(d + e*x)/2])/Sqrt[-a^2 - c^2]])/Sqrt[-
a^2 - c^2] - (B*(a^2 + c^2) + c*(A*c - a*C)*Cosh[d + e*x])/(c*(a + c*Sinh[
d + e*x])))/((a^2 + c^2)*e)
```

Rubi [A] (warning: unable to verify)

Time = 0.62 (sec) , antiderivative size = 111, normalized size of antiderivative = 0.98, number of steps used = 13, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.387$, Rules used = {3042, 4876, 3042, 3147, 17, 3233, 25, 27, 3042, 3139, 1083, 217}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+B\cosh(d+ex)+C\sinh(d+ex)}{(a+c\sinh(d+ex))^2}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{A+B\cos(id+iex)-iC\sin(id+iex)}{(a-ic\sin(id+iex))^2}dx$$

$$\Big\downarrow \text{4876}$$

$$\int \frac{A+C\sinh(d+ex)}{(a+c\sinh(d+ex))^2}dx+B\int \frac{\cosh(d+ex)}{(a+c\sinh(d+ex))^2}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{A-iC\sin(id+iex)}{(a-ic\sin(id+iex))^2}dx+B\int \frac{\cos(id+iex)}{(a-ic\sin(id+iex))^2}dx$$

$$\Big\downarrow \text{3147}$$

$$\frac{B\int \frac{1}{(a+c\sinh(d+ex))^2}d(c\sinh(d+ex))}{ce}+\int \frac{A-iC\sin(id+iex)}{(a-ic\sin(id+iex))^2}dx$$

$$\Big\downarrow \text{17}$$

$$-\frac{B}{ce(a+c\sinh(d+ex))}+\int \frac{A-iC\sin(id+iex)}{(a-ic\sin(id+iex))^2}dx$$

$$\Big\downarrow \ 3233$$

$$-\frac{\int -\frac{aA+cC}{a+c\sinh(d+ex)}dx}{a^2+c^2}-\frac{(Ac-aC)\cosh(d+ex)}{e\left(a^2+c^2\right)(a+c\sinh(d+ex))}-\frac{B}{ce(a+c\sinh(d+ex))}$$

$$\Big\downarrow \ 25$$

$$\frac{\int \frac{aA+cC}{a+c\sinh(d+ex)}dx}{a^2+c^2}-\frac{(Ac-aC)\cosh(d+ex)}{e\left(a^2+c^2\right)(a+c\sinh(d+ex))}-\frac{B}{ce(a+c\sinh(d+ex))}$$

$$\Big\downarrow \ 27$$

$$\frac{(aA+cC)\int \frac{1}{a+c\sinh(d+ex)}dx}{a^2+c^2}-\frac{(Ac-aC)\cosh(d+ex)}{e\left(a^2+c^2\right)(a+c\sinh(d+ex))}-\frac{B}{ce(a+c\sinh(d+ex))}$$

$$\Big\downarrow \ 3042$$

$$\frac{(aA+cC)\int \frac{1}{a-ic\sin(id+iex)}dx}{a^2+c^2}-\frac{(Ac-aC)\cosh(d+ex)}{e\left(a^2+c^2\right)(a+c\sinh(d+ex))}-\frac{B}{ce(a+c\sinh(d+ex))}$$

$$\Big\downarrow \ 3139$$

$$-\frac{2i(aA+cC)\int \frac{1}{-a\tanh^2\left(\frac{1}{2}(d+ex)\right)+2c\tanh\left(\frac{1}{2}(d+ex)\right)+a}d\left(i\tanh\left(\frac{1}{2}(d+ex)\right)\right)}{e\left(a^2+c^2\right)}-\frac{(Ac-aC)\cosh(d+ex)}{e\left(a^2+c^2\right)(a+c\sinh(d+ex))}-\frac{B}{ce(a+c\sinh(d+ex))}$$

$$\Big\downarrow \ 1083$$

$$\frac{4i(aA+cC)\int \frac{1}{\tanh^2\left(\frac{1}{2}(d+ex)\right)-4\left(a^2+c^2\right)}d\left(2ia\tanh\left(\frac{1}{2}(d+ex)\right)-2ic\right)}{e\left(a^2+c^2\right)}-\frac{(Ac-aC)\cosh(d+ex)}{e\left(a^2+c^2\right)(a+c\sinh(d+ex))}-\frac{B}{ce(a+c\sinh(d+ex))}$$

$$\Big\downarrow \ 217$$

$$\frac{2(aA+cC)\text{arctanh}\left(\frac{\tanh\left(\frac{1}{2}(d+ex)\right)}{2\sqrt{a^2+c^2}}\right)}{e\left(a^2+c^2\right)^{3/2}}-\frac{(Ac-aC)\cosh(d+ex)}{e\left(a^2+c^2\right)(a+c\sinh(d+ex))}-\frac{B}{ce(a+c\sinh(d+ex))}$$

input
```
Int[(A + B*Cosh[d + e*x] + C*Sinh[d + e*x])/(a + c*Sinh[d + e*x])^2,x]
```

output
```
(2*(a*A + c*C)*ArcTanh[Tanh[(d + e*x)/2]/(2*Sqrt[a^2 + c^2])])/((a^2 + c^2
)^(3/2)*e) - B/(c*e*(a + c*Sinh[d + e*x])) - ((A*c - a*C)*Cosh[d + e*x])/(
(a^2 + c^2)*e*(a + c*Sinh[d + e*x]))
```

Defintions of rubi rules used

rule 17
```
Int[(c_.)*((a_.) + (b_.)*(x_))^(m_.), x_Symbol] :> Simp[c*((a + b*x)^(m + 1
)/(b*(m + 1))), x] /; FreeQ[{a, b, c, m}, x] && NeQ[m, -1]
```

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 1083
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Simp[-2   Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3139
```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Simp[2*(e/d)   Subst[Int[1/(a + 2*b*e*x + a
*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[a^2 - b^2, 0]
```

rule 3147
```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Simp[1/(b^p*f)   Subst[Int[(a + x)^m*(b^2 - x^2)^((p - 1)
/2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && IntegerQ[(p
 - 1)/2] && NeQ[a^2 - b^2, 0]
```

rule 3233
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Simp[(-(b*c - a*d))*Cos[e + f*x]*((a + b*Sin[e +
 f*x])^(m + 1)/(f*(m + 1)*(a^2 - b^2))), x] + Simp[1/((m + 1)*(a^2 - b^2))
  Int[(a + b*Sin[e + f*x])^(m + 1)*Simp[(a*c - b*d)*(m + 1) - (b*c - a*d)*(
m + 2)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c
- a*d, 0] && NeQ[a^2 - b^2, 0] && LtQ[m, -1] && IntegerQ[2*m]
```

rule 4876
```
Int[(u_)*((v_) + (d_.)*(F_)[(c_.)*((a_.) + (b_.)*(x_))]^(n_.)), x_Symbol] :
> With[{e = FreeFactors[Sin[c*(a + b*x)], x]}, Int[ActivateTrig[u*v], x] +
Simp[d   Int[ActivateTrig[u]*Cos[c*(a + b*x)]^n, x], x] /; FunctionOfQ[Sin[
c*(a + b*x)]/e, u, x]] /; FreeQ[{a, b, c, d}, x] &&  !FreeQ[v, x] && Intege
rQ[(n - 1)/2] && NonsumQ[u] && (EqQ[F, Cos] || EqQ[F, cos])
```

Maple [A] (verified)

Time = 4.59 (sec) , antiderivative size = 151, normalized size of antiderivative = 1.34

method	result
derivativedivides	$\frac{-\frac{2\left(-\frac{\left(A\,c^{2}-B\,a^{2}-B\,c^{2}-C a c\right)\tanh\left(\frac{e x}{2}+\frac{d}{2}\right)}{a\left(a^{2}+c^{2}\right)}-\frac{A c-C a}{a^{2}+c^{2}}\right)}{a\tanh\left(\frac{e x}{2}+\frac{d}{2}\right)^{2}-2 c\tanh\left(\frac{e x}{2}+\frac{d}{2}\right)-a}+\frac{2(A a+c C)\,\operatorname{arctanh}\left(\frac{2 a\tanh\left(\frac{e x}{2}+\frac{d}{2}\right)-2 c}{2\sqrt{a^{2}+c^{2}}}\right)}{\left(a^{2}+c^{2}\right)^{\frac{3}{2}}}}{e}$
default	$\frac{-\frac{2\left(-\frac{\left(A\,c^{2}-B\,a^{2}-B\,c^{2}-C a c\right)\tanh\left(\frac{e x}{2}+\frac{d}{2}\right)}{a\left(a^{2}+c^{2}\right)}-\frac{A c-C a}{a^{2}+c^{2}}\right)}{a\tanh\left(\frac{e x}{2}+\frac{d}{2}\right)^{2}-2 c\tanh\left(\frac{e x}{2}+\frac{d}{2}\right)-a}+\frac{2(A a+c C)\,\operatorname{arctanh}\left(\frac{2 a\tanh\left(\frac{e x}{2}+\frac{d}{2}\right)-2 c}{2\sqrt{a^{2}+c^{2}}}\right)}{\left(a^{2}+c^{2}\right)^{\frac{3}{2}}}}{e}$
parts	$\frac{-\frac{2\left(-\frac{c(A c-C a)\tanh\left(\frac{e x}{2}+\frac{d}{2}\right)}{a\left(a^{2}+c^{2}\right)}-\frac{A c-C a}{a^{2}+c^{2}}\right)}{a\tanh\left(\frac{e x}{2}+\frac{d}{2}\right)^{2}-2 c\tanh\left(\frac{e x}{2}+\frac{d}{2}\right)-a}+\frac{2(A a+c C)\,\operatorname{arctanh}\left(\frac{2 a\tanh\left(\frac{e x}{2}+\frac{d}{2}\right)-2 c}{2\sqrt{a^{2}+c^{2}}}\right)}{\left(a^{2}+c^{2}\right)^{\frac{3}{2}}}}{e}-\frac{B}{c e(a+c\sinh(e x+d))}$
risch	$\frac{2 A a c\,\mathrm{e}^{e x+d}-2 B\,a^{2}\mathrm{e}^{e x+d}-2 B\,c^{2}\mathrm{e}^{e x+d}-2 C\,a^{2}\mathrm{e}^{e x+d}-2 A\,c^{2}+2 C a c}{c e\left(a^{2}+c^{2}\right)\left(c\,\mathrm{e}^{2 e x+2 d}+2 a\,\mathrm{e}^{e x+d}-c\right)}+\frac{\ln\left(\mathrm{e}^{e x+d}+\frac{\left(a^{2}+c^{2}\right)^{\frac{3}{2}} a-a^{4}-2 a^{2} c^{2}-c^{4}}{c\left(a^{2}+c^{2}\right)^{\frac{3}{2}}}\right) A a}{\left(a^{2}+c^{2}\right)^{\frac{3}{2}} e}+$

input
```
int((A+B*cosh(e*x+d)+C*sinh(e*x+d))/(a+c*sinh(e*x+d))^2,x,method=_RETURNVE
RBOSE)
```

output
```
1/e*(-2*(-(A*c^2-B*a^2-B*c^2-C*a*c)/a/(a^2+c^2)*tanh(1/2*e*x+1/2*d)-(A*c-C
*a)/(a^2+c^2))/(a*tanh(1/2*e*x+1/2*d)^2-2*c*tanh(1/2*e*x+1/2*d)-a)+2*(A*a+
C*c)/(a^2+c^2)^(3/2)*arctanh(1/2*(2*a*tanh(1/2*e*x+1/2*d)-2*c)/(a^2+c^2)^(
1/2)))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 570 vs. $2(109) = 218$.

Time = 0.10 (sec) , antiderivative size = 570, normalized size of antiderivative = 5.04

$$\int \frac{A + B\cosh(d+ex) + C\sinh(d+ex)}{(a+c\sinh(d+ex))^2}\,dx = \frac{2\,Ca^3c - 2\,Aa^2c^2 + 2\,Cac^3 - 2\,Ac^4 - \left(Aac^2 + Cc^3 - (Aac^2 + Cc^3)\cosh\left(ex+d\right)^2 - (Aac^2 + Cc^3)\sinh\right.}{}$$

input
```
integrate((A+B*cosh(e*x+d)+C*sinh(e*x+d))/(a+c*sinh(e*x+d))^2,x, algorithm
="fricas")
```

output
```
(2*C*a^3*c - 2*A*a^2*c^2 + 2*C*a*c^3 - 2*A*c^4 - (A*a*c^2 + C*c^3 - (A*a*c
^2 + C*c^3)*cosh(e*x + d)^2 - (A*a*c^2 + C*c^3)*sinh(e*x + d)^2 - 2*(A*a^2
*c + C*a*c^2)*cosh(e*x + d) - 2*(A*a^2*c + C*a*c^2 + (A*a*c^2 + C*c^3)*cos
h(e*x + d))*sinh(e*x + d))*sqrt(a^2 + c^2)*log((c^2*cosh(e*x + d)^2 + c^2*
sinh(e*x + d)^2 + 2*a*c*cosh(e*x + d) + 2*a^2 + c^2 + 2*(c^2*cosh(e*x + d)
 + a*c)*sinh(e*x + d) - 2*sqrt(a^2 + c^2)*(c*cosh(e*x + d) + c*sinh(e*x +
d) + a))/(c*cosh(e*x + d)^2 + c*sinh(e*x + d)^2 + 2*a*cosh(e*x + d) + 2*(c
*cosh(e*x + d) + a)*sinh(e*x + d) - c)) - 2*((B + C)*a^4 - A*a^3*c + (2*B
+ C)*a^2*c^2 - A*a*c^3 + B*c^4)*cosh(e*x + d) - 2*((B + C)*a^4 - A*a^3*c +
 (2*B + C)*a^2*c^2 - A*a*c^3 + B*c^4)*sinh(e*x + d))/((a^4*c^2 + 2*a^2*c^4
 + c^6)*e*cosh(e*x + d)^2 + (a^4*c^2 + 2*a^2*c^4 + c^6)*e*sinh(e*x + d)^2
+ 2*(a^5*c + 2*a^3*c^3 + a*c^5)*e*cosh(e*x + d) - (a^4*c^2 + 2*a^2*c^4 + c
^6)*e + 2*((a^4*c^2 + 2*a^2*c^4 + c^6)*e*cosh(e*x + d) + (a^5*c + 2*a^3*c^
3 + a*c^5)*e)*sinh(e*x + d))
```

Sympy [F(-1)]

Timed out.

$$\int \frac{A+B\cosh(d+ex)+C\sinh(d+ex)}{(a+c\sinh(d+ex))^2}\,dx = \text{Timed out}$$

input
```
integrate((A+B*cosh(e*x+d)+C*sinh(e*x+d))/(a+c*sinh(e*x+d))**2,x)
```

output
```
Timed out
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 339 vs. $2(109) = 218$.

Time = 0.15 (sec) , antiderivative size = 339, normalized size of antiderivative = 3.00

$$\begin{aligned}
&\int \frac{A+B\cosh(d+ex)+C\sinh(d+ex)}{(a+c\sinh(d+ex))^2}\,dx \\
&= A\left(\frac{a\log\left(\frac{ce^{(-ex-d)}-a-\sqrt{a^2+c^2}}{ce^{(-ex-d)}-a+\sqrt{a^2+c^2}}\right)}{(a^2+c^2)^{\frac{3}{2}}e} - \frac{2\left(ae^{(-ex-d)}+c\right)}{(a^2c+c^3+2\,(a^3+ac^2)e^{(-ex-d)}-(a^2c+c^3)e^{(-2\,ex-2\,d)})e}\right) \\
&\quad + C\left(\frac{c\log\left(\frac{ce^{(-ex-d)}-a-\sqrt{a^2+c^2}}{ce^{(-ex-d)}-a+\sqrt{a^2+c^2}}\right)}{(a^2+c^2)^{\frac{3}{2}}e} + \frac{2\left(a^2e^{(-ex-d)}+ac\right)}{(a^2c^2+c^4+2\,(a^3c+ac^3)e^{(-ex-d)}-(a^2c^2+c^4)e^{(-2\,ex-2\,d)})e}\right) \\
&\quad - \frac{2\,Be^{(-ex-d)}}{(2\,ace^{(-ex-d)}-c^2e^{(-2\,ex-2\,d)}+c^2)e}
\end{aligned}$$

input
```
integrate((A+B*cosh(e*x+d)+C*sinh(e*x+d))/(a+c*sinh(e*x+d))^2,x, algorithm
="maxima")
```

output
```
A*(a*log((c*e^(-e*x - d) - a - sqrt(a^2 + c^2))/(c*e^(-e*x - d) - a + sqrt
(a^2 + c^2)))/((a^2 + c^2)^(3/2)*e) - 2*(a*e^(-e*x - d) + c)/((a^2*c + c^3
 + 2*(a^3 + a*c^2)*e^(-e*x - d) - (a^2*c + c^3)*e^(-2*e*x - 2*d))*e)) + C*
(c*log((c*e^(-e*x - d) - a - sqrt(a^2 + c^2))/(c*e^(-e*x - d) - a + sqrt(a
^2 + c^2)))/((a^2 + c^2)^(3/2)*e) + 2*(a^2*e^(-e*x - d) + a*c)/((a^2*c^2 +
 c^4 + 2*(a^3*c + a*c^3)*e^(-e*x - d) - (a^2*c^2 + c^4)*e^(-2*e*x - 2*d))*
e)) - 2*B*e^(-e*x - d)/((2*a*c*e^(-e*x - d) - c^2*e^(-2*e*x - 2*d) + c^2)*
e)
```

Giac [A] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 170, normalized size of antiderivative = 1.50

$$\int \frac{A+B\cosh(d+ex)+C\sinh(d+ex)}{(a+c\sinh(d+ex))^2}\,dx$$
$$=\frac{\frac{(Aa+Cc)\log\left(\frac{\left|2\,ce^{(ex+d)}+2\,a-2\,\sqrt{a^2+c^2}\right|}{\left|2\,ce^{(ex+d)}+2\,a+2\,\sqrt{a^2+c^2}\right|}\right)}{(a^2+c^2)^{\frac{3}{2}}}-\frac{2\,(Ba^2e^{(ex+d)}+Ca^2e^{(ex+d)}-Aace^{(ex+d)}+Bc^2e^{(ex+d)}-Cac+Ac^2)}{(a^2c+c^3)\left(ce^{(2\,ex+2\,d)}+2\,ae^{(ex+d)}-c\right)}}{e}$$

input
```
integrate((A+B*cosh(e*x+d)+C*sinh(e*x+d))/(a+c*sinh(e*x+d))^2,x, algorithm
="giac")
```

output
```
((A*a + C*c)*log(abs(2*c*e^(e*x + d) + 2*a - 2*sqrt(a^2 + c^2))/abs(2*c*e^
(e*x + d) + 2*a + 2*sqrt(a^2 + c^2)))/(a^2 + c^2)^(3/2) - 2*(B*a^2*e^(e*x
+ d) + C*a^2*e^(e*x + d) - A*a*c*e^(e*x + d) + B*c^2*e^(e*x + d) - C*a*c +
 A*c^2)/((a^2*c + c^3)*(c*e^(2*e*x + 2*d) + 2*a*e^(e*x + d) - c)))/e
```

Mupad [B] (verification not implemented)

Time = 2.04 (sec) , antiderivative size = 279, normalized size of antiderivative = 2.47

$$\int \frac{A+B\cosh(d+ex)+C\sinh(d+ex)}{(a+c\sinh(d+ex))^2}\,dx$$
$$=\frac{\ln\left(\frac{2\,(A\,a+C\,c)\,(c-a\,\mathrm{e}^{d+e\,x})}{c\,(a^2+c^2)^{3/2}}-\frac{2\,\mathrm{e}^{d+e\,x}\,(A\,a+C\,c)}{c\,(a^2+c^2)}\right)\,(A\,a+C\,c)}{e\,(a^2+c^2)^{3/2}}$$
$$-\frac{\ln\left(-\frac{2\,\mathrm{e}^{d+e\,x}\,(A\,a+C\,c)}{c\,(a^2+c^2)}-\frac{2\,(A\,a+C\,c)\,(c-a\,\mathrm{e}^{d+e\,x})}{c\,(a^2+c^2)^{3/2}}\right)\,(A\,a+C\,c)}{e\,(a^2+c^2)^{3/2}}$$
$$-\frac{\frac{2\,(A\,c^3-C\,a\,c^2)}{c\,e\,(a^2\,c+c^3)}+\frac{2\,\mathrm{e}^{d+e\,x}\,(B\,c^4+B\,a^2\,c^2+C\,a^2\,c^2-A\,a\,c^3)}{c^2\,e\,(a^2\,c+c^3)}}{2\,a\,\mathrm{e}^{d+e\,x}-c+c\,\mathrm{e}^{2\,d+2\,e\,x}}$$

input
```
int((A + B*cosh(d + e*x) + C*sinh(d + e*x))/(a + c*sinh(d + e*x))^2,x)
```

output
```
(log((2*(A*a + C*c)*(c - a*exp(d + e*x)))/(c*(a^2 + c^2)^(3/2)) - (2*exp(d
 + e*x)*(A*a + C*c))/(c*(a^2 + c^2)))*(A*a + C*c))/(e*(a^2 + c^2)^(3/2)) -
 (log(- (2*exp(d + e*x)*(A*a + C*c))/(c*(a^2 + c^2)) - (2*(A*a + C*c)*(c -
 a*exp(d + e*x)))/(c*(a^2 + c^2)^(3/2)))*(A*a + C*c))/(e*(a^2 + c^2)^(3/2)
) - ((2*(A*c^3 - C*a*c^2))/(c*e*(a^2*c + c^3)) + (2*exp(d + e*x)*(B*c^4 +
B*a^2*c^2 + C*a^2*c^2 - A*a*c^3))/(c^2*e*(a^2*c + c^3)))/(2*a*exp(d + e*x)
 - c + c*exp(2*d + 2*e*x))
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 253, normalized size of antiderivative = 2.24

$$\int \frac{A+B\cosh(d+ex)+C\sinh(d+ex)}{(a+c\sinh(d+ex))^2}\,dx$$
$$=\frac{2e^{2ex+2d}\sqrt{a^2+c^2}\,atan\left(\frac{e^{ex+d}ci+ai}{\sqrt{a^2+c^2}}\right)aci+4e^{ex+d}\sqrt{a^2+c^2}\,atan\left(\frac{e^{ex+d}ci+ai}{\sqrt{a^2+c^2}}\right)a^2i-2\sqrt{a^2+c^2}\,atan\left(\frac{e^{ex+d}ci+a}{\sqrt{a^2+c^2}}\right.}{ae\left(e^{2ex+2d}a^2c+e^{2ex+2d}c^3+2e^{ex+d}a^3+2e^{ex+d}a\,c^2-a^2c-c^3\right.}$$

input
```
int((A+B*cosh(e*x+d)+C*sinh(e*x+d))/(a+c*sinh(e*x+d))^2,x)
```

output
```
(2*e**(2*d + 2*e*x)*sqrt(a**2 + c**2)*atan((e**(d + e*x)*c*i + a*i)/sqrt(a
**2 + c**2))*a*c*i + 4*e**(d + e*x)*sqrt(a**2 + c**2)*atan((e**(d + e*x)*c
*i + a*i)/sqrt(a**2 + c**2))*a**2*i - 2*sqrt(a**2 + c**2)*atan((e**(d + e*
x)*c*i + a*i)/sqrt(a**2 + c**2))*a*c*i + e**(2*d + 2*e*x)*a**2*b + e**(2*d
 + 2*e*x)*b*c**2 - a**2*b - b*c**2)/(a*e*(e**(2*d + 2*e*x)*a**2*c + e**(2*
d + 2*e*x)*c**3 + 2*e**(d + e*x)*a**3 + 2*e**(d + e*x)*a*c**2 - a**2*c - c
**3))
```

3.255 $\int \frac{A+B\cosh(d+ex)+C\sinh(d+ex)}{(a+c\sinh(d+ex))^3}\,dx$

Optimal result

Integrand size = 31, antiderivative size = 180

$$\int \frac{A+B\cosh(d+ex)+C\sinh(d+ex)}{(a+c\sinh(d+ex))^3}\,dx = -\frac{(2a^2A-Ac^2+3acC)\operatorname{arctanh}\left(\frac{c-a\tanh\left(\frac{1}{2}(d+ex)\right)}{\sqrt{a^2+c^2}}\right)}{(a^2+c^2)^{5/2}e} - \frac{B}{2ce(a+c\sinh(d+ex))^2} - \frac{(Ac-aC)\cosh(d+ex)}{2(a^2+c^2)e(a+c\sinh(d+ex))^2} - \frac{(3aAc-a^2C+2c^2C)\cosh(d+ex)}{2(a^2+c^2)^2e(a+c\sinh(d+ex))}$$

output
```
-(2*A*a^2-A*c^2+3*C*a*c)*arctanh((c-a*tanh(1/2*e*x+1/2*d))/(a^2+c^2)^(1/2)
)/(a^2+c^2)^(5/2)/e-1/2*B/c/e/(a+c*sinh(e*x+d))^2-1/2*(A*c-C*a)*cosh(e*x+d
)/(a^2+c^2)/e/(a+c*sinh(e*x+d))^2-1/2*(3*A*a*c-C*a^2+2*C*c^2)*cosh(e*x+d)/
(a^2+c^2)^2/e/(a+c*sinh(e*x+d))
```

Mathematica [A] (verified)

Time = 1.44 (sec) , antiderivative size = 170, normalized size of antiderivative = 0.94

$$\int \frac{A+B\cosh(d+ex)+C\sinh(d+ex)}{(a+c\sinh(d+ex))^3}\,dx = \frac{\frac{2(2a^2A-Ac^2+3acC)\arctan\left(\frac{c-a\tanh\left(\frac{1}{2}(d+ex)\right)}{\sqrt{-a^2-c^2}}\right)}{\sqrt{-a^2-c^2}}-\frac{(a^2+c^2)\left(B(a^2+c^2)+c(Ac-aC)\cosh(d+ex)\right)}{c(a+c\sinh(d+ex))^2}+\frac{(-3aAc+a^2C-2c^2C)\cosh(d+ex)}{a+c\sinh(d+ex)}}{2\left(a^2+c^2\right)^2e}$$

input
```
Integrate[(A + B*Cosh[d + e*x] + C*Sinh[d + e*x])/(a + c*Sinh[d + e*x])^3,
x]
```

output
```
((2*(2*a^2*A - A*c^2 + 3*a*c*C)*ArcTan[(c - a*Tanh[(d + e*x)/2])/Sqrt[-a^2
 - c^2]])/Sqrt[-a^2 - c^2] - ((a^2 + c^2)*(B*(a^2 + c^2) + c*(A*c - a*C)*C
osh[d + e*x]))/(c*(a + c*Sinh[d + e*x])^2) + ((-3*a*A*c + a^2*C - 2*c^2*C)
*Cosh[d + e*x])/(a + c*Sinh[d + e*x]))/(2*(a^2 + c^2)^2*e)
```

Rubi [A] (warning: unable to verify)

Time = 0.90 (sec) , antiderivative size = 190, normalized size of antiderivative = 1.06, number of steps used = 16, number of rules used = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.484$, Rules used = {3042, 4876, 3042, 3147, 17, 3233, 25, 3042, 3233, 25, 27, 3042, 3139, 1083, 217}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+B\cosh(d+ex)+C\sinh(d+ex)}{(a+c\sinh(d+ex))^3}dx$$

$$\Big\downarrow \; 3042$$

$$\int \frac{A+B\cos(id+iex)-iC\sin(id+iex)}{(a-ic\sin(id+iex))^3}dx$$

$$\Big\downarrow \; 4876$$

$$\int \frac{A+C\sinh(d+ex)}{(a+c\sinh(d+ex))^3}dx+B\int \frac{\cosh(d+ex)}{(a+c\sinh(d+ex))^3}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{A-iC\sin(id+iex)}{(a-ic\sin(id+iex))^3}dx+B\int \frac{\cos(id+iex)}{(a-ic\sin(id+iex))^3}dx$$

$$\Big\downarrow \text{3147}$$

$$\frac{B\int \frac{1}{(a+c\sinh(d+ex))^3}d(c\sinh(d+ex))}{ce}+\int \frac{A-iC\sin(id+iex)}{(a-ic\sin(id+iex))^3}dx$$

$$\Big\downarrow \text{17}$$

$$-\frac{B}{2ce(a+c\sinh(d+ex))^2}+\int \frac{A-iC\sin(id+iex)}{(a-ic\sin(id+iex))^3}dx$$

$$\Big\downarrow \text{3233}$$

$$-\frac{\int -\frac{2(aA+cC)-(Ac-aC)\sinh(d+ex)}{(a+c\sinh(d+ex))^2}dx}{2\left(a^2+c^2\right)}-\frac{(Ac-aC)\cosh(d+ex)}{2e\left(a^2+c^2\right)(a+c\sinh(d+ex))^2}-\frac{B}{2ce(a+c\sinh(d+ex))^2}$$

$$\Big\downarrow \text{25}$$

$$\frac{\int \frac{2(aA+cC)-(Ac-aC)\sinh(d+ex)}{(a+c\sinh(d+ex))^2}dx}{2\left(a^2+c^2\right)}-\frac{(Ac-aC)\cosh(d+ex)}{2e\left(a^2+c^2\right)(a+c\sinh(d+ex))^2}-\frac{B}{2ce(a+c\sinh(d+ex))^2}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\int \frac{2(aA+cC)+i(Ac-aC)\sin(id+iex)}{(a-ic\sin(id+iex))^2}dx}{2\left(a^2+c^2\right)}-\frac{(Ac-aC)\cosh(d+ex)}{2e\left(a^2+c^2\right)(a+c\sinh(d+ex))^2}-\frac{B}{2ce(a+c\sinh(d+ex))^2}$$

$$\Big\downarrow \text{3233}$$

$$\frac{-\frac{\int -\frac{2Aa^2+3cCa-Ac^2}{a+c\sinh(d+ex)}dx}{a^2+c^2}-\frac{\left(a^2(-C)+3aAc+2c^2C\right)\cosh(d+ex)}{e\left(a^2+c^2\right)(a+c\sinh(d+ex))}}{2\left(a^2+c^2\right)}-\frac{(Ac-aC)\cosh(d+ex)}{2e\left(a^2+c^2\right)(a+c\sinh(d+ex))^2}-\frac{B}{2ce(a+c\sinh(d+ex))^2}$$

$$\Big\downarrow \text{25}$$

$$\frac{\frac{\int \frac{2Aa^2+3cCa-Ac^2}{a+c\sinh(d+ex)}dx}{a^2+c^2}-\frac{\left(a^2(-C)+3aAc+2c^2C\right)\cosh(d+ex)}{e\left(a^2+c^2\right)(a+c\sinh(d+ex))}}{2\left(a^2+c^2\right)}-\frac{(Ac-aC)\cosh(d+ex)}{2e\left(a^2+c^2\right)(a+c\sinh(d+ex))^2}-\frac{B}{2ce(a+c\sinh(d+ex))^2}$$

$$\Big\downarrow \text{27}$$

$$\frac{\frac{(2a^2A+3acC-Ac^2)\int \frac{1}{a+c\sinh(d+ex)}dx}{a^2+c^2}-\frac{(a^2(-C)+3aAc+2c^2C)\cosh(d+ex)}{e(a^2+c^2)(a+c\sinh(d+ex))}}{2(a^2+c^2)}-\frac{(Ac-aC)\cosh(d+ex)}{2e(a^2+c^2)(a+c\sinh(d+ex))^2}-\frac{B}{2ce(a+c\sinh(d+ex))^2}$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{-\frac{(a^2(-C)+3aAc+2c^2C)\cosh(d+ex)}{e(a^2+c^2)(a+c\sinh(d+ex))}+\frac{(2a^2A+3acC-Ac^2)\int \frac{1}{a-ic\sin(id+iex)}dx}{a^2+c^2}}{2(a^2+c^2)}-\frac{(Ac-aC)\cosh(d+ex)}{2e(a^2+c^2)(a+c\sinh(d+ex))^2}-\frac{B}{2ce(a+c\sinh(d+ex))^2}$$

$$\Big\downarrow \text{ 3139}$$

$$\frac{-\frac{(a^2(-C)+3aAc+2c^2C)\cosh(d+ex)}{e(a^2+c^2)(a+c\sinh(d+ex))}-\frac{2i(2a^2A+3acC-Ac^2)\int \frac{1}{-a\tanh^2\left(\frac{1}{2}(d+ex)\right)+2c\tanh\left(\frac{1}{2}(d+ex)\right)+a}d\left(i\tanh\left(\frac{1}{2}(d+ex)\right)\right)}{e(a^2+c^2)}}{2(a^2+c^2)}-\frac{(Ac-aC)\cosh(d+ex)}{2e(a^2+c^2)(a+c\sinh(d+ex))^2}-\frac{B}{2ce(a+c\sinh(d+ex))^2}$$

$$\Big\downarrow \text{ 1083}$$

$$\frac{-\frac{(a^2(-C)+3aAc+2c^2C)\cosh(d+ex)}{e(a^2+c^2)(a+c\sinh(d+ex))}+\frac{4i(2a^2A+3acC-Ac^2)\int \frac{1}{\tanh^2\left(\frac{1}{2}(d+ex)\right)-4\left(a^2+c^2\right)}d\left(2ia\tanh\left(\frac{1}{2}(d+ex)\right)-2ic\right)}{e(a^2+c^2)}}{2(a^2+c^2)}-\frac{(Ac-aC)\cosh(d+ex)}{2e(a^2+c^2)(a+c\sinh(d+ex))^2}-\frac{B}{2ce(a+c\sinh(d+ex))^2}$$

$$\Big\downarrow \text{ 217}$$

$$\frac{\frac{2(2a^2A+3acC-Ac^2)\text{arctanh}\left(\frac{\tanh\left(\frac{1}{2}(d+ex)\right)}{2\sqrt{a^2+c^2}}\right)}{e(a^2+c^2)^{3/2}}-\frac{(a^2(-C)+3aAc+2c^2C)\cosh(d+ex)}{e(a^2+c^2)(a+c\sinh(d+ex))}}{2(a^2+c^2)}-\frac{(Ac-aC)\cosh(d+ex)}{2e(a^2+c^2)(a+c\sinh(d+ex))^2}-\frac{B}{2ce(a+c\sinh(d+ex))^2}$$

input
```
Int[(A + B*Cosh[d + e*x] + C*Sinh[d + e*x])/(a + c*Sinh[d + e*x])^3,x]
```

output
```
-1/2*B/(c*e*(a + c*Sinh[d + e*x])^2) - ((A*c - a*C)*Cosh[d + e*x])/(2*(a^2
 + c^2)*e*(a + c*Sinh[d + e*x])^2) + ((2*(2*a^2*A - A*c^2 + 3*a*c*C)*ArcTa
nh[Tanh[(d + e*x)/2]/(2*Sqrt[a^2 + c^2])])/((a^2 + c^2)^(3/2)*e) - ((3*a*A
*c - a^2*C + 2*c^2*C)*Cosh[d + e*x])/((a^2 + c^2)*e*(a + c*Sinh[d + e*x]))
)/(2*(a^2 + c^2))
```

Defintions of rubi rules used

rule 17
```
Int[(c_.)*((a_.) + (b_.)*(x_))^(m_.), x_Symbol] :> Simp[c*((a + b*x)^(m + 1
)/(b*(m + 1))), x] /; FreeQ[{a, b, c, m}, x] && NeQ[m, -1]
```

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 1083
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Simp[-2   Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3139
```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Simp[2*(e/d)   Subst[Int[1/(a + 2*b*e*x + a
*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[a^2 - b^2, 0]
```

rule 3147
```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Simp[1/(b^p*f)   Subst[Int[(a + x)^m*(b^2 - x^2)^((p - 1)
/2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && IntegerQ[(p
 - 1)/2] && NeQ[a^2 - b^2, 0]
```

rule 3233
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Simp[(-(b*c - a*d))*Cos[e + f*x]*((a + b*Sin[e +
 f*x])^(m + 1)/(f*(m + 1)*(a^2 - b^2))), x] + Simp[1/((m + 1)*(a^2 - b^2))
  Int[(a + b*Sin[e + f*x])^(m + 1)*Simp[(a*c - b*d)*(m + 1) - (b*c - a*d)*(
m + 2)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c
- a*d, 0] && NeQ[a^2 - b^2, 0] && LtQ[m, -1] && IntegerQ[2*m]
```

rule 4876
```
Int[(u_)*((v_) + (d_.)*(F_)[(c_.)*((a_.) + (b_.)*(x_))]^(n_.)), x_Symbol] :
> With[{e = FreeFactors[Sin[c*(a + b*x)], x]}, Int[ActivateTrig[u*v], x] +
Simp[d   Int[ActivateTrig[u]*Cos[c*(a + b*x)]^n, x], x] /; FunctionOfQ[Sin[
c*(a + b*x)]/e, u, x]] /; FreeQ[{a, b, c, d}, x] &&  !FreeQ[v, x] && Intege
rQ[(n - 1)/2] && NonsumQ[u] && (EqQ[F, Cos] || EqQ[F, cos])
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 369 vs. $2(169) = 338$.

Time = 24.94 (sec) , antiderivative size = 370, normalized size of antiderivative = 2.06

method	result
parts	$-\frac{2\left(-\frac{c\left(5A\,a^{2}c+2A\,c^{3}-3C\,a^{3}\right)\tanh\left(\frac{ex}{2}+\frac{d}{2}\right)^{3}}{2a\left(a^{4}+2a^{2}c^{2}+c^{4}\right)}-\frac{\left(4A\,a^{4}c-7A\,a^{2}c^{3}-2A\,c^{5}-2C\,a^{5}+5C\,a^{3}c^{2}-2Ca\,c^{4}\right)\tanh\left(\frac{ex}{2}+\frac{d}{2}\right)^{2}}{2\left(a^{4}+2a^{2}c^{2}+c^{4}\right)a^{2}}+\frac{c\left(11A\,a^{2}\right.}{}}{\left(a\tanh\left(\frac{ex}{2}+\frac{d}{2}\right)^{2}-2c\tanh\left(\frac{ex}{2}+\frac{d}{2}\right)-a\right)^{2}}$
derivativedivides	$-\frac{2\left(-\frac{\left(5A\,c^{2}a^{2}+2A\,c^{4}-2B\,a^{4}-4B\,a^{2}c^{2}-2B\,c^{4}-3C\,a^{3}c\right)\tanh\left(\frac{ex}{2}+\frac{d}{2}\right)^{3}}{2a\left(a^{4}+2a^{2}c^{2}+c^{4}\right)}-\frac{\left(4A\,a^{4}c-7A\,a^{2}c^{3}-2A\,c^{5}+2B\,a^{4}c+4B\,a^{2}c^{3}+2B\,c^{5}-\right.}{2\left(a^{4}+2a^{2}c^{2}+c^{4}\right.}}{\left(a\tanh\left(\right.\right.}$
default	$-\frac{2\left(-\frac{\left(5A\,c^{2}a^{2}+2A\,c^{4}-2B\,a^{4}-4B\,a^{2}c^{2}-2B\,c^{4}-3C\,a^{3}c\right)\tanh\left(\frac{ex}{2}+\frac{d}{2}\right)^{3}}{2a\left(a^{4}+2a^{2}c^{2}+c^{4}\right)}-\frac{\left(4A\,a^{4}c-7A\,a^{2}c^{3}-2A\,c^{5}+2B\,a^{4}c+4B\,a^{2}c^{3}+2B\,c^{5}-\right.}{2\left(a^{4}+2a^{2}c^{2}+c^{4}\right.}}{\left(a\tanh\left(\right.\right.}$
risch	$\frac{2A\,a^{2}c^{2}e^{3ex+3d}-A\,c^{4}e^{3ex+3d}+3Ca\,c^{3}e^{3ex+3d}+6A\,a^{3}c\,e^{2ex+2d}-3Aa\,c^{3}e^{2ex+2d}-2B\,a^{4}e^{2ex+2d}-4B\,a^{2}c^{2}e^{2ex+2d}-2}{ce(a^{2}}$

input
```
int((A+B*cosh(e*x+d)+C*sinh(e*x+d))/(a+c*sinh(e*x+d))^3,x,method=_RETURNVE
RBOSE)
```

output
```
1/e*(-2*(-1/2*c*(5*A*a^2*c+2*A*c^3-3*C*a^3)/a/(a^4+2*a^2*c^2+c^4)*tanh(1/2
*e*x+1/2*d)^3-1/2*(4*A*a^4*c-7*A*a^2*c^3-2*A*c^5-2*C*a^5+5*C*a^3*c^2-2*C*a
*c^4)/(a^4+2*a^2*c^2+c^4)/a^2*tanh(1/2*e*x+1/2*d)^2+1/2*c*(11*A*a^2*c+2*A*
c^3-5*C*a^3+4*C*a*c^2)/(a^4+2*a^2*c^2+c^4)/a*tanh(1/2*e*x+1/2*d)+1/2*(4*A*
a^2*c+A*c^3-2*C*a^3+C*a*c^2)/(a^4+2*a^2*c^2+c^4))/(a*tanh(1/2*e*x+1/2*d)^2
-2*c*tanh(1/2*e*x+1/2*d)-a)^2+(2*A*a^2-A*c^2+3*C*a*c)/(a^4+2*a^2*c^2+c^4)/
(a^2+c^2)^(1/2)*arctanh(1/2*(2*a*tanh(1/2*e*x+1/2*d)-2*c)/(a^2+c^2)^(1/2))
)-1/2*B/c/e/(a+c*sinh(e*x+d))^2
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1880 vs. $2(170) = 340$.

Time = 0.13 (sec) , antiderivative size = 1880, normalized size of antiderivative = 10.44

$$\int \frac{A+B\cosh(d+ex)+C\sinh(d+ex)}{(a+c\sinh(d+ex))^3}\,dx = \text{Too large to display}$$

input
```
integrate((A+B*cosh(e*x+d)+C*sinh(e*x+d))/(a+c*sinh(e*x+d))^3,x, algorithm
="fricas")
```

output
```
-1/2*(2*C*a^4*c^2 - 6*A*a^3*c^3 - 2*C*a^2*c^4 - 6*A*a*c^5 - 4*C*c^6 - 2*(2
*A*a^4*c^2 + 3*C*a^3*c^3 + A*a^2*c^4 + 3*C*a*c^5 - A*c^6)*cosh(e*x + d)^3
- 2*(2*A*a^4*c^2 + 3*C*a^3*c^3 + A*a^2*c^4 + 3*C*a*c^5 - A*c^6)*sinh(e*x +
 d)^3 + 2*(2*(B + C)*a^6 - 6*A*a^5*c + 3*(2*B - C)*a^4*c^2 - 3*A*a^3*c^3 +
 3*(2*B - C)*a^2*c^4 + 3*A*a*c^5 + 2*(B + C)*c^6)*cosh(e*x + d)^2 + 2*(2*(
B + C)*a^6 - 6*A*a^5*c + 3*(2*B - C)*a^4*c^2 - 3*A*a^3*c^3 + 3*(2*B - C)*a
^2*c^4 + 3*A*a*c^5 + 2*(B + C)*c^6 - 3*(2*A*a^4*c^2 + 3*C*a^3*c^3 + A*a^2*
c^4 + 3*C*a*c^5 - A*c^6)*cosh(e*x + d))*sinh(e*x + d)^2 + (2*A*a^2*c^3 + 3
*C*a*c^4 - A*c^5 + (2*A*a^2*c^3 + 3*C*a*c^4 - A*c^5)*cosh(e*x + d)^4 + (2*
A*a^2*c^3 + 3*C*a*c^4 - A*c^5)*sinh(e*x + d)^4 + 4*(2*A*a^3*c^2 + 3*C*a^2*
c^3 - A*a*c^4)*cosh(e*x + d)^3 + 4*(2*A*a^3*c^2 + 3*C*a^2*c^3 - A*a*c^4 +
(2*A*a^2*c^3 + 3*C*a*c^4 - A*c^5)*cosh(e*x + d))*sinh(e*x + d)^3 + 2*(4*A*
a^4*c + 6*C*a^3*c^2 - 4*A*a^2*c^3 - 3*C*a*c^4 + A*c^5)*cosh(e*x + d)^2 + 2
*(4*A*a^4*c + 6*C*a^3*c^2 - 4*A*a^2*c^3 - 3*C*a*c^4 + A*c^5 + 3*(2*A*a^2*c
^3 + 3*C*a*c^4 - A*c^5)*cosh(e*x + d)^2 + 6*(2*A*a^3*c^2 + 3*C*a^2*c^3 - A
*a*c^4)*cosh(e*x + d))*sinh(e*x + d)^2 - 4*(2*A*a^3*c^2 + 3*C*a^2*c^3 - A*
a*c^4)*cosh(e*x + d) - 4*(2*A*a^3*c^2 + 3*C*a^2*c^3 - A*a*c^4 - (2*A*a^2*c
^3 + 3*C*a*c^4 - A*c^5)*cosh(e*x + d)^3 - 3*(2*A*a^3*c^2 + 3*C*a^2*c^3 - A
*a*c^4)*cosh(e*x + d)^2 - (4*A*a^4*c + 6*C*a^3*c^2 - 4*A*a^2*c^3 - 3*C*a*c
^4 + A*c^5)*cosh(e*x + d))*sinh(e*x + d))*sqrt(a^2 + c^2)*log((c^2*cosh...
```

Sympy [F(-1)]

Timed out.

$$\int \frac{A + B\cosh(d+ex) + C\sinh(d+ex)}{(a+c\sinh(d+ex))^3}\,dx = \text{Timed out}$$

input
```
integrate((A+B*cosh(e*x+d)+C*sinh(e*x+d))/(a+c*sinh(e*x+d))**3,x)
```

output
```
Timed out
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 726 vs. $2(170) = 340$.

Time = 0.17 (sec) , antiderivative size = 726, normalized size of antiderivative = 4.03

$$\int \frac{A + B\cosh(d + ex) + C\sinh(d + ex)}{(a + c\sinh(d + ex))^3}\,dx = \text{Too large to display}$$

input
```
integrate((A+B*cosh(e*x+d)+C*sinh(e*x+d))/(a+c*sinh(e*x+d))^3,x, algorithm
="maxima")
```

output
```
1/2*C*(3*a*c*log((c*e^(-e*x - d) - a - sqrt(a^2 + c^2))/(c*e^(-e*x - d) -
a + sqrt(a^2 + c^2)))/((a^4 + 2*a^2*c^2 + c^4)*sqrt(a^2 + c^2)*e) + 2*(3*a
*c^3*e^(-3*e*x - 3*d) + a^2*c^2 - 2*c^4 + (4*a^3*c - 5*a*c^3)*e^(-e*x - d)
 + (2*a^4 - 5*a^2*c^2 + 2*c^4)*e^(-2*e*x - 2*d))/((a^4*c^3 + 2*a^2*c^5 + c
^7 + 4*(a^5*c^2 + 2*a^3*c^4 + a*c^6)*e^(-e*x - d) + 2*(2*a^6*c + 3*a^4*c^3
 - c^7)*e^(-2*e*x - 2*d) - 4*(a^5*c^2 + 2*a^3*c^4 + a*c^6)*e^(-3*e*x - 3*d
) + (a^4*c^3 + 2*a^2*c^5 + c^7)*e^(-4*e*x - 4*d))*e)) + 1/2*A*((2*a^2 - c^
2)*log((c*e^(-e*x - d) - a - sqrt(a^2 + c^2))/(c*e^(-e*x - d) - a + sqrt(a
^2 + c^2)))/((a^4 + 2*a^2*c^2 + c^4)*sqrt(a^2 + c^2)*e) - 2*(3*a*c^2 + (10
*a^2*c + c^3)*e^(-e*x - d) + 3*(2*a^3 - a*c^2)*e^(-2*e*x - 2*d) - (2*a^2*c
 - c^3)*e^(-3*e*x - 3*d))/((a^4*c^2 + 2*a^2*c^4 + c^6 + 4*(a^5*c + 2*a^3*c
^3 + a*c^5)*e^(-e*x - d) + 2*(2*a^6 + 3*a^4*c^2 - c^6)*e^(-2*e*x - 2*d) -
4*(a^5*c + 2*a^3*c^3 + a*c^5)*e^(-3*e*x - 3*d) + (a^4*c^2 + 2*a^2*c^4 + c^
6)*e^(-4*e*x - 4*d))*e)) - 2*B*e^(-2*e*x - 2*d)/((4*a*c^2*e^(-e*x - d) - 4
*a*c^2*e^(-3*e*x - 3*d) + c^3*e^(-4*e*x - 4*d) + c^3 + 2*(2*a^2*c - c^3)*e
^(-2*e*x - 2*d))*e)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 405 vs. $2(170) = 340$.

Time = 0.17 (sec) , antiderivative size = 405, normalized size of antiderivative = 2.25

$$\int \frac{A + B\cosh(d + ex) + C\sinh(d + ex)}{(a + c\sinh(d + ex))^3}\,dx =$$

$$-\frac{\frac{(2\,Aa^2+3\,Cac-Ac^2)\log\left(\frac{\left|-2\,ce^{(ex+d)}-2\,a-2\,\sqrt{a^2+c^2}\right|}{\left|-2\,ce^{(ex+d)}-2\,a+2\,\sqrt{a^2+c^2}\right|}\right)}{(a^4+2\,a^2c^2+c^4)\sqrt{a^2+c^2}} - \frac{2\,(2\,Aa^2c^2e^{(3\,ex+3\,d)}+3\,Cac^3e^{(3\,ex+3\,d)}-Ac^4e^{(3\,ex+3\,d)}-2\,Ba^4e^{(2\,ex+2\,d)}-2\,\cdots}{\cdots}}{\cdots}$$

input
```
integrate((A+B*cosh(e*x+d)+C*sinh(e*x+d))/(a+c*sinh(e*x+d))^3,x, algorithm
="giac")
```

output
```
-1/2*((2*A*a^2 + 3*C*a*c - A*c^2)*log(abs(-2*c*e^(e*x + d) - 2*a - 2*sqrt(
a^2 + c^2))/abs(-2*c*e^(e*x + d) - 2*a + 2*sqrt(a^2 + c^2)))/((a^4 + 2*a^2
*c^2 + c^4)*sqrt(a^2 + c^2)) - 2*(2*A*a^2*c^2*e^(3*e*x + 3*d) + 3*C*a*c^3*
e^(3*e*x + 3*d) - A*c^4*e^(3*e*x + 3*d) - 2*B*a^4*e^(2*e*x + 2*d) - 2*C*a^
4*e^(2*e*x + 2*d) + 6*A*a^3*c*e^(2*e*x + 2*d) - 4*B*a^2*c^2*e^(2*e*x + 2*d
) + 5*C*a^2*c^2*e^(2*e*x + 2*d) - 3*A*a*c^3*e^(2*e*x + 2*d) - 2*B*c^4*e^(2
*e*x + 2*d) - 2*C*c^4*e^(2*e*x + 2*d) + 4*C*a^3*c*e^(e*x + d) - 10*A*a^2*c
^2*e^(e*x + d) - 5*C*a*c^3*e^(e*x + d) - A*c^4*e^(e*x + d) - C*a^2*c^2 + 3
*A*a*c^3 + 2*C*c^4)/((a^4*c + 2*a^2*c^3 + c^5)*(c*e^(2*e*x + 2*d) + 2*a*e^
(e*x + d) - c)^2))/e
```

Mupad [F(-1)]

Timed out.

$$\int \frac{A+B\cosh(d+ex)+C\sinh(d+ex)}{(a+c\sinh(d+ex))^3}\,dx = \int \frac{A+B\cosh(d+e\,x)+C\sinh(d+e\,x)}{\left(a+c\sinh\left(d+e\,x\right)\right)^3}\,dx$$

input
```
int((A + B*cosh(d + e*x) + C*sinh(d + e*x))/(a + c*sinh(d + e*x))^3,x)
```

output
```
int((A + B*cosh(d + e*x) + C*sinh(d + e*x))/(a + c*sinh(d + e*x))^3, x)
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 689, normalized size of antiderivative = 3.83

$$\int \frac{A+B\cosh(d+ex)+C\sinh(d+ex)}{(a+c\sinh(d+ex))^3}\,dx = \frac{4e^{4ex+4d}\sqrt{a^2+c^2}\,atan\left(\frac{e^{ex+d}ci+ai}{\sqrt{a^2+c^2}}\right)a\,c^3i+16e^{3ex+3d}\sqrt{a^2+c^2}\,atan\left(\frac{e^{ex+d}ci+ai}{\sqrt{a^2+c^2}}\right)a^2c^2i+16e^{2ex+2d}\sqrt{a^2+c^2}}{}$$

input
```
int((A+B*cosh(e*x+d)+C*sinh(e*x+d))/(a+c*sinh(e*x+d))^3,x)
```

output
```
(4*e**(4*d + 4*e*x)*sqrt(a**2 + c**2)*atan((e**(d + e*x)*c*i + a*i)/sqrt(a
**2 + c**2))*a*c**3*i + 16*e**(3*d + 3*e*x)*sqrt(a**2 + c**2)*atan((e**(d
+ e*x)*c*i + a*i)/sqrt(a**2 + c**2))*a**2*c**2*i + 16*e**(2*d + 2*e*x)*sqr
t(a**2 + c**2)*atan((e**(d + e*x)*c*i + a*i)/sqrt(a**2 + c**2))*a**3*c*i -
 8*e**(2*d + 2*e*x)*sqrt(a**2 + c**2)*atan((e**(d + e*x)*c*i + a*i)/sqrt(a
**2 + c**2))*a*c**3*i - 16*e**(d + e*x)*sqrt(a**2 + c**2)*atan((e**(d + e*
x)*c*i + a*i)/sqrt(a**2 + c**2))*a**2*c**2*i + 4*sqrt(a**2 + c**2)*atan((e
**(d + e*x)*c*i + a*i)/sqrt(a**2 + c**2))*a*c**3*i - e**(4*d + 4*e*x)*a**2
*c**3 - e**(4*d + 4*e*x)*c**5 - 4*e**(2*d + 2*e*x)*a**4*b + 4*e**(2*d + 2*
e*x)*a**4*c - 8*e**(2*d + 2*e*x)*a**2*b*c**2 + 2*e**(2*d + 2*e*x)*a**2*c**
3 - 4*e**(2*d + 2*e*x)*b*c**4 - 2*e**(2*d + 2*e*x)*c**5 - 8*e**(d + e*x)*a
**3*c**2 - 8*e**(d + e*x)*a*c**4 + 3*a**2*c**3 + 3*c**5)/(2*c*e*(e**(4*d +
 4*e*x)*a**4*c**2 + 2*e**(4*d + 4*e*x)*a**2*c**4 + e**(4*d + 4*e*x)*c**6 +
 4*e**(3*d + 3*e*x)*a**5*c + 8*e**(3*d + 3*e*x)*a**3*c**3 + 4*e**(3*d + 3*
e*x)*a*c**5 + 4*e**(2*d + 2*e*x)*a**6 + 6*e**(2*d + 2*e*x)*a**4*c**2 - 2*e
**(2*d + 2*e*x)*c**6 - 4*e**(d + e*x)*a**5*c - 8*e**(d + e*x)*a**3*c**3 -
4*e**(d + e*x)*a*c**5 + a**4*c**2 + 2*a**2*c**4 + c**6))
```

3.256 $\int \frac{A+B\cosh(d+ex)+C\sinh(d+ex)}{(a+c\sinh(d+ex))^4}\,dx$

Optimal result

Integrand size = 31, antiderivative size = 250

$$\begin{aligned}
&\int \frac{A+B\cosh(d+ex)+C\sinh(d+ex)}{(a+c\sinh(d+ex))^4}\,dx \\
&= -\frac{(2a^3A-3aAc^2+4a^2cC-c^3C)\,\text{arctanh}\left(\frac{c-a\tanh\left(\frac{1}{2}(d+ex)\right)}{\sqrt{a^2+c^2}}\right)}{(a^2+c^2)^{7/2}\,e} \\
&\quad -\frac{B}{3ce(a+c\sinh(d+ex))^3}-\frac{(Ac-aC)\cosh(d+ex)}{3\,(a^2+c^2)\,e(a+c\sinh(d+ex))^3} \\
&\quad -\frac{(5aAc-2a^2C+3c^2C)\cosh(d+ex)}{6\,(a^2+c^2)^2\,e(a+c\sinh(d+ex))^2} \\
&\quad -\frac{(11a^2Ac-4Ac^3-2a^3C+13ac^2C)\cosh(d+ex)}{6\,(a^2+c^2)^3\,e(a+c\sinh(d+ex))}
\end{aligned}$$

output

```
-(2*A*a^3-3*A*a*c^2+4*C*a^2*c-C*c^3)*arctanh((c-a*tanh(1/2*e*x+1/2*d))/(a^
2+c^2)^(1/2))/(a^2+c^2)^(7/2)/e-1/3*B/c/e/(a+c*sinh(e*x+d))^3-1/3*(A*c-C*a
)*cosh(e*x+d)/(a^2+c^2)/e/(a+c*sinh(e*x+d))^3-1/6*(5*A*a*c-2*C*a^2+3*C*c^2
)*cosh(e*x+d)/(a^2+c^2)^2/e/(a+c*sinh(e*x+d))^2-1/6*(11*A*a^2*c-4*A*c^3-2*
C*a^3+13*C*a*c^2)*cosh(e*x+d)/(a^2+c^2)^3/e/(a+c*sinh(e*x+d))
```

Mathematica [A] (verified)

Time = 1.74 (sec) , antiderivative size = 235, normalized size of antiderivative = 0.94

$$\int \frac{A+B\cosh(d+ex)+C\sinh(d+ex)}{(a+c\sinh(d+ex))^4}\,dx$$
$$=\frac{\frac{6(2a^3A-3aAc^2+4a^2cC-c^3C)\arctan\left(\frac{c-a\tanh\left(\frac{1}{2}(d+ex)\right)}{\sqrt{-a^2-c^2}}\right)}{\sqrt{-a^2-c^2}}-\frac{2(a^2+c^2)^2(B(a^2+c^2)+c(Ac-aC)\cosh(d+ex))}{c(a+c\sinh(d+ex))^3}+\frac{(a^2+c^2)(-5aAc+2a^2C\cdots}{(a+c\sinh(d\cdots}}{6(a^2+c^2)^3e}$$

input
```
Integrate[(A + B*Cosh[d + e*x] + C*Sinh[d + e*x])/(a + c*Sinh[d + e*x])^4,
x]
```

output
```
((6*(2*a^3*A - 3*a*A*c^2 + 4*a^2*c*C - c^3*C)*ArcTan[(c - a*Tanh[(d + e*x)
/2])/Sqrt[-a^2 - c^2]])/Sqrt[-a^2 - c^2] - (2*(a^2 + c^2)^2*(B*(a^2 + c^2)
 + c*(A*c - a*C)*Cosh[d + e*x]))/(c*(a + c*Sinh[d + e*x])^3) + ((a^2 + c^2
)*(-5*a*A*c + 2*a^2*C - 3*c^2*C)*Cosh[d + e*x])/(a + c*Sinh[d + e*x])^2 +
((-11*a^2*A*c + 4*A*c^3 + 2*a^3*C - 13*a*c^2*C)*Cosh[d + e*x])/(a + c*Sinh
[d + e*x]))/(6*(a^2 + c^2)^3*e)
```

Rubi [A] (warning: unable to verify)

Time = 1.22 (sec) , antiderivative size = 274, normalized size of antiderivative = 1.10, number of steps used = 18, number of rules used = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.548, Rules used = {3042, 4876, 3042, 3147, 17, 3233, 25, 3042, 3233, 25, 3042, 3233, 27, 3042, 3139, 1083, 217}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+B\cosh(d+ex)+C\sinh(d+ex)}{(a+c\sinh(d+ex))^4}\,dx$$

$$\Big\downarrow \ 3042$$

$$\int \frac{A+B\cos(id+iex)-iC\sin(id+iex)}{(a-ic\sin(id+iex))^4}\,dx$$

$$\Big\downarrow \ 4876$$

$$\int \frac{A+C\sinh(d+ex)}{(a+c\sinh(d+ex))^4}dx+B\int \frac{\cosh(d+ex)}{(a+c\sinh(d+ex))^4}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{A-iC\sin(id+iex)}{(a-ic\sin(id+iex))^4}dx+B\int \frac{\cos(id+iex)}{(a-ic\sin(id+iex))^4}dx$$

$$\Big\downarrow \text{3147}$$

$$\frac{B\int \frac{1}{(a+c\sinh(d+ex))^4}d(c\sinh(d+ex))}{ce}+\int \frac{A-iC\sin(id+iex)}{(a-ic\sin(id+iex))^4}dx$$

$$\Big\downarrow \text{17}$$

$$-\frac{B}{3ce(a+c\sinh(d+ex))^3}+\int \frac{A-iC\sin(id+iex)}{(a-ic\sin(id+iex))^4}dx$$

$$\Big\downarrow \text{3233}$$

$$-\frac{\int -\frac{3(aA+cC)-2(Ac-aC)\sinh(d+ex)}{(a+c\sinh(d+ex))^3}dx}{3\left(a^2+c^2\right)}-\frac{(Ac-aC)\cosh(d+ex)}{3e\left(a^2+c^2\right)(a+c\sinh(d+ex))^3}-\frac{B}{3ce(a+c\sinh(d+ex))^3}$$

$$\Big\downarrow \text{25}$$

$$\frac{\int \frac{3(aA+cC)-2(Ac-aC)\sinh(d+ex)}{(a+c\sinh(d+ex))^3}dx}{3\left(a^2+c^2\right)}-\frac{(Ac-aC)\cosh(d+ex)}{3e\left(a^2+c^2\right)(a+c\sinh(d+ex))^3}-\frac{B}{3ce(a+c\sinh(d+ex))^3}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\int \frac{3(aA+cC)+2i(Ac-aC)\sin(id+iex)}{(a-ic\sin(id+iex))^3}dx}{3\left(a^2+c^2\right)}-\frac{(Ac-aC)\cosh(d+ex)}{3e\left(a^2+c^2\right)(a+c\sinh(d+ex))^3}-\frac{B}{3ce(a+c\sinh(d+ex))^3}$$

$$\Big\downarrow \text{3233}$$

$$\frac{-\frac{\int -\frac{2\left(3Aa^2+5cCa-2Ac^2\right)-\left(-2Ca^2+5Aca+3c^2C\right)\sinh(d+ex)}{(a+c\sinh(d+ex))^2}dx}{2(a^2+c^2)}-\frac{(-2a^2C+5aAc+3c^2C)\cosh(d+ex)}{2e(a^2+c^2)(a+c\sinh(d+ex))^2}}{3\left(a^2+c^2\right)}-\frac{(Ac-aC)\cosh(d+ex)}{3e\left(a^2+c^2\right)(a+c\sinh(d+ex))^3}-\frac{B}{3ce(a+c\sinh(d+ex))^3}$$

$$\Big\downarrow \text{25}$$

$$\frac{\frac{\int \frac{2\left(3Aa^2+5cCa-2Ac^2\right)-\left(-2Ca^2+5Aca+3c^2C\right)\sinh(d+ex)}{(a+c\sinh(d+ex))^2}dx}{2(a^2+c^2)}-\frac{(-2a^2C+5aAc+3c^2C)\cosh(d+ex)}{2e(a^2+c^2)(a+c\sinh(d+ex))^2}}{3\left(a^2+c^2\right)}-\frac{(Ac-aC)\cosh(d+ex)}{3e\left(a^2+c^2\right)(a+c\sinh(d+ex))^3}-\frac{B}{3ce(a+c\sinh(d+ex))^3}$$

$\Big\downarrow$ 3042

$$\frac{-\frac{(-2a^2C+5aAc+3c^2C)\cosh(d+ex)}{2e(a^2+c^2)(a+c\sinh(d+ex))^2}+\frac{\int \frac{2\left(3Aa^2+5cCa-2Ac^2\right)+i\left(-2Ca^2+5Aca+3c^2C\right)\sin(id+iex)}{(a-ic\sin(id+iex))^2}dx}{2(a^2+c^2)}}{3\left(a^2+c^2\right)}-\frac{(Ac-aC)\cosh(d+ex)}{3e\left(a^2+c^2\right)(a+c\sinh(d+ex))^3}-\frac{B}{3ce(a+c\sinh(d+ex))^3}$$

$\Big\downarrow$ 3233

$$\frac{\frac{-\frac{\int -\frac{3\left(2Aa^3+4cCa^2-3Ac^2a-c^3C\right)}{a+c\sinh(d+ex)}dx}{a^2+c^2}-\frac{\left(-2a^3C+11a^2Ac+13ac^2C-4Ac^3\right)\cosh(d+ex)}{e\left(a^2+c^2\right)(a+c\sinh(d+ex))}}{2(a^2+c^2)}-\frac{(-2a^2C+5aAc+3c^2C)\cosh(d+ex)}{2e(a^2+c^2)(a+c\sinh(d+ex))^2}}{3\left(a^2+c^2\right)}-\frac{(Ac-aC)\cosh(d+ex)}{3e\left(a^2+c^2\right)(a+c\sinh(d+ex))^3}-\frac{B}{3ce(a+c\sinh(d+ex))^3}$$

$\Big\downarrow$ 27

$$\frac{\frac{\frac{3\left(2a^3A+4a^2cC-3aAc^2-c^3C\right)\int \frac{1}{a+c\sinh(d+ex)}dx}{a^2+c^2}-\frac{\left(-2a^3C+11a^2Ac+13ac^2C-4Ac^3\right)\cosh(d+ex)}{e\left(a^2+c^2\right)(a+c\sinh(d+ex))}}{2(a^2+c^2)}-\frac{(-2a^2C+5aAc+3c^2C)\cosh(d+ex)}{2e(a^2+c^2)(a+c\sinh(d+ex))^2}}{3\left(a^2+c^2\right)}-\frac{(Ac-aC)\cosh(d+ex)}{3e\left(a^2+c^2\right)(a+c\sinh(d+ex))^3}-\frac{B}{3ce(a+c\sinh(d+ex))^3}$$

$\Big\downarrow$ 3042

$$\frac{-\frac{(-2a^2C+5aAc+3c^2C)\cosh(d+ex)}{2e(a^2+c^2)(a+c\sinh(d+ex))^2}+\frac{-\frac{\left(-2a^3C+11a^2Ac+13ac^2C-4Ac^3\right)\cosh(d+ex)}{e\left(a^2+c^2\right)(a+c\sinh(d+ex))}+\frac{3\left(2a^3A+4a^2cC-3aAc^2-c^3C\right)\int \frac{1}{a-ic\sin(id+iex)}dx}{a^2+c^2}}{2(a^2+c^2)}}{3\left(a^2+c^2\right)}-\frac{(Ac-aC)\cosh(d+ex)}{3e\left(a^2+c^2\right)(a+c\sinh(d+ex))^3}-\frac{B}{3ce(a+c\sinh(d+ex))^3}$$

$\Big\downarrow$ 3139

$$\frac{-\frac{(-2a^2C+5aAc+3c^2C)\cosh(d+ex)}{2e(a^2+c^2)(a+c\sinh(d+ex))^2}+\frac{-\frac{(-2a^3C+11a^2Ac+13ac^2C-4Ac^3)\cosh(d+ex)}{e(a^2+c^2)(a+c\sinh(d+ex))}-\frac{6i(2a^3A+4a^2cC-3aAc^2-c^3C)\int\frac{1}{-a\tanh^2\left(\frac{1}{2}(d+ex)\right)+2c\,t}}{e(a^2+c^2)}}{2(a^2+c^2)}}{3(a^2+c^2)}$$

$$\frac{(Ac-aC)\cosh(d+ex)}{3e(a^2+c^2)(a+c\sinh(d+ex))^3}-\frac{B}{3ce(a+c\sinh(d+ex))^3}$$

$$\Big\downarrow \text{1083}$$

$$\frac{-\frac{(-2a^2C+5aAc+3c^2C)\cosh(d+ex)}{2e(a^2+c^2)(a+c\sinh(d+ex))^2}+\frac{-\frac{(-2a^3C+11a^2Ac+13ac^2C-4Ac^3)\cosh(d+ex)}{e(a^2+c^2)(a+c\sinh(d+ex))}+\frac{12i(2a^3A+4a^2cC-3aAc^2-c^3C)\int\frac{1}{\tanh^2\left(\frac{1}{2}(d+ex)\right)-4(a^2-}}{e(a^2+c^2)}}{2(a^2+c^2)}}{3(a^2+c^2)}$$

$$\frac{(Ac-aC)\cosh(d+ex)}{3e(a^2+c^2)(a+c\sinh(d+ex))^3}-\frac{B}{3ce(a+c\sinh(d+ex))^3}$$

$$\Big\downarrow \text{217}$$

$$-\frac{(Ac-aC)\cosh(d+ex)}{3e(a^2+c^2)(a+c\sinh(d+ex))^3}+$$

$$\frac{\frac{\frac{6(2a^3A+4a^2cC-3aAc^2-c^3C)\text{arctanh}\left(\frac{\tanh\left(\frac{1}{2}(d+ex)\right)}{2\sqrt{a^2+c^2}}\right)}{e(a^2+c^2)^{3/2}}-\frac{(-2a^3C+11a^2Ac+13ac^2C-4Ac^3)\cosh(d+ex)}{e(a^2+c^2)(a+c\sinh(d+ex))}}{2(a^2+c^2)}-\frac{(-2a^2C+5aAc+3c^2C)\cosh(d+ex)}{2e(a^2+c^2)(a+c\sinh(d+ex))^2}}{3(a^2+c^2)}-$$

$$\frac{B}{3ce(a+c\sinh(d+ex))^3}$$

input
```
Int[(A + B*Cosh[d + e*x] + C*Sinh[d + e*x])/(a + c*Sinh[d + e*x])^4,x]
```

output
```
-1/3*B/(c*e*(a + c*Sinh[d + e*x])^3) - ((A*c - a*C)*Cosh[d + e*x])/(3*(a^2
 + c^2)*e*(a + c*Sinh[d + e*x])^3) + (-1/2*((5*a*A*c - 2*a^2*C + 3*c^2*C)*
Cosh[d + e*x])/((a^2 + c^2)*e*(a + c*Sinh[d + e*x])^2) + ((6*(2*a^3*A - 3*
a*A*c^2 + 4*a^2*c*C - c^3*C)*ArcTanh[Tanh[(d + e*x)/2]/(2*Sqrt[a^2 + c^2])
])/((a^2 + c^2)^(3/2)*e) - ((11*a^2*A*c - 4*A*c^3 - 2*a^3*C + 13*a*c^2*C)*
Cosh[d + e*x])/((a^2 + c^2)*e*(a + c*Sinh[d + e*x])))/(2*(a^2 + c^2)))/(3*
(a^2 + c^2))
```

Defintions of rubi rules used

rule 17
```
Int[(c_.)*((a_.) + (b_.)*(x_))^(m_.), x_Symbol] :> Simp[c*((a + b*x)^(m + 1
)/(b*(m + 1))), x] /; FreeQ[{a, b, c, m}, x] && NeQ[m, -1]
```

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 1083
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Simp[-2   Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3139
```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Simp[2*(e/d)   Subst[Int[1/(a + 2*b*e*x + a
*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[a^2 - b^2, 0]
```

rule 3147
```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Simp[1/(b^p*f)   Subst[Int[(a + x)^m*(b^2 - x^2)^((p - 1)
/2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && IntegerQ[(p
 - 1)/2] && NeQ[a^2 - b^2, 0]
```

rule 3233
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Simp[(-(b*c - a*d))*Cos[e + f*x]*((a + b*Sin[e +
 f*x])^(m + 1)/(f*(m + 1)*(a^2 - b^2))), x] + Simp[1/((m + 1)*(a^2 - b^2))
  Int[(a + b*Sin[e + f*x])^(m + 1)*Simp[(a*c - b*d)*(m + 1) - (b*c - a*d)*(
m + 2)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c
- a*d, 0] && NeQ[a^2 - b^2, 0] && LtQ[m, -1] && IntegerQ[2*m]
```

rule 4876
```
Int[(u_)*((v_) + (d_.)*(F_)[(c_.)*((a_.) + (b_.)*(x_))]^(n_.)), x_Symbol] :
> With[{e = FreeFactors[Sin[c*(a + b*x)], x]}, Int[ActivateTrig[u*v], x] +
Simp[d   Int[ActivateTrig[u]*Cos[c*(a + b*x)]^n, x], x] /; FunctionOfQ[Sin[
c*(a + b*x)]/e, u, x]] /; FreeQ[{a, b, c, d}, x] &&  !FreeQ[v, x] && Intege
rQ[(n - 1)/2] && NonsumQ[u] && (EqQ[F, Cos] || EqQ[F, cos])
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 698 vs. $2(237) = 474$.

Time = 122.99 (sec) , antiderivative size = 699, normalized size of antiderivative = 2.80

method	result
parts	$-\frac{2\left(-\frac{c\left(9A\,a^4c+6A\,a^2c^3+2A\,c^5-4C\,a^5+C\,a^3c^2\right)\tanh\left(\frac{ex}{2}+\frac{d}{2}\right)^5}{2a\left(a^6+3a^4c^2+3a^2c^4+c^6\right)}-\frac{\left(6A\,a^6c-27A\,a^4c^3-12A\,a^2c^5-4A\,c^7-2C\,a^7+14C\,a^5c^2-11C}{2\left(a^6+3a^4c^2+3a^2c^4+c^6\right)a^2}\right.}{}$
derivativedivides	$-\frac{2\left(-\frac{\left(9A\,a^4c^2+6A\,a^2c^4+2A\,c^6-2B\,a^6-6c^2B\,a^4-6c^4B\,a^2-2c^6B-4C\,a^5c+C\,a^3c^3\right)\tanh\left(\frac{ex}{2}+\frac{d}{2}\right)^5}{2a\left(a^6+3a^4c^2+3a^2c^4+c^6\right)}-\frac{\left(6A\,a^6c-27A\,a^4c^3-12A}{}\right.}{}$
default	$-\frac{2\left(-\frac{\left(9A\,a^4c^2+6A\,a^2c^4+2A\,c^6-2B\,a^6-6c^2B\,a^4-6c^4B\,a^2-2c^6B-4C\,a^5c+C\,a^3c^3\right)\tanh\left(\frac{ex}{2}+\frac{d}{2}\right)^5}{2a\left(a^6+3a^4c^2+3a^2c^4+c^6\right)}-\frac{\left(6A\,a^6c-27A\,a^4c^3-12A}{}\right.}{}$
risch	Expression too large to display

input
```
int((A+B*cosh(e*x+d)+C*sinh(e*x+d))/(a+c*sinh(e*x+d))^4,x,method=_RETURNVE
RBOSE)
```

output
```
1/e*(-2*(-1/2*c*(9*A*a^4*c+6*A*a^2*c^3+2*A*c^5-4*C*a^5+C*a^3*c^2)/a/(a^6+3
*a^4*c^2+3*a^2*c^4+c^6)*tanh(1/2*e*x+1/2*d)^5-1/2*(6*A*a^6*c-27*A*a^4*c^3-
12*A*a^2*c^5-4*A*c^7-2*C*a^7+14*C*a^5*c^2-11*C*a^3*c^4-2*C*a*c^6)/(a^6+3*a
^4*c^2+3*a^2*c^4+c^6)/a^2*tanh(1/2*e*x+1/2*d)^4+1/3/a^3*c*(54*A*a^6*c-21*A
*a^4*c^3-4*A*a^2*c^5-4*A*c^7-18*C*a^7+42*C*a^5*c^2-17*C*a^3*c^4-2*C*a*c^6)
/(a^6+3*a^4*c^2+3*a^2*c^4+c^6)*tanh(1/2*e*x+1/2*d)^3+1/a^2*(6*A*a^6*c-20*A
*a^4*c^3-3*A*a^2*c^5-2*A*c^7-2*C*a^7+10*C*a^5*c^2-14*C*a^3*c^4-C*a*c^6)/(a
^6+3*a^4*c^2+3*a^2*c^4+c^6)*tanh(1/2*e*x+1/2*d)^2-1/2/a*c*(27*A*a^4*c+4*A*
a^2*c^3+2*A*c^5-8*C*a^5+19*C*a^3*c^2+2*C*a*c^4)/(a^6+3*a^4*c^2+3*a^2*c^4+c
^6)*tanh(1/2*e*x+1/2*d)-1/6*(18*A*a^4*c+5*A*a^2*c^3+2*A*c^5-6*C*a^5+10*C*a
^3*c^2+C*a*c^4)/(a^6+3*a^4*c^2+3*a^2*c^4+c^6))/(a*tanh(1/2*e*x+1/2*d)^2-2*
c*tanh(1/2*e*x+1/2*d)-a)^3+(2*A*a^3-3*A*a*c^2+4*C*a^2*c-C*c^3)/(a^6+3*a^4*
c^2+3*a^2*c^4+c^6)/(a^2+c^2)^(1/2)*arctanh(1/2*(2*a*tanh(1/2*e*x+1/2*d)-2*
c)/(a^2+c^2)^(1/2)))-1/3*B/c/e/(a+c*sinh(e*x+d))^3
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 4350 vs. 2(239) = 478.

Time = 0.24 (sec) , antiderivative size = 4350, normalized size of antiderivative = 17.40

$$\int \frac{A + B\cosh(d + ex) + C\sinh(d + ex)}{(a + c\sinh(d + ex))^4}\,dx = \text{Too large to display}$$

input
```
integrate((A+B*cosh(e*x+d)+C*sinh(e*x+d))/(a+c*sinh(e*x+d))^4,x, algorithm
="fricas")
```

output
```
Too large to include
```

Sympy [F(-1)]

Timed out.

$$\int \frac{A + B\cosh(d + ex) + C\sinh(d + ex)}{(a + c\sinh(d + ex))^4}\,dx = \text{Timed out}$$

input
```
integrate((A+B*cosh(e*x+d)+C*sinh(e*x+d))/(a+c*sinh(e*x+d))**4,x)
```

output
```
Timed out
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1263 vs. 2(239) = 478.

Time = 0.19 (sec) , antiderivative size = 1263, normalized size of antiderivative = 5.05

$$\int \frac{A + B\cosh(d + ex) + C\sinh(d + ex)}{(a + c\sinh(d + ex))^4}\, dx = \text{Too large to display}$$

input
```
integrate((A+B*cosh(e*x+d)+C*sinh(e*x+d))/(a+c*sinh(e*x+d))^4,x, algorithm
="maxima")
```

output
```
1/6*A*(3*(2*a^2 - 3*c^2)*a*log((c*e^(-e*x - d) - a - sqrt(a^2 + c^2))/(c*e
^(-e*x - d) - a + sqrt(a^2 + c^2)))/((a^6 + 3*a^4*c^2 + 3*a^2*c^4 + c^6)*s
qrt(a^2 + c^2)*e) - 2*(11*a^2*c^3 - 4*c^5 + 15*(4*a^3*c^2 - a*c^4)*e^(-e*x
 - d) + 6*(17*a^4*c - 6*a^2*c^3 + 2*c^5)*e^(-2*e*x - 2*d) + 2*(22*a^5 - 41
*a^3*c^2 + 12*a*c^4)*e^(-3*e*x - 3*d) - 15*(2*a^4*c - 3*a^2*c^3)*e^(-4*e*x
 - 4*d) + 3*(2*a^3*c^2 - 3*a*c^4)*e^(-5*e*x - 5*d))/((a^6*c^3 + 3*a^4*c^5
+ 3*a^2*c^7 + c^9 + 6*(a^7*c^2 + 3*a^5*c^4 + 3*a^3*c^6 + a*c^8)*e^(-e*x -
d) + 3*(4*a^8*c + 11*a^6*c^3 + 9*a^4*c^5 + a^2*c^7 - c^9)*e^(-2*e*x - 2*d)
 + 4*(2*a^9 + 3*a^7*c^2 - 3*a^5*c^4 - 7*a^3*c^6 - 3*a*c^8)*e^(-3*e*x - 3*d
) - 3*(4*a^8*c + 11*a^6*c^3 + 9*a^4*c^5 + a^2*c^7 - c^9)*e^(-4*e*x - 4*d)
+ 6*(a^7*c^2 + 3*a^5*c^4 + 3*a^3*c^6 + a*c^8)*e^(-5*e*x - 5*d) - (a^6*c^3
+ 3*a^4*c^5 + 3*a^2*c^7 + c^9)*e^(-6*e*x - 6*d))*e)) + 1/6*C*(3*(4*a^2*c -
 c^3)*log((c*e^(-e*x - d) - a - sqrt(a^2 + c^2))/(c*e^(-e*x - d) - a + sqr
t(a^2 + c^2)))/((a^6 + 3*a^4*c^2 + 3*a^2*c^4 + c^6)*sqrt(a^2 + c^2)*e) + 2
*(2*a^3*c^3 - 13*a*c^5 + 3*(4*a^4*c^2 - 22*a^2*c^4 - c^6)*e^(-e*x - d) + 6
*(4*a^5*c - 17*a^3*c^3 + 4*a*c^5)*e^(-2*e*x - 2*d) + 2*(4*a^6 - 32*a^4*c^2
 + 39*a^2*c^4)*e^(-3*e*x - 3*d) + 15*(4*a^3*c^3 - a*c^5)*e^(-4*e*x - 4*d)
- 3*(4*a^2*c^4 - c^6)*e^(-5*e*x - 5*d))/((a^6*c^4 + 3*a^4*c^6 + 3*a^2*c^8
+ c^10 + 6*(a^7*c^3 + 3*a^5*c^5 + 3*a^3*c^7 + a*c^9)*e^(-e*x - d) + 3*(4*a
^8*c^2 + 11*a^6*c^4 + 9*a^4*c^6 + a^2*c^8 - c^10)*e^(-2*e*x - 2*d) + 4*...
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 685 vs. $2(239) = 478$.

Time = 0.20 (sec) , antiderivative size = 685, normalized size of antiderivative = 2.74

$$\int \frac{A + B\cosh(d + ex) + C\sinh(d + ex)}{(a + c\sinh(d + ex))^4}\, dx = \text{Too large to display}$$

input
```
integrate((A+B*cosh(e*x+d)+C*sinh(e*x+d))/(a+c*sinh(e*x+d))^4,x, algorithm
="giac")
```

output
```
1/6*(3*(2*A*a^3 + 4*C*a^2*c - 3*A*a*c^2 - C*c^3)*log(abs(2*c*e^(e*x + d) +
 2*a - 2*sqrt(a^2 + c^2))/abs(2*c*e^(e*x + d) + 2*a + 2*sqrt(a^2 + c^2)))/
((a^6 + 3*a^4*c^2 + 3*a^2*c^4 + c^6)*sqrt(a^2 + c^2)) + 2*(6*A*a^3*c^3*e^(
5*e*x + 5*d) + 12*C*a^2*c^4*e^(5*e*x + 5*d) - 9*A*a*c^5*e^(5*e*x + 5*d) -
3*C*c^6*e^(5*e*x + 5*d) + 30*A*a^4*c^2*e^(4*e*x + 4*d) + 60*C*a^3*c^3*e^(4
*e*x + 4*d) - 45*A*a^2*c^4*e^(4*e*x + 4*d) - 15*C*a*c^5*e^(4*e*x + 4*d) -
8*B*a^6*e^(3*e*x + 3*d) - 8*C*a^6*e^(3*e*x + 3*d) + 44*A*a^5*c*e^(3*e*x +
3*d) - 24*B*a^4*c^2*e^(3*e*x + 3*d) + 64*C*a^4*c^2*e^(3*e*x + 3*d) - 82*A*
a^3*c^3*e^(3*e*x + 3*d) - 24*B*a^2*c^4*e^(3*e*x + 3*d) - 78*C*a^2*c^4*e^(3
*e*x + 3*d) + 24*A*a*c^5*e^(3*e*x + 3*d) - 8*B*c^6*e^(3*e*x + 3*d) + 24*C*
a^5*c*e^(2*e*x + 2*d) - 102*A*a^4*c^2*e^(2*e*x + 2*d) - 102*C*a^3*c^3*e^(2
*e*x + 2*d) + 36*A*a^2*c^4*e^(2*e*x + 2*d) + 24*C*a*c^5*e^(2*e*x + 2*d) -
12*A*c^6*e^(2*e*x + 2*d) - 12*C*a^4*c^2*e^(e*x + d) + 60*A*a^3*c^3*e^(e*x
+ d) + 66*C*a^2*c^4*e^(e*x + d) - 15*A*a*c^5*e^(e*x + d) + 3*C*c^6*e^(e*x
+ d) + 2*C*a^3*c^3 - 11*A*a^2*c^4 - 13*C*a*c^5 + 4*A*c^6)/((a^6*c + 3*a^4*
c^3 + 3*a^2*c^5 + c^7)*(c*e^(2*e*x + 2*d) + 2*a*e^(e*x + d) - c)^3))/e
```

Mupad [F(-1)]

Timed out.

$$\int \frac{A + B\cosh(d + ex) + C\sinh(d + ex)}{(a + c\sinh(d + ex))^4}\, dx$$
$$= \int \frac{A + B\cosh(d + e\,x) + C\sinh(d + e\,x)}{\left(a + c\sinh\left(d + e\,x\right)\right)^4}\, dx$$

input
```
int((A + B*cosh(d + e*x) + C*sinh(d + e*x))/(a + c*sinh(d + e*x))^4,x)
```

output
```
int((A + B*cosh(d + e*x) + C*sinh(d + e*x))/(a + c*sinh(d + e*x))^4, x)
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 1750, normalized size of antiderivative = 7.00

$$\int \frac{A + B\cosh(d + ex) + C\sinh(d + ex)}{(a + c\sinh(d + ex))^4}\, dx = \text{Too large to display}$$

input
```
int((A+B*cosh(e*x+d)+C*sinh(e*x+d))/(a+c*sinh(e*x+d))^4,x)
```

output
```
(12*e**(6*d + 6*e*x)*sqrt(a**2 + c**2)*atan((e**(d + e*x)*c*i + a*i)/sqrt(
a**2 + c**2))*a**3*c**4*i - 6*e**(6*d + 6*e*x)*sqrt(a**2 + c**2)*atan((e**
(d + e*x)*c*i + a*i)/sqrt(a**2 + c**2))*a*c**6*i + 72*e**(5*d + 5*e*x)*sqr
t(a**2 + c**2)*atan((e**(d + e*x)*c*i + a*i)/sqrt(a**2 + c**2))*a**4*c**3*
i - 36*e**(5*d + 5*e*x)*sqrt(a**2 + c**2)*atan((e**(d + e*x)*c*i + a*i)/sq
rt(a**2 + c**2))*a**2*c**5*i + 144*e**(4*d + 4*e*x)*sqrt(a**2 + c**2)*atan
((e**(d + e*x)*c*i + a*i)/sqrt(a**2 + c**2))*a**5*c**2*i - 108*e**(4*d + 4
*e*x)*sqrt(a**2 + c**2)*atan((e**(d + e*x)*c*i + a*i)/sqrt(a**2 + c**2))*a
**3*c**4*i + 18*e**(4*d + 4*e*x)*sqrt(a**2 + c**2)*atan((e**(d + e*x)*c*i
+ a*i)/sqrt(a**2 + c**2))*a*c**6*i + 96*e**(3*d + 3*e*x)*sqrt(a**2 + c**2)
*atan((e**(d + e*x)*c*i + a*i)/sqrt(a**2 + c**2))*a**6*c*i - 192*e**(3*d +
 3*e*x)*sqrt(a**2 + c**2)*atan((e**(d + e*x)*c*i + a*i)/sqrt(a**2 + c**2))
*a**4*c**3*i + 72*e**(3*d + 3*e*x)*sqrt(a**2 + c**2)*atan((e**(d + e*x)*c*
i + a*i)/sqrt(a**2 + c**2))*a**2*c**5*i - 144*e**(2*d + 2*e*x)*sqrt(a**2 +
 c**2)*atan((e**(d + e*x)*c*i + a*i)/sqrt(a**2 + c**2))*a**5*c**2*i + 108*
e**(2*d + 2*e*x)*sqrt(a**2 + c**2)*atan((e**(d + e*x)*c*i + a*i)/sqrt(a**2
 + c**2))*a**3*c**4*i - 18*e**(2*d + 2*e*x)*sqrt(a**2 + c**2)*atan((e**(d
+ e*x)*c*i + a*i)/sqrt(a**2 + c**2))*a*c**6*i + 72*e**(d + e*x)*sqrt(a**2
+ c**2)*atan((e**(d + e*x)*c*i + a*i)/sqrt(a**2 + c**2))*a**4*c**3*i - 36*
e**(d + e*x)*sqrt(a**2 + c**2)*atan((e**(d + e*x)*c*i + a*i)/sqrt(a**2 ...
```

3.257 $\int \frac{x^3}{a+b\sinh^2(x)}\,dx$

Optimal result

Integrand size = 14, antiderivative size = 439

$$\begin{aligned}\int \frac{x^3}{a+b\sinh^2(x)}\,dx &= \frac{x^3\log\left(1+\frac{be^{2x}}{2a-2\sqrt{a}\sqrt{a-b}-b}\right)}{2\sqrt{a}\sqrt{a-b}} - \frac{x^3\log\left(1+\frac{be^{2x}}{2a+2\sqrt{a}\sqrt{a-b}-b}\right)}{2\sqrt{a}\sqrt{a-b}} \\ &+ \frac{3x^2\operatorname{PolyLog}\left(2,-\frac{be^{2x}}{2a-2\sqrt{a}\sqrt{a-b}-b}\right)}{4\sqrt{a}\sqrt{a-b}} \\ &- \frac{3x^2\operatorname{PolyLog}\left(2,-\frac{be^{2x}}{2a+2\sqrt{a}\sqrt{a-b}-b}\right)}{4\sqrt{a}\sqrt{a-b}} \\ &- \frac{3x\operatorname{PolyLog}\left(3,-\frac{be^{2x}}{2a-2\sqrt{a}\sqrt{a-b}-b}\right)}{4\sqrt{a}\sqrt{a-b}} \\ &+ \frac{3x\operatorname{PolyLog}\left(3,-\frac{be^{2x}}{2a+2\sqrt{a}\sqrt{a-b}-b}\right)}{4\sqrt{a}\sqrt{a-b}} \\ &+ \frac{3\operatorname{PolyLog}\left(4,-\frac{be^{2x}}{2a-2\sqrt{a}\sqrt{a-b}-b}\right)}{8\sqrt{a}\sqrt{a-b}} \\ &- \frac{3\operatorname{PolyLog}\left(4,-\frac{be^{2x}}{2a+2\sqrt{a}\sqrt{a-b}-b}\right)}{8\sqrt{a}\sqrt{a-b}}\end{aligned}$$

output
```
1/2*x^3*ln(1+b*exp(2*x)/(2*a-2*a^(1/2)*(a-b)^(1/2)-b))/a^(1/2)/(a-b)^(1/2)
-1/2*x^3*ln(1+b*exp(2*x)/(2*a+2*a^(1/2)*(a-b)^(1/2)-b))/a^(1/2)/(a-b)^(1/2
)+3/4*x^2*polylog(2,-b*exp(2*x)/(2*a-2*a^(1/2)*(a-b)^(1/2)-b))/a^(1/2)/(a-
b)^(1/2)-3/4*x^2*polylog(2,-b*exp(2*x)/(2*a+2*a^(1/2)*(a-b)^(1/2)-b))/a^(1
/2)/(a-b)^(1/2)-3/4*x*polylog(3,-b*exp(2*x)/(2*a-2*a^(1/2)*(a-b)^(1/2)-b))
/a^(1/2)/(a-b)^(1/2)+3/4*x*polylog(3,-b*exp(2*x)/(2*a+2*a^(1/2)*(a-b)^(1/2
)-b))/a^(1/2)/(a-b)^(1/2)+3/8*polylog(4,-b*exp(2*x)/(2*a-2*a^(1/2)*(a-b)^(
1/2)-b))/a^(1/2)/(a-b)^(1/2)-3/8*polylog(4,-b*exp(2*x)/(2*a+2*a^(1/2)*(a-b
)^(1/2)-b))/a^(1/2)/(a-b)^(1/2)
```

Mathematica [A] (verified)

Time = 0.48 (sec) , antiderivative size = 598, normalized size of antiderivative = 1.36

$$\int \frac{x^3}{a+b\sinh^2(x)}\,dx = \frac{-x^3\log\left(1-\frac{e^x}{\sqrt{\frac{-2a-2\sqrt{a(a-b)}+b}{b}}}\right)-x^3\log\left(1+\frac{e^x}{\sqrt{\frac{-2a-2\sqrt{a(a-b)}+b}{b}}}\right)+x^3\log\left(1-\frac{e^x}{\sqrt{\frac{-2a+2\sqrt{a(a-b)}+b}{b}}}\right)+x^3\log}{}$$

input
```
Integrate[x^3/(a + b*Sinh[x]^2),x]
```

output
```
(-(x^3*Log[1 - E^x/Sqrt[(-2*a - 2*Sqrt[a*(a - b)] + b)/b]]) - x^3*Log[1 +
E^x/Sqrt[(-2*a - 2*Sqrt[a*(a - b)] + b)/b]] + x^3*Log[1 - E^x/Sqrt[(-2*a +
 2*Sqrt[a*(a - b)] + b)/b]] + x^3*Log[1 + E^x/Sqrt[(-2*a + 2*Sqrt[a*(a - b
)] + b)/b]] - 3*x^2*PolyLog[2, -(E^x/Sqrt[(-2*a - 2*Sqrt[a*(a - b)] + b)/b
])] - 3*x^2*PolyLog[2, E^x/Sqrt[(-2*a - 2*Sqrt[a*(a - b)] + b)/b]] + 3*x^2
*PolyLog[2, -(E^x/Sqrt[(-2*a + 2*Sqrt[a*(a - b)] + b)/b])] + 3*x^2*PolyLog
[2, E^x/Sqrt[(-2*a + 2*Sqrt[a*(a - b)] + b)/b]] + 6*x*PolyLog[3, -(E^x/Sqr
t[(-2*a - 2*Sqrt[a*(a - b)] + b)/b])] + 6*x*PolyLog[3, E^x/Sqrt[(-2*a - 2*
Sqrt[a*(a - b)] + b)/b]] - 6*x*PolyLog[3, -(E^x/Sqrt[(-2*a + 2*Sqrt[a*(a -
 b)] + b)/b])] - 6*x*PolyLog[3, E^x/Sqrt[(-2*a + 2*Sqrt[a*(a - b)] + b)/b]
] - 6*PolyLog[4, -(E^x/Sqrt[(-2*a - 2*Sqrt[a*(a - b)] + b)/b])] - 6*PolyLo
g[4, E^x/Sqrt[(-2*a - 2*Sqrt[a*(a - b)] + b)/b]] + 6*PolyLog[4, -(E^x/Sqrt
[(-2*a + 2*Sqrt[a*(a - b)] + b)/b])] + 6*PolyLog[4, E^x/Sqrt[(-2*a + 2*Sqr
t[a*(a - b)] + b)/b]])/(2*Sqrt[a*(a - b)])
```

Rubi [A] (verified)

Time = 1.78 (sec) , antiderivative size = 391, normalized size of antiderivative = 0.89, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.714, Rules used = {6163, 3042, 3801, 2694, 27, 2620, 3011, 7163, 2720, 7143}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^3}{a+b\sinh^2(x)}\,dx$$

$$\Big\downarrow \text{6163}$$

$$2\int \frac{x^3}{2a-b+b\cosh(2x)}dx$$

$$\Big\downarrow \text{3042}$$

$$2\int \frac{x^3}{2a-b+b\sin\left(2ix+\frac{\pi}{2}\right)}dx$$

$$\Big\downarrow \text{3801}$$

$$4\int \frac{e^{2x}x^3}{2e^{2x}(2a-b)+be^{4x}+b}dx$$

$$\Big\downarrow \text{2694}$$

$$4\left(\frac{b\int \frac{e^{2x}x^3}{2\left(2a-2\sqrt{a-b}\sqrt{a}+be^{2x}-b\right)}dx}{2\sqrt{a}\sqrt{a-b}}-\frac{b\int \frac{e^{2x}x^3}{2\left(2a+2\sqrt{a-b}\sqrt{a}+be^{2x}-b\right)}dx}{2\sqrt{a}\sqrt{a-b}}\right)$$

$$\Big\downarrow \text{27}$$

$$4\left(\frac{b\int \frac{e^{2x}x^3}{2a-2\sqrt{a-b}\sqrt{a}+be^{2x}-b}dx}{4\sqrt{a}\sqrt{a-b}}-\frac{b\int \frac{e^{2x}x^3}{2a+2\sqrt{a-b}\sqrt{a}+be^{2x}-b}dx}{4\sqrt{a}\sqrt{a-b}}\right)$$

$$\Big\downarrow \text{2620}$$

$$4\left(\frac{b\left(\frac{x^3\log\left(\frac{be^{2x}}{-2\sqrt{a}\sqrt{a-b}+2a-b}+1\right)}{2b}-\frac{3\int x^2\log\left(\frac{e^{2x}b}{2a-2\sqrt{a-b}\sqrt{a}-b}+1\right)dx}{2b}\right)}{4\sqrt{a}\sqrt{a-b}}-\frac{b\left(\frac{x^3\log\left(\frac{be^{2x}}{2\sqrt{a}\sqrt{a-b}+2a-b}+1\right)}{2b}-\frac{3\int x^2\log\left(\frac{e^{2x}b}{2a+2\sqrt{a-b}\sqrt{a-}}\right.}{2b}\right.}{4\sqrt{a}\sqrt{a-b}}\right.$$

$\Big\downarrow$ 3011

$$4\left(\frac{b\left(\frac{x^3\log\left(\frac{be^{2x}}{-2\sqrt{a}\sqrt{a-b}+2a-b}+1\right)}{2b}-\frac{3\left(\int x\,\text{PolyLog}\left(2,-\frac{be^{2x}}{2a-2\sqrt{a-b}\sqrt{a}-b}\right)dx-\frac{1}{2}x^2\,\text{PolyLog}\left(2,-\frac{be^{2x}}{2a-2\sqrt{a-b}\sqrt{a}-b}\right)\right)}{2b}\right)}{4\sqrt{a}\sqrt{a-b}}-\frac{b\left(\frac{x^3\log\left(\frac{}{2\sqrt{a}\sqrt{}}\right.}{}\right.}{}\right.$$

$\Big\downarrow$ 7163

$$4\left(\frac{b\left(\frac{x^3\log\left(\frac{be^{2x}}{-2\sqrt{a}\sqrt{a-b}+2a-b}+1\right)}{2b}-\frac{3\left(-\frac{1}{2}\int\text{PolyLog}\left(3,-\frac{be^{2x}}{2a-2\sqrt{a-b}\sqrt{a}-b}\right)dx-\frac{1}{2}x^2\,\text{PolyLog}\left(2,-\frac{be^{2x}}{2a-2\sqrt{a-b}\sqrt{a}-b}\right)+\frac{1}{2}x\,\text{PolyLog}\left(3,-\frac{}{2a-}\right.\right.}{2b}\right.}{4\sqrt{a}\sqrt{a-b}}\right.$$

$\Big\downarrow$ 2720

$$4\left(\frac{b\left(\frac{x^3\log\left(\frac{be^{2x}}{-2\sqrt{a}\sqrt{a-b}+2a-b}+1\right)}{2b}-\frac{3\left(-\frac{1}{4}\int e^{-2x}\,\text{PolyLog}\left(3,-\frac{be^{2x}}{2a-2\sqrt{a-b}\sqrt{a}-b}\right)de^{2x}-\frac{1}{2}x^2\,\text{PolyLog}\left(2,-\frac{be^{2x}}{2a-2\sqrt{a-b}\sqrt{a}-b}\right)+\frac{1}{2}x\,\text{PolyLog}\left(3\right.\right.}{2b}\right.}{4\sqrt{a}\sqrt{a-b}}\right.$$

$\Big\downarrow$ 7143

$$4\left(\frac{b\left(\frac{x^3\log\left(\frac{be^{2x}}{-2\sqrt{a}\sqrt{a-b}+2a-b}+1\right)}{2b}-\frac{3\left(-\frac{1}{2}x^2\,\text{PolyLog}\left(2,-\frac{be^{2x}}{2a-2\sqrt{a-b}\sqrt{a}-b}\right)+\frac{1}{2}x\,\text{PolyLog}\left(3,-\frac{be^{2x}}{2a-2\sqrt{a-b}\sqrt{a}-b}\right)-\frac{1}{4}\,\text{PolyLog}\left(4,-\frac{be^{2}}{2a-2\sqrt{a-}}\right.\right.}{2b}\right.}{4\sqrt{a}\sqrt{a-b}}\right.$$

input
```
Int[x^3/(a + b*Sinh[x]^2),x]
```

output
```
4*((b*((x^3*Log[1 + (b*E^(2*x))/(2*a - 2*Sqrt[a]*Sqrt[a - b] - b)])/(2*b)
- (3*(-1/2*(x^2*PolyLog[2, -((b*E^(2*x))/(2*a - 2*Sqrt[a]*Sqrt[a - b] - b)
)]) + (x*PolyLog[3, -((b*E^(2*x))/(2*a - 2*Sqrt[a]*Sqrt[a - b] - b))])/2 -
 PolyLog[4, -((b*E^(2*x))/(2*a - 2*Sqrt[a]*Sqrt[a - b] - b))]/4))/(2*b)))/
(4*Sqrt[a]*Sqrt[a - b]) - (b*((x^3*Log[1 + (b*E^(2*x))/(2*a + 2*Sqrt[a]*Sq
rt[a - b] - b)])/(2*b) - (3*(-1/2*(x^2*PolyLog[2, -((b*E^(2*x))/(2*a + 2*S
qrt[a]*Sqrt[a - b] - b))]) + (x*PolyLog[3, -((b*E^(2*x))/(2*a + 2*Sqrt[a]*
Sqrt[a - b] - b))])/2 - PolyLog[4, -((b*E^(2*x))/(2*a + 2*Sqrt[a]*Sqrt[a -
 b] - b))]/4))/(2*b)))/(4*Sqrt[a]*Sqrt[a - b]))
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2620
```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Si
mp[d*(m/(b*f*g*n*Log[F]))   Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x
)))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

rule 2694
```
Int[((F_)^(u_)*((f_.) + (g_.)*(x_))^(m_.))/((a_.) + (b_.)*(F_)^(u_) + (c_.)
*(F_)^(v_)), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Simp[2*(c/q)   Int
[(f + g*x)^m*(F^u/(b - q + 2*c*F^u)), x], x] - Simp[2*(c/q)   Int[(f + g*x)
^m*(F^u/(b + q + 2*c*F^u)), x], x]] /; FreeQ[{F, a, b, c, f, g}, x] && EqQ[
v, 2*u] && LinearQ[u, x] && NeQ[b^2 - 4*a*c, 0] && IGtQ[m, 0]
```

rule 2720
```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

rule 3011
```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Simp[g*(m/(b*c*n*Log[F]))   Int[(f + g*x)^(
m - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e
, f, g, n}, x] && GtQ[m, 0]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3801
```
Int[((c_.) + (d_.)*(x_))^(m_.)/((a_) + (b_.)*sin[(e_.) + Pi*(k_.) + (Comple
x[0, fz_])*(f_.)*(x_)]), x_Symbol] :> Simp[2   Int[((c + d*x)^m*(E^((-I)*e
+ f*fz*x)/(b + (2*a*E^((-I)*e + f*fz*x))/E^(I*Pi*(k - 1/2)) - (b*E^(2*((-I)
*e + f*fz*x)))/E^(2*I*k*Pi))))/E^(I*Pi*(k - 1/2)), x], x] /; FreeQ[{a, b, c
, d, e, f, fz}, x] && IntegerQ[2*k] && NeQ[a^2 - b^2, 0] && IGtQ[m, 0]
```

rule 6163
```
Int[(x_)^(m_.)*((a_) + (b_.)*Sinh[(c_.) + (d_.)*(x_)]^2)^(n_), x_Symbol] :>
 Simp[1/2^n   Int[x^m*(2*a - b + b*Cosh[2*c + 2*d*x])^n, x], x] /; FreeQ[{a
, b, c, d}, x] && NeQ[a - b, 0] && IGtQ[m, 0] && ILtQ[n, 0] && (EqQ[n, -1]
|| (EqQ[m, 1] && EqQ[n, -2]))
```

rule 7143
```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

rule 7163
```
Int[((e_.) + (f_.)*(x_))^(m_.)*PolyLog[n_, (d_.)*((F_)^((c_.)*((a_.) + (b_.
)*(x_))))^(p_.)], x_Symbol] :> Simp[(e + f*x)^m*(PolyLog[n + 1, d*(F^(c*(a
+ b*x)))^p]/(b*c*p*Log[F])), x] - Simp[f*(m/(b*c*p*Log[F]))   Int[(e + f*x)
^(m - 1)*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p], x], x] /; FreeQ[{F, a, b, c
, d, e, f, n, p}, x] && GtQ[m, 0]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 918 vs. $2(351) = 702$.

Time = 0.57 (sec) , antiderivative size = 919, normalized size of antiderivative = 2.09

method	result	size
risch	Expression too large to display	919

input
```
int(x^3/(a+b*sinh(x)^2),x,method=_RETURNVERBOSE)
```

output
```
1/2/((a-b)*a)^(1/2)*x^3*ln(1-b*exp(2*x)/(2*((a-b)*a)^(1/2)-2*a+b))-1/4/((a
-b)*a)^(1/2)*x^4+3/4/((a-b)*a)^(1/2)*x^2*polylog(2,b*exp(2*x)/(2*((a-b)*a)
^(1/2)-2*a+b))-3/4/((a-b)*a)^(1/2)*x*polylog(3,b*exp(2*x)/(2*((a-b)*a)^(1/
2)-2*a+b))+3/8/((a-b)*a)^(1/2)*polylog(4,b*exp(2*x)/(2*((a-b)*a)^(1/2)-2*a
+b))+1/(-2*((a-b)*a)^(1/2)-2*a+b)*ln(1-b*exp(2*x)/(-2*((a-b)*a)^(1/2)-2*a+
b))*x^3+1/((a-b)*a)^(1/2)/(-2*((a-b)*a)^(1/2)-2*a+b)*ln(1-b*exp(2*x)/(-2*(
(a-b)*a)^(1/2)-2*a+b))*a*x^3-1/2/((a-b)*a)^(1/2)/(-2*((a-b)*a)^(1/2)-2*a+b
)*ln(1-b*exp(2*x)/(-2*((a-b)*a)^(1/2)-2*a+b))*b*x^3-1/2/(-2*((a-b)*a)^(1/2
)-2*a+b)*x^4-1/2/((a-b)*a)^(1/2)/(-2*((a-b)*a)^(1/2)-2*a+b)*a*x^4+1/4/((a-
b)*a)^(1/2)/(-2*((a-b)*a)^(1/2)-2*a+b)*b*x^4+3/2/(-2*((a-b)*a)^(1/2)-2*a+b
)*polylog(2,b*exp(2*x)/(-2*((a-b)*a)^(1/2)-2*a+b))*x^2+3/2/((a-b)*a)^(1/2)
/(-2*((a-b)*a)^(1/2)-2*a+b)*polylog(2,b*exp(2*x)/(-2*((a-b)*a)^(1/2)-2*a+b
))*a*x^2-3/4/((a-b)*a)^(1/2)/(-2*((a-b)*a)^(1/2)-2*a+b)*polylog(2,b*exp(2*
x)/(-2*((a-b)*a)^(1/2)-2*a+b))*b*x^2-3/2/(-2*((a-b)*a)^(1/2)-2*a+b)*polylo
g(3,b*exp(2*x)/(-2*((a-b)*a)^(1/2)-2*a+b))*x-3/2/((a-b)*a)^(1/2)/(-2*((a-b
)*a)^(1/2)-2*a+b)*polylog(3,b*exp(2*x)/(-2*((a-b)*a)^(1/2)-2*a+b))*a*x+3/4
/((a-b)*a)^(1/2)/(-2*((a-b)*a)^(1/2)-2*a+b)*polylog(3,b*exp(2*x)/(-2*((a-b
)*a)^(1/2)-2*a+b))*b*x+3/4/(-2*((a-b)*a)^(1/2)-2*a+b)*polylog(4,b*exp(2*x)
/(-2*((a-b)*a)^(1/2)-2*a+b))+3/4/((a-b)*a)^(1/2)/(-2*((a-b)*a)^(1/2)-2*a+b
)*polylog(4,b*exp(2*x)/(-2*((a-b)*a)^(1/2)-2*a+b))*a-3/8/((a-b)*a)^(1/2...
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1655 vs. $2(339) = 678$.

Time = 0.14 (sec) , antiderivative size = 1655, normalized size of antiderivative = 3.77

$$\int \frac{x^3}{a + b\sinh^2(x)}\,dx = \text{Too large to display}$$

input
```
integrate(x^3/(a+b*sinh(x)^2),x, algorithm="fricas")
```

output
```
-1/2*(b*x^3*sqrt((a^2 - a*b)/b^2)*log((((2*a - b)*cosh(x) + (2*a - b)*sinh
(x) - 2*(b*cosh(x) + b*sinh(x))*sqrt((a^2 - a*b)/b^2))*sqrt(-(2*b*sqrt((a^
2 - a*b)/b^2) + 2*a - b)/b) + b)/b) + b*x^3*sqrt((a^2 - a*b)/b^2)*log(-(((
2*a - b)*cosh(x) + (2*a - b)*sinh(x) - 2*(b*cosh(x) + b*sinh(x))*sqrt((a^2
 - a*b)/b^2))*sqrt(-(2*b*sqrt((a^2 - a*b)/b^2) + 2*a - b)/b) - b)/b) - b*x
^3*sqrt((a^2 - a*b)/b^2)*log((((2*a - b)*cosh(x) + (2*a - b)*sinh(x) + 2*(
b*cosh(x) + b*sinh(x))*sqrt((a^2 - a*b)/b^2))*sqrt((2*b*sqrt((a^2 - a*b)/b
^2) - 2*a + b)/b) + b)/b) - b*x^3*sqrt((a^2 - a*b)/b^2)*log(-(((2*a - b)*c
osh(x) + (2*a - b)*sinh(x) + 2*(b*cosh(x) + b*sinh(x))*sqrt((a^2 - a*b)/b^
2))*sqrt((2*b*sqrt((a^2 - a*b)/b^2) - 2*a + b)/b) - b)/b) + 3*b*x^2*sqrt((
a^2 - a*b)/b^2)*dilog(-(((2*a - b)*cosh(x) + (2*a - b)*sinh(x) - 2*(b*cosh
(x) + b*sinh(x))*sqrt((a^2 - a*b)/b^2))*sqrt(-(2*b*sqrt((a^2 - a*b)/b^2) +
 2*a - b)/b) + b)/b + 1) + 3*b*x^2*sqrt((a^2 - a*b)/b^2)*dilog((((2*a - b)
*cosh(x) + (2*a - b)*sinh(x) - 2*(b*cosh(x) + b*sinh(x))*sqrt((a^2 - a*b)/
b^2))*sqrt(-(2*b*sqrt((a^2 - a*b)/b^2) + 2*a - b)/b) - b)/b + 1) - 3*b*x^2
*sqrt((a^2 - a*b)/b^2)*dilog(-(((2*a - b)*cosh(x) + (2*a - b)*sinh(x) + 2*
(b*cosh(x) + b*sinh(x))*sqrt((a^2 - a*b)/b^2))*sqrt((2*b*sqrt((a^2 - a*b)/
b^2) - 2*a + b)/b) + b)/b + 1) - 3*b*x^2*sqrt((a^2 - a*b)/b^2)*dilog((((2*
a - b)*cosh(x) + (2*a - b)*sinh(x) + 2*(b*cosh(x) + b*sinh(x))*sqrt((a^2 -
 a*b)/b^2))*sqrt((2*b*sqrt((a^2 - a*b)/b^2) - 2*a + b)/b) - b)/b + 1) -...
```

Sympy [F]

$$\int \frac{x^3}{a+b\sinh^2(x)}\,dx = \int \frac{x^3}{a+b\sinh^2{(x)}}\,dx$$

input
```
integrate(x**3/(a+b*sinh(x)**2),x)
```

output
```
Integral(x**3/(a + b*sinh(x)**2), x)
```

Maxima [F]

$$\int \frac{x^3}{a+b\sinh^2(x)}\,dx = \int \frac{x^3}{b\sinh{(x)}^2+a}\,dx$$

input
```
integrate(x^3/(a+b*sinh(x)^2),x, algorithm="maxima")
```

output
```
integrate(x^3/(b*sinh(x)^2 + a), x)
```

Giac [F]

$$\int \frac{x^3}{a+b\sinh^2(x)}\,dx = \int \frac{x^3}{b\sinh{(x)}^2+a}\,dx$$

input
```
integrate(x^3/(a+b*sinh(x)^2),x, algorithm="giac")
```

output
```
integrate(x^3/(b*sinh(x)^2 + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^3}{a + b\sinh^2(x)}\,dx = \int \frac{x^3}{b\sinh{(x)}^2 + a}\,dx$$

input
```
int(x^3/(a + b*sinh(x)^2),x)
```

output
```
int(x^3/(a + b*sinh(x)^2), x)
```

Reduce [F]

$$\int \frac{x^3}{a + b\sinh^2(x)}\,dx = \int \frac{x^3}{\sinh{(x)}^2 b + a}dx$$

input
```
int(x^3/(a+b*sinh(x)^2),x)
```

output
```
int(x**3/(sinh(x)**2*b + a),x)
```

3.258 $\int \frac{x^2}{a+b\sinh^2(x)}\,dx$

Optimal result

Integrand size = 14, antiderivative size = 327

$$\int \frac{x^2}{a+b\sinh^2(x)}\,dx = \frac{x^2\log\left(1+\frac{be^{2x}}{2a-2\sqrt{a}\sqrt{a-b}-b}\right)}{2\sqrt{a}\sqrt{a-b}} - \frac{x^2\log\left(1+\frac{be^{2x}}{2a+2\sqrt{a}\sqrt{a-b}-b}\right)}{2\sqrt{a}\sqrt{a-b}} + \frac{x\,\text{PolyLog}\left(2,-\frac{be^{2x}}{2a-2\sqrt{a}\sqrt{a-b}-b}\right)}{2\sqrt{a}\sqrt{a-b}} - \frac{x\,\text{PolyLog}\left(2,-\frac{be^{2x}}{2a+2\sqrt{a}\sqrt{a-b}-b}\right)}{2\sqrt{a}\sqrt{a-b}} - \frac{\text{PolyLog}\left(3,-\frac{be^{2x}}{2a-2\sqrt{a}\sqrt{a-b}-b}\right)}{4\sqrt{a}\sqrt{a-b}} + \frac{\text{PolyLog}\left(3,-\frac{be^{2x}}{2a+2\sqrt{a}\sqrt{a-b}-b}\right)}{4\sqrt{a}\sqrt{a-b}}$$

output
```
1/2*x^2*ln(1+b*exp(2*x)/(2*a-2*a^(1/2)*(a-b)^(1/2)-b))/a^(1/2)/(a-b)^(1/2)
-1/2*x^2*ln(1+b*exp(2*x)/(2*a+2*a^(1/2)*(a-b)^(1/2)-b))/a^(1/2)/(a-b)^(1/2
)+1/2*x*polylog(2,-b*exp(2*x)/(2*a-2*a^(1/2)*(a-b)^(1/2)-b))/a^(1/2)/(a-b)
^(1/2)-1/2*x*polylog(2,-b*exp(2*x)/(2*a+2*a^(1/2)*(a-b)^(1/2)-b))/a^(1/2)/
(a-b)^(1/2)-1/4*polylog(3,-b*exp(2*x)/(2*a-2*a^(1/2)*(a-b)^(1/2)-b))/a^(1/
2)/(a-b)^(1/2)+1/4*polylog(3,-b*exp(2*x)/(2*a+2*a^(1/2)*(a-b)^(1/2)-b))/a^
(1/2)/(a-b)^(1/2)
```

Mathematica [A] (verified)

Time = 0.31 (sec) , antiderivative size = 448, normalized size of antiderivative = 1.37

$$\int \frac{x^2}{a+b\sinh^2(x)}\,dx = \frac{-x^2\log\left(1-\frac{e^x}{\sqrt{\frac{-2a-2\sqrt{a(a-b)}+b}{b}}}\right)-x^2\log\left(1+\frac{e^x}{\sqrt{\frac{-2a-2\sqrt{a(a-b)}+b}{b}}}\right)+x^2\log\left(1-\frac{e^x}{\sqrt{\frac{-2a+2\sqrt{a(a-b)}+b}{b}}}\right)+x^2\log}{}$$

input
```
Integrate[x^2/(a + b*Sinh[x]^2),x]
```

output
```
(-(x^2*Log[1 - E^x/Sqrt[(-2*a - 2*Sqrt[a*(a - b)] + b)/b]]) - x^2*Log[1 +
E^x/Sqrt[(-2*a - 2*Sqrt[a*(a - b)] + b)/b]] + x^2*Log[1 - E^x/Sqrt[(-2*a +
 2*Sqrt[a*(a - b)] + b)/b]] + x^2*Log[1 + E^x/Sqrt[(-2*a + 2*Sqrt[a*(a - b
)] + b)/b]] - 2*x*PolyLog[2, -(E^x/Sqrt[(-2*a - 2*Sqrt[a*(a - b)] + b)/b])
] - 2*x*PolyLog[2, E^x/Sqrt[(-2*a - 2*Sqrt[a*(a - b)] + b)/b]] + 2*x*PolyL
og[2, -(E^x/Sqrt[(-2*a + 2*Sqrt[a*(a - b)] + b)/b])] + 2*x*PolyLog[2, E^x/
Sqrt[(-2*a + 2*Sqrt[a*(a - b)] + b)/b]] + 2*PolyLog[3, -(E^x/Sqrt[(-2*a -
2*Sqrt[a*(a - b)] + b)/b])] + 2*PolyLog[3, E^x/Sqrt[(-2*a - 2*Sqrt[a*(a -
b)] + b)/b]] - 2*PolyLog[3, -(E^x/Sqrt[(-2*a + 2*Sqrt[a*(a - b)] + b)/b])]
 - 2*PolyLog[3, E^x/Sqrt[(-2*a + 2*Sqrt[a*(a - b)] + b)/b]])/(2*Sqrt[a*(a
- b)])
```

Rubi [A] (verified)

Time = 1.31 (sec) , antiderivative size = 303, normalized size of antiderivative = 0.93, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.643$, Rules used = {6163, 3042, 3801, 2694, 27, 2620, 3011, 2720, 7143}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2}{a+b\sinh^2(x)}\,dx$$

$$\Big\downarrow \text{6163}$$

$$2\int \frac{x^2}{2a-b+b\cosh(2x)}dx$$

$\big\downarrow$ 3042

$$2\int \frac{x^2}{2a-b+b\sin\left(2ix+\frac{\pi}{2}\right)}dx$$

$\big\downarrow$ 3801

$$4\int \frac{e^{2x}x^2}{2e^{2x}(2a-b)+be^{4x}+b}dx$$

$\big\downarrow$ 2694

$$4\left(\frac{b\int \frac{e^{2x}x^2}{2\left(2a-2\sqrt{a-b}\sqrt{a}+be^{2x}-b\right)}dx}{2\sqrt{a}\sqrt{a-b}}-\frac{b\int \frac{e^{2x}x^2}{2\left(2a+2\sqrt{a-b}\sqrt{a}+be^{2x}-b\right)}dx}{2\sqrt{a}\sqrt{a-b}}\right)$$

$\big\downarrow$ 27

$$4\left(\frac{b\int \frac{e^{2x}x^2}{2a-2\sqrt{a-b}\sqrt{a}+be^{2x}-b}dx}{4\sqrt{a}\sqrt{a-b}}-\frac{b\int \frac{e^{2x}x^2}{2a+2\sqrt{a-b}\sqrt{a}+be^{2x}-b}dx}{4\sqrt{a}\sqrt{a-b}}\right)$$

$\big\downarrow$ 2620

$$4\left(\frac{b\left(\frac{x^2\log\left(\frac{be^{2x}}{-2\sqrt{a}\sqrt{a-b}+2a-b}+1\right)}{2b}-\frac{\int x\log\left(\frac{e^{2x}b}{2a-2\sqrt{a-b}\sqrt{a-b}}+1\right)dx}{b}\right)}{4\sqrt{a}\sqrt{a-b}}-\frac{b\left(\frac{x^2\log\left(\frac{be^{2x}}{2\sqrt{a}\sqrt{a-b}+2a-b}+1\right)}{2b}-\frac{\int x\log\left(\frac{e^{2x}b}{2a+2\sqrt{a-b}\sqrt{a-b}}+1\right)}{b}\right.}{4\sqrt{a}\sqrt{a-b}}\right.$$

$\big\downarrow$ 3011

$$4\left(\frac{b\left(\frac{x^2\log\left(\frac{be^{2x}}{-2\sqrt{a}\sqrt{a-b}+2a-b}+1\right)}{2b}-\frac{\frac{1}{2}\int \text{PolyLog}\left(2,-\frac{be^{2x}}{2a-2\sqrt{a-b}\sqrt{a-b}}\right)dx-\frac{1}{2}x\,\text{PolyLog}\left(2,-\frac{be^{2x}}{2a-2\sqrt{a-b}\sqrt{a-b}}\right)}{b}\right)}{4\sqrt{a}\sqrt{a-b}}-\frac{b\left(\frac{x^2\log\left(\frac{be^{2x}}{2\sqrt{a}\sqrt{a-b}+\cdots}\right.}{2b}\right.}{}\right.$$

$\big\downarrow$ 2720

$$4\left(\frac{b\left(\frac{x^2\log\left(\frac{be^{2x}}{-2\sqrt{a}\sqrt{a-b}+2a-b}+1\right)}{2b}-\frac{\frac{1}{4}\int e^{-2x}\,\text{PolyLog}\left(2,-\frac{be^{2x}}{2a-2\sqrt{a-b}\sqrt{a-b}}\right)de^{2x}-\frac{1}{2}x\,\text{PolyLog}\left(2,-\frac{be^{2x}}{2a-2\sqrt{a-b}\sqrt{a-b}}\right)}{b}\right)}{4\sqrt{a}\sqrt{a-b}}-\frac{b\left(\frac{x^2\log\left(\frac{\cdots}{2\sqrt{\cdots}}\right.}{}\right.}{}\right.$$

$$\Big\downarrow \; 7143$$

$$4\left(\frac{b\left(\frac{x^2\log\left(\frac{be^{2x}}{-2\sqrt{a}\sqrt{a-b}+2a-b}+1\right)}{2b}-\frac{\frac{1}{4}\text{PolyLog}\left(3,-\frac{be^{2x}}{2a-2\sqrt{a-b}\sqrt{a}-b}\right)-\frac{1}{2}x\,\text{PolyLog}\left(2,-\frac{be^{2x}}{2a-2\sqrt{a-b}\sqrt{a}-b}\right)}{b}\right)}{4\sqrt{a}\sqrt{a-b}}-\frac{b\left(\frac{x^2\log\left(\frac{be^{2x}}{2\sqrt{a}\sqrt{a-b}+2a-}\right.}{2b}\right.}{}\right.$$

input
```
Int[x^2/(a + b*Sinh[x]^2),x]
```

output
```
4*((b*((x^2*Log[1 + (b*E^(2*x))/(2*a - 2*Sqrt[a]*Sqrt[a - b] - b)])/(2*b)
- (-1/2*(x*PolyLog[2, -((b*E^(2*x))/(2*a - 2*Sqrt[a]*Sqrt[a - b] - b))]) +
 PolyLog[3, -((b*E^(2*x))/(2*a - 2*Sqrt[a]*Sqrt[a - b] - b))]/4)/b))/(4*Sq
rt[a]*Sqrt[a - b]) - (b*((x^2*Log[1 + (b*E^(2*x))/(2*a + 2*Sqrt[a]*Sqrt[a
- b] - b)])/(2*b) - (-1/2*(x*PolyLog[2, -((b*E^(2*x))/(2*a + 2*Sqrt[a]*Sqr
t[a - b] - b))]) + PolyLog[3, -((b*E^(2*x))/(2*a + 2*Sqrt[a]*Sqrt[a - b] -
 b))]/4)/b))/(4*Sqrt[a]*Sqrt[a - b]))
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2620
```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Si
mp[d*(m/(b*f*g*n*Log[F]))   Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x
)))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

rule 2694
```
Int[((F_)^(u_)*((f_.) + (g_.)*(x_))^(m_.))/((a_.) + (b_.)*(F_)^(u_) + (c_.)
*(F_)^(v_)), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Simp[2*(c/q)   Int
[(f + g*x)^m*(F^u/(b - q + 2*c*F^u)), x], x] - Simp[2*(c/q)   Int[(f + g*x)
^m*(F^u/(b + q + 2*c*F^u)), x], x]] /; FreeQ[{F, a, b, c, f, g}, x] && EqQ[
v, 2*u] && LinearQ[u, x] && NeQ[b^2 - 4*a*c, 0] && IGtQ[m, 0]
```

rule 2720
```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

rule 3011
```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Simp[g*(m/(b*c*n*Log[F]))   Int[(f + g*x)^(
m - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e
, f, g, n}, x] && GtQ[m, 0]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3801
```
Int[((c_.) + (d_.)*(x_))^(m_.)/((a_) + (b_.)*sin[(e_.) + Pi*(k_.) + (Comple
x[0, fz_])*(f_.)*(x_)]), x_Symbol] :> Simp[2   Int[((c + d*x)^m*(E^((-I)*e
+ f*fz*x)/(b + (2*a*E^((-I)*e + f*fz*x))/E^(I*Pi*(k - 1/2)) - (b*E^(2*((-I)
*e + f*fz*x)))/E^(2*I*k*Pi))))/E^(I*Pi*(k - 1/2)), x], x] /; FreeQ[{a, b, c
, d, e, f, fz}, x] && IntegerQ[2*k] && NeQ[a^2 - b^2, 0] && IGtQ[m, 0]
```

rule 6163
```
Int[(x_)^(m_.)*((a_) + (b_.)*Sinh[(c_.) + (d_.)*(x_)]^2)^(n_), x_Symbol] :>
 Simp[1/2^n   Int[x^m*(2*a - b + b*Cosh[2*c + 2*d*x])^n, x], x] /; FreeQ[{a
, b, c, d}, x] && NeQ[a - b, 0] && IGtQ[m, 0] && ILtQ[n, 0] && (EqQ[n, -1]
|| (EqQ[m, 1] && EqQ[n, -2]))
```

rule 7143
```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 709 vs. $2(261) = 522$.

Time = 0.25 (sec) , antiderivative size = 710, normalized size of antiderivative = 2.17

method	result
risch	$-\frac{x^3}{3\sqrt{(a-b)a}}+\frac{x^2\ln\left(1-\frac{b\,e^{2x}}{2\sqrt{(a-b)a}-2a+b}\right)}{2\sqrt{(a-b)a}}+\frac{x\,\mathrm{polylog}\left(2,\frac{b\,e^{2x}}{2\sqrt{(a-b)a}-2a+b}\right)}{2\sqrt{(a-b)a}}-\frac{\mathrm{polylog}\left(3,\frac{b\,e^{2x}}{2\sqrt{(a-b)a}-2a+b}\right)}{4\sqrt{(a-b)a}}-\frac{2}{3\left(-2\sqrt{(a-\right.}}$

input
```
int(x^2/(a+b*sinh(x)^2),x,method=_RETURNVERBOSE)
```

output
```
-1/3/((a-b)*a)^(1/2)*x^3+1/2/((a-b)*a)^(1/2)*x^2*ln(1-b*exp(2*x)/(2*((a-b)
*a)^(1/2)-2*a+b))+1/2/((a-b)*a)^(1/2)*x*polylog(2,b*exp(2*x)/(2*((a-b)*a)^
(1/2)-2*a+b))-1/4/((a-b)*a)^(1/2)*polylog(3,b*exp(2*x)/(2*((a-b)*a)^(1/2)-
2*a+b))-2/3/(-2*((a-b)*a)^(1/2)-2*a+b)*x^3+1/(-2*((a-b)*a)^(1/2)-2*a+b)*ln
(1-b*exp(2*x)/(-2*((a-b)*a)^(1/2)-2*a+b))*x^2+1/(-2*((a-b)*a)^(1/2)-2*a+b)
*polylog(2,b*exp(2*x)/(-2*((a-b)*a)^(1/2)-2*a+b))*x-1/2/(-2*((a-b)*a)^(1/2
)-2*a+b)*polylog(3,b*exp(2*x)/(-2*((a-b)*a)^(1/2)-2*a+b))-2/3/((a-b)*a)^(1
/2)/(-2*((a-b)*a)^(1/2)-2*a+b)*a*x^3+1/((a-b)*a)^(1/2)/(-2*((a-b)*a)^(1/2)
-2*a+b)*ln(1-b*exp(2*x)/(-2*((a-b)*a)^(1/2)-2*a+b))*a*x^2+1/((a-b)*a)^(1/2
)/(-2*((a-b)*a)^(1/2)-2*a+b)*polylog(2,b*exp(2*x)/(-2*((a-b)*a)^(1/2)-2*a+
b))*a*x-1/2/((a-b)*a)^(1/2)/(-2*((a-b)*a)^(1/2)-2*a+b)*polylog(3,b*exp(2*x
)/(-2*((a-b)*a)^(1/2)-2*a+b))*a+1/3/((a-b)*a)^(1/2)/(-2*((a-b)*a)^(1/2)-2*
a+b)*b*x^3-1/2/((a-b)*a)^(1/2)/(-2*((a-b)*a)^(1/2)-2*a+b)*ln(1-b*exp(2*x)/
(-2*((a-b)*a)^(1/2)-2*a+b))*b*x^2-1/2/((a-b)*a)^(1/2)/(-2*((a-b)*a)^(1/2)-
2*a+b)*polylog(2,b*exp(2*x)/(-2*((a-b)*a)^(1/2)-2*a+b))*b*x+1/4/((a-b)*a)^
(1/2)/(-2*((a-b)*a)^(1/2)-2*a+b)*polylog(3,b*exp(2*x)/(-2*((a-b)*a)^(1/2)-
2*a+b))*b
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1247 vs. $2(252) = 504$.

Time = 0.12 (sec) , antiderivative size = 1247, normalized size of antiderivative = 3.81

$$\int \frac{x^2}{a+b\sinh^2(x)}\,dx = \text{Too large to display}$$

input
```
integrate(x^2/(a+b*sinh(x)^2),x, algorithm="fricas")
```

output
```
-1/2*(b*x^2*sqrt((a^2 - a*b)/b^2)*log((((2*a - b)*cosh(x) + (2*a - b)*sinh
(x) - 2*(b*cosh(x) + b*sinh(x))*sqrt((a^2 - a*b)/b^2))*sqrt(-(2*b*sqrt((a^
2 - a*b)/b^2) + 2*a - b)/b) + b)/b) + b*x^2*sqrt((a^2 - a*b)/b^2)*log(-(((
2*a - b)*cosh(x) + (2*a - b)*sinh(x) - 2*(b*cosh(x) + b*sinh(x))*sqrt((a^2
 - a*b)/b^2))*sqrt(-(2*b*sqrt((a^2 - a*b)/b^2) + 2*a - b)/b) - b)/b) - b*x
^2*sqrt((a^2 - a*b)/b^2)*log((((2*a - b)*cosh(x) + (2*a - b)*sinh(x) + 2*(
b*cosh(x) + b*sinh(x))*sqrt((a^2 - a*b)/b^2))*sqrt((2*b*sqrt((a^2 - a*b)/b
^2) - 2*a + b)/b) + b)/b) - b*x^2*sqrt((a^2 - a*b)/b^2)*log(-(((2*a - b)*c
osh(x) + (2*a - b)*sinh(x) + 2*(b*cosh(x) + b*sinh(x))*sqrt((a^2 - a*b)/b^
2))*sqrt((2*b*sqrt((a^2 - a*b)/b^2) - 2*a + b)/b) - b)/b) + 2*b*x*sqrt((a^
2 - a*b)/b^2)*dilog(-(((2*a - b)*cosh(x) + (2*a - b)*sinh(x) - 2*(b*cosh(x
) + b*sinh(x))*sqrt((a^2 - a*b)/b^2))*sqrt(-(2*b*sqrt((a^2 - a*b)/b^2) + 2
*a - b)/b) + b)/b + 1) + 2*b*x*sqrt((a^2 - a*b)/b^2)*dilog((((2*a - b)*cos
h(x) + (2*a - b)*sinh(x) - 2*(b*cosh(x) + b*sinh(x))*sqrt((a^2 - a*b)/b^2)
)*sqrt(-(2*b*sqrt((a^2 - a*b)/b^2) + 2*a - b)/b) - b)/b + 1) - 2*b*x*sqrt(
(a^2 - a*b)/b^2)*dilog(-(((2*a - b)*cosh(x) + (2*a - b)*sinh(x) + 2*(b*cos
h(x) + b*sinh(x))*sqrt((a^2 - a*b)/b^2))*sqrt((2*b*sqrt((a^2 - a*b)/b^2) -
 2*a + b)/b) + b)/b + 1) - 2*b*x*sqrt((a^2 - a*b)/b^2)*dilog((((2*a - b)*c
osh(x) + (2*a - b)*sinh(x) + 2*(b*cosh(x) + b*sinh(x))*sqrt((a^2 - a*b)/b^
2))*sqrt((2*b*sqrt((a^2 - a*b)/b^2) - 2*a + b)/b) - b)/b + 1) - 2*b*sqr...
```

Sympy [F]

$$\int \frac{x^2}{a+b\sinh^2(x)}\,dx=\int \frac{x^2}{a+b\sinh^2{(x)}}\,dx$$

input
```
integrate(x**2/(a+b*sinh(x)**2),x)
```

output
```
Integral(x**2/(a + b*sinh(x)**2), x)
```

Maxima [F]

$$\int \frac{x^2}{a + b\sinh^2(x)}\,dx = \int \frac{x^2}{b\sinh(x)^2 + a}\,dx$$

input
```
integrate(x^2/(a+b*sinh(x)^2),x, algorithm="maxima")
```

output
```
integrate(x^2/(b*sinh(x)^2 + a), x)
```

Giac [F]

$$\int \frac{x^2}{a + b\sinh^2(x)}\,dx = \int \frac{x^2}{b\sinh(x)^2 + a}\,dx$$

input
```
integrate(x^2/(a+b*sinh(x)^2),x, algorithm="giac")
```

output
```
integrate(x^2/(b*sinh(x)^2 + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^2}{a + b\sinh^2(x)}\,dx = \int \frac{x^2}{b\sinh(x)^2 + a}\,dx$$

input
```
int(x^2/(a + b*sinh(x)^2),x)
```

output
```
int(x^2/(a + b*sinh(x)^2), x)
```

Reduce [F]

$$\int \frac{x^2}{a+b\sinh^2(x)}\,dx = \int \frac{x^2}{\sinh(x)^2 b + a}dx$$

input
```
int(x^2/(a+b*sinh(x)^2),x)
```

output
```
int(x**2/(sinh(x)**2*b + a),x)
```

3.259 $\int \frac{x}{a+b\sinh^2(x)}\,dx$

Optimal result

Integrand size = 12, antiderivative size = 215

$$\int \frac{x}{a+b\sinh^2(x)}\,dx = \frac{x\log\left(1+\frac{be^{2x}}{2a-2\sqrt{a}\sqrt{a-b}-b}\right)}{2\sqrt{a}\sqrt{a-b}} - \frac{x\log\left(1+\frac{be^{2x}}{2a+2\sqrt{a}\sqrt{a-b}-b}\right)}{2\sqrt{a}\sqrt{a-b}} + \frac{\text{PolyLog}\left(2,-\frac{be^{2x}}{2a-2\sqrt{a}\sqrt{a-b}-b}\right)}{4\sqrt{a}\sqrt{a-b}} - \frac{\text{PolyLog}\left(2,-\frac{be^{2x}}{2a+2\sqrt{a}\sqrt{a-b}-b}\right)}{4\sqrt{a}\sqrt{a-b}}$$

output
```
1/2*x*ln(1+b*exp(2*x)/(2*a-2*a^(1/2)*(a-b)^(1/2)-b))/a^(1/2)/(a-b)^(1/2)-1
/2*x*ln(1+b*exp(2*x)/(2*a+2*a^(1/2)*(a-b)^(1/2)-b))/a^(1/2)/(a-b)^(1/2)+1/
4*polylog(2,-b*exp(2*x)/(2*a-2*a^(1/2)*(a-b)^(1/2)-b))/a^(1/2)/(a-b)^(1/2)
-1/4*polylog(2,-b*exp(2*x)/(2*a+2*a^(1/2)*(a-b)^(1/2)-b))/a^(1/2)/(a-b)^(1
/2)
```

Mathematica [A] (verified)

Time = 0.23 (sec) , antiderivative size = 294, normalized size of antiderivative = 1.37

$$\int \frac{x}{a+b\sinh^2(x)}\,dx$$
$$= \frac{-x\log\left(1-\frac{e^x}{\sqrt{\frac{-2a-2\sqrt{a(a-b)}+b}{b}}}\right)-x\log\left(1+\frac{e^x}{\sqrt{\frac{-2a-2\sqrt{a(a-b)}+b}{b}}}\right)+x\log\left(1-\frac{e^x}{\sqrt{\frac{-2a+2\sqrt{a(a-b)}+b}{b}}}\right)+x\log\left(1-\right.}{}$$

input
```
Integrate[x/(a + b*Sinh[x]^2),x]
```

output
```
(-(x*Log[1 - E^x/Sqrt[(-2*a - 2*Sqrt[a*(a - b)] + b)/b]]) - x*Log[1 + E^x/
Sqrt[(-2*a - 2*Sqrt[a*(a - b)] + b)/b]] + x*Log[1 - E^x/Sqrt[(-2*a + 2*Sqr
t[a*(a - b)] + b)/b]] + x*Log[1 + E^x/Sqrt[(-2*a + 2*Sqrt[a*(a - b)] + b)/
b]] - PolyLog[2, -(E^x/Sqrt[(-2*a - 2*Sqrt[a*(a - b)] + b)/b])] - PolyLog[
2, E^x/Sqrt[(-2*a - 2*Sqrt[a*(a - b)] + b)/b]] + PolyLog[2, -(E^x/Sqrt[(-2
*a + 2*Sqrt[a*(a - b)] + b)/b])] + PolyLog[2, E^x/Sqrt[(-2*a + 2*Sqrt[a*(a
 - b)] + b)/b]])/(2*Sqrt[a*(a - b)])
```

Rubi [A] (verified)

Time = 0.79 (sec) , antiderivative size = 213, normalized size of antiderivative = 0.99, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.667$, Rules used = {6163, 3042, 3801, 2694, 27, 2620, 2715, 2838}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x}{a+b\sinh^2(x)}\,dx$$
$$\Big\downarrow \text{6163}$$
$$2\int \frac{x}{2a-b+b\cosh(2x)}dx$$
$$\Big\downarrow \text{3042}$$

$$2\int \frac{x}{2a-b+b\sin\left(2ix+\frac{\pi}{2}\right)}dx$$

$\Big\downarrow$ 3801

$$4\int \frac{e^{2x}x}{2e^{2x}(2a-b)+be^{4x}+b}dx$$

$\Big\downarrow$ 2694

$$4\left(\frac{b\int \frac{e^{2x}x}{2\left(2a-2\sqrt{a-b}\sqrt{a}+be^{2x}-b\right)}dx}{2\sqrt{a}\sqrt{a-b}}-\frac{b\int \frac{e^{2x}x}{2\left(2a+2\sqrt{a-b}\sqrt{a}+be^{2x}-b\right)}dx}{2\sqrt{a}\sqrt{a-b}}\right)$$

$\Big\downarrow$ 27

$$4\left(\frac{b\int \frac{e^{2x}x}{2a-2\sqrt{a-b}\sqrt{a}+be^{2x}-b}dx}{4\sqrt{a}\sqrt{a-b}}-\frac{b\int \frac{e^{2x}x}{2a+2\sqrt{a-b}\sqrt{a}+be^{2x}-b}dx}{4\sqrt{a}\sqrt{a-b}}\right)$$

$\Big\downarrow$ 2620

$$4\left(\frac{b\left(\frac{x\log\left(\frac{be^{2x}}{-2\sqrt{a}\sqrt{a-b}+2a-b}+1\right)}{2b}-\frac{\int \log\left(\frac{e^{2x}b}{2a-2\sqrt{a-b}\sqrt{a-b}}+1\right)dx}{2b}\right)}{4\sqrt{a}\sqrt{a-b}}-\frac{b\left(\frac{x\log\left(\frac{be^{2x}}{2\sqrt{a}\sqrt{a-b}+2a-b}+1\right)}{2b}-\frac{\int \log\left(\frac{e^{2x}b}{2a+2\sqrt{a-b}\sqrt{a-b}}+1\right)dx}{2b}\right)}{4\sqrt{a}\sqrt{a-b}}\right.$$

$\Big\downarrow$ 2715

$$4\left(\frac{b\left(\frac{x\log\left(\frac{be^{2x}}{-2\sqrt{a}\sqrt{a-b}+2a-b}+1\right)}{2b}-\frac{\int e^{-2x}\log\left(\frac{e^{2x}b}{2a-2\sqrt{a-b}\sqrt{a-b}}+1\right)de^{2x}}{4b}\right)}{4\sqrt{a}\sqrt{a-b}}-\frac{b\left(\frac{x\log\left(\frac{be^{2x}}{2\sqrt{a}\sqrt{a-b}+2a-b}+1\right)}{2b}-\frac{\int e^{-2x}\log\left(\frac{e^{2x}b}{2a+2\sqrt{a-b}\sqrt{a}}\right.}{4b}\right.}{4\sqrt{a}\sqrt{a-b}}\right.$$

$\Big\downarrow$ 2838

$$4\left(\frac{b\left(\frac{\text{PolyLog}\left(2,-\frac{be^{2x}}{2a-2\sqrt{a-b}\sqrt{a-b}}\right)}{4b}+\frac{x\log\left(\frac{be^{2x}}{-2\sqrt{a}\sqrt{a-b}+2a-b}+1\right)}{2b}\right)}{4\sqrt{a}\sqrt{a-b}}-\frac{b\left(\frac{\text{PolyLog}\left(2,-\frac{be^{2x}}{2a+2\sqrt{a-b}\sqrt{a-b}}\right)}{4b}+\frac{x\log\left(\frac{be^{2x}}{2\sqrt{a}\sqrt{a-b}+2a-b}+1\right)}{2b}\right.}{4\sqrt{a}\sqrt{a-b}}\right.$$

input
```
Int[x/(a + b*Sinh[x]^2),x]
```

output
```
4*((b*((x*Log[1 + (b*E^(2*x))/(2*a - 2*Sqrt[a]*Sqrt[a - b] - b)])/(2*b) +
PolyLog[2, -((b*E^(2*x))/(2*a - 2*Sqrt[a]*Sqrt[a - b] - b))]/(4*b)))/(4*Sq
rt[a]*Sqrt[a - b]) - (b*((x*Log[1 + (b*E^(2*x))/(2*a + 2*Sqrt[a]*Sqrt[a -
b] - b)])/(2*b) + PolyLog[2, -((b*E^(2*x))/(2*a + 2*Sqrt[a]*Sqrt[a - b] -
b))]/(4*b)))/(4*Sqrt[a]*Sqrt[a - b]))
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2620
```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Si
mp[d*(m/(b*f*g*n*Log[F]))   Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x
)))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

rule 2694
```
Int[((F_)^(u_)*((f_.) + (g_.)*(x_))^(m_.))/((a_.) + (b_.)*(F_)^(u_) + (c_.)
*(F_)^(v_)), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Simp[2*(c/q)   Int
[(f + g*x)^m*(F^u/(b - q + 2*c*F^u)), x], x] - Simp[2*(c/q)   Int[(f + g*x)
^m*(F^u/(b + q + 2*c*F^u)), x], x]] /; FreeQ[{F, a, b, c, f, g}, x] && EqQ[
v, 2*u] && LinearQ[u, x] && NeQ[b^2 - 4*a*c, 0] && IGtQ[m, 0]
```

rule 2715
```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Simp[1/(d*e*n*Log[F])   Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x)
))^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

rule 2838
```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3801
```
Int[((c_.) + (d_.)*(x_))^(m_.)/((a_) + (b_.)*sin[(e_.) + Pi*(k_.) + (Comple
x[0, fz_])*(f_.)*(x_)]), x_Symbol] :> Simp[2   Int[((c + d*x)^m*(E^((-I)*e
+ f*fz*x)/(b + (2*a*E^((-I)*e + f*fz*x))/E^(I*Pi*(k - 1/2)) - (b*E^(2*((-I)
*e + f*fz*x)))/E^(2*I*k*Pi))))/E^(I*Pi*(k - 1/2)), x], x] /; FreeQ[{a, b, c
, d, e, f, fz}, x] && IntegerQ[2*k] && NeQ[a^2 - b^2, 0] && IGtQ[m, 0]
```

rule 6163
```
Int[(x_)^(m_.)*((a_) + (b_.)*Sinh[(c_.) + (d_.)*(x_)]^2)^(n_), x_Symbol] :>
 Simp[1/2^n   Int[x^m*(2*a - b + b*Cosh[2*c + 2*d*x])^n, x], x] /; FreeQ[{a
, b, c, d}, x] && NeQ[a - b, 0] && IGtQ[m, 0] && ILtQ[n, 0] && (EqQ[n, -1]
|| (EqQ[m, 1] && EqQ[n, -2]))
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 504 vs. $2(171) = 342$.

Time = 0.21 (sec) , antiderivative size = 505, normalized size of antiderivative = 2.35

method	result
risch	$\frac{x \ln\left(1-\frac{b\,\mathrm{e}^{2x}}{2\sqrt{(a-b)a}-2a+b}\right)}{2\sqrt{(a-b)a}} - \frac{x^{2}}{2\sqrt{(a-b)a}} + \frac{\mathrm{polylog}\left(2, \frac{b\,\mathrm{e}^{2x}}{2\sqrt{(a-b)a}-2a+b}\right)}{4\sqrt{(a-b)a}} + \frac{\ln\left(1-\frac{b\,\mathrm{e}^{2x}}{-2\sqrt{(a-b)a}-2a+b}\right)x}{-2\sqrt{(a-b)a}-2a+b} + \frac{\ln\left(1-\frac{b\,\mathrm{e}^{2}}{-2\sqrt{(a-b}}\right.}{\sqrt{(a-b)a}\left(-2\sqrt{(}\right.}$

input
```
int(x/(a+b*sinh(x)^2),x,method=_RETURNVERBOSE)
```

output
```
1/2/((a-b)*a)^(1/2)*x*ln(1-b*exp(2*x)/(2*((a-b)*a)^(1/2)-2*a+b))-1/2/((a-b
)*a)^(1/2)*x^2+1/4/((a-b)*a)^(1/2)*polylog(2,b*exp(2*x)/(2*((a-b)*a)^(1/2)
-2*a+b))+1/(-2*((a-b)*a)^(1/2)-2*a+b)*ln(1-b*exp(2*x)/(-2*((a-b)*a)^(1/2)-
2*a+b))*x+1/((a-b)*a)^(1/2)/(-2*((a-b)*a)^(1/2)-2*a+b)*ln(1-b*exp(2*x)/(-2
*((a-b)*a)^(1/2)-2*a+b))*a*x-1/2/((a-b)*a)^(1/2)/(-2*((a-b)*a)^(1/2)-2*a+b
)*ln(1-b*exp(2*x)/(-2*((a-b)*a)^(1/2)-2*a+b))*b*x-1/(-2*((a-b)*a)^(1/2)-2*
a+b)*x^2-1/((a-b)*a)^(1/2)/(-2*((a-b)*a)^(1/2)-2*a+b)*a*x^2+1/2/((a-b)*a)^
(1/2)/(-2*((a-b)*a)^(1/2)-2*a+b)*b*x^2+1/2/(-2*((a-b)*a)^(1/2)-2*a+b)*poly
log(2,b*exp(2*x)/(-2*((a-b)*a)^(1/2)-2*a+b))+1/2/((a-b)*a)^(1/2)/(-2*((a-b
)*a)^(1/2)-2*a+b)*polylog(2,b*exp(2*x)/(-2*((a-b)*a)^(1/2)-2*a+b))*a-1/4/(
(a-b)*a)^(1/2)/(-2*((a-b)*a)^(1/2)-2*a+b)*polylog(2,b*exp(2*x)/(-2*((a-b)*
a)^(1/2)-2*a+b))*b
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 837 vs. $2(165) = 330$.

Time = 0.17 (sec) , antiderivative size = 837, normalized size of antiderivative = 3.89

$$\int \frac{x}{a + b\sinh^2(x)}\,dx = \text{Too large to display}$$

input
```
integrate(x/(a+b*sinh(x)^2),x, algorithm="fricas")
```

output
```
-1/2*(b*x*sqrt((a^2 - a*b)/b^2)*log((((2*a - b)*cosh(x) + (2*a - b)*sinh(x
) - 2*(b*cosh(x) + b*sinh(x))*sqrt((a^2 - a*b)/b^2))*sqrt(-(2*b*sqrt((a^2
- a*b)/b^2) + 2*a - b)/b) + b)/b) + b*x*sqrt((a^2 - a*b)/b^2)*log(-(((2*a
- b)*cosh(x) + (2*a - b)*sinh(x) - 2*(b*cosh(x) + b*sinh(x))*sqrt((a^2 - a
*b)/b^2))*sqrt(-(2*b*sqrt((a^2 - a*b)/b^2) + 2*a - b)/b) - b)/b) - b*x*sqr
t((a^2 - a*b)/b^2)*log((((2*a - b)*cosh(x) + (2*a - b)*sinh(x) + 2*(b*cosh
(x) + b*sinh(x))*sqrt((a^2 - a*b)/b^2))*sqrt((2*b*sqrt((a^2 - a*b)/b^2) -
2*a + b)/b) + b)/b) - b*x*sqrt((a^2 - a*b)/b^2)*log(-(((2*a - b)*cosh(x) +
 (2*a - b)*sinh(x) + 2*(b*cosh(x) + b*sinh(x))*sqrt((a^2 - a*b)/b^2))*sqrt
((2*b*sqrt((a^2 - a*b)/b^2) - 2*a + b)/b) - b)/b) + b*sqrt((a^2 - a*b)/b^2
)*dilog(-(((2*a - b)*cosh(x) + (2*a - b)*sinh(x) - 2*(b*cosh(x) + b*sinh(x
))*sqrt((a^2 - a*b)/b^2))*sqrt(-(2*b*sqrt((a^2 - a*b)/b^2) + 2*a - b)/b) +
 b)/b + 1) + b*sqrt((a^2 - a*b)/b^2)*dilog((((2*a - b)*cosh(x) + (2*a - b)
*sinh(x) - 2*(b*cosh(x) + b*sinh(x))*sqrt((a^2 - a*b)/b^2))*sqrt(-(2*b*sqr
t((a^2 - a*b)/b^2) + 2*a - b)/b) - b)/b + 1) - b*sqrt((a^2 - a*b)/b^2)*dil
og(-(((2*a - b)*cosh(x) + (2*a - b)*sinh(x) + 2*(b*cosh(x) + b*sinh(x))*sq
rt((a^2 - a*b)/b^2))*sqrt((2*b*sqrt((a^2 - a*b)/b^2) - 2*a + b)/b) + b)/b
+ 1) - b*sqrt((a^2 - a*b)/b^2)*dilog((((2*a - b)*cosh(x) + (2*a - b)*sinh(
x) + 2*(b*cosh(x) + b*sinh(x))*sqrt((a^2 - a*b)/b^2))*sqrt((2*b*sqrt((a^2
- a*b)/b^2) - 2*a + b)/b) - b)/b + 1))/(a^2 - a*b)
```

Sympy [F]

$$\int \frac{x}{a+b\sinh^2(x)}\,dx=\int \frac{x}{a+b\sinh^2{(x)}}\,dx$$

input
```
integrate(x/(a+b*sinh(x)**2),x)
```

output
```
Integral(x/(a + b*sinh(x)**2), x)
```

Maxima [F]

$$\int \frac{x}{a+b\sinh^2(x)}\,dx=\int \frac{x}{b\sinh{(x)}^2+a}\,dx$$

input
```
integrate(x/(a+b*sinh(x)^2),x, algorithm="maxima")
```

output
```
integrate(x/(b*sinh(x)^2 + a), x)
```

Giac [F]

$$\int \frac{x}{a+b\sinh^2(x)}\,dx=\int \frac{x}{b\sinh{(x)}^2+a}\,dx$$

input
```
integrate(x/(a+b*sinh(x)^2),x, algorithm="giac")
```

output
```
integrate(x/(b*sinh(x)^2 + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x}{a+b\sinh^2(x)}\,dx=\int \frac{x}{b\sinh{(x)}^2+a}\,dx$$

input
```
int(x/(a + b*sinh(x)^2),x)
```

output
```
int(x/(a + b*sinh(x)^2), x)
```

Reduce [F]

$$\int \frac{x}{a+b\sinh^2(x)}\,dx=\int \frac{x}{\sinh{(x)}^2 b+a}dx$$

input
```
int(x/(a+b*sinh(x)^2),x)
```

output
```
int(x/(sinh(x)**2*b + a),x)
```

3.260 $\int \frac{\cosh(a+bx)(-2+\sinh^2(a+bx))}{x}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 47

$$\int \frac{\cosh(a+bx)\left(-2+\sinh^2(a+bx)\right)}{x}\,dx = -\frac{9}{4}\cosh(a)\text{Chi}(bx)+\frac{1}{4}\cosh(3a)\text{Chi}(3bx) - \frac{9}{4}\sinh(a)\text{Shi}(bx)+\frac{1}{4}\sinh(3a)\text{Shi}(3bx)$$

output

```
-9/4*cosh(a)*Chi(b*x)+1/4*cosh(3*a)*Chi(3*b*x)-9/4*sinh(a)*Shi(b*x)+1/4*si
nh(3*a)*Shi(3*b*x)
```

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.87

$$\int \frac{\cosh(a+bx)\left(-2+\sinh^2(a+bx)\right)}{x}\,dx = \frac{1}{4}(-9\cosh(a)\text{Chi}(bx)+\cosh(3a)\text{Chi}(3bx) - 9\sinh(a)\text{Shi}(bx)+\sinh(3a)\text{Shi}(3bx))$$

input

```
Integrate[(Cosh[a + b*x]*(-2 + Sinh[a + b*x]^2))/x,x]
```

output
```
(-9*Cosh[a]*CoshIntegral[b*x] + Cosh[3*a]*CoshIntegral[3*b*x] - 9*Sinh[a]*
SinhIntegral[b*x] + Sinh[3*a]*SinhIntegral[3*b*x])/4
```

Rubi [A] (verified)

Time = 0.63 (sec) , antiderivative size = 47, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.100$, Rules used = {7293, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(\sinh^2(a+bx)-2)\cosh(a+bx)}{x}\,dx$$

$$\Big\downarrow \text{7293}$$

$$\int \left(\frac{\sinh^2(a+bx)\cosh(a+bx)}{x}-\frac{2\cosh(a+bx)}{x}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$-\frac{9}{4}\cosh(a)\text{Chi}(bx)+\frac{1}{4}\cosh(3a)\text{Chi}(3bx)-\frac{9}{4}\sinh(a)\text{Shi}(bx)+\frac{1}{4}\sinh(3a)\text{Shi}(3bx)$$

input
```
Int[(Cosh[a + b*x]*(-2 + Sinh[a + b*x]^2))/x,x]
```

output
```
(-9*Cosh[a]*CoshIntegral[b*x])/4 + (Cosh[3*a]*CoshIntegral[3*b*x])/4 - (9*
Sinh[a]*SinhIntegral[b*x])/4 + (Sinh[3*a]*SinhIntegral[3*b*x])/4
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 7293
```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Maple [A] (verified)

Time = 3.21 (sec) , antiderivative size = 47, normalized size of antiderivative = 1.00

method	result	size
risch	$-\frac{e^{-3a}\,\mathrm{expIntegral}_1(3bx)}{8}+\frac{9\,e^{-a}\,\mathrm{expIntegral}_1(bx)}{8}+\frac{9\,e^{a}\,\mathrm{expIntegral}_1(-bx)}{8}-\frac{e^{3a}\,\mathrm{expIntegral}_1(-3bx)}{8}$	47

input
```
int(cosh(b*x+a)*(-2+sinh(b*x+a)^2)/x,x,method=_RETURNVERBOSE)
```

output
```
-1/8*exp(-3*a)*Ei(1,3*b*x)+9/8*exp(-a)*Ei(1,b*x)+9/8*exp(a)*Ei(1,-b*x)-1/8
*exp(3*a)*Ei(1,-3*b*x)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 67, normalized size of antiderivative = 1.43

$$\int \frac{\cosh(a+bx)\left(-2+\sinh^2(a+bx)\right)}{x}\,dx=\frac{1}{8}\left(\mathrm{Ei}(3\,bx)+\mathrm{Ei}(-3\,bx)\right)\cosh\left(3\,a\right)-\frac{9}{8}\left(\mathrm{Ei}(bx)+\mathrm{Ei}(-bx)\right)\cosh\left(a\right)+\frac{1}{8}\left(\mathrm{Ei}(3\,bx)-\mathrm{Ei}(-3\,bx)\right)\sinh\left(3\,a\right)-\frac{9}{8}\left(\mathrm{Ei}(bx)-\mathrm{Ei}(-bx)\right)\sinh\left(a\right)$$

input
```
integrate(cosh(b*x+a)*(-2+sinh(b*x+a)^2)/x,x, algorithm="fricas")
```

output
```
1/8*(Ei(3*b*x) + Ei(-3*b*x))*cosh(3*a) - 9/8*(Ei(b*x) + Ei(-b*x))*cosh(a)
+ 1/8*(Ei(3*b*x) - Ei(-3*b*x))*sinh(3*a) - 9/8*(Ei(b*x) - Ei(-b*x))*sinh(a
)
```

Sympy [F]

$$\int \frac{\cosh(a+bx)\left(-2+\sinh^2(a+bx)\right)}{x}\,dx = \int \frac{\left(\sinh^2\left(a+bx\right)-2\right)\cosh\left(a+bx\right)}{x}\,dx$$

input
```
integrate(cosh(b*x+a)*(-2+sinh(b*x+a)**2)/x,x)
```

output
```
Integral((sinh(a + b*x)**2 - 2)*cosh(a + b*x)/x, x)
```

Maxima [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.89

$$\int \frac{\cosh(a+bx)\left(-2+\sinh^2(a+bx)\right)}{x}\,dx = \frac{1}{8}\operatorname{Ei}(3\,bx)\,e^{(3\,a)} - \frac{9}{8}\operatorname{Ei}(-bx)\,e^{(-a)} + \frac{1}{8}\operatorname{Ei}(-3\,bx)\,e^{(-3\,a)} - \frac{9}{8}\operatorname{Ei}(bx)\,e^{a}$$

input
```
integrate(cosh(b*x+a)*(-2+sinh(b*x+a)^2)/x,x, algorithm="maxima")
```

output
```
1/8*Ei(3*b*x)*e^(3*a) - 9/8*Ei(-b*x)*e^(-a) + 1/8*Ei(-3*b*x)*e^(-3*a) - 9/
8*Ei(b*x)*e^a
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 43, normalized size of antiderivative = 0.91

$$\int \frac{\cosh(a+bx)\left(-2+\sinh^2(a+bx)\right)}{x}\,dx = \frac{1}{8}\left(\operatorname{Ei}(3\,bx)\,e^{(6\,a)} - 9\operatorname{Ei}(bx)\,e^{(4\,a)} - 9\operatorname{Ei}(-bx)\,e^{(2\,a)} + \operatorname{Ei}(-3\,bx)\right)e^{(-3\,a)}$$

input
```
integrate(cosh(b*x+a)*(-2+sinh(b*x+a)^2)/x,x, algorithm="giac")
```

output
```
1/8*(Ei(3*b*x)*e^(6*a) - 9*Ei(b*x)*e^(4*a) - 9*Ei(-b*x)*e^(2*a) + Ei(-3*b*
x))*e^(-3*a)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\cosh(a+bx)\left(-2+\sinh^2(a+bx)\right)}{x}\,dx = \int \frac{\cosh(a+b\,x)\,\left(\sinh(a+b\,x)^2-2\right)}{x}\,dx$$

input
```
int((cosh(a + b*x)*(sinh(a + b*x)^2 - 2))/x,x)
```

output
```
int((cosh(a + b*x)*(sinh(a + b*x)^2 - 2))/x, x)
```

Reduce [F]

$$\int \frac{\cosh(a+bx)\left(-2+\sinh^2(a+bx)\right)}{x}\,dx = -2\left(\int \frac{\cosh(bx+a)}{x}dx\right) + \int \frac{\cosh(bx+a)\sinh(bx+a)^2}{x}dx$$

input
```
int(cosh(b*x+a)*(-2+sinh(b*x+a)^2)/x,x)
```

output
```
- 2*int(cosh(a + b*x)/x,x) + int((cosh(a + b*x)*sinh(a + b*x)**2)/x,x)
```

3.261 $\int \frac{\sinh^3\left(\frac{\sqrt{1-ax}}{\sqrt{1+ax}}\right)}{1-a^2x^2}\,dx$

Optimal result

Integrand size = 36, antiderivative size = 58

$$\int \frac{\sinh^3\left(\frac{\sqrt{1-ax}}{\sqrt{1+ax}}\right)}{1-a^2x^2}\,dx = \frac{3\mathrm{Shi}\left(\frac{\sqrt{1-ax}}{\sqrt{1+ax}}\right)}{4a} - \frac{\mathrm{Shi}\left(\frac{3\sqrt{1-ax}}{\sqrt{1+ax}}\right)}{4a}$$

output
```
3/4*Shi((-a*x+1)^(1/2)/(a*x+1)^(1/2))/a-1/4*Shi(3*(-a*x+1)^(1/2)/(a*x+1)^(
1/2))/a
```

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 55, normalized size of antiderivative = 0.95

$$\int \frac{\sinh^3\left(\frac{\sqrt{1-ax}}{\sqrt{1+ax}}\right)}{1-a^2x^2}\,dx = \frac{3\mathrm{Shi}\left(\frac{\sqrt{1-ax}}{\sqrt{1+ax}}\right) - \mathrm{Shi}\left(\frac{3\sqrt{1-ax}}{\sqrt{1+ax}}\right)}{4a}$$

input
```
Integrate[Sinh[Sqrt[1 - a*x]/Sqrt[1 + a*x]]^3/(1 - a^2*x^2),x]
```

output
```
(3*SinhIntegral[Sqrt[1 - a*x]/Sqrt[1 + a*x]] - SinhIntegral[(3*Sqrt[1 - a*
x])/Sqrt[1 + a*x]])/(4*a)
```

Rubi [C] (verified)

Result contains complex when optimal does not.

Time = 0.36 (sec) , antiderivative size = 63, normalized size of antiderivative = 1.09, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.139$, Rules used = {7232, 3042, 26, 3793, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sinh^3\left(\frac{\sqrt{1-ax}}{\sqrt{ax+1}}\right)}{1-a^2x^2}dx$$

$$\Big\downarrow \text{7232}$$

$$-\frac{\int \frac{\sqrt{ax+1}\sinh^3\left(\frac{\sqrt{1-ax}}{\sqrt{ax+1}}\right)}{\sqrt{1-ax}}d\frac{\sqrt{1-ax}}{\sqrt{ax+1}}}{a}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{\int \frac{i\sqrt{ax+1}\sin\left(\frac{i\sqrt{1-ax}}{\sqrt{ax+1}}\right)^3}{\sqrt{1-ax}}d\frac{\sqrt{1-ax}}{\sqrt{ax+1}}}{a}$$

$$\Big\downarrow \text{26}$$

$$-\frac{i\int \frac{\sqrt{ax+1}\sin\left(\frac{i\sqrt{1-ax}}{\sqrt{ax+1}}\right)^3}{\sqrt{1-ax}}d\frac{\sqrt{1-ax}}{\sqrt{ax+1}}}{a}$$

$$\Big\downarrow \text{3793}$$

$$-\frac{i\int \left(\frac{3i\sqrt{ax+1}\sinh\left(\frac{\sqrt{1-ax}}{\sqrt{ax+1}}\right)}{4\sqrt{1-ax}}-\frac{i\sqrt{ax+1}\sinh\left(\frac{3\sqrt{1-ax}}{\sqrt{ax+1}}\right)}{4\sqrt{1-ax}}\right)d\frac{\sqrt{1-ax}}{\sqrt{ax+1}}}{a}$$

$$\Big\downarrow \text{2009}$$

$$-\frac{i\left(\frac{3}{4}i\text{Shi}\left(\frac{\sqrt{1-ax}}{\sqrt{ax+1}}\right)-\frac{1}{4}i\text{Shi}\left(\frac{3\sqrt{1-ax}}{\sqrt{ax+1}}\right)\right)}{a}$$

input
```
Int[Sinh[Sqrt[1 - a*x]/Sqrt[1 + a*x]]^3/(1 - a^2*x^2),x]
```

output
```
((-I)*(((3*I)/4)*SinhIntegral[Sqrt[1 - a*x]/Sqrt[1 + a*x]] - (I/4)*SinhInt
egral[(3*Sqrt[1 - a*x])/Sqrt[1 + a*x]]))/a
```

Defintions of rubi rules used

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3793
```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> In
t[ExpandTrigReduce[(c + d*x)^m, Sin[e + f*x]^n, x], x] /; FreeQ[{c, d, e, f
, m}, x] && IGtQ[n, 1] && ( !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))
```

rule 7232
```
Int[((a_.) + (b_.)*(F_)[((c_.)*Sqrt[(d_.) + (e_.)*(x_)])/Sqrt[(f_.) + (g_.)
*(x_)]])^(n_.)/((A_.) + (C_.)*(x_)^2), x_Symbol] :> Simp[2*e*(g/(C*(e*f - d
*g)))   Subst[Int[(a + b*F[c*x])^n/x, x], x, Sqrt[d + e*x]/Sqrt[f + g*x]],
x] /; FreeQ[{a, b, c, d, e, f, g, A, C, F}, x] && EqQ[C*d*f - A*e*g, 0] &&
EqQ[e*f + d*g, 0] && IGtQ[n, 0]
```

Maple [F]

$$\int \frac{\sinh\left(\frac{\sqrt{-ax+1}}{\sqrt{ax+1}}\right)^3}{-a^2x^2+1}dx$$

input
```
int(sinh((-a*x+1)^(1/2)/(a*x+1)^(1/2))^3/(-a^2*x^2+1),x)
```

output
```
int(sinh((-a*x+1)^(1/2)/(a*x+1)^(1/2))^3/(-a^2*x^2+1),x)
```

Fricas [F]

$$\int \frac{\sinh^3\left(\frac{\sqrt{1-ax}}{\sqrt{1+ax}}\right)}{1-a^2x^2}\,dx = \int -\frac{\sinh\left(\frac{\sqrt{-ax+1}}{\sqrt{ax+1}}\right)^3}{a^2x^2-1}\,dx$$

input
```
integrate(sinh((-a*x+1)^(1/2)/(a*x+1)^(1/2))^3/(-a^2*x^2+1),x, algorithm="
fricas")
```

output
```
integral(-sinh(sqrt(-a*x + 1)/sqrt(a*x + 1))^3/(a^2*x^2 - 1), x)
```

Sympy [F]

$$\int \frac{\sinh^3\left(\frac{\sqrt{1-ax}}{\sqrt{1+ax}}\right)}{1-a^2x^2}\,dx = -\int \frac{\sinh^3\left(\frac{\sqrt{-ax+1}}{\sqrt{ax+1}}\right)}{a^2x^2-1}\,dx$$

input
```
integrate(sinh((-a*x+1)**(1/2)/(a*x+1)**(1/2))**3/(-a**2*x**2+1),x)
```

output
```
-Integral(sinh(sqrt(-a*x + 1)/sqrt(a*x + 1))**3/(a**2*x**2 - 1), x)
```

Maxima [F]

$$\int \frac{\sinh^3\left(\frac{\sqrt{1-ax}}{\sqrt{1+ax}}\right)}{1-a^2x^2}\,dx = \int -\frac{\sinh\left(\frac{\sqrt{-ax+1}}{\sqrt{ax+1}}\right)^3}{a^2x^2-1}\,dx$$

input
```
integrate(sinh((-a*x+1)^(1/2)/(a*x+1)^(1/2))^3/(-a^2*x^2+1),x, algorithm="
maxima")
```

output
```
-integrate(sinh(sqrt(-a*x + 1)/sqrt(a*x + 1))^3/(a^2*x^2 - 1), x)
```

Giac [F]

$$\int \frac{\sinh^3\left(\frac{\sqrt{1-ax}}{\sqrt{1+ax}}\right)}{1-a^2x^2}\,dx = \int -\frac{\sinh\left(\frac{\sqrt{-ax+1}}{\sqrt{ax+1}}\right)^3}{a^2x^2-1}\,dx$$

input
```
integrate(sinh((-a*x+1)^(1/2)/(a*x+1)^(1/2))^3/(-a^2*x^2+1),x, algorithm="
giac")
```

output
```
integrate(-sinh(sqrt(-a*x + 1)/sqrt(a*x + 1))^3/(a^2*x^2 - 1), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sinh^3\left(\frac{\sqrt{1-ax}}{\sqrt{1+ax}}\right)}{1-a^2x^2}\,dx = -\int \frac{\sinh\left(\frac{\sqrt{1-a\,x}}{\sqrt{a\,x+1}}\right)^3}{a^2\,x^2-1}\,dx$$

input
```
int(-sinh((1 - a*x)^(1/2)/(a*x + 1)^(1/2))^3/(a^2*x^2 - 1),x)
```

output
```
-int(sinh((1 - a*x)^(1/2)/(a*x + 1)^(1/2))^3/(a^2*x^2 - 1), x)
```

Reduce [F]

$$\int \frac{\sinh^3\left(\frac{\sqrt{1-ax}}{\sqrt{1+ax}}\right)}{1-a^2x^2}\,dx = -\left(\int \frac{\sinh\left(\frac{\sqrt{-ax+1}}{\sqrt{ax+1}}\right)^3}{a^2x^2-1}dx\right)$$

input
```
int(sinh((-a*x+1)^(1/2)/(a*x+1)^(1/2))^3/(-a^2*x^2+1),x)
```

output
```
- int(sinh(sqrt( - a*x + 1)/sqrt(a*x + 1))**3/(a**2*x**2 - 1),x)
```

3.262 $\int \frac{\sinh^2\left(\frac{\sqrt{1-ax}}{\sqrt{1+ax}}\right)}{1-a^2x^2}\,dx$

Optimal result

Integrand size = 36, antiderivative size = 58

$$\int \frac{\sinh^2\left(\frac{\sqrt{1-ax}}{\sqrt{1+ax}}\right)}{1-a^2x^2}\,dx=-\frac{\operatorname{Chi}\left(\frac{2\sqrt{1-ax}}{\sqrt{1+ax}}\right)}{2a}+\frac{\log\left(\frac{\sqrt{1-ax}}{\sqrt{1+ax}}\right)}{2a}$$

output
```
-1/2*Chi(2*(-a*x+1)^(1/2)/(a*x+1)^(1/2))/a+1/2*ln((-a*x+1)^(1/2)/(a*x+1)^(
1/2))/a
```

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.98

$$\int \frac{\sinh^2\left(\frac{\sqrt{1-ax}}{\sqrt{1+ax}}\right)}{1-a^2x^2}\,dx=-\frac{\operatorname{Chi}\left(\frac{2\sqrt{1-ax}}{\sqrt{1+ax}}\right)}{2a}+\frac{\log(1-ax)}{4a}-\frac{\log(1+ax)}{4a}$$

input
```
Integrate[Sinh[Sqrt[1 - a*x]/Sqrt[1 + a*x]]^2/(1 - a^2*x^2),x]
```

output
```
-1/2*CoshIntegral[(2*Sqrt[1 - a*x])/Sqrt[1 + a*x]]/a + Log[1 - a*x]/(4*a)
- Log[1 + a*x]/(4*a)
```

Rubi [A] (verified)

Time = 0.33 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.98, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.139, Rules used = {7232, 3042, 25, 3793, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sinh^2\left(\frac{\sqrt{1-ax}}{\sqrt{ax+1}}\right)}{1-a^2x^2}dx$$
$$\Big\downarrow \text{7232}$$
$$-\frac{\int \frac{\sqrt{ax+1}\sinh^2\left(\frac{\sqrt{1-ax}}{\sqrt{ax+1}}\right)}{\sqrt{1-ax}}d\frac{\sqrt{1-ax}}{\sqrt{ax+1}}}{a}$$
$$\Big\downarrow \text{3042}$$
$$-\frac{\int -\frac{\sqrt{ax+1}\sin\left(\frac{i\sqrt{1-ax}}{\sqrt{ax+1}}\right)^2}{\sqrt{1-ax}}d\frac{\sqrt{1-ax}}{\sqrt{ax+1}}}{a}$$
$$\Big\downarrow \text{25}$$
$$\frac{\int \frac{\sqrt{ax+1}\sin\left(\frac{i\sqrt{1-ax}}{\sqrt{ax+1}}\right)^2}{\sqrt{1-ax}}d\frac{\sqrt{1-ax}}{\sqrt{ax+1}}}{a}$$
$$\Big\downarrow \text{3793}$$
$$\frac{\int \left(\frac{\sqrt{ax+1}}{2\sqrt{1-ax}}-\frac{\sqrt{ax+1}\cosh\left(\frac{2\sqrt{1-ax}}{\sqrt{ax+1}}\right)}{2\sqrt{1-ax}}\right)d\frac{\sqrt{1-ax}}{\sqrt{ax+1}}}{a}$$
$$\Big\downarrow \text{2009}$$
$$-\frac{\frac{1}{2}\text{Chi}\left(\frac{2\sqrt{1-ax}}{\sqrt{ax+1}}\right)-\frac{1}{2}\log\left(\frac{\sqrt{1-ax}}{\sqrt{ax+1}}\right)}{a}$$

input
```
Int[Sinh[Sqrt[1 - a*x]/Sqrt[1 + a*x]]^2/(1 - a^2*x^2),x]
```

output
```
-((CoshIntegral[(2*Sqrt[1 - a*x])/Sqrt[1 + a*x]]/2 - Log[Sqrt[1 - a*x]/Sqr
t[1 + a*x]]/2)/a)
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3793
```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> In
t[ExpandTrigReduce[(c + d*x)^m, Sin[e + f*x]^n, x], x] /; FreeQ[{c, d, e, f
, m}, x] && IGtQ[n, 1] && ( !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))
```

rule 7232
```
Int[((a_.) + (b_.)*(F_)[((c_.)*Sqrt[(d_.) + (e_.)*(x_)])/Sqrt[(f_.) + (g_.)
*(x_)]])^(n_.)/((A_.) + (C_.)*(x_)^2), x_Symbol] :> Simp[2*e*(g/(C*(e*f - d
*g)))   Subst[Int[(a + b*F[c*x])^n/x, x], x, Sqrt[d + e*x]/Sqrt[f + g*x]],
x] /; FreeQ[{a, b, c, d, e, f, g, A, C, F}, x] && EqQ[C*d*f - A*e*g, 0] &&
EqQ[e*f + d*g, 0] && IGtQ[n, 0]
```

Maple [F]

$$\int \frac{\sinh\left(\frac{\sqrt{-ax+1}}{\sqrt{ax+1}}\right)^{2}}{-a^{2}x^{2}+1}dx$$

input
```
int(sinh((-a*x+1)^(1/2)/(a*x+1)^(1/2))^2/(-a^2*x^2+1),x)
```

output
```
int(sinh((-a*x+1)^(1/2)/(a*x+1)^(1/2))^2/(-a^2*x^2+1),x)
```

Fricas [F]

$$\int \frac{\sinh^2\left(\frac{\sqrt{1-ax}}{\sqrt{1+ax}}\right)}{1-a^2x^2}\,dx = \int -\frac{\sinh\left(\frac{\sqrt{-ax+1}}{\sqrt{ax+1}}\right)^2}{a^2x^2-1}\,dx$$

input
```
integrate(sinh((-a*x+1)^(1/2)/(a*x+1)^(1/2))^2/(-a^2*x^2+1),x, algorithm="
fricas")
```

output
```
integral(-sinh(sqrt(-a*x + 1)/sqrt(a*x + 1))^2/(a^2*x^2 - 1), x)
```

Sympy [F]

$$\int \frac{\sinh^2\left(\frac{\sqrt{1-ax}}{\sqrt{1+ax}}\right)}{1-a^2x^2}\,dx = -\int \frac{\sinh^2\left(\frac{\sqrt{-ax+1}}{\sqrt{ax+1}}\right)}{a^2x^2-1}\,dx$$

input
```
integrate(sinh((-a*x+1)**(1/2)/(a*x+1)**(1/2))**2/(-a**2*x**2+1),x)
```

output
```
-Integral(sinh(sqrt(-a*x + 1)/sqrt(a*x + 1))**2/(a**2*x**2 - 1), x)
```

Maxima [F]

$$\int \frac{\sinh^2\left(\frac{\sqrt{1-ax}}{\sqrt{1+ax}}\right)}{1-a^2x^2}\,dx = \int -\frac{\sinh\left(\frac{\sqrt{-ax+1}}{\sqrt{ax+1}}\right)^2}{a^2x^2-1}\,dx$$

input
```
integrate(sinh((-a*x+1)^(1/2)/(a*x+1)^(1/2))^2/(-a^2*x^2+1),x, algorithm="
maxima")
```

output
```
-1/4*log(a*x + 1)/a + 1/4*log(a*x - 1)/a - 1/4*integrate(e^(2*sqrt(-a*x +
1)/sqrt(a*x + 1))/(a^2*x^2 - 1), x) - 1/4*integrate(e^(-2*sqrt(-a*x + 1)/s
qrt(a*x + 1))/(a^2*x^2 - 1), x)
```

Giac [F]

$$\int \frac{\sinh^2\left(\frac{\sqrt{1-ax}}{\sqrt{1+ax}}\right)}{1-a^2x^2}\,dx = \int -\frac{\sinh\left(\frac{\sqrt{-ax+1}}{\sqrt{ax+1}}\right)^2}{a^2x^2-1}\,dx$$

input
```
integrate(sinh((-a*x+1)^(1/2)/(a*x+1)^(1/2))^2/(-a^2*x^2+1),x, algorithm="
giac")
```

output
```
integrate(-sinh(sqrt(-a*x + 1)/sqrt(a*x + 1))^2/(a^2*x^2 - 1), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sinh^2\left(\frac{\sqrt{1-ax}}{\sqrt{1+ax}}\right)}{1-a^2x^2}\,dx = -\int \frac{\sinh\left(\frac{\sqrt{1-a\,x}}{\sqrt{a\,x+1}}\right)^2}{a^2\,x^2-1}\,dx$$

input
```
int(-sinh((1 - a*x)^(1/2)/(a*x + 1)^(1/2))^2/(a^2*x^2 - 1),x)
```

output
```
-int(sinh((1 - a*x)^(1/2)/(a*x + 1)^(1/2))^2/(a^2*x^2 - 1), x)
```

Reduce [F]

$$\int \frac{\sinh^2\left(\frac{\sqrt{1-ax}}{\sqrt{1+ax}}\right)}{1-a^2x^2}\,dx = -\left(\int \frac{\sinh\left(\frac{\sqrt{-ax+1}}{\sqrt{ax+1}}\right)^2}{a^2x^2-1}dx\right)$$

input
```
int(sinh((-a*x+1)^(1/2)/(a*x+1)^(1/2))^2/(-a^2*x^2+1),x)
```

output
```
- int(sinh(sqrt( - a*x + 1)/sqrt(a*x + 1))**2/(a**2*x**2 - 1),x)
```

3.263 $\int \frac{\sinh\left(\frac{\sqrt{1-ax}}{\sqrt{1+ax}}\right)}{1-a^2x^2}\,dx$

Optimal result

Integrand size = 34, antiderivative size = 26

$$\int \frac{\sinh\left(\frac{\sqrt{1-ax}}{\sqrt{1+ax}}\right)}{1-a^2x^2}\,dx=-\frac{\text{Shi}\left(\frac{\sqrt{1-ax}}{\sqrt{1+ax}}\right)}{a}$$

output
```
-Shi((-a*x+1)^(1/2)/(a*x+1)^(1/2))/a
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.00

$$\int \frac{\sinh\left(\frac{\sqrt{1-ax}}{\sqrt{1+ax}}\right)}{1-a^2x^2}\,dx=-\frac{\text{Shi}\left(\frac{\sqrt{1-ax}}{\sqrt{1+ax}}\right)}{a}$$

input
```
Integrate[Sinh[Sqrt[1 - a*x]/Sqrt[1 + a*x]]/(1 - a^2*x^2),x]
```

output
```
-(SinhIntegral[Sqrt[1 - a*x]/Sqrt[1 + a*x]]/a)
```

Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.118$, Rules used = {7232, 3042, 26, 3779}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sinh\left(\frac{\sqrt{1-ax}}{\sqrt{ax+1}}\right)}{1-a^2x^2}dx$$
$$\Big\downarrow \text{7232}$$
$$-\frac{\int \frac{\sqrt{ax+1}\sinh\left(\frac{\sqrt{1-ax}}{\sqrt{ax+1}}\right)}{\sqrt{1-ax}}d\frac{\sqrt{1-ax}}{\sqrt{ax+1}}}{a}$$
$$\Big\downarrow \text{3042}$$
$$-\frac{\int -\frac{i\sqrt{ax+1}\sin\left(\frac{i\sqrt{1-ax}}{\sqrt{ax+1}}\right)}{\sqrt{1-ax}}d\frac{\sqrt{1-ax}}{\sqrt{ax+1}}}{a}$$
$$\Big\downarrow \text{26}$$
$$\frac{i\int \frac{\sqrt{ax+1}\sin\left(\frac{i\sqrt{1-ax}}{\sqrt{ax+1}}\right)}{\sqrt{1-ax}}d\frac{\sqrt{1-ax}}{\sqrt{ax+1}}}{a}$$
$$\Big\downarrow \text{3779}$$
$$-\frac{\text{Shi}\left(\frac{\sqrt{1-ax}}{\sqrt{ax+1}}\right)}{a}$$

input
```
Int[Sinh[Sqrt[1 - a*x]/Sqrt[1 + a*x]]/(1 - a^2*x^2),x]
```

output
```
-(SinhIntegral[Sqrt[1 - a*x]/Sqrt[1 + a*x]]/a)
```

Defintions of rubi rules used

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3779
```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[I*(SinhIntegral[c*f*(fz/d) + f*fz*x]/d), x] /; FreeQ[{c, d, e, f
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

rule 7232
```
Int[((a_.) + (b_.)*(F_)[((c_.)*Sqrt[(d_.) + (e_.)*(x_)])/Sqrt[(f_.) + (g_.)
*(x_)]])^(n_.)/((A_.) + (C_.)*(x_)^2), x_Symbol] :> Simp[2*e*(g/(C*(e*f - d
*g)))   Subst[Int[(a + b*F[c*x])^n/x, x], x, Sqrt[d + e*x]/Sqrt[f + g*x]],
x] /; FreeQ[{a, b, c, d, e, f, g, A, C, F}, x] && EqQ[C*d*f - A*e*g, 0] &&
EqQ[e*f + d*g, 0] && IGtQ[n, 0]
```

Maple [F]

$$\int \frac{\sinh\left(\frac{\sqrt{-ax+1}}{\sqrt{ax+1}}\right)}{-a^2x^2+1}dx$$

input
```
int(sinh((-a*x+1)^(1/2)/(a*x+1)^(1/2))/(-a^2*x^2+1),x)
```

output
```
int(sinh((-a*x+1)^(1/2)/(a*x+1)^(1/2))/(-a^2*x^2+1),x)
```

Fricas [F]

$$\int \frac{\sinh\left(\frac{\sqrt{1-ax}}{\sqrt{1+ax}}\right)}{1-a^2x^2}\,dx = \int -\frac{\sinh\left(\frac{\sqrt{-ax+1}}{\sqrt{ax+1}}\right)}{a^2x^2-1}\,dx$$

input
```
integrate(sinh((-a*x+1)^(1/2)/(a*x+1)^(1/2))/(-a^2*x^2+1),x, algorithm="fr
icas")
```

output
```
integral(-sinh(sqrt(-a*x + 1)/sqrt(a*x + 1))/(a^2*x^2 - 1), x)
```

Sympy [F]

$$\int \frac{\sinh\left(\frac{\sqrt{1-ax}}{\sqrt{1+ax}}\right)}{1-a^2x^2}\,dx = -\int \frac{\sinh\left(\frac{\sqrt{-ax+1}}{\sqrt{ax+1}}\right)}{a^2x^2-1}\,dx$$

input
```
integrate(sinh((-a*x+1)**(1/2)/(a*x+1)**(1/2))/(-a**2*x**2+1),x)
```

output
```
-Integral(sinh(sqrt(-a*x + 1)/sqrt(a*x + 1))/(a**2*x**2 - 1), x)
```

Maxima [F]

$$\int \frac{\sinh\left(\frac{\sqrt{1-ax}}{\sqrt{1+ax}}\right)}{1-a^2x^2}\,dx = \int -\frac{\sinh\left(\frac{\sqrt{-ax+1}}{\sqrt{ax+1}}\right)}{a^2x^2-1}\,dx$$

input
```
integrate(sinh((-a*x+1)^(1/2)/(a*x+1)^(1/2))/(-a^2*x^2+1),x, algorithm="ma
xima")
```

output
```
-integrate(sinh(sqrt(-a*x + 1)/sqrt(a*x + 1))/(a^2*x^2 - 1), x)
```

Giac [F]

$$\int \frac{\sinh\left(\frac{\sqrt{1-ax}}{\sqrt{1+ax}}\right)}{1-a^2x^2}\,dx = \int -\frac{\sinh\left(\frac{\sqrt{-ax+1}}{\sqrt{ax+1}}\right)}{a^2x^2-1}\,dx$$

input
```
integrate(sinh((-a*x+1)^(1/2)/(a*x+1)^(1/2))/(-a^2*x^2+1),x, algorithm="gi
ac")
```

output
```
integrate(-sinh(sqrt(-a*x + 1)/sqrt(a*x + 1))/(a^2*x^2 - 1), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sinh\left(\frac{\sqrt{1-ax}}{\sqrt{1+ax}}\right)}{1-a^2x^2}\,dx = -\int \frac{\sinh\left(\frac{\sqrt{1-a\,x}}{\sqrt{a\,x+1}}\right)}{a^2\,x^2-1}\,dx$$

input
```
int(-sinh((1 - a*x)^(1/2)/(a*x + 1)^(1/2))/(a^2*x^2 - 1),x)
```

output
```
-int(sinh((1 - a*x)^(1/2)/(a*x + 1)^(1/2))/(a^2*x^2 - 1), x)
```

Reduce [F]

$$\int \frac{\sinh\left(\frac{\sqrt{1-ax}}{\sqrt{1+ax}}\right)}{1-a^2x^2}\,dx = -\left(\int \frac{\sinh\left(\frac{\sqrt{-ax+1}}{\sqrt{ax+1}}\right)}{a^2x^2-1}dx\right)$$

input
```
int(sinh((-a*x+1)^(1/2)/(a*x+1)^(1/2))/(-a^2*x^2+1),x)
```

output
```
 - int(sinh(sqrt( - a*x + 1)/sqrt(a*x + 1))/(a**2*x**2 - 1),x)
```

3.264 $\int \frac{\text{csch}\left(\frac{\sqrt{1-ax}}{\sqrt{1+ax}}\right)}{1-a^2x^2}\,dx$

Optimal result

Integrand size = 34, antiderivative size = 34

$$\int \frac{\text{csch}\left(\frac{\sqrt{1-ax}}{\sqrt{1+ax}}\right)}{1-a^2x^2}\,dx=\text{Int}\left(\frac{\text{csch}\left(\frac{\sqrt{1-ax}}{\sqrt{1+ax}}\right)}{(1-ax)(1+ax)},x\right)$$

output
```
Defer(Int)(csch((-a*x+1)^(1/2)/(a*x+1)^(1/2))/(-a*x+1)/(a*x+1),x)
```

Mathematica [N/A]

Not integrable

Time = 11.92 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.06

$$\int \frac{\text{csch}\left(\frac{\sqrt{1-ax}}{\sqrt{1+ax}}\right)}{1-a^2x^2}\,dx=\int \frac{\text{csch}\left(\frac{\sqrt{1-ax}}{\sqrt{1+ax}}\right)}{1-a^2x^2}\,dx$$

input
```
Integrate[Csch[Sqrt[1 - a*x]/Sqrt[1 + a*x]]/(1 - a^2*x^2),x]
```

output
```
Integrate[Csch[Sqrt[1 - a*x]/Sqrt[1 + a*x]]/(1 - a^2*x^2), x]
```

Rubi [N/A]

Not integrable

Time = 0.28 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\operatorname{csch}\left(\frac{\sqrt{1-ax}}{\sqrt{ax+1}}\right)}{1-a^2x^2}dx$$

$$\Big\downarrow \text{7232}$$

$$-\frac{\int \frac{\sqrt{ax+1}\operatorname{csch}\left(\frac{\sqrt{1-ax}}{\sqrt{ax+1}}\right)}{\sqrt{1-ax}}d\frac{\sqrt{1-ax}}{\sqrt{ax+1}}}{a}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{\int \frac{i\sqrt{ax+1}\csc\left(\frac{i\sqrt{1-ax}}{\sqrt{ax+1}}\right)}{\sqrt{1-ax}}d\frac{\sqrt{1-ax}}{\sqrt{ax+1}}}{a}$$

$$\Big\downarrow \text{26}$$

$$-\frac{i\int \frac{\sqrt{ax+1}\csc\left(\frac{i\sqrt{1-ax}}{\sqrt{ax+1}}\right)}{\sqrt{1-ax}}d\frac{\sqrt{1-ax}}{\sqrt{ax+1}}}{a}$$

$$\Big\downarrow \text{4680}$$

$$-\frac{\int \frac{\sqrt{ax+1}\operatorname{csch}\left(\frac{\sqrt{1-ax}}{\sqrt{ax+1}}\right)}{\sqrt{1-ax}}d\frac{\sqrt{1-ax}}{\sqrt{ax+1}}}{a}$$

input
```
Int[Csch[Sqrt[1 - a*x]/Sqrt[1 + a*x]]/(1 - a^2*x^2),x]
```

output
```
$Aborted
```

Maple [N/A]

Not integrable

Time = 0.14 (sec) , antiderivative size = 30, normalized size of antiderivative = 0.88

$$\int \frac{\operatorname{csch}\left(\frac{\sqrt{-ax+1}}{\sqrt{ax+1}}\right)}{-a^2x^2+1}dx$$

input
```
int(csch((-a*x+1)^(1/2)/(a*x+1)^(1/2))/(-a^2*x^2+1),x)
```

output
```
int(csch((-a*x+1)^(1/2)/(a*x+1)^(1/2))/(-a^2*x^2+1),x)
```

Fricas [N/A]

Not integrable

Time = 0.09 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.94

$$\int \frac{\operatorname{csch}\left(\frac{\sqrt{1-ax}}{\sqrt{1+ax}}\right)}{1-a^2x^2}\,dx = \int -\frac{\operatorname{csch}\left(\frac{\sqrt{-ax+1}}{\sqrt{ax+1}}\right)}{a^2x^2-1}\,dx$$

input
```
integrate(csch((-a*x+1)^(1/2)/(a*x+1)^(1/2))/(-a^2*x^2+1),x, algorithm="fr
icas")
```

output
```
integral(-csch(sqrt(-a*x + 1)/sqrt(a*x + 1))/(a^2*x^2 - 1), x)
```

Sympy [N/A]

Not integrable

Time = 1.43 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.85

$$\int \frac{\operatorname{csch}\left(\frac{\sqrt{1-ax}}{\sqrt{1+ax}}\right)}{1-a^2x^2}\,dx = -\int \frac{\operatorname{csch}\left(\frac{\sqrt{-ax+1}}{\sqrt{ax+1}}\right)}{a^2x^2-1}\,dx$$

input
```
integrate(csch((-a*x+1)**(1/2)/(a*x+1)**(1/2))/(-a**2*x**2+1),x)
```

output
```
-Integral(csch(sqrt(-a*x + 1)/sqrt(a*x + 1))/(a**2*x**2 - 1), x)
```

Maxima [N/A]

Not integrable

Time = 0.19 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.97

$$\int \frac{\operatorname{csch}\left(\frac{\sqrt{1-ax}}{\sqrt{1+ax}}\right)}{1-a^2x^2}\,dx = \int -\frac{\operatorname{csch}\left(\frac{\sqrt{-ax+1}}{\sqrt{ax+1}}\right)}{a^2x^2-1}\,dx$$

input
```
integrate(csch((-a*x+1)^(1/2)/(a*x+1)^(1/2))/(-a^2*x^2+1),x, algorithm="ma
xima")
```

output
```
-integrate(csch(sqrt(-a*x + 1)/sqrt(a*x + 1))/(a^2*x^2 - 1), x)
```

Giac [N/A]

Not integrable

Time = 0.20 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.94

$$\int \frac{\operatorname{csch}\left(\frac{\sqrt{1-ax}}{\sqrt{1+ax}}\right)}{1-a^2x^2}\,dx = \int -\frac{\operatorname{csch}\left(\frac{\sqrt{-ax+1}}{\sqrt{ax+1}}\right)}{a^2x^2-1}\,dx$$

input
```
integrate(csch((-a*x+1)^(1/2)/(a*x+1)^(1/2))/(-a^2*x^2+1),x, algorithm="gi
ac")
```

output
```
integrate(-csch(sqrt(-a*x + 1)/sqrt(a*x + 1))/(a^2*x^2 - 1), x)
```

Mupad [N/A]

Not integrable

Time = 1.54 (sec) , antiderivative size = 35, normalized size of antiderivative = 1.03

$$\int \frac{\operatorname{csch}\left(\frac{\sqrt{1-ax}}{\sqrt{1+ax}}\right)}{1-a^2x^2}\,dx = -\int \frac{1}{\sinh\left(\frac{\sqrt{1-a\,x}}{\sqrt{a\,x+1}}\right)\,\left(a^2\,x^2-1\right)}\,dx$$

input
```
int(-1/(sinh((1 - a*x)^(1/2)/(a*x + 1)^(1/2))*(a^2*x^2 - 1)),x)
```

output
```
-int(1/(sinh((1 - a*x)^(1/2)/(a*x + 1)^(1/2))*(a^2*x^2 - 1)), x)
```

Reduce [N/A]

Not integrable

Time = 0.18 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.97

$$\int \frac{\operatorname{csch}\left(\frac{\sqrt{1-ax}}{\sqrt{1+ax}}\right)}{1-a^2x^2}\,dx = -\left(\int \frac{\operatorname{csch}\left(\frac{\sqrt{-ax+1}}{\sqrt{ax+1}}\right)}{a^2x^2-1}dx\right)$$

input
```
int(csch((-a*x+1)^(1/2)/(a*x+1)^(1/2))/(-a^2*x^2+1),x)
```

output
```
- int(csch(sqrt( - a*x + 1)/sqrt(a*x + 1))/(a**2*x**2 - 1),x)
```

3.265 $\int \frac{\operatorname{csch}^2\left(\frac{\sqrt{1-ax}}{\sqrt{1+ax}}\right)}{1-a^2x^2}\,dx$

Optimal result

Integrand size = 36, antiderivative size = 36

$$\int \frac{\operatorname{csch}^2\left(\frac{\sqrt{1-ax}}{\sqrt{1+ax}}\right)}{1-a^2x^2}\,dx = \operatorname{Int}\left(\frac{\operatorname{csch}^2\left(\frac{\sqrt{1-ax}}{\sqrt{1+ax}}\right)}{(1-ax)(1+ax)},x\right)$$

output
```
Defer(Int)(csch((-a*x+1)^(1/2)/(a*x+1)^(1/2))^2/(-a*x+1)/(a*x+1),x)
```

Mathematica [N/A]

Not integrable

Time = 38.99 (sec) , antiderivative size = 38, normalized size of antiderivative = 1.06

$$\int \frac{\operatorname{csch}^2\left(\frac{\sqrt{1-ax}}{\sqrt{1+ax}}\right)}{1-a^2x^2}\,dx = \int \frac{\operatorname{csch}^2\left(\frac{\sqrt{1-ax}}{\sqrt{1+ax}}\right)}{1-a^2x^2}\,dx$$

input
```
Integrate[Csch[Sqrt[1 - a*x]/Sqrt[1 + a*x]]^2/(1 - a^2*x^2),x]
```

output
```
Integrate[Csch[Sqrt[1 - a*x]/Sqrt[1 + a*x]]^2/(1 - a^2*x^2), x]
```

Rubi [N/A]

Not integrable

Time = 0.31 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\operatorname{csch}^2\left(\frac{\sqrt{1-ax}}{\sqrt{ax+1}}\right)}{1-a^2x^2}dx$$

$$\Big\downarrow \text{7232}$$

$$-\frac{\int \frac{\sqrt{ax+1}\operatorname{csch}^2\left(\frac{\sqrt{1-ax}}{\sqrt{ax+1}}\right)}{\sqrt{1-ax}}d\frac{\sqrt{1-ax}}{\sqrt{ax+1}}}{a}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{\int -\frac{\sqrt{ax+1}\csc\left(\frac{i\sqrt{1-ax}}{\sqrt{ax+1}}\right)^2}{\sqrt{1-ax}}d\frac{\sqrt{1-ax}}{\sqrt{ax+1}}}{a}$$

$$\Big\downarrow \text{25}$$

$$\frac{\int \frac{\sqrt{ax+1}\csc\left(\frac{i\sqrt{1-ax}}{\sqrt{ax+1}}\right)^2}{\sqrt{1-ax}}d\frac{\sqrt{1-ax}}{\sqrt{ax+1}}}{a}$$

$$\Big\downarrow \text{4680}$$

$$-\frac{\int \frac{\sqrt{ax+1}\operatorname{csch}^2\left(\frac{\sqrt{1-ax}}{\sqrt{ax+1}}\right)}{\sqrt{1-ax}}d\frac{\sqrt{1-ax}}{\sqrt{ax+1}}}{a}$$

input
```
Int[Csch[Sqrt[1 - a*x]/Sqrt[1 + a*x]]^2/(1 - a^2*x^2),x]
```

output
```
$Aborted
```

Maple [N/A]

Not integrable

Time = 0.16 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.89

$$\int \frac{\operatorname{csch}\left(\frac{\sqrt{-ax+1}}{\sqrt{ax+1}}\right)^{2}}{-a^{2}x^{2}+1}dx$$

input
```
int(csch((-a*x+1)^(1/2)/(a*x+1)^(1/2))^2/(-a^2*x^2+1),x)
```

output
```
int(csch((-a*x+1)^(1/2)/(a*x+1)^(1/2))^2/(-a^2*x^2+1),x)
```

Fricas [N/A]

Not integrable

Time = 0.10 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.94

$$\int \frac{\operatorname{csch}^2\left(\frac{\sqrt{1-ax}}{\sqrt{1+ax}}\right)}{1-a^2x^2}\,dx = \int -\frac{\operatorname{csch}\left(\frac{\sqrt{-ax+1}}{\sqrt{ax+1}}\right)^{2}}{a^{2}x^{2}-1}\,dx$$

input
```
integrate(csch((-a*x+1)^(1/2)/(a*x+1)^(1/2))^2/(-a^2*x^2+1),x, algorithm="
fricas")
```

output
```
integral(-csch(sqrt(-a*x + 1)/sqrt(a*x + 1))^2/(a^2*x^2 - 1), x)
```

Sympy [N/A]

Not integrable

Time = 1.43 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.86

$$\int \frac{\operatorname{csch}^2\left(\frac{\sqrt{1-ax}}{\sqrt{1+ax}}\right)}{1-a^2x^2}\,dx = -\int \frac{\operatorname{csch}^{2}\left(\frac{\sqrt{-ax+1}}{\sqrt{ax+1}}\right)}{a^{2}x^{2}-1}\,dx$$

input
```
integrate(csch((-a*x+1)**(1/2)/(a*x+1)**(1/2))**2/(-a**2*x**2+1),x)
```

output
```
-Integral(csch(sqrt(-a*x + 1)/sqrt(a*x + 1))**2/(a**2*x**2 - 1), x)
```

Maxima [N/A]

Not integrable

Time = 0.26 (sec) , antiderivative size = 191, normalized size of antiderivative = 5.31

$$\int \frac{\operatorname{csch}^2\left(\frac{\sqrt{1-ax}}{\sqrt{1+ax}}\right)}{1-a^2x^2}\,dx=\int -\frac{\operatorname{csch}\left(\frac{\sqrt{-ax+1}}{\sqrt{ax+1}}\right)^2}{a^2x^2-1}\,dx$$

input
```
integrate(csch((-a*x+1)^(1/2)/(a*x+1)^(1/2))^2/(-a^2*x^2+1),x, algorithm="
maxima")
```

output
```
2*sqrt(a*x + 1)/(sqrt(-a*x + 1)*a*e^(2*sqrt(-a*x + 1)/sqrt(a*x + 1)) - sqr
t(-a*x + 1)*a) - 4*integrate(1/4*sqrt(a*x + 1)/((a^2*x^2 - 1)*sqrt(-a*x +
1)*e^(sqrt(-a*x + 1)/sqrt(a*x + 1)) + (a^2*x^2 - 1)*sqrt(-a*x + 1)), x) +
4*integrate(1/4*sqrt(a*x + 1)/((a^2*x^2 - 1)*sqrt(-a*x + 1)*e^(sqrt(-a*x +
 1)/sqrt(a*x + 1)) - (a^2*x^2 - 1)*sqrt(-a*x + 1)), x)
```

Giac [N/A]

Not integrable

Time = 0.27 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.94

$$\int \frac{\operatorname{csch}^2\left(\frac{\sqrt{1-ax}}{\sqrt{1+ax}}\right)}{1-a^2x^2}\,dx=\int -\frac{\operatorname{csch}\left(\frac{\sqrt{-ax+1}}{\sqrt{ax+1}}\right)^2}{a^2x^2-1}\,dx$$

input
```
integrate(csch((-a*x+1)^(1/2)/(a*x+1)^(1/2))^2/(-a^2*x^2+1),x, algorithm="
giac")
```

output
```
integrate(-csch(sqrt(-a*x + 1)/sqrt(a*x + 1))^2/(a^2*x^2 - 1), x)
```

Mupad [N/A]

Not integrable

Time = 1.62 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.97

$$\int \frac{\operatorname{csch}^2\left(\frac{\sqrt{1-ax}}{\sqrt{1+ax}}\right)}{1-a^2x^2}\,dx = -\int \frac{1}{\sinh\left(\frac{\sqrt{1-a\,x}}{\sqrt{a\,x+1}}\right)^2\,\left(a^2\,x^2-1\right)}\,dx$$

input
```
int(-1/(sinh((1 - a*x)^(1/2)/(a*x + 1)^(1/2))^2*(a^2*x^2 - 1)),x)
```

output
```
-int(1/(sinh((1 - a*x)^(1/2)/(a*x + 1)^(1/2))^2*(a^2*x^2 - 1)), x)
```

Reduce [N/A]

Not integrable

Time = 0.22 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.97

$$\int \frac{\operatorname{csch}^2\left(\frac{\sqrt{1-ax}}{\sqrt{1+ax}}\right)}{1-a^2x^2}\,dx = -\left(\int \frac{\operatorname{csch}\left(\frac{\sqrt{-ax+1}}{\sqrt{ax+1}}\right)^2}{a^2x^2-1}dx\right)$$

input
```
int(csch((-a*x+1)^(1/2)/(a*x+1)^(1/2))^2/(-a^2*x^2+1),x)
```

output
```
- int(csch(sqrt( - a*x + 1)/sqrt(a*x + 1))**2/(a**2*x**2 - 1),x)
```

3.266 $\int \sinh(a+b\log(cx^n))\,dx$

Optimal result

Integrand size = 11, antiderivative size = 54

$$\int \sinh(a+b\log(cx^n))\,dx = -\frac{bnx\cosh(a+b\log(cx^n))}{1-b^2n^2}+\frac{x\sinh(a+b\log(cx^n))}{1-b^2n^2}$$

output
```
-b*n*x*cosh(a+b*ln(c*x^n))/(-b^2*n^2+1)+x*sinh(a+b*ln(c*x^n))/(-b^2*n^2+1)
```

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.76

$$\int \sinh(a+b\log(cx^n))\,dx = \frac{x(bn\cosh(a+b\log(cx^n))-\sinh(a+b\log(cx^n)))}{-1+b^2n^2}$$

input
```
Integrate[Sinh[a + b*Log[c*x^n]],x]
```

output
```
(x*(b*n*Cosh[a + b*Log[c*x^n]] - Sinh[a + b*Log[c*x^n]]))/(-1 + b^2*n^2)
```

Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 54, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {6043}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sinh(a+b\log(cx^n))\,dx$$

$$\Big\downarrow \ 6043$$

$$\frac{x\sinh(a+b\log(cx^n))}{1-b^2n^2}-\frac{bnx\cosh(a+b\log(cx^n))}{1-b^2n^2}$$

input
```
Int[Sinh[a + b*Log[c*x^n]],x]
```

output
```
-((b*n*x*Cosh[a + b*Log[c*x^n]])/(1 - b^2*n^2)) + (x*Sinh[a + b*Log[c*x^n]
])/(1 - b^2*n^2)
```

Defintions of rubi rules used

rule 6043
```
Int[Sinh[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(d_.)], x_Symbol] :> Simp[(-
x)*(Sinh[d*(a + b*Log[c*x^n])]/(b^2*d^2*n^2 - 1)), x] + Simp[b*d*n*x*(Cosh[
d*(a + b*Log[c*x^n])]/(b^2*d^2*n^2 - 1)), x] /; FreeQ[{a, b, c, d, n}, x] &
& NeQ[b^2*d^2*n^2 - 1, 0]
```

Maple [A] (verified)

Time = 0.47 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.78

method	result	size
parallelrisch	$\frac{x(bn\cosh(a+b\ln(cx^n))-\sinh(a+b\ln(cx^n)))}{b^2n^2-1}$	42

input
```
int(sinh(a+b*ln(c*x^n)),x,method=_RETURNVERBOSE)
```

output
```
x*(b*n*cosh(a+b*ln(c*x^n))-sinh(a+b*ln(c*x^n)))/(b^2*n^2-1)
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.81

$$\int \sinh \left(a+b\log \left(cx^{n}\right)\right)\,dx = \frac{bnx\cosh \left(bn\log \left(x\right)+b\log \left(c\right)+a\right)-x\sinh \left(bn\log \left(x\right)+b\log \left(c\right)+a\right)}{b^{2}n^{2}-1}$$

input
```
integrate(sinh(a+b*log(c*x^n)),x, algorithm="fricas")
```

output
```
(b*n*x*cosh(b*n*log(x) + b*log(c) + a) - x*sinh(b*n*log(x) + b*log(c) + a)
)/(b^2*n^2 - 1)
```

Sympy [F]

$$\int \sinh \left(a+b\log \left(cx^{n}\right)\right)\,dx = \begin{cases} \int \sinh \left(a-\frac{\log \left(cx^{n}\right)}{n}\right)\,dx & \text{for}\: b=-\frac{1}{n} \\ \int \sinh \left(a+\frac{\log \left(cx^{n}\right)}{n}\right)\,dx & \text{for}\: b=\frac{1}{n} \\ \frac{bnx\cosh \left(a+b\log \left(cx^{n}\right)\right)}{b^{2}n^{2}-1}-\frac{x\sinh \left(a+b\log \left(cx^{n}\right)\right)}{b^{2}n^{2}-1} & \text{otherwise} \end{cases}$$

input
```
integrate(sinh(a+b*ln(c*x**n)),x)
```

output
```
Piecewise((Integral(sinh(a - log(c*x**n)/n), x), Eq(b, -1/n)), (Integral(s
inh(a + log(c*x**n)/n), x), Eq(b, 1/n)), (b*n*x*cosh(a + b*log(c*x**n))/(b
**2*n**2 - 1) - x*sinh(a + b*log(c*x**n))/(b**2*n**2 - 1), True))
```

Maxima [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 52, normalized size of antiderivative = 0.96

$$\int \sinh \left(a+b\log \left(cx^n\right)\right)\,dx=\frac{c^bxe^{\left(b\log\left(x^n\right)+a\right)}}{2\,(bn+1)}+\frac{xe^{\left(-b\log\left(x^n\right)-a\right)}}{2\,(bc^bn-c^b)}$$

input
```
integrate(sinh(a+b*log(c*x^n)),x, algorithm="maxima")
```

output
```
1/2*c^b*x*e^(b*log(x^n) + a)/(b*n + 1) + 1/2*x*e^(-b*log(x^n) - a)/(b*c^b*
n - c^b)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.87

$$\int \sinh \left(a+b\log \left(cx^n\right)\right)\,dx=\frac{c^bxx^{bn}e^a}{2\,(bn+1)}+\frac{xe^{(-a)}}{2\,(bn-1)c^bx^{bn}}$$

input
```
integrate(sinh(a+b*log(c*x^n)),x, algorithm="giac")
```

output
```
1/2*c^b*x*x^(b*n)*e^a/(b*n + 1) + 1/2*x*e^(-a)/((b*n - 1)*c^b*x^(b*n))
```

Mupad [B] (verification not implemented)

Time = 1.53 (sec) , antiderivative size = 43, normalized size of antiderivative = 0.80

$$\int \sinh \left(a+b\log \left(cx^n\right)\right)\,dx=\frac{x\,\mathrm{e}^{-a}}{{\left(c\,x^n\right)}^b\,\left(2\,b\,n-2\right)}+\frac{x\,\mathrm{e}^a\,{\left(c\,x^n\right)}^b}{2\,b\,n+2}$$

input
```
int(sinh(a + b*log(c*x^n)),x)
```

output
```
(x*exp(-a))/((c*x^n)^b*(2*b*n - 2)) + (x*exp(a)*(c*x^n)^b)/(2*b*n + 2)
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.76

$$\int \sinh \left(a+b\log \left(cx^n\right)\right) \, dx=\frac{x(\cosh \left(\log(x^n c) \, b+a\right) bn-\sinh \left(\log(x^n c) \, b+a\right))}{b^2n^2-1}$$

input
```
int(sinh(a+b*log(c*x^n)),x)
```

output
```
(x*(cosh(log(x**n*c)*b + a)*b*n - sinh(log(x**n*c)*b + a)))/(b**2*n**2 - 1
)
```

3.267 $\int \sinh^2(a+b\log(cx^n))\,dx$

Optimal result

Integrand size = 13, antiderivative size = 88

$$\int \sinh^2(a+b\log(cx^n))\,dx = \frac{2b^2n^2x}{1-4b^2n^2} - \frac{2bnx\cosh(a+b\log(cx^n))\sinh(a+b\log(cx^n))}{1-4b^2n^2} + \frac{x\sinh^2(a+b\log(cx^n))}{1-4b^2n^2}$$

output
```
2*b^2*n^2*x/(-4*b^2*n^2+1)-2*b*n*x*cosh(a+b*ln(c*x^n))*sinh(a+b*ln(c*x^n))
/(-4*b^2*n^2+1)+x*sinh(a+b*ln(c*x^n))^2/(-4*b^2*n^2+1)
```

Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 55, normalized size of antiderivative = 0.62

$$\int \sinh^2(a+b\log(cx^n))\,dx = -\frac{x(-1+4b^2n^2+\cosh(2(a+b\log(cx^n)))-2bn\sinh(2(a+b\log(cx^n))))}{-2+8b^2n^2}$$

input
```
Integrate[Sinh[a + b*Log[c*x^n]]^2,x]
```

output
```
-((x*(-1 + 4*b^2*n^2 + Cosh[2*(a + b*Log[c*x^n])] - 2*b*n*Sinh[2*(a + b*Lo
g[c*x^n])]))/(-2 + 8*b^2*n^2))
```

Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 88, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {6045, 24}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sinh^2(a+b\log(cx^n))\,dx$$

$$\Big\downarrow \text{6045}$$

$$\frac{2b^2n^2\int 1dx}{1-4b^2n^2}+\frac{x\sinh^2(a+b\log(cx^n))}{1-4b^2n^2}-\frac{2bnx\sinh(a+b\log(cx^n))\cosh(a+b\log(cx^n))}{1-4b^2n^2}$$

$$\Big\downarrow \text{24}$$

$$\frac{x\sinh^2(a+b\log(cx^n))}{1-4b^2n^2}-\frac{2bnx\sinh(a+b\log(cx^n))\cosh(a+b\log(cx^n))}{1-4b^2n^2}+\frac{2b^2n^2x}{1-4b^2n^2}$$

input
```
Int[Sinh[a + b*Log[c*x^n]]^2,x]
```

output
```
(2*b^2*n^2*x)/(1 - 4*b^2*n^2) - (2*b*n*x*Cosh[a + b*Log[c*x^n]]*Sinh[a + b
*Log[c*x^n]])/(1 - 4*b^2*n^2) + (x*Sinh[a + b*Log[c*x^n]]^2)/(1 - 4*b^2*n^
2)
```

Defintions of rubi rules used

rule 24
```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

rule 6045
```
Int[Sinh[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(d_.)]^(p_), x_Symbol] :> Si
mp[(-x)*(Sinh[d*(a + b*Log[c*x^n])]^p/(b^2*d^2*n^2*p^2 - 1)), x] + (Simp[b*
d*n*p*x*Cosh[d*(a + b*Log[c*x^n])]*(Sinh[d*(a + b*Log[c*x^n])]^(p - 1)/(b^2
*d^2*n^2*p^2 - 1)), x] - Simp[b^2*d^2*n^2*p*((p - 1)/(b^2*d^2*n^2*p^2 - 1))
   Int[Sinh[d*(a + b*Log[c*x^n])]^(p - 2), x], x]) /; FreeQ[{a, b, c, d, n}
, x] && IGtQ[p, 1] && NeQ[b^2*d^2*n^2*p^2 - 1, 0]
```

Maple [A] (verified)

Time = 1.85 (sec) , antiderivative size = 58, normalized size of antiderivative = 0.66

method	result	size
parallelrisch	$-\frac{x\left(-1+\cosh\left(2b\ln\left(c\,x^{n}\right)+2a\right)+4b^{2}n^{2}-2bn\sinh\left(2b\ln\left(c\,x^{n}\right)+2a\right)\right)}{8b^{2}n^{2}-2}$	58

input
```
int(sinh(a+b*ln(c*x^n))^2,x,method=_RETURNVERBOSE)
```

output
```
-x*(-1+cosh(2*b*ln(c*x^n)+2*a)+4*b^2*n^2-2*b*n*sinh(2*b*ln(c*x^n)+2*a))/(8
*b^2*n^2-2)
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 91, normalized size of antiderivative = 1.03

$$\int \sinh^2\left(a+b\log\left(cx^n\right)\right)\,dx$$
$$=\frac{4\,bnx\cosh\left(bn\log\left(x\right)+b\log\left(c\right)+a\right)\sinh\left(bn\log\left(x\right)+b\log\left(c\right)+a\right)-x\cosh\left(bn\log\left(x\right)+b\log\left(c\right)+a\right)^2}{2\left(4\,b^2n^2-1\right)}$$

input
```
integrate(sinh(a+b*log(c*x^n))^2,x, algorithm="fricas")
```

output
```
1/2*(4*b*n*x*cosh(b*n*log(x) + b*log(c) + a)*sinh(b*n*log(x) + b*log(c) +
a) - x*cosh(b*n*log(x) + b*log(c) + a)^2 - x*sinh(b*n*log(x) + b*log(c) +
a)^2 - (4*b^2*n^2 - 1)*x)/(4*b^2*n^2 - 1)
```

Sympy [F]

$$\int \sinh^2\left(a+b\log\left(cx^n\right)\right)\,dx$$
$$=\begin{cases}\int \sinh^2\left(a-\frac{\log\left(cx^n\right)}{2n}\right)dx\\ \int \sinh^2\left(a+\frac{\log\left(cx^n\right)}{2n}\right)dx\\ \frac{2b^2n^2x\sinh^2\left(a+b\log\left(cx^n\right)\right)}{4b^2n^2-1}-\frac{2b^2n^2x\cosh^2\left(a+b\log\left(cx^n\right)\right)}{4b^2n^2-1}+\frac{2bnx\sinh\left(a+b\log\left(cx^n\right)\right)\cosh\left(a+b\log\left(cx^n\right)\right)}{4b^2n^2-1}-\frac{x\sinh^2\left(a+b\log\left(cx^n\right)\right)}{4b^2n^2-1}\end{cases}$$

input
```
integrate(sinh(a+b*ln(c*x**n))**2,x)
```

output
```
Piecewise((Integral(sinh(a - log(c*x**n)/(2*n))**2, x), Eq(b, -1/(2*n))),
(Integral(sinh(a + log(c*x**n)/(2*n))**2, x), Eq(b, 1/(2*n))), (2*b**2*n**
2*x*sinh(a + b*log(c*x**n))**2/(4*b**2*n**2 - 1) - 2*b**2*n**2*x*cosh(a +
b*log(c*x**n))**2/(4*b**2*n**2 - 1) + 2*b*n*x*sinh(a + b*log(c*x**n))*cosh
(a + b*log(c*x**n))/(4*b**2*n**2 - 1) - x*sinh(a + b*log(c*x**n))**2/(4*b*
*2*n**2 - 1), True))
```

Maxima [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 67, normalized size of antiderivative = 0.76

$$\int \sinh^2\left(a+b\log\left(cx^n\right)\right)\,dx=\frac{c^{2\,b}xe^{\left(2\,b\log\left(x^n\right)+2\,a\right)}}{4\,(2\,bn+1)}-\frac{1}{2}\,x-\frac{xe^{\left(-2\,a\right)}}{4\,(2\,bc^{2\,b}n-c^{2\,b})(x^n)^{2\,b}}$$

input
```
integrate(sinh(a+b*log(c*x^n))^2,x, algorithm="maxima")
```

output
```
1/4*c^(2*b)*x*e^(2*b*log(x^n) + 2*a)/(2*b*n + 1) - 1/2*x - 1/4*x*e^(-2*a)/
((2*b*c^(2*b)*n - c^(2*b))*(x^n)^(2*b))
```

Giac [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 169, normalized size of antiderivative = 1.92

$$\int \sinh^2(a+b\log(cx^n))\,dx = \frac{bc^{2b}nxx^{2bn}e^{(2a)}}{2(4b^2n^2-1)} - \frac{2b^2n^2x}{4b^2n^2-1} - \frac{c^{2b}xx^{2bn}e^{(2a)}}{4(4b^2n^2-1)} - \frac{bnxe^{(-2a)}}{2(4b^2n^2-1)c^{2b}x^{2bn}} + \frac{x}{2(4b^2n^2-1)} - \frac{xe^{(-2a)}}{4(4b^2n^2-1)c^{2b}x^{2bn}}$$

input
```
integrate(sinh(a+b*log(c*x^n))^2,x, algorithm="giac")
```

output
```
1/2*b*c^(2*b)*n*x*x^(2*b*n)*e^(2*a)/(4*b^2*n^2 - 1) - 2*b^2*n^2*x/(4*b^2*n
^2 - 1) - 1/4*c^(2*b)*x*x^(2*b*n)*e^(2*a)/(4*b^2*n^2 - 1) - 1/2*b*n*x*e^(-
2*a)/((4*b^2*n^2 - 1)*c^(2*b)*x^(2*b*n)) + 1/2*x/(4*b^2*n^2 - 1) - 1/4*x*e
^(-2*a)/((4*b^2*n^2 - 1)*c^(2*b)*x^(2*b*n))
```

Mupad [B] (verification not implemented)

Time = 1.50 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.60

$$\int \sinh^2(a+b\log(cx^n))\,dx = \frac{x\,e^{2a}\,(c\,x^n)^{2b}}{8\,b\,n+4} - \frac{x\,e^{-2a}}{(c\,x^n)^{2b}\,(8\,b\,n-4)} - \frac{x}{2}$$

input
```
int(sinh(a + b*log(c*x^n))^2,x)
```

output
```
(x*exp(2*a)*(c*x^n)^(2*b))/(8*b*n + 4) - (x*exp(-2*a))/((c*x^n)^(2*b)*(8*b
*n - 4)) - x/2
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 123, normalized size of antiderivative = 1.40

$$\int \sinh^2\left(a+b\log\left(cx^n\right)\right)\,dx = \frac{x\left(2x^{4bn}e^{4a}c^{4b}bn - x^{4bn}e^{4a}c^{4b} - 8x^{2bn}e^{2a}c^{2b}b^2n^2 + 2x^{2bn}e^{2a}c^{2b} - 2bn - 1\right)}{4x^{2bn}e^{2a}c^{2b}\left(4b^2n^2-1\right)}$$

input
```
int(sinh(a+b*log(c*x^n))^2,x)
```

output
```
(x*(2*x**(4*b*n)*e**(4*a)*c**(4*b)*b*n - x**(4*b*n)*e**(4*a)*c**(4*b) - 8*
x**(2*b*n)*e**(2*a)*c**(2*b)*b**2*n**2 + 2*x**(2*b*n)*e**(2*a)*c**(2*b) -
2*b*n - 1))/(4*x**(2*b*n)*e**(2*a)*c**(2*b)*(4*b**2*n**2 - 1))
```

3.268 $\int \sinh^3(a+b\log(cx^n))\,dx$

Optimal result

Integrand size = 13, antiderivative size = 149

$$\int \sinh^3(a+b\log(cx^n))\,dx = -\frac{6b^3n^3x\cosh(a+b\log(cx^n))}{1-10b^2n^2+9b^4n^4} + \frac{6b^2n^2x\sinh(a+b\log(cx^n))}{1-10b^2n^2+9b^4n^4} - \frac{3bnx\cosh(a+b\log(cx^n))\sinh^2(a+b\log(cx^n))}{1-9b^2n^2} + \frac{x\sinh^3(a+b\log(cx^n))}{1-9b^2n^2}$$

output
```
-6*b^3*n^3*x*cosh(a+b*ln(c*x^n))/(9*b^4*n^4-10*b^2*n^2+1)+6*b^2*n^2*x*sinh
(a+b*ln(c*x^n))/(9*b^4*n^4-10*b^2*n^2+1)-3*b*n*x*cosh(a+b*ln(c*x^n))*sinh(
a+b*ln(c*x^n))^2/(-9*b^2*n^2+1)+x*sinh(a+b*ln(c*x^n))^3/(-9*b^2*n^2+1)
```

Mathematica [A] (verified)

Time = 0.31 (sec) , antiderivative size = 120, normalized size of antiderivative = 0.81

$$\int \sinh^3(a+b\log(cx^n))\,dx$$
$$=\frac{x(-3bn(-1+9b^2n^2)\cosh(a+b\log(cx^n))+3bn(-1+b^2n^2)\cosh(3(a+b\log(cx^n)))-2(1-13b^2n^2+}{4-40b^2n^2+36b^4n^4}$$

input
```
Integrate[Sinh[a + b*Log[c*x^n]]^3,x]
```

output
```
(x*(-3*b*n*(-1 + 9*b^2*n^2)*Cosh[a + b*Log[c*x^n]] + 3*b*n*(-1 + b^2*n^2)*
Cosh[3*(a + b*Log[c*x^n])] - 2*(1 - 13*b^2*n^2 + (-1 + b^2*n^2)*Cosh[2*(a
+ b*Log[c*x^n])])*Sinh[a + b*Log[c*x^n]]))/(4 - 40*b^2*n^2 + 36*b^4*n^4)
```

Rubi [A] (verified)

Time = 0.34 (sec) , antiderivative size = 143, normalized size of antiderivative = 0.96, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {6045, 6043}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sinh^3(a+b\log(cx^n))\,dx$$
$$\Big\downarrow \text{6045}$$
$$\frac{6b^2n^2\int \sinh(a+b\log(cx^n))dx}{1-9b^2n^2}+\frac{x\sinh^3(a+b\log(cx^n))}{1-9b^2n^2}-\frac{3bnx\sinh^2(a+b\log(cx^n))\cosh(a+b\log(cx^n))}{1-9b^2n^2}$$
$$\Big\downarrow \text{6043}$$
$$\frac{x\sinh^3(a+b\log(cx^n))}{1-9b^2n^2}-\frac{3bnx\sinh^2(a+b\log(cx^n))\cosh(a+b\log(cx^n))}{1-9b^2n^2}+\frac{6b^2n^2\left(\frac{x\sinh(a+b\log(cx^n))}{1-b^2n^2}-\frac{bnx\cosh(a+b\log(cx^n))}{1-b^2n^2}\right)}{1-9b^2n^2}$$

input
```
Int[Sinh[a + b*Log[c*x^n]]^3,x]
```

output
```
(-3*b*n*x*Cosh[a + b*Log[c*x^n]]*Sinh[a + b*Log[c*x^n]]^2)/(1 - 9*b^2*n^2)
 + (x*Sinh[a + b*Log[c*x^n]]^3)/(1 - 9*b^2*n^2) + (6*b^2*n^2*(-((b*n*x*Cos
h[a + b*Log[c*x^n]])/(1 - b^2*n^2)) + (x*Sinh[a + b*Log[c*x^n]])/(1 - b^2*
n^2)))/(1 - 9*b^2*n^2)
```

Defintions of rubi rules used

rule 6043
```
Int[Sinh[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(d_.)], x_Symbol] :> Simp[(-
x)*(Sinh[d*(a + b*Log[c*x^n])]/(b^2*d^2*n^2 - 1)), x] + Simp[b*d*n*x*(Cosh[
d*(a + b*Log[c*x^n])]/(b^2*d^2*n^2 - 1)), x] /; FreeQ[{a, b, c, d, n}, x] &
& NeQ[b^2*d^2*n^2 - 1, 0]
```

rule 6045
```
Int[Sinh[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(d_.)]^(p_), x_Symbol] :> Si
mp[(-x)*(Sinh[d*(a + b*Log[c*x^n])]^p/(b^2*d^2*n^2*p^2 - 1)), x] + (Simp[b*
d*n*p*x*Cosh[d*(a + b*Log[c*x^n])]*(Sinh[d*(a + b*Log[c*x^n])]^(p - 1)/(b^2
*d^2*n^2*p^2 - 1)), x] - Simp[b^2*d^2*n^2*p*((p - 1)/(b^2*d^2*n^2*p^2 - 1))
   Int[Sinh[d*(a + b*Log[c*x^n])]^(p - 2), x], x]) /; FreeQ[{a, b, c, d, n}
, x] && IGtQ[p, 1] && NeQ[b^2*d^2*n^2*p^2 - 1, 0]
```

Maple [A] (verified)

Time = 9.10 (sec) , antiderivative size = 254, normalized size of antiderivative = 1.70

method	result
parallelrisch	$\frac{6x\left(b^3n^3\tanh\left(\frac{a}{2}+b\ln\left(\sqrt{c x^n}\right)\right)^6-2b^2n^2\tanh\left(\frac{a}{2}+b\ln\left(\sqrt{c x^n}\right)\right)^5+\left(-3b^3n^3+2bn\right)\tanh\left(\frac{a}{2}+b\ln\left(\sqrt{c x^n}\right)\right)^4+\frac{4\left(4b^2n^2-1\right)\tanh\left(\right.}{3}\right.}{\left(9b^4n^4-10b^2n^2+1\right)\left(\tanh\left(\frac{a}{2}+b\ln\left(\sqrt{c x^n}\right)\right)^6-3\tanh\left(\frac{a}{2}+b\ln\left(\sqrt{c\right.\right.\right.}}$

input
```
int(sinh(a+b*ln(c*x^n))^3,x,method=_RETURNVERBOSE)
```

output
```
6*x*(b^3*n^3*tanh(1/2*a+b*ln((c*x^n)^(1/2)))^6-2*b^2*n^2*tanh(1/2*a+b*ln((
c*x^n)^(1/2)))^5+(-3*b^3*n^3+2*b*n)*tanh(1/2*a+b*ln((c*x^n)^(1/2)))^4+4/3*
(4*b^2*n^2-1)*tanh(1/2*a+b*ln((c*x^n)^(1/2)))^3+(-3*b^3*n^3+2*b*n)*tanh(1/
2*a+b*ln((c*x^n)^(1/2)))^2-2*b^2*n^2*tanh(1/2*a+b*ln((c*x^n)^(1/2)))+b^3*n
^3)/(9*b^4*n^4-10*b^2*n^2+1)/(tanh(1/2*a+b*ln((c*x^n)^(1/2)))^6-3*tanh(1/2
*a+b*ln((c*x^n)^(1/2)))^4+3*tanh(1/2*a+b*ln((c*x^n)^(1/2)))^2-1)
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 200, normalized size of antiderivative = 1.34

$$\int \sinh^3(a + b\log(cx^n)) \, dx = \frac{3(b^3n^3 - bn)x\cosh(bn\log(x) + b\log(c) + a)^3 + 9(b^3n^3 - bn)x\cosh(bn\log(x) + b\log(c) + a)\sinh(bn$$

input
```
integrate(sinh(a+b*log(c*x^n))^3,x, algorithm="fricas")
```

output
```
1/4*(3*(b^3*n^3 - b*n)*x*cosh(b*n*log(x) + b*log(c) + a)^3 + 9*(b^3*n^3 -
b*n)*x*cosh(b*n*log(x) + b*log(c) + a)*sinh(b*n*log(x) + b*log(c) + a)^2 -
 (b^2*n^2 - 1)*x*sinh(b*n*log(x) + b*log(c) + a)^3 - 3*(9*b^3*n^3 - b*n)*x
*cosh(b*n*log(x) + b*log(c) + a) - 3*((b^2*n^2 - 1)*x*cosh(b*n*log(x) + b*
log(c) + a)^2 - (9*b^2*n^2 - 1)*x)*sinh(b*n*log(x) + b*log(c) + a))/(9*b^4
*n^4 - 10*b^2*n^2 + 1)
```

Sympy [F]

$$\int \sinh^3(a+b\log(cx^n))\,dx$$
$$=\begin{cases}\int \sinh^3\left(a-\frac{\log(cx^n)}{n}\right)dx\\ \int \sinh^3\left(a-\frac{\log(cx^n)}{3n}\right)dx\\ \int \sinh^3\left(a+\frac{\log(cx^n)}{3n}\right)dx\\ \int \sinh^3\left(a+\frac{\log(cx^n)}{n}\right)dx\\ \frac{9b^3n^3x\sinh^2(a+b\log(cx^n))\cosh(a+b\log(cx^n))}{9b^4n^4-10b^2n^2+1}-\frac{6b^3n^3x\cosh^3(a+b\log(cx^n))}{9b^4n^4-10b^2n^2+1}-\frac{7b^2n^2x\sinh^3(a+b\log(cx^n))}{9b^4n^4-10b^2n^2+1}+\frac{6b^2n^2x\sinh(a+b\log}{9b^4}\end{cases}$$

input
```
integrate(sinh(a+b*ln(c*x**n))**3,x)
```

output
```
Piecewise((Integral(sinh(a - log(c*x**n)/n)**3, x), Eq(b, -1/n)), (Integra
l(sinh(a - log(c*x**n)/(3*n))**3, x), Eq(b, -1/(3*n))), (Integral(sinh(a +
 log(c*x**n)/(3*n))**3, x), Eq(b, 1/(3*n))), (Integral(sinh(a + log(c*x**n
)/n)**3, x), Eq(b, 1/n)), (9*b**3*n**3*x*sinh(a + b*log(c*x**n))**2*cosh(a
 + b*log(c*x**n))/(9*b**4*n**4 - 10*b**2*n**2 + 1) - 6*b**3*n**3*x*cosh(a
+ b*log(c*x**n))**3/(9*b**4*n**4 - 10*b**2*n**2 + 1) - 7*b**2*n**2*x*sinh(
a + b*log(c*x**n))**3/(9*b**4*n**4 - 10*b**2*n**2 + 1) + 6*b**2*n**2*x*sin
h(a + b*log(c*x**n))*cosh(a + b*log(c*x**n))**2/(9*b**4*n**4 - 10*b**2*n**
2 + 1) - 3*b*n*x*sinh(a + b*log(c*x**n))**2*cosh(a + b*log(c*x**n))/(9*b**
4*n**4 - 10*b**2*n**2 + 1) + x*sinh(a + b*log(c*x**n))**3/(9*b**4*n**4 - 1
0*b**2*n**2 + 1), True))
```

Maxima [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 114, normalized size of antiderivative = 0.77

$$\int \sinh^3(a+b\log(cx^n))\,dx=\frac{c^{3b}xe^{(3b\log(x^n)+3a)}}{8(3bn+1)}-\frac{3c^bxe^{(b\log(x^n)+a)}}{8(bn+1)}-\frac{3xe^{(-b\log(x^n)-a)}}{8(bc^bn-c^b)}+\frac{xe^{(-3b\log(x^n)-3a)}}{8(3bc^{3b}n-c^{3b})}$$

input
```
integrate(sinh(a+b*log(c*x^n))^3,x, algorithm="maxima")
```

output
```
1/8*c^(3*b)*x*e^(3*b*log(x^n) + 3*a)/(3*b*n + 1) - 3/8*c^b*x*e^(b*log(x^n)
 + a)/(b*n + 1) - 3/8*x*e^(-b*log(x^n) - a)/(b*c^b*n - c^b) + 1/8*x*e^(-3*
b*log(x^n) - 3*a)/(3*b*c^(3*b)*n - c^(3*b))
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 665 vs. $2(150) = 300$.

Time = 0.16 (sec) , antiderivative size = 665, normalized size of antiderivative = 4.46

$$\begin{aligned}\int \sinh^3(a+b\log(cx^n))\,dx = {} & \frac{3\,b^3c^{3\,b}n^3xx^{3\,bn}e^{(3\,a)}}{8\,(9\,b^4n^4-10\,b^2n^2+1)} - \frac{27\,b^3c^bn^3xx^{bn}e^a}{8\,(9\,b^4n^4-10\,b^2n^2+1)} \\ & - \frac{b^2c^{3\,b}n^2xx^{3\,bn}e^{(3\,a)}}{8\,(9\,b^4n^4-10\,b^2n^2+1)} + \frac{27\,b^2c^bn^2xx^{bn}e^a}{8\,(9\,b^4n^4-10\,b^2n^2+1)} \\ & - \frac{3\,bc^{3\,b}nxx^{3\,bn}e^{(3\,a)}}{8\,(9\,b^4n^4-10\,b^2n^2+1)} - \frac{27\,b^3n^3xe^{(-a)}}{8\,(9\,b^4n^4-10\,b^2n^2+1)c^bx^{bn}} \\ & + \frac{3\,b^3n^3xe^{(-3\,a)}}{8\,(9\,b^4n^4-10\,b^2n^2+1)c^{3\,b}x^{3\,bn}} \\ & + \frac{3\,bc^bnxx^{bn}e^a}{8\,(9\,b^4n^4-10\,b^2n^2+1)} + \frac{c^{3\,b}xx^{3\,bn}e^{(3\,a)}}{8\,(9\,b^4n^4-10\,b^2n^2+1)} \\ & - \frac{27\,b^2n^2xe^{(-a)}}{8\,(9\,b^4n^4-10\,b^2n^2+1)c^bx^{bn}} \\ & + \frac{b^2n^2xe^{(-3\,a)}}{8\,(9\,b^4n^4-10\,b^2n^2+1)c^{3\,b}x^{3\,bn}} \\ & - \frac{3\,c^bxx^{bn}e^a}{8\,(9\,b^4n^4-10\,b^2n^2+1)} + \frac{3\,bnxe^{(-a)}}{8\,(9\,b^4n^4-10\,b^2n^2+1)c^bx^{bn}} \\ & - \frac{3\,bnxe^{(-3\,a)}}{8\,(9\,b^4n^4-10\,b^2n^2+1)c^{3\,b}x^{3\,bn}} \\ & + \frac{3\,xe^{(-a)}}{8\,(9\,b^4n^4-10\,b^2n^2+1)c^bx^{bn}} \\ & - \frac{xe^{(-3\,a)}}{8\,(9\,b^4n^4-10\,b^2n^2+1)c^{3\,b}x^{3\,bn}}\end{aligned}$$

input
```
integrate(sinh(a+b*log(c*x^n))^3,x, algorithm="giac")
```

output
```
3/8*b^3*c^(3*b)*n^3*x*x^(3*b*n)*e^(3*a)/(9*b^4*n^4 - 10*b^2*n^2 + 1) - 27/
8*b^3*c^b*n^3*x*x^(b*n)*e^a/(9*b^4*n^4 - 10*b^2*n^2 + 1) - 1/8*b^2*c^(3*b)
*n^2*x*x^(3*b*n)*e^(3*a)/(9*b^4*n^4 - 10*b^2*n^2 + 1) + 27/8*b^2*c^b*n^2*x
*x^(b*n)*e^a/(9*b^4*n^4 - 10*b^2*n^2 + 1) - 3/8*b*c^(3*b)*n*x*x^(3*b*n)*e^
(3*a)/(9*b^4*n^4 - 10*b^2*n^2 + 1) - 27/8*b^3*n^3*x*e^(-a)/((9*b^4*n^4 - 1
0*b^2*n^2 + 1)*c^b*x^(b*n)) + 3/8*b^3*n^3*x*e^(-3*a)/((9*b^4*n^4 - 10*b^2*
n^2 + 1)*c^(3*b)*x^(3*b*n)) + 3/8*b*c^b*n*x*x^(b*n)*e^a/(9*b^4*n^4 - 10*b^
2*n^2 + 1) + 1/8*c^(3*b)*x*x^(3*b*n)*e^(3*a)/(9*b^4*n^4 - 10*b^2*n^2 + 1)
- 27/8*b^2*n^2*x*e^(-a)/((9*b^4*n^4 - 10*b^2*n^2 + 1)*c^b*x^(b*n)) + 1/8*b
^2*n^2*x*e^(-3*a)/((9*b^4*n^4 - 10*b^2*n^2 + 1)*c^(3*b)*x^(3*b*n)) - 3/8*c
^b*x*x^(b*n)*e^a/(9*b^4*n^4 - 10*b^2*n^2 + 1) + 3/8*b*n*x*e^(-a)/((9*b^4*n
^4 - 10*b^2*n^2 + 1)*c^b*x^(b*n)) - 3/8*b*n*x*e^(-3*a)/((9*b^4*n^4 - 10*b^
2*n^2 + 1)*c^(3*b)*x^(3*b*n)) + 3/8*x*e^(-a)/((9*b^4*n^4 - 10*b^2*n^2 + 1)
*c^b*x^(b*n)) - 1/8*x*e^(-3*a)/((9*b^4*n^4 - 10*b^2*n^2 + 1)*c^(3*b)*x^(3*
b*n))
```

Mupad [B] (verification not implemented)

Time = 1.57 (sec) , antiderivative size = 93, normalized size of antiderivative = 0.62

$$\int \sinh^3(a+b\log(cx^n))\,dx = \frac{x\,\mathrm{e}^{-3\,a}}{(c\,x^n)^{3\,b}\,(24\,b\,n-8)} - \frac{3\,x\,\mathrm{e}^{-a}}{(c\,x^n)^b\,(8\,b\,n-8)} + \frac{x\,\mathrm{e}^{3\,a}\,(c\,x^n)^{3\,b}}{24\,b\,n+8} - \frac{3\,x\,\mathrm{e}^{a}\,(c\,x^n)^b}{8\,b\,n+8}$$

input
```
int(sinh(a + b*log(c*x^n))^3,x)
```

output
```
(x*exp(-3*a))/((c*x^n)^(3*b)*(24*b*n - 8)) - (3*x*exp(-a))/((c*x^n)^b*(8*b
*n - 8)) + (x*exp(3*a)*(c*x^n)^(3*b))/(24*b*n + 8) - (3*x*exp(a)*(c*x^n)^b
)/(8*b*n + 8)
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 323, normalized size of antiderivative = 2.17

$$\int \sinh^3(a + b\log(cx^n))\, dx$$
$$= \frac{x(3x^{6bn}e^{6a}c^{6b}b^3n^3 - x^{6bn}e^{6a}c^{6b}b^2n^2 - 3x^{6bn}e^{6a}c^{6b}bn + x^{6bn}e^{6a}c^{6b} - 27x^{4bn}e^{4a}c^{4b}b^3n^3 + 27x^{4bn}e^{4a}c^{4b}b^2n^2 -}{8x^{3bn}}$$

input
```
int(sinh(a+b*log(c*x^n))^3,x)
```

output
```
(x*(3*x**(6*b*n)*e**(6*a)*c**(6*b)*b**3*n**3 - x**(6*b*n)*e**(6*a)*c**(6*b
)*b**2*n**2 - 3*x**(6*b*n)*e**(6*a)*c**(6*b)*b*n + x**(6*b*n)*e**(6*a)*c**
(6*b) - 27*x**(4*b*n)*e**(4*a)*c**(4*b)*b**3*n**3 + 27*x**(4*b*n)*e**(4*a)
*c**(4*b)*b**2*n**2 + 3*x**(4*b*n)*e**(4*a)*c**(4*b)*b*n - 3*x**(4*b*n)*e*
*(4*a)*c**(4*b) - 27*x**(2*b*n)*e**(2*a)*c**(2*b)*b**3*n**3 - 27*x**(2*b*n
)*e**(2*a)*c**(2*b)*b**2*n**2 + 3*x**(2*b*n)*e**(2*a)*c**(2*b)*b*n + 3*x**
(2*b*n)*e**(2*a)*c**(2*b) + 3*b**3*n**3 + b**2*n**2 - 3*b*n - 1))/(8*x**(3
*b*n)*e**(3*a)*c**(3*b)*(9*b**4*n**4 - 10*b**2*n**2 + 1))
```

3.269 $\int \sinh^4(a+b\log(cx^n))\,dx$

Optimal result

Integrand size = 13, antiderivative size = 191

$$\int \sinh^4(a+b\log(cx^n))\,dx = \frac{24b^4n^4x}{1-20b^2n^2+64b^4n^4} - \frac{24b^3n^3x\cosh(a+b\log(cx^n))\sinh(a+b\log(cx^n))}{1-20b^2n^2+64b^4n^4} + \frac{12b^2n^2x\sinh^2(a+b\log(cx^n))}{1-20b^2n^2+64b^4n^4} - \frac{4bnx\cosh(a+b\log(cx^n))\sinh^3(a+b\log(cx^n))}{1-16b^2n^2} + \frac{x\sinh^4(a+b\log(cx^n))}{1-16b^2n^2}$$

output
```
24*b^4*n^4*x/(64*b^4*n^4-20*b^2*n^2+1)-24*b^3*n^3*x*cosh(a+b*ln(c*x^n))*si
nh(a+b*ln(c*x^n))/(64*b^4*n^4-20*b^2*n^2+1)+12*b^2*n^2*x*sinh(a+b*ln(c*x^n
))^2/(64*b^4*n^4-20*b^2*n^2+1)-4*b*n*x*cosh(a+b*ln(c*x^n))*sinh(a+b*ln(c*x
^n))^3/(-16*b^2*n^2+1)+x*sinh(a+b*ln(c*x^n))^4/(-16*b^2*n^2+1)
```

Mathematica [A] (verified)

Time = 0.28 (sec) , antiderivative size = 167, normalized size of antiderivative = 0.87

$$\int \sinh^4(a+b\log(cx^n))\,dx$$
$$=\frac{x(3-60b^2n^2+192b^4n^4+(-4+64b^2n^2)\cosh(2(a+b\log(cx^n)))+(1-4b^2n^2)\cosh(4(a+b\log(cx^n)))}{}$$

input
```
Integrate[Sinh[a + b*Log[c*x^n]]^4,x]
```

output
```
(x*(3 - 60*b^2*n^2 + 192*b^4*n^4 + (-4 + 64*b^2*n^2)*Cosh[2*(a + b*Log[c*x
^n])] + (1 - 4*b^2*n^2)*Cosh[4*(a + b*Log[c*x^n])] + 8*b*n*Sinh[2*(a + b*L
og[c*x^n])] - 128*b^3*n^3*Sinh[2*(a + b*Log[c*x^n])] - 4*b*n*Sinh[4*(a + b
*Log[c*x^n])] + 16*b^3*n^3*Sinh[4*(a + b*Log[c*x^n])]))/(8*(1 - 20*b^2*n^2
 + 64*b^4*n^4))
```

Rubi [A] (verified)

Time = 0.37 (sec) , antiderivative size = 177, normalized size of antiderivative = 0.93, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.231$, Rules used = {6045, 6045, 24}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sinh^4(a+b\log(cx^n))\,dx$$
$$\Big\downarrow \text{6045}$$
$$\frac{12b^2n^2\int \sinh^2(a+b\log(cx^n))\,dx}{1-16b^2n^2}+\frac{x\sinh^4(a+b\log(cx^n))}{1-16b^2n^2}-\frac{4bnx\sinh^3(a+b\log(cx^n))\cosh(a+b\log(cx^n))}{1-16b^2n^2}$$
$$\Big\downarrow \text{6045}$$

$$\frac{12b^2n^2\left(\frac{2b^2n^2\int 1dx}{1-4b^2n^2}+\frac{x\sinh^2(a+b\log(cx^n))}{1-4b^2n^2}-\frac{2bnx\sinh(a+b\log(cx^n))\cosh(a+b\log(cx^n))}{1-4b^2n^2}\right)}{1-16b^2n^2}+\frac{x\sinh^4(a+b\log(cx^n))}{1-16b^2n^2}-\frac{4bnx\sinh^3(a+b\log(cx^n))\cosh(a+b\log(cx^n))}{1-16b^2n^2}$$

$$\Big\downarrow \text{24}$$

$$\frac{x\sinh^4(a+b\log(cx^n))}{1-16b^2n^2}-\frac{4bnx\sinh^3(a+b\log(cx^n))\cosh(a+b\log(cx^n))}{1-16b^2n^2}+\frac{12b^2n^2\left(\frac{x\sinh^2(a+b\log(cx^n))}{1-4b^2n^2}-\frac{2bnx\sinh(a+b\log(cx^n))\cosh(a+b\log(cx^n))}{1-4b^2n^2}+\frac{2b^2n^2x}{1-4b^2n^2}\right)}{1-16b^2n^2}$$

input
```
Int[Sinh[a + b*Log[c*x^n]]^4,x]
```

output
```
(-4*b*n*x*Cosh[a + b*Log[c*x^n]]*Sinh[a + b*Log[c*x^n]]^3)/(1 - 16*b^2*n^2
) + (x*Sinh[a + b*Log[c*x^n]]^4)/(1 - 16*b^2*n^2) + (12*b^2*n^2*((2*b^2*n^
2*x)/(1 - 4*b^2*n^2) - (2*b*n*x*Cosh[a + b*Log[c*x^n]]*Sinh[a + b*Log[c*x^
n]])/(1 - 4*b^2*n^2) + (x*Sinh[a + b*Log[c*x^n]]^2)/(1 - 4*b^2*n^2)))/(1 -
 16*b^2*n^2)
```

Defintions of rubi rules used

rule 24
```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

rule 6045
```
Int[Sinh[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(d_.)]^(p_), x_Symbol] :> Si
mp[(-x)*(Sinh[d*(a + b*Log[c*x^n])]^p/(b^2*d^2*n^2*p^2 - 1)), x] + (Simp[b*
d*n*p*x*Cosh[d*(a + b*Log[c*x^n])]*(Sinh[d*(a + b*Log[c*x^n])]^(p - 1)/(b^2
*d^2*n^2*p^2 - 1)), x] - Simp[b^2*d^2*n^2*p*((p - 1)/(b^2*d^2*n^2*p^2 - 1))
   Int[Sinh[d*(a + b*Log[c*x^n])]^(p - 2), x], x]) /; FreeQ[{a, b, c, d, n}
, x] && IGtQ[p, 1] && NeQ[b^2*d^2*n^2*p^2 - 1, 0]
```

Maple [A] (verified)

Time = 20.38 (sec) , antiderivative size = 184, normalized size of antiderivative = 0.96

method	result
parallelrisch	$\frac{x(4(16b^2n^2-1)\cosh(2b\ln(c\,x^n)+2a)+192b^4n^4-128b^3n^3\sinh(2b\ln(c\,x^n)+2a)+16b^3n^3\sinh(4b\ln(c\,x^n)+4a)-4b^2n^2\cosh(4}{512b^4n^4-160b^2n^2+8}$

input
```
int(sinh(a+b*ln(c*x^n))^4,x,method=_RETURNVERBOSE)
```

output
```
1/8*x*(4*(16*b^2*n^2-1)*cosh(2*b*ln(c*x^n)+2*a)+192*b^4*n^4-128*b^3*n^3*si
nh(2*b*ln(c*x^n)+2*a)+16*b^3*n^3*sinh(4*b*ln(c*x^n)+4*a)-4*b^2*n^2*cosh(4*
b*ln(c*x^n)+4*a)-60*b^2*n^2+8*b*n*sinh(2*b*ln(c*x^n)+2*a)-4*b*n*sinh(4*b*l
n(c*x^n)+4*a)+cosh(4*b*ln(c*x^n)+4*a)+3)/(64*b^4*n^4-20*b^2*n^2+1)
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 294, normalized size of antiderivative = 1.54

$$\int \sinh^4(a+b\log(cx^n))\,dx =$$
$$-\frac{(4\,b^2n^2-1)x\cosh(bn\log(x)+b\log(c)+a)^4-16\,(4\,b^3n^3-bn)x\cosh(bn\log(x)+b\log(c)+a)\sinh}{}$$

input
```
integrate(sinh(a+b*log(c*x^n))^4,x, algorithm="fricas")
```

output
```
-1/8*((4*b^2*n^2 - 1)*x*cosh(b*n*log(x) + b*log(c) + a)^4 - 16*(4*b^3*n^3
 - b*n)*x*cosh(b*n*log(x) + b*log(c) + a)*sinh(b*n*log(x) + b*log(c) + a)^3
 + (4*b^2*n^2 - 1)*x*sinh(b*n*log(x) + b*log(c) + a)^4 - 4*(16*b^2*n^2 - 1
)*x*cosh(b*n*log(x) + b*log(c) + a)^2 + 2*(3*(4*b^2*n^2 - 1)*x*cosh(b*n*lo
g(x) + b*log(c) + a)^2 - 2*(16*b^2*n^2 - 1)*x)*sinh(b*n*log(x) + b*log(c)
+ a)^2 - 3*(64*b^4*n^4 - 20*b^2*n^2 + 1)*x - 16*((4*b^3*n^3 - b*n)*x*cosh(
b*n*log(x) + b*log(c) + a)^3 - (16*b^3*n^3 - b*n)*x*cosh(b*n*log(x) + b*lo
g(c) + a))*sinh(b*n*log(x) + b*log(c) + a))/(64*b^4*n^4 - 20*b^2*n^2 + 1)
```

Sympy [F]

$$\int \sinh^4(a + b\log(cx^n)) \, dx$$

$$= \begin{cases} \int \sinh^4\left(a - \frac{\log(cx^n)}{2n}\right) dx \\ \int \sinh^4\left(a - \frac{\log(cx^n)}{4n}\right) dx \\ \int \sinh^4\left(a + \frac{\log(cx^n)}{4n}\right) dx \\ \int \sinh^4\left(a + \frac{\log(cx^n)}{2n}\right) dx \\ \frac{24b^4n^4x\sinh^4(a+b\log(cx^n))}{64b^4n^4-20b^2n^2+1} - \frac{48b^4n^4x\sinh^2(a+b\log(cx^n))\cosh^2(a+b\log(cx^n))}{64b^4n^4-20b^2n^2+1} + \frac{24b^4n^4x\cosh^4(a+b\log(cx^n))}{64b^4n^4-20b^2n^2+1} + \frac{40b^3n^3x\sinh^3}{} \end{cases}$$

input
```
integrate(sinh(a+b*ln(c*x**n))**4,x)
```

output
```
Piecewise((Integral(sinh(a - log(c*x**n)/(2*n))**4, x), Eq(b, -1/(2*n))),
(Integral(sinh(a - log(c*x**n)/(4*n))**4, x), Eq(b, -1/(4*n))), (Integral(
sinh(a + log(c*x**n)/(4*n))**4, x), Eq(b, 1/(4*n))), (Integral(sinh(a + lo
g(c*x**n)/(2*n))**4, x), Eq(b, 1/(2*n))), (24*b**4*n**4*x*sinh(a + b*log(c
*x**n))**4/(64*b**4*n**4 - 20*b**2*n**2 + 1) - 48*b**4*n**4*x*sinh(a + b*l
og(c*x**n))**2*cosh(a + b*log(c*x**n))**2/(64*b**4*n**4 - 20*b**2*n**2 + 1
) + 24*b**4*n**4*x*cosh(a + b*log(c*x**n))**4/(64*b**4*n**4 - 20*b**2*n**2
 + 1) + 40*b**3*n**3*x*sinh(a + b*log(c*x**n))**3*cosh(a + b*log(c*x**n))/
(64*b**4*n**4 - 20*b**2*n**2 + 1) - 24*b**3*n**3*x*sinh(a + b*log(c*x**n))
*cosh(a + b*log(c*x**n))**3/(64*b**4*n**4 - 20*b**2*n**2 + 1) - 16*b**2*n*
*2*x*sinh(a + b*log(c*x**n))**4/(64*b**4*n**4 - 20*b**2*n**2 + 1) + 12*b**
2*n**2*x*sinh(a + b*log(c*x**n))**2*cosh(a + b*log(c*x**n))**2/(64*b**4*n*
*4 - 20*b**2*n**2 + 1) - 4*b*n*x*sinh(a + b*log(c*x**n))**3*cosh(a + b*log
(c*x**n))/(64*b**4*n**4 - 20*b**2*n**2 + 1) + x*sinh(a + b*log(c*x**n))**4
/(64*b**4*n**4 - 20*b**2*n**2 + 1), True))
```

Maxima [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 129, normalized size of antiderivative = 0.68

$$\int \sinh^4(a+b\log(cx^n))\,dx = \frac{c^{4\,b}xe^{(4\,b\log(x^n)+4\,a)}}{16\,(4\,bn+1)} - \frac{c^{2\,b}xe^{(2\,b\log(x^n)+2\,a)}}{4\,(2\,bn+1)} + \frac{3}{8}\,x + \frac{xe^{(-2\,b\log(x^n)-2\,a)}}{4\,(2\,bc^{2\,b}n-c^{2\,b})} - \frac{xe^{(-4\,a)}}{16\,(4\,bc^{4\,b}n-c^{4\,b})(x^n)^{4\,b}}$$

input
```
integrate(sinh(a+b*log(c*x^n))^4,x, algorithm="maxima")
```

output
```
1/16*c^(4*b)*x*e^(4*b*log(x^n) + 4*a)/(4*b*n + 1) - 1/4*c^(2*b)*x*e^(2*b*l
og(x^n) + 2*a)/(2*b*n + 1) + 3/8*x + 1/4*x*e^(-2*b*log(x^n) - 2*a)/(2*b*c^
(2*b)*n - c^(2*b)) - 1/16*x*e^(-4*a)/((4*b*c^(4*b)*n - c^(4*b))*(x^n)^(4*b
))
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 777 vs. 2(192) = 384.

Time = 0.15 (sec) , antiderivative size = 777, normalized size of antiderivative = 4.07

$$\int \sinh^4(a+b\log(cx^n))\,dx = \text{Too large to display}$$

input
```
integrate(sinh(a+b*log(c*x^n))^4,x, algorithm="giac")
```

output
```
b^3*c^(4*b)*n^3*x*x^(4*b*n)*e^(4*a)/(64*b^4*n^4 - 20*b^2*n^2 + 1) - 8*b^3*
c^(2*b)*n^3*x*x^(2*b*n)*e^(2*a)/(64*b^4*n^4 - 20*b^2*n^2 + 1) + 24*b^4*n^4
*x/(64*b^4*n^4 - 20*b^2*n^2 + 1) - 1/4*b^2*c^(4*b)*n^2*x*x^(4*b*n)*e^(4*a)
/(64*b^4*n^4 - 20*b^2*n^2 + 1) + 4*b^2*c^(2*b)*n^2*x*x^(2*b*n)*e^(2*a)/(64
*b^4*n^4 - 20*b^2*n^2 + 1) - 1/4*b*c^(4*b)*n*x*x^(4*b*n)*e^(4*a)/(64*b^4*n
^4 - 20*b^2*n^2 + 1) + 1/2*b*c^(2*b)*n*x*x^(2*b*n)*e^(2*a)/(64*b^4*n^4 - 2
0*b^2*n^2 + 1) + 8*b^3*n^3*x*e^(-2*a)/((64*b^4*n^4 - 20*b^2*n^2 + 1)*c^(2*
b)*x^(2*b*n)) - b^3*n^3*x*e^(-4*a)/((64*b^4*n^4 - 20*b^2*n^2 + 1)*c^(4*b)*
x^(4*b*n)) - 15/2*b^2*n^2*x/(64*b^4*n^4 - 20*b^2*n^2 + 1) + 1/16*c^(4*b)*x
*x^(4*b*n)*e^(4*a)/(64*b^4*n^4 - 20*b^2*n^2 + 1) - 1/4*c^(2*b)*x*x^(2*b*n)
*e^(2*a)/(64*b^4*n^4 - 20*b^2*n^2 + 1) + 4*b^2*n^2*x*e^(-2*a)/((64*b^4*n^4
 - 20*b^2*n^2 + 1)*c^(2*b)*x^(2*b*n)) - 1/4*b^2*n^2*x*e^(-4*a)/((64*b^4*n^
4 - 20*b^2*n^2 + 1)*c^(4*b)*x^(4*b*n)) - 1/2*b*n*x*e^(-2*a)/((64*b^4*n^4 -
 20*b^2*n^2 + 1)*c^(2*b)*x^(2*b*n)) + 1/4*b*n*x*e^(-4*a)/((64*b^4*n^4 - 20
*b^2*n^2 + 1)*c^(4*b)*x^(4*b*n)) + 3/8*x/(64*b^4*n^4 - 20*b^2*n^2 + 1) - 1
/4*x*e^(-2*a)/((64*b^4*n^4 - 20*b^2*n^2 + 1)*c^(2*b)*x^(2*b*n)) + 1/16*x*e
^(-4*a)/((64*b^4*n^4 - 20*b^2*n^2 + 1)*c^(4*b)*x^(4*b*n))
```

Mupad [B] (verification not implemented)

Time = 1.60 (sec) , antiderivative size = 102, normalized size of antiderivative = 0.53

$$\int \sinh^4(a+b\log(cx^n))\,dx = \frac{3\,x}{8}+\frac{x\,\mathrm{e}^{-2\,a}}{{\left(c\,x^n\right)}^{2\,b}\,\left(8\,b\,n-4\right)}-\frac{x\,\mathrm{e}^{2\,a}\,{\left(c\,x^n\right)}^{2\,b}}{8\,b\,n+4}-\frac{x\,\mathrm{e}^{-4\,a}}{{\left(c\,x^n\right)}^{4\,b}\,\left(64\,b\,n-16\right)}+\frac{x\,\mathrm{e}^{4\,a}\,{\left(c\,x^n\right)}^{4\,b}}{64\,b\,n+16}$$

input
```
int(sinh(a + b*log(c*x^n))^4,x)
```

output
```
(3*x)/8 + (x*exp(-2*a))/((c*x^n)^(2*b)*(8*b*n - 4)) - (x*exp(2*a)*(c*x^n)^
(2*b))/(8*b*n + 4) - (x*exp(-4*a))/((c*x^n)^(4*b)*(64*b*n - 16)) + (x*exp(
4*a)*(c*x^n)^(4*b))/(64*b*n + 16)
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 390, normalized size of antiderivative = 2.04

$$\int \sinh^4(a+b\log(cx^n))\,dx$$

$$= \frac{x\left(16x^{8bn}e^{8a}c^{8b}b^3n^3 - 4x^{8bn}e^{8a}c^{8b}b^2n^2 - 4x^{8bn}e^{8a}c^{8b}bn + x^{8bn}e^{8a}c^{8b} - 128x^{6bn}e^{6a}c^{6b}b^3n^3 + 64x^{6bn}e^{6a}c^{6b}b^2\right.}{}$$

input
```
int(sinh(a+b*log(c*x^n))^4,x)
```

output
```
(x*(16*x**(8*b*n)*e**(8*a)*c**(8*b)*b**3*n**3 - 4*x**(8*b*n)*e**(8*a)*c**(
8*b)*b**2*n**2 - 4*x**(8*b*n)*e**(8*a)*c**(8*b)*b*n + x**(8*b*n)*e**(8*a)*
c**(8*b) - 128*x**(6*b*n)*e**(6*a)*c**(6*b)*b**3*n**3 + 64*x**(6*b*n)*e**(
6*a)*c**(6*b)*b**2*n**2 + 8*x**(6*b*n)*e**(6*a)*c**(6*b)*b*n - 4*x**(6*b*n
)*e**(6*a)*c**(6*b) + 384*x**(4*b*n)*e**(4*a)*c**(4*b)*b**4*n**4 - 120*x**
(4*b*n)*e**(4*a)*c**(4*b)*b**2*n**2 + 6*x**(4*b*n)*e**(4*a)*c**(4*b) + 128
*x**(2*b*n)*e**(2*a)*c**(2*b)*b**3*n**3 + 64*x**(2*b*n)*e**(2*a)*c**(2*b)*
b**2*n**2 - 8*x**(2*b*n)*e**(2*a)*c**(2*b)*b*n - 4*x**(2*b*n)*e**(2*a)*c**
(2*b) - 16*b**3*n**3 - 4*b**2*n**2 + 4*b*n + 1))/(16*x**(4*b*n)*e**(4*a)*c
**(4*b)*(64*b**4*n**4 - 20*b**2*n**2 + 1))
```

3.270 $\int x^m \sinh(a+b\log(cx^n))\,dx$

Optimal result

Integrand size = 15, antiderivative size = 73

$$\int x^m \sinh(a+b\log(cx^n))\,dx = -\frac{bnx^{1+m}\cosh(a+b\log(cx^n))}{(1+m)^2-b^2n^2} + \frac{(1+m)x^{1+m}\sinh(a+b\log(cx^n))}{(1+m)^2-b^2n^2}$$

output
```
-b*n*x^(1+m)*cosh(a+b*ln(c*x^n))/((1+m)^2-b^2*n^2)+(1+m)*x^(1+m)*sinh(a+b*
ln(c*x^n))/((1+m)^2-b^2*n^2)
```

Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.74

$$\int x^m \sinh(a+b\log(cx^n))\,dx = \frac{x^{1+m}(-bn\cosh(a+b\log(cx^n))+(1+m)\sinh(a+b\log(cx^n)))}{(1+m-bn)(1+m+bn)}$$

input
```
Integrate[x^m*Sinh[a + b*Log[c*x^n]],x]
```

output
```
(x^(1 + m)*(-(b*n*Cosh[a + b*Log[c*x^n]]) + (1 + m)*Sinh[a + b*Log[c*x^n]]
))/((1 + m - b*n)*(1 + m + b*n))
```

Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 74, normalized size of antiderivative = 1.01, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.067$, Rules used = {6053}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^m \sinh (a+b \log (c x^n)) \, dx$$

$$\Big\downarrow \text{ 6053}$$

$$\frac{(m+1)x^{m+1}\sinh (a+b\log (cx^n))}{(-bn+m+1)(bn+m+1)}-\frac{bnx^{m+1}\cosh (a+b\log (cx^n))}{(m+1)^2-b^2n^2}$$

input
```
Int[x^m*Sinh[a + b*Log[c*x^n]],x]
```

output
```
-((b*n*x^(1 + m)*Cosh[a + b*Log[c*x^n]])/((1 + m)^2 - b^2*n^2)) + ((1 + m)
*x^(1 + m)*Sinh[a + b*Log[c*x^n]])/((1 + m - b*n)*(1 + m + b*n))
```

Defintions of rubi rules used

rule 6053
```
Int[((e_.)*(x_))^(m_.)*Sinh[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(d_.)], x
_Symbol] :> Simp[(-(m + 1))*(e*x)^(m + 1)*(Sinh[d*(a + b*Log[c*x^n])]/(b^2*
d^2*e*n^2 - e*(m + 1)^2)), x] + Simp[b*d*n*(e*x)^(m + 1)*(Cosh[d*(a + b*Log
[c*x^n])]/(b^2*d^2*e*n^2 - e*(m + 1)^2)), x] /; FreeQ[{a, b, c, d, e, m, n}
, x] && NeQ[b^2*d^2*n^2 - (m + 1)^2, 0]
```

Maple [F]

$$\int x^{m} \sinh\left(a+b\ln\left(c\,x^{n}\right)\right) dx$$

input
```
int(x^m*sinh(a+b*ln(c*x^n)),x)
```

output
```
int(x^m*sinh(a+b*ln(c*x^n)),x)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 98, normalized size of antiderivative = 1.34

$$\int x^m \sinh\left(a+b\log\left(cx^n\right)\right)\,dx$$
$$= \frac{bnx\cosh\left(bn\log\left(x\right)+b\log\left(c\right)+a\right)\cosh\left(m\log\left(x\right)\right)+bnx\cosh\left(bn\log\left(x\right)+b\log\left(c\right)+a\right)\sinh\left(m\log\left(x\right.\right.}{b^2n^2-m^2-2m}$$

input
```
integrate(x^m*sinh(a+b*log(c*x^n)),x, algorithm="fricas")
```

output
```
(b*n*x*cosh(b*n*log(x) + b*log(c) + a)*cosh(m*log(x)) + b*n*x*cosh(b*n*log
(x) + b*log(c) + a)*sinh(m*log(x)) - ((m + 1)*x*cosh(m*log(x)) + (m + 1)*x
*sinh(m*log(x)))*sinh(b*n*log(x) + b*log(c) + a))/(b^2*n^2 - m^2 - 2*m - 1
)
```

Sympy [F]

$$\int x^m \sinh\left(a+b\log\left(cx^n\right)\right)\,dx$$
$$= \begin{cases} \log\left(x\right)\sinh\left(a\right) & \text{for}\ b=0\wedge m=-1 \\ -\int x^m\sinh\left(-a+\frac{m\log\left(cx^n\right)}{n}+\frac{\log\left(cx^n\right)}{n}\right)dx & \text{for}\ b=-\frac{m+1}{n} \\ \int x^m\sinh\left(a+\frac{m\log\left(cx^n\right)}{n}+\frac{\log\left(cx^n\right)}{n}\right)dx & \text{for}\ b=\frac{m+1}{n} \\ \frac{bnxx^m\cosh\left(a+b\log\left(cx^n\right)\right)}{b^2n^2-m^2-2m-1}-\frac{mxx^m\sinh\left(a+b\log\left(cx^n\right)\right)}{b^2n^2-m^2-2m-1}-\frac{xx^m\sinh\left(a+b\log\left(cx^n\right)\right)}{b^2n^2-m^2-2m-1} & \text{otherwise} \end{cases}$$

input
```
integrate(x**m*sinh(a+b*ln(c*x**n)),x)
```

output
```
Piecewise((log(x)*sinh(a), Eq(b, 0) & Eq(m, -1)), (-Integral(x**m*sinh(-a
+ m*log(c*x**n)/n + log(c*x**n)/n), x), Eq(b, -(m + 1)/n)), (Integral(x**m
*sinh(a + m*log(c*x**n)/n + log(c*x**n)/n), x), Eq(b, (m + 1)/n)), (b*n*x*
x**m*cosh(a + b*log(c*x**n))/(b**2*n**2 - m**2 - 2*m - 1) - m*x*x**m*sinh(
a + b*log(c*x**n))/(b**2*n**2 - m**2 - 2*m - 1) - x*x**m*sinh(a + b*log(c*
x**n))/(b**2*n**2 - m**2 - 2*m - 1), True))
```

Maxima [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 64, normalized size of antiderivative = 0.88

$$\int x^m \sinh\left(a+b\log\left(cx^n\right)\right)\,dx=\frac{c^b x e^{\left(b\log\left(x^n\right)+m\log\left(x\right)+a\right)}}{2\left(bn+m+1\right)}+\frac{x e^{\left(-b\log\left(x^n\right)+m\log\left(x\right)-a\right)}}{2\left(bc^b n-c^b(m+1)\right)}$$

input
```
integrate(x^m*sinh(a+b*log(c*x^n)),x, algorithm="maxima")
```

output
```
1/2*c^b*x*e^(b*log(x^n) + m*log(x) + a)/(b*n + m + 1) + 1/2*x*e^(-b*log(x^
n) + m*log(x) - a)/(b*c^b*n - c^b*(m + 1))
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 235 vs. 2(75) = 150.

Time = 0.14 (sec) , antiderivative size = 235, normalized size of antiderivative = 3.22

$$\begin{aligned}\int x^m \sinh\left(a+b\log\left(cx^n\right)\right)\,dx=&\frac{bc^b nx x^{bn} x^m e^a}{2\left(b^2n^2-m^2-2m-1\right)}-\frac{c^b mx x^{bn} x^m e^a}{2\left(b^2n^2-m^2-2m-1\right)}\\&-\frac{c^b x x^{bn} x^m e^a}{2\left(b^2n^2-m^2-2m-1\right)}\\&+\frac{bnx x^m e^{(-a)}}{2\left(b^2n^2-m^2-2m-1\right)c^b x^{bn}}\\&+\frac{mx x^m e^{(-a)}}{2\left(b^2n^2-m^2-2m-1\right)c^b x^{bn}}\\&+\frac{x x^m e^{(-a)}}{2\left(b^2n^2-m^2-2m-1\right)c^b x^{bn}}\end{aligned}$$

input
```
integrate(x^m*sinh(a+b*log(c*x^n)),x, algorithm="giac")
```

output
```
1/2*b*c^b*n*x*x^(b*n)*x^m*e^a/(b^2*n^2 - m^2 - 2*m - 1) - 1/2*c^b*m*x*x^(b
*n)*x^m*e^a/(b^2*n^2 - m^2 - 2*m - 1) - 1/2*c^b*x*x^(b*n)*x^m*e^a/(b^2*n^2
 - m^2 - 2*m - 1) + 1/2*b*n*x*x^m*e^(-a)/((b^2*n^2 - m^2 - 2*m - 1)*c^b*x^
(b*n)) + 1/2*m*x*x^m*e^(-a)/((b^2*n^2 - m^2 - 2*m - 1)*c^b*x^(b*n)) + 1/2*
x*x^m*e^(-a)/((b^2*n^2 - m^2 - 2*m - 1)*c^b*x^(b*n))
```

Mupad [B] (verification not implemented)

Time = 1.64 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.77

$$\int x^m \sinh\left(a + b\log\left(cx^n\right)\right)\,dx = \frac{x\,x^m\,\mathrm{e}^a\,\left(c\,x^n\right)^b}{2\,m+2\,b\,n+2} - \frac{x\,x^m\,\mathrm{e}^{-a}}{\left(c\,x^n\right)^b\,\left(2\,m-2\,b\,n+2\right)}$$

input
```
int(x^m*sinh(a + b*log(c*x^n)),x)
```

output
```
(x*x^m*exp(a)*(c*x^n)^b)/(2*m + 2*b*n + 2) - (x*x^m*exp(-a))/((c*x^n)^b*(2
*m - 2*b*n + 2))
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 66, normalized size of antiderivative = 0.90

$$\int x^m \sinh\left(a + b\log\left(cx^n\right)\right)\,dx = \frac{x^m x(\cosh\left(\log(x^n c)\,b + a\right) bn - \sinh\left(\log(x^n c)\,b + a\right) m - \sinh\left(\log(x^n c)\,b + a\right))}{b^2n^2 - m^2 - 2m - 1}$$

input
```
int(x^m*sinh(a+b*log(c*x^n)),x)
```

output
```
(x**m*x*(cosh(log(x**n*c)*b + a)*b*n - sinh(log(x**n*c)*b + a)*m - sinh(lo
g(x**n*c)*b + a)))/(b**2*n**2 - m**2 - 2*m - 1)
```

3.271 $\int x^m \sinh^2(a+b\log(cx^n))\,dx$

Optimal result

Integrand size = 17, antiderivative size = 120

$$\int x^m \sinh^2(a+b\log(cx^n))\,dx = \frac{2b^2n^2x^{1+m}}{(1+m)\left((1+m)^2-4b^2n^2\right)} - \frac{2bnx^{1+m}\cosh(a+b\log(cx^n))\sinh(a+b\log(cx^n))}{(1+m)^2-4b^2n^2} + \frac{(1+m)x^{1+m}\sinh^2(a+b\log(cx^n))}{(1+m)^2-4b^2n^2}$$

output
```
2*b^2*n^2*x^(1+m)/(1+m)/((1+m)^2-4*b^2*n^2)-2*b*n*x^(1+m)*cosh(a+b*ln(c*x^
n))*sinh(a+b*ln(c*x^n))/((1+m)^2-4*b^2*n^2)+(1+m)*x^(1+m)*sinh(a+b*ln(c*x^
n))^2/((1+m)^2-4*b^2*n^2)
```

Mathematica [A] (verified)

Time = 0.18 (sec) , antiderivative size = 89, normalized size of antiderivative = 0.74

$$\int x^m \sinh^2(a+b\log(cx^n))\,dx = \frac{x^{1+m}\left(-1-2m-m^2+4b^2n^2+(1+m)^2\cosh(2(a+b\log(cx^n)))-2b(1+m)n\sinh(2(a+b\log(cx^n))\right)}{2(1+m)(1+m-2bn)(1+m+2bn)}$$

input
```
Integrate[x^m*Sinh[a + b*Log[c*x^n]]^2,x]
```

output
```
(x^(1 + m)*(-1 - 2*m - m^2 + 4*b^2*n^2 + (1 + m)^2*Cosh[2*(a + b*Log[c*x^n
])] - 2*b*(1 + m)*n*Sinh[2*(a + b*Log[c*x^n])]))/(2*(1 + m)*(1 + m - 2*b*n
)*(1 + m + 2*b*n))
```

Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 122, normalized size of antiderivative = 1.02, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.118$, Rules used = {6055, 15}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^m \sinh^2(a+b\log(cx^n))\,dx$$

$$\Big\downarrow \text{6055}$$

$$\frac{2b^2n^2\int x^m dx}{(m+1)^2-4b^2n^2}+\frac{(m+1)x^{m+1}\sinh^2(a+b\log(cx^n))}{-4b^2n^2+m^2+2m+1}-\frac{2bnx^{m+1}\sinh(a+b\log(cx^n))\cosh(a+b\log(cx^n))}{(m+1)^2-4b^2n^2}$$

$$\Big\downarrow \text{15}$$

$$\frac{(m+1)x^{m+1}\sinh^2(a+b\log(cx^n))}{-4b^2n^2+m^2+2m+1}-\frac{2bnx^{m+1}\sinh(a+b\log(cx^n))\cosh(a+b\log(cx^n))}{(m+1)^2-4b^2n^2}+\frac{2b^2n^2x^{m+1}}{(m+1)\left((m+1)^2-4b^2n^2\right)}$$

input
```
Int[x^m*Sinh[a + b*Log[c*x^n]]^2,x]
```

output
```
(2*b^2*n^2*x^(1 + m))/((1 + m)*((1 + m)^2 - 4*b^2*n^2)) - (2*b*n*x^(1 + m)
*Cosh[a + b*Log[c*x^n]]*Sinh[a + b*Log[c*x^n]])/((1 + m)^2 - 4*b^2*n^2) +
((1 + m)*x^(1 + m)*Sinh[a + b*Log[c*x^n]]^2)/(1 + 2*m + m^2 - 4*b^2*n^2)
```

Defintions of rubi rules used

rule 15
```
Int[(a_.)*(x_)^(m_.), x_Symbol] :> Simp[a*(x^(m + 1)/(m + 1)), x] /; FreeQ[
{a, m}, x] && NeQ[m, -1]
```

rule 6055
```
Int[((e_.)*(x_))^(m_.)*Sinh[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(d_.)]^(p
_), x_Symbol] :> Simp[(-(m + 1))*(e*x)^(m + 1)*(Sinh[d*(a + b*Log[c*x^n])]^
p/(b^2*d^2*e*n^2*p^2 - e*(m + 1)^2)), x] + (Simp[b*d*n*p*(e*x)^(m + 1)*Cosh
[d*(a + b*Log[c*x^n])]*(Sinh[d*(a + b*Log[c*x^n])]^(p - 1)/(b^2*d^2*e*n^2*p
^2 - e*(m + 1)^2)), x] - Simp[b^2*d^2*n^2*p*((p - 1)/(b^2*d^2*n^2*p^2 - (m
+ 1)^2))   Int[(e*x)^m*Sinh[d*(a + b*Log[c*x^n])]^(p - 2), x], x]) /; FreeQ
[{a, b, c, d, e, m, n}, x] && IGtQ[p, 1] && NeQ[b^2*d^2*n^2*p^2 - (m + 1)^2
, 0]
```

Maple [F]

$$\int x^m \sinh\left(a+b\ln\left(c\,x^n\right)\right)^2 dx$$

input
```
int(x^m*sinh(a+b*ln(c*x^n))^2,x)
```

output
```
int(x^m*sinh(a+b*ln(c*x^n))^2,x)
```

Fricas [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 248, normalized size of antiderivative = 2.07

$$\int x^m \sinh^2(a+b\log(cx^n))\,dx = \frac{(m^2+2\,m+1)x\cosh(bn\log(x)+b\log(c)+a)^2\cosh(m\log(x))+(4\,b^2n^2-m^2-2\,m-1)x\cosh(m\log(x))}{}$$

input
```
integrate(x^m*sinh(a+b*log(c*x^n))^2,x, algorithm="fricas")
```

output
```
1/2*((m^2 + 2*m + 1)*x*cosh(b*n*log(x) + b*log(c) + a)^2*cosh(m*log(x)) +
(4*b^2*n^2 - m^2 - 2*m - 1)*x*cosh(m*log(x)) + ((m^2 + 2*m + 1)*x*cosh(m*l
og(x)) + (m^2 + 2*m + 1)*x*sinh(m*log(x)))*sinh(b*n*log(x) + b*log(c) + a)
^2 - 4*((b*m + b)*n*x*cosh(b*n*log(x) + b*log(c) + a)*cosh(m*log(x)) + (b*
m + b)*n*x*cosh(b*n*log(x) + b*log(c) + a)*sinh(m*log(x)))*sinh(b*n*log(x)
 + b*log(c) + a) + ((m^2 + 2*m + 1)*x*cosh(b*n*log(x) + b*log(c) + a)^2 +
(4*b^2*n^2 - m^2 - 2*m - 1)*x)*sinh(m*log(x)))/(m^3 - 4*(b^2*m + b^2)*n^2
+ 3*m^2 + 3*m + 1)
```

Sympy [F]

$$\int x^m \sinh^2(a + b\log(cx^n))\,dx$$
$$= \begin{cases} \log(x)\sinh^2(a) \\ \int x^m \sinh^2\left(-a + \frac{m\log(cx^n)}{2n} + \frac{\log(cx^n)}{2n}\right) dx \\ \int x^m \sinh^2\left(a + \frac{m\log(cx^n)}{2n} + \frac{\log(cx^n)}{2n}\right) dx \\ \int \frac{\sinh^2(a+b\log(cx^n))}{x}\,dx \\ \frac{2b^2n^2xx^m\sinh^2(a+b\log(cx^n))}{4b^2mn^2+4b^2n^2-m^3-3m^2-3m-1} - \frac{2b^2n^2xx^m\cosh^2(a+b\log(cx^n))}{4b^2mn^2+4b^2n^2-m^3-3m^2-3m-1} + \frac{2bmnxx^m\sinh(a+b\log(cx^n))\cosh(a+b\log(cx^n))}{4b^2mn^2+4b^2n^2-m^3-3m^2-3m-1} + \frac{2bn}{} \end{cases}$$

input
```
integrate(x**m*sinh(a+b*ln(c*x**n))**2,x)
```

output
```
Piecewise((log(x)*sinh(a)**2, Eq(b, 0) & Eq(m, -1)), (Integral(x**m*sinh(-
a + m*log(c*x**n)/(2*n) + log(c*x**n)/(2*n))**2, x), Eq(b, -(m + 1)/(2*n))
), (Integral(x**m*sinh(a + m*log(c*x**n)/(2*n) + log(c*x**n)/(2*n))**2, x)
, Eq(b, (m + 1)/(2*n))), (Integral(sinh(a + b*log(c*x**n))**2/x, x), Eq(m,
 -1)), (2*b**2*n**2*x*x**m*sinh(a + b*log(c*x**n))**2/(4*b**2*m*n**2 + 4*b
**2*n**2 - m**3 - 3*m**2 - 3*m - 1) - 2*b**2*n**2*x*x**m*cosh(a + b*log(c*
x**n))**2/(4*b**2*m*n**2 + 4*b**2*n**2 - m**3 - 3*m**2 - 3*m - 1) + 2*b*m*
n*x*x**m*sinh(a + b*log(c*x**n))*cosh(a + b*log(c*x**n))/(4*b**2*m*n**2 +
4*b**2*n**2 - m**3 - 3*m**2 - 3*m - 1) + 2*b*n*x*x**m*sinh(a + b*log(c*x**
n))*cosh(a + b*log(c*x**n))/(4*b**2*m*n**2 + 4*b**2*n**2 - m**3 - 3*m**2 -
 3*m - 1) - m**2*x*x**m*sinh(a + b*log(c*x**n))**2/(4*b**2*m*n**2 + 4*b**2
*n**2 - m**3 - 3*m**2 - 3*m - 1) - 2*m*x*x**m*sinh(a + b*log(c*x**n))**2/(
4*b**2*m*n**2 + 4*b**2*n**2 - m**3 - 3*m**2 - 3*m - 1) - x*x**m*sinh(a + b
*log(c*x**n))**2/(4*b**2*m*n**2 + 4*b**2*n**2 - m**3 - 3*m**2 - 3*m - 1),
True))
```

Maxima [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 87, normalized size of antiderivative = 0.72

$$\int x^m \sinh^2(a + b\log(cx^n))\,dx = \frac{c^{2\,b}xe^{(2\,b\log(x^n)+m\log(x)+2\,a)}}{4\,(2\,bn+m+1)} - \frac{xe^{(-2\,b\log(x^n)+m\log(x)-2\,a)}}{4\,(2\,bc^{2\,b}n-c^{2\,b}(m+1))} - \frac{x^{m+1}}{2\,(m+1)}$$

input
```
integrate(x^m*sinh(a+b*log(c*x^n))^2,x, algorithm="maxima")
```

output
```
1/4*c^(2*b)*x*e^(2*b*log(x^n) + m*log(x) + 2*a)/(2*b*n + m + 1) - 1/4*x*e^
(-2*b*log(x^n) + m*log(x) - 2*a)/(2*b*c^(2*b)*n - c^(2*b)*(m + 1)) - 1/2*x
^(m + 1)/(m + 1)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 758 vs. $2(127) = 254$.

Time = 0.17 (sec) , antiderivative size = 758, normalized size of antiderivative = 6.32

$$\int x^m \sinh^2(a + b\log(cx^n))\, dx = \text{Too large to display}$$

input
```
integrate(x^m*sinh(a+b*log(c*x^n))^2,x, algorithm="giac")
```

output
```
1/2*b*c^(2*b)*m*n*x*x^(2*b*n)*x^m*e^(2*a)/(4*b^2*m*n^2 + 4*b^2*n^2 - m^3 -
 3*m^2 - 3*m - 1) - 1/4*c^(2*b)*m^2*x*x^(2*b*n)*x^m*e^(2*a)/(4*b^2*m*n^2 +
 4*b^2*n^2 - m^3 - 3*m^2 - 3*m - 1) + 1/2*b*c^(2*b)*n*x*x^(2*b*n)*x^m*e^(2
*a)/(4*b^2*m*n^2 + 4*b^2*n^2 - m^3 - 3*m^2 - 3*m - 1) - 2*b^2*n^2*x*x^m/(4
*b^2*m*n^2 + 4*b^2*n^2 - m^3 - 3*m^2 - 3*m - 1) - 1/2*c^(2*b)*m*x*x^(2*b*n
)*x^m*e^(2*a)/(4*b^2*m*n^2 + 4*b^2*n^2 - m^3 - 3*m^2 - 3*m - 1) - 1/4*c^(2
*b)*x*x^(2*b*n)*x^m*e^(2*a)/(4*b^2*m*n^2 + 4*b^2*n^2 - m^3 - 3*m^2 - 3*m -
 1) + 1/2*m^2*x*x^m/(4*b^2*m*n^2 + 4*b^2*n^2 - m^3 - 3*m^2 - 3*m - 1) - 1/
2*b*m*n*x*x^m*e^(-2*a)/((4*b^2*m*n^2 + 4*b^2*n^2 - m^3 - 3*m^2 - 3*m - 1)*
c^(2*b)*x^(2*b*n)) + m*x*x^m/(4*b^2*m*n^2 + 4*b^2*n^2 - m^3 - 3*m^2 - 3*m
- 1) - 1/4*m^2*x*x^m*e^(-2*a)/((4*b^2*m*n^2 + 4*b^2*n^2 - m^3 - 3*m^2 - 3*
m - 1)*c^(2*b)*x^(2*b*n)) - 1/2*b*n*x*x^m*e^(-2*a)/((4*b^2*m*n^2 + 4*b^2*n
^2 - m^3 - 3*m^2 - 3*m - 1)*c^(2*b)*x^(2*b*n)) + 1/2*x*x^m/(4*b^2*m*n^2 +
4*b^2*n^2 - m^3 - 3*m^2 - 3*m - 1) - 1/2*m*x*x^m*e^(-2*a)/((4*b^2*m*n^2 +
4*b^2*n^2 - m^3 - 3*m^2 - 3*m - 1)*c^(2*b)*x^(2*b*n)) - 1/4*x*x^m*e^(-2*a)
/((4*b^2*m*n^2 + 4*b^2*n^2 - m^3 - 3*m^2 - 3*m - 1)*c^(2*b)*x^(2*b*n))
```

Mupad [B] (verification not implemented)

Time = 1.68 (sec) , antiderivative size = 74, normalized size of antiderivative = 0.62

$$\int x^m \sinh^2(a+b\log(cx^n))\, dx = \frac{x\,x^m\,\mathrm{e}^{-2\,a}}{(c\,x^n)^{2\,b}\,(4\,m-8\,b\,n+4)} - \frac{x\,x^m}{2\,m+2} + \frac{x\,x^m\,\mathrm{e}^{2\,a}\,(c\,x^n)^{2\,b}}{4\,m+8\,b\,n+4}$$

input
```
int(x^m*sinh(a + b*log(c*x^n))^2,x)
```

output
```
(x*x^m*exp(-2*a))/((c*x^n)^(2*b)*(4*m - 8*b*n + 4)) - (x*x^m)/(2*m + 2) +
(x*x^m*exp(2*a)*(c*x^n)^(2*b))/(4*m + 8*b*n + 4)
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 262, normalized size of antiderivative = 2.18

$$\int x^m \sinh^2(a + b\log(cx^n))\,dx$$
$$= \frac{x^m x(2x^{4bn}e^{4a}c^{4b}bmn + 2x^{4bn}e^{4a}c^{4b}bn - x^{4bn}e^{4a}c^{4b}m^2 - 2x^{4bn}e^{4a}c^{4b}m - x^{4bn}e^{4a}c^{4b} - 8x^{2bn}e^{2a}c^{2b}b^2n^2 + 2}{4x^{2bn}e^{2a}c^{2b}(4b^2mn^2 + 4b^2n^2 - m^3 - 3m}$$

input
```
int(x^m*sinh(a+b*log(c*x^n))^2,x)
```

output
```
(x**m*x*(2*x**(4*b*n)*e**(4*a)*c**(4*b)*b*m*n + 2*x**(4*b*n)*e**(4*a)*c**(
4*b)*b*n - x**(4*b*n)*e**(4*a)*c**(4*b)*m**2 - 2*x**(4*b*n)*e**(4*a)*c**(4
*b)*m - x**(4*b*n)*e**(4*a)*c**(4*b) - 8*x**(2*b*n)*e**(2*a)*c**(2*b)*b**2
*n**2 + 2*x**(2*b*n)*e**(2*a)*c**(2*b)*m**2 + 4*x**(2*b*n)*e**(2*a)*c**(2*
b)*m + 2*x**(2*b*n)*e**(2*a)*c**(2*b) - 2*b*m*n - 2*b*n - m**2 - 2*m - 1))
/(4*x**(2*b*n)*e**(2*a)*c**(2*b)*(4*b**2*m*n**2 + 4*b**2*n**2 - m**3 - 3*m
**2 - 3*m - 1))
```

3.272 $\int x^m \sinh^3(a+b\log(cx^n))\,dx$

Optimal result

Integrand size = 17, antiderivative size = 203

$$\begin{aligned}\int x^m \sinh^3(a+b\log(cx^n))\,dx = &-\frac{6b^3n^3x^{1+m}\cosh(a+b\log(cx^n))}{((1+m)^2-9b^2n^2)((1+m)^2-b^2n^2)} \\ &+\frac{6b^2(1+m)n^2x^{1+m}\sinh(a+b\log(cx^n))}{((1+m)^2-9b^2n^2)((1+m)^2-b^2n^2)} \\ &-\frac{3bnx^{1+m}\cosh(a+b\log(cx^n))\sinh^2(a+b\log(cx^n))}{(1+m)^2-9b^2n^2} \\ &+\frac{(1+m)x^{1+m}\sinh^3(a+b\log(cx^n))}{(1+m)^2-9b^2n^2}\end{aligned}$$

output
```
-6*b^3*n^3*x^(1+m)*cosh(a+b*ln(c*x^n))/((1+m)^2-9*b^2*n^2)/((1+m)^2-b^2*n^
2)+6*b^2*(1+m)*n^2*x^(1+m)*sinh(a+b*ln(c*x^n))/((1+m)^2-9*b^2*n^2)/((1+m)^
2-b^2*n^2)-3*b*n*x^(1+m)*cosh(a+b*ln(c*x^n))*sinh(a+b*ln(c*x^n))^2/((1+m)^
2-9*b^2*n^2)+(1+m)*x^(1+m)*sinh(a+b*ln(c*x^n))^3/((1+m)^2-9*b^2*n^2)
```

Mathematica [A] (verified)

Time = 0.92 (sec) , antiderivative size = 292, normalized size of antiderivative = 1.44

$$\int x^m \sinh^3(a+b\log(cx^n))\,dx$$
$$=\frac{1}{4}x^{1+m}\left(-\frac{3\cosh(bn\log(x))\left(-bn\cosh\left(a-bn\log(x)+b\log\left(cx^n\right)\right)+(1+m)\sinh\left(a-bn\log(x)+b\log\right.\right.}{(1+m-bn)(1+m+bn)}\right.$$
$$-\frac{3\sinh(bn\log(x))\left((1+m)\cosh\left(a-bn\log(x)+b\log\left(cx^n\right)\right)-bn\sinh\left(a-bn\log(x)+b\log\left(cx^n\right)\right)\right)}{(1+m-bn)(1+m+bn)}$$
$$+\frac{\cosh(3bn\log(x))\left(-3bn\cosh\left(3(a-bn\log(x)+b\log\left(cx^n\right))\right)+(1+m)\sinh\left(3(a-bn\log(x)+b\log\left(\right.\right.\right.\right.}{(1+m-3bn)(1+m+3bn)}$$
$$+\frac{\sinh(3bn\log(x))\left((1+m)\cosh\left(3(a-bn\log(x)+b\log\left(cx^n\right))\right)-3bn\sinh\left(3(a-bn\log(x)+b\log\left(cx\right.\right.\right.\right.}{(1+m-3bn)(1+m+3bn)}$$

input
```
Integrate[x^m*Sinh[a + b*Log[c*x^n]]^3,x]
```

output
```
(x^(1 + m)*((-3*Cosh[b*n*Log[x]]*(-(b*n*Cosh[a - b*n*Log[x] + b*Log[c*x^n]
]) + (1 + m)*Sinh[a - b*n*Log[x] + b*Log[c*x^n]]))/((1 + m - b*n)*(1 + m +
 b*n)) - (3*Sinh[b*n*Log[x]]*((1 + m)*Cosh[a - b*n*Log[x] + b*Log[c*x^n]]
- b*n*Sinh[a - b*n*Log[x] + b*Log[c*x^n]]))/((1 + m - b*n)*(1 + m + b*n))
+ (Cosh[3*b*n*Log[x]]*(-3*b*n*Cosh[3*(a - b*n*Log[x] + b*Log[c*x^n])] + (1
 + m)*Sinh[3*(a - b*n*Log[x] + b*Log[c*x^n])]))/((1 + m - 3*b*n)*(1 + m +
3*b*n)) + (Sinh[3*b*n*Log[x]]*((1 + m)*Cosh[3*(a - b*n*Log[x] + b*Log[c*x^
n])] - 3*b*n*Sinh[3*(a - b*n*Log[x] + b*Log[c*x^n])]))/((1 + m - 3*b*n)*(1
 + m + 3*b*n))))/4
```

Rubi [A] (verified)

Time = 0.41 (sec) , antiderivative size = 188, normalized size of antiderivative = 0.93, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {6055, 6053}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^m \sinh^3(a+b\log(cx^n))\,dx$$

$$\Big\downarrow \text{6055}$$

$$\frac{6b^2n^2\int x^m \sinh(a+b\log(cx^n))\,dx}{(m+1)^2-9b^2n^2}+\frac{(m+1)x^{m+1}\sinh^3(a+b\log(cx^n))}{-9b^2n^2+m^2+2m+1}-\frac{3bnx^{m+1}\sinh^2(a+b\log(cx^n))\cosh(a+b\log(cx^n))}{(m+1)^2-9b^2n^2}$$

$$\Big\downarrow \text{6053}$$

$$\frac{(m+1)x^{m+1}\sinh^3(a+b\log(cx^n))}{-9b^2n^2+m^2+2m+1}-\frac{3bnx^{m+1}\sinh^2(a+b\log(cx^n))\cosh(a+b\log(cx^n))}{(m+1)^2-9b^2n^2}+\frac{6b^2n^2\left(\frac{(m+1)x^{m+1}\sinh(a+b\log(cx^n))}{(-bn+m+1)(bn+m+1)}-\frac{bnx^{m+1}\cosh(a+b\log(cx^n))}{(m+1)^2-b^2n^2}\right)}{(m+1)^2-9b^2n^2}$$

input
```
Int[x^m*Sinh[a + b*Log[c*x^n]]^3,x]
```

output
```
(-3*b*n*x^(1 + m)*Cosh[a + b*Log[c*x^n]]*Sinh[a + b*Log[c*x^n]]^2)/((1 + m
)^2 - 9*b^2*n^2) + ((1 + m)*x^(1 + m)*Sinh[a + b*Log[c*x^n]]^3)/(1 + 2*m +
 m^2 - 9*b^2*n^2) + (6*b^2*n^2*(-((b*n*x^(1 + m)*Cosh[a + b*Log[c*x^n]])/(
(1 + m)^2 - b^2*n^2)) + ((1 + m)*x^(1 + m)*Sinh[a + b*Log[c*x^n]])/((1 + m
 - b*n)*(1 + m + b*n))))/((1 + m)^2 - 9*b^2*n^2)
```

Defintions of rubi rules used

rule 6053
```
Int[((e_.)*(x_))^(m_.)*Sinh[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(d_.)], x
_Symbol] :> Simp[(-(m + 1))*(e*x)^(m + 1)*(Sinh[d*(a + b*Log[c*x^n])]/(b^2*
d^2*e*n^2 - e*(m + 1)^2)), x] + Simp[b*d*n*(e*x)^(m + 1)*(Cosh[d*(a + b*Log
[c*x^n])]/(b^2*d^2*e*n^2 - e*(m + 1)^2)), x] /; FreeQ[{a, b, c, d, e, m, n}
, x] && NeQ[b^2*d^2*n^2 - (m + 1)^2, 0]
```

rule 6055
```
Int[((e_.)*(x_))^(m_.)*Sinh[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(d_.)]^(p
_), x_Symbol] :> Simp[(-(m + 1))*(e*x)^(m + 1)*(Sinh[d*(a + b*Log[c*x^n])]^
p/(b^2*d^2*e*n^2*p^2 - e*(m + 1)^2)), x] + (Simp[b*d*n*p*(e*x)^(m + 1)*Cosh
[d*(a + b*Log[c*x^n])]*(Sinh[d*(a + b*Log[c*x^n])]^(p - 1)/(b^2*d^2*e*n^2*p
^2 - e*(m + 1)^2)), x] - Simp[b^2*d^2*n^2*p*((p - 1)/(b^2*d^2*n^2*p^2 - (m
+ 1)^2))   Int[(e*x)^m*Sinh[d*(a + b*Log[c*x^n])]^(p - 2), x], x]) /; FreeQ
[{a, b, c, d, e, m, n}, x] && IGtQ[p, 1] && NeQ[b^2*d^2*n^2*p^2 - (m + 1)^2
, 0]
```

Maple [F]

$$\int x^{m}\sinh\left(a+b\ln\left(c\,x^{n}\right)\right)^{3}dx$$

input
```
int(x^m*sinh(a+b*ln(c*x^n))^3,x)
```

output
```
int(x^m*sinh(a+b*ln(c*x^n))^3,x)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 585 vs. $2(214) = 428$.

Time = 0.12 (sec) , antiderivative size = 585, normalized size of antiderivative = 2.88

$$\int x^m \sinh^3(a+b\log(cx^n))\,dx = \text{Too large to display}$$

input
```
integrate(x^m*sinh(a+b*log(c*x^n))^3,x, algorithm="fricas")
```

output
```
1/4*(3*(b^3*n^3 - (b*m^2 + 2*b*m + b)*n)*x*cosh(b*n*log(x) + b*log(c) + a)
^3*cosh(m*log(x)) - 3*(9*b^3*n^3 - (b*m^2 + 2*b*m + b)*n)*x*cosh(b*n*log(x
) + b*log(c) + a)*cosh(m*log(x)) + ((m^3 - (b^2*m + b^2)*n^2 + 3*m^2 + 3*m
 + 1)*x*cosh(m*log(x)) + (m^3 - (b^2*m + b^2)*n^2 + 3*m^2 + 3*m + 1)*x*sin
h(m*log(x)))*sinh(b*n*log(x) + b*log(c) + a)^3 + 9*((b^3*n^3 - (b*m^2 + 2*
b*m + b)*n)*x*cosh(b*n*log(x) + b*log(c) + a)*cosh(m*log(x)) + (b^3*n^3 -
(b*m^2 + 2*b*m + b)*n)*x*cosh(b*n*log(x) + b*log(c) + a)*sinh(m*log(x)))*s
inh(b*n*log(x) + b*log(c) + a)^2 + 3*((m^3 - (b^2*m + b^2)*n^2 + 3*m^2 + 3
*m + 1)*x*cosh(b*n*log(x) + b*log(c) + a)^2*cosh(m*log(x)) - (m^3 - 9*(b^2
*m + b^2)*n^2 + 3*m^2 + 3*m + 1)*x*cosh(m*log(x)) + ((m^3 - (b^2*m + b^2)*
n^2 + 3*m^2 + 3*m + 1)*x*cosh(b*n*log(x) + b*log(c) + a)^2 - (m^3 - 9*(b^2
*m + b^2)*n^2 + 3*m^2 + 3*m + 1)*x)*sinh(m*log(x)))*sinh(b*n*log(x) + b*lo
g(c) + a) + 3*((b^3*n^3 - (b*m^2 + 2*b*m + b)*n)*x*cosh(b*n*log(x) + b*log
(c) + a)^3 - (9*b^3*n^3 - (b*m^2 + 2*b*m + b)*n)*x*cosh(b*n*log(x) + b*log
(c) + a))*sinh(m*log(x)))/(9*b^4*n^4 + m^4 + 4*m^3 - 10*(b^2*m^2 + 2*b^2*m
 + b^2)*n^2 + 6*m^2 + 4*m + 1)
```

Sympy [F(-1)]

Timed out.

$$\int x^m \sinh^3(a + b\log(cx^n))\,dx = \text{Timed out}$$

input
```
integrate(x**m*sinh(a+b*ln(c*x**n))**3,x)
```

output
```
Timed out
```

Maxima [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 138, normalized size of antiderivative = 0.68

$$\int x^m \sinh^3(a + b\log(cx^n))\,dx = \frac{c^{3\,b}xe^{(3\,b\log(x^n)+m\log(x)+3\,a)}}{8\,(3\,bn+m+1)} - \frac{3\,c^bxe^{(b\log(x^n)+m\log(x)+a)}}{8\,(bn+m+1)} - \frac{3\,xe^{(-b\log(x^n)+m\log(x)-a)}}{8\,(bc^bn-c^b(m+1))} + \frac{xe^{(-3\,b\log(x^n)+m\log(x)-3\,a)}}{8\,(3\,bc^{3\,b}n-c^{3\,b}(m+1))}$$

input
```
integrate(x^m*sinh(a+b*log(c*x^n))^3,x, algorithm="maxima")
```

output
```
1/8*c^(3*b)*x*e^(3*b*log(x^n) + m*log(x) + 3*a)/(3*b*n + m + 1) - 3/8*c^b*
x*e^(b*log(x^n) + m*log(x) + a)/(b*n + m + 1) - 3/8*x*e^(-b*log(x^n) + m*l
og(x) - a)/(b*c^b*n - c^b*(m + 1)) + 1/8*x*e^(-3*b*log(x^n) + m*log(x) - 3
*a)/(3*b*c^(3*b)*n - c^(3*b)*(m + 1))
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 3225 vs. $2(214) = 428$.

Time = 0.19 (sec) , antiderivative size = 3225, normalized size of antiderivative = 15.89

$$\int x^m \sinh^3(a + b\log(cx^n))\,dx = \text{Too large to display}$$

input
```
integrate(x^m*sinh(a+b*log(c*x^n))^3,x, algorithm="giac")
```

output
```
3/8*b^3*c^(3*b)*n^3*x*x^(3*b*n)*x^m*e^(3*a)/(9*b^4*n^4 - 10*b^2*m^2*n^2 -
20*b^2*m*n^2 + m^4 - 10*b^2*n^2 + 4*m^3 + 6*m^2 + 4*m + 1) - 27/8*b^3*c^b*
n^3*x*x^(b*n)*x^m*e^a/(9*b^4*n^4 - 10*b^2*m^2*n^2 - 20*b^2*m*n^2 + m^4 - 1
0*b^2*n^2 + 4*m^3 + 6*m^2 + 4*m + 1) - 1/8*b^2*c^(3*b)*m*n^2*x*x^(3*b*n)*x
^m*e^(3*a)/(9*b^4*n^4 - 10*b^2*m^2*n^2 - 20*b^2*m*n^2 + m^4 - 10*b^2*n^2 +
 4*m^3 + 6*m^2 + 4*m + 1) + 27/8*b^2*c^b*m*n^2*x*x^(b*n)*x^m*e^a/(9*b^4*n^
4 - 10*b^2*m^2*n^2 - 20*b^2*m*n^2 + m^4 - 10*b^2*n^2 + 4*m^3 + 6*m^2 + 4*m
 + 1) - 3/8*b*c^(3*b)*m^2*n*x*x^(3*b*n)*x^m*e^(3*a)/(9*b^4*n^4 - 10*b^2*m^
2*n^2 - 20*b^2*m*n^2 + m^4 - 10*b^2*n^2 + 4*m^3 + 6*m^2 + 4*m + 1) - 1/8*b
^2*c^(3*b)*n^2*x*x^(3*b*n)*x^m*e^(3*a)/(9*b^4*n^4 - 10*b^2*m^2*n^2 - 20*b^
2*m*n^2 + m^4 - 10*b^2*n^2 + 4*m^3 + 6*m^2 + 4*m + 1) + 3/8*b*c^b*m^2*n*x*
x^(b*n)*x^m*e^a/(9*b^4*n^4 - 10*b^2*m^2*n^2 - 20*b^2*m*n^2 + m^4 - 10*b^2*
n^2 + 4*m^3 + 6*m^2 + 4*m + 1) + 27/8*b^2*c^b*n^2*x*x^(b*n)*x^m*e^a/(9*b^4
*n^4 - 10*b^2*m^2*n^2 - 20*b^2*m*n^2 + m^4 - 10*b^2*n^2 + 4*m^3 + 6*m^2 +
4*m + 1) + 1/8*c^(3*b)*m^3*x*x^(3*b*n)*x^m*e^(3*a)/(9*b^4*n^4 - 10*b^2*m^2
*n^2 - 20*b^2*m*n^2 + m^4 - 10*b^2*n^2 + 4*m^3 + 6*m^2 + 4*m + 1) - 3/4*b*
c^(3*b)*m*n*x*x^(3*b*n)*x^m*e^(3*a)/(9*b^4*n^4 - 10*b^2*m^2*n^2 - 20*b^2*m
*n^2 + m^4 - 10*b^2*n^2 + 4*m^3 + 6*m^2 + 4*m + 1) - 3/8*c^b*m^3*x*x^(b*n)
*x^m*e^a/(9*b^4*n^4 - 10*b^2*m^2*n^2 - 20*b^2*m*n^2 + m^4 - 10*b^2*n^2 + 4
*m^3 + 6*m^2 + 4*m + 1) + 3/4*b*c^b*m*n*x*x^(b*n)*x^m*e^a/(9*b^4*n^4 - ...
```

Mupad [B] (verification not implemented)

Time = 1.79 (sec) , antiderivative size = 118, normalized size of antiderivative = 0.58

$$\int x^m \sinh^3(a+b\log(cx^n))\,dx = \frac{3\,x\,x^m\,\mathrm{e}^{-a}}{(c\,x^n)^b\,(8\,m-8\,b\,n+8)} - \frac{x\,x^m\,\mathrm{e}^{-3\,a}}{(c\,x^n)^{3\,b}\,(8\,m-24\,b\,n+8)} + \frac{x\,x^m\,\mathrm{e}^{3\,a}\,(c\,x^n)^{3\,b}}{8\,m+24\,b\,n+8} - \frac{3\,x\,x^m\,\mathrm{e}^{a}\,(c\,x^n)^b}{8\,m+8\,b\,n+8}$$

input
```
int(x^m*sinh(a + b*log(c*x^n))^3,x)
```

output
```
(3*x*x^m*exp(-a))/((c*x^n)^b*(8*m - 8*b*n + 8)) - (x*x^m*exp(-3*a))/((c*x^
n)^(3*b)*(8*m - 24*b*n + 8)) + (x*x^m*exp(3*a)*(c*x^n)^(3*b))/(8*m + 24*b*
n + 8) - (3*x*x^m*exp(a)*(c*x^n)^b)/(8*m + 8*b*n + 8)
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 784, normalized size of antiderivative = 3.86

$$\int x^m \sinh^3(a + b\log(cx^n))\,dx$$
$$= \frac{x^m x\left(-1 + 3x^{2bn}e^{2a}c^{2b} - 3m - x^{6bn}e^{6a}c^{6b}b^2n^2 - 3x^{4bn}e^{4a}c^{4b} - 3m^2 + 9x^{2bn}e^{2a}c^{2b}m^2 + 9x^{2bn}e^{2a}c^{2b}m + 3\right.}{}$$

input
```
int(x^m*sinh(a+b*log(c*x^n))^3,x)
```

output
```
(x**m*x*(3*x**(6*b*n)*e**(6*a)*c**(6*b)*b**3*n**3 - x**(6*b*n)*e**(6*a)*c*
*(6*b)*b**2*m*n**2 - x**(6*b*n)*e**(6*a)*c**(6*b)*b**2*n**2 - 3*x**(6*b*n)
*e**(6*a)*c**(6*b)*b*m**2*n - 6*x**(6*b*n)*e**(6*a)*c**(6*b)*b*m*n - 3*x**
(6*b*n)*e**(6*a)*c**(6*b)*b*n + x**(6*b*n)*e**(6*a)*c**(6*b)*m**3 + 3*x**(
6*b*n)*e**(6*a)*c**(6*b)*m**2 + 3*x**(6*b*n)*e**(6*a)*c**(6*b)*m + x**(6*b
*n)*e**(6*a)*c**(6*b) - 27*x**(4*b*n)*e**(4*a)*c**(4*b)*b**3*n**3 + 27*x**
(4*b*n)*e**(4*a)*c**(4*b)*b**2*m*n**2 + 27*x**(4*b*n)*e**(4*a)*c**(4*b)*b*
*2*n**2 + 3*x**(4*b*n)*e**(4*a)*c**(4*b)*b*m**2*n + 6*x**(4*b*n)*e**(4*a)*
c**(4*b)*b*m*n + 3*x**(4*b*n)*e**(4*a)*c**(4*b)*b*n - 3*x**(4*b*n)*e**(4*a
)*c**(4*b)*m**3 - 9*x**(4*b*n)*e**(4*a)*c**(4*b)*m**2 - 9*x**(4*b*n)*e**(4
*a)*c**(4*b)*m - 3*x**(4*b*n)*e**(4*a)*c**(4*b) - 27*x**(2*b*n)*e**(2*a)*c
**(2*b)*b**3*n**3 - 27*x**(2*b*n)*e**(2*a)*c**(2*b)*b**2*m*n**2 - 27*x**(2
*b*n)*e**(2*a)*c**(2*b)*b**2*n**2 + 3*x**(2*b*n)*e**(2*a)*c**(2*b)*b*m**2*
n + 6*x**(2*b*n)*e**(2*a)*c**(2*b)*b*m*n + 3*x**(2*b*n)*e**(2*a)*c**(2*b)*
b*n + 3*x**(2*b*n)*e**(2*a)*c**(2*b)*m**3 + 9*x**(2*b*n)*e**(2*a)*c**(2*b)
*m**2 + 9*x**(2*b*n)*e**(2*a)*c**(2*b)*m + 3*x**(2*b*n)*e**(2*a)*c**(2*b)
+ 3*b**3*n**3 + b**2*m*n**2 + b**2*n**2 - 3*b*m**2*n - 6*b*m*n - 3*b*n - m
**3 - 3*m**2 - 3*m - 1))/(8*x**(3*b*n)*e**(3*a)*c**(3*b)*(9*b**4*n**4 - 10
*b**2*m**2*n**2 - 20*b**2*m*n**2 - 10*b**2*n**2 + m**4 + 4*m**3 + 6*m**2 +
 4*m + 1))
```

3.273 $\int x^m \sinh^4(a+b\log(cx^n))\,dx$

Optimal result

Integrand size = 17, antiderivative size = 266

$$\begin{aligned}\int x^m \sinh^4(a+b\log(cx^n))\,dx &= \frac{24b^4n^4x^{1+m}}{(1+m)\left((1+m)^2-16b^2n^2\right)\left((1+m)^2-4b^2n^2\right)} \\ &\quad - \frac{24b^3n^3x^{1+m}\cosh(a+b\log(cx^n))\sinh(a+b\log(cx^n))}{\left((1+m)^2-16b^2n^2\right)\left((1+m)^2-4b^2n^2\right)} \\ &\quad + \frac{12b^2(1+m)n^2x^{1+m}\sinh^2(a+b\log(cx^n))}{\left((1+m)^2-16b^2n^2\right)\left((1+m)^2-4b^2n^2\right)} \\ &\quad - \frac{4bnx^{1+m}\cosh(a+b\log(cx^n))\sinh^3(a+b\log(cx^n))}{(1+m)^2-16b^2n^2} \\ &\quad + \frac{(1+m)x^{1+m}\sinh^4(a+b\log(cx^n))}{(1+m)^2-16b^2n^2}\end{aligned}$$

output
```
24*b^4*n^4*x^(1+m)/(1+m)/((1+m)^2-16*b^2*n^2)/((1+m)^2-4*b^2*n^2)-24*b^3*n
^3*x^(1+m)*cosh(a+b*ln(c*x^n))*sinh(a+b*ln(c*x^n))/((1+m)^2-16*b^2*n^2)/((
1+m)^2-4*b^2*n^2)+12*b^2*(1+m)*n^2*x^(1+m)*sinh(a+b*ln(c*x^n))^2/((1+m)^2-
16*b^2*n^2)/((1+m)^2-4*b^2*n^2)-4*b*n*x^(1+m)*cosh(a+b*ln(c*x^n))*sinh(a+b
*ln(c*x^n))^3/((1+m)^2-16*b^2*n^2)+(1+m)*x^(1+m)*sinh(a+b*ln(c*x^n))^4/((1
+m)^2-16*b^2*n^2)
```

Mathematica [A] (verified)

Time = 2.35 (sec) , antiderivative size = 311, normalized size of antiderivative = 1.17

$$\int x^m \sinh^4(a+b\log(cx^n))\,dx = \frac{1}{8}x^{1+m}\Bigg(\frac{3}{1+m}$$
$$-\frac{4\sinh(2bn\log(x))(-2bn\cosh(2(a-bn\log(x)+b\log(cx^n)))+(1+m)\sinh(2(a-bn\log(x)+b\log}{(1+m-2bn)(1+m+2bn)}$$
$$-\frac{4\cosh(2bn\log(x))((1+m)\cosh(2(a-bn\log(x)+b\log(cx^n)))-2bn\sinh(2(a-bn\log(x)+b\log(c}{(1+m-2bn)(1+m+2bn)}$$
$$+\frac{\sinh(4bn\log(x))(-4bn\cosh(4(a-bn\log(x)+b\log(cx^n)))+(1+m)\sinh(4(a-bn\log(x)+b\log(c}{(1+m-4bn)(1+m+4bn)}$$
$$+\frac{\cosh(4bn\log(x))((1+m)\cosh(4(a-bn\log(x)+b\log(cx^n)))-4bn\sinh(4(a-bn\log(x)+b\log(cx}{(1+m-4bn)(1+m+4bn)}$$

input
```
Integrate[x^m*Sinh[a + b*Log[c*x^n]]^4,x]
```

output
```
(x^(1 + m)*(3/(1 + m) - (4*Sinh[2*b*n*Log[x]]*(-2*b*n*Cosh[2*(a - b*n*Log[
x] + b*Log[c*x^n])] + (1 + m)*Sinh[2*(a - b*n*Log[x] + b*Log[c*x^n])]))/((
1 + m - 2*b*n)*(1 + m + 2*b*n)) - (4*Cosh[2*b*n*Log[x]]*((1 + m)*Cosh[2*(a
 - b*n*Log[x] + b*Log[c*x^n])] - 2*b*n*Sinh[2*(a - b*n*Log[x] + b*Log[c*x^
n])]))/((1 + m - 2*b*n)*(1 + m + 2*b*n)) + (Sinh[4*b*n*Log[x]]*(-4*b*n*Cos
h[4*(a - b*n*Log[x] + b*Log[c*x^n])] + (1 + m)*Sinh[4*(a - b*n*Log[x] + b*
Log[c*x^n])]))/((1 + m - 4*b*n)*(1 + m + 4*b*n)) + (Cosh[4*b*n*Log[x]]*((1
 + m)*Cosh[4*(a - b*n*Log[x] + b*Log[c*x^n])] - 4*b*n*Sinh[4*(a - b*n*Log[
x] + b*Log[c*x^n])]))/((1 + m - 4*b*n)*(1 + m + 4*b*n))))/8
```

Rubi [A] (verified)

Time = 0.47 (sec) , antiderivative size = 236, normalized size of antiderivative = 0.89, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.176$, Rules used = {6055, 6055, 15}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^m \sinh^4(a+b\log(cx^n))\,dx$$

$$\Big\downarrow \; 6055$$

$$\frac{12b^2n^2\int x^m\sinh^2(a+b\log(cx^n))\,dx}{(m+1)^2-16b^2n^2}+\frac{(m+1)x^{m+1}\sinh^4(a+b\log(cx^n))}{-16b^2n^2+m^2+2m+1}-\frac{4bnx^{m+1}\sinh^3(a+b\log(cx^n))\cosh(a+b\log(cx^n))}{(m+1)^2-16b^2n^2}$$

$$\Big\downarrow \; 6055$$

$$\frac{12b^2n^2\left(\frac{2b^2n^2\int x^m dx}{(m+1)^2-4b^2n^2}+\frac{(m+1)x^{m+1}\sinh^2(a+b\log(cx^n))}{-4b^2n^2+m^2+2m+1}-\frac{2bnx^{m+1}\sinh(a+b\log(cx^n))\cosh(a+b\log(cx^n))}{(m+1)^2-4b^2n^2}\right)}{(m+1)^2-16b^2n^2}+\frac{(m+1)x^{m+1}\sinh^4(a+b\log(cx^n))}{-16b^2n^2+m^2+2m+1}-\frac{4bnx^{m+1}\sinh^3(a+b\log(cx^n))\cosh(a+b\log(cx^n))}{(m+1)^2-16b^2n^2}$$

$$\Big\downarrow \; 15$$

$$\frac{(m+1)x^{m+1}\sinh^4(a+b\log(cx^n))}{-16b^2n^2+m^2+2m+1}+\frac{12b^2n^2\left(\frac{(m+1)x^{m+1}\sinh^2(a+b\log(cx^n))}{-4b^2n^2+m^2+2m+1}-\frac{2bnx^{m+1}\sinh(a+b\log(cx^n))\cosh(a+b\log(cx^n))}{(m+1)^2-4b^2n^2}+\frac{2b^2n^2x^{m+1}}{(m+1)((m+1)^2-4b^2n^2)}\right)}{(m+1)^2-16b^2n^2}-\frac{4bnx^{m+1}\sinh^3(a+b\log(cx^n))\cosh(a+b\log(cx^n))}{(m+1)^2-16b^2n^2}$$

input
```
Int[x^m*Sinh[a + b*Log[c*x^n]]^4,x]
```

output
```
(-4*b*n*x^(1 + m)*Cosh[a + b*Log[c*x^n]]*Sinh[a + b*Log[c*x^n]]^3)/((1 + m
)^2 - 16*b^2*n^2) + ((1 + m)*x^(1 + m)*Sinh[a + b*Log[c*x^n]]^4)/(1 + 2*m
+ m^2 - 16*b^2*n^2) + (12*b^2*n^2*((2*b^2*n^2*x^(1 + m))/((1 + m)*((1 + m)
^2 - 4*b^2*n^2)) - (2*b*n*x^(1 + m)*Cosh[a + b*Log[c*x^n]]*Sinh[a + b*Log[
c*x^n]])/((1 + m)^2 - 4*b^2*n^2) + ((1 + m)*x^(1 + m)*Sinh[a + b*Log[c*x^n
]]^2)/(1 + 2*m + m^2 - 4*b^2*n^2)))/((1 + m)^2 - 16*b^2*n^2)
```

Defintions of rubi rules used

rule 15
```
Int[(a_.)*(x_)^(m_.), x_Symbol] :> Simp[a*(x^(m + 1)/(m + 1)), x] /; FreeQ[
{a, m}, x] && NeQ[m, -1]
```

rule 6055
```
Int[((e_.)*(x_))^(m_.)*Sinh[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(d_.)]^(p
_), x_Symbol] :> Simp[(-(m + 1))*(e*x)^(m + 1)*(Sinh[d*(a + b*Log[c*x^n])]^
p/(b^2*d^2*e*n^2*p^2 - e*(m + 1)^2)), x] + (Simp[b*d*n*p*(e*x)^(m + 1)*Cosh
[d*(a + b*Log[c*x^n])]*(Sinh[d*(a + b*Log[c*x^n])]^(p - 1)/(b^2*d^2*e*n^2*p
^2 - e*(m + 1)^2)), x] - Simp[b^2*d^2*n^2*p*((p - 1)/(b^2*d^2*n^2*p^2 - (m
+ 1)^2))   Int[(e*x)^m*Sinh[d*(a + b*Log[c*x^n])]^(p - 2), x], x]) /; FreeQ
[{a, b, c, d, e, m, n}, x] && IGtQ[p, 1] && NeQ[b^2*d^2*n^2*p^2 - (m + 1)^2
, 0]
```

Maple [F]

$$\int x^m \sinh\left(a+b\ln\left(c\,x^n\right)\right)^4 dx$$

input
```
int(x^m*sinh(a+b*ln(c*x^n))^4,x)
```

output
```
int(x^m*sinh(a+b*ln(c*x^n))^4,x)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1125 vs. 2(283) = 566.

Time = 0.12 (sec) , antiderivative size = 1125, normalized size of antiderivative = 4.23

$$\int x^m \sinh^4\left(a+b\log\left(cx^n\right)\right)\,dx = \text{Too large to display}$$

input
```
integrate(x^m*sinh(a+b*log(c*x^n))^4,x, algorithm="fricas")
```

output
```
1/8*((m^4 + 4*m^3 - 4*(b^2*m^2 + 2*b^2*m + b^2)*n^2 + 6*m^2 + 4*m + 1)*x*c
osh(b*n*log(x) + b*log(c) + a)^4*cosh(m*log(x)) - 4*(m^4 + 4*m^3 - 16*(b^2
*m^2 + 2*b^2*m + b^2)*n^2 + 6*m^2 + 4*m + 1)*x*cosh(b*n*log(x) + b*log(c)
+ a)^2*cosh(m*log(x)) + ((m^4 + 4*m^3 - 4*(b^2*m^2 + 2*b^2*m + b^2)*n^2 +
6*m^2 + 4*m + 1)*x*cosh(m*log(x)) + (m^4 + 4*m^3 - 4*(b^2*m^2 + 2*b^2*m +
b^2)*n^2 + 6*m^2 + 4*m + 1)*x*sinh(m*log(x)))*sinh(b*n*log(x) + b*log(c) +
 a)^4 + 16*((4*(b^3*m + b^3)*n^3 - (b*m^3 + 3*b*m^2 + 3*b*m + b)*n)*x*cosh
(b*n*log(x) + b*log(c) + a)*cosh(m*log(x)) + (4*(b^3*m + b^3)*n^3 - (b*m^3
 + 3*b*m^2 + 3*b*m + b)*n)*x*cosh(b*n*log(x) + b*log(c) + a)*sinh(m*log(x)
))*sinh(b*n*log(x) + b*log(c) + a)^3 + 3*(64*b^4*n^4 + m^4 + 4*m^3 - 20*(b
^2*m^2 + 2*b^2*m + b^2)*n^2 + 6*m^2 + 4*m + 1)*x*cosh(m*log(x)) + 2*(3*(m^
4 + 4*m^3 - 4*(b^2*m^2 + 2*b^2*m + b^2)*n^2 + 6*m^2 + 4*m + 1)*x*cosh(b*n*
log(x) + b*log(c) + a)^2*cosh(m*log(x)) - 2*(m^4 + 4*m^3 - 16*(b^2*m^2 + 2
*b^2*m + b^2)*n^2 + 6*m^2 + 4*m + 1)*x*cosh(m*log(x)) + (3*(m^4 + 4*m^3 -
4*(b^2*m^2 + 2*b^2*m + b^2)*n^2 + 6*m^2 + 4*m + 1)*x*cosh(b*n*log(x) + b*l
og(c) + a)^2 - 2*(m^4 + 4*m^3 - 16*(b^2*m^2 + 2*b^2*m + b^2)*n^2 + 6*m^2 +
 4*m + 1)*x)*sinh(m*log(x)))*sinh(b*n*log(x) + b*log(c) + a)^2 + 16*((4*(b
^3*m + b^3)*n^3 - (b*m^3 + 3*b*m^2 + 3*b*m + b)*n)*x*cosh(b*n*log(x) + b*l
og(c) + a)^3*cosh(m*log(x)) - (16*(b^3*m + b^3)*n^3 - (b*m^3 + 3*b*m^2 + 3
*b*m + b)*n)*x*cosh(b*n*log(x) + b*log(c) + a)*cosh(m*log(x)) + ((4*(b^...
```

Sympy [F(-1)]

Timed out.

$$\int x^m \sinh^4(a + b\log(cx^n)) \, dx = \text{Timed out}$$

input
```
integrate(x**m*sinh(a+b*ln(c*x**n))**4,x)
```

output
```
Timed out
```

Maxima [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 161, normalized size of antiderivative = 0.61

$$\int x^m \sinh^4(a+b\log(cx^n))\,dx = \frac{c^{4b}xe^{(4b\log(x^n)+m\log(x)+4a)}}{16(4bn+m+1)} - \frac{c^{2b}xe^{(2b\log(x^n)+m\log(x)+2a)}}{4(2bn+m+1)} + \frac{xe^{(-2b\log(x^n)+m\log(x)-2a)}}{4(2bc^{2b}n-c^{2b}(m+1))} - \frac{xe^{(-4b\log(x^n)+m\log(x)-4a)}}{16(4bc^{4b}n-c^{4b}(m+1))} + \frac{3x^{m+1}}{8(m+1)}$$

input
```
integrate(x^m*sinh(a+b*log(c*x^n))^4,x, algorithm="maxima")
```

output
```
1/16*c^(4*b)*x*e^(4*b*log(x^n) + m*log(x) + 4*a)/(4*b*n + m + 1) - 1/4*c^(
2*b)*x*e^(2*b*log(x^n) + m*log(x) + 2*a)/(2*b*n + m + 1) + 1/4*x*e^(-2*b*l
og(x^n) + m*log(x) - 2*a)/(2*b*c^(2*b)*n - c^(2*b)*(m + 1)) - 1/16*x*e^(-4
*b*log(x^n) + m*log(x) - 4*a)/(4*b*c^(4*b)*n - c^(4*b)*(m + 1)) + 3/8*x^(m
 + 1)/(m + 1)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 6884 vs. 2(283) = 566.

Time = 0.21 (sec) , antiderivative size = 6884, normalized size of antiderivative = 25.88

$$\int x^m \sinh^4(a+b\log(cx^n))\,dx = \text{Too large to display}$$

input
```
integrate(x^m*sinh(a+b*log(c*x^n))^4,x, algorithm="giac")
```

output

```
b^3*c^(4*b)*m*n^3*x*x^(4*b*n)*x^m*e^(4*a)/(64*b^4*m*n^4 + 64*b^4*n^4 - 20*
b^2*m^3*n^2 - 60*b^2*m^2*n^2 + m^5 - 60*b^2*m*n^2 + 5*m^4 - 20*b^2*n^2 + 1
0*m^3 + 10*m^2 + 5*m + 1) - 8*b^3*c^(2*b)*m*n^3*x*x^(2*b*n)*x^m*e^(2*a)/(6
4*b^4*m*n^4 + 64*b^4*n^4 - 20*b^2*m^3*n^2 - 60*b^2*m^2*n^2 + m^5 - 60*b^2*
m*n^2 + 5*m^4 - 20*b^2*n^2 + 10*m^3 + 10*m^2 + 5*m + 1) - 1/4*b^2*c^(4*b)*
m^2*n^2*x*x^(4*b*n)*x^m*e^(4*a)/(64*b^4*m*n^4 + 64*b^4*n^4 - 20*b^2*m^3*n^
2 - 60*b^2*m^2*n^2 + m^5 - 60*b^2*m*n^2 + 5*m^4 - 20*b^2*n^2 + 10*m^3 + 10
*m^2 + 5*m + 1) + b^3*c^(4*b)*n^3*x*x^(4*b*n)*x^m*e^(4*a)/(64*b^4*m*n^4 +
64*b^4*n^4 - 20*b^2*m^3*n^2 - 60*b^2*m^2*n^2 + m^5 - 60*b^2*m*n^2 + 5*m^4
- 20*b^2*n^2 + 10*m^3 + 10*m^2 + 5*m + 1) + 4*b^2*c^(2*b)*m^2*n^2*x*x^(2*b
*n)*x^m*e^(2*a)/(64*b^4*m*n^4 + 64*b^4*n^4 - 20*b^2*m^3*n^2 - 60*b^2*m^2*n
^2 + m^5 - 60*b^2*m*n^2 + 5*m^4 - 20*b^2*n^2 + 10*m^3 + 10*m^2 + 5*m + 1)
- 8*b^3*c^(2*b)*n^3*x*x^(2*b*n)*x^m*e^(2*a)/(64*b^4*m*n^4 + 64*b^4*n^4 - 2
0*b^2*m^3*n^2 - 60*b^2*m^2*n^2 + m^5 - 60*b^2*m*n^2 + 5*m^4 - 20*b^2*n^2 +
 10*m^3 + 10*m^2 + 5*m + 1) + 24*b^4*n^4*x*x^m/(64*b^4*m*n^4 + 64*b^4*n^4
- 20*b^2*m^3*n^2 - 60*b^2*m^2*n^2 + m^5 - 60*b^2*m*n^2 + 5*m^4 - 20*b^2*n^
2 + 10*m^3 + 10*m^2 + 5*m + 1) - 1/4*b*c^(4*b)*m^3*n*x*x^(4*b*n)*x^m*e^(4*
a)/(64*b^4*m*n^4 + 64*b^4*n^4 - 20*b^2*m^3*n^2 - 60*b^2*m^2*n^2 + m^5 - 60
*b^2*m*n^2 + 5*m^4 - 20*b^2*n^2 + 10*m^3 + 10*m^2 + 5*m + 1) - 1/2*b^2*c^(
4*b)*m*n^2*x*x^(4*b*n)*x^m*e^(4*a)/(64*b^4*m*n^4 + 64*b^4*n^4 - 20*b^2*...
```

Mupad [B] (verification not implemented)

Time = 1.72 (sec) , antiderivative size = 136, normalized size of antiderivative = 0.51

$$\int x^m \sinh^4(a+b\log(cx^n))\,dx = \frac{3\,x\,x^m}{8\,m+8} - \frac{x\,x^m\,\mathrm{e}^{-2\,a}}{(c\,x^n)^{2\,b}\,(4\,m-8\,b\,n+4)} - \frac{x\,x^m\,\mathrm{e}^{2\,a}\,(c\,x^n)^{2\,b}}{4\,m+8\,b\,n+4} + \frac{x\,x^m\,\mathrm{e}^{-4\,a}}{(c\,x^n)^{4\,b}\,(16\,m-64\,b\,n+16)} + \frac{x\,x^m\,\mathrm{e}^{4\,a}\,(c\,x^n)^{4\,b}}{16\,m+64\,b\,n+16}$$

input

```
int(x^m*sinh(a + b*log(c*x^n))^4,x)
```

output

```
(3*x*x^m)/(8*m + 8) - (x*x^m*exp(-2*a))/((c*x^n)^(2*b)*(4*m - 8*b*n + 4))
- (x*x^m*exp(2*a)*(c*x^n)^(2*b))/(4*m + 8*b*n + 4) + (x*x^m*exp(-4*a))/((c
*x^n)^(4*b)*(16*m - 64*b*n + 16)) + (x*x^m*exp(4*a)*(c*x^n)^(4*b))/(16*m +
 64*b*n + 16)
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 1329, normalized size of antiderivative = 5.00

$$\int x^m \sinh^4(a + b\log(cx^n))\,dx = \text{Too large to display}$$

input
```
int(x^m*sinh(a+b*log(c*x^n))^4,x)
```

output
```
(x**m*x*(16*x**(8*b*n)*e**(8*a)*c**(8*b)*b**3*m*n**3 + 16*x**(8*b*n)*e**(8
*a)*c**(8*b)*b**3*n**3 - 4*x**(8*b*n)*e**(8*a)*c**(8*b)*b**2*m**2*n**2 - 8
*x**(8*b*n)*e**(8*a)*c**(8*b)*b**2*m*n**2 - 4*x**(8*b*n)*e**(8*a)*c**(8*b)
*b**2*n**2 - 4*x**(8*b*n)*e**(8*a)*c**(8*b)*b*m**3*n - 12*x**(8*b*n)*e**(8
*a)*c**(8*b)*b*m**2*n - 12*x**(8*b*n)*e**(8*a)*c**(8*b)*b*m*n - 4*x**(8*b*
n)*e**(8*a)*c**(8*b)*b*n + x**(8*b*n)*e**(8*a)*c**(8*b)*m**4 + 4*x**(8*b*n
)*e**(8*a)*c**(8*b)*m**3 + 6*x**(8*b*n)*e**(8*a)*c**(8*b)*m**2 + 4*x**(8*b
*n)*e**(8*a)*c**(8*b)*m + x**(8*b*n)*e**(8*a)*c**(8*b) - 128*x**(6*b*n)*e*
*(6*a)*c**(6*b)*b**3*m*n**3 - 128*x**(6*b*n)*e**(6*a)*c**(6*b)*b**3*n**3 +
 64*x**(6*b*n)*e**(6*a)*c**(6*b)*b**2*m**2*n**2 + 128*x**(6*b*n)*e**(6*a)*
c**(6*b)*b**2*m*n**2 + 64*x**(6*b*n)*e**(6*a)*c**(6*b)*b**2*n**2 + 8*x**(6
*b*n)*e**(6*a)*c**(6*b)*b*m**3*n + 24*x**(6*b*n)*e**(6*a)*c**(6*b)*b*m**2*
n + 24*x**(6*b*n)*e**(6*a)*c**(6*b)*b*m*n + 8*x**(6*b*n)*e**(6*a)*c**(6*b)
*b*n - 4*x**(6*b*n)*e**(6*a)*c**(6*b)*m**4 - 16*x**(6*b*n)*e**(6*a)*c**(6*
b)*m**3 - 24*x**(6*b*n)*e**(6*a)*c**(6*b)*m**2 - 16*x**(6*b*n)*e**(6*a)*c*
*(6*b)*m - 4*x**(6*b*n)*e**(6*a)*c**(6*b) + 384*x**(4*b*n)*e**(4*a)*c**(4*
b)*b**4*n**4 - 120*x**(4*b*n)*e**(4*a)*c**(4*b)*b**2*m**2*n**2 - 240*x**(4
*b*n)*e**(4*a)*c**(4*b)*b**2*m*n**2 - 120*x**(4*b*n)*e**(4*a)*c**(4*b)*b**
2*n**2 + 6*x**(4*b*n)*e**(4*a)*c**(4*b)*m**4 + 24*x**(4*b*n)*e**(4*a)*c**(
4*b)*m**3 + 36*x**(4*b*n)*e**(4*a)*c**(4*b)*m**2 + 24*x**(4*b*n)*e**(4*...
```

3.274 $\int \frac{\sinh(a+b\log(cx^n))}{x} dx$

Optimal result

Integrand size = 15, antiderivative size = 18

$$\int \frac{\sinh(a+b\log(cx^n))}{x} dx = \frac{\cosh(a+b\log(cx^n))}{bn}$$

output
```
cosh(a+b*ln(c*x^n))/b/n
```

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 37 vs. 2(18) = 36.

Time = 0.01 (sec) , antiderivative size = 37, normalized size of antiderivative = 2.06

$$\int \frac{\sinh(a+b\log(cx^n))}{x} dx = \frac{\cosh(a)\cosh(b\log(cx^n))}{bn} + \frac{\sinh(a)\sinh(b\log(cx^n))}{bn}$$

input
```
Integrate[Sinh[a + b*Log[c*x^n]]/x,x]
```

output
```
(Cosh[a]*Cosh[b*Log[c*x^n]])/(b*n) + (Sinh[a]*Sinh[b*Log[c*x^n]])/(b*n)
```

Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.267, Rules used = {3039, 3042, 26, 3118}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sinh (a+b \log (c x^n))}{x} \, dx$$
$$\Big\downarrow \text{ 3039}$$
$$\frac{\int \sinh (a+b \log (c x^n))d\log (c x^n)}{n}$$
$$\Big\downarrow \text{ 3042}$$
$$\frac{\int -i \sin (i a+i b \log (c x^n))d\log (c x^n)}{n}$$
$$\Big\downarrow \text{ 26}$$
$$-\frac{i \int \sin (i a+i b \log (c x^n))d\log (c x^n)}{n}$$
$$\Big\downarrow \text{ 3118}$$
$$\frac{\cosh (a+b \log (c x^n))}{b n}$$

input
```
Int[Sinh[a + b*Log[c*x^n]]/x,x]
```

output
```
Cosh[a + b*Log[c*x^n]]/(b*n)
```

Defintions of rubi rules used

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 3039
```
Int[u_, x_Symbol] :> With[{lst = FunctionOfLog[Cancel[x*u], x]}, Simp[1/lst
[[3]]   Subst[Int[lst[[1]], x], x, Log[lst[[2]]]], x] /;  !FalseQ[lst]] /;
NonsumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3118
```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Maple [A] (verified)

Time = 0.45 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.06

method	result	size
derivativedivides	$\frac{\cosh(a+b\ln(c\,x^{n}))}{bn}$	19
default	$\frac{\cosh(a+b\ln(c\,x^{n}))}{bn}$	19
parallelrisch	$\frac{\cosh\left(a+2b\ln\left(\sqrt{c\,x^{n}}\right)\right)+1}{bn}$	24

input
```
int(sinh(a+b*ln(c*x^n))/x,x,method=_RETURNVERBOSE)
```

output
```
cosh(a+b*ln(c*x^n))/b/n
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.06

$$\int \frac{\sinh(a + b\log(cx^n))}{x}\, dx = \frac{\cosh(bn\log(x) + b\log(c) + a)}{bn}$$

input
```
integrate(sinh(a+b*log(c*x^n))/x,x, algorithm="fricas")
```

output
```
cosh(b*n*log(x) + b*log(c) + a)/(b*n)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 34 vs. 2(14) = 28.

Time = 0.25 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.89

$$\int \frac{\sinh(a + b\log(cx^n))}{x}\, dx = \begin{cases} \log(x)\sinh(a) & \text{for } b = 0 \wedge (b = 0 \vee n = 0) \\ \log(x)\sinh(a + b\log(c)) & \text{for } n = 0 \\ \frac{\cosh(a+b\log(cx^n))}{bn} & \text{otherwise} \end{cases}$$

input
```
integrate(sinh(a+b*ln(c*x**n))/x,x)
```

output
```
Piecewise((log(x)*sinh(a), Eq(b, 0) & (Eq(b, 0) | Eq(n, 0))), (log(x)*sinh
(a + b*log(c)), Eq(n, 0)), (cosh(a + b*log(c*x**n))/(b*n), True))
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00

$$\int \frac{\sinh(a + b\log(cx^n))}{x}\, dx = \frac{\cosh(b\log(cx^n) + a)}{bn}$$

input
```
integrate(sinh(a+b*log(c*x^n))/x,x, algorithm="maxima")
```

output
```
cosh(b*log(c*x^n) + a)/(b*n)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 40 vs. 2(18) = 36.

Time = 0.12 (sec) , antiderivative size = 40, normalized size of antiderivative = 2.22

$$\int \frac{\sinh (a + b \log (cx^n))}{x} \, dx = \frac{\left(c^{2\,b} x^{bn} e^{(2\,a)} + \frac{1}{x^{bn}}\right) e^{(-a)}}{2\,bc^b n}$$

input
```
integrate(sinh(a+b*log(c*x^n))/x,x, algorithm="giac")
```

output
```
1/2*(c^(2*b)*x^(b*n)*e^(2*a) + 1/x^(b*n))*e^(-a)/(b*c^b*n)
```

Mupad [B] (verification not implemented)

Time = 1.52 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00

$$\int \frac{\sinh (a + b \log (cx^n))}{x} \, dx = \frac{\cosh(a + b\, \ln (c\, x^n))}{b\, n}$$

input
```
int(sinh(a + b*log(c*x^n))/x,x)
```

output
```
cosh(a + b*log(c*x^n))/(b*n)
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00

$$\int \frac{\sinh (a + b \log (cx^n))}{x} \, dx = \frac{\cosh (\log(x^n c)\, b + a)}{bn}$$

input
```
int(sinh(a+b*log(c*x^n))/x,x)
```

output
```
cosh(log(x**n*c)*b + a)/(b*n)
```

3.275 $\int \frac{\sinh^2(a+b\log(cx^n))}{x}\,dx$

Optimal result

Integrand size = 17, antiderivative size = 39

$$\int \frac{\sinh^2(a+b\log(cx^n))}{x}\,dx = -\frac{\log(x)}{2} + \frac{\cosh(a+b\log(cx^n))\sinh(a+b\log(cx^n))}{2bn}$$

output
```
-1/2*ln(x)+1/2*cosh(a+b*ln(c*x^n))*sinh(a+b*ln(c*x^n))/b/n
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.92

$$\int \frac{\sinh^2(a+b\log(cx^n))}{x}\,dx = \frac{-2(a+b\log(cx^n))+\sinh(2(a+b\log(cx^n)))}{4bn}$$

input
```
Integrate[Sinh[a + b*Log[c*x^n]]^2/x,x]
```

output
```
(-2*(a + b*Log[c*x^n]) + Sinh[2*(a + b*Log[c*x^n])])/(4*b*n)
```

Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 44, normalized size of antiderivative = 1.13, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.294$, Rules used = {3039, 3042, 25, 3115, 24}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sinh^2(a+b\log(cx^n))}{x}dx$$
$$\Big\downarrow \text{3039}$$
$$\frac{\int \sinh^2(a+b\log(cx^n))d\log(cx^n)}{n}$$
$$\Big\downarrow \text{3042}$$
$$\frac{\int -\sin(ia+ib\log(cx^n))^2d\log(cx^n)}{n}$$
$$\Big\downarrow \text{25}$$
$$-\frac{\int \sin(ia+ib\log(cx^n))^2d\log(cx^n)}{n}$$
$$\Big\downarrow \text{3115}$$
$$\frac{\frac{\sinh(a+b\log(cx^n))\cosh(a+b\log(cx^n))}{2b}-\frac{1}{2}\int 1d\log(cx^n)}{n}$$
$$\Big\downarrow \text{24}$$
$$\frac{\frac{\sinh(a+b\log(cx^n))\cosh(a+b\log(cx^n))}{2b}-\frac{1}{2}\log(cx^n)}{n}$$

input
```
Int[Sinh[a + b*Log[c*x^n]]^2/x,x]
```

output
```
(-1/2*Log[c*x^n] + (Cosh[a + b*Log[c*x^n]]*Sinh[a + b*Log[c*x^n]])/(2*b))/
n
```

Defintions of rubi rules used

rule 24
```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 3039
```
Int[u_, x_Symbol] :> With[{lst = FunctionOfLog[Cancel[x*u], x]}, Simp[1/lst
[[3]]   Subst[Int[lst[[1]], x], x, Log[lst[[2]]]], x] /;  !FalseQ[lst]] /;
NonsumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3115
```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Simp[b^2*((n - 1)/n)   Int[(b*Sin
[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[
2*n]
```

Maple [A] (verified)

Time = 1.75 (sec) , antiderivative size = 30, normalized size of antiderivative = 0.77

method	result	size
parallelrisch	$\frac{-2\ln(x)bn+\sinh(2b\ln(c\,x^{n})+2a)}{4bn}$	30
derivativedivides	$\frac{\frac{\cosh(a+b\ln(c\,x^{n}))\sinh(a+b\ln(c\,x^{n}))}{2}-\frac{b\ln(c\,x^{n})}{2}-\frac{a}{2}}{nb}$	45
default	$\frac{\frac{\cosh(a+b\ln(c\,x^{n}))\sinh(a+b\ln(c\,x^{n}))}{2}-\frac{b\ln(c\,x^{n})}{2}-\frac{a}{2}}{nb}$	45

input
```
int(sinh(a+b*ln(c*x^n))^2/x,x,method=_RETURNVERBOSE)
```

output
```
1/4*(-2*ln(x)*b*n+sinh(2*b*ln(c*x^n)+2*a))/b/n
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 40, normalized size of antiderivative = 1.03

$$\int \frac{\sinh^2(a+b\log(cx^n))}{x}\,dx = -\frac{bn\log(x) - \cosh(bn\log(x)+b\log(c)+a)\sinh(bn\log(x)+b\log(c)+a)}{2\,bn}$$

input
```
integrate(sinh(a+b*log(c*x^n))^2/x,x, algorithm="fricas")
```

output
```
-1/2*(b*n*log(x) - cosh(b*n*log(x) + b*log(c) + a)*sinh(b*n*log(x) + b*log
(c) + a))/(b*n)
```

Sympy [F]

$$\int \frac{\sinh^2(a+b\log(cx^n))}{x}\,dx = \int \frac{\sinh^2(a+b\log(cx^n))}{x}\,dx$$

input
```
integrate(sinh(a+b*ln(c*x**n))**2/x,x)
```

output
```
Integral(sinh(a + b*log(c*x**n))**2/x, x)
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 49, normalized size of antiderivative = 1.26

$$\int \frac{\sinh^2(a+b\log(cx^n))}{x}\,dx = \frac{e^{(2\,b\log(cx^n)+2\,a)}}{8\,bn} - \frac{e^{(-2\,b\log(cx^n)-2\,a)}}{8\,bn} - \frac{1}{2}\log(x)$$

input
```
integrate(sinh(a+b*log(c*x^n))^2/x,x, algorithm="maxima")
```

output
```
1/8*e^(2*b*log(c*x^n) + 2*a)/(b*n) - 1/8*e^(-2*b*log(c*x^n) - 2*a)/(b*n) -
 1/2*log(x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 81 vs. $2(35) = 70$.

Time = 0.13 (sec) , antiderivative size = 81, normalized size of antiderivative = 2.08

$$\int \frac{\sinh^2(a + b\log(cx^n))}{x}\,dx = \frac{\left(c^{4\,b}x^{2\,bn}e^{(4\,a)} - 4\,c^{2\,b}e^{(2\,a)}\log\left(x^{bn}\right) + \frac{2\,c^{2\,b}x^{2\,bn}e^{(2\,a)}-1}{x^{2\,bn}}\right)e^{(-2\,a)}}{8\,bc^{2\,b}n}$$

input
```
integrate(sinh(a+b*log(c*x^n))^2/x,x, algorithm="giac")
```

output
```
1/8*(c^(4*b)*x^(2*b*n)*e^(4*a) - 4*c^(2*b)*e^(2*a)*log(x^(b*n)) + (2*c^(2*
b)*x^(2*b*n)*e^(2*a) - 1)/x^(2*b*n))*e^(-2*a)/(b*c^(2*b)*n)
```

Mupad [B] (verification not implemented)

Time = 1.56 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.82

$$\int \frac{\sinh^2(a + b\log(cx^n))}{x}\,dx = \frac{\sinh(2\,a + 2\,b\,\ln(c\,x^n))}{4\,b\,n} - \frac{\ln(x^n)}{2\,n}$$

input
```
int(sinh(a + b*log(c*x^n))^2/x,x)
```

output
```
sinh(2*a + 2*b*log(c*x^n))/(4*b*n) - log(x^n)/(2*n)
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 71, normalized size of antiderivative = 1.82

$$\int \frac{\sinh^2(a + b\log(cx^n))}{x}\,dx = \frac{x^{4bn}e^{4a}c^{4b} - 4x^{2bn}e^{2a}c^{2b}\log(x)\,bn - 1}{8x^{2bn}e^{2a}c^{2b}bn}$$

input
```
int(sinh(a+b*log(c*x^n))^2/x,x)
```

output
```
(x**(4*b*n)*e**(4*a)*c**(4*b) - 4*x**(2*b*n)*e**(2*a)*c**(2*b)*log(x)*b*n
- 1)/(8*x**(2*b*n)*e**(2*a)*c**(2*b)*b*n)
```

3.276 $\int \frac{\sinh^3(a+b\log(cx^n))}{x} dx$

Optimal result

Integrand size = 17, antiderivative size = 43

$$\int \frac{\sinh^3(a+b\log(cx^n))}{x} dx = -\frac{\cosh(a+b\log(cx^n))}{bn} + \frac{\cosh^3(a+b\log(cx^n))}{3bn}$$

output
```
-cosh(a+b*ln(c*x^n))/b/n+1/3*cosh(a+b*ln(c*x^n))^3/b/n
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 45, normalized size of antiderivative = 1.05

$$\int \frac{\sinh^3(a+b\log(cx^n))}{x} dx = -\frac{3\cosh(a+b\log(cx^n))}{4bn} + \frac{\cosh(3(a+b\log(cx^n)))}{12bn}$$

input
```
Integrate[Sinh[a + b*Log[c*x^n]]^3/x,x]
```

output
```
(-3*Cosh[a + b*Log[c*x^n]])/(4*b*n) + Cosh[3*(a + b*Log[c*x^n])]/(12*b*n)
```

Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.86, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.294, Rules used = {3039, 3042, 26, 3113, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sinh^3(a+b\log(cx^n))}{x}dx$$

$$\Big\downarrow \text{3039}$$

$$\frac{\int \sinh^3(a+b\log(cx^n))d\log(cx^n)}{n}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\int i\sin(ia+ib\log(cx^n))^3d\log(cx^n)}{n}$$

$$\Big\downarrow \text{26}$$

$$\frac{i\int \sin(ia+ib\log(cx^n))^3d\log(cx^n)}{n}$$

$$\Big\downarrow \text{3113}$$

$$-\frac{\int (1-\cosh^2(a+b\log(cx^n)))d\cosh(a+b\log(cx^n))}{bn}$$

$$\Big\downarrow \text{2009}$$

$$-\frac{\cosh(a+b\log(cx^n))-\frac{1}{3}\cosh^3(a+b\log(cx^n))}{bn}$$

input
```
Int[Sinh[a + b*Log[c*x^n]]^3/x,x]
```

output
```
-((Cosh[a + b*Log[c*x^n]] - Cosh[a + b*Log[c*x^n]]^3/3)/(b*n))
```

Defintions of rubi rules used

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3039
```
Int[u_, x_Symbol] :> With[{lst = FunctionOfLog[Cancel[x*u], x]}, Simp[1/lst
[[3]]   Subst[Int[lst[[1]], x], x, Log[lst[[2]]]], x] /;  !FalseQ[lst]] /;
NonsumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3113
```
Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Simp[-d^(-1)   Subst[Int[Exp
and[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x]
 && IGtQ[(n - 1)/2, 0]
```

Maple [A] (verified)

Time = 8.92 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.84

method	result	size
derivativedivides	$\frac{\left(-\frac{2}{3}+\frac{\sinh(a+b\ln(c\,x^{n}))^{2}}{3}\right)\cosh(a+b\ln(c\,x^{n}))}{nb}$	36
default	$\frac{\left(-\frac{2}{3}+\frac{\sinh(a+b\ln(c\,x^{n}))^{2}}{3}\right)\cosh(a+b\ln(c\,x^{n}))}{nb}$	36
parallelrisch	$\frac{-8+\cosh(3b\ln(c\,x^{n})+3a)-9\cosh(a+b\ln(c\,x^{n}))}{12bn}$	38

input
```
int(sinh(a+b*ln(c*x^n))^3/x,x,method=_RETURNVERBOSE)
```

output
```
1/n/b*(-2/3+1/3*sinh(a+b*ln(c*x^n))^2)*cosh(a+b*ln(c*x^n))
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 65, normalized size of antiderivative = 1.51

$$\int \frac{\sinh^3(a+b\log(cx^n))}{x}\,dx = \frac{\cosh(bn\log(x)+b\log(c)+a)^3+3\cosh(bn\log(x)+b\log(c)+a)\sinh(bn\log(x)+b\log(c)+a)^2-9}{12\,bn}$$

input
```
integrate(sinh(a+b*log(c*x^n))^3/x,x, algorithm="fricas")
```

output
```
1/12*(cosh(b*n*log(x) + b*log(c) + a)^3 + 3*cosh(b*n*log(x) + b*log(c) + a
)*sinh(b*n*log(x) + b*log(c) + a)^2 - 9*cosh(b*n*log(x) + b*log(c) + a))/(
b*n)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 71 vs. $2(32) = 64$.

Time = 1.41 (sec) , antiderivative size = 71, normalized size of antiderivative = 1.65

$$\int \frac{\sinh^3(a+b\log(cx^n))}{x}\,dx = \begin{cases} \log(x)\sinh^3(a) & \text{for } b=0 \wedge (b=0 \vee n=0) \\ \log(x)\sinh^3(a+b\log(c)) & \text{for } n=0 \\ \frac{\sinh^2(a+b\log(cx^n))\cosh(a+b\log(cx^n))}{bn} - \frac{2\cosh^3(a+b\log(cx^n))}{3bn} & \text{otherwise} \end{cases}$$

input
```
integrate(sinh(a+b*ln(c*x**n))**3/x,x)
```

output
```
Piecewise((log(x)*sinh(a)**3, Eq(b, 0) & (Eq(b, 0) | Eq(n, 0))), (log(x)*s
inh(a + b*log(c))**3, Eq(n, 0)), (sinh(a + b*log(c*x**n))**2*cosh(a + b*lo
g(c*x**n))/(b*n) - 2*cosh(a + b*log(c*x**n))**3/(3*b*n), True))
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 86 vs. $2(41) = 82$.

Time = 0.03 (sec) , antiderivative size = 86, normalized size of antiderivative = 2.00

$$\int \frac{\sinh^3(a+b\log(cx^n))}{x}\,dx = \frac{e^{(3\,b\log(cx^n)+3\,a)}}{24\,bn} - \frac{3\,e^{(b\log(cx^n)+a)}}{8\,bn} - \frac{3\,e^{(-b\log(cx^n)-a)}}{8\,bn} + \frac{e^{(-3\,b\log(cx^n)-3\,a)}}{24\,bn}$$

input
```
integrate(sinh(a+b*log(c*x^n))^3/x,x, algorithm="maxima")
```

output
```
1/24*e^(3*b*log(c*x^n) + 3*a)/(b*n) - 3/8*e^(b*log(c*x^n) + a)/(b*n) - 3/8
*e^(-b*log(c*x^n) - a)/(b*n) + 1/24*e^(-3*b*log(c*x^n) - 3*a)/(b*n)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 81, normalized size of antiderivative = 1.88

$$\int \frac{\sinh^3(a+b\log(cx^n))}{x}\,dx = \frac{\left(c^{6\,b}x^{3\,bn}e^{(6\,a)} - 9\,c^{4\,b}x^{bn}e^{(4\,a)} - \frac{9\,c^{2\,b}x^{2\,bn}e^{(2\,a)}-1}{x^{3\,bn}}\right)e^{(-3\,a)}}{24\,bc^{3\,b}n}$$

input
```
integrate(sinh(a+b*log(c*x^n))^3/x,x, algorithm="giac")
```

output
```
1/24*(c^(6*b)*x^(3*b*n)*e^(6*a) - 9*c^(4*b)*x^(b*n)*e^(4*a) - (9*c^(2*b)*x
^(2*b*n)*e^(2*a) - 1)/x^(3*b*n))*e^(-3*a)/(b*c^(3*b)*n)
```

Mupad [B] (verification not implemented)

Time = 1.58 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.86

$$\int \frac{\sinh^3(a+b\log(cx^n))}{x}\,dx = -\frac{3\cosh(a+b\,\ln(c\,x^n)) - \cosh(a+b\,\ln(c\,x^n))^3}{3\,b\,n}$$

input
```
int(sinh(a + b*log(c*x^n))^3/x,x)
```

output
```
-(3*cosh(a + b*log(c*x^n)) - cosh(a + b*log(c*x^n))^3)/(3*b*n)
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 85, normalized size of antiderivative = 1.98

$$\int \frac{\sinh^3(a+b\log(cx^n))}{x}\,dx = \frac{x^{6bn}e^{6a}c^{6b} - 9x^{4bn}e^{4a}c^{4b} - 9x^{2bn}e^{2a}c^{2b} + 1}{24x^{3bn}e^{3a}c^{3b}bn}$$

input
```
int(sinh(a+b*log(c*x^n))^3/x,x)
```

output
```
(x**(6*b*n)*e**(6*a)*c**(6*b) - 9*x**(4*b*n)*e**(4*a)*c**(4*b) - 9*x**(2*b
*n)*e**(2*a)*c**(2*b) + 1)/(24*x**(3*b*n)*e**(3*a)*c**(3*b)*b*n)
```

3.277 $\int \frac{\sinh^4(a+b\log(cx^n))}{x} dx$

Optimal result

Integrand size = 17, antiderivative size = 73

$$\int \frac{\sinh^4(a+b\log(cx^n))}{x}\,dx=\frac{3\log(x)}{8}-\frac{3\cosh(a+b\log(cx^n))\sinh(a+b\log(cx^n))}{8bn}+\frac{\cosh(a+b\log(cx^n))\sinh^3(a+b\log(cx^n))}{4bn}$$

output
```
3/8*ln(x)-3/8*cosh(a+b*ln(c*x^n))*sinh(a+b*ln(c*x^n))/b/n+1/4*cosh(a+b*ln(
c*x^n))*sinh(a+b*ln(c*x^n))^3/b/n
```

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.70

$$\int \frac{\sinh^4(a+b\log(cx^n))}{x}\,dx=\frac{12(a+b\log(cx^n))-8\sinh(2(a+b\log(cx^n)))+\sinh(4(a+b\log(cx^n)))}{32bn}$$

input
```
Integrate[Sinh[a + b*Log[c*x^n]]^4/x,x]
```

output
```
(12*(a + b*Log[c*x^n]) - 8*Sinh[2*(a + b*Log[c*x^n])] + Sinh[4*(a + b*Log[
c*x^n])])/(32*b*n)
```

Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 80, normalized size of antiderivative = 1.10, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.471$, Rules used = {3039, 3042, 3115, 25, 3042, 25, 3115, 24}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sinh^4(a+b\log(cx^n))}{x}dx$$

$$\Big\downarrow \text{3039}$$

$$\frac{\int \sinh^4(a+b\log(cx^n))d\log(cx^n)}{n}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\int \sin(ia+ib\log(cx^n))^4d\log(cx^n)}{n}$$

$$\Big\downarrow \text{3115}$$

$$\frac{\frac{3}{4}\int -\sinh^2(a+b\log(cx^n))d\log(cx^n)+\frac{\sinh^3(a+b\log(cx^n))\cosh(a+b\log(cx^n))}{4b}}{n}$$

$$\Big\downarrow \text{25}$$

$$\frac{\frac{\sinh^3(a+b\log(cx^n))\cosh(a+b\log(cx^n))}{4b}-\frac{3}{4}\int \sinh^2(a+b\log(cx^n))d\log(cx^n)}{n}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\frac{\sinh^3(a+b\log(cx^n))\cosh(a+b\log(cx^n))}{4b}-\frac{3}{4}\int -\sin(ia+ib\log(cx^n))^2d\log(cx^n)}{n}$$

$$\Big\downarrow \text{25}$$

$$\frac{\frac{\sinh^3(a+b\log(cx^n))\cosh(a+b\log(cx^n))}{4b}+\frac{3}{4}\int \sin(ia+ib\log(cx^n))^2d\log(cx^n)}{n}$$

$$\Big\downarrow \text{3115}$$

$$\frac{\frac{3}{4}\left(\frac{1}{2}\int 1d\log\left(cx^n\right)-\frac{\sinh(a+b\log(cx^n))\cosh(a+b\log(cx^n))}{2b}\right)+\frac{\sinh^3(a+b\log(cx^n))\cosh(a+b\log(cx^n))}{4b}}{n}$$

$$\Big\downarrow \; 24$$

$$\frac{\frac{\sinh^3(a+b\log(cx^n))\cosh(a+b\log(cx^n))}{4b}+\frac{3}{4}\left(\frac{1}{2}\log\left(cx^n\right)-\frac{\sinh(a+b\log(cx^n))\cosh(a+b\log(cx^n))}{2b}\right)}{n}$$

input
```
Int[Sinh[a + b*Log[c*x^n]]^4/x,x]
```

output
```
((Cosh[a + b*Log[c*x^n]]*Sinh[a + b*Log[c*x^n]]^3)/(4*b) + (3*(Log[c*x^n]/
2 - (Cosh[a + b*Log[c*x^n]]*Sinh[a + b*Log[c*x^n]])/(2*b)))/4)/n
```

Defintions of rubi rules used

rule 24
```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 3039
```
Int[u_, x_Symbol] :> With[{lst = FunctionOfLog[Cancel[x*u], x]}, Simp[1/lst
[[3]]   Subst[Int[lst[[1]], x], x, Log[lst[[2]]]], x] /;  !FalseQ[lst]] /;
NonsumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3115
```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Simp[b^2*((n - 1)/n)   Int[(b*Sin
[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[
2*n]
```

Maple [A] (verified)

Time = 30.22 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.63

method	result	size
parallelrisch	$\frac{-8\sinh(2b\ln(c\,x^{n})+2a)+\sinh(4b\ln(c\,x^{n})+4a)+12\ln(x)bn}{32bn}$	46
derivativedivides	$\frac{\left(\frac{\sinh(a+b\ln(c\,x^{n}))^{3}}{4}-\frac{3\sinh(a+b\ln(c\,x^{n}))}{8}\right)\cosh(a+b\ln(c\,x^{n}))+\frac{3b\ln(c\,x^{n})}{8}+\frac{3a}{8}}{nb}$	62
default	$\frac{\left(\frac{\sinh(a+b\ln(c\,x^{n}))^{3}}{4}-\frac{3\sinh(a+b\ln(c\,x^{n}))}{8}\right)\cosh(a+b\ln(c\,x^{n}))+\frac{3b\ln(c\,x^{n})}{8}+\frac{3a}{8}}{nb}$	62

input
```
int(sinh(a+b*ln(c*x^n))^4/x,x,method=_RETURNVERBOSE)
```

output
```
1/32*(-8*sinh(2*b*ln(c*x^n)+2*a)+sinh(4*b*ln(c*x^n)+4*a)+12*ln(x)*b*n)/b/n
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 84, normalized size of antiderivative = 1.15

$$\int \frac{\sinh^{4}(a+b\log(cx^{n}))}{x}\,dx = \frac{\cosh(bn\log(x)+b\log(c)+a)\sinh(bn\log(x)+b\log(c)+a)^{3}+3\,bn\log(x)+\big(\cosh(bn\log(x)+b\log}{8\,bn}$$

input
```
integrate(sinh(a+b*log(c*x^n))^4/x,x, algorithm="fricas")
```

output
```
1/8*(cosh(b*n*log(x) + b*log(c) + a)*sinh(b*n*log(x) + b*log(c) + a)^3 + 3
*b*n*log(x) + (cosh(b*n*log(x) + b*log(c) + a)^3 - 4*cosh(b*n*log(x) + b*l
og(c) + a))*sinh(b*n*log(x) + b*log(c) + a))/(b*n)
```

Sympy [F]

$$\int \frac{\sinh^4(a + b\log(cx^n))}{x}\,dx = \int \frac{\sinh^4(a + b\log(cx^n))}{x}\,dx$$

input
```
integrate(sinh(a+b*ln(c*x**n))**4/x,x)
```

output
```
Integral(sinh(a + b*log(c*x**n))**4/x, x)
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 93, normalized size of antiderivative = 1.27

$$\int \frac{\sinh^4(a + b\log(cx^n))}{x}\,dx = \frac{e^{(4\,b\log(cx^n)+4\,a)}}{64\,bn} - \frac{e^{(2\,b\log(cx^n)+2\,a)}}{8\,bn} + \frac{e^{(-2\,b\log(cx^n)-2\,a)}}{8\,bn} - \frac{e^{(-4\,b\log(cx^n)-4\,a)}}{64\,bn} + \frac{3}{8}\log(x)$$

input
```
integrate(sinh(a+b*log(c*x^n))^4/x,x, algorithm="maxima")
```

output
```
1/64*e^(4*b*log(c*x^n) + 4*a)/(b*n) - 1/8*e^(2*b*log(c*x^n) + 2*a)/(b*n) +
 1/8*e^(-2*b*log(c*x^n) - 2*a)/(b*n) - 1/64*e^(-4*b*log(c*x^n) - 4*a)/(b*n
) + 3/8*log(x)
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 116, normalized size of antiderivative = 1.59

$$\int \frac{\sinh^4(a + b\log(cx^n))}{x}\,dx = \frac{\left(c^{8\,b}x^{4\,bn}e^{(8\,a)} - 8\,c^{6\,b}x^{2\,bn}e^{(6\,a)} + 24\,c^{4\,b}e^{(4\,a)}\log\left(x^{bn}\right) - \frac{18\,c^{4\,b}x^{4\,bn}e^{(4\,a)} - 8\,c^{2\,b}x^{2\,bn}e^{(2\,a)} + 1}{x^{4\,bn}}\right)e^{(-4\,a)}}{64\,bc^{4\,b}n}$$

input
```
integrate(sinh(a+b*log(c*x^n))^4/x,x, algorithm="giac")
```

output
```
1/64*(c^(8*b)*x^(4*b*n)*e^(8*a) - 8*c^(6*b)*x^(2*b*n)*e^(6*a) + 24*c^(4*b)
*e^(4*a)*log(x^(b*n)) - (18*c^(4*b)*x^(4*b*n)*e^(4*a) - 8*c^(2*b)*x^(2*b*n
)*e^(2*a) + 1)/x^(4*b*n))*e^(-4*a)/(b*c^(4*b)*n)
```

Mupad [B] (verification not implemented)

Time = 1.65 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.70

$$\int \frac{\sinh^4(a+b\log(cx^n))}{x}\,dx = \frac{3\,\ln(x^n)}{8\,n} - \frac{\frac{\sinh(2\,a+2\,b\,\ln(c\,x^n))}{4} - \frac{\sinh(4\,a+4\,b\,\ln(c\,x^n))}{32}}{b\,n}$$

input
```
int(sinh(a + b*log(c*x^n))^4/x,x)
```

output
```
(3*log(x^n))/(8*n) - (sinh(2*a + 2*b*log(c*x^n))/4 - sinh(4*a + 4*b*log(c*
x^n))/32)/(b*n)
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 107, normalized size of antiderivative = 1.47

$$\int \frac{\sinh^4(a+b\log(cx^n))}{x}\,dx = \frac{x^{8bn}e^{8a}c^{8b} - 8x^{6bn}e^{6a}c^{6b} + 24x^{4bn}e^{4a}c^{4b}\log(x)\,bn + 8x^{2bn}e^{2a}c^{2b} - 1}{64x^{4bn}e^{4a}c^{4b}bn}$$

input
```
int(sinh(a+b*log(c*x^n))^4/x,x)
```

output
```
(x**(8*b*n)*e**(8*a)*c**(8*b) - 8*x**(6*b*n)*e**(6*a)*c**(6*b) + 24*x**(4*
b*n)*e**(4*a)*c**(4*b)*log(x)*b*n + 8*x**(2*b*n)*e**(2*a)*c**(2*b) - 1)/(6
4*x**(4*b*n)*e**(4*a)*c**(4*b)*b*n)
```

3.278 $\int \frac{\sinh^5(a+b\log(cx^n))}{x}\,dx$

Optimal result

Integrand size = 17, antiderivative size = 65

$$\int \frac{\sinh^5(a+b\log(cx^n))}{x}\,dx = \frac{\cosh(a+b\log(cx^n))}{bn} - \frac{2\cosh^3(a+b\log(cx^n))}{3bn} + \frac{\cosh^5(a+b\log(cx^n))}{5bn}$$

output
```
cosh(a+b*ln(c*x^n))/b/n-2/3*cosh(a+b*ln(c*x^n))^3/b/n+1/5*cosh(a+b*ln(c*x^
n))^5/b/n
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 68, normalized size of antiderivative = 1.05

$$\int \frac{\sinh^5(a+b\log(cx^n))}{x}\,dx = \frac{5\cosh(a+b\log(cx^n))}{8bn} - \frac{5\cosh(3(a+b\log(cx^n)))}{48bn} + \frac{\cosh(5(a+b\log(cx^n)))}{80bn}$$

input
```
Integrate[Sinh[a + b*Log[c*x^n]]^5/x,x]
```

output
```
(5*Cosh[a + b*Log[c*x^n]])/(8*b*n) - (5*Cosh[3*(a + b*Log[c*x^n])])/(48*b*
n) + Cosh[5*(a + b*Log[c*x^n])]/(80*b*n)
```

Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.82, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.294$, Rules used = {3039, 3042, 26, 3113, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sinh^5(a+b\log(cx^n))}{x}dx$$

$$\Big\downarrow \; 3039$$

$$\frac{\int \sinh^5(a+b\log(cx^n))d\log(cx^n)}{n}$$

$$\Big\downarrow \; 3042$$

$$\frac{\int -i\sin(ia+ib\log(cx^n))^5d\log(cx^n)}{n}$$

$$\Big\downarrow \; 26$$

$$-\frac{i\int \sin(ia+ib\log(cx^n))^5d\log(cx^n)}{n}$$

$$\Big\downarrow \; 3113$$

$$\frac{\int (\cosh^4(a+b\log(cx^n))-2\cosh^2(a+b\log(cx^n))+1)d\cosh(a+b\log(cx^n))}{bn}$$

$$\Big\downarrow \; 2009$$

$$\frac{\frac{1}{5}\cosh^5(a+b\log(cx^n))-\frac{2}{3}\cosh^3(a+b\log(cx^n))+\cosh(a+b\log(cx^n))}{bn}$$

input
```
Int[Sinh[a + b*Log[c*x^n]]^5/x,x]
```

output
```
(Cosh[a + b*Log[c*x^n]] - (2*Cosh[a + b*Log[c*x^n]]^3)/3 + Cosh[a + b*Log[
c*x^n]]^5/5)/(b*n)
```

Defintions of rubi rules used

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3039
```
Int[u_, x_Symbol] :> With[{lst = FunctionOfLog[Cancel[x*u], x]}, Simp[1/lst
[[3]]   Subst[Int[lst[[1]], x], x, Log[lst[[2]]]], x] /;  !FalseQ[lst]] /;
NonsumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3113
```
Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Simp[-d^(-1)   Subst[Int[Exp
and[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x]
 && IGtQ[(n - 1)/2, 0]
```

Maple [A] (verified)

Time = 74.14 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.78

method	result	size
derivativedivides	$\frac{\left(\frac{8}{15}+\frac{\sinh(a+b\ln(c\,x^{n}))^{4}}{5}-\frac{4\sinh(a+b\ln(c\,x^{n}))^{2}}{15}\right)\cosh(a+b\ln(c\,x^{n}))}{nb}$	51
default	$\frac{\left(\frac{8}{15}+\frac{\sinh(a+b\ln(c\,x^{n}))^{4}}{5}-\frac{4\sinh(a+b\ln(c\,x^{n}))^{2}}{15}\right)\cosh(a+b\ln(c\,x^{n}))}{nb}$	51
parallelrisch	$\frac{128-25\cosh(3b\ln(c\,x^{n})+3a)+150\cosh(a+b\ln(c\,x^{n}))+3\cosh(5b\ln(c\,x^{n})+5a)}{240bn}$	56

input
```
int(sinh(a+b*ln(c*x^n))^5/x,x,method=_RETURNVERBOSE)
```

output
```
1/n/b*(8/15+1/5*sinh(a+b*ln(c*x^n))^4-4/15*sinh(a+b*ln(c*x^n))^2)*cosh(a+b
*ln(c*x^n))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 130 vs. $2(61) = 122$.

Time = 0.08 (sec) , antiderivative size = 130, normalized size of antiderivative = 2.00

$$\int \frac{\sinh^5(a+b\log(cx^n))}{x}\,dx = \frac{3\cosh(bn\log(x)+b\log(c)+a)^5+15\cosh(bn\log(x)+b\log(c)+a)\sinh(bn\log(x)+b\log(c)+a)^4}{}$$

input
```
integrate(sinh(a+b*log(c*x^n))^5/x,x, algorithm="fricas")
```

output
```
1/240*(3*cosh(b*n*log(x) + b*log(c) + a)^5 + 15*cosh(b*n*log(x) + b*log(c)
 + a)*sinh(b*n*log(x) + b*log(c) + a)^4 - 25*cosh(b*n*log(x) + b*log(c) +
a)^3 + 15*(2*cosh(b*n*log(x) + b*log(c) + a)^3 - 5*cosh(b*n*log(x) + b*log
(c) + a))*sinh(b*n*log(x) + b*log(c) + a)^2 + 150*cosh(b*n*log(x) + b*log(
c) + a))/(b*n)
```

Sympy [A] (verification not implemented)

Time = 9.59 (sec) , antiderivative size = 105, normalized size of antiderivative = 1.62

$$\int \frac{\sinh^5(a+b\log(cx^n))}{x}\,dx = \begin{cases} \log(x)\sinh^5(a) & \text{for } b=0 \wedge (b \\ \log(x)\sinh^5(a+b\log(c)) & \text{for } n=0 \\ \frac{\sinh^4(a+b\log(cx^n))\cosh(a+b\log(cx^n))}{bn} - \frac{4\sinh^2(a+b\log(cx^n))\cosh^3(a+b\log(cx^n))}{3bn} + \frac{8\cosh^5(a+b\log(cx^n))}{15bn} & \text{otherwise} \end{cases}$$

input
```
integrate(sinh(a+b*ln(c*x**n))**5/x,x)
```

output
```
Piecewise((log(x)*sinh(a)**5, Eq(b, 0) & (Eq(b, 0) | Eq(n, 0))), (log(x)*s
inh(a + b*log(c))**5, Eq(n, 0)), (sinh(a + b*log(c*x**n))**4*cosh(a + b*lo
g(c*x**n))/(b*n) - 4*sinh(a + b*log(c*x**n))**2*cosh(a + b*log(c*x**n))**3
/(3*b*n) + 8*cosh(a + b*log(c*x**n))**5/(15*b*n), True))
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 130 vs. $2(61) = 122$.

Time = 0.04 (sec) , antiderivative size = 130, normalized size of antiderivative = 2.00

$$\int \frac{\sinh^5(a+b\log(cx^n))}{x}\,dx = \frac{e^{(5\,b\log(cx^n)+5\,a)}}{160\,bn} - \frac{5\,e^{(3\,b\log(cx^n)+3\,a)}}{96\,bn} + \frac{5\,e^{(b\log(cx^n)+a)}}{16\,bn} + \frac{5\,e^{(-b\log(cx^n)-a)}}{16\,bn} - \frac{5\,e^{(-3\,b\log(cx^n)-3\,a)}}{96\,bn} + \frac{e^{(-5\,b\log(cx^n)-5\,a)}}{160\,bn}$$

input
```
integrate(sinh(a+b*log(c*x^n))^5/x,x, algorithm="maxima")
```

output
```
1/160*e^(5*b*log(c*x^n) + 5*a)/(b*n) - 5/96*e^(3*b*log(c*x^n) + 3*a)/(b*n)
 + 5/16*e^(b*log(c*x^n) + a)/(b*n) + 5/16*e^(-b*log(c*x^n) - a)/(b*n) - 5/
96*e^(-3*b*log(c*x^n) - 3*a)/(b*n) + 1/160*e^(-5*b*log(c*x^n) - 5*a)/(b*n)
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 115, normalized size of antiderivative = 1.77

$$\int \frac{\sinh^5(a+b\log(cx^n))}{x}\,dx = \frac{\left(3\,c^{10\,b}x^{5\,bn}e^{(10\,a)} - 25\,c^{8\,b}x^{3\,bn}e^{(8\,a)} + 150\,c^{6\,b}x^{bn}e^{(6\,a)} + \frac{150\,c^{4\,b}x^{4\,bn}e^{(4\,a)} - 25\,c^{2\,b}x^{2\,bn}e^{(2\,a)} + 3}{x^{5\,bn}}\right)e^{(-5\,a)}}{480\,bc^{5\,b}n}$$

input
```
integrate(sinh(a+b*log(c*x^n))^5/x,x, algorithm="giac")
```

output
```
1/480*(3*c^(10*b)*x^(5*b*n)*e^(10*a) - 25*c^(8*b)*x^(3*b*n)*e^(8*a) + 150*
c^(6*b)*x^(b*n)*e^(6*a) + (150*c^(4*b)*x^(4*b*n)*e^(4*a) - 25*c^(2*b)*x^(2
*b*n)*e^(2*a) + 3)/x^(5*b*n))*e^(-5*a)/(b*c^(5*b)*n)
```

Mupad [B] (verification not implemented)

Time = 1.71 (sec) , antiderivative size = 49, normalized size of antiderivative = 0.75

$$\int \frac{\sinh^5(a+b\log(cx^n))}{x}\,dx = \frac{\frac{\cosh(a+b\,\ln(c\,x^n))^5}{5}-\frac{2\cosh(a+b\,\ln(c\,x^n))^3}{3}+\cosh(a+b\,\ln(c\,x^n))}{b\,n}$$

input
```
int(sinh(a + b*log(c*x^n))^5/x,x)
```

output
```
(cosh(a + b*log(c*x^n)) - (2*cosh(a + b*log(c*x^n))^3)/3 + cosh(a + b*log(
c*x^n))^5/5)/(b*n)
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 122, normalized size of antiderivative = 1.88

$$\int \frac{\sinh^5(a+b\log(cx^n))}{x}\,dx = \frac{3x^{10bn}e^{10a}c^{10b}-25x^{8bn}e^{8a}c^{8b}+150x^{6bn}e^{6a}c^{6b}+150x^{4bn}e^{4a}c^{4b}-25x^{2bn}e^{2a}c^{2b}+3}{480x^{5bn}e^{5a}c^{5b}bn}$$

input
```
int(sinh(a+b*log(c*x^n))^5/x,x)
```

output
```
(3*x**(10*b*n)*e**(10*a)*c**(10*b) - 25*x**(8*b*n)*e**(8*a)*c**(8*b) + 150
*x**(6*b*n)*e**(6*a)*c**(6*b) + 150*x**(4*b*n)*e**(4*a)*c**(4*b) - 25*x**(
2*b*n)*e**(2*a)*c**(2*b) + 3)/(480*x**(5*b*n)*e**(5*a)*c**(5*b)*b*n)
```

3.279 $\int \frac{\sinh^{\frac{5}{2}}(a+b\log(cx^n))}{x}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 109

$$\begin{aligned}\int \frac{\sinh^{\frac{5}{2}}(a+b\log(cx^n))}{x}\,dx &= \frac{6iE\left(\frac{1}{4}(2ia-\pi+2ib\log(cx^n))\middle|2\right)\sqrt{\sinh(a+b\log(cx^n))}}{5bn\sqrt{i\sinh(a+b\log(cx^n))}} \\ &\quad+\frac{2\cosh(a+b\log(cx^n))\sinh^{\frac{3}{2}}(a+b\log(cx^n))}{5bn}\end{aligned}$$

output
```
-6/5*I*EllipticE(cos(1/2*I*a+1/4*Pi+1/2*I*b*ln(c*x^n)),2^(1/2))*sinh(a+b*l
n(c*x^n))^(1/2)/b/n/(I*sinh(a+b*ln(c*x^n)))^(1/2)+2/5*cosh(a+b*ln(c*x^n))*
sinh(a+b*ln(c*x^n))^(3/2)/b/n
```

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 96, normalized size of antiderivative = 0.88

$$\int \frac{\sinh^{\frac{5}{2}}(a+b\log(cx^n))}{x}\,dx$$
$$= \frac{-6E\left(\frac{1}{4}(-2ia+\pi-2ib\log(cx^n))\middle|2\right)\sqrt{i\sinh(a+b\log(cx^n))}+\sinh(a+b\log(cx^n))\sinh(2(a+b\log($$
$$5bn\sqrt{\sinh(a+b\log(cx^n))}$$

input
```
Integrate[Sinh[a + b*Log[c*x^n]]^(5/2)/x,x]
```

output
```
(-6*EllipticE[((-2*I)*a + Pi - (2*I)*b*Log[c*x^n])/4, 2]*Sqrt[I*Sinh[a + b
*Log[c*x^n]]] + Sinh[a + b*Log[c*x^n]]*Sinh[2*(a + b*Log[c*x^n])])/(5*b*n*
Sqrt[Sinh[a + b*Log[c*x^n]]])
```

Rubi [A] (verified)

Time = 0.39 (sec) , antiderivative size = 109, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.368$, Rules used = {3039, 3042, 3115, 3042, 3121, 3042, 3119}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sinh^{\frac{5}{2}}(a+b\log(cx^n))}{x}\,dx$$
$$\Big\downarrow \text{ 3039}$$
$$\frac{\int \sinh^{\frac{5}{2}}(a+b\log(cx^n))\,d\log(cx^n)}{n}$$
$$\Big\downarrow \text{ 3042}$$
$$\frac{\int (-i\sin(ia+ib\log(cx^n)))^{5/2}\,d\log(cx^n)}{n}$$
$$\Big\downarrow \text{ 3115}$$

$$\frac{\frac{2\sinh^{\frac{3}{2}}(a+b\log(cx^n))\cosh(a+b\log(cx^n))}{5b}-\frac{3}{5}\int\sqrt{\sinh\left(a+b\log\left(cx^n\right)\right)}d\log\left(cx^n\right)}{n}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\frac{2\sinh^{\frac{3}{2}}(a+b\log(cx^n))\cosh(a+b\log(cx^n))}{5b}-\frac{3}{5}\int\sqrt{-i\sin\left(ia+ib\log\left(cx^n\right)\right)}d\log\left(cx^n\right)}{n}$$

$$\Big\downarrow \text{3121}$$

$$\frac{\frac{2\sinh^{\frac{3}{2}}(a+b\log(cx^n))\cosh(a+b\log(cx^n))}{5b}-\frac{3\sqrt{\sinh(a+b\log(cx^n))}\int\sqrt{i\sinh(a+b\log(cx^n))}d\log(cx^n)}{5\sqrt{i\sinh(a+b\log(cx^n))}}}{n}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\frac{2\sinh^{\frac{3}{2}}(a+b\log(cx^n))\cosh(a+b\log(cx^n))}{5b}-\frac{3\sqrt{\sinh(a+b\log(cx^n))}\int\sqrt{\sin(ia+ib\log(cx^n))}d\log(cx^n)}{5\sqrt{i\sinh(a+b\log(cx^n))}}}{n}$$

$$\Big\downarrow \text{3119}$$

$$\frac{\frac{2\sinh^{\frac{3}{2}}(a+b\log(cx^n))\cosh(a+b\log(cx^n))}{5b}+\frac{6i\sqrt{\sinh(a+b\log(cx^n))}E\left(\frac{1}{2}\left(ia+ib\log(cx^n)-\frac{\pi}{2}\right)|2\right)}{5b\sqrt{i\sinh(a+b\log(cx^n))}}}{n}$$

input
```
Int[Sinh[a + b*Log[c*x^n]]^(5/2)/x,x]
```

output
```
((((6*I)/5)*EllipticE[(I*a - Pi/2 + I*b*Log[c*x^n])/2, 2]*Sqrt[Sinh[a + b*
Log[c*x^n]]])/(b*Sqrt[I*Sinh[a + b*Log[c*x^n]]]) + (2*Cosh[a + b*Log[c*x^n
]]*Sinh[a + b*Log[c*x^n]]^(3/2))/(5*b))/n
```

Defintions of rubi rules used

rule 3039
```
Int[u_, x_Symbol] :> With[{lst = FunctionOfLog[Cancel[x*u], x]}, Simp[1/lst
[[3]]   Subst[Int[lst[[1]], x], x, Log[lst[[2]]]], x] /;  !FalseQ[lst]] /;
NonsumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3115
```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Simp[b^2*((n - 1)/n)   Int[(b*Sin
[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[
2*n]
```

rule 3119
```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3121
```
Int[((b_)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*Sin[c + d*x])
^n/Sin[c + d*x]^n   Int[Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] && Lt
Q[-1, n, 1] && IntegerQ[2*n]
```

Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 226 vs. $2(94) = 188$.

Time = 2.04 (sec) , antiderivative size = 227, normalized size of antiderivative = 2.08

method	result
derivativedivides	$\frac{-\frac{6\sqrt{1-i\sinh(a+b\ln(c\,x^n))}\,\sqrt{2}\,\sqrt{1+i\sinh(a+b\ln(c\,x^n))}\,\sqrt{i\sinh(a+b\ln(c\,x^n))}\ \mathrm{EllipticE}\left(\sqrt{1-i\sinh(a+b\ln(c\,x^n))},\frac{\sqrt{2}}{2}\right)}{5}+\frac{3\sqrt{1-i\,s}}{}}{n\cosh(a-}$
default	$\frac{-\frac{6\sqrt{1-i\sinh(a+b\ln(c\,x^n))}\,\sqrt{2}\,\sqrt{1+i\sinh(a+b\ln(c\,x^n))}\,\sqrt{i\sinh(a+b\ln(c\,x^n))}\ \mathrm{EllipticE}\left(\sqrt{1-i\sinh(a+b\ln(c\,x^n))},\frac{\sqrt{2}}{2}\right)}{5}+\frac{3\sqrt{1-i\,s}}{}}{n\cosh(a-}$

input
```
int(sinh(a+b*ln(c*x^n))^(5/2)/x,x,method=_RETURNVERBOSE)
```

output
```
1/n*(-6/5*(1-I*sinh(a+b*ln(c*x^n)))^(1/2)*2^(1/2)*(1+I*sinh(a+b*ln(c*x^n))
)^(1/2)*(I*sinh(a+b*ln(c*x^n)))^(1/2)*EllipticE((1-I*sinh(a+b*ln(c*x^n)))^
(1/2),1/2*2^(1/2))+3/5*(1-I*sinh(a+b*ln(c*x^n)))^(1/2)*2^(1/2)*(1+I*sinh(a
+b*ln(c*x^n)))^(1/2)*(I*sinh(a+b*ln(c*x^n)))^(1/2)*EllipticF((1-I*sinh(a+b
*ln(c*x^n)))^(1/2),1/2*2^(1/2))+2/5*cosh(a+b*ln(c*x^n))^4-2/5*cosh(a+b*ln(
c*x^n))^2)/cosh(a+b*ln(c*x^n))/sinh(a+b*ln(c*x^n))^(1/2)/b
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 331 vs. $2(90) = 180$.

Time = 0.10 (sec) , antiderivative size = 331, normalized size of antiderivative = 3.04

$$\int \frac{\sinh^{\frac{5}{2}}(a + b\log(cx^n))}{x}\,dx = \frac{12\left(\sqrt{2}\cosh\left(bn\log\left(x\right) + b\log\left(c\right) + a\right)^2 + 2\sqrt{2}\cosh\left(bn\log\left(x\right) + b\log\left(c\right) + a\right)\sinh\left(bn\log\left(x\right) + b\log\left(c\right.\right.}{}$$

input
```
integrate(sinh(a+b*log(c*x^n))^(5/2)/x,x, algorithm="fricas")
```

output
```
1/10*(12*(sqrt(2)*cosh(b*n*log(x) + b*log(c) + a)^2 + 2*sqrt(2)*cosh(b*n*l
og(x) + b*log(c) + a)*sinh(b*n*log(x) + b*log(c) + a) + sqrt(2)*sinh(b*n*l
og(x) + b*log(c) + a)^2)*weierstrassZeta(4, 0, weierstrassPInverse(4, 0, c
osh(b*n*log(x) + b*log(c) + a) + sinh(b*n*log(x) + b*log(c) + a))) + (cosh
(b*n*log(x) + b*log(c) + a)^4 + 4*cosh(b*n*log(x) + b*log(c) + a)*sinh(b*n
*log(x) + b*log(c) + a)^3 + sinh(b*n*log(x) + b*log(c) + a)^4 + 6*(cosh(b*
n*log(x) + b*log(c) + a)^2 + 2)*sinh(b*n*log(x) + b*log(c) + a)^2 + 12*cos
h(b*n*log(x) + b*log(c) + a)^2 + 4*(cosh(b*n*log(x) + b*log(c) + a)^3 + 6*
cosh(b*n*log(x) + b*log(c) + a))*sinh(b*n*log(x) + b*log(c) + a) - 1)*sqrt
(sinh(b*n*log(x) + b*log(c) + a)))/(b*n*cosh(b*n*log(x) + b*log(c) + a)^2
+ 2*b*n*cosh(b*n*log(x) + b*log(c) + a)*sinh(b*n*log(x) + b*log(c) + a) +
b*n*sinh(b*n*log(x) + b*log(c) + a)^2)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\sinh^{\frac{5}{2}}(a + b\log(cx^n))}{x}\,dx = \text{Timed out}$$

input
```
integrate(sinh(a+b*ln(c*x**n))**(5/2)/x,x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{\sinh^{\frac{5}{2}}(a+b\log(cx^n))}{x}\,dx = \int \frac{\sinh(b\log(cx^n)+a)^{\frac{5}{2}}}{x}\,dx$$

input
```
integrate(sinh(a+b*log(c*x^n))^(5/2)/x,x, algorithm="maxima")
```

output
```
integrate(sinh(b*log(c*x^n) + a)^(5/2)/x, x)
```

Giac [F]

$$\int \frac{\sinh^{\frac{5}{2}}(a+b\log(cx^n))}{x}\,dx = \int \frac{\sinh(b\log(cx^n)+a)^{\frac{5}{2}}}{x}\,dx$$

input
```
integrate(sinh(a+b*log(c*x^n))^(5/2)/x,x, algorithm="giac")
```

output
```
integrate(sinh(b*log(c*x^n) + a)^(5/2)/x, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sinh^{\frac{5}{2}}(a+b\log(cx^n))}{x}\,dx = \int \frac{\sinh(a+b\,\ln(c\,x^n))^{5/2}}{x}\,dx$$

input
```
int(sinh(a + b*log(c*x^n))^(5/2)/x,x)
```

output
```
int(sinh(a + b*log(c*x^n))^(5/2)/x, x)
```

Reduce [F]

$$\int \frac{\sinh^{\frac{5}{2}}(a + b\log(cx^n))}{x}\,dx = \int \frac{\sqrt{\sinh(\log(x^n c)\,b + a)}\,\sinh(\log(x^n c)\,b + a)^2}{x}dx$$

input
```
int(sinh(a+b*log(c*x^n))^(5/2)/x,x)
```

output
```
int((sqrt(sinh(log(x**n*c)*b + a))*sinh(log(x**n*c)*b + a)**2)/x,x)
```

3.280 $\int \frac{\sinh^{\frac{3}{2}}(a+b\log(cx^n))}{x}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 109

$$\int \frac{\sinh^{\frac{3}{2}}(a+b\log(cx^n))}{x}\,dx = \frac{2i\,\text{EllipticF}\left(\frac{1}{4}(2ia-\pi+2ib\log(cx^n)),2\right)\sqrt{i\sinh(a+b\log(cx^n))}}{3bn\sqrt{\sinh(a+b\log(cx^n))}} + \frac{2\cosh(a+b\log(cx^n))\sqrt{\sinh(a+b\log(cx^n))}}{3bn}$$

output
```
2/3*I*InverseJacobiAM(1/2*I*a-1/4*Pi+1/2*I*b*ln(c*x^n),2^(1/2))*(I*sinh(a+
b*ln(c*x^n)))^(1/2)/b/n/sinh(a+b*ln(c*x^n))^(1/2)+2/3*cosh(a+b*ln(c*x^n))*
sinh(a+b*ln(c*x^n))^(1/2)/b/n
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 0.08 (sec) , antiderivative size = 114, normalized size of antiderivative = 1.05

$$\int \frac{\sinh^{\frac{3}{2}}(a+b\log(cx^n))}{x}\,dx = \frac{-2\,\text{Hypergeometric2F1}\left(\frac{1}{4},\frac{1}{2},\frac{5}{4},\cosh(2(a+b\log(cx^n)))+\sinh(2(a+b\log(cx^n)))\right)\sqrt{1-\cosh(2(a+b\log(cx^n)))-\sinh(2(a+b\log(cx^n)))}\dots}{3bn\sqrt{\sinh(a+b\log(cx^n))}}$$

input
```
Integrate[Sinh[a + b*Log[c*x^n]]^(3/2)/x,x]
```

output
```
(-2*Hypergeometric2F1[1/4, 1/2, 5/4, Cosh[2*(a + b*Log[c*x^n])] + Sinh[2*(
a + b*Log[c*x^n])]]*Sqrt[1 - Cosh[2*(a + b*Log[c*x^n])] - Sinh[2*(a + b*Lo
g[c*x^n])]] + Sinh[2*(a + b*Log[c*x^n])])/(3*b*n*Sqrt[Sinh[a + b*Log[c*x^n
]]])
```

Rubi [A] (verified)

Time = 0.38 (sec) , antiderivative size = 109, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.368$, Rules used = {3039, 3042, 3115, 3042, 3121, 3042, 3120}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sinh^{\frac{3}{2}}(a+b\log(cx^n))}{x}\,dx$$

$$\Big\downarrow \text{3039}$$

$$\frac{\int \sinh^{\frac{3}{2}}(a+b\log(cx^n))\,d\log(cx^n)}{n}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\int (-i\sin(ia+ib\log(cx^n)))^{3/2}\,d\log(cx^n)}{n}$$

$$\Big\downarrow \text{3115}$$

$$\frac{\frac{2\sqrt{\sinh(a+b\log(cx^n))}\cosh(a+b\log(cx^n))}{3b}-\frac{1}{3}\int\frac{1}{\sqrt{\sinh(a+b\log(cx^n))}}d\log\left(cx^n\right)}{n}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\frac{2\sqrt{\sinh(a+b\log(cx^n))}\cosh(a+b\log(cx^n))}{3b}-\frac{1}{3}\int\frac{1}{\sqrt{-i\sin(ia+ib\log(cx^n))}}d\log\left(cx^n\right)}{n}$$

$$\Big\downarrow \text{3121}$$

$$\frac{\frac{2\sqrt{\sinh(a+b\log(cx^n))}\cosh(a+b\log(cx^n))}{3b}-\frac{\sqrt{i\sinh(a+b\log(cx^n))}\int\frac{1}{\sqrt{i\sinh(a+b\log(cx^n))}}d\log(cx^n)}{3\sqrt{\sinh(a+b\log(cx^n))}}}{n}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\frac{2\sqrt{\sinh(a+b\log(cx^n))}\cosh(a+b\log(cx^n))}{3b}-\frac{\sqrt{i\sinh(a+b\log(cx^n))}\int\frac{1}{\sqrt{\sin(ia+ib\log(cx^n))}}d\log(cx^n)}{3\sqrt{\sinh(a+b\log(cx^n))}}}{n}$$

$$\Big\downarrow \text{3120}$$

$$\frac{\frac{2\sqrt{\sinh(a+b\log(cx^n))}\cosh(a+b\log(cx^n))}{3b}+\frac{2i\sqrt{i\sinh(a+b\log(cx^n))}\,\text{EllipticF}\left(\frac{1}{2}\left(ia+ib\log(cx^n)-\frac{\pi}{2}\right),2\right)}{3b\sqrt{\sinh(a+b\log(cx^n))}}}{n}$$

input
```
Int[Sinh[a + b*Log[c*x^n]]^(3/2)/x,x]
```

output
```
((((2*I)/3)*EllipticF[(I*a - Pi/2 + I*b*Log[c*x^n])/2, 2]*Sqrt[I*Sinh[a +
b*Log[c*x^n]]])/(b*Sqrt[Sinh[a + b*Log[c*x^n]]]) + (2*Cosh[a + b*Log[c*x^n
]]*Sqrt[Sinh[a + b*Log[c*x^n]]])/(3*b))/n
```

Defintions of rubi rules used

rule 3039
```
Int[u_, x_Symbol] :> With[{lst = FunctionOfLog[Cancel[x*u], x]}, Simp[1/lst
[[3]]   Subst[Int[lst[[1]], x], x, Log[lst[[2]]]], x] /;  !FalseQ[lst]] /;
NonsumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3115
```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Simp[b^2*((n - 1)/n)   Int[(b*Sin
[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[
2*n]
```

rule 3120
```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3121
```
Int[((b_)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*Sin[c + d*x])
^n/Sin[c + d*x]^n   Int[Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] && Lt
Q[-1, n, 1] && IntegerQ[2*n]
```

Maple [A] (verified)

Time = 0.20 (sec) , antiderivative size = 143, normalized size of antiderivative = 1.31

method	result
derivativedivides	$\frac{-\frac{i\sqrt{1-i\sinh(a+b\ln(c\,x^n))}\,\sqrt{2}\,\sqrt{1+i\sinh(a+b\ln(c\,x^n))}\,\sqrt{i\sinh(a+b\ln(c\,x^n))}\ \operatorname{EllipticF}\left(\sqrt{1-i\sinh(a+b\ln(c\,x^n))},\,\frac{\sqrt{2}}{2}\right)}{3}+\frac{2\cosh(a}{}}{n\cosh(a+b\ln(c\,x^n))\sqrt{\sinh(a+b\ln(c\,x^n))}\,b}$
default	$\frac{-\frac{i\sqrt{1-i\sinh(a+b\ln(c\,x^n))}\,\sqrt{2}\,\sqrt{1+i\sinh(a+b\ln(c\,x^n))}\,\sqrt{i\sinh(a+b\ln(c\,x^n))}\ \operatorname{EllipticF}\left(\sqrt{1-i\sinh(a+b\ln(c\,x^n))},\,\frac{\sqrt{2}}{2}\right)}{3}+\frac{2\cosh(a}{}}{n\cosh(a+b\ln(c\,x^n))\sqrt{\sinh(a+b\ln(c\,x^n))}\,b}$

input
```
int(sinh(a+b*ln(c*x^n))^(3/2)/x,x,method=_RETURNVERBOSE)
```

output
```
1/n*(-1/3*I*(1-I*sinh(a+b*ln(c*x^n)))^(1/2)*2^(1/2)*(1+I*sinh(a+b*ln(c*x^n
)))^(1/2)*(I*sinh(a+b*ln(c*x^n)))^(1/2)*EllipticF((1-I*sinh(a+b*ln(c*x^n))
)^(1/2),1/2*2^(1/2))+2/3*cosh(a+b*ln(c*x^n))^2*sinh(a+b*ln(c*x^n)))/cosh(a
+b*ln(c*x^n))/sinh(a+b*ln(c*x^n))^(1/2)/b
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 171, normalized size of antiderivative = 1.57

$$\int \frac{\sinh^{\frac{3}{2}}(a+b\log(cx^n))}{x}\,dx = -\frac{2\left(\sqrt{2}\cosh(bn\log(x)+b\log(c)+a)+\sqrt{2}\sinh(bn\log(x)+b\log(c)+a)\right)\text{weierstrassPInverse}(4,0,\cdots$$

input
```
integrate(sinh(a+b*log(c*x^n))^(3/2)/x,x, algorithm="fricas")
```

output
```
-1/3*(2*(sqrt(2)*cosh(b*n*log(x) + b*log(c) + a) + sqrt(2)*sinh(b*n*log(x)
 + b*log(c) + a))*weierstrassPInverse(4, 0, cosh(b*n*log(x) + b*log(c) + a
) + sinh(b*n*log(x) + b*log(c) + a)) - (cosh(b*n*log(x) + b*log(c) + a)^2
+ 2*cosh(b*n*log(x) + b*log(c) + a)*sinh(b*n*log(x) + b*log(c) + a) + sinh
(b*n*log(x) + b*log(c) + a)^2 + 1)*sqrt(sinh(b*n*log(x) + b*log(c) + a)))/
(b*n*cosh(b*n*log(x) + b*log(c) + a) + b*n*sinh(b*n*log(x) + b*log(c) + a)
)
```

Sympy [F]

$$\int \frac{\sinh^{\frac{3}{2}}(a+b\log(cx^n))}{x}\,dx = \int \frac{\sinh^{\frac{3}{2}}(a+b\log(cx^n))}{x}\,dx$$

input
```
integrate(sinh(a+b*ln(c*x**n))**(3/2)/x,x)
```

output
```
Integral(sinh(a + b*log(c*x**n))**(3/2)/x, x)
```

Maxima [F]

$$\int \frac{\sinh^{\frac{3}{2}}(a+b\log(cx^n))}{x}\,dx = \int \frac{\sinh(b\log(cx^n)+a)^{\frac{3}{2}}}{x}\,dx$$

input
```
integrate(sinh(a+b*log(c*x^n))^(3/2)/x,x, algorithm="maxima")
```

output
```
integrate(sinh(b*log(c*x^n) + a)^(3/2)/x, x)
```

Giac [F]

$$\int \frac{\sinh^{\frac{3}{2}}(a+b\log(cx^n))}{x}\,dx = \int \frac{\sinh(b\log(cx^n)+a)^{\frac{3}{2}}}{x}\,dx$$

input
```
integrate(sinh(a+b*log(c*x^n))^(3/2)/x,x, algorithm="giac")
```

output
```
integrate(sinh(b*log(c*x^n) + a)^(3/2)/x, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sinh^{\frac{3}{2}}(a+b\log(cx^n))}{x}\,dx = \int \frac{\sinh(a+b\ln(c\,x^n))^{3/2}}{x}\,dx$$

input
```
int(sinh(a + b*log(c*x^n))^(3/2)/x,x)
```

output
```
int(sinh(a + b*log(c*x^n))^(3/2)/x, x)
```

Reduce [F]

$$\int \frac{\sinh^{\frac{3}{2}}(a+b\log(cx^n))}{x}\,dx = \int \frac{\sqrt{\sinh(\log(x^n c)\,b+a)}\,\sinh(\log(x^n c)\,b+a)}{x}dx$$

input
```
int(sinh(a+b*log(c*x^n))^(3/2)/x,x)
```

output
```
int((sqrt(sinh(log(x**n*c)*b + a))*sinh(log(x**n*c)*b + a))/x,x)
```

3.281 $\int \frac{\sqrt{\sinh(a+b\log(cx^n))}}{x}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 70

$$\int \frac{\sqrt{\sinh(a+b\log(cx^n))}}{x}\,dx=-\frac{2iE\left(\frac{1}{4}(2ia-\pi+2ib\log(cx^n))\middle|2\right)\sqrt{\sinh(a+b\log(cx^n))}}{bn\sqrt{i\sinh(a+b\log(cx^n))}}$$

output
```
2*I*EllipticE(cos(1/2*I*a+1/4*Pi+1/2*I*b*ln(c*x^n)),2^(1/2))*sinh(a+b*ln(c
*x^n))^(1/2)/b/n/(I*sinh(a+b*ln(c*x^n)))^(1/2)
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 68, normalized size of antiderivative = 0.97

$$\int \frac{\sqrt{\sinh(a+b\log(cx^n))}}{x}\,dx=\frac{2E\left(\frac{1}{2}\left(\frac{\pi}{2}-i(a+b\log(cx^n))\right)\middle|2\right)\sqrt{i\sinh(a+b\log(cx^n))}}{bn\sqrt{\sinh(a+b\log(cx^n))}}$$

input
```
Integrate[Sqrt[Sinh[a + b*Log[c*x^n]]]/x,x]
```

output
```
(2*EllipticE[(Pi/2 - I*(a + b*Log[c*x^n]))/2, 2]*Sqrt[I*Sinh[a + b*Log[c*x
^n]]])/(b*n*Sqrt[Sinh[a + b*Log[c*x^n]]])
```

Rubi [A] (verified)

Time = 0.34 (sec) , antiderivative size = 72, normalized size of antiderivative = 1.03, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.263$, Rules used = {3039, 3042, 3121, 3042, 3119}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{\sinh(a+b\log(cx^n))}}{x}dx$$

$$\Big\downarrow \text{3039}$$

$$\frac{\int \sqrt{\sinh(a+b\log(cx^n))}d\log(cx^n)}{n}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\int \sqrt{-i\sin(ia+ib\log(cx^n))}d\log(cx^n)}{n}$$

$$\Big\downarrow \text{3121}$$

$$\frac{\sqrt{\sinh(a+b\log(cx^n))}\int \sqrt{i\sinh(a+b\log(cx^n))}d\log(cx^n)}{n\sqrt{i\sinh(a+b\log(cx^n))}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\sqrt{\sinh(a+b\log(cx^n))}\int \sqrt{\sin(ia+ib\log(cx^n))}d\log(cx^n)}{n\sqrt{i\sinh(a+b\log(cx^n))}}$$

$$\Big\downarrow \text{3119}$$

$$-\frac{2i\sqrt{\sinh(a+b\log(cx^n))}E\left(\frac{1}{2}\left(ia+ib\log(cx^n)-\frac{\pi}{2}\right)|2\right)}{bn\sqrt{i\sinh(a+b\log(cx^n))}}$$

input
```
Int[Sqrt[Sinh[a + b*Log[c*x^n]]]/x,x]
```

output
```
((-2*I)*EllipticE[(I*a - Pi/2 + I*b*Log[c*x^n])/2, 2]*Sqrt[Sinh[a + b*Log[
c*x^n]]])/(b*n*Sqrt[I*Sinh[a + b*Log[c*x^n]]])
```

Defintions of rubi rules used

rule 3039
```
Int[u_, x_Symbol] :> With[{lst = FunctionOfLog[Cancel[x*u], x]}, Simp[1/lst
[[3]]   Subst[Int[lst[[1]], x], x, Log[lst[[2]]]], x] /;  !FalseQ[lst]] /;
NonsumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3119
```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3121
```
Int[((b_)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*Sin[c + d*x])
^n/Sin[c + d*x]^n   Int[Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] && Lt
Q[-1, n, 1] && IntegerQ[2*n]
```

Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 145 vs. $2(61) = 122$.

Time = 0.19 (sec) , antiderivative size = 146, normalized size of antiderivative = 2.09

method	result
derivativedivides	$\frac{\sqrt{-i(\sinh(a+b\ln(c\,x^n))+i)}\,\sqrt{2}\,\sqrt{-i(-\sinh(a+b\ln(c\,x^n))+i)}\,\sqrt{i\sinh(a+b\ln(c\,x^n))}\,\left(2\,\mathrm{EllipticE}\left(\sqrt{1-i\sinh(a+b\ln(\right.\right.}{n\cosh(a+b\ln(c\,x^n))\sqrt{\sinh(a+b\ln(c\,x^n))}\,b}$
default	$\frac{\sqrt{-i(\sinh(a+b\ln(c\,x^n))+i)}\,\sqrt{2}\,\sqrt{-i(-\sinh(a+b\ln(c\,x^n))+i)}\,\sqrt{i\sinh(a+b\ln(c\,x^n))}\,\left(2\,\mathrm{EllipticE}\left(\sqrt{1-i\sinh(a+b\ln(\right.\right.}{n\cosh(a+b\ln(c\,x^n))\sqrt{\sinh(a+b\ln(c\,x^n))}\,b}$

input
```
int(sinh(a+b*ln(c*x^n))^(1/2)/x,x,method=_RETURNVERBOSE)
```

output
```
1/n*(-I*(sinh(a+b*ln(c*x^n))+I))^(1/2)*2^(1/2)*(-I*(-sinh(a+b*ln(c*x^n))+I
))^(1/2)*(I*sinh(a+b*ln(c*x^n)))^(1/2)*(2*EllipticE((1-I*sinh(a+b*ln(c*x^n
)))^(1/2),1/2*2^(1/2))-EllipticF((1-I*sinh(a+b*ln(c*x^n)))^(1/2),1/2*2^(1/
2)))/cosh(a+b*ln(c*x^n))/sinh(a+b*ln(c*x^n))^(1/2)/b
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 58, normalized size of antiderivative = 0.83

$$\int \frac{\sqrt{\sinh(a + b\log(cx^n))}}{x}\,dx = -\frac{2\left(\sqrt{2}\text{weierstrassZeta}(4, 0, \text{weierstrassPInverse}(4, 0, \cosh(bn\log(x) + b\log(c) + a) + \sinh(bn\log(x)\right.}{bn}$$

input
```
integrate(sinh(a+b*log(c*x^n))^(1/2)/x,x, algorithm="fricas")
```

output
```
-2*(sqrt(2)*weierstrassZeta(4, 0, weierstrassPInverse(4, 0, cosh(b*n*log(x
) + b*log(c) + a) + sinh(b*n*log(x) + b*log(c) + a))) + sqrt(sinh(b*n*log(
x) + b*log(c) + a)))/(b*n)
```

Sympy [F]

$$\int \frac{\sqrt{\sinh(a + b\log(cx^n))}}{x}\,dx = \int \frac{\sqrt{\sinh(a + b\log(cx^n))}}{x}\,dx$$

input
```
integrate(sinh(a+b*ln(c*x**n))**(1/2)/x,x)
```

output
```
Integral(sqrt(sinh(a + b*log(c*x**n)))/x, x)
```

Maxima [F]

$$\int \frac{\sqrt{\sinh\left(a+b\log\left(cx^n\right)\right)}}{x}\,dx=\int \frac{\sqrt{\sinh\left(b\log\left(cx^n\right)+a\right)}}{x}\,dx$$

input
```
integrate(sinh(a+b*log(c*x^n))^(1/2)/x,x, algorithm="maxima")
```

output
```
integrate(sqrt(sinh(b*log(c*x^n) + a))/x, x)
```

Giac [F]

$$\int \frac{\sqrt{\sinh\left(a+b\log\left(cx^n\right)\right)}}{x}\,dx=\int \frac{\sqrt{\sinh\left(b\log\left(cx^n\right)+a\right)}}{x}\,dx$$

input
```
integrate(sinh(a+b*log(c*x^n))^(1/2)/x,x, algorithm="giac")
```

output
```
integrate(sqrt(sinh(b*log(c*x^n) + a))/x, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{\sinh\left(a+b\log\left(cx^n\right)\right)}}{x}\,dx=\int \frac{\sqrt{\sinh\left(a+b\,\ln\left(c\,x^n\right)\right)}}{x}\,dx$$

input
```
int(sinh(a + b*log(c*x^n))^(1/2)/x,x)
```

output
```
int(sinh(a + b*log(c*x^n))^(1/2)/x, x)
```

Reduce [F]

$$\int \frac{\sqrt{\sinh\left(a+b\log\left(cx^n\right)\right)}}{x}\,dx=\int \frac{\sqrt{\sinh\left(\log\left(x^nc\right)b+a\right)}}{x}dx$$

input
```
int(sinh(a+b*log(c*x^n))^(1/2)/x,x)
```

output
```
int(sqrt(sinh(log(x**n*c)*b + a))/x,x)
```

3.282 $\int \frac{1}{x\sqrt{\sinh(a+b\log(cx^n))}}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 70

$$\int \frac{1}{x\sqrt{\sinh(a+b\log(cx^n))}}\,dx = -\frac{2i\,\text{EllipticF}\left(\frac{1}{4}(2ia-\pi+2ib\log(cx^n)),2\right)\sqrt{i\sinh(a+b\log(cx^n))}}{bn\sqrt{\sinh(a+b\log(cx^n))}}$$

output
```
-2*I*InverseJacobiAM(1/2*I*a-1/4*Pi+1/2*I*b*ln(c*x^n),2^(1/2))*(I*sinh(a+b
*ln(c*x^n)))^(1/2)/b/n/sinh(a+b*ln(c*x^n))^(1/2)
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 66, normalized size of antiderivative = 0.94

$$\int \frac{1}{x\sqrt{\sinh(a+b\log(cx^n))}}\,dx = -\frac{2\,\text{EllipticF}\left(\frac{1}{4}(-2ia+\pi-2ib\log(cx^n)),2\right)\sqrt{\sinh(a+b\log(cx^n))}}{bn\sqrt{i\sinh(a+b\log(cx^n))}}$$

input
```
Integrate[1/(x*Sqrt[Sinh[a + b*Log[c*x^n]]]),x]
```

output
```
(-2*EllipticF[((-2*I)*a + Pi - (2*I)*b*Log[c*x^n])/4, 2]*Sqrt[Sinh[a + b*L
og[c*x^n]]])/(b*n*Sqrt[I*Sinh[a + b*Log[c*x^n]]])
```

Rubi [A] (verified)

Time = 0.32 (sec) , antiderivative size = 72, normalized size of antiderivative = 1.03, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.263$, Rules used = {3039, 3042, 3121, 3042, 3120}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x\sqrt{\sinh(a+b\log(cx^n))}}dx$$

$$\Big\downarrow \text{3039}$$

$$\frac{\int \frac{1}{\sqrt{\sinh(a+b\log(cx^n))}}d\log(cx^n)}{n}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\int \frac{1}{\sqrt{-i\sin(ia+ib\log(cx^n))}}d\log(cx^n)}{n}$$

$$\Big\downarrow \text{3121}$$

$$\frac{\sqrt{i\sinh(a+b\log(cx^n))}\int \frac{1}{\sqrt{i\sinh(a+b\log(cx^n))}}d\log(cx^n)}{n\sqrt{\sinh(a+b\log(cx^n))}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\sqrt{i\sinh(a+b\log(cx^n))}\int \frac{1}{\sqrt{\sin(ia+ib\log(cx^n))}}d\log(cx^n)}{n\sqrt{\sinh(a+b\log(cx^n))}}$$

$$\Big\downarrow \text{3120}$$

$$-\frac{2i\sqrt{i\sinh(a+b\log(cx^n))}\,\text{EllipticF}\left(\frac{1}{2}\left(ia+ib\log(cx^n)-\frac{\pi}{2}\right),2\right)}{bn\sqrt{\sinh(a+b\log(cx^n))}}$$

input
```
Int[1/(x*Sqrt[Sinh[a + b*Log[c*x^n]]]),x]
```

output
```
((-2*I)*EllipticF[(I*a - Pi/2 + I*b*Log[c*x^n])/2, 2]*Sqrt[I*Sinh[a + b*Lo
g[c*x^n]]])/(b*n*Sqrt[Sinh[a + b*Log[c*x^n]]])
```

Defintions of rubi rules used

rule 3039
```
Int[u_, x_Symbol] :> With[{lst = FunctionOfLog[Cancel[x*u], x]}, Simp[1/lst
[[3]]   Subst[Int[lst[[1]], x], x, Log[lst[[2]]]], x] /;  !FalseQ[lst]] /;
NonsumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3120
```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3121
```
Int[((b_)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*Sin[c + d*x])
^n/Sin[c + d*x]^n   Int[Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] && Lt
Q[-1, n, 1] && IntegerQ[2*n]
```

Maple [A] (verified)

Time = 0.18 (sec) , antiderivative size = 120, normalized size of antiderivative = 1.71

method	result
derivativedivides	$\frac{i\sqrt{-i(\sinh(a+b\ln(c\,x^n))+i)}\,\sqrt{2}\,\sqrt{-i(-\sinh(a+b\ln(c\,x^n))+i)}\,\sqrt{i\sinh(a+b\ln(c\,x^n))}\,\operatorname{EllipticF}\left(\sqrt{-i(\sinh(a+b\ln(c\,x\right.}{n\cosh(a+b\ln(c\,x^n))\sqrt{\sinh(a+b\ln(c\,x^n))}\,b}$
default	$\frac{i\sqrt{-i(\sinh(a+b\ln(c\,x^n))+i)}\,\sqrt{2}\,\sqrt{-i(-\sinh(a+b\ln(c\,x^n))+i)}\,\sqrt{i\sinh(a+b\ln(c\,x^n))}\,\operatorname{EllipticF}\left(\sqrt{-i(\sinh(a+b\ln(c\,x\right.}{n\cosh(a+b\ln(c\,x^n))\sqrt{\sinh(a+b\ln(c\,x^n))}\,b}$

input
```
int(1/x/sinh(a+b*ln(c*x^n))^(1/2),x,method=_RETURNVERBOSE)
```

output
```
I/n*(-I*(sinh(a+b*ln(c*x^n))+I))^(1/2)*2^(1/2)*(-I*(-sinh(a+b*ln(c*x^n))+I
))^(1/2)*(I*sinh(a+b*ln(c*x^n)))^(1/2)*EllipticF((-I*(sinh(a+b*ln(c*x^n))+
I))^(1/2),1/2*2^(1/2))/cosh(a+b*ln(c*x^n))/sinh(a+b*ln(c*x^n))^(1/2)/b
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.56

$$\int \frac{1}{x\sqrt{\sinh\left(a+b\log\left(cx^n\right)\right)}}\,dx = \frac{2\sqrt{2}\text{weierstrassPInverse}(4,0,\cosh\left(bn\log\left(x\right)+b\log\left(c\right)+a\right)+\sinh\left(bn\log\left(x\right)+b\log\left(c\right)+a\right))}{bn}$$

input
```
integrate(1/x/sinh(a+b*log(c*x^n))^(1/2),x, algorithm="fricas")
```

output
```
2*sqrt(2)*weierstrassPInverse(4, 0, cosh(b*n*log(x) + b*log(c) + a) + sinh
(b*n*log(x) + b*log(c) + a))/(b*n)
```

Sympy [F]

$$\int \frac{1}{x\sqrt{\sinh\left(a+b\log\left(cx^n\right)\right)}}\,dx = \int \frac{1}{x\sqrt{\sinh\left(a+b\log\left(cx^n\right)\right)}}\,dx$$

input
```
integrate(1/x/sinh(a+b*ln(c*x**n))**(1/2),x)
```

output
```
Integral(1/(x*sqrt(sinh(a + b*log(c*x**n)))), x)
```

Maxima [F]

$$\int \frac{1}{x\sqrt{\sinh\left(a+b\log\left(cx^n\right)\right)}}\,dx = \int \frac{1}{x\sqrt{\sinh\left(b\log\left(cx^n\right)+a\right)}}\,dx$$

input
```
integrate(1/x/sinh(a+b*log(c*x^n))^(1/2),x, algorithm="maxima")
```

output
```
integrate(1/(x*sqrt(sinh(b*log(c*x^n) + a))), x)
```

Giac [F]

$$\int \frac{1}{x\sqrt{\sinh\left(a+b\log\left(cx^n\right)\right)}}\,dx = \int \frac{1}{x\sqrt{\sinh\left(b\log\left(cx^n\right)+a\right)}}\,dx$$

input
```
integrate(1/x/sinh(a+b*log(c*x^n))^(1/2),x, algorithm="giac")
```

output
```
integrate(1/(x*sqrt(sinh(b*log(c*x^n) + a))), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{x\sqrt{\sinh\left(a+b\log\left(cx^n\right)\right)}}\,dx = \int \frac{1}{x\,\sqrt{\sinh\left(a+b\,\ln\left(c\,x^n\right)\right)}}\,dx$$

input
```
int(1/(x*sinh(a + b*log(c*x^n))^(1/2)),x)
```

output
```
int(1/(x*sinh(a + b*log(c*x^n))^(1/2)), x)
```

Reduce [F]

$$\int \frac{1}{x\sqrt{\sinh\left(a+b\log\left(cx^n\right)\right)}}\,dx = \int \frac{\sqrt{\sinh\left(\log\left(x^n c\right)b+a\right)}}{\sinh\left(\log\left(x^n c\right)b+a\right)x}dx$$

input
```
int(1/x/sinh(a+b*log(c*x^n))^(1/2),x)
```

output
```
int(sqrt(sinh(log(x**n*c)*b + a))/(sinh(log(x**n*c)*b + a)*x),x)
```

3.283 $\int \frac{1}{x\sinh^{\frac{3}{2}}(a+b\log(cx^n))}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 105

$$\int \frac{1}{x\sinh^{\frac{3}{2}}(a+b\log(cx^n))}\,dx = -\frac{2\cosh(a+b\log(cx^n))}{bn\sqrt{\sinh(a+b\log(cx^n))}} - \frac{2iE\left(\frac{1}{4}(2ia-\pi+2ib\log(cx^n))\middle|2\right)\sqrt{\sinh(a+b\log(cx^n))}}{bn\sqrt{i\sinh(a+b\log(cx^n))}}$$

output

```
-2*cosh(a+b*ln(c*x^n))/b/n/sinh(a+b*ln(c*x^n))^(1/2)+2*I*EllipticE(cos(1/2
*I*a+1/4*Pi+1/2*I*b*ln(c*x^n)),2^(1/2))*sinh(a+b*ln(c*x^n))^(1/2)/b/n/(I*s
inh(a+b*ln(c*x^n)))^(1/2)
```

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 80, normalized size of antiderivative = 0.76

$$\int \frac{1}{x\sinh^{\frac{3}{2}}(a+b\log(cx^n))}\,dx = -\frac{2\left(\cosh(a+b\log(cx^n))-E\left(\frac{1}{4}(-2ia+\pi-2ib\log(cx^n))\middle|2\right)\sqrt{i\sinh(a+b\log(cx^n))}\right)}{bn\sqrt{\sinh(a+b\log(cx^n))}}$$

input
```
Integrate[1/(x*Sinh[a + b*Log[c*x^n]]^(3/2)),x]
```

output
```
(-2*(Cosh[a + b*Log[c*x^n]] - EllipticE[((-2*I)*a + Pi - (2*I)*b*Log[c*x^n
])/4, 2]*Sqrt[I*Sinh[a + b*Log[c*x^n]]]))/(b*n*Sqrt[Sinh[a + b*Log[c*x^n]]
])
```

Rubi [A] (verified)

Time = 0.42 (sec) , antiderivative size = 105, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.368$, Rules used = {3039, 3042, 3116, 3042, 3121, 3042, 3119}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x\sinh^{\frac{3}{2}}(a+b\log(cx^n))}\,dx$$

$$\Big\downarrow \; 3039$$

$$\frac{\int \frac{1}{\sinh^{\frac{3}{2}}(a+b\log(cx^n))}d\log(cx^n)}{n}$$

$$\Big\downarrow \; 3042$$

$$\frac{\int \frac{1}{(-i\sin(ia+ib\log(cx^n)))^{3/2}}d\log(cx^n)}{n}$$

$$\Big\downarrow \; 3116$$

$$\frac{\int \sqrt{\sinh\left(a+b\log\left(cx^n\right)\right)}d\log\left(cx^n\right)-\frac{2\cosh(a+b\log(cx^n))}{b\sqrt{\sinh(a+b\log(cx^n))}}}{n}$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{-\frac{2\cosh(a+b\log(cx^n))}{b\sqrt{\sinh(a+b\log(cx^n))}}+\int \sqrt{-i\sin\left(ia+ib\log\left(cx^n\right)\right)}d\log\left(cx^n\right)}{n}$$

$$\Big\downarrow \text{ 3121}$$

$$\frac{-\frac{2\cosh(a+b\log(cx^n))}{b\sqrt{\sinh(a+b\log(cx^n))}}+\frac{\sqrt{\sinh(a+b\log(cx^n))}\int \sqrt{i\sinh(a+b\log(cx^n))}d\log(cx^n)}{\sqrt{i\sinh(a+b\log(cx^n))}}}{n}$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{-\frac{2\cosh(a+b\log(cx^n))}{b\sqrt{\sinh(a+b\log(cx^n))}}+\frac{\sqrt{\sinh(a+b\log(cx^n))}\int \sqrt{\sin(ia+ib\log(cx^n))}d\log(cx^n)}{\sqrt{i\sinh(a+b\log(cx^n))}}}{n}$$

$$\Big\downarrow \text{ 3119}$$

$$\frac{-\frac{2\cosh(a+b\log(cx^n))}{b\sqrt{\sinh(a+b\log(cx^n))}}-\frac{2i\sqrt{\sinh(a+b\log(cx^n))}E\left(\frac{1}{2}\left(ia+ib\log(cx^n)-\frac{\pi}{2}\right)|2\right)}{b\sqrt{i\sinh(a+b\log(cx^n))}}}{n}$$

input
```
Int[1/(x*Sinh[a + b*Log[c*x^n]]^(3/2)),x]
```

output
```
((-2*Cosh[a + b*Log[c*x^n]])/(b*Sqrt[Sinh[a + b*Log[c*x^n]]]) - ((2*I)*Ell
ipticE[(I*a - Pi/2 + I*b*Log[c*x^n])/2, 2]*Sqrt[Sinh[a + b*Log[c*x^n]]])/(
b*Sqrt[I*Sinh[a + b*Log[c*x^n]]]))/n
```

Defintions of rubi rules used

rule 3039
```
Int[u_, x_Symbol] :> With[{lst = FunctionOfLog[Cancel[x*u], x]}, Simp[1/lst
[[3]]   Subst[Int[lst[[1]], x], x, Log[lst[[2]]]], x] /;  !FalseQ[lst]] /;
NonsumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3116
```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1))), x] + Simp[(n + 2)/(b^2*(n + 1))   I
nt[(b*Sin[c + d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] &&
 IntegerQ[2*n]
```

rule 3119
```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3121
```
Int[((b_)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*Sin[c + d*x])
^n/Sin[c + d*x]^n   Int[Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] && Lt
Q[-1, n, 1] && IntegerQ[2*n]
```

Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 211 vs. $2(94) = 188$.

Time = 0.20 (sec) , antiderivative size = 212, normalized size of antiderivative = 2.02

method	result
derivativedivides	$\frac{-\sqrt{1-i\sinh(a+b\ln(c\,x^{n}))}\,\sqrt{2}\,\sqrt{1+i\sinh(a+b\ln(c\,x^{n}))}\,\sqrt{i\sinh(a+b\ln(c\,x^{n}))}\,\operatorname{EllipticF}\left(\sqrt{1-i\sinh(a+b\ln(c\,x^{n}))},\frac{\sqrt{}}{2}\right.}{n\cosh}$
default	$\frac{-\sqrt{1-i\sinh(a+b\ln(c\,x^{n}))}\,\sqrt{2}\,\sqrt{1+i\sinh(a+b\ln(c\,x^{n}))}\,\sqrt{i\sinh(a+b\ln(c\,x^{n}))}\,\operatorname{EllipticF}\left(\sqrt{1-i\sinh(a+b\ln(c\,x^{n}))},\frac{\sqrt{}}{2}\right.}{n\cosh}$

input
```
int(1/x/sinh(a+b*ln(c*x^n))^(3/2),x,method=_RETURNVERBOSE)
```

output
```
1/n*(-(1-I*sinh(a+b*ln(c*x^n)))^(1/2)*2^(1/2)*(1+I*sinh(a+b*ln(c*x^n)))^(1
/2)*(I*sinh(a+b*ln(c*x^n)))^(1/2)*EllipticF((1-I*sinh(a+b*ln(c*x^n)))^(1/2
),1/2*2^(1/2))+2*(1-I*sinh(a+b*ln(c*x^n)))^(1/2)*2^(1/2)*(1+I*sinh(a+b*ln(
c*x^n)))^(1/2)*(I*sinh(a+b*ln(c*x^n)))^(1/2)*EllipticE((1-I*sinh(a+b*ln(c*
x^n)))^(1/2),1/2*2^(1/2))-2*cosh(a+b*ln(c*x^n))^2)/cosh(a+b*ln(c*x^n))/sin
h(a+b*ln(c*x^n))^(1/2)/b
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 246 vs. 2(90) = 180.

Time = 0.10 (sec) , antiderivative size = 246, normalized size of antiderivative = 2.34

$$\int \frac{1}{x \sinh^{\frac{3}{2}}(a+b\log(cx^n))}\,dx = -\frac{2\left(\left(\sqrt{2}\cosh(bn\log(x)+b\log(c)+a)^2+2\sqrt{2}\cosh(bn\log(x)+b\log(c)+a)\sinh(bn\log(x)+b\log(c)+a)\right.\right.}{}$$

input
```
integrate(1/x/sinh(a+b*log(c*x^n))^(3/2),x, algorithm="fricas")
```

output
```
-2*((sqrt(2)*cosh(b*n*log(x) + b*log(c) + a)^2 + 2*sqrt(2)*cosh(b*n*log(x)
 + b*log(c) + a)*sinh(b*n*log(x) + b*log(c) + a) + sqrt(2)*sinh(b*n*log(x)
 + b*log(c) + a)^2 - sqrt(2))*weierstrassZeta(4, 0, weierstrassPInverse(4,
 0, cosh(b*n*log(x) + b*log(c) + a) + sinh(b*n*log(x) + b*log(c) + a))) +
2*(cosh(b*n*log(x) + b*log(c) + a)^2 + 2*cosh(b*n*log(x) + b*log(c) + a)*s
inh(b*n*log(x) + b*log(c) + a) + sinh(b*n*log(x) + b*log(c) + a)^2)*sqrt(s
inh(b*n*log(x) + b*log(c) + a)))/(b*n*cosh(b*n*log(x) + b*log(c) + a)^2 +
2*b*n*cosh(b*n*log(x) + b*log(c) + a)*sinh(b*n*log(x) + b*log(c) + a) + b*
n*sinh(b*n*log(x) + b*log(c) + a)^2 - b*n)
```

Sympy [F]

$$\int \frac{1}{x \sinh^{\frac{3}{2}}(a+b\log(cx^n))}\,dx = \int \frac{1}{x \sinh^{\frac{3}{2}}(a+b\log(cx^n))}\,dx$$

input
```
integrate(1/x/sinh(a+b*ln(c*x**n))**(3/2),x)
```

output
```
Integral(1/(x*sinh(a + b*log(c*x**n))**(3/2)), x)
```

Maxima [F]

$$\int \frac{1}{x \sinh^{\frac{3}{2}}(a + b \log(cx^n))} dx = \int \frac{1}{x \sinh(b \log(cx^n) + a)^{\frac{3}{2}}} dx$$

input
```
integrate(1/x/sinh(a+b*log(c*x^n))^(3/2),x, algorithm="maxima")
```

output
```
integrate(1/(x*sinh(b*log(c*x^n) + a)^(3/2)), x)
```

Giac [F]

$$\int \frac{1}{x \sinh^{\frac{3}{2}}(a + b \log(cx^n))} dx = \int \frac{1}{x \sinh(b \log(cx^n) + a)^{\frac{3}{2}}} dx$$

input
```
integrate(1/x/sinh(a+b*log(c*x^n))^(3/2),x, algorithm="giac")
```

output
```
integrate(1/(x*sinh(b*log(c*x^n) + a)^(3/2)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{x \sinh^{\frac{3}{2}}(a + b \log(cx^n))} dx = \int \frac{1}{x \sinh(a + b \ln(c x^n))^{3/2}} dx$$

input
```
int(1/(x*sinh(a + b*log(c*x^n))^(3/2)),x)
```

output
```
int(1/(x*sinh(a + b*log(c*x^n))^(3/2)), x)
```

Reduce [F]

$$\int \frac{1}{x \sinh^{\frac{3}{2}}(a+b\log(cx^n))}\,dx = \int \frac{\sqrt{\sinh(\log(x^n c)\,b+a)}}{\sinh(\log(x^n c)\,b+a)^2 x}dx$$

input
```
int(1/x/sinh(a+b*log(c*x^n))^(3/2),x)
```

output
```
int(sqrt(sinh(log(x**n*c)*b + a))/(sinh(log(x**n*c)*b + a)**2*x),x)
```

3.284 $\int \frac{1}{x \sinh^{\frac{5}{2}}(a+b\log(cx^n))}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 109

$$\int \frac{1}{x \sinh^{\frac{5}{2}}(a+b\log(cx^n))}\,dx = -\frac{2\cosh(a+b\log(cx^n))}{3bn\sinh^{\frac{3}{2}}(a+b\log(cx^n))} + \frac{2i\operatorname{EllipticF}\left(\frac{1}{4}(2ia-\pi+2ib\log(cx^n)),2\right)\sqrt{i\sinh(a+b\log(cx^n))}}{3bn\sqrt{\sinh(a+b\log(cx^n))}}$$

output
```
-2/3*cosh(a+b*ln(c*x^n))/b/n/sinh(a+b*ln(c*x^n))^(3/2)+2/3*I*InverseJacobi
AM(1/2*I*a-1/4*Pi+1/2*I*b*ln(c*x^n),2^(1/2))*(I*sinh(a+b*ln(c*x^n)))^(1/2)
/b/n/sinh(a+b*ln(c*x^n))^(1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 0.08 (sec) , antiderivative size = 122, normalized size of antiderivative = 1.12

$$\int \frac{1}{x\sinh^{\frac{5}{2}}(a+b\log(cx^n))}\,dx =$$
$$-\frac{2\Big(\cosh(a+b\log(cx^n)) + \text{Hypergeometric2F1}\left(\frac{1}{4},\frac{1}{2},\frac{5}{4},\cosh(2(a+b\log(cx^n))) + \sinh(2(a+b\log(c\right.}{3bn\sinh^{\frac{3}{2}}(a+b\log}$$

input
```
Integrate[1/(x*Sinh[a + b*Log[c*x^n]]^(5/2)),x]
```

output
```
(-2*(Cosh[a + b*Log[c*x^n]] + Hypergeometric2F1[1/4, 1/2, 5/4, Cosh[2*(a +
 b*Log[c*x^n])] + Sinh[2*(a + b*Log[c*x^n])]]*Sinh[a + b*Log[c*x^n]]*Sqrt[
1 - Cosh[2*(a + b*Log[c*x^n])] - Sinh[2*(a + b*Log[c*x^n])]]))/(3*b*n*Sinh
[a + b*Log[c*x^n]]^(3/2))
```

Rubi [A] (verified)

Time = 0.42 (sec) , antiderivative size = 109, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.368$, Rules used = {3039, 3042, 3116, 3042, 3121, 3042, 3120}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x\sinh^{\frac{5}{2}}(a+b\log(cx^n))}\,dx$$
$$\Big\downarrow \text{3039}$$
$$\frac{\int \frac{1}{\sinh^{\frac{5}{2}}(a+b\log(cx^n))}d\log(cx^n)}{n}$$
$$\Big\downarrow \text{3042}$$
$$\frac{\int \frac{1}{(-i\sin(ia+ib\log(cx^n)))^{5/2}}d\log(cx^n)}{n}$$

$$\Big\downarrow \text{3116}$$

$$\frac{-\frac{1}{3}\int \frac{1}{\sqrt{\sinh(a+b\log(cx^n))}}d\log(cx^n)-\frac{2\cosh(a+b\log(cx^n))}{3b\sinh^{\frac{3}{2}}(a+b\log(cx^n))}}{n}$$

$$\Big\downarrow \text{3042}$$

$$\frac{-\frac{2\cosh(a+b\log(cx^n))}{3b\sinh^{\frac{3}{2}}(a+b\log(cx^n))}-\frac{1}{3}\int \frac{1}{\sqrt{-i\sin(ia+ib\log(cx^n))}}d\log(cx^n)}{n}$$

$$\Big\downarrow \text{3121}$$

$$\frac{-\frac{2\cosh(a+b\log(cx^n))}{3b\sinh^{\frac{3}{2}}(a+b\log(cx^n))}-\frac{\sqrt{i\sinh(a+b\log(cx^n))}\int \frac{1}{\sqrt{i\sinh(a+b\log(cx^n))}}d\log(cx^n)}{3\sqrt{\sinh(a+b\log(cx^n))}}}{n}$$

$$\Big\downarrow \text{3042}$$

$$\frac{-\frac{2\cosh(a+b\log(cx^n))}{3b\sinh^{\frac{3}{2}}(a+b\log(cx^n))}-\frac{\sqrt{i\sinh(a+b\log(cx^n))}\int \frac{1}{\sqrt{\sin(ia+ib\log(cx^n))}}d\log(cx^n)}{3\sqrt{\sinh(a+b\log(cx^n))}}}{n}$$

$$\Big\downarrow \text{3120}$$

$$\frac{-\frac{2\cosh(a+b\log(cx^n))}{3b\sinh^{\frac{3}{2}}(a+b\log(cx^n))}+\frac{2i\sqrt{i\sinh(a+b\log(cx^n))}\,\text{EllipticF}\left(\frac{1}{2}\left(ia+ib\log(cx^n)-\frac{\pi}{2}\right),2\right)}{3b\sqrt{\sinh(a+b\log(cx^n))}}}{n}$$

input
```
Int[1/(x*Sinh[a + b*Log[c*x^n]]^(5/2)),x]
```

output
```
((-2*Cosh[a + b*Log[c*x^n]])/(3*b*Sinh[a + b*Log[c*x^n]]^(3/2)) + (((2*I)/
3)*EllipticF[(I*a - Pi/2 + I*b*Log[c*x^n])/2, 2]*Sqrt[I*Sinh[a + b*Log[c*x
^n]]])/(b*Sqrt[Sinh[a + b*Log[c*x^n]]]))/n
```

Defintions of rubi rules used

rule 3039
```
Int[u_, x_Symbol] :> With[{lst = FunctionOfLog[Cancel[x*u], x]}, Simp[1/lst
[[3]]   Subst[Int[lst[[1]], x], x, Log[lst[[2]]]], x] /;  !FalseQ[lst]] /;
NonsumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3116
```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1))), x] + Simp[(n + 2)/(b^2*(n + 1))   I
nt[(b*Sin[c + d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] &&
 IntegerQ[2*n]
```

rule 3120
```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3121
```
Int[((b_)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*Sin[c + d*x])
^n/Sin[c + d*x]^n   Int[Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] && Lt
Q[-1, n, 1] && IntegerQ[2*n]
```

Maple [A] (verified)

Time = 0.19 (sec) , antiderivative size = 144, normalized size of antiderivative = 1.32

method	result
derivativedivides	$-\frac{i\sqrt{1-i\sinh(a+b\ln(c\,x^n))}\,\sqrt{2}\,\sqrt{1+i\sinh(a+b\ln(c\,x^n))}\,\sqrt{i\sinh(a+b\ln(c\,x^n))}\,\operatorname{EllipticF}\left(\sqrt{1-i\sinh(a+b\ln(c\,x^n))},\right.}{3n\sinh(a+b\ln(c\,x^n))^{\frac{3}{2}}\cosh(a+b\ln(c\,x^n))b}$
default	$-\frac{i\sqrt{1-i\sinh(a+b\ln(c\,x^n))}\,\sqrt{2}\,\sqrt{1+i\sinh(a+b\ln(c\,x^n))}\,\sqrt{i\sinh(a+b\ln(c\,x^n))}\,\operatorname{EllipticF}\left(\sqrt{1-i\sinh(a+b\ln(c\,x^n))},\right.}{3n\sinh(a+b\ln(c\,x^n))^{\frac{3}{2}}\cosh(a+b\ln(c\,x^n))b}$

input
```
int(1/x/sinh(a+b*ln(c*x^n))^(5/2),x,method=_RETURNVERBOSE)
```

output
```
-1/3/n/sinh(a+b*ln(c*x^n))^(3/2)*(I*(1-I*sinh(a+b*ln(c*x^n)))^(1/2)*2^(1/2
)*(1+I*sinh(a+b*ln(c*x^n)))^(1/2)*(I*sinh(a+b*ln(c*x^n)))^(1/2)*EllipticF(
(1-I*sinh(a+b*ln(c*x^n)))^(1/2),1/2*2^(1/2))*sinh(a+b*ln(c*x^n))+2*cosh(a+
b*ln(c*x^n))^2)/cosh(a+b*ln(c*x^n))/b
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 504 vs. $2(89) = 178$.

Time = 0.13 (sec) , antiderivative size = 504, normalized size of antiderivative = 4.62

$$\int \frac{1}{x\sinh^{\frac{5}{2}}(a+b\log(cx^n))}\,dx = \text{Too large to display}$$

input
```
integrate(1/x/sinh(a+b*log(c*x^n))^(5/2),x, algorithm="fricas")
```

output
```
-2/3*((sqrt(2)*cosh(b*n*log(x) + b*log(c) + a)^4 + 4*sqrt(2)*cosh(b*n*log(
x) + b*log(c) + a)*sinh(b*n*log(x) + b*log(c) + a)^3 + sqrt(2)*sinh(b*n*lo
g(x) + b*log(c) + a)^4 + 2*(3*sqrt(2)*cosh(b*n*log(x) + b*log(c) + a)^2 -
sqrt(2))*sinh(b*n*log(x) + b*log(c) + a)^2 - 2*sqrt(2)*cosh(b*n*log(x) + b
*log(c) + a)^2 + 4*(sqrt(2)*cosh(b*n*log(x) + b*log(c) + a)^3 - sqrt(2)*co
sh(b*n*log(x) + b*log(c) + a))*sinh(b*n*log(x) + b*log(c) + a) + sqrt(2))*
weierstrassPInverse(4, 0, cosh(b*n*log(x) + b*log(c) + a) + sinh(b*n*log(x
) + b*log(c) + a)) + 2*(cosh(b*n*log(x) + b*log(c) + a)^3 + 3*cosh(b*n*log
(x) + b*log(c) + a)*sinh(b*n*log(x) + b*log(c) + a)^2 + sinh(b*n*log(x) +
b*log(c) + a)^3 + (3*cosh(b*n*log(x) + b*log(c) + a)^2 + 1)*sinh(b*n*log(x
) + b*log(c) + a) + cosh(b*n*log(x) + b*log(c) + a))*sqrt(sinh(b*n*log(x)
+ b*log(c) + a)))/(b*n*cosh(b*n*log(x) + b*log(c) + a)^4 + 4*b*n*cosh(b*n*
log(x) + b*log(c) + a)*sinh(b*n*log(x) + b*log(c) + a)^3 + b*n*sinh(b*n*lo
g(x) + b*log(c) + a)^4 - 2*b*n*cosh(b*n*log(x) + b*log(c) + a)^2 + 2*(3*b*
n*cosh(b*n*log(x) + b*log(c) + a)^2 - b*n)*sinh(b*n*log(x) + b*log(c) + a)
^2 + b*n + 4*(b*n*cosh(b*n*log(x) + b*log(c) + a)^3 - b*n*cosh(b*n*log(x)
+ b*log(c) + a))*sinh(b*n*log(x) + b*log(c) + a))
```

Sympy [F(-1)]

Timed out.

$$\int \frac{1}{x\sinh^{\frac{5}{2}}(a+b\log(cx^n))}\,dx = \text{Timed out}$$

input
```
integrate(1/x/sinh(a+b*ln(c*x**n))**(5/2),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{1}{x \sinh^{\frac{5}{2}}(a + b \log(cx^n))} dx = \int \frac{1}{x \sinh(b \log(cx^n) + a)^{\frac{5}{2}}} dx$$

input
```
integrate(1/x/sinh(a+b*log(c*x^n))^(5/2),x, algorithm="maxima")
```

output
```
integrate(1/(x*sinh(b*log(c*x^n) + a)^(5/2)), x)
```

Giac [F]

$$\int \frac{1}{x \sinh^{\frac{5}{2}}(a + b \log(cx^n))} dx = \int \frac{1}{x \sinh(b \log(cx^n) + a)^{\frac{5}{2}}} dx$$

input
```
integrate(1/x/sinh(a+b*log(c*x^n))^(5/2),x, algorithm="giac")
```

output
```
integrate(1/(x*sinh(b*log(c*x^n) + a)^(5/2)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{x \sinh^{\frac{5}{2}}(a + b \log(cx^n))} dx = \int \frac{1}{x \sinh(a + b \ln(c x^n))^{5/2}} dx$$

input
```
int(1/(x*sinh(a + b*log(c*x^n))^(5/2)),x)
```

output
```
int(1/(x*sinh(a + b*log(c*x^n))^(5/2)), x)
```

Reduce [F]

$$\int \frac{1}{x\sinh^{\frac{5}{2}}(a+b\log(cx^n))}\,dx=\int \frac{\sqrt{\sinh(\log(x^n c)\,b+a)}}{\sinh(\log(x^n c)\,b+a)^3 x}dx$$

input
```
int(1/x/sinh(a+b*log(c*x^n))^(5/2),x)
```

output
```
int(sqrt(sinh(log(x**n*c)*b + a))/(sinh(log(x**n*c)*b + a)**3*x),x)
```

3.285 $\int \sinh^{\frac{5}{2}}\left(a+\frac{2\log(cx^n)}{n}\right)\,dx$

Optimal result

Integrand size = 18, antiderivative size = 209

$$\int \sinh^{\frac{5}{2}}\left(a+\frac{2\log(cx^n)}{n}\right)\,dx = -\frac{1}{4}x\sinh^{\frac{5}{2}}\left(a+\frac{2\log(cx^n)}{n}\right) - \frac{5e^{-2a}x(cx^n)^{-4/n}\sinh^{\frac{5}{2}}\left(a+\frac{2\log(cx^n)}{n}\right)}{4\left(1-e^{-2a}(cx^n)^{-4/n}\right)^2} + \frac{5x\sinh^{\frac{5}{2}}\left(a+\frac{2\log(cx^n)}{n}\right)}{12\left(1-e^{-2a}(cx^n)^{-4/n}\right)} - \frac{5e^{-3a}x(cx^n)^{-6/n}\csc^{-1}\left(e^a(cx^n)^{2/n}\right)\sinh^{\frac{5}{2}}\left(a+\frac{2\log(cx^n)}{n}\right)}{4\left(1-e^{-2a}(cx^n)^{-4/n}\right)^{5/2}}$$

output
```
-1/4*x*sinh(a+2*ln(c*x^n)/n)^(5/2)-5/4*x*sinh(a+2*ln(c*x^n)/n)^(5/2)/exp(2
*a)/((c*x^n)^(4/n))/(1-1/exp(2*a)/((c*x^n)^(4/n)))^2+5*x*sinh(a+2*ln(c*x^n
)/n)^(5/2)/(12-12/exp(2*a)/((c*x^n)^(4/n)))-5/4*x*arccsc(exp(a)*(c*x^n)^(2
/n))*sinh(a+2*ln(c*x^n)/n)^(5/2)/exp(3*a)/((c*x^n)^(6/n))/(1-1/exp(2*a)/((
c*x^n)^(4/n)))^(5/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 0.21 (sec) , antiderivative size = 86, normalized size of antiderivative = 0.41

$$\int \sinh^{\frac{5}{2}}\left(a+\frac{2\log\left(cx^n\right)}{n}\right)\,dx = \frac{1}{14}e^{2a}x\left(cx^n\right)^{4/n}\left(-1+e^{2a}\left(cx^n\right)^{4/n}\right)\text{Hypergeometric2F1}\left(2,\frac{7}{2},\frac{9}{2},1-e^{2a}\left(cx^n\right)^{4/n}\right)\sinh^{\frac{5}{2}}\left(a+\frac{2\log\left(cx^n\right)}{n}\right)$$

input
```
Integrate[Sinh[a + (2*Log[c*x^n])/n]^(5/2),x]
```

output
```
(E^(2*a)*x*(c*x^n)^(4/n)*(-1 + E^(2*a)*(c*x^n)^(4/n))*Hypergeometric2F1[2,
 7/2, 9/2, 1 - E^(2*a)*(c*x^n)^(4/n)]*Sinh[a + (2*Log[c*x^n])/n]^(5/2))/14
```

Rubi [A] (warning: unable to verify)

Time = 0.52 (sec) , antiderivative size = 208, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.444$, Rules used = {6051, 6059, 876, 872, 868, 773, 247, 223}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sinh^{\frac{5}{2}}\left(a+\frac{2\log\left(cx^n\right)}{n}\right)\,dx$$

$$\Big\downarrow \text{ 6051}$$

$$\frac{x\left(cx^n\right)^{-1/n}\int \left(cx^n\right)^{\frac{1}{n}-1}\sinh^{\frac{5}{2}}\left(a+\frac{2\log(cx^n)}{n}\right)d(cx^n)}{n}$$

$$\Big\downarrow \text{ 6059}$$

$$\frac{x\left(cx^n\right)^{-6/n}\sinh^{\frac{5}{2}}\left(a+\frac{2\log(cx^n)}{n}\right)\int \left(cx^n\right)^{\frac{6}{n}-1}\left(1-e^{-2a}\left(cx^n\right)^{-4/n}\right)^{5/2}d(cx^n)}{n\left(1-e^{-2a}\left(cx^n\right)^{-4/n}\right)^{5/2}}$$

$$\Big\downarrow \text{ 876}$$

$$\frac{x(cx^n)^{-6/n}\sinh^{\frac{5}{2}}\left(a+\frac{2\log(cx^n)}{n}\right)\left(\frac{5}{2}\int (cx^n)^{\frac{6}{n}-1}\left(1-e^{-2a}(cx^n)^{-4/n}\right)^{3/2}d(cx^n)-\frac{1}{4}n(cx^n)^{6/n}\left(1-e^{-2a}(cx^n)^{-4}\right.\right.}{n\left(1-e^{-2a}\left(cx^n\right)^{-4/n}\right)^{5/2}}$$

$$\Big\downarrow \text{872}$$

$$\frac{x(cx^n)^{-6/n}\sinh^{\frac{5}{2}}\left(a+\frac{2\log(cx^n)}{n}\right)\left(\frac{5}{2}\left(\frac{1}{6}n(cx^n)^{6/n}\left(1-e^{-2a}(cx^n)^{-4/n}\right)^{3/2}-e^{-2a}\int (cx^n)^{\frac{2}{n}-1}\sqrt{1-e^{-2a}\left(cx^n\right)^{-}}\right.\right.}{n\left(1-e^{-2a}\left(cx^n\right)^{-4/n}\right)^{5/2}}$$

$$\Big\downarrow \text{868}$$

$$\frac{x(cx^n)^{-6/n}\sinh^{\frac{5}{2}}\left(a+\frac{2\log(cx^n)}{n}\right)\left(\frac{5}{2}\left(\frac{1}{6}n(cx^n)^{6/n}\left(1-e^{-2a}(cx^n)^{-4/n}\right)^{3/2}-\frac{1}{2}e^{-2a}n\int\sqrt{1-\frac{e^{-2a}x^{-2n}}{c^2}}d(cx^n)^{2/n}\right.\right.}{n\left(1-e^{-2a}\left(cx^n\right)^{-4/n}\right)^{5/2}}$$

$$\Big\downarrow \text{773}$$

$$\frac{x(cx^n)^{-6/n}\sinh^{\frac{5}{2}}\left(a+\frac{2\log(cx^n)}{n}\right)\left(\frac{5}{2}\left(\frac{1}{2}e^{-2a}n\int\frac{x^{-2n}\sqrt{1-c^2e^{-2a}x^{2n}}}{c^2}d\frac{x^{-n}}{c}+\frac{1}{6}n(cx^n)^{6/n}\left(1-e^{-2a}(cx^n)^{-4/n}\right)^{3/2}\right)-\right.}{n\left(1-e^{-2a}\left(cx^n\right)^{-4/n}\right)^{5/2}}$$

$$\Big\downarrow \text{247}$$

$$\frac{x(cx^n)^{-6/n}\sinh^{\frac{5}{2}}\left(a+\frac{2\log(cx^n)}{n}\right)\left(\frac{5}{2}\left(\frac{1}{2}e^{-2a}n\left(-e^{-2a}\int\frac{1}{\sqrt{1-c^2e^{-2a}x^{2n}}}d\frac{x^{-n}}{c}-\frac{x^{-n}\sqrt{1-e^{-2a}c^2x^{2n}}}{c}\right)+\frac{1}{6}n(cx^n)^{6/n}\left(1\right.\right.\right.}{n\left(1-e^{-2a}\left(cx^n\right)^{-4/n}\right)^{5/2}}$$

$$\Big\downarrow \text{223}$$

$$\frac{x(cx^n)^{-6/n}\left(\frac{5}{2}\left(\frac{1}{2}e^{-2a}n\left(-e^{-a}\arcsin\left(\frac{e^{-a}x^{-n}}{c}\right)-\frac{x^{-n}\sqrt{1-e^{-2a}c^2x^{2n}}}{c}\right)+\frac{1}{6}n(cx^n)^{6/n}\left(1-e^{-2a}(cx^n)^{-4/n}\right)^{3/2}\right)-\frac{1}{4}\right.}{n\left(1-e^{-2a}\left(cx^n\right)^{-4/n}\right)^{5/2}}$$

input
```
Int[Sinh[a + (2*Log[c*x^n])/n]^(5/2),x]
```

output

```
(x*(-1/4*(n*(c*x^n)^(6/n)*(1 - 1/(E^(2*a)*(c*x^n)^(4/n)))^(5/2)) + (5*((n*
(c*x^n)^(6/n)*(1 - 1/(E^(2*a)*(c*x^n)^(4/n)))^(3/2))/6 + (n*(-(Sqrt[1 - (c
^2*x^(2*n))/E^(2*a)]/(c*x^n)) - ArcSin[1/(c*E^a*x^n)]/E^a))/(2*E^(2*a))))/
2)*Sinh[a + (2*Log[c*x^n])/n]^(5/2))/(n*(c*x^n)^(6/n)*(1 - 1/(E^(2*a)*(c*x
^n)^(4/n)))^(5/2))
```

Defintions of rubi rules used

rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

rule 247

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(c*x)^
(m + 1)*((a + b*x^2)^p/(c*(m + 1))), x] - Simp[2*b*(p/(c^2*(m + 1)))   Int[
(c*x)^(m + 2)*(a + b*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && GtQ[p,
0] && LtQ[m, -1] &&  !ILtQ[(m + 2*p + 3)/2, 0] && IntBinomialQ[a, b, c, 2,
m, p, x]
```

rule 773

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Subst[Int[(a + b/x^n)^p/x^
2, x], x, 1/x] /; FreeQ[{a, b, p}, x] && ILtQ[n, 0] &&  !IntegerQ[p]
```

rule 868

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[1/(m + 1)
 Subst[Int[(a + b*x^Simplify[n/(m + 1)])^p, x], x, x^(m + 1)], x] /; FreeQ[
{a, b, m, n, p}, x] && IntegerQ[Simplify[n/(m + 1)]] &&  !IntegerQ[n]
```

rule 872

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x^(m + 1)*(
(a + b*x^n)^p/(m + 1)), x] - Simp[b*n*(p/(m + 1))   Int[x^(m + n)*(a + b*x^
n)^(p - 1), x], x] /; FreeQ[{a, b, m, n}, x] && EqQ[(m + 1)/n + p, 0] && Gt
Q[p, 0]
```

rule 876

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*
x)^(m + 1)*((a + b*x^n)^p/(c*(m + n*p + 1))), x] + Simp[a*n*(p/(m + n*p + 1
))   Int[(c*x)^m*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c, m, n}, x] &
& IntegerQ[p + Simplify[(m + 1)/n]] && GtQ[p, 0] && NeQ[m + n*p + 1, 0]
```

rule 6051
```
Int[Sinh[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(d_.)]^(p_.), x_Symbol] :> S
imp[x/(n*(c*x^n)^(1/n))   Subst[Int[x^(1/n - 1)*Sinh[d*(a + b*Log[x])]^p, x
], x, c*x^n], x] /; FreeQ[{a, b, c, d, n, p}, x] && (NeQ[c, 1] || NeQ[n, 1]
)
```

rule 6059
```
Int[((e_.)*(x_))^(m_.)*Sinh[((a_.) + Log[x_]*(b_.))*(d_.)]^(p_), x_Symbol]
:> Simp[Sinh[d*(a + b*Log[x])]^p/(x^(b*d*p)*(1 - 1/(E^(2*a*d)*x^(2*b*d)))^p
)   Int[(e*x)^m*x^(b*d*p)*(1 - 1/(E^(2*a*d)*x^(2*b*d)))^p, x], x] /; FreeQ[
{a, b, d, e, m, p}, x] &&  !IntegerQ[p]
```

Maple [F]

$$\int \sinh\left(a+\frac{2\ln\left(c\,x^{n}\right)}{n}\right)^{\frac{5}{2}}dx$$

input
```
int(sinh(a+2*ln(c*x^n)/n)^(5/2),x)
```

output
```
int(sinh(a+2*ln(c*x^n)/n)^(5/2),x)
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 162, normalized size of antiderivative = 0.78

$$\int \sinh^{\frac{5}{2}}\left(a+\frac{2\log\left(cx^{n}\right)}{n}\right)\,dx = \frac{\left(15\,\sqrt{2}x^{3}\arctan\left(\sqrt{2}\sqrt{\frac{1}{2}}x\sqrt{\frac{x^{4}e^{\left(\frac{2\,(an+2\,\log(c))}{n}\right)}-1}{x^{2}}}\right)e^{\left(\frac{3\,(an+2\,\log(c))}{2\,n}\right)}+2\,\sqrt{\frac{1}{2}}\left(2\,x^{8}e^{\left(\frac{4\,(an+2\,\log(c))}{n}\right)}-14\,x^{4}e^{\left(2\right.}\right.\right.}{96\,x^{3}}$$

input
```
integrate(sinh(a+2*log(c*x^n)/n)^(5/2),x, algorithm="fricas")
```

output
```
1/96*(15*sqrt(2)*x^3*arctan(sqrt(2)*sqrt(1/2)*x*sqrt((x^4*e^(2*(a*n + 2*lo
g(c))/n) - 1)/x^2))*e^(3/2*(a*n + 2*log(c))/n) + 2*sqrt(1/2)*(2*x^8*e^(4*(
a*n + 2*log(c))/n) - 14*x^4*e^(2*(a*n + 2*log(c))/n) - 3)*sqrt((x^4*e^(2*(
a*n + 2*log(c))/n) - 1)/x^2)*e^(-1/2*(a*n + 2*log(c))/n))*e^(-2*(a*n + 2*l
og(c))/n)/x^3
```

Sympy [F(-1)]

Timed out.

$$\int \sinh^{\frac{5}{2}}\left(a+\frac{2\log\left(cx^n\right)}{n}\right)\,dx=\text{Timed out}$$

input
```
integrate(sinh(a+2*ln(c*x**n)/n)**(5/2),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \sinh^{\frac{5}{2}}\left(a+\frac{2\log\left(cx^n\right)}{n}\right)\,dx=\int \sinh\left(a+\frac{2\,\log\left(cx^n\right)}{n}\right)^{\frac{5}{2}}\,dx$$

input
```
integrate(sinh(a+2*log(c*x^n)/n)^(5/2),x, algorithm="maxima")
```

output
```
integrate(sinh(a + 2*log(c*x^n)/n)^(5/2), x)
```

Giac [A] (verification not implemented)

Time = 0.43 (sec) , antiderivative size = 171, normalized size of antiderivative = 0.82

$$\int \sinh^{\frac{5}{2}}\left(a+\frac{2\log(cx^n)}{n}\right)\,dx = \frac{1}{48}\sqrt{2}\sqrt{c^{\frac{6}{n}}x^6e^{(3a)}-c^{\frac{2}{n}}x^2e^a}\,c^{\frac{2}{n}}x^3e^a + \frac{\sqrt{2}\left(15\arctan\left(\sqrt{c^{\frac{4}{n}}x^4e^{(3a)}-e^a}\,e^{\left(-\frac{1}{2}a\right)}\right)e^{\left(-\frac{3}{2}a\right)}-14\sqrt{c^{\frac{4}{n}}x^4e^{(3a)}-e^a}\,e^{(-2a)}-\frac{3\sqrt{c^{\frac{4}{n}}x^4e^{(3a)}-e^a}\,e^{(-4a)}}{c^{\frac{4}{n}}x^4}\right)e}{96\,c^{\left(\frac{1}{n}\right)}\mathrm{sgn}(x)}$$

input
```
integrate(sinh(a+2*log(c*x^n)/n)^(5/2),x, algorithm="giac")
```

output
```
1/48*sqrt(2)*sqrt(c^(6/n)*x^6*e^(3*a) - c^(2/n)*x^2*e^a)*c^(2/n)*x^3*e^a +
 1/96*sqrt(2)*(15*arctan(sqrt(c^(4/n)*x^4*e^(3*a) - e^a)*e^(-1/2*a))*e^(-3
/2*a) - 14*sqrt(c^(4/n)*x^4*e^(3*a) - e^a)*e^(-2*a) - 3*sqrt(c^(4/n)*x^4*e
^(3*a) - e^a)*e^(-4*a)/(c^(4/n)*x^4))*e^a/(c^(1/n)*sgn(x))
```

Mupad [F(-1)]

Timed out.

$$\int \sinh^{\frac{5}{2}}\left(a+\frac{2\log(cx^n)}{n}\right)\,dx = \int \sinh\left(a+\frac{2\,\ln(c\,x^n)}{n}\right)^{5/2}\,dx$$

input
```
int(sinh(a + (2*log(c*x^n))/n)^(5/2),x)
```

output
```
int(sinh(a + (2*log(c*x^n))/n)^(5/2), x)
```

Reduce [F]

$$\int \sinh^{\frac{5}{2}}\left(a+\frac{2\log\left(cx^n\right)}{n}\right)\,dx = \sqrt{\sinh\left(\frac{2\log\left(x^nc\right)+an}{n}\right)}\sinh\left(\frac{2\log(x^nc)+an}{n}\right)^2 x - 5\left(\int\sqrt{\sinh\left(\frac{2\log\left(x^nc\right)+an}{n}\right)}\cosh\left(\frac{2\log(x^nc)+an}{n}\right)\sinh\left(\frac{2\log(x^nc)+an}{n}\right)dx\right)$$

input
```
int(sinh(a+2*log(c*x^n)/n)^(5/2),x)
```

output
```
sqrt(sinh((2*log(x**n*c) + a*n)/n))*sinh((2*log(x**n*c) + a*n)/n)**2*x - 5
*int(sqrt(sinh((2*log(x**n*c) + a*n)/n))*cosh((2*log(x**n*c) + a*n)/n)*sin
h((2*log(x**n*c) + a*n)/n),x)
```

3.286 $\int \sqrt{\sinh \left(a+\frac{2 \log(cx^n)}{n}\right)}\,dx$

Optimal result

Integrand size = 18, antiderivative size = 103

$$\int \sqrt{\sinh \left(a+\frac{2 \log (cx^n)}{n}\right)}\,dx = \frac{1}{2}x\sqrt{\sinh \left(a+\frac{2 \log (cx^n)}{n}\right)} + \frac{e^{-a}x(cx^n)^{-2/n}\csc^{-1}\left(e^a(cx^n)^{2/n}\right)\sqrt{\sinh \left(a+\frac{2\log(cx^n)}{n}\right)}}{2\sqrt{1-e^{-2a}(cx^n)^{-4/n}}}$$

output
```
1/2*x*sinh(a+2*ln(c*x^n)/n)^(1/2)+1/2*x*arccsc(exp(a)*(c*x^n)^(2/n))*sinh(
a+2*ln(c*x^n)/n)^(1/2)/exp(a)/((c*x^n)^(2/n))/(1-1/exp(2*a)/((c*x^n)^(4/n)
))^(1/2)
```

Mathematica [A] (verified)

Time = 0.17 (sec) , antiderivative size = 74, normalized size of antiderivative = 0.72

$$\int \sqrt{\sinh \left(a+\frac{2 \log (c x^n)}{n}\right)} \, dx = \frac{1}{2} x \left(1-\frac{\arctan \left(\sqrt{-1+e^{2 a} (c x^n)^{4/n}}\right)}{\sqrt{-1+e^{2 a} (c x^n)^{4/n}}}\right) \sqrt{\sinh \left(a+\frac{2 \log (c x^n)}{n}\right)}$$

input
```
Integrate[Sqrt[Sinh[a + (2*Log[c*x^n])/n]],x]
```

output
```
(x*(1 - ArcTan[Sqrt[-1 + E^(2*a)*(c*x^n)^(4/n)]]/Sqrt[-1 + E^(2*a)*(c*x^n)
^(4/n)])*Sqrt[Sinh[a + (2*Log[c*x^n])/n]])/2
```

Rubi [A] (warning: unable to verify)

Time = 0.40 (sec) , antiderivative size = 112, normalized size of antiderivative = 1.09, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {6051, 6059, 868, 773, 247, 223}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{\sinh \left(a+\frac{2 \log (c x^n)}{n}\right)} \, dx$$

$$\Big\downarrow \text{6051}$$

$$\frac{x (c x^n)^{-1/n} \int (c x^n)^{\frac{1}{n}-1} \sqrt{\sinh \left(a+\frac{2 \log (c x^n)}{n}\right)} d(c x^n)}{n}$$

$$\Big\downarrow \text{6059}$$

$$\frac{x (c x^n)^{-2/n} \sqrt{\sinh \left(a+\frac{2 \log (c x^n)}{n}\right)} \int (c x^n)^{\frac{2}{n}-1} \sqrt{1-e^{-2 a} (c x^n)^{-4/n}} d(c x^n)}{n \sqrt{1-e^{-2 a} (c x^n)^{-4/n}}}$$

$$\Big\downarrow \text{868}$$

$$\frac{x(cx^n)^{-2/n}\sqrt{\sinh\left(a+\frac{2\log(cx^n)}{n}\right)}\int\sqrt{1-\frac{e^{-2a}x^{-2n}}{c^2}}d(cx^n)^{2/n}}{2\sqrt{1-e^{-2a}\left(cx^n\right)^{-4/n}}}$$

$$\Big\downarrow \text{773}$$

$$-\frac{x(cx^n)^{-2/n}\sqrt{\sinh\left(a+\frac{2\log(cx^n)}{n}\right)}\int\frac{x^{-2n}\sqrt{1-c^2e^{-2a}x^{2n}}}{c^2}d\frac{x^{-n}}{c}}{2\sqrt{1-e^{-2a}\left(cx^n\right)^{-4/n}}}$$

$$\Big\downarrow \text{247}$$

$$-\frac{x(cx^n)^{-2/n}\sqrt{\sinh\left(a+\frac{2\log(cx^n)}{n}\right)}\left(-e^{-2a}\int\frac{1}{\sqrt{1-c^2e^{-2a}x^{2n}}}d\frac{x^{-n}}{c}-\frac{x^{-n}\sqrt{1-e^{-2a}c^2x^{2n}}}{c}\right)}{2\sqrt{1-e^{-2a}\left(cx^n\right)^{-4/n}}}$$

$$\Big\downarrow \text{223}$$

$$-\frac{x(cx^n)^{-2/n}\left(-e^{-a}\arcsin\left(\frac{e^{-a}x^{-n}}{c}\right)-\frac{x^{-n}\sqrt{1-e^{-2a}c^2x^{2n}}}{c}\right)\sqrt{\sinh\left(a+\frac{2\log(cx^n)}{n}\right)}}{2\sqrt{1-e^{-2a}\left(cx^n\right)^{-4/n}}}$$

input
```
Int[Sqrt[Sinh[a + (2*Log[c*x^n])/n]],x]
```

output
```
-1/2*(x*(-(Sqrt[1 - (c^2*x^(2*n))/E^(2*a)]/(c*x^n)) - ArcSin[1/(c*E^a*x^n)
]/E^a)*Sqrt[Sinh[a + (2*Log[c*x^n])/n]])/((c*x^n)^(2/n)*Sqrt[1 - 1/(E^(2*a
)*(c*x^n)^(4/n))])
```

Defintions of rubi rules used

rule 223
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

rule 247
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(c*x)^
(m + 1)*((a + b*x^2)^p/(c*(m + 1))), x] - Simp[2*b*(p/(c^2*(m + 1)))   Int[
(c*x)^(m + 2)*(a + b*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && GtQ[p,
0] && LtQ[m, -1] &&  !ILtQ[(m + 2*p + 3)/2, 0] && IntBinomialQ[a, b, c, 2,
m, p, x]
```

rule 773
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Subst[Int[(a + b/x^n)^p/x^
2, x], x, 1/x] /; FreeQ[{a, b, p}, x] && ILtQ[n, 0] &&  !IntegerQ[p]
```

rule 868
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[1/(m + 1)
 Subst[Int[(a + b*x^Simplify[n/(m + 1)])^p, x], x, x^(m + 1)], x] /; FreeQ[
{a, b, m, n, p}, x] && IntegerQ[Simplify[n/(m + 1)]] &&  !IntegerQ[n]
```

rule 6051
```
Int[Sinh[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(d_.)]^(p_.), x_Symbol] :> S
imp[x/(n*(c*x^n)^(1/n))   Subst[Int[x^(1/n - 1)*Sinh[d*(a + b*Log[x])]^p, x
], x, c*x^n], x] /; FreeQ[{a, b, c, d, n, p}, x] && (NeQ[c, 1] || NeQ[n, 1]
)
```

rule 6059
```
Int[((e_.)*(x_))^(m_.)*Sinh[((a_.) + Log[x_]*(b_.))*(d_.)]^(p_), x_Symbol]
:> Simp[Sinh[d*(a + b*Log[x])]^p/(x^(b*d*p)*(1 - 1/(E^(2*a*d)*x^(2*b*d)))^p
)   Int[(e*x)^m*x^(b*d*p)*(1 - 1/(E^(2*a*d)*x^(2*b*d)))^p, x], x] /; FreeQ[
{a, b, d, e, m, p}, x] &&  !IntegerQ[p]
```

Maple [F]

$$\int \sqrt{\sinh \left(a+\frac{2 \ln \left(c\, x^{n}\right)}{n}\right)}d x$$

input
```
int(sinh(a+2*ln(c*x^n)/n)^(1/2),x)
```

output
```
int(sinh(a+2*ln(c*x^n)/n)^(1/2),x)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 117, normalized size of antiderivative = 1.14

$$\int \sqrt{\sinh \left(a+\frac{2 \log \left(c x^{n}\right)}{n}\right)}\, dx$$
$$=\frac{1}{4}\left(2 \sqrt{\frac{1}{2}} x \sqrt{\frac{x^{4} e^{\left(\frac{2\,(a n+2\,\log (c))}{n}\right)}-1}{x^{2}}}\, e^{\left(\frac{a n+2\,\log (c)}{2\,n}\right)}-\sqrt{2} \arctan \left(\sqrt{2} \sqrt{\frac{1}{2}} x \sqrt{\frac{x^{4} e^{\left(\frac{2\,(a n+2\,\log (c))}{n}\right)}-1}{x^{2}}}\right) e^{\left(\frac{a n+2\,\log}{2\,n}\right.}$$

input
```
integrate(sinh(a+2*log(c*x^n)/n)^(1/2),x, algorithm="fricas")
```

output
```
1/4*(2*sqrt(1/2)*x*sqrt((x^4*e^(2*(a*n + 2*log(c))/n) - 1)/x^2)*e^(1/2*(a*
n + 2*log(c))/n) - sqrt(2)*arctan(sqrt(2)*sqrt(1/2)*x*sqrt((x^4*e^(2*(a*n
+ 2*log(c))/n) - 1)/x^2))*e^(1/2*(a*n + 2*log(c))/n))*e^(-(a*n + 2*log(c))
/n)
```

Sympy [F]

$$\int \sqrt{\sinh \left(a+\frac{2 \log \left(c x^{n}\right)}{n}\right)}\, dx=\int \sqrt{\sinh \left(a+\frac{2 \log \left(c x^{n}\right)}{n}\right)}\, dx$$

input
```
integrate(sinh(a+2*ln(c*x**n)/n)**(1/2),x)
```

output
```
Integral(sqrt(sinh(a + 2*log(c*x**n)/n)), x)
```

Maxima [F]

$$\int \sqrt{\sinh \left(a + \frac{2 \log \left(cx^n\right)}{n}\right)}\, dx = \int \sqrt{\sinh \left(a + \frac{2 \log \left(cx^n\right)}{n}\right)}\, dx$$

input
```
integrate(sinh(a+2*log(c*x^n)/n)^(1/2),x, algorithm="maxima")
```

output
```
integrate(sqrt(sinh(a + 2*log(c*x^n)/n)), x)
```

Giac [F]

$$\int \sqrt{\sinh \left(a + \frac{2 \log \left(cx^n\right)}{n}\right)}\, dx = \int \sqrt{\sinh \left(a + \frac{2 \log \left(cx^n\right)}{n}\right)}\, dx$$

input
```
integrate(sinh(a+2*log(c*x^n)/n)^(1/2),x, algorithm="giac")
```

output
```
sage0*x
```

Mupad [F(-1)]

Timed out.

$$\int \sqrt{\sinh \left(a + \frac{2 \log \left(cx^n\right)}{n}\right)}\, dx = \int \sqrt{\sinh \left(a + \frac{2 \ln \left(c\, x^n\right)}{n}\right)}\, dx$$

input
```
int(sinh(a + (2*log(c*x^n))/n)^(1/2),x)
```

output
```
int(sinh(a + (2*log(c*x^n))/n)^(1/2), x)
```

Reduce [F]

$$\int \sqrt{\sinh \left(a+\frac{2\log \left(cx^{n}\right)}{n}\right)}\,dx = \sqrt{\sinh \left(\frac{2\log \left(x^{n}c\right)+an}{n}\right)}\,x - \left(\int \frac{\sqrt{\sinh \left(\frac{2\log(x^{n}c)+an}{n}\right)}\cosh \left(\frac{2\log(x^{n}c)+an}{n}\right)}{\sinh \left(\frac{2\log(x^{n}c)+an}{n}\right)}dx\right)$$

input
```
int(sinh(a+2*log(c*x^n)/n)^(1/2),x)
```

output
```
sqrt(sinh((2*log(x**n*c) + a*n)/n))*x - int((sqrt(sinh((2*log(x**n*c) + a*
n)/n))*cosh((2*log(x**n*c) + a*n)/n))/sinh((2*log(x**n*c) + a*n)/n),x)
```

3.287 $\int \frac{1}{\sinh^{\frac{3}{2}}\left(a+\frac{2\log(cx^n)}{n}\right)}\,dx$

Optimal result

Integrand size = 18, antiderivative size = 43

$$\int \frac{1}{\sinh^{\frac{3}{2}}\left(a+\frac{2\log(cx^n)}{n}\right)}\,dx = -\frac{x\left(1-e^{-2a}(cx^n)^{-4/n}\right)}{2\sinh^{\frac{3}{2}}\left(a+\frac{2\log(cx^n)}{n}\right)}$$

output
```
-1/2*x*(1-1/exp(2*a)/((c*x^n)^(4/n)))/sinh(a+2*ln(c*x^n)/n)^(3/2)
```

Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 61, normalized size of antiderivative = 1.42

$$\int \frac{1}{\sinh^{\frac{3}{2}}\left(a+\frac{2\log(cx^n)}{n}\right)}\,dx = \frac{-\cosh\left(a-2\log(x)+\frac{2\log(cx^n)}{n}\right)+\sinh\left(a-2\log(x)+\frac{2\log(cx^n)}{n}\right)}{x\sqrt{\sinh\left(a+\frac{2\log(cx^n)}{n}\right)}}$$

input
```
Integrate[Sinh[a + (2*Log[c*x^n])/n]^(-3/2),x]
```

output
```
(-Cosh[a - 2*Log[x] + (2*Log[c*x^n])/n] + Sinh[a - 2*Log[x] + (2*Log[c*x^n
])/n])/(x*Sqrt[Sinh[a + (2*Log[c*x^n])/n]])
```

Rubi [A] (verified)

Time = 0.47 (sec) , antiderivative size = 43, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {6051, 6059, 796}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sinh^{\frac{3}{2}}\left(a+\frac{2\log(cx^n)}{n}\right)}dx$$

$$\Big\downarrow \text{6051}$$

$$\frac{x(cx^n)^{-1/n}\int \frac{(cx^n)^{\frac{1}{n}-1}}{\sinh^{\frac{3}{2}}\left(a+\frac{2\log(cx^n)}{n}\right)}d(cx^n)}{n}$$

$$\Big\downarrow \text{6059}$$

$$\frac{x(cx^n)^{2/n}\left(1-e^{-2a}(cx^n)^{-4/n}\right)^{3/2}\int \frac{(cx^n)^{-1-\frac{2}{n}}}{\left(1-e^{-2a}(cx^n)^{-4/n}\right)^{3/2}}d(cx^n)}{n\sinh^{\frac{3}{2}}\left(a+\frac{2\log(cx^n)}{n}\right)}$$

$$\Big\downarrow \text{796}$$

$$-\frac{x\left(1-e^{-2a}(cx^n)^{-4/n}\right)}{2\sinh^{\frac{3}{2}}\left(a+\frac{2\log(cx^n)}{n}\right)}$$

input
```
Int[Sinh[a + (2*Log[c*x^n])/n]^(-3/2),x]
```

output
```
-1/2*(x*(1 - 1/(E^(2*a)*(c*x^n)^(4/n))))/Sinh[a + (2*Log[c*x^n])/n]^(3/2)
```

Defintions of rubi rules used

rule 796
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*
x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

rule 6051
```
Int[Sinh[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(d_.)]^(p_.), x_Symbol] :> S
imp[x/(n*(c*x^n)^(1/n))   Subst[Int[x^(1/n - 1)*Sinh[d*(a + b*Log[x])]^p, x
], x, c*x^n], x] /; FreeQ[{a, b, c, d, n, p}, x] && (NeQ[c, 1] || NeQ[n, 1]
)
```

rule 6059
```
Int[((e_.)*(x_))^(m_.)*Sinh[((a_.) + Log[x_]*(b_.))*(d_.)]^(p_), x_Symbol]
:> Simp[Sinh[d*(a + b*Log[x])]^p/(x^(b*d*p)*(1 - 1/(E^(2*a*d)*x^(2*b*d)))^p
)   Int[(e*x)^m*x^(b*d*p)*(1 - 1/(E^(2*a*d)*x^(2*b*d)))^p, x], x] /; FreeQ[
{a, b, d, e, m, p}, x] &&  !IntegerQ[p]
```

Maple [F]

$$\int \frac{1}{\sinh\left(a+\frac{2\ln(c\,x^{n})}{n}\right)^{\frac{3}{2}}}dx$$

input
```
int(1/sinh(a+2*ln(c*x^n)/n)^(3/2),x)
```

output
```
int(1/sinh(a+2*ln(c*x^n)/n)^(3/2),x)
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 68, normalized size of antiderivative = 1.58

$$\int \frac{1}{\sinh^{\frac{3}{2}}\left(a+\frac{2\log(cx^n)}{n}\right)}\,dx = -\frac{2\,\sqrt{\frac{1}{2}}x\sqrt{\frac{x^4e^{\left(\frac{2\,(an+2\,\log(c))}{n}\right)}-1}{x^2}}e^{\left(-\frac{an+2\,\log(c)}{2\,n}\right)}}{x^4e^{\left(\frac{2\,(an+2\,\log(c))}{n}\right)}-1}$$

input
```
integrate(1/sinh(a+2*log(c*x^n)/n)^(3/2),x, algorithm="fricas")
```

output
```
-2*sqrt(1/2)*x*sqrt((x^4*e^(2*(a*n + 2*log(c))/n) - 1)/x^2)*e^(-1/2*(a*n +
 2*log(c))/n)/(x^4*e^(2*(a*n + 2*log(c))/n) - 1)
```

Sympy [F]

$$\int \frac{1}{\sinh^{\frac{3}{2}}\left(a+\frac{2\log(cx^n)}{n}\right)}\,dx=\int \frac{1}{\sinh^{\frac{3}{2}}\left(a+\frac{2\log{(cx^n)}}{n}\right)}\,dx$$

input
```
integrate(1/sinh(a+2*ln(c*x**n)/n)**(3/2),x)
```

output
```
Integral(sinh(a + 2*log(c*x**n)/n)**(-3/2), x)
```

Maxima [F]

$$\int \frac{1}{\sinh^{\frac{3}{2}}\left(a+\frac{2\log(cx^n)}{n}\right)}\,dx=\int \frac{1}{\sinh\left(a+\frac{2\log(cx^n)}{n}\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/sinh(a+2*log(c*x^n)/n)^(3/2),x, algorithm="maxima")
```

output
```
integrate(sinh(a + 2*log(c*x^n)/n)^(-3/2), x)
```

Giac [A] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.95

$$\int \frac{1}{\sinh^{\frac{3}{2}}\left(a+\frac{2\log(cx^n)}{n}\right)}\,dx=-\frac{\sqrt{2}}{\sqrt{c^{\frac{4}{n}}e^{(3\,a)}-\frac{e^a}{x^4}}c^{\left(\frac{1}{n}\right)}x^2\mathrm{sgn}\left(x\right)}$$

input
```
integrate(1/sinh(a+2*log(c*x^n)/n)^(3/2),x, algorithm="giac")
```

output
```
-sqrt(2)/(sqrt(c^(4/n)*e^(3*a) - e^a/x^4)*c^(1/n)*x^2*sgn(x))
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\sinh^{\frac{3}{2}}\left(a+\frac{2\log(cx^n)}{n}\right)}\,dx=\int \frac{1}{{\mathrm{sinh}\left(a+\frac{2\,\ln\left(c\,x^n\right)}{n}\right)}^{3/2}}\,dx$$

input
```
int(1/sinh(a + (2*log(c*x^n))/n)^(3/2),x)
```

output
```
int(1/sinh(a + (2*log(c*x^n))/n)^(3/2), x)
```

Reduce [F]

$$\int \frac{1}{\sinh^{\frac{3}{2}}\left(a+\frac{2\log(cx^n)}{n}\right)}\,dx=\int \frac{\sqrt{\sinh\left(\frac{2\log(x^n c)+an}{n}\right)}}{\sinh\left(\frac{2\log(x^n c)+an}{n}\right)^2}dx$$

input
```
int(1/sinh(a+2*log(c*x^n)/n)^(3/2),x)
```

output
```
int(sqrt(sinh((2*log(x**n*c) + a*n)/n))/sinh((2*log(x**n*c) + a*n)/n)**2,x
)
```

3.288 $\int \frac{1}{\sinh^{\frac{7}{2}}\left(a+\frac{2\log(cx^n)}{n}\right)}\,dx$

Optimal result

Integrand size = 18, antiderivative size = 89

$$\int \frac{1}{\sinh^{\frac{7}{2}}\left(a+\frac{2\log(cx^n)}{n}\right)}\,dx=-\frac{x\left(1-e^{-2a}(cx^n)^{-4/n}\right)}{10\sinh^{\frac{7}{2}}\left(a+\frac{2\log(cx^n)}{n}\right)}-\frac{x\left(1-e^{-2a}(cx^n)^{-4/n}\right)^2}{15\sinh^{\frac{7}{2}}\left(a+\frac{2\log(cx^n)}{n}\right)}$$

output
```
-1/10*x*(1-1/exp(2*a)/((c*x^n)^(4/n)))/sinh(a+2*ln(c*x^n)/n)^(7/2)-1/15*x*
(1-1/exp(2*a)/((c*x^n)^(4/n)))^2/sinh(a+2*ln(c*x^n)/n)^(7/2)
```

Mathematica [A] (verified)

Time = 0.15 (sec) , antiderivative size = 121, normalized size of antiderivative = 1.36

$$\int \frac{1}{\sinh^{\frac{7}{2}}\left(a+\frac{2\log(cx^n)}{n}\right)}\,dx$$

$$=\frac{\left((-2+5x^4)\cosh\left(a-2\log(x)+\frac{2\log(cx^n)}{n}\right)+(2+5x^4)\sinh\left(a-2\log(x)+\frac{2\log(cx^n)}{n}\right)\right)\left(-\cosh\left(2a-\right.\right.}{15x^5\sinh^{\frac{5}{2}}\left(a+\frac{2\log(cx^n)}{n}\right)}$$

input
```
Integrate[Sinh[a + (2*Log[c*x^n])/n]^(-7/2),x]
```

output
```
(((-2 + 5*x^4)*Cosh[a - 2*Log[x] + (2*Log[c*x^n])/n] + (2 + 5*x^4)*Sinh[a
- 2*Log[x] + (2*Log[c*x^n])/n])*(-Cosh[2*a - 4*Log[x] + (4*Log[c*x^n])/n]
+ Sinh[2*a - 4*Log[x] + (4*Log[c*x^n])/n]))/(15*x^5*Sinh[a + (2*Log[c*x^n]
)/n]^(5/2))
```

Rubi [A] (verified)

Time = 0.42 (sec) , antiderivative size = 144, normalized size of antiderivative = 1.62, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {6051, 6059, 803, 796}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sinh^{\frac{7}{2}}\left(a+\frac{2\log(cx^n)}{n}\right)}dx$$

$$\Big\downarrow \text{ 6051}$$

$$\frac{x(cx^n)^{-1/n}\int \frac{(cx^n)^{\frac{1}{n}-1}}{\sinh^{\frac{7}{2}}\left(a+\frac{2\log(cx^n)}{n}\right)}d(cx^n)}{n}$$

$$\Big\downarrow \text{ 6059}$$

$$\frac{x(cx^n)^{6/n}\left(1-e^{-2a}(cx^n)^{-4/n}\right)^{7/2}\int \frac{(cx^n)^{-1-\frac{6}{n}}}{\left(1-e^{-2a}(cx^n)^{-4/n}\right)^{7/2}}d(cx^n)}{n\sinh^{\frac{7}{2}}\left(a+\frac{2\log(cx^n)}{n}\right)}$$

$$\Big\downarrow \text{ 803}$$

$$\frac{x(cx^n)^{6/n}\left(1-e^{-2a}(cx^n)^{-4/n}\right)^{7/2}\left(-\frac{2}{3}e^{-2a}\int \frac{(cx^n)^{-1-\frac{10}{n}}}{\left(1-e^{-2a}(cx^n)^{-4/n}\right)^{7/2}}d(cx^n)-\frac{n(cx^n)^{-6/n}}{6\left(1-e^{-2a}(cx^n)^{-4/n}\right)^{5/2}}\right)}{n\sinh^{\frac{7}{2}}\left(a+\frac{2\log(cx^n)}{n}\right)}$$

$$\Big\downarrow \text{ 796}$$

$$\frac{x(cx^n)^{6/n}\left(1-e^{-2a}(cx^n)^{-4/n}\right)^{7/2}\left(\frac{e^{-2a}n(cx^n)^{-10/n}}{15\left(1-e^{-2a}(cx^n)^{-4/n}\right)^{5/2}}-\frac{n(cx^n)^{-6/n}}{6\left(1-e^{-2a}(cx^n)^{-4/n}\right)^{5/2}}\right)}{n\sinh^{\frac{7}{2}}\left(a+\frac{2\log(cx^n)}{n}\right)}$$

input
```
Int[Sinh[a + (2*Log[c*x^n])/n]^(-7/2),x]
```

output
```
(x*(c*x^n)^(6/n)*(1 - 1/(E^(2*a)*(c*x^n)^(4/n)))^(7/2)*(n/(15*E^(2*a)*(c*x
^n)^(10/n)*(1 - 1/(E^(2*a)*(c*x^n)^(4/n)))^(5/2)) - n/(6*(c*x^n)^(6/n)*(1
- 1/(E^(2*a)*(c*x^n)^(4/n)))^(5/2))))/(n*Sinh[a + (2*Log[c*x^n])/n]^(7/2))
```

Defintions of rubi rules used

rule 796
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*
x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

rule 803
```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x^(m + 1)*((
a + b*x^n)^(p + 1)/(a*(m + 1))), x] - Simp[b*((m + n*(p + 1) + 1)/(a*(m + 1
)))   Int[x^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, m, n, p}, x] && I
LtQ[Simplify[(m + 1)/n + p + 1], 0] && NeQ[m, -1]
```

rule 6051
```
Int[Sinh[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(d_.)]^(p_.), x_Symbol] :> S
imp[x/(n*(c*x^n)^(1/n))   Subst[Int[x^(1/n - 1)*Sinh[d*(a + b*Log[x])]^p, x
], x, c*x^n], x] /; FreeQ[{a, b, c, d, n, p}, x] && (NeQ[c, 1] || NeQ[n, 1]
)
```

rule 6059
```
Int[((e_.)*(x_))^(m_.)*Sinh[((a_.) + Log[x_]*(b_.))*(d_.)]^(p_), x_Symbol]
:> Simp[Sinh[d*(a + b*Log[x])]^p/(x^(b*d*p)*(1 - 1/(E^(2*a*d)*x^(2*b*d)))^p
)   Int[(e*x)^m*x^(b*d*p)*(1 - 1/(E^(2*a*d)*x^(2*b*d)))^p, x], x] /; FreeQ[
{a, b, d, e, m, p}, x] &&  !IntegerQ[p]
```

Maple [F]

$$\int \frac{1}{\sinh\left(a+\frac{2\ln(c\,x^n)}{n}\right)^{\frac{7}{2}}}dx$$

input
```
int(1/sinh(a+2*ln(c*x^n)/n)^(7/2),x)
```

output
```
int(1/sinh(a+2*ln(c*x^n)/n)^(7/2),x)
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 128, normalized size of antiderivative = 1.44

$$\int \frac{1}{\sinh^{\frac{7}{2}}\left(a+\frac{2\log(cx^n)}{n}\right)}\,dx = -\frac{8\,\sqrt{\frac{1}{2}}\left(5\,x^5e^{\left(\frac{2\,(an+2\,\log(c))}{n}\right)}-2\,x\right)\sqrt{\frac{x^4e^{\left(\frac{2\,(an+2\,\log(c))}{n}\right)}-1}{x^2}}\,e^{\left(-\frac{an+2\,\log(c)}{2\,n}\right)}}{15\left(x^{12}e^{\left(\frac{6\,(an+2\,\log(c))}{n}\right)}-3\,x^8e^{\left(\frac{4\,(an+2\,\log(c))}{n}\right)}+3\,x^4e^{\left(\frac{2\,(an+2\,\log(c))}{n}\right)}-1\right)}$$

input
```
integrate(1/sinh(a+2*log(c*x^n)/n)^(7/2),x, algorithm="fricas")
```

output
```
-8/15*sqrt(1/2)*(5*x^5*e^(2*(a*n + 2*log(c))/n) - 2*x)*sqrt((x^4*e^(2*(a*n
 + 2*log(c))/n) - 1)/x^2)*e^(-1/2*(a*n + 2*log(c))/n)/(x^12*e^(6*(a*n + 2*
log(c))/n) - 3*x^8*e^(4*(a*n + 2*log(c))/n) + 3*x^4*e^(2*(a*n + 2*log(c))/
n) - 1)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{1}{\sinh^{\frac{7}{2}}\left(a+\frac{2\log(cx^n)}{n}\right)}\,dx = \text{Timed out}$$

input
```
integrate(1/sinh(a+2*ln(c*x**n)/n)**(7/2),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{1}{\sinh^{\frac{7}{2}}\left(a+\frac{2\log(cx^n)}{n}\right)}\,dx = \int \frac{1}{\sinh\left(a+\frac{2\log(cx^n)}{n}\right)^{\frac{7}{2}}}\,dx$$

input
```
integrate(1/sinh(a+2*log(c*x^n)/n)^(7/2),x, algorithm="maxima")
```

output
```
integrate(sinh(a + 2*log(c*x^n)/n)^(-7/2), x)
```

Giac [A] (verification not implemented)

Time = 0.39 (sec) , antiderivative size = 81, normalized size of antiderivative = 0.91

$$\int \frac{1}{\sinh^{\frac{7}{2}}\left(a+\frac{2\log(cx^n)}{n}\right)}\,dx = -\frac{4\sqrt{2}c^{\frac{7}{n}}\left(\frac{5e^a}{c^{\frac{4}{n}}\operatorname{sgn}(x)}-\frac{2e^{(-a)}}{c^{\frac{8}{n}}x^4\operatorname{sgn}(x)}\right)e^{(3a)}}{15\left(c^{\frac{4}{n}}e^{(3a)}-\frac{e^a}{x^4}\right)^{\frac{5}{2}}x^6}$$

input
```
integrate(1/sinh(a+2*log(c*x^n)/n)^(7/2),x, algorithm="giac")
```

output
```
-4/15*sqrt(2)*c^(7/n)*(5*e^a/(c^(4/n)*sgn(x)) - 2*e^(-a)/(c^(8/n)*x^4*sgn(
x)))*e^(3*a)/((c^(4/n)*e^(3*a) - e^a/x^4)^(5/2)*x^6)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\sinh^{\frac{7}{2}}\left(a+\frac{2\log(cx^n)}{n}\right)}\,dx=\int \frac{1}{\sinh\left(a+\frac{2\ln(c\,x^n)}{n}\right)^{7/2}}\,dx$$

input
```
int(1/sinh(a + (2*log(c*x^n))/n)^(7/2),x)
```

output
```
int(1/sinh(a + (2*log(c*x^n))/n)^(7/2), x)
```

Reduce [F]

$$\int \frac{1}{\sinh^{\frac{7}{2}}\left(a+\frac{2\log(cx^n)}{n}\right)}\,dx=\int \frac{\sqrt{\sinh\left(\frac{2\log(x^nc)+an}{n}\right)}}{\sinh\left(\frac{2\log(x^nc)+an}{n}\right)^4}dx$$

input
```
int(1/sinh(a+2*log(c*x^n)/n)^(7/2),x)
```

output
```
int(sqrt(sinh((2*log(x**n*c) + a*n)/n))/sinh((2*log(x**n*c) + a*n)/n)**4,x
)
```

3.289 $\int \sinh\left(\frac{a}{c+dx}\right) dx$

Optimal result

Integrand size = 10, antiderivative size = 36

$$\int \sinh\left(\frac{a}{c+dx}\right) dx = -\frac{a\text{Chi}\left(\frac{a}{c+dx}\right)}{d} + \frac{(c+dx)\sinh\left(\frac{a}{c+dx}\right)}{d}$$

output
```
-a*Chi(a/(d*x+c))/d+(d*x+c)*sinh(a/(d*x+c))/d
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.00

$$\int \sinh\left(\frac{a}{c+dx}\right) dx = -\frac{a\text{Chi}\left(\frac{a}{c+dx}\right)}{d} + \frac{(c+dx)\sinh\left(\frac{a}{c+dx}\right)}{d}$$

input
```
Integrate[Sinh[a/(c + d*x)],x]
```

output
```
-((a*CoshIntegral[a/(c + d*x)])/d) + ((c + d*x)*Sinh[a/(c + d*x)])/d
```

Rubi [C] (verified)

Result contains complex when optimal does not.

Time = 0.35 (sec) , antiderivative size = 42, normalized size of antiderivative = 1.17, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.700$, Rules used = {5833, 5825, 3042, 26, 3778, 3042, 3782}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sinh\left(\frac{a}{c+dx}\right)dx$$

$$\Big\downarrow \text{5833}$$

$$\frac{\int \sinh\left(\frac{a}{c+dx}\right)d(c+dx)}{d}$$

$$\Big\downarrow \text{5825}$$

$$-\frac{\int (c+dx)^2\sinh\left(\frac{a}{c+dx}\right)d\frac{1}{c+dx}}{d}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{\int -i(c+dx)^2\sin\left(\frac{ia}{c+dx}\right)d\frac{1}{c+dx}}{d}$$

$$\Big\downarrow \text{26}$$

$$\frac{i\int (c+dx)^2\sin\left(\frac{ia}{c+dx}\right)d\frac{1}{c+dx}}{d}$$

$$\Big\downarrow \text{3778}$$

$$\frac{i\left(ia\int (c+dx)\cosh\left(\frac{a}{c+dx}\right)d\frac{1}{c+dx}-i(c+dx)\sinh\left(\frac{a}{c+dx}\right)\right)}{d}$$

$$\Big\downarrow \text{3042}$$

$$\frac{i\left(ia\int (c+dx)\sin\left(\frac{ia}{c+dx}+\frac{\pi}{2}\right)d\frac{1}{c+dx}-i(c+dx)\sinh\left(\frac{a}{c+dx}\right)\right)}{d}$$

$$\Big\downarrow \text{3782}$$

$$\frac{i\left(ia\text{Chi}\left(\frac{a}{c+dx}\right)-i(c+dx)\sinh\left(\frac{a}{c+dx}\right)\right)}{d}$$

input
```
Int[Sinh[a/(c + d*x)],x]
```

output
```
(I*(I*a*CoshIntegral[a/(c + d*x)] - I*(c + d*x)*Sinh[a/(c + d*x)]))/d
```

Defintions of rubi rules used

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3778
```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(c
 + d*x)^(m + 1)*(Sin[e + f*x]/(d*(m + 1))), x] - Simp[f/(d*(m + 1))   Int[(
c + d*x)^(m + 1)*Cos[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && LtQ[m, -
1]
```

rule 3782
```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[CoshIntegral[c*f*(fz/d) + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

rule 5825
```
Int[((a_.) + (b_.)*Sinh[(c_.) + (d_.)*(x_)^(n_)])^(p_.), x_Symbol] :> -Subs
t[Int[(a + b*Sinh[c + d/x^n])^p/x^2, x], x, 1/x] /; FreeQ[{a, b, c, d}, x]
&& ILtQ[n, 0] && IntegerQ[p]
```

rule 5833
```
Int[((a_.) + (b_.)*Sinh[(c_.) + (d_.)*(u_)^(n_)])^(p_.), x_Symbol] :> Simp[
1/Coefficient[u, x, 1]   Subst[Int[(a + b*Sinh[c + d*x^n])^p, x], x, u], x]
 /; FreeQ[{a, b, c, d, n}, x] && IntegerQ[p] && LinearQ[u, x] && NeQ[u, x]
```

Maple [A] (verified)

Time = 0.35 (sec) , antiderivative size = 38, normalized size of antiderivative = 1.06

method	result	size
derivativedivides	$-\frac{a\left(-\frac{(dx+c)\sinh\left(\frac{a}{dx+c}\right)}{a}+\operatorname{Chi}\left(\frac{a}{dx+c}\right)\right)}{d}$	38
default	$-\frac{a\left(-\frac{(dx+c)\sinh\left(\frac{a}{dx+c}\right)}{a}+\operatorname{Chi}\left(\frac{a}{dx+c}\right)\right)}{d}$	38
risch	$-\frac{\mathrm{e}^{-\frac{a}{dx+c}}x}{2}-\frac{\mathrm{e}^{-\frac{a}{dx+c}}c}{2d}+\frac{a\,\mathrm{expIntegral}_1\left(\frac{a}{dx+c}\right)}{2d}+\frac{\mathrm{e}^{\frac{a}{dx+c}}x}{2}+\frac{\mathrm{e}^{\frac{a}{dx+c}}c}{2d}+\frac{a\,\mathrm{expIntegral}_1\left(-\frac{a}{dx+c}\right)}{2d}$	99

input
```
int(sinh(a/(d*x+c)),x,method=_RETURNVERBOSE)
```

output
```
-1/d*a*(-1/a*(d*x+c)*sinh(a/(d*x+c))+Chi(a/(d*x+c)))
```

Fricas [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 48, normalized size of antiderivative = 1.33

$$\int \sinh\left(\frac{a}{c+dx}\right)\,dx=-\frac{a\mathrm{Ei}\left(\frac{a}{dx+c}\right)+a\mathrm{Ei}\left(-\frac{a}{dx+c}\right)-2\,(dx+c)\sinh\left(\frac{a}{dx+c}\right)}{2\,d}$$

input
```
integrate(sinh(a/(d*x+c)),x, algorithm="fricas")
```

output
```
-1/2*(a*Ei(a/(d*x + c)) + a*Ei(-a/(d*x + c)) - 2*(d*x + c)*sinh(a/(d*x + c
)))/d
```

Sympy [F]

$$\int \sinh\left(\frac{a}{c+dx}\right)\,dx = \int \sinh\left(\frac{a}{c+dx}\right)\,dx$$

input
```
integrate(sinh(a/(d*x+c)),x)
```

output
```
Integral(sinh(a/(c + d*x)), x)
```

Maxima [F]

$$\int \sinh\left(\frac{a}{c+dx}\right)\,dx = \int \sinh\left(\frac{a}{dx+c}\right)\,dx$$

input
```
integrate(sinh(a/(d*x+c)),x, algorithm="maxima")
```

output
```
1/2*a*d*integrate(x*e^(a/(d*x + c))/(d^2*x^2 + 2*c*d*x + c^2), x) + 1/2*a*
d*integrate(x*e^(-a/(d*x + c))/(d^2*x^2 + 2*c*d*x + c^2), x) + 1/2*x*e^(a/
(d*x + c)) - 1/2*x*e^(-a/(d*x + c))
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 102 vs. 2(36) = 72.

Time = 0.14 (sec) , antiderivative size = 102, normalized size of antiderivative = 2.83

$$\int \sinh\left(\frac{a}{c+dx}\right)\,dx = -\frac{\left(\frac{a^3\operatorname{Ei}\left(\frac{a}{dx+c}\right)}{dx+c} - a^2e^{\left(\frac{a}{dx+c}\right)}\right)(dx+c)}{2\,a^2d} - \frac{\left(\frac{a^3\operatorname{Ei}\left(-\frac{a}{dx+c}\right)}{dx+c} + a^2e^{\left(-\frac{a}{dx+c}\right)}\right)(dx+c)}{2\,a^2d}$$

input
```
integrate(sinh(a/(d*x+c)),x, algorithm="giac")
```

output
```
-1/2*(a^3*Ei(a/(d*x + c))/(d*x + c) - a^2*e^(a/(d*x + c)))*(d*x + c)/(a^2*
d) - 1/2*(a^3*Ei(-a/(d*x + c))/(d*x + c) + a^2*e^(-a/(d*x + c)))*(d*x + c)
/(a^2*d)
```

Mupad [F(-1)]

Timed out.

$$\int \sinh\left(\frac{a}{c+dx}\right)\,dx=\int \sinh\left(\frac{a}{c+d\,x}\right)\,dx$$

input
```
int(sinh(a/(c + d*x)),x)
```

output
```
int(sinh(a/(c + d*x)), x)
```

Reduce [F]

$$\int \sinh\left(\frac{a}{c+dx}\right)\,dx$$
$$=\frac{e^{\frac{2a}{dx+c}}a\,d^2x^2-e^{\frac{2a}{dx+c}}c^3-e^{\frac{2a}{dx+c}}c^2dx+e^{\frac{a}{dx+c}}\left(\int\frac{x^2}{e^{\frac{a}{dx+c}}c^3+3e^{\frac{a}{dx+c}}c^2dx+3e^{\frac{a}{dx+c}}c\,d^2x^2+e^{\frac{a}{dx+c}}d^3x^3}dx\right)a^2c\,d^3+e^{\frac{a}{dx+c}}\left(\cdots\right.}{\cdots}$$

input
```
int(sinh(a/(d*x+c)),x)
```

output

```
(e**((2*a)/(c + d*x))*a*d**2*x**2 - e**((2*a)/(c + d*x))*c**3 - e**((2*a)/
(c + d*x))*c**2*d*x + e**(a/(c + d*x))*int(x**2/(e**(a/(c + d*x))*c**3 + 3
*e**(a/(c + d*x))*c**2*d*x + 3*e**(a/(c + d*x))*c*d**2*x**2 + e**(a/(c + d
*x))*d**3*x**3),x)*a**2*c*d**3 + e**(a/(c + d*x))*int(x**2/(e**(a/(c + d*x
))*c**3 + 3*e**(a/(c + d*x))*c**2*d*x + 3*e**(a/(c + d*x))*c*d**2*x**2 + e
**(a/(c + d*x))*d**3*x**3),x)*a**2*d**4*x + e**(a/(c + d*x))*int((e**(a/(c
 + d*x))*x**2)/(c**3 + 3*c**2*d*x + 3*c*d**2*x**2 + d**3*x**3),x)*a**2*c*d
**3 + e**(a/(c + d*x))*int((e**(a/(c + d*x))*x**2)/(c**3 + 3*c**2*d*x + 3*
c*d**2*x**2 + d**3*x**3),x)*a**2*d**4*x - a*d**2*x**2 - c**3 - c**2*d*x)/(
2*e**(a/(c + d*x))*a*d*(c + d*x))
```

3.290 $\int \sinh^2\left(\frac{a}{c+dx}\right)\,dx$

Optimal result

Integrand size = 12, antiderivative size = 39

$$\int \sinh^2\left(\frac{a}{c+dx}\right)\,dx=\frac{(c+dx)\sinh^2\left(\frac{a}{c+dx}\right)}{d}-\frac{a\mathrm{Shi}\left(\frac{2a}{c+dx}\right)}{d}$$

output
```
(d*x+c)*sinh(a/(d*x+c))^2/d-a*Shi(2*a/(d*x+c))/d
```

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.95

$$\int \sinh^2\left(\frac{a}{c+dx}\right)\,dx=\frac{(c+dx)\sinh^2\left(\frac{a}{c+dx}\right)-a\mathrm{Shi}\left(\frac{2a}{c+dx}\right)}{d}$$

input
```
Integrate[Sinh[a/(c + d*x)]^2,x]
```

output
```
((c + d*x)*Sinh[a/(c + d*x)]^2 - a*SinhIntegral[(2*a)/(c + d*x)])/d
```

Rubi [A] (verified)

Time = 0.40 (sec) , antiderivative size = 38, normalized size of antiderivative = 0.97, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.750$, Rules used = {5833, 5825, 3042, 25, 3794, 27, 3042, 26, 3779}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sinh^2\left(\frac{a}{c+dx}\right) dx$$

$$\Big\downarrow \text{5833}$$

$$\frac{\int \sinh^2\left(\frac{a}{c+dx}\right) d(c+dx)}{d}$$

$$\Big\downarrow \text{5825}$$

$$-\frac{\int (c+dx)^2 \sinh^2\left(\frac{a}{c+dx}\right) d\frac{1}{c+dx}}{d}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{\int -(c+dx)^2 \sin\left(\frac{ia}{c+dx}\right)^2 d\frac{1}{c+dx}}{d}$$

$$\Big\downarrow \text{25}$$

$$\frac{\int (c+dx)^2 \sin\left(\frac{ia}{c+dx}\right)^2 d\frac{1}{c+dx}}{d}$$

$$\Big\downarrow \text{3794}$$

$$-\frac{-\left((c+dx)\sinh^2\left(\frac{a}{c+dx}\right)\right) - 2ia\int \frac{1}{2}i(c+dx)\sinh\left(\frac{2a}{c+dx}\right) d\frac{1}{c+dx}}{d}$$

$$\Big\downarrow \text{27}$$

$$-\frac{a\int (c+dx)\sinh\left(\frac{2a}{c+dx}\right) d\frac{1}{c+dx} - (c+dx)\sinh^2\left(\frac{a}{c+dx}\right)}{d}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{-(c+dx)\sinh^2\left(\frac{a}{c+dx}\right) + a\int -i(c+dx)\sin\left(\frac{2ia}{c+dx}\right) d\frac{1}{c+dx}}{d}$$

$$\Big\downarrow 26$$

$$-\frac{-\left((c+dx)\sinh^2\left(\frac{a}{c+dx}\right)\right)-ia\int(c+dx)\sin\left(\frac{2ia}{c+dx}\right)d\frac{1}{c+dx}}{d}$$

$$\Big\downarrow 3779$$

$$-\frac{a\text{Shi}\left(\frac{2a}{c+dx}\right)-(c+dx)\sinh^2\left(\frac{a}{c+dx}\right)}{d}$$

input
```
Int[Sinh[a/(c + d*x)]^2,x]
```

output
```
-((-((c + d*x)*Sinh[a/(c + d*x)]^2) + a*SinhIntegral[(2*a)/(c + d*x)])/d)
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3779
```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[I*(SinhIntegral[c*f*(fz/d) + f*fz*x]/d), x] /; FreeQ[{c, d, e, f
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

rule 3794
```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> Si
mp[(c + d*x)^(m + 1)*(Sin[e + f*x]^n/(d*(m + 1))), x] - Simp[f*(n/(d*(m + 1
)))   Int[ExpandTrigReduce[(c + d*x)^(m + 1), Cos[e + f*x]*Sin[e + f*x]^(n
- 1), x], x], x] /; FreeQ[{c, d, e, f, m}, x] && IGtQ[n, 1] && GeQ[m, -2] &
& LtQ[m, -1]
```

rule 5825
```
Int[((a_.) + (b_.)*Sinh[(c_.) + (d_.)*(x_)^(n_)])^(p_.), x_Symbol] :> -Subs
t[Int[(a + b*Sinh[c + d/x^n])^p/x^2, x], x, 1/x] /; FreeQ[{a, b, c, d}, x]
&& ILtQ[n, 0] && IntegerQ[p]
```

rule 5833
```
Int[((a_.) + (b_.)*Sinh[(c_.) + (d_.)*(u_)^(n_)])^(p_.), x_Symbol] :> Simp[
1/Coefficient[u, x, 1]   Subst[Int[(a + b*Sinh[c + d*x^n])^p, x], x, u], x]
 /; FreeQ[{a, b, c, d, n}, x] && IntegerQ[p] && LinearQ[u, x] && NeQ[u, x]
```

Maple [A] (verified)

Time = 0.34 (sec) , antiderivative size = 50, normalized size of antiderivative = 1.28

method	result
derivativedivides	$-\frac{a\left(\frac{dx+c}{2a}-\frac{(dx+c)\cosh\left(\frac{2a}{dx+c}\right)}{2a}+\operatorname{Shi}\left(\frac{2a}{dx+c}\right)\right)}{d}$
default	$-\frac{a\left(\frac{dx+c}{2a}-\frac{(dx+c)\cosh\left(\frac{2a}{dx+c}\right)}{2a}+\operatorname{Shi}\left(\frac{2a}{dx+c}\right)\right)}{d}$
risch	$-\frac{x}{2}+\frac{\mathrm{e}^{-\frac{2a}{dx+c}}x}{4}+\frac{\mathrm{e}^{-\frac{2a}{dx+c}}c}{4d}-\frac{a\,\mathrm{expIntegral}_1\left(\frac{2a}{dx+c}\right)}{2d}+\frac{\mathrm{e}^{\frac{2a}{dx+c}}x}{4}+\frac{\mathrm{e}^{\frac{2a}{dx+c}}c}{4d}+\frac{a\,\mathrm{expIntegral}_1\left(-\frac{2a}{dx+c}\right)}{2d}$

input
```
int(sinh(a/(d*x+c))^2,x,method=_RETURNVERBOSE)
```

output
```
-1/d*a*(1/2/a*(d*x+c)-1/2/a*(d*x+c)*cosh(2*a/(d*x+c))+Shi(2*a/(d*x+c)))
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 73, normalized size of antiderivative = 1.87

$$\int \sinh^2\left(\frac{a}{c+dx}\right)\,dx = \frac{(dx+c)\cosh\left(\frac{a}{dx+c}\right)^2 + (dx+c)\sinh\left(\frac{a}{dx+c}\right)^2 - dx - a\text{Ei}\left(\frac{2a}{dx+c}\right) + a\text{Ei}\left(-\frac{2a}{dx+c}\right)}{2d}$$

input
```
integrate(sinh(a/(d*x+c))^2,x, algorithm="fricas")
```

output
```
1/2*((d*x + c)*cosh(a/(d*x + c))^2 + (d*x + c)*sinh(a/(d*x + c))^2 - d*x -
 a*Ei(2*a/(d*x + c)) + a*Ei(-2*a/(d*x + c)))/d
```

Sympy [F]

$$\int \sinh^2\left(\frac{a}{c+dx}\right)\,dx = \int \sinh^2\left(\frac{a}{c+dx}\right)\,dx$$

input
```
integrate(sinh(a/(d*x+c))**2,x)
```

output
```
Integral(sinh(a/(c + d*x))**2, x)
```

Maxima [F]

$$\int \sinh^2\left(\frac{a}{c+dx}\right)\,dx = \int \sinh\left(\frac{a}{dx+c}\right)^2\,dx$$

input
```
integrate(sinh(a/(d*x+c))^2,x, algorithm="maxima")
```

output
```
1/2*a*d*integrate(x*e^(2*a/(d*x + c))/(d^2*x^2 + 2*c*d*x + c^2), x) - 1/2*
a*d*integrate(x*e^(-2*a/(d*x + c))/(d^2*x^2 + 2*c*d*x + c^2), x) + 1/4*x*e
^(2*a/(d*x + c)) + 1/4*x*e^(-2*a/(d*x + c)) - 1/2*x
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 130 vs. $2(39) = 78$.

Time = 0.15 (sec) , antiderivative size = 130, normalized size of antiderivative = 3.33

$$\int \sinh^2\left(\frac{a}{c+dx}\right)\,dx = -\frac{\left(\frac{2\,a^3\mathrm{Ei}\left(\frac{2\,a}{dx+c}\right)e^{\left(\frac{2\,a}{dx+c}\right)}}{dx+c} - \frac{2\,a^3\mathrm{Ei}\left(-\frac{2\,a}{dx+c}\right)e^{\left(\frac{2\,a}{dx+c}\right)}}{dx+c} - a^2e^{\left(\frac{4\,a}{dx+c}\right)} + 2\,a^2e^{\left(\frac{2\,a}{dx+c}\right)} - a^2\right)(dx+c)e^{\left(-\frac{2\,a}{dx+c}\right)}}{4\,a^2d}$$

input
```
integrate(sinh(a/(d*x+c))^2,x, algorithm="giac")
```

output
```
-1/4*(2*a^3*Ei(2*a/(d*x + c))*e^(2*a/(d*x + c))/(d*x + c) - 2*a^3*Ei(-2*a/
(d*x + c))*e^(2*a/(d*x + c))/(d*x + c) - a^2*e^(4*a/(d*x + c)) + 2*a^2*e^(
2*a/(d*x + c)) - a^2)*(d*x + c)*e^(-2*a/(d*x + c))/(a^2*d)
```

Mupad [F(-1)]

Timed out.

$$\int \sinh^2\left(\frac{a}{c+dx}\right)\,dx = \int \sinh\left(\frac{a}{c+d\,x}\right)^2\,dx$$

input
```
int(sinh(a/(c + d*x))^2,x)
```

output
```
int(sinh(a/(c + d*x))^2, x)
```

Reduce [F]

$$\int \sinh^2\left(\frac{a}{c+dx}\right)\,dx$$
$$= \frac{2e^{\frac{4a}{dx+c}}a\,d^2x^2 - e^{\frac{4a}{dx+c}}c^3 - e^{\frac{4a}{dx+c}}c^2dx - 4e^{\frac{2a}{dx+c}}\left(\int \frac{x^2}{e^{\frac{2a}{dx+c}}c^3+3e^{\frac{2a}{dx+c}}c^2dx+3e^{\frac{2a}{dx+c}}c\,d^2x^2+e^{\frac{2a}{dx+c}}d^3x^3}dx\right)a^2c\,d^3 - 4e^{\frac{2a}{dx+c}}}{}$$

input
```
int(sinh(a/(d*x+c))^2,x)
```

output
```
(2*e**((4*a)/(c + d*x))*a*d**2*x**2 - e**((4*a)/(c + d*x))*c**3 - e**((4*a
)/(c + d*x))*c**2*d*x - 4*e**((2*a)/(c + d*x))*int(x**2/(e**((2*a)/(c + d*
x))*c**3 + 3*e**((2*a)/(c + d*x))*c**2*d*x + 3*e**((2*a)/(c + d*x))*c*d**2
*x**2 + e**((2*a)/(c + d*x))*d**3*x**3),x)*a**2*c*d**3 - 4*e**((2*a)/(c +
d*x))*int(x**2/(e**((2*a)/(c + d*x))*c**3 + 3*e**((2*a)/(c + d*x))*c**2*d*
x + 3*e**((2*a)/(c + d*x))*c*d**2*x**2 + e**((2*a)/(c + d*x))*d**3*x**3),x
)*a**2*d**4*x + 4*e**((2*a)/(c + d*x))*int((e**((2*a)/(c + d*x))*x**2)/(c*
*3 + 3*c**2*d*x + 3*c*d**2*x**2 + d**3*x**3),x)*a**2*c*d**3 + 4*e**((2*a)/
(c + d*x))*int((e**((2*a)/(c + d*x))*x**2)/(c**3 + 3*c**2*d*x + 3*c*d**2*x
**2 + d**3*x**3),x)*a**2*d**4*x - 4*e**((2*a)/(c + d*x))*a*c*d*x - 4*e**((
2*a)/(c + d*x))*a*d**2*x**2 + 2*a*d**2*x**2 + c**3 + c**2*d*x)/(8*e**((2*a
)/(c + d*x))*a*d*(c + d*x))
```

3.291 $\int \sinh^3\left(\frac{a}{c+dx}\right) dx$

Optimal result

Integrand size = 12, antiderivative size = 59

$$\int \sinh^3\left(\frac{a}{c+dx}\right) dx = \frac{3a\text{Chi}\left(\frac{a}{c+dx}\right)}{4d} - \frac{3a\text{Chi}\left(\frac{3a}{c+dx}\right)}{4d} + \frac{(c+dx)\sinh^3\left(\frac{a}{c+dx}\right)}{d}$$

output
```
3/4*a*Chi(a/(d*x+c))/d-3/4*a*Chi(3*a/(d*x+c))/d+(d*x+c)*sinh(a/(d*x+c))^3/
d
```

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.92

$$\int \sinh^3\left(\frac{a}{c+dx}\right) dx = \frac{3a\text{Chi}\left(\frac{a}{c+dx}\right) - 3a\text{Chi}\left(\frac{3a}{c+dx}\right) + 4(c+dx)\sinh^3\left(\frac{a}{c+dx}\right)}{4d}$$

input
```
Integrate[Sinh[a/(c + d*x)]^3,x]
```

output
```
(3*a*CoshIntegral[a/(c + d*x)] - 3*a*CoshIntegral[(3*a)/(c + d*x)] + 4*(c
+ d*x)*Sinh[a/(c + d*x)]^3)/(4*d)
```

Rubi [C] (verified)

Result contains complex when optimal does not.

Time = 0.40 (sec) , antiderivative size = 64, normalized size of antiderivative = 1.08, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {5833, 5825, 3042, 26, 3794, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sinh^3\left(\frac{a}{c+dx}\right) dx$$

$$\Big\downarrow \text{5833}$$

$$\frac{\int \sinh^3\left(\frac{a}{c+dx}\right)d(c+dx)}{d}$$

$$\Big\downarrow \text{5825}$$

$$-\frac{\int (c+dx)^2\sinh^3\left(\frac{a}{c+dx}\right)d\frac{1}{c+dx}}{d}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{\int i(c+dx)^2\sin\left(\frac{ia}{c+dx}\right)^3 d\frac{1}{c+dx}}{d}$$

$$\Big\downarrow \text{26}$$

$$-\frac{i\int (c+dx)^2\sin\left(\frac{ia}{c+dx}\right)^3 d\frac{1}{c+dx}}{d}$$

$$\Big\downarrow \text{3794}$$

$$-\frac{i\left(3ia\int \left(\frac{1}{4}(c+dx)\cosh\left(\frac{a}{c+dx}\right)-\frac{1}{4}(c+dx)\cosh\left(\frac{3a}{c+dx}\right)\right)d\frac{1}{c+dx}+i(c+dx)\sinh^3\left(\frac{a}{c+dx}\right)\right)}{d}$$

$$\Big\downarrow \text{2009}$$

$$-\frac{i\left(3ia\left(\frac{1}{4}\text{Chi}\left(\frac{a}{c+dx}\right)-\frac{1}{4}\text{Chi}\left(\frac{3a}{c+dx}\right)\right)+i(c+dx)\sinh^3\left(\frac{a}{c+dx}\right)\right)}{d}$$

input
```
Int[Sinh[a/(c + d*x)]^3,x]
```

output
```
((-I)*((3*I)*a*(CoshIntegral[a/(c + d*x)]/4 - CoshIntegral[(3*a)/(c + d*x)
]/4) + I*(c + d*x)*Sinh[a/(c + d*x)]^3))/d
```

Defintions of rubi rules used

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3794
```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> Si
mp[(c + d*x)^(m + 1)*(Sin[e + f*x]^n/(d*(m + 1))), x] - Simp[f*(n/(d*(m + 1
)))   Int[ExpandTrigReduce[(c + d*x)^(m + 1), Cos[e + f*x]*Sin[e + f*x]^(n
- 1), x], x], x] /; FreeQ[{c, d, e, f, m}, x] && IGtQ[n, 1] && GeQ[m, -2] &
& LtQ[m, -1]
```

rule 5825
```
Int[((a_.) + (b_.)*Sinh[(c_.) + (d_.)*(x_)^(n_)])^(p_.), x_Symbol] :> -Subs
t[Int[(a + b*Sinh[c + d/x^n])^p/x^2, x], x, 1/x] /; FreeQ[{a, b, c, d}, x]
&& ILtQ[n, 0] && IntegerQ[p]
```

rule 5833
```
Int[((a_.) + (b_.)*Sinh[(c_.) + (d_.)*(u_)^(n_)])^(p_.), x_Symbol] :> Simp[
1/Coefficient[u, x, 1]   Subst[Int[(a + b*Sinh[c + d*x^n])^p, x], x, u], x]
 /; FreeQ[{a, b, c, d, n}, x] && IntegerQ[p] && LinearQ[u, x] && NeQ[u, x]
```

Maple [A] (verified)

Time = 0.52 (sec) , antiderivative size = 74, normalized size of antiderivative = 1.25

method	result
derivativedivides	$-\frac{a\left(\frac{3(dx+c)\sinh\left(\frac{a}{dx+c}\right)}{4a}-\frac{3\,\mathrm{Chi}\left(\frac{a}{dx+c}\right)}{4}-\frac{(dx+c)\sinh\left(\frac{3a}{dx+c}\right)}{4a}+\frac{3\,\mathrm{Chi}\left(\frac{3a}{dx+c}\right)}{4}\right)}{d}$
default	$-\frac{a\left(\frac{3(dx+c)\sinh\left(\frac{a}{dx+c}\right)}{4a}-\frac{3\,\mathrm{Chi}\left(\frac{a}{dx+c}\right)}{4}-\frac{(dx+c)\sinh\left(\frac{3a}{dx+c}\right)}{4a}+\frac{3\,\mathrm{Chi}\left(\frac{3a}{dx+c}\right)}{4}\right)}{d}$
risch	$-\frac{e^{-\frac{3a}{dx+c}}x}{8}-\frac{e^{-\frac{3a}{dx+c}}c}{8d}+\frac{3a\,\mathrm{expIntegral}_1\left(\frac{3a}{dx+c}\right)}{8d}+\frac{3\,e^{-\frac{a}{dx+c}}x}{8}+\frac{3\,e^{-\frac{a}{dx+c}}c}{8d}-\frac{3a\,\mathrm{expIntegral}_1\left(\frac{a}{dx+c}\right)}{8d}-$

input
```
int(sinh(a/(d*x+c))^3,x,method=_RETURNVERBOSE)
```

output
```
-1/d*a*(3/4/a*(d*x+c)*sinh(a/(d*x+c))-3/4*Chi(a/(d*x+c))-1/4/a*(d*x+c)*sin
h(3*a/(d*x+c))+3/4*Chi(3*a/(d*x+c)))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 118 vs. 2(55) = 110.

Time = 0.10 (sec) , antiderivative size = 118, normalized size of antiderivative = 2.00

$$\int \sinh^3\left(\frac{a}{c+dx}\right)\,dx$$
$$=\frac{2\,(dx+c)\sinh\left(\frac{a}{dx+c}\right)^3-3\,a\mathrm{Ei}\left(\frac{3\,a}{dx+c}\right)+3\,a\mathrm{Ei}\left(\frac{a}{dx+c}\right)+3\,a\mathrm{Ei}\left(-\frac{a}{dx+c}\right)-3\,a\mathrm{Ei}\left(-\frac{3\,a}{dx+c}\right)+6\left((dx+c)\cosh\right.}{8\,d}$$

input
```
integrate(sinh(a/(d*x+c))^3,x, algorithm="fricas")
```

output
```
1/8*(2*(d*x + c)*sinh(a/(d*x + c))^3 - 3*a*Ei(3*a/(d*x + c)) + 3*a*Ei(a/(d
*x + c)) + 3*a*Ei(-a/(d*x + c)) - 3*a*Ei(-3*a/(d*x + c)) + 6*((d*x + c)*co
sh(a/(d*x + c))^2 - d*x - c)*sinh(a/(d*x + c)))/d
```

Sympy [F(-1)]

Timed out.

$$\int \sinh^3\left(\frac{a}{c+dx}\right)\,dx = \text{Timed out}$$

input
```
integrate(sinh(a/(d*x+c))**3,x)
```

output
```
Timed out
```

Maxima [F]

$$\int \sinh^3\left(\frac{a}{c+dx}\right)\,dx = \int \sinh\left(\frac{a}{dx+c}\right)^3\,dx$$

input
```
integrate(sinh(a/(d*x+c))^3,x, algorithm="maxima")
```

output
```
3/8*a*d*integrate(x*e^(3*a/(d*x + c))/(d^2*x^2 + 2*c*d*x + c^2), x) - 3/8*
a*d*integrate(x*e^(a/(d*x + c))/(d^2*x^2 + 2*c*d*x + c^2), x) - 3/8*a*d*in
tegrate(x*e^(-a/(d*x + c))/(d^2*x^2 + 2*c*d*x + c^2), x) + 3/8*a*d*integra
te(x*e^(-3*a/(d*x + c))/(d^2*x^2 + 2*c*d*x + c^2), x) + 1/8*x*e^(3*a/(d*x
+ c)) - 3/8*x*e^(a/(d*x + c)) + 3/8*x*e^(-a/(d*x + c)) - 1/8*x*e^(-3*a/(d*
x + c))
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 211 vs. $2(55) = 110$.

Time = 0.17 (sec) , antiderivative size = 211, normalized size of antiderivative = 3.58

$$\int \sinh^3\left(\frac{a}{c+dx}\right)\,dx = -\frac{\left(\frac{3a^3\mathrm{Ei}\left(\frac{3a}{dx+c}\right)e^{\left(\frac{3a}{dx+c}\right)}}{dx+c} - \frac{3a^3\mathrm{Ei}\left(\frac{a}{dx+c}\right)e^{\left(\frac{3a}{dx+c}\right)}}{dx+c} - \frac{3a^3\mathrm{Ei}\left(-\frac{a}{dx+c}\right)e^{\left(\frac{3a}{dx+c}\right)}}{dx+c} + \frac{3a^3\mathrm{Ei}\left(-\frac{3a}{dx+c}\right)e^{\left(\frac{3a}{dx+c}\right)}}{dx+c} - a^2e^{\left(\frac{6a}{dx+c}\right)} + 3a\right.}{8a^2d}$$

input
```
integrate(sinh(a/(d*x+c))^3,x, algorithm="giac")
```

output
```
-1/8*(3*a^3*Ei(3*a/(d*x + c))*e^(3*a/(d*x + c))/(d*x + c) - 3*a^3*Ei(a/(d*
x + c))*e^(3*a/(d*x + c))/(d*x + c) - 3*a^3*Ei(-a/(d*x + c))*e^(3*a/(d*x +
 c))/(d*x + c) + 3*a^3*Ei(-3*a/(d*x + c))*e^(3*a/(d*x + c))/(d*x + c) - a^
2*e^(6*a/(d*x + c)) + 3*a^2*e^(4*a/(d*x + c)) - 3*a^2*e^(2*a/(d*x + c)) +
a^2)*(d*x + c)*e^(-3*a/(d*x + c))/(a^2*d)
```

Mupad [F(-1)]

Timed out.

$$\int \sinh^3\left(\frac{a}{c+dx}\right)\,dx=\int \sinh\left(\frac{a}{c+d\,x}\right)^3\,dx$$

input
```
int(sinh(a/(c + d*x))^3,x)
```

output
```
int(sinh(a/(c + d*x))^3, x)
```

Reduce [F]

$$\int \sinh^3\left(\frac{a}{c+dx}\right)\,dx$$
$$= \frac{3e^{\frac{6a}{dx+c}}a\,d^2x^2-e^{\frac{6a}{dx+c}}c^3-e^{\frac{6a}{dx+c}}c^2dx-9e^{\frac{4a}{dx+c}}a\,d^2x^2+9e^{\frac{4a}{dx+c}}c^3+9e^{\frac{4a}{dx+c}}c^2dx+9e^{\frac{3a}{dx+c}}\left(\int\frac{}{e^{\frac{3a}{dx+c}}c^3+3e^{\frac{3a}{dx+c}}c^2 \cdots}\right.}{}$$

input
```
int(sinh(a/(d*x+c))^3,x)
```

output
```
(3*e**((6*a)/(c + d*x))*a*d**2*x**2 - e**((6*a)/(c + d*x))*c**3 - e**((6*a
)/(c + d*x))*c**2*d*x - 9*e**((4*a)/(c + d*x))*a*d**2*x**2 + 9*e**((4*a)/(
c + d*x))*c**3 + 9*e**((4*a)/(c + d*x))*c**2*d*x + 9*e**((3*a)/(c + d*x))*
int(x**2/(e**((3*a)/(c + d*x))*c**3 + 3*e**((3*a)/(c + d*x))*c**2*d*x + 3*
e**((3*a)/(c + d*x))*c*d**2*x**2 + e**((3*a)/(c + d*x))*d**3*x**3),x)*a**2
*c*d**3 + 9*e**((3*a)/(c + d*x))*int(x**2/(e**((3*a)/(c + d*x))*c**3 + 3*e
**((3*a)/(c + d*x))*c**2*d*x + 3*e**((3*a)/(c + d*x))*c*d**2*x**2 + e**((3
*a)/(c + d*x))*d**3*x**3),x)*a**2*d**4*x - 9*e**((3*a)/(c + d*x))*int(x**2
/(e**(a/(c + d*x))*c**3 + 3*e**(a/(c + d*x))*c**2*d*x + 3*e**(a/(c + d*x))
*c*d**2*x**2 + e**(a/(c + d*x))*d**3*x**3),x)*a**2*c*d**3 - 9*e**((3*a)/(c
 + d*x))*int(x**2/(e**(a/(c + d*x))*c**3 + 3*e**(a/(c + d*x))*c**2*d*x + 3
*e**(a/(c + d*x))*c*d**2*x**2 + e**(a/(c + d*x))*d**3*x**3),x)*a**2*d**4*x
 + 9*e**((3*a)/(c + d*x))*int((e**((3*a)/(c + d*x))*x**2)/(c**3 + 3*c**2*d
*x + 3*c*d**2*x**2 + d**3*x**3),x)*a**2*c*d**3 + 9*e**((3*a)/(c + d*x))*in
t((e**((3*a)/(c + d*x))*x**2)/(c**3 + 3*c**2*d*x + 3*c*d**2*x**2 + d**3*x*
*3),x)*a**2*d**4*x - 9*e**((3*a)/(c + d*x))*int((e**(a/(c + d*x))*x**2)/(c
**3 + 3*c**2*d*x + 3*c*d**2*x**2 + d**3*x**3),x)*a**2*c*d**3 - 9*e**((3*a)
/(c + d*x))*int((e**(a/(c + d*x))*x**2)/(c**3 + 3*c**2*d*x + 3*c*d**2*x**2
 + d**3*x**3),x)*a**2*d**4*x + 9*e**((2*a)/(c + d*x))*a*d**2*x**2 + 9*e**(
(2*a)/(c + d*x))*c**3 + 9*e**((2*a)/(c + d*x))*c**2*d*x - 3*a*d**2*x**2...
```

3.292 $\int \sinh\left(\frac{bx}{c+dx}\right)\,dx$

Optimal result

Integrand size = 11, antiderivative size = 74

$$\int \sinh\left(\frac{bx}{c+dx}\right)\,dx=\frac{bc\cosh\left(\frac{b}{d}\right)\text{Chi}\left(\frac{bc}{d(c+dx)}\right)}{d^2}+\frac{(c+dx)\sinh\left(\frac{bx}{c+dx}\right)}{d}-\frac{bc\sinh\left(\frac{b}{d}\right)\text{Shi}\left(\frac{bc}{d(c+dx)}\right)}{d^2}$$

output
```
b*c*cosh(b/d)*Chi(b*c/d/(d*x+c))/d^2+(d*x+c)*sinh(b*x/(d*x+c))/d-b*c*sinh(
b/d)*Shi(b*c/d/(d*x+c))/d^2
```

Mathematica [A] (verified)

Time = 0.16 (sec) , antiderivative size = 93, normalized size of antiderivative = 1.26

$$\int \sinh\left(\frac{bx}{c+dx}\right)\,dx=\frac{de^{-\frac{bx}{c+dx}}\left(-1+e^{\frac{2bx}{c+dx}}\right)(c+dx)+2bc\cosh\left(\frac{b}{d}\right)\text{Chi}\left(\frac{bc}{cd+d^2x}\right)-2bc\sinh\left(\frac{b}{d}\right)\text{Shi}\left(\frac{bc}{cd+d^2x}\right)}{2d^2}$$

input
```
Integrate[Sinh[(b*x)/(c + d*x)],x]
```

output
```
((d*(-1 + E^((2*b*x)/(c + d*x)))*(c + d*x))/E^((b*x)/(c + d*x)) + 2*b*c*Co
sh[b/d]*CoshIntegral[(b*c)/(c*d + d^2*x)] - 2*b*c*Sinh[b/d]*SinhIntegral[(
b*c)/(c*d + d^2*x)])/(2*d^2)
```

Rubi [C] (verified)

Result contains complex when optimal does not.

Time = 0.58 (sec) , antiderivative size = 91, normalized size of antiderivative = 1.23, number of steps used = 12, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 1.000, Rules used = {6141, 3042, 26, 3778, 3042, 3784, 26, 3042, 26, 3779, 3782}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sinh\left(\frac{bx}{c+dx}\right)dx$$

$$\Big\downarrow \text{6141}$$

$$-\frac{\int (c+dx)^2\sinh\left(\frac{b}{d}-\frac{bc}{d(c+dx)}\right)d\frac{1}{c+dx}}{d}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{\int -i(c+dx)^2\sin\left(\frac{ib}{d}-\frac{ibc}{d(c+dx)}\right)d\frac{1}{c+dx}}{d}$$

$$\Big\downarrow \text{26}$$

$$\frac{i\int (c+dx)^2\sin\left(\frac{ib}{d}-\frac{ibc}{d(c+dx)}\right)d\frac{1}{c+dx}}{d}$$

$$\Big\downarrow \text{3778}$$

$$\frac{i\left(-\frac{ibc\int (c+dx)\cosh\left(\frac{b}{d}-\frac{bc}{d(c+dx)}\right)d\frac{1}{c+dx}}{d}-i(c+dx)\sinh\left(\frac{b}{d}-\frac{bc}{d(c+dx)}\right)\right)}{d}$$

$$\Big\downarrow \text{3042}$$

$$\frac{i\left(-\frac{ibc\int (c+dx)\sin\left(\frac{ib}{d}-\frac{icb}{d(c+dx)}+\frac{\pi}{2}\right)d\frac{1}{c+dx}}{d}-i(c+dx)\sinh\left(\frac{b}{d}-\frac{bc}{d(c+dx)}\right)\right)}{d}$$

$\Big\downarrow$ 3784

$$\frac{i\left(-\frac{ibc\left(\cosh\left(\frac{b}{d}\right)\int(c+dx)\cosh\left(\frac{bc}{d(c+dx)}\right)d\frac{1}{c+dx}-i\sinh\left(\frac{b}{d}\right)\int-i(c+dx)\sinh\left(\frac{bc}{d(c+dx)}\right)d\frac{1}{c+dx}\right)}{d}-i(c+dx)\sinh\left(\frac{b}{d}-\frac{bc}{d(c+dx)}\right)\right)}{d}$$

$\Big\downarrow$ 26

$$\frac{i\left(-\frac{ibc\left(\cosh\left(\frac{b}{d}\right)\int(c+dx)\cosh\left(\frac{bc}{d(c+dx)}\right)d\frac{1}{c+dx}-\sinh\left(\frac{b}{d}\right)\int(c+dx)\sinh\left(\frac{bc}{d(c+dx)}\right)d\frac{1}{c+dx}\right)}{d}-i(c+dx)\sinh\left(\frac{b}{d}-\frac{bc}{d(c+dx)}\right)\right)}{d}$$

$\Big\downarrow$ 3042

$$\frac{i\left(-\frac{ibc\left(\cosh\left(\frac{b}{d}\right)\int(c+dx)\sin\left(\frac{ibc}{d(c+dx)}+\frac{\pi}{2}\right)d\frac{1}{c+dx}-\sinh\left(\frac{b}{d}\right)\int-i(c+dx)\sin\left(\frac{ibc}{d(c+dx)}\right)d\frac{1}{c+dx}\right)}{d}-i(c+dx)\sinh\left(\frac{b}{d}-\frac{bc}{d(c+dx)}\right)\right)}{d}$$

$\Big\downarrow$ 26

$$\frac{i\left(-\frac{ibc\left(i\sinh\left(\frac{b}{d}\right)\int(c+dx)\sin\left(\frac{ibc}{d(c+dx)}\right)d\frac{1}{c+dx}+\cosh\left(\frac{b}{d}\right)\int(c+dx)\sin\left(\frac{ibc}{d(c+dx)}+\frac{\pi}{2}\right)d\frac{1}{c+dx}\right)}{d}-i(c+dx)\sinh\left(\frac{b}{d}-\frac{bc}{d(c+dx)}\right)\right)}{d}$$

$\Big\downarrow$ 3779

$$\frac{i\left(-\frac{ibc\left(-\sinh\left(\frac{b}{d}\right)\text{Shi}\left(\frac{bc}{d(c+dx)}\right)+\cosh\left(\frac{b}{d}\right)\int(c+dx)\sin\left(\frac{ibc}{d(c+dx)}+\frac{\pi}{2}\right)d\frac{1}{c+dx}\right)}{d}-i(c+dx)\sinh\left(\frac{b}{d}-\frac{bc}{d(c+dx)}\right)\right)}{d}$$

$\Big\downarrow$ 3782

$$\frac{i\left(-\frac{ibc\left(\cosh\left(\frac{b}{d}\right)\text{Chi}\left(\frac{bc}{d(c+dx)}\right)-\sinh\left(\frac{b}{d}\right)\text{Shi}\left(\frac{bc}{d(c+dx)}\right)\right)}{d}-i(c+dx)\sinh\left(\frac{b}{d}-\frac{bc}{d(c+dx)}\right)\right)}{d}$$

input
```
Int[Sinh[(b*x)/(c + d*x)],x]
```

output
```
(I*((-I)*(c + d*x)*Sinh[b/d - (b*c)/(d*(c + d*x))] - (I*b*c*(Cosh[b/d]*Cos
hIntegral[(b*c)/(d*(c + d*x))] - Sinh[b/d]*SinhIntegral[(b*c)/(d*(c + d*x)
)]))/d))/d
```

Defintions of rubi rules used

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3778
```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(c
 + d*x)^(m + 1)*(Sin[e + f*x]/(d*(m + 1))), x] - Simp[f/(d*(m + 1))   Int[(
c + d*x)^(m + 1)*Cos[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && LtQ[m, -
1]
```

rule 3779
```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[I*(SinhIntegral[c*f*(fz/d) + f*fz*x]/d), x] /; FreeQ[{c, d, e, f
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

rule 3782
```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[CoshIntegral[c*f*(fz/d) + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

rule 3784
```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[Cos[(d*
e - c*f)/d]   Int[Sin[c*(f/d) + f*x]/(c + d*x), x], x] + Simp[Sin[(d*e - c*
f)/d]   Int[Cos[c*(f/d) + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x]
&& NeQ[d*e - c*f, 0]
```

rule 6141
```
Int[Sinh[((e_.)*((a_.) + (b_.)*(x_)))/((c_.) + (d_.)*(x_))]^(n_.), x_Symbol
] :> Simp[-d^(-1)   Subst[Int[Sinh[b*(e/d) - e*(b*c - a*d)*(x/d)]^n/x^2, x]
, x, 1/(c + d*x)], x] /; FreeQ[{a, b, c, d}, x] && IGtQ[n, 0] && NeQ[b*c -
a*d, 0]
```

Maple [A] (verified)

Time = 0.32 (sec) , antiderivative size = 113, normalized size of antiderivative = 1.53

method	result	size
risch	$-\frac{{\mathrm e}^{-\frac{b x}{d x +c}} \left(d x +c \right)}{2 d}-\frac{b c \,{\mathrm e}^{-\frac{b}{d}} \operatorname{expIntegral}_{1}\left(-\frac{b c}{d \left(d x +c \right)}\right)}{2 d^{2}}+\frac{{\mathrm e}^{\frac{b x}{d x +c}} x}{2}+\frac{c \,{\mathrm e}^{\frac{b x}{d x +c}}}{2 d}-\frac{b c \,{\mathrm e}^{\frac{b}{d}} \operatorname{expIntegral}_{1}\left(\frac{b c}{d \left(d x +c \right)}\right)}{2 d^{2}}$	113

input
```
int(sinh(b*x/(d*x+c)),x,method=_RETURNVERBOSE)
```

output
```
-1/2/d*exp(-b*x/(d*x+c))*(d*x+c)-1/2*b*c/d^2*exp(-b/d)*Ei(1,-b*c/d/(d*x+c)
)+1/2*exp(b*x/(d*x+c))*x+1/2*c/d*exp(b*x/(d*x+c))-1/2*b*c/d^2*exp(b/d)*Ei(
1,b*c/d/(d*x+c))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 253 vs. 2(74) = 148.

Time = 0.10 (sec) , antiderivative size = 253, normalized size of antiderivative = 3.42

$$\int \sinh\left(\frac{bx}{c+dx}\right)\,dx =$$
$$-\frac{bc\mathrm{Ei}\left(-\frac{bc}{d^2x+cd}\right)\cosh\left(\frac{b}{d}\right)\sinh\left(\frac{bx}{dx+c}\right)^2-\left(bc\mathrm{Ei}\left(-\frac{bc}{d^2x+cd}\right)\cosh\left(\frac{bx}{dx+c}\right)^2+bc\mathrm{Ei}\left(\frac{bc}{d^2x+cd}\right)\right)\cosh\left(\frac{b}{d}\right)-2\left(d^2\right.}{2\left(d^2\cosh\left(\frac{bx}{dx+c}\right.\right.}$$

input
```
integrate(sinh(b*x/(d*x+c)),x, algorithm="fricas")
```

output
```
-1/2*(b*c*Ei(-b*c/(d^2*x + c*d))*cosh(b/d)*sinh(b*x/(d*x + c))^2 - (b*c*Ei
(-b*c/(d^2*x + c*d))*cosh(b*x/(d*x + c))^2 + b*c*Ei(b*c/(d^2*x + c*d)))*co
sh(b/d) - 2*(d^2*x + c*d)*sinh(b*x/(d*x + c)) - (b*c*Ei(-b*c/(d^2*x + c*d)
)*cosh(b*x/(d*x + c))^2 - b*c*Ei(-b*c/(d^2*x + c*d))*sinh(b*x/(d*x + c))^2
 - b*c*Ei(b*c/(d^2*x + c*d)))*sinh(b/d))/(d^2*cosh(b*x/(d*x + c))^2 - d^2*
sinh(b*x/(d*x + c))^2)
```

Sympy [F]

$$\int \sinh\left(\frac{bx}{c+dx}\right)\,dx = \int \sinh\left(\frac{bx}{c+dx}\right)\,dx$$

input
```
integrate(sinh(b*x/(d*x+c)),x)
```

output
```
Integral(sinh(b*x/(c + d*x)), x)
```

Maxima [F]

$$\int \sinh\left(\frac{bx}{c+dx}\right)\,dx = \int \sinh\left(\frac{bx}{dx+c}\right)\,dx$$

input
```
integrate(sinh(b*x/(d*x+c)),x, algorithm="maxima")
```

output
```
-1/2*b*c*integrate(x*e^(b*c/(d^2*x + c*d))/(d^2*x^2*e^(b/d) + 2*c*d*x*e^(b
/d) + c^2*e^(b/d)), x) - 1/2*b*c*integrate(x*e^(-b*c/(d^2*x + c*d) + b/d)/
(d^2*x^2 + 2*c*d*x + c^2), x) - 1/2*(x*e^(b*c/(d^2*x + c*d)) - x*e^(-b*c/(
d^2*x + c*d) + 2*b/d))*e^(-b/d)
```

Giac [F]

$$\int \sinh\left(\frac{bx}{c+dx}\right)\,dx = \int \sinh\left(\frac{bx}{dx+c}\right)\,dx$$

input
```
integrate(sinh(b*x/(d*x+c)),x, algorithm="giac")
```

output
```
integrate(sinh(b*x/(d*x + c)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \sinh\left(\frac{bx}{c+dx}\right)\,dx = \int \sinh\left(\frac{b\,x}{c+d\,x}\right)\,dx$$

input
```
int(sinh((b*x)/(c + d*x)),x)
```

output
```
int(sinh((b*x)/(c + d*x)), x)
```

Reduce [F]

$$\int \sinh\left(\frac{bx}{c+dx}\right)\,dx$$
$$= \frac{e^{\frac{2bx}{dx+c}} bd\,x^2 + e^{\frac{2bx}{dx+c}} c^2 + e^{\frac{2bx}{dx+c}} cdx - e^{\frac{bx}{dx+c}} \left(\int \frac{x^2}{e^{\frac{bx}{dx+c}} c^3 + 3e^{\frac{bx}{dx+c}} c^2 dx + 3e^{\frac{bx}{dx+c}} c\,d^2 x^2 + e^{\frac{bx}{dx+c}} d^3 x^3} dx\right) b^2 c^2 d - e^{\frac{bx}{dx+c}} \left(\int \right.}{}$$

input
```
int(sinh(b*x/(d*x+c)),x)
```

output
```
(e**((2*b*x)/(c + d*x))*b*d*x**2 + e**((2*b*x)/(c + d*x))*c**2 + e**((2*b*
x)/(c + d*x))*c*d*x - e**((b*x)/(c + d*x))*int(x**2/(e**((b*x)/(c + d*x))*
c**3 + 3*e**((b*x)/(c + d*x))*c**2*d*x + 3*e**((b*x)/(c + d*x))*c*d**2*x**
2 + e**((b*x)/(c + d*x))*d**3*x**3),x)*b**2*c**2*d - e**((b*x)/(c + d*x))*
int(x**2/(e**((b*x)/(c + d*x))*c**3 + 3*e**((b*x)/(c + d*x))*c**2*d*x + 3*
e**((b*x)/(c + d*x))*c*d**2*x**2 + e**((b*x)/(c + d*x))*d**3*x**3),x)*b**2
*c*d**2*x - e**((b*x)/(c + d*x))*int((e**((b*x)/(c + d*x))*x**2)/(c**3 + 3
*c**2*d*x + 3*c*d**2*x**2 + d**3*x**3),x)*b**2*c**2*d - e**((b*x)/(c + d*x
))*int((e**((b*x)/(c + d*x))*x**2)/(c**3 + 3*c**2*d*x + 3*c*d**2*x**2 + d*
*3*x**3),x)*b**2*c*d**2*x - b*d*x**2 + c**2 + c*d*x)/(2*e**((b*x)/(c + d*x
))*b*(c + d*x))
```

3.293 $\int \sinh^2\left(\frac{bx}{c+dx}\right) dx$

Optimal result

Integrand size = 13, antiderivative size = 80

$$\int \sinh^2\left(\frac{bx}{c+dx}\right) dx=\frac{bc\text{Chi}\left(\frac{2bc}{d(c+dx)}\right)\sinh\left(\frac{2b}{d}\right)}{d^2}+\frac{(c+dx)\sinh^2\left(\frac{bx}{c+dx}\right)}{d}-\frac{bc\cosh\left(\frac{2b}{d}\right)\text{Shi}\left(\frac{2bc}{d(c+dx)}\right)}{d^2}$$

output
```
b*c*Chi(2*b*c/d/(d*x+c))*sinh(2*b/d)/d^2+(d*x+c)*sinh(b*x/(d*x+c))^2/d-b*c
*cosh(2*b/d)*Shi(2*b*c/d/(d*x+c))/d^2
```

Mathematica [A] (verified)

Time = 0.21 (sec) , antiderivative size = 115, normalized size of antiderivative = 1.44

$$\int \sinh^2\left(\frac{bx}{c+dx}\right) dx = \frac{de^{-\frac{2bx}{c+dx}}\left(c\left(1+e^{\frac{4bx}{c+dx}}\right)+d\left(-1+e^{\frac{2bx}{c+dx}}\right)^2x\right)+4bc\text{Chi}\left(\frac{2bc}{cd+d^2x}\right)\sinh\left(\frac{2b}{d}\right)-4bc\cosh\left(\frac{2b}{d}\right)\text{Shi}\left(\frac{2bc}{cd+d^2x}\right)}{4d^2}$$

input
```
Integrate[Sinh[(b*x)/(c + d*x)]^2,x]
```

output
```
((d*(c*(1 + E^((4*b*x)/(c + d*x))) + d*(-1 + E^((2*b*x)/(c + d*x)))^2*x))/
E^((2*b*x)/(c + d*x)) + 4*b*c*CoshIntegral[(2*b*c)/(c*d + d^2*x)]*Sinh[(2*
b)/d] - 4*b*c*Cosh[(2*b)/d]*SinhIntegral[(2*b*c)/(c*d + d^2*x)])/(4*d^2)
```

Rubi [C] (verified)

Result contains complex when optimal does not.

Time = 0.58 (sec) , antiderivative size = 98, normalized size of antiderivative = 1.22, number of steps used = 14, number of rules used = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 1.000$, Rules used = {6141, 3042, 25, 3794, 27, 3042, 26, 3784, 26, 3042, 26, 3779, 3782}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sinh^2\left(\frac{bx}{c+dx}\right)dx$$

$$\Big\downarrow \text{6141}$$

$$-\frac{\int (c+dx)^2\sinh^2\left(\frac{b}{d}-\frac{bc}{d(c+dx)}\right)d\frac{1}{c+dx}}{d}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{\int -(c+dx)^2\sin\left(\frac{ib}{d}-\frac{ibc}{d(c+dx)}\right)^2d\frac{1}{c+dx}}{d}$$

$$\Big\downarrow \text{25}$$

$$\frac{\int (c+dx)^2\sin\left(\frac{ib}{d}-\frac{ibc}{d(c+dx)}\right)^2d\frac{1}{c+dx}}{d}$$

$$\Big\downarrow \text{3794}$$

$$-\frac{-(c+dx)\sinh^2\left(\frac{b}{d}-\frac{bc}{d(c+dx)}\right)+\frac{2ibc\int \frac{1}{2}i(c+dx)\sinh\left(\frac{2b}{d}-\frac{2bc}{d(c+dx)}\right)d\frac{1}{c+dx}}{d}}{d}$$

$$\Big\downarrow \text{27}$$

$$-\frac{-\frac{bc\int (c+dx)\sinh\left(\frac{2b}{d}-\frac{2bc}{d(c+dx)}\right)d\frac{1}{c+dx}}{d}-\left((c+dx)\sinh^2\left(\frac{b}{d}-\frac{bc}{d(c+dx)}\right)\right)}{d}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{-\left((c+dx)\sinh^2\left(\frac{b}{d}-\frac{bc}{d(c+dx)}\right)\right)-\frac{bc\int -i(c+dx)\sin\left(\frac{2ib}{d}-\frac{2ibc}{d(c+dx)}\right)d\frac{1}{c+dx}}{d}}{d}$$

$$\Big\downarrow \text{26}$$

$$-\frac{-(c+dx)\sinh^2\left(\frac{b}{d}-\frac{bc}{d(c+dx)}\right)+\frac{ibc\int (c+dx)\sin\left(\frac{2ib}{d}-\frac{2ibc}{d(c+dx)}\right)d\frac{1}{c+dx}}{d}}{d}$$

$$\Big\downarrow \text{3784}$$

$$-\frac{-(c+dx)\sinh^2\left(\frac{b}{d}-\frac{bc}{d(c+dx)}\right)+\frac{ibc\left(i\sinh\left(\frac{2b}{d}\right)\int (c+dx)\cosh\left(\frac{2bc}{d(c+dx)}\right)d\frac{1}{c+dx}+\cosh\left(\frac{2b}{d}\right)\int -i(c+dx)\sinh\left(\frac{2bc}{d(c+dx)}\right)d\frac{1}{c+dx}\right)}{d}}{d}$$

$$\Big\downarrow \text{26}$$

$$-\frac{-(c+dx)\sinh^2\left(\frac{b}{d}-\frac{bc}{d(c+dx)}\right)+\frac{ibc\left(i\sinh\left(\frac{2b}{d}\right)\int (c+dx)\cosh\left(\frac{2bc}{d(c+dx)}\right)d\frac{1}{c+dx}-i\cosh\left(\frac{2b}{d}\right)\int (c+dx)\sinh\left(\frac{2bc}{d(c+dx)}\right)d\frac{1}{c+dx}\right)}{d}}{d}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{-(c+dx)\sinh^2\left(\frac{b}{d}-\frac{bc}{d(c+dx)}\right)+\frac{ibc\left(i\sinh\left(\frac{2b}{d}\right)\int (c+dx)\sin\left(\frac{2ibc}{d(c+dx)}+\frac{\pi}{2}\right)d\frac{1}{c+dx}-i\cosh\left(\frac{2b}{d}\right)\int -i(c+dx)\sin\left(\frac{2ibc}{d(c+dx)}\right)d\frac{1}{c+dx}\right)}{d}}{d}$$

$$\Big\downarrow \text{26}$$

$$-\frac{-(c+dx)\sinh^2\left(\frac{b}{d}-\frac{bc}{d(c+dx)}\right)+\frac{ibc\left(i\sinh\left(\frac{2b}{d}\right)\int (c+dx)\sin\left(\frac{2ibc}{d(c+dx)}+\frac{\pi}{2}\right)d\frac{1}{c+dx}-\cosh\left(\frac{2b}{d}\right)\int (c+dx)\sin\left(\frac{2ibc}{d(c+dx)}\right)d\frac{1}{c+dx}\right)}{d}}{d}$$

$$\Big\downarrow \text{3779}$$

$$-\frac{-(c+dx)\sinh^2\left(\frac{b}{d}-\frac{bc}{d(c+dx)}\right)+\frac{ibc\left(i\sinh\left(\frac{2b}{d}\right)\int (c+dx)\sin\left(\frac{2ibc}{d(c+dx)}+\frac{\pi}{2}\right)d\frac{1}{c+dx}-i\cosh\left(\frac{2b}{d}\right)\text{Shi}\left(\frac{2bc}{d(c+dx)}\right)\right)}{d}}{d}$$

$$\Big\downarrow \text{3782}$$

$$-\frac{-(c+dx)\sinh^2\left(\frac{b}{d}-\frac{bc}{d(c+dx)}\right)+\frac{ibc\left(i\sinh\left(\frac{2b}{d}\right)\text{Chi}\left(\frac{2bc}{d(c+dx)}\right)-i\cosh\left(\frac{2b}{d}\right)\text{Shi}\left(\frac{2bc}{d(c+dx)}\right)\right)}{d}}{d}$$

input
```
Int[Sinh[(b*x)/(c + d*x)]^2,x]
```

output
```
-((-((c + d*x)*Sinh[b/d - (b*c)/(d*(c + d*x))]^2) + (I*b*c*(I*CoshIntegral
[(2*b*c)/(d*(c + d*x))]*Sinh[(2*b)/d] - I*Cosh[(2*b)/d]*SinhIntegral[(2*b*
c)/(d*(c + d*x))]))/d)/d)
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3779
```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[I*(SinhIntegral[c*f*(fz/d) + f*fz*x]/d), x] /; FreeQ[{c, d, e, f
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

rule 3782
```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[CoshIntegral[c*f*(fz/d) + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

rule 3784
```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[Cos[(d*
e - c*f)/d]   Int[Sin[c*(f/d) + f*x]/(c + d*x), x], x] + Simp[Sin[(d*e - c*
f)/d]   Int[Cos[c*(f/d) + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x]
&& NeQ[d*e - c*f, 0]
```

rule 3794
```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> Si
mp[(c + d*x)^(m + 1)*(Sin[e + f*x]^n/(d*(m + 1))), x] - Simp[f*(n/(d*(m + 1
)))   Int[ExpandTrigReduce[(c + d*x)^(m + 1), Cos[e + f*x]*Sin[e + f*x]^(n
- 1), x], x], x] /; FreeQ[{c, d, e, f, m}, x] && IGtQ[n, 1] && GeQ[m, -2] &
& LtQ[m, -1]
```

rule 6141
```
Int[Sinh[((e_.)*((a_.) + (b_.)*(x_)))/((c_.) + (d_.)*(x_))]^(n_.), x_Symbol
] :> Simp[-d^(-1)   Subst[Int[Sinh[b*(e/d) - e*(b*c - a*d)*(x/d)]^n/x^2, x]
, x, 1/(c + d*x)], x] /; FreeQ[{a, b, c, d}, x] && IGtQ[n, 0] && NeQ[b*c -
a*d, 0]
```

Maple [A] (verified)

Time = 2.25 (sec) , antiderivative size = 120, normalized size of antiderivative = 1.50

method	result	siz
risch	$-\frac{x}{2}+\frac{{\mathrm e}^{-\frac{2 b x}{d x +c}} \left(d x +c \right)}{4 d}+\frac{b c \,{\mathrm e}^{-\frac{2 b}{d}} \operatorname{expIntegral}_{1}\left(-\frac{2 b c}{d \left(d x +c \right)}\right)}{2 d^{2}}+\frac{{\mathrm e}^{\frac{2 b x}{d x +c}} x}{4}+\frac{c \,{\mathrm e}^{\frac{2 b x}{d x +c}}}{4 d}-\frac{b c \,{\mathrm e}^{\frac{2 b}{d}} \operatorname{expIntegral}_{1}\left(\frac{2 b c}{d \left(d x +c \right)}\right)}{2 d^{2}}$	120

input
```
int(sinh(b*x/(d*x+c))^2,x,method=_RETURNVERBOSE)
```

output
```
-1/2*x+1/4/d*exp(-2*b*x/(d*x+c))*(d*x+c)+1/2*b*c/d^2*exp(-2*b/d)*Ei(1,-2*b
*c/d/(d*x+c))+1/4*exp(2*b*x/(d*x+c))*x+1/4*c/d*exp(2*b*x/(d*x+c))-1/2*b*c/
d^2*exp(2*b/d)*Ei(1,2*b*c/d/(d*x+c))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 277 vs. 2(80) = 160.

Time = 0.09 (sec) , antiderivative size = 277, normalized size of antiderivative = 3.46

$$\int \sinh^2\left(\frac{bx}{c+dx}\right)\,dx =$$

$$-\frac{d^2x-(d^2x+cd)\cosh\left(\frac{bx}{dx+c}\right)^2+\left(bc\mathrm{Ei}\left(-\frac{2\,bc}{d^2x+cd}\right)\cosh\left(\frac{2\,b}{d}\right)-d^2x-cd\right)\sinh\left(\frac{bx}{dx+c}\right)^2-\left(bc\mathrm{Ei}\left(-\frac{2\,bc}{d^2x+cd}\right.\right.}{2\left(d\right.}$$

input
```
integrate(sinh(b*x/(d*x+c))^2,x, algorithm="fricas")
```

output
```
-1/2*(d^2*x - (d^2*x + c*d)*cosh(b*x/(d*x + c))^2 + (b*c*Ei(-2*b*c/(d^2*x
+ c*d))*cosh(2*b/d) - d^2*x - c*d)*sinh(b*x/(d*x + c))^2 - (b*c*Ei(-2*b*c/
(d^2*x + c*d))*cosh(b*x/(d*x + c))^2 - b*c*Ei(2*b*c/(d^2*x + c*d)))*cosh(2
*b/d) - (b*c*Ei(-2*b*c/(d^2*x + c*d))*cosh(b*x/(d*x + c))^2 - b*c*Ei(-2*b*
c/(d^2*x + c*d))*sinh(b*x/(d*x + c))^2 + b*c*Ei(2*b*c/(d^2*x + c*d)))*sinh
(2*b/d))/(d^2*cosh(b*x/(d*x + c))^2 - d^2*sinh(b*x/(d*x + c))^2)
```

Sympy [F]

$$\int \sinh^2\left(\frac{bx}{c+dx}\right)\,dx = \int \sinh^2\left(\frac{bx}{c+dx}\right)\,dx$$

input
```
integrate(sinh(b*x/(d*x+c))**2,x)
```

output
```
Integral(sinh(b*x/(c + d*x))**2, x)
```

Maxima [F]

$$\int \sinh^2\left(\frac{bx}{c+dx}\right)\,dx = \int \sinh\left(\frac{bx}{dx+c}\right)^2\,dx$$

input
```
integrate(sinh(b*x/(d*x+c))^2,x, algorithm="maxima")
```

output
```
1/2*b*c*integrate(x*e^(2*b*c/(d^2*x + c*d))/(d^2*x^2*e^(2*b/d) + 2*c*d*x*e
^(2*b/d) + c^2*e^(2*b/d)), x) - 1/2*b*c*integrate(x*e^(-2*b*c/(d^2*x + c*d
) + 2*b/d)/(d^2*x^2 + 2*c*d*x + c^2), x) + 1/4*(x*e^(2*b*c/(d^2*x + c*d))
+ x*e^(-2*b*c/(d^2*x + c*d) + 4*b/d))*e^(-2*b/d) - 1/2*x
```

Giac [F]

$$\int \sinh^2\left(\frac{bx}{c+dx}\right)\,dx = \int \sinh\left(\frac{bx}{dx+c}\right)^2\,dx$$

input
```
integrate(sinh(b*x/(d*x+c))^2,x, algorithm="giac")
```

output
```
integrate(sinh(b*x/(d*x + c))^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int \sinh^2\left(\frac{bx}{c+dx}\right)\,dx = \int \sinh\left(\frac{b\,x}{c+d\,x}\right)^2\,dx$$

input
```
int(sinh((b*x)/(c + d*x))^2,x)
```

output
```
int(sinh((b*x)/(c + d*x))^2, x)
```

Reduce [F]

$$\int \sinh^2\left(\frac{bx}{c+dx}\right)\,dx$$
$$= \frac{2e^{\frac{4bx}{dx+c}}bd\,x^2+e^{\frac{4bx}{dx+c}}c^2+e^{\frac{4bx}{dx+c}}cdx+4e^{\frac{2bx}{dx+c}}\left(\int \frac{x^2}{e^{\frac{2bx}{dx+c}}c^3+3e^{\frac{2bx}{dx+c}}c^2dx+3e^{\frac{2bx}{dx+c}}c\,d^2x^2+e^{\frac{2bx}{dx+c}}d^3x^3}dx\right)b^2c^2d+4e^{\frac{2bx}{dx+c}}}{}$$

input
```
int(sinh(b*x/(d*x+c))^2,x)
```

output
```
(2*e**((4*b*x)/(c + d*x))*b*d*x**2 + e**((4*b*x)/(c + d*x))*c**2 + e**((4*
b*x)/(c + d*x))*c*d*x + 4*e**((2*b*x)/(c + d*x))*int(x**2/(e**((2*b*x)/(c
+ d*x))*c**3 + 3*e**((2*b*x)/(c + d*x))*c**2*d*x + 3*e**((2*b*x)/(c + d*x)
)*c*d**2*x**2 + e**((2*b*x)/(c + d*x))*d**3*x**3),x)*b**2*c**2*d + 4*e**((
2*b*x)/(c + d*x))*int(x**2/(e**((2*b*x)/(c + d*x))*c**3 + 3*e**((2*b*x)/(c
 + d*x))*c**2*d*x + 3*e**((2*b*x)/(c + d*x))*c*d**2*x**2 + e**((2*b*x)/(c
+ d*x))*d**3*x**3),x)*b**2*c*d**2*x - 4*e**((2*b*x)/(c + d*x))*int((e**((2
*b*x)/(c + d*x))*x**2)/(c**3 + 3*c**2*d*x + 3*c*d**2*x**2 + d**3*x**3),x)*
b**2*c**2*d - 4*e**((2*b*x)/(c + d*x))*int((e**((2*b*x)/(c + d*x))*x**2)/(
c**3 + 3*c**2*d*x + 3*c*d**2*x**2 + d**3*x**3),x)*b**2*c*d**2*x - 4*e**((2
*b*x)/(c + d*x))*b*c*x - 4*e**((2*b*x)/(c + d*x))*b*d*x**2 + 2*b*d*x**2 -
c**2 - c*d*x)/(8*e**((2*b*x)/(c + d*x))*b*(c + d*x))
```

3.294 $\int \sinh^3\left(\frac{bx}{c+dx}\right)\,dx$

Optimal result

Integrand size = 13, antiderivative size = 143

$$\int \sinh^3\left(\frac{bx}{c+dx}\right)\,dx = -\frac{3bc\cosh\left(\frac{b}{d}\right)\mathrm{Chi}\left(\frac{bc}{d(c+dx)}\right)}{4d^2}+\frac{3bc\cosh\left(\frac{3b}{d}\right)\mathrm{Chi}\left(\frac{3bc}{d(c+dx)}\right)}{4d^2}+\frac{(c+dx)\sinh^3\left(\frac{bx}{c+dx}\right)}{d}+\frac{3bc\sinh\left(\frac{b}{d}\right)\mathrm{Shi}\left(\frac{bc}{d(c+dx)}\right)}{4d^2}-\frac{3bc\sinh\left(\frac{3b}{d}\right)\mathrm{Shi}\left(\frac{3bc}{d(c+dx)}\right)}{4d^2}$$

output
```
-3/4*b*c*cosh(b/d)*Chi(b*c/d/(d*x+c))/d^2+3/4*b*c*cosh(3*b/d)*Chi(3*b*c/d/
(d*x+c))/d^2+(d*x+c)*sinh(b*x/(d*x+c))^3/d+3/4*b*c*sinh(b/d)*Shi(b*c/d/(d*
x+c))/d^2-3/4*b*c*sinh(3*b/d)*Shi(3*b*c/d/(d*x+c))/d^2
```

Mathematica [A] (verified)

Time = 0.40 (sec) , antiderivative size = 232, normalized size of antiderivative = 1.62

$$\int \sinh^3\left(\frac{bx}{c+dx}\right)\,dx$$
$$= \frac{-cde^{-\frac{3bx}{c+dx}} + 3cde^{-\frac{bx}{c+dx}} - 3cde^{\frac{bx}{c+dx}} + cde^{\frac{3bx}{c+dx}} - d^2e^{-\frac{3bx}{c+dx}}x + d^2e^{\frac{3bx}{c+dx}}x - 6bc\cosh\left(\frac{b}{d}\right)\mathrm{Chi}\left(\frac{bc}{cd+d^2x}\right) + 6bc\,\cdots}{8d^2}$$

input
```
Integrate[Sinh[(b*x)/(c + d*x)]^3,x]
```

output
```
(-((c*d)/E^((3*b*x)/(c + d*x))) + (3*c*d)/E^((b*x)/(c + d*x)) - 3*c*d*E^((
b*x)/(c + d*x)) + c*d*E^((3*b*x)/(c + d*x)) - (d^2*x)/E^((3*b*x)/(c + d*x)
) + d^2*E^((3*b*x)/(c + d*x))*x - 6*b*c*Cosh[b/d]*CoshIntegral[(b*c)/(c*d
+ d^2*x)] + 6*b*c*Cosh[(3*b)/d]*CoshIntegral[(3*b*c)/(c*d + d^2*x)] - 6*d^
2*x*Sinh[(b*x)/(c + d*x)] + 6*b*c*Sinh[b/d]*SinhIntegral[(b*c)/(c*d + d^2*
x)] - 6*b*c*Sinh[(3*b)/d]*SinhIntegral[(3*b*c)/(c*d + d^2*x)])/(8*d^2)
```

Rubi [C] (verified)

Result contains complex when optimal does not.

Time = 0.48 (sec) , antiderivative size = 150, normalized size of antiderivative = 1.05, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.385$, Rules used = {6141, 3042, 26, 3794, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sinh^3\left(\frac{bx}{c+dx}\right)\,dx$$
$$\Big\downarrow \text{6141}$$
$$-\frac{\int (c+dx)^2\sinh^3\left(\frac{b}{d}-\frac{bc}{d(c+dx)}\right)d\frac{1}{c+dx}}{d}$$
$$\Big\downarrow \text{3042}$$

$$-\frac{\int i(c+dx)^2 \sin\left(\frac{ib}{d}-\frac{ibc}{d(c+dx)}\right)^3 d\frac{1}{c+dx}}{d}$$

$\Big\downarrow$ 26

$$-\frac{i\int (c+dx)^2 \sin\left(\frac{ib}{d}-\frac{ibc}{d(c+dx)}\right)^3 d\frac{1}{c+dx}}{d}$$

$\Big\downarrow$ 3794

$$-\frac{i\left(i(c+dx)\sinh^3\left(\frac{b}{d}-\frac{bc}{d(c+dx)}\right)-\frac{3ibc\int\left(\frac{1}{4}(c+dx)\cosh\left(\frac{b}{d}-\frac{bc}{d(c+dx)}\right)-\frac{1}{4}(c+dx)\cosh\left(\frac{3b}{d}-\frac{3bc}{d(c+dx)}\right)\right)d\frac{1}{c+dx}}{d}\right)}{d}$$

$\Big\downarrow$ 2009

$$-\frac{i\left(i(c+dx)\sinh^3\left(\frac{b}{d}-\frac{bc}{d(c+dx)}\right)-\frac{3ibc\left(\frac{1}{4}\cosh\left(\frac{b}{d}\right)\text{Chi}\left(\frac{bc}{d(c+dx)}\right)-\frac{1}{4}\cosh\left(\frac{3b}{d}\right)\text{Chi}\left(\frac{3bc}{d(c+dx)}\right)-\frac{1}{4}\sinh\left(\frac{b}{d}\right)\text{Shi}\left(\frac{bc}{d(c+dx)}\right)+\frac{1}{4}\sinh\right.}{d}}{d}$$

input
```
Int[Sinh[(b*x)/(c + d*x)]^3,x]
```

output
```
((-I)*(I*(c + d*x)*Sinh[b/d - (b*c)/(d*(c + d*x))]^3 - ((3*I)*b*c*((Cosh[b
/d]*CoshIntegral[(b*c)/(d*(c + d*x))])/4 - (Cosh[(3*b)/d]*CoshIntegral[(3*
b*c)/(d*(c + d*x))])/4 - (Sinh[b/d]*SinhIntegral[(b*c)/(d*(c + d*x))])/4 +
 (Sinh[(3*b)/d]*SinhIntegral[(3*b*c)/(d*(c + d*x))])/4))/d))/d
```

Defintions of rubi rules used

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3794
```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> Si
mp[(c + d*x)^(m + 1)*(Sin[e + f*x]^n/(d*(m + 1))), x] - Simp[f*(n/(d*(m + 1
)))   Int[ExpandTrigReduce[(c + d*x)^(m + 1), Cos[e + f*x]*Sin[e + f*x]^(n
- 1), x], x], x] /; FreeQ[{c, d, e, f, m}, x] && IGtQ[n, 1] && GeQ[m, -2] &
& LtQ[m, -1]
```

rule 6141
```
Int[Sinh[((e_.)*((a_.) + (b_.)*(x_)))/((c_.) + (d_.)*(x_))]^(n_.), x_Symbol
] :> Simp[-d^(-1)   Subst[Int[Sinh[b*(e/d) - e*(b*c - a*d)*(x/d)]^n/x^2, x]
, x, 1/(c + d*x)], x] /; FreeQ[{a, b, c, d}, x] && IGtQ[n, 0] && NeQ[b*c -
a*d, 0]
```

Maple [A] (verified)

Time = 1.84 (sec) , antiderivative size = 250, normalized size of antiderivative = 1.75

method	result
risch	$\frac{3bc\,\mathrm{e}^{\frac{b}{d}}\,\mathrm{expIntegral}_1\left(\frac{bc}{d(dx+c)}\right)}{8d^2}+\frac{3\,\mathrm{e}^{-\frac{bx}{dx+c}}x}{8}+\frac{3bc\,\mathrm{e}^{-\frac{b}{d}}\,\mathrm{expIntegral}_1\left(-\frac{bc}{d(dx+c)}\right)}{8d^2}-\frac{3\,\mathrm{e}^{\frac{bx}{dx+c}}x}{8}+\frac{\mathrm{e}^{\frac{3bx}{dx+c}}x}{8}-\frac{3\,\mathrm{e}^{\frac{3b}{d}}\,\mathrm{expInte}}{}$

input
```
int(sinh(b*x/(d*x+c))^3,x,method=_RETURNVERBOSE)
```

output
```
3/8*b*c/d^2*exp(b/d)*Ei(1,b*c/d/(d*x+c))+3/8*exp(-b*x/(d*x+c))*x+3/8*b*c/d
^2*exp(-b/d)*Ei(1,-b*c/d/(d*x+c))-3/8*exp(b*x/(d*x+c))*x+1/8*exp(3*b*x/(d*
x+c))*x-3/8/d^2*exp(3*b/d)*Ei(1,3*b*c/d/(d*x+c))*b*c-1/8*exp(-3*b*x/(d*x+c
))*x-3/8/d^2*exp(-3*b/d)*Ei(1,-3*b*c/d/(d*x+c))*b*c+3/8/d*exp(-b*x/(d*x+c)
)*c-3/8*c/d*exp(b*x/(d*x+c))+1/8/d*exp(3*b*x/(d*x+c))*c-1/8/d*exp(-3*b*x/(
d*x+c))*c
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 701 vs. $2(135) = 270$.

Time = 0.10 (sec) , antiderivative size = 701, normalized size of antiderivative = 4.90

$$\int \sinh^3\left(\frac{bx}{c+dx}\right) dx = \text{Too large to display}$$

input
```
integrate(sinh(b*x/(d*x+c))^3,x, algorithm="fricas")
```

output
```
1/8*(3*(b*c*Ei(-3*b*c/(d^2*x + c*d))*cosh(3*b/d) - b*c*Ei(-b*c/(d^2*x + c*
d))*cosh(b/d))*sinh(b*x/(d*x + c))^4 + 2*(d^2*x + c*d)*sinh(b*x/(d*x + c))
^3 - 6*(b*c*Ei(-3*b*c/(d^2*x + c*d))*cosh(b*x/(d*x + c))^2*cosh(3*b/d) - b
*c*Ei(-b*c/(d^2*x + c*d))*cosh(b*x/(d*x + c))^2*cosh(b/d))*sinh(b*x/(d*x +
 c))^2 + 3*(b*c*Ei(-3*b*c/(d^2*x + c*d))*cosh(b*x/(d*x + c))^4 + b*c*Ei(3*
b*c/(d^2*x + c*d)))*cosh(3*b/d) - 3*(b*c*Ei(-b*c/(d^2*x + c*d))*cosh(b*x/(
d*x + c))^4 + b*c*Ei(b*c/(d^2*x + c*d)))*cosh(b/d) - 6*(d^2*x - (d^2*x + c
*d)*cosh(b*x/(d*x + c))^2 + c*d)*sinh(b*x/(d*x + c)) + 3*(b*c*Ei(-3*b*c/(d
^2*x + c*d))*cosh(b*x/(d*x + c))^4 - 2*b*c*Ei(-3*b*c/(d^2*x + c*d))*cosh(b
*x/(d*x + c))^2*sinh(b*x/(d*x + c))^2 + b*c*Ei(-3*b*c/(d^2*x + c*d))*sinh(
b*x/(d*x + c))^4 - b*c*Ei(3*b*c/(d^2*x + c*d)))*sinh(3*b/d) - 3*(b*c*Ei(-b
*c/(d^2*x + c*d))*cosh(b*x/(d*x + c))^4 - 2*b*c*Ei(-b*c/(d^2*x + c*d))*cos
h(b*x/(d*x + c))^2*sinh(b*x/(d*x + c))^2 + b*c*Ei(-b*c/(d^2*x + c*d))*sinh
(b*x/(d*x + c))^4 - b*c*Ei(b*c/(d^2*x + c*d)))*sinh(b/d))/(d^2*cosh(b*x/(d
*x + c))^4 - 2*d^2*cosh(b*x/(d*x + c))^2*sinh(b*x/(d*x + c))^2 + d^2*sinh(
b*x/(d*x + c))^4)
```

Sympy [F(-1)]

Timed out.

$$\int \sinh^3\left(\frac{bx}{c+dx}\right) dx = \text{Timed out}$$

input
```
integrate(sinh(b*x/(d*x+c))**3,x)
```

output
```
Timed out
```

Maxima [F]

$$\int \sinh^3\left(\frac{bx}{c+dx}\right) dx = \int \sinh\left(\frac{bx}{dx+c}\right)^3 dx$$

input
```
integrate(sinh(b*x/(d*x+c))^3,x, algorithm="maxima")
```

output
```
-3/8*b*c*integrate(x*e^(3*b*c/(d^2*x + c*d))/(d^2*x^2*e^(3*b/d) + 2*c*d*x*
e^(3*b/d) + c^2*e^(3*b/d)), x) + 3/8*b*c*integrate(x*e^(b*c/(d^2*x + c*d))
/(d^2*x^2*e^(b/d) + 2*c*d*x*e^(b/d) + c^2*e^(b/d)), x) + 3/8*b*c*integrate
(x*e^(-b*c/(d^2*x + c*d) + b/d)/(d^2*x^2 + 2*c*d*x + c^2), x) - 3/8*b*c*in
tegrate(x*e^(-3*b*c/(d^2*x + c*d) + 3*b/d)/(d^2*x^2 + 2*c*d*x + c^2), x) -
 1/8*(x*e^(3*b*c/(d^2*x + c*d)) - 3*x*e^(b*c/(d^2*x + c*d) + 2*b/d) + 3*x*
e^(-b*c/(d^2*x + c*d) + 4*b/d) - x*e^(-3*b*c/(d^2*x + c*d) + 6*b/d))*e^(-3
*b/d)
```

Giac [F]

$$\int \sinh^3\left(\frac{bx}{c+dx}\right) dx = \int \sinh\left(\frac{bx}{dx+c}\right)^3 dx$$

input
```
integrate(sinh(b*x/(d*x+c))^3,x, algorithm="giac")
```

output
```
integrate(sinh(b*x/(d*x + c))^3, x)
```

Mupad [F(-1)]

Timed out.

$$\int \sinh^3\left(\frac{bx}{c+dx}\right) dx = \int \sinh\left(\frac{b\,x}{c+d\,x}\right)^3 dx$$

input
```
int(sinh((b*x)/(c + d*x))^3,x)
```

output
```
int(sinh((b*x)/(c + d*x))^3, x)
```

Reduce [F]

$$\int \sinh^3\left(\frac{bx}{c+dx}\right)dx = \frac{3e^{\frac{6bx}{dx+c}}bd\,x^2 + e^{\frac{6bx}{dx+c}}c^2 + e^{\frac{6bx}{dx+c}}cdx - 9e^{\frac{4bx}{dx+c}}bd\,x^2 - 9e^{\frac{4bx}{dx+c}}c^2 - 9e^{\frac{4bx}{dx+c}}cdx + 9e^{\frac{3bx}{dx+c}}\left(\int \frac{x^2}{e^{\frac{bx}{dx+c}}c^3+3e^{\frac{bx}{dx+c}}c^2dx+3e\cdots}\right.}{}$$

input
```
int(sinh(b*x/(d*x+c))^3,x)
```

output
```
(3*e**((6*b*x)/(c + d*x))*b*d*x**2 + e**((6*b*x)/(c + d*x))*c**2 + e**((6*
b*x)/(c + d*x))*c*d*x - 9*e**((4*b*x)/(c + d*x))*b*d*x**2 - 9*e**((4*b*x)/
(c + d*x))*c**2 - 9*e**((4*b*x)/(c + d*x))*c*d*x + 9*e**((3*b*x)/(c + d*x)
)*int(x**2/(e**((b*x)/(c + d*x))*c**3 + 3*e**((b*x)/(c + d*x))*c**2*d*x +
3*e**((b*x)/(c + d*x))*c*d**2*x**2 + e**((b*x)/(c + d*x))*d**3*x**3),x)*b*
*2*c**2*d + 9*e**((3*b*x)/(c + d*x))*int(x**2/(e**((b*x)/(c + d*x))*c**3 +
 3*e**((b*x)/(c + d*x))*c**2*d*x + 3*e**((b*x)/(c + d*x))*c*d**2*x**2 + e*
*((b*x)/(c + d*x))*d**3*x**3),x)*b**2*c*d**2*x - 9*e**((3*b*x)/(c + d*x))*
int(x**2/(e**((3*b*x)/(c + d*x))*c**3 + 3*e**((3*b*x)/(c + d*x))*c**2*d*x
+ 3*e**((3*b*x)/(c + d*x))*c*d**2*x**2 + e**((3*b*x)/(c + d*x))*d**3*x**3)
,x)*b**2*c**2*d - 9*e**((3*b*x)/(c + d*x))*int(x**2/(e**((3*b*x)/(c + d*x)
)*c**3 + 3*e**((3*b*x)/(c + d*x))*c**2*d*x + 3*e**((3*b*x)/(c + d*x))*c*d*
*2*x**2 + e**((3*b*x)/(c + d*x))*d**3*x**3),x)*b**2*c*d**2*x + 9*e**((3*b*
x)/(c + d*x))*int((e**((b*x)/(c + d*x))*x**2)/(c**3 + 3*c**2*d*x + 3*c*d**
2*x**2 + d**3*x**3),x)*b**2*c**2*d + 9*e**((3*b*x)/(c + d*x))*int((e**((b*
x)/(c + d*x))*x**2)/(c**3 + 3*c**2*d*x + 3*c*d**2*x**2 + d**3*x**3),x)*b**
2*c*d**2*x - 9*e**((3*b*x)/(c + d*x))*int((e**((3*b*x)/(c + d*x))*x**2)/(c
**3 + 3*c**2*d*x + 3*c*d**2*x**2 + d**3*x**3),x)*b**2*c**2*d - 9*e**((3*b*
x)/(c + d*x))*int((e**((3*b*x)/(c + d*x))*x**2)/(c**3 + 3*c**2*d*x + 3*c*d
**2*x**2 + d**3*x**3),x)*b**2*c*d**2*x + 9*e**((2*b*x)/(c + d*x))*b*d*x...
```

3.295 $\int \sinh \left(\frac{a+bx}{c+dx}\right) dx$

Optimal result

Integrand size = 14, antiderivative size = 101

$$\int \sinh \left(\frac{a+bx}{c+dx}\right) dx = \frac{(bc-ad)\cosh\left(\frac{b}{d}\right)\text{Chi}\left(\frac{bc-ad}{d(c+dx)}\right)}{d^2} + \frac{(c+dx)\sinh\left(\frac{a+bx}{c+dx}\right)}{d} - \frac{(bc-ad)\sinh\left(\frac{b}{d}\right)\text{Shi}\left(\frac{bc-ad}{d(c+dx)}\right)}{d^2}$$

output
```
(-a*d+b*c)*cosh(b/d)*Chi((-a*d+b*c)/d/(d*x+c))/d^2+(d*x+c)*sinh((b*x+a)/(d
*x+c))/d-(-a*d+b*c)*sinh(b/d)*Shi((-a*d+b*c)/d/(d*x+c))/d^2
```

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 302 vs. 2(101) = 202.

Time = 0.45 (sec) , antiderivative size = 302, normalized size of antiderivative = 2.99

$$\int \sinh \left(\frac{a+bx}{c+dx}\right) dx$$
$$= \frac{-cde^{-\frac{a+bx}{c+dx}} + cde^{\frac{a+bx}{c+dx}} + 2d^2x\cosh\left(\frac{-bc+ad}{d(c+dx)}\right)\sinh\left(\frac{b}{d}\right) + 2d^2x\cosh\left(\frac{b}{d}\right)\sinh\left(\frac{-bc+ad}{d(c+dx)}\right) + (bc-ad)\left(\text{Chi}\left(\frac{bc}{cd}\right.\right.}{}$$

input
```
Integrate[Sinh[(a + b*x)/(c + d*x)],x]
```

output
```
(-((c*d)/E^((a + b*x)/(c + d*x))) + c*d*E^((a + b*x)/(c + d*x)) + 2*d^2*x*
Cosh[(-(b*c) + a*d)/(d*(c + d*x))]*Sinh[b/d] + 2*d^2*x*Cosh[b/d]*Sinh[(-(b
*c) + a*d)/(d*(c + d*x))] + (b*c - a*d)*(CoshIntegral[(b*c - a*d)/(c*d + d
^2*x)]*(Cosh[b/d] - Sinh[b/d]) + CoshIntegral[(-(b*c) + a*d)/(d*(c + d*x))
]*(Cosh[b/d] + Sinh[b/d]) + Cosh[b/d]*SinhIntegral[(-(b*c) + a*d)/(d*(c +
d*x))] + Sinh[b/d]*SinhIntegral[(-(b*c) + a*d)/(d*(c + d*x))] + Cosh[b/d]*
SinhIntegral[(b*c - a*d)/(c*d + d^2*x)] - Sinh[b/d]*SinhIntegral[(b*c - a*
d)/(c*d + d^2*x)]))/(2*d^2)
```

Rubi [C] (verified)

Result contains complex when optimal does not.

Time = 0.64 (sec) , antiderivative size = 115, normalized size of antiderivative = 1.14, number of steps used = 12, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.786$, Rules used = {6141, 3042, 26, 3778, 3042, 3784, 26, 3042, 26, 3779, 3782}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sinh \left(\frac{a+bx}{c+dx}\right) dx$$

$$\Big\downarrow \text{6141}$$

$$-\frac{\int (c+dx)^2 \sinh \left(\frac{b}{d}-\frac{bc-ad}{d(c+dx)}\right) d\frac{1}{c+dx}}{d}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{\int -i(c+dx)^2 \sin \left(\frac{ib}{d}-\frac{i(bc-ad)}{d(c+dx)}\right) d\frac{1}{c+dx}}{d}$$

$$\Big\downarrow \text{26}$$

$$\frac{i\int (c+dx)^2 \sin \left(\frac{ib}{d}-\frac{i(bc-ad)}{d(c+dx)}\right) d\frac{1}{c+dx}}{d}$$

$$\Big\downarrow \text{3778}$$

$$\frac{i\left(-\frac{i(bc-ad)\int (c+dx)\cosh\left(\frac{b}{d}-\frac{bc-ad}{d(c+dx)}\right)d\frac{1}{c+dx}}{d}-i(c+dx)\sinh\left(\frac{b}{d}-\frac{bc-ad}{d(c+dx)}\right)\right)}{d}$$

$\big\downarrow$ 3042

$$\frac{i\left(-\frac{i(bc-ad)\int (c+dx)\sin\left(\frac{ib}{d}-\frac{i(bc-ad)}{d(c+dx)}+\frac{\pi}{2}\right)d\frac{1}{c+dx}}{d}-i(c+dx)\sinh\left(\frac{b}{d}-\frac{bc-ad}{d(c+dx)}\right)\right)}{d}$$

$\big\downarrow$ 3784

$$\frac{i\left(-\frac{i(bc-ad)\left(\cosh\left(\frac{b}{d}\right)\int (c+dx)\cosh\left(\frac{bc-ad}{d(c+dx)}\right)d\frac{1}{c+dx}-i\sinh\left(\frac{b}{d}\right)\int -i(c+dx)\sinh\left(\frac{bc-ad}{d(c+dx)}\right)d\frac{1}{c+dx}\right)}{d}-i(c+dx)\sinh\left(\frac{b}{d}-\frac{bc-ad}{d(c+dx)}\right)\right)}{d}$$

$\big\downarrow$ 26

$$\frac{i\left(-\frac{i(bc-ad)\left(\cosh\left(\frac{b}{d}\right)\int (c+dx)\cosh\left(\frac{bc-ad}{d(c+dx)}\right)d\frac{1}{c+dx}-\sinh\left(\frac{b}{d}\right)\int (c+dx)\sinh\left(\frac{bc-ad}{d(c+dx)}\right)d\frac{1}{c+dx}\right)}{d}-i(c+dx)\sinh\left(\frac{b}{d}-\frac{bc-ad}{d(c+dx)}\right)\right)}{d}$$

$\big\downarrow$ 3042

$$\frac{i\left(-\frac{i(bc-ad)\left(\cosh\left(\frac{b}{d}\right)\int (c+dx)\sin\left(\frac{i(bc-ad)}{d(c+dx)}+\frac{\pi}{2}\right)d\frac{1}{c+dx}-\sinh\left(\frac{b}{d}\right)\int -i(c+dx)\sin\left(\frac{i(bc-ad)}{d(c+dx)}\right)d\frac{1}{c+dx}\right)}{d}-i(c+dx)\sinh\left(\frac{b}{d}-\frac{bc-ad}{d(c+dx)}\right)\right)}{d}$$

$\big\downarrow$ 26

$$\frac{i\left(-\frac{i(bc-ad)\left(i\sinh\left(\frac{b}{d}\right)\int (c+dx)\sin\left(\frac{i(bc-ad)}{d(c+dx)}\right)d\frac{1}{c+dx}+\cosh\left(\frac{b}{d}\right)\int (c+dx)\sin\left(\frac{i(bc-ad)}{d(c+dx)}+\frac{\pi}{2}\right)d\frac{1}{c+dx}\right)}{d}-i(c+dx)\sinh\left(\frac{b}{d}-\frac{bc-ad}{d(c+dx)}\right)\right)}{d}$$

$\big\downarrow$ 3779

$$\frac{i\left(-\frac{i(bc-ad)\left(-\sinh\left(\frac{b}{d}\right)\text{Shi}\left(\frac{bc-ad}{d(c+dx)}\right)+\cosh\left(\frac{b}{d}\right)\int (c+dx)\sin\left(\frac{i(bc-ad)}{d(c+dx)}+\frac{\pi}{2}\right)d\frac{1}{c+dx}\right)}{d}-i(c+dx)\sinh\left(\frac{b}{d}-\frac{bc-ad}{d(c+dx)}\right)\right)}{d}$$

$\big\downarrow$ 3782

$$\frac{i\left(-\frac{i(bc-ad)\left(\cosh\left(\frac{b}{d}\right)\text{Chi}\left(\frac{bc-ad}{d(c+dx)}\right)-\sinh\left(\frac{b}{d}\right)\text{Shi}\left(\frac{bc-ad}{d(c+dx)}\right)\right)}{d}-i(c+dx)\sinh\left(\frac{b}{d}-\frac{bc-ad}{d(c+dx)}\right)\right)}{d}$$

input
```
Int[Sinh[(a + b*x)/(c + d*x)],x]
```

output
```
(I*((-I)*(c + d*x)*Sinh[b/d - (b*c - a*d)/(d*(c + d*x))] - (I*(b*c - a*d)*
(Cosh[b/d]*CoshIntegral[(b*c - a*d)/(d*(c + d*x))] - Sinh[b/d]*SinhIntegra
l[(b*c - a*d)/(d*(c + d*x))]))/d))/d
```

Defintions of rubi rules used

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3778
```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(c
 + d*x)^(m + 1)*(Sin[e + f*x]/(d*(m + 1))), x] - Simp[f/(d*(m + 1))   Int[(
c + d*x)^(m + 1)*Cos[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && LtQ[m, -
1]
```

rule 3779
```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[I*(SinhIntegral[c*f*(fz/d) + f*fz*x]/d), x] /; FreeQ[{c, d, e, f
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

rule 3782
```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[CoshIntegral[c*f*(fz/d) + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

rule 3784
```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[Cos[(d*
e - c*f)/d]   Int[Sin[c*(f/d) + f*x]/(c + d*x), x], x] + Simp[Sin[(d*e - c*
f)/d]   Int[Cos[c*(f/d) + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x]
&& NeQ[d*e - c*f, 0]
```

rule 6141
```
Int[Sinh[((e_.)*((a_.) + (b_.)*(x_)))/((c_.) + (d_.)*(x_))]^(n_.), x_Symbol
] :> Simp[-d^(-1)   Subst[Int[Sinh[b*(e/d) - e*(b*c - a*d)*(x/d)]^n/x^2, x]
, x, 1/(c + d*x)], x] /; FreeQ[{a, b, c, d}, x] && IGtQ[n, 0] && NeQ[b*c -
a*d, 0]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 346 vs. $2(101) = 202$.

Time = 0.39 (sec) , antiderivative size = 347, normalized size of antiderivative = 3.44

method	result
risch	$-\frac{e^{-\frac{bx+a}{dx+c}} a}{2\left(\frac{da}{dx+c}-\frac{bc}{dx+c}\right)}+\frac{e^{-\frac{bx+a}{dx+c}} bc}{2d\left(\frac{da}{dx+c}-\frac{bc}{dx+c}\right)}+\frac{e^{-\frac{b}{d}} \operatorname{expIntegral}_1\left(\frac{da-bc}{d(dx+c)}\right) a}{2d}-\frac{e^{-\frac{b}{d}} \operatorname{expIntegral}_1\left(\frac{da-bc}{d(dx+c)}\right) bc}{2d^2}+\frac{d\, e^{\frac{bx+a}{dx+c}} xa}{2da-2bc}-$

input
```
int(sinh((b*x+a)/(d*x+c)),x,method=_RETURNVERBOSE)
```

output
```
-1/2*exp(-(b*x+a)/(d*x+c))/(d*a/(d*x+c)-b*c/(d*x+c))*a+1/2/d*exp(-(b*x+a)/
(d*x+c))/(d*a/(d*x+c)-b*c/(d*x+c))*b*c+1/2/d*exp(-b/d)*Ei(1,(a*d-b*c)/d/(d
*x+c))*a-1/2/d^2*exp(-b/d)*Ei(1,(a*d-b*c)/d/(d*x+c))*b*c+1/2*d*exp((b*x+a)
/(d*x+c))/(a*d-b*c)*x*a-1/2*exp((b*x+a)/(d*x+c))/(a*d-b*c)*x*b*c+1/2*exp((
b*x+a)/(d*x+c))/(a*d-b*c)*c*a-1/2/d*exp((b*x+a)/(d*x+c))/(a*d-b*c)*c^2*b+1
/2/d*exp(b/d)*Ei(1,-(a*d-b*c)/d/(d*x+c))*a-1/2/d^2*exp(b/d)*Ei(1,-(a*d-b*c
)/d/(d*x+c))*b*c
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 171, normalized size of antiderivative = 1.69

$$\int \sinh\left(\frac{a+bx}{c+dx}\right) dx = \frac{\left((bc-ad)\text{Ei}\left(\frac{bc-ad}{d^2x+cd}\right)+(bc-ad)\text{Ei}\left(-\frac{bc-ad}{d^2x+cd}\right)\right)\cosh\left(\frac{b}{d}\right)+2\left(d^2x+cd\right)\sinh\left(\frac{bx+a}{dx+c}\right)-\left((bc-ad)\text{Ei}\left(\frac{bc-ad}{d^2x+cd}\right)\right.}{2\,d^2}$$

input
```
integrate(sinh((b*x+a)/(d*x+c)),x, algorithm="fricas")
```

output
```
1/2*(((b*c - a*d)*Ei((b*c - a*d)/(d^2*x + c*d)) + (b*c - a*d)*Ei(-(b*c - a
*d)/(d^2*x + c*d)))*cosh(b/d) + 2*(d^2*x + c*d)*sinh((b*x + a)/(d*x + c))
- ((b*c - a*d)*Ei((b*c - a*d)/(d^2*x + c*d)) - (b*c - a*d)*Ei(-(b*c - a*d)
/(d^2*x + c*d)))*sinh(b/d))/d^2
```

Sympy [F]

$$\int \sinh\left(\frac{a+bx}{c+dx}\right) dx = \int \sinh\left(\frac{a+bx}{c+dx}\right) dx$$

input
```
integrate(sinh((b*x+a)/(d*x+c)),x)
```

output
```
Integral(sinh((a + b*x)/(c + d*x)), x)
```

Maxima [F]

$$\int \sinh\left(\frac{a+bx}{c+dx}\right) dx = \int \sinh\left(\frac{bx+a}{dx+c}\right) dx$$

input
```
integrate(sinh((b*x+a)/(d*x+c)),x, algorithm="maxima")
```

output
```
integrate(sinh((b*x + a)/(d*x + c)), x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 764 vs. $2(101) = 202$.

Time = 1.73 (sec) , antiderivative size = 764, normalized size of antiderivative = 7.56

$$\int \sinh\left(\frac{a+bx}{c+dx}\right)\,dx = \text{Too large to display}$$

input
```
integrate(sinh((b*x+a)/(d*x+c)),x, algorithm="giac")
```

output
```
1/2*(b^3*c^2*Ei(-(b - (b*x + a)*d/(d*x + c))/d)*e^(b/d) - 2*a*b^2*c*d*Ei(-
(b - (b*x + a)*d/(d*x + c))/d)*e^(b/d) - (b*x + a)*b^2*c^2*d*Ei(-(b - (b*x
 + a)*d/(d*x + c))/d)*e^(b/d)/(d*x + c) + a^2*b*d^2*Ei(-(b - (b*x + a)*d/(
d*x + c))/d)*e^(b/d) + 2*(b*x + a)*a*b*c*d^2*Ei(-(b - (b*x + a)*d/(d*x + c
))/d)*e^(b/d)/(d*x + c) - (b*x + a)*a^2*d^3*Ei(-(b - (b*x + a)*d/(d*x + c)
)/d)*e^(b/d)/(d*x + c) + b^2*c^2*d*e^((b*x + a)/(d*x + c)) - 2*a*b*c*d^2*e
^((b*x + a)/(d*x + c)) + a^2*d^3*e^((b*x + a)/(d*x + c)))*(b*c/(b*c - a*d)
^2 - a*d/(b*c - a*d)^2)/(b*d^2 - (b*x + a)*d^3/(d*x + c)) + 1/2*(b^3*c^2*E
i((b - (b*x + a)*d/(d*x + c))/d)*e^(-b/d) - 2*a*b^2*c*d*Ei((b - (b*x + a)*
d/(d*x + c))/d)*e^(-b/d) - (b*x + a)*b^2*c^2*d*Ei((b - (b*x + a)*d/(d*x +
c))/d)*e^(-b/d)/(d*x + c) + a^2*b*d^2*Ei((b - (b*x + a)*d/(d*x + c))/d)*e^
(-b/d) + 2*(b*x + a)*a*b*c*d^2*Ei((b - (b*x + a)*d/(d*x + c))/d)*e^(-b/d)/
(d*x + c) - (b*x + a)*a^2*d^3*Ei((b - (b*x + a)*d/(d*x + c))/d)*e^(-b/d)/(
d*x + c) - b^2*c^2*d*e^(-(b*x + a)/(d*x + c)) + 2*a*b*c*d^2*e^(-(b*x + a)/
(d*x + c)) - a^2*d^3*e^(-(b*x + a)/(d*x + c)))*(b*c/(b*c - a*d)^2 - a*d/(b
*c - a*d)^2)/(b*d^2 - (b*x + a)*d^3/(d*x + c))
```

Mupad [F(-1)]

Timed out.

$$\int \sinh\left(\frac{a+bx}{c+dx}\right)\,dx = \int \sinh\left(\frac{a+b\,x}{c+d\,x}\right)\,dx$$

input
```
int(sinh((a + b*x)/(c + d*x)),x)
```

output
```
int(sinh((a + b*x)/(c + d*x)), x)
```

Reduce [F]

$$\int \sinh\left(\frac{a+bx}{c+dx}\right)\,dx = \text{Too large to display}$$

input
```
int(sinh((b*x+a)/(d*x+c)),x)
```

output
```
(e**((2*a + 2*b*x)/(c + d*x))*a*d**2*x**2 - e**((2*a + 2*b*x)/(c + d*x))*b
*c*d*x**2 - e**((2*a + 2*b*x)/(c + d*x))*c**3 - e**((2*a + 2*b*x)/(c + d*x
))*c**2*d*x + e**((a + b*x)/(c + d*x))*int(x**2/(e**((a + b*x)/(c + d*x))*
c**3 + 3*e**((a + b*x)/(c + d*x))*c**2*d*x + 3*e**((a + b*x)/(c + d*x))*c*
d**2*x**2 + e**((a + b*x)/(c + d*x))*d**3*x**3),x)*a**2*c*d**3 + e**((a +
b*x)/(c + d*x))*int(x**2/(e**((a + b*x)/(c + d*x))*c**3 + 3*e**((a + b*x)/
(c + d*x))*c**2*d*x + 3*e**((a + b*x)/(c + d*x))*c*d**2*x**2 + e**((a + b*
x)/(c + d*x))*d**3*x**3),x)*a**2*d**4*x - 2*e**((a + b*x)/(c + d*x))*int(x
**2/(e**((a + b*x)/(c + d*x))*c**3 + 3*e**((a + b*x)/(c + d*x))*c**2*d*x +
 3*e**((a + b*x)/(c + d*x))*c*d**2*x**2 + e**((a + b*x)/(c + d*x))*d**3*x*
*3),x)*a*b*c**2*d**2 - 2*e**((a + b*x)/(c + d*x))*int(x**2/(e**((a + b*x)/
(c + d*x))*c**3 + 3*e**((a + b*x)/(c + d*x))*c**2*d*x + 3*e**((a + b*x)/(c
 + d*x))*c*d**2*x**2 + e**((a + b*x)/(c + d*x))*d**3*x**3),x)*a*b*c*d**3*x
 + e**((a + b*x)/(c + d*x))*int(x**2/(e**((a + b*x)/(c + d*x))*c**3 + 3*e*
*((a + b*x)/(c + d*x))*c**2*d*x + 3*e**((a + b*x)/(c + d*x))*c*d**2*x**2 +
 e**((a + b*x)/(c + d*x))*d**3*x**3),x)*b**2*c**3*d + e**((a + b*x)/(c + d
*x))*int(x**2/(e**((a + b*x)/(c + d*x))*c**3 + 3*e**((a + b*x)/(c + d*x))*
c**2*d*x + 3*e**((a + b*x)/(c + d*x))*c*d**2*x**2 + e**((a + b*x)/(c + d*x
))*d**3*x**3),x)*b**2*c**2*d**2*x + e**((a + b*x)/(c + d*x))*int((e**((a +
 b*x)/(c + d*x))*x**2)/(c**3 + 3*c**2*d*x + 3*c*d**2*x**2 + d**3*x**3),...
```

3.296 $\int \sinh^2\left(\frac{a+bx}{c+dx}\right)\,dx$

Optimal result

Integrand size = 16, antiderivative size = 107

$$\int \sinh^2\left(\frac{a+bx}{c+dx}\right)dx = \frac{(bc-ad)\text{Chi}\left(\frac{2(bc-ad)}{d(c+dx)}\right)\sinh\left(\frac{2b}{d}\right)}{d^2} + \frac{(c+dx)\sinh^2\left(\frac{a+bx}{c+dx}\right)}{d} - \frac{(bc-ad)\cosh\left(\frac{2b}{d}\right)\text{Shi}\left(\frac{2(bc-ad)}{d(c+dx)}\right)}{d^2}$$

output
```
(-a*d+b*c)*Chi(2*(-a*d+b*c)/d/(d*x+c))*sinh(2*b/d)/d^2+(d*x+c)*sinh((b*x+a
)/(d*x+c))^2/d-(-a*d+b*c)*cosh(2*b/d)*Shi(2*(-a*d+b*c)/d/(d*x+c))/d^2
```

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 475 vs. 2(107) = 214.

Time = 2.73 (sec) , antiderivative size = 475, normalized size of antiderivative = 4.44

$$\int \sinh^2\left(\frac{a+bx}{c+dx}\right)dx$$
$$= \frac{cde^{-\frac{2(a+bx)}{c+dx}} + cde^{\frac{2(a+bx)}{c+dx}} - 2d^2x + 2d^2x\cosh\left(\frac{2b}{d}\right)\cosh\left(\frac{2(-bc+ad)}{d(c+dx)}\right) - 2(bc-ad)\text{Chi}\left(\frac{2bc-2ad}{cd+d^2x}\right)\left(\cosh\left(\frac{2b}{d}\right) - \right.}{}$$

input
```
Integrate[Sinh[(a + b*x)/(c + d*x)]^2,x]
```

output
```
((c*d)/E^((2*(a + b*x))/(c + d*x)) + c*d*E^((2*(a + b*x))/(c + d*x)) - 2*d
^2*x + 2*d^2*x*Cosh[(2*b)/d]*Cosh[(2*(-(b*c) + a*d))/(d*(c + d*x))] - 2*(b
*c - a*d)*CoshIntegral[(2*b*c - 2*a*d)/(c*d + d^2*x)]*(Cosh[(2*b)/d] - Sin
h[(2*b)/d]) + 2*(b*c - a*d)*CoshIntegral[(2*(-(b*c) + a*d))/(d*(c + d*x))]
*(Cosh[(2*b)/d] + Sinh[(2*b)/d]) + 2*d^2*x*Sinh[(2*b)/d]*Sinh[(2*(-(b*c) +
 a*d))/(d*(c + d*x))] + 2*b*c*Cosh[(2*b)/d]*SinhIntegral[(2*(-(b*c) + a*d)
)/(d*(c + d*x))] - 2*a*d*Cosh[(2*b)/d]*SinhIntegral[(2*(-(b*c) + a*d))/(d*
(c + d*x))] + 2*b*c*Sinh[(2*b)/d]*SinhIntegral[(2*(-(b*c) + a*d))/(d*(c +
d*x))] - 2*a*d*Sinh[(2*b)/d]*SinhIntegral[(2*(-(b*c) + a*d))/(d*(c + d*x))
] - 2*b*c*Cosh[(2*b)/d]*SinhIntegral[(2*b*c - 2*a*d)/(c*d + d^2*x)] + 2*a*
d*Cosh[(2*b)/d]*SinhIntegral[(2*b*c - 2*a*d)/(c*d + d^2*x)] + 2*b*c*Sinh[(
2*b)/d]*SinhIntegral[(2*b*c - 2*a*d)/(c*d + d^2*x)] - 2*a*d*Sinh[(2*b)/d]*
SinhIntegral[(2*b*c - 2*a*d)/(c*d + d^2*x)])/(4*d^2)
```

Rubi [C] (verified)

Result contains complex when optimal does not.

Time = 0.69 (sec) , antiderivative size = 122, normalized size of antiderivative = 1.14, number of steps used = 14, number of rules used = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.812$, Rules used = {6141, 3042, 25, 3794, 27, 3042, 26, 3784, 26, 3042, 26, 3779, 3782}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sinh^2\left(\frac{a+bx}{c+dx}\right)dx$$

$$\Big\downarrow \text{6141}$$

$$-\frac{\int (c+dx)^2\sinh^2\left(\frac{b}{d}-\frac{bc-ad}{d(c+dx)}\right)d\frac{1}{c+dx}}{d}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{\int -(c+dx)^2\sin\left(\frac{ib}{d}-\frac{i(bc-ad)}{d(c+dx)}\right)^2d\frac{1}{c+dx}}{d}$$

$$\Big\downarrow \; 25$$

$$\frac{\int (c+dx)^2 \sin \left(\frac{ib}{d}-\frac{i(bc-ad)}{d(c+dx)}\right)^2 d\frac{1}{c+dx}}{d}$$

$$\Big\downarrow \; 3794$$

$$-\frac{-(c+dx)\sinh^2\left(\frac{b}{d}-\frac{bc-ad}{d(c+dx)}\right)+\frac{2i(bc-ad)\int \frac{1}{2}i(c+dx)\sinh\left(\frac{2b}{d}-\frac{2(bc-ad)}{d(c+dx)}\right)d\frac{1}{c+dx}}{d}}{d}$$

$$\Big\downarrow \; 27$$

$$-\frac{-\frac{(bc-ad)\int (c+dx)\sinh\left(\frac{2b}{d}-\frac{2(bc-ad)}{d(c+dx)}\right)d\frac{1}{c+dx}}{d}-\left((c+dx)\sinh^2\left(\frac{b}{d}-\frac{bc-ad}{d(c+dx)}\right)\right)}{d}$$

$$\Big\downarrow \; 3042$$

$$-\frac{-\left((c+dx)\sinh^2\left(\frac{b}{d}-\frac{bc-ad}{d(c+dx)}\right)\right)-\frac{(bc-ad)\int -i(c+dx)\sin\left(\frac{2ib}{d}-\frac{2i(bc-ad)}{d(c+dx)}\right)d\frac{1}{c+dx}}{d}}{d}$$

$$\Big\downarrow \; 26$$

$$-\frac{-(c+dx)\sinh^2\left(\frac{b}{d}-\frac{bc-ad}{d(c+dx)}\right)+\frac{i(bc-ad)\int (c+dx)\sin\left(\frac{2ib}{d}-\frac{2i(bc-ad)}{d(c+dx)}\right)d\frac{1}{c+dx}}{d}}{d}$$

$$\Big\downarrow \; 3784$$

$$-\frac{-(c+dx)\sinh^2\left(\frac{b}{d}-\frac{bc-ad}{d(c+dx)}\right)+\frac{i(bc-ad)\left(i\sinh\left(\frac{2b}{d}\right)\int (c+dx)\cosh\left(\frac{2(bc-ad)}{d(c+dx)}\right)d\frac{1}{c+dx}+\cosh\left(\frac{2b}{d}\right)\int -i(c+dx)\sinh\left(\frac{2(bc-ad)}{d(c+dx)}\right)d\frac{1}{c+d\right.}{d}}{d}$$

$$\Big\downarrow \; 26$$

$$-\frac{-(c+dx)\sinh^2\left(\frac{b}{d}-\frac{bc-ad}{d(c+dx)}\right)+\frac{i(bc-ad)\left(i\sinh\left(\frac{2b}{d}\right)\int (c+dx)\cosh\left(\frac{2(bc-ad)}{d(c+dx)}\right)d\frac{1}{c+dx}-i\cosh\left(\frac{2b}{d}\right)\int (c+dx)\sinh\left(\frac{2(bc-ad)}{d(c+dx)}\right)d\frac{1}{c+dx}\right.}{d}}{d}$$

$$\Big\downarrow \; 3042$$

$$-\frac{-(c+dx)\sinh^2\left(\frac{b}{d}-\frac{bc-ad}{d(c+dx)}\right)+\frac{i(bc-ad)\left(i\sinh\left(\frac{2b}{d}\right)\int (c+dx)\sin\left(\frac{2i(bc-ad)}{d(c+dx)}+\frac{\pi}{2}\right)d\frac{1}{c+dx}-i\cosh\left(\frac{2b}{d}\right)\int -i(c+dx)\sin\left(\frac{2i(bc-ad)}{d(c+dx)}\right)d\right.}{d}}{d}$$

$$\Big\downarrow \; 26$$

$$-\frac{-(c+dx)\sinh^2\left(\frac{b}{d}-\frac{bc-ad}{d(c+dx)}\right)+\frac{i(bc-ad)\left(i\sinh\left(\frac{2b}{d}\right)\int(c+dx)\sin\left(\frac{2i(bc-ad)}{d(c+dx)}+\frac{\pi}{2}\right)d\frac{1}{c+dx}-\cosh\left(\frac{2b}{d}\right)\int(c+dx)\sin\left(\frac{2i(bc-ad)}{d(c+dx)}\right)d\frac{1}{c+dx}\right)}{d}}{d}$$

$$\Big\downarrow \text{3779}$$

$$-\frac{-(c+dx)\sinh^2\left(\frac{b}{d}-\frac{bc-ad}{d(c+dx)}\right)+\frac{i(bc-ad)\left(i\sinh\left(\frac{2b}{d}\right)\int(c+dx)\sin\left(\frac{2i(bc-ad)}{d(c+dx)}+\frac{\pi}{2}\right)d\frac{1}{c+dx}-i\cosh\left(\frac{2b}{d}\right)\text{Shi}\left(\frac{2(bc-ad)}{d(c+dx)}\right)\right)}{d}}{d}$$

$$\Big\downarrow \text{3782}$$

$$-\frac{-(c+dx)\sinh^2\left(\frac{b}{d}-\frac{bc-ad}{d(c+dx)}\right)+\frac{i(bc-ad)\left(i\sinh\left(\frac{2b}{d}\right)\text{Chi}\left(\frac{2(bc-ad)}{d(c+dx)}\right)-i\cosh\left(\frac{2b}{d}\right)\text{Shi}\left(\frac{2(bc-ad)}{d(c+dx)}\right)\right)}{d}}{d}$$

input
```
Int[Sinh[(a + b*x)/(c + d*x)]^2,x]
```

output
```
-((-((c + d*x)*Sinh[b/d - (b*c - a*d)/(d*(c + d*x))]^2) + (I*(b*c - a*d)*(
I*CoshIntegral[(2*(b*c - a*d))/(d*(c + d*x))]*Sinh[(2*b)/d] - I*Cosh[(2*b)
/d]*SinhIntegral[(2*(b*c - a*d))/(d*(c + d*x))]))/d)/d)
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3779
```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[I*(SinhIntegral[c*f*(fz/d) + f*fz*x]/d), x] /; FreeQ[{c, d, e, f
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

rule 3782
```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[CoshIntegral[c*f*(fz/d) + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

rule 3784
```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[Cos[(d*
e - c*f)/d]   Int[Sin[c*(f/d) + f*x]/(c + d*x), x], x] + Simp[Sin[(d*e - c*
f)/d]   Int[Cos[c*(f/d) + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x]
&& NeQ[d*e - c*f, 0]
```

rule 3794
```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> Si
mp[(c + d*x)^(m + 1)*(Sin[e + f*x]^n/(d*(m + 1))), x] - Simp[f*(n/(d*(m + 1
)))   Int[ExpandTrigReduce[(c + d*x)^(m + 1), Cos[e + f*x]*Sin[e + f*x]^(n
- 1), x], x], x] /; FreeQ[{c, d, e, f, m}, x] && IGtQ[n, 1] && GeQ[m, -2] &
& LtQ[m, -1]
```

rule 6141
```
Int[Sinh[((e_.)*((a_.) + (b_.)*(x_)))/((c_.) + (d_.)*(x_))]^(n_.), x_Symbol
] :> Simp[-d^(-1)   Subst[Int[Sinh[b*(e/d) - e*(b*c - a*d)*(x/d)]^n/x^2, x]
, x, 1/(c + d*x)], x] /; FreeQ[{a, b, c, d}, x] && IGtQ[n, 0] && NeQ[b*c -
a*d, 0]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 357 vs. $2(107) = 214$.

Time = 2.61 (sec) , antiderivative size = 358, normalized size of antiderivative = 3.35

method	result
risch	$-\frac{x}{2}+\frac{{\mathrm e}^{-\frac{2(bx+a)}{dx+c}} a}{\frac{4da}{dx+c}-\frac{4bc}{dx+c}}-\frac{{\mathrm e}^{-\frac{2(bx+a)}{dx+c}} bc}{4d\left(\frac{da}{dx+c}-\frac{bc}{dx+c}\right)}-\frac{{\mathrm e}^{-\frac{2b}{d}} \operatorname{expIntegral}_{1}\left(\frac{2da-2bc}{(dx+c)d}\right) a}{2d}+\frac{{\mathrm e}^{-\frac{2b}{d}} \operatorname{expIntegral}_{1}\left(\frac{2da-2bc}{(dx+c)d}\right) bc}{2d^{2}}+\frac{d\,{\mathrm e}^{\frac{2bx+2a}{dx+c}}}{4da-4b}$

input
```
int(sinh((b*x+a)/(d*x+c))^2,x,method=_RETURNVERBOSE)
```

output
```
-1/2*x+1/4*exp(-2*(b*x+a)/(d*x+c))/(d*a/(d*x+c)-b*c/(d*x+c))*a-1/4/d*exp(-
2*(b*x+a)/(d*x+c))/(d*a/(d*x+c)-b*c/(d*x+c))*b*c-1/2/d*exp(-2*b/d)*Ei(1,2*
(a*d-b*c)/d/(d*x+c))*a+1/2/d^2*exp(-2*b/d)*Ei(1,2*(a*d-b*c)/d/(d*x+c))*b*c
+1/4*d*exp(2*(b*x+a)/(d*x+c))/(a*d-b*c)*x*a-1/4*exp(2*(b*x+a)/(d*x+c))/(a*
d-b*c)*x*b*c+1/4*exp(2*(b*x+a)/(d*x+c))/(a*d-b*c)*c*a-1/4/d*exp(2*(b*x+a)/
(d*x+c))/(a*d-b*c)*c^2*b+1/2/d*exp(2*b/d)*Ei(1,-2*(a*d-b*c)/d/(d*x+c))*a-1
/2/d^2*exp(2*b/d)*Ei(1,-2*(a*d-b*c)/d/(d*x+c))*b*c
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 370 vs. $2(107) = 214$.

Time = 0.11 (sec) , antiderivative size = 370, normalized size of antiderivative = 3.46

$$\int \sinh^2\left(\frac{a+bx}{c+dx}\right)\,dx =$$

$$-\frac{d^2x-(d^2x+cd)\cosh\left(\frac{bx+a}{dx+c}\right)^2-\left(d^2x-(bc-ad)\mathrm{Ei}\left(-\frac{2\,(bc-ad)}{d^2x+cd}\right)\cosh\left(\frac{2\,b}{d}\right)+cd\right)\sinh\left(\frac{bx+a}{dx+c}\right)^2-\left((bc\right.}{}$$

input
```
integrate(sinh((b*x+a)/(d*x+c))^2,x, algorithm="fricas")
```

output
```
-1/2*(d^2*x - (d^2*x + c*d)*cosh((b*x + a)/(d*x + c))^2 - (d^2*x - (b*c -
a*d)*Ei(-2*(b*c - a*d)/(d^2*x + c*d))*cosh(2*b/d) + c*d)*sinh((b*x + a)/(d
*x + c))^2 - ((b*c - a*d)*Ei(-2*(b*c - a*d)/(d^2*x + c*d))*cosh((b*x + a)/
(d*x + c))^2 - (b*c - a*d)*Ei(2*(b*c - a*d)/(d^2*x + c*d)))*cosh(2*b/d) -
((b*c - a*d)*Ei(-2*(b*c - a*d)/(d^2*x + c*d))*cosh((b*x + a)/(d*x + c))^2
- (b*c - a*d)*Ei(-2*(b*c - a*d)/(d^2*x + c*d))*sinh((b*x + a)/(d*x + c))^2
 + (b*c - a*d)*Ei(2*(b*c - a*d)/(d^2*x + c*d)))*sinh(2*b/d))/(d^2*cosh((b*
x + a)/(d*x + c))^2 - d^2*sinh((b*x + a)/(d*x + c))^2)
```

Sympy [F(-1)]

Timed out.

$$\int \sinh^2\left(\frac{a+bx}{c+dx}\right) dx = \text{Timed out}$$

input
```
integrate(sinh((b*x+a)/(d*x+c))**2,x)
```

output
```
Timed out
```

Maxima [F]

$$\int \sinh^2\left(\frac{a+bx}{c+dx}\right) dx = \int \sinh\left(\frac{bx+a}{dx+c}\right)^2 dx$$

input
```
integrate(sinh((b*x+a)/(d*x+c))^2,x, algorithm="maxima")
```

output
```
-1/2*x + 1/4*integrate(e^(2*b*c/(d^2*x + c*d) - 2*a/(d*x + c) - 2*b/d), x)
 + 1/4*integrate(e^(-2*b*c/(d^2*x + c*d) + 2*a/(d*x + c) + 2*b/d), x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 749 vs. $2(107) = 214$.

Time = 7.90 (sec) , antiderivative size = 749, normalized size of antiderivative = 7.00

$$\int \sinh^2\left(\frac{a+bx}{c+dx}\right) dx = \text{Too large to display}$$

input
```
integrate(sinh((b*x+a)/(d*x+c))^2,x, algorithm="giac")
```

output
```
1/4*(2*b^3*c^2*Ei(-2*(b - (b*x + a)*d/(d*x + c))/d)*e^(2*b/d) - 4*a*b^2*c*
d*Ei(-2*(b - (b*x + a)*d/(d*x + c))/d)*e^(2*b/d) - 2*(b*x + a)*b^2*c^2*d*E
i(-2*(b - (b*x + a)*d/(d*x + c))/d)*e^(2*b/d)/(d*x + c) + 2*a^2*b*d^2*Ei(-
2*(b - (b*x + a)*d/(d*x + c))/d)*e^(2*b/d) + 4*(b*x + a)*a*b*c*d^2*Ei(-2*(
b - (b*x + a)*d/(d*x + c))/d)*e^(2*b/d)/(d*x + c) - 2*(b*x + a)*a^2*d^3*Ei
(-2*(b - (b*x + a)*d/(d*x + c))/d)*e^(2*b/d)/(d*x + c) - 2*b^3*c^2*Ei(2*(b
 - (b*x + a)*d/(d*x + c))/d)*e^(-2*b/d) + 4*a*b^2*c*d*Ei(2*(b - (b*x + a)*
d/(d*x + c))/d)*e^(-2*b/d) + 2*(b*x + a)*b^2*c^2*d*Ei(2*(b - (b*x + a)*d/(
d*x + c))/d)*e^(-2*b/d)/(d*x + c) - 2*a^2*b*d^2*Ei(2*(b - (b*x + a)*d/(d*x
 + c))/d)*e^(-2*b/d) - 4*(b*x + a)*a*b*c*d^2*Ei(2*(b - (b*x + a)*d/(d*x +
c))/d)*e^(-2*b/d)/(d*x + c) + 2*(b*x + a)*a^2*d^3*Ei(2*(b - (b*x + a)*d/(d
*x + c))/d)*e^(-2*b/d)/(d*x + c) + b^2*c^2*d*e^(2*(b*x + a)/(d*x + c)) - 2
*a*b*c*d^2*e^(2*(b*x + a)/(d*x + c)) + a^2*d^3*e^(2*(b*x + a)/(d*x + c)) +
 b^2*c^2*d*e^(-2*(b*x + a)/(d*x + c)) - 2*a*b*c*d^2*e^(-2*(b*x + a)/(d*x +
 c)) + a^2*d^3*e^(-2*(b*x + a)/(d*x + c)) - 2*b^2*c^2*d + 4*a*b*c*d^2 - 2*
a^2*d^3)*(b*c/(b*c - a*d)^2 - a*d/(b*c - a*d)^2)/(b*d^2 - (b*x + a)*d^3/(d
*x + c))
```

Mupad [F(-1)]

Timed out.

$$\int \sinh^2\left(\frac{a+bx}{c+dx}\right)\,dx=\int \sinh\left(\frac{a+b\,x}{c+d\,x}\right)^2\,dx$$

input
```
int(sinh((a + b*x)/(c + d*x))^2,x)
```

output
```
int(sinh((a + b*x)/(c + d*x))^2, x)
```

Reduce [F]

$$\int \sinh^2\left(\frac{a+bx}{c+dx}\right)\,dx=\text{Too large to display}$$

input
```
int(sinh((b*x+a)/(d*x+c))^2,x)
```

output
```
(2*e**((4*a + 4*b*x)/(c + d*x))*a*d**2*x**2 - 2*e**((4*a + 4*b*x)/(c + d*x
))*b*c*d*x**2 - e**((4*a + 4*b*x)/(c + d*x))*c**3 - e**((4*a + 4*b*x)/(c +
 d*x))*c**2*d*x - 4*e**((2*a + 2*b*x)/(c + d*x))*int(x**2/(e**((2*a + 2*b*
x)/(c + d*x))*c**3 + 3*e**((2*a + 2*b*x)/(c + d*x))*c**2*d*x + 3*e**((2*a
+ 2*b*x)/(c + d*x))*c*d**2*x**2 + e**((2*a + 2*b*x)/(c + d*x))*d**3*x**3),
x)*a**2*c*d**3 - 4*e**((2*a + 2*b*x)/(c + d*x))*int(x**2/(e**((2*a + 2*b*x
)/(c + d*x))*c**3 + 3*e**((2*a + 2*b*x)/(c + d*x))*c**2*d*x + 3*e**((2*a +
 2*b*x)/(c + d*x))*c*d**2*x**2 + e**((2*a + 2*b*x)/(c + d*x))*d**3*x**3),x
)*a**2*d**4*x + 8*e**((2*a + 2*b*x)/(c + d*x))*int(x**2/(e**((2*a + 2*b*x)
/(c + d*x))*c**3 + 3*e**((2*a + 2*b*x)/(c + d*x))*c**2*d*x + 3*e**((2*a +
2*b*x)/(c + d*x))*c*d**2*x**2 + e**((2*a + 2*b*x)/(c + d*x))*d**3*x**3),x)
*a*b*c**2*d**2 + 8*e**((2*a + 2*b*x)/(c + d*x))*int(x**2/(e**((2*a + 2*b*x
)/(c + d*x))*c**3 + 3*e**((2*a + 2*b*x)/(c + d*x))*c**2*d*x + 3*e**((2*a +
 2*b*x)/(c + d*x))*c*d**2*x**2 + e**((2*a + 2*b*x)/(c + d*x))*d**3*x**3),x
)*a*b*c*d**3*x - 4*e**((2*a + 2*b*x)/(c + d*x))*int(x**2/(e**((2*a + 2*b*x
)/(c + d*x))*c**3 + 3*e**((2*a + 2*b*x)/(c + d*x))*c**2*d*x + 3*e**((2*a +
 2*b*x)/(c + d*x))*c*d**2*x**2 + e**((2*a + 2*b*x)/(c + d*x))*d**3*x**3),x
)*b**2*c**3*d - 4*e**((2*a + 2*b*x)/(c + d*x))*int(x**2/(e**((2*a + 2*b*x)
/(c + d*x))*c**3 + 3*e**((2*a + 2*b*x)/(c + d*x))*c**2*d*x + 3*e**((2*a +
2*b*x)/(c + d*x))*c*d**2*x**2 + e**((2*a + 2*b*x)/(c + d*x))*d**3*x**3)...
```

3.297 $\int \sinh^3\left(\frac{a+bx}{c+dx}\right)\,dx$

Optimal result

Integrand size = 16, antiderivative size = 194

$$\int \sinh^3\left(\frac{a+bx}{c+dx}\right)\,dx = -\frac{3(bc-ad)\cosh\left(\frac{b}{d}\right)\text{Chi}\left(\frac{bc-ad}{d(c+dx)}\right)}{4d^2} + \frac{3(bc-ad)\cosh\left(\frac{3b}{d}\right)\text{Chi}\left(\frac{3(bc-ad)}{d(c+dx)}\right)}{4d^2} + \frac{(c+dx)\sinh^3\left(\frac{a+bx}{c+dx}\right)}{d} + \frac{3(bc-ad)\sinh\left(\frac{b}{d}\right)\text{Shi}\left(\frac{bc-ad}{d(c+dx)}\right)}{4d^2} - \frac{3(bc-ad)\sinh\left(\frac{3b}{d}\right)\text{Shi}\left(\frac{3(bc-ad)}{d(c+dx)}\right)}{4d^2}$$

output

```
-3/4*(-a*d+b*c)*cosh(b/d)*Chi((-a*d+b*c)/d/(d*x+c))/d^2+3/4*(-a*d+b*c)*cos
h(3*b/d)*Chi(3*(-a*d+b*c)/d/(d*x+c))/d^2+(d*x+c)*sinh((b*x+a)/(d*x+c))^3/d
+3/4*(-a*d+b*c)*sinh(b/d)*Shi((-a*d+b*c)/d/(d*x+c))/d^2-3/4*(-a*d+b*c)*sin
h(3*b/d)*Shi(3*(-a*d+b*c)/d/(d*x+c))/d^2
```

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 651 vs. $2(194) = 388$.

Time = 5.19 (sec) , antiderivative size = 651, normalized size of antiderivative = 3.36

$$\int \sinh^3\left(\frac{a+bx}{c+dx}\right) dx$$
$$= \frac{-cde^{-\frac{3(a+bx)}{c+dx}} + 3cde^{-\frac{a+bx}{c+dx}} - 3cde^{\frac{a+bx}{c+dx}} + cde^{\frac{3(a+bx)}{c+dx}} - 6d^2x\cosh\left(\frac{-bc+ad}{d(c+dx)}\right)\sinh\left(\frac{b}{d}\right) + 2d^2x\cosh\left(\frac{3(-bc+ad)}{d(c+dx)}\right)}{}$$

input
```
Integrate[Sinh[(a + b*x)/(c + d*x)]^3,x]
```

output
```
(-((c*d)/E^((3*(a + b*x))/(c + d*x))) + (3*c*d)/E^((a + b*x)/(c + d*x)) -
3*c*d*E^((a + b*x)/(c + d*x)) + c*d*E^((3*(a + b*x))/(c + d*x)) - 6*d^2*x*
Cosh[(-(b*c) + a*d)/(d*(c + d*x))]*Sinh[b/d] + 2*d^2*x*Cosh[(3*(-(b*c) + a
*d))/(d*(c + d*x))]*Sinh[(3*b)/d] - 6*d^2*x*Cosh[b/d]*Sinh[(-(b*c) + a*d)/
(d*(c + d*x))] + 2*d^2*x*Cosh[(3*b)/d]*Sinh[(3*(-(b*c) + a*d))/(d*(c + d*x
))] + 3*(b*c - a*d)*(Cosh[(3*b)/d]*CoshIntegral[(3*b*c - 3*a*d)/(c*d + d^2
*x)] - Cosh[b/d]*CoshIntegral[(b*c - a*d)/(c*d + d^2*x)] + CoshIntegral[(b
*c - a*d)/(c*d + d^2*x)]*Sinh[b/d] - CoshIntegral[(-(b*c) + a*d)/(d*(c + d
*x))]*(Cosh[b/d] + Sinh[b/d]) - CoshIntegral[(3*b*c - 3*a*d)/(c*d + d^2*x)
]*Sinh[(3*b)/d] + CoshIntegral[(3*(-(b*c) + a*d))/(d*(c + d*x))]*(Cosh[(3*
b)/d] + Sinh[(3*b)/d]) - Cosh[b/d]*SinhIntegral[(-(b*c) + a*d)/(d*(c + d*x
))] - Sinh[b/d]*SinhIntegral[(-(b*c) + a*d)/(d*(c + d*x))] + Cosh[(3*b)/d]
*SinhIntegral[(3*(-(b*c) + a*d))/(d*(c + d*x))] + Sinh[(3*b)/d]*SinhIntegr
al[(3*(-(b*c) + a*d))/(d*(c + d*x))] + Cosh[(3*b)/d]*SinhIntegral[(3*b*c -
 3*a*d)/(c*d + d^2*x)] - Sinh[(3*b)/d]*SinhIntegral[(3*b*c - 3*a*d)/(c*d +
 d^2*x)] - Cosh[b/d]*SinhIntegral[(b*c - a*d)/(c*d + d^2*x)] + Sinh[b/d]*S
inhIntegral[(b*c - a*d)/(c*d + d^2*x)]))/(8*d^2)
```

Rubi [C] (verified)

Result contains complex when optimal does not.

Time = 0.89 (sec) , antiderivative size = 186, normalized size of antiderivative = 0.96, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.312$, Rules used = {6141, 3042, 26, 3794, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sinh^3\left(\frac{a+bx}{c+dx}\right)dx$$

$$\Big\downarrow \text{6141}$$

$$-\frac{\int (c+dx)^2\sinh^3\left(\frac{b}{d}-\frac{bc-ad}{d(c+dx)}\right)d\frac{1}{c+dx}}{d}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{\int i(c+dx)^2\sin\left(\frac{ib}{d}-\frac{i(bc-ad)}{d(c+dx)}\right)^3d\frac{1}{c+dx}}{d}$$

$$\Big\downarrow \text{26}$$

$$-\frac{i\int (c+dx)^2\sin\left(\frac{ib}{d}-\frac{i(bc-ad)}{d(c+dx)}\right)^3d\frac{1}{c+dx}}{d}$$

$$\Big\downarrow \text{3794}$$

$$-\frac{i\left(i(c+dx)\sinh^3\left(\frac{b}{d}-\frac{bc-ad}{d(c+dx)}\right)-\frac{3i(bc-ad)\int\left(\frac{1}{4}(c+dx)\cosh\left(\frac{b}{d}-\frac{bc-ad}{d(c+dx)}\right)-\frac{1}{4}(c+dx)\cosh\left(\frac{3b}{d}-\frac{3(bc-ad)}{d(c+dx)}\right)\right)d\frac{1}{c+dx}}{d}\right)}{d}$$

$$\Big\downarrow \text{2009}$$

$$-\frac{i\left(i(c+dx)\sinh^3\left(\frac{b}{d}-\frac{bc-ad}{d(c+dx)}\right)-\frac{3i(bc-ad)\left(\frac{1}{4}\cosh\left(\frac{b}{d}\right)\text{Chi}\left(\frac{bc-ad}{d(c+dx)}\right)-\frac{1}{4}\cosh\left(\frac{3b}{d}\right)\text{Chi}\left(\frac{3(bc-ad)}{d(c+dx)}\right)-\frac{1}{4}\sinh\left(\frac{b}{d}\right)\text{Shi}\left(\frac{bc-ad}{d(c+dx)}\right)\right.}{d}}{d}$$

input
```
Int[Sinh[(a + b*x)/(c + d*x)]^3,x]
```

output
```
((-I)*(I*(c + d*x)*Sinh[b/d - (b*c - a*d)/(d*(c + d*x))]^3 - ((3*I)*(b*c -
 a*d)*((Cosh[b/d]*CoshIntegral[(b*c - a*d)/(d*(c + d*x))])/4 - (Cosh[(3*b)
/d]*CoshIntegral[(3*(b*c - a*d))/(d*(c + d*x))])/4 - (Sinh[b/d]*SinhIntegr
al[(b*c - a*d)/(d*(c + d*x))])/4 + (Sinh[(3*b)/d]*SinhIntegral[(3*(b*c - a
*d))/(d*(c + d*x))])/4))/d))/d
```

Defintions of rubi rules used

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3794
```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> Si
mp[(c + d*x)^(m + 1)*(Sin[e + f*x]^n/(d*(m + 1))), x] - Simp[f*(n/(d*(m + 1
)))   Int[ExpandTrigReduce[(c + d*x)^(m + 1), Cos[e + f*x]*Sin[e + f*x]^(n
 - 1), x], x], x] /; FreeQ[{c, d, e, f, m}, x] && IGtQ[n, 1] && GeQ[m, -2] &
& LtQ[m, -1]
```

rule 6141
```
Int[Sinh[((e_.)*((a_.) + (b_.)*(x_)))/((c_.) + (d_.)*(x_))]^(n_.), x_Symbol
] :> Simp[-d^(-1)   Subst[Int[Sinh[b*(e/d) - e*(b*c - a*d)*(x/d)]^n/x^2, x]
, x, 1/(c + d*x)], x] /; FreeQ[{a, b, c, d}, x] && IGtQ[n, 0] && NeQ[b*c -
a*d, 0]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 699 vs. $2(186) = 372$.

Time = 1.32 (sec) , antiderivative size = 700, normalized size of antiderivative = 3.61

method	result
risch	$-\frac{e^{-\frac{3(bx+a)}{dx+c}} a}{8\left(\frac{da}{dx+c}-\frac{bc}{dx+c}\right)}+\frac{e^{-\frac{3(bx+a)}{dx+c}} bc}{8d\left(\frac{da}{dx+c}-\frac{bc}{dx+c}\right)}+\frac{3\,e^{-\frac{3b}{d}} \operatorname{expIntegral}_1\left(\frac{3da-3bc}{d(dx+c)}\right) a}{8d}-\frac{3\,e^{-\frac{3b}{d}} \operatorname{expIntegral}_1\left(\frac{3da-3bc}{d(dx+c)}\right) bc}{8d^2}+\frac{3\,e^{-\frac{b}{d}\cdots}}{8\left(\frac{da}{dx+c}\cdots\right)}$

input
```
int(sinh((b*x+a)/(d*x+c))^3,x,method=_RETURNVERBOSE)
```

output
```
-1/8*exp(-3*(b*x+a)/(d*x+c))/(d*a/(d*x+c)-b*c/(d*x+c))*a+1/8/d*exp(-3*(b*x
+a)/(d*x+c))/(d*a/(d*x+c)-b*c/(d*x+c))*b*c+3/8/d*exp(-3*b/d)*Ei(1,3*(a*d-b
*c)/d/(d*x+c))*a-3/8/d^2*exp(-3*b/d)*Ei(1,3*(a*d-b*c)/d/(d*x+c))*b*c+3/8*e
xp(-(b*x+a)/(d*x+c))/(d*a/(d*x+c)-b*c/(d*x+c))*a-3/8/d*exp(-(b*x+a)/(d*x+c
))/(d*a/(d*x+c)-b*c/(d*x+c))*b*c-3/8/d*exp(-b/d)*Ei(1,(a*d-b*c)/d/(d*x+c))
*a+3/8/d^2*exp(-b/d)*Ei(1,(a*d-b*c)/d/(d*x+c))*b*c+1/8*d*exp(3*(b*x+a)/(d*
x+c))/(a*d-b*c)*x*a-1/8*exp(3*(b*x+a)/(d*x+c))/(a*d-b*c)*x*b*c+1/8*exp(3*(
b*x+a)/(d*x+c))/(a*d-b*c)*c*a-1/8/d*exp(3*(b*x+a)/(d*x+c))/(a*d-b*c)*c^2*b
+3/8/d*exp(3*b/d)*Ei(1,-3*(a*d-b*c)/d/(d*x+c))*a-3/8/d^2*exp(3*b/d)*Ei(1,-
3*(a*d-b*c)/d/(d*x+c))*b*c-3/8*d*exp((b*x+a)/(d*x+c))/(a*d-b*c)*x*a+3/8*ex
p((b*x+a)/(d*x+c))/(a*d-b*c)*x*b*c-3/8*exp((b*x+a)/(d*x+c))/(a*d-b*c)*c*a+
3/8/d*exp((b*x+a)/(d*x+c))/(a*d-b*c)*c^2*b-3/8/d*exp(b/d)*Ei(1,-(a*d-b*c)/
d/(d*x+c))*a+3/8/d^2*exp(b/d)*Ei(1,-(a*d-b*c)/d/(d*x+c))*b*c
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 717 vs. $2(186) = 372$.

Time = 0.11 (sec) , antiderivative size = 717, normalized size of antiderivative = 3.70

$$\int \sinh^3\left(\frac{a+bx}{c+dx}\right)\,dx = \text{Too large to display}$$

input
```
integrate(sinh((b*x+a)/(d*x+c))^3,x, algorithm="fricas")
```

output
```
-1/8*(6*(b*c - a*d)*Ei(-3*(b*c - a*d)/(d^2*x + c*d))*cosh((b*x + a)/(d*x +
 c))^2*cosh(3*b/d)*sinh((b*x + a)/(d*x + c))^2 - 3*(b*c - a*d)*Ei(-3*(b*c
- a*d)/(d^2*x + c*d))*cosh(3*b/d)*sinh((b*x + a)/(d*x + c))^4 - 2*(d^2*x +
 c*d)*sinh((b*x + a)/(d*x + c))^3 - 3*((b*c - a*d)*Ei(-3*(b*c - a*d)/(d^2*
x + c*d))*cosh((b*x + a)/(d*x + c))^4 + (b*c - a*d)*Ei(3*(b*c - a*d)/(d^2*
x + c*d)))*cosh(3*b/d) + 3*((b*c - a*d)*Ei((b*c - a*d)/(d^2*x + c*d)) + (b
*c - a*d)*Ei(-(b*c - a*d)/(d^2*x + c*d)))*cosh(b/d) + 6*(d^2*x - (d^2*x +
c*d)*cosh((b*x + a)/(d*x + c))^2 + c*d)*sinh((b*x + a)/(d*x + c)) - 3*((b*
c - a*d)*Ei(-3*(b*c - a*d)/(d^2*x + c*d))*cosh((b*x + a)/(d*x + c))^4 - 2*
(b*c - a*d)*Ei(-3*(b*c - a*d)/(d^2*x + c*d))*cosh((b*x + a)/(d*x + c))^2*s
inh((b*x + a)/(d*x + c))^2 + (b*c - a*d)*Ei(-3*(b*c - a*d)/(d^2*x + c*d))*
sinh((b*x + a)/(d*x + c))^4 - (b*c - a*d)*Ei(3*(b*c - a*d)/(d^2*x + c*d)))
*sinh(3*b/d) - 3*((b*c - a*d)*Ei((b*c - a*d)/(d^2*x + c*d)) - (b*c - a*d)*
Ei(-(b*c - a*d)/(d^2*x + c*d)))*sinh(b/d))/(d^2*cosh((b*x + a)/(d*x + c))^
4 - 2*d^2*cosh((b*x + a)/(d*x + c))^2*sinh((b*x + a)/(d*x + c))^2 + d^2*si
nh((b*x + a)/(d*x + c))^4)
```

Sympy [F(-1)]

Timed out.

$$\int \sinh^3\left(\frac{a+bx}{c+dx}\right)\,dx = \text{Timed out}$$

input
```
integrate(sinh((b*x+a)/(d*x+c))**3,x)
```

output
```
Timed out
```

Maxima [F]

$$\int \sinh^3\left(\frac{a+bx}{c+dx}\right)\,dx = \int \sinh\left(\frac{bx+a}{dx+c}\right)^3\,dx$$

input
```
integrate(sinh((b*x+a)/(d*x+c))^3,x, algorithm="maxima")
```

output
```
integrate(sinh((b*x + a)/(d*x + c))^3, x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1383 vs. $2(186) = 372$.

Time = 9.49 (sec) , antiderivative size = 1383, normalized size of antiderivative = 7.13

$$\int \sinh^3\left(\frac{a+bx}{c+dx}\right) dx = \text{Too large to display}$$

input
```
integrate(sinh((b*x+a)/(d*x+c))^3,x, algorithm="giac")
```

output
```
1/8*(3*b^3*c^2*Ei(-3*(b - (b*x + a)*d/(d*x + c))/d)*e^(3*b/d) - 6*a*b^2*c*
d*Ei(-3*(b - (b*x + a)*d/(d*x + c))/d)*e^(3*b/d) - 3*(b*x + a)*b^2*c^2*d*E
i(-3*(b - (b*x + a)*d/(d*x + c))/d)*e^(3*b/d)/(d*x + c) + 3*a^2*b*d^2*Ei(-
3*(b - (b*x + a)*d/(d*x + c))/d)*e^(3*b/d) + 6*(b*x + a)*a*b*c*d^2*Ei(-3*(
b - (b*x + a)*d/(d*x + c))/d)*e^(3*b/d)/(d*x + c) - 3*(b*x + a)*a^2*d^3*Ei
(-3*(b - (b*x + a)*d/(d*x + c))/d)*e^(3*b/d)/(d*x + c) - 3*b^3*c^2*Ei(-(b
- (b*x + a)*d/(d*x + c))/d)*e^(b/d) + 6*a*b^2*c*d*Ei(-(b - (b*x + a)*d/(d*
x + c))/d)*e^(b/d) + 3*(b*x + a)*b^2*c^2*d*Ei(-(b - (b*x + a)*d/(d*x + c))
/d)*e^(b/d)/(d*x + c) - 3*a^2*b*d^2*Ei(-(b - (b*x + a)*d/(d*x + c))/d)*e^(
b/d) - 6*(b*x + a)*a*b*c*d^2*Ei(-(b - (b*x + a)*d/(d*x + c))/d)*e^(b/d)/(d
*x + c) + 3*(b*x + a)*a^2*d^3*Ei(-(b - (b*x + a)*d/(d*x + c))/d)*e^(b/d)/(
d*x + c) - 3*b^3*c^2*Ei((b - (b*x + a)*d/(d*x + c))/d)*e^(-b/d) + 6*a*b^2*
c*d*Ei((b - (b*x + a)*d/(d*x + c))/d)*e^(-b/d) + 3*(b*x + a)*b^2*c^2*d*Ei(
(b - (b*x + a)*d/(d*x + c))/d)*e^(-b/d)/(d*x + c) - 3*a^2*b*d^2*Ei((b - (b
*x + a)*d/(d*x + c))/d)*e^(-b/d) - 6*(b*x + a)*a*b*c*d^2*Ei((b - (b*x + a)
*d/(d*x + c))/d)*e^(-b/d)/(d*x + c) + 3*(b*x + a)*a^2*d^3*Ei((b - (b*x + a
)*d/(d*x + c))/d)*e^(-b/d)/(d*x + c) + 3*b^3*c^2*Ei(3*(b - (b*x + a)*d/(d*
x + c))/d)*e^(-3*b/d) - 6*a*b^2*c*d*Ei(3*(b - (b*x + a)*d/(d*x + c))/d)*e^
(-3*b/d) - 3*(b*x + a)*b^2*c^2*d*Ei(3*(b - (b*x + a)*d/(d*x + c))/d)*e^(-3
*b/d)/(d*x + c) + 3*a^2*b*d^2*Ei(3*(b - (b*x + a)*d/(d*x + c))/d)*e^(-3...
```

Mupad [F(-1)]

Timed out.

$$\int \sinh^3\left(\frac{a+bx}{c+dx}\right)\,dx=\int \sinh\left(\frac{a+b\,x}{c+d\,x}\right)^3\,dx$$

input
```
int(sinh((a + b*x)/(c + d*x))^3,x)
```

output
```
int(sinh((a + b*x)/(c + d*x))^3, x)
```

Reduce [F]

$$\int \sinh^3\left(\frac{a+bx}{c+dx}\right)\,dx=\text{too large to display}$$

input
```
int(sinh((b*x+a)/(d*x+c))^3,x)
```

output

```
(3*e**((6*a + 6*b*x)/(c + d*x))*a*d**2*x**2 - 3*e**((6*a + 6*b*x)/(c + d*x
))*b*c*d*x**2 - e**((6*a + 6*b*x)/(c + d*x))*c**3 - e**((6*a + 6*b*x)/(c +
 d*x))*c**2*d*x - 9*e**((4*a + 4*b*x)/(c + d*x))*a*d**2*x**2 + 9*e**((4*a
+ 4*b*x)/(c + d*x))*b*c*d*x**2 + 9*e**((4*a + 4*b*x)/(c + d*x))*c**3 + 9*e
**((4*a + 4*b*x)/(c + d*x))*c**2*d*x + 9*e**((3*a + 3*b*x)/(c + d*x))*int(
x**2/(e**((3*a + 3*b*x)/(c + d*x))*c**3 + 3*e**((3*a + 3*b*x)/(c + d*x))*c
**2*d*x + 3*e**((3*a + 3*b*x)/(c + d*x))*c*d**2*x**2 + e**((3*a + 3*b*x)/(
c + d*x))*d**3*x**3),x)*a**2*c*d**3 + 9*e**((3*a + 3*b*x)/(c + d*x))*int(x
**2/(e**((3*a + 3*b*x)/(c + d*x))*c**3 + 3*e**((3*a + 3*b*x)/(c + d*x))*c*
*2*d*x + 3*e**((3*a + 3*b*x)/(c + d*x))*c*d**2*x**2 + e**((3*a + 3*b*x)/(c
 + d*x))*d**3*x**3),x)*a**2*d**4*x - 18*e**((3*a + 3*b*x)/(c + d*x))*int(x
**2/(e**((3*a + 3*b*x)/(c + d*x))*c**3 + 3*e**((3*a + 3*b*x)/(c + d*x))*c*
*2*d*x + 3*e**((3*a + 3*b*x)/(c + d*x))*c*d**2*x**2 + e**((3*a + 3*b*x)/(c
 + d*x))*d**3*x**3),x)*a*b*c**2*d**2 - 18*e**((3*a + 3*b*x)/(c + d*x))*int
(x**2/(e**((3*a + 3*b*x)/(c + d*x))*c**3 + 3*e**((3*a + 3*b*x)/(c + d*x))*
c**2*d*x + 3*e**((3*a + 3*b*x)/(c + d*x))*c*d**2*x**2 + e**((3*a + 3*b*x)/
(c + d*x))*d**3*x**3),x)*a*b*c*d**3*x + 9*e**((3*a + 3*b*x)/(c + d*x))*int
(x**2/(e**((3*a + 3*b*x)/(c + d*x))*c**3 + 3*e**((3*a + 3*b*x)/(c + d*x))*
c**2*d*x + 3*e**((3*a + 3*b*x)/(c + d*x))*c*d**2*x**2 + e**((3*a + 3*b*x)/
(c + d*x))*d**3*x**3),x)*b**2*c**3*d + 9*e**((3*a + 3*b*x)/(c + d*x))*i...
```

3.298 $\int \sinh\left(e+\frac{f(a+bx)}{c+dx}\right) dx$

Optimal result

Integrand size = 17, antiderivative size = 121

$$\int \sinh\left(e+\frac{f(a+bx)}{c+dx}\right) dx = \frac{(bc-ad)f\cosh\left(e+\frac{bf}{d}\right)\text{Chi}\left(\frac{(bc-ad)f}{d(c+dx)}\right)}{d^2} + \frac{(c+dx)\sinh\left(\frac{ce+af+dex+bfx}{c+dx}\right)}{d} - \frac{(bc-ad)f\sinh\left(e+\frac{bf}{d}\right)\text{Shi}\left(\frac{(bc-ad)f}{d(c+dx)}\right)}{d^2}$$

output
```
(-a*d+b*c)*f*cosh(e+b*f/d)*Chi((-a*d+b*c)*f/d/(d*x+c))/d^2+(d*x+c)*sinh((b
*f*x+d*e*x+a*f+c*e)/(d*x+c))/d-(-a*d+b*c)*f*sinh(e+b*f/d)*Shi((-a*d+b*c)*f
/d/(d*x+c))/d^2
```

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 363 vs. $2(121) = 242$.

Time = 1.52 (sec) , antiderivative size = 363, normalized size of antiderivative = 3.00

$$\int \sinh\left(e+\frac{f(a+bx)}{c+dx}\right)\,dx$$
$$= \frac{-cde^{-\frac{ce+af+dex+bfx}{c+dx}}+cde^{\frac{ce+af+dex+bfx}{c+dx}}+2d^2x\cosh\left(\frac{-bcf+adf}{d(c+dx)}\right)\sinh\left(e+\frac{bf}{d}\right)+2d^2x\cosh\left(e+\frac{bf}{d}\right)\sinh\left(\frac{-b}{d(}\right.}{}$$

input
```
Integrate[Sinh[e + (f*(a + b*x))/(c + d*x)],x]
```

output
```
(-((c*d)/E^((c*e + a*f + d*e*x + b*f*x)/(c + d*x))) + c*d*E^((c*e + a*f +
d*e*x + b*f*x)/(c + d*x)) + 2*d^2*x*Cosh[(-(b*c*f) + a*d*f)/(d*(c + d*x))]
*Sinh[e + (b*f)/d] + 2*d^2*x*Cosh[e + (b*f)/d]*Sinh[(-(b*c*f) + a*d*f)/(d*
(c + d*x))] + (b*c - a*d)*f*(CoshIntegral[((b*c - a*d)*f)/(d*(c + d*x))]*(
Cosh[e + (b*f)/d] - Sinh[e + (b*f)/d]) + CoshIntegral[(-(b*c*f) + a*d*f)/(
d*(c + d*x))]*(Cosh[e + (b*f)/d] + Sinh[e + (b*f)/d]) + Cosh[e + (b*f)/d]*
SinhIntegral[((b*c - a*d)*f)/(d*(c + d*x))] - Sinh[e + (b*f)/d]*SinhIntegr
al[((b*c - a*d)*f)/(d*(c + d*x))] + Cosh[e + (b*f)/d]*SinhIntegral[(-(b*c*
f) + a*d*f)/(d*(c + d*x))] + Sinh[e + (b*f)/d]*SinhIntegral[(-(b*c*f) + a*
d*f)/(d*(c + d*x))]))/(2*d^2)
```

Rubi [C] (verified)

Result contains complex when optimal does not.

Time = 1.21 (sec) , antiderivative size = 127, normalized size of antiderivative = 1.05, number of steps used = 13, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.706$, Rules used = {6143, 6141, 3042, 26, 3778, 3042, 3784, 26, 3042, 26, 3779, 3782}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sinh\left(\frac{f(a+bx)}{c+dx}+e\right)\,dx$$
$$\Big\downarrow \text{6143}$$

$$\int \sinh\left(\frac{af+x(bf+de)+ce}{c+dx}\right)dx$$

$\Big\downarrow$ 6141

$$-\frac{\int (c+dx)^2\sinh\left(e+\frac{bf}{d}-\frac{(bc-ad)f}{d(c+dx)}\right)d\frac{1}{c+dx}}{d}$$

$\Big\downarrow$ 3042

$$-\frac{\int -i(c+dx)^2\sin\left(i\left(e+\frac{bf}{d}\right)-\frac{i(bc-ad)f}{d(c+dx)}\right)d\frac{1}{c+dx}}{d}$$

$\Big\downarrow$ 26

$$\frac{i\int (c+dx)^2\sin\left(i\left(e+\frac{bf}{d}\right)-\frac{i(bc-ad)f}{d(c+dx)}\right)d\frac{1}{c+dx}}{d}$$

$\Big\downarrow$ 3778

$$\frac{i\left(-\frac{if(bc-ad)\int (c+dx)\cosh\left(e+\frac{bf}{d}-\frac{(bc-ad)f}{d(c+dx)}\right)d\frac{1}{c+dx}}{d}-i(c+dx)\sinh\left(-\frac{f(bc-ad)}{d(c+dx)}+\frac{bf}{d}+e\right)\right)}{d}$$

$\Big\downarrow$ 3042

$$\frac{i\left(-\frac{if(bc-ad)\int (c+dx)\sin\left(-\frac{i(bc-ad)f}{d(c+dx)}+i\left(e+\frac{bf}{d}\right)+\frac{\pi}{2}\right)d\frac{1}{c+dx}}{d}-i(c+dx)\sinh\left(-\frac{f(bc-ad)}{d(c+dx)}+\frac{bf}{d}+e\right)\right)}{d}$$

$\Big\downarrow$ 3784

$$\frac{i\left(-\frac{if(bc-ad)\left(\cosh\left(\frac{bf}{d}+e\right)\int (c+dx)\cosh\left(\frac{(bc-ad)f}{d(c+dx)}\right)d\frac{1}{c+dx}-i\sinh\left(\frac{bf}{d}+e\right)\int -i(c+dx)\sinh\left(\frac{(bc-ad)f}{d(c+dx)}\right)d\frac{1}{c+dx}\right)}{d}-i(c+dx)\sinh\left(-\frac{f(}{d}\right.\right.}{d}$$

$\Big\downarrow$ 26

$$\frac{i\left(-\frac{if(bc-ad)\left(\cosh\left(\frac{bf}{d}+e\right)\int (c+dx)\cosh\left(\frac{(bc-ad)f}{d(c+dx)}\right)d\frac{1}{c+dx}-\sinh\left(\frac{bf}{d}+e\right)\int (c+dx)\sinh\left(\frac{(bc-ad)f}{d(c+dx)}\right)d\frac{1}{c+dx}\right)}{d}-i(c+dx)\sinh\left(-\frac{f(bc-}{d(c+}\right.\right.}{d}$$

$\Big\downarrow$ 3042

$$\frac{i\left(-\frac{if(bc-ad)\left(\cosh\left(\frac{bf}{d}+e\right)\int(c+dx)\sin\left(\frac{i(bc-ad)f}{d(c+dx)}+\frac{\pi}{2}\right)d\frac{1}{c+dx}-\sinh\left(\frac{bf}{d}+e\right)\int -i(c+dx)\sin\left(\frac{i(bc-ad)f}{d(c+dx)}\right)d\frac{1}{c+dx}\right)}{d}-i(c+dx)\sinh\left(-\right.\right.}{d}$$

$\big\downarrow$ 26

$$\frac{i\left(-\frac{if(bc-ad)\left(i\sinh\left(\frac{bf}{d}+e\right)\int(c+dx)\sin\left(\frac{i(bc-ad)f}{d(c+dx)}\right)d\frac{1}{c+dx}+\cosh\left(\frac{bf}{d}+e\right)\int(c+dx)\sin\left(\frac{i(bc-ad)f}{d(c+dx)}+\frac{\pi}{2}\right)d\frac{1}{c+dx}\right)}{d}-i(c+dx)\sinh\left(-\frac{f(}{d(}\right.\right.}{d}$$

$\big\downarrow$ 3779

$$\frac{i\left(-\frac{if(bc-ad)\left(-\sinh\left(\frac{bf}{d}+e\right)\text{Shi}\left(\frac{(bc-ad)f}{d(c+dx)}\right)+\cosh\left(\frac{bf}{d}+e\right)\int(c+dx)\sin\left(\frac{i(bc-ad)f}{d(c+dx)}+\frac{\pi}{2}\right)d\frac{1}{c+dx}\right)}{d}-i(c+dx)\sinh\left(-\frac{f(bc-ad)}{d(c+dx)}+\frac{bf}{d}\right.\right.}{d}$$

$\big\downarrow$ 3782

$$\frac{i\left(-\frac{if(bc-ad)\left(\cosh\left(\frac{bf}{d}+e\right)\text{Chi}\left(\frac{(bc-ad)f}{d(c+dx)}\right)-\sinh\left(\frac{bf}{d}+e\right)\text{Shi}\left(\frac{(bc-ad)f}{d(c+dx)}\right)\right)}{d}-i(c+dx)\sinh\left(-\frac{f(bc-ad)}{d(c+dx)}+\frac{bf}{d}+e\right)\right)}{d}$$

input
```
Int[Sinh[e + (f*(a + b*x))/(c + d*x)],x]
```

output
```
(I*((-I)*(c + d*x)*Sinh[e + (b*f)/d - ((b*c - a*d)*f)/(d*(c + d*x))] - (I*
(b*c - a*d)*f*(Cosh[e + (b*f)/d]*CoshIntegral[((b*c - a*d)*f)/(d*(c + d*x)
)] - Sinh[e + (b*f)/d]*SinhIntegral[((b*c - a*d)*f)/(d*(c + d*x))]))/d))/d
```

Defintions of rubi rules used

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3778
```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(c
 + d*x)^(m + 1)*(Sin[e + f*x]/(d*(m + 1))), x] - Simp[f/(d*(m + 1))   Int[(
c + d*x)^(m + 1)*Cos[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && LtQ[m, -
1]
```

rule 3779
```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[I*(SinhIntegral[c*f*(fz/d) + f*fz*x]/d), x] /; FreeQ[{c, d, e, f
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

rule 3782
```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[CoshIntegral[c*f*(fz/d) + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

rule 3784
```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[Cos[(d*
e - c*f)/d]   Int[Sin[c*(f/d) + f*x]/(c + d*x), x], x] + Simp[Sin[(d*e - c*
f)/d]   Int[Cos[c*(f/d) + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x]
&& NeQ[d*e - c*f, 0]
```

rule 6141
```
Int[Sinh[((e_.)*((a_.) + (b_.)*(x_)))/((c_.) + (d_.)*(x_))]^(n_.), x_Symbol
] :> Simp[-d^(-1)   Subst[Int[Sinh[b*(e/d) - e*(b*c - a*d)*(x/d)]^n/x^2, x]
, x, 1/(c + d*x)], x] /; FreeQ[{a, b, c, d}, x] && IGtQ[n, 0] && NeQ[b*c -
a*d, 0]
```

rule 6143
```
Int[Sinh[u_]^(n_.), x_Symbol] :> With[{lst = QuotientOfLinearsParts[u, x]},
 Int[Sinh[(lst[[1]] + lst[[2]]*x)/(lst[[3]] + lst[[4]]*x)]^n, x]] /; IGtQ[n
, 0] && QuotientOfLinearsQ[u, x]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 462 vs. $2(121) = 242$.

Time = 0.61 (sec) , antiderivative size = 463, normalized size of antiderivative = 3.83

method	result
risch	$-\frac{\mathrm{e}^{-\frac{bfx+dex+af+ce}{dx+c}} af}{2\left(\frac{dfa}{dx+c}-\frac{bcf}{dx+c}\right)}+\frac{\mathrm{e}^{-\frac{bfx+dex+af+ce}{dx+c}} bcf}{2d\left(\frac{dfa}{dx+c}-\frac{bcf}{dx+c}\right)}+\frac{\mathrm{e}^{-\frac{bf+de}{d}} \mathrm{expIntegral}_1\left(\frac{daf-bcf}{d(dx+c)}\right) af}{2d}-\frac{\mathrm{e}^{-\frac{bf+de}{d}} \mathrm{expIntegral}_1\left(\frac{daf-bcf}{d(dx+c)}\right.}{2d^2}$

input
```
int(sinh(e+f*(b*x+a)/(d*x+c)),x,method=_RETURNVERBOSE)
```

output
```
-1/2*exp(-(b*f*x+d*e*x+a*f+c*e)/(d*x+c))/(d*f/(d*x+c)*a-1/(d*x+c)*b*c*f)*a
*f+1/2/d*exp(-(b*f*x+d*e*x+a*f+c*e)/(d*x+c))/(d*f/(d*x+c)*a-1/(d*x+c)*b*c*
f)*b*c*f+1/2/d*exp(-(b*f+d*e)/d)*Ei(1,1/d*(a*d*f-b*c*f)/(d*x+c))*a*f-1/2/d
^2*exp(-(b*f+d*e)/d)*Ei(1,1/d*(a*d*f-b*c*f)/(d*x+c))*b*c*f+1/2/d*exp((b*f*
x+d*e*x+a*f+c*e)/(d*x+c))/(f/(d*x+c)*a-1/(d*x+c)/d*b*c*f)*a*f-1/2/d^2*exp(
(b*f*x+d*e*x+a*f+c*e)/(d*x+c))/(f/(d*x+c)*a-1/(d*x+c)/d*b*c*f)*b*c*f+1/2/d
*exp((b*f+d*e)/d)*Ei(1,-1/d*(a*d*f-b*c*f)/(d*x+c)-(b*f+d*e)/d-(-b*f-d*e)/d
)*a*f-1/2/d^2*exp((b*f+d*e)/d)*Ei(1,-1/d*(a*d*f-b*c*f)/(d*x+c)-(b*f+d*e)/d
-(-b*f-d*e)/d)*b*c*f
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 202, normalized size of antiderivative = 1.67

$$\int \sinh\left(e+\frac{f(a+bx)}{c+dx}\right)dx$$
$$=\frac{\left((bc-ad)f\mathrm{Ei}\left(\frac{(bc-ad)f}{d^2x+cd}\right)+(bc-ad)f\mathrm{Ei}\left(-\frac{(bc-ad)f}{d^2x+cd}\right)\right)\cosh\left(\frac{de+bf}{d}\right)+2\left(d^2x+cd\right)\sinh\left(\frac{ce+af+(de+bf)x}{dx+c}\right)}{2\,d^2}$$

input
```
integrate(sinh(e+f*(b*x+a)/(d*x+c)),x, algorithm="fricas")
```

output
```
1/2*(((b*c - a*d)*f*Ei((b*c - a*d)*f/(d^2*x + c*d)) + (b*c - a*d)*f*Ei(-(b
*c - a*d)*f/(d^2*x + c*d)))*cosh((d*e + b*f)/d) + 2*(d^2*x + c*d)*sinh((c*
e + a*f + (d*e + b*f)*x)/(d*x + c)) - ((b*c - a*d)*f*Ei((b*c - a*d)*f/(d^2
*x + c*d)) - (b*c - a*d)*f*Ei(-(b*c - a*d)*f/(d^2*x + c*d)))*sinh((d*e + b
*f)/d))/d^2
```

Sympy [F]

$$\int \sinh \left(e+\frac{f(a+bx)}{c+dx}\right) dx = \int \sinh \left(e+\frac{f(a+bx)}{c+dx}\right) dx$$

input
```
integrate(sinh(e+f*(b*x+a)/(d*x+c)),x)
```

output
```
Integral(sinh(e + f*(a + b*x)/(c + d*x)), x)
```

Maxima [F]

$$\int \sinh \left(e+\frac{f(a+bx)}{c+dx}\right) dx = \int \sinh \left(e+\frac{(bx+a)f}{dx+c}\right) dx$$

input
```
integrate(sinh(e+f*(b*x+a)/(d*x+c)),x, algorithm="maxima")
```

output
```
integrate(sinh(e + (b*x + a)*f/(d*x + c)), x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1624 vs. $2(121) = 242$.

Time = 4.79 (sec) , antiderivative size = 1624, normalized size of antiderivative = 13.42

$$\int \sinh\left(e + \frac{f(a+bx)}{c+dx}\right) dx = \text{Too large to display}$$

input
```
integrate(sinh(e+f*(b*x+a)/(d*x+c)),x, algorithm="giac")
```

output
```
1/2*(b^2*c^2*d*e*f^2*Ei(-(d*e + b*f - (d*e*x + b*f*x + c*e + a*f)*d/(d*x +
 c))/d)*e^((d*e + b*f)/d) - 2*a*b*c*d^2*e*f^2*Ei(-(d*e + b*f - (d*e*x + b*
f*x + c*e + a*f)*d/(d*x + c))/d)*e^((d*e + b*f)/d) + a^2*d^3*e*f^2*Ei(-(d*
e + b*f - (d*e*x + b*f*x + c*e + a*f)*d/(d*x + c))/d)*e^((d*e + b*f)/d) +
b^3*c^2*f^3*Ei(-(d*e + b*f - (d*e*x + b*f*x + c*e + a*f)*d/(d*x + c))/d)*e
^((d*e + b*f)/d) - 2*a*b^2*c*d*f^3*Ei(-(d*e + b*f - (d*e*x + b*f*x + c*e +
 a*f)*d/(d*x + c))/d)*e^((d*e + b*f)/d) + a^2*b*d^2*f^3*Ei(-(d*e + b*f - (
d*e*x + b*f*x + c*e + a*f)*d/(d*x + c))/d)*e^((d*e + b*f)/d) - (d*e*x + b*
f*x + c*e + a*f)*b^2*c^2*d*f^2*Ei(-(d*e + b*f - (d*e*x + b*f*x + c*e + a*f
)*d/(d*x + c))/d)*e^((d*e + b*f)/d)/(d*x + c) + 2*(d*e*x + b*f*x + c*e + a
*f)*a*b*c*d^2*f^2*Ei(-(d*e + b*f - (d*e*x + b*f*x + c*e + a*f)*d/(d*x + c)
)/d)*e^((d*e + b*f)/d)/(d*x + c) - (d*e*x + b*f*x + c*e + a*f)*a^2*d^3*f^2
*Ei(-(d*e + b*f - (d*e*x + b*f*x + c*e + a*f)*d/(d*x + c))/d)*e^((d*e + b*
f)/d)/(d*x + c) + b^2*c^2*d*f^2*e^((d*e*x + b*f*x + c*e + a*f)/(d*x + c))
- 2*a*b*c*d^2*f^2*e^((d*e*x + b*f*x + c*e + a*f)/(d*x + c)) + a^2*d^3*f^2*
e^((d*e*x + b*f*x + c*e + a*f)/(d*x + c)))*((d*e + b*f)*c/(b*c*f - a*d*f)^
2 - (c*e + a*f)*d/(b*c*f - a*d*f)^2)/(d^3*e + b*d^2*f - (d*e*x + b*f*x + c
*e + a*f)*d^3/(d*x + c)) + 1/2*(b^2*c^2*d*e*f^2*Ei((d*e + b*f - (d*e*x + b
*f*x + c*e + a*f)*d/(d*x + c))/d)*e^(-(d*e + b*f)/d) - 2*a*b*c*d^2*e*f^2*E
i((d*e + b*f - (d*e*x + b*f*x + c*e + a*f)*d/(d*x + c))/d)*e^(-(d*e + b...
```

Mupad [F(-1)]

Timed out.

$$\int \sinh\left(e+\frac{f(a+bx)}{c+dx}\right)\,dx=\int \sinh\left(e+\frac{f\,(a+b\,x)}{c+d\,x}\right)\,dx$$

input
```
int(sinh(e + (f*(a + b*x))/(c + d*x)),x)
```

output
```
int(sinh(e + (f*(a + b*x))/(c + d*x)), x)
```

Reduce [F]

$$\int \sinh\left(e+\frac{f(a+bx)}{c+dx}\right)\,dx=\text{too large to display}$$

input
```
int(sinh(e+f*(b*x+a)/(d*x+c)),x)
```

output

```
(e**((2*a*f + 2*b*f*x + 2*c*e + 2*d*e*x)/(c + d*x))*a*d**2*f*x**2 - e**((2
*a*f + 2*b*f*x + 2*c*e + 2*d*e*x)/(c + d*x))*b*c*d*f*x**2 - e**((2*a*f + 2
*b*f*x + 2*c*e + 2*d*e*x)/(c + d*x))*c**3 - e**((2*a*f + 2*b*f*x + 2*c*e +
 2*d*e*x)/(c + d*x))*c**2*d*x + e**((a*f + b*f*x + c*e + d*e*x)/(c + d*x))
*int(x**2/(e**((a*f + b*f*x + c*e + d*e*x)/(c + d*x))*c**3 + 3*e**((a*f +
b*f*x + c*e + d*e*x)/(c + d*x))*c**2*d*x + 3*e**((a*f + b*f*x + c*e + d*e*
x)/(c + d*x))*c*d**2*x**2 + e**((a*f + b*f*x + c*e + d*e*x)/(c + d*x))*d**
3*x**3),x)*a**2*c*d**3*f**2 + e**((a*f + b*f*x + c*e + d*e*x)/(c + d*x))*i
nt(x**2/(e**((a*f + b*f*x + c*e + d*e*x)/(c + d*x))*c**3 + 3*e**((a*f + b*
f*x + c*e + d*e*x)/(c + d*x))*c**2*d*x + 3*e**((a*f + b*f*x + c*e + d*e*x)
/(c + d*x))*c*d**2*x**2 + e**((a*f + b*f*x + c*e + d*e*x)/(c + d*x))*d**3*
x**3),x)*a**2*d**4*f**2*x - 2*e**((a*f + b*f*x + c*e + d*e*x)/(c + d*x))*i
nt(x**2/(e**((a*f + b*f*x + c*e + d*e*x)/(c + d*x))*c**3 + 3*e**((a*f + b*
f*x + c*e + d*e*x)/(c + d*x))*c**2*d*x + 3*e**((a*f + b*f*x + c*e + d*e*x)
/(c + d*x))*c*d**2*x**2 + e**((a*f + b*f*x + c*e + d*e*x)/(c + d*x))*d**3*
x**3),x)*a*b*c**2*d**2*f**2 - 2*e**((a*f + b*f*x + c*e + d*e*x)/(c + d*x))
*int(x**2/(e**((a*f + b*f*x + c*e + d*e*x)/(c + d*x))*c**3 + 3*e**((a*f +
b*f*x + c*e + d*e*x)/(c + d*x))*c**2*d*x + 3*e**((a*f + b*f*x + c*e + d*e*
x)/(c + d*x))*c*d**2*x**2 + e**((a*f + b*f*x + c*e + d*e*x)/(c + d*x))*d**
3*x**3),x)*a*b*c*d**3*f**2*x + e**((a*f + b*f*x + c*e + d*e*x)/(c + d*x...
```

3.299 $\int \sinh^2\left(e+\frac{f(a+bx)}{c+dx}\right)\,dx$

Optimal result

Integrand size = 19, antiderivative size = 129

$$\int \sinh^2\left(e+\frac{f(a+bx)}{c+dx}\right)\,dx = \frac{(bc-ad)f\mathrm{Chi}\left(\frac{2(bc-ad)f}{d(c+dx)}\right)\sinh\left(2\left(e+\frac{bf}{d}\right)\right)}{d^2} + \frac{(c+dx)\sinh^2\left(\frac{ce+af+dex+bfx}{c+dx}\right)}{d} - \frac{(bc-ad)f\cosh\left(2\left(e+\frac{bf}{d}\right)\right)\mathrm{Shi}\left(\frac{2(bc-ad)f}{d(c+dx)}\right)}{d^2}$$

output
```
(-a*d+b*c)*f*Chi(2*(-a*d+b*c)*f/d/(d*x+c))*sinh(2*e+2*b*f/d)/d^2+(d*x+c)*s
inh((b*f*x+d*e*x+a*f+c*e)/(d*x+c))^2/d-(-a*d+b*c)*f*cosh(2*e+2*b*f/d)*Shi(
2*(-a*d+b*c)*f/d/(d*x+c))/d^2
```

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 572 vs. $2(129) = 258$.

Time = 3.35 (sec) , antiderivative size = 572, normalized size of antiderivative = 4.43

$$\int \sinh^2\left(e+\frac{f(a+bx)}{c+dx}\right)\,dx$$
$$= \frac{cde^{-\frac{2(ce+af+dex+bfx)}{c+dx}}+cde^{\frac{2(ce+af+dex+bfx)}{c+dx}}+2d^2x\cosh\left(2\left(e+\frac{bf}{d}\right)\right)\cosh\left(\frac{2(-bcf+adf)}{d(c+dx)}\right)+2d^2x\sinh\left(2\left(e+\frac{bf}{d}\right.\right.}{}$$

input
```
Integrate[Sinh[e + (f*(a + b*x))/(c + d*x)]^2,x]
```

output
```
((c*d)/E^((2*(c*e + a*f + d*e*x + b*f*x))/(c + d*x)) + c*d*E^((2*(c*e + a*
f + d*e*x + b*f*x))/(c + d*x)) + 2*d^2*x*Cosh[2*(e + (b*f)/d)]*Cosh[(2*(-(
b*c*f) + a*d*f))/(d*(c + d*x))] + 2*d^2*x*Sinh[2*(e + (b*f)/d)]*Sinh[(2*(-
(b*c*f) + a*d*f))/(d*(c + d*x))] - 2*(d^2*x + (b*c - a*d)*f*CoshIntegral[(
2*(b*c - a*d)*f)/(d*(c + d*x))]*(Cosh[2*(e + (b*f)/d)] - Sinh[2*(e + (b*f)
/d)]) - (b*c - a*d)*f*CoshIntegral[(2*(-(b*c*f) + a*d*f))/(d*(c + d*x))]*(
Cosh[2*(e + (b*f)/d)] + Sinh[2*(e + (b*f)/d)]) + b*c*f*Cosh[2*(e + (b*f)/d
)]*SinhIntegral[(2*(b*c - a*d)*f)/(d*(c + d*x))] - a*d*f*Cosh[2*(e + (b*f)
/d)]*SinhIntegral[(2*(b*c - a*d)*f)/(d*(c + d*x))] - b*c*f*Sinh[2*(e + (b*
f)/d)]*SinhIntegral[(2*(b*c - a*d)*f)/(d*(c + d*x))] + a*d*f*Sinh[2*(e + (
b*f)/d)]*SinhIntegral[(2*(b*c - a*d)*f)/(d*(c + d*x))] - b*c*f*Cosh[2*(e +
 (b*f)/d)]*SinhIntegral[(2*(-(b*c*f) + a*d*f))/(d*(c + d*x))] + a*d*f*Cosh
[2*(e + (b*f)/d)]*SinhIntegral[(2*(-(b*c*f) + a*d*f))/(d*(c + d*x))] - b*c
*f*Sinh[2*(e + (b*f)/d)]*SinhIntegral[(2*(-(b*c*f) + a*d*f))/(d*(c + d*x))
] + a*d*f*Sinh[2*(e + (b*f)/d)]*SinhIntegral[(2*(-(b*c*f) + a*d*f))/(d*(c
+ d*x))]))/(4*d^2)
```

Rubi [C] (verified)

Result contains complex when optimal does not.

Time = 0.97 (sec) , antiderivative size = 136, normalized size of antiderivative = 1.05, number of steps used = 15, number of rules used = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.737$, Rules used = {6143, 6141, 3042, 25, 3794, 27, 3042, 26, 3784, 26, 3042, 26, 3779, 3782}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sinh^2\left(\frac{f(a+bx)}{c+dx}+e\right)dx$$

$$\Big\downarrow \text{ 6143}$$

$$\int \sinh^2\left(\frac{af+x(bf+de)+ce}{c+dx}\right)dx$$

$$\Big\downarrow \text{ 6141}$$

$$-\frac{\int (c+dx)^2\sinh^2\left(e+\frac{bf}{d}-\frac{(bc-ad)f}{d(c+dx)}\right)d\frac{1}{c+dx}}{d}$$

$$\Big\downarrow \text{ 3042}$$

$$-\frac{\int -(c+dx)^2\sin\left(i\left(e+\frac{bf}{d}\right)-\frac{i(bc-ad)f}{d(c+dx)}\right)^2d\frac{1}{c+dx}}{d}$$

$$\Big\downarrow \text{ 25}$$

$$\frac{\int (c+dx)^2\sin\left(i\left(e+\frac{bf}{d}\right)-\frac{i(bc-ad)f}{d(c+dx)}\right)^2d\frac{1}{c+dx}}{d}$$

$$\Big\downarrow \text{ 3794}$$

$$-\frac{-(c+dx)\sinh^2\left(-\frac{f(bc-ad)}{d(c+dx)}+\frac{bf}{d}+e\right)+\frac{2if(bc-ad)\int \frac{1}{2}i(c+dx)\sinh\left(2\left(e+\frac{bf}{d}\right)-\frac{2(bc-ad)f}{d(c+dx)}\right)d\frac{1}{c+dx}}{d}}{d}$$

$$\Big\downarrow \text{ 27}$$

$$-\frac{-\frac{f(bc-ad)\int (c+dx)\sinh\left(2\left(e+\frac{bf}{d}\right)-\frac{2(bc-ad)f}{d(c+dx)}\right)d\frac{1}{c+dx}}{d}-\left((c+dx)\sinh^2\left(-\frac{f(bc-ad)}{d(c+dx)}+\frac{bf}{d}+e\right)\right)}{d}$$

$$\Big\downarrow \text{ 3042}$$

$$-\frac{-\left((c+dx)\sinh^2\left(-\frac{f(bc-ad)}{d(c+dx)}+\frac{bf}{d}+e\right)\right)-\frac{f(bc-ad)\int -i(c+dx)\sin\left(2i\left(e+\frac{bf}{d}\right)-\frac{2i(bc-ad)f}{d(c+dx)}\right)d\frac{1}{c+dx}}{d}}{d}$$

$$\Big\downarrow \text{ 26}$$

$$-\frac{-(c+dx)\sinh^2\left(-\frac{f(bc-ad)}{d(c+dx)}+\frac{bf}{d}+e\right)+\frac{if(bc-ad)\int (c+dx)\sin\left(2i\left(e+\frac{bf}{d}\right)-\frac{2i(bc-ad)f}{d(c+dx)}\right)d\frac{1}{c+dx}}{d}}{d}$$

$$\Big\downarrow \text{ 3784}$$

$$-\frac{-(c+dx)\sinh^2\left(-\frac{f(bc-ad)}{d(c+dx)}+\frac{bf}{d}+e\right)+\frac{if(bc-ad)\left(i\sinh\left(2\left(\frac{bf}{d}+e\right)\right)\int(c+dx)\cosh\left(\frac{2(bc-ad)f}{d(c+dx)}\right)d\frac{1}{c+dx}+\cosh\left(2\left(\frac{bf}{d}+e\right)\right)\int -i(c}{d}}{d}$$

$\downarrow$ 26

$$-\frac{-(c+dx)\sinh^2\left(-\frac{f(bc-ad)}{d(c+dx)}+\frac{bf}{d}+e\right)+\frac{if(bc-ad)\left(i\sinh\left(2\left(\frac{bf}{d}+e\right)\right)\int(c+dx)\cosh\left(\frac{2(bc-ad)f}{d(c+dx)}\right)d\frac{1}{c+dx}-i\cosh\left(2\left(\frac{bf}{d}+e\right)\right)\int(c+}{d}}{d}$$

$\downarrow$ 3042

$$-\frac{-(c+dx)\sinh^2\left(-\frac{f(bc-ad)}{d(c+dx)}+\frac{bf}{d}+e\right)+\frac{if(bc-ad)\left(i\sinh\left(2\left(\frac{bf}{d}+e\right)\right)\int(c+dx)\sin\left(\frac{2i(bc-ad)f}{d(c+dx)}+\frac{\pi}{2}\right)d\frac{1}{c+dx}-i\cosh\left(2\left(\frac{bf}{d}+e\right)\right)\int}{d}}{d}$$

$\downarrow$ 26

$$-\frac{-(c+dx)\sinh^2\left(-\frac{f(bc-ad)}{d(c+dx)}+\frac{bf}{d}+e\right)+\frac{if(bc-ad)\left(i\sinh\left(2\left(\frac{bf}{d}+e\right)\right)\int(c+dx)\sin\left(\frac{2i(bc-ad)f}{d(c+dx)}+\frac{\pi}{2}\right)d\frac{1}{c+dx}-\cosh\left(2\left(\frac{bf}{d}+e\right)\right)\int(c}{d}}{d}$$

$\downarrow$ 3779

$$-\frac{-(c+dx)\sinh^2\left(-\frac{f(bc-ad)}{d(c+dx)}+\frac{bf}{d}+e\right)+\frac{if(bc-ad)\left(i\sinh\left(2\left(\frac{bf}{d}+e\right)\right)\int(c+dx)\sin\left(\frac{2i(bc-ad)f}{d(c+dx)}+\frac{\pi}{2}\right)d\frac{1}{c+dx}-i\cosh\left(2\left(\frac{bf}{d}+e\right)\right)\text{Sh}}{d}}{d}$$

$\downarrow$ 3782

$$-\frac{-(c+dx)\sinh^2\left(-\frac{f(bc-ad)}{d(c+dx)}+\frac{bf}{d}+e\right)+\frac{if(bc-ad)\left(i\sinh\left(2\left(\frac{bf}{d}+e\right)\right)\text{Chi}\left(\frac{2(bc-ad)f}{d(c+dx)}\right)-i\cosh\left(2\left(\frac{bf}{d}+e\right)\right)\text{Shi}\left(\frac{2(bc-ad)f}{d(c+dx)}\right)\right)}{d}}{d}$$

input
```
Int[Sinh[e + (f*(a + b*x))/(c + d*x)]^2,x]
```

output
```
-((-((c + d*x)*Sinh[e + (b*f)/d - ((b*c - a*d)*f)/(d*(c + d*x))]^2) + (I*(
b*c - a*d)*f*(I*CoshIntegral[(2*(b*c - a*d)*f)/(d*(c + d*x))]*Sinh[2*(e +
(b*f)/d)] - I*Cosh[2*(e + (b*f)/d)]*SinhIntegral[(2*(b*c - a*d)*f)/(d*(c +
 d*x))]))/d)/d)
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3779
```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[I*(SinhIntegral[c*f*(fz/d) + f*fz*x]/d), x] /; FreeQ[{c, d, e, f
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

rule 3782
```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[CoshIntegral[c*f*(fz/d) + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

rule 3784
```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[Cos[(d*
e - c*f)/d]   Int[Sin[c*(f/d) + f*x]/(c + d*x), x], x] + Simp[Sin[(d*e - c*
f)/d]   Int[Cos[c*(f/d) + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x]
&& NeQ[d*e - c*f, 0]
```

rule 3794
```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> Si
mp[(c + d*x)^(m + 1)*(Sin[e + f*x]^n/(d*(m + 1))), x] - Simp[f*(n/(d*(m + 1
)))   Int[ExpandTrigReduce[(c + d*x)^(m + 1), Cos[e + f*x]*Sin[e + f*x]^(n
- 1), x], x], x] /; FreeQ[{c, d, e, f, m}, x] && IGtQ[n, 1] && GeQ[m, -2] &
& LtQ[m, -1]
```

rule 6141
```
Int[Sinh[((e_.)*((a_.) + (b_.)*(x_)))/((c_.) + (d_.)*(x_))]^(n_.), x_Symbol
] :> Simp[-d^(-1)   Subst[Int[Sinh[b*(e/d) - e*(b*c - a*d)*(x/d)]^n/x^2, x]
, x, 1/(c + d*x)], x] /; FreeQ[{a, b, c, d}, x] && IGtQ[n, 0] && NeQ[b*c -
a*d, 0]
```

rule 6143
```
Int[Sinh[u_]^(n_.), x_Symbol] :> With[{lst = QuotientOfLinearsParts[u, x]},
 Int[Sinh[(lst[[1]] + lst[[2]]*x)/(lst[[3]] + lst[[4]]*x)]^n, x]] /; IGtQ[n
, 0] && QuotientOfLinearsQ[u, x]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 471 vs. 2(131) = 262.

Time = 3.48 (sec) , antiderivative size = 472, normalized size of antiderivative = 3.66

method	result
risch	$-\frac{x}{2}+\frac{e^{-\frac{2(bfx+dex+af+ce)}{dx+c}} af}{\frac{4dfa}{dx+c}-\frac{4bcf}{dx+c}}-\frac{e^{-\frac{2(bfx+dex+af+ce)}{dx+c}} bcf}{4d\left(\frac{dfa}{dx+c}-\frac{bcf}{dx+c}\right)}-\frac{e^{-\frac{2(bf+de)}{d}} \operatorname{expIntegral}_1\left(\frac{2daf-2bcf}{d(dx+c)}\right) af}{2d}+\frac{e^{-\frac{2(bf+de)}{d}} \operatorname{expIn}}{}$

input
```
int(sinh(e+f*(b*x+a)/(d*x+c))^2,x,method=_RETURNVERBOSE)
```

output
```
-1/2*x+1/4*exp(-2*(b*f*x+d*e*x+a*f+c*e)/(d*x+c))/(d*f/(d*x+c)*a-1/(d*x+c)*
b*c*f)*a*f-1/4/d*exp(-2*(b*f*x+d*e*x+a*f+c*e)/(d*x+c))/(d*f/(d*x+c)*a-1/(d
*x+c)*b*c*f)*b*c*f-1/2/d*exp(-2*(b*f+d*e)/d)*Ei(1,2/d*(a*d*f-b*c*f)/(d*x+c
))*a*f+1/2/d^2*exp(-2*(b*f+d*e)/d)*Ei(1,2/d*(a*d*f-b*c*f)/(d*x+c))*b*c*f+1
/4/d*exp(2*(b*f*x+d*e*x+a*f+c*e)/(d*x+c))/(f/(d*x+c)*a-1/(d*x+c)/d*b*c*f)*
a*f-1/4/d^2*exp(2*(b*f*x+d*e*x+a*f+c*e)/(d*x+c))/(f/(d*x+c)*a-1/(d*x+c)/d*
b*c*f)*b*c*f+1/2/d*exp(2*(b*f+d*e)/d)*Ei(1,-2/d*(a*d*f-b*c*f)/(d*x+c)-2*(b
*f+d*e)/d-2*(-b*f-d*e)/d)*a*f-1/2/d^2*exp(2*(b*f+d*e)/d)*Ei(1,-2/d*(a*d*f-
b*c*f)/(d*x+c)-2*(b*f+d*e)/d-2*(-b*f-d*e)/d)*b*c*f
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 477 vs. $2(131) = 262$.

Time = 0.09 (sec) , antiderivative size = 477, normalized size of antiderivative = 3.70

$$\int \sinh^2\left(e+\frac{f(a+bx)}{c+dx}\right)\,dx =$$

$$-\frac{d^2x-(d^2x+cd)\cosh\left(\frac{ce+af+(de+bf)x}{dx+c}\right)^2+\left((bc-ad)f\mathrm{Ei}\left(-\frac{2\,(bc-ad)f}{d^2x+cd}\right)\cosh\left(\frac{2\,(de+bf)}{d}\right)-d^2x-cd\right)\mathrm{si}}{}$$

input
```
integrate(sinh(e+f*(b*x+a)/(d*x+c))^2,x, algorithm="fricas")
```

output
```
-1/2*(d^2*x - (d^2*x + c*d)*cosh((c*e + a*f + (d*e + b*f)*x)/(d*x + c))^2
+ ((b*c - a*d)*f*Ei(-2*(b*c - a*d)*f/(d^2*x + c*d))*cosh(2*(d*e + b*f)/d)
- d^2*x - c*d)*sinh((c*e + a*f + (d*e + b*f)*x)/(d*x + c))^2 - ((b*c - a*d
)*f*Ei(-2*(b*c - a*d)*f/(d^2*x + c*d))*cosh((c*e + a*f + (d*e + b*f)*x)/(d
*x + c))^2 - (b*c - a*d)*f*Ei(2*(b*c - a*d)*f/(d^2*x + c*d)))*cosh(2*(d*e
+ b*f)/d) - ((b*c - a*d)*f*Ei(-2*(b*c - a*d)*f/(d^2*x + c*d))*cosh((c*e +
a*f + (d*e + b*f)*x)/(d*x + c))^2 - (b*c - a*d)*f*Ei(-2*(b*c - a*d)*f/(d^2
*x + c*d))*sinh((c*e + a*f + (d*e + b*f)*x)/(d*x + c))^2 + (b*c - a*d)*f*E
i(2*(b*c - a*d)*f/(d^2*x + c*d)))*sinh(2*(d*e + b*f)/d))/(d^2*cosh((c*e +
a*f + (d*e + b*f)*x)/(d*x + c))^2 - d^2*sinh((c*e + a*f + (d*e + b*f)*x)/(
d*x + c))^2)
```

Sympy [F(-1)]

Timed out.

$$\int \sinh^2\left(e+\frac{f(a+bx)}{c+dx}\right)\,dx = \text{Timed out}$$

input
```
integrate(sinh(e+f*(b*x+a)/(d*x+c))**2,x)
```

output
```
Timed out
```

Maxima [F]

$$\int \sinh^2\left(e+\frac{f(a+bx)}{c+dx}\right)\,dx=\int \sinh\left(e+\frac{(bx+a)f}{dx+c}\right)^2\,dx$$

input
```
integrate(sinh(e+f*(b*x+a)/(d*x+c))^2,x, algorithm="maxima")
```

output
```
-1/2*x + 1/4*integrate(e^(2*b*c*f/(d^2*x + c*d) - 2*e - 2*a*f/(d*x + c) -
2*b*f/d), x) + 1/4*integrate(e^(-2*b*c*f/(d^2*x + c*d) + 2*e + 2*a*f/(d*x
+ c) + 2*b*f/d), x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1596 vs. $2(131) = 262$.

Time = 22.72 (sec) , antiderivative size = 1596, normalized size of antiderivative = 12.37

$$\int \sinh^2\left(e+\frac{f(a+bx)}{c+dx}\right)\,dx=\text{Too large to display}$$

input
```
integrate(sinh(e+f*(b*x+a)/(d*x+c))^2,x, algorithm="giac")
```

output
```
1/4*(2*b^2*c^2*d*e*f^2*Ei(-2*(d*e + b*f - (d*e*x + b*f*x + c*e + a*f)*d/(d
*x + c))/d)*e^(2*(d*e + b*f)/d) - 4*a*b*c*d^2*e*f^2*Ei(-2*(d*e + b*f - (d*
e*x + b*f*x + c*e + a*f)*d/(d*x + c))/d)*e^(2*(d*e + b*f)/d) + 2*a^2*d^3*e
*f^2*Ei(-2*(d*e + b*f - (d*e*x + b*f*x + c*e + a*f)*d/(d*x + c))/d)*e^(2*(
d*e + b*f)/d) + 2*b^3*c^2*f^3*Ei(-2*(d*e + b*f - (d*e*x + b*f*x + c*e + a*
f)*d/(d*x + c))/d)*e^(2*(d*e + b*f)/d) - 4*a*b^2*c*d*f^3*Ei(-2*(d*e + b*f
- (d*e*x + b*f*x + c*e + a*f)*d/(d*x + c))/d)*e^(2*(d*e + b*f)/d) + 2*a^2*
b*d^2*f^3*Ei(-2*(d*e + b*f - (d*e*x + b*f*x + c*e + a*f)*d/(d*x + c))/d)*e
^(2*(d*e + b*f)/d) - 2*b^2*c^2*d*e*f^2*Ei(2*(d*e + b*f - (d*e*x + b*f*x +
c*e + a*f)*d/(d*x + c))/d)*e^(-2*(d*e + b*f)/d) + 4*a*b*c*d^2*e*f^2*Ei(2*(
d*e + b*f - (d*e*x + b*f*x + c*e + a*f)*d/(d*x + c))/d)*e^(-2*(d*e + b*f)/
d) - 2*a^2*d^3*e*f^2*Ei(2*(d*e + b*f - (d*e*x + b*f*x + c*e + a*f)*d/(d*x
+ c))/d)*e^(-2*(d*e + b*f)/d) - 2*b^3*c^2*f^3*Ei(2*(d*e + b*f - (d*e*x + b
*f*x + c*e + a*f)*d/(d*x + c))/d)*e^(-2*(d*e + b*f)/d) + 4*a*b^2*c*d*f^3*E
i(2*(d*e + b*f - (d*e*x + b*f*x + c*e + a*f)*d/(d*x + c))/d)*e^(-2*(d*e +
b*f)/d) - 2*a^2*b*d^2*f^3*Ei(2*(d*e + b*f - (d*e*x + b*f*x + c*e + a*f)*d/
(d*x + c))/d)*e^(-2*(d*e + b*f)/d) - 2*(d*e*x + b*f*x + c*e + a*f)*b^2*c^2
*d*f^2*Ei(-2*(d*e + b*f - (d*e*x + b*f*x + c*e + a*f)*d/(d*x + c))/d)*e^(2
*(d*e + b*f)/d)/(d*x + c) + 4*(d*e*x + b*f*x + c*e + a*f)*a*b*c*d^2*f^2*Ei
(-2*(d*e + b*f - (d*e*x + b*f*x + c*e + a*f)*d/(d*x + c))/d)*e^(2*(d*e ...
```

Mupad [F(-1)]

Timed out.

$$\int \sinh^2\left(e+\frac{f(a+bx)}{c+dx}\right)\,dx=\int \sinh\left(e+\frac{f\,(a+b\,x)}{c+d\,x}\right)^2\,dx$$

input
```
int(sinh(e + (f*(a + b*x))/(c + d*x))^2,x)
```

output
```
int(sinh(e + (f*(a + b*x))/(c + d*x))^2, x)
```

Reduce [F]

$$\int \sinh^2\left(e + \frac{f(a+bx)}{c+dx}\right) dx = \text{too large to display}$$

input

```
int(sinh(e+f*(b*x+a)/(d*x+c))^2,x)
```

output

```
(2*e**((4*a*f + 4*b*f*x + 4*c*e + 4*d*e*x)/(c + d*x))*a*d**2*f*x**2 - 2*e*
*((4*a*f + 4*b*f*x + 4*c*e + 4*d*e*x)/(c + d*x))*b*c*d*f*x**2 - e**((4*a*f
 + 4*b*f*x + 4*c*e + 4*d*e*x)/(c + d*x))*c**3 - e**((4*a*f + 4*b*f*x + 4*c
*e + 4*d*e*x)/(c + d*x))*c**2*d*x - 4*e**((2*a*f + 2*b*f*x + 2*c*e + 2*d*e
*x)/(c + d*x))*int(x**2/(e**((2*a*f + 2*b*f*x + 2*c*e + 2*d*e*x)/(c + d*x)
)*c**3 + 3*e**((2*a*f + 2*b*f*x + 2*c*e + 2*d*e*x)/(c + d*x))*c**2*d*x + 3
*e**((2*a*f + 2*b*f*x + 2*c*e + 2*d*e*x)/(c + d*x))*c*d**2*x**2 + e**((2*a
*f + 2*b*f*x + 2*c*e + 2*d*e*x)/(c + d*x))*d**3*x**3),x)*a**2*c*d**3*f**2
- 4*e**((2*a*f + 2*b*f*x + 2*c*e + 2*d*e*x)/(c + d*x))*int(x**2/(e**((2*a*
f + 2*b*f*x + 2*c*e + 2*d*e*x)/(c + d*x))*c**3 + 3*e**((2*a*f + 2*b*f*x +
2*c*e + 2*d*e*x)/(c + d*x))*c**2*d*x + 3*e**((2*a*f + 2*b*f*x + 2*c*e + 2*
d*e*x)/(c + d*x))*c*d**2*x**2 + e**((2*a*f + 2*b*f*x + 2*c*e + 2*d*e*x)/(c
 + d*x))*d**3*x**3),x)*a**2*d**4*f**2*x + 8*e**((2*a*f + 2*b*f*x + 2*c*e +
 2*d*e*x)/(c + d*x))*int(x**2/(e**((2*a*f + 2*b*f*x + 2*c*e + 2*d*e*x)/(c
+ d*x))*c**3 + 3*e**((2*a*f + 2*b*f*x + 2*c*e + 2*d*e*x)/(c + d*x))*c**2*d
*x + 3*e**((2*a*f + 2*b*f*x + 2*c*e + 2*d*e*x)/(c + d*x))*c*d**2*x**2 + e*
*((2*a*f + 2*b*f*x + 2*c*e + 2*d*e*x)/(c + d*x))*d**3*x**3),x)*a*b*c**2*d*
*2*f**2 + 8*e**((2*a*f + 2*b*f*x + 2*c*e + 2*d*e*x)/(c + d*x))*int(x**2/(e
**((2*a*f + 2*b*f*x + 2*c*e + 2*d*e*x)/(c + d*x))*c**3 + 3*e**((2*a*f + 2*
b*f*x + 2*c*e + 2*d*e*x)/(c + d*x))*c**2*d*x + 3*e**((2*a*f + 2*b*f*x +...
```

3.300 $\int \sinh^3\left(e+\frac{f(a+bx)}{c+dx}\right)\,dx$

Optimal result

Integrand size = 19, antiderivative size = 226

$$\int \sinh^3\left(e+\frac{f(a+bx)}{c+dx}\right)\,dx = -\frac{3(bc-ad)f\cosh\left(e+\frac{bf}{d}\right)\text{Chi}\left(\frac{(bc-ad)f}{d(c+dx)}\right)}{4d^2} + \frac{3(bc-ad)f\cosh\left(3\left(e+\frac{bf}{d}\right)\right)\text{Chi}\left(\frac{3(bc-ad)f}{d(c+dx)}\right)}{4d^2} + \frac{(c+dx)\sinh^3\left(\frac{ce+af+dex+bfx}{c+dx}\right)}{d} + \frac{3(bc-ad)f\sinh\left(e+\frac{bf}{d}\right)\text{Shi}\left(\frac{(bc-ad)f}{d(c+dx)}\right)}{4d^2} - \frac{3(bc-ad)f\sinh\left(3\left(e+\frac{bf}{d}\right)\right)\text{Shi}\left(\frac{3(bc-ad)f}{d(c+dx)}\right)}{4d^2}$$

output
```
-3/4*(-a*d+b*c)*f*cosh(e+b*f/d)*Chi((-a*d+b*c)*f/d/(d*x+c))/d^2+3/4*(-a*d+
b*c)*f*cosh(3*e+3*b*f/d)*Chi(3*(-a*d+b*c)*f/d/(d*x+c))/d^2+(d*x+c)*sinh((b
*f*x+d*e*x+a*f+c*e)/(d*x+c))^3/d+3/4*(-a*d+b*c)*f*sinh(e+b*f/d)*Shi((-a*d+
b*c)*f/d/(d*x+c))/d^2-3/4*(-a*d+b*c)*f*sinh(3*e+3*b*f/d)*Shi(3*(-a*d+b*c)*
f/d/(d*x+c))/d^2
```

Mathematica [B] (warning: unable to verify)

Leaf count is larger than twice the leaf count of optimal. 913 vs. $2(226) = 452$.

Time = 6.46 (sec) , antiderivative size = 913, normalized size of antiderivative = 4.04

$$\int \sinh^3\left(e+\frac{f(a+bx)}{c+dx}\right)\,dx = \text{Too large to display}$$

input
```
Integrate[Sinh[e + (f*(a + b*x))/(c + d*x)]^3,x]
```

output
```
-1/8*c/(d*E^((3*(c*e + a*f + d*e*x + b*f*x))/(c + d*x))) + (3*c)/(8*d*E^((
c*e + a*f + d*e*x + b*f*x)/(c + d*x))) - (3*c*E^((c*e + a*f + d*e*x + b*f*
x)/(c + d*x)))/(8*d) + (c*E^((3*(c*e + a*f + d*e*x + b*f*x))/(c + d*x)))/(
8*d) - (3*x*Cosh[(-(b*c*f) + a*d*f)/(d*(c + d*x))]*Sinh[(d*e + b*f)/d])/4
+ (x*Cosh[(3*(-(b*c*f) + a*d*f))/(d*(c + d*x))]*Sinh[(3*(d*e + b*f))/d])/4
 - (3*x*Cosh[(d*e + b*f)/d]*Sinh[(-(b*c*f) + a*d*f)/(d*(c + d*x))])/4 + (x
*Cosh[(3*(d*e + b*f))/d]*Sinh[(3*(-(b*c*f) + a*d*f))/(d*(c + d*x))])/4 - (
3*(-(b*c) + a*d)*f*(Cosh[(3*(d*e + b*f))/d]*CoshIntegral[(3*b*c*f - 3*a*d*
f)/(c*d + d^2*x)] - Cosh[(d*e + b*f)/d]*CoshIntegral[(b*c*f - a*d*f)/(c*d
+ d^2*x)] - Cosh[(d*e + b*f)/d]*CoshIntegral[(-(b*c*f) + a*d*f)/(c*d + d^2
*x)] + Cosh[(3*(d*e + b*f))/d]*CoshIntegral[(-3*b*c*f + 3*a*d*f)/(c*d + d^
2*x)] + CoshIntegral[(b*c*f - a*d*f)/(c*d + d^2*x)]*Sinh[(d*e + b*f)/d] -
CoshIntegral[(-(b*c*f) + a*d*f)/(c*d + d^2*x)]*Sinh[(d*e + b*f)/d] - CoshI
ntegral[(3*b*c*f - 3*a*d*f)/(c*d + d^2*x)]*Sinh[(3*(d*e + b*f))/d] + CoshI
ntegral[(-3*b*c*f + 3*a*d*f)/(c*d + d^2*x)]*Sinh[(3*(d*e + b*f))/d] + Cosh
[(3*(d*e + b*f))/d]*SinhIntegral[(3*b*c*f - 3*a*d*f)/(c*d + d^2*x)] - Sinh
[(3*(d*e + b*f))/d]*SinhIntegral[(3*b*c*f - 3*a*d*f)/(c*d + d^2*x)] - Cosh
[(d*e + b*f)/d]*SinhIntegral[(b*c*f - a*d*f)/(c*d + d^2*x)] + Sinh[(d*e +
b*f)/d]*SinhIntegral[(b*c*f - a*d*f)/(c*d + d^2*x)] - Cosh[(d*e + b*f)/d]*
SinhIntegral[(-(b*c*f) + a*d*f)/(c*d + d^2*x)] - Sinh[(d*e + b*f)/d]*Si...
```

Rubi [C] (verified)

Result contains complex when optimal does not.

Time = 0.70 (sec) , antiderivative size = 208, normalized size of antiderivative = 0.92, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.316$, Rules used = {6143, 6141, 3042, 26, 3794, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sinh^3\left(\frac{f(a+bx)}{c+dx}+e\right)dx$$

$$\Big\downarrow \text{6143}$$

$$\int \sinh^3\left(\frac{af+x(bf+de)+ce}{c+dx}\right)dx$$

$$\Big\downarrow \text{6141}$$

$$-\frac{\int (c+dx)^2\sinh^3\left(e+\frac{bf}{d}-\frac{(bc-ad)f}{d(c+dx)}\right)d\frac{1}{c+dx}}{d}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{\int i(c+dx)^2\sin\left(i\left(e+\frac{bf}{d}\right)-\frac{i(bc-ad)f}{d(c+dx)}\right)^3d\frac{1}{c+dx}}{d}$$

$$\Big\downarrow \text{26}$$

$$-\frac{i\int (c+dx)^2\sin\left(i\left(e+\frac{bf}{d}\right)-\frac{i(bc-ad)f}{d(c+dx)}\right)^3d\frac{1}{c+dx}}{d}$$

$$\Big\downarrow \text{3794}$$

$$-\frac{i\Bigg(i(c+dx)\sinh^3\left(-\frac{f(bc-ad)}{d(c+dx)}+\frac{bf}{d}+e\right)-\frac{3if(bc-ad)\int\Big(\frac{1}{4}(c+dx)\cosh\Big(e+\frac{bf}{d}-\frac{(bc-ad)f}{d(c+dx)}\Big)-\frac{1}{4}(c+dx)\cosh\Big(3\Big(e+\frac{bf}{d}\Big)-\frac{3(bc-ad)f}{d(c+dx)}}{d}}{d}$$

$$\Big\downarrow \text{2009}$$

$$-\frac{i\Bigg(i(c+dx)\sinh^3\left(-\frac{f(bc-ad)}{d(c+dx)}+\frac{bf}{d}+e\right)-\frac{3if(bc-ad)\Big(\frac{1}{4}\cosh\Big(\frac{bf}{d}+e\Big)\text{Chi}\Big(\frac{(bc-ad)f}{d(c+dx)}\Big)-\frac{1}{4}\cosh\Big(3\Big(\frac{bf}{d}+e\Big)\Big)\text{Chi}\Big(\frac{3(bc-ad)f}{d(c+dx)}\Big)-\frac{1}{4}}{d}}{d}$$

input
```
Int[Sinh[e + (f*(a + b*x))/(c + d*x)]^3,x]
```

output
```
((-I)*(I*(c + d*x)*Sinh[e + (b*f)/d - ((b*c - a*d)*f)/(d*(c + d*x))]^3 - (
(3*I)*(b*c - a*d)*f*((Cosh[e + (b*f)/d]*CoshIntegral[((b*c - a*d)*f)/(d*(c
 + d*x))])/4 - (Cosh[3*(e + (b*f)/d)]*CoshIntegral[(3*(b*c - a*d)*f)/(d*(c
 + d*x))])/4 - (Sinh[e + (b*f)/d]*SinhIntegral[((b*c - a*d)*f)/(d*(c + d*x
))])/4 + (Sinh[3*(e + (b*f)/d)]*SinhIntegral[(3*(b*c - a*d)*f)/(d*(c + d*x
))])/4))/d))/d
```

Defintions of rubi rules used

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3794
```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> Si
mp[(c + d*x)^(m + 1)*(Sin[e + f*x]^n/(d*(m + 1))), x] - Simp[f*(n/(d*(m + 1
)))   Int[ExpandTrigReduce[(c + d*x)^(m + 1), Cos[e + f*x]*Sin[e + f*x]^(n
- 1), x], x], x] /; FreeQ[{c, d, e, f, m}, x] && IGtQ[n, 1] && GeQ[m, -2] &
& LtQ[m, -1]
```

rule 6141
```
Int[Sinh[((e_.)*((a_.) + (b_.)*(x_)))/((c_.) + (d_.)*(x_))]^(n_.), x_Symbol
] :> Simp[-d^(-1)   Subst[Int[Sinh[b*(e/d) - e*(b*c - a*d)*(x/d)]^n/x^2, x]
, x, 1/(c + d*x)], x] /; FreeQ[{a, b, c, d}, x] && IGtQ[n, 0] && NeQ[b*c -
a*d, 0]
```

rule 6143
```
Int[Sinh[u_]^(n_.), x_Symbol] :> With[{lst = QuotientOfLinearsParts[u, x]},
 Int[Sinh[(lst[[1]] + lst[[2]]*x)/(lst[[3]] + lst[[4]]*x)]^n, x]] /; IGtQ[n
, 0] && QuotientOfLinearsQ[u, x]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 929 vs. 2(220) = 440.

Time = 5.83 (sec) , antiderivative size = 930, normalized size of antiderivative = 4.12

method	result
risch	$-\frac{e^{-\frac{3(bfx+dex+af+ce)}{dx+c}} af}{8\left(\frac{dfa}{dx+c}-\frac{bcf}{dx+c}\right)}+\frac{e^{-\frac{3(bfx+dex+af+ce)}{dx+c}} bcf}{8d\left(\frac{dfa}{dx+c}-\frac{bcf}{dx+c}\right)}+\frac{3\,e^{-\frac{3(bf+de)}{d}}\,\text{expIntegral}_1\left(\frac{3daf-3bcf}{d(dx+c)}\right)af}{8d}-\frac{3\,e^{-\frac{3(bf+de)}{d}}\,\text{expInteg}}{8c}$

input
```
int(sinh(e+f*(b*x+a)/(d*x+c))^3,x,method=_RETURNVERBOSE)
```

output
```
-1/8*exp(-3*(b*f*x+d*e*x+a*f+c*e)/(d*x+c))/(d*f/(d*x+c)*a-1/(d*x+c)*b*c*f)
*a*f+1/8/d*exp(-3*(b*f*x+d*e*x+a*f+c*e)/(d*x+c))/(d*f/(d*x+c)*a-1/(d*x+c)*
b*c*f)*b*c*f+3/8/d*exp(-3*(b*f+d*e)/d)*Ei(1,3/d*(a*d*f-b*c*f)/(d*x+c))*a*f
-3/8/d^2*exp(-3*(b*f+d*e)/d)*Ei(1,3/d*(a*d*f-b*c*f)/(d*x+c))*b*c*f+3/8*exp
(-(b*f*x+d*e*x+a*f+c*e)/(d*x+c))/(d*f/(d*x+c)*a-1/(d*x+c)*b*c*f)*a*f-3/8/d
*exp(-(b*f*x+d*e*x+a*f+c*e)/(d*x+c))/(d*f/(d*x+c)*a-1/(d*x+c)*b*c*f)*b*c*f
-3/8/d*exp(-(b*f+d*e)/d)*Ei(1,1/d*(a*d*f-b*c*f)/(d*x+c))*a*f+3/8/d^2*exp(-
(b*f+d*e)/d)*Ei(1,1/d*(a*d*f-b*c*f)/(d*x+c))*b*c*f+1/8/d*exp(3*(b*f*x+d*e*
x+a*f+c*e)/(d*x+c))/(f/(d*x+c)*a-1/(d*x+c)/d*b*c*f)*a*f-1/8/d^2*exp(3*(b*f
*x+d*e*x+a*f+c*e)/(d*x+c))/(f/(d*x+c)*a-1/(d*x+c)/d*b*c*f)*b*c*f+3/8/d*exp
(3*(b*f+d*e)/d)*Ei(1,-3/d*(a*d*f-b*c*f)/(d*x+c)-3*(b*f+d*e)/d-3*(-b*f-d*e)
/d)*a*f-3/8/d^2*exp(3*(b*f+d*e)/d)*Ei(1,-3/d*(a*d*f-b*c*f)/(d*x+c)-3*(b*f+
d*e)/d-3*(-b*f-d*e)/d)*b*c*f-3/8/d*exp((b*f*x+d*e*x+a*f+c*e)/(d*x+c))/(f/(
d*x+c)*a-1/(d*x+c)/d*b*c*f)*a*f+3/8/d^2*exp((b*f*x+d*e*x+a*f+c*e)/(d*x+c))
/(f/(d*x+c)*a-1/(d*x+c)/d*b*c*f)*b*c*f-3/8/d*exp((b*f+d*e)/d)*Ei(1,-1/d*(a
*d*f-b*c*f)/(d*x+c)-(b*f+d*e)/d-(-b*f-d*e)/d)*a*f+3/8/d^2*exp((b*f+d*e)/d)
*Ei(1,-1/d*(a*d*f-b*c*f)/(d*x+c)-(b*f+d*e)/d-(-b*f-d*e)/d)*b*c*f
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 942 vs. $2(220) = 440$.

Time = 0.13 (sec) , antiderivative size = 942, normalized size of antiderivative = 4.17

$$\int \sinh^3\left(e+\frac{f(a+bx)}{c+dx}\right)\,dx = \text{Too large to display}$$

input
```
integrate(sinh(e+f*(b*x+a)/(d*x+c))^3,x, algorithm="fricas")
```

output
```
-1/8*(6*(b*c - a*d)*f*Ei(-3*(b*c - a*d)*f/(d^2*x + c*d))*cosh((c*e + a*f +
 (d*e + b*f)*x)/(d*x + c))^2*cosh(3*(d*e + b*f)/d)*sinh((c*e + a*f + (d*e
+ b*f)*x)/(d*x + c))^2 - 3*(b*c - a*d)*f*Ei(-3*(b*c - a*d)*f/(d^2*x + c*d)
)*cosh(3*(d*e + b*f)/d)*sinh((c*e + a*f + (d*e + b*f)*x)/(d*x + c))^4 - 2*
(d^2*x + c*d)*sinh((c*e + a*f + (d*e + b*f)*x)/(d*x + c))^3 - 3*((b*c - a*
d)*f*Ei(-3*(b*c - a*d)*f/(d^2*x + c*d))*cosh((c*e + a*f + (d*e + b*f)*x)/(
d*x + c))^4 + (b*c - a*d)*f*Ei(3*(b*c - a*d)*f/(d^2*x + c*d)))*cosh(3*(d*e
 + b*f)/d) + 3*((b*c - a*d)*f*Ei((b*c - a*d)*f/(d^2*x + c*d)) + (b*c - a*d
)*f*Ei(-(b*c - a*d)*f/(d^2*x + c*d)))*cosh((d*e + b*f)/d) + 6*(d^2*x - (d^
2*x + c*d)*cosh((c*e + a*f + (d*e + b*f)*x)/(d*x + c))^2 + c*d)*sinh((c*e
+ a*f + (d*e + b*f)*x)/(d*x + c)) - 3*((b*c - a*d)*f*Ei(-3*(b*c - a*d)*f/(
d^2*x + c*d))*cosh((c*e + a*f + (d*e + b*f)*x)/(d*x + c))^4 - 2*(b*c - a*d
)*f*Ei(-3*(b*c - a*d)*f/(d^2*x + c*d))*cosh((c*e + a*f + (d*e + b*f)*x)/(d
*x + c))^2*sinh((c*e + a*f + (d*e + b*f)*x)/(d*x + c))^2 + (b*c - a*d)*f*E
i(-3*(b*c - a*d)*f/(d^2*x + c*d))*sinh((c*e + a*f + (d*e + b*f)*x)/(d*x +
c))^4 - (b*c - a*d)*f*Ei(3*(b*c - a*d)*f/(d^2*x + c*d)))*sinh(3*(d*e + b*f
)/d) - 3*((b*c - a*d)*f*Ei((b*c - a*d)*f/(d^2*x + c*d)) - (b*c - a*d)*f*Ei
(-(b*c - a*d)*f/(d^2*x + c*d)))*sinh((d*e + b*f)/d))/(d^2*cosh((c*e + a*f
+ (d*e + b*f)*x)/(d*x + c))^4 - 2*d^2*cosh((c*e + a*f + (d*e + b*f)*x)/(d*
x + c))^2*sinh((c*e + a*f + (d*e + b*f)*x)/(d*x + c))^2 + d^2*sinh((c*e...
```

Sympy [F(-1)]

Timed out.

$$\int \sinh^3\left(e+\frac{f(a+bx)}{c+dx}\right)\,dx = \text{Timed out}$$

input
```
integrate(sinh(e+f*(b*x+a)/(d*x+c))**3,x)
```

output
```
Timed out
```

Maxima [F]

$$\int \sinh^3\left(e+\frac{f(a+bx)}{c+dx}\right)\,dx = \int \sinh\left(e+\frac{(bx+a)f}{dx+c}\right)^3\,dx$$

input
```
integrate(sinh(e+f*(b*x+a)/(d*x+c))^3,x, algorithm="maxima")
```

output
```
integrate(sinh(e + (b*x + a)*f/(d*x + c))^3, x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 3021 vs. $2(220) = 440$.

Time $= 27.91$ (sec) , antiderivative size $= 3021$, normalized size of antiderivative $= 13.37$

$$\int \sinh^3\left(e+\frac{f(a+bx)}{c+dx}\right)\,dx = \text{Too large to display}$$

input
```
integrate(sinh(e+f*(b*x+a)/(d*x+c))^3,x, algorithm="giac")
```

output
```
1/8*(3*b^2*c^2*d*e*f^2*Ei(-3*(d*e + b*f - (d*e*x + b*f*x + c*e + a*f)*d/(d
*x + c))/d)*e^(3*(d*e + b*f)/d) - 6*a*b*c*d^2*e*f^2*Ei(-3*(d*e + b*f - (d*
e*x + b*f*x + c*e + a*f)*d/(d*x + c))/d)*e^(3*(d*e + b*f)/d) + 3*a^2*d^3*e
*f^2*Ei(-3*(d*e + b*f - (d*e*x + b*f*x + c*e + a*f)*d/(d*x + c))/d)*e^(3*(
d*e + b*f)/d) + 3*b^3*c^2*f^3*Ei(-3*(d*e + b*f - (d*e*x + b*f*x + c*e + a*
f)*d/(d*x + c))/d)*e^(3*(d*e + b*f)/d) - 6*a*b^2*c*d*f^3*Ei(-3*(d*e + b*f
 - (d*e*x + b*f*x + c*e + a*f)*d/(d*x + c))/d)*e^(3*(d*e + b*f)/d) + 3*a^2*
b*d^2*f^3*Ei(-3*(d*e + b*f - (d*e*x + b*f*x + c*e + a*f)*d/(d*x + c))/d)*e
^(3*(d*e + b*f)/d) - 3*b^2*c^2*d*e*f^2*Ei(-(d*e + b*f - (d*e*x + b*f*x + c
*e + a*f)*d/(d*x + c))/d)*e^((d*e + b*f)/d) + 6*a*b*c*d^2*e*f^2*Ei(-(d*e +
 b*f - (d*e*x + b*f*x + c*e + a*f)*d/(d*x + c))/d)*e^((d*e + b*f)/d) - 3*a
^2*d^3*e*f^2*Ei(-(d*e + b*f - (d*e*x + b*f*x + c*e + a*f)*d/(d*x + c))/d)*
e^((d*e + b*f)/d) - 3*b^3*c^2*f^3*Ei(-(d*e + b*f - (d*e*x + b*f*x + c*e +
a*f)*d/(d*x + c))/d)*e^((d*e + b*f)/d) + 6*a*b^2*c*d*f^3*Ei(-(d*e + b*f -
(d*e*x + b*f*x + c*e + a*f)*d/(d*x + c))/d)*e^((d*e + b*f)/d) - 3*a^2*b*d^
2*f^3*Ei(-(d*e + b*f - (d*e*x + b*f*x + c*e + a*f)*d/(d*x + c))/d)*e^((d*e
 + b*f)/d) - 3*b^2*c^2*d*e*f^2*Ei((d*e + b*f - (d*e*x + b*f*x + c*e + a*f)
*d/(d*x + c))/d)*e^(-(d*e + b*f)/d) + 6*a*b*c*d^2*e*f^2*Ei((d*e + b*f - (d
*e*x + b*f*x + c*e + a*f)*d/(d*x + c))/d)*e^(-(d*e + b*f)/d) - 3*a^2*d^3*e
*f^2*Ei((d*e + b*f - (d*e*x + b*f*x + c*e + a*f)*d/(d*x + c))/d)*e^(-(d...
```

Mupad [F(-1)]

Timed out.

$$\int \sinh^3\left(e+\frac{f(a+bx)}{c+dx}\right)\,dx=\int \sinh\left(e+\frac{f\,(a+b\,x)}{c+d\,x}\right)^3\,dx$$

input
```
int(sinh(e + (f*(a + b*x))/(c + d*x))^3,x)
```

output
```
int(sinh(e + (f*(a + b*x))/(c + d*x))^3, x)
```

Reduce [F]

$$\int \sinh^3\left(e+\frac{f(a+bx)}{c+dx}\right)\,dx = \text{too large to display}$$

input
```
int(sinh(e+f*(b*x+a)/(d*x+c))^3,x)
```

output
```
(3*e**((6*a*f + 6*b*f*x + 6*c*e + 6*d*e*x)/(c + d*x))*a*d**2*f*x**2 - 3*e*
*((6*a*f + 6*b*f*x + 6*c*e + 6*d*e*x)/(c + d*x))*b*c*d*f*x**2 - e**((6*a*f
 + 6*b*f*x + 6*c*e + 6*d*e*x)/(c + d*x))*c**3 - e**((6*a*f + 6*b*f*x + 6*c
*e + 6*d*e*x)/(c + d*x))*c**2*d*x - 9*e**((4*a*f + 4*b*f*x + 4*c*e + 4*d*e
*x)/(c + d*x))*a*d**2*f*x**2 + 9*e**((4*a*f + 4*b*f*x + 4*c*e + 4*d*e*x)/(
c + d*x))*b*c*d*f*x**2 + 9*e**((4*a*f + 4*b*f*x + 4*c*e + 4*d*e*x)/(c + d*
x))*c**3 + 9*e**((4*a*f + 4*b*f*x + 4*c*e + 4*d*e*x)/(c + d*x))*c**2*d*x -
 9*e**((3*a*f + 3*b*f*x + 3*c*e + 3*d*e*x)/(c + d*x))*int(x**2/(e**((a*f +
 b*f*x + c*e + d*e*x)/(c + d*x))*c**3 + 3*e**((a*f + b*f*x + c*e + d*e*x)/
(c + d*x))*c**2*d*x + 3*e**((a*f + b*f*x + c*e + d*e*x)/(c + d*x))*c*d**2*
x**2 + e**((a*f + b*f*x + c*e + d*e*x)/(c + d*x))*d**3*x**3),x)*a**2*c*d**
3*f**2 - 9*e**((3*a*f + 3*b*f*x + 3*c*e + 3*d*e*x)/(c + d*x))*int(x**2/(e*
*((a*f + b*f*x + c*e + d*e*x)/(c + d*x))*c**3 + 3*e**((a*f + b*f*x + c*e +
 d*e*x)/(c + d*x))*c**2*d*x + 3*e**((a*f + b*f*x + c*e + d*e*x)/(c + d*x))
*c*d**2*x**2 + e**((a*f + b*f*x + c*e + d*e*x)/(c + d*x))*d**3*x**3),x)*a*
*2*d**4*f**2*x + 18*e**((3*a*f + 3*b*f*x + 3*c*e + 3*d*e*x)/(c + d*x))*int
(x**2/(e**((a*f + b*f*x + c*e + d*e*x)/(c + d*x))*c**3 + 3*e**((a*f + b*f*
x + c*e + d*e*x)/(c + d*x))*c**2*d*x + 3*e**((a*f + b*f*x + c*e + d*e*x)/(
c + d*x))*c*d**2*x**2 + e**((a*f + b*f*x + c*e + d*e*x)/(c + d*x))*d**3*x*
*3),x)*a*b*c**2*d**2*f**2 + 18*e**((3*a*f + 3*b*f*x + 3*c*e + 3*d*e*x)/...
```

3.301 $\int e^{a+bx}\sinh^4(a+bx)\,dx$

Optimal result

Integrand size = 16, antiderivative size = 83

$$\int e^{a+bx}\sinh^4(a+bx)\,dx=-\frac{e^{-3a-3bx}}{48b}+\frac{e^{-a-bx}}{4b}+\frac{3e^{a+bx}}{8b}-\frac{e^{3a+3bx}}{12b}+\frac{e^{5a+5bx}}{80b}$$

output
```
-1/48*exp(-3*b*x-3*a)/b+1/4*exp(-b*x-a)/b+3/8*exp(b*x+a)/b-1/12*exp(3*b*x+
3*a)/b+1/80*exp(5*b*x+5*a)/b
```

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 62, normalized size of antiderivative = 0.75

$$\int e^{a+bx}\sinh^4(a+bx)\,dx=\frac{e^{-3(a+bx)}\left(-5+60e^{2(a+bx)}+90e^{4(a+bx)}-20e^{6(a+bx)}+3e^{8(a+bx)}\right)}{240b}$$

input
```
Integrate[E^(a + b*x)*Sinh[a + b*x]^4,x]
```

output
```
(-5 + 60*E^(2*(a + b*x)) + 90*E^(4*(a + b*x)) - 20*E^(6*(a + b*x)) + 3*E^(
8*(a + b*x)))/(240*b*E^(3*(a + b*x)))
```

Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 71, normalized size of antiderivative = 0.86, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {2720, 27, 244, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int e^{a+bx} \sinh^4(a+bx) \, dx$$

$$\Big\downarrow \text{2720}$$

$$\frac{\int \frac{1}{16} e^{-4a-4bx} \left(1-e^{2a+2bx}\right)^4 de^{a+bx}}{b}$$

$$\Big\downarrow \text{27}$$

$$\frac{\int e^{-4a-4bx} \left(1-e^{2a+2bx}\right)^4 de^{a+bx}}{16b}$$

$$\Big\downarrow \text{244}$$

$$\frac{\int \left(6+e^{-4a-4bx}-4e^{-2a-2bx}-4e^{2a+2bx}+e^{4a+4bx}\right) de^{a+bx}}{16b}$$

$$\Big\downarrow \text{2009}$$

$$\frac{-\frac{1}{3}e^{-3a-3bx}+4e^{-a-bx}+6e^{a+bx}-\frac{4}{3}e^{3a+3bx}+\frac{1}{5}e^{5a+5bx}}{16b}$$

input
```
Int[E^(a + b*x)*Sinh[a + b*x]^4,x]
```

output
```
(-1/3*E^(-3*a - 3*b*x) + 4*E^(-a - b*x) + 6*E^(a + b*x) - (4*E^(3*a + 3*b*
x))/3 + E^(5*a + 5*b*x)/5)/(16*b)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 244
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[Expand
Integrand[(c*x)^m*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x] && IGtQ[p
, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2720
```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Maple [A] (verified)

Time = 1.36 (sec) , antiderivative size = 45, normalized size of antiderivative = 0.54

method	result
derivativedivides	$\frac{\left(\frac{8}{15}+\frac{\sinh(bx+a)^4}{5}-\frac{4\sinh(bx+a)^2}{15}\right)\cosh(bx+a)+\frac{\sinh(bx+a)^5}{5}}{b}$
default	$\frac{\left(\frac{8}{15}+\frac{\sinh(bx+a)^4}{5}-\frac{4\sinh(bx+a)^2}{15}\right)\cosh(bx+a)+\frac{\sinh(bx+a)^5}{5}}{b}$
risch	$-\frac{e^{-3bx-3a}}{48b}+\frac{e^{-bx-a}}{4b}+\frac{3\,e^{bx+a}}{8b}-\frac{e^{3bx+3a}}{12b}+\frac{e^{5bx+5a}}{80b}$
parallelrisch	$\frac{16\,e^{bx+a}\left(6\tanh\left(\frac{bx}{2}+\frac{a}{2}\right)^3-2\tanh\left(\frac{bx}{2}+\frac{a}{2}\right)^2-2\tanh\left(\frac{bx}{2}+\frac{a}{2}\right)+1\right)}{15b\left(-1+\tanh\left(\frac{bx}{2}+\frac{a}{2}\right)\right)^4\left(1+\tanh\left(\frac{bx}{2}+\frac{a}{2}\right)\right)^4}$
orering	$\frac{e^{bx+a}\sinh(bx+a)^4}{5b}+\frac{\frac{10b\,e^{bx+a}\sinh(bx+a)^4}{9}+\frac{40\,e^{bx+a}\sinh(bx+a)^3 b\cosh(bx+a)}{9}}{b^2}-\frac{2\left(5b^2e^{bx+a}\sinh(bx+a)^4+8b^2e^{bx-}\right.}{}$

input
```
int(exp(b*x+a)*sinh(b*x+a)^4,x,method=_RETURNVERBOSE)
```

output
```
1/b*((8/15+1/5*sinh(b*x+a)^4-4/15*sinh(b*x+a)^2)*cosh(b*x+a)+1/5*sinh(b*x+
a)^5)
```

Fricas [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 113, normalized size of antiderivative = 1.36

$$\int e^{a+bx}\sinh^4(a+bx)\,dx = -\frac{\cosh{(bx+a)}^4-16\,\cosh{(bx+a)}\sinh{(bx+a)}^3+\sinh{(bx+a)}^4+2\left(3\,\cosh{(bx+a)}^2-10\right)\sinh{(bx}{120\,(b\cosh{(bx+a)}-b\sin}$$

input
```
integrate(exp(b*x+a)*sinh(b*x+a)^4,x, algorithm="fricas")
```

output
```
-1/120*(cosh(b*x + a)^4 - 16*cosh(b*x + a)*sinh(b*x + a)^3 + sinh(b*x + a)
^4 + 2*(3*cosh(b*x + a)^2 - 10)*sinh(b*x + a)^2 - 20*cosh(b*x + a)^2 - 16*
(cosh(b*x + a)^3 - 5*cosh(b*x + a))*sinh(b*x + a) - 45)/(b*cosh(b*x + a) -
 b*sinh(b*x + a))
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 139 vs. 2(65) = 130.

Time = 2.37 (sec) , antiderivative size = 139, normalized size of antiderivative = 1.67

$$\int e^{a+bx}\sinh^4(a+bx)\,dx = \begin{cases} \frac{e^{a}e^{bx}\sinh^4{(a+bx)}}{5b}+\frac{4e^{a}e^{bx}\sinh^3{(a+bx)}\cosh{(a+bx)}}{5b}-\frac{4e^{a}e^{bx}\sinh^2{(a+bx)}\cosh^2{(a+bx)}}{5b}-\frac{8e^{a}e^{bx}\sinh{(a+bx)}\cosh^3{(a+bx)}}{15b}+\frac{8e^{\cdots}}{} \\ xe^{a}\sinh^4{(a)} \end{cases}$$

input
```
integrate(exp(b*x+a)*sinh(b*x+a)**4,x)
```

output
```
Piecewise((exp(a)*exp(b*x)*sinh(a + b*x)**4/(5*b) + 4*exp(a)*exp(b*x)*sinh
(a + b*x)**3*cosh(a + b*x)/(5*b) - 4*exp(a)*exp(b*x)*sinh(a + b*x)**2*cosh
(a + b*x)**2/(5*b) - 8*exp(a)*exp(b*x)*sinh(a + b*x)*cosh(a + b*x)**3/(15*
b) + 8*exp(a)*exp(b*x)*cosh(a + b*x)**4/(15*b), Ne(b, 0)), (x*exp(a)*sinh(
a)**4, True))
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 68, normalized size of antiderivative = 0.82

$$\int e^{a+bx}\sinh^4(a+bx)\,dx = \frac{e^{(5\,bx+5\,a)}}{80\,b} - \frac{e^{(3\,bx+3\,a)}}{12\,b} + \frac{3\,e^{(bx+a)}}{8\,b} + \frac{e^{(-bx-a)}}{4\,b} - \frac{e^{(-3\,bx-3\,a)}}{48\,b}$$

input
```
integrate(exp(b*x+a)*sinh(b*x+a)^4,x, algorithm="maxima")
```

output
```
1/80*e^(5*b*x + 5*a)/b - 1/12*e^(3*b*x + 3*a)/b + 3/8*e^(b*x + a)/b + 1/4*
e^(-b*x - a)/b - 1/48*e^(-3*b*x - 3*a)/b
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 60, normalized size of antiderivative = 0.72

$$\int e^{a+bx}\sinh^4(a+bx)\,dx = \frac{5\,\left(12\,e^{(2\,bx+2\,a)}-1\right)e^{(-3\,bx-3\,a)} + 3\,e^{(5\,bx+5\,a)} - 20\,e^{(3\,bx+3\,a)} + 90\,e^{(bx+a)}}{240\,b}$$

input
```
integrate(exp(b*x+a)*sinh(b*x+a)^4,x, algorithm="giac")
```

output
```
1/240*(5*(12*e^(2*b*x + 2*a) - 1)*e^(-3*b*x - 3*a) + 3*e^(5*b*x + 5*a) - 2
0*e^(3*b*x + 3*a) + 90*e^(b*x + a))/b
```

Mupad [B] (verification not implemented)

Time = 0.53 (sec) , antiderivative size = 58, normalized size of antiderivative = 0.70

$$\int e^{a+bx}\sinh^4(a+bx)\,dx = \frac{90\,\mathrm{e}^{a+b\,x}+60\,\mathrm{e}^{-a-b\,x}-5\,\mathrm{e}^{-3\,a-3\,b\,x}-20\,\mathrm{e}^{3\,a+3\,b\,x}+3\,\mathrm{e}^{5\,a+5\,b\,x}}{240\,b}$$

input
```
int(exp(a + b*x)*sinh(a + b*x)^4,x)
```

output
```
(90*exp(a + b*x) + 60*exp(- a - b*x) - 5*exp(- 3*a - 3*b*x) - 20*exp(3*a +
 3*b*x) + 3*exp(5*a + 5*b*x))/(240*b)
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 67, normalized size of antiderivative = 0.81

$$\int e^{a+bx}\sinh^4(a+bx)\,dx = \frac{3e^{8bx+8a}-20e^{6bx+6a}+90e^{4bx+4a}+60e^{2bx+2a}-5}{240e^{3bx+3a}b}$$

input
```
int(exp(b*x+a)*sinh(b*x+a)^4,x)
```

output
```
(3*e**(8*a + 8*b*x) - 20*e**(6*a + 6*b*x) + 90*e**(4*a + 4*b*x) + 60*e**(2
*a + 2*b*x) - 5)/(240*e**(3*a + 3*b*x)*b)
```

3.302 $\int e^{a+bx}\sinh^3(a+bx)\,dx$

Optimal result

Integrand size = 16, antiderivative size = 57

$$\int e^{a+bx}\sinh^3(a+bx)\,dx=\frac{e^{-2a-2bx}}{16b}-\frac{3e^{2a+2bx}}{16b}+\frac{e^{4a+4bx}}{32b}+\frac{3x}{8}$$

output
```
1/16*exp(-2*b*x-2*a)/b-3/16*exp(2*b*x+2*a)/b+1/32*exp(4*b*x+4*a)/b+3/8*x
```

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 45, normalized size of antiderivative = 0.79

$$\int e^{a+bx}\sinh^3(a+bx)\,dx=\frac{e^{-2(a+bx)}-3e^{2(a+bx)}+\frac{1}{2}e^{4(a+bx)}+6bx}{16b}$$

input
```
Integrate[E^(a + b*x)*Sinh[a + b*x]^3,x]
```

output
```
(E^(-2*(a + b*x)) - 3*E^(2*(a + b*x)) + E^(4*(a + b*x))/2 + 6*b*x)/(16*b)
```

Rubi [A] (warning: unable to verify)

Time = 0.22 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.82, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.312, Rules used = {2720, 27, 243, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int e^{a+bx}\sinh^3(a+bx)\,dx$$

$$\Big\downarrow \text{ 2720}$$

$$\frac{\int -\frac{1}{8}e^{-3a-3bx}\left(1-e^{2a+2bx}\right)^3de^{a+bx}}{b}$$

$$\Big\downarrow \text{ 27}$$

$$-\frac{\int e^{-3a-3bx}\left(1-e^{2a+2bx}\right)^3de^{a+bx}}{8b}$$

$$\Big\downarrow \text{ 243}$$

$$-\frac{\int e^{-2a-2bx}\left(1-e^{2a+2bx}\right)^3de^{2a+2bx}}{16b}$$

$$\Big\downarrow \text{ 49}$$

$$-\frac{\int \left(3+e^{-2a-2bx}-3e^{-a-bx}-e^{2a+2bx}\right)de^{2a+2bx}}{16b}$$

$$\Big\downarrow \text{ 2009}$$

$$-\frac{-e^{-a-bx}+\frac{5}{2}e^{2a+2bx}-3\log\left(e^{2a+2bx}\right)}{16b}$$

input
```
Int[E^(a + b*x)*Sinh[a + b*x]^3,x]
```

output
```
-1/16*(-E^(-a - b*x) + (5*E^(2*a + 2*b*x))/2 - 3*Log[E^(2*a + 2*b*x)])/b
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2720
```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Maple [A] (verified)

Time = 0.56 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.82

method	result
risch	$\frac{{\mathrm e}^{-2 b x -2 a}}{16 b}-\frac{3 \,{\mathrm e}^{2 b x +2 a}}{16 b}+\frac{{\mathrm e}^{4 b x +4 a}}{32 b}+\frac{3 x}{8}$
derivativedivides	$\frac{\left(\frac{\sinh \left(b x +a \right)^{3}}{4}-\frac{3 \sinh \left(b x +a \right)}{8}\right) \cosh \left(b x +a \right)+\frac{3 b x}{8}+\frac{3 a}{8}+\frac{\sinh \left(b x +a \right)^{4}}{4}}{b}$
default	$\frac{\left(\frac{\sinh \left(b x +a \right)^{3}}{4}-\frac{3 \sinh \left(b x +a \right)}{8}\right) \cosh \left(b x +a \right)+\frac{3 b x}{8}+\frac{3 a}{8}+\frac{\sinh \left(b x +a \right)^{4}}{4}}{b}$
orering	$\frac{\left(4 b x +1\right) {\mathrm e}^{b x +a} \sinh \left(b x +a \right)^{3}}{4 b}-\frac{\left(b x -1\right) \left({\mathrm e}^{b x +a} \sinh \left(b x +a \right)^{3} b +3 \,{\mathrm e}^{b x +a} \sinh \left(b x +a \right)^{2} b \cosh \left(b x +a \right)\right)}{4 b^{2}}-\frac{\left(4 b x +1\right) \left(4 \sinh\right.}{}$

input
```
int(exp(b*x+a)*sinh(b*x+a)^3,x,method=_RETURNVERBOSE)
```

output
```
1/16/b*exp(-2*b*x-2*a)-3/16/b*exp(2*b*x+2*a)+1/32/b*exp(4*b*x+4*a)+3/8*x
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 95 vs. 2(46) = 92.

Time = 0.08 (sec) , antiderivative size = 95, normalized size of antiderivative = 1.67

$$\int e^{a+bx}\sinh^3(a+bx)\,dx$$
$$=\frac{3\cosh(bx+a)^3+9\cosh(bx+a)\sinh(bx+a)^2-\sinh(bx+a)^3+6(2bx-1)\cosh(bx+a)-3(4bx}{32(b\cosh(bx+a)-b\sinh(bx+a))}$$

input
```
integrate(exp(b*x+a)*sinh(b*x+a)^3,x, algorithm="fricas")
```

output
```
1/32*(3*cosh(b*x + a)^3 + 9*cosh(b*x + a)*sinh(b*x + a)^2 - sinh(b*x + a)^
3 + 6*(2*b*x - 1)*cosh(b*x + a) - 3*(4*b*x + cosh(b*x + a)^2 + 2)*sinh(b*x
 + a))/(b*cosh(b*x + a) - b*sinh(b*x + a))
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 182 vs. 2(48) = 96.

Time = 0.99 (sec) , antiderivative size = 182, normalized size of antiderivative = 3.19

$$\int e^{a+bx}\sinh^3(a+bx)\,dx$$
$$=\begin{cases}\frac{3xe^ae^{bx}\sinh^3(a+bx)}{8}-\frac{3xe^ae^{bx}\sinh^2(a+bx)\cosh(a+bx)}{8}-\frac{3xe^ae^{bx}\sinh(a+bx)\cosh^2(a+bx)}{8}+\frac{3xe^ae^{bx}\cosh^3(a+bx)}{8}-\frac{e^ae^{bx}\sinh}{8}\\ xe^a\sinh^3(a)\end{cases}$$

input
```
integrate(exp(b*x+a)*sinh(b*x+a)**3,x)
```

output
```
Piecewise((3*x*exp(a)*exp(b*x)*sinh(a + b*x)**3/8 - 3*x*exp(a)*exp(b*x)*si
nh(a + b*x)**2*cosh(a + b*x)/8 - 3*x*exp(a)*exp(b*x)*sinh(a + b*x)*cosh(a
+ b*x)**2/8 + 3*x*exp(a)*exp(b*x)*cosh(a + b*x)**3/8 - exp(a)*exp(b*x)*sin
h(a + b*x)**3/(8*b) + 3*exp(a)*exp(b*x)*sinh(a + b*x)**2*cosh(a + b*x)/(4*
b) - 3*exp(a)*exp(b*x)*cosh(a + b*x)**3/(8*b), Ne(b, 0)), (x*exp(a)*sinh(a
)**3, True))
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.93

$$\int e^{a+bx}\sinh^3(a+bx)\,dx=\frac{3\,(bx+a)}{8\,b}+\frac{e^{(4\,bx+4\,a)}}{32\,b}-\frac{3\,e^{(2\,bx+2\,a)}}{16\,b}+\frac{e^{(-2\,bx-2\,a)}}{16\,b}$$

input
```
integrate(exp(b*x+a)*sinh(b*x+a)^3,x, algorithm="maxima")
```

output
```
3/8*(b*x + a)/b + 1/32*e^(4*b*x + 4*a)/b - 3/16*e^(2*b*x + 2*a)/b + 1/16*e
^(-2*b*x - 2*a)/b
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 57, normalized size of antiderivative = 1.00

$$\int e^{a+bx}\sinh^3(a+bx)\,dx=\frac{12\,bx-2\left(3\,e^{(2\,bx+2\,a)}-1\right)e^{(-2\,bx-2\,a)}+12\,a+e^{(4\,bx+4\,a)}-6\,e^{(2\,bx+2\,a)}}{32\,b}$$

input
```
integrate(exp(b*x+a)*sinh(b*x+a)^3,x, algorithm="giac")
```

output
```
1/32*(12*b*x - 2*(3*e^(2*b*x + 2*a) - 1)*e^(-2*b*x - 2*a) + 12*a + e^(4*b*
x + 4*a) - 6*e^(2*b*x + 2*a))/b
```

Mupad [B] (verification not implemented)

Time = 1.83 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.74

$$\int e^{a+bx}\sinh^3(a+bx)\,dx = \frac{3\,x}{8}+\frac{\frac{\mathrm{e}^{-2\,a-2\,b\,x}}{16}-\frac{3\,\mathrm{e}^{2\,a+2\,b\,x}}{16}+\frac{\mathrm{e}^{4\,a+4\,b\,x}}{32}}{b}$$

input
```
int(exp(a + b*x)*sinh(a + b*x)^3,x)
```

output
```
(3*x)/8 + (exp(- 2*a - 2*b*x)/16 - (3*exp(2*a + 2*b*x))/16 + exp(4*a + 4*b
*x)/32)/b
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 55, normalized size of antiderivative = 0.96

$$\int e^{a+bx}\sinh^3(a+bx)\,dx = \frac{e^{6bx+6a}-6e^{4bx+4a}+12e^{2bx+2a}bx+2}{32e^{2bx+2a}b}$$

input
```
int(exp(b*x+a)*sinh(b*x+a)^3,x)
```

output
```
(e**(6*a + 6*b*x) - 6*e**(4*a + 4*b*x) + 12*e**(2*a + 2*b*x)*b*x + 2)/(32*
e**(2*a + 2*b*x)*b)
```

3.303 $\int e^{a+bx} \sinh^2(a+bx)\,dx$

Optimal result

Integrand size = 16, antiderivative size = 49

$$\int e^{a+bx} \sinh^2(a+bx)\,dx = -\frac{e^{-a-bx}}{4b} - \frac{e^{a+bx}}{2b} + \frac{e^{3a+3bx}}{12b}$$

output
```
-1/4*exp(-b*x-a)/b-1/2*exp(b*x+a)/b+1/12*exp(3*b*x+3*a)/b
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.80

$$\int e^{a+bx} \sinh^2(a+bx)\,dx = \frac{e^{-a-bx}\left(-3-6e^{2(a+bx)}+e^{4(a+bx)}\right)}{12b}$$

input
```
Integrate[E^(a + b*x)*Sinh[a + b*x]^2,x]
```

output
```
(E^(-a - b*x)*(-3 - 6*E^(2*(a + b*x)) + E^(4*(a + b*x))))/(12*b)
```

Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 43, normalized size of antiderivative = 0.88, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {2720, 27, 244, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int e^{a+bx}\sinh^2(a+bx)\,dx$$

$$\Big\downarrow \text{2720}$$

$$\frac{\int \frac{1}{4}e^{-2a-2bx}\left(1-e^{2a+2bx}\right)^2de^{a+bx}}{b}$$

$$\Big\downarrow \text{27}$$

$$\frac{\int e^{-2a-2bx}\left(1-e^{2a+2bx}\right)^2de^{a+bx}}{4b}$$

$$\Big\downarrow \text{244}$$

$$\frac{\int \left(-2+e^{-2a-2bx}+e^{2a+2bx}\right)de^{a+bx}}{4b}$$

$$\Big\downarrow \text{2009}$$

$$\frac{-e^{-a-bx}-2e^{a+bx}+\frac{1}{3}e^{3a+3bx}}{4b}$$

input
```
Int[E^(a + b*x)*Sinh[a + b*x]^2,x]
```

output
```
(-E^(-a - b*x) - 2*E^(a + b*x) + E^(3*a + 3*b*x)/3)/(4*b)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 244
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[Expand
Integrand[(c*x)^m*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x] && IGtQ[p
, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2720
```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Maple [A] (verified)

Time = 0.25 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.71

method	result
derivativedivides	$\frac{\left(-\frac{2}{3}+\frac{\sinh(bx+a)^2}{3}\right)\cosh(bx+a)+\frac{\sinh(bx+a)^3}{3}}{b}$
default	$\frac{\left(-\frac{2}{3}+\frac{\sinh(bx+a)^2}{3}\right)\cosh(bx+a)+\frac{\sinh(bx+a)^3}{3}}{b}$
risch	$-\frac{e^{-bx-a}}{4b}-\frac{e^{bx+a}}{2b}+\frac{e^{3bx+3a}}{12b}$
parallelrisch	$\frac{4\,e^{bx+a}\left(-1+2\tanh\left(\frac{bx}{2}+\frac{a}{2}\right)\right)}{3b\left(-1+\tanh\left(\frac{bx}{2}+\frac{a}{2}\right)\right)^2\left(1+\tanh\left(\frac{bx}{2}+\frac{a}{2}\right)\right)^2}$
orering	$\frac{e^{bx+a}\sinh(bx+a)^2}{3b}+\frac{e^{bx+a}\sinh(bx+a)^2 b+2\,e^{bx+a}\sinh(bx+a)b\cosh(bx+a)}{b^2}-\frac{3\,e^{bx+a}\sinh(bx+a)^2b^2+4\,e^{bx+a}\sin$

input
```
int(exp(b*x+a)*sinh(b*x+a)^2,x,method=_RETURNVERBOSE)
```

output
```
1/b*((-2/3+1/3*sinh(b*x+a)^2)*cosh(b*x+a)+1/3*sinh(b*x+a)^3)
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 54, normalized size of antiderivative = 1.10

$$\int e^{a+bx}\sinh^2(a+bx)\,dx$$
$$=-\frac{\cosh\left(bx+a\right)^2-4\cosh\left(bx+a\right)\sinh\left(bx+a\right)+\sinh\left(bx+a\right)^2+3}{6\left(b\cosh\left(bx+a\right)-b\sinh\left(bx+a\right)\right)}$$

input
```
integrate(exp(b*x+a)*sinh(b*x+a)^2,x, algorithm="fricas")
```

output
```
-1/6*(cosh(b*x + a)^2 - 4*cosh(b*x + a)*sinh(b*x + a) + sinh(b*x + a)^2 +
3)/(b*cosh(b*x + a) - b*sinh(b*x + a))
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 78 vs. 2(34) = 68.

Time = 0.48 (sec) , antiderivative size = 78, normalized size of antiderivative = 1.59

$$\int e^{a+bx}\sinh^2(a+bx)\,dx$$
$$=\begin{cases}\frac{e^ae^{bx}\sinh^2\left(a+bx\right)}{3b}+\frac{2e^ae^{bx}\sinh\left(a+bx\right)\cosh\left(a+bx\right)}{3b}-\frac{2e^ae^{bx}\cosh^2\left(a+bx\right)}{3b} & \text{for } b\neq 0\\ xe^a\sinh^2\left(a\right) & \text{otherwise}\end{cases}$$

input
```
integrate(exp(b*x+a)*sinh(b*x+a)**2,x)
```

output
```
Piecewise((exp(a)*exp(b*x)*sinh(a + b*x)**2/(3*b) + 2*exp(a)*exp(b*x)*sinh
(a + b*x)*cosh(a + b*x)/(3*b) - 2*exp(a)*exp(b*x)*cosh(a + b*x)**2/(3*b),
Ne(b, 0)), (x*exp(a)*sinh(a)**2, True))
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 40, normalized size of antiderivative = 0.82

$$\int e^{a+bx}\sinh^2(a+bx)\,dx = \frac{e^{(3\,bx+3\,a)}}{12\,b} - \frac{e^{(bx+a)}}{2\,b} - \frac{e^{(-bx-a)}}{4\,b}$$

input
```
integrate(exp(b*x+a)*sinh(b*x+a)^2,x, algorithm="maxima")
```

output
```
1/12*e^(3*b*x + 3*a)/b - 1/2*e^(b*x + a)/b - 1/4*e^(-b*x - a)/b
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.69

$$\int e^{a+bx}\sinh^2(a+bx)\,dx = \frac{e^{(3\,bx+3\,a)} - 6\,e^{(bx+a)} - 3\,e^{(-bx-a)}}{12\,b}$$

input
```
integrate(exp(b*x+a)*sinh(b*x+a)^2,x, algorithm="giac")
```

output
```
1/12*(e^(3*b*x + 3*a) - 6*e^(b*x + a) - 3*e^(-b*x - a))/b
```

Mupad [B] (verification not implemented)

Time = 1.62 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.73

$$\int e^{a+bx}\sinh^2(a+bx)\,dx = -\frac{6\,\mathrm{e}^{a+b\,x} + 3\,\mathrm{e}^{-a-b\,x} - \mathrm{e}^{3\,a+3\,b\,x}}{12\,b}$$

input
```
int(exp(a + b*x)*sinh(a + b*x)^2,x)
```

output
```
-(6*exp(a + b*x) + 3*exp(- a - b*x) - exp(3*a + 3*b*x))/(12*b)
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 38, normalized size of antiderivative = 0.78

$$\int e^{a+bx}\sinh^2(a+bx)\,dx=\frac{e^{4bx+4a}-6e^{2bx+2a}-3}{12e^{bx+a}b}$$

input
```
int(exp(b*x+a)*sinh(b*x+a)^2,x)
```

output
```
(e**(4*a + 4*b*x) - 6*e**(2*a + 2*b*x) - 3)/(12*e**(a + b*x)*b)
```

3.304 $\int e^{a+bx} \sinh(a+bx)\,dx$

Optimal result

Integrand size = 14, antiderivative size = 23

$$\int e^{a+bx} \sinh(a+bx)\,dx = \frac{e^{2a+2bx}}{4b} - \frac{x}{2}$$

output
```
1/4*exp(2*b*x+2*a)/b-1/2*x
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00

$$\int e^{a+bx} \sinh(a+bx)\,dx = \frac{e^{2a+2bx}}{4b} - \frac{x}{2}$$

input
```
Integrate[E^(a + b*x)*Sinh[a + b*x],x]
```

output
```
E^(2*a + 2*b*x)/(4*b) - x/2
```

Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.39, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$, Rules used = {2720, 27, 244, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int e^{a+bx} \sinh(a+bx) \, dx$$

$$\Big\downarrow \text{ 2720}$$

$$\frac{\int -\frac{1}{2} e^{-a-bx} \left(1-e^{2a+2bx}\right) de^{a+bx}}{b}$$

$$\Big\downarrow \text{ 27}$$

$$-\frac{\int e^{-a-bx} \left(1-e^{2a+2bx}\right) de^{a+bx}}{2b}$$

$$\Big\downarrow \text{ 244}$$

$$-\frac{\int \left(e^{-a-bx}-e^{a+bx}\right) de^{a+bx}}{2b}$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{\frac{1}{2} e^{2a+2bx}-\log\left(e^{a+bx}\right)}{2b}$$

input
```
Int[E^(a + b*x)*Sinh[a + b*x],x]
```

output
```
(E^(2*a + 2*b*x)/2 - Log[E^(a + b*x)])/(2*b)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 244
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[Expand
Integrand[(c*x)^m*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x] && IGtQ[p
, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2720
```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Maple [A] (verified)

Time = 0.10 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.83

method	result	size
risch	$\frac{e^{2bx+2a}}{4b}-\frac{x}{2}$	19
derivativedivides	$\frac{\frac{\cosh(bx+a)\sinh(bx+a)}{2}-\frac{bx}{2}-\frac{a}{2}+\frac{\cosh(bx+a)^2}{2}}{b}$	37
default	$\frac{\frac{\cosh(bx+a)\sinh(bx+a)}{2}-\frac{bx}{2}-\frac{a}{2}+\frac{\cosh(bx+a)^2}{2}}{b}$	37
orering	$\frac{(2bx+1)e^{bx+a}\sinh(bx+a)}{2b}-\frac{x\left(e^{bx+a}\sinh(bx+a)b+e^{bx+a}b\cosh(bx+a)\right)}{2b}$	60

input
```
int(exp(b*x+a)*sinh(b*x+a),x,method=_RETURNVERBOSE)
```

output
```
1/4/b*exp(2*b*x+2*a)-1/2*x
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 50 vs. $2(18) = 36$.

Time = 0.08 (sec) , antiderivative size = 50, normalized size of antiderivative = 2.17

$$\int e^{a+bx}\sinh(a+bx)\,dx = -\frac{(2\,bx-1)\cosh\left(bx+a\right)-(2\,bx+1)\sinh\left(bx+a\right)}{4\left(b\cosh\left(bx+a\right)-b\sinh\left(bx+a\right)\right)}$$

input
```
integrate(exp(b*x+a)*sinh(b*x+a),x, algorithm="fricas")
```

output
```
-1/4*((2*b*x - 1)*cosh(b*x + a) - (2*b*x + 1)*sinh(b*x + a))/(b*cosh(b*x +
 a) - b*sinh(b*x + a))
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 80 vs. $2(15) = 30$.

Time = 0.23 (sec) , antiderivative size = 80, normalized size of antiderivative = 3.48

$$\int e^{a+bx}\sinh(a+bx)\,dx = \begin{cases} \frac{xe^{a}e^{bx}\sinh\left(a+bx\right)}{2}-\frac{xe^{a}e^{bx}\cosh\left(a+bx\right)}{2}-\frac{e^{a}e^{bx}\sinh\left(a+bx\right)}{2b}+\frac{e^{a}e^{bx}\cosh\left(a+bx\right)}{b} & \text{for } b\neq 0 \\ xe^{a}\sinh\left(a\right) & \text{otherwise} \end{cases}$$

input
```
integrate(exp(b*x+a)*sinh(b*x+a),x)
```

output
```
Piecewise((x*exp(a)*exp(b*x)*sinh(a + b*x)/2 - x*exp(a)*exp(b*x)*cosh(a +
b*x)/2 - exp(a)*exp(b*x)*sinh(a + b*x)/(2*b) + exp(a)*exp(b*x)*cosh(a + b*
x)/b, Ne(b, 0)), (x*exp(a)*sinh(a), True))
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.04

$$\int e^{a+bx}\sinh(a+bx)\,dx = -\frac{1}{2}\,x - \frac{a}{2\,b} + \frac{e^{(2\,bx+2\,a)}}{4\,b}$$

input
```
integrate(exp(b*x+a)*sinh(b*x+a),x, algorithm="maxima")
```

output
```
-1/2*x - 1/2*a/b + 1/4*e^(2*b*x + 2*a)/b
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.04

$$\int e^{a+bx}\sinh(a+bx)\,dx = -\frac{2\,bx + 2\,a - e^{(2\,bx+2\,a)}}{4\,b}$$

input
```
integrate(exp(b*x+a)*sinh(b*x+a),x, algorithm="giac")
```

output
```
-1/4*(2*b*x + 2*a - e^(2*b*x + 2*a))/b
```

Mupad [B] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.78

$$\int e^{a+bx}\sinh(a+bx)\,dx = \frac{\mathrm{e}^{2\,a+2\,b\,x}}{4\,b} - \frac{x}{2}$$

input
```
int(exp(a + b*x)*sinh(a + b*x),x)
```

output
```
exp(2*a + 2*b*x)/(4*b) - x/2
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 38, normalized size of antiderivative = 1.65

$$\int e^{a+bx}\sinh(a+bx)\,dx = \frac{e^{bx+a}(-\cosh(bx+a)\,bx+\cosh(bx+a)+\sinh(bx+a)\,bx)}{2b}$$

input
```
int(exp(b*x+a)*sinh(b*x+a),x)
```

output
```
(e**(a + b*x)*( - cosh(a + b*x)*b*x + cosh(a + b*x) + sinh(a + b*x)*b*x))/
(2*b)
```

3.305 $\int e^{a+bx}\mathbf{csch}(a+bx)\,dx$

Optimal result

Integrand size = 14, antiderivative size = 19

$$\int e^{a+bx}\text{csch}(a+bx)\,dx=\frac{\log\left(1-e^{2a+2bx}\right)}{b}$$

output
```
ln(1-exp(2*b*x+2*a))/b
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.00

$$\int e^{a+bx}\text{csch}(a+bx)\,dx=\frac{\log\left(1-e^{2a+2bx}\right)}{b}$$

input
```
Integrate[E^(a + b*x)*Csch[a + b*x],x]
```

output
```
Log[1 - E^(2*a + 2*b*x)]/b
```

Rubi [A] (verified)

Time = 0.17 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.214, Rules used = {2720, 27, 240}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int e^{a+bx}\text{csch}(a+bx)\,dx$$

$$\Big\downarrow \text{2720}$$

$$\frac{\int -\frac{2e^{a+bx}}{1-e^{2a+2bx}}de^{a+bx}}{b}$$

$$\Big\downarrow \text{27}$$

$$-\frac{2\int \frac{e^{a+bx}}{1-e^{2a+2bx}}de^{a+bx}}{b}$$

$$\Big\downarrow \text{240}$$

$$\frac{\log\left(1-e^{2a+2bx}\right)}{b}$$

input
```
Int[E^(a + b*x)*Csch[a + b*x],x]
```

output
```
Log[1 - E^(2*a + 2*b*x)]/b
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 240
```
Int[(x_)/((a_) + (b_.)*(x_)^2), x_Symbol] :> Simp[Log[RemoveContent[a + b*x
^2, x]]/(2*b), x] /; FreeQ[{a, b}, x]
```

rule 2720
```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Maple [A] (verified)

Time = 0.08 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.89

method	result	size
derivativedivides	$\frac{bx+a+\ln(\sinh(bx+a))}{b}$	17
default	$\frac{bx+a+\ln(\sinh(bx+a))}{b}$	17
risch	$-\frac{2a}{b}+\frac{\ln\left(e^{2bx+2a}-1\right)}{b}$	24

input
```
int(exp(b*x+a)*csch(b*x+a),x,method=_RETURNVERBOSE)
```

output
```
1/b*(b*x+a+ln(sinh(b*x+a)))
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.58

$$\int e^{a+bx}\operatorname{csch}(a+bx)\,dx=\frac{\log\left(\frac{2\,\sinh(bx+a)}{\cosh(bx+a)-\sinh(bx+a)}\right)}{b}$$

input
```
integrate(exp(b*x+a)*csch(b*x+a),x, algorithm="fricas")
```

output
```
log(2*sinh(b*x + a)/(cosh(b*x + a) - sinh(b*x + a)))/b
```

Sympy [F]

$$\int e^{a+bx}\text{csch}(a+bx)\,dx = e^{a}\int e^{bx}\,\text{csch}\,(a+bx)\,dx$$

input
```
integrate(exp(b*x+a)*csch(b*x+a),x)
```

output
```
exp(a)*Integral(exp(b*x)*csch(a + b*x), x)
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.42

$$\int e^{a+bx}\text{csch}(a+bx)\,dx = \frac{\log\left(e^{(bx+a)}+1\right)}{b} + \frac{\log\left(e^{(bx+a)}-1\right)}{b}$$

input
```
integrate(exp(b*x+a)*csch(b*x+a),x, algorithm="maxima")
```

output
```
log(e^(b*x + a) + 1)/b + log(e^(b*x + a) - 1)/b
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.26

$$\int e^{a+bx}\text{csch}(a+bx)\,dx = \frac{\log\left(e^{(bx+a)}+1\right)+\log\left(\left|e^{(bx+a)}-1\right|\right)}{b}$$

input
```
integrate(exp(b*x+a)*csch(b*x+a),x, algorithm="giac")
```

output
```
(log(e^(b*x + a) + 1) + log(abs(e^(b*x + a) - 1)))/b
```

Mupad [B] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.84

$$\int e^{a+bx}\operatorname{csch}(a+bx)\,dx=\frac{\ln\left(\mathrm{e}^{2\,a+2\,b\,x}-1\right)}{b}$$

input
```
int(exp(a + b*x)/sinh(a + b*x),x)
```

output
```
log(exp(2*a + 2*b*x) - 1)/b
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.32

$$\int e^{a+bx}\operatorname{csch}(a+bx)\,dx=\frac{\log\left(e^{bx+a}-1\right)+\log\left(e^{bx+a}+1\right)}{b}$$

input
```
int(exp(b*x+a)*csch(b*x+a),x)
```

output
```
(log(e**(a + b*x) - 1) + log(e**(a + b*x) + 1))/b
```

3.306 $\int e^{a+bx}\text{csch}^2(a+bx)\,dx$

Optimal result

Integrand size = 16, antiderivative size = 42

$$\int e^{a+bx}\text{csch}^2(a+bx)\,dx=\frac{2e^{a+bx}}{b\left(1-e^{2a+2bx}\right)}-\frac{2\text{arctanh}\left(e^{a+bx}\right)}{b}$$

output
```
2*exp(b*x+a)/b/(1-exp(2*b*x+2*a))-2*arctanh(exp(b*x+a))/b
```

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.88

$$\int e^{a+bx}\text{csch}^2(a+bx)\,dx=\frac{-\frac{2e^{a+bx}}{-1+e^{2(a+bx)}}-2\text{arctanh}\left(e^{a+bx}\right)}{b}$$

input
```
Integrate[E^(a + b*x)*Csch[a + b*x]^2,x]
```

output
```
((-2*E^(a + b*x))/(-1 + E^(2*(a + b*x))) - 2*ArcTanh[E^(a + b*x)])/b
```

Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 45, normalized size of antiderivative = 1.07, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {2720, 27, 252, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int e^{a+bx}\operatorname{csch}^2(a+bx)\,dx$$

$$\Big\downarrow \text{2720}$$

$$\frac{\int \frac{4e^{2a+2bx}}{\left(1-e^{2a+2bx}\right)^2}de^{a+bx}}{b}$$

$$\Big\downarrow \text{27}$$

$$\frac{4\int \frac{e^{2a+2bx}}{\left(1-e^{2a+2bx}\right)^2}de^{a+bx}}{b}$$

$$\Big\downarrow \text{252}$$

$$\frac{4\left(\frac{e^{a+bx}}{2\left(1-e^{2a+2bx}\right)}-\frac{1}{2}\int \frac{1}{1-e^{2a+2bx}}de^{a+bx}\right)}{b}$$

$$\Big\downarrow \text{219}$$

$$\frac{4\left(\frac{e^{a+bx}}{2\left(1-e^{2a+2bx}\right)}-\frac{1}{2}\text{arctanh}\left(e^{a+bx}\right)\right)}{b}$$

input
```
Int[E^(a + b*x)*Csch[a + b*x]^2,x]
```

output
```
(4*(E^(a + b*x)/(2*(1 - E^(2*a + 2*b*x))) - ArcTanh[E^(a + b*x)]/2))/b
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 252
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[c*(c*x
)^(m - 1)*((a + b*x^2)^(p + 1)/(2*b*(p + 1))), x] - Simp[c^2*((m - 1)/(2*b*
(p + 1)))   Int[(c*x)^(m - 2)*(a + b*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c
}, x] && LtQ[p, -1] && GtQ[m, 1] &&  !ILtQ[(m + 2*p + 3)/2, 0] && IntBinomi
alQ[a, b, c, 2, m, p, x]
```

rule 2720
```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Maple [A] (verified)

Time = 0.18 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.60

method	result	size
derivativedivides	$\frac{-2\,\operatorname{arctanh}\left({\mathrm e}^{b x +a}\right)-\frac{1}{\sinh\left(b x +a \right)}}{b}$	25
default	$\frac{-2\,\operatorname{arctanh}\left({\mathrm e}^{b x +a}\right)-\frac{1}{\sinh\left(b x +a \right)}}{b}$	25
risch	$-\frac{2\,{\mathrm e}^{b x +a}}{b\left({\mathrm e}^{2 b x +2 a}-1\right)}-\frac{\ln\left({\mathrm e}^{b x +a}+1\right)}{b}+\frac{\ln\left({\mathrm e}^{b x +a}-1\right)}{b}$	53

input
```
int(exp(b*x+a)*csch(b*x+a)^2,x,method=_RETURNVERBOSE)
```

output
```
1/b*(-2*arctanh(exp(b*x+a))-1/sinh(b*x+a))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 157 vs. $2(37) = 74$.

Time = 0.08 (sec) , antiderivative size = 157, normalized size of antiderivative = 3.74

$$\int e^{a+bx}\text{csch}^2(a+bx)\,dx = -\frac{\left(\cosh\left(bx+a\right)^2 + 2\,\cosh\left(bx+a\right)\sinh\left(bx+a\right) + \sinh\left(bx+a\right)^2 - 1\right)\log\left(\cosh\left(bx+a\right) + \sinh\left(bx+a\right.}{b\cosh\left(bx\right.}$$

input
```
integrate(exp(b*x+a)*csch(b*x+a)^2,x, algorithm="fricas")
```

output
```
-((cosh(b*x + a)^2 + 2*cosh(b*x + a)*sinh(b*x + a) + sinh(b*x + a)^2 - 1)*
log(cosh(b*x + a) + sinh(b*x + a) + 1) - (cosh(b*x + a)^2 + 2*cosh(b*x + a
)*sinh(b*x + a) + sinh(b*x + a)^2 - 1)*log(cosh(b*x + a) + sinh(b*x + a) -
 1) + 2*cosh(b*x + a) + 2*sinh(b*x + a))/(b*cosh(b*x + a)^2 + 2*b*cosh(b*x
 + a)*sinh(b*x + a) + b*sinh(b*x + a)^2 - b)
```

Sympy [F]

$$\int e^{a+bx}\text{csch}^2(a+bx)\,dx = e^a \int e^{bx}\,\text{csch}^2\left(a+bx\right)dx$$

input
```
integrate(exp(b*x+a)*csch(b*x+a)**2,x)
```

output
```
exp(a)*Integral(exp(b*x)*csch(a + b*x)**2, x)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 52, normalized size of antiderivative = 1.24

$$\int e^{a+bx}\operatorname{csch}^2(a+bx)\,dx = -\frac{\log\left(e^{(bx+a)}+1\right)}{b} + \frac{\log\left(e^{(bx+a)}-1\right)}{b} - \frac{2\,e^{(bx+a)}}{b(e^{(2\,bx+2\,a)}-1)}$$

input
```
integrate(exp(b*x+a)*csch(b*x+a)^2,x, algorithm="maxima")
```

output
```
-log(e^(b*x + a) + 1)/b + log(e^(b*x + a) - 1)/b - 2*e^(b*x + a)/(b*(e^(2*
b*x + 2*a) - 1))
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 48, normalized size of antiderivative = 1.14

$$\int e^{a+bx}\operatorname{csch}^2(a+bx)\,dx = -\frac{\frac{2\,e^{(bx+a)}}{e^{(2\,bx+2\,a)}-1} + \log\left(e^{(bx+a)}+1\right) - \log\left(\left|e^{(bx+a)}-1\right|\right)}{b}$$

input
```
integrate(exp(b*x+a)*csch(b*x+a)^2,x, algorithm="giac")
```

output
```
-(2*e^(b*x + a)/(e^(2*b*x + 2*a) - 1) + log(e^(b*x + a) + 1) - log(abs(e^(
b*x + a) - 1)))/b
```

Mupad [B] (verification not implemented)

Time = 1.61 (sec) , antiderivative size = 52, normalized size of antiderivative = 1.24

$$\int e^{a+bx}\operatorname{csch}^2(a+bx)\,dx = -\frac{2\,\mathrm{atan}\left(\frac{\mathrm{e}^{b\,x}\,\mathrm{e}^{a}\,\sqrt{-b^2}}{b}\right)}{\sqrt{-b^2}} - \frac{2\,\mathrm{e}^{a+b\,x}}{b\,\left(\mathrm{e}^{2\,a+2\,b\,x}-1\right)}$$

input
```
int(exp(a + b*x)/sinh(a + b*x)^2,x)
```

output
```
- (2*atan((exp(b*x)*exp(a)*(-b^2)^(1/2))/b))/(-b^2)^(1/2) - (2*exp(a + b*x
))/(b*(exp(2*a + 2*b*x) - 1))
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 93, normalized size of antiderivative = 2.21

$$\int e^{a+bx}\text{csch}^2(a+bx)\,dx$$
$$= \frac{e^{2bx+2a}\log\left(e^{bx+a}-1\right)-e^{2bx+2a}\log\left(e^{bx+a}+1\right)-2e^{bx+a}-\log\left(e^{bx+a}-1\right)+\log\left(e^{bx+a}+1\right)}{b\left(e^{2bx+2a}-1\right)}$$

input
```
int(exp(b*x+a)*csch(b*x+a)^2,x)
```

output
```
(e**(2*a + 2*b*x)*log(e**(a + b*x) - 1) - e**(2*a + 2*b*x)*log(e**(a + b*x
) + 1) - 2*e**(a + b*x) - log(e**(a + b*x) - 1) + log(e**(a + b*x) + 1))/(
b*(e**(2*a + 2*b*x) - 1))
```

3.307 $\int e^{a+bx}\mathbf{csch}^3(a+bx)\,dx$

Optimal result

Integrand size = 16, antiderivative size = 31

$$\int e^{a+bx}\operatorname{csch}^3(a+bx)\,dx=-\frac{2e^{4a+4bx}}{b\left(1-e^{2a+2bx}\right)^2}$$

output
```
-2*exp(4*b*x+4*a)/b/(1-exp(2*b*x+2*a))^2
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.94

$$\int e^{a+bx}\operatorname{csch}^3(a+bx)\,dx=-\frac{2e^{4a+4bx}}{b\left(-1+e^{2a+2bx}\right)^2}$$

input
```
Integrate[E^(a + b*x)*Csch[a + b*x]^3,x]
```

output
```
(-2*E^(4*a + 4*b*x))/(b*(-1 + E^(2*a + 2*b*x))^2)
```

Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.188, Rules used = {2720, 27, 242}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int e^{a+bx}\text{csch}^3(a+bx)\,dx$$

$$\Big\downarrow \text{2720}$$

$$\frac{\int -\frac{8e^{3a+3bx}}{(1-e^{2a+2bx})^3}de^{a+bx}}{b}$$

$$\Big\downarrow \text{27}$$

$$-\frac{8\int \frac{e^{3a+3bx}}{(1-e^{2a+2bx})^3}de^{a+bx}}{b}$$

$$\Big\downarrow \text{242}$$

$$-\frac{2e^{4a+4bx}}{b\left(1-e^{2a+2bx}\right)^2}$$

input
```
Int[E^(a + b*x)*Csch[a + b*x]^3,x]
```

output
```
(-2*E^(4*a + 4*b*x))/(b*(1 - E^(2*a + 2*b*x))^2)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 242
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(c*x)^
(m + 1)*((a + b*x^2)^(p + 1)/(a*c*(m + 1))), x] /; FreeQ[{a, b, c, m, p}, x
] && EqQ[m + 2*p + 3, 0] && NeQ[m, -1]
```

rule 2720
```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Maple [A] (verified)

Time = 1.28 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.77

method	result	size
derivativedivides	$\frac{-\coth(bx+a)-\frac{1}{2\sinh(bx+a)^2}}{b}$	24
default	$\frac{-\coth(bx+a)-\frac{1}{2\sinh(bx+a)^2}}{b}$	24
risch	$-\frac{2(2\,e^{2bx+2a}-1)}{b(e^{2bx+2a}-1)^2}$	32
parallelrisch	$-\frac{e^{bx+a}\left(\coth\left(\frac{bx}{2}+\frac{a}{2}\right)+2+\tanh\left(\frac{bx}{2}+\frac{a}{2}\right)\right)\left(\coth\left(\frac{bx}{2}+\frac{a}{2}\right)-\tanh\left(\frac{bx}{2}+\frac{a}{2}\right)\right)}{8b}$	53

input
```
int(exp(b*x+a)*csch(b*x+a)^3,x,method=_RETURNVERBOSE)
```

output
```
1/b*(-coth(b*x+a)-1/2/sinh(b*x+a)^2)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 88 vs. $2(27) = 54$.

Time = 0.08 (sec) , antiderivative size = 88, normalized size of antiderivative = 2.84

$$\int e^{a+bx}\text{csch}^3(a+bx)\,dx =$$

$$-\frac{2\left(\cosh\left(bx+a\right)+3\,\sinh\left(bx+a\right)\right)}{b\cosh\left(bx+a\right)^3+3\,b\cosh\left(bx+a\right)\sinh\left(bx+a\right)^2+b\sinh\left(bx+a\right)^3-b\cosh\left(bx+a\right)+3\left(b\cosh\left(bx-\right.\right.}$$

input
```
integrate(exp(b*x+a)*csch(b*x+a)^3,x, algorithm="fricas")
```

output
```
-2*(cosh(b*x + a) + 3*sinh(b*x + a))/(b*cosh(b*x + a)^3 + 3*b*cosh(b*x + a
)*sinh(b*x + a)^2 + b*sinh(b*x + a)^3 - b*cosh(b*x + a) + 3*(b*cosh(b*x +
a)^2 - b)*sinh(b*x + a))
```

Sympy [F]

$$\int e^{a+bx}\operatorname{csch}^3(a+bx)\,dx = e^a \int e^{bx} \operatorname{csch}^3(a+bx)\,dx$$

input
```
integrate(exp(b*x+a)*csch(b*x+a)**3,x)
```

output
```
exp(a)*Integral(exp(b*x)*csch(a + b*x)**3, x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 68 vs. $2(27) = 54$.

Time = 0.04 (sec) , antiderivative size = 68, normalized size of antiderivative = 2.19

$$\int e^{a+bx}\operatorname{csch}^3(a+bx)\,dx = -\frac{4\,e^{(2\,bx+2\,a)}}{b(e^{(4\,bx+4\,a)}-2\,e^{(2\,bx+2\,a)}+1)}+\frac{2}{b(e^{(4\,bx+4\,a)}-2\,e^{(2\,bx+2\,a)}+1)}$$

input
```
integrate(exp(b*x+a)*csch(b*x+a)^3,x, algorithm="maxima")
```

output
```
-4*e^(2*b*x + 2*a)/(b*(e^(4*b*x + 4*a) - 2*e^(2*b*x + 2*a) + 1)) + 2/(b*(e
^(4*b*x + 4*a) - 2*e^(2*b*x + 2*a) + 1))
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.00

$$\int e^{a+bx}\operatorname{csch}^3(a+bx)\,dx = -\frac{2\left(2\,e^{(2\,bx+2\,a)}-1\right)}{b\left(e^{(2\,bx+2\,a)}-1\right)^2}$$

input
```
integrate(exp(b*x+a)*csch(b*x+a)^3,x, algorithm="giac")
```

output
```
-2*(2*e^(2*b*x + 2*a) - 1)/(b*(e^(2*b*x + 2*a) - 1)^2)
```

Mupad [B] (verification not implemented)

Time = 1.58 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.00

$$\int e^{a+bx}\operatorname{csch}^3(a+bx)\,dx = -\frac{2\left(2\,\mathrm{e}^{2\,a+2\,b\,x}-1\right)}{b\left(\mathrm{e}^{2\,a+2\,b\,x}-1\right)^2}$$

input
```
int(exp(a + b*x)/sinh(a + b*x)^3,x)
```

output
```
-(2*(2*exp(2*a + 2*b*x) - 1))/(b*(exp(2*a + 2*b*x) - 1)^2)
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 41, normalized size of antiderivative = 1.32

$$\int e^{a+bx}\operatorname{csch}^3(a+bx)\,dx = -\frac{2e^{4bx+4a}}{b\left(e^{4bx+4a}-2e^{2bx+2a}+1\right)}$$

input
```
int(exp(b*x+a)*csch(b*x+a)^3,x)
```

output
```
( - 2*e**(4*a + 4*b*x))/(b*(e**(4*a + 4*b*x) - 2*e**(2*a + 2*b*x) + 1))
```

3.308 $\int e^{a+bx}\operatorname{csch}^4(a+bx)\,dx$

Optimal result

Integrand size = 16, antiderivative size = 101

$$\int e^{a+bx}\operatorname{csch}^4(a+bx)\,dx=\frac{8e^{3a+3bx}}{3b\left(1-e^{2a+2bx}\right)^3}-\frac{2e^{a+bx}}{b\left(1-e^{2a+2bx}\right)^2}+\frac{e^{a+bx}}{b\left(1-e^{2a+2bx}\right)}+\frac{\operatorname{arctanh}\left(e^{a+bx}\right)}{b}$$

output
```
8/3*exp(3*b*x+3*a)/b/(1-exp(2*b*x+2*a))^3-2*exp(b*x+a)/b/(1-exp(2*b*x+2*a)
)^2+exp(b*x+a)/b/(1-exp(2*b*x+2*a))+arctanh(exp(b*x+a))/b
```

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 75, normalized size of antiderivative = 0.74

$$\int e^{a+bx}\operatorname{csch}^4(a+bx)\,dx=\frac{3e^{a+bx}-8e^{3(a+bx)}-3e^{5(a+bx)}+3\left(-1+e^{2(a+bx)}\right)^3\operatorname{arctanh}\left(e^{a+bx}\right)}{3b\left(-1+e^{2(a+bx)}\right)^3}$$

input
```
Integrate[E^(a + b*x)*Csch[a + b*x]^4,x]
```

output
```
(3*E^(a + b*x) - 8*E^(3*(a + b*x)) - 3*E^(5*(a + b*x)) + 3*(-1 + E^(2*(a +
 b*x)))^3*ArcTanh[E^(a + b*x)])/(3*b*(-1 + E^(2*(a + b*x)))^3)
```

Rubi [A] (verified)

Time = 0.37 (sec) , antiderivative size = 112, normalized size of antiderivative = 1.11, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.375$, Rules used = {2720, 27, 252, 252, 215, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int e^{a+bx}\operatorname{csch}^4(a+bx)\,dx$$

$$\Big\downarrow \text{2720}$$

$$\frac{\int \frac{16e^{4a+4bx}}{(1-e^{2a+2bx})^4}de^{a+bx}}{b}$$

$$\Big\downarrow \text{27}$$

$$\frac{16\int \frac{e^{4a+4bx}}{(1-e^{2a+2bx})^4}de^{a+bx}}{b}$$

$$\Big\downarrow \text{252}$$

$$\frac{16\left(\frac{e^{3a+3bx}}{6(1-e^{2a+2bx})^3}-\frac{1}{2}\int \frac{e^{2a+2bx}}{(1-e^{2a+2bx})^3}de^{a+bx}\right)}{b}$$

$$\Big\downarrow \text{252}$$

$$\frac{16\left(\frac{1}{2}\left(\frac{1}{4}\int \frac{1}{(1-e^{2a+2bx})^2}de^{a+bx}-\frac{e^{a+bx}}{4(1-e^{2a+2bx})^2}\right)+\frac{e^{3a+3bx}}{6(1-e^{2a+2bx})^3}\right)}{b}$$

$$\Big\downarrow \text{215}$$

$$\frac{16\left(\frac{1}{2}\left(\frac{1}{4}\left(\frac{1}{2}\int \frac{1}{1-e^{2a+2bx}}de^{a+bx}+\frac{e^{a+bx}}{2(1-e^{2a+2bx})}\right)-\frac{e^{a+bx}}{4(1-e^{2a+2bx})^2}\right)+\frac{e^{3a+3bx}}{6(1-e^{2a+2bx})^3}\right)}{b}$$

$$\Big\downarrow \text{219}$$

$$\frac{16\left(\frac{1}{2}\left(\frac{1}{4}\left(\frac{1}{2}\text{arctanh}\left(e^{a+bx}\right)+\frac{e^{a+bx}}{2\left(1-e^{2a+2bx}\right)}\right)-\frac{e^{a+bx}}{4\left(1-e^{2a+2bx}\right)^2}\right)+\frac{e^{3a+3bx}}{6\left(1-e^{2a+2bx}\right)^3}\right)}{b}$$

input
```
Int[E^(a + b*x)*Csch[a + b*x]^4,x]
```

output
```
(16*(E^(3*a + 3*b*x)/(6*(1 - E^(2*a + 2*b*x))^3) + (-1/4*E^(a + b*x)/(1 -
E^(2*a + 2*b*x))^2 + (E^(a + b*x)/(2*(1 - E^(2*a + 2*b*x))) + ArcTanh[E^(a
 + b*x)]/2)/4)/2))/b
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 215
```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(-x)*((a + b*x^2)^(p + 1)
/(2*a*(p + 1))), x] + Simp[(2*p + 3)/(2*a*(p + 1))   Int[(a + b*x^2)^(p + 1
), x], x] /; FreeQ[{a, b}, x] && LtQ[p, -1] && (IntegerQ[4*p] || IntegerQ[6
*p])
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 252
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[c*(c*x
)^(m - 1)*((a + b*x^2)^(p + 1)/(2*b*(p + 1))), x] - Simp[c^2*((m - 1)/(2*b*
(p + 1)))   Int[(c*x)^(m - 2)*(a + b*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c
}, x] && LtQ[p, -1] && GtQ[m, 1] &&  !ILtQ[(m + 2*p + 3)/2, 0] && IntBinomi
alQ[a, b, c, 2, m, p, x]
```

rule 2720
```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Maple [A] (verified)

Time = 1.75 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.37

method	result	size
derivativedivides	$\frac{-\frac{\coth(bx+a)\operatorname{csch}(bx+a)}{2}+\operatorname{arctanh}\left(e^{bx+a}\right)-\frac{1}{3\sinh(bx+a)^3}}{b}$	37
default	$\frac{-\frac{\coth(bx+a)\operatorname{csch}(bx+a)}{2}+\operatorname{arctanh}\left(e^{bx+a}\right)-\frac{1}{3\sinh(bx+a)^3}}{b}$	37
risch	$-\frac{e^{bx+a}\left(3\,e^{4bx+4a}+8\,e^{2bx+2a}-3\right)}{3b\left(e^{2bx+2a}-1\right)^3}+\frac{\ln\left(e^{bx+a}+1\right)}{2b}-\frac{\ln\left(e^{bx+a}-1\right)}{2b}$	78

input
```
int(exp(b*x+a)*csch(b*x+a)^4,x,method=_RETURNVERBOSE)
```

output
```
1/b*(-1/2*coth(b*x+a)*csch(b*x+a)+arctanh(exp(b*x+a))-1/3/sinh(b*x+a)^3)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 705 vs. $2(87) = 174$.

Time = 0.09 (sec) , antiderivative size = 705, normalized size of antiderivative = 6.98

$$\int e^{a+bx}\operatorname{csch}^4(a+bx)\,dx = \text{Too large to display}$$

input
```
integrate(exp(b*x+a)*csch(b*x+a)^4,x, algorithm="fricas")
```

output
```
-1/6*(6*cosh(b*x + a)^5 + 30*cosh(b*x + a)*sinh(b*x + a)^4 + 6*sinh(b*x +
a)^5 + 4*(15*cosh(b*x + a)^2 + 4)*sinh(b*x + a)^3 + 16*cosh(b*x + a)^3 + 1
2*(5*cosh(b*x + a)^3 + 4*cosh(b*x + a))*sinh(b*x + a)^2 - 3*(cosh(b*x + a)
^6 + 6*cosh(b*x + a)*sinh(b*x + a)^5 + sinh(b*x + a)^6 + 3*(5*cosh(b*x + a
)^2 - 1)*sinh(b*x + a)^4 - 3*cosh(b*x + a)^4 + 4*(5*cosh(b*x + a)^3 - 3*co
sh(b*x + a))*sinh(b*x + a)^3 + 3*(5*cosh(b*x + a)^4 - 6*cosh(b*x + a)^2 +
1)*sinh(b*x + a)^2 + 3*cosh(b*x + a)^2 + 6*(cosh(b*x + a)^5 - 2*cosh(b*x +
 a)^3 + cosh(b*x + a))*sinh(b*x + a) - 1)*log(cosh(b*x + a) + sinh(b*x + a
) + 1) + 3*(cosh(b*x + a)^6 + 6*cosh(b*x + a)*sinh(b*x + a)^5 + sinh(b*x +
 a)^6 + 3*(5*cosh(b*x + a)^2 - 1)*sinh(b*x + a)^4 - 3*cosh(b*x + a)^4 + 4*
(5*cosh(b*x + a)^3 - 3*cosh(b*x + a))*sinh(b*x + a)^3 + 3*(5*cosh(b*x + a)
^4 - 6*cosh(b*x + a)^2 + 1)*sinh(b*x + a)^2 + 3*cosh(b*x + a)^2 + 6*(cosh(
b*x + a)^5 - 2*cosh(b*x + a)^3 + cosh(b*x + a))*sinh(b*x + a) - 1)*log(cos
h(b*x + a) + sinh(b*x + a) - 1) + 6*(5*cosh(b*x + a)^4 + 8*cosh(b*x + a)^2
 - 1)*sinh(b*x + a) - 6*cosh(b*x + a))/(b*cosh(b*x + a)^6 + 6*b*cosh(b*x +
 a)*sinh(b*x + a)^5 + b*sinh(b*x + a)^6 - 3*b*cosh(b*x + a)^4 + 3*(5*b*cos
h(b*x + a)^2 - b)*sinh(b*x + a)^4 + 4*(5*b*cosh(b*x + a)^3 - 3*b*cosh(b*x
+ a))*sinh(b*x + a)^3 + 3*b*cosh(b*x + a)^2 + 3*(5*b*cosh(b*x + a)^4 - 6*b
*cosh(b*x + a)^2 + b)*sinh(b*x + a)^2 + 6*(b*cosh(b*x + a)^5 - 2*b*cosh(b*
x + a)^3 + b*cosh(b*x + a))*sinh(b*x + a) - b)
```

Sympy [F]

$$\int e^{a+bx}\operatorname{csch}^4(a+bx)\,dx = e^a\int e^{bx}\operatorname{csch}^4(a+bx)\,dx$$

input
```
integrate(exp(b*x+a)*csch(b*x+a)**4,x)
```

output
```
exp(a)*Integral(exp(b*x)*csch(a + b*x)**4, x)
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 100, normalized size of antiderivative = 0.99

$$\int e^{a+bx}\text{csch}^4(a+bx)\,dx = \frac{\log\left(e^{(bx+a)}+1\right)}{2\,b} - \frac{\log\left(e^{(bx+a)}-1\right)}{2\,b} - \frac{3\,e^{(5\,bx+5\,a)}+8\,e^{(3\,bx+3\,a)}-3\,e^{(bx+a)}}{3\,b\left(e^{(6\,bx+6\,a)}-3\,e^{(4\,bx+4\,a)}+3\,e^{(2\,bx+2\,a)}-1\right)}$$

input
```
integrate(exp(b*x+a)*csch(b*x+a)^4,x, algorithm="maxima")
```

output
```
1/2*log(e^(b*x + a) + 1)/b - 1/2*log(e^(b*x + a) - 1)/b - 1/3*(3*e^(5*b*x
+ 5*a) + 8*e^(3*b*x + 3*a) - 3*e^(b*x + a))/(b*(e^(6*b*x + 6*a) - 3*e^(4*b
*x + 4*a) + 3*e^(2*b*x + 2*a) - 1))
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 75, normalized size of antiderivative = 0.74

$$\int e^{a+bx}\text{csch}^4(a+bx)\,dx = -\frac{\frac{2\left(3\,e^{(5\,bx+5\,a)}+8\,e^{(3\,bx+3\,a)}-3\,e^{(bx+a)}\right)}{\left(e^{(2\,bx+2\,a)}-1\right)^3} - 3\,\log\left(e^{(bx+a)}+1\right) + 3\,\log\left(\left|e^{(bx+a)}-1\right|\right)}{6\,b}$$

input
```
integrate(exp(b*x+a)*csch(b*x+a)^4,x, algorithm="giac")
```

output
```
-1/6*(2*(3*e^(5*b*x + 5*a) + 8*e^(3*b*x + 3*a) - 3*e^(b*x + a))/(e^(2*b*x
+ 2*a) - 1)^3 - 3*log(e^(b*x + a) + 1) + 3*log(abs(e^(b*x + a) - 1)))/b
```

Mupad [B] (verification not implemented)

Time = 1.60 (sec) , antiderivative size = 135, normalized size of antiderivative = 1.34

$$\int e^{a+bx}\operatorname{csch}^4(a+bx)\,dx = \frac{\operatorname{atan}\left(\frac{e^{bx}\,e^{a}\,\sqrt{-b^2}}{b}\right)}{\sqrt{-b^2}} - \frac{2\,e^{a+b\,x}}{b\,\left(e^{4\,a+4\,b\,x}-2\,e^{2\,a+2\,b\,x}+1\right)} - \frac{8\,e^{3\,a+3\,b\,x}}{3\,b\,\left(3\,e^{2\,a+2\,b\,x}-3\,e^{4\,a+4\,b\,x}+e^{6\,a+6\,b\,x}-1\right)} - \frac{e^{a+b\,x}}{b\,\left(e^{2\,a+2\,b\,x}-1\right)}$$

input
```
int(exp(a + b*x)/sinh(a + b*x)^4,x)
```

output
```
atan((exp(b*x)*exp(a)*(-b^2)^(1/2))/b)/(-b^2)^(1/2) - (2*exp(a + b*x))/(b*
(exp(4*a + 4*b*x) - 2*exp(2*a + 2*b*x) + 1)) - (8*exp(3*a + 3*b*x))/(3*b*(
3*exp(2*a + 2*b*x) - 3*exp(4*a + 4*b*x) + exp(6*a + 6*b*x) - 1)) - exp(a +
 b*x)/(b*(exp(2*a + 2*b*x) - 1))
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 233, normalized size of antiderivative = 2.31

$$\int e^{a+bx}\operatorname{csch}^4(a+bx)\,dx = \frac{-3e^{6bx+6a}\log\left(e^{bx+a}-1\right)+3e^{6bx+6a}\log\left(e^{bx+a}+1\right)-6e^{5bx+5a}+9e^{4bx+4a}\log\left(e^{bx+a}-1\right)-9e^{4bx+4a}\log\left(e^{bx\cdots}\right.}{6b\left(e^{6bx+6a}\cdots\right.}$$

input
```
int(exp(b*x+a)*csch(b*x+a)^4,x)
```

output
```
( - 3*e**(6*a + 6*b*x)*log(e**(a + b*x) - 1) + 3*e**(6*a + 6*b*x)*log(e**(
a + b*x) + 1) - 6*e**(5*a + 5*b*x) + 9*e**(4*a + 4*b*x)*log(e**(a + b*x) -
 1) - 9*e**(4*a + 4*b*x)*log(e**(a + b*x) + 1) - 16*e**(3*a + 3*b*x) - 9*e
**(2*a + 2*b*x)*log(e**(a + b*x) - 1) + 9*e**(2*a + 2*b*x)*log(e**(a + b*x
) + 1) + 6*e**(a + b*x) + 3*log(e**(a + b*x) - 1) - 3*log(e**(a + b*x) + 1
))/(6*b*(e**(6*a + 6*b*x) - 3*e**(4*a + 4*b*x) + 3*e**(2*a + 2*b*x) - 1))
```

3.309 $\int e^{a+bx}\operatorname{csch}^5(a+bx)\,dx$

Optimal result

Integrand size = 16, antiderivative size = 66

$$\int e^{a+bx}\operatorname{csch}^5(a+bx)\,dx=-\frac{4}{b\left(1-e^{2a+2bx}\right)^4}+\frac{32}{3b\left(1-e^{2a+2bx}\right)^3}-\frac{8}{b\left(1-e^{2a+2bx}\right)^2}$$

output
```
-4/b/(1-exp(2*b*x+2*a))^4+32/3/b/(1-exp(2*b*x+2*a))^3-8/b/(1-exp(2*b*x+2*a
))^2
```

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.67

$$\int e^{a+bx}\operatorname{csch}^5(a+bx)\,dx=-\frac{4\left(1-4e^{2(a+bx)}+6e^{4(a+bx)}\right)}{3b\left(-1+e^{2(a+bx)}\right)^4}$$

input
```
Integrate[E^(a + b*x)*Csch[a + b*x]^5,x]
```

output
```
(-4*(1 - 4*E^(2*(a + b*x)) + 6*E^(4*(a + b*x))))/(3*b*(-1 + E^(2*(a + b*x)
))^4)
```

Rubi [A] (verified)

Time = 0.38 (sec) , antiderivative size = 66, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.312$, Rules used = {2720, 27, 243, 53, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int e^{a+bx}\text{csch}^5(a+bx)\,dx$$

$$\Big\downarrow \text{2720}$$

$$\frac{\int -\frac{32e^{5a+5bx}}{\left(1-e^{2a+2bx}\right)^5}de^{a+bx}}{b}$$

$$\Big\downarrow \text{27}$$

$$-\frac{32\int \frac{e^{5a+5bx}}{\left(1-e^{2a+2bx}\right)^5}de^{a+bx}}{b}$$

$$\Big\downarrow \text{243}$$

$$-\frac{16\int \frac{e^{2a+2bx}}{\left(1-e^{2a+2bx}\right)^5}de^{2a+2bx}}{b}$$

$$\Big\downarrow \text{53}$$

$$-\frac{16\int \left(-\frac{1}{\left(-1+e^{2a+2bx}\right)^3}-\frac{2}{\left(-1+e^{2a+2bx}\right)^4}-\frac{1}{\left(-1+e^{2a+2bx}\right)^5}\right)de^{2a+2bx}}{b}$$

$$\Big\downarrow \text{2009}$$

$$-\frac{16\left(\frac{1}{2\left(1-e^{2a+2bx}\right)^2}-\frac{2}{3\left(1-e^{2a+2bx}\right)^3}+\frac{1}{4\left(1-e^{2a+2bx}\right)^4}\right)}{b}$$

input
```
Int[E^(a + b*x)*Csch[a + b*x]^5,x]
```

output
```
(-16*(1/(4*(1 - E^(2*a + 2*b*x))^4) - 2/(3*(1 - E^(2*a + 2*b*x))^3) + 1/(2
*(1 - E^(2*a + 2*b*x))^2)))/b
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 53
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && LeQ[7*m + 4*n + 4, 0])
|| LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2720
```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Maple [A] (verified)

Time = 2.34 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.53

method	result
derivativedivides	$\frac{\left(\frac{2}{3}-\frac{\operatorname{csch}(bx+a)^2}{3}\right)\coth(bx+a)-\frac{1}{4\sinh(bx+a)^4}}{b}$
default	$\frac{\left(\frac{2}{3}-\frac{\operatorname{csch}(bx+a)^2}{3}\right)\coth(bx+a)-\frac{1}{4\sinh(bx+a)^4}}{b}$
risch	$-\frac{4\left(6\,e^{4bx+4a}-4\,e^{2bx+2a}+1\right)}{3b\left(e^{2bx+2a}-1\right)^4}$
parallelrisch	$\frac{e^{bx+a}\left(3\tanh\left(\frac{bx}{2}+\frac{a}{2}\right)^4-3\coth\left(\frac{bx}{2}+\frac{a}{2}\right)^4+2\tanh\left(\frac{bx}{2}+\frac{a}{2}\right)^3-2\coth\left(\frac{bx}{2}+\frac{a}{2}\right)^3-22\tanh\left(\frac{bx}{2}+\frac{a}{2}\right)^2+22\coth\left(\frac{bx}{2}+\frac{a}{2}\right)\right.}{192b}$

input
```
int(exp(b*x+a)*csch(b*x+a)^5,x,method=_RETURNVERBOSE)
```

output
```
1/b*((2/3-1/3*csch(b*x+a)^2)*coth(b*x+a)-1/4/sinh(b*x+a)^4)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 233 vs. $2(55) = 110$.

Time = 0.08 (sec) , antiderivative size = 233, normalized size of antiderivative = 3.53

$$\int e^{a+bx}\operatorname{csch}^5(a+bx)\,dx =$$

$$-\frac{}{3\left(b\cosh\left(bx+a\right)^{6}+6\,b\cosh\left(bx+a\right)\sinh\left(bx+a\right)^{5}+b\sinh\left(bx+a\right)^{6}-4\,b\cosh\left(bx+a\right)^{4}+\left(15\,b\cos\right.\right.}$$

input
```
integrate(exp(b*x+a)*csch(b*x+a)^5,x, algorithm="fricas")
```

output
```
-4/3*(7*cosh(b*x + a)^2 + 10*cosh(b*x + a)*sinh(b*x + a) + 7*sinh(b*x + a)
^2 - 4)/(b*cosh(b*x + a)^6 + 6*b*cosh(b*x + a)*sinh(b*x + a)^5 + b*sinh(b*
x + a)^6 - 4*b*cosh(b*x + a)^4 + (15*b*cosh(b*x + a)^2 - 4*b)*sinh(b*x + a
)^4 + 4*(5*b*cosh(b*x + a)^3 - 4*b*cosh(b*x + a))*sinh(b*x + a)^3 + 7*b*co
sh(b*x + a)^2 + (15*b*cosh(b*x + a)^4 - 24*b*cosh(b*x + a)^2 + 7*b)*sinh(b
*x + a)^2 + 2*(3*b*cosh(b*x + a)^5 - 8*b*cosh(b*x + a)^3 + 5*b*cosh(b*x +
a))*sinh(b*x + a) - 4*b)
```

Sympy [F]

$$\int e^{a+bx}\operatorname{csch}^5(a+bx)\,dx = e^{a}\int e^{bx}\operatorname{csch}^{5}\left(a+bx\right)dx$$

input
```
integrate(exp(b*x+a)*csch(b*x+a)**5,x)
```

output
```
exp(a)*Integral(exp(b*x)*csch(a + b*x)**5, x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 172 vs. $2(55) = 110$.

Time = 0.04 (sec) , antiderivative size = 172, normalized size of antiderivative = 2.61

$$\int e^{a+bx}\operatorname{csch}^5(a+bx)\,dx = -\frac{8\,e^{(4\,bx+4\,a)}}{b\left(e^{(8\,bx+8\,a)}-4\,e^{(6\,bx+6\,a)}+6\,e^{(4\,bx+4\,a)}-4\,e^{(2\,bx+2\,a)}+1\right)} + \frac{16\,e^{(2\,bx+2\,a)}}{3\,b\left(e^{(8\,bx+8\,a)}-4\,e^{(6\,bx+6\,a)}+6\,e^{(4\,bx+4\,a)}-4\,e^{(2\,bx+2\,a)}+1\right)} - \frac{4}{3\,b\left(e^{(8\,bx+8\,a)}-4\,e^{(6\,bx+6\,a)}+6\,e^{(4\,bx+4\,a)}-4\,e^{(2\,bx+2\,a)}+1\right)}$$

input
```
integrate(exp(b*x+a)*csch(b*x+a)^5,x, algorithm="maxima")
```

output
```
-8*e^(4*b*x + 4*a)/(b*(e^(8*b*x + 8*a) - 4*e^(6*b*x + 6*a) + 6*e^(4*b*x +
4*a) - 4*e^(2*b*x + 2*a) + 1)) + 16/3*e^(2*b*x + 2*a)/(b*(e^(8*b*x + 8*a)
- 4*e^(6*b*x + 6*a) + 6*e^(4*b*x + 4*a) - 4*e^(2*b*x + 2*a) + 1)) - 4/3/(b
*(e^(8*b*x + 8*a) - 4*e^(6*b*x + 6*a) + 6*e^(4*b*x + 4*a) - 4*e^(2*b*x + 2
*a) + 1))
```

Giac [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.64

$$\int e^{a+bx}\operatorname{csch}^5(a+bx)\,dx = -\frac{4\left(6\,e^{(4\,bx+4\,a)}-4\,e^{(2\,bx+2\,a)}+1\right)}{3\,b\left(e^{(2\,bx+2\,a)}-1\right)^4}$$

input
```
integrate(exp(b*x+a)*csch(b*x+a)^5,x, algorithm="giac")
```

output
```
-4/3*(6*e^(4*b*x + 4*a) - 4*e^(2*b*x + 2*a) + 1)/(b*(e^(2*b*x + 2*a) - 1)^
4)
```

Mupad [B] (verification not implemented)

Time = 1.62 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.64

$$\int e^{a+bx}\operatorname{csch}^5(a+bx)\,dx = -\frac{4\left(6\,\mathrm{e}^{4\,a+4\,b\,x}-4\,\mathrm{e}^{2\,a+2\,b\,x}+1\right)}{3\,b\,\left(\mathrm{e}^{2\,a+2\,b\,x}-1\right)^4}$$

input
```
int(exp(a + b*x)/sinh(a + b*x)^5,x)
```

output
```
-(4*(6*exp(4*a + 4*b*x) - 4*exp(2*a + 2*b*x) + 1))/(3*b*(exp(2*a + 2*b*x)
- 1)^4)
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 81, normalized size of antiderivative = 1.23

$$\int e^{a+bx}\operatorname{csch}^5(a+bx)\,dx = \frac{-8e^{4bx+4a}+\frac{16e^{2bx+2a}}{3}-\frac{4}{3}}{b\left(e^{8bx+8a}-4e^{6bx+6a}+6e^{4bx+4a}-4e^{2bx+2a}+1\right)}$$

input
```
int(exp(b*x+a)*csch(b*x+a)^5,x)
```

output
```
(4*( - 6*e**(4*a + 4*b*x) + 4*e**(2*a + 2*b*x) - 1))/(3*b*(e**(8*a + 8*b*x
) - 4*e**(6*a + 6*b*x) + 6*e**(4*a + 4*b*x) - 4*e**(2*a + 2*b*x) + 1))
```

3.310 $\int e^x \sinh^2(2x)\,dx$

Optimal result

Integrand size = 10, antiderivative size = 26

$$\int e^x \sinh^2(2x)\,dx = -\frac{1}{12}e^{-3x} - \frac{e^x}{2} + \frac{e^{5x}}{20}$$

output
```
-1/12/exp(3*x)-1/2*exp(x)+1/20*exp(5*x)
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.00

$$\int e^x \sinh^2(2x)\,dx = -\frac{1}{12}e^{-3x} - \frac{e^x}{2} + \frac{e^{5x}}{20}$$

input
```
Integrate[E^x*Sinh[2*x]^2,x]
```

output
```
-1/12*1/E^(3*x) - E^x/2 + E^(5*x)/20
```

Rubi [A] (verified)

Time = 0.32 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.08, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.400, Rules used = {2720, 27, 802, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int e^x \sinh^2(2x)\,dx$$

$$\Big\downarrow \ 2720$$

$$\int \frac{1}{4}e^{-4x}\left(1-e^{4x}\right)^2de^x$$

$$\Big\downarrow \ 27$$

$$\frac{1}{4}\int e^{-4x}\left(1-e^{4x}\right)^2de^x$$

$$\Big\downarrow \ 802$$

$$\frac{1}{4}\int \left(-2+e^{-4x}+e^{4x}\right)de^x$$

$$\Big\downarrow \ 2009$$

$$\frac{1}{4}\left(-\frac{1}{3}e^{-3x}-2e^x+\frac{e^{5x}}{5}\right)$$

input
```
Int[E^x*Sinh[2*x]^2,x]
```

output
```
(-1/3*1/E^(3*x) - 2*E^x + E^(5*x)/5)/4
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 802
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2720
```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Maple [A] (verified)

Time = 0.31 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.65

method	result	size
parallelrisch	$-\frac{e^{x} (15+\cosh (4 x)-4 \sinh (4 x))}{30}$	17
risch	$\frac{e^{5 x}}{20}-\frac{e^{x}}{2}-\frac{e^{-3 x}}{12}$	18
default	$-\frac{\sinh (x)}{2}+\frac{\sinh (3 x)}{12}+\frac{\sinh (5 x)}{20}-\frac{\cosh (x)}{2}-\frac{\cosh (3 x)}{12}+\frac{\cosh (5 x)}{20}$	34
orering	$\frac{7 e^{x} \sinh (2 x)^{2}}{15}+\frac{4 e^{x} \sinh (2 x) \cosh (2 x)}{15}-\frac{8 e^{x} \cosh (2 x)^{2}}{15}$	34

input
```
int(exp(x)*sinh(2*x)^2,x,method=_RETURNVERBOSE)
```

output
```
-1/30*exp(x)*(15+cosh(4*x)-4*sinh(4*x))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 47 vs. $2(17) = 34$.

Time = 0.08 (sec) , antiderivative size = 47, normalized size of antiderivative = 1.81

$$\int e^x \sinh^2(2x)\,dx = -\frac{\cosh(x)^4 - 16\cosh(x)^3\sinh(x) + 6\cosh(x)^2\sinh(x)^2 - 16\cosh(x)\sinh(x)^3 + \sinh(x)^4 + 15}{30(\cosh(x) - \sinh(x))}$$

input
```
integrate(exp(x)*sinh(2*x)^2,x, algorithm="fricas")
```

output
```
-1/30*(cosh(x)^4 - 16*cosh(x)^3*sinh(x) + 6*cosh(x)^2*sinh(x)^2 - 16*cosh(
x)*sinh(x)^3 + sinh(x)^4 + 15)/(cosh(x) - sinh(x))
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 42 vs. $2(19) = 38$.

Time = 0.19 (sec) , antiderivative size = 42, normalized size of antiderivative = 1.62

$$\int e^x \sinh^2(2x)\,dx = \frac{7e^x\sinh^2(2x)}{15} + \frac{4e^x\sinh(2x)\cosh(2x)}{15} - \frac{8e^x\cosh^2(2x)}{15}$$

input
```
integrate(exp(x)*sinh(2*x)**2,x)
```

output
```
7*exp(x)*sinh(2*x)**2/15 + 4*exp(x)*sinh(2*x)*cosh(2*x)/15 - 8*exp(x)*cosh
(2*x)**2/15
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.65

$$\int e^x \sinh^2(2x)\,dx = \frac{1}{20}\,e^{(5\,x)} - \frac{1}{12}\,e^{(-3\,x)} - \frac{1}{2}\,e^x$$

input
```
integrate(exp(x)*sinh(2*x)^2,x, algorithm="maxima")
```

output
```
1/20*e^(5*x) - 1/12*e^(-3*x) - 1/2*e^x
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.65

$$\int e^x \sinh^2(2x)\,dx = \frac{1}{20}\,e^{(5\,x)} - \frac{1}{12}\,e^{(-3\,x)} - \frac{1}{2}\,e^x$$

input
```
integrate(exp(x)*sinh(2*x)^2,x, algorithm="giac")
```

output
```
1/20*e^(5*x) - 1/12*e^(-3*x) - 1/2*e^x
```

Mupad [B] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.65

$$\int e^x \sinh^2(2x)\,dx = \frac{\mathrm{e}^{5\,x}}{20} - \frac{\mathrm{e}^{-3\,x}}{12} - \frac{\mathrm{e}^x}{2}$$

input
```
int(sinh(2*x)^2*exp(x),x)
```

output
```
exp(5*x)/20 - exp(-3*x)/12 - exp(x)/2
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.96

$$\int e^x \sinh^2(2x)\,dx = \frac{3e^{8x} - 30e^{4x} - 5}{60e^{3x}}$$

input
```
int(exp(x)*sinh(2*x)^2,x)
```

output
```
(3*e**(8*x) - 30*e**(4*x) - 5)/(60*e**(3*x))
```

3.311 $\int e^x \sinh(2x)\,dx$

Optimal result

Integrand size = 8, antiderivative size = 19

$$\int e^x \sinh(2x)\,dx = \frac{e^{-x}}{2} + \frac{e^{3x}}{6}$$

output
```
1/2/exp(x)+1/6*exp(3*x)
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.84

$$\int e^x \sinh(2x)\,dx = \frac{1}{6}e^{-x}\left(3 + e^{4x}\right)$$

input
```
Integrate[E^x*Sinh[2*x],x]
```

output
```
(3 + E^(4*x))/(6*E^x)
```

Rubi [A] (verified)

Time = 0.31 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {2720, 27, 802, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int e^x \sinh(2x)\,dx$$

$$\Big\downarrow \text{2720}$$

$$\int -\frac{1}{2}e^{-2x}\left(1-e^{4x}\right)de^x$$

$$\Big\downarrow \text{27}$$

$$-\frac{1}{2}\int e^{-2x}\left(1-e^{4x}\right)de^x$$

$$\Big\downarrow \text{802}$$

$$-\frac{1}{2}\int \left(e^{-2x}-e^{2x}\right)de^x$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{2}\left(e^{-x}+\frac{e^{3x}}{3}\right)$$

input
```
Int[E^x*Sinh[2*x],x]
```

output
```
(E^(-x) + E^(3*x)/3)/2
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 802
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2720
```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Maple [A] (verified)

Time = 0.11 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.74

method	result	size
risch	$\frac{{\mathrm e}^{3 x}}{6}+\frac{{\mathrm e}^{-x}}{2}$	14
parallelrisch	$\frac{{\mathrm e}^{x} \left(2 \cosh \left(2 x \right)-\sinh \left(2 x \right)\right)}{3}$	18
orering	$-\frac{{\mathrm e}^{x} \sinh \left(2 x \right)}{3}+\frac{2 \,{\mathrm e}^{x} \cosh \left(2 x \right)}{3}$	18
default	$-\frac{\sinh \left(x \right)}{2}+\frac{\sinh \left(3 x \right)}{6}+\frac{\cosh \left(x \right)}{2}+\frac{\cosh \left(3 x \right)}{6}$	22

input
```
int(exp(x)*sinh(2*x),x,method=_RETURNVERBOSE)
```

output
```
1/6*exp(3*x)+1/2*exp(-x)
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.37

$$\int e^x \sinh(2x)\,dx = \frac{2\left(\cosh(x)^2 - \cosh(x)\sinh(x) + \sinh(x)^2\right)}{3\left(\cosh(x) - \sinh(x)\right)}$$

input
```
integrate(exp(x)*sinh(2*x),x, algorithm="fricas")
```

output
```
2/3*(cosh(x)^2 - cosh(x)*sinh(x) + sinh(x)^2)/(cosh(x) - sinh(x))
```

Sympy [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.05

$$\int e^x \sinh(2x)\,dx = -\frac{e^x \sinh(2x)}{3} + \frac{2e^x \cosh(2x)}{3}$$

input
```
integrate(exp(x)*sinh(2*x),x)
```

output
```
-exp(x)*sinh(2*x)/3 + 2*exp(x)*cosh(2*x)/3
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.68

$$\int e^x \sinh(2x)\,dx = \frac{1}{6}\,e^{(3\,x)} + \frac{1}{2}\,e^{(-x)}$$

input
```
integrate(exp(x)*sinh(2*x),x, algorithm="maxima")
```

output
```
1/6*e^(3*x) + 1/2*e^(-x)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.68

$$\int e^x \sinh(2x)\, dx = \frac{1}{6} e^{(3x)} + \frac{1}{2} e^{(-x)}$$

input
```
integrate(exp(x)*sinh(2*x),x, algorithm="giac")
```

output
```
1/6*e^(3*x) + 1/2*e^(-x)
```

Mupad [B] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.63

$$\int e^x \sinh(2x)\, dx = \frac{e^{-x}\,(e^{4x} + 3)}{6}$$

input
```
int(sinh(2*x)*exp(x),x)
```

output
```
(exp(-x)*(exp(4*x) + 3))/6
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.95

$$\int e^x \sinh(2x)\, dx = \frac{e^x(2\cosh(2x) - \sinh(2x))}{3}$$

input
```
int(exp(x)*sinh(2*x),x)
```

output
```
(e**x*(2*cosh(2*x) - sinh(2*x)))/3
```

3.312 $\int e^x \text{csch}(2x)\,dx$

Optimal result

Integrand size = 8, antiderivative size = 11

$$\int e^x \text{csch}(2x)\,dx = \arctan(e^x) - \text{arctanh}(e^x)$$

output
```
arctan(exp(x))-arctanh(exp(x))
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 11, normalized size of antiderivative = 1.00

$$\int e^x \text{csch}(2x)\,dx = \arctan(e^x) - \text{arctanh}(e^x)$$

input
```
Integrate[E^x*Csch[2*x],x]
```

output
```
ArcTan[E^x] - ArcTanh[E^x]
```

Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.73, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.625$, Rules used = {2720, 27, 827, 216, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int e^x \text{csch}(2x) \, dx$$

$$\Big\downarrow \text{2720}$$

$$\int -\frac{2e^{2x}}{1-e^{4x}}de^x$$

$$\Big\downarrow \text{27}$$

$$-2\int \frac{e^{2x}}{1-e^{4x}}de^x$$

$$\Big\downarrow \text{827}$$

$$-2\left(\frac{1}{2}\int \frac{1}{1-e^{2x}}de^x-\frac{1}{2}\int \frac{1}{1+e^{2x}}de^x\right)$$

$$\Big\downarrow \text{216}$$

$$-2\left(\frac{1}{2}\int \frac{1}{1-e^{2x}}de^x-\frac{\arctan(e^x)}{2}\right)$$

$$\Big\downarrow \text{219}$$

$$-2\left(\frac{\text{arctanh}(e^x)}{2}-\frac{\arctan(e^x)}{2}\right)$$

input
```
Int[E^x*Csch[2*x],x]
```

output
```
-2*(-1/2*ArcTan[E^x] + ArcTanh[E^x]/2)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 827
```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[-a/b,
 2]], s = Denominator[Rt[-a/b, 2]]}, Simp[s/(2*b)   Int[1/(r + s*x^2), x],
x] - Simp[s/(2*b)   Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ
[a/b, 0]
```

rule 2720
```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 0.20 (sec) , antiderivative size = 34, normalized size of antiderivative = 3.09

method	result	size
risch	$-\frac{\ln(e^x+1)}{2}+\frac{i\ln(e^x+i)}{2}-\frac{i\ln(e^x-i)}{2}+\frac{\ln(e^x-1)}{2}$	34

input
```
int(exp(x)*csch(2*x),x,method=_RETURNVERBOSE)
```

output
```
-1/2*ln(exp(x)+1)+1/2*I*ln(exp(x)+I)-1/2*I*ln(exp(x)-I)+1/2*ln(exp(x)-1)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 25 vs. $2(9) = 18$.

Time = 0.10 (sec) , antiderivative size = 25, normalized size of antiderivative = 2.27

$$\int e^x \text{csch}(2x)\,dx = \arctan\left(\cosh\left(x\right) + \sinh\left(x\right)\right) - \frac{1}{2}\log\left(\cosh\left(x\right) + \sinh\left(x\right) + 1\right) + \frac{1}{2}\log\left(\cosh\left(x\right) + \sinh\left(x\right) - 1\right)$$

input
```
integrate(exp(x)*csch(2*x),x, algorithm="fricas")
```

output
```
arctan(cosh(x) + sinh(x)) - 1/2*log(cosh(x) + sinh(x) + 1) + 1/2*log(cosh(
x) + sinh(x) - 1)
```

Sympy [F]

$$\int e^x \text{csch}(2x)\,dx = \int e^{x} \operatorname{csch}\left(2x\right)\,dx$$

input
```
integrate(exp(x)*csch(2*x),x)
```

output
```
Integral(exp(x)*csch(2*x), x)
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.64

$$\int e^x \text{csch}(2x)\,dx = \arctan\left(e^x\right) - \frac{1}{2}\,\log\left(e^x + 1\right) + \frac{1}{2}\,\log\left(e^x - 1\right)$$

input
```
integrate(exp(x)*csch(2*x),x, algorithm="maxima")
```

output
```
arctan(e^x) - 1/2*log(e^x + 1) + 1/2*log(e^x - 1)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 19 vs. $2(9) = 18$.

Time = 0.12 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.73

$$\int e^x \text{csch}(2x)\,dx = \arctan\left(e^x\right) - \frac{1}{2}\,\log\left(e^x + 1\right) + \frac{1}{2}\,\log\left(|e^x - 1|\right)$$

input
```
integrate(exp(x)*csch(2*x),x, algorithm="giac")
```

output
```
arctan(e^x) - 1/2*log(e^x + 1) + 1/2*log(abs(e^x - 1))
```

Mupad [B] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 26, normalized size of antiderivative = 2.36

$$\int e^x \text{csch}(2x)\,dx = \frac{\ln\left(4\,\mathrm{e}^x - 4\right)}{2} - \frac{\ln\left(-4\,\mathrm{e}^x - 4\right)}{2} - \mathrm{atan}\left(\mathrm{e}^{-x}\right)$$

input
```
int(exp(x)/sinh(2*x),x)
```

output
```
log(4*exp(x) - 4)/2 - log(- 4*exp(x) - 4)/2 - atan(exp(-x))
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.91

$$\int e^x \operatorname{csch}(2x)\,dx = atan(e^x) + \frac{\log(e^x-1)}{2} - \frac{\log(e^x+1)}{2}$$

input
```
int(exp(x)*csch(2*x),x)
```

output
```
(2*atan(e**x) + log(e**x - 1) - log(e**x + 1))/2
```

3.313 $\int e^x \operatorname{csch}^2(2x) \, dx$

Optimal result

Integrand size = 10, antiderivative size = 32

$$\int e^x \operatorname{csch}^2(2x) \, dx = \frac{e^x}{1-e^{4x}} - \frac{\arctan(e^x)}{2} - \frac{\operatorname{arctanh}(e^x)}{2}$$

output
```
exp(x)/(1-exp(4*x))-1/2*arctan(exp(x))-1/2*arctanh(exp(x))
```

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.00

$$\int e^x \operatorname{csch}^2(2x) \, dx = \frac{e^x}{1-e^{4x}} - \frac{\arctan(e^x)}{2} - \frac{\operatorname{arctanh}(e^x)}{2}$$

input
```
Integrate[E^x*Csch[2*x]^2,x]
```

output
```
E^x/(1 - E^(4*x)) - ArcTan[E^x]/2 - ArcTanh[E^x]/2
```

Rubi [A] (verified)

Time = 0.33 (sec) , antiderivative size = 42, normalized size of antiderivative = 1.31, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.600, Rules used = {2720, 27, 817, 756, 216, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int e^x \text{csch}^2(2x) \, dx$$

$$\Big\downarrow \text{2720}$$

$$\int \frac{4e^{4x}}{(1-e^{4x})^2}de^x$$

$$\Big\downarrow \text{27}$$

$$4\int \frac{e^{4x}}{(1-e^{4x})^2}de^x$$

$$\Big\downarrow \text{817}$$

$$4\left(\frac{e^x}{4(1-e^{4x})}-\frac{1}{4}\int \frac{1}{1-e^{4x}}de^x\right)$$

$$\Big\downarrow \text{756}$$

$$4\left(\frac{1}{4}\left(-\frac{1}{2}\int \frac{1}{1-e^{2x}}de^x-\frac{1}{2}\int \frac{1}{1+e^{2x}}de^x\right)+\frac{e^x}{4(1-e^{4x})}\right)$$

$$\Big\downarrow \text{216}$$

$$4\left(\frac{1}{4}\left(-\frac{1}{2}\int \frac{1}{1-e^{2x}}de^x-\frac{1}{2}\arctan(e^x)\right)+\frac{e^x}{4(1-e^{4x})}\right)$$

$$\Big\downarrow \text{219}$$

$$4\left(\frac{1}{4}\left(-\frac{1}{2}\arctan(e^x)-\frac{\text{arctanh}(e^x)}{2}\right)+\frac{e^x}{4(1-e^{4x})}\right)$$

input
```
Int[E^x*Csch[2*x]^2,x]
```

output
```
4*(E^x/(4*(1 - E^(4*x))) + (-1/2*ArcTan[E^x] - ArcTanh[E^x]/2)/4)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 756
```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-a/b, 2
]], s = Denominator[Rt[-a/b, 2]]}, Simp[r/(2*a)   Int[1/(r - s*x^2), x], x]
 + Simp[r/(2*a)   Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ[a
/b, 0]
```

rule 817
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(
n - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*n*(p + 1))), x] - Simp[c^n
*((m - n + 1)/(b*n*(p + 1)))   Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x
] /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !
ILtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

rule 2720
```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Maple [A] (verified)

Time = 0.20 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.75

method	result	size
default	$\frac{1}{4\cosh(x)}-\frac{\operatorname{arctanh}(e^x)}{2}-\frac{1}{4\sinh(x)}-\frac{\arctan(e^x)}{2}$	24
risch	$-\frac{e^x}{e^{4x}-1}-\frac{\ln(e^x+1)}{4}+\frac{\ln(e^x-1)}{4}+\frac{i\ln(e^x-i)}{4}-\frac{i\ln(e^x+i)}{4}$	46

input
```
int(exp(x)*csch(2*x)^2,x,method=_RETURNVERBOSE)
```

output
```
1/4/cosh(x)-1/2*arctanh(exp(x))-1/4/sinh(x)-1/2*arctan(exp(x))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 182 vs. 2(23) = 46.

Time = 0.08 (sec) , antiderivative size = 182, normalized size of antiderivative = 5.69

$$\int e^x \operatorname{csch}^2(2x)\,dx =$$
$$-\frac{2\left(\cosh(x)^4+4\cosh(x)^3\sinh(x)+6\cosh(x)^2\sinh(x)^2+4\cosh(x)\sinh(x)^3+\sinh(x)^4-1\right)\operatorname{arcta}}{}$$

input
```
integrate(exp(x)*csch(2*x)^2,x, algorithm="fricas")
```

output
```
-1/4*(2*(cosh(x)^4 + 4*cosh(x)^3*sinh(x) + 6*cosh(x)^2*sinh(x)^2 + 4*cosh(
x)*sinh(x)^3 + sinh(x)^4 - 1)*arctan(cosh(x) + sinh(x)) + (cosh(x)^4 + 4*c
osh(x)^3*sinh(x) + 6*cosh(x)^2*sinh(x)^2 + 4*cosh(x)*sinh(x)^3 + sinh(x)^4
 - 1)*log(cosh(x) + sinh(x) + 1) - (cosh(x)^4 + 4*cosh(x)^3*sinh(x) + 6*co
sh(x)^2*sinh(x)^2 + 4*cosh(x)*sinh(x)^3 + sinh(x)^4 - 1)*log(cosh(x) + sin
h(x) - 1) + 4*cosh(x) + 4*sinh(x))/(cosh(x)^4 + 4*cosh(x)^3*sinh(x) + 6*co
sh(x)^2*sinh(x)^2 + 4*cosh(x)*sinh(x)^3 + sinh(x)^4 - 1)
```

Sympy [F]

$$\int e^x \text{csch}^2(2x)\,dx = \int e^x \operatorname{csch}^2(2x)\,dx$$

input
```
integrate(exp(x)*csch(2*x)**2,x)
```

output
```
Integral(exp(x)*csch(2*x)**2, x)
```

Maxima [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.00

$$\int e^x \text{csch}^2(2x)\,dx = -\frac{e^x}{e^{(4x)}-1} - \frac{1}{2}\arctan(e^x) - \frac{1}{4}\log(e^x+1) + \frac{1}{4}\log(e^x-1)$$

input
```
integrate(exp(x)*csch(2*x)^2,x, algorithm="maxima")
```

output
```
-e^x/(e^(4*x) - 1) - 1/2*arctan(e^x) - 1/4*log(e^x + 1) + 1/4*log(e^x - 1)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 33, normalized size of antiderivative = 1.03

$$\int e^x \text{csch}^2(2x)\,dx = -\frac{e^x}{e^{(4x)}-1} - \frac{1}{2}\arctan(e^x) - \frac{1}{4}\log(e^x+1) + \frac{1}{4}\log(|e^x-1|)$$

input
```
integrate(exp(x)*csch(2*x)^2,x, algorithm="giac")
```

output
```
-e^x/(e^(4*x) - 1) - 1/2*arctan(e^x) - 1/4*log(e^x + 1) + 1/4*log(abs(e^x
- 1))
```

Mupad [B] (verification not implemented)

Time = 1.76 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.12

$$\int e^x \operatorname{csch}^2(2x)\,dx = \frac{\ln(1-e^x)}{4} - \frac{\ln(-e^x-1)}{4} - \frac{\operatorname{atan}(e^x)}{2} - \frac{e^x}{e^{4x}-1}$$

input
```
int(exp(x)/sinh(2*x)^2,x)
```

output
```
log(1 - exp(x))/4 - log(- exp(x) - 1)/4 - atan(exp(x))/2 - exp(x)/(exp(4*x
) - 1)
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 74, normalized size of antiderivative = 2.31

$$\int e^x \operatorname{csch}^2(2x)\,dx = \frac{-2e^{4x}atan(e^x) + 2atan(e^x) + e^{4x}\log(e^x-1) - e^{4x}\log(e^x+1) - 4e^x - \log(e^x-1) + \log(e^x+1)}{4e^{4x}-4}$$

input
```
int(exp(x)*csch(2*x)^2,x)
```

output
```
( - 2*e**(4*x)*atan(e**x) + 2*atan(e**x) + e**(4*x)*log(e**x - 1) - e**(4*
x)*log(e**x + 1) - 4*e**x - log(e**x - 1) + log(e**x + 1))/(4*(e**(4*x) -
1))
```

3.314 $\int e^x \sinh^2(3x)\,dx$

Optimal result

Integrand size = 10, antiderivative size = 26

$$\int e^x \sinh^2(3x)\,dx=-\frac{1}{20}e^{-5x}-\frac{e^x}{2}+\frac{e^{7x}}{28}$$

output
```
-1/20/exp(5*x)-1/2*exp(x)+1/28*exp(7*x)
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.00

$$\int e^x \sinh^2(3x)\,dx=-\frac{1}{20}e^{-5x}-\frac{e^x}{2}+\frac{e^{7x}}{28}$$

input
```
Integrate[E^x*Sinh[3*x]^2,x]
```

output
```
-1/20*1/E^(5*x) - E^x/2 + E^(7*x)/28
```

Rubi [A] (verified)

Time = 0.32 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.08, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.400, Rules used = {2720, 27, 802, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int e^x \sinh^2(3x)\,dx$$

$$\Big\downarrow \text{ 2720}$$

$$\int \frac{1}{4}e^{-6x}\left(1-e^{6x}\right)^2de^x$$

$$\Big\downarrow \text{ 27}$$

$$\frac{1}{4}\int e^{-6x}\left(1-e^{6x}\right)^2de^x$$

$$\Big\downarrow \text{ 802}$$

$$\frac{1}{4}\int \left(-2+e^{-6x}+e^{6x}\right)de^x$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{1}{4}\left(-\frac{1}{5}e^{-5x}-2e^x+\frac{e^{7x}}{7}\right)$$

input
```
Int[E^x*Sinh[3*x]^2,x]
```

output
```
(-1/5*1/E^(5*x) - 2*E^x + E^(7*x)/7)/4
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 802
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2720
```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Maple [A] (verified)

Time = 0.28 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.65

method	result	size
parallelrisch	$-\frac{e^{x} \left(35+\cosh \left(6 x \right)-6 \sinh \left(6 x \right)\right)}{70}$	17
risch	$\frac{e^{7 x}}{28}-\frac{e^{x}}{2}-\frac{e^{-5 x}}{20}$	18
default	$-\frac{\sinh \left(x \right)}{2}+\frac{\sinh \left(5 x \right)}{20}+\frac{\sinh \left(7 x \right)}{28}-\frac{\cosh \left(x \right)}{2}-\frac{\cosh \left(5 x \right)}{20}+\frac{\cosh \left(7 x \right)}{28}$	34
orering	$\frac{17 e^{x} \sinh \left(3 x \right)^{2}}{35}+\frac{6 e^{x} \sinh \left(3 x \right) \cosh \left(3 x \right)}{35}-\frac{18 e^{x} \cosh \left(3 x \right)^{2}}{35}$	34

input
```
int(exp(x)*sinh(3*x)^2,x,method=_RETURNVERBOSE)
```

output
```
-1/70*exp(x)*(35+cosh(6*x)-6*sinh(6*x))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 67 vs. $2(17) = 34$.

Time = 0.11 (sec) , antiderivative size = 67, normalized size of antiderivative = 2.58

$$\int e^x \sinh^2(3x)\,dx =$$
$$-\frac{\cosh(x)^6 - 36\cosh(x)^5\sinh(x) + 15\cosh(x)^4\sinh(x)^2 - 120\cosh(x)^3\sinh(x)^3 + 15\cosh(x)^2\sinh}{70(\cosh(x) - \sinh(x))}$$

input
```
integrate(exp(x)*sinh(3*x)^2,x, algorithm="fricas")
```

output
```
-1/70*(cosh(x)^6 - 36*cosh(x)^5*sinh(x) + 15*cosh(x)^4*sinh(x)^2 - 120*cos
h(x)^3*sinh(x)^3 + 15*cosh(x)^2*sinh(x)^4 - 36*cosh(x)*sinh(x)^5 + sinh(x)
^6 + 35)/(cosh(x) - sinh(x))
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 42 vs. $2(19) = 38$.

Time = 0.20 (sec) , antiderivative size = 42, normalized size of antiderivative = 1.62

$$\int e^x \sinh^2(3x)\,dx = \frac{17e^x\sinh^2(3x)}{35} + \frac{6e^x\sinh(3x)\cosh(3x)}{35} - \frac{18e^x\cosh^2(3x)}{35}$$

input
```
integrate(exp(x)*sinh(3*x)**2,x)
```

output
```
17*exp(x)*sinh(3*x)**2/35 + 6*exp(x)*sinh(3*x)*cosh(3*x)/35 - 18*exp(x)*co
sh(3*x)**2/35
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.65

$$\int e^x \sinh^2(3x)\,dx = \frac{1}{28}\,e^{(7\,x)} - \frac{1}{20}\,e^{(-5\,x)} - \frac{1}{2}\,e^x$$

input
```
integrate(exp(x)*sinh(3*x)^2,x, algorithm="maxima")
```

output
```
1/28*e^(7*x) - 1/20*e^(-5*x) - 1/2*e^x
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.65

$$\int e^x \sinh^2(3x)\,dx = \frac{1}{28}\,e^{(7\,x)} - \frac{1}{20}\,e^{(-5\,x)} - \frac{1}{2}\,e^x$$

input
```
integrate(exp(x)*sinh(3*x)^2,x, algorithm="giac")
```

output
```
1/28*e^(7*x) - 1/20*e^(-5*x) - 1/2*e^x
```

Mupad [B] (verification not implemented)

Time = 1.60 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.65

$$\int e^x \sinh^2(3x)\,dx = \frac{e^{7\,x}}{28} - \frac{e^{-5\,x}}{20} - \frac{e^x}{2}$$

input
```
int(sinh(3*x)^2*exp(x),x)
```

output
```
exp(7*x)/28 - exp(-5*x)/20 - exp(x)/2
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.96

$$\int e^x \sinh^2(3x)\,dx = \frac{5e^{12x} - 70e^{6x} - 7}{140e^{5x}}$$

input
```
int(exp(x)*sinh(3*x)^2,x)
```

output
```
(5*e**(12*x) - 70*e**(6*x) - 7)/(140*e**(5*x))
```

3.315 $\int e^x \sinh(3x)\,dx$

Optimal result

Integrand size = 8, antiderivative size = 19

$$\int e^x \sinh(3x)\,dx = \frac{e^{-2x}}{4} + \frac{e^{4x}}{8}$$

output
```
1/4/exp(2*x)+1/8*exp(4*x)
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.84

$$\int e^x \sinh(3x)\,dx = \frac{1}{8}e^{-2x}\left(2 + e^{6x}\right)$$

input
```
Integrate[E^x*Sinh[3*x],x]
```

output
```
(2 + E^(6*x))/(8*E^(2*x))
```

Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.21, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$, Rules used = {2720, 27, 802, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int e^x \sinh(3x)\,dx$$

$$\Big\downarrow \text{2720}$$

$$\int -\frac{1}{2}e^{-3x}\left(1-e^{6x}\right)de^x$$

$$\Big\downarrow \text{27}$$

$$-\frac{1}{2}\int e^{-3x}\left(1-e^{6x}\right)de^x$$

$$\Big\downarrow \text{802}$$

$$-\frac{1}{2}\int \left(e^{-3x}-e^{3x}\right)de^x$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{2}\left(\frac{e^{-2x}}{2}+\frac{e^{4x}}{4}\right)$$

input
```
Int[E^x*Sinh[3*x],x]
```

output
```
(1/(2*E^(2*x)) + E^(4*x)/4)/2
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 802
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2720
```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Maple [A] (verified)

Time = 0.19 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.74

method	result	size
risch	$\frac{{\mathrm e}^{4 x}}{8}+\frac{{\mathrm e}^{-2 x}}{4}$	14
parallelrisch	$-\frac{{\mathrm e}^{x} \left(-3 \cosh \left(3 x \right)+\sinh \left(3 x \right)\right)}{8}$	16
orering	$-\frac{{\mathrm e}^{x} \sinh \left(3 x \right)}{8}+\frac{3 \,{\mathrm e}^{x} \cosh \left(3 x \right)}{8}$	18
default	$-\frac{\sinh \left(2 x \right)}{4}+\frac{\sinh \left(4 x \right)}{8}+\frac{\cosh \left(2 x \right)}{4}+\frac{\cosh \left(4 x \right)}{8}$	26

input
```
int(exp(x)*sinh(3*x),x,method=_RETURNVERBOSE)
```

output
```
1/8*exp(4*x)+1/4*exp(-2*x)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 40 vs. 2(13) = 26.

Time = 0.07 (sec) , antiderivative size = 40, normalized size of antiderivative = 2.11

$$\int e^x \sinh(3x)\,dx = \frac{3 \cosh(x)^3 - 3 \cosh(x)^2 \sinh(x) + 9 \cosh(x) \sinh(x)^2 - \sinh(x)^3}{8 (\cosh(x) - \sinh(x))}$$

input
```
integrate(exp(x)*sinh(3*x),x, algorithm="fricas")
```

output
```
1/8*(3*cosh(x)^3 - 3*cosh(x)^2*sinh(x) + 9*cosh(x)*sinh(x)^2 - sinh(x)^3)/
(cosh(x) - sinh(x))
```

Sympy [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.05

$$\int e^x \sinh(3x)\,dx = -\frac{e^x \sinh(3x)}{8} + \frac{3e^x \cosh(3x)}{8}$$

input
```
integrate(exp(x)*sinh(3*x),x)
```

output
```
-exp(x)*sinh(3*x)/8 + 3*exp(x)*cosh(3*x)/8
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.68

$$\int e^x \sinh(3x)\,dx = \frac{1}{8}\,e^{(4\,x)} + \frac{1}{4}\,e^{(-2\,x)}$$

input
```
integrate(exp(x)*sinh(3*x),x, algorithm="maxima")
```

output
```
1/8*e^(4*x) + 1/4*e^(-2*x)
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.68

$$\int e^x \sinh(3x)\,dx = \frac{1}{8}\,e^{(4\,x)} + \frac{1}{4}\,e^{(-2\,x)}$$

input
```
integrate(exp(x)*sinh(3*x),x, algorithm="giac")
```

output
```
1/8*e^(4*x) + 1/4*e^(-2*x)
```

Mupad [B] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.63

$$\int e^x \sinh(3x)\,dx = \frac{\mathrm{e}^{-2\,x}\,\left(\mathrm{e}^{6\,x}+2\right)}{8}$$

input
```
int(sinh(3*x)*exp(x),x)
```

output
```
(exp(-2*x)*(exp(6*x) + 2))/8
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.95

$$\int e^x \sinh(3x)\,dx = \frac{e^x(3\cosh(3x) - \sinh(3x))}{8}$$

input
```
int(exp(x)*sinh(3*x),x)
```

output
```
(e**x*(3*cosh(3*x) - sinh(3*x)))/8
```

3.316 $\int e^x \operatorname{csch}(3x) \, dx$

Optimal result

Integrand size = 8, antiderivative size = 54

$$\int e^x \operatorname{csch}(3x) \, dx = \frac{\arctan\left(\frac{1+2e^{2x}}{\sqrt{3}}\right)}{\sqrt{3}} + \frac{1}{3}\log\left(1-e^{2x}\right) - \frac{1}{6}\log\left(1+e^{2x}+e^{4x}\right)$$

output
```
1/3*arctan(1/3*(1+2*exp(2*x))*3^(1/2))*3^(1/2)+1/3*ln(1-exp(2*x))-1/6*ln(1
+exp(2*x)+exp(4*x))
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 0.01 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.41

$$\int e^x \operatorname{csch}(3x) \, dx = -\frac{1}{2}e^{4x} \text{Hypergeometric2F1}\left(\frac{2}{3},1,\frac{5}{3},e^{6x}\right)$$

input
```
Integrate[E^x*Csch[3*x],x]
```

output
```
-1/2*(E^(4*x)*Hypergeometric2F1[2/3, 1, 5/3, E^(6*x)])
```

Rubi [A] (warning: unable to verify)

Time = 0.26 (sec) , antiderivative size = 56, normalized size of antiderivative = 1.04, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 1.125, Rules used = {2720, 27, 807, 821, 16, 1142, 1083, 217, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int e^x \text{csch}(3x)\, dx$$

$$\Big\downarrow \ 2720$$

$$\int -\frac{2e^{3x}}{1-e^{6x}}de^x$$

$$\Big\downarrow \ 27$$

$$-2\int \frac{e^{3x}}{1-e^{6x}}de^x$$

$$\Big\downarrow \ 807$$

$$-\int \frac{e^{2x}}{1-e^{3x}}de^{2x}$$

$$\Big\downarrow \ 821$$

$$\frac{1}{3}\int \frac{1-e^{2x}}{1+2e^{2x}}de^{2x}-\frac{1}{3}\int \frac{1}{1-e^{2x}}de^{2x}$$

$$\Big\downarrow \ 16$$

$$\frac{1}{3}\int \frac{1-e^{2x}}{1+2e^{2x}}de^{2x}+\frac{1}{3}\log\left(1-e^{2x}\right)$$

$$\Big\downarrow \ 1142$$

$$\frac{1}{3}\left(\frac{3}{2}\int \frac{1}{1+2e^{2x}}de^{2x}-\frac{\int 1de^{2x}}{2}\right)+\frac{1}{3}\log\left(1-e^{2x}\right)$$

$$\Big\downarrow \ 1083$$

$$\frac{1}{3}\left(-\frac{1}{2}\int 1de^{2x}-3\int \frac{1}{-4-2e^{2x}}d\left(1+2e^{2x}\right)\right)+\frac{1}{3}\log\left(1-e^{2x}\right)$$

$$\Big\downarrow \ 217$$

$$\frac{1}{3}\left(\sqrt{3}\arctan\left(\frac{2e^{2x}+1}{\sqrt{3}}\right)-\frac{\int 1de^{2x}}{2}\right)+\frac{1}{3}\log\left(1-e^{2x}\right)$$

$$\Big\downarrow \text{1103}$$

$$\frac{1}{3}\left(\sqrt{3}\arctan\left(\frac{2e^{2x}+1}{\sqrt{3}}\right)-\frac{1}{2}\log\left(2e^{2x}+1\right)\right)+\frac{1}{3}\log\left(1-e^{2x}\right)$$

input
```
Int[E^x*Csch[3*x],x]
```

output
```
Log[1 - E^(2*x)]/3 + (Sqrt[3]*ArcTan[(1 + 2*E^(2*x))/Sqrt[3]] - Log[1 + 2*
E^(2*x)]/2)/3
```

Defintions of rubi rules used

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 807
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Simp[1/k   Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x,
x^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

rule 821
```
Int[(x_)/((a_) + (b_.)*(x_)^3), x_Symbol] :> Simp[-(3*Rt[a, 3]*Rt[b, 3])^(-
1)   Int[1/(Rt[a, 3] + Rt[b, 3]*x), x], x] + Simp[1/(3*Rt[a, 3]*Rt[b, 3])
 Int[(Rt[a, 3] + Rt[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2
*x^2), x], x] /; FreeQ[{a, b}, x]
```

rule 1083
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Simp[-2   Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1142
```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(2*c*d - b*e)/(2*c)   Int[1/(a + b*x + c*x^2), x], x] + Simp[e/(2*c)
Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x]
```

rule 2720
```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 0.17 (sec) , antiderivative size = 79, normalized size of antiderivative = 1.46

method	result	size
risch	$-\frac{\ln\left(e^{2x}+\frac{1}{2}+\frac{i\sqrt{3}}{2}\right)}{6}+\frac{i\ln\left(e^{2x}+\frac{1}{2}+\frac{i\sqrt{3}}{2}\right)\sqrt{3}}{6}-\frac{\ln\left(e^{2x}+\frac{1}{2}-\frac{i\sqrt{3}}{2}\right)}{6}-\frac{i\ln\left(e^{2x}+\frac{1}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{3}}{6}+\frac{\ln\left(e^{2x}-1\right)}{3}$	79

input
```
int(exp(x)*csch(3*x),x,method=_RETURNVERBOSE)
```

output
```
-1/6*ln(exp(2*x)+1/2+1/2*I*3^(1/2))+1/6*I*ln(exp(2*x)+1/2+1/2*I*3^(1/2))*3
^(1/2)-1/6*ln(exp(2*x)+1/2-1/2*I*3^(1/2))-1/6*I*ln(exp(2*x)+1/2-1/2*I*3^(1
/2))*3^(1/2)+1/3*ln(exp(2*x)-1)
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 83, normalized size of antiderivative = 1.54

$$\int e^x \operatorname{csch}(3x)\, dx = -\frac{1}{3}\sqrt{3}\arctan\left(-\frac{3\sqrt{3}\cosh(x) + \sqrt{3}\sinh(x)}{3(\cosh(x) - \sinh(x))}\right) - \frac{1}{6}\log\left(\frac{2\cosh(x)^2 + 2\sinh(x)^2 + 1}{\cosh(x)^2 - 2\cosh(x)\sinh(x) + \sinh(x)^2}\right) + \frac{1}{3}\log\left(\frac{2\sinh(x)}{\cosh(x) - \sinh(x)}\right)$$

input
```
integrate(exp(x)*csch(3*x),x, algorithm="fricas")
```

output
```
-1/3*sqrt(3)*arctan(-1/3*(3*sqrt(3)*cosh(x) + sqrt(3)*sinh(x))/(cosh(x) -
sinh(x))) - 1/6*log((2*cosh(x)^2 + 2*sinh(x)^2 + 1)/(cosh(x)^2 - 2*cosh(x)
*sinh(x) + sinh(x)^2)) + 1/3*log(2*sinh(x)/(cosh(x) - sinh(x)))
```

Sympy [F]

$$\int e^x \operatorname{csch}(3x)\, dx = \int e^x \operatorname{csch}(3x)\, dx$$

input
```
integrate(exp(x)*csch(3*x),x)
```

output
```
Integral(exp(x)*csch(3*x), x)
```

Maxima [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 73, normalized size of antiderivative = 1.35

$$\int e^x \operatorname{csch}(3x)\,dx = -\frac{1}{3}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2e^x+1)\right) + \frac{1}{3}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2e^x-1)\right) - \frac{1}{6}\log\left(e^{(2x)}+e^x+1\right) - \frac{1}{6}\log\left(e^{(2x)}-e^x+1\right) + \frac{1}{3}\log\left(e^x+1\right) + \frac{1}{3}\log\left(e^x-1\right)$$

input
```
integrate(exp(x)*csch(3*x),x, algorithm="maxima")
```

output
```
-1/3*sqrt(3)*arctan(1/3*sqrt(3)*(2*e^x + 1)) + 1/3*sqrt(3)*arctan(1/3*sqrt
(3)*(2*e^x - 1)) - 1/6*log(e^(2*x) + e^x + 1) - 1/6*log(e^(2*x) - e^x + 1)
 + 1/3*log(e^x + 1) + 1/3*log(e^x - 1)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 43, normalized size of antiderivative = 0.80

$$\int e^x \operatorname{csch}(3x)\,dx = \frac{1}{3}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}\left(2e^{(2x)}+1\right)\right) - \frac{1}{6}\log\left(e^{(4x)}+e^{(2x)}+1\right) + \frac{1}{3}\log\left(\left|e^{(2x)}-1\right|\right)$$

input
```
integrate(exp(x)*csch(3*x),x, algorithm="giac")
```

output
```
1/3*sqrt(3)*arctan(1/3*sqrt(3)*(2*e^(2*x) + 1)) - 1/6*log(e^(4*x) + e^(2*x
) + 1) + 1/3*log(abs(e^(2*x) - 1))
```

Mupad [B] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 65, normalized size of antiderivative = 1.20

$$\int e^x \operatorname{csch}(3x)\,dx = \frac{\ln\left(8\,\mathrm{e}^{2\,x}-8\right)}{3}+\ln\left(24\,\mathrm{e}^{2\,x}\,\left(-\frac{1}{6}+\frac{\sqrt{3}\,1\mathrm{i}}{6}\right)-8\right)\,\left(-\frac{1}{6}+\frac{\sqrt{3}\,1\mathrm{i}}{6}\right) - \ln\left(-24\,\mathrm{e}^{2\,x}\,\left(\frac{1}{6}+\frac{\sqrt{3}\,1\mathrm{i}}{6}\right)-8\right)\,\left(\frac{1}{6}+\frac{\sqrt{3}\,1\mathrm{i}}{6}\right)$$

input
```
int(exp(x)/sinh(3*x),x)
```

output
```
log(8*exp(2*x) - 8)/3 + log(24*exp(2*x)*((3^(1/2)*1i)/6 - 1/6) - 8)*((3^(1
/2)*1i)/6 - 1/6) - log(- 24*exp(2*x)*((3^(1/2)*1i)/6 + 1/6) - 8)*((3^(1/2)
*1i)/6 + 1/6)
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 79, normalized size of antiderivative = 1.46

$$\int e^x \operatorname{csch}(3x)\,dx = \frac{\sqrt{3}\,atan\left(\frac{2e^x-1}{\sqrt{3}}\right)}{3}-\frac{\sqrt{3}\,atan\left(\frac{2e^x+1}{\sqrt{3}}\right)}{3}-\frac{\log(e^{2x}+e^x+1)}{6} - \frac{\log(e^{2x}-e^x+1)}{6}+\frac{\log(e^x-1)}{3}+\frac{\log(e^x+1)}{3}$$

input
```
int(exp(x)*csch(3*x),x)
```

output
```
(2*sqrt(3)*atan((2*e**x - 1)/sqrt(3)) - 2*sqrt(3)*atan((2*e**x + 1)/sqrt(3
)) - log(e**(2*x) + e**x + 1) - log(e**(2*x) - e**x + 1) + 2*log(e**x - 1)
 + 2*log(e**x + 1))/6
```

3.317 $\int e^x \operatorname{csch}^2(3x)\,dx$

Optimal result

Integrand size = 10, antiderivative size = 91

$$\int e^x \operatorname{csch}^2(3x)\,dx = \frac{2e^x}{3(1-e^{6x})} + \frac{\arctan\left(\frac{1-2e^x}{\sqrt{3}}\right)}{3\sqrt{3}} - \frac{\arctan\left(\frac{1+2e^x}{\sqrt{3}}\right)}{3\sqrt{3}} - \frac{2\operatorname{arctanh}(e^x)}{9} - \frac{1}{9}\operatorname{arctanh}\left(\frac{e^x}{1+e^{2x}}\right)$$

output
```
2*exp(x)/(3-3*exp(6*x))+1/9*arctan(1/3*(1-2*exp(x))*3^(1/2))*3^(1/2)-1/9*a
rctan(1/3*(1+2*exp(x))*3^(1/2))*3^(1/2)-2/9*arctanh(exp(x))-1/9*arctanh(ex
p(x)/(1+exp(2*x)))
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 0.02 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.37

$$\int e^x \operatorname{csch}^2(3x)\,dx = \frac{2}{3}e^x\left(\frac{1}{1-e^{6x}} - \text{Hypergeometric2F1}\left(\frac{1}{6},1,\frac{7}{6},e^{6x}\right)\right)$$

input
```
Integrate[E^x*Csch[3*x]^2,x]
```

output
```
(2*E^x*((1 - E^(6*x))^(-1) - Hypergeometric2F1[1/6, 1, 7/6, E^(6*x)]))/3
```

Rubi [A] (verified)

Time = 0.32 (sec) , antiderivative size = 118, normalized size of antiderivative = 1.30, number of steps used = 12, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 1.100$, Rules used = {2720, 27, 817, 754, 27, 219, 1142, 25, 1083, 217, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int e^x \text{csch}^2(3x)\,dx$$

$$\Big\downarrow \text{2720}$$

$$\int \frac{4e^{6x}}{(1-e^{6x})^2}de^x$$

$$\Big\downarrow \text{27}$$

$$4\int \frac{e^{6x}}{(1-e^{6x})^2}de^x$$

$$\Big\downarrow \text{817}$$

$$4\left(\frac{e^x}{6(1-e^{6x})}-\frac{1}{6}\int \frac{1}{1-e^{6x}}de^x\right)$$

$$\Big\downarrow \text{754}$$

$$4\left(\frac{1}{6}\left(-\frac{1}{3}\int \frac{1}{1-e^{2x}}de^x-\frac{1}{3}\int \frac{2-e^x}{2(1-e^x+e^{2x})}de^x-\frac{1}{3}\int \frac{2+e^x}{2(1+e^x+e^{2x})}de^x\right)+\frac{e^x}{6(1-e^{6x})}\right)$$

$$\Big\downarrow \text{27}$$

$$4\left(\frac{1}{6}\left(-\frac{1}{3}\int \frac{1}{1-e^{2x}}de^x-\frac{1}{6}\int \frac{2-e^x}{1-e^x+e^{2x}}de^x-\frac{1}{6}\int \frac{2+e^x}{1+e^x+e^{2x}}de^x\right)+\frac{e^x}{6(1-e^{6x})}\right)$$

$$\Big\downarrow \text{219}$$

$$4\left(\frac{1}{6}\left(-\frac{1}{6}\int \frac{2-e^x}{1-e^x+e^{2x}}de^x-\frac{1}{6}\int \frac{2+e^x}{1+e^x+e^{2x}}de^x-\frac{1}{3}\text{arctanh}(e^x)\right)+\frac{e^x}{6(1-e^{6x})}\right)$$

$$\Big\downarrow \text{1142}$$

$$4\left(\frac{1}{6}\left(\frac{1}{6}\left(\frac{1}{2}\int -\frac{1-2e^x}{1-e^x+e^{2x}}de^x-\frac{3}{2}\int \frac{1}{1-e^x+e^{2x}}de^x\right)+\frac{1}{6}\left(-\frac{3}{2}\int \frac{1}{1+e^x+e^{2x}}de^x-\frac{1}{2}\int \frac{1+2e^x}{1+e^x+e^{2x}}de^x\right.\right.\right.$$

$\downarrow$ 25

$$4\left(\frac{1}{6}\left(\frac{1}{6}\left(-\frac{3}{2}\int \frac{1}{1-e^x+e^{2x}}de^x-\frac{1}{2}\int \frac{1-2e^x}{1-e^x+e^{2x}}de^x\right)+\frac{1}{6}\left(-\frac{3}{2}\int \frac{1}{1+e^x+e^{2x}}de^x-\frac{1}{2}\int \frac{1+2e^x}{1+e^x+e^{2x}}de^x\right.\right.\right.$$

$\downarrow$ 1083

$$4\left(\frac{1}{6}\left(\frac{1}{6}\left(3\int \frac{1}{-3-e^{2x}}d(-1+2e^x)-\frac{1}{2}\int \frac{1-2e^x}{1-e^x+e^{2x}}de^x\right)+\frac{1}{6}\left(3\int \frac{1}{-3-e^{2x}}d(1+2e^x)-\frac{1}{2}\int \frac{1+2e^x}{1+e^x+e^{2x}}\right.\right.\right.$$

$\downarrow$ 217

$$4\left(\frac{1}{6}\left(\frac{1}{6}\left(-\frac{1}{2}\int \frac{1-2e^x}{1-e^x+e^{2x}}de^x-\sqrt{3}\arctan\left(\frac{2e^x-1}{\sqrt{3}}\right)\right)+\frac{1}{6}\left(-\frac{1}{2}\int \frac{1+2e^x}{1+e^x+e^{2x}}de^x-\sqrt{3}\arctan\left(\frac{2e^x+1}{\sqrt{3}}\right.\right.\right.\right.$$

$\downarrow$ 1103

$$4\left(\frac{1}{6}\left(\frac{1}{6}\left(\frac{1}{2}\log\left(-e^x+e^{2x}+1\right)-\sqrt{3}\arctan\left(\frac{2e^x-1}{\sqrt{3}}\right)\right)+\frac{1}{6}\left(-\sqrt{3}\arctan\left(\frac{2e^x+1}{\sqrt{3}}\right)-\frac{1}{2}\log\left(e^x+e^{2x}+1\right)\right)\right.\right.$$

input
```
Int[E^x*Csch[3*x]^2,x]
```

output
```
4*(E^x/(6*(1 - E^(6*x))) + (-1/3*ArcTanh[E^x] + (-(Sqrt[3]*ArcTan[(-1 + 2*
E^x)/Sqrt[3]]) + Log[1 - E^x + E^(2*x)]/2)/6 + (-(Sqrt[3]*ArcTan[(1 + 2*E^
x)/Sqrt[3]]) - Log[1 + E^x + E^(2*x)]/2)/6)/6)
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 754
```
Int[((a_) + (b_.)*(x_)^(n_))^(-1), x_Symbol] :> Module[{r = Numerator[Rt[-a
/b, n]], s = Denominator[Rt[-a/b, n]], k, u}, Simp[u = Int[(r - s*Cos[(2*k*
Pi)/n]*x)/(r^2 - 2*r*s*Cos[(2*k*Pi)/n]*x + s^2*x^2), x] + Int[(r + s*Cos[(2
*k*Pi)/n]*x)/(r^2 + 2*r*s*Cos[(2*k*Pi)/n]*x + s^2*x^2), x]; 2*(r^2/(a*n))
 Int[1/(r^2 - s^2*x^2), x] + 2*(r/(a*n))   Sum[u, {k, 1, (n - 2)/4}], x]] /
; FreeQ[{a, b}, x] && IGtQ[(n - 2)/4, 0] && NegQ[a/b]
```

rule 817
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(
n - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*n*(p + 1))), x] - Simp[c^n
*((m - n + 1)/(b*n*(p + 1)))   Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x
] /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !
ILtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

rule 1083
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Simp[-2   Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1142
```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(2*c*d - b*e)/(2*c)   Int[1/(a + b*x + c*x^2), x], x] + Simp[e/(2*c)
Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x]
```

rule 2720
```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Maple [A] (verified)

Time = 0.29 (sec) , antiderivative size = 116, normalized size of antiderivative = 1.27

method	result
default	$\frac{\tanh\left(\frac{x}{2}\right)}{18}+\frac{-\frac{2\tanh\left(\frac{x}{2}\right)}{3}+\frac{2}{3}}{9\tanh\left(\frac{x}{2}\right)^{2}+3}+\frac{\ln\left(3\tanh\left(\frac{x}{2}\right)^{2}+1\right)}{18}-\frac{\sqrt{3}\,\arctan\left(\tanh\left(\frac{x}{2}\right)\sqrt{3}\right)}{9}-\frac{1}{18\tanh\left(\frac{x}{2}\right)}+\frac{\ln\left(\tanh\left(\frac{x}{2}\right)\right)}{9}-\frac{-2\tanh\cdots}{9\left(\tanh\cdots\right.}$
risch	$-\frac{2\,\mathrm{e}^{x}}{3\left(-1+\mathrm{e}^{6x}\right)}+\frac{\ln\left(\mathrm{e}^{x}-1\right)}{9}-\frac{\ln\left(\mathrm{e}^{x}+\frac{1}{2}-\frac{i\sqrt{3}}{2}\right)}{18}+\frac{i\sqrt{3}\,\ln\left(\mathrm{e}^{x}+\frac{1}{2}-\frac{i\sqrt{3}}{2}\right)}{18}-\frac{\ln\left(\mathrm{e}^{x}+\frac{1}{2}+\frac{i\sqrt{3}}{2}\right)}{18}-\frac{i\sqrt{3}\,\ln\left(\mathrm{e}^{x}+\frac{1}{2}+\frac{i\sqrt{3}}{2}\right)}{18}+\frac{\ln\left(\cdots\right.}{}$

input
```
int(exp(x)*csch(3*x)^2,x,method=_RETURNVERBOSE)
```

output
```
1/18*tanh(1/2*x)+1/9*(-2/3*tanh(1/2*x)+2/3)/(tanh(1/2*x)^2+1/3)+1/18*ln(3*
tanh(1/2*x)^2+1)-1/9*3^(1/2)*arctan(tanh(1/2*x)*3^(1/2))-1/18/tanh(1/2*x)+
1/9*ln(tanh(1/2*x))-1/9*(-2*tanh(1/2*x)-6)/(tanh(1/2*x)^2+3)-1/18*ln(tanh(
1/2*x)^2+3)-1/9*3^(1/2)*arctan(1/3*tanh(1/2*x)*3^(1/2))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 560 vs. 2(66) = 132.

Time = 0.11 (sec) , antiderivative size = 560, normalized size of antiderivative = 6.15

$$\int e^x \operatorname{csch}^2(3x)\,dx = \text{Too large to display}$$

input
```
integrate(exp(x)*csch(3*x)^2,x, algorithm="fricas")
```

output
```
-1/18*(2*(sqrt(3)*cosh(x)^6 + 6*sqrt(3)*cosh(x)^5*sinh(x) + 15*sqrt(3)*cos
h(x)^4*sinh(x)^2 + 20*sqrt(3)*cosh(x)^3*sinh(x)^3 + 15*sqrt(3)*cosh(x)^2*s
inh(x)^4 + 6*sqrt(3)*cosh(x)*sinh(x)^5 + sqrt(3)*sinh(x)^6 - sqrt(3))*arct
an(2/3*sqrt(3)*cosh(x) + 2/3*sqrt(3)*sinh(x) + 1/3*sqrt(3)) + 2*(sqrt(3)*c
osh(x)^6 + 6*sqrt(3)*cosh(x)^5*sinh(x) + 15*sqrt(3)*cosh(x)^4*sinh(x)^2 +
20*sqrt(3)*cosh(x)^3*sinh(x)^3 + 15*sqrt(3)*cosh(x)^2*sinh(x)^4 + 6*sqrt(3
)*cosh(x)*sinh(x)^5 + sqrt(3)*sinh(x)^6 - sqrt(3))*arctan(2/3*sqrt(3)*cosh
(x) + 2/3*sqrt(3)*sinh(x) - 1/3*sqrt(3)) + (cosh(x)^6 + 6*cosh(x)^5*sinh(x
) + 15*cosh(x)^4*sinh(x)^2 + 20*cosh(x)^3*sinh(x)^3 + 15*cosh(x)^2*sinh(x)
^4 + 6*cosh(x)*sinh(x)^5 + sinh(x)^6 - 1)*log((2*cosh(x) + 1)/(cosh(x) - s
inh(x))) - (cosh(x)^6 + 6*cosh(x)^5*sinh(x) + 15*cosh(x)^4*sinh(x)^2 + 20*
cosh(x)^3*sinh(x)^3 + 15*cosh(x)^2*sinh(x)^4 + 6*cosh(x)*sinh(x)^5 + sinh(
x)^6 - 1)*log((2*cosh(x) - 1)/(cosh(x) - sinh(x))) + 2*(cosh(x)^6 + 6*cosh
(x)^5*sinh(x) + 15*cosh(x)^4*sinh(x)^2 + 20*cosh(x)^3*sinh(x)^3 + 15*cosh(
x)^2*sinh(x)^4 + 6*cosh(x)*sinh(x)^5 + sinh(x)^6 - 1)*log(cosh(x) + sinh(x
) + 1) - 2*(cosh(x)^6 + 6*cosh(x)^5*sinh(x) + 15*cosh(x)^4*sinh(x)^2 + 20*
cosh(x)^3*sinh(x)^3 + 15*cosh(x)^2*sinh(x)^4 + 6*cosh(x)*sinh(x)^5 + sinh(
x)^6 - 1)*log(cosh(x) + sinh(x) - 1) + 12*cosh(x) + 12*sinh(x))/(cosh(x)^6
 + 6*cosh(x)^5*sinh(x) + 15*cosh(x)^4*sinh(x)^2 + 20*cosh(x)^3*sinh(x)^3 +
 15*cosh(x)^2*sinh(x)^4 + 6*cosh(x)*sinh(x)^5 + sinh(x)^6 - 1)
```

Sympy [F]

$$\int e^x \operatorname{csch}^2(3x)\,dx = \int e^x \operatorname{csch}^2(3x)\,dx$$

input
```
integrate(exp(x)*csch(3*x)**2,x)
```

output
```
Integral(exp(x)*csch(3*x)**2, x)
```

Maxima [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 85, normalized size of antiderivative = 0.93

$$\int e^x \operatorname{csch}^2(3x)\,dx = -\frac{1}{9}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2e^x+1)\right) - \frac{1}{9}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2e^x-1)\right) - \frac{2e^x}{3\left(e^{(6x)}-1\right)} - \frac{1}{18}\log\left(e^{(2x)}+e^x+1\right) + \frac{1}{18}\log\left(e^{(2x)}-e^x+1\right) - \frac{1}{9}\log\left(e^x+1\right) + \frac{1}{9}\log\left(e^x-1\right)$$

input
```
integrate(exp(x)*csch(3*x)^2,x, algorithm="maxima")
```

output
```
-1/9*sqrt(3)*arctan(1/3*sqrt(3)*(2*e^x + 1)) - 1/9*sqrt(3)*arctan(1/3*sqrt
(3)*(2*e^x - 1)) - 2/3*e^x/(e^(6*x) - 1) - 1/18*log(e^(2*x) + e^x + 1) + 1
/18*log(e^(2*x) - e^x + 1) - 1/9*log(e^x + 1) + 1/9*log(e^x - 1)
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 86, normalized size of antiderivative = 0.95

$$\int e^x \operatorname{csch}^2(3x)\,dx = -\frac{1}{9}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2e^x+1)\right) - \frac{1}{9}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2e^x-1)\right) - \frac{2e^x}{3\left(e^{(6x)}-1\right)} - \frac{1}{18}\log\left(e^{(2x)}+e^x+1\right) + \frac{1}{18}\log\left(e^{(2x)}-e^x+1\right) - \frac{1}{9}\log\left(e^x+1\right) + \frac{1}{9}\log\left(|e^x-1|\right)$$

input
```
integrate(exp(x)*csch(3*x)^2,x, algorithm="giac")
```

output
```
-1/9*sqrt(3)*arctan(1/3*sqrt(3)*(2*e^x + 1)) - 1/9*sqrt(3)*arctan(1/3*sqrt
(3)*(2*e^x - 1)) - 2/3*e^x/(e^(6*x) - 1) - 1/18*log(e^(2*x) + e^x + 1) + 1
/18*log(e^(2*x) - e^x + 1) - 1/9*log(e^x + 1) + 1/9*log(abs(e^x - 1))
```

Mupad [B] (verification not implemented)

Time = 0.40 (sec) , antiderivative size = 91, normalized size of antiderivative = 1.00

$$\int e^x \text{csch}^2(3x)\,dx = \frac{\ln\left(\frac{2}{3}-\frac{2\,e^x}{3}\right)}{9} - \frac{\ln\left(-\frac{2\,e^x}{3}-\frac{2}{3}\right)}{9} + \frac{\ln\left(\left(\frac{2\,e^x}{3}-\frac{1}{3}\right)^2+\frac{1}{3}\right)}{18} - \frac{\ln\left(\left(\frac{2\,e^x}{3}+\frac{1}{3}\right)^2+\frac{1}{3}\right)}{18} - \frac{2\,e^x}{3\,\left(e^{6\,x}-1\right)} - \frac{\sqrt{3}\,\text{atan}\left(\sqrt{3}\,\left(\frac{2\,e^x}{3}-\frac{1}{3}\right)\right)}{9} - \frac{\sqrt{3}\,\text{atan}\left(\sqrt{3}\,\left(\frac{2\,e^x}{3}+\frac{1}{3}\right)\right)}{9}$$

input
```
int(exp(x)/sinh(3*x)^2,x)
```

output
```
log(2/3 - (2*exp(x))/3)/9 - log(- (2*exp(x))/3 - 2/3)/9 + log(((2*exp(x))/
3 - 1/3)^2 + 1/3)/18 - log(((2*exp(x))/3 + 1/3)^2 + 1/3)/18 - (2*exp(x))/(
3*(exp(6*x) - 1)) - (3^(1/2)*atan(3^(1/2)*((2*exp(x))/3 - 1/3)))/9 - (3^(1
/2)*atan(3^(1/2)*((2*exp(x))/3 + 1/3)))/9
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 201, normalized size of antiderivative = 2.21

$$\int e^x \text{csch}^2(3x)\,dx = \frac{-2e^{6x}\sqrt{3}\,atan\left(\frac{2e^x-1}{\sqrt{3}}\right) + 2\sqrt{3}\,atan\left(\frac{2e^x-1}{\sqrt{3}}\right) - 2e^{6x}\sqrt{3}\,atan\left(\frac{2e^x+1}{\sqrt{3}}\right) + 2\sqrt{3}\,atan\left(\frac{2e^x+1}{\sqrt{3}}\right) - e^{6x}\log(e^{2x}+e$$

input
```
int(exp(x)*csch(3*x)^2,x)
```

output
```
( - 2*e**(6*x)*sqrt(3)*atan((2*e**x - 1)/sqrt(3)) + 2*sqrt(3)*atan((2*e**x
 - 1)/sqrt(3)) - 2*e**(6*x)*sqrt(3)*atan((2*e**x + 1)/sqrt(3)) + 2*sqrt(3)
*atan((2*e**x + 1)/sqrt(3)) - e**(6*x)*log(e**(2*x) + e**x + 1) + e**(6*x)
*log(e**(2*x) - e**x + 1) + 2*e**(6*x)*log(e**x - 1) - 2*e**(6*x)*log(e**x
 + 1) - 12*e**x + log(e**(2*x) + e**x + 1) - log(e**(2*x) - e**x + 1) - 2*
log(e**x - 1) + 2*log(e**x + 1))/(18*(e**(6*x) - 1))
```

3.318 $\int e^x \sinh^2(4x) \, dx$

Optimal result

Integrand size = 10, antiderivative size = 26

$$\int e^x \sinh^2(4x) \, dx = -\frac{1}{28} e^{-7x} - \frac{e^x}{2} + \frac{e^{9x}}{36}$$

output
```
-1/28/exp(7*x)-1/2*exp(x)+1/36*exp(9*x)
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.00

$$\int e^x \sinh^2(4x) \, dx = -\frac{1}{28} e^{-7x} - \frac{e^x}{2} + \frac{e^{9x}}{36}$$

input
```
Integrate[E^x*Sinh[4*x]^2,x]
```

output
```
-1/28*1/E^(7*x) - E^x/2 + E^(9*x)/36
```

Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.08, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.400, Rules used = {2720, 27, 802, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int e^x \sinh^2(4x)\,dx$$

$$\Big\downarrow \text{2720}$$

$$\int \frac{1}{4}e^{-8x}\left(1-e^{8x}\right)^2 de^x$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{4}\int e^{-8x}\left(1-e^{8x}\right)^2 de^x$$

$$\Big\downarrow \text{802}$$

$$\frac{1}{4}\int \left(-2+e^{-8x}+e^{8x}\right) de^x$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{4}\left(-\frac{1}{7}e^{-7x}-2e^x+\frac{e^{9x}}{9}\right)$$

input
```
Int[E^x*Sinh[4*x]^2,x]
```

output
```
(-1/7*1/E^(7*x) - 2*E^x + E^(9*x)/9)/4
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 802
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2720
```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Maple [A] (verified)

Time = 0.35 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.65

method	result	size
parallelrisch	$-\frac{e^{x} \left(\cosh \left(8 x \right)+63-8 \sinh \left(8 x \right)\right)}{126}$	17
risch	$\frac{e^{9 x}}{36}-\frac{e^{x}}{2}-\frac{e^{-7 x}}{28}$	18
default	$-\frac{\sinh \left(x \right)}{2}+\frac{\sinh \left(7 x \right)}{28}+\frac{\sinh \left(9 x \right)}{36}-\frac{\cosh \left(x \right)}{2}-\frac{\cosh \left(7 x \right)}{28}+\frac{\cosh \left(9 x \right)}{36}$	34
orering	$\frac{31 e^{x} \sinh \left(4 x \right)^{2}}{63}+\frac{8 e^{x} \sinh \left(4 x \right) \cosh \left(4 x \right)}{63}-\frac{32 e^{x} \cosh \left(4 x \right)^{2}}{63}$	34

input
```
int(exp(x)*sinh(4*x)^2,x,method=_RETURNVERBOSE)
```

output
```
-1/126*exp(x)*(cosh(8*x)+63-8*sinh(8*x))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 87 vs. $2(17) = 34$.

Time = 0.08 (sec) , antiderivative size = 87, normalized size of antiderivative = 3.35

$$\int e^x \sinh^2(4x)\,dx =$$
$$-\frac{\cosh(x)^8 - 64\cosh(x)^7\sinh(x) + 28\cosh(x)^6\sinh(x)^2 - 448\cosh(x)^5\sinh(x)^3 + 70\cosh(x)^4\sinh$$
$$126(\cosh(x) -$$

input
```
integrate(exp(x)*sinh(4*x)^2,x, algorithm="fricas")
```

output
```
-1/126*(cosh(x)^8 - 64*cosh(x)^7*sinh(x) + 28*cosh(x)^6*sinh(x)^2 - 448*co
sh(x)^5*sinh(x)^3 + 70*cosh(x)^4*sinh(x)^4 - 448*cosh(x)^3*sinh(x)^5 + 28*
cosh(x)^2*sinh(x)^6 - 64*cosh(x)*sinh(x)^7 + sinh(x)^8 + 63)/(cosh(x) - si
nh(x))
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 42 vs. $2(19) = 38$.

Time = 0.19 (sec) , antiderivative size = 42, normalized size of antiderivative = 1.62

$$\int e^x \sinh^2(4x)\,dx = \frac{31e^x\sinh^2(4x)}{63} + \frac{8e^x\sinh(4x)\cosh(4x)}{63} - \frac{32e^x\cosh^2(4x)}{63}$$

input
```
integrate(exp(x)*sinh(4*x)**2,x)
```

output
```
31*exp(x)*sinh(4*x)**2/63 + 8*exp(x)*sinh(4*x)*cosh(4*x)/63 - 32*exp(x)*co
sh(4*x)**2/63
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.65

$$\int e^x \sinh^2(4x)\,dx = \frac{1}{36}\,e^{(9\,x)} - \frac{1}{28}\,e^{(-7\,x)} - \frac{1}{2}\,e^x$$

input
```
integrate(exp(x)*sinh(4*x)^2,x, algorithm="maxima")
```

output
```
1/36*e^(9*x) - 1/28*e^(-7*x) - 1/2*e^x
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.65

$$\int e^x \sinh^2(4x)\,dx = \frac{1}{36}\,e^{(9\,x)} - \frac{1}{28}\,e^{(-7\,x)} - \frac{1}{2}\,e^x$$

input
```
integrate(exp(x)*sinh(4*x)^2,x, algorithm="giac")
```

output
```
1/36*e^(9*x) - 1/28*e^(-7*x) - 1/2*e^x
```

Mupad [B] (verification not implemented)

Time = 1.60 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.65

$$\int e^x \sinh^2(4x)\,dx = \frac{e^{9\,x}}{36} - \frac{e^{-7\,x}}{28} - \frac{e^x}{2}$$

input
```
int(sinh(4*x)^2*exp(x),x)
```

output
```
exp(9*x)/36 - exp(-7*x)/28 - exp(x)/2
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.96

$$\int e^x \sinh^2(4x)\,dx = \frac{7e^{16x} - 126e^{8x} - 9}{252e^{7x}}$$

input
```
int(exp(x)*sinh(4*x)^2,x)
```

output
```
(7*e**(16*x) - 126*e**(8*x) - 9)/(252*e**(7*x))
```

3.319 $\int e^x \sinh(4x)\,dx$

Optimal result

Integrand size = 8, antiderivative size = 19

$$\int e^x \sinh(4x)\,dx = \frac{e^{-3x}}{6} + \frac{e^{5x}}{10}$$

output
```
1/6/exp(3*x)+1/10*exp(5*x)
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.00

$$\int e^x \sinh(4x)\,dx = \frac{e^{-3x}}{6} + \frac{e^{5x}}{10}$$

input
```
Integrate[E^x*Sinh[4*x],x]
```

output
```
1/(6*E^(3*x)) + E^(5*x)/10
```

Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.21, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$, Rules used = {2720, 27, 802, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int e^x \sinh(4x)\,dx$$

$$\Big\downarrow \ 2720$$

$$\int -\frac{1}{2}e^{-4x}\left(1-e^{8x}\right)de^x$$

$$\Big\downarrow \ 27$$

$$-\frac{1}{2}\int e^{-4x}\left(1-e^{8x}\right)de^x$$

$$\Big\downarrow \ 802$$

$$-\frac{1}{2}\int \left(e^{-4x}-e^{4x}\right)de^x$$

$$\Big\downarrow \ 2009$$

$$\frac{1}{2}\left(\frac{e^{-3x}}{3}+\frac{e^{5x}}{5}\right)$$

input
```
Int[E^x*Sinh[4*x],x]
```

output
```
(1/(3*E^(3*x)) + E^(5*x)/5)/2
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 802
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2720
```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Maple [A] (verified)

Time = 0.13 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.74

method	result	size
risch	$\frac{{\mathrm e}^{5 x}}{10}+\frac{{\mathrm e}^{-3 x}}{6}$	14
parallelrisch	$\frac{{\mathrm e}^{x} \left(4 \cosh \left(4 x \right)-\sinh \left(4 x \right)\right)}{15}$	18
orering	$-\frac{{\mathrm e}^{x} \sinh \left(4 x \right)}{15}+\frac{4 \,{\mathrm e}^{x} \cosh \left(4 x \right)}{15}$	18
default	$-\frac{\sinh \left(3 x \right)}{6}+\frac{\sinh \left(5 x \right)}{10}+\frac{\cosh \left(3 x \right)}{6}+\frac{\cosh \left(5 x \right)}{10}$	26

input
```
int(exp(x)*sinh(4*x),x,method=_RETURNVERBOSE)
```

output
```
1/10*exp(5*x)+1/6*exp(-3*x)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 46 vs. $2(13) = 26$.

Time = 0.10 (sec) , antiderivative size = 46, normalized size of antiderivative = 2.42

$$\int e^x \sinh(4x)\,dx = \frac{4\left(\cosh(x)^4 - \cosh(x)^3\sinh(x) + 6\cosh(x)^2\sinh(x)^2 - \cosh(x)\sinh(x)^3 + \sinh(x)^4\right)}{15\left(\cosh(x) - \sinh(x)\right)}$$

input
```
integrate(exp(x)*sinh(4*x),x, algorithm="fricas")
```

output
```
4/15*(cosh(x)^4 - cosh(x)^3*sinh(x) + 6*cosh(x)^2*sinh(x)^2 - cosh(x)*sinh
(x)^3 + sinh(x)^4)/(cosh(x) - sinh(x))
```

Sympy [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.05

$$\int e^x \sinh(4x)\,dx = -\frac{e^x \sinh(4x)}{15} + \frac{4e^x \cosh(4x)}{15}$$

input
```
integrate(exp(x)*sinh(4*x),x)
```

output
```
-exp(x)*sinh(4*x)/15 + 4*exp(x)*cosh(4*x)/15
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.68

$$\int e^x \sinh(4x)\,dx = \frac{1}{10}\,e^{(5\,x)} + \frac{1}{6}\,e^{(-3\,x)}$$

input
```
integrate(exp(x)*sinh(4*x),x, algorithm="maxima")
```

output
```
1/10*e^(5*x) + 1/6*e^(-3*x)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.68

$$\int e^x \sinh(4x)\,dx = \frac{1}{10}\,e^{(5\,x)} + \frac{1}{6}\,e^{(-3\,x)}$$

input
```
integrate(exp(x)*sinh(4*x),x, algorithm="giac")
```

output
```
1/10*e^(5*x) + 1/6*e^(-3*x)
```

Mupad [B] (verification not implemented)

Time = 1.55 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.68

$$\int e^x \sinh(4x)\,dx = \frac{\mathrm{e}^{-3\,x}}{6} + \frac{\mathrm{e}^{5\,x}}{10}$$

input
```
int(sinh(4*x)*exp(x),x)
```

output
```
exp(-3*x)/6 + exp(5*x)/10
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.95

$$\int e^x \sinh(4x)\,dx = \frac{e^x(4\cosh(4x) - \sinh(4x))}{15}$$

input
```
int(exp(x)*sinh(4*x),x)
```

output
```
(e**x*(4*cosh(4*x) - sinh(4*x)))/15
```

3.320 $\int e^x \mathbf{csch}(4x)\,dx$

Optimal result

Integrand size = 8, antiderivative size = 88

$$\int e^x \operatorname{csch}(4x)\,dx = -\frac{1}{2}\arctan\left(e^x\right) - \frac{\arctan\left(1-\sqrt{2}e^x\right)}{2\sqrt{2}} + \frac{\arctan\left(1+\sqrt{2}e^x\right)}{2\sqrt{2}} - \frac{\operatorname{arctanh}(e^x)}{2} + \frac{\operatorname{arctanh}\left(\frac{\sqrt{2}e^x}{1+e^{2x}}\right)}{2\sqrt{2}}$$

output
```
-1/2*arctan(exp(x))+1/4*arctan(-1+2^(1/2)*exp(x))*2^(1/2)+1/4*arctan(1+2^(
1/2)*exp(x))*2^(1/2)-1/2*arctanh(exp(x))+1/4*arctanh(2^(1/2)*exp(x)/(1+exp
(2*x)))*2^(1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 0.01 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.25

$$\int e^x \operatorname{csch}(4x)\,dx = -\frac{2}{5}e^{5x}\,\text{Hypergeometric2F1}\left(\frac{5}{8},1,\frac{13}{8},e^{8x}\right)$$

input
```
Integrate[E^x*Csch[4*x],x]
```

output
```
(-2*E^(5*x)*Hypergeometric2F1[5/8, 1, 13/8, E^(8*x)])/5
```

Rubi [A] (verified)

Time = 0.38 (sec) , antiderivative size = 130, normalized size of antiderivative = 1.48, number of steps used = 15, number of rules used = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 1.750$, Rules used = {2720, 27, 830, 755, 756, 216, 219, 1476, 1082, 217, 1479, 25, 27, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int e^x \text{csch}(4x)\,dx$$

$$\Big\downarrow \text{2720}$$

$$\int -\frac{2e^{4x}}{1-e^{8x}}de^x$$

$$\Big\downarrow \text{27}$$

$$-2\int \frac{e^{4x}}{1-e^{8x}}de^x$$

$$\Big\downarrow \text{830}$$

$$-2\left(\frac{1}{2}\int \frac{1}{1-e^{4x}}de^x-\frac{1}{2}\int \frac{1}{1+e^{4x}}de^x\right)$$

$$\Big\downarrow \text{755}$$

$$-2\left(\frac{1}{2}\int \frac{1}{1-e^{4x}}de^x+\frac{1}{2}\left(-\frac{1}{2}\int \frac{1-e^{2x}}{1+e^{4x}}de^x-\frac{1}{2}\int \frac{1+e^{2x}}{1+e^{4x}}de^x\right)\right)$$

$$\Big\downarrow \text{756}$$

$$-2\left(\frac{1}{2}\left(\frac{1}{2}\int \frac{1}{1-e^{2x}}de^x+\frac{1}{2}\int \frac{1}{1+e^{2x}}de^x\right)+\frac{1}{2}\left(-\frac{1}{2}\int \frac{1-e^{2x}}{1+e^{4x}}de^x-\frac{1}{2}\int \frac{1+e^{2x}}{1+e^{4x}}de^x\right)\right)$$

$$\Big\downarrow \text{216}$$

$$-2\left(\frac{1}{2}\left(\frac{1}{2}\int \frac{1}{1-e^{2x}}de^x+\frac{\arctan(e^x)}{2}\right)+\frac{1}{2}\left(-\frac{1}{2}\int \frac{1-e^{2x}}{1+e^{4x}}de^x-\frac{1}{2}\int \frac{1+e^{2x}}{1+e^{4x}}de^x\right)\right)$$

$$\Big\downarrow \text{219}$$

$$-2\left(\frac{1}{2}\left(-\frac{1}{2}\int \frac{1-e^{2x}}{1+e^{4x}}de^x-\frac{1}{2}\int \frac{1+e^{2x}}{1+e^{4x}}de^x\right)+\frac{1}{2}\left(\frac{\arctan\left(e^x\right)}{2}+\frac{\text{arctanh}(e^x)}{2}\right)\right)$$

$$\Big\downarrow \; 1476$$

$$-2\left(\frac{1}{2}\left(\frac{1}{2}\left(-\frac{1}{2}\int \frac{1}{1-\sqrt{2}e^x+e^{2x}}de^x-\frac{1}{2}\int \frac{1}{1+\sqrt{2}e^x+e^{2x}}de^x\right)-\frac{1}{2}\int \frac{1-e^{2x}}{1+e^{4x}}de^x\right)+\frac{1}{2}\left(\frac{\arctan\left(e^x\right)}{2}+\frac{\text{arct}}{}\right.\right.$$

$$\Big\downarrow \; 1082$$

$$-2\left(\frac{1}{2}\left(\frac{1}{2}\left(\frac{\int \frac{1}{-1-e^{2x}}d\left(1+\sqrt{2}e^x\right)}{\sqrt{2}}-\frac{\int \frac{1}{-1-e^{2x}}d\left(1-\sqrt{2}e^x\right)}{\sqrt{2}}\right)-\frac{1}{2}\int \frac{1-e^{2x}}{1+e^{4x}}de^x\right)+\frac{1}{2}\left(\frac{\arctan\left(e^x\right)}{2}+\frac{\text{arctanh}(e}{2}\right.\right.$$

$$\Big\downarrow \; 217$$

$$-2\left(\frac{1}{2}\left(\frac{1}{2}\left(\frac{\arctan\left(1-\sqrt{2}e^x\right)}{\sqrt{2}}-\frac{\arctan\left(\sqrt{2}e^x+1\right)}{\sqrt{2}}\right)-\frac{1}{2}\int \frac{1-e^{2x}}{1+e^{4x}}de^x\right)+\frac{1}{2}\left(\frac{\arctan\left(e^x\right)}{2}+\frac{\text{arctanh}(e^x)}{2}\right)\right)$$

$$\Big\downarrow \; 1479$$

$$-2\left(\frac{1}{2}\left(\frac{1}{2}\left(\frac{\int -\frac{\sqrt{2}-2e^x}{1-\sqrt{2}e^x+e^{2x}}de^x}{2\sqrt{2}}+\frac{\int -\frac{\sqrt{2}\left(1+\sqrt{2}e^x\right)}{1+\sqrt{2}e^x+e^{2x}}de^x}{2\sqrt{2}}\right)+\frac{1}{2}\left(\frac{\arctan\left(1-\sqrt{2}e^x\right)}{\sqrt{2}}-\frac{\arctan\left(\sqrt{2}e^x+1\right)}{\sqrt{2}}\right)\right)+\frac{1}{2}\right.$$

$$\Big\downarrow \; 25$$

$$-2\left(\frac{1}{2}\left(\frac{1}{2}\left(-\frac{\int \frac{\sqrt{2}-2e^x}{1-\sqrt{2}e^x+e^{2x}}de^x}{2\sqrt{2}}-\frac{\int \frac{\sqrt{2}\left(1+\sqrt{2}e^x\right)}{1+\sqrt{2}e^x+e^{2x}}de^x}{2\sqrt{2}}\right)+\frac{1}{2}\left(\frac{\arctan\left(1-\sqrt{2}e^x\right)}{\sqrt{2}}-\frac{\arctan\left(\sqrt{2}e^x+1\right)}{\sqrt{2}}\right)\right)+\frac{1}{2}\left(\right.\right.$$

$$\Big\downarrow \; 27$$

$$-2\left(\frac{1}{2}\left(\frac{1}{2}\left(-\frac{\int \frac{\sqrt{2}-2e^x}{1-\sqrt{2}e^x+e^{2x}}de^x}{2\sqrt{2}}-\frac{1}{2}\int \frac{1+\sqrt{2}e^x}{1+\sqrt{2}e^x+e^{2x}}de^x\right)+\frac{1}{2}\left(\frac{\arctan\left(1-\sqrt{2}e^x\right)}{\sqrt{2}}-\frac{\arctan\left(\sqrt{2}e^x+1\right)}{\sqrt{2}}\right)\right)\right.$$

$$\Big\downarrow \; 1103$$

$$-2\left(\frac{1}{2}\left(\frac{\arctan\left(e^x\right)}{2}+\frac{\text{arctanh}(e^x)}{2}\right)+\frac{1}{2}\left(\frac{1}{2}\left(\frac{\arctan\left(1-\sqrt{2}e^x\right)}{\sqrt{2}}-\frac{\arctan\left(\sqrt{2}e^x+1\right)}{\sqrt{2}}\right)+\frac{1}{2}\left(\frac{\log\left(-\sqrt{2}e^x+e\right.}{2\sqrt{2}}\right.\right.\right.$$

input
```
Int[E^x*Csch[4*x],x]
```

output
```
-2*((ArcTan[E^x]/2 + ArcTanh[E^x]/2)/2 + ((ArcTan[1 - Sqrt[2]*E^x]/Sqrt[2]
 - ArcTan[1 + Sqrt[2]*E^x]/Sqrt[2])/2 + (Log[1 - Sqrt[2]*E^x + E^(2*x)]/(2
*Sqrt[2]) - Log[1 + Sqrt[2]*E^x + E^(2*x)]/(2*Sqrt[2]))/2)/2)
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 755
```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Simp[1/(2*r)   Int[(r - s*x^2)/(a + b*x^4)
, x], x] + Simp[1/(2*r)   Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

rule 756
```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-a/b, 2
]], s = Denominator[Rt[-a/b, 2]]}, Simp[r/(2*a)   Int[1/(r - s*x^2), x], x]
 + Simp[r/(2*a)   Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ[a
/b, 0]
```

rule 830
```
Int[(x_)^(m_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{r = Numerator[Rt
[-a/b, 2]], s = Denominator[Rt[-a/b, 2]]}, Simp[s/(2*b)   Int[x^(m - n/2)/(
r + s*x^(n/2)), x], x] - Simp[s/(2*b)   Int[x^(m - n/2)/(r - s*x^(n/2)), x]
, x]] /; FreeQ[{a, b}, x] && IGtQ[n/4, 0] && IGtQ[m, 0] && LeQ[n/2, m] && L
tQ[m, n] &&  !GtQ[a/b, 0]
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1476
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Simp[e/(2*c)   Int[1/Simp[d/e + q*x + x^2, x], x], x] + Simp[
e/(2*c)   Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x]
 && EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

rule 1479
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Simp[e/(2*c*q)   Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
 x] + Simp[e/(2*c*q)   Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; F
reeQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

rule 2720
```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.20 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.64

method	result	size
risch	$\frac{\ln(e^x-1)}{4}+\frac{i\ln(e^x-i)}{4}-\frac{i\ln(e^x+i)}{4}-\frac{\ln(e^x+1)}{4}+2\left(\sum_{\_R=\mathrm{RootOf}(4096\_Z^4+1)}\_R\ln\left(e^x+8\_R\right)\right)$	56

input
```
int(exp(x)*csch(4*x),x,method=_RETURNVERBOSE)
```

output
```
1/4*ln(exp(x)-1)+1/4*I*ln(exp(x)-I)-1/4*I*ln(exp(x)+I)-1/4*ln(exp(x)+1)+2*
sum(_R*ln(exp(x)+8*_R),_R=RootOf(4096*_Z^4+1))
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 116, normalized size of antiderivative = 1.32

$$\begin{aligned}\int e^x\mathrm{csch}(4x)\,dx=&\frac{1}{4}\sqrt{2}\arctan\left(\sqrt{2}\cosh(x)+\sqrt{2}\sinh(x)+1\right)\\&+\frac{1}{4}\sqrt{2}\arctan\left(\sqrt{2}\cosh(x)+\sqrt{2}\sinh(x)-1\right)\\&+\frac{1}{8}\sqrt{2}\log\left(\frac{\sqrt{2}+2\cosh(x)}{\cosh(x)-\sinh(x)}\right)\\&-\frac{1}{8}\sqrt{2}\log\left(-\frac{\sqrt{2}-2\cosh(x)}{\cosh(x)-\sinh(x)}\right)-\frac{1}{2}\arctan(\cosh(x)+\sinh(x))\\&-\frac{1}{4}\log(\cosh(x)+\sinh(x)+1)+\frac{1}{4}\log(\cosh(x)+\sinh(x)-1)\end{aligned}$$

input
```
integrate(exp(x)*csch(4*x),x, algorithm="fricas")
```

output
```
1/4*sqrt(2)*arctan(sqrt(2)*cosh(x) + sqrt(2)*sinh(x) + 1) + 1/4*sqrt(2)*ar
ctan(sqrt(2)*cosh(x) + sqrt(2)*sinh(x) - 1) + 1/8*sqrt(2)*log((sqrt(2) + 2
*cosh(x))/(cosh(x) - sinh(x))) - 1/8*sqrt(2)*log(-(sqrt(2) - 2*cosh(x))/(c
osh(x) - sinh(x))) - 1/2*arctan(cosh(x) + sinh(x)) - 1/4*log(cosh(x) + sin
h(x) + 1) + 1/4*log(cosh(x) + sinh(x) - 1)
```

Sympy [F]

$$\int e^x \operatorname{csch}(4x)\,dx = \int e^x \operatorname{csch}(4x)\,dx$$

input
```
integrate(exp(x)*csch(4*x),x)
```

output
```
Integral(exp(x)*csch(4*x), x)
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 95, normalized size of antiderivative = 1.08

$$\begin{aligned}\int e^x \operatorname{csch}(4x)\,dx &= \frac{1}{4}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\left(\sqrt{2}+2e^x\right)\right)\\ &\quad+\frac{1}{4}\sqrt{2}\arctan\left(-\frac{1}{2}\sqrt{2}\left(\sqrt{2}-2e^x\right)\right)\\ &\quad+\frac{1}{8}\sqrt{2}\log\left(\sqrt{2}e^x+e^{(2x)}+1\right)-\frac{1}{8}\sqrt{2}\log\left(-\sqrt{2}e^x+e^{(2x)}+1\right)\\ &\quad-\frac{1}{2}\arctan\left(e^x\right)-\frac{1}{4}\log\left(e^x+1\right)+\frac{1}{4}\log\left(e^x-1\right)\end{aligned}$$

input
```
integrate(exp(x)*csch(4*x),x, algorithm="maxima")
```

output
```
1/4*sqrt(2)*arctan(1/2*sqrt(2)*(sqrt(2) + 2*e^x)) + 1/4*sqrt(2)*arctan(-1/
2*sqrt(2)*(sqrt(2) - 2*e^x)) + 1/8*sqrt(2)*log(sqrt(2)*e^x + e^(2*x) + 1)
- 1/8*sqrt(2)*log(-sqrt(2)*e^x + e^(2*x) + 1) - 1/2*arctan(e^x) - 1/4*log(
e^x + 1) + 1/4*log(e^x - 1)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 96, normalized size of antiderivative = 1.09

$$\int e^x \operatorname{csch}(4x)\,dx = \frac{1}{4}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\left(\sqrt{2}+2e^x\right)\right) + \frac{1}{4}\sqrt{2}\arctan\left(-\frac{1}{2}\sqrt{2}\left(\sqrt{2}-2e^x\right)\right) + \frac{1}{8}\sqrt{2}\log\left(\sqrt{2}e^x+e^{(2x)}+1\right) - \frac{1}{8}\sqrt{2}\log\left(-\sqrt{2}e^x+e^{(2x)}+1\right) - \frac{1}{2}\arctan\left(e^x\right) - \frac{1}{4}\log\left(e^x+1\right) + \frac{1}{4}\log\left(|e^x-1|\right)$$

input
```
integrate(exp(x)*csch(4*x),x, algorithm="giac")
```

output
```
1/4*sqrt(2)*arctan(1/2*sqrt(2)*(sqrt(2) + 2*e^x)) + 1/4*sqrt(2)*arctan(-1/
2*sqrt(2)*(sqrt(2) - 2*e^x)) + 1/8*sqrt(2)*log(sqrt(2)*e^x + e^(2*x) + 1)
- 1/8*sqrt(2)*log(-sqrt(2)*e^x + e^(2*x) + 1) - 1/2*arctan(e^x) - 1/4*log(
e^x + 1) + 1/4*log(abs(e^x - 1))
```

Mupad [B] (verification not implemented)

Time = 0.37 (sec) , antiderivative size = 106, normalized size of antiderivative = 1.20

$$\int e^x \operatorname{csch}(4x)\,dx = \frac{\ln\left(128-128\,\mathrm{e}^x\right)}{4} - \frac{\ln\left(-128\,\mathrm{e}^x-128\right)}{4} - \frac{\mathrm{atan}(\mathrm{e}^x)}{2} + \frac{\sqrt{2}\,\mathrm{atan}\left(\frac{\sqrt{2}\left(128\,\mathrm{e}^x-64\sqrt{2}\right)}{128}\right)}{4} + \frac{\sqrt{2}\,\mathrm{atan}\left(\frac{\sqrt{2}\left(128\,\mathrm{e}^x+64\sqrt{2}\right)}{128}\right)}{4} - \frac{\sqrt{2}\,\ln\left(\left(128\,\mathrm{e}^x-64\sqrt{2}\right)^2+8192\right)}{8} + \frac{\sqrt{2}\,\ln\left(\left(128\,\mathrm{e}^x+64\sqrt{2}\right)^2+8192\right)}{8}$$

input
```
int(exp(x)/sinh(4*x),x)
```

output
```
log(128 - 128*exp(x))/4 - log(- 128*exp(x) - 128)/4 - atan(exp(x))/2 + (2^
(1/2)*atan((2^(1/2)*(128*exp(x) - 64*2^(1/2)))/128))/4 + (2^(1/2)*atan((2^
(1/2)*(128*exp(x) + 64*2^(1/2)))/128))/4 - (2^(1/2)*log((128*exp(x) - 64*2
^(1/2))^2 + 8192))/8 + (2^(1/2)*log((128*exp(x) + 64*2^(1/2))^2 + 8192))/8
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 98, normalized size of antiderivative = 1.11

$$\int e^x \operatorname{csch}(4x)\,dx = -\frac{atan(e^x)}{2} + \frac{\sqrt{2}\,atan\left(\frac{2e^x-\sqrt{2}}{\sqrt{2}}\right)}{4} + \frac{\sqrt{2}\,atan\left(\frac{2e^x+\sqrt{2}}{\sqrt{2}}\right)}{4} - \frac{\sqrt{2}\log\left(e^{2x}-e^x\sqrt{2}+1\right)}{8} + \frac{\sqrt{2}\log\left(e^{2x}+e^x\sqrt{2}+1\right)}{8} + \frac{\log(e^x-1)}{4} - \frac{\log(e^x+1)}{4}$$

input
```
int(exp(x)*csch(4*x),x)
```

output
```
( - 4*atan(e**x) + 2*sqrt(2)*atan((2*e**x - sqrt(2))/sqrt(2)) + 2*sqrt(2)*
atan((2*e**x + sqrt(2))/sqrt(2)) - sqrt(2)*log(e**(2*x) - e**x*sqrt(2) + 1
) + sqrt(2)*log(e**(2*x) + e**x*sqrt(2) + 1) + 2*log(e**x - 1) - 2*log(e**
x + 1))/8
```

3.321 $\int e^x \mathrm{csch}^2(4x)\,dx$

Optimal result

Integrand size = 10, antiderivative size = 106

$$\int e^x \mathrm{csch}^2(4x)\,dx = \frac{e^x}{2(1-e^{8x})} - \frac{\arctan(e^x)}{8} + \frac{\arctan\left(1-\sqrt{2}e^x\right)}{8\sqrt{2}} - \frac{\arctan\left(1+\sqrt{2}e^x\right)}{8\sqrt{2}} - \frac{\mathrm{arctanh}(e^x)}{8} - \frac{\mathrm{arctanh}\left(\frac{\sqrt{2}e^x}{1+e^{2x}}\right)}{8\sqrt{2}}$$

output
```
exp(x)/(2-2*exp(8*x))-1/8*arctan(exp(x))-1/16*arctan(-1+2^(1/2)*exp(x))*2^
(1/2)-1/16*arctan(1+2^(1/2)*exp(x))*2^(1/2)-1/8*arctanh(exp(x))-1/16*arcta
nh(2^(1/2)*exp(x)/(1+exp(2*x)))*2^(1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 0.02 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.32

$$\int e^x \mathrm{csch}^2(4x)\,dx = \frac{1}{2}e^x\left(\frac{1}{1-e^{8x}} - \text{Hypergeometric2F1}\left(\frac{1}{8},1,\frac{9}{8},e^{8x}\right)\right)$$

input
```
Integrate[E^x*Csch[4*x]^2,x]
```

output
```
(E^x*((1 - E^(8*x))^(-1) - Hypergeometric2F1[1/8, 1, 9/8, E^(8*x)]))/2
```

Rubi [A] (verified)

Time = 0.38 (sec) , antiderivative size = 153, normalized size of antiderivative = 1.44, number of steps used = 16, number of rules used = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 1.500, Rules used = {2720, 27, 817, 758, 755, 756, 216, 219, 1476, 1082, 217, 1479, 25, 27, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int e^x \text{csch}^2(4x)\,dx$$

$$\Big\downarrow \text{2720}$$

$$\int \frac{4e^{8x}}{(1-e^{8x})^2}de^x$$

$$\Big\downarrow \text{27}$$

$$4\int \frac{e^{8x}}{(1-e^{8x})^2}de^x$$

$$\Big\downarrow \text{817}$$

$$4\left(\frac{e^x}{8(1-e^{8x})}-\frac{1}{8}\int \frac{1}{1-e^{8x}}de^x\right)$$

$$\Big\downarrow \text{758}$$

$$4\left(\frac{1}{8}\left(-\frac{1}{2}\int \frac{1}{1-e^{4x}}de^x-\frac{1}{2}\int \frac{1}{1+e^{4x}}de^x\right)+\frac{e^x}{8(1-e^{8x})}\right)$$

$$\Big\downarrow \text{755}$$

$$4\left(\frac{1}{8}\left(\frac{1}{2}\left(-\frac{1}{2}\int \frac{1-e^{2x}}{1+e^{4x}}de^x-\frac{1}{2}\int \frac{1+e^{2x}}{1+e^{4x}}de^x\right)-\frac{1}{2}\int \frac{1}{1-e^{4x}}de^x\right)+\frac{e^x}{8(1-e^{8x})}\right)$$

$$\Big\downarrow \text{756}$$

$$4\left(\frac{1}{8}\left(\frac{1}{2}\left(-\frac{1}{2}\int \frac{1}{1-e^{2x}}de^x-\frac{1}{2}\int \frac{1}{1+e^{2x}}de^x\right)+\frac{1}{2}\left(-\frac{1}{2}\int \frac{1-e^{2x}}{1+e^{4x}}de^x-\frac{1}{2}\int \frac{1+e^{2x}}{1+e^{4x}}de^x\right)\right)+\frac{e^x}{8(1-e^{8x})}\right)$$

$$\Big\downarrow \text{216}$$

$$4\left(\frac{1}{8}\left(\frac{1}{2}\left(-\frac{1}{2}\int\frac{1}{1-e^{2x}}de^x-\frac{1}{2}\arctan\left(e^x\right)\right)+\frac{1}{2}\left(-\frac{1}{2}\int\frac{1-e^{2x}}{1+e^{4x}}de^x-\frac{1}{2}\int\frac{1+e^{2x}}{1+e^{4x}}de^x\right)\right)+\frac{e^x}{8\left(1-e^{8x}\right)}\right)$$

$\Big\downarrow$ 219

$$4\left(\frac{1}{8}\left(\frac{1}{2}\left(-\frac{1}{2}\int\frac{1-e^{2x}}{1+e^{4x}}de^x-\frac{1}{2}\int\frac{1+e^{2x}}{1+e^{4x}}de^x\right)+\frac{1}{2}\left(-\frac{1}{2}\arctan\left(e^x\right)-\frac{\text{arctanh}(e^x)}{2}\right)\right)+\frac{e^x}{8\left(1-e^{8x}\right)}\right)$$

$\Big\downarrow$ 1476

$$4\left(\frac{1}{8}\left(\frac{1}{2}\left(\frac{1}{2}\left(-\frac{1}{2}\int\frac{1}{1-\sqrt{2}e^x+e^{2x}}de^x-\frac{1}{2}\int\frac{1}{1+\sqrt{2}e^x+e^{2x}}de^x\right)-\frac{1}{2}\int\frac{1-e^{2x}}{1+e^{4x}}de^x\right)+\frac{1}{2}\left(-\frac{1}{2}\arctan\left(e^x\right)-\right.\right.\right.$$

$\Big\downarrow$ 1082

$$4\left(\frac{1}{8}\left(\frac{1}{2}\left(\frac{1}{2}\left(\frac{\int\frac{1}{-1-e^{2x}}d\left(1+\sqrt{2}e^x\right)}{\sqrt{2}}-\frac{\int\frac{1}{-1-e^{2x}}d\left(1-\sqrt{2}e^x\right)}{\sqrt{2}}\right)-\frac{1}{2}\int\frac{1-e^{2x}}{1+e^{4x}}de^x\right)+\frac{1}{2}\left(-\frac{1}{2}\arctan\left(e^x\right)-\frac{\text{arcta}}{}\right.\right.\right.$$

$\Big\downarrow$ 217

$$4\left(\frac{1}{8}\left(\frac{1}{2}\left(\frac{1}{2}\left(\frac{\arctan\left(1-\sqrt{2}e^x\right)}{\sqrt{2}}-\frac{\arctan\left(\sqrt{2}e^x+1\right)}{\sqrt{2}}\right)-\frac{1}{2}\int\frac{1-e^{2x}}{1+e^{4x}}de^x\right)+\frac{1}{2}\left(-\frac{1}{2}\arctan\left(e^x\right)-\frac{\text{arctanh}(e^x}{2}\right.\right.\right.$$

$\Big\downarrow$ 1479

$$4\left(\frac{1}{8}\left(\frac{1}{2}\left(\frac{1}{2}\left(\frac{\int-\frac{\sqrt{2}-2e^x}{1-\sqrt{2}e^x+e^{2x}}de^x}{2\sqrt{2}}+\frac{\int-\frac{\sqrt{2}\left(1+\sqrt{2}e^x\right)}{1+\sqrt{2}e^x+e^{2x}}de^x}{2\sqrt{2}}\right)+\frac{1}{2}\left(\frac{\arctan\left(1-\sqrt{2}e^x\right)}{\sqrt{2}}-\frac{\arctan\left(\sqrt{2}e^x+1\right)}{\sqrt{2}}\right)\right)+\right.\right.$$

$\Big\downarrow$ 25

$$4\left(\frac{1}{8}\left(\frac{1}{2}\left(\frac{1}{2}\left(-\frac{\int\frac{\sqrt{2}-2e^x}{1-\sqrt{2}e^x+e^{2x}}de^x}{2\sqrt{2}}-\frac{\int\frac{\sqrt{2}\left(1+\sqrt{2}e^x\right)}{1+\sqrt{2}e^x+e^{2x}}de^x}{2\sqrt{2}}\right)+\frac{1}{2}\left(\frac{\arctan\left(1-\sqrt{2}e^x\right)}{\sqrt{2}}-\frac{\arctan\left(\sqrt{2}e^x+1\right)}{\sqrt{2}}\right)\right)+\frac{1}{2}\right.\right.$$

$\Big\downarrow$ 27

$$4\left(\frac{1}{8}\left(\frac{1}{2}\left(\frac{1}{2}\left(-\frac{\int\frac{\sqrt{2}-2e^x}{1-\sqrt{2}e^x+e^{2x}}de^x}{2\sqrt{2}}-\frac{1}{2}\int\frac{1+\sqrt{2}e^x}{1+\sqrt{2}e^x+e^{2x}}de^x\right)+\frac{1}{2}\left(\frac{\arctan\left(1-\sqrt{2}e^x\right)}{\sqrt{2}}-\frac{\arctan\left(\sqrt{2}e^x+1\right)}{\sqrt{2}}\right)\right.\right.\right.$$

$$\Big\downarrow \text{1103}$$

$$4\left(\frac{1}{8}\left(\frac{1}{2}\left(-\frac{1}{2}\arctan\left(e^x\right)-\frac{\text{arctanh}(e^x)}{2}\right)+\frac{1}{2}\left(\frac{1}{2}\left(\frac{\arctan\left(1-\sqrt{2}e^x\right)}{\sqrt{2}}-\frac{\arctan\left(\sqrt{2}e^x+1\right)}{\sqrt{2}}\right)+\frac{1}{2}\left(\frac{\log\left(-\sqrt{2}e^x\right.}{2}\right.\right.\right.\right.$$

input
```
Int[E^x*Csch[4*x]^2,x]
```

output
```
4*(E^x/(8*(1 - E^(8*x))) + ((-1/2*ArcTan[E^x] - ArcTanh[E^x]/2)/2 + ((ArcT
an[1 - Sqrt[2]*E^x]/Sqrt[2] - ArcTan[1 + Sqrt[2]*E^x]/Sqrt[2])/2 + (Log[1
- Sqrt[2]*E^x + E^(2*x)]/(2*Sqrt[2]) - Log[1 + Sqrt[2]*E^x + E^(2*x)]/(2*S
qrt[2]))/2)/2)/8)
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 755
```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Simp[1/(2*r)   Int[(r - s*x^2)/(a + b*x^4)
, x], x] + Simp[1/(2*r)   Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

rule 756
```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-a/b, 2
]], s = Denominator[Rt[-a/b, 2]]}, Simp[r/(2*a)   Int[1/(r - s*x^2), x], x]
 + Simp[r/(2*a)   Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ[a
/b, 0]
```

rule 758
```
Int[((a_) + (b_.)*(x_)^(n_))^(-1), x_Symbol] :> With[{r = Numerator[Rt[-a/b
, 2]], s = Denominator[Rt[-a/b, 2]]}, Simp[r/(2*a)   Int[1/(r - s*x^(n/2)),
 x], x] + Simp[r/(2*a)   Int[1/(r + s*x^(n/2)), x], x]] /; FreeQ[{a, b}, x]
 && IGtQ[n/4, 1] &&  !GtQ[a/b, 0]
```

rule 817
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(
n - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*n*(p + 1))), x] - Simp[c^n
*((m - n + 1)/(b*n*(p + 1)))   Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x
] /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !
ILtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1476
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Simp[e/(2*c)   Int[1/Simp[d/e + q*x + x^2, x], x], x] + Simp[
e/(2*c)   Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x]
 && EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

rule 1479
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Simp[e/(2*c*q)   Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
 x] + Simp[e/(2*c*q)   Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; F
reeQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

rule 2720
```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.26 (sec) , antiderivative size = 68, normalized size of antiderivative = 0.64

method	result
risch	$-\frac{e^{x}}{2(e^{8x}-1)}-\frac{\ln(e^{x}+1)}{16}+\frac{\ln(e^{x}-1)}{16}+4\left(\sum_{\_R=\operatorname{RootOf}(16777216\_Z^{4}+1)}\_R\ln\left(e^{x}-64\_R\right)\right)+\frac{i\ln(e^{x}-i)}{16}-\frac{i}{}$

input
```
int(exp(x)*csch(4*x)^2,x,method=_RETURNVERBOSE)
```

output
```
-1/2*exp(x)/(exp(8*x)-1)-1/16*ln(exp(x)+1)+1/16*ln(exp(x)-1)+4*sum(_R*ln(e
xp(x)-64*_R),_R=RootOf(16777216*_Z^4+1))+1/16*I*ln(exp(x)-I)-1/16*I*ln(exp
(x)+I)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 855 vs. $2(71) = 142$.

Time = 0.11 (sec) , antiderivative size = 855, normalized size of antiderivative = 8.07

$$\int e^x \operatorname{csch}^2(4x)\,dx = \text{Too large to display}$$

input

```
integrate(exp(x)*csch(4*x)^2,x, algorithm="fricas")
```

output

```
-1/32*(2*(sqrt(2)*cosh(x)^8 + 8*sqrt(2)*cosh(x)^7*sinh(x) + 28*sqrt(2)*cos
h(x)^6*sinh(x)^2 + 56*sqrt(2)*cosh(x)^5*sinh(x)^3 + 70*sqrt(2)*cosh(x)^4*s
inh(x)^4 + 56*sqrt(2)*cosh(x)^3*sinh(x)^5 + 28*sqrt(2)*cosh(x)^2*sinh(x)^6
 + 8*sqrt(2)*cosh(x)*sinh(x)^7 + sqrt(2)*sinh(x)^8 - sqrt(2))*arctan(sqrt(
2)*cosh(x) + sqrt(2)*sinh(x) + 1) + 2*(sqrt(2)*cosh(x)^8 + 8*sqrt(2)*cosh(
x)^7*sinh(x) + 28*sqrt(2)*cosh(x)^6*sinh(x)^2 + 56*sqrt(2)*cosh(x)^5*sinh(
x)^3 + 70*sqrt(2)*cosh(x)^4*sinh(x)^4 + 56*sqrt(2)*cosh(x)^3*sinh(x)^5 + 2
8*sqrt(2)*cosh(x)^2*sinh(x)^6 + 8*sqrt(2)*cosh(x)*sinh(x)^7 + sqrt(2)*sinh
(x)^8 - sqrt(2))*arctan(sqrt(2)*cosh(x) + sqrt(2)*sinh(x) - 1) + 4*(cosh(x
)^8 + 8*cosh(x)^7*sinh(x) + 28*cosh(x)^6*sinh(x)^2 + 56*cosh(x)^5*sinh(x)^
3 + 70*cosh(x)^4*sinh(x)^4 + 56*cosh(x)^3*sinh(x)^5 + 28*cosh(x)^2*sinh(x)
^6 + 8*cosh(x)*sinh(x)^7 + sinh(x)^8 - 1)*arctan(cosh(x) + sinh(x)) + (sqr
t(2)*cosh(x)^8 + 8*sqrt(2)*cosh(x)^7*sinh(x) + 28*sqrt(2)*cosh(x)^6*sinh(x
)^2 + 56*sqrt(2)*cosh(x)^5*sinh(x)^3 + 70*sqrt(2)*cosh(x)^4*sinh(x)^4 + 56
*sqrt(2)*cosh(x)^3*sinh(x)^5 + 28*sqrt(2)*cosh(x)^2*sinh(x)^6 + 8*sqrt(2)*
cosh(x)*sinh(x)^7 + sqrt(2)*sinh(x)^8 - sqrt(2))*log((sqrt(2) + 2*cosh(x))
/(cosh(x) - sinh(x))) - (sqrt(2)*cosh(x)^8 + 8*sqrt(2)*cosh(x)^7*sinh(x) +
 28*sqrt(2)*cosh(x)^6*sinh(x)^2 + 56*sqrt(2)*cosh(x)^5*sinh(x)^3 + 70*sqrt
(2)*cosh(x)^4*sinh(x)^4 + 56*sqrt(2)*cosh(x)^3*sinh(x)^5 + 28*sqrt(2)*cosh
(x)^2*sinh(x)^6 + 8*sqrt(2)*cosh(x)*sinh(x)^7 + sqrt(2)*sinh(x)^8 - sqr...
```

Sympy [F]

$$\int e^x \mathrm{csch}^2(4x)\,dx = \int e^x \operatorname{csch}^2(4x)\,dx$$

input
```
integrate(exp(x)*csch(4*x)**2,x)
```

output
```
Integral(exp(x)*csch(4*x)**2, x)
```

Maxima [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 107, normalized size of antiderivative = 1.01

$$\begin{aligned}\int e^x \mathrm{csch}^2(4x)\,dx = &-\frac{1}{16}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\left(\sqrt{2}+2e^x\right)\right)\\ &-\frac{1}{16}\sqrt{2}\arctan\left(-\frac{1}{2}\sqrt{2}\left(\sqrt{2}-2e^x\right)\right)\\ &-\frac{1}{32}\sqrt{2}\log\left(\sqrt{2}e^x+e^{(2x)}+1\right)+\frac{1}{32}\sqrt{2}\log\left(-\sqrt{2}e^x+e^{(2x)}+1\right)\\ &-\frac{e^x}{2\left(e^{(8x)}-1\right)}-\frac{1}{8}\arctan\left(e^x\right)-\frac{1}{16}\log\left(e^x+1\right)+\frac{1}{16}\log\left(e^x-1\right)\end{aligned}$$

input
```
integrate(exp(x)*csch(4*x)^2,x, algorithm="maxima")
```

output
```
-1/16*sqrt(2)*arctan(1/2*sqrt(2)*(sqrt(2) + 2*e^x)) - 1/16*sqrt(2)*arctan(
-1/2*sqrt(2)*(sqrt(2) - 2*e^x)) - 1/32*sqrt(2)*log(sqrt(2)*e^x + e^(2*x) +
 1) + 1/32*sqrt(2)*log(-sqrt(2)*e^x + e^(2*x) + 1) - 1/2*e^x/(e^(8*x) - 1)
 - 1/8*arctan(e^x) - 1/16*log(e^x + 1) + 1/16*log(e^x - 1)
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 108, normalized size of antiderivative = 1.02

$$\int e^x \operatorname{csch}^2(4x)\,dx = -\frac{1}{16}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\left(\sqrt{2}+2e^x\right)\right) - \frac{1}{16}\sqrt{2}\arctan\left(-\frac{1}{2}\sqrt{2}\left(\sqrt{2}-2e^x\right)\right) - \frac{1}{32}\sqrt{2}\log\left(\sqrt{2}e^x+e^{(2x)}+1\right) + \frac{1}{32}\sqrt{2}\log\left(-\sqrt{2}e^x+e^{(2x)}+1\right) - \frac{e^x}{2\left(e^{(8x)}-1\right)} - \frac{1}{8}\arctan\left(e^x\right) - \frac{1}{16}\log\left(e^x+1\right) + \frac{1}{16}\log\left(\left|e^x-1\right|\right)$$

input
```
integrate(exp(x)*csch(4*x)^2,x, algorithm="giac")
```

output
```
-1/16*sqrt(2)*arctan(1/2*sqrt(2)*(sqrt(2) + 2*e^x)) - 1/16*sqrt(2)*arctan(
-1/2*sqrt(2)*(sqrt(2) - 2*e^x)) - 1/32*sqrt(2)*log(sqrt(2)*e^x + e^(2*x) +
 1) + 1/32*sqrt(2)*log(-sqrt(2)*e^x + e^(2*x) + 1) - 1/2*e^x/(e^(8*x) - 1)
 - 1/8*arctan(e^x) - 1/16*log(e^x + 1) + 1/16*log(abs(e^x - 1))
```

Mupad [B] (verification not implemented)

Time = 1.91 (sec) , antiderivative size = 120, normalized size of antiderivative = 1.13

$$\int e^x \operatorname{csch}^2(4x)\,dx = \frac{\ln\left(\frac{1}{2}-\frac{e^x}{2}\right)}{16} - \frac{\ln\left(-\frac{e^x}{2}-\frac{1}{2}\right)}{16} - \frac{\operatorname{atan}(e^x)}{8} - \frac{e^x}{2\left(e^{8x}-1\right)} - \frac{\sqrt{2}\operatorname{atan}\left(2\sqrt{2}\left(\frac{e^x}{2}-\frac{\sqrt{2}}{4}\right)\right)}{16} - \frac{\sqrt{2}\operatorname{atan}\left(2\sqrt{2}\left(\frac{e^x}{2}+\frac{\sqrt{2}}{4}\right)\right)}{16} + \frac{\sqrt{2}\ln\left(\left(\frac{e^x}{2}-\frac{\sqrt{2}}{4}\right)^2+\frac{1}{8}\right)}{32} - \frac{\sqrt{2}\ln\left(\left(\frac{e^x}{2}+\frac{\sqrt{2}}{4}\right)^2+\frac{1}{8}\right)}{32}$$

input
```
int(exp(x)/sinh(4*x)^2,x)
```

output
```
log(1/2 - exp(x)/2)/16 - log(- exp(x)/2 - 1/2)/16 - atan(exp(x))/8 - exp(x
)/(2*(exp(8*x) - 1)) - (2^(1/2)*atan(2*2^(1/2)*(exp(x)/2 - 2^(1/2)/4)))/16
 - (2^(1/2)*atan(2*2^(1/2)*(exp(x)/2 + 2^(1/2)/4)))/16 + (2^(1/2)*log((exp
(x)/2 - 2^(1/2)/4)^2 + 1/8))/32 - (2^(1/2)*log((exp(x)/2 + 2^(1/2)/4)^2 +
1/8))/32
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 245, normalized size of antiderivative = 2.31

$$\int e^x \operatorname{csch}^2(4x)\,dx$$
$$= \frac{-4e^{8x} atan(e^x) + 4atan(e^x) - 2e^{8x}\sqrt{2}\, atan\left(\frac{2e^x-\sqrt{2}}{\sqrt{2}}\right) + 2\sqrt{2}\, atan\left(\frac{2e^x-\sqrt{2}}{\sqrt{2}}\right) - 2e^{8x}\sqrt{2}\, atan\left(\frac{2e^x+\sqrt{2}}{\sqrt{2}}\right) + 2}{}$$

input
```
int(exp(x)*csch(4*x)^2,x)
```

output
```
( - 4*e**(8*x)*atan(e**x) + 4*atan(e**x) - 2*e**(8*x)*sqrt(2)*atan((2*e**x
 - sqrt(2))/sqrt(2)) + 2*sqrt(2)*atan((2*e**x - sqrt(2))/sqrt(2)) - 2*e**(
8*x)*sqrt(2)*atan((2*e**x + sqrt(2))/sqrt(2)) + 2*sqrt(2)*atan((2*e**x + s
qrt(2))/sqrt(2)) + e**(8*x)*sqrt(2)*log(e**(2*x) - e**x*sqrt(2) + 1) - e**
(8*x)*sqrt(2)*log(e**(2*x) + e**x*sqrt(2) + 1) + 2*e**(8*x)*log(e**x - 1)
- 2*e**(8*x)*log(e**x + 1) - 16*e**x - sqrt(2)*log(e**(2*x) - e**x*sqrt(2)
 + 1) + sqrt(2)*log(e**(2*x) + e**x*sqrt(2) + 1) - 2*log(e**x - 1) + 2*log
(e**x + 1))/(32*(e**(8*x) - 1))
```

3.322 $\int F^{c(a+bx)}\sinh^3(d+ex)\,dx$

Optimal result

Integrand size = 18, antiderivative size = 202

$$\int F^{c(a+bx)}\sinh^3(d+ex)\,dx = -\frac{6e^3F^{c(a+bx)}\cosh(d+ex)}{9e^4-10b^2c^2e^2\log^2(F)+b^4c^4\log^4(F)} + \frac{6bce^2F^{c(a+bx)}\log(F)\sinh(d+ex)}{9e^4-10b^2c^2e^2\log^2(F)+b^4c^4\log^4(F)} + \frac{3eF^{c(a+bx)}\cosh(d+ex)\sinh^2(d+ex)}{9e^2-b^2c^2\log^2(F)} - \frac{bcF^{c(a+bx)}\log(F)\sinh^3(d+ex)}{9e^2-b^2c^2\log^2(F)}$$

output
```
-6*e^3*F^(c*(b*x+a))*cosh(e*x+d)/(9*e^4-10*b^2*c^2*e^2*ln(F)^2+b^4*c^4*ln(
F)^4)+6*b*c*e^2*F^(c*(b*x+a))*ln(F)*sinh(e*x+d)/(9*e^4-10*b^2*c^2*e^2*ln(F
)^2+b^4*c^4*ln(F)^4)+3*e*F^(c*(b*x+a))*cosh(e*x+d)*sinh(e*x+d)^2/(9*e^2-b^
2*c^2*ln(F)^2)-b*c*F^(c*(b*x+a))*ln(F)*sinh(e*x+d)^3/(9*e^2-b^2*c^2*ln(F)^
2)
```

Mathematica [A] (verified)

Time = 0.47 (sec) , antiderivative size = 157, normalized size of antiderivative = 0.78

$$\int F^{c(a+bx)}\sinh^3(d+ex)\,dx$$
$$=\frac{F^{c(a+bx)}\left(3\cosh(3(d+ex))\left(e^3-b^2c^2e\log^2(F)\right)+3\cosh(d+ex)\left(-9e^3+b^2c^2e\log^2(F)\right)+2bc\log(F)\left(\ldots\right.\right.}{4\left(9e^4-10b^2c^2e^2\log^2(F)+b^4c^4\log\ldots\right.}$$

input
```
Integrate[F^(c*(a + b*x))*Sinh[d + e*x]^3,x]
```

output
```
(F^(c*(a + b*x))*(3*Cosh[3*(d + e*x)]*(e^3 - b^2*c^2*e*Log[F]^2) + 3*Cosh[
d + e*x]*(-9*e^3 + b^2*c^2*e*Log[F]^2) + 2*b*c*Log[F]*(13*e^2 - b^2*c^2*Lo
g[F]^2 + Cosh[2*(d + e*x)]*(-e^2 + b^2*c^2*Log[F]^2))*Sinh[d + e*x]))/(4*(
9*e^4 - 10*b^2*c^2*e^2*Log[F]^2 + b^4*c^4*Log[F]^4))
```

Rubi [A] (verified)

Time = 0.44 (sec) , antiderivative size = 190, normalized size of antiderivative = 0.94, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {5999, 5997}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sinh^3(d+ex)F^{c(a+bx)}\,dx$$
$$\Big\downarrow \text{5999}$$
$$-\frac{6e^2\int F^{c(a+bx)}\sinh(d+ex)dx}{9e^2-b^2c^2\log^2(F)}-\frac{bc\log(F)\sinh^3(d+ex)F^{c(a+bx)}}{9e^2-b^2c^2\log^2(F)}+\frac{3e\sinh^2(d+ex)\cosh(d+ex)F^{c(a+bx)}}{9e^2-b^2c^2\log^2(F)}$$
$$\Big\downarrow \text{5997}$$

$$-\frac{bc\log(F)\sinh^3(d+ex)F^{c(a+bx)}}{9e^2-b^2c^2\log^2(F)}+\frac{3e\sinh^2(d+ex)\cosh(d+ex)F^{c(a+bx)}}{9e^2-b^2c^2\log^2(F)}-\frac{6e^2\left(\frac{e\cosh(d+ex)F^{c(a+bx)}}{e^2-b^2c^2\log^2(F)}-\frac{bc\log(F)\sinh(d+ex)F^{c(a+bx)}}{e^2-b^2c^2\log^2(F)}\right)}{9e^2-b^2c^2\log^2(F)}$$

input
```
Int[F^(c*(a + b*x))*Sinh[d + e*x]^3,x]
```

output
```
(3*e*F^(c*(a + b*x))*Cosh[d + e*x]*Sinh[d + e*x]^2)/(9*e^2 - b^2*c^2*Log[F
]^2) - (b*c*F^(c*(a + b*x))*Log[F]*Sinh[d + e*x]^3)/(9*e^2 - b^2*c^2*Log[F
]^2) - (6*e^2*((e*F^(c*(a + b*x))*Cosh[d + e*x])/(e^2 - b^2*c^2*Log[F]^2)
- (b*c*F^(c*(a + b*x))*Log[F]*Sinh[d + e*x])/(e^2 - b^2*c^2*Log[F]^2)))/(9
*e^2 - b^2*c^2*Log[F]^2)
```

Defintions of rubi rules used

rule 5997
```
Int[(F_)^((c_.)*((a_.) + (b_.)*(x_)))*Sinh[(d_.) + (e_.)*(x_)], x_Symbol] :
> Simp[(-b)*c*Log[F]*F^(c*(a + b*x))*(Sinh[d + e*x]/(e^2 - b^2*c^2*Log[F]^2
)), x] + Simp[e*F^(c*(a + b*x))*(Cosh[d + e*x]/(e^2 - b^2*c^2*Log[F]^2)), x
] /; FreeQ[{F, a, b, c, d, e}, x] && NeQ[e^2 - b^2*c^2*Log[F]^2, 0]
```

rule 5999
```
Int[(F_)^((c_.)*((a_.) + (b_.)*(x_)))*Sinh[(d_.) + (e_.)*(x_)]^(n_), x_Symb
ol] :> Simp[(-b)*c*Log[F]*F^(c*(a + b*x))*(Sinh[d + e*x]^n/(e^2*n^2 - b^2*c
^2*Log[F]^2)), x] + (Simp[e*n*F^(c*(a + b*x))*Cosh[d + e*x]*(Sinh[d + e*x]^
(n - 1)/(e^2*n^2 - b^2*c^2*Log[F]^2)), x] - Simp[n*(n - 1)*(e^2/(e^2*n^2 -
b^2*c^2*Log[F]^2))   Int[F^(c*(a + b*x))*Sinh[d + e*x]^(n - 2), x], x]) /;
FreeQ[{F, a, b, c, d, e}, x] && NeQ[e^2*n^2 - b^2*c^2*Log[F]^2, 0] && GtQ[n
, 1]
```

Maple [A] (verified)

Time = 1.36 (sec) , antiderivative size = 147, normalized size of antiderivative = 0.73

method	result
parallelrisch	$-\frac{3F^{c(bx+a)}\left(\left(\ln(F)^{2}b^{2}c^{2}e-e^{3}\right)\cosh(3ex+3d)+\frac{\left(-\ln(F)^{3}b^{3}c^{3}+\ln(F)bc\,e^{2}\right)\sinh(3ex+3d)}{3}+(bc\ln(F)-3e)(bc\ln(F)+3e)(\sinh}{4\left(9e^{4}-10b^{2}c^{2}e^{2}\ln(F)^{2}+b^{4}c^{4}\ln(F)^{4}\right)}$
risch	$\left(\ln(F)^{3}b^{3}c^{3}e^{6ex+6d}-3\ln(F)^{3}b^{3}c^{3}e^{4ex+4d}-3\ln(F)^{2}b^{2}c^{2}e\,e^{6ex+6d}+3\ln(F)^{3}b^{3}c^{3}e^{2ex+2d}+3\ln(F)^{2}b^{2}c^{2}e\,e^{4ex+4d}-\ln(F)bc\right.$
orering	$\frac{4\ln(F)bc\left(b^{2}c^{2}\ln(F)^{2}-5e^{2}\right)F^{c(bx+a)}\sinh(ex+d)^{3}}{9e^{4}-10b^{2}c^{2}e^{2}\ln(F)^{2}+b^{4}c^{4}\ln(F)^{4}}-\frac{2\left(3b^{2}c^{2}\ln(F)^{2}-5e^{2}\right)\left(F^{c(bx+a)}bc\ln(F)\sinh(ex+d)^{3}+3F^{c(bx+a)}\,\mathrm{si}\right.}{9e^{4}-10b^{2}c^{2}e^{2}\ln(F)^{2}+b^{4}c^{4}\ln(F)^{4}}$

input
```
int(F^(c*(b*x+a))*sinh(e*x+d)^3,x,method=_RETURNVERBOSE)
```

output
```
-3/4*F^(c*(b*x+a))*((ln(F)^2*b^2*c^2*e-e^3)*cosh(3*e*x+3*d)+1/3*(-ln(F)^3*
b^3*c^3+ln(F)*b*c*e^2)*sinh(3*e*x+3*d)+(b*c*ln(F)-3*e)*(b*c*ln(F)+3*e)*(si
nh(e*x+d)*ln(F)*b*c-e*cosh(e*x+d)))/(9*e^4-10*b^2*c^2*e^2*ln(F)^2+b^4*c^4*
ln(F)^4)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2228 vs. 2(199) = 398.

Time = 0.17 (sec) , antiderivative size = 2228, normalized size of antiderivative = 11.03

$$\int F^{c(a+bx)}\sinh^{3}(d+ex)\,dx = \text{Too large to display}$$

input
```
integrate(F^(c*(b*x+a))*sinh(e*x+d)^3,x, algorithm="fricas")
```

output
```
1/8*((3*e^3*cosh(e*x + d)^6 - 27*e^3*cosh(e*x + d)^4 + (b^3*c^3*log(F)^3 -
 3*b^2*c^2*e*log(F)^2 - b*c*e^2*log(F) + 3*e^3)*sinh(e*x + d)^6 + 6*(b^3*c
^3*cosh(e*x + d)*log(F)^3 - 3*b^2*c^2*e*cosh(e*x + d)*log(F)^2 - b*c*e^2*c
osh(e*x + d)*log(F) + 3*e^3*cosh(e*x + d))*sinh(e*x + d)^5 - 27*e^3*cosh(e
*x + d)^2 + 3*(15*e^3*cosh(e*x + d)^2 + (5*b^3*c^3*cosh(e*x + d)^2 - b^3*c
^3)*log(F)^3 - 9*e^3 - (15*b^2*c^2*e*cosh(e*x + d)^2 - b^2*c^2*e)*log(F)^2
 - (5*b*c*e^2*cosh(e*x + d)^2 - 9*b*c*e^2)*log(F))*sinh(e*x + d)^4 + (b^3*
c^3*cosh(e*x + d)^6 - 3*b^3*c^3*cosh(e*x + d)^4 + 3*b^3*c^3*cosh(e*x + d)^
2 - b^3*c^3)*log(F)^3 + 4*(15*e^3*cosh(e*x + d)^3 - 27*e^3*cosh(e*x + d) +
 (5*b^3*c^3*cosh(e*x + d)^3 - 3*b^3*c^3*cosh(e*x + d))*log(F)^3 - 3*(5*b^2
*c^2*e*cosh(e*x + d)^3 - b^2*c^2*e*cosh(e*x + d))*log(F)^2 - (5*b*c*e^2*co
sh(e*x + d)^3 - 27*b*c*e^2*cosh(e*x + d))*log(F))*sinh(e*x + d)^3 + 3*e^3
- 3*(b^2*c^2*e*cosh(e*x + d)^6 - b^2*c^2*e*cosh(e*x + d)^4 - b^2*c^2*e*cos
h(e*x + d)^2 + b^2*c^2*e)*log(F)^2 + 3*(15*e^3*cosh(e*x + d)^4 - 54*e^3*co
sh(e*x + d)^2 + (5*b^3*c^3*cosh(e*x + d)^4 - 6*b^3*c^3*cosh(e*x + d)^2 + b
^3*c^3)*log(F)^3 - 9*e^3 - (15*b^2*c^2*e*cosh(e*x + d)^4 - 6*b^2*c^2*e*cos
h(e*x + d)^2 - b^2*c^2*e)*log(F)^2 - (5*b*c*e^2*cosh(e*x + d)^4 - 54*b*c*e
^2*cosh(e*x + d)^2 + 9*b*c*e^2)*log(F))*sinh(e*x + d)^2 - (b*c*e^2*cosh(e*
x + d)^6 - 27*b*c*e^2*cosh(e*x + d)^4 + 27*b*c*e^2*cosh(e*x + d)^2 - b*c*e
^2)*log(F) + 6*(3*e^3*cosh(e*x + d)^5 - 18*e^3*cosh(e*x + d)^3 - 9*e^3*...
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1525 vs. $2(199) = 398$.

Time = 3.61 (sec) , antiderivative size = 1525, normalized size of antiderivative = 7.55

$$\int F^{c(a+bx)} \sinh^3(d+ex)\,dx = \text{Too large to display}$$

input
```
integrate(F**(c*(b*x+a))*sinh(e*x+d)**3,x)
```

output
```
Piecewise((x*sinh(d)**3, Eq(F, 1) & Eq(e, 0)), (F**(a*c)*x*sinh(d)**3, Eq(
b, 0) & Eq(e, 0)), (x*sinh(d)**3, Eq(c, 0) & Eq(e, 0)), (-3*F**(a*c + b*c*
x)*x*sinh(b*c*x*log(F) - d)**3/8 + 3*F**(a*c + b*c*x)*x*sinh(b*c*x*log(F)
- d)**2*cosh(b*c*x*log(F) - d)/8 + 3*F**(a*c + b*c*x)*x*sinh(b*c*x*log(F)
- d)*cosh(b*c*x*log(F) - d)**2/8 - 3*F**(a*c + b*c*x)*x*cosh(b*c*x*log(F)
- d)**3/8 + F**(a*c + b*c*x)*sinh(b*c*x*log(F) - d)**3/(8*b*c*log(F)) - 3*
F**(a*c + b*c*x)*sinh(b*c*x*log(F) - d)**2*cosh(b*c*x*log(F) - d)/(4*b*c*l
og(F)) + 3*F**(a*c + b*c*x)*cosh(b*c*x*log(F) - d)**3/(8*b*c*log(F)), Eq(e
, -b*c*log(F))), (-F**(a*c + b*c*x)*x*sinh(b*c*x*log(F)/3 - d)**3/8 + 3*F*
*(a*c + b*c*x)*x*sinh(b*c*x*log(F)/3 - d)**2*cosh(b*c*x*log(F)/3 - d)/8 -
3*F**(a*c + b*c*x)*x*sinh(b*c*x*log(F)/3 - d)*cosh(b*c*x*log(F)/3 - d)**2/
8 + F**(a*c + b*c*x)*x*cosh(b*c*x*log(F)/3 - d)**3/8 - 9*F**(a*c + b*c*x)*
sinh(b*c*x*log(F)/3 - d)**3/(8*b*c*log(F)) + 3*F**(a*c + b*c*x)*sinh(b*c*x
*log(F)/3 - d)**2*cosh(b*c*x*log(F)/3 - d)/(4*b*c*log(F)) - F**(a*c + b*c*
x)*cosh(b*c*x*log(F)/3 - d)**3/(8*b*c*log(F)), Eq(e, -b*c*log(F)/3)), (F**
(a*c + b*c*x)*x*sinh(b*c*x*log(F)/3 + d)**3/8 - 3*F**(a*c + b*c*x)*x*sinh(
b*c*x*log(F)/3 + d)**2*cosh(b*c*x*log(F)/3 + d)/8 + 3*F**(a*c + b*c*x)*x*s
inh(b*c*x*log(F)/3 + d)*cosh(b*c*x*log(F)/3 + d)**2/8 - F**(a*c + b*c*x)*x
*cosh(b*c*x*log(F)/3 + d)**3/8 - F**(a*c + b*c*x)*sinh(b*c*x*log(F)/3 + d)
**3/(8*b*c*log(F)) + 3*F**(a*c + b*c*x)*sinh(b*c*x*log(F)/3 + d)**2*cos...
```

Maxima [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 134, normalized size of antiderivative = 0.66

$$\int F^{c(a+bx)}\sinh^3(d+ex)\,dx = \frac{F^{ac}e^{(bcx\log(F)+3\,ex+3\,d)}}{8\,(bc\log{(F)}+3\,e)} - \frac{3\,F^{ac}e^{(bcx\log(F)+ex+d)}}{8\,(bc\log{(F)}+e)} + \frac{3\,F^{ac}e^{(bcx\log(F)-ex)}}{8\,(bce^{d}\log{(F)}-ee^{d})} - \frac{F^{ac}e^{(bcx\log(F)-3\,ex)}}{8\,(bce^{(3\,d)}\log{(F)}-3\,ee^{(3\,d)})}$$

input
```
integrate(F^(c*(b*x+a))*sinh(e*x+d)^3,x, algorithm="maxima")
```

output
```
1/8*F^(a*c)*e^(b*c*x*log(F) + 3*e*x + 3*d)/(b*c*log(F) + 3*e) - 3/8*F^(a*c
)*e^(b*c*x*log(F) + e*x + d)/(b*c*log(F) + e) + 3/8*F^(a*c)*e^(b*c*x*log(F
) - e*x)/(b*c*e^d*log(F) - e*e^d) - 1/8*F^(a*c)*e^(b*c*x*log(F) - 3*e*x)/(
b*c*e^(3*d)*log(F) - 3*e*e^(3*d))
```

Giac [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.17 (sec) , antiderivative size = 1211, normalized size of antiderivative = 6.00

$$\int F^{c(a+bx)} \sinh^3(d+ex)\,dx = \text{Too large to display}$$

input
```
integrate(F^(c*(b*x+a))*sinh(e*x+d)^3,x, algorithm="giac")
```

output
```
1/4*(2*(b*c*log(abs(F)) + 3*e)*cos(-1/2*pi*b*c*x*sgn(F) + 1/2*pi*b*c*x - 1
/2*pi*a*c*sgn(F) + 1/2*pi*a*c)/((pi*b*c*sgn(F) - pi*b*c)^2 + 4*(b*c*log(ab
s(F)) + 3*e)^2) - (pi*b*c*sgn(F) - pi*b*c)*sin(-1/2*pi*b*c*x*sgn(F) + 1/2*
pi*b*c*x - 1/2*pi*a*c*sgn(F) + 1/2*pi*a*c)/((pi*b*c*sgn(F) - pi*b*c)^2 + 4
*(b*c*log(abs(F)) + 3*e)^2))*e^(a*c*log(abs(F)) + (b*c*log(abs(F)) + 3*e)*
x + 3*d) + I*(I*e^(1/2*I*pi*b*c*x*sgn(F) - 1/2*I*pi*b*c*x + 1/2*I*pi*a*c*s
gn(F) - 1/2*I*pi*a*c)/(8*I*pi*b*c*sgn(F) - 8*I*pi*b*c + 16*b*c*log(abs(F))
 + 48*e) - I*e^(-1/2*I*pi*b*c*x*sgn(F) + 1/2*I*pi*b*c*x - 1/2*I*pi*a*c*sgn
(F) + 1/2*I*pi*a*c)/(-8*I*pi*b*c*sgn(F) + 8*I*pi*b*c + 16*b*c*log(abs(F))
+ 48*e))*e^(a*c*log(abs(F)) + (b*c*log(abs(F)) + 3*e)*x + 3*d) - 3/4*(2*(b
*c*log(abs(F)) + e)*cos(-1/2*pi*b*c*x*sgn(F) + 1/2*pi*b*c*x - 1/2*pi*a*c*s
gn(F) + 1/2*pi*a*c)/((pi*b*c*sgn(F) - pi*b*c)^2 + 4*(b*c*log(abs(F)) + e)^
2) - (pi*b*c*sgn(F) - pi*b*c)*sin(-1/2*pi*b*c*x*sgn(F) + 1/2*pi*b*c*x - 1/
2*pi*a*c*sgn(F) + 1/2*pi*a*c)/((pi*b*c*sgn(F) - pi*b*c)^2 + 4*(b*c*log(abs
(F)) + e)^2))*e^(a*c*log(abs(F)) + (b*c*log(abs(F)) + e)*x + d) + 3*I*(-I*
e^(1/2*I*pi*b*c*x*sgn(F) - 1/2*I*pi*b*c*x + 1/2*I*pi*a*c*sgn(F) - 1/2*I*pi
*a*c)/(8*I*pi*b*c*sgn(F) - 8*I*pi*b*c + 16*b*c*log(abs(F)) + 16*e) + I*e^(
-1/2*I*pi*b*c*x*sgn(F) + 1/2*I*pi*b*c*x - 1/2*I*pi*a*c*sgn(F) + 1/2*I*pi*a
*c)/(-8*I*pi*b*c*sgn(F) + 8*I*pi*b*c + 16*b*c*log(abs(F)) + 16*e))*e^(a*c*
log(abs(F)) + (b*c*log(abs(F)) + e)*x + d) + 3/4*(2*(b*c*log(abs(F)) - ...
```

Mupad [B] (verification not implemented)

Time = 2.58 (sec) , antiderivative size = 166, normalized size of antiderivative = 0.82

$$\int F^{c(a+bx)}\sinh^3(d+ex)\,dx = -\frac{F^{a\,c+b\,c\,x}\left(-b^3\,c^3\,\sinh(d+e\,x)^3\,\ln(F)^3+3\,b^2\,c^2\,e\cosh(d+e\,x)\,\sinh(d+e\,x)^2\ln(F)^2-6\,b\,c\,e^2\cosh(d\right.}{b^4\,c^4\ln(F)^4}$$

input
```
int(F^(c*(a + b*x))*sinh(d + e*x)^3,x)
```

output
```
-(F^(a*c + b*c*x)*(6*e^3*cosh(d + e*x)^3 - 9*e^3*cosh(d + e*x)*sinh(d + e*
x)^2 - b^3*c^3*sinh(d + e*x)^3*log(F)^3 + 7*b*c*e^2*sinh(d + e*x)^3*log(F)
 + 3*b^2*c^2*e*cosh(d + e*x)*sinh(d + e*x)^2*log(F)^2 - 6*b*c*e^2*cosh(d +
 e*x)^2*sinh(d + e*x)*log(F)))/(9*e^4 + b^4*c^4*log(F)^4 - 10*b^2*c^2*e^2*
log(F)^2)
```

Reduce [F]

$$\int F^{c(a+bx)}\sinh^3(d+ex)\,dx = f^{ac}\left(\int f^{bcx}\sinh(ex+d)^3\,dx\right)$$

input
```
int(F^(c*(b*x+a))*sinh(e*x+d)^3,x)
```

output
```
f**(a*c)*int(f**(b*c*x)*sinh(d + e*x)**3,x)
```

3.323 $\int F^{c(a+bx)} \sinh^2(d+ex)\,dx$

Optimal result

Integrand size = 18, antiderivative size = 132

$$\int F^{c(a+bx)} \sinh^2(d+ex)\,dx = -\frac{2e^2F^{c(a+bx)}}{bc\log(F)\left(4e^2-b^2c^2\log^2(F)\right)} + \frac{2eF^{c(a+bx)}\cosh(d+ex)\sinh(d+ex)}{4e^2-b^2c^2\log^2(F)} - \frac{bcF^{c(a+bx)}\log(F)\sinh^2(d+ex)}{4e^2-b^2c^2\log^2(F)}$$

output
```
-2*e^2*F^(c*(b*x+a))/b/c/ln(F)/(4*e^2-b^2*c^2*ln(F)^2)+2*e*F^(c*(b*x+a))*c
osh(e*x+d)*sinh(e*x+d)/(4*e^2-b^2*c^2*ln(F)^2)-b*c*F^(c*(b*x+a))*ln(F)*sin
h(e*x+d)^2/(4*e^2-b^2*c^2*ln(F)^2)
```

Mathematica [A] (verified)

Time = 0.12 (sec) , antiderivative size = 86, normalized size of antiderivative = 0.65

$$\int F^{c(a+bx)} \sinh^2(d+ex)\,dx = \frac{F^{c(a+bx)}\left(4e^2-b^2c^2\log^2(F)+b^2c^2\cosh(2(d+ex))\log^2(F)-2bce\log(F)\sinh(2(d+ex))\right)}{-8bce^2\log(F)+2b^3c^3\log^3(F)}$$

input
```
Integrate[F^(c*(a + b*x))*Sinh[d + e*x]^2,x]
```

output
```
(F^(c*(a + b*x))*(4*e^2 - b^2*c^2*Log[F]^2 + b^2*c^2*Cosh[2*(d + e*x)]*Log
[F]^2 - 2*b*c*e*Log[F]*Sinh[2*(d + e*x)]))/(-8*b*c*e^2*Log[F] + 2*b^3*c^3*
Log[F]^3)
```

Rubi [A] (verified)

Time = 0.33 (sec) , antiderivative size = 132, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {5999, 2624}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sinh^2(d+ex)F^{c(a+bx)}\,dx$$

$$\Big\downarrow \text{5999}$$

$$-\frac{2e^2\int F^{c(a+bx)}dx}{4e^2-b^2c^2\log^2(F)}-\frac{bc\log(F)\sinh^2(d+ex)F^{c(a+bx)}}{4e^2-b^2c^2\log^2(F)}+\frac{2e\sinh(d+ex)\cosh(d+ex)F^{c(a+bx)}}{4e^2-b^2c^2\log^2(F)}$$

$$\Big\downarrow \text{2624}$$

$$-\frac{bc\log(F)\sinh^2(d+ex)F^{c(a+bx)}}{4e^2-b^2c^2\log^2(F)}+\frac{2e\sinh(d+ex)\cosh(d+ex)F^{c(a+bx)}}{4e^2-b^2c^2\log^2(F)}-\frac{2e^2F^{c(a+bx)}}{bc\log(F)\left(4e^2-b^2c^2\log^2(F)\right)}$$

input
```
Int[F^(c*(a + b*x))*Sinh[d + e*x]^2,x]
```

output
```
(-2*e^2*F^(c*(a + b*x)))/(b*c*Log[F]*(4*e^2 - b^2*c^2*Log[F]^2)) + (2*e*F^
(c*(a + b*x))*Cosh[d + e*x]*Sinh[d + e*x])/(4*e^2 - b^2*c^2*Log[F]^2) - (b
*c*F^(c*(a + b*x))*Log[F]*Sinh[d + e*x]^2)/(4*e^2 - b^2*c^2*Log[F]^2)
```

Defintions of rubi rules used

rule 2624
```
Int[((F_)^(v_))^(n_.), x_Symbol] :> Simp[(F^v)^n/(n*Log[F]*D[v, x]), x] /;
FreeQ[{F, n}, x] && LinearQ[v, x]
```

rule 5999
```
Int[(F_)^((c_.)*((a_.) + (b_.)*(x_)))*Sinh[(d_.) + (e_.)*(x_)]^(n_), x_Symb
ol] :> Simp[(-b)*c*Log[F]*F^(c*(a + b*x))*(Sinh[d + e*x]^n/(e^2*n^2 - b^2*c
^2*Log[F]^2)), x] + (Simp[e*n*F^(c*(a + b*x))*Cosh[d + e*x]*(Sinh[d + e*x]^
(n - 1)/(e^2*n^2 - b^2*c^2*Log[F]^2)), x] - Simp[n*(n - 1)*(e^2/(e^2*n^2 -
b^2*c^2*Log[F]^2))   Int[F^(c*(a + b*x))*Sinh[d + e*x]^(n - 2), x], x]) /;
FreeQ[{F, a, b, c, d, e}, x] && NeQ[e^2*n^2 - b^2*c^2*Log[F]^2, 0] && GtQ[n
, 1]
```

Maple [A] (verified)

Time = 0.54 (sec) , antiderivative size = 90, normalized size of antiderivative = 0.68

method	result
parallelrisch	$-\frac{2F^{c(bx+a)}\left(-\frac{b^{2}c^{2}\ln(F)^{2}\cosh(2ex+2d)}{2}+\frac{b^{2}c^{2}\ln(F)^{2}}{2}+ebc\ln(F)\sinh(2ex+2d)-2e^{2}\right)}{2\ln(F)^{3}b^{3}c^{3}-8\ln(F)bc\,e^{2}}$
risch	$\frac{\left(\ln(F)^{2}b^{2}c^{2}\mathrm{e}^{4ex+4d}-2\ln(F)^{2}b^{2}c^{2}\mathrm{e}^{2ex+2d}-2\ln(F)bce\,\mathrm{e}^{4ex+4d}+b^{2}c^{2}\ln(F)^{2}+2ebc\ln(F)+8e^{2}\mathrm{e}^{2ex+2d}\right)\mathrm{e}^{-2ex-2d}F^{c(bx+a)}}{4\ln(F)bc(bc\ln(F)-2e)(2e+bc\ln(F))}$
orering	$\frac{\left(3b^{2}c^{2}\ln(F)^{2}-4e^{2}\right)F^{c(bx+a)}\sinh(ex+d)^{2}}{\ln(F)bc\left(b^{2}c^{2}\ln(F)^{2}-4e^{2}\right)}-\frac{3\left(F^{c(bx+a)}bc\ln(F)\sinh(ex+d)^{2}+2F^{c(bx+a)}\sinh(ex+d)e\cosh(ex+d)\right)}{b^{2}c^{2}\ln(F)^{2}-4e^{2}}+$

input
```
int(F^(c*(b*x+a))*sinh(e*x+d)^2,x,method=_RETURNVERBOSE)
```

output
```
-2*F^(c*(b*x+a))*(-1/2*b^2*c^2*ln(F)^2*cosh(2*e*x+2*d)+1/2*b^2*c^2*ln(F)^2
+e*b*c*ln(F)*sinh(2*e*x+2*d)-2*e^2)/(2*ln(F)^3*b^3*c^3-8*ln(F)*b*c*e^2)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 703 vs. $2(128) = 256$.

Time = 0.09 (sec) , antiderivative size = 703, normalized size of antiderivative = 5.33

$$\int F^{c(a+bx)} \sinh^2(d+ex)\,dx = \text{Too large to display}$$

input
```
integrate(F^(c*(b*x+a))*sinh(e*x+d)^2,x, algorithm="fricas")
```

output
```
1/4*(((b^2*c^2*log(F)^2 - 2*b*c*e*log(F))*sinh(e*x + d)^4 + 8*e^2*cosh(e*x
 + d)^2 + 4*(b^2*c^2*cosh(e*x + d)*log(F)^2 - 2*b*c*e*cosh(e*x + d)*log(F)
)*sinh(e*x + d)^3 + (b^2*c^2*cosh(e*x + d)^4 - 2*b^2*c^2*cosh(e*x + d)^2 +
 b^2*c^2)*log(F)^2 - 2*(6*b*c*e*cosh(e*x + d)^2*log(F) - (3*b^2*c^2*cosh(e
*x + d)^2 - b^2*c^2)*log(F)^2 - 4*e^2)*sinh(e*x + d)^2 - 2*(b*c*e*cosh(e*x
 + d)^4 - b*c*e)*log(F) - 4*(2*b*c*e*cosh(e*x + d)^3*log(F) - 4*e^2*cosh(e
*x + d) - (b^2*c^2*cosh(e*x + d)^3 - b^2*c^2*cosh(e*x + d))*log(F)^2)*sinh
(e*x + d))*cosh((b*c*x + a*c)*log(F)) + ((b^2*c^2*log(F)^2 - 2*b*c*e*log(F
))*sinh(e*x + d)^4 + 8*e^2*cosh(e*x + d)^2 + 4*(b^2*c^2*cosh(e*x + d)*log(
F)^2 - 2*b*c*e*cosh(e*x + d)*log(F))*sinh(e*x + d)^3 + (b^2*c^2*cosh(e*x +
 d)^4 - 2*b^2*c^2*cosh(e*x + d)^2 + b^2*c^2)*log(F)^2 - 2*(6*b*c*e*cosh(e*
x + d)^2*log(F) - (3*b^2*c^2*cosh(e*x + d)^2 - b^2*c^2)*log(F)^2 - 4*e^2)*
sinh(e*x + d)^2 - 2*(b*c*e*cosh(e*x + d)^4 - b*c*e)*log(F) - 4*(2*b*c*e*co
sh(e*x + d)^3*log(F) - 4*e^2*cosh(e*x + d) - (b^2*c^2*cosh(e*x + d)^3 - b^
2*c^2*cosh(e*x + d))*log(F)^2)*sinh(e*x + d))*sinh((b*c*x + a*c)*log(F)))/
(b^3*c^3*cosh(e*x + d)^2*log(F)^3 - 4*b*c*e^2*cosh(e*x + d)^2*log(F) + (b^
3*c^3*log(F)^3 - 4*b*c*e^2*log(F))*sinh(e*x + d)^2 + 2*(b^3*c^3*cosh(e*x +
 d)*log(F)^3 - 4*b*c*e^2*cosh(e*x + d)*log(F))*sinh(e*x + d))
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 707 vs. $2(119) = 238$.

Time = 1.26 (sec) , antiderivative size = 707, normalized size of antiderivative = 5.36

$$\int F^{c(a+bx)} \sinh^2(d+ex)\,dx = \text{Too large to display}$$

input
```
integrate(F**(c*(b*x+a))*sinh(e*x+d)**2,x)
```

output
```
Piecewise((x*sinh(d)**2, Eq(F, 1) & Eq(b, 0) & Eq(c, 0) & Eq(e, 0)), (x*si
nh(d + e*x)**2/2 - x*cosh(d + e*x)**2/2 + sinh(d + e*x)*cosh(d + e*x)/(2*e
), Eq(F, 1)), (F**(a*c)*(x*sinh(d + e*x)**2/2 - x*cosh(d + e*x)**2/2 + sin
h(d + e*x)*cosh(d + e*x)/(2*e)), Eq(b, 0)), (x*sinh(d + e*x)**2/2 - x*cosh
(d + e*x)**2/2 + sinh(d + e*x)*cosh(d + e*x)/(2*e), Eq(c, 0)), (F**(a*c +
b*c*x)*x*sinh(b*c*x*log(F)/2 - d)**2/4 - F**(a*c + b*c*x)*x*sinh(b*c*x*log
(F)/2 - d)*cosh(b*c*x*log(F)/2 - d)/2 + F**(a*c + b*c*x)*x*cosh(b*c*x*log(
F)/2 - d)**2/4 + 3*F**(a*c + b*c*x)*sinh(b*c*x*log(F)/2 - d)*cosh(b*c*x*lo
g(F)/2 - d)/(2*b*c*log(F)) - F**(a*c + b*c*x)*cosh(b*c*x*log(F)/2 - d)**2/
(b*c*log(F)), Eq(e, -b*c*log(F)/2)), (F**(a*c + b*c*x)*x*sinh(b*c*x*log(F)
/2 + d)**2/4 - F**(a*c + b*c*x)*x*sinh(b*c*x*log(F)/2 + d)*cosh(b*c*x*log(
F)/2 + d)/2 + F**(a*c + b*c*x)*x*cosh(b*c*x*log(F)/2 + d)**2/4 + F**(a*c +
 b*c*x)*sinh(b*c*x*log(F)/2 + d)**2/(b*c*log(F)) - F**(a*c + b*c*x)*sinh(b
*c*x*log(F)/2 + d)*cosh(b*c*x*log(F)/2 + d)/(2*b*c*log(F)), Eq(e, b*c*log(
F)/2)), (F**(a*c + b*c*x)*b**2*c**2*log(F)**2*sinh(d + e*x)**2/(b**3*c**3*
log(F)**3 - 4*b*c*e**2*log(F)) - 2*F**(a*c + b*c*x)*b*c*e*log(F)*sinh(d +
e*x)*cosh(d + e*x)/(b**3*c**3*log(F)**3 - 4*b*c*e**2*log(F)) - 2*F**(a*c +
 b*c*x)*e**2*sinh(d + e*x)**2/(b**3*c**3*log(F)**3 - 4*b*c*e**2*log(F)) +
2*F**(a*c + b*c*x)*e**2*cosh(d + e*x)**2/(b**3*c**3*log(F)**3 - 4*b*c*e**2
*log(F)), True))
```

Maxima [A] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 94, normalized size of antiderivative = 0.71

$$\int F^{c(a+bx)}\sinh^2(d+ex)\,dx = \frac{F^{ac}e^{(bcx\log(F)+2\,ex+2\,d)}}{4\,(bc\log{(F)}+2\,e)} + \frac{F^{ac}e^{(bcx\log(F)-2\,ex)}}{4\,(bce^{(2\,d)}\log{(F)}-2\,ee^{(2\,d)})} - \frac{F^{bcx+ac}}{2\,bc\log{(F)}}$$

input
```
integrate(F^(c*(b*x+a))*sinh(e*x+d)^2,x, algorithm="maxima")
```

output
```
1/4*F^(a*c)*e^(b*c*x*log(F) + 2*e*x + 2*d)/(b*c*log(F) + 2*e) + 1/4*F^(a*c
)*e^(b*c*x*log(F) - 2*e*x)/(b*c*e^(2*d)*log(F) - 2*e*e^(2*d)) - 1/2*F^(b*c
*x + a*c)/(b*c*log(F))
```

Giac [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.15 (sec) , antiderivative size = 890, normalized size of antiderivative = 6.74

$$\int F^{c(a+bx)} \sinh^2(d+ex) \, dx = \text{Too large to display}$$

input
```
integrate(F^(c*(b*x+a))*sinh(e*x+d)^2,x, algorithm="giac")
```

output
```
-(2*b*c*cos(-1/2*pi*b*c*x*sgn(F) + 1/2*pi*b*c*x - 1/2*pi*a*c*sgn(F) + 1/2*
pi*a*c)*log(abs(F))/(4*b^2*c^2*log(abs(F))^2 + (pi*b*c*sgn(F) - pi*b*c)^2)
 - (pi*b*c*sgn(F) - pi*b*c)*sin(-1/2*pi*b*c*x*sgn(F) + 1/2*pi*b*c*x - 1/2*
pi*a*c*sgn(F) + 1/2*pi*a*c)/(4*b^2*c^2*log(abs(F))^2 + (pi*b*c*sgn(F) - pi
*b*c)^2))*e^(b*c*x*log(abs(F)) + a*c*log(abs(F))) + I*(-I*e^(1/2*I*pi*b*c*
x*sgn(F) - 1/2*I*pi*b*c*x + 1/2*I*pi*a*c*sgn(F) - 1/2*I*pi*a*c)/(2*I*pi*b*
c*sgn(F) - 2*I*pi*b*c + 4*b*c*log(abs(F))) + I*e^(-1/2*I*pi*b*c*x*sgn(F) +
 1/2*I*pi*b*c*x - 1/2*I*pi*a*c*sgn(F) + 1/2*I*pi*a*c)/(-2*I*pi*b*c*sgn(F)
+ 2*I*pi*b*c + 4*b*c*log(abs(F))))*e^(b*c*x*log(abs(F)) + a*c*log(abs(F)))
 + 1/2*(2*(b*c*log(abs(F)) + 2*e)*cos(-1/2*pi*b*c*x*sgn(F) + 1/2*pi*b*c*x
- 1/2*pi*a*c*sgn(F) + 1/2*pi*a*c)/((pi*b*c*sgn(F) - pi*b*c)^2 + 4*(b*c*log
(abs(F)) + 2*e)^2) - (pi*b*c*sgn(F) - pi*b*c)*sin(-1/2*pi*b*c*x*sgn(F) + 1
/2*pi*b*c*x - 1/2*pi*a*c*sgn(F) + 1/2*pi*a*c)/((pi*b*c*sgn(F) - pi*b*c)^2
+ 4*(b*c*log(abs(F)) + 2*e)^2))*e^(a*c*log(abs(F)) + (b*c*log(abs(F)) + 2*
e)*x + 2*d) + I*(I*e^(1/2*I*pi*b*c*x*sgn(F) - 1/2*I*pi*b*c*x + 1/2*I*pi*a*
c*sgn(F) - 1/2*I*pi*a*c)/(4*I*pi*b*c*sgn(F) - 4*I*pi*b*c + 8*b*c*log(abs(F
)) + 16*e) - I*e^(-1/2*I*pi*b*c*x*sgn(F) + 1/2*I*pi*b*c*x - 1/2*I*pi*a*c*s
gn(F) + 1/2*I*pi*a*c)/(-4*I*pi*b*c*sgn(F) + 4*I*pi*b*c + 8*b*c*log(abs(F))
 + 16*e))*e^(a*c*log(abs(F)) + (b*c*log(abs(F)) + 2*e)*x + 2*d) + 1/2*(2*(
b*c*log(abs(F)) - 2*e)*cos(-1/2*pi*b*c*x*sgn(F) + 1/2*pi*b*c*x - 1/2*pi...
```

Mupad [B] (verification not implemented)

Time = 1.91 (sec) , antiderivative size = 97, normalized size of antiderivative = 0.73

$$\int F^{c(a+bx)} \sinh^2(d+ex)\,dx$$
$$= -\frac{F^{a\,c+b\,c\,x}\,\left(2\,e^2-\frac{b^2\,c^2\,\ln(F)^2}{2}+\frac{b^2\,c^2\,\ln(F)^2\,\cosh(2\,d+2\,e\,x)}{2}-b\,c\,e\,\ln\left(F\right)\,\sinh(2\,d+2\,e\,x)\right)}{b\,c\,\ln\left(F\right)\,\left(4\,e^2-b^2\,c^2\ln\left(F\right)^2\right)}$$

input
```
int(F^(c*(a + b*x))*sinh(d + e*x)^2,x)
```

output
```
-(F^(a*c + b*c*x)*(2*e^2 - (b^2*c^2*log(F)^2)/2 + (b^2*c^2*log(F)^2*cosh(2
*d + 2*e*x))/2 - b*c*e*log(F)*sinh(2*d + 2*e*x)))/(b*c*log(F)*(4*e^2 - b^2
*c^2*log(F)^2))
```

Reduce [F]

$$\int F^{c(a+bx)} \sinh^2(d+ex)\,dx = f^{ac}\left(\int f^{bcx} \sinh\left(ex+d\right)^2 dx\right)$$

input
```
int(F^(c*(b*x+a))*sinh(e*x+d)^2,x)
```

output
```
f**(a*c)*int(f**(b*c*x)*sinh(d + e*x)**2,x)
```

3.324 $\int F^{c(a+bx)} \sinh(d+ex) \, dx$

Optimal result

Integrand size = 16, antiderivative size = 75

$$\int F^{c(a+bx)} \sinh(d+ex) \, dx = \frac{eF^{c(a+bx)} \cosh(d+ex)}{e^2 - b^2c^2 \log^2(F)} - \frac{bcF^{c(a+bx)} \log(F) \sinh(d+ex)}{e^2 - b^2c^2 \log^2(F)}$$

output
```
e*F^(c*(b*x+a))*cosh(e*x+d)/(e^2-b^2*c^2*ln(F)^2)-b*c*F^(c*(b*x+a))*ln(F)*
sinh(e*x+d)/(e^2-b^2*c^2*ln(F)^2)
```

Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 50, normalized size of antiderivative = 0.67

$$\int F^{c(a+bx)} \sinh(d+ex) \, dx = \frac{F^{c(a+bx)}(e \cosh(d+ex) - bc \log(F) \sinh(d+ex))}{(e - bc \log(F))(e + bc \log(F))}$$

input
```
Integrate[F^(c*(a + b*x))*Sinh[d + e*x],x]
```

output
```
(F^(c*(a + b*x))*(e*Cosh[d + e*x] - b*c*Log[F]*Sinh[d + e*x]))/((e - b*c*L
og[F])*(e + b*c*Log[F]))
```

Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 75, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.062$, Rules used = {5997}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sinh(d+ex)F^{c(a+bx)}\,dx$$

$$\Big\downarrow \text{ 5997}$$

$$\frac{e\cosh(d+ex)F^{c(a+bx)}}{e^2-b^2c^2\log^2(F)}-\frac{bc\log(F)\sinh(d+ex)F^{c(a+bx)}}{e^2-b^2c^2\log^2(F)}$$

input
```
Int[F^(c*(a + b*x))*Sinh[d + e*x],x]
```

output
```
(e*F^(c*(a + b*x))*Cosh[d + e*x])/(e^2 - b^2*c^2*Log[F]^2) - (b*c*F^(c*(a
+ b*x))*Log[F]*Sinh[d + e*x])/(e^2 - b^2*c^2*Log[F]^2)
```

Defintions of rubi rules used

rule 5997
```
Int[(F_)^((c_.)*((a_.) + (b_.)*(x_)))*Sinh[(d_.) + (e_.)*(x_)], x_Symbol] :
> Simp[(-b)*c*Log[F]*F^(c*(a + b*x))*(Sinh[d + e*x]/(e^2 - b^2*c^2*Log[F]^2
)), x] + Simp[e*F^(c*(a + b*x))*(Cosh[d + e*x]/(e^2 - b^2*c^2*Log[F]^2)), x
] /; FreeQ[{F, a, b, c, d, e}, x] && NeQ[e^2 - b^2*c^2*Log[F]^2, 0]
```

Maple [A] (verified)

Time = 0.16 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.68

method	result	size
parallelrisch	$\frac{(\sinh(ex+d)\ln(F)bc-e\cosh(ex+d))F^{c(bx+a)}}{b^2c^2\ln(F)^2-e^2}$	51
risch	$\frac{(\ln(F)bc\,\mathrm{e}^{2ex+2d}-bc\ln(F)-e\,\mathrm{e}^{2ex+2d}-e)\,\mathrm{e}^{-ex-d}F^{c(bx+a)}}{2(bc\ln(F)-e)(e+bc\ln(F))}$	77
orering	$\frac{2bc\ln(F)F^{c(bx+a)}\sinh(ex+d)}{b^2c^2\ln(F)^2-e^2}-\frac{F^{c(bx+a)}bc\ln(F)\sinh(ex+d)+F^{c(bx+a)}e\cosh(ex+d)}{b^2c^2\ln(F)^2-e^2}$	101

input
```
int(F^(c*(b*x+a))*sinh(e*x+d),x,method=_RETURNVERBOSE)
```

output
```
(sinh(e*x+d)*ln(F)*b*c-e*cosh(e*x+d))*F^(c*(b*x+a))/(b^2*c^2*ln(F)^2-e^2)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 244 vs. $2(77) = 154$.

Time = 0.12 (sec) , antiderivative size = 244, normalized size of antiderivative = 3.25

$$\int F^{c(a+bx)}\sinh(d+ex)\,dx =$$
$$-\frac{\left(e\cosh\left(ex+d\right)^2-\left(bc\log\left(F\right)-e\right)\sinh\left(ex+d\right)^2-\left(bc\cosh\left(ex+d\right)^2-bc\right)\log\left(F\right)-2\left(bc\cosh\left(ex+\right.\right.\right.}{}$$

input
```
integrate(F^(c*(b*x+a))*sinh(e*x+d),x, algorithm="fricas")
```

output
```
-1/2*((e*cosh(e*x + d)^2 - (b*c*log(F) - e)*sinh(e*x + d)^2 - (b*c*cosh(e*
x + d)^2 - b*c)*log(F) - 2*(b*c*cosh(e*x + d)*log(F) - e*cosh(e*x + d))*si
nh(e*x + d) + e)*cosh((b*c*x + a*c)*log(F)) + (e*cosh(e*x + d)^2 - (b*c*lo
g(F) - e)*sinh(e*x + d)^2 - (b*c*cosh(e*x + d)^2 - b*c)*log(F) - 2*(b*c*co
sh(e*x + d)*log(F) - e*cosh(e*x + d))*sinh(e*x + d) + e)*sinh((b*c*x + a*c
)*log(F)))/(b^2*c^2*cosh(e*x + d)*log(F)^2 - e^2*cosh(e*x + d) + (b^2*c^2*
log(F)^2 - e^2)*sinh(e*x + d))
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 323 vs. $2(68) = 136$.

Time = 0.72 (sec) , antiderivative size = 323, normalized size of antiderivative = 4.31

$$\int F^{c(a+bx)}\sinh(d+ex)\,dx$$
$$= \begin{cases} x\sinh(d) \\ F^{ac}x\sinh(d) \\ x\sinh(d) \\ -\frac{F^{ac+bcx}x\sinh(bcx\log(F)-d)}{2}+\frac{F^{ac+bcx}x\cosh(bcx\log(F)-d)}{2}+\frac{F^{ac+bcx}\sinh(bcx\log(F)-d)}{2bc\log(F)}-\frac{F^{ac+bcx}\cosh(bcx\log(F)-d)}{bc\log(F)} \\ \frac{F^{ac+bcx}x\sinh(bcx\log(F)+d)}{2}-\frac{F^{ac+bcx}x\cosh(bcx\log(F)+d)}{2}-\frac{F^{ac+bcx}\sinh(bcx\log(F)+d)}{2bc\log(F)}+\frac{F^{ac+bcx}\cosh(bcx\log(F)+d)}{bc\log(F)} \\ \frac{F^{ac+bcx}bc\log(F)\sinh(d+ex)}{b^2c^2\log(F)^2-e^2}-\frac{F^{ac+bcx}e\cosh(d+ex)}{b^2c^2\log(F)^2-e^2} \end{cases}$$

input
```
integrate(F**(c*(b*x+a))*sinh(e*x+d),x)
```

output
```
Piecewise((x*sinh(d), Eq(F, 1) & Eq(e, 0)), (F**(a*c)*x*sinh(d), Eq(b, 0)
& Eq(e, 0)), (x*sinh(d), Eq(c, 0) & Eq(e, 0)), (-F**(a*c + b*c*x)*x*sinh(b
*c*x*log(F) - d)/2 + F**(a*c + b*c*x)*x*cosh(b*c*x*log(F) - d)/2 + F**(a*c
 + b*c*x)*sinh(b*c*x*log(F) - d)/(2*b*c*log(F)) - F**(a*c + b*c*x)*cosh(b*
c*x*log(F) - d)/(b*c*log(F)), Eq(e, -b*c*log(F))), (F**(a*c + b*c*x)*x*sin
h(b*c*x*log(F) + d)/2 - F**(a*c + b*c*x)*x*cosh(b*c*x*log(F) + d)/2 - F**(
a*c + b*c*x)*sinh(b*c*x*log(F) + d)/(2*b*c*log(F)) + F**(a*c + b*c*x)*cosh
(b*c*x*log(F) + d)/(b*c*log(F)), Eq(e, b*c*log(F))), (F**(a*c + b*c*x)*b*c
*log(F)*sinh(d + e*x)/(b**2*c**2*log(F)**2 - e**2) - F**(a*c + b*c*x)*e*co
sh(d + e*x)/(b**2*c**2*log(F)**2 - e**2), True))
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 63, normalized size of antiderivative = 0.84

$$\int F^{c(a+bx)}\sinh(d+ex)\,dx = \frac{F^{ac}e^{(bcx\log(F)+ex+d)}}{2\,(bc\log(F)+e)} - \frac{F^{ac}e^{(bcx\log(F)-ex)}}{2\,(bce^d\log(F)-ee^d)}$$

input
```
integrate(F^(c*(b*x+a))*sinh(e*x+d),x, algorithm="maxima")
```

output
```
1/2*F^(a*c)*e^(b*c*x*log(F) + e*x + d)/(b*c*log(F) + e) - 1/2*F^(a*c)*e^(b
*c*x*log(F) - e*x)/(b*c*e^d*log(F) - e*e^d)
```

Giac [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.14 (sec) , antiderivative size = 598, normalized size of antiderivative = 7.97

$$\int F^{c(a+bx)} \sinh(d+ex)\, dx = \text{Too large to display}$$

input
```
integrate(F^(c*(b*x+a))*sinh(e*x+d),x, algorithm="giac")
```

output
```
(2*(b*c*log(abs(F)) + e)*cos(-1/2*pi*b*c*x*sgn(F) + 1/2*pi*b*c*x - 1/2*pi*
a*c*sgn(F) + 1/2*pi*a*c)/((pi*b*c*sgn(F) - pi*b*c)^2 + 4*(b*c*log(abs(F))
+ e)^2) - (pi*b*c*sgn(F) - pi*b*c)*sin(-1/2*pi*b*c*x*sgn(F) + 1/2*pi*b*c*x
 - 1/2*pi*a*c*sgn(F) + 1/2*pi*a*c)/((pi*b*c*sgn(F) - pi*b*c)^2 + 4*(b*c*lo
g(abs(F)) + e)^2))*e^(a*c*log(abs(F)) + (b*c*log(abs(F)) + e)*x + d) + 1/2
*I*(I*e^(1/2*I*pi*b*c*x*sgn(F) - 1/2*I*pi*b*c*x + 1/2*I*pi*a*c*sgn(F) - 1/
2*I*pi*a*c)/(I*pi*b*c*sgn(F) - I*pi*b*c + 2*b*c*log(abs(F)) + 2*e) - I*e^(
-1/2*I*pi*b*c*x*sgn(F) + 1/2*I*pi*b*c*x - 1/2*I*pi*a*c*sgn(F) + 1/2*I*pi*a
*c)/(-I*pi*b*c*sgn(F) + I*pi*b*c + 2*b*c*log(abs(F)) + 2*e))*e^(a*c*log(ab
s(F)) + (b*c*log(abs(F)) + e)*x + d) - (2*(b*c*log(abs(F)) - e)*cos(-1/2*p
i*b*c*x*sgn(F) + 1/2*pi*b*c*x - 1/2*pi*a*c*sgn(F) + 1/2*pi*a*c)/((pi*b*c*s
gn(F) - pi*b*c)^2 + 4*(b*c*log(abs(F)) - e)^2) - (pi*b*c*sgn(F) - pi*b*c)*
sin(-1/2*pi*b*c*x*sgn(F) + 1/2*pi*b*c*x - 1/2*pi*a*c*sgn(F) + 1/2*pi*a*c)/
((pi*b*c*sgn(F) - pi*b*c)^2 + 4*(b*c*log(abs(F)) - e)^2))*e^(a*c*log(abs(F
)) + (b*c*log(abs(F)) - e)*x - d) + 1/2*I*(-I*e^(1/2*I*pi*b*c*x*sgn(F) - 1
/2*I*pi*b*c*x + 1/2*I*pi*a*c*sgn(F) - 1/2*I*pi*a*c)/(I*pi*b*c*sgn(F) - I*p
i*b*c + 2*b*c*log(abs(F)) - 2*e) + I*e^(-1/2*I*pi*b*c*x*sgn(F) + 1/2*I*pi*
b*c*x - 1/2*I*pi*a*c*sgn(F) + 1/2*I*pi*a*c)/(-I*pi*b*c*sgn(F) + I*pi*b*c +
 2*b*c*log(abs(F)) - 2*e))*e^(a*c*log(abs(F)) + (b*c*log(abs(F)) - e)*x -
d)
```

Mupad [B] (verification not implemented)

Time = 1.70 (sec) , antiderivative size = 73, normalized size of antiderivative = 0.97

$$\int F^{c(a+bx)}\sinh(d+ex)\,dx = \frac{F^{a\,c+b\,c\,x}\,\mathrm{e}^{-d-e\,x}\,\left(e+e\,\mathrm{e}^{2\,d+2\,e\,x}+b\,c\,\ln\left(F\right)-b\,c\,\mathrm{e}^{2\,d+2\,e\,x}\,\ln\left(F\right)\right)}{2\,\left(e^2-b^2\,c^2\,\ln\left(F\right)^2\right)}$$

input
```
int(F^(c*(a + b*x))*sinh(d + e*x),x)
```

output
```
(F^(a*c + b*c*x)*exp(- d - e*x)*(e + e*exp(2*d + 2*e*x) + b*c*log(F) - b*c
*exp(2*d + 2*e*x)*log(F)))/(2*(e^2 - b^2*c^2*log(F)^2))
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.68

$$\int F^{c(a+bx)}\sinh(d+ex)\,dx = \frac{f^{bcx+ac}(-\cosh\left(ex+d\right)e+\log(f)\sinh\left(ex+d\right)bc)}{\log\left(f\right)^2b^2c^2-e^2}$$

input
```
int(F^(c*(b*x+a))*sinh(e*x+d),x)
```

output
```
(f**(a*c + b*c*x)*( - cosh(d + e*x)*e + log(f)*sinh(d + e*x)*b*c))/(log(f)
**2*b**2*c**2 - e**2)
```

3.325 $\int F^{c(a+bx)}\text{csch}(d+ex)\,dx$

Optimal result

Integrand size = 16, antiderivative size = 66

$$\int F^{c(a+bx)}\text{csch}(d+ex)\,dx = -\frac{2e^{d+ex}F^{c(a+bx)}\text{Hypergeometric2F1}\left(1,\frac{e+bc\log(F)}{2e},\frac{1}{2}\left(3+\frac{bc\log(F)}{e}\right),e^{2(d+ex)}\right)}{e+bc\log(F)}$$

output
```
-2*exp(e*x+d)*F^(c*(b*x+a))*hypergeom([1, 1/2*(e+b*c*ln(F))/e],[3/2+1/2*b*
c*ln(F)/e],exp(2*e*x+2*d))/(e+b*c*ln(F))
```

Mathematica [A] (verified)

Time = 2.75 (sec) , antiderivative size = 79, normalized size of antiderivative = 1.20

$$\int F^{c(a+bx)}\text{csch}(d+ex)\,dx = \frac{F^{c(a+bx)}\left(\text{Hypergeometric2F1}\left(1,\frac{bc\log(F)}{e},1+\frac{bc\log(F)}{e},-e^{d+ex}\right)-\text{Hypergeometric2F1}\left(1,\frac{bc\log(F)}{e},1+\frac{bc}{}\right.\right.}{bc\log(F)}$$

input
```
Integrate[F^(c*(a + b*x))*Csch[d + e*x],x]
```

output
```
(F^(c*(a + b*x))*(Hypergeometric2F1[1, (b*c*Log[F])/e, 1 + (b*c*Log[F])/e,
 -E^(d + e*x)] - Hypergeometric2F1[1, (b*c*Log[F])/e, 1 + (b*c*Log[F])/e,
E^(d + e*x)]))/(b*c*Log[F])
```

Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 66, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.062$, Rules used = {6016}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \operatorname{csch}(d+ex)F^{c(a+bx)}\,dx$$

$$\Big\downarrow \; 6016$$

$$-\frac{2e^{d+ex}F^{c(a+bx)}\,\text{Hypergeometric2F1}\left(1,\frac{e+bc\log(F)}{2e},\frac{1}{2}\left(\frac{bc\log(F)}{e}+3\right),e^{2(d+ex)}\right)}{bc\log(F)+e}$$

input
```
Int[F^(c*(a + b*x))*Csch[d + e*x],x]
```

output
```
(-2*E^(d + e*x)*F^(c*(a + b*x))*Hypergeometric2F1[1, (e + b*c*Log[F])/(2*e
), (3 + (b*c*Log[F])/e)/2, E^(2*(d + e*x))])/(e + b*c*Log[F])
```

Defintions of rubi rules used

rule 6016
```
Int[Csch[(d_.) + (e_.)*(x_)]^(n_.)*(F_)^((c_.)*((a_.) + (b_.)*(x_))), x_Sym
bol] :> Simp[(-2)^n*E^(n*(d + e*x))*(F^(c*(a + b*x))/(e*n + b*c*Log[F]))*Hy
pergeometric2F1[n, n/2 + b*c*(Log[F]/(2*e)), 1 + n/2 + b*c*(Log[F]/(2*e)),
E^(2*(d + e*x))], x] /; FreeQ[{F, a, b, c, d, e}, x] && IntegerQ[n]
```

Maple [F]

$$\int F^{c(bx+a)} \operatorname{csch}(ex+d)\,dx$$

input
```
int(F^(c*(b*x+a))*csch(e*x+d),x)
```

output
```
int(F^(c*(b*x+a))*csch(e*x+d),x)
```

Fricas [F]

$$\int F^{c(a+bx)}\operatorname{csch}(d+ex)\,dx = \int F^{(bx+a)c} \operatorname{csch}(ex+d)\,dx$$

input
```
integrate(F^(c*(b*x+a))*csch(e*x+d),x, algorithm="fricas")
```

output
```
integral(F^(b*c*x + a*c)*csch(e*x + d), x)
```

Sympy [F]

$$\int F^{c(a+bx)}\operatorname{csch}(d+ex)\,dx = \int F^{c(a+bx)} \operatorname{csch}(d+ex)\,dx$$

input
```
integrate(F**(c*(b*x+a))*csch(e*x+d),x)
```

output
```
Integral(F**(c*(a + b*x))*csch(d + e*x), x)
```

Maxima [F]

$$\int F^{c(a+bx)}\text{csch}(d+ex)\,dx=\int F^{(bx+a)c}\,\text{csch}\,(ex+d)\,\,dx$$

input
```
integrate(F^(c*(b*x+a))*csch(e*x+d),x, algorithm="maxima")
```

output
```
4*F^(a*c)*e*integrate(e^(b*c*x*log(F) + e*x + d)/(b*c*log(F) + (b*c*e^(4*d
)*log(F) - e*e^(4*d))*e^(4*e*x) - 2*(b*c*e^(2*d)*log(F) - e*e^(2*d))*e^(2*
e*x) - e), x) - 2*F^(a*c)*e^(b*c*x*log(F) + e*x + d)/(b*c*log(F) - (b*c*e^
(2*d)*log(F) - e*e^(2*d))*e^(2*e*x) - e)
```

Giac [F]

$$\int F^{c(a+bx)}\text{csch}(d+ex)\,dx=\int F^{(bx+a)c}\,\text{csch}\,(ex+d)\,\,dx$$

input
```
integrate(F^(c*(b*x+a))*csch(e*x+d),x, algorithm="giac")
```

output
```
integrate(F^((b*x + a)*c)*csch(e*x + d), x)
```

Mupad [F(-1)]

Timed out.

$$\int F^{c(a+bx)}\text{csch}(d+ex)\,dx=\int \frac{F^{c\,(a+b\,x)}}{\sinh\left(d+e\,x\right)}\,dx$$

input
```
int(F^(c*(a + b*x))/sinh(d + e*x),x)
```

output
```
int(F^(c*(a + b*x))/sinh(d + e*x), x)
```

Reduce [F]

$$\int F^{c(a+bx)}\operatorname{csch}(d+ex)\,dx = f^{ac}\left(\int f^{bcx}\operatorname{csch}(ex+d)\,dx\right)$$

input
```
int(F^(c*(b*x+a))*csch(e*x+d),x)
```

output
```
f**(a*c)*int(f**(b*c*x)*csch(d + e*x),x)
```

3.326 $\int F^{c(a+bx)}\text{csch}^2(d+ex)\,dx$

Optimal result

Integrand size = 18, antiderivative size = 68

$$\int F^{c(a+bx)}\text{csch}^2(d+ex)\,dx = \frac{4e^{2(d+ex)}F^{c(a+bx)}\,\text{Hypergeometric2F1}\left(2,1+\frac{bc\log(F)}{2e},2+\frac{bc\log(F)}{2e},e^{2(d+ex)}\right)}{2e+bc\log(F)}$$

output
```
4*exp(2*e*x+2*d)*F^(c*(b*x+a))*hypergeom([2, 1+1/2*b*c*ln(F)/e],[2+1/2*b*c
*ln(F)/e],exp(2*e*x+2*d))/(2*e+b*c*ln(F))
```

Mathematica [A] (verified)

Time = 0.82 (sec) , antiderivative size = 87, normalized size of antiderivative = 1.28

$$\int F^{c(a+bx)}\text{csch}^2(d+ex)\,dx = -\frac{2F^{c(a+bx)}\left(\left(-1+e^{2d}\right)\text{Hypergeometric2F1}\left(1,\frac{bc\log(F)}{2e},1+\frac{bc\log(F)}{2e},e^{2(d+ex)}\right)+\text{csch}(d+ex)\sinh(d)(\text{co}}{e\left(-1+e^{2d}\right)}$$

input
```
Integrate[F^(c*(a + b*x))*Csch[d + e*x]^2,x]
```

output
```
(-2*F^(c*(a + b*x))*((-1 + E^(2*d))*Hypergeometric2F1[1, (b*c*Log[F])/(2*e
), 1 + (b*c*Log[F])/(2*e), E^(2*(d + e*x))] + Csch[d + e*x]*Sinh[d]*(Cosh[
e*x] - Sinh[e*x])))/(e*(-1 + E^(2*d)))
```

Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 68, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.056$, Rules used = {6016}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \text{csch}^2(d+ex)F^{c(a+bx)}\,dx$$

$$\Big\downarrow \text{6016}$$

$$\frac{4e^{2(d+ex)}F^{c(a+bx)}\,\text{Hypergeometric2F1}\left(2,\frac{bc\log(F)}{2e}+1,\frac{bc\log(F)}{2e}+2,e^{2(d+ex)}\right)}{bc\log(F)+2e}$$

input
```
Int[F^(c*(a + b*x))*Csch[d + e*x]^2,x]
```

output
```
(4*E^(2*(d + e*x))*F^(c*(a + b*x))*Hypergeometric2F1[2, 1 + (b*c*Log[F])/(
2*e), 2 + (b*c*Log[F])/(2*e), E^(2*(d + e*x))])/(2*e + b*c*Log[F])
```

Defintions of rubi rules used

rule 6016
```
Int[Csch[(d_.) + (e_.)*(x_)]^(n_.)*(F_)^((c_.)*((a_.) + (b_.)*(x_))), x_Sym
bol] :> Simp[(-2)^n*E^(n*(d + e*x))*(F^(c*(a + b*x))/(e*n + b*c*Log[F]))*Hy
pergeometric2F1[n, n/2 + b*c*(Log[F]/(2*e)), 1 + n/2 + b*c*(Log[F]/(2*e)),
E^(2*(d + e*x))], x] /; FreeQ[{F, a, b, c, d, e}, x] && IntegerQ[n]
```

Maple [F]

$$\int F^{c(bx+a)} \operatorname{csch}(ex+d)^2 \, dx$$

input
```
int(F^(c*(b*x+a))*csch(e*x+d)^2,x)
```

output
```
int(F^(c*(b*x+a))*csch(e*x+d)^2,x)
```

Fricas [F]

$$\int F^{c(a+bx)}\operatorname{csch}^2(d+ex)\,dx = \int F^{(bx+a)c} \operatorname{csch}(ex+d)^2 \, dx$$

input
```
integrate(F^(c*(b*x+a))*csch(e*x+d)^2,x, algorithm="fricas")
```

output
```
integral(F^(b*c*x + a*c)*csch(e*x + d)^2, x)
```

Sympy [F]

$$\int F^{c(a+bx)}\operatorname{csch}^2(d+ex)\,dx = \int F^{c(a+bx)} \operatorname{csch}^2(d+ex)\,dx$$

input
```
integrate(F**(c*(b*x+a))*csch(e*x+d)**2,x)
```

output
```
Integral(F**(c*(a + b*x))*csch(d + e*x)**2, x)
```

Maxima [F]

$$\int F^{c(a+bx)}\text{csch}^2(d+ex)\,dx=\int F^{(bx+a)c}\,\text{csch}\,(ex+d)^2\,dx$$

input
```
integrate(F^(c*(b*x+a))*csch(e*x+d)^2,x, algorithm="maxima")
```

output
```
16*F^(a*c)*b*c*e*integrate(-F^(b*c*x)/(b^2*c^2*log(F)^2 - 6*b*c*e*log(F) +
 8*e^2 - (b^2*c^2*e^(6*d)*log(F)^2 - 6*b*c*e*e^(6*d)*log(F) + 8*e^2*e^(6*d
))*e^(6*e*x) + 3*(b^2*c^2*e^(4*d)*log(F)^2 - 6*b*c*e*e^(4*d)*log(F) + 8*e^
2*e^(4*d))*e^(4*e*x) - 3*(b^2*c^2*e^(2*d)*log(F)^2 - 6*b*c*e*e^(2*d)*log(F
) + 8*e^2*e^(2*d))*e^(2*e*x)), x)*log(F) + 4*(4*F^(a*c)*e + (F^(a*c)*b*c*e
^(2*d)*log(F) - 4*F^(a*c)*e*e^(2*d))*e^(2*e*x))*F^(b*c*x)/(b^2*c^2*log(F)^
2 - 6*b*c*e*log(F) + 8*e^2 + (b^2*c^2*e^(4*d)*log(F)^2 - 6*b*c*e*e^(4*d)*l
og(F) + 8*e^2*e^(4*d))*e^(4*e*x) - 2*(b^2*c^2*e^(2*d)*log(F)^2 - 6*b*c*e*e
^(2*d)*log(F) + 8*e^2*e^(2*d))*e^(2*e*x))
```

Giac [F]

$$\int F^{c(a+bx)}\text{csch}^2(d+ex)\,dx=\int F^{(bx+a)c}\,\text{csch}\,(ex+d)^2\,dx$$

input
```
integrate(F^(c*(b*x+a))*csch(e*x+d)^2,x, algorithm="giac")
```

output
```
integrate(F^((b*x + a)*c)*csch(e*x + d)^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int F^{c(a+bx)}\operatorname{csch}^2(d+ex)\,dx=\int \frac{F^{c\,(a+b\,x)}}{\sinh\left(d+e\,x\right)^2}\,dx$$

input
```
int(F^(c*(a + b*x))/sinh(d + e*x)^2,x)
```

output
```
int(F^(c*(a + b*x))/sinh(d + e*x)^2, x)
```

Reduce [F]

$$\int F^{c(a+bx)}\operatorname{csch}^2(d+ex)\,dx=f^{ac}\left(\int f^{bcx}\operatorname{csch}(ex+d)^2\,dx\right)$$

input
```
int(F^(c*(b*x+a))*csch(e*x+d)^2,x)
```

output
```
f**(a*c)*int(f**(b*c*x)*csch(d + e*x)**2,x)
```

3.327 $\int F^{c(a+bx)}\operatorname{csch}^3(d+ex)\,dx$

Optimal result

Integrand size = 18, antiderivative size = 122

$$\int F^{c(a+bx)}\operatorname{csch}^3(d+ex)\,dx = -\frac{F^{c(a+bx)}\coth(d+ex)\operatorname{csch}(d+ex)}{2e} - \frac{bcF^{c(a+bx)}\operatorname{csch}(d+ex)\log(F)}{2e^2} + \frac{e^{d+ex}F^{c(a+bx)}\,\text{Hypergeometric2F1}\left(1,\frac{e+bc\log(F)}{2e},\frac{1}{2}\left(3+\frac{bc\log(F)}{e}\right),e^{2(d+ex)}\right)(e-bc\log(F))}{e^2}$$

output
```
-1/2*F^(c*(b*x+a))*coth(e*x+d)*csch(e*x+d)/e-1/2*b*c*F^(c*(b*x+a))*csch(e*
x+d)*ln(F)/e^2+exp(e*x+d)*F^(c*(b*x+a))*hypergeom([1, 1/2*(e+b*c*ln(F))/e]
,[3/2+1/2*b*c*ln(F)/e],exp(2*e*x+2*d))*(e-b*c*ln(F))/e^2
```

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 281 vs. 2(122) = 244.

Time = 15.84 (sec) , antiderivative size = 281, normalized size of antiderivative = 2.30

$$\int F^{c(a+bx)}\operatorname{csch}^3(d+ex)\,dx = \frac{F^{c(a+bx)}\left(-e\operatorname{csch}^2\left(\frac{1}{2}(d+ex)\right)-4bc\operatorname{csch}(d)\log(F)+\operatorname{csch}(d)\left(-\frac{4e^2}{bc\log(F)}+4bc\log(F)\right)+\frac{4\left(1-(1+e^d)\text{ Hyper}\right.}{}\right.}{}$$

input
```
Integrate[F^(c*(a + b*x))*Csch[d + e*x]^3,x]
```

output
```
(F^(c*(a + b*x))*(-(e*Csch[(d + e*x)/2]^2) - 4*b*c*Csch[d]*Log[F] + Csch[d
]*((-4*e^2)/(b*c*Log[F]) + 4*b*c*Log[F]) + (4*(1 - (1 + E^d)*Hypergeometri
c2F1[1, (b*c*Log[F])/e, 1 + (b*c*Log[F])/e, -E^(d + e*x)])*(e^2 - b^2*c^2*
Log[F]^2))/(b*c*(1 + E^d)*Log[F]) + (4*(1 + (-1 + E^d)*Hypergeometric2F1[1
, (b*c*Log[F])/e, 1 + (b*c*Log[F])/e, E^(d + e*x)])*(e^2 - b^2*c^2*Log[F]^
2))/(b*c*(-1 + E^d)*Log[F]) - e*Sech[(d + e*x)/2]^2 + 2*b*c*Csch[d/2]*Csch
[(d + e*x)/2]*Log[F]*Sinh[(e*x)/2] + 2*b*c*Log[F]*Sech[d/2]*Sech[(d + e*x)
/2]*Sinh[(e*x)/2]))/(8*e^2)
```

Rubi [A] (verified)

Time = 0.35 (sec) , antiderivative size = 137, normalized size of antiderivative = 1.12, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.111$, Rules used = {6014, 6016}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \operatorname{csch}^3(d+ex)F^{c(a+bx)}\,dx$$

$$\Big\downarrow \text{6014}$$

$$-\frac{1}{2}\left(1-\frac{b^2c^2\log^2(F)}{e^2}\right)\int F^{c(a+bx)}\operatorname{csch}(d+ex)dx-\frac{bc\log(F)\operatorname{csch}(d+ex)F^{c(a+bx)}}{2e^2}-\frac{\coth(d+ex)\operatorname{csch}(d+ex)F^{c(a+bx)}}{2e}$$

$$\Big\downarrow \text{6016}$$

$$\frac{e^{d+ex}F^{c(a+bx)}\left(1-\frac{b^2c^2\log^2(F)}{e^2}\right)\text{Hypergeometric2F1}\left(1,\frac{e+bc\log(F)}{2e},\frac{1}{2}\left(\frac{bc\log(F)}{e}+3\right),e^{2(d+ex)}\right)}{bc\log(F)+e}-\frac{bc\log(F)\operatorname{csch}(d+ex)F^{c(a+bx)}}{2e^2}-\frac{\coth(d+ex)\operatorname{csch}(d+ex)F^{c(a+bx)}}{2e}$$

input
```
Int[F^(c*(a + b*x))*Csch[d + e*x]^3,x]
```

output
```
-1/2*(F^(c*(a + b*x))*Coth[d + e*x]*Csch[d + e*x])/e - (b*c*F^(c*(a + b*x)
)*Csch[d + e*x]*Log[F])/(2*e^2) + (E^(d + e*x)*F^(c*(a + b*x))*Hypergeomet
ric2F1[1, (e + b*c*Log[F])/(2*e), (3 + (b*c*Log[F])/e)/2, E^(2*(d + e*x))]
*(1 - (b^2*c^2*Log[F]^2)/e^2))/(e + b*c*Log[F])
```

Defintions of rubi rules used

rule 6014
```
Int[Csch[(d_.) + (e_.)*(x_)]^(n_)*(F_)^((c_.)*((a_.) + (b_.)*(x_))), x_Symb
ol] :> Simp[(-b)*c*Log[F]*F^(c*(a + b*x))*(Csch[d + e*x]^(n - 2)/(e^2*(n -
1)*(n - 2))), x] + (-Simp[F^(c*(a + b*x))*Csch[d + e*x]^(n - 1)*(Cosh[d + e
*x]/(e*(n - 1))), x] - Simp[(e^2*(n - 2)^2 - b^2*c^2*Log[F]^2)/(e^2*(n - 1)
*(n - 2))   Int[F^(c*(a + b*x))*Csch[d + e*x]^(n - 2), x], x]) /; FreeQ[{F,
 a, b, c, d, e}, x] && NeQ[e^2*(n - 2)^2 - b^2*c^2*Log[F]^2, 0] && GtQ[n, 1
] && NeQ[n, 2]
```

rule 6016
```
Int[Csch[(d_.) + (e_.)*(x_)]^(n_.)*(F_)^((c_.)*((a_.) + (b_.)*(x_))), x_Sym
bol] :> Simp[(-2)^n*E^(n*(d + e*x))*(F^(c*(a + b*x))/(e*n + b*c*Log[F]))*Hy
pergeometric2F1[n, n/2 + b*c*(Log[F]/(2*e)), 1 + n/2 + b*c*(Log[F]/(2*e)),
E^(2*(d + e*x))], x] /; FreeQ[{F, a, b, c, d, e}, x] && IntegerQ[n]
```

Maple [F]

$$\int F^{c(bx+a)} \operatorname{csch}(ex+d)^3 \, dx$$

input
```
int(F^(c*(b*x+a))*csch(e*x+d)^3,x)
```

output
```
int(F^(c*(b*x+a))*csch(e*x+d)^3,x)
```

Fricas [F]

$$\int F^{c(a+bx)}\text{csch}^3(d+ex)\,dx=\int F^{(bx+a)c}\,\text{csch}\left(ex+d\right)^3\,dx$$

input
```
integrate(F^(c*(b*x+a))*csch(e*x+d)^3,x, algorithm="fricas")
```

output
```
integral(F^(b*c*x + a*c)*csch(e*x + d)^3, x)
```

Sympy [F]

$$\int F^{c(a+bx)}\text{csch}^3(d+ex)\,dx=\int F^{c(a+bx)}\,\text{csch}^3\left(d+ex\right)dx$$

input
```
integrate(F**(c*(b*x+a))*csch(e*x+d)**3,x)
```

output
```
Integral(F**(c*(a + b*x))*csch(d + e*x)**3, x)
```

Maxima [F]

$$\int F^{c(a+bx)}\text{csch}^3(d+ex)\,dx=\int F^{(bx+a)c}\,\text{csch}\left(ex+d\right)^3\,dx$$

input
```
integrate(F^(c*(b*x+a))*csch(e*x+d)^3,x, algorithm="maxima")
```

output
```
48*(F^(a*c)*b*c*e*e^d*log(F) + F^(a*c)*e^2*e^d)*integrate(e^(b*c*x*log(F)
+ e*x)/(b^2*c^2*log(F)^2 - 8*b*c*e*log(F) + 15*e^2 + (b^2*c^2*e^(8*d)*log(
F)^2 - 8*b*c*e*e^(8*d)*log(F) + 15*e^2*e^(8*d))*e^(8*e*x) - 4*(b^2*c^2*e^(
6*d)*log(F)^2 - 8*b*c*e*e^(6*d)*log(F) + 15*e^2*e^(6*d))*e^(6*e*x) + 6*(b^
2*c^2*e^(4*d)*log(F)^2 - 8*b*c*e*e^(4*d)*log(F) + 15*e^2*e^(4*d))*e^(4*e*x
) - 4*(b^2*c^2*e^(2*d)*log(F)^2 - 8*b*c*e*e^(2*d)*log(F) + 15*e^2*e^(2*d))
*e^(2*e*x)), x) - 8*(6*F^(a*c)*e*e^(e*x + d) + (F^(a*c)*b*c*e^(3*d)*log(F)
 - 5*F^(a*c)*e*e^(3*d))*e^(3*e*x))*F^(b*c*x)/(b^2*c^2*log(F)^2 - 8*b*c*e*l
og(F) + 15*e^2 - (b^2*c^2*e^(6*d)*log(F)^2 - 8*b*c*e*e^(6*d)*log(F) + 15*e
^2*e^(6*d))*e^(6*e*x) + 3*(b^2*c^2*e^(4*d)*log(F)^2 - 8*b*c*e*e^(4*d)*log(
F) + 15*e^2*e^(4*d))*e^(4*e*x) - 3*(b^2*c^2*e^(2*d)*log(F)^2 - 8*b*c*e*e^(
2*d)*log(F) + 15*e^2*e^(2*d))*e^(2*e*x))
```

Giac [F]

$$\int F^{c(a+bx)}\operatorname{csch}^3(d+ex)\,dx=\int F^{(bx+a)c}\operatorname{csch}(ex+d)^3\,dx$$

input
```
integrate(F^(c*(b*x+a))*csch(e*x+d)^3,x, algorithm="giac")
```

output
```
integrate(F^((b*x + a)*c)*csch(e*x + d)^3, x)
```

Mupad [F(-1)]

Timed out.

$$\int F^{c(a+bx)}\operatorname{csch}^3(d+ex)\,dx=\int \frac{F^{c\,(a+b\,x)}}{\sinh\left(d+e\,x\right)^3}\,dx$$

input
```
int(F^(c*(a + b*x))/sinh(d + e*x)^3,x)
```

output
```
int(F^(c*(a + b*x))/sinh(d + e*x)^3, x)
```

Reduce [F]

$$\int F^{c(a+bx)}\operatorname{csch}^3(d+ex)\,dx = f^{ac}\left(\int f^{bcx}\operatorname{csch}(ex+d)^3\,dx\right)$$

input
```
int(F^(c*(b*x+a))*csch(e*x+d)^3,x)
```

output
```
f**(a*c)*int(f**(b*c*x)*csch(d + e*x)**3,x)
```

3.328 $\int F^{c(a+bx)}\operatorname{csch}^4(d+ex)\,dx$

Optimal result

Integrand size = 18, antiderivative size = 131

$$\int F^{c(a+bx)}\operatorname{csch}^4(d+ex)\,dx = -\frac{F^{c(a+bx)}\coth(d+ex)\operatorname{csch}^2(d+ex)}{3e}-\frac{bcF^{c(a+bx)}\operatorname{csch}^2(d+ex)\log(F)}{6e^2}-\frac{2e^{2(d+ex)}F^{c(a+bx)}\,\text{Hypergeometric2F1}\left(2,1+\frac{bc\log(F)}{2e},2+\frac{bc\log(F)}{2e},e^{2(d+ex)}\right)(2e-bc\log(F))}{3e^2}$$

output

```
-1/3*F^(c*(b*x+a))*coth(e*x+d)*csch(e*x+d)^2/e-1/6*b*c*F^(c*(b*x+a))*csch(
e*x+d)^2*ln(F)/e^2-2/3*exp(2*e*x+2*d)*F^(c*(b*x+a))*hypergeom([2, 1+1/2*b*
c*ln(F)/e],[2+1/2*b*c*ln(F)/e],exp(2*e*x+2*d))*(2*e-b*c*ln(F))/e^2
```

Mathematica [A] (verified)

Time = 1.51 (sec) , antiderivative size = 163, normalized size of antiderivative = 1.24

$$\int F^{c(a+bx)}\operatorname{csch}^4(d+ex)\,dx = \frac{F^{c(a+bx)}\left(-e\operatorname{csch}^2(d+ex)(2e\coth(d)+bc\log(F))-\frac{2\left(1+(-1+e^{2d})\,\text{Hypergeometric2F1}\left(1,\frac{bc\log(F)}{2e},1+\frac{bc\log(F)}{2e},e^{2(d+ex}\right.\right.}{-1+e^{2d}}\right.}{}$$

input
```
Integrate[F^(c*(a + b*x))*Csch[d + e*x]^4,x]
```

output
```
(F^(c*(a + b*x))*(-(e*Csch[d + e*x]^2*(2*e*Coth[d] + b*c*Log[F])) - (2*(1
+ (-1 + E^(2*d))*Hypergeometric2F1[1, (b*c*Log[F])/(2*e), 1 + (b*c*Log[F])
/(2*e), E^(2*(d + e*x))])*(-4*e^2 + b^2*c^2*Log[F]^2))/(-1 + E^(2*d)) + 2*
e^2*Csch[d]*Csch[d + e*x]^3*Sinh[e*x] - Csch[d]*Csch[d + e*x]*(4*e^2 - b^2
*c^2*Log[F]^2)*Sinh[e*x]))/(6*e^3)
```

Rubi [A] (verified)

Time = 0.36 (sec) , antiderivative size = 146, normalized size of antiderivative = 1.11, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.111$, Rules used = {6014, 6016}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \operatorname{csch}^4(d+ex)F^{c(a+bx)}\,dx$$

$$\Big\downarrow \text{6014}$$

$$-\frac{1}{6}\left(4-\frac{b^2c^2\log^2(F)}{e^2}\right)\int F^{c(a+bx)}\operatorname{csch}^2(d+ex)dx-\frac{bc\log(F)\operatorname{csch}^2(d+ex)F^{c(a+bx)}}{6e^2}-\frac{\coth(d+ex)\operatorname{csch}^2(d+ex)F^{c(a+bx)}}{3e}$$

$$\Big\downarrow \text{6016}$$

$$-\frac{2e^{2(d+ex)}F^{c(a+bx)}\left(4-\frac{b^2c^2\log^2(F)}{e^2}\right)\text{Hypergeometric2F1}\left(2,\frac{bc\log(F)}{2e}+1,\frac{bc\log(F)}{2e}+2,e^{2(d+ex)}\right)}{3(bc\log(F)+2e)}-\frac{bc\log(F)\operatorname{csch}^2(d+ex)F^{c(a+bx)}}{6e^2}-\frac{\coth(d+ex)\operatorname{csch}^2(d+ex)F^{c(a+bx)}}{3e}$$

input
```
Int[F^(c*(a + b*x))*Csch[d + e*x]^4,x]
```

output
```
-1/3*(F^(c*(a + b*x))*Coth[d + e*x]*Csch[d + e*x]^2)/e - (b*c*F^(c*(a + b*
x))*Csch[d + e*x]^2*Log[F])/(6*e^2) - (2*E^(2*(d + e*x))*F^(c*(a + b*x))*H
ypergeometric2F1[2, 1 + (b*c*Log[F])/(2*e), 2 + (b*c*Log[F])/(2*e), E^(2*(
d + e*x))]*(4 - (b^2*c^2*Log[F]^2)/e^2))/(3*(2*e + b*c*Log[F]))
```

Defintions of rubi rules used

rule 6014
```
Int[Csch[(d_.) + (e_.)*(x_)]^(n_)*(F_)^((c_.)*((a_.) + (b_.)*(x_))), x_Symb
ol] :> Simp[(-b)*c*Log[F]*F^(c*(a + b*x))*(Csch[d + e*x]^(n - 2)/(e^2*(n -
1)*(n - 2))), x] + (-Simp[F^(c*(a + b*x))*Csch[d + e*x]^(n - 1)*(Cosh[d + e
*x]/(e*(n - 1))), x] - Simp[(e^2*(n - 2)^2 - b^2*c^2*Log[F]^2)/(e^2*(n - 1)
*(n - 2))   Int[F^(c*(a + b*x))*Csch[d + e*x]^(n - 2), x], x]) /; FreeQ[{F,
 a, b, c, d, e}, x] && NeQ[e^2*(n - 2)^2 - b^2*c^2*Log[F]^2, 0] && GtQ[n, 1
] && NeQ[n, 2]
```

rule 6016
```
Int[Csch[(d_.) + (e_.)*(x_)]^(n_.)*(F_)^((c_.)*((a_.) + (b_.)*(x_))), x_Sym
bol] :> Simp[(-2)^n*E^(n*(d + e*x))*(F^(c*(a + b*x))/(e*n + b*c*Log[F]))*Hy
pergeometric2F1[n, n/2 + b*c*(Log[F]/(2*e)), 1 + n/2 + b*c*(Log[F]/(2*e)),
E^(2*(d + e*x))], x] /; FreeQ[{F, a, b, c, d, e}, x] && IntegerQ[n]
```

Maple [F]

$$\int F^{c(bx+a)} \operatorname{csch}(ex+d)^4 \, dx$$

input
```
int(F^(c*(b*x+a))*csch(e*x+d)^4,x)
```

output
```
int(F^(c*(b*x+a))*csch(e*x+d)^4,x)
```

Fricas [F]

$$\int F^{c(a+bx)}\operatorname{csch}^4(d+ex)\,dx=\int F^{(bx+a)c}\operatorname{csch}(ex+d)^4\,dx$$

input
```
integrate(F^(c*(b*x+a))*csch(e*x+d)^4,x, algorithm="fricas")
```

output
```
integral(F^(b*c*x + a*c)*csch(e*x + d)^4, x)
```

Sympy [F]

$$\int F^{c(a+bx)}\operatorname{csch}^4(d+ex)\,dx=\int F^{c(a+bx)}\operatorname{csch}^4(d+ex)\,dx$$

input
```
integrate(F**(c*(b*x+a))*csch(e*x+d)**4,x)
```

output
```
Integral(F**(c*(a + b*x))*csch(d + e*x)**4, x)
```

Maxima [F]

$$\int F^{c(a+bx)}\operatorname{csch}^4(d+ex)\,dx=\int F^{(bx+a)c}\operatorname{csch}(ex+d)^4\,dx$$

input
```
integrate(F^(c*(b*x+a))*csch(e*x+d)^4,x, algorithm="maxima")
```

output

```
128*(F^(a*c)*b^2*c^2*e*log(F)^2 + 2*F^(a*c)*b*c*e^2*log(F))*integrate(-F^(
b*c*x)/(b^3*c^3*log(F)^3 - 18*b^2*c^2*e*log(F)^2 + 104*b*c*e^2*log(F) - 19
2*e^3 - (b^3*c^3*e^(10*d)*log(F)^3 - 18*b^2*c^2*e*e^(10*d)*log(F)^2 + 104*
b*c*e^2*e^(10*d)*log(F) - 192*e^3*e^(10*d))*e^(10*e*x) + 5*(b^3*c^3*e^(8*d
)*log(F)^3 - 18*b^2*c^2*e*e^(8*d)*log(F)^2 + 104*b*c*e^2*e^(8*d)*log(F) -
192*e^3*e^(8*d))*e^(8*e*x) - 10*(b^3*c^3*e^(6*d)*log(F)^3 - 18*b^2*c^2*e*e
^(6*d)*log(F)^2 + 104*b*c*e^2*e^(6*d)*log(F) - 192*e^3*e^(6*d))*e^(6*e*x)
+ 10*(b^3*c^3*e^(4*d)*log(F)^3 - 18*b^2*c^2*e*e^(4*d)*log(F)^2 + 104*b*c*e
^2*e^(4*d)*log(F) - 192*e^3*e^(4*d))*e^(4*e*x) - 5*(b^3*c^3*e^(2*d)*log(F)
^3 - 18*b^2*c^2*e*e^(2*d)*log(F)^2 + 104*b*c*e^2*e^(2*d)*log(F) - 192*e^3*
e^(2*d))*e^(2*e*x)), x) + 16*(8*F^(a*c)*b*c*e*log(F) + 16*F^(a*c)*e^2 + (F
^(a*c)*b^2*c^2*e^(4*d)*log(F)^2 - 14*F^(a*c)*b*c*e*e^(4*d)*log(F) + 48*F^(
a*c)*e^2*e^(4*d))*e^(4*e*x) + 8*(F^(a*c)*b*c*e*e^(2*d)*log(F) - 8*F^(a*c)*
e^2*e^(2*d))*e^(2*e*x))*F^(b*c*x)/(b^3*c^3*log(F)^3 - 18*b^2*c^2*e*log(F)^
2 + 104*b*c*e^2*log(F) - 192*e^3 + (b^3*c^3*e^(8*d)*log(F)^3 - 18*b^2*c^2*
e*e^(8*d)*log(F)^2 + 104*b*c*e^2*e^(8*d)*log(F) - 192*e^3*e^(8*d))*e^(8*e*
x) - 4*(b^3*c^3*e^(6*d)*log(F)^3 - 18*b^2*c^2*e*e^(6*d)*log(F)^2 + 104*b*c
*e^2*e^(6*d)*log(F) - 192*e^3*e^(6*d))*e^(6*e*x) + 6*(b^3*c^3*e^(4*d)*log(
F)^3 - 18*b^2*c^2*e*e^(4*d)*log(F)^2 + 104*b*c*e^2*e^(4*d)*log(F) - 192*e^
3*e^(4*d))*e^(4*e*x) - 4*(b^3*c^3*e^(2*d)*log(F)^3 - 18*b^2*c^2*e*e^(2*...
```

Giac [F]

$$\int F^{c(a+bx)}\operatorname{csch}^4(d+ex)\,dx=\int F^{(bx+a)c}\operatorname{csch}\left(ex+d\right)^4\,dx$$

input

```
integrate(F^(c*(b*x+a))*csch(e*x+d)^4,x, algorithm="giac")
```

output

```
integrate(F^((b*x + a)*c)*csch(e*x + d)^4, x)
```

Mupad [F(-1)]

Timed out.

$$\int F^{c(a+bx)}\operatorname{csch}^4(d+ex)\,dx=\int \frac{F^{c\,(a+b\,x)}}{\sinh\left(d+e\,x\right)^4}\,dx$$

input
```
int(F^(c*(a + b*x))/sinh(d + e*x)^4,x)
```

output
```
int(F^(c*(a + b*x))/sinh(d + e*x)^4, x)
```

Reduce [F]

$$\int F^{c(a+bx)}\operatorname{csch}^4(d+ex)\,dx=f^{ac}\left(\int f^{bcx}\operatorname{csch}(ex+d)^4\,dx\right)$$

input
```
int(F^(c*(b*x+a))*csch(e*x+d)^4,x)
```

output
```
f**(a*c)*int(f**(b*c*x)*csch(d + e*x)**4,x)
```

3.329 $\int e^{c(a+bx)}\sinh^2(ac+bcx)^{5/2}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 250

$$\begin{aligned}\int e^{c(a+bx)}\sinh^2(ac+bcx)^{5/2}\,dx =& \frac{e^{-4c(a+bx)}\text{csch}(ac+bcx)\sqrt{\sinh^2(ac+bcx)}}{128bc}\\&-\frac{5e^{-2c(a+bx)}\text{csch}(ac+bcx)\sqrt{\sinh^2(ac+bcx)}}{64bc}\\&+\frac{5e^{2c(a+bx)}\text{csch}(ac+bcx)\sqrt{\sinh^2(ac+bcx)}}{32bc}\\&-\frac{5e^{4c(a+bx)}\text{csch}(ac+bcx)\sqrt{\sinh^2(ac+bcx)}}{128bc}\\&+\frac{e^{6c(a+bx)}\text{csch}(ac+bcx)\sqrt{\sinh^2(ac+bcx)}}{192bc}\\&-\frac{5}{16}x\text{csch}(ac+bcx)\sqrt{\sinh^2(ac+bcx)}\end{aligned}$$

output
```
1/128*csch(b*c*x+a*c)*(sinh(b*c*x+a*c)^2)^(1/2)/b/c/exp(4*c*(b*x+a))-5/64*
csch(b*c*x+a*c)*(sinh(b*c*x+a*c)^2)^(1/2)/b/c/exp(2*c*(b*x+a))+5/32*exp(2*
c*(b*x+a))*csch(b*c*x+a*c)*(sinh(b*c*x+a*c)^2)^(1/2)/b/c-5/128*exp(4*c*(b*
x+a))*csch(b*c*x+a*c)*(sinh(b*c*x+a*c)^2)^(1/2)/b/c+1/192*exp(6*c*(b*x+a))
*csch(b*c*x+a*c)*(sinh(b*c*x+a*c)^2)^(1/2)/b/c-5/16*x*csch(b*c*x+a*c)*(sin
h(b*c*x+a*c)^2)^(1/2)
```

Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 109, normalized size of antiderivative = 0.44

$$\int e^{c(a+bx)}\sinh^2(ac+bcx)^{5/2}\,dx = \frac{\left(\frac{1}{128}e^{-4c(a+bx)}-\frac{5}{64}e^{-2c(a+bx)}+\frac{5}{32}e^{2c(a+bx)}-\frac{5}{128}e^{4c(a+bx)}+\frac{1}{192}e^{6c(a+bx)}-\frac{5bcx}{16}\right)\operatorname{csch}^5(c(a+bx$$

input
```
Integrate[E^(c*(a + b*x))*(Sinh[a*c + b*c*x]^2)^(5/2),x]
```

output
```
((1/(128*E^(4*c*(a + b*x))) - 5/(64*E^(2*c*(a + b*x))) + (5*E^(2*c*(a + b*
x)))/32 - (5*E^(4*c*(a + b*x)))/128 + E^(6*c*(a + b*x))/192 - (5*b*c*x)/16
)*Csch[c*(a + b*x)]^5*(Sinh[c*(a + b*x)]^2)^(5/2))/(b*c)
```

Rubi [A] (warning: unable to verify)

Time = 0.49 (sec) , antiderivative size = 102, normalized size of antiderivative = 0.41, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.240, Rules used = {7271, 2720, 27, 243, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int e^{c(a+bx)}\sinh^2(ac+bcx)^{5/2}\,dx$$

$$\Big\downarrow \; 7271$$

$$\sqrt{\sinh^2(ac+bcx)}\operatorname{csch}(ac+bcx)\int e^{c(a+bx)}\sinh^5(ac+bxc)dx$$

$$\Big\downarrow \; 2720$$

$$\frac{\sqrt{\sinh^2(ac+bcx)}\operatorname{csch}(ac+bcx)\int -\frac{1}{32}e^{-5c(a+bx)}\left(1-e^{2c(a+bx)}\right)^5de^{c(a+bx)}}{bc}$$

$$\Big\downarrow \; 27$$

$$-\frac{\sqrt{\sinh^2(ac+bcx)}\operatorname{csch}(ac+bcx)\int e^{-5c(a+bx)}\left(1-e^{2c(a+bx)}\right)^5de^{c(a+bx)}}{32bc}$$

$$\Big\downarrow \text{243}$$

$$-\frac{\sqrt{\sinh^2(ac+bcx)}\text{csch}(ac+bcx)\int e^{-3c(a+bx)}\left(1-e^{2c(a+bx)}\right)^5de^{2c(a+bx)}}{64bc}$$

$$\Big\downarrow \text{49}$$

$$-\frac{\sqrt{\sinh^2(ac+bcx)}\text{csch}(ac+bcx)\int \left(-10+e^{-3c(a+bx)}-5e^{-2c(a+bx)}+10e^{-c(a+bx)}+4e^{2c(a+bx)}\right)de^{2c(a+bx)}}{64bc}$$

$$\Big\downarrow \text{2009}$$

$$-\frac{\left(-\frac{1}{2}e^{-2c(a+bx)}+5e^{-c(a+bx)}-\frac{15}{2}e^{2c(a+bx)}-\frac{1}{3}e^{3c(a+bx)}+10\log\left(e^{2c(a+bx)}\right)\right)\sqrt{\sinh^2(ac+bcx)}\text{csch}(ac+bcx)}{64bc}$$

input
```
Int[E^(c*(a + b*x))*(Sinh[a*c + b*c*x]^2)^(5/2),x]
```

output
```
-1/64*(Csch[a*c + b*c*x]*(-1/2*1/E^(2*c*(a + b*x)) + 5/E^(c*(a + b*x)) - (
15*E^(2*c*(a + b*x)))/2 - E^(3*c*(a + b*x))/3 + 10*Log[E^(2*c*(a + b*x))])
*Sqrt[Sinh[a*c + b*c*x]^2])/(b*c)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2720
```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

rule 7271
```
Int[(u_.)*((a_.)*(v_)^(m_.))^(p_), x_Symbol] :> Simp[a^IntPart[p]*((a*v^m)^
FracPart[p]/v^(m*FracPart[p]))   Int[u*v^(m*p), x], x] /; FreeQ[{a, m, p},
x] &&  !IntegerQ[p] &&  !FreeQ[v, x] &&  !(EqQ[a, 1] && EqQ[m, 1]) &&  !(Eq
Q[v, x] && EqQ[m, 1])
```

Maple [A] (verified)

Time = 4.52 (sec) , antiderivative size = 326, normalized size of antiderivative = 1.30

method	result
risch	$-\frac{5x\sqrt{\left(e^{2c(bx+a)}-1\right)^{2} e^{-2c(bx+a)}}\, e^{c(bx+a)}}{16\left(e^{2c(bx+a)}-1\right)}+\frac{\sqrt{\left(e^{2c(bx+a)}-1\right)^{2} e^{-2c(bx+a)}}\, e^{7c(bx+a)}}{192bc\left(e^{2c(bx+a)}-1\right)}-\frac{5\sqrt{\left(e^{2c(bx+a)}-1\right)^{2} e^{-2c(bx+a)}}\, e^{5c(bx+}}{128bc\left(e^{2c(bx+a)}-1\right)}$

input
```
int(exp(c*(b*x+a))*(sinh(b*c*x+a*c)^2)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
-5/16*x*((exp(2*c*(b*x+a))-1)^2*exp(-2*c*(b*x+a)))^(1/2)/(exp(2*c*(b*x+a))
-1)*exp(c*(b*x+a))+1/192/b/c*((exp(2*c*(b*x+a))-1)^2*exp(-2*c*(b*x+a)))^(1
/2)/(exp(2*c*(b*x+a))-1)*exp(7*c*(b*x+a))-5/128/b/c*((exp(2*c*(b*x+a))-1)^
2*exp(-2*c*(b*x+a)))^(1/2)/(exp(2*c*(b*x+a))-1)*exp(5*c*(b*x+a))+5/32/b/c*
((exp(2*c*(b*x+a))-1)^2*exp(-2*c*(b*x+a)))^(1/2)/(exp(2*c*(b*x+a))-1)*exp(
3*c*(b*x+a))-5/64/b/c*((exp(2*c*(b*x+a))-1)^2*exp(-2*c*(b*x+a)))^(1/2)/(ex
p(2*c*(b*x+a))-1)*exp(-c*(b*x+a))+1/128/b/c*((exp(2*c*(b*x+a))-1)^2*exp(-2
*c*(b*x+a)))^(1/2)/(exp(2*c*(b*x+a))-1)*exp(-3*c*(b*x+a))
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 218, normalized size of antiderivative = 0.87

$$\int e^{c(a+bx)} \sinh^2(ac+bcx)^{5/2}\,dx = \frac{5 \cosh(bcx+ac)^5 + 25 \cosh(bcx+ac)\sinh(bcx+ac)^4 - \sinh(bcx+ac)^5 - 5\left(2\cosh(bcx\right.}{}$$

input
```
integrate(exp(c*(b*x+a))*(sinh(b*c*x+a*c)^2)^(5/2),x, algorithm="fricas")
```

output
```
1/384*(5*cosh(b*c*x + a*c)^5 + 25*cosh(b*c*x + a*c)*sinh(b*c*x + a*c)^4 -
sinh(b*c*x + a*c)^5 - 5*(2*cosh(b*c*x + a*c)^2 - 3)*sinh(b*c*x + a*c)^3 -
45*cosh(b*c*x + a*c)^3 + 5*(10*cosh(b*c*x + a*c)^3 - 27*cosh(b*c*x + a*c))
*sinh(b*c*x + a*c)^2 - 60*(2*b*c*x - 1)*cosh(b*c*x + a*c) - 5*(cosh(b*c*x
+ a*c)^4 - 24*b*c*x - 9*cosh(b*c*x + a*c)^2 - 12)*sinh(b*c*x + a*c))/(b*c*
cosh(b*c*x + a*c) - b*c*sinh(b*c*x + a*c))
```

Sympy [F(-1)]

Timed out.

$$\int e^{c(a+bx)} \sinh^2(ac+bcx)^{5/2}\,dx = \text{Timed out}$$

input
```
integrate(exp(c*(b*x+a))*(sinh(b*c*x+a*c)**2)**(5/2),x)
```

output
```
Timed out
```

Maxima [A] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 90, normalized size of antiderivative = 0.36

$$\int e^{c(a+bx)}\sinh^2(ac+bcx)^{5/2}\,dx = \frac{\left(2\,e^{(10\,bcx+10\,ac)}-15\,e^{(8\,bcx+8\,ac)}+60\,e^{(6\,bcx+6\,ac)}-30\,e^{(2\,bcx+2\,ac)}+3\right)e^{(-4\,bcx-4\,ac)}}{384\,bc} - \frac{5\,(bcx+ac)}{16\,bc}$$

input
```
integrate(exp(c*(b*x+a))*(sinh(b*c*x+a*c)^2)^(5/2),x, algorithm="maxima")
```

output
```
1/384*(2*e^(10*b*c*x + 10*a*c) - 15*e^(8*b*c*x + 8*a*c) + 60*e^(6*b*c*x +
6*a*c) - 30*e^(2*b*c*x + 2*a*c) + 3)*e^(-4*b*c*x - 4*a*c)/(b*c) - 5/16*(b*
c*x + a*c)/(b*c)
```

Giac [A] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 266, normalized size of antiderivative = 1.06

$$\int e^{c(a+bx)}\sinh^2(ac+bcx)^{5/2}\,dx = -\frac{\left(120\,bcxe^{(4\,ac)}\mathrm{sgn}\left(e^{(bcx+ac)}-e^{(-bcx-ac)}\right)-3\,\left(30\,e^{(4\,bcx+4\,ac)}\mathrm{sgn}\left(e^{(bcx+ac)}-e^{(-bcx-ac)}\right)-10\,e^{(2\,bcx+2\,ac)}\mathrm{sgn}(\right.\right.}{}$$

input
```
integrate(exp(c*(b*x+a))*(sinh(b*c*x+a*c)^2)^(5/2),x, algorithm="giac")
```

output
```
-1/384*(120*b*c*x*e^(4*a*c)*sgn(e^(b*c*x + a*c) - e^(-b*c*x - a*c)) - 3*(3
0*e^(4*b*c*x + 4*a*c)*sgn(e^(b*c*x + a*c) - e^(-b*c*x - a*c)) - 10*e^(2*b*
c*x + 2*a*c)*sgn(e^(b*c*x + a*c) - e^(-b*c*x - a*c)) + sgn(e^(b*c*x + a*c)
 - e^(-b*c*x - a*c)))*e^(-4*b*c*x) - 2*e^(6*b*c*x + 10*a*c)*sgn(e^(b*c*x +
 a*c) - e^(-b*c*x - a*c)) + 15*e^(4*b*c*x + 8*a*c)*sgn(e^(b*c*x + a*c) - e
^(-b*c*x - a*c)) - 60*e^(2*b*c*x + 6*a*c)*sgn(e^(b*c*x + a*c) - e^(-b*c*x
- a*c)))*e^(-4*a*c)/(b*c)
```

Mupad [F(-1)]

Timed out.

$$\int e^{c(a+bx)}\sinh^2(ac+bcx)^{5/2}\,dx=\int e^{c\,(a+b\,x)}\,\left(\sinh(a\,c+b\,c\,x)^2\right)^{5/2}\,dx$$

input
```
int(exp(c*(a + b*x))*(sinh(a*c + b*c*x)^2)^(5/2),x)
```

output
```
int(exp(c*(a + b*x))*(sinh(a*c + b*c*x)^2)^(5/2), x)
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 97, normalized size of antiderivative = 0.39

$$\int e^{c(a+bx)}\sinh^2(ac+bcx)^{5/2}\,dx=\frac{2e^{10bcx+10ac}-15e^{8bcx+8ac}+60e^{6bcx+6ac}-120e^{4bcx+4ac}bcx-30e^{2bcx+2ac}+3}{384e^{4bcx+4ac}bc}$$

input
```
int(exp(c*(b*x+a))*(sinh(b*c*x+a*c)^2)^(5/2),x)
```

output
```
(2*e**(10*a*c + 10*b*c*x) - 15*e**(8*a*c + 8*b*c*x) + 60*e**(6*a*c + 6*b*c
*x) - 120*e**(4*a*c + 4*b*c*x)*b*c*x - 30*e**(2*a*c + 2*b*c*x) + 3)/(384*e
**(4*a*c + 4*b*c*x)*b*c)
```

3.330 $\int e^{c(a+bx)} \sinh^2(ac+bcx)^{3/2}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 162

$$\int e^{c(a+bx)} \sinh^2(ac+bcx)^{3/2}\,dx = \frac{e^{-2c(a+bx)}\operatorname{csch}(ac+bcx)\sqrt{\sinh^2(ac+bcx)}}{16bc} - \frac{3e^{2c(a+bx)}\operatorname{csch}(ac+bcx)\sqrt{\sinh^2(ac+bcx)}}{16bc} + \frac{e^{4c(a+bx)}\operatorname{csch}(ac+bcx)\sqrt{\sinh^2(ac+bcx)}}{32bc} + \frac{3}{8}x\operatorname{csch}(ac+bcx)\sqrt{\sinh^2(ac+bcx)}$$

output
```
1/16*csch(b*c*x+a*c)*(sinh(b*c*x+a*c)^2)^(1/2)/b/c/exp(2*c*(b*x+a))-3/16*e
xp(2*c*(b*x+a))*csch(b*c*x+a*c)*(sinh(b*c*x+a*c)^2)^(1/2)/b/c+1/32*exp(4*c
*(b*x+a))*csch(b*c*x+a*c)*(sinh(b*c*x+a*c)^2)^(1/2)/b/c+3/8*x*csch(b*c*x+a
*c)*(sinh(b*c*x+a*c)^2)^(1/2)
```

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 76, normalized size of antiderivative = 0.47

$$\int e^{c(a+bx)} \sinh^2(ac+bcx)^{3/2}\,dx = \frac{\left(e^{-2c(a+bx)}-3e^{2c(a+bx)}+\frac{1}{2}e^{4c(a+bx)}+6bcx\right)\operatorname{csch}^3(c(a+bx))\sinh^2(c(a+bx))^{3/2}}{16bc}$$

input
```
Integrate[E^(c*(a + b*x))*(Sinh[a*c + b*c*x]^2)^(3/2),x]
```

output
```
((E^(-2*c*(a + b*x)) - 3*E^(2*c*(a + b*x)) + E^(4*c*(a + b*x))/2 + 6*b*c*x
)*Csch[c*(a + b*x)]^3*(Sinh[c*(a + b*x)]^2)^(3/2))/(16*b*c)
```

Rubi [A] (warning: unable to verify)

Time = 0.39 (sec) , antiderivative size = 74, normalized size of antiderivative = 0.46, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.240$, Rules used = {7271, 2720, 27, 243, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int e^{c(a+bx)} \sinh^2(ac+bcx)^{3/2}\,dx$$

$$\Big\downarrow \text{7271}$$

$$\sqrt{\sinh^2(ac+bcx)}\operatorname{csch}(ac+bcx)\int e^{c(a+bx)}\sinh^3(ac+bxc)dx$$

$$\Big\downarrow \text{2720}$$

$$\frac{\sqrt{\sinh^2(ac+bcx)}\operatorname{csch}(ac+bcx)\int -\frac{1}{8}e^{-3c(a+bx)}\left(1-e^{2c(a+bx)}\right)^3de^{c(a+bx)}}{bc}$$

$$\Big\downarrow \text{27}$$

$$-\frac{\sqrt{\sinh^2(ac+bcx)}\operatorname{csch}(ac+bcx)\int e^{-3c(a+bx)}\left(1-e^{2c(a+bx)}\right)^3de^{c(a+bx)}}{8bc}$$

$$\Big\downarrow \text{243}$$

$$-\frac{\sqrt{\sinh^2(ac+bcx)}\text{csch}(ac+bcx)\int e^{-2c(a+bx)}\left(1-e^{2c(a+bx)}\right)^3de^{2c(a+bx)}}{16bc}$$

$$\Big\downarrow \text{49}$$

$$-\frac{\sqrt{\sinh^2(ac+bcx)}\text{csch}(ac+bcx)\int \left(3+e^{-2c(a+bx)}-3e^{-c(a+bx)}-e^{2c(a+bx)}\right)de^{2c(a+bx)}}{16bc}$$

$$\Big\downarrow \text{2009}$$

$$-\frac{\left(-e^{-c(a+bx)}+\frac{5}{2}e^{2c(a+bx)}-3\log\left(e^{2c(a+bx)}\right)\right)\sqrt{\sinh^2(ac+bcx)}\text{csch}(ac+bcx)}{16bc}$$

input
```
Int[E^(c*(a + b*x))*(Sinh[a*c + b*c*x]^2)^(3/2),x]
```

output
```
-1/16*(Csch[a*c + b*c*x]*(-E^(-(c*(a + b*x))) + (5*E^(2*c*(a + b*x)))/2 -
3*Log[E^(2*c*(a + b*x))])*Sqrt[Sinh[a*c + b*c*x]^2])/(b*c)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2720
```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

rule 7271
```
Int[(u_.)*((a_.)*(v_)^(m_.))^(p_), x_Symbol] :> Simp[a^IntPart[p]*((a*v^m)^
FracPart[p]/v^(m*FracPart[p]))   Int[u*v^(m*p), x], x] /; FreeQ[{a, m, p},
x] &&  !IntegerQ[p] &&  !FreeQ[v, x] &&  !(EqQ[a, 1] && EqQ[m, 1]) &&  !(Eq
Q[v, x] && EqQ[m, 1])
```

Maple [A] (verified)

Time = 0.21 (sec) , antiderivative size = 216, normalized size of antiderivative = 1.33

method	result
risch	$\frac{3x\sqrt{\left(\mathrm{e}^{2c(bx+a)}-1\right)^{2}\mathrm{e}^{-2c(bx+a)}}\,\mathrm{e}^{c(bx+a)}}{8\left(\mathrm{e}^{2c(bx+a)}-1\right)}+\frac{\sqrt{\left(\mathrm{e}^{2c(bx+a)}-1\right)^{2}\mathrm{e}^{-2c(bx+a)}}\,\mathrm{e}^{5c(bx+a)}}{32bc\left(\mathrm{e}^{2c(bx+a)}-1\right)}-\frac{3\sqrt{\left(\mathrm{e}^{2c(bx+a)}-1\right)^{2}\mathrm{e}^{-2c(bx+a)}}\,\mathrm{e}^{3c(bx+a)}}{16bc\left(\mathrm{e}^{2c(bx+a)}-1\right)}$

input
```
int(exp(c*(b*x+a))*(sinh(b*c*x+a*c)^2)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
3/8*x*((exp(2*c*(b*x+a))-1)^2*exp(-2*c*(b*x+a)))^(1/2)/(exp(2*c*(b*x+a))-1
)*exp(c*(b*x+a))+1/32/b/c*((exp(2*c*(b*x+a))-1)^2*exp(-2*c*(b*x+a)))^(1/2)
/(exp(2*c*(b*x+a))-1)*exp(5*c*(b*x+a))-3/16/b/c*((exp(2*c*(b*x+a))-1)^2*ex
p(-2*c*(b*x+a)))^(1/2)/(exp(2*c*(b*x+a))-1)*exp(3*c*(b*x+a))+1/16/b/c*((ex
p(2*c*(b*x+a))-1)^2*exp(-2*c*(b*x+a)))^(1/2)/(exp(2*c*(b*x+a))-1)*exp(-c*(
b*x+a))
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 126, normalized size of antiderivative = 0.78

$$\int e^{c(a+bx)}\sinh^2(ac+bcx)^{3/2}\,dx = \frac{3\cosh(bcx+ac)^3+9\cosh(bcx+ac)\sinh(bcx+ac)^2-\sinh(bcx+ac)^3+6(2bcx-1)\cos\ldots}{32(bc\cosh(bcx+ac)-bc\sinh(bcx\ldots}$$

input
```
integrate(exp(c*(b*x+a))*(sinh(b*c*x+a*c)^2)^(3/2),x, algorithm="fricas")
```

output
```
1/32*(3*cosh(b*c*x + a*c)^3 + 9*cosh(b*c*x + a*c)*sinh(b*c*x + a*c)^2 - si
nh(b*c*x + a*c)^3 + 6*(2*b*c*x - 1)*cosh(b*c*x + a*c) - 3*(4*b*c*x + cosh(
b*c*x + a*c)^2 + 2)*sinh(b*c*x + a*c))/(b*c*cosh(b*c*x + a*c) - b*c*sinh(b
*c*x + a*c))
```

Sympy [F(-1)]

Timed out.

$$\int e^{c(a+bx)}\sinh^2(ac+bcx)^{3/2}\,dx = \text{Timed out}$$

input
```
integrate(exp(c*(b*x+a))*(sinh(b*c*x+a*c)**2)**(3/2),x)
```

output
```
Timed out
```

Maxima [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 62, normalized size of antiderivative = 0.38

$$\int e^{c(a+bx)}\sinh^2(ac+bcx)^{3/2}\,dx = \frac{\left(e^{(6\,bcx+6\,ac)}-6\,e^{(4\,bcx+4\,ac)}+2\right)e^{(-2\,bcx-2\,ac)}}{32\,bc} + \frac{3\,(bcx+ac)}{8\,bc}$$

input
```
integrate(exp(c*(b*x+a))*(sinh(b*c*x+a*c)^2)^(3/2),x, algorithm="maxima")
```

output
```
1/32*(e^(6*b*c*x + 6*a*c) - 6*e^(4*b*c*x + 4*a*c) + 2)*e^(-2*b*c*x - 2*a*c
)/(b*c) + 3/8*(b*c*x + a*c)/(b*c)
```

Giac [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 193, normalized size of antiderivative = 1.19

$$\int e^{c(a+bx)}\sinh^2(ac+bcx)^{3/2}\,dx = \frac{\left(12\,bcxe^{(2\,ac)}\mathrm{sgn}\left(e^{(bcx+ac)}-e^{(-bcx-ac)}\right)-2\left(3\,e^{(2\,bcx+2\,ac)}\mathrm{sgn}\left(e^{(bcx+ac)}-e^{(-bcx-ac)}\right)-\mathrm{sgn}\left(e^{(bc}\right.\right.\right.}{}$$

input
```
integrate(exp(c*(b*x+a))*(sinh(b*c*x+a*c)^2)^(3/2),x, algorithm="giac")
```

output
```
1/32*(12*b*c*x*e^(2*a*c)*sgn(e^(b*c*x + a*c) - e^(-b*c*x - a*c)) - 2*(3*e^
(2*b*c*x + 2*a*c)*sgn(e^(b*c*x + a*c) - e^(-b*c*x - a*c)) - sgn(e^(b*c*x +
 a*c) - e^(-b*c*x - a*c)))*e^(-2*b*c*x) + e^(4*b*c*x + 6*a*c)*sgn(e^(b*c*x
 + a*c) - e^(-b*c*x - a*c)) - 6*e^(2*b*c*x + 4*a*c)*sgn(e^(b*c*x + a*c) -
e^(-b*c*x - a*c)))*e^(-2*a*c)/(b*c)
```

Mupad [F(-1)]

Timed out.

$$\int e^{c(a+bx)}\sinh^2(ac+bcx)^{3/2}\,dx = \int e^{c\,(a+b\,x)}\,\left(\sinh(a\,c+b\,c\,x)^2\right)^{3/2}\,dx$$

input
```
int(exp(c*(a + b*x))*(sinh(a*c + b*c*x)^2)^(3/2),x)
```

output
```
int(exp(c*(a + b*x))*(sinh(a*c + b*c*x)^2)^(3/2), x)
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 67, normalized size of antiderivative = 0.41

$$\int e^{c(a+bx)}\sinh^2(ac+bcx)^{3/2}\,dx = \frac{e^{6bcx+6ac}-6e^{4bcx+4ac}+12e^{2bcx+2ac}bcx+2}{32e^{2bcx+2ac}bc}$$

input
```
int(exp(c*(b*x+a))*(sinh(b*c*x+a*c)^2)^(3/2),x)
```

output
```
(e**(6*a*c + 6*b*c*x) - 6*e**(4*a*c + 4*b*c*x) + 12*e**(2*a*c + 2*b*c*x)*b
*c*x + 2)/(32*e**(2*a*c + 2*b*c*x)*b*c)
```

3.331 $\int e^{c(a+bx)} \sqrt{\sinh^2(ac+bcx)}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 74

$$\int e^{c(a+bx)} \sqrt{\sinh^2(ac+bcx)}\,dx = \frac{e^{2c(a+bx)}\operatorname{csch}(ac+bcx)\sqrt{\sinh^2(ac+bcx)}}{4bc} - \frac{1}{2}x\operatorname{csch}(ac+bcx)\sqrt{\sinh^2(ac+bcx)}$$

output
```
1/4*exp(2*c*(b*x+a))*csch(b*c*x+a*c)*(sinh(b*c*x+a*c)^2)^(1/2)/b/c-1/2*x*c
sch(b*c*x+a*c)*(sinh(b*c*x+a*c)^2)^(1/2)
```

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.65

$$\int e^{c(a+bx)} \sqrt{\sinh^2(ac+bcx)}\,dx = \frac{\left(e^{2c(a+bx)} - 2bcx\right)\operatorname{csch}(c(a+bx))\sqrt{\sinh^2(c(a+bx))}}{4bc}$$

input
```
Integrate[E^(c*(a + b*x))*Sqrt[Sinh[a*c + b*c*x]^2],x]
```

output
```
((E^(2*c*(a + b*x)) - 2*b*c*x)*Csch[c*(a + b*x)]*Sqrt[Sinh[c*(a + b*x)]^2]
)/(4*b*c)
```

Rubi [A] (verified)

Time = 0.37 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.82, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {7271, 2720, 27, 244, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int e^{c(a+bx)}\sqrt{\sinh^2(ac+bcx)}\,dx$$

$$\Big\downarrow \text{7271}$$

$$\sqrt{\sinh^2(ac+bcx)}\text{csch}(ac+bcx)\int e^{c(a+bx)}\sinh(ac+bxc)dx$$

$$\Big\downarrow \text{2720}$$

$$\frac{\sqrt{\sinh^2(ac+bcx)}\text{csch}(ac+bcx)\int -\frac{1}{2}e^{-c(a+bx)}\left(1-e^{2c(a+bx)}\right)de^{c(a+bx)}}{bc}$$

$$\Big\downarrow \text{27}$$

$$-\frac{\sqrt{\sinh^2(ac+bcx)}\text{csch}(ac+bcx)\int e^{-c(a+bx)}\left(1-e^{2c(a+bx)}\right)de^{c(a+bx)}}{2bc}$$

$$\Big\downarrow \text{244}$$

$$-\frac{\sqrt{\sinh^2(ac+bcx)}\text{csch}(ac+bcx)\int \left(e^{-c(a+bx)}-e^{c(a+bx)}\right)de^{c(a+bx)}}{2bc}$$

$$\Big\downarrow \text{2009}$$

$$\frac{\left(\frac{1}{2}e^{2c(a+bx)}-\log\left(e^{c(a+bx)}\right)\right)\sqrt{\sinh^2(ac+bcx)}\text{csch}(ac+bcx)}{2bc}$$

input
```
Int[E^(c*(a + b*x))*Sqrt[Sinh[a*c + b*c*x]^2],x]
```

output
```
(Csch[a*c + b*c*x]*(E^(2*c*(a + b*x))/2 - Log[E^(c*(a + b*x))])*Sqrt[Sinh[
a*c + b*c*x]^2])/(2*b*c)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 244
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[Expand
Integrand[(c*x)^m*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x] && IGtQ[p
, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2720
```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

rule 7271
```
Int[(u_.)*((a_.)*(v_)^(m_.))^(p_), x_Symbol] :> Simp[a^IntPart[p]*((a*v^m)^
FracPart[p]/v^(m*FracPart[p]))   Int[u*v^(m*p), x], x] /; FreeQ[{a, m, p},
x] &&  !IntegerQ[p] &&  !FreeQ[v, x] &&  !(EqQ[a, 1] && EqQ[m, 1]) &&  !(Eq
Q[v, x] && EqQ[m, 1])
```

Maple [A] (verified)

Time = 0.20 (sec) , antiderivative size = 106, normalized size of antiderivative = 1.43

method	result	size
risch	$-\frac{x\sqrt{\left(\mathrm{e}^{2c(bx+a)}-1\right)^{2}\mathrm{e}^{-2c(bx+a)}}\,\mathrm{e}^{c(bx+a)}}{2\left(\mathrm{e}^{2c(bx+a)}-1\right)}+\frac{\sqrt{\left(\mathrm{e}^{2c(bx+a)}-1\right)^{2}\mathrm{e}^{-2c(bx+a)}}\,\mathrm{e}^{3c(bx+a)}}{4bc\left(\mathrm{e}^{2c(bx+a)}-1\right)}$	106

input
```
int(exp(c*(b*x+a))*(sinh(b*c*x+a*c)^2)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-1/2*x*((exp(2*c*(b*x+a))-1)^2*exp(-2*c*(b*x+a)))^(1/2)/(exp(2*c*(b*x+a))-
1)*exp(c*(b*x+a))+1/4/b/c*((exp(2*c*(b*x+a))-1)^2*exp(-2*c*(b*x+a)))^(1/2)
/(exp(2*c*(b*x+a))-1)*exp(3*c*(b*x+a))
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 66, normalized size of antiderivative = 0.89

$$\int e^{c(a+bx)}\sqrt{\sinh^2(ac+bcx)}\,dx = -\frac{(2\,bcx-1)\cosh\left(bcx+ac\right)-(2\,bcx+1)\sinh\left(bcx+ac\right)}{4\left(bc\cosh\left(bcx+ac\right)-bc\sinh\left(bcx+ac\right)\right)}$$

input
```
integrate(exp(c*(b*x+a))*(sinh(b*c*x+a*c)^2)^(1/2),x, algorithm="fricas")
```

output
```
-1/4*((2*b*c*x - 1)*cosh(b*c*x + a*c) - (2*b*c*x + 1)*sinh(b*c*x + a*c))/(
b*c*cosh(b*c*x + a*c) - b*c*sinh(b*c*x + a*c))
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 189 vs. $2(68) = 136$.

Time = 2.64 (sec) , antiderivative size = 189, normalized size of antiderivative = 2.55

$$\int e^{c(a+bx)}\sqrt{\sinh^2(ac+bcx)}\,dx = \begin{cases} 0 \\ x\sqrt{\sinh^2\left(ac\right)}e^{ac} \\ 0 \\ \frac{x\sqrt{\sinh^2\left(ac+bcx\right)}e^{ac}e^{bcx}}{2} - \frac{x\sqrt{\sinh^2\left(ac+bcx\right)}e^{ac}e^{bcx}\cosh\left(ac+bcx\right)}{2\sinh\left(ac+bcx\right)} - \frac{\sqrt{\sinh^2\left(ac+bcx\right)}e^{ac}e^{bcx}}{2bc} + \frac{\sqrt{\sinh^2\left(ac+bcx\right)}e^{ac}e^{bcx}\cosh\left(ac\right)}{bc\sinh\left(ac+bcx\right)} \end{cases}$$

input
```
integrate(exp(c*(b*x+a))*(sinh(b*c*x+a*c)**2)**(1/2),x)
```

output
```
Piecewise((0, Eq(a, 0) & Eq(b, 0) & Eq(c, 0)), (x*sqrt(sinh(a*c)**2)*exp(a
*c), Eq(b, 0)), (0, Eq(c, 0) | Eq(a, -b*x)), (x*sqrt(sinh(a*c + b*c*x)**2)
*exp(a*c)*exp(b*c*x)/2 - x*sqrt(sinh(a*c + b*c*x)**2)*exp(a*c)*exp(b*c*x)*
cosh(a*c + b*c*x)/(2*sinh(a*c + b*c*x)) - sqrt(sinh(a*c + b*c*x)**2)*exp(a
*c)*exp(b*c*x)/(2*b*c) + sqrt(sinh(a*c + b*c*x)**2)*exp(a*c)*exp(b*c*x)*co
sh(a*c + b*c*x)/(b*c*sinh(a*c + b*c*x)), True))
```

Maxima [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.49

$$\int e^{c(a+bx)}\sqrt{\sinh^2(ac+bcx)}\,dx = -\frac{bcx+ac}{2\,bc} + \frac{e^{(2\,bcx+2\,ac)}}{4\,bc}$$

input
```
integrate(exp(c*(b*x+a))*(sinh(b*c*x+a*c)^2)^(1/2),x, algorithm="maxima")
```

output
```
-1/2*(b*c*x + a*c)/(b*c) + 1/4*e^(2*b*c*x + 2*a*c)/(b*c)
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 75, normalized size of antiderivative = 1.01

$$\int e^{c(a+bx)}\sqrt{\sinh^2(ac+bcx)}\,dx = -\frac{2\,bcx\mathrm{sgn}\left(e^{(bcx+ac)}-e^{(-bcx-ac)}\right)-e^{(2\,bcx+2\,ac)}\mathrm{sgn}\left(e^{(bcx+ac)}-e^{(-bcx-ac)}\right)}{4\,bc}$$

input
```
integrate(exp(c*(b*x+a))*(sinh(b*c*x+a*c)^2)^(1/2),x, algorithm="giac")
```

output
```
-1/4*(2*b*c*x*sgn(e^(b*c*x + a*c) - e^(-b*c*x - a*c)) - e^(2*b*c*x + 2*a*c
)*sgn(e^(b*c*x + a*c) - e^(-b*c*x - a*c)))/(b*c)
```

Mupad [B] (verification not implemented)

Time = 1.65 (sec) , antiderivative size = 77, normalized size of antiderivative = 1.04

$$\int e^{c(a+bx)}\sqrt{\sinh^2(ac+bcx)}\,dx = -\frac{\left(x\,e^{a\,c+b\,c\,x}-\frac{e^{3\,a\,c+3\,b\,c\,x}}{2\,b\,c}\right)\sqrt{{\left(\frac{e^{a\,c+b\,c\,x}}{2}-\frac{e^{-a\,c-b\,c\,x}}{2}\right)}^2}}{e^{2\,a\,c+2\,b\,c\,x}-1}$$

input
```
int(exp(c*(a + b*x))*(sinh(a*c + b*c*x)^2)^(1/2),x)
```

output
```
-((x*exp(a*c + b*c*x) - exp(3*a*c + 3*b*c*x)/(2*b*c))*((exp(a*c + b*c*x)/2
 - exp(- a*c - b*c*x)/2)^2)^(1/2))/(exp(2*a*c + 2*b*c*x) - 1)
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 55, normalized size of antiderivative = 0.74

$$\int e^{c(a+bx)}\sqrt{\sinh^2(ac+bcx)}\,dx = \frac{e^{bcx+ac}(-\cosh(bcx+ac)\,bcx+\cosh(bcx+ac)+\sinh(bcx+ac)\,bcx)}{2bc}$$

input
```
int(exp(c*(b*x+a))*(sinh(b*c*x+a*c)^2)^(1/2),x)
```

output
```
(e**(a*c + b*c*x)*( - cosh(a*c + b*c*x)*b*c*x + cosh(a*c + b*c*x) + sinh(a
*c + b*c*x)*b*c*x))/(2*b*c)
```

3.332 $\int \frac{e^{c(a+bx)}}{\sqrt{\sinh^2(ac+bcx)}}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 46

$$\int \frac{e^{c(a+bx)}}{\sqrt{\sinh^2(ac+bcx)}}\,dx = \frac{\log\left(1-e^{2c(a+bx)}\right)\sinh(ac+bcx)}{bc\sqrt{\sinh^2(ac+bcx)}}$$

output
```
ln(1-exp(2*c*(b*x+a)))*sinh(b*c*x+a*c)/b/c/(sinh(b*c*x+a*c)^2)^(1/2)
```

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.96

$$\int \frac{e^{c(a+bx)}}{\sqrt{\sinh^2(ac+bcx)}}\,dx = \frac{\log\left(1-e^{2c(a+bx)}\right)\sinh(c(a+bx))}{bc\sqrt{\sinh^2(c(a+bx))}}$$

input
```
Integrate[E^(c*(a + b*x))/Sqrt[Sinh[a*c + b*c*x]^2],x]
```

output
```
(Log[1 - E^(2*c*(a + b*x))]*Sinh[c*(a + b*x)])/(b*c*Sqrt[Sinh[c*(a + b*x)]
^2])
```

Rubi [A] (verified)

Time = 0.32 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.160, Rules used = {7271, 2720, 27, 240}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{e^{c(a+bx)}}{\sqrt{\sinh^2(ac+bcx)}}dx$$

$$\Big\downarrow \text{7271}$$

$$\frac{\sinh(ac+bcx)\int e^{c(a+bx)}\text{csch}(ac+bxc)dx}{\sqrt{\sinh^2(ac+bcx)}}$$

$$\Big\downarrow \text{2720}$$

$$\frac{\sinh(ac+bcx)\int -\frac{2e^{c(a+bx)}}{1-e^{2c(a+bx)}}de^{c(a+bx)}}{bc\sqrt{\sinh^2(ac+bcx)}}$$

$$\Big\downarrow \text{27}$$

$$-\frac{2\sinh(ac+bcx)\int \frac{e^{c(a+bx)}}{1-e^{2c(a+bx)}}de^{c(a+bx)}}{bc\sqrt{\sinh^2(ac+bcx)}}$$

$$\Big\downarrow \text{240}$$

$$\frac{\log\left(1-e^{2c(a+bx)}\right)\sinh(ac+bcx)}{bc\sqrt{\sinh^2(ac+bcx)}}$$

input
```
Int[E^(c*(a + b*x))/Sqrt[Sinh[a*c + b*c*x]^2],x]
```

output
```
(Log[1 - E^(2*c*(a + b*x))]*Sinh[a*c + b*c*x])/(b*c*Sqrt[Sinh[a*c + b*c*x]
^2])
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 240
```
Int[(x_)/((a_) + (b_.)*(x_)^2), x_Symbol] :> Simp[Log[RemoveContent[a + b*x
^2, x]]/(2*b), x] /; FreeQ[{a, b}, x]
```

rule 2720
```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

rule 7271
```
Int[(u_.)*((a_.)*(v_)^(m_.))^(p_), x_Symbol] :> Simp[a^IntPart[p]*((a*v^m)^
FracPart[p]/v^(m*FracPart[p]))   Int[u*v^(m*p), x], x] /; FreeQ[{a, m, p},
x] &&  !IntegerQ[p] &&  !FreeQ[v, x] &&  !(EqQ[a, 1] && EqQ[m, 1]) &&  !(Eq
Q[v, x] && EqQ[m, 1])
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.13 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.63

method	result	size
default	$\operatorname{csgn}\left(\sinh\left(c(bx+a)\right)\right)\left(x+\frac{\ln(\sinh(c(bx+a)))}{cb}\right)$	29
risch	$\frac{\ln\left(e^{2bcx}-e^{-2ac}\right)\left(e^{2c(bx+a)}-1\right)e^{-c(bx+a)}}{bc\sqrt{\left(e^{2c(bx+a)}-1\right)^{2}e^{-2c(bx+a)}}}$	68

input
```
int(exp(c*(b*x+a))/(sinh(b*c*x+a*c)^2)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
csgn(sinh(c*(b*x+a)))*(x+1/c/b*ln(sinh(c*(b*x+a))))
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.91

$$\int \frac{e^{c(a+bx)}}{\sqrt{\sinh^2(ac+bcx)}}\,dx = \frac{\log\left(\frac{2\sinh(bcx+ac)}{\cosh(bcx+ac)-\sinh(bcx+ac)}\right)}{bc}$$

input
```
integrate(exp(c*(b*x+a))/(sinh(b*c*x+a*c)^2)^(1/2),x, algorithm="fricas")
```

output
```
log(2*sinh(b*c*x + a*c)/(cosh(b*c*x + a*c) - sinh(b*c*x + a*c)))/(b*c)
```

Sympy [F]

$$\int \frac{e^{c(a+bx)}}{\sqrt{\sinh^2(ac+bcx)}}\,dx = e^{ac}\int \frac{e^{bcx}}{\sqrt{\sinh^2(ac+bcx)}}\,dx$$

input
```
integrate(exp(c*(b*x+a))/(sinh(b*c*x+a*c)**2)**(1/2),x)
```

output
```
exp(a*c)*Integral(exp(b*c*x)/sqrt(sinh(a*c + b*c*x)**2), x)
```

Maxima [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.85

$$\int \frac{e^{c(a+bx)}}{\sqrt{\sinh^2(ac+bcx)}}\,dx = \frac{\log\left(e^{(bcx+ac)}+1\right)}{bc} + \frac{\log\left(e^{(bcx+ac)}-1\right)}{bc}$$

input
```
integrate(exp(c*(b*x+a))/(sinh(b*c*x+a*c)^2)^(1/2),x, algorithm="maxima")
```

output
```
log(e^(b*c*x + a*c) + 1)/(b*c) + log(e^(b*c*x + a*c) - 1)/(b*c)
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.00

$$\int \frac{e^{c(a+bx)}}{\sqrt{\sinh^2(ac+bcx)}}\,dx = \frac{\log\left(\left|e^{(2\,bcx+2\,ac)}-1\right|\right)\operatorname{sgn}\left(e^{(bcx+ac)}-e^{(-bcx-ac)}\right)}{bc}$$

input
```
integrate(exp(c*(b*x+a))/(sinh(b*c*x+a*c)^2)^(1/2),x, algorithm="giac")
```

output
```
log(abs(e^(2*b*c*x + 2*a*c) - 1))*sgn(e^(b*c*x + a*c) - e^(-b*c*x - a*c))/
(b*c)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{e^{c(a+bx)}}{\sqrt{\sinh^2(ac+bcx)}}\,dx = \int \frac{e^{c\,(a+b\,x)}}{\sqrt{\sinh\left(a\,c+b\,c\,x\right)^2}}\,dx$$

input
```
int(exp(c*(a + b*x))/(sinh(a*c + b*c*x)^2)^(1/2),x)
```

output
```
int(exp(c*(a + b*x))/(sinh(a*c + b*c*x)^2)^(1/2), x)
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.00

$$\int \frac{e^{c(a+bx)}}{\sqrt{\sinh^2(ac+bcx)}}\,dx = \frac{\log\left(e^{bcx+2ac}+e^{ac}\right)+\log\left(e^{bcx+2ac}-e^{ac}\right)}{bc}$$

input
```
int(exp(c*(b*x+a))/(sinh(b*c*x+a*c)^2)^(1/2),x)
```

output
```
(log(e**(2*a*c + b*c*x) + e**(a*c)) + log(e**(2*a*c + b*c*x) - e**(a*c)))/
(b*c)
```

3.333 $\int \frac{e^{c(a+bx)}}{\sinh^2(ac+bcx)^{3/2}}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 58

$$\int \frac{e^{c(a+bx)}}{\sinh^2(ac+bcx)^{3/2}}\,dx = -\frac{2e^{4c(a+bx)}\sinh(ac+bcx)}{bc\left(1-e^{2c(a+bx)}\right)^2\sqrt{\sinh^2(ac+bcx)}}$$

output
```
-2*exp(4*c*(b*x+a))*sinh(b*c*x+a*c)/b/c/(1-exp(2*c*(b*x+a)))^2/(sinh(b*c*x
+a*c)^2)^(1/2)
```

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.79

$$\int \frac{e^{c(a+bx)}}{\sinh^2(ac+bcx)^{3/2}}\,dx = -\frac{4e^{5c(a+bx)}\sqrt{\sinh^2(c(a+bx))}}{bc\left(-1+e^{2c(a+bx)}\right)^3}$$

input
```
Integrate[E^(c*(a + b*x))/(Sinh[a*c + b*c*x]^2)^(3/2),x]
```

output
```
(-4*E^(5*c*(a + b*x))*Sqrt[Sinh[c*(a + b*x)]^2])/(b*c*(-1 + E^(2*c*(a + b*
x))^3)
```

Rubi [A] (verified)

Time = 0.36 (sec) , antiderivative size = 58, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.160$, Rules used = {7271, 2720, 27, 242}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{e^{c(a+bx)}}{\sinh^2(ac+bcx)^{3/2}}dx$$

$$\Big\downarrow \text{ 7271}$$

$$\frac{\sinh(ac+bcx)\int e^{c(a+bx)}\text{csch}^3(ac+bxc)dx}{\sqrt{\sinh^2(ac+bcx)}}$$

$$\Big\downarrow \text{ 2720}$$

$$\frac{\sinh(ac+bcx)\int -\frac{8e^{3c(a+bx)}}{\left(1-e^{2c(a+bx)}\right)^3}de^{c(a+bx)}}{bc\sqrt{\sinh^2(ac+bcx)}}$$

$$\Big\downarrow \text{ 27}$$

$$-\frac{8\sinh(ac+bcx)\int \frac{e^{3c(a+bx)}}{\left(1-e^{2c(a+bx)}\right)^3}de^{c(a+bx)}}{bc\sqrt{\sinh^2(ac+bcx)}}$$

$$\Big\downarrow \text{ 242}$$

$$-\frac{2e^{4c(a+bx)}\sinh(ac+bcx)}{bc\left(1-e^{2c(a+bx)}\right)^2\sqrt{\sinh^2(ac+bcx)}}$$

input
```
Int[E^(c*(a + b*x))/(Sinh[a*c + b*c*x]^2)^(3/2),x]
```

output
```
(-2*E^(4*c*(a + b*x))*Sinh[a*c + b*c*x])/(b*c*(1 - E^(2*c*(a + b*x)))^2*Sq
rt[Sinh[a*c + b*c*x]^2])
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 242
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(c*x)^
(m + 1)*((a + b*x^2)^(p + 1)/(a*c*(m + 1))), x] /; FreeQ[{a, b, c, m, p}, x
] && EqQ[m + 2*p + 3, 0] && NeQ[m, -1]
```

rule 2720
```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

rule 7271
```
Int[(u_.)*((a_.)*(v_)^(m_.))^(p_), x_Symbol] :> Simp[a^IntPart[p]*((a*v^m)^
FracPart[p]/v^(m*FracPart[p]))   Int[u*v^(m*p), x], x] /; FreeQ[{a, m, p},
x] &&  !IntegerQ[p] &&  !FreeQ[v, x] &&  !(EqQ[a, 1] && EqQ[m, 1]) &&  !(Eq
Q[v, x] && EqQ[m, 1])
```

Maple [A] (verified)

Time = 0.20 (sec) , antiderivative size = 69, normalized size of antiderivative = 1.19

method	result	size
risch	$-\frac{2\left(2\,{\mathrm e}^{2 c \left(b x +a \right)}-1\right) {\mathrm e}^{-c \left(b x +a \right)}}{b c \sqrt{\left({\mathrm e}^{2 c \left(b x +a \right)}-1\right)^{2} {\mathrm e}^{-2 c \left(b x +a \right)}}\, \left({\mathrm e}^{2 c \left(b x +a \right)}-1\right)}$	69

input
```
int(exp(c*(b*x+a))/(sinh(b*c*x+a*c)^2)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-2/b/c*(2*exp(2*c*(b*x+a))-1)/((exp(2*c*(b*x+a))-1)^2*exp(-2*c*(b*x+a)))^(
1/2)/(exp(2*c*(b*x+a))-1)*exp(-c*(b*x+a))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 121 vs. $2(52) = 104$.

Time = 0.08 (sec) , antiderivative size = 121, normalized size of antiderivative = 2.09

$$\int \frac{e^{c(a+bx)}}{\sinh^2(ac+bcx)^{3/2}}\,dx =$$

$$-\frac{2\left(\cosh\left(bcx+ac\right)+3\,\sinh\left(bcx+ac\right)\right)}{bc\cosh\left(bcx+ac\right)^{3}+3\,bc\cosh\left(bcx+ac\right)\sinh\left(bcx+ac\right)^{2}+bc\sinh\left(bcx+ac\right)^{3}-bc\cosh\left(bcx+ac\right)+3\,\ldots}$$

input
```
integrate(exp(c*(b*x+a))/(sinh(b*c*x+a*c)^2)^(3/2),x, algorithm="fricas")
```

output
```
-2*(cosh(b*c*x + a*c) + 3*sinh(b*c*x + a*c))/(b*c*cosh(b*c*x + a*c)^3 + 3*
b*c*cosh(b*c*x + a*c)*sinh(b*c*x + a*c)^2 + b*c*sinh(b*c*x + a*c)^3 - b*c*
cosh(b*c*x + a*c) + 3*(b*c*cosh(b*c*x + a*c)^2 - b*c)*sinh(b*c*x + a*c))
```

Sympy [F]

$$\int \frac{e^{c(a+bx)}}{\sinh^2(ac+bcx)^{3/2}}\,dx = e^{ac}\int \frac{e^{bcx}}{\left(\sinh^{2}\left(ac+bcx\right)\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate(exp(c*(b*x+a))/(sinh(b*c*x+a*c)**2)**(3/2),x)
```

output
```
exp(a*c)*Integral(exp(b*c*x)/(sinh(a*c + b*c*x)**2)**(3/2), x)
```

Maxima [A] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 84, normalized size of antiderivative = 1.45

$$\int \frac{e^{c(a+bx)}}{\sinh^2(ac+bcx)^{3/2}}\,dx = -\frac{4\,e^{(2\,bcx+2\,ac)}}{bc(e^{(4\,bcx+4\,ac)}-2\,e^{(2\,bcx+2\,ac)}+1)} + \frac{2}{bc(e^{(4\,bcx+4\,ac)}-2\,e^{(2\,bcx+2\,ac)}+1)}$$

input
```
integrate(exp(c*(b*x+a))/(sinh(b*c*x+a*c)^2)^(3/2),x, algorithm="maxima")
```

output
```
-4*e^(2*b*c*x + 2*a*c)/(b*c*(e^(4*b*c*x + 4*a*c) - 2*e^(2*b*c*x + 2*a*c) +
 1)) + 2/(b*c*(e^(4*b*c*x + 4*a*c) - 2*e^(2*b*c*x + 2*a*c) + 1))
```

Giac [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 62, normalized size of antiderivative = 1.07

$$\int \frac{e^{c(a+bx)}}{\sinh^2(ac+bcx)^{3/2}}\,dx = -\frac{2\left(2\,e^{(2\,bcx+2\,ac)}-1\right)\mathrm{sgn}\left(e^{(bcx+ac)}-e^{(-bcx-ac)}\right)}{bc(e^{(2\,bcx+2\,ac)}-1)^2}$$

input
```
integrate(exp(c*(b*x+a))/(sinh(b*c*x+a*c)^2)^(3/2),x, algorithm="giac")
```

output
```
-2*(2*e^(2*b*c*x + 2*a*c) - 1)*sgn(e^(b*c*x + a*c) - e^(-b*c*x - a*c))/(b*
c*(e^(2*b*c*x + 2*a*c) - 1)^2)
```

Mupad [B] (verification not implemented)

Time = 1.63 (sec) , antiderivative size = 76, normalized size of antiderivative = 1.31

$$\int \frac{e^{c(a+bx)}}{\sinh^2(ac+bcx)^{3/2}}\,dx = -\frac{4\,\mathrm{e}^{a\,c+b\,c\,x}\,\left(2\,\mathrm{e}^{2\,a\,c+2\,b\,c\,x}-1\right)\,\sqrt{{\left(\frac{\mathrm{e}^{a\,c+b\,c\,x}}{2}-\frac{\mathrm{e}^{-a\,c-b\,c\,x}}{2}\right)}^2}}{b\,c\,{\left(\mathrm{e}^{2\,a\,c+2\,b\,c\,x}-1\right)}^3}$$

input
```
int(exp(c*(a + b*x))/(sinh(a*c + b*c*x)^2)^(3/2),x)
```

output
```
-(4*exp(a*c + b*c*x)*(2*exp(2*a*c + 2*b*c*x) - 1)*((exp(a*c + b*c*x)/2 - e
xp(- a*c - b*c*x)/2)^2)^(1/2))/(b*c*(exp(2*a*c + 2*b*c*x) - 1)^3)
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 50, normalized size of antiderivative = 0.86

$$\int \frac{e^{c(a+bx)}}{\sinh^2(ac+bcx)^{3/2}}\,dx = -\frac{2e^{4bcx+4ac}}{bc\,(e^{4bcx+4ac}-2e^{2bcx+2ac}+1)}$$

input
```
int(exp(c*(b*x+a))/(sinh(b*c*x+a*c)^2)^(3/2),x)
```

output
```
( - 2*e**(4*a*c + 4*b*c*x))/(b*c*(e**(4*a*c + 4*b*c*x) - 2*e**(2*a*c + 2*b
*c*x) + 1))
```

3.334 $\int \frac{e^{c(a+bx)}}{\sinh^2(ac+bcx)^{5/2}}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 147

$$\int \frac{e^{c(a+bx)}}{\sinh^2(ac+bcx)^{5/2}}\,dx = -\frac{4\sinh(ac+bcx)}{bc\left(1-e^{2c(a+bx)}\right)^4\sqrt{\sinh^2(ac+bcx)}} + \frac{32\sinh(ac+bcx)}{3bc\left(1-e^{2c(a+bx)}\right)^3\sqrt{\sinh^2(ac+bcx)}} - \frac{8\sinh(ac+bcx)}{bc\left(1-e^{2c(a+bx)}\right)^2\sqrt{\sinh^2(ac+bcx)}}$$

output
```
-4*sinh(b*c*x+a*c)/b/c/(1-exp(2*c*(b*x+a)))^4/(sinh(b*c*x+a*c)^2)^(1/2)+32
/3*sinh(b*c*x+a*c)/b/c/(1-exp(2*c*(b*x+a)))^3/(sinh(b*c*x+a*c)^2)^(1/2)-8*
sinh(b*c*x+a*c)/b/c/(1-exp(2*c*(b*x+a)))^2/(sinh(b*c*x+a*c)^2)^(1/2)
```

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 72, normalized size of antiderivative = 0.49

$$\int \frac{e^{c(a+bx)}}{\sinh^2(ac+bcx)^{5/2}}\,dx = -\frac{4\left(1-4e^{2c(a+bx)}+6e^{4c(a+bx)}\right)\sinh(c(a+bx))}{3bc\left(-1+e^{2c(a+bx)}\right)^4\sqrt{\sinh^2(c(a+bx))}}$$

input
```
Integrate[E^(c*(a + b*x))/(Sinh[a*c + b*c*x]^2)^(5/2),x]
```

output
```
(-4*(1 - 4*E^(2*c*(a + b*x)) + 6*E^(4*c*(a + b*x)))*Sinh[c*(a + b*x)])/(3*
b*c*(-1 + E^(2*c*(a + b*x)))^4*Sqrt[Sinh[c*(a + b*x)]^2])
```

Rubi [A] (verified)

Time = 0.43 (sec) , antiderivative size = 93, normalized size of antiderivative = 0.63, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.240$, Rules used = {7271, 2720, 27, 243, 53, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{e^{c(a+bx)}}{\sinh^2(ac+bcx)^{5/2}}dx$$

$$\Big\downarrow \text{7271}$$

$$\frac{\sinh(ac+bcx)\int e^{c(a+bx)}\text{csch}^5(ac+bxc)dx}{\sqrt{\sinh^2(ac+bcx)}}$$

$$\Big\downarrow \text{2720}$$

$$\frac{\sinh(ac+bcx)\int -\frac{32e^{5c(a+bx)}}{\left(1-e^{2c(a+bx)}\right)^5}de^{c(a+bx)}}{bc\sqrt{\sinh^2(ac+bcx)}}$$

$$\Big\downarrow \text{27}$$

$$-\frac{32\sinh(ac+bcx)\int \frac{e^{5c(a+bx)}}{\left(1-e^{2c(a+bx)}\right)^5}de^{c(a+bx)}}{bc\sqrt{\sinh^2(ac+bcx)}}$$

$$\Big\downarrow \text{243}$$

$$-\frac{16\sinh(ac+bcx)\int \frac{e^{2c(a+bx)}}{\left(1-e^{2c(a+bx)}\right)^5}de^{2c(a+bx)}}{bc\sqrt{\sinh^2(ac+bcx)}}$$

$$\Big\downarrow \text{53}$$

$$-\frac{16\sinh(ac+bcx)\int \left(-\frac{1}{\left(-1+e^{2c(a+bx)}\right)^3}-\frac{2}{\left(-1+e^{2c(a+bx)}\right)^4}-\frac{1}{\left(-1+e^{2c(a+bx)}\right)^5}\right)de^{2c(a+bx)}}{bc\sqrt{\sinh^2(ac+bcx)}}$$

$$\Big\downarrow \text{2009}$$

$$-\frac{16\left(\frac{1}{2\left(1-e^{2c(a+bx)}\right)^2}-\frac{2}{3\left(1-e^{2c(a+bx)}\right)^3}+\frac{1}{4\left(1-e^{2c(a+bx)}\right)^4}\right)\sinh(ac+bcx)}{bc\sqrt{\sinh^2(ac+bcx)}}$$

input
```
Int[E^(c*(a + b*x))/(Sinh[a*c + b*c*x]^2)^(5/2),x]
```

output
```
(-16*(1/(4*(1 - E^(2*c*(a + b*x)))^4) - 2/(3*(1 - E^(2*c*(a + b*x)))^3) +
1/(2*(1 - E^(2*c*(a + b*x)))^2))*Sinh[a*c + b*c*x])/(b*c*Sqrt[Sinh[a*c + b
*c*x]^2])
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 53
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && LeQ[7*m + 4*n + 4, 0])
|| LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2720
```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

rule 7271
```
Int[(u_.)*((a_.)*(v_)^(m_.))^(p_), x_Symbol] :> Simp[a^IntPart[p]*((a*v^m)^
FracPart[p]/v^(m*FracPart[p]))   Int[u*v^(m*p), x], x] /; FreeQ[{a, m, p},
x] &&  !IntegerQ[p] &&  !FreeQ[v, x] &&  !(EqQ[a, 1] && EqQ[m, 1]) &&  !(Eq
Q[v, x] && EqQ[m, 1])
```

Maple [A] (verified)

Time = 0.21 (sec) , antiderivative size = 80, normalized size of antiderivative = 0.54

method	result	size
risch	$-\frac{4\left(6\,\mathrm{e}^{4c(bx+a)}-4\,\mathrm{e}^{2c(bx+a)}+1\right)\mathrm{e}^{-c(bx+a)}}{3bc\sqrt{\left(\mathrm{e}^{2c(bx+a)}-1\right)^{2}\mathrm{e}^{-2c(bx+a)}}\,\left(\mathrm{e}^{2c(bx+a)}-1\right)^{3}}$	80

input
```
int(exp(c*(b*x+a))/(sinh(b*c*x+a*c)^2)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
-4/3/b/c*(6*exp(4*c*(b*x+a))-4*exp(2*c*(b*x+a))+1)/((exp(2*c*(b*x+a))-1)^2
*exp(-2*c*(b*x+a)))^(1/2)/(exp(2*c*(b*x+a))-1)^3*exp(-c*(b*x+a))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 315 vs. 2(130) = 260.

Time = 0.08 (sec) , antiderivative size = 315, normalized size of antiderivative = 2.14

$$\int \frac{e^{c(a+bx)}}{\sinh^2(ac+bcx)^{5/2}}\,dx =$$

$$-\frac{}{3\left(bc\cosh\left(bcx+ac\right)^{6}+6\,bc\cosh\left(bcx+ac\right)\sinh\left(bcx+ac\right)^{5}+bc\sinh\left(bcx+ac\right)^{6}-4\,bc\cosh\left(bcx+ac\right)^{4}\right.}$$

input
```
integrate(exp(c*(b*x+a))/(sinh(b*c*x+a*c)^2)^(5/2),x, algorithm="fricas")
```

output
```
-4/3*(7*cosh(b*c*x + a*c)^2 + 10*cosh(b*c*x + a*c)*sinh(b*c*x + a*c) + 7*s
inh(b*c*x + a*c)^2 - 4)/(b*c*cosh(b*c*x + a*c)^6 + 6*b*c*cosh(b*c*x + a*c)
*sinh(b*c*x + a*c)^5 + b*c*sinh(b*c*x + a*c)^6 - 4*b*c*cosh(b*c*x + a*c)^4
 + (15*b*c*cosh(b*c*x + a*c)^2 - 4*b*c)*sinh(b*c*x + a*c)^4 + 7*b*c*cosh(b
*c*x + a*c)^2 + 4*(5*b*c*cosh(b*c*x + a*c)^3 - 4*b*c*cosh(b*c*x + a*c))*si
nh(b*c*x + a*c)^3 + (15*b*c*cosh(b*c*x + a*c)^4 - 24*b*c*cosh(b*c*x + a*c)
^2 + 7*b*c)*sinh(b*c*x + a*c)^2 - 4*b*c + 2*(3*b*c*cosh(b*c*x + a*c)^5 - 8
*b*c*cosh(b*c*x + a*c)^3 + 5*b*c*cosh(b*c*x + a*c))*sinh(b*c*x + a*c))
```

Sympy [F(-1)]

Timed out.

$$\int \frac{e^{c(a+bx)}}{\sinh^2(ac+bcx)^{5/2}}\,dx = \text{Timed out}$$

input
```
integrate(exp(c*(b*x+a))/(sinh(b*c*x+a*c)**2)**(5/2),x)
```

output
```
Timed out
```

Maxima [A] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 209, normalized size of antiderivative = 1.42

$$\int \frac{e^{c(a+bx)}}{\sinh^2(ac+bcx)^{5/2}}\,dx = -\frac{8\,e^{(4\,bcx+4\,ac)}}{bc(e^{(8\,bcx+8\,ac)}-4\,e^{(6\,bcx+6\,ac)}+6\,e^{(4\,bcx+4\,ac)}-4\,e^{(2\,bcx+2\,ac)}+1)} + \frac{16\,e^{(2\,bcx+2\,ac)}}{3\,bc(e^{(8\,bcx+8\,ac)}-4\,e^{(6\,bcx+6\,ac)}+6\,e^{(4\,bcx+4\,ac)}-4\,e^{(2\,bcx+2\,ac)}+1)} - \frac{4}{3\,bc(e^{(8\,bcx+8\,ac)}-4\,e^{(6\,bcx+6\,ac)}+6\,e^{(4\,bcx+4\,ac)}-4\,e^{(2\,bcx+2\,ac)}+1)}$$

input
```
integrate(exp(c*(b*x+a))/(sinh(b*c*x+a*c)^2)^(5/2),x, algorithm="maxima")
```

output
```
-8*e^(4*b*c*x + 4*a*c)/(b*c*(e^(8*b*c*x + 8*a*c) - 4*e^(6*b*c*x + 6*a*c) +
 6*e^(4*b*c*x + 4*a*c) - 4*e^(2*b*c*x + 2*a*c) + 1)) + 16/3*e^(2*b*c*x + 2
*a*c)/(b*c*(e^(8*b*c*x + 8*a*c) - 4*e^(6*b*c*x + 6*a*c) + 6*e^(4*b*c*x + 4
*a*c) - 4*e^(2*b*c*x + 2*a*c) + 1)) - 4/3/(b*c*(e^(8*b*c*x + 8*a*c) - 4*e^
(6*b*c*x + 6*a*c) + 6*e^(4*b*c*x + 4*a*c) - 4*e^(2*b*c*x + 2*a*c) + 1))
```

Giac [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 75, normalized size of antiderivative = 0.51

$$\int \frac{e^{c(a+bx)}}{\sinh^2(ac+bcx)^{5/2}}\,dx = -\frac{4\left(6\,e^{(4\,bcx+4\,ac)}-4\,e^{(2\,bcx+2\,ac)}+1\right)\operatorname{sgn}\left(e^{(bcx+ac)}-e^{(-bcx-ac)}\right)}{3\,bc\left(e^{(2\,bcx+2\,ac)}-1\right)^4}$$

input
```
integrate(exp(c*(b*x+a))/(sinh(b*c*x+a*c)^2)^(5/2),x, algorithm="giac")
```

output
```
-4/3*(6*e^(4*b*c*x + 4*a*c) - 4*e^(2*b*c*x + 2*a*c) + 1)*sgn(e^(b*c*x + a*
c) - e^(-b*c*x - a*c))/(b*c*(e^(2*b*c*x + 2*a*c) - 1)^4)
```

Mupad [B] (verification not implemented)

Time = 1.64 (sec) , antiderivative size = 89, normalized size of antiderivative = 0.61

$$\int \frac{e^{c(a+bx)}}{\sinh^2(ac+bcx)^{5/2}}\,dx = -\frac{8\,e^{a\,c+b\,c\,x}\,\sqrt{\left(\frac{e^{a\,c+b\,c\,x}}{2}-\frac{e^{-a\,c-b\,c\,x}}{2}\right)^2}\,\left(6\,e^{4\,a\,c+4\,b\,c\,x}-4\,e^{2\,a\,c+2\,b\,c\,x}+1\right)}{3\,b\,c\,\left(e^{2\,a\,c+2\,b\,c\,x}-1\right)^5}$$

input
```
int(exp(c*(a + b*x))/(sinh(a*c + b*c*x)^2)^(5/2),x)
```

output
```
-(8*exp(a*c + b*c*x)*((exp(a*c + b*c*x)/2 - exp(- a*c - b*c*x)/2)^2)^(1/2)
*(6*exp(4*a*c + 4*b*c*x) - 4*exp(2*a*c + 2*b*c*x) + 1))/(3*b*c*(exp(2*a*c
+ 2*b*c*x) - 1)^5)
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 96, normalized size of antiderivative = 0.65

$$\int \frac{e^{c(a+bx)}}{\sinh^2(ac+bcx)^{5/2}}\,dx = \frac{-8e^{4bcx+4ac}+\frac{16e^{2bcx+2ac}}{3}-\frac{4}{3}}{bc\,(e^{8bcx+8ac}-4e^{6bcx+6ac}+6e^{4bcx+4ac}-4e^{2bcx+2ac}+1)}$$

input
```
int(exp(c*(b*x+a))/(sinh(b*c*x+a*c)^2)^(5/2),x)
```

output
```
(4*( - 6*e**(4*a*c + 4*b*c*x) + 4*e**(2*a*c + 2*b*c*x) - 1))/(3*b*c*(e**(8
*a*c + 8*b*c*x) - 4*e**(6*a*c + 6*b*c*x) + 6*e**(4*a*c + 4*b*c*x) - 4*e**(
2*a*c + 2*b*c*x) + 1))
```

3.335 $\int \frac{e^{c(a+bx)}}{\sinh^2(ac+bcx)^{7/2}}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 199

$$\int \frac{e^{c(a+bx)}}{\sinh^2(ac+bcx)^{7/2}}\,dx = -\frac{32\sinh(ac+bcx)}{3bc\left(1-e^{2c(a+bx)}\right)^6\sqrt{\sinh^2(ac+bcx)}} + \frac{192\sinh(ac+bcx)}{5bc\left(1-e^{2c(a+bx)}\right)^5\sqrt{\sinh^2(ac+bcx)}} - \frac{48\sinh(ac+bcx)}{bc\left(1-e^{2c(a+bx)}\right)^4\sqrt{\sinh^2(ac+bcx)}} + \frac{64\sinh(ac+bcx)}{3bc\left(1-e^{2c(a+bx)}\right)^3\sqrt{\sinh^2(ac+bcx)}}$$

output

```
-32/3*sinh(b*c*x+a*c)/b/c/(1-exp(2*c*(b*x+a)))^6/(sinh(b*c*x+a*c)^2)^(1/2)
+192/5*sinh(b*c*x+a*c)/b/c/(1-exp(2*c*(b*x+a)))^5/(sinh(b*c*x+a*c)^2)^(1/2
)-48*sinh(b*c*x+a*c)/b/c/(1-exp(2*c*(b*x+a)))^4/(sinh(b*c*x+a*c)^2)^(1/2)+
64/3*sinh(b*c*x+a*c)/b/c/(1-exp(2*c*(b*x+a)))^3/(sinh(b*c*x+a*c)^2)^(1/2)
```

Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 84, normalized size of antiderivative = 0.42

$$\int \frac{e^{c(a+bx)}}{\sinh^2(ac+bcx)^{7/2}}\,dx = -\frac{16\left(-1+6e^{2c(a+bx)}-15e^{4c(a+bx)}+20e^{6c(a+bx)}\right)\sinh(c(a+bx))}{15bc\left(-1+e^{2c(a+bx)}\right)^6\sqrt{\sinh^2(c(a+bx))}}$$

input
```
Integrate[E^(c*(a + b*x))/(Sinh[a*c + b*c*x]^2)^(7/2),x]
```

output
```
(-16*(-1 + 6*E^(2*c*(a + b*x)) - 15*E^(4*c*(a + b*x)) + 20*E^(6*c*(a + b*x
)))*Sinh[c*(a + b*x)])/(15*b*c*(-1 + E^(2*c*(a + b*x)))^6*Sqrt[Sinh[c*(a +
 b*x)]^2])
```

Rubi [A] (verified)

Time = 0.45 (sec) , antiderivative size = 113, normalized size of antiderivative = 0.57, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.240$, Rules used = {7271, 2720, 27, 243, 53, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{e^{c(a+bx)}}{\sinh^2(ac+bcx)^{7/2}}\,dx$$

$$\Big\downarrow \text{ 7271}$$

$$\frac{\sinh(ac+bcx)\int e^{c(a+bx)}\text{csch}^7(ac+bxc)dx}{\sqrt{\sinh^2(ac+bcx)}}$$

$$\Big\downarrow \text{ 2720}$$

$$\frac{\sinh(ac+bcx)\int -\frac{128e^{7c(a+bx)}}{\left(1-e^{2c(a+bx)}\right)^7}de^{c(a+bx)}}{bc\sqrt{\sinh^2(ac+bcx)}}$$

$$\Big\downarrow \text{ 27}$$

$$-\frac{128\sinh(ac+bcx)\int \frac{e^{7c(a+bx)}}{\left(1-e^{2c(a+bx)}\right)^7}de^{c(a+bx)}}{bc\sqrt{\sinh^2(ac+bcx)}}$$

$$\Big\downarrow \text{ 243}$$

$$-\frac{64\sinh(ac+bcx)\int \frac{e^{3c(a+bx)}}{\left(1-e^{2c(a+bx)}\right)^7}de^{2c(a+bx)}}{bc\sqrt{\sinh^2(ac+bcx)}}$$

$$\Big\downarrow \text{ 53}$$

$$-\frac{64\sinh(ac+bcx)\int \left(-\frac{1}{\left(-1+e^{2c(a+bx)}\right)^4}-\frac{3}{\left(-1+e^{2c(a+bx)}\right)^5}-\frac{3}{\left(-1+e^{2c(a+bx)}\right)^6}-\frac{1}{\left(-1+e^{2c(a+bx)}\right)^7}\right)de^{2c(a+bx)}}{bc\sqrt{\sinh^2(ac+bcx)}}$$

$$\Big\downarrow \text{ 2009}$$

$$-\frac{64\left(-\frac{1}{3\left(1-e^{2c(a+bx)}\right)^3}+\frac{3}{4\left(1-e^{2c(a+bx)}\right)^4}-\frac{3}{5\left(1-e^{2c(a+bx)}\right)^5}+\frac{1}{6\left(1-e^{2c(a+bx)}\right)^6}\right)\sinh(ac+bcx)}{bc\sqrt{\sinh^2(ac+bcx)}}$$

input
```
Int[E^(c*(a + b*x))/(Sinh[a*c + b*c*x]^2)^(7/2),x]
```

output
```
(-64*(1/(6*(1 - E^(2*c*(a + b*x)))^6) - 3/(5*(1 - E^(2*c*(a + b*x)))^5) +
3/(4*(1 - E^(2*c*(a + b*x)))^4) - 1/(3*(1 - E^(2*c*(a + b*x)))^3))*Sinh[a*
c + b*c*x])/(b*c*Sqrt[Sinh[a*c + b*c*x]^2])
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 53
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && LeQ[7*m + 4*n + 4, 0])
|| LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2720
```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

rule 7271
```
Int[(u_.)*((a_.)*(v_)^(m_.))^(p_), x_Symbol] :> Simp[a^IntPart[p]*((a*v^m)^
FracPart[p]/v^(m*FracPart[p]))   Int[u*v^(m*p), x], x] /; FreeQ[{a, m, p},
x] &&  !IntegerQ[p] &&  !FreeQ[v, x] &&  !(EqQ[a, 1] && EqQ[m, 1]) &&  !(Eq
Q[v, x] && EqQ[m, 1])
```

Maple [A] (verified)

Time = 0.21 (sec) , antiderivative size = 91, normalized size of antiderivative = 0.46

method	result	size
risch	$-\frac{16\left(20\,{\mathrm e}^{6c(bx+a)}-15\,{\mathrm e}^{4c(bx+a)}+6\,{\mathrm e}^{2c(bx+a)}-1\right){\mathrm e}^{-c(bx+a)}}{15bc\sqrt{\left({\mathrm e}^{2c(bx+a)}-1\right)^{2}{\mathrm e}^{-2c(bx+a)}}\,\left({\mathrm e}^{2c(bx+a)}-1\right)^{5}}$	91

input
```
int(exp(c*(b*x+a))/(sinh(b*c*x+a*c)^2)^(7/2),x,method=_RETURNVERBOSE)
```

output
```
-16/15/b/c*(20*exp(6*c*(b*x+a))-15*exp(4*c*(b*x+a))+6*exp(2*c*(b*x+a))-1)/
((exp(2*c*(b*x+a))-1)^2*exp(-2*c*(b*x+a)))^(1/2)/(exp(2*c*(b*x+a))-1)^5*ex
p(-c*(b*x+a))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 592 vs. $2(173) = 346$.

Time = 0.11 (sec) , antiderivative size = 592, normalized size of antiderivative = 2.97

$$\int \frac{e^{c(a+bx)}}{\sinh^2(ac+bcx)^{7/2}}\,dx =$$

$$-\frac{}{15\left(bc\cosh\left(bcx+ac\right)^9+9\,bc\cosh\left(bcx+ac\right)\sinh\left(bcx+ac\right)^8+bc\sinh\left(bcx+ac\right)^9-6\,bc\cosh\left(bcx+ac\right)\right.}$$

input
```
integrate(exp(c*(b*x+a))/(sinh(b*c*x+a*c)^2)^(7/2),x, algorithm="fricas")
```

output
```
-16/15*(19*cosh(b*c*x + a*c)^3 + 57*cosh(b*c*x + a*c)*sinh(b*c*x + a*c)^2
+ 21*sinh(b*c*x + a*c)^3 + 21*(3*cosh(b*c*x + a*c)^2 - 1)*sinh(b*c*x + a*c
) - 9*cosh(b*c*x + a*c))/(b*c*cosh(b*c*x + a*c)^9 + 9*b*c*cosh(b*c*x + a*c
)*sinh(b*c*x + a*c)^8 + b*c*sinh(b*c*x + a*c)^9 - 6*b*c*cosh(b*c*x + a*c)^
7 + 6*(6*b*c*cosh(b*c*x + a*c)^2 - b*c)*sinh(b*c*x + a*c)^7 + 15*b*c*cosh(
b*c*x + a*c)^5 + 42*(2*b*c*cosh(b*c*x + a*c)^3 - b*c*cosh(b*c*x + a*c))*si
nh(b*c*x + a*c)^6 + 3*(42*b*c*cosh(b*c*x + a*c)^4 - 42*b*c*cosh(b*c*x + a*
c)^2 + 5*b*c)*sinh(b*c*x + a*c)^5 - 19*b*c*cosh(b*c*x + a*c)^3 + 3*(42*b*c
*cosh(b*c*x + a*c)^5 - 70*b*c*cosh(b*c*x + a*c)^3 + 25*b*c*cosh(b*c*x + a*
c))*sinh(b*c*x + a*c)^4 + 3*(28*b*c*cosh(b*c*x + a*c)^6 - 70*b*c*cosh(b*c*
x + a*c)^4 + 50*b*c*cosh(b*c*x + a*c)^2 - 7*b*c)*sinh(b*c*x + a*c)^3 + 9*b
*c*cosh(b*c*x + a*c) + 3*(12*b*c*cosh(b*c*x + a*c)^7 - 42*b*c*cosh(b*c*x +
 a*c)^5 + 50*b*c*cosh(b*c*x + a*c)^3 - 19*b*c*cosh(b*c*x + a*c))*sinh(b*c*
x + a*c)^2 + 3*(3*b*c*cosh(b*c*x + a*c)^8 - 14*b*c*cosh(b*c*x + a*c)^6 + 2
5*b*c*cosh(b*c*x + a*c)^4 - 21*b*c*cosh(b*c*x + a*c)^2 + 7*b*c)*sinh(b*c*x
 + a*c))
```

Sympy [F(-1)]

Timed out.

$$\int \frac{e^{c(a+bx)}}{\sinh^2(ac+bcx)^{7/2}}\,dx = \text{Timed out}$$

input
```
integrate(exp(c*(b*x+a))/(sinh(b*c*x+a*c)**2)**(7/2),x)
```

output
```
Timed out
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 386 vs. $2(173) = 346$.

Time = 0.16 (sec) , antiderivative size = 386, normalized size of antiderivative = 1.94

$$\int \frac{e^{c(a+bx)}}{\sinh^2(ac+bcx)^{7/2}}\,dx = -\frac{64\,e^{(6\,bcx+6\,ac)}}{3\,bc(e^{(12\,bcx+12\,ac)}-6\,e^{(10\,bcx+10\,ac)}+15\,e^{(8\,bcx+8\,ac)}-20\,e^{(6\,bcx+6\,ac)}+15\,e^{(4\,bcx+4\,ac)}-6\,e^{(2\,bcx+2\,ac)}+1)} + \frac{16\,e^{(4\,bcx+4\,ac)}}{bc(e^{(12\,bcx+12\,ac)}-6\,e^{(10\,bcx+10\,ac)}+15\,e^{(8\,bcx+8\,ac)}-20\,e^{(6\,bcx+6\,ac)}+15\,e^{(4\,bcx+4\,ac)}-6\,e^{(2\,bcx+2\,ac)}+1)} - \frac{32\,e^{(2\,bcx+2\,ac)}}{5\,bc(e^{(12\,bcx+12\,ac)}-6\,e^{(10\,bcx+10\,ac)}+15\,e^{(8\,bcx+8\,ac)}-20\,e^{(6\,bcx+6\,ac)}+15\,e^{(4\,bcx+4\,ac)}-6\,e^{(2\,bcx+2\,ac)}+1)} + \frac{16}{15\,bc(e^{(12\,bcx+12\,ac)}-6\,e^{(10\,bcx+10\,ac)}+15\,e^{(8\,bcx+8\,ac)}-20\,e^{(6\,bcx+6\,ac)}+15\,e^{(4\,bcx+4\,ac)}-6\,e^{(2\,bcx+2\,ac)}+1)}$$

input
```
integrate(exp(c*(b*x+a))/(sinh(b*c*x+a*c)^2)^(7/2),x, algorithm="maxima")
```

output
```
-64/3*e^(6*b*c*x + 6*a*c)/(b*c*(e^(12*b*c*x + 12*a*c) - 6*e^(10*b*c*x + 10
*a*c) + 15*e^(8*b*c*x + 8*a*c) - 20*e^(6*b*c*x + 6*a*c) + 15*e^(4*b*c*x +
4*a*c) - 6*e^(2*b*c*x + 2*a*c) + 1)) + 16*e^(4*b*c*x + 4*a*c)/(b*c*(e^(12*
b*c*x + 12*a*c) - 6*e^(10*b*c*x + 10*a*c) + 15*e^(8*b*c*x + 8*a*c) - 20*e^
(6*b*c*x + 6*a*c) + 15*e^(4*b*c*x + 4*a*c) - 6*e^(2*b*c*x + 2*a*c) + 1)) -
 32/5*e^(2*b*c*x + 2*a*c)/(b*c*(e^(12*b*c*x + 12*a*c) - 6*e^(10*b*c*x + 10
*a*c) + 15*e^(8*b*c*x + 8*a*c) - 20*e^(6*b*c*x + 6*a*c) + 15*e^(4*b*c*x +
4*a*c) - 6*e^(2*b*c*x + 2*a*c) + 1)) + 16/15/(b*c*(e^(12*b*c*x + 12*a*c) -
 6*e^(10*b*c*x + 10*a*c) + 15*e^(8*b*c*x + 8*a*c) - 20*e^(6*b*c*x + 6*a*c)
 + 15*e^(4*b*c*x + 4*a*c) - 6*e^(2*b*c*x + 2*a*c) + 1))
```

Giac [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 88, normalized size of antiderivative = 0.44

$$\int \frac{e^{c(a+bx)}}{\sinh^2(ac+bcx)^{7/2}}\,dx = -\frac{16\left(20\,e^{(6\,bcx+6\,ac)}-15\,e^{(4\,bcx+4\,ac)}+6\,e^{(2\,bcx+2\,ac)}-1\right)\operatorname{sgn}\left(e^{(bcx+ac)}-e^{(-bcx-ac)}\right)}{15\,bc\left(e^{(2\,bcx+2\,ac)}-1\right)^6}$$

input
```
integrate(exp(c*(b*x+a))/(sinh(b*c*x+a*c)^2)^(7/2),x, algorithm="giac")
```

output
```
-16/15*(20*e^(6*b*c*x + 6*a*c) - 15*e^(4*b*c*x + 4*a*c) + 6*e^(2*b*c*x + 2
*a*c) - 1)*sgn(e^(b*c*x + a*c) - e^(-b*c*x - a*c))/(b*c*(e^(2*b*c*x + 2*a*
c) - 1)^6)
```

Mupad [B] (verification not implemented)

Time = 1.65 (sec) , antiderivative size = 353, normalized size of antiderivative = 1.77

$$\int \frac{e^{c(a+bx)}}{\sinh^2(ac+bcx)^{7/2}}\,dx = \frac{128\,e^{2\,a\,c+2\,b\,c\,x}\sqrt{\left(\frac{e^{a\,c+b\,c\,x}}{2}-\frac{e^{-a\,c-b\,c\,x}}{2}\right)^2}}{3\,b\,c\,\left(e^{a\,c+b\,c\,x}-e^{3\,a\,c+3\,b\,c\,x}\right)\,\left(e^{2\,a\,c+2\,b\,c\,x}-1\right)^3} + \frac{96\,e^{2\,a\,c+2\,b\,c\,x}\sqrt{\left(\frac{e^{a\,c+b\,c\,x}}{2}-\frac{e^{-a\,c-b\,c\,x}}{2}\right)^2}}{b\,c\,\left(e^{a\,c+b\,c\,x}-e^{3\,a\,c+3\,b\,c\,x}\right)\,\left(e^{2\,a\,c+2\,b\,c\,x}-1\right)^4} + \frac{384\,e^{2\,a\,c+2\,b\,c\,x}\sqrt{\left(\frac{e^{a\,c+b\,c\,x}}{2}-\frac{e^{-a\,c-b\,c\,x}}{2}\right)^2}}{5\,b\,c\,\left(e^{a\,c+b\,c\,x}-e^{3\,a\,c+3\,b\,c\,x}\right)\,\left(e^{2\,a\,c+2\,b\,c\,x}-1\right)^5} + \frac{64\,e^{2\,a\,c+2\,b\,c\,x}\sqrt{\left(\frac{e^{a\,c+b\,c\,x}}{2}-\frac{e^{-a\,c-b\,c\,x}}{2}\right)^2}}{3\,b\,c\,\left(e^{a\,c+b\,c\,x}-e^{3\,a\,c+3\,b\,c\,x}\right)\,\left(e^{2\,a\,c+2\,b\,c\,x}-1\right)^6}$$

input
```
int(exp(c*(a + b*x))/(sinh(a*c + b*c*x)^2)^(7/2),x)
```

output
```
(128*exp(2*a*c + 2*b*c*x)*((exp(a*c + b*c*x)/2 - exp(- a*c - b*c*x)/2)^2)^
(1/2))/(3*b*c*(exp(a*c + b*c*x) - exp(3*a*c + 3*b*c*x))*(exp(2*a*c + 2*b*c
*x) - 1)^3) + (96*exp(2*a*c + 2*b*c*x)*((exp(a*c + b*c*x)/2 - exp(- a*c -
b*c*x)/2)^2)^(1/2))/(b*c*(exp(a*c + b*c*x) - exp(3*a*c + 3*b*c*x))*(exp(2*
a*c + 2*b*c*x) - 1)^4) + (384*exp(2*a*c + 2*b*c*x)*((exp(a*c + b*c*x)/2 -
exp(- a*c - b*c*x)/2)^2)^(1/2))/(5*b*c*(exp(a*c + b*c*x) - exp(3*a*c + 3*b
*c*x))*(exp(2*a*c + 2*b*c*x) - 1)^5) + (64*exp(2*a*c + 2*b*c*x)*((exp(a*c
+ b*c*x)/2 - exp(- a*c - b*c*x)/2)^2)^(1/2))/(3*b*c*(exp(a*c + b*c*x) - ex
p(3*a*c + 3*b*c*x))*(exp(2*a*c + 2*b*c*x) - 1)^6)
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 138, normalized size of antiderivative = 0.69

$$\int \frac{e^{c(a+bx)}}{\sinh^2(ac+bcx)^{7/2}}\,dx = \frac{-\frac{64e^{6bcx+6ac}}{3}+16e^{4bcx+4ac}-\frac{32e^{2bcx+2ac}}{5}+\frac{16}{15}}{bc\left(e^{12bcx+12ac}-6e^{10bcx+10ac}+15e^{8bcx+8ac}-20e^{6bcx+6ac}+15e^{4bcx+4ac}-6e^{2bcx+2}\right.}$$

input
```
int(exp(c*(b*x+a))/(sinh(b*c*x+a*c)^2)^(7/2),x)
```

output
```
(16*( - 20*e**(6*a*c + 6*b*c*x) + 15*e**(4*a*c + 4*b*c*x) - 6*e**(2*a*c +
2*b*c*x) + 1))/(15*b*c*(e**(12*a*c + 12*b*c*x) - 6*e**(10*a*c + 10*b*c*x)
+ 15*e**(8*a*c + 8*b*c*x) - 20*e**(6*a*c + 6*b*c*x) + 15*e**(4*a*c + 4*b*c
*x) - 6*e**(2*a*c + 2*b*c*x) + 1))
```

3.336 $\int e^x \sinh(a+bx)\,dx$

Optimal result

Integrand size = 10, antiderivative size = 41

$$\int e^x \sinh(a+bx)\,dx = -\frac{be^x \cosh(a+bx)}{1-b^2} + \frac{e^x \sinh(a+bx)}{1-b^2}$$

output
```
-b*exp(x)*cosh(b*x+a)/(-b^2+1)+exp(x)*sinh(b*x+a)/(-b^2+1)
```

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.68

$$\int e^x \sinh(a+bx)\,dx = \frac{e^x(b\cosh(a+bx)-\sinh(a+bx))}{-1+b^2}$$

input
```
Integrate[E^x*Sinh[a + b*x],x]
```

output
```
(E^x*(b*Cosh[a + b*x] - Sinh[a + b*x]))/(-1 + b^2)
```

Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 41, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.100$, Rules used = {5997}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int e^x \sinh(a+bx)\,dx$$

$$\Big\downarrow \text{ 5997}$$

$$\frac{e^x \sinh(a+bx)}{1-b^2}-\frac{be^x \cosh(a+bx)}{1-b^2}$$

input
```
Int[E^x*Sinh[a + b*x],x]
```

output
```
-((b*E^x*Cosh[a + b*x])/(1 - b^2)) + (E^x*Sinh[a + b*x])/(1 - b^2)
```

Defintions of rubi rules used

rule 5997
```
Int[(F_)^((c_.)*((a_.) + (b_.)*(x_)))*Sinh[(d_.) + (e_.)*(x_)], x_Symbol] :
> Simp[(-b)*c*Log[F]*F^(c*(a + b*x))*(Sinh[d + e*x]/(e^2 - b^2*c^2*Log[F]^2
)), x] + Simp[e*F^(c*(a + b*x))*(Cosh[d + e*x]/(e^2 - b^2*c^2*Log[F]^2)), x
] /; FreeQ[{F, a, b, c, d, e}, x] && NeQ[e^2 - b^2*c^2*Log[F]^2, 0]
```

Maple [A] (verified)

Time = 0.16 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.68

method	result	size
parallelrisch	$\frac{(b\cosh(bx+a)-\sinh(bx+a))\,\mathrm{e}^{x}}{b^{2}-1}$	28
risch	$\frac{\mathrm{e}^{bx+a+x}}{2+2b}+\frac{\mathrm{e}^{-bx-a+x}}{2b-2}$	33
orering	$-\frac{2\,\mathrm{e}^{x}\sinh(bx+a)}{b^{2}-1}+\frac{\mathrm{e}^{x}\sinh(bx+a)+\mathrm{e}^{x}b\cosh(bx+a)}{b^{2}-1}$	47
default	$-\frac{\sinh((b-1)x+a)}{2(b-1)}+\frac{\sinh((1+b)x+a)}{2+2b}+\frac{\cosh((b-1)x+a)}{2b-2}+\frac{\cosh((1+b)x+a)}{2+2b}$	62

input
```
int(exp(x)*sinh(b*x+a),x,method=_RETURNVERBOSE)
```

output
```
(b*cosh(b*x+a)-sinh(b*x+a))/(b^2-1)*exp(x)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 42, normalized size of antiderivative = 1.02

$$\int e^x\sinh(a+bx)\,dx = \frac{b\cosh\left(bx+a\right)\cosh\left(x\right)+b\cosh\left(bx+a\right)\sinh\left(x\right)-\left(\cosh\left(x\right)+\sinh\left(x\right)\right)\sinh\left(bx+a\right)}{b^2-1}$$

input
```
integrate(exp(x)*sinh(b*x+a),x, algorithm="fricas")
```

output
```
(b*cosh(b*x + a)*cosh(x) + b*cosh(b*x + a)*sinh(x) - (cosh(x) + sinh(x))*s
inh(b*x + a))/(b^2 - 1)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 99 vs. 2(31) = 62.

Time = 0.29 (sec) , antiderivative size = 99, normalized size of antiderivative = 2.41

$$\int e^x\sinh(a+bx)\,dx = \begin{cases} \frac{xe^{x}\sinh\left(a-x\right)}{2}+\frac{xe^{x}\cosh\left(a-x\right)}{2}-\frac{e^{x}\cosh\left(a-x\right)}{2} & \text{for}\ b=-1 \\ \frac{xe^{x}\sinh\left(a+x\right)}{2}-\frac{xe^{x}\cosh\left(a+x\right)}{2}+\frac{e^{x}\cosh\left(a+x\right)}{2} & \text{for}\ b=1 \\ \frac{be^{x}\cosh\left(a+bx\right)}{b^{2}-1}-\frac{e^{x}\sinh\left(a+bx\right)}{b^{2}-1} & \text{otherwise} \end{cases}$$

input
```
integrate(exp(x)*sinh(b*x+a),x)
```

output
```
Piecewise((x*exp(x)*sinh(a - x)/2 + x*exp(x)*cosh(a - x)/2 - exp(x)*cosh(a
 - x)/2, Eq(b, -1)), (x*exp(x)*sinh(a + x)/2 - x*exp(x)*cosh(a + x)/2 + ex
p(x)*cosh(a + x)/2, Eq(b, 1)), (b*exp(x)*cosh(a + b*x)/(b**2 - 1) - exp(x)
*sinh(a + b*x)/(b**2 - 1), True))
```

Maxima [F(-2)]

Exception generated.

$$\int e^x \sinh(a+bx)\,dx = \text{Exception raised: ValueError}$$

input
```
integrate(exp(x)*sinh(b*x+a),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(-b>0)', see `assume?` for more d
etails)Is
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.78

$$\int e^x \sinh(a+bx)\,dx = \frac{e^{(bx+a+x)}}{2\,(b+1)} + \frac{e^{(-bx-a+x)}}{2\,(b-1)}$$

input
```
integrate(exp(x)*sinh(b*x+a),x, algorithm="giac")
```

output
```
1/2*e^(b*x + a + x)/(b + 1) + 1/2*e^(-b*x - a + x)/(b - 1)
```

Mupad [B] (verification not implemented)

Time = 1.61 (sec) , antiderivative size = 45, normalized size of antiderivative = 1.10

$$\int e^x \sinh(a+bx)\,dx = \frac{e^{x-a-b\,x}\,\left(b-e^{2\,a+2\,b\,x}+b\,e^{2\,a+2\,b\,x}+1\right)}{2\,\left(b^2-1\right)}$$

input
```
int(exp(x)*sinh(a + b*x),x)
```

output
```
(exp(x - a - b*x)*(b - exp(2*a + 2*b*x) + b*exp(2*a + 2*b*x) + 1))/(2*(b^2
 - 1))
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.68

$$\int e^x \sinh(a+bx)\,dx = \frac{e^x(\cosh\left(bx+a\right)b-\sinh\left(bx+a\right))}{b^2-1}$$

input
```
int(exp(x)*sinh(b*x+a),x)
```

output
```
(e**x*(cosh(a + b*x)*b - sinh(a + b*x)))/(b**2 - 1)
```

3.337 $\int e^x \sinh(a + cx^2)\,dx$

Optimal result

Integrand size = 12, antiderivative size = 85

$$\int e^x \sinh\left(a+cx^2\right)\,dx=\frac{e^{-a+\frac{1}{4c}}\sqrt{\pi}\,\mathrm{erf}\left(\frac{1-2cx}{2\sqrt{c}}\right)}{4\sqrt{c}}+\frac{e^{a-\frac{1}{4c}}\sqrt{\pi}\,\mathrm{erfi}\left(\frac{1+2cx}{2\sqrt{c}}\right)}{4\sqrt{c}}$$

output
```
1/4*exp(-a+1/4/c)*Pi^(1/2)*erf(1/2*(-2*c*x+1)/c^(1/2))/c^(1/2)+1/4*exp(a-1
/4/c)*Pi^(1/2)*erfi(1/2*(2*c*x+1)/c^(1/2))/c^(1/2)
```

Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 80, normalized size of antiderivative = 0.94

$$\int e^x \sinh\left(a+cx^2\right)\,dx$$
$$=\frac{e^{-\frac{1}{4}/c}\sqrt{\pi}\left(-e^{\frac{1}{2}/c}\mathrm{erf}\left(\frac{-1+2cx}{2\sqrt{c}}\right)(\cosh(a)-\sinh(a))+\mathrm{erfi}\left(\frac{1+2cx}{2\sqrt{c}}\right)(\cosh(a)+\sinh(a))\right)}{4\sqrt{c}}$$

input
```
Integrate[E^x*Sinh[a + c*x^2],x]
```

output
```
(Sqrt[Pi]*(-(E^(1/(2*c))*Erf[(-1 + 2*c*x)/(2*Sqrt[c])]*(Cosh[a] - Sinh[a])
) + Erfi[(1 + 2*c*x)/(2*Sqrt[c])]*(Cosh[a] + Sinh[a])))/(4*Sqrt[c]*E^(1/(4
*c)))
```

Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 85, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {6038, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int e^x \sinh \left(a+cx^2\right) \, dx$$

$$\Big\downarrow \text{ 6038}$$

$$\int \left(\frac{1}{2}e^{a+cx^2+x}-\frac{1}{2}e^{-a-cx^2+x}\right)dx$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{\sqrt{\pi}e^{\frac{1}{4c}-a}\text{erf}\left(\frac{1-2cx}{2\sqrt{c}}\right)}{4\sqrt{c}}+\frac{\sqrt{\pi}e^{a-\frac{1}{4c}}\text{erfi}\left(\frac{2cx+1}{2\sqrt{c}}\right)}{4\sqrt{c}}$$

input
```
Int[E^x*Sinh[a + c*x^2],x]
```

output
```
(E^(-a + 1/(4*c))*Sqrt[Pi]*Erf[(1 - 2*c*x)/(2*Sqrt[c])])/(4*Sqrt[c]) + (E^
(a - 1/(4*c))*Sqrt[Pi]*Erfi[(1 + 2*c*x)/(2*Sqrt[c])])/(4*Sqrt[c])
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 6038
```
Int[(F_)^(u_)*Sinh[v_]^(n_.), x_Symbol] :> Int[ExpandTrigToExp[F^u, Sinh[v]
^n, x], x] /; FreeQ[F, x] && (LinearQ[u, x] || PolyQ[u, x, 2]) && (LinearQ[
v, x] || PolyQ[v, x, 2]) && IGtQ[n, 0]
```

Maple [A] (verified)

Time = 0.21 (sec) , antiderivative size = 72, normalized size of antiderivative = 0.85

method	result	size
risch	$-\frac{\sqrt{\pi}\,\mathrm{e}^{-\frac{4ac-1}{4c}}\operatorname{erf}\left(\sqrt{c}\,x-\frac{1}{2\sqrt{c}}\right)}{4\sqrt{c}}+\frac{\sqrt{\pi}\,\mathrm{e}^{\frac{4ac-1}{4c}}\operatorname{erf}\left(\sqrt{-c}\,x-\frac{1}{2\sqrt{-c}}\right)}{4\sqrt{-c}}$	72

input
```
int(exp(x)*sinh(c*x^2+a),x,method=_RETURNVERBOSE)
```

output
```
-1/4*Pi^(1/2)*exp(-1/4*(4*a*c-1)/c)/c^(1/2)*erf(c^(1/2)*x-1/2/c^(1/2))+1/4
*Pi^(1/2)*exp(1/4*(4*a*c-1)/c)/(-c)^(1/2)*erf((-c)^(1/2)*x-1/2/(-c)^(1/2))
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 103, normalized size of antiderivative = 1.21

$$\int e^x \sinh\left(a+cx^2\right)\,dx =$$

$$-\frac{\sqrt{\pi}\sqrt{-c}\left(\cosh\left(\frac{4\,ac-1}{4\,c}\right)+\sinh\left(\frac{4\,ac-1}{4\,c}\right)\right)\operatorname{erf}\left(\frac{(2\,cx+1)\sqrt{-c}}{2\,c}\right)+\sqrt{\pi}\sqrt{c}\left(\cosh\left(\frac{4\,ac-1}{4\,c}\right)-\sinh\left(\frac{4\,ac-1}{4\,c}\right)\right)\operatorname{erf}\left(\frac{2}{2}\right.}{4\,c}$$

input
```
integrate(exp(x)*sinh(c*x^2+a),x, algorithm="fricas")
```

output
```
-1/4*(sqrt(pi)*sqrt(-c)*(cosh(1/4*(4*a*c - 1)/c) + sinh(1/4*(4*a*c - 1)/c)
)*erf(1/2*(2*c*x + 1)*sqrt(-c)/c) + sqrt(pi)*sqrt(c)*(cosh(1/4*(4*a*c - 1)
/c) - sinh(1/4*(4*a*c - 1)/c))*erf(1/2*(2*c*x - 1)/sqrt(c)))/c
```

Sympy [F]

$$\int e^x \sinh\left(a+cx^2\right)\,dx=\int e^x \sinh\left(a+cx^2\right)dx$$

input
```
integrate(exp(x)*sinh(c*x**2+a),x)
```

output
```
Integral(exp(x)*sinh(a + c*x**2), x)
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 65, normalized size of antiderivative = 0.76

$$\int e^x \sinh\left(a+cx^2\right)\,dx = \frac{\sqrt{\pi}\,\mathrm{erf}\left(\sqrt{-c}x-\frac{1}{2\sqrt{-c}}\right)e^{\left(a-\frac{1}{4c}\right)}}{4\sqrt{-c}}-\frac{\sqrt{\pi}\,\mathrm{erf}\left(\sqrt{c}x-\frac{1}{2\sqrt{c}}\right)e^{\left(-a+\frac{1}{4c}\right)}}{4\sqrt{c}}$$

input
```
integrate(exp(x)*sinh(c*x^2+a),x, algorithm="maxima")
```

output
```
1/4*sqrt(pi)*erf(sqrt(-c)*x - 1/2/sqrt(-c))*e^(a - 1/4/c)/sqrt(-c) - 1/4*s
qrt(pi)*erf(sqrt(c)*x - 1/2/sqrt(c))*e^(-a + 1/4/c)/sqrt(c)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 73, normalized size of antiderivative = 0.86

$$\int e^x \sinh\left(a+cx^2\right)\,dx = -\frac{\sqrt{\pi}\,\mathrm{erf}\left(-\frac{1}{2}\sqrt{-c}\left(2x+\frac{1}{c}\right)\right)e^{\left(\frac{4ac-1}{4c}\right)}}{4\sqrt{-c}} + \frac{\sqrt{\pi}\,\mathrm{erf}\left(-\frac{1}{2}\sqrt{c}\left(2x-\frac{1}{c}\right)\right)e^{\left(-\frac{4ac-1}{4c}\right)}}{4\sqrt{c}}$$

input
```
integrate(exp(x)*sinh(c*x^2+a),x, algorithm="giac")
```

output
```
-1/4*sqrt(pi)*erf(-1/2*sqrt(-c)*(2*x + 1/c))*e^(1/4*(4*a*c - 1)/c)/sqrt(-c
) + 1/4*sqrt(pi)*erf(-1/2*sqrt(c)*(2*x - 1/c))*e^(-1/4*(4*a*c - 1)/c)/sqrt
(c)
```

Mupad [F(-1)]

Timed out.

$$\int e^x \sinh\left(a+cx^2\right)\,dx = \int e^x \sinh\left(c\,x^2+a\right)\,dx$$

input
```
int(exp(x)*sinh(a + c*x^2),x)
```

output
```
int(exp(x)*sinh(a + c*x^2), x)
```

Reduce [F]

$$\int e^x \sinh\left(a+cx^2\right)\,dx = \int e^x \sinh\left(c\,x^2+a\right)dx$$

input
```
int(exp(x)*sinh(c*x^2+a),x)
```

output
```
int(e**x*sinh(a + c*x**2),x)
```

3.338 $\int e^x \sinh(a+bx+cx^2)\,dx$

Optimal result

Integrand size = 15, antiderivative size = 101

$$\int e^x \sinh\left(a+bx+cx^2\right)\,dx=\frac{e^{-a+\frac{(1-b)^2}{4c}}\sqrt{\pi}\text{erf}\left(\frac{1-b-2cx}{2\sqrt{c}}\right)}{4\sqrt{c}}+\frac{e^{a-\frac{(1+b)^2}{4c}}\sqrt{\pi}\text{erfi}\left(\frac{1+b+2cx}{2\sqrt{c}}\right)}{4\sqrt{c}}$$

output
```
1/4*exp(-a+1/4*(1-b)^2/c)*Pi^(1/2)*erf(1/2*(-2*c*x-b+1)/c^(1/2))/c^(1/2)+1
/4*exp(a-1/4*(1+b)^2/c)*Pi^(1/2)*erfi(1/2*(2*c*x+b+1)/c^(1/2))/c^(1/2)
```

Mathematica [A] (verified)

Time = 0.12 (sec) , antiderivative size = 92, normalized size of antiderivative = 0.91

$$\int e^x \sinh\left(a+bx+cx^2\right)\,dx$$
$$=\frac{e^{-\frac{(1+b)^2}{4c}}\sqrt{\pi}\left(-e^{\frac{1+b^2}{2c}}\text{erf}\left(\frac{-1+b+2cx}{2\sqrt{c}}\right)(\cosh(a)-\sinh(a))+\text{erfi}\left(\frac{1+b+2cx}{2\sqrt{c}}\right)(\cosh(a)+\sinh(a))\right)}{4\sqrt{c}}$$

input
```
Integrate[E^x*Sinh[a + b*x + c*x^2],x]
```

output
```
(Sqrt[Pi]*(-(E^((1 + b^2)/(2*c))*Erf[(-1 + b + 2*c*x)/(2*Sqrt[c])]*(Cosh[a
] - Sinh[a])) + Erfi[(1 + b + 2*c*x)/(2*Sqrt[c])]*(Cosh[a] + Sinh[a])))/(4
*Sqrt[c]*E^((1 + b)^2/(4*c)))
```

Rubi [A] (verified)

Time = 0.35 (sec) , antiderivative size = 101, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {6038, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int e^x \sinh\left(a+bx+cx^2\right) dx$$

$$\Big\downarrow \text{6038}$$

$$\int \left(\frac{1}{2}e^{a+(b+1)x+cx^2}-\frac{1}{2}e^{-a+(1-b)x-cx^2}\right) dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{\sqrt{\pi}e^{\frac{(1-b)^2}{4c}-a}\text{erf}\left(\frac{-b-2cx+1}{2\sqrt{c}}\right)}{4\sqrt{c}}+\frac{\sqrt{\pi}e^{a-\frac{(b+1)^2}{4c}}\text{erfi}\left(\frac{b+2cx+1}{2\sqrt{c}}\right)}{4\sqrt{c}}$$

input
```
Int[E^x*Sinh[a + b*x + c*x^2],x]
```

output
```
(E^(-a + (1 - b)^2/(4*c))*Sqrt[Pi]*Erf[(1 - b - 2*c*x)/(2*Sqrt[c])])/(4*Sq
rt[c]) + (E^(a - (1 + b)^2/(4*c))*Sqrt[Pi]*Erfi[(1 + b + 2*c*x)/(2*Sqrt[c]
)])/(4*Sqrt[c])
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 6038
```
Int[(F_)^(u_)*Sinh[v_]^(n_.), x_Symbol] :> Int[ExpandTrigToExp[F^u, Sinh[v]
^n, x], x] /; FreeQ[F, x] && (LinearQ[u, x] || PolyQ[u, x, 2]) && (LinearQ[
v, x] || PolyQ[v, x, 2]) && IGtQ[n, 0]
```

Maple [A] (verified)

Time = 0.34 (sec) , antiderivative size = 97, normalized size of antiderivative = 0.96

method	result	size
risch	$-\frac{\sqrt{\pi}\, {\mathrm e}^{-\frac{4 a c -b^{2}+2 b -1}{4 c}} \operatorname{erf}\left(\sqrt{c}\, x -\frac{1-b}{2 \sqrt{c}}\right)}{4 \sqrt{c}}-\frac{\sqrt{\pi}\, {\mathrm e}^{\frac{4 a c -b^{2}-2 b -1}{4 c}} \operatorname{erf}\left(-\sqrt{-c}\, x +\frac{1+b}{2 \sqrt{-c}}\right)}{4 \sqrt{-c}}$	97

input
```
int(exp(x)*sinh(c*x^2+b*x+a),x,method=_RETURNVERBOSE)
```

output
```
-1/4*Pi^(1/2)*exp(-1/4*(4*a*c-b^2+2*b-1)/c)/c^(1/2)*erf(c^(1/2)*x-1/2*(1-b
)/c^(1/2))-1/4*Pi^(1/2)*exp(1/4*(4*a*c-b^2-2*b-1)/c)/(-c)^(1/2)*erf(-(-c)^
(1/2)*x+1/2*(1+b)/(-c)^(1/2))
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 129, normalized size of antiderivative = 1.28

$$\int e^x \sinh \left(a+bx+cx^2\right) \, dx =$$

$$-\frac{\sqrt{\pi}\sqrt{-c}\left(\cosh\left(-\frac{b^2-4\,ac+2\,b+1}{4\,c}\right)+\sinh\left(-\frac{b^2-4\,ac+2\,b+1}{4\,c}\right)\right)\operatorname{erf}\left(\frac{(2\,cx+b+1)\sqrt{-c}}{2\,c}\right)+\sqrt{\pi}\sqrt{c}\left(\cosh\left(-\frac{b^2-4\,ac-}{4\,c}\right.\right.}{4\,c}$$

input
```
integrate(exp(x)*sinh(c*x^2+b*x+a),x, algorithm="fricas")
```

output
```
-1/4*(sqrt(pi)*sqrt(-c)*(cosh(-1/4*(b^2 - 4*a*c + 2*b + 1)/c) + sinh(-1/4*
(b^2 - 4*a*c + 2*b + 1)/c))*erf(1/2*(2*c*x + b + 1)*sqrt(-c)/c) + sqrt(pi)
*sqrt(c)*(cosh(-1/4*(b^2 - 4*a*c - 2*b + 1)/c) - sinh(-1/4*(b^2 - 4*a*c -
2*b + 1)/c))*erf(1/2*(2*c*x + b - 1)/sqrt(c)))/c
```

Sympy [F]

$$\int e^x \sinh\left(a + bx + cx^2\right)\,dx = \int e^x \sinh\left(a + bx + cx^2\right)dx$$

input
```
integrate(exp(x)*sinh(c*x**2+b*x+a),x)
```

output
```
Integral(exp(x)*sinh(a + b*x + c*x**2), x)
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 81, normalized size of antiderivative = 0.80

$$\int e^x \sinh\left(a + bx + cx^2\right)\,dx = \frac{\sqrt{\pi}\,\mathrm{erf}\left(\sqrt{-c}x - \frac{b+1}{2\sqrt{-c}}\right)e^{\left(a - \frac{(b+1)^2}{4c}\right)}}{4\sqrt{-c}} - \frac{\sqrt{\pi}\,\mathrm{erf}\left(\sqrt{c}x + \frac{b-1}{2\sqrt{c}}\right)e^{\left(-a + \frac{(b-1)^2}{4c}\right)}}{4\sqrt{c}}$$

input
```
integrate(exp(x)*sinh(c*x^2+b*x+a),x, algorithm="maxima")
```

output
```
1/4*sqrt(pi)*erf(sqrt(-c)*x - 1/2*(b + 1)/sqrt(-c))*e^(a - 1/4*(b + 1)^2/c
)/sqrt(-c) - 1/4*sqrt(pi)*erf(sqrt(c)*x + 1/2*(b - 1)/sqrt(c))*e^(-a + 1/4
*(b - 1)^2/c)/sqrt(c)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 91, normalized size of antiderivative = 0.90

$$\int e^x \sinh\left(a+bx+cx^2\right) dx = -\frac{\sqrt{\pi}\,\mathrm{erf}\left(-\frac{1}{2}\sqrt{-c}\left(2\,x+\frac{b+1}{c}\right)\right)e^{\left(-\frac{b^2-4\,ac+2\,b+1}{4\,c}\right)}}{4\sqrt{-c}} + \frac{\sqrt{\pi}\,\mathrm{erf}\left(-\frac{1}{2}\sqrt{c}\left(2\,x+\frac{b-1}{c}\right)\right)e^{\left(\frac{b^2-4\,ac-2\,b+1}{4\,c}\right)}}{4\sqrt{c}}$$

input
```
integrate(exp(x)*sinh(c*x^2+b*x+a),x, algorithm="giac")
```

output
```
-1/4*sqrt(pi)*erf(-1/2*sqrt(-c)*(2*x + (b + 1)/c))*e^(-1/4*(b^2 - 4*a*c +
2*b + 1)/c)/sqrt(-c) + 1/4*sqrt(pi)*erf(-1/2*sqrt(c)*(2*x + (b - 1)/c))*e^
(1/4*(b^2 - 4*a*c - 2*b + 1)/c)/sqrt(c)
```

Mupad [F(-1)]

Timed out.

$$\int e^x \sinh\left(a+bx+cx^2\right) dx = \int \sinh\left(c\,x^2+b\,x+a\right)\,\mathrm{e}^x\,dx$$

input
```
int(sinh(a + b*x + c*x^2)*exp(x),x)
```

output
```
int(sinh(a + b*x + c*x^2)*exp(x), x)
```

Reduce [F]

$$\int e^x \sinh\left(a+bx+cx^2\right) dx = \int e^x \sinh\left(c\,x^2+bx+a\right) dx$$

input
```
int(exp(x)*sinh(c*x^2+b*x+a),x)
```

output
```
int(e**x*sinh(a + b*x + c*x**2),x)
```

3.339 $\int e^{x^2}\sinh(a+bx)\,dx$

Optimal result

Integrand size = 12, antiderivative size = 65

$$\int e^{x^2}\sinh(a+bx)\,dx=-\frac{1}{4}e^{-a-\frac{b^2}{4}}\sqrt{\pi}\text{erfi}\left(\frac{1}{2}(-b+2x)\right)+\frac{1}{4}e^{a-\frac{b^2}{4}}\sqrt{\pi}\text{erfi}\left(\frac{1}{2}(b+2x)\right)$$

output
```
1/4*exp(-a-1/4*b^2)*Pi^(1/2)*erfi(1/2*b-x)+1/4*exp(a-1/4*b^2)*Pi^(1/2)*erf
i(1/2*b+x)
```

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.78

$$\int e^{x^2}\sinh(a+bx)\,dx=\frac{1}{4}e^{-\frac{b^2}{4}}\sqrt{\pi}\left(\text{erfi}\left(\frac{b}{2}-x\right)(\cosh(a)-\sinh(a))\right.$$
$$\left.+\text{erfi}\left(\frac{b}{2}+x\right)(\cosh(a)+\sinh(a))\right)$$

input
```
Integrate[E^x^2*Sinh[a + b*x],x]
```

output
```
(Sqrt[Pi]*(Erfi[b/2 - x]*(Cosh[a] - Sinh[a]) + Erfi[b/2 + x]*(Cosh[a] + Si
nh[a])))/(4*E^(b^2/4))
```

Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 65, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {6038, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int e^{x^2} \sinh(a+bx) \, dx$$

$$\Big\downarrow \text{ 6038}$$

$$\int \left(\frac{1}{2}e^{a+bx+x^2}-\frac{1}{2}e^{-a-bx+x^2}\right)dx$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{1}{4}\sqrt{\pi}e^{a-\frac{b^2}{4}}\text{erfi}\left(\frac{1}{2}(b+2x)\right)-\frac{1}{4}\sqrt{\pi}e^{-a-\frac{b^2}{4}}\text{erfi}\left(\frac{1}{2}(2x-b)\right)$$

input
```
Int[E^x^2*Sinh[a + b*x],x]
```

output
```
-1/4*(E^(-a - b^2/4)*Sqrt[Pi]*Erfi[(-b + 2*x)/2]) + (E^(a - b^2/4)*Sqrt[Pi
]*Erfi[(b + 2*x)/2])/4
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 6038
```
Int[(F_)^(u_)*Sinh[v_]^(n_.), x_Symbol] :> Int[ExpandTrigToExp[F^u, Sinh[v]
^n, x], x] /; FreeQ[F, x] && (LinearQ[u, x] || PolyQ[u, x, 2]) && (LinearQ[
v, x] || PolyQ[v, x, 2]) && IGtQ[n, 0]
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 0.17 (sec) , antiderivative size = 52, normalized size of antiderivative = 0.80

method	result	size
risch	$-\frac{i\sqrt{\pi}\,\mathrm{e}^{-a-\frac{b^{2}}{4}}\operatorname{erf}\left(-ix+\frac{1}{2}ib\right)}{4}-\frac{i\sqrt{\pi}\,\mathrm{e}^{a-\frac{b^{2}}{4}}\operatorname{erf}\left(ix+\frac{1}{2}ib\right)}{4}$	52

input
```
int(exp(x^2)*sinh(b*x+a),x,method=_RETURNVERBOSE)
```

output
```
-1/4*I*Pi^(1/2)*exp(-a-1/4*b^2)*erf(-I*x+1/2*I*b)-1/4*I*Pi^(1/2)*exp(a-1/4
*b^2)*erf(I*x+1/2*I*b)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 72, normalized size of antiderivative = 1.11

$$\int e^{x^2}\sinh(a+bx)\,dx$$
$$=\frac{1}{4}\sqrt{\pi}\left(\cosh\left(\frac{1}{4}b^2-a\right)\operatorname{erfi}\left(\frac{1}{2}b+x\right)-\cosh\left(\frac{1}{4}b^2+a\right)\operatorname{erfi}\left(-\frac{1}{2}b+x\right)+\operatorname{erfi}\left(-\frac{1}{2}b+x\right)\sinh\left(\frac{1}{4}b\right.\right.$$

input
```
integrate(exp(x^2)*sinh(b*x+a),x, algorithm="fricas")
```

output
```
1/4*sqrt(pi)*(cosh(1/4*b^2 - a)*erfi(1/2*b + x) - cosh(1/4*b^2 + a)*erfi(-
1/2*b + x) + erfi(-1/2*b + x)*sinh(1/4*b^2 + a) - erfi(1/2*b + x)*sinh(1/4
*b^2 - a))
```

Sympy [F]

$$\int e^{x^2} \sinh(a+bx)\,dx = \int e^{x^2} \sinh{(a+bx)}\,dx$$

input
```
integrate(exp(x**2)*sinh(b*x+a),x)
```

output
```
Integral(exp(x**2)*sinh(a + b*x), x)
```

Maxima [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.04 (sec) , antiderivative size = 45, normalized size of antiderivative = 0.69

$$\int e^{x^2} \sinh(a+bx)\,dx = -\frac{1}{4} i \sqrt{\pi} \operatorname{erf}\left(\frac{1}{2} i b + i x\right) e^{\left(-\frac{1}{4} b^2 + a\right)} + \frac{1}{4} i \sqrt{\pi} \operatorname{erf}\left(-\frac{1}{2} i b + i x\right) e^{\left(-\frac{1}{4} b^2 - a\right)}$$

input
```
integrate(exp(x^2)*sinh(b*x+a),x, algorithm="maxima")
```

output
```
-1/4*I*sqrt(pi)*erf(1/2*I*b + I*x)*e^(-1/4*b^2 + a) + 1/4*I*sqrt(pi)*erf(-
1/2*I*b + I*x)*e^(-1/4*b^2 - a)
```

Giac [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.12 (sec) , antiderivative size = 45, normalized size of antiderivative = 0.69

$$\int e^{x^2} \sinh(a+bx)\,dx = \frac{1}{4} i \sqrt{\pi} \operatorname{erf}\left(-\frac{1}{2} i b - i x\right) e^{\left(-\frac{1}{4} b^2 + a\right)} - \frac{1}{4} i \sqrt{\pi} \operatorname{erf}\left(\frac{1}{2} i b - i x\right) e^{\left(-\frac{1}{4} b^2 - a\right)}$$

input
```
integrate(exp(x^2)*sinh(b*x+a),x, algorithm="giac")
```

output
```
1/4*I*sqrt(pi)*erf(-1/2*I*b - I*x)*e^(-1/4*b^2 + a) - 1/4*I*sqrt(pi)*erf(1
/2*I*b - I*x)*e^(-1/4*b^2 - a)
```

Mupad [F(-1)]

Timed out.

$$\int e^{x^2}\sinh(a+bx)\,dx=\int e^{x^2}\sinh(a+b\,x)\,dx$$

input
```
int(exp(x^2)*sinh(a + b*x),x)
```

output
```
int(exp(x^2)*sinh(a + b*x), x)
```

Reduce [F]

$$\int e^{x^2}\sinh(a+bx)\,dx=\int e^{x^2}\sinh(bx+a)\,dx$$

input
```
int(exp(x^2)*sinh(b*x+a),x)
```

output
```
int(e**(x**2)*sinh(a + b*x),x)
```

3.340 $\int e^{x^2} \sinh(a+cx^2)\, dx$

Optimal result

Integrand size = 14, antiderivative size = 65

$$\int e^{x^2} \sinh(a+cx^2)\, dx = -\frac{e^{-a}\sqrt{\pi}\operatorname{erfi}\left(\sqrt{1-c}\,x\right)}{4\sqrt{1-c}} + \frac{e^{a}\sqrt{\pi}\operatorname{erfi}\left(\sqrt{1+c}\,x\right)}{4\sqrt{1+c}}$$

output
```
-1/4*Pi^(1/2)*erfi((1-c)^(1/2)*x)/(1-c)^(1/2)/exp(a)+1/4*exp(a)*Pi^(1/2)*e
rfi((1+c)^(1/2)*x)/(1+c)^(1/2)
```

Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 72, normalized size of antiderivative = 1.11

$$\int e^{x^2} \sinh(a+cx^2)\, dx$$
$$= \frac{\sqrt{\pi}\left(-\sqrt{-1+c}(1+c)\operatorname{erf}\left(\sqrt{-1+c}\,x\right)(\cosh(a)-\sinh(a)) + (-1+c)\sqrt{1+c}\operatorname{erfi}\left(\sqrt{1+c}\,x\right)(\cosh(a)+\si}{4(-1+c^2)}$$

input
```
Integrate[E^x^2*Sinh[a + c*x^2],x]
```

output
```
(Sqrt[Pi]*(-(Sqrt[-1 + c]*(1 + c)*Erf[Sqrt[-1 + c]*x]*(Cosh[a] - Sinh[a]))
 + (-1 + c)*Sqrt[1 + c]*Erfi[Sqrt[1 + c]*x]*(Cosh[a] + Sinh[a])))/(4*(-1 +
 c^2))
```

Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 65, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {6038, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int e^{x^2} \sinh \left(a+cx^2\right) dx$$

$$\Big\downarrow \text{6038}$$

$$\int \left(\frac{1}{2}e^{a+(c+1)x^2}-\frac{1}{2}e^{(1-c)x^2-a}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{\sqrt{\pi}e^a \text{erfi}\left(\sqrt{c+1}x\right)}{4\sqrt{c+1}}-\frac{\sqrt{\pi}e^{-a}\text{erfi}\left(\sqrt{1-c}x\right)}{4\sqrt{1-c}}$$

input
```
Int[E^x^2*Sinh[a + c*x^2],x]
```

output
```
-1/4*(Sqrt[Pi]*Erfi[Sqrt[1 - c]*x])/(Sqrt[1 - c]*E^a) + (E^a*Sqrt[Pi]*Erfi
[Sqrt[1 + c]*x])/(4*Sqrt[1 + c])
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 6038
```
Int[(F_)^(u_)*Sinh[v_]^(n_.), x_Symbol] :> Int[ExpandTrigToExp[F^u, Sinh[v]
^n, x], x] /; FreeQ[F, x] && (LinearQ[u, x] || PolyQ[u, x, 2]) && (LinearQ[
v, x] || PolyQ[v, x, 2]) && IGtQ[n, 0]
```

Maple [A] (verified)

Time = 0.42 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.74

method	result	size
risch	$-\frac{\sqrt{\pi}\,\mathrm{e}^{-a}\operatorname{erf}\left(\sqrt{c-1}\,x\right)}{4\sqrt{c-1}}+\frac{\sqrt{\pi}\,\mathrm{e}^{a}\operatorname{erf}\left(\sqrt{-1-c}\,x\right)}{4\sqrt{-1-c}}$	48

input
```
int(exp(x^2)*sinh(c*x^2+a),x,method=_RETURNVERBOSE)
```

output
```
-1/4*Pi^(1/2)*exp(-a)/(c-1)^(1/2)*erf((c-1)^(1/2)*x)+1/4*Pi^(1/2)*exp(a)/(
-1-c)^(1/2)*erf((-1-c)^(1/2)*x)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 75, normalized size of antiderivative = 1.15

$$\int e^{x^2}\sinh\left(a+cx^2\right)\,dx = -\frac{\sqrt{\pi}((c+1)\cosh(a)-(c+1)\sinh(a))\sqrt{c-1}\operatorname{erf}\left(\sqrt{c-1}x\right)+\sqrt{\pi}((c-1)\cosh(a)+(c-1)\sinh(a)}{4\left(c^2-1\right)}$$

input
```
integrate(exp(x^2)*sinh(c*x^2+a),x, algorithm="fricas")
```

output
```
-1/4*(sqrt(pi)*((c + 1)*cosh(a) - (c + 1)*sinh(a))*sqrt(c - 1)*erf(sqrt(c
- 1)*x) + sqrt(pi)*((c - 1)*cosh(a) + (c - 1)*sinh(a))*sqrt(-c - 1)*erf(sq
rt(-c - 1)*x))/(c^2 - 1)
```

Sympy [F]

$$\int e^{x^2}\sinh\left(a+cx^2\right)\,dx=\int e^{x^2}\sinh\left(a+cx^2\right)dx$$

input
```
integrate(exp(x**2)*sinh(c*x**2+a),x)
```

output
```
Integral(exp(x**2)*sinh(a + c*x**2), x)
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.72

$$\int e^{x^2}\sinh\left(a+cx^2\right)\,dx=-\frac{\sqrt{\pi}\,\mathrm{erf}\left(\sqrt{c-1}x\right)e^{(-a)}}{4\sqrt{c-1}}+\frac{\sqrt{\pi}\,\mathrm{erf}\left(\sqrt{-c-1}x\right)e^{a}}{4\sqrt{-c-1}}$$

input
```
integrate(exp(x^2)*sinh(c*x^2+a),x, algorithm="maxima")
```

output
```
-1/4*sqrt(pi)*erf(sqrt(c - 1)*x)*e^(-a)/sqrt(c - 1) + 1/4*sqrt(pi)*erf(sqr
t(-c - 1)*x)*e^a/sqrt(-c - 1)
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 49, normalized size of antiderivative = 0.75

$$\int e^{x^2}\sinh\left(a+cx^2\right)\,dx=\frac{\sqrt{\pi}\,\mathrm{erf}\left(-\sqrt{c-1}x\right)e^{(-a)}}{4\sqrt{c-1}}-\frac{\sqrt{\pi}\,\mathrm{erf}\left(-\sqrt{-c-1}x\right)e^{a}}{4\sqrt{-c-1}}$$

input
```
integrate(exp(x^2)*sinh(c*x^2+a),x, algorithm="giac")
```

output
```
1/4*sqrt(pi)*erf(-sqrt(c - 1)*x)*e^(-a)/sqrt(c - 1) - 1/4*sqrt(pi)*erf(-sq
rt(-c - 1)*x)*e^a/sqrt(-c - 1)
```

Mupad [F(-1)]

Timed out.

$$\int e^{x^2} \sinh \left(a + cx^2\right) \, dx = \int e^{x^2} \sinh\left(c\,x^2 + a\right) \, dx$$

input
```
int(exp(x^2)*sinh(a + c*x^2),x)
```

output
```
int(exp(x^2)*sinh(a + c*x^2), x)
```

Reduce [F]

$$\int e^{x^2} \sinh \left(a + cx^2\right) \, dx = \int e^{x^2} \sinh \left(c\,x^2 + a\right) dx$$

input
```
int(exp(x^2)*sinh(c*x^2+a),x)
```

output
```
int(e**(x**2)*sinh(a + c*x**2),x)
```

3.341 $\int e^{x^2} \sinh\left(a+bx+cx^2\right) dx$

Optimal result

Integrand size = 17, antiderivative size = 115

$$\int e^{x^2} \sinh\left(a+bx+cx^2\right) dx = \frac{e^{-a-\frac{b^2}{4(1-c)}}\sqrt{\pi}\,\text{erfi}\left(\frac{b-2(1-c)x}{2\sqrt{1-c}}\right)}{4\sqrt{1-c}} + \frac{e^{a-\frac{b^2}{4(1+c)}}\sqrt{\pi}\,\text{erfi}\left(\frac{b+2(1+c)x}{2\sqrt{1+c}}\right)}{4\sqrt{1+c}}$$

output
```
1/4*exp(-a-b^2/(4-4*c))*Pi^(1/2)*erfi(1/2*(b-2*(1-c)*x)/(1-c)^(1/2))/(1-c)
^(1/2)+1/4*exp(a-b^2/(4+4*c))*Pi^(1/2)*erfi(1/2*(b+2*(1+c)*x)/(1+c)^(1/2))
/(1+c)^(1/2)
```

Mathematica [A] (verified)

Time = 0.27 (sec) , antiderivative size = 123, normalized size of antiderivative = 1.07

$$\int e^{x^2} \sinh\left(a+bx+cx^2\right) dx$$
$$= \frac{e^{-\frac{b^2}{4+4c}}\sqrt{\pi}\left(-\sqrt{-1+c}(1+c)e^{\frac{b^2c}{2(-1+c^2)}}\text{erf}\left(\frac{b+2(-1+c)x}{2\sqrt{-1+c}}\right)(\cosh(a)-\sinh(a))+(-1+c)\sqrt{1+c}\,\text{erfi}\left(\frac{b+2(1+c}{2\sqrt{1+}\right.\right.}{4\left(-1+c^2\right)}$$

input
```
Integrate[E^x^2*Sinh[a + b*x + c*x^2],x]
```

output
```
(Sqrt[Pi]*(-(Sqrt[-1 + c]*(1 + c)*E^((b^2*c)/(2*(-1 + c^2)))*Erf[(b + 2*(-
1 + c)*x)/(2*Sqrt[-1 + c])]*(Cosh[a] - Sinh[a])) + (-1 + c)*Sqrt[1 + c]*Er
fi[(b + 2*(1 + c)*x)/(2*Sqrt[1 + c])]*(Cosh[a] + Sinh[a])))/(4*(-1 + c^2)*
E^(b^2/(4 + 4*c)))
```

Rubi [A] (verified)

Time = 0.39 (sec) , antiderivative size = 115, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.118$, Rules used = $\{6038, 2009\}$

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int e^{x^2}\sinh\left(a+bx+cx^2\right)\,dx$$

$$\Big\downarrow \text{6038}$$

$$\int \left(\frac{1}{2}e^{a+bx+(c+1)x^2}-\frac{1}{2}e^{-a-bx+(1-c)x^2}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{\sqrt{\pi}e^{-a-\frac{b^2}{4(1-c)}}\operatorname{erfi}\left(\frac{b-2(1-c)x}{2\sqrt{1-c}}\right)}{4\sqrt{1-c}}+\frac{\sqrt{\pi}e^{a-\frac{b^2}{4(c+1)}}\operatorname{erfi}\left(\frac{b+2(c+1)x}{2\sqrt{c+1}}\right)}{4\sqrt{c+1}}$$

input
```
Int[E^x^2*Sinh[a + b*x + c*x^2],x]
```

output
```
(E^(-a - b^2/(4*(1 - c)))*Sqrt[Pi]*Erfi[(b - 2*(1 - c)*x)/(2*Sqrt[1 - c])]
)/(4*Sqrt[1 - c]) + (E^(a - b^2/(4*(1 + c)))*Sqrt[Pi]*Erfi[(b + 2*(1 + c)*
x)/(2*Sqrt[1 + c])])/(4*Sqrt[1 + c])
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 6038
```
Int[(F_)^(u_)*Sinh[v_]^(n_.), x_Symbol] :> Int[ExpandTrigToExp[F^u, Sinh[v]
^n, x], x] /; FreeQ[F, x] && (LinearQ[u, x] || PolyQ[u, x, 2]) && (LinearQ[
v, x] || PolyQ[v, x, 2]) && IGtQ[n, 0]
```

Maple [A] (verified)

Time = 0.44 (sec) , antiderivative size = 105, normalized size of antiderivative = 0.91

method	result	size
risch	$-\frac{\sqrt{\pi}\,\mathrm{e}^{-\frac{4ac-b^{2}-4a}{4(c-1)}}\operatorname{erf}\left(\sqrt{c-1}\,x+\frac{b}{2\sqrt{c-1}}\right)}{4\sqrt{c-1}}-\frac{\sqrt{\pi}\,\mathrm{e}^{\frac{4ac-b^{2}+4a}{4+4c}}\operatorname{erf}\left(-\sqrt{-1-c}\,x+\frac{b}{2\sqrt{-1-c}}\right)}{4\sqrt{-1-c}}$	105

input
```
int(exp(x^2)*sinh(c*x^2+b*x+a),x,method=_RETURNVERBOSE)
```

output
```
-1/4*Pi^(1/2)*exp(-1/4*(4*a*c-b^2-4*a)/(c-1))/(c-1)^(1/2)*erf((c-1)^(1/2)*
x+1/2*b/(c-1)^(1/2))-1/4*Pi^(1/2)*exp(1/4*(4*a*c-b^2+4*a)/(1+c))/(-1-c)^(1
/2)*erf(-(-1-c)^(1/2)*x+1/2*b/(-1-c)^(1/2))
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 164, normalized size of antiderivative = 1.43

$$\int e^{x^2}\sinh\left(a+bx+cx^2\right)\,dx = -\frac{\sqrt{\pi}\left((c+1)\cosh\left(-\frac{b^2-4\,ac+4\,a}{4\,(c-1)}\right)-(c+1)\sinh\left(-\frac{b^2-4\,ac+4\,a}{4\,(c-1)}\right)\right)\sqrt{c-1}\,\mathrm{erf}\left(\frac{2\,(c-1)x+b}{2\,\sqrt{c-1}}\right)+\sqrt{\pi}\Big((c-1)\,\mathrm{c}}{4\,(c^2-1)}$$

input
```
integrate(exp(x^2)*sinh(c*x^2+b*x+a),x, algorithm="fricas")
```

output
```
-1/4*(sqrt(pi)*((c + 1)*cosh(-1/4*(b^2 - 4*a*c + 4*a)/(c - 1)) - (c + 1)*s
inh(-1/4*(b^2 - 4*a*c + 4*a)/(c - 1)))*sqrt(c - 1)*erf(1/2*(2*(c - 1)*x +
b)/sqrt(c - 1)) + sqrt(pi)*((c - 1)*cosh(-1/4*(b^2 - 4*a*c - 4*a)/(c + 1))
 + (c - 1)*sinh(-1/4*(b^2 - 4*a*c - 4*a)/(c + 1)))*sqrt(-c - 1)*erf(1/2*(2
*(c + 1)*x + b)*sqrt(-c - 1)/(c + 1)))/(c^2 - 1)
```

Sympy [F]

$$\int e^{x^2} \sinh\left(a + bx + cx^2\right) \, dx = \int e^{x^2} \sinh\left(a + bx + cx^2\right) dx$$

input
```
integrate(exp(x**2)*sinh(c*x**2+b*x+a),x)
```

output
```
Integral(exp(x**2)*sinh(a + b*x + c*x**2), x)
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 89, normalized size of antiderivative = 0.77

$$\int e^{x^2} \sinh\left(a + bx + cx^2\right) \, dx = \frac{\sqrt{\pi} \operatorname{erf}\left(\sqrt{-c-1}x - \frac{b}{2\sqrt{-c-1}}\right) e^{\left(a - \frac{b^2}{4(c+1)}\right)}}{4\sqrt{-c-1}} - \frac{\sqrt{\pi} \operatorname{erf}\left(\sqrt{c-1}x + \frac{b}{2\sqrt{c-1}}\right) e^{\left(-a + \frac{b^2}{4(c-1)}\right)}}{4\sqrt{c-1}}$$

input
```
integrate(exp(x^2)*sinh(c*x^2+b*x+a),x, algorithm="maxima")
```

output
```
1/4*sqrt(pi)*erf(sqrt(-c - 1)*x - 1/2*b/sqrt(-c - 1))*e^(a - 1/4*b^2/(c +
1))/sqrt(-c - 1) - 1/4*sqrt(pi)*erf(sqrt(c - 1)*x + 1/2*b/sqrt(c - 1))*e^(
-a + 1/4*b^2/(c - 1))/sqrt(c - 1)
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 101, normalized size of antiderivative = 0.88

$$\int e^{x^2} \sinh \left(a + bx + cx^2\right) \, dx = -\frac{\sqrt{\pi} \operatorname{erf}\left(-\frac{1}{2} \sqrt{-c-1} \left(2 x + \frac{b}{c+1}\right)\right) e^{\left(-\frac{b^2-4 ac-4 a}{4 (c+1)}\right)}}{4 \sqrt{-c-1}} + \frac{\sqrt{\pi} \operatorname{erf}\left(-\frac{1}{2} \sqrt{c-1} \left(2 x + \frac{b}{c-1}\right)\right) e^{\left(\frac{b^2-4 ac+4 a}{4 (c-1)}\right)}}{4 \sqrt{c-1}}$$

input
```
integrate(exp(x^2)*sinh(c*x^2+b*x+a),x, algorithm="giac")
```

output
```
-1/4*sqrt(pi)*erf(-1/2*sqrt(-c - 1)*(2*x + b/(c + 1)))*e^(-1/4*(b^2 - 4*a*
c - 4*a)/(c + 1))/sqrt(-c - 1) + 1/4*sqrt(pi)*erf(-1/2*sqrt(c - 1)*(2*x +
b/(c - 1)))*e^(1/4*(b^2 - 4*a*c + 4*a)/(c - 1))/sqrt(c - 1)
```

Mupad [F(-1)]

Timed out.

$$\int e^{x^2} \sinh \left(a + bx + cx^2\right) \, dx = \int \sinh\left(c\, x^2 + b\, x + a\right) \, \mathrm{e}^{x^2} \,d x$$

input
```
int(sinh(a + b*x + c*x^2)*exp(x^2),x)
```

output
```
int(sinh(a + b*x + c*x^2)*exp(x^2), x)
```

Reduce [F]

$$\int e^{x^2} \sinh \left(a + bx + cx^2\right) \, dx = \int e^{x^2} \sinh \left(c\, x^2 + bx + a\right) dx$$

input
```
int(exp(x^2)*sinh(c*x^2+b*x+a),x)
```

output
```
int(e**(x**2)*sinh(a + b*x + c*x**2),x)
```

3.342 $\int f^{a+bx} \sinh\left(d+fx^2\right) dx$

Optimal result

Integrand size = 16, antiderivative size = 110

$$\int f^{a+bx}\sinh\left(d+fx^2\right)dx = -\frac{1}{4}e^{-d+\frac{b^2\log^2(f)}{4f}}f^{-\frac{1}{2}+a}\sqrt{\pi}\operatorname{erf}\left(\frac{2fx-b\log(f)}{2\sqrt{f}}\right) + \frac{1}{4}e^{d-\frac{b^2\log^2(f)}{4f}}f^{-\frac{1}{2}+a}\sqrt{\pi}\operatorname{erfi}\left(\frac{2fx+b\log(f)}{2\sqrt{f}}\right)$$

output
```
-1/4*exp(-d+1/4*b^2*ln(f)^2/f)*f^(-1/2+a)*Pi^(1/2)*erf(1/2*(2*f*x-b*ln(f))
/f^(1/2))+1/4*exp(d-1/4*b^2*ln(f)^2/f)*f^(-1/2+a)*Pi^(1/2)*erfi(1/2*(2*f*x
+b*ln(f))/f^(1/2))
```

Mathematica [A] (verified)

Time = 0.10 (sec) , antiderivative size = 103, normalized size of antiderivative = 0.94

$$\int f^{a+bx}\sinh\left(d+fx^2\right)dx = \frac{1}{4}e^{-\frac{b^2\log^2(f)}{4f}}f^{-\frac{1}{2}+a}\sqrt{\pi}\left(-e^{\frac{b^2\log^2(f)}{2f}}\operatorname{erf}\left(\frac{2fx-b\log(f)}{2\sqrt{f}}\right)(\cosh(d)-\sinh(d)) + \operatorname{erfi}\left(\frac{2fx+b\log(f)}{2\sqrt{f}}\right)(\cosh(d)+\sinh(d))\right)$$

input
```
Integrate[f^(a + b*x)*Sinh[d + f*x^2],x]
```

output
```
(f^(-1/2 + a)*Sqrt[Pi]*(-(E^((b^2*Log[f]^2)/(2*f))*Erf[(2*f*x - b*Log[f])/
(2*Sqrt[f])]*(Cosh[d] - Sinh[d])) + Erfi[(2*f*x + b*Log[f])/(2*Sqrt[f])]*(
Cosh[d] + Sinh[d])))/(4*E^((b^2*Log[f]^2)/(4*f)))
```

Rubi [A] (verified)

Time = 0.37 (sec) , antiderivative size = 110, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$, Rules used = {6038, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int f^{a+bx}\sinh\left(d+fx^2\right)\,dx$$

$$\Big\downarrow \text{6038}$$

$$\int \left(\frac{1}{2}e^{d+fx^2}f^{a+bx}-\frac{1}{2}e^{-d-fx^2}f^{a+bx}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{4}\sqrt{\pi}f^{a-\frac{1}{2}}e^{d-\frac{b^2\log^2(f)}{4f}}\text{erfi}\left(\frac{b\log(f)+2fx}{2\sqrt{f}}\right)-\frac{1}{4}\sqrt{\pi}f^{a-\frac{1}{2}}e^{\frac{b^2\log^2(f)}{4f}-d}\text{erf}\left(\frac{2fx-b\log(f)}{2\sqrt{f}}\right)$$

input
```
Int[f^(a + b*x)*Sinh[d + f*x^2],x]
```

output
```
-1/4*(E^(-d + (b^2*Log[f]^2)/(4*f))*f^(-1/2 + a)*Sqrt[Pi]*Erf[(2*f*x - b*L
og[f])/(2*Sqrt[f])]) + (E^(d - (b^2*Log[f]^2)/(4*f))*f^(-1/2 + a)*Sqrt[Pi]
*Erfi[(2*f*x + b*Log[f])/(2*Sqrt[f])])/4
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 6038
```
Int[(F_)^(u_)*Sinh[v_]^(n_.), x_Symbol] :> Int[ExpandTrigToExp[F^u, Sinh[v]
^n, x], x] /; FreeQ[F, x] && (LinearQ[u, x] || PolyQ[u, x, 2]) && (LinearQ[
v, x] || PolyQ[v, x, 2]) && IGtQ[n, 0]
```

Maple [A] (verified)

Time = 0.14 (sec) , antiderivative size = 100, normalized size of antiderivative = 0.91

method	result	size
risch	$-\frac{\operatorname{erf}\left(-\sqrt{-f}\, x+\frac{\ln(f) b}{2\sqrt{-f}}\right)\sqrt{\pi}\, f^{a} \mathrm{e}^{-\frac{b^{2} \ln(f)^{2}-4 d f}{4 f}}}{4\sqrt{-f}}+\frac{\operatorname{erf}\left(-\sqrt{f}\, x+\frac{\ln(f) b}{2\sqrt{f}}\right)\sqrt{\pi}\, f^{a} \mathrm{e}^{\frac{b^{2} \ln(f)^{2}-4 d f}{4 f}}}{4\sqrt{f}}$	100

input
```
int(f^(b*x+a)*sinh(f*x^2+d),x,method=_RETURNVERBOSE)
```

output
```
-1/4*erf(-(-f)^(1/2)*x+1/2*ln(f)*b/(-f)^(1/2))/(-f)^(1/2)*Pi^(1/2)*f^a*exp
(-1/4*(b^2*ln(f)^2-4*d*f)/f)+1/4*erf(-f^(1/2)*x+1/2*ln(f)*b/f^(1/2))/f^(1/
2)*Pi^(1/2)*f^a*exp(1/4*(b^2*ln(f)^2-4*d*f)/f)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 213 vs. 2(84) = 168.

Time = 0.09 (sec) , antiderivative size = 213, normalized size of antiderivative = 1.94

$$\int f^{a+bx}\sinh\left(d+fx^2\right)\,dx = -\frac{\sqrt{\pi}\sqrt{-f}\cosh\left(\frac{b^2\log(f)^2-4\,af\log(f)-4\,df}{4\,f}\right)\operatorname{erf}\left(\frac{(2\,fx+b\log(f))\sqrt{-f}}{2\,f}\right)-\sqrt{\pi}\sqrt{f}\cosh\left(\frac{b^2\log(f)^2+4\,af\log(f)-4\,df}{4\,f}\right)\operatorname{er}$$

input
```
integrate(f^(b*x+a)*sinh(f*x^2+d),x, algorithm="fricas")
```

output
```
-1/4*(sqrt(pi)*sqrt(-f)*cosh(1/4*(b^2*log(f)^2 - 4*a*f*log(f) - 4*d*f)/f)*
erf(1/2*(2*f*x + b*log(f))*sqrt(-f)/f) - sqrt(pi)*sqrt(f)*cosh(1/4*(b^2*lo
g(f)^2 + 4*a*f*log(f) - 4*d*f)/f)*erf(-1/2*(2*f*x - b*log(f))/sqrt(f)) - s
qrt(pi)*sqrt(f)*erf(-1/2*(2*f*x - b*log(f))/sqrt(f))*sinh(1/4*(b^2*log(f)^
2 + 4*a*f*log(f) - 4*d*f)/f) - sqrt(pi)*sqrt(-f)*erf(1/2*(2*f*x + b*log(f)
)*sqrt(-f)/f)*sinh(1/4*(b^2*log(f)^2 - 4*a*f*log(f) - 4*d*f)/f))/f
```

Sympy [F]

$$\int f^{a+bx}\sinh\left(d+fx^2\right)\,dx=\int f^{a+bx}\sinh\left(d+fx^2\right)dx$$

input
```
integrate(f**(b*x+a)*sinh(f*x**2+d),x)
```

output
```
Integral(f**(a + b*x)*sinh(d + f*x**2), x)
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 90, normalized size of antiderivative = 0.82

$$\int f^{a+bx}\sinh\left(d+fx^2\right)\,dx=-\frac{1}{4}\sqrt{\pi}f^{a-\frac{1}{2}}\operatorname{erf}\left(\sqrt{f}x-\frac{b\log\left(f\right)}{2\sqrt{f}}\right)e^{\left(\frac{b^2\log(f)^2}{4f}-d\right)}+\frac{\sqrt{\pi}f^a\operatorname{erf}\left(\sqrt{-f}x-\frac{b\log(f)}{2\sqrt{-f}}\right)e^{\left(-\frac{b^2\log(f)^2}{4f}+d\right)}}{4\sqrt{-f}}$$

input
```
integrate(f^(b*x+a)*sinh(f*x^2+d),x, algorithm="maxima")
```

output
```
-1/4*sqrt(pi)*f^(a - 1/2)*erf(sqrt(f)*x - 1/2*b*log(f)/sqrt(f))*e^(1/4*b^2
*log(f)^2/f - d) + 1/4*sqrt(pi)*f^a*erf(sqrt(-f)*x - 1/2*b*log(f)/sqrt(-f)
)*e^(-1/4*b^2*log(f)^2/f + d)/sqrt(-f)
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 106, normalized size of antiderivative = 0.96

$$\int f^{a+bx} \sinh \left(d+fx^2\right) dx = \frac{\sqrt{\pi}\,\mathrm{erf}\left(-\frac{1}{2}\sqrt{f}\left(2x-\frac{b\log(f)}{f}\right)\right)e^{\left(\frac{b^2\log(f)^2+4af\log(f)-4df}{4f}\right)}}{4\sqrt{f}} - \frac{\sqrt{\pi}\,\mathrm{erf}\left(-\frac{1}{2}\sqrt{-f}\left(2x+\frac{b\log(f)}{f}\right)\right)e^{\left(-\frac{b^2\log(f)^2-4af\log(f)-4df}{4f}\right)}}{4\sqrt{-f}}$$

input
```
integrate(f^(b*x+a)*sinh(f*x^2+d),x, algorithm="giac")
```

output
```
1/4*sqrt(pi)*erf(-1/2*sqrt(f)*(2*x - b*log(f)/f))*e^(1/4*(b^2*log(f)^2 + 4
*a*f*log(f) - 4*d*f)/f)/sqrt(f) - 1/4*sqrt(pi)*erf(-1/2*sqrt(-f)*(2*x + b*
log(f)/f))*e^(-1/4*(b^2*log(f)^2 - 4*a*f*log(f) - 4*d*f)/f)/sqrt(-f)
```

Mupad [F(-1)]

Timed out.

$$\int f^{a+bx} \sinh \left(d+fx^2\right) dx = \int f^{a+b\,x}\,\sinh\left(f\,x^2+d\right) dx$$

input
```
int(f^(a + b*x)*sinh(d + f*x^2),x)
```

output
```
int(f^(a + b*x)*sinh(d + f*x^2), x)
```

Reduce [F]

$$\int f^{a+bx}\sinh\left(d+fx^2\right)\,dx=f^a\left(\int f^{bx}\sinh\left(f\,x^2+d\right)dx\right)$$

input
```
int(f^(b*x+a)*sinh(f*x^2+d),x)
```

output
```
f**a*int(f**(b*x)*sinh(d + f*x**2),x)
```

3.343 $\int f^{a+bx} \sinh^2(d + fx^2)\, dx$

Optimal result

Integrand size = 18, antiderivative size = 148

$$\int f^{a+bx}\sinh^2\left(d+fx^2\right)\, dx = \frac{1}{8}e^{-2d+\frac{b^2\log^2(f)}{8f}}f^{-\frac{1}{2}+a}\sqrt{\frac{\pi}{2}}\text{erf}\left(\frac{4fx-b\log(f)}{2\sqrt{2}\sqrt{f}}\right) + \frac{1}{8}e^{2d-\frac{b^2\log^2(f)}{8f}}f^{-\frac{1}{2}+a}\sqrt{\frac{\pi}{2}}\text{erfi}\left(\frac{4fx+b\log(f)}{2\sqrt{2}\sqrt{f}}\right) - \frac{f^{a+bx}}{2b\log(f)}$$

output
```
1/16*exp(-2*d+1/8*b^2*ln(f)^2/f)*f^(-1/2+a)*2^(1/2)*Pi^(1/2)*erf(1/4*(4*f*
x-b*ln(f))*2^(1/2)/f^(1/2))+1/16*exp(2*d-1/8*b^2*ln(f)^2/f)*f^(-1/2+a)*2^(
1/2)*Pi^(1/2)*erfi(1/4*(4*f*x+b*ln(f))*2^(1/2)/f^(1/2))-1/2*f^(b*x+a)/b/ln
(f)
```

Mathematica [A] (verified)

Time = 0.51 (sec) , antiderivative size = 149, normalized size of antiderivative = 1.01

$$\int f^{a+bx}\sinh^2\left(d+fx^2\right)\,dx=\frac{1}{16}f^a\left(-\frac{8f^{bx}}{b\log(f)}+\frac{e^{\frac{b^2\log^2(f)}{8f}}\sqrt{2\pi}\,\mathrm{erf}\left(\frac{4fx-b\log(f)}{2\sqrt{2}\sqrt{f}}\right)(\cosh(2d)-\sinh(2d))}{\sqrt{f}}+\frac{e^{-\frac{b^2\log^2(f)}{8f}}\sqrt{2\pi}\,\mathrm{erfi}\left(\frac{4fx+b\log(f)}{2\sqrt{2}\sqrt{f}}\right)(\cosh(2d)+\sinh(2d))}{\sqrt{f}}\right)$$

input
```
Integrate[f^(a + b*x)*Sinh[d + f*x^2]^2,x]
```

output
```
(f^a*((-8*f^(b*x))/(b*Log[f]) + (E^((b^2*Log[f]^2)/(8*f))*Sqrt[2*Pi]*Erf[(
4*f*x - b*Log[f])/(2*Sqrt[2]*Sqrt[f])]*(Cosh[2*d] - Sinh[2*d]))/Sqrt[f] +
(Sqrt[2*Pi]*Erfi[(4*f*x + b*Log[f])/(2*Sqrt[2]*Sqrt[f])]*(Cosh[2*d] + Sinh
[2*d]))/(E^((b^2*Log[f]^2)/(8*f))*Sqrt[f])))/16
```

Rubi [A] (verified)

Time = 0.42 (sec) , antiderivative size = 148, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {6038, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int f^{a+bx}\sinh^2\left(d+fx^2\right)\,dx$$

$$\Big\downarrow \text{6038}$$

$$\int \left(\frac{1}{4}e^{-2d-2fx^2}f^{a+bx}+\frac{1}{4}e^{2d+2fx^2}f^{a+bx}-\frac{1}{2}f^{a+bx}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{8}\sqrt{\frac{\pi}{2}}f^{a-\frac{1}{2}}e^{\frac{b^2\log^2(f)}{8f}-2d}\text{erf}\left(\frac{4fx-b\log(f)}{2\sqrt{2}\sqrt{f}}\right)+\frac{1}{8}\sqrt{\frac{\pi}{2}}f^{a-\frac{1}{2}}e^{2d-\frac{b^2\log^2(f)}{8f}}\text{erfi}\left(\frac{b\log(f)+4fx}{2\sqrt{2}\sqrt{f}}\right)-\frac{f^{a+bx}}{2b\log(f)}$$

input
```
Int[f^(a + b*x)*Sinh[d + f*x^2]^2,x]
```

output
```
(E^(-2*d + (b^2*Log[f]^2)/(8*f))*f^(-1/2 + a)*Sqrt[Pi/2]*Erf[(4*f*x - b*Lo
g[f])/(2*Sqrt[2]*Sqrt[f])])/8 + (E^(2*d - (b^2*Log[f]^2)/(8*f))*f^(-1/2 +
a)*Sqrt[Pi/2]*Erfi[(4*f*x + b*Log[f])/(2*Sqrt[2]*Sqrt[f])])/8 - f^(a + b*x
)/(2*b*Log[f])
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 6038
```
Int[(F_)^(u_)*Sinh[v_]^(n_.), x_Symbol] :> Int[ExpandTrigToExp[F^u, Sinh[v]
^n, x], x] /; FreeQ[F, x] && (LinearQ[u, x] || PolyQ[u, x, 2]) && (LinearQ[
v, x] || PolyQ[v, x, 2]) && IGtQ[n, 0]
```

Maple [A] (verified)

Time = 0.35 (sec) , antiderivative size = 126, normalized size of antiderivative = 0.85

method	result	size
risch	$-\frac{\text{erf}\left(-\sqrt{2}\,\sqrt{f}\,x+\frac{b\ln(f)\sqrt{2}}{4\sqrt{f}}\right)\sqrt{2}\,\sqrt{\pi}\,f^{a}\,\mathrm{e}^{\frac{b^{2}\ln(f)^{2}-16df}{8f}}}{16\sqrt{f}}-\frac{\text{erf}\left(-\sqrt{-2f}\,x+\frac{b\ln(f)}{2\sqrt{-2f}}\right)\sqrt{\pi}\,f^{a}\,\mathrm{e}^{-\frac{b^{2}\ln(f)^{2}-16df}{8f}}}{8\sqrt{-2f}}-\frac{f^{a}f^{bx}}{2b\ln(f)}$	126

input
```
int(f^(b*x+a)*sinh(f*x^2+d)^2,x,method=_RETURNVERBOSE)
```

output
```
-1/16*erf(-2^(1/2)*f^(1/2)*x+1/4*b*ln(f)*2^(1/2)/f^(1/2))/f^(1/2)*2^(1/2)*
Pi^(1/2)*f^a*exp(1/8*(b^2*ln(f)^2-16*d*f)/f)-1/8*erf(-(-2*f)^(1/2)*x+1/2*b
*ln(f)/(-2*f)^(1/2))/(-2*f)^(1/2)*Pi^(1/2)*f^a*exp(-1/8*(b^2*ln(f)^2-16*d*
f)/f)-1/2*f^a*f^(b*x)/b/ln(f)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 278 vs. $2(114) = 228$.

Time = 0.09 (sec) , antiderivative size = 278, normalized size of antiderivative = 1.88

$$\int f^{a+bx} \sinh^2\left(d+fx^2\right) dx = -\frac{\sqrt{2}\sqrt{\pi}b\sqrt{-f}\cosh\left(\frac{b^2\log(f)^2-8\,af\log(f)-16\,df}{8\,f}\right)\operatorname{erf}\left(\frac{\sqrt{2}(4\,fx+b\log(f))\sqrt{-f}}{4\,f}\right)\log(f)+\sqrt{2}\sqrt{\pi}b\sqrt{f}\cosh\left(\frac{b^2\log(f}{}\right.}{}$$

input
```
integrate(f^(b*x+a)*sinh(f*x^2+d)^2,x, algorithm="fricas")
```

output
```
-1/16*(sqrt(2)*sqrt(pi)*b*sqrt(-f)*cosh(1/8*(b^2*log(f)^2 - 8*a*f*log(f) -
 16*d*f)/f)*erf(1/4*sqrt(2)*(4*f*x + b*log(f))*sqrt(-f)/f)*log(f) + sqrt(2
)*sqrt(pi)*b*sqrt(f)*cosh(1/8*(b^2*log(f)^2 + 8*a*f*log(f) - 16*d*f)/f)*er
f(-1/4*sqrt(2)*(4*f*x - b*log(f))/sqrt(f))*log(f) + sqrt(2)*sqrt(pi)*b*sqr
t(f)*erf(-1/4*sqrt(2)*(4*f*x - b*log(f))/sqrt(f))*log(f)*sinh(1/8*(b^2*log
(f)^2 + 8*a*f*log(f) - 16*d*f)/f) - sqrt(2)*sqrt(pi)*b*sqrt(-f)*erf(1/4*sq
rt(2)*(4*f*x + b*log(f))*sqrt(-f)/f)*log(f)*sinh(1/8*(b^2*log(f)^2 - 8*a*f
*log(f) - 16*d*f)/f) + 8*f*cosh((b*x + a)*log(f)) + 8*f*sinh((b*x + a)*log
(f)))/(b*f*log(f))
```

Sympy [F]

$$\int f^{a+bx} \sinh^2\left(d+fx^2\right) dx = \int f^{a+bx} \sinh^2\left(d+fx^2\right) dx$$

input
```
integrate(f**(b*x+a)*sinh(f*x**2+d)**2,x)
```

output
```
Integral(f**(a + b*x)*sinh(d + f*x**2)**2, x)
```

Maxima [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 127, normalized size of antiderivative = 0.86

$$\int f^{a+bx}\sinh^2\left(d+fx^2\right)\,dx=\frac{\sqrt{2}\sqrt{\pi}f^a\operatorname{erf}\left(\sqrt{2}\sqrt{f}x-\frac{\sqrt{2}b\log(f)}{4\sqrt{f}}\right)e^{\left(\frac{b^2\log(f)^2}{8f}-2d\right)}}{16\sqrt{f}}+\frac{\sqrt{2}\sqrt{\pi}f^a\operatorname{erf}\left(\sqrt{2}\sqrt{-f}x-\frac{\sqrt{2}b\log(f)}{4\sqrt{-f}}\right)e^{\left(-\frac{b^2\log(f)^2}{8f}+2d\right)}}{16\sqrt{-f}}-\frac{f^{bx+a}}{2b\log(f)}$$

input
```
integrate(f^(b*x+a)*sinh(f*x^2+d)^2,x, algorithm="maxima")
```

output
```
1/16*sqrt(2)*sqrt(pi)*f^a*erf(sqrt(2)*sqrt(f)*x - 1/4*sqrt(2)*b*log(f)/sqr
t(f))*e^(1/8*b^2*log(f)^2/f - 2*d)/sqrt(f) + 1/16*sqrt(2)*sqrt(pi)*f^a*erf
(sqrt(2)*sqrt(-f)*x - 1/4*sqrt(2)*b*log(f)/sqrt(-f))*e^(-1/8*b^2*log(f)^2/
f + 2*d)/sqrt(-f) - 1/2*f^(b*x + a)/(b*log(f))
```

Giac [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.14 (sec) , antiderivative size = 356, normalized size of antiderivative = 2.41

$$\int f^{a+bx}\sinh^2\left(d+fx^2\right)\,dx=\text{Too large to display}$$

input
```
integrate(f^(b*x+a)*sinh(f*x^2+d)^2,x, algorithm="giac")
```

output
```
-1/16*sqrt(2)*sqrt(pi)*erf(-1/4*sqrt(2)*sqrt(f)*(4*x - b*log(f)/f))*e^(1/8
*(b^2*log(f)^2 + 8*a*f*log(f) - 16*d*f)/f)/sqrt(f) - 1/16*sqrt(2)*sqrt(pi)
*erf(-1/4*sqrt(2)*sqrt(-f)*(4*x + b*log(f)/f))*e^(-1/8*(b^2*log(f)^2 - 8*a
*f*log(f) - 16*d*f)/f)/sqrt(-f) - (2*b*cos(-1/2*pi*b*x*sgn(f) + 1/2*pi*b*x
 - 1/2*pi*a*sgn(f) + 1/2*pi*a)*log(abs(f))/(4*b^2*log(abs(f))^2 + (pi*b*sg
n(f) - pi*b)^2) - (pi*b*sgn(f) - pi*b)*sin(-1/2*pi*b*x*sgn(f) + 1/2*pi*b*x
 - 1/2*pi*a*sgn(f) + 1/2*pi*a)/(4*b^2*log(abs(f))^2 + (pi*b*sgn(f) - pi*b)
^2))*e^(b*x*log(abs(f)) + a*log(abs(f))) + I*(-I*e^(1/2*I*pi*b*x*sgn(f) -
1/2*I*pi*b*x + 1/2*I*pi*a*sgn(f) - 1/2*I*pi*a)/(2*I*pi*b*sgn(f) - 2*I*pi*b
 + 4*b*log(abs(f))) + I*e^(-1/2*I*pi*b*x*sgn(f) + 1/2*I*pi*b*x - 1/2*I*pi*
a*sgn(f) + 1/2*I*pi*a)/(-2*I*pi*b*sgn(f) + 2*I*pi*b + 4*b*log(abs(f))))*e^
(b*x*log(abs(f)) + a*log(abs(f)))
```

Mupad [F(-1)]

Timed out.

$$\int f^{a+bx} \sinh^2(d+fx^2)\, dx = \int f^{a+b\,x}\,\sinh\left(f\,x^2+d\right)^2 dx$$

input
```
int(f^(a + b*x)*sinh(d + f*x^2)^2,x)
```

output
```
int(f^(a + b*x)*sinh(d + f*x^2)^2, x)
```

Reduce [F]

$$\int f^{a+bx} \sinh^2(d+fx^2)\, dx = f^a \left(\int f^{bx} \sinh\left(f\,x^2+d\right)^2 dx\right)$$

input
```
int(f^(b*x+a)*sinh(f*x^2+d)^2,x)
```

output
```
f**a*int(f**(b*x)*sinh(d + f*x**2)**2,x)
```

3.344 $\int f^{a+bx}\sinh^3(d+fx^2)\,dx$

Optimal result

Integrand size = 18, antiderivative size = 239

$$\begin{aligned}\int f^{a+bx}\sinh^3(d+fx^2)\,dx =& \frac{3}{16}e^{-d+\frac{b^2\log^2(f)}{4f}}f^{-\frac{1}{2}+a}\sqrt{\pi}\,\mathrm{erf}\left(\frac{2fx-b\log(f)}{2\sqrt{f}}\right)\\ &-\frac{1}{16}e^{-3d+\frac{b^2\log^2(f)}{12f}}f^{-\frac{1}{2}+a}\sqrt{\frac{\pi}{3}}\,\mathrm{erf}\left(\frac{6fx-b\log(f)}{2\sqrt{3}\sqrt{f}}\right)\\ &-\frac{3}{16}e^{d-\frac{b^2\log^2(f)}{4f}}f^{-\frac{1}{2}+a}\sqrt{\pi}\,\mathrm{erfi}\left(\frac{2fx+b\log(f)}{2\sqrt{f}}\right)\\ &+\frac{1}{16}e^{3d-\frac{b^2\log^2(f)}{12f}}f^{-\frac{1}{2}+a}\sqrt{\frac{\pi}{3}}\,\mathrm{erfi}\left(\frac{6fx+b\log(f)}{2\sqrt{3}\sqrt{f}}\right)\end{aligned}$$

output
```
3/16*exp(-d+1/4*b^2*ln(f)^2/f)*f^(-1/2+a)*Pi^(1/2)*erf(1/2*(2*f*x-b*ln(f))
/f^(1/2))-1/48*exp(-3*d+1/12*b^2*ln(f)^2/f)*f^(-1/2+a)*3^(1/2)*Pi^(1/2)*er
f(1/6*(6*f*x-b*ln(f))*3^(1/2)/f^(1/2))-3/16*exp(d-1/4*b^2*ln(f)^2/f)*f^(-1
/2+a)*Pi^(1/2)*erfi(1/2*(2*f*x+b*ln(f))/f^(1/2))+1/48*exp(3*d-1/12*b^2*ln(
f)^2/f)*f^(-1/2+a)*3^(1/2)*Pi^(1/2)*erfi(1/6*(6*f*x+b*ln(f))*3^(1/2)/f^(1/
2))
```

Mathematica [A] (verified)

Time = 0.28 (sec) , antiderivative size = 287, normalized size of antiderivative = 1.20

$$\int f^{a+bx}\sinh^3\left(d+fx^2\right)\,dx = \frac{1}{16}e^{-\frac{b^2\log^2(f)}{4f}}f^{-\frac{1}{2}+a}\sqrt{\frac{\pi}{3}}\left(-3\sqrt{3}\cosh(d)\text{erfi}\left(\frac{2fx+b\log(f)}{2\sqrt{f}}\right)\right. + e^{\frac{b^2\log^2(f)}{6f}}\cosh(3d)\text{erfi}\left(\frac{6fx+b\log(f)}{2\sqrt{3}\sqrt{f}}\right) + 3\sqrt{3}e^{\frac{b^2\log^2(f)}{2f}}\text{erf}\left(\frac{2fx-b\log(f)}{2\sqrt{f}}\right)(\cosh(d)-\sinh(d)) - 3\sqrt{3}\text{erfi}\left(\frac{2fx+b\log(f)}{2\sqrt{f}}\right)\sinh(d) - e^{\frac{b^2\log^2(f)}{3f}}\text{erf}\left(\frac{6fx-b\log(f)}{2\sqrt{3}\sqrt{f}}\right)(\cosh(3d)-\sinh(3d)) \left. + e^{\frac{b^2\log^2(f)}{6f}}\text{erfi}\left(\frac{6fx+b\log(f)}{2\sqrt{3}\sqrt{f}}\right)\sinh(3d)\right)$$

input
```
Integrate[f^(a + b*x)*Sinh[d + f*x^2]^3,x]
```

output
```
(f^(-1/2 + a)*Sqrt[Pi/3]*(-3*Sqrt[3]*Cosh[d]*Erfi[(2*f*x + b*Log[f])/(2*Sq
rt[f])] + E^((b^2*Log[f]^2)/(6*f))*Cosh[3*d]*Erfi[(6*f*x + b*Log[f])/(2*Sq
rt[3]*Sqrt[f])] + 3*Sqrt[3]*E^((b^2*Log[f]^2)/(2*f))*Erf[(2*f*x - b*Log[f]
)/(2*Sqrt[f])]*(Cosh[d] - Sinh[d]) - 3*Sqrt[3]*Erfi[(2*f*x + b*Log[f])/(2*
Sqrt[f])]*Sinh[d] - E^((b^2*Log[f]^2)/(3*f))*Erf[(6*f*x - b*Log[f])/(2*Sqr
t[3]*Sqrt[f])]*(Cosh[3*d] - Sinh[3*d]) + E^((b^2*Log[f]^2)/(6*f))*Erfi[(6*
f*x + b*Log[f])/(2*Sqrt[3]*Sqrt[f])]*Sinh[3*d]))/(16*E^((b^2*Log[f]^2)/(4*
f)))
```

Rubi [A] (verified)

Time = 0.55 (sec) , antiderivative size = 239, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.111$, Rules used = {6038, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int f^{a+bx} \sinh^3\left(d+fx^2\right) dx$$

$$\Big\downarrow \text{6038}$$

$$\int \left(-\frac{1}{8}e^{-3d-3fx^2}f^{a+bx}+\frac{3}{8}e^{-d-fx^2}f^{a+bx}-\frac{3}{8}e^{d+fx^2}f^{a+bx}+\frac{1}{8}e^{3d+3fx^2}f^{a+bx}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{3}{16}\sqrt{\pi}f^{a-\frac{1}{2}}e^{\frac{b^2\log^2(f)}{4f}-d}\text{erf}\left(\frac{2fx-b\log(f)}{2\sqrt{f}}\right)-\frac{1}{16}\sqrt{\frac{\pi}{3}}f^{a-\frac{1}{2}}e^{\frac{b^2\log^2(f)}{12f}-3d}\text{erf}\left(\frac{6fx-b\log(f)}{2\sqrt{3}\sqrt{f}}\right)-\frac{3}{16}\sqrt{\pi}f^{a-\frac{1}{2}}e^{d-\frac{b^2\log^2(f)}{4f}}\text{erfi}\left(\frac{b\log(f)+2fx}{2\sqrt{f}}\right)+\frac{1}{16}\sqrt{\frac{\pi}{3}}f^{a-\frac{1}{2}}e^{3d-\frac{b^2\log^2(f)}{12f}}\text{erfi}\left(\frac{b\log(f)+6fx}{2\sqrt{3}\sqrt{f}}\right)$$

input
```
Int[f^(a + b*x)*Sinh[d + f*x^2]^3,x]
```

output
```
(3*E^(-d + (b^2*Log[f]^2)/(4*f))*f^(-1/2 + a)*Sqrt[Pi]*Erf[(2*f*x - b*Log[
f])/(2*Sqrt[f])])/16 - (E^(-3*d + (b^2*Log[f]^2)/(12*f))*f^(-1/2 + a)*Sqrt
[Pi/3]*Erf[(6*f*x - b*Log[f])/(2*Sqrt[3]*Sqrt[f])])/16 - (3*E^(d - (b^2*Lo
g[f]^2)/(4*f))*f^(-1/2 + a)*Sqrt[Pi]*Erfi[(2*f*x + b*Log[f])/(2*Sqrt[f])])
/16 + (E^(3*d - (b^2*Log[f]^2)/(12*f))*f^(-1/2 + a)*Sqrt[Pi/3]*Erfi[(6*f*x
 + b*Log[f])/(2*Sqrt[3]*Sqrt[f])])/16
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 6038
```
Int[(F_)^(u_)*Sinh[v_]^(n_.), x_Symbol] :> Int[ExpandTrigToExp[F^u, Sinh[v]
^n, x], x] /; FreeQ[F, x] && (LinearQ[u, x] || PolyQ[u, x, 2]) && (LinearQ[
v, x] || PolyQ[v, x, 2]) && IGtQ[n, 0]
```

Maple [A] (verified)

Time = 0.89 (sec) , antiderivative size = 207, normalized size of antiderivative = 0.87

method	result
risch	$-\frac{\operatorname{erf}\left(-\sqrt{-3f}\,x+\frac{\ln(f)b}{2\sqrt{-3f}}\right)\sqrt{\pi}\,f^{a}\mathrm{e}^{-\frac{b^{2}\ln(f)^{2}-36df}{12f}}}{16\sqrt{-3f}}+\frac{\operatorname{erf}\left(-\sqrt{3}\,\sqrt{f}\,x+\frac{\ln(f)b\sqrt{3}}{6\sqrt{f}}\right)\sqrt{3}\,\sqrt{\pi}\,f^{a}\mathrm{e}^{\frac{b^{2}\ln(f)^{2}-36df}{12f}}}{48\sqrt{f}}-\frac{3\operatorname{erf}\left(-\sqrt{f}\,x+\frac{\ln(}{2\sqrt{}}\right.}{}$

input
```
int(f^(b*x+a)*sinh(f*x^2+d)^3,x,method=_RETURNVERBOSE)
```

output
```
-1/16*erf(-(-3*f)^(1/2)*x+1/2*ln(f)*b/(-3*f)^(1/2))/(-3*f)^(1/2)*Pi^(1/2)*
f^a*exp(-1/12*(b^2*ln(f)^2-36*d*f)/f)+1/48*erf(-3^(1/2)*f^(1/2)*x+1/6*ln(f
)*b*3^(1/2)/f^(1/2))/f^(1/2)*3^(1/2)*Pi^(1/2)*f^a*exp(1/12*(b^2*ln(f)^2-36
*d*f)/f)-3/16*erf(-f^(1/2)*x+1/2*ln(f)*b/f^(1/2))/f^(1/2)*Pi^(1/2)*f^a*exp
(1/4*(b^2*ln(f)^2-4*d*f)/f)+3/16*erf(-(-f)^(1/2)*x+1/2*ln(f)*b/(-f)^(1/2))
/(-f)^(1/2)*Pi^(1/2)*f^a*exp(-1/4*(b^2*ln(f)^2-4*d*f)/f)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 445 vs. 2(181) = 362.

Time = 0.09 (sec) , antiderivative size = 445, normalized size of antiderivative = 1.86

$$\int f^{a+bx}\sinh^{3}\left(d+fx^{2}\right)\,dx = -\frac{\sqrt{3}\sqrt{\pi}\sqrt{-f}\cosh\left(\frac{b^{2}\log(f)^{2}-12\,af\log(f)-36\,df}{12\,f}\right)\operatorname{erf}\left(\frac{\sqrt{3}(6\,fx+b\log(f))\sqrt{-f}}{6\,f}\right)-\sqrt{3}\sqrt{\pi}\sqrt{f}\cosh\left(\frac{b^{2}\log(f)^{2}+12\,af\log}{12\,f}\right.}{}$$

input
```
integrate(f^(b*x+a)*sinh(f*x^2+d)^3,x, algorithm="fricas")
```

output
```
-1/48*(sqrt(3)*sqrt(pi)*sqrt(-f)*cosh(1/12*(b^2*log(f)^2 - 12*a*f*log(f) -
 36*d*f)/f)*erf(1/6*sqrt(3)*(6*f*x + b*log(f))*sqrt(-f)/f) - sqrt(3)*sqrt(
pi)*sqrt(f)*cosh(1/12*(b^2*log(f)^2 + 12*a*f*log(f) - 36*d*f)/f)*erf(-1/6*
sqrt(3)*(6*f*x - b*log(f))/sqrt(f)) - sqrt(3)*sqrt(pi)*sqrt(f)*erf(-1/6*sq
rt(3)*(6*f*x - b*log(f))/sqrt(f))*sinh(1/12*(b^2*log(f)^2 + 12*a*f*log(f)
- 36*d*f)/f) - sqrt(3)*sqrt(pi)*sqrt(-f)*erf(1/6*sqrt(3)*(6*f*x + b*log(f)
)*sqrt(-f)/f)*sinh(1/12*(b^2*log(f)^2 - 12*a*f*log(f) - 36*d*f)/f) - 9*sqr
t(pi)*sqrt(-f)*cosh(1/4*(b^2*log(f)^2 - 4*a*f*log(f) - 4*d*f)/f)*erf(1/2*(
2*f*x + b*log(f))*sqrt(-f)/f) + 9*sqrt(pi)*sqrt(f)*cosh(1/4*(b^2*log(f)^2
+ 4*a*f*log(f) - 4*d*f)/f)*erf(-1/2*(2*f*x - b*log(f))/sqrt(f)) + 9*sqrt(p
i)*sqrt(f)*erf(-1/2*(2*f*x - b*log(f))/sqrt(f))*sinh(1/4*(b^2*log(f)^2 + 4
*a*f*log(f) - 4*d*f)/f) + 9*sqrt(pi)*sqrt(-f)*erf(1/2*(2*f*x + b*log(f))*s
qrt(-f)/f)*sinh(1/4*(b^2*log(f)^2 - 4*a*f*log(f) - 4*d*f)/f))/f
```

Sympy [F]

$$\int f^{a+bx}\sinh^3\left(d+fx^2\right)\,dx=\int f^{a+bx}\sinh^3\left(d+fx^2\right)dx$$

input
```
integrate(f**(b*x+a)*sinh(f*x**2+d)**3,x)
```

output
```
Integral(f**(a + b*x)*sinh(d + f*x**2)**3, x)
```

Maxima [A] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 200, normalized size of antiderivative = 0.84

$$\int f^{a+bx}\sinh^3\left(d+fx^2\right)\,dx=\frac{3}{16}\sqrt{\pi}f^{a-\frac{1}{2}}\operatorname{erf}\left(\sqrt{f}x-\frac{b\log(f)}{2\sqrt{f}}\right)e^{\left(\frac{b^2\log(f)^2}{4f}-d\right)}-\frac{\sqrt{3}\sqrt{\pi}f^a\operatorname{erf}\left(\sqrt{3}\sqrt{f}x-\frac{\sqrt{3}b\log(f)}{6\sqrt{f}}\right)e^{\left(\frac{b^2\log(f)^2}{12f}-3d\right)}}{48\sqrt{f}}+\frac{\sqrt{3}\sqrt{\pi}f^a\operatorname{erf}\left(\sqrt{3}\sqrt{-f}x-\frac{\sqrt{3}b\log(f)}{6\sqrt{-f}}\right)e^{\left(-\frac{b^2\log(f)^2}{12f}+3d\right)}}{48\sqrt{-f}}-\frac{3\sqrt{\pi}f^a\operatorname{erf}\left(\sqrt{-f}x-\frac{b\log(f)}{2\sqrt{-f}}\right)e^{\left(-\frac{b^2\log(f)^2}{4f}+d\right)}}{16\sqrt{-f}}$$

input
```
integrate(f^(b*x+a)*sinh(f*x^2+d)^3,x, algorithm="maxima")
```

output
```
3/16*sqrt(pi)*f^(a - 1/2)*erf(sqrt(f)*x - 1/2*b*log(f)/sqrt(f))*e^(1/4*b^2
*log(f)^2/f - d) - 1/48*sqrt(3)*sqrt(pi)*f^a*erf(sqrt(3)*sqrt(f)*x - 1/6*s
qrt(3)*b*log(f)/sqrt(f))*e^(1/12*b^2*log(f)^2/f - 3*d)/sqrt(f) + 1/48*sqrt
(3)*sqrt(pi)*f^a*erf(sqrt(3)*sqrt(-f)*x - 1/6*sqrt(3)*b*log(f)/sqrt(-f))*e
^(-1/12*b^2*log(f)^2/f + 3*d)/sqrt(-f) - 3/16*sqrt(pi)*f^a*erf(sqrt(-f)*x
- 1/2*b*log(f)/sqrt(-f))*e^(-1/4*b^2*log(f)^2/f + d)/sqrt(-f)
```

Giac [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 223, normalized size of antiderivative = 0.93

$$\int f^{a+bx}\sinh^3\left(d+fx^2\right)\,dx$$
$$=\frac{\sqrt{3}\sqrt{\pi}\,\mathrm{erf}\left(-\frac{1}{6}\sqrt{3}\sqrt{f}\left(6x-\frac{b\log(f)}{f}\right)\right)e^{\left(\frac{b^2\log(f)^2+12\,af\log(f)-36\,df}{12\,f}\right)}}{48\sqrt{f}}$$
$$-\frac{\sqrt{3}\sqrt{\pi}\,\mathrm{erf}\left(-\frac{1}{6}\sqrt{3}\sqrt{-f}\left(6x+\frac{b\log(f)}{f}\right)\right)e^{\left(-\frac{b^2\log(f)^2-12\,af\log(f)-36\,df}{12\,f}\right)}}{48\sqrt{-f}}$$
$$-\frac{3\sqrt{\pi}\,\mathrm{erf}\left(-\frac{1}{2}\sqrt{f}\left(2x-\frac{b\log(f)}{f}\right)\right)e^{\left(\frac{b^2\log(f)^2+4\,af\log(f)-4\,df}{4\,f}\right)}}{16\sqrt{f}}$$
$$+\frac{3\sqrt{\pi}\,\mathrm{erf}\left(-\frac{1}{2}\sqrt{-f}\left(2x+\frac{b\log(f)}{f}\right)\right)e^{\left(-\frac{b^2\log(f)^2-4\,af\log(f)-4\,df}{4\,f}\right)}}{16\sqrt{-f}}$$

input
```
integrate(f^(b*x+a)*sinh(f*x^2+d)^3,x, algorithm="giac")
```

output
```
1/48*sqrt(3)*sqrt(pi)*erf(-1/6*sqrt(3)*sqrt(f)*(6*x - b*log(f)/f))*e^(1/12
*(b^2*log(f)^2 + 12*a*f*log(f) - 36*d*f)/f)/sqrt(f) - 1/48*sqrt(3)*sqrt(pi
)*erf(-1/6*sqrt(3)*sqrt(-f)*(6*x + b*log(f)/f))*e^(-1/12*(b^2*log(f)^2 - 1
2*a*f*log(f) - 36*d*f)/f)/sqrt(-f) - 3/16*sqrt(pi)*erf(-1/2*sqrt(f)*(2*x -
 b*log(f)/f))*e^(1/4*(b^2*log(f)^2 + 4*a*f*log(f) - 4*d*f)/f)/sqrt(f) + 3/
16*sqrt(pi)*erf(-1/2*sqrt(-f)*(2*x + b*log(f)/f))*e^(-1/4*(b^2*log(f)^2 -
4*a*f*log(f) - 4*d*f)/f)/sqrt(-f)
```

Mupad [F(-1)]

Timed out.

$$\int f^{a+bx}\sinh^3\left(d+fx^2\right)\,dx=\int f^{a+b\,x}\,{\mathrm{sinh}\left(f\,x^2+d\right)}^3\,dx$$

input
```
int(f^(a + b*x)*sinh(d + f*x^2)^3,x)
```

output
```
int(f^(a + b*x)*sinh(d + f*x^2)^3, x)
```

Reduce [F]

$$\int f^{a+bx}\sinh^3\left(d+fx^2\right)\,dx=f^a\left(\int f^{bx}\sinh\left(f\,x^2+d\right)^3dx\right)$$

input
```
int(f^(b*x+a)*sinh(f*x^2+d)^3,x)
```

output
```
f**a*int(f**(b*x)*sinh(d + f*x**2)**3,x)
```

3.345 $\int f^{a+bx} \sinh(d+ex+fx^2)\,dx$

Optimal result

Integrand size = 19, antiderivative size = 115

$$\int f^{a+bx}\sinh\left(d+ex+fx^2\right)\,dx = -\frac{1}{4}e^{-d+\frac{(e-b\log(f))^2}{4f}}f^{-\frac{1}{2}+a}\sqrt{\pi}\,\text{erf}\left(\frac{e+2fx-b\log(f)}{2\sqrt{f}}\right) + \frac{1}{4}e^{d-\frac{(e+b\log(f))^2}{4f}}f^{-\frac{1}{2}+a}\sqrt{\pi}\,\text{erfi}\left(\frac{e+2fx+b\log(f)}{2\sqrt{f}}\right)$$

output
```
-1/4*exp(-d+1/4*(e-b*ln(f))^2/f)*f^(-1/2+a)*Pi^(1/2)*erf(1/2*(e+2*f*x-b*ln
(f))/f^(1/2))+1/4*exp(d-1/4*(e+b*ln(f))^2/f)*f^(-1/2+a)*Pi^(1/2)*erfi(1/2*
(e+2*f*x+b*ln(f))/f^(1/2))
```

Mathematica [A] (verified)

Time = 0.19 (sec) , antiderivative size = 124, normalized size of antiderivative = 1.08

$$\int f^{a+bx}\sinh\left(d+ex+fx^2\right)\,dx = \frac{1}{4}e^{-\frac{e^2+b^2\log^2(f)}{4f}}f^{a-\frac{be+f}{2f}}\sqrt{\pi}\left(-e^{\frac{e^2+b^2\log^2(f)}{2f}}\text{erf}\left(\frac{e+2fx-b\log(f)}{2\sqrt{f}}\right)(\cosh(d)-\sinh(d)) + \text{erfi}\left(\frac{e+2fx+b\log(f)}{2\sqrt{f}}\right)(\cosh(d)+\sinh(d))\right)$$

input
```
Integrate[f^(a + b*x)*Sinh[d + e*x + f*x^2],x]
```

output
```
(f^(a - (b*e + f)/(2*f))*Sqrt[Pi]*(-(E^((e^2 + b^2*Log[f]^2)/(2*f))*Erf[(e
 + 2*f*x - b*Log[f])/(2*Sqrt[f])]*(Cosh[d] - Sinh[d])) + Erfi[(e + 2*f*x +
 b*Log[f])/(2*Sqrt[f])]*(Cosh[d] + Sinh[d])))/(4*E^((e^2 + b^2*Log[f]^2)/(
4*f)))
```

Rubi [A] (verified)

Time = 0.45 (sec) , antiderivative size = 115, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.105$, Rules used = {6038, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int f^{a+bx} \sinh \left(d+ex+fx^2\right) \, dx$$

$$\Big\downarrow \text{6038}$$

$$\int \left(\frac{1}{2} f^{a+bx} e^{d+ex+fx^2}-\frac{1}{2} f^{a+bx} e^{-d-ex-fx^2}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{4}\sqrt{\pi} f^{a-\frac{1}{2}} e^{d-\frac{(b\log(f)+e)^2}{4f}} \text{erfi}\left(\frac{b\log(f)+e+2fx}{2\sqrt{f}}\right)-\frac{1}{4}\sqrt{\pi} f^{a-\frac{1}{2}} e^{\frac{(e-b\log(f))^2}{4f}-d} \text{erf}\left(\frac{-b\log(f)+e+2fx}{2\sqrt{f}}\right)$$

input
```
Int[f^(a + b*x)*Sinh[d + e*x + f*x^2],x]
```

output
```
-1/4*(E^(-d + (e - b*Log[f])^2/(4*f))*f^(-1/2 + a)*Sqrt[Pi]*Erf[(e + 2*f*x
 - b*Log[f])/(2*Sqrt[f])]) + (E^(d - (e + b*Log[f])^2/(4*f))*f^(-1/2 + a)*
Sqrt[Pi]*Erfi[(e + 2*f*x + b*Log[f])/(2*Sqrt[f])])/4
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 6038
```
Int[(F_)^(u_)*Sinh[v_]^(n_.), x_Symbol] :> Int[ExpandTrigToExp[F^u, Sinh[v]
^n, x], x] /; FreeQ[F, x] && (LinearQ[u, x] || PolyQ[u, x, 2]) && (LinearQ[
v, x] || PolyQ[v, x, 2]) && IGtQ[n, 0]
```

Maple [A] (verified)

Time = 0.18 (sec) , antiderivative size = 126, normalized size of antiderivative = 1.10

method	result
risch	$-\frac{\operatorname{erf}\left(-\sqrt{-f}\,x+\frac{e+b\ln(f)}{2\sqrt{-f}}\right)\sqrt{\pi}\,f^{a}\mathrm{e}^{-\frac{b^{2}\ln(f)^{2}+2\ln(f)be-4df+e^{2}}{4f}}}{4\sqrt{-f}}+\frac{\operatorname{erf}\left(-\sqrt{f}\,x+\frac{b\ln(f)-e}{2\sqrt{f}}\right)\sqrt{\pi}\,f^{a}\mathrm{e}^{\frac{b^{2}\ln(f)^{2}-2\ln(f)be-4df+e^{2}}{4f}}}{4\sqrt{f}}$

input
```
int(f^(b*x+a)*sinh(f*x^2+e*x+d),x,method=_RETURNVERBOSE)
```

output
```
-1/4*erf(-(-f)^(1/2)*x+1/2*(e+b*ln(f))/(-f)^(1/2))/(-f)^(1/2)*Pi^(1/2)*f^a
*exp(-1/4*(b^2*ln(f)^2+2*ln(f)*b*e-4*d*f+e^2)/f)+1/4*erf(-f^(1/2)*x+1/2*(b
*ln(f)-e)/f^(1/2))/f^(1/2)*Pi^(1/2)*f^a*exp(1/4*(b^2*ln(f)^2-2*ln(f)*b*e-4
*d*f+e^2)/f)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 253 vs. $2(90) = 180$.

Time = 0.08 (sec) , antiderivative size = 253, normalized size of antiderivative = 2.20

$$\int f^{a+bx}\sinh\left(d+ex+fx^{2}\right)\,dx = -\frac{\sqrt{\pi}\sqrt{-f}\cosh\left(\frac{b^{2}\log(f)^{2}+e^{2}-4\,df+2\,(be-2\,af)\log(f)}{4\,f}\right)\operatorname{erf}\left(\frac{(2\,fx+b\log(f)+e)\sqrt{-f}}{2\,f}\right)-\sqrt{\pi}\sqrt{f}\cosh\left(\frac{b^{2}\log(f)^{2}+e^{2}-4\,df}{4}\right.}{}$$

input
```
integrate(f^(b*x+a)*sinh(f*x^2+e*x+d),x, algorithm="fricas")
```

output
```
-1/4*(sqrt(pi)*sqrt(-f)*cosh(1/4*(b^2*log(f)^2 + e^2 - 4*d*f + 2*(b*e - 2*
a*f)*log(f))/f)*erf(1/2*(2*f*x + b*log(f) + e)*sqrt(-f)/f) - sqrt(pi)*sqrt
(f)*cosh(1/4*(b^2*log(f)^2 + e^2 - 4*d*f - 2*(b*e - 2*a*f)*log(f))/f)*erf(
-1/2*(2*f*x - b*log(f) + e)/sqrt(f)) - sqrt(pi)*sqrt(-f)*erf(1/2*(2*f*x +
b*log(f) + e)*sqrt(-f)/f)*sinh(1/4*(b^2*log(f)^2 + e^2 - 4*d*f + 2*(b*e -
2*a*f)*log(f))/f) - sqrt(pi)*sqrt(f)*erf(-1/2*(2*f*x - b*log(f) + e)/sqrt(
f))*sinh(1/4*(b^2*log(f)^2 + e^2 - 4*d*f - 2*(b*e - 2*a*f)*log(f))/f))/f
```

Sympy [F]

$$\int f^{a+bx}\sinh\left(d+ex+fx^2\right)\,dx=\int f^{a+bx}\sinh\left(d+ex+fx^2\right)dx$$

input
```
integrate(f**(b*x+a)*sinh(f*x**2+e*x+d),x)
```

output
```
Integral(f**(a + b*x)*sinh(d + e*x + f*x**2), x)
```

Maxima [A] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 102, normalized size of antiderivative = 0.89

$$\int f^{a+bx}\sinh\left(d+ex+fx^2\right)\,dx=-\frac{1}{4}\sqrt{\pi}f^{a-\frac{1}{2}}\operatorname{erf}\left(\sqrt{f}x-\frac{b\log\left(f\right)-e}{2\sqrt{f}}\right)e^{\left(-d+\frac{(b\log(f)-e)^2}{4f}\right)}+\frac{\sqrt{\pi}f^a\operatorname{erf}\left(\sqrt{-f}x-\frac{b\log(f)+e}{2\sqrt{-f}}\right)e^{\left(d-\frac{(b\log(f)+e)^2}{4f}\right)}}{4\sqrt{-f}}$$

input
```
integrate(f^(b*x+a)*sinh(f*x^2+e*x+d),x, algorithm="maxima")
```

output
```
-1/4*sqrt(pi)*f^(a - 1/2)*erf(sqrt(f)*x - 1/2*(b*log(f) - e)/sqrt(f))*e^(-
d + 1/4*(b*log(f) - e)^2/f) + 1/4*sqrt(pi)*f^a*erf(sqrt(-f)*x - 1/2*(b*log
(f) + e)/sqrt(-f))*e^(d - 1/4*(b*log(f) + e)^2/f)/sqrt(-f)
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 132, normalized size of antiderivative = 1.15

$$\int f^{a+bx}\sinh\left(d+ex+fx^2\right)\,dx$$
$$=-\frac{\sqrt{\pi}\,\mathrm{erf}\left(-\frac{1}{2}\sqrt{-f}\left(2x+\frac{b\log(f)+e}{f}\right)\right)e^{\left(-\frac{b^2\log(f)^2+2\,be\log(f)-4\,af\log(f)+e^2-4\,df}{4\,f}\right)}}{4\sqrt{-f}}$$
$$+\frac{\sqrt{\pi}\,\mathrm{erf}\left(-\frac{1}{2}\sqrt{f}\left(2x-\frac{b\log(f)-e}{f}\right)\right)e^{\left(\frac{b^2\log(f)^2-2\,be\log(f)+4\,af\log(f)+e^2-4\,df}{4\,f}\right)}}{4\sqrt{f}}$$

input
```
integrate(f^(b*x+a)*sinh(f*x^2+e*x+d),x, algorithm="giac")
```

output
```
-1/4*sqrt(pi)*erf(-1/2*sqrt(-f)*(2*x + (b*log(f) + e)/f))*e^(-1/4*(b^2*log
(f)^2 + 2*b*e*log(f) - 4*a*f*log(f) + e^2 - 4*d*f)/f)/sqrt(-f) + 1/4*sqrt(
pi)*erf(-1/2*sqrt(f)*(2*x - (b*log(f) - e)/f))*e^(1/4*(b^2*log(f)^2 - 2*b*
e*log(f) + 4*a*f*log(f) + e^2 - 4*d*f)/f)/sqrt(f)
```

Mupad [F(-1)]

Timed out.

$$\int f^{a+bx}\sinh\left(d+ex+fx^2\right)\,dx=\int f^{a+b\,x}\,\sinh\left(f\,x^2+e\,x+d\right)\,dx$$

input
```
int(f^(a + b*x)*sinh(d + e*x + f*x^2),x)
```

output
```
int(f^(a + b*x)*sinh(d + e*x + f*x^2), x)
```

Reduce [F]

$$\int f^{a+bx} \sinh \left(d+ex+fx^2\right) \, dx = f^a \left(\int f^{bx} \sinh \left(f\, x^2+ex+d\right) dx \right)$$

input
```
int(f^(b*x+a)*sinh(f*x^2+e*x+d),x)
```

output
```
f**a*int(f**(b*x)*sinh(d + e*x + f*x**2),x)
```

3.346 $\int f^{a+bx}\sinh^2\left(d+ex+fx^2\right)\,dx$

Optimal result

Integrand size = 21, antiderivative size = 161

$$\begin{aligned}\int f^{a+bx}\sinh^2\left(d+ex+fx^2\right)\,dx &= \frac{1}{8}e^{-2d+\frac{(2e-b\log(f))^2}{8f}}f^{-\frac{1}{2}+a}\sqrt{\frac{\pi}{2}}\text{erf}\left(\frac{2e+4fx-b\log(f)}{2\sqrt{2}\sqrt{f}}\right)\\&\quad+\frac{1}{8}e^{2d-\frac{(2e+b\log(f))^2}{8f}}f^{-\frac{1}{2}+a}\sqrt{\frac{\pi}{2}}\text{erfi}\left(\frac{2e+4fx+b\log(f)}{2\sqrt{2}\sqrt{f}}\right)-\frac{f^{a+bx}}{2b\log(f)}\end{aligned}$$

output
```
1/16*exp(-2*d+1/8*(2*e-b*ln(f))^2/f)*f^(-1/2+a)*2^(1/2)*Pi^(1/2)*erf(1/4*(
2*e+4*f*x-b*ln(f))*2^(1/2)/f^(1/2))+1/16*exp(2*d-1/8*(2*e+b*ln(f))^2/f)*f^
(-1/2+a)*2^(1/2)*Pi^(1/2)*erfi(1/4*(2*e+4*f*x+b*ln(f))*2^(1/2)/f^(1/2))-1/
2*f^(b*x+a)/b/ln(f)
```

Mathematica [A] (verified)

Time = 0.42 (sec) , antiderivative size = 220, normalized size of antiderivative = 1.37

$$\int f^{a+bx}\sinh^2\left(d+ex+fx^2\right)\,dx$$
$$=\frac{e^{-\frac{4e^2+b^2\log^2(f)}{8f}}f^{a-\frac{be+f}{2f}}\left(-4\sqrt{2}e^{\frac{4e^2+b^2\log^2(f)}{8f}}f^{\frac{1}{2}+b\left(\frac{e}{2f}+x\right)}+be^{\frac{4e^2+b^2\log^2(f)}{4f}}\sqrt{\pi}\mathrm{erf}\left(\frac{2e+4fx-b\log(f)}{2\sqrt{2}\sqrt{f}}\right)\log(f)(\cosh(2\right.}{8\sqrt{2}b\log(f)}$$

input
```
Integrate[f^(a + b*x)*Sinh[d + e*x + f*x^2]^2,x]
```

output
```
(f^(a - (b*e + f)/(2*f))*(-4*Sqrt[2]*E^((4*e^2 + b^2*Log[f]^2)/(8*f))*f^(1
/2 + b*(e/(2*f) + x)) + b*E^((4*e^2 + b^2*Log[f]^2)/(4*f))*Sqrt[Pi]*Erf[(2
*e + 4*f*x - b*Log[f])/(2*Sqrt[2]*Sqrt[f])]*Log[f]*(Cosh[2*d] - Sinh[2*d])
 + b*Sqrt[Pi]*Erfi[(2*e + 4*f*x + b*Log[f])/(2*Sqrt[2]*Sqrt[f])]*Log[f]*(C
osh[2*d] + Sinh[2*d])))/(8*Sqrt[2]*b*E^((4*e^2 + b^2*Log[f]^2)/(8*f))*Log[
f])
```

Rubi [A] (verified)

Time = 0.57 (sec) , antiderivative size = 161, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {6038, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int f^{a+bx}\sinh^2\left(d+ex+fx^2\right)\,dx$$
$$\Big\downarrow \text{ 6038}$$
$$\int\left(\frac{1}{4}f^{a+bx}e^{-2d-2ex-2fx^2}+\frac{1}{4}f^{a+bx}e^{2d+2ex+2fx^2}-\frac{1}{2}f^{a+bx}\right)dx$$
$$\Big\downarrow \text{ 2009}$$

$$\frac{1}{8}\sqrt{\frac{\pi}{2}}f^{a-\frac{1}{2}}e^{\frac{(2e-b\log(f))^2}{8f}-2d}\operatorname{erf}\left(\frac{-b\log(f)+2e+4fx}{2\sqrt{2}\sqrt{f}}\right)+\frac{1}{8}\sqrt{\frac{\pi}{2}}f^{a-\frac{1}{2}}e^{2d-\frac{(b\log(f)+2e)^2}{8f}}\operatorname{erfi}\left(\frac{b\log(f)+2e+4fx}{2\sqrt{2}\sqrt{f}}\right)-\frac{f^{a+bx}}{2b\log(f)}$$

input
```
Int[f^(a + b*x)*Sinh[d + e*x + f*x^2]^2,x]
```

output
```
(E^(-2*d + (2*e - b*Log[f])^2/(8*f))*f^(-1/2 + a)*Sqrt[Pi/2]*Erf[(2*e + 4*
f*x - b*Log[f])/(2*Sqrt[2]*Sqrt[f])])/8 + (E^(2*d - (2*e + b*Log[f])^2/(8*
f))*f^(-1/2 + a)*Sqrt[Pi/2]*Erfi[(2*e + 4*f*x + b*Log[f])/(2*Sqrt[2]*Sqrt[
f])])/8 - f^(a + b*x)/(2*b*Log[f])
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 6038
```
Int[(F_)^(u_)*Sinh[v_]^(n_.), x_Symbol] :> Int[ExpandTrigToExp[F^u, Sinh[v]
^n, x], x] /; FreeQ[F, x] && (LinearQ[u, x] || PolyQ[u, x, 2]) && (LinearQ[
v, x] || PolyQ[v, x, 2]) && IGtQ[n, 0]
```

Maple [A] (verified)

Time = 0.58 (sec) , antiderivative size = 158, normalized size of antiderivative = 0.98

method	result
risch	$-\frac{\operatorname{erf}\left(-\sqrt{2}\,\sqrt{f}\,x+\frac{(b\ln(f)-2e)\sqrt{2}}{4\sqrt{f}}\right)\sqrt{2}\,\sqrt{\pi}\,f^{a}\mathrm{e}^{\frac{b^{2}\ln(f)^{2}-4\ln(f)be-16df+4e^{2}}{8f}}}{16\sqrt{f}}-\frac{\operatorname{erf}\left(-\sqrt{-2f}\,x+\frac{2e+b\ln(f)}{2\sqrt{-2f}}\right)\sqrt{\pi}\,f^{a}\mathrm{e}^{-\frac{b^{2}\ln(f)^{2}+4\ln(}{8}}}{8\sqrt{-2f}}$

input
```
int(f^(b*x+a)*sinh(f*x^2+e*x+d)^2,x,method=_RETURNVERBOSE)
```

output
```
-1/16*erf(-2^(1/2)*f^(1/2)*x+1/4*(b*ln(f)-2*e)*2^(1/2)/f^(1/2))/f^(1/2)*2^
(1/2)*Pi^(1/2)*f^a*exp(1/8*(b^2*ln(f)^2-4*ln(f)*b*e-16*d*f+4*e^2)/f)-1/8*e
rf(-(-2*f)^(1/2)*x+1/2*(2*e+b*ln(f))/(-2*f)^(1/2))/(-2*f)^(1/2)*Pi^(1/2)*f
^a*exp(-1/8*(b^2*ln(f)^2+4*ln(f)*b*e-16*d*f+4*e^2)/f)-1/2*f^a*f^(b*x)/b/ln
(f)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 334 vs. $2(126) = 252$.

Time = 0.09 (sec) , antiderivative size = 334, normalized size of antiderivative = 2.07

$$\int f^{a+bx}\sinh^2\left(d+ex+fx^2\right)\,dx =$$

$$-\frac{\sqrt{2}\sqrt{\pi}b\sqrt{-f}\cosh\left(\frac{b^2\log(f)^2+4\,e^2-16\,df+4\,(be-2\,af)\log(f)}{8\,f}\right)\operatorname{erf}\left(\frac{\sqrt{2}(4\,fx+b\log(f)+2\,e)\sqrt{-f}}{4\,f}\right)\log\left(f\right)+\sqrt{2}\sqrt{\pi}b\sqrt{f}}{}$$

input
```
integrate(f^(b*x+a)*sinh(f*x^2+e*x+d)^2,x, algorithm="fricas")
```

output
```
-1/16*(sqrt(2)*sqrt(pi)*b*sqrt(-f)*cosh(1/8*(b^2*log(f)^2 + 4*e^2 - 16*d*f
 + 4*(b*e - 2*a*f)*log(f))/f)*erf(1/4*sqrt(2)*(4*f*x + b*log(f) + 2*e)*sqr
t(-f)/f)*log(f) + sqrt(2)*sqrt(pi)*b*sqrt(f)*cosh(1/8*(b^2*log(f)^2 + 4*e^
2 - 16*d*f - 4*(b*e - 2*a*f)*log(f))/f)*erf(-1/4*sqrt(2)*(4*f*x - b*log(f)
 + 2*e)/sqrt(f))*log(f) - sqrt(2)*sqrt(pi)*b*sqrt(-f)*erf(1/4*sqrt(2)*(4*f
*x + b*log(f) + 2*e)*sqrt(-f)/f)*log(f)*sinh(1/8*(b^2*log(f)^2 + 4*e^2 - 1
6*d*f + 4*(b*e - 2*a*f)*log(f))/f) + sqrt(2)*sqrt(pi)*b*sqrt(f)*erf(-1/4*s
qrt(2)*(4*f*x - b*log(f) + 2*e)/sqrt(f))*log(f)*sinh(1/8*(b^2*log(f)^2 + 4
*e^2 - 16*d*f - 4*(b*e - 2*a*f)*log(f))/f) + 8*f*cosh((b*x + a)*log(f)) +
8*f*sinh((b*x + a)*log(f)))/(b*f*log(f))
```

Sympy [F]

$$\int f^{a+bx}\sinh^2\left(d+ex+fx^2\right)\,dx=\int f^{a+bx}\sinh^2\left(d+ex+fx^2\right)dx$$

input
```
integrate(f**(b*x+a)*sinh(f*x**2+e*x+d)**2,x)
```

output
```
Integral(f**(a + b*x)*sinh(d + e*x + f*x**2)**2, x)
```

Maxima [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 143, normalized size of antiderivative = 0.89

$$\int f^{a+bx}\sinh^2\left(d+ex+fx^2\right)\,dx$$
$$=\frac{\sqrt{2}\sqrt{\pi}f^a\operatorname{erf}\left(\sqrt{2}\sqrt{-f}x-\frac{\sqrt{2}(b\log(f)+2e)}{4\sqrt{-f}}\right)e^{\left(2d-\frac{(b\log(f)+2e)^2}{8f}\right)}}{16\sqrt{-f}}+\frac{\sqrt{2}\sqrt{\pi}f^a\operatorname{erf}\left(\sqrt{2}\sqrt{f}x-\frac{\sqrt{2}(b\log(f)-2e)}{4\sqrt{f}}\right)e^{\left(-2d+\frac{(b\log(f)-2e)^2}{8f}\right)}}{16\sqrt{f}}-\frac{f^{bx+a}}{2b\log(f)}$$

input
```
integrate(f^(b*x+a)*sinh(f*x^2+e*x+d)^2,x, algorithm="maxima")
```

output
```
1/16*sqrt(2)*sqrt(pi)*f^a*erf(sqrt(2)*sqrt(-f)*x - 1/4*sqrt(2)*(b*log(f) +
 2*e)/sqrt(-f))*e^(2*d - 1/8*(b*log(f) + 2*e)^2/f)/sqrt(-f) + 1/16*sqrt(2)
*sqrt(pi)*f^a*erf(sqrt(2)*sqrt(f)*x - 1/4*sqrt(2)*(b*log(f) - 2*e)/sqrt(f)
)*e^(-2*d + 1/8*(b*log(f) - 2*e)^2/f)/sqrt(f) - 1/2*f^(b*x + a)/(b*log(f))
```

Giac [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.15 (sec) , antiderivative size = 388, normalized size of antiderivative = 2.41

$$\int f^{a+bx}\sinh^2\left(d+ex+fx^2\right)\,dx=\text{Too large to display}$$

input
```
integrate(f^(b*x+a)*sinh(f*x^2+e*x+d)^2,x, algorithm="giac")
```

output
```
-1/16*sqrt(2)*sqrt(pi)*erf(-1/4*sqrt(2)*sqrt(-f)*(4*x + (b*log(f) + 2*e)/f
))*e^(-1/8*(b^2*log(f)^2 + 4*b*e*log(f) - 8*a*f*log(f) + 4*e^2 - 16*d*f)/f
)/sqrt(-f) - 1/16*sqrt(2)*sqrt(pi)*erf(-1/4*sqrt(2)*sqrt(f)*(4*x - (b*log(
f) - 2*e)/f))*e^(1/8*(b^2*log(f)^2 - 4*b*e*log(f) + 8*a*f*log(f) + 4*e^2 -
 16*d*f)/f)/sqrt(f) - (2*b*cos(-1/2*pi*b*x*sgn(f) + 1/2*pi*b*x - 1/2*pi*a*
sgn(f) + 1/2*pi*a)*log(abs(f))/(4*b^2*log(abs(f))^2 + (pi*b*sgn(f) - pi*b)
^2) - (pi*b*sgn(f) - pi*b)*sin(-1/2*pi*b*x*sgn(f) + 1/2*pi*b*x - 1/2*pi*a*
sgn(f) + 1/2*pi*a)/(4*b^2*log(abs(f))^2 + (pi*b*sgn(f) - pi*b)^2))*e^(b*x*
log(abs(f)) + a*log(abs(f))) + I*(-I*e^(1/2*I*pi*b*x*sgn(f) - 1/2*I*pi*b*x
 + 1/2*I*pi*a*sgn(f) - 1/2*I*pi*a)/(2*I*pi*b*sgn(f) - 2*I*pi*b + 4*b*log(a
bs(f))) + I*e^(-1/2*I*pi*b*x*sgn(f) + 1/2*I*pi*b*x - 1/2*I*pi*a*sgn(f) + 1
/2*I*pi*a)/(-2*I*pi*b*sgn(f) + 2*I*pi*b + 4*b*log(abs(f))))*e^(b*x*log(abs
(f)) + a*log(abs(f)))
```

Mupad [F(-1)]

Timed out.

$$\int f^{a+bx}\sinh^2\left(d+ex+fx^2\right)\,dx=\int f^{a+b\,x}\,{\sinh\left(f\,x^2+e\,x+d\right)}^2\,dx$$

input
```
int(f^(a + b*x)*sinh(d + e*x + f*x^2)^2,x)
```

output
```
int(f^(a + b*x)*sinh(d + e*x + f*x^2)^2, x)
```

Reduce [F]

$$\int f^{a+bx}\sinh^2\left(d+ex+fx^2\right)\,dx=f^a\left(\int f^{bx}\sinh\left(f\,x^2+ex+d\right)^2dx\right)$$

input
```
int(f^(b*x+a)*sinh(f*x^2+e*x+d)^2,x)
```

output
```
f**a*int(f**(b*x)*sinh(d + e*x + f*x**2)**2,x)
```

3.347 $\int f^{a+bx}\sinh^3\left(d+ex+fx^2\right)\,dx$

Optimal result

Integrand size = 21, antiderivative size = 257

$$\begin{aligned}\int f^{a+bx}\sinh^3\left(d+ex+fx^2\right)\,dx &= \frac{3}{16}e^{-d+\frac{(e-b\log(f))^2}{4f}}f^{-\frac{1}{2}+a}\sqrt{\pi}\operatorname{erf}\left(\frac{e+2fx-b\log(f)}{2\sqrt{f}}\right)\\ &\quad-\frac{1}{16}e^{-3d+\frac{(3e-b\log(f))^2}{12f}}f^{-\frac{1}{2}+a}\sqrt{\frac{\pi}{3}}\operatorname{erf}\left(\frac{3e+6fx-b\log(f)}{2\sqrt{3}\sqrt{f}}\right)\\ &\quad-\frac{3}{16}e^{d-\frac{(e+b\log(f))^2}{4f}}f^{-\frac{1}{2}+a}\sqrt{\pi}\operatorname{erfi}\left(\frac{e+2fx+b\log(f)}{2\sqrt{f}}\right)\\ &\quad+\frac{1}{16}e^{3d-\frac{(3e+b\log(f))^2}{12f}}f^{-\frac{1}{2}+a}\sqrt{\frac{\pi}{3}}\operatorname{erfi}\left(\frac{3e+6fx+b\log(f)}{2\sqrt{3}\sqrt{f}}\right)\end{aligned}$$

output
```
3/16*exp(-d+1/4*(e-b*ln(f))^2/f)*f^(-1/2+a)*Pi^(1/2)*erf(1/2*(e+2*f*x-b*ln
(f))/f^(1/2))-1/48*exp(-3*d+1/12*(3*e-b*ln(f))^2/f)*f^(-1/2+a)*3^(1/2)*Pi^
(1/2)*erf(1/6*(3*e+6*f*x-b*ln(f))*3^(1/2)/f^(1/2))-3/16*exp(d-1/4*(e+b*ln(
f))^2/f)*f^(-1/2+a)*Pi^(1/2)*erfi(1/2*(e+2*f*x+b*ln(f))/f^(1/2))+1/48*exp(
3*d-1/12*(3*e+b*ln(f))^2/f)*f^(-1/2+a)*3^(1/2)*Pi^(1/2)*erfi(1/6*(3*e+6*f*
x+b*ln(f))*3^(1/2)/f^(1/2))
```

Mathematica [A] (verified)

Time = 0.53 (sec) , antiderivative size = 354, normalized size of antiderivative = 1.38

$$\int f^{a+bx}\sinh^3\left(d+ex+fx^2\right)\,dx$$
$$=\frac{1}{16}e^{-\frac{3e^2+b^2\log^2(f)}{4f}}f^{a-\frac{be+f}{2f}}\sqrt{\frac{\pi}{3}}\left(-3\sqrt{3}e^{\frac{e^2}{2f}}\cosh(d)\text{erfi}\left(\frac{e+2fx+b\log(f)}{2\sqrt{f}}\right)+e^{\frac{b^2\log^2(f)}{6f}}\cosh(3d)\text{erfi}\left(\frac{3e+6fx+b\log(f)}{2\sqrt{3}\sqrt{f}}\right)+3\sqrt{3}e^{\frac{2e^2+b^2\log^2(f)}{2f}}\text{erf}\left(\frac{e+2fx-b\log(f)}{2\sqrt{f}}\right)(\cosh(d)-\sinh(d))-3\sqrt{3}e^{\frac{e^2}{2f}}\text{erfi}\left(\frac{e+2fx+b\log(f)}{2\sqrt{f}}\right)\sinh(d)-e^{\frac{9e^2+2b^2\log^2(f)}{6f}}\text{erf}\left(\frac{3e+6fx-b\log(f)}{2\sqrt{3}\sqrt{f}}\right)(\cosh(3d)-\sinh(3d))+e^{\frac{b^2\log^2(f)}{6f}}\text{erfi}\left(\frac{3e+6fx+b\log(f)}{2\sqrt{3}\sqrt{f}}\right)\sinh(3d)\right)$$

input
```
Integrate[f^(a + b*x)*Sinh[d + e*x + f*x^2]^3,x]
```

output
```
(f^(a - (b*e + f)/(2*f))*Sqrt[Pi/3]*(-3*Sqrt[3]*E^(e^2/(2*f))*Cosh[d]*Erfi
[(e + 2*f*x + b*Log[f])/(2*Sqrt[f])] + E^((b^2*Log[f]^2)/(6*f))*Cosh[3*d]*
Erfi[(3*e + 6*f*x + b*Log[f])/(2*Sqrt[3]*Sqrt[f])] + 3*Sqrt[3]*E^((2*e^2 +
 b^2*Log[f]^2)/(2*f))*Erf[(e + 2*f*x - b*Log[f])/(2*Sqrt[f])]*(Cosh[d] - S
inh[d]) - 3*Sqrt[3]*E^(e^2/(2*f))*Erfi[(e + 2*f*x + b*Log[f])/(2*Sqrt[f])]
*Sinh[d] - E^((9*e^2 + 2*b^2*Log[f]^2)/(6*f))*Erf[(3*e + 6*f*x - b*Log[f])
/(2*Sqrt[3]*Sqrt[f])]*(Cosh[3*d] - Sinh[3*d]) + E^((b^2*Log[f]^2)/(6*f))*E
rfi[(3*e + 6*f*x + b*Log[f])/(2*Sqrt[3]*Sqrt[f])]*Sinh[3*d]))/(16*E^((3*e^
2 + b^2*Log[f]^2)/(4*f)))
```

Rubi [A] (verified)

Time = 0.85 (sec) , antiderivative size = 257, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {6038, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int f^{a+bx}\sinh^3\left(d+ex+fx^2\right)\,dx$$

$$\Big\downarrow \text{6038}$$

$$\int \left(\frac{3}{8}f^{a+bx}\exp\left(-3\left(d+ex+fx^2\right)+2d+2ex+2fx^2\right)-\frac{3}{8}f^{a+bx}\exp\left(-3\left(d+ex+fx^2\right)+4d+4ex+4fx^2\right)\right.$$

$$\Big\downarrow \text{2009}$$

$$\frac{3}{16}\sqrt{\pi}f^{a-\frac{1}{2}}e^{\frac{(e-b\log(f))^2}{4f}-d}\text{erf}\left(\frac{-b\log(f)+e+2fx}{2\sqrt{f}}\right)-\frac{1}{16}\sqrt{\frac{\pi}{3}}f^{a-\frac{1}{2}}e^{\frac{(3e-b\log(f))^2}{12f}-3d}\text{erf}\left(\frac{-b\log(f)+3e+6fx}{2\sqrt{3}\sqrt{f}}\right)-\frac{3}{16}\sqrt{\pi}f^{a-\frac{1}{2}}e^{d-\frac{(b\log(f)+e)^2}{4f}}\text{erfi}\left(\frac{b\log(f)+e+2fx}{2\sqrt{f}}\right)+\frac{1}{16}\sqrt{\frac{\pi}{3}}f^{a-\frac{1}{2}}e^{3d-\frac{(b\log(f)+3e)^2}{12f}}\text{erfi}\left(\frac{b\log(f)+3e+6fx}{2\sqrt{3}\sqrt{f}}\right)$$

input
```
Int[f^(a + b*x)*Sinh[d + e*x + f*x^2]^3,x]
```

output
```
(3*E^(-d + (e - b*Log[f])^2/(4*f))*f^(-1/2 + a)*Sqrt[Pi]*Erf[(e + 2*f*x -
b*Log[f])/(2*Sqrt[f])])/16 - (E^(-3*d + (3*e - b*Log[f])^2/(12*f))*f^(-1/2
 + a)*Sqrt[Pi/3]*Erf[(3*e + 6*f*x - b*Log[f])/(2*Sqrt[3]*Sqrt[f])])/16 - (
3*E^(d - (e + b*Log[f])^2/(4*f))*f^(-1/2 + a)*Sqrt[Pi]*Erfi[(e + 2*f*x + b
*Log[f])/(2*Sqrt[f])])/16 + (E^(3*d - (3*e + b*Log[f])^2/(12*f))*f^(-1/2 +
 a)*Sqrt[Pi/3]*Erfi[(3*e + 6*f*x + b*Log[f])/(2*Sqrt[3]*Sqrt[f])])/16
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 6038
```
Int[(F_)^(u_)*Sinh[v_]^(n_.), x_Symbol] :> Int[ExpandTrigToExp[F^u, Sinh[v]
^n, x], x] /; FreeQ[F, x] && (LinearQ[u, x] || PolyQ[u, x, 2]) && (LinearQ[
v, x] || PolyQ[v, x, 2]) && IGtQ[n, 0]
```

Maple [A] (verified)

Time = 1.69 (sec) , antiderivative size = 265, normalized size of antiderivative = 1.03

method	result
risch	$-\frac{\operatorname{erf}\left(-\sqrt{-3f}\,x+\frac{3e+b\ln(f)}{2\sqrt{-3f}}\right)\sqrt{\pi}\,f^{a}\mathrm{e}^{-\frac{b^{2}\ln(f)^{2}+6\ln(f)be-36df+9e^{2}}{12f}}}{16\sqrt{-3f}}+\frac{\operatorname{erf}\left(-\sqrt{3}\,\sqrt{f}\,x+\frac{(b\ln(f)-3e)\sqrt{3}}{6\sqrt{f}}\right)\sqrt{3}\,\sqrt{\pi}\,f^{a}\mathrm{e}^{\frac{b^{2}\ln(f)^{2}-6\ln(}{12}}}{48\sqrt{f}}$

input
```
int(f^(b*x+a)*sinh(f*x^2+e*x+d)^3,x,method=_RETURNVERBOSE)
```

output
```
-1/16*erf(-(-3*f)^(1/2)*x+1/2*(3*e+b*ln(f))/(-3*f)^(1/2))/(-3*f)^(1/2)*Pi^
(1/2)*f^a*exp(-1/12*(b^2*ln(f)^2+6*ln(f)*b*e-36*d*f+9*e^2)/f)+1/48*erf(-3^
(1/2)*f^(1/2)*x+1/6*(b*ln(f)-3*e)*3^(1/2)/f^(1/2))/f^(1/2)*3^(1/2)*Pi^(1/2
)*f^a*exp(1/12*(b^2*ln(f)^2-6*ln(f)*b*e-36*d*f+9*e^2)/f)-3/16*erf(-f^(1/2)
*x+1/2*(b*ln(f)-e)/f^(1/2))/f^(1/2)*Pi^(1/2)*f^a*exp(1/4*(b^2*ln(f)^2-2*ln
(f)*b*e-4*d*f+e^2)/f)+3/16*erf(-(-f)^(1/2)*x+1/2*(e+b*ln(f))/(-f)^(1/2))/(
-f)^(1/2)*Pi^(1/2)*f^a*exp(-1/4*(b^2*ln(f)^2+2*ln(f)*b*e-4*d*f+e^2)/f)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 541 vs. 2(199) = 398.

Time = 0.10 (sec) , antiderivative size = 541, normalized size of antiderivative = 2.11

$$\int f^{a+bx}\sinh^{3}\left(d+ex+fx^{2}\right)\,dx=\text{Too large to display}$$

input
```
integrate(f^(b*x+a)*sinh(f*x^2+e*x+d)^3,x, algorithm="fricas")
```

output
```
-1/48*(sqrt(3)*sqrt(pi)*sqrt(-f)*cosh(1/12*(b^2*log(f)^2 + 9*e^2 - 36*d*f
+ 6*(b*e - 2*a*f)*log(f))/f)*erf(1/6*sqrt(3)*(6*f*x + b*log(f) + 3*e)*sqrt
(-f)/f) - sqrt(3)*sqrt(pi)*sqrt(f)*cosh(1/12*(b^2*log(f)^2 + 9*e^2 - 36*d*
f - 6*(b*e - 2*a*f)*log(f))/f)*erf(-1/6*sqrt(3)*(6*f*x - b*log(f) + 3*e)/s
qrt(f)) - sqrt(3)*sqrt(pi)*sqrt(-f)*erf(1/6*sqrt(3)*(6*f*x + b*log(f) + 3*
e)*sqrt(-f)/f)*sinh(1/12*(b^2*log(f)^2 + 9*e^2 - 36*d*f + 6*(b*e - 2*a*f)*
log(f))/f) - sqrt(3)*sqrt(pi)*sqrt(f)*erf(-1/6*sqrt(3)*(6*f*x - b*log(f) +
 3*e)/sqrt(f))*sinh(1/12*(b^2*log(f)^2 + 9*e^2 - 36*d*f - 6*(b*e - 2*a*f)*
log(f))/f) - 9*sqrt(pi)*sqrt(-f)*cosh(1/4*(b^2*log(f)^2 + e^2 - 4*d*f + 2*
(b*e - 2*a*f)*log(f))/f)*erf(1/2*(2*f*x + b*log(f) + e)*sqrt(-f)/f) + 9*sq
rt(pi)*sqrt(f)*cosh(1/4*(b^2*log(f)^2 + e^2 - 4*d*f - 2*(b*e - 2*a*f)*log(
f))/f)*erf(-1/2*(2*f*x - b*log(f) + e)/sqrt(f)) + 9*sqrt(pi)*sqrt(-f)*erf(
1/2*(2*f*x + b*log(f) + e)*sqrt(-f)/f)*sinh(1/4*(b^2*log(f)^2 + e^2 - 4*d*
f + 2*(b*e - 2*a*f)*log(f))/f) + 9*sqrt(pi)*sqrt(f)*erf(-1/2*(2*f*x - b*lo
g(f) + e)/sqrt(f))*sinh(1/4*(b^2*log(f)^2 + e^2 - 4*d*f - 2*(b*e - 2*a*f)*
log(f))/f))/f
```

Sympy [F]

$$\int f^{a+bx}\sinh^3\left(d+ex+fx^2\right)\,dx=\int f^{a+bx}\sinh^3\left(d+ex+fx^2\right)dx$$

input
```
integrate(f**(b*x+a)*sinh(f*x**2+e*x+d)**3,x)
```

output
```
Integral(f**(a + b*x)*sinh(d + e*x + f*x**2)**3, x)
```

Maxima [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 228, normalized size of antiderivative = 0.89

$$\int f^{a+bx}\sinh^3\left(d+ex+fx^2\right)\,dx$$
$$=\frac{\sqrt{3}\sqrt{\pi}f^a\operatorname{erf}\left(\sqrt{3}\sqrt{-f}x-\frac{\sqrt{3}(b\log(f)+3e)}{6\sqrt{-f}}\right)e^{\left(3d-\frac{(b\log(f)+3e)^2}{12f}\right)}}{48\sqrt{-f}}$$
$$+\frac{3}{16}\sqrt{\pi}f^{a-\frac{1}{2}}\operatorname{erf}\left(\sqrt{f}x-\frac{b\log(f)-e}{2\sqrt{f}}\right)e^{\left(-d+\frac{(b\log(f)-e)^2}{4f}\right)}$$
$$-\frac{\sqrt{3}\sqrt{\pi}f^a\operatorname{erf}\left(\sqrt{3}\sqrt{f}x-\frac{\sqrt{3}(b\log(f)-3e)}{6\sqrt{f}}\right)e^{\left(-3d+\frac{(b\log(f)-3e)^2}{12f}\right)}}{48\sqrt{f}}$$
$$-\frac{3\sqrt{\pi}f^a\operatorname{erf}\left(\sqrt{-f}x-\frac{b\log(f)+e}{2\sqrt{-f}}\right)e^{\left(d-\frac{(b\log(f)+e)^2}{4f}\right)}}{16\sqrt{-f}}$$

input
```
integrate(f^(b*x+a)*sinh(f*x^2+e*x+d)^3,x, algorithm="maxima")
```

output
```
1/48*sqrt(3)*sqrt(pi)*f^a*erf(sqrt(3)*sqrt(-f)*x - 1/6*sqrt(3)*(b*log(f) +
 3*e)/sqrt(-f))*e^(3*d - 1/12*(b*log(f) + 3*e)^2/f)/sqrt(-f) + 3/16*sqrt(p
i)*f^(a - 1/2)*erf(sqrt(f)*x - 1/2*(b*log(f) - e)/sqrt(f))*e^(-d + 1/4*(b*
log(f) - e)^2/f) - 1/48*sqrt(3)*sqrt(pi)*f^a*erf(sqrt(3)*sqrt(f)*x - 1/6*s
qrt(3)*(b*log(f) - 3*e)/sqrt(f))*e^(-3*d + 1/12*(b*log(f) - 3*e)^2/f)/sqrt
(f) - 3/16*sqrt(pi)*f^a*erf(sqrt(-f)*x - 1/2*(b*log(f) + e)/sqrt(-f))*e^(d
 - 1/4*(b*log(f) + e)^2/f)/sqrt(-f)
```

Giac [A] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 281, normalized size of antiderivative = 1.09

$$\int f^{a+bx}\sinh^3\left(d+ex+fx^2\right)\,dx$$

$$= -\frac{\sqrt{3}\sqrt{\pi}\operatorname{erf}\left(-\frac{1}{6}\sqrt{3}\sqrt{-f}\left(6\,x+\frac{b\log(f)+3\,e}{f}\right)\right)e^{\left(-\frac{b^2\log(f)^2+6\,be\log(f)-12\,af\log(f)+9\,e^2-36\,df}{12\,f}\right)}}{48\,\sqrt{-f}}$$
$$+\frac{\sqrt{3}\sqrt{\pi}\operatorname{erf}\left(-\frac{1}{6}\sqrt{3}\sqrt{f}\left(6\,x-\frac{b\log(f)-3\,e}{f}\right)\right)e^{\left(\frac{b^2\log(f)^2-6\,be\log(f)+12\,af\log(f)+9\,e^2-36\,df}{12\,f}\right)}}{48\,\sqrt{f}}$$
$$+\frac{3\,\sqrt{\pi}\operatorname{erf}\left(-\frac{1}{2}\sqrt{-f}\left(2\,x+\frac{b\log(f)+e}{f}\right)\right)e^{\left(-\frac{b^2\log(f)^2+2\,be\log(f)-4\,af\log(f)+e^2-4\,df}{4\,f}\right)}}{16\,\sqrt{-f}}$$
$$-\frac{3\,\sqrt{\pi}\operatorname{erf}\left(-\frac{1}{2}\sqrt{f}\left(2\,x-\frac{b\log(f)-e}{f}\right)\right)e^{\left(\frac{b^2\log(f)^2-2\,be\log(f)+4\,af\log(f)+e^2-4\,df}{4\,f}\right)}}{16\,\sqrt{f}}$$

input
```
integrate(f^(b*x+a)*sinh(f*x^2+e*x+d)^3,x, algorithm="giac")
```

output
```
-1/48*sqrt(3)*sqrt(pi)*erf(-1/6*sqrt(3)*sqrt(-f)*(6*x + (b*log(f) + 3*e)/f
))*e^(-1/12*(b^2*log(f)^2 + 6*b*e*log(f) - 12*a*f*log(f) + 9*e^2 - 36*d*f)
/f)/sqrt(-f) + 1/48*sqrt(3)*sqrt(pi)*erf(-1/6*sqrt(3)*sqrt(f)*(6*x - (b*lo
g(f) - 3*e)/f))*e^(1/12*(b^2*log(f)^2 - 6*b*e*log(f) + 12*a*f*log(f) + 9*e
^2 - 36*d*f)/f)/sqrt(f) + 3/16*sqrt(pi)*erf(-1/2*sqrt(-f)*(2*x + (b*log(f)
 + e)/f))*e^(-1/4*(b^2*log(f)^2 + 2*b*e*log(f) - 4*a*f*log(f) + e^2 - 4*d*
f)/f)/sqrt(-f) - 3/16*sqrt(pi)*erf(-1/2*sqrt(f)*(2*x - (b*log(f) - e)/f))*
e^(1/4*(b^2*log(f)^2 - 2*b*e*log(f) + 4*a*f*log(f) + e^2 - 4*d*f)/f)/sqrt(
f)
```

Mupad [F(-1)]

Timed out.

$$\int f^{a+bx}\sinh^3\left(d+ex+fx^2\right)\,dx=\int f^{a+b\,x}\,\sinh\left(f\,x^2+e\,x+d\right)^3\,dx$$

input
```
int(f^(a + b*x)*sinh(d + e*x + f*x^2)^3,x)
```

output
```
int(f^(a + b*x)*sinh(d + e*x + f*x^2)^3, x)
```

Reduce [F]

$$\int f^{a+bx}\sinh^3\left(d+ex+fx^2\right)\,dx=f^a\left(\int f^{bx}\sinh\left(f\,x^2+ex+d\right)^3dx\right)$$

input
```
int(f^(b*x+a)*sinh(f*x^2+e*x+d)^3,x)
```

output
```
f**a*int(f**(b*x)*sinh(d + e*x + f*x**2)**3,x)
```

3.348 $\int f^{a+cx^2} \sinh(d+ex)\,dx$

Optimal result

Integrand size = 16, antiderivative size = 133

$$\int f^{a+cx^2}\sinh(d+ex)\,dx = \frac{e^{-d-\frac{e^2}{4c\log(f)}}f^a\sqrt{\pi}\,\mathrm{erfi}\left(\frac{e-2cx\log(f)}{2\sqrt{c}\sqrt{\log(f)}}\right)}{4\sqrt{c}\sqrt{\log(f)}} + \frac{e^{d-\frac{e^2}{4c\log(f)}}f^a\sqrt{\pi}\,\mathrm{erfi}\left(\frac{e+2cx\log(f)}{2\sqrt{c}\sqrt{\log(f)}}\right)}{4\sqrt{c}\sqrt{\log(f)}}$$

output
```
1/4*exp(-d-1/4*e^2/c/ln(f))*f^a*Pi^(1/2)*erfi(1/2*(e-2*c*x*ln(f))/c^(1/2)/
ln(f)^(1/2))/c^(1/2)/ln(f)^(1/2)+1/4*exp(d-1/4*e^2/c/ln(f))*f^a*Pi^(1/2)*e
rfi(1/2*(e+2*c*x*ln(f))/c^(1/2)/ln(f)^(1/2))/c^(1/2)/ln(f)^(1/2)
```

Mathematica [A] (verified)

Time = 0.11 (sec) , antiderivative size = 104, normalized size of antiderivative = 0.78

$$\int f^{a+cx^2}\sinh(d+ex)\,dx = \frac{e^{-\frac{e^2}{4c\log(f)}}f^a\sqrt{\pi}\left(\mathrm{erfi}\left(\frac{-e+2cx\log(f)}{2\sqrt{c}\sqrt{\log(f)}}\right)(-\cosh(d)+\sinh(d))+\mathrm{erfi}\left(\frac{e+2cx\log(f)}{2\sqrt{c}\sqrt{\log(f)}}\right)(\cosh(d)+\sinh(d))\right)}{4\sqrt{c}\sqrt{\log(f)}}$$

input
```
Integrate[f^(a + c*x^2)*Sinh[d + e*x],x]
```

output
```
(f^a*Sqrt[Pi]*(Erfi[(-e + 2*c*x*Log[f])/(2*Sqrt[c]*Sqrt[Log[f]])]*(-Cosh[d
] + Sinh[d]) + Erfi[(e + 2*c*x*Log[f])/(2*Sqrt[c]*Sqrt[Log[f]])]*(Cosh[d]
+ Sinh[d])))/(4*Sqrt[c]*E^(e^2/(4*c*Log[f]))*Sqrt[Log[f]])
```

Rubi [A] (verified)

Time = 0.48 (sec) , antiderivative size = 133, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {6038, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int f^{a+cx^2}\sinh(d+ex)\,dx$$
$$\Big\downarrow \text{6038}$$
$$\int \left(\frac{1}{2}e^{d+ex}f^{a+cx^2}-\frac{1}{2}e^{-d-ex}f^{a+cx^2}\right)dx$$
$$\Big\downarrow \text{2009}$$
$$\frac{\sqrt{\pi}f^a e^{-\frac{e^2}{4c\log(f)}-d}\text{erfi}\left(\frac{e-2cx\log(f)}{2\sqrt{c}\sqrt{\log(f)}}\right)}{4\sqrt{c}\sqrt{\log(f)}}+\frac{\sqrt{\pi}f^a e^{d-\frac{e^2}{4c\log(f)}}\text{erfi}\left(\frac{2cx\log(f)+e}{2\sqrt{c}\sqrt{\log(f)}}\right)}{4\sqrt{c}\sqrt{\log(f)}}$$

input
```
Int[f^(a + c*x^2)*Sinh[d + e*x],x]
```

output
```
(E^(-d - e^2/(4*c*Log[f]))*f^a*Sqrt[Pi]*Erfi[(e - 2*c*x*Log[f])/(2*Sqrt[c]
*Sqrt[Log[f]])])/(4*Sqrt[c]*Sqrt[Log[f]]) + (E^(d - e^2/(4*c*Log[f]))*f^a*
Sqrt[Pi]*Erfi[(e + 2*c*x*Log[f])/(2*Sqrt[c]*Sqrt[Log[f]])])/(4*Sqrt[c]*Sqr
t[Log[f]])
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 6038
```
Int[(F_)^(u_)*Sinh[v_]^(n_.), x_Symbol] :> Int[ExpandTrigToExp[F^u, Sinh[v]
^n, x], x] /; FreeQ[F, x] && (LinearQ[u, x] || PolyQ[u, x, 2]) && (LinearQ[
v, x] || PolyQ[v, x, 2]) && IGtQ[n, 0]
```

Maple [A] (verified)

Time = 0.12 (sec) , antiderivative size = 117, normalized size of antiderivative = 0.88

method	result	size
risch	$-\frac{\operatorname{erf}\left(-\sqrt{-c\ln(f)}\,x+\frac{e}{2\sqrt{-c\ln(f)}}\right)\sqrt{\pi}\,f^{a}\mathrm{e}^{\frac{4d\ln(f)c-e^{2}}{4\ln(f)c}}}{4\sqrt{-c\ln(f)}}-\frac{\operatorname{erf}\left(\sqrt{-c\ln(f)}\,x+\frac{e}{2\sqrt{-c\ln(f)}}\right)\sqrt{\pi}\,f^{a}\mathrm{e}^{-\frac{4d\ln(f)c+e^{2}}{4\ln(f)c}}}{4\sqrt{-c\ln(f)}}$	117

input
```
int(f^(c*x^2+a)*sinh(e*x+d),x,method=_RETURNVERBOSE)
```

output
```
-1/4*erf(-(-c*ln(f))^(1/2)*x+1/2*e/(-c*ln(f))^(1/2))/(-c*ln(f))^(1/2)*Pi^(
1/2)*f^a*exp(1/4*(4*d*ln(f)*c-e^2)/ln(f)/c)-1/4*erf((-c*ln(f))^(1/2)*x+1/2
*e/(-c*ln(f))^(1/2))/(-c*ln(f))^(1/2)*Pi^(1/2)*f^a*exp(-1/4*(4*d*ln(f)*c+e
^2)/ln(f)/c)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 217 vs. 2(101) = 202.

Time = 0.09 (sec) , antiderivative size = 217, normalized size of antiderivative = 1.63

$$\int f^{a+cx^2}\sinh(d+ex)\,dx = -\frac{\sqrt{-c\log(f)}\left(\sqrt{\pi}\cosh\left(\frac{4\,ac\log(f)^2+4\,cd\log(f)-e^2}{4\,c\log(f)}\right)+\sqrt{\pi}\sinh\left(\frac{4\,ac\log(f)^2+4\,cd\log(f)-e^2}{4\,c\log(f)}\right)\right)\operatorname{erf}\left(\frac{(2\,cx\log(f)+e)\sqrt{\cdots}}{2\,c\log(f)\cdots}\right.\cdots}{\cdots}$$

input
```
integrate(f^(c*x^2+a)*sinh(e*x+d),x, algorithm="fricas")
```

output
```
-1/4*(sqrt(-c*log(f))*(sqrt(pi)*cosh(1/4*(4*a*c*log(f)^2 + 4*c*d*log(f) -
e^2)/(c*log(f))) + sqrt(pi)*sinh(1/4*(4*a*c*log(f)^2 + 4*c*d*log(f) - e^2)
/(c*log(f))))*erf(1/2*(2*c*x*log(f) + e)*sqrt(-c*log(f))/(c*log(f))) - sqr
t(-c*log(f))*(sqrt(pi)*cosh(1/4*(4*a*c*log(f)^2 - 4*c*d*log(f) - e^2)/(c*l
og(f))) + sqrt(pi)*sinh(1/4*(4*a*c*log(f)^2 - 4*c*d*log(f) - e^2)/(c*log(f
))))*erf(1/2*(2*c*x*log(f) - e)*sqrt(-c*log(f))/(c*log(f))))/(c*log(f))
```

Sympy [F]

$$\int f^{a+cx^2} \sinh(d+ex)\,dx = \int f^{a+cx^2} \sinh(d+ex)\,dx$$

input
```
integrate(f**(c*x**2+a)*sinh(e*x+d),x)
```

output
```
Integral(f**(a + c*x**2)*sinh(d + e*x), x)
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 105, normalized size of antiderivative = 0.79

$$\int f^{a+cx^2} \sinh(d+ex)\,dx = \frac{\sqrt{\pi} f^a \operatorname{erf}\left(\sqrt{-c\log(f)}x - \frac{e}{2\sqrt{-c\log(f)}}\right) e^{\left(d-\frac{e^2}{4\,c\log(f)}\right)}}{4\sqrt{-c\log(f)}} - \frac{\sqrt{\pi} f^a \operatorname{erf}\left(\sqrt{-c\log(f)}x + \frac{e}{2\sqrt{-c\log(f)}}\right) e^{\left(-d-\frac{e^2}{4\,c\log(f)}\right)}}{4\sqrt{-c\log(f)}}$$

input
```
integrate(f^(c*x^2+a)*sinh(e*x+d),x, algorithm="maxima")
```

output
```
1/4*sqrt(pi)*f^a*erf(sqrt(-c*log(f))*x - 1/2*e/sqrt(-c*log(f)))*e^(d - 1/4
*e^2/(c*log(f)))/sqrt(-c*log(f)) - 1/4*sqrt(pi)*f^a*erf(sqrt(-c*log(f))*x
+ 1/2*e/sqrt(-c*log(f)))*e^(-d - 1/4*e^2/(c*log(f)))/sqrt(-c*log(f))
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 132, normalized size of antiderivative = 0.99

$$\int f^{a+cx^2}\sinh(d+ex)\,dx$$
$$= -\frac{\sqrt{\pi}\operatorname{erf}\left(-\frac{1}{2}\sqrt{-c\log(f)}\left(2x+\frac{e}{c\log(f)}\right)\right)e^{\left(\frac{4ac\log(f)^2+4cd\log(f)-e^2}{4c\log(f)}\right)}}{4\sqrt{-c\log(f)}}$$
$$+\frac{\sqrt{\pi}\operatorname{erf}\left(-\frac{1}{2}\sqrt{-c\log(f)}\left(2x-\frac{e}{c\log(f)}\right)\right)e^{\left(\frac{4ac\log(f)^2-4cd\log(f)-e^2}{4c\log(f)}\right)}}{4\sqrt{-c\log(f)}}$$

input
```
integrate(f^(c*x^2+a)*sinh(e*x+d),x, algorithm="giac")
```

output
```
-1/4*sqrt(pi)*erf(-1/2*sqrt(-c*log(f))*(2*x + e/(c*log(f))))*e^(1/4*(4*a*c
*log(f)^2 + 4*c*d*log(f) - e^2)/(c*log(f)))/sqrt(-c*log(f)) + 1/4*sqrt(pi)
*erf(-1/2*sqrt(-c*log(f))*(2*x - e/(c*log(f))))*e^(1/4*(4*a*c*log(f)^2 - 4
*c*d*log(f) - e^2)/(c*log(f)))/sqrt(-c*log(f))
```

Mupad [F(-1)]

Timed out.

$$\int f^{a+cx^2}\sinh(d+ex)\,dx = \int f^{c\,x^2+a}\,\sinh(d+e\,x)\,dx$$

input
```
int(f^(a + c*x^2)*sinh(d + e*x),x)
```

output
```
int(f^(a + c*x^2)*sinh(d + e*x), x)
```

Reduce [F]

$$\int f^{a+cx^2} \sinh(d+ex)\,dx = f^a \left(\int f^{c\,x^2} \sinh\left(ex+d\right)dx \right)$$

input
```
int(f^(c*x^2+a)*sinh(e*x+d),x)
```

output
```
f**a*int(f**(c*x**2)*sinh(d + e*x),x)
```

3.349 $\int f^{a+cx^2} \sinh^2(d+ex)\,dx$

Optimal result

Integrand size = 18, antiderivative size = 161

$$\int f^{a+cx^2} \sinh^2(d+ex)\,dx = -\frac{f^a\sqrt{\pi}\operatorname{erfi}\left(\sqrt{c}x\sqrt{\log(f)}\right)}{4\sqrt{c}\sqrt{\log(f)}} - \frac{e^{-2d-\frac{e^2}{c\log(f)}}f^a\sqrt{\pi}\operatorname{erfi}\left(\frac{e-cx\log(f)}{\sqrt{c}\sqrt{\log(f)}}\right)}{8\sqrt{c}\sqrt{\log(f)}} + \frac{e^{2d-\frac{e^2}{c\log(f)}}f^a\sqrt{\pi}\operatorname{erfi}\left(\frac{e+cx\log(f)}{\sqrt{c}\sqrt{\log(f)}}\right)}{8\sqrt{c}\sqrt{\log(f)}}$$

output
```
-1/4*f^a*Pi^(1/2)*erfi(c^(1/2)*x*ln(f)^(1/2))/c^(1/2)/ln(f)^(1/2)-1/8*exp(
-2*d-e^2/c/ln(f))*f^a*Pi^(1/2)*erfi((e-c*x*ln(f))/c^(1/2)/ln(f)^(1/2))/c^(
1/2)/ln(f)^(1/2)+1/8*exp(2*d-e^2/c/ln(f))*f^a*Pi^(1/2)*erfi((e+c*x*ln(f))/
c^(1/2)/ln(f)^(1/2))/c^(1/2)/ln(f)^(1/2)
```

Mathematica [A] (verified)

Time = 0.17 (sec) , antiderivative size = 131, normalized size of antiderivative = 0.81

$$\int f^{a+cx^2}\sinh^2(d+ex)\,dx$$
$$=\frac{e^{-\frac{e^2}{c\log(f)}}f^a\sqrt{\pi}\left(-2e^{\frac{e^2}{c\log(f)}}\text{erfi}\left(\sqrt{c}x\sqrt{\log(f)}\right)+\text{erfi}\left(\frac{-e+cx\log(f)}{\sqrt{c}\sqrt{\log(f)}}\right)(\cosh(2d)-\sinh(2d))+\text{erfi}\left(\frac{e+cx\log(f)}{\sqrt{c}\sqrt{\log(f)}}\right.}{8\sqrt{c}\sqrt{\log(f)}}$$

input
```
Integrate[f^(a + c*x^2)*Sinh[d + e*x]^2,x]
```

output
```
(f^a*Sqrt[Pi]*(-2*E^(e^2/(c*Log[f]))*Erfi[Sqrt[c]*x*Sqrt[Log[f]]] + Erfi[(
-e + c*x*Log[f])/(Sqrt[c]*Sqrt[Log[f]])]*(Cosh[2*d] - Sinh[2*d]) + Erfi[(e
 + c*x*Log[f])/(Sqrt[c]*Sqrt[Log[f]])]*(Cosh[2*d] + Sinh[2*d])))/(8*Sqrt[c
]*E^(e^2/(c*Log[f]))*Sqrt[Log[f]])
```

Rubi [A] (verified)

Time = 0.49 (sec) , antiderivative size = 161, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {6038, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int f^{a+cx^2}\sinh^2(d+ex)\,dx$$
$$\Big\downarrow \ 6038$$
$$\int \left(\frac{1}{4}e^{-2d-2ex}f^{a+cx^2}+\frac{1}{4}e^{2d+2ex}f^{a+cx^2}-\frac{1}{2}f^{a+cx^2}\right)dx$$
$$\Big\downarrow \ 2009$$

$$-\frac{\sqrt{\pi}f^a e^{-\frac{e^2}{c\log(f)}-2d}\operatorname{erfi}\left(\frac{e-cx\log(f)}{\sqrt{c}\sqrt{\log(f)}}\right)}{8\sqrt{c}\sqrt{\log(f)}}+\frac{\sqrt{\pi}f^a e^{2d-\frac{e^2}{c\log(f)}}\operatorname{erfi}\left(\frac{cx\log(f)+e}{\sqrt{c}\sqrt{\log(f)}}\right)}{8\sqrt{c}\sqrt{\log(f)}}-\frac{\sqrt{\pi}f^a\operatorname{erfi}\left(\sqrt{c}x\sqrt{\log(f)}\right)}{4\sqrt{c}\sqrt{\log(f)}}$$

input
```
Int[f^(a + c*x^2)*Sinh[d + e*x]^2,x]
```

output
```
-1/4*(f^a*Sqrt[Pi]*Erfi[Sqrt[c]*x*Sqrt[Log[f]]])/(Sqrt[c]*Sqrt[Log[f]]) -
(E^(-2*d - e^2/(c*Log[f]))*f^a*Sqrt[Pi]*Erfi[(e - c*x*Log[f])/(Sqrt[c]*Sqr
t[Log[f]])])/(8*Sqrt[c]*Sqrt[Log[f]]) + (E^(2*d - e^2/(c*Log[f]))*f^a*Sqrt
[Pi]*Erfi[(e + c*x*Log[f])/(Sqrt[c]*Sqrt[Log[f]])])/(8*Sqrt[c]*Sqrt[Log[f]
])
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 6038
```
Int[(F_)^(u_)*Sinh[v_]^(n_.), x_Symbol] :> Int[ExpandTrigToExp[F^u, Sinh[v]
^n, x], x] /; FreeQ[F, x] && (LinearQ[u, x] || PolyQ[u, x, 2]) && (LinearQ[
v, x] || PolyQ[v, x, 2]) && IGtQ[n, 0]
```

Maple [A] (verified)

Time = 0.28 (sec) , antiderivative size = 139, normalized size of antiderivative = 0.86

method	result
risch	$\frac{\operatorname{erf}\left(\sqrt{-c\ln(f)}\,x+\frac{e}{\sqrt{-c\ln(f)}}\right)\sqrt{\pi}\,f^a\mathrm{e}^{-\frac{2d\ln(f)c+e^2}{\ln(f)c}}}{8\sqrt{-c\ln(f)}}-\frac{\operatorname{erf}\left(-\sqrt{-c\ln(f)}\,x+\frac{e}{\sqrt{-c\ln(f)}}\right)\sqrt{\pi}\,f^a\mathrm{e}^{\frac{2d\ln(f)c-e^2}{\ln(f)c}}}{8\sqrt{-c\ln(f)}}-\frac{f^a\sqrt{\pi}\,\operatorname{erf}\left(\sqrt{-c\right.}}{4\sqrt{-c\ln(}}$

input
```
int(f^(c*x^2+a)*sinh(e*x+d)^2,x,method=_RETURNVERBOSE)
```

output
```
1/8*erf((-c*ln(f))^(1/2)*x+e/(-c*ln(f))^(1/2))/(-c*ln(f))^(1/2)*Pi^(1/2)*f
^a*exp(-(2*d*ln(f)*c+e^2)/ln(f)/c)-1/8*erf(-(-c*ln(f))^(1/2)*x+e/(-c*ln(f)
)^(1/2))/(-c*ln(f))^(1/2)*Pi^(1/2)*f^a*exp((2*d*ln(f)*c-e^2)/ln(f)/c)-1/4*
f^a*Pi^(1/2)/(-c*ln(f))^(1/2)*erf((-c*ln(f))^(1/2)*x)
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 244, normalized size of antiderivative = 1.52

$$\int f^{a+cx^2}\sinh^2(d+ex)\,dx = \frac{2\sqrt{-c\log(f)}\left(\sqrt{\pi}\cosh(a\log(f))+\sqrt{\pi}\sinh(a\log(f))\right)\operatorname{erf}\left(\sqrt{-c\log(f)}x\right)-\sqrt{-c\log(f)}\left(\sqrt{\pi}\cosh\left(\right.\right.}{}$$

input
```
integrate(f^(c*x^2+a)*sinh(e*x+d)^2,x, algorithm="fricas")
```

output
```
1/8*(2*sqrt(-c*log(f))*(sqrt(pi)*cosh(a*log(f)) + sqrt(pi)*sinh(a*log(f)))
*erf(sqrt(-c*log(f))*x) - sqrt(-c*log(f))*(sqrt(pi)*cosh((a*c*log(f)^2 + 2
*c*d*log(f) - e^2)/(c*log(f))) + sqrt(pi)*sinh((a*c*log(f)^2 + 2*c*d*log(f
) - e^2)/(c*log(f))))*erf((c*x*log(f) + e)*sqrt(-c*log(f))/(c*log(f))) - s
qrt(-c*log(f))*(sqrt(pi)*cosh((a*c*log(f)^2 - 2*c*d*log(f) - e^2)/(c*log(f
))) + sqrt(pi)*sinh((a*c*log(f)^2 - 2*c*d*log(f) - e^2)/(c*log(f))))*erf((
c*x*log(f) - e)*sqrt(-c*log(f))/(c*log(f))))/(c*log(f))
```

Sympy [F]

$$\int f^{a+cx^2}\sinh^2(d+ex)\,dx = \int f^{a+cx^2}\sinh^2(d+ex)\,dx$$

input
```
integrate(f**(c*x**2+a)*sinh(e*x+d)**2,x)
```

output
```
Integral(f**(a + c*x**2)*sinh(d + e*x)**2, x)
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 131, normalized size of antiderivative = 0.81

$$\int f^{a+cx^2}\sinh^2(d+ex)\,dx = \frac{\sqrt{\pi}f^a\,\mathrm{erf}\left(\sqrt{-c\log(f)}x - \frac{e}{\sqrt{-c\log(f)}}\right)e^{\left(2\,d-\frac{e^2}{c\log(f)}\right)}}{8\sqrt{-c\log(f)}} + \frac{\sqrt{\pi}f^a\,\mathrm{erf}\left(\sqrt{-c\log(f)}x + \frac{e}{\sqrt{-c\log(f)}}\right)e^{\left(-2\,d-\frac{e^2}{c\log(f)}\right)}}{8\sqrt{-c\log(f)}} - \frac{\sqrt{\pi}f^a\,\mathrm{erf}\left(\sqrt{-c\log(f)}x\right)}{4\sqrt{-c\log(f)}}$$

input
```
integrate(f^(c*x^2+a)*sinh(e*x+d)^2,x, algorithm="maxima")
```

output
```
1/8*sqrt(pi)*f^a*erf(sqrt(-c*log(f))*x - e/sqrt(-c*log(f)))*e^(2*d - e^2/(
c*log(f)))/sqrt(-c*log(f)) + 1/8*sqrt(pi)*f^a*erf(sqrt(-c*log(f))*x + e/sq
rt(-c*log(f)))*e^(-2*d - e^2/(c*log(f)))/sqrt(-c*log(f)) - 1/4*sqrt(pi)*f^
a*erf(sqrt(-c*log(f))*x)/sqrt(-c*log(f))
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 150, normalized size of antiderivative = 0.93

$$\int f^{a+cx^2}\sinh^2(d+ex)\,dx = \frac{\sqrt{\pi}f^a\,\mathrm{erf}\left(-\sqrt{-c\log(f)}x\right)}{4\sqrt{-c\log(f)}} - \frac{\sqrt{\pi}\,\mathrm{erf}\left(-\sqrt{-c\log(f)}\left(x+\frac{e}{c\log(f)}\right)\right)e^{\left(\frac{ac\log(f)^2+2\,cd\log(f)-e^2}{c\log(f)}\right)}}{8\sqrt{-c\log(f)}} - \frac{\sqrt{\pi}\,\mathrm{erf}\left(-\sqrt{-c\log(f)}\left(x-\frac{e}{c\log(f)}\right)\right)e^{\left(\frac{ac\log(f)^2-2\,cd\log(f)-e^2}{c\log(f)}\right)}}{8\sqrt{-c\log(f)}}$$

input
```
integrate(f^(c*x^2+a)*sinh(e*x+d)^2,x, algorithm="giac")
```

output
```
1/4*sqrt(pi)*f^a*erf(-sqrt(-c*log(f))*x)/sqrt(-c*log(f)) - 1/8*sqrt(pi)*er
f(-sqrt(-c*log(f))*(x + e/(c*log(f))))*e^((a*c*log(f)^2 + 2*c*d*log(f) - e
^2)/(c*log(f)))/sqrt(-c*log(f)) - 1/8*sqrt(pi)*erf(-sqrt(-c*log(f))*(x - e
/(c*log(f))))*e^((a*c*log(f)^2 - 2*c*d*log(f) - e^2)/(c*log(f)))/sqrt(-c*l
og(f))
```

Mupad [F(-1)]

Timed out.

$$\int f^{a+cx^2}\sinh^2(d+ex)\,dx=\int f^{c\,x^2+a}\sinh(d+e\,x)^2\,dx$$

input
```
int(f^(a + c*x^2)*sinh(d + e*x)^2,x)
```

output
```
int(f^(a + c*x^2)*sinh(d + e*x)^2, x)
```

Reduce [F]

$$\int f^{a+cx^2}\sinh^2(d+ex)\,dx=f^a\left(\int f^{c\,x^2}\sinh\left(ex+d\right)^2dx\right)$$

input
```
int(f^(c*x^2+a)*sinh(e*x+d)^2,x)
```

output
```
f**a*int(f**(c*x**2)*sinh(d + e*x)**2,x)
```

3.350 $\int f^{a+cx^2} \sinh^3(d+ex)\,dx$

Optimal result

Integrand size = 18, antiderivative size = 271

$$\int f^{a+cx^2}\sinh^3(d+ex)\,dx = -\frac{3e^{-d-\frac{e^2}{4c\log(f)}}f^a\sqrt{\pi}\,\mathrm{erfi}\left(\frac{e-2cx\log(f)}{2\sqrt{c}\sqrt{\log(f)}}\right)}{16\sqrt{c}\sqrt{\log(f)}} + \frac{e^{-3d-\frac{9e^2}{4c\log(f)}}f^a\sqrt{\pi}\,\mathrm{erfi}\left(\frac{3e-2cx\log(f)}{2\sqrt{c}\sqrt{\log(f)}}\right)}{16\sqrt{c}\sqrt{\log(f)}} - \frac{3e^{d-\frac{e^2}{4c\log(f)}}f^a\sqrt{\pi}\,\mathrm{erfi}\left(\frac{e+2cx\log(f)}{2\sqrt{c}\sqrt{\log(f)}}\right)}{16\sqrt{c}\sqrt{\log(f)}} + \frac{e^{3d-\frac{9e^2}{4c\log(f)}}f^a\sqrt{\pi}\,\mathrm{erfi}\left(\frac{3e+2cx\log(f)}{2\sqrt{c}\sqrt{\log(f)}}\right)}{16\sqrt{c}\sqrt{\log(f)}}$$

output
```
-3/16*exp(-d-1/4*e^2/c/ln(f))*f^a*Pi^(1/2)*erfi(1/2*(e-2*c*x*ln(f))/c^(1/2
)/ln(f)^(1/2))/c^(1/2)/ln(f)^(1/2)+1/16*exp(-3*d-9/4*e^2/c/ln(f))*f^a*Pi^(
1/2)*erfi(1/2*(3*e-2*c*x*ln(f))/c^(1/2)/ln(f)^(1/2))/c^(1/2)/ln(f)^(1/2)-3
/16*exp(d-1/4*e^2/c/ln(f))*f^a*Pi^(1/2)*erfi(1/2*(e+2*c*x*ln(f))/c^(1/2)/l
n(f)^(1/2))/c^(1/2)/ln(f)^(1/2)+1/16*exp(3*d-9/4*e^2/c/ln(f))*f^a*Pi^(1/2)
*erfi(1/2*(3*e+2*c*x*ln(f))/c^(1/2)/ln(f)^(1/2))/c^(1/2)/ln(f)^(1/2)
```

Mathematica [A] (verified)

Time = 0.30 (sec) , antiderivative size = 214, normalized size of antiderivative = 0.79

$$\int f^{a+cx^2}\sinh^3(d+ex)\,dx$$

$$=\frac{e^{-\frac{9e^2}{4c\log(f)}}f^a\sqrt{\pi}\left((\cosh(d)+\sinh(d))\left(-3e^{\frac{2e^2}{c\log(f)}}\operatorname{erfi}\left(\frac{e+2cx\log(f)}{2\sqrt{c}\sqrt{\log(f)}}\right)+3e^{\frac{2e^2}{c\log(f)}}\operatorname{erfi}\left(\frac{-e+2cx\log(f)}{2\sqrt{c}\sqrt{\log(f)}}\right)(\cosh(2d)-\right.\right.}{16\sqrt{c}\sqrt{}}$$

input
```
Integrate[f^(a + c*x^2)*Sinh[d + e*x]^3,x]
```

output
```
(f^a*Sqrt[Pi]*((Cosh[d] + Sinh[d])*(-3*E^((2*e^2)/(c*Log[f]))*Erfi[(e + 2*
c*x*Log[f])/(2*Sqrt[c]*Sqrt[Log[f]])] + 3*E^((2*e^2)/(c*Log[f]))*Erfi[(-e
+ 2*c*x*Log[f])/(2*Sqrt[c]*Sqrt[Log[f]])]*(Cosh[2*d] - Sinh[2*d]) + Erfi[(
3*e + 2*c*x*Log[f])/(2*Sqrt[c]*Sqrt[Log[f]])]*(Cosh[2*d] + Sinh[2*d])) + E
rfi[(-3*e + 2*c*x*Log[f])/(2*Sqrt[c]*Sqrt[Log[f]])]*(-Cosh[3*d] + Sinh[3*d
])))/(16*Sqrt[c]*E^((9*e^2)/(4*c*Log[f]))*Sqrt[Log[f]])
```

Rubi [A] (verified)

Time = 0.66 (sec) , antiderivative size = 271, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {6038, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int f^{a+cx^2}\sinh^3(d+ex)\,dx$$

$$\Big\downarrow \; 6038$$

$$\int \left(-\frac{1}{8}e^{-3d-3ex}f^{a+cx^2}+\frac{3}{8}e^{-d-ex}f^{a+cx^2}-\frac{3}{8}e^{d+ex}f^{a+cx^2}+\frac{1}{8}e^{3d+3ex}f^{a+cx^2}\right)dx$$

$$\Big\downarrow \; 2009$$

$$-\frac{3\sqrt{\pi}f^a e^{-\frac{e^2}{4c\log(f)}-d}\operatorname{erfi}\left(\frac{e-2cx\log(f)}{2\sqrt{c}\sqrt{\log(f)}}\right)}{16\sqrt{c}\sqrt{\log(f)}}+\frac{\sqrt{\pi}f^a e^{-\frac{9e^2}{4c\log(f)}-3d}\operatorname{erfi}\left(\frac{3e-2cx\log(f)}{2\sqrt{c}\sqrt{\log(f)}}\right)}{16\sqrt{c}\sqrt{\log(f)}}-\frac{3\sqrt{\pi}f^a e^{d-\frac{e^2}{4c\log(f)}}\operatorname{erfi}\left(\frac{2cx\log(f)+e}{2\sqrt{c}\sqrt{\log(f)}}\right)}{16\sqrt{c}\sqrt{\log(f)}}+\frac{\sqrt{\pi}f^a e^{3d-\frac{9e^2}{4c\log(f)}}\operatorname{erfi}\left(\frac{2cx\log(f)+3e}{2\sqrt{c}\sqrt{\log(f)}}\right)}{16\sqrt{c}\sqrt{\log(f)}}$$

input
```
Int[f^(a + c*x^2)*Sinh[d + e*x]^3,x]
```

output
```
(-3*E^(-d - e^2/(4*c*Log[f]))*f^a*Sqrt[Pi]*Erfi[(e - 2*c*x*Log[f])/(2*Sqrt
[c]*Sqrt[Log[f]])])/(16*Sqrt[c]*Sqrt[Log[f]]) + (E^(-3*d - (9*e^2)/(4*c*Lo
g[f]))*f^a*Sqrt[Pi]*Erfi[(3*e - 2*c*x*Log[f])/(2*Sqrt[c]*Sqrt[Log[f]])])/(
16*Sqrt[c]*Sqrt[Log[f]]) - (3*E^(d - e^2/(4*c*Log[f]))*f^a*Sqrt[Pi]*Erfi[(
e + 2*c*x*Log[f])/(2*Sqrt[c]*Sqrt[Log[f]])])/(16*Sqrt[c]*Sqrt[Log[f]]) + (
E^(3*d - (9*e^2)/(4*c*Log[f]))*f^a*Sqrt[Pi]*Erfi[(3*e + 2*c*x*Log[f])/(2*S
qrt[c]*Sqrt[Log[f]])])/(16*Sqrt[c]*Sqrt[Log[f]])
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 6038
```
Int[(F_)^(u_)*Sinh[v_]^(n_.), x_Symbol] :> Int[ExpandTrigToExp[F^u, Sinh[v]
^n, x], x] /; FreeQ[F, x] && (LinearQ[u, x] || PolyQ[u, x, 2]) && (LinearQ[
v, x] || PolyQ[v, x, 2]) && IGtQ[n, 0]
```

Maple [A] (verified)

Time = 0.80 (sec) , antiderivative size = 234, normalized size of antiderivative = 0.86

method	result
risch	$-\frac{\operatorname{erf}\left(-\sqrt{-c\ln(f)}\,x+\frac{3e}{2\sqrt{-c\ln(f)}}\right)\sqrt{\pi}\,f^a\,\mathrm{e}^{\frac{3d\ln(f)c-\frac{9e^2}{4}}{c\ln(f)}}}{16\sqrt{-c\ln(f)}}-\frac{\operatorname{erf}\left(\sqrt{-c\ln(f)}\,x+\frac{3e}{2\sqrt{-c\ln(f)}}\right)\sqrt{\pi}\,f^a\,\mathrm{e}^{-\frac{3\left(4d\ln(f)c+3e^2\right)}{4\ln(f)c}}}{16\sqrt{-c\ln(f)}}+$ 3 erf(

input
```
int(f^(c*x^2+a)*sinh(e*x+d)^3,x,method=_RETURNVERBOSE)
```

output
```
-1/16*erf(-(-c*ln(f))^(1/2)*x+3/2*e/(-c*ln(f))^(1/2))/(-c*ln(f))^(1/2)*Pi^
(1/2)*f^a*exp(3/4*(4*d*ln(f)*c-3*e^2)/ln(f)/c)-1/16*erf((-c*ln(f))^(1/2)*x
+3/2*e/(-c*ln(f))^(1/2))/(-c*ln(f))^(1/2)*Pi^(1/2)*f^a*exp(-3/4*(4*d*ln(f)
*c+3*e^2)/ln(f)/c)+3/16*erf((-c*ln(f))^(1/2)*x+1/2*e/(-c*ln(f))^(1/2))/(-c
*ln(f))^(1/2)*Pi^(1/2)*f^a*exp(-1/4*(4*d*ln(f)*c+e^2)/ln(f)/c)+3/16*erf(-(
-c*ln(f))^(1/2)*x+1/2*e/(-c*ln(f))^(1/2))/(-c*ln(f))^(1/2)*Pi^(1/2)*f^a*ex
p(1/4*(4*d*ln(f)*c-e^2)/ln(f)/c)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 427 vs. 2(205) = 410.

Time = 0.10 (sec) , antiderivative size = 427, normalized size of antiderivative = 1.58

$$\int f^{a+cx^2}\sinh^3(d+ex)\,dx =$$

$$-\frac{\sqrt{-c\log(f)}\left(\sqrt{\pi}\cosh\left(\frac{4\,ac\log(f)^2+12\,cd\log(f)-9\,e^2}{4\,c\log(f)}\right)+\sqrt{\pi}\sinh\left(\frac{4\,ac\log(f)^2+12\,cd\log(f)-9\,e^2}{4\,c\log(f)}\right)\right)\operatorname{erf}\left(\frac{(2\,cx\log(f)+}{2\,c}\right.}{}$$

input
```
integrate(f^(c*x^2+a)*sinh(e*x+d)^3,x, algorithm="fricas")
```

output
```
-1/16*(sqrt(-c*log(f))*(sqrt(pi)*cosh(1/4*(4*a*c*log(f)^2 + 12*c*d*log(f)
- 9*e^2)/(c*log(f))) + sqrt(pi)*sinh(1/4*(4*a*c*log(f)^2 + 12*c*d*log(f) -
 9*e^2)/(c*log(f))))*erf(1/2*(2*c*x*log(f) + 3*e)*sqrt(-c*log(f))/(c*log(f
))) - 3*sqrt(-c*log(f))*(sqrt(pi)*cosh(1/4*(4*a*c*log(f)^2 + 4*c*d*log(f)
- e^2)/(c*log(f))) + sqrt(pi)*sinh(1/4*(4*a*c*log(f)^2 + 4*c*d*log(f) - e^
2)/(c*log(f))))*erf(1/2*(2*c*x*log(f) + e)*sqrt(-c*log(f))/(c*log(f))) + 3
*sqrt(-c*log(f))*(sqrt(pi)*cosh(1/4*(4*a*c*log(f)^2 - 4*c*d*log(f) - e^2)/
(c*log(f))) + sqrt(pi)*sinh(1/4*(4*a*c*log(f)^2 - 4*c*d*log(f) - e^2)/(c*l
og(f))))*erf(1/2*(2*c*x*log(f) - e)*sqrt(-c*log(f))/(c*log(f))) - sqrt(-c*
log(f))*(sqrt(pi)*cosh(1/4*(4*a*c*log(f)^2 - 12*c*d*log(f) - 9*e^2)/(c*log
(f))) + sqrt(pi)*sinh(1/4*(4*a*c*log(f)^2 - 12*c*d*log(f) - 9*e^2)/(c*log(
f))))*erf(1/2*(2*c*x*log(f) - 3*e)*sqrt(-c*log(f))/(c*log(f))))/(c*log(f))
```

Sympy [F]

$$\int f^{a+cx^2}\sinh^3(d+ex)\,dx = \int f^{a+cx^2}\sinh^3(d+ex)\,dx$$

input
```
integrate(f**(c*x**2+a)*sinh(e*x+d)**3,x)
```

output
```
Integral(f**(a + c*x**2)*sinh(d + e*x)**3, x)
```

Maxima [A] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 211, normalized size of antiderivative = 0.78

$$\int f^{a+cx^2}\sinh^3(d+ex)\,dx = \frac{\sqrt{\pi}f^a\,\mathrm{erf}\left(\sqrt{-c\log(f)}x-\frac{3e}{2\sqrt{-c\log(f)}}\right)e^{\left(3d-\frac{9e^2}{4c\log(f)}\right)}}{16\sqrt{-c\log(f)}} - \frac{3\sqrt{\pi}f^a\,\mathrm{erf}\left(\sqrt{-c\log(f)}x-\frac{e}{2\sqrt{-c\log(f)}}\right)e^{\left(d-\frac{e^2}{4c\log(f)}\right)}}{16\sqrt{-c\log(f)}} + \frac{3\sqrt{\pi}f^a\,\mathrm{erf}\left(\sqrt{-c\log(f)}x+\frac{e}{2\sqrt{-c\log(f)}}\right)e^{\left(-d-\frac{e^2}{4c\log(f)}\right)}}{16\sqrt{-c\log(f)}} - \frac{\sqrt{\pi}f^a\,\mathrm{erf}\left(\sqrt{-c\log(f)}x+\frac{3e}{2\sqrt{-c\log(f)}}\right)e^{\left(-3d-\frac{9e^2}{4c\log(f)}\right)}}{16\sqrt{-c\log(f)}}$$

input
```
integrate(f^(c*x^2+a)*sinh(e*x+d)^3,x, algorithm="maxima")
```

output
```
1/16*sqrt(pi)*f^a*erf(sqrt(-c*log(f))*x - 3/2*e/sqrt(-c*log(f)))*e^(3*d -
9/4*e^2/(c*log(f)))/sqrt(-c*log(f)) - 3/16*sqrt(pi)*f^a*erf(sqrt(-c*log(f)
)*x - 1/2*e/sqrt(-c*log(f)))*e^(d - 1/4*e^2/(c*log(f)))/sqrt(-c*log(f)) +
3/16*sqrt(pi)*f^a*erf(sqrt(-c*log(f))*x + 1/2*e/sqrt(-c*log(f)))*e^(-d - 1
/4*e^2/(c*log(f)))/sqrt(-c*log(f)) - 1/16*sqrt(pi)*f^a*erf(sqrt(-c*log(f))
*x + 3/2*e/sqrt(-c*log(f)))*e^(-3*d - 9/4*e^2/(c*log(f)))/sqrt(-c*log(f))
```

Giac [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 264, normalized size of antiderivative = 0.97

$$\int f^{a+cx^2}\sinh^3(d+ex)\,dx$$
$$= -\frac{\sqrt{\pi}\,\mathrm{erf}\left(-\frac{1}{2}\sqrt{-c\log(f)}\left(2x+\frac{3e}{c\log(f)}\right)\right)e^{\left(\frac{4ac\log(f)^2+12cd\log(f)-9e^2}{4c\log(f)}\right)}}{16\sqrt{-c\log(f)}}$$
$$+\frac{3\sqrt{\pi}\,\mathrm{erf}\left(-\frac{1}{2}\sqrt{-c\log(f)}\left(2x+\frac{e}{c\log(f)}\right)\right)e^{\left(\frac{4ac\log(f)^2+4cd\log(f)-e^2}{4c\log(f)}\right)}}{16\sqrt{-c\log(f)}}$$
$$-\frac{3\sqrt{\pi}\,\mathrm{erf}\left(-\frac{1}{2}\sqrt{-c\log(f)}\left(2x-\frac{e}{c\log(f)}\right)\right)e^{\left(\frac{4ac\log(f)^2-4cd\log(f)-e^2}{4c\log(f)}\right)}}{16\sqrt{-c\log(f)}}$$
$$+\frac{\sqrt{\pi}\,\mathrm{erf}\left(-\frac{1}{2}\sqrt{-c\log(f)}\left(2x-\frac{3e}{c\log(f)}\right)\right)e^{\left(\frac{4ac\log(f)^2-12cd\log(f)-9e^2}{4c\log(f)}\right)}}{16\sqrt{-c\log(f)}}$$

input
```
integrate(f^(c*x^2+a)*sinh(e*x+d)^3,x, algorithm="giac")
```

output
```
-1/16*sqrt(pi)*erf(-1/2*sqrt(-c*log(f))*(2*x + 3*e/(c*log(f))))*e^(1/4*(4*
a*c*log(f)^2 + 12*c*d*log(f) - 9*e^2)/(c*log(f)))/sqrt(-c*log(f)) + 3/16*s
qrt(pi)*erf(-1/2*sqrt(-c*log(f))*(2*x + e/(c*log(f))))*e^(1/4*(4*a*c*log(f
)^2 + 4*c*d*log(f) - e^2)/(c*log(f)))/sqrt(-c*log(f)) - 3/16*sqrt(pi)*erf(
-1/2*sqrt(-c*log(f))*(2*x - e/(c*log(f))))*e^(1/4*(4*a*c*log(f)^2 - 4*c*d*
log(f) - e^2)/(c*log(f)))/sqrt(-c*log(f)) + 1/16*sqrt(pi)*erf(-1/2*sqrt(-c
*log(f))*(2*x - 3*e/(c*log(f))))*e^(1/4*(4*a*c*log(f)^2 - 12*c*d*log(f) -
9*e^2)/(c*log(f)))/sqrt(-c*log(f))
```

Mupad [F(-1)]

Timed out.

$$\int f^{a+cx^2}\sinh^3(d+ex)\,dx=\int f^{c\,x^2+a}\,\sinh(d+e\,x)^3\,dx$$

input
```
int(f^(a + c*x^2)*sinh(d + e*x)^3,x)
```

output
```
int(f^(a + c*x^2)*sinh(d + e*x)^3, x)
```

Reduce [F]

$$\int f^{a+cx^2}\sinh^3(d+ex)\,dx=f^a\left(\int f^{c\,x^2}\sinh\left(ex+d\right)^3dx\right)$$

input
```
int(f^(c*x^2+a)*sinh(e*x+d)^3,x)
```

output
```
f**a*int(f**(c*x**2)*sinh(d + e*x)**3,x)
```

3.351 $\int f^{a+cx^2} \sinh(d+fx^2)\,dx$

Optimal result

Integrand size = 18, antiderivative size = 81

$$\int f^{a+cx^2} \sinh(d+fx^2)\,dx = -\frac{e^{-d} f^a \sqrt{\pi}\,\text{erf}\left(x\sqrt{f-c\log(f)}\right)}{4\sqrt{f-c\log(f)}} + \frac{e^d f^a \sqrt{\pi}\,\text{erfi}\left(x\sqrt{f+c\log(f)}\right)}{4\sqrt{f+c\log(f)}}$$

output
```
-1/4*f^a*Pi^(1/2)*erf(x*(f-c*ln(f))^(1/2))/exp(d)/(f-c*ln(f))^(1/2)+1/4*ex
p(d)*f^a*Pi^(1/2)*erfi(x*(f+c*ln(f))^(1/2))/(f+c*ln(f))^(1/2)
```

Mathematica [A] (verified)

Time = 0.23 (sec) , antiderivative size = 76, normalized size of antiderivative = 0.94

$$\int f^{a+cx^2} \sinh(d+fx^2)\,dx = \frac{1}{4} f^a \sqrt{\pi} \left(-\frac{\text{erf}\left(x\sqrt{f-c\log(f)}\right)(\cosh(d)-\sinh(d))}{\sqrt{f-c\log(f)}} + \frac{\text{erfi}\left(x\sqrt{f+c\log(f)}\right)(\cosh(d)+\sinh(d))}{\sqrt{f+c\log(f)}}\right)$$

input
```
Integrate[f^(a + c*x^2)*Sinh[d + f*x^2],x]
```

output
```
(f^a*Sqrt[Pi]*(-((Erf[x*Sqrt[f - c*Log[f]]]*(Cosh[d] - Sinh[d]))/Sqrt[f -
c*Log[f]]) + (Erfi[x*Sqrt[f + c*Log[f]]]*(Cosh[d] + Sinh[d]))/Sqrt[f + c*L
og[f]]))/4
```

Rubi [A] (verified)

Time = 0.43 (sec) , antiderivative size = 81, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.111$, Rules used = {6038, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int f^{a+cx^2} \sinh \left(d+fx^2\right) dx$$

$$\Big\downarrow \text{6038}$$

$$\int \left(\frac{1}{2} e^{d+fx^2} f^{a+cx^2}-\frac{1}{2} e^{-d-fx^2} f^{a+cx^2}\right) dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{\sqrt{\pi} e^d f^a \text{erfi}\left(x \sqrt{c \log (f)+f}\right)}{4 \sqrt{c \log (f)+f}}-\frac{\sqrt{\pi} e^{-d} f^a \text{erf}\left(x \sqrt{f-c \log (f)}\right)}{4 \sqrt{f-c \log (f)}}$$

input
```
Int[f^(a + c*x^2)*Sinh[d + f*x^2],x]
```

output
```
-1/4*(f^a*Sqrt[Pi]*Erf[x*Sqrt[f - c*Log[f]]])/(E^d*Sqrt[f - c*Log[f]]) + (
E^d*f^a*Sqrt[Pi]*Erfi[x*Sqrt[f + c*Log[f]]])/(4*Sqrt[f + c*Log[f]])
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 6038
```
Int[(F_)^(u_)*Sinh[v_]^(n_.), x_Symbol] :> Int[ExpandTrigToExp[F^u, Sinh[v]
^n, x], x] /; FreeQ[F, x] && (LinearQ[u, x] || PolyQ[u, x, 2]) && (LinearQ[
v, x] || PolyQ[v, x, 2]) && IGtQ[n, 0]
```

Maple [A] (verified)

Time = 0.12 (sec) , antiderivative size = 70, normalized size of antiderivative = 0.86

method	result	size
risch	$\frac{f^{a} \mathrm{e}^{d} \sqrt{\pi}\, \mathrm{erf}\left(\sqrt{-c \ln(f)-f}\, x\right)}{4 \sqrt{-c \ln(f)-f}}-\frac{f^{a} \mathrm{e}^{-d} \sqrt{\pi}\, \mathrm{erf}\left(x \sqrt{f-c \ln(f)}\right)}{4 \sqrt{f-c \ln(f)}}$	70

input
```
int(f^(c*x^2+a)*sinh(f*x^2+d),x,method=_RETURNVERBOSE)
```

output
```
1/4*f^a*exp(d)*Pi^(1/2)/(-c*ln(f)-f)^(1/2)*erf((-c*ln(f)-f)^(1/2)*x)-1/4*f
^a*exp(-d)*Pi^(1/2)/(f-c*ln(f))^(1/2)*erf(x*(f-c*ln(f))^(1/2))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 146 vs. 2(63) = 126.

Time = 0.09 (sec) , antiderivative size = 146, normalized size of antiderivative = 1.80

$$\int f^{a+cx^2} \sinh\left(d+fx^2\right) dx$$
$$= \frac{\left(\sqrt{\pi}(c\log(f)+f)\cosh(a\log(f)-d)+\sqrt{\pi}(c\log(f)+f)\sinh(a\log(f)-d)\right)\sqrt{-c\log(f)+f}\,\mathrm{erf}\left(\sqrt{\right.}}{}$$

input
```
integrate(f^(c*x^2+a)*sinh(f*x^2+d),x, algorithm="fricas")
```

output
```
1/4*((sqrt(pi)*(c*log(f) + f)*cosh(a*log(f) - d) + sqrt(pi)*(c*log(f) + f)
*sinh(a*log(f) - d))*sqrt(-c*log(f) + f)*erf(sqrt(-c*log(f) + f)*x) - (sqr
t(pi)*(c*log(f) - f)*cosh(a*log(f) + d) + sqrt(pi)*(c*log(f) - f)*sinh(a*l
og(f) + d))*sqrt(-c*log(f) - f)*erf(sqrt(-c*log(f) - f)*x))/(c^2*log(f)^2
- f^2)
```

Sympy [F]

$$\int f^{a+cx^2}\sinh\left(d+fx^2\right)\,dx=\int f^{a+cx^2}\sinh\left(d+fx^2\right)dx$$

input
```
integrate(f**(c*x**2+a)*sinh(f*x**2+d),x)
```

output
```
Integral(f**(a + c*x**2)*sinh(d + f*x**2), x)
```

Maxima [A] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 69, normalized size of antiderivative = 0.85

$$\int f^{a+cx^2}\sinh\left(d+fx^2\right)\,dx=-\frac{\sqrt{\pi}f^a\,\mathrm{erf}\left(\sqrt{-c\log\left(f\right)+f}x\right)e^{(-d)}}{4\sqrt{-c\log\left(f\right)+f}}+\frac{\sqrt{\pi}f^a\,\mathrm{erf}\left(\sqrt{-c\log\left(f\right)-f}x\right)e^{d}}{4\sqrt{-c\log\left(f\right)-f}}$$

input
```
integrate(f^(c*x^2+a)*sinh(f*x^2+d),x, algorithm="maxima")
```

output
```
-1/4*sqrt(pi)*f^a*erf(sqrt(-c*log(f) + f)*x)*e^(-d)/sqrt(-c*log(f) + f) +
1/4*sqrt(pi)*f^a*erf(sqrt(-c*log(f) - f)*x)*e^d/sqrt(-c*log(f) - f)
```

Giac [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 75, normalized size of antiderivative = 0.93

$$\int f^{a+cx^2} \sinh\left(d+fx^2\right)\,dx = -\frac{\sqrt{\pi}\,\mathrm{erf}\left(-\sqrt{-c\log\left(f\right)-f}\,x\right)e^{\left(a\log(f)+d\right)}}{4\sqrt{-c\log\left(f\right)-f}} + \frac{\sqrt{\pi}\,\mathrm{erf}\left(-\sqrt{-c\log\left(f\right)+f}\,x\right)e^{\left(a\log(f)-d\right)}}{4\sqrt{-c\log\left(f\right)+f}}$$

input
```
integrate(f^(c*x^2+a)*sinh(f*x^2+d),x, algorithm="giac")
```

output
```
-1/4*sqrt(pi)*erf(-sqrt(-c*log(f) - f)*x)*e^(a*log(f) + d)/sqrt(-c*log(f)
- f) + 1/4*sqrt(pi)*erf(-sqrt(-c*log(f) + f)*x)*e^(a*log(f) - d)/sqrt(-c*l
og(f) + f)
```

Mupad [F(-1)]

Timed out.

$$\int f^{a+cx^2} \sinh\left(d+fx^2\right)\,dx = \int f^{c\,x^2+a}\,\sinh\left(f\,x^2+d\right)\,dx$$

input
```
int(f^(a + c*x^2)*sinh(d + f*x^2),x)
```

output
```
int(f^(a + c*x^2)*sinh(d + f*x^2), x)
```

Reduce [F]

$$\int f^{a+cx^2} \sinh\left(d+fx^2\right)\,dx = f^a\left(\int f^{c\,x^2}\sinh\left(f\,x^2+d\right)dx\right)$$

input
```
int(f^(c*x^2+a)*sinh(f*x^2+d),x)
```

output
```
f**a*int(f**(c*x**2)*sinh(d + f*x**2),x)
```

3.352 $\int f^{a+cx^2}\sinh^2\left(d+fx^2\right)\,dx$

Optimal result

Integrand size = 20, antiderivative size = 128

$$\int f^{a+cx^2}\sinh^2\left(d+fx^2\right)\,dx=-\frac{f^a\sqrt{\pi}\,\text{erfi}\left(\sqrt{c}x\sqrt{\log(f)}\right)}{4\sqrt{c}\sqrt{\log(f)}}+\frac{e^{-2d}f^a\sqrt{\pi}\,\text{erf}\left(x\sqrt{2f-c\log(f)}\right)}{8\sqrt{2f-c\log(f)}}+\frac{e^{2d}f^a\sqrt{\pi}\,\text{erfi}\left(x\sqrt{2f+c\log(f)}\right)}{8\sqrt{2f+c\log(f)}}$$

output
```
-1/4*f^a*Pi^(1/2)*erfi(c^(1/2)*x*ln(f)^(1/2))/c^(1/2)/ln(f)^(1/2)+1/8*f^a*
Pi^(1/2)*erf(x*(2*f-c*ln(f))^(1/2))/exp(2*d)/(2*f-c*ln(f))^(1/2)+1/8*exp(2
*d)*f^a*Pi^(1/2)*erfi(x*(2*f+c*ln(f))^(1/2))/(2*f+c*ln(f))^(1/2)
```

Mathematica [A] (verified)

Time = 0.36 (sec) , antiderivative size = 179, normalized size of antiderivative = 1.40

$$\int f^{a+cx^2} \sinh^2\left(d+fx^2\right)\,dx$$
$$= \frac{f^a\sqrt{\pi}\left(\text{erfi}\left(\sqrt{c}x\sqrt{\log(f)}\right)\left(8f^2-2c^2\log^2(f)\right)+\sqrt{c}\sqrt{\log(f)}\left(\text{erf}\left(x\sqrt{2f-c\log(f)}\right)\sqrt{2f-c\log(f)}(2\right.\right.}{8\sqrt{c}\sqrt{\log}}$$

input
```
Integrate[f^(a + c*x^2)*Sinh[d + f*x^2]^2,x]
```

output
```
(f^a*Sqrt[Pi]*(Erfi[Sqrt[c]*x*Sqrt[Log[f]]]*(8*f^2 - 2*c^2*Log[f]^2) + Sqr
t[c]*Sqrt[Log[f]]*(Erf[x*Sqrt[2*f - c*Log[f]]]*Sqrt[2*f - c*Log[f]]*(2*f +
 c*Log[f])*(-Cosh[2*d] + Sinh[2*d]) - Erfi[x*Sqrt[2*f + c*Log[f]]]*(2*f -
c*Log[f])*Sqrt[2*f + c*Log[f]]*(Cosh[2*d] + Sinh[2*d]))))/(8*Sqrt[c]*Sqrt[
Log[f]]*(-4*f^2 + c^2*Log[f]^2))
```

Rubi [A] (verified)

Time = 0.47 (sec) , antiderivative size = 128, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {6038, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int f^{a+cx^2} \sinh^2\left(d+fx^2\right)\,dx$$
$$\Big\downarrow \text{ 6038}$$
$$\int \left(\frac{1}{4}e^{-2d-2fx^2}f^{a+cx^2}+\frac{1}{4}e^{2d+2fx^2}f^{a+cx^2}-\frac{1}{2}f^{a+cx^2}\right)dx$$
$$\Big\downarrow \text{ 2009}$$
$$\frac{\sqrt{\pi}e^{-2d}f^a\text{erf}\left(x\sqrt{2f-c\log(f)}\right)}{8\sqrt{2f-c\log(f)}}+\frac{\sqrt{\pi}e^{2d}f^a\text{erfi}\left(x\sqrt{c\log(f)+2f}\right)}{8\sqrt{c\log(f)+2f}}-\frac{\sqrt{\pi}f^a\text{erfi}\left(\sqrt{c}x\sqrt{\log(f)}\right)}{4\sqrt{c}\sqrt{\log(f)}}$$

input
```
Int[f^(a + c*x^2)*Sinh[d + f*x^2]^2,x]
```

output
```
-1/4*(f^a*Sqrt[Pi]*Erfi[Sqrt[c]*x*Sqrt[Log[f]]])/(Sqrt[c]*Sqrt[Log[f]]) +
(f^a*Sqrt[Pi]*Erf[x*Sqrt[2*f - c*Log[f]]])/(8*E^(2*d)*Sqrt[2*f - c*Log[f]]
) + (E^(2*d)*f^a*Sqrt[Pi]*Erfi[x*Sqrt[2*f + c*Log[f]]])/(8*Sqrt[2*f + c*Lo
g[f]])
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 6038
```
Int[(F_)^(u_)*Sinh[v_]^(n_.), x_Symbol] :> Int[ExpandTrigToExp[F^u, Sinh[v]
^n, x], x] /; FreeQ[F, x] && (LinearQ[u, x] || PolyQ[u, x, 2]) && (LinearQ[
v, x] || PolyQ[v, x, 2]) && IGtQ[n, 0]
```

Maple [A] (verified)

Time = 0.31 (sec) , antiderivative size = 101, normalized size of antiderivative = 0.79

method	result	size
risch	$\frac{f^{a} \mathrm{e}^{-2 d} \sqrt{\pi}\, \operatorname{erf}\left(x \sqrt{2 f -c \ln \left(f \right)}\right)}{8 \sqrt{2 f -c \ln \left(f \right)}}+\frac{f^{a} \mathrm{e}^{2 d} \sqrt{\pi}\, \operatorname{erf}\left(\sqrt{-c \ln \left(f \right)-2 f}\, x \right)}{8 \sqrt{-c \ln \left(f \right)-2 f}}-\frac{f^{a} \sqrt{\pi}\, \operatorname{erf}\left(\sqrt{-c \ln \left(f \right)}\, x \right)}{4 \sqrt{-c \ln \left(f \right)}}$	101

input
```
int(f^(c*x^2+a)*sinh(f*x^2+d)^2,x,method=_RETURNVERBOSE)
```

output
```
1/8*f^a*exp(-2*d)*Pi^(1/2)/(2*f-c*ln(f))^(1/2)*erf(x*(2*f-c*ln(f))^(1/2))+
1/8*f^a*exp(2*d)*Pi^(1/2)/(-c*ln(f)-2*f)^(1/2)*erf((-c*ln(f)-2*f)^(1/2)*x)
-1/4*f^a*Pi^(1/2)/(-c*ln(f))^(1/2)*erf((-c*ln(f))^(1/2)*x)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 254 vs. $2(98) = 196$.

Time = 0.10 (sec) , antiderivative size = 254, normalized size of antiderivative = 1.98

$$\int f^{a+cx^2}\sinh^2\left(d+fx^2\right)\,dx =$$

$$-\frac{\left(\sqrt{\pi}\left(c^2\log\left(f\right)^2+2\,cf\log\left(f\right)\right)\cosh\left(a\log\left(f\right)-2\,d\right)+\sqrt{\pi}\left(c^2\log\left(f\right)^2+2\,cf\log\left(f\right)\right)\sinh\left(a\log\left(f\right)-\right.\right.}{}$$

input
```
integrate(f^(c*x^2+a)*sinh(f*x^2+d)^2,x, algorithm="fricas")
```

output
```
-1/8*((sqrt(pi)*(c^2*log(f)^2 + 2*c*f*log(f))*cosh(a*log(f) - 2*d) + sqrt(
pi)*(c^2*log(f)^2 + 2*c*f*log(f))*sinh(a*log(f) - 2*d))*sqrt(-c*log(f) + 2
*f)*erf(sqrt(-c*log(f) + 2*f)*x) + (sqrt(pi)*(c^2*log(f)^2 - 2*c*f*log(f))
*cosh(a*log(f) + 2*d) + sqrt(pi)*(c^2*log(f)^2 - 2*c*f*log(f))*sinh(a*log(
f) + 2*d))*sqrt(-c*log(f) - 2*f)*erf(sqrt(-c*log(f) - 2*f)*x) - 2*(sqrt(pi
)*(c^2*log(f)^2 - 4*f^2)*cosh(a*log(f)) + sqrt(pi)*(c^2*log(f)^2 - 4*f^2)*
sinh(a*log(f)))*sqrt(-c*log(f))*erf(sqrt(-c*log(f))*x))/(c^3*log(f)^3 - 4*
c*f^2*log(f))
```

Sympy [F]

$$\int f^{a+cx^2}\sinh^2\left(d+fx^2\right)\,dx=\int f^{a+cx^2}\sinh^2\left(d+fx^2\right)dx$$

input
```
integrate(f**(c*x**2+a)*sinh(f*x**2+d)**2,x)
```

output
```
Integral(f**(a + c*x**2)*sinh(d + f*x**2)**2, x)
```

Maxima [A] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 100, normalized size of antiderivative = 0.78

$$\int f^{a+cx^2}\sinh^2\left(d+fx^2\right)\,dx = \frac{\sqrt{\pi}f^a \operatorname{erf}\left(\sqrt{-c\log\left(f\right)-2\,f}x\right)e^{(2\,d)}}{8\,\sqrt{-c\log\left(f\right)-2\,f}} + \frac{\sqrt{\pi}f^a \operatorname{erf}\left(\sqrt{-c\log\left(f\right)+2\,f}x\right)e^{(-2\,d)}}{8\,\sqrt{-c\log\left(f\right)+2\,f}} - \frac{\sqrt{\pi}f^a \operatorname{erf}\left(\sqrt{-c\log\left(f\right)}x\right)}{4\,\sqrt{-c\log\left(f\right)}}$$

input
```
integrate(f^(c*x^2+a)*sinh(f*x^2+d)^2,x, algorithm="maxima")
```

output
```
1/8*sqrt(pi)*f^a*erf(sqrt(-c*log(f) - 2*f)*x)*e^(2*d)/sqrt(-c*log(f) - 2*f
) + 1/8*sqrt(pi)*f^a*erf(sqrt(-c*log(f) + 2*f)*x)*e^(-2*d)/sqrt(-c*log(f)
+ 2*f) - 1/4*sqrt(pi)*f^a*erf(sqrt(-c*log(f))*x)/sqrt(-c*log(f))
```

Giac [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 107, normalized size of antiderivative = 0.84

$$\int f^{a+cx^2}\sinh^2\left(d+fx^2\right)\,dx = \frac{\sqrt{\pi}f^a \operatorname{erf}\left(-\sqrt{-c\log\left(f\right)}x\right)}{4\,\sqrt{-c\log\left(f\right)}} - \frac{\sqrt{\pi}\operatorname{erf}\left(-\sqrt{-c\log\left(f\right)-2\,f}x\right)e^{(a\log(f)+2\,d)}}{8\,\sqrt{-c\log\left(f\right)-2\,f}} - \frac{\sqrt{\pi}\operatorname{erf}\left(-\sqrt{-c\log\left(f\right)+2\,f}x\right)e^{(a\log(f)-2\,d)}}{8\,\sqrt{-c\log\left(f\right)+2\,f}}$$

input
```
integrate(f^(c*x^2+a)*sinh(f*x^2+d)^2,x, algorithm="giac")
```

output
```
1/4*sqrt(pi)*f^a*erf(-sqrt(-c*log(f))*x)/sqrt(-c*log(f)) - 1/8*sqrt(pi)*er
f(-sqrt(-c*log(f) - 2*f)*x)*e^(a*log(f) + 2*d)/sqrt(-c*log(f) - 2*f) - 1/8
*sqrt(pi)*erf(-sqrt(-c*log(f) + 2*f)*x)*e^(a*log(f) - 2*d)/sqrt(-c*log(f)
+ 2*f)
```

Mupad [F(-1)]

Timed out.

$$\int f^{a+cx^2}\sinh^2\left(d+fx^2\right)\,dx=\int f^{c\,x^2+a}\,\sinh\left(f\,x^2+d\right)^2\,dx$$

input
```
int(f^(a + c*x^2)*sinh(d + f*x^2)^2,x)
```

output
```
int(f^(a + c*x^2)*sinh(d + f*x^2)^2, x)
```

Reduce [F]

$$\int f^{a+cx^2}\sinh^2\left(d+fx^2\right)\,dx=f^a\left(\int f^{c\,x^2}\sinh\left(f\,x^2+d\right)^2dx\right)$$

input
```
int(f^(c*x^2+a)*sinh(f*x^2+d)^2,x)
```

output
```
f**a*int(f**(c*x**2)*sinh(d + f*x**2)**2,x)
```

3.353 $\int f^{a+cx^2}\sinh^3\left(d+fx^2\right)\,dx$

Optimal result

Integrand size = 20, antiderivative size = 171

$$\int f^{a+cx^2}\sinh^3\left(d+fx^2\right)\,dx=\frac{3e^{-d}f^a\sqrt{\pi}\,\text{erf}\left(x\sqrt{f-c\log(f)}\right)}{16\sqrt{f-c\log(f)}}-\frac{e^{-3d}f^a\sqrt{\pi}\,\text{erf}\left(x\sqrt{3f-c\log(f)}\right)}{16\sqrt{3f-c\log(f)}}-\frac{3e^{d}f^a\sqrt{\pi}\,\text{erfi}\left(x\sqrt{f+c\log(f)}\right)}{16\sqrt{f+c\log(f)}}+\frac{e^{3d}f^a\sqrt{\pi}\,\text{erfi}\left(x\sqrt{3f+c\log(f)}\right)}{16\sqrt{3f+c\log(f)}}$$

output
```
3/16*f^a*Pi^(1/2)*erf(x*(f-c*ln(f))^(1/2))/exp(d)/(f-c*ln(f))^(1/2)-1/16*f
^a*Pi^(1/2)*erf(x*(3*f-c*ln(f))^(1/2))/exp(3*d)/(3*f-c*ln(f))^(1/2)-3/16*e
xp(d)*f^a*Pi^(1/2)*erfi(x*(f+c*ln(f))^(1/2))/(f+c*ln(f))^(1/2)+1/16*exp(3*
d)*f^a*Pi^(1/2)*erfi(x*(3*f+c*ln(f))^(1/2))/(3*f+c*ln(f))^(1/2)
```

Mathematica [A] (verified)

Time = 0.85 (sec) , antiderivative size = 272, normalized size of antiderivative = 1.59

$$\int f^{a+cx^2}\sinh^3\left(d+fx^2\right)\,dx$$
$$=\frac{f^a\sqrt{\pi}\left(3\text{erf}\left(x\sqrt{f-c\log(f)}\right)\sqrt{f-c\log(f)}\left(9f^3+9cf^2\log(f)-c^2f\log^2(f)-c^3\log^3(f)\right)(\cosh(d)-s\right.}{}$$

input
```
Integrate[f^(a + c*x^2)*Sinh[d + f*x^2]^3,x]
```

output
```
(f^a*Sqrt[Pi]*(3*Erf[x*Sqrt[f - c*Log[f]]]*Sqrt[f - c*Log[f]]*(9*f^3 + 9*c
*f^2*Log[f] - c^2*f*Log[f]^2 - c^3*Log[f]^3)*(Cosh[d] - Sinh[d]) - (f - c*
Log[f])*(Erf[x*Sqrt[3*f - c*Log[f]]]*Sqrt[3*f - c*Log[f]]*(3*f^2 + 4*c*f*L
og[f] + c^2*Log[f]^2)*(Cosh[3*d] - Sinh[3*d]) + (3*f - c*Log[f])*(3*Erfi[x
*Sqrt[f + c*Log[f]]]*Sqrt[f + c*Log[f]]*(3*f + c*Log[f])*(Cosh[d] + Sinh[d
]) - Erfi[x*Sqrt[3*f + c*Log[f]]]*(f + c*Log[f])*Sqrt[3*f + c*Log[f]]*(Cos
h[3*d] + Sinh[3*d])))))/(16*(9*f^4 - 10*c^2*f^2*Log[f]^2 + c^4*Log[f]^4))
```

Rubi [A] (verified)

Time = 0.54 (sec) , antiderivative size = 171, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {6038, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int f^{a+cx^2}\sinh^3\left(d+fx^2\right)\,dx$$
$$\Big\downarrow \text{ 6038}$$
$$\int\left(-\frac{1}{8}e^{-3d-3fx^2}f^{a+cx^2}+\frac{3}{8}e^{-d-fx^2}f^{a+cx^2}-\frac{3}{8}e^{d+fx^2}f^{a+cx^2}+\frac{1}{8}e^{3d+3fx^2}f^{a+cx^2}\right)dx$$
$$\Big\downarrow \text{ 2009}$$

$$\frac{3\sqrt{\pi}e^{-d}f^a\text{erf}\left(x\sqrt{f-c\log(f)}\right)}{16\sqrt{f-c\log(f)}}-\frac{\sqrt{\pi}e^{-3d}f^a\text{erf}\left(x\sqrt{3f-c\log(f)}\right)}{16\sqrt{3f-c\log(f)}}-\frac{3\sqrt{\pi}e^{d}f^a\text{erfi}\left(x\sqrt{c\log(f)+f}\right)}{16\sqrt{c\log(f)+f}}+\frac{\sqrt{\pi}e^{3d}f^a\text{erfi}\left(x\sqrt{c\log(f)+3f}\right)}{16\sqrt{c\log(f)+3f}}$$

input
```
Int[f^(a + c*x^2)*Sinh[d + f*x^2]^3,x]
```

output
```
(3*f^a*Sqrt[Pi]*Erf[x*Sqrt[f - c*Log[f]]])/(16*E^d*Sqrt[f - c*Log[f]]) - (
f^a*Sqrt[Pi]*Erf[x*Sqrt[3*f - c*Log[f]]])/(16*E^(3*d)*Sqrt[3*f - c*Log[f]]
) - (3*E^d*f^a*Sqrt[Pi]*Erfi[x*Sqrt[f + c*Log[f]]])/(16*Sqrt[f + c*Log[f]]
) + (E^(3*d)*f^a*Sqrt[Pi]*Erfi[x*Sqrt[3*f + c*Log[f]]])/(16*Sqrt[3*f + c*L
og[f]])
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 6038
```
Int[(F_)^(u_)*Sinh[v_]^(n_.), x_Symbol] :> Int[ExpandTrigToExp[F^u, Sinh[v]
^n, x], x] /; FreeQ[F, x] && (LinearQ[u, x] || PolyQ[u, x, 2]) && (LinearQ[
v, x] || PolyQ[v, x, 2]) && IGtQ[n, 0]
```

Maple [A] (verified)

Time = 0.80 (sec) , antiderivative size = 144, normalized size of antiderivative = 0.84

method	result
risch	$\frac{f^{a} \mathrm{e}^{3 d} \sqrt{\pi}\, \mathrm{erf}\left(\sqrt{-c \ln(f)-3 f}\, x\right)}{16 \sqrt{-c \ln(f)-3 f}}-\frac{f^{a} \mathrm{e}^{-3 d} \sqrt{\pi}\, \mathrm{erf}\left(x \sqrt{3 f-c \ln(f)}\right)}{16 \sqrt{3 f-c \ln(f)}}+\frac{3 f^{a} \mathrm{e}^{-d} \sqrt{\pi}\, \mathrm{erf}\left(x \sqrt{f-c \ln(f)}\right)}{16 \sqrt{f-c \ln(f)}}-\frac{3 f^{a} \mathrm{e}^{d} \sqrt{\pi}\, \mathrm{erf}\left(\sqrt{-c\right.}{16 \sqrt{-c \ln(f}$

input
```
int(f^(c*x^2+a)*sinh(f*x^2+d)^3,x,method=_RETURNVERBOSE)
```

output
```
1/16*f^a*exp(3*d)*Pi^(1/2)/(-c*ln(f)-3*f)^(1/2)*erf((-c*ln(f)-3*f)^(1/2)*x
)-1/16*f^a*exp(-3*d)*Pi^(1/2)/(3*f-c*ln(f))^(1/2)*erf(x*(3*f-c*ln(f))^(1/2
))+3/16*f^a*exp(-d)*Pi^(1/2)/(f-c*ln(f))^(1/2)*erf(x*(f-c*ln(f))^(1/2))-3/
16*f^a*exp(d)*Pi^(1/2)/(-c*ln(f)-f)^(1/2)*erf((-c*ln(f)-f)^(1/2)*x)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 492 vs. $2(135) = 270$.

Time = 0.12 (sec) , antiderivative size = 492, normalized size of antiderivative = 2.88

$$\int f^{a+cx^2}\sinh^3\left(d+fx^2\right)\,dx = \text{Too large to display}$$

input
```
integrate(f^(c*x^2+a)*sinh(f*x^2+d)^3,x, algorithm="fricas")
```

output
```
1/16*((sqrt(pi)*(c^3*log(f)^3 + 3*c^2*f*log(f)^2 - c*f^2*log(f) - 3*f^3)*c
osh(a*log(f) - 3*d) + sqrt(pi)*(c^3*log(f)^3 + 3*c^2*f*log(f)^2 - c*f^2*lo
g(f) - 3*f^3)*sinh(a*log(f) - 3*d))*sqrt(-c*log(f) + 3*f)*erf(sqrt(-c*log(
f) + 3*f)*x) - 3*(sqrt(pi)*(c^3*log(f)^3 + c^2*f*log(f)^2 - 9*c*f^2*log(f)
 - 9*f^3)*cosh(a*log(f) - d) + sqrt(pi)*(c^3*log(f)^3 + c^2*f*log(f)^2 - 9
*c*f^2*log(f) - 9*f^3)*sinh(a*log(f) - d))*sqrt(-c*log(f) + f)*erf(sqrt(-c
*log(f) + f)*x) + 3*(sqrt(pi)*(c^3*log(f)^3 - c^2*f*log(f)^2 - 9*c*f^2*log
(f) + 9*f^3)*cosh(a*log(f) + d) + sqrt(pi)*(c^3*log(f)^3 - c^2*f*log(f)^2
- 9*c*f^2*log(f) + 9*f^3)*sinh(a*log(f) + d))*sqrt(-c*log(f) - f)*erf(sqrt
(-c*log(f) - f)*x) - (sqrt(pi)*(c^3*log(f)^3 - 3*c^2*f*log(f)^2 - c*f^2*lo
g(f) + 3*f^3)*cosh(a*log(f) + 3*d) + sqrt(pi)*(c^3*log(f)^3 - 3*c^2*f*log(
f)^2 - c*f^2*log(f) + 3*f^3)*sinh(a*log(f) + 3*d))*sqrt(-c*log(f) - 3*f)*e
rf(sqrt(-c*log(f) - 3*f)*x))/(c^4*log(f)^4 - 10*c^2*f^2*log(f)^2 + 9*f^4)
```

Sympy [F]

$$\int f^{a+cx^2}\sinh^3\left(d+fx^2\right)\,dx=\int f^{a+cx^2}\sinh^3\left(d+fx^2\right)dx$$

input
```
integrate(f**(c*x**2+a)*sinh(f*x**2+d)**3,x)
```

output
```
Integral(f**(a + c*x**2)*sinh(d + f*x**2)**3, x)
```

Maxima [A] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 143, normalized size of antiderivative = 0.84

$$\int f^{a+cx^2}\sinh^3\left(d+fx^2\right)\,dx=\frac{\sqrt{\pi}f^a\operatorname{erf}\left(\sqrt{-c\log\left(f\right)-3\,f}x\right)e^{(3\,d)}}{16\,\sqrt{-c\log\left(f\right)-3\,f}}+\frac{3\,\sqrt{\pi}f^a\operatorname{erf}\left(\sqrt{-c\log\left(f\right)+f}x\right)e^{(-d)}}{16\,\sqrt{-c\log\left(f\right)+f}}-\frac{\sqrt{\pi}f^a\operatorname{erf}\left(\sqrt{-c\log\left(f\right)+3\,f}x\right)e^{(-3\,d)}}{16\,\sqrt{-c\log\left(f\right)+3\,f}}-\frac{3\,\sqrt{\pi}f^a\operatorname{erf}\left(\sqrt{-c\log\left(f\right)-f}x\right)e^{d}}{16\,\sqrt{-c\log\left(f\right)-f}}$$

input
```
integrate(f^(c*x^2+a)*sinh(f*x^2+d)^3,x, algorithm="maxima")
```

output
```
1/16*sqrt(pi)*f^a*erf(sqrt(-c*log(f) - 3*f)*x)*e^(3*d)/sqrt(-c*log(f) - 3*
f) + 3/16*sqrt(pi)*f^a*erf(sqrt(-c*log(f) + f)*x)*e^(-d)/sqrt(-c*log(f) +
f) - 1/16*sqrt(pi)*f^a*erf(sqrt(-c*log(f) + 3*f)*x)*e^(-3*d)/sqrt(-c*log(f
) + 3*f) - 3/16*sqrt(pi)*f^a*erf(sqrt(-c*log(f) - f)*x)*e^d/sqrt(-c*log(f)
 - f)
```

Giac [A] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 155, normalized size of antiderivative = 0.91

$$\int f^{a+cx^2}\sinh^3\left(d+fx^2\right)\,dx=-\frac{\sqrt{\pi}\,\mathrm{erf}\left(-\sqrt{-c\log\left(f\right)-3\,f}x\right)e^{\left(a\log\left(f\right)+3\,d\right)}}{16\,\sqrt{-c\log\left(f\right)-3\,f}}+\frac{3\,\sqrt{\pi}\,\mathrm{erf}\left(-\sqrt{-c\log\left(f\right)-f}x\right)e^{\left(a\log\left(f\right)+d\right)}}{16\,\sqrt{-c\log\left(f\right)-f}}-\frac{3\,\sqrt{\pi}\,\mathrm{erf}\left(-\sqrt{-c\log\left(f\right)+f}x\right)e^{\left(a\log\left(f\right)-d\right)}}{16\,\sqrt{-c\log\left(f\right)+f}}+\frac{\sqrt{\pi}\,\mathrm{erf}\left(-\sqrt{-c\log\left(f\right)+3\,f}x\right)e^{\left(a\log\left(f\right)-3\,d\right)}}{16\,\sqrt{-c\log\left(f\right)+3\,f}}$$

input
```
integrate(f^(c*x^2+a)*sinh(f*x^2+d)^3,x, algorithm="giac")
```

output
```
-1/16*sqrt(pi)*erf(-sqrt(-c*log(f) - 3*f)*x)*e^(a*log(f) + 3*d)/sqrt(-c*lo
g(f) - 3*f) + 3/16*sqrt(pi)*erf(-sqrt(-c*log(f) - f)*x)*e^(a*log(f) + d)/s
qrt(-c*log(f) - f) - 3/16*sqrt(pi)*erf(-sqrt(-c*log(f) + f)*x)*e^(a*log(f)
 - d)/sqrt(-c*log(f) + f) + 1/16*sqrt(pi)*erf(-sqrt(-c*log(f) + 3*f)*x)*e^
(a*log(f) - 3*d)/sqrt(-c*log(f) + 3*f)
```

Mupad [F(-1)]

Timed out.

$$\int f^{a+cx^2}\sinh^3\left(d+fx^2\right)\,dx=\int f^{c\,x^2+a}\,{\mathrm{sinh}\left(f\,x^2+d\right)}^3\,dx$$

input
```
int(f^(a + c*x^2)*sinh(d + f*x^2)^3,x)
```

output
```
int(f^(a + c*x^2)*sinh(d + f*x^2)^3, x)
```

Reduce [F]

$$\int f^{a+cx^2}\sinh^3\left(d+fx^2\right)\,dx = f^a\left(\int f^{c\,x^2}\sinh\left(f\,x^2+d\right)^3dx\right)$$

input
```
int(f^(c*x^2+a)*sinh(f*x^2+d)^3,x)
```

output
```
f**a*int(f**(c*x**2)*sinh(d + f*x**2)**3,x)
```

3.354 $\int f^{a+cx^2} \sinh\left(d+ex+fx^2\right) dx$

Optimal result

Integrand size = 21, antiderivative size = 140

$$\int f^{a+cx^2}\sinh\left(d+ex+fx^2\right)\,dx=-\frac{e^{-d+\frac{e^2}{4f-4c\log(f)}}f^a\sqrt{\pi}\operatorname{erf}\left(\frac{e+2x(f-c\log(f))}{2\sqrt{f-c\log(f)}}\right)}{4\sqrt{f-c\log(f)}}+\frac{e^{d-\frac{e^2}{4(f+c\log(f))}}f^a\sqrt{\pi}\operatorname{erfi}\left(\frac{e+2x(f+c\log(f))}{2\sqrt{f+c\log(f)}}\right)}{4\sqrt{f+c\log(f)}}$$

output
```
-1/4*exp(-d+e^2/(4*f-4*c*ln(f)))*f^a*Pi^(1/2)*erf(1/2*(e+2*x*(f-c*ln(f)))/
(f-c*ln(f))^(1/2))/(f-c*ln(f))^(1/2)+1/4*exp(d-e^2/(4*f+4*c*ln(f)))*f^a*Pi
^(1/2)*erfi(1/2*(e+2*x*(f+c*ln(f)))/(f+c*ln(f))^(1/2))/(f+c*ln(f))^(1/2)
```

Mathematica [A] (verified)

Time = 0.48 (sec) , antiderivative size = 166, normalized size of antiderivative = 1.19

$$\int f^{a+cx^2}\sinh\left(d+ex+fx^2\right)\,dx$$
$$=\frac{e^{-\frac{e^2}{4(f+c\log(f))}}f^a\sqrt{\pi}\left(-e^{\frac{e^2f}{2f^2-2c^2\log^2(f)}}\operatorname{erf}\left(\frac{e+2fx-2cx\log(f)}{2\sqrt{f-c\log(f)}}\right)\sqrt{f+c\log(f)}(\cosh(d)-\sinh(d))+\operatorname{erfi}\left(\frac{e+2fx+2c\ldots}{2\sqrt{f+c\ldots}}\right.\right.}{4\sqrt{f-c\log(f)}\sqrt{f+c\log(f)}}$$

input
```
Integrate[f^(a + c*x^2)*Sinh[d + e*x + f*x^2],x]
```

output
```
(f^a*Sqrt[Pi]*(-(E^((e^2*f)/(2*f^2 - 2*c^2*Log[f]^2))*Erf[(e + 2*f*x - 2*c
*x*Log[f])/(2*Sqrt[f - c*Log[f]])]*Sqrt[f + c*Log[f]]*(Cosh[d] - Sinh[d]))
 + Erfi[(e + 2*f*x + 2*c*x*Log[f])/(2*Sqrt[f + c*Log[f]])]*Sqrt[f - c*Log[
f]]*(Cosh[d] + Sinh[d])))/(4*E^(e^2/(4*(f + c*Log[f])))*Sqrt[f - c*Log[f]]
*Sqrt[f + c*Log[f]])
```

Rubi [A] (verified)

Time = 0.51 (sec) , antiderivative size = 140, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {6038, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int f^{a+cx^2} \sinh\left(d+ex+fx^2\right) dx$$

$$\Big\downarrow \text{6038}$$

$$\int \left(\frac{1}{2} f^{a+cx^2} e^{d+ex+fx^2} - \frac{1}{2} f^{a+cx^2} e^{-d-ex-fx^2}\right) dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{\sqrt{\pi} f^a e^{d-\frac{e^2}{4(c\log(f)+f)}} \text{erfi}\left(\frac{2x(c\log(f)+f)+e}{2\sqrt{c\log(f)+f}}\right)}{4\sqrt{c\log(f)+f}} - \frac{\sqrt{\pi} f^a e^{\frac{e^2}{4f-4c\log(f)}-d} \text{erf}\left(\frac{2x(f-c\log(f))+e}{2\sqrt{f-c\log(f)}}\right)}{4\sqrt{f-c\log(f)}}$$

input
```
Int[f^(a + c*x^2)*Sinh[d + e*x + f*x^2],x]
```

output
```
-1/4*(E^(-d + e^2/(4*f - 4*c*Log[f]))*f^a*Sqrt[Pi]*Erf[(e + 2*x*(f - c*Log
[f]))/(2*Sqrt[f - c*Log[f]])])/Sqrt[f - c*Log[f]] + (E^(d - e^2/(4*(f + c*
Log[f])))*f^a*Sqrt[Pi]*Erfi[(e + 2*x*(f + c*Log[f]))/(2*Sqrt[f + c*Log[f]]
)])/(4*Sqrt[f + c*Log[f]])
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 6038
```
Int[(F_)^(u_)*Sinh[v_]^(n_.), x_Symbol] :> Int[ExpandTrigToExp[F^u, Sinh[v]
^n, x], x] /; FreeQ[F, x] && (LinearQ[u, x] || PolyQ[u, x, 2]) && (LinearQ[
v, x] || PolyQ[v, x, 2]) && IGtQ[n, 0]
```

Maple [A] (verified)

Time = 0.17 (sec) , antiderivative size = 147, normalized size of antiderivative = 1.05

method	result
risch	$-\frac{\operatorname{erf}\left(-\sqrt{-c\ln(f)-f}\,x+\frac{e}{2\sqrt{-c\ln(f)-f}}\right)\sqrt{\pi}\,f^{a}\mathrm{e}^{\frac{4d\ln(f)c+4df-e^{2}}{4f+4c\ln(f)}}}{4\sqrt{-c\ln(f)-f}}-\frac{\operatorname{erf}\left(x\sqrt{f-c\ln(f)}+\frac{e}{2\sqrt{f-c\ln(f)}}\right)\sqrt{\pi}\,f^{a}\mathrm{e}^{-\frac{4d\ln(f)c-4df+e^{2}}{4(c\ln(f)-f)}}}{4\sqrt{f-c\ln(f)}}$

input
```
int(f^(c*x^2+a)*sinh(f*x^2+e*x+d),x,method=_RETURNVERBOSE)
```

output
```
-1/4*erf(-(-c*ln(f)-f)^(1/2)*x+1/2*e/(-c*ln(f)-f)^(1/2))/(-c*ln(f)-f)^(1/2
)*Pi^(1/2)*f^a*exp(1/4*(4*d*ln(f)*c+4*d*f-e^2)/(f+c*ln(f)))-1/4*erf(x*(f-c
*ln(f))^(1/2)+1/2*e/(f-c*ln(f))^(1/2))/(f-c*ln(f))^(1/2)*Pi^(1/2)*f^a*exp(
-1/4*(4*d*ln(f)*c-4*d*f+e^2)/(c*ln(f)-f))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 322 vs. 2(119) = 238.

Time = 0.09 (sec) , antiderivative size = 322, normalized size of antiderivative = 2.30

$$\int f^{a+cx^2}\sinh\left(d+ex+fx^2\right)\,dx$$
$$=\frac{\left(\sqrt{\pi}(c\log(f)+f)\cosh\left(\frac{4\,ac\log(f)^2-e^2+4\,df-4\,(cd+af)\log(f)}{4\,(c\log(f)-f)}\right)+\sqrt{\pi}(c\log(f)+f)\sinh\left(\frac{4\,ac\log(f)^2-e^2+4\,df-4\,(c}{4\,(c\log(f)-f)}\right.\right.}{}$$

input
```
integrate(f^(c*x^2+a)*sinh(f*x^2+e*x+d),x, algorithm="fricas")
```

output
```
1/4*((sqrt(pi)*(c*log(f) + f)*cosh(1/4*(4*a*c*log(f)^2 - e^2 + 4*d*f - 4*(
c*d + a*f)*log(f))/(c*log(f) - f)) + sqrt(pi)*(c*log(f) + f)*sinh(1/4*(4*a
*c*log(f)^2 - e^2 + 4*d*f - 4*(c*d + a*f)*log(f))/(c*log(f) - f)))*sqrt(-c
*log(f) + f)*erf(1/2*(2*c*x*log(f) - 2*f*x - e)*sqrt(-c*log(f) + f)/(c*log
(f) - f)) - (sqrt(pi)*(c*log(f) - f)*cosh(1/4*(4*a*c*log(f)^2 - e^2 + 4*d*
f + 4*(c*d + a*f)*log(f))/(c*log(f) + f)) + sqrt(pi)*(c*log(f) - f)*sinh(1
/4*(4*a*c*log(f)^2 - e^2 + 4*d*f + 4*(c*d + a*f)*log(f))/(c*log(f) + f)))*
sqrt(-c*log(f) - f)*erf(1/2*(2*c*x*log(f) + 2*f*x + e)*sqrt(-c*log(f) - f)
/(c*log(f) + f)))/(c^2*log(f)^2 - f^2)
```

Sympy [F]

$$\int f^{a+cx^2}\sinh\left(d+ex+fx^2\right)\,dx=\int f^{a+cx^2}\sinh\left(d+ex+fx^2\right)dx$$

input
```
integrate(f**(c*x**2+a)*sinh(f*x**2+e*x+d),x)
```

output
```
Integral(f**(a + c*x**2)*sinh(d + e*x + f*x**2), x)
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 127, normalized size of antiderivative = 0.91

$$\int f^{a+cx^2}\sinh\left(d+ex+fx^2\right)\,dx = \frac{\sqrt{\pi}f^a\operatorname{erf}\left(\sqrt{-c\log\left(f\right)-f}x-\frac{e}{2\sqrt{-c\log(f)-f}}\right)e^{\left(d-\frac{e^2}{4\,(c\log(f)+f)}\right)}}{4\sqrt{-c\log\left(f\right)-f}} - \frac{\sqrt{\pi}f^a\operatorname{erf}\left(\sqrt{-c\log\left(f\right)+f}x+\frac{e}{2\sqrt{-c\log(f)+f}}\right)e^{\left(-d-\frac{e^2}{4\,(c\log(f)-f)}\right)}}{4\sqrt{-c\log\left(f\right)+f}}$$

input
```
integrate(f^(c*x^2+a)*sinh(f*x^2+e*x+d),x, algorithm="maxima")
```

output
```
1/4*sqrt(pi)*f^a*erf(sqrt(-c*log(f) - f)*x - 1/2*e/sqrt(-c*log(f) - f))*e^
(d - 1/4*e^2/(c*log(f) + f))/sqrt(-c*log(f) - f) - 1/4*sqrt(pi)*f^a*erf(sq
rt(-c*log(f) + f)*x + 1/2*e/sqrt(-c*log(f) + f))*e^(-d - 1/4*e^2/(c*log(f)
 - f))/sqrt(-c*log(f) + f)
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 172, normalized size of antiderivative = 1.23

$$\int f^{a+cx^2}\sinh\left(d+ex+fx^2\right)\,dx$$
$$=-\frac{\sqrt{\pi}\,\mathrm{erf}\left(-\frac{1}{2}\sqrt{-c\log(f)-f}\left(2x+\frac{e}{c\log(f)+f}\right)\right)e^{\left(\frac{4ac\log(f)^2+4cd\log(f)+4af\log(f)-e^2+4df}{4(c\log(f)+f)}\right)}}{4\sqrt{-c\log(f)-f}}$$
$$+\frac{\sqrt{\pi}\,\mathrm{erf}\left(-\frac{1}{2}\sqrt{-c\log(f)+f}\left(2x-\frac{e}{c\log(f)-f}\right)\right)e^{\left(\frac{4ac\log(f)^2-4cd\log(f)-4af\log(f)-e^2+4df}{4(c\log(f)-f)}\right)}}{4\sqrt{-c\log(f)+f}}$$

input
```
integrate(f^(c*x^2+a)*sinh(f*x^2+e*x+d),x, algorithm="giac")
```

output
```
-1/4*sqrt(pi)*erf(-1/2*sqrt(-c*log(f) - f)*(2*x + e/(c*log(f) + f)))*e^(1/
4*(4*a*c*log(f)^2 + 4*c*d*log(f) + 4*a*f*log(f) - e^2 + 4*d*f)/(c*log(f) +
 f))/sqrt(-c*log(f) - f) + 1/4*sqrt(pi)*erf(-1/2*sqrt(-c*log(f) + f)*(2*x
- e/(c*log(f) - f)))*e^(1/4*(4*a*c*log(f)^2 - 4*c*d*log(f) - 4*a*f*log(f)
- e^2 + 4*d*f)/(c*log(f) - f))/sqrt(-c*log(f) + f)
```

Mupad [F(-1)]

Timed out.

$$\int f^{a+cx^2}\sinh\left(d+ex+fx^2\right)\,dx=\int f^{c\,x^2+a}\,\sinh\left(f\,x^2+e\,x+d\right)\,dx$$

input
```
int(f^(a + c*x^2)*sinh(d + e*x + f*x^2),x)
```

output
```
int(f^(a + c*x^2)*sinh(d + e*x + f*x^2), x)
```

Reduce [F]

$$\int f^{a+cx^2} \sinh \left(d+ex+fx^2\right) \, dx = f^a \left(\int f^{c\,x^2} \sinh \left(f\,x^2+ex+d\right) dx\right)$$

input
```
int(f^(c*x^2+a)*sinh(f*x^2+e*x+d),x)
```

output
```
f**a*int(f**(c*x**2)*sinh(d + e*x + f*x**2),x)
```

3.355 $\int f^{a+cx^2} \sinh^2(d+ex+fx^2)\,dx$

Optimal result

Integrand size = 23, antiderivative size = 183

$$\int f^{a+cx^2}\sinh^2\left(d+ex+fx^2\right)\,dx = -\frac{f^a\sqrt{\pi}\text{erfi}\left(\sqrt{c}x\sqrt{\log(f)}\right)}{4\sqrt{c}\sqrt{\log(f)}} + \frac{e^{-2d+\frac{e^2}{2f-c\log(f)}}f^a\sqrt{\pi}\text{erf}\left(\frac{e+x(2f-c\log(f))}{\sqrt{2f-c\log(f)}}\right)}{8\sqrt{2f-c\log(f)}} + \frac{e^{2d-\frac{e^2}{2f+c\log(f)}}f^a\sqrt{\pi}\text{erfi}\left(\frac{e+x(2f+c\log(f))}{\sqrt{2f+c\log(f)}}\right)}{8\sqrt{2f+c\log(f)}}$$

output
```
-1/4*f^a*Pi^(1/2)*erfi(c^(1/2)*x*ln(f)^(1/2))/c^(1/2)/ln(f)^(1/2)+1/8*exp(
-2*d+e^2/(2*f-c*ln(f)))*f^a*Pi^(1/2)*erf((e+x*(2*f-c*ln(f)))/(2*f-c*ln(f))
^(1/2))/(2*f-c*ln(f))^(1/2)+1/8*exp(2*d-e^2/(2*f+c*ln(f)))*f^a*Pi^(1/2)*er
fi((e+x*(2*f+c*ln(f)))/(2*f+c*ln(f))^(1/2))/(2*f+c*ln(f))^(1/2)
```

Mathematica [A] (verified)

Time = 0.98 (sec) , antiderivative size = 258, normalized size of antiderivative = 1.41

$$\int f^{a+cx^2}\sinh^2\left(d+ex+fx^2\right)\,dx$$
$$=\frac{e^{\frac{e^2}{2f-c\log(f)}}f^a\sqrt{\pi}\left(2e^{\frac{e^2}{-2f+c\log(f)}}\operatorname{erfi}\left(\sqrt{c}x\sqrt{\log(f)}\right)\left(4f^2-c^2\log^2(f)\right)-\sqrt{c}\sqrt{\log(f)}\left(\operatorname{erf}\left(\frac{e+2fx-cx\log(f)}{\sqrt{2f-c\log(f)}}\right)\sqrt{\right.\right.}{}$$

input
```
Integrate[f^(a + c*x^2)*Sinh[d + e*x + f*x^2]^2,x]
```

output
```
(E^(e^2/(2*f - c*Log[f]))*f^a*Sqrt[Pi]*(2*E^(e^2/(-2*f + c*Log[f]))*Erfi[S
qrt[c]*x*Sqrt[Log[f]]]*(4*f^2 - c^2*Log[f]^2) - Sqrt[c]*Sqrt[Log[f]]*(Erf[
(e + 2*f*x - c*x*Log[f])/Sqrt[2*f - c*Log[f]]]*Sqrt[2*f - c*Log[f]]*(2*f +
 c*Log[f])*(Cosh[2*d] - Sinh[2*d]) + E^((4*e^2*f)/(-4*f^2 + c^2*Log[f]^2))
*Erfi[(e + 2*f*x + c*x*Log[f])/Sqrt[2*f + c*Log[f]]]*(2*f - c*Log[f])*Sqrt
[2*f + c*Log[f]]*(Cosh[2*d] + Sinh[2*d]))))/(8*Sqrt[c]*Sqrt[Log[f]]*(-4*f^
2 + c^2*Log[f]^2))
```

Rubi [A] (verified)

Time = 0.61 (sec) , antiderivative size = 183, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.087, Rules used = {6038, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int f^{a+cx^2}\sinh^2\left(d+ex+fx^2\right)\,dx$$
$$\Big\downarrow \text{6038}$$
$$\int \left(\frac{1}{4}f^{a+cx^2}e^{-2d-2ex-2fx^2}+\frac{1}{4}f^{a+cx^2}e^{2d+2ex+2fx^2}-\frac{1}{2}f^{a+cx^2}\right)dx$$
$$\Big\downarrow \text{2009}$$

$$\frac{\sqrt{\pi}f^a e^{\frac{e^2}{2f-c\log(f)}-2d}\operatorname{erf}\left(\frac{x(2f-c\log(f))+e}{\sqrt{2f-c\log(f)}}\right)}{8\sqrt{2f-c\log(f)}}+\frac{\sqrt{\pi}f^a e^{2d-\frac{e^2}{c\log(f)+2f}}\operatorname{erfi}\left(\frac{x(c\log(f)+2f)+e}{\sqrt{c\log(f)+2f}}\right)}{8\sqrt{c\log(f)+2f}}-\frac{\sqrt{\pi}f^a\operatorname{erfi}\left(\sqrt{c}x\sqrt{\log(f)}\right)}{4\sqrt{c}\sqrt{\log(f)}}$$

input
```
Int[f^(a + c*x^2)*Sinh[d + e*x + f*x^2]^2,x]
```

output
```
-1/4*(f^a*Sqrt[Pi]*Erfi[Sqrt[c]*x*Sqrt[Log[f]]])/(Sqrt[c]*Sqrt[Log[f]]) +
(E^(-2*d + e^2/(2*f - c*Log[f]))*f^a*Sqrt[Pi]*Erf[(e + x*(2*f - c*Log[f]))
/Sqrt[2*f - c*Log[f]]])/(8*Sqrt[2*f - c*Log[f]]) + (E^(2*d - e^2/(2*f + c*
Log[f]))*f^a*Sqrt[Pi]*Erfi[(e + x*(2*f + c*Log[f]))/Sqrt[2*f + c*Log[f]]])
/(8*Sqrt[2*f + c*Log[f]])
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 6038
```
Int[(F_)^(u_)*Sinh[v_]^(n_.), x_Symbol] :> Int[ExpandTrigToExp[F^u, Sinh[v]
^n, x], x] /; FreeQ[F, x] && (LinearQ[u, x] || PolyQ[u, x, 2]) && (LinearQ[
v, x] || PolyQ[v, x, 2]) && IGtQ[n, 0]
```

Maple [A] (verified)

Time = 0.53 (sec) , antiderivative size = 177, normalized size of antiderivative = 0.97

method	result
risch	$\frac{\operatorname{erf}\left(x\sqrt{2f-c\ln(f)}+\frac{e}{\sqrt{2f-c\ln(f)}}\right)\sqrt{\pi}\, f^a \mathrm{e}^{-\frac{2d\ln(f)c-4df+e^2}{c\ln(f)-2f}}}{8\sqrt{2f-c\ln(f)}}-\frac{\operatorname{erf}\left(-\sqrt{-c\ln(f)-2f}\,x+\frac{e}{\sqrt{-c\ln(f)-2f}}\right)\sqrt{\pi}\, f^a \mathrm{e}^{\frac{2d\ln(f)c+4df-e^2}{2f+c\ln(f)}}}{8\sqrt{-c\ln(f)-2f}}$

input
```
int(f^(c*x^2+a)*sinh(f*x^2+e*x+d)^2,x,method=_RETURNVERBOSE)
```

output
```
1/8*erf(x*(2*f-c*ln(f))^(1/2)+e/(2*f-c*ln(f))^(1/2))/(2*f-c*ln(f))^(1/2)*P
i^(1/2)*f^a*exp(-(2*d*ln(f)*c-4*d*f+e^2)/(c*ln(f)-2*f))-1/8*erf(-(-c*ln(f)
-2*f)^(1/2)*x+e/(-c*ln(f)-2*f)^(1/2))/(-c*ln(f)-2*f)^(1/2)*Pi^(1/2)*f^a*ex
p((2*d*ln(f)*c+4*d*f-e^2)/(2*f+c*ln(f)))-1/4*f^a*Pi^(1/2)/(-c*ln(f))^(1/2)
*erf((-c*ln(f))^(1/2)*x)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 422 vs. $2(155) = 310$.

Time = 0.11 (sec) , antiderivative size = 422, normalized size of antiderivative = 2.31

$$\int f^{a+cx^2} \sinh^2\left(d+ex+fx^2\right) dx$$
$$= \frac{2\left(\sqrt{\pi}\left(c^2 \log(f)^2 - 4f^2\right)\cosh\left(a\log(f)\right) + \sqrt{\pi}\left(c^2\log(f)^2 - 4f^2\right)\sinh\left(a\log(f)\right)\right)\sqrt{-c\log(f)}\,\mathrm{erf}\left(\sqrt{-\right.}$$

input
```
integrate(f^(c*x^2+a)*sinh(f*x^2+e*x+d)^2,x, algorithm="fricas")
```

output
```
1/8*(2*(sqrt(pi)*(c^2*log(f)^2 - 4*f^2)*cosh(a*log(f)) + sqrt(pi)*(c^2*log
(f)^2 - 4*f^2)*sinh(a*log(f)))*sqrt(-c*log(f))*erf(sqrt(-c*log(f))*x) - (s
qrt(pi)*(c^2*log(f)^2 + 2*c*f*log(f))*cosh((a*c*log(f)^2 - e^2 + 4*d*f - 2
*(c*d + a*f)*log(f))/(c*log(f) - 2*f)) + sqrt(pi)*(c^2*log(f)^2 + 2*c*f*lo
g(f))*sinh((a*c*log(f)^2 - e^2 + 4*d*f - 2*(c*d + a*f)*log(f))/(c*log(f) -
 2*f)))*sqrt(-c*log(f) + 2*f)*erf((c*x*log(f) - 2*f*x - e)*sqrt(-c*log(f)
+ 2*f)/(c*log(f) - 2*f)) - (sqrt(pi)*(c^2*log(f)^2 - 2*c*f*log(f))*cosh((a
*c*log(f)^2 - e^2 + 4*d*f + 2*(c*d + a*f)*log(f))/(c*log(f) + 2*f)) + sqrt
(pi)*(c^2*log(f)^2 - 2*c*f*log(f))*sinh((a*c*log(f)^2 - e^2 + 4*d*f + 2*(c
*d + a*f)*log(f))/(c*log(f) + 2*f)))*sqrt(-c*log(f) - 2*f)*erf((c*x*log(f)
 + 2*f*x + e)*sqrt(-c*log(f) - 2*f)/(c*log(f) + 2*f)))/(c^3*log(f)^3 - 4*c
*f^2*log(f))
```

Sympy [F]

$$\int f^{a+cx^2}\sinh^2\left(d+ex+fx^2\right)\,dx=\int f^{a+cx^2}\sinh^2\left(d+ex+fx^2\right)dx$$

input
```
integrate(f**(c*x**2+a)*sinh(f*x**2+e*x+d)**2,x)
```

output
```
Integral(f**(a + c*x**2)*sinh(d + e*x + f*x**2)**2, x)
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 161, normalized size of antiderivative = 0.88

$$\int f^{a+cx^2}\sinh^2\left(d+ex+fx^2\right)\,dx$$
$$=\frac{\sqrt{\pi}f^a\operatorname{erf}\left(\sqrt{-c\log(f)-2f}\,x-\frac{e}{\sqrt{-c\log(f)-2f}}\right)e^{\left(2d-\frac{e^2}{c\log(f)+2f}\right)}}{8\sqrt{-c\log(f)-2f}}+\frac{\sqrt{\pi}f^a\operatorname{erf}\left(\sqrt{-c\log(f)+2f}\,x+\frac{e}{\sqrt{-c\log(f)+2f}}\right)e^{\left(-2d-\frac{e^2}{c\log(f)-2f}\right)}}{8\sqrt{-c\log(f)+2f}}-\frac{\sqrt{\pi}f^a\operatorname{erf}\left(\sqrt{-c\log(f)}x\right)}{4\sqrt{-c\log(f)}}$$

input
```
integrate(f^(c*x^2+a)*sinh(f*x^2+e*x+d)^2,x, algorithm="maxima")
```

output
```
1/8*sqrt(pi)*f^a*erf(sqrt(-c*log(f) - 2*f)*x - e/sqrt(-c*log(f) - 2*f))*e^
(2*d - e^2/(c*log(f) + 2*f))/sqrt(-c*log(f) - 2*f) + 1/8*sqrt(pi)*f^a*erf(
sqrt(-c*log(f) + 2*f)*x + e/sqrt(-c*log(f) + 2*f))*e^(-2*d - e^2/(c*log(f)
 - 2*f))/sqrt(-c*log(f) + 2*f) - 1/4*sqrt(pi)*f^a*erf(sqrt(-c*log(f))*x)/s
qrt(-c*log(f))
```

Giac [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 198, normalized size of antiderivative = 1.08

$$\int f^{a+cx^2}\sinh^2\left(d+ex+fx^2\right)\,dx = \frac{\sqrt{\pi}f^a\operatorname{erf}\left(-\sqrt{-c\log(f)}x\right)}{4\sqrt{-c\log(f)}} - \frac{\sqrt{\pi}\operatorname{erf}\left(-\sqrt{-c\log(f)-2f}\left(x+\frac{e}{c\log(f)+2f}\right)\right)e^{\left(\frac{ac\log(f)^2+2cd\log(f)+2af\log(f)-e^2+4df}{c\log(f)+2f}\right)}}{8\sqrt{-c\log(f)-2f}} - \frac{\sqrt{\pi}\operatorname{erf}\left(-\sqrt{-c\log(f)+2f}\left(x-\frac{e}{c\log(f)-2f}\right)\right)e^{\left(\frac{ac\log(f)^2-2cd\log(f)-2af\log(f)-e^2+4df}{c\log(f)-2f}\right)}}{8\sqrt{-c\log(f)+2f}}$$

input
```
integrate(f^(c*x^2+a)*sinh(f*x^2+e*x+d)^2,x, algorithm="giac")
```

output
```
1/4*sqrt(pi)*f^a*erf(-sqrt(-c*log(f))*x)/sqrt(-c*log(f)) - 1/8*sqrt(pi)*er
f(-sqrt(-c*log(f) - 2*f)*(x + e/(c*log(f) + 2*f)))*e^((a*c*log(f)^2 + 2*c*
d*log(f) + 2*a*f*log(f) - e^2 + 4*d*f)/(c*log(f) + 2*f))/sqrt(-c*log(f) -
2*f) - 1/8*sqrt(pi)*erf(-sqrt(-c*log(f) + 2*f)*(x - e/(c*log(f) - 2*f)))*e
^((a*c*log(f)^2 - 2*c*d*log(f) - 2*a*f*log(f) - e^2 + 4*d*f)/(c*log(f) - 2
*f))/sqrt(-c*log(f) + 2*f)
```

Mupad [F(-1)]

Timed out.

$$\int f^{a+cx^2}\sinh^2\left(d+ex+fx^2\right)\,dx = \int f^{c\,x^2+a}\,{\sinh\left(f\,x^2+e\,x+d\right)}^2\,dx$$

input
```
int(f^(a + c*x^2)*sinh(d + e*x + f*x^2)^2,x)
```

output
```
int(f^(a + c*x^2)*sinh(d + e*x + f*x^2)^2, x)
```

Reduce [F]

$$\int f^{a+cx^2}\sinh^2\left(d+ex+fx^2\right)\,dx=f^a\left(\int f^{c\,x^2}\sinh\left(f\,x^2+ex+d\right)^2dx\right)$$

input
```
int(f^(c*x^2+a)*sinh(f*x^2+e*x+d)^2,x)
```

output
```
f**a*int(f**(c*x**2)*sinh(d + e*x + f*x**2)**2,x)
```

3.356 $\int f^{a+cx^2}\sinh^3\left(d+ex+fx^2\right)\,dx$

Optimal result

Integrand size = 23, antiderivative size = 300

$$\int f^{a+cx^2}\sinh^3\left(d+ex+fx^2\right)\,dx=\frac{3e^{-d+\frac{e^2}{4f-4c\log(f)}}f^a\sqrt{\pi}\,\mathrm{erf}\left(\frac{e+2x(f-c\log(f))}{2\sqrt{f-c\log(f)}}\right)}{16\sqrt{f-c\log(f)}}-\frac{e^{-3d+\frac{9e^2}{12f-4c\log(f)}}f^a\sqrt{\pi}\,\mathrm{erf}\left(\frac{3e+2x(3f-c\log(f))}{2\sqrt{3f-c\log(f)}}\right)}{16\sqrt{3f-c\log(f)}}-\frac{3e^{d-\frac{e^2}{4(f+c\log(f))}}f^a\sqrt{\pi}\,\mathrm{erfi}\left(\frac{e+2x(f+c\log(f))}{2\sqrt{f+c\log(f)}}\right)}{16\sqrt{f+c\log(f)}}+\frac{e^{3d-\frac{9e^2}{4(3f+c\log(f))}}f^a\sqrt{\pi}\,\mathrm{erfi}\left(\frac{3e+2x(3f+c\log(f))}{2\sqrt{3f+c\log(f)}}\right)}{16\sqrt{3f+c\log(f)}}$$

output
```
3/16*exp(-d+e^2/(4*f-4*c*ln(f)))*f^a*Pi^(1/2)*erf(1/2*(e+2*x*(f-c*ln(f)))/
(f-c*ln(f))^(1/2))/(f-c*ln(f))^(1/2)-1/16*exp(-3*d+9*e^2/(12*f-4*c*ln(f)))
*f^a*Pi^(1/2)*erf(1/2*(3*e+2*x*(3*f-c*ln(f)))/(3*f-c*ln(f))^(1/2))/(3*f-c*
ln(f))^(1/2)-3/16*exp(d-e^2/(4*f+4*c*ln(f)))*f^a*Pi^(1/2)*erfi(1/2*(e+2*x*
(f+c*ln(f)))/(f+c*ln(f))^(1/2))/(f+c*ln(f))^(1/2)+1/16*exp(3*d-9*e^2/(12*f
+4*c*ln(f)))*f^a*Pi^(1/2)*erfi(1/2*(3*e+2*x*(3*f+c*ln(f)))/(3*f+c*ln(f))^(
1/2))/(3*f+c*ln(f))^(1/2)
```

Mathematica [A] (warning: unable to verify)

Time = 4.13 (sec) , antiderivative size = 480, normalized size of antiderivative = 1.60

$$\int f^{a+cx^2}\sinh^3\left(d+ex+fx^2\right)\,dx$$
$$=\frac{e^{-\frac{1}{4}e^2\left(\frac{1}{f+c\log(f)}+\frac{9}{3f+c\log(f)}\right)}f^a\sqrt{\pi}\left(3e^{\frac{1}{4}e^2\left(\frac{1}{f-c\log(f)}+\frac{1}{f+c\log(f)}+\frac{9}{3f+c\log(f)}\right)}\operatorname{erf}\left(\frac{e+2fx-2cx\log(f)}{2\sqrt{f-c\log(f)}}\right)\sqrt{f-c\log(f)}(9f^3}{}$$

input
```
Integrate[f^(a + c*x^2)*Sinh[d + e*x + f*x^2]^3,x]
```

output
```
(f^a*Sqrt[Pi]*(3*E^((e^2*((f - c*Log[f])^(-1) + (f + c*Log[f])^(-1) + 9/(3
*f + c*Log[f])))/4)*Erf[(e + 2*f*x - 2*c*x*Log[f])/(2*Sqrt[f - c*Log[f]])]
*Sqrt[f - c*Log[f]]*(9*f^3 + 9*c*f^2*Log[f] - c^2*f*Log[f]^2 - c^3*Log[f]^
3)*(Cosh[d] - Sinh[d]) - (f - c*Log[f])*(E^((e^2*(9/(3*f - c*Log[f]) + (f
+ c*Log[f])^(-1) + 9/(3*f + c*Log[f])))/4)*Erf[(3*e + 6*f*x - 2*c*x*Log[f]
)/(2*Sqrt[3*f - c*Log[f]])]*Sqrt[3*f - c*Log[f]]*(3*f^2 + 4*c*f*Log[f] + c
^2*Log[f]^2)*(Cosh[3*d] - Sinh[3*d]) + (3*f - c*Log[f])*(3*E^((9*e^2)/(4*(
3*f + c*Log[f])))*Erfi[(e + 2*f*x + 2*c*x*Log[f])/(2*Sqrt[f + c*Log[f]])]*
Sqrt[f + c*Log[f]]*(3*f + c*Log[f])*(Cosh[d] + Sinh[d]) - E^(e^2/(4*(f + c
*Log[f])))*Erfi[(3*e + 6*f*x + 2*c*x*Log[f])/(2*Sqrt[3*f + c*Log[f]])]*(f
+ c*Log[f])*Sqrt[3*f + c*Log[f]]*(Cosh[3*d] + Sinh[3*d])))))/(16*E^((e^2*(
(f + c*Log[f])^(-1) + 9/(3*f + c*Log[f])))/4)*(9*f^4 - 10*c^2*f^2*Log[f]^2
 + c^4*Log[f]^4))
```

Rubi [A] (verified)

Time = 0.90 (sec) , antiderivative size = 300, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.087, Rules used = {6038, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int f^{a+cx^2}\sinh^3\left(d+ex+fx^2\right)\,dx$$

$$\Big\downarrow \text{6038}$$

$$\int \left(\frac{3}{8}f^{a+cx^2}\exp\left(-3(d+ex+fx^2)+2d+2ex+2fx^2\right)-\frac{3}{8}f^{a+cx^2}\exp\left(-3(d+ex+fx^2)+4d+4ex+4fx^2\right)\right.$$

$$\Big\downarrow \text{2009}$$

$$\frac{3\sqrt{\pi}f^a e^{\frac{e^2}{4f-4c\log(f)}-d}\text{erf}\left(\frac{2x(f-c\log(f))+e}{2\sqrt{f-c\log(f)}}\right)}{16\sqrt{f-c\log(f)}}-\frac{\sqrt{\pi}f^a e^{\frac{9e^2}{12f-4c\log(f)}-3d}\text{erf}\left(\frac{2x(3f-c\log(f))+3e}{2\sqrt{3f-c\log(f)}}\right)}{16\sqrt{3f-c\log(f)}}-\frac{3\sqrt{\pi}f^a e^{d-\frac{e^2}{4(c\log(f)+f)}}\text{erfi}\left(\frac{2x(c\log(f)+f)+e}{2\sqrt{c\log(f)+f}}\right)}{16\sqrt{c\log(f)+f}}+\frac{\sqrt{\pi}f^a e^{3d-\frac{9e^2}{4(c\log(f)+3f)}}\text{erfi}\left(\frac{2x(c\log(f)+3f)+3e}{2\sqrt{c\log(f)+3f}}\right)}{16\sqrt{c\log(f)+3f}}$$

input
```
Int[f^(a + c*x^2)*Sinh[d + e*x + f*x^2]^3,x]
```

output
```
(3*E^(-d + e^2/(4*f - 4*c*Log[f]))*f^a*Sqrt[Pi]*Erf[(e + 2*x*(f - c*Log[f]
))/(2*Sqrt[f - c*Log[f]])])/(16*Sqrt[f - c*Log[f]]) - (E^(-3*d + (9*e^2)/(
12*f - 4*c*Log[f]))*f^a*Sqrt[Pi]*Erf[(3*e + 2*x*(3*f - c*Log[f]))/(2*Sqrt[
3*f - c*Log[f]])])/(16*Sqrt[3*f - c*Log[f]]) - (3*E^(d - e^2/(4*(f + c*Log
[f])))*f^a*Sqrt[Pi]*Erfi[(e + 2*x*(f + c*Log[f]))/(2*Sqrt[f + c*Log[f]])])
/(16*Sqrt[f + c*Log[f]]) + (E^(3*d - (9*e^2)/(4*(3*f + c*Log[f])))*f^a*Sqr
t[Pi]*Erfi[(3*e + 2*x*(3*f + c*Log[f]))/(2*Sqrt[3*f + c*Log[f]])])/(16*Sqr
t[3*f + c*Log[f]])
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 6038
```
Int[(F_)^(u_)*Sinh[v_]^(n_.), x_Symbol] :> Int[ExpandTrigToExp[F^u, Sinh[v]
^n, x], x] /; FreeQ[F, x] && (LinearQ[u, x] || PolyQ[u, x, 2]) && (LinearQ[
v, x] || PolyQ[v, x, 2]) && IGtQ[n, 0]
```

Maple [A] (verified)

Time = 1.44 (sec) , antiderivative size = 302, normalized size of antiderivative = 1.01

method	result
risch	$-\frac{\operatorname{erf}\left(-\sqrt{-c\ln(f)-3f}\,x+\frac{3e}{2\sqrt{-c\ln(f)-3f}}\right)\sqrt{\pi}\,f^{a}\mathrm{e}^{\frac{3d\ln(f)c+9df-\frac{9e^{2}}{4}}{3f+c\ln(f)}}}{16\sqrt{-c\ln(f)-3f}}-\frac{\operatorname{erf}\left(x\sqrt{3f-c\ln(f)}+\frac{3e}{2\sqrt{3f-c\ln(f)}}\right)\sqrt{\pi}\,f^{a}\mathrm{e}^{-\frac{3\left(4d\ln(f)c\right.}{4(c\ln}}}{16\sqrt{3f-c\ln(f)}}$

input
```
int(f^(c*x^2+a)*sinh(f*x^2+e*x+d)^3,x,method=_RETURNVERBOSE)
```

output
```
-1/16*erf(-(-c*ln(f)-3*f)^(1/2)*x+3/2*e/(-c*ln(f)-3*f)^(1/2))/(-c*ln(f)-3*
f)^(1/2)*Pi^(1/2)*f^a*exp(3/4*(4*d*ln(f)*c+12*d*f-3*e^2)/(3*f+c*ln(f)))-1/
16*erf(x*(3*f-c*ln(f))^(1/2)+3/2*e/(3*f-c*ln(f))^(1/2))/(3*f-c*ln(f))^(1/2
)*Pi^(1/2)*f^a*exp(-3/4*(4*d*ln(f)*c-12*d*f+3*e^2)/(c*ln(f)-3*f))+3/16*erf
(x*(f-c*ln(f))^(1/2)+1/2*e/(f-c*ln(f))^(1/2))/(f-c*ln(f))^(1/2)*Pi^(1/2)*f
^a*exp(-1/4*(4*d*ln(f)*c-4*d*f+e^2)/(c*ln(f)-f))+3/16*erf(-(-c*ln(f)-f)^(1
/2)*x+1/2*e/(-c*ln(f)-f)^(1/2))/(-c*ln(f)-f)^(1/2)*Pi^(1/2)*f^a*exp(1/4*(4
*d*ln(f)*c+4*d*f-e^2)/(f+c*ln(f)))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 848 vs. 2(253) = 506.

Time = 0.12 (sec) , antiderivative size = 848, normalized size of antiderivative = 2.83

$$\int f^{a+cx^2}\sinh^3\left(d+ex+fx^2\right)\,dx=\text{Too large to display}$$

input
```
integrate(f^(c*x^2+a)*sinh(f*x^2+e*x+d)^3,x, algorithm="fricas")
```

output
```
1/16*((sqrt(pi)*(c^3*log(f)^3 + 3*c^2*f*log(f)^2 - c*f^2*log(f) - 3*f^3)*c
osh(1/4*(4*a*c*log(f)^2 - 9*e^2 + 36*d*f - 12*(c*d + a*f)*log(f))/(c*log(f
) - 3*f)) + sqrt(pi)*(c^3*log(f)^3 + 3*c^2*f*log(f)^2 - c*f^2*log(f) - 3*f
^3)*sinh(1/4*(4*a*c*log(f)^2 - 9*e^2 + 36*d*f - 12*(c*d + a*f)*log(f))/(c*
log(f) - 3*f)))*sqrt(-c*log(f) + 3*f)*erf(1/2*(2*c*x*log(f) - 6*f*x - 3*e)
*sqrt(-c*log(f) + 3*f)/(c*log(f) - 3*f)) - 3*(sqrt(pi)*(c^3*log(f)^3 + c^2
*f*log(f)^2 - 9*c*f^2*log(f) - 9*f^3)*cosh(1/4*(4*a*c*log(f)^2 - e^2 + 4*d
*f - 4*(c*d + a*f)*log(f))/(c*log(f) - f)) + sqrt(pi)*(c^3*log(f)^3 + c^2*
f*log(f)^2 - 9*c*f^2*log(f) - 9*f^3)*sinh(1/4*(4*a*c*log(f)^2 - e^2 + 4*d*
f - 4*(c*d + a*f)*log(f))/(c*log(f) - f)))*sqrt(-c*log(f) + f)*erf(1/2*(2*
c*x*log(f) - 2*f*x - e)*sqrt(-c*log(f) + f)/(c*log(f) - f)) + 3*(sqrt(pi)*
(c^3*log(f)^3 - c^2*f*log(f)^2 - 9*c*f^2*log(f) + 9*f^3)*cosh(1/4*(4*a*c*l
og(f)^2 - e^2 + 4*d*f + 4*(c*d + a*f)*log(f))/(c*log(f) + f)) + sqrt(pi)*(
c^3*log(f)^3 - c^2*f*log(f)^2 - 9*c*f^2*log(f) + 9*f^3)*sinh(1/4*(4*a*c*lo
g(f)^2 - e^2 + 4*d*f + 4*(c*d + a*f)*log(f))/(c*log(f) + f)))*sqrt(-c*log(
f) - f)*erf(1/2*(2*c*x*log(f) + 2*f*x + e)*sqrt(-c*log(f) - f)/(c*log(f) +
 f)) - (sqrt(pi)*(c^3*log(f)^3 - 3*c^2*f*log(f)^2 - c*f^2*log(f) + 3*f^3)*
cosh(1/4*(4*a*c*log(f)^2 - 9*e^2 + 36*d*f + 12*(c*d + a*f)*log(f))/(c*log(
f) + 3*f)) + sqrt(pi)*(c^3*log(f)^3 - 3*c^2*f*log(f)^2 - c*f^2*log(f) + 3*
f^3)*sinh(1/4*(4*a*c*log(f)^2 - 9*e^2 + 36*d*f + 12*(c*d + a*f)*log(f))...
```

Sympy [F]

$$\int f^{a+cx^2}\sinh^3\left(d+ex+fx^2\right)\,dx=\int f^{a+cx^2}\sinh^3\left(d+ex+fx^2\right)dx$$

input
```
integrate(f**(c*x**2+a)*sinh(f*x**2+e*x+d)**3,x)
```

output
```
Integral(f**(a + c*x**2)*sinh(d + e*x + f*x**2)**3, x)
```

Maxima [A] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 263, normalized size of antiderivative = 0.88

$$\int f^{a+cx^2} \sinh^3\left(d+ex+fx^2\right)\,dx$$
$$= \frac{\sqrt{\pi}f^a \operatorname{erf}\left(\sqrt{-c\log\left(f\right)-3\,f}x-\frac{3\,e}{2\,\sqrt{-c\log(f)-3\,f}}\right)e^{\left(3\,d-\frac{9\,e^2}{4\,(c\log(f)+3\,f)}\right)}}{16\,\sqrt{-c\log\left(f\right)-3\,f}}$$
$$- \frac{3\,\sqrt{\pi}f^a \operatorname{erf}\left(\sqrt{-c\log\left(f\right)-f}x-\frac{e}{2\,\sqrt{-c\log(f)-f}}\right)e^{\left(d-\frac{e^2}{4\,(c\log(f)+f)}\right)}}{16\,\sqrt{-c\log\left(f\right)-f}}$$
$$+ \frac{3\,\sqrt{\pi}f^a \operatorname{erf}\left(\sqrt{-c\log\left(f\right)+f}x+\frac{e}{2\,\sqrt{-c\log(f)+f}}\right)e^{\left(-d-\frac{e^2}{4\,(c\log(f)-f)}\right)}}{16\,\sqrt{-c\log\left(f\right)+f}}$$
$$- \frac{\sqrt{\pi}f^a \operatorname{erf}\left(\sqrt{-c\log\left(f\right)+3\,f}x+\frac{3\,e}{2\,\sqrt{-c\log(f)+3\,f}}\right)e^{\left(-3\,d-\frac{9\,e^2}{4\,(c\log(f)-3\,f)}\right)}}{16\,\sqrt{-c\log\left(f\right)+3\,f}}$$

input
```
integrate(f^(c*x^2+a)*sinh(f*x^2+e*x+d)^3,x, algorithm="maxima")
```

output
```
1/16*sqrt(pi)*f^a*erf(sqrt(-c*log(f) - 3*f)*x - 3/2*e/sqrt(-c*log(f) - 3*f
))*e^(3*d - 9/4*e^2/(c*log(f) + 3*f))/sqrt(-c*log(f) - 3*f) - 3/16*sqrt(pi
)*f^a*erf(sqrt(-c*log(f) - f)*x - 1/2*e/sqrt(-c*log(f) - f))*e^(d - 1/4*e^
2/(c*log(f) + f))/sqrt(-c*log(f) - f) + 3/16*sqrt(pi)*f^a*erf(sqrt(-c*log(
f) + f)*x + 1/2*e/sqrt(-c*log(f) + f))*e^(-d - 1/4*e^2/(c*log(f) - f))/sqr
t(-c*log(f) + f) - 1/16*sqrt(pi)*f^a*erf(sqrt(-c*log(f) + 3*f)*x + 3/2*e/s
qrt(-c*log(f) + 3*f))*e^(-3*d - 9/4*e^2/(c*log(f) - 3*f))/sqrt(-c*log(f) +
 3*f)
```

Giac [A] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 352, normalized size of antiderivative = 1.17

$$\int f^{a+cx^2}\sinh^3\left(d+ex+fx^2\right)\,dx =$$

$$-\frac{\sqrt{\pi}\,\mathrm{erf}\left(-\frac{1}{2}\sqrt{-c\log\left(f\right)-3\,f}\left(2\,x+\frac{3\,e}{c\log(f)+3\,f}\right)\right)e^{\left(\frac{4\,ac\log(f)^2+12\,cd\log(f)+12\,af\log(f)-9\,e^2+36\,df}{4\,(c\log(f)+3\,f)}\right)}}{16\,\sqrt{-c\log\left(f\right)-3\,f}}$$
$$+\frac{3\,\sqrt{\pi}\,\mathrm{erf}\left(-\frac{1}{2}\sqrt{-c\log\left(f\right)-f}\left(2\,x+\frac{e}{c\log(f)+f}\right)\right)e^{\left(\frac{4\,ac\log(f)^2+4\,cd\log(f)+4\,af\log(f)-e^2+4\,df}{4\,(c\log(f)+f)}\right)}}{16\,\sqrt{-c\log\left(f\right)-f}}$$
$$-\frac{3\,\sqrt{\pi}\,\mathrm{erf}\left(-\frac{1}{2}\sqrt{-c\log\left(f\right)+f}\left(2\,x-\frac{e}{c\log(f)-f}\right)\right)e^{\left(\frac{4\,ac\log(f)^2-4\,cd\log(f)-4\,af\log(f)-e^2+4\,df}{4\,(c\log(f)-f)}\right)}}{16\,\sqrt{-c\log\left(f\right)+f}}$$
$$+\frac{\sqrt{\pi}\,\mathrm{erf}\left(-\frac{1}{2}\sqrt{-c\log\left(f\right)+3\,f}\left(2\,x-\frac{3\,e}{c\log(f)-3\,f}\right)\right)e^{\left(\frac{4\,ac\log(f)^2-12\,cd\log(f)-12\,af\log(f)-9\,e^2+36\,df}{4\,(c\log(f)-3\,f)}\right)}}{16\,\sqrt{-c\log\left(f\right)+3\,f}}$$

input
```
integrate(f^(c*x^2+a)*sinh(f*x^2+e*x+d)^3,x, algorithm="giac")
```

output
```
-1/16*sqrt(pi)*erf(-1/2*sqrt(-c*log(f) - 3*f)*(2*x + 3*e/(c*log(f) + 3*f))
)*e^(1/4*(4*a*c*log(f)^2 + 12*c*d*log(f) + 12*a*f*log(f) - 9*e^2 + 36*d*f)
/(c*log(f) + 3*f))/sqrt(-c*log(f) - 3*f) + 3/16*sqrt(pi)*erf(-1/2*sqrt(-c*
log(f) - f)*(2*x + e/(c*log(f) + f)))*e^(1/4*(4*a*c*log(f)^2 + 4*c*d*log(f
) + 4*a*f*log(f) - e^2 + 4*d*f)/(c*log(f) + f))/sqrt(-c*log(f) - f) - 3/16
*sqrt(pi)*erf(-1/2*sqrt(-c*log(f) + f)*(2*x - e/(c*log(f) - f)))*e^(1/4*(4
*a*c*log(f)^2 - 4*c*d*log(f) - 4*a*f*log(f) - e^2 + 4*d*f)/(c*log(f) - f))
/sqrt(-c*log(f) + f) + 1/16*sqrt(pi)*erf(-1/2*sqrt(-c*log(f) + 3*f)*(2*x -
 3*e/(c*log(f) - 3*f)))*e^(1/4*(4*a*c*log(f)^2 - 12*c*d*log(f) - 12*a*f*lo
g(f) - 9*e^2 + 36*d*f)/(c*log(f) - 3*f))/sqrt(-c*log(f) + 3*f)
```

Mupad [F(-1)]

Timed out.

$$\int f^{a+cx^2}\sinh^3\left(d+ex+fx^2\right)\,dx=\int f^{c\,x^2+a}\,\sinh\left(f\,x^2+e\,x+d\right)^3\,dx$$

input
```
int(f^(a + c*x^2)*sinh(d + e*x + f*x^2)^3,x)
```

output
```
int(f^(a + c*x^2)*sinh(d + e*x + f*x^2)^3, x)
```

Reduce [F]

$$\int f^{a+cx^2}\sinh^3\left(d+ex+fx^2\right)\,dx=f^a\left(\int f^{c\,x^2}\sinh\left(f\,x^2+ex+d\right)^3dx\right)$$

input
```
int(f^(c*x^2+a)*sinh(f*x^2+e*x+d)^3,x)
```

output
```
f**a*int(f**(c*x**2)*sinh(d + e*x + f*x**2)**3,x)
```

3.357 $\int f^{a+bx+cx^2}\sinh(d+ex)\,dx$

Optimal result

Integrand size = 19, antiderivative size = 153

$$\int f^{a+bx+cx^2}\sinh(d+ex)\,dx=\frac{e^{-d-\frac{(e-b\log(f))^2}{4c\log(f)}}f^a\sqrt{\pi}\operatorname{erfi}\left(\frac{e-b\log(f)-2cx\log(f)}{2\sqrt{c}\sqrt{\log(f)}}\right)}{4\sqrt{c}\sqrt{\log(f)}}+\frac{e^{d-\frac{(e+b\log(f))^2}{4c\log(f)}}f^a\sqrt{\pi}\operatorname{erfi}\left(\frac{e+b\log(f)+2cx\log(f)}{2\sqrt{c}\sqrt{\log(f)}}\right)}{4\sqrt{c}\sqrt{\log(f)}}$$

output
```
1/4*exp(-d-1/4*(e-b*ln(f))^2/c/ln(f))*f^a*Pi^(1/2)*erfi(1/2*(e-b*ln(f)-2*c
*x*ln(f))/c^(1/2)/ln(f)^(1/2))/c^(1/2)/ln(f)^(1/2)+1/4*exp(d-1/4*(e+b*ln(f
))^2/c/ln(f))*f^a*Pi^(1/2)*erfi(1/2*(e+b*ln(f)+2*c*x*ln(f))/c^(1/2)/ln(f)^
(1/2))/c^(1/2)/ln(f)^(1/2)
```

Mathematica [A] (verified)

Time = 0.20 (sec) , antiderivative size = 135, normalized size of antiderivative = 0.88

$$\int f^{a+bx+cx^2}\sinh(d+ex)\,dx$$
$$=\frac{e^{-\frac{e(e+2b\log(f))}{4c\log(f)}}f^{a-\frac{b^2}{4c}}\sqrt{\pi}\left(-e^{\frac{be}{c}}\operatorname{erfi}\left(\frac{-e+(b+2cx)\log(f)}{2\sqrt{c}\sqrt{\log(f)}}\right)(\cosh(d)-\sinh(d))+\operatorname{erfi}\left(\frac{e+(b+2cx)\log(f)}{2\sqrt{c}\sqrt{\log(f)}}\right)(\cosh(d)+\text{s}\right.}{4\sqrt{c}\sqrt{\log(f)}}$$

input
```
Integrate[f^(a + b*x + c*x^2)*Sinh[d + e*x],x]
```

output
```
(f^(a - b^2/(4*c))*Sqrt[Pi]*(-(E^((b*e)/c)*Erfi[(-e + (b + 2*c*x)*Log[f])/
(2*Sqrt[c]*Sqrt[Log[f]])]*(Cosh[d] - Sinh[d])) + Erfi[(e + (b + 2*c*x)*Log
[f])/(2*Sqrt[c]*Sqrt[Log[f]])]*(Cosh[d] + Sinh[d])))/(4*Sqrt[c]*E^((e*(e +
 2*b*Log[f]))/(4*c*Log[f]))*Sqrt[Log[f]])
```

Rubi [A] (verified)

Time = 0.52 (sec) , antiderivative size = 153, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.105$, Rules used = {6038, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sinh(d+ex) f^{a+bx+cx^2} \, dx$$

$$\Big\downarrow \text{6038}$$

$$\int \left(\frac{1}{2} e^{d+ex} f^{a+bx+cx^2} - \frac{1}{2} e^{-d-ex} f^{a+bx+cx^2}\right) dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{\sqrt{\pi} f^a e^{-\frac{(e-b\log(f))^2}{4c\log(f)}-d} \text{erfi}\left(\frac{-b\log(f)-2cx\log(f)+e}{2\sqrt{c}\sqrt{\log(f)}}\right)}{4\sqrt{c}\sqrt{\log(f)}} + \frac{\sqrt{\pi} f^a e^{d-\frac{(b\log(f)+e)^2}{4c\log(f)}} \text{erfi}\left(\frac{b\log(f)+2cx\log(f)+e}{2\sqrt{c}\sqrt{\log(f)}}\right)}{4\sqrt{c}\sqrt{\log(f)}}$$

input
```
Int[f^(a + b*x + c*x^2)*Sinh[d + e*x],x]
```

output
```
(E^(-d - (e - b*Log[f])^2/(4*c*Log[f]))*f^a*Sqrt[Pi]*Erfi[(e - b*Log[f] -
2*c*x*Log[f])/(2*Sqrt[c]*Sqrt[Log[f]])])/(4*Sqrt[c]*Sqrt[Log[f]]) + (E^(d
- (e + b*Log[f])^2/(4*c*Log[f]))*f^a*Sqrt[Pi]*Erfi[(e + b*Log[f] + 2*c*x*L
og[f])/(2*Sqrt[c]*Sqrt[Log[f]])])/(4*Sqrt[c]*Sqrt[Log[f]])
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 6038
```
Int[(F_)^(u_)*Sinh[v_]^(n_.), x_Symbol] :> Int[ExpandTrigToExp[F^u, Sinh[v]
^n, x], x] /; FreeQ[F, x] && (LinearQ[u, x] || PolyQ[u, x, 2]) && (LinearQ[
v, x] || PolyQ[v, x, 2]) && IGtQ[n, 0]
```

Maple [A] (verified)

Time = 0.14 (sec) , antiderivative size = 162, normalized size of antiderivative = 1.06

method	result
risch	$-\frac{\operatorname{erf}\left(-\sqrt{-c\ln(f)}\,x+\frac{e+b\ln(f)}{2\sqrt{-c\ln(f)}}\right)\sqrt{\pi}\,f^{a}f^{-\frac{b^2}{4c}}\mathrm{e}^{-\frac{2\ln(f)be-4d\ln(f)c+e^2}{4\ln(f)c}}}{4\sqrt{-c\ln(f)}}+\frac{\operatorname{erf}\left(-\sqrt{-c\ln(f)}\,x+\frac{b\ln(f)-e}{2\sqrt{-c\ln(f)}}\right)\sqrt{\pi}\,f^{a}f^{-\frac{b^2}{4c}}\mathrm{e}^{\frac{2\ln(f)}{}}}{4\sqrt{-c\ln(f)}}$

input
```
int(f^(c*x^2+b*x+a)*sinh(e*x+d),x,method=_RETURNVERBOSE)
```

output
```
-1/4*erf(-(-c*ln(f))^(1/2)*x+1/2*(e+b*ln(f))/(-c*ln(f))^(1/2))/(-c*ln(f))^
(1/2)*Pi^(1/2)*f^a*f^(-1/4*b^2/c)*exp(-1/4*(2*ln(f)*b*e-4*d*ln(f)*c+e^2)/l
n(f)/c)+1/4*erf(-(-c*ln(f))^(1/2)*x+1/2*(b*ln(f)-e)/(-c*ln(f))^(1/2))/(-c*
ln(f))^(1/2)*Pi^(1/2)*f^a*f^(-1/4*b^2/c)*exp(1/4*(2*ln(f)*b*e-4*d*ln(f)*c-
e^2)/ln(f)/c)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 263 vs. $2(121) = 242$.

Time = 0.10 (sec) , antiderivative size = 263, normalized size of antiderivative = 1.72

$$\int f^{a+bx+cx^2}\sinh(d+ex)\,dx =$$

$$-\frac{\sqrt{-c\log(f)}\left(\sqrt{\pi}\cosh\left(-\frac{(b^2-4ac)\log(f)^2+e^2-2(2cd-be)\log(f)}{4c\log(f)}\right)+\sqrt{\pi}\sinh\left(-\frac{(b^2-4ac)\log(f)^2+e^2-2(2cd-be)\log(}{4c\log(f)}\right.\right.}{}$$

input
```
integrate(f^(c*x^2+b*x+a)*sinh(e*x+d),x, algorithm="fricas")
```

output
```
-1/4*(sqrt(-c*log(f))*(sqrt(pi)*cosh(-1/4*((b^2 - 4*a*c)*log(f)^2 + e^2 -
2*(2*c*d - b*e)*log(f))/(c*log(f))) + sqrt(pi)*sinh(-1/4*((b^2 - 4*a*c)*lo
g(f)^2 + e^2 - 2*(2*c*d - b*e)*log(f))/(c*log(f))))*erf(1/2*((2*c*x + b)*l
og(f) + e)*sqrt(-c*log(f))/(c*log(f))) - sqrt(-c*log(f))*(sqrt(pi)*cosh(-1
/4*((b^2 - 4*a*c)*log(f)^2 + e^2 + 2*(2*c*d - b*e)*log(f))/(c*log(f))) + s
qrt(pi)*sinh(-1/4*((b^2 - 4*a*c)*log(f)^2 + e^2 + 2*(2*c*d - b*e)*log(f))/
(c*log(f))))*erf(1/2*((2*c*x + b)*log(f) - e)*sqrt(-c*log(f))/(c*log(f))))
/(c*log(f))
```

Sympy [F]

$$\int f^{a+bx+cx^2}\sinh(d+ex)\,dx=\int f^{a+bx+cx^2}\sinh{(d+ex)}\,dx$$

input
```
integrate(f**(c*x**2+b*x+a)*sinh(e*x+d),x)
```

output
```
Integral(f**(a + b*x + c*x**2)*sinh(d + e*x), x)
```

Maxima [A] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 129, normalized size of antiderivative = 0.84

$$\int f^{a+bx+cx^2}\sinh(d+ex)\,dx = \frac{\sqrt{\pi}f^a\,\text{erf}\left(\sqrt{-c\log{(f)}}x-\frac{b\log(f)+e}{2\sqrt{-c\log(f)}}\right)e^{\left(d-\frac{(b\log(f)+e)^2}{4\,c\log(f)}\right)}}{4\sqrt{-c\log{(f)}}} - \frac{\sqrt{\pi}f^a\,\text{erf}\left(\sqrt{-c\log{(f)}}x-\frac{b\log(f)-e}{2\sqrt{-c\log(f)}}\right)e^{\left(-d-\frac{(b\log(f)-e)^2}{4\,c\log(f)}\right)}}{4\sqrt{-c\log{(f)}}}$$

input
```
integrate(f^(c*x^2+b*x+a)*sinh(e*x+d),x, algorithm="maxima")
```

output
```
1/4*sqrt(pi)*f^a*erf(sqrt(-c*log(f))*x - 1/2*(b*log(f) + e)/sqrt(-c*log(f)
))*e^(d - 1/4*(b*log(f) + e)^2/(c*log(f)))/sqrt(-c*log(f)) - 1/4*sqrt(pi)*
f^a*erf(sqrt(-c*log(f))*x - 1/2*(b*log(f) - e)/sqrt(-c*log(f)))*e^(-d - 1/
4*(b*log(f) - e)^2/(c*log(f)))/sqrt(-c*log(f))
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 167, normalized size of antiderivative = 1.09

$$\int f^{a+bx+cx^2}\sinh(d+ex)\,dx$$
$$=\frac{\sqrt{\pi}\,\mathrm{erf}\left(-\frac{1}{2}\sqrt{-c\log(f)}\left(2x+\frac{b\log(f)-e}{c\log(f)}\right)\right)e^{\left(-\frac{b^2\log(f)^2-4\,ac\log(f)^2+4\,cd\log(f)-2\,be\log(f)+e^2}{4\,c\log(f)}\right)}}{4\sqrt{-c\log(f)}}$$
$$-\frac{\sqrt{\pi}\,\mathrm{erf}\left(-\frac{1}{2}\sqrt{-c\log(f)}\left(2x+\frac{b\log(f)+e}{c\log(f)}\right)\right)e^{\left(-\frac{b^2\log(f)^2-4\,ac\log(f)^2-4\,cd\log(f)+2\,be\log(f)+e^2}{4\,c\log(f)}\right)}}{4\sqrt{-c\log(f)}}$$

input
```
integrate(f^(c*x^2+b*x+a)*sinh(e*x+d),x, algorithm="giac")
```

output
```
1/4*sqrt(pi)*erf(-1/2*sqrt(-c*log(f))*(2*x + (b*log(f) - e)/(c*log(f))))*e
^(-1/4*(b^2*log(f)^2 - 4*a*c*log(f)^2 + 4*c*d*log(f) - 2*b*e*log(f) + e^2)
/(c*log(f)))/sqrt(-c*log(f)) - 1/4*sqrt(pi)*erf(-1/2*sqrt(-c*log(f))*(2*x
+ (b*log(f) + e)/(c*log(f))))*e^(-1/4*(b^2*log(f)^2 - 4*a*c*log(f)^2 - 4*c
*d*log(f) + 2*b*e*log(f) + e^2)/(c*log(f)))/sqrt(-c*log(f))
```

Mupad [F(-1)]

Timed out.

$$\int f^{a+bx+cx^2}\sinh(d+ex)\,dx=\int f^{c\,x^2+b\,x+a}\,\sinh(d+e\,x)\,dx$$

input
```
int(f^(a + b*x + c*x^2)*sinh(d + e*x),x)
```

output
```
int(f^(a + b*x + c*x^2)*sinh(d + e*x), x)
```

Reduce [F]

$$\int f^{a+bx+cx^2}\sinh(d+ex)\,dx = f^a\left(\int f^{c\,x^2+bx}\sinh(ex+d)\,dx\right)$$

input
```
int(f^(c*x^2+b*x+a)*sinh(e*x+d),x)
```

output
```
f**a*int(f**(b*x + c*x**2)*sinh(d + e*x),x)
```

3.358 $\int f^{a+bx+cx^2}\sinh^2(d+ex)\,dx$

Optimal result

Integrand size = 21, antiderivative size = 219

$$\int f^{a+bx+cx^2}\sinh^2(d+ex)\,dx=-\frac{f^{a-\frac{b^2}{4c}}\sqrt{\pi}\,\text{erfi}\left(\frac{(b+2cx)\sqrt{\log(f)}}{2\sqrt{c}}\right)}{4\sqrt{c}\sqrt{\log(f)}}-\frac{e^{-2d-\frac{(2e-b\log(f))^2}{4c\log(f)}}f^a\sqrt{\pi}\,\text{erfi}\left(\frac{2e-b\log(f)-2cx\log(f)}{2\sqrt{c}\sqrt{\log(f)}}\right)}{8\sqrt{c}\sqrt{\log(f)}}+\frac{e^{2d-\frac{(2e+b\log(f))^2}{4c\log(f)}}f^a\sqrt{\pi}\,\text{erfi}\left(\frac{2e+b\log(f)+2cx\log(f)}{2\sqrt{c}\sqrt{\log(f)}}\right)}{8\sqrt{c}\sqrt{\log(f)}}$$

output
```
-1/4*f^(a-1/4*b^2/c)*Pi^(1/2)*erfi(1/2*(2*c*x+b)*ln(f)^(1/2)/c^(1/2))/c^(1
/2)/ln(f)^(1/2)-1/8*exp(-2*d-1/4*(2*e-b*ln(f))^2/c/ln(f))*f^a*Pi^(1/2)*erf
i(1/2*(2*e-b*ln(f)-2*c*x*ln(f))/c^(1/2)/ln(f)^(1/2))/c^(1/2)/ln(f)^(1/2)+1
/8*exp(2*d-1/4*(2*e+b*ln(f))^2/c/ln(f))*f^a*Pi^(1/2)*erfi(1/2*(2*e+b*ln(f)
+2*c*x*ln(f))/c^(1/2)/ln(f)^(1/2))/c^(1/2)/ln(f)^(1/2)
```

Mathematica [A] (verified)

Time = 0.39 (sec) , antiderivative size = 183, normalized size of antiderivative = 0.84

$$\int f^{a+bx+cx^2}\sinh^2(d+ex)\,dx = \frac{e^{-\frac{e(e+b\log(f))}{c\log(f)}}f^{a-\frac{b^2}{4c}}\sqrt{\pi}\left(-2e^{\frac{e(e+b\log(f))}{c\log(f)}}\operatorname{erfi}\left(\frac{(b+2cx)\sqrt{\log(f)}}{2\sqrt{c}}\right)+e^{\frac{2be}{c}}\operatorname{erfi}\left(\frac{-2e+(b+2cx)\log(f)}{2\sqrt{c}\sqrt{\log(f)}}\right)(\cosh(2d)-\sinh(2d}{8\sqrt{c}\sqrt{\log(f)}}$$

input
```
Integrate[f^(a + b*x + c*x^2)*Sinh[d + e*x]^2,x]
```

output
```
(f^(a - b^2/(4*c))*Sqrt[Pi]*(-2*E^((e*(e + b*Log[f]))/(c*Log[f]))*Erfi[((b
 + 2*c*x)*Sqrt[Log[f]])/(2*Sqrt[c])] + E^((2*b*e)/c)*Erfi[(-2*e + (b + 2*c
*x)*Log[f])/(2*Sqrt[c]*Sqrt[Log[f]])]*(Cosh[2*d] - Sinh[2*d]) + Erfi[(2*e
+ (b + 2*c*x)*Log[f])/(2*Sqrt[c]*Sqrt[Log[f]])]*(Cosh[2*d] + Sinh[2*d])))/
(8*Sqrt[c]*E^((e*(e + b*Log[f]))/(c*Log[f]))*Sqrt[Log[f]])
```

Rubi [A] (verified)

Time = 0.61 (sec) , antiderivative size = 219, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.095$, Rules used = {6038, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sinh^2(d+ex)f^{a+bx+cx^2}\,dx$$

$$\Big\downarrow \text{6038}$$

$$\int \left(\frac{1}{4}e^{-2d-2ex}f^{a+bx+cx^2}+\frac{1}{4}e^{2d+2ex}f^{a+bx+cx^2}-\frac{1}{2}f^{a+bx+cx^2}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$-\frac{\sqrt{\pi} f^{a-\frac{b^2}{4c}} \operatorname{erfi}\left(\frac{\sqrt{\log(f)}(b+2cx)}{2\sqrt{c}}\right)}{4\sqrt{c}\sqrt{\log(f)}} - \frac{\sqrt{\pi} f^a e^{-\frac{(2e-b\log(f))^2}{4c\log(f)}-2d} \operatorname{erfi}\left(\frac{-b\log(f)-2cx\log(f)+2e}{2\sqrt{c}\sqrt{\log(f)}}\right)}{8\sqrt{c}\sqrt{\log(f)}} + \frac{\sqrt{\pi} f^a e^{2d-\frac{(b\log(f)+2e)^2}{4c\log(f)}} \operatorname{erfi}\left(\frac{b\log(f)+2cx\log(f)+2e}{2\sqrt{c}\sqrt{\log(f)}}\right)}{8\sqrt{c}\sqrt{\log(f)}}$$

input
```
Int[f^(a + b*x + c*x^2)*Sinh[d + e*x]^2,x]
```

output
```
-1/4*(f^(a - b^2/(4*c))*Sqrt[Pi]*Erfi[((b + 2*c*x)*Sqrt[Log[f]])/(2*Sqrt[c
])])/(Sqrt[c]*Sqrt[Log[f]]) - (E^(-2*d - (2*e - b*Log[f])^2/(4*c*Log[f]))*
f^a*Sqrt[Pi]*Erfi[(2*e - b*Log[f] - 2*c*x*Log[f])/(2*Sqrt[c]*Sqrt[Log[f]])
])/(8*Sqrt[c]*Sqrt[Log[f]]) + (E^(2*d - (2*e + b*Log[f])^2/(4*c*Log[f]))*f
^a*Sqrt[Pi]*Erfi[(2*e + b*Log[f] + 2*c*x*Log[f])/(2*Sqrt[c]*Sqrt[Log[f]])]
)/(8*Sqrt[c]*Sqrt[Log[f]])
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 6038
```
Int[(F_)^(u_)*Sinh[v_]^(n_.), x_Symbol] :> Int[ExpandTrigToExp[F^u, Sinh[v]
^n, x], x] /; FreeQ[F, x] && (LinearQ[u, x] || PolyQ[u, x, 2]) && (LinearQ[
v, x] || PolyQ[v, x, 2]) && IGtQ[n, 0]
```

Maple [A] (verified)

Time = 0.34 (sec) , antiderivative size = 210, normalized size of antiderivative = 0.96

method	result
risch	$-\frac{\operatorname{erf}\left(-\sqrt{-c\ln(f)}\,x+\frac{b\ln(f)-2e}{2\sqrt{-c\ln(f)}}\right)\sqrt{\pi}\, f^a f^{-\frac{b^2}{4c}} e^{\frac{\ln(f)be-2d\ln(f)c-e^2}{\ln(f)c}}}{8\sqrt{-c\ln(f)}} - \frac{\operatorname{erf}\left(-\sqrt{-c\ln(f)}\,x+\frac{2e+b\ln(f)}{2\sqrt{-c\ln(f)}}\right)\sqrt{\pi}\, f^a f^{-\frac{b^2}{4c}} e^{-\frac{\ln(f)be\cdots}{}}}{8\sqrt{-c\ln(f)}}$

input
```
int(f^(c*x^2+b*x+a)*sinh(e*x+d)^2,x,method=_RETURNVERBOSE)
```

output
```
-1/8*erf(-(-c*ln(f))^(1/2)*x+1/2*(b*ln(f)-2*e)/(-c*ln(f))^(1/2))/(-c*ln(f)
)^(1/2)*Pi^(1/2)*f^a*f^(-1/4*b^2/c)*exp((ln(f)*b*e-2*d*ln(f)*c-e^2)/ln(f)/
c)-1/8*erf(-(-c*ln(f))^(1/2)*x+1/2*(2*e+b*ln(f))/(-c*ln(f))^(1/2))/(-c*ln(
f))^(1/2)*Pi^(1/2)*f^a*f^(-1/4*b^2/c)*exp(-(ln(f)*b*e-2*d*ln(f)*c+e^2)/ln(
f)/c)+1/4*f^a*Pi^(1/2)*f^(-1/4*b^2/c)/(-c*ln(f))^(1/2)*erf(-(-c*ln(f))^(1/
2)*x+1/2*ln(f)*b/(-c*ln(f))^(1/2))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 343 vs. $2(167) = 334$.

Time = 0.10 (sec) , antiderivative size = 343, normalized size of antiderivative = 1.57

$$\int f^{a+bx+cx^2}\sinh^2(d+ex)\,dx$$
$$= \frac{2\sqrt{-c\log(f)}\left(\sqrt{\pi}\cosh\left(-\frac{(b^2-4ac)\log(f)}{4c}\right)+\sqrt{\pi}\sinh\left(-\frac{(b^2-4ac)\log(f)}{4c}\right)\right)\operatorname{erf}\left(\frac{(2cx+b)\sqrt{-c\log(f)}}{2c}\right)-\sqrt{-c\,\mathrm{lo}}}{}$$

input
```
integrate(f^(c*x^2+b*x+a)*sinh(e*x+d)^2,x, algorithm="fricas")
```

output
```
1/8*(2*sqrt(-c*log(f))*(sqrt(pi)*cosh(-1/4*(b^2 - 4*a*c)*log(f)/c) + sqrt(
pi)*sinh(-1/4*(b^2 - 4*a*c)*log(f)/c))*erf(1/2*(2*c*x + b)*sqrt(-c*log(f))
/c) - sqrt(-c*log(f))*(sqrt(pi)*cosh(-1/4*((b^2 - 4*a*c)*log(f)^2 + 4*e^2
- 4*(2*c*d - b*e)*log(f))/(c*log(f))) + sqrt(pi)*sinh(-1/4*((b^2 - 4*a*c)*
log(f)^2 + 4*e^2 - 4*(2*c*d - b*e)*log(f))/(c*log(f))))*erf(1/2*((2*c*x +
b)*log(f) + 2*e)*sqrt(-c*log(f))/(c*log(f))) - sqrt(-c*log(f))*(sqrt(pi)*c
osh(-1/4*((b^2 - 4*a*c)*log(f)^2 + 4*e^2 + 4*(2*c*d - b*e)*log(f))/(c*log(
f))) + sqrt(pi)*sinh(-1/4*((b^2 - 4*a*c)*log(f)^2 + 4*e^2 + 4*(2*c*d - b*e
)*log(f))/(c*log(f))))*erf(1/2*((2*c*x + b)*log(f) - 2*e)*sqrt(-c*log(f))/
(c*log(f))))/(c*log(f))
```

Sympy [F]

$$\int f^{a+bx+cx^2}\sinh^2(d+ex)\,dx=\int f^{a+bx+cx^2}\sinh^2{(d+ex)}\,dx$$

input
```
integrate(f**(c*x**2+b*x+a)*sinh(e*x+d)**2,x)
```

output
```
Integral(f**(a + b*x + c*x**2)*sinh(d + e*x)**2, x)
```

Maxima [A] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 185, normalized size of antiderivative = 0.84

$$\int f^{a+bx+cx^2}\sinh^2(d+ex)\,dx = \frac{\sqrt{\pi}f^a\operatorname{erf}\left(\sqrt{-c\log(f)}x-\frac{b\log(f)+2e}{2\sqrt{-c\log(f)}}\right)e^{\left(2d-\frac{(b\log(f)+2e)^2}{4c\log(f)}\right)}}{8\sqrt{-c\log(f)}} + \frac{\sqrt{\pi}f^a\operatorname{erf}\left(\sqrt{-c\log(f)}x-\frac{b\log(f)-2e}{2\sqrt{-c\log(f)}}\right)e^{\left(-2d-\frac{(b\log(f)-2e)^2}{4c\log(f)}\right)}}{8\sqrt{-c\log(f)}} - \frac{\sqrt{\pi}f^a\operatorname{erf}\left(\sqrt{-c\log(f)}x-\frac{b\log(f)}{2\sqrt{-c\log(f)}}\right)}{4\sqrt{-c\log(f)}f^{\frac{b^2}{4c}}}$$

input
```
integrate(f^(c*x^2+b*x+a)*sinh(e*x+d)^2,x, algorithm="maxima")
```

output
```
1/8*sqrt(pi)*f^a*erf(sqrt(-c*log(f))*x - 1/2*(b*log(f) + 2*e)/sqrt(-c*log(
f)))*e^(2*d - 1/4*(b*log(f) + 2*e)^2/(c*log(f)))/sqrt(-c*log(f)) + 1/8*sqr
t(pi)*f^a*erf(sqrt(-c*log(f))*x - 1/2*(b*log(f) - 2*e)/sqrt(-c*log(f)))*e^
(-2*d - 1/4*(b*log(f) - 2*e)^2/(c*log(f)))/sqrt(-c*log(f)) - 1/4*sqrt(pi)*
f^a*erf(sqrt(-c*log(f))*x - 1/2*b*log(f)/sqrt(-c*log(f)))/(sqrt(-c*log(f))
*f^(1/4*b^2/c))
```

Giac [A] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 223, normalized size of antiderivative = 1.02

$$\int f^{a+bx+cx^2}\sinh^2(d+ex)\,dx = \frac{\sqrt{\pi}\,\mathrm{erf}\left(-\frac{1}{2}\sqrt{-c\log(f)}\left(2x+\frac{b}{c}\right)\right)e^{\left(-\frac{b^2\log(f)-4ac\log(f)}{4c}\right)}}{4\sqrt{-c\log(f)}} - \frac{\sqrt{\pi}\,\mathrm{erf}\left(-\frac{1}{2}\sqrt{-c\log(f)}\left(2x+\frac{b\log(f)-2e}{c\log(f)}\right)\right)e^{\left(-\frac{b^2\log(f)^2-4ac\log(f)^2+8cd\log(f)-4be\log(f)+4e^2}{4c\log(f)}\right)}}{8\sqrt{-c\log(f)}} - \frac{\sqrt{\pi}\,\mathrm{erf}\left(-\frac{1}{2}\sqrt{-c\log(f)}\left(2x+\frac{b\log(f)+2e}{c\log(f)}\right)\right)e^{\left(-\frac{b^2\log(f)^2-4ac\log(f)^2-8cd\log(f)+4be\log(f)+4e^2}{4c\log(f)}\right)}}{8\sqrt{-c\log(f)}}$$

input
```
integrate(f^(c*x^2+b*x+a)*sinh(e*x+d)^2,x, algorithm="giac")
```

output
```
1/4*sqrt(pi)*erf(-1/2*sqrt(-c*log(f))*(2*x + b/c))*e^(-1/4*(b^2*log(f) - 4
*a*c*log(f))/c)/sqrt(-c*log(f)) - 1/8*sqrt(pi)*erf(-1/2*sqrt(-c*log(f))*(2
*x + (b*log(f) - 2*e)/(c*log(f))))*e^(-1/4*(b^2*log(f)^2 - 4*a*c*log(f)^2
+ 8*c*d*log(f) - 4*b*e*log(f) + 4*e^2)/(c*log(f)))/sqrt(-c*log(f)) - 1/8*s
qrt(pi)*erf(-1/2*sqrt(-c*log(f))*(2*x + (b*log(f) + 2*e)/(c*log(f))))*e^(-
1/4*(b^2*log(f)^2 - 4*a*c*log(f)^2 - 8*c*d*log(f) + 4*b*e*log(f) + 4*e^2)/
(c*log(f)))/sqrt(-c*log(f))
```

Mupad [F(-1)]

Timed out.

$$\int f^{a+bx+cx^2}\sinh^2(d+ex)\,dx = \int f^{c\,x^2+b\,x+a}\,\sinh(d+e\,x)^2\,dx$$

input
```
int(f^(a + b*x + c*x^2)*sinh(d + e*x)^2,x)
```

output
```
int(f^(a + b*x + c*x^2)*sinh(d + e*x)^2, x)
```

Reduce [F]

$$\int f^{a+bx+cx^2}\sinh^2(d+ex)\,dx = f^a\left(\int f^{c\,x^2+bx}\sinh\left(ex+d\right)^2dx\right)$$

input
```
int(f^(c*x^2+b*x+a)*sinh(e*x+d)^2,x)
```

output
```
f**a*int(f**(b*x + c*x**2)*sinh(d + e*x)**2,x)
```

3.359 $\int f^{a+bx+cx^2}\sinh^3(d+ex)\,dx$

Optimal result

Integrand size = 21, antiderivative size = 315

$$\int f^{a+bx+cx^2}\sinh^3(d+ex)\,dx = -\frac{3e^{-d-\frac{(e-b\log(f))^2}{4c\log(f)}}f^a\sqrt{\pi}\operatorname{erfi}\left(\frac{e-b\log(f)-2cx\log(f)}{2\sqrt{c}\sqrt{\log(f)}}\right)}{16\sqrt{c}\sqrt{\log(f)}} + \frac{e^{-3d-\frac{(3e-b\log(f))^2}{4c\log(f)}}f^a\sqrt{\pi}\operatorname{erfi}\left(\frac{3e-b\log(f)-2cx\log(f)}{2\sqrt{c}\sqrt{\log(f)}}\right)}{16\sqrt{c}\sqrt{\log(f)}} - \frac{3e^{d-\frac{(e+b\log(f))^2}{4c\log(f)}}f^a\sqrt{\pi}\operatorname{erfi}\left(\frac{e+b\log(f)+2cx\log(f)}{2\sqrt{c}\sqrt{\log(f)}}\right)}{16\sqrt{c}\sqrt{\log(f)}} + \frac{e^{3d-\frac{(3e+b\log(f))^2}{4c\log(f)}}f^a\sqrt{\pi}\operatorname{erfi}\left(\frac{3e+b\log(f)+2cx\log(f)}{2\sqrt{c}\sqrt{\log(f)}}\right)}{16\sqrt{c}\sqrt{\log(f)}}$$

output
```
-3/16*exp(-d-1/4*(e-b*ln(f))^2/c/ln(f))*f^a*Pi^(1/2)*erfi(1/2*(e-b*ln(f)-2
*c*x*ln(f))/c^(1/2)/ln(f)^(1/2))/c^(1/2)/ln(f)^(1/2)+1/16*exp(-3*d-1/4*(3*
e-b*ln(f))^2/c/ln(f))*f^a*Pi^(1/2)*erfi(1/2*(3*e-b*ln(f)-2*c*x*ln(f))/c^(1
/2)/ln(f)^(1/2))/c^(1/2)/ln(f)^(1/2)-3/16*exp(d-1/4*(e+b*ln(f))^2/c/ln(f))
*f^a*Pi^(1/2)*erfi(1/2*(e+b*ln(f)+2*c*x*ln(f))/c^(1/2)/ln(f)^(1/2))/c^(1/2
)/ln(f)^(1/2)+1/16*exp(3*d-1/4*(3*e+b*ln(f))^2/c/ln(f))*f^a*Pi^(1/2)*erfi(
1/2*(3*e+b*ln(f)+2*c*x*ln(f))/c^(1/2)/ln(f)^(1/2))/c^(1/2)/ln(f)^(1/2)
```

Mathematica [A] (verified)

Time = 0.63 (sec) , antiderivative size = 263, normalized size of antiderivative = 0.83

$$\int f^{a+bx+cx^2}\sinh^3(d+ex)\,dx$$
$$= \frac{e^{-\frac{3e(3e+2b\log(f))}{4c\log(f)}}f^{a-\frac{b^2}{4c}}\sqrt{\pi}\left((\cosh(d)+\sinh(d))\left(-3e^{\frac{e(2e+b\log(f))}{c\log(f)}}\operatorname{erfi}\left(\frac{e+(b+2cx)\log(f)}{2\sqrt{c}\sqrt{\log(f)}}\right)+3e^{\frac{2e(e+b\log(f))}{c\log(f)}}\operatorname{erfi}\left(\frac{-e+(\cdots}{2\sqrt{\cdots}}\right.\right.\right.}{}$$

input
```
Integrate[f^(a + b*x + c*x^2)*Sinh[d + e*x]^3,x]
```

output
```
(f^(a - b^2/(4*c))*Sqrt[Pi]*((Cosh[d] + Sinh[d])*(-3*E^((e*(2*e + b*Log[f]
))/(c*Log[f]))*Erfi[(e + (b + 2*c*x)*Log[f])/(2*Sqrt[c]*Sqrt[Log[f]])] + 3
*E^((2*e*(e + b*Log[f]))/(c*Log[f]))*Erfi[(-e + (b + 2*c*x)*Log[f])/(2*Sqr
t[c]*Sqrt[Log[f]])]*(Cosh[2*d] - Sinh[2*d]) + Erfi[(3*e + (b + 2*c*x)*Log[
f])/(2*Sqrt[c]*Sqrt[Log[f]])]*(Cosh[2*d] + Sinh[2*d])) - E^((3*b*e)/c)*Erf
i[(-3*e + (b + 2*c*x)*Log[f])/(2*Sqrt[c]*Sqrt[Log[f]])]*(Cosh[3*d] - Sinh[
3*d])))/(16*Sqrt[c]*E^((3*e*(3*e + 2*b*Log[f]))/(4*c*Log[f]))*Sqrt[Log[f]]
)
```

Rubi [A] (verified)

Time = 0.85 (sec) , antiderivative size = 315, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.095$, Rules used = {6038, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sinh^3(d+ex)f^{a+bx+cx^2}\,dx$$
$$\Big\downarrow \ 6038$$
$$\int \left(-\frac{1}{8}e^{-3d-3ex}f^{a+bx+cx^2}+\frac{3}{8}e^{-d-ex}f^{a+bx+cx^2}-\frac{3}{8}e^{d+ex}f^{a+bx+cx^2}+\frac{1}{8}e^{3d+3ex}f^{a+bx+cx^2}\right)dx$$
$$\Big\downarrow \ 2009$$

$$-\frac{3\sqrt{\pi}f^a e^{-\frac{(e-b\log(f))^2}{4c\log(f)}-d}\operatorname{erfi}\left(\frac{-b\log(f)-2cx\log(f)+e}{2\sqrt{c}\sqrt{\log(f)}}\right)}{16\sqrt{c}\sqrt{\log(f)}}+\frac{\sqrt{\pi}f^a e^{-\frac{(3e-b\log(f))^2}{4c\log(f)}-3d}\operatorname{erfi}\left(\frac{-b\log(f)-2cx\log(f)+3e}{2\sqrt{c}\sqrt{\log(f)}}\right)}{16\sqrt{c}\sqrt{\log(f)}}-\frac{3\sqrt{\pi}f^a e^{d-\frac{(b\log(f)+e)^2}{4c\log(f)}}\operatorname{erfi}\left(\frac{b\log(f)+2cx\log(f)+e}{2\sqrt{c}\sqrt{\log(f)}}\right)}{16\sqrt{c}\sqrt{\log(f)}}+\frac{\sqrt{\pi}f^a e^{3d-\frac{(b\log(f)+3e)^2}{4c\log(f)}}\operatorname{erfi}\left(\frac{b\log(f)+2cx\log(f)+3e}{2\sqrt{c}\sqrt{\log(f)}}\right)}{16\sqrt{c}\sqrt{\log(f)}}$$

input
```
Int[f^(a + b*x + c*x^2)*Sinh[d + e*x]^3,x]
```

output
```
(-3*E^(-d - (e - b*Log[f])^2/(4*c*Log[f]))*f^a*Sqrt[Pi]*Erfi[(e - b*Log[f]
 - 2*c*x*Log[f])/(2*Sqrt[c]*Sqrt[Log[f]])])/(16*Sqrt[c]*Sqrt[Log[f]]) + (E
^(-3*d - (3*e - b*Log[f])^2/(4*c*Log[f]))*f^a*Sqrt[Pi]*Erfi[(3*e - b*Log[f
] - 2*c*x*Log[f])/(2*Sqrt[c]*Sqrt[Log[f]])])/(16*Sqrt[c]*Sqrt[Log[f]]) - (
3*E^(d - (e + b*Log[f])^2/(4*c*Log[f]))*f^a*Sqrt[Pi]*Erfi[(e + b*Log[f] +
2*c*x*Log[f])/(2*Sqrt[c]*Sqrt[Log[f]])])/(16*Sqrt[c]*Sqrt[Log[f]]) + (E^(3
*d - (3*e + b*Log[f])^2/(4*c*Log[f]))*f^a*Sqrt[Pi]*Erfi[(3*e + b*Log[f] +
2*c*x*Log[f])/(2*Sqrt[c]*Sqrt[Log[f]])])/(16*Sqrt[c]*Sqrt[Log[f]])
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 6038
```
Int[(F_)^(u_)*Sinh[v_]^(n_.), x_Symbol] :> Int[ExpandTrigToExp[F^u, Sinh[v]
^n, x], x] /; FreeQ[F, x] && (LinearQ[u, x] || PolyQ[u, x, 2]) && (LinearQ[
v, x] || PolyQ[v, x, 2]) && IGtQ[n, 0]
```

Maple [A] (verified)

Time = 0.87 (sec) , antiderivative size = 326, normalized size of antiderivative = 1.03

method	result
risch	$-\frac{\operatorname{erf}\left(-\sqrt{-c\ln(f)}\,x+\frac{3e+b\ln(f)}{2\sqrt{-c\ln(f)}}\right)\sqrt{\pi}\,f^{a} f^{-\frac{b^{2}}{4c}} \mathrm{e}^{-\frac{3\left(2\ln(f)be-4d\ln(f)c+3e^{2}\right)}{4\ln(f)c}}}{16\sqrt{-c\ln(f)}}+\frac{\operatorname{erf}\left(-\sqrt{-c\ln(f)}\,x+\frac{b\ln(f)-3e}{2\sqrt{-c\ln(f)}}\right)\sqrt{\pi}\,f^{a} f^{-\frac{b^{2}}{4c}} \mathrm{e}^{\frac{3}{}}}{16\sqrt{-c\ln(f)}}$

input
```
int(f^(c*x^2+b*x+a)*sinh(e*x+d)^3,x,method=_RETURNVERBOSE)
```

output
```
-1/16*erf(-(-c*ln(f))^(1/2)*x+1/2*(3*e+b*ln(f))/(-c*ln(f))^(1/2))/(-c*ln(f
))^(1/2)*Pi^(1/2)*f^a*f^(-1/4*b^2/c)*exp(-3/4*(2*ln(f)*b*e-4*d*ln(f)*c+3*e
^2)/ln(f)/c)+1/16*erf(-(-c*ln(f))^(1/2)*x+1/2*(b*ln(f)-3*e)/(-c*ln(f))^(1/
2))/(-c*ln(f))^(1/2)*Pi^(1/2)*f^a*f^(-1/4*b^2/c)*exp(3/4*(2*ln(f)*b*e-4*d*
ln(f)*c-3*e^2)/ln(f)/c)-3/16*erf(-(-c*ln(f))^(1/2)*x+1/2*(b*ln(f)-e)/(-c*l
n(f))^(1/2))/(-c*ln(f))^(1/2)*Pi^(1/2)*f^a*f^(-1/4*b^2/c)*exp(1/4*(2*ln(f)
*b*e-4*d*ln(f)*c-e^2)/ln(f)/c)+3/16*erf(-(-c*ln(f))^(1/2)*x+1/2*(e+b*ln(f)
)/(-c*ln(f))^(1/2))/(-c*ln(f))^(1/2)*Pi^(1/2)*f^a*f^(-1/4*b^2/c)*exp(-1/4*
(2*ln(f)*b*e-4*d*ln(f)*c+e^2)/ln(f)/c)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 527 vs. 2(247) = 494.

Time = 0.12 (sec) , antiderivative size = 527, normalized size of antiderivative = 1.67

$$\int f^{a+bx+cx^2}\sinh^3(d+ex)\,dx = \text{Too large to display}$$

input
```
integrate(f^(c*x^2+b*x+a)*sinh(e*x+d)^3,x, algorithm="fricas")
```

output
```
-1/16*(sqrt(-c*log(f))*(sqrt(pi)*cosh(-1/4*((b^2 - 4*a*c)*log(f)^2 + 9*e^2
 - 6*(2*c*d - b*e)*log(f))/(c*log(f))) + sqrt(pi)*sinh(-1/4*((b^2 - 4*a*c)
*log(f)^2 + 9*e^2 - 6*(2*c*d - b*e)*log(f))/(c*log(f))))*erf(1/2*((2*c*x +
 b)*log(f) + 3*e)*sqrt(-c*log(f))/(c*log(f))) - 3*sqrt(-c*log(f))*(sqrt(pi
)*cosh(-1/4*((b^2 - 4*a*c)*log(f)^2 + e^2 - 2*(2*c*d - b*e)*log(f))/(c*log
(f))) + sqrt(pi)*sinh(-1/4*((b^2 - 4*a*c)*log(f)^2 + e^2 - 2*(2*c*d - b*e)
*log(f))/(c*log(f))))*erf(1/2*((2*c*x + b)*log(f) + e)*sqrt(-c*log(f))/(c*
log(f))) + 3*sqrt(-c*log(f))*(sqrt(pi)*cosh(-1/4*((b^2 - 4*a*c)*log(f)^2 +
 e^2 + 2*(2*c*d - b*e)*log(f))/(c*log(f))) + sqrt(pi)*sinh(-1/4*((b^2 - 4*
a*c)*log(f)^2 + e^2 + 2*(2*c*d - b*e)*log(f))/(c*log(f))))*erf(1/2*((2*c*x
 + b)*log(f) - e)*sqrt(-c*log(f))/(c*log(f))) - sqrt(-c*log(f))*(sqrt(pi)*
cosh(-1/4*((b^2 - 4*a*c)*log(f)^2 + 9*e^2 + 6*(2*c*d - b*e)*log(f))/(c*log
(f))) + sqrt(pi)*sinh(-1/4*((b^2 - 4*a*c)*log(f)^2 + 9*e^2 + 6*(2*c*d - b*
e)*log(f))/(c*log(f))))*erf(1/2*((2*c*x + b)*log(f) - 3*e)*sqrt(-c*log(f))
/(c*log(f))))/(c*log(f))
```

Sympy [F]

$$\int f^{a+bx+cx^2} \sinh^3(d+ex)\,dx = \int f^{a+bx+cx^2} \sinh^3{(d+ex)}\,dx$$

input
```
integrate(f**(c*x**2+b*x+a)*sinh(e*x+d)**3,x)
```

output
```
Integral(f**(a + b*x + c*x**2)*sinh(d + e*x)**3, x)
```

Maxima [A] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 263, normalized size of antiderivative = 0.83

$$\int f^{a+bx+cx^2}\sinh^3(d+ex)\,dx$$
$$= \frac{\sqrt{\pi}f^a\,\mathrm{erf}\left(\sqrt{-c\log(f)}x - \frac{b\log(f)+3\,e}{2\sqrt{-c\log(f)}}\right)e^{\left(3\,d-\frac{(b\log(f)+3\,e)^2}{4\,c\log(f)}\right)}}{16\sqrt{-c\log(f)}} - \frac{3\sqrt{\pi}f^a\,\mathrm{erf}\left(\sqrt{-c\log(f)}x - \frac{b\log(f)+e}{2\sqrt{-c\log(f)}}\right)e^{\left(d-\frac{(b\log(f)+e)^2}{4\,c\log(f)}\right)}}{16\sqrt{-c\log(f)}} + \frac{3\sqrt{\pi}f^a\,\mathrm{erf}\left(\sqrt{-c\log(f)}x - \frac{b\log(f)-e}{2\sqrt{-c\log(f)}}\right)e^{\left(-d-\frac{(b\log(f)-e)^2}{4\,c\log(f)}\right)}}{16\sqrt{-c\log(f)}} - \frac{\sqrt{\pi}f^a\,\mathrm{erf}\left(\sqrt{-c\log(f)}x - \frac{b\log(f)-3\,e}{2\sqrt{-c\log(f)}}\right)e^{\left(-3\,d-\frac{(b\log(f)-3\,e)^2}{4\,c\log(f)}\right)}}{16\sqrt{-c\log(f)}}$$

input
```
integrate(f^(c*x^2+b*x+a)*sinh(e*x+d)^3,x, algorithm="maxima")
```

output
```
1/16*sqrt(pi)*f^a*erf(sqrt(-c*log(f))*x - 1/2*(b*log(f) + 3*e)/sqrt(-c*log
(f)))*e^(3*d - 1/4*(b*log(f) + 3*e)^2/(c*log(f)))/sqrt(-c*log(f)) - 3/16*s
qrt(pi)*f^a*erf(sqrt(-c*log(f))*x - 1/2*(b*log(f) + e)/sqrt(-c*log(f)))*e^
(d - 1/4*(b*log(f) + e)^2/(c*log(f)))/sqrt(-c*log(f)) + 3/16*sqrt(pi)*f^a*
erf(sqrt(-c*log(f))*x - 1/2*(b*log(f) - e)/sqrt(-c*log(f)))*e^(-d - 1/4*(b
*log(f) - e)^2/(c*log(f)))/sqrt(-c*log(f)) - 1/16*sqrt(pi)*f^a*erf(sqrt(-c
*log(f))*x - 1/2*(b*log(f) - 3*e)/sqrt(-c*log(f)))*e^(-3*d - 1/4*(b*log(f)
 - 3*e)^2/(c*log(f)))/sqrt(-c*log(f))
```

Giac [A] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 339, normalized size of antiderivative = 1.08

$$\int f^{a+bx+cx^2}\sinh^3(d+ex)\,dx$$

$$= \frac{\sqrt{\pi}\,\mathrm{erf}\left(-\frac{1}{2}\sqrt{-c\log(f)}\left(2x+\frac{b\log(f)-3e}{c\log(f)}\right)\right)e^{\left(-\frac{b^2\log(f)^2-4ac\log(f)^2+12cd\log(f)-6be\log(f)+9e^2}{4c\log(f)}\right)}}{16\sqrt{-c\log(f)}}$$
$$-\frac{3\sqrt{\pi}\,\mathrm{erf}\left(-\frac{1}{2}\sqrt{-c\log(f)}\left(2x+\frac{b\log(f)-e}{c\log(f)}\right)\right)e^{\left(-\frac{b^2\log(f)^2-4ac\log(f)^2+4cd\log(f)-2be\log(f)+e^2}{4c\log(f)}\right)}}{16\sqrt{-c\log(f)}}$$
$$+\frac{3\sqrt{\pi}\,\mathrm{erf}\left(-\frac{1}{2}\sqrt{-c\log(f)}\left(2x+\frac{b\log(f)+e}{c\log(f)}\right)\right)e^{\left(-\frac{b^2\log(f)^2-4ac\log(f)^2-4cd\log(f)+2be\log(f)+e^2}{4c\log(f)}\right)}}{16\sqrt{-c\log(f)}}$$
$$-\frac{\sqrt{\pi}\,\mathrm{erf}\left(-\frac{1}{2}\sqrt{-c\log(f)}\left(2x+\frac{b\log(f)+3e}{c\log(f)}\right)\right)e^{\left(-\frac{b^2\log(f)^2-4ac\log(f)^2-12cd\log(f)+6be\log(f)+9e^2}{4c\log(f)}\right)}}{16\sqrt{-c\log(f)}}$$

input
```
integrate(f^(c*x^2+b*x+a)*sinh(e*x+d)^3,x, algorithm="giac")
```

output
```
1/16*sqrt(pi)*erf(-1/2*sqrt(-c*log(f))*(2*x + (b*log(f) - 3*e)/(c*log(f)))
)*e^(-1/4*(b^2*log(f)^2 - 4*a*c*log(f)^2 + 12*c*d*log(f) - 6*b*e*log(f) +
9*e^2)/(c*log(f)))/sqrt(-c*log(f)) - 3/16*sqrt(pi)*erf(-1/2*sqrt(-c*log(f)
)*(2*x + (b*log(f) - e)/(c*log(f))))*e^(-1/4*(b^2*log(f)^2 - 4*a*c*log(f)^
2 + 4*c*d*log(f) - 2*b*e*log(f) + e^2)/(c*log(f)))/sqrt(-c*log(f)) + 3/16*
sqrt(pi)*erf(-1/2*sqrt(-c*log(f))*(2*x + (b*log(f) + e)/(c*log(f))))*e^(-1
/4*(b^2*log(f)^2 - 4*a*c*log(f)^2 - 4*c*d*log(f) + 2*b*e*log(f) + e^2)/(c*
log(f)))/sqrt(-c*log(f)) - 1/16*sqrt(pi)*erf(-1/2*sqrt(-c*log(f))*(2*x + (
b*log(f) + 3*e)/(c*log(f))))*e^(-1/4*(b^2*log(f)^2 - 4*a*c*log(f)^2 - 12*c
*d*log(f) + 6*b*e*log(f) + 9*e^2)/(c*log(f)))/sqrt(-c*log(f))
```

Mupad [F(-1)]

Timed out.

$$\int f^{a+bx+cx^2}\sinh^3(d+ex)\,dx=\int f^{c\,x^2+b\,x+a}\,\sinh(d+e\,x)^3\,dx$$

input
```
int(f^(a + b*x + c*x^2)*sinh(d + e*x)^3,x)
```

output
```
int(f^(a + b*x + c*x^2)*sinh(d + e*x)^3, x)
```

Reduce [F]

$$\int f^{a+bx+cx^2}\sinh^3(d+ex)\,dx=f^a\left(\int f^{c\,x^2+bx}\sinh\left(ex+d\right)^3dx\right)$$

input
```
int(f^(c*x^2+b*x+a)*sinh(e*x+d)^3,x)
```

output
```
f**a*int(f**(b*x + c*x**2)*sinh(d + e*x)**3,x)
```

3.360 $\int f^{a+bx+cx^2} \sinh(d+fx^2)\,dx$

Optimal result

Integrand size = 21, antiderivative size = 154

$$\int f^{a+bx+cx^2}\sinh\left(d+fx^2\right)\,dx=\frac{e^{-d+\frac{b^2\log^2(f)}{4f-4c\log(f)}}f^a\sqrt{\pi}\,\text{erf}\left(\frac{b\log(f)-2x(f-c\log(f))}{2\sqrt{f-c\log(f)}}\right)}{4\sqrt{f-c\log(f)}}+\frac{e^{d-\frac{b^2\log^2(f)}{4(f+c\log(f))}}f^a\sqrt{\pi}\,\text{erfi}\left(\frac{b\log(f)+2x(f+c\log(f))}{2\sqrt{f+c\log(f)}}\right)}{4\sqrt{f+c\log(f)}}$$

output
```
1/4*exp(-d+b^2*ln(f)^2/(4*f-4*c*ln(f)))*f^a*Pi^(1/2)*erf(1/2*(b*ln(f)-2*x*
(f-c*ln(f)))/(f-c*ln(f))^(1/2))/(f-c*ln(f))^(1/2)+1/4*exp(d-b^2*ln(f)^2/(4
*f+4*c*ln(f)))*f^a*Pi^(1/2)*erfi(1/2*(b*ln(f)+2*x*(f+c*ln(f)))/(f+c*ln(f))
^(1/2))/(f+c*ln(f))^(1/2)
```

Mathematica [A] (verified)

Time = 0.53 (sec) , antiderivative size = 179, normalized size of antiderivative = 1.16

$$\int f^{a+bx+cx^2}\sinh\left(d+fx^2\right)\,dx$$

$$=\frac{e^{-\frac{b^2\log^2(f)}{4(f+c\log(f))}}f^a\sqrt{\pi}\left(-e^{\frac{b^2f\log^2(f)}{2f^2-2c^2\log^2(f)}}\text{erf}\left(\frac{2fx-(b+2cx)\log(f)}{2\sqrt{f-c\log(f)}}\right)\sqrt{f+c\log(f)}(\cosh(d)-\sinh(d))+\text{erfi}\left(\frac{2fx+(b+}{2\sqrt{f+}}\right.\right.}{4\sqrt{f-c\log(f)}\sqrt{f+c\log(f)}}$$

input
```
Integrate[f^(a + b*x + c*x^2)*Sinh[d + f*x^2],x]
```

output
```
(f^a*Sqrt[Pi]*(-(E^((b^2*f*Log[f]^2)/(2*f^2 - 2*c^2*Log[f]^2))*Erf[(2*f*x
- (b + 2*c*x)*Log[f])/(2*Sqrt[f - c*Log[f]])]*Sqrt[f + c*Log[f]]*(Cosh[d]
- Sinh[d])) + Erfi[(2*f*x + (b + 2*c*x)*Log[f])/(2*Sqrt[f + c*Log[f]])]*Sq
rt[f - c*Log[f]]*(Cosh[d] + Sinh[d])))/(4*E^((b^2*Log[f]^2)/(4*(f + c*Log[
f])))*Sqrt[f - c*Log[f]]*Sqrt[f + c*Log[f]])
```

Rubi [A] (verified)

Time = 0.61 (sec) , antiderivative size = 154, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.095$, Rules used = {6038, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sinh\left(d+fx^2\right) f^{a+bx+cx^2}\,dx$$

$$\Big\downarrow \ 6038$$

$$\int \left(\frac{1}{2}e^{d+fx^2}f^{a+bx+cx^2}-\frac{1}{2}e^{-d-fx^2}f^{a+bx+cx^2}\right)dx$$

$$\Big\downarrow \ 2009$$

$$\frac{\sqrt{\pi}f^a e^{\frac{b^2\log^2(f)}{4f-4c\log(f)}-d}\text{erf}\left(\frac{b\log(f)-2x(f-c\log(f))}{2\sqrt{f-c\log(f)}}\right)}{4\sqrt{f-c\log(f)}}+\frac{\sqrt{\pi}f^a e^{d-\frac{b^2\log^2(f)}{4(c\log(f)+f)}}\text{erfi}\left(\frac{b\log(f)+2x(c\log(f)+f)}{2\sqrt{c\log(f)+f}}\right)}{4\sqrt{c\log(f)+f}}$$

input
```
Int[f^(a + b*x + c*x^2)*Sinh[d + f*x^2],x]
```

output
```
(E^(-d + (b^2*Log[f]^2)/(4*f - 4*c*Log[f]))*f^a*Sqrt[Pi]*Erf[(b*Log[f] - 2
*x*(f - c*Log[f]))/(2*Sqrt[f - c*Log[f]])])/(4*Sqrt[f - c*Log[f]]) + (E^(d
 - (b^2*Log[f]^2)/(4*(f + c*Log[f])))*f^a*Sqrt[Pi]*Erfi[(b*Log[f] + 2*x*(f
 + c*Log[f]))/(2*Sqrt[f + c*Log[f]])])/(4*Sqrt[f + c*Log[f]])
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 6038
```
Int[(F_)^(u_)*Sinh[v_]^(n_.), x_Symbol] :> Int[ExpandTrigToExp[F^u, Sinh[v]
^n, x], x] /; FreeQ[F, x] && (LinearQ[u, x] || PolyQ[u, x, 2]) && (LinearQ[
v, x] || PolyQ[v, x, 2]) && IGtQ[n, 0]
```

Maple [A] (verified)

Time = 0.15 (sec) , antiderivative size = 160, normalized size of antiderivative = 1.04

method	result
risch	$-\frac{\operatorname{erf}\left(-\sqrt{-c\ln(f)-f}\,x+\frac{\ln(f)b}{2\sqrt{-c\ln(f)-f}}\right)\sqrt{\pi}\,f^{a}\mathrm{e}^{-\frac{b^{2}\ln(f)^{2}-4d\ln(f)c-4df}{4(f+c\ln(f))}}}{4\sqrt{-c\ln(f)-f}}+\frac{\operatorname{erf}\left(-x\sqrt{f-c\ln(f)}+\frac{\ln(f)b}{2\sqrt{f-c\ln(f)}}\right)\sqrt{\pi}\,f^{a}\mathrm{e}^{-\frac{b^{2}\ln(f}{4}}}{4\sqrt{f-c\ln(f)}}$

input
```
int(f^(c*x^2+b*x+a)*sinh(f*x^2+d),x,method=_RETURNVERBOSE)
```

output
```
-1/4*erf(-(-c*ln(f)-f)^(1/2)*x+1/2*ln(f)*b/(-c*ln(f)-f)^(1/2))/(-c*ln(f)-f
)^(1/2)*Pi^(1/2)*f^a*exp(-1/4*(b^2*ln(f)^2-4*d*ln(f)*c-4*d*f)/(f+c*ln(f)))
+1/4*erf(-x*(f-c*ln(f))^(1/2)+1/2*ln(f)*b/(f-c*ln(f))^(1/2))/(f-c*ln(f))^(
1/2)*Pi^(1/2)*f^a*exp(-1/4*(b^2*ln(f)^2+4*d*ln(f)*c-4*d*f)/(c*ln(f)-f))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 325 vs. $2(131) = 262$.

Time = 0.12 (sec) , antiderivative size = 325, normalized size of antiderivative = 2.11

$$\int f^{a+bx+cx^2}\sinh\left(d+fx^2\right)\,dx$$
$$=\frac{\left(\sqrt{\pi}(c\log\left(f\right)+f)\cosh\left(-\frac{(b^2-4\,ac)\log(f)^2-4\,df+4\,(cd+af)\log(f)}{4\,(c\log(f)-f)}\right)+\sqrt{\pi}(c\log\left(f\right)+f)\sinh\left(-\frac{(b^2-4\,ac)\log(f)^2-}{4\,(c\log}\right.\right.}{}$$

input
```
integrate(f^(c*x^2+b*x+a)*sinh(f*x^2+d),x, algorithm="fricas")
```

output
```
1/4*((sqrt(pi)*(c*log(f) + f)*cosh(-1/4*((b^2 - 4*a*c)*log(f)^2 - 4*d*f +
4*(c*d + a*f)*log(f))/(c*log(f) - f)) + sqrt(pi)*(c*log(f) + f)*sinh(-1/4*
((b^2 - 4*a*c)*log(f)^2 - 4*d*f + 4*(c*d + a*f)*log(f))/(c*log(f) - f)))*s
qrt(-c*log(f) + f)*erf(-1/2*(2*f*x - (2*c*x + b)*log(f))*sqrt(-c*log(f) +
f)/(c*log(f) - f)) - (sqrt(pi)*(c*log(f) - f)*cosh(-1/4*((b^2 - 4*a*c)*log
(f)^2 - 4*d*f - 4*(c*d + a*f)*log(f))/(c*log(f) + f)) + sqrt(pi)*(c*log(f)
 - f)*sinh(-1/4*((b^2 - 4*a*c)*log(f)^2 - 4*d*f - 4*(c*d + a*f)*log(f))/(c
*log(f) + f)))*sqrt(-c*log(f) - f)*erf(1/2*(2*f*x + (2*c*x + b)*log(f))*sq
rt(-c*log(f) - f)/(c*log(f) + f)))/(c^2*log(f)^2 - f^2)
```

Sympy [F]

$$\int f^{a+bx+cx^2}\sinh\left(d+fx^2\right)\,dx=\int f^{a+bx+cx^2}\sinh\left(d+fx^2\right)dx$$

input
```
integrate(f**(c*x**2+b*x+a)*sinh(f*x**2+d),x)
```

output
```
Integral(f**(a + b*x + c*x**2)*sinh(d + f*x**2), x)
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 139, normalized size of antiderivative = 0.90

$$\int f^{a+bx+cx^2}\sinh\left(d+fx^2\right)\,dx$$
$$=\frac{\sqrt{\pi}f^a\operatorname{erf}\left(\sqrt{-c\log(f)-f}\,x-\frac{b\log(f)}{2\sqrt{-c\log(f)-f}}\right)e^{\left(-\frac{b^2\log(f)^2}{4\,(c\log(f)+f)}+d\right)}}{4\sqrt{-c\log(f)-f}}-\frac{\sqrt{\pi}f^a\operatorname{erf}\left(\sqrt{-c\log(f)+f}\,x-\frac{b\log(f)}{2\sqrt{-c\log(f)+f}}\right)e^{\left(-\frac{b^2\log(f)^2}{4\,(c\log(f)-f)}-d\right)}}{4\sqrt{-c\log(f)+f}}$$

input
```
integrate(f^(c*x^2+b*x+a)*sinh(f*x^2+d),x, algorithm="maxima")
```

output
```
1/4*sqrt(pi)*f^a*erf(sqrt(-c*log(f) - f)*x - 1/2*b*log(f)/sqrt(-c*log(f) -
 f))*e^(-1/4*b^2*log(f)^2/(c*log(f) + f) + d)/sqrt(-c*log(f) - f) - 1/4*sq
rt(pi)*f^a*erf(sqrt(-c*log(f) + f)*x - 1/2*b*log(f)/sqrt(-c*log(f) + f))*e
^(-1/4*b^2*log(f)^2/(c*log(f) - f) - d)/sqrt(-c*log(f) + f)
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 181, normalized size of antiderivative = 1.18

$$\int f^{a+bx+cx^2}\sinh\left(d+fx^2\right)\,dx = -\frac{\sqrt{\pi}\,\mathrm{erf}\left(-\frac{1}{2}\sqrt{-c\log\left(f\right)-f}\left(2\,x+\frac{b\log(f)}{c\log(f)+f}\right)\right)e^{\left(-\frac{b^2\log(f)^2-4\,ac\log(f)^2-4\,cd\log(f)-4\,af\log(f)-4\,df}{4\,(c\log(f)+f)}\right)}}{4\sqrt{-c\log\left(f\right)-f}} + \frac{\sqrt{\pi}\,\mathrm{erf}\left(-\frac{1}{2}\sqrt{-c\log\left(f\right)+f}\left(2\,x+\frac{b\log(f)}{c\log(f)-f}\right)\right)e^{\left(-\frac{b^2\log(f)^2-4\,ac\log(f)^2+4\,cd\log(f)+4\,af\log(f)-4\,df}{4\,(c\log(f)-f)}\right)}}{4\sqrt{-c\log\left(f\right)+f}}$$

input
```
integrate(f^(c*x^2+b*x+a)*sinh(f*x^2+d),x, algorithm="giac")
```

output
```
-1/4*sqrt(pi)*erf(-1/2*sqrt(-c*log(f) - f)*(2*x + b*log(f)/(c*log(f) + f))
)*e^(-1/4*(b^2*log(f)^2 - 4*a*c*log(f)^2 - 4*c*d*log(f) - 4*a*f*log(f) - 4
*d*f)/(c*log(f) + f))/sqrt(-c*log(f) - f) + 1/4*sqrt(pi)*erf(-1/2*sqrt(-c*
log(f) + f)*(2*x + b*log(f)/(c*log(f) - f)))*e^(-1/4*(b^2*log(f)^2 - 4*a*c
*log(f)^2 + 4*c*d*log(f) + 4*a*f*log(f) - 4*d*f)/(c*log(f) - f))/sqrt(-c*l
og(f) + f)
```

Mupad [F(-1)]

Timed out.

$$\int f^{a+bx+cx^2}\sinh\left(d+fx^2\right)\,dx = \int f^{c\,x^2+b\,x+a}\,\sinh\left(f\,x^2+d\right)\,dx$$

input
```
int(f^(a + b*x + c*x^2)*sinh(d + f*x^2),x)
```

output
```
int(f^(a + b*x + c*x^2)*sinh(d + f*x^2), x)
```

Reduce [F]

$$\int f^{a+bx+cx^2}\sinh\left(d+fx^2\right)\,dx=f^{a}\left(\int f^{c\,x^{2}+b x}\sinh\left(f\,x^{2}+d\right)dx\right)$$

input
```
int(f^(c*x^2+b*x+a)*sinh(f*x^2+d),x)
```

output
```
f**a*int(f**(b*x + c*x**2)*sinh(d + f*x**2),x)
```

3.361 $\int f^{a+bx+cx^2}\sinh^2\left(d+fx^2\right)\,dx$

Optimal result

Integrand size = 23, antiderivative size = 225

$$\int f^{a+bx+cx^2}\sinh^2\left(d+fx^2\right)\,dx=-\frac{f^{a-\frac{b^2}{4c}}\sqrt{\pi}\,\text{erfi}\left(\frac{(b+2cx)\sqrt{\log(f)}}{2\sqrt{c}}\right)}{4\sqrt{c}\sqrt{\log(f)}}-\frac{e^{-2d+\frac{b^2\log^2(f)}{8f-4c\log(f)}}f^a\sqrt{\pi}\,\text{erf}\left(\frac{b\log(f)-2x(2f-c\log(f))}{2\sqrt{2f-c\log(f)}}\right)}{8\sqrt{2f-c\log(f)}}+\frac{e^{2d-\frac{b^2\log^2(f)}{8f+4c\log(f)}}f^a\sqrt{\pi}\,\text{erfi}\left(\frac{b\log(f)+2x(2f+c\log(f))}{2\sqrt{2f+c\log(f)}}\right)}{8\sqrt{2f+c\log(f)}}$$

output
```
-1/4*f^(a-1/4*b^2/c)*Pi^(1/2)*erfi(1/2*(2*c*x+b)*ln(f)^(1/2)/c^(1/2))/c^(1
/2)/ln(f)^(1/2)-1/8*exp(-2*d+b^2*ln(f)^2/(8*f-4*c*ln(f)))*f^a*Pi^(1/2)*erf
(1/2*(b*ln(f)-2*x*(2*f-c*ln(f)))/(2*f-c*ln(f))^(1/2))/(2*f-c*ln(f))^(1/2)+
1/8*exp(2*d-b^2*ln(f)^2/(8*f+4*c*ln(f)))*f^a*Pi^(1/2)*erfi(1/2*(b*ln(f)+2*
x*(2*f+c*ln(f)))/(2*f+c*ln(f))^(1/2))/(2*f+c*ln(f))^(1/2)
```

Mathematica [A] (verified)

Time = 1.46 (sec) , antiderivative size = 257, normalized size of antiderivative = 1.14

$$\int f^{a+bx+cx^2}\sinh^2\left(d+fx^2\right)\,dx=\frac{1}{8}f^a\sqrt{\pi}\left(-\frac{2f^{-\frac{b^2}{4c}}\operatorname{erfi}\left(\frac{(b+2cx)\sqrt{\log(f)}}{2\sqrt{c}}\right)}{\sqrt{c}\sqrt{\log(f)}}\right.$$
$$-\frac{e^{-\frac{b^2\log^2(f)}{8f+4c\log(f)}}\left(e^{\frac{b^2f\log^2(f)}{4f^2-c^2\log^2(f)}}\operatorname{erf}\left(\frac{4fx-(b+2cx)\log(f)}{2\sqrt{2f-c\log(f)}}\right)\sqrt{2f-c\log(f)}(2f+c\log(f))(\cosh(2d)-\sinh(2d))+\right.}{-4f^2+c^2\log^2(f)}$$

input
```
Integrate[f^(a + b*x + c*x^2)*Sinh[d + f*x^2]^2,x]
```

output
```
(f^a*Sqrt[Pi]*((-2*Erfi[((b + 2*c*x)*Sqrt[Log[f]])/(2*Sqrt[c])])/(Sqrt[c]*
f^(b^2/(4*c))*Sqrt[Log[f]]) - (E^((b^2*f*Log[f]^2)/(4*f^2 - c^2*Log[f]^2))
*Erf[(4*f*x - (b + 2*c*x)*Log[f])/(2*Sqrt[2*f - c*Log[f]])]*Sqrt[2*f - c*L
og[f]]*(2*f + c*Log[f])*(Cosh[2*d] - Sinh[2*d]) + Erfi[(4*f*x + (b + 2*c*x
)*Log[f])/(2*Sqrt[2*f + c*Log[f]])]*(2*f - c*Log[f])*Sqrt[2*f + c*Log[f]]*
(Cosh[2*d] + Sinh[2*d]))/(E^((b^2*Log[f]^2)/(8*f + 4*c*Log[f]))*(-4*f^2 +
c^2*Log[f]^2))))/8
```

Rubi [A] (verified)

Time = 0.78 (sec) , antiderivative size = 225, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.087, Rules used = {6038, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sinh^2\left(d+fx^2\right)f^{a+bx+cx^2}dx$$

$$\Big\downarrow \text{ 6038}$$

$$\int \left(\frac{1}{4}e^{-2d-2fx^2}f^{a+bx+cx^2}+\frac{1}{4}e^{2d+2fx^2}f^{a+bx+cx^2}-\frac{1}{2}f^{a+bx+cx^2}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$-\frac{\sqrt{\pi}f^a e^{\frac{b^2\log^2(f)}{8f-4c\log(f)}-2d}\operatorname{erf}\left(\frac{b\log(f)-2x(2f-c\log(f))}{2\sqrt{2f-c\log(f)}}\right)}{8\sqrt{2f-c\log(f)}}+\frac{\sqrt{\pi}f^a e^{2d-\frac{b^2\log^2(f)}{4c\log(f)+8f}}\operatorname{erfi}\left(\frac{b\log(f)+2x(c\log(f)+2f)}{2\sqrt{c\log(f)+2f}}\right)}{8\sqrt{c\log(f)+2f}}-\frac{\sqrt{\pi}f^{a-\frac{b^2}{4c}}\operatorname{erfi}\left(\frac{\sqrt{\log(f)}(b+2cx)}{2\sqrt{c}}\right)}{4\sqrt{c}\sqrt{\log(f)}}$$

input
```
Int[f^(a + b*x + c*x^2)*Sinh[d + f*x^2]^2,x]
```

output
```
-1/4*(f^(a - b^2/(4*c))*Sqrt[Pi]*Erfi[((b + 2*c*x)*Sqrt[Log[f]])/(2*Sqrt[c
]))])/(Sqrt[c]*Sqrt[Log[f]]) - (E^(-2*d + (b^2*Log[f]^2)/(8*f - 4*c*Log[f])
)*f^a*Sqrt[Pi]*Erf[(b*Log[f] - 2*x*(2*f - c*Log[f]))/(2*Sqrt[2*f - c*Log[f
]])])/(8*Sqrt[2*f - c*Log[f]]) + (E^(2*d - (b^2*Log[f]^2)/(8*f + 4*c*Log[f
]))*f^a*Sqrt[Pi]*Erfi[(b*Log[f] + 2*x*(2*f + c*Log[f]))/(2*Sqrt[2*f + c*Lo
g[f]])])/(8*Sqrt[2*f + c*Log[f]])
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 6038
```
Int[(F_)^(u_)*Sinh[v_]^(n_.), x_Symbol] :> Int[ExpandTrigToExp[F^u, Sinh[v]
^n, x], x] /; FreeQ[F, x] && (LinearQ[u, x] || PolyQ[u, x, 2]) && (LinearQ[
v, x] || PolyQ[v, x, 2]) && IGtQ[n, 0]
```

Maple [A] (verified)

Time = 0.39 (sec) , antiderivative size = 217, normalized size of antiderivative = 0.96

method	result
risch	$-\frac{\operatorname{erf}\left(-x\sqrt{2f-c\ln(f)}+\frac{\ln(f)b}{2\sqrt{2f-c\ln(f)}}\right)\sqrt{\pi}\,f^a \mathrm{e}^{-\frac{b^2\ln(f)^2+8d\ln(f)c-16df}{4(c\ln(f)-2f)}}}{8\sqrt{2f-c\ln(f)}}-\frac{\operatorname{erf}\left(-\sqrt{-c\ln(f)-2f}\,x+\frac{\ln(f)b}{2\sqrt{-c\ln(f)-2f}}\right)\sqrt{\pi}\,f^a \mathrm{e}^{-b}}{8\sqrt{-c\ln(f)-2f}}$

input
```
int(f^(c*x^2+b*x+a)*sinh(f*x^2+d)^2,x,method=_RETURNVERBOSE)
```

output
```
-1/8*erf(-x*(2*f-c*ln(f))^(1/2)+1/2*ln(f)*b/(2*f-c*ln(f))^(1/2))/(2*f-c*ln
(f))^(1/2)*Pi^(1/2)*f^a*exp(-1/4*(b^2*ln(f)^2+8*d*ln(f)*c-16*d*f)/(c*ln(f)
-2*f))-1/8*erf(-(-c*ln(f)-2*f)^(1/2)*x+1/2*ln(f)*b/(-c*ln(f)-2*f)^(1/2))/(
-c*ln(f)-2*f)^(1/2)*Pi^(1/2)*f^a*exp(-1/4*(b^2*ln(f)^2-8*d*ln(f)*c-16*d*f)
/(2*f+c*ln(f)))+1/4*f^a*Pi^(1/2)*f^(-1/4*b^2/c)/(-c*ln(f))^(1/2)*erf(-(-c*
ln(f))^(1/2)*x+1/2*ln(f)*b/(-c*ln(f))^(1/2))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 466 vs. $2(185) = 370$.

Time = 0.10 (sec) , antiderivative size = 466, normalized size of antiderivative = 2.07

$$\int f^{a+bx+cx^2} \sinh^2(d+fx^2)\,dx = \text{Too large to display}$$

input
```
integrate(f^(c*x^2+b*x+a)*sinh(f*x^2+d)^2,x, algorithm="fricas")
```

output
```
-1/8*((sqrt(pi)*(c^2*log(f)^2 + 2*c*f*log(f))*cosh(-1/4*((b^2 - 4*a*c)*log
(f)^2 - 16*d*f + 8*(c*d + a*f)*log(f))/(c*log(f) - 2*f)) + sqrt(pi)*(c^2*l
og(f)^2 + 2*c*f*log(f))*sinh(-1/4*((b^2 - 4*a*c)*log(f)^2 - 16*d*f + 8*(c*
d + a*f)*log(f))/(c*log(f) - 2*f)))*sqrt(-c*log(f) + 2*f)*erf(-1/2*(4*f*x
- (2*c*x + b)*log(f))*sqrt(-c*log(f) + 2*f)/(c*log(f) - 2*f)) + (sqrt(pi)*
(c^2*log(f)^2 - 2*c*f*log(f))*cosh(-1/4*((b^2 - 4*a*c)*log(f)^2 - 16*d*f -
 8*(c*d + a*f)*log(f))/(c*log(f) + 2*f)) + sqrt(pi)*(c^2*log(f)^2 - 2*c*f*
log(f))*sinh(-1/4*((b^2 - 4*a*c)*log(f)^2 - 16*d*f - 8*(c*d + a*f)*log(f))
/(c*log(f) + 2*f)))*sqrt(-c*log(f) - 2*f)*erf(1/2*(4*f*x + (2*c*x + b)*log
(f))*sqrt(-c*log(f) - 2*f)/(c*log(f) + 2*f)) - 2*(sqrt(pi)*(c^2*log(f)^2 -
 4*f^2)*cosh(-1/4*(b^2 - 4*a*c)*log(f)/c) + sqrt(pi)*(c^2*log(f)^2 - 4*f^2
)*sinh(-1/4*(b^2 - 4*a*c)*log(f)/c))*sqrt(-c*log(f))*erf(1/2*(2*c*x + b)*s
qrt(-c*log(f))/c))/(c^3*log(f)^3 - 4*c*f^2*log(f))
```

Sympy [F]

$$\int f^{a+bx+cx^2}\sinh^2\left(d+fx^2\right)\,dx=\int f^{a+bx+cx^2}\sinh^2\left(d+fx^2\right)dx$$

input
```
integrate(f**(c*x**2+b*x+a)*sinh(f*x**2+d)**2,x)
```

output
```
Integral(f**(a + b*x + c*x**2)*sinh(d + f*x**2)**2, x)
```

Maxima [A] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 199, normalized size of antiderivative = 0.88

$$\int f^{a+bx+cx^2}\sinh^2\left(d+fx^2\right)\,dx$$
$$=\frac{\sqrt{\pi}f^a\operatorname{erf}\left(\sqrt{-c\log(f)-2f}x-\frac{b\log(f)}{2\sqrt{-c\log(f)-2f}}\right)e^{\left(-\frac{b^2\log(f)^2}{4(c\log(f)+2f)}+2d\right)}}{8\sqrt{-c\log(f)-2f}}+\frac{\sqrt{\pi}f^a\operatorname{erf}\left(\sqrt{-c\log(f)+2f}x-\frac{b\log(f)}{2\sqrt{-c\log(f)+2f}}\right)e^{\left(-\frac{b^2\log(f)^2}{4(c\log(f)-2f)}-2d\right)}}{8\sqrt{-c\log(f)+2f}}-\frac{\sqrt{\pi}f^a\operatorname{erf}\left(\sqrt{-c\log(f)}x-\frac{b\log(f)}{2\sqrt{-c\log(f)}}\right)}{4\sqrt{-c\log(f)}f^{\frac{b^2}{4c}}}$$

input
```
integrate(f^(c*x^2+b*x+a)*sinh(f*x^2+d)^2,x, algorithm="maxima")
```

output
```
1/8*sqrt(pi)*f^a*erf(sqrt(-c*log(f) - 2*f)*x - 1/2*b*log(f)/sqrt(-c*log(f)
 - 2*f))*e^(-1/4*b^2*log(f)^2/(c*log(f) + 2*f) + 2*d)/sqrt(-c*log(f) - 2*f
) + 1/8*sqrt(pi)*f^a*erf(sqrt(-c*log(f) + 2*f)*x - 1/2*b*log(f)/sqrt(-c*lo
g(f) + 2*f))*e^(-1/4*b^2*log(f)^2/(c*log(f) - 2*f) - 2*d)/sqrt(-c*log(f) +
 2*f) - 1/4*sqrt(pi)*f^a*erf(sqrt(-c*log(f))*x - 1/2*b*log(f)/sqrt(-c*log(
f)))/(sqrt(-c*log(f))*f^(1/4*b^2/c))
```

Giac [A] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 239, normalized size of antiderivative = 1.06

$$\int f^{a+bx+cx^2}\sinh^2\left(d+fx^2\right)\,dx =$$

$$-\frac{\sqrt{\pi}\operatorname{erf}\left(-\frac{1}{2}\sqrt{-c\log\left(f\right)-2f}\left(2x+\frac{b\log(f)}{c\log(f)+2f}\right)\right)e^{\left(-\frac{b^2\log(f)^2-4ac\log(f)^2-8cd\log(f)-8af\log(f)-16df}{4(c\log(f)+2f)}\right)}}{8\sqrt{-c\log\left(f\right)-2f}}$$
$$-\frac{\sqrt{\pi}\operatorname{erf}\left(-\frac{1}{2}\sqrt{-c\log\left(f\right)+2f}\left(2x+\frac{b\log(f)}{c\log(f)-2f}\right)\right)e^{\left(-\frac{b^2\log(f)^2-4ac\log(f)^2+8cd\log(f)+8af\log(f)-16df}{4(c\log(f)-2f)}\right)}}{8\sqrt{-c\log\left(f\right)+2f}}$$
$$+\frac{\sqrt{\pi}\operatorname{erf}\left(-\frac{1}{2}\sqrt{-c\log\left(f\right)}\left(2x+\frac{b}{c}\right)\right)e^{\left(-\frac{b^2\log(f)-4ac\log(f)}{4c}\right)}}{4\sqrt{-c\log\left(f\right)}}$$

input
```
integrate(f^(c*x^2+b*x+a)*sinh(f*x^2+d)^2,x, algorithm="giac")
```

output
```
-1/8*sqrt(pi)*erf(-1/2*sqrt(-c*log(f) - 2*f)*(2*x + b*log(f)/(c*log(f) + 2
*f)))*e^(-1/4*(b^2*log(f)^2 - 4*a*c*log(f)^2 - 8*c*d*log(f) - 8*a*f*log(f)
 - 16*d*f)/(c*log(f) + 2*f))/sqrt(-c*log(f) - 2*f) - 1/8*sqrt(pi)*erf(-1/2
*sqrt(-c*log(f) + 2*f)*(2*x + b*log(f)/(c*log(f) - 2*f)))*e^(-1/4*(b^2*log
(f)^2 - 4*a*c*log(f)^2 + 8*c*d*log(f) + 8*a*f*log(f) - 16*d*f)/(c*log(f) -
 2*f))/sqrt(-c*log(f) + 2*f) + 1/4*sqrt(pi)*erf(-1/2*sqrt(-c*log(f))*(2*x
+ b/c))*e^(-1/4*(b^2*log(f) - 4*a*c*log(f))/c)/sqrt(-c*log(f))
```

Mupad [F(-1)]

Timed out.

$$\int f^{a+bx+cx^2}\sinh^2\left(d+fx^2\right)\,dx = \int f^{c\,x^2+b\,x+a}\,{\sinh\left(f\,x^2+d\right)}^2\,dx$$

input
```
int(f^(a + b*x + c*x^2)*sinh(d + f*x^2)^2,x)
```

output
```
int(f^(a + b*x + c*x^2)*sinh(d + f*x^2)^2, x)
```

Reduce [F]

$$\int f^{a+bx+cx^2}\sinh^2\left(d+fx^2\right)\,dx=f^a\left(\int f^{c\,x^2+bx}\sinh\left(f\,x^2+d\right)^2dx\right)$$

input
```
int(f^(c*x^2+b*x+a)*sinh(f*x^2+d)^2,x)
```

output
```
f**a*int(f**(b*x + c*x**2)*sinh(d + f*x**2)**2,x)
```

3.362 $\int f^{a+bx+cx^2}\sinh^3\left(d+fx^2\right)\,dx$

Optimal result

Integrand size = 23, antiderivative size = 323

$$\int f^{a+bx+cx^2}\sinh^3\left(d+fx^2\right)\,dx=-\frac{3e^{-d+\frac{b^2\log^2(f)}{4f-4c\log(f)}}f^a\sqrt{\pi}\,\mathrm{erf}\left(\frac{b\log(f)-2x(f-c\log(f))}{2\sqrt{f-c\log(f)}}\right)}{16\sqrt{f-c\log(f)}}+\frac{e^{-3d+\frac{b^2\log^2(f)}{12f-4c\log(f)}}f^a\sqrt{\pi}\,\mathrm{erf}\left(\frac{b\log(f)-2x(3f-c\log(f))}{2\sqrt{3f-c\log(f)}}\right)}{16\sqrt{3f-c\log(f)}}-\frac{3e^{d-\frac{b^2\log^2(f)}{4(f+c\log(f))}}f^a\sqrt{\pi}\,\mathrm{erfi}\left(\frac{b\log(f)+2x(f+c\log(f))}{2\sqrt{f+c\log(f)}}\right)}{16\sqrt{f+c\log(f)}}+\frac{e^{3d-\frac{b^2\log^2(f)}{4(3f+c\log(f))}}f^a\sqrt{\pi}\,\mathrm{erfi}\left(\frac{b\log(f)+2x(3f+c\log(f))}{2\sqrt{3f+c\log(f)}}\right)}{16\sqrt{3f+c\log(f)}}$$

output
```
-3/16*exp(-d+b^2*ln(f)^2/(4*f-4*c*ln(f)))*f^a*Pi^(1/2)*erf(1/2*(b*ln(f)-2*
x*(f-c*ln(f)))/(f-c*ln(f))^(1/2))/(f-c*ln(f))^(1/2)+1/16*exp(-3*d+b^2*ln(f
)^2/(12*f-4*c*ln(f)))*f^a*Pi^(1/2)*erf(1/2*(b*ln(f)-2*x*(3*f-c*ln(f)))/(3*
f-c*ln(f))^(1/2))/(3*f-c*ln(f))^(1/2)-3/16*exp(d-b^2*ln(f)^2/(4*f+4*c*ln(f
)))*f^a*Pi^(1/2)*erfi(1/2*(b*ln(f)+2*x*(f+c*ln(f)))/(f+c*ln(f))^(1/2))/(f+
c*ln(f))^(1/2)+1/16*exp(3*d-b^2*ln(f)^2/(12*f+4*c*ln(f)))*f^a*Pi^(1/2)*erf
i(1/2*(b*ln(f)+2*x*(3*f+c*ln(f)))/(3*f+c*ln(f))^(1/2))/(3*f+c*ln(f))^(1/2)
```

Mathematica [A] (warning: unable to verify)

Time = 4.54 (sec) , antiderivative size = 503, normalized size of antiderivative = 1.56

$$\int f^{a+bx+cx^2}\sinh^3\left(d+fx^2\right)\,dx$$
$$=\frac{e^{-\frac{b^2\log^2(f)(2f+c\log(f))}{2(f+c\log(f))(3f+c\log(f))}}f^a\sqrt{\pi}\left(3e^{\frac{1}{4}b^2\log^2(f)\left(\frac{1}{f-c\log(f)}+\frac{1}{f+c\log(f)}+\frac{1}{3f+c\log(f)}\right)}\operatorname{erf}\left(\frac{2fx-(b+2cx)\log(f)}{2\sqrt{f-c\log(f)}}\right)\sqrt{f-c\log(f)}(\cdots\right.}{\cdots}$$

input

```
Integrate[f^(a + b*x + c*x^2)*Sinh[d + f*x^2]^3,x]
```

output

```
(f^a*Sqrt[Pi]*(3*E^((b^2*Log[f]^2*((f - c*Log[f])^(-1) + (f + c*Log[f])^(-
1) + (3*f + c*Log[f])^(-1)))/4)*Erf[(2*f*x - (b + 2*c*x)*Log[f])/(2*Sqrt[f
 - c*Log[f]])]*Sqrt[f - c*Log[f]]*(9*f^3 + 9*c*f^2*Log[f] - c^2*f*Log[f]^2
 - c^3*Log[f]^3)*(Cosh[d] - Sinh[d]) - (f - c*Log[f])*(E^((b^2*Log[f]^2*((
3*f - c*Log[f])^(-1) + (f + c*Log[f])^(-1) + (3*f + c*Log[f])^(-1)))/4)*Er
f[(6*f*x - (b + 2*c*x)*Log[f])/(2*Sqrt[3*f - c*Log[f]])]*Sqrt[3*f - c*Log[
f]]*(3*f^2 + 4*c*f*Log[f] + c^2*Log[f]^2)*(Cosh[3*d] - Sinh[3*d]) + (3*f -
 c*Log[f])*(3*E^((b^2*Log[f]^2)/(12*f + 4*c*Log[f]))*Erfi[(2*f*x + (b + 2*
c*x)*Log[f])/(2*Sqrt[f + c*Log[f]])]*Sqrt[f + c*Log[f]]*(3*f + c*Log[f])*(
Cosh[d] + Sinh[d]) - E^((b^2*Log[f]^2)/(4*(f + c*Log[f])))*Erfi[(6*f*x + (
b + 2*c*x)*Log[f])/(2*Sqrt[3*f + c*Log[f]])]*(f + c*Log[f])*Sqrt[3*f + c*L
og[f]]*(Cosh[3*d] + Sinh[3*d])))))/(16*E^((b^2*Log[f]^2*(2*f + c*Log[f]))/
(2*(f + c*Log[f])*(3*f + c*Log[f])))*(9*f^4 - 10*c^2*f^2*Log[f]^2 + c^4*Lo
g[f]^4))
```

Rubi [A] (verified)

Time = 0.93 (sec) , antiderivative size = 323, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.087$, Rules used = {6038, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sinh^3\left(d+fx^2\right) f^{a+bx+cx^2}\,dx$$

$$\Big\downarrow \text{6038}$$

$$\int \left(-\frac{1}{8}e^{-3d-3fx^2}f^{a+bx+cx^2}+\frac{3}{8}e^{-d-fx^2}f^{a+bx+cx^2}-\frac{3}{8}e^{d+fx^2}f^{a+bx+cx^2}+\frac{1}{8}e^{3d+3fx^2}f^{a+bx+cx^2}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$-\frac{3\sqrt{\pi}f^a e^{\frac{b^2\log^2(f)}{4f-4c\log(f)}-d}\text{erf}\left(\frac{b\log(f)-2x(f-c\log(f))}{2\sqrt{f-c\log(f)}}\right)}{16\sqrt{f-c\log(f)}}+\frac{\sqrt{\pi}f^a e^{\frac{b^2\log^2(f)}{12f-4c\log(f)}-3d}\text{erf}\left(\frac{b\log(f)-2x(3f-c\log(f))}{2\sqrt{3f-c\log(f)}}\right)}{16\sqrt{3f-c\log(f)}}-\frac{3\sqrt{\pi}f^a e^{d-\frac{b^2\log^2(f)}{4(c\log(f)+f)}}\text{erfi}\left(\frac{b\log(f)+2x(c\log(f)+f)}{2\sqrt{c\log(f)+f}}\right)}{16\sqrt{c\log(f)+f}}+\frac{\sqrt{\pi}f^a e^{3d-\frac{b^2\log^2(f)}{4(c\log(f)+3f)}}\text{erfi}\left(\frac{b\log(f)+2x(c\log(f)+3f)}{2\sqrt{c\log(f)+3f}}\right)}{16\sqrt{c\log(f)+3f}}$$

input
```
Int[f^(a + b*x + c*x^2)*Sinh[d + f*x^2]^3,x]
```

output
```
(-3*E^(-d + (b^2*Log[f]^2)/(4*f - 4*c*Log[f]))*f^a*Sqrt[Pi]*Erf[(b*Log[f]
 - 2*x*(f - c*Log[f]))/(2*Sqrt[f - c*Log[f]])])/(16*Sqrt[f - c*Log[f]]) + (
E^(-3*d + (b^2*Log[f]^2)/(12*f - 4*c*Log[f]))*f^a*Sqrt[Pi]*Erf[(b*Log[f] -
 2*x*(3*f - c*Log[f]))/(2*Sqrt[3*f - c*Log[f]])])/(16*Sqrt[3*f - c*Log[f]]
) - (3*E^(d - (b^2*Log[f]^2)/(4*(f + c*Log[f])))*f^a*Sqrt[Pi]*Erfi[(b*Log[
f] + 2*x*(f + c*Log[f]))/(2*Sqrt[f + c*Log[f]])])/(16*Sqrt[f + c*Log[f]])
 + (E^(3*d - (b^2*Log[f]^2)/(4*(3*f + c*Log[f])))*f^a*Sqrt[Pi]*Erfi[(b*Log[
f] + 2*x*(3*f + c*Log[f]))/(2*Sqrt[3*f + c*Log[f]])])/(16*Sqrt[3*f + c*Log
[f]])
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 6038
```
Int[(F_)^(u_)*Sinh[v_]^(n_.), x_Symbol] :> Int[ExpandTrigToExp[F^u, Sinh[v]
^n, x], x] /; FreeQ[F, x] && (LinearQ[u, x] || PolyQ[u, x, 2]) && (LinearQ[
v, x] || PolyQ[v, x, 2]) && IGtQ[n, 0]
```

Maple [A] (verified)

Time = 1.06 (sec) , antiderivative size = 326, normalized size of antiderivative = 1.01

method	result
risch	$-\frac{\operatorname{erf}\left(-\sqrt{-c\ln(f)-3f}\,x+\frac{\ln(f)b}{2\sqrt{-c\ln(f)-3f}}\right)\sqrt{\pi}\,f^{a}\mathrm{e}^{-\frac{b^{2}\ln(f)^{2}-12d\ln(f)c-36df}{4(3f+c\ln(f))}}}{16\sqrt{-c\ln(f)-3f}}+\frac{\operatorname{erf}\left(-x\sqrt{3f-c\ln(f)}+\frac{\ln(f)b}{2\sqrt{3f-c\ln(f)}}\right)\sqrt{\pi}\,f^{a}\mathrm{e}^{-}}{16\sqrt{3f-c\ln(f)}}$

input
```
int(f^(c*x^2+b*x+a)*sinh(f*x^2+d)^3,x,method=_RETURNVERBOSE)
```

output
```
-1/16*erf(-(-c*ln(f)-3*f)^(1/2)*x+1/2*ln(f)*b/(-c*ln(f)-3*f)^(1/2))/(-c*ln
(f)-3*f)^(1/2)*Pi^(1/2)*f^a*exp(-1/4*(b^2*ln(f)^2-12*d*ln(f)*c-36*d*f)/(3*
f+c*ln(f)))+1/16*erf(-x*(3*f-c*ln(f))^(1/2)+1/2*ln(f)*b/(3*f-c*ln(f))^(1/2
))/(3*f-c*ln(f))^(1/2)*Pi^(1/2)*f^a*exp(-1/4*(b^2*ln(f)^2+12*d*ln(f)*c-36*
d*f)/(c*ln(f)-3*f))-3/16*erf(-x*(f-c*ln(f))^(1/2)+1/2*ln(f)*b/(f-c*ln(f))^
(1/2))/(f-c*ln(f))^(1/2)*Pi^(1/2)*f^a*exp(-1/4*(b^2*ln(f)^2+4*d*ln(f)*c-4*
d*f)/(c*ln(f)-f))+3/16*erf(-(-c*ln(f)-f)^(1/2)*x+1/2*ln(f)*b/(-c*ln(f)-f)^
(1/2))/(-c*ln(f)-f)^(1/2)*Pi^(1/2)*f^a*exp(-1/4*(b^2*ln(f)^2-4*d*ln(f)*c-4
*d*f)/(f+c*ln(f)))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 852 vs. $2(275) = 550$.

Time = 0.10 (sec) , antiderivative size = 852, normalized size of antiderivative = 2.64

$$\int f^{a+bx+cx^2} \sinh^3\left(d+fx^2\right) dx = \text{Too large to display}$$

input
```
integrate(f^(c*x^2+b*x+a)*sinh(f*x^2+d)^3,x, algorithm="fricas")
```

output
```
1/16*((sqrt(pi)*(c^3*log(f)^3 + 3*c^2*f*log(f)^2 - c*f^2*log(f) - 3*f^3)*c
osh(-1/4*((b^2 - 4*a*c)*log(f)^2 - 36*d*f + 12*(c*d + a*f)*log(f))/(c*log(
f) - 3*f)) + sqrt(pi)*(c^3*log(f)^3 + 3*c^2*f*log(f)^2 - c*f^2*log(f) - 3*
f^3)*sinh(-1/4*((b^2 - 4*a*c)*log(f)^2 - 36*d*f + 12*(c*d + a*f)*log(f))/(
c*log(f) - 3*f)))*sqrt(-c*log(f) + 3*f)*erf(-1/2*(6*f*x - (2*c*x + b)*log(
f))*sqrt(-c*log(f) + 3*f)/(c*log(f) - 3*f)) - 3*(sqrt(pi)*(c^3*log(f)^3 +
c^2*f*log(f)^2 - 9*c*f^2*log(f) - 9*f^3)*cosh(-1/4*((b^2 - 4*a*c)*log(f)^2
 - 4*d*f + 4*(c*d + a*f)*log(f))/(c*log(f) - f)) + sqrt(pi)*(c^3*log(f)^3
+ c^2*f*log(f)^2 - 9*c*f^2*log(f) - 9*f^3)*sinh(-1/4*((b^2 - 4*a*c)*log(f)
^2 - 4*d*f + 4*(c*d + a*f)*log(f))/(c*log(f) - f)))*sqrt(-c*log(f) + f)*er
f(-1/2*(2*f*x - (2*c*x + b)*log(f))*sqrt(-c*log(f) + f)/(c*log(f) - f)) +
3*(sqrt(pi)*(c^3*log(f)^3 - c^2*f*log(f)^2 - 9*c*f^2*log(f) + 9*f^3)*cosh(
-1/4*((b^2 - 4*a*c)*log(f)^2 - 4*d*f - 4*(c*d + a*f)*log(f))/(c*log(f) + f
)) + sqrt(pi)*(c^3*log(f)^3 - c^2*f*log(f)^2 - 9*c*f^2*log(f) + 9*f^3)*sin
h(-1/4*((b^2 - 4*a*c)*log(f)^2 - 4*d*f - 4*(c*d + a*f)*log(f))/(c*log(f) +
 f)))*sqrt(-c*log(f) - f)*erf(1/2*(2*f*x + (2*c*x + b)*log(f))*sqrt(-c*log
(f) - f)/(c*log(f) + f)) - (sqrt(pi)*(c^3*log(f)^3 - 3*c^2*f*log(f)^2 - c*
f^2*log(f) + 3*f^3)*cosh(-1/4*((b^2 - 4*a*c)*log(f)^2 - 36*d*f - 12*(c*d +
 a*f)*log(f))/(c*log(f) + 3*f)) + sqrt(pi)*(c^3*log(f)^3 - 3*c^2*f*log(f)^
2 - c*f^2*log(f) + 3*f^3)*sinh(-1/4*((b^2 - 4*a*c)*log(f)^2 - 36*d*f - ...
```

Sympy [F]

$$\int f^{a+bx+cx^2}\sinh^3\left(d+fx^2\right)\,dx=\int f^{a+bx+cx^2}\sinh^3\left(d+fx^2\right)dx$$

input
```
integrate(f**(c*x**2+b*x+a)*sinh(f*x**2+d)**3,x)
```

output
```
Integral(f**(a + b*x + c*x**2)*sinh(d + f*x**2)**3, x)
```

Maxima [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 287, normalized size of antiderivative = 0.89

$$\int f^{a+bx+cx^2}\sinh^3\left(d+fx^2\right)\,dx$$
$$=\frac{\sqrt{\pi}f^a\operatorname{erf}\left(\sqrt{-c\log\left(f\right)-3\,f}x-\frac{b\log(f)}{2\sqrt{-c\log(f)-3\,f}}\right)e^{\left(-\frac{b^2\log(f)^2}{4\,(c\log(f)+3\,f)}+3\,d\right)}}{16\sqrt{-c\log\left(f\right)-3\,f}}-\frac{3\sqrt{\pi}f^a\operatorname{erf}\left(\sqrt{-c\log\left(f\right)-f}x-\frac{b\log(f)}{2\sqrt{-c\log(f)-f}}\right)e^{\left(-\frac{b^2\log(f)^2}{4\,(c\log(f)+f)}+d\right)}}{16\sqrt{-c\log\left(f\right)-f}}+\frac{3\sqrt{\pi}f^a\operatorname{erf}\left(\sqrt{-c\log\left(f\right)+f}x-\frac{b\log(f)}{2\sqrt{-c\log(f)+f}}\right)e^{\left(-\frac{b^2\log(f)^2}{4\,(c\log(f)-f)}-d\right)}}{16\sqrt{-c\log\left(f\right)+f}}-\frac{\sqrt{\pi}f^a\operatorname{erf}\left(\sqrt{-c\log\left(f\right)+3\,f}x-\frac{b\log(f)}{2\sqrt{-c\log(f)+3\,f}}\right)e^{\left(-\frac{b^2\log(f)^2}{4\,(c\log(f)-3\,f)}-3\,d\right)}}{16\sqrt{-c\log\left(f\right)+3\,f}}$$

input
```
integrate(f^(c*x^2+b*x+a)*sinh(f*x^2+d)^3,x, algorithm="maxima")
```

output
```
1/16*sqrt(pi)*f^a*erf(sqrt(-c*log(f) - 3*f)*x - 1/2*b*log(f)/sqrt(-c*log(f
) - 3*f))*e^(-1/4*b^2*log(f)^2/(c*log(f) + 3*f) + 3*d)/sqrt(-c*log(f) - 3*
f) - 3/16*sqrt(pi)*f^a*erf(sqrt(-c*log(f) - f)*x - 1/2*b*log(f)/sqrt(-c*lo
g(f) - f))*e^(-1/4*b^2*log(f)^2/(c*log(f) + f) + d)/sqrt(-c*log(f) - f) +
3/16*sqrt(pi)*f^a*erf(sqrt(-c*log(f) + f)*x - 1/2*b*log(f)/sqrt(-c*log(f)
+ f))*e^(-1/4*b^2*log(f)^2/(c*log(f) - f) - d)/sqrt(-c*log(f) + f) - 1/16*
sqrt(pi)*f^a*erf(sqrt(-c*log(f) + 3*f)*x - 1/2*b*log(f)/sqrt(-c*log(f) + 3
*f))*e^(-1/4*b^2*log(f)^2/(c*log(f) - 3*f) - 3*d)/sqrt(-c*log(f) + 3*f)
```

Giac [A] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 369, normalized size of antiderivative = 1.14

$$\int f^{a+bx+cx^2}\sinh^3\left(d+fx^2\right)\,dx =$$

$$-\frac{\sqrt{\pi}\,\mathrm{erf}\left(-\frac{1}{2}\sqrt{-c\log\left(f\right)-3\,f}\left(2\,x+\frac{b\log(f)}{c\log(f)+3\,f}\right)\right)e^{\left(-\frac{b^2\log(f)^2-4\,ac\log(f)^2-12\,cd\log(f)-12\,af\log(f)-36\,df}{4\,(c\log(f)+3\,f)}\right)}}{16\,\sqrt{-c\log\left(f\right)-3\,f}}$$
$$+\frac{3\,\sqrt{\pi}\,\mathrm{erf}\left(-\frac{1}{2}\sqrt{-c\log\left(f\right)-f}\left(2\,x+\frac{b\log(f)}{c\log(f)+f}\right)\right)e^{\left(-\frac{b^2\log(f)^2-4\,ac\log(f)^2-4\,cd\log(f)-4\,af\log(f)-4\,df}{4\,(c\log(f)+f)}\right)}}{16\,\sqrt{-c\log\left(f\right)-f}}$$
$$-\frac{3\,\sqrt{\pi}\,\mathrm{erf}\left(-\frac{1}{2}\sqrt{-c\log\left(f\right)+f}\left(2\,x+\frac{b\log(f)}{c\log(f)-f}\right)\right)e^{\left(-\frac{b^2\log(f)^2-4\,ac\log(f)^2+4\,cd\log(f)+4\,af\log(f)-4\,df}{4\,(c\log(f)-f)}\right)}}{16\,\sqrt{-c\log\left(f\right)+f}}$$
$$+\frac{\sqrt{\pi}\,\mathrm{erf}\left(-\frac{1}{2}\sqrt{-c\log\left(f\right)+3\,f}\left(2\,x+\frac{b\log(f)}{c\log(f)-3\,f}\right)\right)e^{\left(-\frac{b^2\log(f)^2-4\,ac\log(f)^2+12\,cd\log(f)+12\,af\log(f)-36\,df}{4\,(c\log(f)-3\,f)}\right)}}{16\,\sqrt{-c\log\left(f\right)+3\,f}}$$

input
```
integrate(f^(c*x^2+b*x+a)*sinh(f*x^2+d)^3,x, algorithm="giac")
```

output
```
-1/16*sqrt(pi)*erf(-1/2*sqrt(-c*log(f) - 3*f)*(2*x + b*log(f)/(c*log(f) +
3*f)))*e^(-1/4*(b^2*log(f)^2 - 4*a*c*log(f)^2 - 12*c*d*log(f) - 12*a*f*log
(f) - 36*d*f)/(c*log(f) + 3*f))/sqrt(-c*log(f) - 3*f) + 3/16*sqrt(pi)*erf(
-1/2*sqrt(-c*log(f) - f)*(2*x + b*log(f)/(c*log(f) + f)))*e^(-1/4*(b^2*log
(f)^2 - 4*a*c*log(f)^2 - 4*c*d*log(f) - 4*a*f*log(f) - 4*d*f)/(c*log(f) +
f))/sqrt(-c*log(f) - f) - 3/16*sqrt(pi)*erf(-1/2*sqrt(-c*log(f) + f)*(2*x
+ b*log(f)/(c*log(f) - f)))*e^(-1/4*(b^2*log(f)^2 - 4*a*c*log(f)^2 + 4*c*d
*log(f) + 4*a*f*log(f) - 4*d*f)/(c*log(f) - f))/sqrt(-c*log(f) + f) + 1/16
*sqrt(pi)*erf(-1/2*sqrt(-c*log(f) + 3*f)*(2*x + b*log(f)/(c*log(f) - 3*f))
)*e^(-1/4*(b^2*log(f)^2 - 4*a*c*log(f)^2 + 12*c*d*log(f) + 12*a*f*log(f) -
 36*d*f)/(c*log(f) - 3*f))/sqrt(-c*log(f) + 3*f)
```

Mupad [F(-1)]

Timed out.

$$\int f^{a+bx+cx^2}\sinh^3\left(d+fx^2\right)\,dx=\int f^{c\,x^2+b\,x+a}\,\sinh\left(f\,x^2+d\right)^3\,dx$$

input
```
int(f^(a + b*x + c*x^2)*sinh(d + f*x^2)^3,x)
```

output
```
int(f^(a + b*x + c*x^2)*sinh(d + f*x^2)^3, x)
```

Reduce [F]

$$\int f^{a+bx+cx^2}\sinh^3\left(d+fx^2\right)\,dx=f^a\left(\int f^{c\,x^2+bx}\sinh\left(f\,x^2+d\right)^3dx\right)$$

input
```
int(f^(c*x^2+b*x+a)*sinh(f*x^2+d)^3,x)
```

output
```
f**a*int(f**(b*x + c*x**2)*sinh(d + f*x**2)**3,x)
```

3.363 $\int f^{a+bx+cx^2} \sinh(d+ex+fx^2)\,dx$

Optimal result

Integrand size = 24, antiderivative size = 161

$$\int f^{a+bx+cx^2}\sinh\left(d+ex+fx^2\right)\,dx = -\frac{e^{-d+\frac{(e-b\log(f))^2}{4(f-c\log(f))}}f^a\sqrt{\pi}\operatorname{erf}\left(\frac{e-b\log(f)+2x(f-c\log(f))}{2\sqrt{f-c\log(f)}}\right)}{4\sqrt{f-c\log(f)}} + \frac{e^{d-\frac{(e+b\log(f))^2}{4(f+c\log(f))}}f^a\sqrt{\pi}\operatorname{erfi}\left(\frac{e+b\log(f)+2x(f+c\log(f))}{2\sqrt{f+c\log(f)}}\right)}{4\sqrt{f+c\log(f)}}$$

output

```
-1/4*exp(-d+(e-b*ln(f))^2/(4*f-4*c*ln(f)))*f^a*Pi^(1/2)*erf(1/2*(e-b*ln(f)
+2*x*(f-c*ln(f)))/(f-c*ln(f))^(1/2))/(f-c*ln(f))^(1/2)+1/4*exp(d-(e+b*ln(f
))^2/(4*f+4*c*ln(f)))*f^a*Pi^(1/2)*erfi(1/2*(e+b*ln(f)+2*x*(f+c*ln(f)))/(f
+c*ln(f))^(1/2))/(f+c*ln(f))^(1/2)
```

Mathematica [A] (warning: unable to verify)

Time = 0.98 (sec) , antiderivative size = 252, normalized size of antiderivative = 1.57

$$\int f^{a+bx+cx^2}\sinh\left(d+ex+fx^2\right)\,dx$$

$$= \frac{e^{-\frac{e^2+b^2\log^2(f)}{4(f+c\log(f))}}f^{a+\frac{bef}{-f^2+c^2\log^2(f)}}\sqrt{\pi}\left(-e^{\frac{f\left(e^2+b^2\log^2(f)\right)}{2\left(f^2-c^2\log^2(f)\right)}}f^{\frac{be}{2(f+c\log(f))}}\operatorname{erf}\left(\frac{e+2fx-(b+2cx)\log(f)}{2\sqrt{f-c\log(f)}}\right)\sqrt{f-c\log(f)}(f+c\text{l}}{4\,(f}$$

input
```
Integrate[f^(a + b*x + c*x^2)*Sinh[d + e*x + f*x^2],x]
```

output
```
(f^(a + (b*e*f)/(-f^2 + c^2*Log[f]^2))*Sqrt[Pi]*(-(E^((f*(e^2 + b^2*Log[f]
^2))/(2*(f^2 - c^2*Log[f]^2)))*f^((b*e)/(2*(f + c*Log[f])))*Erf[(e + 2*f*x
 - (b + 2*c*x)*Log[f])/(2*Sqrt[f - c*Log[f]])]*Sqrt[f - c*Log[f]]*(f + c*L
og[f])*(Cosh[d] - Sinh[d])) + f^((b*e)/(2*f - 2*c*Log[f]))*Erfi[(e + 2*f*x
 + (b + 2*c*x)*Log[f])/(2*Sqrt[f + c*Log[f]])]*(f - c*Log[f])*Sqrt[f + c*L
og[f]]*(Cosh[d] + Sinh[d])))/(4*E^((e^2 + b^2*Log[f]^2)/(4*(f + c*Log[f]))
)*(f^2 - c^2*Log[f]^2))
```

Rubi [A] (verified)

Time = 0.64 (sec) , antiderivative size = 161, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {6038, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int f^{a+bx+cx^2} \sinh \left(d+ex+fx^2\right) \, dx$$

$$\Big\downarrow \text{ 6038}$$

$$\int \left(\frac{1}{2} e^{d+ex+fx^2} f^{a+bx+cx^2}-\frac{1}{2} e^{-d-ex-fx^2} f^{a+bx+cx^2}\right)dx$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{\sqrt{\pi } f^a e^{d-\frac{(b \log (f)+e)^2}{4 (c \log (f)+f)}} \text{erfi}\left(\frac{b \log (f)+2 x (c \log (f)+f)+e}{2 \sqrt{c \log (f)+f}}\right)}{4 \sqrt{c \log (f)+f}}-\frac{\sqrt{\pi } f^a e^{\frac{(e-b \log (f))^2}{4 (f-c \log (f))}-d} \text{erf}\left(\frac{-b \log (f)+2 x (f-c \log (f))+e}{2 \sqrt{f-c \log (f)}}\right)}{4 \sqrt{f-c \log (f)}}$$

input
```
Int[f^(a + b*x + c*x^2)*Sinh[d + e*x + f*x^2],x]
```

output
```
-1/4*(E^(-d + (e - b*Log[f])^2/(4*(f - c*Log[f])))*f^a*Sqrt[Pi]*Erf[(e - b
*Log[f] + 2*x*(f - c*Log[f]))/(2*Sqrt[f - c*Log[f]])])/Sqrt[f - c*Log[f]]
+ (E^(d - (e + b*Log[f])^2/(4*(f + c*Log[f])))*f^a*Sqrt[Pi]*Erfi[(e + b*Lo
g[f] + 2*x*(f + c*Log[f]))/(2*Sqrt[f + c*Log[f]])])/(4*Sqrt[f + c*Log[f]])
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 6038
```
Int[(F_)^(u_)*Sinh[v_]^(n_.), x_Symbol] :> Int[ExpandTrigToExp[F^u, Sinh[v]
^n, x], x] /; FreeQ[F, x] && (LinearQ[u, x] || PolyQ[u, x, 2]) && (LinearQ[
v, x] || PolyQ[v, x, 2]) && IGtQ[n, 0]
```

Maple [A] (verified)

Time = 0.19 (sec) , antiderivative size = 186, normalized size of antiderivative = 1.16

method	result
risch	$-\frac{\operatorname{erf}\left(-\sqrt{-c\ln(f)-f}\,x+\frac{e+b\ln(f)}{2\sqrt{-c\ln(f)-f}}\right)\sqrt{\pi}\,f^{a}\mathrm{e}^{-\frac{b^{2}\ln(f)^{2}+2\ln(f)be-4d\ln(f)c-4df+e^{2}}{4(f+c\ln(f))}}}{4\sqrt{-c\ln(f)-f}}+\frac{\operatorname{erf}\left(-x\sqrt{f-c\ln(f)}+\frac{b\ln(f)-e}{2\sqrt{f-c\ln(f)}}\right)\sqrt{\pi}}{4\sqrt{f}}$

input
```
int(f^(c*x^2+b*x+a)*sinh(f*x^2+e*x+d),x,method=_RETURNVERBOSE)
```

output
```
-1/4*erf(-(-c*ln(f)-f)^(1/2)*x+1/2*(e+b*ln(f))/(-c*ln(f)-f)^(1/2))/(-c*ln(
f)-f)^(1/2)*Pi^(1/2)*f^a*exp(-1/4*(b^2*ln(f)^2+2*ln(f)*b*e-4*d*ln(f)*c-4*d
*f+e^2)/(f+c*ln(f)))+1/4*erf(-x*(f-c*ln(f))^(1/2)+1/2*(b*ln(f)-e)/(f-c*ln(
f))^(1/2))/(f-c*ln(f))^(1/2)*Pi^(1/2)*f^a*exp(-1/4*(b^2*ln(f)^2-2*ln(f)*b*
e+4*d*ln(f)*c-4*d*f+e^2)/(c*ln(f)-f))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 363 vs. $2(139) = 278$.

Time = 0.12 (sec) , antiderivative size = 363, normalized size of antiderivative = 2.25

$$\int f^{a+bx+cx^2}\sinh\left(d+ex+fx^2\right)\,dx = \frac{\left(\sqrt{\pi}(c\log\left(f\right)+f)\cosh\left(-\frac{(b^2-4\,ac)\log(f)^2+e^2-4\,df+2\,(2\,cd-be+2\,af)\log(f)}{4\,(c\log(f)-f)}\right)+\sqrt{\pi}(c\log\left(f\right)+f)\sinh\left(-\frac{(b^2-4\,ac}{}\right.\right.}{}$$

input
```
integrate(f^(c*x^2+b*x+a)*sinh(f*x^2+e*x+d),x, algorithm="fricas")
```

output
```
1/4*((sqrt(pi)*(c*log(f) + f)*cosh(-1/4*((b^2 - 4*a*c)*log(f)^2 + e^2 - 4*
d*f + 2*(2*c*d - b*e + 2*a*f)*log(f))/(c*log(f) - f)) + sqrt(pi)*(c*log(f)
 + f)*sinh(-1/4*((b^2 - 4*a*c)*log(f)^2 + e^2 - 4*d*f + 2*(2*c*d - b*e + 2
*a*f)*log(f))/(c*log(f) - f)))*sqrt(-c*log(f) + f)*erf(-1/2*(2*f*x - (2*c*
x + b)*log(f) + e)*sqrt(-c*log(f) + f)/(c*log(f) - f)) - (sqrt(pi)*(c*log(
f) - f)*cosh(-1/4*((b^2 - 4*a*c)*log(f)^2 + e^2 - 4*d*f - 2*(2*c*d - b*e +
 2*a*f)*log(f))/(c*log(f) + f)) + sqrt(pi)*(c*log(f) - f)*sinh(-1/4*((b^2
- 4*a*c)*log(f)^2 + e^2 - 4*d*f - 2*(2*c*d - b*e + 2*a*f)*log(f))/(c*log(f
) + f)))*sqrt(-c*log(f) - f)*erf(1/2*(2*f*x + (2*c*x + b)*log(f) + e)*sqrt
(-c*log(f) - f)/(c*log(f) + f)))/(c^2*log(f)^2 - f^2)
```

Sympy [F]

$$\int f^{a+bx+cx^2}\sinh\left(d+ex+fx^2\right)\,dx = \int f^{a+bx+cx^2}\sinh\left(d+ex+fx^2\right)dx$$

input
```
integrate(f**(c*x**2+b*x+a)*sinh(f*x**2+e*x+d),x)
```

output
```
Integral(f**(a + b*x + c*x**2)*sinh(d + e*x + f*x**2), x)
```

Maxima [A] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 151, normalized size of antiderivative = 0.94

$$\int f^{a+bx+cx^2}\sinh\left(d+ex+fx^2\right)\,dx = \frac{\sqrt{\pi}f^a\operatorname{erf}\left(\sqrt{-c\log(f)-f}x-\frac{b\log(f)+e}{2\sqrt{-c\log(f)-f}}\right)e^{\left(-\frac{(b\log(f)+e)^2}{4(c\log(f)+f)}+d\right)}}{4\sqrt{-c\log(f)-f}} - \frac{\sqrt{\pi}f^a\operatorname{erf}\left(\sqrt{-c\log(f)+f}x-\frac{b\log(f)-e}{2\sqrt{-c\log(f)+f}}\right)e^{\left(-\frac{(b\log(f)-e)^2}{4(c\log(f)-f)}-d\right)}}{4\sqrt{-c\log(f)+f}}$$

input
```
integrate(f^(c*x^2+b*x+a)*sinh(f*x^2+e*x+d),x, algorithm="maxima")
```

output
```
1/4*sqrt(pi)*f^a*erf(sqrt(-c*log(f) - f)*x - 1/2*(b*log(f) + e)/sqrt(-c*lo
g(f) - f))*e^(-1/4*(b*log(f) + e)^2/(c*log(f) + f) + d)/sqrt(-c*log(f) - f
) - 1/4*sqrt(pi)*f^a*erf(sqrt(-c*log(f) + f)*x - 1/2*(b*log(f) - e)/sqrt(-
c*log(f) + f))*e^(-1/4*(b*log(f) - e)^2/(c*log(f) - f) - d)/sqrt(-c*log(f)
 + f)
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 207, normalized size of antiderivative = 1.29

$$\int f^{a+bx+cx^2}\sinh\left(d+ex+fx^2\right)\,dx = -\frac{\sqrt{\pi}\operatorname{erf}\left(-\frac{1}{2}\sqrt{-c\log(f)-f}\left(2x+\frac{b\log(f)+e}{c\log(f)+f}\right)\right)e^{\left(-\frac{b^2\log(f)^2-4ac\log(f)^2-4cd\log(f)+2be\log(f)-4af\log(f)+e^2-4df}{4(c\log(f)+f)}\right)}}{4\sqrt{-c\log(f)-f}} + \frac{\sqrt{\pi}\operatorname{erf}\left(-\frac{1}{2}\sqrt{-c\log(f)+f}\left(2x+\frac{b\log(f)-e}{c\log(f)-f}\right)\right)e^{\left(-\frac{b^2\log(f)^2-4ac\log(f)^2+4cd\log(f)-2be\log(f)+4af\log(f)+e^2-4df}{4(c\log(f)-f)}\right)}}{4\sqrt{-c\log(f)+f}}$$

input
```
integrate(f^(c*x^2+b*x+a)*sinh(f*x^2+e*x+d),x, algorithm="giac")
```

output
```
-1/4*sqrt(pi)*erf(-1/2*sqrt(-c*log(f) - f)*(2*x + (b*log(f) + e)/(c*log(f)
 + f)))*e^(-1/4*(b^2*log(f)^2 - 4*a*c*log(f)^2 - 4*c*d*log(f) + 2*b*e*log(
f) - 4*a*f*log(f) + e^2 - 4*d*f)/(c*log(f) + f))/sqrt(-c*log(f) - f) + 1/4
*sqrt(pi)*erf(-1/2*sqrt(-c*log(f) + f)*(2*x + (b*log(f) - e)/(c*log(f) - f
)))*e^(-1/4*(b^2*log(f)^2 - 4*a*c*log(f)^2 + 4*c*d*log(f) - 2*b*e*log(f) +
 4*a*f*log(f) + e^2 - 4*d*f)/(c*log(f) - f))/sqrt(-c*log(f) + f)
```

Mupad [F(-1)]

Timed out.

$$\int f^{a+bx+cx^2} \sinh\left(d+ex+fx^2\right)\,dx = \int f^{c\,x^2+b\,x+a}\,\sinh\left(f\,x^2+e\,x+d\right)\,dx$$

input
```
int(f^(a + b*x + c*x^2)*sinh(d + e*x + f*x^2),x)
```

output
```
int(f^(a + b*x + c*x^2)*sinh(d + e*x + f*x^2), x)
```

Reduce [F]

$$\int f^{a+bx+cx^2} \sinh\left(d+ex+fx^2\right)\,dx = f^a\left(\int f^{c\,x^2+bx}\sinh\left(f\,x^2+ex+d\right)dx\right)$$

input
```
int(f^(c*x^2+b*x+a)*sinh(f*x^2+e*x+d),x)
```

output
```
f**a*int(f**(b*x + c*x**2)*sinh(d + e*x + f*x**2),x)
```

3.364 $\int f^{a+bx+cx^2}\sinh^2\left(d+ex+fx^2\right)\,dx$

Optimal result

Integrand size = 26, antiderivative size = 239

$$\int f^{a+bx+cx^2}\sinh^2\left(d+ex+fx^2\right)\,dx = -\frac{f^{a-\frac{b^2}{4c}}\sqrt{\pi}\,\mathrm{erfi}\left(\frac{(b+2cx)\sqrt{\log(f)}}{2\sqrt{c}}\right)}{4\sqrt{c}\sqrt{\log(f)}}+\frac{e^{-2d+\frac{(2e-b\log(f))^2}{8f-4c\log(f)}}f^a\sqrt{\pi}\,\mathrm{erf}\left(\frac{2e-b\log(f)+2x(2f-c\log(f))}{2\sqrt{2f-c\log(f)}}\right)}{8\sqrt{2f-c\log(f)}}+\frac{e^{2d-\frac{(2e+b\log(f))^2}{8f+4c\log(f)}}f^a\sqrt{\pi}\,\mathrm{erfi}\left(\frac{2e+b\log(f)+2x(2f+c\log(f))}{2\sqrt{2f+c\log(f)}}\right)}{8\sqrt{2f+c\log(f)}}$$

output
```
-1/4*f^(a-1/4*b^2/c)*Pi^(1/2)*erfi(1/2*(2*c*x+b)*ln(f)^(1/2)/c^(1/2))/c^(1
/2)/ln(f)^(1/2)+1/8*exp(-2*d+(2*e-b*ln(f))^2/(8*f-4*c*ln(f)))*f^a*Pi^(1/2)
*erf(1/2*(2*e-b*ln(f)+2*x*(2*f-c*ln(f)))/(2*f-c*ln(f))^(1/2))/(2*f-c*ln(f)
)^(1/2)+1/8*exp(2*d-(2*e+b*ln(f))^2/(8*f+4*c*ln(f)))*f^a*Pi^(1/2)*erfi(1/2
*(2*e+b*ln(f)+2*x*(2*f+c*ln(f)))/(2*f+c*ln(f))^(1/2))/(2*f+c*ln(f))^(1/2)
```

Mathematica [A] (warning: unable to verify)

Time = 4.02 (sec) , antiderivative size = 339, normalized size of antiderivative = 1.42

$$\int f^{a+bx+cx^2}\sinh^2\left(d+ex+fx^2\right)\,dx=-\frac{f^{a-\frac{b^2}{4c}}\sqrt{\pi}\,\mathrm{erfi}\left(\frac{(b+2cx)\sqrt{\log(f)}}{2\sqrt{c}}\right)}{4\sqrt{c}\sqrt{\log(f)}}$$

$$-\frac{e^{-\frac{4e^2+b^2\log^2(f)}{8f+4c\log(f)}}f^{a+\frac{4bef}{-4f^2+c^2\log^2(f)}}\sqrt{\pi}\left(e^{\frac{f\left(4e^2+b^2\log^2(f)\right)}{4f^2-c^2\log^2(f)}}f^{\frac{be}{2f+c\log(f)}}\mathrm{erf}\left(\frac{2(e+2fx)-(b+2cx)\log(f)}{2\sqrt{2f-c\log(f)}}\right)\sqrt{2f-c\log(f)}(2\ldots}{}$$

input
```
Integrate[f^(a + b*x + c*x^2)*Sinh[d + e*x + f*x^2]^2,x]
```

output
```
-1/4*(f^(a - b^2/(4*c))*Sqrt[Pi]*Erfi[((b + 2*c*x)*Sqrt[Log[f]])/(2*Sqrt[c
])])/(Sqrt[c]*Sqrt[Log[f]]) - (f^(a + (4*b*e*f)/(-4*f^2 + c^2*Log[f]^2))*S
qrt[Pi]*(E^((f*(4*e^2 + b^2*Log[f]^2))/(4*f^2 - c^2*Log[f]^2))*f^((b*e)/(2
*f + c*Log[f]))*Erf[(2*(e + 2*f*x) - (b + 2*c*x)*Log[f])/(2*Sqrt[2*f - c*L
og[f]])]*Sqrt[2*f - c*Log[f]]*(2*f + c*Log[f])*(Cosh[2*d] - Sinh[2*d]) + f
^((b*e)/(2*f - c*Log[f]))*Erfi[(2*(e + 2*f*x) + (b + 2*c*x)*Log[f])/(2*Sqr
t[2*f + c*Log[f]])]*(2*f - c*Log[f])*Sqrt[2*f + c*Log[f]]*(Cosh[2*d] + Sin
h[2*d])))/(8*E^((4*e^2 + b^2*Log[f]^2)/(8*f + 4*c*Log[f]))*(-4*f^2 + c^2*L
og[f]^2))
```

Rubi [A] (verified)

Time = 0.84 (sec) , antiderivative size = 239, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {6038, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int f^{a+bx+cx^2}\sinh^2\left(d+ex+fx^2\right)\,dx$$

$$\Big\downarrow \ 6038$$

$$\int\left(\frac{1}{4}e^{-2d-2ex-2fx^2}f^{a+bx+cx^2}+\frac{1}{4}e^{2d+2ex+2fx^2}f^{a+bx+cx^2}-\frac{1}{2}f^{a+bx+cx^2}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$-\frac{\sqrt{\pi}f^{a-\frac{b^2}{4c}}\operatorname{erfi}\left(\frac{\sqrt{\log(f)}(b+2cx)}{2\sqrt{c}}\right)}{4\sqrt{c}\sqrt{\log(f)}}+\frac{\sqrt{\pi}f^a\exp\left(\frac{(2e-b\log(f))^2}{8f-4c\log(f)}-2d\right)\operatorname{erf}\left(\frac{-b\log(f)+2x(2f-c\log(f))+2e}{2\sqrt{2f-c\log(f)}}\right)}{8\sqrt{2f-c\log(f)}}+\frac{\sqrt{\pi}f^a\exp\left(2d-\frac{(b\log(f)+2e)^2}{4c\log(f)+8f}\right)\operatorname{erfi}\left(\frac{b\log(f)+2x(c\log(f)+2f)+2e}{2\sqrt{c\log(f)+2f}}\right)}{8\sqrt{c\log(f)+2f}}$$

input
```
Int[f^(a + b*x + c*x^2)*Sinh[d + e*x + f*x^2]^2,x]
```

output
```
-1/4*(f^(a - b^2/(4*c))*Sqrt[Pi]*Erfi[((b + 2*c*x)*Sqrt[Log[f]])/(2*Sqrt[c
])])/(Sqrt[c]*Sqrt[Log[f]]) + (E^(-2*d + (2*e - b*Log[f])^2/(8*f - 4*c*Log
[f]))*f^a*Sqrt[Pi]*Erf[(2*e - b*Log[f] + 2*x*(2*f - c*Log[f]))/(2*Sqrt[2*f
 - c*Log[f]])])/(8*Sqrt[2*f - c*Log[f]]) + (E^(2*d - (2*e + b*Log[f])^2/(8
*f + 4*c*Log[f]))*f^a*Sqrt[Pi]*Erfi[(2*e + b*Log[f] + 2*x*(2*f + c*Log[f])
)/(2*Sqrt[2*f + c*Log[f]])])/(8*Sqrt[2*f + c*Log[f]])
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 6038
```
Int[(F_)^(u_)*Sinh[v_]^(n_.), x_Symbol] :> Int[ExpandTrigToExp[F^u, Sinh[v]
^n, x], x] /; FreeQ[F, x] && (LinearQ[u, x] || PolyQ[u, x, 2]) && (LinearQ[
v, x] || PolyQ[v, x, 2]) && IGtQ[n, 0]
```

Maple [A] (verified)

Time = 0.60 (sec) , antiderivative size = 249, normalized size of antiderivative = 1.04

method	result
risch	$-\frac{\operatorname{erf}\left(-x\sqrt{2f-c\ln(f)}+\frac{b\ln(f)-2e}{2\sqrt{2f-c\ln(f)}}\right)\sqrt{\pi}\, f^{a} \mathrm{e}^{-\frac{b^{2}\ln(f)^{2}-4\ln(f)be+8d\ln(f)c-16df+4e^{2}}{4(c\ln(f)-2f)}}}{8\sqrt{2f-c\ln(f)}}-\frac{\operatorname{erf}\left(-\sqrt{-c\ln(f)-2f}\, x+\frac{2e+b\ln(f)}{2\sqrt{-c\ln(f)-2}}\right.}{8}$

input
```
int(f^(c*x^2+b*x+a)*sinh(f*x^2+e*x+d)^2,x,method=_RETURNVERBOSE)
```

output
```
-1/8*erf(-x*(2*f-c*ln(f))^(1/2)+1/2*(b*ln(f)-2*e)/(2*f-c*ln(f))^(1/2))/(2*
f-c*ln(f))^(1/2)*Pi^(1/2)*f^a*exp(-1/4*(b^2*ln(f)^2-4*ln(f)*b*e+8*d*ln(f)*
c-16*d*f+4*e^2)/(c*ln(f)-2*f))-1/8*erf(-(-c*ln(f)-2*f)^(1/2)*x+1/2*(2*e+b*
ln(f))/(-c*ln(f)-2*f)^(1/2))/(-c*ln(f)-2*f)^(1/2)*Pi^(1/2)*f^a*exp(-1/4*(b
^2*ln(f)^2+4*ln(f)*b*e-8*d*ln(f)*c-16*d*f+4*e^2)/(2*f+c*ln(f)))+1/4*f^a*Pi
^(1/2)*f^(-1/4*b^2/c)/(-c*ln(f))^(1/2)*erf(-(-c*ln(f))^(1/2)*x+1/2*ln(f)*b
/(-c*ln(f))^(1/2))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 516 vs. 2(197) = 394.

Time = 0.09 (sec) , antiderivative size = 516, normalized size of antiderivative = 2.16

$$\int f^{a+bx+cx^2}\sinh^2\left(d+ex+fx^2\right)\,dx=\text{Too large to display}$$

input
```
integrate(f^(c*x^2+b*x+a)*sinh(f*x^2+e*x+d)^2,x, algorithm="fricas")
```

output
```
-1/8*((sqrt(pi)*(c^2*log(f)^2 + 2*c*f*log(f))*cosh(-1/4*((b^2 - 4*a*c)*log
(f)^2 + 4*e^2 - 16*d*f + 4*(2*c*d - b*e + 2*a*f)*log(f))/(c*log(f) - 2*f))
 + sqrt(pi)*(c^2*log(f)^2 + 2*c*f*log(f))*sinh(-1/4*((b^2 - 4*a*c)*log(f)^
2 + 4*e^2 - 16*d*f + 4*(2*c*d - b*e + 2*a*f)*log(f))/(c*log(f) - 2*f)))*sq
rt(-c*log(f) + 2*f)*erf(-1/2*(4*f*x - (2*c*x + b)*log(f) + 2*e)*sqrt(-c*lo
g(f) + 2*f)/(c*log(f) - 2*f)) + (sqrt(pi)*(c^2*log(f)^2 - 2*c*f*log(f))*co
sh(-1/4*((b^2 - 4*a*c)*log(f)^2 + 4*e^2 - 16*d*f - 4*(2*c*d - b*e + 2*a*f)
*log(f))/(c*log(f) + 2*f)) + sqrt(pi)*(c^2*log(f)^2 - 2*c*f*log(f))*sinh(-
1/4*((b^2 - 4*a*c)*log(f)^2 + 4*e^2 - 16*d*f - 4*(2*c*d - b*e + 2*a*f)*log
(f))/(c*log(f) + 2*f)))*sqrt(-c*log(f) - 2*f)*erf(1/2*(4*f*x + (2*c*x + b)
*log(f) + 2*e)*sqrt(-c*log(f) - 2*f)/(c*log(f) + 2*f)) - 2*(sqrt(pi)*(c^2*
log(f)^2 - 4*f^2)*cosh(-1/4*(b^2 - 4*a*c)*log(f)/c) + sqrt(pi)*(c^2*log(f)
^2 - 4*f^2)*sinh(-1/4*(b^2 - 4*a*c)*log(f)/c))*sqrt(-c*log(f))*erf(1/2*(2*
c*x + b)*sqrt(-c*log(f))/c))/(c^3*log(f)^3 - 4*c*f^2*log(f))
```

Sympy [F]

$$\int f^{a+bx+cx^2}\sinh^2\left(d+ex+fx^2\right)\,dx=\int f^{a+bx+cx^2}\sinh^2\left(d+ex+fx^2\right)dx$$

input
```
integrate(f**(c*x**2+b*x+a)*sinh(f*x**2+e*x+d)**2,x)
```

output
```
Integral(f**(a + b*x + c*x**2)*sinh(d + e*x + f*x**2)**2, x)
```

Maxima [A] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 215, normalized size of antiderivative = 0.90

$$\int f^{a+bx+cx^2}\sinh^2\left(d+ex+fx^2\right)\,dx$$
$$=\frac{\sqrt{\pi}f^a\operatorname{erf}\left(\sqrt{-c\log\left(f\right)-2\,f}x-\frac{b\log(f)+2\,e}{2\,\sqrt{-c\log(f)-2\,f}}\right)e^{\left(-\frac{(b\log(f)+2\,e)^2}{4\,(c\log(f)+2\,f)}+2\,d\right)}}{8\,\sqrt{-c\log\left(f\right)-2\,f}}$$
$$+\frac{\sqrt{\pi}f^a\operatorname{erf}\left(\sqrt{-c\log\left(f\right)+2\,f}x-\frac{b\log(f)-2\,e}{2\,\sqrt{-c\log(f)+2\,f}}\right)e^{\left(-\frac{(b\log(f)-2\,e)^2}{4\,(c\log(f)-2\,f)}-2\,d\right)}}{8\,\sqrt{-c\log\left(f\right)+2\,f}}$$
$$-\frac{\sqrt{\pi}f^a\operatorname{erf}\left(\sqrt{-c\log\left(f\right)}x-\frac{b\log(f)}{2\,\sqrt{-c\log(f)}}\right)}{4\,\sqrt{-c\log\left(f\right)}f^{\frac{b^2}{4\,c}}}$$

input
```
integrate(f^(c*x^2+b*x+a)*sinh(f*x^2+e*x+d)^2,x, algorithm="maxima")
```

output
```
1/8*sqrt(pi)*f^a*erf(sqrt(-c*log(f) - 2*f)*x - 1/2*(b*log(f) + 2*e)/sqrt(-
c*log(f) - 2*f))*e^(-1/4*(b*log(f) + 2*e)^2/(c*log(f) + 2*f) + 2*d)/sqrt(-
c*log(f) - 2*f) + 1/8*sqrt(pi)*f^a*erf(sqrt(-c*log(f) + 2*f)*x - 1/2*(b*lo
g(f) - 2*e)/sqrt(-c*log(f) + 2*f))*e^(-1/4*(b*log(f) - 2*e)^2/(c*log(f) -
2*f) - 2*d)/sqrt(-c*log(f) + 2*f) - 1/4*sqrt(pi)*f^a*erf(sqrt(-c*log(f))*x
 - 1/2*b*log(f)/sqrt(-c*log(f)))/(sqrt(-c*log(f))*f^(1/4*b^2/c))
```

Giac [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 271, normalized size of antiderivative = 1.13

$$\int f^{a+bx+cx^2}\sinh^2\left(d+ex+fx^2\right)\,dx =$$

$$-\frac{\sqrt{\pi}\,\mathrm{erf}\left(-\frac{1}{2}\sqrt{-c\log(f)-2f}\left(2x+\frac{b\log(f)+2e}{c\log(f)+2f}\right)\right)e^{\left(-\frac{b^2\log(f)^2-4ac\log(f)^2-8cd\log(f)+4be\log(f)-8af\log(f)+4e^2-16df}{4(c\log(f)+2f)}\right.}}{8\sqrt{-c\log(f)-2f}}$$

$$-\frac{\sqrt{\pi}\,\mathrm{erf}\left(-\frac{1}{2}\sqrt{-c\log(f)+2f}\left(2x+\frac{b\log(f)-2e}{c\log(f)-2f}\right)\right)e^{\left(-\frac{b^2\log(f)^2-4ac\log(f)^2+8cd\log(f)-4be\log(f)+8af\log(f)+4e^2-16df}{4(c\log(f)-2f)}\right.}}{8\sqrt{-c\log(f)+2f}}$$

$$+\frac{\sqrt{\pi}\,\mathrm{erf}\left(-\frac{1}{2}\sqrt{-c\log(f)}\left(2x+\frac{b}{c}\right)\right)e^{\left(-\frac{b^2\log(f)-4ac\log(f)}{4c}\right)}}{4\sqrt{-c\log(f)}}$$

input
```
integrate(f^(c*x^2+b*x+a)*sinh(f*x^2+e*x+d)^2,x, algorithm="giac")
```

output
```
-1/8*sqrt(pi)*erf(-1/2*sqrt(-c*log(f) - 2*f)*(2*x + (b*log(f) + 2*e)/(c*lo
g(f) + 2*f)))*e^(-1/4*(b^2*log(f)^2 - 4*a*c*log(f)^2 - 8*c*d*log(f) + 4*b*
e*log(f) - 8*a*f*log(f) + 4*e^2 - 16*d*f)/(c*log(f) + 2*f))/sqrt(-c*log(f)
 - 2*f) - 1/8*sqrt(pi)*erf(-1/2*sqrt(-c*log(f) + 2*f)*(2*x + (b*log(f) - 2
*e)/(c*log(f) - 2*f)))*e^(-1/4*(b^2*log(f)^2 - 4*a*c*log(f)^2 + 8*c*d*log(
f) - 4*b*e*log(f) + 8*a*f*log(f) + 4*e^2 - 16*d*f)/(c*log(f) - 2*f))/sqrt(
-c*log(f) + 2*f) + 1/4*sqrt(pi)*erf(-1/2*sqrt(-c*log(f))*(2*x + b/c))*e^(-
1/4*(b^2*log(f) - 4*a*c*log(f))/c)/sqrt(-c*log(f))
```

Mupad [F(-1)]

Timed out.

$$\int f^{a+bx+cx^2}\sinh^2\left(d+ex+fx^2\right)\,dx = \int f^{c\,x^2+b\,x+a}\,{\sinh\left(f\,x^2+e\,x+d\right)}^2\,dx$$

input
```
int(f^(a + b*x + c*x^2)*sinh(d + e*x + f*x^2)^2,x)
```

output
```
int(f^(a + b*x + c*x^2)*sinh(d + e*x + f*x^2)^2, x)
```

Reduce [F]

$$\int f^{a+bx+cx^2}\sinh^2\left(d+ex+fx^2\right)\,dx=f^a\left(\int f^{c\,x^2+bx}\sinh\left(f\,x^2+ex+d\right)^2dx\right)$$

input
```
int(f^(c*x^2+b*x+a)*sinh(f*x^2+e*x+d)^2,x)
```

output
```
f**a*int(f**(b*x + c*x**2)*sinh(d + e*x + f*x**2)**2,x)
```

3.365 $\int f^{a+bx+cx^2}\sinh^3\left(d+ex+fx^2\right)\,dx$

Optimal result

Integrand size = 26, antiderivative size = 344

$$\begin{aligned}\int f^{a+bx+cx^2}\sinh^3\left(d+ex+fx^2\right)\,dx &= \frac{3e^{-d+\frac{(e-b\log(f))^2}{4(f-c\log(f))}}f^a\sqrt{\pi}\operatorname{erf}\left(\frac{e-b\log(f)+2x(f-c\log(f))}{2\sqrt{f-c\log(f)}}\right)}{16\sqrt{f-c\log(f)}}\\ &\quad-\frac{e^{-3d+\frac{(3e-b\log(f))^2}{12f-4c\log(f)}}f^a\sqrt{\pi}\operatorname{erf}\left(\frac{3e-b\log(f)+2x(3f-c\log(f))}{2\sqrt{3f-c\log(f)}}\right)}{16\sqrt{3f-c\log(f)}}\\ &\quad-\frac{3e^{d-\frac{(e+b\log(f))^2}{4(f+c\log(f))}}f^a\sqrt{\pi}\operatorname{erfi}\left(\frac{e+b\log(f)+2x(f+c\log(f))}{2\sqrt{f+c\log(f)}}\right)}{16\sqrt{f+c\log(f)}}\\ &\quad+\frac{e^{3d-\frac{(3e+b\log(f))^2}{4(3f+c\log(f))}}f^a\sqrt{\pi}\operatorname{erfi}\left(\frac{3e+b\log(f)+2x(3f+c\log(f))}{2\sqrt{3f+c\log(f)}}\right)}{16\sqrt{3f+c\log(f)}}\end{aligned}$$

output
```
3/16*exp(-d+(e-b*ln(f))^2/(4*f-4*c*ln(f)))*f^a*Pi^(1/2)*erf(1/2*(e-b*ln(f)
+2*x*(f-c*ln(f)))/(f-c*ln(f))^(1/2))/(f-c*ln(f))^(1/2)-1/16*exp(-3*d+(3*e-
b*ln(f))^2/(12*f-4*c*ln(f)))*f^a*Pi^(1/2)*erf(1/2*(3*e-b*ln(f)+2*x*(3*f-c*
ln(f)))/(3*f-c*ln(f))^(1/2))/(3*f-c*ln(f))^(1/2)-3/16*exp(d-(e+b*ln(f))^2/
(4*f+4*c*ln(f)))*f^a*Pi^(1/2)*erfi(1/2*(e+b*ln(f)+2*x*(f+c*ln(f)))/(f+c*ln
(f))^(1/2))/(f+c*ln(f))^(1/2)+1/16*exp(3*d-(3*e+b*ln(f))^2/(12*f+4*c*ln(f)
))*f^a*Pi^(1/2)*erfi(1/2*(3*e+b*ln(f)+2*x*(3*f+c*ln(f)))/(3*f+c*ln(f))^(1/
2))/(3*f+c*ln(f))^(1/2)
```

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 2991 vs. $2(344) = 688$.

Time = 6.42 (sec) , antiderivative size = 2991, normalized size of antiderivative = 8.69

$$\int f^{a+bx+cx^2}\sinh^3\left(d+ex+fx^2\right)\,dx = \text{Result too large to show}$$

input
```
Integrate[f^(a + b*x + c*x^2)*Sinh[d + e*x + f*x^2]^3,x]
```

output
```
(f^a*Sqrt[Pi]*((27*f^3*Cosh[d]*Erf[(e + 2*f*x - b*Log[f] - 2*c*x*Log[f])/(
2*Sqrt[f - c*Log[f]])]*Sqrt[f - c*Log[f]])/E^((-e^2 + 2*b*e*Log[f] - b^2*L
og[f]^2)/(4*(f - c*Log[f]))) + (27*c*f^2*Cosh[d]*Erf[(e + 2*f*x - b*Log[f]
 - 2*c*x*Log[f])/(2*Sqrt[f - c*Log[f]])]*Log[f]*Sqrt[f - c*Log[f]])/E^((-e
^2 + 2*b*e*Log[f] - b^2*Log[f]^2)/(4*(f - c*Log[f]))) - (3*c^2*f*Cosh[d]*E
rf[(e + 2*f*x - b*Log[f] - 2*c*x*Log[f])/(2*Sqrt[f - c*Log[f]])]*Log[f]^2*
Sqrt[f - c*Log[f]])/E^((-e^2 + 2*b*e*Log[f] - b^2*Log[f]^2)/(4*(f - c*Log[
f]))) - (3*c^3*Cosh[d]*Erf[(e + 2*f*x - b*Log[f] - 2*c*x*Log[f])/(2*Sqrt[f
 - c*Log[f]])]*Log[f]^3*Sqrt[f - c*Log[f]])/E^((-e^2 + 2*b*e*Log[f] - b^2*
Log[f]^2)/(4*(f - c*Log[f]))) - (3*f^3*Cosh[3*d]*Erf[(3*e + 6*f*x - b*Log[
f] - 2*c*x*Log[f])/(2*Sqrt[3*f - c*Log[f]])]*Sqrt[3*f - c*Log[f]])/E^((-9*
e^2 + 6*b*e*Log[f] - b^2*Log[f]^2)/(4*(3*f - c*Log[f]))) - (c*f^2*Cosh[3*d
]*Erf[(3*e + 6*f*x - b*Log[f] - 2*c*x*Log[f])/(2*Sqrt[3*f - c*Log[f]])]*Lo
g[f]*Sqrt[3*f - c*Log[f]])/E^((-9*e^2 + 6*b*e*Log[f] - b^2*Log[f]^2)/(4*(3
*f - c*Log[f]))) + (3*c^2*f*Cosh[3*d]*Erf[(3*e + 6*f*x - b*Log[f] - 2*c*x*
Log[f])/(2*Sqrt[3*f - c*Log[f]])]*Log[f]^2*Sqrt[3*f - c*Log[f]])/E^((-9*e^
2 + 6*b*e*Log[f] - b^2*Log[f]^2)/(4*(3*f - c*Log[f]))) + (c^3*Cosh[3*d]*Er
f[(3*e + 6*f*x - b*Log[f] - 2*c*x*Log[f])/(2*Sqrt[3*f - c*Log[f]])]*Log[f]
^3*Sqrt[3*f - c*Log[f]])/E^((-9*e^2 + 6*b*e*Log[f] - b^2*Log[f]^2)/(4*(3*f
 - c*Log[f]))) - (27*f^3*Cosh[d]*Erfi[(e + 2*f*x + b*Log[f] + 2*c*x*Log...
```

Rubi [A] (verified)

Time = 1.11 (sec) , antiderivative size = 344, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.077$, Rules used = {6038, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int f^{a+bx+cx^2} \sinh^3\left(d+ex+fx^2\right) dx$$

$$\Big\downarrow \text{6038}$$

$$\int \left(\frac{3}{8} f^{a+bx+cx^2} \exp\left(-3\left(d+ex+fx^2\right)+2d+2ex+2fx^2\right)-\frac{3}{8} f^{a+bx+cx^2} \exp\left(-3\left(d+ex+fx^2\right)+4d+4ex+\right.\right.$$

$$\Big\downarrow \text{2009}$$

$$-\frac{\sqrt{\pi}f^a\exp\left(\frac{(3e-b\log(f))^2}{12f-4c\log(f)}-3d\right)\operatorname{erf}\left(\frac{-b\log(f)+2x(3f-c\log(f))+3e}{2\sqrt{3f-c\log(f)}}\right)}{16\sqrt{3f-c\log(f)}}+\frac{3\sqrt{\pi}f^ae^{\frac{(e-b\log(f))^2}{4(f-c\log(f))}-d}\operatorname{erf}\left(\frac{-b\log(f)+2x(f-c\log(f))+e}{2\sqrt{f-c\log(f)}}\right)}{16\sqrt{f-c\log(f)}}+\frac{\sqrt{\pi}f^a\exp\left(3d-\frac{(b\log(f)+3e)^2}{4(c\log(f)+3f)}\right)\operatorname{erfi}\left(\frac{b\log(f)+2x(c\log(f)+3f)+3e}{2\sqrt{c\log(f)+3f}}\right)}{16\sqrt{c\log(f)+3f}}-\frac{3\sqrt{\pi}f^ae^{d-\frac{(b\log(f)+e)^2}{4(c\log(f)+f)}}\operatorname{erfi}\left(\frac{b\log(f)+2x(c\log(f)+f)+e}{2\sqrt{c\log(f)+f}}\right)}{16\sqrt{c\log(f)+f}}$$

input
```
Int[f^(a + b*x + c*x^2)*Sinh[d + e*x + f*x^2]^3,x]
```

output
```
(3*E^(-d + (e - b*Log[f])^2/(4*(f - c*Log[f])))*f^a*Sqrt[Pi]*Erf[(e - b*Lo
g[f] + 2*x*(f - c*Log[f]))/(2*Sqrt[f - c*Log[f]])])/(16*Sqrt[f - c*Log[f]]
) - (E^(-3*d + (3*e - b*Log[f])^2/(12*f - 4*c*Log[f]))*f^a*Sqrt[Pi]*Erf[(3
*e - b*Log[f] + 2*x*(3*f - c*Log[f]))/(2*Sqrt[3*f - c*Log[f]])])/(16*Sqrt[
3*f - c*Log[f]]) - (3*E^(d - (e + b*Log[f])^2/(4*(f + c*Log[f])))*f^a*Sqrt
[Pi]*Erfi[(e + b*Log[f] + 2*x*(f + c*Log[f]))/(2*Sqrt[f + c*Log[f]])])/(16
*Sqrt[f + c*Log[f]]) + (E^(3*d - (3*e + b*Log[f])^2/(4*(3*f + c*Log[f])))*
f^a*Sqrt[Pi]*Erfi[(3*e + b*Log[f] + 2*x*(3*f + c*Log[f]))/(2*Sqrt[3*f + c*
Log[f]])])/(16*Sqrt[3*f + c*Log[f]])
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 6038
```
Int[(F_)^(u_)*Sinh[v_]^(n_.), x_Symbol] :> Int[ExpandTrigToExp[F^u, Sinh[v]
^n, x], x] /; FreeQ[F, x] && (LinearQ[u, x] || PolyQ[u, x, 2]) && (LinearQ[
v, x] || PolyQ[v, x, 2]) && IGtQ[n, 0]
```

Maple [A] (verified)

Time = 1.63 (sec) , antiderivative size = 384, normalized size of antiderivative = 1.12

method	result
risch	$-\frac{\operatorname{erf}\left(-\sqrt{-c\ln(f)-3f}\,x+\frac{3e+b\ln(f)}{2\sqrt{-c\ln(f)-3f}}\right)\sqrt{\pi}\,f^{a}\,\mathrm{e}^{-\frac{b^{2}\ln(f)^{2}+6\ln(f)be-12d\ln(f)c-36df+9e^{2}}{4(3f+c\ln(f))}}}{16\sqrt{-c\ln(f)-3f}}+\frac{\operatorname{erf}\left(-x\sqrt{3f-c\ln(f)}+\frac{b\ln(f)-3}{2\sqrt{3f-c\ln}}\right.}{}$

input
```
int(f^(c*x^2+b*x+a)*sinh(f*x^2+e*x+d)^3,x,method=_RETURNVERBOSE)
```

output
```
-1/16*erf(-(-c*ln(f)-3*f)^(1/2)*x+1/2*(3*e+b*ln(f))/(-c*ln(f)-3*f)^(1/2))/
(-c*ln(f)-3*f)^(1/2)*Pi^(1/2)*f^a*exp(-1/4*(b^2*ln(f)^2+6*ln(f)*b*e-12*d*l
n(f)*c-36*d*f+9*e^2)/(3*f+c*ln(f)))+1/16*erf(-x*(3*f-c*ln(f))^(1/2)+1/2*(b
*ln(f)-3*e)/(3*f-c*ln(f))^(1/2))/(3*f-c*ln(f))^(1/2)*Pi^(1/2)*f^a*exp(-1/4
*(b^2*ln(f)^2-6*ln(f)*b*e+12*d*ln(f)*c-36*d*f+9*e^2)/(c*ln(f)-3*f))-3/16*e
rf(-x*(f-c*ln(f))^(1/2)+1/2*(b*ln(f)-e)/(f-c*ln(f))^(1/2))/(f-c*ln(f))^(1/
2)*Pi^(1/2)*f^a*exp(-1/4*(b^2*ln(f)^2-2*ln(f)*b*e+4*d*ln(f)*c-4*d*f+e^2)/(
c*ln(f)-f))+3/16*erf(-(-c*ln(f)-f)^(1/2)*x+1/2*(e+b*ln(f))/(-c*ln(f)-f)^(1
/2))/(-c*ln(f)-f)^(1/2)*Pi^(1/2)*f^a*exp(-1/4*(b^2*ln(f)^2+2*ln(f)*b*e-4*d
*ln(f)*c-4*d*f+e^2)/(f+c*ln(f)))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 940 vs. 2(295) = 590.

Time = 0.11 (sec) , antiderivative size = 940, normalized size of antiderivative = 2.73

$$\int f^{a+bx+cx^2}\sinh^3\left(d+ex+fx^2\right)\,dx=\text{Too large to display}$$

input
```
integrate(f^(c*x^2+b*x+a)*sinh(f*x^2+e*x+d)^3,x, algorithm="fricas")
```

output
```
1/16*((sqrt(pi)*(c^3*log(f)^3 + 3*c^2*f*log(f)^2 - c*f^2*log(f) - 3*f^3)*c
osh(-1/4*((b^2 - 4*a*c)*log(f)^2 + 9*e^2 - 36*d*f + 6*(2*c*d - b*e + 2*a*f
)*log(f))/(c*log(f) - 3*f)) + sqrt(pi)*(c^3*log(f)^3 + 3*c^2*f*log(f)^2 -
c*f^2*log(f) - 3*f^3)*sinh(-1/4*((b^2 - 4*a*c)*log(f)^2 + 9*e^2 - 36*d*f +
 6*(2*c*d - b*e + 2*a*f)*log(f))/(c*log(f) - 3*f)))*sqrt(-c*log(f) + 3*f)*
erf(-1/2*(6*f*x - (2*c*x + b)*log(f) + 3*e)*sqrt(-c*log(f) + 3*f)/(c*log(f
) - 3*f)) - 3*(sqrt(pi)*(c^3*log(f)^3 + c^2*f*log(f)^2 - 9*c*f^2*log(f) -
9*f^3)*cosh(-1/4*((b^2 - 4*a*c)*log(f)^2 + e^2 - 4*d*f + 2*(2*c*d - b*e +
2*a*f)*log(f))/(c*log(f) - f)) + sqrt(pi)*(c^3*log(f)^3 + c^2*f*log(f)^2 -
 9*c*f^2*log(f) - 9*f^3)*sinh(-1/4*((b^2 - 4*a*c)*log(f)^2 + e^2 - 4*d*f +
 2*(2*c*d - b*e + 2*a*f)*log(f))/(c*log(f) - f)))*sqrt(-c*log(f) + f)*erf(
-1/2*(2*f*x - (2*c*x + b)*log(f) + e)*sqrt(-c*log(f) + f)/(c*log(f) - f))
+ 3*(sqrt(pi)*(c^3*log(f)^3 - c^2*f*log(f)^2 - 9*c*f^2*log(f) + 9*f^3)*cos
h(-1/4*((b^2 - 4*a*c)*log(f)^2 + e^2 - 4*d*f - 2*(2*c*d - b*e + 2*a*f)*log
(f))/(c*log(f) + f)) + sqrt(pi)*(c^3*log(f)^3 - c^2*f*log(f)^2 - 9*c*f^2*l
og(f) + 9*f^3)*sinh(-1/4*((b^2 - 4*a*c)*log(f)^2 + e^2 - 4*d*f - 2*(2*c*d
- b*e + 2*a*f)*log(f))/(c*log(f) + f)))*sqrt(-c*log(f) - f)*erf(1/2*(2*f*x
 + (2*c*x + b)*log(f) + e)*sqrt(-c*log(f) - f)/(c*log(f) + f)) - (sqrt(pi)
*(c^3*log(f)^3 - 3*c^2*f*log(f)^2 - c*f^2*log(f) + 3*f^3)*cosh(-1/4*((b^2
- 4*a*c)*log(f)^2 + 9*e^2 - 36*d*f - 6*(2*c*d - b*e + 2*a*f)*log(f))/(c...
```

Sympy [F(-1)]

Timed out.

$$\int f^{a+bx+cx^2} \sinh^3\left(d+ex+fx^2\right) dx = \text{Timed out}$$

input
```
integrate(f**(c*x**2+b*x+a)*sinh(f*x**2+e*x+d)**3,x)
```

output
```
Timed out
```

Maxima [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 315, normalized size of antiderivative = 0.92

$$\int f^{a+bx+cx^2}\sinh^3\left(d+ex+fx^2\right)\,dx$$
$$=\frac{\sqrt{\pi}f^a\operatorname{erf}\left(\sqrt{-c\log(f)-3f}x-\frac{b\log(f)+3e}{2\sqrt{-c\log(f)-3f}}\right)e^{\left(-\frac{(b\log(f)+3e)^2}{4(c\log(f)+3f)}+3d\right)}}{16\sqrt{-c\log(f)-3f}}$$
$$-\frac{3\sqrt{\pi}f^a\operatorname{erf}\left(\sqrt{-c\log(f)-f}x-\frac{b\log(f)+e}{2\sqrt{-c\log(f)-f}}\right)e^{\left(-\frac{(b\log(f)+e)^2}{4(c\log(f)+f)}+d\right)}}{16\sqrt{-c\log(f)-f}}$$
$$+\frac{3\sqrt{\pi}f^a\operatorname{erf}\left(\sqrt{-c\log(f)+f}x-\frac{b\log(f)-e}{2\sqrt{-c\log(f)+f}}\right)e^{\left(-\frac{(b\log(f)-e)^2}{4(c\log(f)-f)}-d\right)}}{16\sqrt{-c\log(f)+f}}$$
$$-\frac{\sqrt{\pi}f^a\operatorname{erf}\left(\sqrt{-c\log(f)+3f}x-\frac{b\log(f)-3e}{2\sqrt{-c\log(f)+3f}}\right)e^{\left(-\frac{(b\log(f)-3e)^2}{4(c\log(f)-3f)}-3d\right)}}{16\sqrt{-c\log(f)+3f}}$$

input
```
integrate(f^(c*x^2+b*x+a)*sinh(f*x^2+e*x+d)^3,x, algorithm="maxima")
```

output
```
1/16*sqrt(pi)*f^a*erf(sqrt(-c*log(f) - 3*f)*x - 1/2*(b*log(f) + 3*e)/sqrt(
-c*log(f) - 3*f))*e^(-1/4*(b*log(f) + 3*e)^2/(c*log(f) + 3*f) + 3*d)/sqrt(
-c*log(f) - 3*f) - 3/16*sqrt(pi)*f^a*erf(sqrt(-c*log(f) - f)*x - 1/2*(b*lo
g(f) + e)/sqrt(-c*log(f) - f))*e^(-1/4*(b*log(f) + e)^2/(c*log(f) + f) + d
)/sqrt(-c*log(f) - f) + 3/16*sqrt(pi)*f^a*erf(sqrt(-c*log(f) + f)*x - 1/2*
(b*log(f) - e)/sqrt(-c*log(f) + f))*e^(-1/4*(b*log(f) - e)^2/(c*log(f) - f
) - d)/sqrt(-c*log(f) + f) - 1/16*sqrt(pi)*f^a*erf(sqrt(-c*log(f) + 3*f)*x
 - 1/2*(b*log(f) - 3*e)/sqrt(-c*log(f) + 3*f))*e^(-1/4*(b*log(f) - 3*e)^2/
(c*log(f) - 3*f) - 3*d)/sqrt(-c*log(f) + 3*f)
```

Giac [A] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 427, normalized size of antiderivative = 1.24

$$\int f^{a+bx+cx^2}\sinh^3\left(d+ex+fx^2\right)\,dx = -\frac{\sqrt{\pi}\,\mathrm{erf}\left(-\frac{1}{2}\sqrt{-c\log(f)-3f}\left(2x+\frac{b\log(f)+3e}{c\log(f)+3f}\right)\right)e^{\left(-\frac{b^2\log(f)^2-4ac\log(f)^2-12cd\log(f)+6be\log(f)-12af\log(f)+9e^2-36df}{4(c\log(f)+3f)}\right)}}{16\sqrt{-c\log(f)-3f}} + \frac{3\sqrt{\pi}\,\mathrm{erf}\left(-\frac{1}{2}\sqrt{-c\log(f)-f}\left(2x+\frac{b\log(f)+e}{c\log(f)+f}\right)\right)e^{\left(-\frac{b^2\log(f)^2-4ac\log(f)^2-4cd\log(f)+2be\log(f)-4af\log(f)+e^2-4df}{4(c\log(f)+f)}\right)}}{16\sqrt{-c\log(f)-f}} - \frac{3\sqrt{\pi}\,\mathrm{erf}\left(-\frac{1}{2}\sqrt{-c\log(f)+f}\left(2x+\frac{b\log(f)-e}{c\log(f)-f}\right)\right)e^{\left(-\frac{b^2\log(f)^2-4ac\log(f)^2+4cd\log(f)-2be\log(f)+4af\log(f)+e^2-4df}{4(c\log(f)-f)}\right)}}{16\sqrt{-c\log(f)+f}} + \frac{\sqrt{\pi}\,\mathrm{erf}\left(-\frac{1}{2}\sqrt{-c\log(f)+3f}\left(2x+\frac{b\log(f)-3e}{c\log(f)-3f}\right)\right)e^{\left(-\frac{b^2\log(f)^2-4ac\log(f)^2+12cd\log(f)-6be\log(f)+12af\log(f)+9e^2-36df}{4(c\log(f)-3f)}\right)}}{16\sqrt{-c\log(f)+3f}}$$

input
```
integrate(f^(c*x^2+b*x+a)*sinh(f*x^2+e*x+d)^3,x, algorithm="giac")
```

output
```
-1/16*sqrt(pi)*erf(-1/2*sqrt(-c*log(f) - 3*f)*(2*x + (b*log(f) + 3*e)/(c*l
og(f) + 3*f)))*e^(-1/4*(b^2*log(f)^2 - 4*a*c*log(f)^2 - 12*c*d*log(f) + 6*
b*e*log(f) - 12*a*f*log(f) + 9*e^2 - 36*d*f)/(c*log(f) + 3*f))/sqrt(-c*log
(f) - 3*f) + 3/16*sqrt(pi)*erf(-1/2*sqrt(-c*log(f) - f)*(2*x + (b*log(f) +
 e)/(c*log(f) + f)))*e^(-1/4*(b^2*log(f)^2 - 4*a*c*log(f)^2 - 4*c*d*log(f)
 + 2*b*e*log(f) - 4*a*f*log(f) + e^2 - 4*d*f)/(c*log(f) + f))/sqrt(-c*log(
f) - f) - 3/16*sqrt(pi)*erf(-1/2*sqrt(-c*log(f) + f)*(2*x + (b*log(f) - e)
/(c*log(f) - f)))*e^(-1/4*(b^2*log(f)^2 - 4*a*c*log(f)^2 + 4*c*d*log(f) -
2*b*e*log(f) + 4*a*f*log(f) + e^2 - 4*d*f)/(c*log(f) - f))/sqrt(-c*log(f)
+ f) + 1/16*sqrt(pi)*erf(-1/2*sqrt(-c*log(f) + 3*f)*(2*x + (b*log(f) - 3*e
)/(c*log(f) - 3*f)))*e^(-1/4*(b^2*log(f)^2 - 4*a*c*log(f)^2 + 12*c*d*log(f
) - 6*b*e*log(f) + 12*a*f*log(f) + 9*e^2 - 36*d*f)/(c*log(f) - 3*f))/sqrt(
-c*log(f) + 3*f)
```

Mupad [F(-1)]

Timed out.

$$\int f^{a+bx+cx^2} \sinh^3\left(d+ex+fx^2\right)\,dx = \int f^{c\,x^2+b\,x+a}\,\sinh\left(f\,x^2+e\,x+d\right)^3\,dx$$

input
```
int(f^(a + b*x + c*x^2)*sinh(d + e*x + f*x^2)^3,x)
```

output
```
int(f^(a + b*x + c*x^2)*sinh(d + e*x + f*x^2)^3, x)
```

Reduce [F]

$$\int f^{a+bx+cx^2} \sinh^3\left(d+ex+fx^2\right)\,dx = f^a\left(\int f^{c\,x^2+bx}\sinh\left(f\,x^2+ex+d\right)^3dx\right)$$

input
```
int(f^(c*x^2+b*x+a)*sinh(f*x^2+e*x+d)^3,x)
```

output
```
f**a*int(f**(b*x + c*x**2)*sinh(d + e*x + f*x**2)**3,x)
```

3.366 $\int (x+\sinh(x))^2\,dx$

Optimal result

Integrand size = 6, antiderivative size = 30

$$\int (x+\sinh(x))^2\,dx = -\frac{x}{2}+\frac{x^3}{3}+2x\cosh(x)-2\sinh(x)+\frac{1}{2}\cosh(x)\sinh(x)$$

output
```
-1/2*x+1/3*x^3+2*x*cosh(x)-2*sinh(x)+1/2*cosh(x)*sinh(x)
```

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.00

$$\int (x+\sinh(x))^2\,dx = \frac{1}{6}x(-3+2x^2)+2x\cosh(x)-2\sinh(x)+\frac{1}{4}\sinh(2x)$$

input
```
Integrate[(x + Sinh[x])^2,x]
```

output
```
(x*(-3 + 2*x^2))/6 + 2*x*Cosh[x] - 2*Sinh[x] + Sinh[2*x]/4
```

Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {7293, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (x+\sinh (x))^2 \, dx$$

$$\Big\downarrow \text{7293}$$

$$\int \left(x^2+\sinh ^2(x)+2 x \sinh (x)\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{x^3}{3}-\frac{x}{2}-2 \sinh (x)+2 x \cosh (x)+\frac{1}{2} \sinh (x) \cosh (x)$$

input
```
Int[(x + Sinh[x])^2,x]
```

output
```
-1/2*x + x^3/3 + 2*x*Cosh[x] - 2*Sinh[x] + (Cosh[x]*Sinh[x])/2
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 7293
```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Maple [A] (verified)

Time = 0.70 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.83

method	result
default	$-\frac{x}{2}+\frac{x^{3}}{3}+2x\cosh\left(x\right)-2\sinh\left(x\right)+\frac{\cosh(x)\sinh(x)}{2}$
parallelrisch	$\frac{x^{3}}{3}-\frac{x}{2}+2x\cosh\left(x\right)-2\sinh\left(x\right)+\frac{\sinh(2x)}{4}$
parts	$-\frac{x}{2}+\frac{x^{3}}{3}+2x\cosh\left(x\right)-2\sinh\left(x\right)+\frac{\cosh(x)\sinh(x)}{2}$
risch	$\frac{x^{3}}{3}-\frac{x}{2}+\frac{e^{2x}}{8}+\left(-1+x\right)e^{x}+\left(1+x\right)e^{-x}-\frac{e^{-2x}}{8}$
orering	$\frac{x(9x^{2}-85)(x+\sinh(x))^{2}}{27x^{2}+15}+\frac{(69x^{2}-95)(x+\sinh(x))(1+\cosh(x))}{18x^{2}+10}-\frac{5x(9x^{2}-37)\left(2(1+\cosh(x))^{2}+2(x+\sinh(x))\sinh(x)\right)}{12(9x^{2}+5)}$

input
```
int((x+sinh(x))^2,x,method=_RETURNVERBOSE)
```

output
```
-1/2*x+1/3*x^3+2*x*cosh(x)-2*sinh(x)+1/2*cosh(x)*sinh(x)
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.73

$$\int (x+\sinh(x))^2\,dx=\frac{1}{3}\,x^{3}+2\,x\cosh\left(x\right)+\frac{1}{2}\left(\cosh\left(x\right)-4\right)\sinh\left(x\right)-\frac{1}{2}\,x$$

input
```
integrate((x+sinh(x))^2,x, algorithm="fricas")
```

output
```
1/3*x^3 + 2*x*cosh(x) + 1/2*(cosh(x) - 4)*sinh(x) - 1/2*x
```

Sympy [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 41, normalized size of antiderivative = 1.37

$$\int (x+\sinh(x))^2\,dx = \frac{x^3}{3} + \frac{x\sinh^2(x)}{2} - \frac{x\cosh^2(x)}{2} + 2x\cosh(x) + \frac{\sinh(x)\cosh(x)}{2} - 2\sinh(x)$$

input
```
integrate((x+sinh(x))**2,x)
```

output
```
x**3/3 + x*sinh(x)**2/2 - x*cosh(x)**2/2 + 2*x*cosh(x) + sinh(x)*cosh(x)/2
 - 2*sinh(x)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 35, normalized size of antiderivative = 1.17

$$\int (x+\sinh(x))^2\,dx = \frac{1}{3}x^3 + (x+1)e^{(-x)} + (x-1)e^x - \frac{1}{2}x + \frac{1}{8}e^{(2x)} - \frac{1}{8}e^{(-2x)}$$

input
```
integrate((x+sinh(x))^2,x, algorithm="maxima")
```

output
```
1/3*x^3 + (x + 1)*e^(-x) + (x - 1)*e^x - 1/2*x + 1/8*e^(2*x) - 1/8*e^(-2*x
)
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 35, normalized size of antiderivative = 1.17

$$\int (x+\sinh(x))^2\,dx = \frac{1}{3}x^3 + (x+1)e^{(-x)} + (x-1)e^x - \frac{1}{2}x + \frac{1}{8}e^{(2x)} - \frac{1}{8}e^{(-2x)}$$

input
```
integrate((x+sinh(x))^2,x, algorithm="giac")
```

output
```
1/3*x^3 + (x + 1)*e^(-x) + (x - 1)*e^x - 1/2*x + 1/8*e^(2*x) - 1/8*e^(-2*x
)
```

Mupad [B] (verification not implemented)

Time = 1.51 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.80

$$\int (x+\sinh(x))^2\,dx=\frac{\cosh(x)\,\sinh(x)}{2}-2\sinh(x)-\frac{x}{2}+2\,x\cosh(x)+\frac{x^3}{3}$$

input
```
int((x + sinh(x))^2,x)
```

output
```
(cosh(x)*sinh(x))/2 - 2*sinh(x) - x/2 + 2*x*cosh(x) + x^3/3
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 62, normalized size of antiderivative = 2.07

$$\int (x+\sinh(x))^2\,dx=\frac{3e^{4x}+24e^{3x}x-24e^{3x}+8e^{2x}x^3-12e^{2x}x+24e^{x}x+24e^{x}-3}{24e^{2x}}$$

input
```
int((x+sinh(x))^2,x)
```

output
```
(3*e**(4*x) + 24*e**(3*x)*x - 24*e**(3*x) + 8*e**(2*x)*x**3 - 12*e**(2*x)*
x + 24*e***x*x + 24*e***x - 3)/(24*e**(2*x))
```

3.367 $\int (x+\sinh(x))^3 \, dx$

Optimal result

Integrand size = 6, antiderivative size = 56

$$\int (x+\sinh(x))^3 \, dx = -\frac{3x^2}{4}+\frac{x^4}{4}+5\cosh(x)+3x^2\cosh(x)+\frac{\cosh^3(x)}{3} - 6x\sinh(x)+\frac{3}{2}x\cosh(x)\sinh(x)-\frac{3\sinh^2(x)}{4}$$

output
```
-3/4*x^2+1/4*x^4+5*cosh(x)+3*x^2*cosh(x)+1/3*cosh(x)^3-6*x*sinh(x)+3/2*x*c
osh(x)*sinh(x)-3/4*sinh(x)^2
```

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.86

$$\int (x+\sinh(x))^3 \, dx = \frac{1}{24}\left(18\left(7+4x^2\right)\cosh(x)-9\cosh(2x)+2\cosh(3x) + 6x\left(-3x+x^3-24\sinh(x)+3\sinh(2x)\right)\right)$$

input
```
Integrate[(x + Sinh[x])^3,x]
```

output
```
(18*(7 + 4*x^2)*Cosh[x] - 9*Cosh[2*x] + 2*Cosh[3*x] + 6*x*(-3*x + x^3 - 24
*Sinh[x] + 3*Sinh[2*x]))/24
```

Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 56, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {7293, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (x+\sinh(x))^3\,dx$$

$$\Big\downarrow \ 7293$$

$$\int \left(x^3+3x^2\sinh(x)+\sinh^3(x)+3x\sinh^2(x)\right)dx$$

$$\Big\downarrow \ 2009$$

$$\frac{x^4}{4}-\frac{3x^2}{4}+3x^2\cosh(x)-\frac{3\sinh^2(x)}{4}-6x\sinh(x)+\frac{\cosh^3(x)}{3}+5\cosh(x)+\frac{3}{2}x\sinh(x)\cosh(x)$$

input
```
Int[(x + Sinh[x])^3,x]
```

output
```
(-3*x^2)/4 + x^4/4 + 5*Cosh[x] + 3*x^2*Cosh[x] + Cosh[x]^3/3 - 6*x*Sinh[x]
 + (3*x*Cosh[x]*Sinh[x])/2 - (3*Sinh[x]^2)/4
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 7293
```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Maple [A] (verified)

Time = 0.04 (sec) , antiderivative size = 52, normalized size of antiderivative = 0.93

$$\left(-\frac{2}{3}+\frac{\sinh\left(x\right)^{2}}{3}\right)\cosh\left(x\right)+\frac{3x\cosh\left(x\right)\sinh\left(x\right)}{2}-\frac{3x^{2}}{4}-\frac{3\cosh\left(x\right)^{2}}{4}+3x^{2}\cosh\left(x\right)-6x\sinh\left(x\right)+6\cosh\left(x\right)$$

input
```
int((x+sinh(x))^3,x)
```

output
```
(-2/3+1/3*sinh(x)^2)*cosh(x)+3/2*x*cosh(x)*sinh(x)-3/4*x^2-3/4*cosh(x)^2+3
*x^2*cosh(x)-6*x*sinh(x)+6*cosh(x)+1/4*x^4
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 58, normalized size of antiderivative = 1.04

$$\int (x+\sinh(x))^3\,dx = \frac{1}{4}\,x^4 + \frac{1}{12}\,\cosh\left(x\right)^3 + \frac{1}{8}\,(2\,\cosh\left(x\right) - 3)\sinh\left(x\right)^2 - \frac{3}{4}\,x^2 + \frac{3}{4}\,(4\,x^2 + 7)\cosh\left(x\right) - \frac{3}{8}\,\cosh\left(x\right)^2 + \frac{3}{2}\,(x\cosh\left(x\right) - 4\,x)\sinh\left(x\right)$$

input
```
integrate((x+sinh(x))^3,x, algorithm="fricas")
```

output
```
1/4*x^4 + 1/12*cosh(x)^3 + 1/8*(2*cosh(x) - 3)*sinh(x)^2 - 3/4*x^2 + 3/4*(
4*x^2 + 7)*cosh(x) - 3/8*cosh(x)^2 + 3/2*(x*cosh(x) - 4*x)*sinh(x)
```

Sympy [A] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 85, normalized size of antiderivative = 1.52

$$\int (x+\sinh(x))^3\,dx = \frac{x^4}{4} + \frac{3x^2\sinh^2(x)}{4} - \frac{3x^2\cosh^2(x)}{4} + 3x^2\cosh(x) + \frac{3x\sinh(x)\cosh(x)}{2} - 6x\sinh(x) + \sinh^2(x)\cosh(x) - \frac{3\sinh^2(x)}{4} - \frac{2\cosh^3(x)}{3} + 6\cosh(x)$$

input
```
integrate((x+sinh(x))**3,x)
```

output
```
x**4/4 + 3*x**2*sinh(x)**2/4 - 3*x**2*cosh(x)**2/4 + 3*x**2*cosh(x) + 3*x*
sinh(x)*cosh(x)/2 - 6*x*sinh(x) + sinh(x)**2*cosh(x) - 3*sinh(x)**2/4 - 2*
cosh(x)**3/3 + 6*cosh(x)
```

Maxima [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 81, normalized size of antiderivative = 1.45

$$\int (x+\sinh(x))^3\,dx = \frac{1}{4}x^4 - \frac{3}{4}x^2 + \frac{3}{16}(2x-1)e^{(2x)} + \frac{3}{2}(x^2+2x+2)e^{(-x)} - \frac{3}{16}(2x+1)e^{(-2x)} + \frac{3}{2}(x^2-2x+2)e^x + \frac{1}{24}e^{(3x)} - \frac{3}{8}e^{(-x)} + \frac{1}{24}e^{(-3x)} - \frac{3}{8}e^x$$

input
```
integrate((x+sinh(x))^3,x, algorithm="maxima")
```

output
```
1/4*x^4 - 3/4*x^2 + 3/16*(2*x - 1)*e^(2*x) + 3/2*(x^2 + 2*x + 2)*e^(-x) -
3/16*(2*x + 1)*e^(-2*x) + 3/2*(x^2 - 2*x + 2)*e^x + 1/24*e^(3*x) - 3/8*e^(
-x) + 1/24*e^(-3*x) - 3/8*e^x
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 75, normalized size of antiderivative = 1.34

$$\int (x+\sinh(x))^3\,dx = \frac{1}{4}\,x^4 - \frac{3}{4}\,x^2 + \frac{3}{16}\,(2\,x-1)e^{(2\,x)} + \frac{3}{8}\,(4\,x^2+8\,x+7)e^{(-x)} - \frac{3}{16}\,(2\,x+1)e^{(-2\,x)} + \frac{3}{8}\,(4\,x^2-8\,x+7)e^{x} + \frac{1}{24}\,e^{(3\,x)} + \frac{1}{24}\,e^{(-3\,x)}$$

input
```
integrate((x+sinh(x))^3,x, algorithm="giac")
```

output
```
1/4*x^4 - 3/4*x^2 + 3/16*(2*x - 1)*e^(2*x) + 3/8*(4*x^2 + 8*x + 7)*e^(-x)
- 3/16*(2*x + 1)*e^(-2*x) + 3/8*(4*x^2 - 8*x + 7)*e^x + 1/24*e^(3*x) + 1/2
4*e^(-3*x)
```

Mupad [B] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.82

$$\int (x+\sinh(x))^3\,dx = 5\cosh(x) + 3\,x^2\cosh(x) - \frac{3\cosh(x)^2}{4} + \frac{\cosh(x)^3}{3} - 6\,x\sinh(x) - \frac{3\,x^2}{4} + \frac{x^4}{4} + \frac{3\,x\cosh(x)\,\sinh(x)}{2}$$

input
```
int((x + sinh(x))^3,x)
```

output
```
5*cosh(x) + 3*x^2*cosh(x) - (3*cosh(x)^2)/4 + cosh(x)^3/3 - 6*x*sinh(x) -
(3*x^2)/4 + x^4/4 + (3*x*cosh(x)*sinh(x))/2
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 114, normalized size of antiderivative = 2.04

$$\int (x+\sinh(x))^3\,dx$$
$$= \frac{2e^{6x}+18e^{5x}x-9e^{5x}+72e^{4x}x^2-144e^{4x}x+126e^{4x}+12e^{3x}x^4-36e^{3x}x^2+72e^{2x}x^2+144e^{2x}x+126e^{2x}}{48e^{3x}}$$

input
```
int((x+sinh(x))^3,x)
```

output
```
(2*e**(6*x) + 18*e**(5*x)*x - 9*e**(5*x) + 72*e**(4*x)*x**2 - 144*e**(4*x)
*x + 126*e**(4*x) + 12*e**(3*x)*x**4 - 36*e**(3*x)*x**2 + 72*e**(2*x)*x**2
 + 144*e**(2*x)*x + 126*e**(2*x) - 18*e**x*x - 9*e**x + 2)/(48*e**(3*x))
```

3.368 $\int \frac{\sinh(a+bx)}{c+dx^2} dx$

Optimal result

Integrand size = 16, antiderivative size = 213

$$\begin{aligned}\int \frac{\sinh(a+bx)}{c+dx^2}\,dx =& -\frac{\text{Chi}\left(\frac{b\sqrt{-c}}{\sqrt{d}}+bx\right)\sinh\left(a-\frac{b\sqrt{-c}}{\sqrt{d}}\right)}{2\sqrt{-c}\sqrt{d}} \\ &+ \frac{\text{Chi}\left(\frac{b\sqrt{-c}}{\sqrt{d}}-bx\right)\sinh\left(a+\frac{b\sqrt{-c}}{\sqrt{d}}\right)}{2\sqrt{-c}\sqrt{d}} \\ &- \frac{\cosh\left(a+\frac{b\sqrt{-c}}{\sqrt{d}}\right)\text{Shi}\left(\frac{b\sqrt{-c}}{\sqrt{d}}-bx\right)}{2\sqrt{-c}\sqrt{d}} \\ &- \frac{\cosh\left(a-\frac{b\sqrt{-c}}{\sqrt{d}}\right)\text{Shi}\left(\frac{b\sqrt{-c}}{\sqrt{d}}+bx\right)}{2\sqrt{-c}\sqrt{d}}\end{aligned}$$

output

```
-1/2*Chi(b*(-c)^(1/2)/d^(1/2)+b*x)*sinh(a-b*(-c)^(1/2)/d^(1/2))/(-c)^(1/2)
/d^(1/2)+1/2*Chi(b*(-c)^(1/2)/d^(1/2)-b*x)*sinh(a+b*(-c)^(1/2)/d^(1/2))/(-
c)^(1/2)/d^(1/2)+1/2*cosh(a+b*(-c)^(1/2)/d^(1/2))*Shi(-b*(-c)^(1/2)/d^(1/2
)+b*x)/(-c)^(1/2)/d^(1/2)-1/2*cosh(a-b*(-c)^(1/2)/d^(1/2))*Shi(b*(-c)^(1/2
)/d^(1/2)+b*x)/(-c)^(1/2)/d^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.49 (sec) , antiderivative size = 167, normalized size of antiderivative = 0.78

$$\int \frac{\sinh(a+bx)}{c+dx^2}\,dx = -\frac{ie^{-a-\frac{ib\sqrt{c}}{\sqrt{d}}}\left(e^{2a+\frac{2ib\sqrt{c}}{\sqrt{d}}}\operatorname{ExpIntegralEi}\left(b\left(-\frac{i\sqrt{c}}{\sqrt{d}}+x\right)\right)-e^{2a}\operatorname{ExpIntegralEi}\left(b\left(\frac{i\sqrt{c}}{\sqrt{d}}+x\right)\right)+e^{\frac{2ib\sqrt{c}}{\sqrt{d}}}\operatorname{ExpIn}\right.}{4\sqrt{c}\sqrt{d}}$$

input
```
Integrate[Sinh[a + b*x]/(c + d*x^2),x]
```

output
```
((-1/4*I)*E^(-a - (I*b*Sqrt[c])/Sqrt[d])*(E^(2*a + ((2*I)*b*Sqrt[c])/Sqrt[
d])*ExpIntegralEi[b*(((-I)*Sqrt[c])/Sqrt[d] + x)] - E^(2*a)*ExpIntegralEi[
b*((I*Sqrt[c])/Sqrt[d] + x)] + E^(((2*I)*b*Sqrt[c])/Sqrt[d])*ExpIntegralEi
[((-I)*b*Sqrt[c])/Sqrt[d] - b*x] - ExpIntegralEi[(I*b*Sqrt[c])/Sqrt[d] - b
*x]))/(Sqrt[c]*Sqrt[d])
```

Rubi [A] (verified)

Time = 0.75 (sec) , antiderivative size = 213, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$, Rules used = {5803, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sinh(a+bx)}{c+dx^2}\,dx$$

$$\Big\downarrow \text{5803}$$

$$\int \left(\frac{\sqrt{-c}\sinh(a+bx)}{2c\left(\sqrt{-c}-\sqrt{d}x\right)}+\frac{\sqrt{-c}\sinh(a+bx)}{2c\left(\sqrt{-c}+\sqrt{d}x\right)}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$-\frac{\sinh\left(a-\frac{b\sqrt{-c}}{\sqrt{d}}\right)\text{Chi}\left(xb+\frac{\sqrt{-c}b}{\sqrt{d}}\right)}{2\sqrt{-c}\sqrt{d}}+\frac{\sinh\left(a+\frac{b\sqrt{-c}}{\sqrt{d}}\right)\text{Chi}\left(\frac{b\sqrt{-c}}{\sqrt{d}}-bx\right)}{2\sqrt{-c}\sqrt{d}}-\frac{\cosh\left(a+\frac{b\sqrt{-c}}{\sqrt{d}}\right)\text{Shi}\left(\frac{b\sqrt{-c}}{\sqrt{d}}-bx\right)}{2\sqrt{-c}\sqrt{d}}-\frac{\cosh\left(a-\frac{b\sqrt{-c}}{\sqrt{d}}\right)\text{Shi}\left(xb+\frac{\sqrt{-c}b}{\sqrt{d}}\right)}{2\sqrt{-c}\sqrt{d}}$$

input
```
Int[Sinh[a + b*x]/(c + d*x^2),x]
```

output
```
-1/2*(CoshIntegral[(b*Sqrt[-c])/Sqrt[d] + b*x]*Sinh[a - (b*Sqrt[-c])/Sqrt[
d]])/(Sqrt[-c]*Sqrt[d]) + (CoshIntegral[(b*Sqrt[-c])/Sqrt[d] - b*x]*Sinh[a
 + (b*Sqrt[-c])/Sqrt[d]])/(2*Sqrt[-c]*Sqrt[d]) - (Cosh[a + (b*Sqrt[-c])/Sq
rt[d]]*SinhIntegral[(b*Sqrt[-c])/Sqrt[d] - b*x])/(2*Sqrt[-c]*Sqrt[d]) - (C
osh[a - (b*Sqrt[-c])/Sqrt[d]]*SinhIntegral[(b*Sqrt[-c])/Sqrt[d] + b*x])/(2
*Sqrt[-c]*Sqrt[d])
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 5803
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*Sinh[(c_.) + (d_.)*(x_)], x_Symbol] :> In
t[ExpandIntegrand[Sinh[c + d*x], (a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d
}, x] && ILtQ[p, 0] && IGtQ[n, 0] && (EqQ[n, 2] || EqQ[p, -1])
```

Maple [A] (verified)

Time = 0.19 (sec) , antiderivative size = 212, normalized size of antiderivative = 1.00

method	result
risch	$-\frac{e^{\frac{b\sqrt{-cd}+da}{d}}\,\text{expIntegral}_1\left(\frac{b\sqrt{-cd}-(bx+a)d+da}{d}\right)}{4\sqrt{-cd}}+\frac{e^{\frac{-b\sqrt{-cd}+da}{d}}\,\text{expIntegral}_1\left(-\frac{b\sqrt{-cd}+(bx+a)d-da}{d}\right)}{4\sqrt{-cd}}+\frac{e^{-\frac{b\sqrt{-cd}+da}{d}}\,\text{expInt}}{}$

input
```
int(sinh(b*x+a)/(d*x^2+c),x,method=_RETURNVERBOSE)
```

output
```
-1/4/(-c*d)^(1/2)*exp((b*(-c*d)^(1/2)+d*a)/d)*Ei(1,(b*(-c*d)^(1/2)-(b*x+a)
*d+d*a)/d)+1/4/(-c*d)^(1/2)*exp((-b*(-c*d)^(1/2)+d*a)/d)*Ei(1,-(b*(-c*d)^(
1/2)+(b*x+a)*d-d*a)/d)+1/4/(-c*d)^(1/2)*exp(-(b*(-c*d)^(1/2)+d*a)/d)*Ei(1,
-(b*(-c*d)^(1/2)-(b*x+a)*d+d*a)/d)-1/4/(-c*d)^(1/2)*exp(-(-b*(-c*d)^(1/2)+
d*a)/d)*Ei(1,(b*(-c*d)^(1/2)+(b*x+a)*d-d*a)/d)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 316 vs. 2(157) = 314.

Time = 0.10 (sec) , antiderivative size = 316, normalized size of antiderivative = 1.48

$$\int \frac{\sinh(a+bx)}{c+dx^2}\,dx = -\frac{\left(\sqrt{-\frac{b^2c}{d}}\,\text{Ei}\left(bx-\sqrt{-\frac{b^2c}{d}}\right)-\sqrt{-\frac{b^2c}{d}}\,\text{Ei}\left(-bx+\sqrt{-\frac{b^2c}{d}}\right)\right)\cosh\left(a+\sqrt{-\frac{b^2c}{d}}\right)-\left(\sqrt{-\frac{b^2c}{d}}\,\text{Ei}\left(bx+\sqrt{\right.\right.}{}$$

input
```
integrate(sinh(b*x+a)/(d*x^2+c),x, algorithm="fricas")
```

output
```
-1/4*((sqrt(-b^2*c/d)*Ei(b*x - sqrt(-b^2*c/d)) - sqrt(-b^2*c/d)*Ei(-b*x +
sqrt(-b^2*c/d)))*cosh(a + sqrt(-b^2*c/d)) - (sqrt(-b^2*c/d)*Ei(b*x + sqrt(
-b^2*c/d)) - sqrt(-b^2*c/d)*Ei(-b*x - sqrt(-b^2*c/d)))*cosh(-a + sqrt(-b^2
*c/d)) + (sqrt(-b^2*c/d)*Ei(b*x - sqrt(-b^2*c/d)) + sqrt(-b^2*c/d)*Ei(-b*x
 + sqrt(-b^2*c/d)))*sinh(a + sqrt(-b^2*c/d)) + (sqrt(-b^2*c/d)*Ei(b*x + sq
rt(-b^2*c/d)) + sqrt(-b^2*c/d)*Ei(-b*x - sqrt(-b^2*c/d)))*sinh(-a + sqrt(-
b^2*c/d)))/(b*c)
```

Sympy [F]

$$\int \frac{\sinh(a+bx)}{c+dx^2}\,dx = \int \frac{\sinh(a+bx)}{c+dx^2}\,dx$$

input
```
integrate(sinh(b*x+a)/(d*x**2+c),x)
```

output
```
Integral(sinh(a + b*x)/(c + d*x**2), x)
```

Maxima [F]

$$\int \frac{\sinh(a+bx)}{c+dx^2}\,dx=\int \frac{\sinh\left(bx+a\right)}{dx^2+c}\,dx$$

input
```
integrate(sinh(b*x+a)/(d*x^2+c),x, algorithm="maxima")
```

output
```
integrate(sinh(b*x + a)/(d*x^2 + c), x)
```

Giac [F]

$$\int \frac{\sinh(a+bx)}{c+dx^2}\,dx=\int \frac{\sinh\left(bx+a\right)}{dx^2+c}\,dx$$

input
```
integrate(sinh(b*x+a)/(d*x^2+c),x, algorithm="giac")
```

output
```
integrate(sinh(b*x + a)/(d*x^2 + c), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sinh(a+bx)}{c+dx^2}\,dx=\int \frac{\sinh(a+b\,x)}{d\,x^2+c}\,dx$$

input
```
int(sinh(a + b*x)/(c + d*x^2),x)
```

output
```
int(sinh(a + b*x)/(c + d*x^2), x)
```

Reduce [F]

$$\int \frac{\sinh(a+bx)}{c+dx^2}\,dx = \int \frac{\sinh\left(bx+a\right)}{d\,x^2+c}dx$$

input
```
int(sinh(b*x+a)/(d*x^2+c),x)
```

output
```
int(sinh(a + b*x)/(c + d*x**2),x)
```

3.369 $\int \frac{\sinh(a+bx)}{c+dx+ex^2}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 271

$$\int \frac{\sinh(a+bx)}{c+dx+ex^2}\,dx = \frac{\text{Chi}\left(\frac{b\left(d-\sqrt{d^2-4ce}\right)}{2e}+bx\right)\sinh\left(a-\frac{b\left(d-\sqrt{d^2-4ce}\right)}{2e}\right)}{\sqrt{d^2-4ce}} - \frac{\text{Chi}\left(\frac{b\left(d+\sqrt{d^2-4ce}\right)}{2e}+bx\right)\sinh\left(a-\frac{b\left(d+\sqrt{d^2-4ce}\right)}{2e}\right)}{\sqrt{d^2-4ce}} + \frac{\cosh\left(a-\frac{b\left(d-\sqrt{d^2-4ce}\right)}{2e}\right)\text{Shi}\left(\frac{b\left(d-\sqrt{d^2-4ce}\right)}{2e}+bx\right)}{\sqrt{d^2-4ce}} - \frac{\cosh\left(a-\frac{b\left(d+\sqrt{d^2-4ce}\right)}{2e}\right)\text{Shi}\left(\frac{b\left(d+\sqrt{d^2-4ce}\right)}{2e}+bx\right)}{\sqrt{d^2-4ce}}$$

output
```
Chi(1/2*b*(d-(-4*c*e+d^2)^(1/2))/e+b*x)*sinh(a-1/2*b*(d-(-4*c*e+d^2)^(1/2)
)/e)/(-4*c*e+d^2)^(1/2)-Chi(1/2*b*(d+(-4*c*e+d^2)^(1/2))/e+b*x)*sinh(a-1/2
*b*(d+(-4*c*e+d^2)^(1/2))/e)/(-4*c*e+d^2)^(1/2)+cosh(a-1/2*b*(d-(-4*c*e+d^
2)^(1/2))/e)*Shi(1/2*b*(d-(-4*c*e+d^2)^(1/2))/e+b*x)/(-4*c*e+d^2)^(1/2)-co
sh(a-1/2*b*(d+(-4*c*e+d^2)^(1/2))/e)*Shi(1/2*b*(d+(-4*c*e+d^2)^(1/2))/e+b*
x)/(-4*c*e+d^2)^(1/2)
```

Mathematica [A] (verified)

Time = 1.03 (sec) , antiderivative size = 220, normalized size of antiderivative = 0.81

$$\int \frac{\sinh(a+bx)}{c+dx+ex^2}\,dx = \frac{e^{-a-\frac{b\left(d+\sqrt{d^2-4ce}\right)}{2e}}\left(-e^{\frac{bd}{e}}\,\text{ExpIntegralEi}\left(-\frac{b\left(d-\sqrt{d^2-4ce}+2ex\right)}{2e}\right)+e^{2a+\frac{b\sqrt{d^2-4ce}}{e}}\,\text{ExpIntegralEi}\left(\frac{b\left(d-\sqrt{d^2-4ce}+\cdots\right.}{2e}\right.\right.}{2\sqrt{d^2-\cdots}}$$

input
```
Integrate[Sinh[a + b*x]/(c + d*x + e*x^2),x]
```

output
```
(E^(-a - (b*(d + Sqrt[d^2 - 4*c*e]))/(2*e))*(-(E^((b*d)/e)*ExpIntegralEi[-
1/2*(b*(d - Sqrt[d^2 - 4*c*e] + 2*e*x))/e]) + E^(2*a + (b*Sqrt[d^2 - 4*c*e
])/e)*ExpIntegralEi[(b*(d - Sqrt[d^2 - 4*c*e] + 2*e*x))/(2*e)] + E^((b*(d
+ Sqrt[d^2 - 4*c*e]))/e)*ExpIntegralEi[-1/2*(b*(d + Sqrt[d^2 - 4*c*e] + 2*
e*x))/e] - E^(2*a)*ExpIntegralEi[(b*(d + Sqrt[d^2 - 4*c*e] + 2*e*x))/(2*e)
]))/(2*Sqrt[d^2 - 4*c*e])
```

Rubi [A] (verified)

Time = 1.06 (sec) , antiderivative size = 271, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.105$, Rules used = {7279, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sinh(a+bx)}{c+dx+ex^2}\,dx$$

$$\Big\downarrow \; 7279$$

$$\int \left(\frac{2e\sinh(a+bx)}{\sqrt{d^2-4ce}\left(-\sqrt{d^2-4ce}+d+2ex\right)}-\frac{2e\sinh(a+bx)}{\sqrt{d^2-4ce}\left(\sqrt{d^2-4ce}+d+2ex\right)}\right)dx$$

$$\Big\downarrow \; 2009$$

$$\frac{\sinh\left(a-\frac{b\left(d-\sqrt{d^2-4ce}\right)}{2e}\right)\text{Chi}\left(\frac{b\left(d-\sqrt{d^2-4ce}\right)}{2e}+bx\right)}{\sqrt{d^2-4ce}}-\frac{\sinh\left(a-\frac{b\left(\sqrt{d^2-4ce}+d\right)}{2e}\right)\text{Chi}\left(\frac{b\left(d+\sqrt{d^2-4ce}\right)}{2e}+bx\right)}{\sqrt{d^2-4ce}}+\frac{\cosh\left(a-\frac{b\left(d-\sqrt{d^2-4ce}\right)}{2e}\right)\text{Shi}\left(\frac{b\left(d-\sqrt{d^2-4ce}\right)}{2e}+bx\right)}{\sqrt{d^2-4ce}}-\frac{\cosh\left(a-\frac{b\left(\sqrt{d^2-4ce}+d\right)}{2e}\right)\text{Shi}\left(\frac{b\left(d+\sqrt{d^2-4ce}\right)}{2e}+bx\right)}{\sqrt{d^2-4ce}}$$

input
```
Int[Sinh[a + b*x]/(c + d*x + e*x^2),x]
```

output
```
(CoshIntegral[(b*(d - Sqrt[d^2 - 4*c*e]))/(2*e) + b*x]*Sinh[a - (b*(d - Sq
rt[d^2 - 4*c*e]))/(2*e)])/Sqrt[d^2 - 4*c*e] - (CoshIntegral[(b*(d + Sqrt[d
^2 - 4*c*e]))/(2*e) + b*x]*Sinh[a - (b*(d + Sqrt[d^2 - 4*c*e]))/(2*e)])/Sq
rt[d^2 - 4*c*e] + (Cosh[a - (b*(d - Sqrt[d^2 - 4*c*e]))/(2*e)]*SinhIntegra
l[(b*(d - Sqrt[d^2 - 4*c*e]))/(2*e) + b*x])/Sqrt[d^2 - 4*c*e] - (Cosh[a -
(b*(d + Sqrt[d^2 - 4*c*e]))/(2*e)]*SinhIntegral[(b*(d + Sqrt[d^2 - 4*c*e])
)/(2*e) + b*x])/Sqrt[d^2 - 4*c*e]
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 7279
```
Int[(u_)/((a_.) + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(n2_.)), x_Symbol] :> With[
{v = RationalFunctionExpand[u/(a + b*x^n + c*x^(2*n)), x]}, Int[v, x] /; Su
mQ[v]] /; FreeQ[{a, b, c}, x] && EqQ[n2, 2*n] && IGtQ[n, 0]
```

Maple [A] (verified)

Time = 0.37 (sec) , antiderivative size = 376, normalized size of antiderivative = 1.39

method	result
risch	$\frac{b\,\mathrm{e}^{-\frac{2ea-db+\sqrt{-4b^2ce+b^2d^2}}{2e}}\,\mathrm{expIntegral}_1\left(-\frac{-2e(bx+a)+2ea-db+\sqrt{-4b^2ce+b^2d^2}}{2e}\right)}{2\sqrt{-4b^2ce+b^2d^2}} - \frac{b\,\mathrm{e}^{-\frac{2ea-db-\sqrt{-4b^2ce+b^2d^2}}{2e}}\,\mathrm{expIntegral}_1\left(\frac{2e}{}\right.}{2\sqrt{-4b^2ce+b^2}}$

input
```
int(sinh(b*x+a)/(e*x^2+d*x+c),x,method=_RETURNVERBOSE)
```

output
```
1/2*b/(-4*b^2*c*e+b^2*d^2)^(1/2)*exp(-1/2/e*(2*e*a-d*b+(-4*b^2*c*e+b^2*d^2
)^(1/2)))*Ei(1,-1/2*(-2*e*(b*x+a)+2*e*a-d*b+(-4*b^2*c*e+b^2*d^2)^(1/2))/e)
-1/2*b/(-4*b^2*c*e+b^2*d^2)^(1/2)*exp(-1/2/e*(2*e*a-d*b-(-4*b^2*c*e+b^2*d^
2)^(1/2)))*Ei(1,1/2*(2*e*(b*x+a)-2*e*a+d*b+(-4*b^2*c*e+b^2*d^2)^(1/2))/e)-
1/2*b/(-4*b^2*c*e+b^2*d^2)^(1/2)*exp(1/2/e*(2*e*a-d*b+(-4*b^2*c*e+b^2*d^2)
^(1/2)))*Ei(1,1/2*(-2*e*(b*x+a)+2*e*a-d*b+(-4*b^2*c*e+b^2*d^2)^(1/2))/e)+1
/2*b/(-4*b^2*c*e+b^2*d^2)^(1/2)*exp(1/2/e*(2*e*a-d*b-(-4*b^2*c*e+b^2*d^2)^
(1/2)))*Ei(1,-1/2*(2*e*(b*x+a)-2*e*a+d*b+(-4*b^2*c*e+b^2*d^2)^(1/2))/e)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 671 vs. $2(231) = 462$.

Time = 0.11 (sec) , antiderivative size = 671, normalized size of antiderivative = 2.48

$$\int \frac{\sinh(a+bx)}{c+dx+ex^2}\,dx = -\frac{\left(e\sqrt{\frac{b^2d^2-4\,b^2ce}{e^2}}\,\mathrm{Ei}\left(\frac{2\,bex+bd+e\sqrt{\frac{b^2d^2-4\,b^2ce}{e^2}}}{2\,e}\right) - e\sqrt{\frac{b^2d^2-4\,b^2ce}{e^2}}\,\mathrm{Ei}\left(-\frac{2\,bex+bd+e\sqrt{\frac{b^2d^2-4\,b^2ce}{e^2}}}{2\,e}\right)\right)\cosh\left(\frac{bd-2\,ae+e\sqrt{}}{2}\right.}{}$$

input
```
integrate(sinh(b*x+a)/(e*x^2+d*x+c),x, algorithm="fricas")
```

output
```
-1/2*((e*sqrt((b^2*d^2 - 4*b^2*c*e)/e^2)*Ei(1/2*(2*b*e*x + b*d + e*sqrt((b
^2*d^2 - 4*b^2*c*e)/e^2))/e) - e*sqrt((b^2*d^2 - 4*b^2*c*e)/e^2)*Ei(-1/2*(
2*b*e*x + b*d + e*sqrt((b^2*d^2 - 4*b^2*c*e)/e^2))/e))*cosh(1/2*(b*d - 2*a
*e + e*sqrt((b^2*d^2 - 4*b^2*c*e)/e^2))/e) - (e*sqrt((b^2*d^2 - 4*b^2*c*e)
/e^2)*Ei(1/2*(2*b*e*x + b*d - e*sqrt((b^2*d^2 - 4*b^2*c*e)/e^2))/e) - e*sq
rt((b^2*d^2 - 4*b^2*c*e)/e^2)*Ei(-1/2*(2*b*e*x + b*d - e*sqrt((b^2*d^2 - 4
*b^2*c*e)/e^2))/e))*cosh(-1/2*(b*d - 2*a*e - e*sqrt((b^2*d^2 - 4*b^2*c*e)/
e^2))/e) - (e*sqrt((b^2*d^2 - 4*b^2*c*e)/e^2)*Ei(1/2*(2*b*e*x + b*d + e*sq
rt((b^2*d^2 - 4*b^2*c*e)/e^2))/e) + e*sqrt((b^2*d^2 - 4*b^2*c*e)/e^2)*Ei(-
1/2*(2*b*e*x + b*d + e*sqrt((b^2*d^2 - 4*b^2*c*e)/e^2))/e))*sinh(1/2*(b*d
- 2*a*e + e*sqrt((b^2*d^2 - 4*b^2*c*e)/e^2))/e) - (e*sqrt((b^2*d^2 - 4*b^2
*c*e)/e^2)*Ei(1/2*(2*b*e*x + b*d - e*sqrt((b^2*d^2 - 4*b^2*c*e)/e^2))/e) +
 e*sqrt((b^2*d^2 - 4*b^2*c*e)/e^2)*Ei(-1/2*(2*b*e*x + b*d - e*sqrt((b^2*d^
2 - 4*b^2*c*e)/e^2))/e))*sinh(-1/2*(b*d - 2*a*e - e*sqrt((b^2*d^2 - 4*b^2*
c*e)/e^2))/e))/(b*d^2 - 4*b*c*e)
```

Sympy [F]

$$\int \frac{\sinh(a+bx)}{c+dx+ex^2}\,dx=\int \frac{\sinh{(a+bx)}}{c+dx+ex^2}\,dx$$

input
```
integrate(sinh(b*x+a)/(e*x**2+d*x+c),x)
```

output
```
Integral(sinh(a + b*x)/(c + d*x + e*x**2), x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{\sinh(a+bx)}{c+dx+ex^2}\,dx=\text{Exception raised: ValueError}$$

input
```
integrate(sinh(b*x+a)/(e*x^2+d*x+c),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(4*c*e-d^2>0)', see `assume?` for
 more deta
```

Giac [F]

$$\int \frac{\sinh(a+bx)}{c+dx+ex^2}\,dx=\int \frac{\sinh(bx+a)}{ex^2+dx+c}\,dx$$

input
```
integrate(sinh(b*x+a)/(e*x^2+d*x+c),x, algorithm="giac")
```

output
```
integrate(sinh(b*x + a)/(e*x^2 + d*x + c), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sinh(a+bx)}{c+dx+ex^2}\,dx=\int \frac{\sinh(a+b\,x)}{e\,x^2+d\,x+c}\,dx$$

input
```
int(sinh(a + b*x)/(c + d*x + e*x^2),x)
```

output
```
int(sinh(a + b*x)/(c + d*x + e*x^2), x)
```

Reduce [F]

$$\int \frac{\sinh(a+bx)}{c+dx+ex^2}\,dx=\int \frac{\sinh(bx+a)}{e\,x^2+dx+c}dx$$

input
```
int(sinh(b*x+a)/(e*x^2+d*x+c),x)
```

output
```
int(sinh(a + b*x)/(c + d*x + e*x**2),x)
```

CHAPTER 4

Appendix

4.1 Listing of Grading functions

The following are the current version of the grading functions used for grading the quality of the antiderivative with reference to the optimal antiderivative included in the test suite.

There is a version for Maple and for Mathematica/Rubi. There is a version for grading Sympy and version for use with Sagemath.

The following are links to the current source code.

The following are the listings of source code of the grading functions.

Mathematica and Rubi grading function

```
(* Original version thanks to Albert Rich emailed on 03/21/2017 *)
(* ::Package:: *)

(* Nasser: April 7,2022. add second output which gives reason for the grade *)
(*                       Small rewrite of logic in main function to make it*)
(*                       match Maple's logic. No change in functionality otherwise*)

(* ::Subsection:: *)
(*GradeAntiderivative[result,optimal]*)
```

```
(* ::Text:: *)
(*If result and optimal are mathematical expressions, *)
(*          GradeAntiderivative[result,optimal] returns*)
(*  "F" if the result fails to integrate an expression that*)
(*      is integrable*)
(*  "C" if result involves higher level functions than necessary*)
(*  "B" if result is more than twice the size of the optimal*)
(*      antiderivative*)
(*  "A" if result can be considered optimal*)


GradeAntiderivative[result_,optimal_] := Module[{expnResult,expnOptimal,leafCountResult,leaf
    expnResult  = ExpnType[result];
    expnOptimal = ExpnType[optimal];
    leafCountResult = LeafCount[result];
    leafCountOptimal = LeafCount[optimal];

    (*Print["expnResult=",expnResult," expnOptimal=",expnOptimal];*)
    If[expnResult<=expnOptimal,
        If[Not[FreeQ[result,Complex]], (*result contains complex*)
            If[Not[FreeQ[optimal,Complex]], (*optimal contains complex*)
                If[leafCountResult<=2*leafCountOptimal,
                    finalresult={"A",""}
                    ,(*ELSE*)
                    finalresult={"B","Both result and optimal contain complex but leaf count
                  ]
            ,(*ELSE*)
                finalresult={"C","Result contains complex when optimal does not."}
            ]
        ,(*ELSE*)(*result does not contains complex*)
            If[leafCountResult<=2*leafCountOptimal,
               finalresult={"A",""}
            ,(*ELSE*)
               finalresult={"B","Leaf count is larger than twice the leaf count of optimal.
              ]
        ]
    ,(*ELSE*) (*expnResult>expnOptimal*)
        If[FreeQ[result,Integrate] && FreeQ[result,Int],
            finalresult={"C","Result contains higher order function than in optimal. Order "
            ,
```

```
            finalresult={"F","Contains unresolved integral."}
        ]
    ];

    finalresult
]

(* ::Text:: *)
(*The following summarizes the type number assigned an *)
(*expression based on the functions it involves*)
(*1 = rational function*)
(*2 = algebraic function*)
(*3 = elementary function*)
(*4 = special function*)
(*5 = hyperpergeometric function*)
(*6 = appell function*)
(*7 = rootsum function*)
(*8 = integrate function*)
(*9 = unknown function*)


ExpnType[expn_] :=
  If[AtomQ[expn],
    1,
  If[ListQ[expn],
    Max[Map[ExpnType,expn]],
  If[Head[expn]===Power,
    If[IntegerQ[expn[[2]]],
      ExpnType[expn[[1]]],
    If[Head[expn[[2]]]===Rational,
      If[IntegerQ[expn[[1]]] || Head[expn[[1]]]===Rational,
        1,
      Max[ExpnType[expn[[1]]],2]],
    Max[ExpnType[expn[[1]]],ExpnType[expn[[2]]],3]]],
  If[Head[expn]===Plus || Head[expn]===Times,
    Max[ExpnType[First[expn]],ExpnType[Rest[expn]]],
  If[ElementaryFunctionQ[Head[expn]],
    Max[3,ExpnType[expn[[1]]]],
  If[SpecialFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],4]],
  If[HypergeometricFunctionQ[Head[expn]],
```

```
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],5]],
  If[AppellFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],6]],
  If[Head[expn]===RootSum,
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],7]],
  If[Head[expn]===Integrate || Head[expn]===Int,
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],8]],
  9]]]]]]]]]]


ElementaryFunctionQ[func_] :=
  MemberQ[{
  Exp,Log,
  Sin,Cos,Tan,Cot,Sec,Csc,
  ArcSin,ArcCos,ArcTan,ArcCot,ArcSec,ArcCsc,
  Sinh,Cosh,Tanh,Coth,Sech,Csch,
  ArcSinh,ArcCosh,ArcTanh,ArcCoth,ArcSech,ArcCsch
},func]


SpecialFunctionQ[func_] :=
  MemberQ[{
  Erf, Erfc, Erfi,
  FresnelS, FresnelC,
  ExpIntegralE, ExpIntegralEi, LogIntegral,
  SinIntegral, CosIntegral, SinhIntegral, CoshIntegral,
  Gamma, LogGamma, PolyGamma,
  Zeta, PolyLog, ProductLog,
  EllipticF, EllipticE, EllipticPi
},func]


HypergeometricFunctionQ[func_] :=
  MemberQ[{Hypergeometric1F1,Hypergeometric2F1,HypergeometricPFQ},func]


AppellFunctionQ[func_] :=
  MemberQ[{AppellF1},func]
```

Maple grading function

```
# File: GradeAntiderivative.mpl
# Original version thanks to Albert Rich emailed on 03/21/2017

#Nasser 03/22/2017  Use Maple leaf count instead since buildin
#Nasser 03/23/2017  missing 'ln' for ElementaryFunctionQ added
#Nasser 03/24/2017  corrected the check for complex result
#Nasser 10/27/2017  check for leafsize and do not call ExpnType()
#                   if leaf size is "too large". Set at 500,000
#Nasser 12/22/2019  Added debug flag, added 'dilog' to special functions
#                   see problem 156, file Apostol_Problems
#Nasser 4/07/2022   add second output which gives reason for the grade

GradeAntiderivative := proc(result,optimal)
local leaf_count_result,
        leaf_count_optimal,
        ExpnType_result,
        ExpnType_optimal,
        debug:=false;

        leaf_count_result:=leafcount(result);
        #do NOT call ExpnType() if leaf size is too large. Recursion problem
        if leaf_count_result > 500000 then
            return "B","result has leaf size over 500,000. Avoiding possible recursion issue
        fi;

        leaf_count_optimal := leafcount(optimal);
        ExpnType_result  := ExpnType(result);
        ExpnType_optimal := ExpnType(optimal);

        if debug then
            print("ExpnType_result",ExpnType_result," ExpnType_optimal=",ExpnType_optimal);
        fi;

# If result and optimal are mathematical expressions,
#  GradeAntiderivative[result,optimal] returns
#   "F" if the result fails to integrate an expression that
#       is integrable
#   "C" if result involves higher level functions than necessary
#   "B" if result is more than twice the size of the optimal
```

```
#       antiderivative
#   "A" if result can be considered optimal

    #This check below actually is not needed, since I only
    #call this grading only for passed integrals. i.e. I check
    #for "F" before calling this. But no harm of keeping it here.
    #just in case.


    if not type(result,freeof('int')) then
        return "F","Result contains unresolved integral";
    fi;


    if ExpnType_result<=ExpnType_optimal then
        if debug then
             print("ExpnType_result<=ExpnType_optimal");
        fi;
        if is_contains_complex(result) then
            if is_contains_complex(optimal) then
                if debug then
                        print("both result and optimal complex");
                fi;
                if leaf_count_result<=2*leaf_count_optimal then
                    return "A"," ";
                else
                   return "B",cat("Both result and optimal contain complex but leaf count of
                                  convert(leaf_count_result,string)," vs. $2 (",
                                  convert(leaf_count_optimal,string)," ) = ",convert(2*leaf
                end if
            else #result contains complex but optimal is not
                if debug then
                        print("result contains complex but optimal is not");
                fi;
                return "C","Result contains complex when optimal does not.";
            fi;
        else # result do not contain complex
             # this assumes optimal do not as well. No check is needed here.
            if debug then
                   print("result do not contain complex, this assumes optimal do not as well
            fi;
```

```
            if leaf_count_result<=2*leaf_count_optimal then
                if debug then
                    print("leaf_count_result<=2*leaf_count_optimal");
                fi;
                return "A"," ";
            else
                if debug then
                    print("leaf_count_result>2*leaf_count_optimal");
                fi;
                return "B",cat("Leaf count of result is larger than twice the leaf count of
                                   convert(leaf_count_result,string),"$ vs. $2(",
                                   convert(leaf_count_optimal,string),")=",convert(2*leaf_co
            fi;
        fi;
    else #ExpnType(result) > ExpnType(optimal)
        if debug then
            print("ExpnType(result) > ExpnType(optimal)");
        fi;
        return "C",cat("Result contains higher order function than in optimal. Order ",
                       convert(ExpnType_result,string)," vs. order ",
                       convert(ExpnType_optimal,string),".");
    fi;

end proc:

#
# is_contains_complex(result)
# takes expressions and returns true if it contains "I" else false
#
#Nasser 032417
is_contains_complex:= proc(expression)
  return (has(expression,I));
end proc:

# The following summarizes the type number assigned an expression
# based on the functions it involves
# 1 = rational function
# 2 = algebraic function
# 3 = elementary function
# 4 = special function
# 5 = hyperpergeometric function
```

```
# 6 = appell function
# 7 = rootsum function
# 8 = integrate function
# 9 = unknown function

ExpnType := proc(expn)
  if type(expn,'atomic') then
    1
  elif type(expn,'list') then
    apply(max,map(ExpnType,expn))
  elif type(expn,'sqrt') then
    if type(op(1,expn),'rational') then
       1
    else
       max(2,ExpnType(op(1,expn)))
    end if
  elif type(expn,'`^`') then
    if type(op(2,expn),'integer') then
      ExpnType(op(1,expn))
    elif type(op(2,expn),'rational') then
      if type(op(1,expn),'rational') then
         1
      else
         max(2,ExpnType(op(1,expn)))
      end if
    else
         max(3,ExpnType(op(1,expn)),ExpnType(op(2,expn)))
    end if
  elif type(expn,'`+`') or type(expn,'`*`') then
    max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
  elif ElementaryFunctionQ(op(0,expn)) then
    max(3,ExpnType(op(1,expn)))
  elif SpecialFunctionQ(op(0,expn)) then
    max(4,apply(max,map(ExpnType,[op(expn)])))
  elif HypergeometricFunctionQ(op(0,expn)) then
    max(5,apply(max,map(ExpnType,[op(expn)])))
  elif AppellFunctionQ(op(0,expn)) then
    max(6,apply(max,map(ExpnType,[op(expn)])))
  elif op(0,expn)='int' then
    max(8,apply(max,map(ExpnType,[op(expn)]))) else
  9
```

```
  end if
end proc:


ElementaryFunctionQ := proc(func)
  member(func,[
        exp,log,ln,
        sin,cos,tan,cot,sec,csc,
        arcsin,arccos,arctan,arccot,arcsec,arccsc,
        sinh,cosh,tanh,coth,sech,csch,
        arcsinh,arccosh,arctanh,arccoth,arcsech,arccsch])
end proc:

SpecialFunctionQ := proc(func)
  member(func,[
        erf,erfc,erfi,
        FresnelS,FresnelC,
        Ei,Ei,Li,Si,Ci,Shi,Chi,
        GAMMA,lnGAMMA,Psi,Zeta,polylog,dilog,LambertW,
        EllipticF,EllipticE,EllipticPi])
end proc:

HypergeometricFunctionQ := proc(func)
  member(func,[Hypergeometric1F1,hypergeom,HypergeometricPFQ])
end proc:

AppellFunctionQ := proc(func)
  member(func,[AppellF1])
end proc:

# u is a sum or product.  rest(u) returns all but the
# first term or factor of u.
rest := proc(u) local v;
  if nops(u)=2 then
     op(2,u)
  else
     apply(op(0,u),op(2..nops(u),u))
  end if
end proc:

#leafcount(u) returns the number of nodes in u.
```

```
#Nasser 3/23/17  Replaced by build-in leafCount from package in Maple
leafcount := proc(u)
   MmaTranslator[Mma][LeafCount](u);
end proc:
```

Sympy grading function

```
#Dec 24, 2019. Nasser M. Abbasi:
#              Port of original Maple grading function by
#              Albert Rich to use with Sympy/Python
#Dec 27, 2019  Nasser. Added `RootSum`. See problem 177, Timofeev file
#              added 'exp_polar'
from sympy import *

def leaf_count(expr):
   #sympy do not have leaf count function. This is approximation
   return round(1.7*count_ops(expr))

def is_sqrt(expr):
   if isinstance(expr,Pow):
      if expr.args[1] == Rational(1,2):
         return True
      else:
         return False
   else:
      return False

def is_elementary_function(func):
   return func in [exp,log,ln,sin,cos,tan,cot,sec,csc,
         asin,acos,atan,acot,asec,acsc,sinh,cosh,tanh,coth,sech,csch,
         asinh,acosh,atanh,acoth,asech,acsch
      ]

def is_special_function(func):
   return func in [ erf,erfc,erfi,
          fresnels,fresnelc,Ei,Ei,Li,Si,Ci,Shi,Chi,
          gamma,loggamma,digamma,zeta,polylog,LambertW,
          elliptic_f,elliptic_e,elliptic_pi,exp_polar
       ]
```

```
def is_hypergeometric_function(func):
    return func in [hyper]

def is_appell_function(func):
    return func in [appellf1]

def is_atom(expn):
    try:
        if expn.isAtom or isinstance(expn,int) or isinstance(expn,float):
          return True
        else:
          return False

    except AttributeError as error:
        return False

def expnType(expn):
    debug=False
    if debug:
        print("expn=",expn,"type(expn)=",type(expn))

    if is_atom(expn):
      return 1
    elif isinstance(expn,list):
        return max(map(expnType, expn))   #apply(max,map(ExpnType,expn))
    elif  is_sqrt(expn):
        if isinstance(expn.args[0],Rational): #type(op(1,expn),'rational')
            return 1
        else:
            return max(2,expnType(expn.args[0]))  #max(2,ExpnType(op(1,expn)))
    elif isinstance(expn,Pow):   #type(expn,'`^`')
        if isinstance(expn.args[1],Integer):  #type(op(2,expn),'integer')
            return expnType(expn.args[0])   #ExpnType(op(1,expn))
        elif isinstance(expn.args[1],Rational):  #type(op(2,expn),'rational')
            if isinstance(expn.args[0],Rational): #type(op(1,expn),'rational')
                return 1
            else:
                return max(2,expnType(expn.args[0]))  #max(2,ExpnType(op(1,expn)))
        else:
            return max(3,expnType(expn.args[0]),expnType(expn.args[1])) #max(3,ExpnType(op(1,expn)),Expr
    elif isinstance(expn,Add) or isinstance(expn,Mul): #type(expn,'`+`') or type(expn,'`*`')
```

```
        m1 = expnType(expn.args[0])
        m2 = expnType(list(expn.args[1:]))
        return max(m1,m2)  #max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
    elif is_elementary_function(expn.func):  #ElementaryFunctionQ(op(0,expn))
        return max(3,expnType(expn.args[0]))  #max(3,ExpnType(op(1,expn)))
    elif is_special_function(expn.func): #SpecialFunctionQ(op(0,expn))
        m1 = max(map(expnType, list(expn.args)))
        return max(4,m1)  #max(4,apply(max,map(ExpnType,[op(expn)])))
    elif is_hypergeometric_function(expn.func): #HypergeometricFunctionQ(op(0,expn))
        m1 = max(map(expnType, list(expn.args)))
        return max(5,m1)  #max(5,apply(max,map(ExpnType,[op(expn)])))
    elif is_appell_function(expn.func):
        m1 = max(map(expnType, list(expn.args)))
        return max(6,m1)  #max(5,apply(max,map(ExpnType,[op(expn)])))
    elif isinstance(expn,RootSum):
        m1 = max(map(expnType, list(expn.args))) #Apply[Max,Append[Map[ExpnType,Apply[List,expn]],7]],
        return max(7,m1)
    elif str(expn).find("Integral") != -1:
        m1 = max(map(expnType, list(expn.args)))
        return max(8,m1)  #max(5,apply(max,map(ExpnType,[op(expn)])))
    else:
        return 9

#main function
def grade_antiderivative(result,optimal):

    #print ("Enter grade_antiderivative for sagemath")
    #print("Enter grade_antiderivative, result=",result," optimal=",optimal)

    leaf_count_result  = leaf_count(result)
    leaf_count_optimal = leaf_count(optimal)

    #print("leaf_count_result=",leaf_count_result)
    #print("leaf_count_optimal=",leaf_count_optimal)

    expnType_result  = expnType(result)
    expnType_optimal = expnType(optimal)

    if str(result).find("Integral") != -1:
        grade = "F"
        grade_annotation =""
```

```
else:
    if expnType_result <= expnType_optimal:
        if result.has(I):
            if optimal.has(I): #both result and optimal complex
                if leaf_count_result <= 2*leaf_count_optimal:
                    grade = "A"
                    grade_annotation =""
                else:
                    grade = "B"
                    grade_annotation ="Both result and optimal contain complex but leaf count of result is lar
            else: #result contains complex but optimal is not
                grade = "C"
                grade_annotation ="Result contains complex when optimal does not."
        else: # result do not contain complex, this assumes optimal do not as well
            if leaf_count_result <= 2*leaf_count_optimal:
                grade = "A"
                grade_annotation =""
            else:
                grade = "B"
                grade_annotation ="Leaf count of result is larger than twice the leaf count of optimal. "+str(
    else:
        grade = "C"
        grade_annotation ="Result contains higher order function than in optimal. Order "+str(ExpnType


#print("Before returning. grade=",grade, " grade_annotation=",grade_annotation)

return grade, grade_annotation
```

SageMath grading function

```
#Dec 24, 2019. Nasser: Ported original Maple grading function by
#              Albert Rich to use with Sagemath. This is used to
#              grade Fricas, Giac and Maxima results.
#Dec 24, 2019. Nasser: Added 'exp_integral_e' and 'sng', 'sin_integral'
#              'arctan2','floor','abs','log_integral'
#June 4, 2022  Made default grade_annotation "none" instead of "" due
#              issue later when reading the file.
#July 14, 2022. Added ellipticF. This is until they fix sagemath, then remove it.
```

```
from sage.all import *
from sage.symbolic.operators import add_vararg, mul_vararg

debug=False;

def tree_size(expr):
    r"""
    Return the tree size of this expression.
    """
    #print("Enter tree_size, expr is ",expr)

    if expr not in SR:
        # deal with lists, tuples, vectors
        return 1 + sum(tree_size(a) for a in expr)
    expr = SR(expr)
    x, aa = expr.operator(), expr.operands()
    if x is None:
        return 1
    else:
        return 1 + sum(tree_size(a) for a in aa)

def is_sqrt(expr):
    if expr.operator() == operator.pow:   #isinstance(expr,Pow):
        if expr.operands()[1]==1/2: #expr.args[1] == Rational(1,2):
            if debug: print ("expr is sqrt")
            return True
        else:
            return False
    else:
        return False

def is_elementary_function(func):
    #debug=False
    m = func.name() in ['exp','log','ln',
            'sin','cos','tan','cot','sec','csc',
            'arcsin','arccos','arctan','arccot','arcsec','arccsc',
            'sinh','cosh','tanh','coth','sech','csch',
            'arcsinh','arccosh','arctanh','arccoth','arcsech','arccsch','sgn',
        'arctan2','floor','abs'
        ]
    if debug:
```

```
        if m:
            print ("func ", func , " is elementary_function")
        else:
            print ("func ", func , " is NOT elementary_function")


    return m

def is_special_function(func):
    #debug=False
    if debug:
        print ("type(func)=", type(func))

    m= func.name() in ['erf','erfc','erfi','fresnel_sin','fresnel_cos','Ei',
            'Ei','Li','Si','sin_integral','Ci','cos_integral','Shi','sinh_integral'
            'Chi','cosh_integral','gamma','log_gamma','psi','zeta',
            'polylog','lambert_w','elliptic_f','elliptic_e','ellipticF',
            'elliptic_pi','exp_integral_e','log_integral',
            'weierstrassPInverse','weierstrass','weierstrassP','weierstrassZeta',
            'weierstrassPPrime','weierstrassSigma']

    if debug:
        print ("m=",m)
        if m:
            print ("func ", func ," is special_function")
        else:
            print ("func ", func ," is NOT special_function")


    return m


def is_hypergeometric_function(func):
    return func.name() in ['hypergeometric','hypergeometric_M','hypergeometric_U']

def is_appell_function(func):
    return func.name() in ['hypergeometric']   #[appellf1] can't find this in sagemath

def is_atom(expn):

    #debug=False
```

```
    if debug:
         print ("Enter is_atom, expn=",expn)

    if not hasattr(expn, 'parent'):
        return False


    #thanks to answer at https://ask.sagemath.org/question/49179/what-is-sagemath-equivalent-to-atomic
    try:
        if expn.parent() is SR:
            return expn.operator() is None
        if expn.parent() in (ZZ, QQ, AA, QQbar):
            return expn in expn.parent() # Should always return True
        if hasattr(expn.parent(),"base_ring") and hasattr(expn.parent(),"gens"):
            return expn in expn.parent().base_ring() or expn in expn.parent().gens()

        return False

    except AttributeError as error:
        print("Exception,AttributeError in  is_atom")
        print ("cought exception" , type(error).__name__ )
        return False


def expnType(expn):

    if debug:
        print (">>>>>Enter expnType, expn=", expn)
        print (">>>>>is_atom(expn)=", is_atom(expn))

    if is_atom(expn):
        return 1
    elif type(expn)==list:   #isinstance(expn,list):
        return max(map(expnType, expn))   #apply(max,map(ExpnType,expn))
    elif  is_sqrt(expn):
        if  type(expn.operands()[0])==Rational: #type(isinstance(expn.args[0],Rational):
            return 1
        else:
            return max(2,expnType(expn.operands()[0]))  #max(2,expnType(expn.args[0]))
    elif expn.operator() == operator.pow:   #isinstance(expn,Pow)
        if type(expn.operands()[1])==Integer:  #isinstance(expn.args[1],Integer)
```

```
            return expnType(expn.operands()[0])    #expnType(expn.args[0])
        elif type(expn.operands()[1])==Rational:  #isinstance(expn.args[1],Rational)
            if type(expn.operands()[0])==Rational: #isinstance(expn.args[0],Rational)
                return 1
            else:
                return max(2,expnType(expn.operands()[0]))  #max(2,expnType(expn.args[0]))
        else:
            return max(3,expnType(expn.operands()[0]),expnType(expn.operands()[1])) #max(3,expnType(expn
    elif expn.operator() == add_vararg or expn.operator() == mul_vararg: #isinstance(expn,Add) or isins
        m1 = expnType(expn.operands()[0]) #expnType(expn.args[0])
        m2 = expnType(expn.operands()[1:]) #expnType(list(expn.args[1:]))
        return max(m1,m2)  #max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
    elif is_elementary_function(expn.operator()):  #is_elementary_function(expn.func)
        return max(3,expnType(expn.operands()[0]))
    elif is_special_function(expn.operator()): #is_special_function(expn.func)
        m1 = max(map(expnType, expn.operands()))      #max(map(expnType, list(expn.args)))
        return max(4,m1)   #max(4,m1)
    elif is_hypergeometric_function(expn.operator()): #is_hypergeometric_function(expn.func)
        m1 = max(map(expnType, expn.operands()))       #max(map(expnType, list(expn.args)))
        return max(5,m1)   #max(5,m1)
    elif is_appell_function(expn.operator()):
        m1 = max(map(expnType, expn.operands()))       #max(map(expnType, list(expn.args)))
        return max(6,m1)   #max(6,m1)
    elif str(expn).find("Integral") != -1: #this will never happen, since it
            #is checked before calling the grading function that is passed.
            #but kept it here.
        m1 = max(map(expnType, expn.operands()))       #max(map(expnType, list(expn.args)))
        return max(8,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
    else:
        return 9

#main function
def grade_antiderivative(result,optimal):


    if debug:
        print ("Enter grade_antiderivative for sagemath")
        print("Enter grade_antiderivative, result=",result)
        print("Enter grade_antiderivative, optimal=",optimal)
        print("type(anti)=",type(result))
        print("type(optimal)=",type(optimal))
```

```
leaf_count_result  = tree_size(result) #leaf_count(result)
leaf_count_optimal = tree_size(optimal) #leaf_count(optimal)

#if debug: print ("leaf_count_result=", leaf_count_result, "leaf_count_optimal=",leaf_count_optimal)


expnType_result  = expnType(result)
expnType_optimal = expnType(optimal)

if debug: print ("expnType_result=", expnType_result, "expnType_optimal=",expnType_optimal)

if expnType_result <= expnType_optimal:
    if result.has(I):
        if optimal.has(I): #both result and optimal complex
            if leaf_count_result <= 2*leaf_count_optimal:
                grade = "A"
                grade_annotation ="none"
            else:
                grade = "B"
                grade_annotation ="Both result and optimal contain complex but leaf count of result is larger
        else: #result contains complex but optimal is not
            grade = "C"
            grade_annotation ="Result contains complex when optimal does not."
    else: # result do not contain complex, this assumes optimal do not as well
        if leaf_count_result <= 2*leaf_count_optimal:
            grade = "A"
            grade_annotation ="none"
        else:
            grade = "B"
            grade_annotation ="Leaf count of result is larger than twice the leaf count of optimal. "+str(lea
else:
    grade = "C"
    grade_annotation ="Result contains higher order function than in optimal. Order "+str(expnType_re


print("Before returning. grade=",grade, " grade_annotation=",grade_annotation)

return grade, grade_annotation
```

4.2 Links to plain text integration problems used in this report for each CAS

1. Mathematica integration problems as .m file
2. Maple integration problems as .txt file
3. Sagemath integration problems as .sage file
4. Reduce integration problems as .txt file
5. Mupad integration problems as .txt file
6. Sympy integration problems as .py file