Computer Algebra
Independent Integration Tests

Summer 2024

6-Hyperbolic-functions/6.1-Hyperbolic-sine/298-6.1.7.2

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Contents

CHAPTER 1

Introduction

This report gives the result of running the computer algebra independent integration test.The download section in on the main webpage contains links to download the problems in plain text format used for all CAS systems. The number of integrals in this report is [**489**]. This is test number [298].

1.1 Listing of CAS systems tested

The following are the CAS systems tested:

1. Mathematica 14 (January 9, 2024) on windows 10 pro.
2. Rubi 4.17.3 (Sept 25, 2023) on Mathematica 14 on windows 10m pro.
3. Maple 2024 (March 1, 2024) on windows 10 pro.
4. Maxima 5.47 (June 1, 2023) using Lisp SBCL 2.4.0 on Linux Manjaro 23.1.2 KDE via sagemath 10.3.
5. FriCAS 1.3.10 built with sbcl 2.3.11 (January 10, 2024) on Linux Manjaro 23.1.2 KDE via sagemath 10.3.
6. Giac/Xcas 1.9.0-99 on Linux via sagemath 10.3.
7. Sympy 1.12 using Python 3.11.6 (Nov 14 2023, 09:36:21) [GCC 13.2.1 20230801] on Linux Manjaro 23.1.2 KDE.
8. Mupad using Matlab 2021a with Symbolic Math Toolbox Version 8.7 on windows 10.
9. Reduce CSL rev 6687 (January 9, 2024) on Linux Manjaro 23.1.2 KDE.

Maxima and Fricas and Giac are called using Sagemath. This was done using Sagemath `integrate` command by changing the name of the algorithm to use the different CAS systems.

Sympy was run directly in Python not via sagemath.

Reduce was called directly.

1.2 Results

Important note: A number of problems in this test suite have no antiderivative in closed form. This means the antiderivative of these integrals can not be expressed in terms of elementary, special functions or `Hypergeometric2F1` functions. `RootSum` and `RootOf` are not allowed. If a CAS returns the above integral unevaluated within the time limit, then the result is counted as passed and assigned an A grade.

However, if CAS times out, then it is assigned an F grade even if the integral is not integrable, as this implies CAS could not determine that the integral is not integrable in the time limit.

If a CAS returns an antiderivative to such an integral, it is assigned an A grade automatically and this special result is listed in the introduction section of each individual test report to make it easy to identify as this can be important result to investigate.

The results given in in the table below reflects the above.

System	% solved	% Failed
Rubi	100.00 (489)	0.00 (0)
Mathematica	95.30 (466)	4.70 (23)
Maple	92.23 (451)	7.77 (38)
Fricas	85.48 (418)	14.52 (71)
Mupad	47.24 (231)	52.76 (258)
Giac	45.81 (224)	54.19 (265)
Reduce	43.56 (213)	56.44 (276)
Maxima	37.42 (183)	62.58 (306)
Sympy	13.70 (67)	86.30 (422)

Table 1.1: Percentage solved for each CAS

The table below gives additional break down of the grading of quality of the antiderivatives generated by each CAS. The grading is given using the letters A,B,C and F with A being the best quality. The grading is accomplished by comparing the antiderivative generated with the optimal antiderivatives included in the test suite. The following table describes the meaning of these grades.

grade	description
A	Integral was solved and antiderivative is optimal in quality and leaf size.
B	Integral was solved and antiderivative is optimal in quality but leaf size is larger than twice the optimal antiderivatives leaf size.
C	Integral was solved and antiderivative is non-optimal in quality. This can be due to one or more of the following reasons 1. antiderivative contains a hypergeometric function and the optimal antiderivative does not. 2. antiderivative contains a special function and the optimal antiderivative does not. 3. antiderivative contains the imaginary unit and the optimal antiderivative does not.
F	Integral was not solved. Either the integral was returned unevaluated within the time limit, or it timed out, or CAS hanged or crashed or an exception was raised.

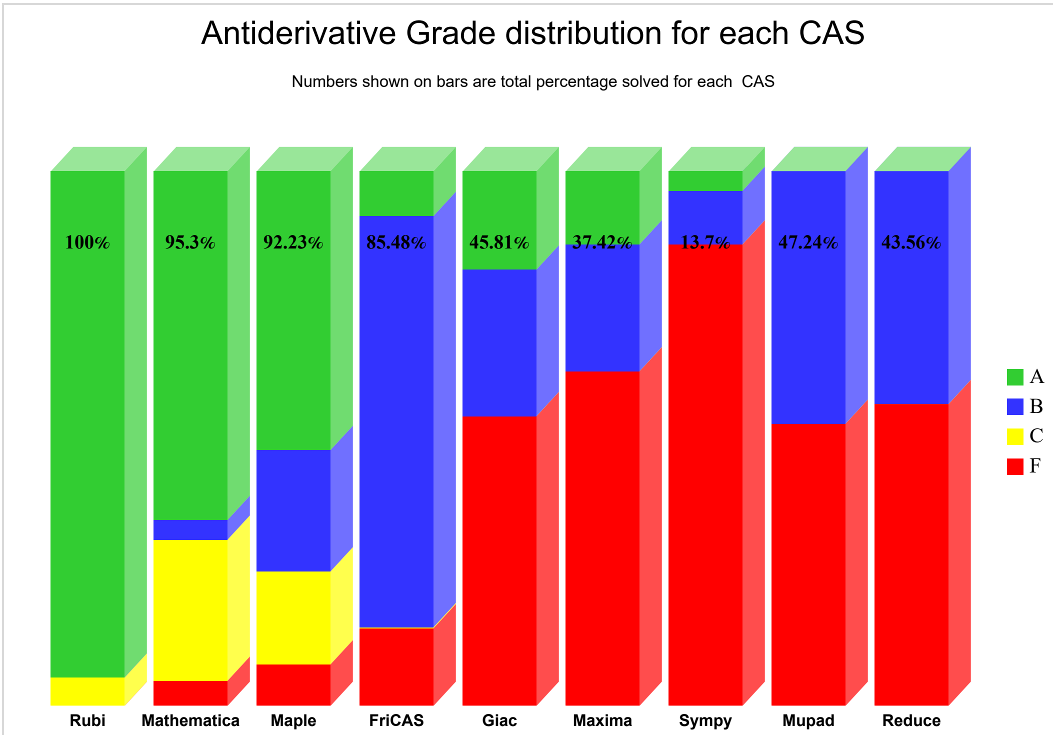
Table 1.2: Description of grading applied to integration result

Grading is implemented for all CAS systems. Based on the above, the following table summarizes the grading for this test suite.

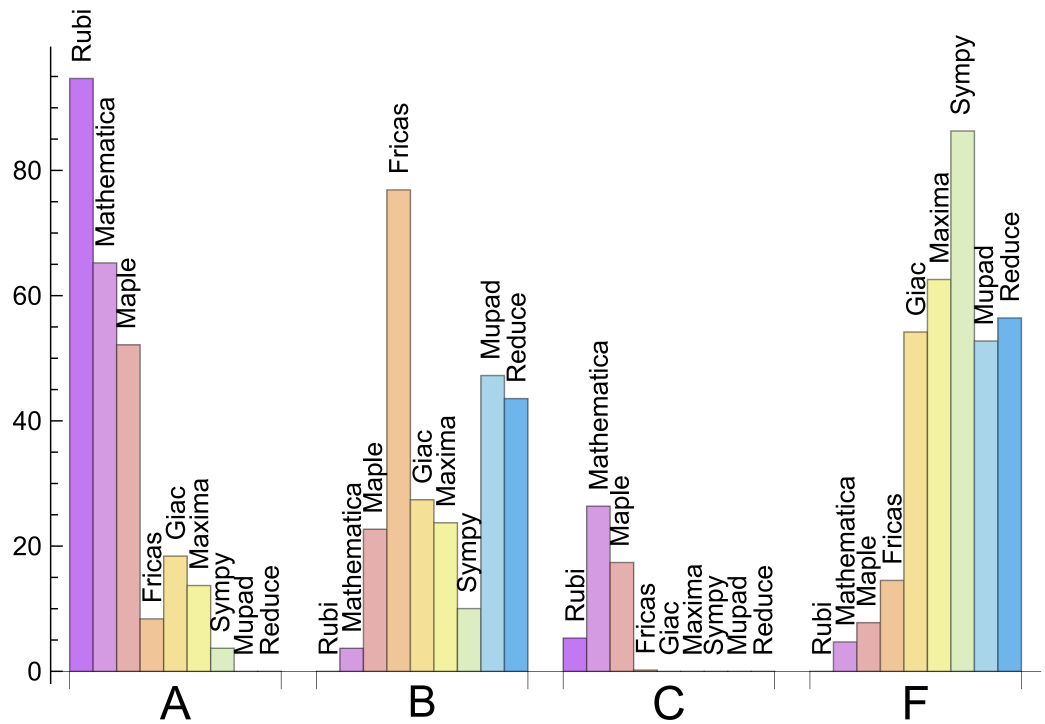
System	% A grade	% B grade	% C grade	% F grade
Rubi	94.683	0.000	5.317	0.000
Mathematica	65.235	3.681	26.380	4.703
Maple	52.147	22.699	17.382	7.771
Giac	18.405	27.403	0.000	54.192
Maxima	13.701	23.722	0.000	62.577
Fricas	8.384	76.892	0.204	14.519
Sympy	3.681	10.020	0.000	86.299
Mupad	0.000	47.239	0.000	52.761
Reduce	0.000	43.558	0.000	56.442

Table 1.3: Antiderivative Grade distribution of each CAS

The following is a Bar chart illustration of the data in the above table.



The figure below compares the grades of the CAS systems.



The following table shows the distribution of the different types of failures for each CAS. There are 3 types failures. The first is when CAS returns the input within the time limit, which means it could not solve it. This is the typical failure and given as **F**.

The second failure is due to time out. CAS could not solve the integral within the 3 minutes time limit which is assigned. This is assigned **F(-1)**.

The third is due to an exception generated, indicated as **F(-2)**. This most likely indicates an interface problem between sagemath and the CAS (applicable only to FriCAS, Maxima and Giac) or it could be an indication of an internal error in the CAS itself. This type of error requires more investigation to determine the cause.

System	Number failed	Percentage normal failure	Percentage timeout failure	Percentage exception failure
Rubi	0	0.00	0.00	0.00
Mathematica	23	100.00	0.00	0.00
Maple	38	100.00	0.00	0.00
Fricas	71	98.59	0.00	1.41
Mupad	258	0.00	100.00	0.00
Giac	265	75.09	0.00	24.91
Reduce	276	100.00	0.00	0.00
Maxima	306	88.89	0.33	10.78
Sympy	422	38.15	61.85	0.00

Table 1.4: Failure statistics for each CAS

1.3 Time and leaf size Performance

The table below summarizes the performance of each CAS system in terms of time used and leaf size of results.

Mean size is the average leaf size produced by the CAS (before any normalization). The Normalized mean is relative to the mean size of the optimal anti-derivative given in the input files.

For example, if CAS has **Normalized mean** of 3, then the mean size of its leaf size is 3 times as large as the mean size of the optimal leaf size.

Median size is value of leaf size where half the values are larger than this and half are smaller (before any normalization). i.e. The Middle value.

Similarly the **Normalized median** is relative to the median leaf size of the optimal.

For example, if a CAS has Normalized median of 1.2, then its median is 1.2 as large as the median leaf size of the optimal.

System	Mean time (sec)
Maxima	0.08
Reduce	0.22
Giac	0.26
Fricas	0.29
Rubi	0.43
Mathematica	1.73
Sympy	1.83
Mupad	3.39
Maple	9.70

Table 1.5: Time performance for each CAS

System	Mean size	Normalized mean	Median size	Normalized median
Rubi	147.57	1.07	117.00	1.01
Mathematica	160.78	1.22	119.50	0.99
Maple	195.57	1.39	130.00	1.19
Giac	312.86	2.79	196.00	1.98
Maxima	352.67	3.71	217.00	2.38
Mupad	495.38	4.08	191.00	2.61
Sympy	537.42	9.98	212.00	2.48
Reduce	1557.84	12.14	363.00	3.96
Fricas	3834.03	22.22	1528.00	14.80

Table 1.6: Leaf size performance for each CAS

1.4 Performance based on number of rules Rubi used

This section shows how each CAS performed based on the number of rules Rubi needed to solve the same integral. One diagram is given for each CAS.

On the y axis is the percentage solved which Rubi itself needed the number of rules given the x axis. These plots show that as more rules are needed then most CAS system percentage of solving decreases which indicates the integral is becoming more complicated to solve.

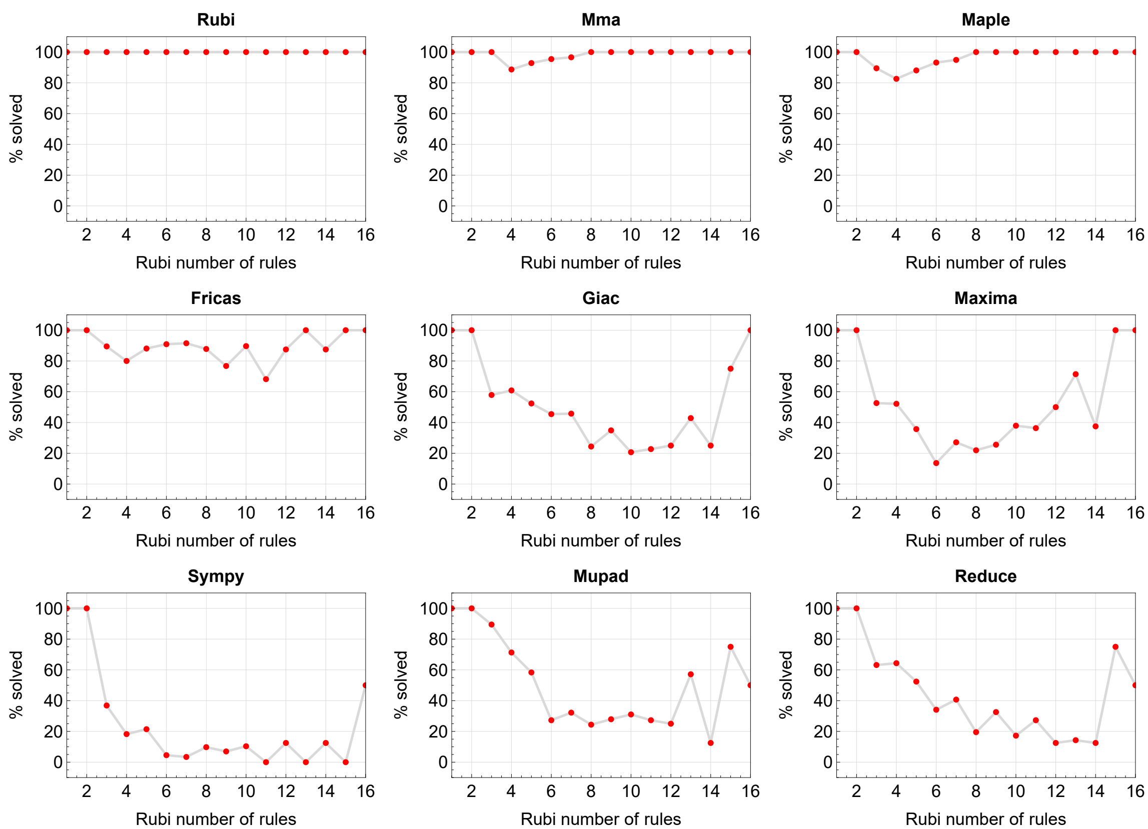


Figure 1.1: Solving statistics per number of Rubi rules used

1.5 Performance based on number of steps Rubi used

This section shows how each CAS performed based on the number of steps Rubi needed to solve the same integral. Note that the number of steps Rubi needed can be much higher than the number of rules, as the same rule could be used more than once.

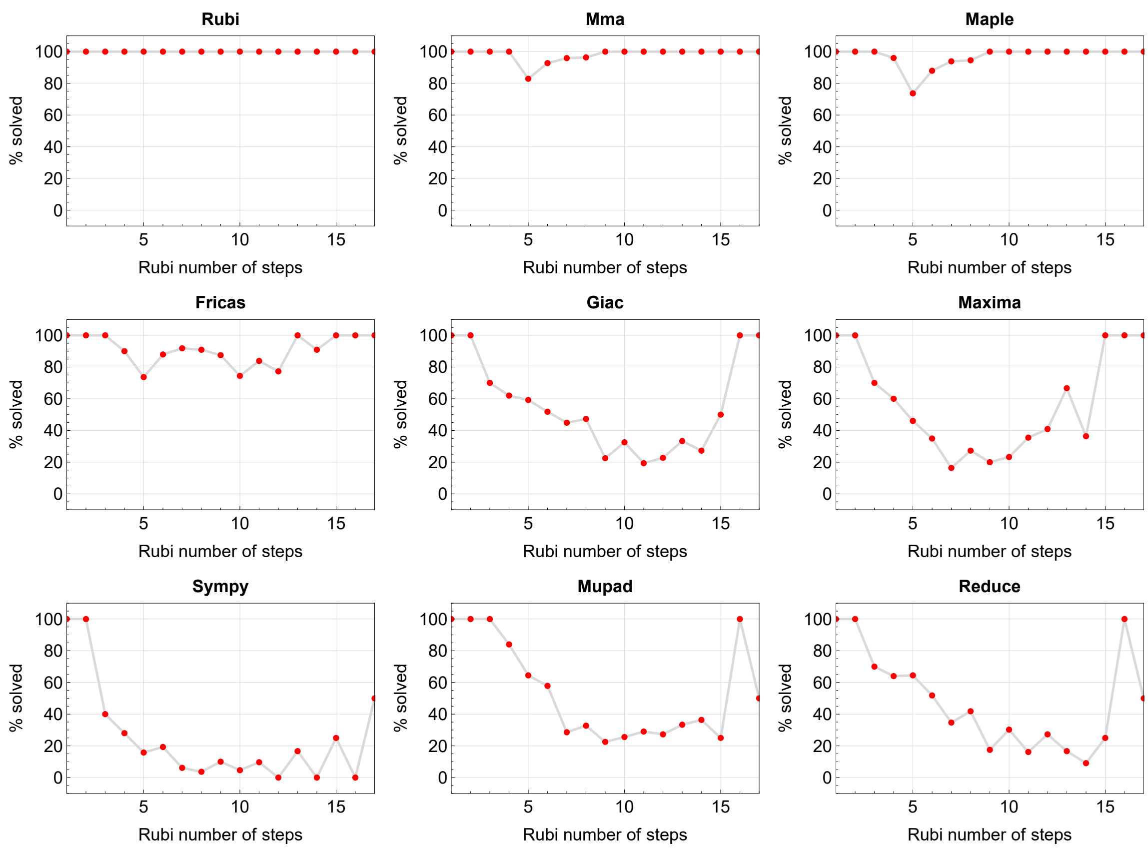


Figure 1.2: Solving statistics per number of Rubi steps used

The above diagram show that the precentage of solved intergals decreases for most CAS systems as the number of steps increases. As expected, for integrals that required less steps by Rubi, CAS systems had more success which indicates the integral was not as hard to solve. As Rubi needed more steps to solve the integral, the solved percentage decreased for most CAS systems which indicates the integral is becoming harder to solve.

1.6 Solved integrals histogram based on leaf size of result

The following shows the distribution of solved integrals for each CAS system based on leaf size of the antiderivatives produced by each CAS. It shows that most integrals solved produced leaf size less than about 100 to 150. The bin size used is 40.

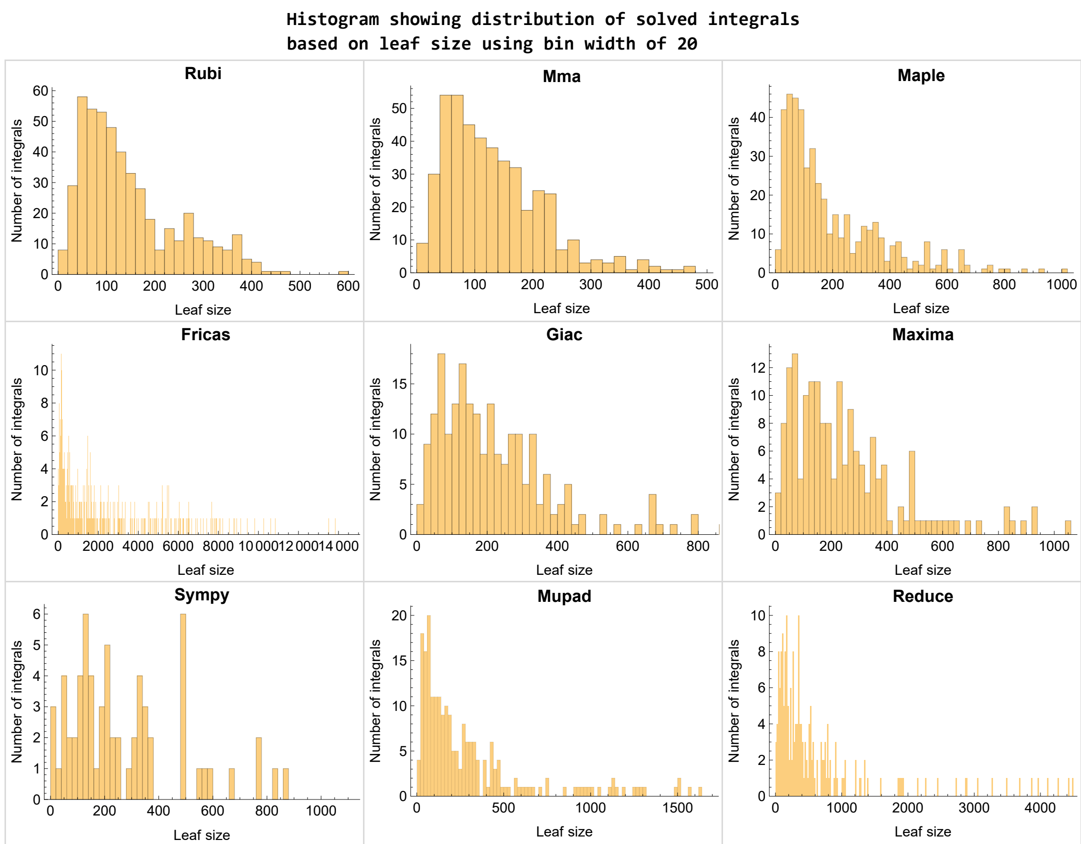


Figure 1.3: Solved integrals based on leaf size distribution

1.7 Solved integrals histogram based on CPU time used

The following shows the distribution of solved integrals for each CAS system based on CPU time used in seconds. The bin size used is 0.1 second.

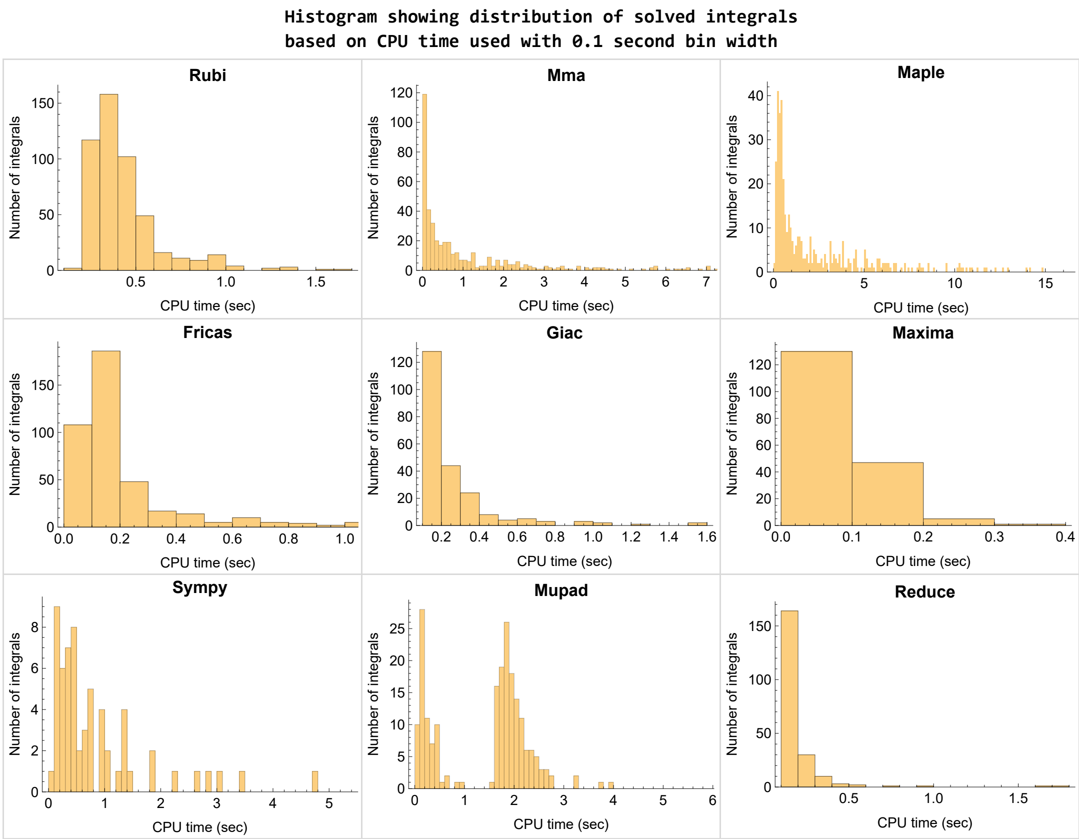


Figure 1.4: Solved integrals histogram based on CPU time used

1.8 Leaf size vs. CPU time used

The following shows the relation between the CPU time used to solve an integral and the leaf size of the antiderivative.

The result for Fricas, Maxima and Giac is shifted more to the right than the other CAS system due to the use of sagemath to call them, which causes an initial slight delay in the timing to start the integration due to overhead of starting a new process each time. This should also be taken into account when looking at the timing of these three CAS systems. Direct calls not using sagemath would result in faster timings, but current implementation uses sagemath as this makes testing much easier to do.

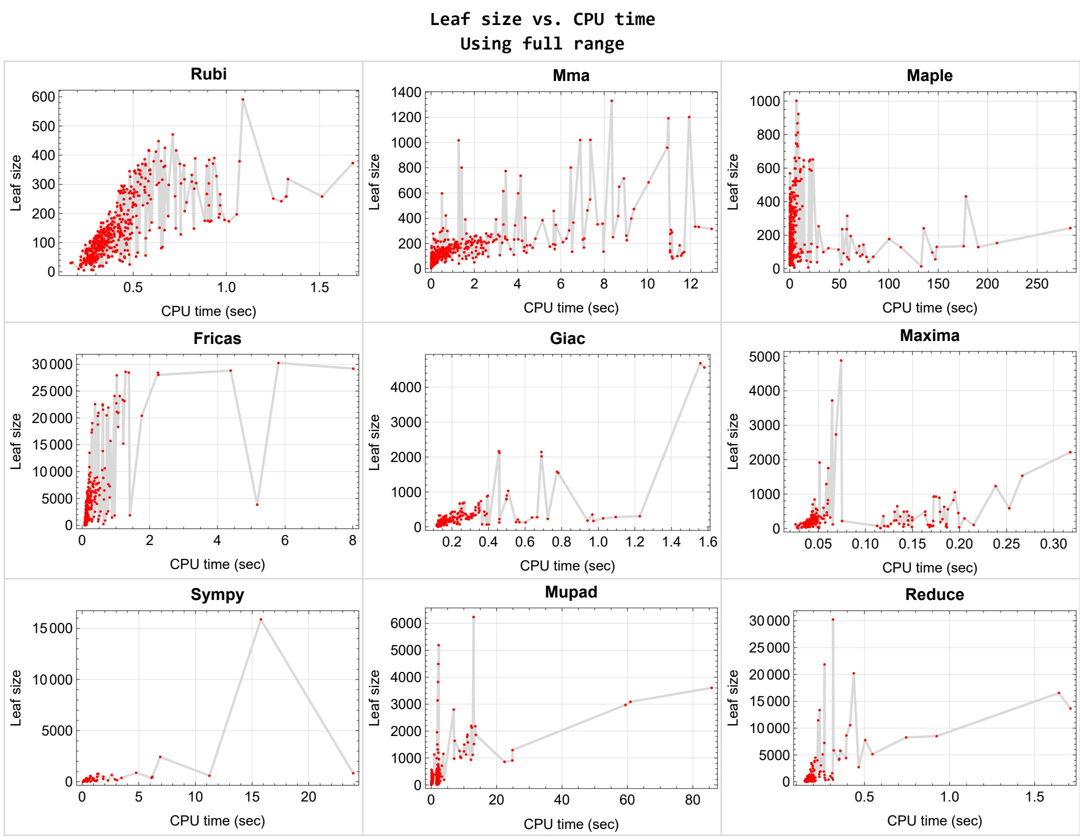


Figure 1.5: Leaf size vs. CPU time. Full range

1.9 list of integrals with no known antiderivative

{}

1.10 List of integrals solved by CAS but has no known antiderivative

Rubi {}

Mathematica {}

Maple {}

Maxima {}

Fricas {}

Sympy {}

Giac {}

Reduce {}

Mupad {}

1.11 list of integrals solved by CAS but failed verification

The following are integrals solved by CAS but the verification phase failed to verify the anti-derivative produced is correct. This does not necessarily mean that the anti-derivative is wrong as additional methods of verification might be needed, or more time is needed (3 minutes time limit was used). These integrals are listed here to make it possible to do further investigation to determine why the result could not be verified.

Rubi {}

Mathematica {263, 265, 267, 274, 276, 278, 320, 331, 340, 348, 349, 358}

Maple {}

Maxima Verification phase not currently implemented.

Fricas Verification phase not currently implemented.

Sympy Verification phase not currently implemented.

Giac Verification phase not currently implemented.

Reduce Verification phase not currently implemented.

Mupad Verification phase not currently implemented.

1.12 Timing

The command `AbsoluteTiming[]` was used in Mathematica to obtain the elapsed time for each integrate call. In Maple, the command `Usage` was used as in the following example

```
cpu_time := Usage(assign ('result_of_int',int(expr,x)),output='realtime'
```

For all other CAS systems, the elapsed time to complete each integral was found by taking the difference between the time after the call completed from the time before the call was made. This was done using Python's `time.time()` call.

All elapsed times shown are in seconds. A time limit of 3 CPU minutes was used for each integral. If the integrate command did not complete within this time limit, the integral was aborted and considered to have failed and assigned an F grade. The time used by failed integrals due to time out was not counted in the final statistics.

1.13 Verification

A verification phase was applied on the result of integration for `Rubi` and `Mathematica`.

Future version of this report will implement verification for the other CAS systems. For the integrals whose result was not run through a verification phase, it is assumed that the antiderivative was correct.

Verification phase also had 3 minutes time out. An integral whose result was not verified could still be correct, but further investigation is needed on those integrals. These integrals were marked in the summary table below and also in each integral separate section so they are easy to identify and locate.

1.14 Important notes about some of the results

Important note about Maxima results

Since tests were run in a batch mode, and using an automated script, then any integral where Maxima needed an interactive response from the user to answer a question during the evaluation of the integral will fail.

The exception raised is `ValueError`. Therefore Maxima results is lower than what would result if Maxima was run directly and each question was answered correctly.

The percentage of such failures were not counted for each test file, but for an example, for the `Timofeev` test file, there were about 14 such integrals out of total 705, or about 2 percent. This percentage can be higher or lower depending on the specific input test file.

Such integrals can be identified by looking at the output of the integration in each section for Maxima. The exception message will indicate the cause of error.

Maxima integrate was run using SageMath with the following settings set by default

```
'besselexpand : true'
'display2d : false'
'domain : complex'
'keepfloat : true'
'load(to_poly_solve)'
'load(simplify_sum)'
'load(abs_integrate)' 'load(diag)'
```

SageMath automatic loading of Maxima `abs_integrate` was found to cause some problems. So the following code was added to disable this effect.

```
from sage.interfaces.maxima_lib import maxima_lib
maxima_lib.set('extra_definite_integration_methods', '[]')
maxima_lib.set('extra_integration_methods', '[]')
```

See `https://ask.sagemath.org/question/43088/integrate-results-that-are-different-from-using-maxima/` for reference.

Important note about FriCAS result

There were few integrals which failed due to SageMath interface and not because FriCAS system could not do the integration.

These will fail With error `Exception raised: NotImplementedError`.

The number of such cases seems to be very small. About 1 or 2 percent of all integrals. These can be identified by looking at the exception message given in the result.

Important note about finding leaf size of antiderivative

For Mathematica, Rubi, and Maple, the builtin system function `LeafSize` was used to find the leaf size of each antiderivative.

The other CAS systems (SageMath and Sympy) do not have special builtin function for this purpose at this time. Therefore the leaf size for Fricas and Sympy antiderivative was determined using the following function, thanks to user `slelievre` at `https://ask.sagemath.org/question/57123/could-we-have-a-leaf_count-function-in-base-sagemath/`

```
def tree_size(expr):
    r"""
    Return the tree size of this expression.
    """
    if expr not in SR:
        # deal with lists, tuples, vectors
        return 1 + sum(tree_size(a) for a in expr)
    expr = SR(expr)
    x, aa = expr.operator(), expr.operands()
    if x is None:
        return 1
    else:
        return 1 + sum(tree_size(a) for a in aa)
```

For Sympy, which was called directly from Python, the following code was used to obtain the leafsize of its result

```
try:
  # 1.7 is a fudge factor since it is low side from actual leaf count
  leafCount = round(1.7*count_ops(anti))

  except Exception as ee:
        leafCount =1
```

Important note about Mupad results

Matlab's symbolic toolbox does not have a leaf count function to measure the size of the antiderivative. Maple was used to determine the leaf size of Mupad output by post processing Mupad result.

Currently no grading of the antiderivative for Mupad is implemented. If it can integrate the problem, it was assigned a B grade automatically as a placeholder. In the future, when grading function is implemented for Mupad, the tests will be rerun again.

The following is an example of using Matlab's symbolic toolbox (Mupad) to solve an integral

```
integrand = evalin(symengine,'cos(x)*sin(x)')
the_variable = evalin(symengine,'x')
anti = int(integrand,the_variable)
```

Which gives `sin(x)^2/2`

1.15 Current tree layout of integration tests

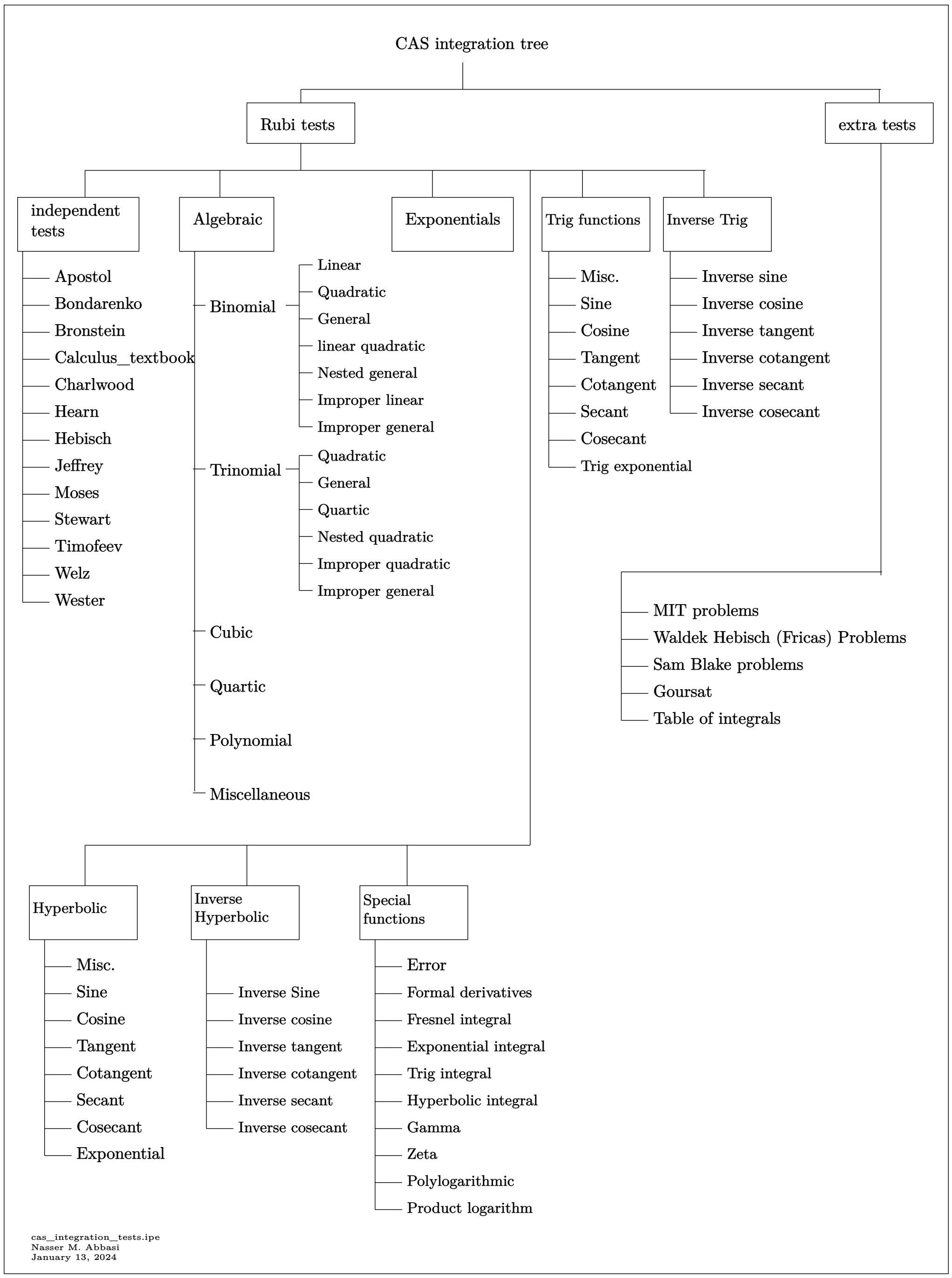
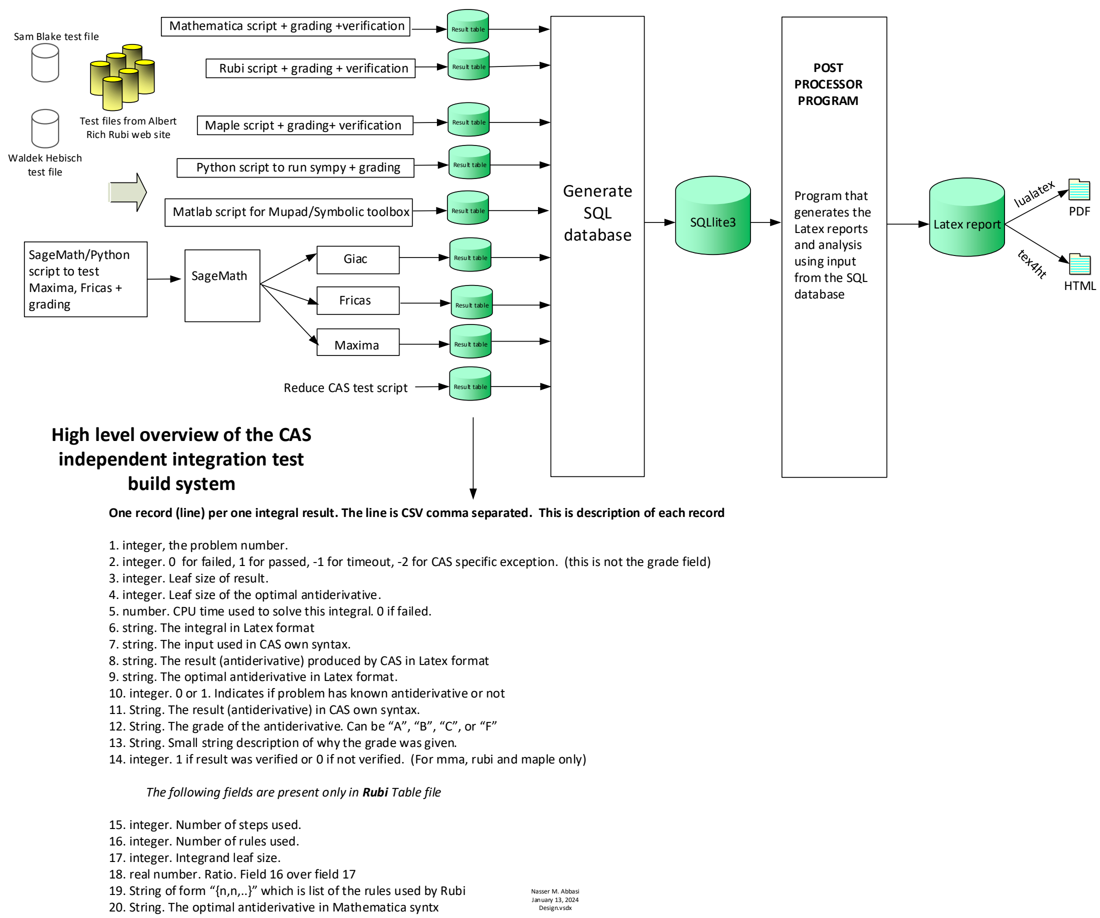


Figure 1.6: CAS integration tests tree

1.16 Design of the test system

The following diagram gives a high level view of the current test build system.



CHAPTER 2

DETAILED SUMMARY TABLES OF RESULTS

2.1 List of integrals sorted by grade for each CAS

Rubi

A grade { 1, 2, 3, 4, 5, 7, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 120, 122, 124, 126, 128, 130, 132, 134, 136, 138, 140, 142, 144, 146, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 196, 197, 198, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 219, 220, 221, 222, 223, 224, 225, 226, 227, 228, 229, 230, 231, 232, 233, 234, 235, 236, 237, 238, 239, 241, 242, 243, 244, 245, 246, 247, 248, 249, 250, 251, 252, 253, 254, 255, 256, 257, 258, 259, 260, 261, 262, 263, 264, 265, 266, 267, 268, 269, 270, 271, 272, 273, 274, 275, 276, 277, 278, 279, 280, 281, 282, 283, 284, 285, 286, 287, 288, 289, 290, 291, 292, 293, 294, 295, 296, 297, 298, 299, 300, 301, 302, 303, 304, 305, 306, 307, 308, 309, 310, 311, 312, 313, 314, 315, 316, 317, 318, 319, 320, 321, 322, 323, 324, 325, 326, 327, 328, 329, 330, 331, 332, 333, 334, 335, 336, 337, 338, 339, 340, 341, 342, 343, 344, 345, 346, 347, 348, 349, 350, 351, 352, 353, 354, 355, 356, 357, 358, 359, 360, 361, 362, 363, 364, 365, 366, 367, 368, 369, 370, 371, 372, 373, 374, 375, 376, 377, 378, 379, 380, 381, 382, 383, 384, 385, 386, 387, 388, 389, 390, 391, 392, 393, 394, 395, 396, 397, 401, 402, 403, 404, 405, 406, 407, 408, 411, 412, 413, 414, 415, 416, 418, 421, 422, 423, 424, 425, 426, 427, 428, 429, 430, 431, 432, 433, 434, 435, 436, 437, 438, 439, 440, 441, 442, 443, 444, 445, 446, 447, 448, 449, 450, 451, 452, 453, 454, 455, 456, 457, 458, 459, 460, 461, 462, 463, 464, 465, 466, 467, 468, 469, 470, 471, 472, 473, 474, 475, 476, 477, 478, 479, 480, 481, 482, 483, 484, 485, 486, 487, 488, 489 }

B grade { }

C grade { 6, 8, 119, 121, 123, 125, 127, 129, 131, 133, 135, 137, 139, 141, 143, 145, 147, 240, 398, 399, 400, 409, 410, 417, 419, 420 }

F normal fail { }

F(-1) timedout fail { }

F(-2) exception fail { }

Mma

A grade { 1, 2, 3, 4, 5, 6, 7, 9, 10, 11, 12, 13, 14, 15, 16, 19, 20, 21, 22, 23, 24, 25, 27, 29, 31, 33, 35, 37, 39, 41, 42, 44, 46, 48, 50, 51, 53, 55, 57, 59, 60, 61, 62, 63, 64, 66, 67, 70, 71, 72, 73, 74, 75, 77, 78, 81, 82, 83, 84, 86, 87, 90, 91, 92, 93, 96, 97, 99, 100, 101, 102, 105, 106, 111, 118, 119, 120, 121, 122, 123, 124, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 159, 160, 161, 162, 163, 164, 165, 167, 169, 171, 172, 173, 174, 175, 176, 177, 178, 179, 180, 182, 183, 184, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 196, 197, 198, 199, 210, 211, 212, 213, 214, 215, 222, 223, 224, 225, 226, 227, 234, 235, 236, 237, 238, 239, 240, 241, 242, 243, 244, 245, 246, 247, 248, 249, 250, 251, 252, 253, 254, 255, 256, 257, 258, 259, 260, 261, 262, 266, 268, 269, 270, 271, 272, 273, 275, 277, 280, 281, 282, 283, 284, 285, 286, 287, 288, 289, 290, 291, 292, 293, 294, 295, 296, 297, 298, 299, 300, 301, 302, 303, 304, 305, 306, 307, 308, 310, 311, 312, 313, 314, 315, 316, 317, 318, 323, 326, 327, 328, 329, 334, 337, 338, 339, 343, 346, 347, 353, 355, 356, 357, 362, 366, 367, 375, 376, 377, 379, 380, 381, 383, 384, 385, 386, 387, 388, 389, 390, 391, 392, 393, 395, 396, 397, 398, 399, 400, 401, 402, 403, 404, 405, 406, 407, 408, 409, 410, 411, 412, 413, 415, 416, 418, 419, 420, 421, 422, 423, 424, 425, 426, 429, 432, 433, 434, 435, 436, 437, 440, 443, 444, 445, 446, 447, 448, 451, 462, 473, 477, 478, 479, 480, 485, 486, 487, 488, 489 }

B grade { 8, 17, 18, 26, 125, 136, 166, 168, 170, 181, 200, 201, 202, 203, 264, 279, 309, 319 }

C grade { 28, 30, 32, 34, 36, 38, 40, 43, 45, 47, 49, 52, 54, 56, 58, 65, 68, 69, 76, 79, 80, 85, 88, 89, 94, 95, 98, 103, 104, 107, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 204, 205, 206, 207, 208, 209, 216, 217, 218, 219, 220, 221, 228, 229, 230, 231, 232, 233, 263, 265, 267, 274, 276, 278, 320, 321, 322, 324, 325, 330, 331, 332, 333, 335, 336, 340, 341, 342, 344, 345, 348, 349, 350, 351, 352, 354, 358, 359, 360, 361, 363, 378, 382, 394, 414, 417, 427, 428, 430, 431, 438, 439, 441, 442, 449, 450, 452, 453, 454, 455, 456, 457, 458, 459, 460, 461, 463, 464, 465, 466, 467, 468, 469, 470, 471, 472, 474, 475 }

F normal fail { 108, 109, 110, 112, 113, 114, 115, 116, 117, 364, 365, 368, 369, 370, 371, 372, 373, 374, 476, 481, 482, 483, 484 }

F(-1) timedout fail { }

F(-2) exception fail { }

Maple

A grade { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 32, 34, 36, 38, 40, 43, 47, 49, 58, 65, 68, 69, 79, 80, 81, 84, 85, 86, 87, 88, 89, 90, 91, 94, 95, 96, 98, 99, 100, 101, 102, 106, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 196, 197, 198, 199, 200, 201, 202, 203, 209, 220, 221, 240, 241, 242, 243, 244, 246, 247, 248, 249, 250, 251, 252, 253, 254, 255, 256, 257, 258, 259, 260, 261, 262, 267, 268, 269, 270, 271, 272, 273, 275, 280, 282, 284, 286, 294, 296, 298, 304, 306, 308, 314, 316, 317, 321, 322, 325, 326, 327, 333, 335, 336, 337, 338, 341, 342, 343, 344, 345, 346, 347, 350, 351, 352, 354, 355, 356, 357, 362, 375, 376, 379, 380, 389, 390, 391, 392, 393, 394, 395, 396, 397, 398, 399, 400, 401, 402, 403, 404, 405, 406, 408, 409, 410, 413, 416, 418, 419, 420, 427, 428, 430, 431, 438, 439, 441, 442, 449, 450, 451, 452, 460, 461, 463, 464, 471, 473, 488, 489 }

B grade { 28, 29, 30, 31, 33, 35, 37, 39, 41, 42, 44, 45, 46, 48, 50, 51, 52, 53, 54, 55, 56, 57, 59, 60, 61, 62, 63, 64, 66, 67, 70, 71, 72, 73, 74, 75, 76, 77, 78, 82, 83, 92, 93, 97, 103, 104, 105, 107, 216, 217, 218, 219, 228, 229, 230, 231, 232, 233, 245, 263, 264, 265, 266, 274, 276, 277, 278, 279, 281, 283, 285, 288, 290, 291, 292, 293, 295, 297, 299, 300, 301, 302, 303, 305, 307, 309, 310, 311, 312, 313, 315, 323, 324, 332, 334, 353, 359, 360, 361, 363, 377, 381, 407, 417, 429, 440, 453, 462, 472, 474, 475 }

C grade { 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 204, 205, 206, 207, 208, 210, 211, 212, 213, 214, 215, 222, 223, 224, 225, 226, 227, 234, 235, 236, 237, 238, 239, 287, 289, 318, 319, 320, 328, 329, 330, 331, 339, 340, 348, 349, 358, 378, 382, 411, 412, 414, 415, 421, 422, 423, 424, 425, 426, 432, 433, 434, 435, 436, 437, 443, 444, 445, 446, 447, 448, 454, 455, 456, 457, 458, 459, 465, 466, 467, 468, 469, 470, 485 }

F normal fail { 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 364, 365, 366, 367, 368, 369, 370, 371, 372, 373, 374, 383, 384, 385, 386, 387, 388, 476, 477, 478, 479, 480, 481, 482, 483, 484, 486, 487 }

F(-1) timedout fail { }

F(-2) exception fail { }

Fricas

A grade { 1, 3, 4, 5, 10, 12, 14, 16, 21, 23, 118, 119, 120, 121, 122, 124, 128, 129, 130, 132, 139, 159, 161, 163, 165, 174, 240, 241, 242, 243, 244, 247, 248, 249, 250, 257, 259, 262, 270, 488, 489 }

B grade { 2, 6, 7, 8, 9, 11, 13, 15, 17, 18, 19, 20, 22, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 68, 69, 70, 71, 72, 73, 74, 75, 80, 81, 82, 83, 84, 87, 88, 89, 90, 91, 92, 93, 96, 97, 98, 99, 100, 101, 102, 104, 105, 106, 107, 123, 125, 126, 127, 131, 133, 134, 135, 136, 137, 138, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 160, 162, 164, 166, 167, 168, 169, 170, 171, 172, 173, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 196, 197, 198, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 219, 220, 221, 222, 223, 224, 225, 226, 227, 228, 229, 230, 231, 232, 233, 234, 235, 236, 237, 238, 239, 245, 246, 251, 252, 253, 254, 255, 256, 258, 260, 261, 263, 264, 265, 266, 267, 268, 269, 271, 272, 273, 274, 275, 276, 277, 278, 279, 280, 281, 282, 283, 284, 285, 286, 287, 288, 289, 290, 291, 292, 293, 294, 295, 296, 297, 298, 299, 300, 301, 302, 303, 304, 305, 306, 307, 308, 309, 310, 311, 312, 313, 314, 315, 316, 317, 318, 319, 320, 324, 325, 326, 327, 328, 329, 330, 331, 336, 337, 338, 339, 340, 343, 344, 345, 346, 347, 348, 349, 352, 353, 354, 355, 356, 357, 358, 360, 361, 362, 363, 375, 376, 377, 378, 379, 380, 381, 382, 389, 390, 391, 392, 393, 394, 395, 396, 397, 398, 399, 400, 401, 402, 403, 404, 405, 406, 407, 408, 409, 410, 411, 412, 413, 414, 415, 416, 417, 418, 419, 420, 421, 422, 423, 424, 425, 426, 432, 433, 434, 435, 436, 437, 443, 444, 445, 446, 447, 448, 449, 450, 451, 452, 453, 454, 455, 456, 457, 458, 459, 460, 461, 462, 463, 464, 465, 466, 467, 468, 469, 470, 471, 472, 473, 474, 475, 487 }

C grade { 485 }

F normal fail { 65, 66, 67, 76, 77, 78, 79, 85, 86, 94, 95, 103, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 321, 322, 323, 332, 333, 334, 335, 341, 342, 350, 351, 359, 364, 365, 366, 367, 368, 369, 370, 371, 372, 373, 374, 383, 384, 385, 386, 387, 388, 427, 428, 429, 430, 431, 438, 439, 440, 441, 442, 476, 477, 478, 479, 480, 481, 482, 483, 484 }

F(-1) timedout fail { }

F(-2) exception fail { 486 }

Maxima

A grade { 1, 3, 5, 6, 7, 10, 12, 14, 16, 21, 23, 25, 118, 119, 120, 121, 122, 123, 124, 127, 128, 129, 130, 131, 132, 138, 139, 140, 141, 142, 143, 145, 159, 161, 163, 164, 165, 172, 174, 176, 178, 192, 193, 194, 195, 241, 243, 244, 247, 249, 250, 251, 257, 259, 260, 268, 270, 271, 392, 393, 394, 397, 403, 404, 405, 414, 415 }

B grade { 2, 4, 8, 9, 11, 13, 15, 17, 18, 19, 20, 22, 24, 26, 27, 91, 100, 101, 125, 126, 133, 134, 135, 136, 137, 144, 146, 147, 160, 162, 166, 167, 168, 169, 170, 171, 173, 175, 177, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 191, 196, 197, 198, 199, 200, 201, 202, 203, 240, 242, 245, 246, 248, 252, 253, 254, 255, 256, 258, 261, 262, 263, 264, 265, 266, 267, 269, 272, 273, 274, 275, 276, 277, 278, 279, 313, 314, 315, 347, 356, 357, 389, 390, 391, 395, 396, 398, 399, 400, 401, 402, 406, 407, 408, 409, 410, 411, 412, 413, 416, 417, 418, 419, 420 }

C grade { }

F normal fail { 28, 30, 32, 34, 36, 38, 40, 43, 45, 47, 49, 52, 54, 56, 58, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 92, 93, 94, 95, 96, 97, 98, 99, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 219, 220, 221, 222, 223, 224, 225, 226, 227, 228, 229, 230, 231, 232, 233, 234, 235, 236, 237, 238, 239, 280, 282, 284, 286, 287, 289, 291, 294, 296, 298, 299, 301, 304, 306, 308, 309, 311, 316, 317, 318, 319, 320, 321, 322, 323, 324, 325, 326, 327, 328, 329, 330, 331, 332, 333, 334, 335, 336, 337, 338, 339, 340, 341, 342, 343, 344, 345, 346, 348, 349, 350, 351, 352, 353, 354, 355, 358, 359, 360, 361, 362, 363, 364, 365, 366, 367, 368, 369, 370, 371, 372, 373, 374, 375, 376, 377, 378, 380, 381, 382, 383, 384, 385, 386, 387, 388, 421, 422, 423, 424, 425, 426, 427, 428, 429, 430, 431, 432, 433, 434, 435, 436, 437, 438, 439, 440, 441, 442, 443, 444, 445, 446, 447, 448, 449, 450, 451, 452, 453, 454, 455, 456, 457, 458, 459, 460, 461, 462, 463, 464, 465, 466, 467, 468, 469, 470, 471, 472, 473, 474, 475, 476, 477, 478, 479, 480, 481, 482, 483, 484, 485, 486, 487, 488, 489 }

F(-1) timedout fail { 379 }

F(-2) exception fail { 29, 31, 33, 35, 37, 39, 41, 42, 44, 46, 48, 50, 51, 53, 55, 57, 59, 281, 283, 285, 288, 290, 292, 293, 295, 297, 300, 302, 303, 305, 307, 310, 312 }

Giac

A grade { 1, 3, 5, 6, 7, 9, 10, 12, 14, 19, 21, 23, 29, 31, 33, 35, 37, 39, 42, 44, 46, 50, 118, 119, 120, 121, 122, 123, 126, 127, 128, 129, 130, 131, 132, 136, 137, 138, 139, 140, 141, 142, 159, 161, 163, 164, 167, 172, 174, 176, 178, 179, 192, 193, 195, 240, 241, 243, 244, 247, 249, 251, 252, 254, 256, 257, 259, 261, 268, 270, 283, 285, 288, 290, 295, 297, 339, 389, 390, 391, 392, 393, 394, 395, 396, 397, 398, 399, 400, 485 }

B grade { 2, 4, 8, 11, 13, 15, 16, 17, 18, 20, 22, 24, 25, 26, 27, 41, 48, 51, 53, 55, 57, 59, 60, 61, 63, 64, 70, 71, 74, 75, 81, 83, 84, 91, 92, 93, 100, 101, 102, 124, 125, 133, 134, 135, 143, 144, 145, 146, 147, 160, 162, 165, 166, 168, 169, 170, 171, 173, 175, 177, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 191, 194, 196, 197, 198, 199, 200, 201, 202, 203, 242, 245, 246, 248, 250, 253, 255, 258, 260, 262, 263, 264, 265, 266, 267, 269, 271, 272, 273, 274, 275, 276, 277, 278, 279, 281, 292, 293, 300, 302, 303, 305, 307, 310, 312, 313, 314, 315, 316, 317, 319, 320, 326, 327, 330, 331, 337, 340, 347, 348, 349, 356, 357 }

C grade { }

F normal fail { 65, 66, 67, 68, 69, 76, 77, 78, 79, 80, 85, 86, 87, 88, 89, 94, 95, 96, 97, 98, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 148, 149, 150, 151, 153, 154, 155, 156, 157, 158, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 219, 220, 221, 222, 223, 224, 225, 226, 227, 228, 229, 230, 231, 232, 233, 234, 235, 236, 237, 238, 239, 321, 322, 323, 324, 325, 332, 333, 334, 335, 336, 341, 342, 343, 344, 345, 350, 351, 352, 353, 354, 359, 360, 361, 362, 363, 364, 365, 366, 367, 368, 369, 370, 371, 372, 373, 374, 375, 376, 377, 378, 379, 380, 381, 382, 383, 384, 385, 386, 387, 388, 421, 422, 423, 424, 425, 426, 427, 428, 429, 430, 431, 432, 433, 434, 435, 436, 437, 438, 439, 440, 441, 442, 443, 444, 445, 446, 447, 448, 449, 450, 451, 452, 453, 454, 455, 456, 457, 458, 459, 460, 461, 462, 463, 464, 465, 466, 467, 468, 469, 470, 471, 472, 473, 474, 475, 476, 477, 478, 479, 480, 481, 482, 483, 484, 486, 487, 488, 489 }

F(-1) timedout fail { }

F(-2) exception fail { 28, 30, 32, 34, 36, 38, 40, 43, 45, 47, 49, 52, 54, 56, 58, 62, 72, 73, 82, 90, 99, 152, 280, 282, 284, 286, 287, 289, 291, 294, 296, 298, 299, 301, 304, 306, 308, 309, 311, 318, 328, 329, 338, 346, 355, 358, 401, 402, 403, 404, 405, 406, 407, 408, 409, 410, 411, 412, 413, 414, 415, 416, 417, 418, 419, 420 }

Mupad

A grade { }

B grade { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 91, 100, 101, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 196, 197, 198, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 240, 241, 242, 243, 244, 245, 246, 247, 248, 249, 250, 251, 252, 253, 254, 255, 256, 257, 258, 259, 260, 261, 262, 263, 264, 265, 266, 267, 268, 269, 270, 271, 272, 273, 274, 275, 276, 277, 278, 279, 280, 281, 282, 283, 284, 285, 286, 287, 288, 289, 290, 291, 292, 298, 308, 313, 314, 315, 317, 327, 338, 347, 356, 357, 367, 377, 381, 385, 388, 389, 390, 391, 392, 398, 399, 400, 401, 402, 403, 408, 409, 410, 411, 412, 413, 418, 419, 420, 485 }

C grade { }

F normal fail { }

F(-1) timedout fail { 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 92, 93, 94, 95, 96, 97, 98, 99, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 216, 217, 218, 219, 220, 221, 222, 223, 224, 225, 226, 227, 228, 229, 230, 231, 232, 233, 234, 235, 236, 237, 238, 239, 293, 294, 295, 296, 297, 299, 300, 301, 302, 303, 304, 305, 306, 307, 309, 310, 311, 312, 316, 318, 319, 320, 321, 322, 323, 324, 325, 326, 328, 329, 330, 331, 332, 333, 334, 335, 336, 337, 339, 340, 341, 342, 343, 344, 345, 346, 348, 349, 350, 351, 352, 353, 354, 355, 358, 359, 360, 361, 362, 363, 364, 365, 366, 368, 369, 370, 371, 372, 373, 374, 375, 376, 378, 379, 380, 382, 383, 384, 386, 387, 393, 394, 395, 396, 397, 404, 405, 406, 407, 414, 415, 416, 417, 421, 422, 423, 424, 425, 426, 427, 428, 429, 430, 431, 432, 433, 434, 435, 436, 437, 438, 439, 440, 441, 442, 443, 444, 445, 446, 447, 448, 449, 450, 451, 452, 453, 454, 455, 456, 457, 458, 459, 460, 461, 462, 463, 464, 465, 466, 467, 468, 469, 470, 471, 472, 473, 474, 475, 476, 477, 478, 479, 480, 481, 482, 483, 484, 486, 487, 488, 489 }

F(-2) exception fail { }

Sympy

A grade { 5, 118, 120, 122, 127, 128, 129, 130, 138, 139, 140, 163, 243, 244, 250, 260, 271, 377 }

B grade { 1, 2, 3, 4, 10, 11, 12, 13, 14, 19, 20, 21, 22, 23, 35, 119, 121, 159, 160, 161, 162, 171, 172, 173, 174, 182, 183, 184, 192, 193, 240, 241, 242, 247, 248, 249, 257, 258, 259, 268, 269, 270, 286, 298, 308, 313, 314, 315, 381 }

C grade { }

F normal fail { 6, 7, 8, 15, 36, 37, 38, 39, 61, 62, 63, 66, 67, 68, 78, 82, 83, 84, 86, 87, 88, 89, 91, 92, 93, 97, 98, 106, 123, 124, 148, 149, 150, 151, 152, 153, 154, 164, 245, 246, 251, 252, 253, 254, 255, 261, 262, 263, 272, 287, 288, 289, 290, 291, 299, 300, 301, 302, 317, 318, 319, 320, 322, 323, 324, 325, 328, 334, 338, 339, 340, 342, 343, 344, 345, 347, 348, 349, 353, 354, 358, 362, 363, 378, 382, 389, 390, 391, 392, 393, 394, 395, 396, 397, 398, 399, 401, 402, 403, 404, 405, 406, 407, 408, 409, 410, 411, 412, 413, 414, 415, 416, 417, 418, 421, 422, 423, 424, 425, 427, 428, 429, 430, 431, 434, 439, 440, 443, 444, 445, 446, 447, 448, 449, 450, 451, 452, 453, 454, 455, 456, 457, 458, 459, 460, 461, 462, 463, 464, 465, 466, 467, 471, 472, 473, 478, 485, 486, 487, 488, 489 }

F(-1) timedout fail { 9, 16, 17, 18, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 64, 65, 69, 70, 71, 72, 73, 74, 75, 76, 77, 79, 80, 81, 85, 90, 94, 95, 96, 99, 100, 101, 102, 103, 104, 105, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 125, 126, 131, 132, 133, 134, 135, 136, 137, 141, 142, 143, 144, 145, 146, 147, 155, 156, 157, 158, 165, 166, 167, 168, 169, 170, 175, 176, 177, 178, 179, 180, 181, 185, 186, 187, 188, 189, 190, 191, 194, 195, 196, 197, 198, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 219, 220, 221, 222, 223, 224, 225, 226, 227, 228, 229, 230, 231, 232, 233, 234, 235, 236, 237, 238, 239, 256, 264, 265, 266, 267, 273, 274, 275, 276, 277, 278, 279, 280, 281, 282, 283, 284, 285, 292, 293, 294, 295, 296, 297, 303, 304, 305, 306, 307, 309, 310, 311, 312, 316, 321, 326, 327, 329, 330, 331, 332, 333, 335, 336, 337, 341, 346, 350, 351, 352, 355, 356, 357, 359, 360, 361, 364, 365, 366, 367, 368, 369, 370, 371, 372, 373, 374, 375, 376, 379, 380, 383, 384, 385, 386, 387, 388, 400, 419, 420, 426, 432, 433, 435, 436, 437, 438, 441, 442, 468, 469, 470, 474, 475, 476, 477, 479, 480, 481, 482, 483, 484 }

F(-2) exception fail { }

Reduce

A grade { }

B grade { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 196, 197, 198, 199, 200, 201, 202, 203, 240, 241, 242, 243, 244, 245, 246, 247, 248, 249, 250, 251, 252, 253, 254, 255, 256, 257, 258, 259, 260, 261, 262, 263, 264, 265, 266, 267, 268, 269, 270, 271, 272, 273, 274, 275, 276, 277, 278, 279, 280, 281, 282, 283, 284, 285, 286, 287, 288, 289, 290, 291, 292, 293, 294, 295, 296, 297, 298, 299, 300, 301, 302, 303, 304, 305, 306, 307, 308, 309, 310, 311, 312, 313, 314, 315, 347, 356, 389 }

C grade { }

F normal fail { 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 219, 220, 221, 222, 223, 224, 225, 226, 227, 228, 229, 230, 231, 232, 233, 234, 235, 236, 237, 238, 239, 316, 317, 318, 319, 320, 321, 322, 323, 324, 325, 326, 327, 328, 329, 330, 331, 332, 333, 334, 335, 336, 337, 338, 339, 340, 341, 342, 343, 344, 345, 346, 348, 349, 350, 351, 352, 353, 354, 355, 357, 358, 359, 360, 361, 362, 363, 364, 365, 366, 367, 368, 369, 370, 371, 372, 373, 374, 375, 376, 377, 378, 379, 380, 381, 382, 383, 384, 385, 386, 387, 388, 390, 391, 392, 393, 394, 395, 396, 397, 398, 399, 400, 401, 402, 403, 404, 405, 406, 407, 408, 409, 410, 411, 412, 413, 414, 415, 416, 417, 418, 419, 420, 421, 422, 423, 424, 425, 426, 427, 428, 429, 430, 431, 432, 433, 434, 435, 436, 437, 438, 439, 440, 441, 442, 443, 444, 445, 446, 447, 448, 449, 450, 451, 452, 453, 454, 455, 456, 457, 458, 459, 460, 461, 462, 463, 464, 465, 466, 467, 468, 469, 470, 471, 472, 473, 474, 475, 476, 477, 478, 479, 480, 481, 482, 483, 484, 485, 486, 487, 488, 489 }

F(-1) timedout fail { }

F(-2) exception fail { }

2.2 Detailed conclusion table per each integral for all CAS systems

Detailed conclusion table per each integral is given by the table below. The elapsed time is in seconds. For failed result it is given as **F(-1)** if the failure was due to timeout. It is given as **F(-2)** if the failure was due to an exception being raised, which could indicate a bug in the system. If the failure was due to integral not being evaluated within the time limit, then it is given as **F**.

In this table, the column `N.S.` means **normalized size** and is defined as $\frac{\text{antiderivative leaf size}}{\text{optimal antiderivative leaf size}}$. To make the table fit the page, the name `Mathematica` was abbreviated to `MMA`.

Problem 1	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	89	85	68	61	150	122	258	125	167	76
N.S.	1	0.96	0.76	0.69	1.69	1.37	2.90	1.40	1.88	0.85
time (sec)	N/A	0.400	0.155	3.373	0.041	0.101	0.378	0.135	0.151	0.234

Problem 2	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	B	B	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	53	48	77	56	141	102	105	112	138	57
N.S.	1	0.91	1.45	1.06	2.66	1.92	1.98	2.11	2.60	1.08
time (sec)	N/A	0.289	0.098	2.260	0.045	0.084	0.234	0.126	0.166	1.756

Problem 3	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	61	59	47	44	97	64	158	79	115	50
N.S.	1	0.97	0.77	0.72	1.59	1.05	2.59	1.30	1.89	0.82
time (sec)	N/A	0.343	0.123	1.118	0.030	0.088	0.179	0.119	0.150	0.113

Problem 4	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	A	B	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	32	30	53	34	67	48	56	70	83	34
N.S.	1	0.94	1.66	1.06	2.09	1.50	1.75	2.19	2.59	1.06
time (sec)	N/A	0.249	0.046	0.639	0.038	0.079	0.124	0.127	0.155	1.705

Problem 5	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	30	30	36	25	38	30	51	38	63	23
N.S.	1	1.00	1.20	0.83	1.27	1.00	1.70	1.27	2.10	0.77
time (sec)	N/A	0.165	0.054	0.288	0.049	0.085	0.110	0.120	0.152	0.072

Problem 6	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	C	A	A	A	B	F	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	25	34	36	24	43	126	0	50	68	66
N.S.	1	1.36	1.44	0.96	1.72	5.04	0.00	2.00	2.72	2.64
time (sec)	N/A	0.267	0.012	0.251	0.037	0.099	0.000	0.126	0.159	1.736

Problem 7	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	B	F	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	16	16	16	22	23	36	0	28	51	23
N.S.	1	1.00	1.00	1.38	1.44	2.25	0.00	1.75	3.19	1.44
time (sec)	N/A	0.221	0.065	0.293	0.037	0.088	0.000	0.133	0.151	1.658

Problem 8	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	C	B	A	B	B	F	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	40	48	92	40	125	484	0	96	289	131
N.S.	1	1.20	2.30	1.00	3.12	12.10	0.00	2.40	7.22	3.28
time (sec)	N/A	0.295	0.059	0.372	0.034	0.083	0.000	0.132	0.157	0.137

Problem 9	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	B	F(-1)	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	43	43	49	35	113	159	0	61	89	61
N.S.	1	1.00	1.14	0.81	2.63	3.70	0.00	1.42	2.07	1.42
time (sec)	N/A	0.291	0.096	0.375	0.037	0.080	0.000	0.136	0.157	1.724

Problem 10	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	144	185	133	101	267	238	490	215	322	149
N.S.	1	1.28	0.92	0.70	1.85	1.65	3.40	1.49	2.24	1.03
time (sec)	N/A	0.400	0.302	20.866	0.045	0.087	0.769	0.143	0.152	2.047

Problem 11	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	B	B	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	85	77	154	95	247	213	204	196	272	112
N.S.	1	0.91	1.81	1.12	2.91	2.51	2.40	2.31	3.20	1.32
time (sec)	N/A	0.302	0.184	11.796	0.042	0.102	0.516	0.149	0.154	1.817

Problem 12	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	110	122	99	82	189	149	332	159	233	108
N.S.	1	1.11	0.90	0.75	1.72	1.35	3.02	1.45	2.12	0.98
time (sec)	N/A	0.400	0.267	4.580	0.037	0.074	0.400	0.139	0.154	0.245

Problem 13	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	B	B	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	57	52	111	70	157	122	128	138	184	76
N.S.	1	0.91	1.95	1.23	2.75	2.14	2.25	2.42	3.23	1.33
time (sec)	N/A	0.266	0.142	3.666	0.034	0.099	0.262	0.131	0.152	1.681

Problem 14	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	72	72	60	57	105	80	168	101	145	67
N.S.	1	1.00	0.83	0.79	1.46	1.11	2.33	1.40	2.01	0.93
time (sec)	N/A	0.228	0.214	1.503	0.036	0.095	0.200	0.126	0.152	0.103

Problem 15	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	B	F	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	52	48	76	50	102	492	0	110	143	116
N.S.	1	0.92	1.46	0.96	1.96	9.46	0.00	2.12	2.75	2.23
time (sec)	N/A	0.262	0.089	0.437	0.038	0.096	0.000	0.136	0.155	0.167

Problem 16	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F(-1)	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	50	82	56	52	63	89	0	135	145	67
N.S.	1	1.64	1.12	1.04	1.26	1.78	0.00	2.70	2.90	1.34
time (sec)	N/A	0.279	0.214	0.386	0.040	0.093	0.000	0.146	0.152	1.769

Problem 17	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	A	B	B	F(-1)	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	56	59	127	53	157	902	0	125	398	179
N.S.	1	1.05	2.27	0.95	2.80	16.11	0.00	2.23	7.11	3.20
time (sec)	N/A	0.273	0.105	0.422	0.047	0.123	0.000	0.154	0.154	1.752

Problem 18	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	A	B	B	F(-1)	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	40	44	85	47	121	174	0	81	154	166
N.S.	1	1.10	2.12	1.18	3.02	4.35	0.00	2.02	3.85	4.15
time (sec)	N/A	0.274	0.636	0.335	0.042	0.097	0.000	0.144	0.153	1.742

Problem 19	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	B	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	206	308	162	153	405	406	777	325	512	239
N.S.	1	1.50	0.79	0.74	1.97	1.97	3.77	1.58	2.49	1.16
time (sec)	N/A	0.525	1.659	209.421	0.044	0.099	1.420	0.168	0.157	2.310

Problem 20	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	B	B	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	115	104	127	143	376	373	330	296	442	185
N.S.	1	0.90	1.10	1.24	3.27	3.24	2.87	2.57	3.84	1.61
time (sec)	N/A	0.346	1.882	74.434	0.056	0.095	1.011	0.163	0.155	2.052

Problem 21	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	181	191	130	129	306	269	561	251	388	181
N.S.	1	1.06	0.72	0.71	1.69	1.49	3.10	1.39	2.14	1.00
time (sec)	N/A	0.688	0.670	27.811	0.048	0.085	0.733	0.155	0.157	0.471

Problem 22	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	B	B	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	79	71	94	116	263	234	221	222	318	129
N.S.	1	0.90	1.19	1.47	3.33	2.96	2.80	2.81	4.03	1.63
time (sec)	N/A	0.338	0.861	10.991	0.046	0.097	0.498	0.150	0.154	1.877

Problem 23	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	128	133	95	131	197	165	350	177	263	123
N.S.	1	1.04	0.74	1.02	1.54	1.29	2.73	1.38	2.05	0.96
time (sec)	N/A	0.430	0.328	0.148	0.044	0.086	0.400	0.125	0.152	1.867

Problem 24	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	B	F(-1)	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	83	76	95	86	193	1128	0	202	244	184
N.S.	1	0.92	1.14	1.04	2.33	13.59	0.00	2.43	2.94	2.22
time (sec)	N/A	0.302	2.636	1.256	0.049	0.109	0.000	0.165	0.155	0.307

Problem 25	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	B	F(-1)	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	87	156	113	94	130	169	0	177	263	121
N.S.	1	1.79	1.30	1.08	1.49	1.94	0.00	2.03	3.02	1.39
time (sec)	N/A	0.420	2.156	0.920	0.042	0.084	0.000	0.168	0.157	1.843

Problem 26	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	A	B	B	F(-1)	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	83	83	210	79	217	1814	0	174	521	229
N.S.	1	1.00	2.53	0.95	2.61	21.86	0.00	2.10	6.28	2.76
time (sec)	N/A	0.346	6.091	0.872	0.049	0.109	0.000	0.170	0.155	0.229

Problem 27	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	B	F(-1)	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	74	119	107	77	161	281	0	154	339	222
N.S.	1	1.61	1.45	1.04	2.18	3.80	0.00	2.08	4.58	3.00
time (sec)	N/A	0.428	3.175	0.818	0.036	0.088	0.000	0.170	0.158	1.757

Problem 28	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	B	F	B	F(-1)	F(-2)	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	109	102	165	296	0	3241	0	0	908	415
N.S.	1	0.94	1.51	2.72	0.00	29.73	0.00	0.00	8.33	3.81
time (sec)	N/A	0.374	1.352	3.556	0.000	0.186	0.000	0.000	0.177	2.734

Problem 29	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F(-2)	B	F(-1)	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	121	151	97	246	0	1725	0	208	356	266
N.S.	1	1.25	0.80	2.03	0.00	14.26	0.00	1.72	2.94	2.20
time (sec)	N/A	0.463	0.734	2.267	0.000	0.140	0.000	0.560	0.171	2.155

Problem 30	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	B	F	B	F(-1)	F(-2)	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	79	74	134	179	0	1667	0	0	772	348
N.S.	1	0.94	1.70	2.27	0.00	21.10	0.00	0.00	9.77	4.41
time (sec)	N/A	0.305	0.776	1.358	0.000	0.127	0.000	0.000	0.169	2.552

Problem 31	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F(-2)	B	F(-1)	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	79	97	71	164	0	859	0	126	264	216
N.S.	1	1.23	0.90	2.08	0.00	10.87	0.00	1.59	3.34	2.73
time (sec)	N/A	0.324	0.482	0.809	0.000	0.117	0.000	0.602	0.167	2.138

Problem 32	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	B	F(-1)	F(-2)	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	56	55	107	93	0	745	0	0	540	293
N.S.	1	0.98	1.91	1.66	0.00	13.30	0.00	0.00	9.64	5.23
time (sec)	N/A	0.270	0.567	0.523	0.000	0.120	0.000	0.000	0.159	2.362

Problem 33	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F(-2)	B	F(-1)	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	50	50	50	120	0	464	0	64	143	473
N.S.	1	1.00	1.00	2.40	0.00	9.28	0.00	1.28	2.86	9.46
time (sec)	N/A	0.312	1.995	0.325	0.000	0.108	0.000	0.390	0.162	2.421

Problem 34	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	B	F(-1)	F(-2)	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	40	40	91	51	0	501	0	0	489	116
N.S.	1	1.00	2.28	1.28	0.00	12.52	0.00	0.00	12.22	2.90
time (sec)	N/A	0.236	2.077	0.254	0.000	0.095	0.000	0.000	0.158	2.159

Problem 35	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F(-2)	B	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	40	40	40	154	0	430	15870	47	114	146
N.S.	1	1.00	1.00	3.85	0.00	10.75	396.75	1.18	2.85	3.65
time (sec)	N/A	0.219	1.284	0.270	0.000	0.104	15.777	0.172	0.163	0.492

Problem 36	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	B	F	F(-2)	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	60	57	135	72	0	586	0	0	566	323
N.S.	1	0.95	2.25	1.20	0.00	9.77	0.00	0.00	9.43	5.38
time (sec)	N/A	0.256	7.969	0.405	0.000	0.115	0.000	0.000	0.166	2.257

Problem 37	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F(-2)	B	F	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	57	55	57	183	0	675	0	72	339	176
N.S.	1	0.96	1.00	3.21	0.00	11.84	0.00	1.26	5.95	3.09
time (sec)	N/A	0.264	0.678	0.515	0.000	0.102	0.000	0.242	0.178	0.495

Problem 38	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	B	F	F(-2)	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	88	97	220	113	0	1837	0	0	2149	571
N.S.	1	1.10	2.50	1.28	0.00	20.88	0.00	0.00	24.42	6.49
time (sec)	N/A	0.309	2.009	0.694	0.000	0.132	0.000	0.000	0.206	2.609

Problem 39	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F(-2)	B	F	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	78	73	126	230	0	1972	0	118	781	350
N.S.	1	0.94	1.62	2.95	0.00	25.28	0.00	1.51	10.01	4.49
time (sec)	N/A	0.315	0.972	0.859	0.000	0.126	0.000	0.232	0.211	2.324

Problem 40	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	B	F(-1)	F(-2)	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	130	150	649	156	0	5809	0	0	4116	1639
N.S.	1	1.15	4.99	1.20	0.00	44.68	0.00	0.00	31.66	12.61
time (sec)	N/A	0.381	8.692	1.167	0.000	0.174	0.000	0.000	0.350	7.184

Problem 41	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F(-2)	B	F(-1)	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	110	102	155	325	0	4540	0	213	1278	479
N.S.	1	0.93	1.41	2.95	0.00	41.27	0.00	1.94	11.62	4.35
time (sec)	N/A	0.353	3.670	1.456	0.000	0.133	0.000	0.228	0.266	2.478

Problem 42	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F(-2)	B	F(-1)	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	102	125	99	315	0	1772	0	168	1353	0
N.S.	1	1.23	0.97	3.09	0.00	17.37	0.00	1.65	13.26	0.00
time (sec)	N/A	0.354	11.387	0.915	0.000	0.129	0.000	0.974	0.177	0.000

Problem 43	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	B	F(-1)	F(-2)	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	90	89	141	155	0	1888	0	0	3499	0
N.S.	1	0.99	1.57	1.72	0.00	20.98	0.00	0.00	38.88	0.00
time (sec)	N/A	0.291	2.071	0.785	0.000	0.110	0.000	0.000	0.187	0.000

Problem 44	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F(-2)	B	F(-1)	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	84	84	81	248	0	1523	0	135	743	0
N.S.	1	1.00	0.96	2.95	0.00	18.13	0.00	1.61	8.85	0.00
time (sec)	N/A	0.373	11.163	0.677	0.000	0.119	0.000	0.569	0.169	0.000

Problem 45	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	B	F	B	F(-1)	F(-2)	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	81	79	130	146	0	1627	0	0	2279	0
N.S.	1	0.98	1.60	1.80	0.00	20.09	0.00	0.00	28.14	0.00
time (sec)	N/A	0.252	0.491	0.655	0.000	0.144	0.000	0.000	0.182	0.000

Problem 46	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F(-2)	B	F(-1)	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	95	95	96	285	0	1617	0	144	1205	0
N.S.	1	1.00	1.01	3.00	0.00	17.02	0.00	1.52	12.68	0.00
time (sec)	N/A	0.347	11.127	0.660	0.000	0.127	0.000	0.194	0.174	0.000

Problem 47	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	B	F(-1)	F(-2)	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	110	123	189	171	0	2529	0	0	3972	0
N.S.	1	1.12	1.72	1.55	0.00	22.99	0.00	0.00	36.11	0.00
time (sec)	N/A	0.371	5.742	0.943	0.000	0.144	0.000	0.000	0.223	0.000

Problem 48	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F(-2)	B	F(-1)	B	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	111	142	170	316	0	2988	0	229	1845	0
N.S.	1	1.28	1.53	2.85	0.00	26.92	0.00	2.06	16.62	0.00
time (sec)	N/A	0.388	1.205	1.072	0.000	0.175	0.000	0.276	0.237	0.000

Problem 49	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	B	F(-1)	F(-2)	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	161	178	391	213	0	8059	0	0	7758	0
N.S.	1	1.11	2.43	1.32	0.00	50.06	0.00	0.00	48.19	0.00
time (sec)	N/A	0.473	3.001	1.309	0.000	0.212	0.000	0.000	0.504	0.000

Problem 50	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F(-2)	B	F(-1)	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	135	169	210	371	0	7110	0	220	4270	0
N.S.	1	1.25	1.56	2.75	0.00	52.67	0.00	1.63	31.63	0.00
time (sec)	N/A	0.485	2.346	1.707	0.000	0.181	0.000	0.293	0.353	0.000

Problem 51	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F(-2)	B	F(-1)	B	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	124	131	104	357	0	5186	0	282	2880	0
N.S.	1	1.06	0.84	2.88	0.00	41.82	0.00	2.27	23.23	0.00
time (sec)	N/A	0.347	11.565	1.658	0.000	0.162	0.000	1.095	0.196	0.000

Problem 52	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	B	F	B	F(-1)	F(-2)	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	135	138	170	281	0	6086	0	0	7235	0
N.S.	1	1.02	1.26	2.08	0.00	45.08	0.00	0.00	53.59	0.00
time (sec)	N/A	0.355	2.082	1.683	0.000	0.148	0.000	0.000	0.264	0.000

Problem 53	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F(-2)	B	F(-1)	B	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	139	151	121	401	0	5519	0	277	3699	0
N.S.	1	1.09	0.87	2.88	0.00	39.71	0.00	1.99	26.61	0.00
time (sec)	N/A	0.641	11.621	1.470	0.000	0.169	0.000	0.666	0.207	0.000

Problem 54	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	B	F	B	F(-1)	F(-2)	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	118	125	149	237	0	5151	0	0	5102	0
N.S.	1	1.06	1.26	2.01	0.00	43.65	0.00	0.00	43.24	0.00
time (sec)	N/A	0.307	0.769	1.466	0.000	0.170	0.000	0.000	0.255	0.000

Problem 55	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F(-2)	B	F(-1)	B	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	154	169	132	418	0	5925	0	302	4496	0
N.S.	1	1.10	0.86	2.71	0.00	38.47	0.00	1.96	29.19	0.00
time (sec)	N/A	0.599	11.698	1.526	0.000	0.150	0.000	0.242	0.212	0.000

Problem 56	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	B	F	B	F(-1)	F(-2)	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	166	193	250	308	0	9815	0	0	10544	0
N.S.	1	1.16	1.51	1.86	0.00	59.13	0.00	0.00	63.52	0.00
time (sec)	N/A	0.427	8.405	2.063	0.000	0.246	0.000	0.000	0.416	0.000

Problem 57	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F(-2)	B	F(-1)	B	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	171	232	225	442	0	9102	0	331	5759	0
N.S.	1	1.36	1.32	2.58	0.00	53.23	0.00	1.94	33.68	0.00
time (sec)	N/A	0.450	2.922	2.241	0.000	0.225	0.000	0.381	0.358	0.000

Problem 58	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	B	F(-1)	F(-2)	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	224	261	462	350	0	22563	0	0	16558	0
N.S.	1	1.17	2.06	1.56	0.00	100.73	0.00	0.00	73.92	0.00
time (sec)	N/A	0.526	7.225	2.523	0.000	0.381	0.000	0.000	1.642	0.000

Problem 59	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F(-2)	B	F(-1)	B	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	195	272	167	497	0	17294	0	378	8270	0
N.S.	1	1.39	0.86	2.55	0.00	88.69	0.00	1.94	42.41	0.00
time (sec)	N/A	0.565	7.065	3.173	0.000	0.282	0.000	0.391	0.745	0.000

Problem 60	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F	B	F(-1)	B	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	130	126	114	339	0	3037	0	890	24	0
N.S.	1	0.97	0.88	2.61	0.00	23.36	0.00	6.85	0.18	0.00
time (sec)	N/A	0.319	0.676	0.469	0.000	0.228	0.000	0.396	0.163	0.000

Problem 61	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F	B	F	B	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	82	80	97	200	0	2130	0	424	22	0
N.S.	1	0.98	1.18	2.44	0.00	25.98	0.00	5.17	0.27	0.00
time (sec)	N/A	0.261	0.227	0.227	0.000	0.147	0.000	0.254	0.164	0.000

Problem 62	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F	B	F	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	84	83	97	174	0	4197	0	0	22	0
N.S.	1	0.99	1.15	2.07	0.00	49.96	0.00	0.00	0.26	0.00
time (sec)	N/A	0.324	0.286	0.303	0.000	0.209	0.000	0.000	0.161	0.000

Problem 63	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F	B	F	B	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	88	94	104	230	0	1165	0	668	24	0
N.S.	1	1.07	1.18	2.61	0.00	13.24	0.00	7.59	0.27	0.00
time (sec)	N/A	0.298	0.586	0.389	0.000	0.125	0.000	0.242	0.163	0.000

Problem 64	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F	B	F(-1)	B	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	143	154	129	381	0	3283	0	2167	24	0
N.S.	1	1.08	0.90	2.66	0.00	22.96	0.00	15.15	0.17	0.00
time (sec)	N/A	0.326	0.633	0.172	0.000	0.271	0.000	0.457	0.160	0.000

Problem 65	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	300	350	210	512	0	0	0	0	24	0
N.S.	1	1.17	0.70	1.71	0.00	0.00	0.00	0.00	0.08	0.00
time (sec)	N/A	0.528	1.135	5.173	0.000	0.000	0.000	0.000	0.161	0.000

Problem 66	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	179	178	170	355	0	0	0	0	24	0
N.S.	1	0.99	0.95	1.98	0.00	0.00	0.00	0.00	0.13	0.00
time (sec)	N/A	0.990	0.768	3.422	0.000	0.000	0.000	0.000	0.163	0.000

Problem 67	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	61	60	69	140	0	0	0	0	15	0
N.S.	1	0.98	1.13	2.30	0.00	0.00	0.00	0.00	0.25	0.00
time (sec)	N/A	0.328	0.097	2.063	0.000	0.000	0.000	0.000	0.159	0.000

Problem 68	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	B	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	179	269	151	167	0	542	0	0	24	0
N.S.	1	1.50	0.84	0.93	0.00	3.03	0.00	0.00	0.13	0.00
time (sec)	N/A	0.493	0.674	2.323	0.000	0.116	0.000	0.000	0.161	0.000

Problem 69	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	B	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	243	335	208	436	0	2144	0	0	24	0
N.S.	1	1.38	0.86	1.79	0.00	8.82	0.00	0.00	0.10	0.00
time (sec)	N/A	0.592	2.154	5.844	0.000	0.141	0.000	0.000	0.159	0.000

Problem 70	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F	B	F(-1)	B	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	177	165	151	483	0	4608	0	1578	53	0
N.S.	1	0.93	0.85	2.73	0.00	26.03	0.00	8.92	0.30	0.00
time (sec)	N/A	0.380	0.632	0.367	0.000	0.306	0.000	0.774	0.178	0.000

Problem 71	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F	B	F(-1)	B	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	121	119	111	336	0	2977	0	897	51	0
N.S.	1	0.98	0.92	2.78	0.00	24.60	0.00	7.41	0.42	0.00
time (sec)	N/A	0.316	0.343	0.259	0.000	0.219	0.000	0.500	0.171	0.000

Problem 72	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F	B	F(-1)	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	127	126	136	268	0	5342	0	0	57	0
N.S.	1	0.99	1.07	2.11	0.00	42.06	0.00	0.00	0.45	0.00
time (sec)	N/A	0.431	0.540	0.305	0.000	0.258	0.000	0.000	0.191	0.000

Problem 73	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F	B	F(-1)	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	130	136	143	297	0	6398	0	0	61	0
N.S.	1	1.05	1.10	2.28	0.00	49.22	0.00	0.00	0.47	0.00
time (sec)	N/A	0.369	0.860	0.351	0.000	0.303	0.000	0.000	0.192	0.000

Problem 74	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F	B	F(-1)	B	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	135	148	123	379	0	3021	0	2146	61	0
N.S.	1	1.10	0.91	2.81	0.00	22.38	0.00	15.90	0.45	0.00
time (sec)	N/A	0.345	0.701	0.519	0.000	0.267	0.000	0.689	0.192	0.000

Problem 75	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F	B	F(-1)	B	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	199	206	174	569	0	7257	0	4685	61	0
N.S.	1	1.04	0.87	2.86	0.00	36.47	0.00	23.54	0.31	0.00
time (sec)	N/A	0.375	1.629	0.258	0.000	0.815	0.000	1.558	0.198	0.000

Problem 76	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	B	F	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	367	425	262	743	0	0	0	0	53	0
N.S.	1	1.16	0.71	2.02	0.00	0.00	0.00	0.00	0.14	0.00
time (sec)	N/A	0.670	2.084	7.406	0.000	0.000	0.000	0.000	0.179	0.000

Problem 77	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	238	242	213	535	0	0	0	0	53	0
N.S.	1	1.02	0.89	2.25	0.00	0.00	0.00	0.00	0.22	0.00
time (sec)	N/A	1.295	1.140	5.791	0.000	0.000	0.000	0.000	0.187	0.000

Problem 78	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	176	175	169	428	0	0	0	0	44	0
N.S.	1	0.99	0.96	2.43	0.00	0.00	0.00	0.00	0.25	0.00
time (sec)	N/A	0.884	0.558	4.019	0.000	0.000	0.000	0.000	0.171	0.000

Problem 79	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	204	273	155	244	0	0	0	0	61	0
N.S.	1	1.34	0.76	1.20	0.00	0.00	0.00	0.00	0.30	0.00
time (sec)	N/A	0.452	1.048	3.417	0.000	0.000	0.000	0.000	0.187	0.000

Problem 80	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	B	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	239	336	213	454	0	2222	0	0	61	0
N.S.	1	1.41	0.89	1.90	0.00	9.30	0.00	0.00	0.26	0.00
time (sec)	N/A	0.534	2.860	4.203	0.000	0.122	0.000	0.000	0.189	0.000

Problem 81	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	B	F(-1)	B	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	83	82	98	133	0	2116	0	430	38	0
N.S.	1	0.99	1.18	1.60	0.00	25.49	0.00	5.18	0.46	0.00
time (sec)	N/A	0.324	0.435	0.314	0.000	0.161	0.000	0.203	0.156	0.000

Problem 82	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F	B	F	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	41	41	49	108	0	1654	0	0	36	0
N.S.	1	1.00	1.20	2.63	0.00	40.34	0.00	0.00	0.88	0.00
time (sec)	N/A	0.283	0.195	0.188	0.000	0.134	0.000	0.000	0.156	0.000

Problem 83	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F	B	F	B	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	42	42	49	113	0	460	0	78	36	0
N.S.	1	1.00	1.17	2.69	0.00	10.95	0.00	1.86	0.86	0.00
time (sec)	N/A	0.295	0.295	0.294	0.000	0.094	0.000	0.154	0.155	0.000

Problem 84	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	B	F	B	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	89	95	102	149	0	1173	0	672	38	0
N.S.	1	1.07	1.15	1.67	0.00	13.18	0.00	7.55	0.43	0.00
time (sec)	N/A	0.353	0.611	3.105	0.000	0.133	0.000	0.260	0.155	0.000

Problem 85	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	229	282	168	356	0	0	0	0	38	0
N.S.	1	1.23	0.73	1.55	0.00	0.00	0.00	0.00	0.17	0.00
time (sec)	N/A	0.504	0.827	1.474	0.000	0.000	0.000	0.000	0.152	0.000

Problem 86	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	130	128	89	113	0	0	0	0	38	0
N.S.	1	0.98	0.68	0.87	0.00	0.00	0.00	0.00	0.29	0.00
time (sec)	N/A	0.756	0.374	0.902	0.000	0.000	0.000	0.000	0.154	0.000

Problem 87	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	B	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	61	60	68	86	0	147	0	0	30	0
N.S.	1	0.98	1.11	1.41	0.00	2.41	0.00	0.00	0.49	0.00
time (sec)	N/A	0.339	0.091	0.555	0.000	0.095	0.000	0.000	0.152	0.000

Problem 88	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	B	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	134	193	150	189	0	541	0	0	38	0
N.S.	1	1.44	1.12	1.41	0.00	4.04	0.00	0.00	0.28	0.00
time (sec)	N/A	0.415	0.627	1.383	0.000	0.100	0.000	0.000	0.153	0.000

Problem 89	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	B	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	238	334	201	456	0	2131	0	0	38	0
N.S.	1	1.40	0.84	1.92	0.00	8.95	0.00	0.00	0.16	0.00
time (sec)	N/A	0.538	2.559	1.962	0.000	0.145	0.000	0.000	0.155	0.000

Problem 90	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	B	F(-1)	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	83	82	98	146	0	3038	0	0	54	0
N.S.	1	0.99	1.18	1.76	0.00	36.60	0.00	0.00	0.65	0.00
time (sec)	N/A	0.336	0.551	0.326	0.000	0.181	0.000	0.000	0.156	0.000

Problem 91	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	B	F	B	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	36	36	43	32	236	296	0	110	52	191
N.S.	1	1.00	1.19	0.89	6.56	8.22	0.00	3.06	1.44	5.31
time (sec)	N/A	0.245	0.242	0.258	0.150	0.105	0.000	0.211	0.158	1.889

Problem 92	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F	B	F	B	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	84	81	98	154	0	1529	0	212	52	0
N.S.	1	0.96	1.17	1.83	0.00	18.20	0.00	2.52	0.62	0.00
time (sec)	N/A	0.296	0.469	0.342	0.000	0.137	0.000	0.251	0.160	0.000

Problem 93	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F	B	F	B	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	139	145	134	251	0	4329	0	798	54	0
N.S.	1	1.04	0.96	1.81	0.00	31.14	0.00	5.74	0.39	0.00
time (sec)	N/A	0.374	0.892	0.269	0.000	0.382	0.000	0.497	0.156	0.000

Problem 94	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	341	370	211	500	0	0	0	0	54	0
N.S.	1	1.09	0.62	1.47	0.00	0.00	0.00	0.00	0.16	0.00
time (sec)	N/A	0.566	1.149	5.921	0.000	0.000	0.000	0.000	0.157	0.000

Problem 95	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	256	295	156	313	0	0	0	0	54	0
N.S.	1	1.15	0.61	1.22	0.00	0.00	0.00	0.00	0.21	0.00
time (sec)	N/A	0.453	0.883	4.854	0.000	0.000	0.000	0.000	0.156	0.000

Problem 96	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	B	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	175	186	151	127	0	1155	0	0	54	0
N.S.	1	1.06	0.86	0.73	0.00	6.60	0.00	0.00	0.31	0.00
time (sec)	N/A	0.959	0.534	2.434	0.000	0.115	0.000	0.000	0.153	0.000

Problem 97	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F	B	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	116	115	100	252	0	1464	0	0	46	0
N.S.	1	0.99	0.86	2.17	0.00	12.62	0.00	0.00	0.40	0.00
time (sec)	N/A	0.486	0.154	1.098	0.000	0.117	0.000	0.000	0.152	0.000

Problem 98	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	B	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	234	347	185	284	0	2829	0	0	54	0
N.S.	1	1.48	0.79	1.21	0.00	12.09	0.00	0.00	0.23	0.00
time (sec)	N/A	0.539	1.136	4.647	0.000	0.153	0.000	0.000	0.157	0.000

Problem 99	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	B	F(-1)	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	137	160	130	230	0	7938	0	0	70	0
N.S.	1	1.17	0.95	1.68	0.00	57.94	0.00	0.00	0.51	0.00
time (sec)	N/A	0.369	1.043	0.819	0.000	0.493	0.000	0.000	0.156	0.000

Problem 100	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	B	F(-1)	B	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	91	90	67	64	927	1214	0	333	70	148
N.S.	1	0.99	0.74	0.70	10.19	13.34	0.00	3.66	0.77	1.63
time (sec)	N/A	0.290	0.492	0.535	0.175	0.235	0.000	0.319	0.159	2.616

Problem 101	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	B	F(-1)	B	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	79	77	63	57	485	1186	0	293	68	133
N.S.	1	0.97	0.80	0.72	6.14	15.01	0.00	3.71	0.86	1.68
time (sec)	N/A	0.248	0.284	1.770	0.179	0.220	0.000	0.306	0.153	2.239

Problem 102	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	B	F(-1)	B	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	136	149	130	236	0	5230	0	470	68	0
N.S.	1	1.10	0.96	1.74	0.00	38.46	0.00	3.46	0.50	0.00
time (sec)	N/A	0.367	0.790	52.927	0.000	0.372	0.000	0.351	0.158	0.000

Problem 103	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	B	F	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	325	386	207	868	0	0	0	0	70	0
N.S.	1	1.19	0.64	2.67	0.00	0.00	0.00	0.00	0.22	0.00
time (sec)	N/A	0.562	1.852	7.690	0.000	0.000	0.000	0.000	0.155	0.000

Problem 104	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	B	F	B	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	259	295	198	659	0	4985	0	0	70	0
N.S.	1	1.14	0.76	2.54	0.00	19.25	0.00	0.00	0.27	0.00
time (sec)	N/A	0.438	1.374	5.072	0.000	0.223	0.000	0.000	0.153	0.000

Problem 105	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F	B	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	243	251	187	598	0	4684	0	0	70	0
N.S.	1	1.03	0.77	2.46	0.00	19.28	0.00	0.00	0.29	0.00
time (sec)	N/A	1.252	1.250	5.064	0.000	0.234	0.000	0.000	0.156	0.000

Problem 106	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	B	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	253	258	190	406	0	5442	0	0	62	0
N.S.	1	1.02	0.75	1.60	0.00	21.51	0.00	0.00	0.25	0.00
time (sec)	N/A	1.320	0.944	1.497	0.000	0.223	0.000	0.000	0.157	0.000

Problem 107	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	B	F	B	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	311	448	234	747	0	8769	0	0	70	0
N.S.	1	1.44	0.75	2.40	0.00	28.20	0.00	0.00	0.23	0.00
time (sec)	N/A	0.636	1.896	6.300	0.000	0.376	0.000	0.000	0.162	0.000

Problem 108	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	F	F	F	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	128	128	0	0	0	0	0	0	29	0
N.S.	1	1.00	0.00	0.00	0.00	0.00	0.00	0.00	0.23	0.00
time (sec)	N/A	0.349	0.000	0.000	0.000	0.000	0.000	0.000	0.164	0.000

Problem 109	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	F	F	F	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	221	224	0	0	0	0	0	0	0	0
N.S.	1	1.01	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.414	0.000	0.000	0.000	0.000	0.000	0.000	22.993	0.000

Problem 110	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	F	F	F	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	137	136	0	0	0	0	0	0	0	0
N.S.	1	0.99	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.318	0.000	0.000	0.000	0.000	0.000	0.000	6.572	0.000

Problem 111	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	78	78	78	0	0	0	0	0	0	0
N.S.	1	1.00	1.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.261	0.679	0.000	0.000	0.000	0.000	0.000	0.949	0.000

Problem 112	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	F	F	F	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	88	88	0	0	0	0	0	0	23	0
N.S.	1	1.00	0.00	0.00	0.00	0.00	0.00	0.00	0.26	0.00
time (sec)	N/A	0.278	0.000	0.000	0.000	0.000	0.000	0.000	0.194	0.000

Problem 113	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	F	F	F	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	87	87	0	0	0	0	0	0	25	0
N.S.	1	1.00	0.00	0.00	0.00	0.00	0.00	0.00	0.29	0.00
time (sec)	N/A	0.282	0.000	0.000	0.000	0.000	0.000	0.000	200.029	0.000

Problem 114	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	F	F	F	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	103	103	0	0	0	0	0	0	0	0
N.S.	1	1.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.299	0.000	0.000	0.000	0.000	0.000	0.000	3.977	0.000

Problem 115	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	F	F	F	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	101	101	0	0	0	0	0	0	0	0
N.S.	1	1.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.372	0.000	0.000	0.000	0.000	0.000	0.000	1.087	0.000

Problem 116	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	F	F	F	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	99	99	0	0	0	0	0	0	25	0
N.S.	1	1.00	0.00	0.00	0.00	0.00	0.00	0.00	0.25	0.00
time (sec)	N/A	0.287	0.000	0.000	0.000	0.000	0.000	0.000	200.025	0.000

Problem 117	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	F	F	F	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	103	103	0	0	0	0	0	0	25	0
N.S.	1	1.00	0.00	0.00	0.00	0.00	0.00	0.00	0.24	0.00
time (sec)	N/A	0.284	0.000	0.000	0.000	0.000	0.000	0.000	200.028	0.000

Problem 118	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	106	106	81	82	164	188	192	182	179	85
N.S.	1	1.00	0.76	0.77	1.55	1.77	1.81	1.72	1.69	0.80
time (sec)	N/A	0.324	0.119	7.263	0.037	0.090	0.482	0.136	0.193	0.254

Problem 119	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	C	A	A	A	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	99	115	66	71	143	135	194	152	152	67
N.S.	1	1.16	0.67	0.72	1.44	1.36	1.96	1.54	1.54	0.68
time (sec)	N/A	0.328	0.074	4.135	0.031	0.087	0.351	0.130	0.192	0.457

Problem 120	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	70	70	79	60	120	105	117	122	127	55
N.S.	1	1.00	1.13	0.86	1.71	1.50	1.67	1.74	1.81	0.79
time (sec)	N/A	0.280	0.054	2.661	0.026	0.093	0.235	0.132	0.188	0.120

Problem 121	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	C	A	A	A	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	60	73	45	47	74	63	121	92	100	42
N.S.	1	1.22	0.75	0.78	1.23	1.05	2.02	1.53	1.67	0.70
time (sec)	N/A	0.267	0.077	2.144	0.037	0.089	0.164	0.128	0.186	1.679

Problem 122	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	32	32	34	28	59	47	41	59	72	29
N.S.	1	1.00	1.06	0.88	1.84	1.47	1.28	1.84	2.25	0.91
time (sec)	N/A	0.173	0.003	0.947	0.036	0.097	0.120	0.121	0.188	0.068

Problem 123	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	C	A	A	A	B	F	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	40	50	46	36	50	258	0	62	94	73
N.S.	1	1.25	1.15	0.90	1.25	6.45	0.00	1.55	2.35	1.82
time (sec)	N/A	0.256	0.028	0.407	0.045	0.118	0.000	0.128	0.193	0.129

Problem 124	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	24	24	35	23	47	40	0	59	57	47
N.S.	1	1.00	1.46	0.96	1.96	1.67	0.00	2.46	2.38	1.96
time (sec)	N/A	0.247	0.013	0.431	0.051	0.093	0.000	0.128	0.190	0.092

Problem 125	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	C	B	A	B	B	F(-1)	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	39	50	82	37	91	521	0	73	206	102
N.S.	1	1.28	2.10	0.95	2.33	13.36	0.00	1.87	5.28	2.62
time (sec)	N/A	0.256	0.006	0.422	0.036	0.097	0.000	0.365	0.190	1.553

Problem 126	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	B	F(-1)	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	41	41	50	36	131	652	0	62	224	110
N.S.	1	1.00	1.22	0.88	3.20	15.90	0.00	1.51	5.46	2.68
time (sec)	N/A	0.253	0.009	0.495	0.042	0.086	0.000	0.139	0.202	1.605

Problem 127	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	C	A	A	A	B	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	192	219	125	128	272	355	325	301	318	149
N.S.	1	1.14	0.65	0.67	1.42	1.85	1.69	1.57	1.66	0.78
time (sec)	N/A	0.434	0.569	112.258	0.043	0.090	0.993	0.159	0.195	1.866

Problem 128	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	180	180	133	122	237	274	340	260	276	126
N.S.	1	1.00	0.74	0.68	1.32	1.52	1.89	1.44	1.53	0.70
time (sec)	N/A	0.444	0.082	38.993	0.048	0.088	0.695	0.150	0.190	2.539

Problem 129	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	C	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	130	152	92	96	180	220	219	219	230	104
N.S.	1	1.17	0.71	0.74	1.38	1.69	1.68	1.68	1.77	0.80
time (sec)	N/A	0.360	0.282	12.936	0.049	0.089	0.474	0.151	0.185	1.734

Problem 130	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	114	114	94	85	151	160	212	178	187	85
N.S.	1	1.00	0.82	0.75	1.32	1.40	1.86	1.56	1.64	0.75
time (sec)	N/A	0.334	0.223	0.217	0.035	0.082	0.357	0.128	0.180	0.477

Problem 131	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	C	A	A	A	B	F(-1)	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	88	106	73	76	140	1052	0	154	192	177
N.S.	1	1.20	0.83	0.86	1.59	11.95	0.00	1.75	2.18	2.01
time (sec)	N/A	0.331	0.213	1.097	0.040	0.109	0.000	0.156	0.185	0.211

Problem 132	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F(-1)	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	82	82	92	65	113	142	0	149	171	123
N.S.	1	1.00	1.12	0.79	1.38	1.73	0.00	1.82	2.09	1.50
time (sec)	N/A	0.334	0.225	0.887	0.047	0.103	0.000	0.173	0.185	1.671

Problem 133	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	C	A	A	B	B	F(-1)	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	77	91	126	63	152	1616	0	162	349	175
N.S.	1	1.18	1.64	0.82	1.97	20.99	0.00	2.10	4.53	2.27
time (sec)	N/A	0.323	0.014	0.917	0.052	0.124	0.000	0.171	0.193	1.644

Problem 134	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	B	F(-1)	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	76	76	61	65	170	1748	0	151	411	163
N.S.	1	1.00	0.80	0.86	2.24	23.00	0.00	1.99	5.41	2.14
time (sec)	N/A	0.313	0.261	0.837	0.043	0.123	0.000	0.168	0.193	0.137

Problem 135	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	C	A	A	B	B	F(-1)	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	90	105	170	66	188	2119	0	172	502	355
N.S.	1	1.17	1.89	0.73	2.09	23.54	0.00	1.91	5.58	3.94
time (sec)	N/A	0.352	0.148	0.904	0.047	0.109	0.000	0.176	0.199	0.144

Problem 136	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	A	B	B	F(-1)	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	88	88	216	73	303	2310	0	141	516	351
N.S.	1	1.00	2.45	0.83	3.44	26.25	0.00	1.60	5.86	3.99
time (sec)	N/A	0.332	0.594	0.795	0.043	0.104	0.000	0.171	0.197	1.640

Problem 137	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	C	A	A	B	B	F(-1)	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	133	151	228	90	316	3607	0	204	887	434
N.S.	1	1.14	1.71	0.68	2.38	27.12	0.00	1.53	6.67	3.26
time (sec)	N/A	0.398	0.153	0.947	0.063	0.109	0.000	0.189	0.192	0.165

Problem 138	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	B	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	291	291	194	188	387	568	498	431	477	231
N.S.	1	1.00	0.67	0.65	1.33	1.95	1.71	1.48	1.64	0.79
time (sec)	N/A	0.562	5.649	0.279	0.043	0.095	1.876	0.201	0.193	2.063

Problem 139	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	C	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	267	300	184	168	318	453	496	379	415	189
N.S.	1	1.12	0.69	0.63	1.19	1.70	1.86	1.42	1.55	0.71
time (sec)	N/A	0.539	4.432	0.279	0.045	0.093	1.332	0.194	0.195	3.920

Problem 140	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	B	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	204	204	159	141	280	380	340	327	353	164
N.S.	1	1.00	0.78	0.69	1.37	1.86	1.67	1.60	1.73	0.80
time (sec)	N/A	0.405	0.405	0.277	0.041	0.095	0.971	0.129	0.186	1.863

Problem 141	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	C	A	A	A	B	F(-1)	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	201	227	174	138	257	2609	0	279	341	315
N.S.	1	1.13	0.87	0.69	1.28	12.98	0.00	1.39	1.70	1.57
time (sec)	N/A	0.499	5.489	4.509	0.038	0.125	0.000	0.211	0.189	0.507

Problem 142	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	B	F(-1)	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	152	152	140	111	220	302	0	276	305	252
N.S.	1	1.00	0.92	0.73	1.45	1.99	0.00	1.82	2.01	1.66
time (sec)	N/A	0.459	4.292	2.374	0.041	0.087	0.000	0.214	0.191	0.385

Problem 143	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	C	A	A	A	B	F(-1)	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	156	179	166	115	244	3627	0	289	534	290
N.S.	1	1.15	1.06	0.74	1.56	23.25	0.00	1.85	3.42	1.86
time (sec)	N/A	0.485	4.691	2.296	0.046	0.157	0.000	0.244	0.193	1.840

Problem 144	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	B	F(-1)	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	129	129	186	101	260	3801	0	285	556	267
N.S.	1	1.00	1.44	0.78	2.02	29.47	0.00	2.21	4.31	2.07
time (sec)	N/A	0.398	4.580	2.024	0.046	0.157	0.000	0.241	0.187	1.798

Problem 145	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	C	A	A	A	B	F(-1)	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	148	168	239	108	255	4541	0	329	732	451
N.S.	1	1.14	1.61	0.73	1.72	30.68	0.00	2.22	4.95	3.05
time (sec)	N/A	0.461	7.079	1.945	0.038	0.148	0.000	0.250	0.201	1.770

Problem 146	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	B	F(-1)	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	131	131	238	99	365	4629	0	270	719	432
N.S.	1	1.00	1.82	0.76	2.79	35.34	0.00	2.06	5.49	3.30
time (sec)	N/A	0.439	3.426	1.857	0.043	0.131	0.000	0.233	0.198	1.742

Problem 147	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	C	A	A	B	B	F(-1)	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	166	188	269	119	355	6210	0	327	1205	486
N.S.	1	1.13	1.62	0.72	2.14	37.41	0.00	1.97	7.26	2.93
time (sec)	N/A	0.499	3.099	1.833	0.045	0.148	0.000	0.249	0.200	0.289

Problem 148	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	C	F	B	F	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	328	328	168	236	0	28816	0	0	413	1579
N.S.	1	1.00	0.51	0.72	0.00	87.85	0.00	0.00	1.26	4.81
time (sec)	N/A	0.947	0.856	2.692	0.000	4.391	0.000	0.000	0.239	11.241

Problem 149	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	C	F	B	F	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	295	309	299	190	0	28427	0	0	612	1114
N.S.	1	1.05	1.01	0.64	0.00	96.36	0.00	0.00	2.07	3.78
time (sec)	N/A	0.815	0.751	1.806	0.000	2.241	0.000	0.000	0.235	12.703

Problem 150	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	C	F	B	F	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	303	303	214	123	0	20941	0	0	299	906
N.S.	1	1.00	0.71	0.41	0.00	69.11	0.00	0.00	0.99	2.99
time (sec)	N/A	0.906	0.443	1.226	0.000	1.070	0.000	0.000	0.216	24.792

Problem 151	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	C	F	B	F	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	294	303	145	124	0	27931	0	0	25	1498
N.S.	1	1.03	0.49	0.42	0.00	95.00	0.00	0.00	0.09	5.10
time (sec)	N/A	0.756	11.048	0.805	0.000	1.024	0.000	0.000	0.198	10.112

Problem 152	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	C	F	B	F	F(-2)	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	262	262	275	78	0	24063	0	0	25	932
N.S.	1	1.00	1.05	0.30	0.00	91.84	0.00	0.00	0.10	3.56
time (sec)	N/A	0.585	11.055	0.765	0.000	1.111	0.000	0.000	0.193	12.161

Problem 153	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	C	F	B	F	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	290	298	199	82	0	18312	0	0	282	857
N.S.	1	1.03	0.69	0.28	0.00	63.14	0.00	0.00	0.97	2.96
time (sec)	N/A	0.627	11.049	0.754	0.000	1.063	0.000	0.000	0.206	22.414

Problem 154	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	C	F	B	F	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	280	280	131	87	0	24084	0	0	16	1261
N.S.	1	1.00	0.47	0.31	0.00	86.01	0.00	0.00	0.06	4.50
time (sec)	N/A	0.561	11.045	0.656	0.000	0.957	0.000	0.000	0.192	8.677

Problem 155	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	C	F	B	F(-1)	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	286	298	307	98	0	28005	0	0	23	2970
N.S.	1	1.04	1.07	0.34	0.00	97.92	0.00	0.00	0.08	10.38
time (sec)	N/A	0.691	11.088	0.959	0.000	2.253	0.000	0.000	0.210	59.382

Problem 156	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	C	F	B	F(-1)	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	304	304	230	118	0	21133	0	0	1088	1293
N.S.	1	1.00	0.76	0.39	0.00	69.52	0.00	0.00	3.58	4.25
time (sec)	N/A	0.838	0.790	1.128	0.000	1.027	0.000	0.000	0.289	24.871

Problem 157	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	C	F	B	F(-1)	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	322	332	191	138	0	29179	0	0	0	3605
N.S.	1	1.03	0.59	0.43	0.00	90.62	0.00	0.00	0.00	11.20
time (sec)	N/A	0.815	0.847	1.516	0.000	8.013	0.000	0.000	0.323	85.724

Problem 158	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	C	F	B	F(-1)	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	317	317	384	164	0	30233	0	0	0	3086
N.S.	1	1.00	1.21	0.52	0.00	95.37	0.00	0.00	0.00	9.74
time (sec)	N/A	0.775	5.149	1.783	0.000	5.803	0.000	0.000	0.378	60.904

Problem 159	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	111	150	82	74	175	174	306	155	194	88
N.S.	1	1.35	0.74	0.67	1.58	1.57	2.76	1.40	1.75	0.79
time (sec)	N/A	0.420	0.142	8.541	0.041	0.104	0.661	0.139	0.185	0.303

Problem 160	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	B	B	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	67	59	93	66	157	155	128	142	164	66
N.S.	1	0.88	1.39	0.99	2.34	2.31	1.91	2.12	2.45	0.99
time (sec)	N/A	0.285	0.010	6.013	0.042	0.112	0.457	0.139	0.186	0.182

Problem 161	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	83	114	63	56	122	109	206	113	141	64
N.S.	1	1.37	0.76	0.67	1.47	1.31	2.48	1.36	1.70	0.77
time (sec)	N/A	0.344	0.042	2.994	0.043	0.081	0.345	0.130	0.184	1.626

Problem 162	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	B	B	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	46	41	69	44	97	91	80	100	112	46
N.S.	1	0.89	1.50	0.96	2.11	1.98	1.74	2.17	2.43	1.00
time (sec)	N/A	0.240	0.008	2.051	0.041	0.075	0.226	0.126	0.199	0.103

Problem 163	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	52	52	49	36	66	59	100	66	89	38
N.S.	1	1.00	0.94	0.69	1.27	1.13	1.92	1.27	1.71	0.73
time (sec)	N/A	0.219	0.032	1.302	0.044	0.095	0.170	0.117	0.208	1.608

Problem 164	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	B	F	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	42	36	44	36	71	395	0	78	103	96
N.S.	1	0.86	1.05	0.86	1.69	9.40	0.00	1.86	2.45	2.29
time (sec)	N/A	0.268	0.006	0.520	0.044	0.124	0.000	0.123	0.202	0.113

Problem 165	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F(-1)	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	39	54	45	39	54	70	0	88	101	54
N.S.	1	1.38	1.15	1.00	1.38	1.79	0.00	2.26	2.59	1.38
time (sec)	N/A	0.302	0.075	0.537	0.035	0.090	0.000	0.132	0.198	1.619

Problem 166	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	A	B	B	F(-1)	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	47	53	101	38	115	690	0	107	236	126
N.S.	1	1.13	2.15	0.81	2.45	14.68	0.00	2.28	5.02	2.68
time (sec)	N/A	0.282	0.012	0.547	0.065	0.099	0.000	0.139	0.185	0.120

Problem 167	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	B	F(-1)	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	31	35	40	33	97	129	0	45	110	81
N.S.	1	1.13	1.29	1.06	3.13	4.16	0.00	1.45	3.55	2.61
time (sec)	N/A	0.286	0.007	0.494	0.040	0.091	0.000	0.136	0.190	0.113

Problem 168	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	A	B	B	F(-1)	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	64	79	132	54	174	1476	0	124	525	242
N.S.	1	1.23	2.06	0.84	2.72	23.06	0.00	1.94	8.20	3.78
time (sec)	N/A	0.295	0.066	0.604	0.040	0.105	0.000	0.144	0.181	1.668

Problem 169	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	B	F(-1)	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	47	42	71	45	228	333	0	97	152	337
N.S.	1	0.89	1.51	0.96	4.85	7.09	0.00	2.06	3.23	7.17
time (sec)	N/A	0.306	0.077	0.697	0.042	0.081	0.000	0.139	0.189	1.602

Problem 170	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	A	B	B	F(-1)	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	92	107	237	78	268	3115	0	207	834	472
N.S.	1	1.16	2.58	0.85	2.91	33.86	0.00	2.25	9.07	5.13
time (sec)	N/A	0.310	0.124	0.853	0.049	0.112	0.000	0.153	0.176	1.615

Problem 171	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	B	B	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	120	106	207	130	307	404	280	278	360	150
N.S.	1	0.88	1.72	1.08	2.56	3.37	2.33	2.32	3.00	1.25
time (sec)	N/A	0.394	0.222	148.509	0.046	0.103	1.862	0.159	0.179	0.363

Problem 172	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	B	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	161	208	139	114	260	305	484	241	322	149
N.S.	1	1.29	0.86	0.71	1.61	1.89	3.01	1.50	2.00	0.93
time (sec)	N/A	0.776	0.339	49.203	0.043	0.094	1.345	0.167	0.188	0.408

Problem 173	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	B	B	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	92	81	164	100	226	279	204	220	272	111
N.S.	1	0.88	1.78	1.09	2.46	3.03	2.22	2.39	2.96	1.21
time (sec)	N/A	0.313	0.021	18.247	0.039	0.096	0.941	0.168	0.187	0.276

Problem 174	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	125	164	92	111	183	205	332	183	234	108
N.S.	1	1.31	0.74	0.89	1.46	1.64	2.66	1.46	1.87	0.86
time (sec)	N/A	0.492	0.249	0.193	0.039	0.105	0.703	0.126	0.188	1.755

Problem 175	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	B	F(-1)	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	92	80	118	82	177	1575	0	196	234	198
N.S.	1	0.87	1.28	0.89	1.92	17.12	0.00	2.13	2.54	2.15
time (sec)	N/A	0.313	0.013	2.438	0.048	0.098	0.000	0.175	0.185	0.348

Problem 176	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	B	F(-1)	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	103	134	77	91	146	217	0	179	247	148
N.S.	1	1.30	0.75	0.88	1.42	2.11	0.00	1.74	2.40	1.44
time (sec)	N/A	0.534	0.203	1.535	0.046	0.118	0.000	0.206	0.213	1.724

Problem 177	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	B	F(-1)	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	92	92	165	74	204	2272	0	182	390	214
N.S.	1	1.00	1.79	0.80	2.22	24.70	0.00	1.98	4.24	2.33
time (sec)	N/A	0.412	0.023	1.443	0.047	0.122	0.000	0.186	0.315	0.240

Problem 178	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	B	F(-1)	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	91	111	68	75	165	300	0	142	313	164
N.S.	1	1.22	0.75	0.82	1.81	3.30	0.00	1.56	3.44	1.80
time (sec)	N/A	0.474	0.223	1.331	0.035	0.096	0.000	0.181	0.271	1.638

Problem 179	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	B	F(-1)	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	101	112	179	79	234	3356	0	179	725	328
N.S.	1	1.11	1.77	0.78	2.32	33.23	0.00	1.77	7.18	3.25
time (sec)	N/A	0.455	0.019	1.352	0.043	0.140	0.000	0.196	0.310	0.220

Problem 180	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	B	F(-1)	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	84	93	67	74	267	457	0	166	392	397
N.S.	1	1.11	0.80	0.88	3.18	5.44	0.00	1.98	4.67	4.73
time (sec)	N/A	0.465	0.496	1.287	0.047	0.096	0.000	0.193	0.289	1.619

Problem 181	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	A	B	B	F(-1)	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	111	133	278	92	299	4500	0	243	1047	535
N.S.	1	1.20	2.50	0.83	2.69	40.54	0.00	2.19	9.43	4.82
time (sec)	N/A	0.494	0.093	1.322	0.043	0.158	0.000	0.205	0.300	1.663

Problem 182	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	B	B	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	220	197	288	258	600	1030	592	520	752	319
N.S.	1	0.90	1.31	1.17	2.73	4.68	2.69	2.36	3.42	1.45
time (sec)	N/A	0.470	11.132	0.349	0.055	0.101	11.238	0.252	0.266	2.573

Problem 183	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	B	B	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	183	163	185	218	501	795	484	446	628	266
N.S.	1	0.89	1.01	1.19	2.74	4.34	2.64	2.44	3.43	1.45
time (sec)	N/A	0.423	11.491	0.333	0.044	0.099	6.194	0.247	0.204	2.187

Problem 184	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	B	B	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	143	126	157	176	399	594	377	372	504	211
N.S.	1	0.88	1.10	1.23	2.79	4.15	2.64	2.60	3.52	1.48
time (sec)	N/A	0.382	1.679	0.282	0.046	0.100	3.482	0.230	0.184	0.466

Problem 185	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	B	F(-1)	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	158	140	155	143	327	3824	0	377	430	326
N.S.	1	0.89	0.98	0.91	2.07	24.20	0.00	2.39	2.72	2.06
time (sec)	N/A	0.394	5.711	14.876	0.045	0.134	0.000	0.272	0.189	0.646

Problem 186	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	B	F(-1)	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	148	142	171	130	334	4895	0	300	586	326
N.S.	1	0.96	1.16	0.88	2.26	33.07	0.00	2.03	3.96	2.20
time (sec)	N/A	0.588	4.314	5.476	0.052	0.156	0.000	0.287	0.192	2.231

Problem 187	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	B	F(-1)	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	142	149	206	125	340	6441	0	271	936	421
N.S.	1	1.05	1.45	0.88	2.39	45.36	0.00	1.91	6.59	2.96
time (sec)	N/A	0.728	3.281	4.460	0.048	0.167	0.000	0.296	0.195	2.695

Problem 188	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	B	F(-1)	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	156	172	256	128	390	8547	0	321	1290	633
N.S.	1	1.10	1.64	0.82	2.50	54.79	0.00	2.06	8.27	4.06
time (sec)	N/A	0.911	3.168	3.836	0.044	0.201	0.000	0.318	0.199	2.266

Problem 189	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	B	F(-1)	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	171	198	233	143	463	10848	0	335	1920	759
N.S.	1	1.16	1.36	0.84	2.71	63.44	0.00	1.96	11.23	4.44
time (sec)	N/A	0.964	4.027	3.285	0.059	0.212	0.000	0.330	0.196	2.175

Problem 190	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	B	F(-1)	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	189	228	279	166	573	13503	0	477	2453	1194
N.S.	1	1.21	1.48	0.88	3.03	71.44	0.00	2.52	12.98	6.32
time (sec)	N/A	0.906	2.544	3.207	0.047	0.218	0.000	0.328	0.198	2.188

Problem 191	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	B	F(-1)	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	220	267	260	202	720	17811	0	537	3279	1314
N.S.	1	1.21	1.18	0.92	3.27	80.96	0.00	2.44	14.90	5.97
time (sec)	N/A	0.895	2.067	3.152	0.060	0.291	0.000	0.348	0.192	2.008

Problem 192	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	B	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	255	318	189	240	442	627	877	401	574	393
N.S.	1	1.25	0.74	0.94	1.73	2.46	3.44	1.57	2.25	1.54
time (sec)	N/A	1.331	1.886	0.252	0.048	0.103	4.770	0.236	0.196	0.808

Problem 193	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	B	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	211	266	156	193	344	461	666	327	450	210
N.S.	1	1.26	0.74	0.91	1.63	2.18	3.16	1.55	2.13	1.00
time (sec)	N/A	0.966	0.599	0.248	0.044	0.093	2.617	0.135	0.179	2.068

Problem 194	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	B	F(-1)	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	181	228	134	151	284	474	0	355	468	265
N.S.	1	1.26	0.74	0.83	1.57	2.62	0.00	1.96	2.59	1.46
time (sec)	N/A	0.967	6.516	6.199	0.055	0.098	0.000	0.277	0.194	2.014

Problem 195	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	B	F(-1)	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	161	197	131	134	282	567	0	285	565	269
N.S.	1	1.22	0.81	0.83	1.75	3.52	0.00	1.77	3.51	1.67
time (sec)	N/A	1.056	2.392	5.059	0.045	0.090	0.000	0.288	0.185	0.491

Problem 196	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	B	F(-1)	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	148	173	110	126	359	768	0	286	682	511
N.S.	1	1.17	0.74	0.85	2.43	5.19	0.00	1.93	4.61	3.45
time (sec)	N/A	1.014	3.824	4.139	0.047	0.099	0.000	0.299	0.185	1.965

Problem 197	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	B	F(-1)	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	133	148	106	121	537	928	0	253	695	749
N.S.	1	1.11	0.80	0.91	4.04	6.98	0.00	1.90	5.23	5.63
time (sec)	N/A	0.826	1.741	3.485	0.049	0.089	0.000	0.308	0.194	1.921

Problem 198	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	B	F(-1)	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	140	143	115	130	842	1314	0	360	784	1500
N.S.	1	1.02	0.82	0.93	6.01	9.39	0.00	2.57	5.60	10.71
time (sec)	N/A	0.668	2.388	2.977	0.046	0.109	0.000	0.317	0.195	1.898

Problem 199	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	B	F(-1)	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	147	139	239	145	1291	1607	0	359	682	1955
N.S.	1	0.95	1.63	0.99	8.78	10.93	0.00	2.44	4.64	13.30
time (sec)	N/A	0.418	6.253	2.631	0.058	0.105	0.000	0.311	0.190	1.821

Problem 200	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	A	B	B	F(-1)	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	144	127	350	177	1916	2323	0	563	744	3138
N.S.	1	0.88	2.43	1.23	13.31	16.13	0.00	3.91	5.17	21.79
time (sec)	N/A	0.363	3.342	3.540	0.051	0.107	0.000	0.329	0.188	1.966

Problem 201	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	A	B	B	F(-1)	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	182	162	404	218	2731	2967	0	621	830	3823
N.S.	1	0.89	2.22	1.20	15.01	16.30	0.00	3.41	4.56	21.01
time (sec)	N/A	0.419	4.351	5.510	0.069	0.109	0.000	0.346	0.199	2.096

Problem 202	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	A	B	B	F(-1)	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	221	198	458	258	3719	3585	0	679	916	4490
N.S.	1	0.90	2.07	1.17	16.83	16.22	0.00	3.07	4.14	20.32
time (sec)	N/A	0.451	5.668	6.608	0.065	0.128	0.000	0.346	0.191	2.199

Problem 203	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	A	B	B	F(-1)	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	248	222	548	298	4883	4259	0	737	1002	5190
N.S.	1	0.90	2.21	1.20	19.69	17.17	0.00	2.97	4.04	20.93
time (sec)	N/A	0.496	7.350	9.522	0.074	0.111	0.000	0.359	0.201	2.283

Problem 204	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	C	F	B	F(-1)	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	148	142	390	172	0	1617	0	0	634	1124
N.S.	1	0.96	2.64	1.16	0.00	10.93	0.00	0.00	4.28	7.59
time (sec)	N/A	0.433	1.579	3.898	0.000	0.132	0.000	0.000	0.249	10.660

Problem 205	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	C	F	B	F(-1)	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	139	134	235	142	0	1247	0	0	45	1046
N.S.	1	0.96	1.69	1.02	0.00	8.97	0.00	0.00	0.32	7.53
time (sec)	N/A	0.366	1.443	1.556	0.000	0.157	0.000	0.000	0.205	8.883

Problem 206	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	C	F	B	F(-1)	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	115	114	365	89	0	975	0	0	29	975
N.S.	1	0.99	3.17	0.77	0.00	8.48	0.00	0.00	0.25	8.48
time (sec)	N/A	0.309	6.576	0.841	0.000	0.133	0.000	0.000	0.194	7.010

Problem 207	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	C	F	B	F(-1)	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	125	123	221	98	0	979	0	0	27	1007
N.S.	1	0.98	1.77	0.78	0.00	7.83	0.00	0.00	0.22	8.06
time (sec)	N/A	0.299	4.202	0.791	0.000	0.121	0.000	0.000	0.197	9.136

Problem 208	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	C	F	B	F(-1)	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	136	131	397	130	0	1067	0	0	27	1243
N.S.	1	0.96	2.92	0.96	0.00	7.85	0.00	0.00	0.20	9.14
time (sec)	N/A	0.376	9.256	1.008	0.000	0.171	0.000	0.000	0.220	10.026

Problem 209	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	B	F(-1)	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	164	173	278	181	0	1954	0	0	29	1517
N.S.	1	1.05	1.70	1.10	0.00	11.91	0.00	0.00	0.18	9.25
time (sec)	N/A	0.389	1.635	1.686	0.000	0.169	0.000	0.000	0.235	13.072

Problem 210	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	F	B	F(-1)	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	155	173	158	162	0	1441	0	0	0	2191
N.S.	1	1.12	1.02	1.05	0.00	9.30	0.00	0.00	0.00	14.14
time (sec)	N/A	0.459	1.549	2.372	0.000	0.159	0.000	0.000	0.250	12.254

Problem 211	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	F	B	F(-1)	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	127	131	143	121	0	1009	0	0	29	1861
N.S.	1	1.03	1.13	0.95	0.00	7.94	0.00	0.00	0.23	14.65
time (sec)	N/A	0.398	2.519	0.870	0.000	0.144	0.000	0.000	0.197	11.122

Problem 212	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	F	B	F(-1)	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	125	168	127	94	0	975	0	0	414	1859
N.S.	1	1.34	1.02	0.75	0.00	7.80	0.00	0.00	3.31	14.87
time (sec)	N/A	0.338	4.409	0.746	0.000	0.118	0.000	0.000	0.220	13.665

Problem 213	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	F	B	F(-1)	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	115	168	128	102	0	975	0	0	20	1787
N.S.	1	1.46	1.11	0.89	0.00	8.48	0.00	0.00	0.17	15.54
time (sec)	N/A	0.339	1.120	0.683	0.000	0.174	0.000	0.000	0.190	10.987

Problem 214	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	F	B	F(-1)	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	139	134	143	130	0	1305	0	0	1467	2128
N.S.	1	0.96	1.03	0.94	0.00	9.39	0.00	0.00	10.55	15.31
time (sec)	N/A	0.427	1.225	1.190	0.000	0.142	0.000	0.000	0.345	12.432

Problem 215	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	F	B	F(-1)	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	148	140	165	168	0	2206	0	0	0	2178
N.S.	1	0.95	1.11	1.14	0.00	14.91	0.00	0.00	0.00	14.72
time (sec)	N/A	0.456	2.311	2.007	0.000	0.149	0.000	0.000	0.481	13.505

Problem 216	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	B	F	B	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	235	259	615	373	0	7664	0	0	0	0
N.S.	1	1.10	2.62	1.59	0.00	32.61	0.00	0.00	0.00	0.00
time (sec)	N/A	0.725	3.337	12.629	0.000	0.331	0.000	0.000	0.325	0.000

Problem 217	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	B	F	B	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	210	228	737	434	0	6266	0	0	0	0
N.S.	1	1.09	3.51	2.07	0.00	29.84	0.00	0.00	0.00	0.00
time (sec)	N/A	0.552	4.136	8.108	0.000	0.238	0.000	0.000	0.330	0.000

Problem 218	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	B	F	B	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	217	227	597	347	0	6250	0	0	0	0
N.S.	1	1.05	2.75	1.60	0.00	28.80	0.00	0.00	0.00	0.00
time (sec)	N/A	0.515	4.022	7.224	0.000	0.248	0.000	0.000	0.320	0.000

Problem 219	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	B	F	B	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	186	211	422	322	0	5238	0	0	0	0
N.S.	1	1.13	2.27	1.73	0.00	28.16	0.00	0.00	0.00	0.00
time (sec)	N/A	0.438	0.676	6.114	0.000	0.211	0.000	0.000	0.288	0.000

Problem 220	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	B	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	221	231	597	339	0	6018	0	0	0	0
N.S.	1	1.05	2.70	1.53	0.00	27.23	0.00	0.00	0.00	0.00
time (sec)	N/A	0.511	0.499	6.628	0.000	0.269	0.000	0.000	0.262	0.000

Problem 221	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	B	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	231	311	774	364	0	7793	0	0	0	0
N.S.	1	1.35	3.35	1.58	0.00	33.74	0.00	0.00	0.00	0.00
time (sec)	N/A	0.582	3.445	4.292	0.000	0.459	0.000	0.000	0.625	0.000

Problem 222	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	F	B	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	221	389	262	331	0	6944	0	0	0	0
N.S.	1	1.76	1.19	1.50	0.00	31.42	0.00	0.00	0.00	0.00
time (sec)	N/A	0.833	9.024	5.104	0.000	0.530	0.000	0.000	0.343	0.000

Problem 223	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	F	B	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	222	273	238	305	0	6045	0	0	0	0
N.S.	1	1.23	1.07	1.37	0.00	27.23	0.00	0.00	0.00	0.00
time (sec)	N/A	0.520	4.820	3.855	0.000	0.412	0.000	0.000	0.398	0.000

Problem 224	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	F	B	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	186	251	225	263	0	5658	0	0	0	0
N.S.	1	1.35	1.21	1.41	0.00	30.42	0.00	0.00	0.00	0.00
time (sec)	N/A	0.483	9.051	3.161	0.000	0.257	0.000	0.000	0.327	0.000

Problem 225	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	F	B	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	220	258	253	297	0	6525	0	0	0	0
N.S.	1	1.17	1.15	1.35	0.00	29.66	0.00	0.00	0.00	0.00
time (sec)	N/A	0.503	1.815	3.379	0.000	0.484	0.000	0.000	0.386	0.000

Problem 226	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	F	B	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	210	261	230	307	0	6522	0	0	0	0
N.S.	1	1.24	1.10	1.46	0.00	31.06	0.00	0.00	0.00	0.00
time (sec)	N/A	0.532	7.016	3.148	0.000	0.399	0.000	0.000	0.331	0.000

Problem 227	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	F	B	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	237	238	272	326	0	8824	0	0	0	0
N.S.	1	1.00	1.15	1.38	0.00	37.23	0.00	0.00	0.00	0.00
time (sec)	N/A	0.786	2.784	4.402	0.000	0.626	0.000	0.000	0.894	0.000

Problem 228	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	B	F	B	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	315	370	1021	652	0	21541	0	0	0	0
N.S.	1	1.17	3.24	2.07	0.00	68.38	0.00	0.00	0.00	0.00
time (sec)	N/A	0.751	7.361	23.331	0.000	0.618	0.000	0.000	0.572	0.000

Problem 229	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	B	F	B	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	290	305	802	585	0	20362	0	0	0	0
N.S.	1	1.05	2.77	2.02	0.00	70.21	0.00	0.00	0.00	0.00
time (sec)	N/A	0.638	6.459	21.815	0.000	0.463	0.000	0.000	0.480	0.000

Problem 230	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	B	F	B	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	313	360	1019	650	0	22506	0	0	0	0
N.S.	1	1.15	3.26	2.08	0.00	71.90	0.00	0.00	0.00	0.00
time (sec)	N/A	0.664	6.892	20.535	0.000	0.605	0.000	0.000	0.457	0.000

Problem 231	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	B	F	B	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	288	316	802	593	0	20961	0	0	0	0
N.S.	1	1.10	2.78	2.06	0.00	72.78	0.00	0.00	0.00	0.00
time (sec)	N/A	0.656	1.405	20.071	0.000	0.482	0.000	0.000	0.419	0.000

Problem 232	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	B	F	B	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	313	363	1018	641	0	22332	0	0	0	0
N.S.	1	1.16	3.25	2.05	0.00	71.35	0.00	0.00	0.00	0.00
time (sec)	N/A	0.671	1.270	19.664	0.000	0.608	0.000	0.000	0.375	0.000

Problem 233	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	B	F	B	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	319	591	1202	647	0	28586	0	0	0	0
N.S.	1	1.85	3.77	2.03	0.00	89.61	0.00	0.00	0.00	0.00
time (sec)	N/A	1.089	11.927	14.157	0.000	1.285	0.000	0.000	2.643	0.000

Problem 234	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	F	B	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	307	365	331	535	0	20486	0	0	0	0
N.S.	1	1.19	1.08	1.74	0.00	66.73	0.00	0.00	0.00	0.00
time (sec)	N/A	0.773	12.374	11.693	0.000	0.726	0.000	0.000	19.494	0.000

Problem 235	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	F	B	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	345	384	351	594	0	22729	0	0	0	0
N.S.	1	1.11	1.02	1.72	0.00	65.88	0.00	0.00	0.00	0.00
time (sec)	N/A	0.908	7.693	10.859	0.000	1.011	0.000	0.000	32.390	0.000

Problem 236	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	F	B	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	314	371	316	523	0	21932	0	0	0	0
N.S.	1	1.18	1.01	1.67	0.00	69.85	0.00	0.00	0.00	0.00
time (sec)	N/A	0.928	12.969	10.176	0.000	0.771	0.000	0.000	15.515	0.000

Problem 237	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	F	B	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	348	390	343	583	0	23355	0	0	0	0
N.S.	1	1.12	0.99	1.68	0.00	67.11	0.00	0.00	0.00	0.00
time (sec)	N/A	0.935	3.882	10.679	0.000	1.200	0.000	0.000	32.171	0.000

Problem 238	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	F	B	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	320	363	333	577	0	23125	0	0	0	0
N.S.	1	1.13	1.04	1.80	0.00	72.27	0.00	0.00	0.00	0.00
time (sec)	N/A	0.895	12.212	10.467	0.000	1.251	0.000	0.000	18.960	0.000

Problem 239	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	F	B	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	357	373	357	608	0	28429	0	0	0	0
N.S.	1	1.04	1.00	1.70	0.00	79.63	0.00	0.00	0.00	0.00
time (sec)	N/A	1.679	7.933	13.986	0.000	1.382	0.000	0.000	80.972	0.000

Problem 240	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	C	A	A	B	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	18	24	15	14	34	20	124	29	33	35
N.S.	1	1.33	0.83	0.78	1.89	1.11	6.89	1.61	1.83	1.94
time (sec)	N/A	0.274	0.005	132.785	0.033	0.085	2.279	0.120	0.166	1.954

Problem 241	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	20	18	18	26	25	12	153	28	27	25
N.S.	1	0.90	0.90	1.30	1.25	0.60	7.65	1.40	1.35	1.25
time (sec)	N/A	0.292	0.007	52.294	0.032	0.076	1.326	0.122	0.164	2.016

Problem 242	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	A	B	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	6	6	6	7	17	6	17	14	17	6
N.S.	1	1.00	1.00	1.17	2.83	1.00	2.83	2.33	2.83	1.00
time (sec)	N/A	0.285	0.001	18.701	0.051	0.085	0.741	0.138	0.160	1.971

Problem 243	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	5	5	5	6	5	5	2	5	5	5
N.S.	1	1.00	1.00	1.20	1.00	1.00	0.40	1.00	1.00	1.00
time (sec)	N/A	0.236	0.000	4.565	0.028	0.084	0.422	0.125	0.165	0.030

Problem 244	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	7	7	8	8	10	11	5	8	7	7
N.S.	1	1.00	1.14	1.14	1.43	1.57	0.71	1.14	1.00	1.00
time (sec)	N/A	0.276	0.001	4.559	0.116	0.080	0.085	0.122	0.165	0.078

Problem 245	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	B	B	F	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	22	20	20	40	40	151	0	52	56	54
N.S.	1	0.91	0.91	1.82	1.82	6.86	0.00	2.36	2.55	2.45
time (sec)	N/A	0.336	0.004	21.135	0.149	0.090	0.000	0.121	0.159	1.992

Problem 246	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	B	F	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	35	35	30	56	69	488	0	67	112	118
N.S.	1	1.00	0.86	1.60	1.97	13.94	0.00	1.91	3.20	3.37
time (sec)	N/A	0.443	0.004	147.095	0.120	0.092	0.000	0.133	0.164	1.924

Problem 247	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	89	106	63	95	152	117	250	121	167	76
N.S.	1	1.19	0.71	1.07	1.71	1.31	2.81	1.36	1.88	0.85
time (sec)	N/A	0.269	0.157	67.624	0.038	0.102	0.360	0.127	0.159	0.206

Problem 248	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	A	B	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	46	41	48	38	136	82	85	108	138	48
N.S.	1	0.89	1.04	0.83	2.96	1.78	1.85	2.35	3.00	1.04
time (sec)	N/A	0.243	0.099	27.520	0.036	0.101	0.224	0.127	0.165	0.123

Problem 249	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	61	77	43	70	76	59	150	71	89	38
N.S.	1	1.26	0.70	1.15	1.25	0.97	2.46	1.16	1.46	0.62
time (sec)	N/A	0.257	0.072	10.333	0.036	0.093	0.166	0.131	0.167	0.106

Problem 250	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	28	26	39	25	26	41	36	70	25	25
N.S.	1	0.93	1.39	0.89	0.93	1.46	1.29	2.50	0.89	0.89
time (sec)	N/A	0.218	0.006	3.693	0.027	0.096	0.101	0.125	0.160	0.071

Problem 251	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	B	F	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	28	26	38	34	56	101	0	40	66	88
N.S.	1	0.93	1.36	1.21	2.00	3.61	0.00	1.43	2.36	3.14
time (sec)	N/A	0.219	0.013	2.878	0.124	0.110	0.000	0.125	0.170	2.004

Problem 252	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	B	F	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	19	26	36	29	47	41	0	32	63	27
N.S.	1	1.37	1.89	1.53	2.47	2.16	0.00	1.68	3.32	1.42
time (sec)	N/A	0.222	0.012	1.098	0.040	0.075	0.000	0.132	0.167	0.081

Problem 253	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	B	F	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	42	46	71	70	136	324	0	105	177	127
N.S.	1	1.10	1.69	1.67	3.24	7.71	0.00	2.50	4.21	3.02
time (sec)	N/A	0.237	0.017	6.708	0.136	0.097	0.000	0.135	0.169	0.131

Problem 254	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	B	F	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	32	30	44	48	185	159	0	47	85	47
N.S.	1	0.94	1.38	1.50	5.78	4.97	0.00	1.47	2.66	1.47
time (sec)	N/A	0.229	0.007	17.007	0.038	0.070	0.000	0.135	0.168	0.096

Problem 255	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	B	F	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	70	75	115	99	228	1046	0	153	341	280
N.S.	1	1.07	1.64	1.41	3.26	14.94	0.00	2.19	4.87	4.00
time (sec)	N/A	0.246	0.022	33.955	0.131	0.116	0.000	0.144	0.166	1.992

Problem 256	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	B	F(-1)	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	54	49	102	84	486	343	0	83	139	298
N.S.	1	0.91	1.89	1.56	9.00	6.35	0.00	1.54	2.57	5.52
time (sec)	N/A	0.253	0.204	73.427	0.043	0.089	0.000	0.141	0.167	1.987

Problem 257	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	144	168	98	172	225	212	481	191	262	121
N.S.	1	1.17	0.68	1.19	1.56	1.47	3.34	1.33	1.82	0.84
time (sec)	N/A	0.328	1.605	0.144	0.038	0.089	0.699	0.139	0.167	0.376

Problem 258	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	B	B	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	74	66	64	65	242	188	136	196	272	80
N.S.	1	0.89	0.86	0.88	3.27	2.54	1.84	2.65	3.68	1.08
time (sec)	N/A	0.289	0.088	0.120	0.042	0.097	0.455	0.140	0.165	0.224

Problem 259	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	104	137	79	134	171	143	314	149	205	95
N.S.	1	1.32	0.76	1.29	1.64	1.38	3.02	1.43	1.97	0.91
time (sec)	N/A	0.342	1.444	175.691	0.057	0.098	0.348	0.133	0.164	2.186

Problem 260	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	B	A	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	49	44	44	41	45	106	58	134	42	42
N.S.	1	0.90	0.90	0.84	0.92	2.16	1.18	2.73	0.86	0.86
time (sec)	N/A	0.238	0.046	79.278	0.039	0.089	0.208	0.137	0.162	2.025

Problem 261	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	B	F	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	55	50	70	70	133	446	0	102	163	182
N.S.	1	0.91	1.27	1.27	2.42	8.11	0.00	1.85	2.96	3.31
time (sec)	N/A	0.267	0.180	77.582	0.127	0.099	0.000	0.144	0.162	0.182

Problem 262	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	A	F	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	53	65	50	71	119	97	0	131	161	75
N.S.	1	1.23	0.94	1.34	2.25	1.83	0.00	2.47	3.04	1.42
time (sec)	N/A	0.299	0.632	84.339	0.042	0.090	0.000	0.147	0.170	0.162

Problem 263	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	B	B	B	F	B	B	B
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	64	65	233	135	234	759	0	163	343	220
N.S.	1	1.02	3.64	2.11	3.66	11.86	0.00	2.55	5.36	3.44
time (sec)	N/A	0.291	3.022	70.117	0.133	0.100	0.000	0.147	0.165	2.094

Problem 264	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	B	B	B	F(-1)	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	47	51	95	92	267	200	0	98	170	194
N.S.	1	1.09	2.02	1.96	5.68	4.26	0.00	2.09	3.62	4.13
time (sec)	N/A	0.275	0.078	54.501	0.046	0.079	0.000	0.152	0.167	1.920

Problem 265	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	B	B	B	F(-1)	B	B	B
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	89	106	303	177	347	1472	0	218	533	327
N.S.	1	1.19	3.40	1.99	3.90	16.54	0.00	2.45	5.99	3.67
time (sec)	N/A	0.287	6.378	100.400	0.137	0.088	0.000	0.155	0.169	1.916

Problem 266	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	B	B	F(-1)	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	57	52	69	129	698	403	0	128	194	464
N.S.	1	0.91	1.21	2.26	12.25	7.07	0.00	2.25	3.40	8.14
time (sec)	N/A	0.287	0.978	190.416	0.044	0.086	0.000	0.146	0.168	1.930

Problem 267	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	B	B	F(-1)	B	B	B
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	121	134	715	208	483	2824	0	291	781	582
N.S.	1	1.11	5.91	1.72	3.99	23.34	0.00	2.40	6.45	4.81
time (sec)	N/A	0.346	8.913	0.297	0.131	0.109	0.000	0.161	0.173	1.961

Problem 268	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	B	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	198	237	144	267	363	376	774	293	448	209
N.S.	1	1.20	0.73	1.35	1.83	1.90	3.91	1.48	2.26	1.06
time (sec)	N/A	0.499	2.526	0.149	0.050	0.087	1.345	0.155	0.167	0.600

Problem 269	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	B	B	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	98	87	83	92	349	324	182	286	412	112
N.S.	1	0.89	0.85	0.94	3.56	3.31	1.86	2.92	4.20	1.14
time (sec)	N/A	0.312	0.176	0.115	0.047	0.098	0.916	0.164	0.175	0.304

Problem 270	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	163	211	120	216	287	257	559	231	356	166
N.S.	1	1.29	0.74	1.33	1.76	1.58	3.43	1.42	2.18	1.02
time (sec)	N/A	0.474	2.224	0.123	0.054	0.095	0.719	0.152	0.167	0.426

Problem 271	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	B	A	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	67	59	59	56	63	209	75	222	58	58
N.S.	1	0.88	0.88	0.84	0.94	3.12	1.12	3.31	0.87	0.87
time (sec)	N/A	0.285	0.091	0.047	0.028	0.092	0.470	0.460	0.165	0.162

Problem 272	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	B	F	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	86	78	100	114	233	1114	0	204	288	294
N.S.	1	0.91	1.16	1.33	2.71	12.95	0.00	2.37	3.35	3.42
time (sec)	N/A	0.302	0.390	0.159	0.145	0.097	0.000	0.153	0.176	2.016

Problem 273	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	B	F(-1)	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	92	107	78	131	215	178	0	197	281	141
N.S.	1	1.16	0.85	1.42	2.34	1.93	0.00	2.14	3.05	1.53
time (sec)	N/A	0.359	1.621	0.162	0.075	0.084	0.000	0.176	0.167	1.907

Problem 274	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	B	B	B	F(-1)	B	B	B
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	91	89	347	219	357	1679	0	247	531	308
N.S.	1	0.98	3.81	2.41	3.92	18.45	0.00	2.71	5.84	3.38
time (sec)	N/A	0.339	5.762	0.171	0.119	0.103	0.000	0.167	0.166	3.248

Problem 275	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	B	F(-1)	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	82	91	84	148	382	321	0	208	355	273
N.S.	1	1.11	1.02	1.80	4.66	3.91	0.00	2.54	4.33	3.33
time (sec)	N/A	0.327	1.928	0.198	0.059	0.107	0.000	0.176	0.172	0.161

Problem 276	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	B	B	B	F(-1)	B	B	B
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	103	105	472	273	489	2245	0	301	791	430
N.S.	1	1.02	4.58	2.65	4.75	21.80	0.00	2.92	7.68	4.17
time (sec)	N/A	0.346	9.375	0.178	0.144	0.106	0.000	0.203	0.168	1.819

Problem 277	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	B	B	F(-1)	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	74	75	103	199	824	530	0	213	347	563
N.S.	1	1.01	1.39	2.69	11.14	7.16	0.00	2.88	4.69	7.61
time (sec)	N/A	0.318	1.296	0.164	0.060	0.094	0.000	0.183	0.167	0.118

Problem 278	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	B	B	B	F(-1)	B	B	B
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	135	170	1192	314	646	3675	0	383	1058	601
N.S.	1	1.26	8.83	2.33	4.79	27.22	0.00	2.84	7.84	4.45
time (sec)	N/A	0.398	10.971	0.273	0.134	0.119	0.000	0.192	0.164	1.790

Problem 279	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	B	B	B	F(-1)	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	80	72	163	289	1754	814	0	260	358	994
N.S.	1	0.90	2.04	3.61	21.92	10.18	0.00	3.25	4.48	12.42
time (sec)	N/A	0.300	2.413	0.296	0.060	0.090	0.000	0.190	0.170	1.732

Problem 280	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	B	F(-1)	F(-2)	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	108	100	117	123	0	3069	0	0	1879	954
N.S.	1	0.93	1.08	1.14	0.00	28.42	0.00	0.00	17.40	8.83
time (sec)	N/A	0.331	0.358	0.112	0.000	0.128	0.000	0.000	0.188	2.390

Problem 281	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F(-2)	B	F(-1)	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	121	149	106	431	0	1817	0	226	630	264
N.S.	1	1.23	0.88	3.56	0.00	15.02	0.00	1.87	5.21	2.18
time (sec)	N/A	0.428	0.411	177.875	0.000	0.115	0.000	0.724	0.187	2.258

Problem 282	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	B	F(-1)	F(-2)	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	77	72	79	74	0	1493	0	0	1346	668
N.S.	1	0.94	1.03	0.96	0.00	19.39	0.00	0.00	17.48	8.68
time (sec)	N/A	0.301	0.193	71.187	0.000	0.119	0.000	0.000	0.188	2.170

Problem 283	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F(-2)	B	F(-1)	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	81	99	80	253	0	875	0	138	376	300
N.S.	1	1.22	0.99	3.12	0.00	10.80	0.00	1.70	4.64	3.70
time (sec)	N/A	0.331	0.246	29.166	0.000	0.109	0.000	0.550	0.177	2.500

Problem 284	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	B	F(-1)	F(-2)	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	52	50	50	44	0	662	0	0	807	426
N.S.	1	0.96	0.96	0.85	0.00	12.73	0.00	0.00	15.52	8.19
time (sec)	N/A	0.258	0.037	10.221	0.000	0.137	0.000	0.000	0.182	2.013

Problem 285	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F(-2)	B	F(-1)	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	50	57	50	112	0	443	0	68	134	166
N.S.	1	1.14	1.00	2.24	0.00	8.86	0.00	1.36	2.68	3.32
time (sec)	N/A	0.280	0.196	3.126	0.000	0.127	0.000	0.402	0.170	0.479

Problem 286	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	B	B	F(-2)	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	32	32	32	24	0	464	107	0	31	23
N.S.	1	1.00	1.00	0.75	0.00	14.50	3.34	0.00	0.97	0.72
time (sec)	N/A	0.235	0.010	2.767	0.000	0.095	1.094	0.000	0.161	1.741

Problem 287	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	F	B	F	F(-2)	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	59	57	54	148	0	511	0	0	364	648
N.S.	1	0.97	0.92	2.51	0.00	8.66	0.00	0.00	6.17	10.98
time (sec)	N/A	0.257	0.124	21.899	0.000	0.128	0.000	0.000	0.184	2.192

Problem 288	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F(-2)	B	F	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	60	58	60	195	0	709	0	80	363	265
N.S.	1	0.97	1.00	3.25	0.00	11.82	0.00	1.33	6.05	4.42
time (sec)	N/A	0.274	0.295	61.497	0.000	0.123	0.000	0.228	0.198	2.353

Problem 289	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	F	B	F	F(-2)	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	92	103	91	241	0	1644	0	0	1396	2797
N.S.	1	1.12	0.99	2.62	0.00	17.87	0.00	0.00	15.17	30.40
time (sec)	N/A	0.316	0.201	135.317	0.000	0.146	0.000	0.000	0.234	6.895

Problem 290	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F(-2)	B	F	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	88	83	84	242	0	2444	0	138	911	710
N.S.	1	0.94	0.95	2.75	0.00	27.77	0.00	1.57	10.35	8.07
time (sec)	N/A	0.315	0.503	283.409	0.000	0.117	0.000	0.231	0.236	3.261

Problem 291	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F	B	F	F(-2)	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	138	161	139	352	0	5500	0	0	2728	6237
N.S.	1	1.17	1.01	2.55	0.00	39.86	0.00	0.00	19.77	45.20
time (sec)	N/A	0.385	0.379	0.252	0.000	0.203	0.000	0.000	0.466	12.964

Problem 292	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F(-2)	B	F(-1)	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	126	118	119	346	0	6046	0	253	1587	1152
N.S.	1	0.94	0.94	2.75	0.00	47.98	0.00	2.01	12.60	9.14
time (sec)	N/A	0.367	0.765	0.237	0.000	0.157	0.000	0.227	0.310	3.753

Problem 293	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F(-2)	B	F(-1)	B	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	158	177	118	425	0	3629	0	305	1910	0
N.S.	1	1.12	0.75	2.69	0.00	22.97	0.00	1.93	12.09	0.00
time (sec)	N/A	0.480	0.689	0.239	0.000	0.142	0.000	1.227	0.242	0.000

Problem 294	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	B	F(-1)	F(-2)	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	104	99	106	97	0	2741	0	0	3876	0
N.S.	1	0.95	1.02	0.93	0.00	26.36	0.00	0.00	37.27	0.00
time (sec)	N/A	0.357	0.233	143.924	0.000	0.131	0.000	0.000	0.207	0.000

Problem 295	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F(-2)	B	F(-1)	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	100	115	108	315	0	1527	0	178	1033	0
N.S.	1	1.15	1.08	3.15	0.00	15.27	0.00	1.78	10.33	0.00
time (sec)	N/A	0.380	0.664	58.012	0.000	0.127	0.000	0.941	0.179	0.000

Problem 296	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	B	F(-1)	F(-2)	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	77	75	75	69	0	1617	0	0	2867	0
N.S.	1	0.97	0.97	0.90	0.00	21.00	0.00	0.00	37.23	0.00
time (sec)	N/A	0.302	0.236	58.035	0.000	0.114	0.000	0.000	0.196	0.000

Problem 297	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F(-2)	B	F(-1)	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	79	77	78	236	0	1421	0	126	700	0
N.S.	1	0.97	0.99	2.99	0.00	17.99	0.00	1.59	8.86	0.00
time (sec)	N/A	0.303	0.356	57.577	0.000	0.160	0.000	0.459	0.178	0.000

Problem 298	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	B	B	F(-2)	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	66	64	64	55	0	1324	377	0	89	54
N.S.	1	0.97	0.97	0.83	0.00	20.06	5.71	0.00	1.35	0.82
time (sec)	N/A	0.254	0.038	59.599	0.000	0.104	6.128	0.000	0.163	1.761

Problem 299	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F	B	F	F(-2)	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	106	116	174	305	0	2143	0	0	3045	0
N.S.	1	1.09	1.64	2.88	0.00	20.22	0.00	0.00	28.73	0.00
time (sec)	N/A	0.334	0.308	0.239	0.000	0.154	0.000	0.000	0.245	0.000

Problem 300	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F(-2)	B	F	B	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	114	109	105	307	0	3147	0	221	1893	0
N.S.	1	0.96	0.92	2.69	0.00	27.61	0.00	1.94	16.61	0.00
time (sec)	N/A	0.422	0.843	0.261	0.000	0.144	0.000	0.289	0.261	0.000

Problem 301	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F	B	F	F(-2)	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	157	171	230	368	0	6548	0	0	5140	0
N.S.	1	1.09	1.46	2.34	0.00	41.71	0.00	0.00	32.74	0.00
time (sec)	N/A	0.437	0.641	0.249	0.000	0.206	0.000	0.000	0.547	0.000

Problem 302	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F(-2)	B	F	B	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	143	135	130	356	0	7894	0	270	4421	0
N.S.	1	0.94	0.91	2.49	0.00	55.20	0.00	1.89	30.92	0.00
time (sec)	N/A	0.435	1.528	0.272	0.000	0.191	0.000	0.284	0.392	0.000

Problem 303	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F(-2)	B	F(-1)	B	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	160	181	164	429	0	5511	0	353	20217	0
N.S.	1	1.13	1.02	2.68	0.00	34.44	0.00	2.21	126.36	0.00
time (sec)	N/A	0.429	1.242	0.253	0.000	0.166	0.000	0.968	0.437	0.000

Problem 304	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	B	F(-1)	F(-2)	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	136	143	149	124	0	5846	0	0	30251	0
N.S.	1	1.05	1.10	0.91	0.00	42.99	0.00	0.00	222.43	0.00
time (sec)	N/A	0.327	0.276	0.106	0.000	0.162	0.000	0.000	0.315	0.000

Problem 305	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F(-2)	B	F(-1)	B	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	114	117	102	300	0	4486	0	244	11451	0
N.S.	1	1.03	0.89	2.63	0.00	39.35	0.00	2.14	100.45	0.00
time (sec)	N/A	0.304	0.809	0.240	0.000	0.132	0.000	1.027	0.228	0.000

Problem 306	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	B	F(-1)	F(-2)	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	117	115	114	91	0	4911	0	0	21865	0
N.S.	1	0.98	0.97	0.78	0.00	41.97	0.00	0.00	186.88	0.00
time (sec)	N/A	0.290	0.411	0.106	0.000	0.162	0.000	0.000	0.265	0.000

Problem 307	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F(-2)	B	F(-1)	B	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	143	139	124	376	0	5183	0	269	13353	0
N.S.	1	0.97	0.87	2.63	0.00	36.24	0.00	1.88	93.38	0.00
time (sec)	N/A	0.311	0.995	0.188	0.000	0.174	0.000	0.637	0.237	0.000

Problem 308	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	B	B	F(-2)	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	96	99	79	85	0	3937	835	0	161	87
N.S.	1	1.03	0.82	0.89	0.00	41.01	8.70	0.00	1.68	0.91
time (sec)	N/A	0.269	0.204	0.054	0.000	0.137	23.920	0.000	0.164	1.830

Problem 309	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	B	F	B	F(-1)	F(-2)	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	159	183	321	414	0	8083	0	0	8621	0
N.S.	1	1.15	2.02	2.60	0.00	50.84	0.00	0.00	54.22	0.00
time (sec)	N/A	0.384	0.550	0.250	0.000	0.243	0.000	0.000	0.394	0.000

Problem 310	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F(-2)	B	F(-1)	B	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	172	164	165	394	0	9442	0	367	5857	0
N.S.	1	0.95	0.96	2.29	0.00	54.90	0.00	2.13	34.05	0.00
time (sec)	N/A	0.475	1.203	0.252	0.000	0.206	0.000	0.304	0.319	0.000

Problem 311	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F	B	F(-1)	F(-2)	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	217	252	232	477	0	18765	0	0	13647	0
N.S.	1	1.16	1.07	2.20	0.00	86.47	0.00	0.00	62.89	0.00
time (sec)	N/A	0.484	1.037	0.248	0.000	0.463	0.000	0.000	1.710	0.000

Problem 312	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F(-2)	B	F(-1)	B	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	203	192	169	445	0	19032	0	436	8497	0
N.S.	1	0.95	0.83	2.19	0.00	93.75	0.00	2.15	41.86	0.00
time (sec)	N/A	0.518	1.726	0.271	0.000	0.297	0.000	0.377	0.924	0.000

Problem 313	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	B	B	B	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	19	21	24	39	64	70	238	41	56	56
N.S.	1	1.11	1.26	2.05	3.37	3.68	12.53	2.16	2.95	2.95
time (sec)	N/A	0.218	0.012	0.909	0.123	0.131	2.890	0.116	0.161	0.133

Problem 314	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	B	B	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	10	10	14	18	39	71	129	37	72	39
N.S.	1	1.00	1.40	1.80	3.90	7.10	12.90	3.70	7.20	3.90
time (sec)	N/A	0.207	0.008	3.804	0.041	0.102	0.563	0.133	0.167	0.060

Problem 315	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	B	B	B	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	30	43	32	50	75	163	2431	61	98	66
N.S.	1	1.43	1.07	1.67	2.50	5.43	81.03	2.03	3.27	2.20
time (sec)	N/A	0.239	0.043	11.235	0.112	0.106	6.889	0.128	0.165	1.700

Problem 316	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	B	F(-1)	B	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	117	112	124	146	0	3169	0	871	24	0
N.S.	1	0.96	1.06	1.25	0.00	27.09	0.00	7.44	0.21	0.00
time (sec)	N/A	0.299	0.458	0.411	0.000	0.195	0.000	0.392	0.169	0.000

Problem 317	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	B	F	B	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	72	70	96	60	0	2307	0	407	65	61
N.S.	1	0.97	1.33	0.83	0.00	32.04	0.00	5.65	0.90	0.85
time (sec)	N/A	0.255	0.173	0.179	0.000	0.154	0.000	0.259	0.165	1.839

Problem 318	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	F	B	F	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	85	83	130	51	0	4915	0	0	22	0
N.S.	1	0.98	1.53	0.60	0.00	57.82	0.00	0.00	0.26	0.00
time (sec)	N/A	0.321	0.506	0.281	0.000	0.211	0.000	0.000	0.166	0.000

Problem 319	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	C	F	B	F	B	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	86	90	175	35	0	1327	0	668	24	0
N.S.	1	1.05	2.03	0.41	0.00	15.43	0.00	7.77	0.28	0.00
time (sec)	N/A	0.320	1.053	0.398	0.000	0.161	0.000	0.244	0.164	0.000

Problem 320	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	C	F	B	F	B	F	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	144	154	684	35	0	3727	0	2122	24	0
N.S.	1	1.07	4.75	0.24	0.00	25.88	0.00	14.74	0.17	0.00
time (sec)	N/A	0.360	10.054	0.191	0.000	0.283	0.000	0.460	0.168	0.000

Problem 321	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	300	348	211	521	0	0	0	0	24	0
N.S.	1	1.16	0.70	1.74	0.00	0.00	0.00	0.00	0.08	0.00
time (sec)	N/A	0.587	1.208	6.219	0.000	0.000	0.000	0.000	0.162	0.000

Problem 322	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	223	276	168	351	0	0	0	0	24	0
N.S.	1	1.24	0.75	1.57	0.00	0.00	0.00	0.00	0.11	0.00
time (sec)	N/A	0.485	0.731	3.963	0.000	0.000	0.000	0.000	0.162	0.000

Problem 323	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	61	60	69	140	0	0	0	0	15	0
N.S.	1	0.98	1.13	2.30	0.00	0.00	0.00	0.00	0.25	0.00
time (sec)	N/A	0.341	0.044	2.050	0.000	0.000	0.000	0.000	0.162	0.000

Problem 324	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	B	F	B	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	70	100	148	177	0	535	0	0	24	0
N.S.	1	1.43	2.11	2.53	0.00	7.64	0.00	0.00	0.34	0.00
time (sec)	N/A	0.336	0.554	2.543	0.000	0.107	0.000	0.000	0.165	0.000

Problem 325	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	B	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	206	249	204	318	0	2257	0	0	24	0
N.S.	1	1.21	0.99	1.54	0.00	10.96	0.00	0.00	0.12	0.00
time (sec)	N/A	0.450	2.142	6.081	0.000	0.124	0.000	0.000	0.166	0.000

Problem 326	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	B	F(-1)	B	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	157	144	149	215	0	4491	0	1546	61	0
N.S.	1	0.92	0.95	1.37	0.00	28.61	0.00	9.85	0.39	0.00
time (sec)	N/A	0.343	0.860	0.318	0.000	0.264	0.000	0.781	0.201	0.000

Problem 327	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	B	F(-1)	B	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	104	102	93	90	0	3049	0	791	94	60
N.S.	1	0.98	0.89	0.87	0.00	29.32	0.00	7.61	0.90	0.58
time (sec)	N/A	0.267	0.319	0.184	0.000	0.202	0.000	0.499	0.182	1.864

Problem 328	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	F	B	F	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	125	123	142	63	0	6113	0	0	57	0
N.S.	1	0.98	1.14	0.50	0.00	48.90	0.00	0.00	0.46	0.00
time (sec)	N/A	0.353	0.336	0.276	0.000	0.357	0.000	0.000	0.201	0.000

Problem 329	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	F	B	F(-1)	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	133	145	150	63	0	7126	0	0	61	0
N.S.	1	1.09	1.13	0.47	0.00	53.58	0.00	0.00	0.46	0.00
time (sec)	N/A	0.380	0.517	0.370	0.000	0.329	0.000	0.000	0.211	0.000

Problem 330	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	C	F	B	F(-1)	B	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	126	134	66	63	0	3089	0	2018	61	0
N.S.	1	1.06	0.52	0.50	0.00	24.52	0.00	16.02	0.48	0.00
time (sec)	N/A	0.324	0.095	0.510	0.000	0.276	0.000	0.690	0.205	0.000

Problem 331	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	C	F	B	F(-1)	B	F	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	205	198	959	63	0	7633	0	4563	61	0
N.S.	1	0.97	4.68	0.31	0.00	37.23	0.00	22.26	0.30	0.00
time (sec)	N/A	0.370	10.912	0.197	0.000	0.776	0.000	1.580	0.209	0.000

Problem 332	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	B	F	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	355	415	256	730	0	0	0	0	61	0
N.S.	1	1.17	0.72	2.06	0.00	0.00	0.00	0.00	0.17	0.00
time (sec)	N/A	0.585	2.014	7.959	0.000	0.000	0.000	0.000	0.196	0.000

Problem 333	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	294	347	213	535	0	0	0	0	61	0
N.S.	1	1.18	0.72	1.82	0.00	0.00	0.00	0.00	0.21	0.00
time (sec)	N/A	0.508	1.146	5.872	0.000	0.000	0.000	0.000	0.203	0.000

Problem 334	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	176	175	169	428	0	0	0	0	44	0
N.S.	1	0.99	0.96	2.43	0.00	0.00	0.00	0.00	0.25	0.00
time (sec)	N/A	0.919	0.523	3.862	0.000	0.000	0.000	0.000	0.174	0.000

Problem 335	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	174	278	160	334	0	0	0	0	61	0
N.S.	1	1.60	0.92	1.92	0.00	0.00	0.00	0.00	0.35	0.00
time (sec)	N/A	0.436	0.930	3.853	0.000	0.000	0.000	0.000	0.218	0.000

Problem 336	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	B	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	193	237	197	324	0	2071	0	0	61	0
N.S.	1	1.23	1.02	1.68	0.00	10.73	0.00	0.00	0.32	0.00
time (sec)	N/A	0.414	1.601	5.040	0.000	0.122	0.000	0.000	0.211	0.000

Problem 337	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	B	F(-1)	B	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	79	77	77	93	0	2367	0	433	38	0
N.S.	1	0.97	0.97	1.18	0.00	29.96	0.00	5.48	0.48	0.00
time (sec)	N/A	0.299	0.072	0.312	0.000	0.149	0.000	0.215	0.166	0.000

Problem 338	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	B	F	F(-2)	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	38	38	38	34	0	1878	0	0	36	33
N.S.	1	1.00	1.00	0.89	0.00	49.42	0.00	0.00	0.95	0.87
time (sec)	N/A	0.255	0.013	0.166	0.000	0.139	0.000	0.000	0.160	1.957

Problem 339	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	F	B	F	A	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	46	46	46	35	0	598	0	80	36	0
N.S.	1	1.00	1.00	0.76	0.00	13.00	0.00	1.74	0.78	0.00
time (sec)	N/A	0.306	0.026	0.308	0.000	0.106	0.000	0.157	0.178	0.000

Problem 340	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	C	F	B	F	B	F	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	97	101	417	35	0	1503	0	694	38	0
N.S.	1	1.04	4.30	0.36	0.00	15.49	0.00	7.15	0.39	0.00
time (sec)	N/A	0.364	8.644	2.229	0.000	0.157	0.000	0.266	0.160	0.000

Problem 341	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	241	292	179	356	0	0	0	0	38	0
N.S.	1	1.21	0.74	1.48	0.00	0.00	0.00	0.00	0.16	0.00
time (sec)	N/A	0.490	0.843	3.116	0.000	0.000	0.000	0.000	0.162	0.000

Problem 342	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	177	228	95	86	0	0	0	0	38	0
N.S.	1	1.29	0.54	0.49	0.00	0.00	0.00	0.00	0.21	0.00
time (sec)	N/A	0.432	0.336	1.516	0.000	0.000	0.000	0.000	0.158	0.000

Problem 343	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	B	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	61	60	68	86	0	147	0	0	30	0
N.S.	1	0.98	1.11	1.41	0.00	2.41	0.00	0.00	0.49	0.00
time (sec)	N/A	0.345	0.037	0.588	0.000	0.109	0.000	0.000	0.165	0.000

Problem 344	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	B	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	160	283	159	133	0	576	0	0	38	0
N.S.	1	1.77	0.99	0.83	0.00	3.60	0.00	0.00	0.24	0.00
time (sec)	N/A	0.467	0.670	1.449	0.000	0.104	0.000	0.000	0.167	0.000

Problem 345	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	B	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	219	270	219	343	0	2443	0	0	38	0
N.S.	1	1.23	1.00	1.57	0.00	11.16	0.00	0.00	0.17	0.00
time (sec)	N/A	0.477	1.742	2.040	0.000	0.148	0.000	0.000	0.166	0.000

Problem 346	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	B	F(-1)	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	77	75	89	85	0	3014	0	0	54	0
N.S.	1	0.97	1.16	1.10	0.00	39.14	0.00	0.00	0.70	0.00
time (sec)	N/A	0.331	0.118	0.325	0.000	0.206	0.000	0.000	0.165	0.000

Problem 347	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	B	F	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	29	29	29	28	236	245	0	131	40	191
N.S.	1	1.00	1.00	0.97	8.14	8.45	0.00	4.52	1.38	6.59
time (sec)	N/A	0.265	0.020	0.233	0.145	0.118	0.000	0.215	0.163	2.107

Problem 348	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	C	F	B	F	B	F	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	85	83	254	101	0	1717	0	323	52	0
N.S.	1	0.98	2.99	1.19	0.00	20.20	0.00	3.80	0.61	0.00
time (sec)	N/A	0.302	3.424	0.332	0.000	0.189	0.000	0.261	0.163	0.000

Problem 349	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	C	F	B	F	B	F	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	142	149	231	95	0	4845	0	1031	54	0
N.S.	1	1.05	1.63	0.67	0.00	34.12	0.00	7.26	0.38	0.00
time (sec)	N/A	0.376	3.837	0.198	0.000	0.372	0.000	0.507	0.162	0.000

Problem 350	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	296	365	196	498	0	0	0	0	54	0
N.S.	1	1.23	0.66	1.68	0.00	0.00	0.00	0.00	0.18	0.00
time (sec)	N/A	0.526	1.088	5.704	0.000	0.000	0.000	0.000	0.169	0.000

Problem 351	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	226	291	155	334	0	0	0	0	54	0
N.S.	1	1.29	0.69	1.48	0.00	0.00	0.00	0.00	0.24	0.00
time (sec)	N/A	0.445	0.654	5.441	0.000	0.000	0.000	0.000	0.169	0.000

Problem 352	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	B	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	91	119	143	181	0	1068	0	0	54	0
N.S.	1	1.31	1.57	1.99	0.00	11.74	0.00	0.00	0.59	0.00
time (sec)	N/A	0.311	0.419	2.883	0.000	0.114	0.000	0.000	0.162	0.000

Problem 353	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F	B	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	116	115	100	252	0	1464	0	0	46	0
N.S.	1	0.99	0.86	2.17	0.00	12.62	0.00	0.00	0.40	0.00
time (sec)	N/A	0.467	0.097	1.034	0.000	0.135	0.000	0.000	0.164	0.000

Problem 354	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	B	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	234	277	178	345	0	2786	0	0	54	0
N.S.	1	1.18	0.76	1.47	0.00	11.91	0.00	0.00	0.23	0.00
time (sec)	N/A	0.422	1.082	4.537	0.000	0.155	0.000	0.000	0.167	0.000

Problem 355	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	B	F(-1)	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	126	137	126	208	0	6662	0	0	70	0
N.S.	1	1.09	1.00	1.65	0.00	52.87	0.00	0.00	0.56	0.00
time (sec)	N/A	0.337	0.621	0.741	0.000	0.480	0.000	0.000	0.167	0.000

Problem 356	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	B	F(-1)	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	81	73	50	138	927	945	0	408	110	144
N.S.	1	0.90	0.62	1.70	11.44	11.67	0.00	5.04	1.36	1.78
time (sec)	N/A	0.271	0.074	0.444	0.173	0.212	0.000	0.346	0.169	2.754

Problem 357	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	B	F(-1)	B	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	65	63	47	56	485	912	0	380	68	129
N.S.	1	0.97	0.72	0.86	7.46	14.03	0.00	5.85	1.05	1.98
time (sec)	N/A	0.246	0.035	2.108	0.141	0.200	0.000	0.315	0.168	2.449

Problem 358	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	C	F	B	F	F(-2)	F	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	134	144	1331	169	0	5396	0	0	68	0
N.S.	1	1.07	9.93	1.26	0.00	40.27	0.00	0.00	0.51	0.00
time (sec)	N/A	0.361	8.351	3.688	0.000	0.395	0.000	0.000	0.173	0.000

Problem 359	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	B	F	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	307	370	206	812	0	0	0	0	70	0
N.S.	1	1.21	0.67	2.64	0.00	0.00	0.00	0.00	0.23	0.00
time (sec)	N/A	0.549	1.949	8.078	0.000	0.000	0.000	0.000	0.170	0.000

Problem 360	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	B	F	B	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	238	270	178	597	0	4355	0	0	70	0
N.S.	1	1.13	0.75	2.51	0.00	18.30	0.00	0.00	0.29	0.00
time (sec)	N/A	0.431	1.262	5.799	0.000	0.193	0.000	0.000	0.171	0.000

Problem 361	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	B	F	B	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	246	273	193	662	0	4770	0	0	70	0
N.S.	1	1.11	0.78	2.69	0.00	19.39	0.00	0.00	0.28	0.00
time (sec)	N/A	0.456	1.202	5.210	0.000	0.218	0.000	0.000	0.166	0.000

Problem 362	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	B	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	253	258	190	406	0	5442	0	0	62	0
N.S.	1	1.02	0.75	1.60	0.00	21.51	0.00	0.00	0.25	0.00
time (sec)	N/A	1.514	0.883	1.447	0.000	0.248	0.000	0.000	0.158	0.000

Problem 363	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	B	F	B	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	309	365	260	1002	0	8928	0	0	70	0
N.S.	1	1.18	0.84	3.24	0.00	28.89	0.00	0.00	0.23	0.00
time (sec)	N/A	0.567	2.636	6.717	0.000	0.441	0.000	0.000	0.172	0.000

Problem 364	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	F	F	F	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	117	117	0	0	0	0	0	0	29	0
N.S.	1	1.00	0.00	0.00	0.00	0.00	0.00	0.00	0.25	0.00
time (sec)	N/A	0.387	0.000	0.000	0.000	0.000	0.000	0.000	0.186	0.000

Problem 365	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	F	F	F	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	208	210	0	0	0	0	0	0	0	0
N.S.	1	1.01	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.436	0.000	0.000	0.000	0.000	0.000	0.000	18.770	0.000

Problem 366	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	125	117	120	0	0	0	0	0	0	0
N.S.	1	0.94	0.96	0.00	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.298	0.148	0.000	0.000	0.000	0.000	0.000	4.461	0.000

Problem 367	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F(-1)	F	F	B
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	67	67	67	0	0	0	0	0	151	64
N.S.	1	1.00	1.00	0.00	0.00	0.00	0.00	0.00	2.25	0.96
time (sec)	N/A	0.244	0.020	0.000	0.000	0.000	0.000	0.000	0.287	2.461

Problem 368	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	F	F	F	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	78	78	0	0	0	0	0	0	23	0
N.S.	1	1.00	0.00	0.00	0.00	0.00	0.00	0.00	0.29	0.00
time (sec)	N/A	0.271	0.000	0.000	0.000	0.000	0.000	0.000	0.210	0.000

Problem 369	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	F	F	F	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	78	78	0	0	0	0	0	0	25	0
N.S.	1	1.00	0.00	0.00	0.00	0.00	0.00	0.00	0.32	0.00
time (sec)	N/A	0.275	0.000	0.000	0.000	0.000	0.000	0.000	200.020	0.000

Problem 370	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	F	F	F	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	92	92	0	0	0	0	0	0	0	0
N.S.	1	1.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.282	0.000	0.000	0.000	0.000	0.000	0.000	2.558	0.000

Problem 371	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	F	F	F	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	92	92	0	0	0	0	0	0	0	0
N.S.	1	1.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.283	0.000	0.000	0.000	0.000	0.000	0.000	0.741	0.000

Problem 372	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	F	F	F	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	92	92	0	0	0	0	0	0	16	0
N.S.	1	1.00	0.00	0.00	0.00	0.00	0.00	0.00	0.17	0.00
time (sec)	N/A	0.244	0.000	0.000	0.000	0.000	0.000	0.000	0.157	0.000

Problem 373	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	F	F	F	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	92	92	0	0	0	0	0	0	25	0
N.S.	1	1.00	0.00	0.00	0.00	0.00	0.00	0.00	0.27	0.00
time (sec)	N/A	0.284	0.000	0.000	0.000	0.000	0.000	0.000	200.025	0.000

Problem 374	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	F	F	F	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	92	92	0	0	0	0	0	0	25	0
N.S.	1	1.00	0.00	0.00	0.00	0.00	0.00	0.00	0.27	0.00
time (sec)	N/A	0.287	0.000	0.000	0.000	0.000	0.000	0.000	200.023	0.000

Problem 375	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	B	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	259	235	220	262	0	2595	0	0	471	0
N.S.	1	0.91	0.85	1.01	0.00	10.02	0.00	0.00	1.82	0.00
time (sec)	N/A	0.576	0.304	0.732	0.000	0.767	0.000	0.000	0.213	0.000

Problem 376	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	B	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	136	124	117	117	0	879	0	0	260	0
N.S.	1	0.91	0.86	0.86	0.00	6.46	0.00	0.00	1.91	0.00
time (sec)	N/A	0.446	0.106	0.363	0.000	0.679	0.000	0.000	0.170	0.000

Problem 377	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F	B	A	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	43	41	41	80	0	225	68	0	86	39
N.S.	1	0.95	0.95	1.86	0.00	5.23	1.58	0.00	2.00	0.91
time (sec)	N/A	0.268	0.021	0.225	0.000	0.629	1.228	0.000	0.156	1.942

Problem 378	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	C	F	B	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	230	285	229	195	0	1811	0	0	163	0
N.S.	1	1.24	1.00	0.85	0.00	7.87	0.00	0.00	0.71	0.00
time (sec)	N/A	0.827	0.189	0.496	0.000	0.963	0.000	0.000	0.175	0.000

Problem 379	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F(-1)	B	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	270	249	288	255	0	5181	0	0	1171	0
N.S.	1	0.92	1.07	0.94	0.00	19.19	0.00	0.00	4.34	0.00
time (sec)	N/A	0.595	0.408	0.688	0.000	0.816	0.000	0.000	0.219	0.000

Problem 380	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	B	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	142	129	123	118	0	2137	0	0	0	0
N.S.	1	0.91	0.87	0.83	0.00	15.05	0.00	0.00	0.00	0.00
time (sec)	N/A	0.483	0.310	0.369	0.000	0.723	0.000	0.000	0.201	0.000

Problem 381	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F	B	B	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	49	46	42	127	0	564	151	0	233	45
N.S.	1	0.94	0.86	2.59	0.00	11.51	3.08	0.00	4.76	0.92
time (sec)	N/A	0.311	0.047	0.240	0.000	0.126	3.075	0.000	0.160	2.348

Problem 382	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	C	F	B	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	322	379	300	376	0	3834	0	0	1277	0
N.S.	1	1.18	0.93	1.17	0.00	11.91	0.00	0.00	3.97	0.00
time (sec)	N/A	1.071	0.492	0.789	0.000	5.175	0.000	0.000	0.219	0.000

Problem 383	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	130	125	119	0	0	0	0	0	360	0
N.S.	1	0.96	0.92	0.00	0.00	0.00	0.00	0.00	2.77	0.00
time (sec)	N/A	0.441	0.108	0.000	0.000	0.000	0.000	0.000	0.382	0.000

Problem 384	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	84	82	82	0	0	0	0	0	241	0
N.S.	1	0.98	0.98	0.00	0.00	0.00	0.00	0.00	2.87	0.00
time (sec)	N/A	0.384	0.043	0.000	0.000	0.000	0.000	0.000	0.291	0.000

Problem 385	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F(-1)	F	F	B
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	37	37	37	0	0	0	0	0	120	38
N.S.	1	1.00	1.00	0.00	0.00	0.00	0.00	0.00	3.24	1.03
time (sec)	N/A	0.273	0.008	0.000	0.000	0.000	0.000	0.000	0.196	1.869

Problem 386	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	130	125	119	0	0	0	0	0	25	0
N.S.	1	0.96	0.92	0.00	0.00	0.00	0.00	0.00	0.19	0.00
time (sec)	N/A	0.448	0.087	0.000	0.000	0.000	0.000	0.000	200.022	0.000

Problem 387	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	84	82	82	0	0	0	0	0	0	0
N.S.	1	0.98	0.98	0.00	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.374	0.041	0.000	0.000	0.000	0.000	0.000	0.949	0.000

Problem 388	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F(-1)	F	F	B
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	37	37	37	0	0	0	0	0	0	38
N.S.	1	1.00	1.00	0.00	0.00	0.00	0.00	0.00	0.00	1.03
time (sec)	N/A	0.241	0.008	0.000	0.000	0.000	0.000	0.000	0.549	1.926

Problem 389	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	B	F	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	17	21	23	24	45	47	0	25	57	27
N.S.	1	1.24	1.35	1.41	2.65	2.76	0.00	1.47	3.35	1.59
time (sec)	N/A	0.215	0.012	0.486	0.037	0.092	0.000	0.126	0.170	0.088

Problem 390	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	B	F	A	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	63	70	44	49	292	875	0	70	25	252
N.S.	1	1.11	0.70	0.78	4.63	13.89	0.00	1.11	0.40	4.00
time (sec)	N/A	0.402	0.195	0.565	0.205	0.097	0.000	0.166	0.172	1.917

Problem 391	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	B	F	A	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	38	49	29	28	106	311	0	44	25	67
N.S.	1	1.29	0.76	0.74	2.79	8.18	0.00	1.16	0.66	1.76
time (sec)	N/A	0.367	0.129	0.469	0.166	0.095	0.000	0.154	0.168	1.887

Problem 392	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	B	F	A	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	18	18	18	19	32	139	0	24	23	18
N.S.	1	1.00	1.00	1.06	1.78	7.72	0.00	1.33	1.28	1.00
time (sec)	N/A	0.317	0.033	0.344	0.187	0.094	0.000	0.120	0.165	1.807

Problem 393	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	B	F	A	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	50	52	49	54	68	200	0	47	23	0
N.S.	1	1.04	0.98	1.08	1.36	4.00	0.00	0.94	0.46	0.00
time (sec)	N/A	0.330	0.071	0.455	0.171	0.099	0.000	0.126	0.172	0.000

Problem 394	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	A	B	F	A	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	87	90	40	102	126	764	0	108	25	0
N.S.	1	1.03	0.46	1.17	1.45	8.78	0.00	1.24	0.29	0.00
time (sec)	N/A	0.360	0.106	0.519	0.186	0.111	0.000	0.162	0.169	0.000

Problem 395	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	B	F	A	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	120	99	75	169	891	1645	0	124	25	0
N.S.	1	0.82	0.62	1.41	7.42	13.71	0.00	1.03	0.21	0.00
time (sec)	N/A	0.463	0.174	0.599	0.179	0.127	0.000	0.160	0.182	0.000

Problem 396	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	B	F	A	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	91	69	55	141	387	742	0	100	25	0
N.S.	1	0.76	0.60	1.55	4.25	8.15	0.00	1.10	0.27	0.00
time (sec)	N/A	0.410	0.137	0.525	0.164	0.107	0.000	0.146	0.176	0.000

Problem 397	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	B	F	A	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	57	41	40	102	50	182	0	35	25	0
N.S.	1	0.72	0.70	1.79	0.88	3.19	0.00	0.61	0.44	0.00
time (sec)	N/A	0.427	0.038	0.479	0.140	0.102	0.000	0.136	0.169	0.000

Problem 398	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	C	A	A	B	B	F	A	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	56	48	35	42	125	317	0	48	25	67
N.S.	1	0.86	0.62	0.75	2.23	5.66	0.00	0.86	0.45	1.20
time (sec)	N/A	0.456	0.052	0.448	0.141	0.104	0.000	0.140	0.177	0.110

Problem 399	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	C	A	A	B	B	F	A	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	91	62	47	75	487	885	0	74	25	281
N.S.	1	0.68	0.52	0.82	5.35	9.73	0.00	0.81	0.27	3.09
time (sec)	N/A	0.434	0.057	0.499	0.164	0.134	0.000	0.146	0.168	1.868

Problem 400	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	C	A	A	B	B	F(-1)	A	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	124	74	67	85	1050	1696	0	96	25	427
N.S.	1	0.60	0.54	0.69	8.47	13.68	0.00	0.77	0.20	3.44
time (sec)	N/A	0.455	0.134	0.515	0.195	0.113	0.000	0.154	0.177	1.871

Problem 401	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	B	F	F(-2)	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	66	72	43	48	446	1387	0	0	40	381
N.S.	1	1.09	0.65	0.73	6.76	21.02	0.00	0.00	0.61	5.77
time (sec)	N/A	0.405	0.158	0.418	0.199	0.112	0.000	0.000	0.169	1.879

Problem 402	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	B	F	F(-2)	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	42	51	31	38	184	641	0	0	40	82
N.S.	1	1.21	0.74	0.90	4.38	15.26	0.00	0.00	0.95	1.95
time (sec)	N/A	0.371	0.117	0.396	0.180	0.091	0.000	0.000	0.164	0.124

Problem 403	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	B	F	F(-2)	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	19	19	19	20	33	168	0	0	38	30
N.S.	1	1.00	1.00	1.05	1.74	8.84	0.00	0.00	2.00	1.58
time (sec)	N/A	0.338	0.024	0.286	0.200	0.079	0.000	0.000	0.162	1.820

Problem 404	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	B	F	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	31	31	32	31	40	174	0	0	38	0
N.S.	1	1.00	1.03	1.00	1.29	5.61	0.00	0.00	1.23	0.00
time (sec)	N/A	0.356	0.021	0.337	0.168	0.095	0.000	0.000	0.168	0.000

Problem 405	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	B	F	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	66	68	56	83	100	529	0	0	40	0
N.S.	1	1.03	0.85	1.26	1.52	8.02	0.00	0.00	0.61	0.00
time (sec)	N/A	0.406	0.163	0.416	0.169	0.097	0.000	0.000	0.161	0.000

Problem 406	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	B	F	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	91	81	66	145	628	1328	0	0	40	0
N.S.	1	0.89	0.73	1.59	6.90	14.59	0.00	0.00	0.44	0.00
time (sec)	N/A	0.651	0.091	0.480	0.184	0.117	0.000	0.000	0.163	0.000

Problem 407	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	B	B	F	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	62	56	44	116	217	504	0	0	40	0
N.S.	1	0.90	0.71	1.87	3.50	8.13	0.00	0.00	0.65	0.00
time (sec)	N/A	0.566	0.039	0.433	0.172	0.100	0.000	0.000	0.165	0.000

Problem 408	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	B	F	F(-2)	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	25	25	25	32	101	170	0	0	40	76
N.S.	1	1.00	1.00	1.28	4.04	6.80	0.00	0.00	1.60	3.04
time (sec)	N/A	0.481	0.028	0.370	0.146	0.086	0.000	0.000	0.164	0.117

Problem 409	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	C	A	A	B	B	F	F(-2)	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	61	52	37	64	556	647	0	0	40	95
N.S.	1	0.85	0.61	1.05	9.11	10.61	0.00	0.00	0.66	1.56
time (sec)	N/A	0.515	0.045	0.402	0.187	0.090	0.000	0.000	0.168	1.842

Problem 410	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	C	A	A	B	B	F	F(-2)	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	96	66	49	74	1231	1399	0	0	40	381
N.S.	1	0.69	0.51	0.77	12.82	14.57	0.00	0.00	0.42	3.97
time (sec)	N/A	0.524	0.064	0.435	0.238	0.107	0.000	0.000	0.162	1.851

Problem 411	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	B	B	F	F(-2)	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	68	74	51	44	586	2507	0	0	50	457
N.S.	1	1.09	0.75	0.65	8.62	36.87	0.00	0.00	0.74	6.72
time (sec)	N/A	0.414	0.187	0.447	0.253	0.136	0.000	0.000	0.166	0.170

Problem 412	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	B	B	F	F(-2)	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	44	53	34	44	268	1400	0	0	50	305
N.S.	1	1.20	0.77	1.00	6.09	31.82	0.00	0.00	1.14	6.93
time (sec)	N/A	0.387	0.170	0.397	0.182	0.113	0.000	0.000	0.162	1.887

Problem 413	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	B	F	F(-2)	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	21	21	21	20	61	608	0	0	48	58
N.S.	1	1.00	1.00	0.95	2.90	28.95	0.00	0.00	2.29	2.76
time (sec)	N/A	0.323	0.030	0.318	0.186	0.107	0.000	0.000	0.166	1.863

Problem 414	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	C	A	B	F	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	53	55	37	44	76	271	0	0	48	0
N.S.	1	1.04	0.70	0.83	1.43	5.11	0.00	0.00	0.91	0.00
time (sec)	N/A	0.365	0.080	0.403	0.172	0.095	0.000	0.000	0.164	0.000

Problem 415	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	A	B	F	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	66	70	61	36	100	565	0	0	50	0
N.S.	1	1.06	0.92	0.55	1.52	8.56	0.00	0.00	0.76	0.00
time (sec)	N/A	0.393	0.106	0.364	0.215	0.093	0.000	0.000	0.166	0.000

Problem 416	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	B	F	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	106	85	58	69	369	1423	0	0	50	0
N.S.	1	0.80	0.55	0.65	3.48	13.42	0.00	0.00	0.47	0.00
time (sec)	N/A	0.657	0.075	0.456	0.146	0.125	0.000	0.000	0.164	0.000

Problem 417	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	C	C	B	B	B	F	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	64	52	46	135	321	254	0	0	50	0
N.S.	1	0.81	0.72	2.11	5.02	3.97	0.00	0.00	0.78	0.00
time (sec)	N/A	0.453	0.059	0.477	0.150	0.094	0.000	0.000	0.163	0.000

Problem 418	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	B	F	F(-2)	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	38	38	29	35	823	612	0	0	50	71
N.S.	1	1.00	0.76	0.92	21.66	16.11	0.00	0.00	1.32	1.87
time (sec)	N/A	0.431	0.035	0.406	0.194	0.087	0.000	0.000	0.167	1.877

Problem 419	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	C	A	A	B	B	F(-1)	F(-2)	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	77	59	41	47	1531	1410	0	0	50	305
N.S.	1	0.77	0.53	0.61	19.88	18.31	0.00	0.00	0.65	3.96
time (sec)	N/A	0.478	0.079	0.422	0.267	0.104	0.000	0.000	0.170	0.168

Problem 420	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	C	A	A	B	B	F(-1)	F(-2)	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	115	73	51	57	2216	2511	0	0	50	457
N.S.	1	0.63	0.44	0.50	19.27	21.83	0.00	0.00	0.43	3.97
time (sec)	N/A	0.470	0.101	0.457	0.318	0.134	0.000	0.000	0.169	2.069

Problem 421	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	F	B	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	187	186	151	43	0	4809	0	0	24	0
N.S.	1	0.99	0.81	0.23	0.00	25.72	0.00	0.00	0.13	0.00
time (sec)	N/A	0.398	0.405	0.572	0.000	0.642	0.000	0.000	0.169	0.000

Problem 422	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	F	B	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	126	119	88	43	0	1774	0	0	24	0
N.S.	1	0.94	0.70	0.34	0.00	14.08	0.00	0.00	0.19	0.00
time (sec)	N/A	0.307	0.289	0.423	0.000	0.507	0.000	0.000	0.168	0.000

Problem 423	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	F	B	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	62	64	65	41	0	728	0	0	22	0
N.S.	1	1.03	1.05	0.66	0.00	11.74	0.00	0.00	0.35	0.00
time (sec)	N/A	0.286	0.041	0.335	0.000	0.475	0.000	0.000	0.175	0.000

Problem 424	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	F	B	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	54	56	53	46	0	718	0	0	22	0
N.S.	1	1.04	0.98	0.85	0.00	13.30	0.00	0.00	0.41	0.00
time (sec)	N/A	0.304	0.034	0.331	0.000	0.285	0.000	0.000	0.167	0.000

Problem 425	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	F	B	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	106	98	69	58	0	1558	0	0	24	0
N.S.	1	0.92	0.65	0.55	0.00	14.70	0.00	0.00	0.23	0.00
time (sec)	N/A	0.329	0.248	0.436	0.000	0.348	0.000	0.000	0.170	0.000

Problem 426	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	F	B	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	167	154	102	80	0	3993	0	0	24	0
N.S.	1	0.92	0.61	0.48	0.00	23.91	0.00	0.00	0.14	0.00
time (sec)	N/A	0.402	0.389	0.532	0.000	0.437	0.000	0.000	0.184	0.000

Problem 427	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	235	350	214	366	0	0	0	0	24	0
N.S.	1	1.49	0.91	1.56	0.00	0.00	0.00	0.00	0.10	0.00
time (sec)	N/A	0.591	1.641	5.807	0.000	0.000	0.000	0.000	0.169	0.000

Problem 428	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	168	267	150	233	0	0	0	0	24	0
N.S.	1	1.59	0.89	1.39	0.00	0.00	0.00	0.00	0.14	0.00
time (sec)	N/A	0.504	0.655	3.838	0.000	0.000	0.000	0.000	0.165	0.000

Problem 429	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	61	60	69	140	0	0	0	0	15	0
N.S.	1	0.98	1.13	2.30	0.00	0.00	0.00	0.00	0.25	0.00
time (sec)	N/A	0.345	0.049	2.061	0.000	0.000	0.000	0.000	0.157	0.000

Problem 430	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	202	273	154	214	0	0	0	0	24	0
N.S.	1	1.35	0.76	1.06	0.00	0.00	0.00	0.00	0.12	0.00
time (sec)	N/A	0.508	0.711	3.633	0.000	0.000	0.000	0.000	0.162	0.000

Problem 431	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	270	338	210	522	0	0	0	0	24	0
N.S.	1	1.25	0.78	1.93	0.00	0.00	0.00	0.00	0.09	0.00
time (sec)	N/A	0.513	2.310	5.000	0.000	0.000	0.000	0.000	0.173	0.000

Problem 432	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	F	B	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	232	213	169	71	0	6484	0	0	61	0
N.S.	1	0.92	0.73	0.31	0.00	27.95	0.00	0.00	0.26	0.00
time (sec)	N/A	0.437	1.093	0.618	0.000	0.657	0.000	0.000	0.281	0.000

Problem 433	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	F	B	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	156	146	122	71	0	2558	0	0	61	0
N.S.	1	0.94	0.78	0.46	0.00	16.40	0.00	0.00	0.39	0.00
time (sec)	N/A	0.341	0.369	0.470	0.000	0.545	0.000	0.000	0.246	0.000

Problem 434	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	F	B	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	90	91	86	69	0	1156	0	0	57	0
N.S.	1	1.01	0.96	0.77	0.00	12.84	0.00	0.00	0.63	0.00
time (sec)	N/A	0.286	0.106	0.370	0.000	0.482	0.000	0.000	0.196	0.000

Problem 435	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	F	B	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	78	79	69	62	0	1113	0	0	57	0
N.S.	1	1.01	0.88	0.79	0.00	14.27	0.00	0.00	0.73	0.00
time (sec)	N/A	0.290	0.083	0.360	0.000	0.296	0.000	0.000	0.199	0.000

Problem 436	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	F	B	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	140	123	90	84	0	2519	0	0	61	0
N.S.	1	0.88	0.64	0.60	0.00	17.99	0.00	0.00	0.44	0.00
time (sec)	N/A	0.323	0.328	0.452	0.000	0.380	0.000	0.000	0.238	0.000

Problem 437	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	F	B	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	203	174	123	113	0	5622	0	0	61	0
N.S.	1	0.86	0.61	0.56	0.00	27.69	0.00	0.00	0.30	0.00
time (sec)	N/A	0.360	0.543	0.553	0.000	0.510	0.000	0.000	0.285	0.000

Problem 438	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	264	379	224	389	0	0	0	0	61	0
N.S.	1	1.44	0.85	1.47	0.00	0.00	0.00	0.00	0.23	0.00
time (sec)	N/A	0.610	2.202	7.044	0.000	0.000	0.000	0.000	0.271	0.000

Problem 439	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	260	326	188	414	0	0	0	0	61	0
N.S.	1	1.25	0.72	1.59	0.00	0.00	0.00	0.00	0.23	0.00
time (sec)	N/A	0.484	2.333	5.142	0.000	0.000	0.000	0.000	0.218	0.000

Problem 440	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	176	175	169	428	0	0	0	0	44	0
N.S.	1	0.99	0.96	2.43	0.00	0.00	0.00	0.00	0.25	0.00
time (sec)	N/A	0.901	0.510	3.731	0.000	0.000	0.000	0.000	0.170	0.000

Problem 441	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	256	324	184	327	0	0	0	0	61	0
N.S.	1	1.27	0.72	1.28	0.00	0.00	0.00	0.00	0.24	0.00
time (sec)	N/A	0.545	1.848	5.325	0.000	0.000	0.000	0.000	0.219	0.000

Problem 442	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	306	380	229	540	0	0	0	0	61	0
N.S.	1	1.24	0.75	1.76	0.00	0.00	0.00	0.00	0.20	0.00
time (sec)	N/A	0.650	3.509	6.224	0.000	0.000	0.000	0.000	0.262	0.000

Problem 443	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	F	B	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	142	155	116	43	0	4205	0	0	38	0
N.S.	1	1.09	0.82	0.30	0.00	29.61	0.00	0.00	0.27	0.00
time (sec)	N/A	0.426	0.343	0.622	0.000	0.260	0.000	0.000	0.160	0.000

Problem 444	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	F	B	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	89	89	85	43	0	1425	0	0	38	0
N.S.	1	1.00	0.96	0.48	0.00	16.01	0.00	0.00	0.43	0.00
time (sec)	N/A	0.342	0.085	0.552	0.000	0.182	0.000	0.000	0.157	0.000

Problem 445	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	F	B	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	41	41	44	41	0	537	0	0	36	0
N.S.	1	1.00	1.07	1.00	0.00	13.10	0.00	0.00	0.88	0.00
time (sec)	N/A	0.296	0.026	0.428	0.000	0.147	0.000	0.000	0.158	0.000

Problem 446	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	F	B	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	33	33	33	35	0	523	0	0	36	0
N.S.	1	1.00	1.00	1.06	0.00	15.85	0.00	0.00	1.09	0.00
time (sec)	N/A	0.300	0.027	0.428	0.000	0.136	0.000	0.000	0.156	0.000

Problem 447	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	F	B	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	77	74	72	44	0	1257	0	0	38	0
N.S.	1	0.96	0.94	0.57	0.00	16.32	0.00	0.00	0.49	0.00
time (sec)	N/A	0.359	0.134	0.533	0.000	0.155	0.000	0.000	0.160	0.000

Problem 448	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	F	B	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	126	128	100	54	0	3199	0	0	38	0
N.S.	1	1.02	0.79	0.43	0.00	25.39	0.00	0.00	0.30	0.00
time (sec)	N/A	0.383	0.246	0.598	0.000	0.194	0.000	0.000	0.159	0.000

Problem 449	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	B	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	219	270	206	369	0	2755	0	0	38	0
N.S.	1	1.23	0.94	1.68	0.00	12.58	0.00	0.00	0.17	0.00
time (sec)	N/A	0.445	1.680	3.376	0.000	0.125	0.000	0.000	0.159	0.000

Problem 450	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	B	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	156	282	109	239	0	703	0	0	38	0
N.S.	1	1.81	0.70	1.53	0.00	4.51	0.00	0.00	0.24	0.00
time (sec)	N/A	0.449	0.571	2.331	0.000	0.112	0.000	0.000	0.159	0.000

Problem 451	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	B	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	61	60	68	86	0	147	0	0	30	0
N.S.	1	0.98	1.11	1.41	0.00	2.41	0.00	0.00	0.49	0.00
time (sec)	N/A	0.317	0.036	0.646	0.000	0.089	0.000	0.000	0.155	0.000

Problem 452	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	B	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	211	274	105	216	0	674	0	0	38	0
N.S.	1	1.30	0.50	1.02	0.00	3.19	0.00	0.00	0.18	0.00
time (sec)	N/A	0.436	0.545	1.823	0.000	0.103	0.000	0.000	0.163	0.000

Problem 453	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	B	F	B	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	252	349	208	522	0	2584	0	0	38	0
N.S.	1	1.38	0.83	2.07	0.00	10.25	0.00	0.00	0.15	0.00
time (sec)	N/A	0.531	2.657	3.344	0.000	0.139	0.000	0.000	0.160	0.000

Problem 454	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	C	F	B	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	187	193	113	103	0	10273	0	0	54	0
N.S.	1	1.03	0.60	0.55	0.00	54.94	0.00	0.00	0.29	0.00
time (sec)	N/A	0.415	0.339	12.204	0.000	0.594	0.000	0.000	0.161	0.000

Problem 455	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	C	F	B	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	122	124	79	103	0	4155	0	0	54	0
N.S.	1	1.02	0.65	0.84	0.00	34.06	0.00	0.00	0.44	0.00
time (sec)	N/A	0.332	0.081	1.226	0.000	0.274	0.000	0.000	0.165	0.000

Problem 456	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	C	F	B	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	69	71	58	93	0	1474	0	0	52	0
N.S.	1	1.03	0.84	1.35	0.00	21.36	0.00	0.00	0.75	0.00
time (sec)	N/A	0.272	0.062	0.448	0.000	0.158	0.000	0.000	0.158	0.000

Problem 457	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	C	F	B	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	57	59	46	35	0	1250	0	0	52	0
N.S.	1	1.04	0.81	0.61	0.00	21.93	0.00	0.00	0.91	0.00
time (sec)	N/A	0.288	0.045	0.432	0.000	0.147	0.000	0.000	0.158	0.000

Problem 458	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	C	F	B	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	110	103	69	43	0	3341	0	0	54	0
N.S.	1	0.94	0.63	0.39	0.00	30.37	0.00	0.00	0.49	0.00
time (sec)	N/A	0.337	0.074	1.202	0.000	0.203	0.000	0.000	0.158	0.000

Problem 459	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	C	F	B	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	167	157	94	43	0	7675	0	0	54	0
N.S.	1	0.94	0.56	0.26	0.00	45.96	0.00	0.00	0.32	0.00
time (sec)	N/A	0.364	0.227	8.215	0.000	0.414	0.000	0.000	0.160	0.000

Problem 460	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	B	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	296	345	212	354	0	7400	0	0	54	0
N.S.	1	1.17	0.72	1.20	0.00	25.00	0.00	0.00	0.18	0.00
time (sec)	N/A	0.481	1.853	10.207	0.000	0.307	0.000	0.000	0.158	0.000

Problem 461	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	B	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	235	275	158	256	0	2612	0	0	54	0
N.S.	1	1.17	0.67	1.09	0.00	11.11	0.00	0.00	0.23	0.00
time (sec)	N/A	0.429	1.221	4.443	0.000	0.133	0.000	0.000	0.160	0.000

Problem 462	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F	B	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	116	115	100	252	0	1464	0	0	46	0
N.S.	1	0.99	0.86	2.17	0.00	12.62	0.00	0.00	0.40	0.00
time (sec)	N/A	0.567	0.099	1.043	0.000	0.130	0.000	0.000	0.151	0.000

Problem 463	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	B	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	213	320	153	218	0	2436	0	0	54	0
N.S.	1	1.50	0.72	1.02	0.00	11.44	0.00	0.00	0.25	0.00
time (sec)	N/A	0.566	0.903	4.127	0.000	0.124	0.000	0.000	0.153	0.000

Problem 464	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	B	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	308	416	214	522	0	6862	0	0	54	0
N.S.	1	1.35	0.69	1.69	0.00	22.28	0.00	0.00	0.18	0.00
time (sec)	N/A	0.731	2.465	8.046	0.000	0.226	0.000	0.000	0.152	0.000

Problem 465	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	C	F	B	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	232	229	114	213	0	20403	0	0	70	0
N.S.	1	0.99	0.49	0.92	0.00	87.94	0.00	0.00	0.30	0.00
time (sec)	N/A	0.501	0.400	2.854	0.000	1.764	0.000	0.000	0.163	0.000

Problem 466	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	C	F	B	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	163	162	82	213	0	10611	0	0	70	0
N.S.	1	0.99	0.50	1.31	0.00	65.10	0.00	0.00	0.43	0.00
time (sec)	N/A	0.389	0.086	1.323	0.000	0.670	0.000	0.000	0.158	0.000

Problem 467	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	C	F	B	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	99	107	60	173	0	4305	0	0	68	0
N.S.	1	1.08	0.61	1.75	0.00	43.48	0.00	0.00	0.69	0.00
time (sec)	N/A	0.334	0.052	0.842	0.000	0.264	0.000	0.000	0.155	0.000

Problem 468	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	C	F	B	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	83	87	49	65	0	3197	0	0	68	0
N.S.	1	1.05	0.59	0.78	0.00	38.52	0.00	0.00	0.82	0.00
time (sec)	N/A	0.320	0.041	0.846	0.000	0.214	0.000	0.000	0.159	0.000

Problem 469	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	C	F	B	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	143	131	69	73	0	7707	0	0	70	0
N.S.	1	0.92	0.48	0.51	0.00	53.90	0.00	0.00	0.49	0.00
time (sec)	N/A	0.338	0.194	1.226	0.000	0.439	0.000	0.000	0.159	0.000

Problem 470	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	C	F	B	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	208	185	117	73	0	15215	0	0	70	0
N.S.	1	0.89	0.56	0.35	0.00	73.15	0.00	0.00	0.34	0.00
time (sec)	N/A	0.397	0.282	2.802	0.000	1.215	0.000	0.000	0.159	0.000

Problem 471	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	B	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	356	417	252	661	0	15718	0	0	70	0
N.S.	1	1.17	0.71	1.86	0.00	44.15	0.00	0.00	0.20	0.00
time (sec)	N/A	0.583	2.737	9.513	0.000	0.839	0.000	0.000	0.162	0.000

Problem 472	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	B	F	B	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	292	344	215	797	0	8226	0	0	70	0
N.S.	1	1.18	0.74	2.73	0.00	28.17	0.00	0.00	0.24	0.00
time (sec)	N/A	0.506	2.157	6.424	0.000	0.361	0.000	0.000	0.167	0.000

Problem 473	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	B	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	253	258	190	406	0	5442	0	0	62	0
N.S.	1	1.02	0.75	1.60	0.00	21.51	0.00	0.00	0.25	0.00
time (sec)	N/A	1.322	0.928	1.520	0.000	0.218	0.000	0.000	0.168	0.000

Problem 474	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	B	F	B	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	281	412	226	642	0	7847	0	0	70	0
N.S.	1	1.47	0.80	2.28	0.00	27.93	0.00	0.00	0.25	0.00
time (sec)	N/A	0.622	2.297	6.314	0.000	0.309	0.000	0.000	0.177	0.000

Problem 475	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	B	F	B	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	336	471	247	923	0	13823	0	0	70	0
N.S.	1	1.40	0.74	2.75	0.00	41.14	0.00	0.00	0.21	0.00
time (sec)	N/A	0.713	2.280	8.571	0.000	0.611	0.000	0.000	0.169	0.000

Problem 476	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	F	F	F	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	122	149	0	0	0	0	0	0	29	0
N.S.	1	1.22	0.00	0.00	0.00	0.00	0.00	0.00	0.24	0.00
time (sec)	N/A	0.472	0.000	0.000	0.000	0.000	0.000	0.000	0.190	0.000

Problem 477	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	110	110	90	0	0	0	0	0	25	0
N.S.	1	1.00	0.82	0.00	0.00	0.00	0.00	0.00	0.23	0.00
time (sec)	N/A	0.366	0.182	0.000	0.000	0.000	0.000	0.000	200.022	0.000

Problem 478	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	63	63	65	0	0	0	0	0	897	0
N.S.	1	1.00	1.03	0.00	0.00	0.00	0.00	0.00	14.24	0.00
time (sec)	N/A	0.285	0.063	0.000	0.000	0.000	0.000	0.000	0.915	0.000

Problem 479	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	54	54	54	0	0	0	0	0	509	0
N.S.	1	1.00	1.00	0.00	0.00	0.00	0.00	0.00	9.43	0.00
time (sec)	N/A	0.279	0.046	0.000	0.000	0.000	0.000	0.000	0.627	0.000

Problem 480	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	94	91	71	0	0	0	0	0	25	0
N.S.	1	0.97	0.76	0.00	0.00	0.00	0.00	0.00	0.27	0.00
time (sec)	N/A	0.313	0.269	0.000	0.000	0.000	0.000	0.000	200.037	0.000

Problem 481	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	F	F	F	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	103	103	0	0	0	0	0	0	25	0
N.S.	1	1.00	0.00	0.00	0.00	0.00	0.00	0.00	0.24	0.00
time (sec)	N/A	0.385	0.000	0.000	0.000	0.000	0.000	0.000	200.023	0.000

Problem 482	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	F	F	F	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	103	103	0	0	0	0	0	0	25	0
N.S.	1	1.00	0.00	0.00	0.00	0.00	0.00	0.00	0.24	0.00
time (sec)	N/A	0.351	0.000	0.000	0.000	0.000	0.000	0.000	200.025	0.000

Problem 483	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	F	F	F	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	99	99	0	0	0	0	0	0	25	0
N.S.	1	1.00	0.00	0.00	0.00	0.00	0.00	0.00	0.25	0.00
time (sec)	N/A	0.351	0.000	0.000	0.000	0.000	0.000	0.000	200.028	0.000

Problem 484	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	F	F	F	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	103	103	0	0	0	0	0	0	25	0
N.S.	1	1.00	0.00	0.00	0.00	0.00	0.00	0.00	0.24	0.00
time (sec)	N/A	0.349	0.000	0.000	0.000	0.000	0.000	0.000	200.025	0.000

Problem 485	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	F	C	F	A	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	152	152	136	91	0	1432	0	209	0	1129
N.S.	1	1.00	0.89	0.60	0.00	9.42	0.00	1.38	0.00	7.43
time (sec)	N/A	0.520	0.229	8.800	0.000	0.883	0.000	0.138	0.192	0.951

Problem 486	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F(-2)	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	28	28	28	0	0	0	0	0	24	0
N.S.	1	1.00	1.00	0.00	0.00	0.00	0.00	0.00	0.86	0.00
time (sec)	N/A	0.271	0.018	0.000	0.000	0.000	0.000	0.000	0.155	0.000

Problem 487	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	B	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	45	45	45	0	0	1860	0	0	14	0
N.S.	1	1.00	1.00	0.00	0.00	41.33	0.00	0.00	0.31	0.00
time (sec)	N/A	0.288	0.018	0.000	0.000	1.411	0.000	0.000	0.165	0.000

Problem 488	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	29	29	29	24	0	110	0	0	24	0
N.S.	1	1.00	1.00	0.83	0.00	3.79	0.00	0.00	0.83	0.00
time (sec)	N/A	0.280	0.019	0.370	0.000	0.109	0.000	0.000	0.153	0.000

Problem 489	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	47	45	45	38	0	153	0	0	14	0
N.S.	1	0.96	0.96	0.81	0.00	3.26	0.00	0.00	0.30	0.00
time (sec)	N/A	0.282	0.017	0.294	0.000	0.097	0.000	0.000	0.149	0.000

2.3 Detailed conclusion table specific for Rubi results

The following table is specific to Rubi only. It gives additional statistics for each integral. the column **steps** is the number of steps used by Rubi to obtain the antiderivative. The **rules** column is the number of unique rules used. The **integrand size** column is the leaf size of the integrand. Finally the ratio $\frac{\text{number of rules}}{\text{integrand size}}$ is also given. The larger this ratio is, the harder the integral is to solve. In this test file, problem number [193] had the largest ratio of [1]

Table 2.1: Rubi specific breakdown of results for each integral

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
1	A	9	9	0.96	21	0.429
2	A	6	5	0.91	21	0.238
3	A	8	8	0.97	21	0.381
4	A	5	4	0.94	19	0.211
5	A	1	1	1.00	12	0.083
6	C	7	7	1.36	19	0.368
7	A	4	4	1.00	21	0.190
8	C	7	7	1.20	21	0.333
9	A	8	7	1.00	21	0.333
10	A	11	10	1.28	23	0.435
11	A	6	5	0.91	23	0.217
12	A	5	5	1.11	23	0.217
13	A	6	5	0.91	21	0.238
14	A	2	2	1.00	14	0.143
15	A	6	5	0.92	21	0.238
16	A	7	6	1.64	23	0.261
17	A	6	5	1.05	23	0.217
18	A	5	4	1.10	23	0.174
19	A	11	10	1.50	23	0.435
20	A	6	5	0.90	23	0.217
21	A	7	7	1.06	23	0.304

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
22	A	6	5	0.90	21	0.238
23	A	4	4	1.04	14	0.286
24	A	6	5	0.92	21	0.238
25	A	8	7	1.79	23	0.304
26	A	6	5	1.00	23	0.217
27	A	6	5	1.61	23	0.217
28	A	6	5	0.94	23	0.217
29	A	10	9	1.25	23	0.391
30	A	6	5	0.94	23	0.217
31	A	7	6	1.23	23	0.261
32	A	6	5	0.98	23	0.217
33	A	7	6	1.00	23	0.261
34	A	5	4	1.00	21	0.190
35	A	4	3	1.00	14	0.214
36	A	7	6	0.95	21	0.286
37	A	6	5	0.96	23	0.217
38	A	8	7	1.10	23	0.304
39	A	5	4	0.94	23	0.174
40	A	9	8	1.15	23	0.348
41	A	6	5	0.93	23	0.217
42	A	7	6	1.23	23	0.261
43	A	6	5	0.99	23	0.217
44	A	8	7	1.00	23	0.304
45	A	6	5	0.98	21	0.238
46	A	8	7	1.00	14	0.500
47	A	9	8	1.12	21	0.381
48	A	7	6	1.28	23	0.261
49	A	10	9	1.11	23	0.391
50	A	6	5	1.25	23	0.217
51	A	6	5	1.06	23	0.217
52	A	7	6	1.02	23	0.261
53	A	10	9	1.09	23	0.391

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
54	A	7	6	1.06	21	0.286
55	A	10	9	1.10	14	0.643
56	A	11	10	1.16	21	0.476
57	A	8	7	1.36	23	0.304
58	A	12	11	1.17	23	0.478
59	A	7	6	1.39	23	0.261
60	A	8	7	0.97	25	0.280
61	A	7	6	0.98	23	0.261
62	A	9	8	0.99	23	0.348
63	A	7	6	1.07	25	0.240
64	A	8	7	1.08	25	0.280
65	A	10	9	1.17	25	0.360
66	A	12	12	0.99	25	0.480
67	A	4	4	0.98	16	0.250
68	A	10	9	1.50	25	0.360
69	A	12	11	1.38	25	0.440
70	A	9	8	0.93	25	0.320
71	A	8	7	0.98	23	0.304
72	A	11	10	0.99	23	0.435
73	A	11	10	1.05	25	0.400
74	A	8	7	1.10	25	0.280
75	A	9	8	1.04	25	0.320
76	A	12	11	1.16	25	0.440
77	A	14	14	1.02	25	0.560
78	A	11	11	0.99	16	0.688
79	A	10	9	1.34	25	0.360
80	A	12	11	1.41	25	0.440
81	A	7	6	0.99	25	0.240
82	A	6	5	1.00	23	0.217
83	A	6	5	1.00	23	0.217
84	A	7	6	1.07	25	0.240
85	A	8	7	1.23	25	0.280

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
86	A	10	10	0.98	25	0.400
87	A	4	4	0.98	16	0.250
88	A	8	7	1.44	25	0.280
89	A	12	11	1.40	25	0.440
90	A	7	6	0.99	25	0.240
91	A	5	4	1.00	23	0.174
92	A	7	6	0.96	23	0.261
93	A	10	9	1.04	25	0.360
94	A	10	9	1.09	25	0.360
95	A	8	7	1.15	25	0.280
96	A	12	12	1.06	25	0.480
97	A	7	7	0.99	16	0.438
98	A	11	10	1.48	25	0.400
99	A	9	8	1.17	25	0.320
100	A	6	5	0.99	25	0.200
101	A	6	5	0.97	23	0.217
102	A	10	9	1.10	23	0.391
103	A	11	10	1.19	25	0.400
104	A	7	6	1.14	25	0.240
105	A	14	14	1.03	25	0.560
106	A	14	14	1.02	16	0.875
107	A	12	11	1.44	25	0.440
108	A	5	4	1.00	25	0.160
109	A	8	7	1.01	23	0.304
110	A	7	6	0.99	23	0.261
111	A	6	5	1.00	21	0.238
112	A	6	5	1.00	21	0.238
113	A	6	5	1.00	23	0.217
114	A	5	4	1.00	23	0.174
115	A	7	6	1.00	23	0.261
116	A	6	5	1.00	23	0.217
117	A	5	4	1.00	23	0.174

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
118	A	3	3	1.00	21	0.143
119	C	4	4	1.16	21	0.190
120	A	4	4	1.00	21	0.190
121	C	4	4	1.22	19	0.211
122	A	1	1	1.00	12	0.083
123	C	4	4	1.25	19	0.211
124	A	4	4	1.00	21	0.190
125	C	4	4	1.28	21	0.190
126	A	3	3	1.00	21	0.143
127	C	4	4	1.14	23	0.174
128	A	4	4	1.00	23	0.174
129	C	4	4	1.17	21	0.190
130	A	3	3	1.00	14	0.214
131	C	4	4	1.20	21	0.190
132	A	4	4	1.00	23	0.174
133	C	4	4	1.18	23	0.174
134	A	3	3	1.00	23	0.130
135	C	4	4	1.17	23	0.174
136	A	4	4	1.00	23	0.174
137	C	4	4	1.14	23	0.174
138	A	4	4	1.00	23	0.174
139	C	4	4	1.12	21	0.190
140	A	3	3	1.00	14	0.214
141	C	4	4	1.13	21	0.190
142	A	4	4	1.00	23	0.174
143	C	4	4	1.15	23	0.174
144	A	3	3	1.00	23	0.130
145	C	4	4	1.14	23	0.174
146	A	4	4	1.00	23	0.174
147	C	4	4	1.13	23	0.174
148	A	4	4	1.00	23	0.174
149	A	4	4	1.05	23	0.174

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
150	A	3	3	1.00	23	0.130
151	A	4	4	1.03	23	0.174
152	A	4	4	1.00	23	0.174
153	A	4	4	1.03	21	0.190
154	A	3	3	1.00	14	0.214
155	A	4	4	1.04	21	0.190
156	A	4	4	1.00	23	0.174
157	A	4	4	1.03	23	0.174
158	A	3	3	1.00	23	0.130
159	A	9	8	1.35	21	0.381
160	A	6	5	0.88	21	0.238
161	A	10	9	1.37	21	0.429
162	A	5	4	0.89	19	0.211
163	A	1	1	1.00	12	0.083
164	A	6	5	0.86	19	0.263
165	A	8	7	1.38	21	0.333
166	A	8	7	1.13	21	0.333
167	A	5	4	1.13	21	0.190
168	A	8	7	1.23	21	0.333
169	A	6	5	0.89	21	0.238
170	A	9	8	1.16	21	0.381
171	A	6	5	0.88	23	0.217
172	A	13	12	1.29	23	0.522
173	A	6	5	0.88	21	0.238
174	A	11	10	1.31	14	0.714
175	A	6	5	0.87	21	0.238
176	A	12	11	1.30	23	0.478
177	A	8	7	1.00	23	0.304
178	A	8	7	1.22	23	0.304
179	A	10	9	1.11	23	0.391
180	A	8	7	1.11	23	0.304
181	A	12	11	1.20	23	0.478

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
182	A	6	5	0.90	23	0.217
183	A	6	5	0.89	23	0.217
184	A	6	5	0.88	21	0.238
185	A	6	5	0.89	21	0.238
186	A	8	7	0.96	23	0.304
187	A	10	9	1.05	23	0.391
188	A	12	11	1.10	23	0.478
189	A	14	13	1.16	23	0.565
190	A	16	15	1.21	23	0.652
191	A	16	15	1.21	23	0.652
192	A	17	16	1.25	23	0.696
193	A	15	14	1.26	14	1.000
194	A	16	15	1.26	23	0.652
195	A	12	11	1.22	23	0.478
196	A	12	11	1.17	23	0.478
197	A	8	7	1.11	23	0.304
198	A	8	7	1.02	23	0.304
199	A	5	4	0.95	23	0.174
200	A	6	5	0.88	23	0.217
201	A	5	4	0.89	23	0.174
202	A	6	5	0.90	23	0.217
203	A	5	4	0.90	23	0.174
204	A	6	5	0.96	24	0.208
205	A	6	5	0.96	24	0.208
206	A	7	6	0.99	24	0.250
207	A	7	6	0.98	22	0.273
208	A	6	5	0.96	22	0.227
209	A	6	5	1.05	24	0.208
210	A	6	5	1.12	24	0.208
211	A	5	4	1.03	24	0.167
212	A	6	5	1.34	24	0.208
213	A	5	4	1.46	15	0.267

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	number of rules / integrand leaf size
214	A	6	5	0.96	24	0.208
215	A	5	4	0.95	24	0.167
216	A	8	7	1.10	24	0.292
217	A	9	8	1.09	24	0.333
218	A	9	8	1.05	24	0.333
219	A	9	8	1.13	24	0.333
220	A	9	8	1.05	22	0.364
221	A	6	5	1.35	22	0.227
222	A	13	12	1.76	24	0.500
223	A	10	9	1.23	24	0.375
224	A	9	8	1.35	24	0.333
225	A	8	7	1.17	24	0.292
226	A	7	6	1.24	15	0.400
227	A	8	7	1.00	24	0.292
228	A	11	10	1.17	24	0.417
229	A	11	10	1.05	24	0.417
230	A	11	10	1.15	24	0.417
231	A	11	10	1.10	24	0.417
232	A	11	10	1.16	22	0.455
233	A	6	5	1.85	22	0.227
234	A	14	13	1.19	24	0.542
235	A	10	9	1.11	24	0.375
236	A	9	8	1.18	24	0.333
237	A	10	9	1.12	24	0.375
238	A	9	8	1.13	15	0.533
239	A	10	9	1.04	24	0.375
240	C	6	5	1.33	15	0.333
241	A	5	5	0.90	15	0.333
242	A	4	4	1.00	15	0.267
243	A	3	3	1.00	15	0.200
244	A	4	4	1.00	13	0.308
245	A	6	6	0.91	13	0.462

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
246	A	8	8	1.00	15	0.533
247	A	7	6	1.19	21	0.286
248	A	5	4	0.89	21	0.190
249	A	6	5	1.26	21	0.238
250	A	4	3	0.93	19	0.158
251	A	5	4	0.93	19	0.211
252	A	5	4	1.37	21	0.190
253	A	5	4	1.10	21	0.190
254	A	4	3	0.94	21	0.143
255	A	6	5	1.07	21	0.238
256	A	5	4	0.91	21	0.190
257	A	9	8	1.17	23	0.348
258	A	5	4	0.89	23	0.174
259	A	8	7	1.32	23	0.304
260	A	5	4	0.90	21	0.190
261	A	5	4	0.91	21	0.190
262	A	5	4	1.23	23	0.174
263	A	5	4	1.02	23	0.174
264	A	5	4	1.09	23	0.174
265	A	6	5	1.19	23	0.217
266	A	5	4	0.91	23	0.174
267	A	7	6	1.11	23	0.261
268	A	10	9	1.20	23	0.391
269	A	5	4	0.89	23	0.174
270	A	9	8	1.29	23	0.348
271	A	5	4	0.88	21	0.190
272	A	5	4	0.91	21	0.190
273	A	5	4	1.16	23	0.174
274	A	5	4	0.98	23	0.174
275	A	5	4	1.11	23	0.174
276	A	5	4	1.02	23	0.174
277	A	5	4	1.01	23	0.174

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
278	A	8	7	1.26	23	0.304
279	A	5	4	0.90	23	0.174
280	A	5	4	0.93	23	0.174
281	A	10	9	1.23	23	0.391
282	A	5	4	0.94	23	0.174
283	A	8	7	1.22	23	0.304
284	A	5	4	0.96	23	0.174
285	A	6	5	1.14	23	0.217
286	A	4	3	1.00	21	0.143
287	A	6	5	0.97	21	0.238
288	A	5	4	0.97	23	0.174
289	A	8	7	1.12	23	0.304
290	A	5	4	0.94	23	0.174
291	A	10	9	1.17	23	0.391
292	A	5	4	0.94	23	0.174
293	A	10	9	1.12	23	0.391
294	A	5	4	0.95	23	0.174
295	A	8	7	1.15	23	0.304
296	A	5	4	0.97	23	0.174
297	A	5	4	0.97	23	0.174
298	A	5	4	0.97	21	0.190
299	A	7	6	1.09	21	0.286
300	A	5	4	0.96	23	0.174
301	A	10	9	1.09	23	0.391
302	A	5	4	0.94	23	0.174
303	A	10	9	1.13	23	0.391
304	A	6	5	1.05	23	0.217
305	A	6	5	1.03	23	0.217
306	A	6	5	0.98	23	0.217
307	A	6	5	0.97	23	0.217
308	A	6	5	1.03	21	0.238
309	A	8	7	1.15	21	0.333

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
310	A	5	4	0.95	23	0.174
311	A	11	10	1.16	23	0.435
312	A	5	4	0.95	23	0.174
313	A	5	4	1.11	15	0.267
314	A	5	4	1.00	15	0.267
315	A	7	6	1.43	15	0.400
316	A	7	6	0.96	25	0.240
317	A	6	5	0.97	23	0.217
318	A	8	7	0.98	23	0.304
319	A	6	5	1.05	25	0.200
320	A	7	6	1.07	25	0.240
321	A	10	9	1.16	25	0.360
322	A	9	8	1.24	25	0.320
323	A	4	4	0.98	16	0.250
324	A	4	3	1.43	25	0.120
325	A	8	7	1.21	25	0.280
326	A	8	7	0.92	25	0.280
327	A	7	6	0.98	23	0.261
328	A	9	8	0.98	23	0.348
329	A	9	8	1.09	25	0.320
330	A	7	6	1.06	25	0.240
331	A	8	7	0.97	25	0.280
332	A	12	11	1.17	25	0.440
333	A	10	9	1.18	25	0.360
334	A	11	11	0.99	16	0.688
335	A	9	8	1.60	25	0.320
336	A	7	6	1.23	25	0.240
337	A	6	5	0.97	25	0.200
338	A	5	4	1.00	23	0.174
339	A	5	4	1.00	23	0.174
340	A	6	5	1.04	25	0.200
341	A	9	8	1.21	25	0.320

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
342	A	7	6	1.29	25	0.240
343	A	4	4	0.98	16	0.250
344	A	9	8	1.77	25	0.320
345	A	8	7	1.23	25	0.280
346	A	6	5	0.97	25	0.200
347	A	4	3	1.00	23	0.130
348	A	6	5	0.98	23	0.217
349	A	9	8	1.05	25	0.320
350	A	10	9	1.23	25	0.360
351	A	8	7	1.29	25	0.280
352	A	4	3	1.31	25	0.120
353	A	7	7	0.99	16	0.438
354	A	8	7	1.18	25	0.280
355	A	7	6	1.09	25	0.240
356	A	5	4	0.90	25	0.160
357	A	5	4	0.97	23	0.174
358	A	8	7	1.07	23	0.304
359	A	10	9	1.21	25	0.360
360	A	7	6	1.13	25	0.240
361	A	8	7	1.11	25	0.280
362	A	14	14	1.02	16	0.875
363	A	9	8	1.18	25	0.320
364	A	5	4	1.00	25	0.160
365	A	8	7	1.01	23	0.304
366	A	6	5	0.94	23	0.217
367	A	5	4	1.00	21	0.190
368	A	5	4	1.00	21	0.190
369	A	5	4	1.00	23	0.174
370	A	5	4	1.00	23	0.174
371	A	5	4	1.00	23	0.174
372	A	5	4	1.00	14	0.286
373	A	5	4	1.00	23	0.174

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
374	A	5	4	1.00	23	0.174
375	A	6	5	0.91	25	0.200
376	A	6	5	0.91	25	0.200
377	A	6	5	0.95	23	0.217
378	A	6	5	1.24	23	0.217
379	A	6	5	0.92	25	0.200
380	A	6	5	0.91	25	0.200
381	A	6	5	0.94	23	0.217
382	A	6	5	1.18	23	0.217
383	A	5	4	0.96	23	0.174
384	A	5	4	0.98	23	0.174
385	A	4	3	1.00	21	0.143
386	A	5	4	0.96	23	0.174
387	A	5	4	0.98	23	0.174
388	A	4	3	1.00	21	0.143
389	A	7	6	1.24	13	0.462
390	A	11	10	1.11	25	0.400
391	A	11	10	1.29	25	0.400
392	A	10	9	1.00	23	0.391
393	A	11	10	1.04	23	0.435
394	A	13	12	1.03	25	0.480
395	A	17	16	0.82	25	0.640
396	A	10	9	0.76	25	0.360
397	A	15	14	0.72	25	0.560
398	C	14	13	0.86	25	0.520
399	C	9	8	0.68	25	0.320
400	C	14	13	0.60	25	0.520
401	A	11	10	1.09	25	0.400
402	A	11	10	1.21	25	0.400
403	A	10	9	1.00	23	0.391
404	A	10	9	1.00	23	0.391
405	A	12	11	1.03	25	0.440

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
406	A	12	12	0.89	25	0.480
407	A	13	13	0.90	25	0.520
408	A	12	11	1.00	25	0.440
409	C	9	8	0.85	25	0.320
410	C	13	12	0.69	25	0.480
411	A	11	10	1.09	25	0.400
412	A	11	10	1.20	25	0.400
413	A	10	9	1.00	23	0.391
414	A	11	10	1.04	23	0.435
415	A	12	11	1.06	25	0.440
416	A	15	15	0.80	25	0.600
417	C	15	14	0.81	25	0.560
418	A	9	8	1.00	25	0.320
419	C	14	13	0.77	25	0.520
420	C	10	9	0.63	25	0.360
421	A	10	9	0.99	25	0.360
422	A	8	7	0.94	25	0.280
423	A	7	6	1.03	23	0.261
424	A	7	6	1.04	23	0.261
425	A	8	7	0.92	25	0.280
426	A	10	9	0.92	25	0.360
427	A	9	8	1.49	25	0.320
428	A	9	8	1.59	25	0.320
429	A	4	4	0.98	16	0.250
430	A	10	9	1.35	25	0.360
431	A	10	9	1.25	25	0.360
432	A	11	10	0.92	25	0.400
433	A	9	8	0.94	25	0.320
434	A	8	7	1.01	23	0.304
435	A	8	7	1.01	23	0.304
436	A	9	8	0.88	25	0.320
437	A	11	10	0.86	25	0.400

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
438	A	12	11	1.44	25	0.440
439	A	10	9	1.25	25	0.360
440	A	11	11	0.99	16	0.688
441	A	11	10	1.27	25	0.400
442	A	12	11	1.24	25	0.440
443	A	9	8	1.09	25	0.320
444	A	7	6	1.00	25	0.240
445	A	6	5	1.00	23	0.217
446	A	6	5	1.00	23	0.217
447	A	7	6	0.96	25	0.240
448	A	9	8	1.02	25	0.320
449	A	7	6	1.23	25	0.240
450	A	9	8	1.81	25	0.320
451	A	4	4	0.98	16	0.250
452	A	9	8	1.30	25	0.320
453	A	10	9	1.38	25	0.360
454	A	10	9	1.03	25	0.360
455	A	8	7	1.02	25	0.280
456	A	7	6	1.03	23	0.261
457	A	7	6	1.04	23	0.261
458	A	8	7	0.94	25	0.280
459	A	10	9	0.94	25	0.360
460	A	9	8	1.17	25	0.320
461	A	8	7	1.17	25	0.280
462	A	7	7	0.99	16	0.438
463	A	12	11	1.50	25	0.440
464	A	12	11	1.35	25	0.440
465	A	11	10	0.99	25	0.400
466	A	9	8	0.99	25	0.320
467	A	8	7	1.08	23	0.304
468	A	8	7	1.05	23	0.304
469	A	9	8	0.92	25	0.320

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
470	A	11	10	0.89	25	0.400
471	A	11	10	1.17	25	0.400
472	A	10	9	1.18	25	0.360
473	A	14	14	1.02	16	0.875
474	A	13	12	1.47	25	0.480
475	A	14	13	1.40	25	0.520
476	A	6	5	1.22	25	0.200
477	A	6	5	1.00	23	0.217
478	A	5	4	1.00	21	0.190
479	A	5	4	1.00	21	0.190
480	A	6	5	0.97	23	0.217
481	A	5	4	1.00	23	0.174
482	A	6	5	1.00	23	0.217
483	A	6	5	1.00	23	0.217
484	A	5	4	1.00	23	0.174
485	A	6	5	1.00	15	0.333
486	A	7	6	1.00	15	0.400
487	A	8	7	1.00	15	0.467
488	A	7	6	1.00	15	0.400
489	A	8	7	0.96	15	0.467

CHAPTER 3

LISTING OF INTEGRALS

3.1 $\int \sinh^4(c+dx)\left(a+b\sinh^2(c+dx)\right)\,dx$

Optimal result

Integrand size = 21, antiderivative size = 89

$$\int \sinh^4(c+dx)\left(a+b\sinh^2(c+dx)\right)\,dx=\frac{1}{16}(6a-5b)x-\frac{(6a-5b)\cosh(c+dx)\sinh(c+dx)}{16d}+\frac{(6a-5b)\cosh(c+dx)\sinh^3(c+dx)}{24d}+\frac{b\cosh(c+dx)\sinh^5(c+dx)}{6d}$$

output
```
1/16*(6*a-5*b)*x-1/16*(6*a-5*b)*cosh(d*x+c)*sinh(d*x+c)/d+1/24*(6*a-5*b)*c
osh(d*x+c)*sinh(d*x+c)^3/d+1/6*b*cosh(d*x+c)*sinh(d*x+c)^5/d
```

Mathematica [A] (verified)

Time = 0.15 (sec) , antiderivative size = 68, normalized size of antiderivative = 0.76

$$\int \sinh^4(c+dx)\left(a+b\sinh^2(c+dx)\right)\,dx=\frac{72ac-60bc+72adx-60bdx+(-48a+45b)\sinh(2(c+dx))+(6a-9b)\sinh(4(c+dx))+b\sinh(6(c}{192d}$$

input
```
Integrate[Sinh[c + d*x]^4*(a + b*Sinh[c + d*x]^2),x]
```

output
```
(72*a*c - 60*b*c + 72*a*d*x - 60*b*d*x + (-48*a + 45*b)*Sinh[2*(c + d*x)]
+ (6*a - 9*b)*Sinh[4*(c + d*x)] + b*Sinh[6*(c + d*x)])/(192*d)
```

Rubi [A] (verified)

Time = 0.40 (sec) , antiderivative size = 85, normalized size of antiderivative = 0.96, number of steps used = 9, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.429$, Rules used = {3042, 3493, 3042, 3115, 25, 3042, 25, 3115, 24}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sinh^4(c+dx)\left(a+b\sinh^2(c+dx)\right)\,dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \sin(ic+idx)^4\left(a-b\sin(ic+idx)^2\right)dx$$

$$\Big\downarrow \text{ 3493}$$

$$\frac{1}{6}(6a-5b)\int \sinh^4(c+dx)dx+\frac{b\sinh^5(c+dx)\cosh(c+dx)}{6d}$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{b\sinh^5(c+dx)\cosh(c+dx)}{6d}+\frac{1}{6}(6a-5b)\int \sin(ic+idx)^4dx$$

$$\Big\downarrow \text{ 3115}$$

$$\frac{1}{6}(6a-5b)\left(\frac{3}{4}\int -\sinh^2(c+dx)dx+\frac{\sinh^3(c+dx)\cosh(c+dx)}{4d}\right)+\frac{b\sinh^5(c+dx)\cosh(c+dx)}{6d}$$

$$\Big\downarrow \text{ 25}$$

$$\frac{1}{6}(6a-5b)\left(\frac{\sinh^3(c+dx)\cosh(c+dx)}{4d}-\frac{3}{4}\int \sinh^2(c+dx)dx\right)+\frac{b\sinh^5(c+dx)\cosh(c+dx)}{6d}$$

$\Big\downarrow$ 3042

$$\frac{b\sinh^5(c+dx)\cosh(c+dx)}{6d}+\frac{1}{6}(6a-5b)\left(\frac{\sinh^3(c+dx)\cosh(c+dx)}{4d}-\frac{3}{4}\int -\sin(ic+idx)^2dx\right)$$

$\Big\downarrow$ 25

$$\frac{b\sinh^5(c+dx)\cosh(c+dx)}{6d}+\frac{1}{6}(6a-5b)\left(\frac{\sinh^3(c+dx)\cosh(c+dx)}{4d}+\frac{3}{4}\int \sin(ic+idx)^2dx\right)$$

$\Big\downarrow$ 3115

$$\frac{1}{6}(6a-5b)\left(\frac{3}{4}\left(\frac{\int 1dx}{2}-\frac{\sinh(c+dx)\cosh(c+dx)}{2d}\right)+\frac{\sinh^3(c+dx)\cosh(c+dx)}{4d}\right)+\frac{b\sinh^5(c+dx)\cosh(c+dx)}{6d}$$

$\Big\downarrow$ 24

$$\frac{1}{6}(6a-5b)\left(\frac{\sinh^3(c+dx)\cosh(c+dx)}{4d}+\frac{3}{4}\left(\frac{x}{2}-\frac{\sinh(c+dx)\cosh(c+dx)}{2d}\right)\right)+\frac{b\sinh^5(c+dx)\cosh(c+dx)}{6d}$$

input
```
Int[Sinh[c + d*x]^4*(a + b*Sinh[c + d*x]^2),x]
```

output
```
(b*Cosh[c + d*x]*Sinh[c + d*x]^5)/(6*d) + ((6*a - 5*b)*((Cosh[c + d*x]*Sin
h[c + d*x]^3)/(4*d) + (3*(x/2 - (Cosh[c + d*x]*Sinh[c + d*x])/(2*d)))/4))/
6
```

Defintions of rubi rules used

rule 24
```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3115
```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Simp[b^2*((n - 1)/n)   Int[(b*Sin
[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[
2*n]
```

rule 3493
```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_) + (C_.)*sin[(e_.) + (f_.)*(
x_)]^2), x_Symbol] :> Simp[(-C)*Cos[e + f*x]*((b*Sin[e + f*x])^(m + 1)/(b*f
*(m + 2))), x] + Simp[(A*(m + 2) + C*(m + 1))/(m + 2)   Int[(b*Sin[e + f*x]
)^m, x], x] /; FreeQ[{b, e, f, A, C, m}, x] &&  !LtQ[m, -1]
```

Maple [A] (verified)

Time = 3.37 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.69

method	result
parallelrisch	$\frac{(-48a+45b)\sinh(2dx+2c)+(6a-9b)\sinh(4dx+4c)+b\sinh(6dx+6c)+72x\left(a-\frac{5b}{6}\right)d}{192d}$
derivativedivides	$\frac{a\left(\left(\frac{\sinh(dx+c)^3}{4}-\frac{3\sinh(dx+c)}{8}\right)\cosh(dx+c)+\frac{3dx}{8}+\frac{3c}{8}\right)+b\left(\left(\frac{\sinh(dx+c)^5}{6}-\frac{5\sinh(dx+c)^3}{24}+\frac{5\sinh(dx+c)}{16}\right)\cosh(dx+c)}{d}$
default	$\frac{a\left(\left(\frac{\sinh(dx+c)^3}{4}-\frac{3\sinh(dx+c)}{8}\right)\cosh(dx+c)+\frac{3dx}{8}+\frac{3c}{8}\right)+b\left(\left(\frac{\sinh(dx+c)^5}{6}-\frac{5\sinh(dx+c)^3}{24}+\frac{5\sinh(dx+c)}{16}\right)\cosh(dx+c)}{d}$
parts	$\frac{a\left(\left(\frac{\sinh(dx+c)^3}{4}-\frac{3\sinh(dx+c)}{8}\right)\cosh(dx+c)+\frac{3dx}{8}+\frac{3c}{8}\right)}{d}+\frac{b\left(\left(\frac{\sinh(dx+c)^5}{6}-\frac{5\sinh(dx+c)^3}{24}+\frac{5\sinh(dx+c)}{16}\right)\cosh(dx+}{d}$
risch	$\frac{3ax}{8}-\frac{5bx}{16}+\frac{b\,e^{6dx+6c}}{384d}+\frac{e^{4dx+4c}a}{64d}-\frac{3\,e^{4dx+4c}b}{128d}-\frac{e^{2dx+2c}a}{8d}+\frac{15\,e^{2dx+2c}b}{128d}+\frac{e^{-2dx-2c}a}{8d}-\frac{15\,e^{-2dx-2}}{128d}$
orering	$x\sinh(dx+c)^4\left(a+b\sinh(dx+c)^2\right)+\frac{\frac{49\sinh(dx+c)^3\left(a+b\sinh(dx+c)^2\right)d\cosh(dx+c)}{36}+\frac{49\sinh(dx+c)^5bd}{72}}{d^2}$

input
```
int(sinh(d*x+c)^4*(a+b*sinh(d*x+c)^2),x,method=_RETURNVERBOSE)
```

output
```
1/192*((-48*a+45*b)*sinh(2*d*x+2*c)+(6*a-9*b)*sinh(4*d*x+4*c)+b*sinh(6*d*x
+6*c)+72*x*(a-5/6*b)*d)/d
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 122, normalized size of antiderivative = 1.37

$$\int \sinh^4(c+dx)\left(a+b\sinh^2(c+dx)\right)\,dx$$
$$= \frac{3\,b\cosh\left(dx+c\right)\sinh\left(dx+c\right)^5+2\left(5\,b\cosh\left(dx+c\right)^3+3\left(2\,a-3\,b\right)\cosh\left(dx+c\right)\right)\sinh\left(dx+c\right)^3+6\,(6}{9}$$

input
```
integrate(sinh(d*x+c)^4*(a+b*sinh(d*x+c)^2),x, algorithm="fricas")
```

output
```
1/96*(3*b*cosh(d*x + c)*sinh(d*x + c)^5 + 2*(5*b*cosh(d*x + c)^3 + 3*(2*a
- 3*b)*cosh(d*x + c))*sinh(d*x + c)^3 + 6*(6*a - 5*b)*d*x + 3*(b*cosh(d*x
+ c)^5 + 2*(2*a - 3*b)*cosh(d*x + c)^3 - (16*a - 15*b)*cosh(d*x + c))*sinh
(d*x + c))/d
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 258 vs. $2(82) = 164$.

Time = 0.38 (sec) , antiderivative size = 258, normalized size of antiderivative = 2.90

$$\int \sinh^4(c+dx)\left(a+b\sinh^2(c+dx)\right)\,dx$$
$$= \begin{cases} \frac{3ax\sinh^4(c+dx)}{8} - \frac{3ax\sinh^2(c+dx)\cosh^2(c+dx)}{4} + \frac{3ax\cosh^4(c+dx)}{8} + \frac{5a\sinh^3(c+dx)\cosh(c+dx)}{8d} - \frac{3a\sinh(c+dx)\cosh^3(c+dx)}{8d} \\ x\left(a+b\sinh^2(c)\right)\sinh^4(c) \end{cases}$$

input
```
integrate(sinh(d*x+c)**4*(a+b*sinh(d*x+c)**2),x)
```

output
```
Piecewise((3*a*x*sinh(c + d*x)**4/8 - 3*a*x*sinh(c + d*x)**2*cosh(c + d*x)
**2/4 + 3*a*x*cosh(c + d*x)**4/8 + 5*a*sinh(c + d*x)**3*cosh(c + d*x)/(8*d
) - 3*a*sinh(c + d*x)*cosh(c + d*x)**3/(8*d) + 5*b*x*sinh(c + d*x)**6/16 -
 15*b*x*sinh(c + d*x)**4*cosh(c + d*x)**2/16 + 15*b*x*sinh(c + d*x)**2*cos
h(c + d*x)**4/16 - 5*b*x*cosh(c + d*x)**6/16 + 11*b*sinh(c + d*x)**5*cosh(
c + d*x)/(16*d) - 5*b*sinh(c + d*x)**3*cosh(c + d*x)**3/(6*d) + 5*b*sinh(c
 + d*x)*cosh(c + d*x)**5/(16*d), Ne(d, 0)), (x*(a + b*sinh(c)**2)*sinh(c)*
*4, True))
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 150, normalized size of antiderivative = 1.69

$$\int \sinh^4(c+dx)\left(a+b\sinh^2(c+dx)\right)\,dx$$
$$= \frac{1}{64}a\left(24x + \frac{e^{(4dx+4c)}}{d} - \frac{8e^{(2dx+2c)}}{d} + \frac{8e^{(-2dx-2c)}}{d} - \frac{e^{(-4dx-4c)}}{d}\right) - \frac{1}{384}b\left(\frac{\left(9e^{(-2dx-2c)} - 45e^{(-4dx-4c)} - 1\right)e^{(6dx+6c)}}{d} + \frac{120(dx+c)}{d} + \frac{45e^{(-2dx-2c)} - 9e^{(-4dx-4c)} + e^{(-}}{d}\right.$$

input
```
integrate(sinh(d*x+c)^4*(a+b*sinh(d*x+c)^2),x, algorithm="maxima")
```

output
```
1/64*a*(24*x + e^(4*d*x + 4*c)/d - 8*e^(2*d*x + 2*c)/d + 8*e^(-2*d*x - 2*c
)/d - e^(-4*d*x - 4*c)/d) - 1/384*b*((9*e^(-2*d*x - 2*c) - 45*e^(-4*d*x -
4*c) - 1)*e^(6*d*x + 6*c)/d + 120*(d*x + c)/d + (45*e^(-2*d*x - 2*c) - 9*e
^(-4*d*x - 4*c) + e^(-6*d*x - 6*c))/d)
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 125, normalized size of antiderivative = 1.40

$$\int \sinh^4(c+dx)\left(a+b\sinh^2(c+dx)\right)\,dx$$
$$= \frac{1}{16}(6a-5b)x + \frac{be^{(6dx+6c)}}{384d} + \frac{(2a-3b)e^{(4dx+4c)}}{128d} - \frac{(16a-15b)e^{(2dx+2c)}}{128d} + \frac{(16a-15b)e^{(-2dx-2c)}}{128d} - \frac{(2a-3b)e^{(-4dx-4c)}}{128d} - \frac{be^{(-6dx-6c)}}{384d}$$

input
```
integrate(sinh(d*x+c)^4*(a+b*sinh(d*x+c)^2),x, algorithm="giac")
```

output
```
1/16*(6*a - 5*b)*x + 1/384*b*e^(6*d*x + 6*c)/d + 1/128*(2*a - 3*b)*e^(4*d*
x + 4*c)/d - 1/128*(16*a - 15*b)*e^(2*d*x + 2*c)/d + 1/128*(16*a - 15*b)*e
^(-2*d*x - 2*c)/d - 1/128*(2*a - 3*b)*e^(-4*d*x - 4*c)/d - 1/384*b*e^(-6*d
*x - 6*c)/d
```

Mupad [B] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 76, normalized size of antiderivative = 0.85

$$\int \sinh^4(c+dx)\left(a+b\sinh^2(c+dx)\right)\,dx = \frac{\frac{3\,a\sinh(4\,c+4\,d\,x)}{2} - 12\,a\sinh(2\,c+2\,d\,x) + \frac{45\,b\sinh(2\,c+2\,d\,x)}{4} - \frac{9\,b\sinh(4\,c+4\,d\,x)}{4} + \frac{b\sinh(6\,c+6\,d\,x)}{4} + 18\,a\,d\,x - 15\,b\,$$

$$48\,d$$

input
```
int(sinh(c + d*x)^4*(a + b*sinh(c + d*x)^2),x)
```

output
```
((3*a*sinh(4*c + 4*d*x))/2 - 12*a*sinh(2*c + 2*d*x) + (45*b*sinh(2*c + 2*d
*x))/4 - (9*b*sinh(4*c + 4*d*x))/4 + (b*sinh(6*c + 6*d*x))/4 + 18*a*d*x -
15*b*d*x)/(48*d)
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 167, normalized size of antiderivative = 1.88

$$\int \sinh^4(c+dx)\left(a+b\sinh^2(c+dx)\right)\,dx = \frac{e^{12dx+12c}b + 6e^{10dx+10c}a - 9e^{10dx+10c}b - 48e^{8dx+8c}a + 45e^{8dx+8c}b + 144e^{6dx+6c}adx - 120e^{6dx+6c}bdx + 48e}{384e^{6dx+6c}d}$$

input
```
int(sinh(d*x+c)^4*(a+b*sinh(d*x+c)^2),x)
```

output
```
(e**(12*c + 12*d*x)*b + 6*e**(10*c + 10*d*x)*a - 9*e**(10*c + 10*d*x)*b -
48*e**(8*c + 8*d*x)*a + 45*e**(8*c + 8*d*x)*b + 144*e**(6*c + 6*d*x)*a*d*x
 - 120*e**(6*c + 6*d*x)*b*d*x + 48*e**(4*c + 4*d*x)*a - 45*e**(4*c + 4*d*x
)*b - 6*e**(2*c + 2*d*x)*a + 9*e**(2*c + 2*d*x)*b - b)/(384*e**(6*c + 6*d*
x)*d)
```

3.2 $\int \sinh^3(c+dx)\left(a+b\sinh^2(c+dx)\right)\,dx$

Optimal result

Integrand size = 21, antiderivative size = 53

$$\int \sinh^3(c+dx)\left(a+b\sinh^2(c+dx)\right)\,dx=-\frac{(a-b)\cosh(c+dx)}{d}+\frac{(a-2b)\cosh^3(c+dx)}{3d}+\frac{b\cosh^5(c+dx)}{5d}$$

output
```
-(a-b)*cosh(d*x+c)/d+1/3*(a-2*b)*cosh(d*x+c)^3/d+1/5*b*cosh(d*x+c)^5/d
```

Mathematica [A] (verified)

Time = 0.10 (sec) , antiderivative size = 77, normalized size of antiderivative = 1.45

$$\int \sinh^3(c+dx)\left(a+b\sinh^2(c+dx)\right)\,dx=-\frac{3a\cosh(c+dx)}{4d}+\frac{5b\cosh(c+dx)}{8d}+\frac{a\cosh(3(c+dx))}{12d}-\frac{5b\cosh(3(c+dx))}{48d}+\frac{b\cosh(5(c+dx))}{80d}$$

input
```
Integrate[Sinh[c + d*x]^3*(a + b*Sinh[c + d*x]^2),x]
```

output
```
(-3*a*Cosh[c + d*x])/(4*d) + (5*b*Cosh[c + d*x])/(8*d) + (a*Cosh[3*(c + d*
x)])/(12*d) - (5*b*Cosh[3*(c + d*x)])/(48*d) + (b*Cosh[5*(c + d*x)])/(80*d
)
```

Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.91, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.238, Rules used = {3042, 26, 3492, 290, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sinh^3(c+dx)\left(a+b\sinh^2(c+dx)\right)\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int i\sin(ic+idx)^3\left(a-b\sin(ic+idx)^2\right)dx$$

$$\Big\downarrow \text{26}$$

$$i\int \sin(ic+idx)^3\left(a-b\sin(ic+idx)^2\right)dx$$

$$\Big\downarrow \text{3492}$$

$$-\frac{\int \left(1-\cosh^2(c+dx)\right)\left(b\cosh^2(c+dx)+a-b\right)d\cosh(c+dx)}{d}$$

$$\Big\downarrow \text{290}$$

$$-\frac{\int \left(-b\cosh^4(c+dx)-(a-2b)\cosh^2(c+dx)+a\left(1-\frac{b}{a}\right)\right)d\cosh(c+dx)}{d}$$

$$\Big\downarrow \text{2009}$$

$$-\frac{-\frac{1}{3}(a-2b)\cosh^3(c+dx)+(a-b)\cosh(c+dx)-\frac{1}{5}b\cosh^5(c+dx)}{d}$$

input
```
Int[Sinh[c + d*x]^3*(a + b*Sinh[c + d*x]^2),x]
```

output
```
-(((a - b)*Cosh[c + d*x] - ((a - 2*b)*Cosh[c + d*x]^3)/3 - (b*Cosh[c + d*x
]^5)/5)/d)
```

Defintions of rubi rules used

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 290
```
Int[((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.), x_Symbol] :> I
nt[ExpandIntegrand[(a + b*x^2)^p*(c + d*x^2)^q, x], x] /; FreeQ[{a, b, c, d
}, x] && NeQ[b*c - a*d, 0] && IGtQ[p, 0] && IGtQ[q, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3492
```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((A_) + (C_.)*sin[(e_.) + (f_.)*(x_)]^2),
 x_Symbol] :> Simp[-f^(-1)   Subst[Int[(1 - x^2)^((m - 1)/2)*(A + C - C*x^2
), x], x, Cos[e + f*x]], x] /; FreeQ[{e, f, A, C}, x] && IGtQ[(m + 1)/2, 0]
```

Maple [A] (verified)

Time = 2.26 (sec) , antiderivative size = 56, normalized size of antiderivative = 1.06

method	result
derivativedivides	$\frac{a\left(-\frac{2}{3}+\frac{\sinh(dx+c)^2}{3}\right)\cosh(dx+c)+b\left(\frac{8}{15}+\frac{\sinh(dx+c)^4}{5}-\frac{4\sinh(dx+c)^2}{15}\right)\cosh(dx+c)}{d}$
default	$\frac{a\left(-\frac{2}{3}+\frac{\sinh(dx+c)^2}{3}\right)\cosh(dx+c)+b\left(\frac{8}{15}+\frac{\sinh(dx+c)^4}{5}-\frac{4\sinh(dx+c)^2}{15}\right)\cosh(dx+c)}{d}$
parallelrisch	$\frac{(20a-25b)\cosh(3dx+3c)+3b\cosh(5dx+5c)+(-180a+150b)\cosh(dx+c)-160a+128b}{240d}$
parts	$\frac{a\left(-\frac{2}{3}+\frac{\sinh(dx+c)^2}{3}\right)\cosh(dx+c)}{d}+\frac{b\left(\frac{8}{15}+\frac{\sinh(dx+c)^4}{5}-\frac{4\sinh(dx+c)^2}{15}\right)\cosh(dx+c)}{d}$
risch	$\frac{b\,e^{5dx+5c}}{160d}+\frac{e^{3dx+3c}a}{24d}-\frac{5\,e^{3dx+3c}b}{96d}-\frac{3\,e^{dx+c}a}{8d}+\frac{5\,e^{dx+c}b}{16d}-\frac{3\,e^{-dx-c}a}{8d}+\frac{5\,e^{-dx-c}b}{16d}+\frac{e^{-3dx-3c}a}{24d}-5$
orering	$\frac{\frac{259\sinh(dx+c)^2\left(a+b\sinh(dx+c)^2\right)d\cosh(dx+c)}{75}+\frac{518\sinh(dx+c)^4bd\cosh(dx+c)}{225}}{d^2}-\frac{7\left(6d^3\cosh(dx+c)^3\left(a+b\sinh(dx+c)^2\right)\right.}{}$

input
```
int(sinh(d*x+c)^3*(a+b*sinh(d*x+c)^2),x,method=_RETURNVERBOSE)
```

output
```
1/d*(a*(-2/3+1/3*sinh(d*x+c)^2)*cosh(d*x+c)+b*(8/15+1/5*sinh(d*x+c)^4-4/15
*sinh(d*x+c)^2)*cosh(d*x+c))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 102 vs. 2(49) = 98.

Time = 0.08 (sec) , antiderivative size = 102, normalized size of antiderivative = 1.92

$$\int \sinh^3(c+dx)\left(a+b\sinh^2(c+dx)\right)\,dx$$
$$=\frac{3\,b\cosh(dx+c)^5+15\,b\cosh(dx+c)\sinh(dx+c)^4+5\,(4\,a-5\,b)\cosh(dx+c)^3+15\,(2\,b\cosh(dx+c)}{240\,d}$$

input
```
integrate(sinh(d*x+c)^3*(a+b*sinh(d*x+c)^2),x, algorithm="fricas")
```

output
```
1/240*(3*b*cosh(d*x + c)^5 + 15*b*cosh(d*x + c)*sinh(d*x + c)^4 + 5*(4*a -
 5*b)*cosh(d*x + c)^3 + 15*(2*b*cosh(d*x + c)^3 + (4*a - 5*b)*cosh(d*x + c
))*sinh(d*x + c)^2 - 30*(6*a - 5*b)*cosh(d*x + c))/d
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 105 vs. $2(42) = 84$.

Time = 0.23 (sec) , antiderivative size = 105, normalized size of antiderivative = 1.98

$$\int \sinh^3(c+dx)\left(a+b\sinh^2(c+dx)\right)\,dx$$
$$= \begin{cases} \frac{a\sinh^2(c+dx)\cosh(c+dx)}{d} - \frac{2a\cosh^3(c+dx)}{3d} + \frac{b\sinh^4(c+dx)\cosh(c+dx)}{d} - \frac{4b\sinh^2(c+dx)\cosh^3(c+dx)}{3d} + \frac{8b\cosh^5(c+dx)}{15d} & \text{fo} \\ x\left(a+b\sinh^2(c)\right)\sinh^3(c) & \text{ot} \end{cases}$$

input
```
integrate(sinh(d*x+c)**3*(a+b*sinh(d*x+c)**2),x)
```

output
```
Piecewise((a*sinh(c + d*x)**2*cosh(c + d*x)/d - 2*a*cosh(c + d*x)**3/(3*d)
 + b*sinh(c + d*x)**4*cosh(c + d*x)/d - 4*b*sinh(c + d*x)**2*cosh(c + d*x)
**3/(3*d) + 8*b*cosh(c + d*x)**5/(15*d), Ne(d, 0)), (x*(a + b*sinh(c)**2)*
sinh(c)**3, True))
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 141 vs. $2(49) = 98$.

Time = 0.05 (sec) , antiderivative size = 141, normalized size of antiderivative = 2.66

$$\int \sinh^3(c+dx)\left(a+b\sinh^2(c+dx)\right)\,dx$$
$$= \frac{1}{480}b\left(\frac{3\,e^{(5\,dx+5\,c)}}{d} - \frac{25\,e^{(3\,dx+3\,c)}}{d} + \frac{150\,e^{(dx+c)}}{d} + \frac{150\,e^{(-dx-c)}}{d} - \frac{25\,e^{(-3\,dx-3\,c)}}{d} + \frac{3\,e^{(-5\,dx-5\,c)}}{d}\right)$$
$$+ \frac{1}{24}a\left(\frac{e^{(3\,dx+3\,c)}}{d} - \frac{9\,e^{(dx+c)}}{d} - \frac{9\,e^{(-dx-c)}}{d} + \frac{e^{(-3\,dx-3\,c)}}{d}\right)$$

input
```
integrate(sinh(d*x+c)^3*(a+b*sinh(d*x+c)^2),x, algorithm="maxima")
```

output
```
1/480*b*(3*e^(5*d*x + 5*c)/d - 25*e^(3*d*x + 3*c)/d + 150*e^(d*x + c)/d +
150*e^(-d*x - c)/d - 25*e^(-3*d*x - 3*c)/d + 3*e^(-5*d*x - 5*c)/d) + 1/24*
a*(e^(3*d*x + 3*c)/d - 9*e^(d*x + c)/d - 9*e^(-d*x - c)/d + e^(-3*d*x - 3*
c)/d)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 112 vs. $2(49) = 98$.

Time = 0.13 (sec) , antiderivative size = 112, normalized size of antiderivative = 2.11

$$\int \sinh^3(c+dx)\left(a+b\sinh^2(c+dx)\right)\,dx = \frac{be^{(5\,dx+5\,c)}}{160\,d}+\frac{(4\,a-5\,b)e^{(3\,dx+3\,c)}}{96\,d}-\frac{(6\,a-5\,b)e^{(dx+c)}}{16\,d}-\frac{(6\,a-5\,b)e^{(-dx-c)}}{16\,d}+\frac{(4\,a-5\,b)e^{(-3\,dx-3\,c)}}{96\,d}+\frac{be^{(-5\,dx-5\,c)}}{160\,d}$$

input
```
integrate(sinh(d*x+c)^3*(a+b*sinh(d*x+c)^2),x, algorithm="giac")
```

output
```
1/160*b*e^(5*d*x + 5*c)/d + 1/96*(4*a - 5*b)*e^(3*d*x + 3*c)/d - 1/16*(6*a
 - 5*b)*e^(d*x + c)/d - 1/16*(6*a - 5*b)*e^(-d*x - c)/d + 1/96*(4*a - 5*b)
*e^(-3*d*x - 3*c)/d + 1/160*b*e^(-5*d*x - 5*c)/d
```

Mupad [B] (verification not implemented)

Time = 1.76 (sec) , antiderivative size = 57, normalized size of antiderivative = 1.08

$$\int \sinh^3(c+dx)\left(a+b\sinh^2(c+dx)\right)\,dx = \frac{15\,b\cosh(c+d\,x)-15\,a\cosh(c+d\,x)+5\,a\cosh(c+d\,x)^3-10\,b\cosh(c+d\,x)^3+3\,b\cosh(c+d\,x)^5}{15\,d}$$

input
```
int(sinh(c + d*x)^3*(a + b*sinh(c + d*x)^2),x)
```

output
```
(15*b*cosh(c + d*x) - 15*a*cosh(c + d*x) + 5*a*cosh(c + d*x)^3 - 10*b*cosh
(c + d*x)^3 + 3*b*cosh(c + d*x)^5)/(15*d)
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 138, normalized size of antiderivative = 2.60

$$\int \sinh^3(c+dx)\left(a+b\sinh^2(c+dx)\right)\,dx$$
$$= \frac{3e^{10dx+10c}b + 20e^{8dx+8c}a - 25e^{8dx+8c}b - 180e^{6dx+6c}a + 150e^{6dx+6c}b - 180e^{4dx+4c}a + 150e^{4dx+4c}b + 20e^{2dx}}{480e^{5dx+5c}d}$$

input
```
int(sinh(d*x+c)^3*(a+b*sinh(d*x+c)^2),x)
```

output
```
(3*e**(10*c + 10*d*x)*b + 20*e**(8*c + 8*d*x)*a - 25*e**(8*c + 8*d*x)*b -
180*e**(6*c + 6*d*x)*a + 150*e**(6*c + 6*d*x)*b - 180*e**(4*c + 4*d*x)*a +
 150*e**(4*c + 4*d*x)*b + 20*e**(2*c + 2*d*x)*a - 25*e**(2*c + 2*d*x)*b +
3*b)/(480*e**(5*c + 5*d*x)*d)
```

3.3 $\int \sinh^2(c+dx)\left(a+b\sinh^2(c+dx)\right)\,dx$

Optimal result

Integrand size = 21, antiderivative size = 61

$$\int \sinh^2(c+dx)\left(a+b\sinh^2(c+dx)\right)\,dx = -\frac{1}{8}(4a-3b)x + \frac{(4a-3b)\cosh(c+dx)\sinh(c+dx)}{8d} + \frac{b\cosh(c+dx)\sinh^3(c+dx)}{4d}$$

output
```
-1/8*(4*a-3*b)*x+1/8*(4*a-3*b)*cosh(d*x+c)*sinh(d*x+c)/d+1/4*b*cosh(d*x+c)
*sinh(d*x+c)^3/d
```

Mathematica [A] (verified)

Time = 0.12 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.77

$$\int \sinh^2(c+dx)\left(a+b\sinh^2(c+dx)\right)\,dx = \frac{-4(4a-3b)(c+dx)+8(a-b)\sinh(2(c+dx))+b\sinh(4(c+dx))}{32d}$$

input
```
Integrate[Sinh[c + d*x]^2*(a + b*Sinh[c + d*x]^2),x]
```

output
```
(-4*(4*a - 3*b)*(c + d*x) + 8*(a - b)*Sinh[2*(c + d*x)] + b*Sinh[4*(c + d*
x)])/(32*d)
```

Rubi [A] (verified)

Time = 0.34 (sec) , antiderivative size = 59, normalized size of antiderivative = 0.97, number of steps used = 8, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.381, Rules used = {3042, 25, 3493, 25, 3042, 25, 3115, 24}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sinh^2(c+dx)\left(a+b\sinh^2(c+dx)\right)\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int -\sin(ic+idx)^2\left(a-b\sin(ic+idx)^2\right)dx$$

$$\Big\downarrow \text{25}$$

$$-\int \sin(ic+idx)^2\left(a-b\sin(ic+idx)^2\right)dx$$

$$\Big\downarrow \text{3493}$$

$$\frac{b\sinh^3(c+dx)\cosh(c+dx)}{4d}-\frac{1}{4}(4a-3b)\int -\sinh^2(c+dx)dx$$

$$\Big\downarrow \text{25}$$

$$\frac{1}{4}(4a-3b)\int \sinh^2(c+dx)dx+\frac{b\sinh^3(c+dx)\cosh(c+dx)}{4d}$$

$$\Big\downarrow \text{3042}$$

$$\frac{b\sinh^3(c+dx)\cosh(c+dx)}{4d}+\frac{1}{4}(4a-3b)\int -\sin(ic+idx)^2dx$$

$$\Big\downarrow \text{25}$$

$$\frac{b\sinh^3(c+dx)\cosh(c+dx)}{4d}-\frac{1}{4}(4a-3b)\int \sin(ic+idx)^2dx$$

$$\Big\downarrow \text{3115}$$

$$\frac{b\sinh^3(c+dx)\cosh(c+dx)}{4d}-\frac{1}{4}(4a-3b)\left(\frac{\int 1dx}{2}-\frac{\sinh(c+dx)\cosh(c+dx)}{2d}\right)$$

$$\Big\downarrow \; 24$$

$$\frac{b\sinh^3(c+dx)\cosh(c+dx)}{4d}-\frac{1}{4}(4a-3b)\left(\frac{x}{2}-\frac{\sinh(c+dx)\cosh(c+dx)}{2d}\right)$$

input
```
Int[Sinh[c + d*x]^2*(a + b*Sinh[c + d*x]^2),x]
```

output
```
(b*Cosh[c + d*x]*Sinh[c + d*x]^3)/(4*d) - ((4*a - 3*b)*(x/2 - (Cosh[c + d*
x]*Sinh[c + d*x])/(2*d)))/4
```

Defintions of rubi rules used

rule 24
```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3115
```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Simp[b^2*((n - 1)/n)   Int[(b*Sin
[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[
2*n]
```

rule 3493
```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_) + (C_.)*sin[(e_.) + (f_.)*(
x_)]^2), x_Symbol] :> Simp[(-C)*Cos[e + f*x]*((b*Sin[e + f*x])^(m + 1)/(b*f
*(m + 2))), x] + Simp[(A*(m + 2) + C*(m + 1))/(m + 2)   Int[(b*Sin[e + f*x]
)^m, x], x] /; FreeQ[{b, e, f, A, C, m}, x] &&  !LtQ[m, -1]
```

Maple [A] (verified)

Time = 1.12 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.72

method	result
parallelrisch	$\frac{(8a-8b)\sinh(2dx+2c)+b\sinh(4dx+4c)-16x\left(a-\frac{3b}{4}\right)d}{32d}$
derivativedivides	$\frac{a\left(\frac{\cosh(dx+c)\sinh(dx+c)}{2}-\frac{dx}{2}-\frac{c}{2}\right)+b\left(\left(\frac{\sinh(dx+c)^3}{4}-\frac{3\sinh(dx+c)}{8}\right)\cosh(dx+c)+\frac{3dx}{8}+\frac{3c}{8}\right)}{d}$
default	$\frac{a\left(\frac{\cosh(dx+c)\sinh(dx+c)}{2}-\frac{dx}{2}-\frac{c}{2}\right)+b\left(\left(\frac{\sinh(dx+c)^3}{4}-\frac{3\sinh(dx+c)}{8}\right)\cosh(dx+c)+\frac{3dx}{8}+\frac{3c}{8}\right)}{d}$
parts	$\frac{a\left(\frac{\cosh(dx+c)\sinh(dx+c)}{2}-\frac{dx}{2}-\frac{c}{2}\right)}{d}+\frac{b\left(\left(\frac{\sinh(dx+c)^3}{4}-\frac{3\sinh(dx+c)}{8}\right)\cosh(dx+c)+\frac{3dx}{8}+\frac{3c}{8}\right)}{d}$
risch	$-\frac{ax}{2}+\frac{3bx}{8}+\frac{e^{4dx+4c}b}{64d}+\frac{e^{2dx+2c}a}{8d}-\frac{e^{2dx+2c}b}{8d}-\frac{e^{-2dx-2c}a}{8d}+\frac{e^{-2dx-2c}b}{8d}-\frac{e^{-4dx-4c}b}{64d}$
orering	$x\sinh\left(dx+c\right)^2\left(a+b\sinh\left(dx+c\right)^2\right)+\frac{\frac{5\sinh(dx+c)\left(a+b\sinh(dx+c)^2\right)d\cosh(dx+c)}{8}+\frac{5\sinh(dx+c)^3bd\cosh}{8}}{d^2}$

input
```
int(sinh(d*x+c)^2*(a+b*sinh(d*x+c)^2),x,method=_RETURNVERBOSE)
```

output
```
1/32*((8*a-8*b)*sinh(2*d*x+2*c)+b*sinh(4*d*x+4*c)-16*x*(a-3/4*b)*d)/d
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 64, normalized size of antiderivative = 1.05

$$\int \sinh^2(c+dx)\left(a+b\sinh^2(c+dx)\right)\,dx$$
$$=\frac{b\cosh\left(dx+c\right)\sinh\left(dx+c\right)^3-(4a-3b)dx+\left(b\cosh\left(dx+c\right)^3+4(a-b)\cosh\left(dx+c\right)\right)\sinh\left(dx+c\right)}{8d}$$

input
```
integrate(sinh(d*x+c)^2*(a+b*sinh(d*x+c)^2),x, algorithm="fricas")
```

output
```
1/8*(b*cosh(d*x + c)*sinh(d*x + c)^3 - (4*a - 3*b)*d*x + (b*cosh(d*x + c)^
3 + 4*(a - b)*cosh(d*x + c))*sinh(d*x + c))/d
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 158 vs. $2(53) = 106$.

Time = 0.18 (sec) , antiderivative size = 158, normalized size of antiderivative = 2.59

$$\int \sinh^2(c+dx)\left(a+b\sinh^2(c+dx)\right)\,dx$$
$$= \begin{cases} \frac{ax\sinh^2(c+dx)}{2} - \frac{ax\cosh^2(c+dx)}{2} + \frac{a\sinh(c+dx)\cosh(c+dx)}{2d} + \frac{3bx\sinh^4(c+dx)}{8} - \frac{3bx\sinh^2(c+dx)\cosh^2(c+dx)}{4} + \frac{3bx\cosh^4(\cdot}{8} \\ x\left(a+b\sinh^2(c)\right)\sinh^2(c) \end{cases}$$

input
```
integrate(sinh(d*x+c)**2*(a+b*sinh(d*x+c)**2),x)
```

output
```
Piecewise((a*x*sinh(c + d*x)**2/2 - a*x*cosh(c + d*x)**2/2 + a*sinh(c + d*
x)*cosh(c + d*x)/(2*d) + 3*b*x*sinh(c + d*x)**4/8 - 3*b*x*sinh(c + d*x)**2
*cosh(c + d*x)**2/4 + 3*b*x*cosh(c + d*x)**4/8 + 5*b*sinh(c + d*x)**3*cosh
(c + d*x)/(8*d) - 3*b*sinh(c + d*x)*cosh(c + d*x)**3/(8*d), Ne(d, 0)), (x*
(a + b*sinh(c)**2)*sinh(c)**2, True))
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 97, normalized size of antiderivative = 1.59

$$\int \sinh^2(c+dx)\left(a+b\sinh^2(c+dx)\right)\,dx$$
$$= \frac{1}{64}b\left(24x + \frac{e^{(4dx+4c)}}{d} - \frac{8e^{(2dx+2c)}}{d} + \frac{8e^{(-2dx-2c)}}{d} - \frac{e^{(-4dx-4c)}}{d}\right) - \frac{1}{8}a\left(4x - \frac{e^{(2dx+2c)}}{d} + \frac{e^{(-2dx-2c)}}{d}\right)$$

input
```
integrate(sinh(d*x+c)^2*(a+b*sinh(d*x+c)^2),x, algorithm="maxima")
```

output
```
1/64*b*(24*x + e^(4*d*x + 4*c)/d - 8*e^(2*d*x + 2*c)/d + 8*e^(-2*d*x - 2*c
)/d - e^(-4*d*x - 4*c)/d) - 1/8*a*(4*x - e^(2*d*x + 2*c)/d + e^(-2*d*x - 2
*c)/d)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 79, normalized size of antiderivative = 1.30

$$\int \sinh^2(c+dx)\left(a+b\sinh^2(c+dx)\right)\,dx=-\frac{1}{8}\,(4\,a-3\,b)x+\frac{be^{(4\,dx+4\,c)}}{64\,d}+\frac{(a-b)e^{(2\,dx+2\,c)}}{8\,d}-\frac{(a-b)e^{(-2\,dx-2\,c)}}{8\,d}-\frac{be^{(-4\,dx-4\,c)}}{64\,d}$$

input
```
integrate(sinh(d*x+c)^2*(a+b*sinh(d*x+c)^2),x, algorithm="giac")
```

output
```
-1/8*(4*a - 3*b)*x + 1/64*b*e^(4*d*x + 4*c)/d + 1/8*(a - b)*e^(2*d*x + 2*c
)/d - 1/8*(a - b)*e^(-2*d*x - 2*c)/d - 1/64*b*e^(-4*d*x - 4*c)/d
```

Mupad [B] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 50, normalized size of antiderivative = 0.82

$$\int \sinh^2(c+dx)\left(a+b\sinh^2(c+dx)\right)\,dx=\frac{\frac{a\sinh(2\,c+2\,d\,x)}{4}-\frac{b\sinh(2\,c+2\,d\,x)}{4}+\frac{b\sinh(4\,c+4\,d\,x)}{32}}{d}-\frac{a\,x}{2}+\frac{3\,b\,x}{8}$$

input
```
int(sinh(c + d*x)^2*(a + b*sinh(c + d*x)^2),x)
```

output
```
((a*sinh(2*c + 2*d*x))/4 - (b*sinh(2*c + 2*d*x))/4 + (b*sinh(4*c + 4*d*x))
/32)/d - (a*x)/2 + (3*b*x)/8
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 115, normalized size of antiderivative = 1.89

$$\int \sinh^2(c+dx)\left(a+b\sinh^2(c+dx)\right)\,dx$$
$$=\frac{e^{8dx+8c}b+8e^{6dx+6c}a-8e^{6dx+6c}b-32e^{4dx+4c}adx+24e^{4dx+4c}bdx-8e^{2dx+2c}a+8e^{2dx+2c}b-b}{64e^{4dx+4c}d}$$

input
```
int(sinh(d*x+c)^2*(a+b*sinh(d*x+c)^2),x)
```

output
```
(e**(8*c + 8*d*x)*b + 8*e**(6*c + 6*d*x)*a - 8*e**(6*c + 6*d*x)*b - 32*e**
(4*c + 4*d*x)*a*d*x + 24*e**(4*c + 4*d*x)*b*d*x - 8*e**(2*c + 2*d*x)*a + 8
*e**(2*c + 2*d*x)*b - b)/(64*e**(4*c + 4*d*x)*d)
```

3.4 $\int \sinh(c+dx)\left(a+b\sinh^2(c+dx)\right)\,dx$

Optimal result

Integrand size = 19, antiderivative size = 32

$$\int \sinh(c+dx)\left(a+b\sinh^2(c+dx)\right)\,dx=\frac{(a-b)\cosh(c+dx)}{d}+\frac{b\cosh^3(c+dx)}{3d}$$

output
```
(a-b)*cosh(d*x+c)/d+1/3*b*cosh(d*x+c)^3/d
```

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 53, normalized size of antiderivative = 1.66

$$\int \sinh(c+dx)\left(a+b\sinh^2(c+dx)\right)\,dx=\frac{a\cosh(c)\cosh(dx)}{d}-\frac{3b\cosh(c+dx)}{4d}+\frac{b\cosh(3(c+dx))}{12d}+\frac{a\sinh(c)\sinh(dx)}{d}$$

input
```
Integrate[Sinh[c + d*x]*(a + b*Sinh[c + d*x]^2),x]
```

output
```
(a*Cosh[c]*Cosh[d*x])/d - (3*b*Cosh[c + d*x])/(4*d) + (b*Cosh[3*(c + d*x)]
)/(12*d) + (a*Sinh[c]*Sinh[d*x])/d
```

Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 30, normalized size of antiderivative = 0.94, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.211, Rules used = {3042, 26, 3492, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sinh(c+dx)\left(a+b\sinh^2(c+dx)\right)\,dx$$
$$\Big\downarrow \ 3042$$
$$\int -i\sin(ic+idx)\left(a-b\sin(ic+idx)^2\right)dx$$
$$\Big\downarrow \ 26$$
$$-i\int \sin(ic+idx)\left(a-b\sin(ic+idx)^2\right)dx$$
$$\Big\downarrow \ 3492$$
$$\frac{\int \left(b\cosh^2(c+dx)+a-b\right)d\cosh(c+dx)}{d}$$
$$\Big\downarrow \ 2009$$
$$\frac{(a-b)\cosh(c+dx)+\frac{1}{3}b\cosh^3(c+dx)}{d}$$

input
```
Int[Sinh[c + d*x]*(a + b*Sinh[c + d*x]^2),x]
```

output
```
((a - b)*Cosh[c + d*x] + (b*Cosh[c + d*x]^3)/3)/d
```

Defintions of rubi rules used

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3492
```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((A_) + (C_.)*sin[(e_.) + (f_.)*(x_)]^2),
 x_Symbol] :> Simp[-f^(-1)   Subst[Int[(1 - x^2)^((m - 1)/2)*(A + C - C*x^2
), x], x, Cos[e + f*x]], x] /; FreeQ[{e, f, A, C}, x] && IGtQ[(m + 1)/2, 0]
```

Maple [A] (verified)

Time = 0.64 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.06

method	result
derivativedivides	$\frac{a\cosh(dx+c)+b\left(-\frac{2}{3}+\frac{\sinh(dx+c)^2}{3}\right)\cosh(dx+c)}{d}$
default	$\frac{a\cosh(dx+c)+b\left(-\frac{2}{3}+\frac{\sinh(dx+c)^2}{3}\right)\cosh(dx+c)}{d}$
parts	$\frac{b\left(-\frac{2}{3}+\frac{\sinh(dx+c)^2}{3}\right)\cosh(dx+c)}{d}+\frac{a\cosh(dx+c)}{d}$
parallelrisch	$\frac{b\cosh(3dx+3c)+(12a-9b)\cosh(dx+c)+12a-8b}{12d}$
risch	$\frac{e^{3dx+3c}b}{24d}+\frac{e^{dx+c}a}{2d}-\frac{3\,e^{dx+c}b}{8d}+\frac{e^{-dx-c}a}{2d}-\frac{3\,e^{-dx-c}b}{8d}+\frac{e^{-3dx-3c}b}{24d}$
orering	$\frac{\frac{10d\cosh(dx+c)\left(a+b\sinh(dx+c)^2\right)}{9}+\frac{20\sinh(dx+c)^2bd\cosh(dx+c)}{9}}{d^2}-\frac{d^3\cosh(dx+c)\left(a+b\sinh(dx+c)^2\right)+20d^3\sinh(dx+c}{9d^4}$

input
```
int(sinh(d*x+c)*(a+b*sinh(d*x+c)^2),x,method=_RETURNVERBOSE)
```

output
```
1/d*(a*cosh(d*x+c)+b*(-2/3+1/3*sinh(d*x+c)^2)*cosh(d*x+c))
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 48, normalized size of antiderivative = 1.50

$$\int \sinh(c+dx)\left(a+b\sinh^2(c+dx)\right)\,dx$$
$$=\frac{b\cosh\left(dx+c\right)^3+3\,b\cosh\left(dx+c\right)\sinh\left(dx+c\right)^2+3\left(4\,a-3\,b\right)\cosh\left(dx+c\right)}{12\,d}$$

input
```
integrate(sinh(d*x+c)*(a+b*sinh(d*x+c)^2),x, algorithm="fricas")
```

output
```
1/12*(b*cosh(d*x + c)^3 + 3*b*cosh(d*x + c)*sinh(d*x + c)^2 + 3*(4*a - 3*b
)*cosh(d*x + c))/d
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 56 vs. 2(24) = 48.

Time = 0.12 (sec) , antiderivative size = 56, normalized size of antiderivative = 1.75

$$\int \sinh(c+dx)\left(a+b\sinh^2(c+dx)\right)\,dx$$
$$=\begin{cases}\frac{a\cosh\left(c+dx\right)}{d}+\frac{b\sinh^2\left(c+dx\right)\cosh\left(c+dx\right)}{d}-\frac{2b\cosh^3\left(c+dx\right)}{3d} & \text{for } d\neq 0\\ x\left(a+b\sinh^2\left(c\right)\right)\sinh\left(c\right) & \text{otherwise}\end{cases}$$

input
```
integrate(sinh(d*x+c)*(a+b*sinh(d*x+c)**2),x)
```

output
```
Piecewise((a*cosh(c + d*x)/d + b*sinh(c + d*x)**2*cosh(c + d*x)/d - 2*b*co
sh(c + d*x)**3/(3*d), Ne(d, 0)), (x*(a + b*sinh(c)**2)*sinh(c), True))
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 67 vs. $2(30) = 60$.

Time = 0.04 (sec) , antiderivative size = 67, normalized size of antiderivative = 2.09

$$\int \sinh(c+dx)\left(a+b\sinh^2(c+dx)\right)\,dx = \frac{1}{24}\,b\left(\frac{e^{(3\,dx+3\,c)}}{d}-\frac{9\,e^{(dx+c)}}{d}-\frac{9\,e^{(-dx-c)}}{d}+\frac{e^{(-3\,dx-3\,c)}}{d}\right)+\frac{a\cosh\left(dx+c\right)}{d}$$

input
```
integrate(sinh(d*x+c)*(a+b*sinh(d*x+c)^2),x, algorithm="maxima")
```

output
```
1/24*b*(e^(3*d*x + 3*c)/d - 9*e^(d*x + c)/d - 9*e^(-d*x - c)/d + e^(-3*d*x
 - 3*c)/d) + a*cosh(d*x + c)/d
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 70 vs. $2(30) = 60$.

Time = 0.13 (sec) , antiderivative size = 70, normalized size of antiderivative = 2.19

$$\int \sinh(c+dx)\left(a+b\sinh^2(c+dx)\right)\,dx = \frac{be^{(3\,dx+3\,c)}}{24\,d}+\frac{(4\,a-3\,b)e^{(dx+c)}}{8\,d}+\frac{(4\,a-3\,b)e^{(-dx-c)}}{8\,d}+\frac{be^{(-3\,dx-3\,c)}}{24\,d}$$

input
```
integrate(sinh(d*x+c)*(a+b*sinh(d*x+c)^2),x, algorithm="giac")
```

output
```
1/24*b*e^(3*d*x + 3*c)/d + 1/8*(4*a - 3*b)*e^(d*x + c)/d + 1/8*(4*a - 3*b)
*e^(-d*x - c)/d + 1/24*b*e^(-3*d*x - 3*c)/d
```

Mupad [B] (verification not implemented)

Time = 1.70 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.06

$$\int \sinh(c+dx)\left(a+b\sinh^2(c+dx)\right)\,dx = \frac{3\,a\cosh(c+d\,x)-3\,b\cosh(c+d\,x)+b\cosh(c+d\,x)^3}{3\,d}$$

input
```
int(sinh(c + d*x)*(a + b*sinh(c + d*x)^2),x)
```

output
```
(3*a*cosh(c + d*x) - 3*b*cosh(c + d*x) + b*cosh(c + d*x)^3)/(3*d)
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 83, normalized size of antiderivative = 2.59

$$\int \sinh(c+dx)\left(a+b\sinh^2(c+dx)\right)\,dx = \frac{e^{6dx+6c}b+12e^{4dx+4c}a-9e^{4dx+4c}b+12e^{2dx+2c}a-9e^{2dx+2c}b+b}{24e^{3dx+3c}d}$$

input
```
int(sinh(d*x+c)*(a+b*sinh(d*x+c)^2),x)
```

output
```
(e**(6*c + 6*d*x)*b + 12*e**(4*c + 4*d*x)*a - 9*e**(4*c + 4*d*x)*b + 12*e*
*(2*c + 2*d*x)*a - 9*e**(2*c + 2*d*x)*b + b)/(24*e**(3*c + 3*d*x)*d)
```

3.5 $\int \left(a+b\sinh^2(c+dx)\right)\,dx$

Optimal result

Integrand size = 12, antiderivative size = 30

$$\int \left(a+b\sinh^2(c+dx)\right)\,dx=ax-\frac{bx}{2}+\frac{b\cosh(c+dx)\sinh(c+dx)}{2d}$$

output
```
a*x-1/2*b*x+1/2*b*cosh(d*x+c)*sinh(d*x+c)/d
```

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.20

$$\int \left(a+b\sinh^2(c+dx)\right)\,dx=ax+\frac{b(-c-dx)}{2d}+\frac{b\sinh(2(c+dx))}{4d}$$

input
```
Integrate[a + b*Sinh[c + d*x]^2,x]
```

output
```
a*x + (b*(-c - d*x))/(2*d) + (b*Sinh[2*(c + d*x)])/(4*d)
```

Rubi [A] (verified)

Time = 0.16 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(a+b\sinh^2(c+dx)\right)\,dx$$

$$\Big\downarrow \text{2009}$$

$$ax+\frac{b\sinh(c+dx)\cosh(c+dx)}{2d}-\frac{bx}{2}$$

input
```
Int[a + b*Sinh[c + d*x]^2,x]
```

output
```
a*x - (b*x)/2 + (b*Cosh[c + d*x]*Sinh[c + d*x])/(2*d)
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.29 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.83

method	result	size
parallelrisch	$\frac{b(-2dx+\sinh(2dx+2c))}{4d}+ax$	25
default	$ax+\frac{b\left(\frac{\cosh(dx+c)\sinh(dx+c)}{2}-\frac{dx}{2}-\frac{c}{2}\right)}{d}$	32
parts	$ax+\frac{b\left(\frac{\cosh(dx+c)\sinh(dx+c)}{2}-\frac{dx}{2}-\frac{c}{2}\right)}{d}$	32
derivativedivides	$\frac{(dx+c)a+b\left(\frac{\cosh(dx+c)\sinh(dx+c)}{2}-\frac{dx}{2}-\frac{c}{2}\right)}{d}$	37
risch	$ax-\frac{bx}{2}+\frac{e^{2dx+2c}b}{8d}-\frac{e^{-2dx-2c}b}{8d}$	39
orering	$x\left(a+b\sinh\left(dx+c\right)^{2}\right)+\frac{b\cosh(dx+c)\sinh(dx+c)}{2d}-\frac{x\left(2b\,d^{2}\cosh(dx+c)^{2}+2b\sinh(dx+c)^{2}d^{2}\right)}{4d^{2}}$	69

input
```
int(a+b*sinh(d*x+c)^2,x,method=_RETURNVERBOSE)
```

output
```
1/4*b*(-2*d*x+sinh(2*d*x+2*c))/d+a*x
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.00

$$\int \left(a+b\sinh^2(c+dx)\right)\,dx=\frac{(2\,a-b)dx+b\cosh\left(dx+c\right)\sinh\left(dx+c\right)}{2\,d}$$

input
```
integrate(a+b*sinh(d*x+c)^2,x, algorithm="fricas")
```

output
```
1/2*((2*a - b)*d*x + b*cosh(d*x + c)*sinh(d*x + c))/d
```

Sympy [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 51, normalized size of antiderivative = 1.70

$$\int \left(a+b\sinh^2(c+dx)\right)\,dx = ax+b\left(\begin{cases}\frac{x\sinh^2(c+dx)}{2}-\frac{x\cosh^2(c+dx)}{2}+\frac{\sinh(c+dx)\cosh(c+dx)}{2d} & \text{for } d\neq 0\\ x\sinh^2(c) & \text{otherwise}\end{cases}\right)$$

input
```
integrate(a+b*sinh(d*x+c)**2,x)
```

output
```
a*x + b*Piecewise((x*sinh(c + d*x)**2/2 - x*cosh(c + d*x)**2/2 + sinh(c +
d*x)*cosh(c + d*x)/(2*d), Ne(d, 0)), (x*sinh(c)**2, True))
```

Maxima [A] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 38, normalized size of antiderivative = 1.27

$$\int \left(a+b\sinh^2(c+dx)\right)\,dx = -\frac{1}{8}\,b\left(4\,x-\frac{e^{(2\,dx+2\,c)}}{d}+\frac{e^{(-2\,dx-2\,c)}}{d}\right)+ax$$

input
```
integrate(a+b*sinh(d*x+c)^2,x, algorithm="maxima")
```

output
```
-1/8*b*(4*x - e^(2*d*x + 2*c)/d + e^(-2*d*x - 2*c)/d) + a*x
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 38, normalized size of antiderivative = 1.27

$$\int \left(a+b\sinh^2(c+dx)\right)\,dx = -\frac{1}{8}\,b\left(4\,x-\frac{e^{(2\,dx+2\,c)}}{d}+\frac{e^{(-2\,dx-2\,c)}}{d}\right)+ax$$

input
```
integrate(a+b*sinh(d*x+c)^2,x, algorithm="giac")
```

output
```
-1/8*b*(4*x - e^(2*d*x + 2*c)/d + e^(-2*d*x - 2*c)/d) + a*x
```

Mupad [B] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.77

$$\int \left(a+b\sinh^2(c+dx)\right)\,dx=a\,x-\frac{b\,x}{2}+\frac{b\sinh(2\,c+2\,d\,x)}{4\,d}$$

input
```
int(a + b*sinh(c + d*x)^2,x)
```

output
```
a*x - (b*x)/2 + (b*sinh(2*c + 2*d*x))/(4*d)
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 63, normalized size of antiderivative = 2.10

$$\int \left(a+b\sinh^2(c+dx)\right)\,dx=\frac{e^{4dx+4c}b+8e^{2dx+2c}adx-4e^{2dx+2c}bdx-b}{8e^{2dx+2c}d}$$

input
```
int(a+b*sinh(d*x+c)^2,x)
```

output
```
(e**(4*c + 4*d*x)*b + 8*e**(2*c + 2*d*x)*a*d*x - 4*e**(2*c + 2*d*x)*b*d*x
- b)/(8*e**(2*c + 2*d*x)*d)
```

3.6 $\int \operatorname{csch}(c+dx)\left(a+b\sinh^2(c+dx)\right)\,dx$

Optimal result

Integrand size = 19, antiderivative size = 25

$$\int \operatorname{csch}(c+dx)\left(a+b\sinh^2(c+dx)\right)\,dx=-\frac{a\operatorname{arctanh}(\cosh(c+dx))}{d}+\frac{b\cosh(c+dx)}{d}$$

output
```
-a*arctanh(cosh(d*x+c))/d+b*cosh(d*x+c)/d
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.44

$$\int \operatorname{csch}(c+dx)\left(a+b\sinh^2(c+dx)\right)\,dx=-\frac{a\operatorname{arctanh}(\cosh(c+dx))}{d}+\frac{b\cosh(c)\cosh(dx)}{d}+\frac{b\sinh(c)\sinh(dx)}{d}$$

input
```
Integrate[Csch[c + d*x]*(a + b*Sinh[c + d*x]^2),x]
```

output
```
-((a*ArcTanh[Cosh[c + d*x]])/d) + (b*Cosh[c]*Cosh[d*x])/d + (b*Sinh[c]*Sin
h[d*x])/d
```

Rubi [C] (verified)

Result contains complex when optimal does not.

Time = 0.27 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.36, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.368, Rules used = {3042, 26, 3493, 26, 3042, 26, 4257}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \operatorname{csch}(c+dx)\left(a+b\sinh^2(c+dx)\right)\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{i\left(a-b\sin(ic+idx)^2\right)}{\sin(ic+idx)}dx$$

$$\Big\downarrow \text{26}$$

$$i\int \frac{a-b\sin(ic+idx)^2}{\sin(ic+idx)}dx$$

$$\Big\downarrow \text{3493}$$

$$i\left(a\int -i\operatorname{csch}(c+dx)dx-\frac{ib\cosh(c+dx)}{d}\right)$$

$$\Big\downarrow \text{26}$$

$$i\left(-ia\int \operatorname{csch}(c+dx)dx-\frac{ib\cosh(c+dx)}{d}\right)$$

$$\Big\downarrow \text{3042}$$

$$i\left(-ia\int i\csc(ic+idx)dx-\frac{ib\cosh(c+dx)}{d}\right)$$

$$\Big\downarrow \text{26}$$

$$i\left(a\int \csc(ic+idx)dx-\frac{ib\cosh(c+dx)}{d}\right)$$

$$\Big\downarrow \text{4257}$$

$$i\left(\frac{ia\text{arctanh}(\cosh(c+dx))}{d}-\frac{ib\cosh(c+dx)}{d}\right)$$

input
```
Int[Csch[c + d*x]*(a + b*Sinh[c + d*x]^2),x]
```

output
```
I*((I*a*ArcTanh[Cosh[c + d*x]])/d - (I*b*Cosh[c + d*x])/d)
```

Defintions of rubi rules used

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3493
```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_) + (C_.)*sin[(e_.) + (f_.)*(
x_)]^2), x_Symbol] :> Simp[(-C)*Cos[e + f*x]*((b*Sin[e + f*x])^(m + 1)/(b*f
*(m + 2))), x] + Simp[(A*(m + 2) + C*(m + 1))/(m + 2)   Int[(b*Sin[e + f*x]
)^m, x], x] /; FreeQ[{b, e, f, A, C, m}, x] &&  !LtQ[m, -1]
```

rule 4257
```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Maple [A] (verified)

Time = 0.25 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.96

method	result	size
derivativedivides	$\frac{-2a\ \operatorname{arctanh}\left(\mathrm{e}^{dx+c}\right)+b\cosh\left(dx+c\right)}{d}$	24
default	$\frac{-2a\ \operatorname{arctanh}\left(\mathrm{e}^{dx+c}\right)+b\cosh\left(dx+c\right)}{d}$	24
parallelrisch	$\frac{b\cosh\left(dx+c\right)+a\ln\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+b}{d}$	27
risch	$\frac{\mathrm{e}^{dx+c}b}{2d}+\frac{\mathrm{e}^{-dx-c}b}{2d}+\frac{a\ln\left(\mathrm{e}^{dx+c}-1\right)}{d}-\frac{a\ln\left(\mathrm{e}^{dx+c}+1\right)}{d}$	58

input
```
int(csch(d*x+c)*(a+b*sinh(d*x+c)^2),x,method=_RETURNVERBOSE)
```

output
```
1/d*(-2*a*arctanh(exp(d*x+c))+b*cosh(d*x+c))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 126 vs. $2(25) = 50$.

Time = 0.10 (sec) , antiderivative size = 126, normalized size of antiderivative = 5.04

$$\int \operatorname{csch}(c+dx)\left(a+b\sinh^2(c+dx)\right)\,dx = \frac{b\cosh(dx+c)^2 + 2\,b\cosh(dx+c)\sinh(dx+c) + b\sinh(dx+c)^2 - 2\,(a\cosh(dx+c) + a\sinh(dx+c)}{2\,(d\cosh}$$

input
```
integrate(csch(d*x+c)*(a+b*sinh(d*x+c)^2),x, algorithm="fricas")
```

output
```
1/2*(b*cosh(d*x + c)^2 + 2*b*cosh(d*x + c)*sinh(d*x + c) + b*sinh(d*x + c)
^2 - 2*(a*cosh(d*x + c) + a*sinh(d*x + c))*log(cosh(d*x + c) + sinh(d*x +
c) + 1) + 2*(a*cosh(d*x + c) + a*sinh(d*x + c))*log(cosh(d*x + c) + sinh(d
*x + c) - 1) + b)/(d*cosh(d*x + c) + d*sinh(d*x + c))
```

Sympy [F]

$$\int \operatorname{csch}(c+dx)\left(a+b\sinh^2(c+dx)\right)\,dx = \int \left(a+b\sinh^2(c+dx)\right)\operatorname{csch}(c+dx)\,dx$$

input
```
integrate(csch(d*x+c)*(a+b*sinh(d*x+c)**2),x)
```

output
```
Integral((a + b*sinh(c + d*x)**2)*csch(c + d*x), x)
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 43, normalized size of antiderivative = 1.72

$$\int \operatorname{csch}(c+dx)\left(a+b\sinh^2(c+dx)\right)\,dx = \frac{1}{2}b\left(\frac{e^{(dx+c)}}{d}+\frac{e^{(-dx-c)}}{d}\right)+\frac{a\log\left(\tanh\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)}{d}$$

input
```
integrate(csch(d*x+c)*(a+b*sinh(d*x+c)^2),x, algorithm="maxima")
```

output
```
1/2*b*(e^(d*x + c)/d + e^(-d*x - c)/d) + a*log(tanh(1/2*d*x + 1/2*c))/d
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 50, normalized size of antiderivative = 2.00

$$\int \operatorname{csch}(c+dx)\left(a+b\sinh^2(c+dx)\right)\,dx = \frac{be^{(dx+c)}+be^{(-dx-c)}-2\,a\log\left(e^{(dx+c)}+1\right)+2\,a\log\left(\left|e^{(dx+c)}-1\right|\right)}{2\,d}$$

input
```
integrate(csch(d*x+c)*(a+b*sinh(d*x+c)^2),x, algorithm="giac")
```

output
```
1/2*(b*e^(d*x + c) + b*e^(-d*x - c) - 2*a*log(e^(d*x + c) + 1) + 2*a*log(a
bs(e^(d*x + c) - 1)))/d
```

Mupad [B] (verification not implemented)

Time = 1.74 (sec) , antiderivative size = 66, normalized size of antiderivative = 2.64

$$\int \operatorname{csch}(c+dx)\left(a+b\sinh^2(c+dx)\right)\,dx = \frac{b\,\mathrm{e}^{-c-d\,x}}{2\,d}+\frac{b\,\mathrm{e}^{c+d\,x}}{2\,d}-\frac{2\,\mathrm{atan}\left(\frac{a\,\mathrm{e}^{d\,x}\,\mathrm{e}^{c}\,\sqrt{-d^2}}{d\,\sqrt{a^2}}\right)\,\sqrt{a^2}}{\sqrt{-d^2}}$$

input
```
int((a + b*sinh(c + d*x)^2)/sinh(c + d*x),x)
```

output
```
(b*exp(- c - d*x))/(2*d) + (b*exp(c + d*x))/(2*d) - (2*atan((a*exp(d*x)*ex
p(c)*(-d^2)^(1/2))/(d*(a^2)^(1/2)))*(a^2)^(1/2))/(-d^2)^(1/2)
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 68, normalized size of antiderivative = 2.72

$$\int \operatorname{csch}(c+dx)\left(a+b\sinh^2(c+dx)\right)\,dx = \frac{e^{2dx+2c}b+2e^{dx+c}\log\left(e^{dx+c}-1\right)a-2e^{dx+c}\log\left(e^{dx+c}+1\right)a+b}{2e^{dx+c}d}$$

input
```
int(csch(d*x+c)*(a+b*sinh(d*x+c)^2),x)
```

output
```
(e**(2*c + 2*d*x)*b + 2*e**(c + d*x)*log(e**(c + d*x) - 1)*a - 2*e**(c + d
*x)*log(e**(c + d*x) + 1)*a + b)/(2*e**(c + d*x)*d)
```

3.7 $\int \text{csch}^2(c+dx)\left(a+b\sinh^2(c+dx)\right)\,dx$

Optimal result

Integrand size = 21, antiderivative size = 16

$$\int \text{csch}^2(c+dx)\left(a+b\sinh^2(c+dx)\right)\,dx = bx-\frac{a\coth(c+dx)}{d}$$

output
```
b*x-a*coth(d*x+c)/d
```

Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00

$$\int \text{csch}^2(c+dx)\left(a+b\sinh^2(c+dx)\right)\,dx = bx-\frac{a\coth(c+dx)}{d}$$

input
```
Integrate[Csch[c + d*x]^2*(a + b*Sinh[c + d*x]^2),x]
```

output
```
b*x - (a*Coth[c + d*x])/d
```

Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.190, Rules used = {3042, 25, 3491, 24}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \operatorname{csch}^2(c+dx)\left(a+b\sinh^2(c+dx)\right)dx$$

$$\Big\downarrow \; 3042$$

$$\int -\frac{a-b\sin(ic+idx)^2}{\sin(ic+idx)^2}dx$$

$$\Big\downarrow \; 25$$

$$-\int \frac{a-b\sin(ic+idx)^2}{\sin(ic+idx)^2}dx$$

$$\Big\downarrow \; 3491$$

$$b\int 1dx-\frac{a\coth(c+dx)}{d}$$

$$\Big\downarrow \; 24$$

$$bx-\frac{a\coth(c+dx)}{d}$$

input
```
Int[Csch[c + d*x]^2*(a + b*Sinh[c + d*x]^2),x]
```

output
```
b*x - (a*Coth[c + d*x])/d
```

Defintions of rubi rules used

rule 24
```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3491
```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_) + (C_.)*sin[(e_.) + (f_.)*(x
_)]^2), x_Symbol] :> Simp[A*Cos[e + f*x]*((b*Sin[e + f*x])^(m + 1)/(b*f*(m
+ 1))), x] + Simp[(A*(m + 2) + C*(m + 1))/(b^2*(m + 1))   Int[(b*Sin[e + f*
x])^(m + 2), x], x] /; FreeQ[{b, e, f, A, C}, x] && LtQ[m, -1]
```

Maple [A] (verified)

Time = 0.29 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.38

method	result	size
derivativedivides	$\frac{-\coth(dx+c)a+b(dx+c)}{d}$	22
default	$\frac{-\coth(dx+c)a+b(dx+c)}{d}$	22
risch	$bx-\frac{2a}{d\left(e^{2dx+2c}-1\right)}$	24
parallelrisch	$-\frac{-2dxb+a\left(\coth\left(\frac{dx}{2}+\frac{c}{2}\right)+\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{2d}$	33

input
```
int(csch(d*x+c)^2*(a+b*sinh(d*x+c)^2),x,method=_RETURNVERBOSE)
```

output
```
1/d*(-coth(d*x+c)*a+b*(d*x+c))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 36 vs. $2(16) = 32$.

Time = 0.09 (sec) , antiderivative size = 36, normalized size of antiderivative = 2.25

$$\int \operatorname{csch}^2(c+dx) \left(a+b\sinh^2(c+dx)\right) \, dx = -\frac{a\cosh{(dx+c)} - (bdx+a)\sinh{(dx+c)}}{d\sinh{(dx+c)}}$$

input
```
integrate(csch(d*x+c)^2*(a+b*sinh(d*x+c)^2),x, algorithm="fricas")
```

output
```
-(a*cosh(d*x + c) - (b*d*x + a)*sinh(d*x + c))/(d*sinh(d*x + c))
```

Sympy [F]

$$\int \operatorname{csch}^2(c+dx) \left(a+b\sinh^2(c+dx)\right) \, dx = \int \left(a+b\sinh^2{(c+dx)}\right)\operatorname{csch}^2{(c+dx)}\, dx$$

input
```
integrate(csch(d*x+c)**2*(a+b*sinh(d*x+c)**2),x)
```

output
```
Integral((a + b*sinh(c + d*x)**2)*csch(c + d*x)**2, x)
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.44

$$\int \operatorname{csch}^2(c+dx) \left(a+b\sinh^2(c+dx)\right) \, dx = bx + \frac{2\,a}{d(e^{(-2\,dx-2\,c)} - 1)}$$

input
```
integrate(csch(d*x+c)^2*(a+b*sinh(d*x+c)^2),x, algorithm="maxima")
```

output
```
b*x + 2*a/(d*(e^(-2*d*x - 2*c) - 1))
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.75

$$\int \operatorname{csch}^2(c+dx)\left(a+b\sinh^2(c+dx)\right)\,dx=\frac{(dx+c)b-\frac{2a}{e^{(2dx+2c)}-1}}{d}$$

input
```
integrate(csch(d*x+c)^2*(a+b*sinh(d*x+c)^2),x, algorithm="giac")
```

output
```
((d*x + c)*b - 2*a/(e^(2*d*x + 2*c) - 1))/d
```

Mupad [B] (verification not implemented)

Time = 1.66 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.44

$$\int \operatorname{csch}^2(c+dx)\left(a+b\sinh^2(c+dx)\right)\,dx=b\,x-\frac{2\,a}{d\,\left(\mathrm{e}^{2\,c+2\,d\,x}-1\right)}$$

input
```
int((a + b*sinh(c + d*x)^2)/sinh(c + d*x)^2,x)
```

output
```
b*x - (2*a)/(d*(exp(2*c + 2*d*x) - 1))
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 51, normalized size of antiderivative = 3.19

$$\int \operatorname{csch}^2(c+dx)\left(a+b\sinh^2(c+dx)\right)\,dx=\frac{-2e^{2dx+2c}a+e^{2dx+2c}bdx-bdx}{d\left(e^{2dx+2c}-1\right)}$$

input
```
int(csch(d*x+c)^2*(a+b*sinh(d*x+c)^2),x)
```

output
```
( - 2*e**(2*c + 2*d*x)*a + e**(2*c + 2*d*x)*b*d*x - b*d*x)/(d*(e**(2*c + 2
*d*x) - 1))
```

3.8 $\int \operatorname{csch}^3(c+dx)\left(a+b\sinh^2(c+dx)\right)\,dx$

Optimal result

Integrand size = 21, antiderivative size = 40

$$\int \operatorname{csch}^3(c+dx)\left(a+b\sinh^2(c+dx)\right)\,dx=\frac{(a-2b)\operatorname{arctanh}(\cosh(c+dx))}{2d}-\frac{a\coth(c+dx)\operatorname{csch}(c+dx)}{2d}$$

output
```
1/2*(a-2*b)*arctanh(cosh(d*x+c))/d-1/2*a*coth(d*x+c)*csch(d*x+c)/d
```

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 92 vs. 2(40) = 80.

Time = 0.06 (sec) , antiderivative size = 92, normalized size of antiderivative = 2.30

$$\int \operatorname{csch}^3(c+dx)\left(a+b\sinh^2(c+dx)\right)\,dx=-\frac{b\operatorname{arctanh}(\cosh(c+dx))}{d}-\frac{a\operatorname{csch}^2\left(\frac{1}{2}(c+dx)\right)}{8d}+\frac{a\log\left(\cosh\left(\frac{1}{2}(c+dx)\right)\right)}{2d}-\frac{a\log\left(\sinh\left(\frac{1}{2}(c+dx)\right)\right)}{2d}-\frac{a\operatorname{sech}^2\left(\frac{1}{2}(c+dx)\right)}{8d}$$

input
```
Integrate[Csch[c + d*x]^3*(a + b*Sinh[c + d*x]^2),x]
```

output
```
-((b*ArcTanh[Cosh[c + d*x]])/d) - (a*Csch[(c + d*x)/2]^2)/(8*d) + (a*Log[C
osh[(c + d*x)/2]])/(2*d) - (a*Log[Sinh[(c + d*x)/2]])/(2*d) - (a*Sech[(c +
 d*x)/2]^2)/(8*d)
```

Rubi [C] (verified)

Result contains complex when optimal does not.

Time = 0.30 (sec) , antiderivative size = 48, normalized size of antiderivative = 1.20, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {3042, 26, 3491, 26, 3042, 26, 4257}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \operatorname{csch}^3(c+dx)\left(a+b\sinh^2(c+dx)\right)\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int -\frac{i\left(a-b\sin(ic+idx)^2\right)}{\sin(ic+idx)^3}dx$$

$$\Big\downarrow \text{26}$$

$$-i\int \frac{a-b\sin(ic+idx)^2}{\sin(ic+idx)^3}dx$$

$$\Big\downarrow \text{3491}$$

$$-i\left(\frac{1}{2}(a-2b)\int -i\operatorname{csch}(c+dx)dx-\frac{ia\coth(c+dx)\operatorname{csch}(c+dx)}{2d}\right)$$

$$\Big\downarrow \text{26}$$

$$-i\left(-\frac{1}{2}i(a-2b)\int \operatorname{csch}(c+dx)dx-\frac{ia\coth(c+dx)\operatorname{csch}(c+dx)}{2d}\right)$$

$$\Big\downarrow \text{3042}$$

$$-i\left(-\frac{1}{2}i(a-2b)\int i\csc(ic+idx)dx-\frac{ia\coth(c+dx)\operatorname{csch}(c+dx)}{2d}\right)$$

$$\Big\downarrow \text{26}$$

$$-i\left(\frac{1}{2}(a-2b)\int \csc(ic+idx)dx-\frac{ia\coth(c+dx)\text{csch}(c+dx)}{2d}\right)$$

$$\Big\downarrow \text{4257}$$

$$-i\left(\frac{i(a-2b)\text{arctanh}(\cosh(c+dx))}{2d}-\frac{ia\coth(c+dx)\text{csch}(c+dx)}{2d}\right)$$

input
```
Int[Csch[c + d*x]^3*(a + b*Sinh[c + d*x]^2),x]
```

output
```
(-I)*(((I/2)*(a - 2*b)*ArcTanh[Cosh[c + d*x]])/d - ((I/2)*a*Coth[c + d*x]*
Csch[c + d*x])/d)
```

Defintions of rubi rules used

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3491
```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_) + (C_.)*sin[(e_.) + (f_.)*(x
_)]^2), x_Symbol] :> Simp[A*Cos[e + f*x]*((b*Sin[e + f*x])^(m + 1)/(b*f*(m
+ 1))), x] + Simp[(A*(m + 2) + C*(m + 1))/(b^2*(m + 1))   Int[(b*Sin[e + f*
x])^(m + 2), x], x] /; FreeQ[{b, e, f, A, C}, x] && LtQ[m, -1]
```

rule 4257
```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Maple [A] (verified)

Time = 0.37 (sec) , antiderivative size = 40, normalized size of antiderivative = 1.00

method	result	size
derivativedivides	$\frac{a\left(-\frac{\operatorname{csch}(dx+c)\coth(dx+c)}{2}+\operatorname{arctanh}\left(\mathrm{e}^{dx+c}\right)\right)-2b\,\operatorname{arctanh}\left(\mathrm{e}^{dx+c}\right)}{d}$	40
default	$\frac{a\left(-\frac{\operatorname{csch}(dx+c)\coth(dx+c)}{2}+\operatorname{arctanh}\left(\mathrm{e}^{dx+c}\right)\right)-2b\,\operatorname{arctanh}\left(\mathrm{e}^{dx+c}\right)}{d}$	40
parallelrisch	$\frac{(-4a+8b)\ln\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-\coth\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}a+\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}a}{8d}$	52
risch	$-\frac{a\,\mathrm{e}^{dx+c}\left(\mathrm{e}^{2dx+2c}+1\right)}{d\left(\mathrm{e}^{2dx+2c}-1\right)^{2}}-\frac{a\ln\left(\mathrm{e}^{dx+c}-1\right)}{2d}+\frac{\ln\left(\mathrm{e}^{dx+c}-1\right)b}{d}+\frac{a\ln\left(\mathrm{e}^{dx+c}+1\right)}{2d}-\frac{\ln\left(\mathrm{e}^{dx+c}+1\right)b}{d}$	97

input
```
int(csch(d*x+c)^3*(a+b*sinh(d*x+c)^2),x,method=_RETURNVERBOSE)
```

output
```
1/d*(a*(-1/2*csch(d*x+c)*coth(d*x+c)+arctanh(exp(d*x+c)))-2*b*arctanh(exp(
d*x+c)))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 484 vs. 2(36) = 72.

Time = 0.08 (sec) , antiderivative size = 484, normalized size of antiderivative = 12.10

$$\int \operatorname{csch}^3(c+dx)\left(a+b\sinh^2(c+dx)\right)\,dx =$$
$$-\frac{2\,a\cosh\left(dx+c\right)^{3}+6\,a\cosh\left(dx+c\right)\sinh\left(dx+c\right)^{2}+2\,a\sinh\left(dx+c\right)^{3}+2\,a\cosh\left(dx+c\right)-\left((a-2\right.}{}$$

input
```
integrate(csch(d*x+c)^3*(a+b*sinh(d*x+c)^2),x, algorithm="fricas")
```

output
```
-1/2*(2*a*cosh(d*x + c)^3 + 6*a*cosh(d*x + c)*sinh(d*x + c)^2 + 2*a*sinh(d
*x + c)^3 + 2*a*cosh(d*x + c) - ((a - 2*b)*cosh(d*x + c)^4 + 4*(a - 2*b)*c
osh(d*x + c)*sinh(d*x + c)^3 + (a - 2*b)*sinh(d*x + c)^4 - 2*(a - 2*b)*cos
h(d*x + c)^2 + 2*(3*(a - 2*b)*cosh(d*x + c)^2 - a + 2*b)*sinh(d*x + c)^2 +
 4*((a - 2*b)*cosh(d*x + c)^3 - (a - 2*b)*cosh(d*x + c))*sinh(d*x + c) + a
 - 2*b)*log(cosh(d*x + c) + sinh(d*x + c) + 1) + ((a - 2*b)*cosh(d*x + c)^
4 + 4*(a - 2*b)*cosh(d*x + c)*sinh(d*x + c)^3 + (a - 2*b)*sinh(d*x + c)^4
- 2*(a - 2*b)*cosh(d*x + c)^2 + 2*(3*(a - 2*b)*cosh(d*x + c)^2 - a + 2*b)*
sinh(d*x + c)^2 + 4*((a - 2*b)*cosh(d*x + c)^3 - (a - 2*b)*cosh(d*x + c))*
sinh(d*x + c) + a - 2*b)*log(cosh(d*x + c) + sinh(d*x + c) - 1) + 2*(3*a*c
osh(d*x + c)^2 + a)*sinh(d*x + c))/(d*cosh(d*x + c)^4 + 4*d*cosh(d*x + c)*
sinh(d*x + c)^3 + d*sinh(d*x + c)^4 - 2*d*cosh(d*x + c)^2 + 2*(3*d*cosh(d*
x + c)^2 - d)*sinh(d*x + c)^2 + 4*(d*cosh(d*x + c)^3 - d*cosh(d*x + c))*si
nh(d*x + c) + d)
```

Sympy [F]

$$\int \operatorname{csch}^3(c+dx)\left(a+b\sinh^2(c+dx)\right)\,dx=\int \left(a+b\sinh^2\left(c+dx\right)\right)\operatorname{csch}^3\left(c+dx\right)dx$$

input
```
integrate(csch(d*x+c)**3*(a+b*sinh(d*x+c)**2),x)
```

output
```
Integral((a + b*sinh(c + d*x)**2)*csch(c + d*x)**3, x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 125 vs. $2(36) = 72$.

Time = 0.03 (sec) , antiderivative size = 125, normalized size of antiderivative = 3.12

$$\int \operatorname{csch}^3(c+dx)\left(a+b\sinh^2(c+dx)\right)\,dx$$
$$=\frac{1}{2}a\left(\frac{\log\left(e^{(-dx-c)}+1\right)}{d}-\frac{\log\left(e^{(-dx-c)}-1\right)}{d}+\frac{2\left(e^{(-dx-c)}+e^{(-3\,dx-3\,c)}\right)}{d(2\,e^{(-2\,dx-2\,c)}-e^{(-4\,dx-4\,c)}-1)}\right)$$
$$-b\left(\frac{\log\left(e^{(-dx-c)}+1\right)}{d}-\frac{\log\left(e^{(-dx-c)}-1\right)}{d}\right)$$

input
```
integrate(csch(d*x+c)^3*(a+b*sinh(d*x+c)^2),x, algorithm="maxima")
```

output
```
1/2*a*(log(e^(-d*x - c) + 1)/d - log(e^(-d*x - c) - 1)/d + 2*(e^(-d*x - c)
 + e^(-3*d*x - 3*c))/(d*(2*e^(-2*d*x - 2*c) - e^(-4*d*x - 4*c) - 1))) - b*
(log(e^(-d*x - c) + 1)/d - log(e^(-d*x - c) - 1)/d)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 96 vs. $2(36) = 72$.

Time = 0.13 (sec) , antiderivative size = 96, normalized size of antiderivative = 2.40

$$\int \operatorname{csch}^3(c+dx)\left(a+b\sinh^2(c+dx)\right)\,dx = \frac{(a-2b)\log\left(e^{(dx+c)}+e^{(-dx-c)}+2\right)-(a-2b)\log\left(e^{(dx+c)}+e^{(-dx-c)}-2\right)-\frac{4a\left(e^{(dx+c)}+e^{(-dx-c)}\right)}{\left(e^{(dx+c)}+e^{(-dx-c)}\right)^2-4}}{4d}$$

input
```
integrate(csch(d*x+c)^3*(a+b*sinh(d*x+c)^2),x, algorithm="giac")
```

output
```
1/4*((a - 2*b)*log(e^(d*x + c) + e^(-d*x - c) + 2) - (a - 2*b)*log(e^(d*x
+ c) + e^(-d*x - c) - 2) - 4*a*(e^(d*x + c) + e^(-d*x - c))/((e^(d*x + c)
+ e^(-d*x - c))^2 - 4))/d
```

Mupad [B] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 131, normalized size of antiderivative = 3.28

$$\int \operatorname{csch}^3(c+dx)\left(a+b\sinh^2(c+dx)\right)\,dx = \frac{\operatorname{atan}\left(\frac{e^{dx}\,e^{c}\,\left(a\sqrt{-d^2}-2b\sqrt{-d^2}\right)}{d\sqrt{a^2-4ab+4b^2}}\right)\sqrt{a^2-4ab+4b^2}}{\sqrt{-d^2}} - \frac{a\,e^{c+dx}}{d\,\left(e^{2c+2dx}-1\right)} - \frac{2a\,e^{c+dx}}{d\,\left(e^{4c+4dx}-2\,e^{2c+2dx}+1\right)}$$

input
```
int((a + b*sinh(c + d*x)^2)/sinh(c + d*x)^3,x)
```

output
```
(atan((exp(d*x)*exp(c)*(a*(-d^2)^(1/2) - 2*b*(-d^2)^(1/2)))/(d*(a^2 - 4*a*
b + 4*b^2)^(1/2)))*(a^2 - 4*a*b + 4*b^2)^(1/2))/(-d^2)^(1/2) - (a*exp(c +
d*x))/(d*(exp(2*c + 2*d*x) - 1)) - (2*a*exp(c + d*x))/(d*(exp(4*c + 4*d*x)
 - 2*exp(2*c + 2*d*x) + 1))
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 289, normalized size of antiderivative = 7.22

$$\int \operatorname{csch}^3(c+dx)\left(a+b\sinh^2(c+dx)\right)\,dx$$
$$= \frac{-e^{4dx+4c}\log\left(e^{dx+c}-1\right)a+2e^{4dx+4c}\log\left(e^{dx+c}-1\right)b+e^{4dx+4c}\log\left(e^{dx+c}+1\right)a-2e^{4dx+4c}\log\left(e^{dx+c}+1\right)b}{}$$

input
```
int(csch(d*x+c)^3*(a+b*sinh(d*x+c)^2),x)
```

output
```
( - e**(4*c + 4*d*x)*log(e**(c + d*x) - 1)*a + 2*e**(4*c + 4*d*x)*log(e**(
c + d*x) - 1)*b + e**(4*c + 4*d*x)*log(e**(c + d*x) + 1)*a - 2*e**(4*c + 4
*d*x)*log(e**(c + d*x) + 1)*b - 2*e**(3*c + 3*d*x)*a + 2*e**(2*c + 2*d*x)*
log(e**(c + d*x) - 1)*a - 4*e**(2*c + 2*d*x)*log(e**(c + d*x) - 1)*b - 2*e
**(2*c + 2*d*x)*log(e**(c + d*x) + 1)*a + 4*e**(2*c + 2*d*x)*log(e**(c + d
*x) + 1)*b - 2*e**(c + d*x)*a - log(e**(c + d*x) - 1)*a + 2*log(e**(c + d*
x) - 1)*b + log(e**(c + d*x) + 1)*a - 2*log(e**(c + d*x) + 1)*b)/(2*d*(e**
(4*c + 4*d*x) - 2*e**(2*c + 2*d*x) + 1))
```

3.9 $\int \operatorname{csch}^4(c+dx)\left(a+b\sinh^2(c+dx)\right)\,dx$

Optimal result

Integrand size = 21, antiderivative size = 43

$$\int \operatorname{csch}^4(c+dx)\left(a+b\sinh^2(c+dx)\right)\,dx=\frac{(2a-3b)\coth(c+dx)}{3d}-\frac{a\coth(c+dx)\operatorname{csch}^2(c+dx)}{3d}$$

output
```
1/3*(2*a-3*b)*coth(d*x+c)/d-1/3*a*coth(d*x+c)*csch(d*x+c)^2/d
```

Mathematica [A] (verified)

Time = 0.10 (sec) , antiderivative size = 49, normalized size of antiderivative = 1.14

$$\int \operatorname{csch}^4(c+dx)\left(a+b\sinh^2(c+dx)\right)\,dx=\frac{2a\coth(c+dx)}{3d}-\frac{b\coth(c+dx)}{d}-\frac{a\coth(c+dx)\operatorname{csch}^2(c+dx)}{3d}$$

input
```
Integrate[Csch[c + d*x]^4*(a + b*Sinh[c + d*x]^2),x]
```

output
```
(2*a*Coth[c + d*x])/(3*d) - (b*Coth[c + d*x])/d - (a*Coth[c + d*x]*Csch[c
+ d*x]^2)/(3*d)
```

Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 43, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {3042, 3491, 25, 3042, 25, 4254, 24}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \operatorname{csch}^4(c+dx)\left(a+b\sinh^2(c+dx)\right)\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{a-b\sin(ic+idx)^2}{\sin(ic+idx)^4}dx$$

$$\Big\downarrow \text{3491}$$

$$\frac{1}{3}(2a-3b)\int -\operatorname{csch}^2(c+dx)dx-\frac{a\coth(c+dx)\operatorname{csch}^2(c+dx)}{3d}$$

$$\Big\downarrow \text{25}$$

$$-\frac{1}{3}(2a-3b)\int \operatorname{csch}^2(c+dx)dx-\frac{a\coth(c+dx)\operatorname{csch}^2(c+dx)}{3d}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{a\coth(c+dx)\operatorname{csch}^2(c+dx)}{3d}-\frac{1}{3}(2a-3b)\int -\csc(ic+idx)^2dx$$

$$\Big\downarrow \text{25}$$

$$-\frac{a\coth(c+dx)\operatorname{csch}^2(c+dx)}{3d}+\frac{1}{3}(2a-3b)\int \csc(ic+idx)^2dx$$

$$\Big\downarrow \text{4254}$$

$$-\frac{a\coth(c+dx)\operatorname{csch}^2(c+dx)}{3d}+\frac{i(2a-3b)\int 1d(-i\coth(c+dx))}{3d}$$

$$\Big\downarrow \text{24}$$

$$\frac{(2a-3b)\coth(c+dx)}{3d}-\frac{a\coth(c+dx)\operatorname{csch}^2(c+dx)}{3d}$$

input
```
Int[Csch[c + d*x]^4*(a + b*Sinh[c + d*x]^2),x]
```

output
```
((2*a - 3*b)*Coth[c + d*x])/(3*d) - (a*Coth[c + d*x]*Csch[c + d*x]^2)/(3*d
)
```

Defintions of rubi rules used

rule 24
```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3491
```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_) + (C_.)*sin[(e_.) + (f_.)*(x
_)]^2), x_Symbol] :> Simp[A*Cos[e + f*x]*((b*Sin[e + f*x])^(m + 1)/(b*f*(m
+ 1))), x] + Simp[(A*(m + 2) + C*(m + 1))/(b^2*(m + 1))   Int[(b*Sin[e + f*
x])^(m + 2), x], x] /; FreeQ[{b, e, f, A, C}, x] && LtQ[m, -1]
```

rule 4254
```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Simp[-d^(-1)   Subst[Int[Exp
andIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
 d}, x] && IGtQ[n/2, 0]
```

Maple [A] (verified)

Time = 0.38 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.81

method	result
derivativedivides	$\frac{a\left(\frac{2}{3}-\frac{\operatorname{csch}(dx+c)^{2}}{3}\right)\coth(dx+c)-b\coth(dx+c)}{d}$
default	$\frac{a\left(\frac{2}{3}-\frac{\operatorname{csch}(dx+c)^{2}}{3}\right)\coth(dx+c)-b\coth(dx+c)}{d}$
risch	$-\frac{2\left(3b\,{\mathrm e}^{4dx+4c}+6\,{\mathrm e}^{2dx+2c}a-6\,{\mathrm e}^{2dx+2c}b-2a+3b\right)}{3d\left({\mathrm e}^{2dx+2c}-1\right)^{3}}$
parallelrisch	$-\frac{\left(\coth\left(\frac{dx}{2}+\frac{c}{2}\right)+\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)\right)\left(\coth\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}a-\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)\coth\left(\frac{dx}{2}+\frac{c}{2}\right)a+\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}a-9a+12b\right)}{24d}$

input
```
int(csch(d*x+c)^4*(a+b*sinh(d*x+c)^2),x,method=_RETURNVERBOSE)
```

output
```
1/d*(a*(2/3-1/3*csch(d*x+c)^2)*coth(d*x+c)-b*coth(d*x+c))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 159 vs. $2(39) = 78$.

Time = 0.08 (sec) , antiderivative size = 159, normalized size of antiderivative = 3.70

$$\int \operatorname{csch}^4(c+dx)\left(a+b\sinh^2(c+dx)\right)\,dx$$
$$=\frac{4\left((a-3\,b)\cosh\left(dx+c\right)^{2}-2\,a\cosh\left(dx+c\right)\sinh\left(dx\right.\right.}{3\left(d\cosh\left(dx+c\right)^{4}+4\,d\cosh\left(dx+c\right)\sinh\left(dx+c\right)^{3}+d\sinh\left(dx+c\right)^{4}-4\,d\cosh\left(dx+c\right)^{2}+2\left(3\,d\cosh\right.\right.}$$

input
```
integrate(csch(d*x+c)^4*(a+b*sinh(d*x+c)^2),x, algorithm="fricas")
```

output
```
4/3*((a - 3*b)*cosh(d*x + c)^2 - 2*a*cosh(d*x + c)*sinh(d*x + c) + (a - 3*
b)*sinh(d*x + c)^2 - 3*a + 3*b)/(d*cosh(d*x + c)^4 + 4*d*cosh(d*x + c)*sin
h(d*x + c)^3 + d*sinh(d*x + c)^4 - 4*d*cosh(d*x + c)^2 + 2*(3*d*cosh(d*x +
 c)^2 - 2*d)*sinh(d*x + c)^2 + 4*(d*cosh(d*x + c)^3 - d*cosh(d*x + c))*sin
h(d*x + c) + 3*d)
```

Sympy [F(-1)]

Timed out.

$$\int \operatorname{csch}^4(c+dx) \left(a+b\sinh^2(c+dx)\right)\, dx = \text{Timed out}$$

input
```
integrate(csch(d*x+c)**4*(a+b*sinh(d*x+c)**2),x)
```

output
```
Timed out
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 113 vs. $2(39) = 78$.

Time = 0.04 (sec) , antiderivative size = 113, normalized size of antiderivative = 2.63

$$\int \operatorname{csch}^4(c+dx) \left(a+b\sinh^2(c+dx)\right)\, dx = \frac{4}{3}\,a\left(\frac{3\,e^{(-2\,dx-2\,c)}}{d(3\,e^{(-2\,dx-2\,c)}-3\,e^{(-4\,dx-4\,c)}+e^{(-6\,dx-6\,c)}-1)}-\frac{1}{d(3\,e^{(-2\,dx-2\,c)}-3\,e^{(-4\,dx-4\,c)}+e^{(-6\,dx-6\,c)}-1)}\right) + \frac{2\,b}{d(e^{(-2\,dx-2\,c)}-1)}$$

input
```
integrate(csch(d*x+c)^4*(a+b*sinh(d*x+c)^2),x, algorithm="maxima")
```

output
```
4/3*a*(3*e^(-2*d*x - 2*c)/(d*(3*e^(-2*d*x - 2*c) - 3*e^(-4*d*x - 4*c) + e^
(-6*d*x - 6*c) - 1)) - 1/(d*(3*e^(-2*d*x - 2*c) - 3*e^(-4*d*x - 4*c) + e^(
-6*d*x - 6*c) - 1))) + 2*b/(d*(e^(-2*d*x - 2*c) - 1))
```

Giac [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 61, normalized size of antiderivative = 1.42

$$\int \operatorname{csch}^4(c+dx)\left(a+b\sinh^2(c+dx)\right)\,dx = -\frac{2\left(3\,be^{(4\,dx+4\,c)}+6\,ae^{(2\,dx+2\,c)}-6\,be^{(2\,dx+2\,c)}-2\,a+3\,b\right)}{3\,d(e^{(2\,dx+2\,c)}-1)^3}$$

input
```
integrate(csch(d*x+c)^4*(a+b*sinh(d*x+c)^2),x, algorithm="giac")
```

output
```
-2/3*(3*b*e^(4*d*x + 4*c) + 6*a*e^(2*d*x + 2*c) - 6*b*e^(2*d*x + 2*c) - 2*
a + 3*b)/(d*(e^(2*d*x + 2*c) - 1)^3)
```

Mupad [B] (verification not implemented)

Time = 1.72 (sec) , antiderivative size = 61, normalized size of antiderivative = 1.42

$$\int \operatorname{csch}^4(c+dx)\left(a+b\sinh^2(c+dx)\right)\,dx = -\frac{2\left(3\,b-2\,a+6\,a\,\mathrm{e}^{2\,c+2\,d\,x}-6\,b\,\mathrm{e}^{2\,c+2\,d\,x}+3\,b\,\mathrm{e}^{4\,c+4\,d\,x}\right)}{3\,d\left(\mathrm{e}^{2\,c+2\,d\,x}-1\right)^3}$$

input
```
int((a + b*sinh(c + d*x)^2)/sinh(c + d*x)^4,x)
```

output
```
-(2*(3*b - 2*a + 6*a*exp(2*c + 2*d*x) - 6*b*exp(2*c + 2*d*x) + 3*b*exp(4*c
 + 4*d*x)))/(3*d*(exp(2*c + 2*d*x) - 1)^3)
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 89, normalized size of antiderivative = 2.07

$$\int \operatorname{csch}^4(c+dx)\left(a+b\sinh^2(c+dx)\right)\,dx=\frac{-\frac{2e^{6dx+6c}b}{3}-4e^{2dx+2c}a+2e^{2dx+2c}b+\frac{4a}{3}-\frac{4b}{3}}{d\left(e^{6dx+6c}-3e^{4dx+4c}+3e^{2dx+2c}-1\right)}$$

input
```
int(csch(d*x+c)^4*(a+b*sinh(d*x+c)^2),x)
```

output
```
(2*( - e**(6*c + 6*d*x)*b - 6*e**(2*c + 2*d*x)*a + 3*e**(2*c + 2*d*x)*b +
2*a - 2*b))/(3*d*(e**(6*c + 6*d*x) - 3*e**(4*c + 4*d*x) + 3*e**(2*c + 2*d*
x) - 1))
```

3.10 $\int \sinh^4(c+dx) \left(a+b\sinh^2(c+dx)\right)^2 \, dx$

Optimal result

Integrand size = 23, antiderivative size = 144

$$\begin{aligned}\int \sinh^4(c+dx) \left(a+b\sinh^2(c+dx)\right)^2 \, dx &= \frac{1}{128}\left(48a^2-80ab+35b^2\right)x - \frac{(80a^2-176ab+93b^2)\cosh(c+dx)\sinh(c+dx)}{128d} \\ &\quad + \frac{(48a^2-208ab+163b^2)\cosh^3(c+dx)\sinh(c+dx)}{192d} \\ &\quad + \frac{(16a-25b)b\cosh^5(c+dx)\sinh(c+dx)}{48d} + \frac{b^2\cosh^7(c+dx)\sinh(c+dx)}{8d}\end{aligned}$$

output
```
1/128*(48*a^2-80*a*b+35*b^2)*x-1/128*(80*a^2-176*a*b+93*b^2)*cosh(d*x+c)*s
inh(d*x+c)/d+1/192*(48*a^2-208*a*b+163*b^2)*cosh(d*x+c)^3*sinh(d*x+c)/d+1/
48*(16*a-25*b)*b*cosh(d*x+c)^5*sinh(d*x+c)/d+1/8*b^2*cosh(d*x+c)^7*sinh(d*
x+c)/d
```

Mathematica [A] (verified)

Time = 0.30 (sec) , antiderivative size = 133, normalized size of antiderivative = 0.92

$$\int \sinh^4(c+dx)\left(a+b\sinh^2(c+dx)\right)^2\,dx$$
$$=\frac{1152a^2c-1920abc+840b^2c+1152a^2dx-1920abdx+840b^2dx-96(8a^2-15ab+7b^2)\sinh(2(c+dx))}{}$$

input
```
Integrate[Sinh[c + d*x]^4*(a + b*Sinh[c + d*x]^2)^2,x]
```

output
```
(1152*a^2*c - 1920*a*b*c + 840*b^2*c + 1152*a^2*d*x - 1920*a*b*d*x + 840*b
^2*d*x - 96*(8*a^2 - 15*a*b + 7*b^2)*Sinh[2*(c + d*x)] + 24*(4*a^2 - 12*a*
b + 7*b^2)*Sinh[4*(c + d*x)] + 32*a*b*Sinh[6*(c + d*x)] - 32*b^2*Sinh[6*(c
 + d*x)] + 3*b^2*Sinh[8*(c + d*x)])/(3072*d)
```

Rubi [A] (verified)

Time = 0.40 (sec) , antiderivative size = 185, normalized size of antiderivative = 1.28, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.435, Rules used = {3042, 3666, 366, 25, 360, 25, 1471, 27, 298, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sinh^4(c+dx)\left(a+b\sinh^2(c+dx)\right)^2\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \sin(ic+idx)^4\left(a-b\sin(ic+idx)^2\right)^2\,dx$$

$$\Big\downarrow \text{3666}$$

$$\frac{\int \frac{\tanh^4(c+dx)\left(a-(a-b)\tanh^2(c+dx)\right)^2}{\left(1-\tanh^2(c+dx)\right)^5}d\tanh(c+dx)}{d}$$

$$\Big\downarrow \text{366}$$

$$\frac{\frac{b^2\tanh^5(c+dx)}{8\left(1-\tanh^2(c+dx)\right)^4}-\frac{1}{8}\int -\frac{\tanh^4(c+dx)\left(8a^2-5b^2-8(a-b)^2\tanh^2(c+dx)\right)}{\left(1-\tanh^2(c+dx)\right)^4}d\tanh(c+dx)}{d}$$

$\Big\downarrow$ 25

$$\frac{\frac{1}{8}\int \frac{\tanh^4(c+dx)\left(8a^2-5b^2-8(a-b)^2\tanh^2(c+dx)\right)}{\left(1-\tanh^2(c+dx)\right)^4}d\tanh(c+dx)+\frac{b^2\tanh^5(c+dx)}{8\left(1-\tanh^2(c+dx)\right)^4}}{d}$$

$\Big\downarrow$ 360

$$\frac{\frac{1}{8}\left(\frac{1}{6}\int -\frac{-48(a-b)^2\tanh^4(c+dx)+6(16a-13b)b\tanh^2(c+dx)+(16a-13b)b}{\left(1-\tanh^2(c+dx)\right)^3}d\tanh(c+dx)+\frac{b(16a-13b)\tanh(c+dx)}{6\left(1-\tanh^2(c+dx)\right)^3}\right)+\frac{b^2\tanh^5(c}{8\left(1-\tanh^2(c\right.}}{d}$$

$\Big\downarrow$ 25

$$\frac{\frac{1}{8}\left(\frac{b(16a-13b)\tanh(c+dx)}{6\left(1-\tanh^2(c+dx)\right)^3}-\frac{1}{6}\int \frac{-48(a-b)^2\tanh^4(c+dx)+6(16a-13b)b\tanh^2(c+dx)+(16a-13b)b}{\left(1-\tanh^2(c+dx)\right)^3}d\tanh(c+dx)\right)+\frac{b^2\tanh^5(c+}{8\left(1-\tanh^2(c+\right.}}{d}$$

$\Big\downarrow$ 1471

$$\frac{\frac{1}{8}\left(\frac{1}{6}\left(\frac{1}{4}\int -\frac{3\left(16a^2-48ba+29b^2+64(a-b)^2\tanh^2(c+dx)\right)}{\left(1-\tanh^2(c+dx)\right)^2}d\tanh(c+dx)+\frac{\left(48a^2-208ab+139b^2\right)\tanh(c+dx)}{4\left(1-\tanh^2(c+dx)\right)^2}\right)+\frac{b(16a-13b)\tanh(c}{6\left(1-\tanh^2(c+d\right.}}{d}$$

$\Big\downarrow$ 27

$$\frac{\frac{1}{8}\left(\frac{1}{6}\left(\frac{\left(48a^2-208ab+139b^2\right)\tanh(c+dx)}{4\left(1-\tanh^2(c+dx)\right)^2}-\frac{3}{4}\int \frac{16a^2-48ba+29b^2+64(a-b)^2\tanh^2(c+dx)}{\left(1-\tanh^2(c+dx)\right)^2}d\tanh(c+dx)\right)+\frac{b(16a-13b)\tanh(c+dx)}{6\left(1-\tanh^2(c+dx)\right)^3}\right.}{d}$$

$\Big\downarrow$ 298

$$\frac{\frac{1}{8}\left(\frac{1}{6}\left(\frac{\left(48a^2-208ab+139b^2\right)\tanh(c+dx)}{4\left(1-\tanh^2(c+dx)\right)^2}-\frac{3}{4}\left(\frac{\left(80a^2-176ab+93b^2\right)\tanh(c+dx)}{2\left(1-\tanh^2(c+dx)\right)}-\frac{1}{2}\left(48a^2-80ab+35b^2\right)\int \frac{1}{1-\tanh^2(c+dx)}d\tanh\right.\right.\right.}{d}$$

$\Big\downarrow$ 219

$$\frac{\frac{1}{8}\left(\frac{1}{6}\left(\frac{\left(48a^2-208ab+139b^2\right)\tanh(c+dx)}{4\left(1-\tanh^2(c+dx)\right)^2}-\frac{3}{4}\left(\frac{\left(80a^2-176ab+93b^2\right)\tanh(c+dx)}{2\left(1-\tanh^2(c+dx)\right)}-\frac{1}{2}\left(48a^2-80ab+35b^2\right)\text{arctanh}(\tanh(c+dx\right.\right.\right.}{d}$$

input
```
Int[Sinh[c + d*x]^4*(a + b*Sinh[c + d*x]^2)^2,x]
```

output
```
((b^2*Tanh[c + d*x]^5)/(8*(1 - Tanh[c + d*x]^2)^4) + (((16*a - 13*b)*b*Tan
h[c + d*x])/(6*(1 - Tanh[c + d*x]^2)^3) + (((48*a^2 - 208*a*b + 139*b^2)*T
anh[c + d*x])/(4*(1 - Tanh[c + d*x]^2)^2) - (3*(-1/2*((48*a^2 - 80*a*b + 3
5*b^2)*ArcTanh[Tanh[c + d*x]]) + ((80*a^2 - 176*a*b + 93*b^2)*Tanh[c + d*x
])/(2*(1 - Tanh[c + d*x]^2))))/4)/6)/8)/d
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 298
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2), x_Symbol] :> Simp[(-(
b*c - a*d))*x*((a + b*x^2)^(p + 1)/(2*a*b*(p + 1))), x] - Simp[(a*d - b*c*(
2*p + 3))/(2*a*b*(p + 1))   Int[(a + b*x^2)^(p + 1), x], x] /; FreeQ[{a, b,
 c, d, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/2 + p, 0])
```

rule 360
```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2), x_Symbol] :
> Simp[(-a)^(m/2 - 1)*(b*c - a*d)*x*((a + b*x^2)^(p + 1)/(2*b^(m/2 + 1)*(p
+ 1))), x] + Simp[1/(2*b^(m/2 + 1)*(p + 1))   Int[(a + b*x^2)^(p + 1)*Expan
dToSum[2*b*(p + 1)*x^2*Together[(b^(m/2)*x^(m - 2)*(c + d*x^2) - (-a)^(m/2
- 1)*(b*c - a*d))/(a + b*x^2)] - (-a)^(m/2 - 1)*(b*c - a*d), x], x], x] /;
FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && IGtQ[m/2, 0] &
& (IntegerQ[p] || EqQ[m + 2*p + 1, 0])
```

rule 366
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^2,
x_Symbol] :> Simp[(-(b*c - a*d)^2)*(e*x)^(m + 1)*((a + b*x^2)^(p + 1)/(2*a*
b^2*e*(p + 1))), x] + Simp[1/(2*a*b^2*(p + 1))   Int[(e*x)^m*(a + b*x^2)^(p
 + 1)*Simp[(b*c - a*d)^2*(m + 1) + 2*b^2*c^2*(p + 1) + 2*a*b*d^2*(p + 1)*x^
2, x], x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b*c - a*d, 0] && LtQ[p
, -1]
```

rule 1471
```
Int[((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.),
x_Symbol] :> With[{Qx = PolynomialQuotient[(a + b*x^2 + c*x^4)^p, d + e*x^2
, x], R = Coeff[PolynomialRemainder[(a + b*x^2 + c*x^4)^p, d + e*x^2, x], x
, 0]}, Simp[(-R)*x*((d + e*x^2)^(q + 1)/(2*d*(q + 1))), x] + Simp[1/(2*d*(q
 + 1))   Int[(d + e*x^2)^(q + 1)*ExpandToSum[2*d*(q + 1)*Qx + R*(2*q + 3),
x], x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^
2 - b*d*e + a*e^2, 0] && IGtQ[p, 0] && LtQ[q, -1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3666
```
Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(
p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Simp[ff^(m + 1
)/f   Subst[Int[x^m*((a + (a + b)*ff^2*x^2)^p/(1 + ff^2*x^2)^(m/2 + p + 1))
, x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f}, x] && IntegerQ[m/2] &
& IntegerQ[p]
```

Maple [A] (verified)

Time = 20.87 (sec) , antiderivative size = 101, normalized size of antiderivative = 0.70

method	result
parallelrisch	$\frac{-768(a-b)\left(a-\frac{7b}{8}\right)\sinh(2dx+2c)+(96a^2-288ab+168b^2)\sinh(4dx+4c)+32b(a-b)\sinh(6dx+6c)+3b^2\sinh(8dx+8c)}{3072d}$
derivativedivides	$\frac{a^2\left(\left(\frac{\sinh(dx+c)^3}{4}-\frac{3\sinh(dx+c)}{8}\right)\cosh(dx+c)+\frac{3dx}{8}+\frac{3c}{8}\right)+2ab\left(\left(\frac{\sinh(dx+c)^5}{6}-\frac{5\sinh(dx+c)^3}{24}+\frac{5\sinh(dx+c)}{16}\right)\cosh(dx}{d}$
default	$\frac{a^2\left(\left(\frac{\sinh(dx+c)^3}{4}-\frac{3\sinh(dx+c)}{8}\right)\cosh(dx+c)+\frac{3dx}{8}+\frac{3c}{8}\right)+2ab\left(\left(\frac{\sinh(dx+c)^5}{6}-\frac{5\sinh(dx+c)^3}{24}+\frac{5\sinh(dx+c)}{16}\right)\cosh(dx}{d}$
parts	$\frac{a^2\left(\left(\frac{\sinh(dx+c)^3}{4}-\frac{3\sinh(dx+c)}{8}\right)\cosh(dx+c)+\frac{3dx}{8}+\frac{3c}{8}\right)}{d}+\frac{b^2\left(\left(\left(\frac{\sinh(dx+c)^7}{8}-\frac{7\sinh(dx+c)^5}{48}+\frac{35\sinh(dx+c)^3}{192}-\frac{35\sin}{}\right.\right.\right.}{d}$
risch	$\frac{3a^2x}{8}-\frac{5abx}{8}+\frac{35b^2x}{128}+\frac{b^2e^{8dx+8c}}{2048d}+\frac{b\,e^{6dx+6c}a}{192d}-\frac{b^2e^{6dx+6c}}{192d}+\frac{e^{4dx+4c}a^2}{64d}-\frac{3\,e^{4dx+4c}ab}{64d}+\frac{7\,e^{4dx+4c}b^2}{256d}$
orering	Expression too large to display

input
```
int(sinh(d*x+c)^4*(a+b*sinh(d*x+c)^2)^2,x,method=_RETURNVERBOSE)
```

output
```
1/3072*(-768*(a-b)*(a-7/8*b)*sinh(2*d*x+2*c)+(96*a^2-288*a*b+168*b^2)*sinh
(4*d*x+4*c)+32*b*(a-b)*sinh(6*d*x+6*c)+3*b^2*sinh(8*d*x+8*c)+1152*(a^2-5/3
*a*b+35/48*b^2)*x*d)/d
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 238, normalized size of antiderivative = 1.65

$$\int \sinh^4(c+dx)\left(a+b\sinh^2(c+dx)\right)^2\,dx$$
$$=\frac{3\,b^2\cosh(dx+c)\sinh(dx+c)^7+3\left(7\,b^2\cosh(dx+c)^3+8\,(ab-b^2)\cosh(dx+c)\right)\sinh(dx+c)^5+(21}{}$$

input
```
integrate(sinh(d*x+c)^4*(a+b*sinh(d*x+c)^2)^2,x, algorithm="fricas")
```

output
```
1/384*(3*b^2*cosh(d*x + c)*sinh(d*x + c)^7 + 3*(7*b^2*cosh(d*x + c)^3 + 8*
(a*b - b^2)*cosh(d*x + c))*sinh(d*x + c)^5 + (21*b^2*cosh(d*x + c)^5 + 80*
(a*b - b^2)*cosh(d*x + c)^3 + 12*(4*a^2 - 12*a*b + 7*b^2)*cosh(d*x + c))*s
inh(d*x + c)^3 + 3*(48*a^2 - 80*a*b + 35*b^2)*d*x + 3*(b^2*cosh(d*x + c)^7
 + 8*(a*b - b^2)*cosh(d*x + c)^5 + 4*(4*a^2 - 12*a*b + 7*b^2)*cosh(d*x + c
)^3 - 8*(8*a^2 - 15*a*b + 7*b^2)*cosh(d*x + c))*sinh(d*x + c))/d
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 490 vs. $2(141) = 282$.

Time = 0.77 (sec) , antiderivative size = 490, normalized size of antiderivative = 3.40

$$\int \sinh^4(c+dx)\left(a+b\sinh^2(c+dx)\right)^2\,dx$$
$$= \begin{cases} \frac{3a^2x\sinh^4(c+dx)}{8} - \frac{3a^2x\sinh^2(c+dx)\cosh^2(c+dx)}{4} + \frac{3a^2x\cosh^4(c+dx)}{8} + \frac{5a^2\sinh^3(c+dx)\cosh(c+dx)}{8d} - \frac{3a^2\sinh(c+dx)\cosh^3(}{8d} \\ x\left(a+b\sinh^2(c)\right)^2\sinh^4(c) \end{cases}$$

input
```
integrate(sinh(d*x+c)**4*(a+b*sinh(d*x+c)**2)**2,x)
```

output
```
Piecewise((3*a**2*x*sinh(c + d*x)**4/8 - 3*a**2*x*sinh(c + d*x)**2*cosh(c
+ d*x)**2/4 + 3*a**2*x*cosh(c + d*x)**4/8 + 5*a**2*sinh(c + d*x)**3*cosh(c
 + d*x)/(8*d) - 3*a**2*sinh(c + d*x)*cosh(c + d*x)**3/(8*d) + 5*a*b*x*sinh
(c + d*x)**6/8 - 15*a*b*x*sinh(c + d*x)**4*cosh(c + d*x)**2/8 + 15*a*b*x*s
inh(c + d*x)**2*cosh(c + d*x)**4/8 - 5*a*b*x*cosh(c + d*x)**6/8 + 11*a*b*s
inh(c + d*x)**5*cosh(c + d*x)/(8*d) - 5*a*b*sinh(c + d*x)**3*cosh(c + d*x)
**3/(3*d) + 5*a*b*sinh(c + d*x)*cosh(c + d*x)**5/(8*d) + 35*b**2*x*sinh(c
+ d*x)**8/128 - 35*b**2*x*sinh(c + d*x)**6*cosh(c + d*x)**2/32 + 105*b**2*
x*sinh(c + d*x)**4*cosh(c + d*x)**4/64 - 35*b**2*x*sinh(c + d*x)**2*cosh(c
 + d*x)**6/32 + 35*b**2*x*cosh(c + d*x)**8/128 + 93*b**2*sinh(c + d*x)**7*
cosh(c + d*x)/(128*d) - 511*b**2*sinh(c + d*x)**5*cosh(c + d*x)**3/(384*d)
 + 385*b**2*sinh(c + d*x)**3*cosh(c + d*x)**5/(384*d) - 35*b**2*sinh(c + d
*x)*cosh(c + d*x)**7/(128*d), Ne(d, 0)), (x*(a + b*sinh(c)**2)**2*sinh(c)*
*4, True))
```

Maxima [A] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 267, normalized size of antiderivative = 1.85

$$\int \sinh^4(c+dx)\left(a+b\sinh^2(c+dx)\right)^2\,dx$$
$$= \frac{1}{64}a^2\left(24x + \frac{e^{(4dx+4c)}}{d} - \frac{8e^{(2dx+2c)}}{d} + \frac{8e^{(-2dx-2c)}}{d} - \frac{e^{(-4dx-4c)}}{d}\right)$$
$$- \frac{1}{6144}b^2\left(\frac{\left(32e^{(-2dx-2c)} - 168e^{(-4dx-4c)} + 672e^{(-6dx-6c)} - 3\right)e^{(8dx+8c)}}{d} - \frac{1680(dx+c)}{d} - \frac{672e^{(-2dx}}{}\right.$$
$$- \frac{1}{192}ab\left(\frac{\left(9e^{(-2dx-2c)} - 45e^{(-4dx-4c)} - 1\right)e^{(6dx+6c)}}{d} + \frac{120(dx+c)}{d} + \frac{45e^{(-2dx-2c)} - 9e^{(-4dx-4c)} + e}{d}\right.$$

input
```
integrate(sinh(d*x+c)^4*(a+b*sinh(d*x+c)^2)^2,x, algorithm="maxima")
```

output
```
1/64*a^2*(24*x + e^(4*d*x + 4*c)/d - 8*e^(2*d*x + 2*c)/d + 8*e^(-2*d*x - 2
*c)/d - e^(-4*d*x - 4*c)/d) - 1/6144*b^2*((32*e^(-2*d*x - 2*c) - 168*e^(-4
*d*x - 4*c) + 672*e^(-6*d*x - 6*c) - 3)*e^(8*d*x + 8*c)/d - 1680*(d*x + c)
/d - (672*e^(-2*d*x - 2*c) - 168*e^(-4*d*x - 4*c) + 32*e^(-6*d*x - 6*c) -
3*e^(-8*d*x - 8*c))/d) - 1/192*a*b*((9*e^(-2*d*x - 2*c) - 45*e^(-4*d*x - 4
*c) - 1)*e^(6*d*x + 6*c)/d + 120*(d*x + c)/d + (45*e^(-2*d*x - 2*c) - 9*e^
(-4*d*x - 4*c) + e^(-6*d*x - 6*c))/d)
```

Giac [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 215, normalized size of antiderivative = 1.49

$$\int \sinh^4(c+dx)\left(a+b\sinh^2(c+dx)\right)^2\,dx$$
$$= \frac{1}{128}\left(48a^2 - 80ab + 35b^2\right)x + \frac{b^2e^{(8dx+8c)}}{2048d} - \frac{b^2e^{(-8dx-8c)}}{2048d}$$
$$+ \frac{(ab-b^2)e^{(6dx+6c)}}{192d} + \frac{(4a^2-12ab+7b^2)e^{(4dx+4c)}}{256d}$$
$$- \frac{(8a^2-15ab+7b^2)e^{(2dx+2c)}}{64d} + \frac{(8a^2-15ab+7b^2)e^{(-2dx-2c)}}{64d}$$
$$- \frac{(4a^2-12ab+7b^2)e^{(-4dx-4c)}}{256d} - \frac{(ab-b^2)e^{(-6dx-6c)}}{192d}$$

input
```
integrate(sinh(d*x+c)^4*(a+b*sinh(d*x+c)^2)^2,x, algorithm="giac")
```

output
```
1/128*(48*a^2 - 80*a*b + 35*b^2)*x + 1/2048*b^2*e^(8*d*x + 8*c)/d - 1/2048
*b^2*e^(-8*d*x - 8*c)/d + 1/192*(a*b - b^2)*e^(6*d*x + 6*c)/d + 1/256*(4*a
^2 - 12*a*b + 7*b^2)*e^(4*d*x + 4*c)/d - 1/64*(8*a^2 - 15*a*b + 7*b^2)*e^(
2*d*x + 2*c)/d + 1/64*(8*a^2 - 15*a*b + 7*b^2)*e^(-2*d*x - 2*c)/d - 1/256*
(4*a^2 - 12*a*b + 7*b^2)*e^(-4*d*x - 4*c)/d - 1/192*(a*b - b^2)*e^(-6*d*x
- 6*c)/d
```

Mupad [B] (verification not implemented)

Time = 2.05 (sec) , antiderivative size = 149, normalized size of antiderivative = 1.03

$$\int \sinh^4(c+dx) \left(a+b\sinh^2(c+dx)\right)^2 \, dx$$
$$= \frac{12\,a^2\sinh(4\,c+4\,d\,x)-96\,a^2\sinh(2\,c+2\,d\,x)-84\,b^2\sinh(2\,c+2\,d\,x)+21\,b^2\sinh(4\,c+4\,d\,x)-4\,b^2\sin}{}$$

input
```
int(sinh(c + d*x)^4*(a + b*sinh(c + d*x)^2)^2,x)
```

output
```
(12*a^2*sinh(4*c + 4*d*x) - 96*a^2*sinh(2*c + 2*d*x) - 84*b^2*sinh(2*c + 2
*d*x) + 21*b^2*sinh(4*c + 4*d*x) - 4*b^2*sinh(6*c + 6*d*x) + (3*b^2*sinh(8
*c + 8*d*x))/8 + 180*a*b*sinh(2*c + 2*d*x) - 36*a*b*sinh(4*c + 4*d*x) + 4*
a*b*sinh(6*c + 6*d*x) + 144*a^2*d*x + 105*b^2*d*x - 240*a*b*d*x)/(384*d)
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 322, normalized size of antiderivative = 2.24

$$\int \sinh^4(c+dx) \left(a+b\sinh^2(c+dx)\right)^2 \, dx$$
$$= \frac{3e^{16dx+16c}b^2+32e^{14dx+14c}ab-32e^{14dx+14c}b^2+96e^{12dx+12c}a^2-288e^{12dx+12c}ab+168e^{12dx+12c}b^2-768e^{10d}}{}$$

input
```
int(sinh(d*x+c)^4*(a+b*sinh(d*x+c)^2)^2,x)
```

output
```
(3*e**(16*c + 16*d*x)*b**2 + 32*e**(14*c + 14*d*x)*a*b - 32*e**(14*c + 14*
d*x)*b**2 + 96*e**(12*c + 12*d*x)*a**2 - 288*e**(12*c + 12*d*x)*a*b + 168*
e**(12*c + 12*d*x)*b**2 - 768*e**(10*c + 10*d*x)*a**2 + 1440*e**(10*c + 10
*d*x)*a*b - 672*e**(10*c + 10*d*x)*b**2 + 2304*e**(8*c + 8*d*x)*a**2*d*x -
 3840*e**(8*c + 8*d*x)*a*b*d*x + 1680*e**(8*c + 8*d*x)*b**2*d*x + 768*e**(
6*c + 6*d*x)*a**2 - 1440*e**(6*c + 6*d*x)*a*b + 672*e**(6*c + 6*d*x)*b**2
- 96*e**(4*c + 4*d*x)*a**2 + 288*e**(4*c + 4*d*x)*a*b - 168*e**(4*c + 4*d*
x)*b**2 - 32*e**(2*c + 2*d*x)*a*b + 32*e**(2*c + 2*d*x)*b**2 - 3*b**2)/(61
44*e**(8*c + 8*d*x)*d)
```

3.11 $\int \sinh^3(c+dx)\left(a+b\sinh^2(c+dx)\right)^2\,dx$

Optimal result

Integrand size = 23, antiderivative size = 85

$$\int \sinh^3(c+dx)\left(a+b\sinh^2(c+dx)\right)^2\,dx = -\frac{(a-b)^2\cosh(c+dx)}{d} + \frac{(a-3b)(a-b)\cosh^3(c+dx)}{3d} + \frac{(2a-3b)b\cosh^5(c+dx)}{5d} + \frac{b^2\cosh^7(c+dx)}{7d}$$

output
```
-(a-b)^2*cosh(d*x+c)/d+1/3*(a-3*b)*(a-b)*cosh(d*x+c)^3/d+1/5*(2*a-3*b)*b*c
osh(d*x+c)^5/d+1/7*b^2*cosh(d*x+c)^7/d
```

Mathematica [A] (verified)

Time = 0.18 (sec) , antiderivative size = 154, normalized size of antiderivative = 1.81

$$\int \sinh^3(c+dx)\left(a+b\sinh^2(c+dx)\right)^2\,dx = -\frac{3a^2\cosh(c+dx)}{4d}+\frac{5ab\cosh(c+dx)}{4d}-\frac{35b^2\cosh(c+dx)}{64d}+\frac{a^2\cosh(3(c+dx))}{12d}-\frac{5ab\cosh(3(c+dx))}{24d}+\frac{7b^2\cosh(3(c+dx))}{64d}+\frac{ab\cosh(5(c+dx))}{40d}-\frac{7b^2\cosh(5(c+dx))}{320d}+\frac{b^2\cosh(7(c+dx))}{448d}$$

input
```
Integrate[Sinh[c + d*x]^3*(a + b*Sinh[c + d*x]^2)^2,x]
```

output
```
(-3*a^2*Cosh[c + d*x])/(4*d) + (5*a*b*Cosh[c + d*x])/(4*d) - (35*b^2*Cosh[
c + d*x])/(64*d) + (a^2*Cosh[3*(c + d*x)])/(12*d) - (5*a*b*Cosh[3*(c + d*x
)])/(24*d) + (7*b^2*Cosh[3*(c + d*x)])/(64*d) + (a*b*Cosh[5*(c + d*x)])/(4
0*d) - (7*b^2*Cosh[5*(c + d*x)])/(320*d) + (b^2*Cosh[7*(c + d*x)])/(448*d)
```

Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 77, normalized size of antiderivative = 0.91, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.217$, Rules used = {3042, 26, 3665, 290, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sinh^3(c+dx)\left(a+b\sinh^2(c+dx)\right)^2\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int i\sin(ic+idx)^3\left(a-b\sin(ic+idx)^2\right)^2dx$$

$$\Big\downarrow \text{26}$$

$$i \int \sin(ic+idx)^3 \left(a-b\sin(ic+idx)^2\right)^2 dx$$

$$\Big\downarrow \text{3665}$$

$$-\frac{\int \left(1-\cosh^2(c+dx)\right)\left(b\cosh^2(c+dx)+a-b\right)^2 d\cosh(c+dx)}{d}$$

$$\Big\downarrow \text{290}$$

$$-\frac{\int \left(-b^2\cosh^6(c+dx)-(2a-3b)b\cosh^4(c+dx)+(a-3b)(b-a)\cosh^2(c+dx)+(a-b)^2\right) d\cosh(c+dx)}{d}$$

$$\Big\downarrow \text{2009}$$

$$-\frac{-\frac{1}{5}b(2a-3b)\cosh^5(c+dx)-\frac{1}{3}(a-3b)(a-b)\cosh^3(c+dx)+(a-b)^2\cosh(c+dx)-\frac{1}{7}b^2\cosh^7(c+dx)}{d}$$

input
```
Int[Sinh[c + d*x]^3*(a + b*Sinh[c + d*x]^2)^2,x]
```

output
```
-(((a - b)^2*Cosh[c + d*x] - ((a - 3*b)*(a - b)*Cosh[c + d*x]^3)/3 - ((2*a
 - 3*b)*b*Cosh[c + d*x]^5)/5 - (b^2*Cosh[c + d*x]^7)/7)/d)
```

Defintions of rubi rules used

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 290
```
Int[((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.), x_Symbol] :> I
nt[ExpandIntegrand[(a + b*x^2)^p*(c + d*x^2)^q, x], x] /; FreeQ[{a, b, c, d
}, x] && NeQ[b*c - a*d, 0] && IGtQ[p, 0] && IGtQ[q, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3665
```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^
(p_.), x_Symbol] :> With[{ff = FreeFactors[Cos[e + f*x], x]}, Simp[-ff/f
Subst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a + b - b*ff^2*x^2)^p, x], x, Cos[e +
 f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m - 1)/2]
```

Maple [A] (verified)

Time = 11.80 (sec) , antiderivative size = 95, normalized size of antiderivative = 1.12

method	result
parallelrisch	$\frac{560\left(a-\frac{7b}{4}\right)\left(a-\frac{3b}{4}\right)\cosh(3dx+3c)+168\left(a-\frac{7b}{8}\right)b\cosh(5dx+5c)+15b^2\cosh(7dx+7c)+\left(-5040a^2+8400ab-3675b^2\right) c}{6720d}$
derivativedivides	$\frac{a^2\left(-\frac{2}{3}+\frac{\sinh(dx+c)^2}{3}\right)\cosh(dx+c)+2ab\left(\frac{8}{15}+\frac{\sinh(dx+c)^4}{5}-\frac{4\sinh(dx+c)^2}{15}\right)\cosh(dx+c)+b^2\left(-\frac{16}{35}+\frac{\sinh(dx+c)^6}{7}-\frac{6 \sin}{}\right.}{d}$
default	$\frac{a^2\left(-\frac{2}{3}+\frac{\sinh(dx+c)^2}{3}\right)\cosh(dx+c)+2ab\left(\frac{8}{15}+\frac{\sinh(dx+c)^4}{5}-\frac{4\sinh(dx+c)^2}{15}\right)\cosh(dx+c)+b^2\left(-\frac{16}{35}+\frac{\sinh(dx+c)^6}{7}-\frac{6 \sin}{}\right.}{d}$
parts	$\frac{a^2\left(-\frac{2}{3}+\frac{\sinh(dx+c)^2}{3}\right)\cosh(dx+c)}{d}+\frac{b^2\left(-\frac{16}{35}+\frac{\sinh(dx+c)^6}{7}-\frac{6\sinh(dx+c)^4}{35}+\frac{8\sinh(dx+c)^2}{35}\right)\cosh(dx+c)}{d}+\frac{2ab\left(\frac{8}{1}\right.}{}$
risch	$\frac{b^2 e^{7dx+7c}}{896d}+\frac{b\,e^{5dx+5c}a}{80d}-\frac{7b^2 e^{5dx+5c}}{640d}+\frac{e^{3dx+3c}a^2}{24d}-\frac{5\,e^{3dx+3c}ab}{48d}+\frac{7\,e^{3dx+3c}b^2}{128d}-\frac{3\,e^{dx+c}a^2}{8d}+\frac{5\,e^{dx+c}a}{8d}$
orering	$\frac{\frac{12916\sinh(dx+c)^2\left(a+b\sinh(dx+c)^2\right)^2 d\cosh(dx+c)}{3675}+\frac{51664\sinh(dx+c)^4\left(a+b\sinh(dx+c)^2\right)bd\cosh(dx+c)}{11025}}{d^2}-\frac{94\left(6d^3\cosh(dx\right.}{}$

input
```
int(sinh(d*x+c)^3*(a+b*sinh(d*x+c)^2)^2,x,method=_RETURNVERBOSE)
```

output
```
1/6720*(560*(a-7/4*b)*(a-3/4*b)*cosh(3*d*x+3*c)+168*(a-7/8*b)*b*cosh(5*d*x
+5*c)+15*b^2*cosh(7*d*x+7*c)+(-5040*a^2+8400*a*b-3675*b^2)*cosh(d*x+c)-448
0*a^2+7168*a*b-3072*b^2)/d
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 213 vs. $2(79) = 158$.

Time = 0.10 (sec) , antiderivative size = 213, normalized size of antiderivative = 2.51

$$\int \sinh^3(c+dx)\left(a+b\sinh^2(c+dx)\right)^2\,dx = \frac{15\,b^2\cosh\left(dx+c\right)^7+105\,b^2\cosh\left(dx+c\right)\sinh\left(dx+c\right)^6+21\left(8\,ab-7\,b^2\right)\cosh\left(dx+c\right)^5+105\left(5\,b^2\cos\right.}{}$$

input
```
integrate(sinh(d*x+c)^3*(a+b*sinh(d*x+c)^2)^2,x, algorithm="fricas")
```

output
```
1/6720*(15*b^2*cosh(d*x + c)^7 + 105*b^2*cosh(d*x + c)*sinh(d*x + c)^6 + 2
1*(8*a*b - 7*b^2)*cosh(d*x + c)^5 + 105*(5*b^2*cosh(d*x + c)^3 + (8*a*b -
7*b^2)*cosh(d*x + c))*sinh(d*x + c)^4 + 35*(16*a^2 - 40*a*b + 21*b^2)*cosh
(d*x + c)^3 + 105*(3*b^2*cosh(d*x + c)^5 + 2*(8*a*b - 7*b^2)*cosh(d*x + c)
^3 + (16*a^2 - 40*a*b + 21*b^2)*cosh(d*x + c))*sinh(d*x + c)^2 - 105*(48*a
^2 - 80*a*b + 35*b^2)*cosh(d*x + c))/d
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 204 vs. $2(68) = 136$.

Time = 0.52 (sec) , antiderivative size = 204, normalized size of antiderivative = 2.40

$$\int \sinh^3(c+dx)\left(a+b\sinh^2(c+dx)\right)^2\,dx = \begin{cases} \frac{a^2\sinh^2(c+dx)\cosh(c+dx)}{d} - \frac{2a^2\cosh^3(c+dx)}{3d} + \frac{2ab\sinh^4(c+dx)\cosh(c+dx)}{d} - \frac{8ab\sinh^2(c+dx)\cosh^3(c+dx)}{3d} + \frac{16ab\cosh^5(c+d}{15d} \\ x\left(a+b\sinh^2(c)\right)^2\sinh^3(c) \end{cases}$$

input
```
integrate(sinh(d*x+c)**3*(a+b*sinh(d*x+c)**2)**2,x)
```

output
```
Piecewise((a**2*sinh(c + d*x)**2*cosh(c + d*x)/d - 2*a**2*cosh(c + d*x)**3
/(3*d) + 2*a*b*sinh(c + d*x)**4*cosh(c + d*x)/d - 8*a*b*sinh(c + d*x)**2*c
osh(c + d*x)**3/(3*d) + 16*a*b*cosh(c + d*x)**5/(15*d) + b**2*sinh(c + d*x
)**6*cosh(c + d*x)/d - 2*b**2*sinh(c + d*x)**4*cosh(c + d*x)**3/d + 8*b**2
*sinh(c + d*x)**2*cosh(c + d*x)**5/(5*d) - 16*b**2*cosh(c + d*x)**7/(35*d)
, Ne(d, 0)), (x*(a + b*sinh(c)**2)**2*sinh(c)**3, True))
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 247 vs. $2(79) = 158$.

Time = 0.04 (sec) , antiderivative size = 247, normalized size of antiderivative = 2.91

$$\int \sinh^3(c+dx)\left(a+b\sinh^2(c+dx)\right)^2\,dx =$$

$$-\frac{1}{4480}b^2\left(\frac{\left(49\,e^{(-2\,dx-2\,c)}-245\,e^{(-4\,dx-4\,c)}+1225\,e^{(-6\,dx-6\,c)}-5\right)e^{(7\,dx+7\,c)}}{d}+\frac{1225\,e^{(-dx-c)}-245\,e^{(-3\,dx}\cdots}{}\right.$$
$$+\frac{1}{240}ab\left(\frac{3\,e^{(5\,dx+5\,c)}}{d}-\frac{25\,e^{(3\,dx+3\,c)}}{d}+\frac{150\,e^{(dx+c)}}{d}+\frac{150\,e^{(-dx-c)}}{d}-\frac{25\,e^{(-3\,dx-3\,c)}}{d}+\frac{3\,e^{(-5\,dx-5\,c)}}{d}\right)$$
$$+\frac{1}{24}a^2\left(\frac{e^{(3\,dx+3\,c)}}{d}-\frac{9\,e^{(dx+c)}}{d}-\frac{9\,e^{(-dx-c)}}{d}+\frac{e^{(-3\,dx-3\,c)}}{d}\right)$$

input
```
integrate(sinh(d*x+c)^3*(a+b*sinh(d*x+c)^2)^2,x, algorithm="maxima")
```

output
```
-1/4480*b^2*((49*e^(-2*d*x - 2*c) - 245*e^(-4*d*x - 4*c) + 1225*e^(-6*d*x
- 6*c) - 5)*e^(7*d*x + 7*c)/d + (1225*e^(-d*x - c) - 245*e^(-3*d*x - 3*c)
+ 49*e^(-5*d*x - 5*c) - 5*e^(-7*d*x - 7*c))/d) + 1/240*a*b*(3*e^(5*d*x + 5
*c)/d - 25*e^(3*d*x + 3*c)/d + 150*e^(d*x + c)/d + 150*e^(-d*x - c)/d - 25
*e^(-3*d*x - 3*c)/d + 3*e^(-5*d*x - 5*c)/d) + 1/24*a^2*(e^(3*d*x + 3*c)/d
- 9*e^(d*x + c)/d - 9*e^(-d*x - c)/d + e^(-3*d*x - 3*c)/d)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 196 vs. $2(79) = 158$.

Time = 0.15 (sec) , antiderivative size = 196, normalized size of antiderivative = 2.31

$$\int \sinh^3(c+dx)\left(a+b\sinh^2(c+dx)\right)^2\,dx = \frac{b^2e^{(7\,dx+7\,c)}}{896\,d} + \frac{b^2e^{(-7\,dx-7\,c)}}{896\,d} + \frac{(8\,ab-7\,b^2)e^{(5\,dx+5\,c)}}{640\,d} + \frac{(16\,a^2-40\,ab+21\,b^2)e^{(3\,dx+3\,c)}}{384\,d} - \frac{(48\,a^2-80\,ab+35\,b^2)e^{(dx+c)}}{128\,d} - \frac{(48\,a^2-80\,ab+35\,b^2)e^{(-dx-c)}}{128\,d} + \frac{(16\,a^2-40\,ab+21\,b^2)e^{(-3\,dx-3\,c)}}{384\,d} + \frac{(8\,ab-7\,b^2)e^{(-5\,dx-5\,c)}}{640\,d}$$

input
```
integrate(sinh(d*x+c)^3*(a+b*sinh(d*x+c)^2)^2,x, algorithm="giac")
```

output
```
1/896*b^2*e^(7*d*x + 7*c)/d + 1/896*b^2*e^(-7*d*x - 7*c)/d + 1/640*(8*a*b
- 7*b^2)*e^(5*d*x + 5*c)/d + 1/384*(16*a^2 - 40*a*b + 21*b^2)*e^(3*d*x + 3
*c)/d - 1/128*(48*a^2 - 80*a*b + 35*b^2)*e^(d*x + c)/d - 1/128*(48*a^2 - 8
0*a*b + 35*b^2)*e^(-d*x - c)/d + 1/384*(16*a^2 - 40*a*b + 21*b^2)*e^(-3*d*
x - 3*c)/d + 1/640*(8*a*b - 7*b^2)*e^(-5*d*x - 5*c)/d
```

Mupad [B] (verification not implemented)

Time = 1.82 (sec) , antiderivative size = 112, normalized size of antiderivative = 1.32

$$\int \sinh^3(c+dx)\left(a+b\sinh^2(c+dx)\right)^2\,dx = \frac{\frac{a^2\cosh(c+d\,x)^3}{3} - a^2\cosh(c+d\,x) + \frac{2\,a\,b\cosh(c+d\,x)^5}{5} - \frac{4\,a\,b\cosh(c+d\,x)^3}{3} + 2\,a\,b\cosh(c+d\,x) + \frac{b^2\cosh(c+d\,x)^7}{7} - \frac{3}{}}{d}$$

input
```
int(sinh(c + d*x)^3*(a + b*sinh(c + d*x)^2)^2,x)
```

output
```
((a^2*cosh(c + d*x)^3)/3 - b^2*cosh(c + d*x) - a^2*cosh(c + d*x) + b^2*cos
h(c + d*x)^3 - (3*b^2*cosh(c + d*x)^5)/5 + (b^2*cosh(c + d*x)^7)/7 + 2*a*b
*cosh(c + d*x) - (4*a*b*cosh(c + d*x)^3)/3 + (2*a*b*cosh(c + d*x)^5)/5)/d
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 272, normalized size of antiderivative = 3.20

$$\int \sinh^3(c+dx)\left(a+b\sinh^2(c+dx)\right)^2\,dx$$
$$= \frac{15e^{14dx+14c}b^2+168e^{12dx+12c}ab-147e^{12dx+12c}b^2+560e^{10dx+10c}a^2-1400e^{10dx+10c}ab+735e^{10dx+10c}b^2-5}{}$$

input
```
int(sinh(d*x+c)^3*(a+b*sinh(d*x+c)^2)^2,x)
```

output
```
(15*e**(14*c + 14*d*x)*b**2 + 168*e**(12*c + 12*d*x)*a*b - 147*e**(12*c +
12*d*x)*b**2 + 560*e**(10*c + 10*d*x)*a**2 - 1400*e**(10*c + 10*d*x)*a*b +
 735*e**(10*c + 10*d*x)*b**2 - 5040*e**(8*c + 8*d*x)*a**2 + 8400*e**(8*c +
 8*d*x)*a*b - 3675*e**(8*c + 8*d*x)*b**2 - 5040*e**(6*c + 6*d*x)*a**2 + 84
00*e**(6*c + 6*d*x)*a*b - 3675*e**(6*c + 6*d*x)*b**2 + 560*e**(4*c + 4*d*x
)*a**2 - 1400*e**(4*c + 4*d*x)*a*b + 735*e**(4*c + 4*d*x)*b**2 + 168*e**(2
*c + 2*d*x)*a*b - 147*e**(2*c + 2*d*x)*b**2 + 15*b**2)/(13440*e**(7*c + 7*
d*x)*d)
```

3.12 $\int \sinh^2(c+dx)\left(a+b\sinh^2(c+dx)\right)^2\,dx$

Optimal result

Integrand size = 23, antiderivative size = 110

$$\int \sinh^2(c+dx)\left(a+b\sinh^2(c+dx)\right)^2\,dx = -\frac{1}{16}\left(8a^2-12ab+5b^2\right)x+\frac{(8a^2-20ab+11b^2)\cosh(c+dx)\sinh(c+dx)}{16d}+\frac{(4a-3b)b\cosh^3(c+dx)\sinh(c+dx)}{8d}+\frac{b^2\cosh^3(c+dx)\sinh^3(c+dx)}{6d}$$

output
```
-1/16*(8*a^2-12*a*b+5*b^2)*x+1/16*(8*a^2-20*a*b+11*b^2)*cosh(d*x+c)*sinh(d
*x+c)/d+1/8*(4*a-3*b)*b*cosh(d*x+c)^3*sinh(d*x+c)/d+1/6*b^2*cosh(d*x+c)^3*
sinh(d*x+c)^3/d
```

Mathematica [A] (verified)

Time = 0.27 (sec) , antiderivative size = 99, normalized size of antiderivative = 0.90

$$\int \sinh^2(c+dx)\left(a+b\sinh^2(c+dx)\right)^2\,dx = \frac{-96a^2c+144abc-60b^2c-96a^2dx+144abdx-60b^2dx+(48a^2-96ab+45b^2)\sinh(2(c+dx))+3(4a}{192d}$$

input
```
Integrate[Sinh[c + d*x]^2*(a + b*Sinh[c + d*x]^2)^2,x]
```

output
```
(-96*a^2*c + 144*a*b*c - 60*b^2*c - 96*a^2*d*x + 144*a*b*d*x - 60*b^2*d*x
+ (48*a^2 - 96*a*b + 45*b^2)*Sinh[2*(c + d*x)] + 3*(4*a - 3*b)*b*Sinh[4*(c
 + d*x)] + b^2*Sinh[6*(c + d*x)])/(192*d)
```

Rubi [A] (verified)

Time = 0.40 (sec) , antiderivative size = 122, normalized size of antiderivative = 1.11, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.217, Rules used = {3042, 25, 3649, 3042, 3648}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sinh^2(c+dx)\left(a+b\sinh^2(c+dx)\right)^2dx$$

$$\Big\downarrow \text{3042}$$

$$\int -\sin(ic+idx)^2\left(a-b\sin(ic+idx)^2\right)^2dx$$

$$\Big\downarrow \text{25}$$

$$-\int \sin(ic+idx)^2\left(a-b\sin(ic+idx)^2\right)^2dx$$

$$\Big\downarrow \text{3649}$$

$$\frac{\sinh(c+dx)\cosh(c+dx)\left(a+b\sinh^2(c+dx)\right)^2}{6d}-\frac{1}{6}\int \left(a-(4a-5b)\sinh^2(c+dx)\right)\left(b\sinh^2(c+dx)+a\right)dx$$

$$\Big\downarrow \text{3042}$$

$$\frac{\sinh(c+dx)\cosh(c+dx)\left(a+b\sinh^2(c+dx)\right)^2}{6d}-\frac{1}{6}\int \left((4a-5b)\sin(ic+idx)^2+a\right)\left(a-b\sin(ic+idx)^2\right)dx$$

$$\Big\downarrow \text{3648}$$

$$\frac{1}{6}\left(\frac{(16a^2-36ab+15b^2)\sinh(c+dx)\cosh(c+dx)}{8d}-\frac{3}{8}x(8a^2-12ab+5b^2)+\frac{b(4a-5b)\sinh^3(c+dx)\cosh(c+dx)}{4d}\right)+\frac{\sinh(c+dx)\cosh(c+dx)\left(a+b\sinh^2(c+dx)\right)^2}{6d}$$

input
```
Int[Sinh[c + d*x]^2*(a + b*Sinh[c + d*x]^2)^2,x]
```

output
```
(Cosh[c + d*x]*Sinh[c + d*x]*(a + b*Sinh[c + d*x]^2)^2)/(6*d) + ((-3*(8*a^
2 - 12*a*b + 5*b^2)*x)/8 + ((16*a^2 - 36*a*b + 15*b^2)*Cosh[c + d*x]*Sinh[
c + d*x])/(8*d) + ((4*a - 5*b)*b*Cosh[c + d*x]*Sinh[c + d*x]^3)/(4*d))/6
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3648
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)*((A_.) + (B_.)*sin[(e_.) + (f_
.)*(x_)]^2), x_Symbol] :> Simp[(4*A*(2*a + b) + B*(4*a + 3*b))*(x/8), x] +
(-Simp[b*B*Cos[e + f*x]*(Sin[e + f*x]^3/(4*f)), x] - Simp[(4*A*b + B*(4*a +
 3*b))*Cos[e + f*x]*(Sin[e + f*x]/(8*f)), x]) /; FreeQ[{a, b, e, f, A, B},
x]
```

rule 3649
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)]^2), x_Symbol] :> Simp[(-B)*Cos[e + f*x]*Sin[e + f*x]*((a + b*
Sin[e + f*x]^2)^p/(2*f*(p + 1))), x] + Simp[1/(2*(p + 1))   Int[(a + b*Sin[
e + f*x]^2)^(p - 1)*Simp[a*B + 2*a*A*(p + 1) + (2*A*b*(p + 1) + B*(b + 2*a*
p + 2*b*p))*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, e, f, A, B}, x] && G
tQ[p, 0]
```

Maple [A] (verified)

Time = 4.58 (sec) , antiderivative size = 82, normalized size of antiderivative = 0.75

method	result
parallelrisch	$\frac{(48a^2-96ab+45b^2)\sinh(2dx+2c)+(12ab-9b^2)\sinh(4dx+4c)+b^2\sinh(6dx+6c)-96\left(a^2-\frac{3}{2}ab+\frac{5}{8}b^2\right)xd}{192d}$
derivativedivides	$\frac{a^2\left(\frac{\cosh(dx+c)\sinh(dx+c)}{2}-\frac{dx}{2}-\frac{c}{2}\right)+2ab\left(\left(\frac{\sinh(dx+c)^3}{4}-\frac{3\sinh(dx+c)}{8}\right)\cosh(dx+c)+\frac{3dx}{8}+\frac{3c}{8}\right)+b^2\left(\left(\frac{\sinh(dx+c)^5}{6}-\frac{5}{}\right.\right.}{d}$
default	$\frac{a^2\left(\frac{\cosh(dx+c)\sinh(dx+c)}{2}-\frac{dx}{2}-\frac{c}{2}\right)+2ab\left(\left(\frac{\sinh(dx+c)^3}{4}-\frac{3\sinh(dx+c)}{8}\right)\cosh(dx+c)+\frac{3dx}{8}+\frac{3c}{8}\right)+b^2\left(\left(\frac{\sinh(dx+c)^5}{6}-\frac{5}{}\right.\right.}{d}$
parts	$\frac{a^2\left(\frac{\cosh(dx+c)\sinh(dx+c)}{2}-\frac{dx}{2}-\frac{c}{2}\right)}{d}+\frac{b^2\left(\left(\frac{\sinh(dx+c)^5}{6}-\frac{5\sinh(dx+c)^3}{24}+\frac{5\sinh(dx+c)}{16}\right)\cosh(dx+c)-\frac{5dx}{16}-\frac{5c}{16}\right)}{d}+\frac{2}{}$
risch	$-\frac{a^2x}{2}+\frac{3abx}{4}-\frac{5b^2x}{16}+\frac{b^2e^{6dx+6c}}{384d}+\frac{e^{4dx+4c}ab}{32d}-\frac{3e^{4dx+4c}b^2}{128d}+\frac{e^{2dx+2c}a^2}{8d}-\frac{e^{2dx+2c}ab}{4d}+\frac{15e^{2dx+2c}b}{128d}$
orering	Expression too large to display

input
```
int(sinh(d*x+c)^2*(a+b*sinh(d*x+c)^2)^2,x,method=_RETURNVERBOSE)
```

output
```
1/192*((48*a^2-96*a*b+45*b^2)*sinh(2*d*x+2*c)+(12*a*b-9*b^2)*sinh(4*d*x+4*
c)+b^2*sinh(6*d*x+6*c)-96*(a^2-3/2*a*b+5/8*b^2)*x*d)/d
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 149, normalized size of antiderivative = 1.35

$$\int \sinh^2(c+dx)\left(a+b\sinh^2(c+dx)\right)^2\,dx$$
$$=\frac{3b^2\cosh(dx+c)\sinh(dx+c)^5+2\left(5b^2\cosh(dx+c)^3+3\left(4ab-3b^2\right)\cosh(dx+c)\right)\sinh(dx+c)^3-}{}$$

input
```
integrate(sinh(d*x+c)^2*(a+b*sinh(d*x+c)^2)^2,x, algorithm="fricas")
```

output
```
1/96*(3*b^2*cosh(d*x + c)*sinh(d*x + c)^5 + 2*(5*b^2*cosh(d*x + c)^3 + 3*(
4*a*b - 3*b^2)*cosh(d*x + c))*sinh(d*x + c)^3 - 6*(8*a^2 - 12*a*b + 5*b^2)
*d*x + 3*(b^2*cosh(d*x + c)^5 + 2*(4*a*b - 3*b^2)*cosh(d*x + c)^3 + (16*a^
2 - 32*a*b + 15*b^2)*cosh(d*x + c))*sinh(d*x + c))/d
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 332 vs. $2(104) = 208$.

Time = 0.40 (sec) , antiderivative size = 332, normalized size of antiderivative = 3.02

$$\int \sinh^2(c+dx) \left(a+b\sinh^2(c+dx)\right)^2 dx$$
$$= \begin{cases} \frac{a^2x\sinh^2(c+dx)}{2} - \frac{a^2x\cosh^2(c+dx)}{2} + \frac{a^2\sinh(c+dx)\cosh(c+dx)}{2d} + \frac{3abx\sinh^4(c+dx)}{4} - \frac{3abx\sinh^2(c+dx)\cosh^2(c+dx)}{2} + \frac{3abx}{} \\ x\left(a+b\sinh^2(c)\right)^2\sinh^2(c) \end{cases}$$

input
```
integrate(sinh(d*x+c)**2*(a+b*sinh(d*x+c)**2)**2,x)
```

output
```
Piecewise((a**2*x*sinh(c + d*x)**2/2 - a**2*x*cosh(c + d*x)**2/2 + a**2*si
nh(c + d*x)*cosh(c + d*x)/(2*d) + 3*a*b*x*sinh(c + d*x)**4/4 - 3*a*b*x*sin
h(c + d*x)**2*cosh(c + d*x)**2/2 + 3*a*b*x*cosh(c + d*x)**4/4 + 5*a*b*sinh
(c + d*x)**3*cosh(c + d*x)/(4*d) - 3*a*b*sinh(c + d*x)*cosh(c + d*x)**3/(4
*d) + 5*b**2*x*sinh(c + d*x)**6/16 - 15*b**2*x*sinh(c + d*x)**4*cosh(c + d
*x)**2/16 + 15*b**2*x*sinh(c + d*x)**2*cosh(c + d*x)**4/16 - 5*b**2*x*cosh
(c + d*x)**6/16 + 11*b**2*sinh(c + d*x)**5*cosh(c + d*x)/(16*d) - 5*b**2*s
inh(c + d*x)**3*cosh(c + d*x)**3/(6*d) + 5*b**2*sinh(c + d*x)*cosh(c + d*x
)**5/(16*d), Ne(d, 0)), (x*(a + b*sinh(c)**2)**2*sinh(c)**2, True))
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 189, normalized size of antiderivative = 1.72

$$\int \sinh^2(c+dx) \left(a+b\sinh^2(c+dx)\right)^2 dx$$
$$= \frac{1}{32}ab\left(24x + \frac{e^{(4dx+4c)}}{d} - \frac{8e^{(2dx+2c)}}{d} + \frac{8e^{(-2dx-2c)}}{d} - \frac{e^{(-4dx-4c)}}{d}\right) - \frac{1}{8}a^2\left(4x - \frac{e^{(2dx+2c)}}{d} + \frac{e^{(-2dx-2c)}}{d}\right) - \frac{1}{384}b^2\left(\frac{\left(9e^{(-2dx-2c)} - 45e^{(-4dx-4c)} - 1\right)e^{(6dx+6c)}}{d} + \frac{120(dx+c)}{d} + \frac{45e^{(-2dx-2c)} - 9e^{(-4dx-4c)} + e}{d}\right.$$

input
```
integrate(sinh(d*x+c)^2*(a+b*sinh(d*x+c)^2)^2,x, algorithm="maxima")
```

output
```
1/32*a*b*(24*x + e^(4*d*x + 4*c)/d - 8*e^(2*d*x + 2*c)/d + 8*e^(-2*d*x - 2
*c)/d - e^(-4*d*x - 4*c)/d) - 1/8*a^2*(4*x - e^(2*d*x + 2*c)/d + e^(-2*d*x
 - 2*c)/d) - 1/384*b^2*((9*e^(-2*d*x - 2*c) - 45*e^(-4*d*x - 4*c) - 1)*e^(
6*d*x + 6*c)/d + 120*(d*x + c)/d + (45*e^(-2*d*x - 2*c) - 9*e^(-4*d*x - 4*
c) + e^(-6*d*x - 6*c))/d)
```

Giac [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 159, normalized size of antiderivative = 1.45

$$\int \sinh^2(c+dx)\left(a+b\sinh^2(c+dx)\right)^2\,dx = -\frac{1}{16}\left(8a^2-12ab+5b^2\right)x+\frac{b^2e^{(6dx+6c)}}{384d}-\frac{b^2e^{(-6dx-6c)}}{384d}+\frac{(4ab-3b^2)e^{(4dx+4c)}}{128d}+\frac{(16a^2-32ab+15b^2)e^{(2dx+2c)}}{128d}-\frac{(16a^2-32ab+15b^2)e^{(-2dx-2c)}}{128d}-\frac{(4ab-3b^2)e^{(-4dx-4c)}}{128d}$$

input
```
integrate(sinh(d*x+c)^2*(a+b*sinh(d*x+c)^2)^2,x, algorithm="giac")
```

output
```
-1/16*(8*a^2 - 12*a*b + 5*b^2)*x + 1/384*b^2*e^(6*d*x + 6*c)/d - 1/384*b^2
*e^(-6*d*x - 6*c)/d + 1/128*(4*a*b - 3*b^2)*e^(4*d*x + 4*c)/d + 1/128*(16*
a^2 - 32*a*b + 15*b^2)*e^(2*d*x + 2*c)/d - 1/128*(16*a^2 - 32*a*b + 15*b^2
)*e^(-2*d*x - 2*c)/d - 1/128*(4*a*b - 3*b^2)*e^(-4*d*x - 4*c)/d
```

Mupad [B] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 108, normalized size of antiderivative = 0.98

$$\int \sinh^2(c+dx)\left(a+b\sinh^2(c+dx)\right)^2\,dx = \frac{12a^2\sinh(2c+2dx)+\frac{45b^2\sinh(2c+2dx)}{4}-\frac{9b^2\sinh(4c+4dx)}{4}+\frac{b^2\sinh(6c+6dx)}{4}-24ab\sinh(2c+2dx)+3a}{48d}$$

input
```
int(sinh(c + d*x)^2*(a + b*sinh(c + d*x)^2)^2,x)
```

output
```
(12*a^2*sinh(2*c + 2*d*x) + (45*b^2*sinh(2*c + 2*d*x))/4 - (9*b^2*sinh(4*c
 + 4*d*x))/4 + (b^2*sinh(6*c + 6*d*x))/4 - 24*a*b*sinh(2*c + 2*d*x) + 3*a*
b*sinh(4*c + 4*d*x) - 24*a^2*d*x - 15*b^2*d*x + 36*a*b*d*x)/(48*d)
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 233, normalized size of antiderivative = 2.12

$$\int \sinh^2(c+dx)\left(a+b\sinh^2(c+dx)\right)^2\,dx$$
$$=\frac{e^{12dx+12c}b^2+12e^{10dx+10c}ab-9e^{10dx+10c}b^2+48e^{8dx+8c}a^2-96e^{8dx+8c}ab+45e^{8dx+8c}b^2-192e^{6dx+6c}a^2dx+}{384}$$

input
```
int(sinh(d*x+c)^2*(a+b*sinh(d*x+c)^2)^2,x)
```

output
```
(e**(12*c + 12*d*x)*b**2 + 12*e**(10*c + 10*d*x)*a*b - 9*e**(10*c + 10*d*x
)*b**2 + 48*e**(8*c + 8*d*x)*a**2 - 96*e**(8*c + 8*d*x)*a*b + 45*e**(8*c +
 8*d*x)*b**2 - 192*e**(6*c + 6*d*x)*a**2*d*x + 288*e**(6*c + 6*d*x)*a*b*d*
x - 120*e**(6*c + 6*d*x)*b**2*d*x - 48*e**(4*c + 4*d*x)*a**2 + 96*e**(4*c
+ 4*d*x)*a*b - 45*e**(4*c + 4*d*x)*b**2 - 12*e**(2*c + 2*d*x)*a*b + 9*e**(
2*c + 2*d*x)*b**2 - b**2)/(384*e**(6*c + 6*d*x)*d)
```

3.13 $\int \sinh(c+dx)\left(a+b\sinh^2(c+dx)\right)^2\,dx$

Optimal result

Integrand size = 21, antiderivative size = 57

$$\int \sinh(c+dx)\left(a+b\sinh^2(c+dx)\right)^2\,dx = \frac{(a-b)^2\cosh(c+dx)}{d} + \frac{2(a-b)b\cosh^3(c+dx)}{3d} + \frac{b^2\cosh^5(c+dx)}{5d}$$

output

```
(a-b)^2*cosh(d*x+c)/d+2/3*(a-b)*b*cosh(d*x+c)^3/d+1/5*b^2*cosh(d*x+c)^5/d
```

Mathematica [A] (verified)

Time = 0.14 (sec) , antiderivative size = 111, normalized size of antiderivative = 1.95

$$\int \sinh(c+dx)\left(a+b\sinh^2(c+dx)\right)^2\,dx = \frac{a^2\cosh(c)\cosh(dx)}{d} - \frac{3ab\cosh(c+dx)}{2d} + \frac{5b^2\cosh(c+dx)}{8d} + \frac{ab\cosh(3(c+dx))}{6d} - \frac{5b^2\cosh(3(c+dx))}{48d} + \frac{b^2\cosh(5(c+dx))}{80d} + \frac{a^2\sinh(c)\sinh(dx)}{d}$$

input
```
Integrate[Sinh[c + d*x]*(a + b*Sinh[c + d*x]^2)^2,x]
```

output
```
(a^2*Cosh[c]*Cosh[d*x])/d - (3*a*b*Cosh[c + d*x])/(2*d) + (5*b^2*Cosh[c +
d*x])/(8*d) + (a*b*Cosh[3*(c + d*x)])/(6*d) - (5*b^2*Cosh[3*(c + d*x)])/(4
8*d) + (b^2*Cosh[5*(c + d*x)])/(80*d) + (a^2*Sinh[c]*Sinh[d*x])/d
```

Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 52, normalized size of antiderivative = 0.91, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.238$, Rules used = {3042, 26, 3665, 210, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sinh(c+dx)\left(a+b\sinh^2(c+dx)\right)^2 dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int -i\sin(ic+idx)\left(a-b\sin(ic+idx)^2\right)^2 dx$$

$$\Big\downarrow \text{ 26}$$

$$-i\int \sin(ic+idx)\left(a-b\sin(ic+idx)^2\right)^2 dx$$

$$\Big\downarrow \text{ 3665}$$

$$\frac{\int \left(b\cosh^2(c+dx)+a-b\right)^2 d\cosh(c+dx)}{d}$$

$$\Big\downarrow \text{ 210}$$

$$\frac{\int \left(b^2\cosh^4(c+dx)+2ab\left(1-\frac{b}{a}\right)\cosh^2(c+dx)+a^2\left(\frac{b(b-2a)}{a^2}+1\right)\right) d\cosh(c+dx)}{d}$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{\frac{2}{3}b(a-b)\cosh^3(c+dx)+(a-b)^2\cosh(c+dx)+\frac{1}{5}b^2\cosh^5(c+dx)}{d}$$

input
```
Int[Sinh[c + d*x]*(a + b*Sinh[c + d*x]^2)^2,x]
```

output
```
((a - b)^2*Cosh[c + d*x] + (2*(a - b)*b*Cosh[c + d*x]^3)/3 + (b^2*Cosh[c +
 d*x]^5)/5)/d
```

Defintions of rubi rules used

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 210
```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Int[ExpandIntegrand[(a + b*x^2
)^p, x], x] /; FreeQ[{a, b}, x] && IGtQ[p, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3665
```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^
(p_.), x_Symbol] :> With[{ff = FreeFactors[Cos[e + f*x], x]}, Simp[-ff/f
Subst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a + b - b*ff^2*x^2)^p, x], x, Cos[e +
 f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m - 1)/2]
```

Maple [A] (verified)

Time = 3.67 (sec) , antiderivative size = 70, normalized size of antiderivative = 1.23

method	result
derivativedivides	$\frac{a^{2}\cosh(dx+c)+2ab\left(-\frac{2}{3}+\frac{\sinh(dx+c)^{2}}{3}\right)\cosh(dx+c)+b^{2}\left(\frac{8}{15}+\frac{\sinh(dx+c)^{4}}{5}-\frac{4\sinh(dx+c)^{2}}{15}\right)\cosh(dx+c)}{d}$
default	$\frac{a^{2}\cosh(dx+c)+2ab\left(-\frac{2}{3}+\frac{\sinh(dx+c)^{2}}{3}\right)\cosh(dx+c)+b^{2}\left(\frac{8}{15}+\frac{\sinh(dx+c)^{4}}{5}-\frac{4\sinh(dx+c)^{2}}{15}\right)\cosh(dx+c)}{d}$
parts	$\frac{b^{2}\left(\frac{8}{15}+\frac{\sinh(dx+c)^{4}}{5}-\frac{4\sinh(dx+c)^{2}}{15}\right)\cosh(dx+c)}{d}+\frac{a^{2}\cosh(dx+c)}{d}+\frac{2ab\left(-\frac{2}{3}+\frac{\sinh(dx+c)^{2}}{3}\right)\cosh(dx+c)}{d}$
parallelrisch	$\frac{(40ab-25b^{2})\cosh(3dx+3c)+3b^{2}\cosh(5dx+5c)+(240a^{2}-360ab+150b^{2})\cosh(dx+c)+240a^{2}-320ab+128b^{2}}{240d}$
risch	$\frac{b^{2}e^{5dx+5c}}{160d}+\frac{e^{3dx+3c}ab}{12d}-\frac{5\,e^{3dx+3c}b^{2}}{96d}+\frac{e^{dx+c}a^{2}}{2d}-\frac{3\,e^{dx+c}ab}{4d}+\frac{5\,e^{dx+c}b^{2}}{16d}+\frac{e^{-dx-c}a^{2}}{2d}-\frac{3\,e^{-dx-c}ab}{4d}+$
orering	$\frac{\frac{259d\cosh(dx+c)\left(a+b\sinh(dx+c)^{2}\right)^{2}}{225}+\frac{1036\sinh(dx+c)^{2}\left(a+b\sinh(dx+c)^{2}\right)bd\cosh(dx+c)}{225}}{d^{2}}-\frac{7\left(d^{3}\cosh(dx+c)\left(a+b\sinh(dx\right.\right.}{}$

input

```
int(sinh(d*x+c)*(a+b*sinh(d*x+c)^2)^2,x,method=_RETURNVERBOSE)
```

output

```
1/d*(a^2*cosh(d*x+c)+2*a*b*(-2/3+1/3*sinh(d*x+c)^2)*cosh(d*x+c)+b^2*(8/15+
1/5*sinh(d*x+c)^4-4/15*sinh(d*x+c)^2)*cosh(d*x+c))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 122 vs. 2(53) = 106.

Time = 0.10 (sec) , antiderivative size = 122, normalized size of antiderivative = 2.14

$$\int \sinh(c+dx)\left(a+b\sinh^{2}(c+dx)\right)^{2}\,dx$$
$$=\frac{3\,b^{2}\cosh(dx+c)^{5}+15\,b^{2}\cosh(dx+c)\sinh(dx+c)^{4}+5\,(8\,ab-5\,b^{2})\cosh(dx+c)^{3}+15\,(2\,b^{2}\cosh(dx}{240\,d}$$

input

```
integrate(sinh(d*x+c)*(a+b*sinh(d*x+c)^2)^2,x, algorithm="fricas")
```

output
```
1/240*(3*b^2*cosh(d*x + c)^5 + 15*b^2*cosh(d*x + c)*sinh(d*x + c)^4 + 5*(8
*a*b - 5*b^2)*cosh(d*x + c)^3 + 15*(2*b^2*cosh(d*x + c)^3 + (8*a*b - 5*b^2
)*cosh(d*x + c))*sinh(d*x + c)^2 + 30*(8*a^2 - 12*a*b + 5*b^2)*cosh(d*x +
c))/d
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 128 vs. $2(49) = 98$.

Time = 0.26 (sec) , antiderivative size = 128, normalized size of antiderivative = 2.25

$$\int \sinh(c+dx)\left(a+b\sinh^2(c+dx)\right)^2\,dx$$
$$=\begin{cases}\frac{a^2\cosh(c+dx)}{d}+\frac{2ab\sinh^2(c+dx)\cosh(c+dx)}{d}-\frac{4ab\cosh^3(c+dx)}{3d}+\frac{b^2\sinh^4(c+dx)\cosh(c+dx)}{d}-\frac{4b^2\sinh^2(c+dx)\cosh^3(c+dx)}{3d} \\ x\left(a+b\sinh^2(c)\right)^2\sinh(c)\end{cases}$$

input
```
integrate(sinh(d*x+c)*(a+b*sinh(d*x+c)**2)**2,x)
```

output
```
Piecewise((a**2*cosh(c + d*x)/d + 2*a*b*sinh(c + d*x)**2*cosh(c + d*x)/d -
 4*a*b*cosh(c + d*x)**3/(3*d) + b**2*sinh(c + d*x)**4*cosh(c + d*x)/d - 4*
b**2*sinh(c + d*x)**2*cosh(c + d*x)**3/(3*d) + 8*b**2*cosh(c + d*x)**5/(15
*d), Ne(d, 0)), (x*(a + b*sinh(c)**2)**2*sinh(c), True))
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 157 vs. $2(53) = 106$.

Time = 0.03 (sec) , antiderivative size = 157, normalized size of antiderivative = 2.75

$$\int \sinh(c+dx)\left(a+b\sinh^2(c+dx)\right)^2\,dx$$
$$=\frac{1}{480}b^2\left(\frac{3\,e^{(5\,dx+5\,c)}}{d}-\frac{25\,e^{(3\,dx+3\,c)}}{d}+\frac{150\,e^{(dx+c)}}{d}+\frac{150\,e^{(-dx-c)}}{d}-\frac{25\,e^{(-3\,dx-3\,c)}}{d}+\frac{3\,e^{(-5\,dx-5\,c)}}{d}\right)$$
$$+\frac{1}{12}ab\left(\frac{e^{(3\,dx+3\,c)}}{d}-\frac{9\,e^{(dx+c)}}{d}-\frac{9\,e^{(-dx-c)}}{d}+\frac{e^{(-3\,dx-3\,c)}}{d}\right)+\frac{a^2\cosh(dx+c)}{d}$$

input
```
integrate(sinh(d*x+c)*(a+b*sinh(d*x+c)^2)^2,x, algorithm="maxima")
```

output
```
1/480*b^2*(3*e^(5*d*x + 5*c)/d - 25*e^(3*d*x + 3*c)/d + 150*e^(d*x + c)/d
+ 150*e^(-d*x - c)/d - 25*e^(-3*d*x - 3*c)/d + 3*e^(-5*d*x - 5*c)/d) + 1/1
2*a*b*(e^(3*d*x + 3*c)/d - 9*e^(d*x + c)/d - 9*e^(-d*x - c)/d + e^(-3*d*x
- 3*c)/d) + a^2*cosh(d*x + c)/d
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 138 vs. $2(53) = 106$.

Time = 0.13 (sec) , antiderivative size = 138, normalized size of antiderivative = 2.42

$$\int \sinh(c+dx)\left(a+b\sinh^2(c+dx)\right)^2\,dx = \frac{b^2e^{(5\,dx+5\,c)}}{160\,d} + \frac{b^2e^{(-5\,dx-5\,c)}}{160\,d} + \frac{(8\,ab-5\,b^2)e^{(3\,dx+3\,c)}}{96\,d} + \frac{(8\,a^2-12\,ab+5\,b^2)e^{(dx+c)}}{16\,d} + \frac{(8\,a^2-12\,ab+5\,b^2)e^{(-dx-c)}}{16\,d} + \frac{(8\,ab-5\,b^2)e^{(-3\,dx-3\,c)}}{96\,d}$$

input
```
integrate(sinh(d*x+c)*(a+b*sinh(d*x+c)^2)^2,x, algorithm="giac")
```

output
```
1/160*b^2*e^(5*d*x + 5*c)/d + 1/160*b^2*e^(-5*d*x - 5*c)/d + 1/96*(8*a*b -
 5*b^2)*e^(3*d*x + 3*c)/d + 1/16*(8*a^2 - 12*a*b + 5*b^2)*e^(d*x + c)/d +
1/16*(8*a^2 - 12*a*b + 5*b^2)*e^(-d*x - c)/d + 1/96*(8*a*b - 5*b^2)*e^(-3*
d*x - 3*c)/d
```

Mupad [B] (verification not implemented)

Time = 1.68 (sec) , antiderivative size = 76, normalized size of antiderivative = 1.33

$$\int \sinh(c+dx)\left(a+b\sinh^2(c+dx)\right)^2\,dx$$
$$=\frac{15\,a^2\cosh(c+d\,x)+10\,a\,b\cosh(c+d\,x)^3-30\,a\,b\cosh(c+d\,x)+3\,b^2\cosh(c+d\,x)^5-10\,b^2\cosh(c+d}{15\,d}$$

input
```
int(sinh(c + d*x)*(a + b*sinh(c + d*x)^2)^2,x)
```

output
```
(15*a^2*cosh(c + d*x) + 15*b^2*cosh(c + d*x) - 10*b^2*cosh(c + d*x)^3 + 3*
b^2*cosh(c + d*x)^5 - 30*a*b*cosh(c + d*x) + 10*a*b*cosh(c + d*x)^3)/(15*d
)
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 184, normalized size of antiderivative = 3.23

$$\int \sinh(c+dx)\left(a+b\sinh^2(c+dx)\right)^2\,dx$$
$$=\frac{3e^{10dx+10c}b^2+40e^{8dx+8c}ab-25e^{8dx+8c}b^2+240e^{6dx+6c}a^2-360e^{6dx+6c}ab+150e^{6dx+6c}b^2+240e^{4dx+4c}a^2-}{480e^{5dx+5c}d}$$

input
```
int(sinh(d*x+c)*(a+b*sinh(d*x+c)^2)^2,x)
```

output
```
(3*e**(10*c + 10*d*x)*b**2 + 40*e**(8*c + 8*d*x)*a*b - 25*e**(8*c + 8*d*x)
*b**2 + 240*e**(6*c + 6*d*x)*a**2 - 360*e**(6*c + 6*d*x)*a*b + 150*e**(6*c
 + 6*d*x)*b**2 + 240*e**(4*c + 4*d*x)*a**2 - 360*e**(4*c + 4*d*x)*a*b + 15
0*e**(4*c + 4*d*x)*b**2 + 40*e**(2*c + 2*d*x)*a*b - 25*e**(2*c + 2*d*x)*b*
*2 + 3*b**2)/(480*e**(5*c + 5*d*x)*d)
```

3.14 $\int \left(a+b\sinh^2(c+dx)\right)^2\,dx$

Optimal result

Integrand size = 14, antiderivative size = 72

$$\int \left(a+b\sinh^2(c+dx)\right)^2\,dx=\frac{1}{8}\left(8a^2-8ab+3b^2\right)x+\frac{(8a-3b)b\cosh(c+dx)\sinh(c+dx)}{8d}+\frac{b^2\cosh(c+dx)\sinh^3(c+dx)}{4d}$$

output
```
1/8*(8*a^2-8*a*b+3*b^2)*x+1/8*(8*a-3*b)*b*cosh(d*x+c)*sinh(d*x+c)/d+1/4*b^
2*cosh(d*x+c)*sinh(d*x+c)^3/d
```

Mathematica [A] (verified)

Time = 0.21 (sec) , antiderivative size = 60, normalized size of antiderivative = 0.83

$$\int \left(a+b\sinh^2(c+dx)\right)^2\,dx=\frac{4\left(8a^2-8ab+3b^2\right)(c+dx)+8(2a-b)b\sinh(2(c+dx))+b^2\sinh(4(c+dx))}{32d}$$

input
```
Integrate[(a + b*Sinh[c + d*x]^2)^2,x]
```

output
```
(4*(8*a^2 - 8*a*b + 3*b^2)*(c + d*x) + 8*(2*a - b)*b*Sinh[2*(c + d*x)] + b
^2*Sinh[4*(c + d*x)])/(32*d)
```

Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 72, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {3042, 3658}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(a+b\sinh^2(c+dx)\right)^2 dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \left(a-b\sin(ic+idx)^2\right)^2 dx$$

$$\Big\downarrow \text{ 3658}$$

$$\frac{1}{8}x\left(8a^2-8ab+3b^2\right)+\frac{b(8a-3b)\sinh(c+dx)\cosh(c+dx)}{8d}+\frac{b^2\sinh^3(c+dx)\cosh(c+dx)}{4d}$$

input
```
Int[(a + b*Sinh[c + d*x]^2)^2,x]
```

output
```
((8*a^2 - 8*a*b + 3*b^2)*x)/8 + ((8*a - 3*b)*b*Cosh[c + d*x]*Sinh[c + d*x]
)/(8*d) + (b^2*Cosh[c + d*x]*Sinh[c + d*x]^3)/(4*d)
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3658
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^2, x_Symbol] :> Simp[(8*a^2 +
8*a*b + 3*b^2)*(x/8), x] + (-Simp[b^2*Cos[e + f*x]*(Sin[e + f*x]^3/(4*f)),
x] - Simp[b*(8*a + 3*b)*Cos[e + f*x]*(Sin[e + f*x]/(8*f)), x]) /; FreeQ[{a,
 b, e, f}, x]
```

Maple [A] (verified)

Time = 1.50 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.79

method	result
parallelrisch	$\frac{(16ab-8b^2)\sinh(2dx+2c)+b^2\sinh(4dx+4c)+32(a^2-ab+\frac{3}{8}b^2)xd}{32d}$
parts	$a^2x+\frac{b^2\left(\left(\frac{\sinh(dx+c)^3}{4}-\frac{3\sinh(dx+c)}{8}\right)\cosh(dx+c)+\frac{3dx}{8}+\frac{3c}{8}\right)}{d}+\frac{2ab\left(\frac{\cosh(dx+c)\sinh(dx+c)}{2}-\frac{dx}{2}-\frac{c}{2}\right)}{d}$
derivativedivides	$\frac{b^2\left(\left(\frac{\sinh(dx+c)^3}{4}-\frac{3\sinh(dx+c)}{8}\right)\cosh(dx+c)+\frac{3dx}{8}+\frac{3c}{8}\right)+2ab\left(\frac{\cosh(dx+c)\sinh(dx+c)}{2}-\frac{dx}{2}-\frac{c}{2}\right)+a^2(dx+c)}{d}$
default	$\frac{b^2\left(\left(\frac{\sinh(dx+c)^3}{4}-\frac{3\sinh(dx+c)}{8}\right)\cosh(dx+c)+\frac{3dx}{8}+\frac{3c}{8}\right)+2ab\left(\frac{\cosh(dx+c)\sinh(dx+c)}{2}-\frac{dx}{2}-\frac{c}{2}\right)+a^2(dx+c)}{d}$
risch	$a^2x-abx+\frac{3b^2x}{8}+\frac{e^{4dx+4c}b^2}{64d}+\frac{e^{2dx+2c}ab}{4d}-\frac{e^{2dx+2c}b^2}{8d}-\frac{e^{-2dx-2c}ab}{4d}+\frac{e^{-2dx-2c}b^2}{8d}-\frac{e^{-4dx-4c}b^2}{64d}$
orering	$x\left(a+b\sinh\left(dx+c\right)^2\right)^2+\frac{5\left(a+b\sinh(dx+c)^2\right)b\sinh(dx+c)\cosh(dx+c)}{4d}-\frac{5x\left(8b^2\sinh(dx+c)^2d^2\cosh(d\right.}{}$

input
```
int((a+b*sinh(d*x+c)^2)^2,x,method=_RETURNVERBOSE)
```

output
```
1/32*((16*a*b-8*b^2)*sinh(2*d*x+2*c)+b^2*sinh(4*d*x+4*c)+32*(a^2-a*b+3/8*b
^2)*x*d)/d
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 80, normalized size of antiderivative = 1.11

$$\int \left(a+b\sinh^2(c+dx)\right)^2\,dx$$
$$=\frac{b^2\cosh\left(dx+c\right)\sinh\left(dx+c\right)^3+\left(8\,a^2-8\,ab+3\,b^2\right)dx+\left(b^2\cosh\left(dx+c\right)^3+4\left(2\,ab-b^2\right)\cosh\left(dx+c\right)\right.}{8\,d}$$

input
```
integrate((a+b*sinh(d*x+c)^2)^2,x, algorithm="fricas")
```

output
```
1/8*(b^2*cosh(d*x + c)*sinh(d*x + c)^3 + (8*a^2 - 8*a*b + 3*b^2)*d*x + (b^
2*cosh(d*x + c)^3 + 4*(2*a*b - b^2)*cosh(d*x + c))*sinh(d*x + c))/d
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 168 vs. $2(60) = 120$.

Time = 0.20 (sec) , antiderivative size = 168, normalized size of antiderivative = 2.33

$$\int \left(a+b\sinh^2(c+dx)\right)^2\,dx$$
$$= \begin{cases} a^2x + abx\sinh^2{(c+dx)} - abx\cosh^2{(c+dx)} + \frac{ab\sinh{(c+dx)}\cosh{(c+dx)}}{d} + \frac{3b^2x\sinh^4{(c+dx)}}{8} - \frac{3b^2x\sinh^2{(c+dx)}\,c}{4} \\ x\left(a+b\sinh^2{(c)}\right)^2 \end{cases}$$

input
```
integrate((a+b*sinh(d*x+c)**2)**2,x)
```

output
```
Piecewise((a**2*x + a*b*x*sinh(c + d*x)**2 - a*b*x*cosh(c + d*x)**2 + a*b*
sinh(c + d*x)*cosh(c + d*x)/d + 3*b**2*x*sinh(c + d*x)**4/8 - 3*b**2*x*sin
h(c + d*x)**2*cosh(c + d*x)**2/4 + 3*b**2*x*cosh(c + d*x)**4/8 + 5*b**2*si
nh(c + d*x)**3*cosh(c + d*x)/(8*d) - 3*b**2*sinh(c + d*x)*cosh(c + d*x)**3
/(8*d), Ne(d, 0)), (x*(a + b*sinh(c)**2)**2, True))
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 105, normalized size of antiderivative = 1.46

$$\int \left(a+b\sinh^2(c+dx)\right)^2\,dx$$
$$= \frac{1}{64}b^2\left(24\,x + \frac{e^{(4\,dx+4\,c)}}{d} - \frac{8\,e^{(2\,dx+2\,c)}}{d} + \frac{8\,e^{(-2\,dx-2\,c)}}{d} - \frac{e^{(-4\,dx-4\,c)}}{d}\right)$$
$$- \frac{1}{4}ab\left(4\,x - \frac{e^{(2\,dx+2\,c)}}{d} + \frac{e^{(-2\,dx-2\,c)}}{d}\right) + a^2x$$

input
```
integrate((a+b*sinh(d*x+c)^2)^2,x, algorithm="maxima")
```

output
```
1/64*b^2*(24*x + e^(4*d*x + 4*c)/d - 8*e^(2*d*x + 2*c)/d + 8*e^(-2*d*x - 2
*c)/d - e^(-4*d*x - 4*c)/d) - 1/4*a*b*(4*x - e^(2*d*x + 2*c)/d + e^(-2*d*x
 - 2*c)/d) + a^2*x
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 101, normalized size of antiderivative = 1.40

$$\int \left(a+b\sinh^2(c+dx)\right)^2\,dx=\frac{1}{8}\left(8\,a^2-8\,ab+3\,b^2\right)x+\frac{b^2e^{(4\,dx+4\,c)}}{64\,d}-\frac{b^2e^{(-4\,dx-4\,c)}}{64\,d}+\frac{(2\,ab-b^2)e^{(2\,dx+2\,c)}}{8\,d}-\frac{(2\,ab-b^2)e^{(-2\,dx-2\,c)}}{8\,d}$$

input
```
integrate((a+b*sinh(d*x+c)^2)^2,x, algorithm="giac")
```

output
```
1/8*(8*a^2 - 8*a*b + 3*b^2)*x + 1/64*b^2*e^(4*d*x + 4*c)/d - 1/64*b^2*e^(-
4*d*x - 4*c)/d + 1/8*(2*a*b - b^2)*e^(2*d*x + 2*c)/d - 1/8*(2*a*b - b^2)*e
^(-2*d*x - 2*c)/d
```

Mupad [B] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 67, normalized size of antiderivative = 0.93

$$\int \left(a+b\sinh^2(c+dx)\right)^2\,dx=a^2\,x+\frac{3\,b^2\,x}{8}-a\,b\,x-\frac{b^2\sinh(2\,c+2\,d\,x)}{4\,d}+\frac{b^2\sinh(4\,c+4\,d\,x)}{32\,d}+\frac{a\,b\sinh(2\,c+2\,d\,x)}{2\,d}$$

input
```
int((a + b*sinh(c + d*x)^2)^2,x)
```

output
```
a^2*x + (3*b^2*x)/8 - a*b*x - (b^2*sinh(2*c + 2*d*x))/(4*d) + (b^2*sinh(4*
c + 4*d*x))/(32*d) + (a*b*sinh(2*c + 2*d*x))/(2*d)
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 145, normalized size of antiderivative = 2.01

$$\int \left(a+b\sinh^2(c+dx)\right)^2\,dx$$
$$=\frac{e^{8dx+8c}b^2+16e^{6dx+6c}ab-8e^{6dx+6c}b^2+64e^{4dx+4c}a^2dx-64e^{4dx+4c}abdx+24e^{4dx+4c}b^2dx-16e^{2dx+2c}ab+}{64e^{4dx+4c}d}$$

input
```
int((a+b*sinh(d*x+c)^2)^2,x)
```

output
```
(e**(8*c + 8*d*x)*b**2 + 16*e**(6*c + 6*d*x)*a*b - 8*e**(6*c + 6*d*x)*b**2
 + 64*e**(4*c + 4*d*x)*a**2*d*x - 64*e**(4*c + 4*d*x)*a*b*d*x + 24*e**(4*c
 + 4*d*x)*b**2*d*x - 16*e**(2*c + 2*d*x)*a*b + 8*e**(2*c + 2*d*x)*b**2 - b
**2)/(64*e**(4*c + 4*d*x)*d)
```

3.15 $\int \operatorname{csch}(c+dx)\left(a+b\sinh^2(c+dx)\right)^2\,dx$

Optimal result

Integrand size = 21, antiderivative size = 52

$$\int \operatorname{csch}(c+dx)\left(a+b\sinh^2(c+dx)\right)^2\,dx=-\frac{a^2\operatorname{arctanh}(\cosh(c+dx))}{d}+\frac{(2a-b)b\cosh(c+dx)}{d}+\frac{b^2\cosh^3(c+dx)}{3d}$$

output
```
-a^2*arctanh(cosh(d*x+c))/d+(2*a-b)*b*cosh(d*x+c)/d+1/3*b^2*cosh(d*x+c)^3/
d
```

Mathematica [A] (verified)

Time = 0.09 (sec) , antiderivative size = 76, normalized size of antiderivative = 1.46

$$\int \operatorname{csch}(c+dx)\left(a+b\sinh^2(c+dx)\right)^2\,dx=-\frac{a^2\operatorname{arctanh}(\cosh(c+dx))}{d}+\frac{2ab\cosh(c)\cosh(dx)}{d}-\frac{3b^2\cosh(c+dx)}{4d}+\frac{b^2\cosh(3(c+dx))}{12d}+\frac{2ab\sinh(c)\sinh(dx)}{d}$$

input
```
Integrate[Csch[c + d*x]*(a + b*Sinh[c + d*x]^2)^2,x]
```

output
```
-((a^2*ArcTanh[Cosh[c + d*x]])/d) + (2*a*b*Cosh[c]*Cosh[d*x])/d - (3*b^2*C
osh[c + d*x])/(4*d) + (b^2*Cosh[3*(c + d*x)])/(12*d) + (2*a*b*Sinh[c]*Sinh
[d*x])/d
```

Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.92, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.238$, Rules used = {3042, 26, 3665, 300, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \operatorname{csch}(c+dx) \left(a+b\sinh^2(c+dx)\right)^2 dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \frac{i\left(a-b\sin(ic+idx)^2\right)^2}{\sin(ic+idx)}dx$$

$$\Big\downarrow \text{ 26}$$

$$i\int \frac{\left(a-b\sin(ic+idx)^2\right)^2}{\sin(ic+idx)}dx$$

$$\Big\downarrow \text{ 3665}$$

$$-\frac{\int \frac{\left(b\cosh^2(c+dx)+a-b\right)^2}{1-\cosh^2(c+dx)}d\cosh(c+dx)}{d}$$

$$\Big\downarrow \text{ 300}$$

$$-\frac{\int \left(\frac{a^2}{1-\cosh^2(c+dx)}-b^2\cosh^2(c+dx)-(2a-b)b\right)d\cosh(c+dx)}{d}$$

$$\Big\downarrow \text{ 2009}$$

$$-\frac{a^2\text{arctanh}(\cosh(c+dx))-b(2a-b)\cosh(c+dx)-\frac{1}{3}b^2\cosh^3(c+dx)}{d}$$

input
```
Int[Csch[c + d*x]*(a + b*Sinh[c + d*x]^2)^2,x]
```

output
```
-((a^2*ArcTanh[Cosh[c + d*x]] - (2*a - b)*b*Cosh[c + d*x] - (b^2*Cosh[c +
d*x]^3)/3)/d)
```

Defintions of rubi rules used

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 300
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_), x_Symbol] :> Int
[PolynomialDivide[(a + b*x^2)^p, (c + d*x^2)^(-q), x], x] /; FreeQ[{a, b, c
, d}, x] && NeQ[b*c - a*d, 0] && IGtQ[p, 0] && ILtQ[q, 0] && GeQ[p, -q]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3665
```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^
(p_.), x_Symbol] :> With[{ff = FreeFactors[Cos[e + f*x], x]}, Simp[-ff/f
Subst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a + b - b*ff^2*x^2)^p, x], x, Cos[e +
 f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m - 1)/2]
```

Maple [A] (verified)

Time = 0.44 (sec) , antiderivative size = 50, normalized size of antiderivative = 0.96

method	result
derivativedivides	$\frac{-2a^2 \operatorname{arctanh}(e^{dx+c})+2ab\cosh(dx+c)+b^2\left(-\frac{2}{3}+\frac{\sinh(dx+c)^2}{3}\right)\cosh(dx+c)}{d}$
default	$\frac{-2a^2 \operatorname{arctanh}(e^{dx+c})+2ab\cosh(dx+c)+b^2\left(-\frac{2}{3}+\frac{\sinh(dx+c)^2}{3}\right)\cosh(dx+c)}{d}$
parallelrisch	$\frac{a^2\ln\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+2b\left(\frac{b\cosh(3dx+3c)}{24}+\left(a-\frac{3b}{8}\right)\cosh(dx+c)+a-\frac{b}{3}\right)}{d}$
risch	$\frac{e^{3dx+3c}b^2}{24d}+\frac{e^{dx+c}ab}{d}-\frac{3\,e^{dx+c}b^2}{8d}+\frac{e^{-dx-c}ab}{d}-\frac{3\,e^{-dx-c}b^2}{8d}+\frac{e^{-3dx-3c}b^2}{24d}+\frac{a^2\ln(e^{dx+c}-1)}{d}-\frac{a^2\ln(e^{c}}{}$

input
```
int(csch(d*x+c)*(a+b*sinh(d*x+c)^2)^2,x,method=_RETURNVERBOSE)
```

output
```
1/d*(-2*a^2*arctanh(exp(d*x+c))+2*a*b*cosh(d*x+c)+b^2*(-2/3+1/3*sinh(d*x+c
)^2)*cosh(d*x+c))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 492 vs. 2(50) = 100.

Time = 0.10 (sec) , antiderivative size = 492, normalized size of antiderivative = 9.46

$$\int \operatorname{csch}(c+dx)\left(a+b\sinh^2(c+dx)\right)^2\,dx$$
$$=\frac{b^2\cosh(dx+c)^6+6\,b^2\cosh(dx+c)\sinh(dx+c)^5+b^2\sinh(dx+c)^6+3\,(8\,ab-3\,b^2)\cosh(dx+c)^4+}{}$$

input
```
integrate(csch(d*x+c)*(a+b*sinh(d*x+c)^2)^2,x, algorithm="fricas")
```

output
```
1/24*(b^2*cosh(d*x + c)^6 + 6*b^2*cosh(d*x + c)*sinh(d*x + c)^5 + b^2*sinh
(d*x + c)^6 + 3*(8*a*b - 3*b^2)*cosh(d*x + c)^4 + 3*(5*b^2*cosh(d*x + c)^2
 + 8*a*b - 3*b^2)*sinh(d*x + c)^4 + 4*(5*b^2*cosh(d*x + c)^3 + 3*(8*a*b -
3*b^2)*cosh(d*x + c))*sinh(d*x + c)^3 + 3*(8*a*b - 3*b^2)*cosh(d*x + c)^2
+ 3*(5*b^2*cosh(d*x + c)^4 + 6*(8*a*b - 3*b^2)*cosh(d*x + c)^2 + 8*a*b - 3
*b^2)*sinh(d*x + c)^2 + b^2 - 24*(a^2*cosh(d*x + c)^3 + 3*a^2*cosh(d*x + c
)^2*sinh(d*x + c) + 3*a^2*cosh(d*x + c)*sinh(d*x + c)^2 + a^2*sinh(d*x + c
)^3)*log(cosh(d*x + c) + sinh(d*x + c) + 1) + 24*(a^2*cosh(d*x + c)^3 + 3*
a^2*cosh(d*x + c)^2*sinh(d*x + c) + 3*a^2*cosh(d*x + c)*sinh(d*x + c)^2 +
a^2*sinh(d*x + c)^3)*log(cosh(d*x + c) + sinh(d*x + c) - 1) + 6*(b^2*cosh(
d*x + c)^5 + 2*(8*a*b - 3*b^2)*cosh(d*x + c)^3 + (8*a*b - 3*b^2)*cosh(d*x
+ c))*sinh(d*x + c))/(d*cosh(d*x + c)^3 + 3*d*cosh(d*x + c)^2*sinh(d*x + c
) + 3*d*cosh(d*x + c)*sinh(d*x + c)^2 + d*sinh(d*x + c)^3)
```

Sympy [F]

$$\int \operatorname{csch}(c+dx)\left(a+b\sinh^2(c+dx)\right)^2\,dx=\int\left(a+b\sinh^2{(c+dx)}\right)^2\operatorname{csch}{(c+dx)}\,dx$$

input
```
integrate(csch(d*x+c)*(a+b*sinh(d*x+c)**2)**2,x)
```

output
```
Integral((a + b*sinh(c + d*x)**2)**2*csch(c + d*x), x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 102 vs. $2(50) = 100$.

Time = 0.04 (sec) , antiderivative size = 102, normalized size of antiderivative = 1.96

$$\begin{aligned}&\int \operatorname{csch}(c+dx)\left(a+b\sinh^2(c+dx)\right)^2\,dx\\&=\frac{1}{24}b^2\left(\frac{e^{(3\,dx+3\,c)}}{d}-\frac{9\,e^{(dx+c)}}{d}-\frac{9\,e^{(-dx-c)}}{d}+\frac{e^{(-3\,dx-3\,c)}}{d}\right)\\&\quad+ab\left(\frac{e^{(dx+c)}}{d}+\frac{e^{(-dx-c)}}{d}\right)+\frac{a^2\log\left(\tanh\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)}{d}\end{aligned}$$

input
```
integrate(csch(d*x+c)*(a+b*sinh(d*x+c)^2)^2,x, algorithm="maxima")
```

output
```
1/24*b^2*(e^(3*d*x + 3*c)/d - 9*e^(d*x + c)/d - 9*e^(-d*x - c)/d + e^(-3*d
*x - 3*c)/d) + a*b*(e^(d*x + c)/d + e^(-d*x - c)/d) + a^2*log(tanh(1/2*d*x
 + 1/2*c))/d
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 110 vs. $2(50) = 100$.

Time = 0.14 (sec) , antiderivative size = 110, normalized size of antiderivative = 2.12

$$\int \operatorname{csch}(c+dx)\left(a+b\sinh^2(c+dx)\right)^2\,dx = \frac{b^2e^{(3\,dx+3\,c)}+24\,abe^{(dx+c)}-9\,b^2e^{(dx+c)}-24\,a^2\log\left(e^{(dx+c)}+1\right)+24\,a^2\log\left(\left|e^{(dx+c)}-1\right|\right)+\left(24\,abe^{(2\,dx+}\right.}{24\,d}$$

input
```
integrate(csch(d*x+c)*(a+b*sinh(d*x+c)^2)^2,x, algorithm="giac")
```

output
```
1/24*(b^2*e^(3*d*x + 3*c) + 24*a*b*e^(d*x + c) - 9*b^2*e^(d*x + c) - 24*a^
2*log(e^(d*x + c) + 1) + 24*a^2*log(abs(e^(d*x + c) - 1)) + (24*a*b*e^(2*d
*x + 2*c) - 9*b^2*e^(2*d*x + 2*c) + b^2)*e^(-3*d*x - 3*c))/d
```

Mupad [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 116, normalized size of antiderivative = 2.23

$$\int \operatorname{csch}(c+dx)\left(a+b\sinh^2(c+dx)\right)^2\,dx = \frac{b^2\,\mathrm{e}^{-3\,c-3\,d\,x}}{24\,d}-\frac{2\,\mathrm{atan}\left(\frac{a^2\,\mathrm{e}^{d\,x}\,\mathrm{e}^{c}\,\sqrt{-d^2}}{d\,\sqrt{a^4}}\right)\,\sqrt{a^4}}{\sqrt{-d^2}}+\frac{b^2\,\mathrm{e}^{3\,c+3\,d\,x}}{24\,d}+\frac{b\,\mathrm{e}^{-c-d\,x}\,(8\,a-3\,b)}{8\,d}+\frac{b\,\mathrm{e}^{c+d\,x}\,(8\,a-3\,b)}{8\,d}$$

input
```
int((a + b*sinh(c + d*x)^2)^2/sinh(c + d*x),x)
```

output
```
(b^2*exp(- 3*c - 3*d*x))/(24*d) - (2*atan((a^2*exp(d*x)*exp(c)*(-d^2)^(1/2
))/(d*(a^4)^(1/2)))*(a^4)^(1/2))/(-d^2)^(1/2) + (b^2*exp(3*c + 3*d*x))/(24
*d) + (b*exp(- c - d*x)*(8*a - 3*b))/(8*d) + (b*exp(c + d*x)*(8*a - 3*b))/
(8*d)
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 143, normalized size of antiderivative = 2.75

$$\int \operatorname{csch}(c+dx)\left(a+b\sinh^2(c+dx)\right)^2\,dx$$
$$=\frac{e^{6dx+6c}b^2+24e^{4dx+4c}ab-9e^{4dx+4c}b^2+24e^{3dx+3c}\log\left(e^{dx+c}-1\right)a^2-24e^{3dx+3c}\log\left(e^{dx+c}+1\right)a^2+24e^{2dx\cdots}}{24e^{3dx+3c}d}$$

input
```
int(csch(d*x+c)*(a+b*sinh(d*x+c)^2)^2,x)
```

output
```
(e**(6*c + 6*d*x)*b**2 + 24*e**(4*c + 4*d*x)*a*b - 9*e**(4*c + 4*d*x)*b**2
 + 24*e**(3*c + 3*d*x)*log(e**(c + d*x) - 1)*a**2 - 24*e**(3*c + 3*d*x)*lo
g(e**(c + d*x) + 1)*a**2 + 24*e**(2*c + 2*d*x)*a*b - 9*e**(2*c + 2*d*x)*b*
*2 + b**2)/(24*e**(3*c + 3*d*x)*d)
```

3.16 $\int \operatorname{csch}^2(c+dx)\left(a+b\sinh^2(c+dx)\right)^2\,dx$

Optimal result

Integrand size = 23, antiderivative size = 50

$$\int \operatorname{csch}^2(c+dx)\left(a+b\sinh^2(c+dx)\right)^2\,dx=\frac{1}{2}(4a-b)bx-\frac{a^2\coth(c+dx)}{d}+\frac{b^2\cosh(c+dx)\sinh(c+dx)}{2d}$$

output
```
1/2*(4*a-b)*b*x-a^2*coth(d*x+c)/d+1/2*b^2*cosh(d*x+c)*sinh(d*x+c)/d
```

Mathematica [A] (verified)

Time = 0.21 (sec) , antiderivative size = 56, normalized size of antiderivative = 1.12

$$\int \operatorname{csch}^2(c+dx)\left(a+b\sinh^2(c+dx)\right)^2\,dx=2abx+\frac{b^2(-c-dx)}{2d}-\frac{a^2\coth(c+dx)}{d}+\frac{b^2\sinh(2(c+dx))}{4d}$$

input
```
Integrate[Csch[c + d*x]^2*(a + b*Sinh[c + d*x]^2)^2,x]
```

output
```
2*a*b*x + (b^2*(-c - d*x))/(2*d) - (a^2*Coth[c + d*x])/d + (b^2*Sinh[2*(c
+ d*x)])/(4*d)
```

Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 82, normalized size of antiderivative = 1.64, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.261$, Rules used = {3042, 25, 3666, 365, 298, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \operatorname{csch}^2(c+dx)\left(a+b\sinh^2(c+dx)\right)^2\,dx$$

$$\Big\downarrow \ 3042$$

$$\int -\frac{\left(a-b\sin(ic+idx)^2\right)^2}{\sin(ic+idx)^2}dx$$

$$\Big\downarrow \ 25$$

$$-\int \frac{\left(a-b\sin(ic+idx)^2\right)^2}{\sin(ic+idx)^2}dx$$

$$\Big\downarrow \ 3666$$

$$\frac{\int \frac{\coth^2(c+dx)\left(a-(a-b)\tanh^2(c+dx)\right)^2}{\left(1-\tanh^2(c+dx)\right)^2}d\tanh(c+dx)}{d}$$

$$\Big\downarrow \ 365$$

$$\frac{\int \frac{(a-b)^2\tanh^2(c+dx)+a(a+2b)}{\left(1-\tanh^2(c+dx)\right)^2}d\tanh(c+dx)-\frac{a^2\coth(c+dx)}{1-\tanh^2(c+dx)}}{d}$$

$$\Big\downarrow \ 298$$

$$\frac{\frac{1}{2}b(4a-b)\int \frac{1}{1-\tanh^2(c+dx)}d\tanh(c+dx)+\frac{(2a^2+b^2)\tanh(c+dx)}{2(1-\tanh^2(c+dx))}-\frac{a^2\coth(c+dx)}{1-\tanh^2(c+dx)}}{d}$$

$$\Big\downarrow \ 219$$

$$\frac{\frac{(2a^2+b^2)\tanh(c+dx)}{2(1-\tanh^2(c+dx))}-\frac{a^2\coth(c+dx)}{1-\tanh^2(c+dx)}+\frac{1}{2}b(4a-b)\text{arctanh}(\tanh(c+dx))}{d}$$

input
```
Int[Csch[c + d*x]^2*(a + b*Sinh[c + d*x]^2)^2,x]
```

output
```
(((4*a - b)*b*ArcTanh[Tanh[c + d*x]])/2 - (a^2*Coth[c + d*x])/(1 - Tanh[c
+ d*x]^2) + ((2*a^2 + b^2)*Tanh[c + d*x])/(2*(1 - Tanh[c + d*x]^2)))/d
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 298
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2), x_Symbol] :> Simp[(-(
b*c - a*d))*x*((a + b*x^2)^(p + 1)/(2*a*b*(p + 1))), x] - Simp[(a*d - b*c*(
2*p + 3))/(2*a*b*(p + 1))   Int[(a + b*x^2)^(p + 1), x], x] /; FreeQ[{a, b,
 c, d, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/2 + p, 0])
```

rule 365
```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^2, x
_Symbol] :> Simp[c^2*(e*x)^(m + 1)*((a + b*x^2)^(p + 1)/(a*e*(m + 1))), x]
- Simp[1/(a*e^2*(m + 1))   Int[(e*x)^(m + 2)*(a + b*x^2)^p*Simp[2*b*c^2*(p
+ 1) + c*(b*c - 2*a*d)*(m + 1) - a*d^2*(m + 1)*x^2, x], x], x] /; FreeQ[{a,
 b, c, d, e, p}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3666
```
Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(
p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Simp[ff^(m + 1
)/f   Subst[Int[x^m*((a + (a + b)*ff^2*x^2)^p/(1 + ff^2*x^2)^(m/2 + p + 1))
, x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f}, x] && IntegerQ[m/2] &
& IntegerQ[p]
```

Maple [A] (verified)

Time = 0.39 (sec) , antiderivative size = 52, normalized size of antiderivative = 1.04

method	result	size
derivativedivides	$\frac{-\coth(dx+c)a^2+2ab(dx+c)+b^2\left(\frac{\cosh(dx+c)\sinh(dx+c)}{2}-\frac{dx}{2}-\frac{c}{2}\right)}{d}$	52
default	$\frac{-\coth(dx+c)a^2+2ab(dx+c)+b^2\left(\frac{\cosh(dx+c)\sinh(dx+c)}{2}-\frac{dx}{2}-\frac{c}{2}\right)}{d}$	52
risch	$2abx-\frac{b^2x}{2}+\frac{e^{2dx+2c}b^2}{8d}-\frac{e^{-2dx-2c}b^2}{8d}-\frac{2a^2}{d(e^{2dx+2c}-1)}$	68
parallelrisch	$\frac{8abdx-2b^2dx+b^2\sinh(2dx+2c)+2\,\mathrm{sech}\left(\frac{dx}{2}+\frac{c}{2}\right)\mathrm{csch}\left(\frac{dx}{2}+\frac{c}{2}\right)a^2-4\coth\left(\frac{dx}{2}+\frac{c}{2}\right)a^2}{4d}$	70

input
```
int(csch(d*x+c)^2*(a+b*sinh(d*x+c)^2)^2,x,method=_RETURNVERBOSE)
```

output
```
1/d*(-coth(d*x+c)*a^2+2*a*b*(d*x+c)+b^2*(1/2*cosh(d*x+c)*sinh(d*x+c)-1/2*d
*x-1/2*c))
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 89, normalized size of antiderivative = 1.78

$$\int \operatorname{csch}^2(c+dx)\left(a+b\sinh^2(c+dx)\right)^2\,dx$$
$$=\frac{b^2\cosh(dx+c)^3+3\,b^2\cosh(dx+c)\sinh(dx+c)^2-(8\,a^2+b^2)\cosh(dx+c)+4\,((4\,ab-b^2)dx+2\,a^2)}{8\,d\sinh(dx+c)}$$

input
```
integrate(csch(d*x+c)^2*(a+b*sinh(d*x+c)^2)^2,x, algorithm="fricas")
```

output
```
1/8*(b^2*cosh(d*x + c)^3 + 3*b^2*cosh(d*x + c)*sinh(d*x + c)^2 - (8*a^2 +
b^2)*cosh(d*x + c) + 4*((4*a*b - b^2)*d*x + 2*a^2)*sinh(d*x + c))/(d*sinh(
d*x + c))
```

Sympy [F(-1)]

Timed out.

$$\int \operatorname{csch}^2(c+dx)\left(a+b\sinh^2(c+dx)\right)^2\,dx = \text{Timed out}$$

input
```
integrate(csch(d*x+c)**2*(a+b*sinh(d*x+c)**2)**2,x)
```

output
```
Timed out
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 63, normalized size of antiderivative = 1.26

$$\int \operatorname{csch}^2(c+dx)\left(a+b\sinh^2(c+dx)\right)^2\,dx = -\frac{1}{8}\,b^2\left(4\,x-\frac{e^{(2\,dx+2\,c)}}{d}+\frac{e^{(-2\,dx-2\,c)}}{d}\right) + 2\,abx + \frac{2\,a^2}{d(e^{(-2\,dx-2\,c)}-1)}$$

input
```
integrate(csch(d*x+c)^2*(a+b*sinh(d*x+c)^2)^2,x, algorithm="maxima")
```

output
```
-1/8*b^2*(4*x - e^(2*d*x + 2*c)/d + e^(-2*d*x - 2*c)/d) + 2*a*b*x + 2*a^2/
(d*(e^(-2*d*x - 2*c) - 1))
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 135 vs. 2(46) = 92.

Time = 0.15 (sec) , antiderivative size = 135, normalized size of antiderivative = 2.70

$$\int \operatorname{csch}^2(c+dx)\left(a+b\sinh^2(c+dx)\right)^2\,dx = \frac{b^2e^{(2\,dx+2\,c)}+4\,(4\,ab-b^2)(dx+c)-\frac{4\,abe^{(4\,dx+4\,c)}-b^2e^{(4\,dx+4\,c)}+16\,a^2e^{(2\,dx+2\,c)}-4\,abe^{(2\,dx+2\,c)}+2\,b^2e^{(2\,dx+2\,c)}-b^2}{e^{(4\,dx+4\,c)}-e^{(2\,dx+2\,c)}}}{8\,d}$$

input
```
integrate(csch(d*x+c)^2*(a+b*sinh(d*x+c)^2)^2,x, algorithm="giac")
```

output
```
1/8*(b^2*e^(2*d*x + 2*c) + 4*(4*a*b - b^2)*(d*x + c) - (4*a*b*e^(4*d*x + 4
*c) - b^2*e^(4*d*x + 4*c) + 16*a^2*e^(2*d*x + 2*c) - 4*a*b*e^(2*d*x + 2*c)
 + 2*b^2*e^(2*d*x + 2*c) - b^2)/(e^(4*d*x + 4*c) - e^(2*d*x + 2*c)))/d
```

Mupad [B] (verification not implemented)

Time = 1.77 (sec) , antiderivative size = 67, normalized size of antiderivative = 1.34

$$\int \operatorname{csch}^2(c+dx)\left(a+b\sinh^2(c+dx)\right)^2\,dx = \frac{b\,x\,(4\,a-b)}{2}-\frac{2\,a^2}{d\,\left(\mathrm{e}^{2\,c+2\,d\,x}-1\right)}-\frac{b^2\,\mathrm{e}^{-2\,c-2\,d\,x}}{8\,d}+\frac{b^2\,\mathrm{e}^{2\,c+2\,d\,x}}{8\,d}$$

input
```
int((a + b*sinh(c + d*x)^2)^2/sinh(c + d*x)^2,x)
```

output
```
(b*x*(4*a - b))/2 - (2*a^2)/(d*(exp(2*c + 2*d*x) - 1)) - (b^2*exp(- 2*c -
2*d*x))/(8*d) + (b^2*exp(2*c + 2*d*x))/(8*d)
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 145, normalized size of antiderivative = 2.90

$$\int \operatorname{csch}^2(c+dx)\left(a+b\sinh^2(c+dx)\right)^2\,dx = \frac{e^{6dx+6c}b^2-16e^{4dx+4c}a^2+16e^{4dx+4c}abdx-4e^{4dx+4c}b^2dx-2e^{4dx+4c}b^2-16e^{2dx+2c}abdx+4e^{2dx+2c}b^2dx+}{8e^{2dx+2c}d\left(e^{2dx+2c}-1\right)}$$

input
```
int(csch(d*x+c)^2*(a+b*sinh(d*x+c)^2)^2,x)
```

output
```
(e**(6*c + 6*d*x)*b**2 - 16*e**(4*c + 4*d*x)*a**2 + 16*e**(4*c + 4*d*x)*a*
b*d*x - 4*e**(4*c + 4*d*x)*b**2*d*x - 2*e**(4*c + 4*d*x)*b**2 - 16*e**(2*c
 + 2*d*x)*a*b*d*x + 4*e**(2*c + 2*d*x)*b**2*d*x + b**2)/(8*e**(2*c + 2*d*x
)*d*(e**(2*c + 2*d*x) - 1))
```

3.17 $\int \operatorname{csch}^3(c+dx)\left(a+b\sinh^2(c+dx)\right)^2\,dx$

Optimal result

Integrand size = 23, antiderivative size = 56

$$\int \operatorname{csch}^3(c+dx)\left(a+b\sinh^2(c+dx)\right)^2\,dx=\frac{a(a-4b)\operatorname{arctanh}(\cosh(c+dx))}{2d}+\frac{b^2\cosh(c+dx)}{d}-\frac{a^2\coth(c+dx)\operatorname{csch}(c+dx)}{2d}$$

output
```
1/2*a*(a-4*b)*arctanh(cosh(d*x+c))/d+b^2*cosh(d*x+c)/d-1/2*a^2*coth(d*x+c)
*csch(d*x+c)/d
```

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 127 vs. $2(56) = 112$.

Time = 0.11 (sec) , antiderivative size = 127, normalized size of antiderivative = 2.27

$$\int \operatorname{csch}^3(c+dx)\left(a+b\sinh^2(c+dx)\right)^2\,dx = -\frac{2ab\operatorname{arctanh}(\cosh(c+dx))}{d} + \frac{b^2\cosh(c)\cosh(dx)}{d} - \frac{a^2\operatorname{csch}^2\left(\frac{1}{2}(c+dx)\right)}{8d} + \frac{a^2\log\left(\cosh\left(\frac{1}{2}(c+dx)\right)\right)}{2d} - \frac{a^2\log\left(\sinh\left(\frac{1}{2}(c+dx)\right)\right)}{2d} - \frac{a^2\operatorname{sech}^2\left(\frac{1}{2}(c+dx)\right)}{8d} + \frac{b^2\sinh(c)\sinh(dx)}{d}$$

input
```
Integrate[Csch[c + d*x]^3*(a + b*Sinh[c + d*x]^2)^2,x]
```

output
```
(-2*a*b*ArcTanh[Cosh[c + d*x]])/d + (b^2*Cosh[c]*Cosh[d*x])/d - (a^2*Csch[
(c + d*x)/2]^2)/(8*d) + (a^2*Log[Cosh[(c + d*x)/2]])/(2*d) - (a^2*Log[Sinh
[(c + d*x)/2]])/(2*d) - (a^2*Sech[(c + d*x)/2]^2)/(8*d) + (b^2*Sinh[c]*Sin
h[d*x])/d
```

Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 59, normalized size of antiderivative = 1.05, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.217$, Rules used = {3042, 26, 3665, 300, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \operatorname{csch}^3(c+dx)\left(a+b\sinh^2(c+dx)\right)^2\,dx$$

$$\Big\downarrow \ 3042$$

$$\int -\frac{i\left(a-b\sin(ic+idx)^2\right)^2}{\sin(ic+idx)^3}dx$$

$$\Big\downarrow \ 26$$

$$-i\int \frac{\left(a-b\sin(ic+idx)^2\right)^2}{\sin(ic+idx)^3}dx$$

$$\Big\downarrow \ 3665$$

$$\frac{\int \frac{\left(b\cosh^2(c+dx)+a-b\right)^2}{\left(1-\cosh^2(c+dx)\right)^2}d\cosh(c+dx)}{d}$$

$$\Big\downarrow \ 300$$

$$\frac{\int \left(b^2+\frac{2ab\cosh^2(c+dx)+a(a-2b)}{\left(1-\cosh^2(c+dx)\right)^2}\right)d\cosh(c+dx)}{d}$$

$$\Big\downarrow \ 2009$$

$$\frac{\frac{a^2\cosh(c+dx)}{2\left(1-\cosh^2(c+dx)\right)}+\frac{1}{2}a(a-4b)\text{arctanh}(\cosh(c+dx))+b^2\cosh(c+dx)}{d}$$

input
```
Int[Csch[c + d*x]^3*(a + b*Sinh[c + d*x]^2)^2,x]
```

output
```
((a*(a - 4*b)*ArcTanh[Cosh[c + d*x]])/2 + b^2*Cosh[c + d*x] + (a^2*Cosh[c
+ d*x])/(2*(1 - Cosh[c + d*x]^2)))/d
```

Defintions of rubi rules used

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 300
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_), x_Symbol] :> Int
[PolynomialDivide[(a + b*x^2)^p, (c + d*x^2)^(-q), x], x] /; FreeQ[{a, b, c
, d}, x] && NeQ[b*c - a*d, 0] && IGtQ[p, 0] && ILtQ[q, 0] && GeQ[p, -q]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3665
```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^
(p_.), x_Symbol] :> With[{ff = FreeFactors[Cos[e + f*x], x]}, Simp[-ff/f
Subst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a + b - b*ff^2*x^2)^p, x], x, Cos[e +
 f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m - 1)/2]
```

Maple [A] (verified)

Time = 0.42 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.95

method	result
derivativedivides	$\frac{a^2\left(-\frac{\operatorname{csch}(dx+c)\coth(dx+c)}{2}+\operatorname{arctanh}\left(e^{dx+c}\right)\right)-4ab\operatorname{arctanh}\left(e^{dx+c}\right)+b^2\cosh(dx+c)}{d}$
default	$\frac{a^2\left(-\frac{\operatorname{csch}(dx+c)\coth(dx+c)}{2}+\operatorname{arctanh}\left(e^{dx+c}\right)\right)-4ab\operatorname{arctanh}\left(e^{dx+c}\right)+b^2\cosh(dx+c)}{d}$
parallelrisch	$\frac{-4a(a-4b)\ln\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+a^2\left(\operatorname{sech}\left(\frac{dx}{2}+\frac{c}{2}\right)^2-3\right)\operatorname{csch}\left(\frac{dx}{2}+\frac{c}{2}\right)^2+\coth\left(\frac{dx}{2}+\frac{c}{2}\right)^2a^2+8b^2(1+\cosh(dx+c))}{8d}$
risch	$\frac{e^{dx+c}b^2}{2d}+\frac{e^{-dx-c}b^2}{2d}-\frac{a^2e^{dx+c}\left(e^{2dx+2c}+1\right)}{d\left(e^{2dx+2c}-1\right)^2}-\frac{a^2\ln\left(e^{dx+c}-1\right)}{2d}+\frac{2a\ln\left(e^{dx+c}-1\right)b}{d}+\frac{a^2\ln\left(e^{dx+c}+1\right)}{2d}-$ 2

input
```
int(csch(d*x+c)^3*(a+b*sinh(d*x+c)^2)^2,x,method=_RETURNVERBOSE)
```

output
```
1/d*(a^2*(-1/2*csch(d*x+c)*coth(d*x+c)+arctanh(exp(d*x+c)))-4*a*b*arctanh(
exp(d*x+c))+b^2*cosh(d*x+c))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 902 vs. $2(52) = 104$.

Time = 0.12 (sec) , antiderivative size = 902, normalized size of antiderivative = 16.11

$$\int \operatorname{csch}^3(c+dx)\left(a+b\sinh^2(c+dx)\right)^2\,dx = \text{Too large to display}$$

input
```
integrate(csch(d*x+c)^3*(a+b*sinh(d*x+c)^2)^2,x, algorithm="fricas")
```

output
```
1/2*(b^2*cosh(d*x + c)^6 + 6*b^2*cosh(d*x + c)*sinh(d*x + c)^5 + b^2*sinh(
d*x + c)^6 - (2*a^2 + b^2)*cosh(d*x + c)^4 + (15*b^2*cosh(d*x + c)^2 - 2*a
^2 - b^2)*sinh(d*x + c)^4 + 4*(5*b^2*cosh(d*x + c)^3 - (2*a^2 + b^2)*cosh(
d*x + c))*sinh(d*x + c)^3 - (2*a^2 + b^2)*cosh(d*x + c)^2 + (15*b^2*cosh(d
*x + c)^4 - 6*(2*a^2 + b^2)*cosh(d*x + c)^2 - 2*a^2 - b^2)*sinh(d*x + c)^2
 + b^2 + ((a^2 - 4*a*b)*cosh(d*x + c)^5 + 5*(a^2 - 4*a*b)*cosh(d*x + c)*si
nh(d*x + c)^4 + (a^2 - 4*a*b)*sinh(d*x + c)^5 - 2*(a^2 - 4*a*b)*cosh(d*x +
 c)^3 + 2*(5*(a^2 - 4*a*b)*cosh(d*x + c)^2 - a^2 + 4*a*b)*sinh(d*x + c)^3
+ 2*(5*(a^2 - 4*a*b)*cosh(d*x + c)^3 - 3*(a^2 - 4*a*b)*cosh(d*x + c))*sinh
(d*x + c)^2 + (a^2 - 4*a*b)*cosh(d*x + c) + (5*(a^2 - 4*a*b)*cosh(d*x + c)
^4 - 6*(a^2 - 4*a*b)*cosh(d*x + c)^2 + a^2 - 4*a*b)*sinh(d*x + c))*log(cos
h(d*x + c) + sinh(d*x + c) + 1) - ((a^2 - 4*a*b)*cosh(d*x + c)^5 + 5*(a^2
- 4*a*b)*cosh(d*x + c)*sinh(d*x + c)^4 + (a^2 - 4*a*b)*sinh(d*x + c)^5 - 2
*(a^2 - 4*a*b)*cosh(d*x + c)^3 + 2*(5*(a^2 - 4*a*b)*cosh(d*x + c)^2 - a^2
+ 4*a*b)*sinh(d*x + c)^3 + 2*(5*(a^2 - 4*a*b)*cosh(d*x + c)^3 - 3*(a^2 - 4
*a*b)*cosh(d*x + c))*sinh(d*x + c)^2 + (a^2 - 4*a*b)*cosh(d*x + c) + (5*(a
^2 - 4*a*b)*cosh(d*x + c)^4 - 6*(a^2 - 4*a*b)*cosh(d*x + c)^2 + a^2 - 4*a*
b)*sinh(d*x + c))*log(cosh(d*x + c) + sinh(d*x + c) - 1) + 2*(3*b^2*cosh(d
*x + c)^5 - 2*(2*a^2 + b^2)*cosh(d*x + c)^3 - (2*a^2 + b^2)*cosh(d*x + c))
*sinh(d*x + c))/(d*cosh(d*x + c)^5 + 5*d*cosh(d*x + c)*sinh(d*x + c)^4 ...
```

Sympy [F(-1)]

Timed out.

$$\int \operatorname{csch}^3(c+dx) \left(a+b\sinh^2(c+dx)\right)^2 \, dx = \text{Timed out}$$

input
```
integrate(csch(d*x+c)**3*(a+b*sinh(d*x+c)**2)**2,x)
```

output
```
Timed out
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 157 vs. $2(52) = 104$.

Time = 0.05 (sec) , antiderivative size = 157, normalized size of antiderivative = 2.80

$$\int \operatorname{csch}^3(c+dx)\left(a+b\sinh^2(c+dx)\right)^2\,dx$$
$$=\frac{1}{2}b^2\left(\frac{e^{(dx+c)}}{d}+\frac{e^{(-dx-c)}}{d}\right)+\frac{1}{2}a^2\left(\frac{\log\left(e^{(-dx-c)}+1\right)}{d}-\frac{\log\left(e^{(-dx-c)}-1\right)}{d}+\frac{2\left(e^{(-dx-c)}+e^{(-3\,dx-3\,c)}\right)}{d(2\,e^{(-2\,dx-2\,c)}-e^{(-4\,dx-4\,c)}-1)}\right)-2\,ab\left(\frac{\log\left(e^{(-dx-c)}+1\right)}{d}-\frac{\log\left(e^{(-dx-c)}-1\right)}{d}\right)$$

input
```
integrate(csch(d*x+c)^3*(a+b*sinh(d*x+c)^2)^2,x, algorithm="maxima")
```

output
```
1/2*b^2*(e^(d*x + c)/d + e^(-d*x - c)/d) + 1/2*a^2*(log(e^(-d*x - c) + 1)/
d - log(e^(-d*x - c) - 1)/d + 2*(e^(-d*x - c) + e^(-3*d*x - 3*c))/(d*(2*e^
(-2*d*x - 2*c) - e^(-4*d*x - 4*c) - 1))) - 2*a*b*(log(e^(-d*x - c) + 1)/d
- log(e^(-d*x - c) - 1)/d)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 125 vs. $2(52) = 104$.

Time = 0.15 (sec) , antiderivative size = 125, normalized size of antiderivative = 2.23

$$\int \operatorname{csch}^3(c+dx)\left(a+b\sinh^2(c+dx)\right)^2\,dx$$
$$=\frac{2\,b^2\left(e^{(dx+c)}+e^{(-dx-c)}\right)-\frac{4\,a^2\left(e^{(dx+c)}+e^{(-dx-c)}\right)}{\left(e^{(dx+c)}+e^{(-dx-c)}\right)^2-4}+(a^2-4\,ab)\log\left(e^{(dx+c)}+e^{(-dx-c)}+2\right)-(a^2-4\,ab)\log\left(\right.}{4\,d}$$

input
```
integrate(csch(d*x+c)^3*(a+b*sinh(d*x+c)^2)^2,x, algorithm="giac")
```

output
```
1/4*(2*b^2*(e^(d*x + c) + e^(-d*x - c)) - 4*a^2*(e^(d*x + c) + e^(-d*x - c
))/((e^(d*x + c) + e^(-d*x - c))^2 - 4) + (a^2 - 4*a*b)*log(e^(d*x + c) +
e^(-d*x - c) + 2) - (a^2 - 4*a*b)*log(e^(d*x + c) + e^(-d*x - c) - 2))/d
```

Mupad [B] (verification not implemented)

Time = 1.75 (sec) , antiderivative size = 179, normalized size of antiderivative = 3.20

$$\int \operatorname{csch}^3(c+dx)\left(a+b\sinh^2(c+dx)\right)^2 dx = \frac{\operatorname{atan}\left(\frac{e^{dx}\,e^{c}\left(a^2\sqrt{-d^2}-4\,a\,b\sqrt{-d^2}\right)}{d\sqrt{a^4-8\,a^3\,b+16\,a^2\,b^2}}\right)\sqrt{a^4-8\,a^3\,b+16\,a^2\,b^2}}{\sqrt{-d^2}}+\frac{b^2\,e^{c+d\,x}}{2\,d} + \frac{b^2\,e^{-c-d\,x}}{2\,d}-\frac{a^2\,e^{c+d\,x}}{d\,\left(e^{2\,c+2\,d\,x}-1\right)}-\frac{2\,a^2\,e^{c+d\,x}}{d\,\left(e^{4\,c+4\,d\,x}-2\,e^{2\,c+2\,d\,x}+1\right)}$$

input
```
int((a + b*sinh(c + d*x)^2)^2/sinh(c + d*x)^3,x)
```

output
```
(atan((exp(d*x)*exp(c)*(a^2*(-d^2)^(1/2) - 4*a*b*(-d^2)^(1/2)))/(d*(a^4 -
8*a^3*b + 16*a^2*b^2)^(1/2)))*(a^4 - 8*a^3*b + 16*a^2*b^2)^(1/2))/(-d^2)^(
1/2) + (b^2*exp(c + d*x))/(2*d) + (b^2*exp(- c - d*x))/(2*d) - (a^2*exp(c
+ d*x))/(d*(exp(2*c + 2*d*x) - 1)) - (2*a^2*exp(c + d*x))/(d*(exp(4*c + 4*
d*x) - 2*exp(2*c + 2*d*x) + 1))
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 398, normalized size of antiderivative = 7.11

$$\int \operatorname{csch}^3(c+dx)\left(a+b\sinh^2(c+dx)\right)^2 dx = \frac{e^{6dx+6c}b^2-e^{5dx+5c}\log\left(e^{dx+c}-1\right)a^2+4e^{5dx+5c}\log\left(e^{dx+c}-1\right)ab+e^{5dx+5c}\log\left(e^{dx+c}+1\right)a^2-4e^{5dx+5c}\log}{}$$

input
```
int(csch(d*x+c)^3*(a+b*sinh(d*x+c)^2)^2,x)
```

output
```
(e**(6*c + 6*d*x)*b**2 - e**(5*c + 5*d*x)*log(e**(c + d*x) - 1)*a**2 + 4*e
**(5*c + 5*d*x)*log(e**(c + d*x) - 1)*a*b + e**(5*c + 5*d*x)*log(e**(c + d
*x) + 1)*a**2 - 4*e**(5*c + 5*d*x)*log(e**(c + d*x) + 1)*a*b - 2*e**(4*c +
 4*d*x)*a**2 - e**(4*c + 4*d*x)*b**2 + 2*e**(3*c + 3*d*x)*log(e**(c + d*x)
 - 1)*a**2 - 8*e**(3*c + 3*d*x)*log(e**(c + d*x) - 1)*a*b - 2*e**(3*c + 3*
d*x)*log(e**(c + d*x) + 1)*a**2 + 8*e**(3*c + 3*d*x)*log(e**(c + d*x) + 1)
*a*b - 2*e**(2*c + 2*d*x)*a**2 - e**(2*c + 2*d*x)*b**2 - e**(c + d*x)*log(
e**(c + d*x) - 1)*a**2 + 4*e**(c + d*x)*log(e**(c + d*x) - 1)*a*b + e**(c
+ d*x)*log(e**(c + d*x) + 1)*a**2 - 4*e**(c + d*x)*log(e**(c + d*x) + 1)*a
*b + b**2)/(2*e**(c + d*x)*d*(e**(4*c + 4*d*x) - 2*e**(2*c + 2*d*x) + 1))
```

3.18 $\int \operatorname{csch}^4(c+dx) \left(a+b\sinh^2(c+dx)\right)^2 \, dx$

Optimal result

Integrand size = 23, antiderivative size = 40

$$\int \operatorname{csch}^4(c+dx) \left(a+b\sinh^2(c+dx)\right)^2 \, dx = b^2x+\frac{a(a-2b)\coth(c+dx)}{d} -\frac{a^2\coth^3(c+dx)}{3d}$$

output
```
b^2*x+a*(a-2*b)*coth(d*x+c)/d-1/3*a^2*coth(d*x+c)^3/d
```

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 85 vs. 2(40) = 80.

Time = 0.64 (sec) , antiderivative size = 85, normalized size of antiderivative = 2.12

$$\int \operatorname{csch}^4(c+dx) \left(a+b\sinh^2(c+dx)\right)^2 \, dx = \frac{4\left(b+a\operatorname{csch}^2(c+dx)\right)^2 \left(3b^2(c+dx)-a\coth(c+dx)\left(-2a+6b+a\operatorname{csch}^2(c+dx)\right)\right)\sinh^4(c+dx)}{3d(2a-b+b\cosh(2(c+dx)))^2}$$

input
```
Integrate[Csch[c + d*x]^4*(a + b*Sinh[c + d*x]^2)^2,x]
```

output
```
(4*(b + a*Csch[c + d*x]^2)^2*(3*b^2*(c + d*x) - a*Coth[c + d*x]*(-2*a + 6*
b + a*Csch[c + d*x]^2))*Sinh[c + d*x]^4)/(3*d*(2*a - b + b*Cosh[2*(c + d*x
)])^2)
```

Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 44, normalized size of antiderivative = 1.10, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.174$, Rules used = {3042, 3666, 364, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \operatorname{csch}^4(c+dx)\left(a+b\sinh^2(c+dx)\right)^2 dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\left(a-b\sin(ic+idx)^2\right)^2}{\sin(ic+idx)^4}dx$$

$$\Big\downarrow \text{3666}$$

$$\frac{\int \frac{\coth^4(c+dx)\left(a-(a-b)\tanh^2(c+dx)\right)^2}{1-\tanh^2(c+dx)}d\tanh(c+dx)}{d}$$

$$\Big\downarrow \text{364}$$

$$\frac{\int \left(a^2\coth^4(c+dx)-a(a-2b)\coth^2(c+dx)-\frac{b^2}{\tanh^2(c+dx)-1}\right)d\tanh(c+dx)}{d}$$

$$\Big\downarrow \text{2009}$$

$$\frac{-\frac{1}{3}a^2\coth^3(c+dx)+a(a-2b)\coth(c+dx)+b^2\text{arctanh}(\tanh(c+dx))}{d}$$

input
```
Int[Csch[c + d*x]^4*(a + b*Sinh[c + d*x]^2)^2,x]
```

output
```
(b^2*ArcTanh[Tanh[c + d*x]] + a*(a - 2*b)*Coth[c + d*x] - (a^2*Coth[c + d*
x]^3)/3)/d
```

Defintions of rubi rules used

rule 364
```
Int[(((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_))/((c_) + (d_.)*(x_)^2),
x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*((a + b*x^2)^p/(c + d*x^2)), x], x
] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b*c - a*d, 0] && IGtQ[p, 0] && (In
tegerQ[m] || IGtQ[2*(m + 1), 0] ||  !RationalQ[m])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3666
```
Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(
p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Simp[ff^(m + 1
)/f   Subst[Int[x^m*((a + (a + b)*ff^2*x^2)^p/(1 + ff^2*x^2)^(m/2 + p + 1))
, x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f}, x] && IntegerQ[m/2] &
& IntegerQ[p]
```

Maple [A] (verified)

Time = 0.34 (sec) , antiderivative size = 47, normalized size of antiderivative = 1.18

method	result	size
derivativedivides	$\frac{a^{2}\left(\frac{2}{3}-\frac{\operatorname{csch}\left(dx+c\right)^{2}}{3}\right)\coth\left(dx+c\right)-2\coth\left(dx+c\right)ab+b^{2}\left(dx+c\right)}{d}$	47
default	$\frac{a^{2}\left(\frac{2}{3}-\frac{\operatorname{csch}\left(dx+c\right)^{2}}{3}\right)\coth\left(dx+c\right)-2\coth\left(dx+c\right)ab+b^{2}\left(dx+c\right)}{d}$	47
risch	$b^{2}x-\frac{4a\left(3b\,{\mathrm e}^{4dx+4c}+3\,{\mathrm e}^{2dx+2c}a-6\,{\mathrm e}^{2dx+2c}b-a+3b\right)}{3d\left({\mathrm e}^{2dx+2c}-1\right)^{3}}$	69
parallelrisch	$\frac{-\coth\left(\frac{dx}{2}+\frac{c}{2}\right)^{3}a^{2}+\left(9a^{2}-24ab\right)\coth\left(\frac{dx}{2}+\frac{c}{2}\right)-\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^{3}a^{2}+\left(9a^{2}-24ab\right)\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)+24b^{2}dx}{24d}$	86

input
```
int(csch(d*x+c)^4*(a+b*sinh(d*x+c)^2)^2,x,method=_RETURNVERBOSE)
```

output
```
1/d*(a^2*(2/3-1/3*csch(d*x+c)^2)*coth(d*x+c)-2*coth(d*x+c)*a*b+b^2*(d*x+c)
)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 174 vs. $2(38) = 76$.

Time = 0.10 (sec) , antiderivative size = 174, normalized size of antiderivative = 4.35

$$\int \operatorname{csch}^4(c+dx)\left(a+b\sinh^2(c+dx)\right)^2\,dx = \frac{2\left(a^2-3\,ab\right)\cosh\left(dx+c\right)^3+6\left(a^2-3\,ab\right)\cosh\left(dx+c\right)\sinh\left(dx+c\right)^2+\left(3\,b^2dx-2\,a^2+6\,ab\right)\sinh\left(dx\right.}{3\left(d\sinh\left(dx+c\right)^3+3\left(d\right.\right.}$$

input
```
integrate(csch(d*x+c)^4*(a+b*sinh(d*x+c)^2)^2,x, algorithm="fricas")
```

output
```
1/3*(2*(a^2 - 3*a*b)*cosh(d*x + c)^3 + 6*(a^2 - 3*a*b)*cosh(d*x + c)*sinh(
d*x + c)^2 + (3*b^2*d*x - 2*a^2 + 6*a*b)*sinh(d*x + c)^3 - 6*(a^2 - a*b)*c
osh(d*x + c) - 3*(3*b^2*d*x - (3*b^2*d*x - 2*a^2 + 6*a*b)*cosh(d*x + c)^2
- 2*a^2 + 6*a*b)*sinh(d*x + c))/(d*sinh(d*x + c)^3 + 3*(d*cosh(d*x + c)^2
- d)*sinh(d*x + c))
```

Sympy [F(-1)]

Timed out.

$$\int \operatorname{csch}^4(c+dx)\left(a+b\sinh^2(c+dx)\right)^2\,dx = \text{Timed out}$$

input
```
integrate(csch(d*x+c)**4*(a+b*sinh(d*x+c)**2)**2,x)
```

output
```
Timed out
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 121 vs. $2(38) = 76$.

Time = 0.04 (sec) , antiderivative size = 121, normalized size of antiderivative = 3.02

$$\int \operatorname{csch}^4(c+dx)\left(a+b\sinh^2(c+dx)\right)^2 dx = b^2x + \frac{4}{3}a^2\left(\frac{3e^{(-2dx-2c)}}{d(3e^{(-2dx-2c)}-3e^{(-4dx-4c)}+e^{(-6dx-6c)}-1)} - \frac{1}{d(3e^{(-2dx-2c)}-3e^{(-4dx-4c)}+e^{(-6dx-6c)}-1)}\right) + \frac{4ab}{d(e^{(-2dx-2c)}-1)}$$

input
```
integrate(csch(d*x+c)^4*(a+b*sinh(d*x+c)^2)^2,x, algorithm="maxima")
```

output
```
b^2*x + 4/3*a^2*(3*e^(-2*d*x - 2*c)/(d*(3*e^(-2*d*x - 2*c) - 3*e^(-4*d*x -
 4*c) + e^(-6*d*x - 6*c) - 1)) - 1/(d*(3*e^(-2*d*x - 2*c) - 3*e^(-4*d*x -
4*c) + e^(-6*d*x - 6*c) - 1))) + 4*a*b/(d*(e^(-2*d*x - 2*c) - 1))
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 81 vs. $2(38) = 76$.

Time = 0.14 (sec) , antiderivative size = 81, normalized size of antiderivative = 2.02

$$\int \operatorname{csch}^4(c+dx)\left(a+b\sinh^2(c+dx)\right)^2 dx = \frac{3(dx+c)b^2 - \frac{4\left(3abe^{(4dx+4c)}+3a^2e^{(2dx+2c)}-6abe^{(2dx+2c)}-a^2+3ab\right)}{\left(e^{(2dx+2c)}-1\right)^3}}{3d}$$

input
```
integrate(csch(d*x+c)^4*(a+b*sinh(d*x+c)^2)^2,x, algorithm="giac")
```

output
```
1/3*(3*(d*x + c)*b^2 - 4*(3*a*b*e^(4*d*x + 4*c) + 3*a^2*e^(2*d*x + 2*c) -
6*a*b*e^(2*d*x + 2*c) - a^2 + 3*a*b)/(e^(2*d*x + 2*c) - 1)^3)/d
```

Mupad [B] (verification not implemented)

Time = 1.74 (sec) , antiderivative size = 166, normalized size of antiderivative = 4.15

$$\int \operatorname{csch}^4(c+dx)\left(a+b\sinh^2(c+dx)\right)^2\,dx = b^2\,x-\frac{\frac{4\,a\,b}{3\,d}-\frac{8\,{\mathrm{e}}^{2\,c+2\,d\,x}\,\left(a\,b-a^2\right)}{3\,d}+\frac{4\,a\,b\,{\mathrm{e}}^{4\,c+4\,d\,x}}{3\,d}}{3\,{\mathrm{e}}^{2\,c+2\,d\,x}-3\,{\mathrm{e}}^{4\,c+4\,d\,x}+{\mathrm{e}}^{6\,c+6\,d\,x}-1}+\frac{\frac{4\,\left(a\,b-a^2\right)}{3\,d}-\frac{4\,a\,b\,{\mathrm{e}}^{2\,c+2\,d\,x}}{3\,d}}{{\mathrm{e}}^{4\,c+4\,d\,x}-2\,{\mathrm{e}}^{2\,c+2\,d\,x}+1}-\frac{4\,a\,b}{3\,d\,\left({\mathrm{e}}^{2\,c+2\,d\,x}-1\right)}$$

input
```
int((a + b*sinh(c + d*x)^2)^2/sinh(c + d*x)^4,x)
```

output
```
b^2*x - ((4*a*b)/(3*d) - (8*exp(2*c + 2*d*x)*(a*b - a^2))/(3*d) + (4*a*b*e
xp(4*c + 4*d*x))/(3*d))/(3*exp(2*c + 2*d*x) - 3*exp(4*c + 4*d*x) + exp(6*c
 + 6*d*x) - 1) + ((4*(a*b - a^2))/(3*d) - (4*a*b*exp(2*c + 2*d*x))/(3*d))/
(exp(4*c + 4*d*x) - 2*exp(2*c + 2*d*x) + 1) - (4*a*b)/(3*d*(exp(2*c + 2*d*
x) - 1))
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 154, normalized size of antiderivative = 3.85

$$\int \operatorname{csch}^4(c+dx)\left(a+b\sinh^2(c+dx)\right)^2\,dx = \frac{-4e^{6dx+6c}ab+3e^{6dx+6c}b^2dx-9e^{4dx+4c}b^2dx-12e^{2dx+2c}a^2+12e^{2dx+2c}ab+9e^{2dx+2c}b^2dx+4a^2-8ab-3}{3d\left(e^{6dx+6c}-3e^{4dx+4c}+3e^{2dx+2c}-1\right)}$$

input
```
int(csch(d*x+c)^4*(a+b*sinh(d*x+c)^2)^2,x)
```

output
```
( - 4*e**(6*c + 6*d*x)*a*b + 3*e**(6*c + 6*d*x)*b**2*d*x - 9*e**(4*c + 4*d
*x)*b**2*d*x - 12*e**(2*c + 2*d*x)*a**2 + 12*e**(2*c + 2*d*x)*a*b + 9*e**(
2*c + 2*d*x)*b**2*d*x + 4*a**2 - 8*a*b - 3*b**2*d*x)/(3*d*(e**(6*c + 6*d*x
) - 3*e**(4*c + 4*d*x) + 3*e**(2*c + 2*d*x) - 1))
```

3.19 $\int \sinh^4(c+dx)\left(a+b\sinh^2(c+dx)\right)^3\,dx$

Optimal result

Integrand size = 23, antiderivative size = 206

$$\begin{aligned}
&\int \sinh^4(c+dx)\left(a+b\sinh^2(c+dx)\right)^3\,dx \\
&= \frac{3}{256}(4a-3b)\left(8a^2-14ab+7b^2\right)x \\
&\quad - \frac{(160a^3-528a^2b+558ab^2-193b^3)\cosh(c+dx)\sinh(c+dx)}{256d} \\
&\quad + \frac{(32a^3-208a^2b+326ab^2-149b^3)\cosh^3(c+dx)\sinh(c+dx)}{128d} \\
&\quad + \frac{b(80a^2-250ab+171b^2)\cosh^5(c+dx)\sinh(c+dx)}{160d} \\
&\quad + \frac{(30a-41b)b^2\cosh^7(c+dx)\sinh(c+dx)}{80d} + \frac{b^3\cosh^9(c+dx)\sinh(c+dx)}{10d}
\end{aligned}$$

output
```
3/256*(4*a-3*b)*(8*a^2-14*a*b+7*b^2)*x-1/256*(160*a^3-528*a^2*b+558*a*b^2-
193*b^3)*cosh(d*x+c)*sinh(d*x+c)/d+1/128*(32*a^3-208*a^2*b+326*a*b^2-149*b
^3)*cosh(d*x+c)^3*sinh(d*x+c)/d+1/160*b*(80*a^2-250*a*b+171*b^2)*cosh(d*x+
c)^5*sinh(d*x+c)/d+1/80*(30*a-41*b)*b^2*cosh(d*x+c)^7*sinh(d*x+c)/d+1/10*b
^3*cosh(d*x+c)^9*sinh(d*x+c)/d
```

Mathematica [A] (verified)

Time = 1.66 (sec) , antiderivative size = 162, normalized size of antiderivative = 0.79

$$\int \sinh^4(c+dx)\left(a+b\sinh^2(c+dx)\right)^3\,dx$$
$$= \frac{120(4a-3b)\left(8a^2-14ab+7b^2\right)(c+dx)-20\left(128a^3-360a^2b+336ab^2-105b^3\right)\sinh(2(c+dx))+40(}{}$$

input
```
Integrate[Sinh[c + d*x]^4*(a + b*Sinh[c + d*x]^2)^3,x]
```

output
```
(120*(4*a - 3*b)*(8*a^2 - 14*a*b + 7*b^2)*(c + d*x) - 20*(128*a^3 - 360*a^
2*b + 336*a*b^2 - 105*b^3)*Sinh[2*(c + d*x)] + 40*(8*a^3 - 36*a^2*b + 42*a
*b^2 - 15*b^3)*Sinh[4*(c + d*x)] + 10*b*(16*a^2 - 32*a*b + 15*b^2)*Sinh[6*
(c + d*x)] + 5*(6*a - 5*b)*b^2*Sinh[8*(c + d*x)] + 2*b^3*Sinh[10*(c + d*x)
])/(10240*d)
```

Rubi [A] (verified)

Time = 0.53 (sec) , antiderivative size = 308, normalized size of antiderivative = 1.50, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.435$, Rules used = {3042, 3666, 369, 27, 439, 439, 360, 25, 298, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sinh^4(c+dx)\left(a+b\sinh^2(c+dx)\right)^3\,dx$$

$$\Big\downarrow \; 3042$$

$$\int \sin(ic+idx)^4\left(a-b\sin(ic+idx)^2\right)^3dx$$

$$\Big\downarrow \; 3666$$

$$\frac{\int \frac{\tanh^4(c+dx)\left(a-(a-b)\tanh^2(c+dx)\right)^3}{\left(1-\tanh^2(c+dx)\right)^6}d\tanh(c+dx)}{d}$$

$$\Big\downarrow \; 369$$

$$\frac{\frac{\tanh^3(c+dx)\left(a-(a-b)\tanh^2(c+dx)\right)^3}{10\left(1-\tanh^2(c+dx)\right)^5}-\frac{1}{10}\int\frac{3\tanh^2(c+dx)\left(a-3(a-b)\tanh^2(c+dx)\right)\left(a-(a-b)\tanh^2(c+dx)\right)^2}{\left(1-\tanh^2(c+dx)\right)^5}d\tanh(c+dx)}{d}$$

$\downarrow$ 27

$$\frac{\frac{\tanh^3(c+dx)\left(a-(a-b)\tanh^2(c+dx)\right)^3}{10\left(1-\tanh^2(c+dx)\right)^5}-\frac{3}{10}\int\frac{\tanh^2(c+dx)\left(a-3(a-b)\tanh^2(c+dx)\right)\left(a-(a-b)\tanh^2(c+dx)\right)^2}{\left(1-\tanh^2(c+dx)\right)^5}d\tanh(c+dx)}{d}$$

$\downarrow$ 439

$$\frac{\tanh^3(c+dx)\left(a-(a-b)\tanh^2(c+dx)\right)^3}{10\left(1-\tanh^2(c+dx)\right)^5}-\frac{3}{10}\left(\frac{1}{8}\int\frac{\tanh^2(c+dx)\left(a-(a-b)\tanh^2(c+dx)\right)\left(a(14a-9b)-(22a-21b)(a-b)\tanh^2(c+dx)\right)}{\left(1-\tanh^2(c+dx)\right)^4}d\tanh$$

$$d$$

$\downarrow$ 439

$$\frac{\tanh^3(c+dx)\left(a-(a-b)\tanh^2(c+dx)\right)^3}{10\left(1-\tanh^2(c+dx)\right)^5}-\frac{3}{10}\left(\frac{1}{8}\left(\frac{1}{6}\int\frac{\tanh^2(c+dx)\left(3a(14a-9b)(2a-b)-(22a-21b)(6a-5b)(a-b)\tanh^2(c+dx)\right)}{\left(1-\tanh^2(c+dx)\right)^3}d\tanh(c\right.\right.$$

$$d$$

$\downarrow$ 360

$$\frac{\tanh^3(c+dx)\left(a-(a-b)\tanh^2(c+dx)\right)^3}{10\left(1-\tanh^2(c+dx)\right)^5}-\frac{3}{10}\left(\frac{1}{8}\left(\frac{1}{6}\left(-\frac{1}{4}\int-\frac{48a^3-272ba^2+314b^2a-105b^3+4(22a-21b)(6a-5b)(a-b)\tanh^2(c+dx)}{\left(1-\tanh^2(c+dx)\right)^2}d\tan\right.\right.\right.$$

$\downarrow$ 25

$$\frac{\tanh^3(c+dx)\left(a-(a-b)\tanh^2(c+dx)\right)^3}{10\left(1-\tanh^2(c+dx)\right)^5}-\frac{3}{10}\left(\frac{1}{8}\left(\frac{1}{6}\left(\frac{1}{4}\int\frac{48a^3-272ba^2+314b^2a-105b^3+4(22a-21b)(6a-5b)(a-b)\tanh^2(c+dx)}{\left(1-\tanh^2(c+dx)\right)^2}d\tanh(c\right.\right.\right.$$

$\downarrow$ 298

$$\frac{\tanh^3(c+dx)\left(a-(a-b)\tanh^2(c+dx)\right)^3}{10\left(1-\tanh^2(c+dx)\right)^5}-\frac{3}{10}\left(\frac{1}{8}\left(\frac{1}{6}\left(\frac{1}{4}\left(\frac{\left(576a^3-1744a^2b+1678ab^2-525b^3\right)\tanh(c+dx)}{2\left(1-\tanh^2(c+dx)\right)}-\frac{15}{2}(4a-3b)\left(8a^2-14ab\right.\right.\right.\right.\right.$$

$\downarrow$ 219

$$\frac{\tanh^3(c+dx)\left(a-(a-b)\tanh^2(c+dx)\right)^3}{10\left(1-\tanh^2(c+dx)\right)^5}-\frac{3}{10}\left(\frac{1}{8}\left(\frac{1}{6}\left(\frac{1}{4}\left(\frac{\left(576a^3-1744a^2b+1678ab^2-525b^3\right)\tanh(c+dx)}{2\left(1-\tanh^2(c+dx)\right)}-\frac{15}{2}(4a-3b)\left(8a^2-14ab\right.\right.\right.\right.\right.$$

input
```
Int[Sinh[c + d*x]^4*(a + b*Sinh[c + d*x]^2)^3,x]
```

output
```
((Tanh[c + d*x]^3*(a - (a - b)*Tanh[c + d*x]^2)^3)/(10*(1 - Tanh[c + d*x]^
2)^5) - (3*(-1/8*((2*a - 3*b)*Tanh[c + d*x]^3*(a - (a - b)*Tanh[c + d*x]^2
)^2)/(1 - Tanh[c + d*x]^2)^4 + ((b*Tanh[c + d*x]^3*(a*(14*a - 9*b) - (22*a
 - 21*b)*(a - b)*Tanh[c + d*x]^2))/(6*(1 - Tanh[c + d*x]^2)^3) + (-1/4*((4
8*a^3 - 272*a^2*b + 314*a*b^2 - 105*b^3)*Tanh[c + d*x])/(1 - Tanh[c + d*x]
^2)^2 + ((-15*(4*a - 3*b)*(8*a^2 - 14*a*b + 7*b^2)*ArcTanh[Tanh[c + d*x]])
/2 + ((576*a^3 - 1744*a^2*b + 1678*a*b^2 - 525*b^3)*Tanh[c + d*x])/(2*(1 -
 Tanh[c + d*x]^2)))/4)/6)/8))/10)/d
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 298
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2), x_Symbol] :> Simp[(-(
b*c - a*d))*x*((a + b*x^2)^(p + 1)/(2*a*b*(p + 1))), x] - Simp[(a*d - b*c*(
2*p + 3))/(2*a*b*(p + 1))   Int[(a + b*x^2)^(p + 1), x], x] /; FreeQ[{a, b,
 c, d, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/2 + p, 0])
```

rule 360
```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2), x_Symbol] :
> Simp[(-a)^(m/2 - 1)*(b*c - a*d)*x*((a + b*x^2)^(p + 1)/(2*b^(m/2 + 1)*(p
+ 1))), x] + Simp[1/(2*b^(m/2 + 1)*(p + 1))   Int[(a + b*x^2)^(p + 1)*Expan
dToSum[2*b*(p + 1)*x^2*Together[(b^(m/2)*x^(m - 2)*(c + d*x^2) - (-a)^(m/2
- 1)*(b*c - a*d))/(a + b*x^2)] - (-a)^(m/2 - 1)*(b*c - a*d), x], x], x] /;
FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && IGtQ[m/2, 0] &
& (IntegerQ[p] || EqQ[m + 2*p + 1, 0])
```

rule 369
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_
), x_Symbol] :> Simp[e*(e*x)^(m - 1)*(a + b*x^2)^(p + 1)*((c + d*x^2)^q/(2*
b*(p + 1))), x] - Simp[e^2/(2*b*(p + 1))   Int[(e*x)^(m - 2)*(a + b*x^2)^(p
 + 1)*(c + d*x^2)^(q - 1)*Simp[c*(m - 1) + d*(m + 2*q - 1)*x^2, x], x], x]
/; FreeQ[{a, b, c, d, e}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && GtQ[q, 0
] && GtQ[m, 1] && IntBinomialQ[a, b, c, d, e, m, 2, p, q, x]
```

rule 439
```
Int[((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_
.)*((e_) + (f_.)*(x_)^2), x_Symbol] :> Simp[(-(b*e - a*f))*(g*x)^(m + 1)*(a
 + b*x^2)^(p + 1)*((c + d*x^2)^q/(2*a*b*g*(p + 1))), x] + Simp[1/(2*a*b*(p
+ 1))   Int[(g*x)^m*(a + b*x^2)^(p + 1)*(c + d*x^2)^(q - 1)*Simp[c*(2*b*e*(
p + 1) + (b*e - a*f)*(m + 1)) + d*(2*b*e*(p + 1) + (b*e - a*f)*(m + 2*q + 1
))*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, m}, x] && LtQ[p, -1] && G
tQ[q, 0] &&  !(EqQ[q, 1] && SimplerQ[b*c - a*d, b*e - a*f])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3666
```
Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(
p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Simp[ff^(m + 1
)/f   Subst[Int[x^m*((a + (a + b)*ff^2*x^2)^p/(1 + ff^2*x^2)^(m/2 + p + 1))
, x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f}, x] && IntegerQ[m/2] &
& IntegerQ[p]
```

Maple [A] (verified)

Time = 209.42 (sec) , antiderivative size = 153, normalized size of antiderivative = 0.74

method	result
parallelrisch	$\frac{(-2560a^3+7200a^2b-6720b^2a+2100b^3)\sinh(2dx+2c)+(320a^3-1440a^2b+1680b^2a-600b^3)\sinh(4dx+4c)+160\left(a-\frac{3b}{4}\right.}{10240d}$
derivativedivides	$\frac{a^3\left(\left(\frac{\sinh(dx+c)^3}{4}-\frac{3\sinh(dx+c)}{8}\right)\cosh(dx+c)+\frac{3dx}{8}+\frac{3c}{8}\right)+3a^2b\left(\left(\frac{\sinh(dx+c)^5}{6}-\frac{5\sinh(dx+c)^3}{24}+\frac{5\sinh(dx+c)}{16}\right)\cosh(dx\right.}{}$
default	$\frac{a^3\left(\left(\frac{\sinh(dx+c)^3}{4}-\frac{3\sinh(dx+c)}{8}\right)\cosh(dx+c)+\frac{3dx}{8}+\frac{3c}{8}\right)+3a^2b\left(\left(\frac{\sinh(dx+c)^5}{6}-\frac{5\sinh(dx+c)^3}{24}+\frac{5\sinh(dx+c)}{16}\right)\cosh(dx\right.}{}$
parts	$\frac{a^3\left(\left(\frac{\sinh(dx+c)^3}{4}-\frac{3\sinh(dx+c)}{8}\right)\cosh(dx+c)+\frac{3dx}{8}+\frac{3c}{8}\right)}{d}+\frac{b^3\left(\left(\frac{\sinh(dx+c)^9}{10}-\frac{9\sinh(dx+c)^7}{80}+\frac{21\sinh(dx+c)^5}{160}-\frac{21\sin}{}\right.\right.}{d}$
risch	$\frac{b\,e^{6dx+6c}a^2}{128d}-\frac{b^2e^{6dx+6c}a}{64d}-\frac{9\,e^{4dx+4c}a^2b}{128d}+\frac{21\,e^{4dx+4c}b^2a}{256d}+\frac{45\,e^{2dx+2c}a^2b}{128d}-\frac{21\,e^{2dx+2c}b^2a}{64d}-\frac{e^{-4dx-4c}a}{64d}$
orering	Expression too large to display

input
```
int(sinh(d*x+c)^4*(a+b*sinh(d*x+c)^2)^3,x,method=_RETURNVERBOSE)
```

output
```
1/10240*((-2560*a^3+7200*a^2*b-6720*a*b^2+2100*b^3)*sinh(2*d*x+2*c)+(320*a
^3-1440*a^2*b+1680*a*b^2-600*b^3)*sinh(4*d*x+4*c)+160*(a-3/4*b)*b*(a-5/4*b
)*sinh(6*d*x+6*c)+(30*a*b^2-25*b^3)*sinh(8*d*x+8*c)+2*b^3*sinh(10*d*x+10*c
)+3840*(a^2-7/4*a*b+7/8*b^2)*(a-3/4*b)*x*d)/d
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 406 vs. 2(194) = 388.

Time = 0.10 (sec) , antiderivative size = 406, normalized size of antiderivative = 1.97

$$\int \sinh^4(c+dx)\left(a+b\sinh^2(c+dx)\right)^3\,dx$$
$$=\frac{5\,b^3\cosh(dx+c)\sinh(dx+c)^9+10\left(6\,b^3\cosh(dx+c)^3+(6\,ab^2-5\,b^3)\cosh(dx+c)\right)\sinh(dx+c)^7+}{}$$

input
```
integrate(sinh(d*x+c)^4*(a+b*sinh(d*x+c)^2)^3,x, algorithm="fricas")
```

output
```
1/2560*(5*b^3*cosh(d*x + c)*sinh(d*x + c)^9 + 10*(6*b^3*cosh(d*x + c)^3 +
(6*a*b^2 - 5*b^3)*cosh(d*x + c))*sinh(d*x + c)^7 + (126*b^3*cosh(d*x + c)^
5 + 70*(6*a*b^2 - 5*b^3)*cosh(d*x + c)^3 + 15*(16*a^2*b - 32*a*b^2 + 15*b^
3)*cosh(d*x + c))*sinh(d*x + c)^5 + 10*(6*b^3*cosh(d*x + c)^7 + 7*(6*a*b^2
 - 5*b^3)*cosh(d*x + c)^5 + 5*(16*a^2*b - 32*a*b^2 + 15*b^3)*cosh(d*x + c)
^3 + 4*(8*a^3 - 36*a^2*b + 42*a*b^2 - 15*b^3)*cosh(d*x + c))*sinh(d*x + c)
^3 + 30*(32*a^3 - 80*a^2*b + 70*a*b^2 - 21*b^3)*d*x + 5*(b^3*cosh(d*x + c)
^9 + 2*(6*a*b^2 - 5*b^3)*cosh(d*x + c)^7 + 3*(16*a^2*b - 32*a*b^2 + 15*b^3
)*cosh(d*x + c)^5 + 8*(8*a^3 - 36*a^2*b + 42*a*b^2 - 15*b^3)*cosh(d*x + c)
^3 - 2*(128*a^3 - 360*a^2*b + 336*a*b^2 - 105*b^3)*cosh(d*x + c))*sinh(d*x
 + c))/d
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 777 vs. $2(206) = 412$.

Time = 1.42 (sec) , antiderivative size = 777, normalized size of antiderivative = 3.77

$$\int \sinh^4(c+dx)\left(a+b\sinh^2(c+dx)\right)^3\,dx = \text{Too large to display}$$

input
```
integrate(sinh(d*x+c)**4*(a+b*sinh(d*x+c)**2)**3,x)
```

output
```
Piecewise((3*a**3*x*sinh(c + d*x)**4/8 - 3*a**3*x*sinh(c + d*x)**2*cosh(c
+ d*x)**2/4 + 3*a**3*x*cosh(c + d*x)**4/8 + 5*a**3*sinh(c + d*x)**3*cosh(c
 + d*x)/(8*d) - 3*a**3*sinh(c + d*x)*cosh(c + d*x)**3/(8*d) + 15*a**2*b*x*
sinh(c + d*x)**6/16 - 45*a**2*b*x*sinh(c + d*x)**4*cosh(c + d*x)**2/16 + 4
5*a**2*b*x*sinh(c + d*x)**2*cosh(c + d*x)**4/16 - 15*a**2*b*x*cosh(c + d*x
)**6/16 + 33*a**2*b*sinh(c + d*x)**5*cosh(c + d*x)/(16*d) - 5*a**2*b*sinh(
c + d*x)**3*cosh(c + d*x)**3/(2*d) + 15*a**2*b*sinh(c + d*x)*cosh(c + d*x)
**5/(16*d) + 105*a*b**2*x*sinh(c + d*x)**8/128 - 105*a*b**2*x*sinh(c + d*x
)**6*cosh(c + d*x)**2/32 + 315*a*b**2*x*sinh(c + d*x)**4*cosh(c + d*x)**4/
64 - 105*a*b**2*x*sinh(c + d*x)**2*cosh(c + d*x)**6/32 + 105*a*b**2*x*cosh
(c + d*x)**8/128 + 279*a*b**2*sinh(c + d*x)**7*cosh(c + d*x)/(128*d) - 511
*a*b**2*sinh(c + d*x)**5*cosh(c + d*x)**3/(128*d) + 385*a*b**2*sinh(c + d*
x)**3*cosh(c + d*x)**5/(128*d) - 105*a*b**2*sinh(c + d*x)*cosh(c + d*x)**7
/(128*d) + 63*b**3*x*sinh(c + d*x)**10/256 - 315*b**3*x*sinh(c + d*x)**8*c
osh(c + d*x)**2/256 + 315*b**3*x*sinh(c + d*x)**6*cosh(c + d*x)**4/128 - 3
15*b**3*x*sinh(c + d*x)**4*cosh(c + d*x)**6/128 + 315*b**3*x*sinh(c + d*x)
**2*cosh(c + d*x)**8/256 - 63*b**3*x*cosh(c + d*x)**10/256 + 193*b**3*sinh
(c + d*x)**9*cosh(c + d*x)/(256*d) - 237*b**3*sinh(c + d*x)**7*cosh(c + d*
x)**3/(128*d) + 21*b**3*sinh(c + d*x)**5*cosh(c + d*x)**5/(10*d) - 147*b**
3*sinh(c + d*x)**3*cosh(c + d*x)**7/(128*d) + 63*b**3*sinh(c + d*x)*cos...
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 405 vs. $2(194) = 388$.

Time = 0.04 (sec) , antiderivative size = 405, normalized size of antiderivative = 1.97

$$\int \sinh^4(c+dx)\left(a+b\sinh^2(c+dx)\right)^3\,dx$$
$$= \frac{1}{64}a^3\left(24x + \frac{e^{(4dx+4c)}}{d} - \frac{8e^{(2dx+2c)}}{d} + \frac{8e^{(-2dx-2c)}}{d} - \frac{e^{(-4dx-4c)}}{d}\right)$$
$$- \frac{1}{20480}b^3\left(\frac{\left(25e^{(-2dx-2c)} - 150e^{(-4dx-4c)} + 600e^{(-6dx-6c)} - 2100e^{(-8dx-8c)} - 2\right)e^{(10dx+10c)}}{d} + \frac{5040(\cdots}{\cdots}\right.$$
$$- \frac{1}{2048}ab^2\left(\frac{\left(32e^{(-2dx-2c)} - 168e^{(-4dx-4c)} + 672e^{(-6dx-6c)} - 3\right)e^{(8dx+8c)}}{d} - \frac{1680(dx+c)}{d} - \frac{672e^{(-2\cdots}}{\cdots}\right.$$
$$- \frac{1}{128}a^2b\left(\frac{\left(9e^{(-2dx-2c)} - 45e^{(-4dx-4c)} - 1\right)e^{(6dx+6c)}}{d} + \frac{120(dx+c)}{d} + \frac{45e^{(-2dx-2c)} - 9e^{(-4dx-4c)} + \cdots}{d}\right.$$

input
```
integrate(sinh(d*x+c)^4*(a+b*sinh(d*x+c)^2)^3,x, algorithm="maxima")
```

output
```
1/64*a^3*(24*x + e^(4*d*x + 4*c)/d - 8*e^(2*d*x + 2*c)/d + 8*e^(-2*d*x - 2
*c)/d - e^(-4*d*x - 4*c)/d) - 1/20480*b^3*((25*e^(-2*d*x - 2*c) - 150*e^(-
4*d*x - 4*c) + 600*e^(-6*d*x - 6*c) - 2100*e^(-8*d*x - 8*c) - 2)*e^(10*d*x
 + 10*c)/d + 5040*(d*x + c)/d + (2100*e^(-2*d*x - 2*c) - 600*e^(-4*d*x - 4
*c) + 150*e^(-6*d*x - 6*c) - 25*e^(-8*d*x - 8*c) + 2*e^(-10*d*x - 10*c))/d
) - 1/2048*a*b^2*((32*e^(-2*d*x - 2*c) - 168*e^(-4*d*x - 4*c) + 672*e^(-6*
d*x - 6*c) - 3)*e^(8*d*x + 8*c)/d - 1680*(d*x + c)/d - (672*e^(-2*d*x - 2*
c) - 168*e^(-4*d*x - 4*c) + 32*e^(-6*d*x - 6*c) - 3*e^(-8*d*x - 8*c))/d) -
 1/128*a^2*b*((9*e^(-2*d*x - 2*c) - 45*e^(-4*d*x - 4*c) - 1)*e^(6*d*x + 6*
c)/d + 120*(d*x + c)/d + (45*e^(-2*d*x - 2*c) - 9*e^(-4*d*x - 4*c) + e^(-6
*d*x - 6*c))/d)
```

Giac [A] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 325, normalized size of antiderivative = 1.58

$$\int \sinh^4(c+dx)\left(a+b\sinh^2(c+dx)\right)^3\,dx = \frac{b^3e^{(10\,dx+10\,c)}}{10240\,d} - \frac{b^3e^{(-10\,dx-10\,c)}}{10240\,d} + \frac{3}{256}\left(32\,a^3-80\,a^2b+70\,ab^2-21\,b^3\right)x + \frac{(6\,ab^2-5\,b^3)e^{(8\,dx+8\,c)}}{4096\,d} + \frac{(16\,a^2b-32\,ab^2+15\,b^3)e^{(6\,dx+6\,c)}}{2048\,d} + \frac{(8\,a^3-36\,a^2b+42\,ab^2-15\,b^3)e^{(4\,dx+4\,c)}}{512\,d} - \frac{(128\,a^3-360\,a^2b+336\,ab^2-105\,b^3)e^{(2\,dx+2\,c)}}{1024\,d} + \frac{(128\,a^3-360\,a^2b+336\,ab^2-105\,b^3)e^{(-2\,dx-2\,c)}}{1024\,d} - \frac{(8\,a^3-36\,a^2b+42\,ab^2-15\,b^3)e^{(-4\,dx-4\,c)}}{512\,d} - \frac{(16\,a^2b-32\,ab^2+15\,b^3)e^{(-6\,dx-6\,c)}}{2048\,d} - \frac{(6\,ab^2-5\,b^3)e^{(-8\,dx-8\,c)}}{4096\,d}$$

input
```
integrate(sinh(d*x+c)^4*(a+b*sinh(d*x+c)^2)^3,x, algorithm="giac")
```

output
```
1/10240*b^3*e^(10*d*x + 10*c)/d - 1/10240*b^3*e^(-10*d*x - 10*c)/d + 3/256
*(32*a^3 - 80*a^2*b + 70*a*b^2 - 21*b^3)*x + 1/4096*(6*a*b^2 - 5*b^3)*e^(8
*d*x + 8*c)/d + 1/2048*(16*a^2*b - 32*a*b^2 + 15*b^3)*e^(6*d*x + 6*c)/d +
1/512*(8*a^3 - 36*a^2*b + 42*a*b^2 - 15*b^3)*e^(4*d*x + 4*c)/d - 1/1024*(1
28*a^3 - 360*a^2*b + 336*a*b^2 - 105*b^3)*e^(2*d*x + 2*c)/d + 1/1024*(128*
a^3 - 360*a^2*b + 336*a*b^2 - 105*b^3)*e^(-2*d*x - 2*c)/d - 1/512*(8*a^3 -
 36*a^2*b + 42*a*b^2 - 15*b^3)*e^(-4*d*x - 4*c)/d - 1/2048*(16*a^2*b - 32*
a*b^2 + 15*b^3)*e^(-6*d*x - 6*c)/d - 1/4096*(6*a*b^2 - 5*b^3)*e^(-8*d*x -
8*c)/d
```

Mupad [B] (verification not implemented)

Time = 2.31 (sec) , antiderivative size = 239, normalized size of antiderivative = 1.16

$$\int \sinh^4(c+dx)\left(a+b\sinh^2(c+dx)\right)^3\,dx$$
$$= \frac{40\,a^3\sinh(4\,c+4\,d\,x)-320\,a^3\sinh(2\,c+2\,d\,x)+\frac{525\,b^3\sinh(2\,c+2\,d\,x)}{2}-75\,b^3\sinh(4\,c+4\,d\,x)+\frac{75\,b^3\sinh(6\,c+}{4}}{}$$

input
```
int(sinh(c + d*x)^4*(a + b*sinh(c + d*x)^2)^3,x)
```

output
```
(40*a^3*sinh(4*c + 4*d*x) - 320*a^3*sinh(2*c + 2*d*x) + (525*b^3*sinh(2*c
+ 2*d*x))/2 - 75*b^3*sinh(4*c + 4*d*x) + (75*b^3*sinh(6*c + 6*d*x))/4 - (2
5*b^3*sinh(8*c + 8*d*x))/8 + (b^3*sinh(10*c + 10*d*x))/4 - 840*a*b^2*sinh(
2*c + 2*d*x) + 900*a^2*b*sinh(2*c + 2*d*x) + 210*a*b^2*sinh(4*c + 4*d*x) -
 180*a^2*b*sinh(4*c + 4*d*x) - 40*a*b^2*sinh(6*c + 6*d*x) + 20*a^2*b*sinh(
6*c + 6*d*x) + (15*a*b^2*sinh(8*c + 8*d*x))/4 + 480*a^3*d*x - 315*b^3*d*x
+ 1050*a*b^2*d*x - 1200*a^2*b*d*x)/(1280*d)
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 512, normalized size of antiderivative = 2.49

$$\int \sinh^4(c+dx)\left(a+b\sinh^2(c+dx)\right)^3\,dx$$
$$=\frac{-2b^3-19200e^{10dx+10c}a^2bdx+16800e^{10dx+10c}a\,b^2dx+25e^{2dx+2c}b^3+7680e^{10dx+10c}a^3dx-5040e^{10dx+10c}b}{}$$

input
```
int(sinh(d*x+c)^4*(a+b*sinh(d*x+c)^2)^3,x)
```

output
```
(2*e**(20*c + 20*d*x)*b**3 + 30*e**(18*c + 18*d*x)*a*b**2 - 25*e**(18*c +
18*d*x)*b**3 + 160*e**(16*c + 16*d*x)*a**2*b - 320*e**(16*c + 16*d*x)*a*b*
*2 + 150*e**(16*c + 16*d*x)*b**3 + 320*e**(14*c + 14*d*x)*a**3 - 1440*e**(
14*c + 14*d*x)*a**2*b + 1680*e**(14*c + 14*d*x)*a*b**2 - 600*e**(14*c + 14
*d*x)*b**3 - 2560*e**(12*c + 12*d*x)*a**3 + 7200*e**(12*c + 12*d*x)*a**2*b
 - 6720*e**(12*c + 12*d*x)*a*b**2 + 2100*e**(12*c + 12*d*x)*b**3 + 7680*e*
*(10*c + 10*d*x)*a**3*d*x - 19200*e**(10*c + 10*d*x)*a**2*b*d*x + 16800*e*
*(10*c + 10*d*x)*a*b**2*d*x - 5040*e**(10*c + 10*d*x)*b**3*d*x + 2560*e**(
8*c + 8*d*x)*a**3 - 7200*e**(8*c + 8*d*x)*a**2*b + 6720*e**(8*c + 8*d*x)*a
*b**2 - 2100*e**(8*c + 8*d*x)*b**3 - 320*e**(6*c + 6*d*x)*a**3 + 1440*e**(
6*c + 6*d*x)*a**2*b - 1680*e**(6*c + 6*d*x)*a*b**2 + 600*e**(6*c + 6*d*x)*
b**3 - 160*e**(4*c + 4*d*x)*a**2*b + 320*e**(4*c + 4*d*x)*a*b**2 - 150*e**
(4*c + 4*d*x)*b**3 - 30*e**(2*c + 2*d*x)*a*b**2 + 25*e**(2*c + 2*d*x)*b**3
 - 2*b**3)/(20480*e**(10*c + 10*d*x)*d)
```

3.20 $\int \sinh^3(c+dx)\left(a+b\sinh^2(c+dx)\right)^3\,dx$

Optimal result

Integrand size = 23, antiderivative size = 115

$$\int \sinh^3(c+dx)\left(a+b\sinh^2(c+dx)\right)^3\,dx = -\frac{(a-b)^3\cosh(c+dx)}{d} + \frac{(a-4b)(a-b)^2\cosh^3(c+dx)}{3d} + \frac{3(a-2b)(a-b)b\cosh^5(c+dx)}{5d} + \frac{(3a-4b)b^2\cosh^7(c+dx)}{7d} + \frac{b^3\cosh^9(c+dx)}{9d}$$

output
```
-(a-b)^3*cosh(d*x+c)/d+1/3*(a-4*b)*(a-b)^2*cosh(d*x+c)^3/d+3/5*(a-2*b)*(a-
b)*b*cosh(d*x+c)^5/d+1/7*(3*a-4*b)*b^2*cosh(d*x+c)^7/d+1/9*b^3*cosh(d*x+c)
^9/d
```

Mathematica [A] (verified)

Time = 1.88 (sec) , antiderivative size = 127, normalized size of antiderivative = 1.10

$$\int \sinh^3(c+dx)\left(a+b\sinh^2(c+dx)\right)^3\,dx$$
$$=\frac{-1890(4a-3b)\left(8a^2-14ab+7b^2\right)\cosh(c+dx)+420\left(16a^3-60a^2b+63ab^2-21b^3\right)\cosh(3(c+dx))+}{80640d}$$

input
```
Integrate[Sinh[c + d*x]^3*(a + b*Sinh[c + d*x]^2)^3,x]
```

output
```
(-1890*(4*a - 3*b)*(8*a^2 - 14*a*b + 7*b^2)*Cosh[c + d*x] + 420*(16*a^3 -
60*a^2*b + 63*a*b^2 - 21*b^3)*Cosh[3*(c + d*x)] + 756*(4*a - 3*b)*(a - b)*
b*Cosh[5*(c + d*x)] + 135*(4*a - 3*b)*b^2*Cosh[7*(c + d*x)] + 35*b^3*Cosh[
9*(c + d*x)])/(80640*d)
```

Rubi [A] (verified)

Time = 0.35 (sec) , antiderivative size = 104, normalized size of antiderivative = 0.90, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.217$, Rules used = {3042, 26, 3665, 290, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sinh^3(c+dx)\left(a+b\sinh^2(c+dx)\right)^3\,dx$$

$$\Big\downarrow \; 3042$$

$$\int i\sin(ic+idx)^3\left(a-b\sin(ic+idx)^2\right)^3dx$$

$$\Big\downarrow \; 26$$

$$i\int \sin(ic+idx)^3\left(a-b\sin(ic+idx)^2\right)^3dx$$

$$\Big\downarrow \; 3665$$

$$-\frac{\int \left(1-\cosh^2(c+dx)\right)\left(b\cosh^2(c+dx)+a-b\right)^3d\cosh(c+dx)}{d}$$

$$\Big\downarrow \ 290$$

$$-\frac{\int \left(-b^3\cosh^8(c+dx)-(3a-4b)b^2\cosh^6(c+dx)+3(a-2b)b(b-a)\cosh^4(c+dx)-(a-4b)(a-b)^2\cosh^2(c$$

$$d$$

$$\Big\downarrow \ 2009$$

$$-\frac{-\frac{1}{7}b^2(3a-4b)\cosh^7(c+dx)-\frac{3}{5}b(a-2b)(a-b)\cosh^5(c+dx)-\frac{1}{3}(a-4b)(a-b)^2\cosh^3(c+dx)+(a-b)^3c}{d}$$

input
```
Int[Sinh[c + d*x]^3*(a + b*Sinh[c + d*x]^2)^3,x]
```

output
```
-(((a - b)^3*Cosh[c + d*x] - ((a - 4*b)*(a - b)^2*Cosh[c + d*x]^3)/3 - (3*
(a - 2*b)*(a - b)*b*Cosh[c + d*x]^5)/5 - ((3*a - 4*b)*b^2*Cosh[c + d*x]^7)
/7 - (b^3*Cosh[c + d*x]^9)/9)/d)
```

Defintions of rubi rules used

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 290
```
Int[((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.), x_Symbol] :> I
nt[ExpandIntegrand[(a + b*x^2)^p*(c + d*x^2)^q, x], x] /; FreeQ[{a, b, c, d
}, x] && NeQ[b*c - a*d, 0] && IGtQ[p, 0] && IGtQ[q, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3665
```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^
(p_.), x_Symbol] :> With[{ff = FreeFactors[Cos[e + f*x], x]}, Simp[-ff/f
Subst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a + b - b*ff^2*x^2)^p, x], x, Cos[e +
 f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m - 1)/2]
```

Maple [A] (verified)

Time = 74.43 (sec) , antiderivative size = 143, normalized size of antiderivative = 1.24

method	result
parallelrisch	$\frac{(6720a^3-25200a^2b+26460b^2a-8820b^3)\cosh(3dx+3c)+3024\left(a-\frac{3b}{4}\right)(a-b)b\cosh(5dx+5c)+540\left(a-\frac{3b}{4}\right)b^2\cosh(7dx}{8064}$
derivativedivides	$\frac{a^3\left(-\frac{2}{3}+\frac{\sinh(dx+c)^2}{3}\right)\cosh(dx+c)+3a^2b\left(\frac{8}{15}+\frac{\sinh(dx+c)^4}{5}-\frac{4\sinh(dx+c)^2}{15}\right)\cosh(dx+c)+3b^2a\left(-\frac{16}{35}+\frac{\sinh(dx+c)^6}{7}-\right.}{}$
default	$\frac{a^3\left(-\frac{2}{3}+\frac{\sinh(dx+c)^2}{3}\right)\cosh(dx+c)+3a^2b\left(\frac{8}{15}+\frac{\sinh(dx+c)^4}{5}-\frac{4\sinh(dx+c)^2}{15}\right)\cosh(dx+c)+3b^2a\left(-\frac{16}{35}+\frac{\sinh(dx+c)^6}{7}-\right.}{}$
parts	$\frac{a^3\left(-\frac{2}{3}+\frac{\sinh(dx+c)^2}{3}\right)\cosh(dx+c)}{d}+\frac{b^3\left(\frac{128}{315}+\frac{\sinh(dx+c)^8}{9}-\frac{8\sinh(dx+c)^6}{63}+\frac{16\sinh(dx+c)^4}{105}-\frac{64\sinh(dx+c)^2}{315}\right)\cosh(d}{d}$
risch	$-\frac{3\,e^{dx+c}a^3}{8d}+\frac{63\,e^{dx+c}b^3}{256d}-\frac{3\,e^{-dx-c}a^3}{8d}+\frac{63\,e^{-dx-c}b^3}{256d}+\frac{e^{-3dx-3c}a^3}{24d}-\frac{7\,e^{-3dx-3c}b^3}{128d}+\frac{9b^3e^{-5dx-5c}}{640d}-$
orering	Expression too large to display

input
```
int(sinh(d*x+c)^3*(a+b*sinh(d*x+c)^2)^3,x,method=_RETURNVERBOSE)
```

output
```
1/80640*((6720*a^3-25200*a^2*b+26460*a*b^2-8820*b^3)*cosh(3*d*x+3*c)+3024*
(a-3/4*b)*(a-b)*b*cosh(5*d*x+5*c)+540*(a-3/4*b)*b^2*cosh(7*d*x+7*c)+35*b^3
*cosh(9*d*x+9*c)-60480*(a-3/4*b)*(a^2-7/4*a*b+7/8*b^2)*cosh(d*x+c)-53760*a
^3+129024*a^2*b-110592*b^2*a+32768*b^3)/d
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 373 vs. $2(107) = 214$.

Time = 0.09 (sec) , antiderivative size = 373, normalized size of antiderivative = 3.24

$$\int \sinh^3(c+dx)\left(a+b\sinh^2(c+dx)\right)^3\,dx = \frac{35\,b^3\cosh\left(dx+c\right)^9+315\,b^3\cosh\left(dx+c\right)\sinh\left(dx+c\right)^8+135\left(4\,ab^2-3\,b^3\right)\cosh\left(dx+c\right)^7+105\left(28\,b^3\right.}{}$$

input
```
integrate(sinh(d*x+c)^3*(a+b*sinh(d*x+c)^2)^3,x, algorithm="fricas")
```

output
```
1/80640*(35*b^3*cosh(d*x + c)^9 + 315*b^3*cosh(d*x + c)*sinh(d*x + c)^8 +
135*(4*a*b^2 - 3*b^3)*cosh(d*x + c)^7 + 105*(28*b^3*cosh(d*x + c)^3 + 9*(4
*a*b^2 - 3*b^3)*cosh(d*x + c))*sinh(d*x + c)^6 + 756*(4*a^2*b - 7*a*b^2 +
3*b^3)*cosh(d*x + c)^5 + 315*(14*b^3*cosh(d*x + c)^5 + 15*(4*a*b^2 - 3*b^3
)*cosh(d*x + c)^3 + 12*(4*a^2*b - 7*a*b^2 + 3*b^3)*cosh(d*x + c))*sinh(d*x
 + c)^4 + 420*(16*a^3 - 60*a^2*b + 63*a*b^2 - 21*b^3)*cosh(d*x + c)^3 + 31
5*(4*b^3*cosh(d*x + c)^7 + 9*(4*a*b^2 - 3*b^3)*cosh(d*x + c)^5 + 24*(4*a^2
*b - 7*a*b^2 + 3*b^3)*cosh(d*x + c)^3 + 4*(16*a^3 - 60*a^2*b + 63*a*b^2 -
21*b^3)*cosh(d*x + c))*sinh(d*x + c)^2 - 1890*(32*a^3 - 80*a^2*b + 70*a*b^
2 - 21*b^3)*cosh(d*x + c))/d
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 330 vs. $2(100) = 200$.

Time = 1.01 (sec) , antiderivative size = 330, normalized size of antiderivative = 2.87

$$\int \sinh^3(c+dx)\left(a+b\sinh^2(c+dx)\right)^3\,dx = \begin{cases} \frac{a^3\sinh^2(c+dx)\cosh(c+dx)}{d} - \frac{2a^3\cosh^3(c+dx)}{3d} + \frac{3a^2b\sinh^4(c+dx)\cosh(c+dx)}{d} - \frac{4a^2b\sinh^2(c+dx)\cosh^3(c+dx)}{d} + \frac{8a^2b\cosh^5(c\text{-}}{5d} \\ x\left(a+b\sinh^2(c)\right)^3\sinh^3(c) \end{cases}$$

input
```
integrate(sinh(d*x+c)**3*(a+b*sinh(d*x+c)**2)**3,x)
```

output
```
Piecewise((a**3*sinh(c + d*x)**2*cosh(c + d*x)/d - 2*a**3*cosh(c + d*x)**3
/(3*d) + 3*a**2*b*sinh(c + d*x)**4*cosh(c + d*x)/d - 4*a**2*b*sinh(c + d*x
)**2*cosh(c + d*x)**3/d + 8*a**2*b*cosh(c + d*x)**5/(5*d) + 3*a*b**2*sinh(
c + d*x)**6*cosh(c + d*x)/d - 6*a*b**2*sinh(c + d*x)**4*cosh(c + d*x)**3/d
 + 24*a*b**2*sinh(c + d*x)**2*cosh(c + d*x)**5/(5*d) - 48*a*b**2*cosh(c +
d*x)**7/(35*d) + b**3*sinh(c + d*x)**8*cosh(c + d*x)/d - 8*b**3*sinh(c + d
*x)**6*cosh(c + d*x)**3/(3*d) + 16*b**3*sinh(c + d*x)**4*cosh(c + d*x)**5/
(5*d) - 64*b**3*sinh(c + d*x)**2*cosh(c + d*x)**7/(35*d) + 128*b**3*cosh(c
 + d*x)**9/(315*d), Ne(d, 0)), (x*(a + b*sinh(c)**2)**3*sinh(c)**3, True))
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 376 vs. $2(107) = 214$.

Time = 0.06 (sec) , antiderivative size = 376, normalized size of antiderivative = 3.27

$$\int \sinh^3(c+dx)\left(a+b\sinh^2(c+dx)\right)^3\,dx =$$

$$-\frac{1}{161280}b^3\left(\frac{\left(405\,e^{(-2\,dx-2\,c)}-2268\,e^{(-4\,dx-4\,c)}+8820\,e^{(-6\,dx-6\,c)}-39690\,e^{(-8\,dx-8\,c)}-35\right)e^{(9\,dx+9\,c)}}{d}-\right.$$
$$-\frac{3}{4480}ab^2\left(\frac{\left(49\,e^{(-2\,dx-2\,c)}-245\,e^{(-4\,dx-4\,c)}+1225\,e^{(-6\,dx-6\,c)}-5\right)e^{(7\,dx+7\,c)}}{d}+\frac{1225\,e^{(-dx-c)}-245\,e^{(-3\,\ldots}}{}\right.$$
$$+\frac{1}{160}a^2b\left(\frac{3\,e^{(5\,dx+5\,c)}}{d}-\frac{25\,e^{(3\,dx+3\,c)}}{d}+\frac{150\,e^{(dx+c)}}{d}+\frac{150\,e^{(-dx-c)}}{d}-\frac{25\,e^{(-3\,dx-3\,c)}}{d}+\frac{3\,e^{(-5\,dx-5\,c)}}{d}\right)$$
$$+\frac{1}{24}a^3\left(\frac{e^{(3\,dx+3\,c)}}{d}-\frac{9\,e^{(dx+c)}}{d}-\frac{9\,e^{(-dx-c)}}{d}+\frac{e^{(-3\,dx-3\,c)}}{d}\right)$$

input
```
integrate(sinh(d*x+c)^3*(a+b*sinh(d*x+c)^2)^3,x, algorithm="maxima")
```

output
```
-1/161280*b^3*((405*e^(-2*d*x - 2*c) - 2268*e^(-4*d*x - 4*c) + 8820*e^(-6*
d*x - 6*c) - 39690*e^(-8*d*x - 8*c) - 35)*e^(9*d*x + 9*c)/d - (39690*e^(-d
*x - c) - 8820*e^(-3*d*x - 3*c) + 2268*e^(-5*d*x - 5*c) - 405*e^(-7*d*x -
7*c) + 35*e^(-9*d*x - 9*c))/d) - 3/4480*a*b^2*((49*e^(-2*d*x - 2*c) - 245*
e^(-4*d*x - 4*c) + 1225*e^(-6*d*x - 6*c) - 5)*e^(7*d*x + 7*c)/d + (1225*e^
(-d*x - c) - 245*e^(-3*d*x - 3*c) + 49*e^(-5*d*x - 5*c) - 5*e^(-7*d*x - 7*
c))/d) + 1/160*a^2*b*(3*e^(5*d*x + 5*c)/d - 25*e^(3*d*x + 3*c)/d + 150*e^(
d*x + c)/d + 150*e^(-d*x - c)/d - 25*e^(-3*d*x - 3*c)/d + 3*e^(-5*d*x - 5*
c)/d) + 1/24*a^3*(e^(3*d*x + 3*c)/d - 9*e^(d*x + c)/d - 9*e^(-d*x - c)/d +
 e^(-3*d*x - 3*c)/d)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 296 vs. $2(107) = 214$.

Time = 0.16 (sec) , antiderivative size = 296, normalized size of antiderivative = 2.57

$$\begin{aligned}
&\int \sinh^3(c+dx)\left(a+b\sinh^2(c+dx)\right)^3\,dx \\
&= \frac{b^3e^{(9\,dx+9\,c)}}{4608\,d} + \frac{b^3e^{(-9\,dx-9\,c)}}{4608\,d} + \frac{3\,(4\,ab^2-3\,b^3)e^{(7\,dx+7\,c)}}{3584\,d} \\
&\quad + \frac{3\,(4\,a^2b-7\,ab^2+3\,b^3)e^{(5\,dx+5\,c)}}{640\,d} + \frac{(16\,a^3-60\,a^2b+63\,ab^2-21\,b^3)e^{(3\,dx+3\,c)}}{384\,d} \\
&\quad - \frac{3\,(32\,a^3-80\,a^2b+70\,ab^2-21\,b^3)e^{(dx+c)}}{256\,d} \\
&\quad - \frac{3\,(32\,a^3-80\,a^2b+70\,ab^2-21\,b^3)e^{(-dx-c)}}{256\,d} \\
&\quad + \frac{(16\,a^3-60\,a^2b+63\,ab^2-21\,b^3)e^{(-3\,dx-3\,c)}}{384\,d} \\
&\quad + \frac{3\,(4\,a^2b-7\,ab^2+3\,b^3)e^{(-5\,dx-5\,c)}}{640\,d} + \frac{3\,(4\,ab^2-3\,b^3)e^{(-7\,dx-7\,c)}}{3584\,d}
\end{aligned}$$

input
```
integrate(sinh(d*x+c)^3*(a+b*sinh(d*x+c)^2)^3,x, algorithm="giac")
```

output
```
1/4608*b^3*e^(9*d*x + 9*c)/d + 1/4608*b^3*e^(-9*d*x - 9*c)/d + 3/3584*(4*a
*b^2 - 3*b^3)*e^(7*d*x + 7*c)/d + 3/640*(4*a^2*b - 7*a*b^2 + 3*b^3)*e^(5*d
*x + 5*c)/d + 1/384*(16*a^3 - 60*a^2*b + 63*a*b^2 - 21*b^3)*e^(3*d*x + 3*c
)/d - 3/256*(32*a^3 - 80*a^2*b + 70*a*b^2 - 21*b^3)*e^(d*x + c)/d - 3/256*
(32*a^3 - 80*a^2*b + 70*a*b^2 - 21*b^3)*e^(-d*x - c)/d + 1/384*(16*a^3 - 6
0*a^2*b + 63*a*b^2 - 21*b^3)*e^(-3*d*x - 3*c)/d + 3/640*(4*a^2*b - 7*a*b^2
 + 3*b^3)*e^(-5*d*x - 5*c)/d + 3/3584*(4*a*b^2 - 3*b^3)*e^(-7*d*x - 7*c)/d
```

Mupad [B] (verification not implemented)

Time = 2.05 (sec) , antiderivative size = 185, normalized size of antiderivative = 1.61

$$\int \sinh^3(c+dx)\left(a+b\sinh^2(c+dx)\right)^3\,dx = \frac{\frac{a^3\cosh(c+d\,x)^3}{3} - a^3\cosh(c+d\,x) + \frac{3\,a^2\,b\cosh(c+d\,x)^5}{5} - 2\,a^2\,b\cosh(c+d\,x)^3 + 3\,a^2\,b\cosh(c+d\,x) + \frac{3\,a\,b^2\cosh(\cdots}{7}\cdots}{\cdots}$$

input
```
int(sinh(c + d*x)^3*(a + b*sinh(c + d*x)^2)^3,x)
```

output
```
(b^3*cosh(c + d*x) - a^3*cosh(c + d*x) + (a^3*cosh(c + d*x)^3)/3 - (4*b^3*
cosh(c + d*x)^3)/3 + (6*b^3*cosh(c + d*x)^5)/5 - (4*b^3*cosh(c + d*x)^7)/7
 + (b^3*cosh(c + d*x)^9)/9 + 3*a*b^2*cosh(c + d*x)^3 - 2*a^2*b*cosh(c + d*
x)^3 - (9*a*b^2*cosh(c + d*x)^5)/5 + (3*a^2*b*cosh(c + d*x)^5)/5 + (3*a*b^
2*cosh(c + d*x)^7)/7 - 3*a*b^2*cosh(c + d*x) + 3*a^2*b*cosh(c + d*x))/d
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 442, normalized size of antiderivative = 3.84

$$\int \sinh^3(c+dx)\left(a+b\sinh^2(c+dx)\right)^3\,dx = \frac{35b^3 - 405e^{2dx+2c}b^3 - 60480e^{10dx+10c}a^3 + 39690e^{10dx+10c}b^3 + 540e^{16dx+16c}a\,b^2 + 3024e^{14dx+14c}a^2b - 5292\cdots}{\cdots}$$

input
```
int(sinh(d*x+c)^3*(a+b*sinh(d*x+c)^2)^3,x)
```

output

```
(35*e**(18*c + 18*d*x)*b**3 + 540*e**(16*c + 16*d*x)*a*b**2 - 405*e**(16*c
 + 16*d*x)*b**3 + 3024*e**(14*c + 14*d*x)*a**2*b - 5292*e**(14*c + 14*d*x)
*a*b**2 + 2268*e**(14*c + 14*d*x)*b**3 + 6720*e**(12*c + 12*d*x)*a**3 - 25
200*e**(12*c + 12*d*x)*a**2*b + 26460*e**(12*c + 12*d*x)*a*b**2 - 8820*e**
(12*c + 12*d*x)*b**3 - 60480*e**(10*c + 10*d*x)*a**3 + 151200*e**(10*c + 1
0*d*x)*a**2*b - 132300*e**(10*c + 10*d*x)*a*b**2 + 39690*e**(10*c + 10*d*x
)*b**3 - 60480*e**(8*c + 8*d*x)*a**3 + 151200*e**(8*c + 8*d*x)*a**2*b - 13
2300*e**(8*c + 8*d*x)*a*b**2 + 39690*e**(8*c + 8*d*x)*b**3 + 6720*e**(6*c
+ 6*d*x)*a**3 - 25200*e**(6*c + 6*d*x)*a**2*b + 26460*e**(6*c + 6*d*x)*a*b
**2 - 8820*e**(6*c + 6*d*x)*b**3 + 3024*e**(4*c + 4*d*x)*a**2*b - 5292*e**
(4*c + 4*d*x)*a*b**2 + 2268*e**(4*c + 4*d*x)*b**3 + 540*e**(2*c + 2*d*x)*a
*b**2 - 405*e**(2*c + 2*d*x)*b**3 + 35*b**3)/(161280*e**(9*c + 9*d*x)*d)
```

3.21 $\int \sinh^2(c+dx)\left(a+b\sinh^2(c+dx)\right)^3\,dx$

Optimal result

Integrand size = 23, antiderivative size = 181

$$\begin{aligned}\int \sinh^2(c+dx)\left(a+b\sinh^2(c+dx)\right)^3\,dx &= -\frac{1}{128}\left(64a^3-144a^2b+120ab^2-35b^3\right)x\\
&\quad+\frac{(96a^3-376a^2b+360ab^2-105b^3)\cosh(c+dx)\sinh(c+dx)}{384d}\\
&\quad+\frac{b(24a^2-64ab+35b^2)\cosh(c+dx)\sinh^3(c+dx)}{192d}\\
&\quad+\frac{(6a-7b)\cosh(c+dx)\sinh(c+dx)\left(a+b\sinh^2(c+dx)\right)^2}{48d}\\
&\quad+\frac{\cosh(c+dx)\sinh(c+dx)\left(a+b\sinh^2(c+dx)\right)^3}{8d}\end{aligned}$$

output
```
-1/128*(64*a^3-144*a^2*b+120*a*b^2-35*b^3)*x+1/384*(96*a^3-376*a^2*b+360*a
*b^2-105*b^3)*cosh(d*x+c)*sinh(d*x+c)/d+1/192*b*(24*a^2-64*a*b+35*b^2)*cos
h(d*x+c)*sinh(d*x+c)^3/d+1/48*(6*a-7*b)*cosh(d*x+c)*sinh(d*x+c)*(a+b*sinh(
d*x+c)^2)^2/d+1/8*cosh(d*x+c)*sinh(d*x+c)*(a+b*sinh(d*x+c)^2)^3/d
```

Mathematica [A] (verified)

Time = 0.67 (sec) , antiderivative size = 130, normalized size of antiderivative = 0.72

$$\int \sinh^2(c+dx)\left(a+b\sinh^2(c+dx)\right)^3\,dx$$
$$=\frac{-24(64a^3-144a^2b+120ab^2-35b^3)\,(c+dx)+48(16a^3-48a^2b+45ab^2-14b^3)\sinh(2(c+dx))+24b}{3072d}$$

input
```
Integrate[Sinh[c + d*x]^2*(a + b*Sinh[c + d*x]^2)^3,x]
```

output
```
(-24*(64*a^3 - 144*a^2*b + 120*a*b^2 - 35*b^3)*(c + d*x) + 48*(16*a^3 - 48
*a^2*b + 45*a*b^2 - 14*b^3)*Sinh[2*(c + d*x)] + 24*b*(12*a^2 - 18*a*b + 7*
b^2)*Sinh[4*(c + d*x)] + 16*(3*a - 2*b)*b^2*Sinh[6*(c + d*x)] + 3*b^3*Sinh
[8*(c + d*x)])/(3072*d)
```

Rubi [A] (verified)

Time = 0.69 (sec) , antiderivative size = 191, normalized size of antiderivative = 1.06, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.304$, Rules used = {3042, 25, 3649, 3042, 3649, 3042, 3648}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sinh^2(c+dx)\left(a+b\sinh^2(c+dx)\right)^3\,dx$$

$$\Big\downarrow \; 3042$$

$$\int -\sin(ic+idx)^2\left(a-b\sin(ic+idx)^2\right)^3\,dx$$

$$\Big\downarrow \; 25$$

$$-\int \sin(ic+idx)^2\left(a-b\sin(ic+idx)^2\right)^3\,dx$$

$$\Big\downarrow \; 3649$$

$$\frac{\sinh(c+dx)\cosh(c+dx)\left(a+b\sinh^2(c+dx)\right)^3}{8d}-$$
$$\frac{1}{8}\int\left(a-(6a-7b)\sinh^2(c+dx)\right)\left(b\sinh^2(c+dx)+a\right)^2dx$$

$$\Big\downarrow \text{3042}$$

$$\frac{\sinh(c+dx)\cosh(c+dx)\left(a+b\sinh^2(c+dx)\right)^3}{8d}-$$
$$\frac{1}{8}\int\left((6a-7b)\sin(ic+idx)^2+a\right)\left(a-b\sin(ic+idx)^2\right)^2dx$$

$$\Big\downarrow \text{3649}$$

$$\frac{1}{8}\left(\frac{(6a-7b)\sinh(c+dx)\cosh(c+dx)\left(a+b\sinh^2(c+dx)\right)^2}{6d}-\frac{1}{6}\int\left(b\sinh^2(c+dx)+a\right)\left(a(12a-7b)-(24a^2\right.\right.$$
$$\frac{\sinh(c+dx)\cosh(c+dx)\left(a+b\sinh^2(c+dx)\right)^3}{8d}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\sinh(c+dx)\cosh(c+dx)\left(a+b\sinh^2(c+dx)\right)^3}{8d}+$$
$$\frac{1}{8}\left(\frac{(6a-7b)\sinh(c+dx)\cosh(c+dx)\left(a+b\sinh^2(c+dx)\right)^2}{6d}-\frac{1}{6}\int\left(a-b\sin(ic+idx)^2\right)\left((24a^2-64ba+35b\right.\right.$$

$$\Big\downarrow \text{3648}$$

$$\frac{1}{8}\left(\frac{1}{6}\left(\frac{b(24a^2-64ab+35b^2)\sinh^3(c+dx)\cosh(c+dx)}{4d}+\frac{(96a^3-376a^2b+360ab^2-105b^3)\sinh(c+dx)\cosh\right.\right.$$
$$\frac{\sinh(c+dx)\cosh(c+dx)\left(a+b\sinh^2(c+dx)\right)^3}{8d}$$

input
```
Int[Sinh[c + d*x]^2*(a + b*Sinh[c + d*x]^2)^3,x]
```

output
```
(Cosh[c + d*x]*Sinh[c + d*x]*(a + b*Sinh[c + d*x]^2)^3)/(8*d) + (((6*a - 7
*b)*Cosh[c + d*x]*Sinh[c + d*x]*(a + b*Sinh[c + d*x]^2)^2)/(6*d) + ((-3*(6
4*a^3 - 144*a^2*b + 120*a*b^2 - 35*b^3)*x)/8 + ((96*a^3 - 376*a^2*b + 360*
a*b^2 - 105*b^3)*Cosh[c + d*x]*Sinh[c + d*x])/(8*d) + (b*(24*a^2 - 64*a*b
+ 35*b^2)*Cosh[c + d*x]*Sinh[c + d*x]^3)/(4*d))/6)/8
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3648
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)*((A_.) + (B_.)*sin[(e_.) + (f_
.)*(x_)]^2), x_Symbol] :> Simp[(4*A*(2*a + b) + B*(4*a + 3*b))*(x/8), x] +
(-Simp[b*B*Cos[e + f*x]*(Sin[e + f*x]^3/(4*f)), x] - Simp[(4*A*b + B*(4*a +
 3*b))*Cos[e + f*x]*(Sin[e + f*x]/(8*f)), x]) /; FreeQ[{a, b, e, f, A, B},
x]
```

rule 3649
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)]^2), x_Symbol] :> Simp[(-B)*Cos[e + f*x]*Sin[e + f*x]*((a + b*
Sin[e + f*x]^2)^p/(2*f*(p + 1))), x] + Simp[1/(2*(p + 1))   Int[(a + b*Sin[
e + f*x]^2)^(p - 1)*Simp[a*B + 2*a*A*(p + 1) + (2*A*b*(p + 1) + B*(b + 2*a*
p + 2*b*p))*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, e, f, A, B}, x] && G
tQ[p, 0]
```

Maple [A] (verified)

Time = 27.81 (sec) , antiderivative size = 129, normalized size of antiderivative = 0.71

method	result
parallelrisch	$\frac{\left(768a^3-2304a^2b+2160b^2a-672b^3\right)\sinh(2dx+2c)+\left(288a^2b-432b^2a+168b^3\right)\sinh(4dx+4c)+\left(48b^2a-32b^3\right)\sinh(6dx+}{3072d}$
derivativedivides	$\frac{a^3\left(\frac{\cosh(dx+c)\sinh(dx+c)}{2}-\frac{dx}{2}-\frac{c}{2}\right)+3a^2b\left(\left(\frac{\sinh(dx+c)^3}{4}-\frac{3\sinh(dx+c)}{8}\right)\cosh(dx+c)+\frac{3dx}{8}+\frac{3c}{8}\right)+3b^2a\left(\left(\frac{\sinh(dx+c)^5}{6}\right.\right.}{}$
default	$\frac{a^3\left(\frac{\cosh(dx+c)\sinh(dx+c)}{2}-\frac{dx}{2}-\frac{c}{2}\right)+3a^2b\left(\left(\frac{\sinh(dx+c)^3}{4}-\frac{3\sinh(dx+c)}{8}\right)\cosh(dx+c)+\frac{3dx}{8}+\frac{3c}{8}\right)+3b^2a\left(\left(\frac{\sinh(dx+c)^5}{6}\right.\right.}{}$
parts	$\frac{a^3\left(\frac{\cosh(dx+c)\sinh(dx+c)}{2}-\frac{dx}{2}-\frac{c}{2}\right)}{d}+\frac{b^3\left(\left(\frac{\sinh(dx+c)^7}{8}-\frac{7\sinh(dx+c)^5}{48}+\frac{35\sinh(dx+c)^3}{192}-\frac{35\sinh(dx+c)}{128}\right)\cosh(dx+c)-\right.}{d}$
risch	$-\frac{a^3x}{2}+\frac{9a^2bx}{8}-\frac{15a\,b^2x}{16}+\frac{35b^3x}{128}+\frac{b^3e^{8dx+8c}}{2048d}+\frac{b^2e^{6dx+6c}a}{128d}-\frac{b^3e^{6dx+6c}}{192d}+\frac{3\,e^{4dx+4c}a^2b}{64d}-\frac{9\,e^{4dx+4c}}{128d}$
orering	Expression too large to display

input
```
int(sinh(d*x+c)^2*(a+b*sinh(d*x+c)^2)^3,x,method=_RETURNVERBOSE)
```

output
```
1/3072*((768*a^3-2304*a^2*b+2160*a*b^2-672*b^3)*sinh(2*d*x+2*c)+(288*a^2*b
-432*a*b^2+168*b^3)*sinh(4*d*x+4*c)+(48*a*b^2-32*b^3)*sinh(6*d*x+6*c)+3*b^
3*sinh(8*d*x+8*c)-1536*x*d*(a^3-9/4*a^2*b+15/8*b^2*a-35/64*b^3))/d
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 269, normalized size of antiderivative = 1.49

$$\int \sinh^2(c+dx)\left(a+b\sinh^2(c+dx)\right)^3\,dx = \frac{3\,b^3\cosh\left(dx+c\right)\sinh\left(dx+c\right)^7+3\,\left(7\,b^3\cosh\left(dx+c\right)^3+4\,\left(3\,ab^2-2\,b^3\right)\cosh\left(dx+c\right)\right)\sinh\left(dx+c\right)^5+}{}$$

input
```
integrate(sinh(d*x+c)^2*(a+b*sinh(d*x+c)^2)^3,x, algorithm="fricas")
```

output
```
1/384*(3*b^3*cosh(d*x + c)*sinh(d*x + c)^7 + 3*(7*b^3*cosh(d*x + c)^3 + 4*
(3*a*b^2 - 2*b^3)*cosh(d*x + c))*sinh(d*x + c)^5 + (21*b^3*cosh(d*x + c)^5
 + 40*(3*a*b^2 - 2*b^3)*cosh(d*x + c)^3 + 12*(12*a^2*b - 18*a*b^2 + 7*b^3)
*cosh(d*x + c))*sinh(d*x + c)^3 - 3*(64*a^3 - 144*a^2*b + 120*a*b^2 - 35*b
^3)*d*x + 3*(b^3*cosh(d*x + c)^7 + 4*(3*a*b^2 - 2*b^3)*cosh(d*x + c)^5 + 4
*(12*a^2*b - 18*a*b^2 + 7*b^3)*cosh(d*x + c)^3 + 4*(16*a^3 - 48*a^2*b + 45
*a*b^2 - 14*b^3)*cosh(d*x + c))*sinh(d*x + c))/d
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 561 vs. 2(177) = 354.

Time = 0.73 (sec) , antiderivative size = 561, normalized size of antiderivative = 3.10

$$\int \sinh^2(c+dx)\left(a+b\sinh^2(c+dx)\right)^3\,dx = \text{Too large to display}$$

input
```
integrate(sinh(d*x+c)**2*(a+b*sinh(d*x+c)**2)**3,x)
```

output
```
Piecewise((a**3*x*sinh(c + d*x)**2/2 - a**3*x*cosh(c + d*x)**2/2 + a**3*si
nh(c + d*x)*cosh(c + d*x)/(2*d) + 9*a**2*b*x*sinh(c + d*x)**4/8 - 9*a**2*b
*x*sinh(c + d*x)**2*cosh(c + d*x)**2/4 + 9*a**2*b*x*cosh(c + d*x)**4/8 + 1
5*a**2*b*sinh(c + d*x)**3*cosh(c + d*x)/(8*d) - 9*a**2*b*sinh(c + d*x)*cos
h(c + d*x)**3/(8*d) + 15*a*b**2*x*sinh(c + d*x)**6/16 - 45*a*b**2*x*sinh(c
 + d*x)**4*cosh(c + d*x)**2/16 + 45*a*b**2*x*sinh(c + d*x)**2*cosh(c + d*x
)**4/16 - 15*a*b**2*x*cosh(c + d*x)**6/16 + 33*a*b**2*sinh(c + d*x)**5*cos
h(c + d*x)/(16*d) - 5*a*b**2*sinh(c + d*x)**3*cosh(c + d*x)**3/(2*d) + 15*
a*b**2*sinh(c + d*x)*cosh(c + d*x)**5/(16*d) + 35*b**3*x*sinh(c + d*x)**8/
128 - 35*b**3*x*sinh(c + d*x)**6*cosh(c + d*x)**2/32 + 105*b**3*x*sinh(c +
 d*x)**4*cosh(c + d*x)**4/64 - 35*b**3*x*sinh(c + d*x)**2*cosh(c + d*x)**6
/32 + 35*b**3*x*cosh(c + d*x)**8/128 + 93*b**3*sinh(c + d*x)**7*cosh(c + d
*x)/(128*d) - 511*b**3*sinh(c + d*x)**5*cosh(c + d*x)**3/(384*d) + 385*b**
3*sinh(c + d*x)**3*cosh(c + d*x)**5/(384*d) - 35*b**3*sinh(c + d*x)*cosh(c
 + d*x)**7/(128*d), Ne(d, 0)), (x*(a + b*sinh(c)**2)**3*sinh(c)**2, True))
```

Maxima [A] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 306, normalized size of antiderivative = 1.69

$$\int \sinh^2(c+dx)\left(a+b\sinh^2(c+dx)\right)^3\,dx$$
$$= \frac{3}{64}a^2b\left(24x+\frac{e^{(4dx+4c)}}{d}-\frac{8e^{(2dx+2c)}}{d}+\frac{8e^{(-2dx-2c)}}{d}-\frac{e^{(-4dx-4c)}}{d}\right)$$
$$-\frac{1}{8}a^3\left(4x-\frac{e^{(2dx+2c)}}{d}+\frac{e^{(-2dx-2c)}}{d}\right)$$
$$-\frac{1}{6144}b^3\left(\frac{\left(32e^{(-2dx-2c)}-168e^{(-4dx-4c)}+672e^{(-6dx-6c)}-3\right)e^{(8dx+8c)}}{d}-\frac{1680(dx+c)}{d}-\frac{672e^{(-2dx}}{}\right.$$
$$-\frac{1}{128}ab^2\left(\frac{\left(9e^{(-2dx-2c)}-45e^{(-4dx-4c)}-1\right)e^{(6dx+6c)}}{d}+\frac{120(dx+c)}{d}+\frac{45e^{(-2dx-2c)}-9e^{(-4dx-4c)}+}{d}\right.$$

input
```
integrate(sinh(d*x+c)^2*(a+b*sinh(d*x+c)^2)^3,x, algorithm="maxima")
```

output
```
3/64*a^2*b*(24*x + e^(4*d*x + 4*c)/d - 8*e^(2*d*x + 2*c)/d + 8*e^(-2*d*x -
 2*c)/d - e^(-4*d*x - 4*c)/d) - 1/8*a^3*(4*x - e^(2*d*x + 2*c)/d + e^(-2*d
*x - 2*c)/d) - 1/6144*b^3*((32*e^(-2*d*x - 2*c) - 168*e^(-4*d*x - 4*c) + 6
72*e^(-6*d*x - 6*c) - 3)*e^(8*d*x + 8*c)/d - 1680*(d*x + c)/d - (672*e^(-2
*d*x - 2*c) - 168*e^(-4*d*x - 4*c) + 32*e^(-6*d*x - 6*c) - 3*e^(-8*d*x - 8
*c))/d) - 1/128*a*b^2*((9*e^(-2*d*x - 2*c) - 45*e^(-4*d*x - 4*c) - 1)*e^(6
*d*x + 6*c)/d + 120*(d*x + c)/d + (45*e^(-2*d*x - 2*c) - 9*e^(-4*d*x - 4*c
) + e^(-6*d*x - 6*c))/d)
```

Giac [A] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 251, normalized size of antiderivative = 1.39

$$\begin{aligned}\int \sinh^2(c+dx)\left(a+b\sinh^2(c+dx)\right)^3\,dx &= \frac{b^3e^{(8\,dx+8\,c)}}{2048\,d} - \frac{b^3e^{(-8\,dx-8\,c)}}{2048\,d} - \frac{1}{128}\left(64\,a^3-144\,a^2b+120\,ab^2-35\,b^3\right)x \\ &+ \frac{(3\,ab^2-2\,b^3)e^{(6\,dx+6\,c)}}{384\,d} + \frac{(12\,a^2b-18\,ab^2+7\,b^3)e^{(4\,dx+4\,c)}}{256\,d} \\ &+ \frac{(16\,a^3-48\,a^2b+45\,ab^2-14\,b^3)e^{(2\,dx+2\,c)}}{128\,d} \\ &- \frac{(16\,a^3-48\,a^2b+45\,ab^2-14\,b^3)e^{(-2\,dx-2\,c)}}{128\,d} \\ &- \frac{(12\,a^2b-18\,ab^2+7\,b^3)e^{(-4\,dx-4\,c)}}{256\,d} - \frac{(3\,ab^2-2\,b^3)e^{(-6\,dx-6\,c)}}{384\,d}\end{aligned}$$

input
```
integrate(sinh(d*x+c)^2*(a+b*sinh(d*x+c)^2)^3,x, algorithm="giac")
```

output
```
1/2048*b^3*e^(8*d*x + 8*c)/d - 1/2048*b^3*e^(-8*d*x - 8*c)/d - 1/128*(64*a
^3 - 144*a^2*b + 120*a*b^2 - 35*b^3)*x + 1/384*(3*a*b^2 - 2*b^3)*e^(6*d*x
+ 6*c)/d + 1/256*(12*a^2*b - 18*a*b^2 + 7*b^3)*e^(4*d*x + 4*c)/d + 1/128*(
16*a^3 - 48*a^2*b + 45*a*b^2 - 14*b^3)*e^(2*d*x + 2*c)/d - 1/128*(16*a^3 -
 48*a^2*b + 45*a*b^2 - 14*b^3)*e^(-2*d*x - 2*c)/d - 1/256*(12*a^2*b - 18*a
*b^2 + 7*b^3)*e^(-4*d*x - 4*c)/d - 1/384*(3*a*b^2 - 2*b^3)*e^(-6*d*x - 6*c
)/d
```

Mupad [B] (verification not implemented)

Time = 0.47 (sec) , antiderivative size = 181, normalized size of antiderivative = 1.00

$$\int \sinh^2(c+dx)\left(a+b\sinh^2(c+dx)\right)^3\,dx$$
$$=\frac{96\,a^3\sinh(2\,c+2\,d\,x)-84\,b^3\sinh(2\,c+2\,d\,x)+21\,b^3\sinh(4\,c+4\,d\,x)-4\,b^3\sinh(6\,c+6\,d\,x)+\frac{3\,b^3\sinh(8}{8}}{}$$

input
```
int(sinh(c + d*x)^2*(a + b*sinh(c + d*x)^2)^3,x)
```

output
```
(96*a^3*sinh(2*c + 2*d*x) - 84*b^3*sinh(2*c + 2*d*x) + 21*b^3*sinh(4*c + 4
*d*x) - 4*b^3*sinh(6*c + 6*d*x) + (3*b^3*sinh(8*c + 8*d*x))/8 + 270*a*b^2*
sinh(2*c + 2*d*x) - 288*a^2*b*sinh(2*c + 2*d*x) - 54*a*b^2*sinh(4*c + 4*d*
x) + 36*a^2*b*sinh(4*c + 4*d*x) + 6*a*b^2*sinh(6*c + 6*d*x) - 192*a^3*d*x
+ 105*b^3*d*x - 360*a*b^2*d*x + 432*a^2*b*d*x)/(384*d)
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 388, normalized size of antiderivative = 2.14

$$\int \sinh^2(c+dx)\left(a+b\sinh^2(c+dx)\right)^3\,dx$$
$$=\frac{3e^{16dx+16c}b^3+48e^{14dx+14c}a\,b^2-32e^{14dx+14c}b^3+288e^{12dx+12c}a^2b-432e^{12dx+12c}a\,b^2+168e^{12dx+12c}b^3+76}{}$$

input
```
int(sinh(d*x+c)^2*(a+b*sinh(d*x+c)^2)^3,x)
```

output
```
(3*e**(16*c + 16*d*x)*b**3 + 48*e**(14*c + 14*d*x)*a*b**2 - 32*e**(14*c +
14*d*x)*b**3 + 288*e**(12*c + 12*d*x)*a**2*b - 432*e**(12*c + 12*d*x)*a*b*
*2 + 168*e**(12*c + 12*d*x)*b**3 + 768*e**(10*c + 10*d*x)*a**3 - 2304*e**(
10*c + 10*d*x)*a**2*b + 2160*e**(10*c + 10*d*x)*a*b**2 - 672*e**(10*c + 10
*d*x)*b**3 - 3072*e**(8*c + 8*d*x)*a**3*d*x + 6912*e**(8*c + 8*d*x)*a**2*b
*d*x - 5760*e**(8*c + 8*d*x)*a*b**2*d*x + 1680*e**(8*c + 8*d*x)*b**3*d*x -
 768*e**(6*c + 6*d*x)*a**3 + 2304*e**(6*c + 6*d*x)*a**2*b - 2160*e**(6*c +
 6*d*x)*a*b**2 + 672*e**(6*c + 6*d*x)*b**3 - 288*e**(4*c + 4*d*x)*a**2*b +
 432*e**(4*c + 4*d*x)*a*b**2 - 168*e**(4*c + 4*d*x)*b**3 - 48*e**(2*c + 2*
d*x)*a*b**2 + 32*e**(2*c + 2*d*x)*b**3 - 3*b**3)/(6144*e**(8*c + 8*d*x)*d)
```

3.22 $\int \sinh(c+dx)\left(a+b\sinh^2(c+dx)\right)^3\,dx$

Optimal result

Integrand size = 21, antiderivative size = 79

$$\int \sinh(c+dx)\left(a+b\sinh^2(c+dx)\right)^3\,dx = \frac{(a-b)^3\cosh(c+dx)}{d} + \frac{(a-b)^2b\cosh^3(c+dx)}{d} + \frac{3(a-b)b^2\cosh^5(c+dx)}{5d} + \frac{b^3\cosh^7(c+dx)}{7d}$$

output
```
(a-b)^3*cosh(d*x+c)/d+(a-b)^2*b*cosh(d*x+c)^3/d+3/5*(a-b)*b^2*cosh(d*x+c)^
5/d+1/7*b^3*cosh(d*x+c)^7/d
```

Mathematica [A] (verified)

Time = 0.86 (sec) , antiderivative size = 94, normalized size of antiderivative = 1.19

$$\int \sinh(c+dx)\left(a+b\sinh^2(c+dx)\right)^3\,dx = \frac{\cosh(c+dx)\left(1120a^3-2800a^2b+2492ab^2-762b^3+b(560a^2-784ab+299b^2)\cosh(2(c+dx))+6(14\right.}{1120d}$$

input
```
Integrate[Sinh[c + d*x]*(a + b*Sinh[c + d*x]^2)^3,x]
```

output
```
(Cosh[c + d*x]*(1120*a^3 - 2800*a^2*b + 2492*a*b^2 - 762*b^3 + b*(560*a^2
- 784*a*b + 299*b^2)*Cosh[2*(c + d*x)] + 6*(14*a - 9*b)*b^2*Cosh[4*(c + d*
x)] + 5*b^3*Cosh[6*(c + d*x)]))/(1120*d)
```

Rubi [A] (verified)

Time = 0.34 (sec) , antiderivative size = 71, normalized size of antiderivative = 0.90, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.238$, Rules used = {3042, 26, 3665, 210, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sinh(c+dx)\left(a+b\sinh^2(c+dx)\right)^3 dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int -i\sin(ic+idx)\left(a-b\sin(ic+idx)^2\right)^3 dx$$

$$\Big\downarrow \text{ 26}$$

$$-i\int \sin(ic+idx)\left(a-b\sin(ic+idx)^2\right)^3 dx$$

$$\Big\downarrow \text{ 3665}$$

$$\frac{\int \left(b\cosh^2(c+dx)+a-b\right)^3 d\cosh(c+dx)}{d}$$

$$\Big\downarrow \text{ 210}$$

$$\frac{\int \left(b^3\cosh^6(c+dx)+3ab^2\left(1-\frac{b}{a}\right)\cosh^4(c+dx)+3a^2b\left(\frac{b(b-2a)}{a^2}+1\right)\cosh^2(c+dx)+a^3\left(1-\frac{b(3a^2-3ba+b^2)}{a^3}\right)\right) d}{d}$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{\frac{3}{5}b^2(a-b)\cosh^5(c+dx)+b(a-b)^2\cosh^3(c+dx)+(a-b)^3\cosh(c+dx)+\frac{1}{7}b^3\cosh^7(c+dx)}{d}$$

input
```
Int[Sinh[c + d*x]*(a + b*Sinh[c + d*x]^2)^3,x]
```

output
```
((a - b)^3*Cosh[c + d*x] + (a - b)^2*b*Cosh[c + d*x]^3 + (3*(a - b)*b^2*Co
sh[c + d*x]^5)/5 + (b^3*Cosh[c + d*x]^7)/7)/d
```

Defintions of rubi rules used

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 210
```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Int[ExpandIntegrand[(a + b*x^2
)^p, x], x] /; FreeQ[{a, b}, x] && IGtQ[p, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3665
```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^
(p_.), x_Symbol] :> With[{ff = FreeFactors[Cos[e + f*x], x]}, Simp[-ff/f
Subst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a + b - b*ff^2*x^2)^p, x], x, Cos[e +
 f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m - 1)/2]
```

Maple [A] (verified)

Time = 10.99 (sec) , antiderivative size = 116, normalized size of antiderivative = 1.47

method	result
derivativedivides	$\frac{a^3\cosh(dx+c)+3a^2b\left(-\frac{2}{3}+\frac{\sinh(dx+c)^2}{3}\right)\cosh(dx+c)+3b^2a\left(\frac{8}{15}+\frac{\sinh(dx+c)^4}{5}-\frac{4\sinh(dx+c)^2}{15}\right)\cosh(dx+c)+b^3\left(-\frac{16}{35}\right.}{d}$
default	$\frac{a^3\cosh(dx+c)+3a^2b\left(-\frac{2}{3}+\frac{\sinh(dx+c)^2}{3}\right)\cosh(dx+c)+3b^2a\left(\frac{8}{15}+\frac{\sinh(dx+c)^4}{5}-\frac{4\sinh(dx+c)^2}{15}\right)\cosh(dx+c)+b^3\left(-\frac{16}{35}\right.}{d}$
parallelrisch	$\frac{(560a^2b-700b^2a+245b^3)\cosh(3dx+3c)+(84b^2a-49b^3)\cosh(5dx+5c)+5b^3\cosh(7dx+7c)+(2240a^3-5040a^2b+4200b}{2240d}$
parts	$\frac{b^3\left(-\frac{16}{35}+\frac{\sinh(dx+c)^6}{7}-\frac{6\sinh(dx+c)^4}{35}+\frac{8\sinh(dx+c)^2}{35}\right)\cosh(dx+c)}{d}+\frac{a^3\cosh(dx+c)}{d}+\frac{3a^2b\left(-\frac{2}{3}+\frac{\sinh(dx+c)^2}{3}\right)\mathrm{co}}{d}$
risch	$\frac{b^3e^{7dx+7c}}{896d}+\frac{3b^2e^{5dx+5c}a}{160d}-\frac{7b^3e^{5dx+5c}}{640d}+\frac{e^{3dx+3c}a^2b}{8d}-\frac{5\,e^{3dx+3c}b^2a}{32d}+\frac{7\,e^{3dx+3c}b^3}{128d}+\frac{e^{dx+c}a^3}{2d}-\frac{9\,e^{dx+}}{8d}$
orering	$\frac{\frac{12916d\cosh(dx+c)\left(a+b\sinh(dx+c)^2\right)^3}{11025}+\frac{25832\sinh(dx+c)^2\left(a+b\sinh(dx+c)^2\right)^2bd\cosh(dx+c)}{3675}}{d^2}-\frac{94\left(d^3\cosh(dx+c)\left(a+b\sin\right.\right.}{}$

input
```
int(sinh(d*x+c)*(a+b*sinh(d*x+c)^2)^3,x,method=_RETURNVERBOSE)
```

output
```
1/d*(a^3*cosh(d*x+c)+3*a^2*b*(-2/3+1/3*sinh(d*x+c)^2)*cosh(d*x+c)+3*b^2*a*
(8/15+1/5*sinh(d*x+c)^4-4/15*sinh(d*x+c)^2)*cosh(d*x+c)+b^3*(-16/35+1/7*si
nh(d*x+c)^6-6/35*sinh(d*x+c)^4+8/35*sinh(d*x+c)^2)*cosh(d*x+c))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 234 vs. 2(75) = 150.

Time = 0.10 (sec) , antiderivative size = 234, normalized size of antiderivative = 2.96

$$\int \sinh(c+dx)\left(a+b\sinh^2(c+dx)\right)^3\,dx$$
$$=\frac{5\,b^3\cosh(dx+c)^7+35\,b^3\cosh(dx+c)\sinh(dx+c)^6+7\left(12\,ab^2-7\,b^3\right)\cosh(dx+c)^5+35\left(5\,b^3\cosh\left(\right.\right.}{}$$

input
```
integrate(sinh(d*x+c)*(a+b*sinh(d*x+c)^2)^3,x, algorithm="fricas")
```

output
```
1/2240*(5*b^3*cosh(d*x + c)^7 + 35*b^3*cosh(d*x + c)*sinh(d*x + c)^6 + 7*(
12*a*b^2 - 7*b^3)*cosh(d*x + c)^5 + 35*(5*b^3*cosh(d*x + c)^3 + (12*a*b^2
- 7*b^3)*cosh(d*x + c))*sinh(d*x + c)^4 + 35*(16*a^2*b - 20*a*b^2 + 7*b^3)
*cosh(d*x + c)^3 + 35*(3*b^3*cosh(d*x + c)^5 + 2*(12*a*b^2 - 7*b^3)*cosh(d
*x + c)^3 + 3*(16*a^2*b - 20*a*b^2 + 7*b^3)*cosh(d*x + c))*sinh(d*x + c)^2
 + 35*(64*a^3 - 144*a^2*b + 120*a*b^2 - 35*b^3)*cosh(d*x + c))/d
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 221 vs. $2(68) = 136$.

Time = 0.50 (sec) , antiderivative size = 221, normalized size of antiderivative = 2.80

$$\int \sinh(c+dx)\left(a+b\sinh^2(c+dx)\right)^3\,dx$$
$$= \begin{cases} \frac{a^3\cosh(c+dx)}{d} + \frac{3a^2b\sinh^2(c+dx)\cosh(c+dx)}{d} - \frac{2a^2b\cosh^3(c+dx)}{d} + \frac{3ab^2\sinh^4(c+dx)\cosh(c+dx)}{d} - \frac{4ab^2\sinh^2(c+dx)\cosh^3(c-}{d} \\ x\left(a+b\sinh^2(c)\right)^3\sinh(c) \end{cases}$$

input
```
integrate(sinh(d*x+c)*(a+b*sinh(d*x+c)**2)**3,x)
```

output
```
Piecewise((a**3*cosh(c + d*x)/d + 3*a**2*b*sinh(c + d*x)**2*cosh(c + d*x)/
d - 2*a**2*b*cosh(c + d*x)**3/d + 3*a*b**2*sinh(c + d*x)**4*cosh(c + d*x)/
d - 4*a*b**2*sinh(c + d*x)**2*cosh(c + d*x)**3/d + 8*a*b**2*cosh(c + d*x)*
*5/(5*d) + b**3*sinh(c + d*x)**6*cosh(c + d*x)/d - 2*b**3*sinh(c + d*x)**4
*cosh(c + d*x)**3/d + 8*b**3*sinh(c + d*x)**2*cosh(c + d*x)**5/(5*d) - 16*
b**3*cosh(c + d*x)**7/(35*d), Ne(d, 0)), (x*(a + b*sinh(c)**2)**3*sinh(c),
 True))
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 263 vs. $2(75) = 150$.

Time = 0.05 (sec) , antiderivative size = 263, normalized size of antiderivative = 3.33

$$\int \sinh(c+dx)\left(a+b\sinh^2(c+dx)\right)^3\,dx = -\frac{1}{4480}b^3\left(\frac{\left(49\,e^{(-2\,dx-2\,c)}-245\,e^{(-4\,dx-4\,c)}+1225\,e^{(-6\,dx-6\,c)}-5\right)e^{(7\,dx+7\,c)}}{d}+\frac{1225\,e^{(-dx-c)}-245\,e^{(-3\,dx}\cdots\right. + \frac{1}{160}ab^2\left(\frac{3\,e^{(5\,dx+5\,c)}}{d}-\frac{25\,e^{(3\,dx+3\,c)}}{d}+\frac{150\,e^{(dx+c)}}{d}+\frac{150\,e^{(-dx-c)}}{d}-\frac{25\,e^{(-3\,dx-3\,c)}}{d}+\frac{3\,e^{(-5\,dx-5\,c)}}{d}\right) + \frac{1}{8}a^2b\left(\frac{e^{(3\,dx+3\,c)}}{d}-\frac{9\,e^{(dx+c)}}{d}-\frac{9\,e^{(-dx-c)}}{d}+\frac{e^{(-3\,dx-3\,c)}}{d}\right)+\frac{a^3\cosh\left(dx+c\right)}{d}$$

input

```
integrate(sinh(d*x+c)*(a+b*sinh(d*x+c)^2)^3,x, algorithm="maxima")
```

output

```
-1/4480*b^3*((49*e^(-2*d*x - 2*c) - 245*e^(-4*d*x - 4*c) + 1225*e^(-6*d*x
- 6*c) - 5)*e^(7*d*x + 7*c)/d + (1225*e^(-d*x - c) - 245*e^(-3*d*x - 3*c)
+ 49*e^(-5*d*x - 5*c) - 5*e^(-7*d*x - 7*c))/d) + 1/160*a*b^2*(3*e^(5*d*x +
 5*c)/d - 25*e^(3*d*x + 3*c)/d + 150*e^(d*x + c)/d + 150*e^(-d*x - c)/d -
25*e^(-3*d*x - 3*c)/d + 3*e^(-5*d*x - 5*c)/d) + 1/8*a^2*b*(e^(3*d*x + 3*c)
/d - 9*e^(d*x + c)/d - 9*e^(-d*x - c)/d + e^(-3*d*x - 3*c)/d) + a^3*cosh(d
*x + c)/d
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 222 vs. $2(75) = 150$.

Time = 0.15 (sec) , antiderivative size = 222, normalized size of antiderivative = 2.81

$$\int \sinh(c+dx)\left(a+b\sinh^2(c+dx)\right)^3\,dx = \frac{b^3e^{(7\,dx+7\,c)}}{896\,d} + \frac{b^3e^{(-7\,dx-7\,c)}}{896\,d} + \frac{(12\,ab^2-7\,b^3)e^{(5\,dx+5\,c)}}{640\,d} + \frac{(16\,a^2b-20\,ab^2+7\,b^3)e^{(3\,dx+3\,c)}}{128\,d} + \frac{(64\,a^3-144\,a^2b+120\,ab^2-35\,b^3)e^{(dx+c)}}{128\,d} + \frac{(64\,a^3-144\,a^2b+120\,ab^2-35\,b^3)e^{(-dx-c)}}{128\,d} + \frac{(16\,a^2b-20\,ab^2+7\,b^3)e^{(-3\,dx-3\,c)}}{128\,d} + \frac{(12\,ab^2-7\,b^3)e^{(-5\,dx-5\,c)}}{640\,d}$$

input
```
integrate(sinh(d*x+c)*(a+b*sinh(d*x+c)^2)^3,x, algorithm="giac")
```

output
```
1/896*b^3*e^(7*d*x + 7*c)/d + 1/896*b^3*e^(-7*d*x - 7*c)/d + 1/640*(12*a*b
^2 - 7*b^3)*e^(5*d*x + 5*c)/d + 1/128*(16*a^2*b - 20*a*b^2 + 7*b^3)*e^(3*d
*x + 3*c)/d + 1/128*(64*a^3 - 144*a^2*b + 120*a*b^2 - 35*b^3)*e^(d*x + c)/
d + 1/128*(64*a^3 - 144*a^2*b + 120*a*b^2 - 35*b^3)*e^(-d*x - c)/d + 1/128
*(16*a^2*b - 20*a*b^2 + 7*b^3)*e^(-3*d*x - 3*c)/d + 1/640*(12*a*b^2 - 7*b^
3)*e^(-5*d*x - 5*c)/d
```

Mupad [B] (verification not implemented)

Time = 1.88 (sec) , antiderivative size = 129, normalized size of antiderivative = 1.63

$$\int \sinh(c+dx)\left(a+b\sinh^2(c+dx)\right)^3\,dx$$

$$= \frac{a^3\cosh(c+d\,x)+a^2\,b\cosh(c+d\,x)^3-3\,a^2\,b\cosh(c+d\,x)+\frac{3\,a\,b^2\cosh(c+d\,x)^5}{5}-2\,a\,b^2\cosh(c+d\,x)^3+3\,a}{d}$$

input
```
int(sinh(c + d*x)*(a + b*sinh(c + d*x)^2)^3,x)
```

output
```
(a^3*cosh(c + d*x) - b^3*cosh(c + d*x) + b^3*cosh(c + d*x)^3 - (3*b^3*cosh
(c + d*x)^5)/5 + (b^3*cosh(c + d*x)^7)/7 - 2*a*b^2*cosh(c + d*x)^3 + a^2*b
*cosh(c + d*x)^3 + (3*a*b^2*cosh(c + d*x)^5)/5 + 3*a*b^2*cosh(c + d*x) - 3
*a^2*b*cosh(c + d*x))/d
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 318, normalized size of antiderivative = 4.03

$$\int \sinh(c+dx)\left(a+b\sinh^2(c+dx)\right)^3\,dx = \frac{5e^{14dx+14c}b^3+84e^{12dx+12c}a\,b^2-49e^{12dx+12c}b^3+560e^{10dx+10c}a^2b-700e^{10dx+10c}a\,b^2+245e^{10dx+10c}b^3+22}{}$$

input
```
int(sinh(d*x+c)*(a+b*sinh(d*x+c)^2)^3,x)
```

output
```
(5*e**(14*c + 14*d*x)*b**3 + 84*e**(12*c + 12*d*x)*a*b**2 - 49*e**(12*c +
12*d*x)*b**3 + 560*e**(10*c + 10*d*x)*a**2*b - 700*e**(10*c + 10*d*x)*a*b*
*2 + 245*e**(10*c + 10*d*x)*b**3 + 2240*e**(8*c + 8*d*x)*a**3 - 5040*e**(8
*c + 8*d*x)*a**2*b + 4200*e**(8*c + 8*d*x)*a*b**2 - 1225*e**(8*c + 8*d*x)*
b**3 + 2240*e**(6*c + 6*d*x)*a**3 - 5040*e**(6*c + 6*d*x)*a**2*b + 4200*e*
*(6*c + 6*d*x)*a*b**2 - 1225*e**(6*c + 6*d*x)*b**3 + 560*e**(4*c + 4*d*x)*
a**2*b - 700*e**(4*c + 4*d*x)*a*b**2 + 245*e**(4*c + 4*d*x)*b**3 + 84*e**(
2*c + 2*d*x)*a*b**2 - 49*e**(2*c + 2*d*x)*b**3 + 5*b**3)/(4480*e**(7*c + 7
*d*x)*d)
```

3.23 $\int \left(a+b\sinh^2(c+dx)\right)^3\,dx$

Optimal result

Integrand size = 14, antiderivative size = 128

$$\begin{aligned}\int \left(a+b\sinh^2(c+dx)\right)^3\,dx &= \frac{1}{16}(2a-b)\left(8a^2-8ab+5b^2\right)x \\ &+ \frac{b(64a^2-54ab+15b^2)\cosh(c+dx)\sinh(c+dx)}{48d} \\ &+ \frac{5(2a-b)b^2\cosh(c+dx)\sinh^3(c+dx)}{24d} \\ &+ \frac{b\cosh(c+dx)\sinh(c+dx)\left(a+b\sinh^2(c+dx)\right)^2}{6d}\end{aligned}$$

output
```
1/16*(2*a-b)*(8*a^2-8*a*b+5*b^2)*x+1/48*b*(64*a^2-54*a*b+15*b^2)*cosh(d*x+
c)*sinh(d*x+c)/d+5/24*(2*a-b)*b^2*cosh(d*x+c)*sinh(d*x+c)^3/d+1/6*b*cosh(d
*x+c)*sinh(d*x+c)*(a+b*sinh(d*x+c)^2)^2/d
```

Mathematica [A] (verified)

Time = 0.33 (sec) , antiderivative size = 95, normalized size of antiderivative = 0.74

$$\int \left(a+b\sinh^2(c+dx)\right)^3\,dx$$
$$=\frac{12(2a-b)\left(8a^2-8ab+5b^2\right)(c+dx)+9b\left(16a^2-16ab+5b^2\right)\sinh(2(c+dx))+9(2a-b)b^2\sinh(4(c+$$
$$192d$$

input
```
Integrate[(a + b*Sinh[c + d*x]^2)^3,x]
```

output
```
(12*(2*a - b)*(8*a^2 - 8*a*b + 5*b^2)*(c + d*x) + 9*b*(16*a^2 - 16*a*b + 5
*b^2)*Sinh[2*(c + d*x)] + 9*(2*a - b)*b^2*Sinh[4*(c + d*x)] + b^3*Sinh[6*(
c + d*x)])/(192*d)
```

Rubi [A] (verified)

Time = 0.43 (sec) , antiderivative size = 133, normalized size of antiderivative = 1.04, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {3042, 3659, 3042, 3648}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(a+b\sinh^2(c+dx)\right)^3\,dx$$
$$\Big\downarrow \text{3042}$$
$$\int \left(a-b\sin(ic+idx)^2\right)^3dx$$
$$\Big\downarrow \text{3659}$$
$$\frac{1}{6}\int \left(b\sinh^2(c+dx)+a\right)\left(5(2a-b)b\sinh^2(c+dx)+a(6a-b)\right)dx+\frac{b\sinh(c+dx)\cosh(c+dx)\left(a+b\sinh^2(c+dx)\right)^2}{6d}$$
$$\Big\downarrow \text{3042}$$

$$\frac{b\sinh(c+dx)\cosh(c+dx)\left(a+b\sinh^2(c+dx)\right)^2}{6d}+\frac{1}{6}\int\left(a-b\sin(ic+idx)^2\right)\left(a(6a-b)-5(2a-b)b\sin(ic+idx)^2\right)dx$$

$\big\downarrow$ 3648

$$\frac{1}{6}\left(\frac{b(64a^2-54ab+15b^2)\sinh(c+dx)\cosh(c+dx)}{8d}+\frac{3}{8}x(2a-b)\left(8a^2-8ab+5b^2\right)+\frac{5b^2(2a-b)\sinh^3(c+dx)\cdots}{4d}\right.$$
$$\frac{b\sinh(c+dx)\cosh(c+dx)\left(a+b\sinh^2(c+dx)\right)^2}{6d}$$

input
```
Int[(a + b*Sinh[c + d*x]^2)^3,x]
```

output
```
(b*Cosh[c + d*x]*Sinh[c + d*x]*(a + b*Sinh[c + d*x]^2)^2)/(6*d) + ((3*(2*a
 - b)*(8*a^2 - 8*a*b + 5*b^2)*x)/8 + (b*(64*a^2 - 54*a*b + 15*b^2)*Cosh[c
+ d*x]*Sinh[c + d*x])/(8*d) + (5*(2*a - b)*b^2*Cosh[c + d*x]*Sinh[c + d*x]
^3)/(4*d))/6
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3648
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)*((A_.) + (B_.)*sin[(e_.) + (f_
.)*(x_)]^2), x_Symbol] :> Simp[(4*A*(2*a + b) + B*(4*a + 3*b))*(x/8), x] +
(-Simp[b*B*Cos[e + f*x]*(Sin[e + f*x]^3/(4*f)), x] - Simp[(4*A*b + B*(4*a +
 3*b))*Cos[e + f*x]*(Sin[e + f*x]/(8*f)), x]) /; FreeQ[{a, b, e, f, A, B},
x]
```

rule 3659
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> Simp[(-b)*C
os[e + f*x]*Sin[e + f*x]*((a + b*Sin[e + f*x]^2)^(p - 1)/(2*f*p)), x] + Sim
p[1/(2*p)   Int[(a + b*Sin[e + f*x]^2)^(p - 2)*Simp[a*(b + 2*a*p) + b*(2*a
+ b)*(2*p - 1)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, e, f}, x] && NeQ[
a + b, 0] && GtQ[p, 1]
```

Maple [A] (verified)

Time = 0.15 (sec) , antiderivative size = 131, normalized size of antiderivative = 1.02

$$\frac{b^3\left(\left(\frac{\sinh(dx+c)^5}{6}-\frac{5\sinh(dx+c)^3}{24}+\frac{5\sinh(dx+c)}{16}\right)\cosh\left(dx+c\right)-\frac{5dx}{16}-\frac{5c}{16}\right)+3b^2a\left(\left(\frac{\sinh(dx+c)^3}{4}-\frac{3\sinh(dx+c)}{8}\right)\cos\right.}{d}$$

input
```
int((a+b*sinh(d*x+c)^2)^3,x)
```

output
```
1/d*(b^3*((1/6*sinh(d*x+c)^5-5/24*sinh(d*x+c)^3+5/16*sinh(d*x+c))*cosh(d*x
+c)-5/16*d*x-5/16*c)+3*b^2*a*((1/4*sinh(d*x+c)^3-3/8*sinh(d*x+c))*cosh(d*x
+c)+3/8*d*x+3/8*c)+3*a^2*b*(1/2*cosh(d*x+c)*sinh(d*x+c)-1/2*d*x-1/2*c)+a^3
*(d*x+c))
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 165, normalized size of antiderivative = 1.29

$$\int \left(a+b\sinh^2(c+dx)\right)^3\,dx$$
$$=\frac{3\,b^3\cosh\left(dx+c\right)\sinh\left(dx+c\right)^5+2\left(5\,b^3\cosh\left(dx+c\right)^3+9\left(2\,ab^2-b^3\right)\cosh\left(dx+c\right)\right)\sinh\left(dx+c\right)^3+6}{}$$

input
```
integrate((a+b*sinh(d*x+c)^2)^3,x, algorithm="fricas")
```

output
```
1/96*(3*b^3*cosh(d*x + c)*sinh(d*x + c)^5 + 2*(5*b^3*cosh(d*x + c)^3 + 9*(
2*a*b^2 - b^3)*cosh(d*x + c))*sinh(d*x + c)^3 + 6*(16*a^3 - 24*a^2*b + 18*
a*b^2 - 5*b^3)*d*x + 3*(b^3*cosh(d*x + c)^5 + 6*(2*a*b^2 - b^3)*cosh(d*x +
 c)^3 + 3*(16*a^2*b - 16*a*b^2 + 5*b^3)*cosh(d*x + c))*sinh(d*x + c))/d
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 350 vs. $2(119) = 238$.

Time = 0.40 (sec) , antiderivative size = 350, normalized size of antiderivative = 2.73

$$\int \left(a + b\sinh^2(c + dx)\right)^3 dx$$
$$= \begin{cases} a^3x + \frac{3a^2bx\sinh^2(c+dx)}{2} - \frac{3a^2bx\cosh^2(c+dx)}{2} + \frac{3a^2b\sinh(c+dx)\cosh(c+dx)}{2d} + \frac{9ab^2x\sinh^4(c+dx)}{8} - \frac{9ab^2x\sinh^2(c+dx)\cosh^2}{4} \\ x\left(a + b\sinh^2(c)\right)^3 \end{cases}$$

input
```
integrate((a+b*sinh(d*x+c)**2)**3,x)
```

output
```
Piecewise((a**3*x + 3*a**2*b*x*sinh(c + d*x)**2/2 - 3*a**2*b*x*cosh(c + d*
x)**2/2 + 3*a**2*b*sinh(c + d*x)*cosh(c + d*x)/(2*d) + 9*a*b**2*x*sinh(c +
 d*x)**4/8 - 9*a*b**2*x*sinh(c + d*x)**2*cosh(c + d*x)**2/4 + 9*a*b**2*x*c
osh(c + d*x)**4/8 + 15*a*b**2*sinh(c + d*x)**3*cosh(c + d*x)/(8*d) - 9*a*b
**2*sinh(c + d*x)*cosh(c + d*x)**3/(8*d) + 5*b**3*x*sinh(c + d*x)**6/16 -
15*b**3*x*sinh(c + d*x)**4*cosh(c + d*x)**2/16 + 15*b**3*x*sinh(c + d*x)**
2*cosh(c + d*x)**4/16 - 5*b**3*x*cosh(c + d*x)**6/16 + 11*b**3*sinh(c + d*
x)**5*cosh(c + d*x)/(16*d) - 5*b**3*sinh(c + d*x)**3*cosh(c + d*x)**3/(6*d
) + 5*b**3*sinh(c + d*x)*cosh(c + d*x)**5/(16*d), Ne(d, 0)), (x*(a + b*sin
h(c)**2)**3, True))
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 197, normalized size of antiderivative = 1.54

$$\int \left(a + b\sinh^2(c + dx)\right)^3 dx$$
$$= \frac{3}{64}ab^2\left(24x + \frac{e^{(4dx+4c)}}{d} - \frac{8e^{(2dx+2c)}}{d} + \frac{8e^{(-2dx-2c)}}{d} - \frac{e^{(-4dx-4c)}}{d}\right)$$
$$- \frac{3}{8}a^2b\left(4x - \frac{e^{(2dx+2c)}}{d} + \frac{e^{(-2dx-2c)}}{d}\right) + a^3x$$
$$- \frac{1}{384}b^3\left(\frac{\left(9e^{(-2dx-2c)} - 45e^{(-4dx-4c)} - 1\right)e^{(6dx+6c)}}{d} + \frac{120(dx + c)}{d} + \frac{45e^{(-2dx-2c)} - 9e^{(-4dx-4c)} + e}{d}\right.$$

input
```
integrate((a+b*sinh(d*x+c)^2)^3,x, algorithm="maxima")
```

output
```
3/64*a*b^2*(24*x + e^(4*d*x + 4*c)/d - 8*e^(2*d*x + 2*c)/d + 8*e^(-2*d*x -
 2*c)/d - e^(-4*d*x - 4*c)/d) - 3/8*a^2*b*(4*x - e^(2*d*x + 2*c)/d + e^(-2
*d*x - 2*c)/d) + a^3*x - 1/384*b^3*((9*e^(-2*d*x - 2*c) - 45*e^(-4*d*x - 4
*c) - 1)*e^(6*d*x + 6*c)/d + 120*(d*x + c)/d + (45*e^(-2*d*x - 2*c) - 9*e^
(-4*d*x - 4*c) + e^(-6*d*x - 6*c))/d)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 177, normalized size of antiderivative = 1.38

$$\int \left(a+b\sinh^2(c+dx)\right)^3\,dx = \frac{b^3e^{(6\,dx+6\,c)}}{384\,d} - \frac{b^3e^{(-6\,dx-6\,c)}}{384\,d} + \frac{1}{16}\left(16\,a^3-24\,a^2b+18\,ab^2-5\,b^3\right)x + \frac{3\,(2\,ab^2-b^3)e^{(4\,dx+4\,c)}}{128\,d} + \frac{3\,(16\,a^2b-16\,ab^2+5\,b^3)e^{(2\,dx+2\,c)}}{128\,d} - \frac{3\,(16\,a^2b-16\,ab^2+5\,b^3)e^{(-2\,dx-2\,c)}}{128\,d} - \frac{3\,(2\,ab^2-b^3)e^{(-4\,dx-4\,c)}}{128\,d}$$

input
```
integrate((a+b*sinh(d*x+c)^2)^3,x, algorithm="giac")
```

output
```
1/384*b^3*e^(6*d*x + 6*c)/d - 1/384*b^3*e^(-6*d*x - 6*c)/d + 1/16*(16*a^3
- 24*a^2*b + 18*a*b^2 - 5*b^3)*x + 3/128*(2*a*b^2 - b^3)*e^(4*d*x + 4*c)/d
 + 3/128*(16*a^2*b - 16*a*b^2 + 5*b^3)*e^(2*d*x + 2*c)/d - 3/128*(16*a^2*b
 - 16*a*b^2 + 5*b^3)*e^(-2*d*x - 2*c)/d - 3/128*(2*a*b^2 - b^3)*e^(-4*d*x
- 4*c)/d
```

Mupad [B] (verification not implemented)

Time = 1.87 (sec) , antiderivative size = 123, normalized size of antiderivative = 0.96

$$\int \left(a+b\sinh^2(c+dx)\right)^3\,dx$$
$$=\frac{\frac{45\,b^3\sinh(2\,c+2\,d\,x)}{4}-\frac{9\,b^3\sinh(4\,c+4\,d\,x)}{4}+\frac{b^3\sinh(6\,c+6\,d\,x)}{4}-36\,a\,b^2\sinh(2\,c+2\,d\,x)+36\,a^2\,b\sinh(2\,c+2\,d\,x)+\cdots}{48\,d}$$

input
```
int((a + b*sinh(c + d*x)^2)^3,x)
```

output
```
((45*b^3*sinh(2*c + 2*d*x))/4 - (9*b^3*sinh(4*c + 4*d*x))/4 + (b^3*sinh(6*
c + 6*d*x))/4 - 36*a*b^2*sinh(2*c + 2*d*x) + 36*a^2*b*sinh(2*c + 2*d*x) +
(9*a*b^2*sinh(4*c + 4*d*x))/2 + 48*a^3*d*x - 15*b^3*d*x + 54*a*b^2*d*x - 7
2*a^2*b*d*x)/(48*d)
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 263, normalized size of antiderivative = 2.05

$$\int \left(a+b\sinh^2(c+dx)\right)^3\,dx$$
$$=\frac{e^{12dx+12c}b^3+18e^{10dx+10c}a\,b^2-9e^{10dx+10c}b^3+144e^{8dx+8c}a^2b-144e^{8dx+8c}a\,b^2+45e^{8dx+8c}b^3+384e^{6dx+6c}\cdots}{\cdots}$$

input
```
int((a+b*sinh(d*x+c)^2)^3,x)
```

output
```
(e**(12*c + 12*d*x)*b**3 + 18*e**(10*c + 10*d*x)*a*b**2 - 9*e**(10*c + 10*
d*x)*b**3 + 144*e**(8*c + 8*d*x)*a**2*b - 144*e**(8*c + 8*d*x)*a*b**2 + 45
*e**(8*c + 8*d*x)*b**3 + 384*e**(6*c + 6*d*x)*a**3*d*x - 576*e**(6*c + 6*d
*x)*a**2*b*d*x + 432*e**(6*c + 6*d*x)*a*b**2*d*x - 120*e**(6*c + 6*d*x)*b*
*3*d*x - 144*e**(4*c + 4*d*x)*a**2*b + 144*e**(4*c + 4*d*x)*a*b**2 - 45*e*
*(4*c + 4*d*x)*b**3 - 18*e**(2*c + 2*d*x)*a*b**2 + 9*e**(2*c + 2*d*x)*b**3
 - b**3)/(384*e**(6*c + 6*d*x)*d)
```

3.24 $\int \operatorname{csch}(c+dx)\left(a+b\sinh^2(c+dx)\right)^3\,dx$

Optimal result

Integrand size = 21, antiderivative size = 83

$$\int \operatorname{csch}(c+dx)\left(a+b\sinh^2(c+dx)\right)^3\,dx = -\frac{a^3\operatorname{arctanh}(\cosh(c+dx))}{d} + \frac{b(3a^2-3ab+b^2)\cosh(c+dx)}{d} + \frac{(3a-2b)b^2\cosh^3(c+dx)}{3d} + \frac{b^3\cosh^5(c+dx)}{5d}$$

output
```
-a^3*arctanh(cosh(d*x+c))/d+b*(3*a^2-3*a*b+b^2)*cosh(d*x+c)/d+1/3*(3*a-2*b
)*b^2*cosh(d*x+c)^3/d+1/5*b^3*cosh(d*x+c)^5/d
```

Mathematica [A] (verified)

Time = 2.64 (sec) , antiderivative size = 95, normalized size of antiderivative = 1.14

$$\int \operatorname{csch}(c+dx)\left(a+b\sinh^2(c+dx)\right)^3\,dx = \frac{30b(24a^2-18ab+5b^2)\cosh(c+dx)+5(12a-5b)b^2\cosh(3(c+dx))+3b^3\cosh(5(c+dx))+240a^3(-}{240d}$$

input
```
Integrate[Csch[c + d*x]*(a + b*Sinh[c + d*x]^2)^3,x]
```

output
```
(30*b*(24*a^2 - 18*a*b + 5*b^2)*Cosh[c + d*x] + 5*(12*a - 5*b)*b^2*Cosh[3*
(c + d*x)] + 3*b^3*Cosh[5*(c + d*x)] + 240*a^3*(-Log[Cosh[(c + d*x)/2]] +
Log[Sinh[(c + d*x)/2]]))/(240*d)
```

Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 76, normalized size of antiderivative = 0.92, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.238$, Rules used = {3042, 26, 3665, 300, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \operatorname{csch}(c+dx) \left(a+b\sinh^2(c+dx)\right)^3 dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{i\left(a-b\sin(ic+idx)^2\right)^3}{\sin(ic+idx)}dx$$

$$\Big\downarrow \text{26}$$

$$i\int \frac{\left(a-b\sin(ic+idx)^2\right)^3}{\sin(ic+idx)}dx$$

$$\Big\downarrow \text{3665}$$

$$-\frac{\int \frac{\left(b\cosh^2(c+dx)+a-b\right)^3}{1-\cosh^2(c+dx)}d\cosh(c+dx)}{d}$$

$$\Big\downarrow \text{300}$$

$$-\frac{\int \left(-b^3\cosh^4(c+dx)-(3a-2b)b^2\cosh^2(c+dx)-b\left(3a^2-3ba+b^2\right)+\frac{a^3}{1-\cosh^2(c+dx)}\right)d\cosh(c+dx)}{d}$$

$$\Big\downarrow \text{2009}$$

$$-\frac{a^3\text{arctanh}(\cosh(c+dx)) - b(3a^2-3ab+b^2)\cosh(c+dx) - \frac{1}{3}b^2(3a-2b)\cosh^3(c+dx) - \frac{1}{5}b^3\cosh^5(c+dx)}{d}$$

input
```
Int[Csch[c + d*x]*(a + b*Sinh[c + d*x]^2)^3,x]
```

output
```
-((a^3*ArcTanh[Cosh[c + d*x]] - b*(3*a^2 - 3*a*b + b^2)*Cosh[c + d*x] - ((
3*a - 2*b)*b^2*Cosh[c + d*x]^3)/3 - (b^3*Cosh[c + d*x]^5)/5)/d)
```

Defintions of rubi rules used

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 300
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_), x_Symbol] :> Int
[PolynomialDivide[(a + b*x^2)^p, (c + d*x^2)^(-q), x], x] /; FreeQ[{a, b, c
, d}, x] && NeQ[b*c - a*d, 0] && IGtQ[p, 0] && ILtQ[q, 0] && GeQ[p, -q]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3665
```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^
(p_.), x_Symbol] :> With[{ff = FreeFactors[Cos[e + f*x], x]}, Simp[-ff/f
Subst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a + b - b*ff^2*x^2)^p, x], x, Cos[e +
 f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m - 1)/2]
```

Maple [A] (verified)

Time = 1.26 (sec) , antiderivative size = 86, normalized size of antiderivative = 1.04

method	result
derivativedivides	$\frac{-2a^3\,\mathrm{arctanh}\left(e^{dx+c}\right)+3a^2b\cosh(dx+c)+3b^2a\left(-\frac{2}{3}+\frac{\sinh(dx+c)^2}{3}\right)\cosh(dx+c)+b^3\left(\frac{8}{15}+\frac{\sinh(dx+c)^4}{5}-\frac{4\sinh(dx+c)^2}{15}\right.}{d}$
default	$\frac{-2a^3\,\mathrm{arctanh}\left(e^{dx+c}\right)+3a^2b\cosh(dx+c)+3b^2a\left(-\frac{2}{3}+\frac{\sinh(dx+c)^2}{3}\right)\cosh(dx+c)+b^3\left(\frac{8}{15}+\frac{\sinh(dx+c)^4}{5}-\frac{4\sinh(dx+c)^2}{15}\right.}{d}$
parallelrisch	$\frac{a^3\ln\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+3\left(\frac{b\left(a-\frac{5b}{12}\right)\cosh(3dx+3c)}{12}+\frac{b^2\cosh(5dx+5c)}{240}+\left(a^2-\frac{3}{4}ab+\frac{5}{24}b^2\right)\cosh(dx+c)+a^2-\frac{2ab}{3}+\frac{8b^2}{45}\right)b}{d}$
risch	$\frac{b^3e^{5dx+5c}}{160d}+\frac{e^{3dx+3c}b^2a}{8d}-\frac{5\,e^{3dx+3c}b^3}{96d}+\frac{3\,e^{dx+c}a^2b}{2d}-\frac{9\,e^{dx+c}b^2a}{8d}+\frac{5\,e^{dx+c}b^3}{16d}+\frac{3\,e^{-dx-c}a^2b}{2d}-\frac{9\,e^{-dx-}}{8d}$

input
```
int(csch(d*x+c)*(a+b*sinh(d*x+c)^2)^3,x,method=_RETURNVERBOSE)
```

output
```
1/d*(-2*a^3*arctanh(exp(d*x+c))+3*a^2*b*cosh(d*x+c)+3*b^2*a*(-2/3+1/3*sinh
(d*x+c)^2)*cosh(d*x+c)+b^3*(8/15+1/5*sinh(d*x+c)^4-4/15*sinh(d*x+c)^2)*cos
h(d*x+c))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1128 vs. 2(79) = 158.

Time = 0.11 (sec) , antiderivative size = 1128, normalized size of antiderivative = 13.59

$$\int \operatorname{csch}(c+dx)\left(a+b\sinh^2(c+dx)\right)^3\,dx = \text{Too large to display}$$

input
```
integrate(csch(d*x+c)*(a+b*sinh(d*x+c)^2)^3,x, algorithm="fricas")
```

output
```
1/480*(3*b^3*cosh(d*x + c)^10 + 30*b^3*cosh(d*x + c)*sinh(d*x + c)^9 + 3*b
^3*sinh(d*x + c)^10 + 5*(12*a*b^2 - 5*b^3)*cosh(d*x + c)^8 + 5*(27*b^3*cos
h(d*x + c)^2 + 12*a*b^2 - 5*b^3)*sinh(d*x + c)^8 + 40*(9*b^3*cosh(d*x + c)
^3 + (12*a*b^2 - 5*b^3)*cosh(d*x + c))*sinh(d*x + c)^7 + 30*(24*a^2*b - 18
*a*b^2 + 5*b^3)*cosh(d*x + c)^6 + 10*(63*b^3*cosh(d*x + c)^4 + 72*a^2*b -
54*a*b^2 + 15*b^3 + 14*(12*a*b^2 - 5*b^3)*cosh(d*x + c)^2)*sinh(d*x + c)^6
 + 4*(189*b^3*cosh(d*x + c)^5 + 70*(12*a*b^2 - 5*b^3)*cosh(d*x + c)^3 + 45
*(24*a^2*b - 18*a*b^2 + 5*b^3)*cosh(d*x + c))*sinh(d*x + c)^5 + 30*(24*a^2
*b - 18*a*b^2 + 5*b^3)*cosh(d*x + c)^4 + 10*(63*b^3*cosh(d*x + c)^6 + 35*(
12*a*b^2 - 5*b^3)*cosh(d*x + c)^4 + 72*a^2*b - 54*a*b^2 + 15*b^3 + 45*(24*
a^2*b - 18*a*b^2 + 5*b^3)*cosh(d*x + c)^2)*sinh(d*x + c)^4 + 40*(9*b^3*cos
h(d*x + c)^7 + 7*(12*a*b^2 - 5*b^3)*cosh(d*x + c)^5 + 15*(24*a^2*b - 18*a*
b^2 + 5*b^3)*cosh(d*x + c)^3 + 3*(24*a^2*b - 18*a*b^2 + 5*b^3)*cosh(d*x +
c))*sinh(d*x + c)^3 + 3*b^3 + 5*(12*a*b^2 - 5*b^3)*cosh(d*x + c)^2 + 5*(27
*b^3*cosh(d*x + c)^8 + 28*(12*a*b^2 - 5*b^3)*cosh(d*x + c)^6 + 90*(24*a^2*
b - 18*a*b^2 + 5*b^3)*cosh(d*x + c)^4 + 12*a*b^2 - 5*b^3 + 36*(24*a^2*b -
18*a*b^2 + 5*b^3)*cosh(d*x + c)^2)*sinh(d*x + c)^2 - 480*(a^3*cosh(d*x + c
)^5 + 5*a^3*cosh(d*x + c)^4*sinh(d*x + c) + 10*a^3*cosh(d*x + c)^3*sinh(d*
x + c)^2 + 10*a^3*cosh(d*x + c)^2*sinh(d*x + c)^3 + 5*a^3*cosh(d*x + c)*si
nh(d*x + c)^4 + a^3*sinh(d*x + c)^5)*log(cosh(d*x + c) + sinh(d*x + c) ...
```

Sympy [F(-1)]

Timed out.

$$\int \operatorname{csch}(c+dx)\left(a+b\sinh^2(c+dx)\right)^3\,dx=\text{Timed out}$$

input
```
integrate(csch(d*x+c)*(a+b*sinh(d*x+c)**2)**3,x)
```

output
```
Timed out
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 193 vs. $2(79) = 158$.

Time = 0.05 (sec) , antiderivative size = 193, normalized size of antiderivative = 2.33

$$\int \operatorname{csch}(c+dx)\left(a+b\sinh^2(c+dx)\right)^3\,dx$$
$$= \frac{1}{480}b^3\left(\frac{3e^{(5dx+5c)}}{d}-\frac{25e^{(3dx+3c)}}{d}+\frac{150e^{(dx+c)}}{d}+\frac{150e^{(-dx-c)}}{d}-\frac{25e^{(-3dx-3c)}}{d}+\frac{3e^{(-5dx-5c)}}{d}\right)$$
$$+\frac{1}{8}ab^2\left(\frac{e^{(3dx+3c)}}{d}-\frac{9e^{(dx+c)}}{d}-\frac{9e^{(-dx-c)}}{d}+\frac{e^{(-3dx-3c)}}{d}\right)$$
$$+\frac{3}{2}a^2b\left(\frac{e^{(dx+c)}}{d}+\frac{e^{(-dx-c)}}{d}\right)+\frac{a^3\log\left(\tanh\left(\frac{1}{2}dx+\frac{1}{2}c\right)\right)}{d}$$

input
```
integrate(csch(d*x+c)*(a+b*sinh(d*x+c)^2)^3,x, algorithm="maxima")
```

output
```
1/480*b^3*(3*e^(5*d*x + 5*c)/d - 25*e^(3*d*x + 3*c)/d + 150*e^(d*x + c)/d
+ 150*e^(-d*x - c)/d - 25*e^(-3*d*x - 3*c)/d + 3*e^(-5*d*x - 5*c)/d) + 1/8
*a*b^2*(e^(3*d*x + 3*c)/d - 9*e^(d*x + c)/d - 9*e^(-d*x - c)/d + e^(-3*d*x
 - 3*c)/d) + 3/2*a^2*b*(e^(d*x + c)/d + e^(-d*x - c)/d) + a^3*log(tanh(1/2
*d*x + 1/2*c))/d
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 202 vs. $2(79) = 158$.

Time = 0.17 (sec) , antiderivative size = 202, normalized size of antiderivative = 2.43

$$\int \operatorname{csch}(c+dx)\left(a+b\sinh^2(c+dx)\right)^3\,dx$$
$$= \frac{3b^3e^{(5dx+5c)}+60ab^2e^{(3dx+3c)}-25b^3e^{(3dx+3c)}+720a^2be^{(dx+c)}-540ab^2e^{(dx+c)}+150b^3e^{(dx+c)}-480a^3}{}$$

input
```
integrate(csch(d*x+c)*(a+b*sinh(d*x+c)^2)^3,x, algorithm="giac")
```

output
```
1/480*(3*b^3*e^(5*d*x + 5*c) + 60*a*b^2*e^(3*d*x + 3*c) - 25*b^3*e^(3*d*x
+ 3*c) + 720*a^2*b*e^(d*x + c) - 540*a*b^2*e^(d*x + c) + 150*b^3*e^(d*x +
c) - 480*a^3*log(e^(d*x + c) + 1) + 480*a^3*log(abs(e^(d*x + c) - 1)) + (7
20*a^2*b*e^(4*d*x + 4*c) - 540*a*b^2*e^(4*d*x + 4*c) + 150*b^3*e^(4*d*x +
4*c) + 60*a*b^2*e^(2*d*x + 2*c) - 25*b^3*e^(2*d*x + 2*c) + 3*b^3)*e^(-5*d*
x - 5*c))/d
```

Mupad [B] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 184, normalized size of antiderivative = 2.22

$$\int \operatorname{csch}(c+dx)\left(a+b\sinh^2(c+dx)\right)^3\,dx = \frac{e^{c+d\,x}\,\left(24\,a^2\,b-18\,a\,b^2+5\,b^3\right)}{16\,d} - \frac{2\,\mathrm{atan}\left(\frac{a^3\,e^{d\,x}\,e^c\,\sqrt{-d^2}}{d\,\sqrt{a^6}}\right)\,\sqrt{a^6}}{\sqrt{-d^2}} + \frac{e^{-c-d\,x}\,\left(24\,a^2\,b-18\,a\,b^2+5\,b^3\right)}{16\,d} + \frac{b^3\,e^{-5\,c-5\,d\,x}}{160\,d} + \frac{b^3\,e^{5\,c+5\,d\,x}}{160\,d} + \frac{b^2\,e^{-3\,c-3\,d\,x}\,\left(12\,a-5\,b\right)}{96\,d} + \frac{b^2\,e^{3\,c+3\,d\,x}\,\left(12\,a-5\,b\right)}{96\,d}$$

input
```
int((a + b*sinh(c + d*x)^2)^3/sinh(c + d*x),x)
```

output
```
(exp(c + d*x)*(24*a^2*b - 18*a*b^2 + 5*b^3))/(16*d) - (2*atan((a^3*exp(d*x
)*exp(c)*(-d^2)^(1/2))/(d*(a^6)^(1/2)))*(a^6)^(1/2))/(-d^2)^(1/2) + (exp(-
 c - d*x)*(24*a^2*b - 18*a*b^2 + 5*b^3))/(16*d) + (b^3*exp(- 5*c - 5*d*x))
/(160*d) + (b^3*exp(5*c + 5*d*x))/(160*d) + (b^2*exp(- 3*c - 3*d*x)*(12*a
- 5*b))/(96*d) + (b^2*exp(3*c + 3*d*x)*(12*a - 5*b))/(96*d)
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 244, normalized size of antiderivative = 2.94

$$\int \operatorname{csch}(c+dx)\left(a+b\sinh^2(c+dx)\right)^3\,dx$$
$$= \frac{3e^{10dx+10c}b^3+60e^{8dx+8c}a\,b^2-25e^{8dx+8c}b^3+720e^{6dx+6c}a^2b-540e^{6dx+6c}a\,b^2+150e^{6dx+6c}b^3+480e^{5dx+5c}l}{}$$

input
```
int(csch(d*x+c)*(a+b*sinh(d*x+c)^2)^3,x)
```

output
```
(3*e**(10*c + 10*d*x)*b**3 + 60*e**(8*c + 8*d*x)*a*b**2 - 25*e**(8*c + 8*d
*x)*b**3 + 720*e**(6*c + 6*d*x)*a**2*b - 540*e**(6*c + 6*d*x)*a*b**2 + 150
*e**(6*c + 6*d*x)*b**3 + 480*e**(5*c + 5*d*x)*log(e**(c + d*x) - 1)*a**3 -
 480*e**(5*c + 5*d*x)*log(e**(c + d*x) + 1)*a**3 + 720*e**(4*c + 4*d*x)*a*
*2*b - 540*e**(4*c + 4*d*x)*a*b**2 + 150*e**(4*c + 4*d*x)*b**3 + 60*e**(2*
c + 2*d*x)*a*b**2 - 25*e**(2*c + 2*d*x)*b**3 + 3*b**3)/(480*e**(5*c + 5*d*
x)*d)
```

3.25 $\int \operatorname{csch}^2(c+dx)\left(a+b\sinh^2(c+dx)\right)^3\,dx$

Optimal result

Integrand size = 23, antiderivative size = 87

$$\int \operatorname{csch}^2(c+dx)\left(a+b\sinh^2(c+dx)\right)^3\,dx=\frac{3}{8}b\left(8a^2-4ab+b^2\right)x-\frac{a^3\coth(c+dx)}{d}+\frac{(12a-5b)b^2\cosh(c+dx)\sinh(c+dx)}{8d}+\frac{b^3\cosh^3(c+dx)\sinh(c+dx)}{4d}$$

output
```
3/8*b*(8*a^2-4*a*b+b^2)*x-a^3*coth(d*x+c)/d+1/8*(12*a-5*b)*b^2*cosh(d*x+c)
*sinh(d*x+c)/d+1/4*b^3*cosh(d*x+c)^3*sinh(d*x+c)/d
```

Mathematica [A] (verified)

Time = 2.16 (sec) , antiderivative size = 113, normalized size of antiderivative = 1.30

$$\int \operatorname{csch}^2(c+dx)\left(a+b\sinh^2(c+dx)\right)^3\,dx$$

$$=\frac{\left(b+a\operatorname{csch}^2(c+dx)\right)^3\sinh^6(c+dx)\left(12b\left(8a^2-4ab+b^2\right)(c+dx)-32a^3\coth(c+dx)+8(3a-b)b^2\sin\right.}{4d(2a-b+b\cosh(2(c+dx)))^3}$$

input
```
Integrate[Csch[c + d*x]^2*(a + b*Sinh[c + d*x]^2)^3,x]
```

output
```
((b + a*Csch[c + d*x]^2)^3*Sinh[c + d*x]^6*(12*b*(8*a^2 - 4*a*b + b^2)*(c
+ d*x) - 32*a^3*Coth[c + d*x] + 8*(3*a - b)*b^2*Sinh[2*(c + d*x)] + b^3*Si
nh[4*(c + d*x)]))/(4*d*(2*a - b + b*Cosh[2*(c + d*x)])^3)
```

Rubi [A] (verified)

Time = 0.42 (sec) , antiderivative size = 156, normalized size of antiderivative = 1.79, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.304$, Rules used = {3042, 25, 3666, 370, 439, 359, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \operatorname{csch}^2(c+dx)\left(a+b\sinh^2(c+dx)\right)^3dx$$

$$\Big\downarrow \text{3042}$$

$$\int -\frac{\left(a-b\sin(ic+idx)^2\right)^3}{\sin(ic+idx)^2}dx$$

$$\Big\downarrow \text{25}$$

$$-\int \frac{\left(a-b\sin(ic+idx)^2\right)^3}{\sin(ic+idx)^2}dx$$

$$\Big\downarrow \text{3666}$$

$$\frac{\int \frac{\coth^2(c+dx)\left(a-(a-b)\tanh^2(c+dx)\right)^3}{\left(1-\tanh^2(c+dx)\right)^3}d\tanh(c+dx)}{d}$$

$$\Big\downarrow \text{370}$$

$$\frac{\frac{1}{4}\int \frac{\coth^2(c+dx)\left(a-(a-b)\tanh^2(c+dx)\right)\left(a(4a+b)-(4a-3b)(a-b)\tanh^2(c+dx)\right)}{\left(1-\tanh^2(c+dx)\right)^2}d\tanh(c+dx)+\frac{b\coth(c+dx)\left(a-(a-b)\tanh^2(c+dx)\right)^2}{4\left(1-\tanh^2(c+dx)\right)^2}}{d}$$

$$\Big\downarrow \text{439}$$

$$\frac{\frac{1}{4}\left(\frac{1}{2}\int \frac{\coth^2(c+dx)\left(a(2a+b)(4a+b)-(4a-3b)(a-b)(2a-b)\tanh^2(c+dx)\right)}{1-\tanh^2(c+dx)}d\tanh(c+dx)+\frac{b\coth(c+dx)\left(a(4a+b)-(4a-3b)(a-b)\tanh^2\right.}{2\left(1-\tanh^2(c+dx)\right)}\right.}{d}$$

$\downarrow$ 359

$$\frac{\frac{1}{4}\left(\frac{1}{2}\left(3b(8a^2-4ab+b^2)\int \frac{1}{1-\tanh^2(c+dx)}d\tanh(c+dx)-a(2a+b)(4a+b)\coth(c+dx)\right)+\frac{b\coth(c+dx)(a(4a+b)-(}{2(1-\tan}}{d}$$

$\downarrow$ 219

$$\frac{\frac{1}{4}\left(\frac{1}{2}(3b(8a^2-4ab+b^2)\operatorname{arctanh}(\tanh(c+dx))-a(2a+b)(4a+b)\coth(c+dx))+\frac{b\coth(c+dx)(a(4a+b)-(4a-3b)(a}{2(1-\tanh^2(c+dx}}{d}$$

input
```
Int[Csch[c + d*x]^2*(a + b*Sinh[c + d*x]^2)^3,x]
```

output
```
((b*Coth[c + d*x]*(a - (a - b)*Tanh[c + d*x]^2)^2)/(4*(1 - Tanh[c + d*x]^2
)^2) + ((3*b*(8*a^2 - 4*a*b + b^2)*ArcTanh[Tanh[c + d*x]] - a*(2*a + b)*(4
*a + b)*Coth[c + d*x])/2 + (b*Coth[c + d*x]*(a*(4*a + b) - (4*a - 3*b)*(a
- b)*Tanh[c + d*x]^2))/(2*(1 - Tanh[c + d*x]^2)))/4)/d
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 359
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2), x
_Symbol] :> Simp[c*(e*x)^(m + 1)*((a + b*x^2)^(p + 1)/(a*e*(m + 1))), x] +
Simp[(a*d*(m + 1) - b*c*(m + 2*p + 3))/(a*e^2*(m + 1))   Int[(e*x)^(m + 2)*
(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c - a*d, 0]
&& LtQ[m, -1] && !ILtQ[p, -1]
```

rule 370
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_
), x_Symbol] :> Simp[(-(b*c - a*d))*(e*x)^(m + 1)*(a + b*x^2)^(p + 1)*((c +
 d*x^2)^(q - 1)/(a*b*e*2*(p + 1))), x] + Simp[1/(a*b*2*(p + 1))   Int[(e*x)
^m*(a + b*x^2)^(p + 1)*(c + d*x^2)^(q - 2)*Simp[c*(b*c*2*(p + 1) + (b*c - a
*d)*(m + 1)) + d*(b*c*2*(p + 1) + (b*c - a*d)*(m + 2*(q - 1) + 1))*x^2, x],
 x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1]
&& GtQ[q, 1] && IntBinomialQ[a, b, c, d, e, m, 2, p, q, x]
```

rule 439
```
Int[((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_
.)*((e_) + (f_.)*(x_)^2), x_Symbol] :> Simp[(-(b*e - a*f))*(g*x)^(m + 1)*(a
 + b*x^2)^(p + 1)*((c + d*x^2)^q/(2*a*b*g*(p + 1))), x] + Simp[1/(2*a*b*(p
+ 1))   Int[(g*x)^m*(a + b*x^2)^(p + 1)*(c + d*x^2)^(q - 1)*Simp[c*(2*b*e*(
p + 1) + (b*e - a*f)*(m + 1)) + d*(2*b*e*(p + 1) + (b*e - a*f)*(m + 2*q + 1
))*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, m}, x] && LtQ[p, -1] && G
tQ[q, 0] &&  !(EqQ[q, 1] && SimplerQ[b*c - a*d, b*e - a*f])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3666
```
Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(
p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Simp[ff^(m + 1
)/f   Subst[Int[x^m*((a + (a + b)*ff^2*x^2)^p/(1 + ff^2*x^2)^(m/2 + p + 1))
, x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f}, x] && IntegerQ[m/2] &
& IntegerQ[p]
```

Maple [A] (verified)

Time = 0.92 (sec) , antiderivative size = 94, normalized size of antiderivative = 1.08

method	result
derivativedivides	$\frac{-\coth(dx+c)a^3+3a^2b(dx+c)+3b^2a\left(\frac{\cosh(dx+c)\sinh(dx+c)}{2}-\frac{dx}{2}-\frac{c}{2}\right)+b^3\left(\left(\frac{\sinh(dx+c)^3}{4}-\frac{3\sinh(dx+c)}{8}\right)\cosh(dx+c)+\right.}{d}$
default	$\frac{-\coth(dx+c)a^3+3a^2b(dx+c)+3b^2a\left(\frac{\cosh(dx+c)\sinh(dx+c)}{2}-\frac{dx}{2}-\frac{c}{2}\right)+b^3\left(\left(\frac{\sinh(dx+c)^3}{4}-\frac{3\sinh(dx+c)}{8}\right)\cosh(dx+c)+\right.}{d}$
parallelrisch	$\frac{96a^2bdx-48a\,b^2dx+12b^3dx+b^3\sinh(4dx+4c)-8b^3\sinh(2dx+2c)+24b^2a\sinh(2dx+2c)+16\,\mathrm{sech}\left(\frac{dx}{2}+\frac{c}{2}\right)\mathrm{csch}\left(\frac{dx}{2}+\right.}{32d}$
risch	$3a^2bx-\frac{3a\,b^2x}{2}+\frac{3b^3x}{8}+\frac{e^{4dx+4c}b^3}{64d}+\frac{3\,e^{2dx+2c}b^2a}{8d}-\frac{e^{2dx+2c}b^3}{8d}-\frac{3\,e^{-2dx-2c}b^2a}{8d}+\frac{e^{-2dx-2c}b^3}{8d}-$

input
```
int(csch(d*x+c)^2*(a+b*sinh(d*x+c)^2)^3,x,method=_RETURNVERBOSE)
```

output
```
1/d*(-coth(d*x+c)*a^3+3*a^2*b*(d*x+c)+3*b^2*a*(1/2*cosh(d*x+c)*sinh(d*x+c)
-1/2*d*x-1/2*c)+b^3*((1/4*sinh(d*x+c)^3-3/8*sinh(d*x+c))*cosh(d*x+c)+3/8*d
*x+3/8*c))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 169 vs. $2(81) = 162$.

Time = 0.08 (sec) , antiderivative size = 169, normalized size of antiderivative = 1.94

$$\int \mathrm{csch}^2(c+dx)\left(a+b\sinh^2(c+dx)\right)^3\,dx$$
$$=\frac{b^3\cosh(dx+c)^5+5\,b^3\cosh(dx+c)\sinh(dx+c)^4+3\left(8\,ab^2-3\,b^3\right)\cosh(dx+c)^3+\left(10\,b^3\cosh(dx+\right.}{}$$

input
```
integrate(csch(d*x+c)^2*(a+b*sinh(d*x+c)^2)^3,x, algorithm="fricas")
```

output
```
1/64*(b^3*cosh(d*x + c)^5 + 5*b^3*cosh(d*x + c)*sinh(d*x + c)^4 + 3*(8*a*b
^2 - 3*b^3)*cosh(d*x + c)^3 + (10*b^3*cosh(d*x + c)^3 + 9*(8*a*b^2 - 3*b^3
)*cosh(d*x + c))*sinh(d*x + c)^2 - 8*(8*a^3 + 3*a*b^2 - b^3)*cosh(d*x + c)
 + 8*(8*a^3 + 3*(8*a^2*b - 4*a*b^2 + b^3)*d*x)*sinh(d*x + c))/(d*sinh(d*x
+ c))
```

Sympy [F(-1)]

Timed out.

$$\int \operatorname{csch}^2(c+dx)\left(a+b\sinh^2(c+dx)\right)^3\,dx = \text{Timed out}$$

input
```
integrate(csch(d*x+c)**2*(a+b*sinh(d*x+c)**2)**3,x)
```

output
```
Timed out
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 130, normalized size of antiderivative = 1.49

$$\int \operatorname{csch}^2(c+dx)\left(a+b\sinh^2(c+dx)\right)^3\,dx = \frac{1}{64}b^3\left(24x+\frac{e^{(4dx+4c)}}{d}-\frac{8e^{(2dx+2c)}}{d}+\frac{8e^{(-2dx-2c)}}{d}-\frac{e^{(-4dx-4c)}}{d}\right) - \frac{3}{8}ab^2\left(4x-\frac{e^{(2dx+2c)}}{d}+\frac{e^{(-2dx-2c)}}{d}\right)+3a^2bx+\frac{2a^3}{d(e^{(-2dx-2c)}-1)}$$

input
```
integrate(csch(d*x+c)^2*(a+b*sinh(d*x+c)^2)^3,x, algorithm="maxima")
```

output
```
1/64*b^3*(24*x + e^(4*d*x + 4*c)/d - 8*e^(2*d*x + 2*c)/d + 8*e^(-2*d*x - 2
*c)/d - e^(-4*d*x - 4*c)/d) - 3/8*a*b^2*(4*x - e^(2*d*x + 2*c)/d + e^(-2*d
*x - 2*c)/d) + 3*a^2*b*x + 2*a^3/(d*(e^(-2*d*x - 2*c) - 1))
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 177 vs. $2(81) = 162$.

Time = 0.17 (sec) , antiderivative size = 177, normalized size of antiderivative = 2.03

$$\int \operatorname{csch}^2(c+dx)\left(a+b\sinh^2(c+dx)\right)^3\,dx$$
$$= \frac{b^3e^{(4\,dx+4\,c)} + 24\,ab^2e^{(2\,dx+2\,c)} - 8\,b^3e^{(2\,dx+2\,c)} + 24\,(8\,a^2b - 4\,ab^2 + b^3)(dx+c) - \frac{128\,a^3}{e^{(2\,dx+2\,c)}-1} - (144\,a^2be^{(4}}{64\,d}$$

input
```
integrate(csch(d*x+c)^2*(a+b*sinh(d*x+c)^2)^3,x, algorithm="giac")
```

output
```
1/64*(b^3*e^(4*d*x + 4*c) + 24*a*b^2*e^(2*d*x + 2*c) - 8*b^3*e^(2*d*x + 2*
c) + 24*(8*a^2*b - 4*a*b^2 + b^3)*(d*x + c) - 128*a^3/(e^(2*d*x + 2*c) - 1
) - (144*a^2*b*e^(4*d*x + 4*c) - 72*a*b^2*e^(4*d*x + 4*c) + 18*b^3*e^(4*d*
x + 4*c) + 24*a*b^2*e^(2*d*x + 2*c) - 8*b^3*e^(2*d*x + 2*c) + b^3)*e^(-4*d
*x - 4*c))/d
```

Mupad [B] (verification not implemented)

Time = 1.84 (sec) , antiderivative size = 121, normalized size of antiderivative = 1.39

$$\int \operatorname{csch}^2(c+dx)\left(a+b\sinh^2(c+dx)\right)^3\,dx = \frac{3\,b\,x\,(8\,a^2-4\,a\,b+b^2)}{8} - \frac{2\,a^3}{d\,\left(\mathrm{e}^{2\,c+2\,d\,x}-1\right)} - \frac{b^3\,\mathrm{e}^{-4\,c-4\,d\,x}}{64\,d} + \frac{b^3\,\mathrm{e}^{4\,c+4\,d\,x}}{64\,d} - \frac{b^2\,\mathrm{e}^{-2\,c-2\,d\,x}\,(3\,a-b)}{8\,d} + \frac{b^2\,\mathrm{e}^{2\,c+2\,d\,x}\,(3\,a-b)}{8\,d}$$

input
```
int((a + b*sinh(c + d*x)^2)^3/sinh(c + d*x)^2,x)
```

output
```
(3*b*x*(8*a^2 - 4*a*b + b^2))/8 - (2*a^3)/(d*(exp(2*c + 2*d*x) - 1)) - (b^
3*exp(- 4*c - 4*d*x))/(64*d) + (b^3*exp(4*c + 4*d*x))/(64*d) - (b^2*exp(-
2*c - 2*d*x)*(3*a - b))/(8*d) + (b^2*exp(2*c + 2*d*x)*(3*a - b))/(8*d)
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 263, normalized size of antiderivative = 3.02

$$\int \operatorname{csch}^2(c+dx)\left(a+b\sinh^2(c+dx)\right)^3\,dx = \frac{e^{10dx+10c}b^3+24e^{8dx+8c}a\,b^2-9e^{8dx+8c}b^3-128e^{6dx+6c}a^3+192e^{6dx+6c}a^2bdx-96e^{6dx+6c}a\,b^2dx-48e^{6dx+6c}}{64}$$

input
```
int(csch(d*x+c)^2*(a+b*sinh(d*x+c)^2)^3,x)
```

output
```
(e**(10*c + 10*d*x)*b**3 + 24*e**(8*c + 8*d*x)*a*b**2 - 9*e**(8*c + 8*d*x)
*b**3 - 128*e**(6*c + 6*d*x)*a**3 + 192*e**(6*c + 6*d*x)*a**2*b*d*x - 96*e
**(6*c + 6*d*x)*a*b**2*d*x - 48*e**(6*c + 6*d*x)*a*b**2 + 24*e**(6*c + 6*d
*x)*b**3*d*x + 16*e**(6*c + 6*d*x)*b**3 - 192*e**(4*c + 4*d*x)*a**2*b*d*x
+ 96*e**(4*c + 4*d*x)*a*b**2*d*x - 24*e**(4*c + 4*d*x)*b**3*d*x + 24*e**(2
*c + 2*d*x)*a*b**2 - 9*e**(2*c + 2*d*x)*b**3 + b**3)/(64*e**(4*c + 4*d*x)*
d*(e**(2*c + 2*d*x) - 1))
```

3.26 $\int \operatorname{csch}^3(c+dx)\left(a+b\sinh^2(c+dx)\right)^3\,dx$

Optimal result

Integrand size = 23, antiderivative size = 83

$$\int \operatorname{csch}^3(c+dx)\left(a+b\sinh^2(c+dx)\right)^3\,dx = \frac{a^2(a-6b)\operatorname{arctanh}(\cosh(c+dx))}{2d} + \frac{(3a-b)b^2\cosh(c+dx)}{d} + \frac{b^3\cosh^3(c+dx)}{3d} - \frac{a^3\coth(c+dx)\operatorname{csch}(c+dx)}{2d}$$

output
```
1/2*a^2*(a-6*b)*arctanh(cosh(d*x+c))/d+(3*a-b)*b^2*cosh(d*x+c)/d+1/3*b^3*c
osh(d*x+c)^3/d-1/2*a^3*coth(d*x+c)*csch(d*x+c)/d
```

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 210 vs. 2(83) = 166.

Time = 6.09 (sec) , antiderivative size = 210, normalized size of antiderivative = 2.53

$$\int \operatorname{csch}^3(c+dx)\left(a+b\sinh^2(c+dx)\right)^3\,dx =$$

$$-\frac{\left(-18(4a-b)b^2\cosh(c)\cosh(dx) - 2b^3\cosh(3c)\cosh(3dx) + 3a^3\operatorname{csch}^2\left(\frac{1}{2}(c+dx)\right) - 12a^3\log\left(\cosh\left(\frac{1}{2}\right.\right.\right.}{}$$

input
```
Integrate[Csch[c + d*x]^3*(a + b*Sinh[c + d*x]^2)^3,x]
```

output
```
-1/3*((-18*(4*a - b)*b^2*Cosh[c]*Cosh[d*x] - 2*b^3*Cosh[3*c]*Cosh[3*d*x] +
 3*a^3*Csch[(c + d*x)/2]^2 - 12*a^3*Log[Cosh[(c + d*x)/2]] + 72*a^2*b*Log[
Cosh[(c + d*x)/2]] + 12*a^3*Log[Sinh[(c + d*x)/2]] - 72*a^2*b*Log[Sinh[(c
+ d*x)/2]] + 3*a^3*Sech[(c + d*x)/2]^2 - 72*a*b^2*Sinh[c]*Sinh[d*x] + 18*b
^3*Sinh[c]*Sinh[d*x] - 2*b^3*Sinh[3*c]*Sinh[3*d*x])*(a + b*Sinh[c + d*x]^2
)^3)/(d*(2*a - b + b*Cosh[2*(c + d*x)])^3)
```

Rubi [A] (verified)

Time = 0.35 (sec) , antiderivative size = 83, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.217$, Rules used = {3042, 26, 3665, 300, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \operatorname{csch}^3(c+dx)\left(a+b\sinh^2(c+dx)\right)^3 dx$$

$$\Big\downarrow \ 3042$$

$$\int -\frac{i\left(a-b\sin(ic+idx)^2\right)^3}{\sin(ic+idx)^3}dx$$

$$\Big\downarrow \ 26$$

$$-i\int \frac{\left(a-b\sin(ic+idx)^2\right)^3}{\sin(ic+idx)^3}dx$$

$$\Big\downarrow \ 3665$$

$$\frac{\int \frac{\left(b\cosh^2(c+dx)+a-b\right)^3}{\left(1-\cosh^2(c+dx)\right)^2}d\cosh(c+dx)}{d}$$

$$\Big\downarrow \ 300$$

$$\frac{\int \left(\cosh^2(c+dx)b^3+(3a-b)b^2+\frac{3b\cosh^2(c+dx)a^2+(a-3b)a^2}{\left(1-\cosh^2(c+dx)\right)^2}\right)d\cosh(c+dx)}{d}$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{\frac{a^3\cosh(c+dx)}{2(1-\cosh^2(c+dx))}+\frac{1}{2}a^2(a-6b)\text{arctanh}(\cosh(c+dx))+b^2(3a-b)\cosh(c+dx)+\frac{1}{3}b^3\cosh^3(c+dx)}{d}$$

input
```
Int[Csch[c + d*x]^3*(a + b*Sinh[c + d*x]^2)^3,x]
```

output
```
((a^2*(a - 6*b)*ArcTanh[Cosh[c + d*x]])/2 + (3*a - b)*b^2*Cosh[c + d*x] +
(b^3*Cosh[c + d*x]^3)/3 + (a^3*Cosh[c + d*x])/(2*(1 - Cosh[c + d*x]^2)))/d
```

Defintions of rubi rules used

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 300
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_), x_Symbol] :> Int
[PolynomialDivide[(a + b*x^2)^p, (c + d*x^2)^(-q), x], x] /; FreeQ[{a, b, c
, d}, x] && NeQ[b*c - a*d, 0] && IGtQ[p, 0] && ILtQ[q, 0] && GeQ[p, -q]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3665
```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^
(p_.), x_Symbol] :> With[{ff = FreeFactors[Cos[e + f*x], x]}, Simp[-ff/f
Subst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a + b - b*ff^2*x^2)^p, x], x, Cos[e +
 f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m - 1)/2]
```

Maple [A] (verified)

Time = 0.87 (sec) , antiderivative size = 79, normalized size of antiderivative = 0.95

method	result
derivativedivides	$\frac{a^3\left(-\frac{\operatorname{csch}(dx+c)\coth(dx+c)}{2}+\operatorname{arctanh}\left(e^{dx+c}\right)\right)-6a^2b\operatorname{arctanh}\left(e^{dx+c}\right)+3b^2a\cosh(dx+c)+b^3\left(-\frac{2}{3}+\frac{\sinh(dx+c)^2}{3}\right)\cosh}{d}$
default	$\frac{a^3\left(-\frac{\operatorname{csch}(dx+c)\coth(dx+c)}{2}+\operatorname{arctanh}\left(e^{dx+c}\right)\right)-6a^2b\operatorname{arctanh}\left(e^{dx+c}\right)+3b^2a\cosh(dx+c)+b^3\left(-\frac{2}{3}+\frac{\sinh(dx+c)^2}{3}\right)\cosh}{d}$
parallelrisch	$\frac{-4a^2(a-6b)\ln\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+a^3\left(\operatorname{sech}\left(\frac{dx}{2}+\frac{c}{2}\right)^2-5\right)\operatorname{csch}\left(\frac{dx}{2}+\frac{c}{2}\right)^2+3\coth\left(\frac{dx}{2}+\frac{c}{2}\right)^2a^3+24b^2\left(\frac{b\cosh(3dx+3c)}{36}+\right.}{8d}$
risch	$\frac{e^{3dx+3c}b^3}{24d}+\frac{3e^{dx+c}b^2a}{2d}-\frac{3e^{dx+c}b^3}{8d}+\frac{3e^{-dx-c}b^2a}{2d}-\frac{3e^{-dx-c}b^3}{8d}+\frac{e^{-3dx-3c}b^3}{24d}-\frac{a^3e^{dx+c}\left(e^{2dx+2c}+1\right)}{d\left(e^{2dx+2c}-1\right)^2}$

input
```
int(csch(d*x+c)^3*(a+b*sinh(d*x+c)^2)^3,x,method=_RETURNVERBOSE)
```

output
```
1/d*(a^3*(-1/2*csch(d*x+c)*coth(d*x+c)+arctanh(exp(d*x+c)))-6*a^2*b*arctan
h(exp(d*x+c))+3*b^2*a*cosh(d*x+c)+b^3*(-2/3+1/3*sinh(d*x+c)^2)*cosh(d*x+c)
)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1814 vs. 2(77) = 154.

Time = 0.11 (sec) , antiderivative size = 1814, normalized size of antiderivative = 21.86

$$\int \operatorname{csch}^3(c+dx)\left(a+b\sinh^2(c+dx)\right)^3\,dx = \text{Too large to display}$$

input
```
integrate(csch(d*x+c)^3*(a+b*sinh(d*x+c)^2)^3,x, algorithm="fricas")
```

output
```
1/24*(b^3*cosh(d*x + c)^10 + 10*b^3*cosh(d*x + c)*sinh(d*x + c)^9 + b^3*si
nh(d*x + c)^10 + (36*a*b^2 - 11*b^3)*cosh(d*x + c)^8 + (45*b^3*cosh(d*x +
c)^2 + 36*a*b^2 - 11*b^3)*sinh(d*x + c)^8 + 8*(15*b^3*cosh(d*x + c)^3 + (3
6*a*b^2 - 11*b^3)*cosh(d*x + c))*sinh(d*x + c)^7 - 2*(12*a^3 + 18*a*b^2 -
5*b^3)*cosh(d*x + c)^6 + 2*(105*b^3*cosh(d*x + c)^4 - 12*a^3 - 18*a*b^2 +
5*b^3 + 14*(36*a*b^2 - 11*b^3)*cosh(d*x + c)^2)*sinh(d*x + c)^6 + 4*(63*b^
3*cosh(d*x + c)^5 + 14*(36*a*b^2 - 11*b^3)*cosh(d*x + c)^3 - 3*(12*a^3 + 1
8*a*b^2 - 5*b^3)*cosh(d*x + c))*sinh(d*x + c)^5 - 2*(12*a^3 + 18*a*b^2 - 5
*b^3)*cosh(d*x + c)^4 + 2*(105*b^3*cosh(d*x + c)^6 + 35*(36*a*b^2 - 11*b^3
)*cosh(d*x + c)^4 - 12*a^3 - 18*a*b^2 + 5*b^3 - 15*(12*a^3 + 18*a*b^2 - 5*
b^3)*cosh(d*x + c)^2)*sinh(d*x + c)^4 + 8*(15*b^3*cosh(d*x + c)^7 + 7*(36*
a*b^2 - 11*b^3)*cosh(d*x + c)^5 - 5*(12*a^3 + 18*a*b^2 - 5*b^3)*cosh(d*x +
 c)^3 - (12*a^3 + 18*a*b^2 - 5*b^3)*cosh(d*x + c))*sinh(d*x + c)^3 + b^3 +
 (36*a*b^2 - 11*b^3)*cosh(d*x + c)^2 + (45*b^3*cosh(d*x + c)^8 + 28*(36*a*
b^2 - 11*b^3)*cosh(d*x + c)^6 - 30*(12*a^3 + 18*a*b^2 - 5*b^3)*cosh(d*x +
c)^4 + 36*a*b^2 - 11*b^3 - 12*(12*a^3 + 18*a*b^2 - 5*b^3)*cosh(d*x + c)^2)
*sinh(d*x + c)^2 + 12*((a^3 - 6*a^2*b)*cosh(d*x + c)^7 + 7*(a^3 - 6*a^2*b)
*cosh(d*x + c)*sinh(d*x + c)^6 + (a^3 - 6*a^2*b)*sinh(d*x + c)^7 - 2*(a^3
- 6*a^2*b)*cosh(d*x + c)^5 - (2*a^3 - 12*a^2*b - 21*(a^3 - 6*a^2*b)*cosh(d
*x + c)^2)*sinh(d*x + c)^5 + 5*(7*(a^3 - 6*a^2*b)*cosh(d*x + c)^3 - 2*(...
```

Sympy [F(-1)]

Timed out.

$$\int \operatorname{csch}^3(c+dx)\left(a+b\sinh^2(c+dx)\right)^3 dx = \text{Timed out}$$

input
```
integrate(csch(d*x+c)**3*(a+b*sinh(d*x+c)**2)**3,x)
```

output
```
Timed out
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 217 vs. $2(77) = 154$.

Time = 0.05 (sec) , antiderivative size = 217, normalized size of antiderivative = 2.61

$$\int \operatorname{csch}^3(c+dx)\left(a+b\sinh^2(c+dx)\right)^3\,dx$$
$$=\frac{1}{24}\,b^3\left(\frac{e^{(3\,dx+3\,c)}}{d}-\frac{9\,e^{(dx+c)}}{d}-\frac{9\,e^{(-dx-c)}}{d}+\frac{e^{(-3\,dx-3\,c)}}{d}\right)+\frac{3}{2}\,ab^2\left(\frac{e^{(dx+c)}}{d}+\frac{e^{(-dx-c)}}{d}\right)$$
$$+\frac{1}{2}\,a^3\left(\frac{\log\left(e^{(-dx-c)}+1\right)}{d}-\frac{\log\left(e^{(-dx-c)}-1\right)}{d}+\frac{2\left(e^{(-dx-c)}+e^{(-3\,dx-3\,c)}\right)}{d(2\,e^{(-2\,dx-2\,c)}-e^{(-4\,dx-4\,c)}-1)}\right)$$
$$-3\,a^2b\left(\frac{\log\left(e^{(-dx-c)}+1\right)}{d}-\frac{\log\left(e^{(-dx-c)}-1\right)}{d}\right)$$

input
```
integrate(csch(d*x+c)^3*(a+b*sinh(d*x+c)^2)^3,x, algorithm="maxima")
```

output
```
1/24*b^3*(e^(3*d*x + 3*c)/d - 9*e^(d*x + c)/d - 9*e^(-d*x - c)/d + e^(-3*d
*x - 3*c)/d) + 3/2*a*b^2*(e^(d*x + c)/d + e^(-d*x - c)/d) + 1/2*a^3*(log(e
^(-d*x - c) + 1)/d - log(e^(-d*x - c) - 1)/d + 2*(e^(-d*x - c) + e^(-3*d*x
 - 3*c))/(d*(2*e^(-2*d*x - 2*c) - e^(-4*d*x - 4*c) - 1))) - 3*a^2*b*(log(e
^(-d*x - c) + 1)/d - log(e^(-d*x - c) - 1)/d)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 174 vs. $2(77) = 154$.

Time = 0.17 (sec) , antiderivative size = 174, normalized size of antiderivative = 2.10

$$\int \operatorname{csch}^3(c+dx)\left(a+b\sinh^2(c+dx)\right)^3\,dx$$
$$=\frac{b^3\left(e^{(dx+c)}+e^{(-dx-c)}\right)^3+36\,ab^2\left(e^{(dx+c)}+e^{(-dx-c)}\right)-12\,b^3\left(e^{(dx+c)}+e^{(-dx-c)}\right)-\frac{24\,a^3\left(e^{(dx+c)}+e^{(-dx-c)}\right)}{\left(e^{(dx+c)}+e^{(-dx-c)}\right)^2-4}+6}{24\,d}$$

input
```
integrate(csch(d*x+c)^3*(a+b*sinh(d*x+c)^2)^3,x, algorithm="giac")
```

output
```
1/24*(b^3*(e^(d*x + c) + e^(-d*x - c))^3 + 36*a*b^2*(e^(d*x + c) + e^(-d*x
 - c)) - 12*b^3*(e^(d*x + c) + e^(-d*x - c)) - 24*a^3*(e^(d*x + c) + e^(-d
*x - c))/((e^(d*x + c) + e^(-d*x - c))^2 - 4) + 6*(a^3 - 6*a^2*b)*log(e^(d
*x + c) + e^(-d*x - c) + 2) - 6*(a^3 - 6*a^2*b)*log(e^(d*x + c) + e^(-d*x
- c) - 2))/d
```

Mupad [B] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 229, normalized size of antiderivative = 2.76

$$\int \operatorname{csch}^3(c+dx)\left(a+b\sinh^2(c+dx)\right)^3\,dx = \frac{\operatorname{atan}\left(\frac{e^{dx}\,e^{c}\,\left(a^3\,\sqrt{-d^2}-6\,a^2\,b\,\sqrt{-d^2}\right)}{d\,\sqrt{a^6-12\,a^5\,b+36\,a^4\,b^2}}\right)\,\sqrt{a^6-12\,a^5\,b+36\,a^4\,b^2}}{\sqrt{-d^2}}+\frac{b^3\,e^{-3\,c-3\,d\,x}}{24\,d}+\frac{b^3\,e^{3\,c+3\,d\,x}}{24\,d}+\frac{3\,b^2\,e^{c+d\,x}\,(4\,a-b)}{8\,d}+\frac{3\,b^2\,e^{-c-d\,x}\,(4\,a-b)}{8\,d}-\frac{a^3\,e^{c+d\,x}}{d\,\left(e^{2\,c+2\,d\,x}-1\right)}-\frac{2\,a^3\,e^{c+d\,x}}{d\,\left(e^{4\,c+4\,d\,x}-2\,e^{2\,c+2\,d\,x}+1\right)}$$

input
```
int((a + b*sinh(c + d*x)^2)^3/sinh(c + d*x)^3,x)
```

output
```
(atan((exp(d*x)*exp(c)*(a^3*(-d^2)^(1/2) - 6*a^2*b*(-d^2)^(1/2)))/(d*(a^6
- 12*a^5*b + 36*a^4*b^2)^(1/2)))*(a^6 - 12*a^5*b + 36*a^4*b^2)^(1/2))/(-d^
2)^(1/2) + (b^3*exp(- 3*c - 3*d*x))/(24*d) + (b^3*exp(3*c + 3*d*x))/(24*d)
 + (3*b^2*exp(c + d*x)*(4*a - b))/(8*d) + (3*b^2*exp(- c - d*x)*(4*a - b))
/(8*d) - (a^3*exp(c + d*x))/(d*(exp(2*c + 2*d*x) - 1)) - (2*a^3*exp(c + d*
x))/(d*(exp(4*c + 4*d*x) - 2*exp(2*c + 2*d*x) + 1))
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 521, normalized size of antiderivative = 6.28

$$\int \operatorname{csch}^3(c+dx)\left(a+b\sinh^2(c+dx)\right)^3\,dx$$
$$= \frac{e^{10dx+10c}b^3+36e^{8dx+8c}a\,b^2-11e^{8dx+8c}b^3-12e^{7dx+7c}\log\left(e^{dx+c}-1\right)a^3+72e^{7dx+7c}\log\left(e^{dx+c}-1\right)a^2b+1}{}$$

input
```
int(csch(d*x+c)^3*(a+b*sinh(d*x+c)^2)^3,x)
```

output
```
(e**(10*c + 10*d*x)*b**3 + 36*e**(8*c + 8*d*x)*a*b**2 - 11*e**(8*c + 8*d*x
)*b**3 - 12*e**(7*c + 7*d*x)*log(e**(c + d*x) - 1)*a**3 + 72*e**(7*c + 7*d
*x)*log(e**(c + d*x) - 1)*a**2*b + 12*e**(7*c + 7*d*x)*log(e**(c + d*x) +
1)*a**3 - 72*e**(7*c + 7*d*x)*log(e**(c + d*x) + 1)*a**2*b - 24*e**(6*c +
6*d*x)*a**3 - 36*e**(6*c + 6*d*x)*a*b**2 + 10*e**(6*c + 6*d*x)*b**3 + 24*e
**(5*c + 5*d*x)*log(e**(c + d*x) - 1)*a**3 - 144*e**(5*c + 5*d*x)*log(e**(
c + d*x) - 1)*a**2*b - 24*e**(5*c + 5*d*x)*log(e**(c + d*x) + 1)*a**3 + 14
4*e**(5*c + 5*d*x)*log(e**(c + d*x) + 1)*a**2*b - 24*e**(4*c + 4*d*x)*a**3
 - 36*e**(4*c + 4*d*x)*a*b**2 + 10*e**(4*c + 4*d*x)*b**3 - 12*e**(3*c + 3*
d*x)*log(e**(c + d*x) - 1)*a**3 + 72*e**(3*c + 3*d*x)*log(e**(c + d*x) - 1
)*a**2*b + 12*e**(3*c + 3*d*x)*log(e**(c + d*x) + 1)*a**3 - 72*e**(3*c + 3
*d*x)*log(e**(c + d*x) + 1)*a**2*b + 36*e**(2*c + 2*d*x)*a*b**2 - 11*e**(2
*c + 2*d*x)*b**3 + b**3)/(24*e**(3*c + 3*d*x)*d*(e**(4*c + 4*d*x) - 2*e**(
2*c + 2*d*x) + 1))
```

3.27 $\int \operatorname{csch}^4(c+dx)\left(a+b\sinh^2(c+dx)\right)^3\,dx$

Optimal result

Integrand size = 23, antiderivative size = 74

$$\int \operatorname{csch}^4(c+dx)\left(a+b\sinh^2(c+dx)\right)^3\,dx = \frac{1}{2}(6a-b)b^2x+\frac{a^2(a-3b)\coth(c+dx)}{d}-\frac{a^3\coth^3(c+dx)}{3d}+\frac{b^3\cosh(c+dx)\sinh(c+dx)}{2d}$$

output
```
1/2*(6*a-b)*b^2*x+a^2*(a-3*b)*coth(d*x+c)/d-1/3*a^3*coth(d*x+c)^3/d+1/2*b^
3*cosh(d*x+c)*sinh(d*x+c)/d
```

Mathematica [A] (verified)

Time = 3.17 (sec) , antiderivative size = 107, normalized size of antiderivative = 1.45

$$\int \operatorname{csch}^4(c+dx)\left(a+b\sinh^2(c+dx)\right)^3\,dx = \frac{2\left(b+a\operatorname{csch}^2(c+dx)\right)^3\sinh^6(c+dx)\left(-4a^2\coth(c+dx)\left(-2a+9b+a\operatorname{csch}^2(c+dx)\right)+3b^2(2(6a-b)(\right.}{3d(2a-b+b\cosh(2(c+dx)))^3}$$

input
```
Integrate[Csch[c + d*x]^4*(a + b*Sinh[c + d*x]^2)^3,x]
```

output
```
(2*(b + a*Csch[c + d*x]^2)^3*Sinh[c + d*x]^6*(-4*a^2*Coth[c + d*x]*(-2*a +
 9*b + a*Csch[c + d*x]^2) + 3*b^2*(2*(6*a - b)*(c + d*x) + b*Sinh[2*(c + d
*x)])))/(3*d*(2*a - b + b*Cosh[2*(c + d*x)])^3)
```

Rubi [A] (verified)

Time = 0.43 (sec) , antiderivative size = 119, normalized size of antiderivative = 1.61, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.217$, Rules used = {3042, 3666, 370, 437, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \operatorname{csch}^4(c+dx)\left(a+b\sinh^2(c+dx)\right)^3 dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \frac{\left(a-b\sin(ic+idx)^2\right)^3}{\sin(ic+idx)^4}dx$$

$$\Big\downarrow \text{ 3666}$$

$$\frac{\int \frac{\coth^4(c+dx)\left(a-(a-b)\tanh^2(c+dx)\right)^3}{\left(1-\tanh^2(c+dx)\right)^2}d\tanh(c+dx)}{d}$$

$$\Big\downarrow \text{ 370}$$

$$\frac{\frac{1}{2}\int \frac{\coth^4(c+dx)\left(a-(a-b)\tanh^2(c+dx)\right)\left(a(2a+3b)-(a-b)(2a-b)\tanh^2(c+dx)\right)}{1-\tanh^2(c+dx)}d\tanh(c+dx)+\frac{b\coth^3(c+dx)\left(a-(a-b)\tanh^2(c+dx)\right)}{2\left(1-\tanh^2(c+dx)\right)}}{d}$$

$$\Big\downarrow \text{ 437}$$

$$\frac{\frac{1}{2}\int \left(a^2(2a+3b)\coth^4(c+dx)-a\left(2a^2-5ba-2b^2\right)\coth^2(c+dx)+\frac{b^2(b-6a)}{\tanh^2(c+dx)-1}\right)d\tanh(c+dx)+\frac{b\coth^3(c+d}{2}}{d}$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{\frac{1}{2}\left(a\left(2a^2-5ab-2b^2\right)\coth(c+dx)-\frac{1}{3}a^2(2a+3b)\coth^3(c+dx)+b^2(6a-b)\text{arctanh}(\tanh(c+dx))\right)+\frac{b\coth^3(c}{}}{d}$$

input
```
Int[Csch[c + d*x]^4*(a + b*Sinh[c + d*x]^2)^3,x]
```

output
```
(((6*a - b)*b^2*ArcTanh[Tanh[c + d*x]] + a*(2*a^2 - 5*a*b - 2*b^2)*Coth[c
+ d*x] - (a^2*(2*a + 3*b)*Coth[c + d*x]^3)/3)/2 + (b*Coth[c + d*x]^3*(a -
(a - b)*Tanh[c + d*x]^2)^2)/(2*(1 - Tanh[c + d*x]^2)))/d
```

Defintions of rubi rules used

rule 370
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_
), x_Symbol] :> Simp[(-(b*c - a*d))*(e*x)^(m + 1)*(a + b*x^2)^(p + 1)*((c +
 d*x^2)^(q - 1)/(a*b*e*2*(p + 1))), x] + Simp[1/(a*b*2*(p + 1))   Int[(e*x)
^m*(a + b*x^2)^(p + 1)*(c + d*x^2)^(q - 2)*Simp[c*(b*c*2*(p + 1) + (b*c - a
*d)*(m + 1)) + d*(b*c*2*(p + 1) + (b*c - a*d)*(m + 2*(q - 1) + 1))*x^2, x],
 x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1]
&& GtQ[q, 1] && IntBinomialQ[a, b, c, d, e, m, 2, p, q, x]
```

rule 437
```
Int[((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q
_.)*((e_) + (f_.)*(x_)^2)^(r_.), x_Symbol] :> Int[ExpandIntegrand[(g*x)^m*(
a + b*x^2)^p*(c + d*x^2)^q*(e + f*x^2)^r, x], x] /; FreeQ[{a, b, c, d, e, f
, g, m}, x] && IGtQ[p, -2] && IGtQ[q, 0] && IGtQ[r, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3666
```
Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(
p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Simp[ff^(m + 1
)/f   Subst[Int[x^m*((a + (a + b)*ff^2*x^2)^p/(1 + ff^2*x^2)^(m/2 + p + 1))
, x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f}, x] && IntegerQ[m/2] &
& IntegerQ[p]
```

Maple [A] (verified)

Time = 0.82 (sec) , antiderivative size = 77, normalized size of antiderivative = 1.04

method	result
derivativedivides	$\frac{a^3\left(\frac{2}{3}-\frac{\operatorname{csch}(dx+c)^2}{3}\right)\coth(dx+c)-3\coth(dx+c)a^2b+3b^2a(dx+c)+b^3\left(\frac{\cosh(dx+c)\sinh(dx+c)}{2}-\frac{dx}{2}-\frac{c}{2}\right)}{d}$
default	$\frac{a^3\left(\frac{2}{3}-\frac{\operatorname{csch}(dx+c)^2}{3}\right)\coth(dx+c)-3\coth(dx+c)a^2b+3b^2a(dx+c)+b^3\left(\frac{\cosh(dx+c)\sinh(dx+c)}{2}-\frac{dx}{2}-\frac{c}{2}\right)}{d}$
risch	$3a\,b^2x-\frac{b^3x}{2}+\frac{e^{2dx+2c}b^3}{8d}-\frac{e^{-2dx-2c}b^3}{8d}-\frac{2a^2\left(9b\,e^{4dx+4c}+6\,e^{2dx+2c}a-18\,e^{2dx+2c}b-2a+9b\right)}{3d\left(e^{2dx+2c}-1\right)^3}$
parallelrisch	$\frac{-\operatorname{sech}\left(\frac{dx}{2}+\frac{c}{2}\right)^3a^3\left(\cosh(dx+c)-\frac{\cosh(3dx+3c)}{3}\right)\operatorname{csch}\left(\frac{dx}{2}+\frac{c}{2}\right)^3+24\operatorname{sech}\left(\frac{dx}{2}+\frac{c}{2}\right)\operatorname{csch}\left(\frac{dx}{2}+\frac{c}{2}\right)a^2b-48b\left(\coth\left(\frac{dx}{2}+\right.\right.}{16d}$

input
```
int(csch(d*x+c)^4*(a+b*sinh(d*x+c)^2)^3,x,method=_RETURNVERBOSE)
```

output
```
1/d*(a^3*(2/3-1/3*csch(d*x+c)^2)*coth(d*x+c)-3*coth(d*x+c)*a^2*b+3*b^2*a*(
d*x+c)+b^3*(1/2*cosh(d*x+c)*sinh(d*x+c)-1/2*d*x-1/2*c))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 281 vs. 2(68) = 136.

Time = 0.09 (sec) , antiderivative size = 281, normalized size of antiderivative = 3.80

$$\int \operatorname{csch}^4(c+dx)\left(a+b\sinh^2(c+dx)\right)^3\,dx$$
$$=\frac{3\,b^3\cosh(dx+c)^5+15\,b^3\cosh(dx+c)\sinh(dx+c)^4+(16\,a^3-72\,a^2b-9\,b^3)\cosh(dx+c)^3-4\,(4\,a^3}{}$$

input
```
integrate(csch(d*x+c)^4*(a+b*sinh(d*x+c)^2)^3,x, algorithm="fricas")
```

output
```
1/24*(3*b^3*cosh(d*x + c)^5 + 15*b^3*cosh(d*x + c)*sinh(d*x + c)^4 + (16*a
^3 - 72*a^2*b - 9*b^3)*cosh(d*x + c)^3 - 4*(4*a^3 - 18*a^2*b - 3*(6*a*b^2
- b^3)*d*x)*sinh(d*x + c)^3 + 3*(10*b^3*cosh(d*x + c)^3 + (16*a^3 - 72*a^2
*b - 9*b^3)*cosh(d*x + c))*sinh(d*x + c)^2 - 6*(8*a^3 - 12*a^2*b - b^3)*co
sh(d*x + c) + 12*(4*a^3 - 18*a^2*b - 3*(6*a*b^2 - b^3)*d*x - (4*a^3 - 18*a
^2*b - 3*(6*a*b^2 - b^3)*d*x)*cosh(d*x + c)^2)*sinh(d*x + c))/(d*sinh(d*x
+ c)^3 + 3*(d*cosh(d*x + c)^2 - d)*sinh(d*x + c))
```

Sympy [F(-1)]

Timed out.

$$\int \operatorname{csch}^4(c+dx)\left(a+b\sinh^2(c+dx)\right)^3\,dx=\text{Timed out}$$

input
```
integrate(csch(d*x+c)**4*(a+b*sinh(d*x+c)**2)**3,x)
```

output
```
Timed out
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 161 vs. $2(68) = 136$.

Time = 0.04 (sec) , antiderivative size = 161, normalized size of antiderivative = 2.18

$$\begin{aligned}
&\int \operatorname{csch}^4(c+dx)\left(a+b\sinh^2(c+dx)\right)^3\,dx \\
&= -\frac{1}{8}\,b^3\left(4\,x-\frac{e^{(2\,dx+2\,c)}}{d}+\frac{e^{(-2\,dx-2\,c)}}{d}\right)+3\,ab^2x \\
&\quad+\frac{4}{3}\,a^3\left(\frac{3\,e^{(-2\,dx-2\,c)}}{d(3\,e^{(-2\,dx-2\,c)}-3\,e^{(-4\,dx-4\,c)}+e^{(-6\,dx-6\,c)}-1)}-\frac{1}{d(3\,e^{(-2\,dx-2\,c)}-3\,e^{(-4\,dx-4\,c)}+e^{(-6\,dx-6\,c)}-1}\right. \\
&\quad+\frac{6\,a^2b}{d(e^{(-2\,dx-2\,c)}-1)}
\end{aligned}$$

input
```
integrate(csch(d*x+c)^4*(a+b*sinh(d*x+c)^2)^3,x, algorithm="maxima")
```

output
```
-1/8*b^3*(4*x - e^(2*d*x + 2*c)/d + e^(-2*d*x - 2*c)/d) + 3*a*b^2*x + 4/3*
a^3*(3*e^(-2*d*x - 2*c)/(d*(3*e^(-2*d*x - 2*c) - 3*e^(-4*d*x - 4*c) + e^(-
6*d*x - 6*c) - 1)) - 1/(d*(3*e^(-2*d*x - 2*c) - 3*e^(-4*d*x - 4*c) + e^(-6
*d*x - 6*c) - 1))) + 6*a^2*b/(d*(e^(-2*d*x - 2*c) - 1))
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 154 vs. $2(68) = 136$.

Time = 0.17 (sec) , antiderivative size = 154, normalized size of antiderivative = 2.08

$$\int \operatorname{csch}^4(c+dx)\left(a+b\sinh^2(c+dx)\right)^3 dx$$
$$= \frac{3\,b^3e^{(2\,dx+2\,c)} + 12\,(6\,ab^2-b^3)(dx+c) - 3\left(12\,ab^2e^{(2\,dx+2\,c)} - 2\,b^3e^{(2\,dx+2\,c)} + b^3\right)e^{(-2\,dx-2\,c)} - \frac{16\,(9\,a^2be^{(4\,dx+}}{}}{24\,d}$$

input
```
integrate(csch(d*x+c)^4*(a+b*sinh(d*x+c)^2)^3,x, algorithm="giac")
```

output
```
1/24*(3*b^3*e^(2*d*x + 2*c) + 12*(6*a*b^2 - b^3)*(d*x + c) - 3*(12*a*b^2*e
^(2*d*x + 2*c) - 2*b^3*e^(2*d*x + 2*c) + b^3)*e^(-2*d*x - 2*c) - 16*(9*a^2
*b*e^(4*d*x + 4*c) + 6*a^3*e^(2*d*x + 2*c) - 18*a^2*b*e^(2*d*x + 2*c) - 2*
a^3 + 9*a^2*b)/(e^(2*d*x + 2*c) - 1)^3)/d
```

Mupad [B] (verification not implemented)

Time = 1.76 (sec) , antiderivative size = 222, normalized size of antiderivative = 3.00

$$\int \operatorname{csch}^4(c+dx)\left(a+b\sinh^2(c+dx)\right)^3 dx = \frac{\frac{2\,(3\,a^2\,b-2\,a^3)}{3\,d} - \frac{2\,a^2\,b\,e^{2\,c+2\,d\,x}}{d}}{e^{4\,c+4\,d\,x} - 2\,e^{2\,c+2\,d\,x} + 1} - \frac{\frac{2\,a^2\,b}{d} - \frac{4\,e^{2\,c+2\,d\,x}\,(3\,a^2\,b-2\,a^3)}{3\,d} + \frac{2\,a^2\,b\,e^{4\,c+4\,d\,x}}{d}}{3\,e^{2\,c+2\,d\,x} - 3\,e^{4\,c+4\,d\,x} + e^{6\,c+6\,d\,x} - 1} + \frac{b^2\,x\,(6\,a-b)}{2} - \frac{b^3\,e^{-2\,c-2\,d\,x}}{8\,d} + \frac{b^3\,e^{2\,c+2\,d\,x}}{8\,d} - \frac{2\,a^2\,b}{d\,(e^{2\,c+2\,d\,x}-1)}$$

input
```
int((a + b*sinh(c + d*x)^2)^3/sinh(c + d*x)^4,x)
```

output
```
((2*(3*a^2*b - 2*a^3))/(3*d) - (2*a^2*b*exp(2*c + 2*d*x))/d)/(exp(4*c + 4*
d*x) - 2*exp(2*c + 2*d*x) + 1) - ((2*a^2*b)/d - (4*exp(2*c + 2*d*x)*(3*a^2
*b - 2*a^3))/(3*d) + (2*a^2*b*exp(4*c + 4*d*x))/d)/(3*exp(2*c + 2*d*x) - 3
*exp(4*c + 4*d*x) + exp(6*c + 6*d*x) - 1) + (b^2*x*(6*a - b))/2 - (b^3*exp
(- 2*c - 2*d*x))/(8*d) + (b^3*exp(2*c + 2*d*x))/(8*d) - (2*a^2*b)/(d*(exp(
2*c + 2*d*x) - 1))
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 339, normalized size of antiderivative = 4.58

$$\int \operatorname{csch}^4(c+dx)\left(a+b\sinh^2(c+dx)\right)^3\,dx = \frac{3e^{10dx+10c}b^3-48e^{8dx+8c}a^2b+72e^{8dx+8c}a\,b^2dx-12e^{8dx+8c}b^3dx-7e^{8dx+8c}b^3-216e^{6dx+6c}a\,b^2dx+36e^{6dx\cdots}}{}$$

input
```
int(csch(d*x+c)^4*(a+b*sinh(d*x+c)^2)^3,x)
```

output
```
(3*e**(10*c + 10*d*x)*b**3 - 48*e**(8*c + 8*d*x)*a**2*b + 72*e**(8*c + 8*d
*x)*a*b**2*d*x - 12*e**(8*c + 8*d*x)*b**3*d*x - 7*e**(8*c + 8*d*x)*b**3 -
216*e**(6*c + 6*d*x)*a*b**2*d*x + 36*e**(6*c + 6*d*x)*b**3*d*x - 96*e**(4*
c + 4*d*x)*a**3 + 144*e**(4*c + 4*d*x)*a**2*b + 216*e**(4*c + 4*d*x)*a*b**
2*d*x - 36*e**(4*c + 4*d*x)*b**3*d*x + 12*e**(4*c + 4*d*x)*b**3 + 32*e**(2
*c + 2*d*x)*a**3 - 96*e**(2*c + 2*d*x)*a**2*b - 72*e**(2*c + 2*d*x)*a*b**2
*d*x + 12*e**(2*c + 2*d*x)*b**3*d*x - 11*e**(2*c + 2*d*x)*b**3 + 3*b**3)/(
24*e**(2*c + 2*d*x)*d*(e**(6*c + 6*d*x) - 3*e**(4*c + 4*d*x) + 3*e**(2*c +
 2*d*x) - 1))
```

3.28 $\int \frac{\sinh^7(c+dx)}{a+b\sinh^2(c+dx)}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 109

$$\int \frac{\sinh^7(c+dx)}{a+b\sinh^2(c+dx)}\,dx = -\frac{a^3\arctan\left(\frac{\sqrt{b}\cosh(c+dx)}{\sqrt{a-b}}\right)}{\sqrt{a-b}\,b^{7/2}d}+\frac{(a^2+ab+b^2)\cosh(c+dx)}{b^3d} - \frac{(a+2b)\cosh^3(c+dx)}{3b^2d}+\frac{\cosh^5(c+dx)}{5bd}$$

output

```
-a^3*arctan(b^(1/2)*cosh(d*x+c)/(a-b)^(1/2))/(a-b)^(1/2)/b^(7/2)/d+(a^2+a*
b+b^2)*cosh(d*x+c)/b^3/d-1/3*(a+2*b)*cosh(d*x+c)^3/b^2/d+1/5*cosh(d*x+c)^5
/b/d
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 1.35 (sec) , antiderivative size = 165, normalized size of antiderivative = 1.51

$$\int \frac{\sinh^7(c+dx)}{a+b\sinh^2(c+dx)}\,dx = \frac{-\frac{240a^3\left(\arctan\left(\frac{\sqrt{b}-i\sqrt{a}\tanh\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a-b}}\right)+\arctan\left(\frac{\sqrt{b}+i\sqrt{a}\tanh\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a-b}}\right)\right)}{\sqrt{a-b}}+30\sqrt{b}(8a^2+6ab+5b^2)\cosh(c+dx)-5b^{3}}{240b^{7/2}d}$$

input
```
Integrate[Sinh[c + d*x]^7/(a + b*Sinh[c + d*x]^2),x]
```

output
```
((-240*a^3*(ArcTan[(Sqrt[b] - I*Sqrt[a]*Tanh[(c + d*x)/2])/Sqrt[a - b]] +
ArcTan[(Sqrt[b] + I*Sqrt[a]*Tanh[(c + d*x)/2])/Sqrt[a - b]]))/Sqrt[a - b]
+ 30*Sqrt[b]*(8*a^2 + 6*a*b + 5*b^2)*Cosh[c + d*x] - 5*b^(3/2)*(4*a + 5*b)
*Cosh[3*(c + d*x)] + 3*b^(5/2)*Cosh[5*(c + d*x)])/(240*b^(7/2)*d)
```

Rubi [A] (verified)

Time = 0.37 (sec) , antiderivative size = 102, normalized size of antiderivative = 0.94, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.217$, Rules used = {3042, 26, 3665, 300, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sinh^7(c+dx)}{a+b\sinh^2(c+dx)}dx$$

$$\Big\downarrow \ 3042$$

$$\int \frac{i\sin(ic+idx)^7}{a-b\sin(ic+idx)^2}dx$$

$$\Big\downarrow \ 26$$

$$i\int \frac{\sin(ic+idx)^7}{a-b\sin(ic+idx)^2}dx$$

$$\Big\downarrow \ 3665$$

$$-\frac{\int \frac{\left(1-\cosh^2(c+dx)\right)^3}{b\cosh^2(c+dx)+a-b}d\cosh(c+dx)}{d}$$

$$\Big\downarrow \ 300$$

$$-\frac{\int \left(-\frac{\cosh^4(c+dx)}{b}+\frac{(a+2b)\cosh^2(c+dx)}{b^2}-\frac{a^2+ba+b^2}{b^3}+\frac{a^3}{b^3\left(b\cosh^2(c+dx)+a-b\right)}\right)d\cosh(c+dx)}{d}$$

$$\Big\downarrow \ 2009$$

$$-\frac{\frac{a^3\arctan\left(\frac{\sqrt{b}\cosh(c+dx)}{\sqrt{a-b}}\right)}{b^{7/2}\sqrt{a-b}}-\frac{(a^2+ab+b^2)\cosh(c+dx)}{b^3}+\frac{(a+2b)\cosh^3(c+dx)}{3b^2}-\frac{\cosh^5(c+dx)}{5b}}{d}$$

input
```
Int[Sinh[c + d*x]^7/(a + b*Sinh[c + d*x]^2),x]
```

output
```
-(((a^3*ArcTan[(Sqrt[b]*Cosh[c + d*x])/Sqrt[a - b]])/(Sqrt[a - b]*b^(7/2))
 - ((a^2 + a*b + b^2)*Cosh[c + d*x])/b^3 + ((a + 2*b)*Cosh[c + d*x]^3)/(3*
b^2) - Cosh[c + d*x]^5/(5*b))/d)
```

Defintions of rubi rules used

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 300
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_), x_Symbol] :> Int
[PolynomialDivide[(a + b*x^2)^p, (c + d*x^2)^(-q), x], x] /; FreeQ[{a, b, c
, d}, x] && NeQ[b*c - a*d, 0] && IGtQ[p, 0] && ILtQ[q, 0] && GeQ[p, -q]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3665
```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^
(p_.), x_Symbol] :> With[{ff = FreeFactors[Cos[e + f*x], x]}, Simp[-ff/f
Subst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a + b - b*ff^2*x^2)^p, x], x, Cos[e +
 f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m - 1)/2]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 295 vs. $2(97) = 194$.

Time = 3.56 (sec) , antiderivative size = 296, normalized size of antiderivative = 2.72

method	result
derivativedivides	$\frac{-\frac{a^3 \arctan\left(\frac{2\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^2 a-2a+4b}{4\sqrt{ab-b^2}}\right)}{b^3\sqrt{ab-b^2}}-\frac{1}{5b\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)^5}-\frac{1}{2b\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)^4}-\frac{-4a-3b}{8b^2\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)^2}-\frac{}{12b^2(\mathrm{ta}}}{}$
default	$\frac{-\frac{a^3 \arctan\left(\frac{2\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^2 a-2a+4b}{4\sqrt{ab-b^2}}\right)}{b^3\sqrt{ab-b^2}}-\frac{1}{5b\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)^5}-\frac{1}{2b\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)^4}-\frac{-4a-3b}{8b^2\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)^2}-\frac{}{12b^2(\mathrm{ta}}}{}$
risch	$\frac{e^{5dx+5c}}{160bd}-\frac{5\,e^{3dx+3c}}{96bd}-\frac{e^{3dx+3c}a}{24b^2d}+\frac{e^{dx+c}a^2}{2b^3d}+\frac{3\,e^{dx+c}a}{8b^2d}+\frac{5\,e^{dx+c}}{16bd}+\frac{e^{-dx-c}a^2}{2b^3d}+\frac{3\,e^{-dx-c}a}{8b^2d}+\frac{5\,e^{-dx-}}{16bd}$

input
```
int(sinh(d*x+c)^7/(a+b*sinh(d*x+c)^2),x,method=_RETURNVERBOSE)
```

output
```
1/d*(-a^3/b^3/(a*b-b^2)^(1/2)*arctan(1/4*(2*tanh(1/2*d*x+1/2*c)^2*a-2*a+4*
b)/(a*b-b^2)^(1/2))-1/5/b/(tanh(1/2*d*x+1/2*c)-1)^5-1/2/b/(tanh(1/2*d*x+1/
2*c)-1)^4-1/8*(-4*a-3*b)/b^2/(tanh(1/2*d*x+1/2*c)-1)^2-1/12*(-4*a+b)/b^2/(
tanh(1/2*d*x+1/2*c)-1)^3-1/8*(8*a^2+4*a*b+3*b^2)/b^3/(tanh(1/2*d*x+1/2*c)-
1)+1/5/b/(tanh(1/2*d*x+1/2*c)+1)^5-1/2/b/(tanh(1/2*d*x+1/2*c)+1)^4-1/8*(-4
*a-3*b)/b^2/(tanh(1/2*d*x+1/2*c)+1)^2-1/12*(4*a-b)/b^2/(tanh(1/2*d*x+1/2*c
)+1)^3-1/8/b^3*(-8*a^2-4*a*b-3*b^2)/(tanh(1/2*d*x+1/2*c)+1))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1587 vs. $2(97) = 194$.

Time = 0.19 (sec) , antiderivative size = 3241, normalized size of antiderivative = 29.73

$$\int \frac{\sinh^7(c+dx)}{a+b\sinh^2(c+dx)}\,dx = \text{Too large to display}$$

input
```
integrate(sinh(d*x+c)^7/(a+b*sinh(d*x+c)^2),x, algorithm="fricas")
```

output
```
Too large to include
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\sinh^7(c+dx)}{a+b\sinh^2(c+dx)}\,dx = \text{Timed out}$$

input
```
integrate(sinh(d*x+c)**7/(a+b*sinh(d*x+c)**2),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{\sinh^7(c+dx)}{a+b\sinh^2(c+dx)}\,dx = \int \frac{\sinh(dx+c)^7}{b\sinh(dx+c)^2+a}\,dx$$

input
```
integrate(sinh(d*x+c)^7/(a+b*sinh(d*x+c)^2),x, algorithm="maxima")
```

output
```
1/480*(3*b^2*e^(10*d*x + 10*c) + 3*b^2 - 5*(4*a*b*e^(8*c) + 5*b^2*e^(8*c))
*e^(8*d*x) + 30*(8*a^2*e^(6*c) + 6*a*b*e^(6*c) + 5*b^2*e^(6*c))*e^(6*d*x)
+ 30*(8*a^2*e^(4*c) + 6*a*b*e^(4*c) + 5*b^2*e^(4*c))*e^(4*d*x) - 5*(4*a*b*
e^(2*c) + 5*b^2*e^(2*c))*e^(2*d*x))*e^(-5*d*x - 5*c)/(b^3*d) - 1/128*integ
rate(256*(a^3*e^(3*d*x + 3*c) - a^3*e^(d*x + c))/(b^4*e^(4*d*x + 4*c) + b^
4 + 2*(2*a*b^3*e^(2*c) - b^4*e^(2*c))*e^(2*d*x)), x)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{\sinh^7(c+dx)}{a+b\sinh^2(c+dx)}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(sinh(d*x+c)^7/(a+b*sinh(d*x+c)^2),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Limit: Max order reached or unable
to make series expansion Error: Bad Argument Value
```

Mupad [B] (verification not implemented)

Time = 2.73 (sec) , antiderivative size = 415, normalized size of antiderivative = 3.81

$$\int \frac{\sinh^7(c+dx)}{a+b\sinh^2(c+dx)}\,dx = \frac{e^{-5c-5dx}}{160\,b\,d} - \frac{\sqrt{a^6}\left(2\,\mathrm{atan}\left(\frac{a^3\,e^{d\,x}\,e^c\,\sqrt{b^7\,d^2\,(a-b)}}{2\,b^3\,d\,(a-b)\,\sqrt{a^6}}\right) + 2\,\mathrm{atan}\left(\frac{\left(e^{d\,x}\,e^c\left(\frac{2\,a^7}{b^{11}\,d\,(a-b)^2\,\sqrt{a^6}} - \frac{4\left(2\,a^4\,b^4\,d\,\sqrt{a^6}-2\,a^5\,b^3\,d\,\sqrt{a^6}\right)}{a^3\,b^8\,(a-b)\,\sqrt{a\,b^7\,d^2-b^8\,d^2}\,\sqrt{b^7\,d^2\,(a-b)}}\right) + \frac{2\,a^7\,e^3 \cdots}{b^{11}\,d\,(a- \cdots}\cdots\right.}{4\,a^4}\cdots\right.\right.}{2\sqrt{a\,b^7\,d^2-b^8\,d^2}} + \frac{e^{5c+5dx}}{160\,b\,d} + \frac{e^{c+dx}\,(8\,a^2+6\,a\,b+5\,b^2)}{16\,b^3\,d} + \frac{e^{-c-dx}\,(8\,a^2+6\,a\,b+5\,b^2)}{16\,b^3\,d} - \frac{e^{-3c-3dx}\,(4\,a+5\,b)}{96\,b^2\,d} - \frac{e^{3c+3dx}\,(4\,a+5\,b)}{96\,b^2\,d}$$

input
```
int(sinh(c + d*x)^7/(a + b*sinh(c + d*x)^2),x)
```

output
```
exp(- 5*c - 5*d*x)/(160*b*d) - ((a^6)^(1/2)*(2*atan((a^3*exp(d*x)*exp(c)*(
b^7*d^2*(a - b))^(1/2))/(2*b^3*d*(a - b)*(a^6)^(1/2))) + 2*atan(((exp(d*x)
*exp(c)*((2*a^7)/(b^11*d*(a - b)^2*(a^6)^(1/2)) - (4*(2*a^4*b^4*d*(a^6)^(1
/2) - 2*a^5*b^3*d*(a^6)^(1/2)))/(a^3*b^8*(a - b)*(a*b^7*d^2 - b^8*d^2)^(1/
2)*(b^7*d^2*(a - b))^(1/2))) + (2*a^7*exp(3*c)*exp(3*d*x))/(b^11*d*(a - b)
^2*(a^6)^(1/2)))*(b^9*(a*b^7*d^2 - b^8*d^2)^(1/2) - a*b^8*(a*b^7*d^2 - b^8
*d^2)^(1/2)))/(4*a^4))))/(2*(a*b^7*d^2 - b^8*d^2)^(1/2)) + exp(5*c + 5*d*x
)/(160*b*d) + (exp(c + d*x)*(6*a*b + 8*a^2 + 5*b^2))/(16*b^3*d) + (exp(- c
 - d*x)*(6*a*b + 8*a^2 + 5*b^2))/(16*b^3*d) - (exp(- 3*c - 3*d*x)*(4*a + 5
*b))/(96*b^2*d) - (exp(3*c + 3*d*x)*(4*a + 5*b))/(96*b^2*d)
```

Reduce [B] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 908, normalized size of antiderivative = 8.33

$$\int \frac{\sinh^7(c+dx)}{a+b\sinh^2(c+dx)}\,dx = \text{Too large to display}$$

input
```
int(sinh(d*x+c)^7/(a+b*sinh(d*x+c)^2),x)
```

output

```
( - 480*e**(5*c + 5*d*x)*sqrt(b)*sqrt(a)*sqrt(a - b)*sqrt(2*sqrt(a)*sqrt(a
 - b) + 2*a - b)*atan((e**(c + d*x)*b)/(sqrt(b)*sqrt(2*sqrt(a)*sqrt(a - b)
 + 2*a - b)))*a**3 + 480*e**(5*c + 5*d*x)*sqrt(b)*sqrt(2*sqrt(a)*sqrt(a -
b) + 2*a - b)*atan((e**(c + d*x)*b)/(sqrt(b)*sqrt(2*sqrt(a)*sqrt(a - b) +
2*a - b)))*a**4 - 480*e**(5*c + 5*d*x)*sqrt(b)*sqrt(2*sqrt(a)*sqrt(a - b)
+ 2*a - b)*atan((e**(c + d*x)*b)/(sqrt(b)*sqrt(2*sqrt(a)*sqrt(a - b) + 2*a
 - b)))*a**3*b - 240*e**(5*c + 5*d*x)*sqrt(b)*sqrt(a)*sqrt(a - b)*sqrt(2*s
qrt(a)*sqrt(a - b) - 2*a + b)*log( - sqrt(2*sqrt(a)*sqrt(a - b) - 2*a + b)
 + e**(c + d*x)*sqrt(b))*a**3 + 240*e**(5*c + 5*d*x)*sqrt(b)*sqrt(a)*sqrt(
a - b)*sqrt(2*sqrt(a)*sqrt(a - b) - 2*a + b)*log(sqrt(2*sqrt(a)*sqrt(a - b
) - 2*a + b) + e**(c + d*x)*sqrt(b))*a**3 - 240*e**(5*c + 5*d*x)*sqrt(b)*s
qrt(2*sqrt(a)*sqrt(a - b) - 2*a + b)*log( - sqrt(2*sqrt(a)*sqrt(a - b) - 2
*a + b) + e**(c + d*x)*sqrt(b))*a**4 + 240*e**(5*c + 5*d*x)*sqrt(b)*sqrt(2
*sqrt(a)*sqrt(a - b) - 2*a + b)*log( - sqrt(2*sqrt(a)*sqrt(a - b) - 2*a +
b) + e**(c + d*x)*sqrt(b))*a**3*b + 240*e**(5*c + 5*d*x)*sqrt(b)*sqrt(2*sq
rt(a)*sqrt(a - b) - 2*a + b)*log(sqrt(2*sqrt(a)*sqrt(a - b) - 2*a + b) + e
**(c + d*x)*sqrt(b))*a**4 - 240*e**(5*c + 5*d*x)*sqrt(b)*sqrt(2*sqrt(a)*sq
rt(a - b) - 2*a + b)*log(sqrt(2*sqrt(a)*sqrt(a - b) - 2*a + b) + e**(c + d
*x)*sqrt(b))*a**3*b + 3*e**(10*c + 10*d*x)*a*b**4 - 3*e**(10*c + 10*d*x)*b
**5 - 20*e**(8*c + 8*d*x)*a**2*b**3 - 5*e**(8*c + 8*d*x)*a*b**4 + 25*e*...
```

3.29 $\int \frac{\sinh^6(c+dx)}{a+b\sinh^2(c+dx)}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 121

$$\int \frac{\sinh^6(c+dx)}{a+b\sinh^2(c+dx)}\,dx = \frac{(8a^2+4ab+3b^2)\,x}{8b^3} - \frac{a^{5/2}\operatorname{arctanh}\left(\frac{\sqrt{a-b}\tanh(c+dx)}{\sqrt{a}}\right)}{\sqrt{a-b}b^3d} - \frac{(4a+3b)\cosh(c+dx)\sinh(c+dx)}{8b^2d} + \frac{\cosh(c+dx)\sinh^3(c+dx)}{4bd}$$

output
```
1/8*(8*a^2+4*a*b+3*b^2)*x/b^3-a^(5/2)*arctanh((a-b)^(1/2)*tanh(d*x+c)/a^(1
/2))/(a-b)^(1/2)/b^3/d-1/8*(4*a+3*b)*cosh(d*x+c)*sinh(d*x+c)/b^2/d+1/4*cos
h(d*x+c)*sinh(d*x+c)^3/b/d
```

Mathematica [A] (verified)

Time = 0.73 (sec) , antiderivative size = 97, normalized size of antiderivative = 0.80

$$\int \frac{\sinh^6(c+dx)}{a+b\sinh^2(c+dx)}\,dx = \frac{4(8a^2+4ab+3b^2)(c+dx) - \frac{32a^{5/2}\operatorname{arctanh}\left(\frac{\sqrt{a-b}\tanh(c+dx)}{\sqrt{a}}\right)}{\sqrt{a-b}} - 8b(a+b)\sinh(2(c+dx)) + b^2\sinh(4(c+d}{32b^3d}$$

input
```
Integrate[Sinh[c + d*x]^6/(a + b*Sinh[c + d*x]^2),x]
```

output
```
(4*(8*a^2 + 4*a*b + 3*b^2)*(c + d*x) - (32*a^(5/2)*ArcTanh[(Sqrt[a - b]*Ta
nh[c + d*x])/Sqrt[a]])/Sqrt[a - b] - 8*b*(a + b)*Sinh[2*(c + d*x)] + b^2*S
inh[4*(c + d*x)])/(32*b^3*d)
```

Rubi [A] (verified)

Time = 0.46 (sec) , antiderivative size = 151, normalized size of antiderivative = 1.25, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.391$, Rules used = {3042, 25, 3666, 372, 440, 25, 397, 219, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sinh^6(c+dx)}{a+b\sinh^2(c+dx)}dx$$

$$\Big\downarrow \text{3042}$$

$$\int -\frac{\sin(ic+idx)^6}{a-b\sin(ic+idx)^2}dx$$

$$\Big\downarrow \text{25}$$

$$-\int \frac{\sin(ic+idx)^6}{a-b\sin(ic+idx)^2}dx$$

$$\Big\downarrow \text{3666}$$

$$\frac{\int \frac{\tanh^6(c+dx)}{(1-\tanh^2(c+dx))^3(a-(a-b)\tanh^2(c+dx))}d\tanh(c+dx)}{d}$$

$$\Big\downarrow \text{372}$$

$$\frac{\frac{\tanh^3(c+dx)}{4b(1-\tanh^2(c+dx))^2}-\frac{\int \frac{\tanh^2(c+dx)\left((a+3b)\tanh^2(c+dx)+3a\right)}{\left(1-\tanh^2(c+dx)\right)^2\left(a-(a-b)\tanh^2(c+dx)\right)}d\tanh(c+dx)}{4b}}{d}$$

$$\Big\downarrow \text{440}$$

$$\frac{\frac{\tanh^3(c+dx)}{4b(1-\tanh^2(c+dx))^2}-\frac{\frac{\int -\frac{(4a^2+ba+3b^2)\tanh^2(c+dx)+a(4a+3b)}{(1-\tanh^2(c+dx))(a-(a-b)\tanh^2(c+dx))}d\tanh(c+dx)}{2b}+\frac{(4a+3b)\tanh(c+dx)}{2b(1-\tanh^2(c+dx))}}{4b}}{d}$$

$\Big\downarrow$ 25

$$\frac{\frac{\tanh^3(c+dx)}{4b(1-\tanh^2(c+dx))^2}-\frac{\frac{(4a+3b)\tanh(c+dx)}{2b(1-\tanh^2(c+dx))}-\frac{\int \frac{(4a^2+ba+3b^2)\tanh^2(c+dx)+a(4a+3b)}{(1-\tanh^2(c+dx))(a-(a-b)\tanh^2(c+dx))}d\tanh(c+dx)}{2b}}{4b}}{d}$$

$\Big\downarrow$ 397

$$\frac{\frac{\tanh^3(c+dx)}{4b(1-\tanh^2(c+dx))^2}-\frac{\frac{(4a+3b)\tanh(c+dx)}{2b(1-\tanh^2(c+dx))}-\frac{\frac{(8a^2+4ab+3b^2)\int \frac{1}{1-\tanh^2(c+dx)}d\tanh(c+dx)}{b}-\frac{8a^3\int \frac{1}{a-(a-b)\tanh^2(c+dx)}d\tanh(c+dx)}{b}}{2b}}{4b}}{d}$$

$\Big\downarrow$ 219

$$\frac{\frac{\tanh^3(c+dx)}{4b(1-\tanh^2(c+dx))^2}-\frac{\frac{(4a+3b)\tanh(c+dx)}{2b(1-\tanh^2(c+dx))}-\frac{\frac{(8a^2+4ab+3b^2)\text{arctanh}(\tanh(c+dx))}{b}-\frac{8a^3\int \frac{1}{a-(a-b)\tanh^2(c+dx)}d\tanh(c+dx)}{b}}{2b}}{4b}}{d}$$

$\Big\downarrow$ 221

$$\frac{\frac{\tanh^3(c+dx)}{4b(1-\tanh^2(c+dx))^2}-\frac{\frac{(4a+3b)\tanh(c+dx)}{2b(1-\tanh^2(c+dx))}-\frac{\frac{(8a^2+4ab+3b^2)\text{arctanh}(\tanh(c+dx))}{b}-\frac{8a^{5/2}\text{arctanh}\left(\frac{\sqrt{a-b}\tanh(c+dx)}{\sqrt{a}}\right)}{b\sqrt{a-b}}}{2b}}{4b}}{d}$$

input
```
Int[Sinh[c + d*x]^6/(a + b*Sinh[c + d*x]^2),x]
```

output
```
(Tanh[c + d*x]^3/(4*b*(1 - Tanh[c + d*x]^2)^2) - (-1/2*(((8*a^2 + 4*a*b +
3*b^2)*ArcTanh[Tanh[c + d*x]])/b - (8*a^(5/2)*ArcTanh[(Sqrt[a - b]*Tanh[c
+ d*x])/Sqrt[a]])/(Sqrt[a - b]*b))/b + ((4*a + 3*b)*Tanh[c + d*x])/(2*b*(1
 - Tanh[c + d*x]^2)))/(4*b))/d
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 372
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_
), x_Symbol] :> Simp[(-a)*e^3*(e*x)^(m - 3)*(a + b*x^2)^(p + 1)*((c + d*x^2
)^(q + 1)/(2*b*(b*c - a*d)*(p + 1))), x] + Simp[e^4/(2*b*(b*c - a*d)*(p + 1
))   Int[(e*x)^(m - 4)*(a + b*x^2)^(p + 1)*(c + d*x^2)^q*Simp[a*c*(m - 3) +
 (a*d*(m + 2*q - 1) + 2*b*c*(p + 1))*x^2, x], x], x] /; FreeQ[{a, b, c, d,
e, q}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && GtQ[m, 3] && IntBinomialQ[a
, b, c, d, e, m, 2, p, q, x]
```

rule 397
```
Int[((e_) + (f_.)*(x_)^2)/(((a_) + (b_.)*(x_)^2)*((c_) + (d_.)*(x_)^2)), x_
Symbol] :> Simp[(b*e - a*f)/(b*c - a*d)   Int[1/(a + b*x^2), x], x] - Simp[
(d*e - c*f)/(b*c - a*d)   Int[1/(c + d*x^2), x], x] /; FreeQ[{a, b, c, d, e
, f}, x]
```

rule 440
```
Int[((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_
)*((e_) + (f_.)*(x_)^2), x_Symbol] :> Simp[g*(b*e - a*f)*(g*x)^(m - 1)*(a +
 b*x^2)^(p + 1)*((c + d*x^2)^(q + 1)/(2*b*(b*c - a*d)*(p + 1))), x] - Simp[
g^2/(2*b*(b*c - a*d)*(p + 1))   Int[(g*x)^(m - 2)*(a + b*x^2)^(p + 1)*(c +
d*x^2)^q*Simp[c*(b*e - a*f)*(m - 1) + (d*(b*e - a*f)*(m + 2*q + 1) - b*2*(c
*f - d*e)*(p + 1))*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, q}, x] &&
 LtQ[p, -1] && GtQ[m, 1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3666
```
Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(
p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Simp[ff^(m + 1
)/f   Subst[Int[x^m*((a + (a + b)*ff^2*x^2)^p/(1 + ff^2*x^2)^(m/2 + p + 1))
, x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f}, x] && IntegerQ[m/2] &
& IntegerQ[p]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 245 vs. $2(107) = 214$.

Time = 2.27 (sec) , antiderivative size = 246, normalized size of antiderivative = 2.03

method	result
risch	$\frac{x\,a^2}{b^3}+\frac{ax}{2b^2}+\frac{3x}{8b}+\frac{e^{4dx+4c}}{64bd}-\frac{e^{2dx+2c}a}{8b^2d}-\frac{e^{2dx+2c}}{8bd}+\frac{e^{-2dx-2c}a}{8b^2d}+\frac{e^{-2dx-2c}}{8bd}-\frac{e^{-4dx-4c}}{64bd}+\frac{\sqrt{a(a-b)}}{}$
derivativedivides	$-\frac{1}{4b\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^4}+\frac{1}{2b\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^3}-\frac{-4a-b}{8b^2\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^2}-\frac{4a+3b}{8b^2\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)}+\frac{\left(8a^2+4ab+3b^2\right)\ln\left(\tanh\right.}{8b^3}$
default	$-\frac{1}{4b\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^4}+\frac{1}{2b\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^3}-\frac{-4a-b}{8b^2\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^2}-\frac{4a+3b}{8b^2\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)}+\frac{\left(8a^2+4ab+3b^2\right)\ln\left(\tanh\right.}{8b^3}$

input
```
int(sinh(d*x+c)^6/(a+b*sinh(d*x+c)^2),x,method=_RETURNVERBOSE)
```

output
```
x/b^3*a^2+1/2*a*x/b^2+3/8*x/b+1/64/b/d*exp(4*d*x+4*c)-1/8/b^2/d*exp(2*d*x+
2*c)*a-1/8/b/d*exp(2*d*x+2*c)+1/8/b^2/d*exp(-2*d*x-2*c)*a+1/8/b/d*exp(-2*d
*x-2*c)-1/64/b/d*exp(-4*d*x-4*c)+1/2*(a*(a-b))^(1/2)/(a-b)*a^2/d/b^3*ln(ex
p(2*d*x+2*c)+(2*a+2*(a*(a-b))^(1/2)-b)/b)-1/2*(a*(a-b))^(1/2)/(a-b)*a^2/d/
b^3*ln(exp(2*d*x+2*c)-(-2*a+2*(a*(a-b))^(1/2)+b)/b)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 716 vs. $2(107) = 214$.

Time = 0.14 (sec) , antiderivative size = 1725, normalized size of antiderivative = 14.26

$$\int \frac{\sinh^6(c+dx)}{a+b\sinh^2(c+dx)}\,dx = \text{Too large to display}$$

input
```
integrate(sinh(d*x+c)^6/(a+b*sinh(d*x+c)^2),x, algorithm="fricas")
```

output
```
[1/64*(b^2*cosh(d*x + c)^8 + 8*b^2*cosh(d*x + c)*sinh(d*x + c)^7 + b^2*sin
h(d*x + c)^8 + 8*(8*a^2 + 4*a*b + 3*b^2)*d*x*cosh(d*x + c)^4 - 8*(a*b + b^
2)*cosh(d*x + c)^6 + 4*(7*b^2*cosh(d*x + c)^2 - 2*a*b - 2*b^2)*sinh(d*x +
c)^6 + 8*(7*b^2*cosh(d*x + c)^3 - 6*(a*b + b^2)*cosh(d*x + c))*sinh(d*x +
c)^5 + 2*(35*b^2*cosh(d*x + c)^4 + 4*(8*a^2 + 4*a*b + 3*b^2)*d*x - 60*(a*b
 + b^2)*cosh(d*x + c)^2)*sinh(d*x + c)^4 + 8*(7*b^2*cosh(d*x + c)^5 + 4*(8
*a^2 + 4*a*b + 3*b^2)*d*x*cosh(d*x + c) - 20*(a*b + b^2)*cosh(d*x + c)^3)*
sinh(d*x + c)^3 + 8*(a*b + b^2)*cosh(d*x + c)^2 + 4*(7*b^2*cosh(d*x + c)^6
 + 12*(8*a^2 + 4*a*b + 3*b^2)*d*x*cosh(d*x + c)^2 - 30*(a*b + b^2)*cosh(d*
x + c)^4 + 2*a*b + 2*b^2)*sinh(d*x + c)^2 + 32*(a^2*cosh(d*x + c)^4 + 4*a^
2*cosh(d*x + c)^3*sinh(d*x + c) + 6*a^2*cosh(d*x + c)^2*sinh(d*x + c)^2 +
4*a^2*cosh(d*x + c)*sinh(d*x + c)^3 + a^2*sinh(d*x + c)^4)*sqrt(a/(a - b))
*log((b^2*cosh(d*x + c)^4 + 4*b^2*cosh(d*x + c)*sinh(d*x + c)^3 + b^2*sinh
(d*x + c)^4 + 2*(2*a*b - b^2)*cosh(d*x + c)^2 + 2*(3*b^2*cosh(d*x + c)^2 +
 2*a*b - b^2)*sinh(d*x + c)^2 + 8*a^2 - 8*a*b + b^2 + 4*(b^2*cosh(d*x + c)
^3 + (2*a*b - b^2)*cosh(d*x + c))*sinh(d*x + c) + 4*((a*b - b^2)*cosh(d*x
+ c)^2 + 2*(a*b - b^2)*cosh(d*x + c)*sinh(d*x + c) + (a*b - b^2)*sinh(d*x
+ c)^2 + 2*a^2 - 3*a*b + b^2)*sqrt(a/(a - b)))/(b*cosh(d*x + c)^4 + 4*b*co
sh(d*x + c)*sinh(d*x + c)^3 + b*sinh(d*x + c)^4 + 2*(2*a - b)*cosh(d*x + c
)^2 + 2*(3*b*cosh(d*x + c)^2 + 2*a - b)*sinh(d*x + c)^2 + 4*(b*cosh(d*x...
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\sinh^6(c+dx)}{a+b\sinh^2(c+dx)}\,dx = \text{Timed out}$$

input
```
integrate(sinh(d*x+c)**6/(a+b*sinh(d*x+c)**2),x)
```

output
```
Timed out
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{\sinh^6(c+dx)}{a+b\sinh^2(c+dx)}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate(sinh(d*x+c)^6/(a+b*sinh(d*x+c)^2),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(b-a>0)', see `assume?` for more
details)Is
```

Giac [A] (verification not implemented)

Time = 0.56 (sec) , antiderivative size = 208, normalized size of antiderivative = 1.72

$$\int \frac{\sinh^6(c+dx)}{a+b\sinh^2(c+dx)}\,dx = -\frac{\frac{64\,a^3\arctan\left(\frac{be^{(2\,dx+2\,c)}+2\,a-b}{2\,\sqrt{-a^2+ab}}\right)}{\sqrt{-a^2+ab}b^3} - \frac{8\,(8\,a^2+4\,ab+3\,b^2)(dx+c)}{b^3} - \frac{be^{(4\,dx+4\,c)}-8\,ae^{(2\,dx+2\,c)}-8\,be^{(2\,dx+2\,c)}}{b^2} + \frac{(48\,a^2e^{(4\,dx+4\,c)}+24\,abe}{}}{64\,d}$$

input
```
integrate(sinh(d*x+c)^6/(a+b*sinh(d*x+c)^2),x, algorithm="giac")
```

output
```
-1/64*(64*a^3*arctan(1/2*(b*e^(2*d*x + 2*c) + 2*a - b)/sqrt(-a^2 + a*b))/(
sqrt(-a^2 + a*b)*b^3) - 8*(8*a^2 + 4*a*b + 3*b^2)*(d*x + c)/b^3 - (b*e^(4*
d*x + 4*c) - 8*a*e^(2*d*x + 2*c) - 8*b*e^(2*d*x + 2*c))/b^2 + (48*a^2*e^(4
*d*x + 4*c) + 24*a*b*e^(4*d*x + 4*c) + 18*b^2*e^(4*d*x + 4*c) - 8*a*b*e^(2
*d*x + 2*c) - 8*b^2*e^(2*d*x + 2*c) + b^2)*e^(-4*d*x - 4*c)/b^3)/d
```

Mupad [B] (verification not implemented)

Time = 2.15 (sec) , antiderivative size = 266, normalized size of antiderivative = 2.20

$$\int \frac{\sinh^6(c+dx)}{a+b\sinh^2(c+dx)}\,dx = \frac{x\,(8\,a^2+4\,a\,b+3\,b^2)}{8\,b^3} - \frac{e^{-4\,c-4\,d\,x}}{64\,b\,d} + \frac{e^{4\,c+4\,d\,x}}{64\,b\,d} + \frac{e^{-2\,c-2\,d\,x}\,(a+b)}{8\,b^2\,d} - \frac{e^{2\,c+2\,d\,x}\,(a+b)}{8\,b^2\,d} + \frac{a^{5/2}\,\ln\left(\frac{4\,a^3\,e^{2\,c+2\,d\,x}}{b^4} - \frac{2\,a^{5/2}\,(b\,d+2\,a\,d\,e^{2\,c+2\,d\,x}-b\,d\,e^{2\,c+2\,d\,x})}{b^4\,d\,\sqrt{a-b}}\right)}{2\,b^3\,d\,\sqrt{a-b}} - \frac{a^{5/2}\,\ln\left(\frac{4\,a^3\,e^{2\,c+2\,d\,x}}{b^4} + \frac{2\,a^{5/2}\,(b\,d+2\,a\,d\,e^{2\,c+2\,d\,x}-b\,d\,e^{2\,c+2\,d\,x})}{b^4\,d\,\sqrt{a-b}}\right)}{2\,b^3\,d\,\sqrt{a-b}}$$

input
```
int(sinh(c + d*x)^6/(a + b*sinh(c + d*x)^2),x)
```

output
```
(x*(4*a*b + 8*a^2 + 3*b^2))/(8*b^3) - exp(- 4*c - 4*d*x)/(64*b*d) + exp(4*
c + 4*d*x)/(64*b*d) + (exp(- 2*c - 2*d*x)*(a + b))/(8*b^2*d) - (exp(2*c +
2*d*x)*(a + b))/(8*b^2*d) + (a^(5/2)*log((4*a^3*exp(2*c + 2*d*x))/b^4 - (2
*a^(5/2)*(b*d + 2*a*d*exp(2*c + 2*d*x) - b*d*exp(2*c + 2*d*x)))/(b^4*d*(a
- b)^(1/2))))/(2*b^3*d*(a - b)^(1/2)) - (a^(5/2)*log((4*a^3*exp(2*c + 2*d*
x))/b^4 + (2*a^(5/2)*(b*d + 2*a*d*exp(2*c + 2*d*x) - b*d*exp(2*c + 2*d*x))
)/(b^4*d*(a - b)^(1/2))))/(2*b^3*d*(a - b)^(1/2))
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 356, normalized size of antiderivative = 2.94

$$\int \frac{\sinh^6(c+dx)}{a+b\sinh^2(c+dx)}\,dx$$
$$= \frac{-32e^{4dx+4c}\sqrt{a}\,\sqrt{a-b}\log\left(-\sqrt{2\sqrt{a}\,\sqrt{a-b}-2a+b}+e^{dx+c}\sqrt{b}\right)a^2-32e^{4dx+4c}\sqrt{a}\,\sqrt{a-b}\log\left(\sqrt{2\sqrt{a}\right.}{}$$

input
```
int(sinh(d*x+c)^6/(a+b*sinh(d*x+c)^2),x)
```

output
```
( - 32*e**(4*c + 4*d*x)*sqrt(a)*sqrt(a - b)*log( - sqrt(2*sqrt(a)*sqrt(a -
 b) - 2*a + b) + e**(c + d*x)*sqrt(b))*a**2 - 32*e**(4*c + 4*d*x)*sqrt(a)*
sqrt(a - b)*log(sqrt(2*sqrt(a)*sqrt(a - b) - 2*a + b) + e**(c + d*x)*sqrt(
b))*a**2 + 32*e**(4*c + 4*d*x)*sqrt(a)*sqrt(a - b)*log(2*sqrt(a)*sqrt(a -
b) + e**(2*c + 2*d*x)*b + 2*a - b)*a**2 + e**(8*c + 8*d*x)*a*b**2 - e**(8*
c + 8*d*x)*b**3 - 8*e**(6*c + 6*d*x)*a**2*b + 8*e**(6*c + 6*d*x)*b**3 + 64
*e**(4*c + 4*d*x)*a**3*d*x - 32*e**(4*c + 4*d*x)*a**2*b*d*x - 8*e**(4*c +
4*d*x)*a*b**2*d*x - 24*e**(4*c + 4*d*x)*b**3*d*x + 8*e**(2*c + 2*d*x)*a**2
*b - 8*e**(2*c + 2*d*x)*b**3 - a*b**2 + b**3)/(64*e**(4*c + 4*d*x)*b**3*d*
(a - b))
```

3.30 $\int \frac{\sinh^5(c+dx)}{a+b\sinh^2(c+dx)}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 79

$$\int \frac{\sinh^5(c+dx)}{a+b\sinh^2(c+dx)}\,dx=\frac{a^2\arctan\left(\frac{\sqrt{b}\cosh(c+dx)}{\sqrt{a-b}}\right)}{\sqrt{a-b}b^{5/2}d}-\frac{(a+b)\cosh(c+dx)}{b^2d}+\frac{\cosh^3(c+dx)}{3bd}$$

output
```
a^2*arctan(b^(1/2)*cosh(d*x+c)/(a-b)^(1/2))/(a-b)^(1/2)/b^(5/2)/d-(a+b)*co
sh(d*x+c)/b^2/d+1/3*cosh(d*x+c)^3/b/d
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.78 (sec) , antiderivative size = 134, normalized size of antiderivative = 1.70

$$\int \frac{\sinh^5(c+dx)}{a+b\sinh^2(c+dx)}\,dx = \frac{\frac{12a^2\left(\arctan\left(\frac{\sqrt{b}-i\sqrt{a}\tanh\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a-b}}\right)+\arctan\left(\frac{\sqrt{b}+i\sqrt{a}\tanh\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a-b}}\right)\right)}{\sqrt{a-b}}-3\sqrt{b}(4a+3b)\cosh(c+dx)+b^{3/2}\cosh(3(c+}{12b^{5/2}d}$$

input
```
Integrate[Sinh[c + d*x]^5/(a + b*Sinh[c + d*x]^2),x]
```

output
```
((12*a^2*(ArcTan[(Sqrt[b] - I*Sqrt[a]*Tanh[(c + d*x)/2])/Sqrt[a - b]] + Ar
cTan[(Sqrt[b] + I*Sqrt[a]*Tanh[(c + d*x)/2])/Sqrt[a - b]]))/Sqrt[a - b] -
3*Sqrt[b]*(4*a + 3*b)*Cosh[c + d*x] + b^(3/2)*Cosh[3*(c + d*x)])/(12*b^(5/
2)*d)
```

Rubi [A] (verified)

Time = 0.31 (sec) , antiderivative size = 74, normalized size of antiderivative = 0.94, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.217, Rules used = {3042, 26, 3665, 300, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sinh^5(c+dx)}{a+b\sinh^2(c+dx)}dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int -\frac{i\sin(ic+idx)^5}{a-b\sin(ic+idx)^2}dx$$

$$\Big\downarrow \text{ 26}$$

$$-i\int \frac{\sin(ic+idx)^5}{a-b\sin(ic+idx)^2}dx$$

$$\Big\downarrow \text{ 3665}$$

$$\frac{\int \frac{\left(1-\cosh^2(c+dx)\right)^2}{b\cosh^2(c+dx)+a-b}d\cosh(c+dx)}{d}$$

$$\Big\downarrow \text{ 300}$$

$$\frac{\int \left(\frac{a^2}{b^2\left(b\cosh^2(c+dx)+a-b\right)}+\frac{\cosh^2(c+dx)}{b}-\frac{a+b}{b^2}\right)d\cosh(c+dx)}{d}$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{\frac{a^2\arctan\left(\frac{\sqrt{b}\cosh(c+dx)}{\sqrt{a-b}}\right)}{b^{5/2}\sqrt{a-b}}-\frac{(a+b)\cosh(c+dx)}{b^2}+\frac{\cosh^3(c+dx)}{3b}}{d}$$

input
```
Int[Sinh[c + d*x]^5/(a + b*Sinh[c + d*x]^2),x]
```

output
```
((a^2*ArcTan[(Sqrt[b]*Cosh[c + d*x])/Sqrt[a - b]])/(Sqrt[a - b]*b^(5/2)) -
 ((a + b)*Cosh[c + d*x])/b^2 + Cosh[c + d*x]^3/(3*b))/d
```

Defintions of rubi rules used

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 300
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_), x_Symbol] :> Int
[PolynomialDivide[(a + b*x^2)^p, (c + d*x^2)^(-q), x], x] /; FreeQ[{a, b, c
, d}, x] && NeQ[b*c - a*d, 0] && IGtQ[p, 0] && ILtQ[q, 0] && GeQ[p, -q]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3665
```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^
(p_.), x_Symbol] :> With[{ff = FreeFactors[Cos[e + f*x], x]}, Simp[-ff/f
Subst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a + b - b*ff^2*x^2)^p, x], x, Cos[e +
 f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m - 1)/2]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 178 vs. $2(69) = 138$.

Time = 1.36 (sec) , antiderivative size = 179, normalized size of antiderivative = 2.27

method	result
derivativedivides	$\frac{-\frac{1}{3b\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)^{3}}-\frac{1}{2b\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)^{2}}-\frac{-2a-b}{2b^{2}\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}+\frac{a^{2}\arctan\left(\frac{2\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}a-2a+4b}{4\sqrt{ab-b^{2}}}\right)}{b^{2}\sqrt{ab-b^{2}}}+\frac{}{3b\left(\tanh\left(\right.\right.}}{d}$
default	$\frac{-\frac{1}{3b\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)^{3}}-\frac{1}{2b\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)^{2}}-\frac{-2a-b}{2b^{2}\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}+\frac{a^{2}\arctan\left(\frac{2\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}a-2a+4b}{4\sqrt{ab-b^{2}}}\right)}{b^{2}\sqrt{ab-b^{2}}}+\frac{}{3b\left(\tanh\left(\right.\right.}}{d}$
risch	$\frac{e^{3dx+3c}}{24bd}-\frac{e^{dx+c}a}{2b^{2}d}-\frac{3\,e^{dx+c}}{8bd}-\frac{e^{-dx-c}a}{2b^{2}d}-\frac{3\,e^{-dx-c}}{8bd}+\frac{e^{-3dx-3c}}{24bd}-\frac{a^{2}\ln\left(e^{2dx+2c}-\frac{2(a-b)e^{dx+c}}{\sqrt{-ab+b^{2}}}+1\right)}{2\sqrt{-ab+b^{2}}\,d\,b^{2}}+$

input
```
int(sinh(d*x+c)^5/(a+b*sinh(d*x+c)^2),x,method=_RETURNVERBOSE)
```

output
```
1/d*(-1/3/b/(tanh(1/2*d*x+1/2*c)-1)^3-1/2/b/(tanh(1/2*d*x+1/2*c)-1)^2-1/2/
b^2*(-2*a-b)/(tanh(1/2*d*x+1/2*c)-1)+a^2/b^2/(a*b-b^2)^(1/2)*arctan(1/4*(2
*tanh(1/2*d*x+1/2*c)^2*a-2*a+4*b)/(a*b-b^2)^(1/2))+1/3/b/(tanh(1/2*d*x+1/2
*c)+1)^3-1/2/b/(tanh(1/2*d*x+1/2*c)+1)^2-1/2*(2*a+b)/b^2/(tanh(1/2*d*x+1/2
*c)+1))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 779 vs. $2(69) = 138$.

Time = 0.13 (sec) , antiderivative size = 1667, normalized size of antiderivative = 21.10

$$\int \frac{\sinh^5(c+dx)}{a+b\sinh^2(c+dx)}\,dx = \text{Too large to display}$$

input
```
integrate(sinh(d*x+c)^5/(a+b*sinh(d*x+c)^2),x, algorithm="fricas")
```

output
```
[1/24*((a*b^2 - b^3)*cosh(d*x + c)^6 + 6*(a*b^2 - b^3)*cosh(d*x + c)*sinh(
d*x + c)^5 + (a*b^2 - b^3)*sinh(d*x + c)^6 - 3*(4*a^2*b - a*b^2 - 3*b^3)*c
osh(d*x + c)^4 - 3*(4*a^2*b - a*b^2 - 3*b^3 - 5*(a*b^2 - b^3)*cosh(d*x + c
)^2)*sinh(d*x + c)^4 + 4*(5*(a*b^2 - b^3)*cosh(d*x + c)^3 - 3*(4*a^2*b - a
*b^2 - 3*b^3)*cosh(d*x + c))*sinh(d*x + c)^3 + a*b^2 - b^3 - 3*(4*a^2*b -
a*b^2 - 3*b^3)*cosh(d*x + c)^2 + 3*(5*(a*b^2 - b^3)*cosh(d*x + c)^4 - 4*a^
2*b + a*b^2 + 3*b^3 - 6*(4*a^2*b - a*b^2 - 3*b^3)*cosh(d*x + c)^2)*sinh(d*
x + c)^2 - 12*(a^2*cosh(d*x + c)^3 + 3*a^2*cosh(d*x + c)^2*sinh(d*x + c) +
 3*a^2*cosh(d*x + c)*sinh(d*x + c)^2 + a^2*sinh(d*x + c)^3)*sqrt(-a*b + b^
2)*log((b*cosh(d*x + c)^4 + 4*b*cosh(d*x + c)*sinh(d*x + c)^3 + b*sinh(d*x
 + c)^4 - 2*(2*a - 3*b)*cosh(d*x + c)^2 + 2*(3*b*cosh(d*x + c)^2 - 2*a + 3
*b)*sinh(d*x + c)^2 + 4*(b*cosh(d*x + c)^3 - (2*a - 3*b)*cosh(d*x + c))*si
nh(d*x + c) - 4*(cosh(d*x + c)^3 + 3*cosh(d*x + c)*sinh(d*x + c)^2 + sinh(
d*x + c)^3 + (3*cosh(d*x + c)^2 + 1)*sinh(d*x + c) + cosh(d*x + c))*sqrt(-
a*b + b^2) + b)/(b*cosh(d*x + c)^4 + 4*b*cosh(d*x + c)*sinh(d*x + c)^3 + b
*sinh(d*x + c)^4 + 2*(2*a - b)*cosh(d*x + c)^2 + 2*(3*b*cosh(d*x + c)^2 +
2*a - b)*sinh(d*x + c)^2 + 4*(b*cosh(d*x + c)^3 + (2*a - b)*cosh(d*x + c))
*sinh(d*x + c) + b)) + 6*((a*b^2 - b^3)*cosh(d*x + c)^5 - 2*(4*a^2*b - a*b
^2 - 3*b^3)*cosh(d*x + c)^3 - (4*a^2*b - a*b^2 - 3*b^3)*cosh(d*x + c))*sin
h(d*x + c))/((a*b^3 - b^4)*d*cosh(d*x + c)^3 + 3*(a*b^3 - b^4)*d*cosh(d...
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\sinh^5(c+dx)}{a+b\sinh^2(c+dx)}\,dx = \text{Timed out}$$

input
```
integrate(sinh(d*x+c)**5/(a+b*sinh(d*x+c)**2),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{\sinh^5(c+dx)}{a+b\sinh^2(c+dx)}\,dx = \int \frac{\sinh{(dx+c)}^5}{b\sinh{(dx+c)}^2+a}\,dx$$

input
```
integrate(sinh(d*x+c)^5/(a+b*sinh(d*x+c)^2),x, algorithm="maxima")
```

output
```
-1/24*(3*(4*a*e^(4*c) + 3*b*e^(4*c))*e^(4*d*x) + 3*(4*a*e^(2*c) + 3*b*e^(2
*c))*e^(2*d*x) - b*e^(6*d*x + 6*c) - b)*e^(-3*d*x - 3*c)/(b^2*d) + 1/32*in
tegrate(64*(a^2*e^(3*d*x + 3*c) - a^2*e^(d*x + c))/(b^3*e^(4*d*x + 4*c) +
b^3 + 2*(2*a*b^2*e^(2*c) - b^3*e^(2*c))*e^(2*d*x)), x)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{\sinh^5(c+dx)}{a+b\sinh^2(c+dx)}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(sinh(d*x+c)^5/(a+b*sinh(d*x+c)^2),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Limit: Max order reached or unable
to make series expansion Error: Bad Argument Value
```

Mupad [B] (verification not implemented)

Time = 2.55 (sec) , antiderivative size = 348, normalized size of antiderivative = 4.41

$$\int \frac{\sinh^5(c+dx)}{a+b\sinh^2(c+dx)}\,dx$$
$$= \frac{\left(2\,\mathrm{atan}\left(\frac{a^2\,e^{d\,x}\,e^{c}\,\sqrt{b^5\,d^2\,(a-b)}}{2\,b^2\,d\,(a-b)\,\sqrt{a^4}}\right)+2\,\mathrm{atan}\left(\left(e^{d\,x}\,e^{c}\,\left(\frac{2\,a^2}{b^8\,d\,(a-b)^2\,\sqrt{a^4}}-\frac{4\,\left(2\,a^3\,b^3\,d\,\sqrt{a^4}-2\,a^4\,b^2\,d\,\sqrt{a^4}\right)}{a^5\,b^6\,(a-b)\,\sqrt{a\,b^5\,d^2-b^6\,d^2}\,\sqrt{b^5\,d^2\,(a-b)}}\right)+\frac{2\,a^2\,e}{b^8\,d\,(a-}\right.\right.\right.}{2\,\sqrt{a\,b^5\,d^2-b^6\,d^2}}$$
$$+\frac{e^{-3\,c-3\,d\,x}}{24\,b\,d}+\frac{e^{3\,c+3\,d\,x}}{24\,b\,d}-\frac{e^{c+d\,x}\,(4\,a+3\,b)}{8\,b^2\,d}-\frac{e^{-c-d\,x}\,(4\,a+3\,b)}{8\,b^2\,d}$$

input
```
int(sinh(c + d*x)^5/(a + b*sinh(c + d*x)^2),x)
```

output
```
((2*atan((a^2*exp(d*x)*exp(c)*(b^5*d^2*(a - b))^(1/2))/(2*b^2*d*(a - b)*(a
^4)^(1/2))) + 2*atan((exp(d*x)*exp(c)*((2*a^2)/(b^8*d*(a - b)^2*(a^4)^(1/2
)) - (4*(2*a^3*b^3*d*(a^4)^(1/2) - 2*a^4*b^2*d*(a^4)^(1/2)))/(a^5*b^6*(a -
 b)*(a*b^5*d^2 - b^6*d^2)^(1/2)*(b^5*d^2*(a - b))^(1/2))) + (2*a^2*exp(3*c
)*exp(3*d*x))/(b^8*d*(a - b)^2*(a^4)^(1/2)))*((b^7*(a*b^5*d^2 - b^6*d^2)^(
1/2))/4 - (a*b^6*(a*b^5*d^2 - b^6*d^2)^(1/2))/4)))*(a^4)^(1/2))/(2*(a*b^5*
d^2 - b^6*d^2)^(1/2)) + exp(- 3*c - 3*d*x)/(24*b*d) + exp(3*c + 3*d*x)/(24
*b*d) - (exp(c + d*x)*(4*a + 3*b))/(8*b^2*d) - (exp(- c - d*x)*(4*a + 3*b)
)/(8*b^2*d)
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 772, normalized size of antiderivative = 9.77

$$\int \frac{\sinh^5(c+dx)}{a+b\sinh^2(c+dx)}\,dx$$

$$= \frac{24e^{3dx+3c}\sqrt{b}\,\sqrt{a}\,\sqrt{a-b}\,\sqrt{2\sqrt{a}\,\sqrt{a-b}+2a-b}\,atan\left(\frac{e^{dx+c}b}{\sqrt{b}\,\sqrt{2\sqrt{a}\,\sqrt{a-b}+2a-b}}\right)a^2-24e^{3dx+3c}\sqrt{b}\,\sqrt{2\sqrt{a}\,\sqrt{a-}}}{}$$

input
```
int(sinh(d*x+c)^5/(a+b*sinh(d*x+c)^2),x)
```

output
```
(24*e**(3*c + 3*d*x)*sqrt(b)*sqrt(a)*sqrt(a - b)*sqrt(2*sqrt(a)*sqrt(a - b
) + 2*a - b)*atan((e**(c + d*x)*b)/(sqrt(b)*sqrt(2*sqrt(a)*sqrt(a - b) + 2
*a - b)))*a**2 - 24*e**(3*c + 3*d*x)*sqrt(b)*sqrt(2*sqrt(a)*sqrt(a - b) +
2*a - b)*atan((e**(c + d*x)*b)/(sqrt(b)*sqrt(2*sqrt(a)*sqrt(a - b) + 2*a -
 b)))*a**3 + 24*e**(3*c + 3*d*x)*sqrt(b)*sqrt(2*sqrt(a)*sqrt(a - b) + 2*a
- b)*atan((e**(c + d*x)*b)/(sqrt(b)*sqrt(2*sqrt(a)*sqrt(a - b) + 2*a - b))
)*a**2*b + 12*e**(3*c + 3*d*x)*sqrt(b)*sqrt(a)*sqrt(a - b)*sqrt(2*sqrt(a)*
sqrt(a - b) - 2*a + b)*log( - sqrt(2*sqrt(a)*sqrt(a - b) - 2*a + b) + e**(
c + d*x)*sqrt(b))*a**2 - 12*e**(3*c + 3*d*x)*sqrt(b)*sqrt(a)*sqrt(a - b)*s
qrt(2*sqrt(a)*sqrt(a - b) - 2*a + b)*log(sqrt(2*sqrt(a)*sqrt(a - b) - 2*a
+ b) + e**(c + d*x)*sqrt(b))*a**2 + 12*e**(3*c + 3*d*x)*sqrt(b)*sqrt(2*sqr
t(a)*sqrt(a - b) - 2*a + b)*log( - sqrt(2*sqrt(a)*sqrt(a - b) - 2*a + b) +
 e**(c + d*x)*sqrt(b))*a**3 - 12*e**(3*c + 3*d*x)*sqrt(b)*sqrt(2*sqrt(a)*s
qrt(a - b) - 2*a + b)*log( - sqrt(2*sqrt(a)*sqrt(a - b) - 2*a + b) + e**(c
 + d*x)*sqrt(b))*a**2*b - 12*e**(3*c + 3*d*x)*sqrt(b)*sqrt(2*sqrt(a)*sqrt(
a - b) - 2*a + b)*log(sqrt(2*sqrt(a)*sqrt(a - b) - 2*a + b) + e**(c + d*x)
*sqrt(b))*a**3 + 12*e**(3*c + 3*d*x)*sqrt(b)*sqrt(2*sqrt(a)*sqrt(a - b) -
2*a + b)*log(sqrt(2*sqrt(a)*sqrt(a - b) - 2*a + b) + e**(c + d*x)*sqrt(b))
*a**2*b + e**(6*c + 6*d*x)*a*b**3 - e**(6*c + 6*d*x)*b**4 - 12*e**(4*c + 4
*d*x)*a**2*b**2 + 3*e**(4*c + 4*d*x)*a*b**3 + 9*e**(4*c + 4*d*x)*b**4 -...
```

3.31 $\int \frac{\sinh^4(c+dx)}{a+b\sinh^2(c+dx)}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 79

$$\int \frac{\sinh^4(c+dx)}{a+b\sinh^2(c+dx)}\,dx = -\frac{(2a+b)x}{2b^2}+\frac{a^{3/2}\text{arctanh}\left(\frac{\sqrt{a-b}\tanh(c+dx)}{\sqrt{a}}\right)}{\sqrt{a-b}\,b^2d}+\frac{\cosh(c+dx)\sinh(c+dx)}{2bd}$$

output
```
-1/2*(2*a+b)*x/b^2+a^(3/2)*arctanh((a-b)^(1/2)*tanh(d*x+c)/a^(1/2))/(a-b)^
(1/2)/b^2/d+1/2*cosh(d*x+c)*sinh(d*x+c)/b/d
```

Mathematica [A] (verified)

Time = 0.48 (sec) , antiderivative size = 71, normalized size of antiderivative = 0.90

$$\int \frac{\sinh^4(c+dx)}{a+b\sinh^2(c+dx)}\,dx = \frac{-2(2a+b)(c+dx)+\frac{4a^{3/2}\text{arctanh}\left(\frac{\sqrt{a-b}\tanh(c+dx)}{\sqrt{a}}\right)}{\sqrt{a-b}}+b\sinh(2(c+dx))}{4b^2d}$$

input
```
Integrate[Sinh[c + d*x]^4/(a + b*Sinh[c + d*x]^2),x]
```

output
```
(-2*(2*a + b)*(c + d*x) + (4*a^(3/2)*ArcTanh[(Sqrt[a - b]*Tanh[c + d*x])/S
qrt[a]])/Sqrt[a - b] + b*Sinh[2*(c + d*x)])/(4*b^2*d)
```

Rubi [A] (verified)

Time = 0.32 (sec) , antiderivative size = 97, normalized size of antiderivative = 1.23, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.261$, Rules used = {3042, 3666, 372, 397, 219, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sinh^4(c+dx)}{a+b\sinh^2(c+dx)}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\sin(ic+idx)^4}{a-b\sin(ic+idx)^2}dx$$

$$\Big\downarrow \text{3666}$$

$$\frac{\int \frac{\tanh^4(c+dx)}{(1-\tanh^2(c+dx))^2(a-(a-b)\tanh^2(c+dx))}d\tanh(c+dx)}{d}$$

$$\Big\downarrow \text{372}$$

$$\frac{\frac{\tanh(c+dx)}{2b(1-\tanh^2(c+dx))}-\frac{\int \frac{(a+b)\tanh^2(c+dx)+a}{(1-\tanh^2(c+dx))(a-(a-b)\tanh^2(c+dx))}d\tanh(c+dx)}{2b}}{d}$$

$$\Big\downarrow \text{397}$$

$$\frac{\frac{\tanh(c+dx)}{2b(1-\tanh^2(c+dx))}-\frac{\frac{(2a+b)\int \frac{1}{1-\tanh^2(c+dx)}d\tanh(c+dx)}{b}-\frac{2a^2\int \frac{1}{a-(a-b)\tanh^2(c+dx)}d\tanh(c+dx)}{b}}{2b}}{d}$$

$$\Big\downarrow \text{219}$$

$$\frac{\frac{\tanh(c+dx)}{2b(1-\tanh^2(c+dx))}-\frac{\frac{(2a+b)\text{arctanh}(\tanh(c+dx))}{b}-\frac{2a^2\int \frac{1}{a-(a-b)\tanh^2(c+dx)}d\tanh(c+dx)}{b}}{2b}}{d}$$

$$\Big\downarrow \text{221}$$

$$\frac{\frac{\tanh(c+dx)}{2b(1-\tanh^2(c+dx))}-\frac{\frac{(2a+b)\text{arctanh}(\tanh(c+dx))}{b}-\frac{2a^{3/2}\text{arctanh}\left(\frac{\sqrt{a-b}\tanh(c+dx)}{\sqrt{a}}\right)}{b\sqrt{a-b}}}{2b}}{d}$$

input
```
Int[Sinh[c + d*x]^4/(a + b*Sinh[c + d*x]^2),x]
```

output
```
(-1/2*(((2*a + b)*ArcTanh[Tanh[c + d*x]])/b - (2*a^(3/2)*ArcTanh[(Sqrt[a -
 b]*Tanh[c + d*x])/Sqrt[a]])/(Sqrt[a - b]*b))/b + Tanh[c + d*x]/(2*b*(1 -
Tanh[c + d*x]^2)))/d
```

Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 372
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_
), x_Symbol] :> Simp[(-a)*e^3*(e*x)^(m - 3)*(a + b*x^2)^(p + 1)*((c + d*x^2
)^(q + 1)/(2*b*(b*c - a*d)*(p + 1))), x] + Simp[e^4/(2*b*(b*c - a*d)*(p + 1
))   Int[(e*x)^(m - 4)*(a + b*x^2)^(p + 1)*(c + d*x^2)^q*Simp[a*c*(m - 3) +
 (a*d*(m + 2*q - 1) + 2*b*c*(p + 1))*x^2, x], x], x] /; FreeQ[{a, b, c, d,
e, q}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && GtQ[m, 3] && IntBinomialQ[a
, b, c, d, e, m, 2, p, q, x]
```

rule 397
```
Int[((e_) + (f_.)*(x_)^2)/(((a_) + (b_.)*(x_)^2)*((c_) + (d_.)*(x_)^2)), x_
Symbol] :> Simp[(b*e - a*f)/(b*c - a*d)   Int[1/(a + b*x^2), x], x] - Simp[
(d*e - c*f)/(b*c - a*d)   Int[1/(c + d*x^2), x], x] /; FreeQ[{a, b, c, d, e
, f}, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3666
```
Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(
p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Simp[ff^(m + 1
)/f   Subst[Int[x^m*((a + (a + b)*ff^2*x^2)^p/(1 + ff^2*x^2)^(m/2 + p + 1))
, x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f}, x] && IntegerQ[m/2] &
& IntegerQ[p]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 163 vs. $2(67) = 134$.

Time = 0.81 (sec) , antiderivative size = 164, normalized size of antiderivative = 2.08

method	result
risch	$-\frac{ax}{b^2} - \frac{x}{2b} + \frac{e^{2dx+2c}}{8bd} - \frac{e^{-2dx-2c}}{8bd} + \frac{\sqrt{a(a-b)}\,a\ln\left(e^{2dx+2c} - \frac{-2a+2\sqrt{a(a-b)}+b}{b}\right)}{2(a-b)d\,b^2} - \frac{\sqrt{a(a-b)}\,a\ln\left(e^{2dx+2c+}\right.}{2(a-b)d}$
derivativedivides	$-\frac{1}{2b\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^2} + \frac{1}{2b\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)} + \frac{(-2a-b)\ln\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)}{2b^2} - \frac{2a^3\left(\frac{\left(\sqrt{-b(a-b)}+b\right)\operatorname{arctan}\left(\frac{\tanh\left(\frac{dx}{2}\right.}{\sqrt{\left(2\sqrt{-b(a-b}\right.}}\right.}{2a\sqrt{-b(a-b)}\sqrt{\left(2\sqrt{-b(a-b)}-a\right.}}\right.}{}$
default	$-\frac{1}{2b\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^2} + \frac{1}{2b\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)} + \frac{(-2a-b)\ln\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)}{2b^2} - \frac{2a^3\left(\frac{\left(\sqrt{-b(a-b)}+b\right)\operatorname{arctan}\left(\frac{\tanh\left(\frac{dx}{2}\right.}{\sqrt{\left(2\sqrt{-b(a-b}\right.}}\right.}{2a\sqrt{-b(a-b)}\sqrt{\left(2\sqrt{-b(a-b)}-a\right.}}\right.}{}$

input
```
int(sinh(d*x+c)^4/(a+b*sinh(d*x+c)^2),x,method=_RETURNVERBOSE)
```

output
```
-a*x/b^2-1/2*x/b+1/8/b/d*exp(2*d*x+2*c)-1/8/b/d*exp(-2*d*x-2*c)+1/2*(a*(a-
b))^(1/2)/(a-b)*a/d/b^2*ln(exp(2*d*x+2*c)-(-2*a+2*(a*(a-b))^(1/2)+b)/b)-1/
2*(a*(a-b))^(1/2)/(a-b)*a/d/b^2*ln(exp(2*d*x+2*c)+(2*a+2*(a*(a-b))^(1/2)-b
)/b)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 283 vs. $2(67) = 134$.

Time = 0.12 (sec) , antiderivative size = 859, normalized size of antiderivative = 10.87

$$\int \frac{\sinh^4(c+dx)}{a+b\sinh^2(c+dx)}\,dx = \text{Too large to display}$$

input
```
integrate(sinh(d*x+c)^4/(a+b*sinh(d*x+c)^2),x, algorithm="fricas")
```

output
```
[-1/8*(4*(2*a + b)*d*x*cosh(d*x + c)^2 - b*cosh(d*x + c)^4 - 4*b*cosh(d*x
+ c)*sinh(d*x + c)^3 - b*sinh(d*x + c)^4 + 2*(2*(2*a + b)*d*x - 3*b*cosh(d
*x + c)^2)*sinh(d*x + c)^2 - 4*(a*cosh(d*x + c)^2 + 2*a*cosh(d*x + c)*sinh
(d*x + c) + a*sinh(d*x + c)^2)*sqrt(a/(a - b))*log((b^2*cosh(d*x + c)^4 +
4*b^2*cosh(d*x + c)*sinh(d*x + c)^3 + b^2*sinh(d*x + c)^4 + 2*(2*a*b - b^2
)*cosh(d*x + c)^2 + 2*(3*b^2*cosh(d*x + c)^2 + 2*a*b - b^2)*sinh(d*x + c)^
2 + 8*a^2 - 8*a*b + b^2 + 4*(b^2*cosh(d*x + c)^3 + (2*a*b - b^2)*cosh(d*x
+ c))*sinh(d*x + c) - 4*((a*b - b^2)*cosh(d*x + c)^2 + 2*(a*b - b^2)*cosh(
d*x + c)*sinh(d*x + c) + (a*b - b^2)*sinh(d*x + c)^2 + 2*a^2 - 3*a*b + b^2
)*sqrt(a/(a - b)))/(b*cosh(d*x + c)^4 + 4*b*cosh(d*x + c)*sinh(d*x + c)^3
+ b*sinh(d*x + c)^4 + 2*(2*a - b)*cosh(d*x + c)^2 + 2*(3*b*cosh(d*x + c)^2
 + 2*a - b)*sinh(d*x + c)^2 + 4*(b*cosh(d*x + c)^3 + (2*a - b)*cosh(d*x +
c))*sinh(d*x + c) + b)) + 4*(2*(2*a + b)*d*x*cosh(d*x + c) - b*cosh(d*x +
c)^3)*sinh(d*x + c) + b)/(b^2*d*cosh(d*x + c)^2 + 2*b^2*d*cosh(d*x + c)*si
nh(d*x + c) + b^2*d*sinh(d*x + c)^2), -1/8*(4*(2*a + b)*d*x*cosh(d*x + c)^
2 - b*cosh(d*x + c)^4 - 4*b*cosh(d*x + c)*sinh(d*x + c)^3 - b*sinh(d*x + c
)^4 + 2*(2*(2*a + b)*d*x - 3*b*cosh(d*x + c)^2)*sinh(d*x + c)^2 - 8*(a*cos
h(d*x + c)^2 + 2*a*cosh(d*x + c)*sinh(d*x + c) + a*sinh(d*x + c)^2)*sqrt(-
a/(a - b))*arctan(1/2*(b*cosh(d*x + c)^2 + 2*b*cosh(d*x + c)*sinh(d*x + c)
 + b*sinh(d*x + c)^2 + 2*a - b)*sqrt(-a/(a - b))/a) + 4*(2*(2*a + b)*d*...
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\sinh^4(c+dx)}{a+b\sinh^2(c+dx)}\,dx = \text{Timed out}$$

input
```
integrate(sinh(d*x+c)**4/(a+b*sinh(d*x+c)**2),x)
```

output
```
Timed out
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{\sinh^4(c+dx)}{a+b\sinh^2(c+dx)}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate(sinh(d*x+c)^4/(a+b*sinh(d*x+c)^2),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(b-a>0)', see `assume?` for more
details)Is
```

Giac [A] (verification not implemented)

Time = 0.60 (sec) , antiderivative size = 126, normalized size of antiderivative = 1.59

$$\int \frac{\sinh^4(c+dx)}{a+b\sinh^2(c+dx)}\,dx = \frac{\frac{8\,a^2\arctan\left(\frac{be^{(2\,dx+2\,c)}+2\,a-b}{2\,\sqrt{-a^2+ab}}\right)}{\sqrt{-a^2+ab}b^2} - \frac{4\,(dx+c)(2\,a+b)}{b^2} + \frac{e^{(2\,dx+2\,c)}}{b} + \frac{\left(4\,ae^{(2\,dx+2\,c)}+2\,be^{(2\,dx+2\,c)}-b\right)e^{(-2\,dx-2\,c)}}{b^2}}{8\,d}$$

input
```
integrate(sinh(d*x+c)^4/(a+b*sinh(d*x+c)^2),x, algorithm="giac")
```

output
```
1/8*(8*a^2*arctan(1/2*(b*e^(2*d*x + 2*c) + 2*a - b)/sqrt(-a^2 + a*b))/(sqr
t(-a^2 + a*b)*b^2) - 4*(d*x + c)*(2*a + b)/b^2 + e^(2*d*x + 2*c)/b + (4*a*
e^(2*d*x + 2*c) + 2*b*e^(2*d*x + 2*c) - b)*e^(-2*d*x - 2*c)/b^2)/d
```

Mupad [B] (verification not implemented)

Time = 2.14 (sec) , antiderivative size = 216, normalized size of antiderivative = 2.73

$$\int \frac{\sinh^4(c+dx)}{a+b\sinh^2(c+dx)}\,dx = \frac{e^{2c+2dx}}{8bd} - \frac{e^{-2c-2dx}}{8bd} - \frac{x(2a+b)}{2b^2} + \frac{a^{3/2}\ln\left(-\frac{4a^2 e^{2c+2dx}}{b^3} - \frac{2a^{3/2}(bd+2ade^{2c+2dx}-bde^{2c+2dx})}{b^3 d\sqrt{a-b}}\right)}{2b^2 d\sqrt{a-b}} - \frac{a^{3/2}\ln\left(\frac{2a^{3/2}(bd+2ade^{2c+2dx}-bde^{2c+2dx})}{b^3 d\sqrt{a-b}} - \frac{4a^2 e^{2c+2dx}}{b^3}\right)}{2b^2 d\sqrt{a-b}}$$

input
```
int(sinh(c + d*x)^4/(a + b*sinh(c + d*x)^2),x)
```

output
```
exp(2*c + 2*d*x)/(8*b*d) - exp(- 2*c - 2*d*x)/(8*b*d) - (x*(2*a + b))/(2*b
^2) + (a^(3/2)*log(- (4*a^2*exp(2*c + 2*d*x))/b^3 - (2*a^(3/2)*(b*d + 2*a*
d*exp(2*c + 2*d*x) - b*d*exp(2*c + 2*d*x)))/(b^3*d*(a - b)^(1/2))))/(2*b^2
*d*(a - b)^(1/2)) - (a^(3/2)*log((2*a^(3/2)*(b*d + 2*a*d*exp(2*c + 2*d*x)
- b*d*exp(2*c + 2*d*x)))/(b^3*d*(a - b)^(1/2)) - (4*a^2*exp(2*c + 2*d*x))/
b^3))/(2*b^2*d*(a - b)^(1/2))
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 264, normalized size of antiderivative = 3.34

$$\int \frac{\sinh^4(c+dx)}{a+b\sinh^2(c+dx)}\,dx = \frac{4e^{2dx+2c}\sqrt{a}\sqrt{a-b}\log\left(-\sqrt{2\sqrt{a}\sqrt{a-b}-2a+b}+e^{dx+c}\sqrt{b}\right)a + 4e^{2dx+2c}\sqrt{a}\sqrt{a-b}\log\left(\sqrt{2\sqrt{a}\sqrt{a-}\right.}{}$$

input
```
int(sinh(d*x+c)^4/(a+b*sinh(d*x+c)^2),x)
```

output
```
(4*e**(2*c + 2*d*x)*sqrt(a)*sqrt(a - b)*log( - sqrt(2*sqrt(a)*sqrt(a - b)
- 2*a + b) + e**(c + d*x)*sqrt(b))*a + 4*e**(2*c + 2*d*x)*sqrt(a)*sqrt(a -
 b)*log(sqrt(2*sqrt(a)*sqrt(a - b) - 2*a + b) + e**(c + d*x)*sqrt(b))*a -
4*e**(2*c + 2*d*x)*sqrt(a)*sqrt(a - b)*log(2*sqrt(a)*sqrt(a - b) + e**(2*c
 + 2*d*x)*b + 2*a - b)*a + e**(4*c + 4*d*x)*a*b - e**(4*c + 4*d*x)*b**2 -
8*e**(2*c + 2*d*x)*a**2*d*x + 4*e**(2*c + 2*d*x)*a*b*d*x + 4*e**(2*c + 2*d
*x)*b**2*d*x - a*b + b**2)/(8*e**(2*c + 2*d*x)*b**2*d*(a - b))
```

3.32 $\int \frac{\sinh^3(c+dx)}{a+b\sinh^2(c+dx)}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 56

$$\int \frac{\sinh^3(c+dx)}{a+b\sinh^2(c+dx)}\,dx = -\frac{a\arctan\left(\frac{\sqrt{b}\cosh(c+dx)}{\sqrt{a-b}}\right)}{\sqrt{a-b}b^{3/2}d}+\frac{\cosh(c+dx)}{bd}$$

output
```
-a*arctan(b^(1/2)*cosh(d*x+c)/(a-b)^(1/2))/(a-b)^(1/2)/b^(3/2)/d+cosh(d*x+
c)/b/d
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.57 (sec) , antiderivative size = 107, normalized size of antiderivative = 1.91

$$\int \frac{\sinh^3(c+dx)}{a+b\sinh^2(c+dx)}\,dx = \frac{-\frac{a\left(\arctan\left(\frac{\sqrt{b}-i\sqrt{a}\tanh\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a-b}}\right)+\arctan\left(\frac{\sqrt{b}+i\sqrt{a}\tanh\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a-b}}\right)\right)}{\sqrt{a-b}}+\sqrt{b}\cosh(c+dx)}{b^{3/2}d}$$

input
```
Integrate[Sinh[c + d*x]^3/(a + b*Sinh[c + d*x]^2),x]
```

output
```
(-((a*(ArcTan[(Sqrt[b] - I*Sqrt[a]*Tanh[(c + d*x)/2])/Sqrt[a - b]] + ArcTa
n[(Sqrt[b] + I*Sqrt[a]*Tanh[(c + d*x)/2])/Sqrt[a - b]]))/Sqrt[a - b]) + Sq
rt[b]*Cosh[c + d*x])/(b^(3/2)*d)
```

Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 55, normalized size of antiderivative = 0.98, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.217$, Rules used = {3042, 26, 3665, 299, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sinh^3(c+dx)}{a+b\sinh^2(c+dx)}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{i\sin(ic+idx)^3}{a-b\sin(ic+idx)^2}dx$$

$$\Big\downarrow \text{26}$$

$$i\int \frac{\sin(ic+idx)^3}{a-b\sin(ic+idx)^2}dx$$

$$\Big\downarrow \text{3665}$$

$$-\frac{\int \frac{1-\cosh^2(c+dx)}{b\cosh^2(c+dx)+a-b}d\cosh(c+dx)}{d}$$

$$\Big\downarrow \text{299}$$

$$-\frac{\frac{a\int \frac{1}{b\cosh^2(c+dx)+a-b}d\cosh(c+dx)}{b}-\frac{\cosh(c+dx)}{b}}{d}$$

$$\Big\downarrow \text{218}$$

$$-\frac{\frac{a\arctan\left(\frac{\sqrt{b}\cosh(c+dx)}{\sqrt{a-b}}\right)}{b^{3/2}\sqrt{a-b}}-\frac{\cosh(c+dx)}{b}}{d}$$

input
```
Int[Sinh[c + d*x]^3/(a + b*Sinh[c + d*x]^2),x]
```

output
```
-(((a*ArcTan[(Sqrt[b]*Cosh[c + d*x])/Sqrt[a - b]])/(Sqrt[a - b]*b^(3/2)) -
 Cosh[c + d*x]/b)/d)
```

Defintions of rubi rules used

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 299
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2), x_Symbol] :> Simp[d*x
*((a + b*x^2)^(p + 1)/(b*(2*p + 3))), x] - Simp[(a*d - b*c*(2*p + 3))/(b*(2
*p + 3))   Int[(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
 a*d, 0] && NeQ[2*p + 3, 0]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3665
```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^
(p_.), x_Symbol] :> With[{ff = FreeFactors[Cos[e + f*x], x]}, Simp[-ff/f
Subst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a + b - b*ff^2*x^2)^p, x], x, Cos[e +
 f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m - 1)/2]
```

Maple [A] (verified)

Time = 0.52 (sec) , antiderivative size = 93, normalized size of antiderivative = 1.66

method	result	size
derivativedivides	$\frac{\frac{1}{b\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)}-\frac{1}{b\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}-\frac{a\arctan\left(\frac{2\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}a-2a+4b}{4\sqrt{ab-b^{2}}}\right)}{b\sqrt{ab-b^{2}}}}{d}$	93
default	$\frac{\frac{1}{b\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)}-\frac{1}{b\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}-\frac{a\arctan\left(\frac{2\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}a-2a+4b}{4\sqrt{ab-b^{2}}}\right)}{b\sqrt{ab-b^{2}}}}{d}$	93
risch	$\frac{e^{dx+c}}{2bd}+\frac{e^{-dx-c}}{2bd}-\frac{a\ln\left(e^{2dx+2c}+\frac{2(a-b)e^{dx+c}}{\sqrt{-ab+b^{2}}}+1\right)}{2\sqrt{-ab+b^{2}}\,db}+\frac{a\ln\left(e^{2dx+2c}-\frac{2(a-b)e^{dx+c}}{\sqrt{-ab+b^{2}}}+1\right)}{2\sqrt{-ab+b^{2}}\,db}$	141

input
```
int(sinh(d*x+c)^3/(a+b*sinh(d*x+c)^2),x,method=_RETURNVERBOSE)
```

output
```
1/d*(1/b/(tanh(1/2*d*x+1/2*c)+1)-1/b/(tanh(1/2*d*x+1/2*c)-1)-1/b*a/(a*b-b^
2)^(1/2)*arctan(1/4*(2*tanh(1/2*d*x+1/2*c)^2*a-2*a+4*b)/(a*b-b^2)^(1/2)))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 295 vs. 2(48) = 96.

Time = 0.12 (sec) , antiderivative size = 745, normalized size of antiderivative = 13.30

$$\int \frac{\sinh^3(c+dx)}{a+b\sinh^2(c+dx)}\,dx = \text{Too large to display}$$

input
```
integrate(sinh(d*x+c)^3/(a+b*sinh(d*x+c)^2),x, algorithm="fricas")
```

output
```
[1/2*((a*b - b^2)*cosh(d*x + c)^2 + 2*(a*b - b^2)*cosh(d*x + c)*sinh(d*x +
 c) + (a*b - b^2)*sinh(d*x + c)^2 - sqrt(-a*b + b^2)*(a*cosh(d*x + c) + a*
sinh(d*x + c))*log((b*cosh(d*x + c)^4 + 4*b*cosh(d*x + c)*sinh(d*x + c)^3
+ b*sinh(d*x + c)^4 - 2*(2*a - 3*b)*cosh(d*x + c)^2 + 2*(3*b*cosh(d*x + c)
^2 - 2*a + 3*b)*sinh(d*x + c)^2 + 4*(b*cosh(d*x + c)^3 - (2*a - 3*b)*cosh(
d*x + c))*sinh(d*x + c) + 4*(cosh(d*x + c)^3 + 3*cosh(d*x + c)*sinh(d*x +
c)^2 + sinh(d*x + c)^3 + (3*cosh(d*x + c)^2 + 1)*sinh(d*x + c) + cosh(d*x
+ c))*sqrt(-a*b + b^2) + b)/(b*cosh(d*x + c)^4 + 4*b*cosh(d*x + c)*sinh(d*
x + c)^3 + b*sinh(d*x + c)^4 + 2*(2*a - b)*cosh(d*x + c)^2 + 2*(3*b*cosh(d
*x + c)^2 + 2*a - b)*sinh(d*x + c)^2 + 4*(b*cosh(d*x + c)^3 + (2*a - b)*co
sh(d*x + c))*sinh(d*x + c) + b)) + a*b - b^2)/((a*b^2 - b^3)*d*cosh(d*x +
c) + (a*b^2 - b^3)*d*sinh(d*x + c)), 1/2*((a*b - b^2)*cosh(d*x + c)^2 + 2*
(a*b - b^2)*cosh(d*x + c)*sinh(d*x + c) + (a*b - b^2)*sinh(d*x + c)^2 - 2*
sqrt(a*b - b^2)*(a*cosh(d*x + c) + a*sinh(d*x + c))*arctan(-1/2*(b*cosh(d*
x + c)^3 + 3*b*cosh(d*x + c)*sinh(d*x + c)^2 + b*sinh(d*x + c)^3 + (4*a -
3*b)*cosh(d*x + c) + (3*b*cosh(d*x + c)^2 + 4*a - 3*b)*sinh(d*x + c))/sqrt
(a*b - b^2)) + 2*sqrt(a*b - b^2)*(a*cosh(d*x + c) + a*sinh(d*x + c))*arcta
n(2*sqrt(a*b - b^2)/(b*cosh(d*x + c) + b*sinh(d*x + c))) + a*b - b^2)/((a*
b^2 - b^3)*d*cosh(d*x + c) + (a*b^2 - b^3)*d*sinh(d*x + c))]
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\sinh^3(c+dx)}{a+b\sinh^2(c+dx)}\,dx = \text{Timed out}$$

input
```
integrate(sinh(d*x+c)**3/(a+b*sinh(d*x+c)**2),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{\sinh^3(c+dx)}{a+b\sinh^2(c+dx)}\,dx = \int \frac{\sinh{(dx+c)}^3}{b\sinh{(dx+c)}^2+a}\,dx$$

input
```
integrate(sinh(d*x+c)^3/(a+b*sinh(d*x+c)^2),x, algorithm="maxima")
```

output
```
1/2*(e^(2*d*x + 2*c) + 1)*e^(-d*x - c)/(b*d) - 1/8*integrate(16*(a*e^(3*d*
x + 3*c) - a*e^(d*x + c))/(b^2*e^(4*d*x + 4*c) + b^2 + 2*(2*a*b*e^(2*c) -
b^2*e^(2*c))*e^(2*d*x)), x)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{\sinh^3(c+dx)}{a+b\sinh^2(c+dx)}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(sinh(d*x+c)^3/(a+b*sinh(d*x+c)^2),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Limit: Max order reached or unable
to make series expansion Error: Bad Argument Value
```

Mupad [B] (verification not implemented)

Time = 2.36 (sec) , antiderivative size = 293, normalized size of antiderivative = 5.23

$$\int \frac{\sinh^3(c+dx)}{a+b\sinh^2(c+dx)}\,dx = \frac{e^{c+dx}}{2bd} - \frac{\Bigg(2\,\mathrm{atan}\left(\frac{a^3\,e^{dx}\,e^{c}\,\sqrt{b^3\,d^2\,(a-b)}}{2\,b\,d\,(a-b)\,{(a^2)}^{3/2}}\right) + 2\,\mathrm{atan}\Bigg(\Bigg(e^{dx}\,e^{c}\,\Bigg(\frac{2\,a^3}{b^5\,d\,{(a-b)}^2\,{(a^2)}^{3/2}} - \frac{4\,\left(2\,b^2\,d\,{(a^2)}^{3/2}-2\,a\,b\,d\,{(a^2)}^{3/2}\right)}{a^3\,b^4\,(a-b)\,\sqrt{a\,b^3\,d^2-b^4\,d^2}\,\sqrt{b^3\,d^2\,(a-b)}}\Bigg) + \cdots}{2\,\sqrt{a\,b^3\,d^2-b^4\,d^2}} + \frac{e^{-c-dx}}{2bd}$$

input
```
int(sinh(c + d*x)^3/(a + b*sinh(c + d*x)^2),x)
```

output
```
exp(c + d*x)/(2*b*d) - ((2*atan((a^3*exp(d*x)*exp(c)*(b^3*d^2*(a - b))^(1/
2))/(2*b*d*(a - b)*(a^2)^(3/2))) + 2*atan((exp(d*x)*exp(c)*((2*a^3)/(b^5*d
*(a - b)^2*(a^2)^(3/2)) - (4*(2*b^2*d*(a^2)^(3/2) - 2*a*b*d*(a^2)^(3/2)))/
(a^3*b^4*(a - b)*(a*b^3*d^2 - b^4*d^2)^(1/2)*(b^3*d^2*(a - b))^(1/2))) + (
2*a^3*exp(3*c)*exp(3*d*x))/(b^5*d*(a - b)^2*(a^2)^(3/2)))*((b^5*(a*b^3*d^2
 - b^4*d^2)^(1/2))/4 - (a*b^4*(a*b^3*d^2 - b^4*d^2)^(1/2))/4)))*(a^2)^(1/2
))/(2*(a*b^3*d^2 - b^4*d^2)^(1/2)) + exp(- c - d*x)/(2*b*d)
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 540, normalized size of antiderivative = 9.64

$$\int \frac{\sinh^3(c+dx)}{a+b\sinh^2(c+dx)}\,dx$$
$$= \frac{-2\sqrt{b}\,\sqrt{a}\,\sqrt{a-b}\,\sqrt{2\sqrt{a}\,\sqrt{a-b}+2a-b}\,atan\left(\frac{e^{dx+c}b}{\sqrt{b}\,\sqrt{2\sqrt{a}\,\sqrt{a-b}+2a-b}}\right)a+2\sqrt{b}\,\sqrt{2\sqrt{a}\,\sqrt{a-b}+2a-b}\,ata}{}$$

input
```
int(sinh(d*x+c)^3/(a+b*sinh(d*x+c)^2),x)
```

output
```
( - 2*sqrt(b)*sqrt(a)*sqrt(a - b)*sqrt(2*sqrt(a)*sqrt(a - b) + 2*a - b)*at
an((e**(c + d*x)*b)/(sqrt(b)*sqrt(2*sqrt(a)*sqrt(a - b) + 2*a - b)))*a + 2
*sqrt(b)*sqrt(2*sqrt(a)*sqrt(a - b) + 2*a - b)*atan((e**(c + d*x)*b)/(sqrt
(b)*sqrt(2*sqrt(a)*sqrt(a - b) + 2*a - b)))*a**2 - 2*sqrt(b)*sqrt(2*sqrt(a
)*sqrt(a - b) + 2*a - b)*atan((e**(c + d*x)*b)/(sqrt(b)*sqrt(2*sqrt(a)*sqr
t(a - b) + 2*a - b)))*a*b + 2*cosh(c + d*x)*a*b**2 - 2*cosh(c + d*x)*b**3
- sqrt(b)*sqrt(a)*sqrt(a - b)*sqrt(2*sqrt(a)*sqrt(a - b) - 2*a + b)*log( -
 sqrt(2*sqrt(a)*sqrt(a - b) - 2*a + b) + e**(c + d*x)*sqrt(b))*a + sqrt(b)
*sqrt(a)*sqrt(a - b)*sqrt(2*sqrt(a)*sqrt(a - b) - 2*a + b)*log(sqrt(2*sqrt
(a)*sqrt(a - b) - 2*a + b) + e**(c + d*x)*sqrt(b))*a - sqrt(b)*sqrt(2*sqrt
(a)*sqrt(a - b) - 2*a + b)*log( - sqrt(2*sqrt(a)*sqrt(a - b) - 2*a + b) +
e**(c + d*x)*sqrt(b))*a**2 + sqrt(b)*sqrt(2*sqrt(a)*sqrt(a - b) - 2*a + b)
*log( - sqrt(2*sqrt(a)*sqrt(a - b) - 2*a + b) + e**(c + d*x)*sqrt(b))*a*b
+ sqrt(b)*sqrt(2*sqrt(a)*sqrt(a - b) - 2*a + b)*log(sqrt(2*sqrt(a)*sqrt(a
- b) - 2*a + b) + e**(c + d*x)*sqrt(b))*a**2 - sqrt(b)*sqrt(2*sqrt(a)*sqrt
(a - b) - 2*a + b)*log(sqrt(2*sqrt(a)*sqrt(a - b) - 2*a + b) + e**(c + d*x
)*sqrt(b))*a*b)/(2*b**3*d*(a - b))
```

3.33 $\int \frac{\sinh^2(c+dx)}{a+b\sinh^2(c+dx)}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 50

$$\int \frac{\sinh^2(c+dx)}{a+b\sinh^2(c+dx)}\,dx = \frac{x}{b}-\frac{\sqrt{a}\,\mathrm{arctanh}\left(\frac{\sqrt{a-b}\tanh(c+dx)}{\sqrt{a}}\right)}{\sqrt{a-b}\,bd}$$

output
```
x/b-a^(1/2)*arctanh((a-b)^(1/2)*tanh(d*x+c)/a^(1/2))/(a-b)^(1/2)/b/d
```

Mathematica [A] (verified)

Time = 1.99 (sec) , antiderivative size = 50, normalized size of antiderivative = 1.00

$$\int \frac{\sinh^2(c+dx)}{a+b\sinh^2(c+dx)}\,dx = \frac{c+dx-\frac{\sqrt{a}\,\mathrm{arctanh}\left(\frac{\sqrt{a-b}\tanh(c+dx)}{\sqrt{a}}\right)}{\sqrt{a-b}}}{bd}$$

input
```
Integrate[Sinh[c + d*x]^2/(a + b*Sinh[c + d*x]^2),x]
```

output
```
(c + d*x - (Sqrt[a]*ArcTanh[(Sqrt[a - b]*Tanh[c + d*x])/Sqrt[a]])/Sqrt[a -
 b])/(b*d)
```

Rubi [A] (verified)

Time = 0.31 (sec) , antiderivative size = 50, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.261, Rules used = {3042, 25, 3650, 3042, 3660, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sinh^2(c+dx)}{a+b\sinh^2(c+dx)}dx$$

$$\Big\downarrow \textbf{3042}$$

$$\int -\frac{\sin(ic+idx)^2}{a-b\sin(ic+idx)^2}dx$$

$$\Big\downarrow \textbf{25}$$

$$-\int \frac{\sin(ic+idx)^2}{a-b\sin(ic+idx)^2}dx$$

$$\Big\downarrow \textbf{3650}$$

$$\frac{x}{b}-\frac{a\int \frac{1}{b\sinh^2(c+dx)+a}dx}{b}$$

$$\Big\downarrow \textbf{3042}$$

$$\frac{x}{b}-\frac{a\int \frac{1}{a-b\sin(ic+idx)^2}dx}{b}$$

$$\Big\downarrow \textbf{3660}$$

$$\frac{x}{b}-\frac{a\int \frac{1}{a-(a-b)\tanh^2(c+dx)}d\tanh(c+dx)}{bd}$$

$$\Big\downarrow \textbf{221}$$

$$\frac{x}{b}-\frac{\sqrt{a}\text{arctanh}\left(\frac{\sqrt{a-b}\tanh(c+dx)}{\sqrt{a}}\right)}{bd\sqrt{a-b}}$$

input
```
Int[Sinh[c + d*x]^2/(a + b*Sinh[c + d*x]^2),x]
```

output
```
x/b - (Sqrt[a]*ArcTanh[(Sqrt[a - b]*Tanh[c + d*x])/Sqrt[a]])/(Sqrt[a - b]*
b*d)
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3650
```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)]^2)/((a_) + (b_.)*sin[(e_.) + (f_
.)*(x_)]^2), x_Symbol] :> Simp[B*(x/b), x] + Simp[(A*b - a*B)/b   Int[1/(a
+ b*Sin[e + f*x]^2), x], x] /; FreeQ[{a, b, e, f, A, B}, x]
```

rule 3660
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(-1), x_Symbol] :> With[{ff =
FreeFactors[Tan[e + f*x], x]}, Simp[ff/f   Subst[Int[1/(a + (a + b)*ff^2*x^
2), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f}, x]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 119 vs. $2(42) = 84$.

Time = 0.32 (sec) , antiderivative size = 120, normalized size of antiderivative = 2.40

method	result
risch	$\frac{x}{b}+\frac{\sqrt{a(a-b)}\,\ln\left(\mathrm{e}^{2dx+2c}+\frac{2a+2\sqrt{a(a-b)}-b}{b}\right)}{2(a-b)db}-\frac{\sqrt{a(a-b)}\,\ln\left(\mathrm{e}^{2dx+2c}-\frac{-2a+2\sqrt{a(a-b)}+b}{b}\right)}{2(a-b)db}$
derivativedivides	$\frac{-\frac{\ln\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}{b}+\frac{2a^2\left(\frac{\left(\sqrt{-b(a-b)}+b\right)\arctan\left(\frac{\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)a}{\sqrt{\left(2\sqrt{-b(a-b)}-a+2b\right)a}}\right)}{2a\sqrt{-b(a-b)}\sqrt{\left(2\sqrt{-b(a-b)}-a+2b\right)a}}-\frac{\left(\sqrt{-b(a-b)}-b\right)\operatorname{arctanh}\left(\frac{\tanh\left(\frac{dx}{2}+\frac{c}{2}\right.}{\sqrt{\left(2\sqrt{-b(a-b)}+\right.}}\right.}{2a\sqrt{-b(a-b)}\sqrt{\left(2\sqrt{-b(a-b)}+a-2b\right.}}\right.}{b}}{d}$
default	$\frac{-\frac{\ln\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}{b}+\frac{2a^2\left(\frac{\left(\sqrt{-b(a-b)}+b\right)\arctan\left(\frac{\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)a}{\sqrt{\left(2\sqrt{-b(a-b)}-a+2b\right)a}}\right)}{2a\sqrt{-b(a-b)}\sqrt{\left(2\sqrt{-b(a-b)}-a+2b\right)a}}-\frac{\left(\sqrt{-b(a-b)}-b\right)\operatorname{arctanh}\left(\frac{\tanh\left(\frac{dx}{2}+\frac{c}{2}\right.}{\sqrt{\left(2\sqrt{-b(a-b)}+\right.}}\right.}{2a\sqrt{-b(a-b)}\sqrt{\left(2\sqrt{-b(a-b)}+a-2b\right.}}\right.}{b}}{d}$

input
```
int(sinh(d*x+c)^2/(a+b*sinh(d*x+c)^2),x,method=_RETURNVERBOSE)
```

output
```
x/b+1/2*(a*(a-b))^(1/2)/(a-b)/d/b*ln(exp(2*d*x+2*c)+(2*a+2*(a*(a-b))^(1/2)
-b)/b)-1/2*(a*(a-b))^(1/2)/(a-b)/d/b*ln(exp(2*d*x+2*c)-(-2*a+2*(a*(a-b))^(
1/2)+b)/b)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 85 vs. $2(42) = 84$.

Time = 0.11 (sec) , antiderivative size = 464, normalized size of antiderivative = 9.28

$$\int \frac{\sinh^2(c+dx)}{a+b\sinh^2(c+dx)}\,dx$$

$$= \left[\frac{2\,dx+\sqrt{\frac{a}{a-b}}\log\left(\frac{b^2\cosh(dx+c)^4+4\,b^2\cosh(dx+c)\sinh(dx+c)^3+b^2\sinh(dx+c)^4+2\,(2\,ab-b^2)\cosh(dx+c)^2+2\left(3\,b^2\cosh(dx+c)^2+2\right.}{b\cosh(dx+c)^4+4\,b\cosh(dx+c)\sinh(dx+c)^3+b\,\mathrm{s}}\right.}{}\right.$$

input
```
integrate(sinh(d*x+c)^2/(a+b*sinh(d*x+c)^2),x, algorithm="fricas")
```

output
```
[1/2*(2*d*x + sqrt(a/(a - b))*log((b^2*cosh(d*x + c)^4 + 4*b^2*cosh(d*x +
c)*sinh(d*x + c)^3 + b^2*sinh(d*x + c)^4 + 2*(2*a*b - b^2)*cosh(d*x + c)^2
 + 2*(3*b^2*cosh(d*x + c)^2 + 2*a*b - b^2)*sinh(d*x + c)^2 + 8*a^2 - 8*a*b
 + b^2 + 4*(b^2*cosh(d*x + c)^3 + (2*a*b - b^2)*cosh(d*x + c))*sinh(d*x +
c) + 4*((a*b - b^2)*cosh(d*x + c)^2 + 2*(a*b - b^2)*cosh(d*x + c)*sinh(d*x
 + c) + (a*b - b^2)*sinh(d*x + c)^2 + 2*a^2 - 3*a*b + b^2)*sqrt(a/(a - b))
)/(b*cosh(d*x + c)^4 + 4*b*cosh(d*x + c)*sinh(d*x + c)^3 + b*sinh(d*x + c)
^4 + 2*(2*a - b)*cosh(d*x + c)^2 + 2*(3*b*cosh(d*x + c)^2 + 2*a - b)*sinh(
d*x + c)^2 + 4*(b*cosh(d*x + c)^3 + (2*a - b)*cosh(d*x + c))*sinh(d*x + c)
 + b)))/(b*d), (d*x - sqrt(-a/(a - b))*arctan(1/2*(b*cosh(d*x + c)^2 + 2*b
*cosh(d*x + c)*sinh(d*x + c) + b*sinh(d*x + c)^2 + 2*a - b)*sqrt(-a/(a - b
))/a))/(b*d)]
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\sinh^2(c+dx)}{a+b\sinh^2(c+dx)}\,dx = \text{Timed out}$$

input
```
integrate(sinh(d*x+c)**2/(a+b*sinh(d*x+c)**2),x)
```

output
```
Timed out
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{\sinh^2(c+dx)}{a+b\sinh^2(c+dx)}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate(sinh(d*x+c)^2/(a+b*sinh(d*x+c)^2),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(b-a>0)', see `assume?` for more
details)Is
```

Giac [A] (verification not implemented)

Time = 0.39 (sec) , antiderivative size = 64, normalized size of antiderivative = 1.28

$$\int \frac{\sinh^2(c+dx)}{a+b\sinh^2(c+dx)}\,dx = -\frac{\frac{a\arctan\left(\frac{be^{(2\,dx+2\,c)}+2\,a-b}{2\,\sqrt{-a^2+ab}}\right)}{\sqrt{-a^2+ab}b} - \frac{dx+c}{b}}{d}$$

input
```
integrate(sinh(d*x+c)^2/(a+b*sinh(d*x+c)^2),x, algorithm="giac")
```

output
```
-(a*arctan(1/2*(b*e^(2*d*x + 2*c) + 2*a - b)/sqrt(-a^2 + a*b))/(sqrt(-a^2
+ a*b)*b) - (d*x + c)/b)/d
```

Mupad [B] (verification not implemented)

Time = 2.42 (sec) , antiderivative size = 473, normalized size of antiderivative = 9.46

$$\int \frac{\sinh^2(c+dx)}{a+b\sinh^2(c+dx)}\,dx = \frac{x}{b} - \frac{\sqrt{a}\,\mathrm{atan}\left(\frac{\left(b^5\sqrt{b^3\,d^2-a\,b^2\,d^2}-a\,b^4\sqrt{b^3\,d^2-a\,b^2\,d^2}\right)\left(\mathrm{e}^{2\,c}\,\mathrm{e}^{2\,d\,x}\left(\frac{2\left(8\,a^2-8\,a\,b+b^2\right)\left(8\,a^{5/2}\sqrt{b^3\,d^2-a\,b^2\,d^2}-8\,a^{3/2}\,b\sqrt{b^3\,d^2-a\,b^2\,d^2}+\sqrt{a}\,b^2\cdots\right.}{b^8\,d\,(a-b)^2\sqrt{b^3\,d^2-a\,b^2\,d^2}}\cdots\right.\right.}{\cdots}\right.}{\cdots}$$

input
```
int(sinh(c + d*x)^2/(a + b*sinh(c + d*x)^2),x)
```

output
```
x/b - (a^(1/2)*atan(((b^5*(b^3*d^2 - a*b^2*d^2)^(1/2) - a*b^4*(b^3*d^2 - a
*b^2*d^2)^(1/2))*(exp(2*c)*exp(2*d*x)*((2*(8*a^2 - 8*a*b + b^2)*(8*a^(5/2)
*(b^3*d^2 - a*b^2*d^2)^(1/2) - 8*a^(3/2)*b*(b^3*d^2 - a*b^2*d^2)^(1/2) + a
^(1/2)*b^2*(b^3*d^2 - a*b^2*d^2)^(1/2)))/(b^8*d*(a - b)^2*(b^3*d^2 - a*b^2
*d^2)^(1/2)) + (4*a^(1/2)*(4*a - 2*b)*(4*a*b^3*d - 12*a^2*b^2*d + 8*a^3*b*
d))/(b^7*(a - b)*(b^3*d^2 - a*b^2*d^2)^(1/2)*(-b^2*d^2*(a - b))^(1/2))) +
(2*(2*a^(3/2)*b*(b^3*d^2 - a*b^2*d^2)^(1/2) - a^(1/2)*b^2*(b^3*d^2 - a*b^2
*d^2)^(1/2))*(8*a^2 - 8*a*b + b^2))/(b^8*d*(a - b)^2*(b^3*d^2 - a*b^2*d^2)
^(1/2)) + (4*a^(1/2)*(2*a^2*b^2*d - 2*a*b^3*d)*(4*a - 2*b))/(b^7*(a - b)*(
b^3*d^2 - a*b^2*d^2)^(1/2)*(-b^2*d^2*(a - b))^(1/2))))/(4*a)))/(b^3*d^2 -
a*b^2*d^2)^(1/2)
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 143, normalized size of antiderivative = 2.86

$$\int \frac{\sinh^2(c+dx)}{a+b\sinh^2(c+dx)}\,dx$$
$$= \frac{-\sqrt{a}\,\sqrt{a-b}\log\left(-\sqrt{2\sqrt{a}\,\sqrt{a-b}-2a+b}+e^{dx+c}\sqrt{b}\right)-\sqrt{a}\,\sqrt{a-b}\log\left(\sqrt{2\sqrt{a}\,\sqrt{a-b}-2a+b}+e^{c}\right.}{2bd\,(a-b)}$$

input
```
int(sinh(d*x+c)^2/(a+b*sinh(d*x+c)^2),x)
```

output
```
( - sqrt(a)*sqrt(a - b)*log( - sqrt(2*sqrt(a)*sqrt(a - b) - 2*a + b) + e**
(c + d*x)*sqrt(b)) - sqrt(a)*sqrt(a - b)*log(sqrt(2*sqrt(a)*sqrt(a - b) -
2*a + b) + e**(c + d*x)*sqrt(b)) + sqrt(a)*sqrt(a - b)*log(2*sqrt(a)*sqrt(
a - b) + e**(2*c + 2*d*x)*b + 2*a - b) + 2*a*d*x - 2*b*d*x)/(2*b*d*(a - b)
)
```

3.34 $\int \frac{\sinh(c+dx)}{a+b\sinh^2(c+dx)}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 40

$$\int \frac{\sinh(c+dx)}{a+b\sinh^2(c+dx)}\,dx=\frac{\arctan\left(\frac{\sqrt{b}\cosh(c+dx)}{\sqrt{a-b}}\right)}{\sqrt{a-b}\sqrt{b}d}$$

output
```
arctan(b^(1/2)*cosh(d*x+c)/(a-b)^(1/2))/(a-b)^(1/2)/b^(1/2)/d
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 2.08 (sec) , antiderivative size = 91, normalized size of antiderivative = 2.28

$$\int \frac{\sinh(c+dx)}{a+b\sinh^2(c+dx)}\,dx=\frac{\arctan\left(\frac{\sqrt{b}-i\sqrt{a}\tanh\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a-b}}\right)+\arctan\left(\frac{\sqrt{b}+i\sqrt{a}\tanh\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a-b}}\right)}{\sqrt{a-b}\sqrt{b}d}$$

input
```
Integrate[Sinh[c + d*x]/(a + b*Sinh[c + d*x]^2),x]
```

output
```
(ArcTan[(Sqrt[b] - I*Sqrt[a]*Tanh[(c + d*x)/2])/Sqrt[a - b]] + ArcTan[(Sqr
t[b] + I*Sqrt[a]*Tanh[(c + d*x)/2])/Sqrt[a - b]])/(Sqrt[a - b]*Sqrt[b]*d)
```

Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 40, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.190$, Rules used = {3042, 26, 3665, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sinh(c+dx)}{a+b\sinh^2(c+dx)}dx$$

$$\Big\downarrow \text{3042}$$

$$\int -\frac{i\sin(ic+idx)}{a-b\sin(ic+idx)^2}dx$$

$$\Big\downarrow \text{26}$$

$$-i\int \frac{\sin(ic+idx)}{a-b\sin(ic+idx)^2}dx$$

$$\Big\downarrow \text{3665}$$

$$\frac{\int \frac{1}{b\cosh^2(c+dx)+a-b}d\cosh(c+dx)}{d}$$

$$\Big\downarrow \text{218}$$

$$\frac{\arctan\left(\frac{\sqrt{b}\cosh(c+dx)}{\sqrt{a-b}}\right)}{\sqrt{b}d\sqrt{a-b}}$$

input
```
Int[Sinh[c + d*x]/(a + b*Sinh[c + d*x]^2),x]
```

output
```
ArcTan[(Sqrt[b]*Cosh[c + d*x])/Sqrt[a - b]]/(Sqrt[a - b]*Sqrt[b]*d)
```

Defintions of rubi rules used

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3665
```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^
(p_.), x_Symbol] :> With[{ff = FreeFactors[Cos[e + f*x], x]}, Simp[-ff/f
Subst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a + b - b*ff^2*x^2)^p, x], x, Cos[e +
 f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m - 1)/2]
```

Maple [A] (verified)

Time = 0.25 (sec) , antiderivative size = 51, normalized size of antiderivative = 1.28

method	result	size
derivativedivides	$\frac{\arctan\left(\frac{2\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}a-2a+4b}{4\sqrt{ab-b^{2}}}\right)}{d\sqrt{ab-b^{2}}}$	51
default	$\frac{\arctan\left(\frac{2\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}a-2a+4b}{4\sqrt{ab-b^{2}}}\right)}{d\sqrt{ab-b^{2}}}$	51
risch	$-\frac{\ln\left(e^{2dx+2c}-\frac{2(a-b)e^{dx+c}}{\sqrt{-ab+b^{2}}}+1\right)}{2\sqrt{-ab+b^{2}}\,d}+\frac{\ln\left(e^{2dx+2c}+\frac{2(a-b)e^{dx+c}}{\sqrt{-ab+b^{2}}}+1\right)}{2\sqrt{-ab+b^{2}}\,d}$	102

input
```
int(sinh(d*x+c)/(a+b*sinh(d*x+c)^2),x,method=_RETURNVERBOSE)
```

output
```
1/d/(a*b-b^2)^(1/2)*arctan(1/4*(2*tanh(1/2*d*x+1/2*c)^2*a-2*a+4*b)/(a*b-b^
2)^(1/2))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 165 vs. $2(32) = 64$.

Time = 0.10 (sec) , antiderivative size = 501, normalized size of antiderivative = 12.52

$$\int \frac{\sinh(c+dx)}{a+b\sinh^2(c+dx)}\,dx$$
$$= \left[-\frac{\sqrt{-ab+b^2}\log\left(\frac{b\cosh(dx+c)^4+4\,b\cosh(dx+c)\sinh(dx+c)^3+b\sinh(dx+c)^4-2\,(2\,a-3\,b)\cosh(dx+c)^2+2\,\left(3\,b\cosh(dx+c)^2-2\,a+3\,b\right.}{b\cosh(dx+c)^4+4\,b\cosh(dx+c)\sinh(dx+c)^3+b\sinh(}\right.}{}\right.$$

input
```
integrate(sinh(d*x+c)/(a+b*sinh(d*x+c)^2),x, algorithm="fricas")
```

output
```
[-1/2*sqrt(-a*b + b^2)*log((b*cosh(d*x + c)^4 + 4*b*cosh(d*x + c)*sinh(d*x
 + c)^3 + b*sinh(d*x + c)^4 - 2*(2*a - 3*b)*cosh(d*x + c)^2 + 2*(3*b*cosh(
d*x + c)^2 - 2*a + 3*b)*sinh(d*x + c)^2 + 4*(b*cosh(d*x + c)^3 - (2*a - 3*
b)*cosh(d*x + c))*sinh(d*x + c) - 4*(cosh(d*x + c)^3 + 3*cosh(d*x + c)*sin
h(d*x + c)^2 + sinh(d*x + c)^3 + (3*cosh(d*x + c)^2 + 1)*sinh(d*x + c) + c
osh(d*x + c))*sqrt(-a*b + b^2) + b)/(b*cosh(d*x + c)^4 + 4*b*cosh(d*x + c)
*sinh(d*x + c)^3 + b*sinh(d*x + c)^4 + 2*(2*a - b)*cosh(d*x + c)^2 + 2*(3*
b*cosh(d*x + c)^2 + 2*a - b)*sinh(d*x + c)^2 + 4*(b*cosh(d*x + c)^3 + (2*a
 - b)*cosh(d*x + c))*sinh(d*x + c) + b))/((a*b - b^2)*d), (sqrt(a*b - b^2)
*arctan(-1/2*(b*cosh(d*x + c)^3 + 3*b*cosh(d*x + c)*sinh(d*x + c)^2 + b*si
nh(d*x + c)^3 + (4*a - 3*b)*cosh(d*x + c) + (3*b*cosh(d*x + c)^2 + 4*a - 3
*b)*sinh(d*x + c))/sqrt(a*b - b^2)) - sqrt(a*b - b^2)*arctan(2*sqrt(a*b -
b^2)/(b*cosh(d*x + c) + b*sinh(d*x + c))))/((a*b - b^2)*d)]
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\sinh(c+dx)}{a+b\sinh^2(c+dx)}\,dx = \text{Timed out}$$

input
```
integrate(sinh(d*x+c)/(a+b*sinh(d*x+c)**2),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{\sinh(c+dx)}{a+b\sinh^2(c+dx)}\,dx=\int \frac{\sinh\left(dx+c\right)}{b\sinh\left(dx+c\right)^2+a}\,dx$$

input
```
integrate(sinh(d*x+c)/(a+b*sinh(d*x+c)^2),x, algorithm="maxima")
```

output
```
integrate(sinh(d*x + c)/(b*sinh(d*x + c)^2 + a), x)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{\sinh(c+dx)}{a+b\sinh^2(c+dx)}\,dx=\text{Exception raised: TypeError}$$

input
```
integrate(sinh(d*x+c)/(a+b*sinh(d*x+c)^2),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Limit: Max order reached or unable
to make series expansion Error: Bad Argument Value
```

Mupad [B] (verification not implemented)

Time = 2.16 (sec) , antiderivative size = 116, normalized size of antiderivative = 2.90

$$\int \frac{\sinh(c+dx)}{a+b\sinh^2(c+dx)}\,dx=\frac{\ln\left(-\frac{4\,\left(a+a\,{\mathrm{e}}^{2\,c+2\,d\,x}\right)}{b^2\,\left(a-b\right)}-\frac{8\,a\,{\mathrm{e}}^{c+d\,x}}{{\left(-b\right)}^{5/2}\,\sqrt{a-b}}\right)-\ln\left(\frac{8\,a\,{\mathrm{e}}^{c+d\,x}}{{\left(-b\right)}^{5/2}\,\sqrt{a-b}}-\frac{4\,\left(a+a\,{\mathrm{e}}^{2\,c+2\,d\,x}\right)}{b^2\,\left(a-b\right)}\right)}{2\,\sqrt{-b}\,d\,\sqrt{a-b}}$$

input
```
int(sinh(c + d*x)/(a + b*sinh(c + d*x)^2),x)
```

output
```
(log(- (4*(a + a*exp(2*c + 2*d*x)))/(b^2*(a - b)) - (8*a*exp(c + d*x))/((-
b)^(5/2)*(a - b)^(1/2))) - log((8*a*exp(c + d*x))/((-b)^(5/2)*(a - b)^(1/2
)) - (4*(a + a*exp(2*c + 2*d*x)))/(b^2*(a - b))))/(2*(-b)^(1/2)*d*(a - b)^
(1/2))
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 489, normalized size of antiderivative = 12.22

$$\int \frac{\sinh(c+dx)}{a+b\sinh^2(c+dx)}\,dx$$
$$= \frac{\sqrt{b}\left(2\sqrt{a}\,\sqrt{a-b}\,\sqrt{2\sqrt{a}\,\sqrt{a-b}+2a-b}\,atan\left(\frac{e^{dx+c}b}{\sqrt{b}\,\sqrt{2\sqrt{a}\,\sqrt{a-b}+2a-b}}\right)-2\sqrt{2\sqrt{a}\,\sqrt{a-b}+2a-b}\,atan\left(\frac{}{\sqrt{}}\right.\right.}{}$$

input
```
int(sinh(d*x+c)/(a+b*sinh(d*x+c)^2),x)
```

output
```
(sqrt(b)*(2*sqrt(a)*sqrt(a - b)*sqrt(2*sqrt(a)*sqrt(a - b) + 2*a - b)*atan
((e**(c + d*x)*b)/(sqrt(b)*sqrt(2*sqrt(a)*sqrt(a - b) + 2*a - b))) - 2*sqr
t(2*sqrt(a)*sqrt(a - b) + 2*a - b)*atan((e**(c + d*x)*b)/(sqrt(b)*sqrt(2*s
qrt(a)*sqrt(a - b) + 2*a - b)))*a + 2*sqrt(2*sqrt(a)*sqrt(a - b) + 2*a - b
)*atan((e**(c + d*x)*b)/(sqrt(b)*sqrt(2*sqrt(a)*sqrt(a - b) + 2*a - b)))*b
 + sqrt(a)*sqrt(a - b)*sqrt(2*sqrt(a)*sqrt(a - b) - 2*a + b)*log( - sqrt(2
*sqrt(a)*sqrt(a - b) - 2*a + b) + e**(c + d*x)*sqrt(b)) - sqrt(a)*sqrt(a -
 b)*sqrt(2*sqrt(a)*sqrt(a - b) - 2*a + b)*log(sqrt(2*sqrt(a)*sqrt(a - b) -
 2*a + b) + e**(c + d*x)*sqrt(b)) + sqrt(2*sqrt(a)*sqrt(a - b) - 2*a + b)*
log( - sqrt(2*sqrt(a)*sqrt(a - b) - 2*a + b) + e**(c + d*x)*sqrt(b))*a - s
qrt(2*sqrt(a)*sqrt(a - b) - 2*a + b)*log( - sqrt(2*sqrt(a)*sqrt(a - b) - 2
*a + b) + e**(c + d*x)*sqrt(b))*b - sqrt(2*sqrt(a)*sqrt(a - b) - 2*a + b)*
log(sqrt(2*sqrt(a)*sqrt(a - b) - 2*a + b) + e**(c + d*x)*sqrt(b))*a + sqrt
(2*sqrt(a)*sqrt(a - b) - 2*a + b)*log(sqrt(2*sqrt(a)*sqrt(a - b) - 2*a + b
) + e**(c + d*x)*sqrt(b))*b))/(2*b**2*d*(a - b))
```

3.35 $\int \frac{1}{a+b\sinh^2(c+dx)}\,dx$

Optimal result

Integrand size = 14, antiderivative size = 40

$$\int \frac{1}{a+b\sinh^2(c+dx)}\,dx=\frac{\operatorname{arctanh}\left(\frac{\sqrt{a-b}\tanh(c+dx)}{\sqrt{a}}\right)}{\sqrt{a}\sqrt{a-b}d}$$

output
```
arctanh((a-b)^(1/2)*tanh(d*x+c)/a^(1/2))/a^(1/2)/(a-b)^(1/2)/d
```

Mathematica [A] (verified)

Time = 1.28 (sec) , antiderivative size = 40, normalized size of antiderivative = 1.00

$$\int \frac{1}{a+b\sinh^2(c+dx)}\,dx=\frac{\operatorname{arctanh}\left(\frac{\sqrt{a-b}\tanh(c+dx)}{\sqrt{a}}\right)}{\sqrt{a}\sqrt{a-b}d}$$

input
```
Integrate[(a + b*Sinh[c + d*x]^2)^(-1),x]
```

output
```
ArcTanh[(Sqrt[a - b]*Tanh[c + d*x])/Sqrt[a]]/(Sqrt[a]*Sqrt[a - b]*d)
```

Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 40, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.214$, Rules used = {3042, 3660, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{a+b\sinh^2(c+dx)}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{a-b\sin(ic+idx)^2}dx$$

$$\Big\downarrow \text{3660}$$

$$\frac{\int \frac{1}{a-(a-b)\tanh^2(c+dx)}d\tanh(c+dx)}{d}$$

$$\Big\downarrow \text{221}$$

$$\frac{\text{arctanh}\left(\frac{\sqrt{a-b}\tanh(c+dx)}{\sqrt{a}}\right)}{\sqrt{a}d\sqrt{a-b}}$$

input
```
Int[(a + b*Sinh[c + d*x]^2)^(-1),x]
```

output
```
ArcTanh[(Sqrt[a - b]*Tanh[c + d*x])/Sqrt[a]]/(Sqrt[a]*Sqrt[a - b]*d)
```

Defintions of rubi rules used

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3660
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(-1), x_Symbol] :> With[{ff =
FreeFactors[Tan[e + f*x], x]}, Simp[ff/f   Subst[Int[1/(a + (a + b)*ff^2*x^
2), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f}, x]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 153 vs. $2(32) = 64$.

Time = 0.27 (sec) , antiderivative size = 154, normalized size of antiderivative = 3.85

method	result	size
risch	$\frac{\ln\left(\mathrm{e}^{2dx+2c}+\frac{2a\sqrt{a^2-ab}-b\sqrt{a^2-ab}-2a^2+2ab}{b\sqrt{a^2-ab}}\right)}{2\sqrt{a^2-ab}\,d}-\frac{\ln\left(\mathrm{e}^{2dx+2c}+\frac{2a\sqrt{a^2-ab}-b\sqrt{a^2-ab}+2a^2-2ab}{b\sqrt{a^2-ab}}\right)}{2\sqrt{a^2-ab}\,d}$	154
derivativedivides	$\frac{2a\left(\frac{\left(-\sqrt{-b(a-b)}-b\right)\arctan\left(\frac{\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)a}{\sqrt{\left(2\sqrt{-b(a-b)}-a+2b\right)a}}\right)}{2a\sqrt{-b(a-b)}\sqrt{\left(2\sqrt{-b(a-b)}-a+2b\right)a}}-\frac{\left(-\sqrt{-b(a-b)}+b\right)\operatorname{arctanh}\left(\frac{\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)a}{\sqrt{\left(2\sqrt{-b(a-b)}+a-2b\right)a}}\right)}{2a\sqrt{-b(a-b)}\sqrt{\left(2\sqrt{-b(a-b)}+a-2b\right)a}}\right)}{d}$	180
default	$\frac{2a\left(\frac{\left(-\sqrt{-b(a-b)}-b\right)\arctan\left(\frac{\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)a}{\sqrt{\left(2\sqrt{-b(a-b)}-a+2b\right)a}}\right)}{2a\sqrt{-b(a-b)}\sqrt{\left(2\sqrt{-b(a-b)}-a+2b\right)a}}-\frac{\left(-\sqrt{-b(a-b)}+b\right)\operatorname{arctanh}\left(\frac{\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)a}{\sqrt{\left(2\sqrt{-b(a-b)}+a-2b\right)a}}\right)}{2a\sqrt{-b(a-b)}\sqrt{\left(2\sqrt{-b(a-b)}+a-2b\right)a}}\right)}{d}$	180

input
```
int(1/(a+b*sinh(d*x+c)^2),x,method=_RETURNVERBOSE)
```

output
```
1/2/(a^2-a*b)^(1/2)/d*ln(exp(2*d*x+2*c)+(2*a*(a^2-a*b)^(1/2)-b*(a^2-a*b)^(
1/2)-2*a^2+2*a*b)/b/(a^2-a*b)^(1/2))-1/2/(a^2-a*b)^(1/2)/d*ln(exp(2*d*x+2*
c)+(2*a*(a^2-a*b)^(1/2)-b*(a^2-a*b)^(1/2)+2*a^2-2*a*b)/b/(a^2-a*b)^(1/2))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 92 vs. $2(32) = 64$.

Time = 0.10 (sec) , antiderivative size = 430, normalized size of antiderivative = 10.75

$$\int \frac{1}{a+b\sinh^2(c+dx)}\,dx$$
$$= \left[\frac{\log\left(\frac{b^2\cosh(dx+c)^4+4\,b^2\cosh(dx+c)\sinh(dx+c)^3+b^2\sinh(dx+c)^4+2\,(2\,ab-b^2)\cosh(dx+c)^2+2\,\left(3\,b^2\cosh(dx+c)^2+2\,ab-b^2\right)\sinh(dx+c}{b\cosh(dx+c)^4+4\,b\cosh(dx+c)\sinh(dx+c)^3+b\sinh(dx+c)^4+2\,(2\,a-b)\cosh(dx+}\right.}{}\right.$$
$$\left.-\frac{\sqrt{-a^2+ab}\arctan\left(-\frac{\left(b\cosh(dx+c)^2+2\,b\cosh(dx+c)\sinh(dx+c)+b\sinh(dx+c)^2+2\,a-b\right)\sqrt{-a^2+ab}}{2\,(a^2-ab)}\right)}{(a^2-ab)d}\right]$$

input
```
integrate(1/(a+b*sinh(d*x+c)^2),x, algorithm="fricas")
```

output
```
[1/2*log((b^2*cosh(d*x + c)^4 + 4*b^2*cosh(d*x + c)*sinh(d*x + c)^3 + b^2*
sinh(d*x + c)^4 + 2*(2*a*b - b^2)*cosh(d*x + c)^2 + 2*(3*b^2*cosh(d*x + c)
^2 + 2*a*b - b^2)*sinh(d*x + c)^2 + 8*a^2 - 8*a*b + b^2 + 4*(b^2*cosh(d*x
+ c)^3 + (2*a*b - b^2)*cosh(d*x + c))*sinh(d*x + c) - 4*(b*cosh(d*x + c)^2
 + 2*b*cosh(d*x + c)*sinh(d*x + c) + b*sinh(d*x + c)^2 + 2*a - b)*sqrt(a^2
 - a*b))/(b*cosh(d*x + c)^4 + 4*b*cosh(d*x + c)*sinh(d*x + c)^3 + b*sinh(d
*x + c)^4 + 2*(2*a - b)*cosh(d*x + c)^2 + 2*(3*b*cosh(d*x + c)^2 + 2*a - b
)*sinh(d*x + c)^2 + 4*(b*cosh(d*x + c)^3 + (2*a - b)*cosh(d*x + c))*sinh(d
*x + c) + b))/(sqrt(a^2 - a*b)*d), -sqrt(-a^2 + a*b)*arctan(-1/2*(b*cosh(d
*x + c)^2 + 2*b*cosh(d*x + c)*sinh(d*x + c) + b*sinh(d*x + c)^2 + 2*a - b)
*sqrt(-a^2 + a*b)/(a^2 - a*b))/((a^2 - a*b)*d)]
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 15870 vs. $2(32) = 64$.

Time = 15.78 (sec) , antiderivative size = 15870, normalized size of antiderivative = 396.75

$$\int \frac{1}{a + b\sinh^2(c + dx)}\,dx = \text{Too large to display}$$

input
```
integrate(1/(a+b*sinh(d*x+c)**2),x)
```

output
```
Piecewise((zoo*x/sinh(c)**2, Eq(a, 0) & Eq(b, 0) & Eq(d, 0)), ((-tanh(c/2
+ d*x/2)/(2*d) - 1/(2*d*tanh(c/2 + d*x/2)))/b, Eq(a, 0)), (2*tanh(c/2 + d*
x/2)/(b*d*tanh(c/2 + d*x/2)**2 + b*d), Eq(a, b)), (x/a, Eq(b, 0)), (x/(a +
 b*sinh(c)**2), Eq(d, 0)), (-6*a**3*b*sqrt(1 - 2*b/a - 2*sqrt(-a*b + b**2)
/a)*log(-sqrt(1 - 2*b/a + 2*sqrt(-a*b + b**2)/a) + tanh(c/2 + d*x/2))/(10*
a**4*b*d*sqrt(1 - 2*b/a - 2*sqrt(-a*b + b**2)/a)*sqrt(1 - 2*b/a + 2*sqrt(-
a*b + b**2)/a) - 2*a**4*d*sqrt(-a*b + b**2)*sqrt(1 - 2*b/a - 2*sqrt(-a*b +
 b**2)/a)*sqrt(1 - 2*b/a + 2*sqrt(-a*b + b**2)/a) - 50*a**3*b**2*d*sqrt(1
- 2*b/a - 2*sqrt(-a*b + b**2)/a)*sqrt(1 - 2*b/a + 2*sqrt(-a*b + b**2)/a) +
 26*a**3*b*d*sqrt(-a*b + b**2)*sqrt(1 - 2*b/a - 2*sqrt(-a*b + b**2)/a)*sqr
t(1 - 2*b/a + 2*sqrt(-a*b + b**2)/a) + 72*a**2*b**3*d*sqrt(1 - 2*b/a - 2*s
qrt(-a*b + b**2)/a)*sqrt(1 - 2*b/a + 2*sqrt(-a*b + b**2)/a) - 56*a**2*b**2
*d*sqrt(-a*b + b**2)*sqrt(1 - 2*b/a - 2*sqrt(-a*b + b**2)/a)*sqrt(1 - 2*b/
a + 2*sqrt(-a*b + b**2)/a) - 32*a*b**4*d*sqrt(1 - 2*b/a - 2*sqrt(-a*b + b*
*2)/a)*sqrt(1 - 2*b/a + 2*sqrt(-a*b + b**2)/a) + 32*a*b**3*d*sqrt(-a*b + b
**2)*sqrt(1 - 2*b/a - 2*sqrt(-a*b + b**2)/a)*sqrt(1 - 2*b/a + 2*sqrt(-a*b
+ b**2)/a)) + 6*a**3*b*sqrt(1 - 2*b/a - 2*sqrt(-a*b + b**2)/a)*log(sqrt(1
- 2*b/a + 2*sqrt(-a*b + b**2)/a) + tanh(c/2 + d*x/2))/(10*a**4*b*d*sqrt(1
- 2*b/a - 2*sqrt(-a*b + b**2)/a)*sqrt(1 - 2*b/a + 2*sqrt(-a*b + b**2)/a) -
 2*a**4*d*sqrt(-a*b + b**2)*sqrt(1 - 2*b/a - 2*sqrt(-a*b + b**2)/a)*sqr...
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{1}{a+b\sinh^2(c+dx)}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate(1/(a+b*sinh(d*x+c)^2),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(b-a>0)', see `assume?` for more
details)Is
```

Giac [A] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 47, normalized size of antiderivative = 1.18

$$\int \frac{1}{a+b\sinh^2(c+dx)}\,dx = \frac{\arctan\left(\frac{be^{(2\,dx+2\,c)}+2\,a-b}{2\,\sqrt{-a^2+ab}}\right)}{\sqrt{-a^2+ab}d}$$

input
```
integrate(1/(a+b*sinh(d*x+c)^2),x, algorithm="giac")
```

output
```
arctan(1/2*(b*e^(2*d*x + 2*c) + 2*a - b)/sqrt(-a^2 + a*b))/(sqrt(-a^2 + a*
b)*d)
```

Mupad [B] (verification not implemented)

Time = 0.49 (sec) , antiderivative size = 146, normalized size of antiderivative = 3.65

$$\int \frac{1}{a+b\sinh^2(c+dx)}\,dx = \frac{\ln\left(-\frac{4\,e^{2\,c+2\,d\,x}}{b}-\frac{2\,\left(b\,d+2\,a\,d\,e^{2\,c+2\,d\,x}-b\,d\,e^{2\,c+2\,d\,x}\right)}{\sqrt{a}\,b\,d\,\sqrt{a-b}}\right)-\ln\left(\frac{2\,\left(b\,d+2\,a\,d\,e^{2\,c+2\,d\,x}-b\,d\,e^{2\,c+2\,d\,x}\right)}{\sqrt{a}\,b\,d\,\sqrt{a-b}}-\frac{4\,e^{2\,c+2\,d\,x}}{b}\right)}{2\,\sqrt{a}\,d\,\sqrt{a-b}}$$

input
```
int(1/(a + b*sinh(c + d*x)^2),x)
```

output
```
(log(- (4*exp(2*c + 2*d*x))/b - (2*(b*d + 2*a*d*exp(2*c + 2*d*x) - b*d*exp
(2*c + 2*d*x)))/(a^(1/2)*b*d*(a - b)^(1/2))) - log((2*(b*d + 2*a*d*exp(2*c
 + 2*d*x) - b*d*exp(2*c + 2*d*x)))/(a^(1/2)*b*d*(a - b)^(1/2)) - (4*exp(2*
c + 2*d*x))/b))/(2*a^(1/2)*d*(a - b)^(1/2))
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 114, normalized size of antiderivative = 2.85

$$\int \frac{1}{a+b\sinh^2(c+dx)}\,dx = \frac{\sqrt{a}\,\sqrt{a-b}\,\left(\log\left(-\sqrt{2\sqrt{a}\,\sqrt{a-b}-2a+b}+e^{dx+c}\sqrt{b}\right)+\log\left(\sqrt{2\sqrt{a}\,\sqrt{a-b}-2a+b}+e^{dx+c}\sqrt{b}\right)-\right.}{2ad\,(a-b)}$$

input
```
int(1/(a+b*sinh(d*x+c)^2),x)
```

output
```
(sqrt(a)*sqrt(a - b)*(log( - sqrt(2*sqrt(a)*sqrt(a - b) - 2*a + b) + e**(c
 + d*x)*sqrt(b)) + log(sqrt(2*sqrt(a)*sqrt(a - b) - 2*a + b) + e**(c + d*x
)*sqrt(b)) - log(2*sqrt(a)*sqrt(a - b) + e**(2*c + 2*d*x)*b + 2*a - b)))/(
2*a*d*(a - b))
```

3.36 $\int \frac{\operatorname{csch}(c+dx)}{a+b\sinh^2(c+dx)}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 60

$$\int \frac{\operatorname{csch}(c+dx)}{a+b\sinh^2(c+dx)}\,dx=-\frac{\sqrt{b}\arctan\left(\frac{\sqrt{b}\cosh(c+dx)}{\sqrt{a-b}}\right)}{a\sqrt{a-b}d}-\frac{\operatorname{arctanh}(\cosh(c+dx))}{ad}$$

output
```
-b^(1/2)*arctan(b^(1/2)*cosh(d*x+c)/(a-b)^(1/2))/a/(a-b)^(1/2)/d-arctanh(c
osh(d*x+c))/a/d
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 7.97 (sec) , antiderivative size = 135, normalized size of antiderivative = 2.25

$$\int \frac{\operatorname{csch}(c+dx)}{a+b\sinh^2(c+dx)}\,dx = -\frac{\frac{\sqrt{b}\arctan\left(\frac{\sqrt{b}-i\sqrt{a}\tanh\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a-b}}\right)}{\sqrt{a-b}}+\frac{\sqrt{b}\arctan\left(\frac{\sqrt{b}+i\sqrt{a}\tanh\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a-b}}\right)}{\sqrt{a-b}}+\log\left(\cosh\left(\frac{1}{2}(c+dx)\right)\right)-\log\left(\sinh\left(\frac{1}{2}(c+\right.\right.}{ad}$$

input
```
Integrate[Csch[c + d*x]/(a + b*Sinh[c + d*x]^2),x]
```

output
```
-(((Sqrt[b]*ArcTan[(Sqrt[b] - I*Sqrt[a]*Tanh[(c + d*x)/2])/Sqrt[a - b]])/S
qrt[a - b] + (Sqrt[b]*ArcTan[(Sqrt[b] + I*Sqrt[a]*Tanh[(c + d*x)/2])/Sqrt[
a - b]])/Sqrt[a - b] + Log[Cosh[(c + d*x)/2]] - Log[Sinh[(c + d*x)/2]])/(a
*d))
```

Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.95, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$, Rules used = {3042, 26, 3665, 303, 218, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\text{csch}(c+dx)}{a+b\sinh^2(c+dx)}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{i}{\sin(ic+idx)\left(a-b\sin(ic+idx)^2\right)}dx$$

$$\Big\downarrow \text{26}$$

$$i\int \frac{1}{\sin(ic+idx)\left(a-b\sin(ic+idx)^2\right)}dx$$

$$\Big\downarrow \text{3665}$$

$$-\frac{\int \frac{1}{\left(1-\cosh^2(c+dx)\right)\left(b\cosh^2(c+dx)+a-b\right)}d\cosh(c+dx)}{d}$$

$$\Big\downarrow \text{303}$$

$$-\frac{\frac{b\int \frac{1}{b\cosh^2(c+dx)+a-b}d\cosh(c+dx)}{a}+\frac{\int \frac{1}{1-\cosh^2(c+dx)}d\cosh(c+dx)}{a}}{d}$$

$$\Big\downarrow \text{218}$$

$$-\frac{\frac{\int \frac{1}{1-\cosh^2(c+dx)}d\cosh(c+dx)}{a}+\frac{\sqrt{b}\arctan\left(\frac{\sqrt{b}\cosh(c+dx)}{\sqrt{a-b}}\right)}{a\sqrt{a-b}}}{d}$$

$$\Big\downarrow \text{219}$$

$$-\frac{\frac{\sqrt{b}\arctan\left(\frac{\sqrt{b}\cosh(c+dx)}{\sqrt{a-b}}\right)}{a\sqrt{a-b}}+\frac{\operatorname{arctanh}(\cosh(c+dx))}{a}}{d}$$

input
```
Int[Csch[c + d*x]/(a + b*Sinh[c + d*x]^2),x]
```

output
```
-(((Sqrt[b]*ArcTan[(Sqrt[b]*Cosh[c + d*x])/Sqrt[a - b]])/(a*Sqrt[a - b]) +
 ArcTanh[Cosh[c + d*x]]/a)/d)
```

Defintions of rubi rules used

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 303
```
Int[1/(((a_) + (b_.)*(x_)^2)*((c_) + (d_.)*(x_)^2)), x_Symbol] :> Simp[b/(b
*c - a*d)   Int[1/(a + b*x^2), x], x] - Simp[d/(b*c - a*d)   Int[1/(c + d*x
^2), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3665
```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^
(p_.), x_Symbol] :> With[{ff = FreeFactors[Cos[e + f*x], x]}, Simp[-ff/f
Subst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a + b - b*ff^2*x^2)^p, x], x, Cos[e +
 f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m - 1)/2]
```

Maple [A] (verified)

Time = 0.40 (sec) , antiderivative size = 72, normalized size of antiderivative = 1.20

method	result
derivativedivides	$\frac{-\frac{b \arctan\left(\frac{2 \tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^{2} a-2a+4b}{4\sqrt{ab-b^{2}}}\right)}{a\sqrt{ab-b^{2}}}+\frac{\ln\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{a}}{d}$
default	$\frac{-\frac{b \arctan\left(\frac{2 \tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^{2} a-2a+4b}{4\sqrt{ab-b^{2}}}\right)}{a\sqrt{ab-b^{2}}}+\frac{\ln\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{a}}{d}$
risch	$\frac{\ln(e^{dx+c}-1)}{ad}-\frac{\ln(e^{dx+c}+1)}{ad}+\frac{\sqrt{-b(a-b)}\,\ln\left(e^{2dx+2c}-\frac{2\sqrt{-b(a-b)}\,e^{dx+c}}{b}+1\right)}{2(a-b)da}-\frac{\sqrt{-b(a-b)}\,\ln\left(e^{2dx+2c}+\frac{2\sqrt{-}}{}\right.}{2(a-b)da}$

input
```
int(csch(d*x+c)/(a+b*sinh(d*x+c)^2),x,method=_RETURNVERBOSE)
```

output
```
1/d*(-b/a/(a*b-b^2)^(1/2)*arctan(1/4*(2*tanh(1/2*d*x+1/2*c)^2*a-2*a+4*b)/(
a*b-b^2)^(1/2))+1/a*ln(tanh(1/2*d*x+1/2*c)))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 187 vs. $2(52) = 104$.

Time = 0.11 (sec) , antiderivative size = 586, normalized size of antiderivative = 9.77

$$\int \frac{\operatorname{csch}(c+dx)}{a+b\sinh^2(c+dx)}\,dx = \text{Too large to display}$$

input
```
integrate(csch(d*x+c)/(a+b*sinh(d*x+c)^2),x, algorithm="fricas")
```

output
```
[1/2*(sqrt(-b/(a - b))*log((b*cosh(d*x + c)^4 + 4*b*cosh(d*x + c)*sinh(d*x
 + c)^3 + b*sinh(d*x + c)^4 - 2*(2*a - 3*b)*cosh(d*x + c)^2 + 2*(3*b*cosh(
d*x + c)^2 - 2*a + 3*b)*sinh(d*x + c)^2 + 4*(b*cosh(d*x + c)^3 - (2*a - 3*
b)*cosh(d*x + c))*sinh(d*x + c) - 4*((a - b)*cosh(d*x + c)^3 + 3*(a - b)*c
osh(d*x + c)*sinh(d*x + c)^2 + (a - b)*sinh(d*x + c)^3 + (a - b)*cosh(d*x
+ c) + (3*(a - b)*cosh(d*x + c)^2 + a - b)*sinh(d*x + c))*sqrt(-b/(a - b))
 + b)/(b*cosh(d*x + c)^4 + 4*b*cosh(d*x + c)*sinh(d*x + c)^3 + b*sinh(d*x
+ c)^4 + 2*(2*a - b)*cosh(d*x + c)^2 + 2*(3*b*cosh(d*x + c)^2 + 2*a - b)*s
inh(d*x + c)^2 + 4*(b*cosh(d*x + c)^3 + (2*a - b)*cosh(d*x + c))*sinh(d*x
+ c) + b)) - 2*log(cosh(d*x + c) + sinh(d*x + c) + 1) + 2*log(cosh(d*x + c
) + sinh(d*x + c) - 1))/(a*d), -(sqrt(b/(a - b))*arctan(1/2*sqrt(b/(a - b)
)*(cosh(d*x + c) + sinh(d*x + c))) - sqrt(b/(a - b))*arctan(1/2*(b*cosh(d*
x + c)^3 + 3*b*cosh(d*x + c)*sinh(d*x + c)^2 + b*sinh(d*x + c)^3 + (4*a -
3*b)*cosh(d*x + c) + (3*b*cosh(d*x + c)^2 + 4*a - 3*b)*sinh(d*x + c))*sqrt
(b/(a - b))/b) + log(cosh(d*x + c) + sinh(d*x + c) + 1) - log(cosh(d*x + c
) + sinh(d*x + c) - 1))/(a*d)]
```

Sympy [F]

$$\int \frac{\operatorname{csch}(c+dx)}{a+b\sinh^2(c+dx)}\,dx = \int \frac{\operatorname{csch}(c+dx)}{a+b\sinh^2(c+dx)}\,dx$$

input
```
integrate(csch(d*x+c)/(a+b*sinh(d*x+c)**2),x)
```

output
```
Integral(csch(c + d*x)/(a + b*sinh(c + d*x)**2), x)
```

Maxima [F]

$$\int \frac{\operatorname{csch}(c+dx)}{a+b\sinh^2(c+dx)}\,dx = \int \frac{\operatorname{csch}(dx+c)}{b\sinh(dx+c)^2+a}\,dx$$

input
```
integrate(csch(d*x+c)/(a+b*sinh(d*x+c)^2),x, algorithm="maxima")
```

output
```
-log((e^(d*x + c) + 1)*e^(-c))/(a*d) + log((e^(d*x + c) - 1)*e^(-c))/(a*d)
 - 2*integrate((b*e^(3*d*x + 3*c) - b*e^(d*x + c))/(a*b*e^(4*d*x + 4*c) +
a*b + 2*(2*a^2*e^(2*c) - a*b*e^(2*c))*e^(2*d*x)), x)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{\operatorname{csch}(c+dx)}{a+b\sinh^2(c+dx)}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(csch(d*x+c)/(a+b*sinh(d*x+c)^2),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Limit: Max order reached or unable
to make series expansion Error: Bad Argument Value
```

Mupad [B] (verification not implemented)

Time = 2.26 (sec) , antiderivative size = 323, normalized size of antiderivative = 5.38

$$\int \frac{\operatorname{csch}(c+dx)}{a+b\sinh^2(c+dx)}\,dx = -\frac{2\,\mathrm{atan}\left(\frac{\mathrm{e}^{d\,x}\,\mathrm{e}^{c}\,\left(16\,a^2\,\sqrt{-a^2\,d^2}+9\,b^2\,\sqrt{-a^2\,d^2}-24\,a\,b\,\sqrt{-a^2\,d^2}\right)}{16\,d\,a^3-24\,d\,a^2\,b+9\,d\,a\,b^2}\right)}{\sqrt{-a^2\,d^2}} - \frac{\sqrt{b}\,\left(2\,\mathrm{atan}\left(\frac{\sqrt{b}\,\mathrm{e}^{d\,x}\,\mathrm{e}^{c}\,\sqrt{a^2\,d^2\,(a-b)}}{2\,a\,d\,(a-b)}\right)+2\,\mathrm{atan}\left(\frac{4\,a^4\,d^2\,\mathrm{e}^{d\,x}\,\mathrm{e}^{c}+4\,a^2\,b^2\,d^2\,\mathrm{e}^{d\,x}\,\mathrm{e}^{c}+b\,\mathrm{e}^{3\,c}\,\mathrm{e}^{3\,d\,x}\,\sqrt{a^3\,d^2-a^2\,b\,d^2}\,\sqrt{a^2\,d^2\,(a-b)}-8\,a^3\,b}{\sqrt{b}\,d\,(2\,a\,b-2\,a^2)\,\sqrt{a^2\,d^2\,(a-b)}}\right.\right.}{2\,\sqrt{a^3\,d^2-a^2\,b\,d^2}}$$

input
```
int(1/(sinh(c + d*x)*(a + b*sinh(c + d*x)^2)),x)
```

output
```
- (2*atan((exp(d*x)*exp(c)*(16*a^2*(-a^2*d^2)^(1/2) + 9*b^2*(-a^2*d^2)^(1/
2) - 24*a*b*(-a^2*d^2)^(1/2)))/(16*a^3*d + 9*a*b^2*d - 24*a^2*b*d)))/(-a^2
*d^2)^(1/2) - (b^(1/2)*(2*atan((b^(1/2)*exp(d*x)*exp(c)*(a^2*d^2*(a - b))^
(1/2))/(2*a*d*(a - b))) + 2*atan((4*a^4*d^2*exp(d*x)*exp(c) + 4*a^2*b^2*d^
2*exp(d*x)*exp(c) + b*exp(3*c)*exp(3*d*x)*(a^3*d^2 - a^2*b*d^2)^(1/2)*(a^2
*d^2*(a - b))^(1/2) - 8*a^3*b*d^2*exp(d*x)*exp(c) + b*exp(d*x)*exp(c)*(a^3
*d^2 - a^2*b*d^2)^(1/2)*(a^2*d^2*(a - b))^(1/2))/(b^(1/2)*d*(2*a*b - 2*a^2
)*(a^2*d^2*(a - b))^(1/2)))))/(2*(a^3*d^2 - a^2*b*d^2)^(1/2))
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 566, normalized size of antiderivative = 9.43

$$\int \frac{\operatorname{csch}(c+dx)}{a+b\sinh^2(c+dx)}\,dx$$
$$= \frac{-2\sqrt{b}\,\sqrt{a}\,\sqrt{a-b}\,\sqrt{2\sqrt{a}\,\sqrt{a-b}+2a-b}\,\operatorname{atan}\left(\frac{e^{dx+c}b}{\sqrt{b}\,\sqrt{2\sqrt{a}\,\sqrt{a-b}+2a-b}}\right)+2\sqrt{b}\,\sqrt{2\sqrt{a}\,\sqrt{a-b}+2a-b}\,\operatorname{atan}}{}$$

input
```
int(csch(d*x+c)/(a+b*sinh(d*x+c)^2),x)
```

output

```
( - 2*sqrt(b)*sqrt(a)*sqrt(a - b)*sqrt(2*sqrt(a)*sqrt(a - b) + 2*a - b)*at
an((e**(c + d*x)*b)/(sqrt(b)*sqrt(2*sqrt(a)*sqrt(a - b) + 2*a - b))) + 2*s
qrt(b)*sqrt(2*sqrt(a)*sqrt(a - b) + 2*a - b)*atan((e**(c + d*x)*b)/(sqrt(b
)*sqrt(2*sqrt(a)*sqrt(a - b) + 2*a - b)))*a - 2*sqrt(b)*sqrt(2*sqrt(a)*sqr
t(a - b) + 2*a - b)*atan((e**(c + d*x)*b)/(sqrt(b)*sqrt(2*sqrt(a)*sqrt(a -
 b) + 2*a - b)))*b - sqrt(b)*sqrt(a)*sqrt(a - b)*sqrt(2*sqrt(a)*sqrt(a - b
) - 2*a + b)*log( - sqrt(2*sqrt(a)*sqrt(a - b) - 2*a + b) + e**(c + d*x)*s
qrt(b)) + sqrt(b)*sqrt(a)*sqrt(a - b)*sqrt(2*sqrt(a)*sqrt(a - b) - 2*a + b
)*log(sqrt(2*sqrt(a)*sqrt(a - b) - 2*a + b) + e**(c + d*x)*sqrt(b)) - sqrt
(b)*sqrt(2*sqrt(a)*sqrt(a - b) - 2*a + b)*log( - sqrt(2*sqrt(a)*sqrt(a - b
) - 2*a + b) + e**(c + d*x)*sqrt(b))*a + sqrt(b)*sqrt(2*sqrt(a)*sqrt(a - b
) - 2*a + b)*log( - sqrt(2*sqrt(a)*sqrt(a - b) - 2*a + b) + e**(c + d*x)*s
qrt(b))*b + sqrt(b)*sqrt(2*sqrt(a)*sqrt(a - b) - 2*a + b)*log(sqrt(2*sqrt(
a)*sqrt(a - b) - 2*a + b) + e**(c + d*x)*sqrt(b))*a - sqrt(b)*sqrt(2*sqrt(
a)*sqrt(a - b) - 2*a + b)*log(sqrt(2*sqrt(a)*sqrt(a - b) - 2*a + b) + e**(
c + d*x)*sqrt(b))*b + 2*log(e**(c + d*x) - 1)*a*b - 2*log(e**(c + d*x) - 1
)*b**2 - 2*log(e**(c + d*x) + 1)*a*b + 2*log(e**(c + d*x) + 1)*b**2)/(2*a*
b*d*(a - b))
```

3.37 $\int \frac{\operatorname{csch}^2(c+dx)}{a+b\sinh^2(c+dx)}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 57

$$\int \frac{\operatorname{csch}^2(c+dx)}{a+b\sinh^2(c+dx)}\,dx=-\frac{b\operatorname{arctanh}\left(\frac{\sqrt{a-b}\tanh(c+dx)}{\sqrt{a}}\right)}{a^{3/2}\sqrt{a-b}d}-\frac{\coth(c+dx)}{ad}$$

output
```
-b*arctanh((a-b)^(1/2)*tanh(d*x+c)/a^(1/2))/a^(3/2)/(a-b)^(1/2)/d-coth(d*x
+c)/a/d
```

Mathematica [A] (verified)

Time = 0.68 (sec) , antiderivative size = 57, normalized size of antiderivative = 1.00

$$\int \frac{\operatorname{csch}^2(c+dx)}{a+b\sinh^2(c+dx)}\,dx=\frac{-\frac{b\operatorname{arctanh}\left(\frac{\sqrt{a-b}\tanh(c+dx)}{\sqrt{a}}\right)}{\sqrt{a-b}}-\sqrt{a}\coth(c+dx)}{a^{3/2}d}$$

input
```
Integrate[Csch[c + d*x]^2/(a + b*Sinh[c + d*x]^2),x]
```

output
```
(-((b*ArcTanh[(Sqrt[a - b]*Tanh[c + d*x])/Sqrt[a]])/Sqrt[a - b]) - Sqrt[a]
*Coth[c + d*x])/(a^(3/2)*d)
```

Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 55, normalized size of antiderivative = 0.96, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.217, Rules used = {3042, 25, 3666, 359, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\operatorname{csch}^2(c+dx)}{a+b\sinh^2(c+dx)}\,dx$$

$$\Big\downarrow \; 3042$$

$$\int -\frac{1}{\sin(ic+idx)^2\left(a-b\sin(ic+idx)^2\right)}dx$$

$$\Big\downarrow \; 25$$

$$-\int \frac{1}{\sin(ic+idx)^2\left(a-b\sin(ic+idx)^2\right)}dx$$

$$\Big\downarrow \; 3666$$

$$\frac{\int \frac{\coth^2(c+dx)\left(1-\tanh^2(c+dx)\right)}{a-(a-b)\tanh^2(c+dx)}d\tanh(c+dx)}{d}$$

$$\Big\downarrow \; 359$$

$$\frac{-\frac{b\int \frac{1}{a-(a-b)\tanh^2(c+dx)}d\tanh(c+dx)}{a}-\frac{\coth(c+dx)}{a}}{d}$$

$$\Big\downarrow \; 221$$

$$\frac{-\frac{b\operatorname{arctanh}\left(\frac{\sqrt{a-b}\tanh(c+dx)}{\sqrt{a}}\right)}{a^{3/2}\sqrt{a-b}}-\frac{\coth(c+dx)}{a}}{d}$$

input
```
Int[Csch[c + d*x]^2/(a + b*Sinh[c + d*x]^2),x]
```

output
```
(-((b*ArcTanh[(Sqrt[a - b]*Tanh[c + d*x])/Sqrt[a]])/(a^(3/2)*Sqrt[a - b]))
 - Coth[c + d*x]/a)/d
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 359
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2), x
_Symbol] :> Simp[c*(e*x)^(m + 1)*((a + b*x^2)^(p + 1)/(a*e*(m + 1))), x] +
Simp[(a*d*(m + 1) - b*c*(m + 2*p + 3))/(a*e^2*(m + 1))   Int[(e*x)^(m + 2)*
(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c - a*d, 0]
&& LtQ[m, -1] &&  !ILtQ[p, -1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3666
```
Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(
p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Simp[ff^(m + 1
)/f   Subst[Int[x^m*((a + (a + b)*ff^2*x^2)^p/(1 + ff^2*x^2)^(m/2 + p + 1))
, x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f}, x] && IntegerQ[m/2] &
& IntegerQ[p]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 182 vs. $2(49) = 98$.

Time = 0.52 (sec) , antiderivative size = 183, normalized size of antiderivative = 3.21

method	result
risch	$-\frac{2}{ad\left(e^{2dx+2c}-1\right)}+\frac{b\ln\left(e^{2dx+2c}+\frac{2a\sqrt{a^2-ab}-b\sqrt{a^2-ab}+2a^2-2ab}{b\sqrt{a^2-ab}}\right)}{2\sqrt{a^2-ab}\,da}-\frac{b\ln\left(e^{2dx+2c}+\frac{2a\sqrt{a^2-ab}-b\sqrt{a^2-ab}-2a^2+2}{b\sqrt{a^2-ab}}\right.}{2\sqrt{a^2-ab}\,da}$
derivativedivides	$\frac{-\frac{\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)}{2a}+2b\left(\frac{\left(\sqrt{-b(a-b)}+b\right)\arctan\left(\frac{\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)a}{\sqrt{\left(2\sqrt{-b(a-b)}-a+2b\right)a}}\right)}{2a\sqrt{-b(a-b)}\sqrt{\left(2\sqrt{-b(a-b)}-a+2b\right)a}}-\frac{\left(\sqrt{-b(a-b)}-b\right)\operatorname{arctanh}\left(\frac{\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)a}{\sqrt{\left(2\sqrt{-b(a-b)}+a-2b\right)a}}\right)}{2a\sqrt{-b(a-b)}\sqrt{\left(2\sqrt{-b(a-b)}+a-2b\right)a}}\right)}{d}$
default	$\frac{-\frac{\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)}{2a}+2b\left(\frac{\left(\sqrt{-b(a-b)}+b\right)\arctan\left(\frac{\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)a}{\sqrt{\left(2\sqrt{-b(a-b)}-a+2b\right)a}}\right)}{2a\sqrt{-b(a-b)}\sqrt{\left(2\sqrt{-b(a-b)}-a+2b\right)a}}-\frac{\left(\sqrt{-b(a-b)}-b\right)\operatorname{arctanh}\left(\frac{\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)a}{\sqrt{\left(2\sqrt{-b(a-b)}+a-2b\right)a}}\right)}{2a\sqrt{-b(a-b)}\sqrt{\left(2\sqrt{-b(a-b)}+a-2b\right)a}}\right)}{d}$

input
```
int(csch(d*x+c)^2/(a+b*sinh(d*x+c)^2),x,method=_RETURNVERBOSE)
```

output
```
-2/a/d/(exp(2*d*x+2*c)-1)+1/2/(a^2-a*b)^(1/2)*b/d/a*ln(exp(2*d*x+2*c)+(2*a
*(a^2-a*b)^(1/2)-b*(a^2-a*b)^(1/2)+2*a^2-2*a*b)/b/(a^2-a*b)^(1/2))-1/2/(a^
2-a*b)^(1/2)*b/d/a*ln(exp(2*d*x+2*c)+(2*a*(a^2-a*b)^(1/2)-b*(a^2-a*b)^(1/2
)-2*a^2+2*a*b)/b/(a^2-a*b)^(1/2))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 209 vs. $2(49) = 98$.

Time = 0.10 (sec) , antiderivative size = 675, normalized size of antiderivative = 11.84

$$\int \frac{\operatorname{csch}^2(c+dx)}{a+b\sinh^2(c+dx)}\,dx = \text{Too large to display}$$

input
```
integrate(csch(d*x+c)^2/(a+b*sinh(d*x+c)^2),x, algorithm="fricas")
```

output
```
[1/2*((b*cosh(d*x + c)^2 + 2*b*cosh(d*x + c)*sinh(d*x + c) + b*sinh(d*x +
c)^2 - b)*sqrt(a^2 - a*b)*log((b^2*cosh(d*x + c)^4 + 4*b^2*cosh(d*x + c)*s
inh(d*x + c)^3 + b^2*sinh(d*x + c)^4 + 2*(2*a*b - b^2)*cosh(d*x + c)^2 + 2
*(3*b^2*cosh(d*x + c)^2 + 2*a*b - b^2)*sinh(d*x + c)^2 + 8*a^2 - 8*a*b + b
^2 + 4*(b^2*cosh(d*x + c)^3 + (2*a*b - b^2)*cosh(d*x + c))*sinh(d*x + c) +
 4*(b*cosh(d*x + c)^2 + 2*b*cosh(d*x + c)*sinh(d*x + c) + b*sinh(d*x + c)^
2 + 2*a - b)*sqrt(a^2 - a*b))/(b*cosh(d*x + c)^4 + 4*b*cosh(d*x + c)*sinh(
d*x + c)^3 + b*sinh(d*x + c)^4 + 2*(2*a - b)*cosh(d*x + c)^2 + 2*(3*b*cosh
(d*x + c)^2 + 2*a - b)*sinh(d*x + c)^2 + 4*(b*cosh(d*x + c)^3 + (2*a - b)*
cosh(d*x + c))*sinh(d*x + c) + b)) - 4*a^2 + 4*a*b)/((a^3 - a^2*b)*d*cosh(
d*x + c)^2 + 2*(a^3 - a^2*b)*d*cosh(d*x + c)*sinh(d*x + c) + (a^3 - a^2*b)
*d*sinh(d*x + c)^2 - (a^3 - a^2*b)*d), ((b*cosh(d*x + c)^2 + 2*b*cosh(d*x
+ c)*sinh(d*x + c) + b*sinh(d*x + c)^2 - b)*sqrt(-a^2 + a*b)*arctan(-1/2*(
b*cosh(d*x + c)^2 + 2*b*cosh(d*x + c)*sinh(d*x + c) + b*sinh(d*x + c)^2 +
2*a - b)*sqrt(-a^2 + a*b)/(a^2 - a*b)) - 2*a^2 + 2*a*b)/((a^3 - a^2*b)*d*c
osh(d*x + c)^2 + 2*(a^3 - a^2*b)*d*cosh(d*x + c)*sinh(d*x + c) + (a^3 - a^
2*b)*d*sinh(d*x + c)^2 - (a^3 - a^2*b)*d)]
```

Sympy [F]

$$\int \frac{\operatorname{csch}^2(c+dx)}{a+b\sinh^2(c+dx)}\,dx=\int \frac{\operatorname{csch}^2\left(c+dx\right)}{a+b\sinh^2\left(c+dx\right)}\,dx$$

input
```
integrate(csch(d*x+c)**2/(a+b*sinh(d*x+c)**2),x)
```

output
```
Integral(csch(c + d*x)**2/(a + b*sinh(c + d*x)**2), x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{\operatorname{csch}^2(c+dx)}{a+b\sinh^2(c+dx)}\,dx=\text{Exception raised: ValueError}$$

input
```
integrate(csch(d*x+c)^2/(a+b*sinh(d*x+c)^2),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(b-a>0)', see `assume?` for more
details)Is
```

Giac [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 72, normalized size of antiderivative = 1.26

$$\int \frac{\operatorname{csch}^2(c+dx)}{a+b\sinh^2(c+dx)}\,dx = -\frac{\frac{b\arctan\left(\frac{be^{(2\,dx+2\,c)}+2\,a-b}{2\,\sqrt{-a^2+ab}}\right)}{\sqrt{-a^2+ab}a}+\frac{2}{a\left(e^{(2\,dx+2\,c)}-1\right)}}{d}$$

input
```
integrate(csch(d*x+c)^2/(a+b*sinh(d*x+c)^2),x, algorithm="giac")
```

output
```
-(b*arctan(1/2*(b*e^(2*d*x + 2*c) + 2*a - b)/sqrt(-a^2 + a*b))/(sqrt(-a^2
+ a*b)*a) + 2/(a*(e^(2*d*x + 2*c) - 1)))/d
```

Mupad [B] (verification not implemented)

Time = 0.50 (sec) , antiderivative size = 176, normalized size of antiderivative = 3.09

$$\int \frac{\operatorname{csch}^2(c+dx)}{a+b\sinh^2(c+dx)}\,dx = \frac{b\,\ln\left(\frac{4\,e^{2\,c+2\,d\,x}}{a}-\frac{2\,\left(b\,d+2\,a\,d\,e^{2\,c+2\,d\,x}-b\,d\,e^{2\,c+2\,d\,x}\right)}{a^{3/2}\,d\,\sqrt{a-b}}\right)}{2\,a^{3/2}\,d\,\sqrt{a-b}} - \frac{2}{a\,d\,\left(e^{2\,c+2\,d\,x}-1\right)} - \frac{b\,\ln\left(\frac{4\,e^{2\,c+2\,d\,x}}{a}+\frac{2\,\left(b\,d+2\,a\,d\,e^{2\,c+2\,d\,x}-b\,d\,e^{2\,c+2\,d\,x}\right)}{a^{3/2}\,d\,\sqrt{a-b}}\right)}{2\,a^{3/2}\,d\,\sqrt{a-b}}$$

input
```
int(1/(sinh(c + d*x)^2*(a + b*sinh(c + d*x)^2)),x)
```

output
```
(b*log((4*exp(2*c + 2*d*x))/a - (2*(b*d + 2*a*d*exp(2*c + 2*d*x) - b*d*exp
(2*c + 2*d*x)))/(a^(3/2)*d*(a - b)^(1/2))))/(2*a^(3/2)*d*(a - b)^(1/2)) -
2/(a*d*(exp(2*c + 2*d*x) - 1)) - (b*log((4*exp(2*c + 2*d*x))/a + (2*(b*d +
 2*a*d*exp(2*c + 2*d*x) - b*d*exp(2*c + 2*d*x)))/(a^(3/2)*d*(a - b)^(1/2))
))/(2*a^(3/2)*d*(a - b)^(1/2))
```

Reduce [B] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 339, normalized size of antiderivative = 5.95

$$\int \frac{\operatorname{csch}^2(c+dx)}{a+b\sinh^2(c+dx)}\,dx$$
$$= \frac{-e^{2dx+2c}\sqrt{a}\,\sqrt{a-b}\log\left(-\sqrt{2\sqrt{a}\,\sqrt{a-b}-2a+b}+e^{dx+c}\sqrt{b}\right)b-e^{2dx+2c}\sqrt{a}\,\sqrt{a-b}\log\left(\sqrt{2\sqrt{a}\,\sqrt{a-}\right.}{}$$

input
```
int(csch(d*x+c)^2/(a+b*sinh(d*x+c)^2),x)
```

output
```
( - e**(2*c + 2*d*x)*sqrt(a)*sqrt(a - b)*log( - sqrt(2*sqrt(a)*sqrt(a - b)
 - 2*a + b) + e**(c + d*x)*sqrt(b))*b - e**(2*c + 2*d*x)*sqrt(a)*sqrt(a -
b)*log(sqrt(2*sqrt(a)*sqrt(a - b) - 2*a + b) + e**(c + d*x)*sqrt(b))*b + e
**(2*c + 2*d*x)*sqrt(a)*sqrt(a - b)*log(2*sqrt(a)*sqrt(a - b) + e**(2*c +
2*d*x)*b + 2*a - b)*b + sqrt(a)*sqrt(a - b)*log( - sqrt(2*sqrt(a)*sqrt(a -
 b) - 2*a + b) + e**(c + d*x)*sqrt(b))*b + sqrt(a)*sqrt(a - b)*log(sqrt(2*
sqrt(a)*sqrt(a - b) - 2*a + b) + e**(c + d*x)*sqrt(b))*b - sqrt(a)*sqrt(a
- b)*log(2*sqrt(a)*sqrt(a - b) + e**(2*c + 2*d*x)*b + 2*a - b)*b - 4*e**(2
*c + 2*d*x)*a**2 + 4*e**(2*c + 2*d*x)*a*b)/(2*a**2*d*(e**(2*c + 2*d*x)*a -
 e**(2*c + 2*d*x)*b - a + b))
```

3.38 $\int \frac{\operatorname{csch}^3(c+dx)}{a+b\sinh^2(c+dx)}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 88

$$\int \frac{\operatorname{csch}^3(c+dx)}{a+b\sinh^2(c+dx)}\,dx = \frac{b^{3/2}\arctan\left(\frac{\sqrt{b}\cosh(c+dx)}{\sqrt{a-b}}\right)}{a^2\sqrt{a-b}\,d} + \frac{(a+2b)\operatorname{arctanh}(\cosh(c+dx))}{2a^2d} - \frac{\coth(c+dx)\operatorname{csch}(c+dx)}{2ad}$$

output
```
b^(3/2)*arctan(b^(1/2)*cosh(d*x+c)/(a-b)^(1/2))/a^2/(a-b)^(1/2)/d+1/2*(a+2
*b)*arctanh(cosh(d*x+c))/a^2/d-1/2*coth(d*x+c)*csch(d*x+c)/a/d
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 2.01 (sec) , antiderivative size = 220, normalized size of antiderivative = 2.50

$$\int \frac{\operatorname{csch}^3(c+dx)}{a+b\sinh^2(c+dx)}\,dx$$

$$= \frac{(2a-b+b\cosh(2(c+dx)))\operatorname{csch}^2(c+dx)\left(\frac{8b^{3/2}\arctan\left(\frac{\sqrt{b}-i\sqrt{a}\tanh\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a-b}}\right)}{\sqrt{a-b}} + \frac{8b^{3/2}\arctan\left(\frac{\sqrt{b}+i\sqrt{a}\tanh\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a-b}}\right.}{\sqrt{a-b}}\right.}{16a^2d\,(}$$

input
```
Integrate[Csch[c + d*x]^3/(a + b*Sinh[c + d*x]^2),x]
```

output
```
((2*a - b + b*Cosh[2*(c + d*x)])*Csch[c + d*x]^2*((8*b^(3/2)*ArcTan[(Sqrt[
b] - I*Sqrt[a]*Tanh[(c + d*x)/2])/Sqrt[a - b]])/Sqrt[a - b] + (8*b^(3/2)*A
rcTan[(Sqrt[b] + I*Sqrt[a]*Tanh[(c + d*x)/2])/Sqrt[a - b]])/Sqrt[a - b] -
a*Csch[(c + d*x)/2]^2 + 4*(a + 2*b)*Log[Cosh[(c + d*x)/2]] - 4*(a + 2*b)*L
og[Sinh[(c + d*x)/2]] - a*Sech[(c + d*x)/2]^2))/(16*a^2*d*(b + a*Csch[c +
d*x]^2))
```

Rubi [A] (verified)

Time = 0.31 (sec) , antiderivative size = 97, normalized size of antiderivative = 1.10, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.304$, Rules used = {3042, 26, 3665, 316, 397, 218, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\operatorname{csch}^3(c+dx)}{a+b\sinh^2(c+dx)}dx$$

$$\Big\downarrow \text{3042}$$

$$\int -\frac{i}{\sin(ic+idx)^3\left(a-b\sin(ic+idx)^2\right)}dx$$

$$\Big\downarrow \text{26}$$

$$-i\int \frac{1}{\sin(ic+idx)^3\left(a-b\sin(ic+idx)^2\right)}dx$$

$$\Big\downarrow \text{3665}$$

$$\frac{\int \frac{1}{\left(1-\cosh^2(c+dx)\right)^2\left(b\cosh^2(c+dx)+a-b\right)}d\cosh(c+dx)}{d}$$

$$\Big\downarrow \text{316}$$

$$\frac{\frac{\int \frac{b\cosh^2(c+dx)+a+b}{\left(1-\cosh^2(c+dx)\right)\left(b\cosh^2(c+dx)+a-b\right)}d\cosh(c+dx)}{2a}+\frac{\cosh(c+dx)}{2a\left(1-\cosh^2(c+dx)\right)}}{d}$$

$$\Big\downarrow \text{397}$$

$$\frac{\frac{\frac{2b^2\int \frac{1}{b\cosh^2(c+dx)+a-b}d\cosh(c+dx)}{a}+\frac{(a+2b)\int \frac{1}{1-\cosh^2(c+dx)}d\cosh(c+dx)}{a}}{2a}+\frac{\cosh(c+dx)}{2a(1-\cosh^2(c+dx))}}{d}$$

$$\Big\downarrow \text{218}$$

$$\frac{\frac{\frac{(a+2b)\int \frac{1}{1-\cosh^2(c+dx)}d\cosh(c+dx)}{a}+\frac{2b^{3/2}\arctan\left(\frac{\sqrt{b}\cosh(c+dx)}{\sqrt{a-b}}\right)}{a\sqrt{a-b}}}{2a}+\frac{\cosh(c+dx)}{2a(1-\cosh^2(c+dx))}}{d}$$

$$\Big\downarrow \text{219}$$

$$\frac{\frac{\frac{2b^{3/2}\arctan\left(\frac{\sqrt{b}\cosh(c+dx)}{\sqrt{a-b}}\right)}{a\sqrt{a-b}}+\frac{(a+2b)\text{arctanh}(\cosh(c+dx))}{a}}{2a}+\frac{\cosh(c+dx)}{2a(1-\cosh^2(c+dx))}}{d}$$

input
```
Int[Csch[c + d*x]^3/(a + b*Sinh[c + d*x]^2),x]
```

output
```
(((2*b^(3/2)*ArcTan[(Sqrt[b]*Cosh[c + d*x])/Sqrt[a - b]])/(a*Sqrt[a - b])
+ ((a + 2*b)*ArcTanh[Cosh[c + d*x]])/a)/(2*a) + Cosh[c + d*x]/(2*a*(1 - Co
sh[c + d*x]^2)))/d
```

Defintions of rubi rules used

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 316
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_), x_Symbol] :> Sim
p[(-b)*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^(q + 1)/(2*a*(p + 1)*(b*c - a*d))
), x] + Simp[1/(2*a*(p + 1)*(b*c - a*d))   Int[(a + b*x^2)^(p + 1)*(c + d*x
^2)^q*Simp[b*c + 2*(p + 1)*(b*c - a*d) + d*b*(2*(p + q + 2) + 1)*x^2, x], x
], x] /; FreeQ[{a, b, c, d, q}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] &&  !
( !IntegerQ[p] && IntegerQ[q] && LtQ[q, -1]) && IntBinomialQ[a, b, c, d, 2,
 p, q, x]
```

rule 397
```
Int[((e_) + (f_.)*(x_)^2)/(((a_) + (b_.)*(x_)^2)*((c_) + (d_.)*(x_)^2)), x_
Symbol] :> Simp[(b*e - a*f)/(b*c - a*d)   Int[1/(a + b*x^2), x], x] - Simp[
(d*e - c*f)/(b*c - a*d)   Int[1/(c + d*x^2), x], x] /; FreeQ[{a, b, c, d, e
, f}, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3665
```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^
(p_.), x_Symbol] :> With[{ff = FreeFactors[Cos[e + f*x], x]}, Simp[-ff/f
Subst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a + b - b*ff^2*x^2)^p, x], x, Cos[e +
 f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m - 1)/2]
```

Maple [A] (verified)

Time = 0.69 (sec) , antiderivative size = 113, normalized size of antiderivative = 1.28

method	result
derivativedivides	$\frac{\frac{\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^2}{8a}-\frac{1}{8a\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^2}+\frac{(-2a-4b)\ln\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{4a^2}+\frac{b^2\arctan\left(\frac{2\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^2a-2a+4b}{4\sqrt{ab-b^2}}\right)}{a^2\sqrt{ab-b^2}}}{d}$
default	$\frac{\frac{\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^2}{8a}-\frac{1}{8a\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^2}+\frac{(-2a-4b)\ln\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{4a^2}+\frac{b^2\arctan\left(\frac{2\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^2a-2a+4b}{4\sqrt{ab-b^2}}\right)}{a^2\sqrt{ab-b^2}}}{d}$
risch	$-\frac{e^{dx+c}\left(e^{2dx+2c}+1\right)}{da\left(e^{2dx+2c}-1\right)^2}-\frac{\ln\left(e^{dx+c}-1\right)}{2ad}-\frac{\ln\left(e^{dx+c}-1\right)b}{a^2d}+\frac{\ln\left(e^{dx+c}+1\right)}{2ad}+\frac{\ln\left(e^{dx+c}+1\right)b}{a^2d}+\frac{\sqrt{-b(a-b)}\,b\ln\left(e\right.}{}$

input
```
int(csch(d*x+c)^3/(a+b*sinh(d*x+c)^2),x,method=_RETURNVERBOSE)
```

output
```
1/d*(1/8*tanh(1/2*d*x+1/2*c)^2/a-1/8/a/tanh(1/2*d*x+1/2*c)^2+1/4/a^2*(-2*a
-4*b)*ln(tanh(1/2*d*x+1/2*c))+b^2/a^2/(a*b-b^2)^(1/2)*arctan(1/4*(2*tanh(1
/2*d*x+1/2*c)^2*a-2*a+4*b)/(a*b-b^2)^(1/2)))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 865 vs. $2(76) = 152$.

Time = 0.13 (sec) , antiderivative size = 1837, normalized size of antiderivative = 20.88

$$\int \frac{\operatorname{csch}^3(c+dx)}{a+b\sinh^2(c+dx)}\,dx = \text{Too large to display}$$

input
```
integrate(csch(d*x+c)^3/(a+b*sinh(d*x+c)^2),x, algorithm="fricas")
```

output
```
[-1/2*(2*a*cosh(d*x + c)^3 + 6*a*cosh(d*x + c)*sinh(d*x + c)^2 + 2*a*sinh(
d*x + c)^3 - (b*cosh(d*x + c)^4 + 4*b*cosh(d*x + c)*sinh(d*x + c)^3 + b*si
nh(d*x + c)^4 - 2*b*cosh(d*x + c)^2 + 2*(3*b*cosh(d*x + c)^2 - b)*sinh(d*x
 + c)^2 + 4*(b*cosh(d*x + c)^3 - b*cosh(d*x + c))*sinh(d*x + c) + b)*sqrt(
-b/(a - b))*log((b*cosh(d*x + c)^4 + 4*b*cosh(d*x + c)*sinh(d*x + c)^3 + b
*sinh(d*x + c)^4 - 2*(2*a - 3*b)*cosh(d*x + c)^2 + 2*(3*b*cosh(d*x + c)^2
- 2*a + 3*b)*sinh(d*x + c)^2 + 4*(b*cosh(d*x + c)^3 - (2*a - 3*b)*cosh(d*x
 + c))*sinh(d*x + c) + 4*((a - b)*cosh(d*x + c)^3 + 3*(a - b)*cosh(d*x + c
)*sinh(d*x + c)^2 + (a - b)*sinh(d*x + c)^3 + (a - b)*cosh(d*x + c) + (3*(
a - b)*cosh(d*x + c)^2 + a - b)*sinh(d*x + c))*sqrt(-b/(a - b)) + b)/(b*co
sh(d*x + c)^4 + 4*b*cosh(d*x + c)*sinh(d*x + c)^3 + b*sinh(d*x + c)^4 + 2*
(2*a - b)*cosh(d*x + c)^2 + 2*(3*b*cosh(d*x + c)^2 + 2*a - b)*sinh(d*x + c
)^2 + 4*(b*cosh(d*x + c)^3 + (2*a - b)*cosh(d*x + c))*sinh(d*x + c) + b))
+ 2*a*cosh(d*x + c) - ((a + 2*b)*cosh(d*x + c)^4 + 4*(a + 2*b)*cosh(d*x +
c)*sinh(d*x + c)^3 + (a + 2*b)*sinh(d*x + c)^4 - 2*(a + 2*b)*cosh(d*x + c)
^2 + 2*(3*(a + 2*b)*cosh(d*x + c)^2 - a - 2*b)*sinh(d*x + c)^2 + 4*((a + 2
*b)*cosh(d*x + c)^3 - (a + 2*b)*cosh(d*x + c))*sinh(d*x + c) + a + 2*b)*lo
g(cosh(d*x + c) + sinh(d*x + c) + 1) + ((a + 2*b)*cosh(d*x + c)^4 + 4*(a +
 2*b)*cosh(d*x + c)*sinh(d*x + c)^3 + (a + 2*b)*sinh(d*x + c)^4 - 2*(a + 2
*b)*cosh(d*x + c)^2 + 2*(3*(a + 2*b)*cosh(d*x + c)^2 - a - 2*b)*sinh(d*...
```

Sympy [F]

$$\int \frac{\operatorname{csch}^3(c+dx)}{a+b\sinh^2(c+dx)}\,dx = \int \frac{\operatorname{csch}^3(c+dx)}{a+b\sinh^2(c+dx)}\,dx$$

input
```
integrate(csch(d*x+c)**3/(a+b*sinh(d*x+c)**2),x)
```

output
```
Integral(csch(c + d*x)**3/(a + b*sinh(c + d*x)**2), x)
```

Maxima [F]

$$\int \frac{\operatorname{csch}^3(c+dx)}{a+b\sinh^2(c+dx)}\,dx = \int \frac{\operatorname{csch}(dx+c)^3}{b\sinh(dx+c)^2+a}\,dx$$

input
```
integrate(csch(d*x+c)^3/(a+b*sinh(d*x+c)^2),x, algorithm="maxima")
```

output
```
-(e^(3*d*x + 3*c) + e^(d*x + c))/(a*d*e^(4*d*x + 4*c) - 2*a*d*e^(2*d*x + 2
*c) + a*d) + 1/2*(a + 2*b)*log((e^(d*x + c) + 1)*e^(-c))/(a^2*d) - 1/2*(a
+ 2*b)*log((e^(d*x + c) - 1)*e^(-c))/(a^2*d) + 8*integrate(1/4*(b^2*e^(3*d
*x + 3*c) - b^2*e^(d*x + c))/(a^2*b*e^(4*d*x + 4*c) + a^2*b + 2*(2*a^3*e^(
2*c) - a^2*b*e^(2*c))*e^(2*d*x)), x)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{\operatorname{csch}^3(c+dx)}{a+b\sinh^2(c+dx)}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(csch(d*x+c)^3/(a+b*sinh(d*x+c)^2),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Limit: Max order reached or unable
to make series expansion Error: Bad Argument Value
```

Mupad [B] (verification not implemented)

Time = 2.61 (sec) , antiderivative size = 571, normalized size of antiderivative = 6.49

$$\int \frac{\operatorname{csch}^3(c+dx)}{a+b\sinh^2(c+dx)}\,dx$$

$$= \frac{\operatorname{atan}\left(\frac{e^{dx}\,e^{c}\,\left(a^7\,\sqrt{-a^4\,d^2}+18\,b^7\,\sqrt{-a^4\,d^2}-36\,a^2\,b^5\,\sqrt{-a^4\,d^2}-30\,a^3\,b^4\,\sqrt{-a^4\,d^2}+12\,a^4\,b^3\,\sqrt{-a^4\,d^2}+21\,a^5\,b^2\,\sqrt{-a^4\,d^2}+9\,a\,b^6\,\sqrt{-a^4\,d^2}+8}{a^8\,d\,\sqrt{a^2+4\,a\,b+4\,b^2}+9\,a^2\,b^6\,d\,\sqrt{a^2+4\,a\,b+4\,b^2}-18\,a^4\,b^4\,d\,\sqrt{a^2+4\,a\,b+4\,b^2}-6\,a^5\,b^3\,d\,\sqrt{a^2+4\,a\,b+4\,b^2}+9\,a^6\,b^2\,d\,\sqrt{a^2+4\,a\,b+4\,b^2}+6\,a^7\,b}\right.}{\sqrt{-a^4\,d^2}}$$

$$-\frac{e^{c+d\,x}}{a\,d\,\left(e^{2\,c+2\,d\,x}-1\right)}-\frac{2\,e^{c+d\,x}}{a\,d\,\left(e^{4\,c+4\,d\,x}-2\,e^{2\,c+2\,d\,x}+1\right)}$$

$$-\frac{(-b)^{3/2}\,\ln\left(\frac{64\,\left(e^{2\,c+2\,d\,x}+1\right)\,\left(a^3+3\,a^2\,b-3\,b^3\right)}{a^5\,(a-b)^2}-\frac{128\,e^{c+d\,x}\,\left(a^3+3\,a^2\,b-3\,b^3\right)}{a^5\,\sqrt{-b}\,(a-b)^{3/2}}\right)}{2\,a^2\,d\,\sqrt{a-b}}$$

$$+\frac{(-b)^{3/2}\,\ln\left(\frac{64\,\left(e^{2\,c+2\,d\,x}+1\right)\,\left(a^3+3\,a^2\,b-3\,b^3\right)}{a^5\,(a-b)^2}+\frac{128\,e^{c+d\,x}\,\left(a^3+3\,a^2\,b-3\,b^3\right)}{a^5\,\sqrt{-b}\,(a-b)^{3/2}}\right)}{2\,a^2\,d\,\sqrt{a-b}}$$

input
```
int(1/(sinh(c + d*x)^3*(a + b*sinh(c + d*x)^2)),x)
```

output
```
(atan((exp(d*x)*exp(c)*(a^7*(-a^4*d^2)^(1/2) + 18*b^7*(-a^4*d^2)^(1/2) - 3
6*a^2*b^5*(-a^4*d^2)^(1/2) - 30*a^3*b^4*(-a^4*d^2)^(1/2) + 12*a^4*b^3*(-a^
4*d^2)^(1/2) + 21*a^5*b^2*(-a^4*d^2)^(1/2) + 9*a*b^6*(-a^4*d^2)^(1/2) + 8*
a^6*b*(-a^4*d^2)^(1/2)))/(a^8*d*(4*a*b + a^2 + 4*b^2)^(1/2) + 9*a^2*b^6*d*
(4*a*b + a^2 + 4*b^2)^(1/2) - 18*a^4*b^4*d*(4*a*b + a^2 + 4*b^2)^(1/2) - 6
*a^5*b^3*d*(4*a*b + a^2 + 4*b^2)^(1/2) + 9*a^6*b^2*d*(4*a*b + a^2 + 4*b^2)
^(1/2) + 6*a^7*b*d*(4*a*b + a^2 + 4*b^2)^(1/2)))*(4*a*b + a^2 + 4*b^2)^(1/
2))/(-a^4*d^2)^(1/2) - exp(c + d*x)/(a*d*(exp(2*c + 2*d*x) - 1)) - (2*exp(
c + d*x))/(a*d*(exp(4*c + 4*d*x) - 2*exp(2*c + 2*d*x) + 1)) - ((-b)^(3/2)*
log((64*(exp(2*c + 2*d*x) + 1)*(3*a^2*b + a^3 - 3*b^3))/(a^5*(a - b)^2) -
(128*exp(c + d*x)*(3*a^2*b + a^3 - 3*b^3))/(a^5*(-b)^(1/2)*(a - b)^(3/2)))
)/(2*a^2*d*(a - b)^(1/2)) + ((-b)^(3/2)*log((64*(exp(2*c + 2*d*x) + 1)*(3*
a^2*b + a^3 - 3*b^3))/(a^5*(a - b)^2) + (128*exp(c + d*x)*(3*a^2*b + a^3 -
 3*b^3))/(a^5*(-b)^(1/2)*(a - b)^(3/2))))/(2*a^2*d*(a - b)^(1/2))
```

Reduce [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 2149, normalized size of antiderivative = 24.42

$$\int \frac{\operatorname{csch}^3(c+dx)}{a+b\sinh^2(c+dx)}\,dx = \text{Too large to display}$$

input
```
int(csch(d*x+c)^3/(a+b*sinh(d*x+c)^2),x)
```

output
```
(2*e**(4*c + 4*d*x)*sqrt(b)*sqrt(a)*sqrt(a - b)*sqrt(2*sqrt(a)*sqrt(a - b)
 + 2*a - b)*atan((e**(c + d*x)*b)/(sqrt(b)*sqrt(2*sqrt(a)*sqrt(a - b) + 2*
a - b))) - 4*e**(2*c + 2*d*x)*sqrt(b)*sqrt(a)*sqrt(a - b)*sqrt(2*sqrt(a)*s
qrt(a - b) + 2*a - b)*atan((e**(c + d*x)*b)/(sqrt(b)*sqrt(2*sqrt(a)*sqrt(a
 - b) + 2*a - b))) + 2*sqrt(b)*sqrt(a)*sqrt(a - b)*sqrt(2*sqrt(a)*sqrt(a -
 b) + 2*a - b)*atan((e**(c + d*x)*b)/(sqrt(b)*sqrt(2*sqrt(a)*sqrt(a - b) +
 2*a - b))) - 2*e**(4*c + 4*d*x)*sqrt(b)*sqrt(2*sqrt(a)*sqrt(a - b) + 2*a
- b)*atan((e**(c + d*x)*b)/(sqrt(b)*sqrt(2*sqrt(a)*sqrt(a - b) + 2*a - b))
)*a + 2*e**(4*c + 4*d*x)*sqrt(b)*sqrt(2*sqrt(a)*sqrt(a - b) + 2*a - b)*ata
n((e**(c + d*x)*b)/(sqrt(b)*sqrt(2*sqrt(a)*sqrt(a - b) + 2*a - b)))*b + 4*
e**(2*c + 2*d*x)*sqrt(b)*sqrt(2*sqrt(a)*sqrt(a - b) + 2*a - b)*atan((e**(c
 + d*x)*b)/(sqrt(b)*sqrt(2*sqrt(a)*sqrt(a - b) + 2*a - b)))*a - 4*e**(2*c
+ 2*d*x)*sqrt(b)*sqrt(2*sqrt(a)*sqrt(a - b) + 2*a - b)*atan((e**(c + d*x)*
b)/(sqrt(b)*sqrt(2*sqrt(a)*sqrt(a - b) + 2*a - b)))*b - 2*sqrt(b)*sqrt(2*s
qrt(a)*sqrt(a - b) + 2*a - b)*atan((e**(c + d*x)*b)/(sqrt(b)*sqrt(2*sqrt(a
)*sqrt(a - b) + 2*a - b)))*a + 2*sqrt(b)*sqrt(2*sqrt(a)*sqrt(a - b) + 2*a
- b)*atan((e**(c + d*x)*b)/(sqrt(b)*sqrt(2*sqrt(a)*sqrt(a - b) + 2*a - b))
)*b + e**(4*c + 4*d*x)*sqrt(b)*sqrt(a)*sqrt(a - b)*sqrt(2*sqrt(a)*sqrt(a -
 b) - 2*a + b)*log( - sqrt(2*sqrt(a)*sqrt(a - b) - 2*a + b) + e**(c + d*x)
*sqrt(b)) - e**(4*c + 4*d*x)*sqrt(b)*sqrt(a)*sqrt(a - b)*sqrt(2*sqrt(a)...
```

3.39 $\int \frac{\operatorname{csch}^4(c+dx)}{a+b\sinh^2(c+dx)}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 78

$$\int \frac{\operatorname{csch}^4(c+dx)}{a+b\sinh^2(c+dx)}\,dx=\frac{b^2\operatorname{arctanh}\left(\frac{\sqrt{a-b}\tanh(c+dx)}{\sqrt{a}}\right)}{a^{5/2}\sqrt{a-b}d}+\frac{(a+b)\coth(c+dx)}{a^2d}-\frac{\coth^3(c+dx)}{3ad}$$

output
```
b^2*arctanh((a-b)^(1/2)*tanh(d*x+c)/a^(1/2))/a^(5/2)/(a-b)^(1/2)/d+(a+b)*c
oth(d*x+c)/a^2/d-1/3*coth(d*x+c)^3/a/d
```

Mathematica [A] (verified)

Time = 0.97 (sec) , antiderivative size = 126, normalized size of antiderivative = 1.62

$$\int \frac{\operatorname{csch}^4(c+dx)}{a+b\sinh^2(c+dx)}\,dx = -\frac{(2a-b+b\cosh(2(c+dx)))\operatorname{csch}^2(c+dx)\left(-3b^2\operatorname{arctanh}\left(\frac{\sqrt{a-b}\tanh(c+dx)}{\sqrt{a}}\right)+\sqrt{a}\sqrt{a-b}\coth(c+dx)\left(\right.\right.}{6a^{5/2}\sqrt{a-b}d\left(b+a\operatorname{csch}^2(c+dx)\right)}$$

input
```
Integrate[Csch[c + d*x]^4/(a + b*Sinh[c + d*x]^2),x]
```

output
```
-1/6*((2*a - b + b*Cosh[2*(c + d*x)])*Csch[c + d*x]^2*(-3*b^2*ArcTanh[(Sqr
t[a - b]*Tanh[c + d*x])/Sqrt[a]] + Sqrt[a]*Sqrt[a - b]*Coth[c + d*x]*(-2*a
 - 3*b + a*Csch[c + d*x]^2)))/(a^(5/2)*Sqrt[a - b]*d*(b + a*Csch[c + d*x]^
2))
```

Rubi [A] (verified)

Time = 0.32 (sec) , antiderivative size = 73, normalized size of antiderivative = 0.94, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.174$, Rules used = {3042, 3666, 364, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\operatorname{csch}^4(c+dx)}{a+b\sinh^2(c+dx)}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{\sin(ic+idx)^4\left(a-b\sin(ic+idx)^2\right)}dx$$

$$\Big\downarrow \text{3666}$$

$$\frac{\int \frac{\coth^4(c+dx)\left(1-\tanh^2(c+dx)\right)^2}{a-(a-b)\tanh^2(c+dx)}d\tanh(c+dx)}{d}$$

$$\Big\downarrow \text{364}$$

$$\frac{\int \left(\frac{\coth^4(c+dx)}{a}+\frac{(-a-b)\coth^2(c+dx)}{a^2}+\frac{b^2}{a^2\left(a-(a-b)\tanh^2(c+dx)\right)}\right)d\tanh(c+dx)}{d}$$

$$\Big\downarrow \text{2009}$$

$$\frac{\frac{b^2\operatorname{arctanh}\left(\frac{\sqrt{a-b}\tanh(c+dx)}{\sqrt{a}}\right)}{a^{5/2}\sqrt{a-b}}+\frac{(a+b)\coth(c+dx)}{a^2}-\frac{\coth^3(c+dx)}{3a}}{d}$$

input
```
Int[Csch[c + d*x]^4/(a + b*Sinh[c + d*x]^2),x]
```

output
```
((b^2*ArcTanh[(Sqrt[a - b]*Tanh[c + d*x])/Sqrt[a]])/(a^(5/2)*Sqrt[a - b])
+ ((a + b)*Coth[c + d*x])/a^2 - Coth[c + d*x]^3/(3*a))/d
```

Defintions of rubi rules used

rule 364
```
Int[(((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_))/((c_) + (d_.)*(x_)^2),
x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*((a + b*x^2)^p/(c + d*x^2)), x], x
] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b*c - a*d, 0] && IGtQ[p, 0] && (In
tegerQ[m] || IGtQ[2*(m + 1), 0] ||  !RationalQ[m])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3666
```
Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(
p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Simp[ff^(m + 1
)/f   Subst[Int[x^m*((a + (a + b)*ff^2*x^2)^p/(1 + ff^2*x^2)^(m/2 + p + 1))
, x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f}, x] && IntegerQ[m/2] &
& IntegerQ[p]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 229 vs. $2(68) = 136$.

Time = 0.86 (sec) , antiderivative size = 230, normalized size of antiderivative = 2.95

method	result
risch	$-\frac{2(-3b\,\mathrm{e}^{4dx+4c}+6\,\mathrm{e}^{2dx+2c}a+6\,\mathrm{e}^{2dx+2c}b-2a-3b)}{3a^2d(\mathrm{e}^{2dx+2c}-1)^3}+\frac{b^2\ln\left(\mathrm{e}^{2dx+2c}+\frac{2a\sqrt{a^2-ab}-b\sqrt{a^2-ab}-2a^2+2ab}{b\sqrt{a^2-ab}}\right)}{2\sqrt{a^2-ab}\,d\,a^2}-\frac{b^2\ln\left(\mathrm{e}^{2}\right.}{}$
derivativedivides	$\frac{-\frac{\frac{\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^3a}{3}-3\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)a-4b\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)}{8a^2}-\frac{1}{24a\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^3}-\frac{-3a-4b}{8a^2\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)}-\frac{2b^2\left(\frac{\left(\sqrt{-b(a-b)}+b\right)\arctan}{2a\sqrt{-b(a-b)}\sqrt{}}\right.}{}}{d}$
default	$\frac{-\frac{\frac{\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^3a}{3}-3\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)a-4b\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)}{8a^2}-\frac{1}{24a\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^3}-\frac{-3a-4b}{8a^2\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)}-\frac{2b^2\left(\frac{\left(\sqrt{-b(a-b)}+b\right)\arctan}{2a\sqrt{-b(a-b)}\sqrt{}}\right.}{}}{d}$

input
```
int(csch(d*x+c)^4/(a+b*sinh(d*x+c)^2),x,method=_RETURNVERBOSE)
```

output
```
-2/3*(-3*b*exp(4*d*x+4*c)+6*exp(2*d*x+2*c)*a+6*exp(2*d*x+2*c)*b-2*a-3*b)/a
^2/d/(exp(2*d*x+2*c)-1)^3+1/2/(a^2-a*b)^(1/2)*b^2/d/a^2*ln(exp(2*d*x+2*c)+
(2*a*(a^2-a*b)^(1/2)-b*(a^2-a*b)^(1/2)-2*a^2+2*a*b)/b/(a^2-a*b)^(1/2))-1/2
/(a^2-a*b)^(1/2)*b^2/d/a^2*ln(exp(2*d*x+2*c)+(2*a*(a^2-a*b)^(1/2)-b*(a^2-a
*b)^(1/2)+2*a^2-2*a*b)/b/(a^2-a*b)^(1/2))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 858 vs. $2(68) = 136$.

Time = 0.13 (sec) , antiderivative size = 1972, normalized size of antiderivative = 25.28

$$\int \frac{\operatorname{csch}^4(c+dx)}{a+b\sinh^2(c+dx)}\,dx = \text{Too large to display}$$

input
```
integrate(csch(d*x+c)^4/(a+b*sinh(d*x+c)^2),x, algorithm="fricas")
```

output
```
[1/6*(12*(a^2*b - a*b^2)*cosh(d*x + c)^4 + 48*(a^2*b - a*b^2)*cosh(d*x + c
)*sinh(d*x + c)^3 + 12*(a^2*b - a*b^2)*sinh(d*x + c)^4 + 8*a^3 + 4*a^2*b -
 12*a*b^2 - 24*(a^3 - a*b^2)*cosh(d*x + c)^2 - 24*(a^3 - a*b^2 - 3*(a^2*b
- a*b^2)*cosh(d*x + c)^2)*sinh(d*x + c)^2 + 3*(b^2*cosh(d*x + c)^6 + 6*b^2
*cosh(d*x + c)*sinh(d*x + c)^5 + b^2*sinh(d*x + c)^6 - 3*b^2*cosh(d*x + c)
^4 + 3*(5*b^2*cosh(d*x + c)^2 - b^2)*sinh(d*x + c)^4 + 3*b^2*cosh(d*x + c)
^2 + 4*(5*b^2*cosh(d*x + c)^3 - 3*b^2*cosh(d*x + c))*sinh(d*x + c)^3 + 3*(
5*b^2*cosh(d*x + c)^4 - 6*b^2*cosh(d*x + c)^2 + b^2)*sinh(d*x + c)^2 - b^2
 + 6*(b^2*cosh(d*x + c)^5 - 2*b^2*cosh(d*x + c)^3 + b^2*cosh(d*x + c))*sin
h(d*x + c))*sqrt(a^2 - a*b)*log((b^2*cosh(d*x + c)^4 + 4*b^2*cosh(d*x + c)
*sinh(d*x + c)^3 + b^2*sinh(d*x + c)^4 + 2*(2*a*b - b^2)*cosh(d*x + c)^2 +
 2*(3*b^2*cosh(d*x + c)^2 + 2*a*b - b^2)*sinh(d*x + c)^2 + 8*a^2 - 8*a*b +
 b^2 + 4*(b^2*cosh(d*x + c)^3 + (2*a*b - b^2)*cosh(d*x + c))*sinh(d*x + c)
 - 4*(b*cosh(d*x + c)^2 + 2*b*cosh(d*x + c)*sinh(d*x + c) + b*sinh(d*x + c
)^2 + 2*a - b)*sqrt(a^2 - a*b))/(b*cosh(d*x + c)^4 + 4*b*cosh(d*x + c)*sin
h(d*x + c)^3 + b*sinh(d*x + c)^4 + 2*(2*a - b)*cosh(d*x + c)^2 + 2*(3*b*co
sh(d*x + c)^2 + 2*a - b)*sinh(d*x + c)^2 + 4*(b*cosh(d*x + c)^3 + (2*a - b
)*cosh(d*x + c))*sinh(d*x + c) + b)) + 48*((a^2*b - a*b^2)*cosh(d*x + c)^3
 - (a^3 - a*b^2)*cosh(d*x + c))*sinh(d*x + c))/((a^4 - a^3*b)*d*cosh(d*x +
 c)^6 + 6*(a^4 - a^3*b)*d*cosh(d*x + c)*sinh(d*x + c)^5 + (a^4 - a^3*b)...
```

Sympy [F]

$$\int \frac{\operatorname{csch}^4(c+dx)}{a+b\sinh^2(c+dx)}\,dx=\int \frac{\operatorname{csch}^4{\left(c+dx\right)}}{a+b\sinh^2{\left(c+dx\right)}}\,dx$$

input
```
integrate(csch(d*x+c)**4/(a+b*sinh(d*x+c)**2),x)
```

output
```
Integral(csch(c + d*x)**4/(a + b*sinh(c + d*x)**2), x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{\operatorname{csch}^4(c+dx)}{a+b\sinh^2(c+dx)}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate(csch(d*x+c)^4/(a+b*sinh(d*x+c)^2),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(b-a>0)', see `assume?` for more
details)Is
```

Giac [A] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 118, normalized size of antiderivative = 1.51

$$\int \frac{\operatorname{csch}^4(c+dx)}{a+b\sinh^2(c+dx)}\,dx = \frac{\frac{3\,b^2\arctan\left(\frac{be^{(2\,dx+2\,c)}+2\,a-b}{2\,\sqrt{-a^2+ab}}\right)}{\sqrt{-a^2+ab}a^2}+\frac{2\,\left(3\,be^{(4\,dx+4\,c)}-6\,ae^{(2\,dx+2\,c)}-6\,be^{(2\,dx+2\,c)}+2\,a+3\,b\right)}{a^2\left(e^{(2\,dx+2\,c)}-1\right)^3}}{3\,d}$$

input
```
integrate(csch(d*x+c)^4/(a+b*sinh(d*x+c)^2),x, algorithm="giac")
```

output
```
1/3*(3*b^2*arctan(1/2*(b*e^(2*d*x + 2*c) + 2*a - b)/sqrt(-a^2 + a*b))/(sqr
t(-a^2 + a*b)*a^2) + 2*(3*b*e^(4*d*x + 4*c) - 6*a*e^(2*d*x + 2*c) - 6*b*e^
(2*d*x + 2*c) + 2*a + 3*b)/(a^2*(e^(2*d*x + 2*c) - 1)^3))/d
```

Mupad [B] (verification not implemented)

Time = 2.32 (sec) , antiderivative size = 350, normalized size of antiderivative = 4.49

$$\int \frac{\operatorname{csch}^4(c+dx)}{a+b\sinh^2(c+dx)}\,dx = \frac{2\,b}{a^2\,d\,\left(\mathrm{e}^{2\,c+2\,d\,x}-1\right)}-\frac{8}{3\,a\,d\,\left(3\,\mathrm{e}^{2\,c+2\,d\,x}-3\,\mathrm{e}^{4\,c+4\,d\,x}+\mathrm{e}^{6\,c+6\,d\,x}-1\right)}-\frac{4}{a\,d\,\left(\mathrm{e}^{4\,c+4\,d\,x}-2\,\mathrm{e}^{2\,c+2\,d\,x}+1\right)}-\frac{b^2\,\ln\left(\frac{4\,b^2\,\left(2\,a\,b-b^2+8\,a^2\,\mathrm{e}^{2\,c+2\,d\,x}+b^2\,\mathrm{e}^{2\,c+2\,d\,x}-8\,a\,b\,\mathrm{e}^{2\,c+2\,d\,x}\right)}{a^5\,\left(a-b\right)}-\frac{8\,b^2\,\left(b+4\,a\,\mathrm{e}^{2\,c+2\,d\,x}-2\,b\,\mathrm{e}^{2\,c+2\,d\,x}\right)}{a^{9/2}\,\sqrt{a-b}}\right)}{2\,a^{5/2}\,d\,\sqrt{a-b}}+\frac{b^2\,\ln\left(\frac{4\,b^2\,\left(2\,a\,b-b^2+8\,a^2\,\mathrm{e}^{2\,c+2\,d\,x}+b^2\,\mathrm{e}^{2\,c+2\,d\,x}-8\,a\,b\,\mathrm{e}^{2\,c+2\,d\,x}\right)}{a^5\,\left(a-b\right)}+\frac{8\,b^2\,\left(b+4\,a\,\mathrm{e}^{2\,c+2\,d\,x}-2\,b\,\mathrm{e}^{2\,c+2\,d\,x}\right)}{a^{9/2}\,\sqrt{a-b}}\right)}{2\,a^{5/2}\,d\,\sqrt{a-b}}$$

input
```
int(1/(sinh(c + d*x)^4*(a + b*sinh(c + d*x)^2)),x)
```

output
```
(2*b)/(a^2*d*(exp(2*c + 2*d*x) - 1)) - 8/(3*a*d*(3*exp(2*c + 2*d*x) - 3*ex
p(4*c + 4*d*x) + exp(6*c + 6*d*x) - 1)) - 4/(a*d*(exp(4*c + 4*d*x) - 2*exp
(2*c + 2*d*x) + 1)) - (b^2*log((4*b^2*(2*a*b - b^2 + 8*a^2*exp(2*c + 2*d*x
) + b^2*exp(2*c + 2*d*x) - 8*a*b*exp(2*c + 2*d*x)))/(a^5*(a - b)) - (8*b^2
*(b + 4*a*exp(2*c + 2*d*x) - 2*b*exp(2*c + 2*d*x)))/(a^(9/2)*(a - b)^(1/2)
)))/(2*a^(5/2)*d*(a - b)^(1/2)) + (b^2*log((4*b^2*(2*a*b - b^2 + 8*a^2*exp
(2*c + 2*d*x) + b^2*exp(2*c + 2*d*x) - 8*a*b*exp(2*c + 2*d*x)))/(a^5*(a -
b)) + (8*b^2*(b + 4*a*exp(2*c + 2*d*x) - 2*b*exp(2*c + 2*d*x)))/(a^(9/2)*(
a - b)^(1/2))))/(2*a^(5/2)*d*(a - b)^(1/2))
```

Reduce [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 781, normalized size of antiderivative = 10.01

$$\int \frac{\operatorname{csch}^4(c+dx)}{a+b\sinh^2(c+dx)}\,dx = \frac{3e^{6dx+6c}\sqrt{a}\,\sqrt{a-b}\log\left(-\sqrt{2\sqrt{a}\,\sqrt{a-b}-2a+b}+e^{dx+c}\sqrt{b}\right)b^2+3e^{6dx+6c}\sqrt{a}\,\sqrt{a-b}\log\left(\sqrt{2\sqrt{a}\,\sqrt{a-}\right.}{}$$

input
```
int(csch(d*x+c)^4/(a+b*sinh(d*x+c)^2),x)
```

output
```
(3*e**(6*c + 6*d*x)*sqrt(a)*sqrt(a - b)*log( - sqrt(2*sqrt(a)*sqrt(a - b)
- 2*a + b) + e**(c + d*x)*sqrt(b))*b**2 + 3*e**(6*c + 6*d*x)*sqrt(a)*sqrt(
a - b)*log(sqrt(2*sqrt(a)*sqrt(a - b) - 2*a + b) + e**(c + d*x)*sqrt(b))*b
**2 - 3*e**(6*c + 6*d*x)*sqrt(a)*sqrt(a - b)*log(2*sqrt(a)*sqrt(a - b) + e
**(2*c + 2*d*x)*b + 2*a - b)*b**2 - 9*e**(4*c + 4*d*x)*sqrt(a)*sqrt(a - b)
*log( - sqrt(2*sqrt(a)*sqrt(a - b) - 2*a + b) + e**(c + d*x)*sqrt(b))*b**2
 - 9*e**(4*c + 4*d*x)*sqrt(a)*sqrt(a - b)*log(sqrt(2*sqrt(a)*sqrt(a - b) -
 2*a + b) + e**(c + d*x)*sqrt(b))*b**2 + 9*e**(4*c + 4*d*x)*sqrt(a)*sqrt(a
 - b)*log(2*sqrt(a)*sqrt(a - b) + e**(2*c + 2*d*x)*b + 2*a - b)*b**2 + 9*e
**(2*c + 2*d*x)*sqrt(a)*sqrt(a - b)*log( - sqrt(2*sqrt(a)*sqrt(a - b) - 2*
a + b) + e**(c + d*x)*sqrt(b))*b**2 + 9*e**(2*c + 2*d*x)*sqrt(a)*sqrt(a -
b)*log(sqrt(2*sqrt(a)*sqrt(a - b) - 2*a + b) + e**(c + d*x)*sqrt(b))*b**2
- 9*e**(2*c + 2*d*x)*sqrt(a)*sqrt(a - b)*log(2*sqrt(a)*sqrt(a - b) + e**(2
*c + 2*d*x)*b + 2*a - b)*b**2 - 3*sqrt(a)*sqrt(a - b)*log( - sqrt(2*sqrt(a
)*sqrt(a - b) - 2*a + b) + e**(c + d*x)*sqrt(b))*b**2 - 3*sqrt(a)*sqrt(a -
 b)*log(sqrt(2*sqrt(a)*sqrt(a - b) - 2*a + b) + e**(c + d*x)*sqrt(b))*b**2
 + 3*sqrt(a)*sqrt(a - b)*log(2*sqrt(a)*sqrt(a - b) + e**(2*c + 2*d*x)*b +
2*a - b)*b**2 + 4*e**(6*c + 6*d*x)*a**2*b - 4*e**(6*c + 6*d*x)*a*b**2 - 24
*e**(2*c + 2*d*x)*a**3 + 12*e**(2*c + 2*d*x)*a**2*b + 12*e**(2*c + 2*d*x)*
a*b**2 + 8*a**3 - 8*a*b**2)/(6*a**3*d*(e**(6*c + 6*d*x)*a - e**(6*c + 6...
```

3.40 $\int \frac{\operatorname{csch}^5(c+dx)}{a+b\sinh^2(c+dx)}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 130

$$\int \frac{\operatorname{csch}^5(c+dx)}{a+b\sinh^2(c+dx)}\,dx = -\frac{b^{5/2}\arctan\left(\frac{\sqrt{b}\cosh(c+dx)}{\sqrt{a-b}}\right)}{a^3\sqrt{a-b}d} - \frac{(3a^2+4ab+8b^2)\operatorname{arctanh}(\cosh(c+dx))}{8a^3d} + \frac{(3a+4b)\coth(c+dx)\operatorname{csch}(c+dx)}{8a^2d} - \frac{\coth(c+dx)\operatorname{csch}^3(c+dx)}{4ad}$$

output
```
-b^(5/2)*arctan(b^(1/2)*cosh(d*x+c)/(a-b)^(1/2))/a^3/(a-b)^(1/2)/d-1/8*(3*
a^2+4*a*b+8*b^2)*arctanh(cosh(d*x+c))/a^3/d+1/8*(3*a+4*b)*coth(d*x+c)*csch
(d*x+c)/a^2/d-1/4*coth(d*x+c)*csch(d*x+c)^3/a/d
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 8.69 (sec) , antiderivative size = 649, normalized size of antiderivative = 4.99

$$\int \frac{\operatorname{csch}^5(c+dx)}{a+b\sinh^2(c+dx)}\,dx = -\frac{b^{5/2}\arctan\left(\frac{\operatorname{sech}\left(\frac{1}{2}(c+dx)\right)\left(\sqrt{b}\cosh\left(\frac{1}{2}(c+dx)\right)-i\sqrt{a}\sinh\left(\frac{1}{2}(c+dx)\right)\right)}{\sqrt{a-b}}\right)(2a-b+b\cosh(2(c+dx)))\operatorname{csch}^2(c+dx)}{2a^3\sqrt{a-b}d\left(b+a\operatorname{csch}^2(c+dx)\right)}$$
$$-\frac{b^{5/2}\arctan\left(\frac{\operatorname{sech}\left(\frac{1}{2}(c+dx)\right)\left(\sqrt{b}\cosh\left(\frac{1}{2}(c+dx)\right)+i\sqrt{a}\sinh\left(\frac{1}{2}(c+dx)\right)\right)}{\sqrt{a-b}}\right)(2a-b+b\cosh(2(c+dx)))\operatorname{csch}^2(c+dx)}{2a^3\sqrt{a-b}d\left(b+a\operatorname{csch}^2(c+dx)\right)}$$
$$+\frac{(3a+4b)(2a-b+b\cosh(2(c+dx)))\operatorname{csch}^2\left(\frac{1}{2}(c+dx)\right)\operatorname{csch}^2(c+dx)}{64a^2d\left(b+a\operatorname{csch}^2(c+dx)\right)}$$
$$-\frac{(2a-b+b\cosh(2(c+dx)))\operatorname{csch}^4\left(\frac{1}{2}(c+dx)\right)\operatorname{csch}^2(c+dx)}{128ad\left(b+a\operatorname{csch}^2(c+dx)\right)}$$
$$+\frac{(-3a^2-4ab-8b^2)(2a-b+b\cosh(2(c+dx)))\operatorname{csch}^2(c+dx)\log\left(\cosh\left(\frac{1}{2}(c+dx)\right)\right)}{16a^3d\left(b+a\operatorname{csch}^2(c+dx)\right)}$$
$$+\frac{(3a^2+4ab+8b^2)(2a-b+b\cosh(2(c+dx)))\operatorname{csch}^2(c+dx)\log\left(\sinh\left(\frac{1}{2}(c+dx)\right)\right)}{16a^3d\left(b+a\operatorname{csch}^2(c+dx)\right)}$$
$$+\frac{(3a+4b)(2a-b+b\cosh(2(c+dx)))\operatorname{csch}^2(c+dx)\operatorname{sech}^2\left(\frac{1}{2}(c+dx)\right)}{64a^2d\left(b+a\operatorname{csch}^2(c+dx)\right)}$$
$$+\frac{(2a-b+b\cosh(2(c+dx)))\operatorname{csch}^2(c+dx)\operatorname{sech}^4\left(\frac{1}{2}(c+dx)\right)}{128ad\left(b+a\operatorname{csch}^2(c+dx)\right)}$$

input
```
Integrate[Csch[c + d*x]^5/(a + b*Sinh[c + d*x]^2),x]
```

output
```
-1/2*(b^(5/2)*ArcTan[(Sech[(c + d*x)/2]*(Sqrt[b]*Cosh[(c + d*x)/2] - I*Sqr
t[a]*Sinh[(c + d*x)/2]))/Sqrt[a - b]]*(2*a - b + b*Cosh[2*(c + d*x)])*Csch
[c + d*x]^2)/(a^3*Sqrt[a - b]*d*(b + a*Csch[c + d*x]^2)) - (b^(5/2)*ArcTan
[(Sech[(c + d*x)/2]*(Sqrt[b]*Cosh[(c + d*x)/2] + I*Sqrt[a]*Sinh[(c + d*x)/
2]))/Sqrt[a - b]]*(2*a - b + b*Cosh[2*(c + d*x)])*Csch[c + d*x]^2)/(2*a^3*
Sqrt[a - b]*d*(b + a*Csch[c + d*x]^2)) + ((3*a + 4*b)*(2*a - b + b*Cosh[2*
(c + d*x)])*Csch[(c + d*x)/2]^2*Csch[c + d*x]^2)/(64*a^2*d*(b + a*Csch[c +
 d*x]^2)) - ((2*a - b + b*Cosh[2*(c + d*x)])*Csch[(c + d*x)/2]^4*Csch[c +
d*x]^2)/(128*a*d*(b + a*Csch[c + d*x]^2)) + ((-3*a^2 - 4*a*b - 8*b^2)*(2*a
 - b + b*Cosh[2*(c + d*x)])*Csch[c + d*x]^2*Log[Cosh[(c + d*x)/2]])/(16*a^
3*d*(b + a*Csch[c + d*x]^2)) + ((3*a^2 + 4*a*b + 8*b^2)*(2*a - b + b*Cosh[
2*(c + d*x)])*Csch[c + d*x]^2*Log[Sinh[(c + d*x)/2]])/(16*a^3*d*(b + a*Csc
h[c + d*x]^2)) + ((3*a + 4*b)*(2*a - b + b*Cosh[2*(c + d*x)])*Csch[c + d*x
]^2*Sech[(c + d*x)/2]^2)/(64*a^2*d*(b + a*Csch[c + d*x]^2)) + ((2*a - b +
b*Cosh[2*(c + d*x)])*Csch[c + d*x]^2*Sech[(c + d*x)/2]^4)/(128*a*d*(b + a*
Csch[c + d*x]^2))
```

Rubi [A] (verified)

Time = 0.38 (sec) , antiderivative size = 150, normalized size of antiderivative = 1.15, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.348$, Rules used = {3042, 26, 3665, 316, 402, 397, 218, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\text{csch}^5(c+dx)}{a+b\sinh^2(c+dx)}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{i}{\sin(ic+idx)^5\left(a-b\sin(ic+idx)^2\right)}dx$$

$$\Big\downarrow \text{26}$$

$$i\int \frac{1}{\sin(ic+idx)^5\left(a-b\sin(ic+idx)^2\right)}dx$$

$$\Big\downarrow \text{3665}$$

$$-\frac{\int \frac{1}{\left(1-\cosh^2(c+dx)\right)^3\left(b\cosh^2(c+dx)+a-b\right)}d\cosh(c+dx)}{d}$$

$\Big\downarrow$ 316

$$-\frac{\frac{\int \frac{3b\cosh^2(c+dx)+3a+b}{\left(1-\cosh^2(c+dx)\right)^2\left(b\cosh^2(c+dx)+a-b\right)}d\cosh(c+dx)}{4a}+\frac{\cosh(c+dx)}{4a\left(1-\cosh^2(c+dx)\right)^2}}{d}$$

$\Big\downarrow$ 402

$$-\frac{\frac{\frac{\int \frac{3a^2+ba+4b^2+b(3a+4b)\cosh^2(c+dx)}{\left(1-\cosh^2(c+dx)\right)\left(b\cosh^2(c+dx)+a-b\right)}d\cosh(c+dx)}{2a}+\frac{(3a+4b)\cosh(c+dx)}{2a\left(1-\cosh^2(c+dx)\right)}}{4a}+\frac{\cosh(c+dx)}{4a\left(1-\cosh^2(c+dx)\right)^2}}{d}$$

$\Big\downarrow$ 397

$$-\frac{\frac{\frac{\frac{\left(3a^2+4ab+8b^2\right)\int \frac{1}{1-\cosh^2(c+dx)}d\cosh(c+dx)}{a}+\frac{8b^3\int \frac{1}{b\cosh^2(c+dx)+a-b}d\cosh(c+dx)}{a}}{2a}+\frac{(3a+4b)\cosh(c+dx)}{2a\left(1-\cosh^2(c+dx)\right)}}{4a}+\frac{\cosh(c+dx)}{4a\left(1-\cosh^2(c+dx)\right)^2}}{d}$$

$\Big\downarrow$ 218

$$-\frac{\frac{\frac{\frac{\left(3a^2+4ab+8b^2\right)\int \frac{1}{1-\cosh^2(c+dx)}d\cosh(c+dx)}{a}+\frac{8b^{5/2}\arctan\left(\frac{\sqrt{b}\cosh(c+dx)}{\sqrt{a-b}}\right)}{a\sqrt{a-b}}}{2a}+\frac{(3a+4b)\cosh(c+dx)}{2a\left(1-\cosh^2(c+dx)\right)}}{4a}+\frac{\cosh(c+dx)}{4a\left(1-\cosh^2(c+dx)\right)^2}}{d}$$

$\Big\downarrow$ 219

$$-\frac{\frac{\frac{\frac{\left(3a^2+4ab+8b^2\right)\text{arctanh}(\cosh(c+dx))}{a}+\frac{8b^{5/2}\arctan\left(\frac{\sqrt{b}\cosh(c+dx)}{\sqrt{a-b}}\right)}{a\sqrt{a-b}}}{2a}+\frac{(3a+4b)\cosh(c+dx)}{2a\left(1-\cosh^2(c+dx)\right)}}{4a}+\frac{\cosh(c+dx)}{4a\left(1-\cosh^2(c+dx)\right)^2}}{d}$$

input
```
Int[Csch[c + d*x]^5/(a + b*Sinh[c + d*x]^2),x]
```

output
```
-((Cosh[c + d*x]/(4*a*(1 - Cosh[c + d*x]^2)^2) + (((8*b^(5/2)*ArcTan[(Sqrt
[b]*Cosh[c + d*x])/Sqrt[a - b]])/(a*Sqrt[a - b]) + ((3*a^2 + 4*a*b + 8*b^2
)*ArcTanh[Cosh[c + d*x]])/a)/(2*a) + ((3*a + 4*b)*Cosh[c + d*x])/(2*a*(1 -
 Cosh[c + d*x]^2)))/(4*a))/d)
```

Defintions of rubi rules used

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 316
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_), x_Symbol] :> Sim
p[(-b)*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^(q + 1)/(2*a*(p + 1)*(b*c - a*d))
), x] + Simp[1/(2*a*(p + 1)*(b*c - a*d))   Int[(a + b*x^2)^(p + 1)*(c + d*x
^2)^q*Simp[b*c + 2*(p + 1)*(b*c - a*d) + d*b*(2*(p + q + 2) + 1)*x^2, x], x
], x] /; FreeQ[{a, b, c, d, q}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] &&  !
( !IntegerQ[p] && IntegerQ[q] && LtQ[q, -1]) && IntBinomialQ[a, b, c, d, 2,
 p, q, x]
```

rule 397
```
Int[((e_) + (f_.)*(x_)^2)/(((a_) + (b_.)*(x_)^2)*((c_) + (d_.)*(x_)^2)), x_
Symbol] :> Simp[(b*e - a*f)/(b*c - a*d)   Int[1/(a + b*x^2), x], x] - Simp[
(d*e - c*f)/(b*c - a*d)   Int[1/(c + d*x^2), x], x] /; FreeQ[{a, b, c, d, e
, f}, x]
```

rule 402
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_.)*((e_) + (f_.)*(x
_)^2), x_Symbol] :> Simp[(-(b*e - a*f))*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^
(q + 1)/(a*2*(b*c - a*d)*(p + 1))), x] + Simp[1/(a*2*(b*c - a*d)*(p + 1))
 Int[(a + b*x^2)^(p + 1)*(c + d*x^2)^q*Simp[c*(b*e - a*f) + e*2*(b*c - a*d)
*(p + 1) + d*(b*e - a*f)*(2*(p + q + 2) + 1)*x^2, x], x], x] /; FreeQ[{a, b
, c, d, e, f, q}, x] && LtQ[p, -1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3665
```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^
(p_.), x_Symbol] :> With[{ff = FreeFactors[Cos[e + f*x], x]}, Simp[-ff/f
Subst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a + b - b*ff^2*x^2)^p, x], x, Cos[e +
 f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m - 1)/2]
```

Maple [A] (verified)

Time = 1.17 (sec) , antiderivative size = 156, normalized size of antiderivative = 1.20

method	result
derivativedivides	$\frac{\frac{\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^2 a-4a-4b\right)^2}{64a^3}-\frac{1}{64a\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^4}-\frac{-4a-4b}{32a^2\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^2}+\frac{\left(6a^2+8ab+16b^2\right)\ln\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{16a^3}-\frac{b^3\arctan\left(\frac{2\,\mathrm{t}}{}\right.}{}}{d}$
default	$\frac{\frac{\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^2 a-4a-4b\right)^2}{64a^3}-\frac{1}{64a\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^4}-\frac{-4a-4b}{32a^2\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^2}+\frac{\left(6a^2+8ab+16b^2\right)\ln\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{16a^3}-\frac{b^3\arctan\left(\frac{2\,\mathrm{t}}{}\right.}{}}{d}$
risch	$\frac{e^{dx+c}\left(3e^{6dx+6c}a+4e^{6dx+6c}b-11e^{4dx+4c}a-4be^{4dx+4c}-11e^{2dx+2c}a-4e^{2dx+2c}b+3a+4b\right)}{4da^2\left(e^{2dx+2c}-1\right)^4}-\frac{3\ln\left(e^{dx+c}+1\right)}{8ad}-\mathrm{ln}$

input
```
int(csch(d*x+c)^5/(a+b*sinh(d*x+c)^2),x,method=_RETURNVERBOSE)
```

output
```
1/d*(1/64*(tanh(1/2*d*x+1/2*c)^2*a-4*a-4*b)^2/a^3-1/64/a/tanh(1/2*d*x+1/2*
c)^4-1/32*(-4*a-4*b)/a^2/tanh(1/2*d*x+1/2*c)^2+1/16/a^3*(6*a^2+8*a*b+16*b^
2)*ln(tanh(1/2*d*x+1/2*c))-1/a^3*b^3/(a*b-b^2)^(1/2)*arctan(1/4*(2*tanh(1/
2*d*x+1/2*c)^2*a-2*a+4*b)/(a*b-b^2)^(1/2)))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2976 vs. $2(116) = 232$.

Time = 0.17 (sec) , antiderivative size = 5809, normalized size of antiderivative = 44.68

$$\int \frac{\operatorname{csch}^5(c+dx)}{a+b\sinh^2(c+dx)}\,dx = \text{Too large to display}$$

input
```
integrate(csch(d*x+c)^5/(a+b*sinh(d*x+c)^2),x, algorithm="fricas")
```

output
```
Too large to include
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\operatorname{csch}^5(c+dx)}{a+b\sinh^2(c+dx)}\,dx = \text{Timed out}$$

input
```
integrate(csch(d*x+c)**5/(a+b*sinh(d*x+c)**2),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{\operatorname{csch}^5(c+dx)}{a+b\sinh^2(c+dx)}\,dx = \int \frac{\operatorname{csch}\left(dx+c\right)^5}{b\sinh\left(dx+c\right)^2+a}\,dx$$

input
```
integrate(csch(d*x+c)^5/(a+b*sinh(d*x+c)^2),x, algorithm="maxima")
```

output
```
1/4*((3*a*e^(7*c) + 4*b*e^(7*c))*e^(7*d*x) - (11*a*e^(5*c) + 4*b*e^(5*c))*
e^(5*d*x) - (11*a*e^(3*c) + 4*b*e^(3*c))*e^(3*d*x) + (3*a*e^c + 4*b*e^c)*e
^(d*x))/(a^2*d*e^(8*d*x + 8*c) - 4*a^2*d*e^(6*d*x + 6*c) + 6*a^2*d*e^(4*d*
x + 4*c) - 4*a^2*d*e^(2*d*x + 2*c) + a^2*d) - 1/8*(3*a^2 + 4*a*b + 8*b^2)*
log((e^(d*x + c) + 1)*e^(-c))/(a^3*d) + 1/8*(3*a^2 + 4*a*b + 8*b^2)*log((e
^(d*x + c) - 1)*e^(-c))/(a^3*d) - 32*integrate(1/16*(b^3*e^(3*d*x + 3*c) -
 b^3*e^(d*x + c))/(a^3*b*e^(4*d*x + 4*c) + a^3*b + 2*(2*a^4*e^(2*c) - a^3*
b*e^(2*c))*e^(2*d*x)), x)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{\operatorname{csch}^5(c+dx)}{a+b\sinh^2(c+dx)}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(csch(d*x+c)^5/(a+b*sinh(d*x+c)^2),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Limit: Max order reached or unable
to make series expansion Error: Bad Argument Value
```

Mupad [B] (verification not implemented)

Time = 7.18 (sec) , antiderivative size = 1639, normalized size of antiderivative = 12.61

$$\int \frac{\operatorname{csch}^5(c+dx)}{a+b\sinh^2(c+dx)}\,dx = \text{Too large to display}$$

input
```
int(1/(sinh(c + d*x)^5*(a + b*sinh(c + d*x)^2)),x)
```

output
```
(exp(c + d*x)*(4*a*b + 3*a^2))/(4*a^3*d*(exp(2*c + 2*d*x) - 1)) - (atan((e
xp(d*x)*exp(c)*(243*a^12*(-a^6*d^2)^(1/2) + 18432*b^12*(-a^6*d^2)^(1/2) +
6912*a^2*b^10*(-a^6*d^2)^(1/2) - 30720*a^3*b^9*(-a^6*d^2)^(1/2) - 26880*a^
4*b^8*(-a^6*d^2)^(1/2) - 24192*a^5*b^7*(-a^6*d^2)^(1/2) + 5024*a^6*b^6*(-a
^6*d^2)^(1/2) + 13408*a^7*b^5*(-a^6*d^2)^(1/2) + 17160*a^8*b^4*(-a^6*d^2)^
(1/2) + 9540*a^9*b^3*(-a^6*d^2)^(1/2) + 4563*a^10*b^2*(-a^6*d^2)^(1/2) + 9
216*a*b^11*(-a^6*d^2)^(1/2) + 1134*a^11*b*(-a^6*d^2)^(1/2)))/(81*a^13*d*(6
4*a*b^3 + 24*a^3*b + 9*a^4 + 64*b^4 + 64*a^2*b^2)^(1/2) + 2304*a^3*b^10*d*
(64*a*b^3 + 24*a^3*b + 9*a^4 + 64*b^4 + 64*a^2*b^2)^(1/2) - 3840*a^6*b^7*d
*(64*a*b^3 + 24*a^3*b + 9*a^4 + 64*b^4 + 64*a^2*b^2)^(1/2) - 1440*a^7*b^6*
d*(64*a*b^3 + 24*a^3*b + 9*a^4 + 64*b^4 + 64*a^2*b^2)^(1/2) - 864*a^8*b^5*
d*(64*a*b^3 + 24*a^3*b + 9*a^4 + 64*b^4 + 64*a^2*b^2)^(1/2) + 1600*a^9*b^4
*d*(64*a*b^3 + 24*a^3*b + 9*a^4 + 64*b^4 + 64*a^2*b^2)^(1/2) + 1200*a^10*b
^3*d*(64*a*b^3 + 24*a^3*b + 9*a^4 + 64*b^4 + 64*a^2*b^2)^(1/2) + 945*a^11*
b^2*d*(64*a*b^3 + 24*a^3*b + 9*a^4 + 64*b^4 + 64*a^2*b^2)^(1/2) + 270*a^12
*b*d*(64*a*b^3 + 24*a^3*b + 9*a^4 + 64*b^4 + 64*a^2*b^2)^(1/2)))*(64*a*b^3
 + 24*a^3*b + 9*a^4 + 64*b^4 + 64*a^2*b^2)^(1/2))/(4*(-a^6*d^2)^(1/2)) - (
6*exp(c + d*x))/(a*d*(3*exp(2*c + 2*d*x) - 3*exp(4*c + 4*d*x) + exp(6*c +
6*d*x) - 1)) - (4*exp(c + d*x))/(a*d*(6*exp(4*c + 4*d*x) - 4*exp(2*c + 2*d
*x) - 4*exp(6*c + 6*d*x) + exp(8*c + 8*d*x) + 1)) - ((2*atan((b^3*exp(d...
```

Reduce [B] (verification not implemented)

Time = 0.35 (sec) , antiderivative size = 4116, normalized size of antiderivative = 31.66

$$\int \frac{\operatorname{csch}^5(c+dx)}{a+b\sinh^2(c+dx)}\,dx = \text{Too large to display}$$

input
```
int(csch(d*x+c)^5/(a+b*sinh(d*x+c)^2),x)
```

output

```
( - 8*e**(8*c + 8*d*x)*sqrt(b)*sqrt(a)*sqrt(a - b)*sqrt(2*sqrt(a)*sqrt(a -
 b) + 2*a - b)*atan((e**(c + d*x)*b)/(sqrt(b)*sqrt(2*sqrt(a)*sqrt(a - b) +
 2*a - b)))*b + 32*e**(6*c + 6*d*x)*sqrt(b)*sqrt(a)*sqrt(a - b)*sqrt(2*sqr
t(a)*sqrt(a - b) + 2*a - b)*atan((e**(c + d*x)*b)/(sqrt(b)*sqrt(2*sqrt(a)*
sqrt(a - b) + 2*a - b)))*b - 48*e**(4*c + 4*d*x)*sqrt(b)*sqrt(a)*sqrt(a -
b)*sqrt(2*sqrt(a)*sqrt(a - b) + 2*a - b)*atan((e**(c + d*x)*b)/(sqrt(b)*sq
rt(2*sqrt(a)*sqrt(a - b) + 2*a - b)))*b + 32*e**(2*c + 2*d*x)*sqrt(b)*sqrt
(a)*sqrt(a - b)*sqrt(2*sqrt(a)*sqrt(a - b) + 2*a - b)*atan((e**(c + d*x)*b
)/(sqrt(b)*sqrt(2*sqrt(a)*sqrt(a - b) + 2*a - b)))*b - 8*sqrt(b)*sqrt(a)*s
qrt(a - b)*sqrt(2*sqrt(a)*sqrt(a - b) + 2*a - b)*atan((e**(c + d*x)*b)/(sq
rt(b)*sqrt(2*sqrt(a)*sqrt(a - b) + 2*a - b)))*b + 8*e**(8*c + 8*d*x)*sqrt(
b)*sqrt(2*sqrt(a)*sqrt(a - b) + 2*a - b)*atan((e**(c + d*x)*b)/(sqrt(b)*sq
rt(2*sqrt(a)*sqrt(a - b) + 2*a - b)))*a*b - 8*e**(8*c + 8*d*x)*sqrt(b)*sqr
t(2*sqrt(a)*sqrt(a - b) + 2*a - b)*atan((e**(c + d*x)*b)/(sqrt(b)*sqrt(2*s
qrt(a)*sqrt(a - b) + 2*a - b)))*b**2 - 32*e**(6*c + 6*d*x)*sqrt(b)*sqrt(2*
sqrt(a)*sqrt(a - b) + 2*a - b)*atan((e**(c + d*x)*b)/(sqrt(b)*sqrt(2*sqrt(
a)*sqrt(a - b) + 2*a - b)))*a*b + 32*e**(6*c + 6*d*x)*sqrt(b)*sqrt(2*sqrt(
a)*sqrt(a - b) + 2*a - b)*atan((e**(c + d*x)*b)/(sqrt(b)*sqrt(2*sqrt(a)*sq
rt(a - b) + 2*a - b)))*b**2 + 48*e**(4*c + 4*d*x)*sqrt(b)*sqrt(2*sqrt(a)*s
qrt(a - b) + 2*a - b)*atan((e**(c + d*x)*b)/(sqrt(b)*sqrt(2*sqrt(a)*sqr...
```

3.41 $\int \frac{\text{csch}^6(c+dx)}{a+b\sinh^2(c+dx)}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 110

$$\int \frac{\text{csch}^6(c+dx)}{a+b\sinh^2(c+dx)}\,dx=-\frac{b^3\text{arctanh}\left(\frac{\sqrt{a-b}\tanh(c+dx)}{\sqrt{a}}\right)}{a^{7/2}\sqrt{a-b}d}-\frac{(a^2+ab+b^2)\coth(c+dx)}{a^3d}+\frac{(2a+b)\coth^3(c+dx)}{3a^2d}-\frac{\coth^5(c+dx)}{5ad}$$

output
```
-b^3*arctanh((a-b)^(1/2)*tanh(d*x+c)/a^(1/2))/a^(7/2)/(a-b)^(1/2)/d-(a^2+a
*b+b^2)*coth(d*x+c)/a^3/d+1/3*(2*a+b)*coth(d*x+c)^3/a^2/d-1/5*coth(d*x+c)^
5/a/d
```

Mathematica [A] (verified)

Time = 3.67 (sec) , antiderivative size = 155, normalized size of antiderivative = 1.41

$$\int \frac{\text{csch}^6(c+dx)}{a+b\sinh^2(c+dx)}\,dx=-\frac{(2a-b+b\cosh(2(c+dx)))\text{csch}^2(c+dx)\left(15b^3\text{arctanh}\left(\frac{\sqrt{a-b}\tanh(c+dx)}{\sqrt{a}}\right)+\sqrt{a}\sqrt{a-b}\coth(c+dx)\left(8\right.\right.}{30a^{7/2}\sqrt{a-b}d\left(b+a\text{csch}^2(c+dx)\right.}$$

input
```
Integrate[Csch[c + d*x]^6/(a + b*Sinh[c + d*x]^2),x]
```

output
```
-1/30*((2*a - b + b*Cosh[2*(c + d*x)])*Csch[c + d*x]^2*(15*b^3*ArcTanh[(Sq
rt[a - b]*Tanh[c + d*x])/Sqrt[a]] + Sqrt[a]*Sqrt[a - b]*Coth[c + d*x]*(8*a
^2 + 10*a*b + 15*b^2 - a*(4*a + 5*b)*Csch[c + d*x]^2 + 3*a^2*Csch[c + d*x]
^4)))/(a^(7/2)*Sqrt[a - b]*d*(b + a*Csch[c + d*x]^2))
```

Rubi [A] (verified)

Time = 0.35 (sec) , antiderivative size = 102, normalized size of antiderivative = 0.93, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.217$, Rules used = {3042, 25, 3666, 364, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\operatorname{csch}^6(c+dx)}{a+b\sinh^2(c+dx)}dx$$

$$\Big\downarrow \text{3042}$$

$$\int -\frac{1}{\sin(ic+idx)^6\left(a-b\sin(ic+idx)^2\right)}dx$$

$$\Big\downarrow \text{25}$$

$$-\int \frac{1}{\sin(ic+idx)^6\left(a-b\sin(ic+idx)^2\right)}dx$$

$$\Big\downarrow \text{3666}$$

$$\frac{\int \frac{\coth^6(c+dx)\left(1-\tanh^2(c+dx)\right)^3}{a-(a-b)\tanh^2(c+dx)}d\tanh(c+dx)}{d}$$

$$\Big\downarrow \text{364}$$

$$\frac{\int \left(\frac{\coth^6(c+dx)}{a}+\frac{(-2a-b)\coth^4(c+dx)}{a^2}+\frac{\left(a^2+ba+b^2\right)\coth^2(c+dx)}{a^3}+\frac{b^3}{a^3\left((a-b)\tanh^2(c+dx)-a\right)}\right)d\tanh(c+dx)}{d}$$

$$\Big\downarrow \text{2009}$$

$$\frac{-\frac{b^3\text{arctanh}\left(\frac{\sqrt{a-b}\tanh(c+dx)}{\sqrt{a}}\right)}{a^{7/2}\sqrt{a-b}}+\frac{(2a+b)\coth^3(c+dx)}{3a^2}-\frac{(a^2+ab+b^2)\coth(c+dx)}{a^3}-\frac{\coth^5(c+dx)}{5a}}{d}$$

input
```
Int[Csch[c + d*x]^6/(a + b*Sinh[c + d*x]^2),x]
```

output
```
(-((b^3*ArcTanh[(Sqrt[a - b]*Tanh[c + d*x])/Sqrt[a]])/(a^(7/2)*Sqrt[a - b]
)) - ((a^2 + a*b + b^2)*Coth[c + d*x])/a^3 + ((2*a + b)*Coth[c + d*x]^3)/(
3*a^2) - Coth[c + d*x]^5/(5*a))/d
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 364
```
Int[(((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_))/((c_) + (d_.)*(x_)^2),
x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*((a + b*x^2)^p/(c + d*x^2)), x], x
] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b*c - a*d, 0] && IGtQ[p, 0] && (In
tegerQ[m] || IGtQ[2*(m + 1), 0] ||  !RationalQ[m])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3666
```
Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(
p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Simp[ff^(m + 1
)/f   Subst[Int[x^m*((a + (a + b)*ff^2*x^2)^p/(1 + ff^2*x^2)^(m/2 + p + 1))
, x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f}, x] && IntegerQ[m/2] &
& IntegerQ[p]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 324 vs. $2(98) = 196$.

Time = 1.46 (sec) , antiderivative size = 325, normalized size of antiderivative = 2.95

method	result
risch	$-\frac{2\left(15\,e^{8dx+8c}b^{2}-30\,e^{6dx+6c}ab-60\,e^{6dx+6c}b^{2}+80\,e^{4dx+4c}a^{2}+70\,e^{4dx+4c}ab+90\,e^{4dx+4c}b^{2}-40\,e^{2dx+2c}a^{2}-50\,e^{2dx+}\right.}{15d\,a^{3}\left(e^{2dx+2c}-1\right)^{5}}$
derivativedivides	2 $-\frac{\frac{-\frac{\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^{5}a^{2}}{5}-\frac{5\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^{3}a^{2}}{3}-\frac{4\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^{3}ab}{3}+10\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)a^{2}+12\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)ab+16b^{2}\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)}{32a^{3}}+-}{}$
default	2 $-\frac{\frac{-\frac{\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^{5}a^{2}}{5}-\frac{5\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^{3}a^{2}}{3}-\frac{4\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^{3}ab}{3}+10\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)a^{2}+12\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)ab+16b^{2}\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)}{32a^{3}}+-}{}$

input
```
int(csch(d*x+c)^6/(a+b*sinh(d*x+c)^2),x,method=_RETURNVERBOSE)
```

output
```
-2/15*(15*exp(8*d*x+8*c)*b^2-30*exp(6*d*x+6*c)*a*b-60*exp(6*d*x+6*c)*b^2+8
0*exp(4*d*x+4*c)*a^2+70*exp(4*d*x+4*c)*a*b+90*exp(4*d*x+4*c)*b^2-40*exp(2*
d*x+2*c)*a^2-50*exp(2*d*x+2*c)*b*a-60*b^2*exp(2*d*x+2*c)+8*a^2+10*a*b+15*b
^2)/d/a^3/(exp(2*d*x+2*c)-1)^5+1/2/(a^2-a*b)^(1/2)*b^3/d/a^3*ln(exp(2*d*x+
2*c)+(2*a*(a^2-a*b)^(1/2)-b*(a^2-a*b)^(1/2)+2*a^2-2*a*b)/b/(a^2-a*b)^(1/2)
)-1/2/(a^2-a*b)^(1/2)*b^3/d/a^3*ln(exp(2*d*x+2*c)+(2*a*(a^2-a*b)^(1/2)-b*(
a^2-a*b)^(1/2)-2*a^2+2*a*b)/b/(a^2-a*b)^(1/2))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2142 vs. $2(98) = 196$.

Time = 0.13 (sec) , antiderivative size = 4540, normalized size of antiderivative = 41.27

$$\int \frac{\operatorname{csch}^6(c+dx)}{a+b\sinh^2(c+dx)}\,dx = \text{Too large to display}$$

input
```
integrate(csch(d*x+c)^6/(a+b*sinh(d*x+c)^2),x, algorithm="fricas")
```

output
```
Too large to include
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\operatorname{csch}^6(c+dx)}{a+b\sinh^2(c+dx)}\,dx = \text{Timed out}$$

input
```
integrate(csch(d*x+c)**6/(a+b*sinh(d*x+c)**2),x)
```

output
```
Timed out
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{\operatorname{csch}^6(c+dx)}{a+b\sinh^2(c+dx)}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate(csch(d*x+c)^6/(a+b*sinh(d*x+c)^2),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(b-a>0)', see `assume?` for more
details)Is
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 213 vs. $2(98) = 196$.

Time = 0.23 (sec) , antiderivative size = 213, normalized size of antiderivative = 1.94

$$\int \frac{\operatorname{csch}^6(c+dx)}{a+b\sinh^2(c+dx)}\,dx = -\frac{\frac{15\,b^3\arctan\left(\frac{be^{(2\,dx+2\,c)}+2\,a-b}{2\,\sqrt{-a^2+ab}}\right)}{\sqrt{-a^2+ab}a^3}+\frac{2\,(15\,b^2e^{(8\,dx+8\,c)}-30\,abe^{(6\,dx+6\,c)}-60\,b^2e^{(6\,dx+6\,c)}+80\,a^2e^{(4\,dx+4\,c)}+70\,abe^{(4\,dx+4\,c)}+90\,b^2e^{(4\,dx+4}}{a^3\left(e^{(2\,dx+2\,c)}-1\right)^5}}{15\,d}$$

input
```
integrate(csch(d*x+c)^6/(a+b*sinh(d*x+c)^2),x, algorithm="giac")
```

output
```
-1/15*(15*b^3*arctan(1/2*(b*e^(2*d*x + 2*c) + 2*a - b)/sqrt(-a^2 + a*b))/(
sqrt(-a^2 + a*b)*a^3) + 2*(15*b^2*e^(8*d*x + 8*c) - 30*a*b*e^(6*d*x + 6*c)
 - 60*b^2*e^(6*d*x + 6*c) + 80*a^2*e^(4*d*x + 4*c) + 70*a*b*e^(4*d*x + 4*c
) + 90*b^2*e^(4*d*x + 4*c) - 40*a^2*e^(2*d*x + 2*c) - 50*a*b*e^(2*d*x + 2*
c) - 60*b^2*e^(2*d*x + 2*c) + 8*a^2 + 10*a*b + 15*b^2)/(a^3*(e^(2*d*x + 2*
c) - 1)^5))/d
```

Mupad [B] (verification not implemented)

Time = 2.48 (sec) , antiderivative size = 479, normalized size of antiderivative = 4.35

$$\int \frac{\operatorname{csch}^6(c+dx)}{a+b\sinh^2(c+dx)}\,dx$$
$$= \frac{4b}{a^2 d\left(e^{4c+4dx}-2e^{2c+2dx}+1\right)}$$
$$-\frac{32}{5ad\left(5e^{2c+2dx}-10e^{4c+4dx}+10e^{6c+6dx}-5e^{8c+8dx}+e^{10c+10dx}-1\right)}$$
$$-\frac{2b^2}{a^3 d\left(e^{2c+2dx}-1\right)}-\frac{8(4a-b)}{3a^2 d\left(3e^{2c+2dx}-3e^{4c+4dx}+e^{6c+6dx}-1\right)}$$
$$-\frac{16}{ad\left(6e^{4c+4dx}-4e^{2c+2dx}-4e^{6c+6dx}+e^{8c+8dx}+1\right)}$$
$$+\frac{b^3\ln\left(\frac{4b^4\left(2ab-b^2+8a^2e^{2c+2dx}+b^2e^{2c+2dx}-8abe^{2c+2dx}\right)}{a^7(a-b)}-\frac{8b^4\left(b+4ae^{2c+2dx}-2be^{2c+2dx}\right)}{a^{13/2}\sqrt{a-b}}\right)}{2a^{7/2}d\sqrt{a-b}}$$
$$-\frac{b^3\ln\left(\frac{4b^4\left(2ab-b^2+8a^2e^{2c+2dx}+b^2e^{2c+2dx}-8abe^{2c+2dx}\right)}{a^7(a-b)}+\frac{8b^4\left(b+4ae^{2c+2dx}-2be^{2c+2dx}\right)}{a^{13/2}\sqrt{a-b}}\right)}{2a^{7/2}d\sqrt{a-b}}$$

input
```
int(1/(sinh(c + d*x)^6*(a + b*sinh(c + d*x)^2)),x)
```

output
```
(4*b)/(a^2*d*(exp(4*c + 4*d*x) - 2*exp(2*c + 2*d*x) + 1)) - 32/(5*a*d*(5*e
xp(2*c + 2*d*x) - 10*exp(4*c + 4*d*x) + 10*exp(6*c + 6*d*x) - 5*exp(8*c +
8*d*x) + exp(10*c + 10*d*x) - 1)) - (2*b^2)/(a^3*d*(exp(2*c + 2*d*x) - 1))
 - (8*(4*a - b))/(3*a^2*d*(3*exp(2*c + 2*d*x) - 3*exp(4*c + 4*d*x) + exp(6
*c + 6*d*x) - 1)) - 16/(a*d*(6*exp(4*c + 4*d*x) - 4*exp(2*c + 2*d*x) - 4*e
xp(6*c + 6*d*x) + exp(8*c + 8*d*x) + 1)) + (b^3*log((4*b^4*(2*a*b - b^2 +
8*a^2*exp(2*c + 2*d*x) + b^2*exp(2*c + 2*d*x) - 8*a*b*exp(2*c + 2*d*x)))/(
a^7*(a - b)) - (8*b^4*(b + 4*a*exp(2*c + 2*d*x) - 2*b*exp(2*c + 2*d*x)))/(
a^(13/2)*(a - b)^(1/2))))/(2*a^(7/2)*d*(a - b)^(1/2)) - (b^3*log((4*b^4*(2
*a*b - b^2 + 8*a^2*exp(2*c + 2*d*x) + b^2*exp(2*c + 2*d*x) - 8*a*b*exp(2*c
 + 2*d*x)))/(a^7*(a - b)) + (8*b^4*(b + 4*a*exp(2*c + 2*d*x) - 2*b*exp(2*c
 + 2*d*x)))/(a^(13/2)*(a - b)^(1/2))))/(2*a^(7/2)*d*(a - b)^(1/2))
```

Reduce [B] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 1278, normalized size of antiderivative = 11.62

$$\int \frac{\operatorname{csch}^6(c+dx)}{a+b\sinh^2(c+dx)}\,dx = \text{Too large to display}$$

input
```
int(csch(d*x+c)^6/(a+b*sinh(d*x+c)^2),x)
```

output
```
( - 15*e**(10*c + 10*d*x)*sqrt(a)*sqrt(a - b)*log( - sqrt(2*sqrt(a)*sqrt(a
 - b) - 2*a + b) + e**(c + d*x)*sqrt(b))*b**3 - 15*e**(10*c + 10*d*x)*sqrt
(a)*sqrt(a - b)*log(sqrt(2*sqrt(a)*sqrt(a - b) - 2*a + b) + e**(c + d*x)*s
qrt(b))*b**3 + 15*e**(10*c + 10*d*x)*sqrt(a)*sqrt(a - b)*log(2*sqrt(a)*sqr
t(a - b) + e**(2*c + 2*d*x)*b + 2*a - b)*b**3 + 75*e**(8*c + 8*d*x)*sqrt(a
)*sqrt(a - b)*log( - sqrt(2*sqrt(a)*sqrt(a - b) - 2*a + b) + e**(c + d*x)*
sqrt(b))*b**3 + 75*e**(8*c + 8*d*x)*sqrt(a)*sqrt(a - b)*log(sqrt(2*sqrt(a)
*sqrt(a - b) - 2*a + b) + e**(c + d*x)*sqrt(b))*b**3 - 75*e**(8*c + 8*d*x)
*sqrt(a)*sqrt(a - b)*log(2*sqrt(a)*sqrt(a - b) + e**(2*c + 2*d*x)*b + 2*a
- b)*b**3 - 150*e**(6*c + 6*d*x)*sqrt(a)*sqrt(a - b)*log( - sqrt(2*sqrt(a)
*sqrt(a - b) - 2*a + b) + e**(c + d*x)*sqrt(b))*b**3 - 150*e**(6*c + 6*d*x
)*sqrt(a)*sqrt(a - b)*log(sqrt(2*sqrt(a)*sqrt(a - b) - 2*a + b) + e**(c +
d*x)*sqrt(b))*b**3 + 150*e**(6*c + 6*d*x)*sqrt(a)*sqrt(a - b)*log(2*sqrt(a
)*sqrt(a - b) + e**(2*c + 2*d*x)*b + 2*a - b)*b**3 + 150*e**(4*c + 4*d*x)*
sqrt(a)*sqrt(a - b)*log( - sqrt(2*sqrt(a)*sqrt(a - b) - 2*a + b) + e**(c +
 d*x)*sqrt(b))*b**3 + 150*e**(4*c + 4*d*x)*sqrt(a)*sqrt(a - b)*log(sqrt(2*
sqrt(a)*sqrt(a - b) - 2*a + b) + e**(c + d*x)*sqrt(b))*b**3 - 150*e**(4*c
+ 4*d*x)*sqrt(a)*sqrt(a - b)*log(2*sqrt(a)*sqrt(a - b) + e**(2*c + 2*d*x)*
b + 2*a - b)*b**3 - 75*e**(2*c + 2*d*x)*sqrt(a)*sqrt(a - b)*log( - sqrt(2*
sqrt(a)*sqrt(a - b) - 2*a + b) + e**(c + d*x)*sqrt(b))*b**3 - 75*e**(2*...
```

3.42 $\int \frac{\sinh^4(c+dx)}{\left(a+b\sinh^2(c+dx)\right)^2}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 102

$$\int \frac{\sinh^4(c+dx)}{\left(a+b\sinh^2(c+dx)\right)^2}\,dx = \frac{x}{b^2} - \frac{\sqrt{a}(2a-3b)\text{arctanh}\left(\frac{\sqrt{a-b}\tanh(c+dx)}{\sqrt{a}}\right)}{2(a-b)^{3/2}b^2d} - \frac{a\tanh(c+dx)}{2(a-b)bd\left(a-(a-b)\tanh^2(c+dx)\right)}$$

output
```
x/b^2-1/2*a^(1/2)*(2*a-3*b)*arctanh((a-b)^(1/2)*tanh(d*x+c)/a^(1/2))/(a-b)
^(3/2)/b^2/d-1/2*a*tanh(d*x+c)/(a-b)/b/d/(a-(a-b)*tanh(d*x+c)^2)
```

Mathematica [A] (verified)

Time = 11.39 (sec) , antiderivative size = 99, normalized size of antiderivative = 0.97

$$\int \frac{\sinh^4(c+dx)}{\left(a+b\sinh^2(c+dx)\right)^2}\,dx = -\frac{-2(c+dx)+\frac{\sqrt{a}(2a-3b)\text{arctanh}\left(\frac{\sqrt{a-b}\tanh(c+dx)}{\sqrt{a}}\right)}{(a-b)^{3/2}}+\frac{ab\sinh(2(c+dx))}{(a-b)(2a-b+b\cosh(2(c+dx)))}}{2b^2d}$$

input
```
Integrate[Sinh[c + d*x]^4/(a + b*Sinh[c + d*x]^2)^2,x]
```

output
```
-1/2*(-2*(c + d*x) + (Sqrt[a]*(2*a - 3*b)*ArcTanh[(Sqrt[a - b]*Tanh[c + d*
x])/Sqrt[a]])/(a - b)^(3/2) + (a*b*Sinh[2*(c + d*x)])/((a - b)*(2*a - b +
b*Cosh[2*(c + d*x)])))/(b^2*d)
```

Rubi [A] (verified)

Time = 0.35 (sec) , antiderivative size = 125, normalized size of antiderivative = 1.23, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.261$, Rules used = {3042, 3666, 372, 397, 219, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sinh^4(c+dx)}{\left(a+b\sinh^2(c+dx)\right)^2}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\sin(ic+idx)^4}{\left(a-b\sin(ic+idx)^2\right)^2}dx$$

$$\Big\downarrow \text{3666}$$

$$\frac{\int \frac{\tanh^4(c+dx)}{\left(1-\tanh^2(c+dx)\right)\left(a-(a-b)\tanh^2(c+dx)\right)^2}d\tanh(c+dx)}{d}$$

$$\Big\downarrow \text{372}$$

$$\frac{\frac{\int \frac{(a-2b)\tanh^2(c+dx)+a}{\left(1-\tanh^2(c+dx)\right)\left(a-(a-b)\tanh^2(c+dx)\right)}d\tanh(c+dx)}{2b(a-b)}-\frac{a\tanh(c+dx)}{2b(a-b)\left(a-(a-b)\tanh^2(c+dx)\right)}}{d}$$

$$\Big\downarrow \text{397}$$

$$\frac{\frac{\frac{2(a-b)\int \frac{1}{1-\tanh^2(c+dx)}d\tanh(c+dx)}{b}-\frac{a(2a-3b)\int \frac{1}{a-(a-b)\tanh^2(c+dx)}d\tanh(c+dx)}{b}}{2b(a-b)}-\frac{a\tanh(c+dx)}{2b(a-b)\left(a-(a-b)\tanh^2(c+dx)\right)}}{d}$$

$$\Big\downarrow \text{219}$$

$$\frac{\frac{\frac{2(a-b)\text{arctanh}(\tanh(c+dx))}{b}-\frac{a(2a-3b)\int\frac{1}{a-(a-b)\tanh^2(c+dx)}d\tanh(c+dx)}{b}}{2b(a-b)}-\frac{a\tanh(c+dx)}{2b(a-b)\left(a-(a-b)\tanh^2(c+dx)\right)}}{d}$$

$\big\downarrow$ 221

$$\frac{\frac{\frac{2(a-b)\text{arctanh}(\tanh(c+dx))}{b}-\frac{\sqrt{a}(2a-3b)\text{arctanh}\left(\frac{\sqrt{a-b}\tanh(c+dx)}{\sqrt{a}}\right)}{b\sqrt{a-b}}}{2b(a-b)}-\frac{a\tanh(c+dx)}{2b(a-b)\left(a-(a-b)\tanh^2(c+dx)\right)}}{d}$$

input

```
Int[Sinh[c + d*x]^4/(a + b*Sinh[c + d*x]^2)^2,x]
```

output

```
(((2*(a - b)*ArcTanh[Tanh[c + d*x]])/b - (Sqrt[a]*(2*a - 3*b)*ArcTanh[(Sqr
t[a - b]*Tanh[c + d*x])/Sqrt[a]])/(Sqrt[a - b]*b))/(2*(a - b)*b) - (a*Tanh
[c + d*x])/(2*(a - b)*b*(a - (a - b)*Tanh[c + d*x]^2)))/d
```

Defintions of rubi rules used

rule 219

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 221

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 372

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_
), x_Symbol] :> Simp[(-a)*e^3*(e*x)^(m - 3)*(a + b*x^2)^(p + 1)*((c + d*x^2
)^(q + 1)/(2*b*(b*c - a*d)*(p + 1))), x] + Simp[e^4/(2*b*(b*c - a*d)*(p + 1
))   Int[(e*x)^(m - 4)*(a + b*x^2)^(p + 1)*(c + d*x^2)^q*Simp[a*c*(m - 3) +
 (a*d*(m + 2*q - 1) + 2*b*c*(p + 1))*x^2, x], x], x] /; FreeQ[{a, b, c, d,
e, q}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && GtQ[m, 3] && IntBinomialQ[a
, b, c, d, e, m, 2, p, q, x]
```

rule 397
```
Int[((e_) + (f_.)*(x_)^2)/(((a_) + (b_.)*(x_)^2)*((c_) + (d_.)*(x_)^2)), x_
Symbol] :> Simp[(b*e - a*f)/(b*c - a*d)   Int[1/(a + b*x^2), x], x] - Simp[
(d*e - c*f)/(b*c - a*d)   Int[1/(c + d*x^2), x], x] /; FreeQ[{a, b, c, d, e
, f}, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3666
```
Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(
p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Simp[ff^(m + 1
)/f   Subst[Int[x^m*((a + (a + b)*ff^2*x^2)^p/(1 + ff^2*x^2)^(m/2 + p + 1))
, x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f}, x] && IntegerQ[m/2] &
& IntegerQ[p]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 314 vs. $2(90) = 180$.

Time = 0.92 (sec) , antiderivative size = 315, normalized size of antiderivative = 3.09

method	result
risch	$\frac{x}{b^2}+\frac{a(2\,e^{2dx+2c}a-e^{2dx+2c}b+b)}{b^2(a-b)d(b\,e^{4dx+4c}+4\,e^{2dx+2c}a-2\,e^{2dx+2c}b+b)}+\frac{\sqrt{a(a-b)}\,\ln\left(e^{2dx+2c}+\frac{2a+2\sqrt{a(a-b)}-b}{b}\right)a}{2(a-b)^2d\,b^2}-\frac{3\sqrt{a(a-b}}{}$
derivativedivides	$\frac{\frac{\ln\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)}{b^2}+\frac{2a\left(\frac{-\frac{b\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^3}{2(a-b)}-\frac{b\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)}{2(a-b)}}{\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^4a-2\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^2a+4b\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^2+a}+\frac{(2a-3b)a\left(\frac{\left(\sqrt{-b(a-b)}+b\right)\arctan\left(\frac{}{\sqrt{(}}\right.}{2a\sqrt{-b(a-b)}\sqrt{\left(2\sqrt{-}\right.}}\right.}{}\right)}{b^2}}{d}$
default	$\frac{\frac{\ln\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)}{b^2}+\frac{2a\left(\frac{-\frac{b\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^3}{2(a-b)}-\frac{b\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)}{2(a-b)}}{\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^4a-2\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^2a+4b\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^2+a}+\frac{(2a-3b)a\left(\frac{\left(\sqrt{-b(a-b)}+b\right)\arctan\left(\frac{}{\sqrt{(}}\right.}{2a\sqrt{-b(a-b)}\sqrt{\left(2\sqrt{-}\right.}}\right.}{}\right)}{b^2}}{d}$

input
```
int(sinh(d*x+c)^4/(a+b*sinh(d*x+c)^2)^2,x,method=_RETURNVERBOSE)
```

output
```
x/b^2+a*(2*exp(2*d*x+2*c)*a-exp(2*d*x+2*c)*b+b)/b^2/(a-b)/d/(b*exp(4*d*x+4
*c)+4*exp(2*d*x+2*c)*a-2*exp(2*d*x+2*c)*b+b)+1/2*(a*(a-b))^(1/2)/(a-b)^2/d
/b^2*ln(exp(2*d*x+2*c)+(2*a+2*(a*(a-b))^(1/2)-b)/b)*a-3/4*(a*(a-b))^(1/2)/
(a-b)^2/d/b*ln(exp(2*d*x+2*c)+(2*a+2*(a*(a-b))^(1/2)-b)/b)-1/2*(a*(a-b))^(
1/2)/(a-b)^2/d/b^2*ln(exp(2*d*x+2*c)-(-2*a+2*(a*(a-b))^(1/2)+b)/b)*a+3/4*(
a*(a-b))^(1/2)/(a-b)^2/d/b*ln(exp(2*d*x+2*c)-(-2*a+2*(a*(a-b))^(1/2)+b)/b)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 740 vs. $2(91) = 182$.

Time = 0.13 (sec) , antiderivative size = 1772, normalized size of antiderivative = 17.37

$$\int \frac{\sinh^4(c+dx)}{\left(a+b\sinh^2(c+dx)\right)^2}\,dx = \text{Too large to display}$$

input
```
integrate(sinh(d*x+c)^4/(a+b*sinh(d*x+c)^2)^2,x, algorithm="fricas")
```

output
```
[1/4*(4*(a*b - b^2)*d*x*cosh(d*x + c)^4 + 16*(a*b - b^2)*d*x*cosh(d*x + c)
*sinh(d*x + c)^3 + 4*(a*b - b^2)*d*x*sinh(d*x + c)^4 + 4*(a*b - b^2)*d*x +
 4*(2*(2*a^2 - 3*a*b + b^2)*d*x + 2*a^2 - a*b)*cosh(d*x + c)^2 + 4*(6*(a*b
 - b^2)*d*x*cosh(d*x + c)^2 + 2*(2*a^2 - 3*a*b + b^2)*d*x + 2*a^2 - a*b)*s
inh(d*x + c)^2 + ((2*a*b - 3*b^2)*cosh(d*x + c)^4 + 4*(2*a*b - 3*b^2)*cosh
(d*x + c)*sinh(d*x + c)^3 + (2*a*b - 3*b^2)*sinh(d*x + c)^4 + 2*(4*a^2 - 8
*a*b + 3*b^2)*cosh(d*x + c)^2 + 2*(3*(2*a*b - 3*b^2)*cosh(d*x + c)^2 + 4*a
^2 - 8*a*b + 3*b^2)*sinh(d*x + c)^2 + 2*a*b - 3*b^2 + 4*((2*a*b - 3*b^2)*c
osh(d*x + c)^3 + (4*a^2 - 8*a*b + 3*b^2)*cosh(d*x + c))*sinh(d*x + c))*sqr
t(a/(a - b))*log((b^2*cosh(d*x + c)^4 + 4*b^2*cosh(d*x + c)*sinh(d*x + c)^
3 + b^2*sinh(d*x + c)^4 + 2*(2*a*b - b^2)*cosh(d*x + c)^2 + 2*(3*b^2*cosh(
d*x + c)^2 + 2*a*b - b^2)*sinh(d*x + c)^2 + 8*a^2 - 8*a*b + b^2 + 4*(b^2*c
osh(d*x + c)^3 + (2*a*b - b^2)*cosh(d*x + c))*sinh(d*x + c) + 4*((a*b - b^
2)*cosh(d*x + c)^2 + 2*(a*b - b^2)*cosh(d*x + c)*sinh(d*x + c) + (a*b - b^
2)*sinh(d*x + c)^2 + 2*a^2 - 3*a*b + b^2)*sqrt(a/(a - b)))/(b*cosh(d*x + c
)^4 + 4*b*cosh(d*x + c)*sinh(d*x + c)^3 + b*sinh(d*x + c)^4 + 2*(2*a - b)*
cosh(d*x + c)^2 + 2*(3*b*cosh(d*x + c)^2 + 2*a - b)*sinh(d*x + c)^2 + 4*(b
*cosh(d*x + c)^3 + (2*a - b)*cosh(d*x + c))*sinh(d*x + c) + b)) + 4*a*b +
8*(2*(a*b - b^2)*d*x*cosh(d*x + c)^3 + (2*(2*a^2 - 3*a*b + b^2)*d*x + 2*a^
2 - a*b)*cosh(d*x + c))*sinh(d*x + c))/((a*b^3 - b^4)*d*cosh(d*x + c)^4...
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\sinh^4(c+dx)}{\left(a+b\sinh^2(c+dx)\right)^2}\,dx = \text{Timed out}$$

input
```
integrate(sinh(d*x+c)**4/(a+b*sinh(d*x+c)**2)**2,x)
```

output
```
Timed out
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{\sinh^4(c+dx)}{\left(a+b\sinh^2(c+dx)\right)^2}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate(sinh(d*x+c)^4/(a+b*sinh(d*x+c)^2)^2,x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(b-a>0)', see `assume?` for more
details)Is
```

Giac [A] (verification not implemented)

Time = 0.97 (sec) , antiderivative size = 168, normalized size of antiderivative = 1.65

$$\int \frac{\sinh^4(c+dx)}{\left(a+b\sinh^2(c+dx)\right)^2}\,dx = -\frac{\frac{(2a^2-3ab)\arctan\left(\frac{be^{(2dx+2c)}+2a-b}{2\sqrt{-a^2+ab}}\right)}{(ab^2-b^3)\sqrt{-a^2+ab}} - \frac{2\left(2a^2e^{(2dx+2c)}-abe^{(2dx+2c)}+ab\right)}{(ab^2-b^3)\left(be^{(4dx+4c)}+4ae^{(2dx+2c)}-2be^{(2dx+2c)}+b\right)} - \frac{2(dx+c)}{b^2}}{2d}$$

input
```
integrate(sinh(d*x+c)^4/(a+b*sinh(d*x+c)^2)^2,x, algorithm="giac")
```

output
```
-1/2*((2*a^2 - 3*a*b)*arctan(1/2*(b*e^(2*d*x + 2*c) + 2*a - b)/sqrt(-a^2 +
 a*b))/((a*b^2 - b^3)*sqrt(-a^2 + a*b)) - 2*(2*a^2*e^(2*d*x + 2*c) - a*b*e
^(2*d*x + 2*c) + a*b)/((a*b^2 - b^3)*(b*e^(4*d*x + 4*c) + 4*a*e^(2*d*x + 2
*c) - 2*b*e^(2*d*x + 2*c) + b)) - 2*(d*x + c)/b^2)/d
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sinh^4(c+dx)}{\left(a+b\sinh^2(c+dx)\right)^2}\,dx = \int \frac{\sinh(c+d\,x)^4}{\left(b\sinh\left(c+d\,x\right)^2+a\right)^2}\,dx$$

input
```
int(sinh(c + d*x)^4/(a + b*sinh(c + d*x)^2)^2,x)
```

output
```
int(sinh(c + d*x)^4/(a + b*sinh(c + d*x)^2)^2, x)
```

Reduce [B] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 1353, normalized size of antiderivative = 13.26

$$\int \frac{\sinh^4(c+dx)}{\left(a+b\sinh^2(c+dx)\right)^2}\,dx = \text{Too large to display}$$

input
```
int(sinh(d*x+c)^4/(a+b*sinh(d*x+c)^2)^2,x)
```

output
```
( - 2*e**(4*c + 4*d*x)*sqrt(a)*sqrt(a - b)*log( - sqrt(2*sqrt(a)*sqrt(a -
b) - 2*a + b) + e**(c + d*x)*sqrt(b))*a*b + 3*e**(4*c + 4*d*x)*sqrt(a)*sqr
t(a - b)*log( - sqrt(2*sqrt(a)*sqrt(a - b) - 2*a + b) + e**(c + d*x)*sqrt(
b))*b**2 - 2*e**(4*c + 4*d*x)*sqrt(a)*sqrt(a - b)*log(sqrt(2*sqrt(a)*sqrt(
a - b) - 2*a + b) + e**(c + d*x)*sqrt(b))*a*b + 3*e**(4*c + 4*d*x)*sqrt(a)
*sqrt(a - b)*log(sqrt(2*sqrt(a)*sqrt(a - b) - 2*a + b) + e**(c + d*x)*sqrt
(b))*b**2 + 2*e**(4*c + 4*d*x)*sqrt(a)*sqrt(a - b)*log(2*sqrt(a)*sqrt(a -
b) + e**(2*c + 2*d*x)*b + 2*a - b)*a*b - 3*e**(4*c + 4*d*x)*sqrt(a)*sqrt(a
 - b)*log(2*sqrt(a)*sqrt(a - b) + e**(2*c + 2*d*x)*b + 2*a - b)*b**2 - 8*e
**(2*c + 2*d*x)*sqrt(a)*sqrt(a - b)*log( - sqrt(2*sqrt(a)*sqrt(a - b) - 2*
a + b) + e**(c + d*x)*sqrt(b))*a**2 + 16*e**(2*c + 2*d*x)*sqrt(a)*sqrt(a -
 b)*log( - sqrt(2*sqrt(a)*sqrt(a - b) - 2*a + b) + e**(c + d*x)*sqrt(b))*a
*b - 6*e**(2*c + 2*d*x)*sqrt(a)*sqrt(a - b)*log( - sqrt(2*sqrt(a)*sqrt(a -
 b) - 2*a + b) + e**(c + d*x)*sqrt(b))*b**2 - 8*e**(2*c + 2*d*x)*sqrt(a)*s
qrt(a - b)*log(sqrt(2*sqrt(a)*sqrt(a - b) - 2*a + b) + e**(c + d*x)*sqrt(b
))*a**2 + 16*e**(2*c + 2*d*x)*sqrt(a)*sqrt(a - b)*log(sqrt(2*sqrt(a)*sqrt(
a - b) - 2*a + b) + e**(c + d*x)*sqrt(b))*a*b - 6*e**(2*c + 2*d*x)*sqrt(a)
*sqrt(a - b)*log(sqrt(2*sqrt(a)*sqrt(a - b) - 2*a + b) + e**(c + d*x)*sqrt
(b))*b**2 + 8*e**(2*c + 2*d*x)*sqrt(a)*sqrt(a - b)*log(2*sqrt(a)*sqrt(a -
b) + e**(2*c + 2*d*x)*b + 2*a - b)*a**2 - 16*e**(2*c + 2*d*x)*sqrt(a)*s...
```

3.43 $\int \frac{\sinh^3(c+dx)}{\left(a+b\sinh^2(c+dx)\right)^2}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 90

$$\int \frac{\sinh^3(c+dx)}{\left(a+b\sinh^2(c+dx)\right)^2}\,dx=\frac{(a-2b)\arctan\left(\frac{\sqrt{b}\cosh(c+dx)}{\sqrt{a-b}}\right)}{2(a-b)^{3/2}b^{3/2}d}-\frac{a\cosh(c+dx)}{2(a-b)bd\left(a-b+b\cosh^2(c+dx)\right)}$$

output
```
1/2*(a-2*b)*arctan(b^(1/2)*cosh(d*x+c)/(a-b)^(1/2))/(a-b)^(3/2)/b^(3/2)/d-
1/2*a*cosh(d*x+c)/(a-b)/b/d/(a-b+b*cosh(d*x+c)^2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 2.07 (sec) , antiderivative size = 141, normalized size of antiderivative = 1.57

$$\int \frac{\sinh^3(c+dx)}{\left(a+b\sinh^2(c+dx)\right)^2}\,dx=\frac{\frac{(a-2b)\left(\arctan\left(\frac{\sqrt{b}-i\sqrt{a}\tanh\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a-b}}\right)+\arctan\left(\frac{\sqrt{b}+i\sqrt{a}\tanh\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a-b}}\right)\right)}{(a-b)^{3/2}}-\frac{2a\sqrt{b}\cosh(c+dx)}{(a-b)(2a-b+b\cosh(2(c+dx)))}}{2b^{3/2}d}$$

input
```
Integrate[Sinh[c + d*x]^3/(a + b*Sinh[c + d*x]^2)^2,x]
```

output
```
(((a - 2*b)*(ArcTan[(Sqrt[b] - I*Sqrt[a]*Tanh[(c + d*x)/2])/Sqrt[a - b]] +
 ArcTan[(Sqrt[b] + I*Sqrt[a]*Tanh[(c + d*x)/2])/Sqrt[a - b]]))/(a - b)^(3/
2) - (2*a*Sqrt[b]*Cosh[c + d*x])/((a - b)*(2*a - b + b*Cosh[2*(c + d*x)]))
)/(2*b^(3/2)*d)
```

Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 89, normalized size of antiderivative = 0.99, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.217$, Rules used = {3042, 26, 3665, 298, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sinh^3(c+dx)}{\left(a+b\sinh^2(c+dx)\right)^2}dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \frac{i\sin(ic+idx)^3}{\left(a-b\sin(ic+idx)^2\right)^2}dx$$

$$\Big\downarrow \text{ 26}$$

$$i\int \frac{\sin(ic+idx)^3}{\left(a-b\sin(ic+idx)^2\right)^2}dx$$

$$\Big\downarrow \text{ 3665}$$

$$-\frac{\int \frac{1-\cosh^2(c+dx)}{\left(b\cosh^2(c+dx)+a-b\right)^2}d\cosh(c+dx)}{d}$$

$$\Big\downarrow \text{ 298}$$

$$-\frac{\frac{a\cosh(c+dx)}{2b(a-b)\left(a+b\cosh^2(c+dx)-b\right)}-\frac{(a-2b)\int \frac{1}{b\cosh^2(c+dx)+a-b}d\cosh(c+dx)}{2b(a-b)}}{d}$$

$$\Big\downarrow \text{ 218}$$

$$-\frac{\frac{a\cosh(c+dx)}{2b(a-b)\left(a+b\cosh^2(c+dx)-b\right)}-\frac{(a-2b)\arctan\left(\frac{\sqrt{b}\cosh(c+dx)}{\sqrt{a-b}}\right)}{2b^{3/2}(a-b)^{3/2}}}{d}$$

input
```
Int[Sinh[c + d*x]^3/(a + b*Sinh[c + d*x]^2)^2,x]
```

output
```
-((-1/2*((a - 2*b)*ArcTan[(Sqrt[b]*Cosh[c + d*x])/Sqrt[a - b]])/((a - b)^(
3/2)*b^(3/2)) + (a*Cosh[c + d*x])/(2*(a - b)*b*(a - b + b*Cosh[c + d*x]^2)
))/d)
```

Defintions of rubi rules used

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 298
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2), x_Symbol] :> Simp[(-(
b*c - a*d))*x*((a + b*x^2)^(p + 1)/(2*a*b*(p + 1))), x] - Simp[(a*d - b*c*(
2*p + 3))/(2*a*b*(p + 1))   Int[(a + b*x^2)^(p + 1), x], x] /; FreeQ[{a, b,
 c, d, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/2 + p, 0])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3665
```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^
(p_.), x_Symbol] :> With[{ff = FreeFactors[Cos[e + f*x], x]}, Simp[-ff/f
Subst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a + b - b*ff^2*x^2)^p, x], x, Cos[e +
 f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m - 1)/2]
```

Maple [A] (verified)

Time = 0.78 (sec) , antiderivative size = 155, normalized size of antiderivative = 1.72

method	result
derivativedivides	$\frac{\frac{8(2a-4b)\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^2-16a}{\left(16ab-16b^2\right)\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^4 a-2\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^2 a+4b\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^2+a\right)}+\frac{4(2a-4b)\arctan\left(\frac{2\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^2 a-2a+4b}{4\sqrt{ab-b^2}}\right)}{\left(16ab-16b^2\right)\sqrt{ab-b^2}}}{d}$
default	$\frac{\frac{8(2a-4b)\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^2-16a}{\left(16ab-16b^2\right)\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^4 a-2\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^2 a+4b\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^2+a\right)}+\frac{4(2a-4b)\arctan\left(\frac{2\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^2 a-2a+4b}{4\sqrt{ab-b^2}}\right)}{\left(16ab-16b^2\right)\sqrt{ab-b^2}}}{d}$
risch	$-\frac{a\,e^{dx+c}\left(e^{2dx+2c}+1\right)}{bd(a-b)\left(b\,e^{4dx+4c}+4\,e^{2dx+2c}a-2\,e^{2dx+2c}b+b\right)}-\frac{\ln\left(e^{2dx+2c}-\frac{2(a-b)e^{dx+c}}{\sqrt{-ab+b^2}}+1\right)a}{4\sqrt{-ab+b^2}\,(a-b)db}+\frac{\ln\left(e^{2dx+2c}-\frac{2(a-b)e^{dx+c}}{\sqrt{-ab+b^2}}\right.}{2\sqrt{-ab+b^2}\,(a-b)d}$

input
```
int(sinh(d*x+c)^3/(a+b*sinh(d*x+c)^2)^2,x,method=_RETURNVERBOSE)
```

output
```
1/d*(8*((2*a-4*b)*tanh(1/2*d*x+1/2*c)^2-2*a)/(16*a*b-16*b^2)/(tanh(1/2*d*x
+1/2*c)^4*a-2*tanh(1/2*d*x+1/2*c)^2*a+4*b*tanh(1/2*d*x+1/2*c)^2+a)+4*(2*a-
4*b)/(16*a*b-16*b^2)/(a*b-b^2)^(1/2)*arctan(1/4*(2*tanh(1/2*d*x+1/2*c)^2*a
-2*a+4*b)/(a*b-b^2)^(1/2)))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 952 vs. 2(78) = 156.

Time = 0.11 (sec) , antiderivative size = 1888, normalized size of antiderivative = 20.98

$$\int \frac{\sinh^3(c+dx)}{\left(a+b\sinh^2(c+dx)\right)^2}\,dx = \text{Too large to display}$$

input
```
integrate(sinh(d*x+c)^3/(a+b*sinh(d*x+c)^2)^2,x, algorithm="fricas")
```

output
```
[-1/4*(4*(a^2*b - a*b^2)*cosh(d*x + c)^3 + 12*(a^2*b - a*b^2)*cosh(d*x + c
)*sinh(d*x + c)^2 + 4*(a^2*b - a*b^2)*sinh(d*x + c)^3 + ((a*b - 2*b^2)*cos
h(d*x + c)^4 + 4*(a*b - 2*b^2)*cosh(d*x + c)*sinh(d*x + c)^3 + (a*b - 2*b^
2)*sinh(d*x + c)^4 + 2*(2*a^2 - 5*a*b + 2*b^2)*cosh(d*x + c)^2 + 2*(3*(a*b
 - 2*b^2)*cosh(d*x + c)^2 + 2*a^2 - 5*a*b + 2*b^2)*sinh(d*x + c)^2 + a*b -
 2*b^2 + 4*((a*b - 2*b^2)*cosh(d*x + c)^3 + (2*a^2 - 5*a*b + 2*b^2)*cosh(d
*x + c))*sinh(d*x + c))*sqrt(-a*b + b^2)*log((b*cosh(d*x + c)^4 + 4*b*cosh
(d*x + c)*sinh(d*x + c)^3 + b*sinh(d*x + c)^4 - 2*(2*a - 3*b)*cosh(d*x + c
)^2 + 2*(3*b*cosh(d*x + c)^2 - 2*a + 3*b)*sinh(d*x + c)^2 + 4*(b*cosh(d*x
+ c)^3 - (2*a - 3*b)*cosh(d*x + c))*sinh(d*x + c) - 4*(cosh(d*x + c)^3 + 3
*cosh(d*x + c)*sinh(d*x + c)^2 + sinh(d*x + c)^3 + (3*cosh(d*x + c)^2 + 1)
*sinh(d*x + c) + cosh(d*x + c))*sqrt(-a*b + b^2) + b)/(b*cosh(d*x + c)^4 +
 4*b*cosh(d*x + c)*sinh(d*x + c)^3 + b*sinh(d*x + c)^4 + 2*(2*a - b)*cosh(
d*x + c)^2 + 2*(3*b*cosh(d*x + c)^2 + 2*a - b)*sinh(d*x + c)^2 + 4*(b*cosh
(d*x + c)^3 + (2*a - b)*cosh(d*x + c))*sinh(d*x + c) + b)) + 4*(a^2*b - a*
b^2)*cosh(d*x + c) + 4*(a^2*b - a*b^2 + 3*(a^2*b - a*b^2)*cosh(d*x + c)^2)
*sinh(d*x + c))/((a^2*b^3 - 2*a*b^4 + b^5)*d*cosh(d*x + c)^4 + 4*(a^2*b^3
- 2*a*b^4 + b^5)*d*cosh(d*x + c)*sinh(d*x + c)^3 + (a^2*b^3 - 2*a*b^4 + b^
5)*d*sinh(d*x + c)^4 + 2*(2*a^3*b^2 - 5*a^2*b^3 + 4*a*b^4 - b^5)*d*cosh(d*
x + c)^2 + 2*(3*(a^2*b^3 - 2*a*b^4 + b^5)*d*cosh(d*x + c)^2 + (2*a^3*b^...
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\sinh^3(c+dx)}{\left(a+b\sinh^2(c+dx)\right)^2}\,dx = \text{Timed out}$$

input
```
integrate(sinh(d*x+c)**3/(a+b*sinh(d*x+c)**2)**2,x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{\sinh^3(c+dx)}{\left(a+b\sinh^2(c+dx)\right)^2}\,dx = \int \frac{\sinh\left(dx+c\right)^3}{\left(b\sinh\left(dx+c\right)^2+a\right)^2}\,dx$$

input
```
integrate(sinh(d*x+c)^3/(a+b*sinh(d*x+c)^2)^2,x, algorithm="maxima")
```

output
```
-(a*e^(3*d*x + 3*c) + a*e^(d*x + c))/(a*b^2*d - b^3*d + (a*b^2*d*e^(4*c) -
 b^3*d*e^(4*c))*e^(4*d*x) + 2*(2*a^2*b*d*e^(2*c) - 3*a*b^2*d*e^(2*c) + b^3
*d*e^(2*c))*e^(2*d*x)) + 1/8*integrate(8*((a*e^(3*c) - 2*b*e^(3*c))*e^(3*d
*x) - (a*e^c - 2*b*e^c)*e^(d*x))/(a*b^2 - b^3 + (a*b^2*e^(4*c) - b^3*e^(4*
c))*e^(4*d*x) + 2*(2*a^2*b*e^(2*c) - 3*a*b^2*e^(2*c) + b^3*e^(2*c))*e^(2*d
*x)), x)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{\sinh^3(c+dx)}{\left(a+b\sinh^2(c+dx)\right)^2}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(sinh(d*x+c)^3/(a+b*sinh(d*x+c)^2)^2,x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Limit: Max order reached or unable
to make series expansion Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sinh^3(c+dx)}{\left(a+b\sinh^2(c+dx)\right)^2}\,dx=\int \frac{\sinh(c+d\,x)^3}{\left(b\sinh\left(c+d\,x\right)^2+a\right)^2}\,dx$$

input
```
int(sinh(c + d*x)^3/(a + b*sinh(c + d*x)^2)^2,x)
```

output
```
int(sinh(c + d*x)^3/(a + b*sinh(c + d*x)^2)^2, x)
```

Reduce [B] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 3499, normalized size of antiderivative = 38.88

$$\int \frac{\sinh^3(c+dx)}{\left(a+b\sinh^2(c+dx)\right)^2}\,dx=\text{Too large to display}$$

input
```
int(sinh(d*x+c)^3/(a+b*sinh(d*x+c)^2)^2,x)
```

output
```
(2*e**(4*c + 4*d*x)*sqrt(b)*sqrt(a)*sqrt(a - b)*sqrt(2*sqrt(a)*sqrt(a - b)
 + 2*a - b)*atan((e**(c + d*x)*b)/(sqrt(b)*sqrt(2*sqrt(a)*sqrt(a - b) + 2*
a - b)))*a*b - 4*e**(4*c + 4*d*x)*sqrt(b)*sqrt(a)*sqrt(a - b)*sqrt(2*sqrt(
a)*sqrt(a - b) + 2*a - b)*atan((e**(c + d*x)*b)/(sqrt(b)*sqrt(2*sqrt(a)*sq
rt(a - b) + 2*a - b)))*b**2 + 8*e**(2*c + 2*d*x)*sqrt(b)*sqrt(a)*sqrt(a -
b)*sqrt(2*sqrt(a)*sqrt(a - b) + 2*a - b)*atan((e**(c + d*x)*b)/(sqrt(b)*sq
rt(2*sqrt(a)*sqrt(a - b) + 2*a - b)))*a**2 - 20*e**(2*c + 2*d*x)*sqrt(b)*s
qrt(a)*sqrt(a - b)*sqrt(2*sqrt(a)*sqrt(a - b) + 2*a - b)*atan((e**(c + d*x
)*b)/(sqrt(b)*sqrt(2*sqrt(a)*sqrt(a - b) + 2*a - b)))*a*b + 8*e**(2*c + 2*
d*x)*sqrt(b)*sqrt(a)*sqrt(a - b)*sqrt(2*sqrt(a)*sqrt(a - b) + 2*a - b)*ata
n((e**(c + d*x)*b)/(sqrt(b)*sqrt(2*sqrt(a)*sqrt(a - b) + 2*a - b)))*b**2 +
 2*sqrt(b)*sqrt(a)*sqrt(a - b)*sqrt(2*sqrt(a)*sqrt(a - b) + 2*a - b)*atan(
(e**(c + d*x)*b)/(sqrt(b)*sqrt(2*sqrt(a)*sqrt(a - b) + 2*a - b)))*a*b - 4*
sqrt(b)*sqrt(a)*sqrt(a - b)*sqrt(2*sqrt(a)*sqrt(a - b) + 2*a - b)*atan((e*
*(c + d*x)*b)/(sqrt(b)*sqrt(2*sqrt(a)*sqrt(a - b) + 2*a - b)))*b**2 - 2*e*
*(4*c + 4*d*x)*sqrt(b)*sqrt(2*sqrt(a)*sqrt(a - b) + 2*a - b)*atan((e**(c +
 d*x)*b)/(sqrt(b)*sqrt(2*sqrt(a)*sqrt(a - b) + 2*a - b)))*a**2*b + 6*e**(4
*c + 4*d*x)*sqrt(b)*sqrt(2*sqrt(a)*sqrt(a - b) + 2*a - b)*atan((e**(c + d*
x)*b)/(sqrt(b)*sqrt(2*sqrt(a)*sqrt(a - b) + 2*a - b)))*a*b**2 - 4*e**(4*c
+ 4*d*x)*sqrt(b)*sqrt(2*sqrt(a)*sqrt(a - b) + 2*a - b)*atan((e**(c + d*...
```

3.44 $\int \frac{\sinh^2(c+dx)}{\left(a+b\sinh^2(c+dx)\right)^2}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 84

$$\int \frac{\sinh^2(c+dx)}{\left(a+b\sinh^2(c+dx)\right)^2}\,dx = -\frac{\operatorname{arctanh}\left(\frac{\sqrt{a-b}\tanh(c+dx)}{\sqrt{a}}\right)}{2\sqrt{a}(a-b)^{3/2}d}+\frac{\cosh(c+dx)\sinh(c+dx)}{2(a-b)d\left(a+b\sinh^2(c+dx)\right)}$$

output
```
-1/2*arctanh((a-b)^(1/2)*tanh(d*x+c)/a^(1/2))/a^(1/2)/(a-b)^(3/2)/d+1/2*co
sh(d*x+c)*sinh(d*x+c)/(a-b)/d/(a+b*sinh(d*x+c)^2)
```

Mathematica [A] (verified)

Time = 11.16 (sec) , antiderivative size = 81, normalized size of antiderivative = 0.96

$$\int \frac{\sinh^2(c+dx)}{\left(a+b\sinh^2(c+dx)\right)^2}\,dx = \frac{-\frac{\operatorname{arctanh}\left(\frac{\sqrt{a-b}\tanh(c+dx)}{\sqrt{a}}\right)}{\sqrt{a}(a-b)^{3/2}}+\frac{\sinh(2(c+dx))}{(a-b)(2a-b+b\cosh(2(c+dx)))}}{2d}$$

input
```
Integrate[Sinh[c + d*x]^2/(a + b*Sinh[c + d*x]^2)^2,x]
```

output
```
(-(ArcTanh[(Sqrt[a - b]*Tanh[c + d*x])/Sqrt[a]]/(Sqrt[a]*(a - b)^(3/2))) +
 Sinh[2*(c + d*x)]/((a - b)*(2*a - b + b*Cosh[2*(c + d*x)])))/(2*d)
```

Rubi [A] (verified)

Time = 0.37 (sec) , antiderivative size = 84, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.304, Rules used = {3042, 25, 3652, 27, 3042, 3660, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sinh^2(c+dx)}{\left(a+b\sinh^2(c+dx)\right)^2}dx$$

$$\Big\downarrow \text{3042}$$

$$\int -\frac{\sin(ic+idx)^2}{\left(a-b\sin(ic+idx)^2\right)^2}dx$$

$$\Big\downarrow \text{25}$$

$$-\int \frac{\sin(ic+idx)^2}{\left(a-b\sin(ic+idx)^2\right)^2}dx$$

$$\Big\downarrow \text{3652}$$

$$\frac{\sinh(c+dx)\cosh(c+dx)}{2d(a-b)\left(a+b\sinh^2(c+dx)\right)}-\frac{\int \frac{a}{b\sinh^2(c+dx)+a}dx}{2a(a-b)}$$

$$\Big\downarrow \text{27}$$

$$\frac{\sinh(c+dx)\cosh(c+dx)}{2d(a-b)\left(a+b\sinh^2(c+dx)\right)}-\frac{\int \frac{1}{b\sinh^2(c+dx)+a}dx}{2(a-b)}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\sinh(c+dx)\cosh(c+dx)}{2d(a-b)\left(a+b\sinh^2(c+dx)\right)}-\frac{\int \frac{1}{a-b\sin(ic+idx)^2}dx}{2(a-b)}$$

$$\Big\downarrow \text{3660}$$

$$\frac{\sinh(c+dx)\cosh(c+dx)}{2d(a-b)\left(a+b\sinh^2(c+dx)\right)}-\frac{\int \frac{1}{a-(a-b)\tanh^2(c+dx)}d\tanh(c+dx)}{2d(a-b)}$$

$$\Big\downarrow 221$$

$$\frac{\sinh(c+dx)\cosh(c+dx)}{2d(a-b)\left(a+b\sinh^2(c+dx)\right)}-\frac{\operatorname{arctanh}\left(\frac{\sqrt{a-b}\tanh(c+dx)}{\sqrt{a}}\right)}{2\sqrt{a}d(a-b)^{3/2}}$$

input
```
Int[Sinh[c + d*x]^2/(a + b*Sinh[c + d*x]^2)^2,x]
```

output
```
-1/2*ArcTanh[(Sqrt[a - b]*Tanh[c + d*x])/Sqrt[a]]/(Sqrt[a]*(a - b)^(3/2)*d
) + (Cosh[c + d*x]*Sinh[c + d*x])/(2*(a - b)*d*(a + b*Sinh[c + d*x]^2))
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3652
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)]^2), x_Symbol] :> Simp[(-(A*b - a*B))*Cos[e + f*x]*Sin[e + f*x
]*((a + b*Sin[e + f*x]^2)^(p + 1)/(2*a*f*(a + b)*(p + 1))), x] - Simp[1/(2*
a*(a + b)*(p + 1))   Int[(a + b*Sin[e + f*x]^2)^(p + 1)*Simp[a*B - A*(2*a*(
p + 1) + b*(2*p + 3)) + 2*(A*b - a*B)*(p + 2)*Sin[e + f*x]^2, x], x], x] /;
 FreeQ[{a, b, e, f, A, B}, x] && LtQ[p, -1] && NeQ[a + b, 0]
```

rule 3660
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(-1), x_Symbol] :> With[{ff =
FreeFactors[Tan[e + f*x], x]}, Simp[ff/f   Subst[Int[1/(a + (a + b)*ff^2*x^
2), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f}, x]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 247 vs. $2(72) = 144$.

Time = 0.68 (sec) , antiderivative size = 248, normalized size of antiderivative = 2.95

method	result
risch	$-\frac{2\,e^{2dx+2c}a-e^{2dx+2c}b+b}{bd(a-b)\left(b\,e^{4dx+4c}+4\,e^{2dx+2c}a-2\,e^{2dx+2c}b+b\right)}+\frac{\ln\left(e^{2dx+2c}+\frac{2a\sqrt{a^2-ab}-b\sqrt{a^2-ab}+2a^2-2ab}{b\sqrt{a^2-ab}}\right)}{4\sqrt{a^2-ab}\,(a-b)d}-\frac{\ln\left(e^{2dx+2c}\cdots\right.}{\cdots}$
derivativedivides	$\frac{-\frac{8\left(-\frac{\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^3}{8(a-b)}-\frac{\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)}{8(a-b)}\right)}{\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^4a-2\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^2a+4b\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^2+a}-\frac{a\left(\frac{\left(-\sqrt{-b(a-b)}-b\right)\arctan\left(\frac{\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)a}{\sqrt{\left(2\sqrt{-b(a-b)}-a+2b\right)a}}\right)}{2a\sqrt{-b(a-b)}\sqrt{\left(2\sqrt{-b(a-b)}-a+2b\right)a}}-\frac{\left(-\sqrt{-\cdots}\right.}{\cdots}\cdots\right)}{a-b}\cdots}{d}$
default	$\frac{-\frac{8\left(-\frac{\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^3}{8(a-b)}-\frac{\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)}{8(a-b)}\right)}{\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^4a-2\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^2a+4b\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^2+a}-\frac{a\left(\frac{\left(-\sqrt{-b(a-b)}-b\right)\arctan\left(\frac{\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)a}{\sqrt{\left(2\sqrt{-b(a-b)}-a+2b\right)a}}\right)}{2a\sqrt{-b(a-b)}\sqrt{\left(2\sqrt{-b(a-b)}-a+2b\right)a}}-\frac{\left(-\sqrt{-\cdots}\right.}{\cdots}\cdots\right)}{a-b}\cdots}{d}$

input
```
int(sinh(d*x+c)^2/(a+b*sinh(d*x+c)^2)^2,x,method=_RETURNVERBOSE)
```

output
```
-(2*exp(2*d*x+2*c)*a-exp(2*d*x+2*c)*b+b)/b/d/(a-b)/(b*exp(4*d*x+4*c)+4*exp
(2*d*x+2*c)*a-2*exp(2*d*x+2*c)*b+b)+1/4/(a^2-a*b)^(1/2)/(a-b)/d*ln(exp(2*d
*x+2*c)+(2*a*(a^2-a*b)^(1/2)-b*(a^2-a*b)^(1/2)+2*a^2-2*a*b)/b/(a^2-a*b)^(1
/2))-1/4/(a^2-a*b)^(1/2)/(a-b)/d*ln(exp(2*d*x+2*c)+(2*a*(a^2-a*b)^(1/2)-b*
(a^2-a*b)^(1/2)-2*a^2+2*a*b)/b/(a^2-a*b)^(1/2))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 634 vs. $2(72) = 144$.

Time = 0.12 (sec) , antiderivative size = 1523, normalized size of antiderivative = 18.13

$$\int \frac{\sinh^2(c+dx)}{(a+b\sinh^2(c+dx))^2}\,dx = \text{Too large to display}$$

input
```
integrate(sinh(d*x+c)^2/(a+b*sinh(d*x+c)^2)^2,x, algorithm="fricas")
```

output
```
[-1/4*(4*a^2*b - 4*a*b^2 + 4*(2*a^3 - 3*a^2*b + a*b^2)*cosh(d*x + c)^2 + 8
*(2*a^3 - 3*a^2*b + a*b^2)*cosh(d*x + c)*sinh(d*x + c) + 4*(2*a^3 - 3*a^2*
b + a*b^2)*sinh(d*x + c)^2 + (b^2*cosh(d*x + c)^4 + 4*b^2*cosh(d*x + c)*si
nh(d*x + c)^3 + b^2*sinh(d*x + c)^4 + 2*(2*a*b - b^2)*cosh(d*x + c)^2 + 2*
(3*b^2*cosh(d*x + c)^2 + 2*a*b - b^2)*sinh(d*x + c)^2 + b^2 + 4*(b^2*cosh(
d*x + c)^3 + (2*a*b - b^2)*cosh(d*x + c))*sinh(d*x + c))*sqrt(a^2 - a*b)*l
og((b^2*cosh(d*x + c)^4 + 4*b^2*cosh(d*x + c)*sinh(d*x + c)^3 + b^2*sinh(d
*x + c)^4 + 2*(2*a*b - b^2)*cosh(d*x + c)^2 + 2*(3*b^2*cosh(d*x + c)^2 + 2
*a*b - b^2)*sinh(d*x + c)^2 + 8*a^2 - 8*a*b + b^2 + 4*(b^2*cosh(d*x + c)^3
 + (2*a*b - b^2)*cosh(d*x + c))*sinh(d*x + c) - 4*(b*cosh(d*x + c)^2 + 2*b
*cosh(d*x + c)*sinh(d*x + c) + b*sinh(d*x + c)^2 + 2*a - b)*sqrt(a^2 - a*b
))/(b*cosh(d*x + c)^4 + 4*b*cosh(d*x + c)*sinh(d*x + c)^3 + b*sinh(d*x + c
)^4 + 2*(2*a - b)*cosh(d*x + c)^2 + 2*(3*b*cosh(d*x + c)^2 + 2*a - b)*sinh
(d*x + c)^2 + 4*(b*cosh(d*x + c)^3 + (2*a - b)*cosh(d*x + c))*sinh(d*x + c
) + b)))/((a^3*b^2 - 2*a^2*b^3 + a*b^4)*d*cosh(d*x + c)^4 + 4*(a^3*b^2 - 2
*a^2*b^3 + a*b^4)*d*cosh(d*x + c)*sinh(d*x + c)^3 + (a^3*b^2 - 2*a^2*b^3 +
 a*b^4)*d*sinh(d*x + c)^4 + 2*(2*a^4*b - 5*a^3*b^2 + 4*a^2*b^3 - a*b^4)*d*
cosh(d*x + c)^2 + 2*(3*(a^3*b^2 - 2*a^2*b^3 + a*b^4)*d*cosh(d*x + c)^2 + (
2*a^4*b - 5*a^3*b^2 + 4*a^2*b^3 - a*b^4)*d)*sinh(d*x + c)^2 + (a^3*b^2 - 2
*a^2*b^3 + a*b^4)*d + 4*((a^3*b^2 - 2*a^2*b^3 + a*b^4)*d*cosh(d*x + c)^...
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\sinh^2(c+dx)}{(a+b\sinh^2(c+dx))^2}\,dx = \text{Timed out}$$

input
```
integrate(sinh(d*x+c)**2/(a+b*sinh(d*x+c)**2)**2,x)
```

output
```
Timed out
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{\sinh^2(c+dx)}{(a+b\sinh^2(c+dx))^2}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate(sinh(d*x+c)^2/(a+b*sinh(d*x+c)^2)^2,x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(b-a>0)', see `assume?` for more
details)Is
```

Giac [A] (verification not implemented)

Time = 0.57 (sec) , antiderivative size = 135, normalized size of antiderivative = 1.61

$$\int \frac{\sinh^2(c+dx)}{(a+b\sinh^2(c+dx))^2}\,dx = -\frac{\frac{\arctan\left(\frac{be^{(2\,dx+2\,c)}+2\,a-b}{2\,\sqrt{-a^2+ab}}\right)}{\sqrt{-a^2+ab}(a-b)} + \frac{2\,\left(2\,ae^{(2\,dx+2\,c)}-be^{(2\,dx+2\,c)}+b\right)}{(ab-b^2)\left(be^{(4\,dx+4\,c)}+4\,ae^{(2\,dx+2\,c)}-2\,be^{(2\,dx+2\,c)}+b\right)}}{2\,d}$$

input
```
integrate(sinh(d*x+c)^2/(a+b*sinh(d*x+c)^2)^2,x, algorithm="giac")
```

output
```
-1/2*(arctan(1/2*(b*e^(2*d*x + 2*c) + 2*a - b)/sqrt(-a^2 + a*b))/(sqrt(-a^
2 + a*b)*(a - b)) + 2*(2*a*e^(2*d*x + 2*c) - b*e^(2*d*x + 2*c) + b)/((a*b
- b^2)*(b*e^(4*d*x + 4*c) + 4*a*e^(2*d*x + 2*c) - 2*b*e^(2*d*x + 2*c) + b)
))/d
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sinh^2(c+dx)}{\left(a+b\sinh^2(c+dx)\right)^2}\,dx = \int \frac{\sinh(c+d\,x)^2}{\left(b\sinh\left(c+d\,x\right)^2+a\right)^2}\,dx$$

input
```
int(sinh(c + d*x)^2/(a + b*sinh(c + d*x)^2)^2,x)
```

output
```
int(sinh(c + d*x)^2/(a + b*sinh(c + d*x)^2)^2, x)
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 743, normalized size of antiderivative = 8.85

$$\int \frac{\sinh^2(c+dx)}{\left(a+b\sinh^2(c+dx)\right)^2}\,dx$$
$$= \frac{-e^{4dx+4c}\sqrt{a}\,\sqrt{a-b}\log\left(-\sqrt{2\sqrt{a}\,\sqrt{a-b}-2a+b}+e^{dx+c}\sqrt{b}\right)b-e^{4dx+4c}\sqrt{a}\,\sqrt{a-b}\log\left(\sqrt{2\sqrt{a}\,\sqrt{a-}\right.}{}$$

input
```
int(sinh(d*x+c)^2/(a+b*sinh(d*x+c)^2)^2,x)
```

output
```
( - e**(4*c + 4*d*x)*sqrt(a)*sqrt(a - b)*log( - sqrt(2*sqrt(a)*sqrt(a - b)
 - 2*a + b) + e**(c + d*x)*sqrt(b))*b - e**(4*c + 4*d*x)*sqrt(a)*sqrt(a -
b)*log(sqrt(2*sqrt(a)*sqrt(a - b) - 2*a + b) + e**(c + d*x)*sqrt(b))*b + e
**(4*c + 4*d*x)*sqrt(a)*sqrt(a - b)*log(2*sqrt(a)*sqrt(a - b) + e**(2*c +
2*d*x)*b + 2*a - b)*b - 4*e**(2*c + 2*d*x)*sqrt(a)*sqrt(a - b)*log( - sqrt
(2*sqrt(a)*sqrt(a - b) - 2*a + b) + e**(c + d*x)*sqrt(b))*a + 2*e**(2*c +
2*d*x)*sqrt(a)*sqrt(a - b)*log( - sqrt(2*sqrt(a)*sqrt(a - b) - 2*a + b) +
e**(c + d*x)*sqrt(b))*b - 4*e**(2*c + 2*d*x)*sqrt(a)*sqrt(a - b)*log(sqrt(
2*sqrt(a)*sqrt(a - b) - 2*a + b) + e**(c + d*x)*sqrt(b))*a + 2*e**(2*c + 2
*d*x)*sqrt(a)*sqrt(a - b)*log(sqrt(2*sqrt(a)*sqrt(a - b) - 2*a + b) + e**(
c + d*x)*sqrt(b))*b + 4*e**(2*c + 2*d*x)*sqrt(a)*sqrt(a - b)*log(2*sqrt(a)
*sqrt(a - b) + e**(2*c + 2*d*x)*b + 2*a - b)*a - 2*e**(2*c + 2*d*x)*sqrt(a
)*sqrt(a - b)*log(2*sqrt(a)*sqrt(a - b) + e**(2*c + 2*d*x)*b + 2*a - b)*b
- sqrt(a)*sqrt(a - b)*log( - sqrt(2*sqrt(a)*sqrt(a - b) - 2*a + b) + e**(c
 + d*x)*sqrt(b))*b - sqrt(a)*sqrt(a - b)*log(sqrt(2*sqrt(a)*sqrt(a - b) -
2*a + b) + e**(c + d*x)*sqrt(b))*b + sqrt(a)*sqrt(a - b)*log(2*sqrt(a)*sqr
t(a - b) + e**(2*c + 2*d*x)*b + 2*a - b)*b + 2*e**(4*c + 4*d*x)*a**2 - 2*e
**(4*c + 4*d*x)*a*b - 2*a**2 + 2*a*b)/(4*a*d*(e**(4*c + 4*d*x)*a**2*b - 2*
e**(4*c + 4*d*x)*a*b**2 + e**(4*c + 4*d*x)*b**3 + 4*e**(2*c + 2*d*x)*a**3
- 10*e**(2*c + 2*d*x)*a**2*b + 8*e**(2*c + 2*d*x)*a*b**2 - 2*e**(2*c + ...
```

3.45 $\int \frac{\sinh(c+dx)}{\left(a+b\sinh^2(c+dx)\right)^2}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 81

$$\int \frac{\sinh(c+dx)}{\left(a+b\sinh^2(c+dx)\right)^2}\,dx = \frac{\arctan\left(\frac{\sqrt{b}\cosh(c+dx)}{\sqrt{a-b}}\right)}{2(a-b)^{3/2}\sqrt{b}d}+\frac{\cosh(c+dx)}{2(a-b)d\left(a-b+b\cosh^2(c+dx)\right)}$$

output
```
1/2*arctan(b^(1/2)*cosh(d*x+c)/(a-b)^(1/2))/(a-b)^(3/2)/b^(1/2)/d+1/2*cosh
(d*x+c)/(a-b)/d/(a-b+b*cosh(d*x+c)^2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.49 (sec) , antiderivative size = 130, normalized size of antiderivative = 1.60

$$\int \frac{\sinh(c+dx)}{\left(a+b\sinh^2(c+dx)\right)^2}\,dx = \frac{\frac{\arctan\left(\frac{\sqrt{b}-i\sqrt{a}\tanh\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a-b}}\right)+\arctan\left(\frac{\sqrt{b}+i\sqrt{a}\tanh\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a-b}}\right)}{(a-b)^{3/2}\sqrt{b}}+\frac{2\cosh(c+dx)}{(a-b)(2a-b+b\cosh(2(c+dx)))}}{2d}$$

input
```
Integrate[Sinh[c + d*x]/(a + b*Sinh[c + d*x]^2)^2,x]
```

output
```
((ArcTan[(Sqrt[b] - I*Sqrt[a]*Tanh[(c + d*x)/2])/Sqrt[a - b]] + ArcTan[(Sq
rt[b] + I*Sqrt[a]*Tanh[(c + d*x)/2])/Sqrt[a - b]])/((a - b)^(3/2)*Sqrt[b])
 + (2*Cosh[c + d*x])/((a - b)*(2*a - b + b*Cosh[2*(c + d*x)])))/(2*d)
```

Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 79, normalized size of antiderivative = 0.98, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.238$, Rules used = {3042, 26, 3665, 215, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sinh(c+dx)}{\left(a+b\sinh^2(c+dx)\right)^2}dx$$

$$\Big\downarrow \text{3042}$$

$$\int -\frac{i\sin(ic+idx)}{\left(a-b\sin(ic+idx)^2\right)^2}dx$$

$$\Big\downarrow \text{26}$$

$$-i\int \frac{\sin(ic+idx)}{\left(a-b\sin(ic+idx)^2\right)^2}dx$$

$$\Big\downarrow \text{3665}$$

$$\frac{\int \frac{1}{\left(b\cosh^2(c+dx)+a-b\right)^2}d\cosh(c+dx)}{d}$$

$$\Big\downarrow \text{215}$$

$$\frac{\frac{\int \frac{1}{b\cosh^2(c+dx)+a-b}d\cosh(c+dx)}{2(a-b)}+\frac{\cosh(c+dx)}{2(a-b)\left(a+b\cosh^2(c+dx)-b\right)}}{d}$$

$$\Big\downarrow \text{218}$$

$$\frac{\frac{\arctan\left(\frac{\sqrt{b}\cosh(c+dx)}{\sqrt{a-b}}\right)}{2\sqrt{b}(a-b)^{3/2}}+\frac{\cosh(c+dx)}{2(a-b)\left(a+b\cosh^2(c+dx)-b\right)}}{d}$$

input
```
Int[Sinh[c + d*x]/(a + b*Sinh[c + d*x]^2)^2,x]
```

output
```
(ArcTan[(Sqrt[b]*Cosh[c + d*x])/Sqrt[a - b]]/(2*(a - b)^(3/2)*Sqrt[b]) + C
osh[c + d*x]/(2*(a - b)*(a - b + b*Cosh[c + d*x]^2)))/d
```

Defintions of rubi rules used

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 215
```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(-x)*((a + b*x^2)^(p + 1)
/(2*a*(p + 1))), x] + Simp[(2*p + 3)/(2*a*(p + 1))   Int[(a + b*x^2)^(p + 1
), x], x] /; FreeQ[{a, b}, x] && LtQ[p, -1] && (IntegerQ[4*p] || IntegerQ[6
*p])
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3665
```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^
(p_.), x_Symbol] :> With[{ff = FreeFactors[Cos[e + f*x], x]}, Simp[-ff/f
Subst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a + b - b*ff^2*x^2)^p, x], x, Cos[e +
 f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m - 1)/2]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 145 vs. $2(69) = 138$.

Time = 0.66 (sec) , antiderivative size = 146, normalized size of antiderivative = 1.80

method	result
derivativedivides	$\frac{\frac{-\frac{(a-2b)\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}}{(a-b)a}+\frac{2}{2a-2b}}{\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^{4}a-2\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}a+4b\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}+a}+\frac{\arctan\left(\frac{2\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}a-2a+4b}{4\sqrt{ab-b^{2}}}\right)}{2(a-b)\sqrt{ab-b^{2}}}}{d}$
default	$\frac{\frac{-\frac{(a-2b)\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}}{(a-b)a}+\frac{2}{2a-2b}}{\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^{4}a-2\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}a+4b\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}+a}+\frac{\arctan\left(\frac{2\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}a-2a+4b}{4\sqrt{ab-b^{2}}}\right)}{2(a-b)\sqrt{ab-b^{2}}}}{d}$
risch	$\frac{e^{dx+c}\left(e^{2dx+2c}+1\right)}{d(a-b)\left(b\,e^{4dx+4c}+4\,e^{2dx+2c}a-2\,e^{2dx+2c}b+b\right)}-\frac{\ln\left(e^{2dx+2c}-\frac{2(a-b)e^{dx+c}}{\sqrt{-ab+b^{2}}}+1\right)}{4\sqrt{-ab+b^{2}}\,(a-b)d}+\frac{\ln\left(e^{2dx+2c}+\frac{2(a-b)e^{dx+c}}{\sqrt{-ab+b^{2}}}+1\right)}{4\sqrt{-ab+b^{2}}\,(a-b)d}$

input
```
int(sinh(d*x+c)/(a+b*sinh(d*x+c)^2)^2,x,method=_RETURNVERBOSE)
```

output
```
1/d*(2*(-1/2*(a-2*b)/(a-b)/a*tanh(1/2*d*x+1/2*c)^2+1/2/(a-b))/(tanh(1/2*d*
x+1/2*c)^4*a-2*tanh(1/2*d*x+1/2*c)^2*a+4*b*tanh(1/2*d*x+1/2*c)^2+a)+1/2/(a
-b)/(a*b-b^2)^(1/2)*arctan(1/4*(2*tanh(1/2*d*x+1/2*c)^2*a-2*a+4*b)/(a*b-b^
2)^(1/2)))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 786 vs. $2(69) = 138$.

Time = 0.14 (sec) , antiderivative size = 1627, normalized size of antiderivative = 20.09

$$\int \frac{\sinh(c+dx)}{\left(a+b\sinh^2(c+dx)\right)^2}\,dx = \text{Too large to display}$$

input
```
integrate(sinh(d*x+c)/(a+b*sinh(d*x+c)^2)^2,x, algorithm="fricas")
```

output
```
[1/4*(4*(a*b - b^2)*cosh(d*x + c)^3 + 12*(a*b - b^2)*cosh(d*x + c)*sinh(d*
x + c)^2 + 4*(a*b - b^2)*sinh(d*x + c)^3 + (b*cosh(d*x + c)^4 + 4*b*cosh(d
*x + c)*sinh(d*x + c)^3 + b*sinh(d*x + c)^4 + 2*(2*a - b)*cosh(d*x + c)^2
+ 2*(3*b*cosh(d*x + c)^2 + 2*a - b)*sinh(d*x + c)^2 + 4*(b*cosh(d*x + c)^3
 + (2*a - b)*cosh(d*x + c))*sinh(d*x + c) + b)*sqrt(-a*b + b^2)*log((b*cos
h(d*x + c)^4 + 4*b*cosh(d*x + c)*sinh(d*x + c)^3 + b*sinh(d*x + c)^4 - 2*(
2*a - 3*b)*cosh(d*x + c)^2 + 2*(3*b*cosh(d*x + c)^2 - 2*a + 3*b)*sinh(d*x
+ c)^2 + 4*(b*cosh(d*x + c)^3 - (2*a - 3*b)*cosh(d*x + c))*sinh(d*x + c) +
 4*(cosh(d*x + c)^3 + 3*cosh(d*x + c)*sinh(d*x + c)^2 + sinh(d*x + c)^3 +
(3*cosh(d*x + c)^2 + 1)*sinh(d*x + c) + cosh(d*x + c))*sqrt(-a*b + b^2) +
b)/(b*cosh(d*x + c)^4 + 4*b*cosh(d*x + c)*sinh(d*x + c)^3 + b*sinh(d*x + c
)^4 + 2*(2*a - b)*cosh(d*x + c)^2 + 2*(3*b*cosh(d*x + c)^2 + 2*a - b)*sinh
(d*x + c)^2 + 4*(b*cosh(d*x + c)^3 + (2*a - b)*cosh(d*x + c))*sinh(d*x + c
) + b)) + 4*(a*b - b^2)*cosh(d*x + c) + 4*(3*(a*b - b^2)*cosh(d*x + c)^2 +
 a*b - b^2)*sinh(d*x + c))/((a^2*b^2 - 2*a*b^3 + b^4)*d*cosh(d*x + c)^4 +
4*(a^2*b^2 - 2*a*b^3 + b^4)*d*cosh(d*x + c)*sinh(d*x + c)^3 + (a^2*b^2 - 2
*a*b^3 + b^4)*d*sinh(d*x + c)^4 + 2*(2*a^3*b - 5*a^2*b^2 + 4*a*b^3 - b^4)*
d*cosh(d*x + c)^2 + 2*(3*(a^2*b^2 - 2*a*b^3 + b^4)*d*cosh(d*x + c)^2 + (2*
a^3*b - 5*a^2*b^2 + 4*a*b^3 - b^4)*d)*sinh(d*x + c)^2 + (a^2*b^2 - 2*a*b^3
 + b^4)*d + 4*((a^2*b^2 - 2*a*b^3 + b^4)*d*cosh(d*x + c)^3 + (2*a^3*b -...
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\sinh(c+dx)}{\left(a+b\sinh^2(c+dx)\right)^2}\,dx = \text{Timed out}$$

input
```
integrate(sinh(d*x+c)/(a+b*sinh(d*x+c)**2)**2,x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{\sinh(c+dx)}{\left(a+b\sinh^2(c+dx)\right)^2}\,dx = \int \frac{\sinh\left(dx+c\right)}{\left(b\sinh\left(dx+c\right)^2+a\right)^2}\,dx$$

input
```
integrate(sinh(d*x+c)/(a+b*sinh(d*x+c)^2)^2,x, algorithm="maxima")
```

output
```
(e^(3*d*x + 3*c) + e^(d*x + c))/(a*b*d - b^2*d + (a*b*d*e^(4*c) - b^2*d*e^
(4*c))*e^(4*d*x) + 2*(2*a^2*d*e^(2*c) - 3*a*b*d*e^(2*c) + b^2*d*e^(2*c))*e
^(2*d*x)) + 1/2*integrate(2*(e^(3*d*x + 3*c) - e^(d*x + c))/(a*b - b^2 + (
a*b*e^(4*c) - b^2*e^(4*c))*e^(4*d*x) + 2*(2*a^2*e^(2*c) - 3*a*b*e^(2*c) +
b^2*e^(2*c))*e^(2*d*x)), x)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{\sinh(c+dx)}{\left(a+b\sinh^2(c+dx)\right)^2}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(sinh(d*x+c)/(a+b*sinh(d*x+c)^2)^2,x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Limit: Max order reached or unable
to make series expansion Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sinh(c+dx)}{\left(a+b\sinh^2(c+dx)\right)^2}\,dx = \int \frac{\sinh(c+d\,x)}{\left(b\sinh\left(c+d\,x\right)^2+a\right)^2}\,dx$$

input
```
int(sinh(c + d*x)/(a + b*sinh(c + d*x)^2)^2,x)
```

output
```
int(sinh(c + d*x)/(a + b*sinh(c + d*x)^2)^2, x)
```

Reduce [B] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 2279, normalized size of antiderivative = 28.14

$$\int \frac{\sinh(c+dx)}{\left(a+b\sinh^2(c+dx)\right)^2}\,dx = \text{Too large to display}$$

input
```
int(sinh(d*x+c)/(a+b*sinh(d*x+c)^2)^2,x)
```

output
```
(2*e**(4*c + 4*d*x)*sqrt(b)*sqrt(a)*sqrt(a - b)*sqrt(2*sqrt(a)*sqrt(a - b)
 + 2*a - b)*atan((e**(c + d*x)*b)/(sqrt(b)*sqrt(2*sqrt(a)*sqrt(a - b) + 2*
a - b)))*b + 8*e**(2*c + 2*d*x)*sqrt(b)*sqrt(a)*sqrt(a - b)*sqrt(2*sqrt(a)
*sqrt(a - b) + 2*a - b)*atan((e**(c + d*x)*b)/(sqrt(b)*sqrt(2*sqrt(a)*sqrt
(a - b) + 2*a - b)))*a - 4*e**(2*c + 2*d*x)*sqrt(b)*sqrt(a)*sqrt(a - b)*sq
rt(2*sqrt(a)*sqrt(a - b) + 2*a - b)*atan((e**(c + d*x)*b)/(sqrt(b)*sqrt(2*
sqrt(a)*sqrt(a - b) + 2*a - b)))*b + 2*sqrt(b)*sqrt(a)*sqrt(a - b)*sqrt(2*
sqrt(a)*sqrt(a - b) + 2*a - b)*atan((e**(c + d*x)*b)/(sqrt(b)*sqrt(2*sqrt(
a)*sqrt(a - b) + 2*a - b)))*b - 2*e**(4*c + 4*d*x)*sqrt(b)*sqrt(2*sqrt(a)*
sqrt(a - b) + 2*a - b)*atan((e**(c + d*x)*b)/(sqrt(b)*sqrt(2*sqrt(a)*sqrt(
a - b) + 2*a - b)))*a*b + 2*e**(4*c + 4*d*x)*sqrt(b)*sqrt(2*sqrt(a)*sqrt(a
 - b) + 2*a - b)*atan((e**(c + d*x)*b)/(sqrt(b)*sqrt(2*sqrt(a)*sqrt(a - b)
 + 2*a - b)))*b**2 - 8*e**(2*c + 2*d*x)*sqrt(b)*sqrt(2*sqrt(a)*sqrt(a - b)
 + 2*a - b)*atan((e**(c + d*x)*b)/(sqrt(b)*sqrt(2*sqrt(a)*sqrt(a - b) + 2*
a - b)))*a**2 + 12*e**(2*c + 2*d*x)*sqrt(b)*sqrt(2*sqrt(a)*sqrt(a - b) + 2
*a - b)*atan((e**(c + d*x)*b)/(sqrt(b)*sqrt(2*sqrt(a)*sqrt(a - b) + 2*a -
b)))*a*b - 4*e**(2*c + 2*d*x)*sqrt(b)*sqrt(2*sqrt(a)*sqrt(a - b) + 2*a - b
)*atan((e**(c + d*x)*b)/(sqrt(b)*sqrt(2*sqrt(a)*sqrt(a - b) + 2*a - b)))*b
**2 - 2*sqrt(b)*sqrt(2*sqrt(a)*sqrt(a - b) + 2*a - b)*atan((e**(c + d*x)*b
)/(sqrt(b)*sqrt(2*sqrt(a)*sqrt(a - b) + 2*a - b)))*a*b + 2*sqrt(b)*sqrt...
```

3.46 $\int \frac{1}{\left(a+b\sinh^2(c+dx)\right)^2}\,dx$

Optimal result

Integrand size = 14, antiderivative size = 95

$$\int \frac{1}{\left(a+b\sinh^2(c+dx)\right)^2}\,dx = \frac{(2a-b)\text{arctanh}\left(\frac{\sqrt{a-b}\tanh(c+dx)}{\sqrt{a}}\right)}{2a^{3/2}(a-b)^{3/2}d} - \frac{b\cosh(c+dx)\sinh(c+dx)}{2a(a-b)d\left(a+b\sinh^2(c+dx)\right)}$$

output
```
1/2*(2*a-b)*arctanh((a-b)^(1/2)*tanh(d*x+c)/a^(1/2))/a^(3/2)/(a-b)^(3/2)/d
-1/2*b*cosh(d*x+c)*sinh(d*x+c)/a/(a-b)/d/(a+b*sinh(d*x+c)^2)
```

Mathematica [A] (verified)

Time = 11.13 (sec) , antiderivative size = 96, normalized size of antiderivative = 1.01

$$\int \frac{1}{\left(a+b\sinh^2(c+dx)\right)^2}\,dx = \frac{(2a-b)\text{arctanh}\left(\frac{\sqrt{a-b}\tanh(c+dx)}{\sqrt{a}}\right)}{2a^{3/2}(a-b)^{3/2}d} - \frac{b\sinh(2(c+dx))}{2a(a-b)d(2a-b+b\cosh(2(c+dx)))}$$

input
```
Integrate[(a + b*Sinh[c + d*x]^2)^(-2),x]
```

output
```
((2*a - b)*ArcTanh[(Sqrt[a - b]*Tanh[c + d*x])/Sqrt[a]])/(2*a^(3/2)*(a - b
)^(3/2)*d) - (b*Sinh[2*(c + d*x)])/(2*a*(a - b)*d*(2*a - b + b*Cosh[2*(c +
 d*x)]))
```

Rubi [A] (verified)

Time = 0.35 (sec) , antiderivative size = 95, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$, Rules used = {3042, 3663, 25, 27, 3042, 3660, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\left(a+b\sinh^2(c+dx)\right)^2}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{(a-b\sin(ic+idx)^2)^2}dx$$

$$\Big\downarrow \text{3663}$$

$$-\frac{\int -\frac{2a-b}{b\sinh^2(c+dx)+a}dx}{2a(a-b)}-\frac{b\sinh(c+dx)\cosh(c+dx)}{2ad(a-b)\left(a+b\sinh^2(c+dx)\right)}$$

$$\Big\downarrow \text{25}$$

$$\frac{\int \frac{2a-b}{b\sinh^2(c+dx)+a}dx}{2a(a-b)}-\frac{b\sinh(c+dx)\cosh(c+dx)}{2ad(a-b)\left(a+b\sinh^2(c+dx)\right)}$$

$$\Big\downarrow \text{27}$$

$$\frac{(2a-b)\int \frac{1}{b\sinh^2(c+dx)+a}dx}{2a(a-b)}-\frac{b\sinh(c+dx)\cosh(c+dx)}{2ad(a-b)\left(a+b\sinh^2(c+dx)\right)}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{b\sinh(c+dx)\cosh(c+dx)}{2ad(a-b)\left(a+b\sinh^2(c+dx)\right)}+\frac{(2a-b)\int \frac{1}{a-b\sin(ic+idx)^2}dx}{2a(a-b)}$$

$$\Big\downarrow \text{3660}$$

$$\frac{(2a-b)\int \frac{1}{a-(a-b)\tanh^2(c+dx)}d\tanh(c+dx)}{2ad(a-b)}-\frac{b\sinh(c+dx)\cosh(c+dx)}{2ad(a-b)\left(a+b\sinh^2(c+dx)\right)}$$

$$\Big\downarrow \text{221}$$

$$\frac{(2a-b)\text{arctanh}\left(\frac{\sqrt{a-b}\tanh(c+dx)}{\sqrt{a}}\right)}{2a^{3/2}d(a-b)^{3/2}}-\frac{b\sinh(c+dx)\cosh(c+dx)}{2ad(a-b)\left(a+b\sinh^2(c+dx)\right)}$$

input
```
Int[(a + b*Sinh[c + d*x]^2)^(-2),x]
```

output
```
((2*a - b)*ArcTanh[(Sqrt[a - b]*Tanh[c + d*x])/Sqrt[a]])/(2*a^(3/2)*(a - b
)^(3/2)*d) - (b*Cosh[c + d*x]*Sinh[c + d*x])/(2*a*(a - b)*d*(a + b*Sinh[c
+ d*x]^2))
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3660
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(-1), x_Symbol] :> With[{ff =
FreeFactors[Tan[e + f*x], x]}, Simp[ff/f   Subst[Int[1/(a + (a + b)*ff^2*x^
2), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f}, x]
```

rule 3663
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> Simp[(-b)*C
os[e + f*x]*Sin[e + f*x]*((a + b*Sin[e + f*x]^2)^(p + 1)/(2*a*f*(p + 1)*(a
+ b))), x] + Simp[1/(2*a*(p + 1)*(a + b))   Int[(a + b*Sin[e + f*x]^2)^(p +
 1)*Simp[2*a*(p + 1) + b*(2*p + 3) - 2*b*(p + 2)*Sin[e + f*x]^2, x], x], x]
 /; FreeQ[{a, b, e, f}, x] && NeQ[a + b, 0] && LtQ[p, -1]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 284 vs. $2(83) = 166$.

Time = 0.66 (sec) , antiderivative size = 285, normalized size of antiderivative = 3.00

method	result
derivativedivides	$\frac{-\frac{2\left(\frac{b\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^{3}}{2a(a-b)}+\frac{b\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)}{2a(a-b)}\right)}{\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^{4}a-2\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}a+4b\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}+a}-\frac{(2a-b)\left(\frac{\left(\sqrt{-b(a-b)}+b\right)\arctan\left(\frac{\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)a}{\sqrt{\left(2\sqrt{-b(a-b)}-a+2b\right)a}}\right)}{2a\sqrt{-b(a-b)}\sqrt{\left(2\sqrt{-b(a-b)}-a+2b\right)a}}-\frac{\left(\sqrt{\cdots}\right.}{\cdots}\right)}{a-b}}{d}$
default	$\frac{-\frac{2\left(\frac{b\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^{3}}{2a(a-b)}+\frac{b\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)}{2a(a-b)}\right)}{\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^{4}a-2\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}a+4b\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}+a}-\frac{(2a-b)\left(\frac{\left(\sqrt{-b(a-b)}+b\right)\arctan\left(\frac{\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)a}{\sqrt{\left(2\sqrt{-b(a-b)}-a+2b\right)a}}\right)}{2a\sqrt{-b(a-b)}\sqrt{\left(2\sqrt{-b(a-b)}-a+2b\right)a}}-\frac{\left(\sqrt{\cdots}\right.}{\cdots}\right)}{a-b}}{d}$
risch	$\frac{2\,e^{2dx+2c}a-e^{2dx+2c}b+b}{da(a-b)\left(b\,e^{4dx+4c}+4\,e^{2dx+2c}a-2\,e^{2dx+2c}b+b\right)}+\frac{\ln\left(e^{2dx+2c}+\frac{2a\sqrt{a^{2}-ab}-b\sqrt{a^{2}-ab}-2a^{2}+2ab}{b\sqrt{a^{2}-ab}}\right)}{2\sqrt{a^{2}-ab}\,(a-b)d}-\frac{\ln\left(e^{2dx+2c}+\cdots\right.}{\cdots}$

input
```
int(1/(a+b*sinh(d*x+c)^2)^2,x,method=_RETURNVERBOSE)
```

output
```
1/d*(-2*(1/2*b/a/(a-b)*tanh(1/2*d*x+1/2*c)^3+1/2*b/a/(a-b)*tanh(1/2*d*x+1/
2*c))/(tanh(1/2*d*x+1/2*c)^4*a-2*tanh(1/2*d*x+1/2*c)^2*a+4*b*tanh(1/2*d*x+
1/2*c)^2+a)-(2*a-b)/(a-b)*(1/2*((-b*(a-b))^(1/2)+b)/a/(-b*(a-b))^(1/2)/((2
*(-b*(a-b))^(1/2)-a+2*b)*a)^(1/2)*arctan(tanh(1/2*d*x+1/2*c)*a/((2*(-b*(a-
b))^(1/2)-a+2*b)*a)^(1/2))-1/2*((-b*(a-b))^(1/2)-b)/a/(-b*(a-b))^(1/2)/((2
*(-b*(a-b))^(1/2)+a-2*b)*a)^(1/2)*arctanh(tanh(1/2*d*x+1/2*c)*a/((2*(-b*(a
-b))^(1/2)+a-2*b)*a)^(1/2))))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 681 vs. $2(83) = 166$.

Time = 0.13 (sec) , antiderivative size = 1617, normalized size of antiderivative = 17.02

$$\int \frac{1}{\left(a+b\sinh^2(c+dx)\right)^2}\,dx = \text{Too large to display}$$

input
```
integrate(1/(a+b*sinh(d*x+c)^2)^2,x, algorithm="fricas")
```

output
```
[1/4*(4*a^2*b - 4*a*b^2 + 4*(2*a^3 - 3*a^2*b + a*b^2)*cosh(d*x + c)^2 + 8*
(2*a^3 - 3*a^2*b + a*b^2)*cosh(d*x + c)*sinh(d*x + c) + 4*(2*a^3 - 3*a^2*b
 + a*b^2)*sinh(d*x + c)^2 + ((2*a*b - b^2)*cosh(d*x + c)^4 + 4*(2*a*b - b^
2)*cosh(d*x + c)*sinh(d*x + c)^3 + (2*a*b - b^2)*sinh(d*x + c)^4 + 2*(4*a^
2 - 4*a*b + b^2)*cosh(d*x + c)^2 + 2*(3*(2*a*b - b^2)*cosh(d*x + c)^2 + 4*
a^2 - 4*a*b + b^2)*sinh(d*x + c)^2 + 2*a*b - b^2 + 4*((2*a*b - b^2)*cosh(d
*x + c)^3 + (4*a^2 - 4*a*b + b^2)*cosh(d*x + c))*sinh(d*x + c))*sqrt(a^2 -
 a*b)*log((b^2*cosh(d*x + c)^4 + 4*b^2*cosh(d*x + c)*sinh(d*x + c)^3 + b^2
*sinh(d*x + c)^4 + 2*(2*a*b - b^2)*cosh(d*x + c)^2 + 2*(3*b^2*cosh(d*x + c
)^2 + 2*a*b - b^2)*sinh(d*x + c)^2 + 8*a^2 - 8*a*b + b^2 + 4*(b^2*cosh(d*x
 + c)^3 + (2*a*b - b^2)*cosh(d*x + c))*sinh(d*x + c) - 4*(b*cosh(d*x + c)^
2 + 2*b*cosh(d*x + c)*sinh(d*x + c) + b*sinh(d*x + c)^2 + 2*a - b)*sqrt(a^
2 - a*b))/(b*cosh(d*x + c)^4 + 4*b*cosh(d*x + c)*sinh(d*x + c)^3 + b*sinh(
d*x + c)^4 + 2*(2*a - b)*cosh(d*x + c)^2 + 2*(3*b*cosh(d*x + c)^2 + 2*a -
b)*sinh(d*x + c)^2 + 4*(b*cosh(d*x + c)^3 + (2*a - b)*cosh(d*x + c))*sinh(
d*x + c) + b)))/((a^4*b - 2*a^3*b^2 + a^2*b^3)*d*cosh(d*x + c)^4 + 4*(a^4*
b - 2*a^3*b^2 + a^2*b^3)*d*cosh(d*x + c)*sinh(d*x + c)^3 + (a^4*b - 2*a^3*
b^2 + a^2*b^3)*d*sinh(d*x + c)^4 + 2*(2*a^5 - 5*a^4*b + 4*a^3*b^2 - a^2*b^
3)*d*cosh(d*x + c)^2 + 2*(3*(a^4*b - 2*a^3*b^2 + a^2*b^3)*d*cosh(d*x + c)^
2 + (2*a^5 - 5*a^4*b + 4*a^3*b^2 - a^2*b^3)*d)*sinh(d*x + c)^2 + (a^4*b...
```

Sympy [F(-1)]

Timed out.

$$\int \frac{1}{\left(a+b\sinh^2(c+dx)\right)^2}\,dx = \text{Timed out}$$

input
```
integrate(1/(a+b*sinh(d*x+c)**2)**2,x)
```

output
```
Timed out
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{1}{\left(a+b\sinh^2(c+dx)\right)^2}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate(1/(a+b*sinh(d*x+c)^2)^2,x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(b-a>0)', see `assume?` for more
details)Is
```

Giac [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 144, normalized size of antiderivative = 1.52

$$\int \frac{1}{\left(a+b\sinh^2(c+dx)\right)^2}\,dx = \frac{\frac{(2a-b)\arctan\left(\frac{be^{(2dx+2c)}+2a-b}{2\sqrt{-a^2+ab}}\right)}{(a^2-ab)\sqrt{-a^2+ab}} + \frac{2\left(2ae^{(2dx+2c)}-be^{(2dx+2c)}+b\right)}{(a^2-ab)\left(be^{(4dx+4c)}+4ae^{(2dx+2c)}-2be^{(2dx+2c)}+b\right)}}{2d}$$

input
```
integrate(1/(a+b*sinh(d*x+c)^2)^2,x, algorithm="giac")
```

output
```
1/2*((2*a - b)*arctan(1/2*(b*e^(2*d*x + 2*c) + 2*a - b)/sqrt(-a^2 + a*b))/
((a^2 - a*b)*sqrt(-a^2 + a*b)) + 2*(2*a*e^(2*d*x + 2*c) - b*e^(2*d*x + 2*c
) + b)/((a^2 - a*b)*(b*e^(4*d*x + 4*c) + 4*a*e^(2*d*x + 2*c) - 2*b*e^(2*d*
x + 2*c) + b)))/d
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\left(a+b\sinh^2(c+dx)\right)^2}\,dx=\int \frac{1}{\left(b\sinh\left(c+d\,x\right)^2+a\right)^2}\,dx$$

input
```
int(1/(a + b*sinh(c + d*x)^2)^2,x)
```

output
```
int(1/(a + b*sinh(c + d*x)^2)^2, x)
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 1205, normalized size of antiderivative = 12.68

$$\int \frac{1}{\left(a+b\sinh^2(c+dx)\right)^2}\,dx=\text{Too large to display}$$

input
```
int(1/(a+b*sinh(d*x+c)^2)^2,x)
```

output
```
(2*e**(4*c + 4*d*x)*sqrt(a)*sqrt(a - b)*log( - sqrt(2*sqrt(a)*sqrt(a - b)
- 2*a + b) + e**(c + d*x)*sqrt(b))*a*b - e**(4*c + 4*d*x)*sqrt(a)*sqrt(a -
 b)*log( - sqrt(2*sqrt(a)*sqrt(a - b) - 2*a + b) + e**(c + d*x)*sqrt(b))*b
**2 + 2*e**(4*c + 4*d*x)*sqrt(a)*sqrt(a - b)*log(sqrt(2*sqrt(a)*sqrt(a - b
) - 2*a + b) + e**(c + d*x)*sqrt(b))*a*b - e**(4*c + 4*d*x)*sqrt(a)*sqrt(a
 - b)*log(sqrt(2*sqrt(a)*sqrt(a - b) - 2*a + b) + e**(c + d*x)*sqrt(b))*b*
*2 - 2*e**(4*c + 4*d*x)*sqrt(a)*sqrt(a - b)*log(2*sqrt(a)*sqrt(a - b) + e*
*(2*c + 2*d*x)*b + 2*a - b)*a*b + e**(4*c + 4*d*x)*sqrt(a)*sqrt(a - b)*log
(2*sqrt(a)*sqrt(a - b) + e**(2*c + 2*d*x)*b + 2*a - b)*b**2 + 8*e**(2*c +
2*d*x)*sqrt(a)*sqrt(a - b)*log( - sqrt(2*sqrt(a)*sqrt(a - b) - 2*a + b) +
e**(c + d*x)*sqrt(b))*a**2 - 8*e**(2*c + 2*d*x)*sqrt(a)*sqrt(a - b)*log( -
 sqrt(2*sqrt(a)*sqrt(a - b) - 2*a + b) + e**(c + d*x)*sqrt(b))*a*b + 2*e**
(2*c + 2*d*x)*sqrt(a)*sqrt(a - b)*log( - sqrt(2*sqrt(a)*sqrt(a - b) - 2*a
+ b) + e**(c + d*x)*sqrt(b))*b**2 + 8*e**(2*c + 2*d*x)*sqrt(a)*sqrt(a - b)
*log(sqrt(2*sqrt(a)*sqrt(a - b) - 2*a + b) + e**(c + d*x)*sqrt(b))*a**2 -
8*e**(2*c + 2*d*x)*sqrt(a)*sqrt(a - b)*log(sqrt(2*sqrt(a)*sqrt(a - b) - 2*
a + b) + e**(c + d*x)*sqrt(b))*a*b + 2*e**(2*c + 2*d*x)*sqrt(a)*sqrt(a - b
)*log(sqrt(2*sqrt(a)*sqrt(a - b) - 2*a + b) + e**(c + d*x)*sqrt(b))*b**2 -
 8*e**(2*c + 2*d*x)*sqrt(a)*sqrt(a - b)*log(2*sqrt(a)*sqrt(a - b) + e**(2*
c + 2*d*x)*b + 2*a - b)*a**2 + 8*e**(2*c + 2*d*x)*sqrt(a)*sqrt(a - b)*l...
```

3.47 $\int \frac{\operatorname{csch}(c+dx)}{\left(a+b\sinh^2(c+dx)\right)^2}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 110

$$\int \frac{\operatorname{csch}(c+dx)}{\left(a+b\sinh^2(c+dx)\right)^2}\,dx = -\frac{(3a-2b)\sqrt{b}\arctan\left(\frac{\sqrt{b}\cosh(c+dx)}{\sqrt{a-b}}\right)}{2a^2(a-b)^{3/2}d} - \frac{\operatorname{arctanh}(\cosh(c+dx))}{a^2d} - \frac{b\cosh(c+dx)}{2a(a-b)d\left(a-b+b\cosh^2(c+dx)\right)}$$

output
```
-1/2*(3*a-2*b)*b^(1/2)*arctan(b^(1/2)*cosh(d*x+c)/(a-b)^(1/2))/a^2/(a-b)^(
3/2)/d-arctanh(cosh(d*x+c))/a^2/d-1/2*b*cosh(d*x+c)/a/(a-b)/d/(a-b+b*cosh(
d*x+c)^2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 5.74 (sec) , antiderivative size = 189, normalized size of antiderivative = 1.72

$$\int \frac{\operatorname{csch}(c+dx)}{\left(a+b\sinh^2(c+dx)\right)^2}\,dx$$
$$= \frac{\frac{\sqrt{b}(-3a+2b)\arctan\left(\frac{\sqrt{b}-i\sqrt{a}\tanh\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a-b}}\right)}{(a-b)^{3/2}} + \frac{\sqrt{b}(-3a+2b)\arctan\left(\frac{\sqrt{b}+i\sqrt{a}\tanh\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a-b}}\right)}{(a-b)^{3/2}} - \frac{2ab\cosh(c+dx)}{(a-b)(2a-b+b\cosh(2(c+dx)))} - 2\,\mathrm{lo}}{2a^2d}$$

input
```
Integrate[Csch[c + d*x]/(a + b*Sinh[c + d*x]^2)^2,x]
```

output
```
((Sqrt[b]*(-3*a + 2*b)*ArcTan[(Sqrt[b] - I*Sqrt[a]*Tanh[(c + d*x)/2])/Sqrt
[a - b]])/(a - b)^(3/2) + (Sqrt[b]*(-3*a + 2*b)*ArcTan[(Sqrt[b] + I*Sqrt[a
]*Tanh[(c + d*x)/2])/Sqrt[a - b]])/(a - b)^(3/2) - (2*a*b*Cosh[c + d*x])/(
(a - b)*(2*a - b + b*Cosh[2*(c + d*x)])) - 2*Log[Cosh[(c + d*x)/2]] + 2*Lo
g[Sinh[(c + d*x)/2]])/(2*a^2*d)
```

Rubi [A] (verified)

Time = 0.37 (sec) , antiderivative size = 123, normalized size of antiderivative = 1.12, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.381$, Rules used = {3042, 26, 3665, 316, 25, 397, 218, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\operatorname{csch}(c+dx)}{\left(a+b\sinh^2(c+dx)\right)^2}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{i}{\sin(ic+idx)\left(a-b\sin(ic+idx)^2\right)^2}dx$$

$$\Big\downarrow \text{26}$$

$$i\int \frac{1}{\sin(ic+idx)\left(a-b\sin(ic+idx)^2\right)^2}dx$$

$\big\downarrow$ 3665

$$-\frac{\int \frac{1}{(1-\cosh^2(c+dx))\left(b\cosh^2(c+dx)+a-b\right)^2}d\cosh(c+dx)}{d}$$

$\big\downarrow$ 316

$$-\frac{\frac{b\cosh(c+dx)}{2a(a-b)\left(a+b\cosh^2(c+dx)-b\right)}-\frac{\int -\frac{-b\cosh^2(c+dx)+2a-b}{\left(1-\cosh^2(c+dx)\right)\left(b\cosh^2(c+dx)+a-b\right)}d\cosh(c+dx)}{2a(a-b)}}{d}$$

$\big\downarrow$ 25

$$-\frac{\frac{\int \frac{-b\cosh^2(c+dx)+2a-b}{\left(1-\cosh^2(c+dx)\right)\left(b\cosh^2(c+dx)+a-b\right)}d\cosh(c+dx)}{2a(a-b)}+\frac{b\cosh(c+dx)}{2a(a-b)\left(a+b\cosh^2(c+dx)-b\right)}}{d}$$

$\big\downarrow$ 397

$$-\frac{\frac{\frac{2(a-b)\int \frac{1}{1-\cosh^2(c+dx)}d\cosh(c+dx)}{a}+\frac{b(3a-2b)\int \frac{1}{b\cosh^2(c+dx)+a-b}d\cosh(c+dx)}{a}}{2a(a-b)}+\frac{b\cosh(c+dx)}{2a(a-b)\left(a+b\cosh^2(c+dx)-b\right)}}{d}$$

$\big\downarrow$ 218

$$-\frac{\frac{\frac{2(a-b)\int \frac{1}{1-\cosh^2(c+dx)}d\cosh(c+dx)}{a}+\frac{\sqrt{b}(3a-2b)\arctan\left(\frac{\sqrt{b}\cosh(c+dx)}{\sqrt{a-b}}\right)}{a\sqrt{a-b}}}{2a(a-b)}+\frac{b\cosh(c+dx)}{2a(a-b)\left(a+b\cosh^2(c+dx)-b\right)}}{d}$$

$\big\downarrow$ 219

$$-\frac{\frac{\frac{\sqrt{b}(3a-2b)\arctan\left(\frac{\sqrt{b}\cosh(c+dx)}{\sqrt{a-b}}\right)}{a\sqrt{a-b}}+\frac{2(a-b)\text{arctanh}(\cosh(c+dx))}{a}}{2a(a-b)}+\frac{b\cosh(c+dx)}{2a(a-b)\left(a+b\cosh^2(c+dx)-b\right)}}{d}$$

input
```
Int[Csch[c + d*x]/(a + b*Sinh[c + d*x]^2)^2,x]
```

output
```
-(((((3*a - 2*b)*Sqrt[b]*ArcTan[(Sqrt[b]*Cosh[c + d*x])/Sqrt[a - b]])/(a*S
qrt[a - b]) + (2*(a - b)*ArcTanh[Cosh[c + d*x]])/a)/(2*a*(a - b)) + (b*Cos
h[c + d*x])/(2*a*(a - b)*(a - b + b*Cosh[c + d*x]^2)))/d)
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 316
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_), x_Symbol] :> Sim
p[(-b)*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^(q + 1)/(2*a*(p + 1)*(b*c - a*d))
), x] + Simp[1/(2*a*(p + 1)*(b*c - a*d))   Int[(a + b*x^2)^(p + 1)*(c + d*x
^2)^q*Simp[b*c + 2*(p + 1)*(b*c - a*d) + d*b*(2*(p + q + 2) + 1)*x^2, x], x
], x] /; FreeQ[{a, b, c, d, q}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] &&  !
( !IntegerQ[p] && IntegerQ[q] && LtQ[q, -1]) && IntBinomialQ[a, b, c, d, 2,
 p, q, x]
```

rule 397
```
Int[((e_) + (f_.)*(x_)^2)/(((a_) + (b_.)*(x_)^2)*((c_) + (d_.)*(x_)^2)), x_
Symbol] :> Simp[(b*e - a*f)/(b*c - a*d)   Int[1/(a + b*x^2), x], x] - Simp[
(d*e - c*f)/(b*c - a*d)   Int[1/(c + d*x^2), x], x] /; FreeQ[{a, b, c, d, e
, f}, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3665
```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^
(p_.), x_Symbol] :> With[{ff = FreeFactors[Cos[e + f*x], x]}, Simp[-ff/f
Subst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a + b - b*ff^2*x^2)^p, x], x, Cos[e +
 f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m - 1)/2]
```

Maple [A] (verified)

Time = 0.94 (sec) , antiderivative size = 171, normalized size of antiderivative = 1.55

method	result
derivativedivides	$\frac{-\frac{4b\left(\frac{-\frac{(a-2b)\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^2}{4(a-b)}+\frac{a}{4a-4b}}{\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^4 a-2\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^2 a+4b\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^2+a}+\frac{(3a-2b)\arctan\left(\frac{2\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^2 a-2a+4b}{4\sqrt{ab-b^2}}\right)}{8(a-b)\sqrt{ab-b^2}}\right)}{a^2}+\frac{\ln\left(\tanh\left(\frac{dx}{2}\right.\right.}{a^2}}{d}$
default	$\frac{-\frac{4b\left(\frac{-\frac{(a-2b)\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^2}{4(a-b)}+\frac{a}{4a-4b}}{\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^4 a-2\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^2 a+4b\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^2+a}+\frac{(3a-2b)\arctan\left(\frac{2\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^2 a-2a+4b}{4\sqrt{ab-b^2}}\right)}{8(a-b)\sqrt{ab-b^2}}\right)}{a^2}+\frac{\ln\left(\tanh\left(\frac{dx}{2}\right.\right.}{a^2}}{d}$
risch	$-\frac{b\,e^{dx+c}\left(e^{2dx+2c}+1\right)}{a(a-b)d\left(b\,e^{4dx+4c}+4\,e^{2dx+2c}a-2\,e^{2dx+2c}b+b\right)}-\frac{\ln\left(e^{dx+c}+1\right)}{a^2 d}+\frac{\ln\left(e^{dx+c}-1\right)}{a^2 d}+\frac{3\sqrt{-b(a-b)}\,\ln\left(e^{2dx+2c}-\right.}{4(a-b)^2}$

input
```
int(csch(d*x+c)/(a+b*sinh(d*x+c)^2)^2,x,method=_RETURNVERBOSE)
```

output
```
1/d*(-4*b/a^2*((-1/4*(a-2*b)/(a-b)*tanh(1/2*d*x+1/2*c)^2+1/4*a/(a-b))/(tan
h(1/2*d*x+1/2*c)^4*a-2*tanh(1/2*d*x+1/2*c)^2*a+4*b*tanh(1/2*d*x+1/2*c)^2+a
)+1/8*(3*a-2*b)/(a-b)/(a*b-b^2)^(1/2)*arctan(1/4*(2*tanh(1/2*d*x+1/2*c)^2*
a-2*a+4*b)/(a*b-b^2)^(1/2)))+1/a^2*ln(tanh(1/2*d*x+1/2*c)))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1256 vs. $2(98) = 196$.

Time = 0.14 (sec) , antiderivative size = 2529, normalized size of antiderivative = 22.99

$$\int \frac{\operatorname{csch}(c+dx)}{\left(a+b\sinh^2(c+dx)\right)^2}\,dx = \text{Too large to display}$$

input
```
integrate(csch(d*x+c)/(a+b*sinh(d*x+c)^2)^2,x, algorithm="fricas")
```

output
```
[-1/4*(4*a*b*cosh(d*x + c)^3 + 12*a*b*cosh(d*x + c)*sinh(d*x + c)^2 + 4*a*
b*sinh(d*x + c)^3 + 4*a*b*cosh(d*x + c) - ((3*a*b - 2*b^2)*cosh(d*x + c)^4
 + 4*(3*a*b - 2*b^2)*cosh(d*x + c)*sinh(d*x + c)^3 + (3*a*b - 2*b^2)*sinh(
d*x + c)^4 + 2*(6*a^2 - 7*a*b + 2*b^2)*cosh(d*x + c)^2 + 2*(3*(3*a*b - 2*b
^2)*cosh(d*x + c)^2 + 6*a^2 - 7*a*b + 2*b^2)*sinh(d*x + c)^2 + 3*a*b - 2*b
^2 + 4*((3*a*b - 2*b^2)*cosh(d*x + c)^3 + (6*a^2 - 7*a*b + 2*b^2)*cosh(d*x
 + c))*sinh(d*x + c))*sqrt(-b/(a - b))*log((b*cosh(d*x + c)^4 + 4*b*cosh(d
*x + c)*sinh(d*x + c)^3 + b*sinh(d*x + c)^4 - 2*(2*a - 3*b)*cosh(d*x + c)^
2 + 2*(3*b*cosh(d*x + c)^2 - 2*a + 3*b)*sinh(d*x + c)^2 + 4*(b*cosh(d*x +
c)^3 - (2*a - 3*b)*cosh(d*x + c))*sinh(d*x + c) - 4*((a - b)*cosh(d*x + c)
^3 + 3*(a - b)*cosh(d*x + c)*sinh(d*x + c)^2 + (a - b)*sinh(d*x + c)^3 + (
a - b)*cosh(d*x + c) + (3*(a - b)*cosh(d*x + c)^2 + a - b)*sinh(d*x + c))*
sqrt(-b/(a - b)) + b)/(b*cosh(d*x + c)^4 + 4*b*cosh(d*x + c)*sinh(d*x + c)
^3 + b*sinh(d*x + c)^4 + 2*(2*a - b)*cosh(d*x + c)^2 + 2*(3*b*cosh(d*x + c
)^2 + 2*a - b)*sinh(d*x + c)^2 + 4*(b*cosh(d*x + c)^3 + (2*a - b)*cosh(d*x
 + c))*sinh(d*x + c) + b)) + 4*((a*b - b^2)*cosh(d*x + c)^4 + 4*(a*b - b^2
)*cosh(d*x + c)*sinh(d*x + c)^3 + (a*b - b^2)*sinh(d*x + c)^4 + 2*(2*a^2 -
 3*a*b + b^2)*cosh(d*x + c)^2 + 2*(3*(a*b - b^2)*cosh(d*x + c)^2 + 2*a^2 -
 3*a*b + b^2)*sinh(d*x + c)^2 + a*b - b^2 + 4*((a*b - b^2)*cosh(d*x + c)^3
 + (2*a^2 - 3*a*b + b^2)*cosh(d*x + c))*sinh(d*x + c))*log(cosh(d*x + c...
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\operatorname{csch}(c+dx)}{\left(a+b\sinh^2(c+dx)\right)^2}\,dx = \text{Timed out}$$

input
```
integrate(csch(d*x+c)/(a+b*sinh(d*x+c)**2)**2,x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{\operatorname{csch}(c+dx)}{\left(a+b\sinh^2(c+dx)\right)^2}\,dx = \int \frac{\operatorname{csch}\left(dx+c\right)}{\left(b\sinh\left(dx+c\right)^2+a\right)^2}\,dx$$

input
```
integrate(csch(d*x+c)/(a+b*sinh(d*x+c)^2)^2,x, algorithm="maxima")
```

output
```
-(b*e^(3*d*x + 3*c) + b*e^(d*x + c))/(a^2*b*d - a*b^2*d + (a^2*b*d*e^(4*c)
 - a*b^2*d*e^(4*c))*e^(4*d*x) + 2*(2*a^3*d*e^(2*c) - 3*a^2*b*d*e^(2*c) + a
*b^2*d*e^(2*c))*e^(2*d*x)) - log((e^(d*x + c) + 1)*e^(-c))/(a^2*d) + log((
e^(d*x + c) - 1)*e^(-c))/(a^2*d) - 2*integrate(1/2*((3*a*b*e^(3*c) - 2*b^2
*e^(3*c))*e^(3*d*x) - (3*a*b*e^c - 2*b^2*e^c)*e^(d*x))/(a^3*b - a^2*b^2 +
(a^3*b*e^(4*c) - a^2*b^2*e^(4*c))*e^(4*d*x) + 2*(2*a^4*e^(2*c) - 3*a^3*b*e
^(2*c) + a^2*b^2*e^(2*c))*e^(2*d*x)), x)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{\operatorname{csch}(c+dx)}{\left(a+b\sinh^2(c+dx)\right)^2}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(csch(d*x+c)/(a+b*sinh(d*x+c)^2)^2,x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Limit: Max order reached or unable
to make series expansion Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\operatorname{csch}(c+dx)}{\left(a+b\sinh^2(c+dx)\right)^2}\,dx = \int \frac{1}{\sinh\left(c+d\,x\right)\,\left(b\sinh\left(c+d\,x\right)^2+a\right)^2}\,dx$$

input
```
int(1/(sinh(c + d*x)*(a + b*sinh(c + d*x)^2)^2),x)
```

output
```
int(1/(sinh(c + d*x)*(a + b*sinh(c + d*x)^2)^2), x)
```

Reduce [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 3972, normalized size of antiderivative = 36.11

$$\int \frac{\operatorname{csch}(c+dx)}{\left(a+b\sinh^2(c+dx)\right)^2}\,dx = \text{Too large to display}$$

input
```
int(csch(d*x+c)/(a+b*sinh(d*x+c)^2)^2,x)
```

output

```
( - 6*e**(4*c + 4*d*x)*sqrt(b)*sqrt(a)*sqrt(a - b)*sqrt(2*sqrt(a)*sqrt(a -
 b) + 2*a - b)*atan((e**(c + d*x)*b)/(sqrt(b)*sqrt(2*sqrt(a)*sqrt(a - b) +
 2*a - b)))*a*b + 4*e**(4*c + 4*d*x)*sqrt(b)*sqrt(a)*sqrt(a - b)*sqrt(2*sq
rt(a)*sqrt(a - b) + 2*a - b)*atan((e**(c + d*x)*b)/(sqrt(b)*sqrt(2*sqrt(a)
*sqrt(a - b) + 2*a - b)))*b**2 - 24*e**(2*c + 2*d*x)*sqrt(b)*sqrt(a)*sqrt(
a - b)*sqrt(2*sqrt(a)*sqrt(a - b) + 2*a - b)*atan((e**(c + d*x)*b)/(sqrt(b
)*sqrt(2*sqrt(a)*sqrt(a - b) + 2*a - b)))*a**2 + 28*e**(2*c + 2*d*x)*sqrt(
b)*sqrt(a)*sqrt(a - b)*sqrt(2*sqrt(a)*sqrt(a - b) + 2*a - b)*atan((e**(c +
 d*x)*b)/(sqrt(b)*sqrt(2*sqrt(a)*sqrt(a - b) + 2*a - b)))*a*b - 8*e**(2*c
+ 2*d*x)*sqrt(b)*sqrt(a)*sqrt(a - b)*sqrt(2*sqrt(a)*sqrt(a - b) + 2*a - b)
*atan((e**(c + d*x)*b)/(sqrt(b)*sqrt(2*sqrt(a)*sqrt(a - b) + 2*a - b)))*b*
*2 - 6*sqrt(b)*sqrt(a)*sqrt(a - b)*sqrt(2*sqrt(a)*sqrt(a - b) + 2*a - b)*a
tan((e**(c + d*x)*b)/(sqrt(b)*sqrt(2*sqrt(a)*sqrt(a - b) + 2*a - b)))*a*b
+ 4*sqrt(b)*sqrt(a)*sqrt(a - b)*sqrt(2*sqrt(a)*sqrt(a - b) + 2*a - b)*atan
((e**(c + d*x)*b)/(sqrt(b)*sqrt(2*sqrt(a)*sqrt(a - b) + 2*a - b)))*b**2 +
6*e**(4*c + 4*d*x)*sqrt(b)*sqrt(2*sqrt(a)*sqrt(a - b) + 2*a - b)*atan((e**
(c + d*x)*b)/(sqrt(b)*sqrt(2*sqrt(a)*sqrt(a - b) + 2*a - b)))*a**2*b - 10*
e**(4*c + 4*d*x)*sqrt(b)*sqrt(2*sqrt(a)*sqrt(a - b) + 2*a - b)*atan((e**(c
 + d*x)*b)/(sqrt(b)*sqrt(2*sqrt(a)*sqrt(a - b) + 2*a - b)))*a*b**2 + 4*e**
(4*c + 4*d*x)*sqrt(b)*sqrt(2*sqrt(a)*sqrt(a - b) + 2*a - b)*atan((e**(c...
```

3.48 $\int \frac{\operatorname{csch}^2(c+dx)}{\left(a+b\sinh^2(c+dx)\right)^2}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 111

$$\int \frac{\operatorname{csch}^2(c+dx)}{\left(a+b\sinh^2(c+dx)\right)^2}\,dx=-\frac{(4a-3b)b\operatorname{arctanh}\left(\frac{\sqrt{a-b}\tanh(c+dx)}{\sqrt{a}}\right)}{2a^{5/2}(a-b)^{3/2}d}-\frac{\coth(c+dx)}{a^2d}+\frac{b^2\tanh(c+dx)}{2a^2(a-b)d\left(a-(a-b)\tanh^2(c+dx)\right)}$$

output
```
-1/2*(4*a-3*b)*b*arctanh((a-b)^(1/2)*tanh(d*x+c)/a^(1/2))/a^(5/2)/(a-b)^(3
/2)/d-coth(d*x+c)/a^2/d+1/2*b^2*tanh(d*x+c)/a^2/(a-b)/d/(a-(a-b)*tanh(d*x+
c)^2)
```

Mathematica [A] (verified)

Time = 1.20 (sec) , antiderivative size = 170, normalized size of antiderivative = 1.53

$$\int \frac{\operatorname{csch}^2(c+dx)}{\left(a+b\sinh^2(c+dx)\right)^2}\,dx=$$

$$-\frac{(2a-b+b\cosh(2(c+dx)))\operatorname{csch}^5(c+dx)\left(2\sqrt{a}\sqrt{a-b}\cosh(c+dx)\left(4a^2-6ab+3b^2+(2a-3b)b\operatorname{co}\right.\right.}{16a^{5/2}(a-b)^{3/2}d\left(b+\right.}$$

input
```
Integrate[Csch[c + d*x]^2/(a + b*Sinh[c + d*x]^2)^2,x]
```

output
```
-1/16*((2*a - b + b*Cosh[2*(c + d*x)])*Csch[c + d*x]^5*(2*Sqrt[a]*Sqrt[a -
 b]*Cosh[c + d*x]*(4*a^2 - 6*a*b + 3*b^2 + (2*a - 3*b)*b*Cosh[2*(c + d*x)]
) + 2*(4*a - 3*b)*b*ArcTanh[(Sqrt[a - b]*Tanh[c + d*x])/Sqrt[a]]*(2*a - b
+ b*Cosh[2*(c + d*x)])*Sinh[c + d*x]))/(a^(5/2)*(a - b)^(3/2)*d*(b + a*Csc
h[c + d*x]^2)^2)
```

Rubi [A] (verified)

Time = 0.39 (sec) , antiderivative size = 142, normalized size of antiderivative = 1.28, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.261$, Rules used = {3042, 25, 3666, 365, 298, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\text{csch}^2(c+dx)}{(a+b\sinh^2(c+dx))^2}\,dx$$

$$\Big\downarrow \; 3042$$

$$\int -\frac{1}{\sin(ic+idx)^2\,(a-b\sin(ic+idx)^2)^2}dx$$

$$\Big\downarrow \; 25$$

$$-\int \frac{1}{\sin(ic+idx)^2\,(a-b\sin(ic+idx)^2)^2}dx$$

$$\Big\downarrow \; 3666$$

$$\frac{\int \frac{\coth^2(c+dx)(1-\tanh^2(c+dx))^2}{(a-(a-b)\tanh^2(c+dx))^2}d\tanh(c+dx)}{d}$$

$$\Big\downarrow \; 365$$

$$\frac{\frac{\int \frac{a\tanh^2(c+dx)+a-3b}{(a-(a-b)\tanh^2(c+dx))^2}d\tanh(c+dx)}{a}-\frac{\coth(c+dx)}{a(a-(a-b)\tanh^2(c+dx))}}{d}$$

$$\Big\downarrow \; 298$$

$$\frac{\frac{\frac{\left(2a^2-4ab+3b^2\right)\tanh(c+dx)}{2a(a-b)\left(a-(a-b)\tanh^2(c+dx)\right)}-\frac{b(4a-3b)\int\frac{1}{a-(a-b)\tanh^2(c+dx)}d\tanh(c+dx)}{2a(a-b)}}{a}-\frac{\coth(c+dx)}{a(a-(a-b)\tanh^2(c+dx))}}{d}$$

$$\Big\downarrow \ 221$$

$$\frac{\frac{\frac{\left(2a^2-4ab+3b^2\right)\tanh(c+dx)}{2a(a-b)\left(a-(a-b)\tanh^2(c+dx)\right)}-\frac{b(4a-3b)\text{arctanh}\left(\frac{\sqrt{a-b}\tanh(c+dx)}{\sqrt{a}}\right)}{2a^{3/2}(a-b)^{3/2}}}{a}-\frac{\coth(c+dx)}{a(a-(a-b)\tanh^2(c+dx))}}{d}$$

input
```
Int[Csch[c + d*x]^2/(a + b*Sinh[c + d*x]^2)^2,x]
```

output
```
(-(Coth[c + d*x]/(a*(a - (a - b)*Tanh[c + d*x]^2))) + (-1/2*((4*a - 3*b)*b
*ArcTanh[(Sqrt[a - b]*Tanh[c + d*x])/Sqrt[a]])/(a^(3/2)*(a - b)^(3/2)) + (
(2*a^2 - 4*a*b + 3*b^2)*Tanh[c + d*x])/(2*a*(a - b)*(a - (a - b)*Tanh[c +
d*x]^2)))/a)/d
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 298
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2), x_Symbol] :> Simp[(-(
b*c - a*d))*x*((a + b*x^2)^(p + 1)/(2*a*b*(p + 1))), x] - Simp[(a*d - b*c*(
2*p + 3))/(2*a*b*(p + 1))   Int[(a + b*x^2)^(p + 1), x], x] /; FreeQ[{a, b,
 c, d, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/2 + p, 0])
```

rule 365
```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^2, x
_Symbol] :> Simp[c^2*(e*x)^(m + 1)*((a + b*x^2)^(p + 1)/(a*e*(m + 1))), x]
- Simp[1/(a*e^2*(m + 1))   Int[(e*x)^(m + 2)*(a + b*x^2)^p*Simp[2*b*c^2*(p
+ 1) + c*(b*c - 2*a*d)*(m + 1) - a*d^2*(m + 1)*x^2, x], x], x] /; FreeQ[{a,
 b, c, d, e, p}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3666
```
Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(
p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Simp[ff^(m + 1
)/f   Subst[Int[x^m*((a + (a + b)*ff^2*x^2)^p/(1 + ff^2*x^2)^(m/2 + p + 1))
, x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f}, x] && IntegerQ[m/2] &
& IntegerQ[p]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 315 vs. $2(99) = 198$.

Time = 1.07 (sec) , antiderivative size = 316, normalized size of antiderivative = 2.85

method	result
derivativedivides	$\frac{-\frac{\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)}{2a^2}-\frac{1}{2a^2\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)}+\frac{4b\left(\frac{\frac{\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^3 b}{4a-4b}+\frac{b\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)}{4a-4b}}{\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^4 a-2\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^2 a+4b\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^2+a}+\frac{(4a-3b)a\left(\frac{\left(\sqrt{-b(a-b)}+\cdots\right.}{2a\sqrt{-\cdots}}\cdots\right.}{\cdots}\right)}{\cdots}}{d}$
default	$\frac{-\frac{\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)}{2a^2}-\frac{1}{2a^2\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)}+\frac{4b\left(\frac{\frac{\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^3 b}{4a-4b}+\frac{b\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)}{4a-4b}}{\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^4 a-2\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^2 a+4b\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^2+a}+\frac{(4a-3b)a\left(\frac{\left(\sqrt{-b(a-b)}+\cdots\right.}{2a\sqrt{-\cdots}}\cdots\right.}{\cdots}\right)}{\cdots}}{d}$
risch	$-\frac{4\,e^{4dx+4c}ab-3\,e^{4dx+4c}b^2+8\,e^{2dx+2c}a^2-14\,e^{2dx+2c}ba+6b^2e^{2dx+2c}+2ab-3b^2}{a^2(a-b)d\left(b\,e^{4dx+4c}+4\,e^{2dx+2c}a-2\,e^{2dx+2c}b+b\right)\left(e^{2dx+2c}-1\right)}+\frac{\ln\left(e^{2dx+2c}+\frac{2a\sqrt{a^2-ab}-b\sqrt{a^2-\cdots}}{b\sqrt{a^2-a\cdots}}\cdots\right.}{\sqrt{a^2-ab}\,(a-b)da\cdots}$

input
```
int(csch(d*x+c)^2/(a+b*sinh(d*x+c)^2)^2,x,method=_RETURNVERBOSE)
```

output
```
1/d*(-1/2/a^2*tanh(1/2*d*x+1/2*c)-1/2/a^2/tanh(1/2*d*x+1/2*c)+4*b/a^2*((1/
4*b/(a-b)*tanh(1/2*d*x+1/2*c)^3+1/4*b/(a-b)*tanh(1/2*d*x+1/2*c))/(tanh(1/2
*d*x+1/2*c)^4*a-2*tanh(1/2*d*x+1/2*c)^2*a+4*b*tanh(1/2*d*x+1/2*c)^2+a)+1/4
*(4*a-3*b)/(a-b)*a*(1/2*((-b*(a-b))^(1/2)+b)/a/(-b*(a-b))^(1/2)/((2*(-b*(a
-b))^(1/2)-a+2*b)*a)^(1/2)*arctan(tanh(1/2*d*x+1/2*c)*a/((2*(-b*(a-b))^(1/
2)-a+2*b)*a)^(1/2))-1/2*((-b*(a-b))^(1/2)-b)/a/(-b*(a-b))^(1/2)/((2*(-b*(a
-b))^(1/2)+a-2*b)*a)^(1/2)*arctanh(tanh(1/2*d*x+1/2*c)*a/((2*(-b*(a-b))^(1
/2)+a-2*b)*a)^(1/2)))))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1366 vs. $2(100) = 200$.

Time = 0.18 (sec) , antiderivative size = 2988, normalized size of antiderivative = 26.92

$$\int \frac{\operatorname{csch}^2(c+dx)}{\left(a+b\sinh^2(c+dx)\right)^2}\,dx = \text{Too large to display}$$

input
```
integrate(csch(d*x+c)^2/(a+b*sinh(d*x+c)^2)^2,x, algorithm="fricas")
```

output
```
[-1/4*(4*(4*a^3*b - 7*a^2*b^2 + 3*a*b^3)*cosh(d*x + c)^4 + 16*(4*a^3*b - 7
*a^2*b^2 + 3*a*b^3)*cosh(d*x + c)*sinh(d*x + c)^3 + 4*(4*a^3*b - 7*a^2*b^2
 + 3*a*b^3)*sinh(d*x + c)^4 + 8*a^3*b - 20*a^2*b^2 + 12*a*b^3 + 8*(4*a^4 -
 11*a^3*b + 10*a^2*b^2 - 3*a*b^3)*cosh(d*x + c)^2 + 8*(4*a^4 - 11*a^3*b +
10*a^2*b^2 - 3*a*b^3 + 3*(4*a^3*b - 7*a^2*b^2 + 3*a*b^3)*cosh(d*x + c)^2)*
sinh(d*x + c)^2 - ((4*a*b^2 - 3*b^3)*cosh(d*x + c)^6 + 6*(4*a*b^2 - 3*b^3)
*cosh(d*x + c)*sinh(d*x + c)^5 + (4*a*b^2 - 3*b^3)*sinh(d*x + c)^6 + (16*a
^2*b - 24*a*b^2 + 9*b^3)*cosh(d*x + c)^4 + (16*a^2*b - 24*a*b^2 + 9*b^3 +
15*(4*a*b^2 - 3*b^3)*cosh(d*x + c)^2)*sinh(d*x + c)^4 + 4*(5*(4*a*b^2 - 3*
b^3)*cosh(d*x + c)^3 + (16*a^2*b - 24*a*b^2 + 9*b^3)*cosh(d*x + c))*sinh(d
*x + c)^3 - 4*a*b^2 + 3*b^3 - (16*a^2*b - 24*a*b^2 + 9*b^3)*cosh(d*x + c)^
2 + (15*(4*a*b^2 - 3*b^3)*cosh(d*x + c)^4 - 16*a^2*b + 24*a*b^2 - 9*b^3 +
6*(16*a^2*b - 24*a*b^2 + 9*b^3)*cosh(d*x + c)^2)*sinh(d*x + c)^2 + 2*(3*(4
*a*b^2 - 3*b^3)*cosh(d*x + c)^5 + 2*(16*a^2*b - 24*a*b^2 + 9*b^3)*cosh(d*x
 + c)^3 - (16*a^2*b - 24*a*b^2 + 9*b^3)*cosh(d*x + c))*sinh(d*x + c))*sqrt
(a^2 - a*b)*log((b^2*cosh(d*x + c)^4 + 4*b^2*cosh(d*x + c)*sinh(d*x + c)^3
 + b^2*sinh(d*x + c)^4 + 2*(2*a*b - b^2)*cosh(d*x + c)^2 + 2*(3*b^2*cosh(d
*x + c)^2 + 2*a*b - b^2)*sinh(d*x + c)^2 + 8*a^2 - 8*a*b + b^2 + 4*(b^2*co
sh(d*x + c)^3 + (2*a*b - b^2)*cosh(d*x + c))*sinh(d*x + c) + 4*(b*cosh(d*x
 + c)^2 + 2*b*cosh(d*x + c)*sinh(d*x + c) + b*sinh(d*x + c)^2 + 2*a - b...
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\operatorname{csch}^2(c+dx)}{(a+b\sinh^2(c+dx))^2}\,dx = \text{Timed out}$$

input
```
integrate(csch(d*x+c)**2/(a+b*sinh(d*x+c)**2)**2,x)
```

output
```
Timed out
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{\operatorname{csch}^2(c+dx)}{(a+b\sinh^2(c+dx))^2}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate(csch(d*x+c)^2/(a+b*sinh(d*x+c)^2)^2,x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(b-a>0)', see `assume?` for more
details)Is
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 229 vs. 2(100) = 200.

Time = 0.28 (sec) , antiderivative size = 229, normalized size of antiderivative = 2.06

$$\int \frac{\operatorname{csch}^2(c+dx)}{(a+b\sinh^2(c+dx))^2}\,dx = -\frac{\frac{(4ab-3b^2)\arctan\left(\frac{be^{(2dx+2c)}+2a-b}{2\sqrt{-a^2+ab}}\right)}{(a^3-a^2b)\sqrt{-a^2+ab}} + \frac{2\left(4abe^{(4dx+4c)}-3b^2e^{(4dx+4c)}+8a^2e^{(2dx+2c)}-14abe^{(2dx+2c)}+6b^2e^{(2dx+2c)}+2ab-3b^2\right)}{(a^3-a^2b)\left(be^{(6dx+6c)}+4ae^{(4dx+4c)}-3be^{(4dx+4c)}-4ae^{(2dx+2c)}+3be^{(2dx+2c)}-b\right)}}{2d}$$

input
```
integrate(csch(d*x+c)^2/(a+b*sinh(d*x+c)^2)^2,x, algorithm="giac")
```

output
```
-1/2*((4*a*b - 3*b^2)*arctan(1/2*(b*e^(2*d*x + 2*c) + 2*a - b)/sqrt(-a^2 +
 a*b))/((a^3 - a^2*b)*sqrt(-a^2 + a*b)) + 2*(4*a*b*e^(4*d*x + 4*c) - 3*b^2
*e^(4*d*x + 4*c) + 8*a^2*e^(2*d*x + 2*c) - 14*a*b*e^(2*d*x + 2*c) + 6*b^2*
e^(2*d*x + 2*c) + 2*a*b - 3*b^2)/((a^3 - a^2*b)*(b*e^(6*d*x + 6*c) + 4*a*e
^(4*d*x + 4*c) - 3*b*e^(4*d*x + 4*c) - 4*a*e^(2*d*x + 2*c) + 3*b*e^(2*d*x
+ 2*c) - b)))/d
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\operatorname{csch}^2(c+dx)}{\left(a+b\sinh^2(c+dx)\right)^2}\,dx=\int \frac{1}{\sinh\left(c+d\,x\right)^2\,\left(b\sinh\left(c+d\,x\right)^2+a\right)^2}\,dx$$

input
```
int(1/(sinh(c + d*x)^2*(a + b*sinh(c + d*x)^2)^2),x)
```

output
```
int(1/(sinh(c + d*x)^2*(a + b*sinh(c + d*x)^2)^2), x)
```

Reduce [B] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 1845, normalized size of antiderivative = 16.62

$$\int \frac{\operatorname{csch}^2(c+dx)}{\left(a+b\sinh^2(c+dx)\right)^2}\,dx=\text{Too large to display}$$

input
```
int(csch(d*x+c)^2/(a+b*sinh(d*x+c)^2)^2,x)
```

output
```
( - 4*e**(6*c + 6*d*x)*sqrt(a)*sqrt(a - b)*log( - sqrt(2*sqrt(a)*sqrt(a -
b) - 2*a + b) + e**(c + d*x)*sqrt(b))*a*b**2 + 3*e**(6*c + 6*d*x)*sqrt(a)*
sqrt(a - b)*log( - sqrt(2*sqrt(a)*sqrt(a - b) - 2*a + b) + e**(c + d*x)*sq
rt(b))*b**3 - 4*e**(6*c + 6*d*x)*sqrt(a)*sqrt(a - b)*log(sqrt(2*sqrt(a)*sq
rt(a - b) - 2*a + b) + e**(c + d*x)*sqrt(b))*a*b**2 + 3*e**(6*c + 6*d*x)*s
qrt(a)*sqrt(a - b)*log(sqrt(2*sqrt(a)*sqrt(a - b) - 2*a + b) + e**(c + d*x
)*sqrt(b))*b**3 + 4*e**(6*c + 6*d*x)*sqrt(a)*sqrt(a - b)*log(2*sqrt(a)*sqr
t(a - b) + e**(2*c + 2*d*x)*b + 2*a - b)*a*b**2 - 3*e**(6*c + 6*d*x)*sqrt(
a)*sqrt(a - b)*log(2*sqrt(a)*sqrt(a - b) + e**(2*c + 2*d*x)*b + 2*a - b)*b
**3 - 16*e**(4*c + 4*d*x)*sqrt(a)*sqrt(a - b)*log( - sqrt(2*sqrt(a)*sqrt(a
 - b) - 2*a + b) + e**(c + d*x)*sqrt(b))*a**2*b + 24*e**(4*c + 4*d*x)*sqrt
(a)*sqrt(a - b)*log( - sqrt(2*sqrt(a)*sqrt(a - b) - 2*a + b) + e**(c + d*x
)*sqrt(b))*a*b**2 - 9*e**(4*c + 4*d*x)*sqrt(a)*sqrt(a - b)*log( - sqrt(2*s
qrt(a)*sqrt(a - b) - 2*a + b) + e**(c + d*x)*sqrt(b))*b**3 - 16*e**(4*c +
4*d*x)*sqrt(a)*sqrt(a - b)*log(sqrt(2*sqrt(a)*sqrt(a - b) - 2*a + b) + e**
(c + d*x)*sqrt(b))*a**2*b + 24*e**(4*c + 4*d*x)*sqrt(a)*sqrt(a - b)*log(sq
rt(2*sqrt(a)*sqrt(a - b) - 2*a + b) + e**(c + d*x)*sqrt(b))*a*b**2 - 9*e**
(4*c + 4*d*x)*sqrt(a)*sqrt(a - b)*log(sqrt(2*sqrt(a)*sqrt(a - b) - 2*a + b
) + e**(c + d*x)*sqrt(b))*b**3 + 16*e**(4*c + 4*d*x)*sqrt(a)*sqrt(a - b)*l
og(2*sqrt(a)*sqrt(a - b) + e**(2*c + 2*d*x)*b + 2*a - b)*a**2*b - 24*e*...
```

3.49 $\int \frac{\operatorname{csch}^3(c+dx)}{\left(a+b\sinh^2(c+dx)\right)^2}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 161

$$\begin{aligned}\int \frac{\operatorname{csch}^3(c+dx)}{\left(a+b\sinh^2(c+dx)\right)^2}\,dx &= \frac{(5a-4b)b^{3/2}\arctan\left(\frac{\sqrt{b}\cosh(c+dx)}{\sqrt{a-b}}\right)}{2a^3(a-b)^{3/2}d} \\ &+ \frac{(a+4b)\operatorname{arctanh}(\cosh(c+dx))}{2a^3d} \\ &- \frac{(a-2b)b\cosh(c+dx)}{2a^2(a-b)d\left(a-b+b\cosh^2(c+dx)\right)} \\ &- \frac{\coth(c+dx)\operatorname{csch}(c+dx)}{2ad\left(a-b+b\cosh^2(c+dx)\right)}\end{aligned}$$

output
```
1/2*(5*a-4*b)*b^(3/2)*arctan(b^(1/2)*cosh(d*x+c)/(a-b)^(1/2))/a^3/(a-b)^(3
/2)/d+1/2*(a+4*b)*arctanh(cosh(d*x+c))/a^3/d-1/2*(a-2*b)*b*cosh(d*x+c)/a^2
/(a-b)/d/(a-b+b*cosh(d*x+c)^2)-1/2*coth(d*x+c)*csch(d*x+c)/a/d/(a-b+b*cosh
(d*x+c)^2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 3.00 (sec) , antiderivative size = 391, normalized size of antiderivative = 2.43

$$\int \frac{\text{csch}^3(c+dx)}{\left(a+b\sinh^2(c+dx)\right)^2}\,dx$$

$$= \frac{(2a-b+b\cosh(2(c+dx)))\text{csch}^3(c+dx)\left(\frac{8ab^2\coth(c+dx)}{a-b}+\frac{4(5a-4b)b^{3/2}\arctan\left(\frac{\sqrt{b}-i\sqrt{a}\tanh\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a-b}}\right)(2a-b+b\cos}{(a-b)^{3/2}}\right.}{}$$

input
```
Integrate[Csch[c + d*x]^3/(a + b*Sinh[c + d*x]^2)^2,x]
```

output
```
((2*a - b + b*Cosh[2*(c + d*x)])*Csch[c + d*x]^3*((8*a*b^2*Coth[c + d*x])/
(a - b) + (4*(5*a - 4*b)*b^(3/2)*ArcTan[(Sqrt[b] - I*Sqrt[a]*Tanh[(c + d*x
)/2])/Sqrt[a - b]]*(2*a - b + b*Cosh[2*(c + d*x)])*Csch[c + d*x])/(a - b)^
(3/2) + (4*(5*a - 4*b)*b^(3/2)*ArcTan[(Sqrt[b] + I*Sqrt[a]*Tanh[(c + d*x)/
2])/Sqrt[a - b]]*(2*a - b + b*Cosh[2*(c + d*x)])*Csch[c + d*x])/(a - b)^(3
/2) - a*(2*a - b + b*Cosh[2*(c + d*x)])*Csch[(c + d*x)/2]^2*Csch[c + d*x]
+ 4*(a + 4*b)*(2*a - b + b*Cosh[2*(c + d*x)])*Csch[c + d*x]*Log[Cosh[(c +
d*x)/2]] - 4*(a + 4*b)*(2*a - b + b*Cosh[2*(c + d*x)])*Csch[c + d*x]*Log[S
inh[(c + d*x)/2]] - a*(2*a - b + b*Cosh[2*(c + d*x)])*Csch[c + d*x]*Sech[(
c + d*x)/2]^2))/(32*a^3*d*(b + a*Csch[c + d*x]^2)^2)
```

Rubi [A] (verified)

Time = 0.47 (sec) , antiderivative size = 178, normalized size of antiderivative = 1.11, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.391$, Rules used = {3042, 26, 3665, 316, 402, 27, 397, 218, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\operatorname{csch}^3(c+dx)}{\left(a+b\sinh^2(c+dx)\right)^2}dx$$

$$\Big\downarrow \ 3042$$

$$\int -\frac{i}{\sin(ic+idx)^3\left(a-b\sin(ic+idx)^2\right)^2}dx$$

$$\Big\downarrow \ 26$$

$$-i\int \frac{1}{\sin(ic+idx)^3\left(a-b\sin(ic+idx)^2\right)^2}dx$$

$$\Big\downarrow \ 3665$$

$$\frac{\int \frac{1}{\left(1-\cosh^2(c+dx)\right)^2\left(b\cosh^2(c+dx)+a-b\right)^2}d\cosh(c+dx)}{d}$$

$$\Big\downarrow \ 316$$

$$\frac{\frac{\int \frac{3b\cosh^2(c+dx)+a+b}{\left(1-\cosh^2(c+dx)\right)\left(b\cosh^2(c+dx)+a-b\right)^2}d\cosh(c+dx)}{2a}+\frac{\cosh(c+dx)}{2a\left(1-\cosh^2(c+dx)\right)\left(a+b\cosh^2(c+dx)-b\right)}}{d}$$

$$\Big\downarrow \ 402$$

$$\frac{\frac{-\frac{\int -\frac{2\left(a^2+2ba-2b^2+(a-2b)b\cosh^2(c+dx)\right)}{\left(1-\cosh^2(c+dx)\right)\left(b\cosh^2(c+dx)+a-b\right)}d\cosh(c+dx)}{2a(a-b)}-\frac{b(a-2b)\cosh(c+dx)}{a(a-b)\left(a+b\cosh^2(c+dx)-b\right)}}{2a}+\frac{\cosh(c+dx)}{2a\left(1-\cosh^2(c+dx)\right)\left(a+b\cosh^2(c+dx)-b\right)}}{d}$$

$$\Big\downarrow \ 27$$

$$\frac{\frac{\frac{\int \frac{a^2+2ba-2b^2+(a-2b)b\cosh^2(c+dx)}{\left(1-\cosh^2(c+dx)\right)\left(b\cosh^2(c+dx)+a-b\right)}d\cosh(c+dx)}{a(a-b)}-\frac{b(a-2b)\cosh(c+dx)}{a(a-b)\left(a+b\cosh^2(c+dx)-b\right)}}{2a}+\frac{\cosh(c+dx)}{2a\left(1-\cosh^2(c+dx)\right)\left(a+b\cosh^2(c+dx)-b\right)}}{d}$$

$$\Big\downarrow \ 397$$

$$\frac{\frac{\frac{\frac{b^2(5a-4b)\int \frac{1}{b\cosh^2(c+dx)+a-b}d\cosh(c+dx)}{a}+\frac{(a-b)(a+4b)\int \frac{1}{1-\cosh^2(c+dx)}d\cosh(c+dx)}{a}}{a(a-b)}-\frac{b(a-2b)\cosh(c+dx)}{a(a-b)\left(a+b\cosh^2(c+dx)-b\right)}}{2a}+\frac{\cosh(c+dx}{2a\left(1-\cosh^2(c+dx)\right)(a+b}}{d}$$

$$\Big\downarrow \ 218$$

$$\frac{\frac{\frac{\frac{(a-b)(a+4b)\int \frac{1}{1-\cosh^2(c+dx)}d\cosh(c+dx)}{a}+\frac{b^{3/2}(5a-4b)\arctan\left(\frac{\sqrt{b}\cosh(c+dx)}{\sqrt{a-b}}\right)}{a\sqrt{a-b}}}{a(a-b)}-\frac{b(a-2b)\cosh(c+dx)}{a(a-b)\left(a+b\cosh^2(c+dx)-b\right)}}{2a}+\frac{\cosh(c+dx)}{2a(1-\cosh^2(c+dx))\left(a+b\cosh^2(c+dx)-b\right)}}{d}$$

$$\Big\downarrow \text{219}$$

$$\frac{\frac{\frac{\frac{b^{3/2}(5a-4b)\arctan\left(\frac{\sqrt{b}\cosh(c+dx)}{\sqrt{a-b}}\right)}{a\sqrt{a-b}}+\frac{(a-b)(a+4b)\text{arctanh}(\cosh(c+dx))}{a}}{a(a-b)}-\frac{b(a-2b)\cosh(c+dx)}{a(a-b)\left(a+b\cosh^2(c+dx)-b\right)}}{2a}+\frac{\cosh(c+dx)}{2a(1-\cosh^2(c+dx))\left(a+b\cosh^2(c+dx)-b\right)}}{d}$$

input
```
Int[Csch[c + d*x]^3/(a + b*Sinh[c + d*x]^2)^2,x]
```

output
```
(Cosh[c + d*x]/(2*a*(1 - Cosh[c + d*x]^2)*(a - b + b*Cosh[c + d*x]^2)) + (
(((5*a - 4*b)*b^(3/2)*ArcTan[(Sqrt[b]*Cosh[c + d*x])/Sqrt[a - b]])/(a*Sqrt
[a - b]) + ((a - b)*(a + 4*b)*ArcTanh[Cosh[c + d*x]])/a)/(a*(a - b)) - ((a
 - 2*b)*b*Cosh[c + d*x])/(a*(a - b)*(a - b + b*Cosh[c + d*x]^2)))/(2*a))/d
```

Defintions of rubi rules used

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 316
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_), x_Symbol] :> Sim
p[(-b)*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^(q + 1)/(2*a*(p + 1)*(b*c - a*d))
), x] + Simp[1/(2*a*(p + 1)*(b*c - a*d))   Int[(a + b*x^2)^(p + 1)*(c + d*x
^2)^q*Simp[b*c + 2*(p + 1)*(b*c - a*d) + d*b*(2*(p + q + 2) + 1)*x^2, x], x
], x] /; FreeQ[{a, b, c, d, q}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] &&  !
( !IntegerQ[p] && IntegerQ[q] && LtQ[q, -1]) && IntBinomialQ[a, b, c, d, 2,
 p, q, x]
```

rule 397
```
Int[((e_) + (f_.)*(x_)^2)/(((a_) + (b_.)*(x_)^2)*((c_) + (d_.)*(x_)^2)), x_
Symbol] :> Simp[(b*e - a*f)/(b*c - a*d)   Int[1/(a + b*x^2), x], x] - Simp[
(d*e - c*f)/(b*c - a*d)   Int[1/(c + d*x^2), x], x] /; FreeQ[{a, b, c, d, e
, f}, x]
```

rule 402
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_.)*((e_) + (f_.)*(x
_)^2), x_Symbol] :> Simp[(-(b*e - a*f))*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^
(q + 1)/(a*2*(b*c - a*d)*(p + 1))), x] + Simp[1/(a*2*(b*c - a*d)*(p + 1))
 Int[(a + b*x^2)^(p + 1)*(c + d*x^2)^q*Simp[c*(b*e - a*f) + e*2*(b*c - a*d)
*(p + 1) + d*(b*e - a*f)*(2*(p + q + 2) + 1)*x^2, x], x], x] /; FreeQ[{a, b
, c, d, e, f, q}, x] && LtQ[p, -1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3665
```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^
(p_.), x_Symbol] :> With[{ff = FreeFactors[Cos[e + f*x], x]}, Simp[-ff/f
Subst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a + b - b*ff^2*x^2)^p, x], x, Cos[e +
 f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m - 1)/2]
```

Maple [A] (verified)

Time = 1.31 (sec) , antiderivative size = 213, normalized size of antiderivative = 1.32

method	result
derivativedivides	$\frac{\frac{\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^2}{8a^2}+\frac{2b^2\left(\frac{-\frac{(a-2b)\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^2}{2(a-b)}+\frac{a}{2a-2b}}{\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^4a-2\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^2a+4b\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^2+a}+\frac{(5a-4b)\arctan\left(\frac{2\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^2a-2a+4b}{4\sqrt{ab-b^2}}\right)}{4(a-b)\sqrt{ab-b^2}}\right.}{a^3}}{d}$
default	$\frac{\frac{\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^2}{8a^2}+\frac{2b^2\left(\frac{-\frac{(a-2b)\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^2}{2(a-b)}+\frac{a}{2a-2b}}{\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^4a-2\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^2a+4b\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^2+a}+\frac{(5a-4b)\arctan\left(\frac{2\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^2a-2a+4b}{4\sqrt{ab-b^2}}\right)}{4(a-b)\sqrt{ab-b^2}}\right.}{a^3}}{d}$
risch	$-\frac{e^{dx+c}\left(e^{6dx+6c}ab-2e^{6dx+6c}b^2+4e^{4dx+4c}a^2-5e^{4dx+4c}ab+2e^{4dx+4c}b^2+4e^{2dx+2c}a^2-5e^{2dx+2c}ba+2b^2e^{2dx+2c}+c\right.}{d\,a^2\left(e^{2dx+2c}-1\right)^2(a-b)\left(b\,e^{4dx+4c}+4\,e^{2dx+2c}a-2\,e^{2dx+2c}b+b\right)}$

input
```
int(csch(d*x+c)^3/(a+b*sinh(d*x+c)^2)^2,x,method=_RETURNVERBOSE)
```

output
```
1/d*(1/8*tanh(1/2*d*x+1/2*c)^2/a^2+2*b^2/a^3*((-1/2*(a-2*b)/(a-b)*tanh(1/2
*d*x+1/2*c)^2+1/2*a/(a-b))/(tanh(1/2*d*x+1/2*c)^4*a-2*tanh(1/2*d*x+1/2*c)^
2*a+4*b*tanh(1/2*d*x+1/2*c)^2+a)+1/4*(5*a-4*b)/(a-b)/(a*b-b^2)^(1/2)*arcta
n(1/4*(2*tanh(1/2*d*x+1/2*c)^2*a-2*a+4*b)/(a*b-b^2)^(1/2)))-1/8/a^2/tanh(1
/2*d*x+1/2*c)^2+1/4/a^3*(-2*a-8*b)*ln(tanh(1/2*d*x+1/2*c)))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 4252 vs. 2(145) = 290.

Time = 0.21 (sec) , antiderivative size = 8059, normalized size of antiderivative = 50.06

$$\int \frac{\operatorname{csch}^3(c+dx)}{\left(a+b\sinh^2(c+dx)\right)^2}\,dx = \text{Too large to display}$$

input
```
integrate(csch(d*x+c)^3/(a+b*sinh(d*x+c)^2)^2,x, algorithm="fricas")
```

output
```
Too large to include
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\operatorname{csch}^3(c+dx)}{(a+b\sinh^2(c+dx))^2}\,dx = \text{Timed out}$$

input
```
integrate(csch(d*x+c)**3/(a+b*sinh(d*x+c)**2)**2,x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{\operatorname{csch}^3(c+dx)}{(a+b\sinh^2(c+dx))^2}\,dx = \int \frac{\operatorname{csch}(dx+c)^3}{(b\sinh(dx+c)^2+a)^2}\,dx$$

input
```
integrate(csch(d*x+c)^3/(a+b*sinh(d*x+c)^2)^2,x, algorithm="maxima")
```

output
```
-((a*b*e^(7*c) - 2*b^2*e^(7*c))*e^(7*d*x) + (4*a^2*e^(5*c) - 5*a*b*e^(5*c)
 + 2*b^2*e^(5*c))*e^(5*d*x) + (4*a^2*e^(3*c) - 5*a*b*e^(3*c) + 2*b^2*e^(3*
c))*e^(3*d*x) + (a*b*e^c - 2*b^2*e^c)*e^(d*x))/(a^3*b*d - a^2*b^2*d + (a^3
*b*d*e^(8*c) - a^2*b^2*d*e^(8*c))*e^(8*d*x) + 4*(a^4*d*e^(6*c) - 2*a^3*b*d
*e^(6*c) + a^2*b^2*d*e^(6*c))*e^(6*d*x) - 2*(4*a^4*d*e^(4*c) - 7*a^3*b*d*e
^(4*c) + 3*a^2*b^2*d*e^(4*c))*e^(4*d*x) + 4*(a^4*d*e^(2*c) - 2*a^3*b*d*e^(
2*c) + a^2*b^2*d*e^(2*c))*e^(2*d*x)) + 1/2*(a + 4*b)*log((e^(d*x + c) + 1)
*e^(-c))/(a^3*d) - 1/2*(a + 4*b)*log((e^(d*x + c) - 1)*e^(-c))/(a^3*d) + 8
*integrate(1/8*((5*a*b^2*e^(3*c) - 4*b^3*e^(3*c))*e^(3*d*x) - (5*a*b^2*e^c
 - 4*b^3*e^c)*e^(d*x))/(a^4*b - a^3*b^2 + (a^4*b*e^(4*c) - a^3*b^2*e^(4*c)
)*e^(4*d*x) + 2*(2*a^5*e^(2*c) - 3*a^4*b*e^(2*c) + a^3*b^2*e^(2*c))*e^(2*d
*x)), x)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{\operatorname{csch}^3(c+dx)}{\left(a+b\sinh^2(c+dx)\right)^2}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(csch(d*x+c)^3/(a+b*sinh(d*x+c)^2)^2,x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Limit: Max order reached or unable
to make series expansion Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\operatorname{csch}^3(c+dx)}{\left(a+b\sinh^2(c+dx)\right)^2}\,dx = \int \frac{1}{\sinh\left(c+d\,x\right)^3\,\left(b\,\sinh\left(c+d\,x\right)^2+a\right)^2}\,dx$$

input
```
int(1/(sinh(c + d*x)^3*(a + b*sinh(c + d*x)^2)^2),x)
```

output
```
int(1/(sinh(c + d*x)^3*(a + b*sinh(c + d*x)^2)^2), x)
```

Reduce [B] (verification not implemented)

Time = 0.50 (sec) , antiderivative size = 7758, normalized size of antiderivative = 48.19

$$\int \frac{\operatorname{csch}^3(c+dx)}{\left(a+b\sinh^2(c+dx)\right)^2}\,dx = \text{Too large to display}$$

input
```
int(csch(d*x+c)^3/(a+b*sinh(d*x+c)^2)^2,x)
```

output
```
(10*e**(8*c + 8*d*x)*sqrt(b)*sqrt(a)*sqrt(a - b)*sqrt(2*sqrt(a)*sqrt(a - b
) + 2*a - b)*atan((e**(c + d*x)*b)/(sqrt(b)*sqrt(2*sqrt(a)*sqrt(a - b) + 2
*a - b)))*a*b - 8*e**(8*c + 8*d*x)*sqrt(b)*sqrt(a)*sqrt(a - b)*sqrt(2*sqrt
(a)*sqrt(a - b) + 2*a - b)*atan((e**(c + d*x)*b)/(sqrt(b)*sqrt(2*sqrt(a)*s
qrt(a - b) + 2*a - b)))*b**2 + 40*e**(6*c + 6*d*x)*sqrt(b)*sqrt(a)*sqrt(a
- b)*sqrt(2*sqrt(a)*sqrt(a - b) + 2*a - b)*atan((e**(c + d*x)*b)/(sqrt(b)*
sqrt(2*sqrt(a)*sqrt(a - b) + 2*a - b)))*a**2 - 72*e**(6*c + 6*d*x)*sqrt(b)
*sqrt(a)*sqrt(a - b)*sqrt(2*sqrt(a)*sqrt(a - b) + 2*a - b)*atan((e**(c + d
*x)*b)/(sqrt(b)*sqrt(2*sqrt(a)*sqrt(a - b) + 2*a - b)))*a*b + 32*e**(6*c +
 6*d*x)*sqrt(b)*sqrt(a)*sqrt(a - b)*sqrt(2*sqrt(a)*sqrt(a - b) + 2*a - b)*
atan((e**(c + d*x)*b)/(sqrt(b)*sqrt(2*sqrt(a)*sqrt(a - b) + 2*a - b)))*b**
2 - 80*e**(4*c + 4*d*x)*sqrt(b)*sqrt(a)*sqrt(a - b)*sqrt(2*sqrt(a)*sqrt(a
- b) + 2*a - b)*atan((e**(c + d*x)*b)/(sqrt(b)*sqrt(2*sqrt(a)*sqrt(a - b)
+ 2*a - b)))*a**2 + 124*e**(4*c + 4*d*x)*sqrt(b)*sqrt(a)*sqrt(a - b)*sqrt(
2*sqrt(a)*sqrt(a - b) + 2*a - b)*atan((e**(c + d*x)*b)/(sqrt(b)*sqrt(2*sqr
t(a)*sqrt(a - b) + 2*a - b)))*a*b - 48*e**(4*c + 4*d*x)*sqrt(b)*sqrt(a)*sq
rt(a - b)*sqrt(2*sqrt(a)*sqrt(a - b) + 2*a - b)*atan((e**(c + d*x)*b)/(sqr
t(b)*sqrt(2*sqrt(a)*sqrt(a - b) + 2*a - b)))*b**2 + 40*e**(2*c + 2*d*x)*sq
rt(b)*sqrt(a)*sqrt(a - b)*sqrt(2*sqrt(a)*sqrt(a - b) + 2*a - b)*atan((e**(
c + d*x)*b)/(sqrt(b)*sqrt(2*sqrt(a)*sqrt(a - b) + 2*a - b)))*a**2 - 72*...
```

3.50 $\int \frac{\operatorname{csch}^4(c+dx)}{(a+b\sinh^2(c+dx))^2}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 135

$$\int \frac{\operatorname{csch}^4(c+dx)}{\left(a+b\sinh^2(c+dx)\right)^2}\,dx = \frac{(6a-5b)b^2\operatorname{arctanh}\left(\frac{\sqrt{a-b}\tanh(c+dx)}{\sqrt{a}}\right)}{2a^{7/2}(a-b)^{3/2}d} + \frac{(a+2b)\coth(c+dx)}{a^3d} - \frac{\coth^3(c+dx)}{3a^2d} - \frac{b^3\tanh(c+dx)}{2a^3(a-b)d\left(a-(a-b)\tanh^2(c+dx)\right)}$$

output
```
1/2*(6*a-5*b)*b^2*arctanh((a-b)^(1/2)*tanh(d*x+c)/a^(1/2))/a^(7/2)/(a-b)^(
3/2)/d+(a+2*b)*coth(d*x+c)/a^3/d-1/3*coth(d*x+c)^3/a^2/d-1/2*b^3*tanh(d*x+
c)/a^3/(a-b)/d/(a-(a-b)*tanh(d*x+c)^2)
```

Mathematica [A] (verified)

Time = 2.35 (sec) , antiderivative size = 210, normalized size of antiderivative = 1.56

$$\int \frac{\operatorname{csch}^4(c+dx)}{\left(a+b\sinh^2(c+dx)\right)^2}\,dx = \frac{(2a-b+b\cosh(2(c+dx)))\operatorname{csch}^4(c+dx)\left(\frac{3(6a-5b)b^2\operatorname{arctanh}\left(\frac{\sqrt{a-b}\tanh(c+dx)}{\sqrt{a}}\right)(2a-b+b\cosh(2(c+dx)))}{(a-b)^{3/2}}+4\sqrt{a}(a\cdots\right.}{2\cdots}$$

input
```
Integrate[Csch[c + d*x]^4/(a + b*Sinh[c + d*x]^2)^2,x]
```

output
```
((2*a - b + b*Cosh[2*(c + d*x)])*Csch[c + d*x]^4*((3*(6*a - 5*b)*b^2*ArcTa
nh[(Sqrt[a - b]*Tanh[c + d*x])/Sqrt[a]]*(2*a - b + b*Cosh[2*(c + d*x)]))/(
a - b)^(3/2) + 4*Sqrt[a]*(a + 3*b)*(2*a - b + b*Cosh[2*(c + d*x)])*Coth[c
+ d*x] - 2*a^(3/2)*(2*a - b + b*Cosh[2*(c + d*x)])*Coth[c + d*x]*Csch[c +
d*x]^2 - (3*Sqrt[a]*b^3*Sinh[2*(c + d*x)])/(a - b)))/(24*a^(7/2)*d*(b + a*
Csch[c + d*x]^2)^2)
```

Rubi [A] (verified)

Time = 0.49 (sec) , antiderivative size = 169, normalized size of antiderivative = 1.25, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.217, Rules used = {3042, 3666, 370, 437, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\operatorname{csch}^4(c+dx)}{\left(a+b\sinh^2(c+dx)\right)^2}dx$$

$$\Big\downarrow \; 3042$$

$$\int \frac{1}{\sin(ic+idx)^4\left(a-b\sin(ic+idx)^2\right)^2}dx$$

$$\Big\downarrow \; 3666$$

$$\frac{\int \frac{\coth^4(c+dx)(1-\tanh^2(c+dx))^3}{(a-(a-b)\tanh^2(c+dx))^2}d\tanh(c+dx)}{d}$$

$$\Big\downarrow \; 370$$

$$\frac{\frac{\int \frac{\coth^4(c+dx)\left(1-\tanh^2(c+dx)\right)\left(-\left((2a-b)\tanh^2(c+dx)\right)+2a-5b\right)}{a-(a-b)\tanh^2(c+dx)}d\tanh(c+dx)}{2a(a-b)}-\frac{b(1-\tanh^2(c+dx))^2\coth^3(c+dx)}{2a(a-b)(a-(a-b)\tanh^2(c+dx))}}{d}$$

$$\Big\downarrow \; 437$$

$$\frac{\frac{\int \left(\frac{(2a-5b)\coth^4(c+dx)}{a}+\frac{\left(-2a^2-ba+5b^2\right)\coth^2(c+dx)}{a^2}+\frac{(6a-5b)b^2}{a^2\left(a-(a-b)\tanh^2(c+dx)\right)}\right)d\tanh(c+dx)}{2a(a-b)}-\frac{b(1-\tanh^2(c+dx))^2\coth^3(c+dx)}{2a(a-b)(a-(a-b)\tanh^2(c+dx))}}{d}$$

$$\Big\downarrow \; 2009$$

$$\frac{\frac{\frac{b^2(6a-5b)\text{arctanh}\left(\frac{\sqrt{a-b}\tanh(c+dx)}{\sqrt{a}}\right)}{a^{5/2}\sqrt{a-b}}+\frac{\left(2a^2+ab-5b^2\right)\coth(c+dx)}{a^2}-\frac{(2a-5b)\coth^3(c+dx)}{3a}}{2a(a-b)}-\frac{b(1-\tanh^2(c+dx))^2\coth^3(c+dx)}{2a(a-b)(a-(a-b)\tanh^2(c+dx))}}{d}$$

input
```
Int[Csch[c + d*x]^4/(a + b*Sinh[c + d*x]^2)^2,x]
```

output
```
((((6*a - 5*b)*b^2*ArcTanh[(Sqrt[a - b]*Tanh[c + d*x])/Sqrt[a]])/(a^(5/2)*
Sqrt[a - b]) + ((2*a^2 + a*b - 5*b^2)*Coth[c + d*x])/a^2 - ((2*a - 5*b)*Co
th[c + d*x]^3)/(3*a))/(2*a*(a - b)) - (b*Coth[c + d*x]^3*(1 - Tanh[c + d*x
]^2)^2)/(2*a*(a - b)*(a - (a - b)*Tanh[c + d*x]^2)))/d
```

Defintions of rubi rules used

rule 370
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_
), x_Symbol] :> Simp[(-(b*c - a*d))*(e*x)^(m + 1)*(a + b*x^2)^(p + 1)*((c +
 d*x^2)^(q - 1)/(a*b*e*2*(p + 1))), x] + Simp[1/(a*b*2*(p + 1))   Int[(e*x)
^m*(a + b*x^2)^(p + 1)*(c + d*x^2)^(q - 2)*Simp[c*(b*c*2*(p + 1) + (b*c - a
*d)*(m + 1)) + d*(b*c*2*(p + 1) + (b*c - a*d)*(m + 2*(q - 1) + 1))*x^2, x],
 x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1]
&& GtQ[q, 1] && IntBinomialQ[a, b, c, d, e, m, 2, p, q, x]
```

rule 437
```
Int[((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q
_.)*((e_) + (f_.)*(x_)^2)^(r_.), x_Symbol] :> Int[ExpandIntegrand[(g*x)^m*(
a + b*x^2)^p*(c + d*x^2)^q*(e + f*x^2)^r, x], x] /; FreeQ[{a, b, c, d, e, f
, g, m}, x] && IGtQ[p, -2] && IGtQ[q, 0] && IGtQ[r, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3666
```
Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(
p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Simp[ff^(m + 1
)/f   Subst[Int[x^m*((a + (a + b)*ff^2*x^2)^p/(1 + ff^2*x^2)^(m/2 + p + 1))
, x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f}, x] && IntegerQ[m/2] &
& IntegerQ[p]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 370 vs. $2(121) = 242$.

Time = 1.71 (sec) , antiderivative size = 371, normalized size of antiderivative = 2.75

method	result
derivativedivides	$\frac{-\frac{\frac{\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^3 a}{3}-3\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)a-8b\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)}{8a^3}-\frac{1}{24a^2\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^3}-\frac{-3a-8b}{8a^3\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)}-\frac{2b^2\left(\frac{\frac{\tanh\left(\frac{dx}{2}\right.}{2a-}}{\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^4 a-2\,\mathrm{ta}}\right.}{}}{}$
default	$\frac{-\frac{\frac{\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^3 a}{3}-3\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)a-8b\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)}{8a^3}-\frac{1}{24a^2\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^3}-\frac{-3a-8b}{8a^3\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)}-\frac{2b^2\left(\frac{\frac{\tanh\left(\frac{dx}{2}\right.}{2a-}}{\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^4 a-2\,\mathrm{ta}}\right.}{}}{}$
risch	$-\frac{-18a\,b^2e^{8dx+8c}+15b^3e^{8dx+8c}-36a^2b\,e^{6dx+6c}+102a\,b^2e^{6dx+6c}-60b^3e^{6dx+6c}+48a^3e^{4dx+4c}+20a^2b\,e^{4dx+4c}-158a}{3d\,a^3\left(e^{2dx+2c}-1\right)^3(a-b)\left(b\,e^{4dx+4c}+\right.}$

input

```
int(csch(d*x+c)^4/(a+b*sinh(d*x+c)^2)^2,x,method=_RETURNVERBOSE)
```

output

```
1/d*(-1/8/a^3*(1/3*tanh(1/2*d*x+1/2*c)^3*a-3*tanh(1/2*d*x+1/2*c)*a-8*b*tan
h(1/2*d*x+1/2*c))-1/24/a^2/tanh(1/2*d*x+1/2*c)^3-1/8/a^3*(-3*a-8*b)/tanh(1
/2*d*x+1/2*c)-2*b^2/a^3*((1/2*b/(a-b)*tanh(1/2*d*x+1/2*c)^3+1/2*b/(a-b)*ta
nh(1/2*d*x+1/2*c))/(tanh(1/2*d*x+1/2*c)^4*a-2*tanh(1/2*d*x+1/2*c)^2*a+4*b*
tanh(1/2*d*x+1/2*c)^2+a)+1/2*(6*a-5*b)/(a-b)*a*(1/2*((-b*(a-b))^(1/2)+b)/a
/(-b*(a-b))^(1/2)/((2*(-b*(a-b))^(1/2)-a+2*b)*a)^(1/2)*arctan(tanh(1/2*d*x
+1/2*c)*a/((2*(-b*(a-b))^(1/2)-a+2*b)*a)^(1/2))-1/2*((-b*(a-b))^(1/2)-b)/a
/(-b*(a-b))^(1/2)/((2*(-b*(a-b))^(1/2)+a-2*b)*a)^(1/2)*arctanh(tanh(1/2*d*
x+1/2*c)*a/((2*(-b*(a-b))^(1/2)+a-2*b)*a)^(1/2)))))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 3427 vs. $2(122) = 244$.

Time = 0.18 (sec) , antiderivative size = 7110, normalized size of antiderivative = 52.67

$$\int \frac{\operatorname{csch}^4(c+dx)}{\left(a+b\sinh^2(c+dx)\right)^2}\,dx = \text{Too large to display}$$

input
```
integrate(csch(d*x+c)^4/(a+b*sinh(d*x+c)^2)^2,x, algorithm="fricas")
```

output
```
Too large to include
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\operatorname{csch}^4(c+dx)}{\left(a+b\sinh^2(c+dx)\right)^2}\,dx = \text{Timed out}$$

input
```
integrate(csch(d*x+c)**4/(a+b*sinh(d*x+c)**2)**2,x)
```

output
```
Timed out
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{\operatorname{csch}^4(c+dx)}{\left(a+b\sinh^2(c+dx)\right)^2}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate(csch(d*x+c)^4/(a+b*sinh(d*x+c)^2)^2,x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(b-a>0)', see `assume?` for more
details)Is
```

Giac [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 220, normalized size of antiderivative = 1.63

$$\int \frac{\operatorname{csch}^4(c+dx)}{\left(a+b\sinh^2(c+dx)\right)^2}\,dx = \frac{\frac{3\left(6\,ab^2-5\,b^3\right)\arctan\left(\frac{be^{(2\,dx+2\,c)}+2\,a-b}{2\,\sqrt{-a^2+ab}}\right)}{\left(a^4-a^3b\right)\sqrt{-a^2+ab}}+\frac{6\left(2\,ab^2e^{(2\,dx+2\,c)}-b^3e^{(2\,dx+2\,c)}+b^3\right)}{\left(a^4-a^3b\right)\left(be^{(4\,dx+4\,c)}+4\,ae^{(2\,dx+2\,c)}-2\,be^{(2\,dx+2\,c)}+b\right)}+\frac{8\left(3\,be^{(4\,dx+4\,c)}-3\,ae^{(2\,dx+2\,c)}-6\,b\right.}{a^3\left(e^{(2\,dx+2\,c)}-1\right)}}{6\,d}$$

input
```
integrate(csch(d*x+c)^4/(a+b*sinh(d*x+c)^2)^2,x, algorithm="giac")
```

output
```
1/6*(3*(6*a*b^2 - 5*b^3)*arctan(1/2*(b*e^(2*d*x + 2*c) + 2*a - b)/sqrt(-a^
2 + a*b))/((a^4 - a^3*b)*sqrt(-a^2 + a*b)) + 6*(2*a*b^2*e^(2*d*x + 2*c) -
b^3*e^(2*d*x + 2*c) + b^3)/((a^4 - a^3*b)*(b*e^(4*d*x + 4*c) + 4*a*e^(2*d*
x + 2*c) - 2*b*e^(2*d*x + 2*c) + b)) + 8*(3*b*e^(4*d*x + 4*c) - 3*a*e^(2*d
*x + 2*c) - 6*b*e^(2*d*x + 2*c) + a + 3*b)/(a^3*(e^(2*d*x + 2*c) - 1)^3))/
d
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\operatorname{csch}^4(c+dx)}{\left(a+b\sinh^2(c+dx)\right)^2}\,dx = \int \frac{1}{{\mathrm{sinh}\left(c+d\,x\right)}^4\,{\left(b\,{\mathrm{sinh}\left(c+d\,x\right)}^2+a\right)}^2} \,d x$$

input
```
int(1/(sinh(c + d*x)^4*(a + b*sinh(c + d*x)^2)^2),x)
```

output
```
int(1/(sinh(c + d*x)^4*(a + b*sinh(c + d*x)^2)^2), x)
```

Reduce [B] (verification not implemented)

Time = 0.35 (sec) , antiderivative size = 4270, normalized size of antiderivative = 31.63

$$\int \frac{\operatorname{csch}^4(c+dx)}{\left(a+b\sinh^2(c+dx)\right)^2}\,dx = \text{Too large to display}$$

input
```
int(csch(d*x+c)^4/(a+b*sinh(d*x+c)^2)^2,x)
```

output
```
(72*e**(10*c + 10*d*x)*sqrt(a)*sqrt(a - b)*log( - sqrt(2*sqrt(a)*sqrt(a -
b) - 2*a + b) + e**(c + d*x)*sqrt(b))*a**2*b**3 - 150*e**(10*c + 10*d*x)*s
qrt(a)*sqrt(a - b)*log( - sqrt(2*sqrt(a)*sqrt(a - b) - 2*a + b) + e**(c +
d*x)*sqrt(b))*a*b**4 + 75*e**(10*c + 10*d*x)*sqrt(a)*sqrt(a - b)*log( - sq
rt(2*sqrt(a)*sqrt(a - b) - 2*a + b) + e**(c + d*x)*sqrt(b))*b**5 + 72*e**(
10*c + 10*d*x)*sqrt(a)*sqrt(a - b)*log(sqrt(2*sqrt(a)*sqrt(a - b) - 2*a +
b) + e**(c + d*x)*sqrt(b))*a**2*b**3 - 150*e**(10*c + 10*d*x)*sqrt(a)*sqrt
(a - b)*log(sqrt(2*sqrt(a)*sqrt(a - b) - 2*a + b) + e**(c + d*x)*sqrt(b))*
a*b**4 + 75*e**(10*c + 10*d*x)*sqrt(a)*sqrt(a - b)*log(sqrt(2*sqrt(a)*sqrt
(a - b) - 2*a + b) + e**(c + d*x)*sqrt(b))*b**5 - 72*e**(10*c + 10*d*x)*sq
rt(a)*sqrt(a - b)*log(2*sqrt(a)*sqrt(a - b) + e**(2*c + 2*d*x)*b + 2*a - b
)*a**2*b**3 + 150*e**(10*c + 10*d*x)*sqrt(a)*sqrt(a - b)*log(2*sqrt(a)*sqr
t(a - b) + e**(2*c + 2*d*x)*b + 2*a - b)*a*b**4 - 75*e**(10*c + 10*d*x)*sq
rt(a)*sqrt(a - b)*log(2*sqrt(a)*sqrt(a - b) + e**(2*c + 2*d*x)*b + 2*a - b
)*b**5 + 288*e**(8*c + 8*d*x)*sqrt(a)*sqrt(a - b)*log( - sqrt(2*sqrt(a)*sq
rt(a - b) - 2*a + b) + e**(c + d*x)*sqrt(b))*a**3*b**2 - 960*e**(8*c + 8*d
*x)*sqrt(a)*sqrt(a - b)*log( - sqrt(2*sqrt(a)*sqrt(a - b) - 2*a + b) + e**
(c + d*x)*sqrt(b))*a**2*b**3 + 1050*e**(8*c + 8*d*x)*sqrt(a)*sqrt(a - b)*l
og( - sqrt(2*sqrt(a)*sqrt(a - b) - 2*a + b) + e**(c + d*x)*sqrt(b))*a*b**4
 - 375*e**(8*c + 8*d*x)*sqrt(a)*sqrt(a - b)*log( - sqrt(2*sqrt(a)*sqrt(...
```

3.51 $\int \frac{\sinh^4(c+dx)}{\left(a+b\sinh^2(c+dx)\right)^3}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 124

$$\int \frac{\sinh^4(c+dx)}{\left(a+b\sinh^2(c+dx)\right)^3}\,dx = \frac{3\text{arctanh}\left(\frac{\sqrt{a-b}\tanh(c+dx)}{\sqrt{a}}\right)}{8\sqrt{a}(a-b)^{5/2}d} + \frac{\tanh^3(c+dx)}{4(a-b)d\left(a-(a-b)\tanh^2(c+dx)\right)^2} - \frac{3\tanh(c+dx)}{8(a-b)^2d\left(a-(a-b)\tanh^2(c+dx)\right)}$$

output
```
3/8*arctanh((a-b)^(1/2)*tanh(d*x+c)/a^(1/2))/a^(1/2)/(a-b)^(5/2)/d+1/4*tan
h(d*x+c)^3/(a-b)/d/(a-(a-b)*tanh(d*x+c)^2)^2-3/8*tanh(d*x+c)/(a-b)^2/d/(a-
(a-b)*tanh(d*x+c)^2)
```

Mathematica [A] (verified)

Time = 11.56 (sec) , antiderivative size = 104, normalized size of antiderivative = 0.84

$$\int \frac{\sinh^4(c+dx)}{\left(a+b\sinh^2(c+dx)\right)^3}\,dx = \frac{\frac{3\operatorname{arctanh}\left(\frac{\sqrt{a-b}\tanh(c+dx)}{\sqrt{a}}\right)}{\sqrt{a}(a-b)^{5/2}}+\frac{(-8a+5b+(2a-5b)\cosh(2(c+dx)))\sinh(2(c+dx))}{(a-b)^2(2a-b+b\cosh(2(c+dx)))^2}}{8d}$$

input
```
Integrate[Sinh[c + d*x]^4/(a + b*Sinh[c + d*x]^2)^3,x]
```

output
```
((3*ArcTanh[(Sqrt[a - b]*Tanh[c + d*x])/Sqrt[a]])/(Sqrt[a]*(a - b)^(5/2))
+ ((-8*a + 5*b + (2*a - 5*b)*Cosh[2*(c + d*x)])*Sinh[2*(c + d*x)])/((a - b
)^2*(2*a - b + b*Cosh[2*(c + d*x)])^2))/(8*d)
```

Rubi [A] (verified)

Time = 0.35 (sec) , antiderivative size = 131, normalized size of antiderivative = 1.06, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.217$, Rules used = {3042, 3666, 252, 252, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sinh^4(c+dx)}{\left(a+b\sinh^2(c+dx)\right)^3}\,dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \frac{\sin(ic+idx)^4}{(a-b\sin(ic+idx)^2)^3}dx$$

$$\Big\downarrow \text{ 3666}$$

$$\frac{\int \frac{\tanh^4(c+dx)}{\left(a-(a-b)\tanh^2(c+dx)\right)^3}d\tanh(c+dx)}{d}$$

$$\Big\downarrow \text{ 252}$$

$$\frac{\frac{\tanh^3(c+dx)}{4(a-b)\left(a-(a-b)\tanh^2(c+dx)\right)^2}-\frac{3\int \frac{\tanh^2(c+dx)}{\left(a-(a-b)\tanh^2(c+dx)\right)^2}d\tanh(c+dx)}{4(a-b)}}{d}$$

$\Big\downarrow$ 252

$$\frac{\frac{\tanh^3(c+dx)}{4(a-b)\left(a-(a-b)\tanh^2(c+dx)\right)^2}-\frac{3\left(\frac{\tanh(c+dx)}{2(a-b)\left(a-(a-b)\tanh^2(c+dx)\right)}-\frac{\int \frac{1}{a-(a-b)\tanh^2(c+dx)}d\tanh(c+dx)}{2(a-b)}\right)}{4(a-b)}}{d}$$

$\Big\downarrow$ 221

$$\frac{\frac{\tanh^3(c+dx)}{4(a-b)\left(a-(a-b)\tanh^2(c+dx)\right)^2}-\frac{3\left(\frac{\tanh(c+dx)}{2(a-b)\left(a-(a-b)\tanh^2(c+dx)\right)}-\frac{\text{arctanh}\left(\frac{\sqrt{a-b}\tanh(c+dx)}{\sqrt{a}}\right)}{2\sqrt{a}(a-b)^{3/2}}\right)}{4(a-b)}}{d}$$

input
```
Int[Sinh[c + d*x]^4/(a + b*Sinh[c + d*x]^2)^3,x]
```

output
```
(Tanh[c + d*x]^3/(4*(a - b)*(a - (a - b)*Tanh[c + d*x]^2)^2) - (3*(-1/2*Ar
cTanh[(Sqrt[a - b]*Tanh[c + d*x])/Sqrt[a]]/(Sqrt[a]*(a - b)^(3/2)) + Tanh[
c + d*x]/(2*(a - b)*(a - (a - b)*Tanh[c + d*x]^2))))/(4*(a - b)))/d
```

Defintions of rubi rules used

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 252
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[c*(c*x
)^(m - 1)*((a + b*x^2)^(p + 1)/(2*b*(p + 1))), x] - Simp[c^2*((m - 1)/(2*b*
(p + 1)))   Int[(c*x)^(m - 2)*(a + b*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c
}, x] && LtQ[p, -1] && GtQ[m, 1] &&  !ILtQ[(m + 2*p + 3)/2, 0] && IntBinomi
alQ[a, b, c, 2, m, p, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3666
```
Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(
p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Simp[ff^(m + 1
)/f   Subst[Int[x^m*((a + (a + b)*ff^2*x^2)^p/(1 + ff^2*x^2)^(m/2 + p + 1))
, x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f}, x] && IntegerQ[m/2] &
& IntegerQ[p]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 356 vs. $2(110) = 220$.

Time = 1.66 (sec) , antiderivative size = 357, normalized size of antiderivative = 2.88

method	result
derivativedivides	$\frac{-\frac{32\left(\frac{3a\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^{7}}{128\left(a^{2}-2ab+b^{2}\right)}-\frac{(11a-20b)\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^{5}}{128\left(a^{2}-2ab+b^{2}\right)}-\frac{(11a-20b)\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^{3}}{128\left(a^{2}-2ab+b^{2}\right)}+\frac{3a\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)}{128\left(a^{2}-2ab+b^{2}\right)}\right)}{\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^{4}a-2\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}a+4b\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}+a\right)^{2}}-\frac{3a\left(\frac{\left(\sqrt{-b(a-b)}+b\right)\mathrm{ar}\ldots}{2a\sqrt{-b(a-b}\ldots}\ldots\right.}{\ldots}}{d}$
default	$\frac{-\frac{32\left(\frac{3a\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^{7}}{128\left(a^{2}-2ab+b^{2}\right)}-\frac{(11a-20b)\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^{5}}{128\left(a^{2}-2ab+b^{2}\right)}-\frac{(11a-20b)\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^{3}}{128\left(a^{2}-2ab+b^{2}\right)}+\frac{3a\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)}{128\left(a^{2}-2ab+b^{2}\right)}\right)}{\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^{4}a-2\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}a+4b\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}+a\right)^{2}}-\frac{3a\left(\frac{\left(\sqrt{-b(a-b)}+b\right)\mathrm{ar}\ldots}{2a\sqrt{-b(a-b}\ldots}\ldots\right.}{\ldots}}{d}$
risch	$-\frac{8a^{2}b\,e^{6dx+6c}-16a\,b^{2}e^{6dx+6c}+5b^{3}e^{6dx+6c}+16a^{3}e^{4dx+4c}-56a^{2}b\,e^{4dx+4c}+46a\,b^{2}e^{4dx+4c}-15b^{3}e^{4dx+4c}+8\,e^{2dx+2c}\ldots}{4b^{2}d(a-b)^{2}\left(b\,e^{4dx+4c}+4\,e^{2dx+2c}a-2\,e^{2dx+2c}b+b\right)^{2}}$

input
```
int(sinh(d*x+c)^4/(a+b*sinh(d*x+c)^2)^3,x,method=_RETURNVERBOSE)
```

output
```
1/d*(-32*(3/128*a/(a^2-2*a*b+b^2)*tanh(1/2*d*x+1/2*c)^7-1/128*(11*a-20*b)/
(a^2-2*a*b+b^2)*tanh(1/2*d*x+1/2*c)^5-1/128*(11*a-20*b)/(a^2-2*a*b+b^2)*ta
nh(1/2*d*x+1/2*c)^3+3/128*a/(a^2-2*a*b+b^2)*tanh(1/2*d*x+1/2*c))/(tanh(1/2
*d*x+1/2*c)^4*a-2*tanh(1/2*d*x+1/2*c)^2*a+4*b*tanh(1/2*d*x+1/2*c)^2+a)^2-3
/4/(a^2-2*a*b+b^2)*a*(1/2*((-b*(a-b))^(1/2)+b)/a/(-b*(a-b))^(1/2)/((2*(-b*
(a-b))^(1/2)-a+2*b)*a)^(1/2)*arctan(tanh(1/2*d*x+1/2*c)*a/((2*(-b*(a-b))^(
1/2)-a+2*b)*a)^(1/2))-1/2*((-b*(a-b))^(1/2)-b)/a/(-b*(a-b))^(1/2)/((2*(-b*
(a-b))^(1/2)+a-2*b)*a)^(1/2)*arctanh(tanh(1/2*d*x+1/2*c)*a/((2*(-b*(a-b))^
(1/2)+a-2*b)*a)^(1/2))))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2465 vs. $2(112) = 224$.

Time = 0.16 (sec) , antiderivative size = 5186, normalized size of antiderivative = 41.82

$$\int \frac{\sinh^4(c+dx)}{\left(a+b\sinh^2(c+dx)\right)^3}\,dx = \text{Too large to display}$$

input
```
integrate(sinh(d*x+c)^4/(a+b*sinh(d*x+c)^2)^3,x, algorithm="fricas")
```

output
```
Too large to include
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\sinh^4(c+dx)}{\left(a+b\sinh^2(c+dx)\right)^3}\,dx = \text{Timed out}$$

input
```
integrate(sinh(d*x+c)**4/(a+b*sinh(d*x+c)**2)**3,x)
```

output
```
Timed out
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{\sinh^4(c+dx)}{\left(a+b\sinh^2(c+dx)\right)^3}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate(sinh(d*x+c)^4/(a+b*sinh(d*x+c)^2)^3,x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(b-a>0)', see `assume?` for more
details)Is
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 282 vs. $2(112) = 224$.

Time = 1.10 (sec) , antiderivative size = 282, normalized size of antiderivative = 2.27

$$\int \frac{\sinh^4(c+dx)}{\left(a+b\sinh^2(c+dx)\right)^3}\,dx$$
$$= \frac{\frac{3\arctan\left(\frac{be^{(2\,dx+2\,c)}+2\,a-b}{2\,\sqrt{-a^2+ab}}\right)}{(a^2-2\,ab+b^2)\sqrt{-a^2+ab}} - \frac{2\,(8\,a^2be^{(6\,dx+6\,c)}-16\,ab^2e^{(6\,dx+6\,c)}+5\,b^3e^{(6\,dx+6\,c)}+16\,a^3e^{(4\,dx+4\,c)}-56\,a^2be^{(4\,dx+4\,c)}+46\,ab^2e^{(4\,dx+4\,c)}-}{(a^2b^2-2\,ab^3+b^4)\left(be^{(4\,dx+4\,c)}+4\,ae^{(2\,dx+2\,c)}-\right.}}{8\,d}$$

input
```
integrate(sinh(d*x+c)^4/(a+b*sinh(d*x+c)^2)^3,x, algorithm="giac")
```

output
```
1/8*(3*arctan(1/2*(b*e^(2*d*x + 2*c) + 2*a - b)/sqrt(-a^2 + a*b))/((a^2 -
2*a*b + b^2)*sqrt(-a^2 + a*b)) - 2*(8*a^2*b*e^(6*d*x + 6*c) - 16*a*b^2*e^(
6*d*x + 6*c) + 5*b^3*e^(6*d*x + 6*c) + 16*a^3*e^(4*d*x + 4*c) - 56*a^2*b*e
^(4*d*x + 4*c) + 46*a*b^2*e^(4*d*x + 4*c) - 15*b^3*e^(4*d*x + 4*c) + 8*a^2
*b*e^(2*d*x + 2*c) - 32*a*b^2*e^(2*d*x + 2*c) + 15*b^3*e^(2*d*x + 2*c) + 2
*a*b^2 - 5*b^3)/((a^2*b^2 - 2*a*b^3 + b^4)*(b*e^(4*d*x + 4*c) + 4*a*e^(2*d
*x + 2*c) - 2*b*e^(2*d*x + 2*c) + b)^2))/d
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sinh^4(c+dx)}{\left(a+b\sinh^2(c+dx)\right)^3}\,dx = \int \frac{\sinh(c+d\,x)^4}{\left(b\sinh\left(c+d\,x\right)^2+a\right)^3}\,dx$$

input
```
int(sinh(c + d*x)^4/(a + b*sinh(c + d*x)^2)^3,x)
```

output
```
int(sinh(c + d*x)^4/(a + b*sinh(c + d*x)^2)^3, x)
```

Reduce [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 2880, normalized size of antiderivative = 23.23

$$\int \frac{\sinh^4(c+dx)}{\left(a+b\sinh^2(c+dx)\right)^3}\,dx = \text{Too large to display}$$

input
```
int(sinh(d*x+c)^4/(a+b*sinh(d*x+c)^2)^3,x)
```

output
```
(6*e**(8*c + 8*d*x)*sqrt(a)*sqrt(a - b)*log( - sqrt(2*sqrt(a)*sqrt(a - b)
- 2*a + b) + e**(c + d*x)*sqrt(b))*a*b**3 - 3*e**(8*c + 8*d*x)*sqrt(a)*sqr
t(a - b)*log( - sqrt(2*sqrt(a)*sqrt(a - b) - 2*a + b) + e**(c + d*x)*sqrt(
b))*b**4 + 6*e**(8*c + 8*d*x)*sqrt(a)*sqrt(a - b)*log(sqrt(2*sqrt(a)*sqrt(
a - b) - 2*a + b) + e**(c + d*x)*sqrt(b))*a*b**3 - 3*e**(8*c + 8*d*x)*sqrt
(a)*sqrt(a - b)*log(sqrt(2*sqrt(a)*sqrt(a - b) - 2*a + b) + e**(c + d*x)*s
qrt(b))*b**4 - 6*e**(8*c + 8*d*x)*sqrt(a)*sqrt(a - b)*log(2*sqrt(a)*sqrt(a
 - b) + e**(2*c + 2*d*x)*b + 2*a - b)*a*b**3 + 3*e**(8*c + 8*d*x)*sqrt(a)*
sqrt(a - b)*log(2*sqrt(a)*sqrt(a - b) + e**(2*c + 2*d*x)*b + 2*a - b)*b**4
 + 48*e**(6*c + 6*d*x)*sqrt(a)*sqrt(a - b)*log( - sqrt(2*sqrt(a)*sqrt(a -
b) - 2*a + b) + e**(c + d*x)*sqrt(b))*a**2*b**2 - 48*e**(6*c + 6*d*x)*sqrt
(a)*sqrt(a - b)*log( - sqrt(2*sqrt(a)*sqrt(a - b) - 2*a + b) + e**(c + d*x
)*sqrt(b))*a*b**3 + 12*e**(6*c + 6*d*x)*sqrt(a)*sqrt(a - b)*log( - sqrt(2*
sqrt(a)*sqrt(a - b) - 2*a + b) + e**(c + d*x)*sqrt(b))*b**4 + 48*e**(6*c +
 6*d*x)*sqrt(a)*sqrt(a - b)*log(sqrt(2*sqrt(a)*sqrt(a - b) - 2*a + b) + e*
*(c + d*x)*sqrt(b))*a**2*b**2 - 48*e**(6*c + 6*d*x)*sqrt(a)*sqrt(a - b)*lo
g(sqrt(2*sqrt(a)*sqrt(a - b) - 2*a + b) + e**(c + d*x)*sqrt(b))*a*b**3 + 1
2*e**(6*c + 6*d*x)*sqrt(a)*sqrt(a - b)*log(sqrt(2*sqrt(a)*sqrt(a - b) - 2*
a + b) + e**(c + d*x)*sqrt(b))*b**4 - 48*e**(6*c + 6*d*x)*sqrt(a)*sqrt(a -
 b)*log(2*sqrt(a)*sqrt(a - b) + e**(2*c + 2*d*x)*b + 2*a - b)*a**2*b**2...
```

3.52 $\int \frac{\sinh^3(c+dx)}{\left(a+b\sinh^2(c+dx)\right)^3}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 135

$$\int \frac{\sinh^3(c+dx)}{\left(a+b\sinh^2(c+dx)\right)^3}\,dx=\frac{(a-4b)\arctan\left(\frac{\sqrt{b}\cosh(c+dx)}{\sqrt{a-b}}\right)}{8(a-b)^{5/2}b^{3/2}d}-\frac{a\cosh(c+dx)}{4(a-b)bd\left(a-b+b\cosh^2(c+dx)\right)^2}+\frac{(a-4b)\cosh(c+dx)}{8(a-b)^2bd\left(a-b+b\cosh^2(c+dx)\right)}$$

output
```
1/8*(a-4*b)*arctan(b^(1/2)*cosh(d*x+c)/(a-b)^(1/2))/(a-b)^(5/2)/b^(3/2)/d-
1/4*a*cosh(d*x+c)/(a-b)/b/d/(a-b+b*cosh(d*x+c)^2)^2+1/8*(a-4*b)*cosh(d*x+c
)/(a-b)^2/b/d/(a-b+b*cosh(d*x+c)^2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 2.08 (sec) , antiderivative size = 170, normalized size of antiderivative = 1.26

$$\int \frac{\sinh^3(c+dx)}{\left(a+b\sinh^2(c+dx)\right)^3}\,dx = \frac{\frac{(a-4b)\left(\arctan\left(\frac{\sqrt{b}-i\sqrt{a}\tanh\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a-b}}\right)+\arctan\left(\frac{\sqrt{b}+i\sqrt{a}\tanh\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a-b}}\right)\right)}{(a-b)^{5/2}}+\frac{2\sqrt{b}\cosh(c+dx)\left(-2a^2-5ab+4b^2+(a-4b)b\cosh(2(c+dx))\right)}{(a-b)^2(2a-b+b\cosh(2(c+dx)))^2}}{8b^{3/2}d}$$

input
```
Integrate[Sinh[c + d*x]^3/(a + b*Sinh[c + d*x]^2)^3,x]
```

output
```
(((a - 4*b)*(ArcTan[(Sqrt[b] - I*Sqrt[a]*Tanh[(c + d*x)/2])/Sqrt[a - b]] +
 ArcTan[(Sqrt[b] + I*Sqrt[a]*Tanh[(c + d*x)/2])/Sqrt[a - b]]))/(a - b)^(5/
2) + (2*Sqrt[b]*Cosh[c + d*x]*(-2*a^2 - 5*a*b + 4*b^2 + (a - 4*b)*b*Cosh[2
*(c + d*x)]))/((a - b)^2*(2*a - b + b*Cosh[2*(c + d*x)])^2))/(8*b^(3/2)*d)
```

Rubi [A] (verified)

Time = 0.35 (sec) , antiderivative size = 138, normalized size of antiderivative = 1.02, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.261$, Rules used = {3042, 26, 3665, 298, 215, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sinh^3(c+dx)}{\left(a+b\sinh^2(c+dx)\right)^3}\,dx$$

$$\Big\downarrow \; 3042$$

$$\int \frac{i\sin(ic+idx)^3}{\left(a-b\sin(ic+idx)^2\right)^3}dx$$

$$\Big\downarrow \; 26$$

$$i\int \frac{\sin(ic+idx)^3}{\left(a-b\sin(ic+idx)^2\right)^3}dx$$

$$\Big\downarrow \; 3665$$

$$-\frac{\int \frac{1-\cosh^2(c+dx)}{\left(b\cosh^2(c+dx)+a-b\right)^3}d\cosh(c+dx)}{d}$$

$$\Big\downarrow \; 298$$

$$-\frac{\frac{a\cosh(c+dx)}{4b(a-b)\left(a+b\cosh^2(c+dx)-b\right)^2}-\frac{(a-4b)\int \frac{1}{\left(b\cosh^2(c+dx)+a-b\right)^2}d\cosh(c+dx)}{4b(a-b)}}{d}$$

$$\Big\downarrow \; 215$$

$$-\frac{\frac{a\cosh(c+dx)}{4b(a-b)\left(a+b\cosh^2(c+dx)-b\right)^2}-\frac{(a-4b)\left(\frac{\int \frac{1}{b\cosh^2(c+dx)+a-b}d\cosh(c+dx)}{2(a-b)}+\frac{\cosh(c+dx)}{2(a-b)\left(a+b\cosh^2(c+dx)-b\right)}\right)}{4b(a-b)}}{d}$$

$$\Big\downarrow \; 218$$

$$-\frac{\frac{a\cosh(c+dx)}{4b(a-b)\left(a+b\cosh^2(c+dx)-b\right)^2}-\frac{(a-4b)\left(\frac{\arctan\left(\frac{\sqrt{b}\cosh(c+dx)}{\sqrt{a-b}}\right)}{2\sqrt{b}(a-b)^{3/2}}+\frac{\cosh(c+dx)}{2(a-b)\left(a+b\cosh^2(c+dx)-b\right)}\right)}{4b(a-b)}}{d}$$

input
```
Int[Sinh[c + d*x]^3/(a + b*Sinh[c + d*x]^2)^3,x]
```

output
```
-(((a*Cosh[c + d*x])/(4*(a - b)*b*(a - b + b*Cosh[c + d*x]^2)^2) - ((a - 4
*b)*(ArcTan[(Sqrt[b]*Cosh[c + d*x])/Sqrt[a - b]]/(2*(a - b)^(3/2)*Sqrt[b])
 + Cosh[c + d*x]/(2*(a - b)*(a - b + b*Cosh[c + d*x]^2))))/(4*(a - b)*b))/
d)
```

Defintions of rubi rules used

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 215
```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(-x)*((a + b*x^2)^(p + 1)
/(2*a*(p + 1))), x] + Simp[(2*p + 3)/(2*a*(p + 1))   Int[(a + b*x^2)^(p + 1
), x], x] /; FreeQ[{a, b}, x] && LtQ[p, -1] && (IntegerQ[4*p] || IntegerQ[6
*p])
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 298
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2), x_Symbol] :> Simp[(-(
b*c - a*d))*x*((a + b*x^2)^(p + 1)/(2*a*b*(p + 1))), x] - Simp[(a*d - b*c*(
2*p + 3))/(2*a*b*(p + 1))   Int[(a + b*x^2)^(p + 1), x], x] /; FreeQ[{a, b,
 c, d, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/2 + p, 0])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3665
```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^
(p_.), x_Symbol] :> With[{ff = FreeFactors[Cos[e + f*x], x]}, Simp[-ff/f
Subst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a + b - b*ff^2*x^2)^p, x], x, Cos[e +
 f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m - 1)/2]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 280 vs. $2(121) = 242$.

Time $= 1.68$ (sec) , antiderivative size $= 281$, normalized size of antiderivative $= 2.08$

method	result
derivativedivides	$\frac{\frac{\frac{a(a-4b)\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^{6}}{4b\left(a^{2}-2ab+b^{2}\right)}-\frac{\left(3a^{3}-2a^{2}b-8b^{2}a+16b^{3}\right)\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^{4}}{4ab\left(a^{2}-2ab+b^{2}\right)}+\frac{\left(3a^{2}+4ab-16b^{2}\right)\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}}{4b\left(a^{2}-2ab+b^{2}\right)}-\frac{(a+2b)a}{4b\left(a^{2}-2ab+b^{2}\right)}}{\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^{4}a-2\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}a+4b\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}+a\right)^{2}}+\frac{(a-\cdots}{\cdots}}{d}$
default	$\frac{\frac{\frac{a(a-4b)\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^{6}}{4b\left(a^{2}-2ab+b^{2}\right)}-\frac{\left(3a^{3}-2a^{2}b-8b^{2}a+16b^{3}\right)\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^{4}}{4ab\left(a^{2}-2ab+b^{2}\right)}+\frac{\left(3a^{2}+4ab-16b^{2}\right)\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}}{4b\left(a^{2}-2ab+b^{2}\right)}-\frac{(a+2b)a}{4b\left(a^{2}-2ab+b^{2}\right)}}{\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^{4}a-2\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}a+4b\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}+a\right)^{2}}+\frac{(a-\cdots}{\cdots}}{d}$
risch	$-\frac{e^{dx+c}\left(-e^{6dx+6c}ab+4e^{6dx+6c}b^{2}+4e^{4dx+4c}a^{2}+9e^{4dx+4c}ab-4e^{4dx+4c}b^{2}+4e^{2dx+2c}a^{2}+9e^{2dx+2c}ba-4b^{2}e^{2dx+2c}\cdots\right.}{4b(a-b)^{2}d\left(b e^{4dx+4c}+4e^{2dx+2c}a-2e^{2dx+2c}b+b\right)^{2}}$

input
```
int(sinh(d*x+c)^3/(a+b*sinh(d*x+c)^2)^3,x,method=_RETURNVERBOSE)
```

output
```
1/d*(8*(1/32*a*(a-4*b)/b/(a^2-2*a*b+b^2)*tanh(1/2*d*x+1/2*c)^6-1/32*(3*a^3
-2*a^2*b-8*a*b^2+16*b^3)/a/b/(a^2-2*a*b+b^2)*tanh(1/2*d*x+1/2*c)^4+1/32*(3
*a^2+4*a*b-16*b^2)/b/(a^2-2*a*b+b^2)*tanh(1/2*d*x+1/2*c)^2-1/32*(a+2*b)*a/
b/(a^2-2*a*b+b^2))/(tanh(1/2*d*x+1/2*c)^4*a-2*tanh(1/2*d*x+1/2*c)^2*a+4*b*
tanh(1/2*d*x+1/2*c)^2+a)^2+1/8*(a-4*b)/b/(a^2-2*a*b+b^2)/(a*b-b^2)^(1/2)*a
rctan(1/4*(2*tanh(1/2*d*x+1/2*c)^2*a-2*a+4*b)/(a*b-b^2)^(1/2)))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 3292 vs. $2(121) = 242$.

Time = 0.15 (sec) , antiderivative size = 6086, normalized size of antiderivative = 45.08

$$\int \frac{\sinh^3(c+dx)}{\left(a+b\sinh^2(c+dx)\right)^3}\,dx = \text{Too large to display}$$

input
```
integrate(sinh(d*x+c)^3/(a+b*sinh(d*x+c)^2)^3,x, algorithm="fricas")
```

output
```
Too large to include
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\sinh^3(c+dx)}{\left(a+b\sinh^2(c+dx)\right)^3}\,dx = \text{Timed out}$$

input
```
integrate(sinh(d*x+c)**3/(a+b*sinh(d*x+c)**2)**3,x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{\sinh^3(c+dx)}{\left(a+b\sinh^2(c+dx)\right)^3}\,dx = \int \frac{\sinh\left(dx+c\right)^3}{\left(b\sinh\left(dx+c\right)^2+a\right)^3}\,dx$$

input
```
integrate(sinh(d*x+c)^3/(a+b*sinh(d*x+c)^2)^3,x, algorithm="maxima")
```

output
```
1/4*((a*b*e^(7*c) - 4*b^2*e^(7*c))*e^(7*d*x) - (4*a^2*e^(5*c) + 9*a*b*e^(5
*c) - 4*b^2*e^(5*c))*e^(5*d*x) - (4*a^2*e^(3*c) + 9*a*b*e^(3*c) - 4*b^2*e^
(3*c))*e^(3*d*x) + (a*b*e^c - 4*b^2*e^c)*e^(d*x))/(a^2*b^3*d - 2*a*b^4*d +
 b^5*d + (a^2*b^3*d*e^(8*c) - 2*a*b^4*d*e^(8*c) + b^5*d*e^(8*c))*e^(8*d*x)
 + 4*(2*a^3*b^2*d*e^(6*c) - 5*a^2*b^3*d*e^(6*c) + 4*a*b^4*d*e^(6*c) - b^5*
d*e^(6*c))*e^(6*d*x) + 2*(8*a^4*b*d*e^(4*c) - 24*a^3*b^2*d*e^(4*c) + 27*a^
2*b^3*d*e^(4*c) - 14*a*b^4*d*e^(4*c) + 3*b^5*d*e^(4*c))*e^(4*d*x) + 4*(2*a
^3*b^2*d*e^(2*c) - 5*a^2*b^3*d*e^(2*c) + 4*a*b^4*d*e^(2*c) - b^5*d*e^(2*c)
)*e^(2*d*x)) + 1/8*integrate(2*((a*e^(3*c) - 4*b*e^(3*c))*e^(3*d*x) - (a*e
^c - 4*b*e^c)*e^(d*x))/(a^2*b^2 - 2*a*b^3 + b^4 + (a^2*b^2*e^(4*c) - 2*a*b
^3*e^(4*c) + b^4*e^(4*c))*e^(4*d*x) + 2*(2*a^3*b*e^(2*c) - 5*a^2*b^2*e^(2*
c) + 4*a*b^3*e^(2*c) - b^4*e^(2*c))*e^(2*d*x)), x)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{\sinh^3(c+dx)}{\left(a+b\sinh^2(c+dx)\right)^3}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(sinh(d*x+c)^3/(a+b*sinh(d*x+c)^2)^3,x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Limit: Max order reached or unable
to make series expansion Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sinh^3(c+dx)}{\left(a+b\sinh^2(c+dx)\right)^3}\,dx = \int \frac{\sinh(c+d\,x)^3}{\left(b\sinh\left(c+d\,x\right)^2+a\right)^3}\,dx$$

input
```
int(sinh(c + d*x)^3/(a + b*sinh(c + d*x)^2)^3,x)
```

output
```
int(sinh(c + d*x)^3/(a + b*sinh(c + d*x)^2)^3, x)
```

Reduce [B] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 7235, normalized size of antiderivative = 53.59

$$\int \frac{\sinh^3(c+dx)}{\left(a+b\sinh^2(c+dx)\right)^3}\,dx = \text{Too large to display}$$

input
```
int(sinh(d*x+c)^3/(a+b*sinh(d*x+c)^2)^3,x)
```

output

```
(2*e**(8*c + 8*d*x)*sqrt(b)*sqrt(a)*sqrt(a - b)*sqrt(2*sqrt(a)*sqrt(a - b)
 + 2*a - b)*atan((e**(c + d*x)*b)/(sqrt(b)*sqrt(2*sqrt(a)*sqrt(a - b) + 2*
a - b)))*a*b**2 - 8*e**(8*c + 8*d*x)*sqrt(b)*sqrt(a)*sqrt(a - b)*sqrt(2*sq
rt(a)*sqrt(a - b) + 2*a - b)*atan((e**(c + d*x)*b)/(sqrt(b)*sqrt(2*sqrt(a)
*sqrt(a - b) + 2*a - b)))*b**3 + 16*e**(6*c + 6*d*x)*sqrt(b)*sqrt(a)*sqrt(
a - b)*sqrt(2*sqrt(a)*sqrt(a - b) + 2*a - b)*atan((e**(c + d*x)*b)/(sqrt(b
)*sqrt(2*sqrt(a)*sqrt(a - b) + 2*a - b)))*a**2*b - 72*e**(6*c + 6*d*x)*sqr
t(b)*sqrt(a)*sqrt(a - b)*sqrt(2*sqrt(a)*sqrt(a - b) + 2*a - b)*atan((e**(c
 + d*x)*b)/(sqrt(b)*sqrt(2*sqrt(a)*sqrt(a - b) + 2*a - b)))*a*b**2 + 32*e*
*(6*c + 6*d*x)*sqrt(b)*sqrt(a)*sqrt(a - b)*sqrt(2*sqrt(a)*sqrt(a - b) + 2*
a - b)*atan((e**(c + d*x)*b)/(sqrt(b)*sqrt(2*sqrt(a)*sqrt(a - b) + 2*a - b
)))*b**3 + 32*e**(4*c + 4*d*x)*sqrt(b)*sqrt(a)*sqrt(a - b)*sqrt(2*sqrt(a)*
sqrt(a - b) + 2*a - b)*atan((e**(c + d*x)*b)/(sqrt(b)*sqrt(2*sqrt(a)*sqrt(
a - b) + 2*a - b)))*a**3 - 160*e**(4*c + 4*d*x)*sqrt(b)*sqrt(a)*sqrt(a - b
)*sqrt(2*sqrt(a)*sqrt(a - b) + 2*a - b)*atan((e**(c + d*x)*b)/(sqrt(b)*sqr
t(2*sqrt(a)*sqrt(a - b) + 2*a - b)))*a**2*b + 140*e**(4*c + 4*d*x)*sqrt(b)
*sqrt(a)*sqrt(a - b)*sqrt(2*sqrt(a)*sqrt(a - b) + 2*a - b)*atan((e**(c + d
*x)*b)/(sqrt(b)*sqrt(2*sqrt(a)*sqrt(a - b) + 2*a - b)))*a*b**2 - 48*e**(4*
c + 4*d*x)*sqrt(b)*sqrt(a)*sqrt(a - b)*sqrt(2*sqrt(a)*sqrt(a - b) + 2*a -
b)*atan((e**(c + d*x)*b)/(sqrt(b)*sqrt(2*sqrt(a)*sqrt(a - b) + 2*a - b)...
```

3.53 $\int \frac{\sinh^2(c+dx)}{\left(a+b\sinh^2(c+dx)\right)^3}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 139

$$\int \frac{\sinh^2(c+dx)}{\left(a+b\sinh^2(c+dx)\right)^3}\,dx = -\frac{(4a-b)\operatorname{arctanh}\left(\frac{\sqrt{a-b}\tanh(c+dx)}{\sqrt{a}}\right)}{8a^{3/2}(a-b)^{5/2}d} + \frac{\cosh(c+dx)\sinh(c+dx)}{4(a-b)d\left(a+b\sinh^2(c+dx)\right)^2} + \frac{(2a+b)\cosh(c+dx)\sinh(c+dx)}{8a(a-b)^2d\left(a+b\sinh^2(c+dx)\right)}$$

output
```
-1/8*(4*a-b)*arctanh((a-b)^(1/2)*tanh(d*x+c)/a^(1/2))/a^(3/2)/(a-b)^(5/2)/
d+1/4*cosh(d*x+c)*sinh(d*x+c)/(a-b)/d/(a+b*sinh(d*x+c)^2)^2+1/8*(2*a+b)*co
sh(d*x+c)*sinh(d*x+c)/a/(a-b)^2/d/(a+b*sinh(d*x+c)^2)
```

Mathematica [A] (verified)

Time = 11.62 (sec) , antiderivative size = 121, normalized size of antiderivative = 0.87

$$\int \frac{\sinh^2(c+dx)}{\left(a+b\sinh^2(c+dx)\right)^3}\,dx = \frac{-\frac{(4a-b)\text{arctanh}\left(\frac{\sqrt{a-b}\tanh(c+dx)}{\sqrt{a}}\right)}{a^{3/2}(a-b)^{5/2}}+\frac{(8a^2-4ab-b^2+b(2a+b)\cosh(2(c+dx)))\sinh(2(c+dx))}{a(a-b)^2(2a-b+b\cosh(2(c+dx)))^2}}{8d}$$

input
```
Integrate[Sinh[c + d*x]^2/(a + b*Sinh[c + d*x]^2)^3,x]
```

output
```
(-(((4*a - b)*ArcTanh[(Sqrt[a - b]*Tanh[c + d*x])/Sqrt[a]])/(a^(3/2)*(a -
b)^(5/2))) + ((8*a^2 - 4*a*b - b^2 + b*(2*a + b)*Cosh[2*(c + d*x)])*Sinh[2
*(c + d*x)])/(a*(a - b)^2*(2*a - b + b*Cosh[2*(c + d*x)])^2))/(8*d)
```

Rubi [A] (verified)

Time = 0.64 (sec) , antiderivative size = 151, normalized size of antiderivative = 1.09, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.391$, Rules used = {3042, 25, 3652, 3042, 3652, 27, 3042, 3660, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sinh^2(c+dx)}{\left(a+b\sinh^2(c+dx)\right)^3}\,dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int -\frac{\sin(ic+idx)^2}{\left(a-b\sin(ic+idx)^2\right)^3}dx$$

$$\Big\downarrow \text{ 25}$$

$$-\int \frac{\sin(ic+idx)^2}{\left(a-b\sin(ic+idx)^2\right)^3}dx$$

$$\Big\downarrow \text{ 3652}$$

$$\frac{\sinh(c+dx)\cosh(c+dx)}{4d(a-b)\left(a+b\sinh^2(c+dx)\right)^2}-\frac{\int\frac{a-2a\sinh^2(c+dx)}{\left(b\sinh^2(c+dx)+a\right)^2}dx}{4a(a-b)}$$

$$\Big\downarrow \ 3042$$

$$\frac{\sinh(c+dx)\cosh(c+dx)}{4d(a-b)\left(a+b\sinh^2(c+dx)\right)^2}-\frac{\int\frac{2a\sin(ic+idx)^2+a}{(a-b\sin(ic+idx)^2)^2}dx}{4a(a-b)}$$

$$\Big\downarrow \ 3652$$

$$\frac{\sinh(c+dx)\cosh(c+dx)}{4d(a-b)\left(a+b\sinh^2(c+dx)\right)^2}-\frac{\frac{\int\frac{a(4a-b)}{b\sinh^2(c+dx)+a}dx}{2a(a-b)}-\frac{(2a+b)\sinh(c+dx)\cosh(c+dx)}{2d(a-b)\left(a+b\sinh^2(c+dx)\right)}}{4a(a-b)}$$

$$\Big\downarrow \ 27$$

$$\frac{\sinh(c+dx)\cosh(c+dx)}{4d(a-b)\left(a+b\sinh^2(c+dx)\right)^2}-\frac{\frac{(4a-b)\int\frac{1}{b\sinh^2(c+dx)+a}dx}{2(a-b)}-\frac{(2a+b)\sinh(c+dx)\cosh(c+dx)}{2d(a-b)\left(a+b\sinh^2(c+dx)\right)}}{4a(a-b)}$$

$$\Big\downarrow \ 3042$$

$$\frac{\sinh(c+dx)\cosh(c+dx)}{4d(a-b)\left(a+b\sinh^2(c+dx)\right)^2}-\frac{-\frac{(2a+b)\sinh(c+dx)\cosh(c+dx)}{2d(a-b)\left(a+b\sinh^2(c+dx)\right)}+\frac{(4a-b)\int\frac{1}{a-b\sin(ic+idx)^2}dx}{2(a-b)}}{4a(a-b)}$$

$$\Big\downarrow \ 3660$$

$$\frac{\sinh(c+dx)\cosh(c+dx)}{4d(a-b)\left(a+b\sinh^2(c+dx)\right)^2}-\frac{\frac{(4a-b)\int\frac{1}{a-(a-b)\tanh^2(c+dx)}d\tanh(c+dx)}{2d(a-b)}-\frac{(2a+b)\sinh(c+dx)\cosh(c+dx)}{2d(a-b)\left(a+b\sinh^2(c+dx)\right)}}{4a(a-b)}$$

$$\Big\downarrow \ 221$$

$$\frac{\sinh(c+dx)\cosh(c+dx)}{4d(a-b)\left(a+b\sinh^2(c+dx)\right)^2}-\frac{\frac{(4a-b)\text{arctanh}\left(\frac{\sqrt{a-b}\tanh(c+dx)}{\sqrt{a}}\right)}{2\sqrt{a}d(a-b)^{3/2}}-\frac{(2a+b)\sinh(c+dx)\cosh(c+dx)}{2d(a-b)\left(a+b\sinh^2(c+dx)\right)}}{4a(a-b)}$$

input
```
Int[Sinh[c + d*x]^2/(a + b*Sinh[c + d*x]^2)^3,x]
```

output
```
(Cosh[c + d*x]*Sinh[c + d*x])/(4*(a - b)*d*(a + b*Sinh[c + d*x]^2)^2) - ((
(4*a - b)*ArcTanh[(Sqrt[a - b]*Tanh[c + d*x])/Sqrt[a]])/(2*Sqrt[a]*(a - b)
^(3/2)*d) - ((2*a + b)*Cosh[c + d*x]*Sinh[c + d*x])/(2*(a - b)*d*(a + b*Si
nh[c + d*x]^2)))/(4*a*(a - b))
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3652
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)]^2), x_Symbol] :> Simp[(-(A*b - a*B))*Cos[e + f*x]*Sin[e + f*x
]*((a + b*Sin[e + f*x]^2)^(p + 1)/(2*a*f*(a + b)*(p + 1))), x] - Simp[1/(2*
a*(a + b)*(p + 1))   Int[(a + b*Sin[e + f*x]^2)^(p + 1)*Simp[a*B - A*(2*a*(
p + 1) + b*(2*p + 3)) + 2*(A*b - a*B)*(p + 2)*Sin[e + f*x]^2, x], x], x] /;
 FreeQ[{a, b, e, f, A, B}, x] && LtQ[p, -1] && NeQ[a + b, 0]
```

rule 3660
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(-1), x_Symbol] :> With[{ff =
FreeFactors[Tan[e + f*x], x]}, Simp[ff/f   Subst[Int[1/(a + (a + b)*ff^2*x^
2), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f}, x]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 400 vs. $2(125) = 250$.

Time = 1.47 (sec) , antiderivative size = 401, normalized size of antiderivative = 2.88

method	result
derivativedivides	$\frac{-\frac{8\left(-\frac{(4a-b)\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^{7}}{32\left(a^{2}-2ab+b^{2}\right)}+\frac{\left(4a^{2}-9ab-4b^{2}\right)\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^{5}}{32a\left(a^{2}-2ab+b^{2}\right)}+\frac{\left(4a^{2}-9ab-4b^{2}\right)\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^{3}}{32a\left(a^{2}-2ab+b^{2}\right)}-\frac{(4a-b)\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)}{32\left(a^{2}-2ab+b^{2}\right)}\right)}{\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^{4}a-2\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}a+4b\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}+a\right)^{2}}-\frac{(4}{}}{d}$
default	$\frac{-\frac{8\left(-\frac{(4a-b)\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^{7}}{32\left(a^{2}-2ab+b^{2}\right)}+\frac{\left(4a^{2}-9ab-4b^{2}\right)\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^{5}}{32a\left(a^{2}-2ab+b^{2}\right)}+\frac{\left(4a^{2}-9ab-4b^{2}\right)\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^{3}}{32a\left(a^{2}-2ab+b^{2}\right)}-\frac{(4a-b)\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)}{32\left(a^{2}-2ab+b^{2}\right)}\right)}{\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^{4}a-2\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}a+4b\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}+a\right)^{2}}-\frac{(4}{}}{d}$
risch	$-\frac{4a\,b^{2}e^{6dx+6c}-b^{3}e^{6dx+6c}+16a^{3}e^{4dx+4c}-8a^{2}b\,e^{4dx+4c}-2a\,b^{2}e^{4dx+4c}+3b^{3}e^{4dx+4c}+16\,e^{2dx+2c}a^{2}b-4\,e^{2dx+2c}a\,b^{2}-}{4bda(a-b)^{2}\left(b\,e^{4dx+4c}+4\,e^{2dx+2c}a-2\,e^{2dx+2c}b+b\right)^{2}}$

input
```
int(sinh(d*x+c)^2/(a+b*sinh(d*x+c)^2)^3,x,method=_RETURNVERBOSE)
```

output
```
1/d*(-8*(-1/32*(4*a-b)/(a^2-2*a*b+b^2)*tanh(1/2*d*x+1/2*c)^7+1/32*(4*a^2-9
*a*b-4*b^2)/a/(a^2-2*a*b+b^2)*tanh(1/2*d*x+1/2*c)^5+1/32*(4*a^2-9*a*b-4*b^
2)/a/(a^2-2*a*b+b^2)*tanh(1/2*d*x+1/2*c)^3-1/32*(4*a-b)/(a^2-2*a*b+b^2)*ta
nh(1/2*d*x+1/2*c))/(tanh(1/2*d*x+1/2*c)^4*a-2*tanh(1/2*d*x+1/2*c)^2*a+4*b*
tanh(1/2*d*x+1/2*c)^2+a)^2-1/4*(4*a-b)/(a^2-2*a*b+b^2)*(1/2*(-(-b*(a-b))^(
1/2)-b)/a/(-b*(a-b))^(1/2)/((2*(-b*(a-b))^(1/2)-a+2*b)*a)^(1/2)*arctan(tan
h(1/2*d*x+1/2*c)*a/((2*(-b*(a-b))^(1/2)-a+2*b)*a)^(1/2))-1/2*(-(-b*(a-b))^
(1/2)+b)/a/(-b*(a-b))^(1/2)/((2*(-b*(a-b))^(1/2)+a-2*b)*a)^(1/2)*arctanh(t
anh(1/2*d*x+1/2*c)*a/((2*(-b*(a-b))^(1/2)+a-2*b)*a)^(1/2))))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2632 vs. $2(125) = 250$.

Time = 0.17 (sec) , antiderivative size = 5519, normalized size of antiderivative = 39.71

$$\int \frac{\sinh^2(c+dx)}{(a+b\sinh^2(c+dx))^3}\,dx = \text{Too large to display}$$

input
```
integrate(sinh(d*x+c)^2/(a+b*sinh(d*x+c)^2)^3,x, algorithm="fricas")
```

output
```
Too large to include
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\sinh^2(c+dx)}{(a+b\sinh^2(c+dx))^3}\,dx = \text{Timed out}$$

input
```
integrate(sinh(d*x+c)**2/(a+b*sinh(d*x+c)**2)**3,x)
```

output
```
Timed out
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{\sinh^2(c+dx)}{(a+b\sinh^2(c+dx))^3}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate(sinh(d*x+c)^2/(a+b*sinh(d*x+c)^2)^3,x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(b-a>0)', see `assume?` for more
details)Is
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 277 vs. $2(125) = 250$.

Time = 0.67 (sec) , antiderivative size = 277, normalized size of antiderivative = 1.99

$$\int \frac{\sinh^2(c+dx)}{\left(a+b\sinh^2(c+dx)\right)^3}\,dx = -\frac{\frac{(4a-b)\arctan\left(\frac{be^{(2\,dx+2\,c)}+2\,a-b}{2\sqrt{-a^2+ab}}\right)}{(a^3-2\,a^2b+ab^2)\sqrt{-a^2+ab}} + \frac{2\left(4\,ab^2e^{(6\,dx+6\,c)}-b^3e^{(6\,dx+6\,c)}+16\,a^3e^{(4\,dx+4\,c)}-8\,a^2be^{(4\,dx+4\,c)}-2\,ab^2e^{(4\,dx+4\,c)}+3\,b^3e^{(4\,dx+4\,c}\right.}{(a^3b-2\,a^2b^2+ab^3)\left(be^{(4\,dx+4\,c)}+4\,ae^{(2\,dx+2\,c)}-2\,b\right.}}{8\,d}$$

input
```
integrate(sinh(d*x+c)^2/(a+b*sinh(d*x+c)^2)^3,x, algorithm="giac")
```

output
```
-1/8*((4*a - b)*arctan(1/2*(b*e^(2*d*x + 2*c) + 2*a - b)/sqrt(-a^2 + a*b))
/((a^3 - 2*a^2*b + a*b^2)*sqrt(-a^2 + a*b)) + 2*(4*a*b^2*e^(6*d*x + 6*c) -
 b^3*e^(6*d*x + 6*c) + 16*a^3*e^(4*d*x + 4*c) - 8*a^2*b*e^(4*d*x + 4*c) -
2*a*b^2*e^(4*d*x + 4*c) + 3*b^3*e^(4*d*x + 4*c) + 16*a^2*b*e^(2*d*x + 2*c)
 - 4*a*b^2*e^(2*d*x + 2*c) - 3*b^3*e^(2*d*x + 2*c) + 2*a*b^2 + b^3)/((a^3*
b - 2*a^2*b^2 + a*b^3)*(b*e^(4*d*x + 4*c) + 4*a*e^(2*d*x + 2*c) - 2*b*e^(2
*d*x + 2*c) + b)^2))/d
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sinh^2(c+dx)}{\left(a+b\sinh^2(c+dx)\right)^3}\,dx = \int \frac{{\sinh\left(c+d\,x\right)}^2}{{\left(b\,{\sinh\left(c+d\,x\right)}^2+a\right)}^3}\,dx$$

input
```
int(sinh(c + d*x)^2/(a + b*sinh(c + d*x)^2)^3,x)
```

output
```
int(sinh(c + d*x)^2/(a + b*sinh(c + d*x)^2)^3, x)
```

Reduce [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 3699, normalized size of antiderivative = 26.61

$$\int \frac{\sinh^2(c+dx)}{\left(a+b\sinh^2(c+dx)\right)^3}\,dx = \text{Too large to display}$$

input
```
int(sinh(d*x+c)^2/(a+b*sinh(d*x+c)^2)^3,x)
```

output
```
( - 8*e**(8*c + 8*d*x)*sqrt(a)*sqrt(a - b)*log( - sqrt(2*sqrt(a)*sqrt(a -
b) - 2*a + b) + e**(c + d*x)*sqrt(b))*a**2*b**3 + 6*e**(8*c + 8*d*x)*sqrt(
a)*sqrt(a - b)*log( - sqrt(2*sqrt(a)*sqrt(a - b) - 2*a + b) + e**(c + d*x)
*sqrt(b))*a*b**4 - e**(8*c + 8*d*x)*sqrt(a)*sqrt(a - b)*log( - sqrt(2*sqrt
(a)*sqrt(a - b) - 2*a + b) + e**(c + d*x)*sqrt(b))*b**5 - 8*e**(8*c + 8*d*
x)*sqrt(a)*sqrt(a - b)*log(sqrt(2*sqrt(a)*sqrt(a - b) - 2*a + b) + e**(c +
 d*x)*sqrt(b))*a**2*b**3 + 6*e**(8*c + 8*d*x)*sqrt(a)*sqrt(a - b)*log(sqrt
(2*sqrt(a)*sqrt(a - b) - 2*a + b) + e**(c + d*x)*sqrt(b))*a*b**4 - e**(8*c
 + 8*d*x)*sqrt(a)*sqrt(a - b)*log(sqrt(2*sqrt(a)*sqrt(a - b) - 2*a + b) +
e**(c + d*x)*sqrt(b))*b**5 + 8*e**(8*c + 8*d*x)*sqrt(a)*sqrt(a - b)*log(2*
sqrt(a)*sqrt(a - b) + e**(2*c + 2*d*x)*b + 2*a - b)*a**2*b**3 - 6*e**(8*c
+ 8*d*x)*sqrt(a)*sqrt(a - b)*log(2*sqrt(a)*sqrt(a - b) + e**(2*c + 2*d*x)*
b + 2*a - b)*a*b**4 + e**(8*c + 8*d*x)*sqrt(a)*sqrt(a - b)*log(2*sqrt(a)*s
qrt(a - b) + e**(2*c + 2*d*x)*b + 2*a - b)*b**5 - 64*e**(6*c + 6*d*x)*sqrt
(a)*sqrt(a - b)*log( - sqrt(2*sqrt(a)*sqrt(a - b) - 2*a + b) + e**(c + d*x
)*sqrt(b))*a**3*b**2 + 80*e**(6*c + 6*d*x)*sqrt(a)*sqrt(a - b)*log( - sqrt
(2*sqrt(a)*sqrt(a - b) - 2*a + b) + e**(c + d*x)*sqrt(b))*a**2*b**3 - 32*e
**(6*c + 6*d*x)*sqrt(a)*sqrt(a - b)*log( - sqrt(2*sqrt(a)*sqrt(a - b) - 2*
a + b) + e**(c + d*x)*sqrt(b))*a*b**4 + 4*e**(6*c + 6*d*x)*sqrt(a)*sqrt(a
- b)*log( - sqrt(2*sqrt(a)*sqrt(a - b) - 2*a + b) + e**(c + d*x)*sqrt(b...
```

3.54 $\int \frac{\sinh(c+dx)}{\left(a+b\sinh^2(c+dx)\right)^3}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 118

$$\int \frac{\sinh(c+dx)}{\left(a+b\sinh^2(c+dx)\right)^3}\,dx = \frac{3\arctan\left(\frac{\sqrt{b}\cosh(c+dx)}{\sqrt{a-b}}\right)}{8(a-b)^{5/2}\sqrt{b}d} + \frac{\cosh(c+dx)}{4(a-b)d\left(a-b+b\cosh^2(c+dx)\right)^2} + \frac{3\cosh(c+dx)}{8(a-b)^2d\left(a-b+b\cosh^2(c+dx)\right)}$$

output
```
3/8*arctan(b^(1/2)*cosh(d*x+c)/(a-b)^(1/2))/(a-b)^(5/2)/b^(1/2)/d+1/4*cosh
(d*x+c)/(a-b)/d/(a-b+b*cosh(d*x+c)^2)^2+3/8*cosh(d*x+c)/(a-b)^2/d/(a-b+b*c
osh(d*x+c)^2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.77 (sec) , antiderivative size = 149, normalized size of antiderivative = 1.26

$$\int \frac{\sinh(c+dx)}{\left(a+b\sinh^2(c+dx)\right)^3}\,dx = \frac{\frac{3\left(\arctan\left(\frac{\sqrt{b}-i\sqrt{a}\tanh\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a-b}}\right)+\arctan\left(\frac{\sqrt{b}+i\sqrt{a}\tanh\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a-b}}\right)\right)}{(a-b)^{5/2}\sqrt{b}}+\frac{2\cosh(c+dx)(10a-7b+3b\cosh(2(c+dx)))}{(a-b)^2(2a-b+b\cosh(2(c+dx)))^2}}{8d}$$

input
```
Integrate[Sinh[c + d*x]/(a + b*Sinh[c + d*x]^2)^3,x]
```

output
```
((3*(ArcTan[(Sqrt[b] - I*Sqrt[a]*Tanh[(c + d*x)/2])/Sqrt[a - b]] + ArcTan[
(Sqrt[b] + I*Sqrt[a]*Tanh[(c + d*x)/2])/Sqrt[a - b]]))/((a - b)^(5/2)*Sqrt
[b]) + (2*Cosh[c + d*x]*(10*a - 7*b + 3*b*Cosh[2*(c + d*x)]))/((a - b)^2*(
2*a - b + b*Cosh[2*(c + d*x)])^2))/(8*d)
```

Rubi [A] (verified)

Time = 0.31 (sec) , antiderivative size = 125, normalized size of antiderivative = 1.06, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {3042, 26, 3665, 215, 215, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sinh(c+dx)}{\left(a+b\sinh^2(c+dx)\right)^3}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int -\frac{i\sin(ic+idx)}{\left(a-b\sin(ic+idx)^2\right)^3}dx$$

$$\Big\downarrow \text{26}$$

$$-i\int \frac{\sin(ic+idx)}{\left(a-b\sin(ic+idx)^2\right)^3}dx$$

$$\Big\downarrow \; 3665$$

$$\frac{\int \frac{1}{\left(b\cosh^2(c+dx)+a-b\right)^3}d\cosh(c+dx)}{d}$$

$$\Big\downarrow \; 215$$

$$\frac{\frac{3\int \frac{1}{\left(b\cosh^2(c+dx)+a-b\right)^2}d\cosh(c+dx)}{4(a-b)}+\frac{\cosh(c+dx)}{4(a-b)\left(a+b\cosh^2(c+dx)-b\right)^2}}{d}$$

$$\Big\downarrow \; 215$$

$$\frac{\frac{3\left(\frac{\int \frac{1}{b\cosh^2(c+dx)+a-b}d\cosh(c+dx)}{2(a-b)}+\frac{\cosh(c+dx)}{2(a-b)\left(a+b\cosh^2(c+dx)-b\right)}\right)}{4(a-b)}+\frac{\cosh(c+dx)}{4(a-b)\left(a+b\cosh^2(c+dx)-b\right)^2}}{d}$$

$$\Big\downarrow \; 218$$

$$\frac{\frac{3\left(\frac{\arctan\left(\frac{\sqrt{b}\cosh(c+dx)}{\sqrt{a-b}}\right)}{2\sqrt{b}(a-b)^{3/2}}+\frac{\cosh(c+dx)}{2(a-b)\left(a+b\cosh^2(c+dx)-b\right)}\right)}{4(a-b)}+\frac{\cosh(c+dx)}{4(a-b)\left(a+b\cosh^2(c+dx)-b\right)^2}}{d}$$

input
```
Int[Sinh[c + d*x]/(a + b*Sinh[c + d*x]^2)^3,x]
```

output
```
(Cosh[c + d*x]/(4*(a - b)*(a - b + b*Cosh[c + d*x]^2)^2) + (3*(ArcTan[(Sqr
t[b]*Cosh[c + d*x])/Sqrt[a - b]]/(2*(a - b)^(3/2)*Sqrt[b]) + Cosh[c + d*x]
/(2*(a - b)*(a - b + b*Cosh[c + d*x]^2))))/(4*(a - b)))/d
```

Defintions of rubi rules used

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 215
```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(-x)*((a + b*x^2)^(p + 1)
/(2*a*(p + 1))), x] + Simp[(2*p + 3)/(2*a*(p + 1))   Int[(a + b*x^2)^(p + 1
), x], x] /; FreeQ[{a, b}, x] && LtQ[p, -1] && (IntegerQ[4*p] || IntegerQ[6
*p])
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3665
```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^
(p_.), x_Symbol] :> With[{ff = FreeFactors[Cos[e + f*x], x]}, Simp[-ff/f
Subst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a + b - b*ff^2*x^2)^p, x], x, Cos[e +
 f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m - 1)/2]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 236 vs. $2(104) = 208$.

Time = 1.47 (sec) , antiderivative size = 237, normalized size of antiderivative = 2.01

method	result
risch	$\frac{e^{dx+c}\left(3\,e^{6dx+6c}b+20\,e^{4dx+4c}a-11b\,e^{4dx+4c}+20\,e^{2dx+2c}a-11\,e^{2dx+2c}b+3b\right)}{4(a-b)^2d\left(b\,e^{4dx+4c}+4\,e^{2dx+2c}a-2\,e^{2dx+2c}b+b\right)^2}-\frac{3\ln\left(e^{2dx+2c}-\frac{2(a-b)e^{dx+c}}{\sqrt{-ab+b^2}}+1\right)}{16\sqrt{-ab+b^2}\,(a-b)^2d}+\cdots$
derivativedivides	$\frac{\frac{-\frac{\left(5a^2-16ab+8b^2\right)\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^6}{4a\left(a^2-2ab+b^2\right)}+\frac{\left(15a^3-46a^2b+56b^2a-16b^3\right)\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^4}{4a^2\left(a^2-2ab+b^2\right)}-\frac{\left(15a^2-32ab+8b^2\right)\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^2}{4a\left(a^2-2ab+b^2\right)}+\frac{2(5a-\cdots}{8a^2-16\cdots}}{\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^4a-2\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^2a+4b\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^2+a\right)^2}}{d}$
default	$\frac{\frac{-\frac{\left(5a^2-16ab+8b^2\right)\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^6}{4a\left(a^2-2ab+b^2\right)}+\frac{\left(15a^3-46a^2b+56b^2a-16b^3\right)\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^4}{4a^2\left(a^2-2ab+b^2\right)}-\frac{\left(15a^2-32ab+8b^2\right)\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^2}{4a\left(a^2-2ab+b^2\right)}+\frac{2(5a-\cdots}{8a^2-16\cdots}}{\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^4a-2\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^2a+4b\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^2+a\right)^2}}{d}$

input
```
int(sinh(d*x+c)/(a+b*sinh(d*x+c)^2)^3,x,method=_RETURNVERBOSE)
```

output
```
1/4*exp(d*x+c)*(3*exp(6*d*x+6*c)*b+20*exp(4*d*x+4*c)*a-11*b*exp(4*d*x+4*c)
+20*exp(2*d*x+2*c)*a-11*exp(2*d*x+2*c)*b+3*b)/(a-b)^2/d/(b*exp(4*d*x+4*c)+
4*exp(2*d*x+2*c)*a-2*exp(2*d*x+2*c)*b+b)^2-3/16/(-a*b+b^2)^(1/2)/(a-b)^2/d
*ln(exp(2*d*x+2*c)-2*(a-b)/(-a*b+b^2)^(1/2)*exp(d*x+c)+1)+3/16/(-a*b+b^2)^
(1/2)/(a-b)^2/d*ln(exp(2*d*x+2*c)+2*(a-b)/(-a*b+b^2)^(1/2)*exp(d*x+c)+1)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2725 vs. $2(104) = 208$.

Time = 0.17 (sec) , antiderivative size = 5151, normalized size of antiderivative = 43.65

$$\int \frac{\sinh(c+dx)}{\left(a+b\sinh^2(c+dx)\right)^3}\,dx = \text{Too large to display}$$

input
```
integrate(sinh(d*x+c)/(a+b*sinh(d*x+c)^2)^3,x, algorithm="fricas")
```

output
```
Too large to include
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\sinh(c+dx)}{\left(a+b\sinh^2(c+dx)\right)^3}\,dx = \text{Timed out}$$

input
```
integrate(sinh(d*x+c)/(a+b*sinh(d*x+c)**2)**3,x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{\sinh(c+dx)}{\left(a+b\sinh^2(c+dx)\right)^3}\,dx = \int \frac{\sinh\left(dx+c\right)}{\left(b\sinh\left(dx+c\right)^2+a\right)^3}\,dx$$

input
```
integrate(sinh(d*x+c)/(a+b*sinh(d*x+c)^2)^3,x, algorithm="maxima")
```

output
```
1/4*((20*a*e^(5*c) - 11*b*e^(5*c))*e^(5*d*x) + (20*a*e^(3*c) - 11*b*e^(3*c
))*e^(3*d*x) + 3*b*e^(7*d*x + 7*c) + 3*b*e^(d*x + c))/(a^2*b^2*d - 2*a*b^3
*d + b^4*d + (a^2*b^2*d*e^(8*c) - 2*a*b^3*d*e^(8*c) + b^4*d*e^(8*c))*e^(8*
d*x) + 4*(2*a^3*b*d*e^(6*c) - 5*a^2*b^2*d*e^(6*c) + 4*a*b^3*d*e^(6*c) - b^
4*d*e^(6*c))*e^(6*d*x) + 2*(8*a^4*d*e^(4*c) - 24*a^3*b*d*e^(4*c) + 27*a^2*
b^2*d*e^(4*c) - 14*a*b^3*d*e^(4*c) + 3*b^4*d*e^(4*c))*e^(4*d*x) + 4*(2*a^3
*b*d*e^(2*c) - 5*a^2*b^2*d*e^(2*c) + 4*a*b^3*d*e^(2*c) - b^4*d*e^(2*c))*e^
(2*d*x)) + 1/2*integrate(3/2*(e^(3*d*x + 3*c) - e^(d*x + c))/(a^2*b - 2*a*
b^2 + b^3 + (a^2*b*e^(4*c) - 2*a*b^2*e^(4*c) + b^3*e^(4*c))*e^(4*d*x) + 2*
(2*a^3*e^(2*c) - 5*a^2*b*e^(2*c) + 4*a*b^2*e^(2*c) - b^3*e^(2*c))*e^(2*d*x
)), x)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{\sinh(c+dx)}{\left(a+b\sinh^2(c+dx)\right)^3}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(sinh(d*x+c)/(a+b*sinh(d*x+c)^2)^3,x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Limit: Max order reached or unable
to make series expansion Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sinh(c+dx)}{\left(a+b\sinh^2(c+dx)\right)^3}\,dx = \int \frac{\sinh(c+d\,x)}{{\left(b\,{\sinh\left(c+d\,x\right)}^2+a\right)}^3}\,dx$$

input
```
int(sinh(c + d*x)/(a + b*sinh(c + d*x)^2)^3,x)
```

output
```
int(sinh(c + d*x)/(a + b*sinh(c + d*x)^2)^3, x)
```

Reduce [B] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 5102, normalized size of antiderivative = 43.24

$$\int \frac{\sinh(c+dx)}{\left(a+b\sinh^2(c+dx)\right)^3}\,dx = \text{Too large to display}$$

input
```
int(sinh(d*x+c)/(a+b*sinh(d*x+c)^2)^3,x)
```

output
```
(6*e**(8*c + 8*d*x)*sqrt(b)*sqrt(a)*sqrt(a - b)*sqrt(2*sqrt(a)*sqrt(a - b)
 + 2*a - b)*atan((e**(c + d*x)*b)/(sqrt(b)*sqrt(2*sqrt(a)*sqrt(a - b) + 2*
a - b)))*b**2 + 48*e**(6*c + 6*d*x)*sqrt(b)*sqrt(a)*sqrt(a - b)*sqrt(2*sqr
t(a)*sqrt(a - b) + 2*a - b)*atan((e**(c + d*x)*b)/(sqrt(b)*sqrt(2*sqrt(a)*
sqrt(a - b) + 2*a - b)))*a*b - 24*e**(6*c + 6*d*x)*sqrt(b)*sqrt(a)*sqrt(a
- b)*sqrt(2*sqrt(a)*sqrt(a - b) + 2*a - b)*atan((e**(c + d*x)*b)/(sqrt(b)*
sqrt(2*sqrt(a)*sqrt(a - b) + 2*a - b)))*b**2 + 96*e**(4*c + 4*d*x)*sqrt(b)
*sqrt(a)*sqrt(a - b)*sqrt(2*sqrt(a)*sqrt(a - b) + 2*a - b)*atan((e**(c + d
*x)*b)/(sqrt(b)*sqrt(2*sqrt(a)*sqrt(a - b) + 2*a - b)))*a**2 - 96*e**(4*c
+ 4*d*x)*sqrt(b)*sqrt(a)*sqrt(a - b)*sqrt(2*sqrt(a)*sqrt(a - b) + 2*a - b)
*atan((e**(c + d*x)*b)/(sqrt(b)*sqrt(2*sqrt(a)*sqrt(a - b) + 2*a - b)))*a*
b + 36*e**(4*c + 4*d*x)*sqrt(b)*sqrt(a)*sqrt(a - b)*sqrt(2*sqrt(a)*sqrt(a
- b) + 2*a - b)*atan((e**(c + d*x)*b)/(sqrt(b)*sqrt(2*sqrt(a)*sqrt(a - b)
+ 2*a - b)))*b**2 + 48*e**(2*c + 2*d*x)*sqrt(b)*sqrt(a)*sqrt(a - b)*sqrt(2
*sqrt(a)*sqrt(a - b) + 2*a - b)*atan((e**(c + d*x)*b)/(sqrt(b)*sqrt(2*sqrt
(a)*sqrt(a - b) + 2*a - b)))*a*b - 24*e**(2*c + 2*d*x)*sqrt(b)*sqrt(a)*sqr
t(a - b)*sqrt(2*sqrt(a)*sqrt(a - b) + 2*a - b)*atan((e**(c + d*x)*b)/(sqrt
(b)*sqrt(2*sqrt(a)*sqrt(a - b) + 2*a - b)))*b**2 + 6*sqrt(b)*sqrt(a)*sqrt(
a - b)*sqrt(2*sqrt(a)*sqrt(a - b) + 2*a - b)*atan((e**(c + d*x)*b)/(sqrt(b
)*sqrt(2*sqrt(a)*sqrt(a - b) + 2*a - b)))*b**2 - 6*e**(8*c + 8*d*x)*sqr...
```

3.55 $\int \frac{1}{\left(a+b\sinh^2(c+dx)\right)^3}\,dx$

Optimal result

Integrand size = 14, antiderivative size = 154

$$\int \frac{1}{\left(a+b\sinh^2(c+dx)\right)^3}\,dx = \frac{(8a^2-8ab+3b^2)\operatorname{arctanh}\left(\frac{\sqrt{a-b}\tanh(c+dx)}{\sqrt{a}}\right)}{8a^{5/2}(a-b)^{5/2}d} - \frac{b\cosh(c+dx)\sinh(c+dx)}{4a(a-b)d\left(a+b\sinh^2(c+dx)\right)^2} - \frac{3(2a-b)b\cosh(c+dx)\sinh(c+dx)}{8a^2(a-b)^2d\left(a+b\sinh^2(c+dx)\right)}$$

output
```
1/8*(8*a^2-8*a*b+3*b^2)*arctanh((a-b)^(1/2)*tanh(d*x+c)/a^(1/2))/a^(5/2)/(
a-b)^(5/2)/d-1/4*b*cosh(d*x+c)*sinh(d*x+c)/a/(a-b)/d/(a+b*sinh(d*x+c)^2)^2
-3/8*(2*a-b)*b*cosh(d*x+c)*sinh(d*x+c)/a^2/(a-b)^2/d/(a+b*sinh(d*x+c)^2)
```

Mathematica [A] (verified)

Time = 11.70 (sec) , antiderivative size = 132, normalized size of antiderivative = 0.86

$$\int \frac{1}{\left(a+b\sinh^2(c+dx)\right)^3}\,dx = \frac{\frac{(8a^2-8ab+3b^2)\operatorname{arctanh}\left(\frac{\sqrt{a-b}\tanh(c+dx)}{\sqrt{a}}\right)}{(a-b)^{5/2}}+\frac{\sqrt{a}b(-16a^2+16ab-3b^2+3b(-2a+b)\cosh(2(c+dx)))\sinh(2(c+dx))}{(a-b)^2(2a-b+b\cosh(2(c+dx)))^2}}{8a^{5/2}d}$$

input
```
Integrate[(a + b*Sinh[c + d*x]^2)^(-3),x]
```

output
```
(((8*a^2 - 8*a*b + 3*b^2)*ArcTanh[(Sqrt[a - b]*Tanh[c + d*x])/Sqrt[a]])/(a
 - b)^(5/2) + (Sqrt[a]*b*(-16*a^2 + 16*a*b - 3*b^2 + 3*b*(-2*a + b)*Cosh[2
*(c + d*x)])*Sinh[2*(c + d*x)])/((a - b)^2*(2*a - b + b*Cosh[2*(c + d*x)])
^2))/(8*a^(5/2)*d)
```

Rubi [A] (verified)

Time = 0.60 (sec) , antiderivative size = 169, normalized size of antiderivative = 1.10, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.643$, Rules used = {3042, 3663, 25, 3042, 3652, 27, 3042, 3660, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\left(a+b\sinh^2(c+dx)\right)^3}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{(a-b\sin(ic+idx)^2)^3}dx$$

$$\Big\downarrow \text{3663}$$

$$-\frac{\int -\frac{-2b\sinh^2(c+dx)+4a-3b}{\left(b\sinh^2(c+dx)+a\right)^2}dx}{4a(a-b)}-\frac{b\sinh(c+dx)\cosh(c+dx)}{4ad(a-b)\left(a+b\sinh^2(c+dx)\right)^2}$$

$$\Big\downarrow \; 25$$

$$\frac{\int \frac{-2b\sinh^2(c+dx)+4a-3b}{\left(b\sinh^2(c+dx)+a\right)^2}dx}{4a(a-b)}-\frac{b\sinh(c+dx)\cosh(c+dx)}{4ad(a-b)\left(a+b\sinh^2(c+dx)\right)^2}$$

$$\Big\downarrow \; 3042$$

$$-\frac{b\sinh(c+dx)\cosh(c+dx)}{4ad(a-b)\left(a+b\sinh^2(c+dx)\right)^2}+\frac{\int \frac{2b\sin(ic+idx)^2+4a-3b}{\left(a-b\sin(ic+idx)^2\right)^2}dx}{4a(a-b)}$$

$$\Big\downarrow \; 3652$$

$$\frac{\frac{\int \frac{8a^2-8ba+3b^2}{b\sinh^2(c+dx)+a}dx}{2a(a-b)}-\frac{3b(2a-b)\sinh(c+dx)\cosh(c+dx)}{2ad(a-b)\left(a+b\sinh^2(c+dx)\right)}}{4a(a-b)}-\frac{b\sinh(c+dx)\cosh(c+dx)}{4ad(a-b)\left(a+b\sinh^2(c+dx)\right)^2}$$

$$\Big\downarrow \; 27$$

$$\frac{\frac{(8a^2-8ab+3b^2)\int \frac{1}{b\sinh^2(c+dx)+a}dx}{2a(a-b)}-\frac{3b(2a-b)\sinh(c+dx)\cosh(c+dx)}{2ad(a-b)\left(a+b\sinh^2(c+dx)\right)}}{4a(a-b)}-\frac{b\sinh(c+dx)\cosh(c+dx)}{4ad(a-b)\left(a+b\sinh^2(c+dx)\right)^2}$$

$$\Big\downarrow \; 3042$$

$$-\frac{b\sinh(c+dx)\cosh(c+dx)}{4ad(a-b)\left(a+b\sinh^2(c+dx)\right)^2}+\frac{-\frac{3b(2a-b)\sinh(c+dx)\cosh(c+dx)}{2ad(a-b)\left(a+b\sinh^2(c+dx)\right)}+\frac{(8a^2-8ab+3b^2)\int \frac{1}{a-b\sin(ic+idx)^2}dx}{2a(a-b)}}{4a(a-b)}$$

$$\Big\downarrow \; 3660$$

$$\frac{\frac{(8a^2-8ab+3b^2)\int \frac{1}{a-(a-b)\tanh^2(c+dx)}d\tanh(c+dx)}{2ad(a-b)}-\frac{3b(2a-b)\sinh(c+dx)\cosh(c+dx)}{2ad(a-b)\left(a+b\sinh^2(c+dx)\right)}}{4a(a-b)}-\frac{b\sinh(c+dx)\cosh(c+dx)}{4ad(a-b)\left(a+b\sinh^2(c+dx)\right)^2}$$

$$\Big\downarrow \; 221$$

$$\frac{\frac{(8a^2-8ab+3b^2)\text{arctanh}\left(\frac{\sqrt{a-b}\tanh(c+dx)}{\sqrt{a}}\right)}{2a^{3/2}d(a-b)^{3/2}}-\frac{3b(2a-b)\sinh(c+dx)\cosh(c+dx)}{2ad(a-b)\left(a+b\sinh^2(c+dx)\right)}}{4a(a-b)}-\frac{b\sinh(c+dx)\cosh(c+dx)}{4ad(a-b)\left(a+b\sinh^2(c+dx)\right)^2}$$

input
```
Int[(a + b*Sinh[c + d*x]^2)^(-3),x]
```

output
```
-1/4*(b*Cosh[c + d*x]*Sinh[c + d*x])/(a*(a - b)*d*(a + b*Sinh[c + d*x]^2)^
2) + (((8*a^2 - 8*a*b + 3*b^2)*ArcTanh[(Sqrt[a - b]*Tanh[c + d*x])/Sqrt[a]
])/(2*a^(3/2)*(a - b)^(3/2)*d) - (3*(2*a - b)*b*Cosh[c + d*x]*Sinh[c + d*x
])/(2*a*(a - b)*d*(a + b*Sinh[c + d*x]^2)))/(4*a*(a - b))
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3652
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)]^2), x_Symbol] :> Simp[(-(A*b - a*B))*Cos[e + f*x]*Sin[e + f*x
]*((a + b*Sin[e + f*x]^2)^(p + 1)/(2*a*f*(a + b)*(p + 1))), x] - Simp[1/(2*
a*(a + b)*(p + 1))   Int[(a + b*Sin[e + f*x]^2)^(p + 1)*Simp[a*B - A*(2*a*(
p + 1) + b*(2*p + 3)) + 2*(A*b - a*B)*(p + 2)*Sin[e + f*x]^2, x], x], x] /;
 FreeQ[{a, b, e, f, A, B}, x] && LtQ[p, -1] && NeQ[a + b, 0]
```

rule 3660
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(-1), x_Symbol] :> With[{ff =
FreeFactors[Tan[e + f*x], x]}, Simp[ff/f   Subst[Int[1/(a + (a + b)*ff^2*x^
2), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f}, x]
```

rule 3663
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> Simp[(-b)*C
os[e + f*x]*Sin[e + f*x]*((a + b*Sin[e + f*x]^2)^(p + 1)/(2*a*f*(p + 1)*(a
+ b))), x] + Simp[1/(2*a*(p + 1)*(a + b))   Int[(a + b*Sin[e + f*x]^2)^(p +
 1)*Simp[2*a*(p + 1) + b*(2*p + 3) - 2*b*(p + 2)*Sin[e + f*x]^2, x], x], x]
 /; FreeQ[{a, b, e, f}, x] && NeQ[a + b, 0] && LtQ[p, -1]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 417 vs. $2(140) = 280$.

Time = 1.53 (sec) , antiderivative size = 418, normalized size of antiderivative = 2.71

method	result
derivativedivides	$-\frac{2\left(\frac{b(8a-5b)\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^{7}}{8a\left(a^{2}-2ab+b^{2}\right)}-\frac{\left(8a^{2}-29ab+12b^{2}\right)b\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^{5}}{8a^{2}\left(a^{2}-2ab+b^{2}\right)}-\frac{\left(8a^{2}-29ab+12b^{2}\right)b\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^{3}}{8a^{2}\left(a^{2}-2ab+b^{2}\right)}+\frac{b(8a-5b)\tanh\left(\frac{dx}{2}+\frac{c}{2}\right.}{8a\left(a^{2}-2ab+b^{2}\right)}\right.}{\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^{4}a-2\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}a+4b\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}+a\right)^{2}}$
default	$-\frac{2\left(\frac{b(8a-5b)\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^{7}}{8a\left(a^{2}-2ab+b^{2}\right)}-\frac{\left(8a^{2}-29ab+12b^{2}\right)b\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^{5}}{8a^{2}\left(a^{2}-2ab+b^{2}\right)}-\frac{\left(8a^{2}-29ab+12b^{2}\right)b\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^{3}}{8a^{2}\left(a^{2}-2ab+b^{2}\right)}+\frac{b(8a-5b)\tanh\left(\frac{dx}{2}+\frac{c}{2}\right.}{8a\left(a^{2}-2ab+b^{2}\right)}\right.}{\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^{4}a-2\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}a+4b\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}+a\right)^{2}}$
risch	$\frac{8a^{2}b\,e^{6dx+6c}-8a\,b^{2}e^{6dx+6c}+3b^{3}e^{6dx+6c}+48a^{3}e^{4dx+4c}-72a^{2}b\,e^{4dx+4c}+42a\,b^{2}e^{4dx+4c}-9b^{3}e^{4dx+4c}+40\,e^{2dx+2c}a^{2}b-}{4d\,a^{2}(a-b)^{2}\left(b\,e^{4dx+4c}+4\,e^{2dx+2c}a-2\,e^{2dx+2c}b+b\right)^{2}}$

input
```
int(1/(a+b*sinh(d*x+c)^2)^3,x,method=_RETURNVERBOSE)
```

output
```
1/d*(-2*(1/8*b*(8*a-5*b)/a/(a^2-2*a*b+b^2)*tanh(1/2*d*x+1/2*c)^7-1/8*(8*a^
2-29*a*b+12*b^2)/a^2*b/(a^2-2*a*b+b^2)*tanh(1/2*d*x+1/2*c)^5-1/8*(8*a^2-29
*a*b+12*b^2)/a^2*b/(a^2-2*a*b+b^2)*tanh(1/2*d*x+1/2*c)^3+1/8*b*(8*a-5*b)/a
/(a^2-2*a*b+b^2)*tanh(1/2*d*x+1/2*c))/(tanh(1/2*d*x+1/2*c)^4*a-2*tanh(1/2*
d*x+1/2*c)^2*a+4*b*tanh(1/2*d*x+1/2*c)^2+a)^2-1/4/a*(8*a^2-8*a*b+3*b^2)/(a
^2-2*a*b+b^2)*(1/2*((-b*(a-b))^(1/2)+b)/a/(-b*(a-b))^(1/2)/((2*(-b*(a-b))^
(1/2)-a+2*b)*a)^(1/2)*arctan(tanh(1/2*d*x+1/2*c)*a/((2*(-b*(a-b))^(1/2)-a+
2*b)*a)^(1/2))-1/2*((-b*(a-b))^(1/2)-b)/a/(-b*(a-b))^(1/2)/((2*(-b*(a-b))^
(1/2)+a-2*b)*a)^(1/2)*arctanh(tanh(1/2*d*x+1/2*c)*a/((2*(-b*(a-b))^(1/2)+a
-2*b)*a)^(1/2))))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2835 vs. $2(140) = 280$.

Time = 0.15 (sec) , antiderivative size = 5925, normalized size of antiderivative = 38.47

$$\int \frac{1}{\left(a+b\sinh^2(c+dx)\right)^3}\,dx = \text{Too large to display}$$

input
```
integrate(1/(a+b*sinh(d*x+c)^2)^3,x, algorithm="fricas")
```

output
```
Too large to include
```

Sympy [F(-1)]

Timed out.

$$\int \frac{1}{\left(a+b\sinh^2(c+dx)\right)^3}\,dx = \text{Timed out}$$

input
```
integrate(1/(a+b*sinh(d*x+c)**2)**3,x)
```

output
```
Timed out
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{1}{\left(a+b\sinh^2(c+dx)\right)^3}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate(1/(a+b*sinh(d*x+c)^2)^3,x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(b-a>0)', see `assume?` for more
details)Is
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 302 vs. $2(140) = 280$.

Time = 0.24 (sec) , antiderivative size = 302, normalized size of antiderivative = 1.96

$$\int \frac{1}{\left(a+b\sinh^2(c+dx)\right)^3}\,dx = \frac{\frac{\left(8\,a^2-8\,ab+3\,b^2\right)\arctan\left(\frac{be^{(2\,dx+2\,c)}+2\,a-b}{2\,\sqrt{-a^2+ab}}\right)}{\left(a^4-2\,a^3b+a^2b^2\right)\sqrt{-a^2+ab}} + \frac{2\,\left(8\,a^2be^{(6\,dx+6\,c)}-8\,ab^2e^{(6\,dx+6\,c)}+3\,b^3e^{(6\,dx+6\,c)}+48\,a^3e^{(4\,dx+4\,c)}-72\,a^2be^{(4\,dx+4\,c)}+42\,a\cdots}{\left(a^4-2\,a^3b+a^2b^2\right)\left(be^{(4\,dx+4\,c)}+\cdots}}{8\,d}$$

input
```
integrate(1/(a+b*sinh(d*x+c)^2)^3,x, algorithm="giac")
```

output
```
1/8*((8*a^2 - 8*a*b + 3*b^2)*arctan(1/2*(b*e^(2*d*x + 2*c) + 2*a - b)/sqrt
(-a^2 + a*b))/((a^4 - 2*a^3*b + a^2*b^2)*sqrt(-a^2 + a*b)) + 2*(8*a^2*b*e^
(6*d*x + 6*c) - 8*a*b^2*e^(6*d*x + 6*c) + 3*b^3*e^(6*d*x + 6*c) + 48*a^3*e
^(4*d*x + 4*c) - 72*a^2*b*e^(4*d*x + 4*c) + 42*a*b^2*e^(4*d*x + 4*c) - 9*b
^3*e^(4*d*x + 4*c) + 40*a^2*b*e^(2*d*x + 2*c) - 40*a*b^2*e^(2*d*x + 2*c) +
 9*b^3*e^(2*d*x + 2*c) + 6*a*b^2 - 3*b^3)/((a^4 - 2*a^3*b + a^2*b^2)*(b*e^
(4*d*x + 4*c) + 4*a*e^(2*d*x + 2*c) - 2*b*e^(2*d*x + 2*c) + b)^2))/d
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\left(a+b\sinh^2(c+dx)\right)^3}\,dx=\int \frac{1}{\left(b\sinh\left(c+d\,x\right)^2+a\right)^3}\,dx$$

input
```
int(1/(a + b*sinh(c + d*x)^2)^3,x)
```

output
```
int(1/(a + b*sinh(c + d*x)^2)^3, x)
```

Reduce [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 4496, normalized size of antiderivative = 29.19

$$\int \frac{1}{\left(a+b\sinh^2(c+dx)\right)^3}\,dx=\text{Too large to display}$$

input
```
int(1/(a+b*sinh(d*x+c)^2)^3,x)
```

output

```
(16*e**(8*c + 8*d*x)*sqrt(a)*sqrt(a - b)*log( - sqrt(2*sqrt(a)*sqrt(a - b)
 - 2*a + b) + e**(c + d*x)*sqrt(b))*a**3*b**2 - 24*e**(8*c + 8*d*x)*sqrt(a
)*sqrt(a - b)*log( - sqrt(2*sqrt(a)*sqrt(a - b) - 2*a + b) + e**(c + d*x)*
sqrt(b))*a**2*b**3 + 14*e**(8*c + 8*d*x)*sqrt(a)*sqrt(a - b)*log( - sqrt(2
*sqrt(a)*sqrt(a - b) - 2*a + b) + e**(c + d*x)*sqrt(b))*a*b**4 - 3*e**(8*c
 + 8*d*x)*sqrt(a)*sqrt(a - b)*log( - sqrt(2*sqrt(a)*sqrt(a - b) - 2*a + b)
 + e**(c + d*x)*sqrt(b))*b**5 + 16*e**(8*c + 8*d*x)*sqrt(a)*sqrt(a - b)*lo
g(sqrt(2*sqrt(a)*sqrt(a - b) - 2*a + b) + e**(c + d*x)*sqrt(b))*a**3*b**2
- 24*e**(8*c + 8*d*x)*sqrt(a)*sqrt(a - b)*log(sqrt(2*sqrt(a)*sqrt(a - b) -
 2*a + b) + e**(c + d*x)*sqrt(b))*a**2*b**3 + 14*e**(8*c + 8*d*x)*sqrt(a)*
sqrt(a - b)*log(sqrt(2*sqrt(a)*sqrt(a - b) - 2*a + b) + e**(c + d*x)*sqrt(
b))*a*b**4 - 3*e**(8*c + 8*d*x)*sqrt(a)*sqrt(a - b)*log(sqrt(2*sqrt(a)*sqr
t(a - b) - 2*a + b) + e**(c + d*x)*sqrt(b))*b**5 - 16*e**(8*c + 8*d*x)*sqr
t(a)*sqrt(a - b)*log(2*sqrt(a)*sqrt(a - b) + e**(2*c + 2*d*x)*b + 2*a - b)
*a**3*b**2 + 24*e**(8*c + 8*d*x)*sqrt(a)*sqrt(a - b)*log(2*sqrt(a)*sqrt(a
- b) + e**(2*c + 2*d*x)*b + 2*a - b)*a**2*b**3 - 14*e**(8*c + 8*d*x)*sqrt(
a)*sqrt(a - b)*log(2*sqrt(a)*sqrt(a - b) + e**(2*c + 2*d*x)*b + 2*a - b)*a
*b**4 + 3*e**(8*c + 8*d*x)*sqrt(a)*sqrt(a - b)*log(2*sqrt(a)*sqrt(a - b) +
 e**(2*c + 2*d*x)*b + 2*a - b)*b**5 + 128*e**(6*c + 6*d*x)*sqrt(a)*sqrt(a
- b)*log( - sqrt(2*sqrt(a)*sqrt(a - b) - 2*a + b) + e**(c + d*x)*sqrt(b...
```

3.56 $\int \frac{\operatorname{csch}(c+dx)}{\left(a+b\sinh^2(c+dx)\right)^3}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 166

$$\int \frac{\operatorname{csch}(c+dx)}{\left(a+b\sinh^2(c+dx)\right)^3}\,dx = -\frac{\sqrt{b}(15a^2-20ab+8b^2)\arctan\left(\frac{\sqrt{b}\cosh(c+dx)}{\sqrt{a-b}}\right)}{8a^3(a-b)^{5/2}d} - \frac{\operatorname{arctanh}(\cosh(c+dx))}{a^3d} - \frac{b\cosh(c+dx)}{4a(a-b)d\left(a-b+b\cosh^2(c+dx)\right)^2} - \frac{(7a-4b)b\cosh(c+dx)}{8a^2(a-b)^2d\left(a-b+b\cosh^2(c+dx)\right)}$$

output
```
-1/8*b^(1/2)*(15*a^2-20*a*b+8*b^2)*arctan(b^(1/2)*cosh(d*x+c)/(a-b)^(1/2))
/a^3/(a-b)^(5/2)/d-arctanh(cosh(d*x+c))/a^3/d-1/4*b*cosh(d*x+c)/a/(a-b)/d/
(a-b+b*cosh(d*x+c)^2)^2-1/8*(7*a-4*b)*b*cosh(d*x+c)/a^2/(a-b)^2/d/(a-b+b*c
osh(d*x+c)^2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 8.41 (sec) , antiderivative size = 250, normalized size of antiderivative = 1.51

$$\int \frac{\operatorname{csch}(c+dx)}{\left(a+b\sinh^2(c+dx)\right)^3}\,dx = -\frac{\frac{\sqrt{b}(15a^2-20ab+8b^2)\arctan\left(\frac{\sqrt{b}-i\sqrt{a}\tanh\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a-b}}\right)}{(a-b)^{5/2}}+\frac{\sqrt{b}(15a^2-20ab+8b^2)\arctan\left(\frac{\sqrt{b}+i\sqrt{a}\tanh\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a-b}}\right)}{(a-b)^{5/2}}+\frac{8a^2b\cosh(c}{(a-b)(2a-b+b\cosh}}{8a^3d}$$

input
```
Integrate[Csch[c + d*x]/(a + b*Sinh[c + d*x]^2)^3,x]
```

output
```
-1/8*((Sqrt[b]*(15*a^2 - 20*a*b + 8*b^2)*ArcTan[(Sqrt[b] - I*Sqrt[a]*Tanh[
(c + d*x)/2])/Sqrt[a - b]])/(a - b)^(5/2) + (Sqrt[b]*(15*a^2 - 20*a*b + 8*
b^2)*ArcTan[(Sqrt[b] + I*Sqrt[a]*Tanh[(c + d*x)/2])/Sqrt[a - b]])/(a - b)^
(5/2) + (8*a^2*b*Cosh[c + d*x])/((a - b)*(2*a - b + b*Cosh[2*(c + d*x)])^2
) + (2*a*(7*a - 4*b)*b*Cosh[c + d*x])/((a - b)^2*(2*a - b + b*Cosh[2*(c +
d*x)])) + 8*Log[Cosh[(c + d*x)/2]] - 8*Log[Sinh[(c + d*x)/2]])/(a^3*d)
```

Rubi [A] (verified)

Time = 0.43 (sec) , antiderivative size = 193, normalized size of antiderivative = 1.16, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.476$, Rules used = {3042, 26, 3665, 316, 25, 402, 25, 397, 218, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\operatorname{csch}(c+dx)}{\left(a+b\sinh^2(c+dx)\right)^3}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{i}{\sin(ic+idx)\left(a-b\sin(ic+idx)^2\right)^3}dx$$

$$\Big\downarrow \text{26}$$

$$i\int \frac{1}{\sin(ic+idx)\left(a-b\sin(ic+idx)^2\right)^3}dx$$

$$\Big\downarrow \ 3665$$

$$-\frac{\int \frac{1}{(1-\cosh^2(c+dx))\left(b\cosh^2(c+dx)+a-b\right)^3}d\cosh(c+dx)}{d}$$

$$\Big\downarrow \ 316$$

$$-\frac{\frac{b\cosh(c+dx)}{4a(a-b)\left(a+b\cosh^2(c+dx)-b\right)^2}-\frac{\int -\frac{-3b\cosh^2(c+dx)+4a-b}{\left(1-\cosh^2(c+dx)\right)\left(b\cosh^2(c+dx)+a-b\right)^2}d\cosh(c+dx)}{4a(a-b)}}{d}$$

$$\Big\downarrow \ 25$$

$$-\frac{\frac{\int \frac{-3b\cosh^2(c+dx)+4a-b}{\left(1-\cosh^2(c+dx)\right)\left(b\cosh^2(c+dx)+a-b\right)^2}d\cosh(c+dx)}{4a(a-b)}+\frac{b\cosh(c+dx)}{4a(a-b)\left(a+b\cosh^2(c+dx)-b\right)^2}}{d}$$

$$\Big\downarrow \ 402$$

$$-\frac{\frac{\frac{b(7a-4b)\cosh(c+dx)}{2a(a-b)\left(a+b\cosh^2(c+dx)-b\right)}-\frac{\int -\frac{8a^2-9ba+4b^2-(7a-4b)b\cosh^2(c+dx)}{\left(1-\cosh^2(c+dx)\right)\left(b\cosh^2(c+dx)+a-b\right)}d\cosh(c+dx)}{2a(a-b)}}{4a(a-b)}+\frac{b\cosh(c+dx)}{4a(a-b)\left(a+b\cosh^2(c+dx)-b\right)^2}}{d}$$

$$\Big\downarrow \ 25$$

$$-\frac{\frac{\frac{\int \frac{8a^2-9ba+4b^2-(7a-4b)b\cosh^2(c+dx)}{\left(1-\cosh^2(c+dx)\right)\left(b\cosh^2(c+dx)+a-b\right)}d\cosh(c+dx)}{2a(a-b)}+\frac{b(7a-4b)\cosh(c+dx)}{2a(a-b)\left(a+b\cosh^2(c+dx)-b\right)}}{4a(a-b)}+\frac{b\cosh(c+dx)}{4a(a-b)\left(a+b\cosh^2(c+dx)-b\right)^2}}{d}$$

$$\Big\downarrow \ 397$$

$$-\frac{\frac{\frac{\frac{b\left(15a^2-20ab+8b^2\right)\int \frac{1}{b\cosh^2(c+dx)+a-b}d\cosh(c+dx)}{a}+\frac{8(a-b)^2\int \frac{1}{1-\cosh^2(c+dx)}d\cosh(c+dx)}{a}}{2a(a-b)}+\frac{b(7a-4b)\cosh(c+dx)}{2a(a-b)\left(a+b\cosh^2(c+dx)-b\right)}}{4a(a-b)}+\frac{b\cosh(c+}{4a(a-b)(a+b\cosh}}{d}$$

$$\Big\downarrow \ 218$$

$$-\frac{\frac{\frac{\frac{8(a-b)^2\int \frac{1}{1-\cosh^2(c+dx)}d\cosh(c+dx)}{a}+\frac{\sqrt{b}\left(15a^2-20ab+8b^2\right)\arctan\left(\frac{\sqrt{b}\cosh(c+dx)}{\sqrt{a-b}}\right)}{a\sqrt{a-b}}}{2a(a-b)}+\frac{b(7a-4b)\cosh(c+dx)}{2a(a-b)\left(a+b\cosh^2(c+dx)-b\right)}}{4a(a-b)}+\frac{b\cosh(c+dx)}{4a(a-b)\left(a+b\cosh^2(c+dx\right.}}{d}$$

$\Big\downarrow$ 219

$$-\frac{\frac{\frac{\frac{\sqrt{b}\left(15a^2-20ab+8b^2\right)\arctan\left(\frac{\sqrt{b}\cosh(c+dx)}{\sqrt{a-b}}\right)}{a\sqrt{a-b}}+\frac{8(a-b)^2\text{arctanh}(\cosh(c+dx))}{a}}{2a(a-b)}+\frac{b(7a-4b)\cosh(c+dx)}{2a(a-b)\left(a+b\cosh^2(c+dx)-b\right)}}{4a(a-b)}+\frac{b\cosh(c+dx)}{4a(a-b)\left(a+b\cosh^2(c+dx)-b\right)^2}}{d}$$

input
```
Int[Csch[c + d*x]/(a + b*Sinh[c + d*x]^2)^3,x]
```

output
```
-(((b*Cosh[c + d*x])/(4*a*(a - b)*(a - b + b*Cosh[c + d*x]^2)^2) + (((Sqrt
[b]*(15*a^2 - 20*a*b + 8*b^2)*ArcTan[(Sqrt[b]*Cosh[c + d*x])/Sqrt[a - b]])
/(a*Sqrt[a - b]) + (8*(a - b)^2*ArcTanh[Cosh[c + d*x]])/a)/(2*a*(a - b)) +
 ((7*a - 4*b)*b*Cosh[c + d*x])/(2*a*(a - b)*(a - b + b*Cosh[c + d*x]^2)))/
(4*a*(a - b)))/d)
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 316
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_), x_Symbol] :> Sim
p[(-b)*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^(q + 1)/(2*a*(p + 1)*(b*c - a*d))
), x] + Simp[1/(2*a*(p + 1)*(b*c - a*d))   Int[(a + b*x^2)^(p + 1)*(c + d*x
^2)^q*Simp[b*c + 2*(p + 1)*(b*c - a*d) + d*b*(2*(p + q + 2) + 1)*x^2, x], x
], x] /; FreeQ[{a, b, c, d, q}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] &&  !
( !IntegerQ[p] && IntegerQ[q] && LtQ[q, -1]) && IntBinomialQ[a, b, c, d, 2,
 p, q, x]
```

rule 397
```
Int[((e_) + (f_.)*(x_)^2)/(((a_) + (b_.)*(x_)^2)*((c_) + (d_.)*(x_)^2)), x_
Symbol] :> Simp[(b*e - a*f)/(b*c - a*d)   Int[1/(a + b*x^2), x], x] - Simp[
(d*e - c*f)/(b*c - a*d)   Int[1/(c + d*x^2), x], x] /; FreeQ[{a, b, c, d, e
, f}, x]
```

rule 402
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_.)*((e_) + (f_.)*(x
_)^2), x_Symbol] :> Simp[(-(b*e - a*f))*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^
(q + 1)/(a*2*(b*c - a*d)*(p + 1))), x] + Simp[1/(a*2*(b*c - a*d)*(p + 1))
 Int[(a + b*x^2)^(p + 1)*(c + d*x^2)^q*Simp[c*(b*e - a*f) + e*2*(b*c - a*d)
*(p + 1) + d*(b*e - a*f)*(2*(p + q + 2) + 1)*x^2, x], x], x] /; FreeQ[{a, b
, c, d, e, f, q}, x] && LtQ[p, -1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3665
```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^
(p_.), x_Symbol] :> With[{ff = FreeFactors[Cos[e + f*x], x]}, Simp[-ff/f
Subst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a + b - b*ff^2*x^2)^p, x], x, Cos[e +
 f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m - 1)/2]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 307 vs. $2(152) = 304$.

Time = 2.06 (sec) , antiderivative size = 308, normalized size of antiderivative = 1.86

method	result
derivativedivides	$\frac{\frac{\ln\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{a^3}-\frac{2b\left(\frac{-\frac{\left(9a^2-28ab+16b^2\right)a\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^6}{8\left(a^2-2ab+b^2\right)}+\frac{3\left(9a^3-30a^2b+40b^2a-16b^3\right)\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^4}{8\left(a^2-2ab+b^2\right)}-\frac{a\left(27a^2-68ab+32\right.}{8\left(a^2-\right.}}{\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^4a-2\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^2a+4b\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^2+a\right)^2}\right.}{a^3}}{d}$
default	$\frac{\frac{\ln\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{a^3}-\frac{2b\left(\frac{-\frac{\left(9a^2-28ab+16b^2\right)a\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^6}{8\left(a^2-2ab+b^2\right)}+\frac{3\left(9a^3-30a^2b+40b^2a-16b^3\right)\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^4}{8\left(a^2-2ab+b^2\right)}-\frac{a\left(27a^2-68ab+32\right.}{8\left(a^2-\right.}}{\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^4a-2\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^2a+4b\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^2+a\right)^2}\right.}{a^3}}{d}$
risch	$-\frac{e^{dx+c}b\left(7e^{6dx+6c}ab-4e^{6dx+6c}b^2+36e^{4dx+4c}a^2-31e^{4dx+4c}ab+4e^{4dx+4c}b^2+36e^{2dx+2c}a^2-31e^{2dx+2c}ba+4b^2e^{2d}\right.}{4d\,a^2(a-b)^2\left(b\,e^{4dx+4c}+4\,e^{2dx+2c}a-2\,e^{2dx+2c}b+b\right)^2}$

input
```
int(csch(d*x+c)/(a+b*sinh(d*x+c)^2)^3,x,method=_RETURNVERBOSE)
```

output
```
1/d*(1/a^3*ln(tanh(1/2*d*x+1/2*c))-2*b/a^3*((-1/8*(9*a^2-28*a*b+16*b^2)*a/
(a^2-2*a*b+b^2)*tanh(1/2*d*x+1/2*c)^6+3/8*(9*a^3-30*a^2*b+40*a*b^2-16*b^3)
/(a^2-2*a*b+b^2)*tanh(1/2*d*x+1/2*c)^4-1/8*a*(27*a^2-68*a*b+32*b^2)/(a^2-2
*a*b+b^2)*tanh(1/2*d*x+1/2*c)^2+3/8*a^2*(3*a-2*b)/(a^2-2*a*b+b^2))/(tanh(1
/2*d*x+1/2*c)^4*a-2*tanh(1/2*d*x+1/2*c)^2*a+4*b*tanh(1/2*d*x+1/2*c)^2+a)^2
+1/16*(15*a^2-20*a*b+8*b^2)/(a^2-2*a*b+b^2)/(a*b-b^2)^(1/2)*arctan(1/4*(2*
tanh(1/2*d*x+1/2*c)^2*a-2*a+4*b)/(a*b-b^2)^(1/2))))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 5242 vs. $2(152) = 304$.

Time = 0.25 (sec) , antiderivative size = 9815, normalized size of antiderivative = 59.13

$$\int \frac{\operatorname{csch}(c+dx)}{\left(a+b\sinh^2(c+dx)\right)^3}\,dx = \text{Too large to display}$$

input
```
integrate(csch(d*x+c)/(a+b*sinh(d*x+c)^2)^3,x, algorithm="fricas")
```

output
```
Too large to include
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\operatorname{csch}(c+dx)}{\left(a+b\sinh^2(c+dx)\right)^3}\,dx = \text{Timed out}$$

input
```
integrate(csch(d*x+c)/(a+b*sinh(d*x+c)**2)**3,x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{\operatorname{csch}(c+dx)}{\left(a+b\sinh^2(c+dx)\right)^3}\,dx = \int \frac{\operatorname{csch}\left(dx+c\right)}{\left(b\sinh\left(dx+c\right)^2+a\right)^3}\,dx$$

input
```
integrate(csch(d*x+c)/(a+b*sinh(d*x+c)^2)^3,x, algorithm="maxima")
```

output
```
-1/4*((7*a*b^2*e^(7*c) - 4*b^3*e^(7*c))*e^(7*d*x) + (36*a^2*b*e^(5*c) - 31
*a*b^2*e^(5*c) + 4*b^3*e^(5*c))*e^(5*d*x) + (36*a^2*b*e^(3*c) - 31*a*b^2*e
^(3*c) + 4*b^3*e^(3*c))*e^(3*d*x) + (7*a*b^2*e^c - 4*b^3*e^c)*e^(d*x))/(a^
4*b^2*d - 2*a^3*b^3*d + a^2*b^4*d + (a^4*b^2*d*e^(8*c) - 2*a^3*b^3*d*e^(8*
c) + a^2*b^4*d*e^(8*c))*e^(8*d*x) + 4*(2*a^5*b*d*e^(6*c) - 5*a^4*b^2*d*e^(
6*c) + 4*a^3*b^3*d*e^(6*c) - a^2*b^4*d*e^(6*c))*e^(6*d*x) + 2*(8*a^6*d*e^(
4*c) - 24*a^5*b*d*e^(4*c) + 27*a^4*b^2*d*e^(4*c) - 14*a^3*b^3*d*e^(4*c) +
3*a^2*b^4*d*e^(4*c))*e^(4*d*x) + 4*(2*a^5*b*d*e^(2*c) - 5*a^4*b^2*d*e^(2*c
) + 4*a^3*b^3*d*e^(2*c) - a^2*b^4*d*e^(2*c))*e^(2*d*x)) - log((e^(d*x + c)
 + 1)*e^(-c))/(a^3*d) + log((e^(d*x + c) - 1)*e^(-c))/(a^3*d) - 2*integrat
e(1/8*((15*a^2*b*e^(3*c) - 20*a*b^2*e^(3*c) + 8*b^3*e^(3*c))*e^(3*d*x) - (
15*a^2*b*e^c - 20*a*b^2*e^c + 8*b^3*e^c)*e^(d*x))/(a^5*b - 2*a^4*b^2 + a^3
*b^3 + (a^5*b*e^(4*c) - 2*a^4*b^2*e^(4*c) + a^3*b^3*e^(4*c))*e^(4*d*x) + 2
*(2*a^6*e^(2*c) - 5*a^5*b*e^(2*c) + 4*a^4*b^2*e^(2*c) - a^3*b^3*e^(2*c))*e
^(2*d*x)), x)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{\operatorname{csch}(c+dx)}{\left(a+b\sinh^2(c+dx)\right)^3}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(csch(d*x+c)/(a+b*sinh(d*x+c)^2)^3,x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Limit: Max order reached or unable
to make series expansion Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\operatorname{csch}(c+dx)}{\left(a+b\sinh^2(c+dx)\right)^3}\,dx=\int \frac{1}{\sinh\left(c+d\,x\right)\,\left(b\,{\sinh\left(c+d\,x\right)}^2+a\right)^3}\,dx$$

input
```
int(1/(sinh(c + d*x)*(a + b*sinh(c + d*x)^2)^3),x)
```

output
```
int(1/(sinh(c + d*x)*(a + b*sinh(c + d*x)^2)^3), x)
```

Reduce [B] (verification not implemented)

Time = 0.42 (sec) , antiderivative size = 10544, normalized size of antiderivative = 63.52

$$\int \frac{\operatorname{csch}(c+dx)}{\left(a+b\sinh^2(c+dx)\right)^3}\,dx=\text{Too large to display}$$

input
```
int(csch(d*x+c)/(a+b*sinh(d*x+c)^2)^3,x)
```

output
```
( - 30*e**(8*c + 8*d*x)*sqrt(b)*sqrt(a)*sqrt(a - b)*sqrt(2*sqrt(a)*sqrt(a
- b) + 2*a - b)*atan((e**(c + d*x)*b)/(sqrt(b)*sqrt(2*sqrt(a)*sqrt(a - b)
+ 2*a - b)))*a**2*b**2 + 40*e**(8*c + 8*d*x)*sqrt(b)*sqrt(a)*sqrt(a - b)*s
qrt(2*sqrt(a)*sqrt(a - b) + 2*a - b)*atan((e**(c + d*x)*b)/(sqrt(b)*sqrt(2
*sqrt(a)*sqrt(a - b) + 2*a - b)))*a*b**3 - 16*e**(8*c + 8*d*x)*sqrt(b)*sqr
t(a)*sqrt(a - b)*sqrt(2*sqrt(a)*sqrt(a - b) + 2*a - b)*atan((e**(c + d*x)*
b)/(sqrt(b)*sqrt(2*sqrt(a)*sqrt(a - b) + 2*a - b)))*b**4 - 240*e**(6*c + 6
*d*x)*sqrt(b)*sqrt(a)*sqrt(a - b)*sqrt(2*sqrt(a)*sqrt(a - b) + 2*a - b)*at
an((e**(c + d*x)*b)/(sqrt(b)*sqrt(2*sqrt(a)*sqrt(a - b) + 2*a - b)))*a**3*
b + 440*e**(6*c + 6*d*x)*sqrt(b)*sqrt(a)*sqrt(a - b)*sqrt(2*sqrt(a)*sqrt(a
 - b) + 2*a - b)*atan((e**(c + d*x)*b)/(sqrt(b)*sqrt(2*sqrt(a)*sqrt(a - b)
 + 2*a - b)))*a**2*b**2 - 288*e**(6*c + 6*d*x)*sqrt(b)*sqrt(a)*sqrt(a - b)
*sqrt(2*sqrt(a)*sqrt(a - b) + 2*a - b)*atan((e**(c + d*x)*b)/(sqrt(b)*sqrt
(2*sqrt(a)*sqrt(a - b) + 2*a - b)))*a*b**3 + 64*e**(6*c + 6*d*x)*sqrt(b)*s
qrt(a)*sqrt(a - b)*sqrt(2*sqrt(a)*sqrt(a - b) + 2*a - b)*atan((e**(c + d*x
)*b)/(sqrt(b)*sqrt(2*sqrt(a)*sqrt(a - b) + 2*a - b)))*b**4 - 480*e**(4*c +
 4*d*x)*sqrt(b)*sqrt(a)*sqrt(a - b)*sqrt(2*sqrt(a)*sqrt(a - b) + 2*a - b)*
atan((e**(c + d*x)*b)/(sqrt(b)*sqrt(2*sqrt(a)*sqrt(a - b) + 2*a - b)))*a**
4 + 1120*e**(4*c + 4*d*x)*sqrt(b)*sqrt(a)*sqrt(a - b)*sqrt(2*sqrt(a)*sqrt(
a - b) + 2*a - b)*atan((e**(c + d*x)*b)/(sqrt(b)*sqrt(2*sqrt(a)*sqrt(a ...
```

3.57 $\int \frac{\operatorname{csch}^2(c+dx)}{\left(a+b\sinh^2(c+dx)\right)^3}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 171

$$\begin{aligned}\int \frac{\operatorname{csch}^2(c+dx)}{\left(a+b\sinh^2(c+dx)\right)^3}\,dx &= -\frac{3b(8a^2-12ab+5b^2)\operatorname{arctanh}\left(\frac{\sqrt{a-b}\tanh(c+dx)}{\sqrt{a}}\right)}{8a^{7/2}(a-b)^{5/2}d} \\ &\quad -\frac{\coth(c+dx)}{a^3d} \\ &\quad -\frac{b^3\tanh(c+dx)}{4a^2(a-b)^2d\left(a-(a-b)\tanh^2(c+dx)\right)^2} \\ &\quad +\frac{(12a-7b)b^2\tanh(c+dx)}{8a^3(a-b)^2d\left(a-(a-b)\tanh^2(c+dx)\right)}\end{aligned}$$

output
```
-3/8*b*(8*a^2-12*a*b+5*b^2)*arctanh((a-b)^(1/2)*tanh(d*x+c)/a^(1/2))/a^(7/
2)/(a-b)^(5/2)/d-coth(d*x+c)/a^3/d-1/4*b^3*tanh(d*x+c)/a^2/(a-b)^2/d/(a-(a
-b)*tanh(d*x+c)^2)^2+1/8*(12*a-7*b)*b^2*tanh(d*x+c)/a^3/(a-b)^2/d/(a-(a-b)
*tanh(d*x+c)^2)
```

Mathematica [A] (verified)

Time = 2.92 (sec) , antiderivative size = 225, normalized size of antiderivative = 1.32

$$\int \frac{\operatorname{csch}^2(c+dx)}{\left(a+b\sinh^2(c+dx)\right)^3}\,dx$$
$$= \frac{(2a-b+b\cosh(2(c+dx)))\operatorname{csch}^6(c+dx)\left(-\frac{3b(8a^2-12ab+5b^2)\operatorname{arctanh}\left(\frac{\sqrt{a-b}\tanh(c+dx)}{\sqrt{a}}\right)(2a-b+b\cosh(2(c+dx)))^2}{(a-b)^{5/2}}-\right.}{64a^{7/2}d\,(b}$$

input
```
Integrate[Csch[c + d*x]^2/(a + b*Sinh[c + d*x]^2)^3,x]
```

output
```
((2*a - b + b*Cosh[2*(c + d*x)])*Csch[c + d*x]^6*((-3*b*(8*a^2 - 12*a*b +
5*b^2)*ArcTanh[(Sqrt[a - b]*Tanh[c + d*x])/Sqrt[a]]*(2*a - b + b*Cosh[2*(c
 + d*x)])^2)/(a - b)^(5/2) - 8*Sqrt[a]*(2*a - b + b*Cosh[2*(c + d*x)])^2*C
oth[c + d*x] + (4*a^(3/2)*b^2*Sinh[2*(c + d*x)])/(a - b) + (Sqrt[a]*(10*a
- 7*b)*b^2*(2*a - b + b*Cosh[2*(c + d*x)])*Sinh[2*(c + d*x)])/(a - b)^2))/
(64*a^(7/2)*d*(b + a*Csch[c + d*x]^2)^3)
```

Rubi [A] (verified)

Time = 0.45 (sec) , antiderivative size = 232, normalized size of antiderivative = 1.36, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.304, Rules used = {3042, 25, 3666, 370, 439, 359, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\operatorname{csch}^2(c+dx)}{\left(a+b\sinh^2(c+dx)\right)^3}\,dx$$
$$\Big\downarrow \; 3042$$
$$\int -\frac{1}{\sin(ic+idx)^2\left(a-b\sin(ic+idx)^2\right)^3}dx$$
$$\Big\downarrow \; 25$$

$$-\int \frac{1}{\sin(ic+idx)^2\left(a-b\sin(ic+idx)^2\right)^3}dx$$

$$\Big\downarrow \ 3666$$

$$\frac{\int \frac{\coth^2(c+dx)\left(1-\tanh^2(c+dx)\right)^3}{\left(a-(a-b)\tanh^2(c+dx)\right)^3}d\tanh(c+dx)}{d}$$

$$\Big\downarrow \ 370$$

$$\frac{\frac{\int \frac{\coth^2(c+dx)\left(1-\tanh^2(c+dx)\right)\left(-\left((4a-b)\tanh^2(c+dx)\right)+4a-5b\right)}{\left(a-(a-b)\tanh^2(c+dx)\right)^2}d\tanh(c+dx)}{4a(a-b)}-\frac{b\left(1-\tanh^2(c+dx)\right)^2\coth(c+dx)}{4a(a-b)\left(a-(a-b)\tanh^2(c+dx)\right)^2}}{d}$$

$$\Big\downarrow \ 439$$

$$\frac{\frac{\frac{\int \frac{\coth^2(c+dx)\left((4a-5b)(2a-3b)-(2a-b)(4a-b)\tanh^2(c+dx)\right)}{a-(a-b)\tanh^2(c+dx)}d\tanh(c+dx)}{2a(a-b)}-\frac{b\coth(c+dx)\left(-\left((4a-b)\tanh^2(c+dx)\right)+4a-5b\right)}{2a(a-b)\left(a-(a-b)\tanh^2(c+dx)\right)}}{4a(a-b)}-\frac{b\left(1-\tanh^2(c+dx)\right)^2\text{cot}}{4a(a-b)\left(a-(a-b)\tanh^2\right.}}{d}$$

$$\Big\downarrow \ 359$$

$$\frac{\frac{\frac{-\frac{3b\left(8a^2-12ab+5b^2\right)\int \frac{1}{a-(a-b)\tanh^2(c+dx)}d\tanh(c+dx)}{a}-\frac{(4a-5b)(2a-3b)\coth(c+dx)}{a}}{2a(a-b)}-\frac{b\coth(c+dx)\left(-\left((4a-b)\tanh^2(c+dx)\right)+4a-5b\right)}{2a(a-b)\left(a-(a-b)\tanh^2(c+dx)\right)}}{4a(a-b)}-\frac{b\left(1-\tanh^2\right.}{4a(a-b)(a-}}{d}$$

$$\Big\downarrow \ 221$$

$$\frac{\frac{\frac{-\frac{3b\left(8a^2-12ab+5b^2\right)\text{arctanh}\left(\frac{\sqrt{a-b}\tanh(c+dx)}{\sqrt{a}}\right)}{a^{3/2}\sqrt{a-b}}-\frac{(4a-5b)(2a-3b)\coth(c+dx)}{a}}{2a(a-b)}-\frac{b\coth(c+dx)\left(-\left((4a-b)\tanh^2(c+dx)\right)+4a-5b\right)}{2a(a-b)\left(a-(a-b)\tanh^2(c+dx)\right)}}{4a(a-b)}-\frac{b\left(1-\tanh^2(c+\right.}{4a(a-b)(a-(a-}}{d}$$

input
```
Int[Csch[c + d*x]^2/(a + b*Sinh[c + d*x]^2)^3,x]
```

output
```
(-1/4*(b*Coth[c + d*x]*(1 - Tanh[c + d*x]^2)^2)/(a*(a - b)*(a - (a - b)*Ta
nh[c + d*x]^2)^2) + (((-3*b*(8*a^2 - 12*a*b + 5*b^2)*ArcTanh[(Sqrt[a - b]*
Tanh[c + d*x])/Sqrt[a]])/(a^(3/2)*Sqrt[a - b]) - ((4*a - 5*b)*(2*a - 3*b)*
Coth[c + d*x])/a)/(2*a*(a - b)) - (b*Coth[c + d*x]*(4*a - 5*b - (4*a - b)*
Tanh[c + d*x]^2))/(2*a*(a - b)*(a - (a - b)*Tanh[c + d*x]^2)))/(4*a*(a - b
)))/d
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 359
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2), x
_Symbol] :> Simp[c*(e*x)^(m + 1)*((a + b*x^2)^(p + 1)/(a*e*(m + 1))), x] +
Simp[(a*d*(m + 1) - b*c*(m + 2*p + 3))/(a*e^2*(m + 1))   Int[(e*x)^(m + 2)*
(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c - a*d, 0]
&& LtQ[m, -1] &&  !ILtQ[p, -1]
```

rule 370
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_
), x_Symbol] :> Simp[(-(b*c - a*d))*(e*x)^(m + 1)*(a + b*x^2)^(p + 1)*((c +
 d*x^2)^(q - 1)/(a*b*e*2*(p + 1))), x] + Simp[1/(a*b*2*(p + 1))   Int[(e*x)
^m*(a + b*x^2)^(p + 1)*(c + d*x^2)^(q - 2)*Simp[c*(b*c*2*(p + 1) + (b*c - a
*d)*(m + 1)) + d*(b*c*2*(p + 1) + (b*c - a*d)*(m + 2*(q - 1) + 1))*x^2, x],
 x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1]
&& GtQ[q, 1] && IntBinomialQ[a, b, c, d, e, m, 2, p, q, x]
```

rule 439
```
Int[((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_
.)*((e_) + (f_.)*(x_)^2), x_Symbol] :> Simp[(-(b*e - a*f))*(g*x)^(m + 1)*(a
 + b*x^2)^(p + 1)*((c + d*x^2)^q/(2*a*b*g*(p + 1))), x] + Simp[1/(2*a*b*(p
+ 1))   Int[(g*x)^m*(a + b*x^2)^(p + 1)*(c + d*x^2)^(q - 1)*Simp[c*(2*b*e*(
p + 1) + (b*e - a*f)*(m + 1)) + d*(2*b*e*(p + 1) + (b*e - a*f)*(m + 2*q + 1
))*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, m}, x] && LtQ[p, -1] && G
tQ[q, 0] &&  !(EqQ[q, 1] && SimplerQ[b*c - a*d, b*e - a*f])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3666
```
Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(
p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Simp[ff^(m + 1
)/f   Subst[Int[x^m*((a + (a + b)*ff^2*x^2)^p/(1 + ff^2*x^2)^(m/2 + p + 1))
, x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f}, x] && IntegerQ[m/2] &
& IntegerQ[p]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 441 vs. $2(157) = 314$.

Time = 2.24 (sec) , antiderivative size = 442, normalized size of antiderivative = 2.58

method	result
derivativedivides	$\dfrac{-\frac{\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)}{2a^3}-\frac{1}{2a^3\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)}+\frac{2b\left(\frac{\frac{3ab(4a-3b)\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^7}{8\left(a^2-2ab+b^2\right)}-\frac{\left(12a^2-49ab+28b^2\right)b\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^5}{8\left(a^2-2ab+b^2\right)}-\frac{\left(12a^2-49ab+28b\right.}{8\left(a^2-\right.}}{\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^4a-2\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^2a+4b\tanh\left(\frac{dx}{}\right.\right.}\right.}{}}{}$
default	$\dfrac{-\frac{\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)}{2a^3}-\frac{1}{2a^3\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)}+\frac{2b\left(\frac{\frac{3ab(4a-3b)\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^7}{8\left(a^2-2ab+b^2\right)}-\frac{\left(12a^2-49ab+28b^2\right)b\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^5}{8\left(a^2-2ab+b^2\right)}-\frac{\left(12a^2-49ab+28b\right.}{8\left(a^2-\right.}}{\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^4a-2\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^2a+4b\tanh\left(\frac{dx}{}\right.\right.}\right.}{}}{}$
risch	$-\frac{24\,\mathrm{e}^{8dx+8c}a^2b^2-36\,\mathrm{e}^{8dx+8c}a\,b^3+15\,\mathrm{e}^{8dx+8c}b^4+144b\,a^3\mathrm{e}^{6dx+6c}-312a^2b^2\mathrm{e}^{6dx+6c}+234a\,b^3\mathrm{e}^{6dx+6c}-60\,\mathrm{e}^{6dx+6c}b^4+}{4d\,a^3}$

input

```
int(csch(d*x+c)^2/(a+b*sinh(d*x+c)^2)^3,x,method=_RETURNVERBOSE)
```

output

```
1/d*(-1/2/a^3*tanh(1/2*d*x+1/2*c)-1/2/a^3/tanh(1/2*d*x+1/2*c)+2*b/a^3*((3/
8*a*b*(4*a-3*b)/(a^2-2*a*b+b^2)*tanh(1/2*d*x+1/2*c)^7-1/8*(12*a^2-49*a*b+2
8*b^2)*b/(a^2-2*a*b+b^2)*tanh(1/2*d*x+1/2*c)^5-1/8*(12*a^2-49*a*b+28*b^2)*
b/(a^2-2*a*b+b^2)*tanh(1/2*d*x+1/2*c)^3+3/8*a*b*(4*a-3*b)/(a^2-2*a*b+b^2)*
tanh(1/2*d*x+1/2*c))/(tanh(1/2*d*x+1/2*c)^4*a-2*tanh(1/2*d*x+1/2*c)^2*a+4*
b*tanh(1/2*d*x+1/2*c)^2+a)^2+3/8*(8*a^2-12*a*b+5*b^2)/(a^2-2*a*b+b^2)*a*(1
/2*((-b*(a-b))^(1/2)+b)/a/(-b*(a-b))^(1/2)/((2*(-b*(a-b))^(1/2)-a+2*b)*a)^
(1/2)*arctan(tanh(1/2*d*x+1/2*c)*a/((2*(-b*(a-b))^(1/2)-a+2*b)*a)^(1/2))-1
/2*((-b*(a-b))^(1/2)-b)/a/(-b*(a-b))^(1/2)/((2*(-b*(a-b))^(1/2)+a-2*b)*a)^
(1/2)*arctanh(tanh(1/2*d*x+1/2*c)*a/((2*(-b*(a-b))^(1/2)+a-2*b)*a)^(1/2)))
))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 4423 vs. $2(159) = 318$.

Time = 0.22 (sec) , antiderivative size = 9102, normalized size of antiderivative = 53.23

$$\int \frac{\operatorname{csch}^2(c+dx)}{\left(a+b\sinh^2(c+dx)\right)^3}\,dx = \text{Too large to display}$$

input
```
integrate(csch(d*x+c)^2/(a+b*sinh(d*x+c)^2)^3,x, algorithm="fricas")
```

output
```
Too large to include
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\operatorname{csch}^2(c+dx)}{\left(a+b\sinh^2(c+dx)\right)^3}\,dx = \text{Timed out}$$

input
```
integrate(csch(d*x+c)**2/(a+b*sinh(d*x+c)**2)**3,x)
```

output
```
Timed out
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{\operatorname{csch}^2(c+dx)}{\left(a+b\sinh^2(c+dx)\right)^3}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate(csch(d*x+c)^2/(a+b*sinh(d*x+c)^2)^3,x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(b-a>0)', see `assume?` for more
details)Is
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 331 vs. $2(159) = 318$.

Time = 0.38 (sec) , antiderivative size = 331, normalized size of antiderivative = 1.94

$$\int \frac{\operatorname{csch}^2(c+dx)}{(a+b\sinh^2(c+dx))^3}\,dx =$$

$$-\frac{\frac{3\,(8\,a^2b-12\,ab^2+5\,b^3)\arctan\left(\frac{be^{(2\,dx+2\,c)}+2\,a-b}{2\,\sqrt{-a^2+ab}}\right)}{(a^5-2\,a^4b+a^3b^2)\sqrt{-a^2+ab}}+\frac{2\,(16\,a^2b^2e^{(6\,dx+6\,c)}-20\,ab^3e^{(6\,dx+6\,c)}+7\,b^4e^{(6\,dx+6\,c)}+80\,a^3be^{(4\,dx+4\,c)}-136\,a^2b^2e}{(a^5-2\,a^4b+a^3b^2}}{8\,d}$$

input
```
integrate(csch(d*x+c)^2/(a+b*sinh(d*x+c)^2)^3,x, algorithm="giac")
```

output
```
-1/8*(3*(8*a^2*b - 12*a*b^2 + 5*b^3)*arctan(1/2*(b*e^(2*d*x + 2*c) + 2*a -
 b)/sqrt(-a^2 + a*b))/((a^5 - 2*a^4*b + a^3*b^2)*sqrt(-a^2 + a*b)) + 2*(16
*a^2*b^2*e^(6*d*x + 6*c) - 20*a*b^3*e^(6*d*x + 6*c) + 7*b^4*e^(6*d*x + 6*c
) + 80*a^3*b*e^(4*d*x + 4*c) - 136*a^2*b^2*e^(4*d*x + 4*c) + 86*a*b^3*e^(4
*d*x + 4*c) - 21*b^4*e^(4*d*x + 4*c) + 64*a^2*b^2*e^(2*d*x + 2*c) - 76*a*b
^3*e^(2*d*x + 2*c) + 21*b^4*e^(2*d*x + 2*c) + 10*a*b^3 - 7*b^4)/((a^5 - 2*
a^4*b + a^3*b^2)*(b*e^(4*d*x + 4*c) + 4*a*e^(2*d*x + 2*c) - 2*b*e^(2*d*x +
 2*c) + b)^2) + 16/(a^3*(e^(2*d*x + 2*c) - 1)))/d
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\operatorname{csch}^2(c+dx)}{(a+b\sinh^2(c+dx))^3}\,dx = \int \frac{1}{\sinh(c+d\,x)^2\,\left(b\sinh(c+d\,x)^2+a\right)^3}\,dx$$

input
```
int(1/(sinh(c + d*x)^2*(a + b*sinh(c + d*x)^2)^3),x)
```

output
```
int(1/(sinh(c + d*x)^2*(a + b*sinh(c + d*x)^2)^3), x)
```

Reduce [B] (verification not implemented)

Time = 0.36 (sec) , antiderivative size = 5759, normalized size of antiderivative = 33.68

$$\int \frac{\operatorname{csch}^2(c+dx)}{\left(a+b\sinh^2(c+dx)\right)^3}\,dx = \text{Too large to display}$$

input
```
int(csch(d*x+c)^2/(a+b*sinh(d*x+c)^2)^3,x)
```

output
```
( - 192*e**(10*c + 10*d*x)*sqrt(a)*sqrt(a - b)*log( - sqrt(2*sqrt(a)*sqrt(
a - b) - 2*a + b) + e**(c + d*x)*sqrt(b))*a**3*b**3 + 408*e**(10*c + 10*d*
x)*sqrt(a)*sqrt(a - b)*log( - sqrt(2*sqrt(a)*sqrt(a - b) - 2*a + b) + e**(
c + d*x)*sqrt(b))*a**2*b**4 - 300*e**(10*c + 10*d*x)*sqrt(a)*sqrt(a - b)*l
og( - sqrt(2*sqrt(a)*sqrt(a - b) - 2*a + b) + e**(c + d*x)*sqrt(b))*a*b**5
 + 75*e**(10*c + 10*d*x)*sqrt(a)*sqrt(a - b)*log( - sqrt(2*sqrt(a)*sqrt(a
- b) - 2*a + b) + e**(c + d*x)*sqrt(b))*b**6 - 192*e**(10*c + 10*d*x)*sqrt
(a)*sqrt(a - b)*log(sqrt(2*sqrt(a)*sqrt(a - b) - 2*a + b) + e**(c + d*x)*s
qrt(b))*a**3*b**3 + 408*e**(10*c + 10*d*x)*sqrt(a)*sqrt(a - b)*log(sqrt(2*
sqrt(a)*sqrt(a - b) - 2*a + b) + e**(c + d*x)*sqrt(b))*a**2*b**4 - 300*e**
(10*c + 10*d*x)*sqrt(a)*sqrt(a - b)*log(sqrt(2*sqrt(a)*sqrt(a - b) - 2*a +
 b) + e**(c + d*x)*sqrt(b))*a*b**5 + 75*e**(10*c + 10*d*x)*sqrt(a)*sqrt(a
- b)*log(sqrt(2*sqrt(a)*sqrt(a - b) - 2*a + b) + e**(c + d*x)*sqrt(b))*b**
6 + 192*e**(10*c + 10*d*x)*sqrt(a)*sqrt(a - b)*log(2*sqrt(a)*sqrt(a - b) +
 e**(2*c + 2*d*x)*b + 2*a - b)*a**3*b**3 - 408*e**(10*c + 10*d*x)*sqrt(a)*
sqrt(a - b)*log(2*sqrt(a)*sqrt(a - b) + e**(2*c + 2*d*x)*b + 2*a - b)*a**2
*b**4 + 300*e**(10*c + 10*d*x)*sqrt(a)*sqrt(a - b)*log(2*sqrt(a)*sqrt(a -
b) + e**(2*c + 2*d*x)*b + 2*a - b)*a*b**5 - 75*e**(10*c + 10*d*x)*sqrt(a)*
sqrt(a - b)*log(2*sqrt(a)*sqrt(a - b) + e**(2*c + 2*d*x)*b + 2*a - b)*b**6
 - 1536*e**(8*c + 8*d*x)*sqrt(a)*sqrt(a - b)*log( - sqrt(2*sqrt(a)*sqrt...
```

3.58 $\int \frac{\operatorname{csch}^3(c+dx)}{\left(a+b\sinh^2(c+dx)\right)^3}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 224

$$\int \frac{\operatorname{csch}^3(c+dx)}{\left(a+b\sinh^2(c+dx)\right)^3}\,dx = \frac{b^{3/2}(35a^2-56ab+24b^2)\arctan\left(\frac{\sqrt{b}\cosh(c+dx)}{\sqrt{a-b}}\right)}{8a^4(a-b)^{5/2}d} + \frac{(a+6b)\text{arctanh}(\cosh(c+dx))}{2a^4d} - \frac{(2a-3b)b\cosh(c+dx)}{4a^2(a-b)d\left(a-b+b\cosh^2(c+dx)\right)^2} - \frac{(a-4b)(4a-3b)b\cosh(c+dx)}{8a^3(a-b)^2d\left(a-b+b\cosh^2(c+dx)\right)} - \frac{\coth(c+dx)\operatorname{csch}(c+dx)}{2ad\left(a-b+b\cosh^2(c+dx)\right)^2}$$

output
```
1/8*b^(3/2)*(35*a^2-56*a*b+24*b^2)*arctan(b^(1/2)*cosh(d*x+c)/(a-b)^(1/2))
/a^4/(a-b)^(5/2)/d+1/2*(a+6*b)*arctanh(cosh(d*x+c))/a^4/d-1/4*(2*a-3*b)*b*
cosh(d*x+c)/a^2/(a-b)/d/(a-b+b*cosh(d*x+c)^2)^2-1/8*(a-4*b)*(4*a-3*b)*b*co
sh(d*x+c)/a^3/(a-b)^2/d/(a-b+b*cosh(d*x+c)^2)-1/2*coth(d*x+c)*csch(d*x+c)/
a/d/(a-b+b*cosh(d*x+c)^2)^2
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 7.22 (sec) , antiderivative size = 462, normalized size of antiderivative = 2.06

$$\int \frac{\operatorname{csch}^3(c+dx)}{\left(a+b\sinh^2(c+dx)\right)^3}\,dx$$

$$= \frac{(2a-b+b\cosh(2(c+dx)))\operatorname{csch}^5(c+dx)\left(\frac{8a^2b^2\coth(c+dx)}{a-b}+\frac{2a(11a-8b)b^2(2a-b+b\cosh(2(c+dx)))\coth(c+dx)}{(a-b)^2}+\frac{b^{3/}}{}\right.}{}$$

input
```
Integrate[Csch[c + d*x]^3/(a + b*Sinh[c + d*x]^2)^3,x]
```

output
```
((2*a - b + b*Cosh[2*(c + d*x)])*Csch[c + d*x]^5*((8*a^2*b^2*Coth[c + d*x]
)/(a - b) + (2*a*(11*a - 8*b)*b^2*(2*a - b + b*Cosh[2*(c + d*x)])*Coth[c +
 d*x])/(a - b)^2 + (b^(3/2)*(35*a^2 - 56*a*b + 24*b^2)*ArcTan[(Sqrt[b] - I
*Sqrt[a]*Tanh[(c + d*x)/2])/Sqrt[a - b]]*(2*a - b + b*Cosh[2*(c + d*x)])^2
*Csch[c + d*x])/(a - b)^(5/2) + (b^(3/2)*(35*a^2 - 56*a*b + 24*b^2)*ArcTan
[(Sqrt[b] + I*Sqrt[a]*Tanh[(c + d*x)/2])/Sqrt[a - b]]*(2*a - b + b*Cosh[2*
(c + d*x)])^2*Csch[c + d*x])/(a - b)^(5/2) - a*(2*a - b + b*Cosh[2*(c + d*
x)])^2*Csch[(c + d*x)/2]^2*Csch[c + d*x] + 4*(a + 6*b)*(2*a - b + b*Cosh[2
*(c + d*x)])^2*Csch[c + d*x]*Log[Cosh[(c + d*x)/2]] - 4*(a + 6*b)*(2*a - b
 + b*Cosh[2*(c + d*x)])^2*Csch[c + d*x]*Log[Sinh[(c + d*x)/2]] - a*(2*a -
b + b*Cosh[2*(c + d*x)])^2*Csch[c + d*x]*Sech[(c + d*x)/2]^2))/(64*a^4*d*(
b + a*Csch[c + d*x]^2)^3)
```

Rubi [A] (verified)

Time = 0.53 (sec) , antiderivative size = 261, normalized size of antiderivative = 1.17, number of steps used = 12, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.478$, Rules used = {3042, 26, 3665, 316, 402, 27, 402, 25, 397, 218, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\operatorname{csch}^3(c+dx)}{\left(a+b\sinh^2(c+dx)\right)^3}dx$$

$\Big\downarrow$ 3042

$$\int -\frac{i}{\sin(ic+idx)^3\left(a-b\sin(ic+idx)^2\right)^3}dx$$

$\Big\downarrow$ 26

$$-i\int \frac{1}{\sin(ic+idx)^3\left(a-b\sin(ic+idx)^2\right)^3}dx$$

$\Big\downarrow$ 3665

$$\frac{\int \frac{1}{\left(1-\cosh^2(c+dx)\right)^2\left(b\cosh^2(c+dx)+a-b\right)^3}d\cosh(c+dx)}{d}$$

$\Big\downarrow$ 316

$$\frac{\frac{\int \frac{5b\cosh^2(c+dx)+a+b}{\left(1-\cosh^2(c+dx)\right)\left(b\cosh^2(c+dx)+a-b\right)^3}d\cosh(c+dx)}{2a}+\frac{\cosh(c+dx)}{2a\left(1-\cosh^2(c+dx)\right)\left(a+b\cosh^2(c+dx)-b\right)^2}}{d}$$

$\Big\downarrow$ 402

$$\frac{\frac{-\frac{\int -\frac{2\left(2a^2+4ba-3b^2+3(2a-3b)b\cosh^2(c+dx)\right)}{\left(1-\cosh^2(c+dx)\right)\left(b\cosh^2(c+dx)+a-b\right)^2}d\cosh(c+dx)}{4a(a-b)}-\frac{b(2a-3b)\cosh(c+dx)}{2a(a-b)\left(a+b\cosh^2(c+dx)-b\right)^2}}{2a}+\frac{\cosh(c+dx)}{2a\left(1-\cosh^2(c+dx)\right)\left(a+b\cosh^2(c+dx)-b\right)^2}}{d}$$

$\Big\downarrow$ 27

$$\frac{\frac{\frac{\int \frac{2a^2+4ba-3b^2+3(2a-3b)b\cosh^2(c+dx)}{\left(1-\cosh^2(c+dx)\right)\left(b\cosh^2(c+dx)+a-b\right)^2}d\cosh(c+dx)}{2a(a-b)}-\frac{b(2a-3b)\cosh(c+dx)}{2a(a-b)\left(a+b\cosh^2(c+dx)-b\right)^2}}{2a}+\frac{\cosh(c+dx)}{2a\left(1-\cosh^2(c+dx)\right)\left(a+b\cosh^2(c+dx)-b\right)^2}}{d}$$

$\Big\downarrow$ 402

$$\frac{\frac{-\frac{\frac{\int -\frac{4a^3+12ba^2-25b^2a+12b^3+(a-4b)(4a-3b)b\cosh^2(c+dx)}{\left(1-\cosh^2(c+dx)\right)\left(b\cosh^2(c+dx)+a-b\right)}d\cosh(c+dx)}{2a(a-b)}-\frac{b(a-4b)(4a-3b)\cosh(c+dx)}{2a(a-b)\left(a+b\cosh^2(c+dx)-b\right)}}{2a(a-b)}-\frac{b(2a-3b)\cosh(c+dx)}{2a(a-b)\left(a+b\cosh^2(c+dx)-b\right)^2}}{2a}+\frac{\cdots}{2a(1-\cosh\cdots}}{d}$$

$\big\downarrow$ 25

$$\frac{\frac{\frac{\int \frac{4a^3+12ba^2-25b^2a+12b^3+(a-4b)(4a-3b)b\cosh^2(c+dx)}{\left(1-\cosh^2(c+dx)\right)\left(b\cosh^2(c+dx)+a-b\right)}d\cosh(c+dx)}{2a(a-b)}-\frac{b(a-4b)(4a-3b)\cosh(c+dx)}{2a(a-b)\left(a+b\cosh^2(c+dx)-b\right)}}{2a(a-b)}-\frac{b(2a-3b)\cosh(c+dx)}{2a(a-b)\left(a+b\cosh^2(c+dx)-b\right)^2}}{2a}+\frac{\cdots}{2a\left(1-\cosh^2(\cdots\right.}}{d}$$

$\big\downarrow$ 397

$$\frac{\frac{\frac{\frac{b^2\left(35a^2-56ab+24b^2\right)\int\frac{1}{b\cosh^2(c+dx)+a-b}d\cosh(c+dx)}{a}+\frac{4(a+6b)(a-b)^2\int\frac{1}{1-\cosh^2(c+dx)}d\cosh(c+dx)}{a}}{2a(a-b)}-\frac{b(a-4b)(4a-3b)\cosh(c+dx)}{2a(a-b)\left(a+b\cosh^2(c+dx)-b\right)}}{2a(a-b)}-\frac{b(2a-3b)\cdots}{2a(a-b)\left(a+b\cosh\cdots\right.}}{2a}\cdots}{d}$$

$\big\downarrow$ 218

$$\frac{\frac{\frac{\frac{4(a+6b)(a-b)^2\int\frac{1}{1-\cosh^2(c+dx)}d\cosh(c+dx)}{a}+\frac{b^{3/2}\left(35a^2-56ab+24b^2\right)\arctan\left(\frac{\sqrt{b}\cosh(c+dx)}{\sqrt{a-b}}\right)}{a\sqrt{a-b}}}{2a(a-b)}-\frac{b(a-4b)(4a-3b)\cosh(c+dx)}{2a(a-b)\left(a+b\cosh^2(c+dx)-b\right)}}{2a(a-b)}-\frac{b(2a-3b)\cosh(c+\cdots}{2a(a-b)\left(a+b\cosh^2(c+\cdots\right.}}{2a}\cdots}{d}$$

$\big\downarrow$ 219

$$\frac{\frac{\frac{\frac{b^{3/2}\left(35a^2-56ab+24b^2\right)\arctan\left(\frac{\sqrt{b}\cosh(c+dx)}{\sqrt{a-b}}\right)}{a\sqrt{a-b}}+\frac{4(a+6b)(a-b)^2\text{arctanh}(\cosh(c+dx))}{a}}{2a(a-b)}-\frac{b(a-4b)(4a-3b)\cosh(c+dx)}{2a(a-b)\left(a+b\cosh^2(c+dx)-b\right)}}{2a(a-b)}-\frac{b(2a-3b)\cosh(c+dx)}{2a(a-b)\left(a+b\cosh^2(c+dx)-b\right)\cdots}}{2a}\cdots}{d}$$

input

```
Int[Csch[c + d*x]^3/(a + b*Sinh[c + d*x]^2)^3,x]
```

output

```
(Cosh[c + d*x]/(2*a*(1 - Cosh[c + d*x]^2)*(a - b + b*Cosh[c + d*x]^2)^2) +
 (-1/2*((2*a - 3*b)*b*Cosh[c + d*x])/(a*(a - b)*(a - b + b*Cosh[c + d*x]^2
)^2) + (((b^(3/2)*(35*a^2 - 56*a*b + 24*b^2)*ArcTan[(Sqrt[b]*Cosh[c + d*x]
)/Sqrt[a - b]])/(a*Sqrt[a - b]) + (4*(a - b)^2*(a + 6*b)*ArcTanh[Cosh[c +
d*x]])/a)/(2*a*(a - b)) - ((a - 4*b)*(4*a - 3*b)*b*Cosh[c + d*x])/(2*a*(a
- b)*(a - b + b*Cosh[c + d*x]^2)))/(2*a*(a - b)))/(2*a))/d
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 316
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_), x_Symbol] :> Sim
p[(-b)*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^(q + 1)/(2*a*(p + 1)*(b*c - a*d))
), x] + Simp[1/(2*a*(p + 1)*(b*c - a*d))   Int[(a + b*x^2)^(p + 1)*(c + d*x
^2)^q*Simp[b*c + 2*(p + 1)*(b*c - a*d) + d*b*(2*(p + q + 2) + 1)*x^2, x], x
], x] /; FreeQ[{a, b, c, d, q}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] &&  !
( !IntegerQ[p] && IntegerQ[q] && LtQ[q, -1]) && IntBinomialQ[a, b, c, d, 2,
 p, q, x]
```

rule 397
```
Int[((e_) + (f_.)*(x_)^2)/(((a_) + (b_.)*(x_)^2)*((c_) + (d_.)*(x_)^2)), x_
Symbol] :> Simp[(b*e - a*f)/(b*c - a*d)   Int[1/(a + b*x^2), x], x] - Simp[
(d*e - c*f)/(b*c - a*d)   Int[1/(c + d*x^2), x], x] /; FreeQ[{a, b, c, d, e
, f}, x]
```

rule 402
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_.)*((e_) + (f_.)*(x
_)^2), x_Symbol] :> Simp[(-(b*e - a*f))*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^
(q + 1)/(a*2*(b*c - a*d)*(p + 1))), x] + Simp[1/(a*2*(b*c - a*d)*(p + 1))
 Int[(a + b*x^2)^(p + 1)*(c + d*x^2)^q*Simp[c*(b*e - a*f) + e*2*(b*c - a*d)
*(p + 1) + d*(b*e - a*f)*(2*(p + q + 2) + 1)*x^2, x], x], x] /; FreeQ[{a, b
, c, d, e, f, q}, x] && LtQ[p, -1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3665
```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^
(p_.), x_Symbol] :> With[{ff = FreeFactors[Cos[e + f*x], x]}, Simp[-ff/f
Subst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a + b - b*ff^2*x^2)^p, x], x, Cos[e +
 f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m - 1)/2]
```

Maple [A] (verified)

Time = 2.52 (sec) , antiderivative size = 350, normalized size of antiderivative = 1.56

method	result
derivativedivides	$\frac{\frac{\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^2}{8a^3}-\frac{1}{8a^3\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^2}+\frac{(-2a-12b)\ln\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{4a^4}+\frac{4b^2\left(\frac{-\frac{\left(13a^2-40ab+24b^2\right)a\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^6}{16\left(a^2-2ab+b^2\right)}+\frac{(39a^3-13}{}}{\left(\tanh\left(\frac{d}{}\right.\right.}\right.}{}}{}$
default	$\frac{\frac{\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^2}{8a^3}-\frac{1}{8a^3\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^2}+\frac{(-2a-12b)\ln\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{4a^4}+\frac{4b^2\left(\frac{-\frac{\left(13a^2-40ab+24b^2\right)a\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^6}{16\left(a^2-2ab+b^2\right)}+\frac{(39a^3-13}{}}{\left(\tanh\left(\frac{d}{}\right.\right.}\right.}{}}{}$
risch	$-\frac{e^{dx+c}\left(4a^2b^2e^{10dx+10c}-19ab^3e^{10dx+10c}+12e^{10dx+10c}b^4+32e^{8dx+8c}a^3b-128e^{8dx+8c}a^2b^2+129e^{8dx+8c}ab^3-36e\right.}{}$

input
```
int(csch(d*x+c)^3/(a+b*sinh(d*x+c)^2)^3,x,method=_RETURNVERBOSE)
```

output
```
1/d*(1/8*tanh(1/2*d*x+1/2*c)^2/a^3-1/8/a^3/tanh(1/2*d*x+1/2*c)^2+1/4/a^4*(
-2*a-12*b)*ln(tanh(1/2*d*x+1/2*c))+4*b^2/a^4*((-1/16*(13*a^2-40*a*b+24*b^2
)*a/(a^2-2*a*b+b^2)*tanh(1/2*d*x+1/2*c)^6+1/16*(39*a^3-134*a^2*b+184*a*b^2
-80*b^3)/(a^2-2*a*b+b^2)*tanh(1/2*d*x+1/2*c)^4-1/16*a*(39*a^2-104*a*b+56*b
^2)/(a^2-2*a*b+b^2)*tanh(1/2*d*x+1/2*c)^2+1/16*a^2*(13*a-10*b)/(a^2-2*a*b+
b^2))/(tanh(1/2*d*x+1/2*c)^4*a-2*tanh(1/2*d*x+1/2*c)^2*a+4*b*tanh(1/2*d*x+
1/2*c)^2+a)^2+1/32*(35*a^2-56*a*b+24*b^2)/(a^2-2*a*b+b^2)/(a*b-b^2)^(1/2)*
arctan(1/4*(2*tanh(1/2*d*x+1/2*c)^2*a-2*a+4*b)/(a*b-b^2)^(1/2))))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 12128 vs. $2(206) = 412$.

Time = 0.38 (sec) , antiderivative size = 22563, normalized size of antiderivative = 100.73

$$\int \frac{\operatorname{csch}^3(c+dx)}{(a+b\sinh^2(c+dx))^3}\,dx = \text{Too large to display}$$

input
```
integrate(csch(d*x+c)^3/(a+b*sinh(d*x+c)^2)^3,x, algorithm="fricas")
```

output
```
Too large to include
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\operatorname{csch}^3(c+dx)}{(a+b\sinh^2(c+dx))^3}\,dx = \text{Timed out}$$

input
```
integrate(csch(d*x+c)**3/(a+b*sinh(d*x+c)**2)**3,x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{\operatorname{csch}^3(c+dx)}{\left(a+b\sinh^2(c+dx)\right)^3}\,dx = \int \frac{\operatorname{csch}\left(dx+c\right)^3}{\left(b\sinh\left(dx+c\right)^2+a\right)^3}\,dx$$

input
```
integrate(csch(d*x+c)^3/(a+b*sinh(d*x+c)^2)^3,x, algorithm="maxima")
```

output
```
-1/4*((4*a^2*b^2*e^(11*c) - 19*a*b^3*e^(11*c) + 12*b^4*e^(11*c))*e^(11*d*x
) + (32*a^3*b*e^(9*c) - 128*a^2*b^2*e^(9*c) + 129*a*b^3*e^(9*c) - 36*b^4*e
^(9*c))*e^(9*d*x) + 2*(32*a^4*e^(7*c) - 80*a^3*b*e^(7*c) + 94*a^2*b^2*e^(7
*c) - 55*a*b^3*e^(7*c) + 12*b^4*e^(7*c))*e^(7*d*x) + 2*(32*a^4*e^(5*c) - 8
0*a^3*b*e^(5*c) + 94*a^2*b^2*e^(5*c) - 55*a*b^3*e^(5*c) + 12*b^4*e^(5*c))*
e^(5*d*x) + (32*a^3*b*e^(3*c) - 128*a^2*b^2*e^(3*c) + 129*a*b^3*e^(3*c) -
36*b^4*e^(3*c))*e^(3*d*x) + (4*a^2*b^2*e^c - 19*a*b^3*e^c + 12*b^4*e^c)*e^
(d*x))/(a^5*b^2*d - 2*a^4*b^3*d + a^3*b^4*d + (a^5*b^2*d*e^(12*c) - 2*a^4*
b^3*d*e^(12*c) + a^3*b^4*d*e^(12*c))*e^(12*d*x) + 2*(4*a^6*b*d*e^(10*c) -
11*a^5*b^2*d*e^(10*c) + 10*a^4*b^3*d*e^(10*c) - 3*a^3*b^4*d*e^(10*c))*e^(1
0*d*x) + (16*a^7*d*e^(8*c) - 64*a^6*b*d*e^(8*c) + 95*a^5*b^2*d*e^(8*c) - 6
2*a^4*b^3*d*e^(8*c) + 15*a^3*b^4*d*e^(8*c))*e^(8*d*x) - 4*(8*a^7*d*e^(6*c)
 - 28*a^6*b*d*e^(6*c) + 37*a^5*b^2*d*e^(6*c) - 22*a^4*b^3*d*e^(6*c) + 5*a^
3*b^4*d*e^(6*c))*e^(6*d*x) + (16*a^7*d*e^(4*c) - 64*a^6*b*d*e^(4*c) + 95*a
^5*b^2*d*e^(4*c) - 62*a^4*b^3*d*e^(4*c) + 15*a^3*b^4*d*e^(4*c))*e^(4*d*x)
+ 2*(4*a^6*b*d*e^(2*c) - 11*a^5*b^2*d*e^(2*c) + 10*a^4*b^3*d*e^(2*c) - 3*a
^3*b^4*d*e^(2*c))*e^(2*d*x)) + 1/2*(a + 6*b)*log((e^(d*x + c) + 1)*e^(-c))
/(a^4*d) - 1/2*(a + 6*b)*log((e^(d*x + c) - 1)*e^(-c))/(a^4*d) + 8*integra
te(1/32*((35*a^2*b^2*e^(3*c) - 56*a*b^3*e^(3*c) + 24*b^4*e^(3*c))*e^(3*d*x
) - (35*a^2*b^2*e^c - 56*a*b^3*e^c + 24*b^4*e^c)*e^(d*x))/(a^6*b - 2*a^...
```

Giac [F(-2)]

Exception generated.

$$\int \frac{\operatorname{csch}^3(c+dx)}{\left(a+b\sinh^2(c+dx)\right)^3}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(csch(d*x+c)^3/(a+b*sinh(d*x+c)^2)^3,x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Limit: Max order reached or unable
to make series expansion Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\operatorname{csch}^3(c+dx)}{\left(a+b\sinh^2(c+dx)\right)^3}\,dx = \int \frac{1}{\sinh\left(c+d\,x\right)^3\,\left(b\sinh\left(c+d\,x\right)^2+a\right)^3}\,dx$$

input
```
int(1/(sinh(c + d*x)^3*(a + b*sinh(c + d*x)^2)^3),x)
```

output
```
int(1/(sinh(c + d*x)^3*(a + b*sinh(c + d*x)^2)^3), x)
```

Reduce [B] (verification not implemented)

Time = 1.64 (sec) , antiderivative size = 16558, normalized size of antiderivative = 73.92

$$\int \frac{\operatorname{csch}^3(c+dx)}{\left(a+b\sinh^2(c+dx)\right)^3}\,dx = \text{Too large to display}$$

input
```
int(csch(d*x+c)^3/(a+b*sinh(d*x+c)^2)^3,x)
```

output
```
(70*e**(12*c + 12*d*x)*sqrt(b)*sqrt(a)*sqrt(a - b)*sqrt(2*sqrt(a)*sqrt(a -
 b) + 2*a - b)*atan((e**(c + d*x)*b)/(sqrt(b)*sqrt(2*sqrt(a)*sqrt(a - b) +
 2*a - b)))*a**2*b**2 - 112*e**(12*c + 12*d*x)*sqrt(b)*sqrt(a)*sqrt(a - b)
*sqrt(2*sqrt(a)*sqrt(a - b) + 2*a - b)*atan((e**(c + d*x)*b)/(sqrt(b)*sqrt
(2*sqrt(a)*sqrt(a - b) + 2*a - b)))*a*b**3 + 48*e**(12*c + 12*d*x)*sqrt(b)
*sqrt(a)*sqrt(a - b)*sqrt(2*sqrt(a)*sqrt(a - b) + 2*a - b)*atan((e**(c + d
*x)*b)/(sqrt(b)*sqrt(2*sqrt(a)*sqrt(a - b) + 2*a - b)))*b**4 + 560*e**(10*
c + 10*d*x)*sqrt(b)*sqrt(a)*sqrt(a - b)*sqrt(2*sqrt(a)*sqrt(a - b) + 2*a -
 b)*atan((e**(c + d*x)*b)/(sqrt(b)*sqrt(2*sqrt(a)*sqrt(a - b) + 2*a - b)))
*a**3*b - 1316*e**(10*c + 10*d*x)*sqrt(b)*sqrt(a)*sqrt(a - b)*sqrt(2*sqrt(
a)*sqrt(a - b) + 2*a - b)*atan((e**(c + d*x)*b)/(sqrt(b)*sqrt(2*sqrt(a)*sq
rt(a - b) + 2*a - b)))*a**2*b**2 + 1056*e**(10*c + 10*d*x)*sqrt(b)*sqrt(a)
*sqrt(a - b)*sqrt(2*sqrt(a)*sqrt(a - b) + 2*a - b)*atan((e**(c + d*x)*b)/(
sqrt(b)*sqrt(2*sqrt(a)*sqrt(a - b) + 2*a - b)))*a*b**3 - 288*e**(10*c + 10
*d*x)*sqrt(b)*sqrt(a)*sqrt(a - b)*sqrt(2*sqrt(a)*sqrt(a - b) + 2*a - b)*at
an((e**(c + d*x)*b)/(sqrt(b)*sqrt(2*sqrt(a)*sqrt(a - b) + 2*a - b)))*b**4
+ 1120*e**(8*c + 8*d*x)*sqrt(b)*sqrt(a)*sqrt(a - b)*sqrt(2*sqrt(a)*sqrt(a
- b) + 2*a - b)*atan((e**(c + d*x)*b)/(sqrt(b)*sqrt(2*sqrt(a)*sqrt(a - b)
+ 2*a - b)))*a**4 - 4032*e**(8*c + 8*d*x)*sqrt(b)*sqrt(a)*sqrt(a - b)*sqrt
(2*sqrt(a)*sqrt(a - b) + 2*a - b)*atan((e**(c + d*x)*b)/(sqrt(b)*sqrt(2...
```

3.59 $\int \frac{\operatorname{csch}^4(c+dx)}{\left(a+b\sinh^2(c+dx)\right)^3}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 195

$$\int \frac{\operatorname{csch}^4(c+dx)}{\left(a+b\sinh^2(c+dx)\right)^3}\,dx = \frac{b^2(48a^2-80ab+35b^2)\operatorname{arctanh}\left(\frac{\sqrt{a-b}\tanh(c+dx)}{\sqrt{a}}\right)}{8a^{9/2}(a-b)^{5/2}d} + \frac{(a+3b)\coth(c+dx)}{a^4d} - \frac{\coth^3(c+dx)}{3a^3d} + \frac{b^4\tanh(c+dx)}{4a^3(a-b)^2d\left(a-(a-b)\tanh^2(c+dx)\right)^2} - \frac{(16a-11b)b^3\tanh(c+dx)}{8a^4(a-b)^2d\left(a-(a-b)\tanh^2(c+dx)\right)}$$

output
```
1/8*b^2*(48*a^2-80*a*b+35*b^2)*arctanh((a-b)^(1/2)*tanh(d*x+c)/a^(1/2))/a^
(9/2)/(a-b)^(5/2)/d+(a+3*b)*coth(d*x+c)/a^4/d-1/3*coth(d*x+c)^3/a^3/d+1/4*
b^4*tanh(d*x+c)/a^3/(a-b)^2/d/(a-(a-b)*tanh(d*x+c)^2)^2-1/8*(16*a-11*b)*b^
3*tanh(d*x+c)/a^4/(a-b)^2/d/(a-(a-b)*tanh(d*x+c)^2)
```

Mathematica [A] (verified)

Time = 7.06 (sec) , antiderivative size = 167, normalized size of antiderivative = 0.86

$$\int \frac{\operatorname{csch}^4(c+dx)}{\left(a+b\sinh^2(c+dx)\right)^3}\,dx$$
$$=\frac{\frac{3b^2(48a^2-80ab+35b^2)\operatorname{arctanh}\left(\frac{\sqrt{a-b}\tanh(c+dx)}{\sqrt{a}}\right)}{(a-b)^{5/2}}+\sqrt{a}\Big(-8\coth(c+dx)\left(-2a-9b+a\operatorname{csch}^2(c+dx)\right)+\frac{3b^3(-32a^2}{}}{24a^{9/2}d}$$

input
```
Integrate[Csch[c + d*x]^4/(a + b*Sinh[c + d*x]^2)^3,x]
```

output
```
((3*b^2*(48*a^2 - 80*a*b + 35*b^2)*ArcTanh[(Sqrt[a - b]*Tanh[c + d*x])/Sqr
t[a]])/(a - b)^(5/2) + Sqrt[a]*(-8*Coth[c + d*x]*(-2*a - 9*b + a*Csch[c +
d*x]^2) + (3*b^3*(-32*a^2 + 40*a*b - 11*b^2 + b*(-14*a + 11*b)*Cosh[2*(c +
 d*x)])*Sinh[2*(c + d*x)])/((a - b)^2*(2*a - b + b*Cosh[2*(c + d*x)])^2)))
/(24*a^(9/2)*d)
```

Rubi [A] (verified)

Time = 0.56 (sec) , antiderivative size = 272, normalized size of antiderivative = 1.39, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.261, Rules used = {3042, 3666, 370, 439, 437, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\operatorname{csch}^4(c+dx)}{\left(a+b\sinh^2(c+dx)\right)^3}\,dx$$
$$\Big\downarrow \text{3042}$$
$$\int \frac{1}{\sin(ic+idx)^4\left(a-b\sin(ic+idx)^2\right)^3}dx$$
$$\Big\downarrow \text{3666}$$
$$\frac{\int \frac{\coth^4(c+dx)\left(1-\tanh^2(c+dx)\right)^4}{\left(a-(a-b)\tanh^2(c+dx)\right)^3}d\tanh(c+dx)}{d}$$

$\Big\downarrow$ 370

$$\frac{\frac{\int \frac{\coth^4(c+dx)\left(1-\tanh^2(c+dx)\right)^2\left(-\left((4a-b)\tanh^2(c+dx)\right)+4a-7b\right)}{\left(a-(a-b)\tanh^2(c+dx)\right)^2}d\tanh(c+dx)}{4a(a-b)}-\frac{b\left(1-\tanh^2(c+dx)\right)^3\coth^3(c+dx)}{4a(a-b)\left(a-(a-b)\tanh^2(c+dx)\right)^2}}{d}$$

$\Big\downarrow$ 439

$$\frac{\frac{\frac{\int \frac{\coth^4(c+dx)\left(1-\tanh^2(c+dx)\right)\left(8a^2-52ba+35b^2-\left(8a^2-12ba+7b^2\right)\tanh^2(c+dx)\right)}{a-(a-b)\tanh^2(c+dx)}d\tanh(c+dx)}{2a(a-b)}-\frac{b(10a-7b)\left(1-\tanh^2(c+dx)\right)^2\coth^3(c+dx)}{2a(a-b)\left(a-(a-b)\tanh^2(c+dx)\right)}}{4a(a-b)}-\frac{b(1-\tanh}{4a(a-b)(}}{d}$$

$\Big\downarrow$ 437

$$\frac{\frac{\frac{\int \left(\frac{\left(8a^2-52ba+35b^2\right)\coth^4(c+dx)}{a}+\frac{\left(-8a^3+4ba^2+45b^2a-35b^3\right)\coth^2(c+dx)}{a^2}+\frac{b^2\left(48a^2-80ba+35b^2\right)}{a^2\left(a-(a-b)\tanh^2(c+dx)\right)}\right)d\tanh(c+dx)}{2a(a-b)}-\frac{b(10a-7b)\left(1-\tanh^2(c+dx)\right)^2\cot}{2a(a-b)\left(a-(a-b)\tanh^2(c+\right.}}{4a(a-b)}}{d}$$

$\Big\downarrow$ 2009

$$\frac{\frac{\frac{-\frac{\left(8a^2-52ab+35b^2\right)\coth^3(c+dx)}{3a}+\frac{b^2\left(48a^2-80ab+35b^2\right)\text{arctanh}\left(\frac{\sqrt{a-b}\tanh(c+dx)}{\sqrt{a}}\right)}{a^{5/2}\sqrt{a-b}}+\frac{\left(8a^3-4a^2b-45ab^2+35b^3\right)\coth(c+dx)}{a^2}}{2a(a-b)}-\frac{b(10a-7b)\left(1-\tanh^2(c+\right.}{2a(a-b)\left(a-(a-b)\right.}}{4a(a-b)}}{d}$$

input
```
Int[Csch[c + d*x]^4/(a + b*Sinh[c + d*x]^2)^3,x]
```

output
```
(-1/4*(b*Coth[c + d*x]^3*(1 - Tanh[c + d*x]^2)^3)/(a*(a - b)*(a - (a - b)*
Tanh[c + d*x]^2)^2) + (((b^2*(48*a^2 - 80*a*b + 35*b^2)*ArcTanh[(Sqrt[a -
b]*Tanh[c + d*x])/Sqrt[a]])/(a^(5/2)*Sqrt[a - b]) + ((8*a^3 - 4*a^2*b - 45
*a*b^2 + 35*b^3)*Coth[c + d*x])/a^2 - ((8*a^2 - 52*a*b + 35*b^2)*Coth[c +
d*x]^3)/(3*a))/(2*a*(a - b)) - ((10*a - 7*b)*b*Coth[c + d*x]^3*(1 - Tanh[c
 + d*x]^2)^2)/(2*a*(a - b)*(a - (a - b)*Tanh[c + d*x]^2)))/(4*a*(a - b)))/
d
```

Defintions of rubi rules used

rule 370
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_
), x_Symbol] :> Simp[(-(b*c - a*d))*(e*x)^(m + 1)*(a + b*x^2)^(p + 1)*((c +
 d*x^2)^(q - 1)/(a*b*e*2*(p + 1))), x] + Simp[1/(a*b*2*(p + 1))   Int[(e*x)
^m*(a + b*x^2)^(p + 1)*(c + d*x^2)^(q - 2)*Simp[c*(b*c*2*(p + 1) + (b*c - a
*d)*(m + 1)) + d*(b*c*2*(p + 1) + (b*c - a*d)*(m + 2*(q - 1) + 1))*x^2, x],
 x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1]
&& GtQ[q, 1] && IntBinomialQ[a, b, c, d, e, m, 2, p, q, x]
```

rule 437
```
Int[((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q
_.)*((e_) + (f_.)*(x_)^2)^(r_.), x_Symbol] :> Int[ExpandIntegrand[(g*x)^m*(
a + b*x^2)^p*(c + d*x^2)^q*(e + f*x^2)^r, x], x] /; FreeQ[{a, b, c, d, e, f
, g, m}, x] && IGtQ[p, -2] && IGtQ[q, 0] && IGtQ[r, 0]
```

rule 439
```
Int[((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_
.)*((e_) + (f_.)*(x_)^2), x_Symbol] :> Simp[(-(b*e - a*f))*(g*x)^(m + 1)*(a
 + b*x^2)^(p + 1)*((c + d*x^2)^q/(2*a*b*g*(p + 1))), x] + Simp[1/(2*a*b*(p
+ 1))   Int[(g*x)^m*(a + b*x^2)^(p + 1)*(c + d*x^2)^(q - 1)*Simp[c*(2*b*e*(
p + 1) + (b*e - a*f)*(m + 1)) + d*(2*b*e*(p + 1) + (b*e - a*f)*(m + 2*q + 1
))*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, m}, x] && LtQ[p, -1] && G
tQ[q, 0] &&  !(EqQ[q, 1] && SimplerQ[b*c - a*d, b*e - a*f])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3666
```
Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(
p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Simp[ff^(m + 1
)/f   Subst[Int[x^m*((a + (a + b)*ff^2*x^2)^p/(1 + ff^2*x^2)^(m/2 + p + 1))
, x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f}, x] && IntegerQ[m/2] &
& IntegerQ[p]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 496 vs. $2(179) = 358$.

Time = 3.17 (sec) , antiderivative size = 497, normalized size of antiderivative = 2.55

method	result
derivativedivides	$\frac{-\frac{\frac{\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^{3}a}{3}-3\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)a-12b\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)}{8a^{4}}-\frac{1}{24a^{3}\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^{3}}-\frac{-3a-12b}{8a^{4}\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)}-\frac{4b^{2}\left(\frac{\frac{ab(16a-13b)\tanh\left(\frac{d}{\ldots}\right.}{16a^{2}-32ab+16\ldots}}{\ldots}\right.}{\ldots}}{\ldots}$
default	$\frac{-\frac{\frac{\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^{3}a}{3}-3\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)a-12b\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)}{8a^{4}}-\frac{1}{24a^{3}\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^{3}}-\frac{-3a-12b}{8a^{4}\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)}-\frac{4b^{2}\left(\frac{\frac{ab(16a-13b)\tanh\left(\frac{d}{\ldots}\right.}{16a^{2}-32ab+16\ldots}}{\ldots}\right.}{\ldots}}{\ldots}$
risch	Expression too large to display

input
```
int(csch(d*x+c)^4/(a+b*sinh(d*x+c)^2)^3,x,method=_RETURNVERBOSE)
```

output
```
1/d*(-1/8/a^4*(1/3*tanh(1/2*d*x+1/2*c)^3*a-3*tanh(1/2*d*x+1/2*c)*a-12*b*ta
nh(1/2*d*x+1/2*c))-1/24/a^3/tanh(1/2*d*x+1/2*c)^3-1/8/a^4*(-3*a-12*b)/tanh
(1/2*d*x+1/2*c)-4*b^2/a^4*((1/16*a*b*(16*a-13*b)/(a^2-2*a*b+b^2)*tanh(1/2*
d*x+1/2*c)^7-1/16*(16*a^2-69*a*b+44*b^2)*b/(a^2-2*a*b+b^2)*tanh(1/2*d*x+1/
2*c)^5-1/16*(16*a^2-69*a*b+44*b^2)*b/(a^2-2*a*b+b^2)*tanh(1/2*d*x+1/2*c)^3
+1/16*a*b*(16*a-13*b)/(a^2-2*a*b+b^2)*tanh(1/2*d*x+1/2*c))/(tanh(1/2*d*x+1
/2*c)^4*a-2*tanh(1/2*d*x+1/2*c)^2*a+4*b*tanh(1/2*d*x+1/2*c)^2+a)^2+1/16*(4
8*a^2-80*a*b+35*b^2)/(a^2-2*a*b+b^2)*a*(1/2*((-b*(a-b))^(1/2)+b)/a/(-b*(a-
b))^(1/2)/((2*(-b*(a-b))^(1/2)-a+2*b)*a)^(1/2)*arctan(tanh(1/2*d*x+1/2*c)*
a/((2*(-b*(a-b))^(1/2)-a+2*b)*a)^(1/2))-1/2*((-b*(a-b))^(1/2)-b)/a/(-b*(a-
b))^(1/2)/((2*(-b*(a-b))^(1/2)+a-2*b)*a)^(1/2)*arctanh(tanh(1/2*d*x+1/2*c)
*a/((2*(-b*(a-b))^(1/2)+a-2*b)*a)^(1/2)))))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 8519 vs. $2(181) = 362$.

Time = 0.28 (sec) , antiderivative size = 17294, normalized size of antiderivative = 88.69

$$\int \frac{\operatorname{csch}^4(c+dx)}{(a+b\sinh^2(c+dx))^3}\,dx = \text{Too large to display}$$

input
```
integrate(csch(d*x+c)^4/(a+b*sinh(d*x+c)^2)^3,x, algorithm="fricas")
```

output
```
Too large to include
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\operatorname{csch}^4(c+dx)}{(a+b\sinh^2(c+dx))^3}\,dx = \text{Timed out}$$

input
```
integrate(csch(d*x+c)**4/(a+b*sinh(d*x+c)**2)**3,x)
```

output
```
Timed out
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{\operatorname{csch}^4(c+dx)}{(a+b\sinh^2(c+dx))^3}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate(csch(d*x+c)^4/(a+b*sinh(d*x+c)^2)^3,x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(b-a>0)', see `assume?` for more
details)Is
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 378 vs. $2(181) = 362$.

Time = 0.39 (sec) , antiderivative size = 378, normalized size of antiderivative = 1.94

$$\int \frac{\operatorname{csch}^4(c+dx)}{(a+b\sinh^2(c+dx))^3}\,dx = \frac{\frac{3\,(48\,a^2b^2-80\,ab^3+35\,b^4)\arctan\left(\frac{be^{(2\,dx+2\,c)}+2\,a-b}{2\,\sqrt{-a^2+ab}}\right)}{(a^6-2\,a^5b+a^4b^2)\sqrt{-a^2+ab}} + \frac{6\,(24\,a^2b^3e^{(6\,dx+6\,c)}-32\,ab^4e^{(6\,dx+6\,c)}+11\,b^5e^{(6\,dx+6\,c)}+112\,a^3b^2e^{(4\,dx+4\,c)}-200\,a}{(a^6-2\,a^5b+a}}{}$$

input
```
integrate(csch(d*x+c)^4/(a+b*sinh(d*x+c)^2)^3,x, algorithm="giac")
```

output
```
1/24*(3*(48*a^2*b^2 - 80*a*b^3 + 35*b^4)*arctan(1/2*(b*e^(2*d*x + 2*c) + 2
*a - b)/sqrt(-a^2 + a*b))/((a^6 - 2*a^5*b + a^4*b^2)*sqrt(-a^2 + a*b)) + 6
*(24*a^2*b^3*e^(6*d*x + 6*c) - 32*a*b^4*e^(6*d*x + 6*c) + 11*b^5*e^(6*d*x
+ 6*c) + 112*a^3*b^2*e^(4*d*x + 4*c) - 200*a^2*b^3*e^(4*d*x + 4*c) + 130*a
*b^4*e^(4*d*x + 4*c) - 33*b^5*e^(4*d*x + 4*c) + 88*a^2*b^3*e^(2*d*x + 2*c)
 - 112*a*b^4*e^(2*d*x + 2*c) + 33*b^5*e^(2*d*x + 2*c) + 14*a*b^4 - 11*b^5)
/((a^6 - 2*a^5*b + a^4*b^2)*(b*e^(4*d*x + 4*c) + 4*a*e^(2*d*x + 2*c) - 2*b
*e^(2*d*x + 2*c) + b)^2) + 16*(9*b*e^(4*d*x + 4*c) - 6*a*e^(2*d*x + 2*c) -
 18*b*e^(2*d*x + 2*c) + 2*a + 9*b)/(a^4*(e^(2*d*x + 2*c) - 1)^3))/d
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\operatorname{csch}^4(c+dx)}{\left(a+b\sinh^2(c+dx)\right)^3}\,dx = \int \frac{1}{\sinh\left(c+d\,x\right)^4\,\left(b\sinh\left(c+d\,x\right)^2+a\right)^3}\,dx$$

input
```
int(1/(sinh(c + d*x)^4*(a + b*sinh(c + d*x)^2)^3),x)
```

output
```
int(1/(sinh(c + d*x)^4*(a + b*sinh(c + d*x)^2)^3), x)
```

Reduce [B] (verification not implemented)

Time = 0.74 (sec) , antiderivative size = 8270, normalized size of antiderivative = 42.41

$$\int \frac{\operatorname{csch}^4(c+dx)}{\left(a+b\sinh^2(c+dx)\right)^3}\,dx = \text{Too large to display}$$

input
```
int(csch(d*x+c)^4/(a+b*sinh(d*x+c)^2)^3,x)
```

output
```
(1152*e**(14*c + 14*d*x)*sqrt(a)*sqrt(a - b)*log( - sqrt(2*sqrt(a)*sqrt(a
- b) - 2*a + b) + e**(c + d*x)*sqrt(b))*a**3*b**4 - 2928*e**(14*c + 14*d*x
)*sqrt(a)*sqrt(a - b)*log( - sqrt(2*sqrt(a)*sqrt(a - b) - 2*a + b) + e**(c
 + d*x)*sqrt(b))*a**2*b**5 + 2520*e**(14*c + 14*d*x)*sqrt(a)*sqrt(a - b)*l
og( - sqrt(2*sqrt(a)*sqrt(a - b) - 2*a + b) + e**(c + d*x)*sqrt(b))*a*b**6
 - 735*e**(14*c + 14*d*x)*sqrt(a)*sqrt(a - b)*log( - sqrt(2*sqrt(a)*sqrt(a
 - b) - 2*a + b) + e**(c + d*x)*sqrt(b))*b**7 + 1152*e**(14*c + 14*d*x)*sq
rt(a)*sqrt(a - b)*log(sqrt(2*sqrt(a)*sqrt(a - b) - 2*a + b) + e**(c + d*x)
*sqrt(b))*a**3*b**4 - 2928*e**(14*c + 14*d*x)*sqrt(a)*sqrt(a - b)*log(sqrt
(2*sqrt(a)*sqrt(a - b) - 2*a + b) + e**(c + d*x)*sqrt(b))*a**2*b**5 + 2520
*e**(14*c + 14*d*x)*sqrt(a)*sqrt(a - b)*log(sqrt(2*sqrt(a)*sqrt(a - b) - 2
*a + b) + e**(c + d*x)*sqrt(b))*a*b**6 - 735*e**(14*c + 14*d*x)*sqrt(a)*sq
rt(a - b)*log(sqrt(2*sqrt(a)*sqrt(a - b) - 2*a + b) + e**(c + d*x)*sqrt(b)
)*b**7 - 1152*e**(14*c + 14*d*x)*sqrt(a)*sqrt(a - b)*log(2*sqrt(a)*sqrt(a
- b) + e**(2*c + 2*d*x)*b + 2*a - b)*a**3*b**4 + 2928*e**(14*c + 14*d*x)*s
qrt(a)*sqrt(a - b)*log(2*sqrt(a)*sqrt(a - b) + e**(2*c + 2*d*x)*b + 2*a -
b)*a**2*b**5 - 2520*e**(14*c + 14*d*x)*sqrt(a)*sqrt(a - b)*log(2*sqrt(a)*s
qrt(a - b) + e**(2*c + 2*d*x)*b + 2*a - b)*a*b**6 + 735*e**(14*c + 14*d*x)
*sqrt(a)*sqrt(a - b)*log(2*sqrt(a)*sqrt(a - b) + e**(2*c + 2*d*x)*b + 2*a
- b)*b**7 + 9216*e**(12*c + 12*d*x)*sqrt(a)*sqrt(a - b)*log( - sqrt(2*s...
```

3.60 $\int \sinh^3(e+fx)\sqrt{a+b\sinh^2(e+fx)}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 130

$$\begin{aligned}\int \sinh^3(e+fx)\sqrt{a+b\sinh^2(e+fx)}\,dx &= -\frac{(a-b)(a+3b)\text{arctanh}\left(\frac{\sqrt{b}\cosh(e+fx)}{\sqrt{a-b+b\cosh^2(e+fx)}}\right)}{8b^{3/2}f} \\ &\quad -\frac{(a+3b)\cosh(e+fx)\sqrt{a-b+b\cosh^2(e+fx)}}{8bf} \\ &\quad +\frac{\cosh(e+fx)\left(a-b+b\cosh^2(e+fx)\right)^{3/2}}{4bf}\end{aligned}$$

output
```
-1/8*(a-b)*(a+3*b)*arctanh(b^(1/2)*cosh(f*x+e)/(a-b+b*cosh(f*x+e)^2)^(1/2)
)/b^(3/2)/f-1/8*(a+3*b)*cosh(f*x+e)*(a-b+b*cosh(f*x+e)^2)^(1/2)/b/f+1/4*co
sh(f*x+e)*(a-b+b*cosh(f*x+e)^2)^(3/2)/b/f
```

Mathematica [A] (verified)

Time = 0.68 (sec) , antiderivative size = 114, normalized size of antiderivative = 0.88

$$\int \sinh^3(e+fx)\sqrt{a+b\sinh^2(e+fx)}\,dx$$
$$=\frac{\frac{\cosh(e+fx)(a-4b+b\cosh(2(e+fx)))\sqrt{4a-2b+2b\cosh(2(e+fx))}}{2b}+\frac{(-a+b)(a+3b)\log\left(\sqrt{2}\sqrt{b}\cosh(e+fx)+\sqrt{2a-b+b\cosh(2(e+fx))}\right)}{b^{3/2}}}{8f}$$

input
```
Integrate[Sinh[e + f*x]^3*Sqrt[a + b*Sinh[e + f*x]^2],x]
```

output
```
((Cosh[e + f*x]*(a - 4*b + b*Cosh[2*(e + f*x)])*Sqrt[4*a - 2*b + 2*b*Cosh[
2*(e + f*x)]])/(2*b) + ((-a + b)*(a + 3*b)*Log[Sqrt[2]*Sqrt[b]*Cosh[e + f*
x] + Sqrt[2*a - b + b*Cosh[2*(e + f*x)]]])/b^(3/2))/(8*f)
```

Rubi [A] (verified)

Time = 0.32 (sec) , antiderivative size = 126, normalized size of antiderivative = 0.97, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.280$, Rules used = {3042, 26, 3665, 299, 211, 224, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sinh^3(e+fx)\sqrt{a+b\sinh^2(e+fx)}\,dx$$
$$\Big\downarrow \text{3042}$$
$$\int i\sin(ie+ifx)^3\sqrt{a-b\sin(ie+ifx)^2}dx$$
$$\Big\downarrow \text{26}$$
$$i\int \sin(ie+ifx)^3\sqrt{a-b\sin(ie+ifx)^2}dx$$
$$\Big\downarrow \text{3665}$$
$$-\frac{\int (1-\cosh^2(e+fx))\sqrt{b\cosh^2(e+fx)+a-b}d\cosh(e+fx)}{f}$$

$$\Big\downarrow \; 299$$

$$-\frac{\frac{(a+3b)\int \sqrt{b\cosh^2(e+fx)+a-b}d\cosh(e+fx)}{4b}-\frac{\cosh(e+fx)\left(a+b\cosh^2(e+fx)-b\right)^{3/2}}{4b}}{f}$$

$$\Big\downarrow \; 211$$

$$-\frac{\frac{(a+3b)\left(\frac{1}{2}(a-b)\int \frac{1}{\sqrt{b\cosh^2(e+fx)+a-b}}d\cosh(e+fx)+\frac{1}{2}\cosh(e+fx)\sqrt{a+b\cosh^2(e+fx)-b}\right)}{4b}-\frac{\cosh(e+fx)\left(a+b\cosh^2(e+fx)-b\right)^{3/2}}{4b}}{f}$$

$$\Big\downarrow \; 224$$

$$-\frac{\frac{(a+3b)\left(\frac{1}{2}(a-b)\int \frac{1}{1-\frac{b\cosh^2(e+fx)}{b\cosh^2(e+fx)+a-b}}d\frac{\cosh(e+fx)}{\sqrt{b\cosh^2(e+fx)+a-b}}+\frac{1}{2}\cosh(e+fx)\sqrt{a+b\cosh^2(e+fx)-b}\right)}{4b}-\frac{\cosh(e+fx)\left(a+b\cosh^2(e+fx)-b\right.}{4b}}{f}$$

$$\Big\downarrow \; 219$$

$$-\frac{\frac{(a+3b)\left(\frac{(a-b)\text{arctanh}\left(\frac{\sqrt{b}\cosh(e+fx)}{\sqrt{a+b\cosh^2(e+fx)-b}}\right)}{2\sqrt{b}}+\frac{1}{2}\cosh(e+fx)\sqrt{a+b\cosh^2(e+fx)-b}\right)}{4b}-\frac{\cosh(e+fx)\left(a+b\cosh^2(e+fx)-b\right)^{3/2}}{4b}}{f}$$

input
```
Int[Sinh[e + f*x]^3*Sqrt[a + b*Sinh[e + f*x]^2],x]
```

output
```
-((-1/4*(Cosh[e + f*x]*(a - b + b*Cosh[e + f*x]^2)^(3/2))/b + ((a + 3*b)*(
((a - b)*ArcTanh[(Sqrt[b]*Cosh[e + f*x])/Sqrt[a - b + b*Cosh[e + f*x]^2]])
/(2*Sqrt[b]) + (Cosh[e + f*x]*Sqrt[a - b + b*Cosh[e + f*x]^2])/2))/(4*b))/
f)
```

Defintions of rubi rules used

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 211
```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[x*((a + b*x^2)^p/(2*p + 1
)), x] + Simp[2*a*(p/(2*p + 1))   Int[(a + b*x^2)^(p - 1), x], x] /; FreeQ[
{a, b}, x] && GtQ[p, 0] && (IntegerQ[4*p] || IntegerQ[6*p])
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

rule 299
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2), x_Symbol] :> Simp[d*x
*((a + b*x^2)^(p + 1)/(b*(2*p + 3))), x] - Simp[(a*d - b*c*(2*p + 3))/(b*(2
*p + 3))   Int[(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
 a*d, 0] && NeQ[2*p + 3, 0]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3665
```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^
(p_.), x_Symbol] :> With[{ff = FreeFactors[Cos[e + f*x], x]}, Simp[-ff/f
Subst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a + b - b*ff^2*x^2)^p, x], x, Cos[e +
 f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m - 1)/2]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 338 vs. $2(114) = 228$.

Time = 0.47 (sec) , antiderivative size = 339, normalized size of antiderivative = 2.61

method	result
default	$\frac{\sqrt{\left(a+b\sinh(fx+e)^2\right)\cosh(fx+e)^2}\left(4b^{\frac{5}{2}}\sqrt{b\cosh(fx+e)^4+(a-b)\cosh(fx+e)^2}\,\cosh(fx+e)^2-10\sqrt{b\cosh(fx+e)^4+(a-b)\cosh(f}\right.}{}$

input
```
int(sinh(f*x+e)^3*(a+b*sinh(f*x+e)^2)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/16*((a+b*sinh(f*x+e)^2)*cosh(f*x+e)^2)^(1/2)*(4*b^(5/2)*(b*cosh(f*x+e)^4
+(a-b)*cosh(f*x+e)^2)^(1/2)*cosh(f*x+e)^2-10*(b*cosh(f*x+e)^4+(a-b)*cosh(f
*x+e)^2)^(1/2)*b^(5/2)+2*a*(b*cosh(f*x+e)^4+(a-b)*cosh(f*x+e)^2)^(1/2)*b^(
3/2)-ln(1/2*(2*b*cosh(f*x+e)^2+2*(b*cosh(f*x+e)^4+(a-b)*cosh(f*x+e)^2)^(1/
2)*b^(1/2)+a-b)/b^(1/2))*a^2*b-2*a*ln(1/2*(2*b*cosh(f*x+e)^2+2*(b*cosh(f*x
+e)^4+(a-b)*cosh(f*x+e)^2)^(1/2)*b^(1/2)+a-b)/b^(1/2))*b^2+3*b^3*ln(1/2*(2
*b*cosh(f*x+e)^2+2*(b*cosh(f*x+e)^4+(a-b)*cosh(f*x+e)^2)^(1/2)*b^(1/2)+a-b
)/b^(1/2)))/b^(5/2)/cosh(f*x+e)/(a+b*sinh(f*x+e)^2)^(1/2)/f
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1180 vs. $2(114) = 228$.

Time = 0.23 (sec) , antiderivative size = 3037, normalized size of antiderivative = 23.36

$$\int \sinh^3(e+fx)\sqrt{a+b\sinh^2(e+fx)}\,dx = \text{Too large to display}$$

input
```
integrate(sinh(f*x+e)^3*(a+b*sinh(f*x+e)^2)^(1/2),x, algorithm="fricas")
```

output
```
Too large to include
```

Sympy [F(-1)]

Timed out.

$$\int \sinh^3(e+fx)\sqrt{a+b\sinh^2(e+fx)}\,dx = \text{Timed out}$$

input
```
integrate(sinh(f*x+e)**3*(a+b*sinh(f*x+e)**2)**(1/2),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \sinh^3(e+fx)\sqrt{a+b\sinh^2(e+fx)}\,dx = \int \sqrt{b\sinh(fx+e)^2+a}\sinh(fx+e)^3\,dx$$

input
```
integrate(sinh(f*x+e)^3*(a+b*sinh(f*x+e)^2)^(1/2),x, algorithm="maxima")
```

output
```
integrate(sqrt(b*sinh(f*x + e)^2 + a)*sinh(f*x + e)^3, x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 890 vs. $2(114) = 228$.

Time = 0.40 (sec) , antiderivative size = 890, normalized size of antiderivative = 6.85

$$\int \sinh^3(e+fx)\sqrt{a+b\sinh^2(e+fx)}\,dx = \text{Too large to display}$$

input
```
integrate(sinh(f*x+e)^3*(a+b*sinh(f*x+e)^2)^(1/2),x, algorithm="giac")
```

output
```
1/64*(sqrt(b*e^(4*f*x + 4*e) + 4*a*e^(2*f*x + 2*e) - 2*b*e^(2*f*x + 2*e) +
 b)*((2*a*e^(6*e) - 7*b*e^(6*e))*e^(-2*e)/b + e^(2*f*x + 6*e)) + 8*(a^2*e^
(4*e) + 2*a*b*e^(4*e) - 3*b^2*e^(4*e))*arctan(-(sqrt(b)*e^(2*f*x + 2*e) -
sqrt(b*e^(4*f*x + 4*e) + 4*a*e^(2*f*x + 2*e) - 2*b*e^(2*f*x + 2*e) + b))/s
qrt(-b))/(sqrt(-b)*b) + 4*(a^2*sqrt(b)*e^(4*e) + 2*a*b^(3/2)*e^(4*e) - 3*b
^(5/2)*e^(4*e))*log(abs(-(sqrt(b)*e^(2*f*x + 2*e) - sqrt(b*e^(4*f*x + 4*e)
 + 4*a*e^(2*f*x + 2*e) - 2*b*e^(2*f*x + 2*e) + b))*b - 2*a*sqrt(b) + b^(3/
2)))/b^2 - 4*(2*(sqrt(b)*e^(2*f*x + 2*e) - sqrt(b*e^(4*f*x + 4*e) + 4*a*e^
(2*f*x + 2*e) - 2*b*e^(2*f*x + 2*e) + b))^3*a^2*e^(4*e) - 8*(sqrt(b)*e^(2*
f*x + 2*e) - sqrt(b*e^(4*f*x + 4*e) + 4*a*e^(2*f*x + 2*e) - 2*b*e^(2*f*x +
 2*e) + b))^3*a*b*e^(4*e) + 4*(sqrt(b)*e^(2*f*x + 2*e) - sqrt(b*e^(4*f*x +
 4*e) + 4*a*e^(2*f*x + 2*e) - 2*b*e^(2*f*x + 2*e) + b))^3*b^2*e^(4*e) + 4*
(sqrt(b)*e^(2*f*x + 2*e) - sqrt(b*e^(4*f*x + 4*e) + 4*a*e^(2*f*x + 2*e) -
2*b*e^(2*f*x + 2*e) + b))^2*a*b^(3/2)*e^(4*e) - 5*(sqrt(b)*e^(2*f*x + 2*e)
 - sqrt(b*e^(4*f*x + 4*e) + 4*a*e^(2*f*x + 2*e) - 2*b*e^(2*f*x + 2*e) + b)
)^2*b^(5/2)*e^(4*e) + 2*(sqrt(b)*e^(2*f*x + 2*e) - sqrt(b*e^(4*f*x + 4*e)
+ 4*a*e^(2*f*x + 2*e) - 2*b*e^(2*f*x + 2*e) + b))*a^2*b*e^(4*e) + 4*(sqrt(
b)*e^(2*f*x + 2*e) - sqrt(b*e^(4*f*x + 4*e) + 4*a*e^(2*f*x + 2*e) - 2*b*e^
(2*f*x + 2*e) + b))*a*b^2*e^(4*e) - 2*(sqrt(b)*e^(2*f*x + 2*e) - sqrt(b*e^
(4*f*x + 4*e) + 4*a*e^(2*f*x + 2*e) - 2*b*e^(2*f*x + 2*e) + b))*b^3*e^(...
```

Mupad [F(-1)]

Timed out.

$$\int \sinh^3(e+fx)\sqrt{a+b\sinh^2(e+fx)}\,dx = \int \sinh(e+f\,x)^3\,\sqrt{b\sinh\left(e+f\,x\right)^2+a}\,dx$$

input
```
int(sinh(e + f*x)^3*(a + b*sinh(e + f*x)^2)^(1/2),x)
```

output
```
int(sinh(e + f*x)^3*(a + b*sinh(e + f*x)^2)^(1/2), x)
```

Reduce [F]

$$\int \sinh^3(e+fx)\sqrt{a+b\sinh^2(e+fx)}\,dx = \int \sqrt{\sinh(fx+e)^2 b+a}\,\sinh(fx+e)^3\,dx$$

input
```
int(sinh(f*x+e)^3*(a+b*sinh(f*x+e)^2)^(1/2),x)
```

output
```
int(sqrt(sinh(e + f*x)**2*b + a)*sinh(e + f*x)**3,x)
```

3.61 $\int \sinh(e+fx)\sqrt{a+b\sinh^2(e+fx)}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 82

$$\int \sinh(e+fx)\sqrt{a+b\sinh^2(e+fx)}\,dx = \frac{(a-b)\text{arctanh}\left(\frac{\sqrt{b}\cosh(e+fx)}{\sqrt{a-b+b\cosh^2(e+fx)}}\right)}{2\sqrt{b}f} + \frac{\cosh(e+fx)\sqrt{a-b+b\cosh^2(e+fx)}}{2f}$$

output
```
1/2*(a-b)*arctanh(b^(1/2)*cosh(f*x+e)/(a-b+b*cosh(f*x+e)^2)^(1/2))/b^(1/2)
/f+1/2*cosh(f*x+e)*(a-b+b*cosh(f*x+e)^2)^(1/2)/f
```

Mathematica [A] (verified)

Time = 0.23 (sec) , antiderivative size = 97, normalized size of antiderivative = 1.18

$$\int \sinh(e+fx)\sqrt{a+b\sinh^2(e+fx)}\,dx = \frac{\cosh(e+fx)\sqrt{2a-b+b\cosh(2(e+fx))}}{2\sqrt{2}f} + \frac{(a-b)\log\left(\sqrt{2}\sqrt{b}\cosh(e+fx)+\sqrt{2a-b+b\cosh(2(e+fx))}\right)}{2\sqrt{b}f}$$

input
```
Integrate[Sinh[e + f*x]*Sqrt[a + b*Sinh[e + f*x]^2],x]
```

output
```
(Cosh[e + f*x]*Sqrt[2*a - b + b*Cosh[2*(e + f*x)]])/(2*Sqrt[2]*f) + ((a -
b)*Log[Sqrt[2]*Sqrt[b]*Cosh[e + f*x] + Sqrt[2*a - b + b*Cosh[2*(e + f*x)]]
])/(2*Sqrt[b]*f)
```

Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 80, normalized size of antiderivative = 0.98, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.261, Rules used = {3042, 26, 3665, 211, 224, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sinh(e+fx)\sqrt{a+b\sinh^2(e+fx)}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int -i\sin(ie+ifx)\sqrt{a-b\sin(ie+ifx)^2}dx$$

$$\Big\downarrow \text{26}$$

$$-i\int \sin(ie+ifx)\sqrt{a-b\sin(ie+ifx)^2}dx$$

$$\Big\downarrow \text{3665}$$

$$\frac{\int \sqrt{b\cosh^2(e+fx)+a-b}d\cosh(e+fx)}{f}$$

$$\Big\downarrow \text{211}$$

$$\frac{\frac{1}{2}(a-b)\int \frac{1}{\sqrt{b\cosh^2(e+fx)+a-b}}d\cosh(e+fx)+\frac{1}{2}\cosh(e+fx)\sqrt{a+b\cosh^2(e+fx)-b}}{f}$$

$$\Big\downarrow \text{224}$$

$$\frac{\frac{1}{2}(a-b)\int \frac{1}{1-\frac{b\cosh^2(e+fx)}{b\cosh^2(e+fx)+a-b}}d\frac{\cosh(e+fx)}{\sqrt{b\cosh^2(e+fx)+a-b}}+\frac{1}{2}\cosh(e+fx)\sqrt{a+b\cosh^2(e+fx)-b}}{f}$$

$$\Big\downarrow \text{219}$$

$$\frac{\frac{(a-b)\operatorname{arctanh}\left(\frac{\sqrt{b}\cosh(e+fx)}{\sqrt{a+b\cosh^2(e+fx)-b}}\right)}{2\sqrt{b}}+\frac{1}{2}\cosh(e+fx)\sqrt{a+b\cosh^2(e+fx)-b}}{f}$$

input
```
Int[Sinh[e + f*x]*Sqrt[a + b*Sinh[e + f*x]^2],x]
```

output
```
(((a - b)*ArcTanh[(Sqrt[b]*Cosh[e + f*x])/Sqrt[a - b + b*Cosh[e + f*x]^2]]
)/(2*Sqrt[b]) + (Cosh[e + f*x]*Sqrt[a - b + b*Cosh[e + f*x]^2])/2)/f
```

Defintions of rubi rules used

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 211
```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[x*((a + b*x^2)^p/(2*p + 1
)), x] + Simp[2*a*(p/(2*p + 1))   Int[(a + b*x^2)^(p - 1), x], x] /; FreeQ[
{a, b}, x] && GtQ[p, 0] && (IntegerQ[4*p] || IntegerQ[6*p])
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3665
```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^
(p_.), x_Symbol] :> With[{ff = FreeFactors[Cos[e + f*x], x]}, Simp[-ff/f
Subst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a + b - b*ff^2*x^2)^p, x], x, Cos[e +
 f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m - 1)/2]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 199 vs. $2(70) = 140$.

Time = 0.23 (sec) , antiderivative size = 200, normalized size of antiderivative = 2.44

method	result
default	$\frac{\sqrt{\left(a+b\sinh(fx+e)^2\right)\cosh(fx+e)^2}\left(-b\ln\left(\frac{2b\cosh(fx+e)^2+2\sqrt{b\cosh(fx+e)^4+(a-b)\cosh(fx+e)^2}\,\sqrt{b}+a-b}{2\sqrt{b}}\right)+2\sqrt{b\cosh(fx+e)^4+(a-}\right.}{4\sqrt{b}\,\cosh(fx+e)\sqrt{a+b\sinh(fx+e)^2}\,f}$

input
```
int(sinh(f*x+e)*(a+b*sinh(f*x+e)^2)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/4*((a+b*sinh(f*x+e)^2)*cosh(f*x+e)^2)^(1/2)*(-b*ln(1/2*(2*b*cosh(f*x+e)^
2+2*(b*cosh(f*x+e)^4+(a-b)*cosh(f*x+e)^2)^(1/2)*b^(1/2)+a-b)/b^(1/2))+2*(b
*cosh(f*x+e)^4+(a-b)*cosh(f*x+e)^2)^(1/2)*b^(1/2)+a*ln(1/2*(2*b*cosh(f*x+e
)^2+2*(b*cosh(f*x+e)^4+(a-b)*cosh(f*x+e)^2)^(1/2)*b^(1/2)+a-b)/b^(1/2)))/b
^(1/2)/cosh(f*x+e)/(a+b*sinh(f*x+e)^2)^(1/2)/f
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 727 vs. $2(70) = 140$.

Time = 0.15 (sec) , antiderivative size = 2130, normalized size of antiderivative = 25.98

$$\int \sinh(e+fx)\sqrt{a+b\sinh^2(e+fx)}\,dx = \text{Too large to display}$$

input
```
integrate(sinh(f*x+e)*(a+b*sinh(f*x+e)^2)^(1/2),x, algorithm="fricas")
```

output
```
[-1/8*(((a - b)*cosh(f*x + e)^2 + 2*(a - b)*cosh(f*x + e)*sinh(f*x + e) +
(a - b)*sinh(f*x + e)^2)*sqrt(b)*log((a^2*b*cosh(f*x + e)^8 + 8*a^2*b*cosh
(f*x + e)*sinh(f*x + e)^7 + a^2*b*sinh(f*x + e)^8 + 2*(a^3 + a^2*b)*cosh(f
*x + e)^6 + 2*(14*a^2*b*cosh(f*x + e)^2 + a^3 + a^2*b)*sinh(f*x + e)^6 + 4
*(14*a^2*b*cosh(f*x + e)^3 + 3*(a^3 + a^2*b)*cosh(f*x + e))*sinh(f*x + e)^
5 + (9*a^2*b - 4*a*b^2 + b^3)*cosh(f*x + e)^4 + (70*a^2*b*cosh(f*x + e)^4
+ 9*a^2*b - 4*a*b^2 + b^3 + 30*(a^3 + a^2*b)*cosh(f*x + e)^2)*sinh(f*x + e
)^4 + 4*(14*a^2*b*cosh(f*x + e)^5 + 10*(a^3 + a^2*b)*cosh(f*x + e)^3 + (9*
a^2*b - 4*a*b^2 + b^3)*cosh(f*x + e))*sinh(f*x + e)^3 + b^3 + 2*(3*a*b^2 -
 b^3)*cosh(f*x + e)^2 + 2*(14*a^2*b*cosh(f*x + e)^6 + 15*(a^3 + a^2*b)*cos
h(f*x + e)^4 + 3*a*b^2 - b^3 + 3*(9*a^2*b - 4*a*b^2 + b^3)*cosh(f*x + e)^2
)*sinh(f*x + e)^2 - sqrt(2)*(a^2*cosh(f*x + e)^6 + 6*a^2*cosh(f*x + e)*sin
h(f*x + e)^5 + a^2*sinh(f*x + e)^6 + 3*a^2*cosh(f*x + e)^4 + 3*(5*a^2*cosh
(f*x + e)^2 + a^2)*sinh(f*x + e)^4 + 4*(5*a^2*cosh(f*x + e)^3 + 3*a^2*cosh
(f*x + e))*sinh(f*x + e)^3 + (4*a*b - b^2)*cosh(f*x + e)^2 + (15*a^2*cosh(
f*x + e)^4 + 18*a^2*cosh(f*x + e)^2 + 4*a*b - b^2)*sinh(f*x + e)^2 + b^2 +
 2*(3*a^2*cosh(f*x + e)^5 + 6*a^2*cosh(f*x + e)^3 + (4*a*b - b^2)*cosh(f*x
 + e))*sinh(f*x + e))*sqrt(b)*sqrt((b*cosh(f*x + e)^2 + b*sinh(f*x + e)^2
+ 2*a - b)/(cosh(f*x + e)^2 - 2*cosh(f*x + e)*sinh(f*x + e) + sinh(f*x + e
)^2)) + 4*(2*a^2*b*cosh(f*x + e)^7 + 3*(a^3 + a^2*b)*cosh(f*x + e)^5 + ...
```

Sympy [F]

$$\int \sinh(e+fx)\sqrt{a+b\sinh^2(e+fx)}\,dx=\int \sqrt{a+b\sinh^2{(e+fx)}}\sinh{(e+fx)}\,dx$$

input
```
integrate(sinh(f*x+e)*(a+b*sinh(f*x+e)**2)**(1/2),x)
```

output
```
Integral(sqrt(a + b*sinh(e + f*x)**2)*sinh(e + f*x), x)
```

Maxima [F]

$$\int \sinh(e+fx)\sqrt{a+b\sinh^2(e+fx)}\,dx = \int \sqrt{b\sinh\left(fx+e\right)^2+a}\sinh\left(fx+e\right)\,dx$$

input
```
integrate(sinh(f*x+e)*(a+b*sinh(f*x+e)^2)^(1/2),x, algorithm="maxima")
```

output
```
integrate(sqrt(b*sinh(f*x + e)^2 + a)*sinh(f*x + e), x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 424 vs. $2(70) = 140$.

Time = 0.25 (sec) , antiderivative size = 424, normalized size of antiderivative = 5.17

$$\int \sinh(e+fx)\sqrt{a+b\sinh^2(e+fx)}\,dx =$$

$$-\frac{\left(\frac{4\left(ae^{(2e)}-be^{(2e)}\right)\arctan\left(-\frac{\sqrt{b}e^{(2fx+2e)}-\sqrt{be^{(4fx+4e)}+4ae^{(2fx+2e)}-2be^{(2fx+2e)}+b}}{\sqrt{-b}}\right)}{\sqrt{-b}} - \sqrt{be^{(4fx+4e)}+4ae^{(2fx+2e)}-2}\right.}{}$$

input
```
integrate(sinh(f*x+e)*(a+b*sinh(f*x+e)^2)^(1/2),x, algorithm="giac")
```

output
```
-1/8*(4*(a*e^(2*e) - b*e^(2*e))*arctan(-(sqrt(b)*e^(2*f*x + 2*e) - sqrt(b*
e^(4*f*x + 4*e) + 4*a*e^(2*f*x + 2*e) - 2*b*e^(2*f*x + 2*e) + b))/sqrt(-b)
)/sqrt(-b) - sqrt(b*e^(4*f*x + 4*e) + 4*a*e^(2*f*x + 2*e) - 2*b*e^(2*f*x +
 2*e) + b)*e^(2*e) + 2*(a*e^(2*e) - b*e^(2*e))*log(abs((sqrt(b)*e^(2*f*x +
 2*e) - sqrt(b*e^(4*f*x + 4*e) + 4*a*e^(2*f*x + 2*e) - 2*b*e^(2*f*x + 2*e)
 + b))*sqrt(b) + 2*a - b))/sqrt(b) + 2*(2*(sqrt(b)*e^(2*f*x + 2*e) - sqrt(
b*e^(4*f*x + 4*e) + 4*a*e^(2*f*x + 2*e) - 2*b*e^(2*f*x + 2*e) + b))*a*e^(2
*e) - (sqrt(b)*e^(2*f*x + 2*e) - sqrt(b*e^(4*f*x + 4*e) + 4*a*e^(2*f*x + 2
*e) - 2*b*e^(2*f*x + 2*e) + b))*b*e^(2*e) + b^(3/2)*e^(2*e))/((sqrt(b)*e^(
2*f*x + 2*e) - sqrt(b*e^(4*f*x + 4*e) + 4*a*e^(2*f*x + 2*e) - 2*b*e^(2*f*x
 + 2*e) + b))^2 - b))*e^(-2*e)/f
```

Mupad [F(-1)]

Timed out.

$$\int \sinh(e+fx)\sqrt{a+b\sinh^2(e+fx)}\,dx = \int \sinh(e+f\,x)\,\sqrt{b\sinh\left(e+f\,x\right)^2+a}\,dx$$

input
```
int(sinh(e + f*x)*(a + b*sinh(e + f*x)^2)^(1/2),x)
```

output
```
int(sinh(e + f*x)*(a + b*sinh(e + f*x)^2)^(1/2), x)
```

Reduce [F]

$$\int \sinh(e+fx)\sqrt{a+b\sinh^2(e+fx)}\,dx = \int \sqrt{\sinh\left(fx+e\right)^2 b+a}\,\sinh\left(fx+e\right)dx$$

input
```
int(sinh(f*x+e)*(a+b*sinh(f*x+e)^2)^(1/2),x)
```

output
```
int(sqrt(sinh(e + f*x)**2*b + a)*sinh(e + f*x),x)
```

3.62 $\int \operatorname{csch}(e+fx)\sqrt{a+b\sinh^2(e+fx)}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 84

$$\int \operatorname{csch}(e+fx)\sqrt{a+b\sinh^2(e+fx)}\,dx = -\frac{\sqrt{a}\operatorname{arctanh}\left(\frac{\sqrt{a}\cosh(e+fx)}{\sqrt{a-b+b\cosh^2(e+fx)}}\right)}{f} + \frac{\sqrt{b}\operatorname{arctanh}\left(\frac{\sqrt{b}\cosh(e+fx)}{\sqrt{a-b+b\cosh^2(e+fx)}}\right)}{f}$$

output
```
-a^(1/2)*arctanh(a^(1/2)*cosh(f*x+e)/(a-b+b*cosh(f*x+e)^2)^(1/2))/f+b^(1/2
)*arctanh(b^(1/2)*cosh(f*x+e)/(a-b+b*cosh(f*x+e)^2)^(1/2))/f
```

Mathematica [A] (verified)

Time = 0.29 (sec) , antiderivative size = 97, normalized size of antiderivative = 1.15

$$\int \operatorname{csch}(e+fx)\sqrt{a+b\sinh^2(e+fx)}\,dx = \frac{-\sqrt{a}\operatorname{arctanh}\left(\frac{\sqrt{2}\sqrt{a}\cosh(e+fx)}{\sqrt{2a-b+b\cosh(2(e+fx))}}\right)+\sqrt{b}\log\left(\sqrt{2}\sqrt{b}\cosh(e+fx)+\sqrt{2a-b+b\cosh(2(e+fx))}\right)}{f}$$

input
```
Integrate[Csch[e + f*x]*Sqrt[a + b*Sinh[e + f*x]^2],x]
```

output
```
(-(Sqrt[a]*ArcTanh[(Sqrt[2]*Sqrt[a]*Cosh[e + f*x])/Sqrt[2*a - b + b*Cosh[2
*(e + f*x)]]]) + Sqrt[b]*Log[Sqrt[2]*Sqrt[b]*Cosh[e + f*x] + Sqrt[2*a - b
+ b*Cosh[2*(e + f*x)]]])/f
```

Rubi [A] (verified)

Time = 0.32 (sec) , antiderivative size = 83, normalized size of antiderivative = 0.99, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.348$, Rules used = {3042, 26, 3665, 301, 224, 219, 291, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \text{csch}(e+fx)\sqrt{a+b\sinh^2(e+fx)}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{i\sqrt{a-b\sin(ie+ifx)^2}}{\sin(ie+ifx)}dx$$

$$\Big\downarrow \text{26}$$

$$i\int \frac{\sqrt{a-b\sin(ie+ifx)^2}}{\sin(ie+ifx)}dx$$

$$\Big\downarrow \text{3665}$$

$$-\frac{\int \frac{\sqrt{b\cosh^2(e+fx)+a-b}}{1-\cosh^2(e+fx)}d\cosh(e+fx)}{f}$$

$$\Big\downarrow \text{301}$$

$$-\frac{a\int \frac{1}{(1-\cosh^2(e+fx))\sqrt{b\cosh^2(e+fx)+a-b}}d\cosh(e+fx)-b\int \frac{1}{\sqrt{b\cosh^2(e+fx)+a-b}}d\cosh(e+fx)}{f}$$

$$\Big\downarrow \text{224}$$

$$-\frac{a\int \frac{1}{\left(1-\cosh^2(e+fx)\right)\sqrt{b\cosh^2(e+fx)+a-b}}d\cosh(e+fx)-b\int \frac{1}{1-\frac{b\cosh^2(e+fx)}{b\cosh^2(e+fx)+a-b}}d\frac{\cosh(e+fx)}{\sqrt{b\cosh^2(e+fx)+a-b}}}{f}$$

$$\Big\downarrow \text{219}$$

$$-\frac{a\int \frac{1}{\left(1-\cosh^2(e+fx)\right)\sqrt{b\cosh^2(e+fx)+a-b}}d\cosh(e+fx)-\sqrt{b}\text{arctanh}\left(\frac{\sqrt{b}\cosh(e+fx)}{\sqrt{a+b\cosh^2(e+fx)-b}}\right)}{f}$$

$$\Big\downarrow \text{291}$$

$$-\frac{a\int \frac{1}{1-\frac{a\cosh^2(e+fx)}{b\cosh^2(e+fx)+a-b}}d\frac{\cosh(e+fx)}{\sqrt{b\cosh^2(e+fx)+a-b}}-\sqrt{b}\text{arctanh}\left(\frac{\sqrt{b}\cosh(e+fx)}{\sqrt{a+b\cosh^2(e+fx)-b}}\right)}{f}$$

$$\Big\downarrow \text{219}$$

$$-\frac{\sqrt{a}\text{arctanh}\left(\frac{\sqrt{a}\cosh(e+fx)}{\sqrt{a+b\cosh^2(e+fx)-b}}\right)-\sqrt{b}\text{arctanh}\left(\frac{\sqrt{b}\cosh(e+fx)}{\sqrt{a+b\cosh^2(e+fx)-b}}\right)}{f}$$

input

```
Int[Csch[e + f*x]*Sqrt[a + b*Sinh[e + f*x]^2],x]
```

output

```
-((Sqrt[a]*ArcTanh[(Sqrt[a]*Cosh[e + f*x])/Sqrt[a - b + b*Cosh[e + f*x]^2]
] - Sqrt[b]*ArcTanh[(Sqrt[b]*Cosh[e + f*x])/Sqrt[a - b + b*Cosh[e + f*x]^2
]])/f)
```

Defintions of rubi rules used

rule 26

```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 219

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

rule 291
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*((c_) + (d_.)*(x_)^2)), x_Symbol] :> Subst
[Int[1/(c - (b*c - a*d)*x^2), x], x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b, c,
d}, x] && NeQ[b*c - a*d, 0]
```

rule 301
```
Int[((a_) + (b_.)*(x_)^2)^(p_.)/((c_) + (d_.)*(x_)^2), x_Symbol] :> Simp[b/
d   Int[(a + b*x^2)^(p - 1), x], x] - Simp[(b*c - a*d)/d   Int[(a + b*x^2)^
(p - 1)/(c + d*x^2), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0]
&& GtQ[p, 0] && (EqQ[p, 1/2] || EqQ[Denominator[p], 4] || (EqQ[p, 2/3] && E
qQ[b*c + 3*a*d, 0]))
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3665
```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^
(p_.), x_Symbol] :> With[{ff = FreeFactors[Cos[e + f*x], x]}, Simp[-ff/f
Subst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a + b - b*ff^2*x^2)^p, x], x, Cos[e +
 f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m - 1)/2]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 173 vs. $2(72) = 144$.

Time = 0.30 (sec) , antiderivative size = 174, normalized size of antiderivative = 2.07

method	result
default	$-\frac{\sqrt{\left(a+b\sinh(fx+e)^2\right)\cosh(fx+e)^2}\left(\sqrt{a}\,\ln\left(\frac{(a+b)\cosh(fx+e)^2+2\sqrt{a}\,\sqrt{b\cosh(fx+e)^4+(a-b)\cosh(fx+e)^2}+a-b}{\sinh(fx+e)^2}\right)-\sqrt{b}\,\ln\left(\frac{2b\cosh(f}{}\right.\right.}{2\cosh(fx+e)\sqrt{a+b\sinh(fx+e)^2}\,f}$

input
```
int(csch(f*x+e)*(a+b*sinh(f*x+e)^2)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-1/2*((a+b*sinh(f*x+e)^2)*cosh(f*x+e)^2)^(1/2)*(a^(1/2)*ln(((a+b)*cosh(f*x
+e)^2+2*a^(1/2)*(b*cosh(f*x+e)^4+(a-b)*cosh(f*x+e)^2)^(1/2)+a-b)/sinh(f*x+
e)^2)-b^(1/2)*ln(1/2*(2*b*cosh(f*x+e)^2+2*(b*cosh(f*x+e)^4+(a-b)*cosh(f*x+
e)^2)^(1/2)*b^(1/2)+a-b)/b^(1/2)))/cosh(f*x+e)/(a+b*sinh(f*x+e)^2)^(1/2)/f
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 605 vs. $2(72) = 144$.

Time = 0.21 (sec) , antiderivative size = 4197, normalized size of antiderivative = 49.96

$$\int \operatorname{csch}(e+fx)\sqrt{a+b\sinh^2(e+fx)}\,dx = \text{Too large to display}$$

input
```
integrate(csch(f*x+e)*(a+b*sinh(f*x+e)^2)^(1/2),x, algorithm="fricas")
```

output
```
Too large to include
```

Sympy [F]

$$\int \operatorname{csch}(e+fx)\sqrt{a+b\sinh^2(e+fx)}\,dx = \int \sqrt{a+b\sinh^2(e+fx)}\operatorname{csch}(e+fx)\,dx$$

input
```
integrate(csch(f*x+e)*(a+b*sinh(f*x+e)**2)**(1/2),x)
```

output
```
Integral(sqrt(a + b*sinh(e + f*x)**2)*csch(e + f*x), x)
```

Maxima [F]

$$\int \operatorname{csch}(e+fx)\sqrt{a+b\sinh^2(e+fx)}\,dx = \int \sqrt{b\sinh(fx+e)^2+a}\operatorname{csch}(fx+e)\,dx$$

input
```
integrate(csch(f*x+e)*(a+b*sinh(f*x+e)^2)^(1/2),x, algorithm="maxima")
```

output
```
integrate(sqrt(b*sinh(f*x + e)^2 + a)*csch(f*x + e), x)
```

Giac [F(-2)]

Exception generated.

$$\int \operatorname{csch}(e+fx)\sqrt{a+b\sinh^2(e+fx)}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(csch(f*x+e)*(a+b*sinh(f*x+e)^2)^(1/2),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Degree mismatch inside factorisatio
n over extensionNot implemented, e.g. for multivariate mod/approx polynomi
alsError:
```

Mupad [F(-1)]

Timed out.

$$\int \operatorname{csch}(e+fx)\sqrt{a+b\sinh^2(e+fx)}\,dx = \int \frac{\sqrt{b\sinh(e+f\,x)^2+a}}{\sinh(e+f\,x)}\,dx$$

input
```
int((a + b*sinh(e + f*x)^2)^(1/2)/sinh(e + f*x),x)
```

output
```
int((a + b*sinh(e + f*x)^2)^(1/2)/sinh(e + f*x), x)
```

Reduce [F]

$$\int \operatorname{csch}(e+fx)\sqrt{a+b\sinh^2(e+fx)}\,dx=\int \sqrt{\sinh(fx+e)^2 b+a}\operatorname{csch}(fx+e)\,dx$$

input
```
int(csch(f*x+e)*(a+b*sinh(f*x+e)^2)^(1/2),x)
```

output
```
int(sqrt(sinh(e + f*x)**2*b + a)*csch(e + f*x),x)
```

3.63 $\int \operatorname{csch}^3(e+fx)\sqrt{a+b\sinh^2(e+fx)}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 88

$$\int \operatorname{csch}^3(e+fx)\sqrt{a+b\sinh^2(e+fx)}\,dx = \frac{(a-b)\operatorname{arctanh}\left(\frac{\sqrt{a}\cosh(e+fx)}{\sqrt{a-b+b\cosh^2(e+fx)}}\right)}{2\sqrt{a}f} - \frac{\sqrt{a-b+b\cosh^2(e+fx)}\coth(e+fx)\operatorname{csch}(e+fx)}{2f}$$

output
```
1/2*(a-b)*arctanh(a^(1/2)*cosh(f*x+e)/(a-b+b*cosh(f*x+e)^2)^(1/2))/a^(1/2)
/f-1/2*(a-b+b*cosh(f*x+e)^2)^(1/2)*coth(f*x+e)*csch(f*x+e)/f
```

Mathematica [A] (verified)

Time = 0.59 (sec) , antiderivative size = 104, normalized size of antiderivative = 1.18

$$\int \operatorname{csch}^3(e+fx)\sqrt{a+b\sinh^2(e+fx)}\,dx = \frac{2(a-b)\operatorname{arctanh}\left(\frac{\sqrt{2}\sqrt{a}\cosh(e+fx)}{\sqrt{2a-b+b\cosh(2(e+fx))}}\right) - \sqrt{2}\sqrt{a}\sqrt{2a-b+b\cosh(2(e+fx))}\coth(e+fx)\operatorname{csch}(e+fx)}{4\sqrt{a}f}$$

input
```
Integrate[Csch[e + f*x]^3*Sqrt[a + b*Sinh[e + f*x]^2],x]
```

output
```
(2*(a - b)*ArcTanh[(Sqrt[2]*Sqrt[a]*Cosh[e + f*x])/Sqrt[2*a - b + b*Cosh[2
*(e + f*x)]]] - Sqrt[2]*Sqrt[a]*Sqrt[2*a - b + b*Cosh[2*(e + f*x)]]*Coth[e
 + f*x]*Csch[e + f*x])/(4*Sqrt[a]*f)
```

Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 94, normalized size of antiderivative = 1.07, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.240$, Rules used = {3042, 26, 3665, 292, 291, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \operatorname{csch}^3(e+fx)\sqrt{a+b\sinh^2(e+fx)}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int -\frac{i\sqrt{a-b\sin(ie+ifx)^2}}{\sin(ie+ifx)^3}dx$$

$$\Big\downarrow \text{26}$$

$$-i\int \frac{\sqrt{a-b\sin(ie+ifx)^2}}{\sin(ie+ifx)^3}dx$$

$$\Big\downarrow \text{3665}$$

$$\frac{\int \frac{\sqrt{b\cosh^2(e+fx)+a-b}}{\left(1-\cosh^2(e+fx)\right)^2}d\cosh(e+fx)}{f}$$

$$\Big\downarrow \text{292}$$

$$\frac{\frac{1}{2}(a-b)\int \frac{1}{\left(1-\cosh^2(e+fx)\right)\sqrt{b\cosh^2(e+fx)+a-b}}d\cosh(e+fx)+\frac{\cosh(e+fx)\sqrt{a+b\cosh^2(e+fx)-b}}{2\left(1-\cosh^2(e+fx)\right)}}{f}$$

$$\Big\downarrow \text{291}$$

$$\frac{\frac{1}{2}(a-b)\int \frac{1}{1-\frac{a\cosh^2(e+fx)}{b\cosh^2(e+fx)+a-b}}d\frac{\cosh(e+fx)}{\sqrt{b\cosh^2(e+fx)+a-b}}+\frac{\cosh(e+fx)\sqrt{a+b\cosh^2(e+fx)-b}}{2(1-\cosh^2(e+fx))}}{f}$$

$$\Big\downarrow \text{219}$$

$$\frac{\frac{(a-b)\text{arctanh}\left(\frac{\sqrt{a}\cosh(e+fx)}{\sqrt{a+b\cosh^2(e+fx)-b}}\right)}{2\sqrt{a}}+\frac{\cosh(e+fx)\sqrt{a+b\cosh^2(e+fx)-b}}{2(1-\cosh^2(e+fx))}}{f}$$

input
```
Int[Csch[e + f*x]^3*Sqrt[a + b*Sinh[e + f*x]^2],x]
```

output
```
(((a - b)*ArcTanh[(Sqrt[a]*Cosh[e + f*x])/Sqrt[a - b + b*Cosh[e + f*x]^2]]
)/(2*Sqrt[a]) + (Cosh[e + f*x]*Sqrt[a - b + b*Cosh[e + f*x]^2])/(2*(1 - Co
sh[e + f*x]^2)))/f
```

Defintions of rubi rules used

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 291
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*((c_) + (d_.)*(x_)^2)), x_Symbol] :> Subst
[Int[1/(c - (b*c - a*d)*x^2), x], x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b, c,
d}, x] && NeQ[b*c - a*d, 0]
```

rule 292
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_.), x_Symbol] :> Si
mp[(-x)*(a + b*x^2)^(p + 1)*((c + d*x^2)^q/(2*a*(p + 1))), x] - Simp[c*(q/(
a*(p + 1)))   Int[(a + b*x^2)^(p + 1)*(c + d*x^2)^(q - 1), x], x] /; FreeQ[
{a, b, c, d, p}, x] && NeQ[b*c - a*d, 0] && EqQ[2*(p + q + 1) + 1, 0] && Gt
Q[q, 0] && NeQ[p, -1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3665
```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^
(p_.), x_Symbol] :> With[{ff = FreeFactors[Cos[e + f*x], x]}, Simp[-ff/f
Subst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a + b - b*ff^2*x^2)^p, x], x, Cos[e +
 f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m - 1)/2]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 229 vs. $2(76) = 152$.

Time = 0.39 (sec) , antiderivative size = 230, normalized size of antiderivative = 2.61

method	result
default	$-\frac{\sqrt{\left(a+b\sinh(fx+e)^2\right)\cosh(fx+e)^2}\,\left(-a\ln\left(\frac{(a+b)\cosh(fx+e)^2+2\sqrt{a}\,\sqrt{b\cosh(fx+e)^4+(a-b)\cosh(fx+e)^2}+a-b}{\sinh(fx+e)^2}\right)\sinh(fx+e)^2+b\ln}{4\sinh(fx+e)^2\sqrt{a}\,\cosh(fx+}$

input
```
int(csch(f*x+e)^3*(a+b*sinh(f*x+e)^2)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-1/4*((a+b*sinh(f*x+e)^2)*cosh(f*x+e)^2)^(1/2)*(-a*ln(((a+b)*cosh(f*x+e)^2
+2*a^(1/2)*(b*cosh(f*x+e)^4+(a-b)*cosh(f*x+e)^2)^(1/2)+a-b)/sinh(f*x+e)^2)
*sinh(f*x+e)^2+b*ln(((a+b)*cosh(f*x+e)^2+2*a^(1/2)*(b*cosh(f*x+e)^4+(a-b)*
cosh(f*x+e)^2)^(1/2)+a-b)/sinh(f*x+e)^2)*sinh(f*x+e)^2+2*a^(1/2)*((a+b*sin
h(f*x+e)^2)*cosh(f*x+e)^2)^(1/2))/sinh(f*x+e)^2/a^(1/2)/cosh(f*x+e)/(a+b*s
inh(f*x+e)^2)^(1/2)/f
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 475 vs. $2(76) = 152$.

Time = 0.12 (sec) , antiderivative size = 1165, normalized size of antiderivative = 13.24

$$\int \operatorname{csch}^3(e+fx)\sqrt{a+b\sinh^2(e+fx)}\,dx = \text{Too large to display}$$

input
```
integrate(csch(f*x+e)^3*(a+b*sinh(f*x+e)^2)^(1/2),x, algorithm="fricas")
```

output
```
[-1/4*(((a - b)*cosh(f*x + e)^4 + 4*(a - b)*cosh(f*x + e)*sinh(f*x + e)^3
+ (a - b)*sinh(f*x + e)^4 - 2*(a - b)*cosh(f*x + e)^2 + 2*(3*(a - b)*cosh(
f*x + e)^2 - a + b)*sinh(f*x + e)^2 + 4*((a - b)*cosh(f*x + e)^3 - (a - b)
*cosh(f*x + e))*sinh(f*x + e) + a - b)*sqrt(a)*log(-((a + b)*cosh(f*x + e)
^4 + 4*(a + b)*cosh(f*x + e)*sinh(f*x + e)^3 + (a + b)*sinh(f*x + e)^4 + 2
*(3*a - b)*cosh(f*x + e)^2 + 2*(3*(a + b)*cosh(f*x + e)^2 + 3*a - b)*sinh(
f*x + e)^2 - 2*sqrt(2)*(cosh(f*x + e)^2 + 2*cosh(f*x + e)*sinh(f*x + e) +
sinh(f*x + e)^2 + 1)*sqrt(a)*sqrt((b*cosh(f*x + e)^2 + b*sinh(f*x + e)^2 +
 2*a - b)/(cosh(f*x + e)^2 - 2*cosh(f*x + e)*sinh(f*x + e) + sinh(f*x + e)
^2)) + 4*((a + b)*cosh(f*x + e)^3 + (3*a - b)*cosh(f*x + e))*sinh(f*x + e)
 + a + b)/(cosh(f*x + e)^4 + 4*cosh(f*x + e)*sinh(f*x + e)^3 + sinh(f*x +
e)^4 + 2*(3*cosh(f*x + e)^2 - 1)*sinh(f*x + e)^2 - 2*cosh(f*x + e)^2 + 4*(
cosh(f*x + e)^3 - cosh(f*x + e))*sinh(f*x + e) + 1)) + 2*sqrt(2)*(a*cosh(f
*x + e)^2 + 2*a*cosh(f*x + e)*sinh(f*x + e) + a*sinh(f*x + e)^2 + a)*sqrt(
(b*cosh(f*x + e)^2 + b*sinh(f*x + e)^2 + 2*a - b)/(cosh(f*x + e)^2 - 2*cos
h(f*x + e)*sinh(f*x + e) + sinh(f*x + e)^2)))/(a*f*cosh(f*x + e)^4 + 4*a*f
*cosh(f*x + e)*sinh(f*x + e)^3 + a*f*sinh(f*x + e)^4 - 2*a*f*cosh(f*x + e)
^2 + 2*(3*a*f*cosh(f*x + e)^2 - a*f)*sinh(f*x + e)^2 + a*f + 4*(a*f*cosh(f
*x + e)^3 - a*f*cosh(f*x + e))*sinh(f*x + e)), -1/2*(((a - b)*cosh(f*x + e
)^4 + 4*(a - b)*cosh(f*x + e)*sinh(f*x + e)^3 + (a - b)*sinh(f*x + e)^4...
```

Sympy [F]

$$\int \operatorname{csch}^3(e+fx)\sqrt{a+b\sinh^2(e+fx)}\,dx = \int \sqrt{a+b\sinh^2(e+fx)}\operatorname{csch}^3(e+fx)\,dx$$

input
```
integrate(csch(f*x+e)**3*(a+b*sinh(f*x+e)**2)**(1/2),x)
```

output
```
Integral(sqrt(a + b*sinh(e + f*x)**2)*csch(e + f*x)**3, x)
```

Maxima [F]

$$\int \operatorname{csch}^3(e+fx)\sqrt{a+b\sinh^2(e+fx)}\,dx = \int \sqrt{b\sinh(fx+e)^2+a}\operatorname{csch}(fx+e)^3\,dx$$

input
```
integrate(csch(f*x+e)^3*(a+b*sinh(f*x+e)^2)^(1/2),x, algorithm="maxima")
```

output
```
integrate(sqrt(b*sinh(f*x + e)^2 + a)*csch(f*x + e)^3, x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 668 vs. $2(76) = 152$.

Time = 0.24 (sec) , antiderivative size = 668, normalized size of antiderivative = 7.59

$$\int \operatorname{csch}^3(e+fx)\sqrt{a+b\sinh^2(e+fx)}\,dx = \text{Too large to display}$$

input
```
integrate(csch(f*x+e)^3*(a+b*sinh(f*x+e)^2)^(1/2),x, algorithm="giac")
```

output
```
-((a - b)*arctan(-1/2*(sqrt(b)*e^(2*f*x + 2*e) - sqrt(b*e^(4*f*x + 4*e) +
4*a*e^(2*f*x + 2*e) - 2*b*e^(2*f*x + 2*e) + b) - sqrt(b))/sqrt(-a))*e^(-2*
e)/(sqrt(-a)*f) - 2*((sqrt(b)*e^(2*f*x + 2*e) - sqrt(b*e^(4*f*x + 4*e) + 4
*a*e^(2*f*x + 2*e) - 2*b*e^(2*f*x + 2*e) + b))^3*a + (sqrt(b)*e^(2*f*x + 2
*e) - sqrt(b*e^(4*f*x + 4*e) + 4*a*e^(2*f*x + 2*e) - 2*b*e^(2*f*x + 2*e) +
 b))^3*b + 5*(sqrt(b)*e^(2*f*x + 2*e) - sqrt(b*e^(4*f*x + 4*e) + 4*a*e^(2*
f*x + 2*e) - 2*b*e^(2*f*x + 2*e) + b))^2*a*sqrt(b) - 3*(sqrt(b)*e^(2*f*x +
 2*e) - sqrt(b*e^(4*f*x + 4*e) + 4*a*e^(2*f*x + 2*e) - 2*b*e^(2*f*x + 2*e)
 + b))^2*b^(3/2) + 4*(sqrt(b)*e^(2*f*x + 2*e) - sqrt(b*e^(4*f*x + 4*e) + 4
*a*e^(2*f*x + 2*e) - 2*b*e^(2*f*x + 2*e) + b))*a^2 - 9*(sqrt(b)*e^(2*f*x +
 2*e) - sqrt(b*e^(4*f*x + 4*e) + 4*a*e^(2*f*x + 2*e) - 2*b*e^(2*f*x + 2*e)
 + b))*a*b + 3*(sqrt(b)*e^(2*f*x + 2*e) - sqrt(b*e^(4*f*x + 4*e) + 4*a*e^(
2*f*x + 2*e) - 2*b*e^(2*f*x + 2*e) + b))*b^2 - 4*a^2*sqrt(b) + 3*a*b^(3/2)
 - b^(5/2))*e^(-2*e)/(((sqrt(b)*e^(2*f*x + 2*e) - sqrt(b*e^(4*f*x + 4*e) +
 4*a*e^(2*f*x + 2*e) - 2*b*e^(2*f*x + 2*e) + b))^2 - 2*(sqrt(b)*e^(2*f*x +
 2*e) - sqrt(b*e^(4*f*x + 4*e) + 4*a*e^(2*f*x + 2*e) - 2*b*e^(2*f*x + 2*e)
 + b))*sqrt(b) - 4*a + b)^2*f))*e^(2*e)
```

Mupad [F(-1)]

Timed out.

$$\int \operatorname{csch}^3(e+fx)\sqrt{a+b\sinh^2(e+fx)}\,dx=\int \frac{\sqrt{b\sinh\left(e+f\,x\right)^2+a}}{\sinh\left(e+f\,x\right)^3}\,dx$$

input
```
int((a + b*sinh(e + f*x)^2)^(1/2)/sinh(e + f*x)^3,x)
```

output
```
int((a + b*sinh(e + f*x)^2)^(1/2)/sinh(e + f*x)^3, x)
```

Reduce [F]

$$\int \operatorname{csch}^3(e+fx)\sqrt{a+b\sinh^2(e+fx)}\,dx = \int \sqrt{\sinh(fx+e)^2 b+a}\,\operatorname{csch}(fx+e)^3\,dx$$

input
```
int(csch(f*x+e)^3*(a+b*sinh(f*x+e)^2)^(1/2),x)
```

output
```
int(sqrt(sinh(e + f*x)**2*b + a)*csch(e + f*x)**3,x)
```

3.64 $\int \operatorname{csch}^5(e+fx)\sqrt{a+b\sinh^2(e+fx)}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 143

$$\int \operatorname{csch}^5(e+fx)\sqrt{a+b\sinh^2(e+fx)}\,dx = -\frac{(a-b)(3a+b)\operatorname{arctanh}\left(\frac{\sqrt{a}\cosh(e+fx)}{\sqrt{a-b+b\cosh^2(e+fx)}}\right)}{8a^{3/2}f} + \frac{(3a-b)\sqrt{a-b+b\cosh^2(e+fx)}\coth(e+fx)\operatorname{csch}(e+fx)}{8af} - \frac{\sqrt{a-b+b\cosh^2(e+fx)}\coth(e+fx)\operatorname{csch}^3(e+fx)}{4f}$$

output
```
-1/8*(a-b)*(3*a+b)*arctanh(a^(1/2)*cosh(f*x+e)/(a-b+b*cosh(f*x+e)^2)^(1/2)
)/a^(3/2)/f+1/8*(3*a-b)*(a-b+b*cosh(f*x+e)^2)^(1/2)*coth(f*x+e)*csch(f*x+e
)/a/f-1/4*(a-b+b*cosh(f*x+e)^2)^(1/2)*coth(f*x+e)*csch(f*x+e)^3/f
```

Mathematica [A] (verified)

Time = 0.63 (sec) , antiderivative size = 129, normalized size of antiderivative = 0.90

$$\int \operatorname{csch}^5(e+fx)\sqrt{a+b\sinh^2(e+fx)}\,dx$$
$$=\frac{(-6a^2+4ab+2b^2)\operatorname{arctanh}\left(\frac{\sqrt{2}\sqrt{a}\cosh(e+fx)}{\sqrt{2a-b+b\cosh(2(e+fx))}}\right)-\sqrt{2}\sqrt{a}\sqrt{2a-b+b\cosh(2(e+fx))}\coth(e+fx)\text{cs}}{16a^{3/2}f}$$

input
```
Integrate[Csch[e + f*x]^5*Sqrt[a + b*Sinh[e + f*x]^2],x]
```

output
```
((-6*a^2 + 4*a*b + 2*b^2)*ArcTanh[(Sqrt[2]*Sqrt[a]*Cosh[e + f*x])/Sqrt[2*a
 - b + b*Cosh[2*(e + f*x)]]] - Sqrt[2]*Sqrt[a]*Sqrt[2*a - b + b*Cosh[2*(e
+ f*x)]]*Coth[e + f*x]*Csch[e + f*x]*(-3*a + b + 2*a*Csch[e + f*x]^2))/(16
*a^(3/2)*f)
```

Rubi [A] (verified)

Time = 0.33 (sec) , antiderivative size = 154, normalized size of antiderivative = 1.08, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.280$, Rules used = {3042, 26, 3665, 296, 292, 291, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \operatorname{csch}^5(e+fx)\sqrt{a+b\sinh^2(e+fx)}\,dx$$
$$\Big\downarrow \text{3042}$$
$$\int \frac{i\sqrt{a-b\sin(ie+ifx)^2}}{\sin(ie+ifx)^5}dx$$
$$\Big\downarrow \text{26}$$
$$i\int \frac{\sqrt{a-b\sin(ie+ifx)^2}}{\sin(ie+ifx)^5}dx$$
$$\Big\downarrow \text{3665}$$

$$-\frac{\int \frac{\sqrt{b\cosh^2(e+fx)+a-b}}{(1-\cosh^2(e+fx))^3}d\cosh(e+fx)}{f}$$

$\Big\downarrow$ 296

$$-\frac{\frac{(3a+b)\int \frac{\sqrt{b\cosh^2(e+fx)+a-b}}{\left(1-\cosh^2(e+fx)\right)^2}d\cosh(e+fx)}{4a}+\frac{\cosh(e+fx)\left(a+b\cosh^2(e+fx)-b\right)^{3/2}}{4a\left(1-\cosh^2(e+fx)\right)^2}}{f}$$

$\Big\downarrow$ 292

$$-\frac{\frac{(3a+b)\left(\frac{1}{2}(a-b)\int \frac{1}{\left(1-\cosh^2(e+fx)\right)\sqrt{b\cosh^2(e+fx)+a-b}}d\cosh(e+fx)+\frac{\cosh(e+fx)\sqrt{a+b\cosh^2(e+fx)-b}}{2\left(1-\cosh^2(e+fx)\right)}\right)}{4a}+\frac{\cosh(e+fx)\left(a+b\cosh^2(e+fx)-b\right.}{4a\left(1-\cosh^2(e+fx)\right)^2}}{f}$$

$\Big\downarrow$ 291

$$-\frac{\frac{(3a+b)\left(\frac{1}{2}(a-b)\int \frac{1}{1-\frac{a\cosh^2(e+fx)}{b\cosh^2(e+fx)+a-b}}d\frac{\cosh(e+fx)}{\sqrt{b\cosh^2(e+fx)+a-b}}+\frac{\cosh(e+fx)\sqrt{a+b\cosh^2(e+fx)-b}}{2\left(1-\cosh^2(e+fx)\right)}\right)}{4a}+\frac{\cosh(e+fx)\left(a+b\cosh^2(e+fx)-b\right)^{3/2}}{4a\left(1-\cosh^2(e+fx)\right)^2}}{f}$$

$\Big\downarrow$ 219

$$-\frac{\frac{(3a+b)\left(\frac{(a-b)\text{arctanh}\left(\frac{\sqrt{a}\cosh(e+fx)}{\sqrt{a+b\cosh^2(e+fx)-b}}\right)}{2\sqrt{a}}+\frac{\cosh(e+fx)\sqrt{a+b\cosh^2(e+fx)-b}}{2\left(1-\cosh^2(e+fx)\right)}\right)}{4a}+\frac{\cosh(e+fx)\left(a+b\cosh^2(e+fx)-b\right)^{3/2}}{4a\left(1-\cosh^2(e+fx)\right)^2}}{f}$$

input

```
Int[Csch[e + f*x]^5*Sqrt[a + b*Sinh[e + f*x]^2],x]
```

output

```
-(((Cosh[e + f*x]*(a - b + b*Cosh[e + f*x]^2)^(3/2))/(4*a*(1 - Cosh[e + f*
x]^2)^2) + ((3*a + b)*(((a - b)*ArcTanh[(Sqrt[a]*Cosh[e + f*x])/Sqrt[a - b
 + b*Cosh[e + f*x]^2]])/(2*Sqrt[a]) + (Cosh[e + f*x]*Sqrt[a - b + b*Cosh[e
 + f*x]^2])/(2*(1 - Cosh[e + f*x]^2))))/(4*a))/f)
```

Defintions of rubi rules used

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 291
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*((c_) + (d_.)*(x_)^2)), x_Symbol] :> Subst
[Int[1/(c - (b*c - a*d)*x^2), x], x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b, c,
d}, x] && NeQ[b*c - a*d, 0]
```

rule 292
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_.), x_Symbol] :> Si
mp[(-x)*(a + b*x^2)^(p + 1)*((c + d*x^2)^q/(2*a*(p + 1))), x] - Simp[c*(q/(
a*(p + 1)))   Int[(a + b*x^2)^(p + 1)*(c + d*x^2)^(q - 1), x], x] /; FreeQ[
{a, b, c, d, p}, x] && NeQ[b*c - a*d, 0] && EqQ[2*(p + q + 1) + 1, 0] && Gt
Q[q, 0] && NeQ[p, -1]
```

rule 296
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_), x_Symbol] :> Sim
p[(-b)*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^(q + 1)/(2*a*(p + 1)*(b*c - a*d))
), x] + Simp[(b*c + 2*(p + 1)*(b*c - a*d))/(2*a*(p + 1)*(b*c - a*d))   Int[
(a + b*x^2)^(p + 1)*(c + d*x^2)^q, x], x] /; FreeQ[{a, b, c, d, q}, x] && N
eQ[b*c - a*d, 0] && EqQ[2*(p + q + 2) + 1, 0] && (LtQ[p, -1] ||  !LtQ[q, -1
]) && NeQ[p, -1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3665
```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^
(p_.), x_Symbol] :> With[{ff = FreeFactors[Cos[e + f*x], x]}, Simp[-ff/f
Subst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a + b - b*ff^2*x^2)^p, x], x, Cos[e +
 f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m - 1)/2]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 380 vs. $2(127) = 254$.

Time = 0.17 (sec) , antiderivative size = 381, normalized size of antiderivative = 2.66

$$\frac{\sqrt{\left(a+b\sinh\left(fx+e\right)^{2}\right)\cosh\left(fx+e\right)^{2}}\,\left(-3a^{3}\ln\left(\frac{(a+b)\cosh(fx+e)^{2}+2\sqrt{a}\,\sqrt{b\cosh(fx+e)^{4}+(a-b)\cosh(fx+e)^{2}}+a-b}{\sinh(fx+e)^{2}}\right)\mathrm{si}\right.}{}$$

input
```
int(csch(f*x+e)^5*(a+b*sinh(f*x+e)^2)^(1/2),x)
```

output
```
1/16*((a+b*sinh(f*x+e)^2)*cosh(f*x+e)^2)^(1/2)*(-3*a^3*ln(((a+b)*cosh(f*x+
e)^2+2*a^(1/2)*(b*cosh(f*x+e)^4+(a-b)*cosh(f*x+e)^2)^(1/2)+a-b)/sinh(f*x+e
)^2)*sinh(f*x+e)^4+2*b*ln(((a+b)*cosh(f*x+e)^2+2*a^(1/2)*(b*cosh(f*x+e)^4+
(a-b)*cosh(f*x+e)^2)^(1/2)+a-b)/sinh(f*x+e)^2)*sinh(f*x+e)^4*a^2+ln(((a+b)
*cosh(f*x+e)^2+2*a^(1/2)*(b*cosh(f*x+e)^4+(a-b)*cosh(f*x+e)^2)^(1/2)+a-b)/
sinh(f*x+e)^2)*b^2*sinh(f*x+e)^4*a+6*sinh(f*x+e)^2*((a+b*sinh(f*x+e)^2)*co
sh(f*x+e)^2)^(1/2)*a^(5/2)-2*sinh(f*x+e)^2*((a+b*sinh(f*x+e)^2)*cosh(f*x+e
)^2)^(1/2)*a^(3/2)*b-4*((a+b*sinh(f*x+e)^2)*cosh(f*x+e)^2)^(1/2)*a^(5/2))/
sinh(f*x+e)^4/a^(5/2)/cosh(f*x+e)/(a+b*sinh(f*x+e)^2)^(1/2)/f
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1534 vs. $2(127) = 254$.

Time = 0.27 (sec) , antiderivative size = 3283, normalized size of antiderivative = 22.96

$$\int \operatorname{csch}^5(e+fx)\sqrt{a+b\sinh^2(e+fx)}\,dx = \text{Too large to display}$$

input
```
integrate(csch(f*x+e)^5*(a+b*sinh(f*x+e)^2)^(1/2),x, algorithm="fricas")
```

output
```
Too large to include
```

Sympy [F(-1)]

Timed out.

$$\int \operatorname{csch}^5(e+fx)\sqrt{a+b\sinh^2(e+fx)}\,dx = \text{Timed out}$$

input
```
integrate(csch(f*x+e)**5*(a+b*sinh(f*x+e)**2)**(1/2),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \operatorname{csch}^5(e+fx)\sqrt{a+b\sinh^2(e+fx)}\,dx = \int \sqrt{b\sinh(fx+e)^2+a}\,\operatorname{csch}(fx+e)^5\,dx$$

input
```
integrate(csch(f*x+e)^5*(a+b*sinh(f*x+e)^2)^(1/2),x, algorithm="maxima")
```

output
```
integrate(sqrt(b*sinh(f*x + e)^2 + a)*csch(f*x + e)^5, x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2167 vs. $2(127) = 254$.

Time = 0.46 (sec) , antiderivative size = 2167, normalized size of antiderivative = 15.15

$$\int \operatorname{csch}^5(e+fx)\sqrt{a+b\sinh^2(e+fx)}\,dx = \text{Too large to display}$$

input
```
integrate(csch(f*x+e)^5*(a+b*sinh(f*x+e)^2)^(1/2),x, algorithm="giac")
```

output
```
1/4*((3*a^2 - 2*a*b - b^2)*arctan(-1/2*(sqrt(b)*e^(2*f*x + 2*e) - sqrt(b*e
^(4*f*x + 4*e) + 4*a*e^(2*f*x + 2*e) - 2*b*e^(2*f*x + 2*e) + b) - sqrt(b))
/sqrt(-a))*e^(-4*e)/(sqrt(-a)*a*f) - 2*(3*(sqrt(b)*e^(2*f*x + 2*e) - sqrt(
b*e^(4*f*x + 4*e) + 4*a*e^(2*f*x + 2*e) - 2*b*e^(2*f*x + 2*e) + b))^7*a^2
- 2*(sqrt(b)*e^(2*f*x + 2*e) - sqrt(b*e^(4*f*x + 4*e) + 4*a*e^(2*f*x + 2*e
) - 2*b*e^(2*f*x + 2*e) + b))^7*a*b - (sqrt(b)*e^(2*f*x + 2*e) - sqrt(b*e^
(4*f*x + 4*e) + 4*a*e^(2*f*x + 2*e) - 2*b*e^(2*f*x + 2*e) + b))^7*b^2 - 21
*(sqrt(b)*e^(2*f*x + 2*e) - sqrt(b*e^(4*f*x + 4*e) + 4*a*e^(2*f*x + 2*e) -
 2*b*e^(2*f*x + 2*e) + b))^6*a^2*sqrt(b) - 18*(sqrt(b)*e^(2*f*x + 2*e) - s
qrt(b*e^(4*f*x + 4*e) + 4*a*e^(2*f*x + 2*e) - 2*b*e^(2*f*x + 2*e) + b))^6*
a*b^(3/2) + 7*(sqrt(b)*e^(2*f*x + 2*e) - sqrt(b*e^(4*f*x + 4*e) + 4*a*e^(2
*f*x + 2*e) - 2*b*e^(2*f*x + 2*e) + b))^6*b^(5/2) - 44*(sqrt(b)*e^(2*f*x +
 2*e) - sqrt(b*e^(4*f*x + 4*e) + 4*a*e^(2*f*x + 2*e) - 2*b*e^(2*f*x + 2*e)
 + b))^5*a^3 - 121*(sqrt(b)*e^(2*f*x + 2*e) - sqrt(b*e^(4*f*x + 4*e) + 4*a
*e^(2*f*x + 2*e) - 2*b*e^(2*f*x + 2*e) + b))^5*a^2*b + 122*(sqrt(b)*e^(2*f
*x + 2*e) - sqrt(b*e^(4*f*x + 4*e) + 4*a*e^(2*f*x + 2*e) - 2*b*e^(2*f*x +
2*e) + b))^5*a*b^2 - 21*(sqrt(b)*e^(2*f*x + 2*e) - sqrt(b*e^(4*f*x + 4*e)
+ 4*a*e^(2*f*x + 2*e) - 2*b*e^(2*f*x + 2*e) + b))^5*b^3 - 292*(sqrt(b)*e^(
2*f*x + 2*e) - sqrt(b*e^(4*f*x + 4*e) + 4*a*e^(2*f*x + 2*e) - 2*b*e^(2*f*x
 + 2*e) + b))^4*a^3*sqrt(b) + 559*(sqrt(b)*e^(2*f*x + 2*e) - sqrt(b*e^(...
```

Mupad [F(-1)]

Timed out.

$$\int \operatorname{csch}^5(e+fx)\sqrt{a+b\sinh^2(e+fx)}\,dx=\int \frac{\sqrt{b\sinh\left(e+f\,x\right)^2+a}}{\sinh\left(e+f\,x\right)^5}\,dx$$

input
```
int((a + b*sinh(e + f*x)^2)^(1/2)/sinh(e + f*x)^5,x)
```

output
```
int((a + b*sinh(e + f*x)^2)^(1/2)/sinh(e + f*x)^5, x)
```

Reduce [F]

$$\int \operatorname{csch}^5(e+fx)\sqrt{a+b\sinh^2(e+fx)}\,dx=\int \sqrt{\sinh(fx+e)^2 b+a}\,\operatorname{csch}(fx+e)^5\,dx$$

input
```
int(csch(f*x+e)^5*(a+b*sinh(f*x+e)^2)^(1/2),x)
```

output
```
int(sqrt(sinh(e + f*x)**2*b + a)*csch(e + f*x)**5,x)
```

3.65 $\int \sinh^4(e+fx)\sqrt{a+b\sinh^2(e+fx)}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 300

$$\begin{aligned}\int \sinh^4(e+fx)\sqrt{a+b\sinh^2(e+fx)}\,dx &= \frac{(a-4b)\cosh(e+fx)\sinh(e+fx)\sqrt{a+b\sinh^2(e+fx)}}{15bf} \\ &+ \frac{\cosh(e+fx)\sinh^3(e+fx)\sqrt{a+b\sinh^2(e+fx)}}{5f} \\ &+ \frac{(2a^2+3ab-8b^2)\,E\left(\arctan(\sinh(e+fx))\,\middle|\,1-\frac{b}{a}\right)\operatorname{sech}(e+fx)\sqrt{a+b\sinh^2(e+fx)}}{15b^2f\sqrt{\frac{\operatorname{sech}^2(e+fx)(a+b\sinh^2(e+fx))}{a}}} \\ &- \frac{(a-4b)\operatorname{EllipticF}\left(\arctan(\sinh(e+fx)),1-\frac{b}{a}\right)\operatorname{sech}(e+fx)\sqrt{a+b\sinh^2(e+fx)}}{15bf\sqrt{\frac{\operatorname{sech}^2(e+fx)(a+b\sinh^2(e+fx))}{a}}} \\ &- \frac{(2a^2+3ab-8b^2)\sqrt{a+b\sinh^2(e+fx)}\tanh(e+fx)}{15b^2f}\end{aligned}$$

output
```
1/15*(a-4*b)*cosh(f*x+e)*sinh(f*x+e)*(a+b*sinh(f*x+e)^2)^(1/2)/b/f+1/5*cos
h(f*x+e)*sinh(f*x+e)^3*(a+b*sinh(f*x+e)^2)^(1/2)/f+1/15*(2*a^2+3*a*b-8*b^2
)*EllipticE(sinh(f*x+e)/(1+sinh(f*x+e)^2)^(1/2),(1-b/a)^(1/2))*sech(f*x+e)
*(a+b*sinh(f*x+e)^2)^(1/2)/b^2/f/(sech(f*x+e)^2*(a+b*sinh(f*x+e)^2)/a)^(1/
2)-1/15*(a-4*b)*InverseJacobiAM(arctan(sinh(f*x+e)),(1-b/a)^(1/2))*sech(f*
x+e)*(a+b*sinh(f*x+e)^2)^(1/2)/b/f/(sech(f*x+e)^2*(a+b*sinh(f*x+e)^2)/a)^(
1/2)-1/15*(2*a^2+3*a*b-8*b^2)*(a+b*sinh(f*x+e)^2)^(1/2)*tanh(f*x+e)/b^2/f
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 1.13 (sec) , antiderivative size = 210, normalized size of antiderivative = 0.70

$$\int \sinh^4(e+fx)\sqrt{a+b\sinh^2(e+fx)}\,dx$$

$$= \frac{16ia(2a^2+3ab-8b^2)\sqrt{\frac{2a-b+b\cosh(2(e+fx))}{a}}E\left(i(e+fx)\,\middle|\,\frac{b}{a}\right)-32ia(a^2+ab-2b^2)\sqrt{\frac{2a-b+b\cosh(2(e+fx))}{a}}\,\mathrm{E}}{240b^2f\sqrt{2}}$$

input
```
Integrate[Sinh[e + f*x]^4*Sqrt[a + b*Sinh[e + f*x]^2],x]
```

output
```
((16*I)*a*(2*a^2 + 3*a*b - 8*b^2)*Sqrt[(2*a - b + b*Cosh[2*(e + f*x)])/a]*
EllipticE[I*(e + f*x), b/a] - (32*I)*a*(a^2 + a*b - 2*b^2)*Sqrt[(2*a - b +
 b*Cosh[2*(e + f*x)])/a]*EllipticF[I*(e + f*x), b/a] + Sqrt[2]*b*(8*a^2 -
48*a*b + 25*b^2 + 4*(4*a - 7*b)*b*Cosh[2*(e + f*x)] + 3*b^2*Cosh[4*(e + f*
x)])*Sinh[2*(e + f*x)])/(240*b^2*f*Sqrt[2*a - b + b*Cosh[2*(e + f*x)]])
```

Rubi [A] (verified)

Time = 0.53 (sec) , antiderivative size = 350, normalized size of antiderivative = 1.17, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.360$, Rules used = {3042, 3667, 380, 444, 25, 406, 320, 388, 313}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sinh^4(e+fx)\sqrt{a+b\sinh^2(e+fx)}\,dx$$

$\downarrow$ 3042

$$\int \sin(ie+ifx)^4\sqrt{a-b\sin(ie+ifx)^2}dx$$

$\downarrow$ 3667

$$\frac{\sqrt{\cosh^2(e+fx)}\text{sech}(e+fx)\int \frac{\sinh^4(e+fx)\sqrt{b\sinh^2(e+fx)+a}}{\sqrt{\sinh^2(e+fx)+1}}d\sinh(e+fx)}{f}$$

$\downarrow$ 380

$$\frac{\sqrt{\cosh^2(e+fx)}\text{sech}(e+fx)\left(\frac{1}{5}\sinh^3(e+fx)\sqrt{\sinh^2(e+fx)+1}\sqrt{a+b\sinh^2(e+fx)}-\frac{1}{5}\int \frac{\sinh^2(e+fx)(3a-(a\cdots}{\sqrt{\sinh^2(e+fx)+1}\sqrt{\cdots}}\right.}{f}$$

$\downarrow$ 444

$$\frac{\sqrt{\cosh^2(e+fx)}\text{sech}(e+fx)\left(\frac{1}{5}\left(\frac{\int -\frac{(2a^2+3ba-8b^2)\sinh^2(e+fx)+a(a-4b)}{\sqrt{\sinh^2(e+fx)+1}\sqrt{b\sinh^2(e+fx)+a}}d\sinh(e+fx)}{3b}+\frac{(a-4b)\sqrt{\sinh^2(e+fx)+1}\sinh(e+fx)\sqrt{a\cdots}}{3b}\right.\right.}{f}$$

$\downarrow$ 25

$$\frac{\sqrt{\cosh^2(e+fx)}\text{sech}(e+fx)\left(\frac{1}{5}\left(\frac{(a-4b)\sinh(e+fx)\sqrt{\sinh^2(e+fx)+1}\sqrt{a+b\sinh^2(e+fx)}}{3b}-\frac{\int \frac{(2a^2+3ba-8b^2)\sinh^2(e+fx)+a(a-4b\cdots}{\sqrt{\sinh^2(e+fx)+1}\sqrt{b\sinh^2(e+fx)+a}}}{3b}\right.\right.}{f}$$

$\downarrow$ 406

$$\sqrt{\cosh^2(e+fx)}\text{sech}(e+fx)\left(\frac{1}{5}\left(\frac{(a-4b)\sinh(e+fx)\sqrt{\sinh^2(e+fx)+1}\sqrt{a+b\sinh^2(e+fx)}}{3b}-\frac{(2a^2+3ab-8b^2)\int \frac{\sinh^2(e\cdots}{\sqrt{\sinh^2(e+fx)+1}\sqrt{b\cdots}}}{\cdots}\right.\right.$$

$\downarrow$ 320

$$\sqrt{\cosh^2(e+fx)}\operatorname{sech}(e+fx)\left(\frac{1}{5}\left(\left(\frac{(a-4b)\sinh(e+fx)\sqrt{\sinh^2(e+fx)+1}\sqrt{a+b\sinh^2(e+fx)}}{3b}-\frac{(2a^2+3ab-8b^2)\int\frac{\sinh^2(e+}{\sqrt{\sinh^2(e+fx)+1}\sqrt{b}}}{}\right.\right.\right.$$

$\downarrow$ 388

$$\sqrt{\cosh^2(e+fx)}\operatorname{sech}(e+fx)\left(\frac{1}{5}\left(\left(\frac{(a-4b)\sinh(e+fx)\sqrt{\sinh^2(e+fx)+1}\sqrt{a+b\sinh^2(e+fx)}}{3b}-\frac{(2a^2+3ab-8b^2)\left(\frac{\sinh(e+fx)\sqrt{a+b\sin}}{b\sqrt{\sinh^2(e+fx}}\right.}{}\right.\right.\right.$$

$\downarrow$ 313

$$\sqrt{\cosh^2(e+fx)}\operatorname{sech}(e+fx)\left(\frac{1}{5}\left(\left(\frac{(a-4b)\sinh(e+fx)\sqrt{\sinh^2(e+fx)+1}\sqrt{a+b\sinh^2(e+fx)}}{3b}-\frac{(2a^2+3ab-8b^2)\left(\frac{\sinh(e+fx)\sqrt{a+b\sin}}{b\sqrt{\sinh^2(e+fx}}\right.}{}\right.\right.\right.$$

input

```
Int[Sinh[e + f*x]^4*Sqrt[a + b*Sinh[e + f*x]^2],x]
```

output

```
(Sqrt[Cosh[e + f*x]^2]*Sech[e + f*x]*((Sinh[e + f*x]^3*Sqrt[1 + Sinh[e + f
*x]^2]*Sqrt[a + b*Sinh[e + f*x]^2])/5 + (((a - 4*b)*Sinh[e + f*x]*Sqrt[1 +
 Sinh[e + f*x]^2]*Sqrt[a + b*Sinh[e + f*x]^2])/(3*b) - (((a - 4*b)*Ellipti
cF[ArcTan[Sinh[e + f*x]], 1 - b/a]*Sqrt[a + b*Sinh[e + f*x]^2])/(Sqrt[1 +
Sinh[e + f*x]^2]*Sqrt[(a + b*Sinh[e + f*x]^2)/(a*(1 + Sinh[e + f*x]^2))])
+ (2*a^2 + 3*a*b - 8*b^2)*((Sinh[e + f*x]*Sqrt[a + b*Sinh[e + f*x]^2])/(b*
Sqrt[1 + Sinh[e + f*x]^2]) - (EllipticE[ArcTan[Sinh[e + f*x]], 1 - b/a]*Sq
rt[a + b*Sinh[e + f*x]^2])/(b*Sqrt[1 + Sinh[e + f*x]^2]*Sqrt[(a + b*Sinh[e
 + f*x]^2)/(a*(1 + Sinh[e + f*x]^2))])))/(3*b))/5))/f
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 313
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/((c_) + (d_.)*(x_)^2)^(3/2), x_Symbol] :> Sim
p[(Sqrt[a + b*x^2]/(c*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(c
+ d*x^2)))]))*EllipticE[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; FreeQ
[{a, b, c, d}, x] && PosQ[b/a] && PosQ[d/c]
```

rule 320
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(Sqrt[a + b*x^2]/(a*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(
c + d*x^2)))]))*EllipticF[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; Fre
eQ[{a, b, c, d}, x] && PosQ[d/c] && PosQ[b/a] &&  !SimplerSqrtQ[b/a, d/c]
```

rule 380
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_
), x_Symbol] :> Simp[e*(e*x)^(m - 1)*(a + b*x^2)^(p + 1)*((c + d*x^2)^q/(b*
(m + 2*(p + q) + 1))), x] - Simp[e^2/(b*(m + 2*(p + q) + 1))   Int[(e*x)^(m
 - 2)*(a + b*x^2)^p*(c + d*x^2)^(q - 1)*Simp[a*c*(m - 1) + (a*d*(m - 1) - 2
*q*(b*c - a*d))*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c
- a*d, 0] && GtQ[q, 0] && GtQ[m, 1] && IntBinomialQ[a, b, c, d, e, m, 2, p,
 q, x]
```

rule 388
```
Int[(x_)^2/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol]
 :> Simp[x*(Sqrt[a + b*x^2]/(b*Sqrt[c + d*x^2])), x] - Simp[c/b   Int[Sqrt[
a + b*x^2]/(c + d*x^2)^(3/2), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
 a*d, 0] && PosQ[b/a] && PosQ[d/c] &&  !SimplerSqrtQ[b/a, d/c]
```

rule 406
```
Int[((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.)*((e_) + (f_.)*(
x_)^2), x_Symbol] :> Simp[e   Int[(a + b*x^2)^p*(c + d*x^2)^q, x], x] + Sim
p[f   Int[x^2*(a + b*x^2)^p*(c + d*x^2)^q, x], x] /; FreeQ[{a, b, c, d, e,
f, p, q}, x]
```

rule 444
```
Int[((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q
_.)*((e_) + (f_.)*(x_)^2), x_Symbol] :> Simp[f*g*(g*x)^(m - 1)*(a + b*x^2)^
(p + 1)*((c + d*x^2)^(q + 1)/(b*d*(m + 2*(p + q + 1) + 1))), x] - Simp[g^2/
(b*d*(m + 2*(p + q + 1) + 1))   Int[(g*x)^(m - 2)*(a + b*x^2)^p*(c + d*x^2)
^q*Simp[a*f*c*(m - 1) + (a*f*d*(m + 2*q + 1) + b*(f*c*(m + 2*p + 1) - e*d*(
m + 2*(p + q + 1) + 1)))*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p,
q}, x] && GtQ[m, 1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3667
```
Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(
p_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Simp[ff^(m + 1
)*(Sqrt[Cos[e + f*x]^2]/(f*Cos[e + f*x]))   Subst[Int[x^m*((a + b*ff^2*x^2)
^p/Sqrt[1 - ff^2*x^2]), x], x, Sin[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f,
p}, x] && IntegerQ[m/2] &&  !IntegerQ[p]
```

Maple [A] (verified)

Time = 5.17 (sec) , antiderivative size = 512, normalized size of antiderivative = 1.71

method	result
default	$\frac{3\sqrt{-\frac{b}{a}}\, b^2 \sinh(fx+e)^7+4\sqrt{-\frac{b}{a}}\, ab \sinh(fx+e)^5-\sqrt{-\frac{b}{a}}\, b^2 \sinh(fx+e)^5+\sqrt{-\frac{b}{a}}\, a^2 \sinh(fx+e)^3-4\sqrt{-\frac{b}{a}}\, b^2 \sinh(fx+e)^3+a^2\sqrt{\frac{a+b}{}}}{}$

input
```
int(sinh(f*x+e)^4*(a+b*sinh(f*x+e)^2)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/15*(3*(-b/a)^(1/2)*b^2*sinh(f*x+e)^7+4*(-b/a)^(1/2)*a*b*sinh(f*x+e)^5-(-
b/a)^(1/2)*b^2*sinh(f*x+e)^5+(-b/a)^(1/2)*a^2*sinh(f*x+e)^3-4*(-b/a)^(1/2)
*b^2*sinh(f*x+e)^3+a^2*((a+b*sinh(f*x+e)^2)/a)^(1/2)*(cosh(f*x+e)^2)^(1/2)
*EllipticF(sinh(f*x+e)*(-b/a)^(1/2),(1/b*a)^(1/2))+7*a*((a+b*sinh(f*x+e)^2
)/a)^(1/2)*(cosh(f*x+e)^2)^(1/2)*EllipticF(sinh(f*x+e)*(-b/a)^(1/2),(1/b*a
)^(1/2))*b-8*((a+b*sinh(f*x+e)^2)/a)^(1/2)*(cosh(f*x+e)^2)^(1/2)*EllipticF
(sinh(f*x+e)*(-b/a)^(1/2),(1/b*a)^(1/2))*b^2-2*((a+b*sinh(f*x+e)^2)/a)^(1/
2)*(cosh(f*x+e)^2)^(1/2)*EllipticE(sinh(f*x+e)*(-b/a)^(1/2),(1/b*a)^(1/2))
*a^2-3*((a+b*sinh(f*x+e)^2)/a)^(1/2)*(cosh(f*x+e)^2)^(1/2)*EllipticE(sinh(
f*x+e)*(-b/a)^(1/2),(1/b*a)^(1/2))*a*b+8*((a+b*sinh(f*x+e)^2)/a)^(1/2)*(co
sh(f*x+e)^2)^(1/2)*EllipticE(sinh(f*x+e)*(-b/a)^(1/2),(1/b*a)^(1/2))*b^2+(
-b/a)^(1/2)*a^2*sinh(f*x+e)-4*(-b/a)^(1/2)*a*b*sinh(f*x+e))/b/(-b/a)^(1/2)
/cosh(f*x+e)/(a+b*sinh(f*x+e)^2)^(1/2)/f
```

Fricas [F]

$$\int \sinh^4(e+fx)\sqrt{a+b\sinh^2(e+fx)}\,dx = \int \sqrt{b\sinh\left(fx+e\right)^2+a}\sinh\left(fx+e\right)^4\,dx$$

input
```
integrate(sinh(f*x+e)^4*(a+b*sinh(f*x+e)^2)^(1/2),x, algorithm="fricas")
```

output
```
integral(sqrt(b*sinh(f*x + e)^2 + a)*sinh(f*x + e)^4, x)
```

Sympy [F(-1)]

Timed out.

$$\int \sinh^4(e+fx)\sqrt{a+b\sinh^2(e+fx)}\,dx = \text{Timed out}$$

input
```
integrate(sinh(f*x+e)**4*(a+b*sinh(f*x+e)**2)**(1/2),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \sinh^4(e+fx)\sqrt{a+b\sinh^2(e+fx)}\,dx=\int \sqrt{b\sinh\left(fx+e\right)^2+a}\sinh\left(fx+e\right)^4\,dx$$

input
```
integrate(sinh(f*x+e)^4*(a+b*sinh(f*x+e)^2)^(1/2),x, algorithm="maxima")
```

output
```
integrate(sqrt(b*sinh(f*x + e)^2 + a)*sinh(f*x + e)^4, x)
```

Giac [F]

$$\int \sinh^4(e+fx)\sqrt{a+b\sinh^2(e+fx)}\,dx=\int \sqrt{b\sinh\left(fx+e\right)^2+a}\sinh\left(fx+e\right)^4\,dx$$

input
```
integrate(sinh(f*x+e)^4*(a+b*sinh(f*x+e)^2)^(1/2),x, algorithm="giac")
```

output
```
integrate(sqrt(b*sinh(f*x + e)^2 + a)*sinh(f*x + e)^4, x)
```

Mupad [F(-1)]

Timed out.

$$\int \sinh^4(e+fx)\sqrt{a+b\sinh^2(e+fx)}\,dx=\int \sinh(e+f\,x)^4\,\sqrt{b\sinh\left(e+f\,x\right)^2+a}\,dx$$

input
```
int(sinh(e + f*x)^4*(a + b*sinh(e + f*x)^2)^(1/2),x)
```

output
```
int(sinh(e + f*x)^4*(a + b*sinh(e + f*x)^2)^(1/2), x)
```

Reduce [F]

$$\int \sinh^4(e+fx)\sqrt{a+b\sinh^2(e+fx)}\,dx=\int \sqrt{\sinh\left(fx+e\right)^2b+a}\,\sinh\left(fx+e\right)^4dx$$

input
```
int(sinh(f*x+e)^4*(a+b*sinh(f*x+e)^2)^(1/2),x)
```

output
```
int(sqrt(sinh(e + f*x)**2*b + a)*sinh(e + f*x)**4,x)
```

3.66 $\int \sinh^2(e+fx)\sqrt{a+b\sinh^2(e+fx)}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 179

$$\int \sinh^2(e+fx)\sqrt{a+b\sinh^2(e+fx)}\,dx = \frac{\cosh(e+fx)\sinh(e+fx)\sqrt{a+b\sinh^2(e+fx)}}{3f} - \frac{i(a-2b)E\left(ie+ifx\middle|\frac{b}{a}\right)\sqrt{a+b\sinh^2(e+fx)}}{3bf\sqrt{\frac{a+b\sinh^2(e+fx)}{a}}} + \frac{ia(a-b)\operatorname{EllipticF}\left(ie+ifx,\frac{b}{a}\right)\sqrt{\frac{a+b\sinh^2(e+fx)}{a}}}{3bf\sqrt{a+b\sinh^2(e+fx)}}$$

output
```
1/3*cosh(f*x+e)*sinh(f*x+e)*(a+b*sinh(f*x+e)^2)^(1/2)/f-1/3*I*(a-2*b)*Elli
pticE(sin(I*e+I*f*x),(b/a)^(1/2))*(a+b*sinh(f*x+e)^2)^(1/2)/b/f/((a+b*sinh
(f*x+e)^2)/a)^(1/2)+1/3*I*a*(a-b)*InverseJacobiAM(I*e+I*f*x,(b/a)^(1/2))*(
(a+b*sinh(f*x+e)^2)/a)^(1/2)/b/f/(a+b*sinh(f*x+e)^2)^(1/2)
```

Mathematica [A] (verified)

Time = 0.77 (sec) , antiderivative size = 170, normalized size of antiderivative = 0.95

$$\int \sinh^2(e+fx)\sqrt{a+b\sinh^2(e+fx)}\,dx$$
$$= \frac{-2i\sqrt{2}a(a-2b)\sqrt{\frac{2a-b+b\cosh(2(e+fx))}{a}}E\left(i(e+fx)\left|\frac{b}{a}\right.\right)+2i\sqrt{2}a(a-b)\sqrt{\frac{2a-b+b\cosh(2(e+fx))}{a}}\,\text{EllipticF}\left(i(e\right.}{6bf\sqrt{4a-2b+2b\cosh(2(e+fx))}}$$

input
```
Integrate[Sinh[e + f*x]^2*Sqrt[a + b*Sinh[e + f*x]^2],x]
```

output
```
((-2*I)*Sqrt[2]*a*(a - 2*b)*Sqrt[(2*a - b + b*Cosh[2*(e + f*x)])/a]*Ellipt
icE[I*(e + f*x), b/a] + (2*I)*Sqrt[2]*a*(a - b)*Sqrt[(2*a - b + b*Cosh[2*(
e + f*x)])/a]*EllipticF[I*(e + f*x), b/a] + b*(2*a - b + b*Cosh[2*(e + f*x
)])*Sinh[2*(e + f*x)])/(6*b*f*Sqrt[4*a - 2*b + 2*b*Cosh[2*(e + f*x)]])
```

Rubi [A] (verified)

Time = 0.99 (sec) , antiderivative size = 178, normalized size of antiderivative = 0.99, number of steps used = 12, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.480$, Rules used = {3042, 25, 3649, 3042, 3651, 3042, 3657, 3042, 3656, 3662, 3042, 3661}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sinh^2(e+fx)\sqrt{a+b\sinh^2(e+fx)}\,dx$$
$$\Big\downarrow \text{3042}$$
$$\int -\sin(ie+ifx)^2\sqrt{a-b\sin(ie+ifx)^2}dx$$
$$\Big\downarrow \text{25}$$
$$-\int \sin(ie+ifx)^2\sqrt{a-b\sin(ie+ifx)^2}dx$$
$$\Big\downarrow \text{3649}$$

$$\frac{\sinh(e+fx)\cosh(e+fx)\sqrt{a+b\sinh^2(e+fx)}}{3f}-\frac{1}{3}\int\frac{a-(a-2b)\sinh^2(e+fx)}{\sqrt{b\sinh^2(e+fx)+a}}dx$$

$$\Big\downarrow \text{3042}$$

$$\frac{\sinh(e+fx)\cosh(e+fx)\sqrt{a+b\sinh^2(e+fx)}}{3f}-\frac{1}{3}\int\frac{(a-2b)\sin(ie+ifx)^2+a}{\sqrt{a-b\sin(ie+ifx)^2}}dx$$

$$\Big\downarrow \text{3651}$$

$$\frac{1}{3}\left(\frac{(a-2b)\int\sqrt{b\sinh^2(e+fx)+a}dx}{b}-\frac{a(a-b)\int\frac{1}{\sqrt{b\sinh^2(e+fx)+a}}dx}{b}\right)+\frac{\sinh(e+fx)\cosh(e+fx)\sqrt{a+b\sinh^2(e+fx)}}{3f}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\sinh(e+fx)\cosh(e+fx)\sqrt{a+b\sinh^2(e+fx)}}{3f}+\frac{1}{3}\left(\frac{(a-2b)\int\sqrt{a-b\sin(ie+ifx)^2}dx}{b}-\frac{a(a-b)\int\frac{1}{\sqrt{a-b\sin(ie+ifx)^2}}dx}{b}\right)$$

$$\Big\downarrow \text{3657}$$

$$\frac{\sinh(e+fx)\cosh(e+fx)\sqrt{a+b\sinh^2(e+fx)}}{3f}+\frac{1}{3}\left(\frac{(a-2b)\sqrt{a+b\sinh^2(e+fx)}\int\sqrt{\frac{b\sinh^2(e+fx)}{a}+1}dx}{b\sqrt{\frac{b\sinh^2(e+fx)}{a}+1}}-\frac{a(a-b)\int\frac{1}{\sqrt{a-b\sin(ie+ifx)^2}}dx}{b}\right)$$

$$\Big\downarrow \text{3042}$$

$$\frac{\sinh(e+fx)\cosh(e+fx)\sqrt{a+b\sinh^2(e+fx)}}{3f}+\frac{1}{3}\left(\frac{(a-2b)\sqrt{a+b\sinh^2(e+fx)}\int\sqrt{1-\frac{b\sin(ie+ifx)^2}{a}}dx}{b\sqrt{\frac{b\sinh^2(e+fx)}{a}+1}}-\frac{a(a-b)\int\frac{1}{\sqrt{a-b\sin(ie+ifx)^2}}dx}{b}\right)$$

$$\Big\downarrow \text{3656}$$

$$\frac{\sinh(e+fx)\cosh(e+fx)\sqrt{a+b\sinh^2(e+fx)}}{3f}+\frac{1}{3}\left(-\frac{a(a-b)\int\frac{1}{\sqrt{a-b\sin(ie+ifx)^2}}dx}{b}-\frac{i(a-2b)\sqrt{a+b\sinh^2(e+fx)}E\left(ie+ifx|\frac{b}{a}\right)}{bf\sqrt{\frac{b\sinh^2(e+fx)}{a}+1}}\right)$$

$$\Big\downarrow \text{3662}$$

$$\frac{\sinh(e+fx)\cosh(e+fx)\sqrt{a+b\sinh^2(e+fx)}}{3f}+\frac{1}{3}\left(-\frac{a(a-b)\sqrt{\frac{b\sinh^2(e+fx)}{a}+1}\int\frac{1}{\sqrt{\frac{b\sinh^2(e+fx)}{a}+1}}dx}{b\sqrt{a+b\sinh^2(e+fx)}}-\frac{i(a-2b)\sqrt{a+b\sinh^2(e+fx)}E\left(ie+ifx|\frac{b}{a}\right)}{bf\sqrt{\frac{b\sinh^2(e+fx)}{a}+1}}\right)$$

$$\Big\downarrow \text{3042}$$

$$\frac{\sinh(e+fx)\cosh(e+fx)\sqrt{a+b\sinh^2(e+fx)}}{3f}+\frac{1}{3}\left(-\frac{a(a-b)\sqrt{\frac{b\sinh^2(e+fx)}{a}+1}\int\frac{1}{\sqrt{1-\frac{b\sin(ie+ifx)^2}{a}}}dx}{b\sqrt{a+b\sinh^2(e+fx)}}-\frac{i(a-2b)\sqrt{a+b\sinh^2(e+fx)}E\left(ie+ifx|\frac{b}{a}\right)}{bf\sqrt{\frac{b\sinh^2(e+fx)}{a}+1}}\right)$$

$$\Big\downarrow \text{3661}$$

$$\frac{\sinh(e+fx)\cosh(e+fx)\sqrt{a+b\sinh^2(e+fx)}}{3f}+\frac{1}{3}\left(\frac{ia(a-b)\sqrt{\frac{b\sinh^2(e+fx)}{a}+1}\,\text{EllipticF}\left(ie+ifx,\frac{b}{a}\right)}{bf\sqrt{a+b\sinh^2(e+fx)}}-\frac{i(a-2b)\sqrt{a+b\sinh^2(e+fx)}E\left(ie+ifx|\frac{b}{a}\right)}{bf\sqrt{\frac{b\sinh^2(e+fx)}{a}+1}}\right)$$

input
```
Int[Sinh[e + f*x]^2*Sqrt[a + b*Sinh[e + f*x]^2],x]
```

output
```
(Cosh[e + f*x]*Sinh[e + f*x]*Sqrt[a + b*Sinh[e + f*x]^2])/(3*f) + (((-I)*(
a - 2*b)*EllipticE[I*e + I*f*x, b/a]*Sqrt[a + b*Sinh[e + f*x]^2])/(b*f*Sqr
t[1 + (b*Sinh[e + f*x]^2)/a]) + (I*a*(a - b)*EllipticF[I*e + I*f*x, b/a]*S
qrt[1 + (b*Sinh[e + f*x]^2)/a])/(b*f*Sqrt[a + b*Sinh[e + f*x]^2]))/3
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3649
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)]^2), x_Symbol] :> Simp[(-B)*Cos[e + f*x]*Sin[e + f*x]*((a + b*
Sin[e + f*x]^2)^p/(2*f*(p + 1))), x] + Simp[1/(2*(p + 1))   Int[(a + b*Sin[
e + f*x]^2)^(p - 1)*Simp[a*B + 2*a*A*(p + 1) + (2*A*b*(p + 1) + B*(b + 2*a*
p + 2*b*p))*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, e, f, A, B}, x] && G
tQ[p, 0]
```

rule 3651
```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)]^2)/Sqrt[(a_) + (b_.)*sin[(e_.) +
 (f_.)*(x_)]^2], x_Symbol] :> Simp[B/b   Int[Sqrt[a + b*Sin[e + f*x]^2], x]
, x] + Simp[(A*b - a*B)/b   Int[1/Sqrt[a + b*Sin[e + f*x]^2], x], x] /; Fre
eQ[{a, b, e, f, A, B}, x]
```

rule 3656
```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2], x_Symbol] :> Simp[(Sqrt[a
]/f)*EllipticE[e + f*x, -b/a], x] /; FreeQ[{a, b, e, f}, x] && GtQ[a, 0]
```

rule 3657
```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2], x_Symbol] :> Simp[Sqrt[a
+ b*Sin[e + f*x]^2]/Sqrt[1 + b*(Sin[e + f*x]^2/a)]   Int[Sqrt[1 + (b*Sin[e
+ f*x]^2)/a], x], x] /; FreeQ[{a, b, e, f}, x] &&  !GtQ[a, 0]
```

rule 3661
```
Int[1/Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2], x_Symbol] :> Simp[(1/(S
qrt[a]*f))*EllipticF[e + f*x, -b/a], x] /; FreeQ[{a, b, e, f}, x] && GtQ[a,
 0]
```

rule 3662
```
Int[1/Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2], x_Symbol] :> Simp[Sqrt[
1 + b*(Sin[e + f*x]^2/a)]/Sqrt[a + b*Sin[e + f*x]^2]   Int[1/Sqrt[1 + (b*Si
n[e + f*x]^2)/a], x], x] /; FreeQ[{a, b, e, f}, x] &&  !GtQ[a, 0]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 354 vs. 2(162) = 324.

Time = 3.42 (sec) , antiderivative size = 355, normalized size of antiderivative = 1.98

method	result
default	$-\frac{-\sqrt{-\frac{b}{a}}\, b\cosh(fx+e)^4\sinh(fx+e)-\sqrt{-\frac{b}{a}}\, a\cosh(fx+e)^2\sinh(fx+e)+\sqrt{-\frac{b}{a}}\, b\cosh(fx+e)^2\sinh(fx+e)+2a\sqrt{\frac{b\cosh(fx+e)^2}{a}+}}{}$

input
```
int(sinh(f*x+e)^2*(a+b*sinh(f*x+e)^2)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-1/3*(-(-b/a)^(1/2)*b*cosh(f*x+e)^4*sinh(f*x+e)-(-b/a)^(1/2)*a*cosh(f*x+e)
^2*sinh(f*x+e)+(-b/a)^(1/2)*b*cosh(f*x+e)^2*sinh(f*x+e)+2*a*(b/a*cosh(f*x+
e)^2+(a-b)/a)^(1/2)*(cosh(f*x+e)^2)^(1/2)*EllipticF(sinh(f*x+e)*(-b/a)^(1/
2),(1/b*a)^(1/2))-2*(b/a*cosh(f*x+e)^2+(a-b)/a)^(1/2)*(cosh(f*x+e)^2)^(1/2
)*EllipticF(sinh(f*x+e)*(-b/a)^(1/2),(1/b*a)^(1/2))*b-(b/a*cosh(f*x+e)^2+(
a-b)/a)^(1/2)*(cosh(f*x+e)^2)^(1/2)*EllipticE(sinh(f*x+e)*(-b/a)^(1/2),(1/
b*a)^(1/2))*a+2*(b/a*cosh(f*x+e)^2+(a-b)/a)^(1/2)*(cosh(f*x+e)^2)^(1/2)*El
lipticE(sinh(f*x+e)*(-b/a)^(1/2),(1/b*a)^(1/2))*b)/(-b/a)^(1/2)/cosh(f*x+e
)/(a+b*sinh(f*x+e)^2)^(1/2)/f
```

Fricas [F]

$$\int \sinh^2(e+fx)\sqrt{a+b\sinh^2(e+fx)}\,dx=\int \sqrt{b\sinh{(fx+e)}^2+a}\sinh{(fx+e)}^2\,dx$$

input
```
integrate(sinh(f*x+e)^2*(a+b*sinh(f*x+e)^2)^(1/2),x, algorithm="fricas")
```

output
```
integral(sqrt(b*sinh(f*x + e)^2 + a)*sinh(f*x + e)^2, x)
```

Sympy [F]

$$\int \sinh^2(e+fx)\sqrt{a+b\sinh^2(e+fx)}\,dx=\int \sqrt{a+b\sinh^2(e+fx)}\sinh^2(e+fx)\,dx$$

input
```
integrate(sinh(f*x+e)**2*(a+b*sinh(f*x+e)**2)**(1/2),x)
```

output
```
Integral(sqrt(a + b*sinh(e + f*x)**2)*sinh(e + f*x)**2, x)
```

Maxima [F]

$$\int \sinh^2(e+fx)\sqrt{a+b\sinh^2(e+fx)}\,dx=\int \sqrt{b\sinh(fx+e)^2+a}\sinh(fx+e)^2\,dx$$

input
```
integrate(sinh(f*x+e)^2*(a+b*sinh(f*x+e)^2)^(1/2),x, algorithm="maxima")
```

output
```
integrate(sqrt(b*sinh(f*x + e)^2 + a)*sinh(f*x + e)^2, x)
```

Giac [F]

$$\int \sinh^2(e+fx)\sqrt{a+b\sinh^2(e+fx)}\,dx=\int \sqrt{b\sinh(fx+e)^2+a}\sinh(fx+e)^2\,dx$$

input
```
integrate(sinh(f*x+e)^2*(a+b*sinh(f*x+e)^2)^(1/2),x, algorithm="giac")
```

output
```
integrate(sqrt(b*sinh(f*x + e)^2 + a)*sinh(f*x + e)^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int \sinh^2(e+fx)\sqrt{a+b\sinh^2(e+fx)}\,dx = \int \sinh(e+f\,x)^2\,\sqrt{b\sinh\left(e+f\,x\right)^2+a}\,dx$$

input
```
int(sinh(e + f*x)^2*(a + b*sinh(e + f*x)^2)^(1/2),x)
```

output
```
int(sinh(e + f*x)^2*(a + b*sinh(e + f*x)^2)^(1/2), x)
```

Reduce [F]

$$\int \sinh^2(e+fx)\sqrt{a+b\sinh^2(e+fx)}\,dx = \int \sqrt{\sinh\left(fx+e\right)^2 b+a}\,\sinh\left(fx+e\right)^2 dx$$

input
```
int(sinh(f*x+e)^2*(a+b*sinh(f*x+e)^2)^(1/2),x)
```

output
```
int(sqrt(sinh(e + f*x)**2*b + a)*sinh(e + f*x)**2,x)
```

3.67 $\int \sqrt{a+b\sinh^2(e+fx)}\,dx$

Optimal result

Integrand size = 16, antiderivative size = 61

$$\int \sqrt{a+b\sinh^2(e+fx)}\,dx = -\frac{iE\left(ie+ifx\middle|\frac{b}{a}\right)\sqrt{a+b\sinh^2(e+fx)}}{f\sqrt{\frac{a+b\sinh^2(e+fx)}{a}}}$$

output
```
-I*EllipticE(sin(I*e+I*f*x),(b/a)^(1/2))*(a+b*sinh(f*x+e)^2)^(1/2)/f/((a+b
*sinh(f*x+e)^2)/a)^(1/2)
```

Mathematica [A] (verified)

Time = 0.10 (sec) , antiderivative size = 69, normalized size of antiderivative = 1.13

$$\int \sqrt{a+b\sinh^2(e+fx)}\,dx = -\frac{ia\sqrt{\frac{2a-b+b\cosh(2(e+fx))}{a}}E\left(i(e+fx)\middle|\frac{b}{a}\right)}{f\sqrt{2a-b+b\cosh(2(e+fx))}}$$

input
```
Integrate[Sqrt[a + b*Sinh[e + f*x]^2],x]
```

output
```
((-I)*a*Sqrt[(2*a - b + b*Cosh[2*(e + f*x)])/a]*EllipticE[I*(e + f*x), b/a
])/(f*Sqrt[2*a - b + b*Cosh[2*(e + f*x)]])
```

Rubi [A] (verified)

Time = 0.33 (sec) , antiderivative size = 60, normalized size of antiderivative = 0.98, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {3042, 3657, 3042, 3656}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{a+b\sinh^2(e+fx)}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \sqrt{a-b\sin(ie+ifx)^2}dx$$

$$\Big\downarrow \text{3657}$$

$$\frac{\sqrt{a+b\sinh^2(e+fx)}\int \sqrt{\frac{b\sinh^2(e+fx)}{a}+1}dx}{\sqrt{\frac{b\sinh^2(e+fx)}{a}+1}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\sqrt{a+b\sinh^2(e+fx)}\int \sqrt{1-\frac{b\sin(ie+ifx)^2}{a}}dx}{\sqrt{\frac{b\sinh^2(e+fx)}{a}+1}}$$

$$\Big\downarrow \text{3656}$$

$$-\frac{i\sqrt{a+b\sinh^2(e+fx)}E\left(ie+ifx|\frac{b}{a}\right)}{f\sqrt{\frac{b\sinh^2(e+fx)}{a}+1}}$$

input
```
Int[Sqrt[a + b*Sinh[e + f*x]^2],x]
```

output
```
((-I)*EllipticE[I*e + I*f*x, b/a]*Sqrt[a + b*Sinh[e + f*x]^2])/(f*Sqrt[1 +
 (b*Sinh[e + f*x]^2)/a])
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3656
```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2], x_Symbol] :> Simp[(Sqrt[a
]/f)*EllipticE[e + f*x, -b/a], x] /; FreeQ[{a, b, e, f}, x] && GtQ[a, 0]
```

rule 3657
```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2], x_Symbol] :> Simp[Sqrt[a
+ b*Sin[e + f*x]^2]/Sqrt[1 + b*(Sin[e + f*x]^2/a)]   Int[Sqrt[1 + (b*Sin[e
+ f*x]^2)/a], x], x] /; FreeQ[{a, b, e, f}, x] &&  !GtQ[a, 0]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 139 vs. $2(57) = 114$.

Time = 2.06 (sec) , antiderivative size = 140, normalized size of antiderivative = 2.30

method	result
default	$\frac{\sqrt{\frac{a+b\sinh(fx+e)^2}{a}}\,\sqrt{\frac{\cosh(2fx+2e)}{2}+\frac{1}{2}}\,\left(a\,\mathrm{EllipticF}\left(\sinh(fx+e)\sqrt{-\frac{b}{a}},\sqrt{\frac{a}{b}}\right)-b\,\mathrm{EllipticF}\left(\sinh(fx+e)\sqrt{-\frac{b}{a}},\sqrt{\frac{a}{b}}\right)+b\,\mathrm{EllipticE}\Big(\mathrm{si}\right.}{\sqrt{-\frac{b}{a}}\,\cosh(fx+e)\sqrt{a+b\sinh(fx+e)^2}\,f}$

input
```
int((a+b*sinh(f*x+e)^2)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
((a+b*sinh(f*x+e)^2)/a)^(1/2)*(cosh(f*x+e)^2)^(1/2)*(a*EllipticF(sinh(f*x+
e)*(-b/a)^(1/2),(1/b*a)^(1/2))-b*EllipticF(sinh(f*x+e)*(-b/a)^(1/2),(1/b*a
)^(1/2))+b*EllipticE(sinh(f*x+e)*(-b/a)^(1/2),(1/b*a)^(1/2)))/(-b/a)^(1/2)
/cosh(f*x+e)/(a+b*sinh(f*x+e)^2)^(1/2)/f
```

Fricas [F]

$$\int \sqrt{a+b\sinh^2(e+fx)}\,dx = \int \sqrt{b\sinh{(fx+e)}^2+a}\,dx$$

input
```
integrate((a+b*sinh(f*x+e)^2)^(1/2),x, algorithm="fricas")
```

output
```
integral(sqrt(b*sinh(f*x + e)^2 + a), x)
```

Sympy [F]

$$\int \sqrt{a+b\sinh^2(e+fx)}\,dx = \int \sqrt{a+b\sinh^2{(e+fx)}}\,dx$$

input
```
integrate((a+b*sinh(f*x+e)**2)**(1/2),x)
```

output
```
Integral(sqrt(a + b*sinh(e + f*x)**2), x)
```

Maxima [F]

$$\int \sqrt{a+b\sinh^2(e+fx)}\,dx = \int \sqrt{b\sinh{(fx+e)}^2+a}\,dx$$

input
```
integrate((a+b*sinh(f*x+e)^2)^(1/2),x, algorithm="maxima")
```

output
```
integrate(sqrt(b*sinh(f*x + e)^2 + a), x)
```

Giac [F]

$$\int \sqrt{a+b\sinh^2(e+fx)}\,dx=\int \sqrt{b\sinh{(fx+e)}^2+a}\,dx$$

input
```
integrate((a+b*sinh(f*x+e)^2)^(1/2),x, algorithm="giac")
```

output
```
integrate(sqrt(b*sinh(f*x + e)^2 + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int \sqrt{a+b\sinh^2(e+fx)}\,dx=\int \sqrt{b\sinh{(e+f\,x)}^2+a}\,dx$$

input
```
int((a + b*sinh(e + f*x)^2)^(1/2),x)
```

output
```
int((a + b*sinh(e + f*x)^2)^(1/2), x)
```

Reduce [F]

$$\int \sqrt{a+b\sinh^2(e+fx)}\,dx=\int \sqrt{\sinh{(fx+e)}^2 b+a}dx$$

input
```
int((a+b*sinh(f*x+e)^2)^(1/2),x)
```

output
```
int(sqrt(sinh(e + f*x)**2*b + a),x)
```

3.68 $\int \operatorname{csch}^2(e+fx)\sqrt{a+b\sinh^2(e+fx)}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 179

$$\begin{aligned}&\int \operatorname{csch}^2(e+fx)\sqrt{a+b\sinh^2(e+fx)}\,dx\\&=-\frac{\operatorname{csch}(e+fx)\operatorname{sech}(e+fx)\sqrt{a+b\sinh^2(e+fx)}}{f}\\&\quad-\frac{E\left(\arctan(\sinh(e+fx))\left|1-\frac{b}{a}\right.\right)\operatorname{sech}(e+fx)\sqrt{a+b\sinh^2(e+fx)}}{f\sqrt{\frac{\operatorname{sech}^2(e+fx)\left(a+b\sinh^2(e+fx)\right)}{a}}}\\&\quad+\frac{b\operatorname{EllipticF}\left(\arctan(\sinh(e+fx)),1-\frac{b}{a}\right)\operatorname{sech}(e+fx)\sqrt{a+b\sinh^2(e+fx)}}{af\sqrt{\frac{\operatorname{sech}^2(e+fx)\left(a+b\sinh^2(e+fx)\right)}{a}}}\end{aligned}$$

output
```
-csch(f*x+e)*sech(f*x+e)*(a+b*sinh(f*x+e)^2)^(1/2)/f-EllipticE(sinh(f*x+e)
/(1+sinh(f*x+e)^2)^(1/2),(1-b/a)^(1/2))*sech(f*x+e)*(a+b*sinh(f*x+e)^2)^(1
/2)/f/(sech(f*x+e)^2*(a+b*sinh(f*x+e)^2)/a)^(1/2)+b*InverseJacobiAM(arctan
(sinh(f*x+e)),(1-b/a)^(1/2))*sech(f*x+e)*(a+b*sinh(f*x+e)^2)^(1/2)/a/f/(se
ch(f*x+e)^2*(a+b*sinh(f*x+e)^2)/a)^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.67 (sec) , antiderivative size = 151, normalized size of antiderivative = 0.84

$$\int \operatorname{csch}^2(e+fx)\sqrt{a+b\sinh^2(e+fx)}\,dx$$
$$= \frac{\sqrt{2}(-2a+b-b\cosh(2(e+fx)))\coth(e+fx) - 2ia\sqrt{\frac{2a-b+b\cosh(2(e+fx))}{a}}E\left(i(e+fx)\,\middle|\,\frac{b}{a}\right) + 2i(a-b)\sqrt{}}{2f\sqrt{2a-b+b\cosh(2(e+fx))}}$$

input
```
Integrate[Csch[e + f*x]^2*Sqrt[a + b*Sinh[e + f*x]^2],x]
```

output
```
(Sqrt[2]*(-2*a + b - b*Cosh[2*(e + f*x)])*Coth[e + f*x] - (2*I)*a*Sqrt[(2*
a - b + b*Cosh[2*(e + f*x)])/a]*EllipticE[I*(e + f*x), b/a] + (2*I)*(a - b
)*Sqrt[(2*a - b + b*Cosh[2*(e + f*x)])/a]*EllipticF[I*(e + f*x), b/a])/(2*
f*Sqrt[2*a - b + b*Cosh[2*(e + f*x)]])
```

Rubi [A] (verified)

Time = 0.49 (sec) , antiderivative size = 269, normalized size of antiderivative = 1.50, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.360$, Rules used = {3042, 25, 3667, 377, 27, 324, 320, 388, 313}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \operatorname{csch}^2(e+fx)\sqrt{a+b\sinh^2(e+fx)}\,dx$$
$$\Big\downarrow \text{3042}$$
$$\int -\frac{\sqrt{a-b\sin(ie+ifx)^2}}{\sin(ie+ifx)^2}dx$$
$$\Big\downarrow \text{25}$$
$$-\int \frac{\sqrt{a-b\sin(ie+ifx)^2}}{\sin(ie+ifx)^2}dx$$

$\downarrow$ 3667

$$\frac{\sqrt{\cosh^2(e+fx)}\text{sech}(e+fx)\int \frac{\text{csch}^2(e+fx)\sqrt{b\sinh^2(e+fx)+a}}{\sqrt{\sinh^2(e+fx)+1}}d\sinh(e+fx)}{f}$$

$\downarrow$ 377

$$\frac{\sqrt{\cosh^2(e+fx)}\text{sech}(e+fx)\left(\int \frac{b\sqrt{\sinh^2(e+fx)+1}}{\sqrt{b\sinh^2(e+fx)+a}}d\sinh(e+fx)-\sqrt{\sinh^2(e+fx)+1}\text{csch}(e+fx)\sqrt{a+b\sinh^2}\right.}{f}$$

$\downarrow$ 27

$$\frac{\sqrt{\cosh^2(e+fx)}\text{sech}(e+fx)\left(b\int \frac{\sqrt{\sinh^2(e+fx)+1}}{\sqrt{b\sinh^2(e+fx)+a}}d\sinh(e+fx)-\sqrt{\sinh^2(e+fx)+1}\text{csch}(e+fx)\sqrt{a+b\sin}\right.}{f}$$

$\downarrow$ 324

$$\frac{\sqrt{\cosh^2(e+fx)}\text{sech}(e+fx)\left(b\left(\int \frac{1}{\sqrt{\sinh^2(e+fx)+1}\sqrt{b\sinh^2(e+fx)+a}}d\sinh(e+fx)+\int \frac{\sinh^2(e+fx)}{\sqrt{\sinh^2(e+fx)+1}\sqrt{b\sinh^2(e+fx}}\right.\right.}{f}$$

$\downarrow$ 320

$$\frac{\sqrt{\cosh^2(e+fx)}\text{sech}(e+fx)\left(b\left(\int \frac{\sinh^2(e+fx)}{\sqrt{\sinh^2(e+fx)+1}\sqrt{b\sinh^2(e+fx)+a}}d\sinh(e+fx)+\frac{\sqrt{a+b\sinh^2(e+fx)}\text{EllipticF}\left(\text{arcta}\right.}{a\sqrt{\sinh^2(e+fx)+1}\sqrt{\frac{a+b}{a\left(\sin\right.}}}\right.\right.}{f}$$

$\downarrow$ 388

$$\frac{\sqrt{\cosh^2(e+fx)}\text{sech}(e+fx)\left(b\left(-\frac{\int \frac{\sqrt{b\sinh^2(e+fx)+a}}{\left(\sinh^2(e+fx)+1\right)^{3/2}}d\sinh(e+fx)}{b}+\frac{\sqrt{a+b\sinh^2(e+fx)}\text{EllipticF}\left(\arctan(\sinh(e+fx)),1-\frac{b}{a}\right)}{a\sqrt{\sinh^2(e+fx)+1}\sqrt{\frac{a+b\sinh^2(e+fx)}{a\left(\sinh^2(e+fx)+1\right)}}}\right.\right.}{f}$$

$\downarrow$ 313

$$\frac{\sqrt{\cosh^2(e+fx)}\operatorname{sech}(e+fx)\left(b\left(\frac{\sqrt{a+b\sinh^2(e+fx)}\operatorname{EllipticF}\left(\arctan(\sinh(e+fx)),1-\frac{b}{a}\right)}{a\sqrt{\sinh^2(e+fx)+1}\sqrt{\frac{a+b\sinh^2(e+fx)}{a\left(\sinh^2(e+fx)+1\right)}}}-\frac{\sqrt{a+b\sinh^2(e+fx)}E\left(\arctan(\sinh(e-\right.}{b\sqrt{\sinh^2(e+fx)+1}\sqrt{\frac{a+b\sinh^2(}{a\left(\sinh^2(e+\right.}}}\right.\right.}{f}$$

input
```
Int[Csch[e + f*x]^2*Sqrt[a + b*Sinh[e + f*x]^2],x]
```

output
```
(Sqrt[Cosh[e + f*x]^2]*Sech[e + f*x]*(-(Csch[e + f*x]*Sqrt[1 + Sinh[e + f*
x]^2]*Sqrt[a + b*Sinh[e + f*x]^2]) + b*((Sinh[e + f*x]*Sqrt[a + b*Sinh[e +
 f*x]^2])/(b*Sqrt[1 + Sinh[e + f*x]^2]) - (EllipticE[ArcTan[Sinh[e + f*x]]
, 1 - b/a]*Sqrt[a + b*Sinh[e + f*x]^2])/(b*Sqrt[1 + Sinh[e + f*x]^2]*Sqrt[
(a + b*Sinh[e + f*x]^2)/(a*(1 + Sinh[e + f*x]^2))]) + (EllipticF[ArcTan[Si
nh[e + f*x]], 1 - b/a]*Sqrt[a + b*Sinh[e + f*x]^2])/(a*Sqrt[1 + Sinh[e + f
*x]^2]*Sqrt[(a + b*Sinh[e + f*x]^2)/(a*(1 + Sinh[e + f*x]^2))]))))/f
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 313
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/((c_) + (d_.)*(x_)^2)^(3/2), x_Symbol] :> Sim
p[(Sqrt[a + b*x^2]/(c*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(c
 + d*x^2)))]))*EllipticE[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; FreeQ
[{a, b, c, d}, x] && PosQ[b/a] && PosQ[d/c]
```

rule 320
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(Sqrt[a + b*x^2]/(a*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(
c + d*x^2)))]))*EllipticF[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; Fre
eQ[{a, b, c, d}, x] && PosQ[d/c] && PosQ[b/a] &&  !SimplerSqrtQ[b/a, d/c]
```

rule 324
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
a   Int[1/(Sqrt[a + b*x^2]*Sqrt[c + d*x^2]), x], x] + Simp[b   Int[x^2/(Sqr
t[a + b*x^2]*Sqrt[c + d*x^2]), x], x] /; FreeQ[{a, b, c, d}, x] && PosQ[d/c
] && PosQ[b/a]
```

rule 377
```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_)
, x_Symbol] :> Simp[(e*x)^(m + 1)*(a + b*x^2)^(p + 1)*((c + d*x^2)^q/(a*e*(
m + 1))), x] - Simp[1/(a*e^2*(m + 1))   Int[(e*x)^(m + 2)*(a + b*x^2)^p*(c
+ d*x^2)^(q - 1)*Simp[b*c*(m + 1) + 2*(b*c*(p + 1) + a*d*q) + d*(b*(m + 1)
+ 2*b*(p + q + 1))*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b
*c - a*d, 0] && LtQ[0, q, 1] && LtQ[m, -1] && IntBinomialQ[a, b, c, d, e, m
, 2, p, q, x]
```

rule 388
```
Int[(x_)^2/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol]
 :> Simp[x*(Sqrt[a + b*x^2]/(b*Sqrt[c + d*x^2])), x] - Simp[c/b   Int[Sqrt[
a + b*x^2]/(c + d*x^2)^(3/2), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
 a*d, 0] && PosQ[b/a] && PosQ[d/c] &&  !SimplerSqrtQ[b/a, d/c]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3667
```
Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(
p_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Simp[ff^(m + 1
)*(Sqrt[Cos[e + f*x]^2]/(f*Cos[e + f*x]))   Subst[Int[x^m*((a + b*ff^2*x^2)
^p/Sqrt[1 - ff^2*x^2]), x], x, Sin[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f,
p}, x] && IntegerQ[m/2] &&  !IntegerQ[p]
```

Maple [A] (verified)

Time = 2.32 (sec) , antiderivative size = 167, normalized size of antiderivative = 0.93

method	result
default	$-\frac{\sqrt{-\frac{b}{a}}\,b\cosh(fx+e)^4-b\sqrt{\frac{b\cosh(fx+e)^2}{a}+\frac{a-b}{a}}\,\sqrt{\frac{\cosh(2fx+2e)}{2}+\frac{1}{2}}\,\sinh(fx+e)\,\text{EllipticE}\left(\sinh(fx+e)\sqrt{-\frac{b}{a}},\sqrt{\frac{a}{b}}\right)+\sqrt{-\frac{b}{a}}\,a\cosh}{\sinh(fx+e)\sqrt{-\frac{b}{a}}\,\cosh(fx+e)\sqrt{a+b\sinh(fx+e)^2}\,f}$

input
```
int(csch(f*x+e)^2*(a+b*sinh(f*x+e)^2)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-((-b/a)^(1/2)*b*cosh(f*x+e)^4-b*(b/a*cosh(f*x+e)^2+(a-b)/a)^(1/2)*(cosh(f
*x+e)^2)^(1/2)*sinh(f*x+e)*EllipticE(sinh(f*x+e)*(-b/a)^(1/2),(1/b*a)^(1/2
))+(-b/a)^(1/2)*a*cosh(f*x+e)^2-(-b/a)^(1/2)*b*cosh(f*x+e)^2)/sinh(f*x+e)/
(-b/a)^(1/2)/cosh(f*x+e)/(a+b*sinh(f*x+e)^2)^(1/2)/f
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 542 vs. 2(185) = 370.

Time = 0.12 (sec) , antiderivative size = 542, normalized size of antiderivative = 3.03

$$\int \operatorname{csch}^2(e+fx)\sqrt{a+b\sinh^2(e+fx)}\,dx$$

$$= \frac{\Big((2\,a-b)\cosh\left(fx+e\right)^2+2\,(2\,a-b)\cosh\left(fx+e\right)\sinh\left(fx+e\right)+(2\,a-b)\sinh\left(fx+e\right)^2-2\,\big(b\cosh}{}$$

input
```
integrate(csch(f*x+e)^2*(a+b*sinh(f*x+e)^2)^(1/2),x, algorithm="fricas")
```

output
```
(((2*a - b)*cosh(f*x + e)^2 + 2*(2*a - b)*cosh(f*x + e)*sinh(f*x + e) + (2
*a - b)*sinh(f*x + e)^2 - 2*(b*cosh(f*x + e)^2 + 2*b*cosh(f*x + e)*sinh(f*
x + e) + b*sinh(f*x + e)^2 - b)*sqrt((a^2 - a*b)/b^2) - 2*a + b)*sqrt(b)*s
qrt((2*b*sqrt((a^2 - a*b)/b^2) - 2*a + b)/b)*elliptic_e(arcsin(sqrt((2*b*s
qrt((a^2 - a*b)/b^2) - 2*a + b)/b)*(cosh(f*x + e) + sinh(f*x + e))), (8*a^
2 - 8*a*b + b^2 + 4*(2*a*b - b^2)*sqrt((a^2 - a*b)/b^2))/b^2) - 2*((2*a -
b)*cosh(f*x + e)^2 + 2*(2*a - b)*cosh(f*x + e)*sinh(f*x + e) + (2*a - b)*s
inh(f*x + e)^2 - 2*a + b)*sqrt(b)*sqrt((2*b*sqrt((a^2 - a*b)/b^2) - 2*a +
b)/b)*elliptic_f(arcsin(sqrt((2*b*sqrt((a^2 - a*b)/b^2) - 2*a + b)/b)*(cos
h(f*x + e) + sinh(f*x + e))), (8*a^2 - 8*a*b + b^2 + 4*(2*a*b - b^2)*sqrt(
(a^2 - a*b)/b^2))/b^2) - sqrt(2)*(b*cosh(f*x + e) + b*sinh(f*x + e))*sqrt(
(b*cosh(f*x + e)^2 + b*sinh(f*x + e)^2 + 2*a - b)/(cosh(f*x + e)^2 - 2*cos
h(f*x + e)*sinh(f*x + e) + sinh(f*x + e)^2)))/(b*f*cosh(f*x + e)^2 + 2*b*f
*cosh(f*x + e)*sinh(f*x + e) + b*f*sinh(f*x + e)^2 - b*f)
```

Sympy [F]

$$\int \operatorname{csch}^2(e+fx)\sqrt{a+b\sinh^2(e+fx)}\,dx=\int \sqrt{a+b\sinh^2(e+fx)}\operatorname{csch}^2(e+fx)\,dx$$

input
```
integrate(csch(f*x+e)**2*(a+b*sinh(f*x+e)**2)**(1/2),x)
```

output
```
Integral(sqrt(a + b*sinh(e + f*x)**2)*csch(e + f*x)**2, x)
```

Maxima [F]

$$\int \operatorname{csch}^2(e+fx)\sqrt{a+b\sinh^2(e+fx)}\,dx=\int \sqrt{b\sinh(fx+e)^2+a}\operatorname{csch}(fx+e)^2\,dx$$

input
```
integrate(csch(f*x+e)^2*(a+b*sinh(f*x+e)^2)^(1/2),x, algorithm="maxima")
```

output
```
integrate(sqrt(b*sinh(f*x + e)^2 + a)*csch(f*x + e)^2, x)
```

Giac [F]

$$\int \operatorname{csch}^2(e+fx)\sqrt{a+b\sinh^2(e+fx)}\,dx=\int \sqrt{b\sinh(fx+e)^2+a}\operatorname{csch}(fx+e)^2\,dx$$

input
```
integrate(csch(f*x+e)^2*(a+b*sinh(f*x+e)^2)^(1/2),x, algorithm="giac")
```

output
```
integrate(sqrt(b*sinh(f*x + e)^2 + a)*csch(f*x + e)^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int \operatorname{csch}^2(e+fx)\sqrt{a+b\sinh^2(e+fx)}\,dx=\int \frac{\sqrt{b\sinh\left(e+f\,x\right)^2+a}}{\sinh\left(e+f\,x\right)^2}\,dx$$

input
```
int((a + b*sinh(e + f*x)^2)^(1/2)/sinh(e + f*x)^2,x)
```

output
```
int((a + b*sinh(e + f*x)^2)^(1/2)/sinh(e + f*x)^2, x)
```

Reduce [F]

$$\int \operatorname{csch}^2(e+fx)\sqrt{a+b\sinh^2(e+fx)}\,dx=\int \sqrt{\sinh\left(fx+e\right)^2 b+a}\,\operatorname{csch}(fx+e)^2\,dx$$

input
```
int(csch(f*x+e)^2*(a+b*sinh(f*x+e)^2)^(1/2),x)
```

output
```
int(sqrt(sinh(e + f*x)**2*b + a)*csch(e + f*x)**2,x)
```

3.69 $\int \operatorname{csch}^4(e+fx)\sqrt{a+b\sinh^2(e+fx)}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 243

$$\int \operatorname{csch}^4(e+fx)\sqrt{a+b\sinh^2(e+fx)}\,dx = -\frac{\coth(e+fx)\operatorname{csch}^2(e+fx)\sqrt{a+b\sinh^2(e+fx)}}{3f} + \frac{(2a-b)\operatorname{csch}(e+fx)\operatorname{sech}(e+fx)\sqrt{a+b\sinh^2(e+fx)}}{3af} + \frac{(2a-b)E\left(\arctan(\sinh(e+fx))\,\middle|\,1-\frac{b}{a}\right)\operatorname{sech}(e+fx)\sqrt{a+b\sinh^2(e+fx)}}{3af\sqrt{\frac{\operatorname{sech}^2(e+fx)(a+b\sinh^2(e+fx))}{a}}} - \frac{b\,\operatorname{EllipticF}\left(\arctan(\sinh(e+fx)),1-\frac{b}{a}\right)\operatorname{sech}(e+fx)\sqrt{a+b\sinh^2(e+fx)}}{3af\sqrt{\frac{\operatorname{sech}^2(e+fx)(a+b\sinh^2(e+fx))}{a}}}$$

output
```
-1/3*coth(f*x+e)*csch(f*x+e)^2*(a+b*sinh(f*x+e)^2)^(1/2)/f+1/3*(2*a-b)*csc
h(f*x+e)*sech(f*x+e)*(a+b*sinh(f*x+e)^2)^(1/2)/a/f+1/3*(2*a-b)*EllipticE(s
inh(f*x+e)/(1+sinh(f*x+e)^2)^(1/2),(1-b/a)^(1/2))*sech(f*x+e)*(a+b*sinh(f*
x+e)^2)^(1/2)/a/f/(sech(f*x+e)^2*(a+b*sinh(f*x+e)^2)/a)^(1/2)-1/3*b*Invers
eJacobiAM(arctan(sinh(f*x+e)),(1-b/a)^(1/2))*sech(f*x+e)*(a+b*sinh(f*x+e)^
2)^(1/2)/a/f/(sech(f*x+e)^2*(a+b*sinh(f*x+e)^2)/a)^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 2.15 (sec) , antiderivative size = 208, normalized size of antiderivative = 0.86

$$\int \operatorname{csch}^4(e+fx)\sqrt{a+b\sinh^2(e+fx)}\,dx$$
$$= \frac{\frac{(4(2a^2-4ab+b^2)\cosh(2(e+fx))-(2a-b)(8a-3b-b\cosh(4(e+fx))))\coth(e+fx)\operatorname{csch}^2(e+fx)}{2\sqrt{2}} + 2ia(2a-b)\sqrt{\frac{2a-b+b\cosh(2(e+fx))}{a}}}{6af\sqrt{2a-b+b\cosh(2(e+fx}}$$

input
```
Integrate[Csch[e + f*x]^4*Sqrt[a + b*Sinh[e + f*x]^2],x]
```

output
```
(((4*(2*a^2 - 4*a*b + b^2)*Cosh[2*(e + f*x)] - (2*a - b)*(8*a - 3*b - b*Co
sh[4*(e + f*x)]))*Coth[e + f*x]*Csch[e + f*x]^2)/(2*Sqrt[2]) + (2*I)*a*(2*
a - b)*Sqrt[(2*a - b + b*Cosh[2*(e + f*x)])/a]*EllipticE[I*(e + f*x), b/a]
 - (4*I)*a*(a - b)*Sqrt[(2*a - b + b*Cosh[2*(e + f*x)])/a]*EllipticF[I*(e
+ f*x), b/a])/(6*a*f*Sqrt[2*a - b + b*Cosh[2*(e + f*x)]])
```

Rubi [A] (verified)

Time = 0.59 (sec) , antiderivative size = 335, normalized size of antiderivative = 1.38, number of steps used = 12, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.440, Rules used = {3042, 3667, 377, 25, 445, 25, 27, 406, 320, 388, 313}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \operatorname{csch}^4(e+fx)\sqrt{a+b\sinh^2(e+fx)}\,dx$$

$$\Big\downarrow \ 3042$$

$$\int \frac{\sqrt{a-b\sin(ie+ifx)^2}}{\sin(ie+ifx)^4}dx$$

$$\Big\downarrow \ 3667$$

$$\frac{\sqrt{\cosh^2(e+fx)}\text{sech}(e+fx)\int \frac{\text{csch}^4(e+fx)\sqrt{b\sinh^2(e+fx)+a}}{\sqrt{\sinh^2(e+fx)+1}}d\sinh(e+fx)}{f}$$

$\Big\downarrow$ 377

$$\frac{\sqrt{\cosh^2(e+fx)}\text{sech}(e+fx)\left(\frac{1}{3}\int -\frac{\text{csch}^2(e+fx)(b\sinh^2(e+fx)+2a-b)}{\sqrt{\sinh^2(e+fx)+1}\sqrt{b\sinh^2(e+fx)+a}}d\sinh(e+fx)-\frac{1}{3}\sqrt{\sinh^2(e+fx)+1}\text{csch}^3(\right.}{f}$$

$\Big\downarrow$ 25

$$\frac{\sqrt{\cosh^2(e+fx)}\text{sech}(e+fx)\left(-\frac{1}{3}\int \frac{\text{csch}^2(e+fx)(b\sinh^2(e+fx)+2a-b)}{\sqrt{\sinh^2(e+fx)+1}\sqrt{b\sinh^2(e+fx)+a}}d\sinh(e+fx)-\frac{1}{3}\sqrt{\sinh^2(e+fx)+1}\text{csch}^3(\right.}{f}$$

$\Big\downarrow$ 445

$$\frac{\sqrt{\cosh^2(e+fx)}\text{sech}(e+fx)\left(\frac{1}{3}\left(\frac{\int -\frac{b\left((2a-b)\sinh^2(e+fx)+a\right)}{\sqrt{\sinh^2(e+fx)+1}\sqrt{b\sinh^2(e+fx)+a}}d\sinh(e+fx)}{a}+\frac{(2a-b)\sqrt{\sinh^2(e+fx)+1}\text{csch}(e+fx)\sqrt{a+}}{a}\right.\right.}{f}$$

$\Big\downarrow$ 25

$$\frac{\sqrt{\cosh^2(e+fx)}\text{sech}(e+fx)\left(\frac{1}{3}\left(\frac{(2a-b)\sqrt{\sinh^2(e+fx)+1}\text{csch}(e+fx)\sqrt{a+b\sinh^2(e+fx)}}{a}-\frac{\int \frac{b\left((2a-b)\sinh^2(e+fx)+a\right)}{\sqrt{\sinh^2(e+fx)+1}\sqrt{b\sinh^2(e+fx)+a}}d}{a}\right.\right.}{f}$$

$\Big\downarrow$ 27

$$\frac{\sqrt{\cosh^2(e+fx)}\text{sech}(e+fx)\left(\frac{1}{3}\left(\frac{(2a-b)\sqrt{\sinh^2(e+fx)+1}\text{csch}(e+fx)\sqrt{a+b\sinh^2(e+fx)}}{a}-\frac{b\int \frac{(2a-b)\sinh^2(e+fx)+a}{\sqrt{\sinh^2(e+fx)+1}\sqrt{b\sinh^2(e+fx)+a}}}{a}\right.\right.}{f}$$

$\Big\downarrow$ 406

$$\sqrt{\cosh^2(e+fx)}\operatorname{sech}(e+fx)\left(\frac{1}{3}\left(\frac{(2a-b)\sqrt{\sinh^2(e+fx)+1}\operatorname{csch}(e+fx)\sqrt{a+b\sinh^2(e+fx)}}{a}-\frac{b\left(a\int\frac{1}{\sqrt{\sinh^2(e+fx)+1}\sqrt{b\sinh^2(e+fx}}\right.}{}\right.\right.$$

$\big\downarrow$ 320

$$\sqrt{\cosh^2(e+fx)}\operatorname{sech}(e+fx)\left(\frac{1}{3}\left(\frac{(2a-b)\sqrt{\sinh^2(e+fx)+1}\operatorname{csch}(e+fx)\sqrt{a+b\sinh^2(e+fx)}}{a}-\frac{b\left((2a-b)\int\frac{\sinh^2(e+fx)}{\sqrt{\sinh^2(e+fx)+1}\sqrt{b\sinh^2}}\right.}{}\right.\right.$$

$\big\downarrow$ 388

$$\sqrt{\cosh^2(e+fx)}\operatorname{sech}(e+fx)\left(\frac{1}{3}\left(\frac{(2a-b)\sqrt{\sinh^2(e+fx)+1}\operatorname{csch}(e+fx)\sqrt{a+b\sinh^2(e+fx)}}{a}-\frac{b\left((2a-b)\left(\frac{\sinh(e+fx)\sqrt{a+b\sinh^2(e+}}{b\sqrt{\sinh^2(e+fx)+1}}\right.\right.}{}\right.\right.$$

$\big\downarrow$ 313

$$\sqrt{\cosh^2(e+fx)}\operatorname{sech}(e+fx)\left(\frac{1}{3}\left(\frac{(2a-b)\sqrt{\sinh^2(e+fx)+1}\operatorname{csch}(e+fx)\sqrt{a+b\sinh^2(e+fx)}}{a}-\frac{b\left(\frac{\sqrt{a+b\sinh^2(e+fx)}\operatorname{EllipticF}\left(\arctan(\right.}{\sqrt{\sinh^2(e+fx)+1}\sqrt{\frac{a+b\sin}{a\left(\sinh^2(\right.}}}\right.}{}\right.\right.$$

input
```
Int[Csch[e + f*x]^4*Sqrt[a + b*Sinh[e + f*x]^2],x]
```

output
```
(Sqrt[Cosh[e + f*x]^2]*Sech[e + f*x]*(-1/3*(Csch[e + f*x]^3*Sqrt[1 + Sinh[
e + f*x]^2]*Sqrt[a + b*Sinh[e + f*x]^2]) + (((2*a - b)*Csch[e + f*x]*Sqrt[
1 + Sinh[e + f*x]^2]*Sqrt[a + b*Sinh[e + f*x]^2])/a - (b*((EllipticF[ArcTa
n[Sinh[e + f*x]], 1 - b/a]*Sqrt[a + b*Sinh[e + f*x]^2])/(Sqrt[1 + Sinh[e +
 f*x]^2]*Sqrt[(a + b*Sinh[e + f*x]^2)/(a*(1 + Sinh[e + f*x]^2))]) + (2*a -
 b)*((Sinh[e + f*x]*Sqrt[a + b*Sinh[e + f*x]^2])/(b*Sqrt[1 + Sinh[e + f*x]
^2]) - (EllipticE[ArcTan[Sinh[e + f*x]], 1 - b/a]*Sqrt[a + b*Sinh[e + f*x]
^2])/(b*Sqrt[1 + Sinh[e + f*x]^2]*Sqrt[(a + b*Sinh[e + f*x]^2)/(a*(1 + Sin
h[e + f*x]^2))])))))/a)/3))/f
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 313
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/((c_) + (d_.)*(x_)^2)^(3/2), x_Symbol] :> Sim
p[(Sqrt[a + b*x^2]/(c*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(c
+ d*x^2)))]))*EllipticE[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; FreeQ
[{a, b, c, d}, x] && PosQ[b/a] && PosQ[d/c]
```

rule 320
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(Sqrt[a + b*x^2]/(a*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(
c + d*x^2)))]))*EllipticF[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; Fre
eQ[{a, b, c, d}, x] && PosQ[d/c] && PosQ[b/a] &&  !SimplerSqrtQ[b/a, d/c]
```

rule 377
```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_)
, x_Symbol] :> Simp[(e*x)^(m + 1)*(a + b*x^2)^(p + 1)*((c + d*x^2)^q/(a*e*(
m + 1))), x] - Simp[1/(a*e^2*(m + 1))   Int[(e*x)^(m + 2)*(a + b*x^2)^p*(c
+ d*x^2)^(q - 1)*Simp[b*c*(m + 1) + 2*(b*c*(p + 1) + a*d*q) + d*(b*(m + 1)
+ 2*b*(p + q + 1))*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b
*c - a*d, 0] && LtQ[0, q, 1] && LtQ[m, -1] && IntBinomialQ[a, b, c, d, e, m
, 2, p, q, x]
```

rule 388
```
Int[(x_)^2/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol]
 :> Simp[x*(Sqrt[a + b*x^2]/(b*Sqrt[c + d*x^2])), x] - Simp[c/b   Int[Sqrt[
a + b*x^2]/(c + d*x^2)^(3/2), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
 a*d, 0] && PosQ[b/a] && PosQ[d/c] &&  !SimplerSqrtQ[b/a, d/c]
```

rule 406
```
Int[((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.)*((e_) + (f_.)*(
x_)^2), x_Symbol] :> Simp[e   Int[(a + b*x^2)^p*(c + d*x^2)^q, x], x] + Sim
p[f   Int[x^2*(a + b*x^2)^p*(c + d*x^2)^q, x], x] /; FreeQ[{a, b, c, d, e,
f, p, q}, x]
```

rule 445
```
Int[((g_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_
.)*((e_) + (f_.)*(x_)^2), x_Symbol] :> Simp[e*(g*x)^(m + 1)*(a + b*x^2)^(p
+ 1)*((c + d*x^2)^(q + 1)/(a*c*g*(m + 1))), x] + Simp[1/(a*c*g^2*(m + 1))
 Int[(g*x)^(m + 2)*(a + b*x^2)^p*(c + d*x^2)^q*Simp[a*f*c*(m + 1) - e*(b*c
+ a*d)*(m + 2 + 1) - e*2*(b*c*p + a*d*q) - b*e*d*(m + 2*(p + q + 2) + 1)*x^
2, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p, q}, x] && LtQ[m, -1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3667
```
Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(
p_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Simp[ff^(m + 1
)*(Sqrt[Cos[e + f*x]^2]/(f*Cos[e + f*x]))   Subst[Int[x^m*((a + b*ff^2*x^2)
^p/Sqrt[1 - ff^2*x^2]), x], x, Sin[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f,
p}, x] && IntegerQ[m/2] &&  !IntegerQ[p]
```

Maple [A] (verified)

Time = 5.84 (sec) , antiderivative size = 436, normalized size of antiderivative = 1.79

method	result
default	$\frac{2\sqrt{-\frac{b}{a}}\, ab\sinh(fx+e)^6-\sqrt{-\frac{b}{a}}\, b^2\sinh(fx+e)^6+b\sqrt{\frac{a+b\sinh(fx+e)^2}{a}}\,\sqrt{\frac{\cosh(2fx+2e)}{2}+\frac{1}{2}}\ \mathrm{EllipticF}\left(\sinh(fx+e)\sqrt{-\frac{b}{a}},\sqrt{\frac{a}{b}}\right)a\sinh(\ldots}{\ldots}$
risch	Expression too large to display

input
```
int(csch(f*x+e)^4*(a+b*sinh(f*x+e)^2)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/3*(2*(-b/a)^(1/2)*a*b*sinh(f*x+e)^6-(-b/a)^(1/2)*b^2*sinh(f*x+e)^6+b*((a
+b*sinh(f*x+e)^2)/a)^(1/2)*(cosh(f*x+e)^2)^(1/2)*EllipticF(sinh(f*x+e)*(-b
/a)^(1/2),(1/b*a)^(1/2))*a*sinh(f*x+e)^3-((a+b*sinh(f*x+e)^2)/a)^(1/2)*(co
sh(f*x+e)^2)^(1/2)*EllipticF(sinh(f*x+e)*(-b/a)^(1/2),(1/b*a)^(1/2))*b^2*s
inh(f*x+e)^3-2*((a+b*sinh(f*x+e)^2)/a)^(1/2)*(cosh(f*x+e)^2)^(1/2)*Ellipti
cE(sinh(f*x+e)*(-b/a)^(1/2),(1/b*a)^(1/2))*a*b*sinh(f*x+e)^3+((a+b*sinh(f*
x+e)^2)/a)^(1/2)*(cosh(f*x+e)^2)^(1/2)*EllipticE(sinh(f*x+e)*(-b/a)^(1/2),
(1/b*a)^(1/2))*b^2*sinh(f*x+e)^3+2*(-b/a)^(1/2)*a^2*sinh(f*x+e)^4-(-b/a)^(
1/2)*b^2*sinh(f*x+e)^4+(-b/a)^(1/2)*a^2*sinh(f*x+e)^2-2*(-b/a)^(1/2)*a*b*s
inh(f*x+e)^2-(-b/a)^(1/2)*a^2)/a/sinh(f*x+e)^3/(-b/a)^(1/2)/cosh(f*x+e)/(a
+b*sinh(f*x+e)^2)^(1/2)/f
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2144 vs. $2(239) = 478$.

Time = 0.14 (sec) , antiderivative size = 2144, normalized size of antiderivative = 8.82

$$\int \operatorname{csch}^4(e+fx)\sqrt{a+b\sinh^2(e+fx)}\,dx = \text{Too large to display}$$

input
```
integrate(csch(f*x+e)^4*(a+b*sinh(f*x+e)^2)^(1/2),x, algorithm="fricas")
```

output
```
-1/3*(((4*a^2 - 4*a*b + b^2)*cosh(f*x + e)^6 + 6*(4*a^2 - 4*a*b + b^2)*cos
h(f*x + e)*sinh(f*x + e)^5 + (4*a^2 - 4*a*b + b^2)*sinh(f*x + e)^6 - 3*(4*
a^2 - 4*a*b + b^2)*cosh(f*x + e)^4 + 3*(5*(4*a^2 - 4*a*b + b^2)*cosh(f*x +
 e)^2 - 4*a^2 + 4*a*b - b^2)*sinh(f*x + e)^4 + 4*(5*(4*a^2 - 4*a*b + b^2)*
cosh(f*x + e)^3 - 3*(4*a^2 - 4*a*b + b^2)*cosh(f*x + e))*sinh(f*x + e)^3 +
 3*(4*a^2 - 4*a*b + b^2)*cosh(f*x + e)^2 + 3*(5*(4*a^2 - 4*a*b + b^2)*cosh
(f*x + e)^4 - 6*(4*a^2 - 4*a*b + b^2)*cosh(f*x + e)^2 + 4*a^2 - 4*a*b + b^
2)*sinh(f*x + e)^2 - 4*a^2 + 4*a*b - b^2 + 6*((4*a^2 - 4*a*b + b^2)*cosh(f
*x + e)^5 - 2*(4*a^2 - 4*a*b + b^2)*cosh(f*x + e)^3 + (4*a^2 - 4*a*b + b^2
)*cosh(f*x + e))*sinh(f*x + e) - 2*((2*a*b - b^2)*cosh(f*x + e)^6 + 6*(2*a
*b - b^2)*cosh(f*x + e)*sinh(f*x + e)^5 + (2*a*b - b^2)*sinh(f*x + e)^6 -
3*(2*a*b - b^2)*cosh(f*x + e)^4 + 3*(5*(2*a*b - b^2)*cosh(f*x + e)^2 - 2*a
*b + b^2)*sinh(f*x + e)^4 + 4*(5*(2*a*b - b^2)*cosh(f*x + e)^3 - 3*(2*a*b
- b^2)*cosh(f*x + e))*sinh(f*x + e)^3 + 3*(2*a*b - b^2)*cosh(f*x + e)^2 +
3*(5*(2*a*b - b^2)*cosh(f*x + e)^4 - 6*(2*a*b - b^2)*cosh(f*x + e)^2 + 2*a
*b - b^2)*sinh(f*x + e)^2 - 2*a*b + b^2 + 6*((2*a*b - b^2)*cosh(f*x + e)^5
 - 2*(2*a*b - b^2)*cosh(f*x + e)^3 + (2*a*b - b^2)*cosh(f*x + e))*sinh(f*x
 + e))*sqrt((a^2 - a*b)/b^2))*sqrt(b)*sqrt((2*b*sqrt((a^2 - a*b)/b^2) - 2*
a + b)/b)*elliptic_e(arcsin(sqrt((2*b*sqrt((a^2 - a*b)/b^2) - 2*a + b)/b)*
(cosh(f*x + e) + sinh(f*x + e))), (8*a^2 - 8*a*b + b^2 + 4*(2*a*b - b^2...
```

Sympy [F(-1)]

Timed out.

$$\int \operatorname{csch}^4(e+fx)\sqrt{a+b\sinh^2(e+fx)}\,dx = \text{Timed out}$$

input
```
integrate(csch(f*x+e)**4*(a+b*sinh(f*x+e)**2)**(1/2),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \operatorname{csch}^4(e+fx)\sqrt{a+b\sinh^2(e+fx)}\,dx = \int \sqrt{b\sinh{(fx+e)}^2+a}\operatorname{csch}{(fx+e)}^4\,dx$$

input
```
integrate(csch(f*x+e)^4*(a+b*sinh(f*x+e)^2)^(1/2),x, algorithm="maxima")
```

output
```
integrate(sqrt(b*sinh(f*x + e)^2 + a)*csch(f*x + e)^4, x)
```

Giac [F]

$$\int \operatorname{csch}^4(e+fx)\sqrt{a+b\sinh^2(e+fx)}\,dx = \int \sqrt{b\sinh{(fx+e)}^2+a}\operatorname{csch}{(fx+e)}^4\,dx$$

input
```
integrate(csch(f*x+e)^4*(a+b*sinh(f*x+e)^2)^(1/2),x, algorithm="giac")
```

output
```
integrate(sqrt(b*sinh(f*x + e)^2 + a)*csch(f*x + e)^4, x)
```

Mupad [F(-1)]

Timed out.

$$\int \operatorname{csch}^4(e+fx)\sqrt{a+b\sinh^2(e+fx)}\,dx = \int \frac{\sqrt{b\sinh{(e+f\,x)}^2+a}}{\sinh{(e+f\,x)}^4}\,dx$$

input
```
int((a + b*sinh(e + f*x)^2)^(1/2)/sinh(e + f*x)^4,x)
```

output
```
int((a + b*sinh(e + f*x)^2)^(1/2)/sinh(e + f*x)^4, x)
```

Reduce [F]

$$\int \operatorname{csch}^4(e+fx)\sqrt{a+b\sinh^2(e+fx)}\,dx=\int \sqrt{\sinh{(fx+e)}^2\,b+a}\,\operatorname{csch}(fx+e)^4\,dx$$

input
```
int(csch(f*x+e)^4*(a+b*sinh(f*x+e)^2)^(1/2),x)
```

output
```
int(sqrt(sinh(e + f*x)**2*b + a)*csch(e + f*x)**4,x)
```

3.70 $\int \sinh^3(e+fx) \left(a+b\sinh^2(e+fx)\right)^{3/2} dx$

Optimal result

Integrand size = 25, antiderivative size = 177

$$\int \sinh^3(e+fx) \left(a+b\sinh^2(e+fx)\right)^{3/2} dx = -\frac{(a-b)^2(a+5b)\operatorname{arctanh}\left(\frac{\sqrt{b}\cosh(e+fx)}{\sqrt{a-b+b\cosh^2(e+fx)}}\right)}{16b^{3/2}f} - \frac{(a-b)(a+5b)\cosh(e+fx)\sqrt{a-b+b\cosh^2(e+fx)}}{16bf} - \frac{(a+5b)\cosh(e+fx)\left(a-b+b\cosh^2(e+fx)\right)^{3/2}}{24bf} + \frac{\cosh(e+fx)\left(a-b+b\cosh^2(e+fx)\right)^{5/2}}{6bf}$$

output
```
-1/16*(a-b)^2*(a+5*b)*arctanh(b^(1/2)*cosh(f*x+e)/(a-b+b*cosh(f*x+e)^2)^(1
/2))/b^(3/2)/f-1/16*(a-b)*(a+5*b)*cosh(f*x+e)*(a-b+b*cosh(f*x+e)^2)^(1/2)/
b/f-1/24*(a+5*b)*cosh(f*x+e)*(a-b+b*cosh(f*x+e)^2)^(3/2)/b/f+1/6*cosh(f*x+
e)*(a-b+b*cosh(f*x+e)^2)^(5/2)/b/f
```

Mathematica [A] (verified)

Time = 0.63 (sec) , antiderivative size = 151, normalized size of antiderivative = 0.85

$$\int \sinh^3(e+fx)\left(a+b\sinh^2(e+fx)\right)^{3/2}\,dx = \frac{\sqrt{2}\sqrt{b}\sqrt{2a-b+b\cosh(2(e+fx))}\left(\left(6a^2-51ab+37b^2\right)\cosh(e+fx)+b((7a-8b)\cosh(3\right.}{}$$

input
```
Integrate[Sinh[e + f*x]^3*(a + b*Sinh[e + f*x]^2)^(3/2),x]
```

output
```
(Sqrt[2]*Sqrt[b]*Sqrt[2*a - b + b*Cosh[2*(e + f*x)]]*((6*a^2 - 51*a*b + 37
*b^2)*Cosh[e + f*x] + b*((7*a - 8*b)*Cosh[3*(e + f*x)] + b*Cosh[5*(e + f*x
)])) - 12*(a - b)^2*(a + 5*b)*Log[Sqrt[2]*Sqrt[b]*Cosh[e + f*x] + Sqrt[2*a
 - b + b*Cosh[2*(e + f*x)]]])/(192*b^(3/2)*f)
```

Rubi [A] (verified)

Time = 0.38 (sec) , antiderivative size = 165, normalized size of antiderivative = 0.93, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.320$, Rules used = {3042, 26, 3665, 299, 211, 211, 224, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sinh^3(e+fx)\left(a+b\sinh^2(e+fx)\right)^{3/2}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int i\sin(ie+ifx)^3\left(a-b\sin(ie+ifx)^2\right)^{3/2}dx$$

$$\Big\downarrow \text{26}$$

$$i\int \sin(ie+ifx)^3\left(a-b\sin(ie+ifx)^2\right)^{3/2}dx$$

$$\Big\downarrow \text{3665}$$

$$-\frac{\int \left(1-\cosh^2(e+fx)\right)\left(b\cosh^2(e+fx)+a-b\right)^{3/2}d\cosh(e+fx)}{f}$$

$$\Big\downarrow \; 299$$

$$-\frac{\frac{(a+5b)\int \left(b\cosh^2(e+fx)+a-b\right)^{3/2}d\cosh(e+fx)}{6b}-\frac{\cosh(e+fx)\left(a+b\cosh^2(e+fx)-b\right)^{5/2}}{6b}}{f}$$

$$\Big\downarrow \; 211$$

$$-\frac{\frac{(a+5b)\left(\frac{3}{4}(a-b)\int \sqrt{b\cosh^2(e+fx)+a-b}d\cosh(e+fx)+\frac{1}{4}\cosh(e+fx)\left(a+b\cosh^2(e+fx)-b\right)^{3/2}\right)}{6b}-\frac{\cosh(e+fx)\left(a+b\cosh^2(e+fx)-b\right)^{5/2}}{6b}}{f}$$

$$\Big\downarrow \; 211$$

$$-\frac{\frac{(a+5b)\left(\frac{3}{4}(a-b)\left(\frac{1}{2}(a-b)\int \frac{1}{\sqrt{b\cosh^2(e+fx)+a-b}}d\cosh(e+fx)+\frac{1}{2}\cosh(e+fx)\sqrt{a+b\cosh^2(e+fx)-b}\right)+\frac{1}{4}\cosh(e+fx)\left(a+b\cosh^2(e+fx)-b\right)\right.}{6b}}{f}$$

$$\Big\downarrow \; 224$$

$$-\frac{\frac{(a+5b)\left(\frac{3}{4}(a-b)\left(\frac{1}{2}(a-b)\int \frac{1}{1-\frac{b\cosh^2(e+fx)}{b\cosh^2(e+fx)+a-b}}d\frac{\cosh(e+fx)}{\sqrt{b\cosh^2(e+fx)+a-b}}+\frac{1}{2}\cosh(e+fx)\sqrt{a+b\cosh^2(e+fx)-b}\right)+\frac{1}{4}\cosh(e+fx)\left(a+b\cosh^2(e\right.\right.}{6b}}{f}$$

$$\Big\downarrow \; 219$$

$$-\frac{\frac{(a+5b)\left(\frac{3}{4}(a-b)\left(\frac{(a-b)\text{arctanh}\left(\frac{\sqrt{b}\cosh(e+fx)}{\sqrt{a+b\cosh^2(e+fx)-b}}\right)}{2\sqrt{b}}+\frac{1}{2}\cosh(e+fx)\sqrt{a+b\cosh^2(e+fx)-b}\right)+\frac{1}{4}\cosh(e+fx)\left(a+b\cosh^2(e+fx)-b\right)^{3/2}\right.}{6b}}{f}$$

input
```
Int[Sinh[e + f*x]^3*(a + b*Sinh[e + f*x]^2)^(3/2),x]
```

output
```
-((-1/6*(Cosh[e + f*x]*(a - b + b*Cosh[e + f*x]^2)^(5/2))/b + ((a + 5*b)*(
(Cosh[e + f*x]*(a - b + b*Cosh[e + f*x]^2)^(3/2))/4 + (3*(a - b)*(((a - b)
*ArcTanh[(Sqrt[b]*Cosh[e + f*x])/Sqrt[a - b + b*Cosh[e + f*x]^2]])/(2*Sqrt
[b]) + (Cosh[e + f*x]*Sqrt[a - b + b*Cosh[e + f*x]^2])/2))/4))/(6*b))/f)
```

Defintions of rubi rules used

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 211
```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[x*((a + b*x^2)^p/(2*p + 1
)), x] + Simp[2*a*(p/(2*p + 1))   Int[(a + b*x^2)^(p - 1), x], x] /; FreeQ[
{a, b}, x] && GtQ[p, 0] && (IntegerQ[4*p] || IntegerQ[6*p])
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

rule 299
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2), x_Symbol] :> Simp[d*x
*((a + b*x^2)^(p + 1)/(b*(2*p + 3))), x] - Simp[(a*d - b*c*(2*p + 3))/(b*(2
*p + 3))   Int[(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
 a*d, 0] && NeQ[2*p + 3, 0]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3665
```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^
(p_.), x_Symbol] :> With[{ff = FreeFactors[Cos[e + f*x], x]}, Simp[-ff/f
Subst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a + b - b*ff^2*x^2)^p, x], x, Cos[e +
 f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m - 1)/2]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 482 vs. $2(157) = 314$.

Time = 0.37 (sec), antiderivative size = 483, normalized size of antiderivative = 2.73

method	result
default	$\sqrt{\left(a+b\sinh(fx+e)^2\right)\cosh(fx+e)^2}\,\Big(16b^{\frac{7}{2}}\sqrt{b\cosh(fx+e)^4+(a-b)\cosh(fx+e)^2}\,\cosh(fx+e)^4+4b^{\frac{5}{2}}\sqrt{b\cosh(fx+e)^4+(a-b)\cosh$

input
```
int(sinh(f*x+e)^3*(a+b*sinh(f*x+e)^2)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
1/96*((a+b*sinh(f*x+e)^2)*cosh(f*x+e)^2)^(1/2)*(16*b^(7/2)*(b*cosh(f*x+e)^
4+(a-b)*cosh(f*x+e)^2)^(1/2)*cosh(f*x+e)^4+4*b^(5/2)*(b*cosh(f*x+e)^4+(a-b
)*cosh(f*x+e)^2)^(1/2)*(-13*b+7*a)*cosh(f*x+e)^2+66*b^(7/2)*(b*cosh(f*x+e)
^4+(a-b)*cosh(f*x+e)^2)^(1/2)-72*a*(b*cosh(f*x+e)^4+(a-b)*cosh(f*x+e)^2)^(
1/2)*b^(5/2)+6*a^2*(b*cosh(f*x+e)^4+(a-b)*cosh(f*x+e)^2)^(1/2)*b^(3/2)-3*a
^3*ln(1/2*(2*b*cosh(f*x+e)^2+2*(b*cosh(f*x+e)^4+(a-b)*cosh(f*x+e)^2)^(1/2)
*b^(1/2)+a-b)/b^(1/2))*b-9*a^2*ln(1/2*(2*b*cosh(f*x+e)^2+2*(b*cosh(f*x+e)^
4+(a-b)*cosh(f*x+e)^2)^(1/2)*b^(1/2)+a-b)/b^(1/2))*b^2+27*b^3*ln(1/2*(2*b*
cosh(f*x+e)^2+2*(b*cosh(f*x+e)^4+(a-b)*cosh(f*x+e)^2)^(1/2)*b^(1/2)+a-b)/b
^(1/2))*a-15*b^4*ln(1/2*(2*b*cosh(f*x+e)^2+2*(b*cosh(f*x+e)^4+(a-b)*cosh(f
*x+e)^2)^(1/2)*b^(1/2)+a-b)/b^(1/2)))/b^(5/2)/cosh(f*x+e)/(a+b*sinh(f*x+e)
^2)^(1/2)/f
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1965 vs. $2(157) = 314$.

Time = 0.31 (sec), antiderivative size = 4608, normalized size of antiderivative = 26.03

$$\int \sinh^3(e+fx)\left(a+b\sinh^2(e+fx)\right)^{3/2}\,dx = \text{Too large to display}$$

input
```
integrate(sinh(f*x+e)^3*(a+b*sinh(f*x+e)^2)^(3/2),x, algorithm="fricas")
```

output
```
Too large to include
```

Sympy [F(-1)]

Timed out.

$$\int \sinh^3(e+fx)\left(a+b\sinh^2(e+fx)\right)^{3/2}\,dx=\text{Timed out}$$

input
```
integrate(sinh(f*x+e)**3*(a+b*sinh(f*x+e)**2)**(3/2),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \sinh^3(e+fx)\left(a+b\sinh^2(e+fx)\right)^{3/2}\,dx=\int \left(b\sinh\left(fx+e\right)^2+a\right)^{\frac{3}{2}}\sinh\left(fx+e\right)^3\,dx$$

input
```
integrate(sinh(f*x+e)^3*(a+b*sinh(f*x+e)^2)^(3/2),x, algorithm="maxima")
```

output
```
integrate((b*sinh(f*x + e)^2 + a)^(3/2)*sinh(f*x + e)^3, x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1578 vs. $2(157)=314$.

Time $=0.77$ (sec) , antiderivative size $=1578$, normalized size of antiderivative $=8.92$

$$\int \sinh^3(e+fx)\left(a+b\sinh^2(e+fx)\right)^{3/2}\,dx=\text{Too large to display}$$

input
```
integrate(sinh(f*x+e)^3*(a+b*sinh(f*x+e)^2)^(3/2),x, algorithm="giac")
```

output
```
1/384*(((b*e^(2*f*x + 10*e) + (7*a*b^2*e^(14*e) - 8*b^3*e^(14*e))*e^(-6*e)
/b^2)*e^(2*f*x) + (6*a^2*b*e^(12*e) - 51*a*b^2*e^(12*e) + 37*b^3*e^(12*e))
*e^(-6*e)/b^2)*sqrt(b*e^(4*f*x + 4*e) + 4*a*e^(2*f*x + 2*e) - 2*b*e^(2*f*x
 + 2*e) + b) + 24*(a^3*e^(6*e) + 3*a^2*b*e^(6*e) - 9*a*b^2*e^(6*e) + 5*b^3
*e^(6*e))*arctan(-(sqrt(b)*e^(2*f*x + 2*e) - sqrt(b*e^(4*f*x + 4*e) + 4*a*
e^(2*f*x + 2*e) - 2*b*e^(2*f*x + 2*e) + b))/sqrt(-b))/(sqrt(-b)*b) + 12*(a
^3*e^(6*e) + 3*a^2*b*e^(6*e) - 9*a*b^2*e^(6*e) + 5*b^3*e^(6*e))*log(abs((s
qrt(b)*e^(2*f*x + 2*e) - sqrt(b*e^(4*f*x + 4*e) + 4*a*e^(2*f*x + 2*e) - 2*
b*e^(2*f*x + 2*e) + b))*sqrt(b) + 2*a - b))/b^(3/2) - 2*(12*(sqrt(b)*e^(2*
f*x + 2*e) - sqrt(b*e^(4*f*x + 4*e) + 4*a*e^(2*f*x + 2*e) - 2*b*e^(2*f*x +
 2*e) + b))^5*a^3*e^(6*e) - 108*(sqrt(b)*e^(2*f*x + 2*e) - sqrt(b*e^(4*f*x
 + 4*e) + 4*a*e^(2*f*x + 2*e) - 2*b*e^(2*f*x + 2*e) + b))^5*a^2*b*e^(6*e)
+ 132*(sqrt(b)*e^(2*f*x + 2*e) - sqrt(b*e^(4*f*x + 4*e) + 4*a*e^(2*f*x + 2
*e) - 2*b*e^(2*f*x + 2*e) + b))^5*a*b^2*e^(6*e) - 45*(sqrt(b)*e^(2*f*x + 2
*e) - sqrt(b*e^(4*f*x + 4*e) + 4*a*e^(2*f*x + 2*e) - 2*b*e^(2*f*x + 2*e) +
 b))^5*b^3*e^(6*e) + 48*(sqrt(b)*e^(2*f*x + 2*e) - sqrt(b*e^(4*f*x + 4*e)
+ 4*a*e^(2*f*x + 2*e) - 2*b*e^(2*f*x + 2*e) + b))^4*a^2*b^(3/2)*e^(6*e) -
120*(sqrt(b)*e^(2*f*x + 2*e) - sqrt(b*e^(4*f*x + 4*e) + 4*a*e^(2*f*x + 2*e
) - 2*b*e^(2*f*x + 2*e) + b))^4*a*b^(5/2)*e^(6*e) + 63*(sqrt(b)*e^(2*f*x +
 2*e) - sqrt(b*e^(4*f*x + 4*e) + 4*a*e^(2*f*x + 2*e) - 2*b*e^(2*f*x + 2...
```

Mupad [F(-1)]

Timed out.

$$\int \sinh^3(e+fx) \left(a+b\sinh^2(e+fx)\right)^{3/2} dx = \int \sinh(e+fx)^3 \left(b\sinh(e+fx)^2+a\right)^{3/2} dx$$

input
```
int(sinh(e + f*x)^3*(a + b*sinh(e + f*x)^2)^(3/2),x)
```

output
```
int(sinh(e + f*x)^3*(a + b*sinh(e + f*x)^2)^(3/2), x)
```

Reduce [F]

$$\int \sinh^3(e+fx)\left(a+b\sinh^2(e+fx)\right)^{3/2}\,dx=\left(\int\sqrt{\sinh(fx+e)^2b+a}\,\sinh(fx+e)^5\,dx\right)b+\left(\int\sqrt{\sinh(fx+e)^2b+a}\,\sinh(fx+e)^3\,dx\right)a$$

input
```
int(sinh(f*x+e)^3*(a+b*sinh(f*x+e)^2)^(3/2),x)
```

output
```
int(sqrt(sinh(e + f*x)**2*b + a)*sinh(e + f*x)**5,x)*b + int(sqrt(sinh(e +
 f*x)**2*b + a)*sinh(e + f*x)**3,x)*a
```

3.71 $\int \sinh(e+fx)\left(a+b\sinh^2(e+fx)\right)^{3/2}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 121

$$\int \sinh(e+fx)\left(a+b\sinh^2(e+fx)\right)^{3/2}\,dx = \frac{3(a-b)^2\operatorname{arctanh}\left(\frac{\sqrt{b}\cosh(e+fx)}{\sqrt{a-b+b\cosh^2(e+fx)}}\right)}{8\sqrt{b}f} + \frac{3(a-b)\cosh(e+fx)\sqrt{a-b+b\cosh^2(e+fx)}}{8f} + \frac{\cosh(e+fx)\left(a-b+b\cosh^2(e+fx)\right)^{3/2}}{4f}$$

output
```
3/8*(a-b)^2*arctanh(b^(1/2)*cosh(f*x+e)/(a-b+b*cosh(f*x+e)^2)^(1/2))/b^(1/
2)/f+3/8*(a-b)*cosh(f*x+e)*(a-b+b*cosh(f*x+e)^2)^(1/2)/f+1/4*cosh(f*x+e)*(
a-b+b*cosh(f*x+e)^2)^(3/2)/f
```

Mathematica [A] (verified)

Time = 0.34 (sec) , antiderivative size = 111, normalized size of antiderivative = 0.92

$$\int \sinh(e+fx)\left(a+b\sinh^2(e+fx)\right)^{3/2}\,dx=\frac{\frac{1}{2}\cosh(e+fx)(5a-4b+b\cosh(2(e+fx)))\sqrt{4a-2b+2b\cosh(2(e+fx))}+\frac{3(a-b)^2\log\left(\sqrt{2}\right.}{}}{8f}$$

input
```
Integrate[Sinh[e + f*x]*(a + b*Sinh[e + f*x]^2)^(3/2),x]
```

output
```
((Cosh[e + f*x]*(5*a - 4*b + b*Cosh[2*(e + f*x)])*Sqrt[4*a - 2*b + 2*b*Cos
h[2*(e + f*x)]])/2 + (3*(a - b)^2*Log[Sqrt[2]*Sqrt[b]*Cosh[e + f*x] + Sqrt
[2*a - b + b*Cosh[2*(e + f*x)]]])/Sqrt[b])/(8*f)
```

Rubi [A] (verified)

Time = 0.32 (sec) , antiderivative size = 119, normalized size of antiderivative = 0.98, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.304, Rules used = {3042, 26, 3665, 211, 211, 224, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sinh(e+fx)\left(a+b\sinh^2(e+fx)\right)^{3/2}dx$$

$$\Big\downarrow \text{3042}$$

$$\int -i\sin(ie+ifx)\left(a-b\sin(ie+ifx)^2\right)^{3/2}dx$$

$$\Big\downarrow \text{26}$$

$$-i\int \sin(ie+ifx)\left(a-b\sin(ie+ifx)^2\right)^{3/2}dx$$

$$\Big\downarrow \text{3665}$$

$$\frac{\int \left(b\cosh^2(e+fx)+a-b\right)^{3/2}d\cosh(e+fx)}{f}$$

$\Big\downarrow$ 211

$$\frac{\frac{3}{4}(a-b)\int\sqrt{b\cosh^2(e+fx)+a-b}d\cosh(e+fx)+\frac{1}{4}\cosh(e+fx)\left(a+b\cosh^2(e+fx)-b\right)^{3/2}}{f}$$

$\Big\downarrow$ 211

$$\frac{\frac{3}{4}(a-b)\left(\frac{1}{2}(a-b)\int\frac{1}{\sqrt{b\cosh^2(e+fx)+a-b}}d\cosh(e+fx)+\frac{1}{2}\cosh(e+fx)\sqrt{a+b\cosh^2(e+fx)-b}\right)+\frac{1}{4}\cosh(e-}{f}$$

$\Big\downarrow$ 224

$$\frac{\frac{3}{4}(a-b)\left(\frac{1}{2}(a-b)\int\frac{1}{1-\frac{b\cosh^2(e+fx)}{b\cosh^2(e+fx)+a-b}}d\frac{\cosh(e+fx)}{\sqrt{b\cosh^2(e+fx)+a-b}}+\frac{1}{2}\cosh(e+fx)\sqrt{a+b\cosh^2(e+fx)-b}\right)+\frac{1}{4}\cosh}{f}$$

$\Big\downarrow$ 219

$$\frac{\frac{3}{4}(a-b)\left(\frac{(a-b)\text{arctanh}\left(\frac{\sqrt{b}\cosh(e+fx)}{\sqrt{a+b\cosh^2(e+fx)-b}}\right)}{2\sqrt{b}}+\frac{1}{2}\cosh(e+fx)\sqrt{a+b\cosh^2(e+fx)-b}\right)+\frac{1}{4}\cosh(e+fx)\,(a+b}{f}$$

input
```
Int[Sinh[e + f*x]*(a + b*Sinh[e + f*x]^2)^(3/2),x]
```

output
```
((Cosh[e + f*x]*(a - b + b*Cosh[e + f*x]^2)^(3/2))/4 + (3*(a - b)*(((a - b
)*ArcTanh[(Sqrt[b]*Cosh[e + f*x])/Sqrt[a - b + b*Cosh[e + f*x]^2]])/(2*Sqr
t[b]) + (Cosh[e + f*x]*Sqrt[a - b + b*Cosh[e + f*x]^2])/2))/4)/f
```

Defintions of rubi rules used

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 211
```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[x*((a + b*x^2)^p/(2*p + 1
)), x] + Simp[2*a*(p/(2*p + 1))   Int[(a + b*x^2)^(p - 1), x], x] /; FreeQ[
{a, b}, x] && GtQ[p, 0] && (IntegerQ[4*p] || IntegerQ[6*p])
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3665
```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^
(p_.), x_Symbol] :> With[{ff = FreeFactors[Cos[e + f*x], x]}, Simp[-ff/f
Subst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a + b - b*ff^2*x^2)^p, x], x, Cos[e +
 f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m - 1)/2]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 335 vs. $2(105) = 210$.

Time = 0.26 (sec) , antiderivative size = 336, normalized size of antiderivative = 2.78

method	result
default	$\frac{\sqrt{\left(a+b\sinh(fx+e)^2\right)\cosh(fx+e)^2}\left(4\sqrt{b\cosh(fx+e)^4+(a-b)\cosh(fx+e)^2}\,b^{\frac{3}{2}}\cosh(fx+e)^2-10b^{\frac{3}{2}}\sqrt{b\cosh(fx+e)^4+(a-b)\cosh\right.}{}$

input
```
int(sinh(f*x+e)*(a+b*sinh(f*x+e)^2)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
1/16*((a+b*sinh(f*x+e)^2)*cosh(f*x+e)^2)^(1/2)*(4*(b*cosh(f*x+e)^4+(a-b)*c
osh(f*x+e)^2)^(1/2)*b^(3/2)*cosh(f*x+e)^2-10*b^(3/2)*(b*cosh(f*x+e)^4+(a-b
)*cosh(f*x+e)^2)^(1/2)+10*a*(b*cosh(f*x+e)^4+(a-b)*cosh(f*x+e)^2)^(1/2)*b^
(1/2)+3*a^2*ln(1/2*(2*b*cosh(f*x+e)^2+2*(b*cosh(f*x+e)^4+(a-b)*cosh(f*x+e)
^2)^(1/2)*b^(1/2)+a-b)/b^(1/2))-6*b*ln(1/2*(2*b*cosh(f*x+e)^2+2*(b*cosh(f*
x+e)^4+(a-b)*cosh(f*x+e)^2)^(1/2)*b^(1/2)+a-b)/b^(1/2))*a+3*b^2*ln(1/2*(2*
b*cosh(f*x+e)^2+2*(b*cosh(f*x+e)^4+(a-b)*cosh(f*x+e)^2)^(1/2)*b^(1/2)+a-b)
/b^(1/2)))/b^(1/2)/cosh(f*x+e)/(a+b*sinh(f*x+e)^2)^(1/2)/f
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1151 vs. $2(105) = 210$.

Time = 0.22 (sec) , antiderivative size = 2977, normalized size of antiderivative = 24.60

$$\int \sinh(e+fx) \left(a+b\sinh^2(e+fx)\right)^{3/2} dx = \text{Too large to display}$$

input
```
integrate(sinh(f*x+e)*(a+b*sinh(f*x+e)^2)^(3/2),x, algorithm="fricas")
```

output
```
[1/64*(6*((a^2 - 2*a*b + b^2)*cosh(f*x + e)^4 + 4*(a^2 - 2*a*b + b^2)*cosh
(f*x + e)^3*sinh(f*x + e) + 6*(a^2 - 2*a*b + b^2)*cosh(f*x + e)^2*sinh(f*x
 + e)^2 + 4*(a^2 - 2*a*b + b^2)*cosh(f*x + e)*sinh(f*x + e)^3 + (a^2 - 2*a
*b + b^2)*sinh(f*x + e)^4)*sqrt(b)*log((a^2*b*cosh(f*x + e)^8 + 8*a^2*b*co
sh(f*x + e)*sinh(f*x + e)^7 + a^2*b*sinh(f*x + e)^8 + 2*(a^3 + a^2*b)*cosh
(f*x + e)^6 + 2*(14*a^2*b*cosh(f*x + e)^2 + a^3 + a^2*b)*sinh(f*x + e)^6 +
 4*(14*a^2*b*cosh(f*x + e)^3 + 3*(a^3 + a^2*b)*cosh(f*x + e))*sinh(f*x + e
)^5 + (9*a^2*b - 4*a*b^2 + b^3)*cosh(f*x + e)^4 + (70*a^2*b*cosh(f*x + e)^
4 + 9*a^2*b - 4*a*b^2 + b^3 + 30*(a^3 + a^2*b)*cosh(f*x + e)^2)*sinh(f*x +
 e)^4 + 4*(14*a^2*b*cosh(f*x + e)^5 + 10*(a^3 + a^2*b)*cosh(f*x + e)^3 + (
9*a^2*b - 4*a*b^2 + b^3)*cosh(f*x + e))*sinh(f*x + e)^3 + b^3 + 2*(3*a*b^2
 - b^3)*cosh(f*x + e)^2 + 2*(14*a^2*b*cosh(f*x + e)^6 + 15*(a^3 + a^2*b)*c
osh(f*x + e)^4 + 3*a*b^2 - b^3 + 3*(9*a^2*b - 4*a*b^2 + b^3)*cosh(f*x + e)
^2)*sinh(f*x + e)^2 + sqrt(2)*(a^2*cosh(f*x + e)^6 + 6*a^2*cosh(f*x + e)*s
inh(f*x + e)^5 + a^2*sinh(f*x + e)^6 + 3*a^2*cosh(f*x + e)^4 + 3*(5*a^2*co
sh(f*x + e)^2 + a^2)*sinh(f*x + e)^4 + 4*(5*a^2*cosh(f*x + e)^3 + 3*a^2*co
sh(f*x + e))*sinh(f*x + e)^3 + (4*a*b - b^2)*cosh(f*x + e)^2 + (15*a^2*cos
h(f*x + e)^4 + 18*a^2*cosh(f*x + e)^2 + 4*a*b - b^2)*sinh(f*x + e)^2 + b^2
 + 2*(3*a^2*cosh(f*x + e)^5 + 6*a^2*cosh(f*x + e)^3 + (4*a*b - b^2)*cosh(f
*x + e))*sinh(f*x + e))*sqrt(b)*sqrt((b*cosh(f*x + e)^2 + b*sinh(f*x + ...
```

Sympy [F(-1)]

Timed out.

$$\int \sinh(e+fx)\left(a+b\sinh^2(e+fx)\right)^{3/2}\,dx=\text{Timed out}$$

input
```
integrate(sinh(f*x+e)*(a+b*sinh(f*x+e)**2)**(3/2),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \sinh(e+fx)\left(a+b\sinh^2(e+fx)\right)^{3/2}\,dx=\int \left(b\sinh\left(fx+e\right)^2+a\right)^{\frac{3}{2}}\sinh\left(fx+e\right)\,dx$$

input
```
integrate(sinh(f*x+e)*(a+b*sinh(f*x+e)^2)^(3/2),x, algorithm="maxima")
```

output
```
integrate((b*sinh(f*x + e)^2 + a)^(3/2)*sinh(f*x + e), x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 897 vs. $2(105) = 210$.

Time = 0.50 (sec) , antiderivative size = 897, normalized size of antiderivative = 7.41

$$\int \sinh(e+fx)\left(a+b\sinh^2(e+fx)\right)^{3/2}\,dx=\text{Too large to display}$$

input
```
integrate(sinh(f*x+e)*(a+b*sinh(f*x+e)^2)^(3/2),x, algorithm="giac")
```

output
```
1/64*(sqrt(b*e^(4*f*x + 4*e) + 4*a*e^(2*f*x + 2*e) - 2*b*e^(2*f*x + 2*e) +
 b)*(b*e^(2*f*x + 6*e) + (10*a*b*e^(6*e) - 7*b^2*e^(6*e))*e^(-2*e)/b) - 24
*(a^2*e^(4*e) - 2*a*b*e^(4*e) + b^2*e^(4*e))*arctan(-(sqrt(b)*e^(2*f*x + 2
*e) - sqrt(b*e^(4*f*x + 4*e) + 4*a*e^(2*f*x + 2*e) - 2*b*e^(2*f*x + 2*e) +
 b))/sqrt(-b))/sqrt(-b) - 12*(a^2*sqrt(b)*e^(4*e) - 2*a*b^(3/2)*e^(4*e) +
b^(5/2)*e^(4*e))*log(abs(-(sqrt(b)*e^(2*f*x + 2*e) - sqrt(b*e^(4*f*x + 4*e
) + 4*a*e^(2*f*x + 2*e) - 2*b*e^(2*f*x + 2*e) + b))*b - 2*a*sqrt(b) + b^(3
/2)))/b - 4*(10*(sqrt(b)*e^(2*f*x + 2*e) - sqrt(b*e^(4*f*x + 4*e) + 4*a*e^
(2*f*x + 2*e) - 2*b*e^(2*f*x + 2*e) + b))^3*a^2*e^(4*e) - 12*(sqrt(b)*e^(2
*f*x + 2*e) - sqrt(b*e^(4*f*x + 4*e) + 4*a*e^(2*f*x + 2*e) - 2*b*e^(2*f*x
+ 2*e) + b))^3*a*b*e^(4*e) + 4*(sqrt(b)*e^(2*f*x + 2*e) - sqrt(b*e^(4*f*x
+ 4*e) + 4*a*e^(2*f*x + 2*e) - 2*b*e^(2*f*x + 2*e) + b))^3*b^2*e^(4*e) + 8
*(sqrt(b)*e^(2*f*x + 2*e) - sqrt(b*e^(4*f*x + 4*e) + 4*a*e^(2*f*x + 2*e) -
 2*b*e^(2*f*x + 2*e) + b))^2*a*b^(3/2)*e^(4*e) - 5*(sqrt(b)*e^(2*f*x + 2*e
) - sqrt(b*e^(4*f*x + 4*e) + 4*a*e^(2*f*x + 2*e) - 2*b*e^(2*f*x + 2*e) + b
))^2*b^(5/2)*e^(4*e) - 6*(sqrt(b)*e^(2*f*x + 2*e) - sqrt(b*e^(4*f*x + 4*e)
 + 4*a*e^(2*f*x + 2*e) - 2*b*e^(2*f*x + 2*e) + b))*a^2*b*e^(4*e) + 8*(sqrt
(b)*e^(2*f*x + 2*e) - sqrt(b*e^(4*f*x + 4*e) + 4*a*e^(2*f*x + 2*e) - 2*b*e
^(2*f*x + 2*e) + b))*a*b^2*e^(4*e) - 2*(sqrt(b)*e^(2*f*x + 2*e) - sqrt(b*e
^(4*f*x + 4*e) + 4*a*e^(2*f*x + 2*e) - 2*b*e^(2*f*x + 2*e) + b))*b^3*e^...
```

Mupad [F(-1)]

Timed out.

$$\int \sinh(e+fx)\left(a+b\sinh^2(e+fx)\right)^{3/2}\,dx=\int \sinh(e+f\,x)\,\left(b\,{\sinh(e+f\,x)}^2+a\right)^{3/2}\,dx$$

input
```
int(sinh(e + f*x)*(a + b*sinh(e + f*x)^2)^(3/2),x)
```

output
```
int(sinh(e + f*x)*(a + b*sinh(e + f*x)^2)^(3/2), x)
```

Reduce [F]

$$\int \sinh(e+fx)\left(a+b\sinh^2(e+fx)\right)^{3/2}\,dx = \left(\int \sqrt{\sinh\left(fx+e\right)^2 b+a}\,\sinh\left(fx+e\right)^3 dx\right) b + \left(\int \sqrt{\sinh\left(fx+e\right)^2 b+a}\,\sinh\left(fx+e\right) dx\right) a$$

input

```
int(sinh(f*x+e)*(a+b*sinh(f*x+e)^2)^(3/2),x)
```

output

```
int(sqrt(sinh(e + f*x)**2*b + a)*sinh(e + f*x)**3,x)*b + int(sqrt(sinh(e +
 f*x)**2*b + a)*sinh(e + f*x),x)*a
```

3.72 $\int \operatorname{csch}(e+fx) \left(a+b\sinh^2(e+fx)\right)^{3/2} dx$

Optimal result

Integrand size = 23, antiderivative size = 127

$$\int \operatorname{csch}(e+fx) \left(a+b\sinh^2(e+fx)\right)^{3/2} dx = -\frac{a^{3/2}\operatorname{arctanh}\left(\frac{\sqrt{a}\cosh(e+fx)}{\sqrt{a-b+b\cosh^2(e+fx)}}\right)}{f} + \frac{(3a-b)\sqrt{b}\operatorname{arctanh}\left(\frac{\sqrt{b}\cosh(e+fx)}{\sqrt{a-b+b\cosh^2(e+fx)}}\right)}{2f} + \frac{b\cosh(e+fx)\sqrt{a-b+b\cosh^2(e+fx)}}{2f}$$

output
```
-a^(3/2)*arctanh(a^(1/2)*cosh(f*x+e)/(a-b+b*cosh(f*x+e)^2)^(1/2))/f+1/2*(3
*a-b)*b^(1/2)*arctanh(b^(1/2)*cosh(f*x+e)/(a-b+b*cosh(f*x+e)^2)^(1/2))/f+1
/2*b*cosh(f*x+e)*(a-b+b*cosh(f*x+e)^2)^(1/2)/f
```

Mathematica [A] (verified)

Time = 0.54 (sec) , antiderivative size = 136, normalized size of antiderivative = 1.07

$$\int \operatorname{csch}(e+fx) \left(a+b\sinh^2(e+fx)\right)^{3/2} dx = \frac{-4a^{3/2}\operatorname{arctanh}\left(\frac{\sqrt{2}\sqrt{a}\cosh(e+fx)}{\sqrt{2a-b+b\cosh(2(e+fx))}}\right) + b\cosh(e+fx)\sqrt{4a-2b+2b\cosh(2(e+fx))} - 2\sqrt{\ldots}}{4f}$$

input
```
Integrate[Csch[e + f*x]*(a + b*Sinh[e + f*x]^2)^(3/2),x]
```

output
```
(-4*a^(3/2)*ArcTanh[(Sqrt[2]*Sqrt[a]*Cosh[e + f*x])/Sqrt[2*a - b + b*Cosh[
2*(e + f*x)]]] + b*Cosh[e + f*x]*Sqrt[4*a - 2*b + 2*b*Cosh[2*(e + f*x)]] -
 2*Sqrt[b]*(-3*a + b)*Log[Sqrt[2]*Sqrt[b]*Cosh[e + f*x] + Sqrt[2*a - b + b
*Cosh[2*(e + f*x)]]])/(4*f)
```

Rubi [A] (verified)

Time = 0.43 (sec) , antiderivative size = 126, normalized size of antiderivative = 0.99, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.435$, Rules used = {3042, 26, 3665, 318, 25, 398, 224, 219, 291, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \text{csch}(e+fx)\left(a+b\sinh^2(e+fx)\right)^{3/2}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{i\left(a-b\sin(ie+ifx)^2\right)^{3/2}}{\sin(ie+ifx)}dx$$

$$\Big\downarrow \text{26}$$

$$i\int \frac{\left(a-b\sin(ie+ifx)^2\right)^{3/2}}{\sin(ie+ifx)}dx$$

$$\Big\downarrow \text{3665}$$

$$-\frac{\int \frac{\left(b\cosh^2(e+fx)+a-b\right)^{3/2}}{1-\cosh^2(e+fx)}d\cosh(e+fx)}{f}$$

$$\Big\downarrow \text{318}$$

$$-\frac{-\frac{1}{2}\int -\frac{(3a-b)b\cosh^2(e+fx)+(a-b)(2a-b)}{\left(1-\cosh^2(e+fx)\right)\sqrt{b\cosh^2(e+fx)+a-b}}d\cosh(e+fx)-\frac{1}{2}b\cosh(e+fx)\sqrt{a+b\cosh^2(e+fx)-b}}{f}$$

$$\Big\downarrow \text{25}$$

$$-\frac{\frac{1}{2}\int \frac{2a^2-3ba+b^2+(3a-b)b\cosh^2(e+fx)}{(1-\cosh^2(e+fx))\sqrt{b\cosh^2(e+fx)+a-b}}d\cosh(e+fx)-\frac{1}{2}b\cosh(e+fx)\sqrt{a+b\cosh^2(e+fx)-b}}{f}$$

$\downarrow$ 398

$$-\frac{\frac{1}{2}\left(2a^2\int \frac{1}{(1-\cosh^2(e+fx))\sqrt{b\cosh^2(e+fx)+a-b}}d\cosh(e+fx)-b(3a-b)\int \frac{1}{\sqrt{b\cosh^2(e+fx)+a-b}}d\cosh(e+fx)\right)-\frac{1}{2}b}{f}$$

$\downarrow$ 224

$$-\frac{\frac{1}{2}\left(2a^2\int \frac{1}{(1-\cosh^2(e+fx))\sqrt{b\cosh^2(e+fx)+a-b}}d\cosh(e+fx)-b(3a-b)\int \frac{1}{1-\frac{b\cosh^2(e+fx)}{b\cosh^2(e+fx)+a-b}}d\frac{\cosh(e+fx)}{\sqrt{b\cosh^2(e+fx)+a-b}}\right)}{f}$$

$\downarrow$ 219

$$-\frac{\frac{1}{2}\left(2a^2\int \frac{1}{(1-\cosh^2(e+fx))\sqrt{b\cosh^2(e+fx)+a-b}}d\cosh(e+fx)-\sqrt{b}(3a-b)\text{arctanh}\left(\frac{\sqrt{b}\cosh(e+fx)}{\sqrt{a+b\cosh^2(e+fx)-b}}\right)\right)-\frac{1}{2}b\cos}{f}$$

$\downarrow$ 291

$$-\frac{\frac{1}{2}\left(2a^2\int \frac{1}{1-\frac{a\cosh^2(e+fx)}{b\cosh^2(e+fx)+a-b}}d\frac{\cosh(e+fx)}{\sqrt{b\cosh^2(e+fx)+a-b}}-\sqrt{b}(3a-b)\text{arctanh}\left(\frac{\sqrt{b}\cosh(e+fx)}{\sqrt{a+b\cosh^2(e+fx)-b}}\right)\right)-\frac{1}{2}b\cosh(e+fx)}{f}$$

$\downarrow$ 219

$$-\frac{\frac{1}{2}\left(2a^{3/2}\text{arctanh}\left(\frac{\sqrt{a}\cosh(e+fx)}{\sqrt{a+b\cosh^2(e+fx)-b}}\right)-\sqrt{b}(3a-b)\text{arctanh}\left(\frac{\sqrt{b}\cosh(e+fx)}{\sqrt{a+b\cosh^2(e+fx)-b}}\right)\right)-\frac{1}{2}b\cosh(e+fx)\sqrt{a+b}}{f}$$

input
```
Int[Csch[e + f*x]*(a + b*Sinh[e + f*x]^2)^(3/2),x]
```

output
```
-(((2*a^(3/2)*ArcTanh[(Sqrt[a]*Cosh[e + f*x])/Sqrt[a - b + b*Cosh[e + f*x]
^2]] - (3*a - b)*Sqrt[b]*ArcTanh[(Sqrt[b]*Cosh[e + f*x])/Sqrt[a - b + b*Co
sh[e + f*x]^2]])/2 - (b*Cosh[e + f*x]*Sqrt[a - b + b*Cosh[e + f*x]^2])/2)/
f)
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

rule 291
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*((c_) + (d_.)*(x_)^2)), x_Symbol] :> Subst
[Int[1/(c - (b*c - a*d)*x^2), x], x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b, c,
d}, x] && NeQ[b*c - a*d, 0]
```

rule 318
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_), x_Symbol] :> Sim
p[d*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^(q - 1)/(b*(2*(p + q) + 1))), x] + S
imp[1/(b*(2*(p + q) + 1))   Int[(a + b*x^2)^p*(c + d*x^2)^(q - 2)*Simp[c*(b
*c*(2*(p + q) + 1) - a*d) + d*(b*c*(2*(p + 2*q - 1) + 1) - a*d*(2*(q - 1) +
 1))*x^2, x], x], x] /; FreeQ[{a, b, c, d, p}, x] && NeQ[b*c - a*d, 0] && G
tQ[q, 1] && NeQ[2*(p + q) + 1, 0] &&  !IGtQ[p, 1] && IntBinomialQ[a, b, c,
d, 2, p, q, x]
```

rule 398
```
Int[((e_) + (f_.)*(x_)^2)/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2])
, x_Symbol] :> Simp[f/b   Int[1/Sqrt[c + d*x^2], x], x] + Simp[(b*e - a*f)/
b   Int[1/((a + b*x^2)*Sqrt[c + d*x^2]), x], x] /; FreeQ[{a, b, c, d, e, f}
, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3665
```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^
(p_.), x_Symbol] :> With[{ff = FreeFactors[Cos[e + f*x], x]}, Simp[-ff/f
Subst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a + b - b*ff^2*x^2)^p, x], x, Cos[e +
 f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m - 1)/2]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 267 vs. $2(109) = 218$.

Time = 0.30 (sec) , antiderivative size = 268, normalized size of antiderivative = 2.11

method	result
default	$-\frac{\sqrt{\left(a+b\sinh(fx+e)^2\right)\cosh(fx+e)^2}\left(b^{\frac{3}{2}}\ln\left(\frac{2b\cosh(fx+e)^2+2\sqrt{b\cosh(fx+e)^4+(a-b)\cosh(fx+e)^2}\,\sqrt{b}+a-b}{2\sqrt{b}}\right)+2a^{\frac{3}{2}}\ln\left(\frac{(a+b)\cosh(f}{}\right.\right.}{}$

input
```
int(csch(f*x+e)*(a+b*sinh(f*x+e)^2)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-1/4*((a+b*sinh(f*x+e)^2)*cosh(f*x+e)^2)^(1/2)*(b^(3/2)*ln(1/2*(2*b*cosh(f
*x+e)^2+2*(b*cosh(f*x+e)^4+(a-b)*cosh(f*x+e)^2)^(1/2)*b^(1/2)+a-b)/b^(1/2)
)+2*a^(3/2)*ln(((a+b)*cosh(f*x+e)^2+2*a^(1/2)*(b*cosh(f*x+e)^4+(a-b)*cosh(
f*x+e)^2)^(1/2)+a-b)/(cosh(f*x+e)^2-1))-3*b^(1/2)*ln(1/2*(2*b*cosh(f*x+e)^
2+2*(b*cosh(f*x+e)^4+(a-b)*cosh(f*x+e)^2)^(1/2)*b^(1/2)+a-b)/b^(1/2))*a-2*
b*(b*cosh(f*x+e)^4+(a-b)*cosh(f*x+e)^2)^(1/2))/cosh(f*x+e)/(a+b*sinh(f*x+e
)^2)^(1/2)/f
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 891 vs. $2(109) = 218$.

Time = 0.26 (sec) , antiderivative size = 5342, normalized size of antiderivative = 42.06

$$\int \operatorname{csch}(e+fx)\left(a+b\sinh^2(e+fx)\right)^{3/2}\,dx=\text{Too large to display}$$

input
```
integrate(csch(f*x+e)*(a+b*sinh(f*x+e)^2)^(3/2),x, algorithm="fricas")
```

output
```
Too large to include
```

Sympy [F(-1)]

Timed out.

$$\int \operatorname{csch}(e+fx)\left(a+b\sinh^2(e+fx)\right)^{3/2}\,dx=\text{Timed out}$$

input
```
integrate(csch(f*x+e)*(a+b*sinh(f*x+e)**2)**(3/2),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \operatorname{csch}(e+fx)\left(a+b\sinh^2(e+fx)\right)^{3/2}\,dx=\int \left(b\sinh\left(fx+e\right)^2+a\right)^{\frac{3}{2}}\operatorname{csch}\left(fx+e\right)\,dx$$

input
```
integrate(csch(f*x+e)*(a+b*sinh(f*x+e)^2)^(3/2),x, algorithm="maxima")
```

output
```
integrate((b*sinh(f*x + e)^2 + a)^(3/2)*csch(f*x + e), x)
```

Giac [F(-2)]

Exception generated.

$$\int \operatorname{csch}(e+fx)\left(a+b\sinh^2(e+fx)\right)^{3/2}\,dx=\text{Exception raised: TypeError}$$

input
```
integrate(csch(f*x+e)*(a+b*sinh(f*x+e)^2)^(3/2),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Error: Bad Argument Type
```

Mupad [F(-1)]

Timed out.

$$\int \operatorname{csch}(e+fx)\left(a+b\sinh^2(e+fx)\right)^{3/2}\,dx=\int \frac{\left(b\sinh(e+f\,x)^2+a\right)^{3/2}}{\sinh\left(e+f\,x\right)}\,dx$$

input
```
int((a + b*sinh(e + f*x)^2)^(3/2)/sinh(e + f*x),x)
```

output
```
int((a + b*sinh(e + f*x)^2)^(3/2)/sinh(e + f*x), x)
```

Reduce [F]

$$\int \operatorname{csch}(e+fx)\left(a+b\sinh^2(e+fx)\right)^{3/2}\,dx=\left(\int \sqrt{\sinh\left(fx+e\right)^2b+a}\,\operatorname{csch}(fx+e)\sinh\left(fx+e\right)^2dx\right)b+\left(\int \sqrt{\sinh\left(fx+e\right)^2b+a}\,\operatorname{csch}(fx+e)\,dx\right)a$$

input
```
int(csch(f*x+e)*(a+b*sinh(f*x+e)^2)^(3/2),x)
```

output

```
int(sqrt(sinh(e + f*x)**2*b + a)*csch(e + f*x)*sinh(e + f*x)**2,x)*b + int
(sqrt(sinh(e + f*x)**2*b + a)*csch(e + f*x),x)*a
```

3.73 $\int \operatorname{csch}^3(e+fx) \left(a+b\sinh^2(e+fx)\right)^{3/2} dx$

Optimal result

Integrand size = 25, antiderivative size = 130

$$\int \operatorname{csch}^3(e+fx) \left(a+b\sinh^2(e+fx)\right)^{3/2} dx = \frac{\sqrt{a}(a-3b)\operatorname{arctanh}\left(\frac{\sqrt{a}\cosh(e+fx)}{\sqrt{a-b+b\cosh^2(e+fx)}}\right)}{2f} + \frac{b^{3/2}\operatorname{arctanh}\left(\frac{\sqrt{b}\cosh(e+fx)}{\sqrt{a-b+b\cosh^2(e+fx)}}\right)}{f} - \frac{a\sqrt{a-b+b\cosh^2(e+fx)}\coth(e+fx)\operatorname{csch}(e+fx)}{2f}$$

output
```
1/2*a^(1/2)*(a-3*b)*arctanh(a^(1/2)*cosh(f*x+e)/(a-b+b*cosh(f*x+e)^2)^(1/2
))/f+b^(3/2)*arctanh(b^(1/2)*cosh(f*x+e)/(a-b+b*cosh(f*x+e)^2)^(1/2))/f-1/
2*a*(a-b+b*cosh(f*x+e)^2)^(1/2)*coth(f*x+e)*csch(f*x+e)/f
```

Mathematica [A] (verified)

Time = 0.86 (sec) , antiderivative size = 143, normalized size of antiderivative = 1.10

$$\int \operatorname{csch}^3(e+fx)\left(a+b\sinh^2(e+fx)\right)^{3/2}\,dx = \frac{2\sqrt{a}(a-3b)\operatorname{arctanh}\left(\frac{\sqrt{2}\sqrt{a}\cosh(e+fx)}{\sqrt{2a-b+b\cosh(2(e+fx))}}\right)-a\sqrt{4a-2b+2b\cosh(2(e+fx))}\coth(e+fx$$
$$4f$$

input
```
Integrate[Csch[e + f*x]^3*(a + b*Sinh[e + f*x]^2)^(3/2),x]
```

output
```
(2*Sqrt[a]*(a - 3*b)*ArcTanh[(Sqrt[2]*Sqrt[a]*Cosh[e + f*x])/Sqrt[2*a - b
+ b*Cosh[2*(e + f*x)]]] - a*Sqrt[4*a - 2*b + 2*b*Cosh[2*(e + f*x)]]*Coth[e
 + f*x]*Csch[e + f*x] + 4*b^(3/2)*Log[Sqrt[2]*Sqrt[b]*Cosh[e + f*x] + Sqrt
[2*a - b + b*Cosh[2*(e + f*x)]]])/(4*f)
```

Rubi [A] (verified)

Time = 0.37 (sec) , antiderivative size = 136, normalized size of antiderivative = 1.05, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.400, Rules used = {3042, 26, 3665, 315, 25, 398, 224, 219, 291, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \operatorname{csch}^3(e+fx)\left(a+b\sinh^2(e+fx)\right)^{3/2}\,dx$$

$$\Big\downarrow \ 3042$$

$$\int -\frac{i\left(a-b\sin(ie+ifx)^2\right)^{3/2}}{\sin(ie+ifx)^3}dx$$

$$\Big\downarrow \ 26$$

$$-i\int \frac{\left(a-b\sin(ie+ifx)^2\right)^{3/2}}{\sin(ie+ifx)^3}dx$$

$$\Big\downarrow \ 3665$$

$$\frac{\int \frac{\left(b\cosh^2(e+fx)+a-b\right)^{3/2}}{\left(1-\cosh^2(e+fx)\right)^2}d\cosh(e+fx)}{f}$$

$$\Big\downarrow \; 315$$

$$\frac{\frac{a\cosh(e+fx)\sqrt{a+b\cosh^2(e+fx)-b}}{2\left(1-\cosh^2(e+fx)\right)}-\frac{1}{2}\int -\frac{(a-2b)(a-b)-2b^2\cosh^2(e+fx)}{\left(1-\cosh^2(e+fx)\right)\sqrt{b\cosh^2(e+fx)+a-b}}d\cosh(e+fx)}{f}$$

$$\Big\downarrow \; 25$$

$$\frac{\frac{1}{2}\int \frac{(a-2b)(a-b)-2b^2\cosh^2(e+fx)}{\left(1-\cosh^2(e+fx)\right)\sqrt{b\cosh^2(e+fx)+a-b}}d\cosh(e+fx)+\frac{a\cosh(e+fx)\sqrt{a+b\cosh^2(e+fx)-b}}{2\left(1-\cosh^2(e+fx)\right)}}{f}$$

$$\Big\downarrow \; 398$$

$$\frac{\frac{1}{2}\left(2b^2\int \frac{1}{\sqrt{b\cosh^2(e+fx)+a-b}}d\cosh(e+fx)+a(a-3b)\int \frac{1}{\left(1-\cosh^2(e+fx)\right)\sqrt{b\cosh^2(e+fx)+a-b}}d\cosh(e+fx)\right)+\frac{a\cos\cdots}{\cdots}}{f}$$

$$\Big\downarrow \; 224$$

$$\frac{\frac{1}{2}\left(2b^2\int \frac{1}{1-\frac{b\cosh^2(e+fx)}{b\cosh^2(e+fx)+a-b}}d\frac{\cosh(e+fx)}{\sqrt{b\cosh^2(e+fx)+a-b}}+a(a-3b)\int \frac{1}{\left(1-\cosh^2(e+fx)\right)\sqrt{b\cosh^2(e+fx)+a-b}}d\cosh(e+fx)\right)+\cdots}{f}$$

$$\Big\downarrow \; 219$$

$$\frac{\frac{1}{2}\left(a(a-3b)\int \frac{1}{\left(1-\cosh^2(e+fx)\right)\sqrt{b\cosh^2(e+fx)+a-b}}d\cosh(e+fx)+2b^{3/2}\text{arctanh}\left(\frac{\sqrt{b}\cosh(e+fx)}{\sqrt{a+b\cosh^2(e+fx)-b}}\right)\right)+\frac{a\cosh(e+\cdots}{2\cdots}}{f}$$

$$\Big\downarrow \; 291$$

$$\frac{\frac{1}{2}\left(a(a-3b)\int \frac{1}{1-\frac{a\cosh^2(e+fx)}{b\cosh^2(e+fx)+a-b}}d\frac{\cosh(e+fx)}{\sqrt{b\cosh^2(e+fx)+a-b}}+2b^{3/2}\text{arctanh}\left(\frac{\sqrt{b}\cosh(e+fx)}{\sqrt{a+b\cosh^2(e+fx)-b}}\right)\right)+\frac{a\cosh(e+fx)\sqrt{a+b\cos\cdots}}{2\left(1-\cosh^2(e+\cdots\right.}}{f}$$

$$\Big\downarrow \; 219$$

$$\frac{\frac{1}{2}\left(2b^{3/2}\text{arctanh}\left(\frac{\sqrt{b}\cosh(e+fx)}{\sqrt{a+b\cosh^2(e+fx)-b}}\right)+\sqrt{a}(a-3b)\text{arctanh}\left(\frac{\sqrt{a}\cosh(e+fx)}{\sqrt{a+b\cosh^2(e+fx)-b}}\right)\right)+\frac{a\cosh(e+fx)\sqrt{a+b\cosh^2(e+fx}}{2(1-\cosh^2(e+fx))}}{f}$$

input
```
Int[Csch[e + f*x]^3*(a + b*Sinh[e + f*x]^2)^(3/2),x]
```

output
```
((Sqrt[a]*(a - 3*b)*ArcTanh[(Sqrt[a]*Cosh[e + f*x])/Sqrt[a - b + b*Cosh[e
+ f*x]^2]] + 2*b^(3/2)*ArcTanh[(Sqrt[b]*Cosh[e + f*x])/Sqrt[a - b + b*Cosh
[e + f*x]^2]])/2 + (a*Cosh[e + f*x]*Sqrt[a - b + b*Cosh[e + f*x]^2])/(2*(1
 - Cosh[e + f*x]^2)))/f
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

rule 291
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*((c_) + (d_.)*(x_)^2)), x_Symbol] :> Subst
[Int[1/(c - (b*c - a*d)*x^2), x], x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b, c,
d}, x] && NeQ[b*c - a*d, 0]
```

rule 315
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_), x_Symbol] :> Sim
p[(a*d - c*b)*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^(q - 1)/(2*a*b*(p + 1))),
x] - Simp[1/(2*a*b*(p + 1))   Int[(a + b*x^2)^(p + 1)*(c + d*x^2)^(q - 2)*S
imp[c*(a*d - c*b*(2*p + 3)) + d*(a*d*(2*(q - 1) + 1) - b*c*(2*(p + q) + 1))
*x^2, x], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -
1] && GtQ[q, 1] && IntBinomialQ[a, b, c, d, 2, p, q, x]
```

rule 398
```
Int[((e_) + (f_.)*(x_)^2)/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2])
, x_Symbol] :> Simp[f/b   Int[1/Sqrt[c + d*x^2], x], x] + Simp[(b*e - a*f)/
b   Int[1/((a + b*x^2)*Sqrt[c + d*x^2]), x], x] /; FreeQ[{a, b, c, d, e, f}
, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3665
```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^
(p_.), x_Symbol] :> With[{ff = FreeFactors[Cos[e + f*x], x]}, Simp[-ff/f
Subst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a + b - b*ff^2*x^2)^p, x], x, Cos[e +
 f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m - 1)/2]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 296 vs. $2(112) = 224$.

Time = 0.35 (sec) , antiderivative size = 297, normalized size of antiderivative = 2.28

method	result
default	$\frac{\sqrt{\left(a+b\sinh(fx+e)^2\right)\cosh(fx+e)^2}\left(2b^{\frac{3}{2}}\ln\left(\frac{2b\cosh(fx+e)^2+2\sqrt{b\cosh(fx+e)^4+(a-b)\cosh(fx+e)^2}\sqrt{b}+a-b}{2\sqrt{b}}\right)\sinh(fx+e)^2+a^{\frac{3}{2}}\ln\left(\right.\right.}{}$

input
```
int(csch(f*x+e)^3*(a+b*sinh(f*x+e)^2)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
1/4*((a+b*sinh(f*x+e)^2)*cosh(f*x+e)^2)^(1/2)*(2*b^(3/2)*ln(1/2*(2*b*cosh(
f*x+e)^2+2*(b*cosh(f*x+e)^4+(a-b)*cosh(f*x+e)^2)^(1/2)*b^(1/2)+a-b)/b^(1/2
))*sinh(f*x+e)^2+a^(3/2)*ln(((a+b)*cosh(f*x+e)^2+2*a^(1/2)*(b*cosh(f*x+e)^
4+(a-b)*cosh(f*x+e)^2)^(1/2)+a-b)/sinh(f*x+e)^2)*sinh(f*x+e)^2-3*a^(1/2)*b
*ln(((a+b)*cosh(f*x+e)^2+2*a^(1/2)*(b*cosh(f*x+e)^4+(a-b)*cosh(f*x+e)^2)^(
1/2)+a-b)/sinh(f*x+e)^2)*sinh(f*x+e)^2-2*a*((a+b*sinh(f*x+e)^2)*cosh(f*x+e
)^2)^(1/2))/sinh(f*x+e)^2/cosh(f*x+e)/(a+b*sinh(f*x+e)^2)^(1/2)/f
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1153 vs. $2(112) = 224$.

Time = 0.30 (sec) , antiderivative size = 6398, normalized size of antiderivative = 49.22

$$\int \operatorname{csch}^3(e+fx)\left(a+b\sinh^2(e+fx)\right)^{3/2}\,dx = \text{Too large to display}$$

input
```
integrate(csch(f*x+e)^3*(a+b*sinh(f*x+e)^2)^(3/2),x, algorithm="fricas")
```

output
```
Too large to include
```

Sympy [F(-1)]

Timed out.

$$\int \operatorname{csch}^3(e+fx)\left(a+b\sinh^2(e+fx)\right)^{3/2}\,dx = \text{Timed out}$$

input
```
integrate(csch(f*x+e)**3*(a+b*sinh(f*x+e)**2)**(3/2),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \operatorname{csch}^3(e+fx)\left(a+b\sinh^2(e+fx)\right)^{3/2}\,dx=\int \left(b\sinh\left(fx+e\right)^2+a\right)^{\frac{3}{2}}\operatorname{csch}\left(fx+e\right)^3\,dx$$

input
```
integrate(csch(f*x+e)^3*(a+b*sinh(f*x+e)^2)^(3/2),x, algorithm="maxima")
```

output
```
integrate((b*sinh(f*x + e)^2 + a)^(3/2)*csch(f*x + e)^3, x)
```

Giac [F(-2)]

Exception generated.

$$\int \operatorname{csch}^3(e+fx)\left(a+b\sinh^2(e+fx)\right)^{3/2}\,dx=\text{Exception raised: TypeError}$$

input
```
integrate(csch(f*x+e)^3*(a+b*sinh(f*x+e)^2)^(3/2),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Error: Bad Argument Type
```

Mupad [F(-1)]

Timed out.

$$\int \operatorname{csch}^3(e+fx)\left(a+b\sinh^2(e+fx)\right)^{3/2}\,dx=\int \frac{\left(b\sinh(e+f\,x)^2+a\right)^{3/2}}{\sinh\left(e+f\,x\right)^3}\,dx$$

input
```
int((a + b*sinh(e + f*x)^2)^(3/2)/sinh(e + f*x)^3,x)
```

output
```
int((a + b*sinh(e + f*x)^2)^(3/2)/sinh(e + f*x)^3, x)
```

Reduce [F]

$$\int \operatorname{csch}^3(e+fx)\left(a+b\sinh^2(e+fx)\right)^{3/2}dx = \left(\int \sqrt{\sinh(fx+e)^2 b+a}\,\operatorname{csch}(fx+e)^3\sinh(fx+e)^2\,dx\right)b + \left(\int \sqrt{\sinh(fx+e)^2 b+a}\,\operatorname{csch}(fx+e)^3\,dx\right)a$$

input
```
int(csch(f*x+e)^3*(a+b*sinh(f*x+e)^2)^(3/2),x)
```

output
```
int(sqrt(sinh(e + f*x)**2*b + a)*csch(e + f*x)**3*sinh(e + f*x)**2,x)*b +
int(sqrt(sinh(e + f*x)**2*b + a)*csch(e + f*x)**3,x)*a
```

3.74 $\int \operatorname{csch}^5(e+fx)\left(a+b\sinh^2(e+fx)\right)^{3/2}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 135

$$\int \operatorname{csch}^5(e+fx)\left(a+b\sinh^2(e+fx)\right)^{3/2}\,dx = -\frac{3(a-b)^2\operatorname{arctanh}\left(\frac{\sqrt{a}\cosh(e+fx)}{\sqrt{a-b+b\cosh^2(e+fx)}}\right)}{8\sqrt{a}f} + \frac{3(a-b)\sqrt{a-b+b\cosh^2(e+fx)}\coth(e+fx)\operatorname{csch}(e+fx)}{8f} - \frac{\left(a-b+b\cosh^2(e+fx)\right)^{3/2}\coth(e+fx)\operatorname{csch}^3(e+fx)}{4f}$$

output

```
-3/8*(a-b)^2*arctanh(a^(1/2)*cosh(f*x+e)/(a-b+b*cosh(f*x+e)^2)^(1/2))/a^(1
/2)/f+3/8*(a-b)*(a-b+b*cosh(f*x+e)^2)^(1/2)*coth(f*x+e)*csch(f*x+e)/f-1/4*
(a-b+b*cosh(f*x+e)^2)^(3/2)*coth(f*x+e)*csch(f*x+e)^3/f
```

Mathematica [A] (verified)

Time = 0.70 (sec) , antiderivative size = 123, normalized size of antiderivative = 0.91

$$\int \operatorname{csch}^5(e+fx)\left(a+b\sinh^2(e+fx)\right)^{3/2}\,dx=\frac{-6(a-b)^2\operatorname{arctanh}\left(\frac{\sqrt{2}\sqrt{a}\cosh(e+fx)}{\sqrt{2a-b+b\cosh(2(e+fx))}}\right)+\sqrt{2}\sqrt{a}\sqrt{2a-b+b\cosh(2(e+fx))}\coth(e+f}{16\sqrt{a}f}$$

input
```
Integrate[Csch[e + f*x]^5*(a + b*Sinh[e + f*x]^2)^(3/2),x]
```

output
```
(-6*(a - b)^2*ArcTanh[(Sqrt[2]*Sqrt[a]*Cosh[e + f*x])/Sqrt[2*a - b + b*Cos
h[2*(e + f*x)]]] + Sqrt[2]*Sqrt[a]*Sqrt[2*a - b + b*Cosh[2*(e + f*x)]]*Cot
h[e + f*x]*Csch[e + f*x]*(3*a - 5*b - 2*a*Csch[e + f*x]^2))/(16*Sqrt[a]*f)
```

Rubi [A] (verified)

Time = 0.35 (sec) , antiderivative size = 148, normalized size of antiderivative = 1.10, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.280, Rules used = {3042, 26, 3665, 292, 292, 291, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \operatorname{csch}^5(e+fx)\left(a+b\sinh^2(e+fx)\right)^{3/2}dx$$

$$\Big\downarrow \; 3042$$

$$\int \frac{i\left(a-b\sin(ie+ifx)^2\right)^{3/2}}{\sin(ie+ifx)^5}dx$$

$$\Big\downarrow \; 26$$

$$i\int \frac{\left(a-b\sin(ie+ifx)^2\right)^{3/2}}{\sin(ie+ifx)^5}dx$$

$$\Big\downarrow \; 3665$$

$$-\frac{\int \frac{\left(b\cosh^2(e+fx)+a-b\right)^{3/2}}{\left(1-\cosh^2(e+fx)\right)^3}d\cosh(e+fx)}{f}$$

$$\Big\downarrow \text{292}$$

$$-\frac{\frac{3}{4}(a-b)\int \frac{\sqrt{b\cosh^2(e+fx)+a-b}}{\left(1-\cosh^2(e+fx)\right)^2}d\cosh(e+fx)+\frac{\cosh(e+fx)\left(a+b\cosh^2(e+fx)-b\right)^{3/2}}{4\left(1-\cosh^2(e+fx)\right)^2}}{f}$$

$$\Big\downarrow \text{292}$$

$$-\frac{\frac{3}{4}(a-b)\left(\frac{1}{2}(a-b)\int \frac{1}{\left(1-\cosh^2(e+fx)\right)\sqrt{b\cosh^2(e+fx)+a-b}}d\cosh(e+fx)+\frac{\cosh(e+fx)\sqrt{a+b\cosh^2(e+fx)-b}}{2\left(1-\cosh^2(e+fx)\right)}\right)+\frac{\cosh(e+}{}}{f}$$

$$\Big\downarrow \text{291}$$

$$-\frac{\frac{3}{4}(a-b)\left(\frac{1}{2}(a-b)\int \frac{1}{1-\frac{a\cosh^2(e+fx)}{b\cosh^2(e+fx)+a-b}}d\frac{\cosh(e+fx)}{\sqrt{b\cosh^2(e+fx)+a-b}}+\frac{\cosh(e+fx)\sqrt{a+b\cosh^2(e+fx)-b}}{2\left(1-\cosh^2(e+fx)\right)}\right)+\frac{\cosh(e+fx)(a+b\cosh}{4(1-\cosh^2(e}}{f}$$

$$\Big\downarrow \text{219}$$

$$-\frac{\frac{3}{4}(a-b)\left(\frac{(a-b)\text{arctanh}\left(\frac{\sqrt{a}\cosh(e+fx)}{\sqrt{a+b\cosh^2(e+fx)-b}}\right)}{2\sqrt{a}}+\frac{\cosh(e+fx)\sqrt{a+b\cosh^2(e+fx)-b}}{2\left(1-\cosh^2(e+fx)\right)}\right)+\frac{\cosh(e+fx)\left(a+b\cosh^2(e+fx)-b\right)^{3/2}}{4\left(1-\cosh^2(e+fx)\right)^2}}{f}$$

input
```
Int[Csch[e + f*x]^5*(a + b*Sinh[e + f*x]^2)^(3/2),x]
```

output
```
-(((Cosh[e + f*x]*(a - b + b*Cosh[e + f*x]^2)^(3/2))/(4*(1 - Cosh[e + f*x]
^2)^2) + (3*(a - b)*(((a - b)*ArcTanh[(Sqrt[a]*Cosh[e + f*x])/Sqrt[a - b +
 b*Cosh[e + f*x]^2]])/(2*Sqrt[a]) + (Cosh[e + f*x]*Sqrt[a - b + b*Cosh[e +
 f*x]^2])/(2*(1 - Cosh[e + f*x]^2))))/4)/f)
```

Defintions of rubi rules used

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 291
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*((c_) + (d_.)*(x_)^2)), x_Symbol] :> Subst
[Int[1/(c - (b*c - a*d)*x^2), x], x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b, c,
d}, x] && NeQ[b*c - a*d, 0]
```

rule 292
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_.), x_Symbol] :> Si
mp[(-x)*(a + b*x^2)^(p + 1)*((c + d*x^2)^q/(2*a*(p + 1))), x] - Simp[c*(q/(
a*(p + 1)))   Int[(a + b*x^2)^(p + 1)*(c + d*x^2)^(q - 1), x], x] /; FreeQ[
{a, b, c, d, p}, x] && NeQ[b*c - a*d, 0] && EqQ[2*(p + q + 1) + 1, 0] && Gt
Q[q, 0] && NeQ[p, -1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3665
```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^
(p_.), x_Symbol] :> With[{ff = FreeFactors[Cos[e + f*x], x]}, Simp[-ff/f
Subst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a + b - b*ff^2*x^2)^p, x], x, Cos[e +
 f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m - 1)/2]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 378 vs. $2(119) = 238$.

Time = 0.52 (sec) , antiderivative size = 379, normalized size of antiderivative = 2.81

method	result
default	$\sqrt{\left(a+b\sinh(fx+e)^2\right)\cosh(fx+e)^2}\left(-3a^2\ln\left(\frac{(a+b)\cosh(fx+e)^2+2\sqrt{a}\,\sqrt{b\cosh(fx+e)^4+(a-b)\cosh(fx+e)^2}+a-b}{\sinh(fx+e)^2}\right)\sinh(fx+e)^4+6ab$

input
```
int(csch(f*x+e)^5*(a+b*sinh(f*x+e)^2)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
1/16*((a+b*sinh(f*x+e)^2)*cosh(f*x+e)^2)^(1/2)*(-3*a^2*ln(((a+b)*cosh(f*x+
e)^2+2*a^(1/2)*(b*cosh(f*x+e)^4+(a-b)*cosh(f*x+e)^2)^(1/2)+a-b)/sinh(f*x+e
)^2)*sinh(f*x+e)^4+6*a*b*ln(((a+b)*cosh(f*x+e)^2+2*a^(1/2)*(b*cosh(f*x+e)^
4+(a-b)*cosh(f*x+e)^2)^(1/2)+a-b)/sinh(f*x+e)^2)*sinh(f*x+e)^4-3*b^2*ln(((
a+b)*cosh(f*x+e)^2+2*a^(1/2)*(b*cosh(f*x+e)^4+(a-b)*cosh(f*x+e)^2)^(1/2)+a
-b)/sinh(f*x+e)^2)*sinh(f*x+e)^4+6*a^(3/2)*((a+b*sinh(f*x+e)^2)*cosh(f*x+e
)^2)^(1/2)*sinh(f*x+e)^2-10*b*((a+b*sinh(f*x+e)^2)*cosh(f*x+e)^2)^(1/2)*si
nh(f*x+e)^2*a^(1/2)-4*a^(3/2)*((a+b*sinh(f*x+e)^2)*cosh(f*x+e)^2)^(1/2))/s
inh(f*x+e)^4/a^(1/2)/cosh(f*x+e)/(a+b*sinh(f*x+e)^2)^(1/2)/f
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1403 vs. $2(119) = 238$.

Time = 0.27 (sec) , antiderivative size = 3021, normalized size of antiderivative = 22.38

$$\int \operatorname{csch}^5(e+fx)\left(a+b\sinh^2(e+fx)\right)^{3/2}\,dx = \text{Too large to display}$$

input
```
integrate(csch(f*x+e)^5*(a+b*sinh(f*x+e)^2)^(3/2),x, algorithm="fricas")
```

output
```
Too large to include
```

Sympy [F(-1)]

Timed out.

$$\int \operatorname{csch}^5(e+fx)\left(a+b\sinh^2(e+fx)\right)^{3/2}\,dx=\text{Timed out}$$

input
```
integrate(csch(f*x+e)**5*(a+b*sinh(f*x+e)**2)**(3/2),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \operatorname{csch}^5(e+fx)\left(a+b\sinh^2(e+fx)\right)^{3/2}\,dx=\int \left(b\sinh\left(fx+e\right)^2+a\right)^{\frac{3}{2}}\operatorname{csch}\left(fx+e\right)^5\,dx$$

input
```
integrate(csch(f*x+e)^5*(a+b*sinh(f*x+e)^2)^(3/2),x, algorithm="maxima")
```

output
```
integrate((b*sinh(f*x + e)^2 + a)^(3/2)*csch(f*x + e)^5, x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2146 vs. $2(119) = 238$.

Time = 0.69 (sec) , antiderivative size = 2146, normalized size of antiderivative = 15.90

$$\int \operatorname{csch}^5(e+fx)\left(a+b\sinh^2(e+fx)\right)^{3/2}\,dx=\text{Too large to display}$$

input
```
integrate(csch(f*x+e)^5*(a+b*sinh(f*x+e)^2)^(3/2),x, algorithm="giac")
```

output
```
3/4*(a^2 - 2*a*b + b^2)*arctan(-1/2*(sqrt(b)*e^(2*f*x + 2*e) - sqrt(b*e^(4
*f*x + 4*e) + 4*a*e^(2*f*x + 2*e) - 2*b*e^(2*f*x + 2*e) + b) - sqrt(b))/sq
rt(-a))/(sqrt(-a)*f) - 1/2*(3*(sqrt(b)*e^(2*f*x + 2*e) - sqrt(b*e^(4*f*x +
 4*e) + 4*a*e^(2*f*x + 2*e) - 2*b*e^(2*f*x + 2*e) + b))^7*a^2 - 6*(sqrt(b)
*e^(2*f*x + 2*e) - sqrt(b*e^(4*f*x + 4*e) + 4*a*e^(2*f*x + 2*e) - 2*b*e^(2
*f*x + 2*e) + b))^7*a*b - 5*(sqrt(b)*e^(2*f*x + 2*e) - sqrt(b*e^(4*f*x + 4
*e) + 4*a*e^(2*f*x + 2*e) - 2*b*e^(2*f*x + 2*e) + b))^7*b^2 - 21*(sqrt(b)*
e^(2*f*x + 2*e) - sqrt(b*e^(4*f*x + 4*e) + 4*a*e^(2*f*x + 2*e) - 2*b*e^(2*
f*x + 2*e) + b))^6*a^2*sqrt(b) - 22*(sqrt(b)*e^(2*f*x + 2*e) - sqrt(b*e^(4
*f*x + 4*e) + 4*a*e^(2*f*x + 2*e) - 2*b*e^(2*f*x + 2*e) + b))^6*a*b^(3/2)
+ 35*(sqrt(b)*e^(2*f*x + 2*e) - sqrt(b*e^(4*f*x + 4*e) + 4*a*e^(2*f*x + 2*
e) - 2*b*e^(2*f*x + 2*e) + b))^6*b^(5/2) - 44*(sqrt(b)*e^(2*f*x + 2*e) - s
qrt(b*e^(4*f*x + 4*e) + 4*a*e^(2*f*x + 2*e) - 2*b*e^(2*f*x + 2*e) + b))^5*
a^3 - 105*(sqrt(b)*e^(2*f*x + 2*e) - sqrt(b*e^(4*f*x + 4*e) + 4*a*e^(2*f*x
 + 2*e) - 2*b*e^(2*f*x + 2*e) + b))^5*a^2*b + 246*(sqrt(b)*e^(2*f*x + 2*e)
 - sqrt(b*e^(4*f*x + 4*e) + 4*a*e^(2*f*x + 2*e) - 2*b*e^(2*f*x + 2*e) + b)
)^5*a*b^2 - 105*(sqrt(b)*e^(2*f*x + 2*e) - sqrt(b*e^(4*f*x + 4*e) + 4*a*e^
(2*f*x + 2*e) - 2*b*e^(2*f*x + 2*e) + b))^5*b^3 - 292*(sqrt(b)*e^(2*f*x +
2*e) - sqrt(b*e^(4*f*x + 4*e) + 4*a*e^(2*f*x + 2*e) - 2*b*e^(2*f*x + 2*e)
+ b))^4*a^3*sqrt(b) + 735*(sqrt(b)*e^(2*f*x + 2*e) - sqrt(b*e^(4*f*x + ...
```

Mupad [F(-1)]

Timed out.

$$\int \operatorname{csch}^5(e+fx)\left(a+b\sinh^2(e+fx)\right)^{3/2}\,dx=\int \frac{\left(b\sinh(e+f\,x)^2+a\right)^{3/2}}{\sinh\left(e+f\,x\right)^5}\,dx$$

input
```
int((a + b*sinh(e + f*x)^2)^(3/2)/sinh(e + f*x)^5,x)
```

output
```
int((a + b*sinh(e + f*x)^2)^(3/2)/sinh(e + f*x)^5, x)
```

Reduce [F]

$$\int \operatorname{csch}^5(e+fx)\left(a+b\sinh^2(e+fx)\right)^{3/2}\,dx=\left(\int \sqrt{\sinh(fx+e)^2 b+a}\,\operatorname{csch}(fx+e)^5\sinh(fx+e)^2\,dx\right)b+\left(\int \sqrt{\sinh(fx+e)^2 b+a}\,\operatorname{csch}(fx+e)^5\,dx\right)a$$

input

```
int(csch(f*x+e)^5*(a+b*sinh(f*x+e)^2)^(3/2),x)
```

output

```
int(sqrt(sinh(e + f*x)**2*b + a)*csch(e + f*x)**5*sinh(e + f*x)**2,x)*b +
int(sqrt(sinh(e + f*x)**2*b + a)*csch(e + f*x)**5,x)*a
```

3.75 $\int \operatorname{csch}^7(e+fx)\left(a+b\sinh^2(e+fx)\right)^{3/2}dx$

Optimal result

Integrand size = 25, antiderivative size = 199

$$\int \operatorname{csch}^7(e+fx)\left(a+b\sinh^2(e+fx)\right)^{3/2}dx=\frac{(a-b)^2(5a+b)\operatorname{arctanh}\left(\frac{\sqrt{a}\cosh(e+fx)}{\sqrt{a-b+b\cosh^2(e+fx)}}\right)}{16a^{3/2}f}-\frac{(a-b)(5a+b)\sqrt{a-b+b\cosh^2(e+fx)}\coth(e+fx)\operatorname{csch}(e+fx)}{16af}+\frac{(5a+b)\left(a-b+b\cosh^2(e+fx)\right)^{3/2}\coth(e+fx)\operatorname{csch}^3(e+fx)}{24af}-\frac{\left(a-b+b\cosh^2(e+fx)\right)^{5/2}\coth(e+fx)\operatorname{csch}^5(e+fx)}{6af}$$

output

```
1/16*(a-b)^2*(5*a+b)*arctanh(a^(1/2)*cosh(f*x+e)/(a-b+b*cosh(f*x+e)^2)^(1/
2))/a^(3/2)/f-1/16*(a-b)*(5*a+b)*(a-b+b*cosh(f*x+e)^2)^(1/2)*coth(f*x+e)*c
sch(f*x+e)/a/f+1/24*(5*a+b)*(a-b+b*cosh(f*x+e)^2)^(3/2)*coth(f*x+e)*csch(f
*x+e)^3/a/f-1/6*(a-b+b*cosh(f*x+e)^2)^(5/2)*coth(f*x+e)*csch(f*x+e)^5/a/f
```

Mathematica [A] (verified)

Time = 1.63 (sec) , antiderivative size = 174, normalized size of antiderivative = 0.87

$$\int \operatorname{csch}^7(e+fx)\left(a+b\sinh^2(e+fx)\right)^{3/2}\,dx=\frac{\frac{(a-b)^2(5a+b)\operatorname{arctanh}\left(\frac{\sqrt{2}\sqrt{a}\cosh(e+fx)}{\sqrt{2a-b+b\cosh(2(e+fx))}}\right)}{a^{3/2}}-\frac{\sqrt{a-\frac{b}{2}+\frac{1}{2}b\cosh(2(e+fx))}\left(149a^2-122ab+9b^2-4\left(25a^2-36ab+3b^2\right)\cdots\right.}{\cdots}}{16f}$$

input
```
Integrate[Csch[e + f*x]^7*(a + b*Sinh[e + f*x]^2)^(3/2),x]
```

output
```
(((a - b)^2*(5*a + b)*ArcTanh[(Sqrt[2]*Sqrt[a]*Cosh[e + f*x])/Sqrt[2*a - b
 + b*Cosh[2*(e + f*x)]]])/a^(3/2) - (Sqrt[a - b/2 + (b*Cosh[2*(e + f*x)])/
2]*(149*a^2 - 122*a*b + 9*b^2 - 4*(25*a^2 - 36*a*b + 3*b^2)*Cosh[2*(e + f*
x)] + (15*a^2 - 22*a*b + 3*b^2)*Cosh[4*(e + f*x)])*Coth[e + f*x]*Csch[e +
f*x]^5)/(24*a))/(16*f)
```

Rubi [A] (verified)

Time = 0.37 (sec) , antiderivative size = 206, normalized size of antiderivative = 1.04, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.320, Rules used = {3042, 26, 3665, 296, 292, 292, 291, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \operatorname{csch}^7(e+fx)\left(a+b\sinh^2(e+fx)\right)^{3/2}\,dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int -\frac{i\left(a-b\sin(ie+ifx)^2\right)^{3/2}}{\sin(ie+ifx)^7}dx$$

$$\Big\downarrow \text{ 26}$$

$$-i\int \frac{\left(a-b\sin(ie+ifx)^2\right)^{3/2}}{\sin(ie+ifx)^7}dx$$

$\big\downarrow$ 3665

$$\frac{\int \frac{\left(b\cosh^2(e+fx)+a-b\right)^{3/2}}{\left(1-\cosh^2(e+fx)\right)^4}d\cosh(e+fx)}{f}$$

$\big\downarrow$ 296

$$\frac{\frac{(5a+b)\int \frac{\left(b\cosh^2(e+fx)+a-b\right)^{3/2}}{\left(1-\cosh^2(e+fx)\right)^3}d\cosh(e+fx)}{6a}+\frac{\cosh(e+fx)\left(a+b\cosh^2(e+fx)-b\right)^{5/2}}{6a\left(1-\cosh^2(e+fx)\right)^3}}{f}$$

$\big\downarrow$ 292

$$\frac{\frac{(5a+b)\left(\frac{3}{4}(a-b)\int \frac{\sqrt{b\cosh^2(e+fx)+a-b}}{\left(1-\cosh^2(e+fx)\right)^2}d\cosh(e+fx)+\frac{\cosh(e+fx)\left(a+b\cosh^2(e+fx)-b\right)^{3/2}}{4\left(1-\cosh^2(e+fx)\right)^2}\right)}{6a}+\frac{\cosh(e+fx)\left(a+b\cosh^2(e+fx)-b\right)^{5/2}}{6a\left(1-\cosh^2(e+fx)\right)^3}}{f}$$

$\big\downarrow$ 292

$$\frac{\frac{(5a+b)\left(\frac{3}{4}(a-b)\left(\frac{1}{2}(a-b)\int \frac{1}{\left(1-\cosh^2(e+fx)\right)\sqrt{b\cosh^2(e+fx)+a-b}}d\cosh(e+fx)+\frac{\cosh(e+fx)\sqrt{a+b\cosh^2(e+fx)-b}}{2\left(1-\cosh^2(e+fx)\right)}\right)+\frac{\cosh(e+fx)\left(a+b\cosh^2(e+fx)-\cdots\right.}{4\left(1-\cosh^2(e+fx)\right)^2}\cdots}{6a}\cdots}{f}$$

$\big\downarrow$ 291

$$\frac{\frac{(5a+b)\left(\frac{3}{4}(a-b)\left(\frac{1}{2}(a-b)\int \frac{1}{1-\frac{a\cosh^2(e+fx)}{b\cosh^2(e+fx)+a-b}}d\frac{\cosh(e+fx)}{\sqrt{b\cosh^2(e+fx)+a-b}}+\frac{\cosh(e+fx)\sqrt{a+b\cosh^2(e+fx)-b}}{2\left(1-\cosh^2(e+fx)\right)}\right)+\frac{\cosh(e+fx)\left(a+b\cosh^2(e+fx)-b\right)^{3/2}}{4\left(1-\cosh^2(e+fx)\right)^2}\cdots}{6a}\cdots}{f}$$

$\big\downarrow$ 219

$$\frac{\frac{(5a+b)\left(\frac{3}{4}(a-b)\left(\frac{(a-b)\text{arctanh}\left(\frac{\sqrt{a}\cosh(e+fx)}{\sqrt{a+b\cosh^2(e+fx)-b}}\right)}{2\sqrt{a}}+\frac{\cosh(e+fx)\sqrt{a+b\cosh^2(e+fx)-b}}{2\left(1-\cosh^2(e+fx)\right)}\right)+\frac{\cosh(e+fx)\left(a+b\cosh^2(e+fx)-b\right)^{3/2}}{4\left(1-\cosh^2(e+fx)\right)^2}\right)}{6a}+\frac{\cosh(e+\cdots}{\cdots}}{f}$$

input
```
Int[Csch[e + f*x]^7*(a + b*Sinh[e + f*x]^2)^(3/2),x]
```

output
```
((Cosh[e + f*x]*(a - b + b*Cosh[e + f*x]^2)^(5/2))/(6*a*(1 - Cosh[e + f*x]
^2)^3) + ((5*a + b)*((Cosh[e + f*x]*(a - b + b*Cosh[e + f*x]^2)^(3/2))/(4*
(1 - Cosh[e + f*x]^2)^2) + (3*(a - b)*(((a - b)*ArcTanh[(Sqrt[a]*Cosh[e +
f*x])/Sqrt[a - b + b*Cosh[e + f*x]^2]])/(2*Sqrt[a]) + (Cosh[e + f*x]*Sqrt[
a - b + b*Cosh[e + f*x]^2])/(2*(1 - Cosh[e + f*x]^2))))/4))/(6*a))/f
```

Defintions of rubi rules used

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 291
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*((c_) + (d_.)*(x_)^2)), x_Symbol] :> Subst
[Int[1/(c - (b*c - a*d)*x^2), x], x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b, c,
d}, x] && NeQ[b*c - a*d, 0]
```

rule 292
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_.), x_Symbol] :> Si
mp[(-x)*(a + b*x^2)^(p + 1)*((c + d*x^2)^q/(2*a*(p + 1))), x] - Simp[c*(q/(
a*(p + 1)))   Int[(a + b*x^2)^(p + 1)*(c + d*x^2)^(q - 1), x], x] /; FreeQ[
{a, b, c, d, p}, x] && NeQ[b*c - a*d, 0] && EqQ[2*(p + q + 1) + 1, 0] && Gt
Q[q, 0] && NeQ[p, -1]
```

rule 296
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_), x_Symbol] :> Sim
p[(-b)*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^(q + 1)/(2*a*(p + 1)*(b*c - a*d))
), x] + Simp[(b*c + 2*(p + 1)*(b*c - a*d))/(2*a*(p + 1)*(b*c - a*d))   Int[
(a + b*x^2)^(p + 1)*(c + d*x^2)^q, x], x] /; FreeQ[{a, b, c, d, q}, x] && N
eQ[b*c - a*d, 0] && EqQ[2*(p + q + 2) + 1, 0] && (LtQ[p, -1] ||  !LtQ[q, -1
]) && NeQ[p, -1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3665
```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^
(p_.), x_Symbol] :> With[{ff = FreeFactors[Cos[e + f*x], x]}, Simp[-ff/f
Subst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a + b - b*ff^2*x^2)^p, x], x, Cos[e +
 f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m - 1)/2]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 568 vs. $2(179) = 358$.

Time = 0.26 (sec) , antiderivative size = 569, normalized size of antiderivative = 2.86

$$-\frac{\sqrt{(a+b\sinh(fx+e)^2)\cosh(fx+e)^2}\left(-15a^4\ln\left(\frac{(a+b)\cosh(fx+e)^2+2\sqrt{a}\sqrt{b\cosh(fx+e)^4+(a-b)\cosh(fx+e)^2}+a-b}{\sinh(fx+e)^2}\right.\right.}{}$$

input
```
int(csch(f*x+e)^7*(a+b*sinh(f*x+e)^2)^(3/2),x)
```

output
```
-1/96*((a+b*sinh(f*x+e)^2)*cosh(f*x+e)^2)^(1/2)*(-15*a^4*ln(((a+b)*cosh(f*
x+e)^2+2*a^(1/2)*(b*cosh(f*x+e)^4+(a-b)*cosh(f*x+e)^2)^(1/2)+a-b)/sinh(f*x
+e)^2)*sinh(f*x+e)^6+27*a^3*b*ln(((a+b)*cosh(f*x+e)^2+2*a^(1/2)*(b*cosh(f*
x+e)^4+(a-b)*cosh(f*x+e)^2)^(1/2)+a-b)/sinh(f*x+e)^2)*sinh(f*x+e)^6-9*b^2*
ln(((a+b)*cosh(f*x+e)^2+2*a^(1/2)*(b*cosh(f*x+e)^4+(a-b)*cosh(f*x+e)^2)^(1
/2)+a-b)/sinh(f*x+e)^2)*sinh(f*x+e)^6*a^2-3*ln(((a+b)*cosh(f*x+e)^2+2*a^(1
/2)*(b*cosh(f*x+e)^4+(a-b)*cosh(f*x+e)^2)^(1/2)+a-b)/sinh(f*x+e)^2)*b^3*si
nh(f*x+e)^6*a+30*sinh(f*x+e)^4*a^(7/2)*((a+b*sinh(f*x+e)^2)*cosh(f*x+e)^2)
^(1/2)-44*sinh(f*x+e)^4*a^(5/2)*((a+b*sinh(f*x+e)^2)*cosh(f*x+e)^2)^(1/2)*
b+6*sinh(f*x+e)^4*a^(3/2)*((a+b*sinh(f*x+e)^2)*cosh(f*x+e)^2)^(1/2)*b^2-20
*sinh(f*x+e)^2*a^(7/2)*((a+b*sinh(f*x+e)^2)*cosh(f*x+e)^2)^(1/2)+28*sinh(f
*x+e)^2*a^(5/2)*((a+b*sinh(f*x+e)^2)*cosh(f*x+e)^2)^(1/2)*b+16*a^(7/2)*((a
+b*sinh(f*x+e)^2)*cosh(f*x+e)^2)^(1/2))/sinh(f*x+e)^6/a^(5/2)/cosh(f*x+e)/
(a+b*sinh(f*x+e)^2)^(1/2)/f
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 3521 vs. $2(179) = 358$.

Time = 0.82 (sec) , antiderivative size = 7257, normalized size of antiderivative = 36.47

$$\int \operatorname{csch}^7(e+fx)\left(a+b\sinh^2(e+fx)\right)^{3/2}\,dx = \text{Too large to display}$$

input
```
integrate(csch(f*x+e)^7*(a+b*sinh(f*x+e)^2)^(3/2),x, algorithm="fricas")
```

output
```
Too large to include
```

Sympy [F(-1)]

Timed out.

$$\int \operatorname{csch}^7(e+fx)\left(a+b\sinh^2(e+fx)\right)^{3/2}\,dx = \text{Timed out}$$

input
```
integrate(csch(f*x+e)**7*(a+b*sinh(f*x+e)**2)**(3/2),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \operatorname{csch}^7(e+fx)\left(a+b\sinh^2(e+fx)\right)^{3/2}\,dx = \int \left(b\sinh\left(fx+e\right)^2+a\right)^{\frac{3}{2}}\operatorname{csch}\left(fx+e\right)^7\,dx$$

input
```
integrate(csch(f*x+e)^7*(a+b*sinh(f*x+e)^2)^(3/2),x, algorithm="maxima")
```

output
```
integrate((b*sinh(f*x + e)^2 + a)^(3/2)*csch(f*x + e)^7, x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 4685 vs. $2(179) = 358$.

Time = 1.56 (sec) , antiderivative size = 4685, normalized size of antiderivative = 23.54

$$\int \operatorname{csch}^7(e+fx) \left(a+b\sinh^2(e+fx)\right)^{3/2} dx = \text{Too large to display}$$

input
```
integrate(csch(f*x+e)^7*(a+b*sinh(f*x+e)^2)^(3/2),x, algorithm="giac")
```

output
```
-1/8*(5*a^3 - 9*a^2*b + 3*a*b^2 + b^3)*arctan(-1/2*(sqrt(b)*e^(2*f*x + 2*e
) - sqrt(b*e^(4*f*x + 4*e) + 4*a*e^(2*f*x + 2*e) - 2*b*e^(2*f*x + 2*e) + b
) - sqrt(b))/sqrt(-a))/(sqrt(-a)*a*f) + 1/12*(15*(sqrt(b)*e^(2*f*x + 2*e)
- sqrt(b*e^(4*f*x + 4*e) + 4*a*e^(2*f*x + 2*e) - 2*b*e^(2*f*x + 2*e) + b))
^11*a^3 - 27*(sqrt(b)*e^(2*f*x + 2*e) - sqrt(b*e^(4*f*x + 4*e) + 4*a*e^(2*
f*x + 2*e) - 2*b*e^(2*f*x + 2*e) + b))^11*a^2*b + 9*(sqrt(b)*e^(2*f*x + 2*
e) - sqrt(b*e^(4*f*x + 4*e) + 4*a*e^(2*f*x + 2*e) - 2*b*e^(2*f*x + 2*e) +
b))^11*a*b^2 + 3*(sqrt(b)*e^(2*f*x + 2*e) - sqrt(b*e^(4*f*x + 4*e) + 4*a*e
^(2*f*x + 2*e) - 2*b*e^(2*f*x + 2*e) + b))^11*b^3 - 165*(sqrt(b)*e^(2*f*x
+ 2*e) - sqrt(b*e^(4*f*x + 4*e) + 4*a*e^(2*f*x + 2*e) - 2*b*e^(2*f*x + 2*e
) + b))^10*a^3*sqrt(b) + 297*(sqrt(b)*e^(2*f*x + 2*e) - sqrt(b*e^(4*f*x +
4*e) + 4*a*e^(2*f*x + 2*e) - 2*b*e^(2*f*x + 2*e) + b))^10*a^2*b^(3/2) + 93
*(sqrt(b)*e^(2*f*x + 2*e) - sqrt(b*e^(4*f*x + 4*e) + 4*a*e^(2*f*x + 2*e) -
 2*b*e^(2*f*x + 2*e) + b))^10*a*b^(5/2) - 33*(sqrt(b)*e^(2*f*x + 2*e) - sq
rt(b*e^(4*f*x + 4*e) + 4*a*e^(2*f*x + 2*e) - 2*b*e^(2*f*x + 2*e) + b))^10*
b^(7/2) - 340*(sqrt(b)*e^(2*f*x + 2*e) - sqrt(b*e^(4*f*x + 4*e) + 4*a*e^(2
*f*x + 2*e) - 2*b*e^(2*f*x + 2*e) + b))^9*a^4 + 1437*(sqrt(b)*e^(2*f*x + 2
*e) - sqrt(b*e^(4*f*x + 4*e) + 4*a*e^(2*f*x + 2*e) - 2*b*e^(2*f*x + 2*e) +
 b))^9*a^3*b + 615*(sqrt(b)*e^(2*f*x + 2*e) - sqrt(b*e^(4*f*x + 4*e) + 4*a
*e^(2*f*x + 2*e) - 2*b*e^(2*f*x + 2*e) + b))^9*a^2*b^2 - 1237*(sqrt(b)*...
```

Mupad [F(-1)]

Timed out.

$$\int \operatorname{csch}^7(e+fx)\left(a+b\sinh^2(e+fx)\right)^{3/2}\,dx=\int \frac{\left(b\sinh(e+f\,x)^2+a\right)^{3/2}}{\sinh\left(e+f\,x\right)^7}\,dx$$

input
```
int((a + b*sinh(e + f*x)^2)^(3/2)/sinh(e + f*x)^7,x)
```

output
```
int((a + b*sinh(e + f*x)^2)^(3/2)/sinh(e + f*x)^7, x)
```

Reduce [F]

$$\int \operatorname{csch}^7(e+fx)\left(a+b\sinh^2(e+fx)\right)^{3/2}\,dx=\left(\int \sqrt{\sinh\left(fx+e\right)^2b+a}\,\operatorname{csch}(fx+e)^7\sinh\left(fx+e\right)^2dx\right)b+\left(\int \sqrt{\sinh\left(fx+e\right)^2b+a}\,\operatorname{csch}(fx+e)^7\,dx\right)a$$

input
```
int(csch(f*x+e)^7*(a+b*sinh(f*x+e)^2)^(3/2),x)
```

output
```
int(sqrt(sinh(e + f*x)**2*b + a)*csch(e + f*x)**7*sinh(e + f*x)**2,x)*b +
int(sqrt(sinh(e + f*x)**2*b + a)*csch(e + f*x)**7,x)*a
```

3.76 $\int \sinh^4(e+fx)\left(a+b\sinh^2(e+fx)\right)^{3/2}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 367

$$\int \sinh^4(e+fx)\left(a+b\sinh^2(e+fx)\right)^{3/2}\,dx = \frac{(a^2-11ab+8b^2)\cosh(e+fx)\sinh(e+fx)\sqrt{a+b\sinh^2(e+fx)}}{35bf} + \frac{2(4a-3b)\cosh(e+fx)\sinh^3(e+fx)\sqrt{a+b\sinh^2(e+fx)}}{35f} + \frac{b\cosh(e+fx)\sinh^5(e+fx)\sqrt{a+b\sinh^2(e+fx)}}{7f} + \frac{2(a-2b)(a^2+4ab-4b^2)E\left(\arctan(\sinh(e+fx))\,\middle|\,1-\frac{b}{a}\right)\operatorname{sech}(e+fx)\sqrt{a+b\sinh^2(e+fx)}}{35b^2f\sqrt{\frac{\operatorname{sech}^2(e+fx)(a+b\sinh^2(e+fx))}{a}}} - \frac{(a^2-11ab+8b^2)\operatorname{EllipticF}\left(\arctan(\sinh(e+fx)),1-\frac{b}{a}\right)\operatorname{sech}(e+fx)\sqrt{a+b\sinh^2(e+fx)}}{35bf\sqrt{\frac{\operatorname{sech}^2(e+fx)(a+b\sinh^2(e+fx))}{a}}} - \frac{2(a-2b)(a^2+4ab-4b^2)\sqrt{a+b\sinh^2(e+fx)}\tanh(e+fx)}{35b^2f}$$

output
```
1/35*(a^2-11*a*b+8*b^2)*cosh(f*x+e)*sinh(f*x+e)*(a+b*sinh(f*x+e)^2)^(1/2)/
b/f+2/35*(4*a-3*b)*cosh(f*x+e)*sinh(f*x+e)^3*(a+b*sinh(f*x+e)^2)^(1/2)/f+1
/7*b*cosh(f*x+e)*sinh(f*x+e)^5*(a+b*sinh(f*x+e)^2)^(1/2)/f+2/35*(a-2*b)*(a
^2+4*a*b-4*b^2)*EllipticE(sinh(f*x+e)/(1+sinh(f*x+e)^2)^(1/2),(1-b/a)^(1/2
))*sech(f*x+e)*(a+b*sinh(f*x+e)^2)^(1/2)/b^2/f/(sech(f*x+e)^2*(a+b*sinh(f*
x+e)^2)/a)^(1/2)-1/35*(a^2-11*a*b+8*b^2)*InverseJacobiAM(arctan(sinh(f*x+e
)),(1-b/a)^(1/2))*sech(f*x+e)*(a+b*sinh(f*x+e)^2)^(1/2)/b/f/(sech(f*x+e)^2
*(a+b*sinh(f*x+e)^2)/a)^(1/2)-2/35*(a-2*b)*(a^2+4*a*b-4*b^2)*(a+b*sinh(f*x
+e)^2)^(1/2)*tanh(f*x+e)/b^2/f
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 2.08 (sec) , antiderivative size = 262, normalized size of antiderivative = 0.71

$$\int \sinh^4(e+fx)\left(a+b\sinh^2(e+fx)\right)^{3/2}\,dx = \frac{128ia(a^3+2a^2b-12ab^2+8b^3)\sqrt{\frac{2a-b+b\cosh(2(e+fx))}{a}}E\left(i(e+fx)\,\middle|\,\frac{b}{a}\right)-64ia(2a^3+3a^2b-1$$

input
```
Integrate[Sinh[e + f*x]^4*(a + b*Sinh[e + f*x]^2)^(3/2),x]
```

output
```
((128*I)*a*(a^3 + 2*a^2*b - 12*a*b^2 + 8*b^3)*Sqrt[(2*a - b + b*Cosh[2*(e
+ f*x)])/a]*EllipticE[I*(e + f*x), b/a] - (64*I)*a*(2*a^3 + 3*a^2*b - 13*a
*b^2 + 8*b^3)*Sqrt[(2*a - b + b*Cosh[2*(e + f*x)])/a]*EllipticF[I*(e + f*x
), b/a] + Sqrt[2]*b*(32*a^3 - 496*a^2*b + 684*a*b^2 - 250*b^3 + b*(144*a^2
 - 480*a*b + 299*b^2)*Cosh[2*(e + f*x)] + 2*(26*a - 27*b)*b^2*Cosh[4*(e +
f*x)] + 5*b^3*Cosh[6*(e + f*x)])*Sinh[2*(e + f*x)])/(2240*b^2*f*Sqrt[2*a -
 b + b*Cosh[2*(e + f*x)]])
```

Rubi [A] (verified)

Time = 0.67 (sec) , antiderivative size = 425, normalized size of antiderivative = 1.16, number of steps used = 12, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.440, Rules used = {3042, 3667, 379, 444, 27, 444, 25, 406, 320, 388, 313}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sinh^4(e+fx)\left(a+b\sinh^2(e+fx)\right)^{3/2}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \sin(ie+ifx)^4\left(a-b\sin(ie+ifx)^2\right)^{3/2}dx$$

$$\Big\downarrow \text{3667}$$

$$\frac{\sqrt{\cosh^2(e+fx)}\text{sech}(e+fx)\int \frac{\sinh^4(e+fx)\left(b\sinh^2(e+fx)+a\right)^{3/2}}{\sqrt{\sinh^2(e+fx)+1}}d\sinh(e+fx)}{f}$$

$$\Big\downarrow \text{379}$$

$$\frac{\sqrt{\cosh^2(e+fx)}\text{sech}(e+fx)\left(\frac{1}{7}\int \frac{\sinh^4(e+fx)\left(2(4a-3b)b\sinh^2(e+fx)+a(7a-5b)\right)}{\sqrt{\sinh^2(e+fx)+1}\sqrt{b\sinh^2(e+fx)+a}}d\sinh(e+fx)+\frac{1}{7}b\sqrt{\sinh^2(e+fx)+\ldots}\right.}{f}$$

$$\Big\downarrow \text{444}$$

$$\sqrt{\cosh^2(e+fx)}\text{sech}(e+fx)\left(\frac{1}{7}\left(\frac{2}{5}(4a-3b)\sinh^3(e+fx)\sqrt{\sinh^2(e+fx)+1}\sqrt{a+b\sinh^2(e+fx)}-\frac{\int \frac{3b\sinh^{\ldots}}{\ldots}}{\ldots}\right.\right.$$

$$\Big\downarrow \text{27}$$

$$\sqrt{\cosh^2(e+fx)}\text{sech}(e+fx)\left(\frac{1}{7}\left(\frac{2}{5}(4a-3b)\sinh^3(e+fx)\sqrt{\sinh^2(e+fx)+1}\sqrt{a+b\sinh^2(e+fx)}-\frac{3}{5}\int \frac{\sinh\ldots}{\ldots}\right.\right.$$

$$\Big\downarrow \text{444}$$

$$\sqrt{\cosh^2(e+fx)}\text{sech}(e+fx)\left(\frac{1}{7}\left(\frac{2}{5}(4a-3b)\sinh^3(e+fx)\sqrt{\sinh^2(e+fx)+1}\sqrt{a+b\sinh^2(e+fx)}-\frac{3}{5}\left(\right.\right.\right.$$

$\downarrow$ 25

$$\sqrt{\cosh^2(e+fx)}\text{sech}(e+fx)\left(\frac{1}{7}\left(\frac{2}{5}(4a-3b)\sinh^3(e+fx)\sqrt{\sinh^2(e+fx)+1}\sqrt{a+b\sinh^2(e+fx)}-\frac{3}{5}\left(\right.\right.\right.$$

$\downarrow$ 406

$$\sqrt{\cosh^2(e+fx)}\text{sech}(e+fx)\left(\frac{1}{7}\left(\frac{2}{5}(4a-3b)\sinh^3(e+fx)\sqrt{\sinh^2(e+fx)+1}\sqrt{a+b\sinh^2(e+fx)}-\frac{3}{5}\left(\right.\right.\right.$$

$\downarrow$ 320

$$\sqrt{\cosh^2(e+fx)}\text{sech}(e+fx)\left(\frac{1}{7}\left(\frac{2}{5}(4a-3b)\sinh^3(e+fx)\sqrt{\sinh^2(e+fx)+1}\sqrt{a+b\sinh^2(e+fx)}-\frac{3}{5}\left(\right.\right.\right.$$

$\downarrow$ 388

$$\sqrt{\cosh^2(e+fx)}\text{sech}(e+fx)\left(\frac{1}{7}\left(\frac{2}{5}(4a-3b)\sinh^3(e+fx)\sqrt{\sinh^2(e+fx)+1}\sqrt{a+b\sinh^2(e+fx)}-\frac{3}{5}\left(\right.\right.\right.$$

$\downarrow$ 313

$$\sqrt{\cosh^2(e+fx)}\operatorname{sech}(e+fx)\left(\frac{1}{7}\left(\frac{2}{5}(4a-3b)\sinh^3(e+fx)\sqrt{\sinh^2(e+fx)+1}\sqrt{a+b\sinh^2(e+fx)}-\frac{3}{5}\left(\frac{(a^2\cdots}{}\right.\right.\right.$$

input
```
Int[Sinh[e + f*x]^4*(a + b*Sinh[e + f*x]^2)^(3/2),x]
```

output
```
(Sqrt[Cosh[e + f*x]^2]*Sech[e + f*x]*((b*Sinh[e + f*x]^5*Sqrt[1 + Sinh[e +
 f*x]^2]*Sqrt[a + b*Sinh[e + f*x]^2])/7 + ((2*(4*a - 3*b)*Sinh[e + f*x]^3*
Sqrt[1 + Sinh[e + f*x]^2]*Sqrt[a + b*Sinh[e + f*x]^2])/5 - (3*(-1/3*((a^2
- 11*a*b + 8*b^2)*Sinh[e + f*x]*Sqrt[1 + Sinh[e + f*x]^2]*Sqrt[a + b*Sinh[
e + f*x]^2])/b + (((a^2 - 11*a*b + 8*b^2)*EllipticF[ArcTan[Sinh[e + f*x]],
 1 - b/a]*Sqrt[a + b*Sinh[e + f*x]^2])/(Sqrt[1 + Sinh[e + f*x]^2]*Sqrt[(a
+ b*Sinh[e + f*x]^2)/(a*(1 + Sinh[e + f*x]^2))]) + 2*(a - 2*b)*(a^2 + 4*a*
b - 4*b^2)*((Sinh[e + f*x]*Sqrt[a + b*Sinh[e + f*x]^2])/(b*Sqrt[1 + Sinh[e
 + f*x]^2]) - (EllipticE[ArcTan[Sinh[e + f*x]], 1 - b/a]*Sqrt[a + b*Sinh[e
 + f*x]^2])/(b*Sqrt[1 + Sinh[e + f*x]^2]*Sqrt[(a + b*Sinh[e + f*x]^2)/(a*(
1 + Sinh[e + f*x]^2))])))/(3*b)))/5)/7))/f
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 313
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/((c_) + (d_.)*(x_)^2)^(3/2), x_Symbol] :> Sim
p[(Sqrt[a + b*x^2]/(c*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(c
+ d*x^2)))]))*EllipticE[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; FreeQ
[{a, b, c, d}, x] && PosQ[b/a] && PosQ[d/c]
```

rule 320
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(Sqrt[a + b*x^2]/(a*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(
c + d*x^2)))]))*EllipticF[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; Fre
eQ[{a, b, c, d}, x] && PosQ[d/c] && PosQ[b/a] && !SimplerSqrtQ[b/a, d/c]
```

rule 379
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_
), x_Symbol] :> Simp[d*(e*x)^(m + 1)*(a + b*x^2)^(p + 1)*((c + d*x^2)^(q -
1)/(b*e*(m + 2*(p + q) + 1))), x] + Simp[1/(b*(m + 2*(p + q) + 1))   Int[(e
*x)^m*(a + b*x^2)^p*(c + d*x^2)^(q - 2)*Simp[c*((b*c - a*d)*(m + 1) + b*c*2
*(p + q)) + (d*(b*c - a*d)*(m + 1) + d*2*(q - 1)*(b*c - a*d) + b*c*d*2*(p +
 q))*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, m, p}, x] && NeQ[b*c - a*d, 0
] && GtQ[q, 1] && IntBinomialQ[a, b, c, d, e, m, 2, p, q, x]
```

rule 388
```
Int[(x_)^2/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol]
 :> Simp[x*(Sqrt[a + b*x^2]/(b*Sqrt[c + d*x^2])), x] - Simp[c/b   Int[Sqrt[
a + b*x^2]/(c + d*x^2)^(3/2), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
 a*d, 0] && PosQ[b/a] && PosQ[d/c] && !SimplerSqrtQ[b/a, d/c]
```

rule 406
```
Int[((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.)*((e_) + (f_.)*(
x_)^2), x_Symbol] :> Simp[e   Int[(a + b*x^2)^p*(c + d*x^2)^q, x], x] + Sim
p[f   Int[x^2*(a + b*x^2)^p*(c + d*x^2)^q, x], x] /; FreeQ[{a, b, c, d, e,
f, p, q}, x]
```

rule 444
```
Int[((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q
_.)*((e_) + (f_.)*(x_)^2), x_Symbol] :> Simp[f*g*(g*x)^(m - 1)*(a + b*x^2)^
(p + 1)*((c + d*x^2)^(q + 1)/(b*d*(m + 2*(p + q + 1) + 1))), x] - Simp[g^2/
(b*d*(m + 2*(p + q + 1) + 1))   Int[(g*x)^(m - 2)*(a + b*x^2)^p*(c + d*x^2)
^q*Simp[a*f*c*(m - 1) + (a*f*d*(m + 2*q + 1) + b*(f*c*(m + 2*p + 1) - e*d*(
m + 2*(p + q + 1) + 1)))*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p,
q}, x] && GtQ[m, 1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3667
```
Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(
p_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Simp[ff^(m + 1
)*(Sqrt[Cos[e + f*x]^2]/(f*Cos[e + f*x]))   Subst[Int[x^m*((a + b*ff^2*x^2)
^p/Sqrt[1 - ff^2*x^2]), x], x, Sin[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f,
p}, x] && IntegerQ[m/2] &&  !IntegerQ[p]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 742 vs. 2(355) = 710.

Time = 7.41 (sec) , antiderivative size = 743, normalized size of antiderivative = 2.02

method	result
default	$\frac{5\sqrt{-\frac{b}{a}}\, b^{3} \cosh(fx+e)^{8} \sinh(fx+e)+\left(13\sqrt{-\frac{b}{a}}\, a\, b^{2}-21\sqrt{-\frac{b}{a}}\, b^{3}\right) \cosh(fx+e)^{6} \sinh(fx+e)+\left(9\sqrt{-\frac{b}{a}}\, a^{2} b-43\sqrt{-\frac{b}{a}}\, a\, b^{2}+35\sqrt{-\frac{b}{a}}\right.}{}$

input
```
int(sinh(f*x+e)^4*(a+b*sinh(f*x+e)^2)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
1/35*(5*(-b/a)^(1/2)*b^3*cosh(f*x+e)^8*sinh(f*x+e)+(13*(-b/a)^(1/2)*a*b^2-
21*(-b/a)^(1/2)*b^3)*cosh(f*x+e)^6*sinh(f*x+e)+(9*(-b/a)^(1/2)*a^2*b-43*(-
b/a)^(1/2)*a*b^2+35*(-b/a)^(1/2)*b^3)*cosh(f*x+e)^4*sinh(f*x+e)+((-b/a)^(1
/2)*a^3-20*(-b/a)^(1/2)*a^2*b+38*(-b/a)^(1/2)*a*b^2-19*(-b/a)^(1/2)*b^3)*c
osh(f*x+e)^2*sinh(f*x+e)+(b/a*cosh(f*x+e)^2+(a-b)/a)^(1/2)*(cosh(f*x+e)^2)
^(1/2)*EllipticF(sinh(f*x+e)*(-b/a)^(1/2),(1/b*a)^(1/2))*a^3+15*(b/a*cosh(
f*x+e)^2+(a-b)/a)^(1/2)*(cosh(f*x+e)^2)^(1/2)*EllipticF(sinh(f*x+e)*(-b/a)
^(1/2),(1/b*a)^(1/2))*a^2*b-32*(b/a*cosh(f*x+e)^2+(a-b)/a)^(1/2)*(cosh(f*x
+e)^2)^(1/2)*EllipticF(sinh(f*x+e)*(-b/a)^(1/2),(1/b*a)^(1/2))*a*b^2+16*(b
/a*cosh(f*x+e)^2+(a-b)/a)^(1/2)*(cosh(f*x+e)^2)^(1/2)*EllipticF(sinh(f*x+e
)*(-b/a)^(1/2),(1/b*a)^(1/2))*b^3-2*(b/a*cosh(f*x+e)^2+(a-b)/a)^(1/2)*(cos
h(f*x+e)^2)^(1/2)*EllipticE(sinh(f*x+e)*(-b/a)^(1/2),(1/b*a)^(1/2))*a^3-4*
(b/a*cosh(f*x+e)^2+(a-b)/a)^(1/2)*(cosh(f*x+e)^2)^(1/2)*EllipticE(sinh(f*x
+e)*(-b/a)^(1/2),(1/b*a)^(1/2))*a^2*b+24*(b/a*cosh(f*x+e)^2+(a-b)/a)^(1/2)
*(cosh(f*x+e)^2)^(1/2)*EllipticE(sinh(f*x+e)*(-b/a)^(1/2),(1/b*a)^(1/2))*a
*b^2-16*(b/a*cosh(f*x+e)^2+(a-b)/a)^(1/2)*(cosh(f*x+e)^2)^(1/2)*EllipticE(
sinh(f*x+e)*(-b/a)^(1/2),(1/b*a)^(1/2))*b^3)/b/(-b/a)^(1/2)/cosh(f*x+e)/(a
+b*sinh(f*x+e)^2)^(1/2)/f
```

Fricas [F]

$$\int \sinh^4(e+fx) \left(a+b\sinh^2(e+fx)\right)^{3/2} \,dx = \int \left(b\sinh\left(fx+e\right)^2+a\right)^{\frac{3}{2}}\sinh\left(fx+e\right)^4 \,dx$$

input
```
integrate(sinh(f*x+e)^4*(a+b*sinh(f*x+e)^2)^(3/2),x, algorithm="fricas")
```

output
```
integral((b*sinh(f*x + e)^6 + a*sinh(f*x + e)^4)*sqrt(b*sinh(f*x + e)^2 +
a), x)
```

Sympy [F(-1)]

Timed out.

$$\int \sinh^4(e+fx) \left(a+b\sinh^2(e+fx)\right)^{3/2} \,dx = \text{Timed out}$$

input
```
integrate(sinh(f*x+e)**4*(a+b*sinh(f*x+e)**2)**(3/2),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \sinh^4(e+fx) \left(a+b\sinh^2(e+fx)\right)^{3/2} \,dx = \int \left(b\sinh\left(fx+e\right)^2+a\right)^{\frac{3}{2}}\sinh\left(fx+e\right)^4 \,dx$$

input
```
integrate(sinh(f*x+e)^4*(a+b*sinh(f*x+e)^2)^(3/2),x, algorithm="maxima")
```

output
```
integrate((b*sinh(f*x + e)^2 + a)^(3/2)*sinh(f*x + e)^4, x)
```

Giac [F]

$$\int \sinh^4(e+fx)\left(a+b\sinh^2(e+fx)\right)^{3/2}\,dx=\int \left(b\sinh\left(fx+e\right)^2+a\right)^{\frac{3}{2}}\sinh\left(fx+e\right)^4\,dx$$

input
```
integrate(sinh(f*x+e)^4*(a+b*sinh(f*x+e)^2)^(3/2),x, algorithm="giac")
```

output
```
integrate((b*sinh(f*x + e)^2 + a)^(3/2)*sinh(f*x + e)^4, x)
```

Mupad [F(-1)]

Timed out.

$$\int \sinh^4(e+fx)\left(a+b\sinh^2(e+fx)\right)^{3/2}\,dx=\int \sinh(e+f\,x)^4\,\left(b\sinh(e+f\,x)^2+a\right)^{3/2}\,dx$$

input
```
int(sinh(e + f*x)^4*(a + b*sinh(e + f*x)^2)^(3/2),x)
```

output
```
int(sinh(e + f*x)^4*(a + b*sinh(e + f*x)^2)^(3/2), x)
```

Reduce [F]

$$\int \sinh^4(e+fx)\left(a+b\sinh^2(e+fx)\right)^{3/2}\,dx=\left(\int \sqrt{\sinh\left(fx+e\right)^2b+a}\,\sinh\left(fx+e\right)^6dx\right)b+\left(\int \sqrt{\sinh\left(fx+e\right)^2b+a}\,\sinh\left(fx+e\right)^4dx\right)a$$

input
```
int(sinh(f*x+e)^4*(a+b*sinh(f*x+e)^2)^(3/2),x)
```

output
```
int(sqrt(sinh(e + f*x)**2*b + a)*sinh(e + f*x)**6,x)*b + int(sqrt(sinh(e +
 f*x)**2*b + a)*sinh(e + f*x)**4,x)*a
```

3.77 $\int \sinh^2(e+fx)\left(a+b\sinh^2(e+fx)\right)^{3/2}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 238

$$\int \sinh^2(e+fx)\left(a+b\sinh^2(e+fx)\right)^{3/2}dx = \frac{(3a-4b)\cosh(e+fx)\sinh(e+fx)\sqrt{a+b\sinh^2(e+fx)}}{15f} + \frac{\cosh(e+fx)\sinh(e+fx)\left(a+b\sinh^2(e+fx)\right)^{3/2}}{5f} - \frac{i(3a^2-13ab+8b^2)\,E\left(ie+ifx\middle|\frac{b}{a}\right)\sqrt{a+b\sinh^2(e+fx)}}{15bf\sqrt{\frac{a+b\sinh^2(e+fx)}{a}}} + \frac{ia(3a-4b)(a-b)\,\text{EllipticF}\left(ie+ifx,\frac{b}{a}\right)\sqrt{\frac{a+b\sinh^2(e+fx)}{a}}}{15bf\sqrt{a+b\sinh^2(e+fx)}}$$

output
```
1/15*(3*a-4*b)*cosh(f*x+e)*sinh(f*x+e)*(a+b*sinh(f*x+e)^2)^(1/2)/f+1/5*cos
h(f*x+e)*sinh(f*x+e)*(a+b*sinh(f*x+e)^2)^(3/2)/f-1/15*I*(3*a^2-13*a*b+8*b^
2)*EllipticE(sin(I*e+I*f*x),(b/a)^(1/2))*(a+b*sinh(f*x+e)^2)^(1/2)/b/f/((a
+b*sinh(f*x+e)^2)/a)^(1/2)+1/15*I*a*(3*a-4*b)*(a-b)*InverseJacobiAM(I*e+I*
f*x,(b/a)^(1/2))*((a+b*sinh(f*x+e)^2)/a)^(1/2)/b/f/(a+b*sinh(f*x+e)^2)^(1/
2)
```

Mathematica [A] (verified)

Time = 1.14 (sec) , antiderivative size = 213, normalized size of antiderivative = 0.89

$$\int \sinh^2(e+fx)\left(a+b\sinh^2(e+fx)\right)^{3/2}\,dx = \frac{-16ia(3a^2-13ab+8b^2)\sqrt{\frac{2a-b+b\cosh(2(e+fx))}{a}}E\left(i(e+fx)\,\middle|\,\frac{b}{a}\right)+16ia(3a^2-7ab+4b^2)\sqrt{\frac{2a}{}}}{}$$

input
```
Integrate[Sinh[e + f*x]^2*(a + b*Sinh[e + f*x]^2)^(3/2),x]
```

output
```
((-16*I)*a*(3*a^2 - 13*a*b + 8*b^2)*Sqrt[(2*a - b + b*Cosh[2*(e + f*x)])/a
]*EllipticE[I*(e + f*x), b/a] + (16*I)*a*(3*a^2 - 7*a*b + 4*b^2)*Sqrt[(2*a
 - b + b*Cosh[2*(e + f*x)])/a]*EllipticF[I*(e + f*x), b/a] + Sqrt[2]*b*(48
*a^2 - 68*a*b + 25*b^2 + 4*(9*a - 7*b)*b*Cosh[2*(e + f*x)] + 3*b^2*Cosh[4*
(e + f*x)])*Sinh[2*(e + f*x)])/(240*b*f*Sqrt[2*a - b + b*Cosh[2*(e + f*x)]
])
```

Rubi [A] (verified)

Time = 1.29 (sec) , antiderivative size = 242, normalized size of antiderivative = 1.02, number of steps used = 14, number of rules used = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.560$, Rules used = {3042, 25, 3649, 3042, 3649, 3042, 3651, 3042, 3657, 3042, 3656, 3662, 3042, 3661}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sinh^2(e+fx)\left(a+b\sinh^2(e+fx)\right)^{3/2}dx$$

$$\Big\downarrow \text{3042}$$

$$\int -\sin(ie+ifx)^2\left(a-b\sin(ie+ifx)^2\right)^{3/2}dx$$

$$\Big\downarrow \text{25}$$

$$-\int \sin(ie+ifx)^2\left(a-b\sin(ie+ifx)^2\right)^{3/2}dx$$

$$\Big\downarrow \text{3649}$$

$$\frac{\sinh(e+fx)\cosh(e+fx)\left(a+b\sinh^2(e+fx)\right)^{3/2}}{5f}-$$
$$\frac{1}{5}\int \left(a-(3a-4b)\sinh^2(e+fx)\right)\sqrt{b\sinh^2(e+fx)+a}dx$$

$$\Big\downarrow \text{3042}$$

$$\frac{\sinh(e+fx)\cosh(e+fx)\left(a+b\sinh^2(e+fx)\right)^{3/2}}{5f}-$$
$$\frac{1}{5}\int \left((3a-4b)\sin(ie+ifx)^2+a\right)\sqrt{a-b\sin(ie+ifx)^2}dx$$

$$\Big\downarrow \text{3649}$$

$$\frac{1}{5}\left(\frac{(3a-4b)\sinh(e+fx)\cosh(e+fx)\sqrt{a+b\sinh^2(e+fx)}}{3f}-\frac{1}{3}\int \frac{2a(3a-2b)-\left(3a^2-13ba+8b^2\right)\sinh^2(e+}{\sqrt{b\sinh^2(e+fx)+a}}\right.$$
$$\frac{\sinh(e+fx)\cosh(e+fx)\left(a+b\sinh^2(e+fx)\right)^{3/2}}{5f}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\sinh(e+fx)\cosh(e+fx)\left(a+b\sinh^2(e+fx)\right)^{3/2}}{5f}+$$
$$\frac{1}{5}\left(\frac{(3a-4b)\sinh(e+fx)\cosh(e+fx)\sqrt{a+b\sinh^2(e+fx)}}{3f}-\frac{1}{3}\int \frac{\left(3a^2-13ba+8b^2\right)\sin(ie+ifx)^2+2a(3a}{\sqrt{a-b\sin(ie+ifx)^2}}\right.$$

$$\Big\downarrow \text{3651}$$

$$\frac{1}{5}\left(\frac{1}{3}\left(\frac{\left(3a^2-13ab+8b^2\right)\int \sqrt{b\sinh^2(e+fx)+a}dx}{b}-\frac{a(3a-4b)(a-b)\int \frac{1}{\sqrt{b\sinh^2(e+fx)+a}}dx}{b}\right)+\frac{(3a-4b)\sin\right.$$
$$\frac{\sinh(e+fx)\cosh(e+fx)\left(a+b\sinh^2(e+fx)\right)^{3/2}}{5f}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\sinh(e+fx)\cosh(e+fx)\left(a+b\sinh^2(e+fx)\right)^{3/2}}{5f}+$$
$$\frac{1}{5}\left(\frac{(3a-4b)\sinh(e+fx)\cosh(e+fx)\sqrt{a+b\sinh^2(e+fx)}}{3f}+\frac{1}{3}\left(\frac{\left(3a^2-13ab+8b^2\right)\int \sqrt{a-b\sin(ie+ifx)^2}}{b}\right.\right.$$

$$\Big\downarrow \ 3657$$

$$\frac{\sinh(e+fx)\cosh(e+fx)\left(a+b\sinh^2(e+fx)\right)^{3/2}}{5f}+$$
$$\frac{1}{5}\left(\frac{(3a-4b)\sinh(e+fx)\cosh(e+fx)\sqrt{a+b\sinh^2(e+fx)}}{3f}+\frac{1}{3}\left(\frac{\left(3a^2-13ab+8b^2\right)\sqrt{a+b\sinh^2(e+fx)}\int}{b\sqrt{\frac{b\sinh^2(e+fx)}{a}+1}}\right.\right.$$

$$\Big\downarrow \ 3042$$

$$\frac{\sinh(e+fx)\cosh(e+fx)\left(a+b\sinh^2(e+fx)\right)^{3/2}}{5f}+$$
$$\frac{1}{5}\left(\frac{(3a-4b)\sinh(e+fx)\cosh(e+fx)\sqrt{a+b\sinh^2(e+fx)}}{3f}+\frac{1}{3}\left(\frac{\left(3a^2-13ab+8b^2\right)\sqrt{a+b\sinh^2(e+fx)}\int}{b\sqrt{\frac{b\sinh^2(e+fx)}{a}+1}}\right.\right.$$

$$\Big\downarrow \ 3656$$

$$\frac{\sinh(e+fx)\cosh(e+fx)\left(a+b\sinh^2(e+fx)\right)^{3/2}}{5f}+$$
$$\frac{1}{5}\left(\frac{(3a-4b)\sinh(e+fx)\cosh(e+fx)\sqrt{a+b\sinh^2(e+fx)}}{3f}+\frac{1}{3}\left(-\frac{a(3a-4b)(a-b)\int\frac{1}{\sqrt{a-b\sin(ie+ifx)^2}}dx}{b}-\right.\right.$$

$$\Big\downarrow \ 3662$$

$$\frac{\sinh(e+fx)\cosh(e+fx)\left(a+b\sinh^2(e+fx)\right)^{3/2}}{5f}+$$
$$\frac{1}{5}\left(\frac{(3a-4b)\sinh(e+fx)\cosh(e+fx)\sqrt{a+b\sinh^2(e+fx)}}{3f}+\frac{1}{3}\left(-\frac{a(3a-4b)(a-b)\sqrt{\frac{b\sinh^2(e+fx)}{a}+1}\int\frac{}{\sqrt{\frac{b\,s}{}}}}{b\sqrt{a+b\sinh^2(e+fx)}}\right.\right.$$

$$\Big\downarrow \ 3042$$

$$\frac{\sinh(e+fx)\cosh(e+fx)\left(a+b\sinh^2(e+fx)\right)^{3/2}}{5f}+$$
$$\frac{1}{5}\left(\frac{(3a-4b)\sinh(e+fx)\cosh(e+fx)\sqrt{a+b\sinh^2(e+fx)}}{3f}+\frac{1}{3}\left(-\frac{a(3a-4b)(a-b)\sqrt{\frac{b\sinh^2(e+fx)}{a}+1}\int\frac{}{\sqrt{1-}}}{b\sqrt{a+b\sinh^2(e+fx)}}\right.\right.$$

$$\Big\downarrow \ 3661$$

$$\frac{\sinh(e+fx)\cosh(e+fx)\left(a+b\sinh^2(e+fx)\right)^{3/2}}{5f}+$$
$$\frac{1}{5}\left(\frac{(3a-4b)\sinh(e+fx)\cosh(e+fx)\sqrt{a+b\sinh^2(e+fx)}}{3f}+\frac{1}{3}\left(\frac{ia(3a-4b)(a-b)\sqrt{\frac{b\sinh^2(e+fx)}{a}+1}\,\text{Ellipti}}{bf\sqrt{a+b\sinh^2(e+fx)}}\right.\right.$$

input
```
Int[Sinh[e + f*x]^2*(a + b*Sinh[e + f*x]^2)^(3/2),x]
```

output
```
(Cosh[e + f*x]*Sinh[e + f*x]*(a + b*Sinh[e + f*x]^2)^(3/2))/(5*f) + (((3*a
 - 4*b)*Cosh[e + f*x]*Sinh[e + f*x]*Sqrt[a + b*Sinh[e + f*x]^2])/(3*f) + (
((-I)*(3*a^2 - 13*a*b + 8*b^2)*EllipticE[I*e + I*f*x, b/a]*Sqrt[a + b*Sinh
[e + f*x]^2])/(b*f*Sqrt[1 + (b*Sinh[e + f*x]^2)/a]) + (I*a*(3*a - 4*b)*(a
- b)*EllipticF[I*e + I*f*x, b/a]*Sqrt[1 + (b*Sinh[e + f*x]^2)/a])/(b*f*Sqr
t[a + b*Sinh[e + f*x]^2]))/3)/5
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3649
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)]^2), x_Symbol] :> Simp[(-B)*Cos[e + f*x]*Sin[e + f*x]*((a + b*
Sin[e + f*x]^2)^p/(2*f*(p + 1))), x] + Simp[1/(2*(p + 1))   Int[(a + b*Sin[
e + f*x]^2)^(p - 1)*Simp[a*B + 2*a*A*(p + 1) + (2*A*b*(p + 1) + B*(b + 2*a*
p + 2*b*p))*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, e, f, A, B}, x] && G
tQ[p, 0]
```

rule 3651
```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)]^2)/Sqrt[(a_) + (b_.)*sin[(e_.) +
 (f_.)*(x_)]^2], x_Symbol] :> Simp[B/b   Int[Sqrt[a + b*Sin[e + f*x]^2], x]
, x] + Simp[(A*b - a*B)/b   Int[1/Sqrt[a + b*Sin[e + f*x]^2], x], x] /; Fre
eQ[{a, b, e, f, A, B}, x]
```

rule 3656
```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2], x_Symbol] :> Simp[(Sqrt[a
]/f)*EllipticE[e + f*x, -b/a], x] /; FreeQ[{a, b, e, f}, x] && GtQ[a, 0]
```

rule 3657
```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2], x_Symbol] :> Simp[Sqrt[a
+ b*Sin[e + f*x]^2]/Sqrt[1 + b*(Sin[e + f*x]^2/a)]   Int[Sqrt[1 + (b*Sin[e
+ f*x]^2)/a], x], x] /; FreeQ[{a, b, e, f}, x] &&  !GtQ[a, 0]
```

rule 3661
```
Int[1/Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2], x_Symbol] :> Simp[(1/(S
qrt[a]*f))*EllipticF[e + f*x, -b/a], x] /; FreeQ[{a, b, e, f}, x] && GtQ[a,
 0]
```

rule 3662
```
Int[1/Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2], x_Symbol] :> Simp[Sqrt[
1 + b*(Sin[e + f*x]^2/a)]/Sqrt[a + b*Sin[e + f*x]^2]   Int[1/Sqrt[1 + (b*Si
n[e + f*x]^2)/a], x], x] /; FreeQ[{a, b, e, f}, x] &&  !GtQ[a, 0]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 534 vs. $2(217) = 434$.

Time = 5.79 (sec) , antiderivative size = 535, normalized size of antiderivative = 2.25

method	result
default	$-\frac{-3\sqrt{-\frac{b}{a}}\,b^2\cosh(fx+e)^6\sinh(fx+e)+\left(-9\sqrt{-\frac{b}{a}}\,ab+10\sqrt{-\frac{b}{a}}\,b^2\right)\cosh(fx+e)^4\sinh(fx+e)+\left(-6\sqrt{-\frac{b}{a}}\,a^2+13\sqrt{-\frac{b}{a}}\,ab-7\sqrt{-\right.}{}$

input
```
int(sinh(f*x+e)^2*(a+b*sinh(f*x+e)^2)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-1/15*(-3*(-b/a)^(1/2)*b^2*cosh(f*x+e)^6*sinh(f*x+e)+(-9*(-b/a)^(1/2)*a*b+
10*(-b/a)^(1/2)*b^2)*cosh(f*x+e)^4*sinh(f*x+e)+(-6*(-b/a)^(1/2)*a^2+13*(-b
/a)^(1/2)*a*b-7*(-b/a)^(1/2)*b^2)*cosh(f*x+e)^2*sinh(f*x+e)+9*a^2*(b/a*cos
h(f*x+e)^2+(a-b)/a)^(1/2)*(cosh(f*x+e)^2)^(1/2)*EllipticF(sinh(f*x+e)*(-b/
a)^(1/2),(1/b*a)^(1/2))-17*a*(b/a*cosh(f*x+e)^2+(a-b)/a)^(1/2)*(cosh(f*x+e
)^2)^(1/2)*EllipticF(sinh(f*x+e)*(-b/a)^(1/2),(1/b*a)^(1/2))*b+8*(b/a*cosh
(f*x+e)^2+(a-b)/a)^(1/2)*(cosh(f*x+e)^2)^(1/2)*EllipticF(sinh(f*x+e)*(-b/a
)^(1/2),(1/b*a)^(1/2))*b^2-3*(b/a*cosh(f*x+e)^2+(a-b)/a)^(1/2)*(cosh(f*x+e
)^2)^(1/2)*EllipticE(sinh(f*x+e)*(-b/a)^(1/2),(1/b*a)^(1/2))*a^2+13*(b/a*c
osh(f*x+e)^2+(a-b)/a)^(1/2)*(cosh(f*x+e)^2)^(1/2)*EllipticE(sinh(f*x+e)*(-
b/a)^(1/2),(1/b*a)^(1/2))*a*b-8*(b/a*cosh(f*x+e)^2+(a-b)/a)^(1/2)*(cosh(f*
x+e)^2)^(1/2)*EllipticE(sinh(f*x+e)*(-b/a)^(1/2),(1/b*a)^(1/2))*b^2)/(-b/a
)^(1/2)/cosh(f*x+e)/(a+b*sinh(f*x+e)^2)^(1/2)/f
```

Fricas [F]

$$\int \sinh^2(e+fx)\left(a+b\sinh^2(e+fx)\right)^{3/2}\,dx=\int \left(b\sinh\left(fx+e\right)^2+a\right)^{\frac{3}{2}}\sinh\left(fx+e\right)^2\,dx$$

input
```
integrate(sinh(f*x+e)^2*(a+b*sinh(f*x+e)^2)^(3/2),x, algorithm="fricas")
```

output
```
integral((b*sinh(f*x + e)^4 + a*sinh(f*x + e)^2)*sqrt(b*sinh(f*x + e)^2 +
a), x)
```

Sympy [F(-1)]

Timed out.

$$\int \sinh^2(e+fx)\left(a+b\sinh^2(e+fx)\right)^{3/2}\,dx=\text{Timed out}$$

input
```
integrate(sinh(f*x+e)**2*(a+b*sinh(f*x+e)**2)**(3/2),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \sinh^2(e+fx)\left(a+b\sinh^2(e+fx)\right)^{3/2}\,dx=\int \left(b\sinh\left(fx+e\right)^2+a\right)^{\frac{3}{2}}\sinh\left(fx+e\right)^2\,dx$$

input
```
integrate(sinh(f*x+e)^2*(a+b*sinh(f*x+e)^2)^(3/2),x, algorithm="maxima")
```

output
```
integrate((b*sinh(f*x + e)^2 + a)^(3/2)*sinh(f*x + e)^2, x)
```

Giac [F]

$$\int \sinh^2(e+fx)\left(a+b\sinh^2(e+fx)\right)^{3/2}\,dx=\int \left(b\sinh\left(fx+e\right)^2+a\right)^{\frac{3}{2}}\sinh\left(fx+e\right)^2\,dx$$

input
```
integrate(sinh(f*x+e)^2*(a+b*sinh(f*x+e)^2)^(3/2),x, algorithm="giac")
```

output
```
integrate((b*sinh(f*x + e)^2 + a)^(3/2)*sinh(f*x + e)^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int \sinh^2(e+fx)\left(a+b\sinh^2(e+fx)\right)^{3/2}\,dx=\int \sinh(e+f\,x)^2\,\left(b\sinh(e+f\,x)^2+a\right)^{3/2}\,dx$$

input
```
int(sinh(e + f*x)^2*(a + b*sinh(e + f*x)^2)^(3/2),x)
```

output
```
int(sinh(e + f*x)^2*(a + b*sinh(e + f*x)^2)^(3/2), x)
```

Reduce [F]

$$\int \sinh^2(e+fx)\left(a+b\sinh^2(e+fx)\right)^{3/2}\,dx = \left(\int \sqrt{\sinh(fx+e)^2 b+a}\,\sinh(fx+e)^4\,dx\right)b + \left(\int \sqrt{\sinh(fx+e)^2 b+a}\,\sinh(fx+e)^2\,dx\right)a$$

input
```
int(sinh(f*x+e)^2*(a+b*sinh(f*x+e)^2)^(3/2),x)
```

output
```
int(sqrt(sinh(e + f*x)**2*b + a)*sinh(e + f*x)**4,x)*b + int(sqrt(sinh(e +
 f*x)**2*b + a)*sinh(e + f*x)**2,x)*a
```

3.78 $\int \left(a+b\sinh^2(e+fx)\right)^{3/2}\,dx$

Optimal result

Integrand size = 16, antiderivative size = 176

$$\int \left(a+b\sinh^2(e+fx)\right)^{3/2}\,dx=\frac{b\cosh(e+fx)\sinh(e+fx)\sqrt{a+b\sinh^2(e+fx)}}{3f}-\frac{2i(2a-b)E\left(ie+ifx\middle|\frac{b}{a}\right)\sqrt{a+b\sinh^2(e+fx)}}{3f\sqrt{\frac{a+b\sinh^2(e+fx)}{a}}}+\frac{ia(a-b)\operatorname{EllipticF}\left(ie+ifx,\frac{b}{a}\right)\sqrt{\frac{a+b\sinh^2(e+fx)}{a}}}{3f\sqrt{a+b\sinh^2(e+fx)}}$$

output
```
1/3*b*cosh(f*x+e)*sinh(f*x+e)*(a+b*sinh(f*x+e)^2)^(1/2)/f-2/3*I*(2*a-b)*El
lipticE(sin(I*e+I*f*x),(b/a)^(1/2))*(a+b*sinh(f*x+e)^2)^(1/2)/f/((a+b*sinh
(f*x+e)^2)/a)^(1/2)+1/3*I*a*(a-b)*InverseJacobiAM(I*e+I*f*x,(b/a)^(1/2))*(
(a+b*sinh(f*x+e)^2)/a)^(1/2)/f/(a+b*sinh(f*x+e)^2)^(1/2)
```

Mathematica [A] (verified)

Time = 0.56 (sec) , antiderivative size = 169, normalized size of antiderivative = 0.96

$$\int \left(a+b\sinh^2(e+fx)\right)^{3/2}\,dx=\frac{-4i\sqrt{2}a(2a-b)\sqrt{\frac{2a-b+b\cosh(2(e+fx))}{a}}E\left(i(e+fx)\left|\frac{b}{a}\right.\right)+2i\sqrt{2}a(a-b)\sqrt{\frac{2a-b+b\cosh(2(e+fx))}{a}}\text{E}}{6f\sqrt{4a-2b+2b\cosh(2(e}}$$

input
```
Integrate[(a + b*Sinh[e + f*x]^2)^(3/2),x]
```

output
```
((-4*I)*Sqrt[2]*a*(2*a - b)*Sqrt[(2*a - b + b*Cosh[2*(e + f*x)])/a]*Ellipt
icE[I*(e + f*x), b/a] + (2*I)*Sqrt[2]*a*(a - b)*Sqrt[(2*a - b + b*Cosh[2*(
e + f*x)])/a]*EllipticF[I*(e + f*x), b/a] + b*(2*a - b + b*Cosh[2*(e + f*x
)])*Sinh[2*(e + f*x)])/(6*f*Sqrt[4*a - 2*b + 2*b*Cosh[2*(e + f*x)]])
```

Rubi [A] (verified)

Time = 0.88 (sec) , antiderivative size = 175, normalized size of antiderivative = 0.99, number of steps used = 11, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.688, Rules used = {3042, 3659, 3042, 3651, 3042, 3657, 3042, 3656, 3662, 3042, 3661}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(a+b\sinh^2(e+fx)\right)^{3/2}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \left(a-b\sin(ie+ifx)^2\right)^{3/2}dx$$

$$\Big\downarrow \text{3659}$$

$$\frac{1}{3}\int \frac{2(2a-b)b\sinh^2(e+fx)+a(3a-b)}{\sqrt{b\sinh^2(e+fx)+a}}dx+\frac{b\sinh(e+fx)\cosh(e+fx)\sqrt{a+b\sinh^2(e+fx)}}{3f}$$

$$\Big\downarrow \text{3042}$$

$$\frac{b\sinh(e+fx)\cosh(e+fx)\sqrt{a+b\sinh^2(e+fx)}}{3f}+\frac{1}{3}\int\frac{a(3a-b)-2(2a-b)b\sin(ie+ifx)^2}{\sqrt{a-b\sin(ie+ifx)^2}}dx$$

$$\Big\downarrow \text{3651}$$

$$\frac{1}{3}\left(2(2a-b)\int\sqrt{b\sinh^2(e+fx)+a}dx-a(a-b)\int\frac{1}{\sqrt{b\sinh^2(e+fx)+a}}dx\right)+\frac{b\sinh(e+fx)\cosh(e+fx)\sqrt{a+b\sinh^2(e+fx)}}{3f}$$

$$\Big\downarrow \text{3042}$$

$$\frac{b\sinh(e+fx)\cosh(e+fx)\sqrt{a+b\sinh^2(e+fx)}}{3f}+\frac{1}{3}\left(2(2a-b)\int\sqrt{a-b\sin(ie+ifx)^2}dx-a(a-b)\int\frac{1}{\sqrt{a-b\sin(ie+ifx)^2}}dx\right)$$

$$\Big\downarrow \text{3657}$$

$$\frac{b\sinh(e+fx)\cosh(e+fx)\sqrt{a+b\sinh^2(e+fx)}}{3f}+\frac{1}{3}\left(\frac{2(2a-b)\sqrt{a+b\sinh^2(e+fx)}\int\sqrt{\frac{b\sinh^2(e+fx)}{a}+1}dx}{\sqrt{\frac{b\sinh^2(e+fx)}{a}+1}}-a(a-b)\int\frac{1}{\sqrt{a-b\sin(ie+ifx)^2}}dx\right)$$

$$\Big\downarrow \text{3042}$$

$$\frac{b\sinh(e+fx)\cosh(e+fx)\sqrt{a+b\sinh^2(e+fx)}}{3f}+\frac{1}{3}\left(\frac{2(2a-b)\sqrt{a+b\sinh^2(e+fx)}\int\sqrt{1-\frac{b\sin(ie+ifx)^2}{a}}dx}{\sqrt{\frac{b\sinh^2(e+fx)}{a}+1}}-a(a-b)\int\frac{1}{\sqrt{a-b\sin(ie+ifx)^2}}dx\right)$$

$$\Big\downarrow \text{3656}$$

$$\frac{b\sinh(e+fx)\cosh(e+fx)\sqrt{a+b\sinh^2(e+fx)}}{3f}+$$
$$\frac{1}{3}\left(-a(a-b)\int\frac{1}{\sqrt{a-b\sin(ie+ifx)^2}}dx-\frac{2i(2a-b)\sqrt{a+b\sinh^2(e+fx)}E\left(ie+ifx|\frac{b}{a}\right)}{f\sqrt{\frac{b\sinh^2(e+fx)}{a}+1}}\right)$$

$$\Big\downarrow \text{3662}$$

$$\frac{b\sinh(e+fx)\cosh(e+fx)\sqrt{a+b\sinh^2(e+fx)}}{3f}+$$
$$\frac{1}{3}\left(-\frac{a(a-b)\sqrt{\frac{b\sinh^2(e+fx)}{a}+1}\int\frac{1}{\sqrt{\frac{b\sinh^2(e+fx)}{a}+1}}dx}{\sqrt{a+b\sinh^2(e+fx)}}-\frac{2i(2a-b)\sqrt{a+b\sinh^2(e+fx)}E\left(ie+ifx|\frac{b}{a}\right)}{f\sqrt{\frac{b\sinh^2(e+fx)}{a}+1}}\right)$$

$$\Big\downarrow \text{3042}$$

$$\frac{b\sinh(e+fx)\cosh(e+fx)\sqrt{a+b\sinh^2(e+fx)}}{3f}+$$
$$\frac{1}{3}\left(-\frac{a(a-b)\sqrt{\frac{b\sinh^2(e+fx)}{a}+1}\int\frac{1}{\sqrt{1-\frac{b\sin(ie+ifx)^2}{a}}}dx}{\sqrt{a+b\sinh^2(e+fx)}}-\frac{2i(2a-b)\sqrt{a+b\sinh^2(e+fx)}E\left(ie+ifx|\frac{b}{a}\right)}{f\sqrt{\frac{b\sinh^2(e+fx)}{a}+1}}\right)$$

$$\Big\downarrow \text{3661}$$

$$\frac{b\sinh(e+fx)\cosh(e+fx)\sqrt{a+b\sinh^2(e+fx)}}{3f}+$$
$$\frac{1}{3}\left(\frac{ia(a-b)\sqrt{\frac{b\sinh^2(e+fx)}{a}+1}\,\text{EllipticF}\left(ie+ifx,\frac{b}{a}\right)}{f\sqrt{a+b\sinh^2(e+fx)}}-\frac{2i(2a-b)\sqrt{a+b\sinh^2(e+fx)}E\left(ie+ifx|\frac{b}{a}\right)}{f\sqrt{\frac{b\sinh^2(e+fx)}{a}+1}}\right)$$

input
```
Int[(a + b*Sinh[e + f*x]^2)^(3/2),x]
```

output
```
(b*Cosh[e + f*x]*Sinh[e + f*x]*Sqrt[a + b*Sinh[e + f*x]^2])/(3*f) + (((-2*
I)*(2*a - b)*EllipticE[I*e + I*f*x, b/a]*Sqrt[a + b*Sinh[e + f*x]^2])/(f*S
qrt[1 + (b*Sinh[e + f*x]^2)/a]) + (I*a*(a - b)*EllipticF[I*e + I*f*x, b/a]
*Sqrt[1 + (b*Sinh[e + f*x]^2)/a])/(f*Sqrt[a + b*Sinh[e + f*x]^2]))/3
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3651
```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)]^2)/Sqrt[(a_) + (b_.)*sin[(e_.) +
 (f_.)*(x_)]^2], x_Symbol] :> Simp[B/b   Int[Sqrt[a + b*Sin[e + f*x]^2], x]
, x] + Simp[(A*b - a*B)/b   Int[1/Sqrt[a + b*Sin[e + f*x]^2], x], x] /; Fre
eQ[{a, b, e, f, A, B}, x]
```

rule 3656
```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2], x_Symbol] :> Simp[(Sqrt[a
]/f)*EllipticE[e + f*x, -b/a], x] /; FreeQ[{a, b, e, f}, x] && GtQ[a, 0]
```

rule 3657
```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2], x_Symbol] :> Simp[Sqrt[a
+ b*Sin[e + f*x]^2]/Sqrt[1 + b*(Sin[e + f*x]^2/a)]   Int[Sqrt[1 + (b*Sin[e
+ f*x]^2)/a], x], x] /; FreeQ[{a, b, e, f}, x] &&  !GtQ[a, 0]
```

rule 3659
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> Simp[(-b)*C
os[e + f*x]*Sin[e + f*x]*((a + b*Sin[e + f*x]^2)^(p - 1)/(2*f*p)), x] + Sim
p[1/(2*p)   Int[(a + b*Sin[e + f*x]^2)^(p - 2)*Simp[a*(b + 2*a*p) + b*(2*a
+ b)*(2*p - 1)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, e, f}, x] && NeQ[
a + b, 0] && GtQ[p, 1]
```

rule 3661
```
Int[1/Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2], x_Symbol] :> Simp[(1/(S
qrt[a]*f))*EllipticF[e + f*x, -b/a], x] /; FreeQ[{a, b, e, f}, x] && GtQ[a,
 0]
```

rule 3662
```
Int[1/Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2], x_Symbol] :> Simp[Sqrt[
1 + b*(Sin[e + f*x]^2/a)]/Sqrt[a + b*Sin[e + f*x]^2]   Int[1/Sqrt[1 + (b*Si
n[e + f*x]^2)/a], x], x] /; FreeQ[{a, b, e, f}, x] &&  !GtQ[a, 0]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 427 vs. $2(159) = 318$.

Time = 4.02 (sec) , antiderivative size = 428, normalized size of antiderivative = 2.43

method	result
default	$\sqrt{-\frac{b}{a}}\, b^2 \cosh(fx+e)^4 \sinh(fx+e)+\sqrt{-\frac{b}{a}}\, ab \cosh(fx+e)^2 \sinh(fx+e)-\sqrt{-\frac{b}{a}}\, b^2 \cosh(fx+e)^2 \sinh(fx+e)+3a^2 \sqrt{\frac{b \cosh(fx+e)^2}{a}+}$

input
```
int((a+b*sinh(f*x+e)^2)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
1/3*((-b/a)^(1/2)*b^2*cosh(f*x+e)^4*sinh(f*x+e)+(-b/a)^(1/2)*a*b*cosh(f*x+
e)^2*sinh(f*x+e)-(-b/a)^(1/2)*b^2*cosh(f*x+e)^2*sinh(f*x+e)+3*a^2*(b/a*cos
h(f*x+e)^2+(a-b)/a)^(1/2)*(cosh(f*x+e)^2)^(1/2)*EllipticF(sinh(f*x+e)*(-b/
a)^(1/2),(1/b*a)^(1/2))-5*a*(b/a*cosh(f*x+e)^2+(a-b)/a)^(1/2)*(cosh(f*x+e)
^2)^(1/2)*EllipticF(sinh(f*x+e)*(-b/a)^(1/2),(1/b*a)^(1/2))*b+2*(b/a*cosh(
f*x+e)^2+(a-b)/a)^(1/2)*(cosh(f*x+e)^2)^(1/2)*EllipticF(sinh(f*x+e)*(-b/a)
^(1/2),(1/b*a)^(1/2))*b^2+4*(b/a*cosh(f*x+e)^2+(a-b)/a)^(1/2)*(cosh(f*x+e)
^2)^(1/2)*EllipticE(sinh(f*x+e)*(-b/a)^(1/2),(1/b*a)^(1/2))*a*b-2*(b/a*cos
h(f*x+e)^2+(a-b)/a)^(1/2)*(cosh(f*x+e)^2)^(1/2)*EllipticE(sinh(f*x+e)*(-b/
a)^(1/2),(1/b*a)^(1/2))*b^2)/(-b/a)^(1/2)/cosh(f*x+e)/(a+b*sinh(f*x+e)^2)^
(1/2)/f
```

Fricas [F]

$$\int \left(a+b\sinh^2(e+fx)\right)^{3/2}\,dx=\int \left(b\sinh\left(fx+e\right)^2+a\right)^{\frac{3}{2}}\,dx$$

input
```
integrate((a+b*sinh(f*x+e)^2)^(3/2),x, algorithm="fricas")
```

output
```
integral((b*sinh(f*x + e)^2 + a)^(3/2), x)
```

Sympy [F]

$$\int \left(a+b\sinh^2(e+fx)\right)^{3/2}\,dx=\int \left(a+b\sinh^2{(e+fx)}\right)^{\frac{3}{2}}\,dx$$

input
```
integrate((a+b*sinh(f*x+e)**2)**(3/2),x)
```

output
```
Integral((a + b*sinh(e + f*x)**2)**(3/2), x)
```

Maxima [F]

$$\int \left(a+b\sinh^2(e+fx)\right)^{3/2}\,dx=\int \left(b\sinh\left(fx+e\right)^2+a\right)^{\frac{3}{2}}\,dx$$

input
```
integrate((a+b*sinh(f*x+e)^2)^(3/2),x, algorithm="maxima")
```

output
```
integrate((b*sinh(f*x + e)^2 + a)^(3/2), x)
```

Giac [F]

$$\int \left(a+b\sinh^2(e+fx)\right)^{3/2}\,dx=\int \left(b\sinh\left(fx+e\right)^2+a\right)^{\frac{3}{2}}\,dx$$

input
```
integrate((a+b*sinh(f*x+e)^2)^(3/2),x, algorithm="giac")
```

output
```
integrate((b*sinh(f*x + e)^2 + a)^(3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \left(a+b\sinh^2(e+fx)\right)^{3/2}\,dx=\int \left(b\sinh(e+f\,x)^2+a\right)^{3/2}\,dx$$

input
```
int((a + b*sinh(e + f*x)^2)^(3/2),x)
```

output
```
int((a + b*sinh(e + f*x)^2)^(3/2), x)
```

Reduce [F]

$$\int \left(a+b\sinh^2(e+fx)\right)^{3/2}\,dx=\left(\int \sqrt{\sinh\left(fx+e\right)^2 b+a}dx\right)a+\left(\int \sqrt{\sinh\left(fx+e\right)^2 b+a}\,\sinh\left(fx+e\right)^2 dx\right)b$$

input
```
int((a+b*sinh(f*x+e)^2)^(3/2),x)
```

output
```
int(sqrt(sinh(e + f*x)**2*b + a),x)*a + int(sqrt(sinh(e + f*x)**2*b + a)*s
inh(e + f*x)**2,x)*b
```

3.79 $\int \operatorname{csch}^2(e+fx)\left(a+b\sinh^2(e+fx)\right)^{3/2}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 204

$$\int \operatorname{csch}^2(e+fx)\left(a+b\sinh^2(e+fx)\right)^{3/2}\,dx = -\frac{a\coth(e+fx)\sqrt{a+b\sinh^2(e+fx)}}{f} - \frac{(a+b)E\left(\arctan(\sinh(e+fx))\,\middle|\,1-\frac{b}{a}\right)\operatorname{sech}(e+fx)\sqrt{a+b\sinh^2(e+fx)}}{f\sqrt{\frac{\operatorname{sech}^2(e+fx)(a+b\sinh^2(e+fx))}{a}}} + \frac{2b\,\text{EllipticF}\left(\arctan(\sinh(e+fx)),1-\frac{b}{a}\right)\operatorname{sech}(e+fx)\sqrt{a+b\sinh^2(e+fx)}}{f\sqrt{\frac{\operatorname{sech}^2(e+fx)(a+b\sinh^2(e+fx))}{a}}} + \frac{(a+b)\sqrt{a+b\sinh^2(e+fx)}\tanh(e+fx)}{f}$$

output
```
-a*coth(f*x+e)*(a+b*sinh(f*x+e)^2)^(1/2)/f-(a+b)*EllipticE(sinh(f*x+e)/(1+
sinh(f*x+e)^2)^(1/2),(1-b/a)^(1/2))*sech(f*x+e)*(a+b*sinh(f*x+e)^2)^(1/2)/
f/(sech(f*x+e)^2*(a+b*sinh(f*x+e)^2)/a)^(1/2)+2*b*InverseJacobiAM(arctan(s
inh(f*x+e)),(1-b/a)^(1/2))*sech(f*x+e)*(a+b*sinh(f*x+e)^2)^(1/2)/f/(sech(f
*x+e)^2*(a+b*sinh(f*x+e)^2)/a)^(1/2)+(a+b)*(a+b*sinh(f*x+e)^2)^(1/2)*tanh(
f*x+e)/f
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 1.05 (sec) , antiderivative size = 155, normalized size of antiderivative = 0.76

$$\int \operatorname{csch}^2(e+fx)\left(a+b\sinh^2(e+fx)\right)^{3/2}\,dx =$$
$$-\frac{a\left(\sqrt{2}(2a-b+b\cosh(2(e+fx)))\coth(e+fx)+2i(a+b)\sqrt{\frac{2a-b+b\cosh(2(e+fx))}{a}}E\left(i(e+fx)\,\middle|\,\frac{b}{a}\right)-2i(a\right.}{2f\sqrt{2a-b+b\cosh(2(e+fx))}}$$

input
```
Integrate[Csch[e + f*x]^2*(a + b*Sinh[e + f*x]^2)^(3/2),x]
```

output
```
-1/2*(a*(Sqrt[2]*(2*a - b + b*Cosh[2*(e + f*x)])*Coth[e + f*x] + (2*I)*(a
+ b)*Sqrt[(2*a - b + b*Cosh[2*(e + f*x)])/a]*EllipticE[I*(e + f*x), b/a] -
 (2*I)*(a - b)*Sqrt[(2*a - b + b*Cosh[2*(e + f*x)])/a]*EllipticF[I*(e + f*
x), b/a]))/(f*Sqrt[2*a - b + b*Cosh[2*(e + f*x)]])
```

Rubi [A] (verified)

Time = 0.45 (sec) , antiderivative size = 273, normalized size of antiderivative = 1.34, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.360, Rules used = {3042, 25, 3667, 376, 27, 406, 320, 388, 313}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \operatorname{csch}^2(e+fx)\left(a+b\sinh^2(e+fx)\right)^{3/2}dx$$
$$\Big\downarrow \text{3042}$$
$$\int -\frac{\left(a-b\sin(ie+ifx)^2\right)^{3/2}}{\sin(ie+ifx)^2}dx$$
$$\Big\downarrow \text{25}$$
$$-\int \frac{\left(a-b\sin(ie+ifx)^2\right)^{3/2}}{\sin(ie+ifx)^2}dx$$

$\Big\downarrow$ 3667

$$\frac{\sqrt{\cosh^2(e+fx)}\text{sech}(e+fx)\int \frac{\text{csch}^2(e+fx)\left(b\sinh^2(e+fx)+a\right)^{3/2}}{\sqrt{\sinh^2(e+fx)+1}}d\sinh(e+fx)}{f}$$

$\Big\downarrow$ 376

$$\frac{\sqrt{\cosh^2(e+fx)}\text{sech}(e+fx)\left(\int \frac{b\left((a+b)\sinh^2(e+fx)+2a\right)}{\sqrt{\sinh^2(e+fx)+1}\sqrt{b\sinh^2(e+fx)+a}}d\sinh(e+fx)-a\sqrt{\sinh^2(e+fx)+1}\text{csch}(e+f}{f}$$

$\Big\downarrow$ 27

$$\frac{\sqrt{\cosh^2(e+fx)}\text{sech}(e+fx)\left(b\int \frac{(a+b)\sinh^2(e+fx)+2a}{\sqrt{\sinh^2(e+fx)+1}\sqrt{b\sinh^2(e+fx)+a}}d\sinh(e+fx)-a\sqrt{\sinh^2(e+fx)+1}\text{csch}(e+\right.}{f}$$

$\Big\downarrow$ 406

$$\frac{\sqrt{\cosh^2(e+fx)}\text{sech}(e+fx)\left(b\left(2a\int \frac{1}{\sqrt{\sinh^2(e+fx)+1}\sqrt{b\sinh^2(e+fx)+a}}d\sinh(e+fx)+(a+b)\int \frac{\sinh^2(e-}{\sqrt{\sinh^2(e+fx)+1}\sqrt{}}\right.\right.}{f}$$

$\Big\downarrow$ 320

$$\frac{\sqrt{\cosh^2(e+fx)}\text{sech}(e+fx)\left(b\left(\left((a+b)\int \frac{\sinh^2(e+fx)}{\sqrt{\sinh^2(e+fx)+1}\sqrt{b\sinh^2(e+fx)+a}}d\sinh(e+fx)+\frac{2\sqrt{a+b\sinh^2(e+fx)}\text{Ellip}}{\sqrt{\sinh^2(e+fx)+}}\right.\right.\right.}{f}$$

$\Big\downarrow$ 388

$$\frac{\sqrt{\cosh^2(e+fx)}\text{sech}(e+fx)\left(b\left(\left((a+b)\left(\frac{\sinh(e+fx)\sqrt{a+b\sinh^2(e+fx)}}{b\sqrt{\sinh^2(e+fx)+1}}-\frac{\int \frac{\sqrt{b\sinh^2(e+fx)+a}}{\left(\sinh^2(e+fx)+1\right)^{3/2}}d\sinh(e+fx)}{b}\right)+\frac{2\sqrt{a+bs}}{}\right.\right.\right.}{f}$$

$\Big\downarrow$ 313

$$\sqrt{\cosh^2(e+fx)}\text{sech}(e+fx)\left(b\left(\frac{2\sqrt{a+b\sinh^2(e+fx)}\,\text{EllipticF}\left(\arctan(\sinh(e+fx)),1-\frac{b}{a}\right)}{\sqrt{\sinh^2(e+fx)+1}\sqrt{\frac{a+b\sinh^2(e+fx)}{a\left(\sinh^2(e+fx)+1\right)}}}+(a+b)\left(\frac{\sinh(e+fx)\sqrt{a+b\sinh^2}}{b\sqrt{\sinh^2(e+fx)+}}\right.\right.\right.$$

input
```
Int[Csch[e + f*x]^2*(a + b*Sinh[e + f*x]^2)^(3/2),x]
```

output
```
(Sqrt[Cosh[e + f*x]^2]*Sech[e + f*x]*(-(a*Csch[e + f*x]*Sqrt[1 + Sinh[e +
f*x]^2]*Sqrt[a + b*Sinh[e + f*x]^2]) + b*((2*EllipticF[ArcTan[Sinh[e + f*x
]], 1 - b/a]*Sqrt[a + b*Sinh[e + f*x]^2])/(Sqrt[1 + Sinh[e + f*x]^2]*Sqrt[
(a + b*Sinh[e + f*x]^2)/(a*(1 + Sinh[e + f*x]^2))]) + (a + b)*((Sinh[e + f
*x]*Sqrt[a + b*Sinh[e + f*x]^2])/(b*Sqrt[1 + Sinh[e + f*x]^2]) - (Elliptic
E[ArcTan[Sinh[e + f*x]], 1 - b/a]*Sqrt[a + b*Sinh[e + f*x]^2])/(b*Sqrt[1 +
 Sinh[e + f*x]^2]*Sqrt[(a + b*Sinh[e + f*x]^2)/(a*(1 + Sinh[e + f*x]^2))])
))))/f
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 313
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/((c_) + (d_.)*(x_)^2)^(3/2), x_Symbol] :> Sim
p[(Sqrt[a + b*x^2]/(c*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(c
+ d*x^2)))]))*EllipticE[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; FreeQ
[{a, b, c, d}, x] && PosQ[b/a] && PosQ[d/c]
```

rule 320
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(Sqrt[a + b*x^2]/(a*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(
c + d*x^2)))]))*EllipticF[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; Fre
eQ[{a, b, c, d}, x] && PosQ[d/c] && PosQ[b/a] &&  !SimplerSqrtQ[b/a, d/c]
```

rule 376
```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_)
, x_Symbol] :> Simp[c*(e*x)^(m + 1)*(a + b*x^2)^(p + 1)*((c + d*x^2)^(q - 1
)/(a*e*(m + 1))), x] - Simp[1/(a*e^2*(m + 1))   Int[(e*x)^(m + 2)*(a + b*x^
2)^p*(c + d*x^2)^(q - 2)*Simp[c*(b*c - a*d)*(m + 1) + 2*c*(b*c*(p + 1) + a*
d*(q - 1)) + d*((b*c - a*d)*(m + 1) + 2*b*c*(p + q))*x^2, x], x], x] /; Fre
eQ[{a, b, c, d, e, p}, x] && NeQ[b*c - a*d, 0] && GtQ[q, 1] && LtQ[m, -1] &
& IntBinomialQ[a, b, c, d, e, m, 2, p, q, x]
```

rule 388
```
Int[(x_)^2/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol]
 :> Simp[x*(Sqrt[a + b*x^2]/(b*Sqrt[c + d*x^2])), x] - Simp[c/b   Int[Sqrt[
a + b*x^2]/(c + d*x^2)^(3/2), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
 a*d, 0] && PosQ[b/a] && PosQ[d/c] &&  !SimplerSqrtQ[b/a, d/c]
```

rule 406
```
Int[((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.)*((e_) + (f_.)*(
x_)^2), x_Symbol] :> Simp[e   Int[(a + b*x^2)^p*(c + d*x^2)^q, x], x] + Sim
p[f   Int[x^2*(a + b*x^2)^p*(c + d*x^2)^q, x], x] /; FreeQ[{a, b, c, d, e,
f, p, q}, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3667
```
Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(
p_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Simp[ff^(m + 1
)*(Sqrt[Cos[e + f*x]^2]/(f*Cos[e + f*x]))   Subst[Int[x^m*((a + b*ff^2*x^2)
^p/Sqrt[1 - ff^2*x^2]), x], x, Sin[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f,
p}, x] && IntegerQ[m/2] &&  !IntegerQ[p]
```

Maple [A] (verified)

Time = 3.42 (sec) , antiderivative size = 244, normalized size of antiderivative = 1.20

method	result
default	$-\frac{\sqrt{-\frac{b}{a}}\,ab\cosh(fx+e)^4+\left(\sqrt{-\frac{b}{a}}\,a^2-\sqrt{-\frac{b}{a}}\,ab\right)\cosh(fx+e)^2-\sinh(fx+e)\sqrt{\frac{b\cosh(fx+e)^2}{a}+\frac{a-b}{a}}\,\sqrt{\frac{\cosh(2fx+2e)}{2}+\frac{1}{2}}\,b\left(a\,\mathrm{Elliptic}\right.}{\sinh(fx+e)\sqrt{-\frac{b}{a}}}$

input
```
int(csch(f*x+e)^2*(a+b*sinh(f*x+e)^2)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-((-b/a)^(1/2)*a*b*cosh(f*x+e)^4+((-b/a)^(1/2)*a^2-(-b/a)^(1/2)*a*b)*cosh(
f*x+e)^2-sinh(f*x+e)*(b/a*cosh(f*x+e)^2+(a-b)/a)^(1/2)*(cosh(f*x+e)^2)^(1/
2)*b*(a*EllipticF(sinh(f*x+e)*(-b/a)^(1/2),(1/b*a)^(1/2))-b*EllipticF(sinh
(f*x+e)*(-b/a)^(1/2),(1/b*a)^(1/2))+EllipticE(sinh(f*x+e)*(-b/a)^(1/2),(1/
b*a)^(1/2))*a+b*EllipticE(sinh(f*x+e)*(-b/a)^(1/2),(1/b*a)^(1/2))))/sinh(f
*x+e)/(-b/a)^(1/2)/cosh(f*x+e)/(a+b*sinh(f*x+e)^2)^(1/2)/f
```

Fricas [F]

$$\int \operatorname{csch}^2(e+fx) \left(a+b\sinh^2(e+fx)\right)^{3/2} dx = \int \left(b\sinh\left(fx+e\right)^2+a\right)^{\frac{3}{2}} \operatorname{csch}\left(fx+e\right)^2 dx$$

input
```
integrate(csch(f*x+e)^2*(a+b*sinh(f*x+e)^2)^(3/2),x, algorithm="fricas")
```

output
```
integral((b*csch(f*x + e)^2*sinh(f*x + e)^2 + a*csch(f*x + e)^2)*sqrt(b*si
nh(f*x + e)^2 + a), x)
```

Sympy [F(-1)]

Timed out.

$$\int \operatorname{csch}^2(e+fx) \left(a+b\sinh^2(e+fx)\right)^{3/2} dx = \text{Timed out}$$

input
```
integrate(csch(f*x+e)**2*(a+b*sinh(f*x+e)**2)**(3/2),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \operatorname{csch}^2(e+fx)\left(a+b\sinh^2(e+fx)\right)^{3/2}\,dx=\int \left(b\sinh\left(fx+e\right)^2+a\right)^{\frac{3}{2}}\operatorname{csch}\left(fx+e\right)^2\,dx$$

input
```
integrate(csch(f*x+e)^2*(a+b*sinh(f*x+e)^2)^(3/2),x, algorithm="maxima")
```

output
```
integrate((b*sinh(f*x + e)^2 + a)^(3/2)*csch(f*x + e)^2, x)
```

Giac [F]

$$\int \operatorname{csch}^2(e+fx)\left(a+b\sinh^2(e+fx)\right)^{3/2}\,dx=\int \left(b\sinh\left(fx+e\right)^2+a\right)^{\frac{3}{2}}\operatorname{csch}\left(fx+e\right)^2\,dx$$

input
```
integrate(csch(f*x+e)^2*(a+b*sinh(f*x+e)^2)^(3/2),x, algorithm="giac")
```

output
```
integrate((b*sinh(f*x + e)^2 + a)^(3/2)*csch(f*x + e)^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int \operatorname{csch}^2(e+fx)\left(a+b\sinh^2(e+fx)\right)^{3/2}\,dx=\int \frac{\left(b\sinh(e+f\,x)^2+a\right)^{3/2}}{\sinh\left(e+f\,x\right)^2}\,dx$$

input
```
int((a + b*sinh(e + f*x)^2)^(3/2)/sinh(e + f*x)^2,x)
```

output
```
int((a + b*sinh(e + f*x)^2)^(3/2)/sinh(e + f*x)^2, x)
```

Reduce [F]

$$\int \operatorname{csch}^2(e+fx)\left(a+b\sinh^2(e+fx)\right)^{3/2}\,dx = \left(\int \sqrt{\sinh(fx+e)^2 b+a}\,\operatorname{csch}(fx+e)^2\sinh(fx+e)^2\,dx\right)b + \left(\int \sqrt{\sinh(fx+e)^2 b+a}\,\operatorname{csch}(fx+e)^2\,dx\right)a$$

input
```
int(csch(f*x+e)^2*(a+b*sinh(f*x+e)^2)^(3/2),x)
```

output
```
int(sqrt(sinh(e + f*x)**2*b + a)*csch(e + f*x)**2*sinh(e + f*x)**2,x)*b +
int(sqrt(sinh(e + f*x)**2*b + a)*csch(e + f*x)**2,x)*a
```

3.80 $\int \operatorname{csch}^4(e+fx)\left(a+b\sinh^2(e+fx)\right)^{3/2}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 239

$$\int \operatorname{csch}^4(e+fx)\left(a+b\sinh^2(e+fx)\right)^{3/2}\,dx = -\frac{a\coth(e+fx)\operatorname{csch}^2(e+fx)\sqrt{a+b\sinh^2(e+fx)}}{3f} + \frac{2(a-2b)\operatorname{csch}(e+fx)\operatorname{sech}(e+fx)\sqrt{a+b\sinh^2(e+fx)}}{3f} + \frac{2(a-2b)E\left(\arctan(\sinh(e+fx))\left|1-\frac{b}{a}\right.\right)\operatorname{sech}(e+fx)\sqrt{a+b\sinh^2(e+fx)}}{3f\sqrt{\frac{\operatorname{sech}^2(e+fx)(a+b\sinh^2(e+fx))}{a}}} - \frac{(a-3b)b\operatorname{EllipticF}\left(\arctan(\sinh(e+fx)),1-\frac{b}{a}\right)\operatorname{sech}(e+fx)\sqrt{a+b\sinh^2(e+fx)}}{3af\sqrt{\frac{\operatorname{sech}^2(e+fx)(a+b\sinh^2(e+fx))}{a}}}$$

output
```
-1/3*a*coth(f*x+e)*csch(f*x+e)^2*(a+b*sinh(f*x+e)^2)^(1/2)/f+2/3*(a-2*b)*c
sch(f*x+e)*sech(f*x+e)*(a+b*sinh(f*x+e)^2)^(1/2)/f+2/3*(a-2*b)*EllipticE(s
inh(f*x+e)/(1+sinh(f*x+e)^2)^(1/2),(1-b/a)^(1/2))*sech(f*x+e)*(a+b*sinh(f*
x+e)^2)^(1/2)/f/(sech(f*x+e)^2*(a+b*sinh(f*x+e)^2)/a)^(1/2)-1/3*(a-3*b)*b*
InverseJacobiAM(arctan(sinh(f*x+e)),(1-b/a)^(1/2))*sech(f*x+e)*(a+b*sinh(f
*x+e)^2)^(1/2)/a/f/(sech(f*x+e)^2*(a+b*sinh(f*x+e)^2)/a)^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 2.86 (sec) , antiderivative size = 213, normalized size of antiderivative = 0.89

$$\int \operatorname{csch}^4(e+fx)\left(a+b\sinh^2(e+fx)\right)^{3/2}dx=\frac{\frac{(-8a^2+13ab-6b^2+2(2a^2-7ab+4b^2)\cosh(2(e+fx))+(a-2b)b\cosh(4(e+fx)))\coth(e+fx)\operatorname{csch}^2(e+fx)}{\sqrt{2}}+4ia(a-2b)}{6f\sqrt{2a-}}$$

input
```
Integrate[Csch[e + f*x]^4*(a + b*Sinh[e + f*x]^2)^(3/2),x]
```

output
```
(((-8*a^2 + 13*a*b - 6*b^2 + 2*(2*a^2 - 7*a*b + 4*b^2)*Cosh[2*(e + f*x)] +
 (a - 2*b)*b*Cosh[4*(e + f*x)])*Coth[e + f*x]*Csch[e + f*x]^2)/Sqrt[2] + (
4*I)*a*(a - 2*b)*Sqrt[(2*a - b + b*Cosh[2*(e + f*x)])/a]*EllipticE[I*(e +
f*x), b/a] - (2*I)*(2*a^2 - 5*a*b + 3*b^2)*Sqrt[(2*a - b + b*Cosh[2*(e + f
*x)])/a]*EllipticF[I*(e + f*x), b/a])/(6*f*Sqrt[2*a - b + b*Cosh[2*(e + f*
x)]])
```

Rubi [A] (verified)

Time = 0.53 (sec) , antiderivative size = 336, normalized size of antiderivative = 1.41, number of steps used = 12, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.440, Rules used = {3042, 3667, 376, 25, 445, 25, 27, 406, 320, 388, 313}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \operatorname{csch}^4(e+fx)\left(a+b\sinh^2(e+fx)\right)^{3/2}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\left(a-b\sin(ie+ifx)^2\right)^{3/2}}{\sin(ie+ifx)^4}dx$$

$$\Big\downarrow \text{3667}$$

$$\frac{\sqrt{\cosh^2(e+fx)}\text{sech}(e+fx)\int \frac{\text{csch}^4(e+fx)\left(b\sinh^2(e+fx)+a\right)^{3/2}}{\sqrt{\sinh^2(e+fx)+1}}d\sinh(e+fx)}{f}$$

$\Big\downarrow$ 376

$$\frac{\sqrt{\cosh^2(e+fx)}\text{sech}(e+fx)\left(\frac{1}{3}\int -\frac{\text{csch}^2(e+fx)\left((a-3b)b\sinh^2(e+fx)+2a(a-2b)\right)}{\sqrt{\sinh^2(e+fx)+1}\sqrt{b\sinh^2(e+fx)+a}}d\sinh(e+fx)-\frac{1}{3}a\sqrt{\sinh^2(e+fx)}\right.}{f}$$

$\Big\downarrow$ 25

$$\frac{\sqrt{\cosh^2(e+fx)}\text{sech}(e+fx)\left(-\frac{1}{3}\int \frac{\text{csch}^2(e+fx)\left((a-3b)b\sinh^2(e+fx)+2a(a-2b)\right)}{\sqrt{\sinh^2(e+fx)+1}\sqrt{b\sinh^2(e+fx)+a}}d\sinh(e+fx)-\frac{1}{3}a\sqrt{\sinh^2(e+fx)}\right.}{f}$$

$\Big\downarrow$ 445

$$\frac{\sqrt{\cosh^2(e+fx)}\text{sech}(e+fx)\left(\frac{1}{3}\left(\frac{\int -\frac{ab\left(2(a-2b)\sinh^2(e+fx)+a-3b\right)}{\sqrt{\sinh^2(e+fx)+1}\sqrt{b\sinh^2(e+fx)+a}}d\sinh(e+fx)}{a}+2(a-2b)\sqrt{\sinh^2(e+fx)+1}\text{csch}(\right.\right.}{f}$$

$\Big\downarrow$ 25

$$\frac{\sqrt{\cosh^2(e+fx)}\text{sech}(e+fx)\left(\frac{1}{3}\left(2(a-2b)\sqrt{\sinh^2(e+fx)+1}\text{csch}(e+fx)\sqrt{a+b\sinh^2(e+fx)}-\frac{\int \frac{ab\left(2(a-2\right.}{\sqrt{\sinh^2(e+}}}{}\right.\right.}{f}$$

$\Big\downarrow$ 27

$$\frac{\sqrt{\cosh^2(e+fx)}\text{sech}(e+fx)\left(\frac{1}{3}\left(2(a-2b)\sqrt{\sinh^2(e+fx)+1}\text{csch}(e+fx)\sqrt{a+b\sinh^2(e+fx)}-b\int \frac{2(a-}{\sqrt{\sinh^2(}}\right.\right.}{f}$$

$\Big\downarrow$ 406

$$\sqrt{\cosh^2(e+fx)}\text{sech}(e+fx)\left(\frac{1}{3}\left(2(a-2b)\sqrt{\sinh^2(e+fx)+1}\text{csch}(e+fx)\sqrt{a+b\sinh^2(e+fx)}-b\left((a-3b\right.\right.\right.$$

$\big\downarrow$ 320

$$\sqrt{\cosh^2(e+fx)}\text{sech}(e+fx)\left(\frac{1}{3}\left(2(a-2b)\sqrt{\sinh^2(e+fx)+1}\text{csch}(e+fx)\sqrt{a+b\sinh^2(e+fx)}-b\left(2(a-2\right.\right.\right.$$

$\big\downarrow$ 388

$$\sqrt{\cosh^2(e+fx)}\text{sech}(e+fx)\left(\frac{1}{3}\left(2(a-2b)\sqrt{\sinh^2(e+fx)+1}\text{csch}(e+fx)\sqrt{a+b\sinh^2(e+fx)}-b\left(2(a-2\right.\right.\right.$$

$\big\downarrow$ 313

$$\sqrt{\cosh^2(e+fx)}\text{sech}(e+fx)\left(\frac{1}{3}\left(2(a-2b)\sqrt{\sinh^2(e+fx)+1}\text{csch}(e+fx)\sqrt{a+b\sinh^2(e+fx)}-b\left(\frac{(a-3b)\sqrt{}}{}\right.\right.\right.$$

input
```
Int[Csch[e + f*x]^4*(a + b*Sinh[e + f*x]^2)^(3/2),x]
```

output
```
(Sqrt[Cosh[e + f*x]^2]*Sech[e + f*x]*(-1/3*(a*Csch[e + f*x]^3*Sqrt[1 + Sin
h[e + f*x]^2]*Sqrt[a + b*Sinh[e + f*x]^2]) + (2*(a - 2*b)*Csch[e + f*x]*Sq
rt[1 + Sinh[e + f*x]^2]*Sqrt[a + b*Sinh[e + f*x]^2] - b*(((a - 3*b)*Ellipt
icF[ArcTan[Sinh[e + f*x]], 1 - b/a]*Sqrt[a + b*Sinh[e + f*x]^2])/(a*Sqrt[1
 + Sinh[e + f*x]^2]*Sqrt[(a + b*Sinh[e + f*x]^2)/(a*(1 + Sinh[e + f*x]^2))
]) + 2*(a - 2*b)*((Sinh[e + f*x]*Sqrt[a + b*Sinh[e + f*x]^2])/(b*Sqrt[1 +
Sinh[e + f*x]^2]) - (EllipticE[ArcTan[Sinh[e + f*x]], 1 - b/a]*Sqrt[a + b*
Sinh[e + f*x]^2])/(b*Sqrt[1 + Sinh[e + f*x]^2]*Sqrt[(a + b*Sinh[e + f*x]^2
)/(a*(1 + Sinh[e + f*x]^2))]))))/3))/f
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 313
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/((c_) + (d_.)*(x_)^2)^(3/2), x_Symbol] :> Sim
p[(Sqrt[a + b*x^2]/(c*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(c
+ d*x^2)))]))*EllipticE[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; FreeQ
[{a, b, c, d}, x] && PosQ[b/a] && PosQ[d/c]
```

rule 320
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(Sqrt[a + b*x^2]/(a*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(
c + d*x^2)))]))*EllipticF[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; Fre
eQ[{a, b, c, d}, x] && PosQ[d/c] && PosQ[b/a] &&  !SimplerSqrtQ[b/a, d/c]
```

rule 376
```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_)
, x_Symbol] :> Simp[c*(e*x)^(m + 1)*(a + b*x^2)^(p + 1)*((c + d*x^2)^(q - 1
)/(a*e*(m + 1))), x] - Simp[1/(a*e^2*(m + 1))   Int[(e*x)^(m + 2)*(a + b*x^
2)^p*(c + d*x^2)^(q - 2)*Simp[c*(b*c - a*d)*(m + 1) + 2*c*(b*c*(p + 1) + a*
d*(q - 1)) + d*((b*c - a*d)*(m + 1) + 2*b*c*(p + q))*x^2, x], x], x] /; Fre
eQ[{a, b, c, d, e, p}, x] && NeQ[b*c - a*d, 0] && GtQ[q, 1] && LtQ[m, -1] &
& IntBinomialQ[a, b, c, d, e, m, 2, p, q, x]
```

rule 388
```
Int[(x_)^2/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol]
 :> Simp[x*(Sqrt[a + b*x^2]/(b*Sqrt[c + d*x^2])), x] - Simp[c/b   Int[Sqrt[
a + b*x^2]/(c + d*x^2)^(3/2), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
 a*d, 0] && PosQ[b/a] && PosQ[d/c] &&  !SimplerSqrtQ[b/a, d/c]
```

rule 406
```
Int[((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.)*((e_) + (f_.)*(
x_)^2), x_Symbol] :> Simp[e   Int[(a + b*x^2)^p*(c + d*x^2)^q, x], x] + Sim
p[f   Int[x^2*(a + b*x^2)^p*(c + d*x^2)^q, x], x] /; FreeQ[{a, b, c, d, e,
f, p, q}, x]
```

rule 445
```
Int[((g_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_
.)*((e_) + (f_.)*(x_)^2), x_Symbol] :> Simp[e*(g*x)^(m + 1)*(a + b*x^2)^(p
+ 1)*((c + d*x^2)^(q + 1)/(a*c*g*(m + 1))), x] + Simp[1/(a*c*g^2*(m + 1))
 Int[(g*x)^(m + 2)*(a + b*x^2)^p*(c + d*x^2)^q*Simp[a*f*c*(m + 1) - e*(b*c
+ a*d)*(m + 2 + 1) - e*2*(b*c*p + a*d*q) - b*e*d*(m + 2*(p + q + 2) + 1)*x^
2, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p, q}, x] && LtQ[m, -1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3667
```
Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(
p_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Simp[ff^(m + 1
)*(Sqrt[Cos[e + f*x]^2]/(f*Cos[e + f*x]))   Subst[Int[x^m*((a + b*ff^2*x^2)
^p/Sqrt[1 - ff^2*x^2]), x], x, Sin[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f,
p}, x] && IntegerQ[m/2] &&  !IntegerQ[p]
```

Maple [A] (verified)

Time = 4.20 (sec) , antiderivative size = 454, normalized size of antiderivative = 1.90

method	result
default	$\frac{2\sqrt{-\frac{b}{a}}\,ab\sinh(fx+e)^6-4\sqrt{-\frac{b}{a}}\,b^2\sinh(fx+e)^6+b\sqrt{\frac{a+b\sinh(fx+e)^2}{a}}\,\sqrt{\frac{\cosh(2fx+2e)}{2}+\frac{1}{2}}\ \mathrm{EllipticF}\left(\sinh(fx+e)\sqrt{-\frac{b}{a}},\sqrt{\frac{a}{b}}\right)a\sinh}{}$

input
```
int(csch(f*x+e)^4*(a+b*sinh(f*x+e)^2)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
1/3*(2*(-b/a)^(1/2)*a*b*sinh(f*x+e)^6-4*(-b/a)^(1/2)*b^2*sinh(f*x+e)^6+b*(
(a+b*sinh(f*x+e)^2)/a)^(1/2)*(cosh(f*x+e)^2)^(1/2)*EllipticF(sinh(f*x+e)*(
-b/a)^(1/2),(1/b*a)^(1/2))*a*sinh(f*x+e)^3-((a+b*sinh(f*x+e)^2)/a)^(1/2)*(
cosh(f*x+e)^2)^(1/2)*EllipticF(sinh(f*x+e)*(-b/a)^(1/2),(1/b*a)^(1/2))*b^2
*sinh(f*x+e)^3-2*((a+b*sinh(f*x+e)^2)/a)^(1/2)*(cosh(f*x+e)^2)^(1/2)*Ellip
ticE(sinh(f*x+e)*(-b/a)^(1/2),(1/b*a)^(1/2))*a*b*sinh(f*x+e)^3+4*((a+b*sin
h(f*x+e)^2)/a)^(1/2)*(cosh(f*x+e)^2)^(1/2)*EllipticE(sinh(f*x+e)*(-b/a)^(1
/2),(1/b*a)^(1/2))*b^2*sinh(f*x+e)^3+2*(-b/a)^(1/2)*a^2*sinh(f*x+e)^4-3*(-
b/a)^(1/2)*a*b*sinh(f*x+e)^4-4*(-b/a)^(1/2)*b^2*sinh(f*x+e)^4+(-b/a)^(1/2)
*a^2*sinh(f*x+e)^2-5*(-b/a)^(1/2)*a*b*sinh(f*x+e)^2-(-b/a)^(1/2)*a^2)/sinh
(f*x+e)^3/(-b/a)^(1/2)/cosh(f*x+e)/(a+b*sinh(f*x+e)^2)^(1/2)/f
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2222 vs. $2(235) = 470$.

Time = 0.12 (sec) , antiderivative size = 2222, normalized size of antiderivative = 9.30

$$\int \operatorname{csch}^4(e+fx)\left(a+b\sinh^2(e+fx)\right)^{3/2}\,dx = \text{Too large to display}$$

input
```
integrate(csch(f*x+e)^4*(a+b*sinh(f*x+e)^2)^(3/2),x, algorithm="fricas")
```

output
```
-2/3*(((2*a^2 - 5*a*b + 2*b^2)*cosh(f*x + e)^6 + 6*(2*a^2 - 5*a*b + 2*b^2)
*cosh(f*x + e)*sinh(f*x + e)^5 + (2*a^2 - 5*a*b + 2*b^2)*sinh(f*x + e)^6 -
 3*(2*a^2 - 5*a*b + 2*b^2)*cosh(f*x + e)^4 + 3*(5*(2*a^2 - 5*a*b + 2*b^2)*
cosh(f*x + e)^2 - 2*a^2 + 5*a*b - 2*b^2)*sinh(f*x + e)^4 + 4*(5*(2*a^2 - 5
*a*b + 2*b^2)*cosh(f*x + e)^3 - 3*(2*a^2 - 5*a*b + 2*b^2)*cosh(f*x + e))*s
inh(f*x + e)^3 + 3*(2*a^2 - 5*a*b + 2*b^2)*cosh(f*x + e)^2 + 3*(5*(2*a^2 -
 5*a*b + 2*b^2)*cosh(f*x + e)^4 - 6*(2*a^2 - 5*a*b + 2*b^2)*cosh(f*x + e)^
2 + 2*a^2 - 5*a*b + 2*b^2)*sinh(f*x + e)^2 - 2*a^2 + 5*a*b - 2*b^2 + 6*((2
*a^2 - 5*a*b + 2*b^2)*cosh(f*x + e)^5 - 2*(2*a^2 - 5*a*b + 2*b^2)*cosh(f*x
 + e)^3 + (2*a^2 - 5*a*b + 2*b^2)*cosh(f*x + e))*sinh(f*x + e) - 2*((a*b -
 2*b^2)*cosh(f*x + e)^6 + 6*(a*b - 2*b^2)*cosh(f*x + e)*sinh(f*x + e)^5 +
(a*b - 2*b^2)*sinh(f*x + e)^6 - 3*(a*b - 2*b^2)*cosh(f*x + e)^4 + 3*(5*(a*
b - 2*b^2)*cosh(f*x + e)^2 - a*b + 2*b^2)*sinh(f*x + e)^4 + 4*(5*(a*b - 2*
b^2)*cosh(f*x + e)^3 - 3*(a*b - 2*b^2)*cosh(f*x + e))*sinh(f*x + e)^3 + 3*
(a*b - 2*b^2)*cosh(f*x + e)^2 + 3*(5*(a*b - 2*b^2)*cosh(f*x + e)^4 - 6*(a*
b - 2*b^2)*cosh(f*x + e)^2 + a*b - 2*b^2)*sinh(f*x + e)^2 - a*b + 2*b^2 +
6*((a*b - 2*b^2)*cosh(f*x + e)^5 - 2*(a*b - 2*b^2)*cosh(f*x + e)^3 + (a*b
- 2*b^2)*cosh(f*x + e))*sinh(f*x + e))*sqrt((a^2 - a*b)/b^2))*sqrt(b)*sqrt
((2*b*sqrt((a^2 - a*b)/b^2) - 2*a + b)/b)*elliptic_e(arcsin(sqrt((2*b*sqrt
((a^2 - a*b)/b^2) - 2*a + b)/b)*(cosh(f*x + e) + sinh(f*x + e))), (8*a^...
```

Sympy [F(-1)]

Timed out.

$$\int \operatorname{csch}^4(e+fx)\left(a+b\sinh^2(e+fx)\right)^{3/2}\,dx=\text{Timed out}$$

input
```
integrate(csch(f*x+e)**4*(a+b*sinh(f*x+e)**2)**(3/2),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \operatorname{csch}^4(e+fx) \left(a+b\sinh^2(e+fx)\right)^{3/2} dx = \int \left(b\sinh\left(fx+e\right)^2+a\right)^{\frac{3}{2}} \operatorname{csch}\left(fx+e\right)^4 dx$$

input
```
integrate(csch(f*x+e)^4*(a+b*sinh(f*x+e)^2)^(3/2),x, algorithm="maxima")
```

output
```
integrate((b*sinh(f*x + e)^2 + a)^(3/2)*csch(f*x + e)^4, x)
```

Giac [F]

$$\int \operatorname{csch}^4(e+fx) \left(a+b\sinh^2(e+fx)\right)^{3/2} dx = \int \left(b\sinh\left(fx+e\right)^2+a\right)^{\frac{3}{2}} \operatorname{csch}\left(fx+e\right)^4 dx$$

input
```
integrate(csch(f*x+e)^4*(a+b*sinh(f*x+e)^2)^(3/2),x, algorithm="giac")
```

output
```
integrate((b*sinh(f*x + e)^2 + a)^(3/2)*csch(f*x + e)^4, x)
```

Mupad [F(-1)]

Timed out.

$$\int \operatorname{csch}^4(e+fx) \left(a+b\sinh^2(e+fx)\right)^{3/2} dx = \int \frac{\left(b\sinh(e+f\,x)^2+a\right)^{3/2}}{\sinh\left(e+f\,x\right)^4} \,dx$$

input
```
int((a + b*sinh(e + f*x)^2)^(3/2)/sinh(e + f*x)^4,x)
```

output
```
int((a + b*sinh(e + f*x)^2)^(3/2)/sinh(e + f*x)^4, x)
```

Reduce [F]

$$\int \operatorname{csch}^4(e+fx)\left(a+b\sinh^2(e+fx)\right)^{3/2}\,dx = \left(\int \sqrt{\sinh(fx+e)^2 b+a}\,\operatorname{csch}(fx+e)^4\sinh(fx+e)^2\,dx\right)b + \left(\int \sqrt{\sinh(fx+e)^2 b+a}\,\operatorname{csch}(fx+e)^4\,dx\right)a$$

input
```
int(csch(f*x+e)^4*(a+b*sinh(f*x+e)^2)^(3/2),x)
```

output
```
int(sqrt(sinh(e + f*x)**2*b + a)*csch(e + f*x)**4*sinh(e + f*x)**2,x)*b +
int(sqrt(sinh(e + f*x)**2*b + a)*csch(e + f*x)**4,x)*a
```

3.81 $\int \frac{\sinh^3(e+fx)}{\sqrt{a+b\sinh^2(e+fx)}}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 83

$$\int \frac{\sinh^3(e+fx)}{\sqrt{a+b\sinh^2(e+fx)}}\,dx=-\frac{(a+b)\text{arctanh}\left(\frac{\sqrt{b}\cosh(e+fx)}{\sqrt{a-b+b\cosh^2(e+fx)}}\right)}{2b^{3/2}f}+\frac{\cosh(e+fx)\sqrt{a-b+b\cosh^2(e+fx)}}{2bf}$$

output
```
-1/2*(a+b)*arctanh(b^(1/2)*cosh(f*x+e)/(a-b+b*cosh(f*x+e)^2)^(1/2))/b^(3/2
)/f+1/2*cosh(f*x+e)*(a-b+b*cosh(f*x+e)^2)^(1/2)/b/f
```

Mathematica [A] (verified)

Time = 0.43 (sec) , antiderivative size = 98, normalized size of antiderivative = 1.18

$$\int \frac{\sinh^3(e+fx)}{\sqrt{a+b\sinh^2(e+fx)}}\,dx = \frac{\cosh(e+fx)\sqrt{2a-b+b\cosh(2(e+fx))}}{2\sqrt{2}bf} - \frac{(a+b)\log\left(\sqrt{2}\sqrt{b}\cosh(e+fx)+\sqrt{2a-b+b\cosh(2(e+fx))}\right)}{2b^{3/2}f}$$

input
```
Integrate[Sinh[e + f*x]^3/Sqrt[a + b*Sinh[e + f*x]^2],x]
```

output
```
(Cosh[e + f*x]*Sqrt[2*a - b + b*Cosh[2*(e + f*x)]])/(2*Sqrt[2]*b*f) - ((a
+ b)*Log[Sqrt[2]*Sqrt[b]*Cosh[e + f*x] + Sqrt[2*a - b + b*Cosh[2*(e + f*x)
]]])/(2*b^(3/2)*f)
```

Rubi [A] (verified)

Time = 0.32 (sec) , antiderivative size = 82, normalized size of antiderivative = 0.99, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.240$, Rules used = {3042, 26, 3665, 299, 224, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sinh^3(e+fx)}{\sqrt{a+b\sinh^2(e+fx)}}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{i\sin(ie+ifx)^3}{\sqrt{a-b\sin(ie+ifx)^2}}dx$$

$$\Big\downarrow \text{26}$$

$$i\int \frac{\sin(ie+ifx)^3}{\sqrt{a-b\sin(ie+ifx)^2}}dx$$

$$\Big\downarrow \text{3665}$$

$$-\frac{\int \frac{1-\cosh^2(e+fx)}{\sqrt{b\cosh^2(e+fx)+a-b}}d\cosh(e+fx)}{f}$$

$$\Big\downarrow \text{299}$$

$$-\frac{\frac{(a+b)\int \frac{1}{\sqrt{b\cosh^2(e+fx)+a-b}}d\cosh(e+fx)}{2b}-\frac{\cosh(e+fx)\sqrt{a+b\cosh^2(e+fx)-b}}{2b}}{f}$$

$$\Big\downarrow \text{224}$$

$$-\frac{\frac{(a+b)\int \frac{1}{1-\frac{b\cosh^2(e+fx)}{b\cosh^2(e+fx)+a-b}}d\frac{\cosh(e+fx)}{\sqrt{b\cosh^2(e+fx)+a-b}}}{2b}-\frac{\cosh(e+fx)\sqrt{a+b\cosh^2(e+fx)-b}}{2b}}{f}$$

$$\Big\downarrow \text{219}$$

$$-\frac{\frac{(a+b)\text{arctanh}\left(\frac{\sqrt{b}\cosh(e+fx)}{\sqrt{a+b\cosh^2(e+fx)-b}}\right)}{2b^{3/2}}-\frac{\cosh(e+fx)\sqrt{a+b\cosh^2(e+fx)-b}}{2b}}{f}$$

input
```
Int[Sinh[e + f*x]^3/Sqrt[a + b*Sinh[e + f*x]^2],x]
```

output
```
-((((a + b)*ArcTanh[(Sqrt[b]*Cosh[e + f*x])/Sqrt[a - b + b*Cosh[e + f*x]^2
]])/(2*b^(3/2)) - (Cosh[e + f*x]*Sqrt[a - b + b*Cosh[e + f*x]^2])/(2*b))/f
)
```

Defintions of rubi rules used

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

rule 299
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2), x_Symbol] :> Simp[d*x
*((a + b*x^2)^(p + 1)/(b*(2*p + 3))), x] - Simp[(a*d - b*c*(2*p + 3))/(b*(2
*p + 3))   Int[(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
 a*d, 0] && NeQ[2*p + 3, 0]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3665
```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^
(p_.), x_Symbol] :> With[{ff = FreeFactors[Cos[e + f*x], x]}, Simp[-ff/f
Subst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a + b - b*ff^2*x^2)^p, x], x, Cos[e +
 f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m - 1)/2]
```

Maple [A] (verified)

Time = 0.31 (sec) , antiderivative size = 133, normalized size of antiderivative = 1.60

method	result
default	$\frac{\sqrt{\left(a+b\sinh\left(fx+e\right)^{2}\right)\cosh\left(fx+e\right)^{2}}\left(\frac{\sqrt{\left(a+b\sinh\left(fx+e\right)^{2}\right)\cosh\left(fx+e\right)^{2}}}{2b}-\frac{\left(a+b\right)\ln\left(\frac{\frac{a}{2}+\frac{b}{2}+b\sinh\left(fx+e\right)^{2}}{\sqrt{b}}+\sqrt{\left(a+b\sinh\left(fx+e\right)^{2}\right)\cosh\left(fx-\right.}\right.}{4b^{\frac{3}{2}}}\right.}{\cosh\left(fx+e\right)\sqrt{a+b\sinh\left(fx+e\right)^{2}}\,f}$

input
```
int(sinh(f*x+e)^3/(a+b*sinh(f*x+e)^2)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
((a+b*sinh(f*x+e)^2)*cosh(f*x+e)^2)^(1/2)*(1/2/b*((a+b*sinh(f*x+e)^2)*cosh
(f*x+e)^2)^(1/2)-1/4*(a+b)/b^(3/2)*ln((1/2*a+1/2*b+b*sinh(f*x+e)^2)/b^(1/2
)+((a+b*sinh(f*x+e)^2)*cosh(f*x+e)^2)^(1/2)))/cosh(f*x+e)/(a+b*sinh(f*x+e)
^2)^(1/2)/f
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 720 vs. $2(71) = 142$.

Time = 0.16 (sec) , antiderivative size = 2116, normalized size of antiderivative = 25.49

$$\int \frac{\sinh^3(e+fx)}{\sqrt{a+b\sinh^2(e+fx)}}\,dx = \text{Too large to display}$$

input
```
integrate(sinh(f*x+e)^3/(a+b*sinh(f*x+e)^2)^(1/2),x, algorithm="fricas")
```

output
```
[1/8*(((a + b)*cosh(f*x + e)^2 + 2*(a + b)*cosh(f*x + e)*sinh(f*x + e) + (
a + b)*sinh(f*x + e)^2)*sqrt(b)*log((a^2*b*cosh(f*x + e)^8 + 8*a^2*b*cosh(
f*x + e)*sinh(f*x + e)^7 + a^2*b*sinh(f*x + e)^8 + 2*(a^3 + a^2*b)*cosh(f*
x + e)^6 + 2*(14*a^2*b*cosh(f*x + e)^2 + a^3 + a^2*b)*sinh(f*x + e)^6 + 4*
(14*a^2*b*cosh(f*x + e)^3 + 3*(a^3 + a^2*b)*cosh(f*x + e))*sinh(f*x + e)^5
 + (9*a^2*b - 4*a*b^2 + b^3)*cosh(f*x + e)^4 + (70*a^2*b*cosh(f*x + e)^4 +
 9*a^2*b - 4*a*b^2 + b^3 + 30*(a^3 + a^2*b)*cosh(f*x + e)^2)*sinh(f*x + e)
^4 + 4*(14*a^2*b*cosh(f*x + e)^5 + 10*(a^3 + a^2*b)*cosh(f*x + e)^3 + (9*a
^2*b - 4*a*b^2 + b^3)*cosh(f*x + e))*sinh(f*x + e)^3 + b^3 + 2*(3*a*b^2 -
b^3)*cosh(f*x + e)^2 + 2*(14*a^2*b*cosh(f*x + e)^6 + 15*(a^3 + a^2*b)*cosh
(f*x + e)^4 + 3*a*b^2 - b^3 + 3*(9*a^2*b - 4*a*b^2 + b^3)*cosh(f*x + e)^2)
*sinh(f*x + e)^2 - sqrt(2)*(a^2*cosh(f*x + e)^6 + 6*a^2*cosh(f*x + e)*sinh
(f*x + e)^5 + a^2*sinh(f*x + e)^6 + 3*a^2*cosh(f*x + e)^4 + 3*(5*a^2*cosh(
f*x + e)^2 + a^2)*sinh(f*x + e)^4 + 4*(5*a^2*cosh(f*x + e)^3 + 3*a^2*cosh(
f*x + e))*sinh(f*x + e)^3 + (4*a*b - b^2)*cosh(f*x + e)^2 + (15*a^2*cosh(f
*x + e)^4 + 18*a^2*cosh(f*x + e)^2 + 4*a*b - b^2)*sinh(f*x + e)^2 + b^2 +
2*(3*a^2*cosh(f*x + e)^5 + 6*a^2*cosh(f*x + e)^3 + (4*a*b - b^2)*cosh(f*x
+ e))*sinh(f*x + e))*sqrt(b)*sqrt((b*cosh(f*x + e)^2 + b*sinh(f*x + e)^2 +
 2*a - b)/(cosh(f*x + e)^2 - 2*cosh(f*x + e)*sinh(f*x + e) + sinh(f*x + e)
^2)) + 4*(2*a^2*b*cosh(f*x + e)^7 + 3*(a^3 + a^2*b)*cosh(f*x + e)^5 + (...
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\sinh^3(e+fx)}{\sqrt{a+b\sinh^2(e+fx)}}\,dx = \text{Timed out}$$

input
```
integrate(sinh(f*x+e)**3/(a+b*sinh(f*x+e)**2)**(1/2),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{\sinh^3(e+fx)}{\sqrt{a+b\sinh^2(e+fx)}}\,dx = \int \frac{\sinh(fx+e)^3}{\sqrt{b\sinh(fx+e)^2+a}}\,dx$$

input
```
integrate(sinh(f*x+e)^3/(a+b*sinh(f*x+e)^2)^(1/2),x, algorithm="maxima")
```

output
```
integrate(sinh(f*x + e)^3/sqrt(b*sinh(f*x + e)^2 + a), x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 430 vs. 2(71) = 142.

Time = 0.20 (sec) , antiderivative size = 430, normalized size of antiderivative = 5.18

$$\int \frac{\sinh^3(e+fx)}{\sqrt{a+b\sinh^2(e+fx)}}\,dx$$
$$= \frac{\left(\frac{4\left(ae^{(2e)}+be^{(2e)}\right)\arctan\left(-\frac{\sqrt{b}e^{(2fx+2e)}-\sqrt{be^{(4fx+4e)}+4ae^{(2fx+2e)}-2be^{(2fx+2e)}+b}}{\sqrt{-b}}\right)}{\sqrt{-b}b}+\frac{\sqrt{be^{(4fx+4e)}+4ae^{(2fx+2e)}-2be^{(2fx+2e)}+\cdots}}{b}\cdots\right.}{\cdots}$$

input
```
integrate(sinh(f*x+e)^3/(a+b*sinh(f*x+e)^2)^(1/2),x, algorithm="giac")
```

output
```
1/8*(4*(a*e^(2*e) + b*e^(2*e))*arctan(-(sqrt(b)*e^(2*f*x + 2*e) - sqrt(b*e
^(4*f*x + 4*e) + 4*a*e^(2*f*x + 2*e) - 2*b*e^(2*f*x + 2*e) + b))/sqrt(-b))
/(sqrt(-b)*b) + sqrt(b*e^(4*f*x + 4*e) + 4*a*e^(2*f*x + 2*e) - 2*b*e^(2*f*
x + 2*e) + b)*e^(2*e)/b + 2*(a*e^(2*e) + b*e^(2*e))*log(abs((sqrt(b)*e^(2*
f*x + 2*e) - sqrt(b*e^(4*f*x + 4*e) + 4*a*e^(2*f*x + 2*e) - 2*b*e^(2*f*x +
 2*e) + b))*sqrt(b) + 2*a - b))/b^(3/2) - 2*(2*(sqrt(b)*e^(2*f*x + 2*e) -
sqrt(b*e^(4*f*x + 4*e) + 4*a*e^(2*f*x + 2*e) - 2*b*e^(2*f*x + 2*e) + b))*a
*e^(2*e) - (sqrt(b)*e^(2*f*x + 2*e) - sqrt(b*e^(4*f*x + 4*e) + 4*a*e^(2*f*
x + 2*e) - 2*b*e^(2*f*x + 2*e) + b))*b*e^(2*e) + b^(3/2)*e^(2*e))/(((sqrt(
b)*e^(2*f*x + 2*e) - sqrt(b*e^(4*f*x + 4*e) + 4*a*e^(2*f*x + 2*e) - 2*b*e^
(2*f*x + 2*e) + b))^2 - b)*b))*e^(-2*e)/f
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sinh^3(e+fx)}{\sqrt{a+b\sinh^2(e+fx)}}\,dx = \int \frac{\sinh(e+f\,x)^3}{\sqrt{b\sinh\left(e+f\,x\right)^2+a}}\,dx$$

input
```
int(sinh(e + f*x)^3/(a + b*sinh(e + f*x)^2)^(1/2),x)
```

output
```
int(sinh(e + f*x)^3/(a + b*sinh(e + f*x)^2)^(1/2), x)
```

Reduce [F]

$$\int \frac{\sinh^3(e+fx)}{\sqrt{a+b\sinh^2(e+fx)}}\,dx = \int \frac{\sqrt{\sinh\left(fx+e\right)^2 b+a}\,\sinh\left(fx+e\right)^3}{\sinh\left(fx+e\right)^2 b+a}dx$$

input
```
int(sinh(f*x+e)^3/(a+b*sinh(f*x+e)^2)^(1/2),x)
```

output
```
int((sqrt(sinh(e + f*x)**2*b + a)*sinh(e + f*x)**3)/(sinh(e + f*x)**2*b +
a),x)
```

3.82 $\int \frac{\sinh(e+fx)}{\sqrt{a+b\sinh^2(e+fx)}}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 41

$$\int \frac{\sinh(e+fx)}{\sqrt{a+b\sinh^2(e+fx)}}\,dx=\frac{\operatorname{arctanh}\left(\frac{\sqrt{b}\cosh(e+fx)}{\sqrt{a-b+b\cosh^2(e+fx)}}\right)}{\sqrt{b}f}$$

output
```
arctanh(b^(1/2)*cosh(f*x+e)/(a-b+b*cosh(f*x+e)^2)^(1/2))/b^(1/2)/f
```

Mathematica [A] (verified)

Time = 0.19 (sec) , antiderivative size = 49, normalized size of antiderivative = 1.20

$$\int \frac{\sinh(e+fx)}{\sqrt{a+b\sinh^2(e+fx)}}\,dx=\frac{\log\left(\sqrt{2}\sqrt{b}\cosh(e+fx)+\sqrt{2a-b+b\cosh(2(e+fx))}\right)}{\sqrt{b}f}$$

input
```
Integrate[Sinh[e + f*x]/Sqrt[a + b*Sinh[e + f*x]^2],x]
```

output
```
Log[Sqrt[2]*Sqrt[b]*Cosh[e + f*x] + Sqrt[2*a - b + b*Cosh[2*(e + f*x)]]]/(
Sqrt[b]*f)
```

Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 41, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.217$, Rules used = {3042, 26, 3665, 224, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sinh(e+fx)}{\sqrt{a+b\sinh^2(e+fx)}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int -\frac{i\sin(ie+ifx)}{\sqrt{a-b\sin(ie+ifx)^2}}dx$$

$$\Big\downarrow \text{26}$$

$$-i\int \frac{\sin(ie+ifx)}{\sqrt{a-b\sin(ie+ifx)^2}}dx$$

$$\Big\downarrow \text{3665}$$

$$\frac{\int \frac{1}{\sqrt{b\cosh^2(e+fx)+a-b}}d\cosh(e+fx)}{f}$$

$$\Big\downarrow \text{224}$$

$$\frac{\int \frac{1}{1-\frac{b\cosh^2(e+fx)}{b\cosh^2(e+fx)+a-b}}d\frac{\cosh(e+fx)}{\sqrt{b\cosh^2(e+fx)+a-b}}}{f}$$

$$\Big\downarrow \text{219}$$

$$\frac{\text{arctanh}\left(\frac{\sqrt{b}\cosh(e+fx)}{\sqrt{a+b\cosh^2(e+fx)-b}}\right)}{\sqrt{b}f}$$

input
```
Int[Sinh[e + f*x]/Sqrt[a + b*Sinh[e + f*x]^2],x]
```

output
```
ArcTanh[(Sqrt[b]*Cosh[e + f*x])/Sqrt[a - b + b*Cosh[e + f*x]^2]]/(Sqrt[b]*
f)
```

Defintions of rubi rules used

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3665
```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^
(p_.), x_Symbol] :> With[{ff = FreeFactors[Cos[e + f*x], x]}, Simp[-ff/f
Subst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a + b - b*ff^2*x^2)^p, x], x, Cos[e +
 f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m - 1)/2]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 107 vs. $2(35) = 70$.

Time = 0.19 (sec) , antiderivative size = 108, normalized size of antiderivative = 2.63

method	result	size
default	$\frac{\sqrt{\left(a+b\sinh(fx+e)^2\right)\cosh(fx+e)^2}\,\ln\left(\frac{2b\cosh(fx+e)^2+2\sqrt{b\cosh(fx+e)^4+(a-b)\cosh(fx+e)^2}\,\sqrt{b}+a-b}{2\sqrt{b}}\right)}{2\sqrt{b}\,\cosh(fx+e)\sqrt{a+b\sinh(fx+e)^2}\,f}$	108

input
```
int(sinh(f*x+e)/(a+b*sinh(f*x+e)^2)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/2*((a+b*sinh(f*x+e)^2)*cosh(f*x+e)^2)^(1/2)*ln(1/2*(2*b*cosh(f*x+e)^2+2*
(b*cosh(f*x+e)^4+(a-b)*cosh(f*x+e)^2)^(1/2)*b^(1/2)+a-b)/b^(1/2))/b^(1/2)/
cosh(f*x+e)/(a+b*sinh(f*x+e)^2)^(1/2)/f
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 489 vs. $2(35) = 70$.

Time = 0.13 (sec) , antiderivative size = 1654, normalized size of antiderivative = 40.34

$$\int \frac{\sinh(e+fx)}{\sqrt{a+b\sinh^2(e+fx)}}\,dx = \text{Too large to display}$$

input
```
integrate(sinh(f*x+e)/(a+b*sinh(f*x+e)^2)^(1/2),x, algorithm="fricas")
```

output
```
[1/4*(sqrt(b)*log((a^2*b*cosh(f*x + e)^8 + 8*a^2*b*cosh(f*x + e)*sinh(f*x
+ e)^7 + a^2*b*sinh(f*x + e)^8 + 2*(a^3 + a^2*b)*cosh(f*x + e)^6 + 2*(14*a
^2*b*cosh(f*x + e)^2 + a^3 + a^2*b)*sinh(f*x + e)^6 + 4*(14*a^2*b*cosh(f*x
 + e)^3 + 3*(a^3 + a^2*b)*cosh(f*x + e))*sinh(f*x + e)^5 + (9*a^2*b - 4*a*
b^2 + b^3)*cosh(f*x + e)^4 + (70*a^2*b*cosh(f*x + e)^4 + 9*a^2*b - 4*a*b^2
 + b^3 + 30*(a^3 + a^2*b)*cosh(f*x + e)^2)*sinh(f*x + e)^4 + 4*(14*a^2*b*c
osh(f*x + e)^5 + 10*(a^3 + a^2*b)*cosh(f*x + e)^3 + (9*a^2*b - 4*a*b^2 + b
^3)*cosh(f*x + e))*sinh(f*x + e)^3 + b^3 + 2*(3*a*b^2 - b^3)*cosh(f*x + e)
^2 + 2*(14*a^2*b*cosh(f*x + e)^6 + 15*(a^3 + a^2*b)*cosh(f*x + e)^4 + 3*a*
b^2 - b^3 + 3*(9*a^2*b - 4*a*b^2 + b^3)*cosh(f*x + e)^2)*sinh(f*x + e)^2 +
 sqrt(2)*(a^2*cosh(f*x + e)^6 + 6*a^2*cosh(f*x + e)*sinh(f*x + e)^5 + a^2*
sinh(f*x + e)^6 + 3*a^2*cosh(f*x + e)^4 + 3*(5*a^2*cosh(f*x + e)^2 + a^2)*
sinh(f*x + e)^4 + 4*(5*a^2*cosh(f*x + e)^3 + 3*a^2*cosh(f*x + e))*sinh(f*x
 + e)^3 + (4*a*b - b^2)*cosh(f*x + e)^2 + (15*a^2*cosh(f*x + e)^4 + 18*a^2
*cosh(f*x + e)^2 + 4*a*b - b^2)*sinh(f*x + e)^2 + b^2 + 2*(3*a^2*cosh(f*x
+ e)^5 + 6*a^2*cosh(f*x + e)^3 + (4*a*b - b^2)*cosh(f*x + e))*sinh(f*x + e
))*sqrt(b)*sqrt((b*cosh(f*x + e)^2 + b*sinh(f*x + e)^2 + 2*a - b)/(cosh(f*
x + e)^2 - 2*cosh(f*x + e)*sinh(f*x + e) + sinh(f*x + e)^2)) + 4*(2*a^2*b*
cosh(f*x + e)^7 + 3*(a^3 + a^2*b)*cosh(f*x + e)^5 + (9*a^2*b - 4*a*b^2 + b
^3)*cosh(f*x + e)^3 + (3*a*b^2 - b^3)*cosh(f*x + e))*sinh(f*x + e))/(co...
```

Sympy [F]

$$\int \frac{\sinh(e+fx)}{\sqrt{a+b\sinh^2(e+fx)}}\,dx = \int \frac{\sinh{(e+fx)}}{\sqrt{a+b\sinh^2{(e+fx)}}}\,dx$$

input
```
integrate(sinh(f*x+e)/(a+b*sinh(f*x+e)**2)**(1/2),x)
```

output
```
Integral(sinh(e + f*x)/sqrt(a + b*sinh(e + f*x)**2), x)
```

Maxima [F]

$$\int \frac{\sinh(e+fx)}{\sqrt{a+b\sinh^2(e+fx)}}\,dx=\int \frac{\sinh(fx+e)}{\sqrt{b\sinh(fx+e)^2+a}}\,dx$$

input
```
integrate(sinh(f*x+e)/(a+b*sinh(f*x+e)^2)^(1/2),x, algorithm="maxima")
```

output
```
integrate(sinh(f*x + e)/sqrt(b*sinh(f*x + e)^2 + a), x)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{\sinh(e+fx)}{\sqrt{a+b\sinh^2(e+fx)}}\,dx=\text{Exception raised: TypeError}$$

input
```
integrate(sinh(f*x+e)/(a+b*sinh(f*x+e)^2)^(1/2),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Error: Bad Argument Type
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sinh(e+fx)}{\sqrt{a+b\sinh^2(e+fx)}}\,dx=\int \frac{\sinh(e+f\,x)}{\sqrt{b\sinh(e+f\,x)^2+a}}\,dx$$

input
```
int(sinh(e + f*x)/(a + b*sinh(e + f*x)^2)^(1/2),x)
```

output
```
int(sinh(e + f*x)/(a + b*sinh(e + f*x)^2)^(1/2), x)
```

Reduce [F]

$$\int \frac{\sinh(e+fx)}{\sqrt{a+b\sinh^2(e+fx)}}\,dx = \int \frac{\sqrt{\sinh\left(fx+e\right)^{2}b+a}\,\sinh\left(fx+e\right)}{\sinh\left(fx+e\right)^{2}b+a}dx$$

input
```
int(sinh(f*x+e)/(a+b*sinh(f*x+e)^2)^(1/2),x)
```

output
```
int((sqrt(sinh(e + f*x)**2*b + a)*sinh(e + f*x))/(sinh(e + f*x)**2*b + a),
x)
```

3.83 $\int \frac{\operatorname{csch}(e+fx)}{\sqrt{a+b\sinh^2(e+fx)}}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 42

$$\int \frac{\operatorname{csch}(e+fx)}{\sqrt{a+b\sinh^2(e+fx)}}\,dx = -\frac{\operatorname{arctanh}\left(\frac{\sqrt{a}\cosh(e+fx)}{\sqrt{a-b+b\cosh^2(e+fx)}}\right)}{\sqrt{a}f}$$

output
```
-arctanh(a^(1/2)*cosh(f*x+e)/(a-b+b*cosh(f*x+e)^2)^(1/2))/a^(1/2)/f
```

Mathematica [A] (verified)

Time = 0.30 (sec) , antiderivative size = 49, normalized size of antiderivative = 1.17

$$\int \frac{\operatorname{csch}(e+fx)}{\sqrt{a+b\sinh^2(e+fx)}}\,dx = -\frac{\operatorname{arctanh}\left(\frac{\sqrt{2}\sqrt{a}\cosh(e+fx)}{\sqrt{2a-b+b\cosh(2(e+fx))}}\right)}{\sqrt{a}f}$$

input
```
Integrate[Csch[e + f*x]/Sqrt[a + b*Sinh[e + f*x]^2],x]
```

output
```
-(ArcTanh[(Sqrt[2]*Sqrt[a]*Cosh[e + f*x])/Sqrt[2*a - b + b*Cosh[2*(e + f*x
)]]]/(Sqrt[a]*f))
```

Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 42, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.217, Rules used = {3042, 26, 3665, 291, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\operatorname{csch}(e+fx)}{\sqrt{a+b\sinh^2(e+fx)}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{i}{\sin(ie+ifx)\sqrt{a-b\sin(ie+ifx)^2}}dx$$

$$\Big\downarrow \text{26}$$

$$i\int \frac{1}{\sin(ie+ifx)\sqrt{a-b\sin(ie+ifx)^2}}dx$$

$$\Big\downarrow \text{3665}$$

$$-\frac{\int \frac{1}{(1-\cosh^2(e+fx))\sqrt{b\cosh^2(e+fx)+a-b}}d\cosh(e+fx)}{f}$$

$$\Big\downarrow \text{291}$$

$$-\frac{\int \frac{1}{1-\frac{a\cosh^2(e+fx)}{b\cosh^2(e+fx)+a-b}}d\frac{\cosh(e+fx)}{\sqrt{b\cosh^2(e+fx)+a-b}}}{f}$$

$$\Big\downarrow \text{219}$$

$$-\frac{\operatorname{arctanh}\left(\frac{\sqrt{a}\cosh(e+fx)}{\sqrt{a+b\cosh^2(e+fx)-b}}\right)}{\sqrt{a}f}$$

input
```
Int[Csch[e + f*x]/Sqrt[a + b*Sinh[e + f*x]^2],x]
```

output
```
-(ArcTanh[(Sqrt[a]*Cosh[e + f*x])/Sqrt[a - b + b*Cosh[e + f*x]^2]]/(Sqrt[a
]*f))
```

Defintions of rubi rules used

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 291
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*((c_) + (d_.)*(x_)^2)), x_Symbol] :> Subst
[Int[1/(c - (b*c - a*d)*x^2), x], x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b, c,
d}, x] && NeQ[b*c - a*d, 0]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3665
```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^
(p_.), x_Symbol] :> With[{ff = FreeFactors[Cos[e + f*x], x]}, Simp[-ff/f
Subst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a + b - b*ff^2*x^2)^p, x], x, Cos[e +
 f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m - 1)/2]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 112 vs. 2(36) = 72.

Time = 0.29 (sec) , antiderivative size = 113, normalized size of antiderivative = 2.69

method	result	size
default	$-\frac{\sqrt{\left(a+b\sinh(fx+e)^2\right)\cosh(fx+e)^2}\,\ln\left(\frac{(a+b)\cosh(fx+e)^2+2\sqrt{a}\,\sqrt{b\cosh(fx+e)^4+(a-b)\cosh(fx+e)^2}+a-b}{\sinh(fx+e)^2}\right)}{2\sqrt{a}\,\cosh(fx+e)\sqrt{a+b\sinh(fx+e)^2}\,f}$	113

input
```
int(csch(f*x+e)/(a+b*sinh(f*x+e)^2)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-1/2*((a+b*sinh(f*x+e)^2)*cosh(f*x+e)^2)^(1/2)/a^(1/2)*ln(((a+b)*cosh(f*x+
e)^2+2*a^(1/2)*(b*cosh(f*x+e)^4+(a-b)*cosh(f*x+e)^2)^(1/2)+a-b)/sinh(f*x+e
)^2)/cosh(f*x+e)/(a+b*sinh(f*x+e)^2)^(1/2)/f
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 124 vs. 2(36) = 72.

Time = 0.09 (sec) , antiderivative size = 460, normalized size of antiderivative = 10.95

$$\int \frac{\operatorname{csch}(e+fx)}{\sqrt{a+b\sinh^2(e+fx)}}\,dx = \left[\frac{\log\left(-\frac{(a+b)\cosh(fx+e)^4+4\,(a+b)\cosh(fx+e)\sinh(fx+e)^3+(a+b)\sinh(fx+e)^4+2\,(3\,a-b)\cosh(fx+e)^2+2\,\left(3\,(a+b)\cosh(fx+e)^2+3\,a\right.}{\cosh(fx+e)^4+4\,\cosh(fx+e)\sinh(fx+e)^3}\right.}{}\right.$$

input
```
integrate(csch(f*x+e)/(a+b*sinh(f*x+e)^2)^(1/2),x, algorithm="fricas")
```

output
```
[1/2*log(-((a + b)*cosh(f*x + e)^4 + 4*(a + b)*cosh(f*x + e)*sinh(f*x + e)
^3 + (a + b)*sinh(f*x + e)^4 + 2*(3*a - b)*cosh(f*x + e)^2 + 2*(3*(a + b)*
cosh(f*x + e)^2 + 3*a - b)*sinh(f*x + e)^2 - 2*sqrt(2)*(cosh(f*x + e)^2 +
2*cosh(f*x + e)*sinh(f*x + e) + sinh(f*x + e)^2 + 1)*sqrt(a)*sqrt((b*cosh(
f*x + e)^2 + b*sinh(f*x + e)^2 + 2*a - b)/(cosh(f*x + e)^2 - 2*cosh(f*x +
e)*sinh(f*x + e) + sinh(f*x + e)^2)) + 4*((a + b)*cosh(f*x + e)^3 + (3*a -
 b)*cosh(f*x + e))*sinh(f*x + e) + a + b)/(cosh(f*x + e)^4 + 4*cosh(f*x +
e)*sinh(f*x + e)^3 + sinh(f*x + e)^4 + 2*(3*cosh(f*x + e)^2 - 1)*sinh(f*x
+ e)^2 - 2*cosh(f*x + e)^2 + 4*(cosh(f*x + e)^3 - cosh(f*x + e))*sinh(f*x
+ e) + 1))/(sqrt(a)*f), sqrt(-a)*arctan(sqrt(2)*sqrt(-a)*sqrt((b*cosh(f*x
+ e)^2 + b*sinh(f*x + e)^2 + 2*a - b)/(cosh(f*x + e)^2 - 2*cosh(f*x + e)*s
inh(f*x + e) + sinh(f*x + e)^2))/(a*cosh(f*x + e)^2 + 2*a*cosh(f*x + e)*si
nh(f*x + e) + a*sinh(f*x + e)^2 + a))/(a*f)]
```

Sympy [F]

$$\int \frac{\operatorname{csch}(e+fx)}{\sqrt{a+b\sinh^2(e+fx)}}\,dx = \int \frac{\operatorname{csch}(e+fx)}{\sqrt{a+b\sinh^2(e+fx)}}\,dx$$

input
```
integrate(csch(f*x+e)/(a+b*sinh(f*x+e)**2)**(1/2),x)
```

output
```
Integral(csch(e + f*x)/sqrt(a + b*sinh(e + f*x)**2), x)
```

Maxima [F]

$$\int \frac{\operatorname{csch}(e+fx)}{\sqrt{a+b\sinh^2(e+fx)}}\,dx = \int \frac{\operatorname{csch}(fx+e)}{\sqrt{b\sinh(fx+e)^2+a}}\,dx$$

input
```
integrate(csch(f*x+e)/(a+b*sinh(f*x+e)^2)^(1/2),x, algorithm="maxima")
```

output
```
integrate(csch(f*x + e)/sqrt(b*sinh(f*x + e)^2 + a), x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 78 vs. $2(36) = 72$.

Time = 0.15 (sec) , antiderivative size = 78, normalized size of antiderivative = 1.86

$$\int \frac{\operatorname{csch}(e+fx)}{\sqrt{a+b\sinh^2(e+fx)}}\,dx = \frac{2\arctan\left(-\frac{\sqrt{b}e^{(2fx+2e)}-\sqrt{be^{(4fx+4e)}+4ae^{(2fx+2e)}-2be^{(2fx+2e)}+b}-\sqrt{b}}{2\sqrt{-a}}\right)}{\sqrt{-a}f}$$

input
```
integrate(csch(f*x+e)/(a+b*sinh(f*x+e)^2)^(1/2),x, algorithm="giac")
```

output
```
2*arctan(-1/2*(sqrt(b)*e^(2*f*x + 2*e) - sqrt(b*e^(4*f*x + 4*e) + 4*a*e^(2
*f*x + 2*e) - 2*b*e^(2*f*x + 2*e) + b) - sqrt(b))/sqrt(-a))/(sqrt(-a)*f)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\operatorname{csch}(e+fx)}{\sqrt{a+b\sinh^2(e+fx)}}\,dx = \int \frac{1}{\sinh(e+fx)\,\sqrt{b\sinh(e+fx)^2+a}}\,dx$$

input
```
int(1/(sinh(e + f*x)*(a + b*sinh(e + f*x)^2)^(1/2)),x)
```

output
```
int(1/(sinh(e + f*x)*(a + b*sinh(e + f*x)^2)^(1/2)), x)
```

Reduce [F]

$$\int \frac{\operatorname{csch}(e+fx)}{\sqrt{a+b\sinh^2(e+fx)}}\,dx=\int \frac{\sqrt{\sinh\left(fx+e\right)^2b+a}\,\operatorname{csch}(fx+e)}{\sinh\left(fx+e\right)^2b+a}dx$$

input
```
int(csch(f*x+e)/(a+b*sinh(f*x+e)^2)^(1/2),x)
```

output
```
int((sqrt(sinh(e + f*x)**2*b + a)*csch(e + f*x))/(sinh(e + f*x)**2*b + a),
x)
```

3.84 $\int \frac{\operatorname{csch}^3(e+fx)}{\sqrt{a+b\sinh^2(e+fx)}}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 89

$$\int \frac{\operatorname{csch}^3(e+fx)}{\sqrt{a+b\sinh^2(e+fx)}}\,dx=\frac{(a+b)\operatorname{arctanh}\left(\frac{\sqrt{a}\cosh(e+fx)}{\sqrt{a-b+b\cosh^2(e+fx)}}\right)}{2a^{3/2}f}-\frac{\sqrt{a-b+b\cosh^2(e+fx)}\coth(e+fx)\operatorname{csch}(e+fx)}{2af}$$

output
```
1/2*(a+b)*arctanh(a^(1/2)*cosh(f*x+e)/(a-b+b*cosh(f*x+e)^2)^(1/2))/a^(3/2)
/f-1/2*(a-b+b*cosh(f*x+e)^2)^(1/2)*coth(f*x+e)*csch(f*x+e)/a/f
```

Mathematica [A] (verified)

Time = 0.61 (sec) , antiderivative size = 102, normalized size of antiderivative = 1.15

$$\int \frac{\operatorname{csch}^3(e+fx)}{\sqrt{a+b\sinh^2(e+fx)}}\,dx=\frac{2(a+b)\operatorname{arctanh}\left(\frac{\sqrt{2}\sqrt{a}\cosh(e+fx)}{\sqrt{2a-b+b\cosh(2(e+fx))}}\right)-\sqrt{2}\sqrt{a}\sqrt{2a-b+b\cosh(2(e+fx))}\coth(e+fx)\operatorname{csch}(e+fx)}{4a^{3/2}f}$$

input
```
Integrate[Csch[e + f*x]^3/Sqrt[a + b*Sinh[e + f*x]^2],x]
```

output
```
(2*(a + b)*ArcTanh[(Sqrt[2]*Sqrt[a]*Cosh[e + f*x])/Sqrt[2*a - b + b*Cosh[2
*(e + f*x)]]] - Sqrt[2]*Sqrt[a]*Sqrt[2*a - b + b*Cosh[2*(e + f*x)]]*Coth[e
 + f*x]*Csch[e + f*x])/(4*a^(3/2)*f)
```

Rubi [A] (verified)

Time = 0.35 (sec) , antiderivative size = 95, normalized size of antiderivative = 1.07, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.240, Rules used = {3042, 26, 3665, 296, 291, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\operatorname{csch}^3(e+fx)}{\sqrt{a+b\sinh^2(e+fx)}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int -\frac{i}{\sin(ie+ifx)^3\sqrt{a-b\sin(ie+ifx)^2}}dx$$

$$\Big\downarrow \text{26}$$

$$-i\int \frac{1}{\sin(ie+ifx)^3\sqrt{a-b\sin(ie+ifx)^2}}dx$$

$$\Big\downarrow \text{3665}$$

$$\frac{\int \frac{1}{\left(1-\cosh^2(e+fx)\right)^2\sqrt{b\cosh^2(e+fx)+a-b}}d\cosh(e+fx)}{f}$$

$$\Big\downarrow \text{296}$$

$$\frac{\frac{(a+b)\int \frac{1}{\left(1-\cosh^2(e+fx)\right)\sqrt{b\cosh^2(e+fx)+a-b}}d\cosh(e+fx)}{2a}+\frac{\cosh(e+fx)\sqrt{a+b\cosh^2(e+fx)-b}}{2a\left(1-\cosh^2(e+fx)\right)}}{f}$$

$$\Big\downarrow \text{291}$$

$$\frac{\frac{(a+b)\int \frac{1}{1-\frac{a\cosh^2(e+fx)}{b\cosh^2(e+fx)+a-b}}d\frac{\cosh(e+fx)}{\sqrt{b\cosh^2(e+fx)+a-b}}}{2a}+\frac{\cosh(e+fx)\sqrt{a+b\cosh^2(e+fx)-b}}{2a(1-\cosh^2(e+fx))}}{f}$$

$\Big\downarrow$ 219

$$\frac{\frac{(a+b)\text{arctanh}\left(\frac{\sqrt{a}\cosh(e+fx)}{\sqrt{a+b\cosh^2(e+fx)-b}}\right)}{2a^{3/2}}+\frac{\cosh(e+fx)\sqrt{a+b\cosh^2(e+fx)-b}}{2a(1-\cosh^2(e+fx))}}{f}$$

input
```
Int[Csch[e + f*x]^3/Sqrt[a + b*Sinh[e + f*x]^2],x]
```

output
```
(((a + b)*ArcTanh[(Sqrt[a]*Cosh[e + f*x])/Sqrt[a - b + b*Cosh[e + f*x]^2]]
)/(2*a^(3/2)) + (Cosh[e + f*x]*Sqrt[a - b + b*Cosh[e + f*x]^2])/(2*a*(1 -
Cosh[e + f*x]^2)))/f
```

Defintions of rubi rules used

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 291
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*((c_) + (d_.)*(x_)^2)), x_Symbol] :> Subst
[Int[1/(c - (b*c - a*d)*x^2), x], x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b, c,
d}, x] && NeQ[b*c - a*d, 0]
```

rule 296
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_), x_Symbol] :> Sim
p[(-b)*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^(q + 1)/(2*a*(p + 1)*(b*c - a*d))
), x] + Simp[(b*c + 2*(p + 1)*(b*c - a*d))/(2*a*(p + 1)*(b*c - a*d))   Int[
(a + b*x^2)^(p + 1)*(c + d*x^2)^q, x], x] /; FreeQ[{a, b, c, d, q}, x] && N
eQ[b*c - a*d, 0] && EqQ[2*(p + q + 2) + 1, 0] && (LtQ[p, -1] ||  !LtQ[q, -1
]) && NeQ[p, -1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3665
```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^
(p_.), x_Symbol] :> With[{ff = FreeFactors[Cos[e + f*x], x]}, Simp[-ff/f
Subst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a + b - b*ff^2*x^2)^p, x], x, Cos[e +
 f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m - 1)/2]
```

Maple [A] (verified)

Time = 3.10 (sec) , antiderivative size = 149, normalized size of antiderivative = 1.67

method	result
default	$\frac{\sqrt{\left(a+b\sinh(fx+e)^2\right)\cosh(fx+e)^2}\left(-\frac{\sqrt{\left(a+b\sinh(fx+e)^2\right)\cosh(fx+e)^2}}{2a\sinh(fx+e)^2}+\frac{(a+b)\ln\left(\frac{2a+(a+b)\sinh(fx+e)^2+2\sqrt{a}\sqrt{\left(a+b\sinh(fx+e)^2\right)c}}{\sinh(fx+e)^2}\right.}{4a^{\frac{3}{2}}}\right.}{\cosh(fx+e)\sqrt{a+b\sinh(fx+e)^2}\,f}$
risch	Expression too large to display

input
```
int(csch(f*x+e)^3/(a+b*sinh(f*x+e)^2)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
((a+b*sinh(f*x+e)^2)*cosh(f*x+e)^2)^(1/2)*(-1/2/a/sinh(f*x+e)^2*((a+b*sinh
(f*x+e)^2)*cosh(f*x+e)^2)^(1/2)+1/4*(a+b)/a^(3/2)*ln((2*a+(a+b)*sinh(f*x+e
)^2+2*a^(1/2)*((a+b*sinh(f*x+e)^2)*cosh(f*x+e)^2)^(1/2))/sinh(f*x+e)^2))/c
osh(f*x+e)/(a+b*sinh(f*x+e)^2)^(1/2)/f
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 479 vs. $2(77) = 154$.

Time = 0.13 (sec) , antiderivative size = 1173, normalized size of antiderivative = 13.18

$$\int \frac{\operatorname{csch}^3(e+fx)}{\sqrt{a+b\sinh^2(e+fx)}}\,dx = \text{Too large to display}$$

input
```
integrate(csch(f*x+e)^3/(a+b*sinh(f*x+e)^2)^(1/2),x, algorithm="fricas")
```

output
```
[1/4*(((a + b)*cosh(f*x + e)^4 + 4*(a + b)*cosh(f*x + e)*sinh(f*x + e)^3 +
 (a + b)*sinh(f*x + e)^4 - 2*(a + b)*cosh(f*x + e)^2 + 2*(3*(a + b)*cosh(f
*x + e)^2 - a - b)*sinh(f*x + e)^2 + 4*((a + b)*cosh(f*x + e)^3 - (a + b)*
cosh(f*x + e))*sinh(f*x + e) + a + b)*sqrt(a)*log(-((a + b)*cosh(f*x + e)^
4 + 4*(a + b)*cosh(f*x + e)*sinh(f*x + e)^3 + (a + b)*sinh(f*x + e)^4 + 2*
(3*a - b)*cosh(f*x + e)^2 + 2*(3*(a + b)*cosh(f*x + e)^2 + 3*a - b)*sinh(f
*x + e)^2 + 2*sqrt(2)*(cosh(f*x + e)^2 + 2*cosh(f*x + e)*sinh(f*x + e) + s
inh(f*x + e)^2 + 1)*sqrt(a)*sqrt((b*cosh(f*x + e)^2 + b*sinh(f*x + e)^2 +
2*a - b)/(cosh(f*x + e)^2 - 2*cosh(f*x + e)*sinh(f*x + e) + sinh(f*x + e)^
2)) + 4*((a + b)*cosh(f*x + e)^3 + (3*a - b)*cosh(f*x + e))*sinh(f*x + e)
+ a + b)/(cosh(f*x + e)^4 + 4*cosh(f*x + e)*sinh(f*x + e)^3 + sinh(f*x + e
)^4 + 2*(3*cosh(f*x + e)^2 - 1)*sinh(f*x + e)^2 - 2*cosh(f*x + e)^2 + 4*(c
osh(f*x + e)^3 - cosh(f*x + e))*sinh(f*x + e) + 1)) - 2*sqrt(2)*(a*cosh(f*
x + e)^2 + 2*a*cosh(f*x + e)*sinh(f*x + e) + a*sinh(f*x + e)^2 + a)*sqrt((
b*cosh(f*x + e)^2 + b*sinh(f*x + e)^2 + 2*a - b)/(cosh(f*x + e)^2 - 2*cosh
(f*x + e)*sinh(f*x + e) + sinh(f*x + e)^2)))/(a^2*f*cosh(f*x + e)^4 + 4*a^
2*f*cosh(f*x + e)*sinh(f*x + e)^3 + a^2*f*sinh(f*x + e)^4 - 2*a^2*f*cosh(f
*x + e)^2 + a^2*f + 2*(3*a^2*f*cosh(f*x + e)^2 - a^2*f)*sinh(f*x + e)^2 +
4*(a^2*f*cosh(f*x + e)^3 - a^2*f*cosh(f*x + e))*sinh(f*x + e)), -1/2*(((a
+ b)*cosh(f*x + e)^4 + 4*(a + b)*cosh(f*x + e)*sinh(f*x + e)^3 + (a + b...
```

Sympy [F]

$$\int \frac{\operatorname{csch}^3(e+fx)}{\sqrt{a+b\sinh^2(e+fx)}}\,dx = \int \frac{\operatorname{csch}^3(e+fx)}{\sqrt{a+b\sinh^2(e+fx)}}\,dx$$

input
```
integrate(csch(f*x+e)**3/(a+b*sinh(f*x+e)**2)**(1/2),x)
```

output
```
Integral(csch(e + f*x)**3/sqrt(a + b*sinh(e + f*x)**2), x)
```

Maxima [F]

$$\int \frac{\operatorname{csch}^3(e+fx)}{\sqrt{a+b\sinh^2(e+fx)}}\,dx = \int \frac{\operatorname{csch}(fx+e)^3}{\sqrt{b\sinh(fx+e)^2+a}}\,dx$$

input
```
integrate(csch(f*x+e)^3/(a+b*sinh(f*x+e)^2)^(1/2),x, algorithm="maxima")
```

output
```
integrate(csch(f*x + e)^3/sqrt(b*sinh(f*x + e)^2 + a), x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 672 vs. $2(77) = 154$.

Time = 0.26 (sec) , antiderivative size = 672, normalized size of antiderivative = 7.55

$$\int \frac{\operatorname{csch}^3(e+fx)}{\sqrt{a+b\sinh^2(e+fx)}}\,dx = \text{Too large to display}$$

input
```
integrate(csch(f*x+e)^3/(a+b*sinh(f*x+e)^2)^(1/2),x, algorithm="giac")
```

output
```
-((a + b)*arctan(-1/2*(sqrt(b)*e^(2*f*x + 2*e) - sqrt(b*e^(4*f*x + 4*e) +
4*a*e^(2*f*x + 2*e) - 2*b*e^(2*f*x + 2*e) + b) - sqrt(b))/sqrt(-a))*e^(-4*
e)/(sqrt(-a)*a*f) - 2*((sqrt(b)*e^(2*f*x + 2*e) - sqrt(b*e^(4*f*x + 4*e) +
 4*a*e^(2*f*x + 2*e) - 2*b*e^(2*f*x + 2*e) + b))^3*a + (sqrt(b)*e^(2*f*x +
 2*e) - sqrt(b*e^(4*f*x + 4*e) + 4*a*e^(2*f*x + 2*e) - 2*b*e^(2*f*x + 2*e)
 + b))^3*b + 5*(sqrt(b)*e^(2*f*x + 2*e) - sqrt(b*e^(4*f*x + 4*e) + 4*a*e^(
2*f*x + 2*e) - 2*b*e^(2*f*x + 2*e) + b))^2*a*sqrt(b) - 3*(sqrt(b)*e^(2*f*x
 + 2*e) - sqrt(b*e^(4*f*x + 4*e) + 4*a*e^(2*f*x + 2*e) - 2*b*e^(2*f*x + 2*
e) + b))^2*b^(3/2) + 4*(sqrt(b)*e^(2*f*x + 2*e) - sqrt(b*e^(4*f*x + 4*e) +
 4*a*e^(2*f*x + 2*e) - 2*b*e^(2*f*x + 2*e) + b))*a^2 - 9*(sqrt(b)*e^(2*f*x
 + 2*e) - sqrt(b*e^(4*f*x + 4*e) + 4*a*e^(2*f*x + 2*e) - 2*b*e^(2*f*x + 2*
e) + b))*a*b + 3*(sqrt(b)*e^(2*f*x + 2*e) - sqrt(b*e^(4*f*x + 4*e) + 4*a*e
^(2*f*x + 2*e) - 2*b*e^(2*f*x + 2*e) + b))*b^2 - 4*a^2*sqrt(b) + 3*a*b^(3/
2) - b^(5/2))*e^(-4*e)/(((sqrt(b)*e^(2*f*x + 2*e) - sqrt(b*e^(4*f*x + 4*e)
 + 4*a*e^(2*f*x + 2*e) - 2*b*e^(2*f*x + 2*e) + b))^2 - 2*(sqrt(b)*e^(2*f*x
 + 2*e) - sqrt(b*e^(4*f*x + 4*e) + 4*a*e^(2*f*x + 2*e) - 2*b*e^(2*f*x + 2*
e) + b))*sqrt(b) - 4*a + b)^2*a*f))*e^(4*e)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\operatorname{csch}^3(e+fx)}{\sqrt{a+b\sinh^2(e+fx)}}\,dx = \int \frac{1}{\sinh\left(e+f\,x\right)^3\,\sqrt{b\,\sinh\left(e+f\,x\right)^2+a}}\,dx$$

input
```
int(1/(sinh(e + f*x)^3*(a + b*sinh(e + f*x)^2)^(1/2)),x)
```

output
```
int(1/(sinh(e + f*x)^3*(a + b*sinh(e + f*x)^2)^(1/2)), x)
```

Reduce [F]

$$\int \frac{\operatorname{csch}^3(e+fx)}{\sqrt{a+b\sinh^2(e+fx)}}\,dx = \int \frac{\sqrt{\sinh\left(fx+e\right)^2 b+a}\,\operatorname{csch}(fx+e)^3}{\sinh\left(fx+e\right)^2 b+a}dx$$

input
```
int(csch(f*x+e)^3/(a+b*sinh(f*x+e)^2)^(1/2),x)
```

output
```
int((sqrt(sinh(e + f*x)**2*b + a)*csch(e + f*x)**3)/(sinh(e + f*x)**2*b +
a),x)
```

3.85 $\int \frac{\sinh^4(e+fx)}{\sqrt{a+b\sinh^2(e+fx)}}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 229

$$\int \frac{\sinh^4(e+fx)}{\sqrt{a+b\sinh^2(e+fx)}}\,dx = \frac{\cosh(e+fx)\sinh(e+fx)\sqrt{a+b\sinh^2(e+fx)}}{3bf} + \frac{2(a+b)E\left(\arctan(\sinh(e+fx))\,\middle|\,1-\frac{b}{a}\right)\operatorname{sech}(e+fx)\sqrt{a+b\sinh^2(e+fx)}}{3b^2f\sqrt{\frac{\operatorname{sech}^2(e+fx)\left(a+b\sinh^2(e+fx)\right)}{a}}} - \frac{\operatorname{EllipticF}\left(\arctan(\sinh(e+fx)),1-\frac{b}{a}\right)\operatorname{sech}(e+fx)\sqrt{a+b\sinh^2(e+fx)}}{3bf\sqrt{\frac{\operatorname{sech}^2(e+fx)\left(a+b\sinh^2(e+fx)\right)}{a}}} - \frac{2(a+b)\sqrt{a+b\sinh^2(e+fx)}\tanh(e+fx)}{3b^2f}$$

output
```
1/3*cosh(f*x+e)*sinh(f*x+e)*(a+b*sinh(f*x+e)^2)^(1/2)/b/f+2/3*(a+b)*Ellipt
icE(sinh(f*x+e)/(1+sinh(f*x+e)^2)^(1/2),(1-b/a)^(1/2))*sech(f*x+e)*(a+b*si
nh(f*x+e)^2)^(1/2)/b^2/f/(sech(f*x+e)^2*(a+b*sinh(f*x+e)^2)/a)^(1/2)-1/3*I
nverseJacobiAM(arctan(sinh(f*x+e)),(1-b/a)^(1/2))*sech(f*x+e)*(a+b*sinh(f*
x+e)^2)^(1/2)/b/f/(sech(f*x+e)^2*(a+b*sinh(f*x+e)^2)/a)^(1/2)-2/3*(a+b)*(a
+b*sinh(f*x+e)^2)^(1/2)*tanh(f*x+e)/b^2/f
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.83 (sec) , antiderivative size = 168, normalized size of antiderivative = 0.73

$$\int \frac{\sinh^4(e+fx)}{\sqrt{a+b\sinh^2(e+fx)}}\,dx$$
$$= \frac{4i\sqrt{2}a(a+b)\sqrt{\frac{2a-b+b\cosh(2(e+fx))}{a}}E\left(i(e+fx)\,\middle|\,\frac{b}{a}\right)-2i\sqrt{2}a(2a+b)\sqrt{\frac{2a-b+b\cosh(2(e+fx))}{a}}\,\text{EllipticF}\,(i(e+}{6b^2f\sqrt{4a-2b+2b\cosh(2(e+fx))}}$$

input
```
Integrate[Sinh[e + f*x]^4/Sqrt[a + b*Sinh[e + f*x]^2],x]
```

output
```
((4*I)*Sqrt[2]*a*(a + b)*Sqrt[(2*a - b + b*Cosh[2*(e + f*x)])/a]*EllipticE
[I*(e + f*x), b/a] - (2*I)*Sqrt[2]*a*(2*a + b)*Sqrt[(2*a - b + b*Cosh[2*(e
 + f*x)])/a]*EllipticF[I*(e + f*x), b/a] + b*(2*a - b + b*Cosh[2*(e + f*x)
])*Sinh[2*(e + f*x)])/(6*b^2*f*Sqrt[4*a - 2*b + 2*b*Cosh[2*(e + f*x)]])
```

Rubi [A] (verified)

Time = 0.50 (sec) , antiderivative size = 282, normalized size of antiderivative = 1.23, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.280$, Rules used = {3042, 3667, 381, 406, 320, 388, 313}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sinh^4(e+fx)}{\sqrt{a+b\sinh^2(e+fx)}}\,dx$$

$\big\downarrow$ 3042

$$\int \frac{\sin(ie+ifx)^4}{\sqrt{a-b\sin(ie+ifx)^2}}dx$$

$\big\downarrow$ 3667

$$\frac{\sqrt{\cosh^2(e+fx)}\text{sech}(e+fx)\int \frac{\sinh^4(e+fx)}{\sqrt{\sinh^2(e+fx)+1}\sqrt{b\sinh^2(e+fx)+a}}d\sinh(e+fx)}{f}$$

$\big\downarrow$ 381

$$\frac{\sqrt{\cosh^2(e+fx)}\text{sech}(e+fx)\left(\frac{\sinh(e+fx)\sqrt{\sinh^2(e+fx)+1}\sqrt{a+b\sinh^2(e+fx)}}{3b}-\frac{\int \frac{2(a+b)\sinh^2(e+fx)+a}{\sqrt{\sinh^2(e+fx)+1}\sqrt{b\sinh^2(e+fx)+a}}d\sinh(e+fx)}{3b}\right.}{f}$$

$\big\downarrow$ 406

$$\frac{\sqrt{\cosh^2(e+fx)}\text{sech}(e+fx)\left(\frac{\sinh(e+fx)\sqrt{\sinh^2(e+fx)+1}\sqrt{a+b\sinh^2(e+fx)}}{3b}-\frac{a\int \frac{1}{\sqrt{\sinh^2(e+fx)+1}\sqrt{b\sinh^2(e+fx)+a}}d\sinh(e+fx}{}\right.}{f}$$

$\big\downarrow$ 320

$$\frac{\sqrt{\cosh^2(e+fx)}\text{sech}(e+fx)\left(\frac{\sinh(e+fx)\sqrt{\sinh^2(e+fx)+1}\sqrt{a+b\sinh^2(e+fx)}}{3b}-\frac{2(a+b)\int \frac{\sinh^2(e+fx)}{\sqrt{\sinh^2(e+fx)+1}\sqrt{b\sinh^2(e+fx)+a}}d\sinh(}{}\right.}{f}$$

$\big\downarrow$ 388

$$\frac{\sqrt{\cosh^2(e+fx)}\operatorname{sech}(e+fx)\left(\frac{\sinh(e+fx)\sqrt{\sinh^2(e+fx)+1}\sqrt{a+b\sinh^2(e+fx)}}{3b}-\frac{2(a+b)\left(\frac{\sinh(e+fx)\sqrt{a+b\sinh^2(e+fx)}}{b\sqrt{\sinh^2(e+fx)+1}}-\frac{\int\frac{\sqrt{b\sinh}}{\left(\sinh^2(e\right.}}{}\right)}{}\right.}{f}$$

$\Big\downarrow$ 313

$$\frac{\sqrt{\cosh^2(e+fx)}\operatorname{sech}(e+fx)\left(\frac{\sinh(e+fx)\sqrt{\sinh^2(e+fx)+1}\sqrt{a+b\sinh^2(e+fx)}}{3b}-\frac{\frac{\sqrt{a+b\sinh^2(e+fx)}\operatorname{EllipticF}\left(\arctan(\sinh(e+fx)),1-\frac{b}{a}\right.}{\sqrt{\sinh^2(e+fx)+1}\sqrt{\frac{a+b\sinh^2(e+fx)}{a\left(\sinh^2(e+fx)+1\right)}}}}{}\right.}{f}$$

input
```
Int[Sinh[e + f*x]^4/Sqrt[a + b*Sinh[e + f*x]^2],x]
```

output
```
(Sqrt[Cosh[e + f*x]^2]*Sech[e + f*x]*((Sinh[e + f*x]*Sqrt[1 + Sinh[e + f*x
]^2]*Sqrt[a + b*Sinh[e + f*x]^2])/(3*b) - ((EllipticF[ArcTan[Sinh[e + f*x]
], 1 - b/a]*Sqrt[a + b*Sinh[e + f*x]^2])/(Sqrt[1 + Sinh[e + f*x]^2]*Sqrt[(
a + b*Sinh[e + f*x]^2)/(a*(1 + Sinh[e + f*x]^2))]) + 2*(a + b)*((Sinh[e +
f*x]*Sqrt[a + b*Sinh[e + f*x]^2])/(b*Sqrt[1 + Sinh[e + f*x]^2]) - (Ellipti
cE[ArcTan[Sinh[e + f*x]], 1 - b/a]*Sqrt[a + b*Sinh[e + f*x]^2])/(b*Sqrt[1
+ Sinh[e + f*x]^2]*Sqrt[(a + b*Sinh[e + f*x]^2)/(a*(1 + Sinh[e + f*x]^2))]
)))/(3*b)))/f
```

Defintions of rubi rules used

rule 313
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/((c_) + (d_.)*(x_)^2)^(3/2), x_Symbol] :> Sim
p[(Sqrt[a + b*x^2]/(c*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(c
 + d*x^2)))]))*EllipticE[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; FreeQ
[{a, b, c, d}, x] && PosQ[b/a] && PosQ[d/c]
```

rule 320
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(Sqrt[a + b*x^2]/(a*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(
c + d*x^2)))]))*EllipticF[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; Fre
eQ[{a, b, c, d}, x] && PosQ[d/c] && PosQ[b/a] &&  !SimplerSqrtQ[b/a, d/c]
```

rule 381
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_
), x_Symbol] :> Simp[e^3*(e*x)^(m - 3)*(a + b*x^2)^(p + 1)*((c + d*x^2)^(q
+ 1)/(b*d*(m + 2*(p + q) + 1))), x] - Simp[e^4/(b*d*(m + 2*(p + q) + 1))
Int[(e*x)^(m - 4)*(a + b*x^2)^p*(c + d*x^2)^q*Simp[a*c*(m - 3) + (a*d*(m +
2*q - 1) + b*c*(m + 2*p - 1))*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, p, q
}, x] && NeQ[b*c - a*d, 0] && GtQ[m, 3] && IntBinomialQ[a, b, c, d, e, m, 2
, p, q, x]
```

rule 388
```
Int[(x_)^2/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol]
 :> Simp[x*(Sqrt[a + b*x^2]/(b*Sqrt[c + d*x^2])), x] - Simp[c/b   Int[Sqrt[
a + b*x^2]/(c + d*x^2)^(3/2), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
 a*d, 0] && PosQ[b/a] && PosQ[d/c] &&  !SimplerSqrtQ[b/a, d/c]
```

rule 406
```
Int[((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.)*((e_) + (f_.)*(
x_)^2), x_Symbol] :> Simp[e   Int[(a + b*x^2)^p*(c + d*x^2)^q, x], x] + Sim
p[f   Int[x^2*(a + b*x^2)^p*(c + d*x^2)^q, x], x] /; FreeQ[{a, b, c, d, e,
f, p, q}, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3667
```
Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(
p_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Simp[ff^(m + 1
)*(Sqrt[Cos[e + f*x]^2]/(f*Cos[e + f*x]))   Subst[Int[x^m*((a + b*ff^2*x^2)
^p/Sqrt[1 - ff^2*x^2]), x], x, Sin[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f,
p}, x] && IntegerQ[m/2] &&  !IntegerQ[p]
```

Maple [A] (verified)

Time = 1.47 (sec) , antiderivative size = 356, normalized size of antiderivative = 1.55

method	result
default	$\frac{\sqrt{-\frac{b}{a}}\, b\cosh(fx+e)^4\sinh(fx+e)+\sqrt{-\frac{b}{a}}\, a\cosh(fx+e)^2\sinh(fx+e)-\sqrt{-\frac{b}{a}}\, b\cosh(fx+e)^2\sinh(fx+e)+a\sqrt{\frac{b\cosh(fx+e)^2}{a}+\frac{a-b}{a}}\ \sqrt{}}{}$

input
```
int(sinh(f*x+e)^4/(a+b*sinh(f*x+e)^2)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/3*((-b/a)^(1/2)*b*cosh(f*x+e)^4*sinh(f*x+e)+(-b/a)^(1/2)*a*cosh(f*x+e)^2
*sinh(f*x+e)-(-b/a)^(1/2)*b*cosh(f*x+e)^2*sinh(f*x+e)+a*(b/a*cosh(f*x+e)^2
+(a-b)/a)^(1/2)*(cosh(f*x+e)^2)^(1/2)*EllipticF(sinh(f*x+e)*(-b/a)^(1/2),(
1/b*a)^(1/2))+2*(b/a*cosh(f*x+e)^2+(a-b)/a)^(1/2)*(cosh(f*x+e)^2)^(1/2)*El
lipticF(sinh(f*x+e)*(-b/a)^(1/2),(1/b*a)^(1/2))*b-2*(b/a*cosh(f*x+e)^2+(a-
b)/a)^(1/2)*(cosh(f*x+e)^2)^(1/2)*EllipticE(sinh(f*x+e)*(-b/a)^(1/2),(1/b*
a)^(1/2))*a-2*(b/a*cosh(f*x+e)^2+(a-b)/a)^(1/2)*(cosh(f*x+e)^2)^(1/2)*Elli
pticE(sinh(f*x+e)*(-b/a)^(1/2),(1/b*a)^(1/2))*b)/b/(-b/a)^(1/2)/cosh(f*x+e
)/(a+b*sinh(f*x+e)^2)^(1/2)/f
```

Fricas [F]

$$\int \frac{\sinh^4(e+fx)}{\sqrt{a+b\sinh^2(e+fx)}}\,dx=\int \frac{\sinh(fx+e)^4}{\sqrt{b\sinh(fx+e)^2+a}}\,dx$$

input
```
integrate(sinh(f*x+e)^4/(a+b*sinh(f*x+e)^2)^(1/2),x, algorithm="fricas")
```

output
```
integral(sinh(f*x + e)^4/sqrt(b*sinh(f*x + e)^2 + a), x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\sinh^4(e+fx)}{\sqrt{a+b\sinh^2(e+fx)}}\,dx = \text{Timed out}$$

input
```
integrate(sinh(f*x+e)**4/(a+b*sinh(f*x+e)**2)**(1/2),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{\sinh^4(e+fx)}{\sqrt{a+b\sinh^2(e+fx)}}\,dx = \int \frac{\sinh{(fx+e)}^4}{\sqrt{b\sinh{(fx+e)}^2+a}}\,dx$$

input
```
integrate(sinh(f*x+e)^4/(a+b*sinh(f*x+e)^2)^(1/2),x, algorithm="maxima")
```

output
```
integrate(sinh(f*x + e)^4/sqrt(b*sinh(f*x + e)^2 + a), x)
```

Giac [F]

$$\int \frac{\sinh^4(e+fx)}{\sqrt{a+b\sinh^2(e+fx)}}\,dx = \int \frac{\sinh{(fx+e)}^4}{\sqrt{b\sinh{(fx+e)}^2+a}}\,dx$$

input
```
integrate(sinh(f*x+e)^4/(a+b*sinh(f*x+e)^2)^(1/2),x, algorithm="giac")
```

output
```
integrate(sinh(f*x + e)^4/sqrt(b*sinh(f*x + e)^2 + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sinh^4(e+fx)}{\sqrt{a+b\sinh^2(e+fx)}}\,dx = \int \frac{\sinh(e+f\,x)^4}{\sqrt{b\sinh\left(e+f\,x\right)^2+a}}\,dx$$

input
```
int(sinh(e + f*x)^4/(a + b*sinh(e + f*x)^2)^(1/2),x)
```

output
```
int(sinh(e + f*x)^4/(a + b*sinh(e + f*x)^2)^(1/2), x)
```

Reduce [F]

$$\int \frac{\sinh^4(e+fx)}{\sqrt{a+b\sinh^2(e+fx)}}\,dx = \int \frac{\sqrt{\sinh\left(fx+e\right)^2 b+a}\,\sinh\left(fx+e\right)^4}{\sinh\left(fx+e\right)^2 b+a}dx$$

input
```
int(sinh(f*x+e)^4/(a+b*sinh(f*x+e)^2)^(1/2),x)
```

output
```
int((sqrt(sinh(e + f*x)**2*b + a)*sinh(e + f*x)**4)/(sinh(e + f*x)**2*b +
a),x)
```

3.86 $\int \frac{\sinh^2(e+fx)}{\sqrt{a+b\sinh^2(e+fx)}}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 130

$$\int \frac{\sinh^2(e+fx)}{\sqrt{a+b\sinh^2(e+fx)}}\,dx = -\frac{iE\left(ie+ifx\middle|\frac{b}{a}\right)\sqrt{a+b\sinh^2(e+fx)}}{bf\sqrt{\frac{a+b\sinh^2(e+fx)}{a}}} + \frac{ia\,\text{EllipticF}\left(ie+ifx,\frac{b}{a}\right)\sqrt{\frac{a+b\sinh^2(e+fx)}{a}}}{bf\sqrt{a+b\sinh^2(e+fx)}}$$

output
```
-I*EllipticE(sin(I*e+I*f*x),(b/a)^(1/2))*(a+b*sinh(f*x+e)^2)^(1/2)/b/f/((a
+b*sinh(f*x+e)^2)/a)^(1/2)+I*a*InverseJacobiAM(I*e+I*f*x,(b/a)^(1/2))*((a+
b*sinh(f*x+e)^2)/a)^(1/2)/b/f/(a+b*sinh(f*x+e)^2)^(1/2)
```

Mathematica [A] (verified)

Time = 0.37 (sec) , antiderivative size = 89, normalized size of antiderivative = 0.68

$$\int \frac{\sinh^2(e+fx)}{\sqrt{a+b\sinh^2(e+fx)}}\,dx = -\frac{i\sqrt{2a-b+b\cosh(2(e+fx))}\left(E\left(i(e+fx)\left|\frac{b}{a}\right.\right)-\text{EllipticF}\left(i(e+fx),\frac{b}{a}\right)\right)}{bf\sqrt{\frac{2a-b+b\cosh(2(e+fx))}{a}}}$$

input
```
Integrate[Sinh[e + f*x]^2/Sqrt[a + b*Sinh[e + f*x]^2],x]
```

output
```
((-I)*Sqrt[2*a - b + b*Cosh[2*(e + f*x)]]*(EllipticE[I*(e + f*x), b/a] - E
llipticF[I*(e + f*x), b/a]))/(b*f*Sqrt[(2*a - b + b*Cosh[2*(e + f*x)])/a])
```

Rubi [A] (verified)

Time = 0.76 (sec) , antiderivative size = 128, normalized size of antiderivative = 0.98, number of steps used = 10, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.400$, Rules used = {3042, 25, 3651, 3042, 3657, 3042, 3656, 3662, 3042, 3661}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sinh^2(e+fx)}{\sqrt{a+b\sinh^2(e+fx)}}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int -\frac{\sin(ie+ifx)^2}{\sqrt{a-b\sin(ie+ifx)^2}}dx$$

$$\Big\downarrow \text{25}$$

$$-\int \frac{\sin(ie+ifx)^2}{\sqrt{a-b\sin(ie+ifx)^2}}dx$$

$$\Big\downarrow \text{3651}$$

$$\frac{\int \sqrt{b\sinh^2(e+fx)+a}dx}{b}-\frac{a\int \frac{1}{\sqrt{b\sinh^2(e+fx)+a}}dx}{b}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\int \sqrt{a-b\sin(ie+ifx)^2}dx}{b}-\frac{a\int \frac{1}{\sqrt{a-b\sin(ie+ifx)^2}}dx}{b}$$

$$\Big\downarrow \text{3657}$$

$$\frac{\sqrt{a+b\sinh^2(e+fx)}\int \sqrt{\frac{b\sinh^2(e+fx)}{a}+1}dx}{b\sqrt{\frac{b\sinh^2(e+fx)}{a}+1}}-\frac{a\int \frac{1}{\sqrt{a-b\sin(ie+ifx)^2}}dx}{b}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\sqrt{a+b\sinh^2(e+fx)}\int \sqrt{1-\frac{b\sin(ie+ifx)^2}{a}}dx}{b\sqrt{\frac{b\sinh^2(e+fx)}{a}+1}}-\frac{a\int \frac{1}{\sqrt{a-b\sin(ie+ifx)^2}}dx}{b}$$

$$\Big\downarrow \text{3656}$$

$$-\frac{a\int \frac{1}{\sqrt{a-b\sin(ie+ifx)^2}}dx}{b}-\frac{i\sqrt{a+b\sinh^2(e+fx)}E\left(ie+ifx|\frac{b}{a}\right)}{bf\sqrt{\frac{b\sinh^2(e+fx)}{a}+1}}$$

$$\Big\downarrow \text{3662}$$

$$-\frac{a\sqrt{\frac{b\sinh^2(e+fx)}{a}+1}\int \frac{1}{\sqrt{\frac{b\sinh^2(e+fx)}{a}+1}}dx}{b\sqrt{a+b\sinh^2(e+fx)}}-\frac{i\sqrt{a+b\sinh^2(e+fx)}E\left(ie+ifx|\frac{b}{a}\right)}{bf\sqrt{\frac{b\sinh^2(e+fx)}{a}+1}}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{a\sqrt{\frac{b\sinh^2(e+fx)}{a}+1}\int \frac{1}{\sqrt{1-\frac{b\sin(ie+ifx)^2}{a}}}dx}{b\sqrt{a+b\sinh^2(e+fx)}}-\frac{i\sqrt{a+b\sinh^2(e+fx)}E\left(ie+ifx|\frac{b}{a}\right)}{bf\sqrt{\frac{b\sinh^2(e+fx)}{a}+1}}$$

$$\Big\downarrow \text{3661}$$

$$\frac{ia\sqrt{\frac{b\sinh^2(e+fx)}{a}+1}\operatorname{EllipticF}\left(ie+ifx,\frac{b}{a}\right)}{bf\sqrt{a+b\sinh^2(e+fx)}}-\frac{i\sqrt{a+b\sinh^2(e+fx)}E\left(ie+ifx|\frac{b}{a}\right)}{bf\sqrt{\frac{b\sinh^2(e+fx)}{a}+1}}$$

input
```
Int[Sinh[e + f*x]^2/Sqrt[a + b*Sinh[e + f*x]^2],x]
```

output
```
((-I)*EllipticE[I*e + I*f*x, b/a]*Sqrt[a + b*Sinh[e + f*x]^2])/(b*f*Sqrt[1
 + (b*Sinh[e + f*x]^2)/a]) + (I*a*EllipticF[I*e + I*f*x, b/a]*Sqrt[1 + (b*
Sinh[e + f*x]^2)/a])/(b*f*Sqrt[a + b*Sinh[e + f*x]^2])
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3651
```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)]^2)/Sqrt[(a_) + (b_.)*sin[(e_.) +
 (f_.)*(x_)]^2], x_Symbol] :> Simp[B/b   Int[Sqrt[a + b*Sin[e + f*x]^2], x]
, x] + Simp[(A*b - a*B)/b   Int[1/Sqrt[a + b*Sin[e + f*x]^2], x], x] /; Fre
eQ[{a, b, e, f, A, B}, x]
```

rule 3656
```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2], x_Symbol] :> Simp[(Sqrt[a
]/f)*EllipticE[e + f*x, -b/a], x] /; FreeQ[{a, b, e, f}, x] && GtQ[a, 0]
```

rule 3657
```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2], x_Symbol] :> Simp[Sqrt[a
+ b*Sin[e + f*x]^2]/Sqrt[1 + b*(Sin[e + f*x]^2/a)]   Int[Sqrt[1 + (b*Sin[e
+ f*x]^2)/a], x], x] /; FreeQ[{a, b, e, f}, x] &&  !GtQ[a, 0]
```

rule 3661
```
Int[1/Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2], x_Symbol] :> Simp[(1/(S
qrt[a]*f))*EllipticF[e + f*x, -b/a], x] /; FreeQ[{a, b, e, f}, x] && GtQ[a,
 0]
```

rule 3662
```
Int[1/Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2], x_Symbol] :> Simp[Sqrt[
1 + b*(Sin[e + f*x]^2/a)]/Sqrt[a + b*Sin[e + f*x]^2]   Int[1/Sqrt[1 + (b*Si
n[e + f*x]^2)/a], x], x] /; FreeQ[{a, b, e, f}, x] &&  !GtQ[a, 0]
```

Maple [A] (verified)

Time = 0.90 (sec) , antiderivative size = 113, normalized size of antiderivative = 0.87

method	result	size
default	$-\frac{\sqrt{\frac{a+b\sinh(fx+e)^2}{a}}\,\sqrt{\frac{\cosh(2fx+2e)}{2}+\frac{1}{2}}\left(\operatorname{EllipticF}\left(\sinh(fx+e)\sqrt{-\frac{b}{a}},\sqrt{\frac{a}{b}}\right)-\operatorname{EllipticE}\left(\sinh(fx+e)\sqrt{-\frac{b}{a}},\sqrt{\frac{a}{b}}\right)\right)}{\sqrt{-\frac{b}{a}}\,\cosh(fx+e)\sqrt{a+b\sinh(fx+e)^2}\,f}$	113

input
```
int(sinh(f*x+e)^2/(a+b*sinh(f*x+e)^2)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-1/(-b/a)^(1/2)*((a+b*sinh(f*x+e)^2)/a)^(1/2)*(cosh(f*x+e)^2)^(1/2)*(Ellip
ticF(sinh(f*x+e)*(-b/a)^(1/2),(1/b*a)^(1/2))-EllipticE(sinh(f*x+e)*(-b/a)^
(1/2),(1/b*a)^(1/2)))/cosh(f*x+e)/(a+b*sinh(f*x+e)^2)^(1/2)/f
```

Fricas [F]

$$\int \frac{\sinh^2(e+fx)}{\sqrt{a+b\sinh^2(e+fx)}}\,dx = \int \frac{\sinh(fx+e)^2}{\sqrt{b\sinh(fx+e)^2+a}}\,dx$$

input
```
integrate(sinh(f*x+e)^2/(a+b*sinh(f*x+e)^2)^(1/2),x, algorithm="fricas")
```

output
```
integral(sinh(f*x + e)^2/sqrt(b*sinh(f*x + e)^2 + a), x)
```

Sympy [F]

$$\int \frac{\sinh^2(e+fx)}{\sqrt{a+b\sinh^2(e+fx)}}\,dx = \int \frac{\sinh^2(e+fx)}{\sqrt{a+b\sinh^2(e+fx)}}\,dx$$

input
```
integrate(sinh(f*x+e)**2/(a+b*sinh(f*x+e)**2)**(1/2),x)
```

output
```
Integral(sinh(e + f*x)**2/sqrt(a + b*sinh(e + f*x)**2), x)
```

Maxima [F]

$$\int \frac{\sinh^2(e+fx)}{\sqrt{a+b\sinh^2(e+fx)}}\,dx = \int \frac{\sinh(fx+e)^2}{\sqrt{b\sinh(fx+e)^2+a}}\,dx$$

input
```
integrate(sinh(f*x+e)^2/(a+b*sinh(f*x+e)^2)^(1/2),x, algorithm="maxima")
```

output
```
integrate(sinh(f*x + e)^2/sqrt(b*sinh(f*x + e)^2 + a), x)
```

Giac [F]

$$\int \frac{\sinh^2(e+fx)}{\sqrt{a+b\sinh^2(e+fx)}}\,dx = \int \frac{\sinh(fx+e)^2}{\sqrt{b\sinh(fx+e)^2+a}}\,dx$$

input
```
integrate(sinh(f*x+e)^2/(a+b*sinh(f*x+e)^2)^(1/2),x, algorithm="giac")
```

output
```
integrate(sinh(f*x + e)^2/sqrt(b*sinh(f*x + e)^2 + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sinh^2(e+fx)}{\sqrt{a+b\sinh^2(e+fx)}}\,dx = \int \frac{\sinh(e+f\,x)^2}{\sqrt{b\sinh(e+f\,x)^2+a}}\,dx$$

input
```
int(sinh(e + f*x)^2/(a + b*sinh(e + f*x)^2)^(1/2),x)
```

output
```
int(sinh(e + f*x)^2/(a + b*sinh(e + f*x)^2)^(1/2), x)
```

Reduce [F]

$$\int \frac{\sinh^2(e+fx)}{\sqrt{a+b\sinh^2(e+fx)}}\,dx = \int \frac{\sqrt{\sinh(fx+e)^2 b+a}\,\sinh(fx+e)^2}{\sinh(fx+e)^2 b+a}dx$$

input
```
int(sinh(f*x+e)^2/(a+b*sinh(f*x+e)^2)^(1/2),x)
```

output
```
int((sqrt(sinh(e + f*x)**2*b + a)*sinh(e + f*x)**2)/(sinh(e + f*x)**2*b +
a),x)
```

3.87 $\int \frac{1}{\sqrt{a+b\sinh^2(e+fx)}}\,dx$

Optimal result

Integrand size = 16, antiderivative size = 61

$$\int \frac{1}{\sqrt{a+b\sinh^2(e+fx)}}\,dx=-\frac{i\,\text{EllipticF}\left(ie+ifx,\frac{b}{a}\right)\sqrt{\frac{a+b\sinh^2(e+fx)}{a}}}{f\sqrt{a+b\sinh^2(e+fx)}}$$

output
```
-I*InverseJacobiAM(I*e+I*f*x,(b/a)^(1/2))*((a+b*sinh(f*x+e)^2)/a)^(1/2)/f/
(a+b*sinh(f*x+e)^2)^(1/2)
```

Mathematica [A] (verified)

Time = 0.09 (sec) , antiderivative size = 68, normalized size of antiderivative = 1.11

$$\int \frac{1}{\sqrt{a+b\sinh^2(e+fx)}}\,dx=-\frac{i\sqrt{\frac{2a-b+b\cosh(2(e+fx))}{a}}\,\text{EllipticF}\left(i(e+fx),\frac{b}{a}\right)}{f\sqrt{2a-b+b\cosh(2(e+fx))}}$$

input
```
Integrate[1/Sqrt[a + b*Sinh[e + f*x]^2],x]
```

output
```
((-I)*Sqrt[(2*a - b + b*Cosh[2*(e + f*x)])/a]*EllipticF[I*(e + f*x), b/a])
/(f*Sqrt[2*a - b + b*Cosh[2*(e + f*x)]])
```

Rubi [A] (verified)

Time = 0.34 (sec) , antiderivative size = 60, normalized size of antiderivative = 0.98, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {3042, 3662, 3042, 3661}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt{a+b\sinh^2(e+fx)}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{\sqrt{a-b\sin(ie+ifx)^2}}dx$$

$$\Big\downarrow \text{3662}$$

$$\frac{\sqrt{\frac{b\sinh^2(e+fx)}{a}+1}\int \frac{1}{\sqrt{\frac{b\sinh^2(e+fx)}{a}+1}}dx}{\sqrt{a+b\sinh^2(e+fx)}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\sqrt{\frac{b\sinh^2(e+fx)}{a}+1}\int \frac{1}{\sqrt{1-\frac{b\sin(ie+ifx)^2}{a}}}dx}{\sqrt{a+b\sinh^2(e+fx)}}$$

$$\Big\downarrow \text{3661}$$

$$-\frac{i\sqrt{\frac{b\sinh^2(e+fx)}{a}+1}\,\text{EllipticF}\left(ie+ifx,\frac{b}{a}\right)}{f\sqrt{a+b\sinh^2(e+fx)}}$$

input
```
Int[1/Sqrt[a + b*Sinh[e + f*x]^2],x]
```

output
```
((-I)*EllipticF[I*e + I*f*x, b/a]*Sqrt[1 + (b*Sinh[e + f*x]^2)/a])/(f*Sqrt
[a + b*Sinh[e + f*x]^2])
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3661
```
Int[1/Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2], x_Symbol] :> Simp[(1/(S
qrt[a]*f))*EllipticF[e + f*x, -b/a], x] /; FreeQ[{a, b, e, f}, x] && GtQ[a,
 0]
```

rule 3662
```
Int[1/Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2], x_Symbol] :> Simp[Sqrt[
1 + b*(Sin[e + f*x]^2/a)]/Sqrt[a + b*Sin[e + f*x]^2]   Int[1/Sqrt[1 + (b*Si
n[e + f*x]^2)/a], x], x] /; FreeQ[{a, b, e, f}, x] &&  !GtQ[a, 0]
```

Maple [A] (verified)

Time = 0.56 (sec) , antiderivative size = 86, normalized size of antiderivative = 1.41

method	result	size
default	$\frac{\sqrt{\frac{a+b\sinh(fx+e)^2}{a}}\sqrt{\frac{\cosh(2fx+2e)}{2}+\frac{1}{2}}\ \mathrm{EllipticF}\left(\sinh(fx+e)\sqrt{-\frac{b}{a}},\sqrt{\frac{a}{b}}\right)}{\sqrt{-\frac{b}{a}}\ \cosh(fx+e)\sqrt{a+b\sinh(fx+e)^2}\ f}$	86

input
```
int(1/(a+b*sinh(f*x+e)^2)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/(-b/a)^(1/2)*((a+b*sinh(f*x+e)^2)/a)^(1/2)*(cosh(f*x+e)^2)^(1/2)*Ellipti
cF(sinh(f*x+e)*(-b/a)^(1/2),(1/b*a)^(1/2))/cosh(f*x+e)/(a+b*sinh(f*x+e)^2)
^(1/2)/f
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 147 vs. $2(53) = 106$.

Time = 0.10 (sec) , antiderivative size = 147, normalized size of antiderivative = 2.41

$$\int \frac{1}{\sqrt{a+b\sinh^2(e+fx)}}\,dx = -\frac{2\left(2\,b\sqrt{\frac{a^2-ab}{b^2}}+2\,a-b\right)\sqrt{\frac{2\,b\sqrt{\frac{a^2-ab}{b^2}}-2\,a+b}{b}}F\left(\arcsin\left(\sqrt{\frac{2\,b\sqrt{\frac{a^2-ab}{b^2}}-2\,a+b}{b}}(\cosh(fx+e)+\sinh(fx+e))\right.\right.}{b^{\frac{3}{2}}f}$$

input
```
integrate(1/(a+b*sinh(f*x+e)^2)^(1/2),x, algorithm="fricas")
```

output
```
-2*(2*b*sqrt((a^2 - a*b)/b^2) + 2*a - b)*sqrt((2*b*sqrt((a^2 - a*b)/b^2) -
 2*a + b)/b)*elliptic_f(arcsin(sqrt((2*b*sqrt((a^2 - a*b)/b^2) - 2*a + b)/
b)*(cosh(f*x + e) + sinh(f*x + e))), (8*a^2 - 8*a*b + b^2 + 4*(2*a*b - b^2
)*sqrt((a^2 - a*b)/b^2))/b^2)/(b^(3/2)*f)
```

Sympy [F]

$$\int \frac{1}{\sqrt{a+b\sinh^2(e+fx)}}\,dx = \int \frac{1}{\sqrt{a+b\sinh^2(e+fx)}}\,dx$$

input
```
integrate(1/(a+b*sinh(f*x+e)**2)**(1/2),x)
```

output
```
Integral(1/sqrt(a + b*sinh(e + f*x)**2), x)
```

Maxima [F]

$$\int \frac{1}{\sqrt{a+b\sinh^2(e+fx)}}\,dx = \int \frac{1}{\sqrt{b\sinh\left(fx+e\right)^2+a}}\,dx$$

input
```
integrate(1/(a+b*sinh(f*x+e)^2)^(1/2),x, algorithm="maxima")
```

output
```
integrate(1/sqrt(b*sinh(f*x + e)^2 + a), x)
```

Giac [F]

$$\int \frac{1}{\sqrt{a+b\sinh^2(e+fx)}}\,dx = \int \frac{1}{\sqrt{b\sinh\left(fx+e\right)^2+a}}\,dx$$

input
```
integrate(1/(a+b*sinh(f*x+e)^2)^(1/2),x, algorithm="giac")
```

output
```
integrate(1/sqrt(b*sinh(f*x + e)^2 + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt{a+b\sinh^2(e+fx)}}\,dx = \int \frac{1}{\sqrt{b\sinh\left(e+f\,x\right)^2+a}}\,dx$$

input
```
int(1/(a + b*sinh(e + f*x)^2)^(1/2),x)
```

output
```
int(1/(a + b*sinh(e + f*x)^2)^(1/2), x)
```

Reduce [F]

$$\int \frac{1}{\sqrt{a+b\sinh^2(e+fx)}}\,dx = \int \frac{\sqrt{\sinh\left(fx+e\right)^2 b+a}}{\sinh\left(fx+e\right)^2 b+a}dx$$

input
```
int(1/(a+b*sinh(f*x+e)^2)^(1/2),x)
```

output
```
int(sqrt(sinh(e + f*x)**2*b + a)/(sinh(e + f*x)**2*b + a),x)
```

3.88 $\int \frac{\operatorname{csch}^2(e+fx)}{\sqrt{a+b\sinh^2(e+fx)}}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 134

$$\int \frac{\operatorname{csch}^2(e+fx)}{\sqrt{a+b\sinh^2(e+fx)}}\,dx = -\frac{\coth(e+fx)\sqrt{a+b\sinh^2(e+fx)}}{af} - \frac{E\left(\arctan(\sinh(e+fx))\left|1-\frac{b}{a}\right.\right)\operatorname{sech}(e+fx)\sqrt{a+b\sinh^2(e+fx)}}{af\sqrt{\frac{\operatorname{sech}^2(e+fx)(a+b\sinh^2(e+fx))}{a}}} + \frac{\sqrt{a+b\sinh^2(e+fx)}\tanh(e+fx)}{af}$$

output
```
-coth(f*x+e)*(a+b*sinh(f*x+e)^2)^(1/2)/a/f-EllipticE(sinh(f*x+e)/(1+sinh(f
*x+e)^2)^(1/2),(1-b/a)^(1/2))*sech(f*x+e)*(a+b*sinh(f*x+e)^2)^(1/2)/a/f/(s
ech(f*x+e)^2*(a+b*sinh(f*x+e)^2)/a)^(1/2)+(a+b*sinh(f*x+e)^2)^(1/2)*tanh(f
*x+e)/a/f
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.63 (sec) , antiderivative size = 150, normalized size of antiderivative = 1.12

$$\int \frac{\operatorname{csch}^2(e+fx)}{\sqrt{a+b\sinh^2(e+fx)}}\,dx$$
$$= \frac{\sqrt{2}(-2a+b-b\cosh(2(e+fx)))\coth(e+fx)-2ia\sqrt{\frac{2a-b+b\cosh(2(e+fx))}{a}}E\left(i(e+fx)\,\middle|\,\frac{b}{a}\right)+2ia\sqrt{\frac{2a-b+}{}}}{2af\sqrt{2a-b+b\cosh(2(e+fx))}}$$

input
```
Integrate[Csch[e + f*x]^2/Sqrt[a + b*Sinh[e + f*x]^2],x]
```

output
```
(Sqrt[2]*(-2*a + b - b*Cosh[2*(e + f*x)])*Coth[e + f*x] - (2*I)*a*Sqrt[(2*
a - b + b*Cosh[2*(e + f*x)])/a]*EllipticE[I*(e + f*x), b/a] + (2*I)*a*Sqrt
[(2*a - b + b*Cosh[2*(e + f*x)])/a]*EllipticF[I*(e + f*x), b/a])/(2*a*f*Sq
rt[2*a - b + b*Cosh[2*(e + f*x)]])
```

Rubi [A] (verified)

Time = 0.41 (sec) , antiderivative size = 193, normalized size of antiderivative = 1.44, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.280, Rules used = {3042, 25, 3667, 382, 27, 388, 313}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\operatorname{csch}^2(e+fx)}{\sqrt{a+b\sinh^2(e+fx)}}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int -\frac{1}{\sin(ie+ifx)^2\sqrt{a-b\sin(ie+ifx)^2}}dx$$

$$\Big\downarrow \text{25}$$

$$-\int \frac{1}{\sin(ie+ifx)^2\sqrt{a-b\sin(ie+ifx)^2}}dx$$

$\big\downarrow$ 3667

$$\frac{\sqrt{\cosh^2(e+fx)}\text{sech}(e+fx)\int \frac{\text{csch}^2(e+fx)}{\sqrt{\sinh^2(e+fx)+1}\sqrt{b\sinh^2(e+fx)+a}}d\sinh(e+fx)}{f}$$

$\big\downarrow$ 382

$$\frac{\sqrt{\cosh^2(e+fx)}\text{sech}(e+fx)\left(\frac{\int \frac{b\sinh^2(e+fx)}{\sqrt{\sinh^2(e+fx)+1}\sqrt{b\sinh^2(e+fx)+a}}d\sinh(e+fx)}{a}-\frac{\sqrt{\sinh^2(e+fx)+1}\text{csch}(e+fx)\sqrt{a+b\sinh^2(e+fx \ldots}}{a}\right.}{f}$$

$\big\downarrow$ 27

$$\frac{\sqrt{\cosh^2(e+fx)}\text{sech}(e+fx)\left(\frac{b\int \frac{\sinh^2(e+fx)}{\sqrt{\sinh^2(e+fx)+1}\sqrt{b\sinh^2(e+fx)+a}}d\sinh(e+fx)}{a}-\frac{\sqrt{\sinh^2(e+fx)+1}\text{csch}(e+fx)\sqrt{a+b\sinh^2(e+f \ldots}}{a}\right.}{f}$$

$\big\downarrow$ 388

$$\frac{\sqrt{\cosh^2(e+fx)}\text{sech}(e+fx)\left(\frac{b\left(\frac{\sinh(e+fx)\sqrt{a+b\sinh^2(e+fx)}}{b\sqrt{\sinh^2(e+fx)+1}}-\frac{\int \frac{\sqrt{b\sinh^2(e+fx)+a}}{\left(\sinh^2(e+fx)+1\right)^{3/2}}d\sinh(e+fx)}{b}\right)}{a}-\frac{\sqrt{\sinh^2(e+fx)+1}\text{csch}(e+fx \ldots}{a}\right.}{f}$$

$\big\downarrow$ 313

$$\frac{\sqrt{\cosh^2(e+fx)}\text{sech}(e+fx)\left(\frac{b\left(\frac{\sinh(e+fx)\sqrt{a+b\sinh^2(e+fx)}}{b\sqrt{\sinh^2(e+fx)+1}}-\frac{\sqrt{a+b\sinh^2(e+fx)}E\left(\arctan(\sinh(e+fx))\middle|1-\frac{b}{a}\right)}{b\sqrt{\sinh^2(e+fx)+1}\sqrt{\frac{a+b\sinh^2(e+fx)}{a\left(\sinh^2(e+fx)+1\right)}}}\right)}{a}-\frac{\sqrt{\sinh^2(e+fx)+1}\text{c} \ldots}{}\right.}{f}$$

input
```
Int[Csch[e + f*x]^2/Sqrt[a + b*Sinh[e + f*x]^2],x]
```

output
```
(Sqrt[Cosh[e + f*x]^2]*Sech[e + f*x]*(-((Csch[e + f*x]*Sqrt[1 + Sinh[e + f
*x]^2]*Sqrt[a + b*Sinh[e + f*x]^2])/a) + (b*((Sinh[e + f*x]*Sqrt[a + b*Sin
h[e + f*x]^2])/(b*Sqrt[1 + Sinh[e + f*x]^2]) - (EllipticE[ArcTan[Sinh[e +
f*x]], 1 - b/a]*Sqrt[a + b*Sinh[e + f*x]^2])/(b*Sqrt[1 + Sinh[e + f*x]^2]*
Sqrt[(a + b*Sinh[e + f*x]^2)/(a*(1 + Sinh[e + f*x]^2))])))/a))/f
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 313
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/((c_) + (d_.)*(x_)^2)^(3/2), x_Symbol] :> Sim
p[(Sqrt[a + b*x^2]/(c*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(c
+ d*x^2)))]))*EllipticE[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; FreeQ
[{a, b, c, d}, x] && PosQ[b/a] && PosQ[d/c]
```

rule 382
```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_)
, x_Symbol] :> Simp[(e*x)^(m + 1)*(a + b*x^2)^(p + 1)*((c + d*x^2)^(q + 1)/
(a*c*e*(m + 1))), x] - Simp[1/(a*c*e^2*(m + 1))   Int[(e*x)^(m + 2)*(a + b*
x^2)^p*(c + d*x^2)^q*Simp[(b*c + a*d)*(m + 3) + 2*(b*c*p + a*d*q) + b*d*(m
+ 2*p + 2*q + 5)*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, p, q}, x] && NeQ[
b*c - a*d, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, d, e, m, 2, p, q, x]
```

rule 388
```
Int[(x_)^2/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol]
 :> Simp[x*(Sqrt[a + b*x^2]/(b*Sqrt[c + d*x^2])), x] - Simp[c/b   Int[Sqrt[
a + b*x^2]/(c + d*x^2)^(3/2), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
 a*d, 0] && PosQ[b/a] && PosQ[d/c] &&  !SimplerSqrtQ[b/a, d/c]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3667
```
Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(
p_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Simp[ff^(m + 1
)*(Sqrt[Cos[e + f*x]^2]/(f*Cos[e + f*x]))   Subst[Int[x^m*((a + b*ff^2*x^2)
^p/Sqrt[1 - ff^2*x^2]), x], x, Sin[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f,
p}, x] && IntegerQ[m/2] &&  !IntegerQ[p]
```

Maple [A] (verified)

Time = 1.38 (sec) , antiderivative size = 189, normalized size of antiderivative = 1.41

method	result
default	$-\frac{\sqrt{-\frac{b}{a}}\, b\cosh(fx+e)^4+\left(\sqrt{-\frac{b}{a}}\, a-\sqrt{-\frac{b}{a}}\, b\right)\cosh(fx+e)^2+\sinh(fx+e)\sqrt{\frac{\cosh(2fx+2e)}{2}+\frac{1}{2}}\,\sqrt{\frac{b\cosh(fx+e)^2}{a}+\frac{a-b}{a}}\, b\left(\text{EllipticF}\left(\sin\right.\right.}{a\sinh(fx+e)\sqrt{-\frac{b}{a}}\,\cosh(fx+e)\sqrt{a+b\sinh(fx+e)^2}\, f}$
risch	Expression too large to display

input
```
int(csch(f*x+e)^2/(a+b*sinh(f*x+e)^2)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-((-b/a)^(1/2)*b*cosh(f*x+e)^4+((-b/a)^(1/2)*a-(-b/a)^(1/2)*b)*cosh(f*x+e)
^2+sinh(f*x+e)*(cosh(f*x+e)^2)^(1/2)*(b/a*cosh(f*x+e)^2+(a-b)/a)^(1/2)*b*(
EllipticF(sinh(f*x+e)*(-b/a)^(1/2),(1/b*a)^(1/2))-EllipticE(sinh(f*x+e)*(-
b/a)^(1/2),(1/b*a)^(1/2))))/a/sinh(f*x+e)/(-b/a)^(1/2)/cosh(f*x+e)/(a+b*si
nh(f*x+e)^2)^(1/2)/f
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 541 vs. $2(140) = 280$.

Time = 0.10 (sec) , antiderivative size = 541, normalized size of antiderivative = 4.04

$$\int \frac{\operatorname{csch}^2(e+fx)}{\sqrt{a+b\sinh^2(e+fx)}}\,dx$$

$$= \frac{4\left(b\cosh\left(fx+e\right)^2+2\,b\cosh\left(fx+e\right)\sinh\left(fx+e\right)+b\sinh\left(fx+e\right)^2-b\right)\sqrt{b}\sqrt{\frac{2\,b\sqrt{\frac{a^2-ab}{b^2}}-2\,a+b}{b}}\sqrt{\frac{a^2-ab}{b^2}}F}{}$$

input
```
integrate(csch(f*x+e)^2/(a+b*sinh(f*x+e)^2)^(1/2),x, algorithm="fricas")
```

output
```
(4*(b*cosh(f*x + e)^2 + 2*b*cosh(f*x + e)*sinh(f*x + e) + b*sinh(f*x + e)^
2 - b)*sqrt(b)*sqrt((2*b*sqrt((a^2 - a*b)/b^2) - 2*a + b)/b)*sqrt((a^2 - a
*b)/b^2)*elliptic_f(arcsin(sqrt((2*b*sqrt((a^2 - a*b)/b^2) - 2*a + b)/b)*(
cosh(f*x + e) + sinh(f*x + e))), (8*a^2 - 8*a*b + b^2 + 4*(2*a*b - b^2)*sq
rt((a^2 - a*b)/b^2))/b^2) + ((2*a - b)*cosh(f*x + e)^2 + 2*(2*a - b)*cosh(
f*x + e)*sinh(f*x + e) + (2*a - b)*sinh(f*x + e)^2 - 2*(b*cosh(f*x + e)^2
+ 2*b*cosh(f*x + e)*sinh(f*x + e) + b*sinh(f*x + e)^2 - b)*sqrt((a^2 - a*b
)/b^2) - 2*a + b)*sqrt(b)*sqrt((2*b*sqrt((a^2 - a*b)/b^2) - 2*a + b)/b)*el
liptic_e(arcsin(sqrt((2*b*sqrt((a^2 - a*b)/b^2) - 2*a + b)/b)*(cosh(f*x +
e) + sinh(f*x + e))), (8*a^2 - 8*a*b + b^2 + 4*(2*a*b - b^2)*sqrt((a^2 - a
*b)/b^2))/b^2) - sqrt(2)*(b*cosh(f*x + e) + b*sinh(f*x + e))*sqrt((b*cosh(
f*x + e)^2 + b*sinh(f*x + e)^2 + 2*a - b)/(cosh(f*x + e)^2 - 2*cosh(f*x +
e)*sinh(f*x + e) + sinh(f*x + e)^2)))/(a*b*f*cosh(f*x + e)^2 + 2*a*b*f*cos
h(f*x + e)*sinh(f*x + e) + a*b*f*sinh(f*x + e)^2 - a*b*f)
```

Sympy [F]

$$\int \frac{\operatorname{csch}^2(e+fx)}{\sqrt{a+b\sinh^2(e+fx)}}\,dx = \int \frac{\operatorname{csch}^2\left(e+fx\right)}{\sqrt{a+b\sinh^2\left(e+fx\right)}}\,dx$$

input
```
integrate(csch(f*x+e)**2/(a+b*sinh(f*x+e)**2)**(1/2),x)
```

output
```
Integral(csch(e + f*x)**2/sqrt(a + b*sinh(e + f*x)**2), x)
```

Maxima [F]

$$\int \frac{\operatorname{csch}^2(e+fx)}{\sqrt{a+b\sinh^2(e+fx)}}\,dx=\int \frac{\operatorname{csch}\left(fx+e\right)^2}{\sqrt{b\sinh\left(fx+e\right)^2+a}}\,dx$$

input
```
integrate(csch(f*x+e)^2/(a+b*sinh(f*x+e)^2)^(1/2),x, algorithm="maxima")
```

output
```
integrate(csch(f*x + e)^2/sqrt(b*sinh(f*x + e)^2 + a), x)
```

Giac [F]

$$\int \frac{\operatorname{csch}^2(e+fx)}{\sqrt{a+b\sinh^2(e+fx)}}\,dx=\int \frac{\operatorname{csch}\left(fx+e\right)^2}{\sqrt{b\sinh\left(fx+e\right)^2+a}}\,dx$$

input
```
integrate(csch(f*x+e)^2/(a+b*sinh(f*x+e)^2)^(1/2),x, algorithm="giac")
```

output
```
integrate(csch(f*x + e)^2/sqrt(b*sinh(f*x + e)^2 + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\operatorname{csch}^2(e+fx)}{\sqrt{a+b\sinh^2(e+fx)}}\,dx=\int \frac{1}{\sinh\left(e+f\,x\right)^2\,\sqrt{b\,\sinh\left(e+f\,x\right)^2+a}}\,dx$$

input
```
int(1/(sinh(e + f*x)^2*(a + b*sinh(e + f*x)^2)^(1/2)),x)
```

output
```
int(1/(sinh(e + f*x)^2*(a + b*sinh(e + f*x)^2)^(1/2)), x)
```

Reduce [F]

$$\int \frac{\operatorname{csch}^2(e+fx)}{\sqrt{a+b\sinh^2(e+fx)}}\,dx = \int \frac{\sqrt{\sinh(fx+e)^2 b+a}\,\operatorname{csch}(fx+e)^2}{\sinh(fx+e)^2 b+a}\,dx$$

input
```
int(csch(f*x+e)^2/(a+b*sinh(f*x+e)^2)^(1/2),x)
```

output
```
int((sqrt(sinh(e + f*x)**2*b + a)*csch(e + f*x)**2)/(sinh(e + f*x)**2*b +
a),x)
```

3.89 $\int \frac{\operatorname{csch}^4(e+fx)}{\sqrt{a+b\sinh^2(e+fx)}}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 238

$$\begin{aligned}\int \frac{\operatorname{csch}^4(e+fx)}{\sqrt{a+b\sinh^2(e+fx)}}\,dx &= -\frac{\coth(e+fx)\operatorname{csch}^2(e+fx)\sqrt{a+b\sinh^2(e+fx)}}{3af} \\ &\quad+\frac{2(a+b)\operatorname{csch}(e+fx)\operatorname{sech}(e+fx)\sqrt{a+b\sinh^2(e+fx)}}{3a^2f} \\ &\quad+\frac{2(a+b)E\left(\arctan(\sinh(e+fx))\,\middle|\,1-\frac{b}{a}\right)\operatorname{sech}(e+fx)\sqrt{a+b\sinh^2(e+fx)}}{3a^2f\sqrt{\frac{\operatorname{sech}^2(e+fx)(a+b\sinh^2(e+fx))}{a}}} \\ &\quad-\frac{b\,\text{EllipticF}\left(\arctan(\sinh(e+fx)),1-\frac{b}{a}\right)\operatorname{sech}(e+fx)\sqrt{a+b\sinh^2(e+fx)}}{3a^2f\sqrt{\frac{\operatorname{sech}^2(e+fx)(a+b\sinh^2(e+fx))}{a}}}\end{aligned}$$

output
```
-1/3*coth(f*x+e)*csch(f*x+e)^2*(a+b*sinh(f*x+e)^2)^(1/2)/a/f+2/3*(a+b)*csc
h(f*x+e)*sech(f*x+e)*(a+b*sinh(f*x+e)^2)^(1/2)/a^2/f+2/3*(a+b)*EllipticE(s
inh(f*x+e)/(1+sinh(f*x+e)^2)^(1/2),(1-b/a)^(1/2))*sech(f*x+e)*(a+b*sinh(f*
x+e)^2)^(1/2)/a^2/f/(sech(f*x+e)^2*(a+b*sinh(f*x+e)^2)/a)^(1/2)-1/3*b*Inve
rseJacobiAM(arctan(sinh(f*x+e)),(1-b/a)^(1/2))*sech(f*x+e)*(a+b*sinh(f*x+e
)^2)^(1/2)/a^2/f/(sech(f*x+e)^2*(a+b*sinh(f*x+e)^2)/a)^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 2.56 (sec) , antiderivative size = 201, normalized size of antiderivative = 0.84

$$\int \frac{\operatorname{csch}^4(e+fx)}{\sqrt{a+b\sinh^2(e+fx)}}\,dx$$
$$= \frac{\frac{(-8a^2+ab+3b^2+(4a^2-2ab-4b^2)\cosh(2(e+fx))+b(a+b)\cosh(4(e+fx)))\coth(e+fx)\operatorname{csch}^2(e+fx)}{\sqrt{2}} + 4ia(a+b)\sqrt{\frac{2a-b+b\cosh(2(e+\ldots}{a}}\ldots}{6a^2f\sqrt{2a-b+b\cosh(2(e+\ldots}}$$

input
```
Integrate[Csch[e + f*x]^4/Sqrt[a + b*Sinh[e + f*x]^2],x]
```

output
```
(((-8*a^2 + a*b + 3*b^2 + (4*a^2 - 2*a*b - 4*b^2)*Cosh[2*(e + f*x)] + b*(a
 + b)*Cosh[4*(e + f*x)])*Coth[e + f*x]*Csch[e + f*x]^2)/Sqrt[2] + (4*I)*a*
(a + b)*Sqrt[(2*a - b + b*Cosh[2*(e + f*x)])/a]*EllipticE[I*(e + f*x), b/a
] - (2*I)*a*(2*a + b)*Sqrt[(2*a - b + b*Cosh[2*(e + f*x)])/a]*EllipticF[I*
(e + f*x), b/a])/(6*a^2*f*Sqrt[2*a - b + b*Cosh[2*(e + f*x)]])
```

Rubi [A] (verified)

Time = 0.54 (sec) , antiderivative size = 334, normalized size of antiderivative = 1.40, number of steps used = 12, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.440$, Rules used = {3042, 3667, 382, 25, 445, 25, 27, 406, 320, 388, 313}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\operatorname{csch}^4(e+fx)}{\sqrt{a+b\sinh^2(e+fx)}}dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \frac{1}{\sin(ie+ifx)^4\sqrt{a-b\sin(ie+ifx)^2}}dx$$

$$\Big\downarrow \text{ 3667}$$

$$\frac{\sqrt{\cosh^2(e+fx)}\operatorname{sech}(e+fx)\int \frac{\operatorname{csch}^4(e+fx)}{\sqrt{\sinh^2(e+fx)+1}\sqrt{b\sinh^2(e+fx)+a}}d\sinh(e+fx)}{f}$$

$$\Big\downarrow \text{ 382}$$

$$\frac{\sqrt{\cosh^2(e+fx)}\operatorname{sech}(e+fx)\left(\frac{\int -\frac{\operatorname{csch}^2(e+fx)\left(b\sinh^2(e+fx)+2(a+b)\right)}{\sqrt{\sinh^2(e+fx)+1}\sqrt{b\sinh^2(e+fx)+a}}d\sinh(e+fx)}{3a}-\frac{\sqrt{\sinh^2(e+fx)+1}\operatorname{csch}^3(e+fx)\sqrt{a+b\sinh^2(}}{3a}\right.}{f}$$

$$\Big\downarrow \text{ 25}$$

$$\frac{\sqrt{\cosh^2(e+fx)}\operatorname{sech}(e+fx)\left(-\frac{\int \frac{\operatorname{csch}^2(e+fx)\left(b\sinh^2(e+fx)+2(a+b)\right)}{\sqrt{\sinh^2(e+fx)+1}\sqrt{b\sinh^2(e+fx)+a}}d\sinh(e+fx)}{3a}-\frac{\sqrt{\sinh^2(e+fx)+1}\operatorname{csch}^3(e+fx)\sqrt{a+b\sinh^2}}{3a}\right.}{f}$$

$$\Big\downarrow \text{ 445}$$

$$\frac{\sqrt{\cosh^2(e+fx)}\operatorname{sech}(e+fx)\left(-\frac{-\frac{\int -\frac{b\left(2(a+b)\sinh^2(e+fx)+a\right)}{\sqrt{\sinh^2(e+fx)+1}\sqrt{b\sinh^2(e+fx)+a}}d\sinh(e+fx)}{a}-\frac{2(a+b)\sqrt{\sinh^2(e+fx)+1}\operatorname{csch}(e+fx)\sqrt{a+b\sinh^2(e+}}{a}}{3a}\right.}{f}$$

$$\Big\downarrow \text{ 25}$$

$$\frac{\sqrt{\cosh^2(e+fx)}\operatorname{sech}(e+fx)\left(-\frac{\frac{\int \frac{b\left(2(a+b)\sinh^2(e+fx)+a\right)}{\sqrt{\sinh^2(e+fx)+1}\sqrt{b\sinh^2(e+fx)+a}}d\sinh(e+fx)}{a}-\frac{2(a+b)\sqrt{\sinh^2(e+fx)+1}\operatorname{csch}(e+fx)\sqrt{a+b\sinh^2(e+fx)}}{a}}{3a}\right.}{f}$$

$\Big\downarrow$ 27

$$\frac{\sqrt{\cosh^2(e+fx)}\text{sech}(e+fx)\left(-\frac{\frac{b\int\frac{2(a+b)\sinh^2(e+fx)+a}{\sqrt{\sinh^2(e+fx)+1}\sqrt{b\sinh^2(e+fx)+a}}d\sinh(e+fx)}{a}-\frac{2(a+b)\sqrt{\sinh^2(e+fx)+1}\text{csch}(e+fx)\sqrt{a+b\sinh^2(e+fx)}}{a}}{3a}\right)}{f}$$

$\Big\downarrow$ 406

$$\frac{\sqrt{\cosh^2(e+fx)}\text{sech}(e+fx)\left(-\frac{\frac{b\left(a\int\frac{1}{\sqrt{\sinh^2(e+fx)+1}\sqrt{b\sinh^2(e+fx)+a}}d\sinh(e+fx)+2(a+b)\int\frac{\sinh^2(e+fx)}{\sqrt{\sinh^2(e+fx)+1}\sqrt{b\sinh^2(e+fx)+a}}d\sinh(e\right)}{a}}{3a}\right)}{f}$$

$\Big\downarrow$ 320

$$\frac{\sqrt{\cosh^2(e+fx)}\text{sech}(e+fx)\left(-\frac{\frac{b\left(2(a+b)\int\frac{\sinh^2(e+fx)}{\sqrt{\sinh^2(e+fx)+1}\sqrt{b\sinh^2(e+fx)+a}}d\sinh(e+fx)+\frac{\sqrt{a+b\sinh^2(e+fx)}\text{EllipticF}\left(\arctan(\sinh(e+fx)\right)}{\sqrt{\sinh^2(e+fx)+1}\sqrt{\frac{a+b\sinh^2(e+fx)}{a\left(\sinh^2(e+fx)+1\right)}}}\right)}{a}}{3a}\right)}{f}$$

$\Big\downarrow$ 388

$$\frac{\sqrt{\cosh^2(e+fx)}\text{sech}(e+fx)\left(-\frac{\frac{b\left(2(a+b)\left(\frac{\sinh(e+fx)\sqrt{a+b\sinh^2(e+fx)}}{b\sqrt{\sinh^2(e+fx)+1}}-\frac{\int\frac{\sqrt{b\sinh^2(e+fx)+a}}{\left(\sinh^2(e+fx)+1\right)^{3/2}}d\sinh(e+fx)}{b}\right)+\frac{\sqrt{a+b\sinh^2(e+fx)}\text{Ellipti}}{\sqrt{\sinh^2(e+fx)+1}}\right)}{a}}{3a}\right)}{f}$$

$\Big\downarrow$ 313

$$\sqrt{\cosh^2(e+fx)}\operatorname{sech}(e+fx)\left(-\frac{\frac{b\left(\frac{\sqrt{a+b\sinh^2(e+fx)}\operatorname{EllipticF}\left(\arctan(\sinh(e+fx)),1-\frac{b}{a}\right)}{\sqrt{\sinh^2(e+fx)+1}\sqrt{\frac{a+b\sinh^2(e+fx)}{a\left(\sinh^2(e+fx)+1\right)}}}+2(a+b)\left(\frac{\sinh(e+fx)\sqrt{a+b\sinh^2(e+fx)}}{b\sqrt{\sinh^2(e+fx)+1}}-\frac{\sqrt{a+b\sin\cdots}}{b\sqrt{\text{si}\cdots}}\right.\right.}{a}}{3a}\right.$$

input
```
Int[Csch[e + f*x]^4/Sqrt[a + b*Sinh[e + f*x]^2],x]
```

output
```
(Sqrt[Cosh[e + f*x]^2]*Sech[e + f*x]*(-1/3*(Csch[e + f*x]^3*Sqrt[1 + Sinh[
e + f*x]^2]*Sqrt[a + b*Sinh[e + f*x]^2])/a - ((-2*(a + b)*Csch[e + f*x]*Sq
rt[1 + Sinh[e + f*x]^2]*Sqrt[a + b*Sinh[e + f*x]^2])/a + (b*((EllipticF[Ar
cTan[Sinh[e + f*x]], 1 - b/a]*Sqrt[a + b*Sinh[e + f*x]^2])/(Sqrt[1 + Sinh[
e + f*x]^2]*Sqrt[(a + b*Sinh[e + f*x]^2)/(a*(1 + Sinh[e + f*x]^2))]) + 2*(
a + b)*((Sinh[e + f*x]*Sqrt[a + b*Sinh[e + f*x]^2])/(b*Sqrt[1 + Sinh[e + f
*x]^2]) - (EllipticE[ArcTan[Sinh[e + f*x]], 1 - b/a]*Sqrt[a + b*Sinh[e + f
*x]^2])/(b*Sqrt[1 + Sinh[e + f*x]^2]*Sqrt[(a + b*Sinh[e + f*x]^2)/(a*(1 +
Sinh[e + f*x]^2))]))))/a)/(3*a)))/f
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 313
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/((c_) + (d_.)*(x_)^2)^(3/2), x_Symbol] :> Sim
p[(Sqrt[a + b*x^2]/(c*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(c
 + d*x^2)))]))*EllipticE[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; FreeQ
[{a, b, c, d}, x] && PosQ[b/a] && PosQ[d/c]
```

rule 320
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(Sqrt[a + b*x^2]/(a*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(
c + d*x^2)))]))*EllipticF[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; Fre
eQ[{a, b, c, d}, x] && PosQ[d/c] && PosQ[b/a] && !SimplerSqrtQ[b/a, d/c]
```

rule 382
```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_)
, x_Symbol] :> Simp[(e*x)^(m + 1)*(a + b*x^2)^(p + 1)*((c + d*x^2)^(q + 1)/
(a*c*e*(m + 1))), x] - Simp[1/(a*c*e^2*(m + 1))   Int[(e*x)^(m + 2)*(a + b*
x^2)^p*(c + d*x^2)^q*Simp[(b*c + a*d)*(m + 3) + 2*(b*c*p + a*d*q) + b*d*(m
+ 2*p + 2*q + 5)*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, p, q}, x] && NeQ[
b*c - a*d, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, d, e, m, 2, p, q, x]
```

rule 388
```
Int[(x_)^2/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol]
 :> Simp[x*(Sqrt[a + b*x^2]/(b*Sqrt[c + d*x^2])), x] - Simp[c/b   Int[Sqrt[
a + b*x^2]/(c + d*x^2)^(3/2), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
 a*d, 0] && PosQ[b/a] && PosQ[d/c] && !SimplerSqrtQ[b/a, d/c]
```

rule 406
```
Int[((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.)*((e_) + (f_.)*(
x_)^2), x_Symbol] :> Simp[e   Int[(a + b*x^2)^p*(c + d*x^2)^q, x], x] + Sim
p[f   Int[x^2*(a + b*x^2)^p*(c + d*x^2)^q, x], x] /; FreeQ[{a, b, c, d, e,
f, p, q}, x]
```

rule 445
```
Int[((g_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_
.)*((e_) + (f_.)*(x_)^2), x_Symbol] :> Simp[e*(g*x)^(m + 1)*(a + b*x^2)^(p
+ 1)*((c + d*x^2)^(q + 1)/(a*c*g*(m + 1))), x] + Simp[1/(a*c*g^2*(m + 1))
 Int[(g*x)^(m + 2)*(a + b*x^2)^p*(c + d*x^2)^q*Simp[a*f*c*(m + 1) - e*(b*c
+ a*d)*(m + 2 + 1) - e*2*(b*c*p + a*d*q) - b*e*d*(m + 2*(p + q + 2) + 1)*x^
2, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p, q}, x] && LtQ[m, -1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3667
```
Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(
p_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Simp[ff^(m + 1
)*(Sqrt[Cos[e + f*x]^2]/(f*Cos[e + f*x]))   Subst[Int[x^m*((a + b*ff^2*x^2)
^p/Sqrt[1 - ff^2*x^2]), x], x, Sin[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f,
p}, x] && IntegerQ[m/2] &&  !IntegerQ[p]
```

Maple [A] (verified)

Time = 1.96 (sec) , antiderivative size = 456, normalized size of antiderivative = 1.92

method	result
default	$\frac{2\sqrt{-\frac{b}{a}}\,ab\sinh(fx+e)^6+2\sqrt{-\frac{b}{a}}\,b^2\sinh(fx+e)^6+b\sqrt{\frac{a+b\sinh(fx+e)^2}{a}}\,\sqrt{\frac{\cosh(2fx+2e)}{2}+\frac{1}{2}}\,\mathrm{EllipticF}\left(\sinh(fx+e)\sqrt{-\frac{b}{a}},\sqrt{\frac{a}{b}}\right)a\sinh}{}$
risch	Expression too large to display

input
```
int(csch(f*x+e)^4/(a+b*sinh(f*x+e)^2)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/3*(2*(-b/a)^(1/2)*a*b*sinh(f*x+e)^6+2*(-b/a)^(1/2)*b^2*sinh(f*x+e)^6+b*(
(a+b*sinh(f*x+e)^2)/a)^(1/2)*(cosh(f*x+e)^2)^(1/2)*EllipticF(sinh(f*x+e)*(
-b/a)^(1/2),(1/b*a)^(1/2))*a*sinh(f*x+e)^3+2*((a+b*sinh(f*x+e)^2)/a)^(1/2)
*(cosh(f*x+e)^2)^(1/2)*EllipticF(sinh(f*x+e)*(-b/a)^(1/2),(1/b*a)^(1/2))*b
^2*sinh(f*x+e)^3-2*((a+b*sinh(f*x+e)^2)/a)^(1/2)*(cosh(f*x+e)^2)^(1/2)*Ell
ipticE(sinh(f*x+e)*(-b/a)^(1/2),(1/b*a)^(1/2))*a*b*sinh(f*x+e)^3-2*((a+b*s
inh(f*x+e)^2)/a)^(1/2)*(cosh(f*x+e)^2)^(1/2)*EllipticE(sinh(f*x+e)*(-b/a)^
(1/2),(1/b*a)^(1/2))*b^2*sinh(f*x+e)^3+2*(-b/a)^(1/2)*a^2*sinh(f*x+e)^4+3*
(-b/a)^(1/2)*a*b*sinh(f*x+e)^4+2*(-b/a)^(1/2)*b^2*sinh(f*x+e)^4+(-b/a)^(1/
2)*a^2*sinh(f*x+e)^2+(-b/a)^(1/2)*a*b*sinh(f*x+e)^2-(-b/a)^(1/2)*a^2)/a^2/
sinh(f*x+e)^3/(-b/a)^(1/2)/cosh(f*x+e)/(a+b*sinh(f*x+e)^2)^(1/2)/f
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2131 vs. $2(234) = 468$.

Time = 0.15 (sec) , antiderivative size = 2131, normalized size of antiderivative = 8.95

$$\int \frac{\operatorname{csch}^4(e+fx)}{\sqrt{a+b\sinh^2(e+fx)}}\,dx = \text{Too large to display}$$

input

```
integrate(csch(f*x+e)^4/(a+b*sinh(f*x+e)^2)^(1/2),x, algorithm="fricas")
```

output

```
-2/3*(((2*a^2 + a*b - b^2)*cosh(f*x + e)^6 + 6*(2*a^2 + a*b - b^2)*cosh(f*
x + e)*sinh(f*x + e)^5 + (2*a^2 + a*b - b^2)*sinh(f*x + e)^6 - 3*(2*a^2 +
a*b - b^2)*cosh(f*x + e)^4 + 3*(5*(2*a^2 + a*b - b^2)*cosh(f*x + e)^2 - 2*
a^2 - a*b + b^2)*sinh(f*x + e)^4 + 4*(5*(2*a^2 + a*b - b^2)*cosh(f*x + e)^
3 - 3*(2*a^2 + a*b - b^2)*cosh(f*x + e))*sinh(f*x + e)^3 + 3*(2*a^2 + a*b
- b^2)*cosh(f*x + e)^2 + 3*(5*(2*a^2 + a*b - b^2)*cosh(f*x + e)^4 - 6*(2*a
^2 + a*b - b^2)*cosh(f*x + e)^2 + 2*a^2 + a*b - b^2)*sinh(f*x + e)^2 - 2*a
^2 - a*b + b^2 + 6*((2*a^2 + a*b - b^2)*cosh(f*x + e)^5 - 2*(2*a^2 + a*b -
 b^2)*cosh(f*x + e)^3 + (2*a^2 + a*b - b^2)*cosh(f*x + e))*sinh(f*x + e) -
 2*((a*b + b^2)*cosh(f*x + e)^6 + 6*(a*b + b^2)*cosh(f*x + e)*sinh(f*x + e
)^5 + (a*b + b^2)*sinh(f*x + e)^6 - 3*(a*b + b^2)*cosh(f*x + e)^4 + 3*(5*(
a*b + b^2)*cosh(f*x + e)^2 - a*b - b^2)*sinh(f*x + e)^4 + 4*(5*(a*b + b^2)
*cosh(f*x + e)^3 - 3*(a*b + b^2)*cosh(f*x + e))*sinh(f*x + e)^3 + 3*(a*b +
 b^2)*cosh(f*x + e)^2 + 3*(5*(a*b + b^2)*cosh(f*x + e)^4 - 6*(a*b + b^2)*c
osh(f*x + e)^2 + a*b + b^2)*sinh(f*x + e)^2 - a*b - b^2 + 6*((a*b + b^2)*c
osh(f*x + e)^5 - 2*(a*b + b^2)*cosh(f*x + e)^3 + (a*b + b^2)*cosh(f*x + e)
)*sinh(f*x + e))*sqrt((a^2 - a*b)/b^2))*sqrt(b)*sqrt((2*b*sqrt((a^2 - a*b)
/b^2) - 2*a + b)/b)*elliptic_e(arcsin(sqrt((2*b*sqrt((a^2 - a*b)/b^2) - 2*
a + b)/b)*(cosh(f*x + e) + sinh(f*x + e))), (8*a^2 - 8*a*b + b^2 + 4*(2*a*
b - b^2)*sqrt((a^2 - a*b)/b^2))/b^2) - ((2*a^2 - a*b)*cosh(f*x + e)^6 +...
```

Sympy [F]

$$\int \frac{\operatorname{csch}^4(e+fx)}{\sqrt{a+b\sinh^2(e+fx)}}\,dx=\int \frac{\operatorname{csch}^4(e+fx)}{\sqrt{a+b\sinh^2(e+fx)}}\,dx$$

input
```
integrate(csch(f*x+e)**4/(a+b*sinh(f*x+e)**2)**(1/2),x)
```

output
```
Integral(csch(e + f*x)**4/sqrt(a + b*sinh(e + f*x)**2), x)
```

Maxima [F]

$$\int \frac{\operatorname{csch}^4(e+fx)}{\sqrt{a+b\sinh^2(e+fx)}}\,dx=\int \frac{\operatorname{csch}(fx+e)^4}{\sqrt{b\sinh(fx+e)^2+a}}\,dx$$

input
```
integrate(csch(f*x+e)^4/(a+b*sinh(f*x+e)^2)^(1/2),x, algorithm="maxima")
```

output
```
integrate(csch(f*x + e)^4/sqrt(b*sinh(f*x + e)^2 + a), x)
```

Giac [F]

$$\int \frac{\operatorname{csch}^4(e+fx)}{\sqrt{a+b\sinh^2(e+fx)}}\,dx=\int \frac{\operatorname{csch}(fx+e)^4}{\sqrt{b\sinh(fx+e)^2+a}}\,dx$$

input
```
integrate(csch(f*x+e)^4/(a+b*sinh(f*x+e)^2)^(1/2),x, algorithm="giac")
```

output
```
integrate(csch(f*x + e)^4/sqrt(b*sinh(f*x + e)^2 + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\operatorname{csch}^4(e+fx)}{\sqrt{a+b\sinh^2(e+fx)}}\,dx=\int \frac{1}{\sinh\left(e+f\,x\right)^4\,\sqrt{b\sinh\left(e+f\,x\right)^2+a}}\,dx$$

input
```
int(1/(sinh(e + f*x)^4*(a + b*sinh(e + f*x)^2)^(1/2)),x)
```

output
```
int(1/(sinh(e + f*x)^4*(a + b*sinh(e + f*x)^2)^(1/2)), x)
```

Reduce [F]

$$\int \frac{\operatorname{csch}^4(e+fx)}{\sqrt{a+b\sinh^2(e+fx)}}\,dx=\int \frac{\sqrt{\sinh\left(fx+e\right)^2 b+a}\,\operatorname{csch}(fx+e)^4}{\sinh\left(fx+e\right)^2 b+a}dx$$

input
```
int(csch(f*x+e)^4/(a+b*sinh(f*x+e)^2)^(1/2),x)
```

output
```
int((sqrt(sinh(e + f*x)**2*b + a)*csch(e + f*x)**4)/(sinh(e + f*x)**2*b +
a),x)
```

3.90 $\int \frac{\sinh^3(e+fx)}{\left(a+b\sinh^2(e+fx)\right)^{3/2}}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 83

$$\int \frac{\sinh^3(e+fx)}{\left(a+b\sinh^2(e+fx)\right)^{3/2}}\,dx = \frac{\operatorname{arctanh}\left(\frac{\sqrt{b}\cosh(e+fx)}{\sqrt{a-b+b\cosh^2(e+fx)}}\right)}{b^{3/2}f} - \frac{a\cosh(e+fx)}{(a-b)bf\sqrt{a-b+b\cosh^2(e+fx)}}$$

output
```
arctanh(b^(1/2)*cosh(f*x+e)/(a-b+b*cosh(f*x+e)^2)^(1/2))/b^(3/2)/f-a*cosh(
f*x+e)/(a-b)/b/f/(a-b+b*cosh(f*x+e)^2)^(1/2)
```

Mathematica [A] (verified)

Time = 0.55 (sec) , antiderivative size = 98, normalized size of antiderivative = 1.18

$$\int \frac{\sinh^3(e+fx)}{\left(a+b\sinh^2(e+fx)\right)^{3/2}}\,dx = -\frac{\sqrt{2}a\cosh(e+fx)}{(a-b)bf\sqrt{2a-b+b\cosh(2(e+fx))}} + \frac{\log\left(\sqrt{2}\sqrt{b}\cosh(e+fx)+\sqrt{2a-b+b\cosh(2(e+fx))}\right)}{b^{3/2}f}$$

input

```
Integrate[Sinh[e + f*x]^3/(a + b*Sinh[e + f*x]^2)^(3/2),x]
```

output

```
-((Sqrt[2]*a*Cosh[e + f*x])/((a - b)*b*f*Sqrt[2*a - b + b*Cosh[2*(e + f*x)
]])) + Log[Sqrt[2]*Sqrt[b]*Cosh[e + f*x] + Sqrt[2*a - b + b*Cosh[2*(e + f*
x)]]]/(b^(3/2)*f)
```

Rubi [A] (verified)

Time = 0.34 (sec) , antiderivative size = 82, normalized size of antiderivative = 0.99, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.240, Rules used = {3042, 26, 3665, 298, 224, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sinh^3(e+fx)}{\left(a+b\sinh^2(e+fx)\right)^{3/2}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{i\sin(ie+ifx)^3}{(a-b\sin(ie+ifx)^2)^{3/2}}dx$$

$$\Big\downarrow \text{26}$$

$$i\int \frac{\sin(ie+ifx)^3}{(a-b\sin(ie+ifx)^2)^{3/2}}dx$$

$$\Big\downarrow \text{3665}$$

$$-\frac{\int \frac{1-\cosh^2(e+fx)}{\left(b\cosh^2(e+fx)+a-b\right)^{3/2}}d\cosh(e+fx)}{f}$$

$$\Big\downarrow \text{298}$$

$$-\frac{\frac{a\cosh(e+fx)}{b(a-b)\sqrt{a+b\cosh^2(e+fx)-b}}-\frac{\int \frac{1}{\sqrt{b\cosh^2(e+fx)+a-b}}d\cosh(e+fx)}{b}}{f}$$

$$\Big\downarrow \text{224}$$

$$-\frac{\frac{a\cosh(e+fx)}{b(a-b)\sqrt{a+b\cosh^2(e+fx)-b}}-\frac{\int \frac{1}{1-\frac{b\cosh^2(e+fx)}{b\cosh^2(e+fx)+a-b}}d\frac{\cosh(e+fx)}{\sqrt{b\cosh^2(e+fx)+a-b}}}{b}}{f}$$

$\Big\downarrow$ 219

$$-\frac{\frac{a\cosh(e+fx)}{b(a-b)\sqrt{a+b\cosh^2(e+fx)-b}}-\frac{\text{arctanh}\left(\frac{\sqrt{b}\cosh(e+fx)}{\sqrt{a+b\cosh^2(e+fx)-b}}\right)}{b^{3/2}}}{f}$$

input
```
Int[Sinh[e + f*x]^3/(a + b*Sinh[e + f*x]^2)^(3/2),x]
```

output
```
-((-(ArcTanh[(Sqrt[b]*Cosh[e + f*x])/Sqrt[a - b + b*Cosh[e + f*x]^2]]/b^(3
/2)) + (a*Cosh[e + f*x])/((a - b)*b*Sqrt[a - b + b*Cosh[e + f*x]^2]))/f)
```

Defintions of rubi rules used

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

rule 298
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2), x_Symbol] :> Simp[(-(
b*c - a*d))*x*((a + b*x^2)^(p + 1)/(2*a*b*(p + 1))), x] - Simp[(a*d - b*c*(
2*p + 3))/(2*a*b*(p + 1))   Int[(a + b*x^2)^(p + 1), x], x] /; FreeQ[{a, b,
 c, d, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/2 + p, 0])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3665
```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^
(p_.), x_Symbol] :> With[{ff = FreeFactors[Cos[e + f*x], x]}, Simp[-ff/f
Subst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a + b - b*ff^2*x^2)^p, x], x, Cos[e +
 f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m - 1)/2]
```

Maple [A] (verified)

Time = 0.33 (sec) , antiderivative size = 146, normalized size of antiderivative = 1.76

method	result
default	$\frac{\sqrt{\left(a+b\sinh(fx+e)^2\right)\cosh(fx+e)^2}\left(\frac{\ln\left(\frac{\frac{a}{2}+\frac{b}{2}+b\sinh(fx+e)^2}{\sqrt{b}}+\sqrt{\left(a+b\sinh(fx+e)^2\right)\cosh(fx+e)^2}\right)}{2b^{\frac{3}{2}}}-\frac{a\cosh(fx+e)^2}{b(a-b)\sqrt{\left(a+b\sinh(fx+e)^2\right)\cosh(f\cdots}}\cdots\right.}{\cosh(fx+e)\sqrt{a+b\sinh(fx+e)^2}\,f}$

input
```
int(sinh(f*x+e)^3/(a+b*sinh(f*x+e)^2)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
((a+b*sinh(f*x+e)^2)*cosh(f*x+e)^2)^(1/2)*(1/2/b^(3/2)*ln((1/2*a+1/2*b+b*s
inh(f*x+e)^2)/b^(1/2)+((a+b*sinh(f*x+e)^2)*cosh(f*x+e)^2)^(1/2))-1/b*a*cos
h(f*x+e)^2/(a-b)/((a+b*sinh(f*x+e)^2)*cosh(f*x+e)^2)^(1/2))/cosh(f*x+e)/(a
+b*sinh(f*x+e)^2)^(1/2)/f
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1181 vs. 2(75) = 150.

Time = 0.18 (sec) , antiderivative size = 3038, normalized size of antiderivative = 36.60

$$\int \frac{\sinh^3(e+fx)}{\left(a+b\sinh^2(e+fx)\right)^{3/2}}\,dx = \text{Too large to display}$$

input
```
integrate(sinh(f*x+e)^3/(a+b*sinh(f*x+e)^2)^(3/2),x, algorithm="fricas")
```

output
```
Too large to include
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\sinh^3(e+fx)}{\left(a+b\sinh^2(e+fx)\right)^{3/2}}\,dx = \text{Timed out}$$

input
```
integrate(sinh(f*x+e)**3/(a+b*sinh(f*x+e)**2)**(3/2),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{\sinh^3(e+fx)}{\left(a+b\sinh^2(e+fx)\right)^{3/2}}\,dx = \int \frac{\sinh\left(fx+e\right)^3}{\left(b\sinh\left(fx+e\right)^2+a\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate(sinh(f*x+e)^3/(a+b*sinh(f*x+e)^2)^(3/2),x, algorithm="maxima")
```

output
```
integrate(sinh(f*x + e)^3/(b*sinh(f*x + e)^2 + a)^(3/2), x)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{\sinh^3(e+fx)}{\left(a+b\sinh^2(e+fx)\right)^{3/2}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(sinh(f*x+e)^3/(a+b*sinh(f*x+e)^2)^(3/2),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Degree mismatch inside factorisatio
n over extensionNot implemented, e.g. for multivariate mod/approx polynomi
alsError:
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sinh^3(e+fx)}{\left(a+b\sinh^2(e+fx)\right)^{3/2}}\,dx=\int \frac{\sinh(e+f\,x)^3}{\left(b\sinh\left(e+f\,x\right)^2+a\right)^{3/2}}\,dx$$

input
```
int(sinh(e + f*x)^3/(a + b*sinh(e + f*x)^2)^(3/2),x)
```

output
```
int(sinh(e + f*x)^3/(a + b*sinh(e + f*x)^2)^(3/2), x)
```

Reduce [F]

$$\int \frac{\sinh^3(e+fx)}{\left(a+b\sinh^2(e+fx)\right)^{3/2}}\,dx=\int \frac{\sqrt{\sinh\left(fx+e\right)^2b+a}\,\sinh\left(fx+e\right)^3}{\sinh\left(fx+e\right)^4b^2+2\sinh\left(fx+e\right)^2ab+a^2}dx$$

input
```
int(sinh(f*x+e)^3/(a+b*sinh(f*x+e)^2)^(3/2),x)
```

output
```
int((sqrt(sinh(e + f*x)**2*b + a)*sinh(e + f*x)**3)/(sinh(e + f*x)**4*b**2
 + 2*sinh(e + f*x)**2*a*b + a**2),x)
```

3.91 $\int \frac{\sinh(e+fx)}{\left(a+b\sinh^2(e+fx)\right)^{3/2}}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 36

$$\int \frac{\sinh(e+fx)}{\left(a+b\sinh^2(e+fx)\right)^{3/2}}\,dx = \frac{\cosh(e+fx)}{(a-b)f\sqrt{a-b+b\cosh^2(e+fx)}}$$

output
```
cosh(f*x+e)/(a-b)/f/(a-b+b*cosh(f*x+e)^2)^(1/2)
```

Mathematica [A] (verified)

Time = 0.24 (sec) , antiderivative size = 43, normalized size of antiderivative = 1.19

$$\int \frac{\sinh(e+fx)}{\left(a+b\sinh^2(e+fx)\right)^{3/2}}\,dx = \frac{\sqrt{2}\cosh(e+fx)}{(a-b)f\sqrt{2a-b+b\cosh(2(e+fx))}}$$

input
```
Integrate[Sinh[e + f*x]/(a + b*Sinh[e + f*x]^2)^(3/2),x]
```

output
```
(Sqrt[2]*Cosh[e + f*x])/((a - b)*f*Sqrt[2*a - b + b*Cosh[2*(e + f*x)]])
```

Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.174, Rules used = {3042, 26, 3665, 208}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sinh(e+fx)}{\left(a+b\sinh^2(e+fx)\right)^{3/2}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int -\frac{i\sin(ie+ifx)}{\left(a-b\sin(ie+ifx)^2\right)^{3/2}}dx$$

$$\Big\downarrow \text{26}$$

$$-i\int \frac{\sin(ie+ifx)}{\left(a-b\sin(ie+ifx)^2\right)^{3/2}}dx$$

$$\Big\downarrow \text{3665}$$

$$\frac{\int \frac{1}{\left(b\cosh^2(e+fx)+a-b\right)^{3/2}}d\cosh(e+fx)}{f}$$

$$\Big\downarrow \text{208}$$

$$\frac{\cosh(e+fx)}{f(a-b)\sqrt{a+b\cosh^2(e+fx)-b}}$$

input
```
Int[Sinh[e + f*x]/(a + b*Sinh[e + f*x]^2)^(3/2),x]
```

output
```
Cosh[e + f*x]/((a - b)*f*Sqrt[a - b + b*Cosh[e + f*x]^2])
```

Defintions of rubi rules used

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 208
```
Int[((a_) + (b_.)*(x_)^2)^(-3/2), x_Symbol] :> Simp[x/(a*Sqrt[a + b*x^2]),
x] /; FreeQ[{a, b}, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3665
```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^
(p_.), x_Symbol] :> With[{ff = FreeFactors[Cos[e + f*x], x]}, Simp[-ff/f
Subst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a + b - b*ff^2*x^2)^p, x], x, Cos[e +
 f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m - 1)/2]
```

Maple [A] (verified)

Time = 0.26 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.89

method	result	size
default	$\frac{\cosh(fx+e)}{(a-b)\sqrt{a+b\sinh(fx+e)^2}\, f}$	32

input
```
int(sinh(f*x+e)/(a+b*sinh(f*x+e)^2)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
cosh(f*x+e)/(a-b)/(a+b*sinh(f*x+e)^2)^(1/2)/f
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 296 vs. $2(34) = 68$.

Time = 0.11 (sec) , antiderivative size = 296, normalized size of antiderivative = 8.22

$$\int \frac{\sinh(e+fx)}{\left(a+b\sinh^2(e+fx)\right)^{3/2}}\,dx = \frac{}{(ab-b^2)f\cosh\left(fx+e\right)^4 + 4\left(ab-b^2\right)f\cosh\left(fx+e\right)\sinh\left(fx+e\right)^3 + (\cdots}$$

input
```
integrate(sinh(f*x+e)/(a+b*sinh(f*x+e)^2)^(3/2),x, algorithm="fricas")
```

output
```
sqrt(2)*(cosh(f*x + e)^2 + 2*cosh(f*x + e)*sinh(f*x + e) + sinh(f*x + e)^2
 + 1)*sqrt((b*cosh(f*x + e)^2 + b*sinh(f*x + e)^2 + 2*a - b)/(cosh(f*x + e
)^2 - 2*cosh(f*x + e)*sinh(f*x + e) + sinh(f*x + e)^2))/((a*b - b^2)*f*cos
h(f*x + e)^4 + 4*(a*b - b^2)*f*cosh(f*x + e)*sinh(f*x + e)^3 + (a*b - b^2)
*f*sinh(f*x + e)^4 + 2*(2*a^2 - 3*a*b + b^2)*f*cosh(f*x + e)^2 + 2*(3*(a*b
 - b^2)*f*cosh(f*x + e)^2 + (2*a^2 - 3*a*b + b^2)*f)*sinh(f*x + e)^2 + (a*
b - b^2)*f + 4*((a*b - b^2)*f*cosh(f*x + e)^3 + (2*a^2 - 3*a*b + b^2)*f*co
sh(f*x + e))*sinh(f*x + e))
```

Sympy [F]

$$\int \frac{\sinh(e+fx)}{\left(a+b\sinh^2(e+fx)\right)^{3/2}}\,dx = \int \frac{\sinh\left(e+fx\right)}{\left(a+b\sinh^2\left(e+fx\right)\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate(sinh(f*x+e)/(a+b*sinh(f*x+e)**2)**(3/2),x)
```

output
```
Integral(sinh(e + f*x)/(a + b*sinh(e + f*x)**2)**(3/2), x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 236 vs. $2(34) = 68$.

Time = 0.15 (sec) , antiderivative size = 236, normalized size of antiderivative = 6.56

$$\int \frac{\sinh(e+fx)}{\left(a+b\sinh^2(e+fx)\right)^{3/2}}\,dx = \frac{b^2e^{(-6\,fx-6\,e)} + 2\,ab - b^2 + (8\,a^2 - 8\,ab + 3\,b^2)e^{(-2\,fx-2\,e)} + 3\,(2\,ab - b^2)e^{(\cdots}}{2\,(a^2 - ab)(2\,(2\,a - b)e^{(-2\,fx-2\,e)} + be^{(-4\,fx-4\,e)} + b)^{\frac{3}{2}}f}$$
$$+ \frac{b^2 + 3\,(2\,ab - b^2)e^{(-2\,fx-2\,e)} + (8\,a^2 - 8\,ab + 3\,b^2)e^{(-4\,fx-4\,e)} + (2\,ab - b^2)e^{(-6\,fx-6\,e)}}{2\,(a^2 - ab)(2\,(2\,a - b)e^{(-2\,fx-2\,e)} + be^{(-4\,fx-4\,e)} + b)^{\frac{3}{2}}f}$$

input
```
integrate(sinh(f*x+e)/(a+b*sinh(f*x+e)^2)^(3/2),x, algorithm="maxima")
```

output
```
1/2*(b^2*e^(-6*f*x - 6*e) + 2*a*b - b^2 + (8*a^2 - 8*a*b + 3*b^2)*e^(-2*f*
x - 2*e) + 3*(2*a*b - b^2)*e^(-4*f*x - 4*e))/((a^2 - a*b)*(2*(2*a - b)*e^(
-2*f*x - 2*e) + b*e^(-4*f*x - 4*e) + b)^(3/2)*f) + 1/2*(b^2 + 3*(2*a*b - b
^2)*e^(-2*f*x - 2*e) + (8*a^2 - 8*a*b + 3*b^2)*e^(-4*f*x - 4*e) + (2*a*b -
 b^2)*e^(-6*f*x - 6*e))/((a^2 - a*b)*(2*(2*a - b)*e^(-2*f*x - 2*e) + b*e^(
-4*f*x - 4*e) + b)^(3/2)*f)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 110 vs. $2(34) = 68$.

Time = 0.21 (sec) , antiderivative size = 110, normalized size of antiderivative = 3.06

$$\int \frac{\sinh(e+fx)}{\left(a+b\sinh^2(e+fx)\right)^{3/2}}\,dx = \frac{\frac{afe^{(2\,fx+4\,e)}}{a^2f^2e^{(2\,e)} - abf^2e^{(2\,e)}} + \frac{afe^{(2\,e)}}{a^2f^2e^{(2\,e)} - abf^2e^{(2\,e)}}}{\sqrt{be^{(4\,fx+4\,e)} + 4\,ae^{(2\,fx+2\,e)} - 2\,be^{(2\,fx+2\,e)} + b}}$$

input
```
integrate(sinh(f*x+e)/(a+b*sinh(f*x+e)^2)^(3/2),x, algorithm="giac")
```

output
```
(a*f*e^(2*f*x + 4*e)/(a^2*f^2*e^(2*e) - a*b*f^2*e^(2*e)) + a*f*e^(2*e)/(a^
2*f^2*e^(2*e) - a*b*f^2*e^(2*e)))/sqrt(b*e^(4*f*x + 4*e) + 4*a*e^(2*f*x +
2*e) - 2*b*e^(2*f*x + 2*e) + b)
```

Mupad [B] (verification not implemented)

Time = 1.89 (sec) , antiderivative size = 191, normalized size of antiderivative = 5.31

$$\int \frac{\sinh(e+fx)}{\left(a+b\sinh^2(e+fx)\right)^{3/2}}\,dx = -\frac{e^{e+fx}\sqrt{b\sinh\left(e+fx\right)^2+a}\left(\frac{2\,e^{e+fx}\sinh(e+fx)\,(b\,(2\,a-b)-b\,(4\,a-2\,b))}{f\,(a\,b^2-a^2\,b)}+\frac{2\,b^2\cosh(e+fx)\,e^{e+fx}}{f\,(a\,b^2-a^2\,b)}+\frac{b\,e^{2\,e+2\,f\,x}\,(4\,a-2\,b)}{f\,(a\,b^2-a^2\,b)}\right)}{4\,a\,e^{2\,e+2\,f\,x}-2\,b\,e^{2\,e+2\,f\,x}+2\,b\,e^{2\,e+2\,f\,x}\cosh\left(2\,e+2\,f\,x\right)}$$

input
```
int(sinh(e + f*x)/(a + b*sinh(e + f*x)^2)^(3/2),x)
```

output
```
-(exp(e + f*x)*(a + b*sinh(e + f*x)^2)^(1/2)*((2*exp(e + f*x)*sinh(e + f*x
)*(b*(2*a - b) - b*(4*a - 2*b)))/(f*(a*b^2 - a^2*b)) + (2*b^2*cosh(e + f*x
)*exp(e + f*x))/(f*(a*b^2 - a^2*b)) + (b*exp(2*e + 2*f*x)*(4*a - 2*b))/(f*
(a*b^2 - a^2*b))))/(4*a*exp(2*e + 2*f*x) - 2*b*exp(2*e + 2*f*x) + 2*b*exp(
2*e + 2*f*x)*cosh(2*e + 2*f*x))
```

Reduce [F]

$$\int \frac{\sinh(e+fx)}{\left(a+b\sinh^2(e+fx)\right)^{3/2}}\,dx = \int \frac{\sqrt{\sinh\left(fx+e\right)^2 b+a}\,\sinh\left(fx+e\right)}{\sinh\left(fx+e\right)^4 b^2+2\sinh\left(fx+e\right)^2 ab+a^2}dx$$

input
```
int(sinh(f*x+e)/(a+b*sinh(f*x+e)^2)^(3/2),x)
```

output
```
int((sqrt(sinh(e + f*x)**2*b + a)*sinh(e + f*x))/(sinh(e + f*x)**4*b**2 +
2*sinh(e + f*x)**2*a*b + a**2),x)
```

3.92 $\int \frac{\operatorname{csch}(e+fx)}{(a+b\sinh^2(e+fx))^{3/2}}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 84

$$\int \frac{\operatorname{csch}(e+fx)}{(a+b\sinh^2(e+fx))^{3/2}}\,dx = -\frac{\operatorname{arctanh}\left(\frac{\sqrt{a}\cosh(e+fx)}{\sqrt{a-b+b\cosh^2(e+fx)}}\right)}{a^{3/2}f} - \frac{b\cosh(e+fx)}{a(a-b)f\sqrt{a-b+b\cosh^2(e+fx)}}$$

output
```
-arctanh(a^(1/2)*cosh(f*x+e)/(a-b+b*cosh(f*x+e)^2)^(1/2))/a^(3/2)/f-b*cosh
(f*x+e)/a/(a-b)/f/(a-b+b*cosh(f*x+e)^2)^(1/2)
```

Mathematica [A] (verified)

Time = 0.47 (sec) , antiderivative size = 98, normalized size of antiderivative = 1.17

$$\int \frac{\operatorname{csch}(e+fx)}{(a+b\sinh^2(e+fx))^{3/2}}\,dx = \frac{-\operatorname{arctanh}\left(\frac{\sqrt{2}\sqrt{a}\cosh(e+fx)}{\sqrt{2a-b+b\cosh(2(e+fx))}}\right) - \frac{\sqrt{2}\sqrt{a}b\cosh(e+fx)}{(a-b)\sqrt{2a-b+b\cosh(2(e+fx))}}}{a^{3/2}f}$$

input
```
Integrate[Csch[e + f*x]/(a + b*Sinh[e + f*x]^2)^(3/2),x]
```

output
```
(-ArcTanh[(Sqrt[2]*Sqrt[a]*Cosh[e + f*x])/Sqrt[2*a - b + b*Cosh[2*(e + f*x
)]]] - (Sqrt[2]*Sqrt[a]*b*Cosh[e + f*x])/((a - b)*Sqrt[2*a - b + b*Cosh[2*
(e + f*x)]]))/(a^(3/2)*f)
```

Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 81, normalized size of antiderivative = 0.96, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.261$, Rules used = {3042, 26, 3665, 296, 291, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\text{csch}(e+fx)}{\left(a+b\sinh^2(e+fx)\right)^{3/2}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{i}{\sin(ie+ifx)\left(a-b\sin(ie+ifx)^2\right)^{3/2}}dx$$

$$\Big\downarrow \text{26}$$

$$i\int \frac{1}{\sin(ie+ifx)\left(a-b\sin(ie+ifx)^2\right)^{3/2}}dx$$

$$\Big\downarrow \text{3665}$$

$$-\frac{\int \frac{1}{\left(1-\cosh^2(e+fx)\right)\left(b\cosh^2(e+fx)+a-b\right)^{3/2}}d\cosh(e+fx)}{f}$$

$$\Big\downarrow \text{296}$$

$$-\frac{\frac{\int \frac{1}{\left(1-\cosh^2(e+fx)\right)\sqrt{b\cosh^2(e+fx)+a-b}}d\cosh(e+fx)}{a}+\frac{b\cosh(e+fx)}{a(a-b)\sqrt{a+b\cosh^2(e+fx)-b}}}{f}$$

$$\Big\downarrow \text{291}$$

$$-\frac{\frac{\int \frac{1}{1-\frac{a\cosh^2(e+fx)}{b\cosh^2(e+fx)+a-b}}d\frac{\cosh(e+fx)}{\sqrt{b\cosh^2(e+fx)+a-b}}}{a}+\frac{b\cosh(e+fx)}{a(a-b)\sqrt{a+b\cosh^2(e+fx)-b}}}{f}$$

$$\Big\downarrow \text{219}$$

$$-\frac{\frac{\operatorname{arctanh}\left(\frac{\sqrt{a}\cosh(e+fx)}{\sqrt{a+b\cosh^2(e+fx)-b}}\right)}{a^{3/2}}+\frac{b\cosh(e+fx)}{a(a-b)\sqrt{a+b\cosh^2(e+fx)-b}}}{f}$$

input
```
Int[Csch[e + f*x]/(a + b*Sinh[e + f*x]^2)^(3/2),x]
```

output
```
-((ArcTanh[(Sqrt[a]*Cosh[e + f*x])/Sqrt[a - b + b*Cosh[e + f*x]^2]]/a^(3/2
) + (b*Cosh[e + f*x])/(a*(a - b)*Sqrt[a - b + b*Cosh[e + f*x]^2]))/f)
```

Defintions of rubi rules used

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 291
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*((c_) + (d_.)*(x_)^2)), x_Symbol] :> Subst
[Int[1/(c - (b*c - a*d)*x^2), x], x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b, c,
d}, x] && NeQ[b*c - a*d, 0]
```

rule 296
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_), x_Symbol] :> Sim
p[(-b)*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^(q + 1)/(2*a*(p + 1)*(b*c - a*d))
), x] + Simp[(b*c + 2*(p + 1)*(b*c - a*d))/(2*a*(p + 1)*(b*c - a*d))   Int[
(a + b*x^2)^(p + 1)*(c + d*x^2)^q, x], x] /; FreeQ[{a, b, c, d, q}, x] && N
eQ[b*c - a*d, 0] && EqQ[2*(p + q + 2) + 1, 0] && (LtQ[p, -1] ||  !LtQ[q, -1
]) && NeQ[p, -1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3665
```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^
(p_.), x_Symbol] :> With[{ff = FreeFactors[Cos[e + f*x], x]}, Simp[-ff/f
Subst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a + b - b*ff^2*x^2)^p, x], x, Cos[e +
 f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m - 1)/2]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 153 vs. $2(76) = 152$.

Time = 0.34 (sec) , antiderivative size = 154, normalized size of antiderivative = 1.83

method	result
default	$\frac{\sqrt{\left(a+b\sinh(fx+e)^2\right)\cosh(fx+e)^2}\left(-\frac{\ln\left(\frac{2a+(a+b)\sinh(fx+e)^2+2\sqrt{a}\sqrt{\left(a+b\sinh(fx+e)^2\right)\cosh(fx+e)^2}}{\sinh(fx+e)^2}\right)}{2a^{\frac{3}{2}}}-\frac{b\cosh(fx+e)}{a(a-b)\sqrt{\left(a+b\sinh(fx+e)\right.}}\right.}{\cosh(fx+e)\sqrt{a+b\sinh(fx+e)^2}\, f}$

input
```
int(csch(f*x+e)/(a+b*sinh(f*x+e)^2)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
((a+b*sinh(f*x+e)^2)*cosh(f*x+e)^2)^(1/2)*(-1/2/a^(3/2)*ln((2*a+(a+b)*sinh
(f*x+e)^2+2*a^(1/2)*((a+b*sinh(f*x+e)^2)*cosh(f*x+e)^2)^(1/2))/sinh(f*x+e)
^2)-b/a*cosh(f*x+e)^2/(a-b)/((a+b*sinh(f*x+e)^2)*cosh(f*x+e)^2)^(1/2))/cos
h(f*x+e)/(a+b*sinh(f*x+e)^2)^(1/2)/f
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 657 vs. $2(76) = 152$.

Time = 0.14 (sec) , antiderivative size = 1529, normalized size of antiderivative = 18.20

$$\int \frac{\operatorname{csch}(e+fx)}{\left(a+b\sinh^2(e+fx)\right)^{3/2}}\,dx = \text{Too large to display}$$

input
```
integrate(csch(f*x+e)/(a+b*sinh(f*x+e)^2)^(3/2),x, algorithm="fricas")
```

output
```
[1/2*(((a*b - b^2)*cosh(f*x + e)^4 + 4*(a*b - b^2)*cosh(f*x + e)*sinh(f*x
+ e)^3 + (a*b - b^2)*sinh(f*x + e)^4 + 2*(2*a^2 - 3*a*b + b^2)*cosh(f*x +
e)^2 + 2*(3*(a*b - b^2)*cosh(f*x + e)^2 + 2*a^2 - 3*a*b + b^2)*sinh(f*x +
e)^2 + a*b - b^2 + 4*((a*b - b^2)*cosh(f*x + e)^3 + (2*a^2 - 3*a*b + b^2)*
cosh(f*x + e))*sinh(f*x + e))*sqrt(a)*log(-((a + b)*cosh(f*x + e)^4 + 4*(a
 + b)*cosh(f*x + e)*sinh(f*x + e)^3 + (a + b)*sinh(f*x + e)^4 + 2*(3*a - b
)*cosh(f*x + e)^2 + 2*(3*(a + b)*cosh(f*x + e)^2 + 3*a - b)*sinh(f*x + e)^
2 - 2*sqrt(2)*(cosh(f*x + e)^2 + 2*cosh(f*x + e)*sinh(f*x + e) + sinh(f*x
+ e)^2 + 1)*sqrt(a)*sqrt((b*cosh(f*x + e)^2 + b*sinh(f*x + e)^2 + 2*a - b)
/(cosh(f*x + e)^2 - 2*cosh(f*x + e)*sinh(f*x + e) + sinh(f*x + e)^2)) + 4*
((a + b)*cosh(f*x + e)^3 + (3*a - b)*cosh(f*x + e))*sinh(f*x + e) + a + b)
/(cosh(f*x + e)^4 + 4*cosh(f*x + e)*sinh(f*x + e)^3 + sinh(f*x + e)^4 + 2*
(3*cosh(f*x + e)^2 - 1)*sinh(f*x + e)^2 - 2*cosh(f*x + e)^2 + 4*(cosh(f*x
+ e)^3 - cosh(f*x + e))*sinh(f*x + e) + 1)) - 2*sqrt(2)*(a*b*cosh(f*x + e)
^2 + 2*a*b*cosh(f*x + e)*sinh(f*x + e) + a*b*sinh(f*x + e)^2 + a*b)*sqrt((
b*cosh(f*x + e)^2 + b*sinh(f*x + e)^2 + 2*a - b)/(cosh(f*x + e)^2 - 2*cosh
(f*x + e)*sinh(f*x + e) + sinh(f*x + e)^2)))/((a^3*b - a^2*b^2)*f*cosh(f*x
 + e)^4 + 4*(a^3*b - a^2*b^2)*f*cosh(f*x + e)*sinh(f*x + e)^3 + (a^3*b - a
^2*b^2)*f*sinh(f*x + e)^4 + 2*(2*a^4 - 3*a^3*b + a^2*b^2)*f*cosh(f*x + e)^
2 + 2*(3*(a^3*b - a^2*b^2)*f*cosh(f*x + e)^2 + (2*a^4 - 3*a^3*b + a^2*b...
```

Sympy [F]

$$\int \frac{\operatorname{csch}(e+fx)}{\left(a+b\sinh^2(e+fx)\right)^{3/2}}\,dx=\int \frac{\operatorname{csch}\left(e+fx\right)}{\left(a+b\sinh^2\left(e+fx\right)\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate(csch(f*x+e)/(a+b*sinh(f*x+e)**2)**(3/2),x)
```

output
```
Integral(csch(e + f*x)/(a + b*sinh(e + f*x)**2)**(3/2), x)
```

Maxima [F]

$$\int \frac{\operatorname{csch}(e+fx)}{\left(a+b\sinh^2(e+fx)\right)^{3/2}}\,dx = \int \frac{\operatorname{csch}\left(fx+e\right)}{\left(b\sinh\left(fx+e\right)^2+a\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate(csch(f*x+e)/(a+b*sinh(f*x+e)^2)^(3/2),x, algorithm="maxima")
```

output
```
integrate(csch(f*x + e)/(b*sinh(f*x + e)^2 + a)^(3/2), x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 212 vs. $2(76) = 152$.

Time = 0.25 (sec) , antiderivative size = 212, normalized size of antiderivative = 2.52

$$\int \frac{\operatorname{csch}(e+fx)}{\left(a+b\sinh^2(e+fx)\right)^{3/2}}\,dx =$$

$$-\left(\frac{\frac{a^2bfe^{(2fx+4e)}}{a^4f^2e^{(6e)}-a^3bf^2e^{(6e)}}+\frac{a^2bfe^{(2e)}}{a^4f^2e^{(6e)}-a^3bf^2e^{(6e)}}}{\sqrt{be^{(4fx+4e)}+4ae^{(2fx+2e)}-2be^{(2fx+2e)}+b}}-\frac{2\arctan\left(-\frac{\sqrt{b}e^{(2fx+2e)}-\sqrt{be^{(4fx+4e)}+4ae^{(2fx+2e)}-2be^{(2fx+2}}{2\sqrt{-a}}\right.}{\sqrt{-a}af}\right.$$

input
```
integrate(csch(f*x+e)/(a+b*sinh(f*x+e)^2)^(3/2),x, algorithm="giac")
```

output
```
-((a^2*b*f*e^(2*f*x + 4*e)/(a^4*f^2*e^(6*e) - a^3*b*f^2*e^(6*e)) + a^2*b*f
*e^(2*e)/(a^4*f^2*e^(6*e) - a^3*b*f^2*e^(6*e)))/sqrt(b*e^(4*f*x + 4*e) + 4
*a*e^(2*f*x + 2*e) - 2*b*e^(2*f*x + 2*e) + b) - 2*arctan(-1/2*(sqrt(b)*e^(
2*f*x + 2*e) - sqrt(b*e^(4*f*x + 4*e) + 4*a*e^(2*f*x + 2*e) - 2*b*e^(2*f*x
 + 2*e) + b) - sqrt(b))/sqrt(-a))*e^(-4*e)/(sqrt(-a)*a*f))*e^(4*e)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\operatorname{csch}(e+fx)}{\left(a+b\sinh^2(e+fx)\right)^{3/2}}\,dx=\int \frac{1}{\sinh\left(e+f\,x\right)\,\left(b\sinh\left(e+f\,x\right)^2+a\right)^{3/2}}\,dx$$

input
```
int(1/(sinh(e + f*x)*(a + b*sinh(e + f*x)^2)^(3/2)),x)
```

output
```
int(1/(sinh(e + f*x)*(a + b*sinh(e + f*x)^2)^(3/2)), x)
```

Reduce [F]

$$\int \frac{\operatorname{csch}(e+fx)}{\left(a+b\sinh^2(e+fx)\right)^{3/2}}\,dx=\int \frac{\sqrt{\sinh\left(fx+e\right)^2 b+a}\,\operatorname{csch}(fx+e)}{\sinh\left(fx+e\right)^4 b^2+2\sinh\left(fx+e\right)^2 ab+a^2}dx$$

input
```
int(csch(f*x+e)/(a+b*sinh(f*x+e)^2)^(3/2),x)
```

output
```
int((sqrt(sinh(e + f*x)**2*b + a)*csch(e + f*x))/(sinh(e + f*x)**4*b**2 +
2*sinh(e + f*x)**2*a*b + a**2),x)
```

3.93 $\int \frac{\operatorname{csch}^3(e+fx)}{\left(a+b\sinh^2(e+fx)\right)^{3/2}}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 139

$$\int \frac{\operatorname{csch}^3(e+fx)}{\left(a+b\sinh^2(e+fx)\right)^{3/2}}\,dx = \frac{(a+3b)\operatorname{arctanh}\left(\frac{\sqrt{a}\cosh(e+fx)}{\sqrt{a-b+b\cosh^2(e+fx)}}\right)}{2a^{5/2}f} - \frac{(a-3b)b\cosh(e+fx)}{2a^2(a-b)f\sqrt{a-b+b\cosh^2(e+fx)}} - \frac{\coth(e+fx)\operatorname{csch}(e+fx)}{2af\sqrt{a-b+b\cosh^2(e+fx)}}$$

output
```
1/2*(a+3*b)*arctanh(a^(1/2)*cosh(f*x+e)/(a-b+b*cosh(f*x+e)^2)^(1/2))/a^(5/
2)/f-1/2*(a-3*b)*b*cosh(f*x+e)/a^2/(a-b)/f/(a-b+b*cosh(f*x+e)^2)^(1/2)-1/2
*coth(f*x+e)*csch(f*x+e)/a/f/(a-b+b*cosh(f*x+e)^2)^(1/2)
```

Mathematica [A] (verified)

Time = 0.89 (sec) , antiderivative size = 134, normalized size of antiderivative = 0.96

$$\int \frac{\operatorname{csch}^3(e+fx)}{\left(a+b\sinh^2(e+fx)\right)^{3/2}}\,dx = \frac{\frac{(a+3b)\operatorname{arctanh}\left(\frac{\sqrt{2}\sqrt{a}\cosh(e+fx)}{\sqrt{2a-b+b\cosh(2(e+fx))}}\right)}{a^{5/2}} - \frac{(2a^2-3ab+3b^2+(a-3b)b\cosh(2(e+fx)))\coth(e+}{a^2(a-b)\sqrt{4a-2b+2b\cosh(2(e+fx)}}}{2f}$$

input
```
Integrate[Csch[e + f*x]^3/(a + b*Sinh[e + f*x]^2)^(3/2),x]
```

output
```
(((a + 3*b)*ArcTanh[(Sqrt[2]*Sqrt[a]*Cosh[e + f*x])/Sqrt[2*a - b + b*Cosh[
2*(e + f*x)]]])/a^(5/2) - ((2*a^2 - 3*a*b + 3*b^2 + (a - 3*b)*b*Cosh[2*(e
+ f*x)])*Coth[e + f*x]*Csch[e + f*x])/(a^2*(a - b)*Sqrt[4*a - 2*b + 2*b*Co
sh[2*(e + f*x)]]))/(2*f)
```

Rubi [A] (verified)

Time = 0.37 (sec) , antiderivative size = 145, normalized size of antiderivative = 1.04, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.360$, Rules used = {3042, 26, 3665, 316, 402, 25, 27, 291, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\operatorname{csch}^3(e+fx)}{\left(a+b\sinh^2(e+fx)\right)^{3/2}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int -\frac{i}{\sin(ie+ifx)^3\left(a-b\sin(ie+ifx)^2\right)^{3/2}}dx$$

$$\Big\downarrow \text{26}$$

$$-i\int \frac{1}{\sin(ie+ifx)^3\left(a-b\sin(ie+ifx)^2\right)^{3/2}}dx$$

$$\Big\downarrow \text{3665}$$

$$\frac{\int \frac{1}{\left(1-\cosh^2(e+fx)\right)^2\left(b\cosh^2(e+fx)+a-b\right)^{3/2}}d\cosh(e+fx)}{f}$$

$$\Big\downarrow \text{316}$$

$$\frac{\frac{\int \frac{2b\cosh^2(e+fx)+a+b}{\left(1-\cosh^2(e+fx)\right)\left(b\cosh^2(e+fx)+a-b\right)^{3/2}}d\cosh(e+fx)}{2a}+\frac{\cosh(e+fx)}{2a\left(1-\cosh^2(e+fx)\right)\sqrt{a+b\cosh^2(e+fx)-b}}}{f}$$

$$\Big\downarrow \text{402}$$

$$\frac{\frac{-\frac{\int -\frac{(a-b)(a+3b)}{\left(1-\cosh^2(e+fx)\right)\sqrt{b\cosh^2(e+fx)+a-b}}d\cosh(e+fx)}{a(a-b)}-\frac{b(a-3b)\cosh(e+fx)}{a(a-b)\sqrt{a+b\cosh^2(e+fx)-b}}}{2a}+\frac{\cosh(e+fx)}{2a(1-\cosh^2(e+fx))\sqrt{a+b\cosh^2(e+fx)-b}}}{f}$$

$$\Big\downarrow \; 25$$

$$\frac{\frac{\frac{\int \frac{(a-b)(a+3b)}{\left(1-\cosh^2(e+fx)\right)\sqrt{b\cosh^2(e+fx)+a-b}}d\cosh(e+fx)}{a(a-b)}-\frac{b(a-3b)\cosh(e+fx)}{a(a-b)\sqrt{a+b\cosh^2(e+fx)-b}}}{2a}+\frac{\cosh(e+fx)}{2a(1-\cosh^2(e+fx))\sqrt{a+b\cosh^2(e+fx)-b}}}{f}$$

$$\Big\downarrow \; 27$$

$$\frac{\frac{\frac{(a+3b)\int \frac{1}{\left(1-\cosh^2(e+fx)\right)\sqrt{b\cosh^2(e+fx)+a-b}}d\cosh(e+fx)}{a}-\frac{b(a-3b)\cosh(e+fx)}{a(a-b)\sqrt{a+b\cosh^2(e+fx)-b}}}{2a}+\frac{\cosh(e+fx)}{2a(1-\cosh^2(e+fx))\sqrt{a+b\cosh^2(e+fx)-b}}}{f}$$

$$\Big\downarrow \; 291$$

$$\frac{\frac{\frac{(a+3b)\int \frac{1}{1-\frac{a\cosh^2(e+fx)}{b\cosh^2(e+fx)+a-b}}d\frac{\cosh(e+fx)}{\sqrt{b\cosh^2(e+fx)+a-b}}}{a}-\frac{b(a-3b)\cosh(e+fx)}{a(a-b)\sqrt{a+b\cosh^2(e+fx)-b}}}{2a}+\frac{\cosh(e+fx)}{2a(1-\cosh^2(e+fx))\sqrt{a+b\cosh^2(e+fx)-b}}}{f}$$

$$\Big\downarrow \; 219$$

$$\frac{\frac{\frac{(a+3b)\text{arctanh}\left(\frac{\sqrt{a}\cosh(e+fx)}{\sqrt{a+b\cosh^2(e+fx)-b}}\right)}{a^{3/2}}-\frac{b(a-3b)\cosh(e+fx)}{a(a-b)\sqrt{a+b\cosh^2(e+fx)-b}}}{2a}+\frac{\cosh(e+fx)}{2a(1-\cosh^2(e+fx))\sqrt{a+b\cosh^2(e+fx)-b}}}{f}$$

input

```
Int[Csch[e + f*x]^3/(a + b*Sinh[e + f*x]^2)^(3/2),x]
```

output
```
(Cosh[e + f*x]/(2*a*(1 - Cosh[e + f*x]^2)*Sqrt[a - b + b*Cosh[e + f*x]^2])
 + (((a + 3*b)*ArcTanh[(Sqrt[a]*Cosh[e + f*x])/Sqrt[a - b + b*Cosh[e + f*x
]^2]])/a^(3/2) - ((a - 3*b)*b*Cosh[e + f*x])/(a*(a - b)*Sqrt[a - b + b*Cos
h[e + f*x]^2]))/(2*a))/f
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 291
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*((c_) + (d_.)*(x_)^2)), x_Symbol] :> Subst
[Int[1/(c - (b*c - a*d)*x^2), x], x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b, c,
d}, x] && NeQ[b*c - a*d, 0]
```

rule 316
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_), x_Symbol] :> Sim
p[(-b)*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^(q + 1)/(2*a*(p + 1)*(b*c - a*d))
), x] + Simp[1/(2*a*(p + 1)*(b*c - a*d))   Int[(a + b*x^2)^(p + 1)*(c + d*x
^2)^q*Simp[b*c + 2*(p + 1)*(b*c - a*d) + d*b*(2*(p + q + 2) + 1)*x^2, x], x
], x] /; FreeQ[{a, b, c, d, q}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] &&  !
( !IntegerQ[p] && IntegerQ[q] && LtQ[q, -1]) && IntBinomialQ[a, b, c, d, 2,
 p, q, x]
```

rule 402
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_.)*((e_) + (f_.)*(x
_)^2), x_Symbol] :> Simp[(-(b*e - a*f))*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^
(q + 1)/(a*2*(b*c - a*d)*(p + 1))), x] + Simp[1/(a*2*(b*c - a*d)*(p + 1))
 Int[(a + b*x^2)^(p + 1)*(c + d*x^2)^q*Simp[c*(b*e - a*f) + e*2*(b*c - a*d)
*(p + 1) + d*(b*e - a*f)*(2*(p + q + 2) + 1)*x^2, x], x], x] /; FreeQ[{a, b
, c, d, e, f, q}, x] && LtQ[p, -1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3665
```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^
(p_.), x_Symbol] :> With[{ff = FreeFactors[Cos[e + f*x], x]}, Simp[-ff/f
Subst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a + b - b*ff^2*x^2)^p, x], x, Cos[e +
 f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m - 1)/2]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 250 vs. $2(123) = 246$.

Time = 0.27 (sec) , antiderivative size = 251, normalized size of antiderivative = 1.81

$$\frac{\sqrt{\left(a+b\sinh\left(fx+e\right)^{2}\right)\cosh\left(fx+e\right)^{2}}\left(-\frac{\sqrt{\left(a+b\sinh(fx+e)^{2}\right)\cosh(fx+e)^{2}}}{2a^{2}\sinh(fx+e)^{2}}+\frac{\ln\left(\frac{2a+(a+b)\sinh(fx+e)^{2}+2\sqrt{a}\sqrt{\left(a+b\sinh(fx-\right.}}{\sinh(fx+e)^{2}}\right.}{4a^{\frac{3}{2}}}\right.}{\cosh\left(fx+e\right)\sqrt{a+}}$$

input
```
int(csch(f*x+e)^3/(a+b*sinh(f*x+e)^2)^(3/2),x)
```

output
```
((a+b*sinh(f*x+e)^2)*cosh(f*x+e)^2)^(1/2)*(-1/2/a^2/sinh(f*x+e)^2*((a+b*si
nh(f*x+e)^2)*cosh(f*x+e)^2)^(1/2)+1/4/a^(3/2)*ln((2*a+(a+b)*sinh(f*x+e)^2+
2*a^(1/2)*((a+b*sinh(f*x+e)^2)*cosh(f*x+e)^2)^(1/2))/sinh(f*x+e)^2)+3/4*b/
a^(5/2)*ln((2*a+(a+b)*sinh(f*x+e)^2+2*a^(1/2)*((a+b*sinh(f*x+e)^2)*cosh(f*
x+e)^2)^(1/2))/sinh(f*x+e)^2)+b^2/a^2*cosh(f*x+e)^2/(a-b)/((a+b*sinh(f*x+e
)^2)*cosh(f*x+e)^2)^(1/2))/cosh(f*x+e)/(a+b*sinh(f*x+e)^2)^(1/2)/f
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2057 vs. $2(123) = 246$.

Time = 0.38 (sec) , antiderivative size = 4329, normalized size of antiderivative = 31.14

$$\int \frac{\operatorname{csch}^3(e+fx)}{(a+b\sinh^2(e+fx))^{3/2}}\,dx = \text{Too large to display}$$

input
```
integrate(csch(f*x+e)^3/(a+b*sinh(f*x+e)^2)^(3/2),x, algorithm="fricas")
```

output
```
Too large to include
```

Sympy [F]

$$\int \frac{\operatorname{csch}^3(e+fx)}{(a+b\sinh^2(e+fx))^{3/2}}\,dx = \int \frac{\operatorname{csch}^3(e+fx)}{(a+b\sinh^2(e+fx))^{\frac{3}{2}}}\,dx$$

input
```
integrate(csch(f*x+e)**3/(a+b*sinh(f*x+e)**2)**(3/2),x)
```

output
```
Integral(csch(e + f*x)**3/(a + b*sinh(e + f*x)**2)**(3/2), x)
```

Maxima [F]

$$\int \frac{\operatorname{csch}^3(e+fx)}{(a+b\sinh^2(e+fx))^{3/2}}\,dx = \int \frac{\operatorname{csch}(fx+e)^3}{(b\sinh(fx+e)^2+a)^{\frac{3}{2}}}\,dx$$

input
```
integrate(csch(f*x+e)^3/(a+b*sinh(f*x+e)^2)^(3/2),x, algorithm="maxima")
```

output
```
integrate(csch(f*x + e)^3/(b*sinh(f*x + e)^2 + a)^(3/2), x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 798 vs. $2(123) = 246$.

Time = 0.50 (sec) , antiderivative size = 798, normalized size of antiderivative = 5.74

$$\int \frac{\operatorname{csch}^3(e+fx)}{\left(a+b\sinh^2(e+fx)\right)^{3/2}}\,dx = \text{Too large to display}$$

input
```
integrate(csch(f*x+e)^3/(a+b*sinh(f*x+e)^2)^(3/2),x, algorithm="giac")
```

output
```
((a^3*b^2*f*e^(2*f*x + 6*e)/(a^6*f^2*e^(10*e) - a^5*b*f^2*e^(10*e)) + a^3*
b^2*f*e^(4*e)/(a^6*f^2*e^(10*e) - a^5*b*f^2*e^(10*e)))/sqrt(b*e^(4*f*x + 4
*e) + 4*a*e^(2*f*x + 2*e) - 2*b*e^(2*f*x + 2*e) + b) - (a + 3*b)*arctan(-1
/2*(sqrt(b)*e^(2*f*x + 2*e) - sqrt(b*e^(4*f*x + 4*e) + 4*a*e^(2*f*x + 2*e)
 - 2*b*e^(2*f*x + 2*e) + b) - sqrt(b))/sqrt(-a))*e^(-6*e)/(sqrt(-a)*a^2*f)
 + 2*((sqrt(b)*e^(2*f*x + 2*e) - sqrt(b*e^(4*f*x + 4*e) + 4*a*e^(2*f*x + 2
*e) - 2*b*e^(2*f*x + 2*e) + b))^3*a + (sqrt(b)*e^(2*f*x + 2*e) - sqrt(b*e^
(4*f*x + 4*e) + 4*a*e^(2*f*x + 2*e) - 2*b*e^(2*f*x + 2*e) + b))^3*b + 5*(s
qrt(b)*e^(2*f*x + 2*e) - sqrt(b*e^(4*f*x + 4*e) + 4*a*e^(2*f*x + 2*e) - 2*
b*e^(2*f*x + 2*e) + b))^2*a*sqrt(b) - 3*(sqrt(b)*e^(2*f*x + 2*e) - sqrt(b*
e^(4*f*x + 4*e) + 4*a*e^(2*f*x + 2*e) - 2*b*e^(2*f*x + 2*e) + b))^2*b^(3/2
) + 4*(sqrt(b)*e^(2*f*x + 2*e) - sqrt(b*e^(4*f*x + 4*e) + 4*a*e^(2*f*x + 2
*e) - 2*b*e^(2*f*x + 2*e) + b))*a^2 - 9*(sqrt(b)*e^(2*f*x + 2*e) - sqrt(b*
e^(4*f*x + 4*e) + 4*a*e^(2*f*x + 2*e) - 2*b*e^(2*f*x + 2*e) + b))*a*b + 3*
(sqrt(b)*e^(2*f*x + 2*e) - sqrt(b*e^(4*f*x + 4*e) + 4*a*e^(2*f*x + 2*e) -
2*b*e^(2*f*x + 2*e) + b))*b^2 - 4*a^2*sqrt(b) + 3*a*b^(3/2) - b^(5/2))*e^(
-6*e)/(((sqrt(b)*e^(2*f*x + 2*e) - sqrt(b*e^(4*f*x + 4*e) + 4*a*e^(2*f*x +
 2*e) - 2*b*e^(2*f*x + 2*e) + b))^2 - 2*(sqrt(b)*e^(2*f*x + 2*e) - sqrt(b*
e^(4*f*x + 4*e) + 4*a*e^(2*f*x + 2*e) - 2*b*e^(2*f*x + 2*e) + b))*sqrt(b)
- 4*a + b)^2*a^2*f))*e^(6*e)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\operatorname{csch}^3(e+fx)}{\left(a+b\sinh^2(e+fx)\right)^{3/2}}\,dx=\int \frac{1}{\sinh\left(e+f\,x\right)^3\,{\left(b\,{\sinh\left(e+f\,x\right)}^2+a\right)}^{3/2}} \,d x$$

input
```
int(1/(sinh(e + f*x)^3*(a + b*sinh(e + f*x)^2)^(3/2)),x)
```

output
```
int(1/(sinh(e + f*x)^3*(a + b*sinh(e + f*x)^2)^(3/2)), x)
```

Reduce [F]

$$\int \frac{\operatorname{csch}^3(e+fx)}{\left(a+b\sinh^2(e+fx)\right)^{3/2}}\,dx=\int \frac{\sqrt{\sinh\left(fx+e\right)^{2} b+a}\,\mathrm{csch}\left(fx+e\right)^{3}}{\sinh\left(fx+e\right)^{4} b^{2}+2\sinh\left(fx+e\right)^{2} a b+a^{2}}d x$$

input
```
int(csch(f*x+e)^3/(a+b*sinh(f*x+e)^2)^(3/2),x)
```

output
```
int((sqrt(sinh(e + f*x)**2*b + a)*csch(e + f*x)**3)/(sinh(e + f*x)**4*b**2
 + 2*sinh(e + f*x)**2*a*b + a**2),x)
```

3.94 $\int \frac{\sinh^6(e+fx)}{\left(a+b\sinh^2(e+fx)\right)^{3/2}}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 341

$$\begin{aligned}\int \frac{\sinh^6(e+fx)}{\left(a+b\sinh^2(e+fx)\right)^{3/2}}\,dx &= -\frac{a\cosh(e+fx)\sinh^3(e+fx)}{(a-b)bf\sqrt{a+b\sinh^2(e+fx)}} \\ &+ \frac{(4a-b)\cosh(e+fx)\sinh(e+fx)\sqrt{a+b\sinh^2(e+fx)}}{3(a-b)b^2f} \\ &+ \frac{(8a^2-3ab-2b^2)\,E\left(\arctan(\sinh(e+fx))\,\middle|\,1-\frac{b}{a}\right)\operatorname{sech}(e+fx)\sqrt{a+b\sinh^2(e+fx)}}{3(a-b)b^3f\sqrt{\frac{\operatorname{sech}^2(e+fx)\left(a+b\sinh^2(e+fx)\right)}{a}}} \\ &- \frac{(4a-b)\operatorname{EllipticF}\left(\arctan(\sinh(e+fx)),1-\frac{b}{a}\right)\operatorname{sech}(e+fx)\sqrt{a+b\sinh^2(e+fx)}}{3(a-b)b^2f\sqrt{\frac{\operatorname{sech}^2(e+fx)\left(a+b\sinh^2(e+fx)\right)}{a}}} \\ &- \frac{(8a^2-3ab-2b^2)\sqrt{a+b\sinh^2(e+fx)}\tanh(e+fx)}{3(a-b)b^3f}\end{aligned}$$

output
```
-a*cosh(f*x+e)*sinh(f*x+e)^3/(a-b)/b/f/(a+b*sinh(f*x+e)^2)^(1/2)+1/3*(4*a-
b)*cosh(f*x+e)*sinh(f*x+e)*(a+b*sinh(f*x+e)^2)^(1/2)/(a-b)/b^2/f+1/3*(8*a^
2-3*a*b-2*b^2)*EllipticE(sinh(f*x+e)/(1+sinh(f*x+e)^2)^(1/2),(1-b/a)^(1/2)
)*sech(f*x+e)*(a+b*sinh(f*x+e)^2)^(1/2)/(a-b)/b^3/f/(sech(f*x+e)^2*(a+b*si
nh(f*x+e)^2)/a)^(1/2)-1/3*(4*a-b)*InverseJacobiAM(arctan(sinh(f*x+e)),(1-b
/a)^(1/2))*sech(f*x+e)*(a+b*sinh(f*x+e)^2)^(1/2)/(a-b)/b^2/f/(sech(f*x+e)^
2*(a+b*sinh(f*x+e)^2)/a)^(1/2)-1/3*(8*a^2-3*a*b-2*b^2)*(a+b*sinh(f*x+e)^2)
^(1/2)*tanh(f*x+e)/(a-b)/b^3/f
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 1.15 (sec) , antiderivative size = 211, normalized size of antiderivative = 0.62

$$\int \frac{\sinh^6(e+fx)}{(a+b\sinh^2(e+fx))^{3/2}}\,dx = \frac{2i\sqrt{2}a(8a^2-3ab-2b^2)\sqrt{\frac{2a-b+b\cosh(2(e+fx))}{a}}E\left(i(e+fx)\left|\frac{b}{a}\right.\right)-2i\sqrt{2}a(8a}{}$$

input
```
Integrate[Sinh[e + f*x]^6/(a + b*Sinh[e + f*x]^2)^(3/2),x]
```

output
```
((2*I)*Sqrt[2]*a*(8*a^2 - 3*a*b - 2*b^2)*Sqrt[(2*a - b + b*Cosh[2*(e + f*x
)])/a]*EllipticE[I*(e + f*x), b/a] - (2*I)*Sqrt[2]*a*(8*a^2 - 7*a*b - b^2)
*Sqrt[(2*a - b + b*Cosh[2*(e + f*x)])/a]*EllipticF[I*(e + f*x), b/a] - b*(
-8*a^2 + 3*a*b - b^2 + b*(-a + b)*Cosh[2*(e + f*x)])*Sinh[2*(e + f*x)])/(6
*(a - b)*b^3*f*Sqrt[4*a - 2*b + 2*b*Cosh[2*(e + f*x)]])
```

Rubi [A] (verified)

Time = 0.57 (sec) , antiderivative size = 370, normalized size of antiderivative = 1.09, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.360$, Rules used = {3042, 25, 3667, 372, 444, 406, 320, 388, 313}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sinh^6(e+fx)}{\left(a+b\sinh^2(e+fx)\right)^{3/2}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int -\frac{\sin(ie+ifx)^6}{\left(a-b\sin(ie+ifx)^2\right)^{3/2}}dx$$

$$\Big\downarrow \text{25}$$

$$-\int \frac{\sin(ie+ifx)^6}{\left(a-b\sin(ie+ifx)^2\right)^{3/2}}dx$$

$$\Big\downarrow \text{3667}$$

$$\frac{\sqrt{\cosh^2(e+fx)}\text{sech}(e+fx)\int \frac{\sinh^6(e+fx)}{\sqrt{\sinh^2(e+fx)+1}\left(b\sinh^2(e+fx)+a\right)^{3/2}}d\sinh(e+fx)}{f}$$

$$\Big\downarrow \text{372}$$

$$\frac{\sqrt{\cosh^2(e+fx)}\text{sech}(e+fx)\left(\frac{\int \frac{\sinh^2(e+fx)\left((4a-b)\sinh^2(e+fx)+3a\right)}{\sqrt{\sinh^2(e+fx)+1}\sqrt{b\sinh^2(e+fx)+a}}d\sinh(e+fx)}{b(a-b)}-\frac{a\sinh^3(e+fx)\sqrt{\sinh^2(e+fx)+1}}{b(a-b)\sqrt{a+b\sinh^2(e+fx)}}\right)}{f}$$

$$\Big\downarrow \text{444}$$

$$\frac{\sqrt{\cosh^2(e+fx)}\text{sech}(e+fx)\left(\frac{\frac{(4a-b)\sinh(e+fx)\sqrt{\sinh^2(e+fx)+1}\sqrt{a+b\sinh^2(e+fx)}}{3b}-\frac{\int \frac{\left(8a^2-3ba-2b^2\right)\sinh^2(e+fx)+a(4a-b)}{\sqrt{\sinh^2(e+fx)+1}\sqrt{b\sinh^2(e+fx)+a}}d\sinh(e+fx)}{3b}}{b(a-b)}\right.}{f}$$

$$\Big\downarrow \text{406}$$

$$\frac{\sqrt{\cosh^2(e+fx)}\text{sech}(e+fx)\left(\frac{\frac{(4a-b)\sinh(e+fx)\sqrt{\sinh^2(e+fx)+1}\sqrt{a+b\sinh^2(e+fx)}}{3b}-\frac{\left(8a^2-3ab-2b^2\right)\int \frac{\sinh^2(e+fx)}{\sqrt{\sinh^2(e+fx)+1}\sqrt{b\sinh^2(e+fx)+a}}}{b(a-b)}\right.}{f}$$

$$\Big\downarrow \text{320}$$

$$\frac{\sqrt{\cosh^2(e+fx)}\operatorname{sech}(e+fx)\left(\frac{\frac{(4a-b)\sinh(e+fx)\sqrt{\sinh^2(e+fx)+1}\sqrt{a+b\sinh^2(e+fx)}}{3b}-\frac{\left(8a^2-3ab-2b^2\right)\int\frac{\sinh^2(e+fx)}{\sqrt{\sinh^2(e+fx)+1}\sqrt{b\sinh^2(e+fx)+a}}\cdots}{b(a-b)}\cdots}{\cdots}\right)}{f}$$

$\downarrow$ 388

$$\frac{\sqrt{\cosh^2(e+fx)}\operatorname{sech}(e+fx)\left(\frac{\frac{(4a-b)\sinh(e+fx)\sqrt{\sinh^2(e+fx)+1}\sqrt{a+b\sinh^2(e+fx)}}{3b}-\frac{\left(8a^2-3ab-2b^2\right)\left(\frac{\sinh(e+fx)\sqrt{a+b\sinh^2(e+fx)}}{b\sqrt{\sinh^2(e+fx)+1}}-\frac{\int\frac{\sqrt{\cdots}}{\left(\sinh\cdots\right)}\cdots}{\cdots}\right)}{b(a-b)}\cdots}{\cdots}\right)}{f}$$

$\downarrow$ 313

$$\frac{\sqrt{\cosh^2(e+fx)}\operatorname{sech}(e+fx)\left(\frac{\frac{(4a-b)\sinh(e+fx)\sqrt{\sinh^2(e+fx)+1}\sqrt{a+b\sinh^2(e+fx)}}{3b}-\frac{\left(8a^2-3ab-2b^2\right)\left(\frac{\sinh(e+fx)\sqrt{a+b\sinh^2(e+fx)}}{b\sqrt{\sinh^2(e+fx)+1}}-\frac{\sqrt{a+\cdots}}{b\cdots}\right)}{b(a-\cdots}\cdots}{\cdots}\right)}{f}$$

input
```
Int[Sinh[e + f*x]^6/(a + b*Sinh[e + f*x]^2)^(3/2),x]
```

output
```
(Sqrt[Cosh[e + f*x]^2]*Sech[e + f*x]*(-((a*Sinh[e + f*x]^3*Sqrt[1 + Sinh[e
 + f*x]^2])/((a - b)*b*Sqrt[a + b*Sinh[e + f*x]^2])) + (((4*a - b)*Sinh[e
+ f*x]*Sqrt[1 + Sinh[e + f*x]^2]*Sqrt[a + b*Sinh[e + f*x]^2])/(3*b) - (((4
*a - b)*EllipticF[ArcTan[Sinh[e + f*x]], 1 - b/a]*Sqrt[a + b*Sinh[e + f*x]
^2])/(Sqrt[1 + Sinh[e + f*x]^2]*Sqrt[(a + b*Sinh[e + f*x]^2)/(a*(1 + Sinh[
e + f*x]^2))]) + (8*a^2 - 3*a*b - 2*b^2)*((Sinh[e + f*x]*Sqrt[a + b*Sinh[e
 + f*x]^2])/(b*Sqrt[1 + Sinh[e + f*x]^2]) - (EllipticE[ArcTan[Sinh[e + f*x
]], 1 - b/a]*Sqrt[a + b*Sinh[e + f*x]^2])/(b*Sqrt[1 + Sinh[e + f*x]^2]*Sqr
t[(a + b*Sinh[e + f*x]^2)/(a*(1 + Sinh[e + f*x]^2))])))/(3*b))/((a - b)*b)
))/f
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 313
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/((c_) + (d_.)*(x_)^2)^(3/2), x_Symbol] :> Sim
p[(Sqrt[a + b*x^2]/(c*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(c
+ d*x^2)))]))*EllipticE[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; FreeQ
[{a, b, c, d}, x] && PosQ[b/a] && PosQ[d/c]
```

rule 320
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(Sqrt[a + b*x^2]/(a*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(
c + d*x^2)))]))*EllipticF[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; Fre
eQ[{a, b, c, d}, x] && PosQ[d/c] && PosQ[b/a] &&  !SimplerSqrtQ[b/a, d/c]
```

rule 372
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_
), x_Symbol] :> Simp[(-a)*e^3*(e*x)^(m - 3)*(a + b*x^2)^(p + 1)*((c + d*x^2
)^(q + 1)/(2*b*(b*c - a*d)*(p + 1))), x] + Simp[e^4/(2*b*(b*c - a*d)*(p + 1
))   Int[(e*x)^(m - 4)*(a + b*x^2)^(p + 1)*(c + d*x^2)^q*Simp[a*c*(m - 3) +
 (a*d*(m + 2*q - 1) + 2*b*c*(p + 1))*x^2, x], x], x] /; FreeQ[{a, b, c, d,
e, q}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && GtQ[m, 3] && IntBinomialQ[a
, b, c, d, e, m, 2, p, q, x]
```

rule 388
```
Int[(x_)^2/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol]
 :> Simp[x*(Sqrt[a + b*x^2]/(b*Sqrt[c + d*x^2])), x] - Simp[c/b   Int[Sqrt[
a + b*x^2]/(c + d*x^2)^(3/2), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
 a*d, 0] && PosQ[b/a] && PosQ[d/c] &&  !SimplerSqrtQ[b/a, d/c]
```

rule 406
```
Int[((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.)*((e_) + (f_.)*(
x_)^2), x_Symbol] :> Simp[e   Int[(a + b*x^2)^p*(c + d*x^2)^q, x], x] + Sim
p[f   Int[x^2*(a + b*x^2)^p*(c + d*x^2)^q, x], x] /; FreeQ[{a, b, c, d, e,
f, p, q}, x]
```

rule 444
```
Int[((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q
_.)*((e_) + (f_.)*(x_)^2), x_Symbol] :> Simp[f*g*(g*x)^(m - 1)*(a + b*x^2)^
(p + 1)*((c + d*x^2)^(q + 1)/(b*d*(m + 2*(p + q + 1) + 1))), x] - Simp[g^2/
(b*d*(m + 2*(p + q + 1) + 1))   Int[(g*x)^(m - 2)*(a + b*x^2)^p*(c + d*x^2)
^q*Simp[a*f*c*(m - 1) + (a*f*d*(m + 2*q + 1) + b*(f*c*(m + 2*p + 1) - e*d*(
m + 2*(p + q + 1) + 1)))*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p,
q}, x] && GtQ[m, 1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3667
```
Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(
p_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Simp[ff^(m + 1
)*(Sqrt[Cos[e + f*x]^2]/(f*Cos[e + f*x]))   Subst[Int[x^m*((a + b*ff^2*x^2)
^p/Sqrt[1 - ff^2*x^2]), x], x, Sin[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f,
p}, x] && IntegerQ[m/2] &&  !IntegerQ[p]
```

Maple [A] (verified)

Time = 5.92 (sec) , antiderivative size = 500, normalized size of antiderivative = 1.47

method	result
default	$\frac{\sqrt{-\frac{b}{a}}\, ab\sinh(fx+e)^5-\sqrt{-\frac{b}{a}}\, b^2\sinh(fx+e)^5+4\sqrt{-\frac{b}{a}}\, a^2\sinh(fx+e)^3-\sqrt{-\frac{b}{a}}\, b^2\sinh(fx+e)^3+4a^2\sqrt{\frac{a+b\sinh(fx+e)^2}{a}}\, \sqrt{\frac{\cosh(2fx+}{2}}}{}$

input
```
int(sinh(f*x+e)^6/(a+b*sinh(f*x+e)^2)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
1/3*((-b/a)^(1/2)*a*b*sinh(f*x+e)^5-(-b/a)^(1/2)*b^2*sinh(f*x+e)^5+4*(-b/a
)^(1/2)*a^2*sinh(f*x+e)^3-(-b/a)^(1/2)*b^2*sinh(f*x+e)^3+4*a^2*((a+b*sinh(
f*x+e)^2)/a)^(1/2)*(cosh(f*x+e)^2)^(1/2)*EllipticF(sinh(f*x+e)*(-b/a)^(1/2
),(1/b*a)^(1/2))-2*a*((a+b*sinh(f*x+e)^2)/a)^(1/2)*(cosh(f*x+e)^2)^(1/2)*E
llipticF(sinh(f*x+e)*(-b/a)^(1/2),(1/b*a)^(1/2))*b-2*((a+b*sinh(f*x+e)^2)/
a)^(1/2)*(cosh(f*x+e)^2)^(1/2)*EllipticF(sinh(f*x+e)*(-b/a)^(1/2),(1/b*a)^
(1/2))*b^2-8*((a+b*sinh(f*x+e)^2)/a)^(1/2)*(cosh(f*x+e)^2)^(1/2)*EllipticE
(sinh(f*x+e)*(-b/a)^(1/2),(1/b*a)^(1/2))*a^2+3*((a+b*sinh(f*x+e)^2)/a)^(1/
2)*(cosh(f*x+e)^2)^(1/2)*EllipticE(sinh(f*x+e)*(-b/a)^(1/2),(1/b*a)^(1/2))
*a*b+2*((a+b*sinh(f*x+e)^2)/a)^(1/2)*(cosh(f*x+e)^2)^(1/2)*EllipticE(sinh(
f*x+e)*(-b/a)^(1/2),(1/b*a)^(1/2))*b^2+4*(-b/a)^(1/2)*a^2*sinh(f*x+e)-(-b/
a)^(1/2)*a*b*sinh(f*x+e))/b^2/(a-b)/(-b/a)^(1/2)/cosh(f*x+e)/(a+b*sinh(f*x
+e)^2)^(1/2)/f
```

Fricas [F]

$$\int \frac{\sinh^6(e+fx)}{\left(a+b\sinh^2(e+fx)\right)^{3/2}}\,dx = \int \frac{\sinh\left(fx+e\right)^6}{\left(b\sinh\left(fx+e\right)^2+a\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate(sinh(f*x+e)^6/(a+b*sinh(f*x+e)^2)^(3/2),x, algorithm="fricas")
```

output
```
integral(sqrt(b*sinh(f*x + e)^2 + a)*sinh(f*x + e)^6/(b^2*sinh(f*x + e)^4
+ 2*a*b*sinh(f*x + e)^2 + a^2), x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\sinh^6(e+fx)}{\left(a+b\sinh^2(e+fx)\right)^{3/2}}\,dx = \text{Timed out}$$

input
```
integrate(sinh(f*x+e)**6/(a+b*sinh(f*x+e)**2)**(3/2),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{\sinh^6(e+fx)}{(a+b\sinh^2(e+fx))^{3/2}}\,dx = \int \frac{\sinh(fx+e)^6}{(b\sinh(fx+e)^2+a)^{\frac{3}{2}}}\,dx$$

input
```
integrate(sinh(f*x+e)^6/(a+b*sinh(f*x+e)^2)^(3/2),x, algorithm="maxima")
```

output
```
integrate(sinh(f*x + e)^6/(b*sinh(f*x + e)^2 + a)^(3/2), x)
```

Giac [F]

$$\int \frac{\sinh^6(e+fx)}{(a+b\sinh^2(e+fx))^{3/2}}\,dx = \int \frac{\sinh(fx+e)^6}{(b\sinh(fx+e)^2+a)^{\frac{3}{2}}}\,dx$$

input
```
integrate(sinh(f*x+e)^6/(a+b*sinh(f*x+e)^2)^(3/2),x, algorithm="giac")
```

output
```
integrate(sinh(f*x + e)^6/(b*sinh(f*x + e)^2 + a)^(3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sinh^6(e+fx)}{(a+b\sinh^2(e+fx))^{3/2}}\,dx = \int \frac{\sinh(e+f\,x)^6}{(b\sinh(e+f\,x)^2+a)^{3/2}}\,dx$$

input
```
int(sinh(e + f*x)^6/(a + b*sinh(e + f*x)^2)^(3/2),x)
```

output
```
int(sinh(e + f*x)^6/(a + b*sinh(e + f*x)^2)^(3/2), x)
```

Reduce [F]

$$\int \frac{\sinh^6(e+fx)}{\left(a+b\sinh^2(e+fx)\right)^{3/2}}\,dx = \int \frac{\sqrt{\sinh\left(fx+e\right)^2 b+a}\,\sinh\left(fx+e\right)^6}{\sinh\left(fx+e\right)^4 b^2+2\sinh\left(fx+e\right)^2 ab+a^2}dx$$

input
```
int(sinh(f*x+e)^6/(a+b*sinh(f*x+e)^2)^(3/2),x)
```

output
```
int((sqrt(sinh(e + f*x)**2*b + a)*sinh(e + f*x)**6)/(sinh(e + f*x)**4*b**2
 + 2*sinh(e + f*x)**2*a*b + a**2),x)
```

3.95 $\int \frac{\sinh^4(e+fx)}{\left(a+b\sinh^2(e+fx)\right)^{3/2}}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 256

$$\begin{aligned}\int \frac{\sinh^4(e+fx)}{\left(a+b\sinh^2(e+fx)\right)^{3/2}}\,dx =& -\frac{a\cosh(e+fx)\sinh(e+fx)}{(a-b)bf\sqrt{a+b\sinh^2(e+fx)}}\\ &-\frac{(2a-b)E\left(\arctan(\sinh(e+fx))\left|1-\frac{b}{a}\right.\right)\operatorname{sech}(e+fx)\sqrt{a+b\sinh^2(e+fx)}}{(a-b)b^2f\sqrt{\frac{\operatorname{sech}^2(e+fx)\left(a+b\sinh^2(e+fx)\right)}{a}}}\\ &+\frac{\operatorname{EllipticF}\left(\arctan(\sinh(e+fx)),1-\frac{b}{a}\right)\operatorname{sech}(e+fx)\sqrt{a+b\sinh^2(e+fx)}}{(a-b)bf\sqrt{\frac{\operatorname{sech}^2(e+fx)\left(a+b\sinh^2(e+fx)\right)}{a}}}\\ &+\frac{(2a-b)\sqrt{a+b\sinh^2(e+fx)}\tanh(e+fx)}{(a-b)b^2f}\end{aligned}$$

output
```
-a*cosh(f*x+e)*sinh(f*x+e)/(a-b)/b/f/(a+b*sinh(f*x+e)^2)^(1/2)-(2*a-b)*Ell
ipticE(sinh(f*x+e)/(1+sinh(f*x+e)^2)^(1/2),(1-b/a)^(1/2))*sech(f*x+e)*(a+b
*sinh(f*x+e)^2)^(1/2)/(a-b)/b^2/f/(sech(f*x+e)^2*(a+b*sinh(f*x+e)^2)/a)^(1
/2)+InverseJacobiAM(arctan(sinh(f*x+e)),(1-b/a)^(1/2))*sech(f*x+e)*(a+b*si
nh(f*x+e)^2)^(1/2)/(a-b)/b/f/(sech(f*x+e)^2*(a+b*sinh(f*x+e)^2)/a)^(1/2)+(
2*a-b)*(a+b*sinh(f*x+e)^2)^(1/2)*tanh(f*x+e)/(a-b)/b^2/f
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.88 (sec) , antiderivative size = 156, normalized size of antiderivative = 0.61

$$\int \frac{\sinh^4(e+fx)}{\left(a+b\sinh^2(e+fx)\right)^{3/2}}\,dx = \frac{a\left(-2i(2a-b)\sqrt{\frac{2a-b+b\cosh(2(e+fx))}{a}}E\left(i(e+fx)\,\middle|\,\frac{b}{a}\right)+4i(a-b)\sqrt{\frac{2a-b+b\mathrm{c}}{}}\right.}{2(a-b)b^2f\sqrt{2a-b+b\,\mathrm{co}}}$$

input
```
Integrate[Sinh[e + f*x]^4/(a + b*Sinh[e + f*x]^2)^(3/2),x]
```

output
```
(a*((-2*I)*(2*a - b)*Sqrt[(2*a - b + b*Cosh[2*(e + f*x)])/a]*EllipticE[I*(
e + f*x), b/a] + (4*I)*(a - b)*Sqrt[(2*a - b + b*Cosh[2*(e + f*x)])/a]*Ell
ipticF[I*(e + f*x), b/a] - Sqrt[2]*b*Sinh[2*(e + f*x)]))/(2*(a - b)*b^2*f*
Sqrt[2*a - b + b*Cosh[2*(e + f*x)]])
```

Rubi [A] (verified)

Time = 0.45 (sec) , antiderivative size = 295, normalized size of antiderivative = 1.15, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.280$, Rules used = {3042, 3667, 372, 406, 320, 388, 313}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sinh^4(e+fx)}{\left(a+b\sinh^2(e+fx)\right)^{3/2}}dx$$

$$\Big\downarrow \; 3042$$

$$\int \frac{\sin(ie+ifx)^4}{\left(a-b\sin(ie+ifx)^2\right)^{3/2}}dx$$

$$\Big\downarrow \; 3667$$

$$\frac{\sqrt{\cosh^2(e+fx)}\operatorname{sech}(e+fx)\int \frac{\sinh^4(e+fx)}{\sqrt{\sinh^2(e+fx)+1}\left(b\sinh^2(e+fx)+a\right)^{3/2}}d\sinh(e+fx)}{f}$$

$\downarrow$ 372

$$\frac{\sqrt{\cosh^2(e+fx)}\text{sech}(e+fx)\left(\frac{\int \frac{(2a-b)\sinh^2(e+fx)+a}{\sqrt{\sinh^2(e+fx)+1}\sqrt{b\sinh^2(e+fx)+a}}d\sinh(e+fx)}{b(a-b)}-\frac{a\sinh(e+fx)\sqrt{\sinh^2(e+fx)+1}}{b(a-b)\sqrt{a+b\sinh^2(e+fx)}}\right)}{f}$$

$\downarrow$ 406

$$\frac{\sqrt{\cosh^2(e+fx)}\text{sech}(e+fx)\left(\frac{a\int \frac{1}{\sqrt{\sinh^2(e+fx)+1}\sqrt{b\sinh^2(e+fx)+a}}d\sinh(e+fx)+(2a-b)\int \frac{\sinh^2(e+fx)}{\sqrt{\sinh^2(e+fx)+1}\sqrt{b\sinh^2(e+fx)+a}}d\sinh(e}{b(a-b)}\right.}{f}$$

$\downarrow$ 320

$$\frac{\sqrt{\cosh^2(e+fx)}\text{sech}(e+fx)\left(\frac{(2a-b)\int \frac{\sinh^2(e+fx)}{\sqrt{\sinh^2(e+fx)+1}\sqrt{b\sinh^2(e+fx)+a}}d\sinh(e+fx)+\frac{\sqrt{a+b\sinh^2(e+fx)}\text{EllipticF}\left(\arctan(\sinh(e+fx)),\right.}{\sqrt{\sinh^2(e+fx)+1}\sqrt{\frac{a+b\sinh^2(e+fx)}{a\left(\sinh^2(e+fx)+1\right)}}}}{b(a-b)}\right.}{f}$$

$\downarrow$ 388

$$\frac{\sqrt{\cosh^2(e+fx)}\text{sech}(e+fx)\left(\frac{(2a-b)\left(\frac{\sinh(e+fx)\sqrt{a+b\sinh^2(e+fx)}}{b\sqrt{\sinh^2(e+fx)+1}}-\frac{\int \frac{\sqrt{b\sinh^2(e+fx)+a}}{\left(\sinh^2(e+fx)+1\right)^{3/2}}d\sinh(e+fx)}{b}\right)+\frac{\sqrt{a+b\sinh^2(e+fx)}\text{EllipticF}\left(\right.}{\sqrt{\sinh^2(e+fx)+1}\sqrt{\frac{}{a}}}}{b(a-b)}\right.}{f}$$

$\downarrow$ 313

$$\frac{\sqrt{\cosh^2(e+fx)}\text{sech}(e+fx)\left(\frac{\frac{\sqrt{a+b\sinh^2(e+fx)}\text{EllipticF}\left(\arctan(\sinh(e+fx)),1-\frac{b}{a}\right)}{\sqrt{\sinh^2(e+fx)+1}\sqrt{\frac{a+b\sinh^2(e+fx)}{a\left(\sinh^2(e+fx)+1\right)}}}+(2a-b)\left(\frac{\sinh(e+fx)\sqrt{a+b\sinh^2(e+fx)}}{b\sqrt{\sinh^2(e+fx)+1}}-\frac{\sqrt{a+b\sinh^2(}}{b\sqrt{\sinh^2}}\right.}{b(a-b)}\right.}{f}$$

input
```
Int[Sinh[e + f*x]^4/(a + b*Sinh[e + f*x]^2)^(3/2),x]
```

output
```
(Sqrt[Cosh[e + f*x]^2]*Sech[e + f*x]*(-((a*Sinh[e + f*x]*Sqrt[1 + Sinh[e +
 f*x]^2])/((a - b)*b*Sqrt[a + b*Sinh[e + f*x]^2])) + ((EllipticF[ArcTan[Si
nh[e + f*x]], 1 - b/a]*Sqrt[a + b*Sinh[e + f*x]^2])/(Sqrt[1 + Sinh[e + f*x
]^2]*Sqrt[(a + b*Sinh[e + f*x]^2)/(a*(1 + Sinh[e + f*x]^2))]) + (2*a - b)*
((Sinh[e + f*x]*Sqrt[a + b*Sinh[e + f*x]^2])/(b*Sqrt[1 + Sinh[e + f*x]^2])
 - (EllipticE[ArcTan[Sinh[e + f*x]], 1 - b/a]*Sqrt[a + b*Sinh[e + f*x]^2])
/(b*Sqrt[1 + Sinh[e + f*x]^2]*Sqrt[(a + b*Sinh[e + f*x]^2)/(a*(1 + Sinh[e
+ f*x]^2))])))/((a - b)*b)))/f
```

Defintions of rubi rules used

rule 313
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/((c_) + (d_.)*(x_)^2)^(3/2), x_Symbol] :> Sim
p[(Sqrt[a + b*x^2]/(c*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(c
+ d*x^2)))]))*EllipticE[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; FreeQ
[{a, b, c, d}, x] && PosQ[b/a] && PosQ[d/c]
```

rule 320
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(Sqrt[a + b*x^2]/(a*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(
c + d*x^2)))]))*EllipticF[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; Fre
eQ[{a, b, c, d}, x] && PosQ[d/c] && PosQ[b/a] &&  !SimplerSqrtQ[b/a, d/c]
```

rule 372
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_
), x_Symbol] :> Simp[(-a)*e^3*(e*x)^(m - 3)*(a + b*x^2)^(p + 1)*((c + d*x^2
)^(q + 1)/(2*b*(b*c - a*d)*(p + 1))), x] + Simp[e^4/(2*b*(b*c - a*d)*(p + 1
))   Int[(e*x)^(m - 4)*(a + b*x^2)^(p + 1)*(c + d*x^2)^q*Simp[a*c*(m - 3) +
 (a*d*(m + 2*q - 1) + 2*b*c*(p + 1))*x^2, x], x], x] /; FreeQ[{a, b, c, d,
e, q}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && GtQ[m, 3] && IntBinomialQ[a
, b, c, d, e, m, 2, p, q, x]
```

rule 388
```
Int[(x_)^2/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol]
 :> Simp[x*(Sqrt[a + b*x^2]/(b*Sqrt[c + d*x^2])), x] - Simp[c/b   Int[Sqrt[
a + b*x^2]/(c + d*x^2)^(3/2), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
 a*d, 0] && PosQ[b/a] && PosQ[d/c] &&  !SimplerSqrtQ[b/a, d/c]
```

rule 406
```
Int[((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.)*((e_) + (f_.)*(
x_)^2), x_Symbol] :> Simp[e   Int[(a + b*x^2)^p*(c + d*x^2)^q, x], x] + Sim
p[f   Int[x^2*(a + b*x^2)^p*(c + d*x^2)^q, x], x] /; FreeQ[{a, b, c, d, e,
f, p, q}, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3667
```
Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(
p_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Simp[ff^(m + 1
)*(Sqrt[Cos[e + f*x]^2]/(f*Cos[e + f*x]))   Subst[Int[x^m*((a + b*ff^2*x^2)
^p/Sqrt[1 - ff^2*x^2]), x], x, Sin[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f,
p}, x] && IntegerQ[m/2] &&  !IntegerQ[p]
```

Maple [A] (verified)

Time = 4.85 (sec) , antiderivative size = 313, normalized size of antiderivative = 1.22

method	result
default	$-\frac{\sqrt{-\frac{b}{a}}\, a \cosh(fx+e)^2 \sinh(fx+e)+a\sqrt{\frac{b\cosh(fx+e)^2}{a}+\frac{a-b}{a}}\,\sqrt{\frac{\cosh(2fx+2e)}{2}+\frac{1}{2}}\ \operatorname{EllipticF}\left(\sinh(fx+e)\sqrt{-\frac{b}{a}},\sqrt{\frac{a}{b}}\right)-\sqrt{\frac{b\cosh(fx+}{a}}}{}$

input
```
int(sinh(f*x+e)^4/(a+b*sinh(f*x+e)^2)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-1/b*((-b/a)^(1/2)*a*cosh(f*x+e)^2*sinh(f*x+e)+a*(b/a*cosh(f*x+e)^2+(a-b)/
a)^(1/2)*(cosh(f*x+e)^2)^(1/2)*EllipticF(sinh(f*x+e)*(-b/a)^(1/2),(1/b*a)^
(1/2))-(b/a*cosh(f*x+e)^2+(a-b)/a)^(1/2)*(cosh(f*x+e)^2)^(1/2)*EllipticF(s
inh(f*x+e)*(-b/a)^(1/2),(1/b*a)^(1/2))*b-2*(b/a*cosh(f*x+e)^2+(a-b)/a)^(1/
2)*(cosh(f*x+e)^2)^(1/2)*EllipticE(sinh(f*x+e)*(-b/a)^(1/2),(1/b*a)^(1/2))
*a+(b/a*cosh(f*x+e)^2+(a-b)/a)^(1/2)*(cosh(f*x+e)^2)^(1/2)*EllipticE(sinh(
f*x+e)*(-b/a)^(1/2),(1/b*a)^(1/2))*b)/(a-b)/(-b/a)^(1/2)/cosh(f*x+e)/(a+b*
sinh(f*x+e)^2)^(1/2)/f
```

Fricas [F]

$$\int \frac{\sinh^4(e+fx)}{\left(a+b\sinh^2(e+fx)\right)^{3/2}}\,dx=\int \frac{\sinh\left(fx+e\right)^4}{\left(b\sinh\left(fx+e\right)^2+a\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate(sinh(f*x+e)^4/(a+b*sinh(f*x+e)^2)^(3/2),x, algorithm="fricas")
```

output
```
integral(sqrt(b*sinh(f*x + e)^2 + a)*sinh(f*x + e)^4/(b^2*sinh(f*x + e)^4
+ 2*a*b*sinh(f*x + e)^2 + a^2), x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\sinh^4(e+fx)}{\left(a+b\sinh^2(e+fx)\right)^{3/2}}\,dx=\text{Timed out}$$

input
```
integrate(sinh(f*x+e)**4/(a+b*sinh(f*x+e)**2)**(3/2),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{\sinh^4(e+fx)}{(a+b\sinh^2(e+fx))^{3/2}}\,dx = \int \frac{\sinh(fx+e)^4}{(b\sinh(fx+e)^2+a)^{\frac{3}{2}}}\,dx$$

input
```
integrate(sinh(f*x+e)^4/(a+b*sinh(f*x+e)^2)^(3/2),x, algorithm="maxima")
```

output
```
integrate(sinh(f*x + e)^4/(b*sinh(f*x + e)^2 + a)^(3/2), x)
```

Giac [F]

$$\int \frac{\sinh^4(e+fx)}{(a+b\sinh^2(e+fx))^{3/2}}\,dx = \int \frac{\sinh(fx+e)^4}{(b\sinh(fx+e)^2+a)^{\frac{3}{2}}}\,dx$$

input
```
integrate(sinh(f*x+e)^4/(a+b*sinh(f*x+e)^2)^(3/2),x, algorithm="giac")
```

output
```
integrate(sinh(f*x + e)^4/(b*sinh(f*x + e)^2 + a)^(3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sinh^4(e+fx)}{(a+b\sinh^2(e+fx))^{3/2}}\,dx = \int \frac{\sinh(e+f\,x)^4}{(b\sinh(e+f\,x)^2+a)^{3/2}}\,dx$$

input
```
int(sinh(e + f*x)^4/(a + b*sinh(e + f*x)^2)^(3/2),x)
```

output
```
int(sinh(e + f*x)^4/(a + b*sinh(e + f*x)^2)^(3/2), x)
```

Reduce [F]

$$\int \frac{\sinh^4(e+fx)}{\left(a+b\sinh^2(e+fx)\right)^{3/2}}\,dx = \int \frac{\sqrt{\sinh\left(fx+e\right)^2 b+a}\,\sinh\left(fx+e\right)^4}{\sinh\left(fx+e\right)^4 b^2+2\sinh\left(fx+e\right)^2 ab+a^2}dx$$

input
```
int(sinh(f*x+e)^4/(a+b*sinh(f*x+e)^2)^(3/2),x)
```

output
```
int((sqrt(sinh(e + f*x)**2*b + a)*sinh(e + f*x)**4)/(sinh(e + f*x)**4*b**2
 + 2*sinh(e + f*x)**2*a*b + a**2),x)
```

3.96 $\int \frac{\sinh^2(e+fx)}{\left(a+b\sinh^2(e+fx)\right)^{3/2}}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 175

$$\int \frac{\sinh^2(e+fx)}{\left(a+b\sinh^2(e+fx)\right)^{3/2}}\,dx = \frac{\cosh(e+fx)\sinh(e+fx)}{(a-b)f\sqrt{a+b\sinh^2(e+fx)}} + \frac{iE\left(ie+ifx\middle|\frac{b}{a}\right)\sqrt{a+b\sinh^2(e+fx)}}{(a-b)bf\sqrt{\frac{a+b\sinh^2(e+fx)}{a}}} - \frac{i\,\text{EllipticF}\left(ie+ifx,\frac{b}{a}\right)\sqrt{\frac{a+b\sinh^2(e+fx)}{a}}}{bf\sqrt{a+b\sinh^2(e+fx)}}$$

output
```
cosh(f*x+e)*sinh(f*x+e)/(a-b)/f/(a+b*sinh(f*x+e)^2)^(1/2)+I*EllipticE(sin(
I*e+I*f*x),(b/a)^(1/2))*(a+b*sinh(f*x+e)^2)^(1/2)/(a-b)/b/f/((a+b*sinh(f*x
+e)^2)/a)^(1/2)-I*InverseJacobiAM(I*e+I*f*x,(b/a)^(1/2))*((a+b*sinh(f*x+e)
^2)/a)^(1/2)/b/f/(a+b*sinh(f*x+e)^2)^(1/2)
```

Mathematica [A] (verified)

Time = 0.53 (sec) , antiderivative size = 151, normalized size of antiderivative = 0.86

$$\int \frac{\sinh^2(e+fx)}{\left(a+b\sinh^2(e+fx)\right)^{3/2}}\,dx = \frac{i\sqrt{2}a\sqrt{\frac{2a-b+b\cosh(2(e+fx))}{a}}E\left(i(e+fx)\,\middle|\,\frac{b}{a}\right) - i\sqrt{2}(a-b)\sqrt{\frac{2a-b+b\cosh(2(e+f}{a}}}{(a-b)bf\sqrt{4a-2b+2b\cosh(2(e}}$$

input
```
Integrate[Sinh[e + f*x]^2/(a + b*Sinh[e + f*x]^2)^(3/2),x]
```

output
```
(I*Sqrt[2]*a*Sqrt[(2*a - b + b*Cosh[2*(e + f*x)])/a]*EllipticE[I*(e + f*x)
, b/a] - I*Sqrt[2]*(a - b)*Sqrt[(2*a - b + b*Cosh[2*(e + f*x)])/a]*Ellipti
cF[I*(e + f*x), b/a] + b*Sinh[2*(e + f*x)])/((a - b)*b*f*Sqrt[4*a - 2*b +
2*b*Cosh[2*(e + f*x)]])
```

Rubi [A] (verified)

Time = 0.96 (sec) , antiderivative size = 186, normalized size of antiderivative = 1.06, number of steps used = 12, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.480$, Rules used = {3042, 25, 3652, 3042, 3651, 3042, 3657, 3042, 3656, 3662, 3042, 3661}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sinh^2(e+fx)}{\left(a+b\sinh^2(e+fx)\right)^{3/2}}dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int -\frac{\sin(ie+ifx)^2}{(a-b\sin(ie+ifx)^2)^{3/2}}dx$$

$$\Big\downarrow \text{ 25}$$

$$-\int \frac{\sin(ie+ifx)^2}{(a-b\sin(ie+ifx)^2)^{3/2}}dx$$

$$\Big\downarrow \text{ 3652}$$

$$\frac{\sinh(e+fx)\cosh(e+fx)}{f(a-b)\sqrt{a+b\sinh^2(e+fx)}}-\frac{\int\frac{a\sinh^2(e+fx)+a}{\sqrt{b\sinh^2(e+fx)+a}}dx}{a(a-b)}$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{\sinh(e+fx)\cosh(e+fx)}{f(a-b)\sqrt{a+b\sinh^2(e+fx)}}-\frac{\int\frac{a-a\sin(ie+ifx)^2}{\sqrt{a-b\sin(ie+ifx)^2}}dx}{a(a-b)}$$

$$\Big\downarrow \text{ 3651}$$

$$\frac{\sinh(e+fx)\cosh(e+fx)}{f(a-b)\sqrt{a+b\sinh^2(e+fx)}}-\frac{\frac{a\int\sqrt{b\sinh^2(e+fx)+a}dx}{b}-\frac{a(a-b)\int\frac{1}{\sqrt{b\sinh^2(e+fx)+a}}dx}{b}}{a(a-b)}$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{\sinh(e+fx)\cosh(e+fx)}{f(a-b)\sqrt{a+b\sinh^2(e+fx)}}-\frac{\frac{a\int\sqrt{a-b\sin(ie+ifx)^2}dx}{b}-\frac{a(a-b)\int\frac{1}{\sqrt{a-b\sin(ie+ifx)^2}}dx}{b}}{a(a-b)}$$

$$\Big\downarrow \text{ 3657}$$

$$\frac{\sinh(e+fx)\cosh(e+fx)}{f(a-b)\sqrt{a+b\sinh^2(e+fx)}}-\frac{\frac{a\sqrt{a+b\sinh^2(e+fx)}\int\sqrt{\frac{b\sinh^2(e+fx)}{a}+1}dx}{b\sqrt{\frac{b\sinh^2(e+fx)}{a}+1}}-\frac{a(a-b)\int\frac{1}{\sqrt{a-b\sin(ie+ifx)^2}}dx}{b}}{a(a-b)}$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{\sinh(e+fx)\cosh(e+fx)}{f(a-b)\sqrt{a+b\sinh^2(e+fx)}}-\frac{\frac{a\sqrt{a+b\sinh^2(e+fx)}\int\sqrt{1-\frac{b\sin(ie+ifx)^2}{a}}dx}{b\sqrt{\frac{b\sinh^2(e+fx)}{a}+1}}-\frac{a(a-b)\int\frac{1}{\sqrt{a-b\sin(ie+ifx)^2}}dx}{b}}{a(a-b)}$$

$$\Big\downarrow \text{ 3656}$$

$$\frac{\sinh(e+fx)\cosh(e+fx)}{f(a-b)\sqrt{a+b\sinh^2(e+fx)}}-\frac{-\frac{a(a-b)\int\frac{1}{\sqrt{a-b\sin(ie+ifx)^2}}dx}{b}-\frac{ia\sqrt{a+b\sinh^2(e+fx)}E\left(ie+ifx\middle|\frac{b}{a}\right)}{bf\sqrt{\frac{b\sinh^2(e+fx)}{a}+1}}}{a(a-b)}$$

$$\Big\downarrow \text{ 3662}$$

$$\frac{\frac{\sinh(e+fx)\cosh(e+fx)}{f(a-b)\sqrt{a+b\sinh^2(e+fx)}}-\frac{-\frac{a(a-b)\sqrt{\frac{b\sinh^2(e+fx)}{a}+1}\int\frac{1}{\sqrt{\frac{b\sinh^2(e+fx)}{a}+1}}dx}{b\sqrt{a+b\sinh^2(e+fx)}}-\frac{ia\sqrt{a+b\sinh^2(e+fx)}E\left(ie+ifx\middle|\frac{b}{a}\right)}{bf\sqrt{\frac{b\sinh^2(e+fx)}{a}+1}}}{a(a-b)}}{}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\sinh(e+fx)\cosh(e+fx)}{f(a-b)\sqrt{a+b\sinh^2(e+fx)}}-\frac{-\frac{a(a-b)\sqrt{\frac{b\sinh^2(e+fx)}{a}+1}\int\frac{1}{\sqrt{1-\frac{b\sin(ie+ifx)^2}{a}}}dx}{b\sqrt{a+b\sinh^2(e+fx)}}-\frac{ia\sqrt{a+b\sinh^2(e+fx)}E\left(ie+ifx\middle|\frac{b}{a}\right)}{bf\sqrt{\frac{b\sinh^2(e+fx)}{a}+1}}}{a(a-b)}$$

$$\Big\downarrow \text{3661}$$

$$\frac{\sinh(e+fx)\cosh(e+fx)}{f(a-b)\sqrt{a+b\sinh^2(e+fx)}}-\frac{\frac{ia(a-b)\sqrt{\frac{b\sinh^2(e+fx)}{a}+1}\,\text{EllipticF}\left(ie+ifx,\frac{b}{a}\right)}{bf\sqrt{a+b\sinh^2(e+fx)}}-\frac{ia\sqrt{a+b\sinh^2(e+fx)}E\left(ie+ifx\middle|\frac{b}{a}\right)}{bf\sqrt{\frac{b\sinh^2(e+fx)}{a}+1}}}{a(a-b)}$$

input
```
Int[Sinh[e + f*x]^2/(a + b*Sinh[e + f*x]^2)^(3/2),x]
```

output
```
(Cosh[e + f*x]*Sinh[e + f*x])/((a - b)*f*Sqrt[a + b*Sinh[e + f*x]^2]) - ((
(-I)*a*EllipticE[I*e + I*f*x, b/a]*Sqrt[a + b*Sinh[e + f*x]^2])/(b*f*Sqrt[
1 + (b*Sinh[e + f*x]^2)/a]) + (I*a*(a - b)*EllipticF[I*e + I*f*x, b/a]*Sqr
t[1 + (b*Sinh[e + f*x]^2)/a])/(b*f*Sqrt[a + b*Sinh[e + f*x]^2]))/(a*(a - b
))
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3651
```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)]^2)/Sqrt[(a_) + (b_.)*sin[(e_.) +
 (f_.)*(x_)]^2], x_Symbol] :> Simp[B/b   Int[Sqrt[a + b*Sin[e + f*x]^2], x]
, x] + Simp[(A*b - a*B)/b   Int[1/Sqrt[a + b*Sin[e + f*x]^2], x], x] /; Fre
eQ[{a, b, e, f, A, B}, x]
```

rule 3652
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)]^2), x_Symbol] :> Simp[(-(A*b - a*B))*Cos[e + f*x]*Sin[e + f*x
]*((a + b*Sin[e + f*x]^2)^(p + 1)/(2*a*f*(a + b)*(p + 1))), x] - Simp[1/(2*
a*(a + b)*(p + 1))   Int[(a + b*Sin[e + f*x]^2)^(p + 1)*Simp[a*B - A*(2*a*(
p + 1) + b*(2*p + 3)) + 2*(A*b - a*B)*(p + 2)*Sin[e + f*x]^2, x], x], x] /;
 FreeQ[{a, b, e, f, A, B}, x] && LtQ[p, -1] && NeQ[a + b, 0]
```

rule 3656
```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2], x_Symbol] :> Simp[(Sqrt[a
]/f)*EllipticE[e + f*x, -b/a], x] /; FreeQ[{a, b, e, f}, x] && GtQ[a, 0]
```

rule 3657
```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2], x_Symbol] :> Simp[Sqrt[a
+ b*Sin[e + f*x]^2]/Sqrt[1 + b*(Sin[e + f*x]^2/a)]   Int[Sqrt[1 + (b*Sin[e
+ f*x]^2)/a], x], x] /; FreeQ[{a, b, e, f}, x] &&  !GtQ[a, 0]
```

rule 3661
```
Int[1/Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2], x_Symbol] :> Simp[(1/(S
qrt[a]*f))*EllipticF[e + f*x, -b/a], x] /; FreeQ[{a, b, e, f}, x] && GtQ[a,
 0]
```

rule 3662
```
Int[1/Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2], x_Symbol] :> Simp[Sqrt[
1 + b*(Sin[e + f*x]^2/a)]/Sqrt[a + b*Sin[e + f*x]^2]   Int[1/Sqrt[1 + (b*Si
n[e + f*x]^2)/a], x], x] /; FreeQ[{a, b, e, f}, x] &&  !GtQ[a, 0]
```

Maple [A] (verified)

Time = 2.43 (sec) , antiderivative size = 127, normalized size of antiderivative = 0.73

method	result	size
default	$-\frac{-\sqrt{-\frac{b}{a}}\, \cosh(fx+e)^{2} \sinh(fx+e)+\sqrt{\frac{b \cosh(fx+e)^{2}}{a}+\frac{a-b}{a}}\, \sqrt{\frac{\cosh(2fx+2e)}{2}+\frac{1}{2}}\, \operatorname{EllipticE}\left(\sinh(fx+e) \sqrt{-\frac{b}{a}}, \sqrt{\frac{a}{b}}\right)}{(a-b) \sqrt{-\frac{b}{a}}\, \cosh(fx+e) \sqrt{a+b \sinh(fx+e)^{2}}\, f}$	127

input
```
int(sinh(f*x+e)^2/(a+b*sinh(f*x+e)^2)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-(-(-b/a)^(1/2)*cosh(f*x+e)^2*sinh(f*x+e)+(b/a*cosh(f*x+e)^2+(a-b)/a)^(1/2
)*(cosh(f*x+e)^2)^(1/2)*EllipticE(sinh(f*x+e)*(-b/a)^(1/2),(1/b*a)^(1/2)))
/(a-b)/(-b/a)^(1/2)/cosh(f*x+e)/(a+b*sinh(f*x+e)^2)^(1/2)/f
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1155 vs. 2(157) = 314.

Time = 0.12 (sec) , antiderivative size = 1155, normalized size of antiderivative = 6.60

$$\int \frac{\sinh^2(e+fx)}{\left(a+b\sinh^2(e+fx)\right)^{3/2}}\,dx = \text{Too large to display}$$

input
```
integrate(sinh(f*x+e)^2/(a+b*sinh(f*x+e)^2)^(3/2),x, algorithm="fricas")
```

output
```
-(((2*a*b - b^2)*cosh(f*x + e)^4 + 4*(2*a*b - b^2)*cosh(f*x + e)*sinh(f*x
+ e)^3 + (2*a*b - b^2)*sinh(f*x + e)^4 + 2*(4*a^2 - 4*a*b + b^2)*cosh(f*x
+ e)^2 + 2*(3*(2*a*b - b^2)*cosh(f*x + e)^2 + 4*a^2 - 4*a*b + b^2)*sinh(f*
x + e)^2 + 2*a*b - b^2 + 4*((2*a*b - b^2)*cosh(f*x + e)^3 + (4*a^2 - 4*a*b
 + b^2)*cosh(f*x + e))*sinh(f*x + e) - 2*(b^2*cosh(f*x + e)^4 + 4*b^2*cosh
(f*x + e)*sinh(f*x + e)^3 + b^2*sinh(f*x + e)^4 + 2*(2*a*b - b^2)*cosh(f*x
 + e)^2 + 2*(3*b^2*cosh(f*x + e)^2 + 2*a*b - b^2)*sinh(f*x + e)^2 + b^2 +
4*(b^2*cosh(f*x + e)^3 + (2*a*b - b^2)*cosh(f*x + e))*sinh(f*x + e))*sqrt(
(a^2 - a*b)/b^2))*sqrt(b)*sqrt((2*b*sqrt((a^2 - a*b)/b^2) - 2*a + b)/b)*el
liptic_e(arcsin(sqrt((2*b*sqrt((a^2 - a*b)/b^2) - 2*a + b)/b)*(cosh(f*x +
e) + sinh(f*x + e))), (8*a^2 - 8*a*b + b^2 + 4*(2*a*b - b^2)*sqrt((a^2 - a
*b)/b^2))/b^2) - 2*((2*a*b - b^2)*cosh(f*x + e)^4 + 4*(2*a*b - b^2)*cosh(f
*x + e)*sinh(f*x + e)^3 + (2*a*b - b^2)*sinh(f*x + e)^4 + 2*(4*a^2 - 4*a*b
 + b^2)*cosh(f*x + e)^2 + 2*(3*(2*a*b - b^2)*cosh(f*x + e)^2 + 4*a^2 - 4*a
*b + b^2)*sinh(f*x + e)^2 + 2*a*b - b^2 + 4*((2*a*b - b^2)*cosh(f*x + e)^3
 + (4*a^2 - 4*a*b + b^2)*cosh(f*x + e))*sinh(f*x + e))*sqrt(b)*sqrt((2*b*s
qrt((a^2 - a*b)/b^2) - 2*a + b)/b)*elliptic_f(arcsin(sqrt((2*b*sqrt((a^2 -
 a*b)/b^2) - 2*a + b)/b)*(cosh(f*x + e) + sinh(f*x + e))), (8*a^2 - 8*a*b
+ b^2 + 4*(2*a*b - b^2)*sqrt((a^2 - a*b)/b^2))/b^2) - sqrt(2)*(b^2*cosh(f*
x + e)^3 + 3*b^2*cosh(f*x + e)*sinh(f*x + e)^2 + b^2*sinh(f*x + e)^3 + ...
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\sinh^2(e+fx)}{\left(a+b\sinh^2(e+fx)\right)^{3/2}}\,dx = \text{Timed out}$$

input
```
integrate(sinh(f*x+e)**2/(a+b*sinh(f*x+e)**2)**(3/2),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{\sinh^2(e+fx)}{(a+b\sinh^2(e+fx))^{3/2}}\,dx = \int \frac{\sinh(fx+e)^2}{(b\sinh(fx+e)^2+a)^{\frac{3}{2}}}\,dx$$

input
```
integrate(sinh(f*x+e)^2/(a+b*sinh(f*x+e)^2)^(3/2),x, algorithm="maxima")
```

output
```
integrate(sinh(f*x + e)^2/(b*sinh(f*x + e)^2 + a)^(3/2), x)
```

Giac [F]

$$\int \frac{\sinh^2(e+fx)}{(a+b\sinh^2(e+fx))^{3/2}}\,dx = \int \frac{\sinh(fx+e)^2}{(b\sinh(fx+e)^2+a)^{\frac{3}{2}}}\,dx$$

input
```
integrate(sinh(f*x+e)^2/(a+b*sinh(f*x+e)^2)^(3/2),x, algorithm="giac")
```

output
```
integrate(sinh(f*x + e)^2/(b*sinh(f*x + e)^2 + a)^(3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sinh^2(e+fx)}{(a+b\sinh^2(e+fx))^{3/2}}\,dx = \int \frac{\sinh(e+f\,x)^2}{(b\sinh(e+f\,x)^2+a)^{3/2}}\,dx$$

input
```
int(sinh(e + f*x)^2/(a + b*sinh(e + f*x)^2)^(3/2),x)
```

output
```
int(sinh(e + f*x)^2/(a + b*sinh(e + f*x)^2)^(3/2), x)
```

Reduce [F]

$$\int \frac{\sinh^2(e+fx)}{\left(a+b\sinh^2(e+fx)\right)^{3/2}}\,dx=\int \frac{\sqrt{\sinh\left(fx+e\right)^2 b+a}\,\sinh\left(fx+e\right)^2}{\sinh\left(fx+e\right)^4 b^2+2\sinh\left(fx+e\right)^2 ab+a^2}dx$$

input
```
int(sinh(f*x+e)^2/(a+b*sinh(f*x+e)^2)^(3/2),x)
```

output
```
int((sqrt(sinh(e + f*x)**2*b + a)*sinh(e + f*x)**2)/(sinh(e + f*x)**4*b**2
 + 2*sinh(e + f*x)**2*a*b + a**2),x)
```

3.97 $\int \frac{1}{(a+b\sinh^2(e+fx))^{3/2}}\,dx$

Optimal result

Integrand size = 16, antiderivative size = 116

$$\int \frac{1}{(a+b\sinh^2(e+fx))^{3/2}}\,dx = -\frac{b\cosh(e+fx)\sinh(e+fx)}{a(a-b)f\sqrt{a+b\sinh^2(e+fx)}} - \frac{iE\left(ie+ifx\middle|\frac{b}{a}\right)\sqrt{a+b\sinh^2(e+fx)}}{a(a-b)f\sqrt{\frac{a+b\sinh^2(e+fx)}{a}}}$$

output
```
-b*cosh(f*x+e)*sinh(f*x+e)/a/(a-b)/f/(a+b*sinh(f*x+e)^2)^(1/2)-I*EllipticE
(sin(I*e+I*f*x),(b/a)^(1/2))*(a+b*sinh(f*x+e)^2)^(1/2)/a/(a-b)/f/((a+b*sin
h(f*x+e)^2)/a)^(1/2)
```

Mathematica [A] (verified)

Time = 0.15 (sec) , antiderivative size = 100, normalized size of antiderivative = 0.86

$$\int \frac{1}{(a+b\sinh^2(e+fx))^{3/2}}\,dx = \frac{-2ia\sqrt{\frac{2a-b+b\cosh(2(e+fx))}{a}}E\left(i(e+fx)\middle|\frac{b}{a}\right)-\sqrt{2}b\sinh(2(e+fx))}{2a(a-b)f\sqrt{2a-b+b\cosh(2(e+fx))}}$$

input
```
Integrate[(a + b*Sinh[e + f*x]^2)^(-3/2),x]
```

output
```
((-2*I)*a*Sqrt[(2*a - b + b*Cosh[2*(e + f*x)])/a]*EllipticE[I*(e + f*x), b
/a] - Sqrt[2]*b*Sinh[2*(e + f*x)])/(2*a*(a - b)*f*Sqrt[2*a - b + b*Cosh[2*
(e + f*x)]])
```

Rubi [A] (verified)

Time = 0.49 (sec) , antiderivative size = 115, normalized size of antiderivative = 0.99, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.438$, Rules used = {3042, 3663, 25, 3042, 3657, 3042, 3656}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\left(a+b\sinh^2(e+fx)\right)^{3/2}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{\left(a-b\sin(ie+ifx)^2\right)^{3/2}}dx$$

$$\Big\downarrow \text{3663}$$

$$-\frac{\int -\sqrt{b\sinh^2(e+fx)+a}dx}{a(a-b)}-\frac{b\sinh(e+fx)\cosh(e+fx)}{af(a-b)\sqrt{a+b\sinh^2(e+fx)}}$$

$$\Big\downarrow \text{25}$$

$$\frac{\int \sqrt{b\sinh^2(e+fx)+a}dx}{a(a-b)}-\frac{b\sinh(e+fx)\cosh(e+fx)}{af(a-b)\sqrt{a+b\sinh^2(e+fx)}}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{b\sinh(e+fx)\cosh(e+fx)}{af(a-b)\sqrt{a+b\sinh^2(e+fx)}}+\frac{\int \sqrt{a-b\sin(ie+ifx)^2}dx}{a(a-b)}$$

$$\Big\downarrow \text{3657}$$

$$\frac{\sqrt{a+b\sinh^2(e+fx)}\int \sqrt{\frac{b\sinh^2(e+fx)}{a}+1}dx}{a(a-b)\sqrt{\frac{b\sinh^2(e+fx)}{a}+1}}-\frac{b\sinh(e+fx)\cosh(e+fx)}{af(a-b)\sqrt{a+b\sinh^2(e+fx)}}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{b\sinh(e+fx)\cosh(e+fx)}{af(a-b)\sqrt{a+b\sinh^2(e+fx)}}+\frac{\sqrt{a+b\sinh^2(e+fx)}\int\sqrt{1-\frac{b\sin(ie+ifx)^2}{a}}dx}{a(a-b)\sqrt{\frac{b\sinh^2(e+fx)}{a}+1}}$$

$$\Big\downarrow \text{3656}$$

$$-\frac{b\sinh(e+fx)\cosh(e+fx)}{af(a-b)\sqrt{a+b\sinh^2(e+fx)}}-\frac{i\sqrt{a+b\sinh^2(e+fx)}E\left(ie+ifx|\frac{b}{a}\right)}{af(a-b)\sqrt{\frac{b\sinh^2(e+fx)}{a}+1}}$$

input
```
Int[(a + b*Sinh[e + f*x]^2)^(-3/2),x]
```

output
```
-((b*Cosh[e + f*x]*Sinh[e + f*x])/(a*(a - b)*f*Sqrt[a + b*Sinh[e + f*x]^2]
)) - (I*EllipticE[I*e + I*f*x, b/a]*Sqrt[a + b*Sinh[e + f*x]^2])/(a*(a - b
)*f*Sqrt[1 + (b*Sinh[e + f*x]^2)/a])
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3656
```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2], x_Symbol] :> Simp[(Sqrt[a
]/f)*EllipticE[e + f*x, -b/a], x] /; FreeQ[{a, b, e, f}, x] && GtQ[a, 0]
```

rule 3657
```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2], x_Symbol] :> Simp[Sqrt[a
+ b*Sin[e + f*x]^2]/Sqrt[1 + b*(Sin[e + f*x]^2/a)]   Int[Sqrt[1 + (b*Sin[e
+ f*x]^2)/a], x], x] /; FreeQ[{a, b, e, f}, x] && !GtQ[a, 0]
```

rule 3663
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> Simp[(-b)*C
os[e + f*x]*Sin[e + f*x]*((a + b*Sin[e + f*x]^2)^(p + 1)/(2*a*f*(p + 1)*(a
+ b))), x] + Simp[1/(2*a*(p + 1)*(a + b))   Int[(a + b*Sin[e + f*x]^2)^(p +
 1)*Simp[2*a*(p + 1) + b*(2*p + 3) - 2*b*(p + 2)*Sin[e + f*x]^2, x], x], x]
 /; FreeQ[{a, b, e, f}, x] && NeQ[a + b, 0] && LtQ[p, -1]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 251 vs. $2(110) = 220$.

Time = 1.10 (sec) , antiderivative size = 252, normalized size of antiderivative = 2.17

method	result
default	$\frac{-\sqrt{-\frac{b}{a}}\, b\cosh(fx+e)^2\sinh(fx+e)+a\sqrt{\frac{b\cosh(fx+e)^2}{a}+\frac{a-b}{a}}\,\sqrt{\frac{\cosh(2fx+2e)}{2}+\frac{1}{2}}\ \mathrm{EllipticF}\left(\sinh(fx+e)\sqrt{-\frac{b}{a}},\sqrt{\frac{a}{b}}\right)-\sqrt{\frac{b\cosh(fx+e}{a}}}{a(a-b)\sqrt{-\frac{b}{a}}\ \mathrm{co}}$

input
```
int(1/(a+b*sinh(f*x+e)^2)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
(-(-b/a)^(1/2)*b*cosh(f*x+e)^2*sinh(f*x+e)+a*(b/a*cosh(f*x+e)^2+(a-b)/a)^(
1/2)*(cosh(f*x+e)^2)^(1/2)*EllipticF(sinh(f*x+e)*(-b/a)^(1/2),(1/b*a)^(1/2
))-(b/a*cosh(f*x+e)^2+(a-b)/a)^(1/2)*(cosh(f*x+e)^2)^(1/2)*EllipticF(sinh(
f*x+e)*(-b/a)^(1/2),(1/b*a)^(1/2))*b+(b/a*cosh(f*x+e)^2+(a-b)/a)^(1/2)*(co
sh(f*x+e)^2)^(1/2)*EllipticE(sinh(f*x+e)*(-b/a)^(1/2),(1/b*a)^(1/2))*b)/a/
(a-b)/(-b/a)^(1/2)/cosh(f*x+e)/(a+b*sinh(f*x+e)^2)^(1/2)/f
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1464 vs. $2(106) = 212$.

Time = 0.12 (sec) , antiderivative size = 1464, normalized size of antiderivative = 12.62

$$\int \frac{1}{\left(a+b\sinh^2(e+fx)\right)^{3/2}}\,dx = \text{Too large to display}$$

input
```
integrate(1/(a+b*sinh(f*x+e)^2)^(3/2),x, algorithm="fricas")
```

output
```
(((2*a*b^2 - b^3)*cosh(f*x + e)^4 + 4*(2*a*b^2 - b^3)*cosh(f*x + e)*sinh(f
*x + e)^3 + (2*a*b^2 - b^3)*sinh(f*x + e)^4 + 2*a*b^2 - b^3 + 2*(4*a^2*b -
 4*a*b^2 + b^3)*cosh(f*x + e)^2 + 2*(4*a^2*b - 4*a*b^2 + b^3 + 3*(2*a*b^2
- b^3)*cosh(f*x + e)^2)*sinh(f*x + e)^2 + 4*((2*a*b^2 - b^3)*cosh(f*x + e)
^3 + (4*a^2*b - 4*a*b^2 + b^3)*cosh(f*x + e))*sinh(f*x + e) - 2*(b^3*cosh(
f*x + e)^4 + 4*b^3*cosh(f*x + e)*sinh(f*x + e)^3 + b^3*sinh(f*x + e)^4 + b
^3 + 2*(2*a*b^2 - b^3)*cosh(f*x + e)^2 + 2*(3*b^3*cosh(f*x + e)^2 + 2*a*b^
2 - b^3)*sinh(f*x + e)^2 + 4*(b^3*cosh(f*x + e)^3 + (2*a*b^2 - b^3)*cosh(f
*x + e))*sinh(f*x + e))*sqrt((a^2 - a*b)/b^2))*sqrt(b)*sqrt((2*b*sqrt((a^2
 - a*b)/b^2) - 2*a + b)/b)*elliptic_e(arcsin(sqrt((2*b*sqrt((a^2 - a*b)/b^
2) - 2*a + b)/b)*(cosh(f*x + e) + sinh(f*x + e))), (8*a^2 - 8*a*b + b^2 +
4*(2*a*b - b^2)*sqrt((a^2 - a*b)/b^2))/b^2) - 2*((2*a^2*b - a*b^2)*cosh(f*
x + e)^4 + 4*(2*a^2*b - a*b^2)*cosh(f*x + e)*sinh(f*x + e)^3 + (2*a^2*b -
a*b^2)*sinh(f*x + e)^4 + 2*a^2*b - a*b^2 + 2*(4*a^3 - 4*a^2*b + a*b^2)*cos
h(f*x + e)^2 + 2*(4*a^3 - 4*a^2*b + a*b^2 + 3*(2*a^2*b - a*b^2)*cosh(f*x +
 e)^2)*sinh(f*x + e)^2 + 4*((2*a^2*b - a*b^2)*cosh(f*x + e)^3 + (4*a^3 - 4
*a^2*b + a*b^2)*cosh(f*x + e))*sinh(f*x + e) + 2*((a*b^2 - b^3)*cosh(f*x +
 e)^4 + 4*(a*b^2 - b^3)*cosh(f*x + e)*sinh(f*x + e)^3 + (a*b^2 - b^3)*sinh
(f*x + e)^4 + a*b^2 - b^3 + 2*(2*a^2*b - 3*a*b^2 + b^3)*cosh(f*x + e)^2 +
2*(2*a^2*b - 3*a*b^2 + b^3 + 3*(a*b^2 - b^3)*cosh(f*x + e)^2)*sinh(f*x ...
```

Sympy [F]

$$\int \frac{1}{\left(a+b\sinh^2(e+fx)\right)^{3/2}}\,dx=\int \frac{1}{\left(a+b\sinh^2\left(e+fx\right)\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/(a+b*sinh(f*x+e)**2)**(3/2),x)
```

output
```
Integral((a + b*sinh(e + f*x)**2)**(-3/2), x)
```

Maxima [F]

$$\int \frac{1}{\left(a+b\sinh^2(e+fx)\right)^{3/2}}\,dx=\int \frac{1}{\left(b\sinh\left(fx+e\right)^2+a\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/(a+b*sinh(f*x+e)^2)^(3/2),x, algorithm="maxima")
```

output
```
integrate((b*sinh(f*x + e)^2 + a)^(-3/2), x)
```

Giac [F]

$$\int \frac{1}{\left(a+b\sinh^2(e+fx)\right)^{3/2}}\,dx=\int \frac{1}{\left(b\sinh\left(fx+e\right)^2+a\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/(a+b*sinh(f*x+e)^2)^(3/2),x, algorithm="giac")
```

output
```
integrate((b*sinh(f*x + e)^2 + a)^(-3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\left(a+b\sinh^2(e+fx)\right)^{3/2}}\,dx=\int \frac{1}{\left(b\sinh\left(e+f\,x\right)^2+a\right)^{3/2}}\,dx$$

input
```
int(1/(a + b*sinh(e + f*x)^2)^(3/2),x)
```

output
```
int(1/(a + b*sinh(e + f*x)^2)^(3/2), x)
```

Reduce [F]

$$\int \frac{1}{\left(a+b\sinh^2(e+fx)\right)^{3/2}}\,dx=\int \frac{\sqrt{\sinh\left(fx+e\right)^2 b+a}}{\sinh\left(fx+e\right)^4 b^2+2\sinh\left(fx+e\right)^2 ab+a^2}dx$$

input
```
int(1/(a+b*sinh(f*x+e)^2)^(3/2),x)
```

output
```
int(sqrt(sinh(e + f*x)**2*b + a)/(sinh(e + f*x)**4*b**2 + 2*sinh(e + f*x)*
*2*a*b + a**2),x)
```

3.98 $\int \frac{\operatorname{csch}^2(e+fx)}{\left(a+b\sinh^2(e+fx)\right)^{3/2}}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 234

$$\int \frac{\operatorname{csch}^2(e+fx)}{\left(a+b\sinh^2(e+fx)\right)^{3/2}}\,dx = -\frac{\coth(e+fx)}{af\sqrt{a+b\sinh^2(e+fx)}} - \frac{(a-2b)\sqrt{b}\cosh(e+fx)E\left(\arctan\left(\frac{\sqrt{b}\sinh(e+fx)}{\sqrt{a}}\right)|1-\frac{a}{b}\right)}{a^{3/2}(a-b)f\sqrt{\frac{a\cosh^2(e+fx)}{a+b\sinh^2(e+fx)}}\sqrt{a+b\sinh^2(e+fx)}} - \frac{\sqrt{b}\cosh(e+fx)\operatorname{EllipticF}\left(\arctan\left(\frac{\sqrt{b}\sinh(e+fx)}{\sqrt{a}}\right),1-\frac{a}{b}\right)}{\sqrt{a}(a-b)f\sqrt{\frac{a\cosh^2(e+fx)}{a+b\sinh^2(e+fx)}}\sqrt{a+b\sinh^2(e+fx)}}$$

output
```
-coth(f*x+e)/a/f/(a+b*sinh(f*x+e)^2)^(1/2)-(a-2*b)*b^(1/2)*cosh(f*x+e)*Ell
ipticE(b^(1/2)*sinh(f*x+e)/a^(1/2)/(1+b*sinh(f*x+e)^2/a)^(1/2),(1-a/b)^(1/
2))/a^(3/2)/(a-b)/f/(a*cosh(f*x+e)^2/(a+b*sinh(f*x+e)^2))^(1/2)/(a+b*sinh(
f*x+e)^2)^(1/2)-b^(1/2)*cosh(f*x+e)*InverseJacobiAM(arctan(b^(1/2)*sinh(f*
x+e)/a^(1/2)),(1-a/b)^(1/2))/a^(1/2)/(a-b)/f/(a*cosh(f*x+e)^2/(a+b*sinh(f*
x+e)^2))^(1/2)/(a+b*sinh(f*x+e)^2)^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 1.14 (sec) , antiderivative size = 185, normalized size of antiderivative = 0.79

$$\int \frac{\operatorname{csch}^2(e+fx)}{\left(a+b\sinh^2(e+fx)\right)^{3/2}}\,dx = \frac{-((2a^2-3ab+2b^2+(a-2b)b\cosh(2(e+fx)))\coth(e+fx)) - i\sqrt{2}a(\ldots}{a^2(a-\ldots}$$

input
```
Integrate[Csch[e + f*x]^2/(a + b*Sinh[e + f*x]^2)^(3/2),x]
```

output
```
(-((2*a^2 - 3*a*b + 2*b^2 + (a - 2*b)*b*Cosh[2*(e + f*x)])*Coth[e + f*x])
- I*Sqrt[2]*a*(a - 2*b)*Sqrt[(2*a - b + b*Cosh[2*(e + f*x)])/a]*EllipticE[
I*(e + f*x), b/a] + I*Sqrt[2]*a*(a - b)*Sqrt[(2*a - b + b*Cosh[2*(e + f*x)
])/a]*EllipticF[I*(e + f*x), b/a])/(a^2*(a - b)*f*Sqrt[4*a - 2*b + 2*b*Cos
h[2*(e + f*x)]])
```

Rubi [A] (verified)

Time = 0.54 (sec) , antiderivative size = 347, normalized size of antiderivative = 1.48, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.400$, Rules used = {3042, 25, 3667, 374, 445, 27, 406, 320, 388, 313}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\operatorname{csch}^2(e+fx)}{\left(a+b\sinh^2(e+fx)\right)^{3/2}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int -\frac{1}{\sin(ie+ifx)^2\left(a-b\sin(ie+ifx)^2\right)^{3/2}}dx$$

$$\Big\downarrow \text{25}$$

$$-\int \frac{1}{\sin(ie+ifx)^2\left(a-b\sin(ie+ifx)^2\right)^{3/2}}dx$$

$$\Big\downarrow \; 3667$$

$$\frac{\sqrt{\cosh^2(e+fx)}\operatorname{sech}(e+fx)\int \frac{\operatorname{csch}^2(e+fx)}{\sqrt{\sinh^2(e+fx)+1}\left(b\sinh^2(e+fx)+a\right)^{3/2}}d\sinh(e+fx)}{f}$$

$$\Big\downarrow \; 374$$

$$\frac{\sqrt{\cosh^2(e+fx)}\operatorname{sech}(e+fx)\left(\frac{\int \frac{\operatorname{csch}^2(e+fx)\left(-b\sinh^2(e+fx)+a-2b\right)}{\sqrt{\sinh^2(e+fx)+1}\sqrt{b\sinh^2(e+fx)+a}}d\sinh(e+fx)}{a(a-b)}-\frac{b\sqrt{\sinh^2(e+fx)+1}\operatorname{csch}(e+fx)}{a(a-b)\sqrt{a+b\sinh^2(e+fx)}}\right)}{f}$$

$$\Big\downarrow \; 445$$

$$\frac{\sqrt{\cosh^2(e+fx)}\operatorname{sech}(e+fx)\left(\frac{-\frac{\int \frac{b\left(a-(a-2b)\sinh^2(e+fx)\right)}{\sqrt{\sinh^2(e+fx)+1}\sqrt{b\sinh^2(e+fx)+a}}d\sinh(e+fx)}{a}-\frac{(a-2b)\sqrt{\sinh^2(e+fx)+1}\operatorname{csch}(e+fx)\sqrt{a+b\sinh^2(e+fx)}}{a}}{a(a-b)}\right.}{f}$$

$$\Big\downarrow \; 27$$

$$\frac{\sqrt{\cosh^2(e+fx)}\operatorname{sech}(e+fx)\left(\frac{-\frac{b\int \frac{a-(a-2b)\sinh^2(e+fx)}{\sqrt{\sinh^2(e+fx)+1}\sqrt{b\sinh^2(e+fx)+a}}d\sinh(e+fx)}{a}-\frac{(a-2b)\sqrt{\sinh^2(e+fx)+1}\operatorname{csch}(e+fx)\sqrt{a+b\sinh^2(e+fx)}}{a}}{a(a-b)}\right.}{f}$$

$$\Big\downarrow \; 406$$

$$\frac{\sqrt{\cosh^2(e+fx)}\operatorname{sech}(e+fx)\left(\frac{-\frac{b\left(a\int \frac{1}{\sqrt{\sinh^2(e+fx)+1}\sqrt{b\sinh^2(e+fx)+a}}d\sinh(e+fx)-(a-2b)\int \frac{\sinh^2(e+fx)}{\sqrt{\sinh^2(e+fx)+1}\sqrt{b\sinh^2(e+fx)+a}}d\sinh(e\right.}{a}}{a(a-b)}\right.}{f}$$

$$\Big\downarrow \; 320$$

$$\frac{\sqrt{\cosh^2(e+fx)}\text{sech}(e+fx)\left(-\frac{-\frac{b\left(\frac{\sqrt{a+b\sinh^2(e+fx)}\text{EllipticF}\left(\arctan(\sinh(e+fx)),1-\frac{b}{a}\right)}{\sqrt{\sinh^2(e+fx)+1}\sqrt{\frac{a+b\sinh^2(e+fx)}{a\left(\sinh^2(e+fx)+1\right)}}}-(a-2b)\int\frac{\sinh^2(e+fx)}{\sqrt{\sinh^2(e+fx)+1}\sqrt{b\sinh^2(e+fx)+a}}d\sinh}{a}}{a(a-b)}\right.}{f}$$

$\downarrow$ 388

$$\frac{\sqrt{\cosh^2(e+fx)}\text{sech}(e+fx)\left(-\frac{-\frac{b\left(\frac{\sqrt{a+b\sinh^2(e+fx)}\text{EllipticF}\left(\arctan(\sinh(e+fx)),1-\frac{b}{a}\right)}{\sqrt{\sinh^2(e+fx)+1}\sqrt{\frac{a+b\sinh^2(e+fx)}{a\left(\sinh^2(e+fx)+1\right)}}}-(a-2b)\left(\frac{\sinh(e+fx)\sqrt{a+b\sinh^2(e+fx)}}{b\sqrt{\sinh^2(e+fx)+1}}-\frac{\int\frac{\sqrt{b\sin}}{\left(\sinh^2(\right.}}{}\right.}{a}}{a(a-b)}\right.}{f}$$

$\downarrow$ 313

$$\frac{\sqrt{\cosh^2(e+fx)}\text{sech}(e+fx)\left(-\frac{-\frac{b\left(\frac{\sqrt{a+b\sinh^2(e+fx)}\text{EllipticF}\left(\arctan(\sinh(e+fx)),1-\frac{b}{a}\right)}{\sqrt{\sinh^2(e+fx)+1}\sqrt{\frac{a+b\sinh^2(e+fx)}{a\left(\sinh^2(e+fx)+1\right)}}}-(a-2b)\left(\frac{\sinh(e+fx)\sqrt{a+b\sinh^2(e+fx)}}{b\sqrt{\sinh^2(e+fx)+1}}-\frac{\sqrt{a+b\sin}}{b\sqrt{\sin}}\right.}{a}}{a(a-b)}\right.}{f}$$

input

```
Int[Csch[e + f*x]^2/(a + b*Sinh[e + f*x]^2)^(3/2),x]
```

output
```
(Sqrt[Cosh[e + f*x]^2]*Sech[e + f*x]*(-((b*Csch[e + f*x]*Sqrt[1 + Sinh[e +
 f*x]^2])/(a*(a - b)*Sqrt[a + b*Sinh[e + f*x]^2])) + (-(((a - 2*b)*Csch[e
+ f*x]*Sqrt[1 + Sinh[e + f*x]^2]*Sqrt[a + b*Sinh[e + f*x]^2])/a) - (b*((El
lipticF[ArcTan[Sinh[e + f*x]], 1 - b/a]*Sqrt[a + b*Sinh[e + f*x]^2])/(Sqrt
[1 + Sinh[e + f*x]^2]*Sqrt[(a + b*Sinh[e + f*x]^2)/(a*(1 + Sinh[e + f*x]^2
))]) - (a - 2*b)*((Sinh[e + f*x]*Sqrt[a + b*Sinh[e + f*x]^2])/(b*Sqrt[1 +
Sinh[e + f*x]^2]) - (EllipticE[ArcTan[Sinh[e + f*x]], 1 - b/a]*Sqrt[a + b*
Sinh[e + f*x]^2])/(b*Sqrt[1 + Sinh[e + f*x]^2]*Sqrt[(a + b*Sinh[e + f*x]^2
)/(a*(1 + Sinh[e + f*x]^2))]))))/a)/(a*(a - b))))/f
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 313
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/((c_) + (d_.)*(x_)^2)^(3/2), x_Symbol] :> Sim
p[(Sqrt[a + b*x^2]/(c*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(c
+ d*x^2)))]))*EllipticE[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; FreeQ
[{a, b, c, d}, x] && PosQ[b/a] && PosQ[d/c]
```

rule 320
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(Sqrt[a + b*x^2]/(a*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(
c + d*x^2)))]))*EllipticF[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; Fre
eQ[{a, b, c, d}, x] && PosQ[d/c] && PosQ[b/a] &&  !SimplerSqrtQ[b/a, d/c]
```

rule 374
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_
), x_Symbol] :> Simp[(-b)*(e*x)^(m + 1)*(a + b*x^2)^(p + 1)*((c + d*x^2)^(q
 + 1)/(a*e*2*(b*c - a*d)*(p + 1))), x] + Simp[1/(a*2*(b*c - a*d)*(p + 1))
 Int[(e*x)^m*(a + b*x^2)^(p + 1)*(c + d*x^2)^q*Simp[b*c*(m + 1) + 2*(b*c -
a*d)*(p + 1) + d*b*(m + 2*(p + q + 2) + 1)*x^2, x], x], x] /; FreeQ[{a, b,
c, d, e, m, q}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && IntBinomialQ[a, b,
 c, d, e, m, 2, p, q, x]
```

rule 388
```
Int[(x_)^2/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol]
 :> Simp[x*(Sqrt[a + b*x^2]/(b*Sqrt[c + d*x^2])), x] - Simp[c/b   Int[Sqrt[
a + b*x^2]/(c + d*x^2)^(3/2), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
 a*d, 0] && PosQ[b/a] && PosQ[d/c] &&  !SimplerSqrtQ[b/a, d/c]
```

rule 406
```
Int[((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.)*((e_) + (f_.)*(
x_)^2), x_Symbol] :> Simp[e   Int[(a + b*x^2)^p*(c + d*x^2)^q, x], x] + Sim
p[f   Int[x^2*(a + b*x^2)^p*(c + d*x^2)^q, x], x] /; FreeQ[{a, b, c, d, e,
f, p, q}, x]
```

rule 445
```
Int[((g_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_
.)*((e_) + (f_.)*(x_)^2), x_Symbol] :> Simp[e*(g*x)^(m + 1)*(a + b*x^2)^(p
+ 1)*((c + d*x^2)^(q + 1)/(a*c*g*(m + 1))), x] + Simp[1/(a*c*g^2*(m + 1))
 Int[(g*x)^(m + 2)*(a + b*x^2)^p*(c + d*x^2)^q*Simp[a*f*c*(m + 1) - e*(b*c
+ a*d)*(m + 2 + 1) - e*2*(b*c*p + a*d*q) - b*e*d*(m + 2*(p + q + 2) + 1)*x^
2, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p, q}, x] && LtQ[m, -1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3667
```
Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(
p_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Simp[ff^(m + 1
)*(Sqrt[Cos[e + f*x]^2]/(f*Cos[e + f*x]))   Subst[Int[x^m*((a + b*ff^2*x^2)
^p/Sqrt[1 - ff^2*x^2]), x], x, Sin[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f,
p}, x] && IntegerQ[m/2] &&  !IntegerQ[p]
```

Maple [A] (verified)

Time = 4.65 (sec) , antiderivative size = 284, normalized size of antiderivative = 1.21

method	result
default	$-\frac{\left(\sqrt{-\frac{b}{a}}\,ab-2\sqrt{-\frac{b}{a}}\,b^{2}\right)\cosh(fx+e)^{4}+\left(\sqrt{-\frac{b}{a}}\,a^{2}-2\sqrt{-\frac{b}{a}}\,ab+2\sqrt{-\frac{b}{a}}\,b^{2}\right)\cosh(fx+e)^{2}+\sinh(fx+e)\sqrt{\frac{\cosh(2fx+2e)}{2}+\frac{1}{2}}\,\sqrt{\frac{b\cos}{}}}{a^{2}\sinh}$
risch	Expression too large to display

input
```
int(csch(f*x+e)^2/(a+b*sinh(f*x+e)^2)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-((((-b/a)^(1/2)*a*b-2*(-b/a)^(1/2)*b^2)*cosh(f*x+e)^4+((-b/a)^(1/2)*a^2-2*
(-b/a)^(1/2)*a*b+2*(-b/a)^(1/2)*b^2)*cosh(f*x+e)^2+sinh(f*x+e)*(cosh(f*x+e
)^2)^(1/2)*(b/a*cosh(f*x+e)^2+(a-b)/a)^(1/2)*b*(2*a*EllipticF(sinh(f*x+e)*
(-b/a)^(1/2),(1/b*a)^(1/2))-2*b*EllipticF(sinh(f*x+e)*(-b/a)^(1/2),(1/b*a)
^(1/2))-EllipticE(sinh(f*x+e)*(-b/a)^(1/2),(1/b*a)^(1/2))*a+2*b*EllipticE(
sinh(f*x+e)*(-b/a)^(1/2),(1/b*a)^(1/2))))/a^2/sinh(f*x+e)/(-b/a)^(1/2)/(a-
b)/cosh(f*x+e)/(a+b*sinh(f*x+e)^2)^(1/2)/f
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2829 vs. $2(228) = 456$.

Time = 0.15 (sec) , antiderivative size = 2829, normalized size of antiderivative = 12.09

$$\int \frac{\operatorname{csch}^2(e+fx)}{\left(a+b\sinh^2(e+fx)\right)^{3/2}}\,dx = \text{Too large to display}$$

input
```
integrate(csch(f*x+e)^2/(a+b*sinh(f*x+e)^2)^(3/2),x, algorithm="fricas")
```

output
```
(((2*a^2*b - 5*a*b^2 + 2*b^3)*cosh(f*x + e)^6 + 6*(2*a^2*b - 5*a*b^2 + 2*b
^3)*cosh(f*x + e)*sinh(f*x + e)^5 + (2*a^2*b - 5*a*b^2 + 2*b^3)*sinh(f*x +
 e)^6 + (8*a^3 - 26*a^2*b + 23*a*b^2 - 6*b^3)*cosh(f*x + e)^4 + (8*a^3 - 2
6*a^2*b + 23*a*b^2 - 6*b^3 + 15*(2*a^2*b - 5*a*b^2 + 2*b^3)*cosh(f*x + e)^
2)*sinh(f*x + e)^4 + 4*(5*(2*a^2*b - 5*a*b^2 + 2*b^3)*cosh(f*x + e)^3 + (8
*a^3 - 26*a^2*b + 23*a*b^2 - 6*b^3)*cosh(f*x + e))*sinh(f*x + e)^3 - 2*a^2
*b + 5*a*b^2 - 2*b^3 - (8*a^3 - 26*a^2*b + 23*a*b^2 - 6*b^3)*cosh(f*x + e)
^2 + (15*(2*a^2*b - 5*a*b^2 + 2*b^3)*cosh(f*x + e)^4 - 8*a^3 + 26*a^2*b -
23*a*b^2 + 6*b^3 + 6*(8*a^3 - 26*a^2*b + 23*a*b^2 - 6*b^3)*cosh(f*x + e)^2
)*sinh(f*x + e)^2 + 2*(3*(2*a^2*b - 5*a*b^2 + 2*b^3)*cosh(f*x + e)^5 + 2*(
8*a^3 - 26*a^2*b + 23*a*b^2 - 6*b^3)*cosh(f*x + e)^3 - (8*a^3 - 26*a^2*b +
 23*a*b^2 - 6*b^3)*cosh(f*x + e))*sinh(f*x + e) - 2*((a*b^2 - 2*b^3)*cosh(
f*x + e)^6 + 6*(a*b^2 - 2*b^3)*cosh(f*x + e)*sinh(f*x + e)^5 + (a*b^2 - 2*
b^3)*sinh(f*x + e)^6 + (4*a^2*b - 11*a*b^2 + 6*b^3)*cosh(f*x + e)^4 + (4*a
^2*b - 11*a*b^2 + 6*b^3 + 15*(a*b^2 - 2*b^3)*cosh(f*x + e)^2)*sinh(f*x + e
)^4 + 4*(5*(a*b^2 - 2*b^3)*cosh(f*x + e)^3 + (4*a^2*b - 11*a*b^2 + 6*b^3)*
cosh(f*x + e))*sinh(f*x + e)^3 - a*b^2 + 2*b^3 - (4*a^2*b - 11*a*b^2 + 6*b
^3)*cosh(f*x + e)^2 + (15*(a*b^2 - 2*b^3)*cosh(f*x + e)^4 - 4*a^2*b + 11*a
*b^2 - 6*b^3 + 6*(4*a^2*b - 11*a*b^2 + 6*b^3)*cosh(f*x + e)^2)*sinh(f*x +
e)^2 + 2*(3*(a*b^2 - 2*b^3)*cosh(f*x + e)^5 + 2*(4*a^2*b - 11*a*b^2 + 6...
```

Sympy [F]

$$\int \frac{\operatorname{csch}^2(e+fx)}{\left(a+b\sinh^2(e+fx)\right)^{3/2}}\,dx=\int \frac{\operatorname{csch}^2\left(e+fx\right)}{\left(a+b\sinh^2\left(e+fx\right)\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate(csch(f*x+e)**2/(a+b*sinh(f*x+e)**2)**(3/2),x)
```

output
```
Integral(csch(e + f*x)**2/(a + b*sinh(e + f*x)**2)**(3/2), x)
```

Maxima [F]

$$\int \frac{\operatorname{csch}^2(e+fx)}{\left(a+b\sinh^2(e+fx)\right)^{3/2}}\,dx=\int \frac{\operatorname{csch}\left(fx+e\right)^2}{\left(b\sinh\left(fx+e\right)^2+a\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate(csch(f*x+e)^2/(a+b*sinh(f*x+e)^2)^(3/2),x, algorithm="maxima")
```

output
```
integrate(csch(f*x + e)^2/(b*sinh(f*x + e)^2 + a)^(3/2), x)
```

Giac [F]

$$\int \frac{\operatorname{csch}^2(e+fx)}{\left(a+b\sinh^2(e+fx)\right)^{3/2}}\,dx=\int \frac{\operatorname{csch}\left(fx+e\right)^2}{\left(b\sinh\left(fx+e\right)^2+a\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate(csch(f*x+e)^2/(a+b*sinh(f*x+e)^2)^(3/2),x, algorithm="giac")
```

output
```
integrate(csch(f*x + e)^2/(b*sinh(f*x + e)^2 + a)^(3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\operatorname{csch}^2(e+fx)}{\left(a+b\sinh^2(e+fx)\right)^{3/2}}\,dx=\int \frac{1}{\sinh\left(e+f\,x\right)^2\,\left(b\sinh\left(e+f\,x\right)^2+a\right)^{3/2}}\,dx$$

input
```
int(1/(sinh(e + f*x)^2*(a + b*sinh(e + f*x)^2)^(3/2)),x)
```

output
```
int(1/(sinh(e + f*x)^2*(a + b*sinh(e + f*x)^2)^(3/2)), x)
```

Reduce [F]

$$\int \frac{\operatorname{csch}^2(e+fx)}{\left(a+b\sinh^2(e+fx)\right)^{3/2}}\,dx=\int \frac{\sqrt{\sinh\left(fx+e\right)^2 b+a}\,\operatorname{csch}(fx+e)^2}{\sinh\left(fx+e\right)^4 b^2+2\sinh\left(fx+e\right)^2 ab+a^2}dx$$

input
```
int(csch(f*x+e)^2/(a+b*sinh(f*x+e)^2)^(3/2),x)
```

output
```
int((sqrt(sinh(e + f*x)**2*b + a)*csch(e + f*x)**2)/(sinh(e + f*x)**4*b**2
 + 2*sinh(e + f*x)**2*a*b + a**2),x)
```

3.99 $\int \frac{\sinh^5(e+fx)}{\left(a+b\sinh^2(e+fx)\right)^{5/2}}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 137

$$\int \frac{\sinh^5(e+fx)}{\left(a+b\sinh^2(e+fx)\right)^{5/2}}\,dx = \frac{\operatorname{arctanh}\left(\frac{\sqrt{b}\cosh(e+fx)}{\sqrt{a-b+b\cosh^2(e+fx)}}\right)}{b^{5/2}f} + \frac{a^2\cosh(e+fx)}{3(a-b)b^2f\left(a-b+b\cosh^2(e+fx)\right)^{3/2}} - \frac{2a(2a-3b)\cosh(e+fx)}{3(a-b)^2b^2f\sqrt{a-b+b\cosh^2(e+fx)}}$$

output
```
arctanh(b^(1/2)*cosh(f*x+e)/(a-b+b*cosh(f*x+e)^2)^(1/2))/b^(5/2)/f+1/3*a^2
*cosh(f*x+e)/(a-b)/b^2/f/(a-b+b*cosh(f*x+e)^2)^(3/2)-2/3*a*(2*a-3*b)*cosh(
f*x+e)/(a-b)^2/b^2/f/(a-b+b*cosh(f*x+e)^2)^(1/2)
```

Mathematica [A] (verified)

Time = 1.04 (sec) , antiderivative size = 130, normalized size of antiderivative = 0.95

$$\int \frac{\sinh^5(e+fx)}{\left(a+b\sinh^2(e+fx)\right)^{5/2}}\,dx = \frac{-\frac{2\sqrt{2}a\cosh(e+fx)\left(3a^2-7ab+3b^2+(2a-3b)b\cosh(2(e+fx))\right)}{3(a-b)^2b^2(2a-b+b\cosh(2(e+fx)))^{3/2}} + \frac{\log\left(\sqrt{2}\sqrt{b}\cosh(e+fx)+\sqrt{2a-b}\right.}{b^{5/2}}}{f}$$

input
```
Integrate[Sinh[e + f*x]^5/(a + b*Sinh[e + f*x]^2)^(5/2),x]
```

output
```
((-2*Sqrt[2]*a*Cosh[e + f*x]*(3*a^2 - 7*a*b + 3*b^2 + (2*a - 3*b)*b*Cosh[2
*(e + f*x)]))/(3*(a - b)^2*b^2*(2*a - b + b*Cosh[2*(e + f*x)])^(3/2)) + Lo
g[Sqrt[2]*Sqrt[b]*Cosh[e + f*x] + Sqrt[2*a - b + b*Cosh[2*(e + f*x)]]]/b^(
5/2))/f
```

Rubi [A] (verified)

Time = 0.37 (sec) , antiderivative size = 160, normalized size of antiderivative = 1.17, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.320, Rules used = {3042, 26, 3665, 315, 25, 298, 224, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sinh^5(e+fx)}{\left(a+b\sinh^2(e+fx)\right)^{5/2}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int -\frac{i\sin(ie+ifx)^5}{\left(a-b\sin(ie+ifx)^2\right)^{5/2}}dx$$

$$\Big\downarrow \text{26}$$

$$-i\int \frac{\sin(ie+ifx)^5}{\left(a-b\sin(ie+ifx)^2\right)^{5/2}}dx$$

$$\Big\downarrow \text{3665}$$

$$\frac{\int \frac{\left(1-\cosh^2(e+fx)\right)^2}{\left(b\cosh^2(e+fx)+a-b\right)^{5/2}}d\cosh(e+fx)}{f}$$

$$\Big\downarrow \text{315}$$

$$\frac{\frac{\int -\frac{-3(a-b)\cosh^2(e+fx)+a-3b}{\left(b\cosh^2(e+fx)+a-b\right)^{3/2}}d\cosh(e+fx)}{3b(a-b)}+\frac{a\cosh(e+fx)\left(1-\cosh^2(e+fx)\right)}{3b(a-b)\left(a+b\cosh^2(e+fx)-b\right)^{3/2}}}{f}$$

$$\Big\downarrow \text{25}$$

$$\frac{\frac{a\cosh(e+fx)\left(1-\cosh^2(e+fx)\right)}{3b(a-b)\left(a+b\cosh^2(e+fx)-b\right)^{3/2}}-\frac{\int\frac{-3(a-b)\cosh^2(e+fx)+a-3b}{\left(b\cosh^2(e+fx)+a-b\right)^{3/2}}d\cosh(e+fx)}{3b(a-b)}}{f}$$

$\Big\downarrow$ 298

$$\frac{\frac{a\cosh(e+fx)\left(1-\cosh^2(e+fx)\right)}{3b(a-b)\left(a+b\cosh^2(e+fx)-b\right)^{3/2}}-\frac{\frac{a(3a-5b)\cosh(e+fx)}{b(a-b)\sqrt{a+b\cosh^2(e+fx)-b}}-\frac{3(a-b)\int\frac{1}{\sqrt{b\cosh^2(e+fx)+a-b}}d\cosh(e+fx)}{b}}{3b(a-b)}}{f}$$

$\Big\downarrow$ 224

$$\frac{\frac{a\cosh(e+fx)\left(1-\cosh^2(e+fx)\right)}{3b(a-b)\left(a+b\cosh^2(e+fx)-b\right)^{3/2}}-\frac{\frac{a(3a-5b)\cosh(e+fx)}{b(a-b)\sqrt{a+b\cosh^2(e+fx)-b}}-\frac{3(a-b)\int\frac{1}{1-\frac{b\cosh^2(e+fx)}{b\cosh^2(e+fx)+a-b}}d\frac{\cosh(e+fx)}{\sqrt{b\cosh^2(e+fx)+a-b}}}{b}}{3b(a-b)}}{f}$$

$\Big\downarrow$ 219

$$\frac{\frac{a\cosh(e+fx)\left(1-\cosh^2(e+fx)\right)}{3b(a-b)\left(a+b\cosh^2(e+fx)-b\right)^{3/2}}-\frac{\frac{a(3a-5b)\cosh(e+fx)}{b(a-b)\sqrt{a+b\cosh^2(e+fx)-b}}-\frac{3(a-b)\text{arctanh}\left(\frac{\sqrt{b}\cosh(e+fx)}{\sqrt{a+b\cosh^2(e+fx)-b}}\right)}{b^{3/2}}}{3b(a-b)}}{f}$$

input
```
Int[Sinh[e + f*x]^5/(a + b*Sinh[e + f*x]^2)^(5/2),x]
```

output
```
((a*Cosh[e + f*x]*(1 - Cosh[e + f*x]^2))/(3*(a - b)*b*(a - b + b*Cosh[e +
f*x]^2)^(3/2)) - ((-3*(a - b)*ArcTanh[(Sqrt[b]*Cosh[e + f*x])/Sqrt[a - b +
 b*Cosh[e + f*x]^2]])/b^(3/2) + (a*(3*a - 5*b)*Cosh[e + f*x])/((a - b)*b*S
qrt[a - b + b*Cosh[e + f*x]^2]))/(3*(a - b)*b))/f
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

rule 298
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2), x_Symbol] :> Simp[(-(
b*c - a*d))*x*((a + b*x^2)^(p + 1)/(2*a*b*(p + 1))), x] - Simp[(a*d - b*c*(
2*p + 3))/(2*a*b*(p + 1))   Int[(a + b*x^2)^(p + 1), x], x] /; FreeQ[{a, b,
 c, d, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/2 + p, 0])
```

rule 315
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_), x_Symbol] :> Sim
p[(a*d - c*b)*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^(q - 1)/(2*a*b*(p + 1))),
x] - Simp[1/(2*a*b*(p + 1))   Int[(a + b*x^2)^(p + 1)*(c + d*x^2)^(q - 2)*S
imp[c*(a*d - c*b*(2*p + 3)) + d*(a*d*(2*(q - 1) + 1) - b*c*(2*(p + q) + 1))
*x^2, x], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -
1] && GtQ[q, 1] && IntBinomialQ[a, b, c, d, 2, p, q, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3665
```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^
(p_.), x_Symbol] :> With[{ff = FreeFactors[Cos[e + f*x], x]}, Simp[-ff/f
Subst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a + b - b*ff^2*x^2)^p, x], x, Cos[e +
 f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m - 1)/2]
```

Maple [A] (verified)

Time = 0.82 (sec) , antiderivative size = 230, normalized size of antiderivative = 1.68

method	result
default	$\frac{\sqrt{\left(a+b\sinh(fx+e)^2\right)\cosh(fx+e)^2}\left(\frac{\ln\left(\frac{\frac{a}{2}+\frac{b}{2}+b\sinh(fx+e)^2}{\sqrt{b}}+\sqrt{\left(a+b\sinh(fx+e)^2\right)\cosh(fx+e)^2}\right)}{2b^{\frac{5}{2}}}+\frac{a^2\left(2b\sinh(fx+e)^2\right.}{3b^2\sqrt{\left(a+b\sinh(fx+e)^2\right)\cosh(fx+}}\right.}{\cosh(fx+e)\sqrt{a+b\sinh(fx+e)^2}\,f}$

input
```
int(sinh(f*x+e)^5/(a+b*sinh(f*x+e)^2)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
((a+b*sinh(f*x+e)^2)*cosh(f*x+e)^2)^(1/2)*(1/2/b^(5/2)*ln((1/2*a+1/2*b+b*s
inh(f*x+e)^2)/b^(1/2)+((a+b*sinh(f*x+e)^2)*cosh(f*x+e)^2)^(1/2))+1/3*a^2/b
^2*(2*b*sinh(f*x+e)^2+3*a-b)*cosh(f*x+e)^2/((a+b*sinh(f*x+e)^2)*cosh(f*x+e
)^2)^(1/2)/(a+b*sinh(f*x+e)^2)/(a^2-2*a*b+b^2)-2*a/b^2*cosh(f*x+e)^2/(a-b)
/((a+b*sinh(f*x+e)^2)*cosh(f*x+e)^2)^(1/2))/cosh(f*x+e)/(a+b*sinh(f*x+e)^2
)^(1/2)/f
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 3631 vs. 2(123) = 246.

Time = 0.49 (sec) , antiderivative size = 7938, normalized size of antiderivative = 57.94

$$\int \frac{\sinh^5(e+fx)}{\left(a+b\sinh^2(e+fx)\right)^{5/2}}\,dx = \text{Too large to display}$$

input
```
integrate(sinh(f*x+e)^5/(a+b*sinh(f*x+e)^2)^(5/2),x, algorithm="fricas")
```

output
```
Too large to include
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\sinh^5(e+fx)}{\left(a+b\sinh^2(e+fx)\right)^{5/2}}\,dx=\text{Timed out}$$

input
```
integrate(sinh(f*x+e)**5/(a+b*sinh(f*x+e)**2)**(5/2),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{\sinh^5(e+fx)}{\left(a+b\sinh^2(e+fx)\right)^{5/2}}\,dx=\int \frac{\sinh\left(fx+e\right)^5}{\left(b\sinh\left(fx+e\right)^2+a\right)^{\frac{5}{2}}}\,dx$$

input
```
integrate(sinh(f*x+e)^5/(a+b*sinh(f*x+e)^2)^(5/2),x, algorithm="maxima")
```

output
```
integrate(sinh(f*x + e)^5/(b*sinh(f*x + e)^2 + a)^(5/2), x)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{\sinh^5(e+fx)}{\left(a+b\sinh^2(e+fx)\right)^{5/2}}\,dx=\text{Exception raised: TypeError}$$

input
```
integrate(sinh(f*x+e)^5/(a+b*sinh(f*x+e)^2)^(5/2),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Degree mismatch inside factorisatio
n over extensionNot implemented, e.g. for multivariate mod/approx polynomi
alsError:
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sinh^5(e+fx)}{\left(a+b\sinh^2(e+fx)\right)^{5/2}}\,dx=\int \frac{\sinh(e+f\,x)^5}{\left(b\sinh\left(e+f\,x\right)^2+a\right)^{5/2}}\,dx$$

input
```
int(sinh(e + f*x)^5/(a + b*sinh(e + f*x)^2)^(5/2),x)
```

output
```
int(sinh(e + f*x)^5/(a + b*sinh(e + f*x)^2)^(5/2), x)
```

Reduce [F]

$$\int \frac{\sinh^5(e+fx)}{\left(a+b\sinh^2(e+fx)\right)^{5/2}}\,dx=\int \frac{\sqrt{\sinh\left(fx+e\right)^2 b+a}\,\sinh\left(fx+e\right)^5}{\sinh\left(fx+e\right)^6 b^3+3\sinh\left(fx+e\right)^4 a\,b^2+3\sinh\left(fx+e\right)^2 a^2b+a^3}dx$$

input
```
int(sinh(f*x+e)^5/(a+b*sinh(f*x+e)^2)^(5/2),x)
```

output
```
int((sqrt(sinh(e + f*x)**2*b + a)*sinh(e + f*x)**5)/(sinh(e + f*x)**6*b**3
 + 3*sinh(e + f*x)**4*a*b**2 + 3*sinh(e + f*x)**2*a**2*b + a**3),x)
```

3.100 $\int \frac{\sinh^3(e+fx)}{\left(a+b\sinh^2(e+fx)\right)^{5/2}}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 91

$$\int \frac{\sinh^3(e+fx)}{\left(a+b\sinh^2(e+fx)\right)^{5/2}}\,dx = -\frac{a\cosh(e+fx)}{3(a-b)bf\left(a-b+b\cosh^2(e+fx)\right)^{3/2}} + \frac{(a-3b)\cosh(e+fx)}{3(a-b)^2bf\sqrt{a-b+b\cosh^2(e+fx)}}$$

output
```
-1/3*a*cosh(f*x+e)/(a-b)/b/f/(a-b+b*cosh(f*x+e)^2)^(3/2)+1/3*(a-3*b)*cosh(
f*x+e)/(a-b)^2/b/f/(a-b+b*cosh(f*x+e)^2)^(1/2)
```

Mathematica [A] (verified)

Time = 0.49 (sec) , antiderivative size = 67, normalized size of antiderivative = 0.74

$$\int \frac{\sinh^3(e+fx)}{\left(a+b\sinh^2(e+fx)\right)^{5/2}}\,dx = \frac{\sqrt{2}\cosh(e+fx)(-5a+3b+(a-3b)\cosh(2(e+fx)))}{3(a-b)^2f(2a-b+b\cosh(2(e+fx)))^{3/2}}$$

input
```
Integrate[Sinh[e + f*x]^3/(a + b*Sinh[e + f*x]^2)^(5/2),x]
```

output
```
(Sqrt[2]*Cosh[e + f*x]*(-5*a + 3*b + (a - 3*b)*Cosh[2*(e + f*x)]))/(3*(a -
 b)^2*f*(2*a - b + b*Cosh[2*(e + f*x)])^(3/2))
```

Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 90, normalized size of antiderivative = 0.99, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {3042, 26, 3665, 292, 208}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sinh^3(e+fx)}{\left(a+b\sinh^2(e+fx)\right)^{5/2}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{i\sin(ie+ifx)^3}{\left(a-b\sin(ie+ifx)^2\right)^{5/2}}dx$$

$$\Big\downarrow \text{26}$$

$$i\int \frac{\sin(ie+ifx)^3}{\left(a-b\sin(ie+ifx)^2\right)^{5/2}}dx$$

$$\Big\downarrow \text{3665}$$

$$-\frac{\int \frac{1-\cosh^2(e+fx)}{\left(b\cosh^2(e+fx)+a-b\right)^{5/2}}d\cosh(e+fx)}{f}$$

$$\Big\downarrow \text{292}$$

$$-\frac{\frac{2\int \frac{1}{\left(b\cosh^2(e+fx)+a-b\right)^{3/2}}d\cosh(e+fx)}{3(a-b)}+\frac{\cosh(e+fx)\left(1-\cosh^2(e+fx)\right)}{3(a-b)\left(a+b\cosh^2(e+fx)-b\right)^{3/2}}}{f}$$

$$\Big\downarrow \text{208}$$

$$-\frac{\frac{2\cosh(e+fx)}{3(a-b)^2\sqrt{a+b\cosh^2(e+fx)-b}}+\frac{\left(1-\cosh^2(e+fx)\right)\cosh(e+fx)}{3(a-b)\left(a+b\cosh^2(e+fx)-b\right)^{3/2}}}{f}$$

input
```
Int[Sinh[e + f*x]^3/(a + b*Sinh[e + f*x]^2)^(5/2),x]
```

output
```
-(((Cosh[e + f*x]*(1 - Cosh[e + f*x]^2))/(3*(a - b)*(a - b + b*Cosh[e + f*
x]^2)^(3/2)) + (2*Cosh[e + f*x])/(3*(a - b)^2*Sqrt[a - b + b*Cosh[e + f*x]
^2]))/f)
```

Defintions of rubi rules used

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 208
```
Int[((a_) + (b_.)*(x_)^2)^(-3/2), x_Symbol] :> Simp[x/(a*Sqrt[a + b*x^2]),
x] /; FreeQ[{a, b}, x]
```

rule 292
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_.), x_Symbol] :> Si
mp[(-x)*(a + b*x^2)^(p + 1)*((c + d*x^2)^q/(2*a*(p + 1))), x] - Simp[c*(q/(
a*(p + 1)))   Int[(a + b*x^2)^(p + 1)*(c + d*x^2)^(q - 1), x], x] /; FreeQ[
{a, b, c, d, p}, x] && NeQ[b*c - a*d, 0] && EqQ[2*(p + q + 1) + 1, 0] && Gt
Q[q, 0] && NeQ[p, -1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3665
```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^
(p_.), x_Symbol] :> With[{ff = FreeFactors[Cos[e + f*x], x]}, Simp[-ff/f
Subst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a + b - b*ff^2*x^2)^p, x], x, Cos[e +
 f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m - 1)/2]
```

Maple [A] (verified)

Time = 0.54 (sec) , antiderivative size = 64, normalized size of antiderivative = 0.70

method	result	size
default	$\frac{\left(a\sinh(fx+e)^{2}-3b\sinh(fx+e)^{2}-2a\right)\cosh(fx+e)}{3\left(a+b\sinh(fx+e)^{2}\right)^{\frac{3}{2}}(a^{2}-2ab+b^{2})f}$	64

input
```
int(sinh(f*x+e)^3/(a+b*sinh(f*x+e)^2)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
1/3*(a*sinh(f*x+e)^2-3*b*sinh(f*x+e)^2-2*a)*cosh(f*x+e)/(a+b*sinh(f*x+e)^2
)^(3/2)/(a^2-2*a*b+b^2)/f
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1214 vs. $2(83) = 166$.

Time = 0.23 (sec) , antiderivative size = 1214, normalized size of antiderivative = 13.34

$$\int \frac{\sinh^3(e+fx)}{\left(a+b\sinh^2(e+fx)\right)^{5/2}}\,dx = \text{Too large to display}$$

input
```
integrate(sinh(f*x+e)^3/(a+b*sinh(f*x+e)^2)^(5/2),x, algorithm="fricas")
```

output
```
1/3*sqrt(2)*((a - 3*b)*cosh(f*x + e)^6 + 6*(a - 3*b)*cosh(f*x + e)*sinh(f*
x + e)^5 + (a - 3*b)*sinh(f*x + e)^6 - 3*(3*a - b)*cosh(f*x + e)^4 + 3*(5*
(a - 3*b)*cosh(f*x + e)^2 - 3*a + b)*sinh(f*x + e)^4 + 4*(5*(a - 3*b)*cosh
(f*x + e)^3 - 3*(3*a - b)*cosh(f*x + e))*sinh(f*x + e)^3 - 3*(3*a - b)*cos
h(f*x + e)^2 + 3*(5*(a - 3*b)*cosh(f*x + e)^4 - 6*(3*a - b)*cosh(f*x + e)^
2 - 3*a + b)*sinh(f*x + e)^2 + 6*((a - 3*b)*cosh(f*x + e)^5 - 2*(3*a - b)*
cosh(f*x + e)^3 - (3*a - b)*cosh(f*x + e))*sinh(f*x + e) + a - 3*b)*sqrt((
b*cosh(f*x + e)^2 + b*sinh(f*x + e)^2 + 2*a - b)/(cosh(f*x + e)^2 - 2*cosh
(f*x + e)*sinh(f*x + e) + sinh(f*x + e)^2))/((a^2*b^2 - 2*a*b^3 + b^4)*f*c
osh(f*x + e)^8 + 8*(a^2*b^2 - 2*a*b^3 + b^4)*f*cosh(f*x + e)*sinh(f*x + e)
^7 + (a^2*b^2 - 2*a*b^3 + b^4)*f*sinh(f*x + e)^8 + 4*(2*a^3*b - 5*a^2*b^2
+ 4*a*b^3 - b^4)*f*cosh(f*x + e)^6 + 4*(7*(a^2*b^2 - 2*a*b^3 + b^4)*f*cosh
(f*x + e)^2 + (2*a^3*b - 5*a^2*b^2 + 4*a*b^3 - b^4)*f)*sinh(f*x + e)^6 + 2
*(8*a^4 - 24*a^3*b + 27*a^2*b^2 - 14*a*b^3 + 3*b^4)*f*cosh(f*x + e)^4 + 8*
(7*(a^2*b^2 - 2*a*b^3 + b^4)*f*cosh(f*x + e)^3 + 3*(2*a^3*b - 5*a^2*b^2 +
4*a*b^3 - b^4)*f*cosh(f*x + e))*sinh(f*x + e)^5 + 2*(35*(a^2*b^2 - 2*a*b^3
 + b^4)*f*cosh(f*x + e)^4 + 30*(2*a^3*b - 5*a^2*b^2 + 4*a*b^3 - b^4)*f*cos
h(f*x + e)^2 + (8*a^4 - 24*a^3*b + 27*a^2*b^2 - 14*a*b^3 + 3*b^4)*f)*sinh(
f*x + e)^4 + 4*(2*a^3*b - 5*a^2*b^2 + 4*a*b^3 - b^4)*f*cosh(f*x + e)^2 + 8
*(7*(a^2*b^2 - 2*a*b^3 + b^4)*f*cosh(f*x + e)^5 + 10*(2*a^3*b - 5*a^2*b...
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\sinh^3(e+fx)}{(a+b\sinh^2(e+fx))^{5/2}}\,dx = \text{Timed out}$$

input
```
integrate(sinh(f*x+e)**3/(a+b*sinh(f*x+e)**2)**(5/2),x)
```

output
```
Timed out
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 927 vs. $2(83) = 166$.

Time = 0.17 (sec) , antiderivative size = 927, normalized size of antiderivative = 10.19

$$\int \frac{\sinh^3(e+fx)}{\left(a+b\sinh^2(e+fx)\right)^{5/2}}\,dx = \text{Too large to display}$$

input
```
integrate(sinh(f*x+e)^3/(a+b*sinh(f*x+e)^2)^(5/2),x, algorithm="maxima")
```

output
```
-1/12*(b^4*e^(-10*f*x - 10*e) - 4*a^3*b + 6*a^2*b^2 - b^4 - (16*a^4 - 32*a
^3*b + 6*a^2*b^2 + 10*a*b^3 - 5*b^4)*e^(-2*f*x - 2*e) + 10*(2*a^3*b - 3*a^
2*b^2 + 3*a*b^3 - b^4)*e^(-4*f*x - 4*e) + 10*(3*a^2*b^2 - 3*a*b^3 + b^4)*e
^(-6*f*x - 6*e) + 5*(2*a*b^3 - b^4)*e^(-8*f*x - 8*e))/((a^4 - 2*a^3*b + a^
2*b^2)*(2*(2*a - b)*e^(-2*f*x - 2*e) + b*e^(-4*f*x - 4*e) + b)^(5/2)*f) -
1/4*(2*a^2*b^2 - 2*a*b^3 + b^4 + 5*(4*a^3*b - 6*a^2*b^2 + 4*a*b^3 - b^4)*e
^(-2*f*x - 2*e) + 2*(24*a^4 - 48*a^3*b + 49*a^2*b^2 - 25*a*b^3 + 5*b^4)*e^
(-4*f*x - 4*e) + 10*(6*a^3*b - 9*a^2*b^2 + 5*a*b^3 - b^4)*e^(-6*f*x - 6*e)
 + 5*(4*a^2*b^2 - 4*a*b^3 + b^4)*e^(-8*f*x - 8*e) + (2*a*b^3 - b^4)*e^(-10
*f*x - 10*e))/((a^4 - 2*a^3*b + a^2*b^2)*(2*(2*a - b)*e^(-2*f*x - 2*e) + b
*e^(-4*f*x - 4*e) + b)^(5/2)*f) - 1/4*(2*a*b^3 - b^4 + 5*(4*a^2*b^2 - 4*a*
b^3 + b^4)*e^(-2*f*x - 2*e) + 10*(6*a^3*b - 9*a^2*b^2 + 5*a*b^3 - b^4)*e^(
-4*f*x - 4*e) + 2*(24*a^4 - 48*a^3*b + 49*a^2*b^2 - 25*a*b^3 + 5*b^4)*e^(-
6*f*x - 6*e) + 5*(4*a^3*b - 6*a^2*b^2 + 4*a*b^3 - b^4)*e^(-8*f*x - 8*e) +
(2*a^2*b^2 - 2*a*b^3 + b^4)*e^(-10*f*x - 10*e))/((a^4 - 2*a^3*b + a^2*b^2)
*(2*(2*a - b)*e^(-2*f*x - 2*e) + b*e^(-4*f*x - 4*e) + b)^(5/2)*f) - 1/12*(
b^4 + 5*(2*a*b^3 - b^4)*e^(-2*f*x - 2*e) + 10*(3*a^2*b^2 - 3*a*b^3 + b^4)*
e^(-4*f*x - 4*e) + 10*(2*a^3*b - 3*a^2*b^2 + 3*a*b^3 - b^4)*e^(-6*f*x - 6*
e) - (16*a^4 - 32*a^3*b + 6*a^2*b^2 + 10*a*b^3 - 5*b^4)*e^(-8*f*x - 8*e) -
 (4*a^3*b - 6*a^2*b^2 + b^4)*e^(-10*f*x - 10*e))/((a^4 - 2*a^3*b + a^2*...
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 333 vs. $2(83) = 166$.

Time = 0.32 (sec) , antiderivative size = 333, normalized size of antiderivative = 3.66

$$\int \frac{\sinh^3(e+fx)}{\left(a+b\sinh^2(e+fx)\right)^{5/2}}\,dx = \frac{\left(\left(\frac{\left(a^3f^3e^{(12\,e)}-3\,a^2bf^3e^{(12\,e)}\right)e^{(2\,fx)}}{a^4f^4e^{(6\,e)}-2\,a^3bf^4e^{(6\,e)}+a^2b^2f^4e^{(6\,e)}} - \frac{3\left(3\,a^3f^3e^{(10\,e)}-a^2bf^3e^{(10\,e)}\right)}{a^4f^4e^{(6\,e)}-2\,a^3bf^4e^{(6\,e)}+a^2b^2f^4e^{(6\,e)}}\right)e^{(2\,fx)} - \right.}{3\left(be^{(4\,fx+4\,e)}+4\,ae^{(2\,fx+2\,e}\right.}$$

input
```
integrate(sinh(f*x+e)^3/(a+b*sinh(f*x+e)^2)^(5/2),x, algorithm="giac")
```

output
```
1/3*((((a^3*f^3*e^(12*e) - 3*a^2*b*f^3*e^(12*e))*e^(2*f*x)/(a^4*f^4*e^(6*e
) - 2*a^3*b*f^4*e^(6*e) + a^2*b^2*f^4*e^(6*e)) - 3*(3*a^3*f^3*e^(10*e) - a
^2*b*f^3*e^(10*e))/(a^4*f^4*e^(6*e) - 2*a^3*b*f^4*e^(6*e) + a^2*b^2*f^4*e^
(6*e)))*e^(2*f*x) - 3*(3*a^3*f^3*e^(8*e) - a^2*b*f^3*e^(8*e))/(a^4*f^4*e^(
6*e) - 2*a^3*b*f^4*e^(6*e) + a^2*b^2*f^4*e^(6*e)))*e^(2*f*x) + (a^3*f^3*e^
(6*e) - 3*a^2*b*f^3*e^(6*e))/(a^4*f^4*e^(6*e) - 2*a^3*b*f^4*e^(6*e) + a^2*
b^2*f^4*e^(6*e)))/(b*e^(4*f*x + 4*e) + 4*a*e^(2*f*x + 2*e) - 2*b*e^(2*f*x
+ 2*e) + b)^(3/2)
```

Mupad [B] (verification not implemented)

Time = 2.62 (sec) , antiderivative size = 148, normalized size of antiderivative = 1.63

$$\int \frac{\sinh^3(e+fx)}{\left(a+b\sinh^2(e+fx)\right)^{5/2}}\,dx = \frac{2\,\mathrm{e}^{e+f\,x}\left(\mathrm{e}^{2\,e+2\,f\,x}+1\right)\sqrt{a+b\left(\frac{\mathrm{e}^{e+f\,x}}{2}-\frac{\mathrm{e}^{-e-f\,x}}{2}\right)^2}\left(a-3\,b-10\,a\,\mathrm{e}^{2\,e+2\,f\,x}\right.}{3\,f\,(a-b)^2\left(b+4\,a\,\mathrm{e}^{2\,e+2\,f\,x}-2\,b\,\mathrm{e}^{2\,e+2\,f\,x}+\right.}$$

input
```
int(sinh(e + f*x)^3/(a + b*sinh(e + f*x)^2)^(5/2),x)
```

output
```
(2*exp(e + f*x)*(exp(2*e + 2*f*x) + 1)*(a + b*(exp(e + f*x)/2 - exp(- e -
f*x)/2)^2)^(1/2)*(a - 3*b - 10*a*exp(2*e + 2*f*x) + a*exp(4*e + 4*f*x) + 6
*b*exp(2*e + 2*f*x) - 3*b*exp(4*e + 4*f*x)))/(3*f*(a - b)^2*(b + 4*a*exp(2
*e + 2*f*x) - 2*b*exp(2*e + 2*f*x) + b*exp(4*e + 4*f*x))^2)
```

Reduce [F]

$$\int \frac{\sinh^3(e+fx)}{\left(a+b\sinh^2(e+fx)\right)^{5/2}}\,dx = \int \frac{\sqrt{\sinh\left(fx+e\right)^2 b+a}\,\sinh\left(fx+e\right)^3}{\sinh\left(fx+e\right)^6 b^3+3\sinh\left(fx+e\right)^4 a\,b^2+3\sinh\left(fx+e\right)^2 a^2b+a^3}dx$$

input
```
int(sinh(f*x+e)^3/(a+b*sinh(f*x+e)^2)^(5/2),x)
```

output
```
int((sqrt(sinh(e + f*x)**2*b + a)*sinh(e + f*x)**3)/(sinh(e + f*x)**6*b**3
 + 3*sinh(e + f*x)**4*a*b**2 + 3*sinh(e + f*x)**2*a**2*b + a**3),x)
```

3.101 $\int \frac{\sinh(e+fx)}{(a+b\sinh^2(e+fx))^{5/2}}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 79

$$\int \frac{\sinh(e+fx)}{\left(a+b\sinh^2(e+fx)\right)^{5/2}}\,dx = \frac{\cosh(e+fx)}{3(a-b)f\left(a-b+b\cosh^2(e+fx)\right)^{3/2}} + \frac{2\cosh(e+fx)}{3(a-b)^2 f\sqrt{a-b+b\cosh^2(e+fx)}}$$

output
```
1/3*cosh(f*x+e)/(a-b)/f/(a-b+b*cosh(f*x+e)^2)^(3/2)+2/3*cosh(f*x+e)/(a-b)^
2/f/(a-b+b*cosh(f*x+e)^2)^(1/2)
```

Mathematica [A] (verified)

Time = 0.28 (sec) , antiderivative size = 63, normalized size of antiderivative = 0.80

$$\int \frac{\sinh(e+fx)}{\left(a+b\sinh^2(e+fx)\right)^{5/2}}\,dx = \frac{2\sqrt{2}\cosh(e+fx)(3a-2b+b\cosh(2(e+fx)))}{3(a-b)^2 f(2a-b+b\cosh(2(e+fx)))^{3/2}}$$

input
```
Integrate[Sinh[e + f*x]/(a + b*Sinh[e + f*x]^2)^(5/2),x]
```

output
```
(2*Sqrt[2]*Cosh[e + f*x]*(3*a - 2*b + b*Cosh[2*(e + f*x)]))/(3*(a - b)^2*f
*(2*a - b + b*Cosh[2*(e + f*x)])^(3/2))
```

Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 77, normalized size of antiderivative = 0.97, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.217$, Rules used = {3042, 26, 3665, 209, 208}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sinh(e+fx)}{\left(a+b\sinh^2(e+fx)\right)^{5/2}}dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int -\frac{i\sin(ie+ifx)}{(a-b\sin(ie+ifx)^2)^{5/2}}dx$$

$$\Big\downarrow \text{ 26}$$

$$-i\int \frac{\sin(ie+ifx)}{(a-b\sin(ie+ifx)^2)^{5/2}}dx$$

$$\Big\downarrow \text{ 3665}$$

$$\frac{\int \frac{1}{\left(b\cosh^2(e+fx)+a-b\right)^{5/2}}d\cosh(e+fx)}{f}$$

$$\Big\downarrow \text{ 209}$$

$$\frac{\frac{2\int \frac{1}{\left(b\cosh^2(e+fx)+a-b\right)^{3/2}}d\cosh(e+fx)}{3(a-b)}+\frac{\cosh(e+fx)}{3(a-b)\left(a+b\cosh^2(e+fx)-b\right)^{3/2}}}{f}$$

$$\Big\downarrow \text{ 208}$$

$$\frac{\frac{2\cosh(e+fx)}{3(a-b)^2\sqrt{a+b\cosh^2(e+fx)-b}}+\frac{\cosh(e+fx)}{3(a-b)\left(a+b\cosh^2(e+fx)-b\right)^{3/2}}}{f}$$

input
```
Int[Sinh[e + f*x]/(a + b*Sinh[e + f*x]^2)^(5/2),x]
```

output
```
(Cosh[e + f*x]/(3*(a - b)*(a - b + b*Cosh[e + f*x]^2)^(3/2)) + (2*Cosh[e +
 f*x])/(3*(a - b)^2*Sqrt[a - b + b*Cosh[e + f*x]^2]))/f
```

Defintions of rubi rules used

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 208
```
Int[((a_) + (b_.)*(x_)^2)^(-3/2), x_Symbol] :> Simp[x/(a*Sqrt[a + b*x^2]),
x] /; FreeQ[{a, b}, x]
```

rule 209
```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(-x)*((a + b*x^2)^(p + 1)
/(2*a*(p + 1))), x] + Simp[(2*p + 3)/(2*a*(p + 1))   Int[(a + b*x^2)^(p + 1
), x], x] /; FreeQ[{a, b}, x] && ILtQ[p + 3/2, 0]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3665
```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^
(p_.), x_Symbol] :> With[{ff = FreeFactors[Cos[e + f*x], x]}, Simp[-ff/f
Subst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a + b - b*ff^2*x^2)^p, x], x, Cos[e +
 f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m - 1)/2]
```

Maple [A] (verified)

Time = 1.77 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.72

method	result	size
default	$\frac{\left(2b\sinh(fx+e)^{2}+3a-b\right)\cosh(fx+e)}{3\left(a+b\sinh(fx+e)^{2}\right)^{\frac{3}{2}}\left(a^{2}-2ab+b^{2}\right)f}$	57
risch	Expression too large to display	352088

input
```
int(sinh(f*x+e)/(a+b*sinh(f*x+e)^2)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
1/3*(2*b*sinh(f*x+e)^2+3*a-b)*cosh(f*x+e)/(a+b*sinh(f*x+e)^2)^(3/2)/(a^2-2
*a*b+b^2)/f
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1186 vs. $2(71) = 142$.

Time = 0.22 (sec) , antiderivative size = 1186, normalized size of antiderivative = 15.01

$$\int \frac{\sinh(e+fx)}{(a+b\sinh^2(e+fx))^{5/2}}\,dx = \text{Too large to display}$$

input
```
integrate(sinh(f*x+e)/(a+b*sinh(f*x+e)^2)^(5/2),x, algorithm="fricas")
```

output
```
2/3*sqrt(2)*(b*cosh(f*x + e)^6 + 6*b*cosh(f*x + e)*sinh(f*x + e)^5 + b*sin
h(f*x + e)^6 + 3*(2*a - b)*cosh(f*x + e)^4 + 3*(5*b*cosh(f*x + e)^2 + 2*a
- b)*sinh(f*x + e)^4 + 4*(5*b*cosh(f*x + e)^3 + 3*(2*a - b)*cosh(f*x + e))
*sinh(f*x + e)^3 + 3*(2*a - b)*cosh(f*x + e)^2 + 3*(5*b*cosh(f*x + e)^4 +
6*(2*a - b)*cosh(f*x + e)^2 + 2*a - b)*sinh(f*x + e)^2 + 6*(b*cosh(f*x + e
)^5 + 2*(2*a - b)*cosh(f*x + e)^3 + (2*a - b)*cosh(f*x + e))*sinh(f*x + e)
 + b)*sqrt((b*cosh(f*x + e)^2 + b*sinh(f*x + e)^2 + 2*a - b)/(cosh(f*x + e
)^2 - 2*cosh(f*x + e)*sinh(f*x + e) + sinh(f*x + e)^2))/((a^2*b^2 - 2*a*b^
3 + b^4)*f*cosh(f*x + e)^8 + 8*(a^2*b^2 - 2*a*b^3 + b^4)*f*cosh(f*x + e)*s
inh(f*x + e)^7 + (a^2*b^2 - 2*a*b^3 + b^4)*f*sinh(f*x + e)^8 + 4*(2*a^3*b
- 5*a^2*b^2 + 4*a*b^3 - b^4)*f*cosh(f*x + e)^6 + 4*(7*(a^2*b^2 - 2*a*b^3 +
 b^4)*f*cosh(f*x + e)^2 + (2*a^3*b - 5*a^2*b^2 + 4*a*b^3 - b^4)*f)*sinh(f*
x + e)^6 + 2*(8*a^4 - 24*a^3*b + 27*a^2*b^2 - 14*a*b^3 + 3*b^4)*f*cosh(f*x
 + e)^4 + 8*(7*(a^2*b^2 - 2*a*b^3 + b^4)*f*cosh(f*x + e)^3 + 3*(2*a^3*b -
5*a^2*b^2 + 4*a*b^3 - b^4)*f*cosh(f*x + e))*sinh(f*x + e)^5 + 2*(35*(a^2*b
^2 - 2*a*b^3 + b^4)*f*cosh(f*x + e)^4 + 30*(2*a^3*b - 5*a^2*b^2 + 4*a*b^3
- b^4)*f*cosh(f*x + e)^2 + (8*a^4 - 24*a^3*b + 27*a^2*b^2 - 14*a*b^3 + 3*b
^4)*f)*sinh(f*x + e)^4 + 4*(2*a^3*b - 5*a^2*b^2 + 4*a*b^3 - b^4)*f*cosh(f*
x + e)^2 + 8*(7*(a^2*b^2 - 2*a*b^3 + b^4)*f*cosh(f*x + e)^5 + 10*(2*a^3*b
- 5*a^2*b^2 + 4*a*b^3 - b^4)*f*cosh(f*x + e)^3 + (8*a^4 - 24*a^3*b + 27...
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\sinh(e+fx)}{\left(a+b\sinh^2(e+fx)\right)^{5/2}}\,dx = \text{Timed out}$$

input
```
integrate(sinh(f*x+e)/(a+b*sinh(f*x+e)**2)**(5/2),x)
```

output
```
Timed out
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 485 vs. 2(71) = 142.

Time = 0.18 (sec) , antiderivative size = 485, normalized size of antiderivative = 6.14

$$\int \frac{\sinh(e+fx)}{\left(a+b\sinh^2(e+fx)\right)^{5/2}}\,dx = \frac{2\,a^2b^2 - 2\,ab^3 + b^4 + 5\,(4\,a^3b - 6\,a^2b^2 + 4\,ab^3 - b^4)e^{(-2\,fx-2\,e)} + 2\,(24\,a^4 -}{}$$
$$+ \frac{2\,ab^3 - b^4 + 5\,(4\,a^2b^2 - 4\,ab^3 + b^4)e^{(-2\,fx-2\,e)} + 10\,(6\,a^3b - 9\,a^2b^2 + 5\,ab^3 - b^4)e^{(-4\,fx-4\,e)} + 2\,(24\,a^4 - 48}{3\,(a^4 - 2\,a^3b + a^2b^2)(2\,(2}$$

input
```
integrate(sinh(f*x+e)/(a+b*sinh(f*x+e)^2)^(5/2),x, algorithm="maxima")
```

output
```
1/3*(2*a^2*b^2 - 2*a*b^3 + b^4 + 5*(4*a^3*b - 6*a^2*b^2 + 4*a*b^3 - b^4)*e
^(-2*f*x - 2*e) + 2*(24*a^4 - 48*a^3*b + 49*a^2*b^2 - 25*a*b^3 + 5*b^4)*e^
(-4*f*x - 4*e) + 10*(6*a^3*b - 9*a^2*b^2 + 5*a*b^3 - b^4)*e^(-6*f*x - 6*e)
 + 5*(4*a^2*b^2 - 4*a*b^3 + b^4)*e^(-8*f*x - 8*e) + (2*a*b^3 - b^4)*e^(-10
*f*x - 10*e))/((a^4 - 2*a^3*b + a^2*b^2)*(2*(2*a - b)*e^(-2*f*x - 2*e) + b
*e^(-4*f*x - 4*e) + b)^(5/2)*f) + 1/3*(2*a*b^3 - b^4 + 5*(4*a^2*b^2 - 4*a*
b^3 + b^4)*e^(-2*f*x - 2*e) + 10*(6*a^3*b - 9*a^2*b^2 + 5*a*b^3 - b^4)*e^(
-4*f*x - 4*e) + 2*(24*a^4 - 48*a^3*b + 49*a^2*b^2 - 25*a*b^3 + 5*b^4)*e^(-
6*f*x - 6*e) + 5*(4*a^3*b - 6*a^2*b^2 + 4*a*b^3 - b^4)*e^(-8*f*x - 8*e) +
(2*a^2*b^2 - 2*a*b^3 + b^4)*e^(-10*f*x - 10*e))/((a^4 - 2*a^3*b + a^2*b^2)
*(2*(2*a - b)*e^(-2*f*x - 2*e) + b*e^(-4*f*x - 4*e) + b)^(5/2)*f)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 293 vs. $2(71) = 142$.

Time = 0.31 (sec) , antiderivative size = 293, normalized size of antiderivative = 3.71

$$\int \frac{\sinh(e+fx)}{\left(a+b\sinh^2(e+fx)\right)^{5/2}}\,dx = \frac{2\left(\frac{a^2bfe^{(6\,e)}}{a^4f^2e^{(6\,e)}-2\,a^3bf^2e^{(6\,e)}+a^2b^2f^2e^{(6\,e)}}+\left(\left(\frac{a^2bfe^{(2\,fx+12\,e)}}{a^4f^2e^{(6\,e)}-2\,a^3bf^2e^{(6\,e)}+a^2b^2f^2e^{(6\,e)}}+\frac{}{a^4f^2}\right.\right.\right.}{3\left(be^{(4\,fx+4\,e)}+4\,ae^{(2\,fx+}\right.}$$

input
```
integrate(sinh(f*x+e)/(a+b*sinh(f*x+e)^2)^(5/2),x, algorithm="giac")
```

output
```
2/3*(a^2*b*f*e^(6*e)/(a^4*f^2*e^(6*e) - 2*a^3*b*f^2*e^(6*e) + a^2*b^2*f^2*
e^(6*e)) + ((a^2*b*f*e^(2*f*x + 12*e)/(a^4*f^2*e^(6*e) - 2*a^3*b*f^2*e^(6*
e) + a^2*b^2*f^2*e^(6*e)) + 3*(2*a^3*f*e^(10*e) - a^2*b*f*e^(10*e))/(a^4*f
^2*e^(6*e) - 2*a^3*b*f^2*e^(6*e) + a^2*b^2*f^2*e^(6*e)))*e^(2*f*x) + 3*(2*
a^3*f*e^(8*e) - a^2*b*f*e^(8*e))/(a^4*f^2*e^(6*e) - 2*a^3*b*f^2*e^(6*e) +
a^2*b^2*f^2*e^(6*e)))*e^(2*f*x))/(b*e^(4*f*x + 4*e) + 4*a*e^(2*f*x + 2*e)
- 2*b*e^(2*f*x + 2*e) + b)^(3/2)
```

Mupad [B] (verification not implemented)

Time = 2.24 (sec) , antiderivative size = 133, normalized size of antiderivative = 1.68

$$\int \frac{\sinh(e+fx)}{\left(a+b\sinh^2(e+fx)\right)^{5/2}}\,dx = \frac{4\,\mathrm{e}^{e+f\,x}\,\left(\mathrm{e}^{2\,e+2\,f\,x}+1\right)\,\sqrt{a+b\,{\left(\frac{\mathrm{e}^{e+f\,x}}{2}-\frac{\mathrm{e}^{-e-f\,x}}{2}\right)}^2}\,\left(b+6\,a\,\mathrm{e}^{2\,e+2\,f\,x}-4\,b\,\mathrm{e}^{2}\right.}{3\,f\,{\left(a-b\right)}^2\,\left(b+4\,a\,\mathrm{e}^{2\,e+2\,f\,x}-2\,b\,\mathrm{e}^{2\,e+2\,f\,x}+b\,\mathrm{e}^{4\,e+4\,f\,x}\right)}$$

input
```
int(sinh(e + f*x)/(a + b*sinh(e + f*x)^2)^(5/2),x)
```

output
```
(4*exp(e + f*x)*(exp(2*e + 2*f*x) + 1)*(a + b*(exp(e + f*x)/2 - exp(- e -
f*x)/2)^2)^(1/2)*(b + 6*a*exp(2*e + 2*f*x) - 4*b*exp(2*e + 2*f*x) + b*exp(
4*e + 4*f*x)))/(3*f*(a - b)^2*(b + 4*a*exp(2*e + 2*f*x) - 2*b*exp(2*e + 2*
f*x) + b*exp(4*e + 4*f*x))^2)
```

Reduce [F]

$$\int \frac{\sinh(e+fx)}{\left(a+b\sinh^2(e+fx)\right)^{5/2}}\,dx = \int \frac{\sqrt{\sinh\left(fx+e\right)^2 b+a}\,\sinh\left(fx+e\right)}{\sinh\left(fx+e\right)^6 b^3+3\sinh\left(fx+e\right)^4 a\,b^2+3\sinh\left(fx+e\right)^2 a^2b+a^3}dx$$

input
```
int(sinh(f*x+e)/(a+b*sinh(f*x+e)^2)^(5/2),x)
```

output
```
int((sqrt(sinh(e + f*x)**2*b + a)*sinh(e + f*x))/(sinh(e + f*x)**6*b**3 +
3*sinh(e + f*x)**4*a*b**2 + 3*sinh(e + f*x)**2*a**2*b + a**3),x)
```

3.102 $\int \frac{\operatorname{csch}(e+fx)}{\left(a+b\sinh^2(e+fx)\right)^{5/2}}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 136

$$\int \frac{\operatorname{csch}(e+fx)}{\left(a+b\sinh^2(e+fx)\right)^{5/2}}\,dx = -\frac{\operatorname{arctanh}\left(\frac{\sqrt{a}\cosh(e+fx)}{\sqrt{a-b+b\cosh^2(e+fx)}}\right)}{a^{5/2}f} - \frac{b\cosh(e+fx)}{3a(a-b)f\left(a-b+b\cosh^2(e+fx)\right)^{3/2}} - \frac{(5a-3b)b\cosh(e+fx)}{3a^2(a-b)^2f\sqrt{a-b+b\cosh^2(e+fx)}}$$

output
```
-arctanh(a^(1/2)*cosh(f*x+e)/(a-b+b*cosh(f*x+e)^2)^(1/2))/a^(5/2)/f-1/3*b*
cosh(f*x+e)/a/(a-b)/f/(a-b+b*cosh(f*x+e)^2)^(3/2)-1/3*(5*a-3*b)*b*cosh(f*x
+e)/a^2/(a-b)^2/f/(a-b+b*cosh(f*x+e)^2)^(1/2)
```

Mathematica [A] (verified)

Time = 0.79 (sec) , antiderivative size = 130, normalized size of antiderivative = 0.96

$$\int \frac{\operatorname{csch}(e+fx)}{\left(a+b\sinh^2(e+fx)\right)^{5/2}}\,dx = \frac{-\frac{\operatorname{arctanh}\left(\frac{\sqrt{2}\sqrt{a}\cosh(e+fx)}{\sqrt{2a-b+b\cosh(2(e+fx))}}\right)}{a^{5/2}} + \frac{\sqrt{2}b\cosh(e+fx)\left(-12a^2+13ab-3b^2+b(-5a+3b)\cosh(2\right.}{3a^2(a-b)^2(2a-b+b\cosh(2(e+fx)))^{3/2}}}{f}$$

input
```
Integrate[Csch[e + f*x]/(a + b*Sinh[e + f*x]^2)^(5/2),x]
```

output
```
(-(ArcTanh[(Sqrt[2]*Sqrt[a]*Cosh[e + f*x])/Sqrt[2*a - b + b*Cosh[2*(e + f*
x)]]]/a^(5/2)) + (Sqrt[2]*b*Cosh[e + f*x]*(-12*a^2 + 13*a*b - 3*b^2 + b*(-
5*a + 3*b)*Cosh[2*(e + f*x)]))/(3*a^2*(a - b)^2*(2*a - b + b*Cosh[2*(e + f
*x)])^(3/2)))/f
```

Rubi [A] (verified)

Time = 0.37 (sec) , antiderivative size = 149, normalized size of antiderivative = 1.10, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.391$, Rules used = {3042, 26, 3665, 316, 25, 402, 27, 291, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\operatorname{csch}(e+fx)}{\left(a+b\sinh^2(e+fx)\right)^{5/2}}dx$$

$$\Big\downarrow \ 3042$$

$$\int \frac{i}{\sin(ie+ifx)\left(a-b\sin(ie+ifx)^2\right)^{5/2}}dx$$

$$\Big\downarrow \ 26$$

$$i\int \frac{1}{\sin(ie+ifx)\left(a-b\sin(ie+ifx)^2\right)^{5/2}}dx$$

$$\Big\downarrow \ 3665$$

$$-\frac{\int \frac{1}{(1-\cosh^2(e+fx))\left(b\cosh^2(e+fx)+a-b\right)^{5/2}}d\cosh(e+fx)}{f}$$

$$\Big\downarrow \ 316$$

$$-\frac{\frac{b\cosh(e+fx)}{3a(a-b)\left(a+b\cosh^2(e+fx)-b\right)^{3/2}}-\frac{\int -\frac{-2b\cosh^2(e+fx)+3a-b}{\left(1-\cosh^2(e+fx)\right)\left(b\cosh^2(e+fx)+a-b\right)^{3/2}}d\cosh(e+fx)}{3a(a-b)}}{f}$$

$$\Big\downarrow \ 25$$

$$-\frac{\frac{\int \frac{-2b\cosh^2(e+fx)+3a-b}{\left(1-\cosh^2(e+fx)\right)\left(b\cosh^2(e+fx)+a-b\right)^{3/2}}d\cosh(e+fx)}{3a(a-b)}+\frac{b\cosh(e+fx)}{3a(a-b)\left(a+b\cosh^2(e+fx)-b\right)^{3/2}}}{f}$$

$$\Big\downarrow \text{402}$$

$$-\frac{\frac{\frac{b(5a-3b)\cosh(e+fx)}{a(a-b)\sqrt{a+b\cosh^2(e+fx)-b}}-\frac{\int -\frac{3(a-b)^2}{\left(1-\cosh^2(e+fx)\right)\sqrt{b\cosh^2(e+fx)+a-b}}d\cosh(e+fx)}{a(a-b)}}{3a(a-b)}+\frac{b\cosh(e+fx)}{3a(a-b)\left(a+b\cosh^2(e+fx)-b\right)^{3/2}}}{f}$$

$$\Big\downarrow \text{27}$$

$$-\frac{\frac{\frac{3(a-b)\int \frac{1}{\left(1-\cosh^2(e+fx)\right)\sqrt{b\cosh^2(e+fx)+a-b}}d\cosh(e+fx)}{a}+\frac{b(5a-3b)\cosh(e+fx)}{a(a-b)\sqrt{a+b\cosh^2(e+fx)-b}}}{3a(a-b)}+\frac{b\cosh(e+fx)}{3a(a-b)\left(a+b\cosh^2(e+fx)-b\right)^{3/2}}}{f}$$

$$\Big\downarrow \text{291}$$

$$-\frac{\frac{\frac{3(a-b)\int \frac{1}{1-\frac{a\cosh^2(e+fx)}{b\cosh^2(e+fx)+a-b}}d\frac{\cosh(e+fx)}{\sqrt{b\cosh^2(e+fx)+a-b}}}{a}+\frac{b(5a-3b)\cosh(e+fx)}{a(a-b)\sqrt{a+b\cosh^2(e+fx)-b}}}{3a(a-b)}+\frac{b\cosh(e+fx)}{3a(a-b)\left(a+b\cosh^2(e+fx)-b\right)^{3/2}}}{f}$$

$$\Big\downarrow \text{219}$$

$$-\frac{\frac{\frac{3(a-b)\text{arctanh}\left(\frac{\sqrt{a}\cosh(e+fx)}{\sqrt{a+b\cosh^2(e+fx)-b}}\right)}{a^{3/2}}+\frac{b(5a-3b)\cosh(e+fx)}{a(a-b)\sqrt{a+b\cosh^2(e+fx)-b}}}{3a(a-b)}+\frac{b\cosh(e+fx)}{3a(a-b)\left(a+b\cosh^2(e+fx)-b\right)^{3/2}}}{f}$$

input
```
Int[Csch[e + f*x]/(a + b*Sinh[e + f*x]^2)^(5/2),x]
```

output
```
-(((b*Cosh[e + f*x])/(3*a*(a - b)*(a - b + b*Cosh[e + f*x]^2)^(3/2)) + ((3
*(a - b)*ArcTanh[(Sqrt[a]*Cosh[e + f*x])/Sqrt[a - b + b*Cosh[e + f*x]^2]])
/a^(3/2) + ((5*a - 3*b)*b*Cosh[e + f*x])/(a*(a - b)*Sqrt[a - b + b*Cosh[e
+ f*x]^2]))/(3*a*(a - b)))/f)
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 291
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*((c_) + (d_.)*(x_)^2)), x_Symbol] :> Subst
[Int[1/(c - (b*c - a*d)*x^2), x], x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b, c,
d}, x] && NeQ[b*c - a*d, 0]
```

rule 316
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_), x_Symbol] :> Sim
p[(-b)*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^(q + 1)/(2*a*(p + 1)*(b*c - a*d))
), x] + Simp[1/(2*a*(p + 1)*(b*c - a*d))   Int[(a + b*x^2)^(p + 1)*(c + d*x
^2)^q*Simp[b*c + 2*(p + 1)*(b*c - a*d) + d*b*(2*(p + q + 2) + 1)*x^2, x], x
], x] /; FreeQ[{a, b, c, d, q}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] &&  !
( !IntegerQ[p] && IntegerQ[q] && LtQ[q, -1]) && IntBinomialQ[a, b, c, d, 2,
 p, q, x]
```

rule 402
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_.)*((e_) + (f_.)*(x
_)^2), x_Symbol] :> Simp[(-(b*e - a*f))*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^
(q + 1)/(a*2*(b*c - a*d)*(p + 1))), x] + Simp[1/(a*2*(b*c - a*d)*(p + 1))
 Int[(a + b*x^2)^(p + 1)*(c + d*x^2)^q*Simp[c*(b*e - a*f) + e*2*(b*c - a*d)
*(p + 1) + d*(b*e - a*f)*(2*(p + q + 2) + 1)*x^2, x], x], x] /; FreeQ[{a, b
, c, d, e, f, q}, x] && LtQ[p, -1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3665
```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^
(p_.), x_Symbol] :> With[{ff = FreeFactors[Cos[e + f*x], x]}, Simp[-ff/f
Subst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a + b - b*ff^2*x^2)^p, x], x, Cos[e +
 f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m - 1)/2]
```

Maple [A] (verified)

Time = 52.93 (sec) , antiderivative size = 236, normalized size of antiderivative = 1.74

method	result
default	$\frac{\sqrt{\left(a+b\sinh(fx+e)^2\right)\cosh(fx+e)^2}\left(-\frac{\ln\left(\frac{2a+(a+b)\sinh(fx+e)^2+2\sqrt{a}\sqrt{\left(a+b\sinh(fx+e)^2\right)\cosh(fx+e)^2}}{\sinh(fx+e)^2}\right)}{2a^{\frac{5}{2}}}-\frac{b\left(2b\sinh(\ldots\right.}{3a\sqrt{\left(a+b\sinh(fx+e)^2\right)\mathrm{co}\ldots}}\right.}{\cosh(fx+e)\sqrt{a+b\sinh(fx+e)^2}\,f}$
risch	Expression too large to display

input
```
int(csch(f*x+e)/(a+b*sinh(f*x+e)^2)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
((a+b*sinh(f*x+e)^2)*cosh(f*x+e)^2)^(1/2)*(-1/2/a^(5/2)*ln((2*a+(a+b)*sinh
(f*x+e)^2+2*a^(1/2)*((a+b*sinh(f*x+e)^2)*cosh(f*x+e)^2)^(1/2))/sinh(f*x+e)
^2)-1/3*b/a*(2*b*sinh(f*x+e)^2+3*a-b)*cosh(f*x+e)^2/((a+b*sinh(f*x+e)^2)*c
osh(f*x+e)^2)^(1/2)/(a+b*sinh(f*x+e)^2)/(a^2-2*a*b+b^2)-b/a^2*cosh(f*x+e)^
2/(a-b)/((a+b*sinh(f*x+e)^2)*cosh(f*x+e)^2)^(1/2))/cosh(f*x+e)/(a+b*sinh(f
*x+e)^2)^(1/2)/f
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2508 vs. 2(122) = 244.

Time = 0.37 (sec) , antiderivative size = 5230, normalized size of antiderivative = 38.46

$$\int \frac{\operatorname{csch}(e+fx)}{\left(a+b\sinh^2(e+fx)\right)^{5/2}}\,dx = \text{Too large to display}$$

input
```
integrate(csch(f*x+e)/(a+b*sinh(f*x+e)^2)^(5/2),x, algorithm="fricas")
```

output
```
Too large to include
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\operatorname{csch}(e+fx)}{\left(a+b\sinh^2(e+fx)\right)^{5/2}}\,dx = \text{Timed out}$$

input
```
integrate(csch(f*x+e)/(a+b*sinh(f*x+e)**2)**(5/2),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{\operatorname{csch}(e+fx)}{\left(a+b\sinh^2(e+fx)\right)^{5/2}}\,dx = \int \frac{\operatorname{csch}\left(fx+e\right)}{\left(b\sinh\left(fx+e\right)^2+a\right)^{\frac{5}{2}}}\,dx$$

input
```
integrate(csch(f*x+e)/(a+b*sinh(f*x+e)^2)^(5/2),x, algorithm="maxima")
```

output
```
integrate(csch(f*x + e)/(b*sinh(f*x + e)^2 + a)^(5/2), x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 470 vs. $2(122) = 244$.

Time = 0.35 (sec) , antiderivative size = 470, normalized size of antiderivative = 3.46

$$\int \frac{\operatorname{csch}(e+fx)}{\left(a+b\sinh^2(e+fx)\right)^{5/2}}\,dx =$$

$$-\frac{1}{3}\left(\frac{\left(\left(\frac{(5a^9b^2f^3e^{(12e)}-3a^8b^3f^3e^{(12e)})e^{(2fx)}}{a^{12}f^4e^{(12e)}-2a^{11}bf^4e^{(12e)}+a^{10}b^2f^4e^{(12e)}}+\frac{3(8a^{10}bf^3e^{(10e)}-7a^9b^2f^3e^{(10e)}+a^8b^3f^3e^{(10e)})}{a^{12}f^4e^{(12e)}-2a^{11}bf^4e^{(12e)}+a^{10}b^2f^4e^{(12e)}}\right)e^{(2fx)}+\frac{3(8a^{10}bf^3e^{(8e)}-7\cdots}{a^{12}f^4e^{(12e)}-2a^{1}\cdots}\right.}{(be^{(4fx+4e)}+4ae^{(2fx+2e)}-2be^{(2fx+2e)}\cdots}\right.$$

input
```
integrate(csch(f*x+e)/(a+b*sinh(f*x+e)^2)^(5/2),x, algorithm="giac")
```

output
```
-1/3*(((((5*a^9*b^2*f^3*e^(12*e) - 3*a^8*b^3*f^3*e^(12*e))*e^(2*f*x)/(a^12
*f^4*e^(12*e) - 2*a^11*b*f^4*e^(12*e) + a^10*b^2*f^4*e^(12*e)) + 3*(8*a^10
*b*f^3*e^(10*e) - 7*a^9*b^2*f^3*e^(10*e) + a^8*b^3*f^3*e^(10*e))/(a^12*f^4
*e^(12*e) - 2*a^11*b*f^4*e^(12*e) + a^10*b^2*f^4*e^(12*e)))*e^(2*f*x) + 3*
(8*a^10*b*f^3*e^(8*e) - 7*a^9*b^2*f^3*e^(8*e) + a^8*b^3*f^3*e^(8*e))/(a^12
*f^4*e^(12*e) - 2*a^11*b*f^4*e^(12*e) + a^10*b^2*f^4*e^(12*e)))*e^(2*f*x)
+ (5*a^9*b^2*f^3*e^(6*e) - 3*a^8*b^3*f^3*e^(6*e))/(a^12*f^4*e^(12*e) - 2*a
^11*b*f^4*e^(12*e) + a^10*b^2*f^4*e^(12*e)))/(b*e^(4*f*x + 4*e) + 4*a*e^(2
*f*x + 2*e) - 2*b*e^(2*f*x + 2*e) + b)^(3/2) - 6*arctan(-1/2*(sqrt(b)*e^(2
*f*x + 2*e) - sqrt(b*e^(4*f*x + 4*e) + 4*a*e^(2*f*x + 2*e) - 2*b*e^(2*f*x
+ 2*e) + b) - sqrt(b))/sqrt(-a))*e^(-6*e)/(sqrt(-a)*a^2*f))*e^(6*e)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\operatorname{csch}(e+fx)}{\left(a+b\sinh^2(e+fx)\right)^{5/2}}\,dx = \int \frac{1}{\sinh(e+f\,x)\,\left(b\sinh(e+f\,x)^2+a\right)^{5/2}}\,dx$$

input
```
int(1/(sinh(e + f*x)*(a + b*sinh(e + f*x)^2)^(5/2)),x)
```

output
```
int(1/(sinh(e + f*x)*(a + b*sinh(e + f*x)^2)^(5/2)), x)
```

Reduce [F]

$$\int \frac{\operatorname{csch}(e+fx)}{\left(a+b\sinh^2(e+fx)\right)^{5/2}}\,dx = \int \frac{\sqrt{\sinh\left(fx+e\right)^2 b+a}\,\operatorname{csch}(fx+e)}{\sinh\left(fx+e\right)^6 b^3+3\sinh\left(fx+e\right)^4 a\,b^2+3\sinh\left(fx+e\right)^2 a^2b+a^3}dx$$

input
```
int(csch(f*x+e)/(a+b*sinh(f*x+e)^2)^(5/2),x)
```

output
```
int((sqrt(sinh(e + f*x)**2*b + a)*csch(e + f*x))/(sinh(e + f*x)**6*b**3 +
3*sinh(e + f*x)**4*a*b**2 + 3*sinh(e + f*x)**2*a**2*b + a**3),x)
```

3.103 $\int \frac{\sinh^6(e+fx)}{(a+b\sinh^2(e+fx))^{5/2}}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 325

$$\int \frac{\sinh^6(e+fx)}{(a+b\sinh^2(e+fx))^{5/2}}\,dx = -\frac{a\cosh(e+fx)\sinh^3(e+fx)}{3(a-b)bf\,(a+b\sinh^2(e+fx))^{3/2}} + \frac{(4a-3b)\cosh(e+fx)\sinh(e+fx)}{3(a-b)b^2f\sqrt{a+b\sinh^2(e+fx)}} - \frac{\sqrt{a}(8a^2-13ab+3b^2)\cosh(e+fx)E\left(\arctan\left(\frac{\sqrt{b}\sinh(e+fx)}{\sqrt{a}}\right)|1-\frac{a}{b}\right)}{3(a-b)^2b^{5/2}f\sqrt{\frac{a\cosh^2(e+fx)}{a+b\sinh^2(e+fx)}}\sqrt{a+b\sinh^2(e+fx)}} + \frac{2a^{3/2}(2a-3b)\cosh(e+fx)\,\mathrm{EllipticF}\left(\arctan\left(\frac{\sqrt{b}\sinh(e+fx)}{\sqrt{a}}\right),1-\frac{a}{b}\right)}{3(a-b)^2b^{5/2}f\sqrt{\frac{a\cosh^2(e+fx)}{a+b\sinh^2(e+fx)}}\sqrt{a+b\sinh^2(e+fx)}}$$

output

```
-1/3*a*cosh(f*x+e)*sinh(f*x+e)^3/(a-b)/b/f/(a+b*sinh(f*x+e)^2)^(3/2)+1/3*(
4*a-3*b)*cosh(f*x+e)*sinh(f*x+e)/(a-b)/b^2/f/(a+b*sinh(f*x+e)^2)^(1/2)-1/3
*a^(1/2)*(8*a^2-13*a*b+3*b^2)*cosh(f*x+e)*EllipticE(b^(1/2)*sinh(f*x+e)/a^
(1/2)/(1+b*sinh(f*x+e)^2/a)^(1/2),(1-a/b)^(1/2))/(a-b)^2/b^(5/2)/f/(a*cosh
(f*x+e)^2/(a+b*sinh(f*x+e)^2))^(1/2)/(a+b*sinh(f*x+e)^2)^(1/2)+2/3*a^(3/2)
*(2*a-3*b)*cosh(f*x+e)*InverseJacobiAM(arctan(b^(1/2)*sinh(f*x+e)/a^(1/2))
,(1-a/b)^(1/2))/(a-b)^2/b^(5/2)/f/(a*cosh(f*x+e)^2/(a+b*sinh(f*x+e)^2))^(1
/2)/(a+b*sinh(f*x+e)^2)^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 1.85 (sec) , antiderivative size = 207, normalized size of antiderivative = 0.64

$$\int \frac{\sinh^6(e+fx)}{\left(a+b\sinh^2(e+fx)\right)^{5/2}}\,dx = \frac{a\left(-2ia(8a^2-13ab+3b^2)\left(\frac{2a-b+b\cosh(2(e+fx))}{a}\right)^{3/2}E\left(i(e+fx)\,\middle|\,\frac{b}{a}\right)+2ia\right.}{}$$

input
```
Integrate[Sinh[e + f*x]^6/(a + b*Sinh[e + f*x]^2)^(5/2),x]
```

output
```
(a*((-2*I)*a*(8*a^2 - 13*a*b + 3*b^2)*((2*a - b + b*Cosh[2*(e + f*x)])/a)^
(3/2)*EllipticE[I*(e + f*x), b/a] + (2*I)*a*(8*a^2 - 17*a*b + 9*b^2)*((2*a
 - b + b*Cosh[2*(e + f*x)])/a)^(3/2)*EllipticF[I*(e + f*x), b/a] + Sqrt[2]
*b*(-8*a^2 + 17*a*b - 7*b^2 + b*(-5*a + 7*b)*Cosh[2*(e + f*x)])*Sinh[2*(e
+ f*x)]))/(6*(a - b)^2*b^3*f*(2*a - b + b*Cosh[2*(e + f*x)])^(3/2))
```

Rubi [A] (verified)

Time = 0.56 (sec) , antiderivative size = 386, normalized size of antiderivative = 1.19, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.400$, Rules used = {3042, 25, 3667, 372, 440, 25, 406, 320, 388, 313}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sinh^6(e+fx)}{\left(a+b\sinh^2(e+fx)\right)^{5/2}}dx$$

$$\Big\downarrow \; 3042$$

$$\int -\frac{\sin(ie+ifx)^6}{\left(a-b\sin(ie+ifx)^2\right)^{5/2}}dx$$

$$\Big\downarrow \; 25$$

$$-\int \frac{\sin(ie+ifx)^6}{\left(a-b\sin(ie+ifx)^2\right)^{5/2}}dx$$

$\downarrow$ 3667

$$\frac{\sqrt{\cosh^2(e+fx)}\text{sech}(e+fx)\int \frac{\sinh^6(e+fx)}{\sqrt{\sinh^2(e+fx)+1}\left(b\sinh^2(e+fx)+a\right)^{5/2}}d\sinh(e+fx)}{f}$$

$\downarrow$ 372

$$\frac{\sqrt{\cosh^2(e+fx)}\text{sech}(e+fx)\left(\frac{\int \frac{\sinh^2(e+fx)\left((4a-3b)\sinh^2(e+fx)+3a\right)}{\sqrt{\sinh^2(e+fx)+1}\left(b\sinh^2(e+fx)+a\right)^{3/2}}d\sinh(e+fx)}{3b(a-b)}-\frac{a\sinh^3(e+fx)\sqrt{\sinh^2(e+fx)+1}}{3b(a-b)\left(a+b\sinh^2(e+fx)\right)^{3/2}}\right)}{f}$$

$\downarrow$ 440

$$\frac{\sqrt{\cosh^2(e+fx)}\text{sech}(e+fx)\left(\frac{-\frac{\int -\frac{\left(8a^2-13ba+3b^2\right)\sinh^2(e+fx)+2a(2a-3b)}{\sqrt{\sinh^2(e+fx)+1}\sqrt{b\sinh^2(e+fx)+a}}d\sinh(e+fx)}{b(a-b)}-\frac{2a(2a-3b)\sqrt{\sinh^2(e+fx)+1}\sinh(e+fx)}{b(a-b)\sqrt{a+b\sinh^2(e+fx)}}}{3b(a-b)}-\frac{a\sin\cdots}{3b(\cdots}\cdots\right.}{f}$$

$\downarrow$ 25

$$\frac{\sqrt{\cosh^2(e+fx)}\text{sech}(e+fx)\left(\frac{\frac{\int \frac{\left(8a^2-13ba+3b^2\right)\sinh^2(e+fx)+2a(2a-3b)}{\sqrt{\sinh^2(e+fx)+1}\sqrt{b\sinh^2(e+fx)+a}}d\sinh(e+fx)}{b(a-b)}-\frac{2a(2a-3b)\sinh(e+fx)\sqrt{\sinh^2(e+fx)+1}}{b(a-b)\sqrt{a+b\sinh^2(e+fx)}}}{3b(a-b)}-\frac{a\sinh^3(\cdots}{3b(a-\cdots}\cdots\right.}{f}$$

$\downarrow$ 406

$$\frac{\sqrt{\cosh^2(e+fx)}\text{sech}(e+fx)\left(\frac{\frac{\left(8a^2-13ab+3b^2\right)\int \frac{\sinh^2(e+fx)}{\sqrt{\sinh^2(e+fx)+1}\sqrt{b\sinh^2(e+fx)+a}}d\sinh(e+fx)+2a(2a-3b)\int \frac{1}{\sqrt{\sinh^2(e+fx)+1}\sqrt{b\sinh^2(e+\cdots}}\cdots}{b(a-b)}}{3b(a-b)}\cdots\right.}{f}$$

$\downarrow$ 320

$$\frac{\sqrt{\cosh^2(e+fx)}\operatorname{sech}(e+fx)\left(\frac{\frac{\left(8a^2-13ab+3b^2\right)\int \frac{\sinh^2(e+fx)}{\sqrt{\sinh^2(e+fx)+1}\sqrt{b\sinh^2(e+fx)+a}}d\sinh(e+fx)+\frac{2(2a-3b)\sqrt{a+b\sinh^2(e+fx)}\operatorname{EllipticF}\left(\operatorname{arct}\right.}{\sqrt{\sinh^2(e+fx)+1}\sqrt{\frac{a+b\sinh}{a\left(\sinh^2(e\right.}}}}{b(a-b)}}{3b(a-b)}\right.}{f}$$

$\downarrow$ 388

$$\frac{\sqrt{\cosh^2(e+fx)}\operatorname{sech}(e+fx)\left(\frac{\frac{\left(8a^2-13ab+3b^2\right)\left(\frac{\sinh(e+fx)\sqrt{a+b\sinh^2(e+fx)}}{b\sqrt{\sinh^2(e+fx)+1}}-\frac{\int \frac{\sqrt{b\sinh^2(e+fx)+a}}{\left(\sinh^2(e+fx)+1\right)^{3/2}}d\sinh(e+fx)}{b}\right)+\frac{2(2a-3b)\sqrt{a+b\sinh^2}}{\sqrt{\sinh^2(e}}}{b(a-b)}}{3b(a-b)}\right.}{f}$$

$\downarrow$ 313

$$\frac{\sqrt{\cosh^2(e+fx)}\operatorname{sech}(e+fx)\left(\frac{\frac{\left(8a^2-13ab+3b^2\right)\left(\frac{\sinh(e+fx)\sqrt{a+b\sinh^2(e+fx)}}{b\sqrt{\sinh^2(e+fx)+1}}-\frac{\sqrt{a+b\sinh^2(e+fx)}E\left(\arctan(\sinh(e+fx))\middle|1-\frac{b}{a}\right)}{b\sqrt{\sinh^2(e+fx)+1}\sqrt{\frac{a+b\sinh^2(e+fx)}{a\left(\sinh^2(e+fx)+1\right)}}}\right)+\frac{2(2a-3b}{}}{b(a-b)}}{3b(a-b)}\right.}{f}$$

input
```
Int[Sinh[e + f*x]^6/(a + b*Sinh[e + f*x]^2)^(5/2),x]
```

output
```
(Sqrt[Cosh[e + f*x]^2]*Sech[e + f*x]*(-1/3*(a*Sinh[e + f*x]^3*Sqrt[1 + Sin
h[e + f*x]^2])/((a - b)*b*(a + b*Sinh[e + f*x]^2)^(3/2)) + ((-2*a*(2*a - 3
*b)*Sinh[e + f*x]*Sqrt[1 + Sinh[e + f*x]^2])/((a - b)*b*Sqrt[a + b*Sinh[e
+ f*x]^2]) + ((2*(2*a - 3*b)*EllipticF[ArcTan[Sinh[e + f*x]], 1 - b/a]*Sqr
t[a + b*Sinh[e + f*x]^2])/(Sqrt[1 + Sinh[e + f*x]^2]*Sqrt[(a + b*Sinh[e +
f*x]^2)/(a*(1 + Sinh[e + f*x]^2))]) + (8*a^2 - 13*a*b + 3*b^2)*((Sinh[e +
f*x]*Sqrt[a + b*Sinh[e + f*x]^2])/(b*Sqrt[1 + Sinh[e + f*x]^2]) - (Ellipti
cE[ArcTan[Sinh[e + f*x]], 1 - b/a]*Sqrt[a + b*Sinh[e + f*x]^2])/(b*Sqrt[1
+ Sinh[e + f*x]^2]*Sqrt[(a + b*Sinh[e + f*x]^2)/(a*(1 + Sinh[e + f*x]^2))]
)))/((a - b)*b))/(3*(a - b)*b)))/f
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 313
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/((c_) + (d_.)*(x_)^2)^(3/2), x_Symbol] :> Sim
p[(Sqrt[a + b*x^2]/(c*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(c
+ d*x^2)))]))*EllipticE[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; FreeQ
[{a, b, c, d}, x] && PosQ[b/a] && PosQ[d/c]
```

rule 320
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(Sqrt[a + b*x^2]/(a*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(
c + d*x^2)))]))*EllipticF[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; Fre
eQ[{a, b, c, d}, x] && PosQ[d/c] && PosQ[b/a] &&  !SimplerSqrtQ[b/a, d/c]
```

rule 372
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_
), x_Symbol] :> Simp[(-a)*e^3*(e*x)^(m - 3)*(a + b*x^2)^(p + 1)*((c + d*x^2
)^(q + 1)/(2*b*(b*c - a*d)*(p + 1))), x] + Simp[e^4/(2*b*(b*c - a*d)*(p + 1
))   Int[(e*x)^(m - 4)*(a + b*x^2)^(p + 1)*(c + d*x^2)^q*Simp[a*c*(m - 3) +
 (a*d*(m + 2*q - 1) + 2*b*c*(p + 1))*x^2, x], x], x] /; FreeQ[{a, b, c, d,
e, q}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && GtQ[m, 3] && IntBinomialQ[a
, b, c, d, e, m, 2, p, q, x]
```

rule 388
```
Int[(x_)^2/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol]
 :> Simp[x*(Sqrt[a + b*x^2]/(b*Sqrt[c + d*x^2])), x] - Simp[c/b   Int[Sqrt[
a + b*x^2]/(c + d*x^2)^(3/2), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
 a*d, 0] && PosQ[b/a] && PosQ[d/c] &&  !SimplerSqrtQ[b/a, d/c]
```

rule 406
```
Int[((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.)*((e_) + (f_.)*(
x_)^2), x_Symbol] :> Simp[e   Int[(a + b*x^2)^p*(c + d*x^2)^q, x], x] + Sim
p[f   Int[x^2*(a + b*x^2)^p*(c + d*x^2)^q, x], x] /; FreeQ[{a, b, c, d, e,
f, p, q}, x]
```

rule 440
```
Int[((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_
)*((e_) + (f_.)*(x_)^2), x_Symbol] :> Simp[g*(b*e - a*f)*(g*x)^(m - 1)*(a +
 b*x^2)^(p + 1)*((c + d*x^2)^(q + 1)/(2*b*(b*c - a*d)*(p + 1))), x] - Simp[
g^2/(2*b*(b*c - a*d)*(p + 1))   Int[(g*x)^(m - 2)*(a + b*x^2)^(p + 1)*(c +
d*x^2)^q*Simp[c*(b*e - a*f)*(m - 1) + (d*(b*e - a*f)*(m + 2*q + 1) - b*2*(c
*f - d*e)*(p + 1))*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, q}, x] &&
 LtQ[p, -1] && GtQ[m, 1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3667
```
Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(
p_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Simp[ff^(m + 1
)*(Sqrt[Cos[e + f*x]^2]/(f*Cos[e + f*x]))   Subst[Int[x^m*((a + b*ff^2*x^2)
^p/Sqrt[1 - ff^2*x^2]), x], x, Sin[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f,
p}, x] && IntegerQ[m/2] &&  !IntegerQ[p]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 867 vs. $2(309) = 618$.

Time = 7.69 (sec) , antiderivative size = 868, normalized size of antiderivative = 2.67

method	result	size
default	Expression too large to display	868

input
```
int(sinh(f*x+e)^6/(a+b*sinh(f*x+e)^2)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
-1/3*((5*(-b/a)^(1/2)*a^2*b-7*(-b/a)^(1/2)*a*b^2)*cosh(f*x+e)^4*sinh(f*x+e
)+(4*(-b/a)^(1/2)*a^3-11*(-b/a)^(1/2)*a^2*b+7*(-b/a)^(1/2)*a*b^2)*cosh(f*x
+e)^2*sinh(f*x+e)+(b/a*cosh(f*x+e)^2+(a-b)/a)^(1/2)*(cosh(f*x+e)^2)^(1/2)*
b*(4*EllipticF(sinh(f*x+e)*(-b/a)^(1/2),(1/b*a)^(1/2))*a^2-7*EllipticF(sin
h(f*x+e)*(-b/a)^(1/2),(1/b*a)^(1/2))*a*b+3*EllipticF(sinh(f*x+e)*(-b/a)^(1
/2),(1/b*a)^(1/2))*b^2-8*EllipticE(sinh(f*x+e)*(-b/a)^(1/2),(1/b*a)^(1/2))
*a^2+13*EllipticE(sinh(f*x+e)*(-b/a)^(1/2),(1/b*a)^(1/2))*a*b-3*EllipticE(
sinh(f*x+e)*(-b/a)^(1/2),(1/b*a)^(1/2))*b^2)*cosh(f*x+e)^2+4*(b/a*cosh(f*x
+e)^2+(a-b)/a)^(1/2)*(cosh(f*x+e)^2)^(1/2)*EllipticF(sinh(f*x+e)*(-b/a)^(1
/2),(1/b*a)^(1/2))*a^3-11*(b/a*cosh(f*x+e)^2+(a-b)/a)^(1/2)*(cosh(f*x+e)^2
)^(1/2)*EllipticF(sinh(f*x+e)*(-b/a)^(1/2),(1/b*a)^(1/2))*a^2*b+10*(b/a*co
sh(f*x+e)^2+(a-b)/a)^(1/2)*(cosh(f*x+e)^2)^(1/2)*EllipticF(sinh(f*x+e)*(-b
/a)^(1/2),(1/b*a)^(1/2))*a*b^2-3*(b/a*cosh(f*x+e)^2+(a-b)/a)^(1/2)*(cosh(f
*x+e)^2)^(1/2)*EllipticF(sinh(f*x+e)*(-b/a)^(1/2),(1/b*a)^(1/2))*b^3-8*(b/
a*cosh(f*x+e)^2+(a-b)/a)^(1/2)*(cosh(f*x+e)^2)^(1/2)*EllipticE(sinh(f*x+e)
*(-b/a)^(1/2),(1/b*a)^(1/2))*a^3+21*(b/a*cosh(f*x+e)^2+(a-b)/a)^(1/2)*(cos
h(f*x+e)^2)^(1/2)*EllipticE(sinh(f*x+e)*(-b/a)^(1/2),(1/b*a)^(1/2))*a^2*b-
16*(b/a*cosh(f*x+e)^2+(a-b)/a)^(1/2)*(cosh(f*x+e)^2)^(1/2)*EllipticE(sinh(
f*x+e)*(-b/a)^(1/2),(1/b*a)^(1/2))*a*b^2+3*(b/a*cosh(f*x+e)^2+(a-b)/a)^(1/
2)*(cosh(f*x+e)^2)^(1/2)*EllipticE(sinh(f*x+e)*(-b/a)^(1/2),(1/b*a)^(1/...
```

Fricas [F]

$$\int \frac{\sinh^6(e+fx)}{(a+b\sinh^2(e+fx))^{5/2}}\,dx=\int \frac{\sinh(fx+e)^6}{(b\sinh(fx+e)^2+a)^{\frac{5}{2}}}\,dx$$

input
```
integrate(sinh(f*x+e)^6/(a+b*sinh(f*x+e)^2)^(5/2),x, algorithm="fricas")
```

output
```
integral(sqrt(b*sinh(f*x + e)^2 + a)*sinh(f*x + e)^6/(b^3*sinh(f*x + e)^6
+ 3*a*b^2*sinh(f*x + e)^4 + 3*a^2*b*sinh(f*x + e)^2 + a^3), x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\sinh^6(e+fx)}{\left(a+b\sinh^2(e+fx)\right)^{5/2}}\,dx = \text{Timed out}$$

input
```
integrate(sinh(f*x+e)**6/(a+b*sinh(f*x+e)**2)**(5/2),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{\sinh^6(e+fx)}{\left(a+b\sinh^2(e+fx)\right)^{5/2}}\,dx = \int \frac{\sinh\left(fx+e\right)^6}{\left(b\sinh\left(fx+e\right)^2+a\right)^{\frac{5}{2}}}\,dx$$

input
```
integrate(sinh(f*x+e)^6/(a+b*sinh(f*x+e)^2)^(5/2),x, algorithm="maxima")
```

output
```
integrate(sinh(f*x + e)^6/(b*sinh(f*x + e)^2 + a)^(5/2), x)
```

Giac [F]

$$\int \frac{\sinh^6(e+fx)}{\left(a+b\sinh^2(e+fx)\right)^{5/2}}\,dx = \int \frac{\sinh\left(fx+e\right)^6}{\left(b\sinh\left(fx+e\right)^2+a\right)^{\frac{5}{2}}}\,dx$$

input
```
integrate(sinh(f*x+e)^6/(a+b*sinh(f*x+e)^2)^(5/2),x, algorithm="giac")
```

output
```
integrate(sinh(f*x + e)^6/(b*sinh(f*x + e)^2 + a)^(5/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sinh^6(e+fx)}{\left(a+b\sinh^2(e+fx)\right)^{5/2}}\,dx = \int \frac{\sinh(e+f\,x)^6}{\left(b\sinh\left(e+f\,x\right)^2+a\right)^{5/2}}\,dx$$

input
```
int(sinh(e + f*x)^6/(a + b*sinh(e + f*x)^2)^(5/2),x)
```

output
```
int(sinh(e + f*x)^6/(a + b*sinh(e + f*x)^2)^(5/2), x)
```

Reduce [F]

$$\int \frac{\sinh^6(e+fx)}{\left(a+b\sinh^2(e+fx)\right)^{5/2}}\,dx = \int \frac{\sqrt{\sinh\left(fx+e\right)^2 b+a}\,\sinh\left(fx+e\right)^6}{\sinh\left(fx+e\right)^6 b^3+3\sinh\left(fx+e\right)^4 a\,b^2+3\sinh\left(fx+e\right)^2 a^2b+a^3}dx$$

input
```
int(sinh(f*x+e)^6/(a+b*sinh(f*x+e)^2)^(5/2),x)
```

output
```
int((sqrt(sinh(e + f*x)**2*b + a)*sinh(e + f*x)**6)/(sinh(e + f*x)**6*b**3
 + 3*sinh(e + f*x)**4*a*b**2 + 3*sinh(e + f*x)**2*a**2*b + a**3),x)
```

3.104 $\int \frac{\sinh^4(e+fx)}{(a+b\sinh^2(e+fx))^{5/2}}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 259

$$\int \frac{\sinh^4(e+fx)}{(a+b\sinh^2(e+fx))^{5/2}}\,dx = -\frac{a\cosh(e+fx)\sinh(e+fx)}{3(a-b)bf\,(a+b\sinh^2(e+fx))^{3/2}} + \frac{2\sqrt{a}(a-2b)\cosh(e+fx)E\left(\arctan\left(\frac{\sqrt{b}\sinh(e+fx)}{\sqrt{a}}\right)|1-\frac{a}{b}\right)}{3(a-b)^2b^{3/2}f\sqrt{\frac{a\cosh^2(e+fx)}{a+b\sinh^2(e+fx)}}\sqrt{a+b\sinh^2(e+fx)}} - \frac{\sqrt{a}(a-3b)\cosh(e+fx)\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{b}\sinh(e+fx)}{\sqrt{a}}\right),1-\frac{a}{b}\right)}{3(a-b)^2b^{3/2}f\sqrt{\frac{a\cosh^2(e+fx)}{a+b\sinh^2(e+fx)}}\sqrt{a+b\sinh^2(e+fx)}}$$

output
```
-1/3*a*cosh(f*x+e)*sinh(f*x+e)/(a-b)/b/f/(a+b*sinh(f*x+e)^2)^(3/2)+2/3*a^(
1/2)*(a-2*b)*cosh(f*x+e)*EllipticE(b^(1/2)*sinh(f*x+e)/a^(1/2)/(1+b*sinh(f
*x+e)^2/a)^(1/2),(1-a/b)^(1/2))/(a-b)^2/b^(3/2)/f/(a*cosh(f*x+e)^2/(a+b*si
nh(f*x+e)^2))^(1/2)/(a+b*sinh(f*x+e)^2)^(1/2)-1/3*a^(1/2)*(a-3*b)*cosh(f*x
+e)*InverseJacobiAM(arctan(b^(1/2)*sinh(f*x+e)/a^(1/2)),(1-a/b)^(1/2))/(a-
b)^2/b^(3/2)/f/(a*cosh(f*x+e)^2/(a+b*sinh(f*x+e)^2))^(1/2)/(a+b*sinh(f*x+e
)^2)^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 1.37 (sec) , antiderivative size = 198, normalized size of antiderivative = 0.76

$$\int \frac{\sinh^4(e+fx)}{(a+b\sinh^2(e+fx))^{5/2}}\,dx = \frac{2ia^2(a-2b)\left(\frac{2a-b+b\cosh(2(e+fx))}{a}\right)^{3/2}E\left(i(e+fx)\,\middle|\,\frac{b}{a}\right)-ia(2a^2-5ab+3b^2)\ldots}{\ldots}$$

input
```
Integrate[Sinh[e + f*x]^4/(a + b*Sinh[e + f*x]^2)^(5/2),x]
```

output
```
((2*I)*a^2*(a - 2*b)*((2*a - b + b*Cosh[2*(e + f*x)])/a)^(3/2)*EllipticE[I
*(e + f*x), b/a] - I*a*(2*a^2 - 5*a*b + 3*b^2)*((2*a - b + b*Cosh[2*(e + f
*x)])/a)^(3/2)*EllipticF[I*(e + f*x), b/a] - Sqrt[2]*b*(-a^2 + 4*a*b - 2*b
^2 - (a - 2*b)*b*Cosh[2*(e + f*x)])*Sinh[2*(e + f*x)])/(3*(a - b)^2*b^2*f*
(2*a - b + b*Cosh[2*(e + f*x)])^(3/2))
```

Rubi [A] (verified)

Time = 0.44 (sec) , antiderivative size = 295, normalized size of antiderivative = 1.14, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.240$, Rules used = {3042, 3667, 372, 400, 313, 320}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sinh^4(e+fx)}{(a+b\sinh^2(e+fx))^{5/2}}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\sin(ie+ifx)^4}{(a-b\sin(ie+ifx)^2)^{5/2}}dx$$

$$\Big\downarrow \text{3667}$$

$$\frac{\sqrt{\cosh^2(e+fx)}\operatorname{sech}(e+fx)\int \frac{\sinh^4(e+fx)}{\sqrt{\sinh^2(e+fx)+1}\left(b\sinh^2(e+fx)+a\right)^{5/2}}d\sinh(e+fx)}{f}$$

$$\Big\downarrow \text{372}$$

$$\frac{\sqrt{\cosh^2(e+fx)}\text{sech}(e+fx)\left(\frac{\int \frac{(2a-3b)\sinh^2(e+fx)+a}{\sqrt{\sinh^2(e+fx)+1}\left(b\sinh^2(e+fx)+a\right)^{3/2}}d\sinh(e+fx)}{3b(a-b)}-\frac{a\sinh(e+fx)\sqrt{\sinh^2(e+fx)+1}}{3b(a-b)\left(a+b\sinh^2(e+fx)\right)^{3/2}}\right)}{f}$$

$$\Big\downarrow \text{400}$$

$$\frac{\sqrt{\cosh^2(e+fx)}\text{sech}(e+fx)\left(\frac{\frac{2a(a-2b)\int \frac{\sqrt{\sinh^2(e+fx)+1}}{\left(b\sinh^2(e+fx)+a\right)^{3/2}}d\sinh(e+fx)}{a-b}-\frac{(a-3b)\int \frac{1}{\sqrt{\sinh^2(e+fx)+1}\sqrt{b\sinh^2(e+fx)+a}}d\sinh(e+fx)}{a-b}}{3b(a-b)}-\right.}{f}$$

$$\Big\downarrow \text{313}$$

$$\frac{\sqrt{\cosh^2(e+fx)}\text{sech}(e+fx)\left(\frac{\frac{2\sqrt{a}(a-2b)\sqrt{\sinh^2(e+fx)+1}E\left(\arctan\left(\frac{\sqrt{b}\sinh(e+fx)}{\sqrt{a}}\right)|1-\frac{a}{b}\right)}{\sqrt{b}(a-b)\sqrt{\frac{a\left(\sinh^2(e+fx)+1\right)}{a+b\sinh^2(e+fx)}}\sqrt{a+b\sinh^2(e+fx)}}-\frac{(a-3b)\int \frac{1}{\sqrt{\sinh^2(e+fx)+1}\sqrt{b\sinh^2(e+fx)+a}}d}{a-b}}{3b(a-b)}\right.}{f}$$

$$\Big\downarrow \text{320}$$

$$\frac{\sqrt{\cosh^2(e+fx)}\text{sech}(e+fx)\left(\frac{\frac{2\sqrt{a}(a-2b)\sqrt{\sinh^2(e+fx)+1}E\left(\arctan\left(\frac{\sqrt{b}\sinh(e+fx)}{\sqrt{a}}\right)|1-\frac{a}{b}\right)}{\sqrt{b}(a-b)\sqrt{\frac{a\left(\sinh^2(e+fx)+1\right)}{a+b\sinh^2(e+fx)}}\sqrt{a+b\sinh^2(e+fx)}}-\frac{(a-3b)\sqrt{a+b\sinh^2(e+fx)}\text{EllipticF}\left(\arctan(\sinh\right.}{a(a-b)\sqrt{\sinh^2(e+fx)+1}\sqrt{\frac{a+b\sinh^2(e}{a\left(\sinh^2(e+f\right.}}}}{3b(a-b)}\right.}{f}$$

input
```
Int[Sinh[e + f*x]^4/(a + b*Sinh[e + f*x]^2)^(5/2),x]
```

output
```
(Sqrt[Cosh[e + f*x]^2]*Sech[e + f*x]*(-1/3*(a*Sinh[e + f*x]*Sqrt[1 + Sinh[
e + f*x]^2])/((a - b)*b*(a + b*Sinh[e + f*x]^2)^(3/2)) + ((2*Sqrt[a]*(a -
2*b)*EllipticE[ArcTan[(Sqrt[b]*Sinh[e + f*x])/Sqrt[a]], 1 - a/b]*Sqrt[1 +
Sinh[e + f*x]^2])/((a - b)*Sqrt[b]*Sqrt[(a*(1 + Sinh[e + f*x]^2))/(a + b*S
inh[e + f*x]^2)]*Sqrt[a + b*Sinh[e + f*x]^2]) - ((a - 3*b)*EllipticF[ArcTa
n[Sinh[e + f*x]], 1 - b/a]*Sqrt[a + b*Sinh[e + f*x]^2])/(a*(a - b)*Sqrt[1
+ Sinh[e + f*x]^2]*Sqrt[(a + b*Sinh[e + f*x]^2)/(a*(1 + Sinh[e + f*x]^2))]
))/(3*(a - b)*b)))/f
```

Defintions of rubi rules used

rule 313
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/((c_) + (d_.)*(x_)^2)^(3/2), x_Symbol] :> Sim
p[(Sqrt[a + b*x^2]/(c*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(c
+ d*x^2)))]))*EllipticE[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; FreeQ
[{a, b, c, d}, x] && PosQ[b/a] && PosQ[d/c]
```

rule 320
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(Sqrt[a + b*x^2]/(a*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(
c + d*x^2)))]))*EllipticF[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; Fre
eQ[{a, b, c, d}, x] && PosQ[d/c] && PosQ[b/a] &&  !SimplerSqrtQ[b/a, d/c]
```

rule 372
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_
), x_Symbol] :> Simp[(-a)*e^3*(e*x)^(m - 3)*(a + b*x^2)^(p + 1)*((c + d*x^2
)^(q + 1)/(2*b*(b*c - a*d)*(p + 1))), x] + Simp[e^4/(2*b*(b*c - a*d)*(p + 1
))   Int[(e*x)^(m - 4)*(a + b*x^2)^(p + 1)*(c + d*x^2)^q*Simp[a*c*(m - 3) +
 (a*d*(m + 2*q - 1) + 2*b*c*(p + 1))*x^2, x], x], x] /; FreeQ[{a, b, c, d,
e, q}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && GtQ[m, 3] && IntBinomialQ[a
, b, c, d, e, m, 2, p, q, x]
```

rule 400
```
Int[((e_) + (f_.)*(x_)^2)/(Sqrt[(a_) + (b_.)*(x_)^2]*((c_) + (d_.)*(x_)^2)^
(3/2)), x_Symbol] :> Simp[(b*e - a*f)/(b*c - a*d)   Int[1/(Sqrt[a + b*x^2]*
Sqrt[c + d*x^2]), x], x] - Simp[(d*e - c*f)/(b*c - a*d)   Int[Sqrt[a + b*x^
2]/(c + d*x^2)^(3/2), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && PosQ[b/a] &
& PosQ[d/c]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3667
```
Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(
p_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Simp[ff^(m + 1
)*(Sqrt[Cos[e + f*x]^2]/(f*Cos[e + f*x]))   Subst[Int[x^m*((a + b*ff^2*x^2)
^p/Sqrt[1 - ff^2*x^2]), x], x, Sin[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f,
p}, x] && IntegerQ[m/2] &&  !IntegerQ[p]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 658 vs. $2(247) = 494$.

Time = 5.07 (sec) , antiderivative size = 659, normalized size of antiderivative = 2.54

method	result
default	$\frac{2\sqrt{-\frac{b}{a}}\, ab\sinh(fx+e)^{5}-4\sqrt{-\frac{b}{a}}\, b^{2}\sinh(fx+e)^{5}+\sqrt{\frac{a+b\sinh(fx+e)^{2}}{a}}\, \sqrt{\frac{\cosh(2fx+2e)}{2}+\frac{1}{2}}\, \text{EllipticF}\left(\sinh(fx+e)\sqrt{-\frac{b}{a}}, \sqrt{\frac{a}{b}}\right) ab\sinh}{}$

input
```
int(sinh(f*x+e)^4/(a+b*sinh(f*x+e)^2)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
1/3*(2*(-b/a)^(1/2)*a*b*sinh(f*x+e)^5-4*(-b/a)^(1/2)*b^2*sinh(f*x+e)^5+((a
+b*sinh(f*x+e)^2)/a)^(1/2)*(cosh(f*x+e)^2)^(1/2)*EllipticF(sinh(f*x+e)*(-b
/a)^(1/2),(1/b*a)^(1/2))*a*b*sinh(f*x+e)^2-((a+b*sinh(f*x+e)^2)/a)^(1/2)*(
cosh(f*x+e)^2)^(1/2)*EllipticF(sinh(f*x+e)*(-b/a)^(1/2),(1/b*a)^(1/2))*b^2
*sinh(f*x+e)^2-2*((a+b*sinh(f*x+e)^2)/a)^(1/2)*(cosh(f*x+e)^2)^(1/2)*Ellip
ticE(sinh(f*x+e)*(-b/a)^(1/2),(1/b*a)^(1/2))*a*b*sinh(f*x+e)^2+4*((a+b*sin
h(f*x+e)^2)/a)^(1/2)*(cosh(f*x+e)^2)^(1/2)*EllipticE(sinh(f*x+e)*(-b/a)^(1
/2),(1/b*a)^(1/2))*b^2*sinh(f*x+e)^2+(-b/a)^(1/2)*a^2*sinh(f*x+e)^3-(-b/a)
^(1/2)*a*b*sinh(f*x+e)^3-4*(-b/a)^(1/2)*b^2*sinh(f*x+e)^3+a^2*((a+b*sinh(f
*x+e)^2)/a)^(1/2)*(cosh(f*x+e)^2)^(1/2)*EllipticF(sinh(f*x+e)*(-b/a)^(1/2)
,(1/b*a)^(1/2))-a*((a+b*sinh(f*x+e)^2)/a)^(1/2)*(cosh(f*x+e)^2)^(1/2)*Elli
pticF(sinh(f*x+e)*(-b/a)^(1/2),(1/b*a)^(1/2))*b-2*((a+b*sinh(f*x+e)^2)/a)^
(1/2)*(cosh(f*x+e)^2)^(1/2)*EllipticE(sinh(f*x+e)*(-b/a)^(1/2),(1/b*a)^(1/
2))*a^2+4*((a+b*sinh(f*x+e)^2)/a)^(1/2)*(cosh(f*x+e)^2)^(1/2)*EllipticE(si
nh(f*x+e)*(-b/a)^(1/2),(1/b*a)^(1/2))*a*b+(-b/a)^(1/2)*a^2*sinh(f*x+e)-3*(
-b/a)^(1/2)*a*b*sinh(f*x+e))/(-b/a)^(1/2)/(a+b*sinh(f*x+e)^2)^(3/2)/(a-b)^
2/b/cosh(f*x+e)/f
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 4985 vs. $2(247) = 494$.

Time = 0.22 (sec) , antiderivative size = 4985, normalized size of antiderivative = 19.25

$$\int \frac{\sinh^4(e+fx)}{\left(a+b\sinh^2(e+fx)\right)^{5/2}}\,dx = \text{Too large to display}$$

input
```
integrate(sinh(f*x+e)^4/(a+b*sinh(f*x+e)^2)^(5/2),x, algorithm="fricas")
```

output
```
Too large to include
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\sinh^4(e+fx)}{\left(a+b\sinh^2(e+fx)\right)^{5/2}}\,dx = \text{Timed out}$$

input
```
integrate(sinh(f*x+e)**4/(a+b*sinh(f*x+e)**2)**(5/2),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{\sinh^4(e+fx)}{\left(a+b\sinh^2(e+fx)\right)^{5/2}}\,dx = \int \frac{\sinh\left(fx+e\right)^4}{\left(b\sinh\left(fx+e\right)^2+a\right)^{\frac{5}{2}}}\,dx$$

input
```
integrate(sinh(f*x+e)^4/(a+b*sinh(f*x+e)^2)^(5/2),x, algorithm="maxima")
```

output
```
integrate(sinh(f*x + e)^4/(b*sinh(f*x + e)^2 + a)^(5/2), x)
```

Giac [F]

$$\int \frac{\sinh^4(e+fx)}{\left(a+b\sinh^2(e+fx)\right)^{5/2}}\,dx = \int \frac{\sinh\left(fx+e\right)^4}{\left(b\sinh\left(fx+e\right)^2+a\right)^{\frac{5}{2}}}\,dx$$

input
```
integrate(sinh(f*x+e)^4/(a+b*sinh(f*x+e)^2)^(5/2),x, algorithm="giac")
```

output
```
integrate(sinh(f*x + e)^4/(b*sinh(f*x + e)^2 + a)^(5/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sinh^4(e+fx)}{\left(a+b\sinh^2(e+fx)\right)^{5/2}}\,dx=\int \frac{\sinh(e+f\,x)^4}{\left(b\sinh\left(e+f\,x\right)^2+a\right)^{5/2}}\,dx$$

input
```
int(sinh(e + f*x)^4/(a + b*sinh(e + f*x)^2)^(5/2),x)
```

output
```
int(sinh(e + f*x)^4/(a + b*sinh(e + f*x)^2)^(5/2), x)
```

Reduce [F]

$$\int \frac{\sinh^4(e+fx)}{\left(a+b\sinh^2(e+fx)\right)^{5/2}}\,dx=\int \frac{\sqrt{\sinh\left(fx+e\right)^2 b+a}\,\sinh\left(fx+e\right)^4}{\sinh\left(fx+e\right)^6 b^3+3\sinh\left(fx+e\right)^4 a\,b^2+3\sinh\left(fx+e\right)^2 a^2 b+a^3}dx$$

input
```
int(sinh(f*x+e)^4/(a+b*sinh(f*x+e)^2)^(5/2),x)
```

output
```
int((sqrt(sinh(e + f*x)**2*b + a)*sinh(e + f*x)**4)/(sinh(e + f*x)**6*b**3
 + 3*sinh(e + f*x)**4*a*b**2 + 3*sinh(e + f*x)**2*a**2*b + a**3),x)
```

3.105 $\int \frac{\sinh^2(e+fx)}{(a+b\sinh^2(e+fx))^{5/2}}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 243

$$\int \frac{\sinh^2(e+fx)}{(a+b\sinh^2(e+fx))^{5/2}}\,dx = \frac{\cosh(e+fx)\sinh(e+fx)}{3(a-b)f\,(a+b\sinh^2(e+fx))^{3/2}} + \frac{(a+b)\cosh(e+fx)\sinh(e+fx)}{3a(a-b)^2f\sqrt{a+b\sinh^2(e+fx)}} + \frac{i(a+b)E\left(ie+ifx\middle|\frac{b}{a}\right)\sqrt{a+b\sinh^2(e+fx)}}{3a(a-b)^2bf\sqrt{\frac{a+b\sinh^2(e+fx)}{a}}} - \frac{i\,\text{EllipticF}\left(ie+ifx,\frac{b}{a}\right)\sqrt{\frac{a+b\sinh^2(e+fx)}{a}}}{3(a-b)bf\sqrt{a+b\sinh^2(e+fx)}}$$

output
```
1/3*cosh(f*x+e)*sinh(f*x+e)/(a-b)/f/(a+b*sinh(f*x+e)^2)^(3/2)+1/3*(a+b)*co
sh(f*x+e)*sinh(f*x+e)/a/(a-b)^2/f/(a+b*sinh(f*x+e)^2)^(1/2)+1/3*I*(a+b)*El
lipticE(sin(I*e+I*f*x),(b/a)^(1/2))*(a+b*sinh(f*x+e)^2)^(1/2)/a/(a-b)^2/b/
f/((a+b*sinh(f*x+e)^2)/a)^(1/2)-1/3*I*InverseJacobiAM(I*e+I*f*x,(b/a)^(1/2
))*((a+b*sinh(f*x+e)^2)/a)^(1/2)/(a-b)/b/f/(a+b*sinh(f*x+e)^2)^(1/2)
```

Mathematica [A] (verified)

Time = 1.25 (sec) , antiderivative size = 187, normalized size of antiderivative = 0.77

$$\int \frac{\sinh^2(e+fx)}{\left(a+b\sinh^2(e+fx)\right)^{5/2}}\,dx = \frac{2ia^2(a+b)\left(\frac{2a-b+b\cosh(2(e+fx))}{a}\right)^{3/2}E\left(i(e+fx)\,|\,\frac{b}{a}\right)-2ia^2(a-b)\left(\frac{2a-b+}{}\right.}{6a(a}$$

input
```
Integrate[Sinh[e + f*x]^2/(a + b*Sinh[e + f*x]^2)^(5/2),x]
```

output
```
((2*I)*a^2*(a + b)*((2*a - b + b*Cosh[2*(e + f*x)])/a)^(3/2)*EllipticE[I*(
e + f*x), b/a] - (2*I)*a^2*(a - b)*((2*a - b + b*Cosh[2*(e + f*x)])/a)^(3/
2)*EllipticF[I*(e + f*x), b/a] + Sqrt[2]*b*(4*a^2 - a*b - b^2 + b*(a + b)*
Cosh[2*(e + f*x)])*Sinh[2*(e + f*x)])/(6*a*(a - b)^2*b*f*(2*a - b + b*Cosh
[2*(e + f*x)])^(3/2))
```

Rubi [A] (verified)

Time = 1.25 (sec) , antiderivative size = 251, normalized size of antiderivative = 1.03, number of steps used = 14, number of rules used = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.560$, Rules used = {3042, 25, 3652, 3042, 3652, 3042, 3651, 3042, 3657, 3042, 3656, 3662, 3042, 3661}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sinh^2(e+fx)}{\left(a+b\sinh^2(e+fx)\right)^{5/2}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int -\frac{\sin(ie+ifx)^2}{(a-b\sin(ie+ifx)^2)^{5/2}}dx$$

$$\Big\downarrow \text{25}$$

$$-\int \frac{\sin(ie+ifx)^2}{(a-b\sin(ie+ifx)^2)^{5/2}}dx$$

$$\Big\downarrow \; 3652$$

$$\frac{\sinh(e+fx)\cosh(e+fx)}{3f(a-b)\left(a+b\sinh^2(e+fx)\right)^{3/2}}-\frac{\int \frac{a-a\sinh^2(e+fx)}{\left(b\sinh^2(e+fx)+a\right)^{3/2}}dx}{3a(a-b)}$$

$$\Big\downarrow \; 3042$$

$$\frac{\sinh(e+fx)\cosh(e+fx)}{3f(a-b)\left(a+b\sinh^2(e+fx)\right)^{3/2}}-\frac{\int \frac{a\sin(ie+ifx)^2+a}{\left(a-b\sin(ie+ifx)^2\right)^{3/2}}dx}{3a(a-b)}$$

$$\Big\downarrow \; 3652$$

$$\frac{\sinh(e+fx)\cosh(e+fx)}{3f(a-b)\left(a+b\sinh^2(e+fx)\right)^{3/2}}-\frac{\frac{\int \frac{2a^2+(a+b)\sinh^2(e+fx)a}{\sqrt{b\sinh^2(e+fx)+a}}dx}{a(a-b)}-\frac{(a+b)\sinh(e+fx)\cosh(e+fx)}{f(a-b)\sqrt{a+b\sinh^2(e+fx)}}}{3a(a-b)}$$

$$\Big\downarrow \; 3042$$

$$\frac{\sinh(e+fx)\cosh(e+fx)}{3f(a-b)\left(a+b\sinh^2(e+fx)\right)^{3/2}}-\frac{-\frac{(a+b)\sinh(e+fx)\cosh(e+fx)}{f(a-b)\sqrt{a+b\sinh^2(e+fx)}}+\frac{\int \frac{2a^2-a(a+b)\sin(ie+ifx)^2}{\sqrt{a-b\sin(ie+ifx)^2}}dx}{a(a-b)}}{3a(a-b)}$$

$$\Big\downarrow \; 3651$$

$$\frac{\sinh(e+fx)\cosh(e+fx)}{3f(a-b)\left(a+b\sinh^2(e+fx)\right)^{3/2}}-\frac{\frac{\frac{a(a+b)\int \sqrt{b\sinh^2(e+fx)+a}dx}{b}-\frac{a^2(a-b)\int \frac{1}{\sqrt{b\sinh^2(e+fx)+a}}dx}{b}}{a(a-b)}-\frac{(a+b)\sinh(e+fx)\cosh(e+fx)}{f(a-b)\sqrt{a+b\sinh^2(e+fx)}}}{3a(a-b)}$$

$$\Big\downarrow \; 3042$$

$$\frac{\sinh(e+fx)\cosh(e+fx)}{3f(a-b)\left(a+b\sinh^2(e+fx)\right)^{3/2}}-\frac{-\frac{(a+b)\sinh(e+fx)\cosh(e+fx)}{f(a-b)\sqrt{a+b\sinh^2(e+fx)}}+\frac{\frac{a(a+b)\int \sqrt{a-b\sin(ie+ifx)^2}dx}{b}-\frac{a^2(a-b)\int \frac{1}{\sqrt{a-b\sin(ie+ifx)^2}}dx}{b}}{a(a-b)}}{3a(a-b)}$$

$$\Big\downarrow \; 3657$$

$$\frac{\sinh(e+fx)\cosh(e+fx)}{3f(a-b)\left(a+b\sinh^2(e+fx)\right)^{3/2}}-\frac{-\frac{(a+b)\sinh(e+fx)\cosh(e+fx)}{f(a-b)\sqrt{a+b\sinh^2(e+fx)}}+\frac{\frac{a(a+b)\sqrt{a+b\sinh^2(e+fx)}\int\sqrt{\frac{b\sinh^2(e+fx)}{a}+1}dx}{b\sqrt{\frac{b\sinh^2(e+fx)}{a}+1}}-\frac{a^2(a-b)\int\frac{1}{\sqrt{a-b\sin(ie+ifx)^2}}dx}{b}}{a(a-b)}}{3a(a-b)}$$

$\Big\downarrow$ 3042

$$\frac{\sinh(e+fx)\cosh(e+fx)}{3f(a-b)\left(a+b\sinh^2(e+fx)\right)^{3/2}}-\frac{-\frac{(a+b)\sinh(e+fx)\cosh(e+fx)}{f(a-b)\sqrt{a+b\sinh^2(e+fx)}}+\frac{\frac{a(a+b)\sqrt{a+b\sinh^2(e+fx)}\int\sqrt{1-\frac{b\sin(ie+ifx)^2}{a}}dx}{b\sqrt{\frac{b\sinh^2(e+fx)}{a}+1}}-\frac{a^2(a-b)\int\frac{1}{\sqrt{a-b\sin(ie+ifx)^2}}dx}{b}}{a(a-b)}}{3a(a-b)}$$

$\Big\downarrow$ 3656

$$\frac{\sinh(e+fx)\cosh(e+fx)}{3f(a-b)\left(a+b\sinh^2(e+fx)\right)^{3/2}}-\frac{-\frac{(a+b)\sinh(e+fx)\cosh(e+fx)}{f(a-b)\sqrt{a+b\sinh^2(e+fx)}}+\frac{-\frac{a^2(a-b)\int\frac{1}{\sqrt{a-b\sin(ie+ifx)^2}}dx}{b}-\frac{ia(a+b)\sqrt{a+b\sinh^2(e+fx)}E\left(ie+ifx\middle|\frac{b}{a}\right)}{bf\sqrt{\frac{b\sinh^2(e+fx)}{a}+1}}}{a(a-b)}}{3a(a-b)}$$

$\Big\downarrow$ 3662

$$\frac{\sinh(e+fx)\cosh(e+fx)}{3f(a-b)\left(a+b\sinh^2(e+fx)\right)^{3/2}}-\frac{-\frac{(a+b)\sinh(e+fx)\cosh(e+fx)}{f(a-b)\sqrt{a+b\sinh^2(e+fx)}}+\frac{-\frac{a^2(a-b)\sqrt{\frac{b\sinh^2(e+fx)}{a}+1}\int\frac{1}{\sqrt{\frac{b\sinh^2(e+fx)}{a}+1}}dx}{b\sqrt{a+b\sinh^2(e+fx)}}-\frac{ia(a+b)\sqrt{a+b\sinh^2(e+fx)}E\left(ie+ifx\middle|\frac{b}{a}\right)}{bf\sqrt{\frac{b\sinh^2(e+fx)}{a}+1}}}{a(a-b)}}{3a(a-b)}$$

$\Big\downarrow$ 3042

$$\frac{\sinh(e+fx)\cosh(e+fx)}{3f(a-b)\left(a+b\sinh^2(e+fx)\right)^{3/2}}-\frac{-\frac{(a+b)\sinh(e+fx)\cosh(e+fx)}{f(a-b)\sqrt{a+b\sinh^2(e+fx)}}+\frac{-\frac{a^2(a-b)\sqrt{\frac{b\sinh^2(e+fx)}{a}+1}\int\frac{1}{\sqrt{1-\frac{b\sin(ie+ifx)^2}{a}}}dx}{b\sqrt{a+b\sinh^2(e+fx)}}-\frac{ia(a+b)\sqrt{a+b\sinh^2(e+fx)}E\left(ie+ifx\middle|\frac{b}{a}\right)}{bf\sqrt{\frac{b\sinh^2(e+fx)}{a}+1}}}{a(a-b)}}{3a(a-b)}$$

$$\Big\downarrow \text{3661}$$

$$\frac{\sinh(e+fx)\cosh(e+fx)}{3f(a-b)\left(a+b\sinh^2(e+fx)\right)^{3/2}}-\frac{-\frac{(a+b)\sinh(e+fx)\cosh(e+fx)}{f(a-b)\sqrt{a+b\sinh^2(e+fx)}}+\frac{\frac{ia^2(a-b)\sqrt{\frac{b\sinh^2(e+fx)}{a}+1}\,\text{EllipticF}\left(ie+ifx,\frac{b}{a}\right)}{bf\sqrt{a+b\sinh^2(e+fx)}}-\frac{ia(a+b)\sqrt{a+b\sinh^2(e+fx)}E\left(ie+ifx\middle|\frac{b}{a}\right)}{bf\sqrt{\frac{b\sinh^2(e+fx)}{a}+1}}}{a(a-b)}}{3a(a-b)}$$

input
```
Int[Sinh[e + f*x]^2/(a + b*Sinh[e + f*x]^2)^(5/2),x]
```

output
```
(Cosh[e + f*x]*Sinh[e + f*x])/(3*(a - b)*f*(a + b*Sinh[e + f*x]^2)^(3/2))
- (-(((a + b)*Cosh[e + f*x]*Sinh[e + f*x])/((a - b)*f*Sqrt[a + b*Sinh[e +
f*x]^2])) + (((-I)*a*(a + b)*EllipticE[I*e + I*f*x, b/a]*Sqrt[a + b*Sinh[e
 + f*x]^2])/(b*f*Sqrt[1 + (b*Sinh[e + f*x]^2)/a]) + (I*a^2*(a - b)*Ellipti
cF[I*e + I*f*x, b/a]*Sqrt[1 + (b*Sinh[e + f*x]^2)/a])/(b*f*Sqrt[a + b*Sinh
[e + f*x]^2]))/(a*(a - b)))/(3*a*(a - b))
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3651
```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)]^2)/Sqrt[(a_) + (b_.)*sin[(e_.) +
 (f_.)*(x_)]^2], x_Symbol] :> Simp[B/b   Int[Sqrt[a + b*Sin[e + f*x]^2], x]
, x] + Simp[(A*b - a*B)/b   Int[1/Sqrt[a + b*Sin[e + f*x]^2], x], x] /; Fre
eQ[{a, b, e, f, A, B}, x]
```

rule 3652
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)]^2), x_Symbol] :> Simp[(-(A*b - a*B))*Cos[e + f*x]*Sin[e + f*x
]*((a + b*Sin[e + f*x]^2)^(p + 1)/(2*a*f*(a + b)*(p + 1))), x] - Simp[1/(2*
a*(a + b)*(p + 1))   Int[(a + b*Sin[e + f*x]^2)^(p + 1)*Simp[a*B - A*(2*a*(
p + 1) + b*(2*p + 3)) + 2*(A*b - a*B)*(p + 2)*Sin[e + f*x]^2, x], x], x] /;
 FreeQ[{a, b, e, f, A, B}, x] && LtQ[p, -1] && NeQ[a + b, 0]
```

rule 3656
```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2], x_Symbol] :> Simp[(Sqrt[a
]/f)*EllipticE[e + f*x, -b/a], x] /; FreeQ[{a, b, e, f}, x] && GtQ[a, 0]
```

rule 3657
```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2], x_Symbol] :> Simp[Sqrt[a
+ b*Sin[e + f*x]^2]/Sqrt[1 + b*(Sin[e + f*x]^2/a)]   Int[Sqrt[1 + (b*Sin[e
+ f*x]^2)/a], x], x] /; FreeQ[{a, b, e, f}, x] &&  !GtQ[a, 0]
```

rule 3661
```
Int[1/Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2], x_Symbol] :> Simp[(1/(S
qrt[a]*f))*EllipticF[e + f*x, -b/a], x] /; FreeQ[{a, b, e, f}, x] && GtQ[a,
 0]
```

rule 3662
```
Int[1/Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2], x_Symbol] :> Simp[Sqrt[
1 + b*(Sin[e + f*x]^2/a)]/Sqrt[a + b*Sin[e + f*x]^2]   Int[1/Sqrt[1 + (b*Si
n[e + f*x]^2)/a], x], x] /; FreeQ[{a, b, e, f}, x] &&  !GtQ[a, 0]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 597 vs. $2(222) = 444$.

Time = 5.06 (sec) , antiderivative size = 598, normalized size of antiderivative = 2.46

method	result
default	$-\frac{\left(-\sqrt{-\frac{b}{a}}\,ab-\sqrt{-\frac{b}{a}}\,b^{2}\right)\cosh(fx+e)^{4}\sinh(fx+e)+\left(-2\sqrt{-\frac{b}{a}}\,a^{2}+\sqrt{-\frac{b}{a}}\,ab+\sqrt{-\frac{b}{a}}\,b^{2}\right)\cosh(fx+e)^{2}\sinh(fx+e)+\sqrt{\frac{\cosh(2fx+}{2}}}{}$
risch	Expression too large to display

input
```
int(sinh(f*x+e)^2/(a+b*sinh(f*x+e)^2)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
-1/3*((-(-b/a)^(1/2)*a*b-(-b/a)^(1/2)*b^2)*cosh(f*x+e)^4*sinh(f*x+e)+(-2*(
-b/a)^(1/2)*a^2+(-b/a)^(1/2)*a*b+(-b/a)^(1/2)*b^2)*cosh(f*x+e)^2*sinh(f*x+
e)+(cosh(f*x+e)^2)^(1/2)*(b/a*cosh(f*x+e)^2+(a-b)/a)^(1/2)*b*(a*EllipticF(
sinh(f*x+e)*(-b/a)^(1/2),(1/b*a)^(1/2))-b*EllipticF(sinh(f*x+e)*(-b/a)^(1/
2),(1/b*a)^(1/2))+EllipticE(sinh(f*x+e)*(-b/a)^(1/2),(1/b*a)^(1/2))*a+b*El
lipticE(sinh(f*x+e)*(-b/a)^(1/2),(1/b*a)^(1/2)))*cosh(f*x+e)^2+a^2*(b/a*co
sh(f*x+e)^2+(a-b)/a)^(1/2)*(cosh(f*x+e)^2)^(1/2)*EllipticF(sinh(f*x+e)*(-b
/a)^(1/2),(1/b*a)^(1/2))-2*a*(b/a*cosh(f*x+e)^2+(a-b)/a)^(1/2)*(cosh(f*x+e
)^2)^(1/2)*EllipticF(sinh(f*x+e)*(-b/a)^(1/2),(1/b*a)^(1/2))*b+(b/a*cosh(f
*x+e)^2+(a-b)/a)^(1/2)*(cosh(f*x+e)^2)^(1/2)*EllipticF(sinh(f*x+e)*(-b/a)^
(1/2),(1/b*a)^(1/2))*b^2+(b/a*cosh(f*x+e)^2+(a-b)/a)^(1/2)*(cosh(f*x+e)^2)
^(1/2)*EllipticE(sinh(f*x+e)*(-b/a)^(1/2),(1/b*a)^(1/2))*a^2-(b/a*cosh(f*x
+e)^2+(a-b)/a)^(1/2)*(cosh(f*x+e)^2)^(1/2)*EllipticE(sinh(f*x+e)*(-b/a)^(1
/2),(1/b*a)^(1/2))*b^2)/(-b/a)^(1/2)/(a+b*sinh(f*x+e)^2)^(3/2)/(a-b)^2/a/c
osh(f*x+e)/f
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 4684 vs. $2(219) = 438$.

Time = 0.23 (sec) , antiderivative size = 4684, normalized size of antiderivative = 19.28

$$\int \frac{\sinh^2(e+fx)}{\left(a+b\sinh^2(e+fx)\right)^{5/2}}\,dx = \text{Too large to display}$$

input
```
integrate(sinh(f*x+e)^2/(a+b*sinh(f*x+e)^2)^(5/2),x, algorithm="fricas")
```

output
```
Too large to include
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\sinh^2(e+fx)}{\left(a+b\sinh^2(e+fx)\right)^{5/2}}\,dx = \text{Timed out}$$

input
```
integrate(sinh(f*x+e)**2/(a+b*sinh(f*x+e)**2)**(5/2),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{\sinh^2(e+fx)}{\left(a+b\sinh^2(e+fx)\right)^{5/2}}\,dx = \int \frac{\sinh\left(fx+e\right)^2}{\left(b\sinh\left(fx+e\right)^2+a\right)^{\frac{5}{2}}}\,dx$$

input
```
integrate(sinh(f*x+e)^2/(a+b*sinh(f*x+e)^2)^(5/2),x, algorithm="maxima")
```

output
```
integrate(sinh(f*x + e)^2/(b*sinh(f*x + e)^2 + a)^(5/2), x)
```

Giac [F]

$$\int \frac{\sinh^2(e+fx)}{\left(a+b\sinh^2(e+fx)\right)^{5/2}}\,dx = \int \frac{\sinh\left(fx+e\right)^2}{\left(b\sinh\left(fx+e\right)^2+a\right)^{\frac{5}{2}}}\,dx$$

input
```
integrate(sinh(f*x+e)^2/(a+b*sinh(f*x+e)^2)^(5/2),x, algorithm="giac")
```

output
```
integrate(sinh(f*x + e)^2/(b*sinh(f*x + e)^2 + a)^(5/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sinh^2(e+fx)}{\left(a+b\sinh^2(e+fx)\right)^{5/2}}\,dx=\int \frac{\sinh(e+f\,x)^2}{\left(b\sinh\left(e+f\,x\right)^2+a\right)^{5/2}}\,dx$$

input
```
int(sinh(e + f*x)^2/(a + b*sinh(e + f*x)^2)^(5/2),x)
```

output
```
int(sinh(e + f*x)^2/(a + b*sinh(e + f*x)^2)^(5/2), x)
```

Reduce [F]

$$\int \frac{\sinh^2(e+fx)}{\left(a+b\sinh^2(e+fx)\right)^{5/2}}\,dx=\int \frac{\sqrt{\sinh\left(fx+e\right)^2b+a}\,\sinh\left(fx+e\right)^2}{\sinh\left(fx+e\right)^6b^3+3\sinh\left(fx+e\right)^4a\,b^2+3\sinh\left(fx+e\right)^2a^2b+a^3}dx$$

input
```
int(sinh(f*x+e)^2/(a+b*sinh(f*x+e)^2)^(5/2),x)
```

output
```
int((sqrt(sinh(e + f*x)**2*b + a)*sinh(e + f*x)**2)/(sinh(e + f*x)**6*b**3
 + 3*sinh(e + f*x)**4*a*b**2 + 3*sinh(e + f*x)**2*a**2*b + a**3),x)
```

3.106 $\int \frac{1}{\left(a+b\sinh^2(e+fx)\right)^{5/2}}\,dx$

Optimal result

Integrand size = 16, antiderivative size = 253

$$\begin{aligned}\int \frac{1}{\left(a+b\sinh^2(e+fx)\right)^{5/2}}\,dx &= -\frac{b\cosh(e+fx)\sinh(e+fx)}{3a(a-b)f\left(a+b\sinh^2(e+fx)\right)^{3/2}} \\ &\quad -\frac{2(2a-b)b\cosh(e+fx)\sinh(e+fx)}{3a^2(a-b)^2f\sqrt{a+b\sinh^2(e+fx)}} \\ &\quad -\frac{2i(2a-b)E\left(ie+ifx\middle|\frac{b}{a}\right)\sqrt{a+b\sinh^2(e+fx)}}{3a^2(a-b)^2f\sqrt{\frac{a+b\sinh^2(e+fx)}{a}}} \\ &\quad +\frac{i\,\text{EllipticF}\left(ie+ifx,\frac{b}{a}\right)\sqrt{\frac{a+b\sinh^2(e+fx)}{a}}}{3a(a-b)f\sqrt{a+b\sinh^2(e+fx)}}\end{aligned}$$

output
```
-1/3*b*cosh(f*x+e)*sinh(f*x+e)/a/(a-b)/f/(a+b*sinh(f*x+e)^2)^(3/2)-2/3*(2*
a-b)*b*cosh(f*x+e)*sinh(f*x+e)/a^2/(a-b)^2/f/(a+b*sinh(f*x+e)^2)^(1/2)-2/3
*I*(2*a-b)*EllipticE(sin(I*e+I*f*x),(b/a)^(1/2))*(a+b*sinh(f*x+e)^2)^(1/2)
/a^2/(a-b)^2/f/((a+b*sinh(f*x+e)^2)/a)^(1/2)+1/3*I*InverseJacobiAM(I*e+I*f
*x,(b/a)^(1/2))*((a+b*sinh(f*x+e)^2)/a)^(1/2)/a/(a-b)/f/(a+b*sinh(f*x+e)^2
)^(1/2)
```

Mathematica [A] (verified)

Time = 0.94 (sec) , antiderivative size = 190, normalized size of antiderivative = 0.75

$$\int \frac{1}{\left(a+b\sinh^2(e+fx)\right)^{5/2}}\,dx = \frac{-2ia^2(2a-b)\left(\frac{2a-b+b\cosh(2(e+fx))}{a}\right)^{3/2}E\left(i(e+fx)\,|\,\frac{b}{a}\right)+ia^2(a-b)\left(\frac{2a-\cdots}{}\right.}{3\cdots}$$

input
```
Integrate[(a + b*Sinh[e + f*x]^2)^(-5/2),x]
```

output
```
((-2*I)*a^2*(2*a - b)*((2*a - b + b*Cosh[2*(e + f*x)])/a)^(3/2)*EllipticE[
I*(e + f*x), b/a] + I*a^2*(a - b)*((2*a - b + b*Cosh[2*(e + f*x)])/a)^(3/2
)*EllipticF[I*(e + f*x), b/a] + Sqrt[2]*b*(-5*a^2 + 5*a*b - b^2 + b*(-2*a
+ b)*Cosh[2*(e + f*x)])*Sinh[2*(e + f*x)])/(3*a^2*(a - b)^2*f*(2*a - b + b
*Cosh[2*(e + f*x)])^(3/2))
```

Rubi [A] (verified)

Time = 1.32 (sec) , antiderivative size = 258, normalized size of antiderivative = 1.02, number of steps used = 14, number of rules used = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.875$, Rules used = {3042, 3663, 25, 3042, 3652, 3042, 3651, 3042, 3657, 3042, 3656, 3662, 3042, 3661}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\left(a+b\sinh^2(e+fx)\right)^{5/2}}dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \frac{1}{(a-b\sin(ie+ifx)^2)^{5/2}}dx$$

$$\Big\downarrow \text{ 3663}$$

$$-\frac{\int -\frac{-b\sinh^2(e+fx)+3a-2b}{\left(b\sinh^2(e+fx)+a\right)^{3/2}}dx}{3a(a-b)}-\frac{b\sinh(e+fx)\cosh(e+fx)}{3af(a-b)\left(a+b\sinh^2(e+fx)\right)^{3/2}}$$

$$\Big\downarrow \ 25$$

$$\frac{\int \frac{-b\sinh^2(e+fx)+3a-2b}{(b\sinh^2(e+fx)+a)^{3/2}}dx}{3a(a-b)}-\frac{b\sinh(e+fx)\cosh(e+fx)}{3af(a-b)\left(a+b\sinh^2(e+fx)\right)^{3/2}}$$

$$\Big\downarrow \ 3042$$

$$-\frac{b\sinh(e+fx)\cosh(e+fx)}{3af(a-b)\left(a+b\sinh^2(e+fx)\right)^{3/2}}+\frac{\int \frac{b\sin(ie+ifx)^2+3a-2b}{(a-b\sin(ie+ifx)^2)^{3/2}}dx}{3a(a-b)}$$

$$\Big\downarrow \ 3652$$

$$\frac{\frac{\int \frac{2(2a-b)b\sinh^2(e+fx)+a(3a-b)}{\sqrt{b\sinh^2(e+fx)+a}}dx}{a(a-b)}-\frac{2b(2a-b)\sinh(e+fx)\cosh(e+fx)}{af(a-b)\sqrt{a+b\sinh^2(e+fx)}}}{3a(a-b)}-\frac{b\sinh(e+fx)\cosh(e+fx)}{3af(a-b)\left(a+b\sinh^2(e+fx)\right)^{3/2}}$$

$$\Big\downarrow \ 3042$$

$$-\frac{b\sinh(e+fx)\cosh(e+fx)}{3af(a-b)\left(a+b\sinh^2(e+fx)\right)^{3/2}}+\frac{-\frac{2b(2a-b)\sinh(e+fx)\cosh(e+fx)}{af(a-b)\sqrt{a+b\sinh^2(e+fx)}}+\frac{\int \frac{a(3a-b)-2(2a-b)b\sin(ie+ifx)^2}{\sqrt{a-b\sin(ie+ifx)^2}}dx}{a(a-b)}}{3a(a-b)}$$

$$\Big\downarrow \ 3651$$

$$\frac{\frac{2(2a-b)\int\sqrt{b\sinh^2(e+fx)+a}dx-a(a-b)\int\frac{1}{\sqrt{b\sinh^2(e+fx)+a}}dx}{a(a-b)}-\frac{2b(2a-b)\sinh(e+fx)\cosh(e+fx)}{af(a-b)\sqrt{a+b\sinh^2(e+fx)}}}{3a(a-b)}-\frac{b\sinh(e+fx)\cosh(e+fx)}{3af(a-b)\left(a+b\sinh^2(e+fx)\right)^{3/2}}$$

$$\Big\downarrow \ 3042$$

$$-\frac{b\sinh(e+fx)\cosh(e+fx)}{3af(a-b)\left(a+b\sinh^2(e+fx)\right)^{3/2}}+\frac{-\frac{2b(2a-b)\sinh(e+fx)\cosh(e+fx)}{af(a-b)\sqrt{a+b\sinh^2(e+fx)}}+\frac{2(2a-b)\int\sqrt{a-b\sin(ie+ifx)^2}dx-a(a-b)\int\frac{1}{\sqrt{a-b\sin(ie+ifx)^2}}dx}{a(a-b)}}{3a(a-b)}$$

$$\Big\downarrow \ 3657$$

$$-\frac{b\sinh(e+fx)\cosh(e+fx)}{3af(a-b)\left(a+b\sinh^2(e+fx)\right)^{3/2}}+\frac{-\frac{2b(2a-b)\sinh(e+fx)\cosh(e+fx)}{af(a-b)\sqrt{a+b\sinh^2(e+fx)}}+\frac{\frac{2(2a-b)\sqrt{a+b\sinh^2(e+fx)}\int\sqrt{\frac{b\sinh^2(e+fx)}{a}+1}dx}{\sqrt{\frac{b\sinh^2(e+fx)}{a}+1}}-a(a-b)\int\frac{1}{\sqrt{a-b\sin(ie+ifx)^2}}dx}{a(a-b)}}{3a(a-b)}$$

$\Big\downarrow$ 3042

$$-\frac{b\sinh(e+fx)\cosh(e+fx)}{3af(a-b)\left(a+b\sinh^2(e+fx)\right)^{3/2}}+\frac{-\frac{2b(2a-b)\sinh(e+fx)\cosh(e+fx)}{af(a-b)\sqrt{a+b\sinh^2(e+fx)}}+\frac{\frac{2(2a-b)\sqrt{a+b\sinh^2(e+fx)}\int\sqrt{1-\frac{b\sin(ie+ifx)^2}{a}}dx}{\sqrt{\frac{b\sinh^2(e+fx)}{a}+1}}-a(a-b)\int\frac{1}{\sqrt{a-b\sin(ie+ifx)^2}}dx}{a(a-b)}}{3a(a-b)}$$

$\Big\downarrow$ 3656

$$-\frac{b\sinh(e+fx)\cosh(e+fx)}{3af(a-b)\left(a+b\sinh^2(e+fx)\right)^{3/2}}+\frac{-\frac{2b(2a-b)\sinh(e+fx)\cosh(e+fx)}{af(a-b)\sqrt{a+b\sinh^2(e+fx)}}+\frac{-a(a-b)\int\frac{1}{\sqrt{a-b\sin(ie+ifx)^2}}dx-\frac{2i(2a-b)\sqrt{a+b\sinh^2(e+fx)}E\left(ie+ifx\middle|\frac{b}{a}\right)}{f\sqrt{\frac{b\sinh^2(e+fx)}{a}+1}}}{a(a-b)}}{3a(a-b)}$$

$\Big\downarrow$ 3662

$$-\frac{b\sinh(e+fx)\cosh(e+fx)}{3af(a-b)\left(a+b\sinh^2(e+fx)\right)^{3/2}}+\frac{-\frac{2b(2a-b)\sinh(e+fx)\cosh(e+fx)}{af(a-b)\sqrt{a+b\sinh^2(e+fx)}}+\frac{-\frac{a(a-b)\sqrt{\frac{b\sinh^2(e+fx)}{a}+1}\int\frac{1}{\sqrt{\frac{b\sinh^2(e+fx)}{a}+1}}dx}{\sqrt{a+b\sinh^2(e+fx)}}-\frac{2i(2a-b)\sqrt{a+b\sinh^2(e+fx)}E\left(ie+ifx\middle|\frac{b}{a}\right)}{f\sqrt{\frac{b\sinh^2(e+fx)}{a}+1}}}{a(a-b)}}{3a(a-b)}$$

$\Big\downarrow$ 3042

$$-\frac{b\sinh(e+fx)\cosh(e+fx)}{3af(a-b)\left(a+b\sinh^2(e+fx)\right)^{3/2}}+\frac{-\frac{2b(2a-b)\sinh(e+fx)\cosh(e+fx)}{af(a-b)\sqrt{a+b\sinh^2(e+fx)}}+\frac{-\frac{a(a-b)\sqrt{\frac{b\sinh^2(e+fx)}{a}+1}\int\frac{1}{\sqrt{1-\frac{b\sin(ie+ifx)^2}{a}}}dx}{\sqrt{a+b\sinh^2(e+fx)}}-\frac{2i(2a-b)\sqrt{a+b\sinh^2(e+fx)}E\left(ie+ifx\middle|\frac{b}{a}\right)}{f\sqrt{\frac{b\sinh^2(e+fx)}{a}+1}}}{a(a-b)}}{3a(a-b)}$$

$$\Big\downarrow \text{3661}$$

$$-\frac{b\sinh(e+fx)\cosh(e+fx)}{3af(a-b)\left(a+b\sinh^2(e+fx)\right)^{3/2}}+\frac{-\frac{2b(2a-b)\sinh(e+fx)\cosh(e+fx)}{af(a-b)\sqrt{a+b\sinh^2(e+fx)}}+\frac{\frac{ia(a-b)\sqrt{\frac{b\sinh^2(e+fx)}{a}+1}\,\text{EllipticF}\left(ie+ifx,\frac{b}{a}\right)}{f\sqrt{a+b\sinh^2(e+fx)}}-\frac{2i(2a-b)\sqrt{a+b\sinh^2(e+fx)}E\left(ie+ifx\middle|\frac{b}{a}\right)}{f\sqrt{\frac{b\sinh^2(e+fx)}{a}+1}}}{a(a-b)}}{3a(a-b)}$$

input
```
Int[(a + b*Sinh[e + f*x]^2)^(-5/2),x]
```

output
```
-1/3*(b*Cosh[e + f*x]*Sinh[e + f*x])/(a*(a - b)*f*(a + b*Sinh[e + f*x]^2)^
(3/2)) + ((-2*(2*a - b)*b*Cosh[e + f*x]*Sinh[e + f*x])/(a*(a - b)*f*Sqrt[a
 + b*Sinh[e + f*x]^2]) + (((-2*I)*(2*a - b)*EllipticE[I*e + I*f*x, b/a]*Sq
rt[a + b*Sinh[e + f*x]^2])/(f*Sqrt[1 + (b*Sinh[e + f*x]^2)/a]) + (I*a*(a -
 b)*EllipticF[I*e + I*f*x, b/a]*Sqrt[1 + (b*Sinh[e + f*x]^2)/a])/(f*Sqrt[a
 + b*Sinh[e + f*x]^2]))/(a*(a - b)))/(3*a*(a - b))
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3651
```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)]^2)/Sqrt[(a_) + (b_.)*sin[(e_.) +
 (f_.)*(x_)]^2], x_Symbol] :> Simp[B/b   Int[Sqrt[a + b*Sin[e + f*x]^2], x]
, x] + Simp[(A*b - a*B)/b   Int[1/Sqrt[a + b*Sin[e + f*x]^2], x], x] /; Fre
eQ[{a, b, e, f, A, B}, x]
```

rule 3652
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)]^2), x_Symbol] :> Simp[(-(A*b - a*B))*Cos[e + f*x]*Sin[e + f*x
]*((a + b*Sin[e + f*x]^2)^(p + 1)/(2*a*f*(a + b)*(p + 1))), x] - Simp[1/(2*
a*(a + b)*(p + 1))   Int[(a + b*Sin[e + f*x]^2)^(p + 1)*Simp[a*B - A*(2*a*(
p + 1) + b*(2*p + 3)) + 2*(A*b - a*B)*(p + 2)*Sin[e + f*x]^2, x], x], x] /;
 FreeQ[{a, b, e, f, A, B}, x] && LtQ[p, -1] && NeQ[a + b, 0]
```

rule 3656
```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2], x_Symbol] :> Simp[(Sqrt[a
]/f)*EllipticE[e + f*x, -b/a], x] /; FreeQ[{a, b, e, f}, x] && GtQ[a, 0]
```

rule 3657
```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2], x_Symbol] :> Simp[Sqrt[a
+ b*Sin[e + f*x]^2]/Sqrt[1 + b*(Sin[e + f*x]^2/a)]   Int[Sqrt[1 + (b*Sin[e
+ f*x]^2)/a], x], x] /; FreeQ[{a, b, e, f}, x] &&  !GtQ[a, 0]
```

rule 3661
```
Int[1/Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2], x_Symbol] :> Simp[(1/(S
qrt[a]*f))*EllipticF[e + f*x, -b/a], x] /; FreeQ[{a, b, e, f}, x] && GtQ[a,
 0]
```

rule 3662
```
Int[1/Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2], x_Symbol] :> Simp[Sqrt[
1 + b*(Sin[e + f*x]^2/a)]/Sqrt[a + b*Sin[e + f*x]^2]   Int[1/Sqrt[1 + (b*Si
n[e + f*x]^2)/a], x], x] /; FreeQ[{a, b, e, f}, x] &&  !GtQ[a, 0]
```

rule 3663
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> Simp[(-b)*C
os[e + f*x]*Sin[e + f*x]*((a + b*Sin[e + f*x]^2)^(p + 1)/(2*a*f*(p + 1)*(a
+ b))), x] + Simp[1/(2*a*(p + 1)*(a + b))   Int[(a + b*Sin[e + f*x]^2)^(p +
 1)*Simp[2*a*(p + 1) + b*(2*p + 3) - 2*b*(p + 2)*Sin[e + f*x]^2, x], x], x]
 /; FreeQ[{a, b, e, f}, x] && NeQ[a + b, 0] && LtQ[p, -1]
```

Maple [A] (verified)

Time = 1.50 (sec) , antiderivative size = 406, normalized size of antiderivative = 1.60

method	result
default	$\frac{\sqrt{\left(a+b\sinh(fx+e)^2\right)\cosh(fx+e)^2}\left(-\frac{\sinh(fx+e)\sqrt{\left(a+b\sinh(fx+e)^2\right)\cosh(fx+e)^2}}{3ba(a-b)\left(\sinh(fx+e)^2+\frac{a}{b}\right)^2}-\frac{2b\cosh(fx+e)^2\sinh(fx+e)(2a-b)}{3a^2(a-b)^2\sqrt{\left(a+b\sinh(fx+e)^2\right)\cosh(fx+e)^2}}+\frac{(3a\cdots}{\cdots}\right.}{\cdots}$
risch	Expression too large to display

input
```
int(1/(a+b*sinh(f*x+e)^2)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
((a+b*sinh(f*x+e)^2)*cosh(f*x+e)^2)^(1/2)*(-1/3/b/a/(a-b)*sinh(f*x+e)*((a+
b*sinh(f*x+e)^2)*cosh(f*x+e)^2)^(1/2)/(sinh(f*x+e)^2+1/b*a)^2-2/3*b*cosh(f
*x+e)^2/a^2/(a-b)^2*sinh(f*x+e)*(2*a-b)/((a+b*sinh(f*x+e)^2)*cosh(f*x+e)^2
)^(1/2)+(3*a-b)/(3*a^3-6*a^2*b+3*a*b^2)/(-b/a)^(1/2)*((a+b*sinh(f*x+e)^2)/
a)^(1/2)*(cosh(f*x+e)^2)^(1/2)/((a+b*sinh(f*x+e)^2)*cosh(f*x+e)^2)^(1/2)*E
llipticF(sinh(f*x+e)*(-b/a)^(1/2),(1/b*a)^(1/2))-2/3*b*(2*a-b)/a^2/(a-b)^2
/(-b/a)^(1/2)*((a+b*sinh(f*x+e)^2)/a)^(1/2)*(cosh(f*x+e)^2)^(1/2)/((a+b*si
nh(f*x+e)^2)*cosh(f*x+e)^2)^(1/2)*(EllipticF(sinh(f*x+e)*(-b/a)^(1/2),(1/b
*a)^(1/2))-EllipticE(sinh(f*x+e)*(-b/a)^(1/2),(1/b*a)^(1/2))))/cosh(f*x+e)
/(a+b*sinh(f*x+e)^2)^(1/2)/f
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 5442 vs. $2(225) = 450$.

Time = 0.22 (sec) , antiderivative size = 5442, normalized size of antiderivative = 21.51

$$\int \frac{1}{\left(a+b\sinh^2(e+fx)\right)^{5/2}}\,dx = \text{Too large to display}$$

input
```
integrate(1/(a+b*sinh(f*x+e)^2)^(5/2),x, algorithm="fricas")
```

output
```
Too large to include
```

Sympy [F]

$$\int \frac{1}{\left(a+b\sinh^2(e+fx)\right)^{5/2}}\,dx = \int \frac{1}{\left(a+b\sinh^2\left(e+fx\right)\right)^{\frac{5}{2}}}\,dx$$

input
```
integrate(1/(a+b*sinh(f*x+e)**2)**(5/2),x)
```

output
```
Integral((a + b*sinh(e + f*x)**2)**(-5/2), x)
```

Maxima [F]

$$\int \frac{1}{\left(a+b\sinh^2(e+fx)\right)^{5/2}}\,dx = \int \frac{1}{\left(b\sinh\left(fx+e\right)^2+a\right)^{\frac{5}{2}}}\,dx$$

input
```
integrate(1/(a+b*sinh(f*x+e)^2)^(5/2),x, algorithm="maxima")
```

output
```
integrate((b*sinh(f*x + e)^2 + a)^(-5/2), x)
```

Giac [F]

$$\int \frac{1}{\left(a+b\sinh^2(e+fx)\right)^{5/2}}\,dx = \int \frac{1}{\left(b\sinh\left(fx+e\right)^2+a\right)^{\frac{5}{2}}}\,dx$$

input
```
integrate(1/(a+b*sinh(f*x+e)^2)^(5/2),x, algorithm="giac")
```

output
```
integrate((b*sinh(f*x + e)^2 + a)^(-5/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\left(a+b\sinh^2(e+fx)\right)^{5/2}}\,dx = \int \frac{1}{\left(b\sinh\left(e+f\,x\right)^2+a\right)^{5/2}}\,dx$$

input
```
int(1/(a + b*sinh(e + f*x)^2)^(5/2),x)
```

output
```
int(1/(a + b*sinh(e + f*x)^2)^(5/2), x)
```

Reduce [F]

$$\int \frac{1}{\left(a+b\sinh^2(e+fx)\right)^{5/2}}\,dx = \int \frac{\sqrt{\sinh\left(fx+e\right)^2 b+a}}{\sinh\left(fx+e\right)^6 b^3+3\sinh\left(fx+e\right)^4 a\,b^2+3\sinh\left(fx+e\right)^2 a^2 b+a^3}dx$$

input
```
int(1/(a+b*sinh(f*x+e)^2)^(5/2),x)
```

output
```
int(sqrt(sinh(e + f*x)**2*b + a)/(sinh(e + f*x)**6*b**3 + 3*sinh(e + f*x)*
*4*a*b**2 + 3*sinh(e + f*x)**2*a**2*b + a**3),x)
```

3.107 $\int \frac{\operatorname{csch}^2(e+fx)}{(a+b\sinh^2(e+fx))^{5/2}}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 311

$$\begin{aligned}\int \frac{\operatorname{csch}^2(e+fx)}{(a+b\sinh^2(e+fx))^{5/2}}\,dx &= -\frac{b\coth(e+fx)}{3a(a-b)f\,(a+b\sinh^2(e+fx))^{3/2}} \\ &\quad - \frac{(3a-4b)\coth(e+fx)}{3a^2(a-b)f\sqrt{a+b\sinh^2(e+fx)}} \\ &\quad - \frac{\sqrt{b}(3a^2-13ab+8b^2)\cosh(e+fx)E\left(\arctan\left(\frac{\sqrt{b}\sinh(e+fx)}{\sqrt{a}}\right)|1-\frac{a}{b}\right)}{3a^{5/2}(a-b)^2f\sqrt{\frac{a\cosh^2(e+fx)}{a+b\sinh^2(e+fx)}}\sqrt{a+b\sinh^2(e+fx)}} \\ &\quad - \frac{2(3a-2b)\sqrt{b}\cosh(e+fx)\operatorname{EllipticF}\left(\arctan\left(\frac{\sqrt{b}\sinh(e+fx)}{\sqrt{a}}\right),1-\frac{a}{b}\right)}{3a^{3/2}(a-b)^2f\sqrt{\frac{a\cosh^2(e+fx)}{a+b\sinh^2(e+fx)}}\sqrt{a+b\sinh^2(e+fx)}}\end{aligned}$$

output
```
-1/3*b*coth(f*x+e)/a/(a-b)/f/(a+b*sinh(f*x+e)^2)^(3/2)-1/3*(3*a-4*b)*coth(
f*x+e)/a^2/(a-b)/f/(a+b*sinh(f*x+e)^2)^(1/2)-1/3*b^(1/2)*(3*a^2-13*a*b+8*b
^2)*cosh(f*x+e)*EllipticE(b^(1/2)*sinh(f*x+e)/a^(1/2)/(1+b*sinh(f*x+e)^2/a
)^(1/2),(1-a/b)^(1/2))/a^(5/2)/(a-b)^2/f/(a*cosh(f*x+e)^2/(a+b*sinh(f*x+e)
^2))^(1/2)/(a+b*sinh(f*x+e)^2)^(1/2)-2/3*(3*a-2*b)*b^(1/2)*cosh(f*x+e)*Inv
erseJacobiAM(arctan(b^(1/2)*sinh(f*x+e)/a^(1/2)),(1-a/b)^(1/2))/a^(3/2)/(a
-b)^2/f/(a*cosh(f*x+e)^2/(a+b*sinh(f*x+e)^2))^(1/2)/(a+b*sinh(f*x+e)^2)^(1
/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 1.90 (sec) , antiderivative size = 234, normalized size of antiderivative = 0.75

$$\int \frac{\operatorname{csch}^2(e+fx)}{\left(a+b\sinh^2(e+fx)\right)^{5/2}}\,dx = \frac{i\left(4a^2\left(\frac{2a-b+b\cosh(2(e+fx))}{a}\right)^{3/2}\left((-3a^2+13ab-8b^2)\,E\left(i(e+fx)\,\middle|\,\frac{b}{a}\right)+(3\right.\right.}{}$$

input
```
Integrate[Csch[e + f*x]^2/(a + b*Sinh[e + f*x]^2)^(5/2),x]
```

output
```
((I/12)*(4*a^2*((2*a - b + b*Cosh[2*(e + f*x)])/a)^(3/2)*((-3*a^2 + 13*a*b
 - 8*b^2)*EllipticE[I*(e + f*x), b/a] + (3*a^2 - 7*a*b + 4*b^2)*EllipticF[
I*(e + f*x), b/a]) + (2*I)*Sqrt[2]*(3*(a - b)^2*(2*a - b + b*Cosh[2*(e + f
*x)])^2*Coth[e + f*x] - 2*a*(a - b)*b^2*Sinh[2*(e + f*x)] - (7*a - 5*b)*b^
2*(2*a - b + b*Cosh[2*(e + f*x)])*Sinh[2*(e + f*x)])))/(a^3*(a - b)^2*f*(2
*a - b + b*Cosh[2*(e + f*x)])^(3/2))
```

Rubi [A] (verified)

Time = 0.64 (sec) , antiderivative size = 448, normalized size of antiderivative = 1.44, number of steps used = 12, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.440$, Rules used = {3042, 25, 3667, 374, 441, 445, 27, 406, 320, 388, 313}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\text{csch}^2(e+fx)}{\left(a+b\sinh^2(e+fx)\right)^{5/2}}dx$$

$\Big\downarrow$ 3042

$$\int -\frac{1}{\sin(ie+ifx)^2\left(a-b\sin(ie+ifx)^2\right)^{5/2}}dx$$

$\Big\downarrow$ 25

$$-\int \frac{1}{\sin(ie+ifx)^2\left(a-b\sin(ie+ifx)^2\right)^{5/2}}dx$$

$\Big\downarrow$ 3667

$$\frac{\sqrt{\cosh^2(e+fx)}\text{sech}(e+fx)\int \frac{\text{csch}^2(e+fx)}{\sqrt{\sinh^2(e+fx)+1}\left(b\sinh^2(e+fx)+a\right)^{5/2}}d\sinh(e+fx)}{f}$$

$\Big\downarrow$ 374

$$\frac{\sqrt{\cosh^2(e+fx)}\text{sech}(e+fx)\left(\frac{\int \frac{\text{csch}^2(e+fx)\left(-3b\sinh^2(e+fx)+3a-4b\right)}{\sqrt{\sinh^2(e+fx)+1}\left(b\sinh^2(e+fx)+a\right)^{3/2}}d\sinh(e+fx)}{3a(a-b)}-\frac{b\sqrt{\sinh^2(e+fx)+1}\text{csch}(e+fx)}{3a(a-b)\left(a+b\sinh^2(e+fx)\right)^{3/2}}\right)}{f}$$

$\Big\downarrow$ 441

$$\frac{\sqrt{\cosh^2(e+fx)}\text{sech}(e+fx)\left(\frac{\frac{\int \frac{\text{csch}^2(e+fx)\left(3a^2-13ba+8b^2-2(3a-2b)b\sinh^2(e+fx)\right)}{\sqrt{\sinh^2(e+fx)+1}\sqrt{b\sinh^2(e+fx)+a}}d\sinh(e+fx)}{a(a-b)}-\frac{2b(3a-2b)\sqrt{\sinh^2(e+fx)+1}\text{csch}(e+f}{a(a-b)\sqrt{a+b\sinh^2(e+fx)}}}{3a(a-b)}\right.}{f}$$

$\Big\downarrow$ 445

$$\frac{\sqrt{\cosh^2(e+fx)}\text{sech}(e+fx)\left(\frac{\frac{-\frac{\int \frac{b\left(2a(3a-2b)-\left(3a^2-13ba+8b^2\right)\sinh^2(e+fx)\right)}{\sqrt{\sinh^2(e+fx)+1}\sqrt{b\sinh^2(e+fx)+a}}d\sinh(e+fx)}{a}-\frac{\left(3a^2-13ab+8b^2\right)\sqrt{\sinh^2(e+fx)+1}\text{csch}(e+fx}{a}}{a(a-b)}}{3a(a-b)}\right.}{f}$$

$$\Big\downarrow 27$$

$$\frac{\sqrt{\cosh^2(e+fx)}\operatorname{sech}(e+fx)\left(\frac{-\frac{b\int \frac{2a(3a-2b)-\left(3a^2-13ba+8b^2\right)\sinh^2(e+fx)}{\sqrt{\sinh^2(e+fx)+1}\sqrt{b\sinh^2(e+fx)+a}}d\sinh(e+fx)}{a}-\frac{\left(3a^2-13ab+8b^2\right)\sqrt{\sinh^2(e+fx)+1}\operatorname{csch}(e+fx)\sqrt{}}{a}}{a(a-b)}}{3a(a-b)}\right.}{f}$$

$$\Big\downarrow 406$$

$$\sqrt{\cosh^2(e+fx)}\operatorname{sech}(e+fx)\left(\frac{-\frac{b\left(2a(3a-2b)\int \frac{1}{\sqrt{\sinh^2(e+fx)+1}\sqrt{b\sinh^2(e+fx)+a}}d\sinh(e+fx)-\left(3a^2-13ab+8b^2\right)\int \frac{\sinh^2(e+fx)}{\sqrt{\sinh^2(e+fx)+1}\sqrt{b\sinh}}\right.}{a}}{a(a-b)}\right.$$

$$\Big\downarrow 320$$

$$\sqrt{\cosh^2(e+fx)}\operatorname{sech}(e+fx)\left(\frac{-\frac{b\left(\frac{2(3a-2b)\sqrt{a+b\sinh^2(e+fx)}\operatorname{EllipticF}\left(\arctan(\sinh(e+fx)),1-\frac{b}{a}\right)}{\sqrt{\sinh^2(e+fx)+1}\sqrt{\frac{a+b\sinh^2(e+fx)}{a\left(\sinh^2(e+fx)+1\right)}}}-\left(3a^2-13ab+8b^2\right)\int \frac{\sinh^2(e+}{\sqrt{\sinh^2(e+fx)+1}\sqrt{b\,s}}\right.}{a}}{a(a-b)}\right.$$

$$\Big\downarrow 388$$

$$\sqrt{\cosh^2(e+fx)}\operatorname{sech}(e+fx)\left(\frac{-\frac{b\left(\frac{2(3a-2b)\sqrt{a+b\sinh^2(e+fx)}\operatorname{EllipticF}\left(\arctan(\sinh(e+fx)),1-\frac{b}{a}\right)}{\sqrt{\sinh^2(e+fx)+1}\sqrt{\frac{a+b\sinh^2(e+fx)}{a\left(\sinh^2(e+fx)+1\right)}}}-\left(3a^2-13ab+8b^2\right)\left(\frac{\sinh(e+fx)\sqrt{a+b\sinh\cdots}}{b\sqrt{\sinh^2(e+fx)\cdots}}\right.\right.}{a}\cdots}{\cdots}\right.$$

$\downarrow$ 313

$$\sqrt{\cosh^2(e+fx)}\operatorname{sech}(e+fx)\left(\frac{-\frac{b\left(\frac{2(3a-2b)\sqrt{a+b\sinh^2(e+fx)}\operatorname{EllipticF}\left(\arctan(\sinh(e+fx)),1-\frac{b}{a}\right)}{\sqrt{\sinh^2(e+fx)+1}\sqrt{\frac{a+b\sinh^2(e+fx)}{a\left(\sinh^2(e+fx)+1\right)}}}-\left(3a^2-13ab+8b^2\right)\left(\frac{\sinh(e+fx)\sqrt{a+b\sinh\cdots}}{b\sqrt{\sinh^2(e+fx)\cdots}}\right.\right.}{a}\cdots}{\cdots}\right.$$

input
```
Int[Csch[e + f*x]^2/(a + b*Sinh[e + f*x]^2)^(5/2),x]
```

output
```
(Sqrt[Cosh[e + f*x]^2]*Sech[e + f*x]*(-1/3*(b*Csch[e + f*x]*Sqrt[1 + Sinh[
e + f*x]^2])/(a*(a - b)*(a + b*Sinh[e + f*x]^2)^(3/2)) + ((-2*(3*a - 2*b)*
b*Csch[e + f*x]*Sqrt[1 + Sinh[e + f*x]^2])/(a*(a - b)*Sqrt[a + b*Sinh[e +
f*x]^2]) + (-(((3*a^2 - 13*a*b + 8*b^2)*Csch[e + f*x]*Sqrt[1 + Sinh[e + f*
x]^2]*Sqrt[a + b*Sinh[e + f*x]^2])/a) - (b*((2*(3*a - 2*b)*EllipticF[ArcTa
n[Sinh[e + f*x]], 1 - b/a]*Sqrt[a + b*Sinh[e + f*x]^2])/(Sqrt[1 + Sinh[e +
 f*x]^2]*Sqrt[(a + b*Sinh[e + f*x]^2)/(a*(1 + Sinh[e + f*x]^2))]) - (3*a^2
 - 13*a*b + 8*b^2)*((Sinh[e + f*x]*Sqrt[a + b*Sinh[e + f*x]^2])/(b*Sqrt[1
+ Sinh[e + f*x]^2]) - (EllipticE[ArcTan[Sinh[e + f*x]], 1 - b/a]*Sqrt[a +
b*Sinh[e + f*x]^2])/(b*Sqrt[1 + Sinh[e + f*x]^2]*Sqrt[(a + b*Sinh[e + f*x]
^2)/(a*(1 + Sinh[e + f*x]^2))]))))/a)/(a*(a - b)))/(3*a*(a - b))))/f
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 313
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/((c_) + (d_.)*(x_)^2)^(3/2), x_Symbol] :> Sim
p[(Sqrt[a + b*x^2]/(c*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(c
+ d*x^2)))]))*EllipticE[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; FreeQ
[{a, b, c, d}, x] && PosQ[b/a] && PosQ[d/c]
```

rule 320
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(Sqrt[a + b*x^2]/(a*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(
c + d*x^2)))]))*EllipticF[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; Fre
eQ[{a, b, c, d}, x] && PosQ[d/c] && PosQ[b/a] &&  !SimplerSqrtQ[b/a, d/c]
```

rule 374
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_
), x_Symbol] :> Simp[(-b)*(e*x)^(m + 1)*(a + b*x^2)^(p + 1)*((c + d*x^2)^(q
 + 1)/(a*e*2*(b*c - a*d)*(p + 1))), x] + Simp[1/(a*2*(b*c - a*d)*(p + 1))
 Int[(e*x)^m*(a + b*x^2)^(p + 1)*(c + d*x^2)^q*Simp[b*c*(m + 1) + 2*(b*c -
a*d)*(p + 1) + d*b*(m + 2*(p + q + 2) + 1)*x^2, x], x], x] /; FreeQ[{a, b,
c, d, e, m, q}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && IntBinomialQ[a, b,
 c, d, e, m, 2, p, q, x]
```

rule 388
```
Int[(x_)^2/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol]
 :> Simp[x*(Sqrt[a + b*x^2]/(b*Sqrt[c + d*x^2])), x] - Simp[c/b   Int[Sqrt[
a + b*x^2]/(c + d*x^2)^(3/2), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
 a*d, 0] && PosQ[b/a] && PosQ[d/c] &&  !SimplerSqrtQ[b/a, d/c]
```

rule 406
```
Int[((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.)*((e_) + (f_.)*(
x_)^2), x_Symbol] :> Simp[e   Int[(a + b*x^2)^p*(c + d*x^2)^q, x], x] + Sim
p[f   Int[x^2*(a + b*x^2)^p*(c + d*x^2)^q, x], x] /; FreeQ[{a, b, c, d, e,
f, p, q}, x]
```

rule 441
```
Int[((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_
)*((e_) + (f_.)*(x_)^2), x_Symbol] :> Simp[(-(b*e - a*f))*(g*x)^(m + 1)*(a
+ b*x^2)^(p + 1)*((c + d*x^2)^(q + 1)/(a*g*2*(b*c - a*d)*(p + 1))), x] + Si
mp[1/(a*2*(b*c - a*d)*(p + 1))   Int[(g*x)^m*(a + b*x^2)^(p + 1)*(c + d*x^2
)^q*Simp[c*(b*e - a*f)*(m + 1) + e*2*(b*c - a*d)*(p + 1) + d*(b*e - a*f)*(m
 + 2*(p + q + 2) + 1)*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, m, q},
 x] && LtQ[p, -1]
```

rule 445
```
Int[((g_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_
.)*((e_) + (f_.)*(x_)^2), x_Symbol] :> Simp[e*(g*x)^(m + 1)*(a + b*x^2)^(p
+ 1)*((c + d*x^2)^(q + 1)/(a*c*g*(m + 1))), x] + Simp[1/(a*c*g^2*(m + 1))
 Int[(g*x)^(m + 2)*(a + b*x^2)^p*(c + d*x^2)^q*Simp[a*f*c*(m + 1) - e*(b*c
+ a*d)*(m + 2 + 1) - e*2*(b*c*p + a*d*q) - b*e*d*(m + 2*(p + q + 2) + 1)*x^
2, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p, q}, x] && LtQ[m, -1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3667
```
Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(
p_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Simp[ff^(m + 1
)*(Sqrt[Cos[e + f*x]^2]/(f*Cos[e + f*x]))   Subst[Int[x^m*((a + b*ff^2*x^2)
^p/Sqrt[1 - ff^2*x^2]), x], x, Sin[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f,
p}, x] && IntegerQ[m/2] &&  !IntegerQ[p]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 746 vs. $2(295) = 590$.

Time = 6.30 (sec) , antiderivative size = 747, normalized size of antiderivative = 2.40

method	result
default	$-\frac{\left(3\sqrt{-\frac{b}{a}}\,a^{2}b^{2}-13\sqrt{-\frac{b}{a}}\,a\,b^{3}+8\sqrt{-\frac{b}{a}}\,b^{4}\right)\cosh(fx+e)^{6}+\left(6\sqrt{-\frac{b}{a}}\,a^{3}b-26\sqrt{-\frac{b}{a}}\,a^{2}b^{2}+38\sqrt{-\frac{b}{a}}\,a\,b^{3}-16\sqrt{-\frac{b}{a}}\,b^{4}\right)\cosh(fx+e)^{4}\cdots}{\cdots}$
risch	Expression too large to display

input
```
int(csch(f*x+e)^2/(a+b*sinh(f*x+e)^2)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
-1/3*((3*(-b/a)^(1/2)*a^2*b^2-13*(-b/a)^(1/2)*a*b^3+8*(-b/a)^(1/2)*b^4)*co
sh(f*x+e)^6+(6*(-b/a)^(1/2)*a^3*b-26*(-b/a)^(1/2)*a^2*b^2+38*(-b/a)^(1/2)*
a*b^3-16*(-b/a)^(1/2)*b^4)*cosh(f*x+e)^4+(cosh(f*x+e)^2)^(1/2)*(b/a*cosh(f
*x+e)^2+(a-b)/a)^(1/2)*b^2*(9*EllipticF(sinh(f*x+e)*(-b/a)^(1/2),(1/b*a)^(
1/2))*a^2-17*EllipticF(sinh(f*x+e)*(-b/a)^(1/2),(1/b*a)^(1/2))*a*b+8*Ellip
ticF(sinh(f*x+e)*(-b/a)^(1/2),(1/b*a)^(1/2))*b^2-3*EllipticE(sinh(f*x+e)*(
-b/a)^(1/2),(1/b*a)^(1/2))*a^2+13*EllipticE(sinh(f*x+e)*(-b/a)^(1/2),(1/b*
a)^(1/2))*a*b-8*EllipticE(sinh(f*x+e)*(-b/a)^(1/2),(1/b*a)^(1/2))*b^2)*cos
h(f*x+e)^2*sinh(f*x+e)+(3*(-b/a)^(1/2)*a^4-12*(-b/a)^(1/2)*a^3*b+26*(-b/a)
^(1/2)*a^2*b^2-25*(-b/a)^(1/2)*a*b^3+8*(-b/a)^(1/2)*b^4)*cosh(f*x+e)^2+(co
sh(f*x+e)^2)^(1/2)*(b/a*cosh(f*x+e)^2+(a-b)/a)^(1/2)*b*(9*EllipticF(sinh(f
*x+e)*(-b/a)^(1/2),(1/b*a)^(1/2))*a^3-26*EllipticF(sinh(f*x+e)*(-b/a)^(1/2
),(1/b*a)^(1/2))*a^2*b+25*EllipticF(sinh(f*x+e)*(-b/a)^(1/2),(1/b*a)^(1/2)
)*a*b^2-8*EllipticF(sinh(f*x+e)*(-b/a)^(1/2),(1/b*a)^(1/2))*b^3-3*Elliptic
E(sinh(f*x+e)*(-b/a)^(1/2),(1/b*a)^(1/2))*a^3+16*EllipticE(sinh(f*x+e)*(-b
/a)^(1/2),(1/b*a)^(1/2))*a^2*b-21*EllipticE(sinh(f*x+e)*(-b/a)^(1/2),(1/b*
a)^(1/2))*a*b^2+8*EllipticE(sinh(f*x+e)*(-b/a)^(1/2),(1/b*a)^(1/2))*b^3)*s
inh(f*x+e))/(a+b*sinh(f*x+e)^2)^(3/2)/(a-b)^2/(-b/a)^(1/2)/sinh(f*x+e)/a^3
/cosh(f*x+e)/f
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 8769 vs. $2(295) = 590$.

Time = 0.38 (sec) , antiderivative size = 8769, normalized size of antiderivative = 28.20

$$\int \frac{\operatorname{csch}^2(e+fx)}{\left(a+b\sinh^2(e+fx)\right)^{5/2}}\,dx = \text{Too large to display}$$

input
```
integrate(csch(f*x+e)^2/(a+b*sinh(f*x+e)^2)^(5/2),x, algorithm="fricas")
```

output
```
Too large to include
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\operatorname{csch}^2(e+fx)}{\left(a+b\sinh^2(e+fx)\right)^{5/2}}\,dx = \text{Timed out}$$

input
```
integrate(csch(f*x+e)**2/(a+b*sinh(f*x+e)**2)**(5/2),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{\operatorname{csch}^2(e+fx)}{\left(a+b\sinh^2(e+fx)\right)^{5/2}}\,dx = \int \frac{\operatorname{csch}(fx+e)^2}{\left(b\sinh(fx+e)^2+a\right)^{\frac{5}{2}}}\,dx$$

input
```
integrate(csch(f*x+e)^2/(a+b*sinh(f*x+e)^2)^(5/2),x, algorithm="maxima")
```

output
```
integrate(csch(f*x + e)^2/(b*sinh(f*x + e)^2 + a)^(5/2), x)
```

Giac [F]

$$\int \frac{\operatorname{csch}^2(e+fx)}{\left(a+b\sinh^2(e+fx)\right)^{5/2}}\,dx = \int \frac{\operatorname{csch}(fx+e)^2}{\left(b\sinh(fx+e)^2+a\right)^{\frac{5}{2}}}\,dx$$

input
```
integrate(csch(f*x+e)^2/(a+b*sinh(f*x+e)^2)^(5/2),x, algorithm="giac")
```

output
```
integrate(csch(f*x + e)^2/(b*sinh(f*x + e)^2 + a)^(5/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\operatorname{csch}^2(e+fx)}{\left(a+b\sinh^2(e+fx)\right)^{5/2}}\,dx = \int \frac{1}{\sinh\left(e+f\,x\right)^2\,\left(b\sinh\left(e+f\,x\right)^2+a\right)^{5/2}}\,dx$$

input
```
int(1/(sinh(e + f*x)^2*(a + b*sinh(e + f*x)^2)^(5/2)),x)
```

output
```
int(1/(sinh(e + f*x)^2*(a + b*sinh(e + f*x)^2)^(5/2)), x)
```

Reduce [F]

$$\int \frac{\operatorname{csch}^2(e+fx)}{\left(a+b\sinh^2(e+fx)\right)^{5/2}}\,dx = \int \frac{\sqrt{\sinh\left(fx+e\right)^2 b+a}\,\operatorname{csch}(fx+e)^2}{\sinh\left(fx+e\right)^6 b^3+3\sinh\left(fx+e\right)^4 a\,b^2+3\sinh\left(fx+e\right)^2 a^2b+a^3}dx$$

input
```
int(csch(f*x+e)^2/(a+b*sinh(f*x+e)^2)^(5/2),x)
```

output
```
int((sqrt(sinh(e + f*x)**2*b + a)*csch(e + f*x)**2)/(sinh(e + f*x)**6*b**3
 + 3*sinh(e + f*x)**4*a*b**2 + 3*sinh(e + f*x)**2*a**2*b + a**3),x)
```

3.108 $\int (d\sinh(e+fx))^m \left(a+b\sinh^2(e+fx)\right)^p \,dx$

Optimal result

Integrand size = 25, antiderivative size = 128

$$\int (d\sinh(e+fx))^m \left(a+b\sinh^2(e+fx)\right)^p \,dx = \frac{d\,\mathrm{AppellF1}\left(\frac{1}{2},\frac{1-m}{2},-p,\frac{3}{2},\cosh^2(e+fx),-\frac{b\cosh^2(e+fx)}{a-b}\right)\cosh(e+fx)\left(a-b+b\cosh^2(e+fx)\right)^p\left(1+\right.}{f}$$

output
```
d*AppellF1(1/2,1/2-1/2*m,-p,3/2,cosh(f*x+e)^2,-b*cosh(f*x+e)^2/(a-b))*cosh
(f*x+e)*(a-b+b*cosh(f*x+e)^2)^p*(d*sinh(f*x+e))^(-1+m)*(-sinh(f*x+e)^2)^(1
/2-1/2*m)/f/((1+b*cosh(f*x+e)^2/(a-b))^p)
```

Mathematica [F]

$$\int (d\sinh(e+fx))^m \left(a+b\sinh^2(e+fx)\right)^p \,dx = \int (d\sinh(e+fx))^m \left(a+b\sinh^2(e+fx)\right)^p \,dx$$

input
```
Integrate[(d*Sinh[e + f*x])^m*(a + b*Sinh[e + f*x]^2)^p,x]
```

output
```
Integrate[(d*Sinh[e + f*x])^m*(a + b*Sinh[e + f*x]^2)^p, x]
```

Rubi [A] (verified)

Time = 0.35 (sec) , antiderivative size = 128, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.160$, Rules used = {3042, 3668, 334, 333}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (d\sinh(e+fx))^m \left(a+b\sinh^2(e+fx)\right)^p dx$$

$$\Big\downarrow \text{3042}$$

$$\int (-id\sin(ie+ifx))^m \left(a-b\sin(ie+ifx)^2\right)^p dx$$

$$\Big\downarrow \text{3668}$$

$$\frac{d\left(-\sinh^2(e+fx)\right)^{\frac{1-m}{2}} (d\sinh(e+fx))^{m-1} \int \left(1-\cosh^2(e+fx)\right)^{\frac{m-1}{2}} \left(b\cosh^2(e+fx)+a-b\right)^p d\cosh(e+fx)}{f}$$

$$\Big\downarrow \text{334}$$

$$\frac{d\left(-\sinh^2(e+fx)\right)^{\frac{1-m}{2}} (d\sinh(e+fx))^{m-1} \left(a+b\cosh^2(e+fx)-b\right)^p \left(\frac{b\cosh^2(e+fx)}{a-b}+1\right)^{-p} \int \left(1-\cosh^2(e+f\right.}{f}$$

$$\Big\downarrow \text{333}$$

$$\frac{d\cosh(e+fx)\left(-\sinh^2(e+fx)\right)^{\frac{1-m}{2}} (d\sinh(e+fx))^{m-1} \left(a+b\cosh^2(e+fx)-b\right)^p \left(\frac{b\cosh^2(e+fx)}{a-b}+1\right)^{-p} \text{Appe}}{f}$$

input
```
Int[(d*Sinh[e + f*x])^m*(a + b*Sinh[e + f*x]^2)^p,x]
```

output
```
(d*AppellF1[1/2, (1 - m)/2, -p, 3/2, Cosh[e + f*x]^2, -((b*Cosh[e + f*x]^2
)/(a - b))]*Cosh[e + f*x]*(a - b + b*Cosh[e + f*x]^2)^p*(d*Sinh[e + f*x])^
(-1 + m)*(-Sinh[e + f*x]^2)^((1 - m)/2))/(f*(1 + (b*Cosh[e + f*x]^2)/(a -
b))^p)
```

Defintions of rubi rules used

rule 333
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_), x_Symbol] :> Sim
p[a^p*c^q*x*AppellF1[1/2, -p, -q, 3/2, (-b)*(x^2/a), (-d)*(x^2/c)], x] /; F
reeQ[{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && (IntegerQ[p] || GtQ[a,
0]) && (IntegerQ[q] || GtQ[c, 0])
```

rule 334
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_), x_Symbol] :> Sim
p[a^IntPart[p]*((a + b*x^2)^FracPart[p]/(1 + b*(x^2/a))^FracPart[p])   Int[
(1 + b*(x^2/a))^p*(c + d*x^2)^q, x], x] /; FreeQ[{a, b, c, d, p, q}, x] &&
NeQ[b*c - a*d, 0] &&  !(IntegerQ[p] || GtQ[a, 0])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3668
```
Int[((d_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)]^2)^(p_.), x_Symbol] :> With[{ff = FreeFactors[Cos[e + f*x], x]}, Simp[(
-ff)*d^(2*IntPart[(m - 1)/2] + 1)*((d*Sin[e + f*x])^(2*FracPart[(m - 1)/2])
/(f*(Sin[e + f*x]^2)^FracPart[(m - 1)/2]))   Subst[Int[(1 - ff^2*x^2)^((m -
 1)/2)*(a + b - b*ff^2*x^2)^p, x], x, Cos[e + f*x]/ff], x]] /; FreeQ[{a, b,
 d, e, f, m, p}, x] &&  !IntegerQ[m]
```

Maple [F]

$$\int (d\sinh(fx+e))^m \left(a+b\sinh(fx+e)^2\right)^p dx$$

input
```
int((d*sinh(f*x+e))^m*(a+b*sinh(f*x+e)^2)^p,x)
```

output
```
int((d*sinh(f*x+e))^m*(a+b*sinh(f*x+e)^2)^p,x)
```

Fricas [F]

$$\int (d\sinh(e+fx))^m \left(a+b\sinh^2(e+fx)\right)^p dx$$
$$= \int \left(b\sinh(fx+e)^2+a\right)^p (d\sinh(fx+e))^m dx$$

input
```
integrate((d*sinh(f*x+e))^m*(a+b*sinh(f*x+e)^2)^p,x, algorithm="fricas")
```

output
```
integral((b*sinh(f*x + e)^2 + a)^p*(d*sinh(f*x + e))^m, x)
```

Sympy [F(-1)]

Timed out.

$$\int (d\sinh(e+fx))^m \left(a+b\sinh^2(e+fx)\right)^p dx = \text{Timed out}$$

input
```
integrate((d*sinh(f*x+e))**m*(a+b*sinh(f*x+e)**2)**p,x)
```

output
```
Timed out
```

Maxima [F]

$$\int (d \sinh(e+fx))^m \left(a+b \sinh^2(e+fx)\right)^p dx$$
$$= \int \left(b \sinh{(fx+e)}^2 + a\right)^p (d \sinh{(fx+e)})^m dx$$

input
```
integrate((d*sinh(f*x+e))^m*(a+b*sinh(f*x+e)^2)^p,x, algorithm="maxima")
```

output
```
integrate((b*sinh(f*x + e)^2 + a)^p*(d*sinh(f*x + e))^m, x)
```

Giac [F]

$$\int (d \sinh(e+fx))^m \left(a+b \sinh^2(e+fx)\right)^p dx$$
$$= \int \left(b \sinh{(fx+e)}^2 + a\right)^p (d \sinh{(fx+e)})^m dx$$

input
```
integrate((d*sinh(f*x+e))^m*(a+b*sinh(f*x+e)^2)^p,x, algorithm="giac")
```

output
```
integrate((b*sinh(f*x + e)^2 + a)^p*(d*sinh(f*x + e))^m, x)
```

Mupad [F(-1)]

Timed out.

$$\int (d \sinh(e+fx))^m \left(a+b \sinh^2(e+fx)\right)^p dx$$
$$= \int (d \sinh(e+f x))^m \left(b \sinh(e+f x)^2 + a\right)^p dx$$

input
```
int((d*sinh(e + f*x))^m*(a + b*sinh(e + f*x)^2)^p,x)
```

output
```
int((d*sinh(e + f*x))^m*(a + b*sinh(e + f*x)^2)^p, x)
```

Reduce [F]

$$\int (d\sinh(e+fx))^m \left(a+b\sinh^2(e+fx)\right)^p \, dx$$
$$= d^m \left(\int \sinh\left(fx+e\right)^m \left(\sinh\left(fx+e\right)^2 b+a\right)^p dx \right)$$

input
```
int((d*sinh(f*x+e))^m*(a+b*sinh(f*x+e)^2)^p,x)
```

output
```
d**m*int(sinh(e + f*x)**m*(sinh(e + f*x)**2*b + a)**p,x)
```

3.109 $\int \sinh^5(e+fx)\left(a+b\sinh^2(e+fx)\right)^p\,dx$

Optimal result

Integrand size = 23, antiderivative size = 221

$$\begin{aligned}
&\int \sinh^5(e+fx)\left(a+b\sinh^2(e+fx)\right)^p\,dx \\
&= -\frac{(3a+b(7+4p))\cosh(e+fx)\left(a-b+b\cosh^2(e+fx)\right)^{1+p}}{b^2f(3+2p)(5+2p)} \\
&\quad+\frac{\cosh^3(e+fx)\left(a-b+b\cosh^2(e+fx)\right)^{1+p}}{bf(5+2p)} \\
&\quad+\frac{(3a^2+4ab(1+p)+4b^2(2+3p+p^2))\cosh(e+fx)\left(a-b+b\cosh^2(e+fx)\right)^p\left(1+\frac{b\cosh^2(e+fx)}{a-b}\right)^{-p}\text{H}}{b^2f(3+2p)(5+2p)}
\end{aligned}$$

output
```
-(3*a+b*(7+4*p))*cosh(f*x+e)*(a-b+b*cosh(f*x+e)^2)^(p+1)/b^2/f/(3+2*p)/(5+
2*p)+cosh(f*x+e)^3*(a-b+b*cosh(f*x+e)^2)^(p+1)/b/f/(5+2*p)+(3*a^2+4*a*b*(p
+1)+4*b^2*(p^2+3*p+2))*cosh(f*x+e)*(a-b+b*cosh(f*x+e)^2)^p*hypergeom([1/2,
 -p],[3/2],-b*cosh(f*x+e)^2/(a-b))/b^2/f/(3+2*p)/(5+2*p)/((1+b*cosh(f*x+e)
^2/(a-b))^p)
```

Mathematica [F]

$$\int \sinh^5(e+fx) \left(a+b\sinh^2(e+fx)\right)^p \, dx = \int \sinh^5(e+fx) \left(a+b\sinh^2(e+fx)\right)^p \, dx$$

input
```
Integrate[Sinh[e + f*x]^5*(a + b*Sinh[e + f*x]^2)^p,x]
```

output
```
Integrate[Sinh[e + f*x]^5*(a + b*Sinh[e + f*x]^2)^p, x]
```

Rubi [A] (verified)

Time = 0.41 (sec) , antiderivative size = 224, normalized size of antiderivative = 1.01, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.304$, Rules used = {3042, 26, 3665, 318, 299, 238, 237}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sinh^5(e+fx) \left(a+b\sinh^2(e+fx)\right)^p \, dx$$

$$\Big\downarrow \text{3042}$$

$$\int -i\sin(ie+ifx)^5 \left(a-b\sin(ie+ifx)^2\right)^p dx$$

$$\Big\downarrow \text{26}$$

$$-i\int \sin(ie+ifx)^5 \left(a-b\sin(ie+ifx)^2\right)^p dx$$

$$\Big\downarrow \text{3665}$$

$$\frac{\int \left(1-\cosh^2(e+fx)\right)^2 \left(b\cosh^2(e+fx)+a-b\right)^p d\cosh(e+fx)}{f}$$

$$\Big\downarrow \text{318}$$

$$\frac{\frac{\int \left(b\cosh^2(e+fx)+a-b\right)^p \left(-\left((3a+2b(p+2))\cosh^2(e+fx)\right)+a+2b(p+2)\right) d\cosh(e+fx)}{b(2p+5)}-\frac{\cosh(e+fx)\left(1-\cosh^2(e+fx)\right)\left(a+b\cosh^2(e+fx)-b\right)^p}{b(2p+5)}}{f}$$

$\Big\downarrow$ 299

$$\frac{\frac{\left(3a^2+4ab(p+1)+4b^2\left(p^2+3p+2\right)\right)\int\left(b\cosh^2(e+fx)+a-b\right)^p d\cosh(e+fx)}{b(2p+3)}-\frac{(3a+2b(p+2))\cosh(e+fx)\left(a+b\cosh^2(e+fx)-b\right)^{p+1}}{b(2p+3)}}{b(2p+5)}-\frac{\cosh(e+fx)(1-\cosh^2(e}{f}$$

$\Big\downarrow$ 238

$$\frac{\frac{\left(3a^2+4ab(p+1)+4b^2\left(p^2+3p+2\right)\right)\left(a+b\cosh^2(e+fx)-b\right)^p\left(\frac{b\cosh^2(e+fx)}{a-b}+1\right)^{-p}\int\left(\frac{b\cosh^2(e+fx)}{a-b}+1\right)^p d\cosh(e+fx)}{b(2p+3)}-\frac{(3a+2b(p+2))\cosh(e+fx)\left(a+b\cosh^2\right.}{b(2p+3)}}{b(2p+5)}}{f}$$

$\Big\downarrow$ 237

$$\frac{\frac{\left(3a^2+4ab(p+1)+4b^2\left(p^2+3p+2\right)\right)\cosh(e+fx)\left(a+b\cosh^2(e+fx)-b\right)^p\left(\frac{b\cosh^2(e+fx)}{a-b}+1\right)^{-p}\text{Hypergeometric2F1}\left(\frac{1}{2},-p,\frac{3}{2},-\frac{b\cosh^2(e+fx)}{a-b}\right)}{b(2p+3)}-\frac{(3a+2b(p+2))}{}}{b(2p+5)}}{f}$$

input
```
Int[Sinh[e + f*x]^5*(a + b*Sinh[e + f*x]^2)^p,x]
```

output
```
(-((Cosh[e + f*x]*(1 - Cosh[e + f*x]^2)*(a - b + b*Cosh[e + f*x]^2)^(1 + p
))/(b*(5 + 2*p))) + (-(((3*a + 2*b*(2 + p))*Cosh[e + f*x]*(a - b + b*Cosh[
e + f*x]^2)^(1 + p))/(b*(3 + 2*p))) + ((3*a^2 + 4*a*b*(1 + p) + 4*b^2*(2 +
 3*p + p^2))*Cosh[e + f*x]*(a - b + b*Cosh[e + f*x]^2)^p*Hypergeometric2F1
[1/2, -p, 3/2, -((b*Cosh[e + f*x]^2)/(a - b))])/(b*(3 + 2*p)*(1 + (b*Cosh[
e + f*x]^2)/(a - b))^p))/(b*(5 + 2*p)))/f
```

Defintions of rubi rules used

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 237
```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[a^p*x*Hypergeometric2F1[-
p, 1/2, 1/2 + 1, (-b)*(x^2/a)], x] /; FreeQ[{a, b, p}, x] &&  !IntegerQ[2*p
] && GtQ[a, 0]
```

rule 238
```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[a^IntPart[p]*((a + b*x^2)
^FracPart[p]/(1 + b*(x^2/a))^FracPart[p])   Int[(1 + b*(x^2/a))^p, x], x] /
; FreeQ[{a, b, p}, x] && !IntegerQ[2*p] && !GtQ[a, 0]
```

rule 299
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2), x_Symbol] :> Simp[d*x
*((a + b*x^2)^(p + 1)/(b*(2*p + 3))), x] - Simp[(a*d - b*c*(2*p + 3))/(b*(2
*p + 3))   Int[(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
 a*d, 0] && NeQ[2*p + 3, 0]
```

rule 318
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_), x_Symbol] :> Sim
p[d*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^(q - 1)/(b*(2*(p + q) + 1))), x] + S
imp[1/(b*(2*(p + q) + 1))   Int[(a + b*x^2)^p*(c + d*x^2)^(q - 2)*Simp[c*(b
*c*(2*(p + q) + 1) - a*d) + d*(b*c*(2*(p + 2*q - 1) + 1) - a*d*(2*(q - 1) +
 1))*x^2, x], x], x] /; FreeQ[{a, b, c, d, p}, x] && NeQ[b*c - a*d, 0] && G
tQ[q, 1] && NeQ[2*(p + q) + 1, 0] && !IGtQ[p, 1] && IntBinomialQ[a, b, c,
d, 2, p, q, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3665
```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^
(p_.), x_Symbol] :> With[{ff = FreeFactors[Cos[e + f*x], x]}, Simp[-ff/f
Subst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a + b - b*ff^2*x^2)^p, x], x, Cos[e +
 f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m - 1)/2]
```

Maple [F]

$$\int \sinh (fx+e)^5 \left(a+b\sinh (fx+e)^2\right)^p dx$$

input
```
int(sinh(f*x+e)^5*(a+b*sinh(f*x+e)^2)^p,x)
```

output
```
int(sinh(f*x+e)^5*(a+b*sinh(f*x+e)^2)^p,x)
```

Fricas [F]

$$\int \sinh^5(e+fx)\left(a+b\sinh^2(e+fx)\right)^p\,dx=\int \left(b\sinh\left(fx+e\right)^2+a\right)^p\sinh\left(fx+e\right)^5\,dx$$

input
```
integrate(sinh(f*x+e)^5*(a+b*sinh(f*x+e)^2)^p,x, algorithm="fricas")
```

output
```
integral((b*sinh(f*x + e)^2 + a)^p*sinh(f*x + e)^5, x)
```

Sympy [F(-1)]

Timed out.

$$\int \sinh^5(e+fx)\left(a+b\sinh^2(e+fx)\right)^p\,dx=\text{Timed out}$$

input
```
integrate(sinh(f*x+e)**5*(a+b*sinh(f*x+e)**2)**p,x)
```

output
```
Timed out
```

Maxima [F]

$$\int \sinh^5(e+fx)\left(a+b\sinh^2(e+fx)\right)^p\,dx=\int \left(b\sinh\left(fx+e\right)^2+a\right)^p\sinh\left(fx+e\right)^5\,dx$$

input
```
integrate(sinh(f*x+e)^5*(a+b*sinh(f*x+e)^2)^p,x, algorithm="maxima")
```

output
```
integrate((b*sinh(f*x + e)^2 + a)^p*sinh(f*x + e)^5, x)
```

Giac [F]

$$\int \sinh^5(e+fx)\left(a+b\sinh^2(e+fx)\right)^p\,dx=\int \left(b\sinh\left(fx+e\right)^2+a\right)^p\sinh\left(fx+e\right)^5\,dx$$

input
```
integrate(sinh(f*x+e)^5*(a+b*sinh(f*x+e)^2)^p,x, algorithm="giac")
```

output
```
integrate((b*sinh(f*x + e)^2 + a)^p*sinh(f*x + e)^5, x)
```

Mupad [F(-1)]

Timed out.

$$\int \sinh^5(e+fx)\left(a+b\sinh^2(e+fx)\right)^p\,dx=\int \sinh(e+f\,x)^5\,\left(b\sinh(e+f\,x)^2+a\right)^p\,dx$$

input
```
int(sinh(e + f*x)^5*(a + b*sinh(e + f*x)^2)^p,x)
```

output
```
int(sinh(e + f*x)^5*(a + b*sinh(e + f*x)^2)^p, x)
```

Reduce [F]

$$\int \sinh^5(e+fx)\left(a+b\sinh^2(e+fx)\right)^p\,dx=\text{too large to display}$$

input
```
int(sinh(f*x+e)^5*(a+b*sinh(f*x+e)^2)^p,x)
```

output
```
(80*e**(10*e + 10*f*x)*(e**(4*e + 4*f*x)*b + 4*e**(2*e + 2*f*x)*a - 2*e**(
2*e + 2*f*x)*b + b)**p*b**4*p**4 - 200*e**(10*e + 10*f*x)*(e**(4*e + 4*f*x
)*b + 4*e**(2*e + 2*f*x)*a - 2*e**(2*e + 2*f*x)*b + b)**p*b**4*p**2 + 45*e
**(10*e + 10*f*x)*(e**(4*e + 4*f*x)*b + 4*e**(2*e + 2*f*x)*a - 2*e**(2*e +
 2*f*x)*b + b)**p*b**4 + 320*e**(8*e + 8*f*x)*(e**(4*e + 4*f*x)*b + 4*e**(
2*e + 2*f*x)*a - 2*e**(2*e + 2*f*x)*b + b)**p*a*b**3*p**4 - 480*e**(8*e +
8*f*x)*(e**(4*e + 4*f*x)*b + 4*e**(2*e + 2*f*x)*a - 2*e**(2*e + 2*f*x)*b +
 b)**p*a*b**3*p**3 - 80*e**(8*e + 8*f*x)*(e**(4*e + 4*f*x)*b + 4*e**(2*e +
 2*f*x)*a - 2*e**(2*e + 2*f*x)*b + b)**p*a*b**3*p**2 + 120*e**(8*e + 8*f*x
)*(e**(4*e + 4*f*x)*b + 4*e**(2*e + 2*f*x)*a - 2*e**(2*e + 2*f*x)*b + b)**
p*a*b**3*p - 560*e**(8*e + 8*f*x)*(e**(4*e + 4*f*x)*b + 4*e**(2*e + 2*f*x)
*a - 2*e**(2*e + 2*f*x)*b + b)**p*b**4*p**4 - 160*e**(8*e + 8*f*x)*(e**(4*
e + 4*f*x)*b + 4*e**(2*e + 2*f*x)*a - 2*e**(2*e + 2*f*x)*b + b)**p*b**4*p*
*3 + 1640*e**(8*e + 8*f*x)*(e**(4*e + 4*f*x)*b + 4*e**(2*e + 2*f*x)*a - 2*
e**(2*e + 2*f*x)*b + b)**p*b**4*p**2 + 40*e**(8*e + 8*f*x)*(e**(4*e + 4*f*
x)*b + 4*e**(2*e + 2*f*x)*a - 2*e**(2*e + 2*f*x)*b + b)**p*b**4*p - 375*e*
*(8*e + 8*f*x)*(e**(4*e + 4*f*x)*b + 4*e**(2*e + 2*f*x)*a - 2*e**(2*e + 2*
f*x)*b + b)**p*b**4 - 1920*e**(6*e + 6*f*x)*(e**(4*e + 4*f*x)*b + 4*e**(2*
e + 2*f*x)*a - 2*e**(2*e + 2*f*x)*b + b)**p*a**2*b**2*p**3 + 3840*e**(6*e
+ 6*f*x)*(e**(4*e + 4*f*x)*b + 4*e**(2*e + 2*f*x)*a - 2*e**(2*e + 2*f*x...
```

3.110 $\int \sinh^3(e+fx)\left(a+b\sinh^2(e+fx)\right)^p\,dx$

Optimal result

Integrand size = 23, antiderivative size = 137

$$\int \sinh^3(e+fx)\left(a+b\sinh^2(e+fx)\right)^p\,dx=\frac{\cosh(e+fx)\left(a-b+b\cosh^2(e+fx)\right)^{1+p}}{bf(3+2p)}-\frac{(a+2b(1+p))\cosh(e+fx)\left(a-b+b\cosh^2(e+fx)\right)^p\left(1+\frac{b\cosh^2(e+fx)}{a-b}\right)^{-p}\text{Hypergeometric2F1}\left(\frac{1}{2},\right.}{bf(3+2p)}$$

output
```
cosh(f*x+e)*(a-b+b*cosh(f*x+e)^2)^(p+1)/b/f/(3+2*p)-(a+2*b*(p+1))*cosh(f*x
+e)*(a-b+b*cosh(f*x+e)^2)^p*hypergeom([1/2, -p],[3/2],-b*cosh(f*x+e)^2/(a-
b))/b/f/(3+2*p)/((1+b*cosh(f*x+e)^2/(a-b))^p)
```

Mathematica [F]

$$\int \sinh^3(e+fx)\left(a+b\sinh^2(e+fx)\right)^p\,dx=\int \sinh^3(e+fx)\left(a+b\sinh^2(e+fx)\right)^p\,dx$$

input
```
Integrate[Sinh[e + f*x]^3*(a + b*Sinh[e + f*x]^2)^p,x]
```

output
```
Integrate[Sinh[e + f*x]^3*(a + b*Sinh[e + f*x]^2)^p, x]
```

Rubi [A] (verified)

Time = 0.32 (sec) , antiderivative size = 136, normalized size of antiderivative = 0.99, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.261, Rules used = {3042, 26, 3665, 299, 238, 237}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sinh^3(e+fx)\left(a+b\sinh^2(e+fx)\right)^p\,dx$$

$$\Big\downarrow \ 3042$$

$$\int i\sin(ie+ifx)^3\left(a-b\sin(ie+ifx)^2\right)^p\,dx$$

$$\Big\downarrow \ 26$$

$$i\int \sin(ie+ifx)^3\left(a-b\sin(ie+ifx)^2\right)^p\,dx$$

$$\Big\downarrow \ 3665$$

$$-\frac{\int \left(1-\cosh^2(e+fx)\right)\left(b\cosh^2(e+fx)+a-b\right)^p d\cosh(e+fx)}{f}$$

$$\Big\downarrow \ 299$$

$$-\frac{\frac{(a+2b(p+1))\int \left(b\cosh^2(e+fx)+a-b\right)^p d\cosh(e+fx)}{b(2p+3)}-\frac{\cosh(e+fx)\left(a+b\cosh^2(e+fx)-b\right)^{p+1}}{b(2p+3)}}{f}$$

$$\Big\downarrow \ 238$$

$$-\frac{\frac{(a+2b(p+1))\left(a+b\cosh^2(e+fx)-b\right)^p\left(\frac{b\cosh^2(e+fx)}{a-b}+1\right)^{-p}\int \left(\frac{b\cosh^2(e+fx)}{a-b}+1\right)^p d\cosh(e+fx)}{b(2p+3)}-\frac{\cosh(e+fx)\left(a+b\cosh^2(e+fx)-b\right)^{p+1}}{b(2p+3)}}{f}$$

$$\Big\downarrow \ 237$$

$$-\frac{\frac{(a+2b(p+1))\cosh(e+fx)\left(a+b\cosh^2(e+fx)-b\right)^p\left(\frac{b\cosh^2(e+fx)}{a-b}+1\right)^{-p}\text{Hypergeometric2F1}\left(\frac{1}{2},-p,\frac{3}{2},-\frac{b\cosh^2(e+fx)}{a-b}\right)}{b(2p+3)}-\frac{\cosh(e+fx)(a+b}{b(2}}{f}$$

input
```
Int[Sinh[e + f*x]^3*(a + b*Sinh[e + f*x]^2)^p,x]
```

output
```
-((-((Cosh[e + f*x]*(a - b + b*Cosh[e + f*x]^2)^(1 + p))/(b*(3 + 2*p))) +
((a + 2*b*(1 + p))*Cosh[e + f*x]*(a - b + b*Cosh[e + f*x]^2)^p*Hypergeomet
ric2F1[1/2, -p, 3/2, -((b*Cosh[e + f*x]^2)/(a - b))])/(b*(3 + 2*p)*(1 + (b
*Cosh[e + f*x]^2)/(a - b))^p))/f)
```

Defintions of rubi rules used

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 237
```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[a^p*x*Hypergeometric2F1[-
p, 1/2, 1/2 + 1, (-b)*(x^2/a)], x] /; FreeQ[{a, b, p}, x] &&  !IntegerQ[2*p
] && GtQ[a, 0]
```

rule 238
```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[a^IntPart[p]*((a + b*x^2)
^FracPart[p]/(1 + b*(x^2/a))^FracPart[p])   Int[(1 + b*(x^2/a))^p, x], x] /
; FreeQ[{a, b, p}, x] &&  !IntegerQ[2*p] &&  !GtQ[a, 0]
```

rule 299
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2), x_Symbol] :> Simp[d*x
*((a + b*x^2)^(p + 1)/(b*(2*p + 3))), x] - Simp[(a*d - b*c*(2*p + 3))/(b*(2
*p + 3))   Int[(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
 a*d, 0] && NeQ[2*p + 3, 0]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3665
```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^
(p_.), x_Symbol] :> With[{ff = FreeFactors[Cos[e + f*x], x]}, Simp[-ff/f
Subst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a + b - b*ff^2*x^2)^p, x], x, Cos[e +
 f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m - 1)/2]
```

Maple [F]

$$\int \sinh{(fx+e)}^3 \left(a+b \sinh{(fx+e)}^2\right)^p \, dx$$

input
```
int(sinh(f*x+e)^3*(a+b*sinh(f*x+e)^2)^p,x)
```

output
```
int(sinh(f*x+e)^3*(a+b*sinh(f*x+e)^2)^p,x)
```

Fricas [F]

$$\int \sinh^3(e+fx) \left(a+b\sinh^2(e+fx)\right)^p \, dx = \int \left(b \sinh{(fx+e)}^2+a\right)^p \sinh{(fx+e)}^3 \, dx$$

input
```
integrate(sinh(f*x+e)^3*(a+b*sinh(f*x+e)^2)^p,x, algorithm="fricas")
```

output
```
integral((b*sinh(f*x + e)^2 + a)^p*sinh(f*x + e)^3, x)
```

Sympy [F(-1)]

Timed out.

$$\int \sinh^3(e+fx) \left(a+b\sinh^2(e+fx)\right)^p \, dx = \text{Timed out}$$

input
```
integrate(sinh(f*x+e)**3*(a+b*sinh(f*x+e)**2)**p,x)
```

output
```
Timed out
```

Maxima [F]

$$\int \sinh^3(e+fx)\left(a+b\sinh^2(e+fx)\right)^p\,dx=\int \left(b\sinh\left(fx+e\right)^2+a\right)^p\sinh\left(fx+e\right)^3\,dx$$

input
```
integrate(sinh(f*x+e)^3*(a+b*sinh(f*x+e)^2)^p,x, algorithm="maxima")
```

output
```
integrate((b*sinh(f*x + e)^2 + a)^p*sinh(f*x + e)^3, x)
```

Giac [F]

$$\int \sinh^3(e+fx)\left(a+b\sinh^2(e+fx)\right)^p\,dx=\int \left(b\sinh\left(fx+e\right)^2+a\right)^p\sinh\left(fx+e\right)^3\,dx$$

input
```
integrate(sinh(f*x+e)^3*(a+b*sinh(f*x+e)^2)^p,x, algorithm="giac")
```

output
```
integrate((b*sinh(f*x + e)^2 + a)^p*sinh(f*x + e)^3, x)
```

Mupad [F(-1)]

Timed out.

$$\int \sinh^3(e+fx)\left(a+b\sinh^2(e+fx)\right)^p\,dx=\int \sinh(e+f\,x)^3\,\left(b\,\sinh(e+f\,x)^2+a\right)^p\,dx$$

input
```
int(sinh(e + f*x)^3*(a + b*sinh(e + f*x)^2)^p,x)
```

output
```
int(sinh(e + f*x)^3*(a + b*sinh(e + f*x)^2)^p, x)
```

Reduce [F]

$$\int \sinh^3(e+fx)\left(a+b\sinh^2(e+fx)\right)^p\,dx = \text{too large to display}$$

input
```
int(sinh(f*x+e)^3*(a+b*sinh(f*x+e)^2)^p,x)
```

output
```
(12*e**(6*e + 6*f*x)*(e**(4*e + 4*f*x)*b + 4*e**(2*e + 2*f*x)*a - 2*e**(2*
e + 2*f*x)*b + b)**p*b**2*p**2 - 3*e**(6*e + 6*f*x)*(e**(4*e + 4*f*x)*b +
4*e**(2*e + 2*f*x)*a - 2*e**(2*e + 2*f*x)*b + b)**p*b**2 + 48*e**(4*e + 4*
f*x)*(e**(4*e + 4*f*x)*b + 4*e**(2*e + 2*f*x)*a - 2*e**(2*e + 2*f*x)*b + b
)**p*a*b*p**2 - 24*e**(4*e + 4*f*x)*(e**(4*e + 4*f*x)*b + 4*e**(2*e + 2*f*
x)*a - 2*e**(2*e + 2*f*x)*b + b)**p*a*b*p - 60*e**(4*e + 4*f*x)*(e**(4*e +
 4*f*x)*b + 4*e**(2*e + 2*f*x)*a - 2*e**(2*e + 2*f*x)*b + b)**p*b**2*p**2
- 24*e**(4*e + 4*f*x)*(e**(4*e + 4*f*x)*b + 4*e**(2*e + 2*f*x)*a - 2*e**(2
*e + 2*f*x)*b + b)**p*b**2*p + 27*e**(4*e + 4*f*x)*(e**(4*e + 4*f*x)*b + 4
*e**(2*e + 2*f*x)*a - 2*e**(2*e + 2*f*x)*b + b)**p*b**2 - 96*e**(2*e + 2*f
*x)*(e**(4*e + 4*f*x)*b + 4*e**(2*e + 2*f*x)*a - 2*e**(2*e + 2*f*x)*b + b)
**p*a**2*p - 144*e**(2*e + 2*f*x)*(e**(4*e + 4*f*x)*b + 4*e**(2*e + 2*f*x)
*a - 2*e**(2*e + 2*f*x)*b + b)**p*a*b*p**2 - 120*e**(2*e + 2*f*x)*(e**(4*e
 + 4*f*x)*b + 4*e**(2*e + 2*f*x)*a - 2*e**(2*e + 2*f*x)*b + b)**p*a*b*p +
132*e**(2*e + 2*f*x)*(e**(4*e + 4*f*x)*b + 4*e**(2*e + 2*f*x)*a - 2*e**(2*
e + 2*f*x)*b + b)**p*b**2*p**2 + 168*e**(2*e + 2*f*x)*(e**(4*e + 4*f*x)*b
+ 4*e**(2*e + 2*f*x)*a - 2*e**(2*e + 2*f*x)*b + b)**p*b**2*p + 27*e**(2*e
+ 2*f*x)*(e**(4*e + 4*f*x)*b + 4*e**(2*e + 2*f*x)*a - 2*e**(2*e + 2*f*x)*b
 + b)**p*b**2 + 12*(e**(4*e + 4*f*x)*b + 4*e**(2*e + 2*f*x)*a - 2*e**(2*e
+ 2*f*x)*b + b)**p*b**2*p**2 - 3*(e**(4*e + 4*f*x)*b + 4*e**(2*e + 2*f*...
```

3.111 $\int \sinh(e+fx)\left(a+b\sinh^2(e+fx)\right)^p\,dx$

Optimal result

Integrand size = 21, antiderivative size = 78

$$\int \sinh(e+fx)\left(a+b\sinh^2(e+fx)\right)^p\,dx$$
$$=\frac{\cosh(e+fx)\left(a-b+b\cosh^2(e+fx)\right)^p\left(1+\frac{b\cosh^2(e+fx)}{a-b}\right)^{-p}\text{Hypergeometric2F1}\left(\frac{1}{2},-p,\frac{3}{2},-\frac{b\cosh^2(e+f}{a-b}\right.}{f}$$

output
```
cosh(f*x+e)*(a-b+b*cosh(f*x+e)^2)^p*hypergeom([1/2, -p],[3/2],-b*cosh(f*x+
e)^2/(a-b))/f/((1+b*cosh(f*x+e)^2/(a-b))^p)
```

Mathematica [A] (verified)

Time = 0.68 (sec) , antiderivative size = 78, normalized size of antiderivative = 1.00

$$\int \sinh(e+fx)\left(a+b\sinh^2(e+fx)\right)^p\,dx$$
$$=\frac{\cosh(e+fx)\left(a-b+b\cosh^2(e+fx)\right)^p\left(1+\frac{b\cosh^2(e+fx)}{a-b}\right)^{-p}\text{Hypergeometric2F1}\left(\frac{1}{2},-p,\frac{3}{2},-\frac{b\cosh^2(e+f}{a-b}\right.}{f}$$

input
```
Integrate[Sinh[e + f*x]*(a + b*Sinh[e + f*x]^2)^p,x]
```

output
```
(Cosh[e + f*x]*(a - b + b*Cosh[e + f*x]^2)^p*Hypergeometric2F1[1/2, -p, 3/
2, -((b*Cosh[e + f*x]^2)/(a - b))])/(f*(1 + (b*Cosh[e + f*x]^2)/(a - b))^p
)
```

Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 78, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.238$, Rules used = {3042, 26, 3665, 238, 237}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sinh(e+fx)\left(a+b\sinh^2(e+fx)\right)^p\,dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int -i\sin(ie+ifx)\left(a-b\sin(ie+ifx)^2\right)^p\,dx$$

$$\Big\downarrow \text{ 26}$$

$$-i\int \sin(ie+ifx)\left(a-b\sin(ie+ifx)^2\right)^p\,dx$$

$$\Big\downarrow \text{ 3665}$$

$$\frac{\int \left(b\cosh^2(e+fx)+a-b\right)^p\,d\cosh(e+fx)}{f}$$

$$\Big\downarrow \text{ 238}$$

$$\frac{\left(a+b\cosh^2(e+fx)-b\right)^p\left(\frac{b\cosh^2(e+fx)}{a-b}+1\right)^{-p}\int \left(\frac{b\cosh^2(e+fx)}{a-b}+1\right)^p\,d\cosh(e+fx)}{f}$$

$$\Big\downarrow \text{ 237}$$

$$\frac{\cosh(e+fx)\left(a+b\cosh^2(e+fx)-b\right)^p\left(\frac{b\cosh^2(e+fx)}{a-b}+1\right)^{-p}\text{Hypergeometric2F1}\left(\frac{1}{2},-p,\frac{3}{2},-\frac{b\cosh^2(e+fx)}{a-b}\right)}{f}$$

input
```
Int[Sinh[e + f*x]*(a + b*Sinh[e + f*x]^2)^p,x]
```

output
```
(Cosh[e + f*x]*(a - b + b*Cosh[e + f*x]^2)^p*Hypergeometric2F1[1/2, -p, 3/
2, -((b*Cosh[e + f*x]^2)/(a - b))])/(f*(1 + (b*Cosh[e + f*x]^2)/(a - b))^p
)
```

Defintions of rubi rules used

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 237
```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[a^p*x*Hypergeometric2F1[-
p, 1/2, 1/2 + 1, (-b)*(x^2/a)], x] /; FreeQ[{a, b, p}, x] &&  !IntegerQ[2*p
] && GtQ[a, 0]
```

rule 238
```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[a^IntPart[p]*((a + b*x^2)
^FracPart[p]/(1 + b*(x^2/a))^FracPart[p])   Int[(1 + b*(x^2/a))^p, x], x] /
; FreeQ[{a, b, p}, x] &&  !IntegerQ[2*p] &&  !GtQ[a, 0]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3665
```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^
(p_.), x_Symbol] :> With[{ff = FreeFactors[Cos[e + f*x], x]}, Simp[-ff/f
Subst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a + b - b*ff^2*x^2)^p, x], x, Cos[e +
 f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m - 1)/2]
```

Maple [F]

$$\int \sinh (fx+e) \left(a+b\sinh (fx+e)^2\right)^p dx$$

input
```
int(sinh(f*x+e)*(a+b*sinh(f*x+e)^2)^p,x)
```

output
```
int(sinh(f*x+e)*(a+b*sinh(f*x+e)^2)^p,x)
```

Fricas [F]

$$\int \sinh(e+fx)\left(a+b\sinh^2(e+fx)\right)^p\,dx=\int \left(b\sinh\left(fx+e\right)^2+a\right)^p\sinh\left(fx+e\right)\,dx$$

input
```
integrate(sinh(f*x+e)*(a+b*sinh(f*x+e)^2)^p,x, algorithm="fricas")
```

output
```
integral((b*sinh(f*x + e)^2 + a)^p*sinh(f*x + e), x)
```

Sympy [F(-1)]

Timed out.

$$\int \sinh(e+fx)\left(a+b\sinh^2(e+fx)\right)^p\,dx=\text{Timed out}$$

input
```
integrate(sinh(f*x+e)*(a+b*sinh(f*x+e)**2)**p,x)
```

output
```
Timed out
```

Maxima [F]

$$\int \sinh(e+fx)\left(a+b\sinh^2(e+fx)\right)^p\,dx=\int \left(b\sinh\left(fx+e\right)^2+a\right)^p\sinh\left(fx+e\right)\,dx$$

input
```
integrate(sinh(f*x+e)*(a+b*sinh(f*x+e)^2)^p,x, algorithm="maxima")
```

output
```
integrate((b*sinh(f*x + e)^2 + a)^p*sinh(f*x + e), x)
```

Giac [F]

$$\int \sinh(e+fx)\left(a+b\sinh^2(e+fx)\right)^p\,dx=\int \left(b\sinh\left(fx+e\right)^2+a\right)^p\sinh\left(fx+e\right)\,dx$$

input
```
integrate(sinh(f*x+e)*(a+b*sinh(f*x+e)^2)^p,x, algorithm="giac")
```

output
```
integrate((b*sinh(f*x + e)^2 + a)^p*sinh(f*x + e), x)
```

Mupad [F(-1)]

Timed out.

$$\int \sinh(e+fx)\left(a+b\sinh^2(e+fx)\right)^p\,dx=\int \sinh(e+f\,x)\,\left(b\sinh(e+f\,x)^2+a\right)^p\,dx$$

input
```
int(sinh(e + f*x)*(a + b*sinh(e + f*x)^2)^p,x)
```

output
```
int(sinh(e + f*x)*(a + b*sinh(e + f*x)^2)^p, x)
```

Reduce [F]

$$\int \sinh(e+fx)\left(a+b\sinh^2(e+fx)\right)^p\,dx=\text{too large to display}$$

input
```
int(sinh(f*x+e)*(a+b*sinh(f*x+e)^2)^p,x)
```

output

```
(2*e**(2*e + 2*f*x)*(e**(4*e + 4*f*x)*b + 4*e**(2*e + 2*f*x)*a - 2*e**(2*e
 + 2*f*x)*b + b)**p*a - e**(2*e + 2*f*x)*(e**(4*e + 4*f*x)*b + 4*e**(2*e +
 2*f*x)*a - 2*e**(2*e + 2*f*x)*b + b)**p*b + 4*(e**(4*e + 4*f*x)*b + 4*e**
(2*e + 2*f*x)*a - 2*e**(2*e + 2*f*x)*b + b)**p*a*p + 2*(e**(4*e + 4*f*x)*b
 + 4*e**(2*e + 2*f*x)*a - 2*e**(2*e + 2*f*x)*b + b)**p*a - 4*(e**(4*e + 4*
f*x)*b + 4*e**(2*e + 2*f*x)*a - 2*e**(2*e + 2*f*x)*b + b)**p*b*p - (e**(4*
e + 4*f*x)*b + 4*e**(2*e + 2*f*x)*a - 2*e**(2*e + 2*f*x)*b + b)**p*b + 64*
e**(4*e + 2*f*p*x + f*x)*int((e**(3*f*x)*(e**(4*e + 4*f*x)*b + 4*e**(2*e +
 2*f*x)*a - 2*e**(2*e + 2*f*x)*b + b)**p)/(4*e**(4*e + 2*f*p*x + 4*f*x)*a*
b*p + 2*e**(4*e + 2*f*p*x + 4*f*x)*a*b - 2*e**(4*e + 2*f*p*x + 4*f*x)*b**2
*p - e**(4*e + 2*f*p*x + 4*f*x)*b**2 + 16*e**(2*e + 2*f*p*x + 2*f*x)*a**2*
p + 8*e**(2*e + 2*f*p*x + 2*f*x)*a**2 - 16*e**(2*e + 2*f*p*x + 2*f*x)*a*b*
p - 8*e**(2*e + 2*f*p*x + 2*f*x)*a*b + 4*e**(2*e + 2*f*p*x + 2*f*x)*b**2*p
 + 2*e**(2*e + 2*f*p*x + 2*f*x)*b**2 + 4*e**(2*f*p*x)*a*b*p + 2*e**(2*f*p*
x)*a*b - 2*e**(2*f*p*x)*b**2*p - e**(2*f*p*x)*b**2),x)*a**3*f*p**2 + 32*e*
*(4*e + 2*f*p*x + f*x)*int((e**(3*f*x)*(e**(4*e + 4*f*x)*b + 4*e**(2*e + 2
*f*x)*a - 2*e**(2*e + 2*f*x)*b + b)**p)/(4*e**(4*e + 2*f*p*x + 4*f*x)*a*b*
p + 2*e**(4*e + 2*f*p*x + 4*f*x)*a*b - 2*e**(4*e + 2*f*p*x + 4*f*x)*b**2*p
 - e**(4*e + 2*f*p*x + 4*f*x)*b**2 + 16*e**(2*e + 2*f*p*x + 2*f*x)*a**2*p
+ 8*e**(2*e + 2*f*p*x + 2*f*x)*a**2 - 16*e**(2*e + 2*f*p*x + 2*f*x)*a*b...
```

3.112 $\int \operatorname{csch}(e+fx)\left(a+b\sinh^2(e+fx)\right)^p\,dx$

Optimal result

Integrand size = 21, antiderivative size = 88

$$\int \operatorname{csch}(e+fx)\left(a+b\sinh^2(e+fx)\right)^p\,dx = -\frac{\operatorname{AppellF1}\left(\frac{1}{2},1,-p,\frac{3}{2},\cosh^2(e+fx),-\frac{b\cosh^2(e+fx)}{a-b}\right)\cosh(e+fx)\left(a-b+b\cosh^2(e+fx)\right)^p\left(1+\frac{b\cosh^2(e+fx)}{a-b}\right)^{-p}}{f}$$

output
```
-AppellF1(1/2,1,-p,3/2,cosh(f*x+e)^2,-b*cosh(f*x+e)^2/(a-b))*cosh(f*x+e)*(
a-b+b*cosh(f*x+e)^2)^p/f/((1+b*cosh(f*x+e)^2/(a-b))^p)
```

Mathematica [F]

$$\int \operatorname{csch}(e+fx)\left(a+b\sinh^2(e+fx)\right)^p\,dx = \int \operatorname{csch}(e+fx)\left(a+b\sinh^2(e+fx)\right)^p\,dx$$

input
```
Integrate[Csch[e + f*x]*(a + b*Sinh[e + f*x]^2)^p,x]
```

output
```
Integrate[Csch[e + f*x]*(a + b*Sinh[e + f*x]^2)^p, x]
```

Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 88, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.238$, Rules used = {3042, 26, 3665, 334, 333}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \operatorname{csch}(e+fx)\left(a+b\sinh^2(e+fx)\right)^p dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{i\left(a-b\sin(ie+ifx)^2\right)^p}{\sin(ie+ifx)}dx$$

$$\Big\downarrow \text{26}$$

$$i\int \frac{\left(a-b\sin(ie+ifx)^2\right)^p}{\sin(ie+ifx)}dx$$

$$\Big\downarrow \text{3665}$$

$$-\frac{\int \frac{\left(b\cosh^2(e+fx)+a-b\right)^p}{1-\cosh^2(e+fx)}d\cosh(e+fx)}{f}$$

$$\Big\downarrow \text{334}$$

$$-\frac{\left(a+b\cosh^2(e+fx)-b\right)^p\left(\frac{b\cosh^2(e+fx)}{a-b}+1\right)^{-p}\int \frac{\left(\frac{b\cosh^2(e+fx)}{a-b}+1\right)^p}{1-\cosh^2(e+fx)}d\cosh(e+fx)}{f}$$

$$\Big\downarrow \text{333}$$

$$-\frac{\cosh(e+fx)\left(a+b\cosh^2(e+fx)-b\right)^p\left(\frac{b\cosh^2(e+fx)}{a-b}+1\right)^{-p}\operatorname{AppellF1}\left(\frac{1}{2},1,-p,\frac{3}{2},\cosh^2(e+fx),-\frac{b\cosh^2(e+}{a-b}\right.}{f}$$

input
```
Int[Csch[e + f*x]*(a + b*Sinh[e + f*x]^2)^p,x]
```

output
```
-((AppellF1[1/2, 1, -p, 3/2, Cosh[e + f*x]^2, -((b*Cosh[e + f*x]^2)/(a - b
))]*Cosh[e + f*x]*(a - b + b*Cosh[e + f*x]^2)^p)/(f*(1 + (b*Cosh[e + f*x]^
2)/(a - b))^p))
```

Defintions of rubi rules used

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 333
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_), x_Symbol] :> Sim
p[a^p*c^q*x*AppellF1[1/2, -p, -q, 3/2, (-b)*(x^2/a), (-d)*(x^2/c)], x] /; F
reeQ[{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && (IntegerQ[p] || GtQ[a,
0]) && (IntegerQ[q] || GtQ[c, 0])
```

rule 334
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_), x_Symbol] :> Sim
p[a^IntPart[p]*((a + b*x^2)^FracPart[p]/(1 + b*(x^2/a))^FracPart[p])   Int[
(1 + b*(x^2/a))^p*(c + d*x^2)^q, x], x] /; FreeQ[{a, b, c, d, p, q}, x] &&
NeQ[b*c - a*d, 0] &&  !(IntegerQ[p] || GtQ[a, 0])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3665
```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^
(p_.), x_Symbol] :> With[{ff = FreeFactors[Cos[e + f*x], x]}, Simp[-ff/f
Subst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a + b - b*ff^2*x^2)^p, x], x, Cos[e +
 f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m - 1)/2]
```

Maple [F]

$$\int \operatorname{csch}(fx+e)\left(a+b\sinh(fx+e)^2\right)^p dx$$

input
```
int(csch(f*x+e)*(a+b*sinh(f*x+e)^2)^p,x)
```

output
```
int(csch(f*x+e)*(a+b*sinh(f*x+e)^2)^p,x)
```

Fricas [F]

$$\int \operatorname{csch}(e+fx)\left(a+b\sinh^2(e+fx)\right)^p\,dx = \int \left(b\sinh(fx+e)^2+a\right)^p \operatorname{csch}(fx+e)\,dx$$

input
```
integrate(csch(f*x+e)*(a+b*sinh(f*x+e)^2)^p,x, algorithm="fricas")
```

output
```
integral((b*sinh(f*x + e)^2 + a)^p*csch(f*x + e), x)
```

Sympy [F(-1)]

Timed out.

$$\int \operatorname{csch}(e+fx)\left(a+b\sinh^2(e+fx)\right)^p\,dx = \text{Timed out}$$

input
```
integrate(csch(f*x+e)*(a+b*sinh(f*x+e)**2)**p,x)
```

output
```
Timed out
```

Maxima [F]

$$\int \operatorname{csch}(e+fx)\left(a+b\sinh^2(e+fx)\right)^p\,dx=\int \left(b\sinh\left(fx+e\right)^2+a\right)^p\operatorname{csch}\left(fx+e\right)\,dx$$

input
```
integrate(csch(f*x+e)*(a+b*sinh(f*x+e)^2)^p,x, algorithm="maxima")
```

output
```
integrate((b*sinh(f*x + e)^2 + a)^p*csch(f*x + e), x)
```

Giac [F]

$$\int \operatorname{csch}(e+fx)\left(a+b\sinh^2(e+fx)\right)^p\,dx=\int \left(b\sinh\left(fx+e\right)^2+a\right)^p\operatorname{csch}\left(fx+e\right)\,dx$$

input
```
integrate(csch(f*x+e)*(a+b*sinh(f*x+e)^2)^p,x, algorithm="giac")
```

output
```
integrate((b*sinh(f*x + e)^2 + a)^p*csch(f*x + e), x)
```

Mupad [F(-1)]

Timed out.

$$\int \operatorname{csch}(e+fx)\left(a+b\sinh^2(e+fx)\right)^p\,dx=\int \frac{\left(b\sinh(e+f\,x)^2+a\right)^p}{\sinh\left(e+f\,x\right)}\,dx$$

input
```
int((a + b*sinh(e + f*x)^2)^p/sinh(e + f*x),x)
```

output
```
int((a + b*sinh(e + f*x)^2)^p/sinh(e + f*x), x)
```

Reduce [F]

$$\int \operatorname{csch}(e+fx)\left(a+b\sinh^2(e+fx)\right)^p\,dx=\int \left(\sinh\left(fx+e\right)^2b+a\right)^p\operatorname{csch}(fx+e)\,dx$$

input
```
int(csch(f*x+e)*(a+b*sinh(f*x+e)^2)^p,x)
```

output
```
int((sinh(e + f*x)**2*b + a)**p*csch(e + f*x),x)
```

3.113 $\int \operatorname{csch}^3(e+fx)\left(a+b\sinh^2(e+fx)\right)^p dx$

Optimal result

Integrand size = 23, antiderivative size = 87

$$\int \operatorname{csch}^3(e+fx)\left(a+b\sinh^2(e+fx)\right)^p dx = \frac{\operatorname{AppellF1}\left(\frac{1}{2},2,-p,\frac{3}{2},\cosh^2(e+fx),-\frac{b\cosh^2(e+fx)}{a-b}\right)\cosh(e+fx)\left(a-b+b\cosh^2(e+fx)\right)^p\left(1+\frac{b\cosh^2}{a}\right.}{f}$$

output
```
AppellF1(1/2,2,-p,3/2,cosh(f*x+e)^2,-b*cosh(f*x+e)^2/(a-b))*cosh(f*x+e)*(a
-b+b*cosh(f*x+e)^2)^p/f/((1+b*cosh(f*x+e)^2/(a-b))^p)
```

Mathematica [F]

$$\int \operatorname{csch}^3(e+fx)\left(a+b\sinh^2(e+fx)\right)^p dx = \int \operatorname{csch}^3(e+fx)\left(a+b\sinh^2(e+fx)\right)^p dx$$

input
```
Integrate[Csch[e + f*x]^3*(a + b*Sinh[e + f*x]^2)^p,x]
```

output
```
Integrate[Csch[e + f*x]^3*(a + b*Sinh[e + f*x]^2)^p, x]
```

Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 87, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.217, Rules used = {3042, 26, 3665, 334, 333}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \operatorname{csch}^3(e+fx)\left(a+b\sinh^2(e+fx)\right)^p\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int -\frac{i\left(a-b\sin(ie+ifx)^2\right)^p}{\sin(ie+ifx)^3}dx$$

$$\Big\downarrow \text{26}$$

$$-i\int \frac{\left(a-b\sin(ie+ifx)^2\right)^p}{\sin(ie+ifx)^3}dx$$

$$\Big\downarrow \text{3665}$$

$$\frac{\int \frac{\left(b\cosh^2(e+fx)+a-b\right)^p}{\left(1-\cosh^2(e+fx)\right)^2}d\cosh(e+fx)}{f}$$

$$\Big\downarrow \text{334}$$

$$\frac{\left(a+b\cosh^2(e+fx)-b\right)^p\left(\frac{b\cosh^2(e+fx)}{a-b}+1\right)^{-p}\int \frac{\left(\frac{b\cosh^2(e+fx)}{a-b}+1\right)^p}{\left(1-\cosh^2(e+fx)\right)^2}d\cosh(e+fx)}{f}$$

$$\Big\downarrow \text{333}$$

$$\frac{\cosh(e+fx)\left(a+b\cosh^2(e+fx)-b\right)^p\left(\frac{b\cosh^2(e+fx)}{a-b}+1\right)^{-p}\text{AppellF1}\left(\frac{1}{2},2,-p,\frac{3}{2},\cosh^2(e+fx),-\frac{b\cosh^2(e+fx)}{a-b}\right)}{f}$$

input
```
Int[Csch[e + f*x]^3*(a + b*Sinh[e + f*x]^2)^p,x]
```

output
```
(AppellF1[1/2, 2, -p, 3/2, Cosh[e + f*x]^2, -((b*Cosh[e + f*x]^2)/(a - b))
]*Cosh[e + f*x]*(a - b + b*Cosh[e + f*x]^2)^p)/(f*(1 + (b*Cosh[e + f*x]^2)
/(a - b))^p)
```

Defintions of rubi rules used

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 333
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_), x_Symbol] :> Sim
p[a^p*c^q*x*AppellF1[1/2, -p, -q, 3/2, (-b)*(x^2/a), (-d)*(x^2/c)], x] /; F
reeQ[{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && (IntegerQ[p] || GtQ[a,
0]) && (IntegerQ[q] || GtQ[c, 0])
```

rule 334
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_), x_Symbol] :> Sim
p[a^IntPart[p]*((a + b*x^2)^FracPart[p]/(1 + b*(x^2/a))^FracPart[p])   Int[
(1 + b*(x^2/a))^p*(c + d*x^2)^q, x], x] /; FreeQ[{a, b, c, d, p, q}, x] &&
NeQ[b*c - a*d, 0] &&  !(IntegerQ[p] || GtQ[a, 0])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3665
```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^
(p_.), x_Symbol] :> With[{ff = FreeFactors[Cos[e + f*x], x]}, Simp[-ff/f
Subst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a + b - b*ff^2*x^2)^p, x], x, Cos[e +
 f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m - 1)/2]
```

Maple [F]

$$\int \operatorname{csch}\left(fx+e\right)^{3}\left(a+b\sinh\left(fx+e\right)^{2}\right)^{p}dx$$

input
```
int(csch(f*x+e)^3*(a+b*sinh(f*x+e)^2)^p,x)
```

output
```
int(csch(f*x+e)^3*(a+b*sinh(f*x+e)^2)^p,x)
```

Fricas [F]

$$\int \operatorname{csch}^3(e+fx)\left(a+b\sinh^2(e+fx)\right)^p\,dx=\int \left(b\sinh\left(fx+e\right)^{2}+a\right)^{p}\operatorname{csch}\left(fx+e\right)^{3}\,dx$$

input
```
integrate(csch(f*x+e)^3*(a+b*sinh(f*x+e)^2)^p,x, algorithm="fricas")
```

output
```
integral((b*sinh(f*x + e)^2 + a)^p*csch(f*x + e)^3, x)
```

Sympy [F(-1)]

Timed out.

$$\int \operatorname{csch}^3(e+fx)\left(a+b\sinh^2(e+fx)\right)^p\,dx=\text{Timed out}$$

input
```
integrate(csch(f*x+e)**3*(a+b*sinh(f*x+e)**2)**p,x)
```

output
```
Timed out
```

Maxima [F]

$$\int \operatorname{csch}^3(e+fx) \left(a+b\sinh^2(e+fx)\right)^p \, dx = \int \left(b\sinh\left(fx+e\right)^2+a\right)^p \operatorname{csch}\left(fx+e\right)^3 \, dx$$

input
```
integrate(csch(f*x+e)^3*(a+b*sinh(f*x+e)^2)^p,x, algorithm="maxima")
```

output
```
integrate((b*sinh(f*x + e)^2 + a)^p*csch(f*x + e)^3, x)
```

Giac [F]

$$\int \operatorname{csch}^3(e+fx) \left(a+b\sinh^2(e+fx)\right)^p \, dx = \int \left(b\sinh\left(fx+e\right)^2+a\right)^p \operatorname{csch}\left(fx+e\right)^3 \, dx$$

input
```
integrate(csch(f*x+e)^3*(a+b*sinh(f*x+e)^2)^p,x, algorithm="giac")
```

output
```
integrate((b*sinh(f*x + e)^2 + a)^p*csch(f*x + e)^3, x)
```

Mupad [F(-1)]

Timed out.

$$\int \operatorname{csch}^3(e+fx) \left(a+b\sinh^2(e+fx)\right)^p \, dx = \int \frac{\left(b\sinh(e+f\,x)^2+a\right)^p}{\sinh\left(e+f\,x\right)^3} \, dx$$

input
```
int((a + b*sinh(e + f*x)^2)^p/sinh(e + f*x)^3,x)
```

output
```
int((a + b*sinh(e + f*x)^2)^p/sinh(e + f*x)^3, x)
```

Reduce [F]

$$\int \operatorname{csch}^3(e+fx) \left(a+b\sinh^2(e+fx)\right)^p \, dx = \int \operatorname{csch}(fx+e)^3 \left(\sinh(fx+e)^2 b+a\right)^p dx$$

input
```
int(csch(f*x+e)^3*(a+b*sinh(f*x+e)^2)^p,x)
```

output
```
int(csch(f*x+e)^3*(a+b*sinh(f*x+e)^2)^p,x)
```

3.114 $\int \sinh^4(e+fx) \left(a+b\sinh^2(e+fx)\right)^p \, dx$

Optimal result

Integrand size = 23, antiderivative size = 103

$$\int \sinh^4(e+fx) \left(a+b\sinh^2(e+fx)\right)^p \, dx = \frac{\operatorname{AppellF1}\left(\frac{5}{2},\frac{1}{2},-p,\frac{7}{2},-\sinh^2(e+fx),-\frac{b\sinh^2(e+fx)}{a}\right)\sqrt{\cosh^2(e+fx)}\sinh^4(e+fx)\left(a+b\sinh^2(e+f\right.}{5f}$$

output
```
1/5*AppellF1(5/2,1/2,-p,7/2,-sinh(f*x+e)^2,-b*sinh(f*x+e)^2/a)*(cosh(f*x+e
)^2)^(1/2)*sinh(f*x+e)^4*(a+b*sinh(f*x+e)^2)^p*tanh(f*x+e)/f/((1+b*sinh(f*
x+e)^2/a)^p)
```

Mathematica [F]

$$\int \sinh^4(e+fx) \left(a+b\sinh^2(e+fx)\right)^p \, dx = \int \sinh^4(e+fx) \left(a+b\sinh^2(e+fx)\right)^p \, dx$$

input
```
Integrate[Sinh[e + f*x]^4*(a + b*Sinh[e + f*x]^2)^p,x]
```

output
```
Integrate[Sinh[e + f*x]^4*(a + b*Sinh[e + f*x]^2)^p, x]
```

Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 103, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.174, Rules used = {3042, 3667, 395, 394}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sinh^4(e+fx)\left(a+b\sinh^2(e+fx)\right)^p\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \sin(ie+ifx)^4\left(a-b\sin(ie+ifx)^2\right)^p\,dx$$

$$\Big\downarrow \text{3667}$$

$$\frac{\sqrt{\cosh^2(e+fx)}\text{sech}(e+fx)\int \frac{\sinh^4(e+fx)\left(b\sinh^2(e+fx)+a\right)^p}{\sqrt{\sinh^2(e+fx)+1}}d\sinh(e+fx)}{f}$$

$$\Big\downarrow \text{395}$$

$$\frac{\sqrt{\cosh^2(e+fx)}\text{sech}(e+fx)\left(a+b\sinh^2(e+fx)\right)^p\left(\frac{b\sinh^2(e+fx)}{a}+1\right)^{-p}\int \frac{\sinh^4(e+fx)\left(\frac{b\sinh^2(e+fx)}{a}+1\right)^p}{\sqrt{\sinh^2(e+fx)+1}}d\sinh(e+$$

$$\Big\downarrow \text{394}$$

$$\frac{\sinh^4(e+fx)\sqrt{\cosh^2(e+fx)}\tanh(e+fx)\left(a+b\sinh^2(e+fx)\right)^p\left(\frac{b\sinh^2(e+fx)}{a}+1\right)^{-p}\text{AppellF1}\left(\frac{5}{2},\frac{1}{2},-p,\frac{7}{2},-\right.}{5f}$$

input
```
Int[Sinh[e + f*x]^4*(a + b*Sinh[e + f*x]^2)^p,x]
```

output
```
(AppellF1[5/2, 1/2, -p, 7/2, -Sinh[e + f*x]^2, -((b*Sinh[e + f*x]^2)/a)]*S
qrt[Cosh[e + f*x]^2]*Sinh[e + f*x]^4*(a + b*Sinh[e + f*x]^2)^p*Tanh[e + f*
x])/(5*f*(1 + (b*Sinh[e + f*x]^2)/a)^p)
```

Defintions of rubi rules used

rule 394
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_
), x_Symbol] :> Simp[a^p*c^q*((e*x)^(m + 1)/(e*(m + 1)))*AppellF1[(m + 1)/2
, -p, -q, 1 + (m + 1)/2, (-b)*(x^2/a), (-d)*(x^2/c)], x] /; FreeQ[{a, b, c,
 d, e, m, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, 1] && (Int
egerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

rule 395
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_
), x_Symbol] :> Simp[a^IntPart[p]*((a + b*x^2)^FracPart[p]/(1 + b*(x^2/a))^
FracPart[p])   Int[(e*x)^m*(1 + b*(x^2/a))^p*(c + d*x^2)^q, x], x] /; FreeQ
[{a, b, c, d, e, m, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m,
1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3667
```
Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(
p_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Simp[ff^(m + 1
)*(Sqrt[Cos[e + f*x]^2]/(f*Cos[e + f*x]))   Subst[Int[x^m*((a + b*ff^2*x^2)
^p/Sqrt[1 - ff^2*x^2]), x], x, Sin[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f,
p}, x] && IntegerQ[m/2] &&  !IntegerQ[p]
```

Maple [F]

$$\int \sinh(fx+e)^4 \left(a+b\sinh(fx+e)^2\right)^p dx$$

input
```
int(sinh(f*x+e)^4*(a+b*sinh(f*x+e)^2)^p,x)
```

output
```
int(sinh(f*x+e)^4*(a+b*sinh(f*x+e)^2)^p,x)
```

Fricas [F]

$$\int \sinh^4(e+fx)\left(a+b\sinh^2(e+fx)\right)^p\,dx=\int \left(b\sinh\left(fx+e\right)^2+a\right)^p\sinh\left(fx+e\right)^4\,dx$$

input
```
integrate(sinh(f*x+e)^4*(a+b*sinh(f*x+e)^2)^p,x, algorithm="fricas")
```

output
```
integral((b*sinh(f*x + e)^2 + a)^p*sinh(f*x + e)^4, x)
```

Sympy [F(-1)]

Timed out.

$$\int \sinh^4(e+fx)\left(a+b\sinh^2(e+fx)\right)^p\,dx=\text{Timed out}$$

input
```
integrate(sinh(f*x+e)**4*(a+b*sinh(f*x+e)**2)**p,x)
```

output
```
Timed out
```

Maxima [F]

$$\int \sinh^4(e+fx)\left(a+b\sinh^2(e+fx)\right)^p\,dx=\int \left(b\sinh\left(fx+e\right)^2+a\right)^p\sinh\left(fx+e\right)^4\,dx$$

input
```
integrate(sinh(f*x+e)^4*(a+b*sinh(f*x+e)^2)^p,x, algorithm="maxima")
```

output
```
integrate((b*sinh(f*x + e)^2 + a)^p*sinh(f*x + e)^4, x)
```

Giac [F]

$$\int \sinh^4(e+fx)\left(a+b\sinh^2(e+fx)\right)^p\,dx=\int \left(b\sinh\left(fx+e\right)^2+a\right)^p\sinh\left(fx+e\right)^4\,dx$$

input
```
integrate(sinh(f*x+e)^4*(a+b*sinh(f*x+e)^2)^p,x, algorithm="giac")
```

output
```
integrate((b*sinh(f*x + e)^2 + a)^p*sinh(f*x + e)^4, x)
```

Mupad [F(-1)]

Timed out.

$$\int \sinh^4(e+fx)\left(a+b\sinh^2(e+fx)\right)^p\,dx=\int \sinh(e+f\,x)^4\left(b\sinh(e+f\,x)^2+a\right)^p\,dx$$

input
```
int(sinh(e + f*x)^4*(a + b*sinh(e + f*x)^2)^p,x)
```

output
```
int(sinh(e + f*x)^4*(a + b*sinh(e + f*x)^2)^p, x)
```

Reduce [F]

$$\int \sinh^4(e+fx)\left(a+b\sinh^2(e+fx)\right)^p\,dx=\text{too large to display}$$

input
```
int(sinh(f*x+e)^4*(a+b*sinh(f*x+e)^2)^p,x)
```

output
```
(e**(8*e + 8*f*x)*(e**(4*e + 4*f*x)*b + 4*e**(2*e + 2*f*x)*a - 2*e**(2*e +
 2*f*x)*b + b)**p*b**2*p**3 - e**(8*e + 8*f*x)*(e**(4*e + 4*f*x)*b + 4*e**
(2*e + 2*f*x)*a - 2*e**(2*e + 2*f*x)*b + b)**p*b**2*p + 4*e**(6*e + 6*f*x)
*(e**(4*e + 4*f*x)*b + 4*e**(2*e + 2*f*x)*a - 2*e**(2*e + 2*f*x)*b + b)**p
*a*b*p**3 - 4*e**(6*e + 6*f*x)*(e**(4*e + 4*f*x)*b + 4*e**(2*e + 2*f*x)*a
- 2*e**(2*e + 2*f*x)*b + b)**p*a*b*p**2 - 6*e**(6*e + 6*f*x)*(e**(4*e + 4*
f*x)*b + 4*e**(2*e + 2*f*x)*a - 2*e**(2*e + 2*f*x)*b + b)**p*b**2*p**3 - 2
*e**(6*e + 6*f*x)*(e**(4*e + 4*f*x)*b + 4*e**(2*e + 2*f*x)*a - 2*e**(2*e +
 2*f*x)*b + b)**p*b**2*p**2 + 8*e**(6*e + 6*f*x)*(e**(4*e + 4*f*x)*b + 4*e
**(2*e + 2*f*x)*a - 2*e**(2*e + 2*f*x)*b + b)**p*b**2*p - 16*e**(4*e + 4*f
*x)*(e**(4*e + 4*f*x)*b + 4*e**(2*e + 2*f*x)*a - 2*e**(2*e + 2*f*x)*b + b)
**p*a**2*p**2 + 16*e**(4*e + 4*f*x)*(e**(4*e + 4*f*x)*b + 4*e**(2*e + 2*f*
x)*a - 2*e**(2*e + 2*f*x)*b + b)**p*a**2*p - 16*e**(4*e + 4*f*x)*(e**(4*e
+ 4*f*x)*b + 4*e**(2*e + 2*f*x)*a - 2*e**(2*e + 2*f*x)*b + b)**p*a*b*p**3
+ 16*e**(4*e + 4*f*x)*(e**(4*e + 4*f*x)*b + 4*e**(2*e + 2*f*x)*a - 2*e**(2
*e + 2*f*x)*b + b)**p*a*b*p + 16*e**(4*e + 4*f*x)*(e**(4*e + 4*f*x)*b + 4*
e**(2*e + 2*f*x)*a - 2*e**(2*e + 2*f*x)*b + b)**p*b**2*p**3 + 16*e**(4*e +
 4*f*x)*(e**(4*e + 4*f*x)*b + 4*e**(2*e + 2*f*x)*a - 2*e**(2*e + 2*f*x)*b
+ b)**p*b**2*p**2 - 20*e**(4*e + 4*f*x)*(e**(4*e + 4*f*x)*b + 4*e**(2*e +
2*f*x)*a - 2*e**(2*e + 2*f*x)*b + b)**p*b**2*p - 12*e**(4*e + 4*f*x)*(e...
```

3.115 $\int \sinh^2(e+fx) \left(a+b\sinh^2(e+fx)\right)^p \, dx$

Optimal result

Integrand size = 23, antiderivative size = 101

$$\int \sinh^2(e+fx) \left(a+b\sinh^2(e+fx)\right)^p \, dx = \frac{\text{AppellF1}\left(\frac{3}{2},2+p,-p,\frac{5}{2},\tanh^2(e+fx),\frac{(a-b)\tanh^2(e+fx)}{a}\right)\text{sech}^2(e+fx)^p \left(a+b\sinh^2(e+fx)\right)^p \tanh^3(}{3f}$$

output
```
1/3*AppellF1(3/2,2+p,-p,5/2,tanh(f*x+e)^2,(a-b)*tanh(f*x+e)^2/a)*(sech(f*x
+e)^2)^p*(a+b*sinh(f*x+e)^2)^p*tanh(f*x+e)^3/f/((1-(a-b)*tanh(f*x+e)^2/a)^
p)
```

Mathematica [F]

$$\int \sinh^2(e+fx) \left(a+b\sinh^2(e+fx)\right)^p \, dx = \int \sinh^2(e+fx) \left(a+b\sinh^2(e+fx)\right)^p \, dx$$

input
```
Integrate[Sinh[e + f*x]^2*(a + b*Sinh[e + f*x]^2)^p,x]
```

output
```
Integrate[Sinh[e + f*x]^2*(a + b*Sinh[e + f*x]^2)^p, x]
```

Rubi [A] (verified)

Time = 0.37 (sec) , antiderivative size = 101, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.261$, Rules used = {3042, 25, 3653, 25, 395, 394}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sinh^2(e+fx)\left(a+b\sinh^2(e+fx)\right)^p\,dx$$

$$\Big\downarrow \ 3042$$

$$\int -\sin(ie+ifx)^2\left(a-b\sin(ie+ifx)^2\right)^p dx$$

$$\Big\downarrow \ 25$$

$$-\int \sin(ie+ifx)^2\left(a-b\sin(ie+ifx)^2\right)^p dx$$

$$\Big\downarrow \ 3653$$

$$-\frac{\operatorname{sech}^2(e+fx)^p\left(a+b\sinh^2(e+fx)\right)^p\left(a-(a-b)\tanh^2(e+fx)\right)^{-p}\int -\tanh^2(e+fx)\left(1-\tanh^2(e+fx)\right)^{-}}{f}$$

$$\Big\downarrow \ 25$$

$$\frac{\operatorname{sech}^2(e+fx)^p\left(a+b\sinh^2(e+fx)\right)^p\left(a-(a-b)\tanh^2(e+fx)\right)^{-p}\int \tanh^2(e+fx)\left(1-\tanh^2(e+fx)\right)^{-p-2}(}{f}$$

$$\Big\downarrow \ 395$$

$$\frac{\operatorname{sech}^2(e+fx)^p\left(a+b\sinh^2(e+fx)\right)^p\left(1-\frac{(a-b)\tanh^2(e+fx)}{a}\right)^{-p}\int \tanh^2(e+fx)\left(1-\tanh^2(e+fx)\right)^{-p-2}\left(1-\right.}{f}$$

$$\Big\downarrow \ 394$$

$$\frac{\tanh^3(e+fx)\operatorname{sech}^2(e+fx)^p\left(a+b\sinh^2(e+fx)\right)^p\left(1-\frac{(a-b)\tanh^2(e+fx)}{a}\right)^{-p}\operatorname{AppellF1}\left(\frac{3}{2},p+2,-p,\frac{5}{2},\tanh^2(e\right.}{3f}$$

input
```
Int[Sinh[e + f*x]^2*(a + b*Sinh[e + f*x]^2)^p,x]
```

output
```
(AppellF1[3/2, 2 + p, -p, 5/2, Tanh[e + f*x]^2, ((a - b)*Tanh[e + f*x]^2)/
a]*(Sech[e + f*x]^2)^p*(a + b*Sinh[e + f*x]^2)^p*Tanh[e + f*x]^3)/(3*f*(1
- ((a - b)*Tanh[e + f*x]^2)/a)^p)
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 394
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_
), x_Symbol] :> Simp[a^p*c^q*((e*x)^(m + 1)/(e*(m + 1)))*AppellF1[(m + 1)/2
, -p, -q, 1 + (m + 1)/2, (-b)*(x^2/a), (-d)*(x^2/c)], x] /; FreeQ[{a, b, c,
 d, e, m, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, 1] && (Int
egerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

rule 395
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_
), x_Symbol] :> Simp[a^IntPart[p]*((a + b*x^2)^FracPart[p]/(1 + b*(x^2/a))^
FracPart[p])   Int[(e*x)^m*(1 + b*(x^2/a))^p*(c + d*x^2)^q, x], x] /; FreeQ
[{a, b, c, d, e, m, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m,
1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3653
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_)*((A_.) + (B_.)*sin[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, S
imp[ff*(a + b*Sin[e + f*x]^2)^p*((Sec[e + f*x]^2)^p/(f*(a + (a + b)*Tan[e +
 f*x]^2)^p))   Subst[Int[(a + (a + b)*ff^2*x^2)^p*((A + (A + B)*ff^2*x^2)/(
1 + ff^2*x^2)^(p + 2)), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f,
A, B}, x] &&  !IntegerQ[p]
```

Maple [F]

$$\int \sinh(fx+e)^2 \left(a+b\sinh(fx+e)^2\right)^p dx$$

input
```
int(sinh(f*x+e)^2*(a+b*sinh(f*x+e)^2)^p,x)
```

output
```
int(sinh(f*x+e)^2*(a+b*sinh(f*x+e)^2)^p,x)
```

Fricas [F]

$$\int \sinh^2(e+fx)\left(a+b\sinh^2(e+fx)\right)^p\,dx=\int \left(b\sinh(fx+e)^2+a\right)^p \sinh(fx+e)^2\,dx$$

input
```
integrate(sinh(f*x+e)^2*(a+b*sinh(f*x+e)^2)^p,x, algorithm="fricas")
```

output
```
integral((b*sinh(f*x + e)^2 + a)^p*sinh(f*x + e)^2, x)
```

Sympy [F(-1)]

Timed out.

$$\int \sinh^2(e+fx)\left(a+b\sinh^2(e+fx)\right)^p\,dx=\text{Timed out}$$

input
```
integrate(sinh(f*x+e)**2*(a+b*sinh(f*x+e)**2)**p,x)
```

output
```
Timed out
```

Maxima [F]

$$\int \sinh^2(e+fx)\left(a+b\sinh^2(e+fx)\right)^p\,dx=\int \left(b\sinh\left(fx+e\right)^2+a\right)^p\sinh\left(fx+e\right)^2\,dx$$

input
```
integrate(sinh(f*x+e)^2*(a+b*sinh(f*x+e)^2)^p,x, algorithm="maxima")
```

output
```
integrate((b*sinh(f*x + e)^2 + a)^p*sinh(f*x + e)^2, x)
```

Giac [F]

$$\int \sinh^2(e+fx)\left(a+b\sinh^2(e+fx)\right)^p\,dx=\int \left(b\sinh\left(fx+e\right)^2+a\right)^p\sinh\left(fx+e\right)^2\,dx$$

input
```
integrate(sinh(f*x+e)^2*(a+b*sinh(f*x+e)^2)^p,x, algorithm="giac")
```

output
```
integrate((b*sinh(f*x + e)^2 + a)^p*sinh(f*x + e)^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int \sinh^2(e+fx)\left(a+b\sinh^2(e+fx)\right)^p\,dx=\int \sinh(e+f\,x)^2\,\left(b\,\sinh(e+f\,x)^2+a\right)^p\,dx$$

input
```
int(sinh(e + f*x)^2*(a + b*sinh(e + f*x)^2)^p,x)
```

output
```
int(sinh(e + f*x)^2*(a + b*sinh(e + f*x)^2)^p, x)
```

Reduce [F]

$$\int \sinh^2(e+fx)\left(a+b\sinh^2(e+fx)\right)^p\,dx = \text{too large to display}$$

input
```
int(sinh(f*x+e)^2*(a+b*sinh(f*x+e)^2)^p,x)
```

output
```
(e**(4*e + 4*f*x)*(e**(4*e + 4*f*x)*b + 4*e**(2*e + 2*f*x)*a - 2*e**(2*e +
 2*f*x)*b + b)**p*b*p**2 - e**(4*e + 4*f*x)*(e**(4*e + 4*f*x)*b + 4*e**(2*
e + 2*f*x)*a - 2*e**(2*e + 2*f*x)*b + b)**p*b*p + 4*e**(2*e + 2*f*x)*(e**(
4*e + 4*f*x)*b + 4*e**(2*e + 2*f*x)*a - 2*e**(2*e + 2*f*x)*b + b)**p*a*p**
2 - 4*e**(2*e + 2*f*x)*(e**(4*e + 4*f*x)*b + 4*e**(2*e + 2*f*x)*a - 2*e**(
2*e + 2*f*x)*b + b)**p*a*p - 4*e**(2*e + 2*f*x)*(e**(4*e + 4*f*x)*b + 4*e*
*(2*e + 2*f*x)*a - 2*e**(2*e + 2*f*x)*b + b)**p*b*p**2 + 2*e**(2*e + 2*f*x
)*(e**(4*e + 4*f*x)*b + 4*e**(2*e + 2*f*x)*a - 2*e**(2*e + 2*f*x)*b + b)**
p*b*p + 2*e**(2*e + 2*f*x)*(e**(4*e + 4*f*x)*b + 4*e**(2*e + 2*f*x)*a - 2*
e**(2*e + 2*f*x)*b + b)**p*b - 8*(e**(4*e + 4*f*x)*b + 4*e**(2*e + 2*f*x)*
a - 2*e**(2*e + 2*f*x)*b + b)**p*a*p**2 - 8*(e**(4*e + 4*f*x)*b + 4*e**(2*
e + 2*f*x)*a - 2*e**(2*e + 2*f*x)*b + b)**p*a*p + 7*(e**(4*e + 4*f*x)*b +
4*e**(2*e + 2*f*x)*a - 2*e**(2*e + 2*f*x)*b + b)**p*b*p**2 + 5*(e**(4*e +
4*f*x)*b + 4*e**(2*e + 2*f*x)*a - 2*e**(2*e + 2*f*x)*b + b)**p*b*p - 64*e*
*(2*e*p + 4*e + 2*f*p*x + 2*f*x)*int((e**(4*e + 4*f*x)*b + 4*e**(2*e + 2*f
*x)*a - 2*e**(2*e + 2*f*x)*b + b)**p/(e**(2*e*p + 6*e + 2*f*p*x + 4*f*x)*b
*p - e**(2*e*p + 6*e + 2*f*p*x + 4*f*x)*b + 4*e**(2*e*p + 4*e + 2*f*p*x +
2*f*x)*a*p - 4*e**(2*e*p + 4*e + 2*f*p*x + 2*f*x)*a - 2*e**(2*e*p + 4*e +
2*f*p*x + 2*f*x)*b*p + 2*e**(2*e*p + 4*e + 2*f*p*x + 2*f*x)*b + e**(2*e*p
+ 2*e + 2*f*p*x)*b*p - e**(2*e*p + 2*e + 2*f*p*x)*b),x)*a**2*f*p**3 + 6...
```

3.116 $\int \operatorname{csch}^2(e+fx) \left(a+b\sinh^2(e+fx)\right)^p \, dx$

Optimal result

Integrand size = 23, antiderivative size = 99

$$\int \operatorname{csch}^2(e+fx) \left(a+b\sinh^2(e+fx)\right)^p \, dx =$$

$$-\frac{\operatorname{AppellF1}\left(-\frac{1}{2},\frac{1}{2},-p,\frac{1}{2},-\sinh^2(e+fx),-\frac{b\sinh^2(e+fx)}{a}\right)\sqrt{\cosh^2(e+fx)}\operatorname{csch}(e+fx)\operatorname{sech}(e+fx)\,(a}{f}$$

output
```
-AppellF1(-1/2,1/2,-p,1/2,-sinh(f*x+e)^2,-b*sinh(f*x+e)^2/a)*(cosh(f*x+e)^
2)^(1/2)*csch(f*x+e)*sech(f*x+e)*(a+b*sinh(f*x+e)^2)^p/f/((1+b*sinh(f*x+e)
^2/a)^p)
```

Mathematica [F]

$$\int \operatorname{csch}^2(e+fx) \left(a+b\sinh^2(e+fx)\right)^p \, dx = \int \operatorname{csch}^2(e+fx) \left(a+b\sinh^2(e+fx)\right)^p \, dx$$

input
```
Integrate[Csch[e + f*x]^2*(a + b*Sinh[e + f*x]^2)^p,x]
```

output
```
Integrate[Csch[e + f*x]^2*(a + b*Sinh[e + f*x]^2)^p, x]
```

Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 99, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.217, Rules used = {3042, 25, 3667, 395, 394}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \text{csch}^2(e+fx)\left(a+b\sinh^2(e+fx)\right)^p\,dx$$

$$\Big\downarrow \ 3042$$

$$\int -\frac{\left(a-b\sin(ie+ifx)^2\right)^p}{\sin(ie+ifx)^2}dx$$

$$\Big\downarrow \ 25$$

$$-\int \frac{\left(a-b\sin(ie+ifx)^2\right)^p}{\sin(ie+ifx)^2}dx$$

$$\Big\downarrow \ 3667$$

$$\frac{\sqrt{\cosh^2(e+fx)}\text{sech}(e+fx)\int \frac{\text{csch}^2(e+fx)\left(b\sinh^2(e+fx)+a\right)^p}{\sqrt{\sinh^2(e+fx)+1}}d\sinh(e+fx)}{f}$$

$$\Big\downarrow \ 395$$

$$\frac{\sqrt{\cosh^2(e+fx)}\text{sech}(e+fx)\left(a+b\sinh^2(e+fx)\right)^p\left(\frac{b\sinh^2(e+fx)}{a}+1\right)^{-p}\int \frac{\text{csch}^2(e+fx)\left(\frac{b\sinh^2(e+fx)}{a}+1\right)^p}{\sqrt{\sinh^2(e+fx)+1}}d\sinh(e}{f}$$

$$\Big\downarrow \ 394$$

$$-\frac{\sqrt{\cosh^2(e+fx)}\text{csch}(e+fx)\text{sech}(e+fx)\left(a+b\sinh^2(e+fx)\right)^p\left(\frac{b\sinh^2(e+fx)}{a}+1\right)^{-p}\text{AppellF1}\left(-\frac{1}{2},\frac{1}{2},-p,\frac{1}{2}\right.}{f}$$

input
```
Int[Csch[e + f*x]^2*(a + b*Sinh[e + f*x]^2)^p,x]
```

output
```
-((AppellF1[-1/2, 1/2, -p, 1/2, -Sinh[e + f*x]^2, -((b*Sinh[e + f*x]^2)/a)
]*Sqrt[Cosh[e + f*x]^2]*Csch[e + f*x]*Sech[e + f*x]*(a + b*Sinh[e + f*x]^2
)^p)/(f*(1 + (b*Sinh[e + f*x]^2)/a)^p))
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 394
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_
), x_Symbol] :> Simp[a^p*c^q*((e*x)^(m + 1)/(e*(m + 1)))*AppellF1[(m + 1)/2
, -p, -q, 1 + (m + 1)/2, (-b)*(x^2/a), (-d)*(x^2/c)], x] /; FreeQ[{a, b, c,
 d, e, m, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, 1] && (Int
egerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

rule 395
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_
), x_Symbol] :> Simp[a^IntPart[p]*((a + b*x^2)^FracPart[p]/(1 + b*(x^2/a))^
FracPart[p])   Int[(e*x)^m*(1 + b*(x^2/a))^p*(c + d*x^2)^q, x], x] /; FreeQ
[{a, b, c, d, e, m, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m,
1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3667
```
Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(
p_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Simp[ff^(m + 1
)*(Sqrt[Cos[e + f*x]^2]/(f*Cos[e + f*x]))   Subst[Int[x^m*((a + b*ff^2*x^2)
^p/Sqrt[1 - ff^2*x^2]), x], x, Sin[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f,
p}, x] && IntegerQ[m/2] &&  !IntegerQ[p]
```

Maple [F]

$$\int \operatorname{csch}(fx+e)^2 \left(a+b\sinh(fx+e)^2\right)^p dx$$

input
```
int(csch(f*x+e)^2*(a+b*sinh(f*x+e)^2)^p,x)
```

output
```
int(csch(f*x+e)^2*(a+b*sinh(f*x+e)^2)^p,x)
```

Fricas [F]

$$\int \operatorname{csch}^2(e+fx)\left(a+b\sinh^2(e+fx)\right)^p dx = \int \left(b\sinh(fx+e)^2+a\right)^p \operatorname{csch}(fx+e)^2 dx$$

input
```
integrate(csch(f*x+e)^2*(a+b*sinh(f*x+e)^2)^p,x, algorithm="fricas")
```

output
```
integral((b*sinh(f*x + e)^2 + a)^p*csch(f*x + e)^2, x)
```

Sympy [F(-1)]

Timed out.

$$\int \operatorname{csch}^2(e+fx)\left(a+b\sinh^2(e+fx)\right)^p dx = \text{Timed out}$$

input
```
integrate(csch(f*x+e)**2*(a+b*sinh(f*x+e)**2)**p,x)
```

output
```
Timed out
```

Maxima [F]

$$\int \operatorname{csch}^2(e+fx) \left(a+b\sinh^2(e+fx)\right)^p \,dx = \int \left(b\sinh\left(fx+e\right)^2+a\right)^p \operatorname{csch}\left(fx+e\right)^2 \,dx$$

input
```
integrate(csch(f*x+e)^2*(a+b*sinh(f*x+e)^2)^p,x, algorithm="maxima")
```

output
```
integrate((b*sinh(f*x + e)^2 + a)^p*csch(f*x + e)^2, x)
```

Giac [F]

$$\int \operatorname{csch}^2(e+fx) \left(a+b\sinh^2(e+fx)\right)^p \,dx = \int \left(b\sinh\left(fx+e\right)^2+a\right)^p \operatorname{csch}\left(fx+e\right)^2 \,dx$$

input
```
integrate(csch(f*x+e)^2*(a+b*sinh(f*x+e)^2)^p,x, algorithm="giac")
```

output
```
integrate((b*sinh(f*x + e)^2 + a)^p*csch(f*x + e)^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int \operatorname{csch}^2(e+fx) \left(a+b\sinh^2(e+fx)\right)^p \,dx = \int \frac{\left(b\sinh(e+f\,x)^2+a\right)^p}{\sinh\left(e+f\,x\right)^2} \,dx$$

input
```
int((a + b*sinh(e + f*x)^2)^p/sinh(e + f*x)^2,x)
```

output
```
int((a + b*sinh(e + f*x)^2)^p/sinh(e + f*x)^2, x)
```

Reduce [F]

$$\int \operatorname{csch}^2(e+fx)\left(a+b\sinh^2(e+fx)\right)^p\,dx=\int \operatorname{csch}(fx+e)^2\left(\sinh(fx+e)^2b+a\right)^p dx$$

input
```
int(csch(f*x+e)^2*(a+b*sinh(f*x+e)^2)^p,x)
```

output
```
int(csch(f*x+e)^2*(a+b*sinh(f*x+e)^2)^p,x)
```

3.117 $\int \operatorname{csch}^4(e+fx)\left(a+b\sinh^2(e+fx)\right)^p\,dx$

Optimal result

Integrand size = 23, antiderivative size = 103

$$\int \operatorname{csch}^4(e+fx)\left(a+b\sinh^2(e+fx)\right)^p\,dx = -\frac{\operatorname{AppellF1}\left(-\frac{3}{2},\frac{1}{2},-p,-\frac{1}{2},-\sinh^2(e+fx),-\frac{b\sinh^2(e+fx)}{a}\right)\sqrt{\cosh^2(e+fx)}\operatorname{csch}^3(e+fx)\operatorname{sech}(e+fx)}{3f}$$

output
```
-1/3*AppellF1(-3/2,1/2,-p,-1/2,-sinh(f*x+e)^2,-b*sinh(f*x+e)^2/a)*(cosh(f*
x+e)^2)^(1/2)*csch(f*x+e)^3*sech(f*x+e)*(a+b*sinh(f*x+e)^2)^p/f/((1+b*sinh
(f*x+e)^2/a)^p)
```

Mathematica [F]

$$\int \operatorname{csch}^4(e+fx)\left(a+b\sinh^2(e+fx)\right)^p\,dx = \int \operatorname{csch}^4(e+fx)\left(a+b\sinh^2(e+fx)\right)^p\,dx$$

input
```
Integrate[Csch[e + f*x]^4*(a + b*Sinh[e + f*x]^2)^p,x]
```

output
```
Integrate[Csch[e + f*x]^4*(a + b*Sinh[e + f*x]^2)^p, x]
```

Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 103, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.174$, Rules used = {3042, 3667, 395, 394}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \operatorname{csch}^4(e+fx)\left(a+b\sinh^2(e+fx)\right)^p\,dx$$

$$\Big\downarrow \; 3042$$

$$\int \frac{\left(a-b\sin(ie+ifx)^2\right)^p}{\sin(ie+ifx)^4}dx$$

$$\Big\downarrow \; 3667$$

$$\frac{\sqrt{\cosh^2(e+fx)}\operatorname{sech}(e+fx)\int \frac{\operatorname{csch}^4(e+fx)\left(b\sinh^2(e+fx)+a\right)^p}{\sqrt{\sinh^2(e+fx)+1}}d\sinh(e+fx)}{f}$$

$$\Big\downarrow \; 395$$

$$\frac{\sqrt{\cosh^2(e+fx)}\operatorname{sech}(e+fx)\left(a+b\sinh^2(e+fx)\right)^p\left(\frac{b\sinh^2(e+fx)}{a}+1\right)^{-p}\int \frac{\operatorname{csch}^4(e+fx)\left(\frac{b\sinh^2(e+fx)}{a}+1\right)^p}{\sqrt{\sinh^2(e+fx)+1}}d\sinh(e}{f}$$

$$\Big\downarrow \; 394$$

$$-\frac{\sqrt{\cosh^2(e+fx)}\operatorname{csch}^3(e+fx)\operatorname{sech}(e+fx)\left(a+b\sinh^2(e+fx)\right)^p\left(\frac{b\sinh^2(e+fx)}{a}+1\right)^{-p}\operatorname{AppellF1}\left(-\frac{3}{2},\frac{1}{2},-p,\right.}{3f}$$

input
```
Int[Csch[e + f*x]^4*(a + b*Sinh[e + f*x]^2)^p,x]
```

output
```
-1/3*(AppellF1[-3/2, 1/2, -p, -1/2, -Sinh[e + f*x]^2, -((b*Sinh[e + f*x]^2
)/a)]*Sqrt[Cosh[e + f*x]^2]*Csch[e + f*x]^3*Sech[e + f*x]*(a + b*Sinh[e +
f*x]^2)^p)/(f*(1 + (b*Sinh[e + f*x]^2)/a)^p)
```

Defintions of rubi rules used

rule 394
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_
), x_Symbol] :> Simp[a^p*c^q*((e*x)^(m + 1)/(e*(m + 1)))*AppellF1[(m + 1)/2
, -p, -q, 1 + (m + 1)/2, (-b)*(x^2/a), (-d)*(x^2/c)], x] /; FreeQ[{a, b, c,
 d, e, m, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, 1] && (Int
egerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

rule 395
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_
), x_Symbol] :> Simp[a^IntPart[p]*((a + b*x^2)^FracPart[p]/(1 + b*(x^2/a))^
FracPart[p])   Int[(e*x)^m*(1 + b*(x^2/a))^p*(c + d*x^2)^q, x], x] /; FreeQ
[{a, b, c, d, e, m, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m,
1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3667
```
Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(
p_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Simp[ff^(m + 1
)*(Sqrt[Cos[e + f*x]^2]/(f*Cos[e + f*x]))   Subst[Int[x^m*((a + b*ff^2*x^2)
^p/Sqrt[1 - ff^2*x^2]), x], x, Sin[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f,
p}, x] && IntegerQ[m/2] &&  !IntegerQ[p]
```

Maple [F]

$$\int \operatorname{csch}(fx+e)^4 \left(a+b\sinh(fx+e)^2\right)^p dx$$

input
```
int(csch(f*x+e)^4*(a+b*sinh(f*x+e)^2)^p,x)
```

output
```
int(csch(f*x+e)^4*(a+b*sinh(f*x+e)^2)^p,x)
```

Fricas [F]

$$\int \operatorname{csch}^4(e+fx) \left(a+b\sinh^2(e+fx)\right)^p \, dx = \int \left(b\sinh\left(fx+e\right)^2+a\right)^p \operatorname{csch}\left(fx+e\right)^4 \, dx$$

input
```
integrate(csch(f*x+e)^4*(a+b*sinh(f*x+e)^2)^p,x, algorithm="fricas")
```

output
```
integral((b*sinh(f*x + e)^2 + a)^p*csch(f*x + e)^4, x)
```

Sympy [F(-1)]

Timed out.

$$\int \operatorname{csch}^4(e+fx) \left(a+b\sinh^2(e+fx)\right)^p \, dx = \text{Timed out}$$

input
```
integrate(csch(f*x+e)**4*(a+b*sinh(f*x+e)**2)**p,x)
```

output
```
Timed out
```

Maxima [F]

$$\int \operatorname{csch}^4(e+fx) \left(a+b\sinh^2(e+fx)\right)^p \, dx = \int \left(b\sinh\left(fx+e\right)^2+a\right)^p \operatorname{csch}\left(fx+e\right)^4 \, dx$$

input
```
integrate(csch(f*x+e)^4*(a+b*sinh(f*x+e)^2)^p,x, algorithm="maxima")
```

output
```
integrate((b*sinh(f*x + e)^2 + a)^p*csch(f*x + e)^4, x)
```

Giac [F]

$$\int \operatorname{csch}^4(e+fx)\left(a+b\sinh^2(e+fx)\right)^p\,dx=\int \left(b\sinh\left(fx+e\right)^2+a\right)^p\operatorname{csch}\left(fx+e\right)^4\,dx$$

input
```
integrate(csch(f*x+e)^4*(a+b*sinh(f*x+e)^2)^p,x, algorithm="giac")
```

output
```
integrate((b*sinh(f*x + e)^2 + a)^p*csch(f*x + e)^4, x)
```

Mupad [F(-1)]

Timed out.

$$\int \operatorname{csch}^4(e+fx)\left(a+b\sinh^2(e+fx)\right)^p\,dx=\int \frac{\left(b\sinh(e+f\,x)^2+a\right)^p}{\sinh\left(e+f\,x\right)^4}\,dx$$

input
```
int((a + b*sinh(e + f*x)^2)^p/sinh(e + f*x)^4,x)
```

output
```
int((a + b*sinh(e + f*x)^2)^p/sinh(e + f*x)^4, x)
```

Reduce [F]

$$\int \operatorname{csch}^4(e+fx)\left(a+b\sinh^2(e+fx)\right)^p\,dx=\int \operatorname{csch}(fx+e)^4\left(\sinh\left(fx+e\right)^2b+a\right)^p dx$$

input
```
int(csch(f*x+e)^4*(a+b*sinh(f*x+e)^2)^p,x)
```

output
```
int(csch(f*x+e)^4*(a+b*sinh(f*x+e)^2)^p,x)
```

3.118 $\int \sinh^4(c+dx)\left(a+b\sinh^3(c+dx)\right)\,dx$

Optimal result

Integrand size = 21, antiderivative size = 106

$$\int \sinh^4(c+dx)\left(a+b\sinh^3(c+dx)\right)\,dx=\frac{3ax}{8}-\frac{b\cosh(c+dx)}{d}+\frac{b\cosh^3(c+dx)}{d}-\frac{3b\cosh^5(c+dx)}{5d}+\frac{b\cosh^7(c+dx)}{7d}-\frac{3a\cosh(c+dx)\sinh(c+dx)}{8d}+\frac{a\cosh(c+dx)\sinh^3(c+dx)}{4d}$$

output
```
3/8*a*x-b*cosh(d*x+c)/d+b*cosh(d*x+c)^3/d-3/5*b*cosh(d*x+c)^5/d+1/7*b*cosh
(d*x+c)^7/d-3/8*a*cosh(d*x+c)*sinh(d*x+c)/d+1/4*a*cosh(d*x+c)*sinh(d*x+c)^
3/d
```

Mathematica [A] (verified)

Time = 0.12 (sec) , antiderivative size = 81, normalized size of antiderivative = 0.76

$$\int \sinh^4(c+dx)\left(a+b\sinh^3(c+dx)\right)\,dx$$
$$=\frac{840ac+840adx-1225b\cosh(c+dx)+245b\cosh(3(c+dx))-49b\cosh(5(c+dx))+5b\cosh(7(c+dx$$
$$2240d$$

input
```
Integrate[Sinh[c + d*x]^4*(a + b*Sinh[c + d*x]^3),x]
```

output
```
(840*a*c + 840*a*d*x - 1225*b*Cosh[c + d*x] + 245*b*Cosh[3*(c + d*x)] - 49
*b*Cosh[5*(c + d*x)] + 5*b*Cosh[7*(c + d*x)] - 560*a*Sinh[2*(c + d*x)] + 7
0*a*Sinh[4*(c + d*x)])/(2240*d)
```

Rubi [A] (verified)

Time = 0.32 (sec) , antiderivative size = 106, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {3042, 3699, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sinh^4(c+dx)\left(a+b\sinh^3(c+dx)\right)\,dx$$

$$\Big\downarrow \; 3042$$

$$\int \sin(ic+idx)^4\left(a+ib\sin(ic+idx)^3\right)dx$$

$$\Big\downarrow \; 3699$$

$$\int \left(a\sinh^4(c+dx)+b\sinh^7(c+dx)\right)dx$$

$$\Big\downarrow \; 2009$$

$$\frac{a\sinh^3(c+dx)\cosh(c+dx)}{4d}-\frac{3a\sinh(c+dx)\cosh(c+dx)}{8d}+\frac{3ax}{8}+\frac{b\cosh^7(c+dx)}{7d}-\frac{3b\cosh^5(c+dx)}{5d}+\frac{b\cosh^3(c+dx)}{d}-\frac{b\cosh(c+dx)}{d}$$

input
```
Int[Sinh[c + d*x]^4*(a + b*Sinh[c + d*x]^3),x]
```

output
```
(3*a*x)/8 - (b*Cosh[c + d*x])/d + (b*Cosh[c + d*x]^3)/d - (3*b*Cosh[c + d*
x]^5)/(5*d) + (b*Cosh[c + d*x]^7)/(7*d) - (3*a*Cosh[c + d*x]*Sinh[c + d*x]
)/(8*d) + (a*Cosh[c + d*x]*Sinh[c + d*x]^3)/(4*d)
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3699
```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^(n_
))^(p_.), x_Symbol] :> Int[ExpandTrig[sin[e + f*x]^m*(a + b*sin[e + f*x]^n)
^p, x], x] /; FreeQ[{a, b, e, f}, x] && IntegersQ[m, p] && (EqQ[n, 4] || Gt
Q[p, 0] || (EqQ[p, -1] && IntegerQ[n]))
```

Maple [A] (verified)

Time = 7.26 (sec) , antiderivative size = 82, normalized size of antiderivative = 0.77

method	result
derivativedivides	$\frac{a\left(\left(\frac{\sinh(dx+c)^3}{4}-\frac{3\sinh(dx+c)}{8}\right)\cosh(dx+c)+\frac{3dx}{8}+\frac{3c}{8}\right)+b\left(-\frac{16}{35}+\frac{\sinh(dx+c)^6}{7}-\frac{6\sinh(dx+c)^4}{35}+\frac{8\sinh(dx+c)^2}{35}\right)\cosh(}{d}$
default	$\frac{a\left(\left(\frac{\sinh(dx+c)^3}{4}-\frac{3\sinh(dx+c)}{8}\right)\cosh(dx+c)+\frac{3dx}{8}+\frac{3c}{8}\right)+b\left(-\frac{16}{35}+\frac{\sinh(dx+c)^6}{7}-\frac{6\sinh(dx+c)^4}{35}+\frac{8\sinh(dx+c)^2}{35}\right)\cosh(}{d}$
parallelrisch	$\frac{840dxa-1024b+70a\sinh(4dx+4c)-560a\sinh(2dx+2c)-49b\cosh(5dx+5c)+245b\cosh(3dx+3c)-1225b\cosh(dx+c)+}{2240d}$
parts	$\frac{a\left(\left(\frac{\sinh(dx+c)^3}{4}-\frac{3\sinh(dx+c)}{8}\right)\cosh(dx+c)+\frac{3dx}{8}+\frac{3c}{8}\right)}{d}+\frac{b\left(-\frac{16}{35}+\frac{\sinh(dx+c)^6}{7}-\frac{6\sinh(dx+c)^4}{35}+\frac{8\sinh(dx+c)^2}{35}\right)\cos}{d}$
risch	$\frac{3ax}{8}+\frac{b\,e^{7dx+7c}}{896d}-\frac{7b\,e^{5dx+5c}}{640d}+\frac{a\,e^{4dx+4c}}{64d}+\frac{7\,e^{3dx+3c}b}{128d}-\frac{e^{2dx+2c}a}{8d}-\frac{35\,e^{dx+c}b}{128d}-\frac{35\,e^{-dx-c}b}{128d}+\frac{e^{-2dx}}{8}$
orering	Expression too large to display

input
```
int(sinh(d*x+c)^4*(a+b*sinh(d*x+c)^3),x,method=_RETURNVERBOSE)
```

output
```
1/d*(a*((1/4*sinh(d*x+c)^3-3/8*sinh(d*x+c))*cosh(d*x+c)+3/8*d*x+3/8*c)+b*(
-16/35+1/7*sinh(d*x+c)^6-6/35*sinh(d*x+c)^4+8/35*sinh(d*x+c)^2)*cosh(d*x+c
))
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 188, normalized size of antiderivative = 1.77

$$\int \sinh^4(c+dx)\left(a+b\sinh^3(c+dx)\right)\,dx$$
$$=\frac{5\,b\cosh(dx+c)^7+35\,b\cosh(dx+c)\sinh(dx+c)^6-49\,b\cosh(dx+c)^5+280\,a\cosh(dx+c)\sinh(dx}{}$$

input
```
integrate(sinh(d*x+c)^4*(a+b*sinh(d*x+c)^3),x, algorithm="fricas")
```

output
```
1/2240*(5*b*cosh(d*x + c)^7 + 35*b*cosh(d*x + c)*sinh(d*x + c)^6 - 49*b*co
sh(d*x + c)^5 + 280*a*cosh(d*x + c)*sinh(d*x + c)^3 + 35*(5*b*cosh(d*x + c
)^3 - 7*b*cosh(d*x + c))*sinh(d*x + c)^4 + 245*b*cosh(d*x + c)^3 + 840*a*d
*x + 35*(3*b*cosh(d*x + c)^5 - 14*b*cosh(d*x + c)^3 + 21*b*cosh(d*x + c))*
sinh(d*x + c)^2 - 1225*b*cosh(d*x + c) + 280*(a*cosh(d*x + c)^3 - 4*a*cosh
(d*x + c))*sinh(d*x + c))/d
```

Sympy [A] (verification not implemented)

Time = 0.48 (sec) , antiderivative size = 192, normalized size of antiderivative = 1.81

$$\int \sinh^4(c+dx)\left(a+b\sinh^3(c+dx)\right)\,dx$$
$$=\begin{cases} \frac{3ax\sinh^4(c+dx)}{8}-\frac{3ax\sinh^2(c+dx)\cosh^2(c+dx)}{4}+\frac{3ax\cosh^4(c+dx)}{8}+\frac{5a\sinh^3(c+dx)\cosh(c+dx)}{8d}-\frac{3a\sinh(c+dx)\cosh^3(c+dx)}{8d} \\ x\left(a+b\sinh^3(c)\right)\sinh^4(c) \end{cases}$$

input
```
integrate(sinh(d*x+c)**4*(a+b*sinh(d*x+c)**3),x)
```

output
```
Piecewise((3*a*x*sinh(c + d*x)**4/8 - 3*a*x*sinh(c + d*x)**2*cosh(c + d*x)
**2/4 + 3*a*x*cosh(c + d*x)**4/8 + 5*a*sinh(c + d*x)**3*cosh(c + d*x)/(8*d
) - 3*a*sinh(c + d*x)*cosh(c + d*x)**3/(8*d) + b*sinh(c + d*x)**6*cosh(c +
 d*x)/d - 2*b*sinh(c + d*x)**4*cosh(c + d*x)**3/d + 8*b*sinh(c + d*x)**2*c
osh(c + d*x)**5/(5*d) - 16*b*cosh(c + d*x)**7/(35*d), Ne(d, 0)), (x*(a + b
*sinh(c)**3)*sinh(c)**4, True))
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 164, normalized size of antiderivative = 1.55

$$\int \sinh^4(c+dx)\left(a+b\sinh^3(c+dx)\right)\,dx$$
$$=\frac{1}{64}a\left(24x+\frac{e^{(4dx+4c)}}{d}-\frac{8e^{(2dx+2c)}}{d}+\frac{8e^{(-2dx-2c)}}{d}-\frac{e^{(-4dx-4c)}}{d}\right)$$
$$-\frac{1}{4480}b\left(\frac{\left(49e^{(-2dx-2c)}-245e^{(-4dx-4c)}+1225e^{(-6dx-6c)}-5\right)e^{(7dx+7c)}}{d}+\frac{1225e^{(-dx-c)}-245e^{(-3dx-}\right.$$

input
```
integrate(sinh(d*x+c)^4*(a+b*sinh(d*x+c)^3),x, algorithm="maxima")
```

output
```
1/64*a*(24*x + e^(4*d*x + 4*c)/d - 8*e^(2*d*x + 2*c)/d + 8*e^(-2*d*x - 2*c
)/d - e^(-4*d*x - 4*c)/d) - 1/4480*b*((49*e^(-2*d*x - 2*c) - 245*e^(-4*d*x
 - 4*c) + 1225*e^(-6*d*x - 6*c) - 5)*e^(7*d*x + 7*c)/d + (1225*e^(-d*x - c
) - 245*e^(-3*d*x - 3*c) + 49*e^(-5*d*x - 5*c) - 5*e^(-7*d*x - 7*c))/d)
```

Giac [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 182, normalized size of antiderivative = 1.72

$$\int \sinh^4(c+dx)\left(a+b\sinh^3(c+dx)\right)\,dx = \frac{3}{8}ax + \frac{be^{(7\,dx+7\,c)}}{896\,d} - \frac{7\,be^{(5\,dx+5\,c)}}{640\,d} + \frac{ae^{(4\,dx+4\,c)}}{64\,d} + \frac{7\,be^{(3\,dx+3\,c)}}{128\,d} - \frac{ae^{(2\,dx+2\,c)}}{8\,d} - \frac{35\,be^{(dx+c)}}{128\,d} - \frac{35\,be^{(-dx-c)}}{128\,d} + \frac{ae^{(-2\,dx-2\,c)}}{8\,d} + \frac{7\,be^{(-3\,dx-3\,c)}}{128\,d} - \frac{ae^{(-4\,dx-4\,c)}}{64\,d} - \frac{7\,be^{(-5\,dx-5\,c)}}{640\,d} + \frac{be^{(-7\,dx-7\,c)}}{896\,d}$$

input
```
integrate(sinh(d*x+c)^4*(a+b*sinh(d*x+c)^3),x, algorithm="giac")
```

output
```
3/8*a*x + 1/896*b*e^(7*d*x + 7*c)/d - 7/640*b*e^(5*d*x + 5*c)/d + 1/64*a*e
^(4*d*x + 4*c)/d + 7/128*b*e^(3*d*x + 3*c)/d - 1/8*a*e^(2*d*x + 2*c)/d - 3
5/128*b*e^(d*x + c)/d - 35/128*b*e^(-d*x - c)/d + 1/8*a*e^(-2*d*x - 2*c)/d
 + 7/128*b*e^(-3*d*x - 3*c)/d - 1/64*a*e^(-4*d*x - 4*c)/d - 7/640*b*e^(-5*
d*x - 5*c)/d + 1/896*b*e^(-7*d*x - 7*c)/d
```

Mupad [B] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 85, normalized size of antiderivative = 0.80

$$\int \sinh^4(c+dx)\left(a+b\sinh^3(c+dx)\right)\,dx$$
$$= \frac{280\,b\cosh(c+d\,x)^3-280\,b\cosh(c+d\,x)-168\,b\cosh(c+d\,x)^5+40\,b\cosh(c+d\,x)^7-175\,a\cosh(c+d}{280\,d}$$

input
```
int(sinh(c + d*x)^4*(a + b*sinh(c + d*x)^3),x)
```

output
```
(280*b*cosh(c + d*x)^3 - 280*b*cosh(c + d*x) - 168*b*cosh(c + d*x)^5 + 40*
b*cosh(c + d*x)^7 - 175*a*cosh(c + d*x)*sinh(c + d*x) + 105*a*d*x + 70*a*c
osh(c + d*x)^3*sinh(c + d*x))/(280*d)
```

Reduce [B] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 179, normalized size of antiderivative = 1.69

$$\int \sinh^4(c+dx)\left(a+b\sinh^3(c+dx)\right)\,dx$$
$$= \frac{5e^{14dx+14c}b-49e^{12dx+12c}b+70e^{11dx+11c}a+245e^{10dx+10c}b-560e^{9dx+9c}a-1225e^{8dx+8c}b+1680e^{7dx+7c}ad}{4480e^{7dx+7c}d}$$

input
```
int(sinh(d*x+c)^4*(a+b*sinh(d*x+c)^3),x)
```

output
```
(5*e**(14*c + 14*d*x)*b - 49*e**(12*c + 12*d*x)*b + 70*e**(11*c + 11*d*x)*
a + 245*e**(10*c + 10*d*x)*b - 560*e**(9*c + 9*d*x)*a - 1225*e**(8*c + 8*d
*x)*b + 1680*e**(7*c + 7*d*x)*a*d*x - 1225*e**(6*c + 6*d*x)*b + 560*e**(5*
c + 5*d*x)*a + 245*e**(4*c + 4*d*x)*b - 70*e**(3*c + 3*d*x)*a - 49*e**(2*c
 + 2*d*x)*b + 5*b)/(4480*e**(7*c + 7*d*x)*d)
```

3.119 $\int \sinh^3(c+dx)\left(a+b\sinh^3(c+dx)\right)\,dx$

Optimal result

Integrand size = 21, antiderivative size = 99

$$\int \sinh^3(c+dx)\left(a+b\sinh^3(c+dx)\right)\,dx = -\frac{5bx}{16}-\frac{a\cosh(c+dx)}{d}+\frac{a\cosh^3(c+dx)}{3d}+\frac{5b\cosh(c+dx)\sinh(c+dx)}{16d}-\frac{5b\cosh(c+dx)\sinh^3(c+dx)}{24d}+\frac{b\cosh(c+dx)\sinh^5(c+dx)}{6d}$$

output
```
-5/16*b*x-a*cosh(d*x+c)/d+1/3*a*cosh(d*x+c)^3/d+5/16*b*cosh(d*x+c)*sinh(d*
x+c)/d-5/24*b*cosh(d*x+c)*sinh(d*x+c)^3/d+1/6*b*cosh(d*x+c)*sinh(d*x+c)^5/
d
```

Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 66, normalized size of antiderivative = 0.67

$$\int \sinh^3(c+dx)\left(a+b\sinh^3(c+dx)\right)\,dx$$
$$=\frac{-144a\cosh(c+dx)+16a\cosh(3(c+dx))+b(-60c-60dx+45\sinh(2(c+dx))-9\sinh(4(c+dx))+}{192d}$$

input
```
Integrate[Sinh[c + d*x]^3*(a + b*Sinh[c + d*x]^3),x]
```

output
```
(-144*a*Cosh[c + d*x] + 16*a*Cosh[3*(c + d*x)] + b*(-60*c - 60*d*x + 45*Si
nh[2*(c + d*x)] - 9*Sinh[4*(c + d*x)] + Sinh[6*(c + d*x)]))/(192*d)
```

Rubi [C] (verified)

Result contains complex when optimal does not.

Time = 0.33 (sec) , antiderivative size = 115, normalized size of antiderivative = 1.16, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.190, Rules used = {3042, 26, 3699, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sinh^3(c+dx)\left(a+b\sinh^3(c+dx)\right)\,dx$$
$$\Big\downarrow \text{3042}$$
$$\int i\sin(ic+idx)^3\left(a+ib\sin(ic+idx)^3\right)dx$$
$$\Big\downarrow \text{26}$$
$$i\int \sin(ic+idx)^3\left(ib\sin(ic+idx)^3+a\right)dx$$
$$\Big\downarrow \text{3699}$$
$$i\int \left(-ib\sinh^6(c+dx)-ia\sinh^3(c+dx)\right)dx$$

$\downarrow$ 2009

$$i\left(-\frac{ia\cosh^3(c+dx)}{3d}+\frac{ia\cosh(c+dx)}{d}-\frac{ib\sinh^5(c+dx)\cosh(c+dx)}{6d}+\frac{5ib\sinh^3(c+dx)\cosh(c+dx)}{24d}-\frac{5ib}{}\right.$$

input
```
Int[Sinh[c + d*x]^3*(a + b*Sinh[c + d*x]^3),x]
```

output
```
I*(((5*I)/16)*b*x + (I*a*Cosh[c + d*x])/d - ((I/3)*a*Cosh[c + d*x]^3)/d -
(((5*I)/16)*b*Cosh[c + d*x]*Sinh[c + d*x])/d + (((5*I)/24)*b*Cosh[c + d*x]
*Sinh[c + d*x]^3)/d - ((I/6)*b*Cosh[c + d*x]*Sinh[c + d*x]^5)/d)
```

Defintions of rubi rules used

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3699
```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^(n_
))^(p_.), x_Symbol] :> Int[ExpandTrig[sin[e + f*x]^m*(a + b*sin[e + f*x]^n)
^p, x], x] /; FreeQ[{a, b, e, f}, x] && IntegersQ[m, p] && (EqQ[n, 4] || Gt
Q[p, 0] || (EqQ[p, -1] && IntegerQ[n]))
```

Maple [A] (verified)

Time = 4.14 (sec) , antiderivative size = 71, normalized size of antiderivative = 0.72

method	result
parallelrisch	$\frac{-60dxb-144a\cosh(dx+c)+b\sinh(6dx+6c)-9b\sinh(4dx+4c)+45b\sinh(2dx+2c)+16a\cosh(3dx+3c)-128a}{192d}$
derivativedivides	$\frac{a\left(-\frac{2}{3}+\frac{\sinh(dx+c)^2}{3}\right)\cosh(dx+c)+b\left(\left(\frac{\sinh(dx+c)^5}{6}-\frac{5\sinh(dx+c)^3}{24}+\frac{5\sinh(dx+c)}{16}\right)\cosh(dx+c)-\frac{5dx}{16}-\frac{5c}{16}\right)}{d}$
default	$\frac{a\left(-\frac{2}{3}+\frac{\sinh(dx+c)^2}{3}\right)\cosh(dx+c)+b\left(\left(\frac{\sinh(dx+c)^5}{6}-\frac{5\sinh(dx+c)^3}{24}+\frac{5\sinh(dx+c)}{16}\right)\cosh(dx+c)-\frac{5dx}{16}-\frac{5c}{16}\right)}{d}$
parts	$\frac{a\left(-\frac{2}{3}+\frac{\sinh(dx+c)^2}{3}\right)\cosh(dx+c)}{d}+\frac{b\left(\left(\frac{\sinh(dx+c)^5}{6}-\frac{5\sinh(dx+c)^3}{24}+\frac{5\sinh(dx+c)}{16}\right)\cosh(dx+c)-\frac{5dx}{16}-\frac{5c}{16}\right)}{d}$
risch	$-\frac{5bx}{16}+\frac{b\,e^{6dx+6c}}{384d}-\frac{3\,e^{4dx+4c}b}{128d}+\frac{e^{3dx+3c}a}{24d}+\frac{15\,e^{2dx+2c}b}{128d}-\frac{3\,e^{dx+c}a}{8d}-\frac{3\,e^{-dx-c}a}{8d}-\frac{15\,e^{-2dx-2c}b}{128d}+$
orering	Expression too large to display

input
```
int(sinh(d*x+c)^3*(a+b*sinh(d*x+c)^3),x,method=_RETURNVERBOSE)
```

output
```
1/192*(-60*d*x*b-144*a*cosh(d*x+c)+b*sinh(6*d*x+6*c)-9*b*sinh(4*d*x+4*c)+4
5*b*sinh(2*d*x+2*c)+16*a*cosh(3*d*x+3*c)-128*a)/d
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 135, normalized size of antiderivative = 1.36

$$\int \sinh^3(c+dx)\left(a+b\sinh^3(c+dx)\right)\,dx$$
$$=\frac{3\,b\cosh\left(dx+c\right)\sinh\left(dx+c\right)^5+8\,a\cosh\left(dx+c\right)^3+24\,a\cosh\left(dx+c\right)\sinh\left(dx+c\right)^2+2\left(5\,b\cosh\left(dx+\right.\right.}{}$$

input
```
integrate(sinh(d*x+c)^3*(a+b*sinh(d*x+c)^3),x, algorithm="fricas")
```

output
```
1/96*(3*b*cosh(d*x + c)*sinh(d*x + c)^5 + 8*a*cosh(d*x + c)^3 + 24*a*cosh(
d*x + c)*sinh(d*x + c)^2 + 2*(5*b*cosh(d*x + c)^3 - 9*b*cosh(d*x + c))*sin
h(d*x + c)^3 - 30*b*d*x - 72*a*cosh(d*x + c) + 3*(b*cosh(d*x + c)^5 - 6*b*
cosh(d*x + c)^3 + 15*b*cosh(d*x + c))*sinh(d*x + c))/d
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 194 vs. $2(92) = 184$.

Time = 0.35 (sec) , antiderivative size = 194, normalized size of antiderivative = 1.96

$$\int \sinh^3(c+dx)\left(a+b\sinh^3(c+dx)\right)\,dx$$
$$= \begin{cases} \frac{a\sinh^2(c+dx)\cosh(c+dx)}{d} - \frac{2a\cosh^3(c+dx)}{3d} + \frac{5bx\sinh^6(c+dx)}{16} - \frac{15bx\sinh^4(c+dx)\cosh^2(c+dx)}{16} + \frac{15bx\sinh^2(c+dx)\cosh^4(c+d}{16} \\ x\left(a+b\sinh^3(c)\right)\sinh^3(c) \end{cases}$$

input
```
integrate(sinh(d*x+c)**3*(a+b*sinh(d*x+c)**3),x)
```

output
```
Piecewise((a*sinh(c + d*x)**2*cosh(c + d*x)/d - 2*a*cosh(c + d*x)**3/(3*d)
 + 5*b*x*sinh(c + d*x)**6/16 - 15*b*x*sinh(c + d*x)**4*cosh(c + d*x)**2/16
 + 15*b*x*sinh(c + d*x)**2*cosh(c + d*x)**4/16 - 5*b*x*cosh(c + d*x)**6/16
 + 11*b*sinh(c + d*x)**5*cosh(c + d*x)/(16*d) - 5*b*sinh(c + d*x)**3*cosh(
c + d*x)**3/(6*d) + 5*b*sinh(c + d*x)*cosh(c + d*x)**5/(16*d), Ne(d, 0)),
(x*(a + b*sinh(c)**3)*sinh(c)**3, True))
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 143, normalized size of antiderivative = 1.44

$$\int \sinh^3(c+dx)\left(a+b\sinh^3(c+dx)\right)\,dx =$$
$$-\frac{1}{384}b\left(\frac{\left(9e^{(-2dx-2c)} - 45e^{(-4dx-4c)} - 1\right)e^{(6dx+6c)}}{d} + \frac{120(dx+c)}{d} + \frac{45e^{(-2dx-2c)} - 9e^{(-4dx-4c)} + e^{(}}{d}\right.$$
$$+\frac{1}{24}a\left(\frac{e^{(3dx+3c)}}{d} - \frac{9e^{(dx+c)}}{d} - \frac{9e^{(-dx-c)}}{d} + \frac{e^{(-3dx-3c)}}{d}\right)$$

input
```
integrate(sinh(d*x+c)^3*(a+b*sinh(d*x+c)^3),x, algorithm="maxima")
```

output
```
-1/384*b*((9*e^(-2*d*x - 2*c) - 45*e^(-4*d*x - 4*c) - 1)*e^(6*d*x + 6*c)/d
 + 120*(d*x + c)/d + (45*e^(-2*d*x - 2*c) - 9*e^(-4*d*x - 4*c) + e^(-6*d*x
 - 6*c))/d) + 1/24*a*(e^(3*d*x + 3*c)/d - 9*e^(d*x + c)/d - 9*e^(-d*x - c)
/d + e^(-3*d*x - 3*c)/d)
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 152, normalized size of antiderivative = 1.54

$$\int \sinh^3(c+dx)\left(a+b\sinh^3(c+dx)\right)\,dx = -\frac{5}{16}\,bx + \frac{be^{(6\,dx+6\,c)}}{384\,d} - \frac{3\,be^{(4\,dx+4\,c)}}{128\,d} + \frac{ae^{(3\,dx+3\,c)}}{24\,d} + \frac{15\,be^{(2\,dx+2\,c)}}{128\,d} - \frac{3\,ae^{(dx+c)}}{8\,d} - \frac{3\,ae^{(-dx-c)}}{8\,d} - \frac{15\,be^{(-2\,dx-2\,c)}}{128\,d} + \frac{ae^{(-3\,dx-3\,c)}}{24\,d} + \frac{3\,be^{(-4\,dx-4\,c)}}{128\,d} - \frac{be^{(-6\,dx-6\,c)}}{384\,d}$$

input
```
integrate(sinh(d*x+c)^3*(a+b*sinh(d*x+c)^3),x, algorithm="giac")
```

output
```
-5/16*b*x + 1/384*b*e^(6*d*x + 6*c)/d - 3/128*b*e^(4*d*x + 4*c)/d + 1/24*a
*e^(3*d*x + 3*c)/d + 15/128*b*e^(2*d*x + 2*c)/d - 3/8*a*e^(d*x + c)/d - 3/
8*a*e^(-d*x - c)/d - 15/128*b*e^(-2*d*x - 2*c)/d + 1/24*a*e^(-3*d*x - 3*c)
/d + 3/128*b*e^(-4*d*x - 4*c)/d - 1/384*b*e^(-6*d*x - 6*c)/d
```

Mupad [B] (verification not implemented)

Time = 0.46 (sec) , antiderivative size = 67, normalized size of antiderivative = 0.68

$$\int \sinh^3(c+dx)\left(a+b\sinh^3(c+dx)\right)\,dx = \frac{\frac{a\cosh(3\,c+3\,d\,x)}{12} - \frac{3\,a\cosh(c+d\,x)}{4} + \frac{15\,b\sinh(2\,c+2\,d\,x)}{64} - \frac{3\,b\sinh(4\,c+4\,d\,x)}{64} + \frac{b\sinh(6\,c+6\,d\,x)}{192}}{d} - \frac{5\,b\,x}{16}$$

input
```
int(sinh(c + d*x)^3*(a + b*sinh(c + d*x)^3),x)
```

output
```
((a*cosh(3*c + 3*d*x))/12 - (3*a*cosh(c + d*x))/4 + (15*b*sinh(2*c + 2*d*x
))/64 - (3*b*sinh(4*c + 4*d*x))/64 + (b*sinh(6*c + 6*d*x))/192)/d - (5*b*x
)/16
```

Reduce [B] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 152, normalized size of antiderivative = 1.54

$$\int \sinh^3(c+dx)\left(a+b\sinh^3(c+dx)\right)\,dx$$
$$=\frac{e^{12dx+12c}b-9e^{10dx+10c}b+16e^{9dx+9c}a+45e^{8dx+8c}b-144e^{7dx+7c}a-120e^{6dx+6c}bdx-144e^{5dx+5c}a-45e^{4d}}{384e^{6dx+6c}d}$$

input
```
int(sinh(d*x+c)^3*(a+b*sinh(d*x+c)^3),x)
```

output
```
(e**(12*c + 12*d*x)*b - 9*e**(10*c + 10*d*x)*b + 16*e**(9*c + 9*d*x)*a + 4
5*e**(8*c + 8*d*x)*b - 144*e**(7*c + 7*d*x)*a - 120*e**(6*c + 6*d*x)*b*d*x
 - 144*e**(5*c + 5*d*x)*a - 45*e**(4*c + 4*d*x)*b + 16*e**(3*c + 3*d*x)*a
+ 9*e**(2*c + 2*d*x)*b - b)/(384*e**(6*c + 6*d*x)*d)
```

3.120 $\int \sinh^2(c+dx)\left(a+b\sinh^3(c+dx)\right)\,dx$

Optimal result

Integrand size = 21, antiderivative size = 70

$$\int \sinh^2(c+dx)\left(a+b\sinh^3(c+dx)\right)\,dx = -\frac{ax}{2}+\frac{b\cosh(c+dx)}{d}-\frac{2b\cosh^3(c+dx)}{3d}+\frac{b\cosh^5(c+dx)}{5d}+\frac{a\cosh(c+dx)\sinh(c+dx)}{2d}$$

output
```
-1/2*a*x+b*cosh(d*x+c)/d-2/3*b*cosh(d*x+c)^3/d+1/5*b*cosh(d*x+c)^5/d+1/2*a
*cosh(d*x+c)*sinh(d*x+c)/d
```

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 79, normalized size of antiderivative = 1.13

$$\int \sinh^2(c+dx)\left(a+b\sinh^3(c+dx)\right)\,dx = \frac{a(-c-dx)}{2d}+\frac{5b\cosh(c+dx)}{8d}-\frac{5b\cosh(3(c+dx))}{48d}+\frac{b\cosh(5(c+dx))}{80d}+\frac{a\sinh(2(c+dx))}{4d}$$

input
```
Integrate[Sinh[c + d*x]^2*(a + b*Sinh[c + d*x]^3),x]
```

output
```
(a*(-c - d*x))/(2*d) + (5*b*Cosh[c + d*x])/(8*d) - (5*b*Cosh[3*(c + d*x)])
/(48*d) + (b*Cosh[5*(c + d*x)])/(80*d) + (a*Sinh[2*(c + d*x)])/(4*d)
```

Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 70, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.190$, Rules used = {3042, 25, 3699, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sinh^2(c+dx)\left(a+b\sinh^3(c+dx)\right)\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int -\sin(ic+idx)^2\left(a+ib\sin(ic+idx)^3\right)dx$$

$$\Big\downarrow \text{25}$$

$$-\int \sin(ic+idx)^2\left(ib\sin(ic+idx)^3+a\right)dx$$

$$\Big\downarrow \text{3699}$$

$$-\int \left(-b\sinh^5(c+dx)-a\sinh^2(c+dx)\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{a\sinh(c+dx)\cosh(c+dx)}{2d}-\frac{ax}{2}+\frac{b\cosh^5(c+dx)}{5d}-\frac{2b\cosh^3(c+dx)}{3d}+\frac{b\cosh(c+dx)}{d}$$

input
```
Int[Sinh[c + d*x]^2*(a + b*Sinh[c + d*x]^3),x]
```

output
```
-1/2*(a*x) + (b*Cosh[c + d*x])/d - (2*b*Cosh[c + d*x]^3)/(3*d) + (b*Cosh[c
 + d*x]^5)/(5*d) + (a*Cosh[c + d*x]*Sinh[c + d*x])/(2*d)
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3699
```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^(n_
))^(p_.), x_Symbol] :> Int[ExpandTrig[sin[e + f*x]^m*(a + b*sin[e + f*x]^n)
^p, x], x] /; FreeQ[{a, b, e, f}, x] && IntegersQ[m, p] && (EqQ[n, 4] || Gt
Q[p, 0] || (EqQ[p, -1] && IntegerQ[n]))
```

Maple [A] (verified)

Time = 2.66 (sec) , antiderivative size = 60, normalized size of antiderivative = 0.86

method	result
derivativedivides	$\frac{a\left(\frac{\cosh(dx+c)\sinh(dx+c)}{2}-\frac{dx}{2}-\frac{c}{2}\right)+b\left(\frac{8}{15}+\frac{\sinh(dx+c)^4}{5}-\frac{4\sinh(dx+c)^2}{15}\right)\cosh(dx+c)}{d}$
default	$\frac{a\left(\frac{\cosh(dx+c)\sinh(dx+c)}{2}-\frac{dx}{2}-\frac{c}{2}\right)+b\left(\frac{8}{15}+\frac{\sinh(dx+c)^4}{5}-\frac{4\sinh(dx+c)^2}{15}\right)\cosh(dx+c)}{d}$
parallelrisch	$\frac{-120dxa+128b+150b\cosh(dx+c)+3b\cosh(5dx+5c)-25b\cosh(3dx+3c)+60a\sinh(2dx+2c)}{240d}$
parts	$\frac{a\left(\frac{\cosh(dx+c)\sinh(dx+c)}{2}-\frac{dx}{2}-\frac{c}{2}\right)}{d}+\frac{b\left(\frac{8}{15}+\frac{\sinh(dx+c)^4}{5}-\frac{4\sinh(dx+c)^2}{15}\right)\cosh(dx+c)}{d}$
risch	$-\frac{ax}{2}+\frac{b\,e^{5dx+5c}}{160d}-\frac{5\,e^{3dx+3c}b}{96d}+\frac{e^{2dx+2c}a}{8d}+\frac{5\,e^{dx+c}b}{16d}+\frac{5\,e^{-dx-c}b}{16d}-\frac{e^{-2dx-2c}a}{8d}-\frac{5\,e^{-3dx-3c}b}{96d}+\frac{b\,e^{-}}{}$
orering	$x\sinh(dx+c)^2\left(a+b\sinh(dx+c)^3\right)+\frac{\frac{1261\sinh(dx+c)\left(a+b\sinh(dx+c)^3\right)d\cosh(dx+c)}{450}+\frac{1261\sinh(dx+c)^{}}{300}}{d^2}$

input
```
int(sinh(d*x+c)^2*(a+b*sinh(d*x+c)^3),x,method=_RETURNVERBOSE)
```

output
```
1/d*(a*(1/2*cosh(d*x+c)*sinh(d*x+c)-1/2*d*x-1/2*c)+b*(8/15+1/5*sinh(d*x+c)
^4-4/15*sinh(d*x+c)^2)*cosh(d*x+c))
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 105, normalized size of antiderivative = 1.50

$$\int \sinh^2(c+dx)\left(a+b\sinh^3(c+dx)\right)\,dx = \frac{3\,b\cosh\left(dx+c\right)^5+15\,b\cosh\left(dx+c\right)\sinh\left(dx+c\right)^4-25\,b\cosh\left(dx+c\right)^3-120\,adx+120\,a\cosh\left(dx+\right.}{240\,d}$$

input
```
integrate(sinh(d*x+c)^2*(a+b*sinh(d*x+c)^3),x, algorithm="fricas")
```

output
```
1/240*(3*b*cosh(d*x + c)^5 + 15*b*cosh(d*x + c)*sinh(d*x + c)^4 - 25*b*cos
h(d*x + c)^3 - 120*a*d*x + 120*a*cosh(d*x + c)*sinh(d*x + c) + 15*(2*b*cos
h(d*x + c)^3 - 5*b*cosh(d*x + c))*sinh(d*x + c)^2 + 150*b*cosh(d*x + c))/d
```

Sympy [A] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 117, normalized size of antiderivative = 1.67

$$\int \sinh^2(c+dx)\left(a+b\sinh^3(c+dx)\right)\,dx = \begin{cases} \frac{ax\sinh^2(c+dx)}{2}-\frac{ax\cosh^2(c+dx)}{2}+\frac{a\sinh(c+dx)\cosh(c+dx)}{2d}+\frac{b\sinh^4(c+dx)\cosh(c+dx)}{d}-\frac{4b\sinh^2(c+dx)\cosh^3(c+dx)}{3d}+\frac{8b}{} \\ x\left(a+b\sinh^3(c)\right)\sinh^2(c) \end{cases}$$

input
```
integrate(sinh(d*x+c)**2*(a+b*sinh(d*x+c)**3),x)
```

output
```
Piecewise((a*x*sinh(c + d*x)**2/2 - a*x*cosh(c + d*x)**2/2 + a*sinh(c + d*
x)*cosh(c + d*x)/(2*d) + b*sinh(c + d*x)**4*cosh(c + d*x)/d - 4*b*sinh(c +
 d*x)**2*cosh(c + d*x)**3/(3*d) + 8*b*cosh(c + d*x)**5/(15*d), Ne(d, 0)),
(x*(a + b*sinh(c)**3)*sinh(c)**2, True))
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 120, normalized size of antiderivative = 1.71

$$\int \sinh^2(c+dx) \left(a+b\sinh^3(c+dx)\right) dx = -\frac{1}{8} a \left(4x - \frac{e^{(2dx+2c)}}{d} + \frac{e^{(-2dx-2c)}}{d}\right) + \frac{1}{480} b \left(\frac{3e^{(5dx+5c)}}{d} - \frac{25e^{(3dx+3c)}}{d} + \frac{150e^{(dx+c)}}{d} + \frac{150e^{(-dx-c)}}{d} - \frac{25e^{(-3dx-3c)}}{d} + \frac{3e^{(-5dx-5c)}}{d}\right)$$

input
```
integrate(sinh(d*x+c)^2*(a+b*sinh(d*x+c)^3),x, algorithm="maxima")
```

output
```
-1/8*a*(4*x - e^(2*d*x + 2*c)/d + e^(-2*d*x - 2*c)/d) + 1/480*b*(3*e^(5*d*
x + 5*c)/d - 25*e^(3*d*x + 3*c)/d + 150*e^(d*x + c)/d + 150*e^(-d*x - c)/d
 - 25*e^(-3*d*x - 3*c)/d + 3*e^(-5*d*x - 5*c)/d)
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 122, normalized size of antiderivative = 1.74

$$\int \sinh^2(c+dx) \left(a+b\sinh^3(c+dx)\right) dx = -\frac{1}{2} ax + \frac{be^{(5dx+5c)}}{160d} - \frac{5be^{(3dx+3c)}}{96d} + \frac{ae^{(2dx+2c)}}{8d} + \frac{5be^{(dx+c)}}{16d} + \frac{5be^{(-dx-c)}}{16d} - \frac{ae^{(-2dx-2c)}}{8d} - \frac{5be^{(-3dx-3c)}}{96d} + \frac{be^{(-5dx-5c)}}{160d}$$

input
```
integrate(sinh(d*x+c)^2*(a+b*sinh(d*x+c)^3),x, algorithm="giac")
```

output
```
-1/2*a*x + 1/160*b*e^(5*d*x + 5*c)/d - 5/96*b*e^(3*d*x + 3*c)/d + 1/8*a*e^
(2*d*x + 2*c)/d + 5/16*b*e^(d*x + c)/d + 5/16*b*e^(-d*x - c)/d - 1/8*a*e^(
-2*d*x - 2*c)/d - 5/96*b*e^(-3*d*x - 3*c)/d + 1/160*b*e^(-5*d*x - 5*c)/d
```

Mupad [B] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 55, normalized size of antiderivative = 0.79

$$\int \sinh^2(c+dx)\left(a+b\sinh^3(c+dx)\right)\,dx = \frac{b\cosh(c+d\,x)-\frac{2\,b\cosh(c+d\,x)^3}{3}+\frac{b\cosh(c+d\,x)^5}{5}+\frac{a\cosh(c+d\,x)\sinh(c+d\,x)}{2}}{d}-\frac{a\,x}{2}$$

input
```
int(sinh(c + d*x)^2*(a + b*sinh(c + d*x)^3),x)
```

output
```
(b*cosh(c + d*x) - (2*b*cosh(c + d*x)^3)/3 + (b*cosh(c + d*x)^5)/5 + (a*co
sh(c + d*x)*sinh(c + d*x))/2)/d - (a*x)/2
```

Reduce [B] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 127, normalized size of antiderivative = 1.81

$$\int \sinh^2(c+dx)\left(a+b\sinh^3(c+dx)\right)\,dx = \frac{3e^{10dx+10c}b-25e^{8dx+8c}b+60e^{7dx+7c}a+150e^{6dx+6c}b-240e^{5dx+5c}adx+150e^{4dx+4c}b-60e^{3dx+3c}a-25e^{2c}}{480e^{5dx+5c}d}$$

input
```
int(sinh(d*x+c)^2*(a+b*sinh(d*x+c)^3),x)
```

output
```
(3*e**(10*c + 10*d*x)*b - 25*e**(8*c + 8*d*x)*b + 60*e**(7*c + 7*d*x)*a +
150*e**(6*c + 6*d*x)*b - 240*e**(5*c + 5*d*x)*a*d*x + 150*e**(4*c + 4*d*x)
*b - 60*e**(3*c + 3*d*x)*a - 25*e**(2*c + 2*d*x)*b + 3*b)/(480*e**(5*c + 5
*d*x)*d)
```

3.121 $\int \sinh(c+dx)\left(a+b\sinh^3(c+dx)\right)\,dx$

Optimal result

Integrand size = 19, antiderivative size = 60

$$\int \sinh(c+dx)\left(a+b\sinh^3(c+dx)\right)\,dx = \frac{3bx}{8}+\frac{a\cosh(c+dx)}{d} - \frac{3b\cosh(c+dx)\sinh(c+dx)}{8d} + \frac{b\cosh(c+dx)\sinh^3(c+dx)}{4d}$$

output
```
3/8*b*x+a*cosh(d*x+c)/d-3/8*b*cosh(d*x+c)*sinh(d*x+c)/d+1/4*b*cosh(d*x+c)*
sinh(d*x+c)^3/d
```

Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 45, normalized size of antiderivative = 0.75

$$\int \sinh(c+dx)\left(a+b\sinh^3(c+dx)\right)\,dx = \frac{32a\cosh(c+dx)+b(12(c+dx)-8\sinh(2(c+dx))+\sinh(4(c+dx)))}{32d}$$

input
```
Integrate[Sinh[c + d*x]*(a + b*Sinh[c + d*x]^3),x]
```

output
```
(32*a*Cosh[c + d*x] + b*(12*(c + d*x) - 8*Sinh[2*(c + d*x)] + Sinh[4*(c +
d*x)]))/(32*d)
```

Rubi [C] (verified)

Result contains complex when optimal does not.

Time = 0.27 (sec) , antiderivative size = 73, normalized size of antiderivative = 1.22, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.211$, Rules used = {3042, 26, 3699, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sinh(c+dx)\left(a+b\sinh^3(c+dx)\right)\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int -i\sin(ic+idx)\left(a+ib\sin(ic+idx)^3\right)dx$$

$$\Big\downarrow \text{26}$$

$$-i\int \sin(ic+idx)\left(ib\sin(ic+idx)^3+a\right)dx$$

$$\Big\downarrow \text{3699}$$

$$-i\int \left(ib\sinh^4(c+dx)+ia\sinh(c+dx)\right)dx$$

$$\Big\downarrow \text{2009}$$

$$-i\left(\frac{ia\cosh(c+dx)}{d}+\frac{ib\sinh^3(c+dx)\cosh(c+dx)}{4d}-\frac{3ib\sinh(c+dx)\cosh(c+dx)}{8d}+\frac{3ibx}{8}\right)$$

input
```
Int[Sinh[c + d*x]*(a + b*Sinh[c + d*x]^3),x]
```

output
```
(-I)*(((3*I)/8)*b*x + (I*a*Cosh[c + d*x])/d - (((3*I)/8)*b*Cosh[c + d*x]*S
inh[c + d*x])/d + ((I/4)*b*Cosh[c + d*x]*Sinh[c + d*x]^3)/d)
```

Defintions of rubi rules used

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3699
```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^(n_
))^(p_.), x_Symbol] :> Int[ExpandTrig[sin[e + f*x]^m*(a + b*sin[e + f*x]^n)
^p, x], x] /; FreeQ[{a, b, e, f}, x] && IntegersQ[m, p] && (EqQ[n, 4] || Gt
Q[p, 0] || (EqQ[p, -1] && IntegerQ[n]))
```

Maple [A] (verified)

Time = 2.14 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.78

method	result
parallelrisch	$\frac{12dxb+32a\cosh(dx+c)+b\sinh(4dx+4c)-8b\sinh(2dx+2c)+32a}{32d}$
derivativedivides	$\frac{a\cosh(dx+c)+b\left(\left(\frac{\sinh(dx+c)^3}{4}-\frac{3\sinh(dx+c)}{8}\right)\cosh(dx+c)+\frac{3dx}{8}+\frac{3c}{8}\right)}{d}$
default	$\frac{a\cosh(dx+c)+b\left(\left(\frac{\sinh(dx+c)^3}{4}-\frac{3\sinh(dx+c)}{8}\right)\cosh(dx+c)+\frac{3dx}{8}+\frac{3c}{8}\right)}{d}$
parts	$\frac{a\cosh(dx+c)}{d}+\frac{b\left(\left(\frac{\sinh(dx+c)^3}{4}-\frac{3\sinh(dx+c)}{8}\right)\cosh(dx+c)+\frac{3dx}{8}+\frac{3c}{8}\right)}{d}$
risch	$\frac{3bx}{8}+\frac{e^{4dx+4c}b}{64d}-\frac{e^{2dx+2c}b}{8d}+\frac{e^{dx+c}a}{2d}+\frac{e^{-dx-c}a}{2d}+\frac{e^{-2dx-2c}b}{8d}-\frac{e^{-4dx-4c}b}{64d}$
orering	$x\sinh(dx+c)\left(a+b\sinh(dx+c)^3\right)+\frac{\frac{21d\cosh(dx+c)\left(a+b\sinh(dx+c)^3\right)}{16}+\frac{63\sinh(dx+c)^3bd\cosh(dx+c)}{16}}{d^2}$ -

input
```
int(sinh(d*x+c)*(a+b*sinh(d*x+c)^3),x,method=_RETURNVERBOSE)
```

output
```
1/32*(12*d*x*b+32*a*cosh(d*x+c)+b*sinh(4*d*x+4*c)-8*b*sinh(2*d*x+2*c)+32*a
)/d
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 63, normalized size of antiderivative = 1.05

$$\int \sinh(c+dx)\left(a+b\sinh^3(c+dx)\right)\,dx$$
$$=\frac{b\cosh\left(dx+c\right)\sinh\left(dx+c\right)^3+3\,bdx+8\,a\cosh\left(dx+c\right)+\left(b\cosh\left(dx+c\right)^3-4\,b\cosh\left(dx+c\right)\right)\sinh\left(d\right.}{8\,d}$$

input
```
integrate(sinh(d*x+c)*(a+b*sinh(d*x+c)^3),x, algorithm="fricas")
```

output
```
1/8*(b*cosh(d*x + c)*sinh(d*x + c)^3 + 3*b*d*x + 8*a*cosh(d*x + c) + (b*co
sh(d*x + c)^3 - 4*b*cosh(d*x + c))*sinh(d*x + c))/d
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 121 vs. 2(56) = 112.

Time = 0.16 (sec) , antiderivative size = 121, normalized size of antiderivative = 2.02

$$\int \sinh(c+dx)\left(a+b\sinh^3(c+dx)\right)\,dx$$
$$=\begin{cases}\frac{a\cosh\left(c+dx\right)}{d}+\frac{3bx\sinh^4\left(c+dx\right)}{8}-\frac{3bx\sinh^2\left(c+dx\right)\cosh^2\left(c+dx\right)}{4}+\frac{3bx\cosh^4\left(c+dx\right)}{8}+\frac{5b\sinh^3\left(c+dx\right)\cosh\left(c+dx\right)}{8d}-\frac{3b\sinh\left(c+\right.}{} \\ x\left(a+b\sinh^3\left(c\right)\right)\sinh\left(c\right)\end{cases}$$

input
```
integrate(sinh(d*x+c)*(a+b*sinh(d*x+c)**3),x)
```

output
```
Piecewise((a*cosh(c + d*x)/d + 3*b*x*sinh(c + d*x)**4/8 - 3*b*x*sinh(c + d
*x)**2*cosh(c + d*x)**2/4 + 3*b*x*cosh(c + d*x)**4/8 + 5*b*sinh(c + d*x)**
3*cosh(c + d*x)/(8*d) - 3*b*sinh(c + d*x)*cosh(c + d*x)**3/(8*d), Ne(d, 0)
), (x*(a + b*sinh(c)**3)*sinh(c), True))
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 74, normalized size of antiderivative = 1.23

$$\int \sinh(c+dx)\left(a+b\sinh^3(c+dx)\right)\,dx$$
$$=\frac{1}{64}b\left(24\,x+\frac{e^{(4\,dx+4\,c)}}{d}-\frac{8\,e^{(2\,dx+2\,c)}}{d}+\frac{8\,e^{(-2\,dx-2\,c)}}{d}-\frac{e^{(-4\,dx-4\,c)}}{d}\right)+\frac{a\cosh\left(dx+c\right)}{d}$$

input
```
integrate(sinh(d*x+c)*(a+b*sinh(d*x+c)^3),x, algorithm="maxima")
```

output
```
1/64*b*(24*x + e^(4*d*x + 4*c)/d - 8*e^(2*d*x + 2*c)/d + 8*e^(-2*d*x - 2*c
)/d - e^(-4*d*x - 4*c)/d) + a*cosh(d*x + c)/d
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 92, normalized size of antiderivative = 1.53

$$\int \sinh(c+dx)\left(a+b\sinh^3(c+dx)\right)\,dx=\frac{3}{8}\,bx+\frac{be^{(4\,dx+4\,c)}}{64\,d}-\frac{be^{(2\,dx+2\,c)}}{8\,d}+\frac{ae^{(dx+c)}}{2\,d}+\frac{ae^{(-dx-c)}}{2\,d}+\frac{be^{(-2\,dx-2\,c)}}{8\,d}-\frac{be^{(-4\,dx-4\,c)}}{64\,d}$$

input
```
integrate(sinh(d*x+c)*(a+b*sinh(d*x+c)^3),x, algorithm="giac")
```

output
```
3/8*b*x + 1/64*b*e^(4*d*x + 4*c)/d - 1/8*b*e^(2*d*x + 2*c)/d + 1/2*a*e^(d*
x + c)/d + 1/2*a*e^(-d*x - c)/d + 1/8*b*e^(-2*d*x - 2*c)/d - 1/64*b*e^(-4*
d*x - 4*c)/d
```

Mupad [B] (verification not implemented)

Time = 1.68 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.70

$$\int \sinh(c+dx)\left(a+b\sinh^3(c+dx)\right)\,dx = \frac{3\,b\,x}{8}+\frac{a\cosh(c+d\,x)-\frac{b\sinh(2\,c+2\,d\,x)}{4}+\frac{b\sinh(4\,c+4\,d\,x)}{32}}{d}$$

input
```
int(sinh(c + d*x)*(a + b*sinh(c + d*x)^3),x)
```

output
```
(3*b*x)/8 + (a*cosh(c + d*x) - (b*sinh(2*c + 2*d*x))/4 + (b*sinh(4*c + 4*d
*x))/32)/d
```

Reduce [B] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 100, normalized size of antiderivative = 1.67

$$\int \sinh(c+dx)\left(a+b\sinh^3(c+dx)\right)\,dx = \frac{e^{8dx+8c}b-8e^{6dx+6c}b+32e^{5dx+5c}a+24e^{4dx+4c}bdx+32e^{3dx+3c}a+8e^{2dx+2c}b-b}{64e^{4dx+4c}d}$$

input
```
int(sinh(d*x+c)*(a+b*sinh(d*x+c)^3),x)
```

output
```
(e**(8*c + 8*d*x)*b - 8*e**(6*c + 6*d*x)*b + 32*e**(5*c + 5*d*x)*a + 24*e*
*(4*c + 4*d*x)*b*d*x + 32*e**(3*c + 3*d*x)*a + 8*e**(2*c + 2*d*x)*b - b)/(
64*e**(4*c + 4*d*x)*d)
```

3.122 $\int \left(a+b\sinh^3(c+dx)\right)\,dx$

Optimal result

Integrand size = 12, antiderivative size = 32

$$\int \left(a+b\sinh^3(c+dx)\right)\,dx=ax-\frac{b\cosh(c+dx)}{d}+\frac{b\cosh^3(c+dx)}{3d}$$

output
```
a*x-b*cosh(d*x+c)/d+1/3*b*cosh(d*x+c)^3/d
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.06

$$\int \left(a+b\sinh^3(c+dx)\right)\,dx=ax-\frac{3b\cosh(c+dx)}{4d}+\frac{b\cosh(3(c+dx))}{12d}$$

input
```
Integrate[a + b*Sinh[c + d*x]^3,x]
```

output
```
a*x - (3*b*Cosh[c + d*x])/(4*d) + (b*Cosh[3*(c + d*x)])/(12*d)
```

Rubi [A] (verified)

Time = 0.17 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.083$, Rules used = {2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(a+b\sinh^3(c+dx)\right)\,dx$$

$$\Big\downarrow \text{ 2009}$$

$$ax+\frac{b\cosh^3(c+dx)}{3d}-\frac{b\cosh(c+dx)}{d}$$

input
```
Int[a + b*Sinh[c + d*x]^3,x]
```

output
```
a*x - (b*Cosh[c + d*x])/d + (b*Cosh[c + d*x]^3)/(3*d)
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.95 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.88

method	result
default	$ax+\frac{b\left(-\frac{2}{3}+\frac{\sinh(dx+c)^2}{3}\right)\cosh(dx+c)}{d}$
parts	$ax+\frac{b\left(-\frac{2}{3}+\frac{\sinh(dx+c)^2}{3}\right)\cosh(dx+c)}{d}$
parallelrisch	$\frac{b(-8+\cosh(3dx+3c)-9\cosh(dx+c))}{12d}+ax$
derivativedivides	$\frac{(dx+c)a+b\left(-\frac{2}{3}+\frac{\sinh(dx+c)^2}{3}\right)\cosh(dx+c)}{d}$
risch	$ax+\frac{e^{3dx+3c}b}{24d}-\frac{3\,e^{dx+c}b}{8d}-\frac{3\,e^{-dx-c}b}{8d}+\frac{e^{-3dx-3c}b}{24d}$
orering	$x\left(a+b\sinh\left(dx+c\right)^3\right)+\frac{10\sinh(dx+c)^2b\cosh(dx+c)}{3d}-\frac{10x\left(6\sinh(dx+c)b\,d^2\cosh(dx+c)^2+3\sinh(dx+\right.}{9d^2}$

input
```
int(a+b*sinh(d*x+c)^3,x,method=_RETURNVERBOSE)
```

output
```
a*x+b/d*(-2/3+1/3*sinh(d*x+c)^2)*cosh(d*x+c)
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 47, normalized size of antiderivative = 1.47

$$\int \left(a+b\sinh^3(c+dx)\right)\,dx = \frac{b\cosh\left(dx+c\right)^3+3\,b\cosh\left(dx+c\right)\sinh\left(dx+c\right)^2+12\,adx-9\,b\cosh\left(dx+c\right)}{12\,d}$$

input
```
integrate(a+b*sinh(d*x+c)^3,x, algorithm="fricas")
```

output
```
1/12*(b*cosh(d*x + c)^3 + 3*b*cosh(d*x + c)*sinh(d*x + c)^2 + 12*a*d*x - 9
*b*cosh(d*x + c))/d
```

Sympy [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 41, normalized size of antiderivative = 1.28

$$\int \left(a + b\sinh^3(c + dx)\right)\,dx = ax + b\left(\begin{cases} \frac{\sinh^2{(c+dx)}\cosh{(c+dx)}}{d} - \frac{2\cosh^3{(c+dx)}}{3d} & \text{for } d \neq 0 \\ x\sinh^3{(c)} & \text{otherwise} \end{cases}\right)$$

input
```
integrate(a+b*sinh(d*x+c)**3,x)
```

output
```
a*x + b*Piecewise((sinh(c + d*x)**2*cosh(c + d*x)/d - 2*cosh(c + d*x)**3/(
3*d), Ne(d, 0)), (x*sinh(c)**3, True))
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 59, normalized size of antiderivative = 1.84

$$\int \left(a+b\sinh^3(c+dx)\right)\,dx = ax + \frac{1}{24}\,b\left(\frac{e^{(3\,dx+3\,c)}}{d} - \frac{9\,e^{(dx+c)}}{d} - \frac{9\,e^{(-dx-c)}}{d} + \frac{e^{(-3\,dx-3\,c)}}{d}\right)$$

input
```
integrate(a+b*sinh(d*x+c)^3,x, algorithm="maxima")
```

output
```
a*x + 1/24*b*(e^(3*d*x + 3*c)/d - 9*e^(d*x + c)/d - 9*e^(-d*x - c)/d + e^(
-3*d*x - 3*c)/d)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 59, normalized size of antiderivative = 1.84

$$\int \left(a+b\sinh^3(c+dx)\right)\,dx = ax + \frac{1}{24}\,b\left(\frac{e^{(3\,dx+3\,c)}}{d} - \frac{9\,e^{(dx+c)}}{d} - \frac{9\,e^{(-dx-c)}}{d} + \frac{e^{(-3\,dx-3\,c)}}{d}\right)$$

input
```
integrate(a+b*sinh(d*x+c)^3,x, algorithm="giac")
```

output
```
a*x + 1/24*b*(e^(3*d*x + 3*c)/d - 9*e^(d*x + c)/d - 9*e^(-d*x - c)/d + e^(
-3*d*x - 3*c)/d)
```

Mupad [B] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.91

$$\int \left(a+b\sinh^3(c+dx)\right)\,dx=a\,x-\frac{b\cosh(c+d\,x)-\frac{b\cosh(c+d\,x)^3}{3}}{d}$$

input
```
int(a + b*sinh(c + d*x)^3,x)
```

output
```
a*x - (b*cosh(c + d*x) - (b*cosh(c + d*x)^3)/3)/d
```

Reduce [B] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 72, normalized size of antiderivative = 2.25

$$\int \left(a+b\sinh^3(c+dx)\right)\,dx=\frac{e^{6dx+6c}b-9e^{4dx+4c}b+24e^{3dx+3c}adx-9e^{2dx+2c}b+b}{24e^{3dx+3c}d}$$

input
```
int(a+b*sinh(d*x+c)^3,x)
```

output
```
(e**(6*c + 6*d*x)*b - 9*e**(4*c + 4*d*x)*b + 24*e**(3*c + 3*d*x)*a*d*x - 9
*e**(2*c + 2*d*x)*b + b)/(24*e**(3*c + 3*d*x)*d)
```

3.123 $\int \operatorname{csch}(c+dx)\left(a+b\sinh^3(c+dx)\right)\,dx$

Optimal result

Integrand size = 19, antiderivative size = 40

$$\int \operatorname{csch}(c+dx)\left(a+b\sinh^3(c+dx)\right)\,dx=-\frac{bx}{2}-\frac{a\operatorname{arctanh}(\cosh(c+dx))}{d}+\frac{b\cosh(c+dx)\sinh(c+dx)}{2d}$$

output
```
-1/2*b*x-a*arctanh(cosh(d*x+c))/d+1/2*b*cosh(d*x+c)*sinh(d*x+c)/d
```

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.15

$$\int \operatorname{csch}(c+dx)\left(a+b\sinh^3(c+dx)\right)\,dx=\frac{b(-c-dx)}{2d}-\frac{a\operatorname{arctanh}(\cosh(c+dx))}{d}+\frac{b\sinh(2(c+dx))}{4d}$$

input
```
Integrate[Csch[c + d*x]*(a + b*Sinh[c + d*x]^3),x]
```

output
```
(b*(-c - d*x))/(2*d) - (a*ArcTanh[Cosh[c + d*x]])/d + (b*Sinh[2*(c + d*x)]
)/(4*d)
```

Rubi [C] (verified)

Result contains complex when optimal does not.

Time = 0.26 (sec) , antiderivative size = 50, normalized size of antiderivative = 1.25, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.211$, Rules used = {3042, 26, 3699, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \operatorname{csch}(c+dx)\left(a+b\sinh^3(c+dx)\right)\,dx$$

$$\Big\downarrow \ 3042$$

$$\int \frac{i\left(a+ib\sin(ic+idx)^3\right)}{\sin(ic+idx)}dx$$

$$\Big\downarrow \ 26$$

$$i\int \frac{ib\sin(ic+idx)^3+a}{\sin(ic+idx)}dx$$

$$\Big\downarrow \ 3699$$

$$i\int \left(-ib\sinh^2(c+dx)-ia\operatorname{csch}(c+dx)\right)dx$$

$$\Big\downarrow \ 2009$$

$$i\left(\frac{ia\operatorname{arctanh}(\cosh(c+dx))}{d}-\frac{ib\sinh(c+dx)\cosh(c+dx)}{2d}+\frac{ibx}{2}\right)$$

input
```
Int[Csch[c + d*x]*(a + b*Sinh[c + d*x]^3),x]
```

output
```
I*((I/2)*b*x + (I*a*ArcTanh[Cosh[c + d*x]])/d - ((I/2)*b*Cosh[c + d*x]*Sin
h[c + d*x])/d)
```

Defintions of rubi rules used

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3699
```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^(n_
))^(p_.), x_Symbol] :> Int[ExpandTrig[sin[e + f*x]^m*(a + b*sin[e + f*x]^n)
^p, x], x] /; FreeQ[{a, b, e, f}, x] && IntegersQ[m, p] && (EqQ[n, 4] || Gt
Q[p, 0] || (EqQ[p, -1] && IntegerQ[n]))
```

Maple [A] (verified)

Time = 0.41 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.90

method	result	size
parallelrisch	$\frac{-2dxb+4a\ln\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+b\sinh(2dx+2c)}{4d}$	36
derivativedivides	$\frac{-2a\ \operatorname{arctanh}\left(e^{dx+c}\right)+b\left(\frac{\cosh(dx+c)\sinh(dx+c)}{2}-\frac{dx}{2}-\frac{c}{2}\right)}{d}$	40
default	$\frac{-2a\ \operatorname{arctanh}\left(e^{dx+c}\right)+b\left(\frac{\cosh(dx+c)\sinh(dx+c)}{2}-\frac{dx}{2}-\frac{c}{2}\right)}{d}$	40
risch	$-\frac{bx}{2}+\frac{e^{2dx+2c}b}{8d}-\frac{e^{-2dx-2c}b}{8d}+\frac{a\ln\left(e^{dx+c}-1\right)}{d}-\frac{a\ln\left(e^{dx+c}+1\right)}{d}$	65

input
```
int(csch(d*x+c)*(a+b*sinh(d*x+c)^3),x,method=_RETURNVERBOSE)
```

output
```
1/4*(-2*d*x*b+4*a*ln(tanh(1/2*d*x+1/2*c))+b*sinh(2*d*x+2*c))/d
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 258 vs. $2(36) = 72$.

Time = 0.12 (sec) , antiderivative size = 258, normalized size of antiderivative = 6.45

$$\int \operatorname{csch}(c+dx)\left(a+b\sinh^3(c+dx)\right)\,dx =$$
$$-\frac{4\,bdx\cosh\left(dx+c\right)^2 - b\cosh\left(dx+c\right)^4 - 4\,b\cosh\left(dx+c\right)\sinh\left(dx+c\right)^3 - b\sinh\left(dx+c\right)^4 + 2\left(2\,bdx\right.}{}$$

input
```
integrate(csch(d*x+c)*(a+b*sinh(d*x+c)^3),x, algorithm="fricas")
```

output
```
-1/8*(4*b*d*x*cosh(d*x + c)^2 - b*cosh(d*x + c)^4 - 4*b*cosh(d*x + c)*sinh
(d*x + c)^3 - b*sinh(d*x + c)^4 + 2*(2*b*d*x - 3*b*cosh(d*x + c)^2)*sinh(d
*x + c)^2 + 8*(a*cosh(d*x + c)^2 + 2*a*cosh(d*x + c)*sinh(d*x + c) + a*sin
h(d*x + c)^2)*log(cosh(d*x + c) + sinh(d*x + c) + 1) - 8*(a*cosh(d*x + c)^
2 + 2*a*cosh(d*x + c)*sinh(d*x + c) + a*sinh(d*x + c)^2)*log(cosh(d*x + c)
 + sinh(d*x + c) - 1) + 4*(2*b*d*x*cosh(d*x + c) - b*cosh(d*x + c)^3)*sinh
(d*x + c) + b)/(d*cosh(d*x + c)^2 + 2*d*cosh(d*x + c)*sinh(d*x + c) + d*si
nh(d*x + c)^2)
```

Sympy [F]

$$\int \operatorname{csch}(c+dx)\left(a+b\sinh^3(c+dx)\right)\,dx = \int \left(a+b\sinh^3\left(c+dx\right)\right)\operatorname{csch}\left(c+dx\right)dx$$

input
```
integrate(csch(d*x+c)*(a+b*sinh(d*x+c)**3),x)
```

output
```
Integral((a + b*sinh(c + d*x)**3)*csch(c + d*x), x)
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 50, normalized size of antiderivative = 1.25

$$\int \operatorname{csch}(c+dx)\left(a+b\sinh^3(c+dx)\right)\,dx=-\frac{1}{8}b\left(4x-\frac{e^{(2dx+2c)}}{d}+\frac{e^{(-2dx-2c)}}{d}\right)+\frac{a\log\left(\tanh\left(\frac{1}{2}dx+\frac{1}{2}c\right)\right)}{d}$$

input
```
integrate(csch(d*x+c)*(a+b*sinh(d*x+c)^3),x, algorithm="maxima")
```

output
```
-1/8*b*(4*x - e^(2*d*x + 2*c)/d + e^(-2*d*x - 2*c)/d) + a*log(tanh(1/2*d*x
 + 1/2*c))/d
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 62, normalized size of antiderivative = 1.55

$$\int \operatorname{csch}(c+dx)\left(a+b\sinh^3(c+dx)\right)\,dx=-\frac{4(dx+c)b-be^{(2dx+2c)}+be^{(-2dx-2c)}+8a\log\left(e^{(dx+c)}+1\right)-8a\log\left(\left|e^{(dx+c)}-1\right|\right)}{8d}$$

input
```
integrate(csch(d*x+c)*(a+b*sinh(d*x+c)^3),x, algorithm="giac")
```

output
```
-1/8*(4*(d*x + c)*b - b*e^(2*d*x + 2*c) + b*e^(-2*d*x - 2*c) + 8*a*log(e^(
d*x + c) + 1) - 8*a*log(abs(e^(d*x + c) - 1)))/d
```

Mupad [B] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 73, normalized size of antiderivative = 1.82

$$\int \operatorname{csch}(c+dx)\left(a+b\sinh^3(c+dx)\right)\,dx=\frac{b\,\mathrm{e}^{2\,c+2\,d\,x}}{8\,d}-\frac{b\,\mathrm{e}^{-2\,c-2\,d\,x}}{8\,d}-\frac{b\,x}{2}-\frac{2\,\mathrm{atan}\left(\frac{a\,\mathrm{e}^{d\,x}\,\mathrm{e}^{c}\,\sqrt{-d^2}}{d\,\sqrt{a^2}}\right)\,\sqrt{a^2}}{\sqrt{-d^2}}$$

input
```
int((a + b*sinh(c + d*x)^3)/sinh(c + d*x),x)
```

output
```
(b*exp(2*c + 2*d*x))/(8*d) - (b*exp(- 2*c - 2*d*x))/(8*d) - (b*x)/2 - (2*a
tan((a*exp(d*x)*exp(c)*(-d^2)^(1/2))/(d*(a^2)^(1/2)))*(a^2)^(1/2))/(-d^2)^
(1/2)
```

Reduce [B] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 94, normalized size of antiderivative = 2.35

$$\int \operatorname{csch}(c+dx)\left(a+b\sinh^3(c+dx)\right)\,dx =\frac{e^{4dx+4c}b+8e^{2dx+2c}\log\left(e^{dx+c}-1\right)a-8e^{2dx+2c}\log\left(e^{dx+c}+1\right)a-4e^{2dx+2c}bdx-b}{8e^{2dx+2c}d}$$

input
```
int(csch(d*x+c)*(a+b*sinh(d*x+c)^3),x)
```

output
```
(e**(4*c + 4*d*x)*b + 8*e**(2*c + 2*d*x)*log(e**(c + d*x) - 1)*a - 8*e**(2
*c + 2*d*x)*log(e**(c + d*x) + 1)*a - 4*e**(2*c + 2*d*x)*b*d*x - b)/(8*e**
(2*c + 2*d*x)*d)
```

3.124 $\int \operatorname{csch}^2(c+dx) \left(a+b\sinh^3(c+dx)\right) \, dx$

Optimal result

Integrand size = 21, antiderivative size = 24

$$\int \operatorname{csch}^2(c+dx) \left(a+b\sinh^3(c+dx)\right) \, dx = \frac{b\cosh(c+dx)}{d} - \frac{a\coth(c+dx)}{d}$$

output
```
b*cosh(d*x+c)/d-a*coth(d*x+c)/d
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 35, normalized size of antiderivative = 1.46

$$\int \operatorname{csch}^2(c+dx) \left(a+b\sinh^3(c+dx)\right) \, dx = \frac{b\cosh(c)\cosh(dx)}{d} - \frac{a\coth(c+dx)}{d} + \frac{b\sinh(c)\sinh(dx)}{d}$$

input
```
Integrate[Csch[c + d*x]^2*(a + b*Sinh[c + d*x]^3),x]
```

output
```
(b*Cosh[c]*Cosh[d*x])/d - (a*Coth[c + d*x])/d + (b*Sinh[c]*Sinh[d*x])/d
```

Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.190, Rules used = {3042, 25, 3699, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \operatorname{csch}^2(c+dx)\left(a+b\sinh^3(c+dx)\right)\,dx$$

$$\Big\downarrow \ 3042$$

$$\int -\frac{a+ib\sin(ic+idx)^3}{\sin(ic+idx)^2}dx$$

$$\Big\downarrow \ 25$$

$$-\int \frac{ib\sin(ic+idx)^3+a}{\sin(ic+idx)^2}dx$$

$$\Big\downarrow \ 3699$$

$$-\int \left(-a\operatorname{csch}^2(c+dx)-b\sinh(c+dx)\right)dx$$

$$\Big\downarrow \ 2009$$

$$\frac{b\cosh(c+dx)}{d}-\frac{a\coth(c+dx)}{d}$$

input
```
Int[Csch[c + d*x]^2*(a + b*Sinh[c + d*x]^3),x]
```

output
```
(b*Cosh[c + d*x])/d - (a*Coth[c + d*x])/d
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3699
```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^(n_
))^(p_.), x_Symbol] :> Int[ExpandTrig[sin[e + f*x]^m*(a + b*sin[e + f*x]^n)
^p, x], x] /; FreeQ[{a, b, e, f}, x] && IntegersQ[m, p] && (EqQ[n, 4] || Gt
Q[p, 0] || (EqQ[p, -1] && IntegerQ[n]))
```

Maple [A] (verified)

Time = 0.43 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.96

method	result	size
derivativedivides	$\frac{-\coth(dx+c)a+b\cosh(dx+c)}{d}$	23
default	$\frac{-\coth(dx+c)a+b\cosh(dx+c)}{d}$	23
risch	$\frac{b\,e^{3dx+3c}-e^{-dx-c}b-4a}{2d\left(e^{2dx+2c}-1\right)}$	46
parallelrisch	$\frac{-\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^{3}a+\coth\left(\frac{dx}{2}+\frac{c}{2}\right)a-4b}{2d\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}-2d}$	51

input
```
int(csch(d*x+c)^2*(a+b*sinh(d*x+c)^3),x,method=_RETURNVERBOSE)
```

output
```
1/d*(-coth(d*x+c)*a+b*cosh(d*x+c))
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 40, normalized size of antiderivative = 1.67

$$\int \operatorname{csch}^2(c+dx)\left(a+b\sinh^3(c+dx)\right)\,dx = -\frac{a\cosh(dx+c)-(b\cosh(dx+c)+a)\sinh(dx+c)}{d\sinh(dx+c)}$$

input
```
integrate(csch(d*x+c)^2*(a+b*sinh(d*x+c)^3),x, algorithm="fricas")
```

output
```
-(a*cosh(d*x + c) - (b*cosh(d*x + c) + a)*sinh(d*x + c))/(d*sinh(d*x + c))
```

Sympy [F]

$$\int \operatorname{csch}^2(c+dx)\left(a+b\sinh^3(c+dx)\right)\,dx = \int \left(a+b\sinh^3(c+dx)\right)\operatorname{csch}^2(c+dx)\,dx$$

input
```
integrate(csch(d*x+c)**2*(a+b*sinh(d*x+c)**3),x)
```

output
```
Integral((a + b*sinh(c + d*x)**3)*csch(c + d*x)**2, x)
```

Maxima [A] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 47, normalized size of antiderivative = 1.96

$$\int \operatorname{csch}^2(c+dx)\left(a+b\sinh^3(c+dx)\right)\,dx = \frac{1}{2}b\left(\frac{e^{(dx+c)}}{d}+\frac{e^{(-dx-c)}}{d}\right)+\frac{2a}{d(e^{(-2dx-2c)}-1)}$$

input
```
integrate(csch(d*x+c)^2*(a+b*sinh(d*x+c)^3),x, algorithm="maxima")
```

output
```
1/2*b*(e^(d*x + c)/d + e^(-d*x - c)/d) + 2*a/(d*(e^(-2*d*x - 2*c) - 1))
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 59 vs. $2(24) = 48$.

Time = 0.13 (sec) , antiderivative size = 59, normalized size of antiderivative = 2.46

$$\int \operatorname{csch}^2(c+dx)\left(a+b\sinh^3(c+dx)\right)\,dx=\frac{be^{(dx+c)}+\frac{be^{(2\,dx+2\,c)}-4\,ae^{(dx+c)}-b}{e^{(3\,dx+3\,c)}-e^{(dx+c)}}}{2\,d}$$

input
```
integrate(csch(d*x+c)^2*(a+b*sinh(d*x+c)^3),x, algorithm="giac")
```

output
```
1/2*(b*e^(d*x + c) + (b*e^(2*d*x + 2*c) - 4*a*e^(d*x + c) - b)/(e^(3*d*x +
 3*c) - e^(d*x + c)))/d
```

Mupad [B] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 47, normalized size of antiderivative = 1.96

$$\int \operatorname{csch}^2(c+dx)\left(a+b\sinh^3(c+dx)\right)\,dx=\frac{b\,\mathrm{e}^{-c-d\,x}}{2\,d}-\frac{2\,a}{d\,\left(\mathrm{e}^{2\,c+2\,d\,x}-1\right)}+\frac{b\,\mathrm{e}^{c+d\,x}}{2\,d}$$

input
```
int((a + b*sinh(c + d*x)^3)/sinh(c + d*x)^2,x)
```

output
```
(b*exp(- c - d*x))/(2*d) - (2*a)/(d*(exp(2*c + 2*d*x) - 1)) + (b*exp(c + d
*x))/(2*d)
```

Reduce [B] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 57, normalized size of antiderivative = 2.38

$$\int \operatorname{csch}^2(c+dx)\left(a+b\sinh^3(c+dx)\right)\,dx=\frac{e^{4dx+4c}b-4e^{3dx+3c}a-b}{2e^{dx+c}d\left(e^{2dx+2c}-1\right)}$$

input
```
int(csch(d*x+c)^2*(a+b*sinh(d*x+c)^3),x)
```

output

```
(e**(4*c + 4*d*x)*b - 4*e**(3*c + 3*d*x)*a - b)/(2*e**(c + d*x)*d*(e**(2*c
 + 2*d*x) - 1))
```

3.125 $\int \operatorname{csch}^3(c+dx)\left(a+b\sinh^3(c+dx)\right)\,dx$

Optimal result

Integrand size = 21, antiderivative size = 39

$$\int \operatorname{csch}^3(c+dx)\left(a+b\sinh^3(c+dx)\right)\,dx=bx+\frac{a\operatorname{arctanh}(\cosh(c+dx))}{2d}-\frac{a\coth(c+dx)\operatorname{csch}(c+dx)}{2d}$$

output
```
b*x+1/2*a*arctanh(cosh(d*x+c))/d-1/2*a*coth(d*x+c)*csch(d*x+c)/d
```

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 82 vs. 2(39) = 78.

Time = 0.01 (sec) , antiderivative size = 82, normalized size of antiderivative = 2.10

$$\int \operatorname{csch}^3(c+dx)\left(a+b\sinh^3(c+dx)\right)\,dx=bx-\frac{a\operatorname{csch}^2\left(\frac{1}{2}(c+dx)\right)}{8d}+\frac{a\log\left(\cosh\left(\frac{1}{2}(c+dx)\right)\right)}{2d}-\frac{a\log\left(\sinh\left(\frac{1}{2}(c+dx)\right)\right)}{2d}-\frac{a\operatorname{sech}^2\left(\frac{1}{2}(c+dx)\right)}{8d}$$

input
```
Integrate[Csch[c + d*x]^3*(a + b*Sinh[c + d*x]^3),x]
```

output
```
b*x - (a*Csch[(c + d*x)/2]^2)/(8*d) + (a*Log[Cosh[(c + d*x)/2]])/(2*d) - (
a*Log[Sinh[(c + d*x)/2]])/(2*d) - (a*Sech[(c + d*x)/2]^2)/(8*d)
```

Rubi [C] (verified)

Result contains complex when optimal does not.

Time = 0.26 (sec) , antiderivative size = 50, normalized size of antiderivative = 1.28, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.190$, Rules used = {3042, 26, 3699, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \operatorname{csch}^3(c+dx)\left(a+b\sinh^3(c+dx)\right)dx$$

$$\Big\downarrow \text{3042}$$

$$\int -\frac{i\left(a+ib\sin(ic+idx)^3\right)}{\sin(ic+idx)^3}dx$$

$$\Big\downarrow \text{26}$$

$$-i\int \frac{ib\sin(ic+idx)^3+a}{\sin(ic+idx)^3}dx$$

$$\Big\downarrow \text{3699}$$

$$-i\int \left(ia\operatorname{csch}^3(c+dx)+ib\right)dx$$

$$\Big\downarrow \text{2009}$$

$$-i\left(\frac{ia\operatorname{arctanh}(\cosh(c+dx))}{2d}-\frac{ia\coth(c+dx)\operatorname{csch}(c+dx)}{2d}+ibx\right)$$

input
```
Int[Csch[c + d*x]^3*(a + b*Sinh[c + d*x]^3),x]
```

output
```
(-I)*(I*b*x + ((I/2)*a*ArcTanh[Cosh[c + d*x]])/d - ((I/2)*a*Coth[c + d*x]*
Csch[c + d*x])/d)
```

Defintions of rubi rules used

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3699
```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^(n_
))^(p_.), x_Symbol] :> Int[ExpandTrig[sin[e + f*x]^m*(a + b*sin[e + f*x]^n)
^p, x], x] /; FreeQ[{a, b, e, f}, x] && IntegersQ[m, p] && (EqQ[n, 4] || Gt
Q[p, 0] || (EqQ[p, -1] && IntegerQ[n]))
```

Maple [A] (verified)

Time = 0.42 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.95

method	result	size
derivativedivides	$\frac{a\left(-\frac{\operatorname{csch}(dx+c)\coth(dx+c)}{2}+\operatorname{arctanh}\left(\mathrm{e}^{dx+c}\right)\right)+b(dx+c)}{d}$	37
default	$\frac{a\left(-\frac{\operatorname{csch}(dx+c)\coth(dx+c)}{2}+\operatorname{arctanh}\left(\mathrm{e}^{dx+c}\right)\right)+b(dx+c)}{d}$	37
parallelrisch	$-\frac{-8dxb+a\left(-\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}+\coth\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}+4\ln\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)\right)\right)}{8d}$	51
risch	$bx-\frac{a\,\mathrm{e}^{dx+c}\left(\mathrm{e}^{2dx+2c}+1\right)}{d\left(\mathrm{e}^{2dx+2c}-1\right)^{2}}+\frac{a\ln\left(\mathrm{e}^{dx+c}+1\right)}{2d}-\frac{a\ln\left(\mathrm{e}^{dx+c}-1\right)}{2d}$	71

input
```
int(csch(d*x+c)^3*(a+b*sinh(d*x+c)^3),x,method=_RETURNVERBOSE)
```

output
```
1/d*(a*(-1/2*csch(d*x+c)*coth(d*x+c)+arctanh(exp(d*x+c)))+b*(d*x+c))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 521 vs. $2(35) = 70$.

Time = 0.10 (sec) , antiderivative size = 521, normalized size of antiderivative = 13.36

$$\int \operatorname{csch}^3(c+dx)\left(a+b\sinh^3(c+dx)\right)\,dx = \frac{2\,bdx\cosh\left(dx+c\right)^4+2\,bdx\sinh\left(dx+c\right)^4-4\,bdx\cosh\left(dx+c\right)^2-2\,a\cosh\left(dx+c\right)^3+2\left(4\,bdx\cosh\left(dx\right.\right.}{}$$

input
```
integrate(csch(d*x+c)^3*(a+b*sinh(d*x+c)^3),x, algorithm="fricas")
```

output
```
1/2*(2*b*d*x*cosh(d*x + c)^4 + 2*b*d*x*sinh(d*x + c)^4 - 4*b*d*x*cosh(d*x
+ c)^2 - 2*a*cosh(d*x + c)^3 + 2*(4*b*d*x*cosh(d*x + c) - a)*sinh(d*x + c)
^3 + 2*b*d*x + 2*(6*b*d*x*cosh(d*x + c)^2 - 2*b*d*x - 3*a*cosh(d*x + c))*s
inh(d*x + c)^2 - 2*a*cosh(d*x + c) + (a*cosh(d*x + c)^4 + 4*a*cosh(d*x + c
)*sinh(d*x + c)^3 + a*sinh(d*x + c)^4 - 2*a*cosh(d*x + c)^2 + 2*(3*a*cosh(
d*x + c)^2 - a)*sinh(d*x + c)^2 + 4*(a*cosh(d*x + c)^3 - a*cosh(d*x + c))*
sinh(d*x + c) + a)*log(cosh(d*x + c) + sinh(d*x + c) + 1) - (a*cosh(d*x +
c)^4 + 4*a*cosh(d*x + c)*sinh(d*x + c)^3 + a*sinh(d*x + c)^4 - 2*a*cosh(d*
x + c)^2 + 2*(3*a*cosh(d*x + c)^2 - a)*sinh(d*x + c)^2 + 4*(a*cosh(d*x + c
)^3 - a*cosh(d*x + c))*sinh(d*x + c) + a)*log(cosh(d*x + c) + sinh(d*x + c
) - 1) + 2*(4*b*d*x*cosh(d*x + c)^3 - 4*b*d*x*cosh(d*x + c) - 3*a*cosh(d*x
 + c)^2 - a)*sinh(d*x + c))/(d*cosh(d*x + c)^4 + 4*d*cosh(d*x + c)*sinh(d*
x + c)^3 + d*sinh(d*x + c)^4 - 2*d*cosh(d*x + c)^2 + 2*(3*d*cosh(d*x + c)^
2 - d)*sinh(d*x + c)^2 + 4*(d*cosh(d*x + c)^3 - d*cosh(d*x + c))*sinh(d*x
+ c) + d)
```

Sympy [F(-1)]

Timed out.

$$\int \operatorname{csch}^3(c+dx)\left(a+b\sinh^3(c+dx)\right)\,dx=\text{Timed out}$$

input
```
integrate(csch(d*x+c)**3*(a+b*sinh(d*x+c)**3),x)
```

output
```
Timed out
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 91 vs. $2(35) = 70$.

Time = 0.04 (sec) , antiderivative size = 91, normalized size of antiderivative = 2.33

$$\int \operatorname{csch}^3(c+dx)\left(a+b\sinh^3(c+dx)\right)\,dx$$
$$=bx+\frac{1}{2}a\left(\frac{\log\left(e^{(-dx-c)}+1\right)}{d}-\frac{\log\left(e^{(-dx-c)}-1\right)}{d}+\frac{2\left(e^{(-dx-c)}+e^{(-3\,dx-3\,c)}\right)}{d(2\,e^{(-2\,dx-2\,c)}-e^{(-4\,dx-4\,c)}-1)}\right)$$

input
```
integrate(csch(d*x+c)^3*(a+b*sinh(d*x+c)^3),x, algorithm="maxima")
```

output
```
b*x + 1/2*a*(log(e^(-d*x - c) + 1)/d - log(e^(-d*x - c) - 1)/d + 2*(e^(-d*
x - c) + e^(-3*d*x - 3*c))/(d*(2*e^(-2*d*x - 2*c) - e^(-4*d*x - 4*c) - 1))
)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 73 vs. $2(35) = 70$.

Time = 0.36 (sec) , antiderivative size = 73, normalized size of antiderivative = 1.87

$$\int \operatorname{csch}^3(c+dx)\left(a+b\sinh^3(c+dx)\right)\,dx = \frac{2\,(dx+c)b + a\log\left(e^{(dx+c)}+1\right) - a\log\left(\left|e^{(dx+c)}-1\right|\right) - \frac{2\,\left(ae^{(3\,dx+3\,c)}+ae^{(dx+c)}\right)}{\left(e^{(2\,dx+2\,c)}-1\right)^2}}{2\,d}$$

input
```
integrate(csch(d*x+c)^3*(a+b*sinh(d*x+c)^3),x, algorithm="giac")
```

output
```
1/2*(2*(d*x + c)*b + a*log(e^(d*x + c) + 1) - a*log(abs(e^(d*x + c) - 1))
- 2*(a*e^(3*d*x + 3*c) + a*e^(d*x + c))/(e^(2*d*x + 2*c) - 1)^2)/d
```

Mupad [B] (verification not implemented)

Time = 1.55 (sec) , antiderivative size = 102, normalized size of antiderivative = 2.62

$$\int \operatorname{csch}^3(c+dx)\left(a+b\sinh^3(c+dx)\right)\,dx = b\,x + \frac{\operatorname{atan}\left(\frac{a\,e^{d\,x}\,e^{c}\,\sqrt{-d^2}}{d\,\sqrt{a^2}}\right)\,\sqrt{a^2}}{\sqrt{-d^2}} - \frac{a\,e^{c+d\,x}}{d\,\left(e^{2\,c+2\,d\,x}-1\right)} - \frac{2\,a\,e^{c+d\,x}}{d\,\left(e^{4\,c+4\,d\,x}-2\,e^{2\,c+2\,d\,x}+1\right)}$$

input
```
int((a + b*sinh(c + d*x)^3)/sinh(c + d*x)^3,x)
```

output
```
b*x + (atan((a*exp(d*x)*exp(c)*(-d^2)^(1/2))/(d*(a^2)^(1/2)))*(a^2)^(1/2))
/(-d^2)^(1/2) - (a*exp(c + d*x))/(d*(exp(2*c + 2*d*x) - 1)) - (2*a*exp(c +
 d*x))/(d*(exp(4*c + 4*d*x) - 2*exp(2*c + 2*d*x) + 1))
```

Reduce [B] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 206, normalized size of antiderivative = 5.28

$$\int \operatorname{csch}^3(c+dx)\left(a+b\sinh^3(c+dx)\right)\,dx$$
$$=\frac{-e^{4dx+4c}\log\left(e^{dx+c}-1\right)a+e^{4dx+4c}\log\left(e^{dx+c}+1\right)a+2e^{4dx+4c}bdx-2e^{3dx+3c}a+2e^{2dx+2c}\log\left(e^{dx+c}-1\right)a}{2d\left(e^{4dx+4c}-2e\right.}$$

input
```
int(csch(d*x+c)^3*(a+b*sinh(d*x+c)^3),x)
```

output
```
( - e**(4*c + 4*d*x)*log(e**(c + d*x) - 1)*a + e**(4*c + 4*d*x)*log(e**(c
+ d*x) + 1)*a + 2*e**(4*c + 4*d*x)*b*d*x - 2*e**(3*c + 3*d*x)*a + 2*e**(2*
c + 2*d*x)*log(e**(c + d*x) - 1)*a - 2*e**(2*c + 2*d*x)*log(e**(c + d*x) +
 1)*a - 4*e**(2*c + 2*d*x)*b*d*x - 2*e**(c + d*x)*a - log(e**(c + d*x) - 1
)*a + log(e**(c + d*x) + 1)*a + 2*b*d*x)/(2*d*(e**(4*c + 4*d*x) - 2*e**(2*
c + 2*d*x) + 1))
```

3.126 $\int \operatorname{csch}^4(c+dx) \left(a+b\sinh^3(c+dx)\right) \, dx$

Optimal result

Integrand size = 21, antiderivative size = 41

$$\int \operatorname{csch}^4(c+dx) \left(a+b\sinh^3(c+dx)\right) \, dx = -\frac{b\operatorname{arctanh}(\cosh(c+dx))}{d} + \frac{a\coth(c+dx)}{d} - \frac{a\coth^3(c+dx)}{3d}$$

output
```
-b*arctanh(cosh(d*x+c))/d+a*coth(d*x+c)/d-1/3*a*coth(d*x+c)^3/d
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 50, normalized size of antiderivative = 1.22

$$\int \operatorname{csch}^4(c+dx) \left(a+b\sinh^3(c+dx)\right) \, dx = -\frac{b\operatorname{arctanh}(\cosh(c+dx))}{d} + \frac{2a\coth(c+dx)}{3d} - \frac{a\coth(c+dx)\operatorname{csch}^2(c+dx)}{3d}$$

input
```
Integrate[Csch[c + d*x]^4*(a + b*Sinh[c + d*x]^3),x]
```

output
```
-((b*ArcTanh[Cosh[c + d*x]])/d) + (2*a*Coth[c + d*x])/(3*d) - (a*Coth[c +
d*x]*Csch[c + d*x]^2)/(3*d)
```

Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 41, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {3042, 3699, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \operatorname{csch}^4(c+dx)\left(a+b\sinh^3(c+dx)\right)\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{a+ib\sin(ic+idx)^3}{\sin(ic+idx)^4}dx$$

$$\Big\downarrow \text{3699}$$

$$\int \left(a\operatorname{csch}^4(c+dx)+b\operatorname{csch}(c+dx)\right)dx$$

$$\Big\downarrow \text{2009}$$

$$-\frac{a\coth^3(c+dx)}{3d}+\frac{a\coth(c+dx)}{d}-\frac{b\operatorname{arctanh}(\cosh(c+dx))}{d}$$

input
```
Int[Csch[c + d*x]^4*(a + b*Sinh[c + d*x]^3),x]
```

output
```
-((b*ArcTanh[Cosh[c + d*x]])/d) + (a*Coth[c + d*x])/d - (a*Coth[c + d*x]^3
)/(3*d)
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3699
```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^(n_
))^(p_.), x_Symbol] :> Int[ExpandTrig[sin[e + f*x]^m*(a + b*sin[e + f*x]^n)
^p, x], x] /; FreeQ[{a, b, e, f}, x] && IntegersQ[m, p] && (EqQ[n, 4] || Gt
Q[p, 0] || (EqQ[p, -1] && IntegerQ[n]))
```

Maple [A] (verified)

Time = 0.50 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.88

method	result	size
derivativedivides	$\frac{a\left(\frac{2}{3}-\frac{\operatorname{csch}(dx+c)^2}{3}\right)\coth(dx+c)-2b\,\operatorname{arctanh}\left(e^{dx+c}\right)}{d}$	36
default	$\frac{a\left(\frac{2}{3}-\frac{\operatorname{csch}(dx+c)^2}{3}\right)\coth(dx+c)-2b\,\operatorname{arctanh}\left(e^{dx+c}\right)}{d}$	36
risch	$-\frac{4a\left(3\,e^{2dx+2c}-1\right)}{3d\left(e^{2dx+2c}-1\right)^3}+\frac{\ln\left(e^{dx+c}-1\right)b}{d}-\frac{\ln\left(e^{dx+c}+1\right)b}{d}$	63
parallelrisch	$-\frac{-24\ln\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)\right)b+a\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^3+\coth\left(\frac{dx}{2}+\frac{c}{2}\right)^3-9\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)-9\coth\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{24d}$	67

input
```
int(csch(d*x+c)^4*(a+b*sinh(d*x+c)^3),x,method=_RETURNVERBOSE)
```

output
```
1/d*(a*(2/3-1/3*csch(d*x+c)^2)*coth(d*x+c)-2*b*arctanh(exp(d*x+c)))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 652 vs. $2(39) = 78$.

Time = 0.09 (sec) , antiderivative size = 652, normalized size of antiderivative = 15.90

$$\int \operatorname{csch}^4(c+dx)\left(a+b\sinh^3(c+dx)\right)\,dx=\text{Too large to display}$$

input
```
integrate(csch(d*x+c)^4*(a+b*sinh(d*x+c)^3),x, algorithm="fricas")
```

output
```
-1/3*(12*a*cosh(d*x + c)^2 + 24*a*cosh(d*x + c)*sinh(d*x + c) + 12*a*sinh(
d*x + c)^2 + 3*(b*cosh(d*x + c)^6 + 6*b*cosh(d*x + c)*sinh(d*x + c)^5 + b*
sinh(d*x + c)^6 - 3*b*cosh(d*x + c)^4 + 3*(5*b*cosh(d*x + c)^2 - b)*sinh(d
*x + c)^4 + 4*(5*b*cosh(d*x + c)^3 - 3*b*cosh(d*x + c))*sinh(d*x + c)^3 +
3*b*cosh(d*x + c)^2 + 3*(5*b*cosh(d*x + c)^4 - 6*b*cosh(d*x + c)^2 + b)*si
nh(d*x + c)^2 + 6*(b*cosh(d*x + c)^5 - 2*b*cosh(d*x + c)^3 + b*cosh(d*x +
c))*sinh(d*x + c) - b)*log(cosh(d*x + c) + sinh(d*x + c) + 1) - 3*(b*cosh(
d*x + c)^6 + 6*b*cosh(d*x + c)*sinh(d*x + c)^5 + b*sinh(d*x + c)^6 - 3*b*c
osh(d*x + c)^4 + 3*(5*b*cosh(d*x + c)^2 - b)*sinh(d*x + c)^4 + 4*(5*b*cosh
(d*x + c)^3 - 3*b*cosh(d*x + c))*sinh(d*x + c)^3 + 3*b*cosh(d*x + c)^2 + 3
*(5*b*cosh(d*x + c)^4 - 6*b*cosh(d*x + c)^2 + b)*sinh(d*x + c)^2 + 6*(b*co
sh(d*x + c)^5 - 2*b*cosh(d*x + c)^3 + b*cosh(d*x + c))*sinh(d*x + c) - b)*
log(cosh(d*x + c) + sinh(d*x + c) - 1) - 4*a)/(d*cosh(d*x + c)^6 + 6*d*cos
h(d*x + c)*sinh(d*x + c)^5 + d*sinh(d*x + c)^6 - 3*d*cosh(d*x + c)^4 + 3*(
5*d*cosh(d*x + c)^2 - d)*sinh(d*x + c)^4 + 4*(5*d*cosh(d*x + c)^3 - 3*d*co
sh(d*x + c))*sinh(d*x + c)^3 + 3*d*cosh(d*x + c)^2 + 3*(5*d*cosh(d*x + c)^
4 - 6*d*cosh(d*x + c)^2 + d)*sinh(d*x + c)^2 + 6*(d*cosh(d*x + c)^5 - 2*d*
cosh(d*x + c)^3 + d*cosh(d*x + c))*sinh(d*x + c) - d)
```

Sympy [F(-1)]

Timed out.

$$\int \operatorname{csch}^4(c+dx) \left(a+b\sinh^3(c+dx)\right)\,dx = \text{Timed out}$$

input
```
integrate(csch(d*x+c)**4*(a+b*sinh(d*x+c)**3),x)
```

output
```
Timed out
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 131 vs. $2(39) = 78$.

Time = 0.04 (sec) , antiderivative size = 131, normalized size of antiderivative = 3.20

$$\int \operatorname{csch}^4(c+dx)\left(a+b\sinh^3(c+dx)\right)\,dx = -b\left(\frac{\log\left(e^{(-dx-c)}+1\right)}{d}-\frac{\log\left(e^{(-dx-c)}-1\right)}{d}\right) + \frac{4}{3}\,a\left(\frac{3\,e^{(-2\,dx-2\,c)}}{d(3\,e^{(-2\,dx-2\,c)}-3\,e^{(-4\,dx-4\,c)}+e^{(-6\,dx-6\,c)}-1)}-\frac{1}{d(3\,e^{(-2\,dx-2\,c)}-3\,e^{(-4\,dx-4\,c)}+e^{(-6\,dx-6\,c)}-1)}\right)$$

input
```
integrate(csch(d*x+c)^4*(a+b*sinh(d*x+c)^3),x, algorithm="maxima")
```

output
```
-b*(log(e^(-d*x - c) + 1)/d - log(e^(-d*x - c) - 1)/d) + 4/3*a*(3*e^(-2*d*
x - 2*c)/(d*(3*e^(-2*d*x - 2*c) - 3*e^(-4*d*x - 4*c) + e^(-6*d*x - 6*c) -
1)) - 1/(d*(3*e^(-2*d*x - 2*c) - 3*e^(-4*d*x - 4*c) + e^(-6*d*x - 6*c) - 1
)))
```

Giac [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 62, normalized size of antiderivative = 1.51

$$\int \operatorname{csch}^4(c+dx)\left(a+b\sinh^3(c+dx)\right)\,dx = -\frac{3\,b\log\left(e^{(dx+c)}+1\right)-3\,b\log\left(\left|e^{(dx+c)}-1\right|\right)+\frac{4\left(3\,ae^{(2\,dx+2\,c)}-a\right)}{\left(e^{(2\,dx+2\,c)}-1\right)^3}}{3\,d}$$

input
```
integrate(csch(d*x+c)^4*(a+b*sinh(d*x+c)^3),x, algorithm="giac")
```

output
```
-1/3*(3*b*log(e^(d*x + c) + 1) - 3*b*log(abs(e^(d*x + c) - 1)) + 4*(3*a*e^
(2*d*x + 2*c) - a)/(e^(2*d*x + 2*c) - 1)^3)/d
```

Mupad [B] (verification not implemented)

Time = 1.60 (sec) , antiderivative size = 110, normalized size of antiderivative = 2.68

$$\int \operatorname{csch}^4(c+dx)\left(a+b\sinh^3(c+dx)\right)\,dx = -\frac{4\,a}{d\,\left(\mathrm{e}^{4\,c+4\,d\,x}-2\,\mathrm{e}^{2\,c+2\,d\,x}+1\right)} - \frac{8\,a}{3\,d\,\left(3\,\mathrm{e}^{2\,c+2\,d\,x}-3\,\mathrm{e}^{4\,c+4\,d\,x}+\mathrm{e}^{6\,c+6\,d\,x}-1\right)} - \frac{2\,\mathrm{atan}\left(\frac{b\,\mathrm{e}^{d\,x}\,\mathrm{e}^{c}\,\sqrt{-d^2}}{d\,\sqrt{b^2}}\right)\,\sqrt{b^2}}{\sqrt{-d^2}}$$

input
```
int((a + b*sinh(c + d*x)^3)/sinh(c + d*x)^4,x)
```

output
```
- (4*a)/(d*(exp(4*c + 4*d*x) - 2*exp(2*c + 2*d*x) + 1)) - (8*a)/(3*d*(3*ex
p(2*c + 2*d*x) - 3*exp(4*c + 4*d*x) + exp(6*c + 6*d*x) - 1)) - (2*atan((b*
exp(d*x)*exp(c)*(-d^2)^(1/2))/(d*(b^2)^(1/2)))*(b^2)^(1/2))/(-d^2)^(1/2)
```

Reduce [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 224, normalized size of antiderivative = 5.46

$$\int \operatorname{csch}^4(c+dx)\left(a+b\sinh^3(c+dx)\right)\,dx = \frac{3e^{6dx+6c}\log\left(e^{dx+c}-1\right)b-3e^{6dx+6c}\log\left(e^{dx+c}+1\right)b-9e^{4dx+4c}\log\left(e^{dx+c}-1\right)b+9e^{4dx+4c}\log\left(e^{dx+c}+1\right)b}{3d\left(e^{6dx+6c}-\right.}$$

input
```
int(csch(d*x+c)^4*(a+b*sinh(d*x+c)^3),x)
```

output
```
(3*e**(6*c + 6*d*x)*log(e**(c + d*x) - 1)*b - 3*e**(6*c + 6*d*x)*log(e**(c
 + d*x) + 1)*b - 9*e**(4*c + 4*d*x)*log(e**(c + d*x) - 1)*b + 9*e**(4*c +
4*d*x)*log(e**(c + d*x) + 1)*b + 9*e**(2*c + 2*d*x)*log(e**(c + d*x) - 1)*
b - 9*e**(2*c + 2*d*x)*log(e**(c + d*x) + 1)*b - 12*e**(2*c + 2*d*x)*a - 3
*log(e**(c + d*x) - 1)*b + 3*log(e**(c + d*x) + 1)*b + 4*a)/(3*d*(e**(6*c
+ 6*d*x) - 3*e**(4*c + 4*d*x) + 3*e**(2*c + 2*d*x) - 1))
```

3.127 $\int \sinh^3(c+dx) \left(a+b\sinh^3(c+dx)\right)^2 \, dx$

Optimal result

Integrand size = 23, antiderivative size = 192

$$\begin{aligned}\int \sinh^3(c+dx) \left(a+b\sinh^3(c+dx)\right)^2 \, dx = &-\frac{5}{8}abx-\frac{a^2\cosh(c+dx)}{d}\\&+\frac{b^2\cosh(c+dx)}{d}+\frac{a^2\cosh^3(c+dx)}{3d}\\&-\frac{4b^2\cosh^3(c+dx)}{3d}+\frac{6b^2\cosh^5(c+dx)}{5d}\\&-\frac{4b^2\cosh^7(c+dx)}{7d}+\frac{b^2\cosh^9(c+dx)}{9d}\\&+\frac{5ab\cosh(c+dx)\sinh(c+dx)}{8d}\\&-\frac{5ab\cosh(c+dx)\sinh^3(c+dx)}{12d}\\&+\frac{ab\cosh(c+dx)\sinh^5(c+dx)}{3d}\end{aligned}$$

output
```
-5/8*a*b*x-a^2*cosh(d*x+c)/d+b^2*cosh(d*x+c)/d+1/3*a^2*cosh(d*x+c)^3/d-4/3
*b^2*cosh(d*x+c)^3/d+6/5*b^2*cosh(d*x+c)^5/d-4/7*b^2*cosh(d*x+c)^7/d+1/9*b
^2*cosh(d*x+c)^9/d+5/8*a*b*cosh(d*x+c)*sinh(d*x+c)/d-5/12*a*b*cosh(d*x+c)*
sinh(d*x+c)^3/d+1/3*a*b*cosh(d*x+c)*sinh(d*x+c)^5/d
```

Mathematica [A] (verified)

Time = 0.57 (sec) , antiderivative size = 125, normalized size of antiderivative = 0.65

$$\int \sinh^3(c+dx)\left(a+b\sinh^3(c+dx)\right)^2\,dx$$
$$= \frac{-1890(32a^2-21b^2)\cosh(c+dx)+420(16a^2-21b^2)\cosh(3(c+dx))+b(2268b\cosh(5(c+dx))-405b}{}$$

input
```
Integrate[Sinh[c + d*x]^3*(a + b*Sinh[c + d*x]^3)^2,x]
```

output
```
(-1890*(32*a^2 - 21*b^2)*Cosh[c + d*x] + 420*(16*a^2 - 21*b^2)*Cosh[3*(c +
 d*x)] + b*(2268*b*Cosh[5*(c + d*x)] - 405*b*Cosh[7*(c + d*x)] + 35*b*Cosh
[9*(c + d*x)] - 840*a*(60*c + 60*d*x - 45*Sinh[2*(c + d*x)] + 9*Sinh[4*(c
+ d*x)] - Sinh[6*(c + d*x)])))/(80640*d)
```

Rubi [C] (verified)

Result contains complex when optimal does not.

Time = 0.43 (sec) , antiderivative size = 219, normalized size of antiderivative = 1.14, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.174$, Rules used = {3042, 26, 3699, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sinh^3(c+dx)\left(a+b\sinh^3(c+dx)\right)^2\,dx$$
$$\Big\downarrow \text{3042}$$
$$\int i\sin(ic+idx)^3\left(a+ib\sin(ic+idx)^3\right)^2dx$$
$$\Big\downarrow \text{26}$$
$$i\int \sin(ic+idx)^3\left(ib\sin(ic+idx)^3+a\right)^2dx$$
$$\Big\downarrow \text{3699}$$

$$i\int \left(-ib^2\sinh^9(c+dx)-2iab\sinh^6(c+dx)-ia^2\sinh^3(c+dx)\right)dx$$

$$\Big\downarrow \text{2009}$$

$$i\left(-\frac{ia^2\cosh^3(c+dx)}{3d}+\frac{ia^2\cosh(c+dx)}{d}-\frac{iab\sinh^5(c+dx)\cosh(c+dx)}{3d}+\frac{5iab\sinh^3(c+dx)\cosh(c+dx)}{12d}-\right.$$

input
```
Int[Sinh[c + d*x]^3*(a + b*Sinh[c + d*x]^3)^2,x]
```

output
```
I*(((5*I)/8)*a*b*x + (I*a^2*Cosh[c + d*x])/d - (I*b^2*Cosh[c + d*x])/d - (
(I/3)*a^2*Cosh[c + d*x]^3)/d + (((4*I)/3)*b^2*Cosh[c + d*x]^3)/d - (((6*I)
/5)*b^2*Cosh[c + d*x]^5)/d + (((4*I)/7)*b^2*Cosh[c + d*x]^7)/d - ((I/9)*b^
2*Cosh[c + d*x]^9)/d - (((5*I)/8)*a*b*Cosh[c + d*x]*Sinh[c + d*x])/d + (((
5*I)/12)*a*b*Cosh[c + d*x]*Sinh[c + d*x]^3)/d - ((I/3)*a*b*Cosh[c + d*x]*S
inh[c + d*x]^5)/d)
```

Defintions of rubi rules used

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3699
```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^(n_
))^(p_.), x_Symbol] :> Int[ExpandTrig[sin[e + f*x]^m*(a + b*sin[e + f*x]^n)
^p, x], x] /; FreeQ[{a, b, e, f}, x] && IntegersQ[m, p] && (EqQ[n, 4] || Gt
Q[p, 0] || (EqQ[p, -1] && IntegerQ[n]))
```

Maple [A] (verified)

Time = 112.26 (sec) , antiderivative size = 128, normalized size of antiderivative = 0.67

method	result
derivativedivides	$\frac{a^2\left(-\frac{2}{3}+\frac{\sinh(dx+c)^2}{3}\right)\cosh(dx+c)+2ab\left(\left(\frac{\sinh(dx+c)^5}{6}-\frac{5\sinh(dx+c)^3}{24}+\frac{5\sinh(dx+c)}{16}\right)\cosh(dx+c)-\frac{5dx}{16}-\frac{5c}{16}\right)+b^2\left(\frac{1}{3}\right.}{d}$
default	$\frac{a^2\left(-\frac{2}{3}+\frac{\sinh(dx+c)^2}{3}\right)\cosh(dx+c)+2ab\left(\left(\frac{\sinh(dx+c)^5}{6}-\frac{5\sinh(dx+c)^3}{24}+\frac{5\sinh(dx+c)}{16}\right)\cosh(dx+c)-\frac{5dx}{16}-\frac{5c}{16}\right)+b^2\left(\frac{1}{3}\right.}{d}$
parts	$\frac{a^2\left(-\frac{2}{3}+\frac{\sinh(dx+c)^2}{3}\right)\cosh(dx+c)}{d}+\frac{b^2\left(\frac{128}{315}+\frac{\sinh(dx+c)^8}{9}-\frac{8\sinh(dx+c)^6}{63}+\frac{16\sinh(dx+c)^4}{105}-\frac{64\sinh(dx+c)^2}{315}\right)\cosh(d}{d}$
parallelrisch	$\frac{-50400abdx-53760a^2+32768b^2+6720a^2\cosh(3dx+3c)-60480a^2\cosh(dx+c)-8820b^2\cosh(3dx+3c)+39690b^2\cosh(}{}$
risch	$-\frac{5abx}{8}+\frac{b^2e^{9dx+9c}}{4608d}-\frac{9b^2e^{7dx+7c}}{3584d}+\frac{b\,e^{6dx+6c}a}{192d}+\frac{9b^2e^{5dx+5c}}{640d}-\frac{3\,e^{4dx+4c}ab}{64d}+\frac{e^{3dx+3c}a^2}{24d}-\frac{7\,e^{3dx+3c}b^2}{128d}$
orering	Expression too large to display

input
```
int(sinh(d*x+c)^3*(a+b*sinh(d*x+c)^3)^2,x,method=_RETURNVERBOSE)
```

output
```
1/d*(a^2*(-2/3+1/3*sinh(d*x+c)^2)*cosh(d*x+c)+2*a*b*((1/6*sinh(d*x+c)^5-5/
24*sinh(d*x+c)^3+5/16*sinh(d*x+c))*cosh(d*x+c)-5/16*d*x-5/16*c)+b^2*(128/3
15+1/9*sinh(d*x+c)^8-8/63*sinh(d*x+c)^6+16/105*sinh(d*x+c)^4-64/315*sinh(d
*x+c)^2)*cosh(d*x+c))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 355 vs. 2(174) = 348.

Time = 0.09 (sec) , antiderivative size = 355, normalized size of antiderivative = 1.85

$$\int \sinh^3(c+dx)\left(a+b\sinh^3(c+dx)\right)^2\,dx$$
$$=\frac{35\,b^2\cosh\left(dx+c\right)^9+315\,b^2\cosh\left(dx+c\right)\sinh\left(dx+c\right)^8-405\,b^2\cosh\left(dx+c\right)^7+5040\,ab\cosh\left(dx+c\right)}{}$$

input
```
integrate(sinh(d*x+c)^3*(a+b*sinh(d*x+c)^3)^2,x, algorithm="fricas")
```

output
```
1/80640*(35*b^2*cosh(d*x + c)^9 + 315*b^2*cosh(d*x + c)*sinh(d*x + c)^8 -
405*b^2*cosh(d*x + c)^7 + 5040*a*b*cosh(d*x + c)*sinh(d*x + c)^5 + 2268*b^
2*cosh(d*x + c)^5 + 105*(28*b^2*cosh(d*x + c)^3 - 27*b^2*cosh(d*x + c))*si
nh(d*x + c)^6 + 315*(14*b^2*cosh(d*x + c)^5 - 45*b^2*cosh(d*x + c)^3 + 36*
b^2*cosh(d*x + c))*sinh(d*x + c)^4 - 50400*a*b*d*x + 420*(16*a^2 - 21*b^2)
*cosh(d*x + c)^3 + 3360*(5*a*b*cosh(d*x + c)^3 - 9*a*b*cosh(d*x + c))*sinh
(d*x + c)^3 + 315*(4*b^2*cosh(d*x + c)^7 - 27*b^2*cosh(d*x + c)^5 + 72*b^2
*cosh(d*x + c)^3 + 4*(16*a^2 - 21*b^2)*cosh(d*x + c))*sinh(d*x + c)^2 - 18
90*(32*a^2 - 21*b^2)*cosh(d*x + c) + 5040*(a*b*cosh(d*x + c)^5 - 6*a*b*cos
h(d*x + c)^3 + 15*a*b*cosh(d*x + c))*sinh(d*x + c))/d
```

Sympy [A] (verification not implemented)

Time = 0.99 (sec) , antiderivative size = 325, normalized size of antiderivative = 1.69

$$\int \sinh^3(c+dx)\left(a+b\sinh^3(c+dx)\right)^2\,dx$$
$$= \begin{cases} \frac{a^2\sinh^2(c+dx)\cosh(c+dx)}{d} - \frac{2a^2\cosh^3(c+dx)}{3d} + \frac{5abx\sinh^6(c+dx)}{8} - \frac{15abx\sinh^4(c+dx)\cosh^2(c+dx)}{8} + \frac{15abx\sinh^2(c+dx)\cosh^4(c+dx)}{8} \\ x\left(a+b\sinh^3(c)\right)^2\sinh^3(c) \end{cases}$$

input
```
integrate(sinh(d*x+c)**3*(a+b*sinh(d*x+c)**3)**2,x)
```

output
```
Piecewise((a**2*sinh(c + d*x)**2*cosh(c + d*x)/d - 2*a**2*cosh(c + d*x)**3
/(3*d) + 5*a*b*x*sinh(c + d*x)**6/8 - 15*a*b*x*sinh(c + d*x)**4*cosh(c + d
*x)**2/8 + 15*a*b*x*sinh(c + d*x)**2*cosh(c + d*x)**4/8 - 5*a*b*x*cosh(c +
 d*x)**6/8 + 11*a*b*sinh(c + d*x)**5*cosh(c + d*x)/(8*d) - 5*a*b*sinh(c +
d*x)**3*cosh(c + d*x)**3/(3*d) + 5*a*b*sinh(c + d*x)*cosh(c + d*x)**5/(8*d
) + b**2*sinh(c + d*x)**8*cosh(c + d*x)/d - 8*b**2*sinh(c + d*x)**6*cosh(c
 + d*x)**3/(3*d) + 16*b**2*sinh(c + d*x)**4*cosh(c + d*x)**5/(5*d) - 64*b*
*2*sinh(c + d*x)**2*cosh(c + d*x)**7/(35*d) + 128*b**2*cosh(c + d*x)**9/(3
15*d), Ne(d, 0)), (x*(a + b*sinh(c)**3)**2*sinh(c)**3, True))
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 272, normalized size of antiderivative = 1.42

$$\int \sinh^3(c+dx)\left(a+b\sinh^3(c+dx)\right)^2\,dx =$$

$$-\frac{1}{161280}b^2\left(\frac{\left(405\,e^{(-2\,dx-2\,c)}-2268\,e^{(-4\,dx-4\,c)}+8820\,e^{(-6\,dx-6\,c)}-39690\,e^{(-8\,dx-8\,c)}-35\right)e^{(9\,dx+9\,c)}}{d}-\right.$$

$$-\frac{1}{192}ab\left(\frac{\left(9\,e^{(-2\,dx-2\,c)}-45\,e^{(-4\,dx-4\,c)}-1\right)e^{(6\,dx+6\,c)}}{d}+\frac{120\,(dx+c)}{d}+\frac{45\,e^{(-2\,dx-2\,c)}-9\,e^{(-4\,dx-4\,c)}+e}{d}\right.$$

$$+\frac{1}{24}a^2\left(\frac{e^{(3\,dx+3\,c)}}{d}-\frac{9\,e^{(dx+c)}}{d}-\frac{9\,e^{(-dx-c)}}{d}+\frac{e^{(-3\,dx-3\,c)}}{d}\right)$$

input
```
integrate(sinh(d*x+c)^3*(a+b*sinh(d*x+c)^3)^2,x, algorithm="maxima")
```

output
```
-1/161280*b^2*((405*e^(-2*d*x - 2*c) - 2268*e^(-4*d*x - 4*c) + 8820*e^(-6*
d*x - 6*c) - 39690*e^(-8*d*x - 8*c) - 35)*e^(9*d*x + 9*c)/d - (39690*e^(-d
*x - c) - 8820*e^(-3*d*x - 3*c) + 2268*e^(-5*d*x - 5*c) - 405*e^(-7*d*x -
7*c) + 35*e^(-9*d*x - 9*c))/d) - 1/192*a*b*((9*e^(-2*d*x - 2*c) - 45*e^(-4
*d*x - 4*c) - 1)*e^(6*d*x + 6*c)/d + 120*(d*x + c)/d + (45*e^(-2*d*x - 2*c
) - 9*e^(-4*d*x - 4*c) + e^(-6*d*x - 6*c))/d) + 1/24*a^2*(e^(3*d*x + 3*c)/
d - 9*e^(d*x + c)/d - 9*e^(-d*x - c)/d + e^(-3*d*x - 3*c)/d)
```

Giac [A] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 301, normalized size of antiderivative = 1.57

$$\int \sinh^3(c+dx)\left(a+b\sinh^3(c+dx)\right)^2\,dx$$

$$= -\frac{5}{8}abx+\frac{b^2e^{(9\,dx+9\,c)}}{4608\,d}-\frac{9\,b^2e^{(7\,dx+7\,c)}}{3584\,d}+\frac{abe^{(6\,dx+6\,c)}}{192\,d}+\frac{9\,b^2e^{(5\,dx+5\,c)}}{640\,d}-\frac{3\,abe^{(4\,dx+4\,c)}}{64\,d}$$
$$+\frac{15\,abe^{(2\,dx+2\,c)}}{64\,d}-\frac{15\,abe^{(-2\,dx-2\,c)}}{64\,d}+\frac{3\,abe^{(-4\,dx-4\,c)}}{64\,d}+\frac{9\,b^2e^{(-5\,dx-5\,c)}}{640\,d}$$
$$-\frac{abe^{(-6\,dx-6\,c)}}{192\,d}-\frac{9\,b^2e^{(-7\,dx-7\,c)}}{3584\,d}+\frac{b^2e^{(-9\,dx-9\,c)}}{4608\,d}+\frac{(16\,a^2-21\,b^2)e^{(3\,dx+3\,c)}}{384\,d}$$
$$-\frac{3\,(32\,a^2-21\,b^2)e^{(dx+c)}}{256\,d}-\frac{3\,(32\,a^2-21\,b^2)e^{(-dx-c)}}{256\,d}+\frac{(16\,a^2-21\,b^2)e^{(-3\,dx-3\,c)}}{384\,d}$$

input
```
integrate(sinh(d*x+c)^3*(a+b*sinh(d*x+c)^3)^2,x, algorithm="giac")
```

output
```
-5/8*a*b*x + 1/4608*b^2*e^(9*d*x + 9*c)/d - 9/3584*b^2*e^(7*d*x + 7*c)/d +
 1/192*a*b*e^(6*d*x + 6*c)/d + 9/640*b^2*e^(5*d*x + 5*c)/d - 3/64*a*b*e^(4
*d*x + 4*c)/d + 15/64*a*b*e^(2*d*x + 2*c)/d - 15/64*a*b*e^(-2*d*x - 2*c)/d
 + 3/64*a*b*e^(-4*d*x - 4*c)/d + 9/640*b^2*e^(-5*d*x - 5*c)/d - 1/192*a*b*
e^(-6*d*x - 6*c)/d - 9/3584*b^2*e^(-7*d*x - 7*c)/d + 1/4608*b^2*e^(-9*d*x
- 9*c)/d + 1/384*(16*a^2 - 21*b^2)*e^(3*d*x + 3*c)/d - 3/256*(32*a^2 - 21*
b^2)*e^(d*x + c)/d - 3/256*(32*a^2 - 21*b^2)*e^(-d*x - c)/d + 1/384*(16*a^
2 - 21*b^2)*e^(-3*d*x - 3*c)/d
```

Mupad [B] (verification not implemented)

Time = 1.87 (sec) , antiderivative size = 149, normalized size of antiderivative = 0.78

$$\int \sinh^3(c+dx)\left(a+b\sinh^3(c+dx)\right)^2\,dx = \frac{\frac{a^2\cosh(c+d\,x)^3}{3}-a^2\cosh(c+d\,x)+\frac{\sinh(c+d\,x)\,a\,b\cosh(c+d\,x)^5}{3}-\frac{13\sinh(c+d\,x)\,a\,b\cosh(c+d\,x)^3}{12}+\frac{11\sinh(c+d\,x)\,a\,b\cosh(c+d\,x)}{8}}{d}$$

input
```
int(sinh(c + d*x)^3*(a + b*sinh(c + d*x)^3)^2,x)
```

output
```
(b^2*cosh(c + d*x) - a^2*cosh(c + d*x) + (a^2*cosh(c + d*x)^3)/3 - (4*b^2*
cosh(c + d*x)^3)/3 + (6*b^2*cosh(c + d*x)^5)/5 - (4*b^2*cosh(c + d*x)^7)/7
 + (b^2*cosh(c + d*x)^9)/9 - (13*a*b*cosh(c + d*x)^3*sinh(c + d*x))/12 + (
a*b*cosh(c + d*x)^5*sinh(c + d*x))/3 + (11*a*b*cosh(c + d*x)*sinh(c + d*x)
)/8 - (5*a*b*d*x)/8)/d
```

Reduce [B] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 318, normalized size of antiderivative = 1.66

$$\int \sinh^3(c+dx)\left(a+b\sinh^3(c+dx)\right)^2\,dx$$

$$= \frac{35e^{18dx+18c}b^2 - 405e^{16dx+16c}b^2 + 840e^{15dx+15c}ab + 2268e^{14dx+14c}b^2 - 7560e^{13dx+13c}ab + 6720e^{12dx+12c}a^2 -}{}$$

input
```
int(sinh(d*x+c)^3*(a+b*sinh(d*x+c)^3)^2,x)
```

output
```
(35*e**(18*c + 18*d*x)*b**2 - 405*e**(16*c + 16*d*x)*b**2 + 840*e**(15*c +
 15*d*x)*a*b + 2268*e**(14*c + 14*d*x)*b**2 - 7560*e**(13*c + 13*d*x)*a*b
+ 6720*e**(12*c + 12*d*x)*a**2 - 8820*e**(12*c + 12*d*x)*b**2 + 37800*e**(
11*c + 11*d*x)*a*b - 60480*e**(10*c + 10*d*x)*a**2 + 39690*e**(10*c + 10*d
*x)*b**2 - 100800*e**(9*c + 9*d*x)*a*b*d*x - 60480*e**(8*c + 8*d*x)*a**2 +
 39690*e**(8*c + 8*d*x)*b**2 - 37800*e**(7*c + 7*d*x)*a*b + 6720*e**(6*c +
 6*d*x)*a**2 - 8820*e**(6*c + 6*d*x)*b**2 + 7560*e**(5*c + 5*d*x)*a*b + 22
68*e**(4*c + 4*d*x)*b**2 - 840*e**(3*c + 3*d*x)*a*b - 405*e**(2*c + 2*d*x)
*b**2 + 35*b**2)/(161280*e**(9*c + 9*d*x)*d)
```

3.128 $\int \sinh^2(c+dx) \left(a+b\sinh^3(c+dx)\right)^2 \, dx$

Optimal result

Integrand size = 23, antiderivative size = 180

$$\begin{aligned}\int \sinh^2(c+dx) \left(a+b\sinh^3(c+dx)\right)^2 \, dx = & -\frac{a^2x}{2}+\frac{35b^2x}{128}+\frac{2ab\cosh(c+dx)}{d} \\ & -\frac{4ab\cosh^3(c+dx)}{3d}+\frac{2ab\cosh^5(c+dx)}{5d} \\ & +\frac{a^2\cosh(c+dx)\sinh(c+dx)}{2d} \\ & -\frac{35b^2\cosh(c+dx)\sinh(c+dx)}{128d} \\ & +\frac{35b^2\cosh(c+dx)\sinh^3(c+dx)}{192d} \\ & -\frac{7b^2\cosh(c+dx)\sinh^5(c+dx)}{48d} \\ & +\frac{b^2\cosh(c+dx)\sinh^7(c+dx)}{8d}\end{aligned}$$

output
```
-1/2*a^2*x+35/128*b^2*x+2*a*b*cosh(d*x+c)/d-4/3*a*b*cosh(d*x+c)^3/d+2/5*a*
b*cosh(d*x+c)^5/d+1/2*a^2*cosh(d*x+c)*sinh(d*x+c)/d-35/128*b^2*cosh(d*x+c)
*sinh(d*x+c)/d+35/192*b^2*cosh(d*x+c)*sinh(d*x+c)^3/d-7/48*b^2*cosh(d*x+c)
*sinh(d*x+c)^5/d+1/8*b^2*cosh(d*x+c)*sinh(d*x+c)^7/d
```

Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 133, normalized size of antiderivative = 0.74

$$\int \sinh^2(c+dx)\left(a+b\sinh^3(c+dx)\right)^2\,dx$$
$$= \frac{-7680a^2c+4200b^2c-7680a^2dx+4200b^2dx+19200ab\cosh(c+dx)-3200ab\cosh(3(c+dx))+384ab}{}$$

input
```
Integrate[Sinh[c + d*x]^2*(a + b*Sinh[c + d*x]^3)^2,x]
```

output
```
(-7680*a^2*c + 4200*b^2*c - 7680*a^2*d*x + 4200*b^2*d*x + 19200*a*b*Cosh[c
 + d*x] - 3200*a*b*Cosh[3*(c + d*x)] + 384*a*b*Cosh[5*(c + d*x)] + 3840*a^
2*Sinh[2*(c + d*x)] - 3360*b^2*Sinh[2*(c + d*x)] + 840*b^2*Sinh[4*(c + d*x
)] - 160*b^2*Sinh[6*(c + d*x)] + 15*b^2*Sinh[8*(c + d*x)])/(15360*d)
```

Rubi [A] (verified)

Time = 0.44 (sec) , antiderivative size = 180, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.174$, Rules used = {3042, 25, 3699, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sinh^2(c+dx)\left(a+b\sinh^3(c+dx)\right)^2\,dx$$
$$\Big\downarrow \; 3042$$
$$\int -\sin(ic+idx)^2\left(a+ib\sin(ic+idx)^3\right)^2dx$$
$$\Big\downarrow \; 25$$
$$-\int \sin(ic+idx)^2\left(ib\sin(ic+idx)^3+a\right)^2dx$$
$$\Big\downarrow \; 3699$$
$$-\int \left(-b^2\sinh^8(c+dx)-2ab\sinh^5(c+dx)-a^2\sinh^2(c+dx)\right)dx$$

$$\downarrow \text{2009}$$

$$\frac{a^2\sinh(c+dx)\cosh(c+dx)}{2d}-\frac{a^2x}{2}+\frac{2ab\cosh^5(c+dx)}{5d}-\frac{4ab\cosh^3(c+dx)}{3d}+\frac{2ab\cosh(c+dx)}{d}+\frac{b^2\sinh^7(c+dx)\cosh(c+dx)}{8d}-\frac{7b^2\sinh^5(c+dx)\cosh(c+dx)}{48d}+\frac{35b^2\sinh^3(c+dx)\cosh(c+dx)}{192d}-\frac{35b^2\sinh(c+dx)\cosh(c+dx)}{128d}+\frac{35b^2x}{128}$$

input
```
Int[Sinh[c + d*x]^2*(a + b*Sinh[c + d*x]^3)^2,x]
```

output
```
-1/2*(a^2*x) + (35*b^2*x)/128 + (2*a*b*Cosh[c + d*x])/d - (4*a*b*Cosh[c +
d*x]^3)/(3*d) + (2*a*b*Cosh[c + d*x]^5)/(5*d) + (a^2*Cosh[c + d*x]*Sinh[c
+ d*x])/(2*d) - (35*b^2*Cosh[c + d*x]*Sinh[c + d*x])/(128*d) + (35*b^2*Cos
h[c + d*x]*Sinh[c + d*x]^3)/(192*d) - (7*b^2*Cosh[c + d*x]*Sinh[c + d*x]^5
)/(48*d) + (b^2*Cosh[c + d*x]*Sinh[c + d*x]^7)/(8*d)
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3699
```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^(n_
))^(p_.), x_Symbol] :> Int[ExpandTrig[sin[e + f*x]^m*(a + b*sin[e + f*x]^n)
^p, x], x] /; FreeQ[{a, b, e, f}, x] && IntegersQ[m, p] && (EqQ[n, 4] || Gt
Q[p, 0] || (EqQ[p, -1] && IntegerQ[n]))
```

Maple [A] (verified)

Time = 38.99 (sec) , antiderivative size = 122, normalized size of antiderivative = 0.68

method	result
derivativedivides	$\frac{a^2\left(\frac{\cosh(dx+c)\sinh(dx+c)}{2}-\frac{dx}{2}-\frac{c}{2}\right)+2ab\left(\frac{8}{15}+\frac{\sinh(dx+c)^4}{5}-\frac{4\sinh(dx+c)^2}{15}\right)\cosh(dx+c)+b^2\left(\left(\frac{\sinh(dx+c)^7}{8}-\frac{7\sinh(dx+}{48}\right.\right.}{d}$
default	$\frac{a^2\left(\frac{\cosh(dx+c)\sinh(dx+c)}{2}-\frac{dx}{2}-\frac{c}{2}\right)+2ab\left(\frac{8}{15}+\frac{\sinh(dx+c)^4}{5}-\frac{4\sinh(dx+c)^2}{15}\right)\cosh(dx+c)+b^2\left(\left(\frac{\sinh(dx+c)^7}{8}-\frac{7\sinh(dx+}{48}\right.\right.}{d}$
parts	$\frac{a^2\left(\frac{\cosh(dx+c)\sinh(dx+c)}{2}-\frac{dx}{2}-\frac{c}{2}\right)}{d}+\frac{b^2\left(\left(\frac{\sinh(dx+c)^7}{8}-\frac{7\sinh(dx+c)^5}{48}+\frac{35\sinh(dx+c)^3}{192}-\frac{35\sinh(dx+c)}{128}\right)\cosh(dx+c)\right.}{d}$
parallelrisch	$\frac{-7680a^2dx+4200b^2dx+19200ab\cosh(dx+c)+384ab\cosh(5dx+5c)-3200ab\cosh(3dx+3c)+15b^2\sinh(8dx+8c)-160b}{15360d}$
risch	$-\frac{a^2x}{2}+\frac{35b^2x}{128}+\frac{b^2e^{8dx+8c}}{2048d}-\frac{b^2e^{6dx+6c}}{192d}+\frac{b\,e^{5dx+5c}a}{80d}+\frac{7\,e^{4dx+4c}b^2}{256d}-\frac{5\,e^{3dx+3c}ab}{48d}+\frac{e^{2dx+2c}a^2}{8d}-\frac{7\,e}{}$
orering	Expression too large to display

input
```
int(sinh(d*x+c)^2*(a+b*sinh(d*x+c)^3)^2,x,method=_RETURNVERBOSE)
```

output
```
1/d*(a^2*(1/2*cosh(d*x+c)*sinh(d*x+c)-1/2*d*x-1/2*c)+2*a*b*(8/15+1/5*sinh(
d*x+c)^4-4/15*sinh(d*x+c)^2)*cosh(d*x+c)+b^2*((1/8*sinh(d*x+c)^7-7/48*sinh
(d*x+c)^5+35/192*sinh(d*x+c)^3-35/128*sinh(d*x+c))*cosh(d*x+c)+35/128*d*x+
35/128*c))
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 274, normalized size of antiderivative = 1.52

$$\int \sinh^2(c+dx)\left(a+b\sinh^3(c+dx)\right)^2\,dx$$

$$=\frac{15\,b^2\cosh(dx+c)\sinh(dx+c)^7+48\,ab\cosh(dx+c)^5+240\,ab\cosh(dx+c)\sinh(dx+c)^4+15\left(7\,b^2\right.}{}$$

input
```
integrate(sinh(d*x+c)^2*(a+b*sinh(d*x+c)^3)^2,x, algorithm="fricas")
```

output
```
1/1920*(15*b^2*cosh(d*x + c)*sinh(d*x + c)^7 + 48*a*b*cosh(d*x + c)^5 + 24
0*a*b*cosh(d*x + c)*sinh(d*x + c)^4 + 15*(7*b^2*cosh(d*x + c)^3 - 8*b^2*co
sh(d*x + c))*sinh(d*x + c)^5 - 400*a*b*cosh(d*x + c)^3 + 5*(21*b^2*cosh(d*
x + c)^5 - 80*b^2*cosh(d*x + c)^3 + 84*b^2*cosh(d*x + c))*sinh(d*x + c)^3
- 15*(64*a^2 - 35*b^2)*d*x + 2400*a*b*cosh(d*x + c) + 240*(2*a*b*cosh(d*x
+ c)^3 - 5*a*b*cosh(d*x + c))*sinh(d*x + c)^2 + 15*(b^2*cosh(d*x + c)^7 -
8*b^2*cosh(d*x + c)^5 + 28*b^2*cosh(d*x + c)^3 + 8*(8*a^2 - 7*b^2)*cosh(d*
x + c))*sinh(d*x + c))/d
```

Sympy [A] (verification not implemented)

Time = 0.69 (sec) , antiderivative size = 340, normalized size of antiderivative = 1.89

$$\int \sinh^2(c+dx)\left(a+b\sinh^3(c+dx)\right)^2\,dx$$
$$= \begin{cases} \frac{a^2x\sinh^2(c+dx)}{2} - \frac{a^2x\cosh^2(c+dx)}{2} + \frac{a^2\sinh(c+dx)\cosh(c+dx)}{2d} + \frac{2ab\sinh^4(c+dx)\cosh(c+dx)}{d} - \frac{8ab\sinh^2(c+dx)\cosh^3(c+dx)}{3d} \\ x\left(a+b\sinh^3(c)\right)^2\sinh^2(c) \end{cases}$$

input
```
integrate(sinh(d*x+c)**2*(a+b*sinh(d*x+c)**3)**2,x)
```

output
```
Piecewise((a**2*x*sinh(c + d*x)**2/2 - a**2*x*cosh(c + d*x)**2/2 + a**2*si
nh(c + d*x)*cosh(c + d*x)/(2*d) + 2*a*b*sinh(c + d*x)**4*cosh(c + d*x)/d -
 8*a*b*sinh(c + d*x)**2*cosh(c + d*x)**3/(3*d) + 16*a*b*cosh(c + d*x)**5/(
15*d) + 35*b**2*x*sinh(c + d*x)**8/128 - 35*b**2*x*sinh(c + d*x)**6*cosh(c
 + d*x)**2/32 + 105*b**2*x*sinh(c + d*x)**4*cosh(c + d*x)**4/64 - 35*b**2*
x*sinh(c + d*x)**2*cosh(c + d*x)**6/32 + 35*b**2*x*cosh(c + d*x)**8/128 +
93*b**2*sinh(c + d*x)**7*cosh(c + d*x)/(128*d) - 511*b**2*sinh(c + d*x)**5
*cosh(c + d*x)**3/(384*d) + 385*b**2*sinh(c + d*x)**3*cosh(c + d*x)**5/(38
4*d) - 35*b**2*sinh(c + d*x)*cosh(c + d*x)**7/(128*d), Ne(d, 0)), (x*(a +
b*sinh(c)**3)**2*sinh(c)**2, True))
```

Maxima [A] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 237, normalized size of antiderivative = 1.32

$$\int \sinh^2(c+dx)\left(a+b\sinh^3(c+dx)\right)^2\,dx = -\frac{1}{8}\,a^2\left(4\,x-\frac{e^{(2\,dx+2\,c)}}{d}+\frac{e^{(-2\,dx-2\,c)}}{d}\right)$$
$$-\frac{1}{6144}\,b^2\left(\frac{\left(32\,e^{(-2\,dx-2\,c)}-168\,e^{(-4\,dx-4\,c)}+672\,e^{(-6\,dx-6\,c)}-3\right)e^{(8\,dx+8\,c)}}{d}-\frac{1680\,(dx+c)}{d}-\frac{672\,e^{(-2\,dx}}{}\right.$$
$$+\frac{1}{240}\,ab\left(\frac{3\,e^{(5\,dx+5\,c)}}{d}-\frac{25\,e^{(3\,dx+3\,c)}}{d}+\frac{150\,e^{(dx+c)}}{d}+\frac{150\,e^{(-dx-c)}}{d}-\frac{25\,e^{(-3\,dx-3\,c)}}{d}+\frac{3\,e^{(-5\,dx-5\,c)}}{d}\right)$$

input
```
integrate(sinh(d*x+c)^2*(a+b*sinh(d*x+c)^3)^2,x, algorithm="maxima")
```

output
```
-1/8*a^2*(4*x - e^(2*d*x + 2*c)/d + e^(-2*d*x - 2*c)/d) - 1/6144*b^2*((32*
e^(-2*d*x - 2*c) - 168*e^(-4*d*x - 4*c) + 672*e^(-6*d*x - 6*c) - 3)*e^(8*d
*x + 8*c)/d - 1680*(d*x + c)/d - (672*e^(-2*d*x - 2*c) - 168*e^(-4*d*x - 4
*c) + 32*e^(-6*d*x - 6*c) - 3*e^(-8*d*x - 8*c))/d) + 1/240*a*b*(3*e^(5*d*x
 + 5*c)/d - 25*e^(3*d*x + 3*c)/d + 150*e^(d*x + c)/d + 150*e^(-d*x - c)/d
- 25*e^(-3*d*x - 3*c)/d + 3*e^(-5*d*x - 5*c)/d)
```

Giac [A] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 260, normalized size of antiderivative = 1.44

$$\int \sinh^2(c+dx)\left(a+b\sinh^3(c+dx)\right)^2\,dx = -\frac{1}{128}\left(64\,a^2-35\,b^2\right)x+\frac{b^2e^{(8\,dx+8\,c)}}{2048\,d}$$
$$-\frac{b^2e^{(6\,dx+6\,c)}}{192\,d}+\frac{abe^{(5\,dx+5\,c)}}{80\,d}+\frac{7\,b^2e^{(4\,dx+4\,c)}}{256\,d}$$
$$-\frac{5\,abe^{(3\,dx+3\,c)}}{48\,d}+\frac{5\,abe^{(dx+c)}}{8\,d}+\frac{5\,abe^{(-dx-c)}}{8\,d}$$
$$-\frac{5\,abe^{(-3\,dx-3\,c)}}{48\,d}-\frac{7\,b^2e^{(-4\,dx-4\,c)}}{256\,d}$$
$$+\frac{abe^{(-5\,dx-5\,c)}}{80\,d}+\frac{b^2e^{(-6\,dx-6\,c)}}{192\,d}$$
$$-\frac{b^2e^{(-8\,dx-8\,c)}}{2048\,d}+\frac{\left(8\,a^2-7\,b^2\right)e^{(2\,dx+2\,c)}}{64\,d}$$
$$-\frac{\left(8\,a^2-7\,b^2\right)e^{(-2\,dx-2\,c)}}{64\,d}$$

input
```
integrate(sinh(d*x+c)^2*(a+b*sinh(d*x+c)^3)^2,x, algorithm="giac")
```

output
```
-1/128*(64*a^2 - 35*b^2)*x + 1/2048*b^2*e^(8*d*x + 8*c)/d - 1/192*b^2*e^(6
*d*x + 6*c)/d + 1/80*a*b*e^(5*d*x + 5*c)/d + 7/256*b^2*e^(4*d*x + 4*c)/d -
 5/48*a*b*e^(3*d*x + 3*c)/d + 5/8*a*b*e^(d*x + c)/d + 5/8*a*b*e^(-d*x - c)
/d - 5/48*a*b*e^(-3*d*x - 3*c)/d - 7/256*b^2*e^(-4*d*x - 4*c)/d + 1/80*a*b
*e^(-5*d*x - 5*c)/d + 1/192*b^2*e^(-6*d*x - 6*c)/d - 1/2048*b^2*e^(-8*d*x
- 8*c)/d + 1/64*(8*a^2 - 7*b^2)*e^(2*d*x + 2*c)/d - 1/64*(8*a^2 - 7*b^2)*e
^(-2*d*x - 2*c)/d
```

Mupad [B] (verification not implemented)

Time = 2.54 (sec) , antiderivative size = 126, normalized size of antiderivative = 0.70

$$\int \sinh^2(c+dx) \left(a+b\sinh^3(c+dx)\right)^2 \, dx$$
$$= \frac{480\,a^2\sinh(2\,c+2\,d\,x)-420\,b^2\sinh(2\,c+2\,d\,x)+105\,b^2\sinh(4\,c+4\,d\,x)-20\,b^2\sinh(6\,c+6\,d\,x)+\frac{15\,b}{}}{}$$

input
```
int(sinh(c + d*x)^2*(a + b*sinh(c + d*x)^3)^2,x)
```

output
```
(480*a^2*sinh(2*c + 2*d*x) - 420*b^2*sinh(2*c + 2*d*x) + 105*b^2*sinh(4*c
+ 4*d*x) - 20*b^2*sinh(6*c + 6*d*x) + (15*b^2*sinh(8*c + 8*d*x))/8 + 2400*
a*b*cosh(c + d*x) - 400*a*b*cosh(3*c + 3*d*x) + 48*a*b*cosh(5*c + 5*d*x) -
 960*a^2*d*x + 525*b^2*d*x)/(1920*d)
```

Reduce [B] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 276, normalized size of antiderivative = 1.53

$$\int \sinh^2(c+dx) \left(a+b\sinh^3(c+dx)\right)^2 \, dx$$
$$= \frac{15e^{16dx+16c}b^2-160e^{14dx+14c}b^2+384e^{13dx+13c}ab+840e^{12dx+12c}b^2-3200e^{11dx+11c}ab+3840e^{10dx+10c}a^2-}{}$$

input
```
int(sinh(d*x+c)^2*(a+b*sinh(d*x+c)^3)^2,x)
```

output

```
(15*e**(16*c + 16*d*x)*b**2 - 160*e**(14*c + 14*d*x)*b**2 + 384*e**(13*c +
 13*d*x)*a*b + 840*e**(12*c + 12*d*x)*b**2 - 3200*e**(11*c + 11*d*x)*a*b +
 3840*e**(10*c + 10*d*x)*a**2 - 3360*e**(10*c + 10*d*x)*b**2 + 19200*e**(9
*c + 9*d*x)*a*b - 15360*e**(8*c + 8*d*x)*a**2*d*x + 8400*e**(8*c + 8*d*x)*
b**2*d*x + 19200*e**(7*c + 7*d*x)*a*b - 3840*e**(6*c + 6*d*x)*a**2 + 3360*
e**(6*c + 6*d*x)*b**2 - 3200*e**(5*c + 5*d*x)*a*b - 840*e**(4*c + 4*d*x)*b
**2 + 384*e**(3*c + 3*d*x)*a*b + 160*e**(2*c + 2*d*x)*b**2 - 15*b**2)/(307
20*e**(8*c + 8*d*x)*d)
```

3.129 $\int \sinh(c+dx)\left(a+b\sinh^3(c+dx)\right)^2\,dx$

Optimal result

Integrand size = 21, antiderivative size = 130

$$\int \sinh(c+dx)\left(a+b\sinh^3(c+dx)\right)^2\,dx = \frac{3abx}{4}+\frac{a^2\cosh(c+dx)}{d} - \frac{b^2\cosh(c+dx)}{d}+\frac{b^2\cosh^3(c+dx)}{d} - \frac{3b^2\cosh^5(c+dx)}{5d}+\frac{b^2\cosh^7(c+dx)}{7d} - \frac{3ab\cosh(c+dx)\sinh(c+dx)}{4d} + \frac{ab\cosh(c+dx)\sinh^3(c+dx)}{2d}$$

output
```
3/4*a*b*x+a^2*cosh(d*x+c)/d-b^2*cosh(d*x+c)/d+b^2*cosh(d*x+c)^3/d-3/5*b^2*
cosh(d*x+c)^5/d+1/7*b^2*cosh(d*x+c)^7/d-3/4*a*b*cosh(d*x+c)*sinh(d*x+c)/d+
1/2*a*b*cosh(d*x+c)*sinh(d*x+c)^3/d
```

Mathematica [A] (verified)

Time = 0.28 (sec) , antiderivative size = 92, normalized size of antiderivative = 0.71

$$\int \sinh(c+dx)\left(a+b\sinh^3(c+dx)\right)^2\,dx$$
$$=\frac{35(64a^2-35b^2)\cosh(c+dx)+b(245b\cosh(3(c+dx))-49b\cosh(5(c+dx))+5b\cosh(7(c+dx))+14}{2240d}$$

input
```
Integrate[Sinh[c + d*x]*(a + b*Sinh[c + d*x]^3)^2,x]
```

output
```
(35*(64*a^2 - 35*b^2)*Cosh[c + d*x] + b*(245*b*Cosh[3*(c + d*x)] - 49*b*Co
sh[5*(c + d*x)] + 5*b*Cosh[7*(c + d*x)] + 140*a*(12*(c + d*x) - 8*Sinh[2*(
c + d*x)] + Sinh[4*(c + d*x)])))/(2240*d)
```

Rubi [C] (verified)

Result contains complex when optimal does not.

Time = 0.36 (sec) , antiderivative size = 152, normalized size of antiderivative = 1.17, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.190, Rules used = {3042, 26, 3699, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sinh(c+dx)\left(a+b\sinh^3(c+dx)\right)^2\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int -i\sin(ic+idx)\left(a+ib\sin(ic+idx)^3\right)^2\,dx$$

$$\Big\downarrow \text{26}$$

$$-i\int \sin(ic+idx)\left(ib\sin(ic+idx)^3+a\right)^2\,dx$$

$$\Big\downarrow \text{3699}$$

$$-i\int \left(ib^2\sinh^7(c+dx)+2iab\sinh^4(c+dx)+ia^2\sinh(c+dx)\right)dx$$

$$\Big\downarrow \text{2009}$$

$$-i\left(\frac{ia^2\cosh(c+dx)}{d}+\frac{iab\sinh^3(c+dx)\cosh(c+dx)}{2d}-\frac{3iab\sinh(c+dx)\cosh(c+dx)}{4d}+\frac{3}{4}iabx+\frac{ib^2\cosh^7(c+\right.$$

input
```
Int[Sinh[c + d*x]*(a + b*Sinh[c + d*x]^3)^2,x]
```

output
```
(-I)*(((3*I)/4)*a*b*x + (I*a^2*Cosh[c + d*x])/d - (I*b^2*Cosh[c + d*x])/d
+ (I*b^2*Cosh[c + d*x]^3)/d - (((3*I)/5)*b^2*Cosh[c + d*x]^5)/d + ((I/7)*b
^2*Cosh[c + d*x]^7)/d - (((3*I)/4)*a*b*Cosh[c + d*x]*Sinh[c + d*x])/d + ((
I/2)*a*b*Cosh[c + d*x]*Sinh[c + d*x]^3)/d)
```

Defintions of rubi rules used

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3699
```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^(n_
))^(p_.), x_Symbol] :> Int[ExpandTrig[sin[e + f*x]^m*(a + b*sin[e + f*x]^n)
^p, x], x] /; FreeQ[{a, b, e, f}, x] && IntegersQ[m, p] && (EqQ[n, 4] || Gt
Q[p, 0] || (EqQ[p, -1] && IntegerQ[n]))
```

Maple [A] (verified)

Time = 12.94 (sec) , antiderivative size = 96, normalized size of antiderivative = 0.74

method	result
derivativedivides	$\frac{a^2\cosh(dx+c)+2ab\left(\left(\frac{\sinh(dx+c)^3}{4}-\frac{3\sinh(dx+c)}{8}\right)\cosh(dx+c)+\frac{3dx}{8}+\frac{3c}{8}\right)+b^2\left(-\frac{16}{35}+\frac{\sinh(dx+c)^6}{7}-\frac{6\sinh(dx+c)^4}{35}+\right.}{d}$
default	$\frac{a^2\cosh(dx+c)+2ab\left(\left(\frac{\sinh(dx+c)^3}{4}-\frac{3\sinh(dx+c)}{8}\right)\cosh(dx+c)+\frac{3dx}{8}+\frac{3c}{8}\right)+b^2\left(-\frac{16}{35}+\frac{\sinh(dx+c)^6}{7}-\frac{6\sinh(dx+c)^4}{35}+\frac{8}{}\right.}{d}$
parts	$\frac{b^2\left(-\frac{16}{35}+\frac{\sinh(dx+c)^6}{7}-\frac{6\sinh(dx+c)^4}{35}+\frac{8\sinh(dx+c)^2}{35}\right)\cosh(dx+c)}{d}+\frac{a^2\cosh(dx+c)}{d}+\frac{2ab\left(\left(\frac{\sinh(dx+c)^3}{4}-\frac{3\sinh(}{8}\right.\right.}{}$
parallelrisch	$\frac{245b^2\cosh(3dx+3c)-49b^2\cosh(5dx+5c)+5b^2\cosh(7dx+7c)-1120ab\sinh(2dx+2c)+140ab\sinh(4dx+4c)+(2240a^2-}{2240d}$
risch	$\frac{3abx}{4}+\frac{b^2e^{7dx+7c}}{896d}-\frac{7b^2e^{5dx+5c}}{640d}+\frac{e^{4dx+4c}ab}{32d}+\frac{7e^{3dx+3c}b^2}{128d}-\frac{e^{2dx+2c}ab}{4d}+\frac{e^{dx+c}a^2}{2d}-\frac{35e^{dx+c}b^2}{128d}+e^{-}$
orering	Expression too large to display

input
```
int(sinh(d*x+c)*(a+b*sinh(d*x+c)^3)^2,x,method=_RETURNVERBOSE)
```

output
```
1/d*(a^2*cosh(d*x+c)+2*a*b*((1/4*sinh(d*x+c)^3-3/8*sinh(d*x+c))*cosh(d*x+c
)+3/8*d*x+3/8*c)+b^2*(-16/35+1/7*sinh(d*x+c)^6-6/35*sinh(d*x+c)^4+8/35*sin
h(d*x+c)^2)*cosh(d*x+c))
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 220, normalized size of antiderivative = 1.69

$$\int \sinh(c+dx)\left(a+b\sinh^3(c+dx)\right)^2\,dx$$
$$=\frac{5\,b^2\cosh\left(dx+c\right)^7+35\,b^2\cosh\left(dx+c\right)\sinh\left(dx+c\right)^6-49\,b^2\cosh\left(dx+c\right)^5+560\,ab\cosh\left(dx+c\right)\sinh}{}$$

input
```
integrate(sinh(d*x+c)*(a+b*sinh(d*x+c)^3)^2,x, algorithm="fricas")
```

output
```
1/2240*(5*b^2*cosh(d*x + c)^7 + 35*b^2*cosh(d*x + c)*sinh(d*x + c)^6 - 49*
b^2*cosh(d*x + c)^5 + 560*a*b*cosh(d*x + c)*sinh(d*x + c)^3 + 245*b^2*cosh
(d*x + c)^3 + 35*(5*b^2*cosh(d*x + c)^3 - 7*b^2*cosh(d*x + c))*sinh(d*x +
c)^4 + 1680*a*b*d*x + 35*(3*b^2*cosh(d*x + c)^5 - 14*b^2*cosh(d*x + c)^3 +
 21*b^2*cosh(d*x + c))*sinh(d*x + c)^2 + 35*(64*a^2 - 35*b^2)*cosh(d*x + c
) + 560*(a*b*cosh(d*x + c)^3 - 4*a*b*cosh(d*x + c))*sinh(d*x + c))/d
```

Sympy [A] (verification not implemented)

Time = 0.47 (sec) , antiderivative size = 219, normalized size of antiderivative = 1.68

$$\int \sinh(c+dx)\left(a+b\sinh^3(c+dx)\right)^2\,dx$$

$$= \begin{cases} \frac{a^2\cosh(c+dx)}{d} + \frac{3abx\sinh^4(c+dx)}{4} - \frac{3abx\sinh^2(c+dx)\cosh^2(c+dx)}{2} + \frac{3abx\cosh^4(c+dx)}{4} + \frac{5ab\sinh^3(c+dx)\cosh(c+dx)}{4d} - \frac{3ab\,}{} \\ x\left(a+b\sinh^3(c)\right)^2\sinh(c) \end{cases}$$

input
```
integrate(sinh(d*x+c)*(a+b*sinh(d*x+c)**3)**2,x)
```

output
```
Piecewise((a**2*cosh(c + d*x)/d + 3*a*b*x*sinh(c + d*x)**4/4 - 3*a*b*x*sin
h(c + d*x)**2*cosh(c + d*x)**2/2 + 3*a*b*x*cosh(c + d*x)**4/4 + 5*a*b*sinh
(c + d*x)**3*cosh(c + d*x)/(4*d) - 3*a*b*sinh(c + d*x)*cosh(c + d*x)**3/(4
*d) + b**2*sinh(c + d*x)**6*cosh(c + d*x)/d - 2*b**2*sinh(c + d*x)**4*cosh
(c + d*x)**3/d + 8*b**2*sinh(c + d*x)**2*cosh(c + d*x)**5/(5*d) - 16*b**2*
cosh(c + d*x)**7/(35*d), Ne(d, 0)), (x*(a + b*sinh(c)**3)**2*sinh(c), True
))
```

Maxima [A] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 180, normalized size of antiderivative = 1.38

$$\int \sinh(c+dx)\left(a+b\sinh^3(c+dx)\right)^2\,dx$$
$$= \frac{1}{32}\,ab\left(24\,x + \frac{e^{(4\,dx+4\,c)}}{d} - \frac{8\,e^{(2\,dx+2\,c)}}{d} + \frac{8\,e^{(-2\,dx-2\,c)}}{d} - \frac{e^{(-4\,dx-4\,c)}}{d}\right)$$
$$- \frac{1}{4480}\,b^2\left(\frac{\left(49\,e^{(-2\,dx-2\,c)} - 245\,e^{(-4\,dx-4\,c)} + 1225\,e^{(-6\,dx-6\,c)} - 5\right)e^{(7\,dx+7\,c)}}{d} + \frac{1225\,e^{(-dx-c)} - 245\,e^{(-3\,dx}$$
$$+ \frac{a^2\cosh\left(dx+c\right)}{d}$$

input
```
integrate(sinh(d*x+c)*(a+b*sinh(d*x+c)^3)^2,x, algorithm="maxima")
```

output
```
1/32*a*b*(24*x + e^(4*d*x + 4*c)/d - 8*e^(2*d*x + 2*c)/d + 8*e^(-2*d*x - 2
*c)/d - e^(-4*d*x - 4*c)/d) - 1/4480*b^2*((49*e^(-2*d*x - 2*c) - 245*e^(-4
*d*x - 4*c) + 1225*e^(-6*d*x - 6*c) - 5)*e^(7*d*x + 7*c)/d + (1225*e^(-d*x
 - c) - 245*e^(-3*d*x - 3*c) + 49*e^(-5*d*x - 5*c) - 5*e^(-7*d*x - 7*c))/d
) + a^2*cosh(d*x + c)/d
```

Giac [A] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 219, normalized size of antiderivative = 1.68

$$\int \sinh(c+dx)\left(a+b\sinh^3(c+dx)\right)^2\,dx = \frac{3}{4}\,abx + \frac{b^2e^{(7\,dx+7\,c)}}{896\,d} - \frac{7\,b^2e^{(5\,dx+5\,c)}}{640\,d} + \frac{abe^{(4\,dx+4\,c)}}{32\,d} + \frac{7\,b^2e^{(3\,dx+3\,c)}}{128\,d} - \frac{abe^{(2\,dx+2\,c)}}{4\,d} + \frac{abe^{(-2\,dx-2\,c)}}{4\,d} + \frac{7\,b^2e^{(-3\,dx-3\,c)}}{128\,d} - \frac{abe^{(-4\,dx-4\,c)}}{32\,d} - \frac{7\,b^2e^{(-5\,dx-5\,c)}}{640\,d} + \frac{b^2e^{(-7\,dx-7\,c)}}{896\,d} + \frac{(64\,a^2-35\,b^2)e^{(dx+c)}}{128\,d} + \frac{(64\,a^2-35\,b^2)e^{(-dx-c)}}{128\,d}$$

input
```
integrate(sinh(d*x+c)*(a+b*sinh(d*x+c)^3)^2,x, algorithm="giac")
```

output
```
3/4*a*b*x + 1/896*b^2*e^(7*d*x + 7*c)/d - 7/640*b^2*e^(5*d*x + 5*c)/d + 1/
32*a*b*e^(4*d*x + 4*c)/d + 7/128*b^2*e^(3*d*x + 3*c)/d - 1/4*a*b*e^(2*d*x
+ 2*c)/d + 1/4*a*b*e^(-2*d*x - 2*c)/d + 7/128*b^2*e^(-3*d*x - 3*c)/d - 1/3
2*a*b*e^(-4*d*x - 4*c)/d - 7/640*b^2*e^(-5*d*x - 5*c)/d + 1/896*b^2*e^(-7*
d*x - 7*c)/d + 1/128*(64*a^2 - 35*b^2)*e^(d*x + c)/d + 1/128*(64*a^2 - 35*
b^2)*e^(-d*x - c)/d
```

Mupad [B] (verification not implemented)

Time = 1.73 (sec) , antiderivative size = 104, normalized size of antiderivative = 0.80

$$\int \sinh(c+dx)\left(a+b\sinh^3(c+dx)\right)^2\,dx = \frac{a^2\cosh(c+d\,x)+\frac{\sinh(c+d\,x)\,a\,b\cosh(c+d\,x)^3}{2}-\frac{5\sinh(c+d\,x)\,a\,b\cosh(c+d\,x)}{4}+\frac{3\,d\,x\,a\,b}{4}+\frac{b^2\cosh(c+d\,x)^7}{7}-\frac{3\,b^2\cosh(c+d\,x)^5}{5}}{d}$$

input
```
int(sinh(c + d*x)*(a + b*sinh(c + d*x)^3)^2,x)
```

output
```
(a^2*cosh(c + d*x) - b^2*cosh(c + d*x) + b^2*cosh(c + d*x)^3 - (3*b^2*cosh
(c + d*x)^5)/5 + (b^2*cosh(c + d*x)^7)/7 + (a*b*cosh(c + d*x)^3*sinh(c + d
*x))/2 - (5*a*b*cosh(c + d*x)*sinh(c + d*x))/4 + (3*a*b*d*x)/4)/d
```

Reduce [B] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 230, normalized size of antiderivative = 1.77

$$\int \sinh(c+dx)\left(a+b\sinh^3(c+dx)\right)^2\,dx = \frac{5e^{14dx+14c}b^2-49e^{12dx+12c}b^2+140e^{11dx+11c}ab+245e^{10dx+10c}b^2-1120e^{9dx+9c}ab+2240e^{8dx+8c}a^2-1225e}{}$$

input
```
int(sinh(d*x+c)*(a+b*sinh(d*x+c)^3)^2,x)
```

output
```
(5*e**(14*c + 14*d*x)*b**2 - 49*e**(12*c + 12*d*x)*b**2 + 140*e**(11*c + 1
1*d*x)*a*b + 245*e**(10*c + 10*d*x)*b**2 - 1120*e**(9*c + 9*d*x)*a*b + 224
0*e**(8*c + 8*d*x)*a**2 - 1225*e**(8*c + 8*d*x)*b**2 + 3360*e**(7*c + 7*d*
x)*a*b*d*x + 2240*e**(6*c + 6*d*x)*a**2 - 1225*e**(6*c + 6*d*x)*b**2 + 112
0*e**(5*c + 5*d*x)*a*b + 245*e**(4*c + 4*d*x)*b**2 - 140*e**(3*c + 3*d*x)*
a*b - 49*e**(2*c + 2*d*x)*b**2 + 5*b**2)/(4480*e**(7*c + 7*d*x)*d)
```

3.130 $\int \left(a+b\sinh^3(c+dx)\right)^2 \, dx$

Optimal result

Integrand size = 14, antiderivative size = 114

$$\begin{aligned}\int \left(a+b\sinh^3(c+dx)\right)^2 \, dx = {} & a^2x-\frac{5b^2x}{16}-\frac{2ab\cosh(c+dx)}{d}+\frac{2ab\cosh^3(c+dx)}{3d} \\ & +\frac{5b^2\cosh(c+dx)\sinh(c+dx)}{16d} \\ & -\frac{5b^2\cosh(c+dx)\sinh^3(c+dx)}{24d} \\ & +\frac{b^2\cosh(c+dx)\sinh^5(c+dx)}{6d}\end{aligned}$$

output
```
a^2*x-5/16*b^2*x-2*a*b*cosh(d*x+c)/d+2/3*a*b*cosh(d*x+c)^3/d+5/16*b^2*cosh
(d*x+c)*sinh(d*x+c)/d-5/24*b^2*cosh(d*x+c)*sinh(d*x+c)^3/d+1/6*b^2*cosh(d*
x+c)*sinh(d*x+c)^5/d
```

Mathematica [A] (verified)

Time = 0.22 (sec) , antiderivative size = 94, normalized size of antiderivative = 0.82

$$\int \left(a+b\sinh^3(c+dx)\right)^2\,dx$$
$$=\frac{192a^2c-60b^2c+192a^2dx-60b^2dx-288ab\cosh(c+dx)+32ab\cosh(3(c+dx))+45b^2\sinh(2(c+dx)}{192d}$$

input
```
Integrate[(a + b*Sinh[c + d*x]^3)^2,x]
```

output
```
(192*a^2*c - 60*b^2*c + 192*a^2*d*x - 60*b^2*d*x - 288*a*b*Cosh[c + d*x] +
 32*a*b*Cosh[3*(c + d*x)] + 45*b^2*Sinh[2*(c + d*x)] - 9*b^2*Sinh[4*(c + d
*x)] + b^2*Sinh[6*(c + d*x)])/(192*d)
```

Rubi [A] (verified)

Time = 0.33 (sec) , antiderivative size = 114, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.214, Rules used = {3042, 3692, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(a+b\sinh^3(c+dx)\right)^2\,dx$$

$$\Big\downarrow \ 3042$$

$$\int \left(a+ib\sin(ic+idx)^3\right)^2dx$$

$$\Big\downarrow \ 3692$$

$$\int \left(a^2+2ab\sinh^3(c+dx)+b^2\sinh^6(c+dx)\right)dx$$

$$\Big\downarrow \ 2009$$

$$a^2x+\frac{2ab\cosh^3(c+dx)}{3d}-\frac{2ab\cosh(c+dx)}{d}+\frac{b^2\sinh^5(c+dx)\cosh(c+dx)}{6d}-\frac{5b^2\sinh^3(c+dx)\cosh(c+dx)}{24d}+\frac{5b^2\sinh(c+dx)\cosh(c+dx)}{16d}-\frac{5b^2x}{16}$$

input
```
Int[(a + b*Sinh[c + d*x]^3)^2,x]
```

output
```
a^2*x - (5*b^2*x)/16 - (2*a*b*Cosh[c + d*x])/d + (2*a*b*Cosh[c + d*x]^3)/(
3*d) + (5*b^2*Cosh[c + d*x]*Sinh[c + d*x])/(16*d) - (5*b^2*Cosh[c + d*x]*S
inh[c + d*x]^3)/(24*d) + (b^2*Cosh[c + d*x]*Sinh[c + d*x]^5)/(6*d)
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3692
```
Int[((a_) + (b_.)*((c_.)*sin[(e_.) + (f_.)*(x_)])^(n_))^(p_), x_Symbol] :>
Int[ExpandTrig[(a + b*(c*sin[e + f*x])^n)^p, x], x] /; FreeQ[{a, b, c, e, f
, n}, x] && (IGtQ[p, 0] || (EqQ[p, -1] && IntegerQ[n]))
```

Maple [A] (verified)

Time = 0.22 (sec) , antiderivative size = 85, normalized size of antiderivative = 0.75

$$\frac{b^2\left(\left(\frac{\sinh(dx+c)^5}{6}-\frac{5\sinh(dx+c)^3}{24}+\frac{5\sinh(dx+c)}{16}\right)\cosh\left(dx+c\right)-\frac{5dx}{16}-\frac{5c}{16}\right)+2ab\left(-\frac{2}{3}+\frac{\sinh(dx+c)^2}{3}\right)\cosh\left(dx+c\right)}{d}$$

input
```
int((a+b*sinh(d*x+c)^3)^2,x)
```

output
```
1/d*(b^2*((1/6*sinh(d*x+c)^5-5/24*sinh(d*x+c)^3+5/16*sinh(d*x+c))*cosh(d*x
+c)-5/16*d*x-5/16*c)+2*a*b*(-2/3+1/3*sinh(d*x+c)^2)*cosh(d*x+c)+a^2*(d*x+c
))
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 160, normalized size of antiderivative = 1.40

$$\int \left(a+b\sinh^3(c+dx)\right)^2\,dx$$
$$=\frac{3\,b^2\cosh\left(dx+c\right)\sinh\left(dx+c\right)^5+16\,ab\cosh\left(dx+c\right)^3+48\,ab\cosh\left(dx+c\right)\sinh\left(dx+c\right)^2+2\left(5\,b^2\cosh\right.}{}$$

input
```
integrate((a+b*sinh(d*x+c)^3)^2,x, algorithm="fricas")
```

output
```
1/96*(3*b^2*cosh(d*x + c)*sinh(d*x + c)^5 + 16*a*b*cosh(d*x + c)^3 + 48*a*
b*cosh(d*x + c)*sinh(d*x + c)^2 + 2*(5*b^2*cosh(d*x + c)^3 - 9*b^2*cosh(d*
x + c))*sinh(d*x + c)^3 + 6*(16*a^2 - 5*b^2)*d*x - 144*a*b*cosh(d*x + c) +
 3*(b^2*cosh(d*x + c)^5 - 6*b^2*cosh(d*x + c)^3 + 15*b^2*cosh(d*x + c))*si
nh(d*x + c))/d
```

Sympy [A] (verification not implemented)

Time = 0.36 (sec) , antiderivative size = 212, normalized size of antiderivative = 1.86

$$\int \left(a+b\sinh^3(c+dx)\right)^2\,dx$$
$$=\begin{cases} a^2x+\frac{2ab\sinh^2(c+dx)\cosh(c+dx)}{d}-\frac{4ab\cosh^3(c+dx)}{3d}+\frac{5b^2x\sinh^6(c+dx)}{16}-\frac{15b^2x\sinh^4(c+dx)\cosh^2(c+dx)}{16}+\frac{15b^2x\sinh^2(c+\cdots}{} \\ x\left(a+b\sinh^3(c)\right)^2 \end{cases}$$

input
```
integrate((a+b*sinh(d*x+c)**3)**2,x)
```

output
```
Piecewise((a**2*x + 2*a*b*sinh(c + d*x)**2*cosh(c + d*x)/d - 4*a*b*cosh(c
+ d*x)**3/(3*d) + 5*b**2*x*sinh(c + d*x)**6/16 - 15*b**2*x*sinh(c + d*x)**
4*cosh(c + d*x)**2/16 + 15*b**2*x*sinh(c + d*x)**2*cosh(c + d*x)**4/16 - 5
*b**2*x*cosh(c + d*x)**6/16 + 11*b**2*sinh(c + d*x)**5*cosh(c + d*x)/(16*d
) - 5*b**2*sinh(c + d*x)**3*cosh(c + d*x)**3/(6*d) + 5*b**2*sinh(c + d*x)*
cosh(c + d*x)**5/(16*d), Ne(d, 0)), (x*(a + b*sinh(c)**3)**2, True))
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 151, normalized size of antiderivative = 1.32

$$\int \left(a+b\sinh^3(c+dx)\right)^2\,dx=a^2x-\frac{1}{384}\,b^2\left(\frac{\left(9\,e^{(-2\,dx-2\,c)}-45\,e^{(-4\,dx-4\,c)}-1\right)e^{(6\,dx+6\,c)}}{d}+\frac{120\,(dx+c)}{d}+\frac{45\,e^{(-2\,dx-2\,c)}-9\,e^{(-4\,dx-4\,c)}+e}{d}\right.$$
$$+\frac{1}{12}\,ab\left(\frac{e^{(3\,dx+3\,c)}}{d}-\frac{9\,e^{(dx+c)}}{d}-\frac{9\,e^{(-dx-c)}}{d}+\frac{e^{(-3\,dx-3\,c)}}{d}\right)$$

input
```
integrate((a+b*sinh(d*x+c)^3)^2,x, algorithm="maxima")
```

output
```
a^2*x - 1/384*b^2*((9*e^(-2*d*x - 2*c) - 45*e^(-4*d*x - 4*c) - 1)*e^(6*d*x
 + 6*c)/d + 120*(d*x + c)/d + (45*e^(-2*d*x - 2*c) - 9*e^(-4*d*x - 4*c) +
e^(-6*d*x - 6*c))/d) + 1/12*a*b*(e^(3*d*x + 3*c)/d - 9*e^(d*x + c)/d - 9*e
^(-d*x - c)/d + e^(-3*d*x - 3*c)/d)
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 178, normalized size of antiderivative = 1.56

$$\int \left(a+b\sinh^3(c+dx)\right)^2\,dx=\frac{1}{16}\left(16\,a^2-5\,b^2\right)x+\frac{b^2e^{(6\,dx+6\,c)}}{384\,d}-\frac{3\,b^2e^{(4\,dx+4\,c)}}{128\,d}+\frac{abe^{(3\,dx+3\,c)}}{12\,d}+\frac{15\,b^2e^{(2\,dx+2\,c)}}{128\,d}-\frac{3\,abe^{(dx+c)}}{4\,d}-\frac{3\,abe^{(-dx-c)}}{4\,d}-\frac{15\,b^2e^{(-2\,dx-2\,c)}}{128\,d}+\frac{abe^{(-3\,dx-3\,c)}}{12\,d}+\frac{3\,b^2e^{(-4\,dx-4\,c)}}{128\,d}-\frac{b^2e^{(-6\,dx-6\,c)}}{384\,d}$$

input
```
integrate((a+b*sinh(d*x+c)^3)^2,x, algorithm="giac")
```

output
```
1/16*(16*a^2 - 5*b^2)*x + 1/384*b^2*e^(6*d*x + 6*c)/d - 3/128*b^2*e^(4*d*x
 + 4*c)/d + 1/12*a*b*e^(3*d*x + 3*c)/d + 15/128*b^2*e^(2*d*x + 2*c)/d - 3/
4*a*b*e^(d*x + c)/d - 3/4*a*b*e^(-d*x - c)/d - 15/128*b^2*e^(-2*d*x - 2*c)
/d + 1/12*a*b*e^(-3*d*x - 3*c)/d + 3/128*b^2*e^(-4*d*x - 4*c)/d - 1/384*b^
2*e^(-6*d*x - 6*c)/d
```

Mupad [B] (verification not implemented)

Time = 0.48 (sec) , antiderivative size = 85, normalized size of antiderivative = 0.75

$$\int \left(a+b\sinh^3(c+dx)\right)^2\,dx$$
$$=\frac{\frac{45\,b^2\sinh(2\,c+2\,d\,x)}{4}-\frac{9\,b^2\sinh(4\,c+4\,d\,x)}{4}+\frac{b^2\sinh(6\,c+6\,d\,x)}{4}-72\,a\,b\cosh(c+d\,x)+8\,a\,b\cosh(3\,c+3\,d\,x)+48\,a^2\,d}{48\,d}$$

input
```
int((a + b*sinh(c + d*x)^3)^2,x)
```

output
```
((45*b^2*sinh(2*c + 2*d*x))/4 - (9*b^2*sinh(4*c + 4*d*x))/4 + (b^2*sinh(6*
c + 6*d*x))/4 - 72*a*b*cosh(c + d*x) + 8*a*b*cosh(3*c + 3*d*x) + 48*a^2*d*
x - 15*b^2*d*x)/(48*d)
```

Reduce [B] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 187, normalized size of antiderivative = 1.64

$$\int \left(a+b\sinh^3(c+dx)\right)^2\,dx$$
$$=\frac{e^{12dx+12c}b^2-9e^{10dx+10c}b^2+32e^{9dx+9c}ab+45e^{8dx+8c}b^2-288e^{7dx+7c}ab+384e^{6dx+6c}a^2dx-120e^{6dx+6c}b^2d}{384e^{6dx+6c}d}$$

input
```
int((a+b*sinh(d*x+c)^3)^2,x)
```

output
```
(e**(12*c + 12*d*x)*b**2 - 9*e**(10*c + 10*d*x)*b**2 + 32*e**(9*c + 9*d*x)
*a*b + 45*e**(8*c + 8*d*x)*b**2 - 288*e**(7*c + 7*d*x)*a*b + 384*e**(6*c +
 6*d*x)*a**2*d*x - 120*e**(6*c + 6*d*x)*b**2*d*x - 288*e**(5*c + 5*d*x)*a*
b - 45*e**(4*c + 4*d*x)*b**2 + 32*e**(3*c + 3*d*x)*a*b + 9*e**(2*c + 2*d*x
)*b**2 - b**2)/(384*e**(6*c + 6*d*x)*d)
```

3.131 $\int \operatorname{csch}(c+dx) \left(a+b\sinh^3(c+dx)\right)^2 \, dx$

Optimal result

Integrand size = 21, antiderivative size = 88

$$\int \operatorname{csch}(c+dx) \left(a+b\sinh^3(c+dx)\right)^2 \, dx = -abx - \frac{a^2 \operatorname{arctanh}(\cosh(c+dx))}{d} + \frac{b^2\cosh(c+dx)}{d} - \frac{2b^2\cosh^3(c+dx)}{3d} + \frac{b^2\cosh^5(c+dx)}{5d} + \frac{ab\cosh(c+dx)\sinh(c+dx)}{d}$$

output
```
-a*b*x-a^2*arctanh(cosh(d*x+c))/d+b^2*cosh(d*x+c)/d-2/3*b^2*cosh(d*x+c)^3/
d+1/5*b^2*cosh(d*x+c)^5/d+a*b*cosh(d*x+c)*sinh(d*x+c)/d
```

Mathematica [A] (verified)

Time = 0.21 (sec) , antiderivative size = 73, normalized size of antiderivative = 0.83

$$\int \operatorname{csch}(c+dx) \left(a+b\sinh^3(c+dx)\right)^2 \, dx = \frac{-240a^2 \operatorname{arctanh}(\cosh(c+dx)) + b(150b\cosh(c+dx) - 25b\cosh(3(c+dx)) + 3b\cosh(5(c+dx)) + 120a}{240d}$$

input
```
Integrate[Csch[c + d*x]*(a + b*Sinh[c + d*x]^3)^2,x]
```

output
```
(-240*a^2*ArcTanh[Cosh[c + d*x]] + b*(150*b*Cosh[c + d*x] - 25*b*Cosh[3*(c
 + d*x)] + 3*b*Cosh[5*(c + d*x)] + 120*a*(-2*(c + d*x) + Sinh[2*(c + d*x)]
)))/(240*d)
```

Rubi [C] (verified)

Result contains complex when optimal does not.

Time = 0.33 (sec) , antiderivative size = 106, normalized size of antiderivative = 1.20, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.190$, Rules used = {3042, 26, 3699, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \text{csch}(c+dx)\left(a+b\sinh^3(c+dx)\right)^2\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{i\left(a+ib\sin(ic+idx)^3\right)^2}{\sin(ic+idx)}dx$$

$$\Big\downarrow \text{26}$$

$$i\int \frac{\left(ib\sin(ic+idx)^3+a\right)^2}{\sin(ic+idx)}dx$$

$$\Big\downarrow \text{3699}$$

$$i\int \left(-ib^2\sinh^5(c+dx)-2iab\sinh^2(c+dx)-ia^2\text{csch}(c+dx)\right)dx$$

$$\Big\downarrow \text{2009}$$

$$i\left(\frac{ia^2\text{arctanh}(\cosh(c+dx))}{d}-\frac{iab\sinh(c+dx)\cosh(c+dx)}{d}+iabx-\frac{ib^2\cosh^5(c+dx)}{5d}+\frac{2ib^2\cosh^3(c+dx)}{3d}-\right.$$

input
```
Int[Csch[c + d*x]*(a + b*Sinh[c + d*x]^3)^2,x]
```

output
```
I*(I*a*b*x + (I*a^2*ArcTanh[Cosh[c + d*x]])/d - (I*b^2*Cosh[c + d*x])/d +
(((2*I)/3)*b^2*Cosh[c + d*x]^3)/d - ((I/5)*b^2*Cosh[c + d*x]^5)/d - (I*a*b
*Cosh[c + d*x]*Sinh[c + d*x])/d)
```

Defintions of rubi rules used

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3699
```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^(n_
))^(p_.), x_Symbol] :> Int[ExpandTrig[sin[e + f*x]^m*(a + b*sin[e + f*x]^n)
^p, x], x] /; FreeQ[{a, b, e, f}, x] && IntegersQ[m, p] && (EqQ[n, 4] || Gt
Q[p, 0] || (EqQ[p, -1] && IntegerQ[n]))
```

Maple [A] (verified)

Time = 1.10 (sec) , antiderivative size = 76, normalized size of antiderivative = 0.86

method	result
derivativedivides	$\frac{-2a^2\operatorname{arctanh}\left(\mathrm{e}^{dx+c}\right)+2ab\left(\frac{\cosh(dx+c)\sinh(dx+c)}{2}-\frac{dx}{2}-\frac{c}{2}\right)+b^2\left(\frac{8}{15}+\frac{\sinh(dx+c)^4}{5}-\frac{4\sinh(dx+c)^2}{15}\right)\cosh(dx+c)}{d}$
default	$\frac{-2a^2\operatorname{arctanh}\left(\mathrm{e}^{dx+c}\right)+2ab\left(\frac{\cosh(dx+c)\sinh(dx+c)}{2}-\frac{dx}{2}-\frac{c}{2}\right)+b^2\left(\frac{8}{15}+\frac{\sinh(dx+c)^4}{5}-\frac{4\sinh(dx+c)^2}{15}\right)\cosh(dx+c)}{d}$
parallelrisch	$\frac{-240abdx+128b^2+240a^2\ln\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+150b^2\cosh(dx+c)+3b^2\cosh(5dx+5c)-25b^2\cosh(3dx+3c)+120ab\sinh}{240d}$
risch	$-abx+\frac{b^2\mathrm{e}^{5dx+5c}}{160d}-\frac{5\,\mathrm{e}^{3dx+3c}b^2}{96d}+\frac{\mathrm{e}^{2dx+2c}ab}{4d}+\frac{5\,\mathrm{e}^{dx+c}b^2}{16d}+\frac{5\,\mathrm{e}^{-dx-c}b^2}{16d}-\frac{\mathrm{e}^{-2dx-2c}ab}{4d}-\frac{5\,\mathrm{e}^{-3dx-3c}b^2}{96d}$

input
```
int(csch(d*x+c)*(a+b*sinh(d*x+c)^3)^2,x,method=_RETURNVERBOSE)
```

output
```
1/d*(-2*a^2*arctanh(exp(d*x+c))+2*a*b*(1/2*cosh(d*x+c)*sinh(d*x+c)-1/2*d*x
-1/2*c)+b^2*(8/15+1/5*sinh(d*x+c)^4-4/15*sinh(d*x+c)^2)*cosh(d*x+c))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1052 vs. $2(84) = 168$.

Time = 0.11 (sec) , antiderivative size = 1052, normalized size of antiderivative = 11.95

$$\int \operatorname{csch}(c+dx)\left(a+b\sinh^3(c+dx)\right)^2\,dx = \text{Too large to display}$$

input
```
integrate(csch(d*x+c)*(a+b*sinh(d*x+c)^3)^2,x, algorithm="fricas")
```

output
```
1/480*(3*b^2*cosh(d*x + c)^10 + 30*b^2*cosh(d*x + c)*sinh(d*x + c)^9 + 3*b
^2*sinh(d*x + c)^10 - 25*b^2*cosh(d*x + c)^8 - 480*a*b*d*x*cosh(d*x + c)^5
 + 120*a*b*cosh(d*x + c)^7 + 5*(27*b^2*cosh(d*x + c)^2 - 5*b^2)*sinh(d*x +
 c)^8 + 150*b^2*cosh(d*x + c)^6 + 40*(9*b^2*cosh(d*x + c)^3 - 5*b^2*cosh(d
*x + c) + 3*a*b)*sinh(d*x + c)^7 + 10*(63*b^2*cosh(d*x + c)^4 - 70*b^2*cos
h(d*x + c)^2 + 84*a*b*cosh(d*x + c) + 15*b^2)*sinh(d*x + c)^6 + 150*b^2*co
sh(d*x + c)^4 + 4*(189*b^2*cosh(d*x + c)^5 - 350*b^2*cosh(d*x + c)^3 - 120
*a*b*d*x + 630*a*b*cosh(d*x + c)^2 + 225*b^2*cosh(d*x + c))*sinh(d*x + c)^
5 - 120*a*b*cosh(d*x + c)^3 + 10*(63*b^2*cosh(d*x + c)^6 - 175*b^2*cosh(d*
x + c)^4 - 240*a*b*d*x*cosh(d*x + c) + 420*a*b*cosh(d*x + c)^3 + 225*b^2*c
osh(d*x + c)^2 + 15*b^2)*sinh(d*x + c)^4 - 25*b^2*cosh(d*x + c)^2 + 40*(9*
b^2*cosh(d*x + c)^7 - 35*b^2*cosh(d*x + c)^5 - 120*a*b*d*x*cosh(d*x + c)^2
 + 105*a*b*cosh(d*x + c)^4 + 75*b^2*cosh(d*x + c)^3 + 15*b^2*cosh(d*x + c)
 - 3*a*b)*sinh(d*x + c)^3 + 5*(27*b^2*cosh(d*x + c)^8 - 140*b^2*cosh(d*x +
 c)^6 - 960*a*b*d*x*cosh(d*x + c)^3 + 504*a*b*cosh(d*x + c)^5 + 450*b^2*co
sh(d*x + c)^4 + 180*b^2*cosh(d*x + c)^2 - 72*a*b*cosh(d*x + c) - 5*b^2)*si
nh(d*x + c)^2 + 3*b^2 - 480*(a^2*cosh(d*x + c)^5 + 5*a^2*cosh(d*x + c)^4*s
inh(d*x + c) + 10*a^2*cosh(d*x + c)^3*sinh(d*x + c)^2 + 10*a^2*cosh(d*x +
c)^2*sinh(d*x + c)^3 + 5*a^2*cosh(d*x + c)*sinh(d*x + c)^4 + a^2*sinh(d*x
+ c)^5)*log(cosh(d*x + c) + sinh(d*x + c) + 1) + 480*(a^2*cosh(d*x + c)...
```

Sympy [F(-1)]

Timed out.

$$\int \operatorname{csch}(c+dx)\left(a+b\sinh^3(c+dx)\right)^2\,dx=\text{Timed out}$$

input
```
integrate(csch(d*x+c)*(a+b*sinh(d*x+c)**3)**2,x)
```

output
```
Timed out
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 140, normalized size of antiderivative = 1.59

$$\int \operatorname{csch}(c+dx)\left(a+b\sinh^3(c+dx)\right)^2\,dx=-\frac{1}{4}\,ab\left(4\,x-\frac{e^{(2\,dx+2\,c)}}{d}+\frac{e^{(-2\,dx-2\,c)}}{d}\right)+\frac{1}{480}\,b^2\left(\frac{3\,e^{(5\,dx+5\,c)}}{d}-\frac{25\,e^{(3\,dx+3\,c)}}{d}+\frac{150\,e^{(dx+c)}}{d}+\frac{150\,e^{(-dx-c)}}{d}-\frac{25\,e^{(-3\,dx-3\,c)}}{d}+\frac{3\,e^{(-5\,dx-5\,c)}}{d}\right)+\frac{a^2\log\left(\tanh\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)}{d}$$

input
```
integrate(csch(d*x+c)*(a+b*sinh(d*x+c)^3)^2,x, algorithm="maxima")
```

output
```
-1/4*a*b*(4*x - e^(2*d*x + 2*c)/d + e^(-2*d*x - 2*c)/d) + 1/480*b^2*(3*e^(
5*d*x + 5*c)/d - 25*e^(3*d*x + 3*c)/d + 150*e^(d*x + c)/d + 150*e^(-d*x -
c)/d - 25*e^(-3*d*x - 3*c)/d + 3*e^(-5*d*x - 5*c)/d) + a^2*log(tanh(1/2*d*
x + 1/2*c))/d
```

Giac [A] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 154, normalized size of antiderivative = 1.75

$$\int \operatorname{csch}(c+dx)\left(a+b\sinh^3(c+dx)\right)^2\,dx = -\frac{480\,(dx+c)ab-3\,b^2e^{(5\,dx+5\,c)}+25\,b^2e^{(3\,dx+3\,c)}-120\,abe^{(2\,dx+2\,c)}-150\,b^2e^{(dx+c)}+480\,a^2\log\left(e^{(dx+c)}+\right.}{48}$$

input
```
integrate(csch(d*x+c)*(a+b*sinh(d*x+c)^3)^2,x, algorithm="giac")
```

output
```
-1/480*(480*(d*x + c)*a*b - 3*b^2*e^(5*d*x + 5*c) + 25*b^2*e^(3*d*x + 3*c)
 - 120*a*b*e^(2*d*x + 2*c) - 150*b^2*e^(d*x + c) + 480*a^2*log(e^(d*x + c)
 + 1) - 480*a^2*log(abs(e^(d*x + c) - 1)) - (150*b^2*e^(4*d*x + 4*c) - 120
*a*b*e^(3*d*x + 3*c) - 25*b^2*e^(2*d*x + 2*c) + 3*b^2)*e^(-5*d*x - 5*c))/d
```

Mupad [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 177, normalized size of antiderivative = 2.01

$$\int \operatorname{csch}(c+dx)\left(a+b\sinh^3(c+dx)\right)^2\,dx = \frac{5\,b^2\,\mathrm{e}^{c+d\,x}}{16\,d}-\frac{2\,\mathrm{atan}\left(\frac{a^2\,\mathrm{e}^{d\,x}\,\mathrm{e}^c\,\sqrt{-d^2}}{d\,\sqrt{a^4}}\right)\,\sqrt{a^4}}{\sqrt{-d^2}}-a\,b\,x+\frac{5\,b^2\,\mathrm{e}^{-c-d\,x}}{16\,d}-\frac{5\,b^2\,\mathrm{e}^{-3\,c-3\,d\,x}}{96\,d}-\frac{5\,b^2\,\mathrm{e}^{3\,c+3\,d\,x}}{96\,d}+\frac{b^2\,\mathrm{e}^{-5\,c-5\,d\,x}}{160\,d}+\frac{b^2\,\mathrm{e}^{5\,c+5\,d\,x}}{160\,d}-\frac{a\,b\,\mathrm{e}^{-2\,c-2\,d\,x}}{4\,d}+\frac{a\,b\,\mathrm{e}^{2\,c+2\,d\,x}}{4\,d}$$

input
```
int((a + b*sinh(c + d*x)^3)^2/sinh(c + d*x),x)
```

output
```
(5*b^2*exp(c + d*x))/(16*d) - (2*atan((a^2*exp(d*x)*exp(c)*(-d^2)^(1/2))/(
d*(a^4)^(1/2)))*(a^4)^(1/2))/(-d^2)^(1/2) - a*b*x + (5*b^2*exp(- c - d*x))
/(16*d) - (5*b^2*exp(- 3*c - 3*d*x))/(96*d) - (5*b^2*exp(3*c + 3*d*x))/(96
*d) + (b^2*exp(- 5*c - 5*d*x))/(160*d) + (b^2*exp(5*c + 5*d*x))/(160*d) -
(a*b*exp(- 2*c - 2*d*x))/(4*d) + (a*b*exp(2*c + 2*d*x))/(4*d)
```

Reduce [B] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 192, normalized size of antiderivative = 2.18

$$\int \operatorname{csch}(c+dx)\left(a+b\sinh^3(c+dx)\right)^2\,dx$$
$$=\frac{3e^{10dx+10c}b^2-25e^{8dx+8c}b^2+120e^{7dx+7c}ab+150e^{6dx+6c}b^2+480e^{5dx+5c}\log\left(e^{dx+c}-1\right)a^2-480e^{5dx+5c}\log}{480e^{5dx+5c}d}$$

input
```
int(csch(d*x+c)*(a+b*sinh(d*x+c)^3)^2,x)
```

output
```
(3*e**(10*c + 10*d*x)*b**2 - 25*e**(8*c + 8*d*x)*b**2 + 120*e**(7*c + 7*d*
x)*a*b + 150*e**(6*c + 6*d*x)*b**2 + 480*e**(5*c + 5*d*x)*log(e**(c + d*x)
 - 1)*a**2 - 480*e**(5*c + 5*d*x)*log(e**(c + d*x) + 1)*a**2 - 480*e**(5*c
 + 5*d*x)*a*b*d*x + 150*e**(4*c + 4*d*x)*b**2 - 120*e**(3*c + 3*d*x)*a*b -
 25*e**(2*c + 2*d*x)*b**2 + 3*b**2)/(480*e**(5*c + 5*d*x)*d)
```

3.132 $\int \operatorname{csch}^2(c+dx)\left(a+b\sinh^3(c+dx)\right)^2\,dx$

Optimal result

Integrand size = 23, antiderivative size = 82

$$\int \operatorname{csch}^2(c+dx)\left(a+b\sinh^3(c+dx)\right)^2\,dx=\frac{3b^2x}{8}+\frac{2ab\cosh(c+dx)}{d}-\frac{a^2\coth(c+dx)}{d}-\frac{3b^2\cosh(c+dx)\sinh(c+dx)}{8d}+\frac{b^2\cosh(c+dx)\sinh^3(c+dx)}{4d}$$

output
```
3/8*b^2*x+2*a*b*cosh(d*x+c)/d-a^2*coth(d*x+c)/d-3/8*b^2*cosh(d*x+c)*sinh(d
*x+c)/d+1/4*b^2*cosh(d*x+c)*sinh(d*x+c)^3/d
```

Mathematica [A] (verified)

Time = 0.22 (sec) , antiderivative size = 92, normalized size of antiderivative = 1.12

$$\int \operatorname{csch}^2(c+dx)\left(a+b\sinh^3(c+dx)\right)^2\,dx=\frac{3b^2(c+dx)}{8d}+\frac{2ab\cosh(c)\cosh(dx)}{d}-\frac{a^2\coth(c+dx)}{d}+\frac{2ab\sinh(c)\sinh(dx)}{d}-\frac{b^2\sinh(2(c+dx))}{4d}+\frac{b^2\sinh(4(c+dx))}{32d}$$

input
```
Integrate[Csch[c + d*x]^2*(a + b*Sinh[c + d*x]^3)^2,x]
```

output
```
(3*b^2*(c + d*x))/(8*d) + (2*a*b*Cosh[c]*Cosh[d*x])/d - (a^2*Coth[c + d*x]
)/d + (2*a*b*Sinh[c]*Sinh[d*x])/d - (b^2*Sinh[2*(c + d*x)])/(4*d) + (b^2*S
inh[4*(c + d*x)])/(32*d)
```

Rubi [A] (verified)

Time = 0.33 (sec) , antiderivative size = 82, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.174, Rules used = {3042, 25, 3699, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \text{csch}^2(c+dx) \left(a+b\sinh^3(c+dx)\right)^2 dx$$

$$\Big\downarrow \text{3042}$$

$$\int -\frac{\left(a+ib\sin(ic+idx)^3\right)^2}{\sin(ic+idx)^2}dx$$

$$\Big\downarrow \text{25}$$

$$-\int \frac{\left(ib\sin(ic+idx)^3+a\right)^2}{\sin(ic+idx)^2}dx$$

$$\Big\downarrow \text{3699}$$

$$-\int \left(-b^2\sinh^4(c+dx)-2ab\sinh(c+dx)-a^2\text{csch}^2(c+dx)\right)dx$$

$$\Big\downarrow \text{2009}$$

$$-\frac{a^2\coth(c+dx)}{d}+\frac{2ab\cosh(c+dx)}{d}+\frac{b^2\sinh^3(c+dx)\cosh(c+dx)}{4d}-\frac{3b^2\sinh(c+dx)\cosh(c+dx)}{8d}+\frac{3b^2x}{8}$$

input
```
Int[Csch[c + d*x]^2*(a + b*Sinh[c + d*x]^3)^2,x]
```

output
```
(3*b^2*x)/8 + (2*a*b*Cosh[c + d*x])/d - (a^2*Coth[c + d*x])/d - (3*b^2*Cos
h[c + d*x]*Sinh[c + d*x])/(8*d) + (b^2*Cosh[c + d*x]*Sinh[c + d*x]^3)/(4*d
)
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3699
```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^(n_
))^(p_.), x_Symbol] :> Int[ExpandTrig[sin[e + f*x]^m*(a + b*sin[e + f*x]^n)
^p, x], x] /; FreeQ[{a, b, e, f}, x] && IntegersQ[m, p] && (EqQ[n, 4] || Gt
Q[p, 0] || (EqQ[p, -1] && IntegerQ[n]))
```

Maple [A] (verified)

Time = 0.89 (sec) , antiderivative size = 65, normalized size of antiderivative = 0.79

method	result
derivativedivides	$\frac{-\coth(dx+c)a^2+2ab\cosh(dx+c)+b^2\left(\left(\frac{\sinh(dx+c)^3}{4}-\frac{3\sinh(dx+c)}{8}\right)\cosh(dx+c)+\frac{3dx}{8}+\frac{3c}{8}\right)}{d}$
default	$\frac{-\coth(dx+c)a^2+2ab\cosh(dx+c)+b^2\left(\left(\frac{\sinh(dx+c)^3}{4}-\frac{3\sinh(dx+c)}{8}\right)\cosh(dx+c)+\frac{3dx}{8}+\frac{3c}{8}\right)}{d}$
parallelrisch	$\frac{-4\coth\left(\frac{dx}{2}+\frac{c}{2}\right)a^2+2\,\mathrm{sech}\left(\frac{dx}{2}+\frac{c}{2}\right)\mathrm{csch}\left(\frac{dx}{2}+\frac{c}{2}\right)a^2-b^2\sinh(2dx+2c)+8\left(\frac{3dxb}{16}+a\cosh(dx+c)+\frac{b\sinh(4dx+4c)}{64}+a\right)b}{4d}$
risch	$\frac{3b^2x}{8}+\frac{e^{4dx+4c}b^2}{64d}-\frac{e^{2dx+2c}b^2}{8d}+\frac{e^{dx+c}ab}{d}+\frac{e^{-dx-c}ab}{d}+\frac{e^{-2dx-2c}b^2}{8d}-\frac{e^{-4dx-4c}b^2}{64d}-\frac{2a^2}{d(e^{2dx+2c}-1)}$

input
```
int(csch(d*x+c)^2*(a+b*sinh(d*x+c)^3)^2,x,method=_RETURNVERBOSE)
```

output
```
1/d*(-coth(d*x+c)*a^2+2*a*b*cosh(d*x+c)+b^2*((1/4*sinh(d*x+c)^3-3/8*sinh(d
*x+c))*cosh(d*x+c)+3/8*d*x+3/8*c))
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 142, normalized size of antiderivative = 1.73

$$\int \operatorname{csch}^2(c+dx) \left(a+b\sinh^3(c+dx)\right)^2 \, dx$$
$$= \frac{b^2\cosh\left(dx+c\right)^5 + 5\,b^2\cosh\left(dx+c\right)\sinh\left(dx+c\right)^4 - 9\,b^2\cosh\left(dx+c\right)^3 + \left(10\,b^2\cosh\left(dx+c\right)^3 - 27\,b^2 \cdots\right.}{64\,d\,s\cdots}$$

input
```
integrate(csch(d*x+c)^2*(a+b*sinh(d*x+c)^3)^2,x, algorithm="fricas")
```

output
```
1/64*(b^2*cosh(d*x + c)^5 + 5*b^2*cosh(d*x + c)*sinh(d*x + c)^4 - 9*b^2*co
sh(d*x + c)^3 + (10*b^2*cosh(d*x + c)^3 - 27*b^2*cosh(d*x + c))*sinh(d*x +
 c)^2 - 8*(8*a^2 - b^2)*cosh(d*x + c) + 8*(3*b^2*d*x + 16*a*b*cosh(d*x + c
) + 8*a^2)*sinh(d*x + c))/(d*sinh(d*x + c))
```

Sympy [F(-1)]

Timed out.

$$\int \operatorname{csch}^2(c+dx) \left(a+b\sinh^3(c+dx)\right)^2 \, dx = \text{Timed out}$$

input
```
integrate(csch(d*x+c)**2*(a+b*sinh(d*x+c)**3)**2,x)
```

output
```
Timed out
```

Maxima [A] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 113, normalized size of antiderivative = 1.38

$$\int \operatorname{csch}^2(c+dx)\left(a+b\sinh^3(c+dx)\right)^2\,dx$$
$$=\frac{1}{64}b^2\left(24x+\frac{e^{(4dx+4c)}}{d}-\frac{8e^{(2dx+2c)}}{d}+\frac{8e^{(-2dx-2c)}}{d}-\frac{e^{(-4dx-4c)}}{d}\right)+ab\left(\frac{e^{(dx+c)}}{d}+\frac{e^{(-dx-c)}}{d}\right)+\frac{2a^2}{d(e^{(-2dx-2c)}-1)}$$

input
```
integrate(csch(d*x+c)^2*(a+b*sinh(d*x+c)^3)^2,x, algorithm="maxima")
```

output
```
1/64*b^2*(24*x + e^(4*d*x + 4*c)/d - 8*e^(2*d*x + 2*c)/d + 8*e^(-2*d*x - 2
*c)/d - e^(-4*d*x - 4*c)/d) + a*b*(e^(d*x + c)/d + e^(-d*x - c)/d) + 2*a^2
/(d*(e^(-2*d*x - 2*c) - 1))
```

Giac [A] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 149, normalized size of antiderivative = 1.82

$$\int \operatorname{csch}^2(c+dx)\left(a+b\sinh^3(c+dx)\right)^2\,dx$$
$$=\frac{24(dx+c)b^2+b^2e^{(4dx+4c)}-8b^2e^{(2dx+2c)}+64abe^{(dx+c)}+\frac{(64abe^{(5dx+5c)}-64abe^{(3dx+3c)}-9b^2e^{(2dx+2c)}+b^2-8(16}{(e^{(dx+c)}+1)(e^{(dx+c)}-1)}}{64d}$$

input
```
integrate(csch(d*x+c)^2*(a+b*sinh(d*x+c)^3)^2,x, algorithm="giac")
```

output
```
1/64*(24*(d*x + c)*b^2 + b^2*e^(4*d*x + 4*c) - 8*b^2*e^(2*d*x + 2*c) + 64*
a*b*e^(d*x + c) + (64*a*b*e^(5*d*x + 5*c) - 64*a*b*e^(3*d*x + 3*c) - 9*b^2
*e^(2*d*x + 2*c) + b^2 - 8*(16*a^2 - b^2)*e^(4*d*x + 4*c))*e^(-4*d*x - 4*c
)/((e^(d*x + c) + 1)*(e^(d*x + c) - 1)))/d
```

Mupad [B] (verification not implemented)

Time = 1.67 (sec) , antiderivative size = 123, normalized size of antiderivative = 1.50

$$\int \operatorname{csch}^2(c+dx)\left(a+b\sinh^3(c+dx)\right)^2\,dx=\frac{3\,b^2\,x}{8}-\frac{2\,a^2}{d\,\left(\mathrm{e}^{2\,c+2\,d\,x}-1\right)}+\frac{b^2\,\mathrm{e}^{-2\,c-2\,d\,x}}{8\,d}-\frac{b^2\,\mathrm{e}^{2\,c+2\,d\,x}}{8\,d}-\frac{b^2\,\mathrm{e}^{-4\,c-4\,d\,x}}{64\,d}+\frac{b^2\,\mathrm{e}^{4\,c+4\,d\,x}}{64\,d}+\frac{a\,b\,\mathrm{e}^{c+d\,x}}{d}+\frac{a\,b\,\mathrm{e}^{-c-d\,x}}{d}$$

input
```
int((a + b*sinh(c + d*x)^3)^2/sinh(c + d*x)^2,x)
```

output
```
(3*b^2*x)/8 - (2*a^2)/(d*(exp(2*c + 2*d*x) - 1)) + (b^2*exp(- 2*c - 2*d*x)
)/(8*d) - (b^2*exp(2*c + 2*d*x))/(8*d) - (b^2*exp(- 4*c - 4*d*x))/(64*d) +
 (b^2*exp(4*c + 4*d*x))/(64*d) + (a*b*exp(c + d*x))/d + (a*b*exp(- c - d*x
))/d
```

Reduce [B] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 171, normalized size of antiderivative = 2.09

$$\int \operatorname{csch}^2(c+dx)\left(a+b\sinh^3(c+dx)\right)^2\,dx$$
$$=\frac{e^{10dx+10c}b^2-9e^{8dx+8c}b^2+64e^{7dx+7c}ab-128e^{6dx+6c}a^2+24e^{6dx+6c}b^2dx+16e^{6dx+6c}b^2-24e^{4dx+4c}b^2dx-}{64e^{4dx+4c}d\left(e^{2dx+2c}-1\right)}$$

input
```
int(csch(d*x+c)^2*(a+b*sinh(d*x+c)^3)^2,x)
```

output
```
(e**(10*c + 10*d*x)*b**2 - 9*e**(8*c + 8*d*x)*b**2 + 64*e**(7*c + 7*d*x)*a
*b - 128*e**(6*c + 6*d*x)*a**2 + 24*e**(6*c + 6*d*x)*b**2*d*x + 16*e**(6*c
 + 6*d*x)*b**2 - 24*e**(4*c + 4*d*x)*b**2*d*x - 64*e**(3*c + 3*d*x)*a*b -
9*e**(2*c + 2*d*x)*b**2 + b**2)/(64*e**(4*c + 4*d*x)*d*(e**(2*c + 2*d*x) -
 1))
```

3.133 $\int \operatorname{csch}^3(c+dx) \left(a+b\sinh^3(c+dx)\right)^2 \, dx$

Optimal result

Integrand size = 23, antiderivative size = 77

$$\int \operatorname{csch}^3(c+dx) \left(a+b\sinh^3(c+dx)\right)^2 \, dx = 2abx + \frac{a^2 \operatorname{arctanh}(\cosh(c+dx))}{2d} - \frac{b^2\cosh(c+dx)}{d} + \frac{b^2\cosh^3(c+dx)}{3d} - \frac{a^2\coth(c+dx)\operatorname{csch}(c+dx)}{2d}$$

output
```
2*a*b*x+1/2*a^2*arctanh(cosh(d*x+c))/d-b^2*cosh(d*x+c)/d+1/3*b^2*cosh(d*x+
c)^3/d-1/2*a^2*coth(d*x+c)*csch(d*x+c)/d
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 126, normalized size of antiderivative = 1.64

$$\int \operatorname{csch}^3(c+dx) \left(a+b\sinh^3(c+dx)\right)^2 \, dx = 2abx - \frac{3b^2\cosh(c+dx)}{4d} + \frac{b^2\cosh(3(c+dx))}{12d} - \frac{a^2\operatorname{csch}^2\left(\frac{1}{2}(c+dx)\right)}{8d} + \frac{a^2\log\left(\cosh\left(\frac{1}{2}(c+dx)\right)\right)}{2d} - \frac{a^2\log\left(\sinh\left(\frac{1}{2}(c+dx)\right)\right)}{2d} - \frac{a^2\operatorname{sech}^2\left(\frac{1}{2}(c+dx)\right)}{8d}$$

input
```
Integrate[Csch[c + d*x]^3*(a + b*Sinh[c + d*x]^3)^2,x]
```

output
```
2*a*b*x - (3*b^2*Cosh[c + d*x])/(4*d) + (b^2*Cosh[3*(c + d*x)])/(12*d) - (
a^2*Csch[(c + d*x)/2]^2)/(8*d) + (a^2*Log[Cosh[(c + d*x)/2]])/(2*d) - (a^2
*Log[Sinh[(c + d*x)/2]])/(2*d) - (a^2*Sech[(c + d*x)/2]^2)/(8*d)
```

Rubi [C] (verified)

Result contains complex when optimal does not.

Time = 0.32 (sec) , antiderivative size = 91, normalized size of antiderivative = 1.18, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.174$, Rules used = {3042, 26, 3699, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \operatorname{csch}^3(c+dx)\left(a+b\sinh^3(c+dx)\right)^2\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int -\frac{i\left(a+ib\sin(ic+idx)^3\right)^2}{\sin(ic+idx)^3}dx$$

$$\Big\downarrow \text{26}$$

$$-i\int \frac{\left(ib\sin(ic+idx)^3+a\right)^2}{\sin(ic+idx)^3}dx$$

$$\Big\downarrow \text{3699}$$

$$-i\int \left(ia^2\operatorname{csch}^3(c+dx)+ib^2\sinh^3(c+dx)+2iab\right)dx$$

$$\Big\downarrow \text{2009}$$

$$-i\left(\frac{ia^2\text{arctanh}(\cosh(c+dx))}{2d}-\frac{ia^2\coth(c+dx)\operatorname{csch}(c+dx)}{2d}+2iabx+\frac{ib^2\cosh^3(c+dx)}{3d}-\frac{ib^2\cosh(c+dx)}{d}\right)$$

input
```
Int[Csch[c + d*x]^3*(a + b*Sinh[c + d*x]^3)^2,x]
```

output
```
(-I)*((2*I)*a*b*x + ((I/2)*a^2*ArcTanh[Cosh[c + d*x]])/d - (I*b^2*Cosh[c +
 d*x])/d + ((I/3)*b^2*Cosh[c + d*x]^3)/d - ((I/2)*a^2*Coth[c + d*x]*Csch[c
 + d*x])/d)
```

Defintions of rubi rules used

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3699
```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^(n_
))^(p_.), x_Symbol] :> Int[ExpandTrig[sin[e + f*x]^m*(a + b*sin[e + f*x]^n)
^p, x], x] /; FreeQ[{a, b, e, f}, x] && IntegersQ[m, p] && (EqQ[n, 4] || Gt
Q[p, 0] || (EqQ[p, -1] && IntegerQ[n]))
```

Maple [A] (verified)

Time = 0.92 (sec) , antiderivative size = 63, normalized size of antiderivative = 0.82

method	result
derivativedivides	$\frac{a^2\left(-\frac{\operatorname{csch}(dx+c)\coth(dx+c)}{2}+\operatorname{arctanh}\left(e^{dx+c}\right)\right)+2ab(dx+c)+b^2\left(-\frac{2}{3}+\frac{\sinh(dx+c)^2}{3}\right)\cosh(dx+c)}{d}$
default	$\frac{a^2\left(-\frac{\operatorname{csch}(dx+c)\coth(dx+c)}{2}+\operatorname{arctanh}\left(e^{dx+c}\right)\right)+2ab(dx+c)+b^2\left(-\frac{2}{3}+\frac{\sinh(dx+c)^2}{3}\right)\cosh(dx+c)}{d}$
parallelrisch	$\frac{-12a^2\ln\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+3a^2\left(\operatorname{sech}\left(\frac{dx}{2}+\frac{c}{2}\right)^2-5\right)\operatorname{csch}\left(\frac{dx}{2}+\frac{c}{2}\right)^2+9\coth\left(\frac{dx}{2}+\frac{c}{2}\right)^2a^2+48abdx-18b^2\cosh(dx+c)+}{24d}$
risch	$2abx+\frac{e^{3dx+3c}b^2}{24d}-\frac{3\,e^{dx+c}b^2}{8d}-\frac{3\,e^{-dx-c}b^2}{8d}+\frac{e^{-3dx-3c}b^2}{24d}-\frac{a^2e^{dx+c}\left(e^{2dx+2c}+1\right)}{d\left(e^{2dx+2c}-1\right)^2}+\frac{a^2\ln\left(e^{dx+c}+1\right)}{2d}-$

input
```
int(csch(d*x+c)^3*(a+b*sinh(d*x+c)^3)^2,x,method=_RETURNVERBOSE)
```

output
```
1/d*(a^2*(-1/2*csch(d*x+c)*coth(d*x+c)+arctanh(exp(d*x+c)))+2*a*b*(d*x+c)+
b^2*(-2/3+1/3*sinh(d*x+c)^2)*cosh(d*x+c))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1616 vs. $2(71) = 142$.

Time = 0.12 (sec) , antiderivative size = 1616, normalized size of antiderivative = 20.99

$$\int \operatorname{csch}^3(c+dx)\left(a+b\sinh^3(c+dx)\right)^2\,dx = \text{Too large to display}$$

input
```
integrate(csch(d*x+c)^3*(a+b*sinh(d*x+c)^3)^2,x, algorithm="fricas")
```

output
```
1/24*(b^2*cosh(d*x + c)^10 + 10*b^2*cosh(d*x + c)*sinh(d*x + c)^9 + b^2*si
nh(d*x + c)^10 + 48*a*b*d*x*cosh(d*x + c)^7 - 11*b^2*cosh(d*x + c)^8 - 96*
a*b*d*x*cosh(d*x + c)^5 + (45*b^2*cosh(d*x + c)^2 - 11*b^2)*sinh(d*x + c)^
8 + 8*(15*b^2*cosh(d*x + c)^3 + 6*a*b*d*x - 11*b^2*cosh(d*x + c))*sinh(d*x
 + c)^7 + 48*a*b*d*x*cosh(d*x + c)^3 - 2*(12*a^2 - 5*b^2)*cosh(d*x + c)^6
+ 2*(105*b^2*cosh(d*x + c)^4 + 168*a*b*d*x*cosh(d*x + c) - 154*b^2*cosh(d*
x + c)^2 - 12*a^2 + 5*b^2)*sinh(d*x + c)^6 + 4*(63*b^2*cosh(d*x + c)^5 + 2
52*a*b*d*x*cosh(d*x + c)^2 - 154*b^2*cosh(d*x + c)^3 - 24*a*b*d*x - 3*(12*
a^2 - 5*b^2)*cosh(d*x + c))*sinh(d*x + c)^5 - 2*(12*a^2 - 5*b^2)*cosh(d*x
+ c)^4 + 2*(105*b^2*cosh(d*x + c)^6 + 840*a*b*d*x*cosh(d*x + c)^3 - 385*b^
2*cosh(d*x + c)^4 - 240*a*b*d*x*cosh(d*x + c) - 15*(12*a^2 - 5*b^2)*cosh(d
*x + c)^2 - 12*a^2 + 5*b^2)*sinh(d*x + c)^4 - 11*b^2*cosh(d*x + c)^2 + 8*(
15*b^2*cosh(d*x + c)^7 + 210*a*b*d*x*cosh(d*x + c)^4 - 77*b^2*cosh(d*x + c
)^5 - 120*a*b*d*x*cosh(d*x + c)^2 + 6*a*b*d*x - 5*(12*a^2 - 5*b^2)*cosh(d*
x + c)^3 - (12*a^2 - 5*b^2)*cosh(d*x + c))*sinh(d*x + c)^3 + (45*b^2*cosh(
d*x + c)^8 + 1008*a*b*d*x*cosh(d*x + c)^5 - 308*b^2*cosh(d*x + c)^6 - 960*
a*b*d*x*cosh(d*x + c)^3 + 144*a*b*d*x*cosh(d*x + c) - 30*(12*a^2 - 5*b^2)*
cosh(d*x + c)^4 - 12*(12*a^2 - 5*b^2)*cosh(d*x + c)^2 - 11*b^2)*sinh(d*x +
 c)^2 + b^2 + 12*(a^2*cosh(d*x + c)^7 + 7*a^2*cosh(d*x + c)*sinh(d*x + c)^
6 + a^2*sinh(d*x + c)^7 - 2*a^2*cosh(d*x + c)^5 + (21*a^2*cosh(d*x + c)...
```

Sympy [F(-1)]

Timed out.

$$\int \operatorname{csch}^3(c+dx)\left(a+b\sinh^3(c+dx)\right)^2\,dx=\text{Timed out}$$

input
```
integrate(csch(d*x+c)**3*(a+b*sinh(d*x+c)**3)**2,x)
```

output
```
Timed out
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 152 vs. $2(71) = 142$.

Time = 0.05 (sec) , antiderivative size = 152, normalized size of antiderivative = 1.97

$$\int \operatorname{csch}^3(c+dx)\left(a+b\sinh^3(c+dx)\right)^2\,dx = 2\,abx+\frac{1}{24}\,b^2\left(\frac{e^{(3\,dx+3\,c)}}{d}-\frac{9\,e^{(dx+c)}}{d}-\frac{9\,e^{(-dx-c)}}{d}+\frac{e^{(-3\,dx-3\,c)}}{d}\right)+\frac{1}{2}\,a^2\left(\frac{\log\left(e^{(-dx-c)}+1\right)}{d}-\frac{\log\left(e^{(-dx-c)}-1\right)}{d}+\frac{2\left(e^{(-dx-c)}+e^{(-3\,dx-3\,c)}\right)}{d(2\,e^{(-2\,dx-2\,c)}-e^{(-4\,dx-4\,c)}-1)}\right)$$

input
```
integrate(csch(d*x+c)^3*(a+b*sinh(d*x+c)^3)^2,x, algorithm="maxima")
```

output
```
2*a*b*x + 1/24*b^2*(e^(3*d*x + 3*c)/d - 9*e^(d*x + c)/d - 9*e^(-d*x - c)/d
 + e^(-3*d*x - 3*c)/d) + 1/2*a^2*(log(e^(-d*x - c) + 1)/d - log(e^(-d*x -
c) - 1)/d + 2*(e^(-d*x - c) + e^(-3*d*x - 3*c))/(d*(2*e^(-2*d*x - 2*c) - e
^(-4*d*x - 4*c) - 1)))
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 162 vs. $2(71) = 142$.

Time = 0.17 (sec) , antiderivative size = 162, normalized size of antiderivative = 2.10

$$\int \operatorname{csch}^3(c+dx)\left(a+b\sinh^3(c+dx)\right)^2\,dx = \frac{48\,(dx+c)ab+b^2e^{(3\,dx+3\,c)}-9\,b^2e^{(dx+c)}+12\,a^2\log\left(e^{(dx+c)}+1\right)-12\,a^2\log\left(\left|e^{(dx+c)}-1\right|\right)-\frac{(11\,b^2e^{(2\,dx+2}}{}}{24\,d}$$

input
```
integrate(csch(d*x+c)^3*(a+b*sinh(d*x+c)^3)^2,x, algorithm="giac")
```

output
```
1/24*(48*(d*x + c)*a*b + b^2*e^(3*d*x + 3*c) - 9*b^2*e^(d*x + c) + 12*a^2*
log(e^(d*x + c) + 1) - 12*a^2*log(abs(e^(d*x + c) - 1)) - (11*b^2*e^(2*d*x
 + 2*c) - b^2 + 3*(8*a^2 + 3*b^2)*e^(6*d*x + 6*c) + (24*a^2 - 19*b^2)*e^(4
*d*x + 4*c))*e^(-3*d*x - 3*c)/((e^(d*x + c) + 1)^2*(e^(d*x + c) - 1)^2))/d
```

Mupad [B] (verification not implemented)

Time = 1.64 (sec) , antiderivative size = 175, normalized size of antiderivative = 2.27

$$\int \operatorname{csch}^3(c+dx)\left(a+b\sinh^3(c+dx)\right)^2\,dx = \frac{\operatorname{atan}\left(\frac{a^2\,e^{d\,x}\,e^{c}\,\sqrt{-d^2}}{d\,\sqrt{a^4}}\right)\,\sqrt{a^4}}{\sqrt{-d^2}} - \frac{3\,b^2\,e^{c+d\,x}}{8\,d} + 2\,a\,b\,x - \frac{3\,b^2\,e^{-c-d\,x}}{8\,d} + \frac{b^2\,e^{-3\,c-3\,d\,x}}{24\,d} + \frac{b^2\,e^{3\,c+3\,d\,x}}{24\,d} - \frac{a^2\,e^{c+d\,x}}{d\,\left(e^{2\,c+2\,d\,x}-1\right)} - \frac{2\,a^2\,e^{c+d\,x}}{d\,\left(e^{4\,c+4\,d\,x}-2\,e^{2\,c+2\,d\,x}+1\right)}$$

input
```
int((a + b*sinh(c + d*x)^3)^2/sinh(c + d*x)^3,x)
```

output
```
(atan((a^2*exp(d*x)*exp(c)*(-d^2)^(1/2))/(d*(a^4)^(1/2)))*(a^4)^(1/2))/(-d
^2)^(1/2) - (3*b^2*exp(c + d*x))/(8*d) + 2*a*b*x - (3*b^2*exp(- c - d*x))/
(8*d) + (b^2*exp(- 3*c - 3*d*x))/(24*d) + (b^2*exp(3*c + 3*d*x))/(24*d) -
(a^2*exp(c + d*x))/(d*(exp(2*c + 2*d*x) - 1)) - (2*a^2*exp(c + d*x))/(d*(e
xp(4*c + 4*d*x) - 2*exp(2*c + 2*d*x) + 1))
```

Reduce [B] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 349, normalized size of antiderivative = 4.53

$$\int \operatorname{csch}^3(c+dx)\left(a+b\sinh^3(c+dx)\right)^2\,dx$$
$$= \frac{e^{10dx+10c}b^2 - 11e^{8dx+8c}b^2 - 12e^{7dx+7c}\log\left(e^{dx+c}-1\right)a^2 + 12e^{7dx+7c}\log\left(e^{dx+c}+1\right)a^2 + 48e^{7dx+7c}abdx - 2}{}$$

input
```
int(csch(d*x+c)^3*(a+b*sinh(d*x+c)^3)^2,x)
```

output
```
(e**(10*c + 10*d*x)*b**2 - 11*e**(8*c + 8*d*x)*b**2 - 12*e**(7*c + 7*d*x)*
log(e**(c + d*x) - 1)*a**2 + 12*e**(7*c + 7*d*x)*log(e**(c + d*x) + 1)*a**
2 + 48*e**(7*c + 7*d*x)*a*b*d*x - 24*e**(6*c + 6*d*x)*a**2 + 10*e**(6*c +
6*d*x)*b**2 + 24*e**(5*c + 5*d*x)*log(e**(c + d*x) - 1)*a**2 - 24*e**(5*c
+ 5*d*x)*log(e**(c + d*x) + 1)*a**2 - 96*e**(5*c + 5*d*x)*a*b*d*x - 24*e**
(4*c + 4*d*x)*a**2 + 10*e**(4*c + 4*d*x)*b**2 - 12*e**(3*c + 3*d*x)*log(e*
*(c + d*x) - 1)*a**2 + 12*e**(3*c + 3*d*x)*log(e**(c + d*x) + 1)*a**2 + 48
*e**(3*c + 3*d*x)*a*b*d*x - 11*e**(2*c + 2*d*x)*b**2 + b**2)/(24*e**(3*c +
 3*d*x)*d*(e**(4*c + 4*d*x) - 2*e**(2*c + 2*d*x) + 1))
```

3.134 $\int \operatorname{csch}^4(c+dx)\left(a+b\sinh^3(c+dx)\right)^2\,dx$

Optimal result

Integrand size = 23, antiderivative size = 76

$$\int \operatorname{csch}^4(c+dx)\left(a+b\sinh^3(c+dx)\right)^2\,dx=-\frac{b^2x}{2}-\frac{2ab\operatorname{arctanh}(\cosh(c+dx))}{d}+\frac{a^2\coth(c+dx)}{d}-\frac{a^2\coth^3(c+dx)}{3d}+\frac{b^2\cosh(c+dx)\sinh(c+dx)}{2d}$$

output
```
-1/2*b^2*x-2*a*b*arctanh(cosh(d*x+c))/d+a^2*coth(d*x+c)/d-1/3*a^2*coth(d*x
+c)^3/d+1/2*b^2*cosh(d*x+c)*sinh(d*x+c)/d
```

Mathematica [A] (verified)

Time = 0.26 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.80

$$\int \operatorname{csch}^4(c+dx)\left(a+b\sinh^3(c+dx)\right)^2\,dx=\frac{-24ab\operatorname{arctanh}(\cosh(c+dx))-4a^2\coth(c+dx)\left(-2+\operatorname{csch}^2(c+dx)\right)+3b^2(-2(c+dx)+\sinh(2(c+d}{12d}$$

input
```
Integrate[Csch[c + d*x]^4*(a + b*Sinh[c + d*x]^3)^2,x]
```

output
```
(-24*a*b*ArcTanh[Cosh[c + d*x]] - 4*a^2*Coth[c + d*x]*(-2 + Csch[c + d*x]^
2) + 3*b^2*(-2*(c + d*x) + Sinh[2*(c + d*x)]))/(12*d)
```

Rubi [A] (verified)

Time = 0.31 (sec) , antiderivative size = 76, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.130$, Rules used = {3042, 3699, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \text{csch}^4(c+dx)\left(a+b\sinh^3(c+dx)\right)^2\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\left(a+ib\sin(ic+idx)^3\right)^2}{\sin(ic+idx)^4}dx$$

$$\Big\downarrow \text{3699}$$

$$\int \left(a^2\text{csch}^4(c+dx)+2ab\text{csch}(c+dx)+b^2\sinh^2(c+dx)\right)dx$$

$$\Big\downarrow \text{2009}$$

$$-\frac{a^2\coth^3(c+dx)}{3d}+\frac{a^2\coth(c+dx)}{d}-\frac{2ab\text{arctanh}(\cosh(c+dx))}{d}+\frac{b^2\sinh(c+dx)\cosh(c+dx)}{2d}-\frac{b^2x}{2}$$

input
```
Int[Csch[c + d*x]^4*(a + b*Sinh[c + d*x]^3)^2,x]
```

output
```
-1/2*(b^2*x) - (2*a*b*ArcTanh[Cosh[c + d*x]])/d + (a^2*Coth[c + d*x])/d -
(a^2*Coth[c + d*x]^3)/(3*d) + (b^2*Cosh[c + d*x]*Sinh[c + d*x])/(2*d)
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3699
```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^(n_
))^(p_.), x_Symbol] :> Int[ExpandTrig[sin[e + f*x]^m*(a + b*sin[e + f*x]^n)
^p, x], x] /; FreeQ[{a, b, e, f}, x] && IntegersQ[m, p] && (EqQ[n, 4] || Gt
Q[p, 0] || (EqQ[p, -1] && IntegerQ[n]))
```

Maple [A] (verified)

Time = 0.84 (sec) , antiderivative size = 65, normalized size of antiderivative = 0.86

method	result	s
derivativedivides	$\frac{a^2\left(\frac{2}{3}-\frac{\operatorname{csch}(dx+c)^2}{3}\right)\coth(dx+c)-4ab\,\operatorname{arctanh}(e^{dx+c})+b^2\left(\frac{\cosh(dx+c)\sinh(dx+c)}{2}-\frac{dx}{2}-\frac{c}{2}\right)}{d}$	6
default	$\frac{a^2\left(\frac{2}{3}-\frac{\operatorname{csch}(dx+c)^2}{3}\right)\coth(dx+c)-4ab\,\operatorname{arctanh}(e^{dx+c})+b^2\left(\frac{\cosh(dx+c)\sinh(dx+c)}{2}-\frac{dx}{2}-\frac{c}{2}\right)}{d}$	6
parallelrisch	$\frac{32\ln\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)\right)ab-a^2\left(\cosh(dx+c)-\frac{\cosh(3dx+3c)}{3}\right)\operatorname{sech}\left(\frac{dx}{2}+\frac{c}{2}\right)^3\operatorname{csch}\left(\frac{dx}{2}+\frac{c}{2}\right)^3-8\left(dx-\frac{\sinh(2dx+2c)}{2}\right)b^2}{16d}$	8
risch	$-\frac{b^2x}{2}+\frac{e^{2dx+2c}b^2}{8d}-\frac{e^{-2dx-2c}b^2}{8d}-\frac{4a^2\left(3\,e^{2dx+2c}-1\right)}{3d\left(e^{2dx+2c}-1\right)^3}+\frac{2ab\ln\left(e^{dx+c}-1\right)}{d}-\frac{2ab\ln\left(e^{dx+c}+1\right)}{d}$	1

input
```
int(csch(d*x+c)^4*(a+b*sinh(d*x+c)^3)^2,x,method=_RETURNVERBOSE)
```

output
```
1/d*(a^2*(2/3-1/3*csch(d*x+c)^2)*coth(d*x+c)-4*a*b*arctanh(exp(d*x+c))+b^2
*(1/2*cosh(d*x+c)*sinh(d*x+c)-1/2*d*x-1/2*c))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1748 vs. $2(70) = 140$.

Time = 0.12 (sec) , antiderivative size = 1748, normalized size of antiderivative = 23.00

$$\int \operatorname{csch}^4(c+dx)\left(a+b\sinh^3(c+dx)\right)^2\,dx = \text{Too large to display}$$

input
```
integrate(csch(d*x+c)^4*(a+b*sinh(d*x+c)^3)^2,x, algorithm="fricas")
```

output
```
1/24*(3*b^2*cosh(d*x + c)^10 + 30*b^2*cosh(d*x + c)*sinh(d*x + c)^9 + 3*b^
2*sinh(d*x + c)^10 - 3*(4*b^2*d*x + 3*b^2)*cosh(d*x + c)^8 - 3*(4*b^2*d*x
- 45*b^2*cosh(d*x + c)^2 + 3*b^2)*sinh(d*x + c)^8 + 24*(15*b^2*cosh(d*x +
c)^3 - (4*b^2*d*x + 3*b^2)*cosh(d*x + c))*sinh(d*x + c)^7 + 6*(6*b^2*d*x +
 b^2)*cosh(d*x + c)^6 + 6*(105*b^2*cosh(d*x + c)^4 + 6*b^2*d*x - 14*(4*b^2
*d*x + 3*b^2)*cosh(d*x + c)^2 + b^2)*sinh(d*x + c)^6 + 12*(63*b^2*cosh(d*x
 + c)^5 - 14*(4*b^2*d*x + 3*b^2)*cosh(d*x + c)^3 + 3*(6*b^2*d*x + b^2)*cos
h(d*x + c))*sinh(d*x + c)^5 - 6*(6*b^2*d*x + 16*a^2 - b^2)*cosh(d*x + c)^4
 + 6*(105*b^2*cosh(d*x + c)^6 - 35*(4*b^2*d*x + 3*b^2)*cosh(d*x + c)^4 - 6
*b^2*d*x + 15*(6*b^2*d*x + b^2)*cosh(d*x + c)^2 - 16*a^2 + b^2)*sinh(d*x +
 c)^4 + 24*(15*b^2*cosh(d*x + c)^7 - 7*(4*b^2*d*x + 3*b^2)*cosh(d*x + c)^5
 + 5*(6*b^2*d*x + b^2)*cosh(d*x + c)^3 - (6*b^2*d*x + 16*a^2 - b^2)*cosh(d
*x + c))*sinh(d*x + c)^3 + (12*b^2*d*x + 32*a^2 - 9*b^2)*cosh(d*x + c)^2 +
 (135*b^2*cosh(d*x + c)^8 - 84*(4*b^2*d*x + 3*b^2)*cosh(d*x + c)^6 + 90*(6
*b^2*d*x + b^2)*cosh(d*x + c)^4 + 12*b^2*d*x - 36*(6*b^2*d*x + 16*a^2 - b^
2)*cosh(d*x + c)^2 + 32*a^2 - 9*b^2)*sinh(d*x + c)^2 + 3*b^2 - 48*(a*b*cos
h(d*x + c)^8 + 8*a*b*cosh(d*x + c)*sinh(d*x + c)^7 + a*b*sinh(d*x + c)^8 -
 3*a*b*cosh(d*x + c)^6 + (28*a*b*cosh(d*x + c)^2 - 3*a*b)*sinh(d*x + c)^6
+ 3*a*b*cosh(d*x + c)^4 + 2*(28*a*b*cosh(d*x + c)^3 - 9*a*b*cosh(d*x + c))
*sinh(d*x + c)^5 + (70*a*b*cosh(d*x + c)^4 - 45*a*b*cosh(d*x + c)^2 + 3...
```

Sympy [F(-1)]

Timed out.

$$\int \operatorname{csch}^4(c+dx)\left(a+b\sinh^3(c+dx)\right)^2\,dx=\text{Timed out}$$

input
```
integrate(csch(d*x+c)**4*(a+b*sinh(d*x+c)**3)**2,x)
```

output
```
Timed out
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 170 vs. 2(70) = 140.

Time = 0.04 (sec) , antiderivative size = 170, normalized size of antiderivative = 2.24

$$\int \operatorname{csch}^4(c+dx)\left(a+b\sinh^3(c+dx)\right)^2\,dx=-\frac{1}{8}\,b^2\left(4\,x-\frac{e^{(2\,dx+2\,c)}}{d}+\frac{e^{(-2\,dx-2\,c)}}{d}\right)-2\,ab\left(\frac{\log\left(e^{(-dx-c)}+1\right)}{d}-\frac{\log\left(e^{(-dx-c)}-1\right)}{d}\right)+\frac{4}{3}\,a^2\left(\frac{3\,e^{(-2\,dx-2\,c)}}{d(3\,e^{(-2\,dx-2\,c)}-3\,e^{(-4\,dx-4\,c)}+e^{(-6\,dx-6\,c)}-1)}-\frac{1}{d(3\,e^{(-2\,dx-2\,c)}-3\,e^{(-4\,dx-4\,c)}+e^{(-6\,dx-6\,c)}-1}\right.$$

input
```
integrate(csch(d*x+c)^4*(a+b*sinh(d*x+c)^3)^2,x, algorithm="maxima")
```

output
```
-1/8*b^2*(4*x - e^(2*d*x + 2*c)/d + e^(-2*d*x - 2*c)/d) - 2*a*b*(log(e^(-d
*x - c) + 1)/d - log(e^(-d*x - c) - 1)/d) + 4/3*a^2*(3*e^(-2*d*x - 2*c)/(d
*(3*e^(-2*d*x - 2*c) - 3*e^(-4*d*x - 4*c) + e^(-6*d*x - 6*c) - 1)) - 1/(d*
(3*e^(-2*d*x - 2*c) - 3*e^(-4*d*x - 4*c) + e^(-6*d*x - 6*c) - 1)))
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 151 vs. $2(70) = 140$.

Time = 0.17 (sec) , antiderivative size = 151, normalized size of antiderivative = 1.99

$$\int \operatorname{csch}^4(c+dx)\left(a+b\sinh^3(c+dx)\right)^2\,dx = -\frac{12\,(dx+c)b^2 - 3\,b^2e^{(2\,dx+2\,c)} + 48\,ab\log\left(e^{(dx+c)}+1\right) - 48\,ab\log\left(\left|e^{(dx+c)}-1\right|\right) + \frac{(3\,b^2e^{(6\,dx+6\,c)} - 3\,b^2 + 3\,(3}{}}{24\,d}$$

input
```
integrate(csch(d*x+c)^4*(a+b*sinh(d*x+c)^3)^2,x, algorithm="giac")
```

output
```
-1/24*(12*(d*x + c)*b^2 - 3*b^2*e^(2*d*x + 2*c) + 48*a*b*log(e^(d*x + c) +
 1) - 48*a*b*log(abs(e^(d*x + c) - 1)) + (3*b^2*e^(6*d*x + 6*c) - 3*b^2 +
3*(32*a^2 - 3*b^2)*e^(4*d*x + 4*c) - (32*a^2 - 9*b^2)*e^(2*d*x + 2*c))*e^(
-2*d*x - 2*c)/((e^(d*x + c) + 1)^3*(e^(d*x + c) - 1)^3))/d
```

Mupad [B] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 163, normalized size of antiderivative = 2.14

$$\int \operatorname{csch}^4(c+dx)\left(a+b\sinh^3(c+dx)\right)^2\,dx = \frac{b^2\,\mathrm{e}^{2\,c+2\,d\,x}}{8\,d} - \frac{4\,a^2}{d\,\left(\mathrm{e}^{4\,c+4\,d\,x}-2\,\mathrm{e}^{2\,c+2\,d\,x}+1\right)} - \frac{4\,\mathrm{atan}\left(\frac{a\,b\,\mathrm{e}^{d\,x}\,\mathrm{e}^{c}\,\sqrt{-d^2}}{d\,\sqrt{a^2\,b^2}}\right)\,\sqrt{a^2\,b^2}}{\sqrt{-d^2}} - \frac{b^2\,\mathrm{e}^{-2\,c-2\,d\,x}}{8\,d} - \frac{b^2\,x}{2} - \frac{8\,a^2}{3\,d\,\left(3\,\mathrm{e}^{2\,c+2\,d\,x}-3\,\mathrm{e}^{4\,c+4\,d\,x}+\mathrm{e}^{6\,c+6\,d\,x}-1\right)}$$

input
```
int((a + b*sinh(c + d*x)^3)^2/sinh(c + d*x)^4,x)
```

output
```
(b^2*exp(2*c + 2*d*x))/(8*d) - (4*a^2)/(d*(exp(4*c + 4*d*x) - 2*exp(2*c +
2*d*x) + 1)) - (4*atan((a*b*exp(d*x)*exp(c)*(-d^2)^(1/2))/(d*(a^2*b^2)^(1/
2)))*(a^2*b^2)^(1/2))/(-d^2)^(1/2) - (b^2*exp(- 2*c - 2*d*x))/(8*d) - (b^2
*x)/2 - (8*a^2)/(3*d*(3*exp(2*c + 2*d*x) - 3*exp(4*c + 4*d*x) + exp(6*c +
6*d*x) - 1))
```

Reduce [B] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 411, normalized size of antiderivative = 5.41

$$\int \operatorname{csch}^4(c+dx)\left(a+b\sinh^3(c+dx)\right)^2\,dx$$
$$=\frac{3e^{10dx+10c}b^2+48e^{8dx+8c}\log\left(e^{dx+c}-1\right)ab-48e^{8dx+8c}\log\left(e^{dx+c}+1\right)ab-12e^{8dx+8c}b^2dx-7e^{8dx+8c}b^2-1}{}$$

input
```
int(csch(d*x+c)^4*(a+b*sinh(d*x+c)^3)^2,x)
```

output
```
(3*e**(10*c + 10*d*x)*b**2 + 48*e**(8*c + 8*d*x)*log(e**(c + d*x) - 1)*a*b
 - 48*e**(8*c + 8*d*x)*log(e**(c + d*x) + 1)*a*b - 12*e**(8*c + 8*d*x)*b**
2*d*x - 7*e**(8*c + 8*d*x)*b**2 - 144*e**(6*c + 6*d*x)*log(e**(c + d*x) -
1)*a*b + 144*e**(6*c + 6*d*x)*log(e**(c + d*x) + 1)*a*b + 36*e**(6*c + 6*d
*x)*b**2*d*x + 144*e**(4*c + 4*d*x)*log(e**(c + d*x) - 1)*a*b - 144*e**(4*
c + 4*d*x)*log(e**(c + d*x) + 1)*a*b - 96*e**(4*c + 4*d*x)*a**2 - 36*e**(4
*c + 4*d*x)*b**2*d*x + 12*e**(4*c + 4*d*x)*b**2 - 48*e**(2*c + 2*d*x)*log(
e**(c + d*x) - 1)*a*b + 48*e**(2*c + 2*d*x)*log(e**(c + d*x) + 1)*a*b + 32
*e**(2*c + 2*d*x)*a**2 + 12*e**(2*c + 2*d*x)*b**2*d*x - 11*e**(2*c + 2*d*x
)*b**2 + 3*b**2)/(24*e**(2*c + 2*d*x)*d*(e**(6*c + 6*d*x) - 3*e**(4*c + 4*
d*x) + 3*e**(2*c + 2*d*x) - 1))
```

3.135 $\int \operatorname{csch}^5(c+dx) \left(a+b\sinh^3(c+dx)\right)^2 dx$

Optimal result

Integrand size = 23, antiderivative size = 90

$$\int \operatorname{csch}^5(c+dx) \left(a+b\sinh^3(c+dx)\right)^2 dx = -\frac{3a^2\operatorname{arctanh}(\cosh(c+dx))}{8d} + \frac{b^2\cosh(c+dx)}{d} - \frac{2ab\coth(c+dx)}{d} + \frac{3a^2\coth(c+dx)\operatorname{csch}(c+dx)}{8d} - \frac{a^2\coth(c+dx)\operatorname{csch}^3(c+dx)}{4d}$$

output
```
-3/8*a^2*arctanh(cosh(d*x+c))/d+b^2*cosh(d*x+c)/d-2*a*b*coth(d*x+c)/d+3/8*
a^2*coth(d*x+c)*csch(d*x+c)/d-1/4*a^2*coth(d*x+c)*csch(d*x+c)^3/d
```

Mathematica [A] (verified)

Time = 0.15 (sec) , antiderivative size = 170, normalized size of antiderivative = 1.89

$$\int \operatorname{csch}^5(c+dx)\left(a+b\sinh^3(c+dx)\right)^2\,dx$$
$$=\frac{b^2\cosh(c)\cosh(dx)}{d}-\frac{2ab\coth(c+dx)}{d}+\frac{3a^2\operatorname{csch}^2\left(\frac{1}{2}(c+dx)\right)}{32d}-\frac{a^2\operatorname{csch}^4\left(\frac{1}{2}(c+dx)\right)}{64d}-\frac{3a^2\log\left(\cosh\left(\frac{1}{2}(c+dx)\right)\right)}{8d}+\frac{3a^2\log\left(\sinh\left(\frac{1}{2}(c+dx)\right)\right)}{8d}+\frac{3a^2\operatorname{sech}^2\left(\frac{1}{2}(c+dx)\right)}{32d}+\frac{a^2\operatorname{sech}^4\left(\frac{1}{2}(c+dx)\right)}{64d}+\frac{b^2\sinh(c)\sinh(dx)}{d}$$

input
```
Integrate[Csch[c + d*x]^5*(a + b*Sinh[c + d*x]^3)^2,x]
```

output
```
(b^2*Cosh[c]*Cosh[d*x])/d - (2*a*b*Coth[c + d*x])/d + (3*a^2*Csch[(c + d*x
)/2]^2)/(32*d) - (a^2*Csch[(c + d*x)/2]^4)/(64*d) - (3*a^2*Log[Cosh[(c + d
*x)/2]])/(8*d) + (3*a^2*Log[Sinh[(c + d*x)/2]])/(8*d) + (3*a^2*Sech[(c + d
*x)/2]^2)/(32*d) + (a^2*Sech[(c + d*x)/2]^4)/(64*d) + (b^2*Sinh[c]*Sinh[d*
x])/d
```

Rubi [C] (verified)

Result contains complex when optimal does not.

Time = 0.35 (sec) , antiderivative size = 105, normalized size of antiderivative = 1.17, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.174, Rules used = {3042, 26, 3699, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \operatorname{csch}^5(c+dx)\left(a+b\sinh^3(c+dx)\right)^2\,dx$$

$$\Big\downarrow \ 3042$$

$$\int \frac{i\left(a+ib\sin(ic+idx)^3\right)^2}{\sin(ic+idx)^5}dx$$

$$\Big\downarrow \text{ 26}$$

$$i\int \frac{(ib\sin(ic+idx)^3+a)^2}{\sin(ic+idx)^5}dx$$

$$\Big\downarrow \text{ 3699}$$

$$i\int \left(-ia^2\text{csch}^5(c+dx)-2iab\text{csch}^2(c+dx)-ib^2\sinh(c+dx)\right)dx$$

$$\Big\downarrow \text{ 2009}$$

$$i\left(\frac{3ia^2\text{arctanh}(\cosh(c+dx))}{8d}+\frac{ia^2\coth(c+dx)\text{csch}^3(c+dx)}{4d}-\frac{3ia^2\coth(c+dx)\text{csch}(c+dx)}{8d}+\frac{2iab\coth(c+\right.$$

input
```
Int[Csch[c + d*x]^5*(a + b*Sinh[c + d*x]^3)^2,x]
```

output
```
I*((((3*I)/8)*a^2*ArcTanh[Cosh[c + d*x]])/d - (I*b^2*Cosh[c + d*x])/d + ((
2*I)*a*b*Coth[c + d*x])/d - (((3*I)/8)*a^2*Coth[c + d*x]*Csch[c + d*x])/d
+ ((I/4)*a^2*Coth[c + d*x]*Csch[c + d*x]^3)/d)
```

Defintions of rubi rules used

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3699
```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^(n_
))^(p_.), x_Symbol] :> Int[ExpandTrig[sin[e + f*x]^m*(a + b*sin[e + f*x]^n)
^p, x], x] /; FreeQ[{a, b, e, f}, x] && IntegersQ[m, p] && (EqQ[n, 4] || Gt
Q[p, 0] || (EqQ[p, -1] && IntegerQ[n]))
```

Maple [A] (verified)

Time = 0.90 (sec) , antiderivative size = 66, normalized size of antiderivative = 0.73

method	result
derivativedivides	$\frac{a^{2}\left(\left(-\frac{\operatorname{csch}(dx+c)^{3}}{4}+\frac{3\operatorname{csch}(dx+c)}{8}\right)\coth(dx+c)-\frac{3\operatorname{arctanh}\left(e^{dx+c}\right)}{4}\right)-2\coth(dx+c)ab+b^{2}\cosh(dx+c)}{d}$
default	$\frac{a^{2}\left(\left(-\frac{\operatorname{csch}(dx+c)^{3}}{4}+\frac{3\operatorname{csch}(dx+c)}{8}\right)\coth(dx+c)-\frac{3\operatorname{arctanh}\left(e^{dx+c}\right)}{4}\right)-2\coth(dx+c)ab+b^{2}\cosh(dx+c)}{d}$
parallelrisch	$\frac{512a^{2}\ln\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^{\frac{3}{8}}\right)-11\left(\cosh(dx+c)-\frac{2\cosh(2dx+2c)}{11}-\frac{3\cosh(3dx+3c)}{11}+\frac{\cosh(4dx+4c)}{22}+\frac{3}{22}\right)a^{2}\operatorname{sech}\left(\frac{dx}{2}+\frac{c}{2}\right)^{4}\operatorname{cs}}{512d}$
risch	$\frac{e^{dx+c}b^{2}}{2d}+\frac{e^{-dx-c}b^{2}}{2d}+\frac{a\left(3a\,e^{7dx+7c}-16\,e^{6dx+6c}b-11a\,e^{5dx+5c}+48b\,e^{4dx+4c}-11a\,e^{3dx+3c}-48\,e^{2dx+2c}b+3\,e^{dx+c}\right.}{4d\left(e^{2dx+2c}-1\right)^{4}}$

input
```
int(csch(d*x+c)^5*(a+b*sinh(d*x+c)^3)^2,x,method=_RETURNVERBOSE)
```

output
```
1/d*(a^2*((-1/4*csch(d*x+c)^3+3/8*csch(d*x+c))*coth(d*x+c)-3/4*arctanh(exp
(d*x+c)))-2*coth(d*x+c)*a*b+b^2*cosh(d*x+c))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2119 vs. 2(84) = 168.

Time = 0.11 (sec) , antiderivative size = 2119, normalized size of antiderivative = 23.54

$$\int \operatorname{csch}^5(c+dx)\left(a+b\sinh^3(c+dx)\right)^2\,dx = \text{Too large to display}$$

input
```
integrate(csch(d*x+c)^5*(a+b*sinh(d*x+c)^3)^2,x, algorithm="fricas")
```

output
```
1/8*(4*b^2*cosh(d*x + c)^10 + 40*b^2*cosh(d*x + c)*sinh(d*x + c)^9 + 4*b^2
*sinh(d*x + c)^10 - 32*a*b*cosh(d*x + c)^7 + 6*(a^2 - 2*b^2)*cosh(d*x + c)
^8 + 6*(30*b^2*cosh(d*x + c)^2 + a^2 - 2*b^2)*sinh(d*x + c)^8 + 16*(30*b^2
*cosh(d*x + c)^3 - 2*a*b + 3*(a^2 - 2*b^2)*cosh(d*x + c))*sinh(d*x + c)^7
+ 96*a*b*cosh(d*x + c)^5 - 2*(11*a^2 - 4*b^2)*cosh(d*x + c)^6 + 2*(420*b^2
*cosh(d*x + c)^4 - 112*a*b*cosh(d*x + c) + 84*(a^2 - 2*b^2)*cosh(d*x + c)^
2 - 11*a^2 + 4*b^2)*sinh(d*x + c)^6 + 12*(84*b^2*cosh(d*x + c)^5 - 56*a*b*
cosh(d*x + c)^2 + 28*(a^2 - 2*b^2)*cosh(d*x + c)^3 + 8*a*b - (11*a^2 - 4*b
^2)*cosh(d*x + c))*sinh(d*x + c)^5 - 96*a*b*cosh(d*x + c)^3 - 2*(11*a^2 -
4*b^2)*cosh(d*x + c)^4 + 2*(420*b^2*cosh(d*x + c)^6 - 560*a*b*cosh(d*x + c
)^3 + 210*(a^2 - 2*b^2)*cosh(d*x + c)^4 + 240*a*b*cosh(d*x + c) - 15*(11*a
^2 - 4*b^2)*cosh(d*x + c)^2 - 11*a^2 + 4*b^2)*sinh(d*x + c)^4 + 8*(60*b^2*
cosh(d*x + c)^7 - 140*a*b*cosh(d*x + c)^4 + 42*(a^2 - 2*b^2)*cosh(d*x + c)
^5 + 120*a*b*cosh(d*x + c)^2 - 5*(11*a^2 - 4*b^2)*cosh(d*x + c)^3 - 12*a*b
 - (11*a^2 - 4*b^2)*cosh(d*x + c))*sinh(d*x + c)^3 + 32*a*b*cosh(d*x + c)
+ 6*(a^2 - 2*b^2)*cosh(d*x + c)^2 + 6*(30*b^2*cosh(d*x + c)^8 - 112*a*b*co
sh(d*x + c)^5 + 28*(a^2 - 2*b^2)*cosh(d*x + c)^6 + 160*a*b*cosh(d*x + c)^3
 - 5*(11*a^2 - 4*b^2)*cosh(d*x + c)^4 - 48*a*b*cosh(d*x + c) - 2*(11*a^2 -
 4*b^2)*cosh(d*x + c)^2 + a^2 - 2*b^2)*sinh(d*x + c)^2 + 4*b^2 - 3*(a^2*co
sh(d*x + c)^9 + 9*a^2*cosh(d*x + c)*sinh(d*x + c)^8 + a^2*sinh(d*x + c)...
```

Sympy [F(-1)]

Timed out.

$$\int \operatorname{csch}^5(c+dx)\left(a+b\sinh^3(c+dx)\right)^2\,dx=\text{Timed out}$$

input
```
integrate(csch(d*x+c)**5*(a+b*sinh(d*x+c)**3)**2,x)
```

output
```
Timed out
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 188 vs. $2(84) = 168$.

Time = 0.05 (sec) , antiderivative size = 188, normalized size of antiderivative = 2.09

$$\int \operatorname{csch}^5(c+dx)\left(a+b\sinh^3(c+dx)\right)^2\,dx = \frac{1}{2}b^2\left(\frac{e^{(dx+c)}}{d}+\frac{e^{(-dx-c)}}{d}\right) - \frac{1}{8}a^2\left(\frac{3\log\left(e^{(-dx-c)}+1\right)}{d}-\frac{3\log\left(e^{(-dx-c)}-1\right)}{d}+\frac{2\left(3e^{(-dx-c)}-11e^{(-3dx-3c)}-11e^{(-5dx-5c)}+3e^{(-7dx-7c)}\right)}{d(4e^{(-2dx-2c)}-6e^{(-4dx-4c)}+4e^{(-6dx-6c)}-e^{(-8dx-8c)}-1)}\right) + \frac{4ab}{d(e^{(-2dx-2c)}-1)}$$

input
```
integrate(csch(d*x+c)^5*(a+b*sinh(d*x+c)^3)^2,x, algorithm="maxima")
```

output
```
1/2*b^2*(e^(d*x + c)/d + e^(-d*x - c)/d) - 1/8*a^2*(3*log(e^(-d*x - c) + 1
)/d - 3*log(e^(-d*x - c) - 1)/d + 2*(3*e^(-d*x - c) - 11*e^(-3*d*x - 3*c)
- 11*e^(-5*d*x - 5*c) + 3*e^(-7*d*x - 7*c))/(d*(4*e^(-2*d*x - 2*c) - 6*e^(
-4*d*x - 4*c) + 4*e^(-6*d*x - 6*c) - e^(-8*d*x - 8*c) - 1))) + 4*a*b/(d*(e
^(-2*d*x - 2*c) - 1))
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 172 vs. $2(84) = 168$.

Time = 0.18 (sec) , antiderivative size = 172, normalized size of antiderivative = 1.91

$$\int \operatorname{csch}^5(c+dx)\left(a+b\sinh^3(c+dx)\right)^2\,dx = \frac{4b^2e^{(dx+c)}+4b^2e^{(-dx-c)}-3a^2\log\left(e^{(dx+c)}+1\right)+3a^2\log\left(\left|e^{(dx+c)}-1\right|\right)+\frac{2\left(3a^2e^{(7dx+7c)}-16abe^{(6dx+6c)}-11\right.}{}}{8d}$$

input
```
integrate(csch(d*x+c)^5*(a+b*sinh(d*x+c)^3)^2,x, algorithm="giac")
```

output
```
1/8*(4*b^2*e^(d*x + c) + 4*b^2*e^(-d*x - c) - 3*a^2*log(e^(d*x + c) + 1) +
 3*a^2*log(abs(e^(d*x + c) - 1)) + 2*(3*a^2*e^(7*d*x + 7*c) - 16*a*b*e^(6*
d*x + 6*c) - 11*a^2*e^(5*d*x + 5*c) + 48*a*b*e^(4*d*x + 4*c) - 11*a^2*e^(3
*d*x + 3*c) - 48*a*b*e^(2*d*x + 2*c) + 3*a^2*e^(d*x + c) + 16*a*b)/(e^(2*d
*x + 2*c) - 1)^4)/d
```

Mupad [B] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 355, normalized size of antiderivative = 3.94

$$\int \operatorname{csch}^5(c+dx)\left(a+b\sinh^3(c+dx)\right)^2\,dx$$
$$=\frac{\frac{3a^2e^{c+dx}}{4d}-\frac{2ab}{d}}{e^{2c+2dx}-1}-\frac{\frac{4a^2e^{3c+3dx}}{d}-\frac{ab}{d}+\frac{3abe^{2c+2dx}}{d}-\frac{3abe^{4c+4dx}}{d}+\frac{abe^{6c+6dx}}{d}}{6e^{4c+4dx}-4e^{2c+2dx}-4e^{6c+6dx}+e^{8c+8dx}+1}$$
$$-\frac{\frac{2a^2e^{c+dx}}{d}+\frac{ab}{d}-\frac{2abe^{2c+2dx}}{d}+\frac{abe^{4c+4dx}}{d}}{3e^{2c+2dx}-3e^{4c+4dx}+e^{6c+6dx}-1}+\frac{b^2e^{c+dx}}{2d}$$
$$-\frac{3\operatorname{atan}\left(\frac{a^2e^{dx}e^{c}\sqrt{-d^2}}{d\sqrt{a^4}}\right)\sqrt{a^4}}{4\sqrt{-d^2}}+\frac{b^2e^{-c-dx}}{2d}-\frac{a^2e^{c+dx}}{2d\left(e^{4c+4dx}-2e^{2c+2dx}+1\right)}$$

input
```
int((a + b*sinh(c + d*x)^3)^2/sinh(c + d*x)^5,x)
```

output
```
((3*a^2*exp(c + d*x))/(4*d) - (2*a*b)/d)/(exp(2*c + 2*d*x) - 1) - ((4*a^2*
exp(3*c + 3*d*x))/d - (a*b)/d + (3*a*b*exp(2*c + 2*d*x))/d - (3*a*b*exp(4*
c + 4*d*x))/d + (a*b*exp(6*c + 6*d*x))/d)/(6*exp(4*c + 4*d*x) - 4*exp(2*c
+ 2*d*x) - 4*exp(6*c + 6*d*x) + exp(8*c + 8*d*x) + 1) - ((2*a^2*exp(c + d*
x))/d + (a*b)/d - (2*a*b*exp(2*c + 2*d*x))/d + (a*b*exp(4*c + 4*d*x))/d)/(
3*exp(2*c + 2*d*x) - 3*exp(4*c + 4*d*x) + exp(6*c + 6*d*x) - 1) + (b^2*exp
(c + d*x))/(2*d) - (3*atan((a^2*exp(d*x)*exp(c)*(-d^2)^(1/2))/(d*(a^4)^(1/
2)))*(a^4)^(1/2))/(4*(-d^2)^(1/2)) + (b^2*exp(- c - d*x))/(2*d) - (a^2*exp
(c + d*x))/(2*d*(exp(4*c + 4*d*x) - 2*exp(2*c + 2*d*x) + 1))
```

Reduce [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 502, normalized size of antiderivative = 5.58

$$\int \operatorname{csch}^5(c+dx) \left(a+b\sinh^3(c+dx)\right)^2 \, dx$$
$$= \frac{4e^{10dx+10c}b^2 + 3e^{9dx+9c}\log\left(e^{dx+c}-1\right)a^2 - 3e^{9dx+9c}\log\left(e^{dx+c}+1\right)a^2 - 8e^{9dx+9c}ab + 6e^{8dx+8c}a^2 - 12e^{8dx-}}{}$$

input
```
int(csch(d*x+c)^5*(a+b*sinh(d*x+c)^3)^2,x)
```

output
```
(4*e**(10*c + 10*d*x)*b**2 + 3*e**(9*c + 9*d*x)*log(e**(c + d*x) - 1)*a**2
 - 3*e**(9*c + 9*d*x)*log(e**(c + d*x) + 1)*a**2 - 8*e**(9*c + 9*d*x)*a*b
+ 6*e**(8*c + 8*d*x)*a**2 - 12*e**(8*c + 8*d*x)*b**2 - 12*e**(7*c + 7*d*x)
*log(e**(c + d*x) - 1)*a**2 + 12*e**(7*c + 7*d*x)*log(e**(c + d*x) + 1)*a*
*2 - 22*e**(6*c + 6*d*x)*a**2 + 8*e**(6*c + 6*d*x)*b**2 + 18*e**(5*c + 5*d
*x)*log(e**(c + d*x) - 1)*a**2 - 18*e**(5*c + 5*d*x)*log(e**(c + d*x) + 1)
*a**2 + 48*e**(5*c + 5*d*x)*a*b - 22*e**(4*c + 4*d*x)*a**2 + 8*e**(4*c + 4
*d*x)*b**2 - 12*e**(3*c + 3*d*x)*log(e**(c + d*x) - 1)*a**2 + 12*e**(3*c +
 3*d*x)*log(e**(c + d*x) + 1)*a**2 - 64*e**(3*c + 3*d*x)*a*b + 6*e**(2*c +
 2*d*x)*a**2 - 12*e**(2*c + 2*d*x)*b**2 + 3*e**(c + d*x)*log(e**(c + d*x)
- 1)*a**2 - 3*e**(c + d*x)*log(e**(c + d*x) + 1)*a**2 + 24*e**(c + d*x)*a*
b + 4*b**2)/(8*e**(c + d*x)*d*(e**(8*c + 8*d*x) - 4*e**(6*c + 6*d*x) + 6*e
**(4*c + 4*d*x) - 4*e**(2*c + 2*d*x) + 1))
```

3.136 $\int \operatorname{csch}^6(c+dx)\left(a+b\sinh^3(c+dx)\right)^2\,dx$

Optimal result

Integrand size = 23, antiderivative size = 88

$$\int \operatorname{csch}^6(c+dx)\left(a+b\sinh^3(c+dx)\right)^2\,dx = b^2x+\frac{ab\operatorname{arctanh}(\cosh(c+dx))}{d}-\frac{a^2\coth(c+dx)}{d}+\frac{2a^2\coth^3(c+dx)}{3d}-\frac{a^2\coth^5(c+dx)}{5d}-\frac{ab\coth(c+dx)\operatorname{csch}(c+dx)}{d}$$

output
```
b^2*x+a*b*arctanh(cosh(d*x+c))/d-a^2*coth(d*x+c)/d+2/3*a^2*coth(d*x+c)^3/d
-1/5*a^2*coth(d*x+c)^5/d-a*b*coth(d*x+c)*csch(d*x+c)/d
```

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 216 vs. $2(88) = 176$.

Time = 0.59 (sec) , antiderivative size = 216, normalized size of antiderivative = 2.45

$$\int \operatorname{csch}^6(c+dx)\left(a+b\sinh^3(c+dx)\right)^2\,dx$$

$$= \frac{-128a^2\coth\left(\frac{1}{2}(c+dx)\right)-120ab\operatorname{csch}^2\left(\frac{1}{2}(c+dx)\right)+\frac{19}{2}a^2\operatorname{csch}^4\left(\frac{1}{2}(c+dx)\right)\sinh(c+dx)-\frac{3}{2}a^2\operatorname{csch}^6\left(\frac{1}{2}(c\right.}{}$$

input
```
Integrate[Csch[c + d*x]^6*(a + b*Sinh[c + d*x]^3)^2,x]
```

output
```
(-128*a^2*Coth[(c + d*x)/2] - 120*a*b*Csch[(c + d*x)/2]^2 + (19*a^2*Csch[(
c + d*x)/2]^4*Sinh[c + d*x])/2 - (3*a^2*Csch[(c + d*x)/2]^6*Sinh[c + d*x])
/2 + 8*(60*b^2*c + 60*b^2*d*x + 60*a*b*Log[Cosh[(c + d*x)/2]] - 60*a*b*Log
[Sinh[(c + d*x)/2]] - 15*a*b*Sech[(c + d*x)/2]^2 - 19*a^2*Csch[c + d*x]^3*
Sinh[(c + d*x)/2]^4 - 12*a^2*Csch[c + d*x]^5*Sinh[(c + d*x)/2]^6 - 16*a^2*
Tanh[(c + d*x)/2]))/(480*d)
```

Rubi [A] (verified)

Time = 0.33 (sec) , antiderivative size = 88, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.174$, Rules used = {3042, 25, 3699, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \operatorname{csch}^6(c+dx) \left(a+b\sinh^3(c+dx)\right)^2 \, dx$$

$$\Big\downarrow \text{3042}$$

$$\int -\frac{\left(a+ib\sin(ic+idx)^3\right)^2}{\sin(ic+idx)^6}dx$$

$$\Big\downarrow \text{25}$$

$$-\int \frac{\left(ib\sin(ic+idx)^3+a\right)^2}{\sin(ic+idx)^6}dx$$

$$\Big\downarrow \text{3699}$$

$$-\int \left(-a^2\operatorname{csch}^6(c+dx)-2ab\operatorname{csch}^3(c+dx)-b^2\right)dx$$

$$\Big\downarrow \text{2009}$$

$$-\frac{a^2\coth^5(c+dx)}{5d}+\frac{2a^2\coth^3(c+dx)}{3d}-\frac{a^2\coth(c+dx)}{d}+\frac{ab\text{arctanh}(\cosh(c+dx))}{d}-\frac{ab\coth(c+dx)\operatorname{csch}(c+dx)}{d}+b^2x$$

input
```
Int[Csch[c + d*x]^6*(a + b*Sinh[c + d*x]^3)^2,x]
```

output
```
b^2*x + (a*b*ArcTanh[Cosh[c + d*x]])/d - (a^2*Coth[c + d*x])/d + (2*a^2*Co
th[c + d*x]^3)/(3*d) - (a^2*Coth[c + d*x]^5)/(5*d) - (a*b*Coth[c + d*x]*Cs
ch[c + d*x])/d
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3699
```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^(n_
))^(p_.), x_Symbol] :> Int[ExpandTrig[sin[e + f*x]^m*(a + b*sin[e + f*x]^n)
^p, x], x] /; FreeQ[{a, b, e, f}, x] && IntegersQ[m, p] && (EqQ[n, 4] || Gt
Q[p, 0] || (EqQ[p, -1] && IntegerQ[n]))
```

Maple [A] (verified)

Time = 0.80 (sec) , antiderivative size = 73, normalized size of antiderivative = 0.83

method	result
derivativedivides	$\frac{a^{2}\left(-\frac{8}{15}-\frac{\operatorname{csch}(dx+c)^{4}}{5}+\frac{4\operatorname{csch}(dx+c)^{2}}{15}\right)\coth(dx+c)+2ab\left(-\frac{\operatorname{csch}(dx+c)\coth(dx+c)}{2}+\operatorname{arctanh}\left(\mathrm{e}^{dx+c}\right)\right)+b^{2}(dx+c)}{d}$
default	$\frac{a^{2}\left(-\frac{8}{15}-\frac{\operatorname{csch}(dx+c)^{4}}{5}+\frac{4\operatorname{csch}(dx+c)^{2}}{15}\right)\coth(dx+c)+2ab\left(-\frac{\operatorname{csch}(dx+c)\coth(dx+c)}{2}+\operatorname{arctanh}\left(\mathrm{e}^{dx+c}\right)\right)+b^{2}(dx+c)}{d}$
risch	$b^{2}x-\frac{2a\left(15b\,\mathrm{e}^{9dx+9c}-30b\,\mathrm{e}^{7dx+7c}+80\,\mathrm{e}^{4dx+4c}a+30b\,\mathrm{e}^{3dx+3c}-40\,\mathrm{e}^{2dx+2c}a-15\,\mathrm{e}^{dx+c}b+8a\right)}{15d\left(\mathrm{e}^{2dx+2c}-1\right)^{5}}+\frac{ab\ln\left(\mathrm{e}^{dx+c}+1\right)}{d}$
parallelrisch	$\frac{-3\coth\left(\frac{dx}{2}+\frac{c}{2}\right)^{5}a^{2}-3\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^{5}a^{2}+25\coth\left(\frac{dx}{2}+\frac{c}{2}\right)^{3}a^{2}+25\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^{3}a^{2}-120\coth\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}ab+120\tan}{480d}$

input
```
int(csch(d*x+c)^6*(a+b*sinh(d*x+c)^3)^2,x,method=_RETURNVERBOSE)
```

output
```
1/d*(a^2*(-8/15-1/5*csch(d*x+c)^4+4/15*csch(d*x+c)^2)*coth(d*x+c)+2*a*b*(-
1/2*csch(d*x+c)*coth(d*x+c)+arctanh(exp(d*x+c)))+b^2*(d*x+c))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2310 vs. 2(84) = 168.

Time = 0.10 (sec) , antiderivative size = 2310, normalized size of antiderivative = 26.25

$$\int \operatorname{csch}^6(c+dx)\left(a+b\sinh^3(c+dx)\right)^2\,dx = \text{Too large to display}$$

input
```
integrate(csch(d*x+c)^6*(a+b*sinh(d*x+c)^3)^2,x, algorithm="fricas")
```

output
```
1/15*(15*b^2*d*x*cosh(d*x + c)^10 + 15*b^2*d*x*sinh(d*x + c)^10 - 75*b^2*d
*x*cosh(d*x + c)^8 - 30*a*b*cosh(d*x + c)^9 + 150*b^2*d*x*cosh(d*x + c)^6
+ 30*(5*b^2*d*x*cosh(d*x + c) - a*b)*sinh(d*x + c)^9 + 60*a*b*cosh(d*x + c
)^7 + 15*(45*b^2*d*x*cosh(d*x + c)^2 - 5*b^2*d*x - 18*a*b*cosh(d*x + c))*s
inh(d*x + c)^8 + 60*(30*b^2*d*x*cosh(d*x + c)^3 - 10*b^2*d*x*cosh(d*x + c)
 - 18*a*b*cosh(d*x + c)^2 + a*b)*sinh(d*x + c)^7 + 30*(105*b^2*d*x*cosh(d*
x + c)^4 - 70*b^2*d*x*cosh(d*x + c)^2 - 84*a*b*cosh(d*x + c)^3 + 5*b^2*d*x
 + 14*a*b*cosh(d*x + c))*sinh(d*x + c)^6 + 60*(63*b^2*d*x*cosh(d*x + c)^5
- 70*b^2*d*x*cosh(d*x + c)^3 - 63*a*b*cosh(d*x + c)^4 + 15*b^2*d*x*cosh(d*
x + c) + 21*a*b*cosh(d*x + c)^2)*sinh(d*x + c)^5 - 60*a*b*cosh(d*x + c)^3
- 10*(15*b^2*d*x + 16*a^2)*cosh(d*x + c)^4 + 10*(315*b^2*d*x*cosh(d*x + c)
^6 - 525*b^2*d*x*cosh(d*x + c)^4 - 378*a*b*cosh(d*x + c)^5 + 225*b^2*d*x*c
osh(d*x + c)^2 + 210*a*b*cosh(d*x + c)^3 - 15*b^2*d*x - 16*a^2)*sinh(d*x +
 c)^4 - 15*b^2*d*x + 20*(90*b^2*d*x*cosh(d*x + c)^7 - 210*b^2*d*x*cosh(d*x
 + c)^5 - 126*a*b*cosh(d*x + c)^6 + 150*b^2*d*x*cosh(d*x + c)^3 + 105*a*b*
cosh(d*x + c)^4 - 3*a*b - 2*(15*b^2*d*x + 16*a^2)*cosh(d*x + c))*sinh(d*x
+ c)^3 + 30*a*b*cosh(d*x + c) + 5*(15*b^2*d*x + 16*a^2)*cosh(d*x + c)^2 +
5*(135*b^2*d*x*cosh(d*x + c)^8 - 420*b^2*d*x*cosh(d*x + c)^6 - 216*a*b*cos
h(d*x + c)^7 + 450*b^2*d*x*cosh(d*x + c)^4 + 252*a*b*cosh(d*x + c)^5 + 15*
b^2*d*x - 36*a*b*cosh(d*x + c) - 12*(15*b^2*d*x + 16*a^2)*cosh(d*x + c)...
```

Sympy [F(-1)]

Timed out.

$$\int \operatorname{csch}^6(c+dx)\left(a+b\sinh^3(c+dx)\right)^2\,dx = \text{Timed out}$$

input
```
integrate(csch(d*x+c)**6*(a+b*sinh(d*x+c)**3)**2,x)
```

output
```
Timed out
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 303 vs. $2(84) = 168$.

Time = 0.04 (sec) , antiderivative size = 303, normalized size of antiderivative = 3.44

$$\int \operatorname{csch}^6(c+dx)\left(a+b\sinh^3(c+dx)\right)^2\,dx$$
$$= b^2x + ab\left(\frac{\log\left(e^{(-dx-c)}+1\right)}{d} - \frac{\log\left(e^{(-dx-c)}-1\right)}{d} + \frac{2\left(e^{(-dx-c)}+e^{(-3\,dx-3\,c)}\right)}{d(2\,e^{(-2\,dx-2\,c)}-e^{(-4\,dx-4\,c)}-1)}\right)$$
$$-\frac{16}{15}a^2\left(\frac{5\,e^{(-2\,dx-2\,c)}}{d(5\,e^{(-2\,dx-2\,c)}-10\,e^{(-4\,dx-4\,c)}+10\,e^{(-6\,dx-6\,c)}-5\,e^{(-8\,dx-8\,c)}+e^{(-10\,dx-10\,c)}-1)} - \frac{}{d(5\,e^{(-2\,dx-}}\right.$$

input
```
integrate(csch(d*x+c)^6*(a+b*sinh(d*x+c)^3)^2,x, algorithm="maxima")
```

output
```
b^2*x + a*b*(log(e^(-d*x - c) + 1)/d - log(e^(-d*x - c) - 1)/d + 2*(e^(-d*
x - c) + e^(-3*d*x - 3*c))/(d*(2*e^(-2*d*x - 2*c) - e^(-4*d*x - 4*c) - 1))
) - 16/15*a^2*(5*e^(-2*d*x - 2*c)/(d*(5*e^(-2*d*x - 2*c) - 10*e^(-4*d*x -
4*c) + 10*e^(-6*d*x - 6*c) - 5*e^(-8*d*x - 8*c) + e^(-10*d*x - 10*c) - 1))
 - 10*e^(-4*d*x - 4*c)/(d*(5*e^(-2*d*x - 2*c) - 10*e^(-4*d*x - 4*c) + 10*e
^(-6*d*x - 6*c) - 5*e^(-8*d*x - 8*c) + e^(-10*d*x - 10*c) - 1)) - 1/(d*(5*
e^(-2*d*x - 2*c) - 10*e^(-4*d*x - 4*c) + 10*e^(-6*d*x - 6*c) - 5*e^(-8*d*x
 - 8*c) + e^(-10*d*x - 10*c) - 1)))
```

Giac [A] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 141, normalized size of antiderivative = 1.60

$$\int \operatorname{csch}^6(c+dx)\left(a+b\sinh^3(c+dx)\right)^2\,dx$$
$$=\frac{15\,(dx+c)b^2+15\,ab\log\left(e^{(dx+c)}+1\right)-15\,ab\log\left(\left|e^{(dx+c)}-1\right|\right)-\frac{2\,(15\,abe^{(9\,dx+9\,c)}-30\,abe^{(7\,dx+7\,c)}+80\,a^2e^{(4\,dx+4}}{(e^{(2\,c}}}{15\,d}$$

input
```
integrate(csch(d*x+c)^6*(a+b*sinh(d*x+c)^3)^2,x, algorithm="giac")
```

output
```
1/15*(15*(d*x + c)*b^2 + 15*a*b*log(e^(d*x + c) + 1) - 15*a*b*log(abs(e^(d
*x + c) - 1)) - 2*(15*a*b*e^(9*d*x + 9*c) - 30*a*b*e^(7*d*x + 7*c) + 80*a^
2*e^(4*d*x + 4*c) + 30*a*b*e^(3*d*x + 3*c) - 40*a^2*e^(2*d*x + 2*c) - 15*a
*b*e^(d*x + c) + 8*a^2)/(e^(2*d*x + 2*c) - 1)^5)/d
```

Mupad [B] (verification not implemented)

Time = 1.64 (sec) , antiderivative size = 351, normalized size of antiderivative = 3.99

$$\int \operatorname{csch}^6(c+dx)\left(a+b\sinh^3(c+dx)\right)^2\,dx$$
$$=b^2\,x-\frac{\frac{32\,a^2\,e^{4\,c+4\,d\,x}}{5\,d}-\frac{8\,a\,b\,e^{c+d\,x}}{5\,d}+\frac{24\,a\,b\,e^{3\,c+3\,d\,x}}{5\,d}-\frac{24\,a\,b\,e^{5\,c+5\,d\,x}}{5\,d}+\frac{8\,a\,b\,e^{7\,c+7\,d\,x}}{5\,d}}{5\,e^{2\,c+2\,d\,x}-10\,e^{4\,c+4\,d\,x}+10\,e^{6\,c+6\,d\,x}-5\,e^{8\,c+8\,d\,x}+e^{10\,c+10\,d\,x}-1}$$
$$+\frac{2\,\mathrm{atan}\left(\frac{a\,b\,e^{d\,x}\,e^c\,\sqrt{-d^2}}{d\,\sqrt{a^2\,b^2}}\right)\,\sqrt{a^2\,b^2}}{\sqrt{-d^2}}-\frac{64\,a^2}{15\,d\,\left(3\,e^{2\,c+2\,d\,x}-3\,e^{4\,c+4\,d\,x}+e^{6\,c+6\,d\,x}-1\right)}$$
$$-\frac{16\,a^2}{5\,d\,\left(6\,e^{4\,c+4\,d\,x}-4\,e^{2\,c+2\,d\,x}-4\,e^{6\,c+6\,d\,x}+e^{8\,c+8\,d\,x}+1\right)}$$
$$-\frac{2\,a\,b\,e^{c+d\,x}}{d\,\left(e^{2\,c+2\,d\,x}-1\right)}-\frac{12\,a\,b\,e^{c+d\,x}}{5\,d\,\left(e^{4\,c+4\,d\,x}-2\,e^{2\,c+2\,d\,x}+1\right)}$$

input
```
int((a + b*sinh(c + d*x)^3)^2/sinh(c + d*x)^6,x)
```

output
```
b^2*x - ((32*a^2*exp(4*c + 4*d*x))/(5*d) - (8*a*b*exp(c + d*x))/(5*d) + (2
4*a*b*exp(3*c + 3*d*x))/(5*d) - (24*a*b*exp(5*c + 5*d*x))/(5*d) + (8*a*b*e
xp(7*c + 7*d*x))/(5*d))/(5*exp(2*c + 2*d*x) - 10*exp(4*c + 4*d*x) + 10*exp
(6*c + 6*d*x) - 5*exp(8*c + 8*d*x) + exp(10*c + 10*d*x) - 1) + (2*atan((a*
b*exp(d*x)*exp(c)*(-d^2)^(1/2))/(d*(a^2*b^2)^(1/2)))*(a^2*b^2)^(1/2))/(-d^
2)^(1/2) - (64*a^2)/(15*d*(3*exp(2*c + 2*d*x) - 3*exp(4*c + 4*d*x) + exp(6
*c + 6*d*x) - 1)) - (16*a^2)/(5*d*(6*exp(4*c + 4*d*x) - 4*exp(2*c + 2*d*x)
 - 4*exp(6*c + 6*d*x) + exp(8*c + 8*d*x) + 1)) - (2*a*b*exp(c + d*x))/(d*(
exp(2*c + 2*d*x) - 1)) - (12*a*b*exp(c + d*x))/(5*d*(exp(4*c + 4*d*x) - 2*
exp(2*c + 2*d*x) + 1))
```

Reduce [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 516, normalized size of antiderivative = 5.86

$$\int \operatorname{csch}^6(c+dx)\left(a+b\sinh^3(c+dx)\right)^2\,dx$$
$$= \frac{-30e^{9dx+9c}ab-150e^{4dx+4c}b^2dx-75e^{8dx+8c}b^2dx-150e^{4dx+4c}\log\left(e^{dx+c}+1\right)ab-16a^2-60e^{3dx+3c}ab+3}{}$$

input
```
int(csch(d*x+c)^6*(a+b*sinh(d*x+c)^3)^2,x)
```

output
```
( - 15*e**(10*c + 10*d*x)*log(e**(c + d*x) - 1)*a*b + 15*e**(10*c + 10*d*x
)*log(e**(c + d*x) + 1)*a*b + 15*e**(10*c + 10*d*x)*b**2*d*x - 30*e**(9*c
+ 9*d*x)*a*b + 75*e**(8*c + 8*d*x)*log(e**(c + d*x) - 1)*a*b - 75*e**(8*c
+ 8*d*x)*log(e**(c + d*x) + 1)*a*b - 75*e**(8*c + 8*d*x)*b**2*d*x + 60*e**
(7*c + 7*d*x)*a*b - 150*e**(6*c + 6*d*x)*log(e**(c + d*x) - 1)*a*b + 150*e
**(6*c + 6*d*x)*log(e**(c + d*x) + 1)*a*b + 150*e**(6*c + 6*d*x)*b**2*d*x
+ 150*e**(4*c + 4*d*x)*log(e**(c + d*x) - 1)*a*b - 150*e**(4*c + 4*d*x)*lo
g(e**(c + d*x) + 1)*a*b - 160*e**(4*c + 4*d*x)*a**2 - 150*e**(4*c + 4*d*x)
*b**2*d*x - 60*e**(3*c + 3*d*x)*a*b - 75*e**(2*c + 2*d*x)*log(e**(c + d*x)
 - 1)*a*b + 75*e**(2*c + 2*d*x)*log(e**(c + d*x) + 1)*a*b + 80*e**(2*c + 2
*d*x)*a**2 + 75*e**(2*c + 2*d*x)*b**2*d*x + 30*e**(c + d*x)*a*b + 15*log(e
**(c + d*x) - 1)*a*b - 15*log(e**(c + d*x) + 1)*a*b - 16*a**2 - 15*b**2*d*
x)/(15*d*(e**(10*c + 10*d*x) - 5*e**(8*c + 8*d*x) + 10*e**(6*c + 6*d*x) -
10*e**(4*c + 4*d*x) + 5*e**(2*c + 2*d*x) - 1))
```

3.137 $\int \operatorname{csch}^7(c+dx) \left(a+b\sinh^3(c+dx)\right)^2 \, dx$

Optimal result

Integrand size = 23, antiderivative size = 133

$$\int \operatorname{csch}^7(c+dx) \left(a+b\sinh^3(c+dx)\right)^2 \, dx = \frac{5a^2\operatorname{arctanh}(\cosh(c+dx))}{16d} - \frac{b^2\operatorname{arctanh}(\cosh(c+dx))}{d} + \frac{2ab\coth(c+dx)}{d} - \frac{2ab\coth^3(c+dx)}{3d} - \frac{5a^2\coth(c+dx)\operatorname{csch}(c+dx)}{16d} + \frac{5a^2\coth(c+dx)\operatorname{csch}^3(c+dx)}{24d} - \frac{a^2\coth(c+dx)\operatorname{csch}^5(c+dx)}{6d}$$

output
```
5/16*a^2*arctanh(cosh(d*x+c))/d-b^2*arctanh(cosh(d*x+c))/d+2*a*b*coth(d*x+
c)/d-2/3*a*b*coth(d*x+c)^3/d-5/16*a^2*coth(d*x+c)*csch(d*x+c)/d+5/24*a^2*c
oth(d*x+c)*csch(d*x+c)^3/d-1/6*a^2*coth(d*x+c)*csch(d*x+c)^5/d
```

Mathematica [A] (verified)

Time = 0.15 (sec) , antiderivative size = 228, normalized size of antiderivative = 1.71

$$\int \operatorname{csch}^7(c+dx)\left(a+b\sinh^3(c+dx)\right)^2 dx$$
$$= -\frac{b^2\operatorname{arctanh}(\cosh(c+dx))}{d}+\frac{4ab\coth(c+dx)}{3d}-\frac{5a^2\operatorname{csch}^2\left(\frac{1}{2}(c+dx)\right)}{64d}+\frac{a^2\operatorname{csch}^4\left(\frac{1}{2}(c+dx)\right)}{64d}-\frac{a^2\operatorname{csch}^6\left(\frac{1}{2}(c+dx)\right)}{384d}-\frac{2ab\coth(c+dx)\operatorname{csch}^2(c+dx)}{3d}+\frac{5a^2\log\left(\cosh\left(\frac{1}{2}(c+dx)\right)\right)}{16d}-\frac{5a^2\log\left(\sinh\left(\frac{1}{2}(c+dx)\right)\right)}{16d}-\frac{5a^2\operatorname{sech}^2\left(\frac{1}{2}(c+dx)\right)}{64d}-\frac{a^2\operatorname{sech}^4\left(\frac{1}{2}(c+dx)\right)}{64d}-\frac{a^2\operatorname{sech}^6\left(\frac{1}{2}(c+dx)\right)}{384d}$$

input
```
Integrate[Csch[c + d*x]^7*(a + b*Sinh[c + d*x]^3)^2,x]
```

output
```
-((b^2*ArcTanh[Cosh[c + d*x]])/d) + (4*a*b*Coth[c + d*x])/(3*d) - (5*a^2*C
sch[(c + d*x)/2]^2)/(64*d) + (a^2*Csch[(c + d*x)/2]^4)/(64*d) - (a^2*Csch[
(c + d*x)/2]^6)/(384*d) - (2*a*b*Coth[c + d*x]*Csch[c + d*x]^2)/(3*d) + (5
*a^2*Log[Cosh[(c + d*x)/2]])/(16*d) - (5*a^2*Log[Sinh[(c + d*x)/2]])/(16*d
) - (5*a^2*Sech[(c + d*x)/2]^2)/(64*d) - (a^2*Sech[(c + d*x)/2]^4)/(64*d)
- (a^2*Sech[(c + d*x)/2]^6)/(384*d)
```

Rubi [C] (verified)

Result contains complex when optimal does not.

Time = 0.40 (sec) , antiderivative size = 151, normalized size of antiderivative = 1.14, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.174$, Rules used = {3042, 26, 3699, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \operatorname{csch}^7(c+dx)\left(a+b\sinh^3(c+dx)\right)^2 dx$$

$$\Big\downarrow \ 3042$$

$$\int -\frac{i\left(a+ib\sin(ic+idx)^3\right)^2}{\sin(ic+idx)^7}dx$$

$$\Big\downarrow \text{26}$$

$$-i\int \frac{\left(ib\sin(ic+idx)^3+a\right)^2}{\sin(ic+idx)^7}dx$$

$$\Big\downarrow \text{3699}$$

$$-i\int \left(ia^2\text{csch}^7(c+dx)+2iab\text{csch}^4(c+dx)+ib^2\text{csch}(c+dx)\right)dx$$

$$\Big\downarrow \text{2009}$$

$$-i\left(\frac{5ia^2\text{arctanh}(\cosh(c+dx))}{16d}-\frac{ia^2\coth(c+dx)\text{csch}^5(c+dx)}{6d}+\frac{5ia^2\coth(c+dx)\text{csch}^3(c+dx)}{24d}-\frac{5ia^2\coth(\right.$$

input
```
Int[Csch[c + d*x]^7*(a + b*Sinh[c + d*x]^3)^2,x]
```

output
```
(-I)*((((5*I)/16)*a^2*ArcTanh[Cosh[c + d*x]])/d - (I*b^2*ArcTanh[Cosh[c +
d*x]])/d + ((2*I)*a*b*Coth[c + d*x])/d - (((2*I)/3)*a*b*Coth[c + d*x]^3)/d
 - (((5*I)/16)*a^2*Coth[c + d*x]*Csch[c + d*x])/d + (((5*I)/24)*a^2*Coth[c
 + d*x]*Csch[c + d*x]^3)/d - ((I/6)*a^2*Coth[c + d*x]*Csch[c + d*x]^5)/d)
```

Defintions of rubi rules used

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3699
```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^(n_
))^(p_.), x_Symbol] :> Int[ExpandTrig[sin[e + f*x]^m*(a + b*sin[e + f*x]^n)
^p, x], x] /; FreeQ[{a, b, e, f}, x] && IntegersQ[m, p] && (EqQ[n, 4] || Gt
Q[p, 0] || (EqQ[p, -1] && IntegerQ[n]))
```

Maple [A] (verified)

Time = 0.95 (sec) , antiderivative size = 90, normalized size of antiderivative = 0.68

method	result
derivativedivides	$\frac{a^2\left(\left(-\frac{\operatorname{csch}(dx+c)^5}{6}+\frac{5\operatorname{csch}(dx+c)^3}{24}-\frac{5\operatorname{csch}(dx+c)}{16}\right)\coth(dx+c)+\frac{5\operatorname{arctanh}\left(e^{dx+c}\right)}{8}\right)+2ab\left(\frac{2}{3}-\frac{\operatorname{csch}(dx+c)^2}{3}\right)\coth(dx+}{d}$
default	$\frac{a^2\left(\left(-\frac{\operatorname{csch}(dx+c)^5}{6}+\frac{5\operatorname{csch}(dx+c)^3}{24}-\frac{5\operatorname{csch}(dx+c)}{16}\right)\coth(dx+c)+\frac{5\operatorname{arctanh}\left(e^{dx+c}\right)}{8}\right)+2ab\left(\frac{2}{3}-\frac{\operatorname{csch}(dx+c)^2}{3}\right)\coth(dx+}{d}$
parallelrisch	$\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^6a^2-\coth\left(\frac{dx}{2}+\frac{c}{2}\right)^6a^2-9\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^4a^2+9\coth\left(\frac{dx}{2}+\frac{c}{2}\right)^4a^2-32ab\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^3-32\coth\left(\frac{dx}{2}+\frac{c}{2}\right.$
risch	$-\frac{a\left(15a\,e^{11dx+11c}-85a\,e^{9dx+9c}+192\,e^{8dx+8c}b+198a\,e^{7dx+7c}-640\,e^{6dx+6c}b+198a\,e^{5dx+5c}+768b\,e^{4dx+4c}-85a\,e^{3dx}\right.}{24d\left(e^{2dx+2c}-1\right)^6}$

input
```
int(csch(d*x+c)^7*(a+b*sinh(d*x+c)^3)^2,x,method=_RETURNVERBOSE)
```

output
```
1/d*(a^2*((-1/6*csch(d*x+c)^5+5/24*csch(d*x+c)^3-5/16*csch(d*x+c))*coth(d*
x+c)+5/8*arctanh(exp(d*x+c)))+2*a*b*(2/3-1/3*csch(d*x+c)^2)*coth(d*x+c)-2*
b^2*arctanh(exp(d*x+c)))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 3607 vs. 2(123) = 246.

Time = 0.11 (sec) , antiderivative size = 3607, normalized size of antiderivative = 27.12

$$\int \operatorname{csch}^7(c+dx)\left(a+b\sinh^3(c+dx)\right)^2\,dx=\text{Too large to display}$$

input
```
integrate(csch(d*x+c)^7*(a+b*sinh(d*x+c)^3)^2,x, algorithm="fricas")
```

output
```
Too large to include
```

Sympy [F(-1)]

Timed out.

$$\int \operatorname{csch}^7(c+dx)\left(a+b\sinh^3(c+dx)\right)^2\,dx=\text{Timed out}$$

input
```
integrate(csch(d*x+c)**7*(a+b*sinh(d*x+c)**3)**2,x)
```

output
```
Timed out
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 316 vs. 2(123) = 246.

Time = 0.06 (sec) , antiderivative size = 316, normalized size of antiderivative = 2.38

$$\int \operatorname{csch}^7(c+dx)\left(a+b\sinh^3(c+dx)\right)^2\,dx$$
$$=\frac{1}{48}a^2\left(\frac{15\log\left(e^{(-dx-c)}+1\right)}{d}-\frac{15\log\left(e^{(-dx-c)}-1\right)}{d}+\frac{2\left(15e^{(-dx-c)}-85e^{(-3dx-3c)}+198e^{(-5dx-5c)}+\right.}{d(6e^{(-2dx-2c)}-15e^{(-4dx-4c)}+20e^{(-6dx-6c)}-\right.$$
$$-b^2\left(\frac{\log\left(e^{(-dx-c)}+1\right)}{d}-\frac{\log\left(e^{(-dx-c)}-1\right)}{d}\right)$$
$$+\frac{8}{3}ab\left(\frac{3e^{(-2dx-2c)}}{d(3e^{(-2dx-2c)}-3e^{(-4dx-4c)}+e^{(-6dx-6c)}-1)}-\frac{1}{d(3e^{(-2dx-2c)}-3e^{(-4dx-4c)}+e^{(-6dx-6c)}-1\right.$$

input
```
integrate(csch(d*x+c)^7*(a+b*sinh(d*x+c)^3)^2,x, algorithm="maxima")
```

output
```
1/48*a^2*(15*log(e^(-d*x - c) + 1)/d - 15*log(e^(-d*x - c) - 1)/d + 2*(15*
e^(-d*x - c) - 85*e^(-3*d*x - 3*c) + 198*e^(-5*d*x - 5*c) + 198*e^(-7*d*x
- 7*c) - 85*e^(-9*d*x - 9*c) + 15*e^(-11*d*x - 11*c))/(d*(6*e^(-2*d*x - 2*
c) - 15*e^(-4*d*x - 4*c) + 20*e^(-6*d*x - 6*c) - 15*e^(-8*d*x - 8*c) + 6*e
^(-10*d*x - 10*c) - e^(-12*d*x - 12*c) - 1))) - b^2*(log(e^(-d*x - c) + 1)
/d - log(e^(-d*x - c) - 1)/d) + 8/3*a*b*(3*e^(-2*d*x - 2*c)/(d*(3*e^(-2*d*
x - 2*c) - 3*e^(-4*d*x - 4*c) + e^(-6*d*x - 6*c) - 1)) - 1/(d*(3*e^(-2*d*x
 - 2*c) - 3*e^(-4*d*x - 4*c) + e^(-6*d*x - 6*c) - 1)))
```

Giac [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 204, normalized size of antiderivative = 1.53

$$\int \operatorname{csch}^7(c+dx)\left(a+b\sinh^3(c+dx)\right)^2\,dx$$
$$= \frac{3\,(5\,a^2-16\,b^2)\log\left(e^{(dx+c)}+1\right)-3\,(5\,a^2-16\,b^2)\log\left(\left|e^{(dx+c)}-1\right|\right)-\frac{2\,(15\,a^2e^{(11\,dx+11\,c)}-85\,a^2e^{(9\,dx+9\,c)}+192\,ab}{}}{48}$$

input
```
integrate(csch(d*x+c)^7*(a+b*sinh(d*x+c)^3)^2,x, algorithm="giac")
```

output
```
1/48*(3*(5*a^2 - 16*b^2)*log(e^(d*x + c) + 1) - 3*(5*a^2 - 16*b^2)*log(abs
(e^(d*x + c) - 1)) - 2*(15*a^2*e^(11*d*x + 11*c) - 85*a^2*e^(9*d*x + 9*c)
+ 192*a*b*e^(8*d*x + 8*c) + 198*a^2*e^(7*d*x + 7*c) - 640*a*b*e^(6*d*x + 6
*c) + 198*a^2*e^(5*d*x + 5*c) + 768*a*b*e^(4*d*x + 4*c) - 85*a^2*e^(3*d*x
+ 3*c) - 384*a*b*e^(2*d*x + 2*c) + 15*a^2*e^(d*x + c) + 64*a*b)/(e^(2*d*x
+ 2*c) - 1)^6)/d
```

Mupad [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 434, normalized size of antiderivative = 3.26

$$\int \operatorname{csch}^7(c+dx)\left(a+b\sinh^3(c+dx)\right)^2\,dx$$

$$= \frac{\frac{5a^2e^{c+dx}}{12d}-\frac{8ab}{d}}{e^{4c+4dx}-2e^{2c+2dx}+1} - \frac{\frac{a^2e^{c+dx}}{3d}+\frac{16ab}{3d}}{3e^{2c+2dx}-3e^{4c+4dx}+e^{6c+6dx}-1}$$
$$+ \frac{\operatorname{atan}\left(\frac{e^{dx}e^{c}\left(5a^2\sqrt{-d^2}-16b^2\sqrt{-d^2}\right)}{d\sqrt{25a^4-160a^2b^2+256b^4}}\right)\sqrt{25a^4-160a^2b^2+256b^4}}{8\sqrt{-d^2}}$$
$$- \frac{18a^2e^{c+dx}}{d\left(6e^{4c+4dx}-4e^{2c+2dx}-4e^{6c+6dx}+e^{8c+8dx}+1\right)}$$
$$- \frac{80a^2e^{c+dx}}{3d\left(5e^{2c+2dx}-10e^{4c+4dx}+10e^{6c+6dx}-5e^{8c+8dx}+e^{10c+10dx}-1\right)}$$
$$- \frac{32a^2e^{c+dx}}{3d\left(15e^{4c+4dx}-6e^{2c+2dx}-20e^{6c+6dx}+15e^{8c+8dx}-6e^{10c+10dx}+e^{12c+12dx}+1\right)}$$
$$- \frac{5a^2e^{c+dx}}{8d\left(e^{2c+2dx}-1\right)}$$

input
```
int((a + b*sinh(c + d*x)^3)^2/sinh(c + d*x)^7,x)
```

output
```
((5*a^2*exp(c + d*x))/(12*d) - (8*a*b)/d)/(exp(4*c + 4*d*x) - 2*exp(2*c +
2*d*x) + 1) - ((a^2*exp(c + d*x))/(3*d) + (16*a*b)/(3*d))/(3*exp(2*c + 2*d
*x) - 3*exp(4*c + 4*d*x) + exp(6*c + 6*d*x) - 1) + (atan((exp(d*x)*exp(c)*
(5*a^2*(-d^2)^(1/2) - 16*b^2*(-d^2)^(1/2)))/(d*(25*a^4 + 256*b^4 - 160*a^2
*b^2)^(1/2)))*(25*a^4 + 256*b^4 - 160*a^2*b^2)^(1/2))/(8*(-d^2)^(1/2)) - (
18*a^2*exp(c + d*x))/(d*(6*exp(4*c + 4*d*x) - 4*exp(2*c + 2*d*x) - 4*exp(6
*c + 6*d*x) + exp(8*c + 8*d*x) + 1)) - (80*a^2*exp(c + d*x))/(3*d*(5*exp(2
*c + 2*d*x) - 10*exp(4*c + 4*d*x) + 10*exp(6*c + 6*d*x) - 5*exp(8*c + 8*d*
x) + exp(10*c + 10*d*x) - 1)) - (32*a^2*exp(c + d*x))/(3*d*(15*exp(4*c + 4
*d*x) - 6*exp(2*c + 2*d*x) - 20*exp(6*c + 6*d*x) + 15*exp(8*c + 8*d*x) - 6
*exp(10*c + 10*d*x) + exp(12*c + 12*d*x) + 1)) - (5*a^2*exp(c + d*x))/(8*d
*(exp(2*c + 2*d*x) - 1))
```

Reduce [B] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 887, normalized size of antiderivative = 6.67

$$\int \operatorname{csch}^7(c+dx) \left(a+b\sinh^3(c+dx)\right)^2 dx = \text{Too large to display}$$

input
```
int(csch(d*x+c)^7*(a+b*sinh(d*x+c)^3)^2,x)
```

output
```
( - 15*e**(12*c + 12*d*x)*log(e**(c + d*x) - 1)*a**2 + 48*e**(12*c + 12*d*
x)*log(e**(c + d*x) - 1)*b**2 + 15*e**(12*c + 12*d*x)*log(e**(c + d*x) + 1
)*a**2 - 48*e**(12*c + 12*d*x)*log(e**(c + d*x) + 1)*b**2 - 30*e**(11*c +
11*d*x)*a**2 + 90*e**(10*c + 10*d*x)*log(e**(c + d*x) - 1)*a**2 - 288*e**(
10*c + 10*d*x)*log(e**(c + d*x) - 1)*b**2 - 90*e**(10*c + 10*d*x)*log(e**(
c + d*x) + 1)*a**2 + 288*e**(10*c + 10*d*x)*log(e**(c + d*x) + 1)*b**2 + 1
70*e**(9*c + 9*d*x)*a**2 - 225*e**(8*c + 8*d*x)*log(e**(c + d*x) - 1)*a**2
 + 720*e**(8*c + 8*d*x)*log(e**(c + d*x) - 1)*b**2 + 225*e**(8*c + 8*d*x)*
log(e**(c + d*x) + 1)*a**2 - 720*e**(8*c + 8*d*x)*log(e**(c + d*x) + 1)*b*
*2 - 384*e**(8*c + 8*d*x)*a*b - 396*e**(7*c + 7*d*x)*a**2 + 300*e**(6*c +
6*d*x)*log(e**(c + d*x) - 1)*a**2 - 960*e**(6*c + 6*d*x)*log(e**(c + d*x)
- 1)*b**2 - 300*e**(6*c + 6*d*x)*log(e**(c + d*x) + 1)*a**2 + 960*e**(6*c
+ 6*d*x)*log(e**(c + d*x) + 1)*b**2 + 1280*e**(6*c + 6*d*x)*a*b - 396*e**(
5*c + 5*d*x)*a**2 - 225*e**(4*c + 4*d*x)*log(e**(c + d*x) - 1)*a**2 + 720*
e**(4*c + 4*d*x)*log(e**(c + d*x) - 1)*b**2 + 225*e**(4*c + 4*d*x)*log(e**
(c + d*x) + 1)*a**2 - 720*e**(4*c + 4*d*x)*log(e**(c + d*x) + 1)*b**2 - 15
36*e**(4*c + 4*d*x)*a*b + 170*e**(3*c + 3*d*x)*a**2 + 90*e**(2*c + 2*d*x)*
log(e**(c + d*x) - 1)*a**2 - 288*e**(2*c + 2*d*x)*log(e**(c + d*x) - 1)*b*
*2 - 90*e**(2*c + 2*d*x)*log(e**(c + d*x) + 1)*a**2 + 288*e**(2*c + 2*d*x)
*log(e**(c + d*x) + 1)*b**2 + 768*e**(2*c + 2*d*x)*a*b - 30*e**(c + d*x...
```

3.138 $\int \sinh^2(c+dx) \left(a+b\sinh^3(c+dx)\right)^3 dx$

Optimal result

Integrand size = 23, antiderivative size = 291

$$\begin{aligned}\int \sinh^2(c+dx) \left(a+b\sinh^3(c+dx)\right)^3 dx = & -\frac{a^3x}{2}+\frac{105}{128}ab^2x+\frac{3a^2b\cosh(c+dx)}{d} \\ & -\frac{b^3\cosh(c+dx)}{d}-\frac{2a^2b\cosh^3(c+dx)}{d} \\ & +\frac{5b^3\cosh^3(c+dx)}{3d}+\frac{3a^2b\cosh^5(c+dx)}{5d} \\ & -\frac{2b^3\cosh^5(c+dx)}{d}+\frac{10b^3\cosh^7(c+dx)}{7d} \\ & -\frac{5b^3\cosh^9(c+dx)}{9d}+\frac{b^3\cosh^{11}(c+dx)}{11d} \\ & +\frac{a^3\cosh(c+dx)\sinh(c+dx)}{2d} \\ & -\frac{105ab^2\cosh(c+dx)\sinh(c+dx)}{128d} \\ & +\frac{35ab^2\cosh(c+dx)\sinh^3(c+dx)}{64d} \\ & -\frac{7ab^2\cosh(c+dx)\sinh^5(c+dx)}{16d} \\ & +\frac{3ab^2\cosh(c+dx)\sinh^7(c+dx)}{8d}\end{aligned}$$

output
```
-1/2*a^3*x+105/128*a*b^2*x+3*a^2*b*cosh(d*x+c)/d-b^3*cosh(d*x+c)/d-2*a^2*b
*cosh(d*x+c)^3/d+5/3*b^3*cosh(d*x+c)^3/d+3/5*a^2*b*cosh(d*x+c)^5/d-2*b^3*c
osh(d*x+c)^5/d+10/7*b^3*cosh(d*x+c)^7/d-5/9*b^3*cosh(d*x+c)^9/d+1/11*b^3*c
osh(d*x+c)^11/d+1/2*a^3*cosh(d*x+c)*sinh(d*x+c)/d-105/128*a*b^2*cosh(d*x+c
)*sinh(d*x+c)/d+35/64*a*b^2*cosh(d*x+c)*sinh(d*x+c)^3/d-7/16*a*b^2*cosh(d*
x+c)*sinh(d*x+c)^5/d+3/8*a*b^2*cosh(d*x+c)*sinh(d*x+c)^7/d
```

Mathematica [A] (verified)

Time = 5.65 (sec) , antiderivative size = 194, normalized size of antiderivative = 0.67

$$\int \sinh^2(c+dx)\left(a+b\sinh^3(c+dx)\right)^3\,dx$$
$$=\frac{-27720a(64a^2-105b^2)(c+dx)-20790b(-320a^2+77b^2)\cosh(c+dx)+34650b(-32a^2+11b^2)\cosh(}{}$$

input
```
Integrate[Sinh[c + d*x]^2*(a + b*Sinh[c + d*x]^3)^3,x]
```

output
```
(-27720*a*(64*a^2 - 105*b^2)*(c + d*x) - 20790*b*(-320*a^2 + 77*b^2)*Cosh[
c + d*x] + 34650*b*(-32*a^2 + 11*b^2)*Cosh[3*(c + d*x)] - 2079*b*(-64*a^2
+ 55*b^2)*Cosh[5*(c + d*x)] + 27225*b^3*Cosh[7*(c + d*x)] - 4235*b^3*Cosh[
9*(c + d*x)] + 315*b^3*Cosh[11*(c + d*x)] + 110880*a*(8*a^2 - 21*b^2)*Sinh
[2*(c + d*x)] + 582120*a*b^2*Sinh[4*(c + d*x)] - 110880*a*b^2*Sinh[6*(c +
d*x)] + 10395*a*b^2*Sinh[8*(c + d*x)])/(3548160*d)
```

Rubi [A] (verified)

Time = 0.56 (sec) , antiderivative size = 291, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.174$, Rules used = {3042, 25, 3699, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sinh^2(c+dx)\left(a+b\sinh^3(c+dx)\right)^3\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int -\sin(ic+idx)^2\left(a+ib\sin(ic+idx)^3\right)^3dx$$

$$\Big\downarrow \text{25}$$

$$-\int \sin(ic+idx)^2\left(ib\sin(ic+idx)^3+a\right)^3dx$$

$$\Big\downarrow \text{3699}$$

$$-\int \left(-b^3\sinh^{11}(c+dx)-3ab^2\sinh^8(c+dx)-3a^2b\sinh^5(c+dx)-a^3\sinh^2(c+dx)\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{a^3\sinh(c+dx)\cosh(c+dx)}{2d}-\frac{a^3x}{2}+\frac{3a^2b\cosh^5(c+dx)}{5d}-\frac{2a^2b\cosh^3(c+dx)}{d}+\frac{3a^2b\cosh(c+dx)}{d}+\frac{3ab^2\sinh^7(c+dx)\cosh(c+dx)}{8d}-\frac{7ab^2\sinh^5(c+dx)\cosh(c+dx)}{16d}+\frac{35ab^2\sinh^3(c+dx)\cosh(c+dx)}{64d}-\frac{105ab^2\sinh(c+dx)\cosh(c+dx)}{128d}+\frac{105}{128}ab^2x+\frac{b^3\cosh^{11}(c+dx)}{11d}-\frac{5b^3\cosh^9(c+dx)}{9d}+\frac{10b^3\cosh^7(c+dx)}{7d}-\frac{2b^3\cosh^5(c+dx)}{d}+\frac{5b^3\cosh^3(c+dx)}{3d}-\frac{b^3\cosh(c+dx)}{d}$$

input
```
Int[Sinh[c + d*x]^2*(a + b*Sinh[c + d*x]^3)^3,x]
```

output
```
-1/2*(a^3*x) + (105*a*b^2*x)/128 + (3*a^2*b*Cosh[c + d*x])/d - (b^3*Cosh[c
 + d*x])/d - (2*a^2*b*Cosh[c + d*x]^3)/d + (5*b^3*Cosh[c + d*x]^3)/(3*d) +
 (3*a^2*b*Cosh[c + d*x]^5)/(5*d) - (2*b^3*Cosh[c + d*x]^5)/d + (10*b^3*Cos
h[c + d*x]^7)/(7*d) - (5*b^3*Cosh[c + d*x]^9)/(9*d) + (b^3*Cosh[c + d*x]^1
1)/(11*d) + (a^3*Cosh[c + d*x]*Sinh[c + d*x])/(2*d) - (105*a*b^2*Cosh[c +
d*x]*Sinh[c + d*x])/(128*d) + (35*a*b^2*Cosh[c + d*x]*Sinh[c + d*x]^3)/(64
*d) - (7*a*b^2*Cosh[c + d*x]*Sinh[c + d*x]^5)/(16*d) + (3*a*b^2*Cosh[c + d
*x]*Sinh[c + d*x]^7)/(8*d)
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3699
```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^(n_
))^(p_.), x_Symbol] :> Int[ExpandTrig[sin[e + f*x]^m*(a + b*sin[e + f*x]^n)
^p, x], x] /; FreeQ[{a, b, e, f}, x] && IntegersQ[m, p] && (EqQ[n, 4] || Gt
Q[p, 0] || (EqQ[p, -1] && IntegerQ[n]))
```

Maple [A] (verified)

Time = 0.28 (sec) , antiderivative size = 188, normalized size of antiderivative = 0.65

$$a^3\left(\frac{\cosh(dx+c)\sinh(dx+c)}{2}-\frac{dx}{2}-\frac{c}{2}\right)+3a^2b\left(\frac{8}{15}+\frac{\sinh(dx+c)^4}{5}-\frac{4\sinh(dx+c)^2}{15}\right)\cosh\left(dx+c\right)+3b^2a\left(\left(\frac{\sinh(dx+c)^7}{8}-\right.\right.$$

input
```
int(sinh(d*x+c)^2*(a+b*sinh(d*x+c)^3)^3,x)
```

output
```
1/d*(a^3*(1/2*cosh(d*x+c)*sinh(d*x+c)-1/2*d*x-1/2*c)+3*a^2*b*(8/15+1/5*sin
h(d*x+c)^4-4/15*sinh(d*x+c)^2)*cosh(d*x+c)+3*b^2*a*((1/8*sinh(d*x+c)^7-7/4
8*sinh(d*x+c)^5+35/192*sinh(d*x+c)^3-35/128*sinh(d*x+c))*cosh(d*x+c)+35/12
8*d*x+35/128*c)+b^3*(-256/693+1/11*sinh(d*x+c)^10-10/99*sinh(d*x+c)^8+80/6
93*sinh(d*x+c)^6-32/231*sinh(d*x+c)^4+128/693*sinh(d*x+c)^2)*cosh(d*x+c))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 568 vs. $2(267) = 534$.

Time = 0.09 (sec) , antiderivative size = 568, normalized size of antiderivative = 1.95

$$\int \sinh^2(c+dx)\left(a+b\sinh^3(c+dx)\right)^3\,dx = \text{Too large to display}$$

input
```
integrate(sinh(d*x+c)^2*(a+b*sinh(d*x+c)^3)^3,x, algorithm="fricas")
```

output
```
1/3548160*(315*b^3*cosh(d*x + c)^11 + 3465*b^3*cosh(d*x + c)*sinh(d*x + c)
^10 - 4235*b^3*cosh(d*x + c)^9 + 83160*a*b^2*cosh(d*x + c)*sinh(d*x + c)^7
 + 27225*b^3*cosh(d*x + c)^7 + 3465*(15*b^3*cosh(d*x + c)^3 - 11*b^3*cosh(
d*x + c))*sinh(d*x + c)^8 + 1155*(126*b^3*cosh(d*x + c)^5 - 308*b^3*cosh(d
*x + c)^3 + 165*b^3*cosh(d*x + c))*sinh(d*x + c)^6 + 2079*(64*a^2*b - 55*b
^3)*cosh(d*x + c)^5 + 83160*(7*a*b^2*cosh(d*x + c)^3 - 8*a*b^2*cosh(d*x +
c))*sinh(d*x + c)^5 + 3465*(30*b^3*cosh(d*x + c)^7 - 154*b^3*cosh(d*x + c)
^5 + 275*b^3*cosh(d*x + c)^3 + 3*(64*a^2*b - 55*b^3)*cosh(d*x + c))*sinh(d
*x + c)^4 - 34650*(32*a^2*b - 11*b^3)*cosh(d*x + c)^3 + 27720*(21*a*b^2*co
sh(d*x + c)^5 - 80*a*b^2*cosh(d*x + c)^3 + 84*a*b^2*cosh(d*x + c))*sinh(d*
x + c)^3 - 27720*(64*a^3 - 105*a*b^2)*d*x + 3465*(5*b^3*cosh(d*x + c)^9 -
44*b^3*cosh(d*x + c)^7 + 165*b^3*cosh(d*x + c)^5 + 6*(64*a^2*b - 55*b^3)*c
osh(d*x + c)^3 - 30*(32*a^2*b - 11*b^3)*cosh(d*x + c))*sinh(d*x + c)^2 + 2
0790*(320*a^2*b - 77*b^3)*cosh(d*x + c) + 27720*(3*a*b^2*cosh(d*x + c)^7 -
 24*a*b^2*cosh(d*x + c)^5 + 84*a*b^2*cosh(d*x + c)^3 + 8*(8*a^3 - 21*a*b^2
)*cosh(d*x + c))*sinh(d*x + c))/d
```

Sympy [A] (verification not implemented)

Time = 1.88 (sec) , antiderivative size = 498, normalized size of antiderivative = 1.71

$$\int \sinh^2(c+dx)\left(a+b\sinh^3(c+dx)\right)^3\,dx$$

$$= \begin{cases} \frac{a^3x\sinh^2(c+dx)}{2} - \frac{a^3x\cosh^2(c+dx)}{2} + \frac{a^3\sinh(c+dx)\cosh(c+dx)}{2d} + \frac{3a^2b\sinh^4(c+dx)\cosh(c+dx)}{d} - \frac{4a^2b\sinh^2(c+dx)\cosh^3(c+d}{d} \\ x\left(a+b\sinh^3(c)\right)^3\sinh^2(c) \end{cases}$$

input
```
integrate(sinh(d*x+c)**2*(a+b*sinh(d*x+c)**3)**3,x)
```

output
```
Piecewise((a**3*x*sinh(c + d*x)**2/2 - a**3*x*cosh(c + d*x)**2/2 + a**3*si
nh(c + d*x)*cosh(c + d*x)/(2*d) + 3*a**2*b*sinh(c + d*x)**4*cosh(c + d*x)/
d - 4*a**2*b*sinh(c + d*x)**2*cosh(c + d*x)**3/d + 8*a**2*b*cosh(c + d*x)*
*5/(5*d) + 105*a*b**2*x*sinh(c + d*x)**8/128 - 105*a*b**2*x*sinh(c + d*x)*
*6*cosh(c + d*x)**2/32 + 315*a*b**2*x*sinh(c + d*x)**4*cosh(c + d*x)**4/64
 - 105*a*b**2*x*sinh(c + d*x)**2*cosh(c + d*x)**6/32 + 105*a*b**2*x*cosh(c
 + d*x)**8/128 + 279*a*b**2*sinh(c + d*x)**7*cosh(c + d*x)/(128*d) - 511*a
*b**2*sinh(c + d*x)**5*cosh(c + d*x)**3/(128*d) + 385*a*b**2*sinh(c + d*x)
**3*cosh(c + d*x)**5/(128*d) - 105*a*b**2*sinh(c + d*x)*cosh(c + d*x)**7/(
128*d) + b**3*sinh(c + d*x)**10*cosh(c + d*x)/d - 10*b**3*sinh(c + d*x)**8
*cosh(c + d*x)**3/(3*d) + 16*b**3*sinh(c + d*x)**6*cosh(c + d*x)**5/(3*d)
- 32*b**3*sinh(c + d*x)**4*cosh(c + d*x)**7/(7*d) + 128*b**3*sinh(c + d*x)
**2*cosh(c + d*x)**9/(63*d) - 256*b**3*cosh(c + d*x)**11/(693*d), Ne(d, 0)
), (x*(a + b*sinh(c)**3)**3*sinh(c)**2, True))
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 387, normalized size of antiderivative = 1.33

$$\int \sinh^2(c+dx)\left(a+b\sinh^3(c+dx)\right)^3\,dx = -\frac{1}{8}a^3\left(4x-\frac{e^{(2dx+2c)}}{d}+\frac{e^{(-2dx-2c)}}{d}\right) - \frac{1}{1419264}b^3\left(\frac{\left(847e^{(-2dx-2c)}-5445e^{(-4dx-4c)}+22869e^{(-6dx-6c)}-76230e^{(-8dx-8c)}+320166e^{(-10d}\right.}{d}\right. - \frac{1}{2048}ab^2\left(\frac{\left(32e^{(-2dx-2c)}-168e^{(-4dx-4c)}+672e^{(-6dx-6c)}-3\right)e^{(8dx+8c)}}{d}-\frac{1680(dx+c)}{d}-\frac{672e^{(-2c}}{}\right. + \frac{1}{160}a^2b\left(\frac{3e^{(5dx+5c)}}{d}-\frac{25e^{(3dx+3c)}}{d}+\frac{150e^{(dx+c)}}{d}+\frac{150e^{(-dx-c)}}{d}-\frac{25e^{(-3dx-3c)}}{d}+\frac{3e^{(-5dx-5c)}}{d}\right)$$

input
```
integrate(sinh(d*x+c)^2*(a+b*sinh(d*x+c)^3)^3,x, algorithm="maxima")
```

output
```
-1/8*a^3*(4*x - e^(2*d*x + 2*c)/d + e^(-2*d*x - 2*c)/d) - 1/1419264*b^3*((
847*e^(-2*d*x - 2*c) - 5445*e^(-4*d*x - 4*c) + 22869*e^(-6*d*x - 6*c) - 76
230*e^(-8*d*x - 8*c) + 320166*e^(-10*d*x - 10*c) - 63)*e^(11*d*x + 11*c)/d
 + (320166*e^(-d*x - c) - 76230*e^(-3*d*x - 3*c) + 22869*e^(-5*d*x - 5*c)
- 5445*e^(-7*d*x - 7*c) + 847*e^(-9*d*x - 9*c) - 63*e^(-11*d*x - 11*c))/d)
 - 1/2048*a*b^2*((32*e^(-2*d*x - 2*c) - 168*e^(-4*d*x - 4*c) + 672*e^(-6*d
*x - 6*c) - 3)*e^(8*d*x + 8*c)/d - 1680*(d*x + c)/d - (672*e^(-2*d*x - 2*c
) - 168*e^(-4*d*x - 4*c) + 32*e^(-6*d*x - 6*c) - 3*e^(-8*d*x - 8*c))/d) +
1/160*a^2*b*(3*e^(5*d*x + 5*c)/d - 25*e^(3*d*x + 3*c)/d + 150*e^(d*x + c)/
d + 150*e^(-d*x - c)/d - 25*e^(-3*d*x - 3*c)/d + 3*e^(-5*d*x - 5*c)/d)
```

Giac [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 431, normalized size of antiderivative = 1.48

$$\int \sinh^2(c+dx)\left(a+b\sinh^3(c+dx)\right)^3\,dx = \frac{b^3e^{(11\,dx+11\,c)}}{22528\,d} - \frac{11\,b^3e^{(9\,dx+9\,c)}}{18432\,d} + \frac{3\,ab^2e^{(8\,dx+8\,c)}}{2048\,d} + \frac{55\,b^3e^{(7\,dx+7\,c)}}{14336\,d} - \frac{ab^2e^{(6\,dx+6\,c)}}{64\,d} + \frac{21\,ab^2e^{(4\,dx+4\,c)}}{256\,d} - \frac{21\,ab^2e^{(-4\,dx-4\,c)}}{256\,d} + \frac{ab^2e^{(-6\,dx-6\,c)}}{64\,d} + \frac{55\,b^3e^{(-7\,dx-7\,c)}}{14336\,d} - \frac{3\,ab^2e^{(-8\,dx-8\,c)}}{2048\,d} - \frac{11\,b^3e^{(-9\,dx-9\,c)}}{18432\,d} + \frac{b^3e^{(-11\,dx-11\,c)}}{22528\,d} - \frac{1}{128}\left(64\,a^3-105\,ab^2\right)x + \frac{3\left(64\,a^2b-55\,b^3\right)e^{(5\,dx+5\,c)}}{10240\,d} - \frac{5\left(32\,a^2b-11\,b^3\right)e^{(3\,dx+3\,c)}}{1024\,d} + \frac{\left(8\,a^3-21\,ab^2\right)e^{(2\,dx+2\,c)}}{64\,d} + \frac{3\left(320\,a^2b-77\,b^3\right)e^{(dx+c)}}{1024\,d} + \frac{3\left(320\,a^2b-77\,b^3\right)e^{(-dx-c)}}{1024\,d} - \frac{\left(8\,a^3-21\,ab^2\right)e^{(-2\,dx-2\,c)}}{64\,d} - \frac{5\left(32\,a^2b-11\,b^3\right)e^{(-3\,dx-3\,c)}}{1024\,d} + \frac{3\left(64\,a^2b-55\,b^3\right)e^{(-5\,dx-5\,c)}}{10240\,d}$$

input
```
integrate(sinh(d*x+c)^2*(a+b*sinh(d*x+c)^3)^3,x, algorithm="giac")
```

output
```
1/22528*b^3*e^(11*d*x + 11*c)/d - 11/18432*b^3*e^(9*d*x + 9*c)/d + 3/2048*
a*b^2*e^(8*d*x + 8*c)/d + 55/14336*b^3*e^(7*d*x + 7*c)/d - 1/64*a*b^2*e^(6
*d*x + 6*c)/d + 21/256*a*b^2*e^(4*d*x + 4*c)/d - 21/256*a*b^2*e^(-4*d*x -
4*c)/d + 1/64*a*b^2*e^(-6*d*x - 6*c)/d + 55/14336*b^3*e^(-7*d*x - 7*c)/d -
 3/2048*a*b^2*e^(-8*d*x - 8*c)/d - 11/18432*b^3*e^(-9*d*x - 9*c)/d + 1/225
28*b^3*e^(-11*d*x - 11*c)/d - 1/128*(64*a^3 - 105*a*b^2)*x + 3/10240*(64*a
^2*b - 55*b^3)*e^(5*d*x + 5*c)/d - 5/1024*(32*a^2*b - 11*b^3)*e^(3*d*x + 3
*c)/d + 1/64*(8*a^3 - 21*a*b^2)*e^(2*d*x + 2*c)/d + 3/1024*(320*a^2*b - 77
*b^3)*e^(d*x + c)/d + 3/1024*(320*a^2*b - 77*b^3)*e^(-d*x - c)/d - 1/64*(8
*a^3 - 21*a*b^2)*e^(-2*d*x - 2*c)/d - 5/1024*(32*a^2*b - 11*b^3)*e^(-3*d*x
 - 3*c)/d + 3/10240*(64*a^2*b - 55*b^3)*e^(-5*d*x - 5*c)/d
```

Mupad [B] (verification not implemented)

Time = 2.06 (sec) , antiderivative size = 231, normalized size of antiderivative = 0.79

$$\int \sinh^2(c+dx)\left(a+b\sinh^3(c+dx)\right)^3\,dx$$

$$= \frac{\frac{\sinh(c+d\,x)\,a^3\cosh(c+d\,x)}{2} - \frac{d\,x\,a^3}{2} + \frac{3\,a^2\,b\cosh(c+d\,x)^5}{5} - 2\,a^2\,b\cosh(c+d\,x)^3 + 3\,a^2\,b\cosh(c+d\,x) + \frac{3\sinh(c+d\,x)\,a}{8}}{}$$

input
```
int(sinh(c + d*x)^2*(a + b*sinh(c + d*x)^3)^3,x)
```

output
```
((5*b^3*cosh(c + d*x)^3)/3 - b^3*cosh(c + d*x) - 2*b^3*cosh(c + d*x)^5 + (
10*b^3*cosh(c + d*x)^7)/7 - (5*b^3*cosh(c + d*x)^9)/9 + (b^3*cosh(c + d*x)
^11)/11 - 2*a^2*b*cosh(c + d*x)^3 + (3*a^2*b*cosh(c + d*x)^5)/5 + 3*a^2*b*
cosh(c + d*x) + (a^3*cosh(c + d*x)*sinh(c + d*x))/2 - (a^3*d*x)/2 - (279*a
*b^2*cosh(c + d*x)*sinh(c + d*x))/128 + (105*a*b^2*d*x)/128 + (163*a*b^2*c
osh(c + d*x)^3*sinh(c + d*x))/64 - (25*a*b^2*cosh(c + d*x)^5*sinh(c + d*x)
)/16 + (3*a*b^2*cosh(c + d*x)^7*sinh(c + d*x))/8)/d
```

Reduce [B] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 477, normalized size of antiderivative = 1.64

$$\int \sinh^2(c+dx)\left(a+b\sinh^3(c+dx)\right)^3\,dx$$
$$=\frac{315b^3+887040e^{13dx+13c}a^3-887040e^{9dx+9c}a^3+5821200e^{11dx+11c}a\,b^2dx+110880e^{5dx+5c}a\,b^2-10395e^{3dx}}{}$$

input
```
int(sinh(d*x+c)^2*(a+b*sinh(d*x+c)^3)^3,x)
```

output
```
(315*e**(22*c + 22*d*x)*b**3 - 4235*e**(20*c + 20*d*x)*b**3 + 10395*e**(19
*c + 19*d*x)*a*b**2 + 27225*e**(18*c + 18*d*x)*b**3 - 110880*e**(17*c + 17
*d*x)*a*b**2 + 133056*e**(16*c + 16*d*x)*a**2*b - 114345*e**(16*c + 16*d*x
)*b**3 + 582120*e**(15*c + 15*d*x)*a*b**2 - 1108800*e**(14*c + 14*d*x)*a**
2*b + 381150*e**(14*c + 14*d*x)*b**3 + 887040*e**(13*c + 13*d*x)*a**3 - 23
28480*e**(13*c + 13*d*x)*a*b**2 + 6652800*e**(12*c + 12*d*x)*a**2*b - 1600
830*e**(12*c + 12*d*x)*b**3 - 3548160*e**(11*c + 11*d*x)*a**3*d*x + 582120
0*e**(11*c + 11*d*x)*a*b**2*d*x + 6652800*e**(10*c + 10*d*x)*a**2*b - 1600
830*e**(10*c + 10*d*x)*b**3 - 887040*e**(9*c + 9*d*x)*a**3 + 2328480*e**(9
*c + 9*d*x)*a*b**2 - 1108800*e**(8*c + 8*d*x)*a**2*b + 381150*e**(8*c + 8*
d*x)*b**3 - 582120*e**(7*c + 7*d*x)*a*b**2 + 133056*e**(6*c + 6*d*x)*a**2*
b - 114345*e**(6*c + 6*d*x)*b**3 + 110880*e**(5*c + 5*d*x)*a*b**2 + 27225*
e**(4*c + 4*d*x)*b**3 - 10395*e**(3*c + 3*d*x)*a*b**2 - 4235*e**(2*c + 2*d
*x)*b**3 + 315*b**3)/(7096320*e**(11*c + 11*d*x)*d)
```

3.139 $\int \sinh(c+dx) \left(a+b\sinh^3(c+dx)\right)^3 dx$

Optimal result

Integrand size = 21, antiderivative size = 267

$$\begin{aligned}\int \sinh(c+dx) \left(a+b\sinh^3(c+dx)\right)^3 dx &= \frac{9}{8}a^2bx - \frac{63b^3x}{256} + \frac{a^3\cosh(c+dx)}{d} \\ &- \frac{3ab^2\cosh(c+dx)}{d} + \frac{3ab^2\cosh^3(c+dx)}{d} \\ &- \frac{9ab^2\cosh^5(c+dx)}{5d} + \frac{3ab^2\cosh^7(c+dx)}{7d} \\ &- \frac{9a^2b\cosh(c+dx)\sinh(c+dx)}{8d} \\ &+ \frac{63b^3\cosh(c+dx)\sinh(c+dx)}{256d} \\ &+ \frac{3a^2b\cosh(c+dx)\sinh^3(c+dx)}{4d} \\ &- \frac{21b^3\cosh(c+dx)\sinh^3(c+dx)}{128d} \\ &+ \frac{21b^3\cosh(c+dx)\sinh^5(c+dx)}{160d} \\ &- \frac{9b^3\cosh(c+dx)\sinh^7(c+dx)}{80d} \\ &+ \frac{b^3\cosh(c+dx)\sinh^9(c+dx)}{10d}\end{aligned}$$

output
```
9/8*a^2*b*x-63/256*b^3*x+a^3*cosh(d*x+c)/d-3*a*b^2*cosh(d*x+c)/d+3*a*b^2*c
osh(d*x+c)^3/d-9/5*a*b^2*cosh(d*x+c)^5/d+3/7*a*b^2*cosh(d*x+c)^7/d-9/8*a^2
*b*cosh(d*x+c)*sinh(d*x+c)/d+63/256*b^3*cosh(d*x+c)*sinh(d*x+c)/d+3/4*a^2*
b*cosh(d*x+c)*sinh(d*x+c)^3/d-21/128*b^3*cosh(d*x+c)*sinh(d*x+c)^3/d+21/16
0*b^3*cosh(d*x+c)*sinh(d*x+c)^5/d-9/80*b^3*cosh(d*x+c)*sinh(d*x+c)^7/d+1/1
0*b^3*cosh(d*x+c)*sinh(d*x+c)^9/d
```

Mathematica [A] (verified)

Time = 4.43 (sec) , antiderivative size = 184, normalized size of antiderivative = 0.69

$$\int \sinh(c+dx)\left(a+b\sinh^3(c+dx)\right)^3 dx$$
$$= \frac{1120a(64a^2-105b^2)\cosh(c+dx)+b(80640a^2c-17640b^2c+80640a^2dx-17640b^2dx+23520ab\cosh(}{}$$

input
```
Integrate[Sinh[c + d*x]*(a + b*Sinh[c + d*x]^3)^3,x]
```

output
```
(1120*a*(64*a^2 - 105*b^2)*Cosh[c + d*x] + b*(80640*a^2*c - 17640*b^2*c +
80640*a^2*d*x - 17640*b^2*d*x + 23520*a*b*Cosh[3*(c + d*x)] - 4704*a*b*Cos
h[5*(c + d*x)] + 480*a*b*Cosh[7*(c + d*x)] - 53760*a^2*Sinh[2*(c + d*x)] +
 14700*b^2*Sinh[2*(c + d*x)] + 6720*a^2*Sinh[4*(c + d*x)] - 4200*b^2*Sinh[
4*(c + d*x)] + 1050*b^2*Sinh[6*(c + d*x)] - 175*b^2*Sinh[8*(c + d*x)] + 14
*b^2*Sinh[10*(c + d*x)]))/(71680*d)
```

Rubi [C] (verified)

Result contains complex when optimal does not.

Time = 0.54 (sec) , antiderivative size = 300, normalized size of antiderivative = 1.12, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.190$, Rules used = {3042, 26, 3699, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sinh(c+dx)\left(a+b\sinh^3(c+dx)\right)^3\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int -i\sin(ic+idx)\left(a+ib\sin(ic+idx)^3\right)^3dx$$

$$\Big\downarrow \text{26}$$

$$-i\int \sin(ic+idx)\left(ib\sin(ic+idx)^3+a\right)^3dx$$

$$\Big\downarrow \text{3699}$$

$$-i\int \left(ib^3\sinh^{10}(c+dx)+3iab^2\sinh^7(c+dx)+3ia^2b\sinh^4(c+dx)+ia^3\sinh(c+dx)\right)dx$$

$$\Big\downarrow \text{2009}$$

$$-i\left(\frac{ia^3\cosh(c+dx)}{d}+\frac{3ia^2b\sinh^3(c+dx)\cosh(c+dx)}{4d}-\frac{9ia^2b\sinh(c+dx)\cosh(c+dx)}{8d}+\frac{9}{8}ia^2bx+\frac{3iab^2\,\text{co}}{}\right.$$

input
```
Int[Sinh[c + d*x]*(a + b*Sinh[c + d*x]^3)^3,x]
```

output
```
(-I)*(((9*I)/8)*a^2*b*x - ((63*I)/256)*b^3*x + (I*a^3*Cosh[c + d*x])/d - (
(3*I)*a*b^2*Cosh[c + d*x])/d + ((3*I)*a*b^2*Cosh[c + d*x]^3)/d - (((9*I)/5
)*a*b^2*Cosh[c + d*x]^5)/d + (((3*I)/7)*a*b^2*Cosh[c + d*x]^7)/d - (((9*I)
/8)*a^2*b*Cosh[c + d*x]*Sinh[c + d*x])/d + (((63*I)/256)*b^3*Cosh[c + d*x]
*Sinh[c + d*x])/d + (((3*I)/4)*a^2*b*Cosh[c + d*x]*Sinh[c + d*x]^3)/d - ((
(21*I)/128)*b^3*Cosh[c + d*x]*Sinh[c + d*x]^3)/d + (((21*I)/160)*b^3*Cosh[
c + d*x]*Sinh[c + d*x]^5)/d - (((9*I)/80)*b^3*Cosh[c + d*x]*Sinh[c + d*x]^
7)/d + ((I/10)*b^3*Cosh[c + d*x]*Sinh[c + d*x]^9)/d)
```

Defintions of rubi rules used

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3699
```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^(n_
))^(p_.), x_Symbol] :> Int[ExpandTrig[sin[e + f*x]^m*(a + b*sin[e + f*x]^n)
^p, x], x] /; FreeQ[{a, b, e, f}, x] && IntegersQ[m, p] && (EqQ[n, 4] || Gt
Q[p, 0] || (EqQ[p, -1] && IntegerQ[n]))
```

Maple [A] (verified)

Time = 0.28 (sec) , antiderivative size = 168, normalized size of antiderivative = 0.63

$$a^3 \cosh(dx+c) + 3a^2 b\left(\left(\frac{\sinh(dx+c)^3}{4} - \frac{3\sinh(dx+c)}{8}\right)\cosh(dx+c) + \frac{3dx}{8} + \frac{3c}{8}\right) + 3b^2 a\left(-\frac{16}{35} + \frac{\sinh(dx+c)^6}{7} - 6\cdots\right.$$

input
```
int(sinh(d*x+c)*(a+b*sinh(d*x+c)^3)^3,x)
```

output
```
1/d*(a^3*cosh(d*x+c)+3*a^2*b*((1/4*sinh(d*x+c)^3-3/8*sinh(d*x+c))*cosh(d*x
+c)+3/8*d*x+3/8*c)+3*b^2*a*(-16/35+1/7*sinh(d*x+c)^6-6/35*sinh(d*x+c)^4+8/
35*sinh(d*x+c)^2)*cosh(d*x+c)+b^3*((1/10*sinh(d*x+c)^9-9/80*sinh(d*x+c)^7+
21/160*sinh(d*x+c)^5-21/128*sinh(d*x+c)^3+63/256*sinh(d*x+c))*cosh(d*x+c)-
63/256*d*x-63/256*c))
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 453, normalized size of antiderivative = 1.70

$$\int \sinh(c+dx)\left(a+b\sinh^3(c+dx)\right)^3 dx = \frac{35\, b^3 \cosh(dx+c)\sinh(dx+c)^9 + 120\, ab^2 \cosh(dx+c)^7 + 840\, ab^2 \cosh(dx+c)\sinh(dx+c)^6 - 1176\cdots}{\cdots}$$

input
```
integrate(sinh(d*x+c)*(a+b*sinh(d*x+c)^3)^3,x, algorithm="fricas")
```

output
```
1/17920*(35*b^3*cosh(d*x + c)*sinh(d*x + c)^9 + 120*a*b^2*cosh(d*x + c)^7
+ 840*a*b^2*cosh(d*x + c)*sinh(d*x + c)^6 - 1176*a*b^2*cosh(d*x + c)^5 + 7
0*(6*b^3*cosh(d*x + c)^3 - 5*b^3*cosh(d*x + c))*sinh(d*x + c)^7 + 5880*a*b
^2*cosh(d*x + c)^3 + 7*(126*b^3*cosh(d*x + c)^5 - 350*b^3*cosh(d*x + c)^3
+ 225*b^3*cosh(d*x + c))*sinh(d*x + c)^5 + 840*(5*a*b^2*cosh(d*x + c)^3 -
7*a*b^2*cosh(d*x + c))*sinh(d*x + c)^4 + 70*(6*b^3*cosh(d*x + c)^7 - 35*b^
3*cosh(d*x + c)^5 + 75*b^3*cosh(d*x + c)^3 + 12*(8*a^2*b - 5*b^3)*cosh(d*x
 + c))*sinh(d*x + c)^3 + 630*(32*a^2*b - 7*b^3)*d*x + 840*(3*a*b^2*cosh(d*
x + c)^5 - 14*a*b^2*cosh(d*x + c)^3 + 21*a*b^2*cosh(d*x + c))*sinh(d*x + c
)^2 + 280*(64*a^3 - 105*a*b^2)*cosh(d*x + c) + 35*(b^3*cosh(d*x + c)^9 - 1
0*b^3*cosh(d*x + c)^7 + 45*b^3*cosh(d*x + c)^5 + 24*(8*a^2*b - 5*b^3)*cosh
(d*x + c)^3 - 6*(128*a^2*b - 35*b^3)*cosh(d*x + c))*sinh(d*x + c))/d
```

Sympy [A] (verification not implemented)

Time = 1.33 (sec) , antiderivative size = 496, normalized size of antiderivative = 1.86

$$\int \sinh(c+dx)\left(a+b\sinh^3(c+dx)\right)^3\,dx$$

$$= \begin{cases} \frac{a^3\cosh(c+dx)}{d} + \frac{9a^2bx\sinh^4(c+dx)}{8} - \frac{9a^2bx\sinh^2(c+dx)\cosh^2(c+dx)}{4} + \frac{9a^2bx\cosh^4(c+dx)}{8} + \frac{15a^2b\sinh^3(c+dx)\cosh(c+dx)}{8d} - \\ x\left(a+b\sinh^3(c)\right)^3\sinh(c) \end{cases}$$

input
```
integrate(sinh(d*x+c)*(a+b*sinh(d*x+c)**3)**3,x)
```

output
```
Piecewise((a**3*cosh(c + d*x)/d + 9*a**2*b*x*sinh(c + d*x)**4/8 - 9*a**2*b
*x*sinh(c + d*x)**2*cosh(c + d*x)**2/4 + 9*a**2*b*x*cosh(c + d*x)**4/8 + 1
5*a**2*b*sinh(c + d*x)**3*cosh(c + d*x)/(8*d) - 9*a**2*b*sinh(c + d*x)*cos
h(c + d*x)**3/(8*d) + 3*a*b**2*sinh(c + d*x)**6*cosh(c + d*x)/d - 6*a*b**2
*sinh(c + d*x)**4*cosh(c + d*x)**3/d + 24*a*b**2*sinh(c + d*x)**2*cosh(c +
 d*x)**5/(5*d) - 48*a*b**2*cosh(c + d*x)**7/(35*d) + 63*b**3*x*sinh(c + d*
x)**10/256 - 315*b**3*x*sinh(c + d*x)**8*cosh(c + d*x)**2/256 + 315*b**3*x
*sinh(c + d*x)**6*cosh(c + d*x)**4/128 - 315*b**3*x*sinh(c + d*x)**4*cosh(
c + d*x)**6/128 + 315*b**3*x*sinh(c + d*x)**2*cosh(c + d*x)**8/256 - 63*b*
*3*x*cosh(c + d*x)**10/256 + 193*b**3*sinh(c + d*x)**9*cosh(c + d*x)/(256*
d) - 237*b**3*sinh(c + d*x)**7*cosh(c + d*x)**3/(128*d) + 21*b**3*sinh(c +
 d*x)**5*cosh(c + d*x)**5/(10*d) - 147*b**3*sinh(c + d*x)**3*cosh(c + d*x)
**7/(128*d) + 63*b**3*sinh(c + d*x)*cosh(c + d*x)**9/(256*d), Ne(d, 0)), (
x*(a + b*sinh(c)**3)**3*sinh(c), True))
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 318, normalized size of antiderivative = 1.19

$$\int \sinh(c+dx)\left(a+b\sinh^3(c+dx)\right)^3\,dx$$
$$= \frac{3}{64}a^2b\left(24x + \frac{e^{(4dx+4c)}}{d} - \frac{8e^{(2dx+2c)}}{d} + \frac{8e^{(-2dx-2c)}}{d} - \frac{e^{(-4dx-4c)}}{d}\right)$$
$$- \frac{1}{20480}b^3\left(\frac{\left(25e^{(-2dx-2c)} - 150e^{(-4dx-4c)} + 600e^{(-6dx-6c)} - 2100e^{(-8dx-8c)} - 2\right)e^{(10dx+10c)}}{d} + \frac{5040(\cdots}{\cdots}\right.$$
$$- \frac{3}{4480}ab^2\left(\frac{\left(49e^{(-2dx-2c)} - 245e^{(-4dx-4c)} + 1225e^{(-6dx-6c)} - 5\right)e^{(7dx+7c)}}{d} + \frac{1225e^{(-dx-c)} - 245e^{(-3\cdots}}{\cdots}\right.$$
$$+ \frac{a^3\cosh(dx+c)}{d}$$

input
```
integrate(sinh(d*x+c)*(a+b*sinh(d*x+c)^3)^3,x, algorithm="maxima")
```

output
```
3/64*a^2*b*(24*x + e^(4*d*x + 4*c)/d - 8*e^(2*d*x + 2*c)/d + 8*e^(-2*d*x -
 2*c)/d - e^(-4*d*x - 4*c)/d) - 1/20480*b^3*((25*e^(-2*d*x - 2*c) - 150*e^
(-4*d*x - 4*c) + 600*e^(-6*d*x - 6*c) - 2100*e^(-8*d*x - 8*c) - 2)*e^(10*d
*x + 10*c)/d + 5040*(d*x + c)/d + (2100*e^(-2*d*x - 2*c) - 600*e^(-4*d*x -
 4*c) + 150*e^(-6*d*x - 6*c) - 25*e^(-8*d*x - 8*c) + 2*e^(-10*d*x - 10*c))
/d) - 3/4480*a*b^2*((49*e^(-2*d*x - 2*c) - 245*e^(-4*d*x - 4*c) + 1225*e^(
-6*d*x - 6*c) - 5)*e^(7*d*x + 7*c)/d + (1225*e^(-d*x - c) - 245*e^(-3*d*x
- 3*c) + 49*e^(-5*d*x - 5*c) - 5*e^(-7*d*x - 7*c))/d) + a^3*cosh(d*x + c)/
d
```

Giac [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 379, normalized size of antiderivative = 1.42

$$\int \sinh(c+dx)\left(a+b\sinh^3(c+dx)\right)^3\,dx$$
$$= \frac{b^3e^{(10\,dx+10\,c)}}{10240\,d} - \frac{5\,b^3e^{(8\,dx+8\,c)}}{4096\,d} + \frac{3\,ab^2e^{(7\,dx+7\,c)}}{896\,d} + \frac{15\,b^3e^{(6\,dx+6\,c)}}{2048\,d} - \frac{21\,ab^2e^{(5\,dx+5\,c)}}{640\,d} + \frac{21\,ab^2e^{(3\,dx+3\,c)}}{128\,d} + \frac{21\,ab^2e^{(-3\,dx-3\,c)}}{128\,d} - \frac{21\,ab^2e^{(-5\,dx-5\,c)}}{640\,d} - \frac{15\,b^3e^{(-6\,dx-6\,c)}}{2048\,d} + \frac{3\,ab^2e^{(-7\,dx-7\,c)}}{896\,d} + \frac{5\,b^3e^{(-8\,dx-8\,c)}}{4096\,d} - \frac{b^3e^{(-10\,dx-10\,c)}}{10240\,d} + \frac{9}{256}\left(32\,a^2b-7\,b^3\right)x + \frac{3\,(8\,a^2b-5\,b^3)e^{(4\,dx+4\,c)}}{512\,d} - \frac{3\,(128\,a^2b-35\,b^3)e^{(2\,dx+2\,c)}}{1024\,d} + \frac{(64\,a^3-105\,ab^2)e^{(dx+c)}}{128\,d} + \frac{(64\,a^3-105\,ab^2)e^{(-dx-c)}}{128\,d} + \frac{3\,(128\,a^2b-35\,b^3)e^{(-2\,dx-2\,c)}}{1024\,d} - \frac{3\,(8\,a^2b-5\,b^3)e^{(-4\,dx-4\,c)}}{512\,d}$$

input
```
integrate(sinh(d*x+c)*(a+b*sinh(d*x+c)^3)^3,x, algorithm="giac")
```

output
```
1/10240*b^3*e^(10*d*x + 10*c)/d - 5/4096*b^3*e^(8*d*x + 8*c)/d + 3/896*a*b
^2*e^(7*d*x + 7*c)/d + 15/2048*b^3*e^(6*d*x + 6*c)/d - 21/640*a*b^2*e^(5*d
*x + 5*c)/d + 21/128*a*b^2*e^(3*d*x + 3*c)/d + 21/128*a*b^2*e^(-3*d*x - 3*
c)/d - 21/640*a*b^2*e^(-5*d*x - 5*c)/d - 15/2048*b^3*e^(-6*d*x - 6*c)/d +
3/896*a*b^2*e^(-7*d*x - 7*c)/d + 5/4096*b^3*e^(-8*d*x - 8*c)/d - 1/10240*b
^3*e^(-10*d*x - 10*c)/d + 9/256*(32*a^2*b - 7*b^3)*x + 3/512*(8*a^2*b - 5*
b^3)*e^(4*d*x + 4*c)/d - 3/1024*(128*a^2*b - 35*b^3)*e^(2*d*x + 2*c)/d + 1
/128*(64*a^3 - 105*a*b^2)*e^(d*x + c)/d + 1/128*(64*a^3 - 105*a*b^2)*e^(-d
*x - c)/d + 3/1024*(128*a^2*b - 35*b^3)*e^(-2*d*x - 2*c)/d - 3/512*(8*a^2*
b - 5*b^3)*e^(-4*d*x - 4*c)/d
```

Mupad [B] (verification not implemented)

Time = 3.92 (sec) , antiderivative size = 189, normalized size of antiderivative = 0.71

$$\int \sinh(c+dx)\left(a+b\sinh^3(c+dx)\right)^3\,dx$$
$$= \frac{8960\,a^3\cosh(c+d\,x)+\frac{3675\,b^3\sinh(2\,c+2\,d\,x)}{2}-525\,b^3\sinh(4\,c+4\,d\,x)+\frac{525\,b^3\sinh(6\,c+6\,d\,x)}{4}-\frac{175\,b^3\sinh(8\,c+8\,d\,x)}{8}}{}$$

input
```
int(sinh(c + d*x)*(a + b*sinh(c + d*x)^3)^3,x)
```

output
```
(8960*a^3*cosh(c + d*x) + (3675*b^3*sinh(2*c + 2*d*x))/2 - 525*b^3*sinh(4*
c + 4*d*x) + (525*b^3*sinh(6*c + 6*d*x))/4 - (175*b^3*sinh(8*c + 8*d*x))/8
 + (7*b^3*sinh(10*c + 10*d*x))/4 + 2940*a*b^2*cosh(3*c + 3*d*x) - 588*a*b^
2*cosh(5*c + 5*d*x) + 60*a*b^2*cosh(7*c + 7*d*x) - 6720*a^2*b*sinh(2*c + 2
*d*x) + 840*a^2*b*sinh(4*c + 4*d*x) - 14700*a*b^2*cosh(c + d*x) - 2205*b^3
*d*x + 10080*a^2*b*d*x)/(8960*d)
```

Reduce [B] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 415, normalized size of antiderivative = 1.55

$$\int \sinh(c+dx)\left(a+b\sinh^3(c+dx)\right)^3 dx$$
$$= \frac{-14b^3 + 71680e^{9dx+9c}a^3 + 161280e^{10dx+10c}a^2bdx - 4704e^{5dx+5c}a\,b^2 + 480e^{3dx+3c}a\,b^2 + 23520e^{7dx+7c}a\,b^2 -}{}$$

input
```
int(sinh(d*x+c)*(a+b*sinh(d*x+c)^3)^3,x)
```

output
```
(14*e**(20*c + 20*d*x)*b**3 - 175*e**(18*c + 18*d*x)*b**3 + 480*e**(17*c +
 17*d*x)*a*b**2 + 1050*e**(16*c + 16*d*x)*b**3 - 4704*e**(15*c + 15*d*x)*a
*b**2 + 6720*e**(14*c + 14*d*x)*a**2*b - 4200*e**(14*c + 14*d*x)*b**3 + 23
520*e**(13*c + 13*d*x)*a*b**2 - 53760*e**(12*c + 12*d*x)*a**2*b + 14700*e*
*(12*c + 12*d*x)*b**3 + 71680*e**(11*c + 11*d*x)*a**3 - 117600*e**(11*c +
11*d*x)*a*b**2 + 161280*e**(10*c + 10*d*x)*a**2*b*d*x - 35280*e**(10*c + 1
0*d*x)*b**3*d*x + 71680*e**(9*c + 9*d*x)*a**3 - 117600*e**(9*c + 9*d*x)*a*
b**2 + 53760*e**(8*c + 8*d*x)*a**2*b - 14700*e**(8*c + 8*d*x)*b**3 + 23520
*e**(7*c + 7*d*x)*a*b**2 - 6720*e**(6*c + 6*d*x)*a**2*b + 4200*e**(6*c + 6
*d*x)*b**3 - 4704*e**(5*c + 5*d*x)*a*b**2 - 1050*e**(4*c + 4*d*x)*b**3 + 4
80*e**(3*c + 3*d*x)*a*b**2 + 175*e**(2*c + 2*d*x)*b**3 - 14*b**3)/(143360*
e**(10*c + 10*d*x)*d)
```

3.140 $\int \left(a+b\sinh^3(c+dx)\right)^3\,dx$

Optimal result

Integrand size = 14, antiderivative size = 204

$$\int \left(a+b\sinh^3(c+dx)\right)^3\,dx = a^3x-\frac{15}{16}ab^2x-\frac{3a^2b\cosh(c+dx)}{d}+\frac{b^3\cosh(c+dx)}{d}+\frac{a^2b\cosh^3(c+dx)}{d}-\frac{4b^3\cosh^3(c+dx)}{3d}+\frac{6b^3\cosh^5(c+dx)}{5d}-\frac{4b^3\cosh^7(c+dx)}{7d}+\frac{b^3\cosh^9(c+dx)}{9d}+\frac{15ab^2\cosh(c+dx)\sinh(c+dx)}{16d}-\frac{5ab^2\cosh(c+dx)\sinh^3(c+dx)}{8d}+\frac{ab^2\cosh(c+dx)\sinh^5(c+dx)}{2d}$$

output
```
a^3*x-15/16*a*b^2*x-3*a^2*b*cosh(d*x+c)/d+b^3*cosh(d*x+c)/d+a^2*b*cosh(d*x
+c)^3/d-4/3*b^3*cosh(d*x+c)^3/d+6/5*b^3*cosh(d*x+c)^5/d-4/7*b^3*cosh(d*x+c
)^7/d+1/9*b^3*cosh(d*x+c)^9/d+15/16*a*b^2*cosh(d*x+c)*sinh(d*x+c)/d-5/8*a*
b^2*cosh(d*x+c)*sinh(d*x+c)^3/d+1/2*a*b^2*cosh(d*x+c)*sinh(d*x+c)^5/d
```

Mathematica [A] (verified)

Time = 0.40 (sec) , antiderivative size = 159, normalized size of antiderivative = 0.78

$$\int \left(a+b\sinh^3(c+dx)\right)^3\,dx$$
$$= \frac{80640a^3c-75600ab^2c+80640a^3dx-75600ab^2dx+5670b(-32a^2+7b^2)\cosh(c+dx)+1260(16a^2b-}{}$$

input
```
Integrate[(a + b*Sinh[c + d*x]^3)^3,x]
```

output
```
(80640*a^3*c - 75600*a*b^2*c + 80640*a^3*d*x - 75600*a*b^2*d*x + 5670*b*(-
32*a^2 + 7*b^2)*Cosh[c + d*x] + 1260*(16*a^2*b - 7*b^3)*Cosh[3*(c + d*x)]
+ 2268*b^3*Cosh[5*(c + d*x)] - 405*b^3*Cosh[7*(c + d*x)] + 35*b^3*Cosh[9*(
c + d*x)] + 56700*a*b^2*Sinh[2*(c + d*x)] - 11340*a*b^2*Sinh[4*(c + d*x)]
+ 1260*a*b^2*Sinh[6*(c + d*x)])/(80640*d)
```

Rubi [A] (verified)

Time = 0.41 (sec) , antiderivative size = 204, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.214$, Rules used = {3042, 3692, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(a+b\sinh^3(c+dx)\right)^3\,dx$$
$$\Big\downarrow \; 3042$$
$$\int \left(a+ib\sin(ic+idx)^3\right)^3dx$$
$$\Big\downarrow \; 3692$$
$$\int \left(a^3+3a^2b\sinh^3(c+dx)+3ab^2\sinh^6(c+dx)+b^3\sinh^9(c+dx)\right)dx$$
$$\Big\downarrow \; 2009$$

$$a^3x+\frac{a^2b\cosh^3(c+dx)}{d}-\frac{3a^2b\cosh(c+dx)}{d}+\frac{ab^2\sinh^5(c+dx)\cosh(c+dx)}{2d}-\frac{5ab^2\sinh^3(c+dx)\cosh(c+dx)}{8d}+\frac{15ab^2\sinh(c+dx)\cosh(c+dx)}{16d}-\frac{15}{16}ab^2x+\frac{b^3\cosh^9(c+dx)}{9d}-\frac{4b^3\cosh^7(c+dx)}{7d}+\frac{6b^3\cosh^5(c+dx)}{5d}-\frac{4b^3\cosh^3(c+dx)}{3d}+\frac{b^3\cosh(c+dx)}{d}$$

input
```
Int[(a + b*Sinh[c + d*x]^3)^3,x]
```

output
```
a^3*x - (15*a*b^2*x)/16 - (3*a^2*b*Cosh[c + d*x])/d + (b^3*Cosh[c + d*x])/
d + (a^2*b*Cosh[c + d*x]^3)/d - (4*b^3*Cosh[c + d*x]^3)/(3*d) + (6*b^3*Cos
h[c + d*x]^5)/(5*d) - (4*b^3*Cosh[c + d*x]^7)/(7*d) + (b^3*Cosh[c + d*x]^9
)/(9*d) + (15*a*b^2*Cosh[c + d*x]*Sinh[c + d*x])/(16*d) - (5*a*b^2*Cosh[c
+ d*x]*Sinh[c + d*x]^3)/(8*d) + (a*b^2*Cosh[c + d*x]*Sinh[c + d*x]^5)/(2*d
)
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3692
```
Int[((a_) + (b_.)*((c_.)*sin[(e_.) + (f_.)*(x_)])^(n_))^(p_), x_Symbol] :>
Int[ExpandTrig[(a + b*(c*sin[e + f*x])^n)^p, x], x] /; FreeQ[{a, b, c, e, f
, n}, x] && (IGtQ[p, 0] || (EqQ[p, -1] && IntegerQ[n]))
```

Maple [A] (verified)

Time = 0.28 (sec) , antiderivative size = 141, normalized size of antiderivative = 0.69

$$\frac{b^3\left(\frac{128}{315}+\frac{\sinh(dx+c)^8}{9}-\frac{8\sinh(dx+c)^6}{63}+\frac{16\sinh(dx+c)^4}{105}-\frac{64\sinh(dx+c)^2}{315}\right)\cosh\left(dx+c\right)+3b^2a\left(\left(\frac{\sinh(dx+c)^5}{6}-\frac{5\sinh(dx}{24}\right.\right.}{d}$$

input
```
int((a+b*sinh(d*x+c)^3)^3,x)
```

output
```
1/d*(b^3*(128/315+1/9*sinh(d*x+c)^8-8/63*sinh(d*x+c)^6+16/105*sinh(d*x+c)^
4-64/315*sinh(d*x+c)^2)*cosh(d*x+c)+3*b^2*a*((1/6*sinh(d*x+c)^5-5/24*sinh(
d*x+c)^3+5/16*sinh(d*x+c))*cosh(d*x+c)-5/16*d*x-5/16*c)+3*a^2*b*(-2/3+1/3*
sinh(d*x+c)^2)*cosh(d*x+c)+a^3*(d*x+c))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 380 vs. 2(188) = 376.

Time = 0.09 (sec) , antiderivative size = 380, normalized size of antiderivative = 1.86

$$\int \left(a+b\sinh^3(c+dx)\right)^3\,dx$$
$$=\frac{35\,b^3\cosh\left(dx+c\right)^9+315\,b^3\cosh\left(dx+c\right)\sinh\left(dx+c\right)^8-405\,b^3\cosh\left(dx+c\right)^7+7560\,ab^2\cosh\left(dx+c\right)}{}$$

input
```
integrate((a+b*sinh(d*x+c)^3)^3,x, algorithm="fricas")
```

output
```
1/80640*(35*b^3*cosh(d*x + c)^9 + 315*b^3*cosh(d*x + c)*sinh(d*x + c)^8 -
405*b^3*cosh(d*x + c)^7 + 7560*a*b^2*cosh(d*x + c)*sinh(d*x + c)^5 + 2268*
b^3*cosh(d*x + c)^5 + 105*(28*b^3*cosh(d*x + c)^3 - 27*b^3*cosh(d*x + c))*
sinh(d*x + c)^6 + 315*(14*b^3*cosh(d*x + c)^5 - 45*b^3*cosh(d*x + c)^3 + 3
6*b^3*cosh(d*x + c))*sinh(d*x + c)^4 + 1260*(16*a^2*b - 7*b^3)*cosh(d*x +
c)^3 + 5040*(5*a*b^2*cosh(d*x + c)^3 - 9*a*b^2*cosh(d*x + c))*sinh(d*x + c
)^3 + 5040*(16*a^3 - 15*a*b^2)*d*x + 315*(4*b^3*cosh(d*x + c)^7 - 27*b^3*c
osh(d*x + c)^5 + 72*b^3*cosh(d*x + c)^3 + 12*(16*a^2*b - 7*b^3)*cosh(d*x +
 c))*sinh(d*x + c)^2 - 5670*(32*a^2*b - 7*b^3)*cosh(d*x + c) + 7560*(a*b^2
*cosh(d*x + c)^5 - 6*a*b^2*cosh(d*x + c)^3 + 15*a*b^2*cosh(d*x + c))*sinh(
d*x + c))/d
```

Sympy [A] (verification not implemented)

Time = 0.97 (sec) , antiderivative size = 340, normalized size of antiderivative = 1.67

$$\int \left(a+b\sinh^3(c+dx)\right)^3\,dx$$
$$=\begin{cases} a^3x+\frac{3a^2b\sinh^2(c+dx)\cosh(c+dx)}{d}-\frac{2a^2b\cosh^3(c+dx)}{d}+\frac{15ab^2x\sinh^6(c+dx)}{16}-\frac{45ab^2x\sinh^4(c+dx)\cosh^2(c+dx)}{16}+\frac{45ab^2x\,\mathrm{si}}{} \\ x\left(a+b\sinh^3(c)\right)^3 \end{cases}$$

input
```
integrate((a+b*sinh(d*x+c)**3)**3,x)
```

output
```
Piecewise((a**3*x + 3*a**2*b*sinh(c + d*x)**2*cosh(c + d*x)/d - 2*a**2*b*c
osh(c + d*x)**3/d + 15*a*b**2*x*sinh(c + d*x)**6/16 - 45*a*b**2*x*sinh(c +
 d*x)**4*cosh(c + d*x)**2/16 + 45*a*b**2*x*sinh(c + d*x)**2*cosh(c + d*x)*
*4/16 - 15*a*b**2*x*cosh(c + d*x)**6/16 + 33*a*b**2*sinh(c + d*x)**5*cosh(
c + d*x)/(16*d) - 5*a*b**2*sinh(c + d*x)**3*cosh(c + d*x)**3/(2*d) + 15*a*
b**2*sinh(c + d*x)*cosh(c + d*x)**5/(16*d) + b**3*sinh(c + d*x)**8*cosh(c
+ d*x)/d - 8*b**3*sinh(c + d*x)**6*cosh(c + d*x)**3/(3*d) + 16*b**3*sinh(c
 + d*x)**4*cosh(c + d*x)**5/(5*d) - 64*b**3*sinh(c + d*x)**2*cosh(c + d*x)
**7/(35*d) + 128*b**3*cosh(c + d*x)**9/(315*d), Ne(d, 0)), (x*(a + b*sinh(
c)**3)**3, True))
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 280, normalized size of antiderivative = 1.37

$$\int \left(a+b\sinh^3(c+dx)\right)^3\,dx=a^3x$$
$$-\frac{1}{161280}b^3\left(\frac{\left(405\,e^{(-2\,dx-2\,c)}-2268\,e^{(-4\,dx-4\,c)}+8820\,e^{(-6\,dx-6\,c)}-39690\,e^{(-8\,dx-8\,c)}-35\right)e^{(9\,dx+9\,c)}}{d}-\right.$$
$$-\frac{1}{128}ab^2\left(\frac{\left(9\,e^{(-2\,dx-2\,c)}-45\,e^{(-4\,dx-4\,c)}-1\right)e^{(6\,dx+6\,c)}}{d}+\frac{120\,(dx+c)}{d}+\frac{45\,e^{(-2\,dx-2\,c)}-9\,e^{(-4\,dx-4\,c)}+}{d}\right.$$
$$+\frac{1}{8}a^2b\left(\frac{e^{(3\,dx+3\,c)}}{d}-\frac{9\,e^{(dx+c)}}{d}-\frac{9\,e^{(-dx-c)}}{d}+\frac{e^{(-3\,dx-3\,c)}}{d}\right)$$

input
```
integrate((a+b*sinh(d*x+c)^3)^3,x, algorithm="maxima")
```

output
```
a^3*x - 1/161280*b^3*((405*e^(-2*d*x - 2*c) - 2268*e^(-4*d*x - 4*c) + 8820
*e^(-6*d*x - 6*c) - 39690*e^(-8*d*x - 8*c) - 35)*e^(9*d*x + 9*c)/d - (3969
0*e^(-d*x - c) - 8820*e^(-3*d*x - 3*c) + 2268*e^(-5*d*x - 5*c) - 405*e^(-7
*d*x - 7*c) + 35*e^(-9*d*x - 9*c))/d) - 1/128*a*b^2*((9*e^(-2*d*x - 2*c) -
 45*e^(-4*d*x - 4*c) - 1)*e^(6*d*x + 6*c)/d + 120*(d*x + c)/d + (45*e^(-2*
d*x - 2*c) - 9*e^(-4*d*x - 4*c) + e^(-6*d*x - 6*c))/d) + 1/8*a^2*b*(e^(3*d
*x + 3*c)/d - 9*e^(d*x + c)/d - 9*e^(-d*x - c)/d + e^(-3*d*x - 3*c)/d)
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 327, normalized size of antiderivative = 1.60

$$\int \left(a+b\sinh^3(c+dx)\right)^3\,dx=\frac{b^3e^{(9\,dx+9\,c)}}{4608\,d}-\frac{9\,b^3e^{(7\,dx+7\,c)}}{3584\,d}+\frac{ab^2e^{(6\,dx+6\,c)}}{128\,d}+\frac{9\,b^3e^{(5\,dx+5\,c)}}{640\,d}$$
$$-\frac{9\,ab^2e^{(4\,dx+4\,c)}}{128\,d}+\frac{45\,ab^2e^{(2\,dx+2\,c)}}{128\,d}-\frac{45\,ab^2e^{(-2\,dx-2\,c)}}{128\,d}$$
$$+\frac{9\,ab^2e^{(-4\,dx-4\,c)}}{128\,d}+\frac{9\,b^3e^{(-5\,dx-5\,c)}}{640\,d}-\frac{ab^2e^{(-6\,dx-6\,c)}}{128\,d}$$
$$-\frac{9\,b^3e^{(-7\,dx-7\,c)}}{3584\,d}+\frac{b^3e^{(-9\,dx-9\,c)}}{4608\,d}+\frac{1}{16}\left(16\,a^3-15\,ab^2\right)x$$
$$+\frac{\left(16\,a^2b-7\,b^3\right)e^{(3\,dx+3\,c)}}{128\,d}-\frac{9\left(32\,a^2b-7\,b^3\right)e^{(dx+c)}}{256\,d}$$
$$-\frac{9\left(32\,a^2b-7\,b^3\right)e^{(-dx-c)}}{256\,d}+\frac{\left(16\,a^2b-7\,b^3\right)e^{(-3\,dx-3\,c)}}{128\,d}$$

input
```
integrate((a+b*sinh(d*x+c)^3)^3,x, algorithm="giac")
```

output
```
1/4608*b^3*e^(9*d*x + 9*c)/d - 9/3584*b^3*e^(7*d*x + 7*c)/d + 1/128*a*b^2*
e^(6*d*x + 6*c)/d + 9/640*b^3*e^(5*d*x + 5*c)/d - 9/128*a*b^2*e^(4*d*x + 4
*c)/d + 45/128*a*b^2*e^(2*d*x + 2*c)/d - 45/128*a*b^2*e^(-2*d*x - 2*c)/d +
 9/128*a*b^2*e^(-4*d*x - 4*c)/d + 9/640*b^3*e^(-5*d*x - 5*c)/d - 1/128*a*b
^2*e^(-6*d*x - 6*c)/d - 9/3584*b^3*e^(-7*d*x - 7*c)/d + 1/4608*b^3*e^(-9*d
*x - 9*c)/d + 1/16*(16*a^3 - 15*a*b^2)*x + 1/128*(16*a^2*b - 7*b^3)*e^(3*d
*x + 3*c)/d - 9/256*(32*a^2*b - 7*b^3)*e^(d*x + c)/d - 9/256*(32*a^2*b - 7
*b^3)*e^(-d*x - c)/d + 1/128*(16*a^2*b - 7*b^3)*e^(-3*d*x - 3*c)/d
```

Mupad [B] (verification not implemented)

Time = 1.86 (sec) , antiderivative size = 164, normalized size of antiderivative = 0.80

$$\int \left(a+b\sinh^3(c+dx)\right)^3\,dx = \frac{d\,x\,a^3+a^2\,b\cosh(c+d\,x)^3-3\,a^2\,b\cosh(c+d\,x)+\frac{\sinh(c+d\,x)\,a\,b^2\cosh(c+d\,x)^5}{2}-\frac{13\sinh(c+d\,x)\,a\,b^2\cosh(c+d\,x)^3}{8}+ \cdots}{}$$

input
```
int((a + b*sinh(c + d*x)^3)^3,x)
```

output
```
(b^3*cosh(c + d*x) - (4*b^3*cosh(c + d*x)^3)/3 + (6*b^3*cosh(c + d*x)^5)/5
 - (4*b^3*cosh(c + d*x)^7)/7 + (b^3*cosh(c + d*x)^9)/9 + a^2*b*cosh(c + d*
x)^3 - 3*a^2*b*cosh(c + d*x) + a^3*d*x + (33*a*b^2*cosh(c + d*x)*sinh(c +
d*x))/16 - (15*a*b^2*d*x)/16 - (13*a*b^2*cosh(c + d*x)^3*sinh(c + d*x))/8
+ (a*b^2*cosh(c + d*x)^5*sinh(c + d*x))/2)/d
```

Reduce [B] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 353, normalized size of antiderivative = 1.73

$$\int \left(a + b\sinh^3(c + dx)\right)^3 dx$$
$$= \frac{35e^{18dx+18c}b^3 - 405e^{16dx+16c}b^3 + 1260e^{15dx+15c}a\,b^2 + 2268e^{14dx+14c}b^3 - 11340e^{13dx+13c}a\,b^2 + 20160e^{12dx+}}{}$$

input
```
int((a+b*sinh(d*x+c)^3)^3,x)
```

output
```
(35*e**(18*c + 18*d*x)*b**3 - 405*e**(16*c + 16*d*x)*b**3 + 1260*e**(15*c
+ 15*d*x)*a*b**2 + 2268*e**(14*c + 14*d*x)*b**3 - 11340*e**(13*c + 13*d*x)
*a*b**2 + 20160*e**(12*c + 12*d*x)*a**2*b - 8820*e**(12*c + 12*d*x)*b**3 +
 56700*e**(11*c + 11*d*x)*a*b**2 - 181440*e**(10*c + 10*d*x)*a**2*b + 3969
0*e**(10*c + 10*d*x)*b**3 + 161280*e**(9*c + 9*d*x)*a**3*d*x - 151200*e**(
9*c + 9*d*x)*a*b**2*d*x - 181440*e**(8*c + 8*d*x)*a**2*b + 39690*e**(8*c +
 8*d*x)*b**3 - 56700*e**(7*c + 7*d*x)*a*b**2 + 20160*e**(6*c + 6*d*x)*a**2
*b - 8820*e**(6*c + 6*d*x)*b**3 + 11340*e**(5*c + 5*d*x)*a*b**2 + 2268*e**
(4*c + 4*d*x)*b**3 - 1260*e**(3*c + 3*d*x)*a*b**2 - 405*e**(2*c + 2*d*x)*b
**3 + 35*b**3)/(161280*e**(9*c + 9*d*x)*d)
```

3.141 $\int \operatorname{csch}(c+dx)\left(a+b\sinh^3(c+dx)\right)^3\,dx$

Optimal result

Integrand size = 21, antiderivative size = 201

$$\begin{aligned}
&\int \operatorname{csch}(c+dx)\left(a+b\sinh^3(c+dx)\right)^3\,dx\\
&=-\frac{3}{2}a^2bx+\frac{35b^3x}{128}-\frac{a^3\operatorname{arctanh}(\cosh(c+dx))}{d}+\frac{3ab^2\cosh(c+dx)}{d}\\
&\quad-\frac{2ab^2\cosh^3(c+dx)}{d}+\frac{3ab^2\cosh^5(c+dx)}{5d}+\frac{3a^2b\cosh(c+dx)\sinh(c+dx)}{2d}\\
&\quad-\frac{35b^3\cosh(c+dx)\sinh(c+dx)}{128d}+\frac{35b^3\cosh(c+dx)\sinh^3(c+dx)}{192d}\\
&\quad-\frac{7b^3\cosh(c+dx)\sinh^5(c+dx)}{48d}+\frac{b^3\cosh(c+dx)\sinh^7(c+dx)}{8d}
\end{aligned}$$

output
```
-3/2*a^2*b*x+35/128*b^3*x-a^3*arctanh(cosh(d*x+c))/d+3*a*b^2*cosh(d*x+c)/d
-2*a*b^2*cosh(d*x+c)^3/d+3/5*a*b^2*cosh(d*x+c)^5/d+3/2*a^2*b*cosh(d*x+c)*s
inh(d*x+c)/d-35/128*b^3*cosh(d*x+c)*sinh(d*x+c)/d+35/192*b^3*cosh(d*x+c)*s
inh(d*x+c)^3/d-7/48*b^3*cosh(d*x+c)*sinh(d*x+c)^5/d+1/8*b^3*cosh(d*x+c)*si
nh(d*x+c)^7/d
```

Mathematica [A] (verified)

Time = 5.49 (sec) , antiderivative size = 174, normalized size of antiderivative = 0.87

$$\int \operatorname{csch}(c+dx)\left(a+b\sinh^3(c+dx)\right)^3\,dx$$
$$=\frac{-23040a^2bc+4200b^3c-23040a^2bdx+4200b^3dx+28800ab^2\cosh(c+dx)-4800ab^2\cosh(3(c+dx))+}{}$$

input
```
Integrate[Csch[c + d*x]*(a + b*Sinh[c + d*x]^3)^3,x]
```

output
```
(-23040*a^2*b*c + 4200*b^3*c - 23040*a^2*b*d*x + 4200*b^3*d*x + 28800*a*b^
2*Cosh[c + d*x] - 4800*a*b^2*Cosh[3*(c + d*x)] + 576*a*b^2*Cosh[5*(c + d*x
)] - 15360*a^3*Log[Cosh[(c + d*x)/2]] + 15360*a^3*Log[Sinh[(c + d*x)/2]] +
 11520*a^2*b*Sinh[2*(c + d*x)] - 3360*b^3*Sinh[2*(c + d*x)] + 840*b^3*Sinh
[4*(c + d*x)] - 160*b^3*Sinh[6*(c + d*x)] + 15*b^3*Sinh[8*(c + d*x)])/(153
60*d)
```

Rubi [C] (verified)

Result contains complex when optimal does not.

Time = 0.50 (sec) , antiderivative size = 227, normalized size of antiderivative = 1.13, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.190, Rules used = {3042, 26, 3699, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \operatorname{csch}(c+dx)\left(a+b\sinh^3(c+dx)\right)^3\,dx$$

$$\Big\downarrow \ 3042$$

$$\int \frac{i\left(a+ib\sin(ic+idx)^3\right)^3}{\sin(ic+idx)}dx$$

$$\Big\downarrow \ 26$$

$$i\int \frac{\left(ib\sin(ic+idx)^3+a\right)^3}{\sin(ic+idx)}dx$$

$\Big\downarrow$ 3699

$$i\int \left(-ib^3\sinh^8(c+dx)-3iab^2\sinh^5(c+dx)-3ia^2b\sinh^2(c+dx)-ia^3\text{csch}(c+dx)\right)dx$$

$\Big\downarrow$ 2009

$$i\left(\frac{ia^3\text{arctanh}(\cosh(c+dx))}{d}-\frac{3ia^2b\sinh(c+dx)\cosh(c+dx)}{2d}+\frac{3}{2}ia^2bx-\frac{3iab^2\cosh^5(c+dx)}{5d}+\frac{2iab^2\cosh^3(c\right.$$

input
```
Int[Csch[c + d*x]*(a + b*Sinh[c + d*x]^3)^3,x]
```

output
```
I*(((3*I)/2)*a^2*b*x - ((35*I)/128)*b^3*x + (I*a^3*ArcTanh[Cosh[c + d*x]])
/d - ((3*I)*a*b^2*Cosh[c + d*x])/d + ((2*I)*a*b^2*Cosh[c + d*x]^3)/d - (((
3*I)/5)*a*b^2*Cosh[c + d*x]^5)/d - (((3*I)/2)*a^2*b*Cosh[c + d*x]*Sinh[c +
 d*x])/d + (((35*I)/128)*b^3*Cosh[c + d*x]*Sinh[c + d*x])/d - (((35*I)/192
)*b^3*Cosh[c + d*x]*Sinh[c + d*x]^3)/d + (((7*I)/48)*b^3*Cosh[c + d*x]*Sin
h[c + d*x]^5)/d - ((I/8)*b^3*Cosh[c + d*x]*Sinh[c + d*x]^7)/d)
```

Defintions of rubi rules used

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3699
```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^(n_
))^(p_.), x_Symbol] :> Int[ExpandTrig[sin[e + f*x]^m*(a + b*sin[e + f*x]^n)
^p, x], x] /; FreeQ[{a, b, e, f}, x] && IntegersQ[m, p] && (EqQ[n, 4] || Gt
Q[p, 0] || (EqQ[p, -1] && IntegerQ[n]))
```

Maple [A] (verified)

Time = 4.51 (sec) , antiderivative size = 138, normalized size of antiderivative = 0.69

method	result
derivativedivides	$\frac{-2a^3\operatorname{arctanh}(e^{dx+c})+3a^2b\left(\frac{\cosh(dx+c)\sinh(dx+c)}{2}-\frac{dx}{2}-\frac{c}{2}\right)+3b^2a\left(\frac{8}{15}+\frac{\sinh(dx+c)^4}{5}-\frac{4\sinh(dx+c)^2}{15}\right)\cosh(dx+c)+b^3}{d}$
default	$\frac{-2a^3\operatorname{arctanh}(e^{dx+c})+3a^2b\left(\frac{\cosh(dx+c)\sinh(dx+c)}{2}-\frac{dx}{2}-\frac{c}{2}\right)+3b^2a\left(\frac{8}{15}+\frac{\sinh(dx+c)^4}{5}-\frac{4\sinh(dx+c)^2}{15}\right)\cosh(dx+c)+b^3}{d}$
parallelrisch	$-23040a^2bdx+4200b^3dx+24576b^2a+15360a^3\ln\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-160b^3\sinh(6dx+6c)+840b^3\sinh(4dx+4c)+15b^3\mathrm{si}$
risch	$-\frac{3a^2bx}{2}+\frac{35b^3x}{128}+\frac{b^3e^{8dx+8c}}{2048d}-\frac{b^3e^{6dx+6c}}{192d}+\frac{3b^2e^{5dx+5c}a}{160d}+\frac{7e^{4dx+4c}b^3}{256d}-\frac{5e^{3dx+3c}b^2a}{32d}+\frac{3e^{2dx+2c}a^2b}{8d}$

input
```
int(csch(d*x+c)*(a+b*sinh(d*x+c)^3)^3,x,method=_RETURNVERBOSE)
```

output
```
1/d*(-2*a^3*arctanh(exp(d*x+c))+3*a^2*b*(1/2*cosh(d*x+c)*sinh(d*x+c)-1/2*d
*x-1/2*c)+3*b^2*a*(8/15+1/5*sinh(d*x+c)^4-4/15*sinh(d*x+c)^2)*cosh(d*x+c)+
b^3*((1/8*sinh(d*x+c)^7-7/48*sinh(d*x+c)^5+35/192*sinh(d*x+c)^3-35/128*sin
h(d*x+c))*cosh(d*x+c)+35/128*d*x+35/128*c))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2609 vs. 2(185) = 370.

Time = 0.12 (sec) , antiderivative size = 2609, normalized size of antiderivative = 12.98

$$\int \operatorname{csch}(c+dx)\left(a+b\sinh^3(c+dx)\right)^3\,dx=\text{Too large to display}$$

input
```
integrate(csch(d*x+c)*(a+b*sinh(d*x+c)^3)^3,x, algorithm="fricas")
```

output
```
1/30720*(15*b^3*cosh(d*x + c)^16 + 240*b^3*cosh(d*x + c)*sinh(d*x + c)^15
+ 15*b^3*sinh(d*x + c)^16 - 160*b^3*cosh(d*x + c)^14 + 576*a*b^2*cosh(d*x
+ c)^13 + 840*b^3*cosh(d*x + c)^12 + 40*(45*b^3*cosh(d*x + c)^2 - 4*b^3)*s
inh(d*x + c)^14 - 4800*a*b^2*cosh(d*x + c)^11 + 16*(525*b^3*cosh(d*x + c)^
3 - 140*b^3*cosh(d*x + c) + 36*a*b^2)*sinh(d*x + c)^13 + 4*(6825*b^3*cosh(
d*x + c)^4 - 3640*b^3*cosh(d*x + c)^2 + 1872*a*b^2*cosh(d*x + c) + 210*b^3
)*sinh(d*x + c)^12 + 28800*a*b^2*cosh(d*x + c)^9 + 16*(4095*b^3*cosh(d*x +
 c)^5 - 3640*b^3*cosh(d*x + c)^3 + 2808*a*b^2*cosh(d*x + c)^2 + 630*b^3*co
sh(d*x + c) - 300*a*b^2)*sinh(d*x + c)^11 - 240*(192*a^2*b - 35*b^3)*d*x*c
osh(d*x + c)^8 + 480*(24*a^2*b - 7*b^3)*cosh(d*x + c)^10 + 8*(15015*b^3*co
sh(d*x + c)^6 - 20020*b^3*cosh(d*x + c)^4 + 20592*a*b^2*cosh(d*x + c)^3 +
6930*b^3*cosh(d*x + c)^2 - 6600*a*b^2*cosh(d*x + c) + 1440*a^2*b - 420*b^3
)*sinh(d*x + c)^10 + 28800*a*b^2*cosh(d*x + c)^7 + 80*(2145*b^3*cosh(d*x +
 c)^7 - 4004*b^3*cosh(d*x + c)^5 + 5148*a*b^2*cosh(d*x + c)^4 + 2310*b^3*c
osh(d*x + c)^3 - 3300*a*b^2*cosh(d*x + c)^2 + 360*a*b^2 + 60*(24*a^2*b - 7
*b^3)*cosh(d*x + c))*sinh(d*x + c)^9 + 6*(32175*b^3*cosh(d*x + c)^8 - 8008
0*b^3*cosh(d*x + c)^6 + 123552*a*b^2*cosh(d*x + c)^5 + 69300*b^3*cosh(d*x
+ c)^4 - 132000*a*b^2*cosh(d*x + c)^3 + 43200*a*b^2*cosh(d*x + c) - 40*(19
2*a^2*b - 35*b^3)*d*x + 3600*(24*a^2*b - 7*b^3)*cosh(d*x + c)^2)*sinh(d*x
+ c)^8 - 4800*a*b^2*cosh(d*x + c)^5 + 48*(3575*b^3*cosh(d*x + c)^9 - 11...
```

Sympy [F(-1)]

Timed out.

$$\int \operatorname{csch}(c+dx)\left(a+b\sinh^3(c+dx)\right)^3\,dx=\text{Timed out}$$

input
```
integrate(csch(d*x+c)*(a+b*sinh(d*x+c)**3)**3,x)
```

output
```
Timed out
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 257, normalized size of antiderivative = 1.28

$$\int \operatorname{csch}(c+dx)\left(a+b\sinh^3(c+dx)\right)^3\,dx = -\frac{3}{8}\,a^2b\left(4\,x-\frac{e^{(2\,dx+2\,c)}}{d}+\frac{e^{(-2\,dx-2\,c)}}{d}\right) - \frac{1}{6144}\,b^3\left(\frac{\left(32\,e^{(-2\,dx-2\,c)}-168\,e^{(-4\,dx-4\,c)}+672\,e^{(-6\,dx-6\,c)}-3\right)e^{(8\,dx+8\,c)}}{d}-\frac{1680\,(dx+c)}{d}-\frac{672\,e^{(-2\,dx}\cdots\right) + \frac{1}{160}\,ab^2\left(\frac{3\,e^{(5\,dx+5\,c)}}{d}-\frac{25\,e^{(3\,dx+3\,c)}}{d}+\frac{150\,e^{(dx+c)}}{d}+\frac{150\,e^{(-dx-c)}}{d}-\frac{25\,e^{(-3\,dx-3\,c)}}{d}+\frac{3\,e^{(-5\,dx-5\,c)}}{d}\right) + \frac{a^3\log\left(\tanh\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)}{d}$$

input
```
integrate(csch(d*x+c)*(a+b*sinh(d*x+c)^3)^3,x, algorithm="maxima")
```

output
```
-3/8*a^2*b*(4*x - e^(2*d*x + 2*c)/d + e^(-2*d*x - 2*c)/d) - 1/6144*b^3*((3
2*e^(-2*d*x - 2*c) - 168*e^(-4*d*x - 4*c) + 672*e^(-6*d*x - 6*c) - 3)*e^(8
*d*x + 8*c)/d - 1680*(d*x + c)/d - (672*e^(-2*d*x - 2*c) - 168*e^(-4*d*x -
 4*c) + 32*e^(-6*d*x - 6*c) - 3*e^(-8*d*x - 8*c))/d) + 1/160*a*b^2*(3*e^(5
*d*x + 5*c)/d - 25*e^(3*d*x + 3*c)/d + 150*e^(d*x + c)/d + 150*e^(-d*x - c
)/d - 25*e^(-3*d*x - 3*c)/d + 3*e^(-5*d*x - 5*c)/d) + a^3*log(tanh(1/2*d*x
 + 1/2*c))/d
```

Giac [A] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 279, normalized size of antiderivative = 1.39

$$\int \operatorname{csch}(c+dx)\left(a+b\sinh^3(c+dx)\right)^3\,dx = \frac{15\,b^3e^{(8\,dx+8\,c)}-160\,b^3e^{(6\,dx+6\,c)}+576\,ab^2e^{(5\,dx+5\,c)}+840\,b^3e^{(4\,dx+4\,c)}-4800\,ab^2e^{(3\,dx+3\,c)}+11520\,a^2be^{(2\,d}\cdots}{\cdots}$$

input
```
integrate(csch(d*x+c)*(a+b*sinh(d*x+c)^3)^3,x, algorithm="giac")
```

output
```
1/30720*(15*b^3*e^(8*d*x + 8*c) - 160*b^3*e^(6*d*x + 6*c) + 576*a*b^2*e^(5
*d*x + 5*c) + 840*b^3*e^(4*d*x + 4*c) - 4800*a*b^2*e^(3*d*x + 3*c) + 11520
*a^2*b*e^(2*d*x + 2*c) - 3360*b^3*e^(2*d*x + 2*c) + 28800*a*b^2*e^(d*x + c
) - 30720*a^3*log(e^(d*x + c) + 1) + 30720*a^3*log(abs(e^(d*x + c) - 1)) -
 240*(192*a^2*b - 35*b^3)*(d*x + c) + (28800*a*b^2*e^(7*d*x + 7*c) - 4800*
a*b^2*e^(5*d*x + 5*c) - 840*b^3*e^(4*d*x + 4*c) + 576*a*b^2*e^(3*d*x + 3*c
) + 160*b^3*e^(2*d*x + 2*c) - 15*b^3 - 480*(24*a^2*b - 7*b^3)*e^(6*d*x + 6
*c))*e^(-8*d*x - 8*c))/d
```

Mupad [B] (verification not implemented)

Time = 0.51 (sec) , antiderivative size = 315, normalized size of antiderivative = 1.57

$$\int \operatorname{csch}(c+dx)\left(a+b\sinh^3(c+dx)\right)^3\,dx = \frac{7\,b^3\,e^{4\,c+4\,d\,x}}{256\,d}-\frac{2\,\mathrm{atan}\left(\frac{a^3\,e^{d\,x}\,e^c\,\sqrt{-d^2}}{d\,\sqrt{a^6}}\right)\,\sqrt{a^6}}{\sqrt{-d^2}}-\frac{7\,b^3\,e^{-4\,c-4\,d\,x}}{256\,d}-x\left(\frac{3\,a^2\,b}{2}-\frac{35\,b^3}{128}\right)+\frac{b^3\,e^{-6\,c-6\,d\,x}}{192\,d}-\frac{b^3\,e^{6\,c+6\,d\,x}}{192\,d}-\frac{b^3\,e^{-8\,c-8\,d\,x}}{2048\,d}+\frac{b^3\,e^{8\,c+8\,d\,x}}{2048\,d}-\frac{e^{-2\,c-2\,d\,x}\,\left(24\,a^2\,b-7\,b^3\right)}{64\,d}+\frac{e^{2\,c+2\,d\,x}\,\left(24\,a^2\,b-7\,b^3\right)}{64\,d}+\frac{15\,a\,b^2\,e^{-c-d\,x}}{16\,d}-\frac{5\,a\,b^2\,e^{-3\,c-3\,d\,x}}{32\,d}-\frac{5\,a\,b^2\,e^{3\,c+3\,d\,x}}{32\,d}+\frac{3\,a\,b^2\,e^{-5\,c-5\,d\,x}}{160\,d}+\frac{3\,a\,b^2\,e^{5\,c+5\,d\,x}}{160\,d}+\frac{15\,a\,b^2\,e^{c+d\,x}}{16\,d}$$

input
```
int((a + b*sinh(c + d*x)^3)^3/sinh(c + d*x),x)
```

output
```
(7*b^3*exp(4*c + 4*d*x))/(256*d) - (2*atan((a^3*exp(d*x)*exp(c)*(-d^2)^(1/
2))/(d*(a^6)^(1/2)))*(a^6)^(1/2))/(-d^2)^(1/2) - (7*b^3*exp(- 4*c - 4*d*x)
)/(256*d) - x*((3*a^2*b)/2 - (35*b^3)/128) + (b^3*exp(- 6*c - 6*d*x))/(192
*d) - (b^3*exp(6*c + 6*d*x))/(192*d) - (b^3*exp(- 8*c - 8*d*x))/(2048*d) +
 (b^3*exp(8*c + 8*d*x))/(2048*d) - (exp(- 2*c - 2*d*x)*(24*a^2*b - 7*b^3))
/(64*d) + (exp(2*c + 2*d*x)*(24*a^2*b - 7*b^3))/(64*d) + (15*a*b^2*exp(- c
 - d*x))/(16*d) - (5*a*b^2*exp(- 3*c - 3*d*x))/(32*d) - (5*a*b^2*exp(3*c +
 3*d*x))/(32*d) + (3*a*b^2*exp(- 5*c - 5*d*x))/(160*d) + (3*a*b^2*exp(5*c
+ 5*d*x))/(160*d) + (15*a*b^2*exp(c + d*x))/(16*d)
```

Reduce [B] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 341, normalized size of antiderivative = 1.70

$$\int \operatorname{csch}(c+dx)\left(a+b\sinh^3(c+dx)\right)^3\,dx = \frac{15e^{16dx+16c}b^3 - 160e^{14dx+14c}b^3 + 576e^{13dx+13c}a\,b^2 + 840e^{12dx+12c}b^3 - 4800e^{11dx+11c}a\,b^2 + 11520e^{10dx+10c}a}{}$$

input
```
int(csch(d*x+c)*(a+b*sinh(d*x+c)^3)^3,x)
```

output
```
(15*e**(16*c + 16*d*x)*b**3 - 160*e**(14*c + 14*d*x)*b**3 + 576*e**(13*c +
 13*d*x)*a*b**2 + 840*e**(12*c + 12*d*x)*b**3 - 4800*e**(11*c + 11*d*x)*a*
b**2 + 11520*e**(10*c + 10*d*x)*a**2*b - 3360*e**(10*c + 10*d*x)*b**3 + 28
800*e**(9*c + 9*d*x)*a*b**2 + 30720*e**(8*c + 8*d*x)*log(e**(c + d*x) - 1)
*a**3 - 30720*e**(8*c + 8*d*x)*log(e**(c + d*x) + 1)*a**3 - 46080*e**(8*c
+ 8*d*x)*a**2*b*d*x + 8400*e**(8*c + 8*d*x)*b**3*d*x + 28800*e**(7*c + 7*d
*x)*a*b**2 - 11520*e**(6*c + 6*d*x)*a**2*b + 3360*e**(6*c + 6*d*x)*b**3 -
4800*e**(5*c + 5*d*x)*a*b**2 - 840*e**(4*c + 4*d*x)*b**3 + 576*e**(3*c + 3
*d*x)*a*b**2 + 160*e**(2*c + 2*d*x)*b**3 - 15*b**3)/(30720*e**(8*c + 8*d*x
)*d)
```

3.142 $\int \operatorname{csch}^2(c+dx) \left(a+b\sinh^3(c+dx)\right)^3 \, dx$

Optimal result

Integrand size = 23, antiderivative size = 152

$$\begin{aligned}\int \operatorname{csch}^2(c+dx) \left(a+b\sinh^3(c+dx)\right)^3 \, dx = {} & \frac{9}{8}ab^2x+\frac{3a^2b\cosh(c+dx)}{d}-\frac{b^3\cosh(c+dx)}{d} \\ & +\frac{b^3\cosh^3(c+dx)}{d}-\frac{3b^3\cosh^5(c+dx)}{5d} \\ & +\frac{b^3\cosh^7(c+dx)}{7d}-\frac{a^3\coth(c+dx)}{d} \\ & -\frac{9ab^2\cosh(c+dx)\sinh(c+dx)}{8d} \\ & +\frac{3ab^2\cosh(c+dx)\sinh^3(c+dx)}{4d}\end{aligned}$$

output
```
9/8*a*b^2*x+3*a^2*b*cosh(d*x+c)/d-b^3*cosh(d*x+c)/d+b^3*cosh(d*x+c)^3/d-3/
5*b^3*cosh(d*x+c)^5/d+1/7*b^3*cosh(d*x+c)^7/d-a^3*coth(d*x+c)/d-9/8*a*b^2*
cosh(d*x+c)*sinh(d*x+c)/d+3/4*a*b^2*cosh(d*x+c)*sinh(d*x+c)^3/d
```

Mathematica [A] (verified)

Time = 4.29 (sec) , antiderivative size = 140, normalized size of antiderivative = 0.92

$$\int \operatorname{csch}^2(c+dx)\left(a+b\sinh^3(c+dx)\right)^3\,dx = \frac{2520ab^2c+2520ab^2dx+35b(192a^2-35b^2)\cosh(c+dx)+245b^3\cosh(3(c+dx))-49b^3\cosh(5(c+dx)) \cdots}{\cdots}$$

input
```
Integrate[Csch[c + d*x]^2*(a + b*Sinh[c + d*x]^3)^3,x]
```

output
```
(2520*a*b^2*c + 2520*a*b^2*d*x + 35*b*(192*a^2 - 35*b^2)*Cosh[c + d*x] + 2
45*b^3*Cosh[3*(c + d*x)] - 49*b^3*Cosh[5*(c + d*x)] + 5*b^3*Cosh[7*(c + d*
x)] - 1120*a^3*Coth[(c + d*x)/2] - 1680*a*b^2*Sinh[2*(c + d*x)] + 210*a*b^
2*Sinh[4*(c + d*x)] - 1120*a^3*Tanh[(c + d*x)/2])/(2240*d)
```

Rubi [A] (verified)

Time = 0.46 (sec) , antiderivative size = 152, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.174$, Rules used = {3042, 25, 3699, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \operatorname{csch}^2(c+dx)\left(a+b\sinh^3(c+dx)\right)^3\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int -\frac{\left(a+ib\sin(ic+idx)^3\right)^3}{\sin(ic+idx)^2}dx$$

$$\Big\downarrow \text{25}$$

$$-\int \frac{\left(ib\sin(ic+idx)^3+a\right)^3}{\sin(ic+idx)^2}dx$$

$$\Big\downarrow \text{3699}$$

$$-\int \left(-b^3 \sinh^7(c+dx) - 3ab^2 \sinh^4(c+dx) - 3a^2 b \sinh(c+dx) - a^3 \text{csch}^2(c+dx)\right) dx$$

$$\Big\downarrow \text{2009}$$

$$-\frac{a^3 \coth(c+dx)}{d} + \frac{3a^2 b \cosh(c+dx)}{d} + \frac{3ab^2 \sinh^3(c+dx)\cosh(c+dx)}{4d} - \frac{9ab^2 \sinh(c+dx)\cosh(c+dx)}{8d} + \frac{9}{8}ab^2 x + \frac{b^3 \cosh^7(c+dx)}{7d} - \frac{3b^3 \cosh^5(c+dx)}{5d} + \frac{b^3 \cosh^3(c+dx)}{d} - \frac{b^3 \cosh(c+dx)}{d}$$

input
```
Int[Csch[c + d*x]^2*(a + b*Sinh[c + d*x]^3)^3,x]
```

output
```
(9*a*b^2*x)/8 + (3*a^2*b*Cosh[c + d*x])/d - (b^3*Cosh[c + d*x])/d + (b^3*C
osh[c + d*x]^3)/d - (3*b^3*Cosh[c + d*x]^5)/(5*d) + (b^3*Cosh[c + d*x]^7)/
(7*d) - (a^3*Coth[c + d*x])/d - (9*a*b^2*Cosh[c + d*x]*Sinh[c + d*x])/(8*d
) + (3*a*b^2*Cosh[c + d*x]*Sinh[c + d*x]^3)/(4*d)
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3699
```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^(n_
))^(p_.), x_Symbol] :> Int[ExpandTrig[sin[e + f*x]^m*(a + b*sin[e + f*x]^n)
^p, x], x] /; FreeQ[{a, b, e, f}, x] && IntegersQ[m, p] && (EqQ[n, 4] || Gt
Q[p, 0] || (EqQ[p, -1] && IntegerQ[n]))
```

Maple [A] (verified)

Time = 2.37 (sec) , antiderivative size = 111, normalized size of antiderivative = 0.73

method	result
derivativedivides	$\frac{-\coth(dx+c)a^3+3a^2b\cosh(dx+c)+3b^2a\left(\left(\frac{\sinh(dx+c)^3}{4}-\frac{3\sinh(dx+c)}{8}\right)\cosh(dx+c)+\frac{3dx}{8}+\frac{3c}{8}\right)+b^3\left(-\frac{16}{35}+\frac{\sinh(dx+}{7}\right.}{d}$
default	$\frac{-\coth(dx+c)a^3+3a^2b\cosh(dx+c)+3b^2a\left(\left(\frac{\sinh(dx+c)^3}{4}-\frac{3\sinh(dx+c)}{8}\right)\cosh(dx+c)+\frac{3dx}{8}+\frac{3c}{8}\right)+b^3\left(-\frac{16}{35}+\frac{\sinh(dx+}{7}\right.}{d}$
parallelrisch	$\frac{-2240\coth\left(\frac{dx}{2}+\frac{c}{2}\right)a^3+1120\,\mathrm{sech}\left(\frac{dx}{2}+\frac{c}{2}\right)\mathrm{csch}\left(\frac{dx}{2}+\frac{c}{2}\right)a^3+245b^3\cosh(3dx+3c)-49b^3\cosh(5dx+5c)+5b^3\cosh(7dx}{2240d}$
risch	$\frac{9ab^2x}{8}+\frac{b^3e^{7dx+7c}}{896d}-\frac{7b^3e^{5dx+5c}}{640d}+\frac{3e^{4dx+4c}b^2a}{64d}+\frac{7e^{3dx+3c}b^3}{128d}-\frac{3e^{2dx+2c}b^2a}{8d}+\frac{3e^{dx+c}a^2b}{2d}-\frac{35e^{dx+}}{128d}$

input
```
int(csch(d*x+c)^2*(a+b*sinh(d*x+c)^3)^3,x,method=_RETURNVERBOSE)
```

output
```
1/d*(-coth(d*x+c)*a^3+3*a^2*b*cosh(d*x+c)+3*b^2*a*((1/4*sinh(d*x+c)^3-3/8*
sinh(d*x+c))*cosh(d*x+c)+3/8*d*x+3/8*c)+b^3*(-16/35+1/7*sinh(d*x+c)^6-6/35
*sinh(d*x+c)^4+8/35*sinh(d*x+c)^2)*cosh(d*x+c))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 302 vs. 2(142) = 284.

Time = 0.09 (sec) , antiderivative size = 302, normalized size of antiderivative = 1.99

$$\int \operatorname{csch}^2(c+dx)\left(a+b\sinh^3(c+dx)\right)^3\,dx$$
$$=\frac{20\,b^3\cosh(dx+c)\sinh(dx+c)^7+105\,ab^2\cosh(dx+c)^5+525\,ab^2\cosh(dx+c)\sinh(dx+c)^4-945\,a}{}$$

input
```
integrate(csch(d*x+c)^2*(a+b*sinh(d*x+c)^3)^3,x, algorithm="fricas")
```

output
```
1/2240*(20*b^3*cosh(d*x + c)*sinh(d*x + c)^7 + 105*a*b^2*cosh(d*x + c)^5 +
 525*a*b^2*cosh(d*x + c)*sinh(d*x + c)^4 - 945*a*b^2*cosh(d*x + c)^3 + 2*(
70*b^3*cosh(d*x + c)^3 - 81*b^3*cosh(d*x + c))*sinh(d*x + c)^5 + 4*(35*b^3
*cosh(d*x + c)^5 - 135*b^3*cosh(d*x + c)^3 + 147*b^3*cosh(d*x + c))*sinh(d
*x + c)^3 + 105*(10*a*b^2*cosh(d*x + c)^3 - 27*a*b^2*cosh(d*x + c))*sinh(d
*x + c)^2 - 280*(8*a^3 - 3*a*b^2)*cosh(d*x + c) + 2*(10*b^3*cosh(d*x + c)^
7 - 81*b^3*cosh(d*x + c)^5 + 294*b^3*cosh(d*x + c)^3 + 1260*a*b^2*d*x + 11
20*a^3 + 105*(32*a^2*b - 7*b^3)*cosh(d*x + c))*sinh(d*x + c))/(d*sinh(d*x
+ c))
```

Sympy [F(-1)]

Timed out.

$$\int \operatorname{csch}^2(c+dx)\left(a+b\sinh^3(c+dx)\right)^3\,dx=\text{Timed out}$$

input
```
integrate(csch(d*x+c)**2*(a+b*sinh(d*x+c)**3)**3,x)
```

output
```
Timed out
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 220, normalized size of antiderivative = 1.45

$$\int \operatorname{csch}^2(c+dx)\left(a+b\sinh^3(c+dx)\right)^3\,dx$$
$$=\frac{3}{64}ab^2\left(24\,x+\frac{e^{(4\,dx+4\,c)}}{d}-\frac{8\,e^{(2\,dx+2\,c)}}{d}+\frac{8\,e^{(-2\,dx-2\,c)}}{d}-\frac{e^{(-4\,dx-4\,c)}}{d}\right)$$
$$-\frac{1}{4480}b^3\left(\frac{\left(49\,e^{(-2\,dx-2\,c)}-245\,e^{(-4\,dx-4\,c)}+1225\,e^{(-6\,dx-6\,c)}-5\right)e^{(7\,dx+7\,c)}}{d}+\frac{1225\,e^{(-dx-c)}-245\,e^{(-3\,da}}{}\right.$$
$$+\frac{3}{2}a^2b\left(\frac{e^{(dx+c)}}{d}+\frac{e^{(-dx-c)}}{d}\right)+\frac{2\,a^3}{d\left(e^{(-2\,dx-2\,c)}-1\right)}$$

input
```
integrate(csch(d*x+c)^2*(a+b*sinh(d*x+c)^3)^3,x, algorithm="maxima")
```

output
```
3/64*a*b^2*(24*x + e^(4*d*x + 4*c)/d - 8*e^(2*d*x + 2*c)/d + 8*e^(-2*d*x -
 2*c)/d - e^(-4*d*x - 4*c)/d) - 1/4480*b^3*((49*e^(-2*d*x - 2*c) - 245*e^(
-4*d*x - 4*c) + 1225*e^(-6*d*x - 6*c) - 5)*e^(7*d*x + 7*c)/d + (1225*e^(-d
*x - c) - 245*e^(-3*d*x - 3*c) + 49*e^(-5*d*x - 5*c) - 5*e^(-7*d*x - 7*c))
/d) + 3/2*a^2*b*(e^(d*x + c)/d + e^(-d*x - c)/d) + 2*a^3/(d*(e^(-2*d*x - 2
*c) - 1))
```

Giac [A] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 276, normalized size of antiderivative = 1.82

$$\int \operatorname{csch}^2(c+dx)\left(a+b\sinh^3(c+dx)\right)^3\,dx$$

$$= \frac{5040\,(dx+c)ab^2 + 5\,b^3e^{(7\,dx+7\,c)} - 49\,b^3e^{(5\,dx+5\,c)} + 210\,ab^2e^{(4\,dx+4\,c)} + 245\,b^3e^{(3\,dx+3\,c)} - 1680\,ab^2e^{(2\,dx+2\,c}}{}$$

input
```
integrate(csch(d*x+c)^2*(a+b*sinh(d*x+c)^3)^3,x, algorithm="giac")
```

output
```
1/4480*(5040*(d*x + c)*a*b^2 + 5*b^3*e^(7*d*x + 7*c) - 49*b^3*e^(5*d*x + 5
*c) + 210*a*b^2*e^(4*d*x + 4*c) + 245*b^3*e^(3*d*x + 3*c) - 1680*a*b^2*e^(
2*d*x + 2*c) + 6720*a^2*b*e^(d*x + c) - 1225*b^3*e^(d*x + c) - (1890*a*b^2
*e^(5*d*x + 5*c) + 294*b^3*e^(4*d*x + 4*c) - 210*a*b^2*e^(3*d*x + 3*c) - 5
4*b^3*e^(2*d*x + 2*c) + 5*b^3 - 35*(192*a^2*b - 35*b^3)*e^(8*d*x + 8*c) +
560*(16*a^3 - 3*a*b^2)*e^(7*d*x + 7*c) + 210*(32*a^2*b - 7*b^3)*e^(6*d*x +
 6*c))*e^(-7*d*x - 7*c)/((e^(d*x + c) + 1)*(e^(d*x + c) - 1)))/d
```

Mupad [B] (verification not implemented)

Time = 0.38 (sec) , antiderivative size = 252, normalized size of antiderivative = 1.66

$$\int \operatorname{csch}^2(c+dx)\left(a+b\sinh^3(c+dx)\right)^3\,dx=\frac{e^{c+dx}\left(192a^2b-35b^3\right)}{128d}-\frac{2a^3}{d\left(e^{2c+2dx}-1\right)}+\frac{7b^3e^{-3c-3dx}}{128d}+\frac{7b^3e^{3c+3dx}}{128d}-\frac{7b^3e^{-5c-5dx}}{640d}-\frac{7b^3e^{5c+5dx}}{640d}+\frac{b^3e^{-7c-7dx}}{896d}+\frac{b^3e^{7c+7dx}}{896d}+\frac{e^{-c-dx}\left(192a^2b-35b^3\right)}{128d}+\frac{9ab^2x}{8}+\frac{3ab^2e^{-2c-2dx}}{8d}-\frac{3ab^2e^{2c+2dx}}{8d}-\frac{3ab^2e^{-4c-4dx}}{64d}+\frac{3ab^2e^{4c+4dx}}{64d}$$

input
```
int((a + b*sinh(c + d*x)^3)^3/sinh(c + d*x)^2,x)
```

output
```
(exp(c + d*x)*(192*a^2*b - 35*b^3))/(128*d) - (2*a^3)/(d*(exp(2*c + 2*d*x)
 - 1)) + (7*b^3*exp(- 3*c - 3*d*x))/(128*d) + (7*b^3*exp(3*c + 3*d*x))/(12
8*d) - (7*b^3*exp(- 5*c - 5*d*x))/(640*d) - (7*b^3*exp(5*c + 5*d*x))/(640*
d) + (b^3*exp(- 7*c - 7*d*x))/(896*d) + (b^3*exp(7*c + 7*d*x))/(896*d) + (
exp(- c - d*x)*(192*a^2*b - 35*b^3))/(128*d) + (9*a*b^2*x)/8 + (3*a*b^2*ex
p(- 2*c - 2*d*x))/(8*d) - (3*a*b^2*exp(2*c + 2*d*x))/(8*d) - (3*a*b^2*exp(
- 4*c - 4*d*x))/(64*d) + (3*a*b^2*exp(4*c + 4*d*x))/(64*d)
```

Reduce [B] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 305, normalized size of antiderivative = 2.01

$$\int \operatorname{csch}^2(c+dx)\left(a+b\sinh^3(c+dx)\right)^3\,dx$$
$$=\frac{5e^{16dx+16c}b^3-54e^{14dx+14c}b^3+210e^{13dx+13c}ab^2+294e^{12dx+12c}b^3-1890e^{11dx+11c}ab^2+6720e^{10dx+10c}a^2b-}{}$$

input
```
int(csch(d*x+c)^2*(a+b*sinh(d*x+c)^3)^3,x)
```

output
```
(5*e**(16*c + 16*d*x)*b**3 - 54*e**(14*c + 14*d*x)*b**3 + 210*e**(13*c + 1
3*d*x)*a*b**2 + 294*e**(12*c + 12*d*x)*b**3 - 1890*e**(11*c + 11*d*x)*a*b*
*2 + 6720*e**(10*c + 10*d*x)*a**2*b - 1470*e**(10*c + 10*d*x)*b**3 - 8960*
e**(9*c + 9*d*x)*a**3 + 5040*e**(9*c + 9*d*x)*a*b**2*d*x + 3360*e**(9*c +
9*d*x)*a*b**2 - 5040*e**(7*c + 7*d*x)*a*b**2*d*x - 6720*e**(6*c + 6*d*x)*a
**2*b + 1470*e**(6*c + 6*d*x)*b**3 - 1890*e**(5*c + 5*d*x)*a*b**2 - 294*e*
*(4*c + 4*d*x)*b**3 + 210*e**(3*c + 3*d*x)*a*b**2 + 54*e**(2*c + 2*d*x)*b*
*3 - 5*b**3)/(4480*e**(7*c + 7*d*x)*d*(e**(2*c + 2*d*x) - 1))
```

3.143 $\int \operatorname{csch}^3(c+dx)\left(a+b\sinh^3(c+dx)\right)^3\,dx$

Optimal result

Integrand size = 23, antiderivative size = 156

$$\begin{aligned}\int \operatorname{csch}^3(c+dx)\left(a+b\sinh^3(c+dx)\right)^3\,dx &= 3a^2bx-\frac{5b^3x}{16}+\frac{a^3\operatorname{arctanh}(\cosh(c+dx))}{2d}\\ &\quad-\frac{3ab^2\cosh(c+dx)}{d}+\frac{ab^2\cosh^3(c+dx)}{d}\\ &\quad-\frac{a^3\coth(c+dx)\operatorname{csch}(c+dx)}{2d}\\ &\quad+\frac{5b^3\cosh(c+dx)\sinh(c+dx)}{16d}\\ &\quad-\frac{5b^3\cosh(c+dx)\sinh^3(c+dx)}{24d}\\ &\quad+\frac{b^3\cosh(c+dx)\sinh^5(c+dx)}{6d}\end{aligned}$$

output

```
3*a^2*b*x-5/16*b^3*x+1/2*a^3*arctanh(cosh(d*x+c))/d-3*a*b^2*cosh(d*x+c)/d+
a*b^2*cosh(d*x+c)^3/d-1/2*a^3*coth(d*x+c)*csch(d*x+c)/d+5/16*b^3*cosh(d*x+
c)*sinh(d*x+c)/d-5/24*b^3*cosh(d*x+c)*sinh(d*x+c)^3/d+1/6*b^3*cosh(d*x+c)*
sinh(d*x+c)^5/d
```

Mathematica [A] (verified)

Time = 4.69 (sec) , antiderivative size = 166, normalized size of antiderivative = 1.06

$$\int \operatorname{csch}^3(c+dx)\left(a+b\sinh^3(c+dx)\right)^3\,dx$$
$$= \frac{576a^2bc-60b^3c+576a^2bdx-60b^3dx-432ab^2\cosh(c+dx)+48ab^2\cosh(3(c+dx))-24a^3\operatorname{csch}^2\left(\frac{1}{2}(c\right.}{}$$

input
```
Integrate[Csch[c + d*x]^3*(a + b*Sinh[c + d*x]^3)^3,x]
```

output
```
(576*a^2*b*c - 60*b^3*c + 576*a^2*b*d*x - 60*b^3*d*x - 432*a*b^2*Cosh[c +
d*x] + 48*a*b^2*Cosh[3*(c + d*x)] - 24*a^3*Csch[(c + d*x)/2]^2 + 96*a^3*Lo
g[Cosh[(c + d*x)/2]] - 96*a^3*Log[Sinh[(c + d*x)/2]] - 24*a^3*Sech[(c + d*
x)/2]^2 + 45*b^3*Sinh[2*(c + d*x)] - 9*b^3*Sinh[4*(c + d*x)] + b^3*Sinh[6*
(c + d*x)])/(192*d)
```

Rubi [C] (verified)

Result contains complex when optimal does not.

Time = 0.49 (sec) , antiderivative size = 179, normalized size of antiderivative = 1.15, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.174, Rules used = {3042, 26, 3699, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \operatorname{csch}^3(c+dx)\left(a+b\sinh^3(c+dx)\right)^3\,dx$$
$$\Big\downarrow \text{3042}$$
$$\int -\frac{i\left(a+ib\sin(ic+idx)^3\right)^3}{\sin(ic+idx)^3}dx$$
$$\Big\downarrow \text{26}$$
$$-i\int \frac{\left(ib\sin(ic+idx)^3+a\right)^3}{\sin(ic+idx)^3}dx$$

$$\Big\downarrow \text{3699}$$

$$-i\int \left(ib^3\sinh^6(c+dx)+3iab^2\sinh^3(c+dx)+ia^3\text{csch}^3(c+dx)+3ia^2b\right)dx$$

$$\Big\downarrow \text{2009}$$

$$-i\left(\frac{ia^3\text{arctanh}(\cosh(c+dx))}{2d}-\frac{ia^3\coth(c+dx)\text{csch}(c+dx)}{2d}+3ia^2bx+\frac{iab^2\cosh^3(c+dx)}{d}-\frac{3iab^2\cosh(c+}{d}\right.$$

input
```
Int[Csch[c + d*x]^3*(a + b*Sinh[c + d*x]^3)^3,x]
```

output
```
(-I)*((3*I)*a^2*b*x - ((5*I)/16)*b^3*x + ((I/2)*a^3*ArcTanh[Cosh[c + d*x]]
)/d - ((3*I)*a*b^2*Cosh[c + d*x])/d + (I*a*b^2*Cosh[c + d*x]^3)/d - ((I/2)
*a^3*Coth[c + d*x]*Csch[c + d*x])/d + (((5*I)/16)*b^3*Cosh[c + d*x]*Sinh[c
 + d*x])/d - (((5*I)/24)*b^3*Cosh[c + d*x]*Sinh[c + d*x]^3)/d + ((I/6)*b^3
*Cosh[c + d*x]*Sinh[c + d*x]^5)/d)
```

Defintions of rubi rules used

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3699
```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^(n_
))^(p_.), x_Symbol] :> Int[ExpandTrig[sin[e + f*x]^m*(a + b*sin[e + f*x]^n)
^p, x], x] /; FreeQ[{a, b, e, f}, x] && IntegersQ[m, p] && (EqQ[n, 4] || Gt
Q[p, 0] || (EqQ[p, -1] && IntegerQ[n]))
```

Maple [A] (verified)

Time = 2.30 (sec) , antiderivative size = 115, normalized size of antiderivative = 0.74

method	result
derivativedivides	$\frac{a^3\left(-\frac{\operatorname{csch}(dx+c)\coth(dx+c)}{2}+\operatorname{arctanh}\left(e^{dx+c}\right)\right)+3a^2b(dx+c)+3b^2a\left(-\frac{2}{3}+\frac{\sinh(dx+c)^2}{3}\right)\cosh(dx+c)+b^3\left(\left(\frac{\sinh(dx+c)^5}{6}\right.\right.}{d}$
default	$\frac{a^3\left(-\frac{\operatorname{csch}(dx+c)\coth(dx+c)}{2}+\operatorname{arctanh}\left(e^{dx+c}\right)\right)+3a^2b(dx+c)+3b^2a\left(-\frac{2}{3}+\frac{\sinh(dx+c)^2}{3}\right)\cosh(dx+c)+b^3\left(\left(\frac{\sinh(dx+c)^5}{6}\right.\right.}{d}$
parallelrisch	$\frac{192a^3\ln\left(\frac{1}{\sqrt{\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)}}\right)+24a^3\left(\operatorname{sech}\left(\frac{dx}{2}+\frac{c}{2}\right)^2+4\right)\operatorname{csch}\left(\frac{dx}{2}+\frac{c}{2}\right)^2-144\coth\left(\frac{dx}{2}+\frac{c}{2}\right)^2a^3+576a^2bdx-60b^3dx-43}{192d}$
risch	$3a^2bx-\frac{5b^3x}{16}+\frac{b^3e^{6dx+6c}}{384d}-\frac{3\,e^{4dx+4c}b^3}{128d}+\frac{e^{3dx+3c}b^2a}{8d}+\frac{15\,e^{2dx+2c}b^3}{128d}-\frac{9\,e^{dx+c}b^2a}{8d}-\frac{9\,e^{-dx-c}b^2a}{8d}-$

input
```
int(csch(d*x+c)^3*(a+b*sinh(d*x+c)^3)^3,x,method=_RETURNVERBOSE)
```

output
```
1/d*(a^3*(-1/2*csch(d*x+c)*coth(d*x+c)+arctanh(exp(d*x+c)))+3*a^2*b*(d*x+c
)+3*b^2*a*(-2/3+1/3*sinh(d*x+c)^2)*cosh(d*x+c)+b^3*((1/6*sinh(d*x+c)^5-5/2
4*sinh(d*x+c)^3+5/16*sinh(d*x+c))*cosh(d*x+c)-5/16*d*x-5/16*c))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 3627 vs. $2(144) = 288$.

Time = 0.16 (sec) , antiderivative size = 3627, normalized size of antiderivative = 23.25

$$\int \operatorname{csch}^3(c+dx)\left(a+b\sinh^3(c+dx)\right)^3\,dx = \text{Too large to display}$$

input
```
integrate(csch(d*x+c)^3*(a+b*sinh(d*x+c)^3)^3,x, algorithm="fricas")
```

output
```
Too large to include
```

Sympy [F(-1)]

Timed out.

$$\int \operatorname{csch}^3(c+dx)\left(a+b\sinh^3(c+dx)\right)^3\,dx=\text{Timed out}$$

input
```
integrate(csch(d*x+c)**3*(a+b*sinh(d*x+c)**3)**3,x)
```

output
```
Timed out
```

Maxima [A] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 244, normalized size of antiderivative = 1.56

$$\int \operatorname{csch}^3(c+dx)\left(a+b\sinh^3(c+dx)\right)^3\,dx=3\,a^2bx$$
$$-\frac{1}{384}\,b^3\left(\frac{\left(9\,e^{(-2\,dx-2\,c)}-45\,e^{(-4\,dx-4\,c)}-1\right)e^{(6\,dx+6\,c)}}{d}+\frac{120\,(dx+c)}{d}+\frac{45\,e^{(-2\,dx-2\,c)}-9\,e^{(-4\,dx-4\,c)}+e}{d}\right.$$
$$+\frac{1}{8}\,ab^2\left(\frac{e^{(3\,dx+3\,c)}}{d}-\frac{9\,e^{(dx+c)}}{d}-\frac{9\,e^{(-dx-c)}}{d}+\frac{e^{(-3\,dx-3\,c)}}{d}\right)$$
$$+\frac{1}{2}\,a^3\left(\frac{\log\left(e^{(-dx-c)}+1\right)}{d}-\frac{\log\left(e^{(-dx-c)}-1\right)}{d}+\frac{2\left(e^{(-dx-c)}+e^{(-3\,dx-3\,c)}\right)}{d(2\,e^{(-2\,dx-2\,c)}-e^{(-4\,dx-4\,c)}-1)}\right)$$

input
```
integrate(csch(d*x+c)^3*(a+b*sinh(d*x+c)^3)^3,x, algorithm="maxima")
```

output
```
3*a^2*b*x - 1/384*b^3*((9*e^(-2*d*x - 2*c) - 45*e^(-4*d*x - 4*c) - 1)*e^(6
*d*x + 6*c)/d + 120*(d*x + c)/d + (45*e^(-2*d*x - 2*c) - 9*e^(-4*d*x - 4*c
) + e^(-6*d*x - 6*c))/d) + 1/8*a*b^2*(e^(3*d*x + 3*c)/d - 9*e^(d*x + c)/d
- 9*e^(-d*x - c)/d + e^(-3*d*x - 3*c)/d) + 1/2*a^3*(log(e^(-d*x - c) + 1)/
d - log(e^(-d*x - c) - 1)/d + 2*(e^(-d*x - c) + e^(-3*d*x - 3*c))/(d*(2*e^
(-2*d*x - 2*c) - e^(-4*d*x - 4*c) - 1)))
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 289 vs. $2(144) = 288$.

Time = 0.24 (sec) , antiderivative size = 289, normalized size of antiderivative = 1.85

$$\int \operatorname{csch}^3(c+dx)\left(a+b\sinh^3(c+dx)\right)^3\,dx$$
$$= \frac{b^3e^{(6\,dx+6\,c)} - 9\,b^3e^{(4\,dx+4\,c)} + 48\,ab^2e^{(3\,dx+3\,c)} + 45\,b^3e^{(2\,dx+2\,c)} - 432\,ab^2e^{(dx+c)} + 192\,a^3\log\left(e^{(dx+c)}+1\right) -}{}$$

input
```
integrate(csch(d*x+c)^3*(a+b*sinh(d*x+c)^3)^3,x, algorithm="giac")
```

output
```
1/384*(b^3*e^(6*d*x + 6*c) - 9*b^3*e^(4*d*x + 4*c) + 48*a*b^2*e^(3*d*x + 3
*c) + 45*b^3*e^(2*d*x + 2*c) - 432*a*b^2*e^(d*x + c) + 192*a^3*log(e^(d*x
+ c) + 1) - 192*a^3*log(abs(e^(d*x + c) - 1)) + 24*(48*a^2*b - 5*b^3)*(d*x
 + c) - (45*b^3*e^(8*d*x + 8*c) - 99*b^3*e^(6*d*x + 6*c) + 528*a*b^2*e^(5*
d*x + 5*c) + 64*b^3*e^(4*d*x + 4*c) - 48*a*b^2*e^(3*d*x + 3*c) - 11*b^3*e^
(2*d*x + 2*c) + b^3 + 48*(8*a^3 + 9*a*b^2)*e^(9*d*x + 9*c) + 48*(8*a^3 - 1
9*a*b^2)*e^(7*d*x + 7*c))*e^(-6*d*x - 6*c)/((e^(d*x + c) + 1)^2*(e^(d*x +
c) - 1)^2))/d
```

Mupad [B] (verification not implemented)

Time = 1.84 (sec) , antiderivative size = 290, normalized size of antiderivative = 1.86

$$\int \operatorname{csch}^3(c+dx)\left(a+b\sinh^3(c+dx)\right)^3\,dx = x\left(3a^2b-\frac{5b^3}{16}\right)+\frac{\operatorname{atan}\left(\frac{a^3\,\mathrm{e}^{dx}\,\mathrm{e}^{c}\sqrt{-d^2}}{d\sqrt{a^6}}\right)\sqrt{a^6}}{\sqrt{-d^2}} - \frac{15\,b^3\,\mathrm{e}^{-2c-2dx}}{128\,d}+\frac{15\,b^3\,\mathrm{e}^{2c+2dx}}{128\,d} + \frac{3\,b^3\,\mathrm{e}^{-4c-4dx}}{128\,d}-\frac{3\,b^3\,\mathrm{e}^{4c+4dx}}{128\,d} - \frac{b^3\,\mathrm{e}^{-6c-6dx}}{384\,d}+\frac{b^3\,\mathrm{e}^{6c+6dx}}{384\,d}-\frac{9\,a\,b^2\,\mathrm{e}^{-c-dx}}{8\,d} + \frac{a\,b^2\,\mathrm{e}^{-3c-3dx}}{8\,d}+\frac{a\,b^2\,\mathrm{e}^{3c+3dx}}{8\,d} - \frac{9\,a\,b^2\,\mathrm{e}^{c+dx}}{8\,d}-\frac{a^3\,\mathrm{e}^{c+dx}}{d\,(\mathrm{e}^{2c+2dx}-1)} - \frac{2\,a^3\,\mathrm{e}^{c+dx}}{d\,(\mathrm{e}^{4c+4dx}-2\,\mathrm{e}^{2c+2dx}+1)}$$

input
```
int((a + b*sinh(c + d*x)^3)^3/sinh(c + d*x)^3,x)
```

output
```
x*(3*a^2*b - (5*b^3)/16) + (atan((a^3*exp(d*x)*exp(c)*(-d^2)^(1/2))/(d*(a^
6)^(1/2)))*(a^6)^(1/2))/(-d^2)^(1/2) - (15*b^3*exp(- 2*c - 2*d*x))/(128*d)
 + (15*b^3*exp(2*c + 2*d*x))/(128*d) + (3*b^3*exp(- 4*c - 4*d*x))/(128*d)
- (3*b^3*exp(4*c + 4*d*x))/(128*d) - (b^3*exp(- 6*c - 6*d*x))/(384*d) + (b
^3*exp(6*c + 6*d*x))/(384*d) - (9*a*b^2*exp(- c - d*x))/(8*d) + (a*b^2*exp
(- 3*c - 3*d*x))/(8*d) + (a*b^2*exp(3*c + 3*d*x))/(8*d) - (9*a*b^2*exp(c +
 d*x))/(8*d) - (a^3*exp(c + d*x))/(d*(exp(2*c + 2*d*x) - 1)) - (2*a^3*exp(
c + d*x))/(d*(exp(4*c + 4*d*x) - 2*exp(2*c + 2*d*x) + 1))
```

Reduce [B] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 534, normalized size of antiderivative = 3.42

$$\int \operatorname{csch}^3(c+dx)\left(a+b\sinh^3(c+dx)\right)^3\,dx$$
$$=\frac{-b^3+240e^{8dx+8c}b^3dx-384e^{9dx+9c}a^3+1152e^{10dx+10c}a^2bdx-528e^{5dx+5c}a\,b^2+48e^{3dx+3c}a\,b^2+480e^{7dx+7}\cdots}{}$$

input
```
int(csch(d*x+c)^3*(a+b*sinh(d*x+c)^3)^3,x)
```

output
```
(e**(16*c + 16*d*x)*b**3 - 11*e**(14*c + 14*d*x)*b**3 + 48*e**(13*c + 13*d
*x)*a*b**2 + 64*e**(12*c + 12*d*x)*b**3 - 528*e**(11*c + 11*d*x)*a*b**2 -
192*e**(10*c + 10*d*x)*log(e**(c + d*x) - 1)*a**3 + 192*e**(10*c + 10*d*x)
*log(e**(c + d*x) + 1)*a**3 + 1152*e**(10*c + 10*d*x)*a**2*b*d*x - 120*e**
(10*c + 10*d*x)*b**3*d*x - 99*e**(10*c + 10*d*x)*b**3 - 384*e**(9*c + 9*d*
x)*a**3 + 480*e**(9*c + 9*d*x)*a*b**2 + 384*e**(8*c + 8*d*x)*log(e**(c + d
*x) - 1)*a**3 - 384*e**(8*c + 8*d*x)*log(e**(c + d*x) + 1)*a**3 - 2304*e**
(8*c + 8*d*x)*a**2*b*d*x + 240*e**(8*c + 8*d*x)*b**3*d*x - 384*e**(7*c + 7
*d*x)*a**3 + 480*e**(7*c + 7*d*x)*a*b**2 - 192*e**(6*c + 6*d*x)*log(e**(c
+ d*x) - 1)*a**3 + 192*e**(6*c + 6*d*x)*log(e**(c + d*x) + 1)*a**3 + 1152*
e**(6*c + 6*d*x)*a**2*b*d*x - 120*e**(6*c + 6*d*x)*b**3*d*x + 99*e**(6*c +
 6*d*x)*b**3 - 528*e**(5*c + 5*d*x)*a*b**2 - 64*e**(4*c + 4*d*x)*b**3 + 48
*e**(3*c + 3*d*x)*a*b**2 + 11*e**(2*c + 2*d*x)*b**3 - b**3)/(384*e**(6*c +
 6*d*x)*d*(e**(4*c + 4*d*x) - 2*e**(2*c + 2*d*x) + 1))
```

3.144 $\int \operatorname{csch}^4(c+dx) \left(a+b\sinh^3(c+dx)\right)^3 dx$

Optimal result

Integrand size = 23, antiderivative size = 129

$$\int \operatorname{csch}^4(c+dx) \left(a+b\sinh^3(c+dx)\right)^3 dx = -\frac{3}{2}ab^2x - \frac{3a^2b\operatorname{arctanh}(\cosh(c+dx))}{d} + \frac{b^3\cosh(c+dx)}{d} - \frac{2b^3\cosh^3(c+dx)}{3d} + \frac{b^3\cosh^5(c+dx)}{5d} + \frac{a^3\coth(c+dx)}{d} - \frac{a^3\coth^3(c+dx)}{3d} + \frac{3ab^2\cosh(c+dx)\sinh(c+dx)}{2d}$$

output
```
-3/2*a*b^2*x-3*a^2*b*arctanh(cosh(d*x+c))/d+b^3*cosh(d*x+c)/d-2/3*b^3*cosh
(d*x+c)^3/d+1/5*b^3*cosh(d*x+c)^5/d+a^3*coth(d*x+c)/d-1/3*a^3*coth(d*x+c)^
3/d+3/2*a*b^2*cosh(d*x+c)*sinh(d*x+c)/d
```

Mathematica [A] (verified)

Time = 4.58 (sec) , antiderivative size = 186, normalized size of antiderivative = 1.44

$$\int \operatorname{csch}^4(c+dx)\left(a+b\sinh^3(c+dx)\right)^3\,dx$$
$$=\frac{-360ab^2c-360ab^2dx+150b^3\cosh(c+dx)-25b^3\cosh(3(c+dx))+3b^3\cosh(5(c+dx))+80a^3\coth\left(\right.}{}$$

input
```
Integrate[Csch[c + d*x]^4*(a + b*Sinh[c + d*x]^3)^3,x]
```

output
```
(-360*a*b^2*c - 360*a*b^2*d*x + 150*b^3*Cosh[c + d*x] - 25*b^3*Cosh[3*(c +
 d*x)] + 3*b^3*Cosh[5*(c + d*x)] + 80*a^3*Coth[(c + d*x)/2] - 720*a^2*b*Lo
g[Cosh[(c + d*x)/2]] + 720*a^2*b*Log[Sinh[(c + d*x)/2]] + 80*a^3*Csch[c +
d*x]^3*Sinh[(c + d*x)/2]^4 - 5*a^3*Csch[(c + d*x)/2]^4*Sinh[c + d*x] + 180
*a*b^2*Sinh[2*(c + d*x)] + 80*a^3*Tanh[(c + d*x)/2])/(240*d)
```

Rubi [A] (verified)

Time = 0.40 (sec) , antiderivative size = 129, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}=0.130$, Rules used = {3042, 3699, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \operatorname{csch}^4(c+dx)\left(a+b\sinh^3(c+dx)\right)^3\,dx$$
$$\Big\downarrow \text{ 3042}$$
$$\int \frac{\left(a+ib\sin(ic+idx)^3\right)^3}{\sin(ic+idx)^4}dx$$
$$\Big\downarrow \text{ 3699}$$
$$\int \left(a^3\operatorname{csch}^4(c+dx)+3a^2b\operatorname{csch}(c+dx)+3ab^2\sinh^2(c+dx)+b^3\sinh^5(c+dx)\right)dx$$
$$\Big\downarrow \text{ 2009}$$

$$-\frac{a^3\coth^3(c+dx)}{3d}+\frac{a^3\coth(c+dx)}{d}-\frac{3a^2b\text{arctanh}(\cosh(c+dx))}{d}+\frac{3ab^2\sinh(c+dx)\cosh(c+dx)}{2d}-\frac{3}{2}ab^2x+\frac{b^3\cosh^5(c+dx)}{5d}-\frac{2b^3\cosh^3(c+dx)}{3d}+\frac{b^3\cosh(c+dx)}{d}$$

input
```
Int[Csch[c + d*x]^4*(a + b*Sinh[c + d*x]^3)^3,x]
```

output
```
(-3*a*b^2*x)/2 - (3*a^2*b*ArcTanh[Cosh[c + d*x]])/d + (b^3*Cosh[c + d*x])/
d - (2*b^3*Cosh[c + d*x]^3)/(3*d) + (b^3*Cosh[c + d*x]^5)/(5*d) + (a^3*Cot
h[c + d*x])/d - (a^3*Coth[c + d*x]^3)/(3*d) + (3*a*b^2*Cosh[c + d*x]*Sinh[
c + d*x])/(2*d)
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3699
```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^(n_
))^(p_.), x_Symbol] :> Int[ExpandTrig[sin[e + f*x]^m*(a + b*sin[e + f*x]^n)
^p, x], x] /; FreeQ[{a, b, e, f}, x] && IntegersQ[m, p] && (EqQ[n, 4] || Gt
Q[p, 0] || (EqQ[p, -1] && IntegerQ[n]))
```

Maple [A] (verified)

Time = 2.02 (sec) , antiderivative size = 101, normalized size of antiderivative = 0.78

method	result
derivativedivides	$\frac{a^3\left(\frac{2}{3}-\frac{\operatorname{csch}(dx+c)^2}{3}\right)\coth(dx+c)-6a^2b\,\operatorname{arctanh}\left(e^{dx+c}\right)+3b^2a\left(\frac{\cosh(dx+c)\sinh(dx+c)}{2}-\frac{dx}{2}-\frac{c}{2}\right)+b^3\left(\frac{8}{15}+\frac{\sinh(dx+c)^4}{5}\right.}{d}$
default	$\frac{a^3\left(\frac{2}{3}-\frac{\operatorname{csch}(dx+c)^2}{3}\right)\coth(dx+c)-6a^2b\,\operatorname{arctanh}\left(e^{dx+c}\right)+3b^2a\left(\frac{\cosh(dx+c)\sinh(dx+c)}{2}-\frac{dx}{2}-\frac{c}{2}\right)+b^3\left(\frac{8}{15}+\frac{\sinh(dx+c)^4}{5}\right.}{d}$
parallelrisch	$\frac{720\ln\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)\right)a^2b-15\,\operatorname{sech}\left(\frac{dx}{2}+\frac{c}{2}\right)^3a^3\left(\cosh(dx+c)-\frac{\cosh(3dx+3c)}{3}\right)\operatorname{csch}\left(\frac{dx}{2}+\frac{c}{2}\right)^3-25b^3\cosh(3dx+3c)+3b}{240d}$
risch	$-\frac{3ab^2x}{2}+\frac{b^3e^{5dx+5c}}{160d}-\frac{5\,e^{3dx+3c}b^3}{96d}+\frac{3\,e^{2dx+2c}b^2a}{8d}+\frac{5\,e^{dx+c}b^3}{16d}+\frac{5\,e^{-dx-c}b^3}{16d}-\frac{3\,e^{-2dx-2c}b^2a}{8d}-\frac{5\,e^{-3}}{}$

input
```
int(csch(d*x+c)^4*(a+b*sinh(d*x+c)^3)^3,x,method=_RETURNVERBOSE)
```

output
```
1/d*(a^3*(2/3-1/3*csch(d*x+c)^2)*coth(d*x+c)-6*a^2*b*arctanh(exp(d*x+c))+3
*b^2*a*(1/2*cosh(d*x+c)*sinh(d*x+c)-1/2*d*x-1/2*c)+b^3*(8/15+1/5*sinh(d*x+
c)^4-4/15*sinh(d*x+c)^2)*cosh(d*x+c))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 3801 vs. $2(119) = 238$.

Time = 0.16 (sec) , antiderivative size = 3801, normalized size of antiderivative = 29.47

$$\int \operatorname{csch}^4(c+dx)\left(a+b\sinh^3(c+dx)\right)^3\,dx = \text{Too large to display}$$

input
```
integrate(csch(d*x+c)^4*(a+b*sinh(d*x+c)^3)^3,x, algorithm="fricas")
```

output
```
Too large to include
```

Sympy [F(-1)]

Timed out.

$$\int \operatorname{csch}^4(c+dx)\left(a+b\sinh^3(c+dx)\right)^3\,dx=\text{Timed out}$$

input
```
integrate(csch(d*x+c)**4*(a+b*sinh(d*x+c)**3)**3,x)
```

output
```
Timed out
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 260 vs. $2(119) = 238$.

Time = 0.05 (sec) , antiderivative size = 260, normalized size of antiderivative = 2.02

$$\int \operatorname{csch}^4(c+dx)\left(a+b\sinh^3(c+dx)\right)^3\,dx=-\frac{3}{8}ab^2\left(4x-\frac{e^{(2dx+2c)}}{d}+\frac{e^{(-2dx-2c)}}{d}\right)+\frac{1}{480}b^3\left(\frac{3e^{(5dx+5c)}}{d}-\frac{25e^{(3dx+3c)}}{d}+\frac{150e^{(dx+c)}}{d}+\frac{150e^{(-dx-c)}}{d}-\frac{25e^{(-3dx-3c)}}{d}+\frac{3e^{(-5dx-5c)}}{d}\right)-3a^2b\left(\frac{\log\left(e^{(-dx-c)}+1\right)}{d}-\frac{\log\left(e^{(-dx-c)}-1\right)}{d}\right)+\frac{4}{3}a^3\left(\frac{3e^{(-2dx-2c)}}{d(3e^{(-2dx-2c)}-3e^{(-4dx-4c)}+e^{(-6dx-6c)}-1)}-\frac{1}{d(3e^{(-2dx-2c)}-3e^{(-4dx-4c)}+e^{(-6dx-6c)}-1}\right.$$

input
```
integrate(csch(d*x+c)^4*(a+b*sinh(d*x+c)^3)^3,x, algorithm="maxima")
```

output
```
-3/8*a*b^2*(4*x - e^(2*d*x + 2*c)/d + e^(-2*d*x - 2*c)/d) + 1/480*b^3*(3*e
^(5*d*x + 5*c)/d - 25*e^(3*d*x + 3*c)/d + 150*e^(d*x + c)/d + 150*e^(-d*x
- c)/d - 25*e^(-3*d*x - 3*c)/d + 3*e^(-5*d*x - 5*c)/d) - 3*a^2*b*(log(e^(-
d*x - c) + 1)/d - log(e^(-d*x - c) - 1)/d) + 4/3*a^3*(3*e^(-2*d*x - 2*c)/(
d*(3*e^(-2*d*x - 2*c) - 3*e^(-4*d*x - 4*c) + e^(-6*d*x - 6*c) - 1)) - 1/(d
*(3*e^(-2*d*x - 2*c) - 3*e^(-4*d*x - 4*c) + e^(-6*d*x - 6*c) - 1)))
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 285 vs. $2(119) = 238$.

Time = 0.24 (sec) , antiderivative size = 285, normalized size of antiderivative = 2.21

$$\int \operatorname{csch}^4(c+dx)\left(a+b\sinh^3(c+dx)\right)^3\,dx = -\frac{720\,(dx+c)ab^2 - 3\,b^3e^{(5\,dx+5\,c)} + 25\,b^3e^{(3\,dx+3\,c)} - 180\,ab^2e^{(2\,dx+2\,c)} - 150\,b^3e^{(dx+c)} + 1440\,a^2b\log\left(e^{(dx}\right.}{}$$

input
```
integrate(csch(d*x+c)^4*(a+b*sinh(d*x+c)^3)^3,x, algorithm="giac")
```

output
```
-1/480*(720*(d*x + c)*a*b^2 - 3*b^3*e^(5*d*x + 5*c) + 25*b^3*e^(3*d*x + 3*
c) - 180*a*b^2*e^(2*d*x + 2*c) - 150*b^3*e^(d*x + c) + 1440*a^2*b*log(e^(d
*x + c) + 1) - 1440*a^2*b*log(abs(e^(d*x + c) - 1)) - (150*b^3*e^(10*d*x +
 10*c) - 180*a*b^2*e^(9*d*x + 9*c) - 475*b^3*e^(8*d*x + 8*c) + 528*b^3*e^(
6*d*x + 6*c) - 234*b^3*e^(4*d*x + 4*c) + 180*a*b^2*e^(3*d*x + 3*c) + 34*b^
3*e^(2*d*x + 2*c) - 3*b^3 - 60*(32*a^3 - 9*a*b^2)*e^(7*d*x + 7*c) + 20*(32
*a^3 - 27*a*b^2)*e^(5*d*x + 5*c))*e^(-5*d*x - 5*c)/((e^(d*x + c) + 1)^3*(e
^(d*x + c) - 1)^3))/d
```

Mupad [B] (verification not implemented)

Time = 1.80 (sec) , antiderivative size = 267, normalized size of antiderivative = 2.07

$$\int \operatorname{csch}^4(c+dx)\left(a+b\sinh^3(c+dx)\right)^3\,dx = \frac{5\,b^3\,e^{c+d\,x}}{16\,d} - \frac{4\,a^3}{d\,\left(e^{4\,c+4\,d\,x}-2\,e^{2\,c+2\,d\,x}+1\right)} + \frac{5\,b^3\,e^{-c-d\,x}}{16\,d} - \frac{5\,b^3\,e^{-3\,c-3\,d\,x}}{96\,d} - \frac{5\,b^3\,e^{3\,c+3\,d\,x}}{96\,d} + \frac{b^3\,e^{-5\,c-5\,d\,x}}{160\,d} + \frac{b^3\,e^{5\,c+5\,d\,x}}{160\,d} - \frac{8\,a^3}{3\,d\,\left(3\,e^{2\,c+2\,d\,x}-3\,e^{4\,c+4\,d\,x}+e^{6\,c+6\,d\,x}-1\right)} - \frac{6\,\mathrm{atan}\left(\frac{a^2\,b\,e^{d\,x}\,e^c\,\sqrt{-d^2}}{d\,\sqrt{a^4\,b^2}}\right)\,\sqrt{a^4\,b^2}}{\sqrt{-d^2}} - \frac{3\,a\,b^2\,x}{2} - \frac{3\,a\,b^2\,e^{-2\,c-2\,d\,x}}{8\,d} + \frac{3\,a\,b^2\,e^{2\,c+2\,d\,x}}{8\,d}$$

input
```
int((a + b*sinh(c + d*x)^3)^3/sinh(c + d*x)^4,x)
```

output
```
(5*b^3*exp(c + d*x))/(16*d) - (4*a^3)/(d*(exp(4*c + 4*d*x) - 2*exp(2*c + 2
*d*x) + 1)) + (5*b^3*exp(- c - d*x))/(16*d) - (5*b^3*exp(- 3*c - 3*d*x))/(
96*d) - (5*b^3*exp(3*c + 3*d*x))/(96*d) + (b^3*exp(- 5*c - 5*d*x))/(160*d)
 + (b^3*exp(5*c + 5*d*x))/(160*d) - (8*a^3)/(3*d*(3*exp(2*c + 2*d*x) - 3*e
xp(4*c + 4*d*x) + exp(6*c + 6*d*x) - 1)) - (6*atan((a^2*b*exp(d*x)*exp(c)*
(-d^2)^(1/2))/(d*(a^4*b^2)^(1/2)))*(a^4*b^2)^(1/2))/(-d^2)^(1/2) - (3*a*b^
2*x)/2 - (3*a*b^2*exp(- 2*c - 2*d*x))/(8*d) + (3*a*b^2*exp(2*c + 2*d*x))/(
8*d)
```

Reduce [B] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 556, normalized size of antiderivative = 4.31

$$\int \operatorname{csch}^4(c+dx)\left(a+b\sinh^3(c+dx)\right)^3\,dx$$
$$= \frac{-3b^3+1440e^{11dx+11c}\log\left(e^{dx+c}-1\right)a^2b-1440e^{11dx+11c}\log\left(e^{dx+c}+1\right)a^2b-4320e^{9dx+9c}\log\left(e^{dx+c}-1\right)a}{}$$

input
```
int(csch(d*x+c)^4*(a+b*sinh(d*x+c)^3)^3,x)
```

output
```
(3*e**(16*c + 16*d*x)*b**3 - 34*e**(14*c + 14*d*x)*b**3 + 180*e**(13*c + 1
3*d*x)*a*b**2 + 234*e**(12*c + 12*d*x)*b**3 + 1440*e**(11*c + 11*d*x)*log(
e**(c + d*x) - 1)*a**2*b - 1440*e**(11*c + 11*d*x)*log(e**(c + d*x) + 1)*a
**2*b - 720*e**(11*c + 11*d*x)*a*b**2*d*x - 420*e**(11*c + 11*d*x)*a*b**2
- 378*e**(10*c + 10*d*x)*b**3 - 4320*e**(9*c + 9*d*x)*log(e**(c + d*x) - 1
)*a**2*b + 4320*e**(9*c + 9*d*x)*log(e**(c + d*x) + 1)*a**2*b + 2160*e**(9
*c + 9*d*x)*a*b**2*d*x + 4320*e**(7*c + 7*d*x)*log(e**(c + d*x) - 1)*a**2*
b - 4320*e**(7*c + 7*d*x)*log(e**(c + d*x) + 1)*a**2*b - 1920*e**(7*c + 7*
d*x)*a**3 - 2160*e**(7*c + 7*d*x)*a*b**2*d*x + 720*e**(7*c + 7*d*x)*a*b**2
 + 378*e**(6*c + 6*d*x)*b**3 - 1440*e**(5*c + 5*d*x)*log(e**(c + d*x) - 1)
*a**2*b + 1440*e**(5*c + 5*d*x)*log(e**(c + d*x) + 1)*a**2*b + 640*e**(5*c
 + 5*d*x)*a**3 + 720*e**(5*c + 5*d*x)*a*b**2*d*x - 660*e**(5*c + 5*d*x)*a*
b**2 - 234*e**(4*c + 4*d*x)*b**3 + 180*e**(3*c + 3*d*x)*a*b**2 + 34*e**(2*
c + 2*d*x)*b**3 - 3*b**3)/(480*e**(5*c + 5*d*x)*d*(e**(6*c + 6*d*x) - 3*e*
*(4*c + 4*d*x) + 3*e**(2*c + 2*d*x) - 1))
```

3.145 $\int \mathrm{csch}^5(c+dx) \left(a+b\sinh^3(c+dx)\right)^3 \, dx$

Optimal result

Integrand size = 23, antiderivative size = 148

$$\int \mathrm{csch}^5(c+dx) \left(a+b\sinh^3(c+dx)\right)^3 \, dx = \frac{3b^3x}{8} - \frac{3a^3\operatorname{arctanh}(\cosh(c+dx))}{8d} + \frac{3ab^2\cosh(c+dx)}{d} - \frac{3a^2b\coth(c+dx)}{d} + \frac{3a^3\coth(c+dx)\mathrm{csch}(c+dx)}{8d} - \frac{a^3\coth(c+dx)\mathrm{csch}^3(c+dx)}{4d} - \frac{3b^3\cosh(c+dx)\sinh(c+dx)}{8d} + \frac{b^3\cosh(c+dx)\sinh^3(c+dx)}{4d}$$

output

```
3/8*b^3*x-3/8*a^3*arctanh(cosh(d*x+c))/d+3*a*b^2*cosh(d*x+c)/d-3*a^2*b*cot
h(d*x+c)/d+3/8*a^3*coth(d*x+c)*csch(d*x+c)/d-1/4*a^3*coth(d*x+c)*csch(d*x+
c)^3/d-3/8*b^3*cosh(d*x+c)*sinh(d*x+c)/d+1/4*b^3*cosh(d*x+c)*sinh(d*x+c)^3
/d
```

Mathematica [A] (verified)

Time = 7.08 (sec) , antiderivative size = 239, normalized size of antiderivative = 1.61

$$\int \operatorname{csch}^5(c+dx)\left(a+b\sinh^3(c+dx)\right)^3\,dx = \frac{3b^3(c+dx)}{8d}+\frac{3ab^2\cosh(c+dx)}{d}-\frac{3a^2b\coth\left(\frac{1}{2}(c+dx)\right)}{2d} + \frac{3a^3\operatorname{csch}^2\left(\frac{1}{2}(c+dx)\right)}{32d}-\frac{a^3\operatorname{csch}^4\left(\frac{1}{2}(c+dx)\right)}{64d}-\frac{3a^3\log\left(\cosh\left(\frac{1}{2}(c+dx)\right)\right)}{8d} + \frac{3a^3\log\left(\sinh\left(\frac{1}{2}(c+dx)\right)\right)}{8d}+\frac{3a^3\operatorname{sech}^2\left(\frac{1}{2}(c+dx)\right)}{32d}+\frac{a^3\operatorname{sech}^4\left(\frac{1}{2}(c+dx)\right)}{64d} - \frac{b^3\sinh(2(c+dx))}{4d}+\frac{b^3\sinh(4(c+dx))}{32d}-\frac{3a^2b\tanh\left(\frac{1}{2}(c+dx)\right)}{2d}$$

input
```
Integrate[Csch[c + d*x]^5*(a + b*Sinh[c + d*x]^3)^3,x]
```

output
```
(3*b^3*(c + d*x))/(8*d) + (3*a*b^2*Cosh[c + d*x])/d - (3*a^2*b*Coth[(c + d
*x)/2])/(2*d) + (3*a^3*Csch[(c + d*x)/2]^2)/(32*d) - (a^3*Csch[(c + d*x)/2
]^4)/(64*d) - (3*a^3*Log[Cosh[(c + d*x)/2]])/(8*d) + (3*a^3*Log[Sinh[(c +
d*x)/2]])/(8*d) + (3*a^3*Sech[(c + d*x)/2]^2)/(32*d) + (a^3*Sech[(c + d*x)
/2]^4)/(64*d) - (b^3*Sinh[2*(c + d*x)])/(4*d) + (b^3*Sinh[4*(c + d*x)])/(3
2*d) - (3*a^2*b*Tanh[(c + d*x)/2])/(2*d)
```

Rubi [C] (verified)

Result contains complex when optimal does not.

Time = 0.46 (sec) , antiderivative size = 168, normalized size of antiderivative = 1.14, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.174, Rules used = {3042, 26, 3699, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \operatorname{csch}^5(c+dx)\left(a+b\sinh^3(c+dx)\right)^3\,dx$$

$$\Big\downarrow \ 3042$$

$$\int \frac{i\left(a+ib\sin(ic+idx)^3\right)^3}{\sin(ic+idx)^5}dx$$

$$\Big\downarrow \; 26$$

$$i\int \frac{\left(ib\sin(ic+idx)^3+a\right)^3}{\sin(ic+idx)^5}dx$$

$$\Big\downarrow \; 3699$$

$$i\int \left(-ia^3\text{csch}^5(c+dx)-3ia^2b\text{csch}^2(c+dx)-ib^3\sinh^4(c+dx)-3iab^2\sinh(c+dx)\right)dx$$

$$\Big\downarrow \; 2009$$

$$i\left(\frac{3ia^3\text{arctanh}(\cosh(c+dx))}{8d}+\frac{ia^3\coth(c+dx)\text{csch}^3(c+dx)}{4d}-\frac{3ia^3\coth(c+dx)\text{csch}(c+dx)}{8d}+\frac{3ia^2b\coth(c}{d}\right.$$

input
```
Int[Csch[c + d*x]^5*(a + b*Sinh[c + d*x]^3)^3,x]
```

output
```
I*(((-3*I)/8)*b^3*x + (((3*I)/8)*a^3*ArcTanh[Cosh[c + d*x]])/d - ((3*I)*a*
b^2*Cosh[c + d*x])/d + ((3*I)*a^2*b*Coth[c + d*x])/d - (((3*I)/8)*a^3*Coth
[c + d*x]*Csch[c + d*x])/d + ((I/4)*a^3*Coth[c + d*x]*Csch[c + d*x]^3)/d +
 (((3*I)/8)*b^3*Cosh[c + d*x]*Sinh[c + d*x])/d - ((I/4)*b^3*Cosh[c + d*x]*
Sinh[c + d*x]^3)/d)
```

Defintions of rubi rules used

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3699
```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^(n_
))^(p_.), x_Symbol] :> Int[ExpandTrig[sin[e + f*x]^m*(a + b*sin[e + f*x]^n)
^p, x], x] /; FreeQ[{a, b, e, f}, x] && IntegersQ[m, p] && (EqQ[n, 4] || Gt
Q[p, 0] || (EqQ[p, -1] && IntegerQ[n]))
```

Maple [A] (verified)

Time = 1.94 (sec) , antiderivative size = 108, normalized size of antiderivative = 0.73

method	result
derivativedivides	$\frac{a^3\left(\left(-\frac{\operatorname{csch}(dx+c)^3}{4}+\frac{3\operatorname{csch}(dx+c)}{8}\right)\coth(dx+c)-\frac{3\operatorname{arctanh}\left(e^{dx+c}\right)}{4}\right)-3a^2b\coth(dx+c)+3b^2a\cosh(dx+c)+b^3\left(\left(\frac{\sinh}{}\right.\right.}{d}$
default	$\frac{a^3\left(\left(-\frac{\operatorname{csch}(dx+c)^3}{4}+\frac{3\operatorname{csch}(dx+c)}{8}\right)\coth(dx+c)-\frac{3\operatorname{arctanh}\left(e^{dx+c}\right)}{4}\right)-3a^2b\coth(dx+c)+3b^2a\cosh(dx+c)+b^3\left(\left(\frac{\sinh}{}\right.\right.}{d}$
parallelrisch	$\frac{512a^3\ln\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^{\frac{3}{8}}\right)-11\left(\cosh(dx+c)-\frac{19\cosh(2dx+2c)}{22}-\frac{3\cosh(3dx+3c)}{11}+\frac{19\cosh(4dx+4c)}{88}+\frac{57}{88}\right)a^3\operatorname{sech}\left(\frac{dx}{2}+\frac{c}{2}\right)}{}$
risch	$\frac{3b^3x}{8}+\frac{e^{4dx+4c}b^3}{64d}-\frac{e^{2dx+2c}b^3}{8d}+\frac{3\,e^{dx+c}b^2a}{2d}+\frac{3\,e^{-dx-c}b^2a}{2d}+\frac{e^{-2dx-2c}b^3}{8d}-\frac{e^{-4dx-4c}b^3}{64d}+\frac{a^2(3a\,e^{7dx+}}{}$

input
```
int(csch(d*x+c)^5*(a+b*sinh(d*x+c)^3)^3,x,method=_RETURNVERBOSE)
```

output
```
1/d*(a^3*((-1/4*csch(d*x+c)^3+3/8*csch(d*x+c))*coth(d*x+c)-3/4*arctanh(exp
(d*x+c)))-3*a^2*b*coth(d*x+c)+3*b^2*a*cosh(d*x+c)+b^3*((1/4*sinh(d*x+c)^3-
3/8*sinh(d*x+c))*cosh(d*x+c)+3/8*d*x+3/8*c))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 4541 vs. 2(136) = 272.

Time = 0.15 (sec) , antiderivative size = 4541, normalized size of antiderivative = 30.68

$$\int \operatorname{csch}^5(c+dx)\left(a+b\sinh^3(c+dx)\right)^3\,dx = \text{Too large to display}$$

input
```
integrate(csch(d*x+c)^5*(a+b*sinh(d*x+c)^3)^3,x, algorithm="fricas")
```

output
```
Too large to include
```

Sympy [F(-1)]

Timed out.

$$\int \operatorname{csch}^5(c+dx)\left(a+b\sinh^3(c+dx)\right)^3\,dx=\text{Timed out}$$

input
```
integrate(csch(d*x+c)**5*(a+b*sinh(d*x+c)**3)**3,x)
```

output
```
Timed out
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 255, normalized size of antiderivative = 1.72

$$\int \operatorname{csch}^5(c+dx)\left(a+b\sinh^3(c+dx)\right)^3\,dx$$
$$=\frac{1}{64}b^3\left(24x+\frac{e^{(4dx+4c)}}{d}-\frac{8e^{(2dx+2c)}}{d}+\frac{8e^{(-2dx-2c)}}{d}-\frac{e^{(-4dx-4c)}}{d}\right)$$
$$+\frac{3}{2}ab^2\left(\frac{e^{(dx+c)}}{d}+\frac{e^{(-dx-c)}}{d}\right)$$
$$-\frac{1}{8}a^3\left(\frac{3\log\left(e^{(-dx-c)}+1\right)}{d}-\frac{3\log\left(e^{(-dx-c)}-1\right)}{d}+\frac{2\left(3e^{(-dx-c)}-11e^{(-3dx-3c)}-11e^{(-5dx-5c)}+3e\ldots\right.}{d(4e^{(-2dx-2c)}-6e^{(-4dx-4c)}+4e^{(-6dx-6c)}-e^{(-\ldots}}\right.$$
$$+\frac{6a^2b}{d\left(e^{(-2dx-2c)}-1\right)}$$

input
```
integrate(csch(d*x+c)^5*(a+b*sinh(d*x+c)^3)^3,x, algorithm="maxima")
```

output
```
1/64*b^3*(24*x + e^(4*d*x + 4*c)/d - 8*e^(2*d*x + 2*c)/d + 8*e^(-2*d*x - 2
*c)/d - e^(-4*d*x - 4*c)/d) + 3/2*a*b^2*(e^(d*x + c)/d + e^(-d*x - c)/d) -
 1/8*a^3*(3*log(e^(-d*x - c) + 1)/d - 3*log(e^(-d*x - c) - 1)/d + 2*(3*e^(
-d*x - c) - 11*e^(-3*d*x - 3*c) - 11*e^(-5*d*x - 5*c) + 3*e^(-7*d*x - 7*c)
)/(d*(4*e^(-2*d*x - 2*c) - 6*e^(-4*d*x - 4*c) + 4*e^(-6*d*x - 6*c) - e^(-8
*d*x - 8*c) - 1))) + 6*a^2*b/(d*(e^(-2*d*x - 2*c) - 1))
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 329 vs. $2(136) = 272$.

Time = 0.25 (sec) , antiderivative size = 329, normalized size of antiderivative = 2.22

$$\int \operatorname{csch}^5(c+dx)\left(a+b\sinh^3(c+dx)\right)^3\,dx$$

$$= \frac{24\,(dx+c)b^3 + b^3e^{(4\,dx+4\,c)} - 8\,b^3e^{(2\,dx+2\,c)} + 96\,ab^2e^{(dx+c)} - 24\,a^3\log\left(e^{(dx+c)}+1\right) + 24\,a^3\log\left(\left|e^{(dx+c)}-\right.\right.}{}$$

input
```
integrate(csch(d*x+c)^5*(a+b*sinh(d*x+c)^3)^3,x, algorithm="giac")
```

output
```
1/64*(24*(d*x + c)*b^3 + b^3*e^(4*d*x + 4*c) - 8*b^3*e^(2*d*x + 2*c) + 96*
a*b^2*e^(d*x + c) - 24*a^3*log(e^(d*x + c) + 1) + 24*a^3*log(abs(e^(d*x +
c) - 1)) + (96*a*b^2*e^(3*d*x + 3*c) + 12*b^3*e^(2*d*x + 2*c) - b^3 + 48*(
a^3 + 2*a*b^2)*e^(11*d*x + 11*c) - 8*(48*a^2*b - b^3)*e^(10*d*x + 10*c) -
16*(11*a^3 + 24*a*b^2)*e^(9*d*x + 9*c) + 3*(384*a^2*b - 11*b^3)*e^(8*d*x +
 8*c) - 16*(11*a^3 - 36*a*b^2)*e^(7*d*x + 7*c) - 4*(288*a^2*b - 13*b^3)*e^
(6*d*x + 6*c) + 48*(a^3 - 8*a*b^2)*e^(5*d*x + 5*c) + 2*(192*a^2*b - 19*b^3
)*e^(4*d*x + 4*c))*e^(-4*d*x - 4*c)/((e^(d*x + c) + 1)^4*(e^(d*x + c) - 1)
^4))/d
```

Mupad [B] (verification not implemented)

Time = 1.77 (sec) , antiderivative size = 451, normalized size of antiderivative = 3.05

$$\int \operatorname{csch}^5(c+dx)\left(a+b\sinh^3(c+dx)\right)^3 dx = \frac{3b^3x}{8} - \frac{\frac{3a^2b}{2d} + \frac{2a^3e^{c+dx}}{d} - \frac{3a^2b\,e^{2c+2dx}}{d} + \frac{3a^2b\,e^{4c+4dx}}{2d}}{3e^{2c+2dx} - 3e^{4c+4dx} + e^{6c+6dx} - 1} - \frac{\frac{4a^3e^{3c+3dx}}{d} - \frac{3a^2b}{2d} + \frac{9a^2b\,e^{2c+2dx}}{2d} - \frac{9a^2b\,e^{4c+4dx}}{2d} + \frac{3a^2b\,e^{6c+6dx}}{2d}}{6e^{4c+4dx} - 4e^{2c+2dx} - 4e^{6c+6dx} + e^{8c+8dx} + 1} - \frac{\frac{3a^2b}{d} - \frac{3a^3e^{c+dx}}{4d}}{e^{2c+2dx} - 1} - \frac{3\,\mathrm{atan}\left(\frac{a^3e^{dx}e^c\sqrt{-d^2}}{d\sqrt{a^6}}\right)\sqrt{a^6}}{4\sqrt{-d^2}} + \frac{b^3e^{-2c-2dx}}{8d} - \frac{b^3e^{2c+2dx}}{8d} - \frac{b^3e^{-4c-4dx}}{64d} + \frac{b^3e^{4c+4dx}}{64d} + \frac{3ab^2e^{-c-dx}}{2d} + \frac{3ab^2e^{c+dx}}{2d} - \frac{a^3e^{c+dx}}{2d\left(e^{4c+4dx} - 2e^{2c+2dx} + 1\right)}$$

input
```
int((a + b*sinh(c + d*x)^3)^3/sinh(c + d*x)^5,x)
```

output
```
(3*b^3*x)/8 - ((3*a^2*b)/(2*d) + (2*a^3*exp(c + d*x))/d - (3*a^2*b*exp(2*c
 + 2*d*x))/d + (3*a^2*b*exp(4*c + 4*d*x))/(2*d))/(3*exp(2*c + 2*d*x) - 3*e
xp(4*c + 4*d*x) + exp(6*c + 6*d*x) - 1) - ((4*a^3*exp(3*c + 3*d*x))/d - (3
*a^2*b)/(2*d) + (9*a^2*b*exp(2*c + 2*d*x))/(2*d) - (9*a^2*b*exp(4*c + 4*d*
x))/(2*d) + (3*a^2*b*exp(6*c + 6*d*x))/(2*d))/(6*exp(4*c + 4*d*x) - 4*exp(
2*c + 2*d*x) - 4*exp(6*c + 6*d*x) + exp(8*c + 8*d*x) + 1) - ((3*a^2*b)/d -
 (3*a^3*exp(c + d*x))/(4*d))/(exp(2*c + 2*d*x) - 1) - (3*atan((a^3*exp(d*x
)*exp(c)*(-d^2)^(1/2))/(d*(a^6)^(1/2)))*(a^6)^(1/2))/(4*(-d^2)^(1/2)) + (b
^3*exp(- 2*c - 2*d*x))/(8*d) - (b^3*exp(2*c + 2*d*x))/(8*d) - (b^3*exp(- 4
*c - 4*d*x))/(64*d) + (b^3*exp(4*c + 4*d*x))/(64*d) + (3*a*b^2*exp(- c - d
*x))/(2*d) + (3*a*b^2*exp(c + d*x))/(2*d) - (a^3*exp(c + d*x))/(2*d*(exp(4
*c + 4*d*x) - 2*exp(2*c + 2*d*x) + 1))
```

Reduce [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 732, normalized size of antiderivative = 4.95

$$\int \operatorname{csch}^5(c+dx)\left(a+b\sinh^3(c+dx)\right)^3\,dx$$
$$= \frac{-b^3+144e^{8dx+8c}b^3dx+24e^{12dx+12c}\log\left(e^{dx+c}-1\right)a^3-24e^{12dx+12c}\log\left(e^{dx+c}+1\right)a^3-176e^{9dx+9c}a^3+24}{}$$

input
```
int(csch(d*x+c)^5*(a+b*sinh(d*x+c)^3)^3,x)
```

output
```
(e**(16*c + 16*d*x)*b**3 - 12*e**(14*c + 14*d*x)*b**3 + 96*e**(13*c + 13*d
*x)*a*b**2 + 24*e**(12*c + 12*d*x)*log(e**(c + d*x) - 1)*a**3 - 24*e**(12*
c + 12*d*x)*log(e**(c + d*x) + 1)*a**3 - 96*e**(12*c + 12*d*x)*a**2*b + 24
*e**(12*c + 12*d*x)*b**3*d*x + 27*e**(12*c + 12*d*x)*b**3 + 48*e**(11*c +
11*d*x)*a**3 - 288*e**(11*c + 11*d*x)*a*b**2 - 96*e**(10*c + 10*d*x)*log(e
**(c + d*x) - 1)*a**3 + 96*e**(10*c + 10*d*x)*log(e**(c + d*x) + 1)*a**3 -
 96*e**(10*c + 10*d*x)*b**3*d*x - 176*e**(9*c + 9*d*x)*a**3 + 192*e**(9*c
+ 9*d*x)*a*b**2 + 144*e**(8*c + 8*d*x)*log(e**(c + d*x) - 1)*a**3 - 144*e*
*(8*c + 8*d*x)*log(e**(c + d*x) + 1)*a**3 + 576*e**(8*c + 8*d*x)*a**2*b +
144*e**(8*c + 8*d*x)*b**3*d*x - 66*e**(8*c + 8*d*x)*b**3 - 176*e**(7*c + 7
*d*x)*a**3 + 192*e**(7*c + 7*d*x)*a*b**2 - 96*e**(6*c + 6*d*x)*log(e**(c +
 d*x) - 1)*a**3 + 96*e**(6*c + 6*d*x)*log(e**(c + d*x) + 1)*a**3 - 768*e**
(6*c + 6*d*x)*a**2*b - 96*e**(6*c + 6*d*x)*b**3*d*x + 88*e**(6*c + 6*d*x)*
b**3 + 48*e**(5*c + 5*d*x)*a**3 - 288*e**(5*c + 5*d*x)*a*b**2 + 24*e**(4*c
 + 4*d*x)*log(e**(c + d*x) - 1)*a**3 - 24*e**(4*c + 4*d*x)*log(e**(c + d*x
) + 1)*a**3 + 288*e**(4*c + 4*d*x)*a**2*b + 24*e**(4*c + 4*d*x)*b**3*d*x -
 49*e**(4*c + 4*d*x)*b**3 + 96*e**(3*c + 3*d*x)*a*b**2 + 12*e**(2*c + 2*d*
x)*b**3 - b**3)/(64*e**(4*c + 4*d*x)*d*(e**(8*c + 8*d*x) - 4*e**(6*c + 6*d
*x) + 6*e**(4*c + 4*d*x) - 4*e**(2*c + 2*d*x) + 1))
```

3.146 $\int \operatorname{csch}^6(c+dx)\left(a+b\sinh^3(c+dx)\right)^3\,dx$

Optimal result

Integrand size = 23, antiderivative size = 131

$$\int \operatorname{csch}^6(c+dx)\left(a+b\sinh^3(c+dx)\right)^3\,dx = 3ab^2x+\frac{3a^2b\operatorname{arctanh}(\cosh(c+dx))}{2d} - \frac{b^3\cosh(c+dx)}{d}+\frac{b^3\cosh^3(c+dx)}{3d} - \frac{a^3\coth(c+dx)}{d}+\frac{2a^3\coth^3(c+dx)}{3d} - \frac{a^3\coth^5(c+dx)}{5d} - \frac{3a^2b\coth(c+dx)\operatorname{csch}(c+dx)}{2d}$$

output
```
3*a*b^2*x+3/2*a^2*b*arctanh(cosh(d*x+c))/d-b^3*cosh(d*x+c)/d+1/3*b^3*cosh(
d*x+c)^3/d-a^3*coth(d*x+c)/d+2/3*a^3*coth(d*x+c)^3/d-1/5*a^3*coth(d*x+c)^5
/d-3/2*a^2*b*coth(d*x+c)*csch(d*x+c)/d
```

Mathematica [A] (verified)

Time = 3.43 (sec) , antiderivative size = 238, normalized size of antiderivative = 1.82

$$\int \operatorname{csch}^6(c+dx)\left(a+b\sinh^3(c+dx)\right)^3\,dx$$
$$=\frac{-360b^3\cosh(c+dx)+40b^3\cosh(3(c+dx))+\frac{1}{2}a\left(-256a^2\coth\left(\frac{1}{2}(c+dx)\right)-360ab\operatorname{csch}^2\left(\frac{1}{2}(c+dx)\right)+\right.}{}$$

input
```
Integrate[Csch[c + d*x]^6*(a + b*Sinh[c + d*x]^3)^3,x]
```

output
```
(-360*b^3*Cosh[c + d*x] + 40*b^3*Cosh[3*(c + d*x)] + (a*(-256*a^2*Coth[(c
+ d*x)/2] - 360*a*b*Csch[(c + d*x)/2]^2 + 19*a^2*Csch[(c + d*x)/2]^4*Sinh[
c + d*x] - 3*a^2*Csch[(c + d*x)/2]^6*Sinh[c + d*x] + 8*(180*b*(2*b*(c + d*
x) + a*Log[Cosh[(c + d*x)/2]] - a*Log[Sinh[(c + d*x)/2]]) - 45*a*b*Sech[(c
 + d*x)/2]^2 - 38*a^2*Csch[c + d*x]^3*Sinh[(c + d*x)/2]^4 - 24*a^2*Csch[c
+ d*x]^5*Sinh[(c + d*x)/2]^6 - 32*a^2*Tanh[(c + d*x)/2])))/2)/(480*d)
```

Rubi [A] (verified)

Time = 0.44 (sec) , antiderivative size = 131, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.174, Rules used = {3042, 25, 3699, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \operatorname{csch}^6(c+dx)\left(a+b\sinh^3(c+dx)\right)^3\,dx$$
$$\Big\downarrow \; 3042$$
$$\int -\frac{\left(a+ib\sin(ic+idx)^3\right)^3}{\sin(ic+idx)^6}dx$$
$$\Big\downarrow \; 25$$
$$-\int \frac{\left(ib\sin(ic+idx)^3+a\right)^3}{\sin(ic+idx)^6}dx$$

$$\Big\downarrow \text{3699}$$

$$-\int \left(-a^3\text{csch}^6(c+dx)-3a^2b\text{csch}^3(c+dx)-b^3\sinh^3(c+dx)-3ab^2\right)dx$$

$$\Big\downarrow \text{2009}$$

$$-\frac{a^3\coth^5(c+dx)}{5d}+\frac{2a^3\coth^3(c+dx)}{3d}-\frac{a^3\coth(c+dx)}{d}+\frac{3a^2b\text{arctanh}(\cosh(c+dx))}{2d}-\frac{3a^2b\coth(c+dx)\text{csch}(c+dx)}{2d}+3ab^2x+\frac{b^3\cosh^3(c+dx)}{3d}-\frac{b^3\cosh(c+dx)}{d}$$

input
```
Int[Csch[c + d*x]^6*(a + b*Sinh[c + d*x]^3)^3,x]
```

output
```
3*a*b^2*x + (3*a^2*b*ArcTanh[Cosh[c + d*x]])/(2*d) - (b^3*Cosh[c + d*x])/d
 + (b^3*Cosh[c + d*x]^3)/(3*d) - (a^3*Coth[c + d*x])/d + (2*a^3*Coth[c + d
*x]^3)/(3*d) - (a^3*Coth[c + d*x]^5)/(5*d) - (3*a^2*b*Coth[c + d*x]*Csch[c
 + d*x])/(2*d)
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3699
```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^(n_
))^(p_.), x_Symbol] :> Int[ExpandTrig[sin[e + f*x]^m*(a + b*sin[e + f*x]^n)
^p, x], x] /; FreeQ[{a, b, e, f}, x] && IntegersQ[m, p] && (EqQ[n, 4] || Gt
Q[p, 0] || (EqQ[p, -1] && IntegerQ[n]))
```

Maple [A] (verified)

Time = 1.86 (sec) , antiderivative size = 99, normalized size of antiderivative = 0.76

method	result
derivativedivides	$\frac{a^3\left(-\frac{8}{15}-\frac{\operatorname{csch}(dx+c)^4}{5}+\frac{4\operatorname{csch}(dx+c)^2}{15}\right)\coth(dx+c)+3a^2b\left(-\frac{\operatorname{csch}(dx+c)\coth(dx+c)}{2}+\operatorname{arctanh}\left(e^{dx+c}\right)\right)+3b^2a(dx+c)+}{d}$
default	$\frac{a^3\left(-\frac{8}{15}-\frac{\operatorname{csch}(dx+c)^4}{5}+\frac{4\operatorname{csch}(dx+c)^2}{15}\right)\coth(dx+c)+3a^2b\left(-\frac{\operatorname{csch}(dx+c)\coth(dx+c)}{2}+\operatorname{arctanh}\left(e^{dx+c}\right)\right)+3b^2a(dx+c)+}{d}$
parallelrisch	$\frac{-144\ln\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)\right)a^2b-\operatorname{sech}\left(\frac{dx}{2}+\frac{c}{2}\right)^5\left(\cosh(dx+c)-\frac{\cosh(3dx+3c)}{2}+\frac{\cosh(5dx+5c)}{10}\right)a^3\operatorname{csch}\left(\frac{dx}{2}+\frac{c}{2}\right)^5+36ba^2\left(\mathrm{se}\right.}{96d}$
risch	$3ab^2x+\frac{e^{3dx+3c}b^3}{24d}-\frac{3e^{dx+c}b^3}{8d}-\frac{3e^{-dx-c}b^3}{8d}+\frac{e^{-3dx-3c}b^3}{24d}-\frac{a^2\left(45b\,e^{9dx+9c}-90b\,e^{7dx+7c}+160\,e^{4dx+4c}\right.}{15d\left(e^{2c}\right.}$

input
```
int(csch(d*x+c)^6*(a+b*sinh(d*x+c)^3)^3,x,method=_RETURNVERBOSE)
```

output
```
1/d*(a^3*(-8/15-1/5*csch(d*x+c)^4+4/15*csch(d*x+c)^2)*coth(d*x+c)+3*a^2*b*
(-1/2*csch(d*x+c)*coth(d*x+c)+arctanh(exp(d*x+c)))+3*b^2*a*(d*x+c)+b^3*(-2
/3+1/3*sinh(d*x+c)^2)*cosh(d*x+c))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 4629 vs. 2(121) = 242.

Time = 0.13 (sec) , antiderivative size = 4629, normalized size of antiderivative = 35.34

$$\int \operatorname{csch}^6(c+dx)\left(a+b\sinh^3(c+dx)\right)^3\,dx=\text{Too large to display}$$

input
```
integrate(csch(d*x+c)^6*(a+b*sinh(d*x+c)^3)^3,x, algorithm="fricas")
```

output
```
Too large to include
```

Sympy [F(-1)]

Timed out.

$$\int \operatorname{csch}^6(c+dx)\left(a+b\sinh^3(c+dx)\right)^3\,dx=\text{Timed out}$$

input
```
integrate(csch(d*x+c)**6*(a+b*sinh(d*x+c)**3)**3,x)
```

output
```
Timed out
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 365 vs. 2(121) = 242.

Time = 0.04 (sec) , antiderivative size = 365, normalized size of antiderivative = 2.79

$$\begin{aligned}&\int \operatorname{csch}^6(c+dx)\left(a+b\sinh^3(c+dx)\right)^3\,dx\\&=3\,ab^2x+\frac{1}{24}\,b^3\left(\frac{e^{(3\,dx+3\,c)}}{d}-\frac{9\,e^{(dx+c)}}{d}-\frac{9\,e^{(-dx-c)}}{d}+\frac{e^{(-3\,dx-3\,c)}}{d}\right)\\&\quad+\frac{3}{2}\,a^2b\left(\frac{\log\left(e^{(-dx-c)}+1\right)}{d}-\frac{\log\left(e^{(-dx-c)}-1\right)}{d}+\frac{2\left(e^{(-dx-c)}+e^{(-3\,dx-3\,c)}\right)}{d(2\,e^{(-2\,dx-2\,c)}-e^{(-4\,dx-4\,c)}-1)}\right)\\&\quad-\frac{16}{15}\,a^3\left(\frac{5\,e^{(-2\,dx-2\,c)}}{d(5\,e^{(-2\,dx-2\,c)}-10\,e^{(-4\,dx-4\,c)}+10\,e^{(-6\,dx-6\,c)}-5\,e^{(-8\,dx-8\,c)}+e^{(-10\,dx-10\,c)}-1)}-\frac{}{d(5\,e^{(-2\,dx-}}\right.\end{aligned}$$

input
```
integrate(csch(d*x+c)^6*(a+b*sinh(d*x+c)^3)^3,x, algorithm="maxima")
```

output
```
3*a*b^2*x + 1/24*b^3*(e^(3*d*x + 3*c)/d - 9*e^(d*x + c)/d - 9*e^(-d*x - c)
/d + e^(-3*d*x - 3*c)/d) + 3/2*a^2*b*(log(e^(-d*x - c) + 1)/d - log(e^(-d*
x - c) - 1)/d + 2*(e^(-d*x - c) + e^(-3*d*x - 3*c))/(d*(2*e^(-2*d*x - 2*c)
 - e^(-4*d*x - 4*c) - 1))) - 16/15*a^3*(5*e^(-2*d*x - 2*c)/(d*(5*e^(-2*d*x
 - 2*c) - 10*e^(-4*d*x - 4*c) + 10*e^(-6*d*x - 6*c) - 5*e^(-8*d*x - 8*c) +
 e^(-10*d*x - 10*c) - 1)) - 10*e^(-4*d*x - 4*c)/(d*(5*e^(-2*d*x - 2*c) - 1
0*e^(-4*d*x - 4*c) + 10*e^(-6*d*x - 6*c) - 5*e^(-8*d*x - 8*c) + e^(-10*d*x
 - 10*c) - 1)) - 1/(d*(5*e^(-2*d*x - 2*c) - 10*e^(-4*d*x - 4*c) + 10*e^(-6
*d*x - 6*c) - 5*e^(-8*d*x - 8*c) + e^(-10*d*x - 10*c) - 1)))
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 270 vs. $2(121) = 242$.

Time = 0.23 (sec) , antiderivative size = 270, normalized size of antiderivative = 2.06

$$\int \operatorname{csch}^6(c+dx)\left(a+b\sinh^3(c+dx)\right)^3\,dx = \frac{360\,(dx+c)ab^2+5\,b^3e^{(3\,dx+3\,c)}-45\,b^3e^{(dx+c)}+180\,a^2b\log\left(e^{(dx+c)}+1\right)-180\,a^2b\log\left(\left|e^{(dx+c)}-1\right|\right)-\left(\cdots\right.}{}$$

input
```
integrate(csch(d*x+c)^6*(a+b*sinh(d*x+c)^3)^3,x, algorithm="giac")
```

output
```
1/120*(360*(d*x + c)*a*b^2 + 5*b^3*e^(3*d*x + 3*c) - 45*b^3*e^(d*x + c) +
180*a^2*b*log(e^(d*x + c) + 1) - 180*a^2*b*log(abs(e^(d*x + c) - 1)) - (47
5*b^3*e^(8*d*x + 8*c) + 1280*a^3*e^(7*d*x + 7*c) - 640*a^3*e^(5*d*x + 5*c)
 + 128*a^3*e^(3*d*x + 3*c) - 70*b^3*e^(2*d*x + 2*c) + 5*b^3 + 45*(8*a^2*b
+ b^3)*e^(12*d*x + 12*c) - 10*(72*a^2*b + 23*b^3)*e^(10*d*x + 10*c) + 20*(
36*a^2*b - 25*b^3)*e^(6*d*x + 6*c) - 5*(72*a^2*b - 55*b^3)*e^(4*d*x + 4*c)
)*e^(-3*d*x - 3*c)/((e^(d*x + c) + 1)^5*(e^(d*x + c) - 1)^5))/d
```

Mupad [B] (verification not implemented)

Time = 1.74 (sec) , antiderivative size = 432, normalized size of antiderivative = 3.30

$$\begin{aligned}\int \operatorname{csch}^6(c+dx)\left(a+b\sinh^3(c+dx)\right)^3\,dx &= \frac{b^3\,\mathrm{e}^{-3\,c-3\,d\,x}}{24\,d}-\frac{3\,b^3\,\mathrm{e}^{c+d\,x}}{8\,d}-\frac{3\,b^3\,\mathrm{e}^{-c-d\,x}}{8\,d}\\ &\quad-\frac{\frac{32\,a^3\,\mathrm{e}^{4\,c+4\,d\,x}}{5\,d}+\frac{36\,a^2\,b\,\mathrm{e}^{3\,c+3\,d\,x}}{5\,d}-\frac{36\,a^2\,b\,\mathrm{e}^{5\,c+5\,d\,x}}{5\,d}+\frac{12\,a^2\,b\,\mathrm{e}^{7\,c+7\,d\,x}}{5\,d}-\frac{12\,a^2\,b\,\mathrm{e}^{c+d\,x}}{5\,d}}{5\,\mathrm{e}^{2\,c+2\,d\,x}-10\,\mathrm{e}^{4\,c+4\,d\,x}+10\,\mathrm{e}^{6\,c+6\,d\,x}-5\,\mathrm{e}^{8\,c+8\,d\,x}+\mathrm{e}^{10\,c+10\,d\,x}-1}\\ &\quad+\frac{b^3\,\mathrm{e}^{3\,c+3\,d\,x}}{24\,d}-\frac{64\,a^3}{15\,d\,\left(3\,\mathrm{e}^{2\,c+2\,d\,x}-3\,\mathrm{e}^{4\,c+4\,d\,x}+\mathrm{e}^{6\,c+6\,d\,x}-1\right)}\\ &\quad-\frac{16\,a^3}{5\,d\,\left(6\,\mathrm{e}^{4\,c+4\,d\,x}-4\,\mathrm{e}^{2\,c+2\,d\,x}-4\,\mathrm{e}^{6\,c+6\,d\,x}+\mathrm{e}^{8\,c+8\,d\,x}+1\right)}\\ &\quad+\frac{3\,\mathrm{atan}\left(\frac{a^2\,b\,\mathrm{e}^{d\,x}\,\mathrm{e}^{c}\,\sqrt{-d^2}}{d\,\sqrt{a^4\,b^2}}\right)\,\sqrt{a^4\,b^2}}{\sqrt{-d^2}}+3\,a\,b^2\,x\\ &\quad-\frac{3\,a^2\,b\,\mathrm{e}^{c+d\,x}}{d\,\left(\mathrm{e}^{2\,c+2\,d\,x}-1\right)}-\frac{18\,a^2\,b\,\mathrm{e}^{c+d\,x}}{5\,d\,\left(\mathrm{e}^{4\,c+4\,d\,x}-2\,\mathrm{e}^{2\,c+2\,d\,x}+1\right)}\end{aligned}$$

input
```
int((a + b*sinh(c + d*x)^3)^3/sinh(c + d*x)^6,x)
```

output
```
(b^3*exp(- 3*c - 3*d*x))/(24*d) - (3*b^3*exp(c + d*x))/(8*d) - (3*b^3*exp(
- c - d*x))/(8*d) - ((32*a^3*exp(4*c + 4*d*x))/(5*d) + (36*a^2*b*exp(3*c +
 3*d*x))/(5*d) - (36*a^2*b*exp(5*c + 5*d*x))/(5*d) + (12*a^2*b*exp(7*c + 7
*d*x))/(5*d) - (12*a^2*b*exp(c + d*x))/(5*d))/(5*exp(2*c + 2*d*x) - 10*exp
(4*c + 4*d*x) + 10*exp(6*c + 6*d*x) - 5*exp(8*c + 8*d*x) + exp(10*c + 10*d
*x) - 1) + (b^3*exp(3*c + 3*d*x))/(24*d) - (64*a^3)/(15*d*(3*exp(2*c + 2*d
*x) - 3*exp(4*c + 4*d*x) + exp(6*c + 6*d*x) - 1)) - (16*a^3)/(5*d*(6*exp(4
*c + 4*d*x) - 4*exp(2*c + 2*d*x) - 4*exp(6*c + 6*d*x) + exp(8*c + 8*d*x) +
 1)) + (3*atan((a^2*b*exp(d*x)*exp(c)*(-d^2)^(1/2))/(d*(a^4*b^2)^(1/2)))*(
a^4*b^2)^(1/2))/(-d^2)^(1/2) + 3*a*b^2*x - (3*a^2*b*exp(c + d*x))/(d*(exp(
2*c + 2*d*x) - 1)) - (18*a^2*b*exp(c + d*x))/(5*d*(exp(4*c + 4*d*x) - 2*ex
p(2*c + 2*d*x) + 1))
```

Reduce [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 719, normalized size of antiderivative = 5.49

$$\int \operatorname{csch}^6(c+dx) \left(a+b\sinh^3(c+dx)\right)^3 \, dx = \frac{-5b^3 + 900e^{11dx+11c}\log\left(e^{dx+c}-1\right)a^2b - 900e^{11dx+11c}\log\left(e^{dx+c}+1\right)a^2b - 1800e^{9dx+9c}\log\left(e^{dx+c}-1\right)a^2b}{}$$

input
```
int(csch(d*x+c)^6*(a+b*sinh(d*x+c)^3)^3,x)
```

output
```
(5*e**(16*c + 16*d*x)*b**3 - 70*e**(14*c + 14*d*x)*b**3 - 180*e**(13*c + 1
3*d*x)*log(e**(c + d*x) - 1)*a**2*b + 180*e**(13*c + 13*d*x)*log(e**(c + d
*x) + 1)*a**2*b + 360*e**(13*c + 13*d*x)*a*b**2*d*x - 360*e**(12*c + 12*d*
x)*a**2*b + 230*e**(12*c + 12*d*x)*b**3 + 900*e**(11*c + 11*d*x)*log(e**(c
 + d*x) - 1)*a**2*b - 900*e**(11*c + 11*d*x)*log(e**(c + d*x) + 1)*a**2*b
- 1800*e**(11*c + 11*d*x)*a*b**2*d*x + 720*e**(10*c + 10*d*x)*a**2*b - 270
*e**(10*c + 10*d*x)*b**3 - 1800*e**(9*c + 9*d*x)*log(e**(c + d*x) - 1)*a**
2*b + 1800*e**(9*c + 9*d*x)*log(e**(c + d*x) + 1)*a**2*b + 3600*e**(9*c +
9*d*x)*a*b**2*d*x + 1800*e**(7*c + 7*d*x)*log(e**(c + d*x) - 1)*a**2*b - 1
800*e**(7*c + 7*d*x)*log(e**(c + d*x) + 1)*a**2*b - 1280*e**(7*c + 7*d*x)*
a**3 - 3600*e**(7*c + 7*d*x)*a*b**2*d*x - 720*e**(6*c + 6*d*x)*a**2*b + 27
0*e**(6*c + 6*d*x)*b**3 - 900*e**(5*c + 5*d*x)*log(e**(c + d*x) - 1)*a**2*
b + 900*e**(5*c + 5*d*x)*log(e**(c + d*x) + 1)*a**2*b + 640*e**(5*c + 5*d*
x)*a**3 + 1800*e**(5*c + 5*d*x)*a*b**2*d*x + 360*e**(4*c + 4*d*x)*a**2*b -
 230*e**(4*c + 4*d*x)*b**3 + 180*e**(3*c + 3*d*x)*log(e**(c + d*x) - 1)*a*
*2*b - 180*e**(3*c + 3*d*x)*log(e**(c + d*x) + 1)*a**2*b - 128*e**(3*c + 3
*d*x)*a**3 - 360*e**(3*c + 3*d*x)*a*b**2*d*x + 70*e**(2*c + 2*d*x)*b**3 -
5*b**3)/(120*e**(3*c + 3*d*x)*d*(e**(10*c + 10*d*x) - 5*e**(8*c + 8*d*x) +
 10*e**(6*c + 6*d*x) - 10*e**(4*c + 4*d*x) + 5*e**(2*c + 2*d*x) - 1))
```

3.147 $\int \operatorname{csch}^7(c+dx) \left(a+b\sinh^3(c+dx)\right)^3 dx$

Optimal result

Integrand size = 23, antiderivative size = 166

$$\int \operatorname{csch}^7(c+dx) \left(a+b\sinh^3(c+dx)\right)^3 dx = -\frac{b^3x}{2} + \frac{5a^3\operatorname{arctanh}(\cosh(c+dx))}{16d} - \frac{3ab^2\operatorname{arctanh}(\cosh(c+dx))}{d} + \frac{3a^2b\coth(c+dx)}{d} - \frac{a^2b\coth^3(c+dx)}{d} - \frac{5a^3\coth(c+dx)\operatorname{csch}(c+dx)}{16d} + \frac{5a^3\coth(c+dx)\operatorname{csch}^3(c+dx)}{24d} - \frac{a^3\coth(c+dx)\operatorname{csch}^5(c+dx)}{6d} + \frac{b^3\cosh(c+dx)\sinh(c+dx)}{2d}$$

output
```
-1/2*b^3*x+5/16*a^3*arctanh(cosh(d*x+c))/d-3*a*b^2*arctanh(cosh(d*x+c))/d+
3*a^2*b*coth(d*x+c)/d-a^2*b*coth(d*x+c)^3/d-5/16*a^3*coth(d*x+c)*csch(d*x+
c)/d+5/24*a^3*coth(d*x+c)*csch(d*x+c)^3/d-1/6*a^3*coth(d*x+c)*csch(d*x+c)^
5/d+1/2*b^3*cosh(d*x+c)*sinh(d*x+c)/d
```

Mathematica [A] (verified)

Time = 3.10 (sec) , antiderivative size = 269, normalized size of antiderivative = 1.62

$$\int \operatorname{csch}^7(c+dx) \left(a+b\sinh^3(c+dx)\right)^3 \, dx = -\frac{192b^3c+192b^3dx-384a^2b\coth\left(\frac{1}{2}(c+dx)\right)+30a^3\operatorname{csch}^2\left(\frac{1}{2}(c+dx)\right)+a^3\operatorname{csch}^6\left(\frac{1}{2}(c+dx)\right)-120a^3 \operatorname{l}}{}$$

input
```
Integrate[Csch[c + d*x]^7*(a + b*Sinh[c + d*x]^3)^3,x]
```

output
```
-1/384*(192*b^3*c + 192*b^3*d*x - 384*a^2*b*Coth[(c + d*x)/2] + 30*a^3*Csc
h[(c + d*x)/2]^2 + a^3*Csch[(c + d*x)/2]^6 - 120*a^3*Log[Cosh[(c + d*x)/2]
] + 1152*a*b^2*Log[Cosh[(c + d*x)/2]] + 120*a^3*Log[Sinh[(c + d*x)/2]] - 1
152*a*b^2*Log[Sinh[(c + d*x)/2]] + 30*a^3*Sech[(c + d*x)/2]^2 + 6*a^3*Sech
[(c + d*x)/2]^4 + a^3*Sech[(c + d*x)/2]^6 - 384*a^2*b*Csch[c + d*x]^3*Sinh
[(c + d*x)/2]^4 - 6*a^2*Csch[(c + d*x)/2]^4*(a - 4*b*Sinh[c + d*x]) - 96*b
^3*Sinh[2*(c + d*x)] - 384*a^2*b*Tanh[(c + d*x)/2])/d
```

Rubi [C] (verified)

Result contains complex when optimal does not.

Time = 0.50 (sec) , antiderivative size = 188, normalized size of antiderivative = 1.13, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.174$, Rules used = {3042, 26, 3699, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \operatorname{csch}^7(c+dx) \left(a+b\sinh^3(c+dx)\right)^3 \, dx$$

$$\Big\downarrow \; 3042$$

$$\int -\frac{i\left(a+ib\sin(ic+idx)^3\right)^3}{\sin(ic+idx)^7}dx$$

$$\Big\downarrow \; 26$$

$$-i\int \frac{(ib\sin(ic+idx)^3+a)^3}{\sin(ic+idx)^7}dx$$

$$\Big\downarrow \text{3699}$$

$$-i\int \left(ia^3\text{csch}^7(c+dx)+3ia^2b\text{csch}^4(c+dx)+3iab^2\text{csch}(c+dx)+ib^3\sinh^2(c+dx)\right)dx$$

$$\Big\downarrow \text{2009}$$

$$-i\left(\frac{5ia^3\text{arctanh}(\cosh(c+dx))}{16d}-\frac{ia^3\coth(c+dx)\text{csch}^5(c+dx)}{6d}+\frac{5ia^3\coth(c+dx)\text{csch}^3(c+dx)}{24d}-\frac{5ia^3\coth(\right.$$

input
```
Int[Csch[c + d*x]^7*(a + b*Sinh[c + d*x]^3)^3,x]
```

output
```
(-I)*((-1/2*I)*b^3*x + (((5*I)/16)*a^3*ArcTanh[Cosh[c + d*x]])/d - ((3*I)*
a*b^2*ArcTanh[Cosh[c + d*x]])/d + ((3*I)*a^2*b*Coth[c + d*x])/d - (I*a^2*b
*Coth[c + d*x]^3)/d - (((5*I)/16)*a^3*Coth[c + d*x]*Csch[c + d*x])/d + (((
5*I)/24)*a^3*Coth[c + d*x]*Csch[c + d*x]^3)/d - ((I/6)*a^3*Coth[c + d*x]*C
sch[c + d*x]^5)/d + ((I/2)*b^3*Cosh[c + d*x]*Sinh[c + d*x])/d)
```

Defintions of rubi rules used

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3699
```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^(n_
))^(p_.), x_Symbol] :> Int[ExpandTrig[sin[e + f*x]^m*(a + b*sin[e + f*x]^n)
^p, x], x] /; FreeQ[{a, b, e, f}, x] && IntegersQ[m, p] && (EqQ[n, 4] || Gt
Q[p, 0] || (EqQ[p, -1] && IntegerQ[n]))
```

Maple [A] (verified)

Time = 1.83 (sec) , antiderivative size = 119, normalized size of antiderivative = 0.72

method	result
derivativedivides	$\frac{a^{3}\left(\left(-\frac{\operatorname{csch}(dx+c)^{5}}{6}+\frac{5\operatorname{csch}(dx+c)^{3}}{24}-\frac{5\operatorname{csch}(dx+c)}{16}\right)\coth(dx+c)+\frac{5\operatorname{arctanh}\left(e^{dx+c}\right)}{8}\right)+3a^{2}b\left(\frac{2}{3}-\frac{\operatorname{csch}(dx+c)^{2}}{3}\right)\coth(dx}{d}$
default	$\frac{a^{3}\left(\left(-\frac{\operatorname{csch}(dx+c)^{5}}{6}+\frac{5\operatorname{csch}(dx+c)^{3}}{24}-\frac{5\operatorname{csch}(dx+c)}{16}\right)\coth(dx+c)+\frac{5\operatorname{arctanh}\left(e^{dx+c}\right)}{8}\right)+3a^{2}b\left(\frac{2}{3}-\frac{\operatorname{csch}(dx+c)^{2}}{3}\right)\coth(dx}{d}$
parallelrisch	$\frac{\left(-245760a^{3}+2359296b^{2}a\right)\ln\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-240\left(\cosh(5dx+5c)-\frac{33\cosh(6dx+6c)}{80}+\frac{66\cosh(dx+c)}{5}-\frac{99\cosh(2dx+2c)}{16}\right.}{}$
risch	$-\frac{b^{3}x}{2}+\frac{e^{2dx+2c}b^{3}}{8d}-\frac{e^{-2dx-2c}b^{3}}{8d}-\frac{a^{2}\left(15a\,e^{11dx+11c}-85a\,e^{9dx+9c}+288\,e^{8dx+8c}b+198a\,e^{7dx+7c}-960\,e^{6dx+6c}\right.}{24d\left(e^{2c}\right.}$

input
```
int(csch(d*x+c)^7*(a+b*sinh(d*x+c)^3)^3,x,method=_RETURNVERBOSE)
```

output
```
1/d*(a^3*((-1/6*csch(d*x+c)^5+5/24*csch(d*x+c)^3-5/16*csch(d*x+c))*coth(d*
x+c)+5/8*arctanh(exp(d*x+c)))+3*a^2*b*(2/3-1/3*csch(d*x+c)^2)*coth(d*x+c)-
6*b^2*a*arctanh(exp(d*x+c))+b^3*(1/2*cosh(d*x+c)*sinh(d*x+c)-1/2*d*x-1/2*c
))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 6210 vs. $2(154) = 308$.

Time = 0.15 (sec) , antiderivative size = 6210, normalized size of antiderivative = 37.41

$$\int \operatorname{csch}^7(c+dx)\left(a+b\sinh^3(c+dx)\right)^3\,dx = \text{Too large to display}$$

input
```
integrate(csch(d*x+c)^7*(a+b*sinh(d*x+c)^3)^3,x, algorithm="fricas")
```

output
```
Too large to include
```

Sympy [F(-1)]

Timed out.

$$\int \operatorname{csch}^7(c+dx)\left(a+b\sinh^3(c+dx)\right)^3\,dx=\text{Timed out}$$

input
```
integrate(csch(d*x+c)**7*(a+b*sinh(d*x+c)**3)**3,x)
```

output
```
Timed out
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 355 vs. $2(154) = 308$.

Time = 0.04 (sec) , antiderivative size = 355, normalized size of antiderivative = 2.14

$$\int \operatorname{csch}^7(c+dx)\left(a+b\sinh^3(c+dx)\right)^3\,dx = -\frac{1}{8}b^3\left(4x-\frac{e^{(2dx+2c)}}{d}+\frac{e^{(-2dx-2c)}}{d}\right) + \frac{1}{48}a^3\left(\frac{15\log\left(e^{(-dx-c)}+1\right)}{d}-\frac{15\log\left(e^{(-dx-c)}-1\right)}{d}+\frac{2\left(15e^{(-dx-c)}-85e^{(-3dx-3c)}+198e^{(-5dx-5c)}\right.}{d(6e^{(-2dx-2c)}-15e^{(-4dx-4c)}+20e^{(-6dx-6c)}}\right. - 3ab^2\left(\frac{\log\left(e^{(-dx-c)}+1\right)}{d}-\frac{\log\left(e^{(-dx-c)}-1\right)}{d}\right) + 4a^2b\left(\frac{3e^{(-2dx-2c)}}{d(3e^{(-2dx-2c)}-3e^{(-4dx-4c)}+e^{(-6dx-6c)}-1)}-\frac{1}{d(3e^{(-2dx-2c)}-3e^{(-4dx-4c)}+e^{(-6dx-6c)}-1}\right.$$

input
```
integrate(csch(d*x+c)^7*(a+b*sinh(d*x+c)^3)^3,x, algorithm="maxima")
```

output
```
-1/8*b^3*(4*x - e^(2*d*x + 2*c)/d + e^(-2*d*x - 2*c)/d) + 1/48*a^3*(15*log
(e^(-d*x - c) + 1)/d - 15*log(e^(-d*x - c) - 1)/d + 2*(15*e^(-d*x - c) - 8
5*e^(-3*d*x - 3*c) + 198*e^(-5*d*x - 5*c) + 198*e^(-7*d*x - 7*c) - 85*e^(-
9*d*x - 9*c) + 15*e^(-11*d*x - 11*c))/(d*(6*e^(-2*d*x - 2*c) - 15*e^(-4*d*
x - 4*c) + 20*e^(-6*d*x - 6*c) - 15*e^(-8*d*x - 8*c) + 6*e^(-10*d*x - 10*c
) - e^(-12*d*x - 12*c) - 1))) - 3*a*b^2*(log(e^(-d*x - c) + 1)/d - log(e^(
-d*x - c) - 1)/d) + 4*a^2*b*(3*e^(-2*d*x - 2*c)/(d*(3*e^(-2*d*x - 2*c) - 3
*e^(-4*d*x - 4*c) + e^(-6*d*x - 6*c) - 1)) - 1/(d*(3*e^(-2*d*x - 2*c) - 3*
e^(-4*d*x - 4*c) + e^(-6*d*x - 6*c) - 1)))
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 327 vs. $2(154) = 308$.

Time = 0.25 (sec) , antiderivative size = 327, normalized size of antiderivative = 1.97

$$\int \operatorname{csch}^7(c+dx)\left(a+b\sinh^3(c+dx)\right)^3\,dx =$$

$$-\frac{24\,(dx+c)b^3-6\,b^3e^{(2\,dx+2\,c)}-3\,(5\,a^3-48\,ab^2)\log\left(e^{(dx+c)}+1\right)+3\,(5\,a^3-48\,ab^2)\log\left(\left|e^{(dx+c)}-1\right|\right)}{}$$

input
```
integrate(csch(d*x+c)^7*(a+b*sinh(d*x+c)^3)^3,x, algorithm="giac")
```

output
```
-1/48*(24*(d*x + c)*b^3 - 6*b^3*e^(2*d*x + 2*c) - 3*(5*a^3 - 48*a*b^2)*log
(e^(d*x + c) + 1) + 3*(5*a^3 - 48*a*b^2)*log(abs(e^(d*x + c) - 1)) + 2*(15
*a^3*e^(13*d*x + 13*c) + 3*b^3*e^(12*d*x + 12*c) - 85*a^3*e^(11*d*x + 11*c
) + 198*a^3*e^(9*d*x + 9*c) + 198*a^3*e^(7*d*x + 7*c) - 85*a^3*e^(5*d*x +
5*c) + 15*a^3*e^(3*d*x + 3*c) + 3*b^3 + 18*(16*a^2*b - b^3)*e^(10*d*x + 10
*c) - 15*(64*a^2*b - 3*b^3)*e^(8*d*x + 8*c) + 12*(96*a^2*b - 5*b^3)*e^(6*d
*x + 6*c) - 9*(64*a^2*b - 5*b^3)*e^(4*d*x + 4*c) + 6*(16*a^2*b - 3*b^3)*e^
(2*d*x + 2*c))*e^(-2*d*x - 2*c)/((e^(d*x + c) + 1)^6*(e^(d*x + c) - 1)^6))
/d
```

Mupad [B] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 486, normalized size of antiderivative = 2.93

$$\int \operatorname{csch}^7(c+dx)\left(a+b\sinh^3(c+dx)\right)^3\,dx$$
$$= \frac{\operatorname{atan}\left(\frac{e^{dx}e^{c}\left(5a^3\sqrt{-d^2}-48ab^2\sqrt{-d^2}\right)}{d\sqrt{25a^6-480a^4b^2+2304a^2b^4}}\right)\sqrt{25a^6-480a^4b^2+2304a^2b^4}}{8\sqrt{-d^2}}-\frac{b^3x}{2}$$
$$-\frac{\frac{12a^2b}{d}-\frac{5a^3e^{c+dx}}{12d}}{e^{4c+4dx}-2e^{2c+2dx}+1}-\frac{\frac{8a^2b}{d}+\frac{a^3e^{c+dx}}{3d}}{3e^{2c+2dx}-3e^{4c+4dx}+e^{6c+6dx}-1}-\frac{b^3e^{-2c-2dx}}{8d}$$
$$+\frac{b^3e^{2c+2dx}}{8d}-\frac{18a^3e^{c+dx}}{d\left(6e^{4c+4dx}-4e^{2c+2dx}-4e^{6c+6dx}+e^{8c+8dx}+1\right)}$$
$$-\frac{80a^3e^{c+dx}}{3d\left(5e^{2c+2dx}-10e^{4c+4dx}+10e^{6c+6dx}-5e^{8c+8dx}+e^{10c+10dx}-1\right)}$$
$$-\frac{32a^3e^{c+dx}}{3d\left(15e^{4c+4dx}-6e^{2c+2dx}-20e^{6c+6dx}+15e^{8c+8dx}-6e^{10c+10dx}+e^{12c+12dx}+1\right)}$$
$$-\frac{5a^3e^{c+dx}}{8d\left(e^{2c+2dx}-1\right)}$$

input

```
int((a + b*sinh(c + d*x)^3)^3/sinh(c + d*x)^7,x)
```

output

```
(atan((exp(d*x)*exp(c)*(5*a^3*(-d^2)^(1/2) - 48*a*b^2*(-d^2)^(1/2)))/(d*(2
5*a^6 + 2304*a^2*b^4 - 480*a^4*b^2)^(1/2)))*(25*a^6 + 2304*a^2*b^4 - 480*a
^4*b^2)^(1/2))/(8*(-d^2)^(1/2)) - (b^3*x)/2 - ((12*a^2*b)/d - (5*a^3*exp(c
 + d*x))/(12*d))/(exp(4*c + 4*d*x) - 2*exp(2*c + 2*d*x) + 1) - ((8*a^2*b)/
d + (a^3*exp(c + d*x))/(3*d))/(3*exp(2*c + 2*d*x) - 3*exp(4*c + 4*d*x) + e
xp(6*c + 6*d*x) - 1) - (b^3*exp(- 2*c - 2*d*x))/(8*d) + (b^3*exp(2*c + 2*d
*x))/(8*d) - (18*a^3*exp(c + d*x))/(d*(6*exp(4*c + 4*d*x) - 4*exp(2*c + 2*
d*x) - 4*exp(6*c + 6*d*x) + exp(8*c + 8*d*x) + 1)) - (80*a^3*exp(c + d*x))
/(3*d*(5*exp(2*c + 2*d*x) - 10*exp(4*c + 4*d*x) + 10*exp(6*c + 6*d*x) - 5*
exp(8*c + 8*d*x) + exp(10*c + 10*d*x) - 1)) - (32*a^3*exp(c + d*x))/(3*d*(
15*exp(4*c + 4*d*x) - 6*exp(2*c + 2*d*x) - 20*exp(6*c + 6*d*x) + 15*exp(8*
c + 8*d*x) - 6*exp(10*c + 10*d*x) + exp(12*c + 12*d*x) + 1)) - (5*a^3*exp(
c + d*x))/(8*d*(exp(2*c + 2*d*x) - 1))
```

Reduce [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 1205, normalized size of antiderivative = 7.26

$$\int \operatorname{csch}^7(c+dx) \left(a+b\sinh^3(c+dx)\right)^3 dx = \text{Too large to display}$$

input
```
int(csch(d*x+c)^7*(a+b*sinh(d*x+c)^3)^3,x)
```

output
```
(6*e**(16*c + 16*d*x)*b**3 - 15*e**(14*c + 14*d*x)*log(e**(c + d*x) - 1)*a
**3 + 144*e**(14*c + 14*d*x)*log(e**(c + d*x) - 1)*a*b**2 + 15*e**(14*c +
14*d*x)*log(e**(c + d*x) + 1)*a**3 - 144*e**(14*c + 14*d*x)*log(e**(c + d*
x) + 1)*a*b**2 - 24*e**(14*c + 14*d*x)*b**3*d*x - 22*e**(14*c + 14*d*x)*b*
*3 - 30*e**(13*c + 13*d*x)*a**3 + 90*e**(12*c + 12*d*x)*log(e**(c + d*x) -
 1)*a**3 - 864*e**(12*c + 12*d*x)*log(e**(c + d*x) - 1)*a*b**2 - 90*e**(12
*c + 12*d*x)*log(e**(c + d*x) + 1)*a**3 + 864*e**(12*c + 12*d*x)*log(e**(c
 + d*x) + 1)*a*b**2 + 144*e**(12*c + 12*d*x)*b**3*d*x + 170*e**(11*c + 11*
d*x)*a**3 - 225*e**(10*c + 10*d*x)*log(e**(c + d*x) - 1)*a**3 + 2160*e**(1
0*c + 10*d*x)*log(e**(c + d*x) - 1)*a*b**2 + 225*e**(10*c + 10*d*x)*log(e*
*(c + d*x) + 1)*a**3 - 2160*e**(10*c + 10*d*x)*log(e**(c + d*x) + 1)*a*b**
2 - 576*e**(10*c + 10*d*x)*a**2*b - 360*e**(10*c + 10*d*x)*b**3*d*x + 126*
e**(10*c + 10*d*x)*b**3 - 396*e**(9*c + 9*d*x)*a**3 + 300*e**(8*c + 8*d*x)
*log(e**(c + d*x) - 1)*a**3 - 2880*e**(8*c + 8*d*x)*log(e**(c + d*x) - 1)*
a*b**2 - 300*e**(8*c + 8*d*x)*log(e**(c + d*x) + 1)*a**3 + 2880*e**(8*c +
8*d*x)*log(e**(c + d*x) + 1)*a*b**2 + 1920*e**(8*c + 8*d*x)*a**2*b + 480*e
**(8*c + 8*d*x)*b**3*d*x - 280*e**(8*c + 8*d*x)*b**3 - 396*e**(7*c + 7*d*x
)*a**3 - 225*e**(6*c + 6*d*x)*log(e**(c + d*x) - 1)*a**3 + 2160*e**(6*c +
6*d*x)*log(e**(c + d*x) - 1)*a*b**2 + 225*e**(6*c + 6*d*x)*log(e**(c + d*x
) + 1)*a**3 - 2160*e**(6*c + 6*d*x)*log(e**(c + d*x) + 1)*a*b**2 - 2304...
```

3.148 $\int \frac{\sinh^6(c+dx)}{a+b\sinh^3(c+dx)}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 328

$$\begin{aligned}
\int \frac{\sinh^6(c+dx)}{a+b\sinh^3(c+dx)}\,dx &= -\frac{ax}{b^2}-\frac{2(-1)^{2/3}a^{4/3}\arctan\left(\frac{\sqrt[6]{-1}\left(\sqrt[6]{-1}\sqrt[3]{b}+i\sqrt[3]{a}\tanh\left(\frac{1}{2}(c+dx)\right)\right)}{\sqrt{\sqrt[3]{-1}a^{2/3}-(-1)^{2/3}b^{2/3}}}\right)}{3\sqrt{\sqrt[3]{-1}a^{2/3}-(-1)^{2/3}b^{2/3}}\,b^2d} \\
&\quad -\frac{2(-1)^{2/3}a^{4/3}\arctan\left(\frac{\sqrt[6]{-1}\left((-1)^{5/6}\sqrt[3]{b}+i\sqrt[3]{a}\tanh\left(\frac{1}{2}(c+dx)\right)\right)}{\sqrt{\sqrt[3]{-1}a^{2/3}-b^{2/3}}}\right)}{3\sqrt{\sqrt[3]{-1}a^{2/3}-b^{2/3}}\,b^2d} \\
&\quad -\frac{2a^{4/3}\text{arctanh}\left(\frac{\sqrt[3]{b}-\sqrt[3]{a}\tanh\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^{2/3}+b^{2/3}}}\right)}{3\sqrt{a^{2/3}+b^{2/3}}\,b^2d}-\frac{\cosh(c+dx)}{bd}+\frac{\cosh^3(c+dx)}{3bd}
\end{aligned}$$

output
```
-a*x/b^2-2/3*(-1)^(2/3)*a^(4/3)*arctan((-1)^(1/6)*((-1)^(1/6)*b^(1/3)+I*a^
(1/3)*tanh(1/2*d*x+1/2*c))/((-1)^(1/3)*a^(2/3)-(-1)^(2/3)*b^(2/3))^(1/2))/
((-1)^(1/3)*a^(2/3)-(-1)^(2/3)*b^(2/3))^(1/2)/b^2/d-2/3*(-1)^(2/3)*a^(4/3)
*arctan((-1)^(1/6)*((-1)^(5/6)*b^(1/3)+I*a^(1/3)*tanh(1/2*d*x+1/2*c))/((-1
)^(1/3)*a^(2/3)-b^(2/3))^(1/2))/((-1)^(1/3)*a^(2/3)-b^(2/3))^(1/2)/b^2/d-2
/3*a^(4/3)*arctanh((b^(1/3)-a^(1/3)*tanh(1/2*d*x+1/2*c))/(a^(2/3)+b^(2/3))
^(1/2))/(a^(2/3)+b^(2/3))^(1/2)/b^2/d-cosh(d*x+c)/b/d+1/3*cosh(d*x+c)^3/b/
d
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

Time = 0.86 (sec) , antiderivative size = 168, normalized size of antiderivative = 0.51

$$\int \frac{\sinh^6(c+dx)}{a+b\sinh^3(c+dx)}\,dx$$
$$= \frac{-12ac-12adx-9b\cosh(c+dx)+b\cosh(3(c+dx))+8a^2\text{RootSum}\left[-b+3b\#1^2+8a\#1^3-3b\#1^4+\right.}{12b^2d}$$

input
```
Integrate[Sinh[c + d*x]^6/(a + b*Sinh[c + d*x]^3),x]
```

output
```
(-12*a*c - 12*a*d*x - 9*b*Cosh[c + d*x] + b*Cosh[3*(c + d*x)] + 8*a^2*Root
Sum[-b + 3*b*#1^2 + 8*a*#1^3 - 3*b*#1^4 + b*#1^6 & , (c*#1 + d*x*#1 + 2*Lo
g[-Cosh[(c + d*x)/2] - Sinh[(c + d*x)/2] + Cosh[(c + d*x)/2]*#1 - Sinh[(c
+ d*x)/2]*#1]*#1)/(b + 4*a*#1 - 2*b*#1^2 + b*#1^4) & ])/(12*b^2*d)
```

Rubi [A] (verified)

Time = 0.95 (sec) , antiderivative size = 328, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.174$, Rules used = {3042, 25, 3699, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sinh^6(c+dx)}{a+b\sinh^3(c+dx)}dx$$

$\Big\downarrow$ 3042

$$\int -\frac{\sin(ic+idx)^6}{a+ib\sin(ic+idx)^3}dx$$

$\Big\downarrow$ 25

$$-\int \frac{\sin(ic+idx)^6}{ib\sin(ic+idx)^3+a}dx$$

$\Big\downarrow$ 3699

$$-\int \left(-\frac{\sinh^3(c+dx)}{b}-\frac{a^2}{b^2\left(b\sinh^3(c+dx)+a\right)}+\frac{a}{b^2}\right)dx$$

$\Big\downarrow$ 2009

$$-\frac{2(-1)^{2/3}a^{4/3}\arctan\left(\frac{\sqrt[6]{-1}\left(\sqrt[6]{-1}\sqrt[3]{b}+i\sqrt[3]{a}\tanh\left(\frac{1}{2}(c+dx)\right)\right)}{\sqrt{\sqrt[3]{-1}a^{2/3}-(-1)^{2/3}b^{2/3}}}\right)}{3b^2d\sqrt{\sqrt[3]{-1}a^{2/3}-(-1)^{2/3}b^{2/3}}}-\frac{2(-1)^{2/3}a^{4/3}\arctan\left(\frac{\sqrt[6]{-1}\left((-1)^{5/6}\sqrt[3]{b}+i\sqrt[3]{a}\tanh\left(\frac{1}{2}(c+dx)\right)\right)}{\sqrt{\sqrt[3]{-1}a^{2/3}-b^{2/3}}}\right)}{3b^2d\sqrt{\sqrt[3]{-1}a^{2/3}-b^{2/3}}}-\frac{2a^{4/3}\text{arctanh}\left(\frac{\sqrt[3]{b}-\sqrt[3]{a}\tanh\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^{2/3}+b^{2/3}}}\right)}{3b^2d\sqrt{a^{2/3}+b^{2/3}}}-\frac{ax}{b^2}+\frac{\cosh^3(c+dx)}{3bd}-\frac{\cosh(c+dx)}{bd}$$

input
```
Int[Sinh[c + d*x]^6/(a + b*Sinh[c + d*x]^3),x]
```

output
```
-((a*x)/b^2) - (2*(-1)^(2/3)*a^(4/3)*ArcTan[((-1)^(1/6)*((-1)^(1/6)*b^(1/3
) + I*a^(1/3)*Tanh[(c + d*x)/2]))/Sqrt[(-1)^(1/3)*a^(2/3) - (-1)^(2/3)*b^(
2/3)]])/(3*Sqrt[(-1)^(1/3)*a^(2/3) - (-1)^(2/3)*b^(2/3)]*b^2*d) - (2*(-1)^
(2/3)*a^(4/3)*ArcTan[((-1)^(1/6)*((-1)^(5/6)*b^(1/3) + I*a^(1/3)*Tanh[(c +
 d*x)/2]))/Sqrt[(-1)^(1/3)*a^(2/3) - b^(2/3)]])/(3*Sqrt[(-1)^(1/3)*a^(2/3)
 - b^(2/3)]*b^2*d) - (2*a^(4/3)*ArcTanh[(b^(1/3) - a^(1/3)*Tanh[(c + d*x)/
2])/Sqrt[a^(2/3) + b^(2/3)]])/(3*Sqrt[a^(2/3) + b^(2/3)]*b^2*d) - Cosh[c +
 d*x]/(b*d) + Cosh[c + d*x]^3/(3*b*d)
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3699
```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^(n_
))^(p_.), x_Symbol] :> Int[ExpandTrig[sin[e + f*x]^m*(a + b*sin[e + f*x]^n)
^p, x], x] /; FreeQ[{a, b, e, f}, x] && IntegersQ[m, p] && (EqQ[n, 4] || Gt
Q[p, 0] || (EqQ[p, -1] && IntegerQ[n]))
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 2.69 (sec) , antiderivative size = 236, normalized size of antiderivative = 0.72

method	result
derivativedivides	$-\frac{a^{2}\left(\sum_{\_R=\mathrm{RootOf}\left(a\_Z^{6}-3a\_Z^{4}-8b\_Z^{3}+3a\_Z^{2}-a\right)}\frac{\left(\_R^{4}-2\_R^{2}+1\right)\ln\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)-\_R\right)}{\_R^{5}a-2\_R^{3}a-4\_R^{2}b+\_Ra}\right)}{3b^{2}}-\frac{1}{3b\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right.\right.}$
default	$-\frac{a^{2}\left(\sum_{\_R=\mathrm{RootOf}\left(a\_Z^{6}-3a\_Z^{4}-8b\_Z^{3}+3a\_Z^{2}-a\right)}\frac{\left(\_R^{4}-2\_R^{2}+1\right)\ln\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)-\_R\right)}{\_R^{5}a-2\_R^{3}a-4\_R^{2}b+\_Ra}\right)}{3b^{2}}-\frac{1}{3b\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right.\right.}$
risch	$-\frac{ax}{b^{2}}+\frac{e^{3dx+3c}}{24bd}-\frac{3\,e^{dx+c}}{8bd}-\frac{3\,e^{-dx-c}}{8bd}+\frac{e^{-3dx-3c}}{24bd}+\left(\sum_{\_R=\mathrm{RootOf}\left(\left(729a^{2}b^{12}d^{6}+729b^{14}d^{6}\right)\_Z^{6}-243a^{4}b^{8}\right.}\right.$

input
```
int(sinh(d*x+c)^6/(a+b*sinh(d*x+c)^3),x,method=_RETURNVERBOSE)
```

output
```
1/d*(-1/3*a^2/b^2*sum((_R^4-2*_R^2+1)/(_R^5*a-2*_R^3*a-4*_R^2*b+_R*a)*ln(t
anh(1/2*d*x+1/2*c)-_R),_R=RootOf(_Z^6*a-3*_Z^4*a-8*_Z^3*b+3*_Z^2*a-a))-1/3
/b/(tanh(1/2*d*x+1/2*c)-1)^3-1/2/b/(tanh(1/2*d*x+1/2*c)-1)^2+1/2/b/(tanh(1
/2*d*x+1/2*c)-1)+a/b^2*ln(tanh(1/2*d*x+1/2*c)-1)+1/3/b/(tanh(1/2*d*x+1/2*c
)+1)^3-1/2/b/(tanh(1/2*d*x+1/2*c)+1)^2-1/2/b/(tanh(1/2*d*x+1/2*c)+1)-a/b^2
*ln(tanh(1/2*d*x+1/2*c)+1))
```

Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 28816 vs. $2(237) = 474$.

Time = 4.39 (sec) , antiderivative size = 28816, normalized size of antiderivative = 87.85

$$\int \frac{\sinh^6(c+dx)}{a+b\sinh^3(c+dx)}\,dx = \text{Too large to display}$$

input
```
integrate(sinh(d*x+c)^6/(a+b*sinh(d*x+c)^3),x, algorithm="fricas")
```

output
```
Too large to include
```

Sympy [F]

$$\int \frac{\sinh^6(c+dx)}{a+b\sinh^3(c+dx)}\,dx = \int \frac{\sinh^6{(c+dx)}}{a+b\sinh^3{(c+dx)}}\,dx$$

input
```
integrate(sinh(d*x+c)**6/(a+b*sinh(d*x+c)**3),x)
```

output
```
Integral(sinh(c + d*x)**6/(a + b*sinh(c + d*x)**3), x)
```

Maxima [F]

$$\int \frac{\sinh^6(c+dx)}{a+b\sinh^3(c+dx)}\,dx = \int \frac{\sinh(dx+c)^6}{b\sinh(dx+c)^3+a}\,dx$$

input
```
integrate(sinh(d*x+c)^6/(a+b*sinh(d*x+c)^3),x, algorithm="maxima")
```

output
```
8*a^2*integrate(e^(3*d*x + 3*c)/(b^3*e^(6*d*x + 6*c) - 3*b^3*e^(4*d*x + 4*
c) + 8*a*b^2*e^(3*d*x + 3*c) + 3*b^3*e^(2*d*x + 2*c) - b^3), x) - 1/24*(24
*a*d*x*e^(3*d*x + 3*c) - b*e^(6*d*x + 6*c) + 9*b*e^(4*d*x + 4*c) + 9*b*e^(
2*d*x + 2*c) - b)*e^(-3*d*x - 3*c)/(b^2*d)
```

Giac [F]

$$\int \frac{\sinh^6(c+dx)}{a+b\sinh^3(c+dx)}\,dx = \int \frac{\sinh(dx+c)^6}{b\sinh(dx+c)^3+a}\,dx$$

input
```
integrate(sinh(d*x+c)^6/(a+b*sinh(d*x+c)^3),x, algorithm="giac")
```

output
```
sage0*x
```

Mupad [B] (verification not implemented)

Time = 11.24 (sec) , antiderivative size = 1579, normalized size of antiderivative = 4.81

$$\int \frac{\sinh^6(c+dx)}{a+b\sinh^3(c+dx)}\,dx = \text{Too large to display}$$

input
```
int(sinh(c + d*x)^6/(a + b*sinh(c + d*x)^3),x)
```

output
```
symsum(log((294912*root(729*a^2*b^12*d^6*z^6 + 729*b^14*d^6*z^6 - 243*a^4*
b^8*d^4*z^4 + 27*a^6*b^4*d^2*z^2 - a^8, z, k)^2*a^7*b^5*d^2 - 98304*a^10*e
xp(d*x)*exp(root(729*a^2*b^12*d^6*z^6 + 729*b^14*d^6*z^6 - 243*a^4*b^8*d^4
*z^4 + 27*a^6*b^4*d^2*z^2 - a^8, z, k)) + 1327104*root(729*a^2*b^12*d^6*z^
6 + 729*b^14*d^6*z^6 - 243*a^4*b^8*d^4*z^4 + 27*a^6*b^4*d^2*z^2 - a^8, z,
k)^3*a^6*b^7*d^3 + 2654208*root(729*a^2*b^12*d^6*z^6 + 729*b^14*d^6*z^6 -
243*a^4*b^8*d^4*z^4 + 27*a^6*b^4*d^2*z^2 - a^8, z, k)^4*a^5*b^9*d^4 + 1990
656*root(729*a^2*b^12*d^6*z^6 + 729*b^14*d^6*z^6 - 243*a^4*b^8*d^4*z^4 + 2
7*a^6*b^4*d^2*z^2 - a^8, z, k)^5*a^4*b^11*d^5 + 24576*root(729*a^2*b^12*d^
6*z^6 + 729*b^14*d^6*z^6 - 243*a^4*b^8*d^4*z^4 + 27*a^6*b^4*d^2*z^2 - a^8,
 z, k)*a^8*b^3*d + 589824*root(729*a^2*b^12*d^6*z^6 + 729*b^14*d^6*z^6 - 2
43*a^4*b^8*d^4*z^4 + 27*a^6*b^4*d^2*z^2 - a^8, z, k)^2*a^8*b^4*d^2*exp(d*x
)*exp(root(729*a^2*b^12*d^6*z^6 + 729*b^14*d^6*z^6 - 243*a^4*b^8*d^4*z^4 +
 27*a^6*b^4*d^2*z^2 - a^8, z, k)) + 5308416*root(729*a^2*b^12*d^6*z^6 + 72
9*b^14*d^6*z^6 - 243*a^4*b^8*d^4*z^4 + 27*a^6*b^4*d^2*z^2 - a^8, z, k)^3*a
^7*b^6*d^3*exp(d*x)*exp(root(729*a^2*b^12*d^6*z^6 + 729*b^14*d^6*z^6 - 243
*a^4*b^8*d^4*z^4 + 27*a^6*b^4*d^2*z^2 - a^8, z, k)) - 663552*root(729*a^2*
b^12*d^6*z^6 + 729*b^14*d^6*z^6 - 243*a^4*b^8*d^4*z^4 + 27*a^6*b^4*d^2*z^2
 - a^8, z, k)^4*a^4*b^10*d^4*exp(d*x)*exp(root(729*a^2*b^12*d^6*z^6 + 729*
b^14*d^6*z^6 - 243*a^4*b^8*d^4*z^4 + 27*a^6*b^4*d^2*z^2 - a^8, z, k)) +...
```

Reduce [F]

$$\int \frac{\sinh^6(c+dx)}{a+b\sinh^3(c+dx)}\,dx$$

$$= \frac{e^{6dx+6c}b^2 - 9e^{4dx+4c}b^2 + 576e^{3dx+4c}\left(\int \frac{e^{dx}}{e^{6dx+6c}b-3e^{4dx+4c}b+8e^{3dx+3c}a+3e^{2dx+2c}b-b}dx\right)a^2bd + 192e^{3dx+3c}\left(\int \frac{}{e^{9dx+}}\right.}{}$$

input
```
int(sinh(d*x+c)^6/(a+b*sinh(d*x+c)^3),x)
```

output
```
(e**(6*c + 6*d*x)*b**2 - 9*e**(4*c + 4*d*x)*b**2 + 576*e**(4*c + 3*d*x)*in
t(e**(d*x)/(e**(6*c + 6*d*x)*b - 3*e**(4*c + 4*d*x)*b + 8*e**(3*c + 3*d*x)
*a + 3*e**(2*c + 2*d*x)*b - b),x)*a**2*b*d + 192*e**(3*c + 3*d*x)*int(1/(e
**(9*c + 9*d*x)*b - 3*e**(7*c + 7*d*x)*b + 8*e**(6*c + 6*d*x)*a + 3*e**(5*
c + 5*d*x)*b - e**(3*c + 3*d*x)*b),x)*a**2*b*d - 1536*e**(3*c + 3*d*x)*int
(1/(e**(6*c + 6*d*x)*b - 3*e**(4*c + 4*d*x)*b + 8*e**(3*c + 3*d*x)*a + 3*e
**(2*c + 2*d*x)*b - b),x)*a**3*d - 24*e**(3*c + 3*d*x)*a*b*d*x - 576*e**(2
*c + 3*d*x)*int(1/(e**(6*c + 7*d*x)*b - 3*e**(4*c + 5*d*x)*b + 8*e**(3*c +
 4*d*x)*a + 3*e**(2*c + 3*d*x)*b - e**(d*x)*b),x)*a**2*b*d - 9*e**(2*c + 2
*d*x)*b**2 - 64*a**2 + b**2)/(24*e**(3*c + 3*d*x)*b**3*d)
```

3.149 $\int \frac{\sinh^5(c+dx)}{a+b\sinh^3(c+dx)}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 295

$$\int \frac{\sinh^5(c+dx)}{a+b\sinh^3(c+dx)}\,dx = -\frac{x}{2b}+\frac{2a\arctan\left(\frac{(-1)^{5/6}\left(\sqrt[6]{-1}\sqrt[3]{b}+i\sqrt[3]{a}\tanh\left(\frac{1}{2}(c+dx)\right)\right)}{\sqrt{-(-1)^{2/3}a^{2/3}-b^{2/3}}}\right)}{3\sqrt{-(-1)^{2/3}a^{2/3}-b^{2/3}}\,b^{5/3}d}+\frac{2a\arctan\left(\frac{\sqrt[6]{-1}\left((-1)^{5/6}\sqrt[3]{b}+i\sqrt[3]{a}\tanh\left(\frac{1}{2}(c+dx)\right)\right)}{\sqrt{\sqrt[3]{-1}a^{2/3}-b^{2/3}}}\right)}{3\sqrt{\sqrt[3]{-1}a^{2/3}-b^{2/3}}\,b^{5/3}d}+\frac{2a\,\mathrm{arctanh}\left(\frac{\sqrt[3]{b}-\sqrt[3]{a}\tanh\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^{2/3}+b^{2/3}}}\right)}{3\sqrt{a^{2/3}+b^{2/3}}\,b^{5/3}d}+\frac{\cosh(c+dx)\sinh(c+dx)}{2bd}$$

output
```
-1/2*x/b+2/3*a*arctan((-1)^(5/6)*((-1)^(1/6)*b^(1/3)+I*a^(1/3)*tanh(1/2*d*
x+1/2*c))/(-(-1)^(2/3)*a^(2/3)-b^(2/3))^(1/2))/(-(-1)^(2/3)*a^(2/3)-b^(2/3
))^(1/2)/b^(5/3)/d+2/3*a*arctan((-1)^(1/6)*((-1)^(5/6)*b^(1/3)+I*a^(1/3)*t
anh(1/2*d*x+1/2*c))/((-1)^(1/3)*a^(2/3)-b^(2/3))^(1/2))/((-1)^(1/3)*a^(2/3
)-b^(2/3))^(1/2)/b^(5/3)/d+2/3*a*arctanh((b^(1/3)-a^(1/3)*tanh(1/2*d*x+1/2
*c))/(a^(2/3)+b^(2/3))^(1/2))/(a^(2/3)+b^(2/3))^(1/2)/b^(5/3)/d+1/2*cosh(d
*x+c)*sinh(d*x+c)/b/d
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

Time = 0.75 (sec) , antiderivative size = 299, normalized size of antiderivative = 1.01

$$\int \frac{\sinh^5(c+dx)}{a+b\sinh^3(c+dx)}\,dx$$
$$= \frac{-6(c+dx) - 2a\text{RootSum}\left[-b+3b\#1^2+8a\#1^3-3b\#1^4+b\#1^6\&, \frac{c+dx+2\log\left(-\cosh\left(\frac{1}{2}(c+dx)\right)-\sinh\left(\frac{1}{2}(c+dx)\right)\right.}{}\right.}{}$$

input
```
Integrate[Sinh[c + d*x]^5/(a + b*Sinh[c + d*x]^3),x]
```

output
```
(-6*(c + d*x) - 2*a*RootSum[-b + 3*b*#1^2 + 8*a*#1^3 - 3*b*#1^4 + b*#1^6 &
 , (c + d*x + 2*Log[-Cosh[(c + d*x)/2] - Sinh[(c + d*x)/2] + Cosh[(c + d*x
)/2]*#1 - Sinh[(c + d*x)/2]*#1] - 2*c*#1^2 - 2*d*x*#1^2 - 4*Log[-Cosh[(c +
 d*x)/2] - Sinh[(c + d*x)/2] + Cosh[(c + d*x)/2]*#1 - Sinh[(c + d*x)/2]*#1
]*#1^2 + c*#1^4 + d*x*#1^4 + 2*Log[-Cosh[(c + d*x)/2] - Sinh[(c + d*x)/2]
+ Cosh[(c + d*x)/2]*#1 - Sinh[(c + d*x)/2]*#1]*#1^4)/(b*#1 + 4*a*#1^2 - 2*
b*#1^3 + b*#1^5) & ] + 3*Sinh[2*(c + d*x)])/(12*b*d)
```

Rubi [A] (verified)

Time = 0.81 (sec) , antiderivative size = 309, normalized size of antiderivative = 1.05, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.174$, Rules used = {3042, 26, 3699, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sinh^5(c+dx)}{a+b\sinh^3(c+dx)}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int -\frac{i\sin(ic+idx)^5}{a+ib\sin(ic+idx)^3}dx$$

$$\Big\downarrow \text{26}$$

$$-i\int \frac{\sin(ic+idx)^5}{ib\sin(ic+idx)^3+a}dx$$

$$\Big\downarrow \text{3699}$$

$$-i\int \left(\frac{i\sinh^2(c+dx)}{b}-\frac{ia\sinh^2(c+dx)}{b\left(b\sinh^3(c+dx)+a\right)}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$-i\left(\frac{2ia\arctan\left(\frac{(-1)^{5/6}\left(\sqrt[6]{-1}\sqrt[3]{b}+i\sqrt[3]{a}\tanh\left(\frac{1}{2}(c+dx)\right)\right)}{\sqrt{-(-1)^{2/3}a^{2/3}-b^{2/3}}}\right)}{3b^{5/3}d\sqrt{-(-1)^{2/3}a^{2/3}-b^{2/3}}}+\frac{2ia\arctan\left(\frac{\sqrt[6]{-1}\left((-1)^{5/6}\sqrt[3]{b}+i\sqrt[3]{a}\tanh\left(\frac{1}{2}(c+dx)\right)\right)}{\sqrt{\sqrt[3]{-1}a^{2/3}-b^{2/3}}}\right)}{3b^{5/3}d\sqrt{\sqrt[3]{-1}a^{2/3}-b^{2/3}}}+\right.$$

input
```
Int[Sinh[c + d*x]^5/(a + b*Sinh[c + d*x]^3),x]
```

output
```
(-I)*(((-1/2*I)*x)/b + (((2*I)/3)*a*ArcTan[((-1)^(5/6)*((-1)^(1/6)*b^(1/3)
 + I*a^(1/3)*Tanh[(c + d*x)/2]))/Sqrt[-((-1)^(2/3)*a^(2/3)) - b^(2/3)]])/(
Sqrt[-((-1)^(2/3)*a^(2/3)) - b^(2/3)]*b^(5/3)*d) + (((2*I)/3)*a*ArcTan[((-
1)^(1/6)*((-1)^(5/6)*b^(1/3) + I*a^(1/3)*Tanh[(c + d*x)/2]))/Sqrt[(-1)^(1/
3)*a^(2/3) - b^(2/3)]])/(Sqrt[(-1)^(1/3)*a^(2/3) - b^(2/3)]*b^(5/3)*d) + (
((2*I)/3)*a*ArcTanh[(b^(1/3) - a^(1/3)*Tanh[(c + d*x)/2])/Sqrt[a^(2/3) + b
^(2/3)]])/(Sqrt[a^(2/3) + b^(2/3)]*b^(5/3)*d) + ((I/2)*Cosh[c + d*x]*Sinh[
c + d*x])/(b*d))
```

Defintions of rubi rules used

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3699
```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^(n_
))^(p_.), x_Symbol] :> Int[ExpandTrig[sin[e + f*x]^m*(a + b*sin[e + f*x]^n)
^p, x], x] /; FreeQ[{a, b, e, f}, x] && IntegersQ[m, p] && (EqQ[n, 4] || Gt
Q[p, 0] || (EqQ[p, -1] && IntegerQ[n]))
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 1.81 (sec) , antiderivative size = 190, normalized size of antiderivative = 0.64

method	result
derivativedivides	$\frac{\frac{1}{2b\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)^2}+\frac{1}{2b\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}+\frac{\ln\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}{2b}+\frac{4a\left(\sum_{\_R=\mathrm{RootOf}\left(a\_Z^6-3a\_Z^4-8b\_Z^3+3a\_Z^2-a\right.}\right.}{3b}}{d}$
default	$\frac{\frac{1}{2b\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)^2}+\frac{1}{2b\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}+\frac{\ln\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}{2b}+\frac{4a\left(\sum_{\_R=\mathrm{RootOf}\left(a\_Z^6-3a\_Z^4-8b\_Z^3+3a\_Z^2-a\right.}\right.}{3b}}{d}$
risch	$-\frac{x}{2b}+\frac{e^{2dx+2c}}{8bd}-\frac{e^{-2dx-2c}}{8bd}+\left(\sum_{\_R=\mathrm{RootOf}\left(\left(729a^2b^{10}d^6+729b^{12}d^6\right)\_Z^6-243a^2b^8d^4\_Z^4+27a^4b^4d^2\_Z^2-a\right.}\right.$

input
```
int(sinh(d*x+c)^5/(a+b*sinh(d*x+c)^3),x,method=_RETURNVERBOSE)
```

output
```
1/d*(1/2/b/(tanh(1/2*d*x+1/2*c)-1)^2+1/2/b/(tanh(1/2*d*x+1/2*c)-1)+1/2/b*l
n(tanh(1/2*d*x+1/2*c)-1)+4/3/b*a*sum(_R^2/(_R^5*a-2*_R^3*a-4*_R^2*b+_R*a)*
ln(tanh(1/2*d*x+1/2*c)-_R),_R=RootOf(_Z^6*a-3*_Z^4*a-8*_Z^3*b+3*_Z^2*a-a))
-1/2/b/(tanh(1/2*d*x+1/2*c)+1)^2+1/2/b/(tanh(1/2*d*x+1/2*c)+1)-1/2/b*ln(ta
nh(1/2*d*x+1/2*c)+1))
```

Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 28427 vs. 2(210) = 420.

Time = 2.24 (sec) , antiderivative size = 28427, normalized size of antiderivative = 96.36

$$\int \frac{\sinh^5(c+dx)}{a+b\sinh^3(c+dx)}\,dx = \text{Too large to display}$$

input
```
integrate(sinh(d*x+c)^5/(a+b*sinh(d*x+c)^3),x, algorithm="fricas")
```

output
```
Too large to include
```

Sympy [F]

$$\int \frac{\sinh^5(c+dx)}{a+b\sinh^3(c+dx)}\,dx = \int \frac{\sinh^5{(c+dx)}}{a+b\sinh^3{(c+dx)}}\,dx$$

input
```
integrate(sinh(d*x+c)**5/(a+b*sinh(d*x+c)**3),x)
```

output
```
Integral(sinh(c + d*x)**5/(a + b*sinh(c + d*x)**3), x)
```

Maxima [F]

$$\int \frac{\sinh^5(c+dx)}{a+b\sinh^3(c+dx)}\,dx = \int \frac{\sinh{(dx+c)}^5}{b\sinh{(dx+c)}^3+a}\,dx$$

input
```
integrate(sinh(d*x+c)^5/(a+b*sinh(d*x+c)^3),x, algorithm="maxima")
```

output
```
-1/8*(4*d*x*e^(2*d*x + 2*c) - e^(4*d*x + 4*c) + 1)*e^(-2*d*x - 2*c)/(b*d)
- 1/32*integrate(64*(a*e^(5*d*x + 5*c) - 2*a*e^(3*d*x + 3*c) + a*e^(d*x +
c))/(b^2*e^(6*d*x + 6*c) - 3*b^2*e^(4*d*x + 4*c) + 8*a*b*e^(3*d*x + 3*c) +
 3*b^2*e^(2*d*x + 2*c) - b^2), x)
```

Giac [F]

$$\int \frac{\sinh^5(c+dx)}{a+b\sinh^3(c+dx)}\,dx = \int \frac{\sinh{(dx+c)}^5}{b\sinh{(dx+c)}^3+a}\,dx$$

input
```
integrate(sinh(d*x+c)^5/(a+b*sinh(d*x+c)^3),x, algorithm="giac")
```

output
```
sage0*x
```

Mupad [B] (verification not implemented)

Time = 12.70 (sec) , antiderivative size = 1114, normalized size of antiderivative = 3.78

$$\int \frac{\sinh^5(c+dx)}{a+b\sinh^3(c+dx)}\,dx = \text{Too large to display}$$

input
```
int(sinh(c + d*x)^5/(a + b*sinh(c + d*x)^3),x)
```

output
```
symsum(log(- root(729*a^2*b^10*d^6*z^6 + 729*b^12*d^6*z^6 - 243*a^2*b^8*d^
4*z^4 + 27*a^4*b^4*d^2*z^2 - a^6, z, k)*(root(729*a^2*b^10*d^6*z^6 + 729*b
^12*d^6*z^6 - 243*a^2*b^8*d^4*z^4 + 27*a^4*b^4*d^2*z^2 - a^6, z, k)*(root(
729*a^2*b^10*d^6*z^6 + 729*b^12*d^6*z^6 - 243*a^2*b^8*d^4*z^4 + 27*a^4*b^4
*d^2*z^2 - a^6, z, k)*(root(729*a^2*b^10*d^6*z^6 + 729*b^12*d^6*z^6 - 243*
a^2*b^8*d^4*z^4 + 27*a^4*b^4*d^2*z^2 - a^6, z, k)*((663552*(8*a^6*d^4 + 4*
a^4*b^2*d^4 - 5*a^5*b*d^4*exp(d*x)*exp(root(729*a^2*b^10*d^6*z^6 + 729*b^1
2*d^6*z^6 - 243*a^2*b^8*d^4*z^4 + 27*a^4*b^4*d^2*z^2 - a^6, z, k))))/b^7 +
 (1990656*root(729*a^2*b^10*d^6*z^6 + 729*b^12*d^6*z^6 - 243*a^2*b^8*d^4*z
^4 + 27*a^4*b^4*d^2*z^2 - a^6, z, k)*(4*a^5*d^5*exp(d*x)*exp(root(729*a^2*
b^10*d^6*z^6 + 729*b^12*d^6*z^6 - 243*a^2*b^8*d^4*z^4 + 27*a^4*b^4*d^2*z^2
 - a^6, z, k)) - a^4*b*d^5 + 5*a^3*b^2*d^5*exp(d*x)*exp(root(729*a^2*b^10*
d^6*z^6 + 729*b^12*d^6*z^6 - 243*a^2*b^8*d^4*z^4 + 27*a^4*b^4*d^2*z^2 - a^
6, z, k))))/b^5) + (442368*(4*a^6*b*d^3 + 8*a^7*d^3*exp(d*x)*exp(root(729*
a^2*b^10*d^6*z^6 + 729*b^12*d^6*z^6 - 243*a^2*b^8*d^4*z^4 + 27*a^4*b^4*d^2
*z^2 - a^6, z, k)) - 5*a^5*b^2*d^3*exp(d*x)*exp(root(729*a^2*b^10*d^6*z^6
+ 729*b^12*d^6*z^6 - 243*a^2*b^8*d^4*z^4 + 27*a^4*b^4*d^2*z^2 - a^6, z, k)
)))/b^9) - (294912*a^6*d^2*(2*b - 5*a*exp(d*x)*exp(root(729*a^2*b^10*d^6*z
^6 + 729*b^12*d^6*z^6 - 243*a^2*b^8*d^4*z^4 + 27*a^4*b^4*d^2*z^2 - a^6, z,
 k))))/b^10) - (24576*a^7*d*(8*a - 5*b*exp(d*x)*exp(root(729*a^2*b^10*d...
```

Reduce [F]

$$\int \frac{\sinh^5(c+dx)}{a+b\sinh^3(c+dx)}\,dx$$
$$= \frac{3e^{4dx+4c}b+384e^{2dx+4c}\left(\int \frac{e^{2dx}}{e^{6dx+6c}b-3e^{4dx+4c}b+8e^{3dx+3c}a+3e^{2dx+2c}b-b}dx\right)a^2d-12e^{2dx+4c}\left(\int \frac{e^{2dx}}{e^{6dx+6c}b-3e^{4dx+4c}b+8e^{3}\cdots}\right.}{}$$

input

```
int(sinh(d*x+c)^5/(a+b*sinh(d*x+c)^3),x)
```

output

```
(3*e**(4*c + 4*d*x)*b + 384*e**(4*c + 2*d*x)*int(e**(2*d*x)/(e**(6*c + 6*d
*x)*b - 3*e**(4*c + 4*d*x)*b + 8*e**(3*c + 3*d*x)*a + 3*e**(2*c + 2*d*x)*b
 - b),x)*a**2*d - 12*e**(4*c + 2*d*x)*int(e**(2*d*x)/(e**(6*c + 6*d*x)*b -
 3*e**(4*c + 4*d*x)*b + 8*e**(3*c + 3*d*x)*a + 3*e**(2*c + 2*d*x)*b - b),x
)*b**2*d + 192*e**(3*c + 2*d*x)*int(e**(d*x)/(e**(6*c + 6*d*x)*b - 3*e**(4
*c + 4*d*x)*b + 8*e**(3*c + 3*d*x)*a + 3*e**(2*c + 2*d*x)*b - b),x)*a*b*d
- 12*e**(2*c + 2*d*x)*int(1/(e**(8*c + 8*d*x)*b - 3*e**(6*c + 6*d*x)*b + 8
*e**(5*c + 5*d*x)*a + 3*e**(4*c + 4*d*x)*b - e**(2*c + 2*d*x)*b),x)*b**2*d
 + 24*e**(2*c + 2*d*x)*int(1/(e**(6*c + 6*d*x)*b - 3*e**(4*c + 4*d*x)*b +
8*e**(3*c + 3*d*x)*a + 3*e**(2*c + 2*d*x)*b - b),x)*b**2*d + 2*e**(2*c + 2
*d*x)*log(e**(6*c + 6*d*x)*b - 3*e**(4*c + 4*d*x)*b + 8*e**(3*c + 3*d*x)*a
 + 3*e**(2*c + 2*d*x)*b - b)*b - 24*e**(2*c + 2*d*x)*b*d*x - 48*e**(c + 2*
d*x)*int(1/(e**(6*c + 7*d*x)*b - 3*e**(4*c + 5*d*x)*b + 8*e**(3*c + 4*d*x)
*a + 3*e**(2*c + 3*d*x)*b - e**(d*x)*b),x)*a*b*d + 48*e**(c + d*x)*a + 3*b
)/(24*e**(2*c + 2*d*x)*b**2*d)
```

3.150 $\int \frac{\sinh^4(c+dx)}{a+b\sinh^3(c+dx)}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 303

$$\int \frac{\sinh^4(c+dx)}{a+b\sinh^3(c+dx)}\,dx = -\frac{2a^{2/3}\arctan\left(\frac{\sqrt[6]{-1}\left(\sqrt[6]{-1}\sqrt[3]{b}+i\sqrt[3]{a}\tanh\left(\frac{1}{2}(c+dx)\right)\right)}{\sqrt{\sqrt[3]{-1}a^{2/3}-(-1)^{2/3}b^{2/3}}}\right)}{3\sqrt{\sqrt[3]{-1}a^{2/3}-(-1)^{2/3}b^{2/3}}\,b^{4/3}d} + \frac{2\sqrt[3]{-1}a^{2/3}\arctan\left(\frac{\sqrt[6]{-1}\left((-1)^{5/6}\sqrt[3]{b}+i\sqrt[3]{a}\tanh\left(\frac{1}{2}(c+dx)\right)\right)}{\sqrt{\sqrt[3]{-1}a^{2/3}-b^{2/3}}}\right)}{3\sqrt{\sqrt[3]{-1}a^{2/3}-b^{2/3}}\,b^{4/3}d} - \frac{2a^{2/3}\operatorname{arctanh}\left(\frac{\sqrt[3]{b}-\sqrt[3]{a}\tanh\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^{2/3}+b^{2/3}}}\right)}{3\sqrt{a^{2/3}+b^{2/3}}\,b^{4/3}d} + \frac{\cosh(c+dx)}{bd}$$

output

```
-2/3*a^(2/3)*arctan((-1)^(1/6)*((-1)^(1/6)*b^(1/3)+I*a^(1/3)*tanh(1/2*d*x+
1/2*c))/((-1)^(1/3)*a^(2/3)-(-1)^(2/3)*b^(2/3))^(1/2))/((-1)^(1/3)*a^(2/3)
-(-1)^(2/3)*b^(2/3))^(1/2)/b^(4/3)/d+2/3*(-1)^(1/3)*a^(2/3)*arctan((-1)^(1
/6)*((-1)^(5/6)*b^(1/3)+I*a^(1/3)*tanh(1/2*d*x+1/2*c))/((-1)^(1/3)*a^(2/3)
-b^(2/3))^(1/2))/((-1)^(1/3)*a^(2/3)-b^(2/3))^(1/2)/b^(4/3)/d-2/3*a^(2/3)*
arctanh((b^(1/3)-a^(1/3)*tanh(1/2*d*x+1/2*c))/(a^(2/3)+b^(2/3))^(1/2))/(a^
(2/3)+b^(2/3))^(1/2)/b^(4/3)/d+cosh(d*x+c)/b/d
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

Time = 0.44 (sec) , antiderivative size = 214, normalized size of antiderivative = 0.71

$$\int \frac{\sinh^4(c+dx)}{a+b\sinh^3(c+dx)}\,dx$$
$$= \frac{3\cosh(c+dx) - a\text{RootSum}\left[-b+3b\#1^2+8a\#1^3-3b\#1^4+b\#1^6\&, \frac{-c-dx-2\log\left(-\cosh\left(\frac{1}{2}(c+dx)\right)-\sinh\left(\frac{1}{2}(c-\right.\right.}{}\right.}{}$$

input
```
Integrate[Sinh[c + d*x]^4/(a + b*Sinh[c + d*x]^3),x]
```

output
```
(3*Cosh[c + d*x] - a*RootSum[-b + 3*b*#1^2 + 8*a*#1^3 - 3*b*#1^4 + b*#1^6
& , (-c - d*x - 2*Log[-Cosh[(c + d*x)/2] - Sinh[(c + d*x)/2] + Cosh[(c + d
*x)/2]*#1 - Sinh[(c + d*x)/2]*#1] + c*#1^2 + d*x*#1^2 + 2*Log[-Cosh[(c + d
*x)/2] - Sinh[(c + d*x)/2] + Cosh[(c + d*x)/2]*#1 - Sinh[(c + d*x)/2]*#1]*
#1^2)/(b + 4*a*#1 - 2*b*#1^2 + b*#1^4) & ])/(3*b*d)
```

Rubi [A] (verified)

Time = 0.91 (sec) , antiderivative size = 303, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.130, Rules used = {3042, 3699, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sinh^4(c+dx)}{a+b\sinh^3(c+dx)}\,dx$$
$$\Big\downarrow \; 3042$$
$$\int \frac{\sin(ic+idx)^4}{a+ib\sin(ic+idx)^3}dx$$
$$\Big\downarrow \; 3699$$

$$\int \left(\frac{\sinh(c+dx)}{b}-\frac{a\sinh(c+dx)}{b\left(a+b\sinh^3(c+dx)\right)}\right)dx$$

$$\Big\downarrow \text{ 2009}$$

$$-\frac{2a^{2/3}\arctan\left(\frac{\sqrt[6]{-1}\left(\sqrt[6]{-1}\sqrt[3]{b}+i\sqrt[3]{a}\tanh\left(\frac{1}{2}(c+dx)\right)\right)}{\sqrt{\sqrt[3]{-1}a^{2/3}-(-1)^{2/3}b^{2/3}}}\right)}{3b^{4/3}d\sqrt{\sqrt[3]{-1}a^{2/3}-(-1)^{2/3}b^{2/3}}}+\frac{2\sqrt[3]{-1}a^{2/3}\arctan\left(\frac{\sqrt[6]{-1}\left((-1)^{5/6}\sqrt[3]{b}+i\sqrt[3]{a}\tanh\left(\frac{1}{2}(c+dx)\right)\right)}{\sqrt{\sqrt[3]{-1}a^{2/3}-b^{2/3}}}\right)}{3b^{4/3}d\sqrt{\sqrt[3]{-1}a^{2/3}-b^{2/3}}}-\frac{2a^{2/3}\text{arctanh}\left(\frac{\sqrt[3]{b}-\sqrt[3]{a}\tanh\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^{2/3}+b^{2/3}}}\right)}{3b^{4/3}d\sqrt{a^{2/3}+b^{2/3}}}+\frac{\cosh(c+dx)}{bd}$$

input
```
Int[Sinh[c + d*x]^4/(a + b*Sinh[c + d*x]^3),x]
```

output
```
(-2*a^(2/3)*ArcTan[((-1)^(1/6)*((-1)^(1/6)*b^(1/3) + I*a^(1/3)*Tanh[(c + d
*x)/2]))/Sqrt[(-1)^(1/3)*a^(2/3) - (-1)^(2/3)*b^(2/3)]])/(3*Sqrt[(-1)^(1/3
)*a^(2/3) - (-1)^(2/3)*b^(2/3)]*b^(4/3)*d) + (2*(-1)^(1/3)*a^(2/3)*ArcTan[
((-1)^(1/6)*((-1)^(5/6)*b^(1/3) + I*a^(1/3)*Tanh[(c + d*x)/2]))/Sqrt[(-1)^
(1/3)*a^(2/3) - b^(2/3)]])/(3*Sqrt[(-1)^(1/3)*a^(2/3) - b^(2/3)]*b^(4/3)*d
) - (2*a^(2/3)*ArcTanh[(b^(1/3) - a^(1/3)*Tanh[(c + d*x)/2])/Sqrt[a^(2/3)
+ b^(2/3)]])/(3*Sqrt[a^(2/3) + b^(2/3)]*b^(4/3)*d) + Cosh[c + d*x]/(b*d)
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3699
```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^(n_
))^(p_.), x_Symbol] :> Int[ExpandTrig[sin[e + f*x]^m*(a + b*sin[e + f*x]^n)
^p, x], x] /; FreeQ[{a, b, e, f}, x] && IntegersQ[m, p] && (EqQ[n, 4] || Gt
Q[p, 0] || (EqQ[p, -1] && IntegerQ[n]))
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 1.23 (sec) , antiderivative size = 123, normalized size of antiderivative = 0.41

method	result
derivativedivides	$\frac{\frac{1}{b\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)}-\frac{1}{b\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}-\frac{2a\left(\sum_{\_R=\text{RootOf}\left(a\_Z^6-3a\_Z^4-8b\_Z^3+3a\_Z^2-a\right)}\frac{\left(\_R^3-\_R\right)\ln\left(\tanh\right.}{\_R^5a-2\_R^3a-4}\right.}{3b}}{d}$
default	$\frac{\frac{1}{b\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)}-\frac{1}{b\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}-\frac{2a\left(\sum_{\_R=\text{RootOf}\left(a\_Z^6-3a\_Z^4-8b\_Z^3+3a\_Z^2-a\right)}\frac{\left(\_R^3-\_R\right)\ln\left(\tanh\right.}{\_R^5a-2\_R^3a-4}\right.}{3b}}{d}$
risch	$\frac{e^{dx+c}}{2bd}+\frac{e^{-dx-c}}{2bd}+\left(\sum_{\_R=\text{RootOf}\left(\left(729a^2b^8d^6+729b^{10}d^6\right)\_Z^6+243a^2b^6d^4\_Z^4-a^4\right)}\_R\ln\left(e^{dx+c}+\left(-\right.\right.\right.$

input
```
int(sinh(d*x+c)^4/(a+b*sinh(d*x+c)^3),x,method=_RETURNVERBOSE)
```

output
```
1/d*(1/b/(tanh(1/2*d*x+1/2*c)+1)-1/b/(tanh(1/2*d*x+1/2*c)-1)-2/3/b*a*sum((
_R^3-_R)/(_R^5*a-2*_R^3*a-4*_R^2*b+_R*a)*ln(tanh(1/2*d*x+1/2*c)-_R),_R=Roo
tOf(_Z^6*a-3*_Z^4*a-8*_Z^3*b+3*_Z^2*a-a))
```

Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 20941 vs. 2(210) = 420.

Time = 1.07 (sec) , antiderivative size = 20941, normalized size of antiderivative = 69.11

$$\int \frac{\sinh^4(c+dx)}{a+b\sinh^3(c+dx)}\,dx = \text{Too large to display}$$

input
```
integrate(sinh(d*x+c)^4/(a+b*sinh(d*x+c)^3),x, algorithm="fricas")
```

output
```
Too large to include
```

Sympy [F]

$$\int \frac{\sinh^4(c+dx)}{a+b\sinh^3(c+dx)}\,dx = \int \frac{\sinh^4(c+dx)}{a+b\sinh^3(c+dx)}\,dx$$

input
```
integrate(sinh(d*x+c)**4/(a+b*sinh(d*x+c)**3),x)
```

output
```
Integral(sinh(c + d*x)**4/(a + b*sinh(c + d*x)**3), x)
```

Maxima [F]

$$\int \frac{\sinh^4(c+dx)}{a+b\sinh^3(c+dx)}\,dx = \int \frac{\sinh(dx+c)^4}{b\sinh(dx+c)^3+a}\,dx$$

input
```
integrate(sinh(d*x+c)^4/(a+b*sinh(d*x+c)^3),x, algorithm="maxima")
```

output
```
1/2*(e^(2*d*x + 2*c) + 1)*e^(-d*x - c)/(b*d) - 1/16*integrate(64*(a*e^(4*d
*x + 4*c) - a*e^(2*d*x + 2*c))/(b^2*e^(6*d*x + 6*c) - 3*b^2*e^(4*d*x + 4*c
) + 8*a*b*e^(3*d*x + 3*c) + 3*b^2*e^(2*d*x + 2*c) - b^2), x)
```

Giac [F]

$$\int \frac{\sinh^4(c+dx)}{a+b\sinh^3(c+dx)}\,dx = \int \frac{\sinh(dx+c)^4}{b\sinh(dx+c)^3+a}\,dx$$

input
```
integrate(sinh(d*x+c)^4/(a+b*sinh(d*x+c)^3),x, algorithm="giac")
```

output
```
integrate(sinh(d*x + c)^4/(b*sinh(d*x + c)^3 + a), x)
```

Mupad [B] (verification not implemented)

Time = 24.79 (sec) , antiderivative size = 906, normalized size of antiderivative = 2.99

$$\int \frac{\sinh^4(c+dx)}{a+b\sinh^3(c+dx)}\,dx = \text{Too large to display}$$

input
```
int(sinh(c + d*x)^4/(a + b*sinh(c + d*x)^3),x)
```

output
```
symsum(log((8192*a^6*(8*a - b*exp(d*x)*exp(root(729*a^2*b^8*d^6*z^6 + 729*
b^10*d^6*z^6 + 243*a^2*b^6*d^4*z^4 - a^4, z, k))))/b^12 - root(729*a^2*b^8
*d^6*z^6 + 729*b^10*d^6*z^6 + 243*a^2*b^6*d^4*z^4 - a^4, z, k)*(root(729*a
^2*b^8*d^6*z^6 + 729*b^10*d^6*z^6 + 243*a^2*b^6*d^4*z^4 - a^4, z, k)*(root
(729*a^2*b^8*d^6*z^6 + 729*b^10*d^6*z^6 + 243*a^2*b^6*d^4*z^4 - a^4, z, k)
*(root(729*a^2*b^8*d^6*z^6 + 729*b^10*d^6*z^6 + 243*a^2*b^6*d^4*z^4 - a^4,
 z, k)*((663552*(4*a^5*b*d^4 + 16*a^6*d^4*exp(d*x)*exp(root(729*a^2*b^8*d^
6*z^6 + 729*b^10*d^6*z^6 + 243*a^2*b^6*d^4*z^4 - a^4, z, k)) + 11*a^4*b^2*
d^4*exp(d*x)*exp(root(729*a^2*b^8*d^6*z^6 + 729*b^10*d^6*z^6 + 243*a^2*b^6
*d^4*z^4 - a^4, z, k))))/b^7 + (1990656*root(729*a^2*b^8*d^6*z^6 + 729*b^1
0*d^6*z^6 + 243*a^2*b^6*d^4*z^4 - a^4, z, k)*(4*a^5*d^5*exp(d*x)*exp(root(
729*a^2*b^8*d^6*z^6 + 729*b^10*d^6*z^6 + 243*a^2*b^6*d^4*z^4 - a^4, z, k))
 - a^4*b*d^5 + 5*a^3*b^2*d^5*exp(d*x)*exp(root(729*a^2*b^8*d^6*z^6 + 729*b
^10*d^6*z^6 + 243*a^2*b^6*d^4*z^4 - a^4, z, k))))/b^5) - (221184*(8*a^6*d^
3 + a^4*b^2*d^3 - 25*a^5*b*d^3*exp(d*x)*exp(root(729*a^2*b^8*d^6*z^6 + 729
*b^10*d^6*z^6 + 243*a^2*b^6*d^4*z^4 - a^4, z, k))))/b^8) - (294912*a^5*d^2
*(b - 7*a*exp(d*x)*exp(root(729*a^2*b^8*d^6*z^6 + 729*b^10*d^6*z^6 + 243*a
^2*b^6*d^4*z^4 - a^4, z, k))))/b^9) - (196608*a^6*d*(b - 2*a*exp(d*x)*exp(
root(729*a^2*b^8*d^6*z^6 + 729*b^10*d^6*z^6 + 243*a^2*b^6*d^4*z^4 - a^4, z
, k))))/b^11))*root(729*a^2*b^8*d^6*z^6 + 729*b^10*d^6*z^6 + 243*a^2*b^...
```

Reduce [F]

$$\int \frac{\sinh^4(c+dx)}{a+b\sinh^3(c+dx)}\,dx$$
$$= \frac{3\cosh(dx+c)\,b-48e^{3c}\left(\int \frac{e^{3dx}}{e^{6dx+6c}b-3e^{4dx+4c}b+8e^{3dx+3c}a+3e^{2dx+2c}b-b}dx\right)a^2d-12e^{2c}\left(\int \frac{e^{2dx}}{e^{6dx+6c}b-3e^{4dx+4c}b+8e^{3dx+\cdots}}\right.}{}$$

input
```
int(sinh(d*x+c)^4/(a+b*sinh(d*x+c)^3),x)
```

output

```
(3*cosh(c + d*x)*b - 48*e**(3*c)*int(e**(3*d*x)/(e**(6*c + 6*d*x)*b - 3*e*
*(4*c + 4*d*x)*b + 8*e**(3*c + 3*d*x)*a + 3*e**(2*c + 2*d*x)*b - b),x)*a**
2*d - 12*e**(2*c)*int(e**(2*d*x)/(e**(6*c + 6*d*x)*b - 3*e**(4*c + 4*d*x)*
b + 8*e**(3*c + 3*d*x)*a + 3*e**(2*c + 2*d*x)*b - b),x)*a*b*d + 12*int(1/(
e**(6*c + 6*d*x)*b - 3*e**(4*c + 4*d*x)*b + 8*e**(3*c + 3*d*x)*a + 3*e**(2
*c + 2*d*x)*b - b),x)*a*b*d - 2*log(e**(6*c + 6*d*x)*b - 3*e**(4*c + 4*d*x
)*b + 8*e**(3*c + 3*d*x)*a + 3*e**(2*c + 2*d*x)*b - b)*a + 12*a*d*x)/(3*b*
*2*d)
```

3.151 $\int \frac{\sinh^3(c+dx)}{a+b\sinh^3(c+dx)}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 294

$$\int \frac{\sinh^3(c+dx)}{a+b\sinh^3(c+dx)}\,dx = \frac{x}{b} + \frac{2(-1)^{2/3}\sqrt[3]{a}\arctan\left(\frac{\sqrt[6]{-1}\left(\sqrt[6]{-1}\sqrt[3]{b}+i\sqrt[3]{a}\tanh\left(\frac{1}{2}(c+dx)\right)\right)}{\sqrt{\sqrt[3]{-1}a^{2/3}-(-1)^{2/3}b^{2/3}}}\right)}{3\sqrt{\sqrt[3]{-1}a^{2/3}-(-1)^{2/3}b^{2/3}}\,bd} + \frac{2(-1)^{2/3}\sqrt[3]{a}\arctan\left(\frac{\sqrt[6]{-1}\left((-1)^{5/6}\sqrt[3]{b}+i\sqrt[3]{a}\tanh\left(\frac{1}{2}(c+dx)\right)\right)}{\sqrt{\sqrt[3]{-1}a^{2/3}-b^{2/3}}}\right)}{3\sqrt{\sqrt[3]{-1}a^{2/3}-b^{2/3}}\,bd} + \frac{2\sqrt[3]{a}\operatorname{arctanh}\left(\frac{\sqrt[3]{b}-\sqrt[3]{a}\tanh\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^{2/3}+b^{2/3}}}\right)}{3\sqrt{a^{2/3}+b^{2/3}}\,bd}$$

output
```
x/b+2/3*(-1)^(2/3)*a^(1/3)*arctan((-1)^(1/6)*((-1)^(1/6)*b^(1/3)+I*a^(1/3)
*tanh(1/2*d*x+1/2*c))/((-1)^(1/3)*a^(2/3)-(-1)^(2/3)*b^(2/3))^(1/2))/((-1)
^(1/3)*a^(2/3)-(-1)^(2/3)*b^(2/3))^(1/2)/b/d+2/3*(-1)^(2/3)*a^(1/3)*arctan
((-1)^(1/6)*((-1)^(5/6)*b^(1/3)+I*a^(1/3)*tanh(1/2*d*x+1/2*c))/((-1)^(1/3)
*a^(2/3)-b^(2/3))^(1/2))/((-1)^(1/3)*a^(2/3)-b^(2/3))^(1/2)/b/d+2/3*a^(1/3
)*arctanh((b^(1/3)-a^(1/3)*tanh(1/2*d*x+1/2*c))/(a^(2/3)+b^(2/3))^(1/2))/(
a^(2/3)+b^(2/3))^(1/2)/b/d
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

Time = 11.05 (sec) , antiderivative size = 145, normalized size of antiderivative = 0.49

$$\int \frac{\sinh^3(c+dx)}{a+b\sinh^3(c+dx)}\,dx = \frac{3c+3dx-2a\text{RootSum}\left[-b+3b\#1^2+8a\#1^3-3b\#1^4+b\#1^6\&,\frac{c\#1+dx\#1+2\log\left(-\cosh\left(\frac{1}{2}(c+dx)\right)-\sinh\left(\frac{1}{2}(c\right.\right.}{b+4a\#1-}\right.}{3bd}$$

input
```
Integrate[Sinh[c + d*x]^3/(a + b*Sinh[c + d*x]^3),x]
```

output
```
(3*c + 3*d*x - 2*a*RootSum[-b + 3*b*#1^2 + 8*a*#1^3 - 3*b*#1^4 + b*#1^6 &
, (c*#1 + d*x*#1 + 2*Log[-Cosh[(c + d*x)/2] - Sinh[(c + d*x)/2] + Cosh[(c
+ d*x)/2]*#1 - Sinh[(c + d*x)/2]*#1]*#1)/(b + 4*a*#1 - 2*b*#1^2 + b*#1^4)
& ])/(3*b*d)
```

Rubi [A] (verified)

Time = 0.76 (sec) , antiderivative size = 303, normalized size of antiderivative = 1.03, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.174$, Rules used = {3042, 26, 3699, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sinh^3(c+dx)}{a+b\sinh^3(c+dx)}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{i\sin(ic+idx)^3}{a+ib\sin(ic+idx)^3}dx$$

$$\Big\downarrow \text{26}$$

$$i\int \frac{\sin(ic+idx)^3}{ib\sin(ic+idx)^3+a}dx$$

$$\Big\downarrow \text{3699}$$

$$i\int \left(\frac{ia}{b\left(b\sinh^3(c+dx)+a\right)}-\frac{i}{b}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$i\left(\frac{2\sqrt[6]{-1}\sqrt[3]{a}\arctan\left(\frac{\sqrt[6]{-1}\left(\sqrt[6]{-1}\sqrt[3]{b}+i\sqrt[3]{a}\tanh\left(\frac{1}{2}(c+dx)\right)\right)}{\sqrt{\sqrt[3]{-1}a^{2/3}-(-1)^{2/3}b^{2/3}}}\right)}{3bd\sqrt{\sqrt[3]{-1}a^{2/3}-(-1)^{2/3}b^{2/3}}}+\frac{2\sqrt[6]{-1}\sqrt[3]{a}\arctan\left(\frac{\sqrt[6]{-1}\left((-1)^{5/6}\sqrt[3]{b}+i\sqrt[3]{a}\tanh\left(\frac{1}{2}(c+\right.\right.}{\sqrt{\sqrt[3]{-1}a^{2/3}-b^{2/3}}}\right.}{3bd\sqrt{\sqrt[3]{-1}a^{2/3}-b^{2/3}}}\right.$$

input
```
Int[Sinh[c + d*x]^3/(a + b*Sinh[c + d*x]^3),x]
```

output
```
I*(((-I)*x)/b + (2*(-1)^(1/6)*a^(1/3)*ArcTan[((-1)^(1/6)*((-1)^(1/6)*b^(1/
3) + I*a^(1/3)*Tanh[(c + d*x)/2]))/Sqrt[(-1)^(1/3)*a^(2/3) - (-1)^(2/3)*b^
(2/3)]])/(3*Sqrt[(-1)^(1/3)*a^(2/3) - (-1)^(2/3)*b^(2/3)]*b*d) + (2*(-1)^(
1/6)*a^(1/3)*ArcTan[((-1)^(1/6)*((-1)^(5/6)*b^(1/3) + I*a^(1/3)*Tanh[(c +
d*x)/2]))/Sqrt[(-1)^(1/3)*a^(2/3) - b^(2/3)]])/(3*Sqrt[(-1)^(1/3)*a^(2/3)
- b^(2/3)]*b*d) - (((2*I)/3)*a^(1/3)*ArcTanh[(b^(1/3) - a^(1/3)*Tanh[(c +
d*x)/2])/Sqrt[a^(2/3) + b^(2/3)]])/(Sqrt[a^(2/3) + b^(2/3)]*b*d))
```

Defintions of rubi rules used

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3699
```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^(n_
))^(p_.), x_Symbol] :> Int[ExpandTrig[sin[e + f*x]^m*(a + b*sin[e + f*x]^n)
^p, x], x] /; FreeQ[{a, b, e, f}, x] && IntegersQ[m, p] && (EqQ[n, 4] || Gt
Q[p, 0] || (EqQ[p, -1] && IntegerQ[n]))
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.80 (sec) , antiderivative size = 124, normalized size of antiderivative = 0.42

method	result
derivativedivides	$\frac{\frac{\ln\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)}{b}+\frac{a\left(\sum_{\_R=\mathrm{RootOf}\left(a\_Z^6-3a\_Z^4-8b\_Z^3+3a\_Z^2-a\right)}\frac{\left(\_R^4-2\_R^2+1\right)\ln\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)-\_R\right)}{\_R^5a-2\_R^3a-4\_R^2b+\_Ra}\right.}{3b}}{d}$
default	$\frac{\frac{\ln\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)}{b}+\frac{a\left(\sum_{\_R=\mathrm{RootOf}\left(a\_Z^6-3a\_Z^4-8b\_Z^3+3a\_Z^2-a\right)}\frac{\left(\_R^4-2\_R^2+1\right)\ln\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)-\_R\right)}{\_R^5a-2\_R^3a-4\_R^2b+\_Ra}\right.}{3b}}{d}$
risch	$\frac{x}{b}+\left(\sum_{\_R=\mathrm{RootOf}\left(\left(729a^2b^6d^6+729b^8d^6\right)\_Z^6-243a^2b^4d^4\_Z^4+27a^2d^2\_Z^2b^2-a^2\right)}\_R\ln\left(\mathrm{e}^{dx+c}+\left(486a\right.\right.\right.$

input
```
int(sinh(d*x+c)^3/(a+b*sinh(d*x+c)^3),x,method=_RETURNVERBOSE)
```

output
```
1/d*(1/b*ln(tanh(1/2*d*x+1/2*c)+1)+1/3/b*a*sum((_R^4-2*_R^2+1)/(_R^5*a-2*_
R^3*a-4*_R^2*b+_R*a)*ln(tanh(1/2*d*x+1/2*c)-_R),_R=RootOf(_Z^6*a-3*_Z^4*a-
8*_Z^3*b+3*_Z^2*a-a))-1/b*ln(tanh(1/2*d*x+1/2*c)-1))
```

Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 27931 vs. 2(205) = 410.

Time = 1.02 (sec) , antiderivative size = 27931, normalized size of antiderivative = 95.00

$$\int \frac{\sinh^3(c+dx)}{a+b\sinh^3(c+dx)}\,dx = \text{Too large to display}$$

input
```
integrate(sinh(d*x+c)^3/(a+b*sinh(d*x+c)^3),x, algorithm="fricas")
```

output
```
Too large to include
```

Sympy [F]

$$\int \frac{\sinh^3(c+dx)}{a+b\sinh^3(c+dx)}\,dx = \int \frac{\sinh^3{(c+dx)}}{a+b\sinh^3{(c+dx)}}\,dx$$

input
```
integrate(sinh(d*x+c)**3/(a+b*sinh(d*x+c)**3),x)
```

output
```
Integral(sinh(c + d*x)**3/(a + b*sinh(c + d*x)**3), x)
```

Maxima [F]

$$\int \frac{\sinh^3(c+dx)}{a+b\sinh^3(c+dx)}\,dx = \int \frac{\sinh(dx+c)^3}{b\sinh(dx+c)^3+a}\,dx$$

input
```
integrate(sinh(d*x+c)^3/(a+b*sinh(d*x+c)^3),x, algorithm="maxima")
```

output
```
-8*a*integrate(e^(3*d*x + 3*c)/(b^2*e^(6*d*x + 6*c) - 3*b^2*e^(4*d*x + 4*c
) + 8*a*b*e^(3*d*x + 3*c) + 3*b^2*e^(2*d*x + 2*c) - b^2), x) + x/b
```

Giac [F]

$$\int \frac{\sinh^3(c+dx)}{a+b\sinh^3(c+dx)}\,dx = \int \frac{\sinh(dx+c)^3}{b\sinh(dx+c)^3+a}\,dx$$

input
```
integrate(sinh(d*x+c)^3/(a+b*sinh(d*x+c)^3),x, algorithm="giac")
```

output
```
sage0*x
```

Mupad [B] (verification not implemented)

Time = 10.11 (sec) , antiderivative size = 1498, normalized size of antiderivative = 5.10

$$\int \frac{\sinh^3(c+dx)}{a+b\sinh^3(c+dx)}\,dx = \text{Too large to display}$$

input
```
int(sinh(c + d*x)^3/(a + b*sinh(c + d*x)^3),x)
```

output
```
symsum(log(-(24576*a^3*(405*root(729*a^2*b^6*d^6*z^6 + 729*b^8*d^6*z^6 - 2
43*a^2*b^4*d^4*z^4 + 27*a^2*b^2*d^2*z^2 - a^2, z, k)^5*b^7*d^5*exp(root(72
9*a^2*b^6*d^6*z^6 + 729*b^8*d^6*z^6 - 243*a^2*b^4*d^4*z^4 + 27*a^2*b^2*d^2
*z^2 - a^2, z, k) + d*x) - root(729*a^2*b^6*d^6*z^6 + 729*b^8*d^6*z^6 - 24
3*a^2*b^4*d^4*z^4 + 27*a^2*b^2*d^2*z^2 - a^2, z, k)*a*b^2*d - 27*root(729*
a^2*b^6*d^6*z^6 + 729*b^8*d^6*z^6 - 243*a^2*b^4*d^4*z^4 + 27*a^2*b^2*d^2*z
^2 - a^2, z, k)^4*b^6*d^4*exp(root(729*a^2*b^6*d^6*z^6 + 729*b^8*d^6*z^6 -
 243*a^2*b^4*d^4*z^4 + 27*a^2*b^2*d^2*z^2 - a^2, z, k) + d*x) - 4*a^2*exp(
root(729*a^2*b^6*d^6*z^6 + 729*b^8*d^6*z^6 - 243*a^2*b^4*d^4*z^4 + 27*a^2*
b^2*d^2*z^2 - a^2, z, k) + d*x) + 12*root(729*a^2*b^6*d^6*z^6 + 729*b^8*d^
6*z^6 - 243*a^2*b^4*d^4*z^4 + 27*a^2*b^2*d^2*z^2 - a^2, z, k)^2*a*b^3*d^2
- 54*root(729*a^2*b^6*d^6*z^6 + 729*b^8*d^6*z^6 - 243*a^2*b^4*d^4*z^4 + 27
*a^2*b^2*d^2*z^2 - a^2, z, k)^3*a*b^4*d^3 + 108*root(729*a^2*b^6*d^6*z^6 +
 729*b^8*d^6*z^6 - 243*a^2*b^4*d^4*z^4 + 27*a^2*b^2*d^2*z^2 - a^2, z, k)^4
*a*b^5*d^4 - 81*root(729*a^2*b^6*d^6*z^6 + 729*b^8*d^6*z^6 - 243*a^2*b^4*d
^4*z^4 + 27*a^2*b^2*d^2*z^2 - a^2, z, k)^5*a*b^6*d^5 + 20*root(729*a^2*b^6
*d^6*z^6 + 729*b^8*d^6*z^6 - 243*a^2*b^4*d^4*z^4 + 27*a^2*b^2*d^2*z^2 - a^
2, z, k)*a^2*b*d*exp(root(729*a^2*b^6*d^6*z^6 + 729*b^8*d^6*z^6 - 243*a^2*
b^4*d^4*z^4 + 27*a^2*b^2*d^2*z^2 - a^2, z, k) + d*x) + 24*root(729*a^2*b^6
*d^6*z^6 + 729*b^8*d^6*z^6 - 243*a^2*b^4*d^4*z^4 + 27*a^2*b^2*d^2*z^2 -...
```

Reduce [F]

$$\int \frac{\sinh^3(c+dx)}{a+b\sinh^3(c+dx)}\,dx = \frac{-\left(\int \frac{1}{\sinh(dx+c)^3b+a}dx\right)a+x}{b}$$

input
```
int(sinh(d*x+c)^3/(a+b*sinh(d*x+c)^3),x)
```

output
```
( - int(1/(sinh(c + d*x)**3*b + a),x)*a + x)/b
```

3.152 $\int \frac{\sinh^2(c+dx)}{a+b\sinh^3(c+dx)}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 262

$$\int \frac{\sinh^2(c+dx)}{a+b\sinh^3(c+dx)}\,dx = -\frac{2\arctan\left(\frac{(-1)^{5/6}\left(\sqrt[6]{-1}\sqrt[3]{b}+i\sqrt[3]{a}\tanh\left(\frac{1}{2}(c+dx)\right)\right)}{\sqrt{-(-1)^{2/3}a^{2/3}-b^{2/3}}}\right)}{3\sqrt{-(-1)^{2/3}a^{2/3}-b^{2/3}}\,b^{2/3}d} - \frac{2\arctan\left(\frac{\sqrt[6]{-1}\left((-1)^{5/6}\sqrt[3]{b}+i\sqrt[3]{a}\tanh\left(\frac{1}{2}(c+dx)\right)\right)}{\sqrt{\sqrt[3]{-1}a^{2/3}-b^{2/3}}}\right)}{3\sqrt{\sqrt[3]{-1}a^{2/3}-b^{2/3}}\,b^{2/3}d} - \frac{2\,\text{arctanh}\left(\frac{\sqrt[3]{b}-\sqrt[3]{a}\tanh\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^{2/3}+b^{2/3}}}\right)}{3\sqrt{a^{2/3}+b^{2/3}}\,b^{2/3}d}$$

output

```
-2/3*arctan((-1)^(5/6)*((-1)^(1/6)*b^(1/3)+I*a^(1/3)*tanh(1/2*d*x+1/2*c))/
(-(-1)^(2/3)*a^(2/3)-b^(2/3))^(1/2))/(-(-1)^(2/3)*a^(2/3)-b^(2/3))^(1/2)/b
^(2/3)/d-2/3*arctan((-1)^(1/6)*((-1)^(5/6)*b^(1/3)+I*a^(1/3)*tanh(1/2*d*x+
1/2*c))/((-1)^(1/3)*a^(2/3)-b^(2/3))^(1/2))/((-1)^(1/3)*a^(2/3)-b^(2/3))^(
1/2)/b^(2/3)/d-2/3*arctanh((b^(1/3)-a^(1/3)*tanh(1/2*d*x+1/2*c))/(a^(2/3)+
b^(2/3))^(1/2))/(a^(2/3)+b^(2/3))^(1/2)/b^(2/3)/d
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

Time = 11.06 (sec) , antiderivative size = 275, normalized size of antiderivative = 1.05

$$\int \frac{\sinh^2(c+dx)}{a+b\sinh^3(c+dx)}\,dx$$
$$= \frac{\text{RootSum}\left[-b+3b\#1^2+8a\#1^3-3b\#1^4+b\#1^6\&,\frac{c+dx+2\log\left(-\cosh\left(\frac{1}{2}(c+dx)\right)-\sinh\left(\frac{1}{2}(c+dx)\right)+\cosh\left(\frac{1}{2}(c+dx)\right)\#1\right.}{}\right.}{}$$

input
```
Integrate[Sinh[c + d*x]^2/(a + b*Sinh[c + d*x]^3),x]
```

output
```
RootSum[-b + 3*b*#1^2 + 8*a*#1^3 - 3*b*#1^4 + b*#1^6 & , (c + d*x + 2*Log[
-Cosh[(c + d*x)/2] - Sinh[(c + d*x)/2] + Cosh[(c + d*x)/2]*#1 - Sinh[(c +
d*x)/2]*#1] - 2*c*#1^2 - 2*d*x*#1^2 - 4*Log[-Cosh[(c + d*x)/2] - Sinh[(c +
 d*x)/2] + Cosh[(c + d*x)/2]*#1 - Sinh[(c + d*x)/2]*#1]*#1^2 + c*#1^4 + d*
x*#1^4 + 2*Log[-Cosh[(c + d*x)/2] - Sinh[(c + d*x)/2] + Cosh[(c + d*x)/2]*
#1 - Sinh[(c + d*x)/2]*#1]*#1^4)/(b*#1 + 4*a*#1^2 - 2*b*#1^3 + b*#1^5) & ]
/(6*d)
```

Rubi [A] (verified)

Time = 0.58 (sec) , antiderivative size = 262, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.174$, Rules used = {3042, 25, 3699, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sinh^2(c+dx)}{a+b\sinh^3(c+dx)}\,dx$$
$$\Big\downarrow \ 3042$$
$$\int -\frac{\sin(ic+idx)^2}{a+ib\sin(ic+idx)^3}dx$$

$$\downarrow 25$$

$$-\int \frac{\sin(ic+idx)^2}{ib\sin(ic+idx)^3+a}dx$$

$$\downarrow 3699$$

$$-\int \left(\frac{i}{3\left(-i\sqrt[3]{b}\sinh(c+dx)-i\sqrt[3]{a}\right)b^{2/3}}+\frac{i}{3\left(\sqrt[6]{-1}\sqrt[3]{a}-i\sqrt[3]{b}\sinh(c+dx)\right)b^{2/3}}+\frac{i}{3\left((-1)^{5/6}\sqrt[3]{a}-i\sqrt[3]{b}\sinh(c\right.}\right.$$

$$\downarrow 2009$$

$$-\frac{2\arctan\left(\frac{(-1)^{5/6}\left(\sqrt[6]{-1}\sqrt[3]{b}+i\sqrt[3]{a}\tanh\left(\frac{1}{2}(c+dx)\right)\right)}{\sqrt{-(-1)^{2/3}a^{2/3}-b^{2/3}}}\right)}{3b^{2/3}d\sqrt{-(-1)^{2/3}a^{2/3}-b^{2/3}}}-\frac{2\arctan\left(\frac{\sqrt[6]{-1}\left((-1)^{5/6}\sqrt[3]{b}+i\sqrt[3]{a}\tanh\left(\frac{1}{2}(c+dx)\right)\right)}{\sqrt{\sqrt[3]{-1}a^{2/3}-b^{2/3}}}\right)}{3b^{2/3}d\sqrt{\sqrt[3]{-1}a^{2/3}-b^{2/3}}}-\frac{2\text{arctanh}\left(\frac{\sqrt[3]{b}-\sqrt[3]{a}\tanh\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^{2/3}+b^{2/3}}}\right)}{3b^{2/3}d\sqrt{a^{2/3}+b^{2/3}}}$$

input
```
Int[Sinh[c + d*x]^2/(a + b*Sinh[c + d*x]^3),x]
```

output
```
(-2*ArcTan[((-1)^(5/6)*((-1)^(1/6)*b^(1/3) + I*a^(1/3)*Tanh[(c + d*x)/2]))
/Sqrt[-((-1)^(2/3)*a^(2/3)) - b^(2/3)]])/(3*Sqrt[-((-1)^(2/3)*a^(2/3)) - b
^(2/3)]*b^(2/3)*d) - (2*ArcTan[((-1)^(1/6)*((-1)^(5/6)*b^(1/3) + I*a^(1/3)
*Tanh[(c + d*x)/2]))/Sqrt[(-1)^(1/3)*a^(2/3) - b^(2/3)]])/(3*Sqrt[(-1)^(1/
3)*a^(2/3) - b^(2/3)]*b^(2/3)*d) - (2*ArcTanh[(b^(1/3) - a^(1/3)*Tanh[(c +
 d*x)/2])/Sqrt[a^(2/3) + b^(2/3)]])/(3*Sqrt[a^(2/3) + b^(2/3)]*b^(2/3)*d)
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3699
```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^(n_
))^(p_.), x_Symbol] :> Int[ExpandTrig[sin[e + f*x]^m*(a + b*sin[e + f*x]^n)
^p, x], x] /; FreeQ[{a, b, e, f}, x] && IntegersQ[m, p] && (EqQ[n, 4] || Gt
Q[p, 0] || (EqQ[p, -1] && IntegerQ[n]))
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.76 (sec) , antiderivative size = 78, normalized size of antiderivative = 0.30

method	result
derivativedivides	$-\frac{4\left(\sum_{\_R=\mathrm{RootOf}\left(a\_Z^6-3a\_Z^4-8b\_Z^3+3a\_Z^2-a\right)}\frac{\_R^2\ln\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)-\_R\right)}{\_R^5a-2\_R^3a-4\_R^2b+\_Ra}\right)}{3d}$
default	$-\frac{4\left(\sum_{\_R=\mathrm{RootOf}\left(a\_Z^6-3a\_Z^4-8b\_Z^3+3a\_Z^2-a\right)}\frac{\_R^2\ln\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)-\_R\right)}{\_R^5a-2\_R^3a-4\_R^2b+\_Ra}\right)}{3d}$
risch	$\sum_{\_R=\mathrm{RootOf}\left(-1+\left(729a^2b^4d^6+729b^6d^6\right)\_Z^6-243b^4d^4\_Z^4+27d^2\_Z^2b^2\right)}\_R\ln\left(\mathrm{e}^{dx+c}+\left(-243b^3d^5a^2-\right.\right.$

input
```
int(sinh(d*x+c)^2/(a+b*sinh(d*x+c)^3),x,method=_RETURNVERBOSE)
```

output
```
-4/3/d*sum(_R^2/(_R^5*a-2*_R^3*a-4*_R^2*b+_R*a)*ln(tanh(1/2*d*x+1/2*c)-_R)
,_R=RootOf(_Z^6*a-3*_Z^4*a-8*_Z^3*b+3*_Z^2*a-a))
```

Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 24063 vs. 2(181) = 362.

Time = 1.11 (sec) , antiderivative size = 24063, normalized size of antiderivative = 91.84

$$\int \frac{\sinh^2(c+dx)}{a+b\sinh^3(c+dx)}\,dx = \text{Too large to display}$$

input
```
integrate(sinh(d*x+c)^2/(a+b*sinh(d*x+c)^3),x, algorithm="fricas")
```

output
```
Too large to include
```

Sympy [F]

$$\int \frac{\sinh^2(c+dx)}{a+b\sinh^3(c+dx)}\,dx = \int \frac{\sinh^2(c+dx)}{a+b\sinh^3(c+dx)}\,dx$$

input
```
integrate(sinh(d*x+c)**2/(a+b*sinh(d*x+c)**3),x)
```

output
```
Integral(sinh(c + d*x)**2/(a + b*sinh(c + d*x)**3), x)
```

Maxima [F]

$$\int \frac{\sinh^2(c+dx)}{a+b\sinh^3(c+dx)}\,dx = \int \frac{\sinh(dx+c)^2}{b\sinh(dx+c)^3+a}\,dx$$

input
```
integrate(sinh(d*x+c)^2/(a+b*sinh(d*x+c)^3),x, algorithm="maxima")
```

output
```
integrate(sinh(d*x + c)^2/(b*sinh(d*x + c)^3 + a), x)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{\sinh^2(c+dx)}{a+b\sinh^3(c+dx)}\,dx = \text{Exception raised: AttributeError}$$

input
```
integrate(sinh(d*x+c)^2/(a+b*sinh(d*x+c)^3),x, algorithm="giac")
```

output
```
Exception raised: AttributeError >> type
```

Mupad [B] (verification not implemented)

Time = 12.16 (sec) , antiderivative size = 932, normalized size of antiderivative = 3.56

$$\int \frac{\sinh^2(c+dx)}{a+b\sinh^3(c+dx)}\,dx = \text{Too large to display}$$

input
```
int(sinh(c + d*x)^2/(a + b*sinh(c + d*x)^3),x)
```

output
```
symsum(log(root(729*a^2*b^4*d^6*z^6 + 729*b^6*d^6*z^6 - 243*b^4*d^4*z^4 +
27*b^2*d^2*z^2 - 1, z, k)*(root(729*a^2*b^4*d^6*z^6 + 729*b^6*d^6*z^6 - 24
3*b^4*d^4*z^4 + 27*b^2*d^2*z^2 - 1, z, k)*(root(729*a^2*b^4*d^6*z^6 + 729*
b^6*d^6*z^6 - 243*b^4*d^4*z^4 + 27*b^2*d^2*z^2 - 1, z, k)*(root(729*a^2*b^
4*d^6*z^6 + 729*b^6*d^6*z^6 - 243*b^4*d^4*z^4 + 27*b^2*d^2*z^2 - 1, z, k)*
((663552*(8*a^5*d^4 + 4*a^3*b^2*d^4 - 5*a^4*b*d^4*exp(d*x)*exp(root(729*a^
2*b^4*d^6*z^6 + 729*b^6*d^6*z^6 - 243*b^4*d^4*z^4 + 27*b^2*d^2*z^2 - 1, z,
 k))))/b^6 - (1990656*root(729*a^2*b^4*d^6*z^6 + 729*b^6*d^6*z^6 - 243*b^4
*d^4*z^4 + 27*b^2*d^2*z^2 - 1, z, k)*(4*a^5*d^5*exp(d*x)*exp(root(729*a^2*
b^4*d^6*z^6 + 729*b^6*d^6*z^6 - 243*b^4*d^4*z^4 + 27*b^2*d^2*z^2 - 1, z, k
)) - a^4*b*d^5 + 5*a^3*b^2*d^5*exp(d*x)*exp(root(729*a^2*b^4*d^6*z^6 + 729
*b^6*d^6*z^6 - 243*b^4*d^4*z^4 + 27*b^2*d^2*z^2 - 1, z, k))))/b^5) - (4423
68*(4*a^4*b*d^3 + 8*a^5*d^3*exp(d*x)*exp(root(729*a^2*b^4*d^6*z^6 + 729*b^
6*d^6*z^6 - 243*b^4*d^4*z^4 + 27*b^2*d^2*z^2 - 1, z, k)) - 5*a^3*b^2*d^3*e
xp(d*x)*exp(root(729*a^2*b^4*d^6*z^6 + 729*b^6*d^6*z^6 - 243*b^4*d^4*z^4 +
 27*b^2*d^2*z^2 - 1, z, k))))/b^7) - (294912*a^3*d^2*(2*b - 5*a*exp(d*x)*e
xp(root(729*a^2*b^4*d^6*z^6 + 729*b^6*d^6*z^6 - 243*b^4*d^4*z^4 + 27*b^2*d
^2*z^2 - 1, z, k))))/b^7) + (24576*a^3*d*(8*a - 5*b*exp(d*x)*exp(root(729*
a^2*b^4*d^6*z^6 + 729*b^6*d^6*z^6 - 243*b^4*d^4*z^4 + 27*b^2*d^2*z^2 - 1,
z, k))))/b^8) + (32768*a^3*(b - 4*a*exp(d*x)*exp(root(729*a^2*b^4*d^6*z...
```

Reduce [F]

$$\int \frac{\sinh^2(c+dx)}{a+b\sinh^3(c+dx)}\,dx=\int \frac{\sinh\left(dx+c\right)^2}{\sinh\left(dx+c\right)^3b+a}dx$$

input
```
int(sinh(d*x+c)^2/(a+b*sinh(d*x+c)^3),x)
```

output
```
int(sinh(c + d*x)**2/(sinh(c + d*x)**3*b + a),x)
```

3.153 $\int \frac{\sinh(c+dx)}{a+b\sinh^3(c+dx)}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 290

$$\int \frac{\sinh(c+dx)}{a+b\sinh^3(c+dx)}\,dx = \frac{2\arctan\left(\frac{\sqrt[6]{-1}\left(\sqrt[6]{-1}\sqrt[3]{b}+i\sqrt[3]{a}\tanh\left(\frac{1}{2}(c+dx)\right)\right)}{\sqrt{\sqrt[3]{-1}a^{2/3}-(-1)^{2/3}b^{2/3}}}\right)}{3\sqrt[3]{a}\sqrt{\sqrt[3]{-1}a^{2/3}-(-1)^{2/3}b^{2/3}}\sqrt[3]{b}d} - \frac{2\sqrt[3]{-1}\arctan\left(\frac{\sqrt[6]{-1}\left((-1)^{5/6}\sqrt[3]{b}+i\sqrt[3]{a}\tanh\left(\frac{1}{2}(c+dx)\right)\right)}{\sqrt{\sqrt[3]{-1}a^{2/3}-b^{2/3}}}\right)}{3\sqrt[3]{a}\sqrt{\sqrt[3]{-1}a^{2/3}-b^{2/3}}\sqrt[3]{b}d} + \frac{2\operatorname{arctanh}\left(\frac{\sqrt[3]{b}-\sqrt[3]{a}\tanh\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^{2/3}+b^{2/3}}}\right)}{3\sqrt[3]{a}\sqrt{a^{2/3}+b^{2/3}}\sqrt[3]{b}d}$$

output
```
2/3*arctan((-1)^(1/6)*((-1)^(1/6)*b^(1/3)+I*a^(1/3)*tanh(1/2*d*x+1/2*c))/(
(-1)^(1/3)*a^(2/3)-(-1)^(2/3)*b^(2/3))^(1/2))/a^(1/3)/((-1)^(1/3)*a^(2/3)-
(-1)^(2/3)*b^(2/3))^(1/2)/b^(1/3)/d-2/3*(-1)^(1/3)*arctan((-1)^(1/6)*((-1)
^(5/6)*b^(1/3)+I*a^(1/3)*tanh(1/2*d*x+1/2*c))/((-1)^(1/3)*a^(2/3)-b^(2/3))
^(1/2))/a^(1/3)/((-1)^(1/3)*a^(2/3)-b^(2/3))^(1/2)/b^(1/3)/d+2/3*arctanh((
b^(1/3)-a^(1/3)*tanh(1/2*d*x+1/2*c))/(a^(2/3)+b^(2/3))^(1/2))/a^(1/3)/(a^(
2/3)+b^(2/3))^(1/2)/b^(1/3)/d
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

Time = 11.05 (sec) , antiderivative size = 199, normalized size of antiderivative = 0.69

$$\int \frac{\sinh(c+dx)}{a+b\sinh^3(c+dx)}\,dx = \frac{\text{RootSum}\left[-b+3b\#1^2+8a\#1^3-3b\#1^4+b\#1^6\&,\frac{-c-dx-2\log\left(-\cosh\left(\frac{1}{2}(c+dx)\right)-\sinh\left(\frac{1}{2}(c+dx)\right)+\cosh\left(\frac{1}{2}(c+dx)\right)\#\right.}{}\right.}{3d}$$

input
```
Integrate[Sinh[c + d*x]/(a + b*Sinh[c + d*x]^3),x]
```

output
```
RootSum[-b + 3*b*#1^2 + 8*a*#1^3 - 3*b*#1^4 + b*#1^6 & , (-c - d*x - 2*Log
[-Cosh[(c + d*x)/2] - Sinh[(c + d*x)/2] + Cosh[(c + d*x)/2]*#1 - Sinh[(c +
 d*x)/2]*#1] + c*#1^2 + d*x*#1^2 + 2*Log[-Cosh[(c + d*x)/2] - Sinh[(c + d*
x)/2] + Cosh[(c + d*x)/2]*#1 - Sinh[(c + d*x)/2]*#1]*#1^2)/(b + 4*a*#1 - 2
*b*#1^2 + b*#1^4) & ]/(3*d)
```

Rubi [A] (verified)

Time = 0.63 (sec) , antiderivative size = 298, normalized size of antiderivative = 1.03, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.190$, Rules used = {3042, 26, 3699, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sinh(c+dx)}{a+b\sinh^3(c+dx)}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int -\frac{i\sin(ic+idx)}{a+ib\sin(ic+idx)^3}dx$$

$$\Big\downarrow \text{26}$$

$$-i\int \frac{\sin(ic+idx)}{ib\sin(ic+idx)^3+a}dx$$

$$\Big\downarrow \text{3699}$$

$$-i\int \left(\frac{\sqrt[3]{-1}}{3\sqrt[3]{a}\left(\sqrt[6]{-1}\sqrt[3]{a}-i\sqrt[3]{b}\sinh(c+dx)\right)\sqrt[3]{b}}-\frac{(-1)^{2/3}}{3\sqrt[3]{a}\left(\sqrt[6]{-1}\sqrt[3]{b}\sinh(c+dx)+\sqrt[6]{-1}\sqrt[3]{a}\right)\sqrt[3]{b}}-\frac{}{3\sqrt[3]{a}\left((-1)^{5/6}\right.}$$

$$\Big\downarrow \text{2009}$$

$$-i\left(\frac{2i\arctan\left(\frac{\sqrt[6]{-1}\left(\sqrt[6]{-1}\sqrt[3]{b}+i\sqrt[3]{a}\tanh\left(\frac{1}{2}(c+dx)\right)\right)}{\sqrt{\sqrt[3]{-1}a^{2/3}-(-1)^{2/3}b^{2/3}}}\right)}{3\sqrt[3]{a}\sqrt[3]{b}d\sqrt{\sqrt[3]{-1}a^{2/3}-(-1)^{2/3}b^{2/3}}}-\frac{2(-1)^{5/6}\arctan\left(\frac{\sqrt[6]{-1}\left((-1)^{5/6}\sqrt[3]{b}+i\sqrt[3]{a}\tanh\left(\frac{1}{2}(c+dx)\right)\right)}{\sqrt{\sqrt[3]{-1}a^{2/3}-b^{2/3}}}\right.}{3\sqrt[3]{a}\sqrt[3]{b}d\sqrt{\sqrt[3]{-1}a^{2/3}-b^{2/3}}}\right.$$

input
```
Int[Sinh[c + d*x]/(a + b*Sinh[c + d*x]^3),x]
```

output
```
(-I)*((((2*I)/3)*ArcTan[((-1)^(1/6)*((-1)^(1/6)*b^(1/3) + I*a^(1/3)*Tanh[(
c + d*x)/2]))/Sqrt[(-1)^(1/3)*a^(2/3) - (-1)^(2/3)*b^(2/3)]])/(a^(1/3)*Sqr
t[(-1)^(1/3)*a^(2/3) - (-1)^(2/3)*b^(2/3)]*b^(1/3)*d) - (2*(-1)^(5/6)*ArcT
an[((-1)^(1/6)*((-1)^(5/6)*b^(1/3) + I*a^(1/3)*Tanh[(c + d*x)/2]))/Sqrt[(-
1)^(1/3)*a^(2/3) - b^(2/3)]])/(3*a^(1/3)*Sqrt[(-1)^(1/3)*a^(2/3) - b^(2/3)
]*b^(1/3)*d) + (((2*I)/3)*ArcTanh[(b^(1/3) - a^(1/3)*Tanh[(c + d*x)/2])/Sq
rt[a^(2/3) + b^(2/3)]])/(a^(1/3)*Sqrt[a^(2/3) + b^(2/3)]*b^(1/3)*d))
```

Defintions of rubi rules used

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3699
```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^(n_
))^(p_.), x_Symbol] :> Int[ExpandTrig[sin[e + f*x]^m*(a + b*sin[e + f*x]^n)
^p, x], x] /; FreeQ[{a, b, e, f}, x] && IntegersQ[m, p] && (EqQ[n, 4] || Gt
Q[p, 0] || (EqQ[p, -1] && IntegerQ[n]))
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.75 (sec) , antiderivative size = 82, normalized size of antiderivative = 0.28

method	result
derivativedivides	$\frac{2\left(\sum_{\_R=\operatorname{RootOf}\left(a\_Z^6-3a\_Z^4-8b\_Z^3+3a\_Z^2-a\right)}\frac{\left(\_R^3-\_R\right)\ln\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)-\_R\right)}{\_R^5a-2\_R^3a-4\_R^2b+\_Ra}\right)}{3d}$
default	$\frac{2\left(\sum_{\_R=\operatorname{RootOf}\left(a\_Z^6-3a\_Z^4-8b\_Z^3+3a\_Z^2-a\right)}\frac{\left(\_R^3-\_R\right)\ln\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)-\_R\right)}{\_R^5a-2\_R^3a-4\_R^2b+\_Ra}\right)}{3d}$
risch	$\sum_{\_R=\operatorname{RootOf}\left(-1+\left(729a^4b^2d^6+729a^2b^4d^6\right)\_Z^6+243a^2b^2d^4\_Z^4\right)}\_R\ln\left(\mathrm{e}^{dx+c}+\left(\frac{243d^5b^2a^5}{a^2-b^2}+\frac{243d^5b^4a^3}{a^2-b^2}\right.\right.$

input
```
int(sinh(d*x+c)/(a+b*sinh(d*x+c)^3),x,method=_RETURNVERBOSE)
```

output
```
2/3/d*sum((_R^3-_R)/(_R^5*a-2*_R^3*a-4*_R^2*b+_R*a)*ln(tanh(1/2*d*x+1/2*c)
-_R),_R=RootOf(_Z^6*a-3*_Z^4*a-8*_Z^3*b+3*_Z^2*a-a))
```

Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 18312 vs. 2(197) = 394.

Time = 1.06 (sec) , antiderivative size = 18312, normalized size of antiderivative = 63.14

$$\int \frac{\sinh(c+dx)}{a+b\sinh^3(c+dx)}\,dx = \text{Too large to display}$$

input
```
integrate(sinh(d*x+c)/(a+b*sinh(d*x+c)^3),x, algorithm="fricas")
```

output
```
Too large to include
```

Sympy [F]

$$\int \frac{\sinh(c+dx)}{a+b\sinh^3(c+dx)}\,dx = \int \frac{\sinh{(c+dx)}}{a+b\sinh^3{(c+dx)}}\,dx$$

input
```
integrate(sinh(d*x+c)/(a+b*sinh(d*x+c)**3),x)
```

output
```
Integral(sinh(c + d*x)/(a + b*sinh(c + d*x)**3), x)
```

Maxima [F]

$$\int \frac{\sinh(c+dx)}{a+b\sinh^3(c+dx)}\,dx = \int \frac{\sinh{(dx+c)}}{b\sinh{(dx+c)}^3+a}\,dx$$

input
```
integrate(sinh(d*x+c)/(a+b*sinh(d*x+c)^3),x, algorithm="maxima")
```

output
```
integrate(sinh(d*x + c)/(b*sinh(d*x + c)^3 + a), x)
```

Giac [F]

$$\int \frac{\sinh(c+dx)}{a+b\sinh^3(c+dx)}\,dx = \int \frac{\sinh{(dx+c)}}{b\sinh{(dx+c)}^3+a}\,dx$$

input
```
integrate(sinh(d*x+c)/(a+b*sinh(d*x+c)^3),x, algorithm="giac")
```

output
```
integrate(sinh(d*x + c)/(b*sinh(d*x + c)^3 + a), x)
```

Mupad [B] (verification not implemented)

Time = 22.41 (sec) , antiderivative size = 857, normalized size of antiderivative = 2.96

$$\int \frac{\sinh(c+dx)}{a+b\sinh^3(c+dx)}\,dx = \text{Too large to display}$$

input
```
int(sinh(c + d*x)/(a + b*sinh(c + d*x)^3),x)
```

output
```
symsum(log(root(729*a^4*b^2*d^6*z^6 + 729*a^2*b^4*d^6*z^6 + 243*a^2*b^2*d^
4*z^4 - 1, z, k)*(root(729*a^4*b^2*d^6*z^6 + 729*a^2*b^4*d^6*z^6 + 243*a^2
*b^2*d^4*z^4 - 1, z, k)*(root(729*a^4*b^2*d^6*z^6 + 729*a^2*b^4*d^6*z^6 +
243*a^2*b^2*d^4*z^4 - 1, z, k)*(root(729*a^4*b^2*d^6*z^6 + 729*a^2*b^4*d^6
*z^6 + 243*a^2*b^2*d^4*z^4 - 1, z, k)*((663552*(4*a^4*b*d^4 + 16*a^5*d^4*e
xp(d*x)*exp(root(729*a^4*b^2*d^6*z^6 + 729*a^2*b^4*d^6*z^6 + 243*a^2*b^2*d
^4*z^4 - 1, z, k)) + 11*a^3*b^2*d^4*exp(d*x)*exp(root(729*a^4*b^2*d^6*z^6
+ 729*a^2*b^4*d^6*z^6 + 243*a^2*b^2*d^4*z^4 - 1, z, k))))/b^6 - (1990656*r
oot(729*a^4*b^2*d^6*z^6 + 729*a^2*b^4*d^6*z^6 + 243*a^2*b^2*d^4*z^4 - 1, z
, k)*(4*a^5*d^5*exp(d*x)*exp(root(729*a^4*b^2*d^6*z^6 + 729*a^2*b^4*d^6*z^
6 + 243*a^2*b^2*d^4*z^4 - 1, z, k)) - a^4*b*d^5 + 5*a^3*b^2*d^5*exp(d*x)*e
xp(root(729*a^4*b^2*d^6*z^6 + 729*a^2*b^4*d^6*z^6 + 243*a^2*b^2*d^4*z^4 -
1, z, k))))/b^5) + (221184*(8*a^4*d^3 + a^2*b^2*d^3 - 25*a^3*b*d^3*exp(d*x
)*exp(root(729*a^4*b^2*d^6*z^6 + 729*a^2*b^4*d^6*z^6 + 243*a^2*b^2*d^4*z^4
 - 1, z, k))))/b^6) - (294912*a^2*d^2*(b - 7*a*exp(d*x)*exp(root(729*a^4*b
^2*d^6*z^6 + 729*a^2*b^4*d^6*z^6 + 243*a^2*b^2*d^4*z^4 - 1, z, k))))/b^6)
+ (196608*a^2*d*(b - 2*a*exp(d*x)*exp(root(729*a^4*b^2*d^6*z^6 + 729*a^2*b
^4*d^6*z^6 + 243*a^2*b^2*d^4*z^4 - 1, z, k))))/b^7) - (8192*a*(8*a - b*exp
(d*x)*exp(root(729*a^4*b^2*d^6*z^6 + 729*a^2*b^4*d^6*z^6 + 243*a^2*b^2*d^4
*z^4 - 1, z, k))))/b^7)*root(729*a^4*b^2*d^6*z^6 + 729*a^2*b^4*d^6*z^6 ...
```

Reduce [F]

$$\int \frac{\sinh(c+dx)}{a+b\sinh^3(c+dx)}\,dx$$
$$= \frac{16e^{3c}\left(\int \frac{e^{3dx}}{e^{6dx+6c}b-3e^{4dx+4c}b+8e^{3dx+3c}a+3e^{2dx+2c}b-b}dx\right)ad+4e^{2c}\left(\int \frac{e^{2dx}}{e^{6dx+6c}b-3e^{4dx+4c}b+8e^{3dx+3c}a+3e^{2dx+2c}b-b}dx\right)bd}{bd}$$

input
```
int(sinh(d*x+c)/(a+b*sinh(d*x+c)^3),x)
```

output
```
(2*(24*e**(3*c)*int(e**(3*d*x)/(e**(6*c + 6*d*x)*b - 3*e**(4*c + 4*d*x)*b
+ 8*e**(3*c + 3*d*x)*a + 3*e**(2*c + 2*d*x)*b - b),x)*a*d + 6*e**(2*c)*int
(e**(2*d*x)/(e**(6*c + 6*d*x)*b - 3*e**(4*c + 4*d*x)*b + 8*e**(3*c + 3*d*x
)*a + 3*e**(2*c + 2*d*x)*b - b),x)*b*d - 6*int(1/(e**(6*c + 6*d*x)*b - 3*e
**(4*c + 4*d*x)*b + 8*e**(3*c + 3*d*x)*a + 3*e**(2*c + 2*d*x)*b - b),x)*b*
d + log(e**(6*c + 6*d*x)*b - 3*e**(4*c + 4*d*x)*b + 8*e**(3*c + 3*d*x)*a +
 3*e**(2*c + 2*d*x)*b - b) - 6*d*x))/(3*b*d)
```

3.154 $\int \frac{1}{a+b\sinh^3(c+dx)}\,dx$

Optimal result

Integrand size = 14, antiderivative size = 280

$$\int \frac{1}{a+b\sinh^3(c+dx)}\,dx=-\frac{2(-1)^{2/3}\arctan\left(\frac{\sqrt[6]{-1}\left(\sqrt[6]{-1}\sqrt[3]{b}+i\sqrt[3]{a}\tanh\left(\frac{1}{2}(c+dx)\right)\right)}{\sqrt{\sqrt[3]{-1}a^{2/3}-(-1)^{2/3}b^{2/3}}}\right)}{3a^{2/3}\sqrt{\sqrt[3]{-1}a^{2/3}-(-1)^{2/3}b^{2/3}}d}-\frac{2(-1)^{2/3}\arctan\left(\frac{\sqrt[6]{-1}\left((-1)^{5/6}\sqrt[3]{b}+i\sqrt[3]{a}\tanh\left(\frac{1}{2}(c+dx)\right)\right)}{\sqrt{\sqrt[3]{-1}a^{2/3}-b^{2/3}}}\right)}{3a^{2/3}\sqrt{\sqrt[3]{-1}a^{2/3}-b^{2/3}}d}-\frac{2\text{arctanh}\left(\frac{\sqrt[3]{b}-\sqrt[3]{a}\tanh\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^{2/3}+b^{2/3}}}\right)}{3a^{2/3}\sqrt{a^{2/3}+b^{2/3}}d}$$

output
```
-2/3*(-1)^(2/3)*arctan((-1)^(1/6)*((-1)^(1/6)*b^(1/3)+I*a^(1/3)*tanh(1/2*d
*x+1/2*c))/((-1)^(1/3)*a^(2/3)-(-1)^(2/3)*b^(2/3))^(1/2))/a^(2/3)/((-1)^(1
/3)*a^(2/3)-(-1)^(2/3)*b^(2/3))^(1/2)/d-2/3*(-1)^(2/3)*arctan((-1)^(1/6)*(
(-1)^(5/6)*b^(1/3)+I*a^(1/3)*tanh(1/2*d*x+1/2*c))/((-1)^(1/3)*a^(2/3)-b^(2
/3))^(1/2))/a^(2/3)/((-1)^(1/3)*a^(2/3)-b^(2/3))^(1/2)/d-2/3*arctanh((b^(1
/3)-a^(1/3)*tanh(1/2*d*x+1/2*c))/(a^(2/3)+b^(2/3))^(1/2))/a^(2/3)/(a^(2/3)
+b^(2/3))^(1/2)/d
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

Time = 11.05 (sec) , antiderivative size = 131, normalized size of antiderivative = 0.47

$$\int \frac{1}{a+b\sinh^3(c+dx)}\,dx = \frac{2\text{RootSum}\left[-b+3b\#1^2+8a\#1^3-3b\#1^4+b\#1^6\&,\frac{c\#1+dx\#1+2\log\left(-\cosh\left(\frac{1}{2}(c+dx)\right)-\sinh\left(\frac{1}{2}(c+dx)\right)+\cosh\left(\frac{1}{2}(c-\right.\right.}{b+4a\#1-2b\#1^2+b\#1^4}\right.}{3d}$$

input
```
Integrate[(a + b*Sinh[c + d*x]^3)^(-1),x]
```

output
```
(2*RootSum[-b + 3*b*#1^2 + 8*a*#1^3 - 3*b*#1^4 + b*#1^6 & , (c*#1 + d*x*#1
 + 2*Log[-Cosh[(c + d*x)/2] - Sinh[(c + d*x)/2] + Cosh[(c + d*x)/2]*#1 - S
inh[(c + d*x)/2]*#1]*#1)/(b + 4*a*#1 - 2*b*#1^2 + b*#1^4) & ])/(3*d)
```

Rubi [A] (verified)

Time = 0.56 (sec) , antiderivative size = 280, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.214$, Rules used = {3042, 3692, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{a+b\sinh^3(c+dx)}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{a+ib\sin(ic+idx)^3}dx$$

$$\Big\downarrow \text{3692}$$

$$\int \left(\frac{\sqrt[6]{-1}}{3a^{2/3}\left(\sqrt[6]{-1}\sqrt[3]{a}-i\sqrt[3]{b}\sinh(c+dx)\right)}+\frac{\sqrt[6]{-1}}{3a^{2/3}\left(\sqrt[6]{-1}\sqrt[3]{a}+\sqrt[6]{-1}\sqrt[3]{b}\sinh(c+dx)\right)}+\frac{\sqrt[6]{-}}{3a^{2/3}\left(\sqrt[6]{-1}\sqrt[3]{a}+(-1\right.}\right.$$

$$\Big\downarrow \text{2009}$$

$$-\frac{2(-1)^{2/3}\arctan\left(\frac{\sqrt[6]{-1}\left(\sqrt[6]{-1}\sqrt[3]{b}+i\sqrt[3]{a}\tanh\left(\frac{1}{2}(c+dx)\right)\right)}{\sqrt{\sqrt[3]{-1}a^{2/3}-(-1)^{2/3}b^{2/3}}}\right)}{3a^{2/3}d\sqrt{\sqrt[3]{-1}a^{2/3}-(-1)^{2/3}b^{2/3}}}-\frac{2(-1)^{2/3}\arctan\left(\frac{\sqrt[6]{-1}\left((-1)^{5/6}\sqrt[3]{b}+i\sqrt[3]{a}\tanh\left(\frac{1}{2}(c+dx)\right)\right)}{\sqrt{\sqrt[3]{-1}a^{2/3}-b^{2/3}}}\right)}{3a^{2/3}d\sqrt{\sqrt[3]{-1}a^{2/3}-b^{2/3}}}-\frac{2\text{arctanh}\left(\frac{\sqrt[3]{b}-\sqrt[3]{a}\tanh\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^{2/3}+b^{2/3}}}\right)}{3a^{2/3}d\sqrt{a^{2/3}+b^{2/3}}}$$

input
```
Int[(a + b*Sinh[c + d*x]^3)^(-1),x]
```

output
```
(-2*(-1)^(2/3)*ArcTan[((-1)^(1/6)*((-1)^(1/6)*b^(1/3) + I*a^(1/3)*Tanh[(c
+ d*x)/2]))/Sqrt[(-1)^(1/3)*a^(2/3) - (-1)^(2/3)*b^(2/3)]])/(3*a^(2/3)*Sqr
t[(-1)^(1/3)*a^(2/3) - (-1)^(2/3)*b^(2/3)]*d) - (2*(-1)^(2/3)*ArcTan[((-1)
^(1/6)*((-1)^(5/6)*b^(1/3) + I*a^(1/3)*Tanh[(c + d*x)/2]))/Sqrt[(-1)^(1/3)
*a^(2/3) - b^(2/3)]])/(3*a^(2/3)*Sqrt[(-1)^(1/3)*a^(2/3) - b^(2/3)]*d) - (
2*ArcTanh[(b^(1/3) - a^(1/3)*Tanh[(c + d*x)/2])/Sqrt[a^(2/3) + b^(2/3)]])/
(3*a^(2/3)*Sqrt[a^(2/3) + b^(2/3)]*d)
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3692
```
Int[((a_) + (b_.)*((c_.)*sin[(e_.) + (f_.)*(x_)])^(n_))^(p_), x_Symbol] :>
Int[ExpandTrig[(a + b*(c*sin[e + f*x])^n)^p, x], x] /; FreeQ[{a, b, c, e, f
, n}, x] && (IGtQ[p, 0] || (EqQ[p, -1] && IntegerQ[n]))
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.66 (sec) , antiderivative size = 87, normalized size of antiderivative = 0.31

method	result
derivativedivides	$\frac{\sum_{\_R=\text{RootOf}\left(a\_Z^6-3a\_Z^4-8b\_Z^3+3a\_Z^2-a\right)}\frac{\left(-\_R^4+2\_R^2-1\right)\ln\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)-\_R\right)}{\_R^5a-2\_R^3a-4\_R^2b+\_Ra}}{3d}$
default	$\frac{\sum_{\_R=\text{RootOf}\left(a\_Z^6-3a\_Z^4-8b\_Z^3+3a\_Z^2-a\right)}\frac{\left(-\_R^4+2\_R^2-1\right)\ln\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)-\_R\right)}{\_R^5a-2\_R^3a-4\_R^2b+\_Ra}}{3d}$
risch	$\sum_{\_R=\text{RootOf}\left(-1+(729a^6d^6+729a^4b^2d^6)\_Z^6-243a^4d^4\_Z^4+27a^2d^2\_Z^2\right)}\_R\ln\left(e^{dx+c}+\left(-\frac{486d^5a^6}{b}-48\right.\right.$

input
```
int(1/(a+b*sinh(d*x+c)^3),x,method=_RETURNVERBOSE)
```

output
```
1/3/d*sum((-_R^4+2*_R^2-1)/(_R^5*a-2*_R^3*a-4*_R^2*b+_R*a)*ln(tanh(1/2*d*x
+1/2*c)-_R),_R=RootOf(_Z^6*a-3*_Z^4*a-8*_Z^3*b+3*_Z^2*a-a))
```

Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 24084 vs. 2(191) = 382.

Time = 0.96 (sec) , antiderivative size = 24084, normalized size of antiderivative = 86.01

$$\int \frac{1}{a+b\sinh^3(c+dx)}\,dx = \text{Too large to display}$$

input
```
integrate(1/(a+b*sinh(d*x+c)^3),x, algorithm="fricas")
```

output
```
Too large to include
```

Sympy [F]

$$\int \frac{1}{a+b\sinh^3(c+dx)}\,dx=\int \frac{1}{a+b\sinh^3{(c+dx)}}\,dx$$

input
```
integrate(1/(a+b*sinh(d*x+c)**3),x)
```

output
```
Integral(1/(a + b*sinh(c + d*x)**3), x)
```

Maxima [F]

$$\int \frac{1}{a+b\sinh^3(c+dx)}\,dx=\int \frac{1}{b\sinh{(dx+c)}^3+a}\,dx$$

input
```
integrate(1/(a+b*sinh(d*x+c)^3),x, algorithm="maxima")
```

output
```
integrate(1/(b*sinh(d*x + c)^3 + a), x)
```

Giac [F]

$$\int \frac{1}{a+b\sinh^3(c+dx)}\,dx=\int \frac{1}{b\sinh{(dx+c)}^3+a}\,dx$$

input
```
integrate(1/(a+b*sinh(d*x+c)^3),x, algorithm="giac")
```

output
```
integrate(1/(b*sinh(d*x + c)^3 + a), x)
```

Mupad [B] (verification not implemented)

Time = 8.68 (sec) , antiderivative size = 1261, normalized size of antiderivative = 4.50

$$\int \frac{1}{a + b\sinh^3(c + dx)}\,dx = \text{Too large to display}$$

input
```
int(1/(a + b*sinh(c + d*x)^3),x)
```

output
```
symsum(log((24576*(root(729*a^4*b^2*d^6*z^6 + 729*a^6*d^6*z^6 - 243*a^4*d^
4*z^4 + 27*a^2*d^2*z^2 - 1, z, k)*b*d - 4*exp(root(729*a^4*b^2*d^6*z^6 + 7
29*a^6*d^6*z^6 - 243*a^4*d^4*z^4 + 27*a^2*d^2*z^2 - 1, z, k) + d*x) + 12*r
oot(729*a^4*b^2*d^6*z^6 + 729*a^6*d^6*z^6 - 243*a^4*d^4*z^4 + 27*a^2*d^2*z
^2 - 1, z, k)^2*a*b*d^2 - 20*root(729*a^4*b^2*d^6*z^6 + 729*a^6*d^6*z^6 -
243*a^4*d^4*z^4 + 27*a^2*d^2*z^2 - 1, z, k)*a*d*exp(root(729*a^4*b^2*d^6*z
^6 + 729*a^6*d^6*z^6 - 243*a^4*d^4*z^4 + 27*a^2*d^2*z^2 - 1, z, k) + d*x)
+ 24*root(729*a^4*b^2*d^6*z^6 + 729*a^6*d^6*z^6 - 243*a^4*d^4*z^4 + 27*a^2
*d^2*z^2 - 1, z, k)^2*a^2*d^2*exp(root(729*a^4*b^2*d^6*z^6 + 729*a^6*d^6*z
^6 - 243*a^4*d^4*z^4 + 27*a^2*d^2*z^2 - 1, z, k) + d*x) + 216*root(729*a^4
*b^2*d^6*z^6 + 729*a^6*d^6*z^6 - 243*a^4*d^4*z^4 + 27*a^2*d^2*z^2 - 1, z,
k)^3*a^3*d^3*exp(root(729*a^4*b^2*d^6*z^6 + 729*a^6*d^6*z^6 - 243*a^4*d^4*
z^4 + 27*a^2*d^2*z^2 - 1, z, k) + d*x) + 108*root(729*a^4*b^2*d^6*z^6 + 72
9*a^6*d^6*z^6 - 243*a^4*d^4*z^4 + 27*a^2*d^2*z^2 - 1, z, k)^4*a^4*d^4*exp(
root(729*a^4*b^2*d^6*z^6 + 729*a^6*d^6*z^6 - 243*a^4*d^4*z^4 + 27*a^2*d^2*
z^2 - 1, z, k) + d*x) - 324*root(729*a^4*b^2*d^6*z^6 + 729*a^6*d^6*z^6 - 2
43*a^4*d^4*z^4 + 27*a^2*d^2*z^2 - 1, z, k)^5*a^5*d^5*exp(root(729*a^4*b^2*
d^6*z^6 + 729*a^6*d^6*z^6 - 243*a^4*d^4*z^4 + 27*a^2*d^2*z^2 - 1, z, k) +
d*x) + 54*root(729*a^4*b^2*d^6*z^6 + 729*a^6*d^6*z^6 - 243*a^4*d^4*z^4 + 2
7*a^2*d^2*z^2 - 1, z, k)^3*a^2*b*d^3 + 108*root(729*a^4*b^2*d^6*z^6 + 7...
```

Reduce [F]

$$\int \frac{1}{a + b\sinh^3(c + dx)}\,dx = \int \frac{1}{\sinh{(dx + c)}^3 b + a}dx$$

input
```
int(1/(a+b*sinh(d*x+c)^3),x)
```

output
```
int(1/(sinh(c + d*x)**3*b + a),x)
```

3.155 $\int \frac{\operatorname{csch}(c+dx)}{a+b\sinh^3(c+dx)}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 286

$$\int \frac{\operatorname{csch}(c+dx)}{a+b\sinh^3(c+dx)}\,dx = \frac{2\sqrt[3]{b}\arctan\left(\frac{(-1)^{5/6}\left(\sqrt[6]{-1}\sqrt[3]{b}+i\sqrt[3]{a}\tanh\left(\frac{1}{2}(c+dx)\right)\right)}{\sqrt{-(-1)^{2/3}a^{2/3}-b^{2/3}}}\right)}{3a\sqrt{-(-1)^{2/3}a^{2/3}-b^{2/3}}\,d} + \frac{2\sqrt[3]{b}\arctan\left(\frac{\sqrt[6]{-1}\left((-1)^{5/6}\sqrt[3]{b}+i\sqrt[3]{a}\tanh\left(\frac{1}{2}(c+dx)\right)\right)}{\sqrt{\sqrt[3]{-1}a^{2/3}-b^{2/3}}}\right)}{3a\sqrt{\sqrt[3]{-1}a^{2/3}-b^{2/3}}\,d} - \frac{\operatorname{arctanh}(\cosh(c+dx))}{ad} + \frac{2\sqrt[3]{b}\operatorname{arctanh}\left(\frac{\sqrt[3]{b}-\sqrt[3]{a}\tanh\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^{2/3}+b^{2/3}}}\right)}{3a\sqrt{a^{2/3}+b^{2/3}}\,d}$$

output
```
2/3*b^(1/3)*arctan((-1)^(5/6)*((-1)^(1/6)*b^(1/3)+I*a^(1/3)*tanh(1/2*d*x+1
/2*c))/(-(-1)^(2/3)*a^(2/3)-b^(2/3))^(1/2))/a/(-(-1)^(2/3)*a^(2/3)-b^(2/3)
)^(1/2)/d+2/3*b^(1/3)*arctan((-1)^(1/6)*((-1)^(5/6)*b^(1/3)+I*a^(1/3)*tanh
(1/2*d*x+1/2*c))/((-1)^(1/3)*a^(2/3)-b^(2/3))^(1/2))/a/((-1)^(1/3)*a^(2/3)
-b^(2/3))^(1/2)/d-arctanh(cosh(d*x+c))/a/d+2/3*b^(1/3)*arctanh((b^(1/3)-a^
(1/3)*tanh(1/2*d*x+1/2*c))/(a^(2/3)+b^(2/3))^(1/2))/a/(a^(2/3)+b^(2/3))^(1
/2)/d
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

Time = 11.09 (sec) , antiderivative size = 307, normalized size of antiderivative = 1.07

$$\int \frac{\operatorname{csch}(c+dx)}{a+b\sinh^3(c+dx)}\,dx =$$

$$-\frac{6\log\left(\cosh\left(\frac{1}{2}(c+dx)\right)\right)-6\log\left(\sinh\left(\frac{1}{2}(c+dx)\right)\right)+b\text{RootSum}\left[-b+3b\#1^2+8a\#1^3-3b\#1^4+b\#\right.}{}$$

input
```
Integrate[Csch[c + d*x]/(a + b*Sinh[c + d*x]^3),x]
```

output
```
-1/6*(6*Log[Cosh[(c + d*x)/2]] - 6*Log[Sinh[(c + d*x)/2]] + b*RootSum[-b +
 3*b*#1^2 + 8*a*#1^3 - 3*b*#1^4 + b*#1^6 & , (c + d*x + 2*Log[-Cosh[(c + d
*x)/2] - Sinh[(c + d*x)/2] + Cosh[(c + d*x)/2]*#1 - Sinh[(c + d*x)/2]*#1]
- 2*c*#1^2 - 2*d*x*#1^2 - 4*Log[-Cosh[(c + d*x)/2] - Sinh[(c + d*x)/2] + C
osh[(c + d*x)/2]*#1 - Sinh[(c + d*x)/2]*#1]*#1^2 + c*#1^4 + d*x*#1^4 + 2*L
og[-Cosh[(c + d*x)/2] - Sinh[(c + d*x)/2] + Cosh[(c + d*x)/2]*#1 - Sinh[(c
 + d*x)/2]*#1]*#1^4)/(b*#1 + 4*a*#1^2 - 2*b*#1^3 + b*#1^5) & ])/(a*d)
```

Rubi [A] (verified)

Time = 0.69 (sec) , antiderivative size = 298, normalized size of antiderivative = 1.04, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.190, Rules used = {3042, 26, 3699, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\operatorname{csch}(c+dx)}{a+b\sinh^3(c+dx)}dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \frac{i}{\sin(ic+idx)\left(a+ib\sin(ic+idx)^3\right)}dx$$

$$\Big\downarrow \text{ 26}$$

$$i\int \frac{1}{\sin(ic+idx)\left(ib\sin(ic+idx)^3+a\right)}dx$$

$$\Big\downarrow \text{ 3699}$$

$$i\int \left(\frac{ib\sinh^2(c+dx)}{a\left(b\sinh^3(c+dx)+a\right)}-\frac{i\operatorname{csch}(c+dx)}{a}\right)dx$$

$$\Big\downarrow \text{ 2009}$$

$$i\left(-\frac{2i\sqrt[3]{b}\arctan\left(\frac{(-1)^{5/6}\left(\sqrt[6]{-1}\sqrt[3]{b}+i\sqrt[3]{a}\tanh\left(\frac{1}{2}(c+dx)\right)\right)}{\sqrt{-(-1)^{2/3}a^{2/3}-b^{2/3}}}\right)}{3ad\sqrt{-(-1)^{2/3}a^{2/3}-b^{2/3}}}-\frac{2i\sqrt[3]{b}\arctan\left(\frac{\sqrt[6]{-1}\left((-1)^{5/6}\sqrt[3]{b}+i\sqrt[3]{a}\tanh\left(\frac{1}{2}(c+dx)\right)\right)}{\sqrt{\sqrt[3]{-1}a^{2/3}-b^{2/3}}}\right)}{3ad\sqrt{\sqrt[3]{-1}a^{2/3}-b^{2/3}}}\right.$$

input

```
Int[Csch[c + d*x]/(a + b*Sinh[c + d*x]^3),x]
```

output
```
I*((((-2*I)/3)*b^(1/3)*ArcTan[((-1)^(5/6)*((-1)^(1/6)*b^(1/3) + I*a^(1/3)*
Tanh[(c + d*x)/2]))/Sqrt[-((-1)^(2/3)*a^(2/3)) - b^(2/3)]])/(a*Sqrt[-((-1)
^(2/3)*a^(2/3)) - b^(2/3)]*d) - (((2*I)/3)*b^(1/3)*ArcTan[((-1)^(1/6)*((-1
)^(5/6)*b^(1/3) + I*a^(1/3)*Tanh[(c + d*x)/2]))/Sqrt[(-1)^(1/3)*a^(2/3) -
b^(2/3)]])/(a*Sqrt[(-1)^(1/3)*a^(2/3) - b^(2/3)]*d) + (I*ArcTanh[Cosh[c +
d*x]])/(a*d) - (((2*I)/3)*b^(1/3)*ArcTanh[(b^(1/3) - a^(1/3)*Tanh[(c + d*x
)/2])/Sqrt[a^(2/3) + b^(2/3)]])/(a*Sqrt[a^(2/3) + b^(2/3)]*d))
```

Defintions of rubi rules used

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3699
```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^(n_
))^(p_.), x_Symbol] :> Int[ExpandTrig[sin[e + f*x]^m*(a + b*sin[e + f*x]^n)
^p, x], x] /; FreeQ[{a, b, e, f}, x] && IntegersQ[m, p] && (EqQ[n, 4] || Gt
Q[p, 0] || (EqQ[p, -1] && IntegerQ[n]))
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.96 (sec) , antiderivative size = 98, normalized size of antiderivative = 0.34

method	result
derivativedivides	$\frac{\frac{\ln\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{a}+\frac{4b\left(\sum_{\_R=\mathrm{RootOf}\left(a\_Z^6-3a\_Z^4-8b\_Z^3+3a\_Z^2-a\right)}\frac{\_R^2\ln\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)-\_R\right)}{\_R^5a-2\_R^3a-4\_R^2b+\_Ra}\right)}{3a}}{d}$
default	$\frac{\frac{\ln\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{a}+\frac{4b\left(\sum_{\_R=\mathrm{RootOf}\left(a\_Z^6-3a\_Z^4-8b\_Z^3+3a\_Z^2-a\right)}\frac{\_R^2\ln\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)-\_R\right)}{\_R^5a-2\_R^3a-4\_R^2b+\_Ra}\right)}{3a}}{d}$
risch	$\frac{\ln(\mathrm{e}^{dx+c}-1)}{ad}+2\left(\sum_{\_R=\mathrm{RootOf}\left((46656a^8d^6+46656a^6b^2d^6)\_Z^6-3888a^4b^2d^4\_Z^4+108a^2d^2\_Z^2b^2-b^2\right)}\_R\ln\left(\right.\right.$

input
```
int(csch(d*x+c)/(a+b*sinh(d*x+c)^3),x,method=_RETURNVERBOSE)
```

output
```
1/d*(1/a*ln(tanh(1/2*d*x+1/2*c))+4/3*b/a*sum(_R^2/(_R^5*a-2*_R^3*a-4*_R^2*
b+_R*a)*ln(tanh(1/2*d*x+1/2*c)-_R),_R=RootOf(_Z^6*a-3*_Z^4*a-8*_Z^3*b+3*_Z
^2*a-a)))
```

Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 28005 vs. 2(205) = 410.

Time = 2.25 (sec) , antiderivative size = 28005, normalized size of antiderivative = 97.92

$$\int \frac{\operatorname{csch}(c+dx)}{a+b\sinh^3(c+dx)}\,dx = \text{Too large to display}$$

input
```
integrate(csch(d*x+c)/(a+b*sinh(d*x+c)^3),x, algorithm="fricas")
```

output
```
Too large to include
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\operatorname{csch}(c+dx)}{a+b\sinh^3(c+dx)}\,dx = \text{Timed out}$$

input
```
integrate(csch(d*x+c)/(a+b*sinh(d*x+c)**3),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{\operatorname{csch}(c+dx)}{a+b\sinh^3(c+dx)}\,dx = \int \frac{\operatorname{csch}(dx+c)}{b\sinh(dx+c)^3+a}\,dx$$

input
```
integrate(csch(d*x+c)/(a+b*sinh(d*x+c)^3),x, algorithm="maxima")
```

output
```
-log((e^(d*x + c) + 1)*e^(-c))/(a*d) + log((e^(d*x + c) - 1)*e^(-c))/(a*d)
 - 2*integrate((b*e^(5*d*x + 5*c) - 2*b*e^(3*d*x + 3*c) + b*e^(d*x + c))/(
a*b*e^(6*d*x + 6*c) - 3*a*b*e^(4*d*x + 4*c) + 8*a^2*e^(3*d*x + 3*c) + 3*a*
b*e^(2*d*x + 2*c) - a*b), x)
```

Giac [F]

$$\int \frac{\operatorname{csch}(c+dx)}{a+b\sinh^3(c+dx)}\,dx = \int \frac{\operatorname{csch}(dx+c)}{b\sinh(dx+c)^3+a}\,dx$$

input
```
integrate(csch(d*x+c)/(a+b*sinh(d*x+c)^3),x, algorithm="giac")
```

output
```
sage0*x
```

Mupad [B] (verification not implemented)

Time = 59.38 (sec) , antiderivative size = 2970, normalized size of antiderivative = 10.38

$$\int \frac{\operatorname{csch}(c+dx)}{a+b\sinh^3(c+dx)}\,dx = \text{Too large to display}$$

input
```
int(1/(sinh(c + d*x)*(a + b*sinh(c + d*x)^3)),x)
```

output
```
symsum(log(-(2147483648*a*b*exp(root(729*a^6*b^2*d^6*z^6 + 729*a^8*d^6*z^6
 - 243*a^4*b^2*d^4*z^4 + 27*a^2*b^2*d^2*z^2 - b^2, z, k) + d*x) - 10737418
24*b^2 - 86973087744*root(729*a^6*b^2*d^6*z^6 + 729*a^8*d^6*z^6 - 243*a^4*
b^2*d^4*z^4 + 27*a^2*b^2*d^2*z^2 - b^2, z, k)^4*a^6*d^4 + 86973087744*root
(729*a^6*b^2*d^6*z^6 + 729*a^8*d^6*z^6 - 243*a^4*b^2*d^4*z^4 + 27*a^2*b^2*
d^2*z^2 - b^2, z, k)^6*a^8*d^6 + 134217728*root(729*a^6*b^2*d^6*z^6 + 729*
a^8*d^6*z^6 - 243*a^4*b^2*d^4*z^4 + 27*a^2*b^2*d^2*z^2 - b^2, z, k)*b^3*d
+ 3221225472*root(729*a^6*b^2*d^6*z^6 + 729*a^8*d^6*z^6 - 243*a^4*b^2*d^4*
z^4 + 27*a^2*b^2*d^2*z^2 - b^2, z, k)*a^2*b*d + 18589155328*root(729*a^6*b
^2*d^6*z^6 + 729*a^8*d^6*z^6 - 243*a^4*b^2*d^4*z^4 + 27*a^2*b^2*d^2*z^2 -
b^2, z, k)^2*a^2*b^2*d^2 - 2818572288*root(729*a^6*b^2*d^6*z^6 + 729*a^8*d
^6*z^6 - 243*a^4*b^2*d^4*z^4 + 27*a^2*b^2*d^2*z^2 - b^2, z, k)^3*a^2*b^3*d
^3 - 88181047296*root(729*a^6*b^2*d^6*z^6 + 729*a^8*d^6*z^6 - 243*a^4*b^2*
d^4*z^4 + 27*a^2*b^2*d^2*z^2 - b^2, z, k)^4*a^4*b^2*d^4 + 18119393280*root
(729*a^6*b^2*d^6*z^6 + 729*a^8*d^6*z^6 - 243*a^4*b^2*d^4*z^4 + 27*a^2*b^2*
d^2*z^2 - b^2, z, k)^5*a^4*b^3*d^5 + 70665633792*root(729*a^6*b^2*d^6*z^6
+ 729*a^8*d^6*z^6 - 243*a^4*b^2*d^4*z^4 + 27*a^2*b^2*d^2*z^2 - b^2, z, k)^
6*a^6*b^2*d^6 - 32614907904*root(729*a^6*b^2*d^6*z^6 + 729*a^8*d^6*z^6 - 2
43*a^4*b^2*d^4*z^4 + 27*a^2*b^2*d^2*z^2 - b^2, z, k)^7*a^6*b^3*d^7 - 57982
058496*root(729*a^6*b^2*d^6*z^6 + 729*a^8*d^6*z^6 - 243*a^4*b^2*d^4*z^4...
```

Reduce [F]

$$\int \frac{\operatorname{csch}(c+dx)}{a+b\sinh^3(c+dx)}\,dx = \int \frac{\operatorname{csch}(dx+c)}{\sinh(dx+c)^3 b+a}dx$$

input
```
int(csch(d*x+c)/(a+b*sinh(d*x+c)^3),x)
```

output
```
int(csch(c + d*x)/(sinh(c + d*x)**3*b + a),x)
```

3.156 $\int \frac{\operatorname{csch}^2(c+dx)}{a+b\sinh^3(c+dx)}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 304

$$\int \frac{\operatorname{csch}^2(c+dx)}{a+b\sinh^3(c+dx)}\,dx = -\frac{2b^{2/3}\arctan\left(\frac{\sqrt[6]{-1}\left(\sqrt[6]{-1}\sqrt[3]{b}+i\sqrt[3]{a}\tanh\left(\frac{1}{2}(c+dx)\right)\right)}{\sqrt{\sqrt[3]{-1}a^{2/3}-(-1)^{2/3}b^{2/3}}}\right)}{3a^{4/3}\sqrt{\sqrt[3]{-1}a^{2/3}-(-1)^{2/3}b^{2/3}}\,d} + \frac{2\sqrt[3]{-1}b^{2/3}\arctan\left(\frac{\sqrt[6]{-1}\left((-1)^{5/6}\sqrt[3]{b}+i\sqrt[3]{a}\tanh\left(\frac{1}{2}(c+dx)\right)\right)}{\sqrt{\sqrt[3]{-1}a^{2/3}-b^{2/3}}}\right)}{3a^{4/3}\sqrt{\sqrt[3]{-1}a^{2/3}-b^{2/3}}\,d} - \frac{2b^{2/3}\operatorname{arctanh}\left(\frac{\sqrt[3]{b}-\sqrt[3]{a}\tanh\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^{2/3}+b^{2/3}}}\right)}{3a^{4/3}\sqrt{a^{2/3}+b^{2/3}}\,d} - \frac{\coth(c+dx)}{ad}$$

output

```
-2/3*b^(2/3)*arctan((-1)^(1/6)*((-1)^(1/6)*b^(1/3)+I*a^(1/3)*tanh(1/2*d*x+
1/2*c))/((-1)^(1/3)*a^(2/3)-(-1)^(2/3)*b^(2/3))^(1/2))/a^(4/3)/((-1)^(1/3)
*a^(2/3)-(-1)^(2/3)*b^(2/3))^(1/2)/d+2/3*(-1)^(1/3)*b^(2/3)*arctan((-1)^(1
/6)*((-1)^(5/6)*b^(1/3)+I*a^(1/3)*tanh(1/2*d*x+1/2*c))/((-1)^(1/3)*a^(2/3)
-b^(2/3))^(1/2))/a^(4/3)/((-1)^(1/3)*a^(2/3)-b^(2/3))^(1/2)/d-2/3*b^(2/3)*
arctanh((b^(1/3)-a^(1/3)*tanh(1/2*d*x+1/2*c))/(a^(2/3)+b^(2/3))^(1/2))/a^(
4/3)/(a^(2/3)+b^(2/3))^(1/2)/d-coth(d*x+c)/a/d
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

Time = 0.79 (sec) , antiderivative size = 230, normalized size of antiderivative = 0.76

$$\int \frac{\operatorname{csch}^2(c+dx)}{a+b\sinh^3(c+dx)}\,dx =$$

$$-\frac{3\coth\left(\frac{1}{2}(c+dx)\right)+2b\text{RootSum}\left[-b+3b\#1^2+8a\#1^3-3b\#1^4+b\#1^6\&,\frac{-c-dx-2\log\left(-\cosh\left(\frac{1}{2}(c+dx)\right)-\right.}{}\right.}{}$$

input
```
Integrate[Csch[c + d*x]^2/(a + b*Sinh[c + d*x]^3),x]
```

output
```
-1/6*(3*Coth[(c + d*x)/2] + 2*b*RootSum[-b + 3*b*#1^2 + 8*a*#1^3 - 3*b*#1^
4 + b*#1^6 & , (-c - d*x - 2*Log[-Cosh[(c + d*x)/2] - Sinh[(c + d*x)/2] +
Cosh[(c + d*x)/2]*#1 - Sinh[(c + d*x)/2]*#1] + c*#1^2 + d*x*#1^2 + 2*Log[-
Cosh[(c + d*x)/2] - Sinh[(c + d*x)/2] + Cosh[(c + d*x)/2]*#1 - Sinh[(c + d
*x)/2]*#1]*#1^2)/(b + 4*a*#1 - 2*b*#1^2 + b*#1^4) & ] + 3*Tanh[(c + d*x)/2
])/(a*d)
```

Rubi [A] (verified)

Time = 0.84 (sec) , antiderivative size = 304, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.174$, Rules used = {3042, 25, 3699, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\operatorname{csch}^2(c+dx)}{a+b\sinh^3(c+dx)}dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int -\frac{1}{\sin(ic+idx)^2\left(a+ib\sin(ic+idx)^3\right)}dx$$

$$\Big\downarrow \text{ 25}$$

$$-\int \frac{1}{\sin(ic+idx)^2\left(ib\sin(ic+idx)^3+a\right)}dx$$

$$\Big\downarrow \text{3699}$$

$$-\int \left(\frac{b\sinh(c+dx)}{a\left(b\sinh^3(c+dx)+a\right)}-\frac{\operatorname{csch}^2(c+dx)}{a}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$-\frac{2b^{2/3}\arctan\left(\frac{\sqrt[6]{-1}\left(\sqrt[6]{-1}\sqrt[3]{b}+i\sqrt[3]{a}\tanh\left(\frac{1}{2}(c+dx)\right)\right)}{\sqrt{\sqrt[3]{-1}a^{2/3}-(-1)^{2/3}b^{2/3}}}\right)}{3a^{4/3}d\sqrt{\sqrt[3]{-1}a^{2/3}-(-1)^{2/3}b^{2/3}}}+\frac{2\sqrt[3]{-1}b^{2/3}\arctan\left(\frac{\sqrt[6]{-1}\left((-1)^{5/6}\sqrt[3]{b}+i\sqrt[3]{a}\tanh\left(\frac{1}{2}(c+dx)\right)\right)}{\sqrt{\sqrt[3]{-1}a^{2/3}-b^{2/3}}}\right)}{3a^{4/3}d\sqrt{\sqrt[3]{-1}a^{2/3}-b^{2/3}}}-\frac{2b^{2/3}\text{arctanh}\left(\frac{\sqrt[3]{b}-\sqrt[3]{a}\tanh\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^{2/3}+b^{2/3}}}\right)}{3a^{4/3}d\sqrt{a^{2/3}+b^{2/3}}}-\frac{\coth(c+dx)}{ad}$$

input

```
Int[Csch[c + d*x]^2/(a + b*Sinh[c + d*x]^3),x]
```

output

```
(-2*b^(2/3)*ArcTan[((-1)^(1/6)*((-1)^(1/6)*b^(1/3) + I*a^(1/3)*Tanh[(c + d
*x)/2]))/Sqrt[(-1)^(1/3)*a^(2/3) - (-1)^(2/3)*b^(2/3)]])/(3*a^(4/3)*Sqrt[(
-1)^(1/3)*a^(2/3) - (-1)^(2/3)*b^(2/3)]*d) + (2*(-1)^(1/3)*b^(2/3)*ArcTan[
((-1)^(1/6)*((-1)^(5/6)*b^(1/3) + I*a^(1/3)*Tanh[(c + d*x)/2]))/Sqrt[(-1)^
(1/3)*a^(2/3) - b^(2/3)]])/(3*a^(4/3)*Sqrt[(-1)^(1/3)*a^(2/3) - b^(2/3)]*d
) - (2*b^(2/3)*ArcTanh[(b^(1/3) - a^(1/3)*Tanh[(c + d*x)/2])/Sqrt[a^(2/3)
+ b^(2/3)]])/(3*a^(4/3)*Sqrt[a^(2/3) + b^(2/3)]*d) - Coth[c + d*x]/(a*d)
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3699
```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^(n_
))^(p_.), x_Symbol] :> Int[ExpandTrig[sin[e + f*x]^m*(a + b*sin[e + f*x]^n)
^p, x], x] /; FreeQ[{a, b, e, f}, x] && IntegersQ[m, p] && (EqQ[n, 4] || Gt
Q[p, 0] || (EqQ[p, -1] && IntegerQ[n]))
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 1.13 (sec) , antiderivative size = 118, normalized size of antiderivative = 0.39

method	result
derivativedivides	$\frac{-\frac{\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)}{2a}-\frac{2b\left(\sum_{\_R=\operatorname{RootOf}\left(a\_Z^6-3a\_Z^4-8b\_Z^3+3a\_Z^2-a\right)}\frac{\left(\_R^3-\_R\right)\ln\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)-\_R\right)}{\_R^5a-2\_R^3a-4\_R^2b+\_Ra}\right)}{3a}-\frac{}{2a\,\mathrm{ta}}}{d}$
default	$\frac{-\frac{\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)}{2a}-\frac{2b\left(\sum_{\_R=\operatorname{RootOf}\left(a\_Z^6-3a\_Z^4-8b\_Z^3+3a\_Z^2-a\right)}\frac{\left(\_R^3-\_R\right)\ln\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)-\_R\right)}{\_R^5a-2\_R^3a-4\_R^2b+\_Ra}\right)}{3a}-\frac{}{2a\,\mathrm{ta}}}{d}$
risch	$-\frac{2}{ad\left(e^{2dx+2c}-1\right)}+4\left(\sum_{\_R=\operatorname{RootOf}\left(\left(2985984a^{10}d^6+2985984a^8b^2d^6\right)\_Z^6+62208a^6b^2d^4\_Z^4-b^4\right)}\_R\ln\left(e^{da}\right.\right.$

input
```
int(csch(d*x+c)^2/(a+b*sinh(d*x+c)^3),x,method=_RETURNVERBOSE)
```

output
```
1/d*(-1/2/a*tanh(1/2*d*x+1/2*c)-2/3*b/a*sum((_R^3-_R)/(_R^5*a-2*_R^3*a-4*_
R^2*b+_R*a)*ln(tanh(1/2*d*x+1/2*c)-_R),_R=RootOf(_Z^6*a-3*_Z^4*a-8*_Z^3*b+
3*_Z^2*a-a))-1/2/a/tanh(1/2*d*x+1/2*c))
```

Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 21133 vs. $2(211) = 422$.

Time = 1.03 (sec) , antiderivative size = 21133, normalized size of antiderivative = 69.52

$$\int \frac{\operatorname{csch}^2(c+dx)}{a+b\sinh^3(c+dx)}\,dx = \text{Too large to display}$$

input
```
integrate(csch(d*x+c)^2/(a+b*sinh(d*x+c)^3),x, algorithm="fricas")
```

output
```
Too large to include
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\operatorname{csch}^2(c+dx)}{a+b\sinh^3(c+dx)}\,dx = \text{Timed out}$$

input
```
integrate(csch(d*x+c)**2/(a+b*sinh(d*x+c)**3),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{\operatorname{csch}^2(c+dx)}{a+b\sinh^3(c+dx)}\,dx = \int \frac{\operatorname{csch}(dx+c)^2}{b\sinh(dx+c)^3+a}\,dx$$

input
```
integrate(csch(d*x+c)^2/(a+b*sinh(d*x+c)^3),x, algorithm="maxima")
```

output
```
-2/(a*d*e^(2*d*x + 2*c) - a*d) - 4*integrate((b*e^(4*d*x + 4*c) - b*e^(2*d
*x + 2*c))/(a*b*e^(6*d*x + 6*c) - 3*a*b*e^(4*d*x + 4*c) + 8*a^2*e^(3*d*x +
 3*c) + 3*a*b*e^(2*d*x + 2*c) - a*b), x)
```

Giac [F]

$$\int \frac{\operatorname{csch}^2(c+dx)}{a+b\sinh^3(c+dx)}\,dx = \int \frac{\operatorname{csch}(dx+c)^2}{b\sinh(dx+c)^3+a}\,dx$$

input
```
integrate(csch(d*x+c)^2/(a+b*sinh(d*x+c)^3),x, algorithm="giac")
```

output
```
integrate(csch(d*x + c)^2/(b*sinh(d*x + c)^3 + a), x)
```

Mupad [B] (verification not implemented)

Time = 24.87 (sec) , antiderivative size = 1293, normalized size of antiderivative = 4.25

$$\int \frac{\operatorname{csch}^2(c+dx)}{a+b\sinh^3(c+dx)}\,dx = \text{Too large to display}$$

input
```
int(1/(sinh(c + d*x)^2*(a + b*sinh(c + d*x)^3)),x)
```

output
```
symsum(log(-(8192*b^4*exp(root(729*a^8*b^2*d^6*z^6 + 729*a^10*d^6*z^6 + 24
3*a^6*b^2*d^4*z^4 - b^4, z, k) + d*x) - 65536*a*b^3 - 294912*root(729*a^8*
b^2*d^6*z^6 + 729*a^10*d^6*z^6 + 243*a^6*b^2*d^4*z^4 - b^4, z, k)^2*a^3*b^
3*d^2 - 221184*root(729*a^8*b^2*d^6*z^6 + 729*a^10*d^6*z^6 + 243*a^6*b^2*d
^4*z^4 - b^4, z, k)^3*a^4*b^3*d^3 - 196608*root(729*a^8*b^2*d^6*z^6 + 729*
a^10*d^6*z^6 + 243*a^6*b^2*d^4*z^4 - b^4, z, k)*a^2*b^3*d + 10616832*root(
729*a^8*b^2*d^6*z^6 + 729*a^10*d^6*z^6 + 243*a^6*b^2*d^4*z^4 - b^4, z, k)^
4*a^8*d^4*exp(root(729*a^8*b^2*d^6*z^6 + 729*a^10*d^6*z^6 + 243*a^6*b^2*d^
4*z^4 - b^4, z, k) + d*x) + 7962624*root(729*a^8*b^2*d^6*z^6 + 729*a^10*d^
6*z^6 + 243*a^6*b^2*d^4*z^4 - b^4, z, k)^5*a^9*d^5*exp(root(729*a^8*b^2*d^
6*z^6 + 729*a^10*d^6*z^6 + 243*a^6*b^2*d^4*z^4 - b^4, z, k) + d*x) - 17694
72*root(729*a^8*b^2*d^6*z^6 + 729*a^10*d^6*z^6 + 243*a^6*b^2*d^4*z^4 - b^4
, z, k)^3*a^6*b*d^3 + 2654208*root(729*a^8*b^2*d^6*z^6 + 729*a^10*d^6*z^6
+ 243*a^6*b^2*d^4*z^4 - b^4, z, k)^4*a^7*b*d^4 - 1990656*root(729*a^8*b^2*
d^6*z^6 + 729*a^10*d^6*z^6 + 243*a^6*b^2*d^4*z^4 - b^4, z, k)^5*a^8*b*d^5
+ 2064384*root(729*a^8*b^2*d^6*z^6 + 729*a^10*d^6*z^6 + 243*a^6*b^2*d^4*z^
4 - b^4, z, k)^2*a^4*b^2*d^2*exp(root(729*a^8*b^2*d^6*z^6 + 729*a^10*d^6*z
^6 + 243*a^6*b^2*d^4*z^4 - b^4, z, k) + d*x) + 5529600*root(729*a^8*b^2*d^
6*z^6 + 729*a^10*d^6*z^6 + 243*a^6*b^2*d^4*z^4 - b^4, z, k)^3*a^5*b^2*d^3*
exp(root(729*a^8*b^2*d^6*z^6 + 729*a^10*d^6*z^6 + 243*a^6*b^2*d^4*z^4 -...
```

Reduce [F]

$$\int \frac{\operatorname{csch}^2(c+dx)}{a+b\sinh^3(c+dx)}\,dx = \text{Too large to display}$$

input
```
int(csch(d*x+c)^2/(a+b*sinh(d*x+c)^3),x)
```

output

```
(4*(192*e**(6*c + 2*d*x)*int(e**(4*d*x)/(e**(10*c + 10*d*x)*b - 5*e**(8*c
+ 8*d*x)*b + 8*e**(7*c + 7*d*x)*a + 10*e**(6*c + 6*d*x)*b - 16*e**(5*c + 5
*d*x)*a - 10*e**(4*c + 4*d*x)*b + 8*e**(3*c + 3*d*x)*a + 5*e**(2*c + 2*d*x
)*b - b),x)*a**2*d + 96*e**(5*c + 2*d*x)*int(e**(3*d*x)/(e**(10*c + 10*d*x
)*b - 5*e**(8*c + 8*d*x)*b + 8*e**(7*c + 7*d*x)*a + 10*e**(6*c + 6*d*x)*b
- 16*e**(5*c + 5*d*x)*a - 10*e**(4*c + 4*d*x)*b + 8*e**(3*c + 3*d*x)*a + 5
*e**(2*c + 2*d*x)*b - b),x)*a*b*d - 24*e**(3*c + 2*d*x)*int(e**(d*x)/(e**(
10*c + 10*d*x)*b - 5*e**(8*c + 8*d*x)*b + 8*e**(7*c + 7*d*x)*a + 10*e**(6*
c + 6*d*x)*b - 16*e**(5*c + 5*d*x)*a - 10*e**(4*c + 4*d*x)*b + 8*e**(3*c +
 3*d*x)*a + 5*e**(2*c + 2*d*x)*b - b),x)*a*b*d + 6*e**(2*c + 2*d*x)*log(e*
*(c + d*x) - 1)*a + 3*e**(2*c + 2*d*x)*log(e**(c + d*x) - 1)*b - 6*e**(2*c
 + 2*d*x)*log(e**(c + d*x) + 1)*a + 3*e**(2*c + 2*d*x)*log(e**(c + d*x) +
1)*b - e**(2*c + 2*d*x)*log(e**(6*c + 6*d*x)*b - 3*e**(4*c + 4*d*x)*b + 8*
e**(3*c + 3*d*x)*a + 3*e**(2*c + 2*d*x)*b - b)*b + 12*e**(c + d*x)*a - 192
*e**(4*c)*int(e**(4*d*x)/(e**(10*c + 10*d*x)*b - 5*e**(8*c + 8*d*x)*b + 8*
e**(7*c + 7*d*x)*a + 10*e**(6*c + 6*d*x)*b - 16*e**(5*c + 5*d*x)*a - 10*e*
*(4*c + 4*d*x)*b + 8*e**(3*c + 3*d*x)*a + 5*e**(2*c + 2*d*x)*b - b),x)*a**
2*d - 96*e**(3*c)*int(e**(3*d*x)/(e**(10*c + 10*d*x)*b - 5*e**(8*c + 8*d*x
)*b + 8*e**(7*c + 7*d*x)*a + 10*e**(6*c + 6*d*x)*b - 16*e**(5*c + 5*d*x)*a
 - 10*e**(4*c + 4*d*x)*b + 8*e**(3*c + 3*d*x)*a + 5*e**(2*c + 2*d*x)*b ...
```

3.157 $\int \frac{\operatorname{csch}^3(c+dx)}{a+b\sinh^3(c+dx)}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 322

$$\begin{aligned}\int \frac{\operatorname{csch}^3(c+dx)}{a+b\sinh^3(c+dx)}\,dx &= \frac{2(-1)^{2/3}b\arctan\left(\frac{\sqrt[6]{-1}\left(\sqrt[6]{-1}\sqrt[3]{b}+i\sqrt[3]{a}\tanh\left(\frac{1}{2}(c+dx)\right)\right)}{\sqrt{\sqrt[3]{-1}a^{2/3}-(-1)^{2/3}b^{2/3}}}\right)}{3a^{5/3}\sqrt{\sqrt[3]{-1}a^{2/3}-(-1)^{2/3}b^{2/3}}d} \\ &+ \frac{2(-1)^{2/3}b\arctan\left(\frac{\sqrt[6]{-1}\left((-1)^{5/6}\sqrt[3]{b}+i\sqrt[3]{a}\tanh\left(\frac{1}{2}(c+dx)\right)\right)}{\sqrt{\sqrt[3]{-1}a^{2/3}-b^{2/3}}}\right)}{3a^{5/3}\sqrt{\sqrt[3]{-1}a^{2/3}-b^{2/3}}d} \\ &+ \frac{\operatorname{arctanh}(\cosh(c+dx))}{2ad} \\ &+ \frac{2b\operatorname{arctanh}\left(\frac{\sqrt[3]{b}-\sqrt[3]{a}\tanh\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^{2/3}+b^{2/3}}}\right)}{3a^{5/3}\sqrt{a^{2/3}+b^{2/3}}d} \\ &- \frac{\coth(c+dx)\operatorname{csch}(c+dx)}{2ad}\end{aligned}$$

output
```
2/3*(-1)^(2/3)*b*arctan((-1)^(1/6)*((-1)^(1/6)*b^(1/3)+I*a^(1/3)*tanh(1/2*
d*x+1/2*c))/((-1)^(1/3)*a^(2/3)-(-1)^(2/3)*b^(2/3))^(1/2))/a^(5/3)/((-1)^(
1/3)*a^(2/3)-(-1)^(2/3)*b^(2/3))^(1/2)/d+2/3*(-1)^(2/3)*b*arctan((-1)^(1/6
)*((-1)^(5/6)*b^(1/3)+I*a^(1/3)*tanh(1/2*d*x+1/2*c))/((-1)^(1/3)*a^(2/3)-b
^(2/3))^(1/2))/a^(5/3)/((-1)^(1/3)*a^(2/3)-b^(2/3))^(1/2)/d+1/2*arctanh(co
sh(d*x+c))/a/d+2/3*b*arctanh((b^(1/3)-a^(1/3)*tanh(1/2*d*x+1/2*c))/(a^(2/3
)+b^(2/3))^(1/2))/a^(5/3)/(a^(2/3)+b^(2/3))^(1/2)/d-1/2*coth(d*x+c)*csch(d
*x+c)/a/d
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

Time = 0.85 (sec) , antiderivative size = 191, normalized size of antiderivative = 0.59

$$\int \frac{\operatorname{csch}^3(c+dx)}{a+b\sinh^3(c+dx)}\,dx =$$

$$-\frac{16b\,\text{RootSum}\left[-b+3b\#1^2+8a\#1^3-3b\#1^4+b\#1^6\&,\frac{c\#1+dx\#1+2\log\left(-\cosh\left(\frac{1}{2}(c+dx)\right)-\sinh\left(\frac{1}{2}(c+dx)\right)+\cos\cdots\right.}{b+4a\#1-2b\#1^2+b\cdots}\right.}{\cdots}$$

input
```
Integrate[Csch[c + d*x]^3/(a + b*Sinh[c + d*x]^3),x]
```

output
```
-1/24*(16*b*RootSum[-b + 3*b*#1^2 + 8*a*#1^3 - 3*b*#1^4 + b*#1^6 & , (c*#1
 + d*x*#1 + 2*Log[-Cosh[(c + d*x)/2] - Sinh[(c + d*x)/2] + Cosh[(c + d*x)/
2]*#1 - Sinh[(c + d*x)/2]*#1]*#1)/(b + 4*a*#1 - 2*b*#1^2 + b*#1^4) & ] + 3
*(Csch[(c + d*x)/2]^2 - 4*Log[Cosh[(c + d*x)/2]] + 4*Log[Sinh[(c + d*x)/2]
] + Sech[(c + d*x)/2]^2))/(a*d)
```

Rubi [A] (verified)

Time = 0.81 (sec) , antiderivative size = 332, normalized size of antiderivative = 1.03, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.174, Rules used = {3042, 26, 3699, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\operatorname{csch}^3(c+dx)}{a+b\sinh^3(c+dx)}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int -\frac{i}{\sin(ic+idx)^3\,(a+ib\sin(ic+idx)^3)}dx$$

$$\Big\downarrow \text{26}$$

$$-i\int \frac{1}{\sin(ic+idx)^3\,(ib\sin(ic+idx)^3+a)}dx$$

$$\Big\downarrow \text{3699}$$

$$-i\int \left(\frac{i\operatorname{csch}^3(c+dx)}{a}-\frac{ib}{a\,(b\sinh^3(c+dx)+a)}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$-i\left(-\frac{2\sqrt[6]{-1}b\arctan\left(\frac{\sqrt[6]{-1}\left(\sqrt[6]{-1}\sqrt[3]{b}+i\sqrt[3]{a}\tanh\left(\frac{1}{2}(c+dx)\right)\right)}{\sqrt{\sqrt[3]{-1}a^{2/3}-(-1)^{2/3}b^{2/3}}}\right)}{3a^{5/3}d\sqrt{\sqrt[3]{-1}a^{2/3}-(-1)^{2/3}b^{2/3}}}-\frac{2\sqrt[6]{-1}b\arctan\left(\frac{\sqrt[6]{-1}\left((-1)^{5/6}\sqrt[3]{b}+i\sqrt[3]{a}\tanh\left(\frac{1}{2}(c+d\right.\right.}{\sqrt{\sqrt[3]{-1}a^{2/3}-b^{2/3}}}\right.}{3a^{5/3}d\sqrt{\sqrt[3]{-1}a^{2/3}-b^{2/3}}}\right.$$

input
```
Int[Csch[c + d*x]^3/(a + b*Sinh[c + d*x]^3),x]
```

output
```
(-I)*((-2*(-1)^(1/6)*b*ArcTan[((-1)^(1/6)*((-1)^(1/6)*b^(1/3) + I*a^(1/3)*
Tanh[(c + d*x)/2]))/Sqrt[(-1)^(1/3)*a^(2/3) - (-1)^(2/3)*b^(2/3)]])/(3*a^(
5/3)*Sqrt[(-1)^(1/3)*a^(2/3) - (-1)^(2/3)*b^(2/3)]*d) - (2*(-1)^(1/6)*b*Ar
cTan[((-1)^(1/6)*((-1)^(5/6)*b^(1/3) + I*a^(1/3)*Tanh[(c + d*x)/2]))/Sqrt[
(-1)^(1/3)*a^(2/3) - b^(2/3)]])/(3*a^(5/3)*Sqrt[(-1)^(1/3)*a^(2/3) - b^(2/
3)]*d) + ((I/2)*ArcTanh[Cosh[c + d*x]])/(a*d) + (((2*I)/3)*b*ArcTanh[(b^(1
/3) - a^(1/3)*Tanh[(c + d*x)/2])/Sqrt[a^(2/3) + b^(2/3)]])/(a^(5/3)*Sqrt[a
^(2/3) + b^(2/3)]*d) - ((I/2)*Coth[c + d*x]*Csch[c + d*x])/(a*d))
```

Defintions of rubi rules used

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3699
```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^(n_
))^(p_.), x_Symbol] :> Int[ExpandTrig[sin[e + f*x]^m*(a + b*sin[e + f*x]^n)
^p, x], x] /; FreeQ[{a, b, e, f}, x] && IntegersQ[m, p] && (EqQ[n, 4] || Gt
Q[p, 0] || (EqQ[p, -1] && IntegerQ[n]))
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 1.52 (sec) , antiderivative size = 138, normalized size of antiderivative = 0.43

method	result
derivativedivides	$\frac{\frac{\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^2}{8a}-\frac{1}{8a\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^2}-\frac{\ln\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{2a}+\frac{b\left(\sum_{\_R=\mathrm{RootOf}\left(a\_Z^6-3a\_Z^4-8b\_Z^3+3a\_Z^2-a\right)}\frac{\left(\_R^4-2\_\right.}{\_R^5 a}\right.}{3a}}{d}$
default	$\frac{\frac{\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^2}{8a}-\frac{1}{8a\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^2}-\frac{\ln\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{2a}+\frac{b\left(\sum_{\_R=\mathrm{RootOf}\left(a\_Z^6-3a\_Z^4-8b\_Z^3+3a\_Z^2-a\right)}\frac{\left(\_R^4-2\_\right.}{\_R^5 a}\right.}{3a}}{d}$
risch	$-\frac{e^{dx+c}\left(e^{2dx+2c}+1\right)}{da\left(e^{2dx+2c}-1\right)^2}+8\left(\sum_{\_R=\mathrm{RootOf}\left(\left(191102976a^{12}d^6+191102976a^{10}b^2d^6\right)\_Z^6-995328a^8b^2d^4\_Z^4+1728a^4b\right.}\right.$

input
```
int(csch(d*x+c)^3/(a+b*sinh(d*x+c)^3),x,method=_RETURNVERBOSE)
```

output
```
1/d*(1/8*tanh(1/2*d*x+1/2*c)^2/a-1/8/a/tanh(1/2*d*x+1/2*c)^2-1/2/a*ln(tanh
(1/2*d*x+1/2*c))+1/3*b/a*sum((_R^4-2*_R^2+1)/(_R^5*a-2*_R^3*a-4*_R^2*b+_R*
a)*ln(tanh(1/2*d*x+1/2*c)-_R),_R=RootOf(_Z^6*a-3*_Z^4*a-8*_Z^3*b+3*_Z^2*a-
a)))
```

Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 29179 vs. 2(229) = 458.

Time = 8.01 (sec) , antiderivative size = 29179, normalized size of antiderivative = 90.62

$$\int \frac{\operatorname{csch}^3(c+dx)}{a+b\sinh^3(c+dx)}\,dx = \text{Too large to display}$$

input
```
integrate(csch(d*x+c)^3/(a+b*sinh(d*x+c)^3),x, algorithm="fricas")
```

output
```
Too large to include
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\operatorname{csch}^3(c+dx)}{a+b\sinh^3(c+dx)}\,dx = \text{Timed out}$$

input
```
integrate(csch(d*x+c)**3/(a+b*sinh(d*x+c)**3),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{\operatorname{csch}^3(c+dx)}{a+b\sinh^3(c+dx)}\,dx = \int \frac{\operatorname{csch}(dx+c)^3}{b\sinh(dx+c)^3+a}\,dx$$

input
```
integrate(csch(d*x+c)^3/(a+b*sinh(d*x+c)^3),x, algorithm="maxima")
```

output
```
-8*b*integrate(e^(3*d*x + 3*c)/(a*b*e^(6*d*x + 6*c) - 3*a*b*e^(4*d*x + 4*c
) + 8*a^2*e^(3*d*x + 3*c) + 3*a*b*e^(2*d*x + 2*c) - a*b), x) - (e^(3*d*x +
 3*c) + e^(d*x + c))/(a*d*e^(4*d*x + 4*c) - 2*a*d*e^(2*d*x + 2*c) + a*d) +
 1/2*log((e^(d*x + c) + 1)*e^(-c))/(a*d) - 1/2*log((e^(d*x + c) - 1)*e^(-c
))/(a*d)
```

Giac [F]

$$\int \frac{\operatorname{csch}^3(c+dx)}{a+b\sinh^3(c+dx)}\,dx = \int \frac{\operatorname{csch}(dx+c)^3}{b\sinh(dx+c)^3+a}\,dx$$

input
```
integrate(csch(d*x+c)^3/(a+b*sinh(d*x+c)^3),x, algorithm="giac")
```

output
```
sage0*x
```

Mupad [B] (verification not implemented)

Time = 85.72 (sec) , antiderivative size = 3605, normalized size of antiderivative = 11.20

$$\int \frac{\operatorname{csch}^3(c+dx)}{a+b\sinh^3(c+dx)}\,dx = \text{Too large to display}$$

input
```
int(1/(sinh(c + d*x)^3*(a + b*sinh(c + d*x)^3)),x)
```

output
```
symsum(log(-(16777216*b^7*exp(d*x)*exp(root(729*a^10*b^2*d^6*z^6 + 729*a^1
2*d^6*z^6 - 243*a^8*b^2*d^4*z^4 + 27*a^4*b^4*d^2*z^2 - b^6, z, k)) - 50331
648*a*b^6 + 33554432*root(729*a^10*b^2*d^6*z^6 + 729*a^12*d^6*z^6 - 243*a^
8*b^2*d^4*z^4 + 27*a^4*b^4*d^2*z^2 - b^6, z, k)*a*b^7*d + 671088640*a^2*b^
5*exp(d*x)*exp(root(729*a^10*b^2*d^6*z^6 + 729*a^12*d^6*z^6 - 243*a^8*b^2*
d^4*z^4 + 27*a^4*b^4*d^2*z^2 - b^6, z, k)) + 201326592*root(729*a^10*b^2*d
^6*z^6 + 729*a^12*d^6*z^6 - 243*a^8*b^2*d^4*z^4 + 27*a^4*b^4*d^2*z^2 - b^6
, z, k)^2*a^3*b^6*d^2 - 1509949440*root(729*a^10*b^2*d^6*z^6 + 729*a^12*d^
6*z^6 - 243*a^8*b^2*d^4*z^4 + 27*a^4*b^4*d^2*z^2 - b^6, z, k)^2*a^5*b^4*d^
2 - 2717908992*root(729*a^10*b^2*d^6*z^6 + 729*a^12*d^6*z^6 - 243*a^8*b^2*
d^4*z^4 + 27*a^4*b^4*d^2*z^2 - b^6, z, k)^3*a^5*b^5*d^3 + 2717908992*root(
729*a^10*b^2*d^6*z^6 + 729*a^12*d^6*z^6 - 243*a^8*b^2*d^4*z^4 + 27*a^4*b^4
*d^2*z^2 - b^6, z, k)^3*a^7*b^3*d^3 + 6039797760*root(729*a^10*b^2*d^6*z^6
 + 729*a^12*d^6*z^6 - 243*a^8*b^2*d^4*z^4 + 27*a^4*b^4*d^2*z^2 - b^6, z, k
)^4*a^7*b^4*d^4 - 4076863488*root(729*a^10*b^2*d^6*z^6 + 729*a^12*d^6*z^6
- 243*a^8*b^2*d^4*z^4 + 27*a^4*b^4*d^2*z^2 - b^6, z, k)^4*a^9*b^2*d^4 - 67
9477248*root(729*a^10*b^2*d^6*z^6 + 729*a^12*d^6*z^6 - 243*a^8*b^2*d^4*z^4
 + 27*a^4*b^4*d^2*z^2 - b^6, z, k)^5*a^9*b^3*d^5 + 16307453952*root(729*a^
10*b^2*d^6*z^6 + 729*a^12*d^6*z^6 - 243*a^8*b^2*d^4*z^4 + 27*a^4*b^4*d^2*z
^2 - b^6, z, k)^6*a^11*b^2*d^6 - 32614907904*root(729*a^10*b^2*d^6*z^6 ...
```

Reduce [F]

$$\int \frac{\operatorname{csch}^3(c+dx)}{a+b\sinh^3(c+dx)}\,dx = \text{too large to display}$$

input
```
int(csch(d*x+c)^3/(a+b*sinh(d*x+c)^3),x)
```

output

```
(16*(12*e**(8*c + 4*d*x)*int(e**(4*d*x)/(e**(12*c + 12*d*x)*b - 6*e**(10*c
 + 10*d*x)*b + 8*e**(9*c + 9*d*x)*a + 15*e**(8*c + 8*d*x)*b - 24*e**(7*c +
 7*d*x)*a - 20*e**(6*c + 6*d*x)*b + 24*e**(5*c + 5*d*x)*a + 15*e**(4*c + 4
*d*x)*b - 8*e**(3*c + 3*d*x)*a - 6*e**(2*c + 2*d*x)*b + b),x)*b*d - 32*e**
(7*c + 4*d*x)*int(e**(3*d*x)/(e**(12*c + 12*d*x)*b - 6*e**(10*c + 10*d*x)*
b + 8*e**(9*c + 9*d*x)*a + 15*e**(8*c + 8*d*x)*b - 24*e**(7*c + 7*d*x)*a -
 20*e**(6*c + 6*d*x)*b + 24*e**(5*c + 5*d*x)*a + 15*e**(4*c + 4*d*x)*b - 8
*e**(3*c + 3*d*x)*a - 6*e**(2*c + 2*d*x)*b + b),x)*a*d - 12*e**(6*c + 4*d*
x)*int(e**(2*d*x)/(e**(12*c + 12*d*x)*b - 6*e**(10*c + 10*d*x)*b + 8*e**(9
*c + 9*d*x)*a + 15*e**(8*c + 8*d*x)*b - 24*e**(7*c + 7*d*x)*a - 20*e**(6*c
 + 6*d*x)*b + 24*e**(5*c + 5*d*x)*a + 15*e**(4*c + 4*d*x)*b - 8*e**(3*c +
3*d*x)*a - 6*e**(2*c + 2*d*x)*b + b),x)*b*d + 4*e**(4*c + 4*d*x)*int(1/(e*
*(12*c + 12*d*x)*b - 6*e**(10*c + 10*d*x)*b + 8*e**(9*c + 9*d*x)*a + 15*e*
*(8*c + 8*d*x)*b - 24*e**(7*c + 7*d*x)*a - 20*e**(6*c + 6*d*x)*b + 24*e**(
5*c + 5*d*x)*a + 15*e**(4*c + 4*d*x)*b - 8*e**(3*c + 3*d*x)*a - 6*e**(2*c
+ 2*d*x)*b + b),x)*b*d + 2*e**(4*c + 4*d*x)*log(e**(c + d*x) - 1) + 2*e**(
4*c + 4*d*x)*log(e**(c + d*x) + 1) - 4*e**(4*c + 4*d*x)*d*x + e**(4*c + 4*
d*x) - 24*e**(6*c + 2*d*x)*int(e**(4*d*x)/(e**(12*c + 12*d*x)*b - 6*e**(10
*c + 10*d*x)*b + 8*e**(9*c + 9*d*x)*a + 15*e**(8*c + 8*d*x)*b - 24*e**(7*c
 + 7*d*x)*a - 20*e**(6*c + 6*d*x)*b + 24*e**(5*c + 5*d*x)*a + 15*e**(4*...
```

3.158 $\int \frac{\operatorname{csch}^4(c+dx)}{a+b\sinh^3(c+dx)}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 317

$$\int \frac{\operatorname{csch}^4(c+dx)}{a+b\sinh^3(c+dx)}\,dx = -\frac{2b^{4/3}\arctan\left(\frac{(-1)^{5/6}\left(\sqrt[6]{-1}\sqrt[3]{b}+i\sqrt[3]{a}\tanh\left(\frac{1}{2}(c+dx)\right)\right)}{\sqrt{-(-1)^{2/3}a^{2/3}-b^{2/3}}}\right)}{3a^2\sqrt{-(-1)^{2/3}a^{2/3}-b^{2/3}}\,d} - \frac{2b^{4/3}\arctan\left(\frac{\sqrt[6]{-1}\left((-1)^{5/6}\sqrt[3]{b}+i\sqrt[3]{a}\tanh\left(\frac{1}{2}(c+dx)\right)\right)}{\sqrt{\sqrt[3]{-1}a^{2/3}-b^{2/3}}}\right)}{3a^2\sqrt{\sqrt[3]{-1}a^{2/3}-b^{2/3}}\,d} + \frac{b\,\text{arctanh}(\cosh(c+dx))}{a^2 d} - \frac{2b^{4/3}\text{arctanh}\left(\frac{\sqrt[3]{b}-\sqrt[3]{a}\tanh\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^{2/3}+b^{2/3}}}\right)}{3a^2\sqrt{a^{2/3}+b^{2/3}}\,d} + \frac{\coth(c+dx)}{ad} - \frac{\coth^3(c+dx)}{3ad}$$

output
```
-2/3*b^(4/3)*arctan((-1)^(5/6)*((-1)^(1/6)*b^(1/3)+I*a^(1/3)*tanh(1/2*d*x+
1/2*c))/(-(-1)^(2/3)*a^(2/3)-b^(2/3))^(1/2))/a^2/(-(-1)^(2/3)*a^(2/3)-b^(2
/3))^(1/2)/d-2/3*b^(4/3)*arctan((-1)^(1/6)*((-1)^(5/6)*b^(1/3)+I*a^(1/3)*t
anh(1/2*d*x+1/2*c))/((-1)^(1/3)*a^(2/3)-b^(2/3))^(1/2))/a^2/((-1)^(1/3)*a^
(2/3)-b^(2/3))^(1/2)/d+b*arctanh(cosh(d*x+c))/a^2/d-2/3*b^(4/3)*arctanh((b
^(1/3)-a^(1/3)*tanh(1/2*d*x+1/2*c))/(a^(2/3)+b^(2/3))^(1/2))/a^2/(a^(2/3)+
b^(2/3))^(1/2)/d+coth(d*x+c)/a/d-1/3*coth(d*x+c)^3/a/d
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

Time = 5.15 (sec) , antiderivative size = 384, normalized size of antiderivative = 1.21

$$\int \frac{\operatorname{csch}^4(c+dx)}{a+b\sinh^3(c+dx)}\,dx$$
$$= \frac{8a\coth\left(\frac{1}{2}(c+dx)\right)+24b\log\left(\cosh\left(\frac{1}{2}(c+dx)\right)\right)-24b\log\left(\sinh\left(\frac{1}{2}(c+dx)\right)\right)+4b^2\text{RootSum}\left[-b+3b\#\right.}{}$$

input
```
Integrate[Csch[c + d*x]^4/(a + b*Sinh[c + d*x]^3),x]
```

output
```
(8*a*Coth[(c + d*x)/2] + 24*b*Log[Cosh[(c + d*x)/2]] - 24*b*Log[Sinh[(c +
d*x)/2]] + 4*b^2*RootSum[-b + 3*b*#1^2 + 8*a*#1^3 - 3*b*#1^4 + b*#1^6 & ,
(c + d*x + 2*Log[-Cosh[(c + d*x)/2] - Sinh[(c + d*x)/2] + Cosh[(c + d*x)/2
]*#1 - Sinh[(c + d*x)/2]*#1] - 2*c*#1^2 - 2*d*x*#1^2 - 4*Log[-Cosh[(c + d*
x)/2] - Sinh[(c + d*x)/2] + Cosh[(c + d*x)/2]*#1 - Sinh[(c + d*x)/2]*#1]*#
1^2 + c*#1^4 + d*x*#1^4 + 2*Log[-Cosh[(c + d*x)/2] - Sinh[(c + d*x)/2] + C
osh[(c + d*x)/2]*#1 - Sinh[(c + d*x)/2]*#1]*#1^4)/(b*#1 + 4*a*#1^2 - 2*b*#
1^3 + b*#1^5) & ] + 8*a*Csch[c + d*x]^3*Sinh[(c + d*x)/2]^4 - (a*Csch[(c +
 d*x)/2]^4*Sinh[c + d*x])/2 + 8*a*Tanh[(c + d*x)/2])/(24*a^2*d)
```

Rubi [A] (verified)

Time = 0.77 (sec) , antiderivative size = 317, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.130, Rules used = {3042, 3699, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\operatorname{csch}^4(c+dx)}{a+b\sinh^3(c+dx)}dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \frac{1}{\sin(ic+idx)^4\left(a+ib\sin(ic+idx)^3\right)}dx$$

$$\Big\downarrow \text{ 3699}$$

$$\int \left(-\frac{b^2\sinh^2(c+dx)}{a^2\left(-a-b\sinh^3(c+dx)\right)}-\frac{b\operatorname{csch}(c+dx)}{a^2}+\frac{\operatorname{csch}^4(c+dx)}{a}\right)dx$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{b\operatorname{arctanh}(\cosh(c+dx))}{a^2d}-\frac{2b^{4/3}\arctan\left(\frac{(-1)^{5/6}\left(\sqrt[6]{-1}\sqrt[3]{b}+i\sqrt[3]{a}\tanh\left(\frac{1}{2}(c+dx)\right)\right)}{\sqrt{-(-1)^{2/3}a^{2/3}-b^{2/3}}}\right)}{3a^2d\sqrt{-(-1)^{2/3}a^{2/3}-b^{2/3}}}-\frac{2b^{4/3}\arctan\left(\frac{\sqrt[6]{-1}\left((-1)^{5/6}\sqrt[3]{b}+i\sqrt[3]{a}\tanh\left(\frac{1}{2}(c+dx)\right)\right)}{\sqrt{\sqrt[3]{-1}a^{2/3}-b^{2/3}}}\right)}{3a^2d\sqrt{\sqrt[3]{-1}a^{2/3}-b^{2/3}}}-\frac{2b^{4/3}\operatorname{arctanh}\left(\frac{\sqrt[3]{b}-\sqrt[3]{a}\tanh\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^{2/3}+b^{2/3}}}\right)}{3a^2d\sqrt{a^{2/3}+b^{2/3}}}-\frac{\coth^3(c+dx)}{3ad}+\frac{\coth(c+dx)}{ad}$$

input
```
Int[Csch[c + d*x]^4/(a + b*Sinh[c + d*x]^3),x]
```

output
```
(-2*b^(4/3)*ArcTan[((-1)^(5/6)*((-1)^(1/6)*b^(1/3) + I*a^(1/3)*Tanh[(c + d
*x)/2]))/Sqrt[-((-1)^(2/3)*a^(2/3)) - b^(2/3)]])/(3*a^2*Sqrt[-((-1)^(2/3)*
a^(2/3)) - b^(2/3)]*d) - (2*b^(4/3)*ArcTan[((-1)^(1/6)*((-1)^(5/6)*b^(1/3)
 + I*a^(1/3)*Tanh[(c + d*x)/2]))/Sqrt[(-1)^(1/3)*a^(2/3) - b^(2/3)]])/(3*a
^2*Sqrt[(-1)^(1/3)*a^(2/3) - b^(2/3)]*d) + (b*ArcTanh[Cosh[c + d*x]])/(a^2
*d) - (2*b^(4/3)*ArcTanh[(b^(1/3) - a^(1/3)*Tanh[(c + d*x)/2])/Sqrt[a^(2/3
) + b^(2/3)]])/(3*a^2*Sqrt[a^(2/3) + b^(2/3)]*d) + Coth[c + d*x]/(a*d) - C
oth[c + d*x]^3/(3*a*d)
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3699
```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^(n_
))^(p_.), x_Symbol] :> Int[ExpandTrig[sin[e + f*x]^m*(a + b*sin[e + f*x]^n)
^p, x], x] /; FreeQ[{a, b, e, f}, x] && IntegersQ[m, p] && (EqQ[n, 4] || Gt
Q[p, 0] || (EqQ[p, -1] && IntegerQ[n]))
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 1.78 (sec) , antiderivative size = 164, normalized size of antiderivative = 0.52

method	result
derivativedivides	$\frac{-\frac{1}{24a\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^3}+\frac{3}{8a\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)}-\frac{b\ln\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{a^2}-\frac{\frac{\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^3}{3}-3\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)}{8a}-\frac{4b^2\left(\sum_{\_R=\mathrm{RootOf}\left(a\_Z^6-\cdots\right.}\cdots\right.}{\cdots}}{d}$
default	$\frac{-\frac{1}{24a\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^3}+\frac{3}{8a\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)}-\frac{b\ln\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{a^2}-\frac{\frac{\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^3}{3}-3\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)}{8a}-\frac{4b^2\left(\sum_{\_R=\mathrm{RootOf}\left(a\_Z^6-\cdots\right.}\cdots\right.}{\cdots}}{d}$
risch	$-\frac{4(3\,\mathrm{e}^{2dx+2c}-1)}{3ad(\mathrm{e}^{2dx+2c}-1)^3}+\frac{b\ln(\mathrm{e}^{dx+c}+1)}{d\,a^2}-\frac{b\ln(\mathrm{e}^{dx+c}-1)}{d\,a^2}+16\left(\sum_{\_R=\mathrm{RootOf}((12230590464a^{14}d^6+1223059046\cdots}\cdots\right.$

input
```
int(csch(d*x+c)^4/(a+b*sinh(d*x+c)^3),x,method=_RETURNVERBOSE)
```

output
```
1/d*(-1/24/a/tanh(1/2*d*x+1/2*c)^3+3/8/a/tanh(1/2*d*x+1/2*c)-1/a^2*b*ln(ta
nh(1/2*d*x+1/2*c))-1/8/a*(1/3*tanh(1/2*d*x+1/2*c)^3-3*tanh(1/2*d*x+1/2*c))
-4/3*b^2/a^2*sum(_R^2/(_R^5*a-2*_R^3*a-4*_R^2*b+_R*a)*ln(tanh(1/2*d*x+1/2*
c)-_R),_R=RootOf(_Z^6*a-3*_Z^4*a-8*_Z^3*b+3*_Z^2*a-a)))
```

Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 30233 vs. $2(234) = 468$.

Time = 5.80 (sec) , antiderivative size = 30233, normalized size of antiderivative = 95.37

$$\int \frac{\operatorname{csch}^4(c+dx)}{a+b\sinh^3(c+dx)}\,dx = \text{Too large to display}$$

input
```
integrate(csch(d*x+c)^4/(a+b*sinh(d*x+c)^3),x, algorithm="fricas")
```

output
```
Too large to include
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\operatorname{csch}^4(c+dx)}{a+b\sinh^3(c+dx)}\,dx = \text{Timed out}$$

input
```
integrate(csch(d*x+c)**4/(a+b*sinh(d*x+c)**3),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{\operatorname{csch}^4(c+dx)}{a+b\sinh^3(c+dx)}\,dx = \int \frac{\operatorname{csch}(dx+c)^4}{b\sinh(dx+c)^3+a}\,dx$$

input
```
integrate(csch(d*x+c)^4/(a+b*sinh(d*x+c)^3),x, algorithm="maxima")
```

output
```
-4/3*(3*e^(2*d*x + 2*c) - 1)/(a*d*e^(6*d*x + 6*c) - 3*a*d*e^(4*d*x + 4*c)
+ 3*a*d*e^(2*d*x + 2*c) - a*d) + b*log((e^(d*x + c) + 1)*e^(-c))/(a^2*d) -
 b*log((e^(d*x + c) - 1)*e^(-c))/(a^2*d) + 16*integrate(1/8*(b^2*e^(5*d*x
+ 5*c) - 2*b^2*e^(3*d*x + 3*c) + b^2*e^(d*x + c))/(a^2*b*e^(6*d*x + 6*c) -
 3*a^2*b*e^(4*d*x + 4*c) + 8*a^3*e^(3*d*x + 3*c) + 3*a^2*b*e^(2*d*x + 2*c)
 - a^2*b), x)
```

Giac [F]

$$\int \frac{\operatorname{csch}^4(c+dx)}{a+b\sinh^3(c+dx)}\,dx = \int \frac{\operatorname{csch}(dx+c)^4}{b\sinh(dx+c)^3+a}\,dx$$

input
```
integrate(csch(d*x+c)^4/(a+b*sinh(d*x+c)^3),x, algorithm="giac")
```

output
```
sage0*x
```

Mupad [B] (verification not implemented)

Time = 60.90 (sec) , antiderivative size = 3086, normalized size of antiderivative = 9.74

$$\int \frac{\operatorname{csch}^4(c+dx)}{a+b\sinh^3(c+dx)}\,dx = \text{Too large to display}$$

input
```
int(1/(sinh(c + d*x)^4*(a + b*sinh(c + d*x)^3)),x)
```

output
```
symsum(log((18589155328*root(729*a^12*b^2*d^6*z^6 + 729*a^14*d^6*z^6 - 243
*a^8*b^4*d^4*z^4 + 27*a^4*b^6*d^2*z^2 - b^8, z, k)^2*a^4*b^7*d^2 - 1342177
28*root(729*a^12*b^2*d^6*z^6 + 729*a^14*d^6*z^6 - 243*a^8*b^4*d^4*z^4 + 27
*a^4*b^6*d^2*z^2 - b^8, z, k)*a*b^9*d - 1073741824*b^9 + 2818572288*root(7
29*a^12*b^2*d^6*z^6 + 729*a^14*d^6*z^6 - 243*a^8*b^4*d^4*z^4 + 27*a^4*b^6*
d^2*z^2 - b^8, z, k)^3*a^5*b^7*d^3 + 17716740096*root(729*a^12*b^2*d^6*z^6
 + 729*a^14*d^6*z^6 - 243*a^8*b^4*d^4*z^4 + 27*a^4*b^6*d^2*z^2 - b^8, z, k
)^3*a^7*b^5*d^3 - 88181047296*root(729*a^12*b^2*d^6*z^6 + 729*a^14*d^6*z^6
 - 243*a^8*b^4*d^4*z^4 + 27*a^4*b^6*d^2*z^2 - b^8, z, k)^4*a^8*b^5*d^4 - 8
6973087744*root(729*a^12*b^2*d^6*z^6 + 729*a^14*d^6*z^6 - 243*a^8*b^4*d^4*
z^4 + 27*a^4*b^6*d^2*z^2 - b^8, z, k)^4*a^10*b^3*d^4 - 18119393280*root(72
9*a^12*b^2*d^6*z^6 + 729*a^14*d^6*z^6 - 243*a^8*b^4*d^4*z^4 + 27*a^4*b^6*d
^2*z^2 - b^8, z, k)^5*a^9*b^5*d^5 - 30802968576*root(729*a^12*b^2*d^6*z^6
+ 729*a^14*d^6*z^6 - 243*a^8*b^4*d^4*z^4 + 27*a^4*b^6*d^2*z^2 - b^8, z, k)
^5*a^11*b^3*d^5 + 70665633792*root(729*a^12*b^2*d^6*z^6 + 729*a^14*d^6*z^6
 - 243*a^8*b^4*d^4*z^4 + 27*a^4*b^6*d^2*z^2 - b^8, z, k)^6*a^12*b^3*d^6 +
32614907904*root(729*a^12*b^2*d^6*z^6 + 729*a^14*d^6*z^6 - 243*a^8*b^4*d^4
*z^4 + 27*a^4*b^6*d^2*z^2 - b^8, z, k)^7*a^13*b^3*d^7 - 3221225472*root(72
9*a^12*b^2*d^6*z^6 + 729*a^14*d^6*z^6 - 243*a^8*b^4*d^4*z^4 + 27*a^4*b^6*d
^2*z^2 - b^8, z, k)*a^3*b^7*d + 2147483648*a*b^8*exp(d*x)*exp(root(729*...
```

Reduce [F]

$$\int \frac{\operatorname{csch}^4(c+dx)}{a+b\sinh^3(c+dx)}\,dx = \text{too large to display}$$

input
```
int(csch(d*x+c)^4/(a+b*sinh(d*x+c)^3),x)
```

output

```
( - 12288*e**(10*c + 6*d*x)*int(e**(4*d*x)/(e**(14*c + 14*d*x)*b - 7*e**(1
2*c + 12*d*x)*b + 8*e**(11*c + 11*d*x)*a + 21*e**(10*c + 10*d*x)*b - 32*e*
*(9*c + 9*d*x)*a - 35*e**(8*c + 8*d*x)*b + 48*e**(7*c + 7*d*x)*a + 35*e**(
6*c + 6*d*x)*b - 32*e**(5*c + 5*d*x)*a - 21*e**(4*c + 4*d*x)*b + 8*e**(3*c
 + 3*d*x)*a + 7*e**(2*c + 2*d*x)*b - b),x)*a**4*d + 5184*e**(10*c + 6*d*x)
*int(e**(4*d*x)/(e**(14*c + 14*d*x)*b - 7*e**(12*c + 12*d*x)*b + 8*e**(11*
c + 11*d*x)*a + 21*e**(10*c + 10*d*x)*b - 32*e**(9*c + 9*d*x)*a - 35*e**(8
*c + 8*d*x)*b + 48*e**(7*c + 7*d*x)*a + 35*e**(6*c + 6*d*x)*b - 32*e**(5*c
 + 5*d*x)*a - 21*e**(4*c + 4*d*x)*b + 8*e**(3*c + 3*d*x)*a + 7*e**(2*c + 2
*d*x)*b - b),x)*a**2*b**2*d - 23040*e**(9*c + 6*d*x)*int(e**(3*d*x)/(e**(1
4*c + 14*d*x)*b - 7*e**(12*c + 12*d*x)*b + 8*e**(11*c + 11*d*x)*a + 21*e**
(10*c + 10*d*x)*b - 32*e**(9*c + 9*d*x)*a - 35*e**(8*c + 8*d*x)*b + 48*e**
(7*c + 7*d*x)*a + 35*e**(6*c + 6*d*x)*b - 32*e**(5*c + 5*d*x)*a - 21*e**(4
*c + 4*d*x)*b + 8*e**(3*c + 3*d*x)*a + 7*e**(2*c + 2*d*x)*b - b),x)*a**3*b
*d - 6336*e**(8*c + 6*d*x)*int(e**(2*d*x)/(e**(14*c + 14*d*x)*b - 7*e**(12
*c + 12*d*x)*b + 8*e**(11*c + 11*d*x)*a + 21*e**(10*c + 10*d*x)*b - 32*e**
(9*c + 9*d*x)*a - 35*e**(8*c + 8*d*x)*b + 48*e**(7*c + 7*d*x)*a + 35*e**(6
*c + 6*d*x)*b - 32*e**(5*c + 5*d*x)*a - 21*e**(4*c + 4*d*x)*b + 8*e**(3*c
+ 3*d*x)*a + 7*e**(2*c + 2*d*x)*b - b),x)*a**2*b**2*d + 1536*e**(7*c + 6*d
*x)*int(e**(d*x)/(e**(14*c + 14*d*x)*b - 7*e**(12*c + 12*d*x)*b + 8*e**...
```

3.159 $\int \sinh^4(c+dx)\left(a+b\sinh^4(c+dx)\right)\,dx$

Optimal result

Integrand size = 21, antiderivative size = 111

$$\int \sinh^4(c+dx)\left(a+b\sinh^4(c+dx)\right)\,dx = \frac{1}{128}(48a+35b)x - \frac{(80a+93b)\cosh(c+dx)\sinh(c+dx)}{128d} + \frac{(48a+163b)\cosh^3(c+dx)\sinh(c+dx)}{192d} - \frac{25b\cosh^5(c+dx)\sinh(c+dx)}{48d} + \frac{b\cosh^7(c+dx)\sinh(c+dx)}{8d}$$

output
```
1/128*(48*a+35*b)*x-1/128*(80*a+93*b)*cosh(d*x+c)*sinh(d*x+c)/d+1/192*(48*
a+163*b)*cosh(d*x+c)^3*sinh(d*x+c)/d-25/48*b*cosh(d*x+c)^5*sinh(d*x+c)/d+1
/8*b*cosh(d*x+c)^7*sinh(d*x+c)/d
```

Mathematica [A] (verified)

Time = 0.14 (sec) , antiderivative size = 82, normalized size of antiderivative = 0.74

$$\int \sinh^4(c+dx)\left(a+b\sinh^4(c+dx)\right)\,dx$$
$$=\frac{1152ac+840bc+1152adx+840bdx-96(8a+7b)\sinh(2(c+dx))+24(4a+7b)\sinh(4(c+dx))-32}{3072d}$$

input
```
Integrate[Sinh[c + d*x]^4*(a + b*Sinh[c + d*x]^4),x]
```

output
```
(1152*a*c + 840*b*c + 1152*a*d*x + 840*b*d*x - 96*(8*a + 7*b)*Sinh[2*(c +
d*x)] + 24*(4*a + 7*b)*Sinh[4*(c + d*x)] - 32*b*Sinh[6*(c + d*x)] + 3*b*Si
nh[8*(c + d*x)])/(3072*d)
```

Rubi [A] (verified)

Time = 0.42 (sec) , antiderivative size = 150, normalized size of antiderivative = 1.35, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.381, Rules used = {3042, 3696, 1580, 2345, 1471, 27, 298, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sinh^4(c+dx)\left(a+b\sinh^4(c+dx)\right)\,dx$$
$$\Big\downarrow \text{3042}$$
$$\int \sin(ic+idx)^4\left(a+b\sin(ic+idx)^4\right)dx$$
$$\Big\downarrow \text{3696}$$
$$\frac{\int \frac{\tanh^4(c+dx)\left((a+b)\tanh^4(c+dx)-2a\tanh^2(c+dx)+a\right)}{\left(1-\tanh^2(c+dx)\right)^5}d\tanh(c+dx)}{d}$$
$$\Big\downarrow \text{1580}$$
$$\frac{\frac{b\tanh(c+dx)}{8\left(1-\tanh^2(c+dx)\right)^4}-\frac{1}{8}\int \frac{8(a+b)\tanh^6(c+dx)-8(a-b)\tanh^4(c+dx)+8b\tanh^2(c+dx)+b}{\left(1-\tanh^2(c+dx)\right)^4}d\tanh(c+dx)}{d}$$

$\Big\downarrow$ 2345

$$\frac{\frac{1}{8}\left(\frac{1}{6}\int \frac{48(a+b)\tanh^4(c+dx)+96b\tanh^2(c+dx)+19b}{(1-\tanh^2(c+dx))^3}d\tanh(c+dx)-\frac{25b\tanh(c+dx)}{6(1-\tanh^2(c+dx))^3}\right)+\frac{b\tanh(c+dx)}{8(1-\tanh^2(c+dx))^4}}{d}$$

$\Big\downarrow$ 1471

$$\frac{\frac{1}{8}\left(\frac{1}{6}\left(\frac{(48a+163b)\tanh(c+dx)}{4(1-\tanh^2(c+dx))^2}-\frac{1}{4}\int \frac{3(64(a+b)\tanh^2(c+dx)+16a+29b)}{(1-\tanh^2(c+dx))^2}d\tanh(c+dx)\right)-\frac{25b\tanh(c+dx)}{6(1-\tanh^2(c+dx))^3}\right)+\frac{b\tanh(c+}{8(1-\tanh^2(c+}}{d}$$

$\Big\downarrow$ 27

$$\frac{\frac{1}{8}\left(\frac{1}{6}\left(\frac{(48a+163b)\tanh(c+dx)}{4(1-\tanh^2(c+dx))^2}-\frac{3}{4}\int \frac{64(a+b)\tanh^2(c+dx)+16a+29b}{(1-\tanh^2(c+dx))^2}d\tanh(c+dx)\right)-\frac{25b\tanh(c+dx)}{6(1-\tanh^2(c+dx))^3}\right)+\frac{b\tanh(c+dx)}{8(1-\tanh^2(c+dx}}{d}$$

$\Big\downarrow$ 298

$$\frac{\frac{1}{8}\left(\frac{1}{6}\left(\frac{(48a+163b)\tanh(c+dx)}{4(1-\tanh^2(c+dx))^2}-\frac{3}{4}\left(\frac{(80a+93b)\tanh(c+dx)}{2(1-\tanh^2(c+dx))}-\frac{1}{2}(48a+35b)\int \frac{1}{1-\tanh^2(c+dx)}d\tanh(c+dx)\right)\right)-\frac{25b\tanh(c}{6(1-\tanh^2(c}}{d}$$

$\Big\downarrow$ 219

$$\frac{\frac{1}{8}\left(\frac{1}{6}\left(\frac{(48a+163b)\tanh(c+dx)}{4(1-\tanh^2(c+dx))^2}-\frac{3}{4}\left(\frac{(80a+93b)\tanh(c+dx)}{2(1-\tanh^2(c+dx))}-\frac{1}{2}(48a+35b)\text{arctanh}(\tanh(c+dx))\right)\right)-\frac{25b\tanh(c+dx)}{6(1-\tanh^2(c+dx))^3}\right)}{d}$$

input
```
Int[Sinh[c + d*x]^4*(a + b*Sinh[c + d*x]^4),x]
```

output
```
((b*Tanh[c + d*x])/(8*(1 - Tanh[c + d*x]^2)^4) + ((-25*b*Tanh[c + d*x])/(6
*(1 - Tanh[c + d*x]^2)^3) + (((48*a + 163*b)*Tanh[c + d*x])/(4*(1 - Tanh[c
 + d*x]^2)^2) - (3*(-1/2*((48*a + 35*b)*ArcTanh[Tanh[c + d*x]]) + ((80*a +
 93*b)*Tanh[c + d*x])/(2*(1 - Tanh[c + d*x]^2))))/4)/6)/8)/d
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 298
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2), x_Symbol] :> Simp[(-(
b*c - a*d))*x*((a + b*x^2)^(p + 1)/(2*a*b*(p + 1))), x] - Simp[(a*d - b*c*(
2*p + 3))/(2*a*b*(p + 1))   Int[(a + b*x^2)^(p + 1), x], x] /; FreeQ[{a, b,
 c, d, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/2 + p, 0])
```

rule 1471
```
Int[((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.),
x_Symbol] :> With[{Qx = PolynomialQuotient[(a + b*x^2 + c*x^4)^p, d + e*x^2
, x], R = Coeff[PolynomialRemainder[(a + b*x^2 + c*x^4)^p, d + e*x^2, x], x
, 0]}, Simp[(-R)*x*((d + e*x^2)^(q + 1)/(2*d*(q + 1))), x] + Simp[1/(2*d*(q
 + 1))   Int[(d + e*x^2)^(q + 1)*ExpandToSum[2*d*(q + 1)*Qx + R*(2*q + 3),
x], x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^
2 - b*d*e + a*e^2, 0] && IGtQ[p, 0] && LtQ[q, -1]
```

rule 1580
```
Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)
^4)^(p_.), x_Symbol] :> Simp[(-d)^(m/2 - 1)*(c*d^2 - b*d*e + a*e^2)^p*x*((d
 + e*x^2)^(q + 1)/(2*e^(2*p + m/2)*(q + 1))), x] + Simp[1/(2*e^(2*p + m/2)*
(q + 1))   Int[(d + e*x^2)^(q + 1)*ExpandToSum[Together[(1/(d + e*x^2))*(2*
e^(2*p + m/2)*(q + 1)*x^m*(a + b*x^2 + c*x^4)^p - (-d)^(m/2 - 1)*(c*d^2 - b
*d*e + a*e^2)^p*(d + e*(2*q + 3)*x^2))], x], x], x] /; FreeQ[{a, b, c, d, e
}, x] && NeQ[b^2 - 4*a*c, 0] && IGtQ[p, 0] && ILtQ[q, -1] && IGtQ[m/2, 0]
```

rule 2345
```
Int[(Pq_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{Q = PolynomialQuot
ient[Pq, a + b*x^2, x], f = Coeff[PolynomialRemainder[Pq, a + b*x^2, x], x,
 0], g = Coeff[PolynomialRemainder[Pq, a + b*x^2, x], x, 1]}, Simp[(a*g - b
*f*x)*((a + b*x^2)^(p + 1)/(2*a*b*(p + 1))), x] + Simp[1/(2*a*(p + 1))   In
t[(a + b*x^2)^(p + 1)*ExpandToSum[2*a*(p + 1)*Q + f*(2*p + 3), x], x], x]]
/; FreeQ[{a, b}, x] && PolyQ[Pq, x] && LtQ[p, -1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3696
```
Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^4)^(
p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Simp[ff^(m + 1
)/f   Subst[Int[x^m*((a + 2*a*ff^2*x^2 + (a + b)*ff^4*x^4)^p/(1 + ff^2*x^2)
^(m/2 + 2*p + 1)), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f}, x] &
& IntegerQ[m/2] && IntegerQ[p]
```

Maple [A] (verified)

Time = 8.54 (sec) , antiderivative size = 74, normalized size of antiderivative = 0.67

method	result
parallelrisch	$\frac{(-768a-672b)\sinh(2dx+2c)+(96a+168b)\sinh(4dx+4c)-32b\sinh(6dx+6c)+3b\sinh(8dx+8c)+1152dx\left(a+\frac{35b}{48}\right)}{3072d}$
derivativedivides	$\frac{a\left(\left(\frac{\sinh(dx+c)^3}{4}-\frac{3\sinh(dx+c)}{8}\right)\cosh(dx+c)+\frac{3dx}{8}+\frac{3c}{8}\right)+b\left(\left(\frac{\sinh(dx+c)^7}{8}-\frac{7\sinh(dx+c)^5}{48}+\frac{35\sinh(dx+c)^3}{192}-\frac{35\sinh(dx-}{128}\right.\right.}{d}$
default	$\frac{a\left(\left(\frac{\sinh(dx+c)^3}{4}-\frac{3\sinh(dx+c)}{8}\right)\cosh(dx+c)+\frac{3dx}{8}+\frac{3c}{8}\right)+b\left(\left(\frac{\sinh(dx+c)^7}{8}-\frac{7\sinh(dx+c)^5}{48}+\frac{35\sinh(dx+c)^3}{192}-\frac{35\sinh(dx-}{128}\right.\right.}{d}$
parts	$\frac{a\left(\left(\frac{\sinh(dx+c)^3}{4}-\frac{3\sinh(dx+c)}{8}\right)\cosh(dx+c)+\frac{3dx}{8}+\frac{3c}{8}\right)}{d}+\frac{b\left(\left(\frac{\sinh(dx+c)^7}{8}-\frac{7\sinh(dx+c)^5}{48}+\frac{35\sinh(dx+c)^3}{192}-\frac{35\sinh(}{12}\right.\right.}{d}$
risch	$\frac{3ax}{8}+\frac{35bx}{128}+\frac{b\,e^{8dx+8c}}{2048d}-\frac{b\,e^{6dx+6c}}{192d}+\frac{e^{4dx+4c}a}{64d}+\frac{7\,e^{4dx+4c}b}{256d}-\frac{e^{2dx+2c}a}{8d}-\frac{7\,e^{2dx+2c}b}{64d}+\frac{e^{-2dx-2c}a}{8d}-$
orering	Expression too large to display

input
```
int(sinh(d*x+c)^4*(a+b*sinh(d*x+c)^4),x,method=_RETURNVERBOSE)
```

output
```
1/3072*((-768*a-672*b)*sinh(2*d*x+2*c)+(96*a+168*b)*sinh(4*d*x+4*c)-32*b*s
inh(6*d*x+6*c)+3*b*sinh(8*d*x+8*c)+1152*d*x*(a+35/48*b))/d
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 174, normalized size of antiderivative = 1.57

$$\int \sinh^4(c+dx) \left(a+b\sinh^4(c+dx)\right) \, dx$$
$$= \frac{3\,b\cosh\left(dx+c\right)\sinh\left(dx+c\right)^7+3\left(7\,b\cosh\left(dx+c\right)^3-8\,b\cosh\left(dx+c\right)\right)\sinh\left(dx+c\right)^5+\left(21\,b\cosh\left(dx\right.\right.}{}$$

input
```
integrate(sinh(d*x+c)^4*(a+b*sinh(d*x+c)^4),x, algorithm="fricas")
```

output
```
1/384*(3*b*cosh(d*x + c)*sinh(d*x + c)^7 + 3*(7*b*cosh(d*x + c)^3 - 8*b*co
sh(d*x + c))*sinh(d*x + c)^5 + (21*b*cosh(d*x + c)^5 - 80*b*cosh(d*x + c)^
3 + 12*(4*a + 7*b)*cosh(d*x + c))*sinh(d*x + c)^3 + 3*(48*a + 35*b)*d*x +
3*(b*cosh(d*x + c)^7 - 8*b*cosh(d*x + c)^5 + 4*(4*a + 7*b)*cosh(d*x + c)^3
 - 8*(8*a + 7*b)*cosh(d*x + c))*sinh(d*x + c))/d
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 306 vs. 2(104) = 208.

Time = 0.66 (sec) , antiderivative size = 306, normalized size of antiderivative = 2.76

$$\int \sinh^4(c+dx) \left(a+b\sinh^4(c+dx)\right) \, dx$$
$$= \begin{cases} \frac{3ax\sinh^4(c+dx)}{8} - \frac{3ax\sinh^2(c+dx)\cosh^2(c+dx)}{4} + \frac{3ax\cosh^4(c+dx)}{8} + \frac{5a\sinh^3(c+dx)\cosh(c+dx)}{8d} - \frac{3a\sinh(c+dx)\cosh^3(c+dx)}{8d} \\ x\left(a+b\sinh^4(c)\right)\sinh^4(c) \end{cases}$$

input
```
integrate(sinh(d*x+c)**4*(a+b*sinh(d*x+c)**4),x)
```

output
```
Piecewise((3*a*x*sinh(c + d*x)**4/8 - 3*a*x*sinh(c + d*x)**2*cosh(c + d*x)
**2/4 + 3*a*x*cosh(c + d*x)**4/8 + 5*a*sinh(c + d*x)**3*cosh(c + d*x)/(8*d
) - 3*a*sinh(c + d*x)*cosh(c + d*x)**3/(8*d) + 35*b*x*sinh(c + d*x)**8/128
 - 35*b*x*sinh(c + d*x)**6*cosh(c + d*x)**2/32 + 105*b*x*sinh(c + d*x)**4*
cosh(c + d*x)**4/64 - 35*b*x*sinh(c + d*x)**2*cosh(c + d*x)**6/32 + 35*b*x
*cosh(c + d*x)**8/128 + 93*b*sinh(c + d*x)**7*cosh(c + d*x)/(128*d) - 511*
b*sinh(c + d*x)**5*cosh(c + d*x)**3/(384*d) + 385*b*sinh(c + d*x)**3*cosh(
c + d*x)**5/(384*d) - 35*b*sinh(c + d*x)*cosh(c + d*x)**7/(128*d), Ne(d, 0
)), (x*(a + b*sinh(c)**4)*sinh(c)**4, True))
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 175, normalized size of antiderivative = 1.58

$$\int \sinh^4(c+dx)\left(a+b\sinh^4(c+dx)\right)\,dx$$
$$=\frac{1}{64}a\left(24x+\frac{e^{(4dx+4c)}}{d}-\frac{8e^{(2dx+2c)}}{d}+\frac{8e^{(-2dx-2c)}}{d}-\frac{e^{(-4dx-4c)}}{d}\right)$$
$$-\frac{1}{6144}b\left(\frac{\left(32e^{(-2dx-2c)}-168e^{(-4dx-4c)}+672e^{(-6dx-6c)}-3\right)e^{(8dx+8c)}}{d}-\frac{1680(dx+c)}{d}-\frac{672e^{(-2dx-}}{}\right.$$

input
```
integrate(sinh(d*x+c)^4*(a+b*sinh(d*x+c)^4),x, algorithm="maxima")
```

output
```
1/64*a*(24*x + e^(4*d*x + 4*c)/d - 8*e^(2*d*x + 2*c)/d + 8*e^(-2*d*x - 2*c
)/d - e^(-4*d*x - 4*c)/d) - 1/6144*b*((32*e^(-2*d*x - 2*c) - 168*e^(-4*d*x
 - 4*c) + 672*e^(-6*d*x - 6*c) - 3)*e^(8*d*x + 8*c)/d - 1680*(d*x + c)/d -
 (672*e^(-2*d*x - 2*c) - 168*e^(-4*d*x - 4*c) + 32*e^(-6*d*x - 6*c) - 3*e^
(-8*d*x - 8*c))/d)
```

Giac [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 155, normalized size of antiderivative = 1.40

$$\int \sinh^4(c+dx)\left(a+b\sinh^4(c+dx)\right)\,dx = \frac{1}{128}\,(48\,a+35\,b)x+\frac{be^{(8\,dx+8\,c)}}{2048\,d}-\frac{be^{(6\,dx+6\,c)}}{192\,d} + \frac{(4\,a+7\,b)e^{(4\,dx+4\,c)}}{256\,d}-\frac{(8\,a+7\,b)e^{(2\,dx+2\,c)}}{64\,d} + \frac{(8\,a+7\,b)e^{(-2\,dx-2\,c)}}{64\,d} - \frac{(4\,a+7\,b)e^{(-4\,dx-4\,c)}}{256\,d} + \frac{be^{(-6\,dx-6\,c)}}{192\,d}-\frac{be^{(-8\,dx-8\,c)}}{2048\,d}$$

input
```
integrate(sinh(d*x+c)^4*(a+b*sinh(d*x+c)^4),x, algorithm="giac")
```

output
```
1/128*(48*a + 35*b)*x + 1/2048*b*e^(8*d*x + 8*c)/d - 1/192*b*e^(6*d*x + 6*
c)/d + 1/256*(4*a + 7*b)*e^(4*d*x + 4*c)/d - 1/64*(8*a + 7*b)*e^(2*d*x + 2
*c)/d + 1/64*(8*a + 7*b)*e^(-2*d*x - 2*c)/d - 1/256*(4*a + 7*b)*e^(-4*d*x
- 4*c)/d + 1/192*b*e^(-6*d*x - 6*c)/d - 1/2048*b*e^(-8*d*x - 8*c)/d
```

Mupad [B] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 88, normalized size of antiderivative = 0.79

$$\int \sinh^4(c+dx)\left(a+b\sinh^4(c+dx)\right)\,dx = \frac{12\,a\sinh(4\,c+4\,d\,x)-96\,a\sinh(2\,c+2\,d\,x)-84\,b\sinh(2\,c+2\,d\,x)+21\,b\sinh(4\,c+4\,d\,x)-4\,b\sinh(6\,c}{384\,d}$$

input
```
int(sinh(c + d*x)^4*(a + b*sinh(c + d*x)^4),x)
```

output
```
(12*a*sinh(4*c + 4*d*x) - 96*a*sinh(2*c + 2*d*x) - 84*b*sinh(2*c + 2*d*x)
+ 21*b*sinh(4*c + 4*d*x) - 4*b*sinh(6*c + 6*d*x) + (3*b*sinh(8*c + 8*d*x))
/8 + 144*a*d*x + 105*b*d*x)/(384*d)
```

Reduce [B] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 194, normalized size of antiderivative = 1.75

$$\int \sinh^4(c+dx)\left(a+b\sinh^4(c+dx)\right)\,dx = \frac{3e^{16dx+16c}b-32e^{14dx+14c}b+96e^{12dx+12c}a+168e^{12dx+12c}b-768e^{10dx+10c}a-672e^{10dx+10c}b+2304e^{8dx+8c}a}{6144e^{8dx+8}}$$

input
```
int(sinh(d*x+c)^4*(a+b*sinh(d*x+c)^4),x)
```

output
```
(3*e**(16*c + 16*d*x)*b - 32*e**(14*c + 14*d*x)*b + 96*e**(12*c + 12*d*x)*
a + 168*e**(12*c + 12*d*x)*b - 768*e**(10*c + 10*d*x)*a - 672*e**(10*c + 1
0*d*x)*b + 2304*e**(8*c + 8*d*x)*a*d*x + 1680*e**(8*c + 8*d*x)*b*d*x + 768
*e**(6*c + 6*d*x)*a + 672*e**(6*c + 6*d*x)*b - 96*e**(4*c + 4*d*x)*a - 168
*e**(4*c + 4*d*x)*b + 32*e**(2*c + 2*d*x)*b - 3*b)/(6144*e**(8*c + 8*d*x)*
d)
```

3.160 $\int \sinh^3(c+dx)\left(a+b\sinh^4(c+dx)\right)\,dx$

Optimal result

Integrand size = 21, antiderivative size = 67

$$\int \sinh^3(c+dx)\left(a+b\sinh^4(c+dx)\right)\,dx = -\frac{(a+b)\cosh(c+dx)}{d} + \frac{(a+3b)\cosh^3(c+dx)}{3d} - \frac{3b\cosh^5(c+dx)}{5d} + \frac{b\cosh^7(c+dx)}{7d}$$

output
```
-(a+b)*cosh(d*x+c)/d+1/3*(a+3*b)*cosh(d*x+c)^3/d-3/5*b*cosh(d*x+c)^5/d+1/7
*b*cosh(d*x+c)^7/d
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 93, normalized size of antiderivative = 1.39

$$\int \sinh^3(c+dx)\left(a+b\sinh^4(c+dx)\right)\,dx = -\frac{3a\cosh(c+dx)}{4d} - \frac{35b\cosh(c+dx)}{64d} + \frac{a\cosh(3(c+dx))}{12d} + \frac{7b\cosh(3(c+dx))}{64d} - \frac{7b\cosh(5(c+dx))}{320d} + \frac{b\cosh(7(c+dx))}{448d}$$

input
```
Integrate[Sinh[c + d*x]^3*(a + b*Sinh[c + d*x]^4),x]
```

output
```
(-3*a*Cosh[c + d*x])/(4*d) - (35*b*Cosh[c + d*x])/(64*d) + (a*Cosh[3*(c +
d*x)])/(12*d) + (7*b*Cosh[3*(c + d*x)])/(64*d) - (7*b*Cosh[5*(c + d*x)])/(
320*d) + (b*Cosh[7*(c + d*x)])/(448*d)
```

Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 59, normalized size of antiderivative = 0.88, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.238$, Rules used = {3042, 26, 3694, 1467, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sinh^3(c+dx)\left(a+b\sinh^4(c+dx)\right)dx$$

$$\Big\downarrow \text{3042}$$

$$\int i\sin(ic+idx)^3\left(a+b\sin(ic+idx)^4\right)dx$$

$$\Big\downarrow \text{26}$$

$$i\int \sin(ic+idx)^3\left(b\sin(ic+idx)^4+a\right)dx$$

$$\Big\downarrow \text{3694}$$

$$-\frac{\int \left(1-\cosh^2(c+dx)\right)\left(b\cosh^4(c+dx)-2b\cosh^2(c+dx)+a+b\right)d\cosh(c+dx)}{d}$$

$$\Big\downarrow \text{1467}$$

$$-\frac{\int \left(-b\cosh^6(c+dx)+3b\cosh^4(c+dx)-(a+3b)\cosh^2(c+dx)+a\left(\frac{b}{a}+1\right)\right)d\cosh(c+dx)}{d}$$

$$\Big\downarrow \text{2009}$$

$$-\frac{-\frac{1}{3}(a+3b)\cosh^3(c+dx)+(a+b)\cosh(c+dx)-\frac{1}{7}b\cosh^7(c+dx)+\frac{3}{5}b\cosh^5(c+dx)}{d}$$

input
```
Int[Sinh[c + d*x]^3*(a + b*Sinh[c + d*x]^4),x]
```

output
```
-(((a + b)*Cosh[c + d*x] - ((a + 3*b)*Cosh[c + d*x]^3)/3 + (3*b*Cosh[c + d
*x]^5)/5 - (b*Cosh[c + d*x]^7)/7)/d)
```

Defintions of rubi rules used

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 1467
```
Int[((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.),
 x_Symbol] :> Int[ExpandIntegrand[(d + e*x^2)^q*(a + b*x^2 + c*x^4)^p, x],
x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e
 + a*e^2, 0] && IGtQ[p, 0] && IGtQ[q, -2]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3694
```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^4)^
(p_.), x_Symbol] :> With[{ff = FreeFactors[Cos[e + f*x], x]}, Simp[-ff/f
Subst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a + b - 2*b*ff^2*x^2 + b*ff^4*x^4)^p,
 x], x, Cos[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m -
 1)/2]
```

Maple [A] (verified)

Time = 6.01 (sec) , antiderivative size = 66, normalized size of antiderivative = 0.99

method	result
derivativedivides	$\frac{a\left(-\frac{2}{3}+\frac{\sinh(dx+c)^2}{3}\right)\cosh(dx+c)+b\left(-\frac{16}{35}+\frac{\sinh(dx+c)^6}{7}-\frac{6\sinh(dx+c)^4}{35}+\frac{8\sinh(dx+c)^2}{35}\right)\cosh(dx+c)}{d}$
default	$\frac{a\left(-\frac{2}{3}+\frac{\sinh(dx+c)^2}{3}\right)\cosh(dx+c)+b\left(-\frac{16}{35}+\frac{\sinh(dx+c)^6}{7}-\frac{6\sinh(dx+c)^4}{35}+\frac{8\sinh(dx+c)^2}{35}\right)\cosh(dx+c)}{d}$
parallelrisch	$\frac{(560a+735b)\cosh(3dx+3c)-147b\cosh(5dx+5c)+15b\cosh(7dx+7c)+(-5040a-3675b)\cosh(dx+c)-4480a-3072b}{6720d}$
parts	$\frac{a\left(-\frac{2}{3}+\frac{\sinh(dx+c)^2}{3}\right)\cosh(dx+c)}{d}+\frac{b\left(-\frac{16}{35}+\frac{\sinh(dx+c)^6}{7}-\frac{6\sinh(dx+c)^4}{35}+\frac{8\sinh(dx+c)^2}{35}\right)\cosh(dx+c)}{d}$
risch	$\frac{b\,e^{7dx+7c}}{896d}-\frac{7b\,e^{5dx+5c}}{640d}+\frac{e^{3dx+3c}a}{24d}+\frac{7\,e^{3dx+3c}b}{128d}-\frac{3\,e^{dx+c}a}{8d}-\frac{35\,e^{dx+c}b}{128d}-\frac{3\,e^{-dx-c}a}{8d}-\frac{35\,e^{-dx-c}b}{128d}+$
orering	$\frac{\frac{12916\sinh(dx+c)^2\left(a+b\sinh(dx+c)^4\right)d\cosh(dx+c)}{3675}+\frac{51664\sinh(dx+c)^6bd\cosh(dx+c)}{11025}}{d^2}-\frac{94\left(6d^3\cosh(dx+c)^3\left(a+b\sinh(dx\right.\right.}{}$

input
```
int(sinh(d*x+c)^3*(a+b*sinh(d*x+c)^4),x,method=_RETURNVERBOSE)
```

output
```
1/d*(a*(-2/3+1/3*sinh(d*x+c)^2)*cosh(d*x+c)+b*(-16/35+1/7*sinh(d*x+c)^6-6/
35*sinh(d*x+c)^4+8/35*sinh(d*x+c)^2)*cosh(d*x+c))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 155 vs. 2(61) = 122.

Time = 0.11 (sec) , antiderivative size = 155, normalized size of antiderivative = 2.31

$$\int \sinh^3(c+dx)\left(a+b\sinh^4(c+dx)\right)\,dx$$
$$=\frac{15\,b\cosh\left(dx+c\right)^7+105\,b\cosh\left(dx+c\right)\sinh\left(dx+c\right)^6-147\,b\cosh\left(dx+c\right)^5+105\left(5\,b\cosh\left(dx+c\right)^3-\right.}{}$$

input
```
integrate(sinh(d*x+c)^3*(a+b*sinh(d*x+c)^4),x, algorithm="fricas")
```

output
```
1/6720*(15*b*cosh(d*x + c)^7 + 105*b*cosh(d*x + c)*sinh(d*x + c)^6 - 147*b
*cosh(d*x + c)^5 + 105*(5*b*cosh(d*x + c)^3 - 7*b*cosh(d*x + c))*sinh(d*x
+ c)^4 + 35*(16*a + 21*b)*cosh(d*x + c)^3 + 105*(3*b*cosh(d*x + c)^5 - 14*
b*cosh(d*x + c)^3 + (16*a + 21*b)*cosh(d*x + c))*sinh(d*x + c)^2 - 105*(48
*a + 35*b)*cosh(d*x + c))/d
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 128 vs. $2(56) = 112$.

Time = 0.46 (sec) , antiderivative size = 128, normalized size of antiderivative = 1.91

$$\int \sinh^3(c+dx)\left(a+b\sinh^4(c+dx)\right)\,dx$$
$$= \begin{cases} \frac{a\sinh^2(c+dx)\cosh(c+dx)}{d} - \frac{2a\cosh^3(c+dx)}{3d} + \frac{b\sinh^6(c+dx)\cosh(c+dx)}{d} - \frac{2b\sinh^4(c+dx)\cosh^3(c+dx)}{d} + \frac{8b\sinh^2(c+dx)\cosh^5}{5d} \\ x\left(a+b\sinh^4(c)\right)\sinh^3(c) \end{cases}$$

input
```
integrate(sinh(d*x+c)**3*(a+b*sinh(d*x+c)**4),x)
```

output
```
Piecewise((a*sinh(c + d*x)**2*cosh(c + d*x)/d - 2*a*cosh(c + d*x)**3/(3*d)
 + b*sinh(c + d*x)**6*cosh(c + d*x)/d - 2*b*sinh(c + d*x)**4*cosh(c + d*x)
**3/d + 8*b*sinh(c + d*x)**2*cosh(c + d*x)**5/(5*d) - 16*b*cosh(c + d*x)**
7/(35*d), Ne(d, 0)), (x*(a + b*sinh(c)**4)*sinh(c)**3, True))
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 157 vs. $2(61) = 122$.

Time = 0.04 (sec) , antiderivative size = 157, normalized size of antiderivative = 2.34

$$\int \sinh^3(c+dx)\left(a+b\sinh^4(c+dx)\right)\,dx =$$
$$-\frac{1}{4480}b\left(\frac{\left(49\,e^{(-2\,dx-2\,c)} - 245\,e^{(-4\,dx-4\,c)} + 1225\,e^{(-6\,dx-6\,c)} - 5\right)e^{(7\,dx+7\,c)}}{d} + \frac{1225\,e^{(-dx-c)} - 245\,e^{(-3\,dx-}}{}\right.$$
$$\left. + \frac{1}{24}a\left(\frac{e^{(3\,dx+3\,c)}}{d} - \frac{9\,e^{(dx+c)}}{d} - \frac{9\,e^{(-dx-c)}}{d} + \frac{e^{(-3\,dx-3\,c)}}{d}\right)\right.$$

input
```
integrate(sinh(d*x+c)^3*(a+b*sinh(d*x+c)^4),x, algorithm="maxima")
```

output
```
-1/4480*b*((49*e^(-2*d*x - 2*c) - 245*e^(-4*d*x - 4*c) + 1225*e^(-6*d*x -
6*c) - 5)*e^(7*d*x + 7*c)/d + (1225*e^(-d*x - c) - 245*e^(-3*d*x - 3*c) +
49*e^(-5*d*x - 5*c) - 5*e^(-7*d*x - 7*c))/d) + 1/24*a*(e^(3*d*x + 3*c)/d -
 9*e^(d*x + c)/d - 9*e^(-d*x - c)/d + e^(-3*d*x - 3*c)/d)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 142 vs. $2(61) = 122$.

Time = 0.14 (sec) , antiderivative size = 142, normalized size of antiderivative = 2.12

$$\int \sinh^3(c+dx)\left(a+b\sinh^4(c+dx)\right)\,dx = \frac{be^{(7\,dx+7\,c)}}{896\,d} - \frac{7\,be^{(5\,dx+5\,c)}}{640\,d} + \frac{(16\,a+21\,b)e^{(3\,dx+3\,c)}}{384\,d} - \frac{(48\,a+35\,b)e^{(dx+c)}}{128\,d} - \frac{(48\,a+35\,b)e^{(-dx-c)}}{128\,d} + \frac{(16\,a+21\,b)e^{(-3\,dx-3\,c)}}{384\,d} - \frac{7\,be^{(-5\,dx-5\,c)}}{640\,d} + \frac{be^{(-7\,dx-7\,c)}}{896\,d}$$

input
```
integrate(sinh(d*x+c)^3*(a+b*sinh(d*x+c)^4),x, algorithm="giac")
```

output
```
1/896*b*e^(7*d*x + 7*c)/d - 7/640*b*e^(5*d*x + 5*c)/d + 1/384*(16*a + 21*b
)*e^(3*d*x + 3*c)/d - 1/128*(48*a + 35*b)*e^(d*x + c)/d - 1/128*(48*a + 35
*b)*e^(-d*x - c)/d + 1/384*(16*a + 21*b)*e^(-3*d*x - 3*c)/d - 7/640*b*e^(-
5*d*x - 5*c)/d + 1/896*b*e^(-7*d*x - 7*c)/d
```

Mupad [B] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 66, normalized size of antiderivative = 0.99

$$\int \sinh^3(c+dx)\left(a+b\sinh^4(c+dx)\right)\,dx = -\frac{a\cosh(c+d\,x)+b\cosh(c+d\,x)-\frac{a\cosh(c+d\,x)^3}{3}-b\cosh(c+d\,x)^3+\frac{3\,b\cosh(c+d\,x)^5}{5}-\frac{b\cosh(c+d\,x)^7}{7}}{d}$$

input
```
int(sinh(c + d*x)^3*(a + b*sinh(c + d*x)^4),x)
```

output
```
-(a*cosh(c + d*x) + b*cosh(c + d*x) - (a*cosh(c + d*x)^3)/3 - b*cosh(c + d
*x)^3 + (3*b*cosh(c + d*x)^5)/5 - (b*cosh(c + d*x)^7)/7)/d
```

Reduce [B] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 164, normalized size of antiderivative = 2.45

$$\int \sinh^3(c+dx)\left(a+b\sinh^4(c+dx)\right)\,dx = \frac{15e^{14dx+14c}b-147e^{12dx+12c}b+560e^{10dx+10c}a+735e^{10dx+10c}b-5040e^{8dx+8c}a-3675e^{8dx+8c}b-5040e^{6dx+}}{13440e^{7dx+7c}d}$$

input
```
int(sinh(d*x+c)^3*(a+b*sinh(d*x+c)^4),x)
```

output
```
(15*e**(14*c + 14*d*x)*b - 147*e**(12*c + 12*d*x)*b + 560*e**(10*c + 10*d*
x)*a + 735*e**(10*c + 10*d*x)*b - 5040*e**(8*c + 8*d*x)*a - 3675*e**(8*c +
 8*d*x)*b - 5040*e**(6*c + 6*d*x)*a - 3675*e**(6*c + 6*d*x)*b + 560*e**(4*
c + 4*d*x)*a + 735*e**(4*c + 4*d*x)*b - 147*e**(2*c + 2*d*x)*b + 15*b)/(13
440*e**(7*c + 7*d*x)*d)
```

3.161 $\int \sinh^2(c+dx)\left(a+b\sinh^4(c+dx)\right)\,dx$

Optimal result

Integrand size = 21, antiderivative size = 83

$$\int \sinh^2(c+dx)\left(a+b\sinh^4(c+dx)\right)\,dx = -\frac{1}{16}(8a+5b)x + \frac{(8a+11b)\cosh(c+dx)\sinh(c+dx)}{16d} - \frac{13b\cosh^3(c+dx)\sinh(c+dx)}{24d} + \frac{b\cosh^5(c+dx)\sinh(c+dx)}{6d}$$

output
```
-1/16*(8*a+5*b)*x+1/16*(8*a+11*b)*cosh(d*x+c)*sinh(d*x+c)/d-13/24*b*cosh(d
*x+c)^3*sinh(d*x+c)/d+1/6*b*cosh(d*x+c)^5*sinh(d*x+c)/d
```

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 63, normalized size of antiderivative = 0.76

$$\int \sinh^2(c+dx)\left(a+b\sinh^4(c+dx)\right)\,dx = \frac{-96ac-60bc-96adx-60bdx+(48a+45b)\sinh(2(c+dx))-9b\sinh(4(c+dx))+b\sinh(6(c+dx))}{192d}$$

input
```
Integrate[Sinh[c + d*x]^2*(a + b*Sinh[c + d*x]^4),x]
```

output
```
(-96*a*c - 60*b*c - 96*a*d*x - 60*b*d*x + (48*a + 45*b)*Sinh[2*(c + d*x)]
- 9*b*Sinh[4*(c + d*x)] + b*Sinh[6*(c + d*x)])/(192*d)
```

Rubi [A] (verified)

Time = 0.34 (sec) , antiderivative size = 114, normalized size of antiderivative = 1.37, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.429$, Rules used = {3042, 25, 3696, 1580, 25, 1471, 27, 298, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sinh^2(c+dx)\left(a+b\sinh^4(c+dx)\right)dx$$

$$\Big\downarrow \text{3042}$$

$$\int -\sin(ic+idx)^2\left(a+b\sin(ic+idx)^4\right)dx$$

$$\Big\downarrow \text{25}$$

$$-\int \sin(ic+idx)^2\left(b\sin(ic+idx)^4+a\right)dx$$

$$\Big\downarrow \text{3696}$$

$$\frac{\int \frac{\tanh^2(c+dx)\left((a+b)\tanh^4(c+dx)-2a\tanh^2(c+dx)+a\right)}{\left(1-\tanh^2(c+dx)\right)^4}d\tanh(c+dx)}{d}$$

$$\Big\downarrow \text{1580}$$

$$\frac{\frac{1}{6}\int -\frac{6(a+b)\tanh^4(c+dx)-6(a-b)\tanh^2(c+dx)+b}{\left(1-\tanh^2(c+dx)\right)^3}d\tanh(c+dx)+\frac{b\tanh(c+dx)}{6\left(1-\tanh^2(c+dx)\right)^3}}{d}$$

$$\Big\downarrow \text{25}$$

$$\frac{\frac{b\tanh(c+dx)}{6\left(1-\tanh^2(c+dx)\right)^3}-\frac{1}{6}\int \frac{6(a+b)\tanh^4(c+dx)-6(a-b)\tanh^2(c+dx)+b}{\left(1-\tanh^2(c+dx)\right)^3}d\tanh(c+dx)}{d}$$

$$\Big\downarrow \text{1471}$$

$$\frac{\frac{1}{6}\left(\frac{1}{4}\int \frac{3(8(a+b)\tanh^2(c+dx)+3b)}{(1-\tanh^2(c+dx))^2}d\tanh(c+dx)-\frac{13b\tanh(c+dx)}{4(1-\tanh^2(c+dx))^2}\right)+\frac{b\tanh(c+dx)}{6(1-\tanh^2(c+dx))^3}}{d}$$

$$\Big\downarrow \text{27}$$

$$\frac{\frac{1}{6}\left(\frac{3}{4}\int \frac{8(a+b)\tanh^2(c+dx)+3b}{(1-\tanh^2(c+dx))^2}d\tanh(c+dx)-\frac{13b\tanh(c+dx)}{4(1-\tanh^2(c+dx))^2}\right)+\frac{b\tanh(c+dx)}{6(1-\tanh^2(c+dx))^3}}{d}$$

$$\Big\downarrow \text{298}$$

$$\frac{\frac{1}{6}\left(\frac{3}{4}\left(\frac{(8a+11b)\tanh(c+dx)}{2(1-\tanh^2(c+dx))}-\frac{1}{2}(8a+5b)\int \frac{1}{1-\tanh^2(c+dx)}d\tanh(c+dx)\right)-\frac{13b\tanh(c+dx)}{4(1-\tanh^2(c+dx))^2}\right)+\frac{b\tanh(c+dx)}{6(1-\tanh^2(c+dx))^3}}{d}$$

$$\Big\downarrow \text{219}$$

$$\frac{\frac{1}{6}\left(\frac{3}{4}\left(\frac{(8a+11b)\tanh(c+dx)}{2(1-\tanh^2(c+dx))}-\frac{1}{2}(8a+5b)\text{arctanh}(\tanh(c+dx))\right)-\frac{13b\tanh(c+dx)}{4(1-\tanh^2(c+dx))^2}\right)+\frac{b\tanh(c+dx)}{6(1-\tanh^2(c+dx))^3}}{d}$$

input
```
Int[Sinh[c + d*x]^2*(a + b*Sinh[c + d*x]^4),x]
```

output
```
((b*Tanh[c + d*x])/(6*(1 - Tanh[c + d*x]^2)^3) + ((-13*b*Tanh[c + d*x])/(4
*(1 - Tanh[c + d*x]^2)^2) + (3*(-1/2*((8*a + 5*b)*ArcTanh[Tanh[c + d*x]])
+ ((8*a + 11*b)*Tanh[c + d*x])/(2*(1 - Tanh[c + d*x]^2))))/4)/6)/d
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 298
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2), x_Symbol] :> Simp[(-(
b*c - a*d))*x*((a + b*x^2)^(p + 1)/(2*a*b*(p + 1))), x] - Simp[(a*d - b*c*(
2*p + 3))/(2*a*b*(p + 1))   Int[(a + b*x^2)^(p + 1), x], x] /; FreeQ[{a, b,
 c, d, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/2 + p, 0])
```

rule 1471
```
Int[((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.),
x_Symbol] :> With[{Qx = PolynomialQuotient[(a + b*x^2 + c*x^4)^p, d + e*x^2
, x], R = Coeff[PolynomialRemainder[(a + b*x^2 + c*x^4)^p, d + e*x^2, x], x
, 0]}, Simp[(-R)*x*((d + e*x^2)^(q + 1)/(2*d*(q + 1))), x] + Simp[1/(2*d*(q
 + 1))   Int[(d + e*x^2)^(q + 1)*ExpandToSum[2*d*(q + 1)*Qx + R*(2*q + 3),
x], x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^
2 - b*d*e + a*e^2, 0] && IGtQ[p, 0] && LtQ[q, -1]
```

rule 1580
```
Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)
^4)^(p_.), x_Symbol] :> Simp[(-d)^(m/2 - 1)*(c*d^2 - b*d*e + a*e^2)^p*x*((d
 + e*x^2)^(q + 1)/(2*e^(2*p + m/2)*(q + 1))), x] + Simp[1/(2*e^(2*p + m/2)*
(q + 1))   Int[(d + e*x^2)^(q + 1)*ExpandToSum[Together[(1/(d + e*x^2))*(2*
e^(2*p + m/2)*(q + 1)*x^m*(a + b*x^2 + c*x^4)^p - (-d)^(m/2 - 1)*(c*d^2 - b
*d*e + a*e^2)^p*(d + e*(2*q + 3)*x^2))], x], x], x] /; FreeQ[{a, b, c, d, e
}, x] && NeQ[b^2 - 4*a*c, 0] && IGtQ[p, 0] && ILtQ[q, -1] && IGtQ[m/2, 0]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3696
```
Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^4)^(
p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Simp[ff^(m + 1
)/f   Subst[Int[x^m*((a + 2*a*ff^2*x^2 + (a + b)*ff^4*x^4)^p/(1 + ff^2*x^2)
^(m/2 + 2*p + 1)), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f}, x] &
& IntegerQ[m/2] && IntegerQ[p]
```

Maple [A] (verified)

Time = 2.99 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.67

method	result
parallelrisch	$\frac{(48a+45b)\sinh(2dx+2c)-9b\sinh(4dx+4c)+b\sinh(6dx+6c)-96dx\left(a+\frac{5b}{8}\right)}{192d}$
derivativedivides	$\frac{a\left(\frac{\cosh(dx+c)\sinh(dx+c)}{2}-\frac{dx}{2}-\frac{c}{2}\right)+b\left(\left(\frac{\sinh(dx+c)^5}{6}-\frac{5\sinh(dx+c)^3}{24}+\frac{5\sinh(dx+c)}{16}\right)\cosh(dx+c)-\frac{5dx}{16}-\frac{5c}{16}\right)}{d}$
default	$\frac{a\left(\frac{\cosh(dx+c)\sinh(dx+c)}{2}-\frac{dx}{2}-\frac{c}{2}\right)+b\left(\left(\frac{\sinh(dx+c)^5}{6}-\frac{5\sinh(dx+c)^3}{24}+\frac{5\sinh(dx+c)}{16}\right)\cosh(dx+c)-\frac{5dx}{16}-\frac{5c}{16}\right)}{d}$
parts	$\frac{a\left(\frac{\cosh(dx+c)\sinh(dx+c)}{2}-\frac{dx}{2}-\frac{c}{2}\right)}{d}+\frac{b\left(\left(\frac{\sinh(dx+c)^5}{6}-\frac{5\sinh(dx+c)^3}{24}+\frac{5\sinh(dx+c)}{16}\right)\cosh(dx+c)-\frac{5dx}{16}-\frac{5c}{16}\right)}{d}$
risch	$-\frac{ax}{2}-\frac{5bx}{16}+\frac{b\,e^{6dx+6c}}{384d}-\frac{3\,e^{4dx+4c}b}{128d}+\frac{e^{2dx+2c}a}{8d}+\frac{15\,e^{2dx+2c}b}{128d}-\frac{e^{-2dx-2c}a}{8d}-\frac{15\,e^{-2dx-2c}b}{128d}+\frac{3\,e^{-4c}}{12}$
orering	$x\sinh(dx+c)^2\left(a+b\sinh(dx+c)^4\right)+\frac{\frac{49\sinh(dx+c)\left(a+b\sinh(dx+c)^4\right)d\cosh(dx+c)}{72}+\frac{49\sinh(dx+c)^5bd\,c}{36}}{d^2}$

input
```
int(sinh(d*x+c)^2*(a+b*sinh(d*x+c)^4),x,method=_RETURNVERBOSE)
```

output
```
1/192*((48*a+45*b)*sinh(2*d*x+2*c)-9*b*sinh(4*d*x+4*c)+b*sinh(6*d*x+6*c)-9
6*d*x*(a+5/8*b))/d
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 109, normalized size of antiderivative = 1.31

$$\int \sinh^2(c+dx)\left(a+b\sinh^4(c+dx)\right)\,dx$$
$$=\frac{3\,b\cosh(dx+c)\sinh(dx+c)^5+2\left(5\,b\cosh(dx+c)^3-9\,b\cosh(dx+c)\right)\sinh(dx+c)^3-6\,(8\,a+5\,b)d}{96\,d}$$

input
```
integrate(sinh(d*x+c)^2*(a+b*sinh(d*x+c)^4),x, algorithm="fricas")
```

output
```
1/96*(3*b*cosh(d*x + c)*sinh(d*x + c)^5 + 2*(5*b*cosh(d*x + c)^3 - 9*b*cos
h(d*x + c))*sinh(d*x + c)^3 - 6*(8*a + 5*b)*d*x + 3*(b*cosh(d*x + c)^5 - 6
*b*cosh(d*x + c)^3 + (16*a + 15*b)*cosh(d*x + c))*sinh(d*x + c))/d
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 206 vs. $2(76) = 152$.

Time = 0.34 (sec) , antiderivative size = 206, normalized size of antiderivative = 2.48

$$\int \sinh^2(c+dx)\left(a+b\sinh^4(c+dx)\right)\,dx$$
$$=\begin{cases} \frac{ax\sinh^2(c+dx)}{2}-\frac{ax\cosh^2(c+dx)}{2}+\frac{a\sinh(c+dx)\cosh(c+dx)}{2d}+\frac{5bx\sinh^6(c+dx)}{16}-\frac{15bx\sinh^4(c+dx)\cosh^2(c+dx)}{16}+\frac{15bx\sinh^2}{} \\ x\left(a+b\sinh^4(c)\right)\sinh^2(c) \end{cases}$$

input
```
integrate(sinh(d*x+c)**2*(a+b*sinh(d*x+c)**4),x)
```

output
```
Piecewise((a*x*sinh(c + d*x)**2/2 - a*x*cosh(c + d*x)**2/2 + a*sinh(c + d*
x)*cosh(c + d*x)/(2*d) + 5*b*x*sinh(c + d*x)**6/16 - 15*b*x*sinh(c + d*x)*
*4*cosh(c + d*x)**2/16 + 15*b*x*sinh(c + d*x)**2*cosh(c + d*x)**4/16 - 5*b
*x*cosh(c + d*x)**6/16 + 11*b*sinh(c + d*x)**5*cosh(c + d*x)/(16*d) - 5*b*
sinh(c + d*x)**3*cosh(c + d*x)**3/(6*d) + 5*b*sinh(c + d*x)*cosh(c + d*x)*
*5/(16*d), Ne(d, 0)), (x*(a + b*sinh(c)**4)*sinh(c)**2, True))
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 122, normalized size of antiderivative = 1.47

$$\int \sinh^2(c+dx)\left(a+b\sinh^4(c+dx)\right)\,dx=-\frac{1}{8}a\left(4x-\frac{e^{(2dx+2c)}}{d}+\frac{e^{(-2dx-2c)}}{d}\right)$$
$$-\frac{1}{384}b\left(\frac{\left(9e^{(-2dx-2c)}-45e^{(-4dx-4c)}-1\right)e^{(6dx+6c)}}{d}+\frac{120(dx+c)}{d}+\frac{45e^{(-2dx-2c)}-9e^{(-4dx-4c)}+e^{(}}{d}\right.$$

input
```
integrate(sinh(d*x+c)^2*(a+b*sinh(d*x+c)^4),x, algorithm="maxima")
```

output
```
-1/8*a*(4*x - e^(2*d*x + 2*c)/d + e^(-2*d*x - 2*c)/d) - 1/384*b*((9*e^(-2*
d*x - 2*c) - 45*e^(-4*d*x - 4*c) - 1)*e^(6*d*x + 6*c)/d + 120*(d*x + c)/d
+ (45*e^(-2*d*x - 2*c) - 9*e^(-4*d*x - 4*c) + e^(-6*d*x - 6*c))/d)
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 113, normalized size of antiderivative = 1.36

$$\int \sinh^2(c+dx)\left(a+b\sinh^4(c+dx)\right)\,dx = -\frac{1}{16}(8a+5b)x + \frac{be^{(6dx+6c)}}{384d} - \frac{3be^{(4dx+4c)}}{128d} + \frac{(16a+15b)e^{(2dx+2c)}}{128d} - \frac{(16a+15b)e^{(-2dx-2c)}}{128d} + \frac{3be^{(-4dx-4c)}}{128d} - \frac{be^{(-6dx-6c)}}{384d}$$

input
```
integrate(sinh(d*x+c)^2*(a+b*sinh(d*x+c)^4),x, algorithm="giac")
```

output
```
-1/16*(8*a + 5*b)*x + 1/384*b*e^(6*d*x + 6*c)/d - 3/128*b*e^(4*d*x + 4*c)/
d + 1/128*(16*a + 15*b)*e^(2*d*x + 2*c)/d - 1/128*(16*a + 15*b)*e^(-2*d*x
- 2*c)/d + 3/128*b*e^(-4*d*x - 4*c)/d - 1/384*b*e^(-6*d*x - 6*c)/d
```

Mupad [B] (verification not implemented)

Time = 1.63 (sec) , antiderivative size = 64, normalized size of antiderivative = 0.77

$$\int \sinh^2(c+dx)\left(a+b\sinh^4(c+dx)\right)\,dx = \frac{12a\sinh(2c+2dx) + \frac{45b\sinh(2c+2dx)}{4} - \frac{9b\sinh(4c+4dx)}{4} + \frac{b\sinh(6c+6dx)}{4} - 24adx - 15bdx}{48d}$$

input
```
int(sinh(c + d*x)^2*(a + b*sinh(c + d*x)^4),x)
```

output
```
(12*a*sinh(2*c + 2*d*x) + (45*b*sinh(2*c + 2*d*x))/4 - (9*b*sinh(4*c + 4*d
*x))/4 + (b*sinh(6*c + 6*d*x))/4 - 24*a*d*x - 15*b*d*x)/(48*d)
```

Reduce [B] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 141, normalized size of antiderivative = 1.70

$$\int \sinh^2(c+dx)\left(a+b\sinh^4(c+dx)\right)\,dx$$
$$= \frac{e^{12dx+12c}b-9e^{10dx+10c}b+48e^{8dx+8c}a+45e^{8dx+8c}b-192e^{6dx+6c}adx-120e^{6dx+6c}bdx-48e^{4dx+4c}a-45e}{384e^{6dx+6c}d}$$

input
```
int(sinh(d*x+c)^2*(a+b*sinh(d*x+c)^4),x)
```

output
```
(e**(12*c + 12*d*x)*b - 9*e**(10*c + 10*d*x)*b + 48*e**(8*c + 8*d*x)*a + 4
5*e**(8*c + 8*d*x)*b - 192*e**(6*c + 6*d*x)*a*d*x - 120*e**(6*c + 6*d*x)*b
*d*x - 48*e**(4*c + 4*d*x)*a - 45*e**(4*c + 4*d*x)*b + 9*e**(2*c + 2*d*x)*
b - b)/(384*e**(6*c + 6*d*x)*d)
```

3.162 $\int \sinh(c+dx)\left(a+b\sinh^4(c+dx)\right)\,dx$

Optimal result

Integrand size = 19, antiderivative size = 46

$$\int \sinh(c+dx)\left(a+b\sinh^4(c+dx)\right)\,dx=\frac{(a+b)\cosh(c+dx)}{d}-\frac{2b\cosh^3(c+dx)}{3d}+\frac{b\cosh^5(c+dx)}{5d}$$

output
```
(a+b)*cosh(d*x+c)/d-2/3*b*cosh(d*x+c)^3/d+1/5*b*cosh(d*x+c)^5/d
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 69, normalized size of antiderivative = 1.50

$$\int \sinh(c+dx)\left(a+b\sinh^4(c+dx)\right)\,dx=\frac{a\cosh(c)\cosh(dx)}{d}+\frac{5b\cosh(c+dx)}{8d}-\frac{5b\cosh(3(c+dx))}{48d}+\frac{b\cosh(5(c+dx))}{80d}+\frac{a\sinh(c)\sinh(dx)}{d}$$

input
```
Integrate[Sinh[c + d*x]*(a + b*Sinh[c + d*x]^4),x]
```

output
```
(a*Cosh[c]*Cosh[d*x])/d + (5*b*Cosh[c + d*x])/(8*d) - (5*b*Cosh[3*(c + d*x
)])/(48*d) + (b*Cosh[5*(c + d*x)])/(80*d) + (a*Sinh[c]*Sinh[d*x])/d
```

Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.89, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.211$, Rules used = {3042, 26, 3694, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sinh(c+dx)\left(a+b\sinh^4(c+dx)\right)\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int -i\sin(ic+idx)\left(a+b\sin(ic+idx)^4\right)dx$$

$$\Big\downarrow \text{26}$$

$$-i\int \sin(ic+idx)\left(b\sin(ic+idx)^4+a\right)dx$$

$$\Big\downarrow \text{3694}$$

$$\frac{\int \left(b\cosh^4(c+dx)-2b\cosh^2(c+dx)+a+b\right)d\cosh(c+dx)}{d}$$

$$\Big\downarrow \text{2009}$$

$$\frac{(a+b)\cosh(c+dx)+\frac{1}{5}b\cosh^5(c+dx)-\frac{2}{3}b\cosh^3(c+dx)}{d}$$

input
```
Int[Sinh[c + d*x]*(a + b*Sinh[c + d*x]^4),x]
```

output
```
((a + b)*Cosh[c + d*x] - (2*b*Cosh[c + d*x]^3)/3 + (b*Cosh[c + d*x]^5)/5)/
d
```

Defintions of rubi rules used

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3694
```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^4)^
(p_.), x_Symbol] :> With[{ff = FreeFactors[Cos[e + f*x], x]}, Simp[-ff/f
Subst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a + b - 2*b*ff^2*x^2 + b*ff^4*x^4)^p,
 x], x, Cos[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m -
 1)/2]
```

Maple [A] (verified)

Time = 2.05 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.96

method	result
derivativedivides	$\frac{a\cosh(dx+c)+b\left(\frac{8}{15}+\frac{\sinh(dx+c)^4}{5}-\frac{4\sinh(dx+c)^2}{15}\right)\cosh(dx+c)}{d}$
default	$\frac{a\cosh(dx+c)+b\left(\frac{8}{15}+\frac{\sinh(dx+c)^4}{5}-\frac{4\sinh(dx+c)^2}{15}\right)\cosh(dx+c)}{d}$
parts	$\frac{a\cosh(dx+c)}{d}+\frac{b\left(\frac{8}{15}+\frac{\sinh(dx+c)^4}{5}-\frac{4\sinh(dx+c)^2}{15}\right)\cosh(dx+c)}{d}$
parallelrisch	$\frac{-25b\cosh(3dx+3c)+3b\cosh(5dx+5c)+(240a+150b)\cosh(dx+c)+240a+128b}{240d}$
risch	$\frac{b\,e^{5dx+5c}}{160d}-\frac{5\,e^{3dx+3c}b}{96d}+\frac{e^{dx+c}a}{2d}+\frac{5\,e^{dx+c}b}{16d}+\frac{e^{-dx-c}a}{2d}+\frac{5\,e^{-dx-c}b}{16d}-\frac{5\,e^{-3dx-3c}b}{96d}+\frac{b\,e^{-5dx-5c}}{160d}$
orering	$\frac{\frac{259d\cosh(dx+c)\left(a+b\sinh(dx+c)^4\right)}{225}+\frac{1036\sinh(dx+c)^4bd\cosh(dx+c)}{225}}{d^2}-\frac{7\left(d^3\cosh(dx+c)\left(a+b\sinh(dx+c)^4\right)+64\sinh(dx\right.}{}$

input
```
int(sinh(d*x+c)*(a+b*sinh(d*x+c)^4),x,method=_RETURNVERBOSE)
```

output
```
1/d*(a*cosh(d*x+c)+b*(8/15+1/5*sinh(d*x+c)^4-4/15*sinh(d*x+c)^2)*cosh(d*x+
c))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 91 vs. $2(42) = 84$.

Time = 0.07 (sec) , antiderivative size = 91, normalized size of antiderivative = 1.98

$$\int \sinh(c+dx)\left(a+b\sinh^4(c+dx)\right)\,dx$$
$$= \frac{3\,b\cosh\left(dx+c\right)^5+15\,b\cosh\left(dx+c\right)\sinh\left(dx+c\right)^4-25\,b\cosh\left(dx+c\right)^3+15\,\left(2\,b\cosh\left(dx+c\right)^3-5\,b\,c\right.}{240\,d}$$

input
```
integrate(sinh(d*x+c)*(a+b*sinh(d*x+c)^4),x, algorithm="fricas")
```

output
```
1/240*(3*b*cosh(d*x + c)^5 + 15*b*cosh(d*x + c)*sinh(d*x + c)^4 - 25*b*cos
h(d*x + c)^3 + 15*(2*b*cosh(d*x + c)^3 - 5*b*cosh(d*x + c))*sinh(d*x + c)^
2 + 30*(8*a + 5*b)*cosh(d*x + c))/d
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 80 vs. $2(39) = 78$.

Time = 0.23 (sec) , antiderivative size = 80, normalized size of antiderivative = 1.74

$$\int \sinh(c+dx)\left(a+b\sinh^4(c+dx)\right)\,dx$$
$$= \begin{cases} \frac{a\cosh\left(c+dx\right)}{d}+\frac{b\sinh^4\left(c+dx\right)\cosh\left(c+dx\right)}{d}-\frac{4b\sinh^2\left(c+dx\right)\cosh^3\left(c+dx\right)}{3d}+\frac{8b\cosh^5\left(c+dx\right)}{15d} & \text{for}\; d\neq 0 \\ x\left(a+b\sinh^4\left(c\right)\right)\sinh\left(c\right) & \text{otherwise} \end{cases}$$

input
```
integrate(sinh(d*x+c)*(a+b*sinh(d*x+c)**4),x)
```

output
```
Piecewise((a*cosh(c + d*x)/d + b*sinh(c + d*x)**4*cosh(c + d*x)/d - 4*b*si
nh(c + d*x)**2*cosh(c + d*x)**3/(3*d) + 8*b*cosh(c + d*x)**5/(15*d), Ne(d,
 0)), (x*(a + b*sinh(c)**4)*sinh(c), True))
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 97 vs. $2(42) = 84$.

Time = 0.04 (sec) , antiderivative size = 97, normalized size of antiderivative = 2.11

$$\int \sinh(c+dx)\left(a+b\sinh^4(c+dx)\right)\,dx = \frac{1}{480}\,b\left(\frac{3\,e^{(5\,dx+5\,c)}}{d}-\frac{25\,e^{(3\,dx+3\,c)}}{d}+\frac{150\,e^{(dx+c)}}{d}+\frac{150\,e^{(-dx-c)}}{d}-\frac{25\,e^{(-3\,dx-3\,c)}}{d}+\frac{3\,e^{(-5\,dx-5\,c)}}{d}\right)+\frac{a\cosh\left(dx+c\right)}{d}$$

input
```
integrate(sinh(d*x+c)*(a+b*sinh(d*x+c)^4),x, algorithm="maxima")
```

output
```
1/480*b*(3*e^(5*d*x + 5*c)/d - 25*e^(3*d*x + 3*c)/d + 150*e^(d*x + c)/d +
150*e^(-d*x - c)/d - 25*e^(-3*d*x - 3*c)/d + 3*e^(-5*d*x - 5*c)/d) + a*cos
h(d*x + c)/d
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 100 vs. $2(42) = 84$.

Time = 0.13 (sec) , antiderivative size = 100, normalized size of antiderivative = 2.17

$$\int \sinh(c+dx)\left(a+b\sinh^4(c+dx)\right)\,dx = \frac{be^{(5\,dx+5\,c)}}{160\,d}-\frac{5\,be^{(3\,dx+3\,c)}}{96\,d}+\frac{(8\,a+5\,b)e^{(dx+c)}}{16\,d}+\frac{(8\,a+5\,b)e^{(-dx-c)}}{16\,d}-\frac{5\,be^{(-3\,dx-3\,c)}}{96\,d}+\frac{be^{(-5\,dx-5\,c)}}{160\,d}$$

input
```
integrate(sinh(d*x+c)*(a+b*sinh(d*x+c)^4),x, algorithm="giac")
```

output
```
1/160*b*e^(5*d*x + 5*c)/d - 5/96*b*e^(3*d*x + 3*c)/d + 1/16*(8*a + 5*b)*e^
(d*x + c)/d + 1/16*(8*a + 5*b)*e^(-d*x - c)/d - 5/96*b*e^(-3*d*x - 3*c)/d
+ 1/160*b*e^(-5*d*x - 5*c)/d
```

Mupad [B] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.00

$$\int \sinh(c+dx)\left(a+b\sinh^4(c+dx)\right)\,dx = \frac{15\,a\cosh(c+d\,x)+15\,b\cosh(c+d\,x)-10\,b\cosh(c+d\,x)^3+3\,b\cosh(c+d\,x)^5}{15\,d}$$

input
```
int(sinh(c + d*x)*(a + b*sinh(c + d*x)^4),x)
```

output
```
(15*a*cosh(c + d*x) + 15*b*cosh(c + d*x) - 10*b*cosh(c + d*x)^3 + 3*b*cosh
(c + d*x)^5)/(15*d)
```

Reduce [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 112, normalized size of antiderivative = 2.43

$$\int \sinh(c+dx)\left(a+b\sinh^4(c+dx)\right)\,dx = \frac{3e^{10dx+10c}b-25e^{8dx+8c}b+240e^{6dx+6c}a+150e^{6dx+6c}b+240e^{4dx+4c}a+150e^{4dx+4c}b-25e^{2dx+2c}b+3b}{480e^{5dx+5c}d}$$

input
```
int(sinh(d*x+c)*(a+b*sinh(d*x+c)^4),x)
```

output
```
(3*e**(10*c + 10*d*x)*b - 25*e**(8*c + 8*d*x)*b + 240*e**(6*c + 6*d*x)*a +
 150*e**(6*c + 6*d*x)*b + 240*e**(4*c + 4*d*x)*a + 150*e**(4*c + 4*d*x)*b
- 25*e**(2*c + 2*d*x)*b + 3*b)/(480*e**(5*c + 5*d*x)*d)
```

3.163 $\int \left(a+b\sinh^4(c+dx)\right) dx$

Optimal result

Integrand size = 12, antiderivative size = 52

$$\int \left(a+b\sinh^4(c+dx)\right) dx = ax+\frac{3bx}{8}-\frac{3b\cosh(c+dx)\sinh(c+dx)}{8d}+\frac{b\cosh(c+dx)\sinh^3(c+dx)}{4d}$$

output
```
a*x+3/8*b*x-3/8*b*cosh(d*x+c)*sinh(d*x+c)/d+1/4*b*cosh(d*x+c)*sinh(d*x+c)^
3/d
```

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 49, normalized size of antiderivative = 0.94

$$\int \left(a+b\sinh^4(c+dx)\right) dx = ax+\frac{3b(c+dx)}{8d}-\frac{b\sinh(2(c+dx))}{4d}+\frac{b\sinh(4(c+dx))}{32d}$$

input
```
Integrate[a + b*Sinh[c + d*x]^4,x]
```

output
```
a*x + (3*b*(c + d*x))/(8*d) - (b*Sinh[2*(c + d*x)])/(4*d) + (b*Sinh[4*(c +
 d*x)])/(32*d)
```

Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 52, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(a+b\sinh^4(c+dx)\right)\,dx$$

$$\Big\downarrow \; 2009$$

$$ax+\frac{b\sinh^3(c+dx)\cosh(c+dx)}{4d}-\frac{3b\sinh(c+dx)\cosh(c+dx)}{8d}+\frac{3bx}{8}$$

input
```
Int[a + b*Sinh[c + d*x]^4,x]
```

output
```
a*x + (3*b*x)/8 - (3*b*Cosh[c + d*x]*Sinh[c + d*x])/(8*d) + (b*Cosh[c + d*
x]*Sinh[c + d*x]^3)/(4*d)
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 1.30 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.69

method	result
parallelrisch	$\frac{b(12dx+\sinh(4dx+4c)-8\sinh(2dx+2c))}{32d}+ax$
default	$ax+\frac{b\left(\left(\frac{\sinh(dx+c)^3}{4}-\frac{3\sinh(dx+c)}{8}\right)\cosh(dx+c)+\frac{3dx}{8}+\frac{3c}{8}\right)}{d}$
parts	$ax+\frac{b\left(\left(\frac{\sinh(dx+c)^3}{4}-\frac{3\sinh(dx+c)}{8}\right)\cosh(dx+c)+\frac{3dx}{8}+\frac{3c}{8}\right)}{d}$
derivativedivides	$\frac{(dx+c)a+b\left(\left(\frac{\sinh(dx+c)^3}{4}-\frac{3\sinh(dx+c)}{8}\right)\cosh(dx+c)+\frac{3dx}{8}+\frac{3c}{8}\right)}{d}$
risch	$ax+\frac{3bx}{8}+\frac{e^{4dx+4c}b}{64d}-\frac{e^{2dx+2c}b}{8d}+\frac{e^{-2dx-2c}b}{8d}-\frac{e^{-4dx-4c}b}{64d}$
orering	$x\left(a+b\sinh(dx+c)^4\right)+\frac{5b\cosh(dx+c)\sinh(dx+c)^3}{4d}-\frac{5x\left(12\sinh(dx+c)^2\cosh(dx+c)^2b\,d^2+4\sinh(dx+\right.}{16d^2}$

input
```
int(a+b*sinh(d*x+c)^4,x,method=_RETURNVERBOSE)
```

output
```
1/32*b*(12*d*x+sinh(4*d*x+4*c)-8*sinh(2*d*x+2*c))/d+a*x
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 59, normalized size of antiderivative = 1.13

$$\int\left(a+b\sinh^4(c+dx)\right)\,dx$$
$$=\frac{b\cosh(dx+c)\sinh(dx+c)^3+(8a+3b)dx+\left(b\cosh(dx+c)^3-4b\cosh(dx+c)\right)\sinh(dx+c)}{8d}$$

input
```
integrate(a+b*sinh(d*x+c)^4,x, algorithm="fricas")
```

output
```
1/8*(b*cosh(d*x + c)*sinh(d*x + c)^3 + (8*a + 3*b)*d*x + (b*cosh(d*x + c)^
3 - 4*b*cosh(d*x + c))*sinh(d*x + c))/d
```

Sympy [A] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 100, normalized size of antiderivative = 1.92

$$\int \left(a+b\sinh^4(c+dx)\right)\,dx=ax$$
$$+b\left(\begin{cases}\frac{3x\sinh^4(c+dx)}{8}-\frac{3x\sinh^2(c+dx)\cosh^2(c+dx)}{4}+\frac{3x\cosh^4(c+dx)}{8}+\frac{5\sinh^3(c+dx)\cosh(c+dx)}{8d}-\frac{3\sinh(c+dx)\cosh^3(c+dx)}{8d} \\ x\sinh^4(c)\end{cases}\right.$$

input
```
integrate(a+b*sinh(d*x+c)**4,x)
```

output
```
a*x + b*Piecewise((3*x*sinh(c + d*x)**4/8 - 3*x*sinh(c + d*x)**2*cosh(c +
d*x)**2/4 + 3*x*cosh(c + d*x)**4/8 + 5*sinh(c + d*x)**3*cosh(c + d*x)/(8*d
) - 3*sinh(c + d*x)*cosh(c + d*x)**3/(8*d), Ne(d, 0)), (x*sinh(c)**4, True
))
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 66, normalized size of antiderivative = 1.27

$$\int \left(a+b\sinh^4(c+dx)\right)\,dx$$
$$=\frac{1}{64}\,b\left(24\,x+\frac{e^{(4\,dx+4\,c)}}{d}-\frac{8\,e^{(2\,dx+2\,c)}}{d}+\frac{8\,e^{(-2\,dx-2\,c)}}{d}-\frac{e^{(-4\,dx-4\,c)}}{d}\right)+ax$$

input
```
integrate(a+b*sinh(d*x+c)^4,x, algorithm="maxima")
```

output
```
1/64*b*(24*x + e^(4*d*x + 4*c)/d - 8*e^(2*d*x + 2*c)/d + 8*e^(-2*d*x - 2*c
)/d - e^(-4*d*x - 4*c)/d) + a*x
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 66, normalized size of antiderivative = 1.27

$$\int \left(a+b\sinh^4(c+dx)\right)\,dx$$
$$=\frac{1}{64}\,b\left(24\,x+\frac{e^{(4\,dx+4\,c)}}{d}-\frac{8\,e^{(2\,dx+2\,c)}}{d}+\frac{8\,e^{(-2\,dx-2\,c)}}{d}-\frac{e^{(-4\,dx-4\,c)}}{d}\right)+ax$$

input
```
integrate(a+b*sinh(d*x+c)^4,x, algorithm="giac")
```

output
```
1/64*b*(24*x + e^(4*d*x + 4*c)/d - 8*e^(2*d*x + 2*c)/d + 8*e^(-2*d*x - 2*c
)/d - e^(-4*d*x - 4*c)/d) + a*x
```

Mupad [B] (verification not implemented)

Time = 1.61 (sec) , antiderivative size = 38, normalized size of antiderivative = 0.73

$$\int \left(a+b\sinh^4(c+dx)\right)\,dx=a\,x+\frac{3\,b\,x}{8}-\frac{\frac{b\sinh(2\,c+2\,d\,x)}{4}-\frac{b\sinh(4\,c+4\,d\,x)}{32}}{d}$$

input
```
int(a + b*sinh(c + d*x)^4,x)
```

output
```
a*x + (3*b*x)/8 - ((b*sinh(2*c + 2*d*x))/4 - (b*sinh(4*c + 4*d*x))/32)/d
```

Reduce [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 89, normalized size of antiderivative = 1.71

$$\int \left(a+b\sinh^4(c+dx)\right)\,dx$$
$$=\frac{e^{8dx+8c}b-8e^{6dx+6c}b+64e^{4dx+4c}adx+24e^{4dx+4c}bdx+8e^{2dx+2c}b-b}{64e^{4dx+4c}d}$$

input
```
int(a+b*sinh(d*x+c)^4,x)
```

output
```
(e**(8*c + 8*d*x)*b - 8*e**(6*c + 6*d*x)*b + 64*e**(4*c + 4*d*x)*a*d*x + 2
4*e**(4*c + 4*d*x)*b*d*x + 8*e**(2*c + 2*d*x)*b - b)/(64*e**(4*c + 4*d*x)*
d)
```

3.164 $\int \operatorname{csch}(c+dx)\left(a+b\sinh^4(c+dx)\right)\,dx$

Optimal result

Integrand size = 19, antiderivative size = 42

$$\int \operatorname{csch}(c+dx)\left(a+b\sinh^4(c+dx)\right)\,dx = -\frac{a\,\operatorname{arctanh}(\cosh(c+dx))}{d} - \frac{b\cosh(c+dx)}{d} + \frac{b\cosh^3(c+dx)}{3d}$$

output
```
-a*arctanh(cosh(d*x+c))/d-b*cosh(d*x+c)/d+1/3*b*cosh(d*x+c)^3/d
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 44, normalized size of antiderivative = 1.05

$$\int \operatorname{csch}(c+dx)\left(a+b\sinh^4(c+dx)\right)\,dx = -\frac{a\,\operatorname{arctanh}(\cosh(c+dx))}{d} - \frac{3b\cosh(c+dx)}{4d} + \frac{b\cosh(3(c+dx))}{12d}$$

input
```
Integrate[Csch[c + d*x]*(a + b*Sinh[c + d*x]^4),x]
```

output
```
-((a*ArcTanh[Cosh[c + d*x]])/d) - (3*b*Cosh[c + d*x])/(4*d) + (b*Cosh[3*(c
 + d*x)])/(12*d)
```

Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.86, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.263$, Rules used = {3042, 26, 3694, 1467, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \operatorname{csch}(c+dx)\left(a+b\sinh^4(c+dx)\right)\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{i\left(a+b\sin(ic+idx)^4\right)}{\sin(ic+idx)}dx$$

$$\Big\downarrow \text{26}$$

$$i\int \frac{b\sin(ic+idx)^4+a}{\sin(ic+idx)}dx$$

$$\Big\downarrow \text{3694}$$

$$-\frac{\int \frac{b\cosh^4(c+dx)-2b\cosh^2(c+dx)+a+b}{1-\cosh^2(c+dx)}d\cosh(c+dx)}{d}$$

$$\Big\downarrow \text{1467}$$

$$-\frac{\int \left(-b\cosh^2(c+dx)+b+\frac{a}{1-\cosh^2(c+dx)}\right)d\cosh(c+dx)}{d}$$

$$\Big\downarrow \text{2009}$$

$$-\frac{a\text{arctanh}(\cosh(c+dx))-\frac{1}{3}b\cosh^3(c+dx)+b\cosh(c+dx)}{d}$$

input
```
Int[Csch[c + d*x]*(a + b*Sinh[c + d*x]^4),x]
```

output
```
-((a*ArcTanh[Cosh[c + d*x]] + b*Cosh[c + d*x] - (b*Cosh[c + d*x]^3)/3)/d)
```

Defintions of rubi rules used

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 1467
```
Int[((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.),
 x_Symbol] :> Int[ExpandIntegrand[(d + e*x^2)^q*(a + b*x^2 + c*x^4)^p, x],
x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e
 + a*e^2, 0] && IGtQ[p, 0] && IGtQ[q, -2]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3694
```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^4)^
(p_.), x_Symbol] :> With[{ff = FreeFactors[Cos[e + f*x], x]}, Simp[-ff/f
Subst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a + b - 2*b*ff^2*x^2 + b*ff^4*x^4)^p,
 x], x, Cos[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m -
 1)/2]
```

Maple [A] (verified)

Time = 0.52 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.86

method	result	size
derivativedivides	$\frac{-2a\,\operatorname{arctanh}\left(\mathrm{e}^{dx+c}\right)+b\left(-\frac{2}{3}+\frac{\sinh(dx+c)^{2}}{3}\right)\cosh(dx+c)}{d}$	36
default	$\frac{-2a\,\operatorname{arctanh}\left(\mathrm{e}^{dx+c}\right)+b\left(-\frac{2}{3}+\frac{\sinh(dx+c)^{2}}{3}\right)\cosh(dx+c)}{d}$	36
parallelrisch	$\frac{4a\ln\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-3\left(\cosh(dx+c)-\frac{\cosh(3dx+3c)}{9}+\frac{8}{9}\right)b}{4d}$	42
risch	$\frac{\mathrm{e}^{3dx+3c}b}{24d}-\frac{3\,\mathrm{e}^{dx+c}b}{8d}-\frac{3\,\mathrm{e}^{-dx-c}b}{8d}+\frac{\mathrm{e}^{-3dx-3c}b}{24d}+\frac{a\ln\left(\mathrm{e}^{dx+c}-1\right)}{d}-\frac{a\ln\left(\mathrm{e}^{dx+c}+1\right)}{d}$	88

input
```
int(csch(d*x+c)*(a+b*sinh(d*x+c)^4),x,method=_RETURNVERBOSE)
```

output
```
1/d*(-2*a*arctanh(exp(d*x+c))+b*(-2/3+1/3*sinh(d*x+c)^2)*cosh(d*x+c))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 395 vs. $2(40) = 80$.

Time = 0.12 (sec) , antiderivative size = 395, normalized size of antiderivative = 9.40

$$\int \operatorname{csch}(c+dx)\left(a+b\sinh^4(c+dx)\right)\,dx = \frac{b\cosh\left(dx+c\right)^6+6\,b\cosh\left(dx+c\right)\sinh\left(dx+c\right)^5+b\sinh\left(dx+c\right)^6-9\,b\cosh\left(dx+c\right)^4+3\left(5\,b\cosh\left(dx\right.\right.}{}$$

input
```
integrate(csch(d*x+c)*(a+b*sinh(d*x+c)^4),x, algorithm="fricas")
```

output
```
1/24*(b*cosh(d*x + c)^6 + 6*b*cosh(d*x + c)*sinh(d*x + c)^5 + b*sinh(d*x +
 c)^6 - 9*b*cosh(d*x + c)^4 + 3*(5*b*cosh(d*x + c)^2 - 3*b)*sinh(d*x + c)^
4 + 4*(5*b*cosh(d*x + c)^3 - 9*b*cosh(d*x + c))*sinh(d*x + c)^3 - 9*b*cosh
(d*x + c)^2 + 3*(5*b*cosh(d*x + c)^4 - 18*b*cosh(d*x + c)^2 - 3*b)*sinh(d*
x + c)^2 - 24*(a*cosh(d*x + c)^3 + 3*a*cosh(d*x + c)^2*sinh(d*x + c) + 3*a
*cosh(d*x + c)*sinh(d*x + c)^2 + a*sinh(d*x + c)^3)*log(cosh(d*x + c) + si
nh(d*x + c) + 1) + 24*(a*cosh(d*x + c)^3 + 3*a*cosh(d*x + c)^2*sinh(d*x +
c) + 3*a*cosh(d*x + c)*sinh(d*x + c)^2 + a*sinh(d*x + c)^3)*log(cosh(d*x +
 c) + sinh(d*x + c) - 1) + 6*(b*cosh(d*x + c)^5 - 6*b*cosh(d*x + c)^3 - 3*
b*cosh(d*x + c))*sinh(d*x + c) + b)/(d*cosh(d*x + c)^3 + 3*d*cosh(d*x + c)
^2*sinh(d*x + c) + 3*d*cosh(d*x + c)*sinh(d*x + c)^2 + d*sinh(d*x + c)^3)
```

Sympy [F]

$$\int \operatorname{csch}(c+dx)\left(a+b\sinh^4(c+dx)\right)\,dx=\int \left(a+b\sinh^4(c+dx)\right)\operatorname{csch}(c+dx)\,dx$$

input
```
integrate(csch(d*x+c)*(a+b*sinh(d*x+c)**4),x)
```

output
```
Integral((a + b*sinh(c + d*x)**4)*csch(c + d*x), x)
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 71, normalized size of antiderivative = 1.69

$$\int \operatorname{csch}(c+dx)\left(a+b\sinh^4(c+dx)\right)\,dx = \frac{1}{24}b\left(\frac{e^{(3\,dx+3\,c)}}{d}-\frac{9\,e^{(dx+c)}}{d}-\frac{9\,e^{(-dx-c)}}{d}+\frac{e^{(-3\,dx-3\,c)}}{d}\right)+\frac{a\log\left(\tanh\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)}{d}$$

input
```
integrate(csch(d*x+c)*(a+b*sinh(d*x+c)^4),x, algorithm="maxima")
```

output
```
1/24*b*(e^(3*d*x + 3*c)/d - 9*e^(d*x + c)/d - 9*e^(-d*x - c)/d + e^(-3*d*x
 - 3*c)/d) + a*log(tanh(1/2*d*x + 1/2*c))/d
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 78, normalized size of antiderivative = 1.86

$$\int \operatorname{csch}(c+dx)\left(a+b\sinh^4(c+dx)\right)\,dx = \frac{be^{(3\,dx+3\,c)}-9\,be^{(dx+c)}-\left(9\,be^{(2\,dx+2\,c)}-b\right)e^{(-3\,dx-3\,c)}-24\,a\log\left(e^{(dx+c)}+1\right)+24\,a\log\left(\left|e^{(dx+c)}-1\right|\right)}{24\,d}$$

input
```
integrate(csch(d*x+c)*(a+b*sinh(d*x+c)^4),x, algorithm="giac")
```

output
```
1/24*(b*e^(3*d*x + 3*c) - 9*b*e^(d*x + c) - (9*b*e^(2*d*x + 2*c) - b)*e^(-
3*d*x - 3*c) - 24*a*log(e^(d*x + c) + 1) + 24*a*log(abs(e^(d*x + c) - 1)))
/d
```

Mupad [B] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 96, normalized size of antiderivative = 2.29

$$\int \operatorname{csch}(c+dx)\left(a+b\sinh^4(c+dx)\right)\,dx = \frac{b\,\mathrm{e}^{-3\,c-3\,d\,x}}{24\,d}-\frac{3\,b\,\mathrm{e}^{-c-d\,x}}{8\,d}+\frac{b\,\mathrm{e}^{3\,c+3\,d\,x}}{24\,d}-\frac{3\,b\,\mathrm{e}^{c+d\,x}}{8\,d}-\frac{2\,\mathrm{atan}\left(\frac{a\,\mathrm{e}^{d\,x}\,\mathrm{e}^{c}\,\sqrt{-d^2}}{d\,\sqrt{a^2}}\right)\,\sqrt{a^2}}{\sqrt{-d^2}}$$

input
```
int((a + b*sinh(c + d*x)^4)/sinh(c + d*x),x)
```

output
```
(b*exp(- 3*c - 3*d*x))/(24*d) - (3*b*exp(- c - d*x))/(8*d) + (b*exp(3*c +
3*d*x))/(24*d) - (3*b*exp(c + d*x))/(8*d) - (2*atan((a*exp(d*x)*exp(c)*(-d
^2)^(1/2))/(d*(a^2)^(1/2)))*(a^2)^(1/2))/(-d^2)^(1/2)
```

Reduce [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 103, normalized size of antiderivative = 2.45

$$\int \operatorname{csch}(c+dx)\left(a+b\sinh^4(c+dx)\right)\,dx = \frac{e^{6dx+6c}b-9e^{4dx+4c}b+24e^{3dx+3c}\log\left(e^{dx+c}-1\right)a-24e^{3dx+3c}\log\left(e^{dx+c}+1\right)a-9e^{2dx+2c}b+b}{24e^{3dx+3c}d}$$

input
```
int(csch(d*x+c)*(a+b*sinh(d*x+c)^4),x)
```

output
```
(e**(6*c + 6*d*x)*b - 9*e**(4*c + 4*d*x)*b + 24*e**(3*c + 3*d*x)*log(e**(c
 + d*x) - 1)*a - 24*e**(3*c + 3*d*x)*log(e**(c + d*x) + 1)*a - 9*e**(2*c +
 2*d*x)*b + b)/(24*e**(3*c + 3*d*x)*d)
```

3.165 $\int \operatorname{csch}^2(c+dx) \left(a+b\sinh^4(c+dx)\right) dx$

Optimal result

Integrand size = 21, antiderivative size = 39

$$\int \operatorname{csch}^2(c+dx) \left(a+b\sinh^4(c+dx)\right) dx = -\frac{bx}{2}-\frac{a\coth(c+dx)}{d}+\frac{b\cosh(c+dx)\sinh(c+dx)}{2d}$$

output
```
-1/2*b*x-a*coth(d*x+c)/d+1/2*b*cosh(d*x+c)*sinh(d*x+c)/d
```

Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 45, normalized size of antiderivative = 1.15

$$\int \operatorname{csch}^2(c+dx) \left(a+b\sinh^4(c+dx)\right) dx = \frac{b(-c-dx)}{2d}-\frac{a\coth(c+dx)}{d}+\frac{b\sinh(2(c+dx))}{4d}$$

input
```
Integrate[Csch[c + d*x]^2*(a + b*Sinh[c + d*x]^4),x]
```

output
```
(b*(-c - d*x))/(2*d) - (a*Coth[c + d*x])/d + (b*Sinh[2*(c + d*x)])/(4*d)
```

Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 54, normalized size of antiderivative = 1.38, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {3042, 25, 3696, 1582, 25, 359, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \operatorname{csch}^2(c+dx)\left(a+b\sinh^4(c+dx)\right)dx$$

$$\Big\downarrow \text{3042}$$

$$\int -\frac{a+b\sin(ic+idx)^4}{\sin(ic+idx)^2}dx$$

$$\Big\downarrow \text{25}$$

$$-\int \frac{b\sin(ic+idx)^4+a}{\sin(ic+idx)^2}dx$$

$$\Big\downarrow \text{3696}$$

$$\frac{\int \frac{\coth^2(c+dx)\left((a+b)\tanh^4(c+dx)-2a\tanh^2(c+dx)+a\right)}{\left(1-\tanh^2(c+dx)\right)^2}d\tanh(c+dx)}{d}$$

$$\Big\downarrow \text{1582}$$

$$\frac{\frac{b\tanh(c+dx)}{2(1-\tanh^2(c+dx))}-\frac{1}{2}\int -\frac{\coth^2(c+dx)\left(2a-(2a+b)\tanh^2(c+dx)\right)}{1-\tanh^2(c+dx)}d\tanh(c+dx)}{d}$$

$$\Big\downarrow \text{25}$$

$$\frac{\frac{1}{2}\int \frac{\coth^2(c+dx)\left(2a-(2a+b)\tanh^2(c+dx)\right)}{1-\tanh^2(c+dx)}d\tanh(c+dx)+\frac{b\tanh(c+dx)}{2(1-\tanh^2(c+dx))}}{d}$$

$$\Big\downarrow \text{359}$$

$$\frac{\frac{1}{2}\left(-b\int \frac{1}{1-\tanh^2(c+dx)}d\tanh(c+dx)-2a\coth(c+dx)\right)+\frac{b\tanh(c+dx)}{2(1-\tanh^2(c+dx))}}{d}$$

$$\Big\downarrow \text{219}$$

$$\frac{\frac{1}{2}\left(-2a\coth(c+dx)-b\operatorname{arctanh}(\tanh(c+dx))\right)+\frac{b\tanh(c+dx)}{2(1-\tanh^2(c+dx))}}{d}$$

input
```
Int[Csch[c + d*x]^2*(a + b*Sinh[c + d*x]^4),x]
```

output
```
((-(b*ArcTanh[Tanh[c + d*x]]) - 2*a*Coth[c + d*x])/2 + (b*Tanh[c + d*x])/(
2*(1 - Tanh[c + d*x]^2)))/d
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 359
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2), x
_Symbol] :> Simp[c*(e*x)^(m + 1)*((a + b*x^2)^(p + 1)/(a*e*(m + 1))), x] +
Simp[(a*d*(m + 1) - b*c*(m + 2*p + 3))/(a*e^2*(m + 1))   Int[(e*x)^(m + 2)*
(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c - a*d, 0]
&& LtQ[m, -1] &&  !ILtQ[p, -1]
```

rule 1582
```
Int[(x_)^(m_)*((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^
4)^(p_.), x_Symbol] :> Simp[(-d)^(m/2 - 1)*(c*d^2 - b*d*e + a*e^2)^p*x*((d
+ e*x^2)^(q + 1)/(2*e^(2*p + m/2)*(q + 1))), x] + Simp[(-d)^(m/2 - 1)/(2*e^
(2*p)*(q + 1))   Int[x^m*(d + e*x^2)^(q + 1)*ExpandToSum[Together[(1/(d + e
*x^2))*(2*(-d)^(-m/2 + 1)*e^(2*p)*(q + 1)*(a + b*x^2 + c*x^4)^p - ((c*d^2 -
 b*d*e + a*e^2)^p/(e^(m/2)*x^m))*(d + e*(2*q + 3)*x^2))], x], x], x] /; Fre
eQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && IGtQ[p, 0] && ILtQ[q, -1]
&& ILtQ[m/2, 0]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3696
```
Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^4)^(
p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Simp[ff^(m + 1
)/f   Subst[Int[x^m*((a + 2*a*ff^2*x^2 + (a + b)*ff^4*x^4)^p/(1 + ff^2*x^2)
^(m/2 + 2*p + 1)), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f}, x] &
& IntegerQ[m/2] && IntegerQ[p]
```

Maple [A] (verified)

Time = 0.54 (sec) , antiderivative size = 39, normalized size of antiderivative = 1.00

method	result	size
derivativedivides	$\frac{-\coth(dx+c)a+b\left(\frac{\cosh(dx+c)\sinh(dx+c)}{2}-\frac{dx}{2}-\frac{c}{2}\right)}{d}$	39
default	$\frac{-\coth(dx+c)a+b\left(\frac{\cosh(dx+c)\sinh(dx+c)}{2}-\frac{dx}{2}-\frac{c}{2}\right)}{d}$	39
risch	$-\frac{bx}{2}+\frac{e^{2dx+2c}b}{8d}-\frac{e^{-2dx-2c}b}{8d}-\frac{2a}{d(e^{2dx+2c}-1)}$	55
parallelrisch	$\frac{(-2b\cosh(dx+c)+b\cosh(2dx+2c)-4a+b)\coth\left(\frac{dx}{2}+\frac{c}{2}\right)+2\,\mathrm{sech}\left(\frac{dx}{2}+\frac{c}{2}\right)\mathrm{csch}\left(\frac{dx}{2}+\frac{c}{2}\right)a-2dxb}{4d}$	68

input
```
int(csch(d*x+c)^2*(a+b*sinh(d*x+c)^4),x,method=_RETURNVERBOSE)
```

output
```
1/d*(-coth(d*x+c)*a+b*(1/2*cosh(d*x+c)*sinh(d*x+c)-1/2*d*x-1/2*c))
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 70, normalized size of antiderivative = 1.79

$$\int \operatorname{csch}^2(c+dx)\left(a+b\sinh^4(c+dx)\right)\,dx = \frac{b\cosh(dx+c)^3+3\,b\cosh(dx+c)\sinh(dx+c)^2-(8\,a+b)\cosh(dx+c)-4\,(bdx-2\,a)\sinh(dx+c)}{8\,d\sinh(dx+c)}$$

input
```
integrate(csch(d*x+c)^2*(a+b*sinh(d*x+c)^4),x, algorithm="fricas")
```

output
```
1/8*(b*cosh(d*x + c)^3 + 3*b*cosh(d*x + c)*sinh(d*x + c)^2 - (8*a + b)*cos
h(d*x + c) - 4*(b*d*x - 2*a)*sinh(d*x + c))/(d*sinh(d*x + c))
```

Sympy [F(-1)]

Timed out.

$$\int \operatorname{csch}^2(c+dx) \left(a+b\sinh^4(c+dx)\right) \, dx = \text{Timed out}$$

input
```
integrate(csch(d*x+c)**2*(a+b*sinh(d*x+c)**4),x)
```

output
```
Timed out
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 54, normalized size of antiderivative = 1.38

$$\int \operatorname{csch}^2(c+dx) \left(a+b\sinh^4(c+dx)\right) \, dx = -\frac{1}{8}\, b \left(4\, x - \frac{e^{(2\, dx+2\, c)}}{d} + \frac{e^{(-2\, dx-2\, c)}}{d}\right) + \frac{2\, a}{d(e^{(-2\, dx-2\, c)} - 1)}$$

input
```
integrate(csch(d*x+c)^2*(a+b*sinh(d*x+c)^4),x, algorithm="maxima")
```

output
```
-1/8*b*(4*x - e^(2*d*x + 2*c)/d + e^(-2*d*x - 2*c)/d) + 2*a/(d*(e^(-2*d*x
- 2*c) - 1))
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 88 vs. $2(35) = 70$.

Time = 0.13 (sec) , antiderivative size = 88, normalized size of antiderivative = 2.26

$$\int \operatorname{csch}^2(c+dx) \left(a+b\sinh^4(c+dx)\right) \, dx = -\frac{4\,(dx+c)b - be^{(2\,dx+2\,c)} - \frac{be^{(4\,dx+4\,c)} - 16\,ae^{(2\,dx+2\,c)} - 2\,be^{(2\,dx+2\,c)} + b}{e^{(4\,dx+4\,c)} - e^{(2\,dx+2\,c)}}}{8\,d}$$

input
```
integrate(csch(d*x+c)^2*(a+b*sinh(d*x+c)^4),x, algorithm="giac")
```

output
```
-1/8*(4*(d*x + c)*b - b*e^(2*d*x + 2*c) - (b*e^(4*d*x + 4*c) - 16*a*e^(2*d
*x + 2*c) - 2*b*e^(2*d*x + 2*c) + b)/(e^(4*d*x + 4*c) - e^(2*d*x + 2*c)))/
d
```

Mupad [B] (verification not implemented)

Time = 1.62 (sec) , antiderivative size = 54, normalized size of antiderivative = 1.38

$$\int \operatorname{csch}^2(c+dx) \left(a+b\sinh^4(c+dx)\right) \, dx = \frac{b\,\mathrm{e}^{2\,c+2\,d\,x}}{8\,d} - \frac{2\,a}{d\,\left(\mathrm{e}^{2\,c+2\,d\,x}-1\right)} - \frac{b\,\mathrm{e}^{-2\,c-2\,d\,x}}{8\,d} - \frac{b\,x}{2}$$

input
```
int((a + b*sinh(c + d*x)^4)/sinh(c + d*x)^2,x)
```

output
```
(b*exp(2*c + 2*d*x))/(8*d) - (2*a)/(d*(exp(2*c + 2*d*x) - 1)) - (b*exp(- 2
*c - 2*d*x))/(8*d) - (b*x)/2
```

Reduce [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 101, normalized size of antiderivative = 2.59

$$\int \operatorname{csch}^2(c+dx)\left(a+b\sinh^4(c+dx)\right)\,dx = \frac{e^{6dx+6c}b-16e^{4dx+4c}a-4e^{4dx+4c}bdx-2e^{4dx+4c}b+4e^{2dx+2c}bdx+b}{8e^{2dx+2c}d\left(e^{2dx+2c}-1\right)}$$

input
```
int(csch(d*x+c)^2*(a+b*sinh(d*x+c)^4),x)
```

output
```
(e**(6*c + 6*d*x)*b - 16*e**(4*c + 4*d*x)*a - 4*e**(4*c + 4*d*x)*b*d*x - 2
*e**(4*c + 4*d*x)*b + 4*e**(2*c + 2*d*x)*b*d*x + b)/(8*e**(2*c + 2*d*x)*d*
(e**(2*c + 2*d*x) - 1))
```

3.166 $\int \operatorname{csch}^3(c+dx)\left(a+b\sinh^4(c+dx)\right)\,dx$

Optimal result

Integrand size = 21, antiderivative size = 47

$$\int \operatorname{csch}^3(c+dx)\left(a+b\sinh^4(c+dx)\right)\,dx=\frac{a\operatorname{arctanh}(\cosh(c+dx))}{2d}+\frac{b\cosh(c+dx)}{d}-\frac{a\coth(c+dx)\operatorname{csch}(c+dx)}{2d}$$

output
```
1/2*a*arctanh(cosh(d*x+c))/d+b*cosh(d*x+c)/d-1/2*a*coth(d*x+c)*csch(d*x+c)
/d
```

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 101 vs. 2(47) = 94.

Time = 0.01 (sec) , antiderivative size = 101, normalized size of antiderivative = 2.15

$$\int \operatorname{csch}^3(c+dx)\left(a+b\sinh^4(c+dx)\right)\,dx=\frac{b\cosh(c)\cosh(dx)}{d}-\frac{a\operatorname{csch}^2\left(\frac{1}{2}(c+dx)\right)}{8d}+\frac{a\log\left(\cosh\left(\frac{1}{2}(c+dx)\right)\right)}{2d}-\frac{a\log\left(\sinh\left(\frac{1}{2}(c+dx)\right)\right)}{2d}-\frac{a\operatorname{sech}^2\left(\frac{1}{2}(c+dx)\right)}{8d}+\frac{b\sinh(c)\sinh(dx)}{d}$$

input
```
Integrate[Csch[c + d*x]^3*(a + b*Sinh[c + d*x]^4),x]
```

output
```
(b*Cosh[c]*Cosh[d*x])/d - (a*Csch[(c + d*x)/2]^2)/(8*d) + (a*Log[Cosh[(c +
 d*x)/2]])/(2*d) - (a*Log[Sinh[(c + d*x)/2]])/(2*d) - (a*Sech[(c + d*x)/2]
^2)/(8*d) + (b*Sinh[c]*Sinh[d*x])/d
```

Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 53, normalized size of antiderivative = 1.13, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {3042, 26, 3694, 1471, 25, 299, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \operatorname{csch}^3(c+dx)\left(a+b\sinh^4(c+dx)\right)dx$$

$$\Big\downarrow \text{3042}$$

$$\int -\frac{i\left(a+b\sin(ic+idx)^4\right)}{\sin(ic+idx)^3}dx$$

$$\Big\downarrow \text{26}$$

$$-i\int \frac{b\sin(ic+idx)^4+a}{\sin(ic+idx)^3}dx$$

$$\Big\downarrow \text{3694}$$

$$\frac{\int \frac{b\cosh^4(c+dx)-2b\cosh^2(c+dx)+a+b}{\left(1-\cosh^2(c+dx)\right)^2}d\cosh(c+dx)}{d}$$

$$\Big\downarrow \text{1471}$$

$$\frac{\frac{a\cosh(c+dx)}{2\left(1-\cosh^2(c+dx)\right)}-\frac{1}{2}\int -\frac{-2b\cosh^2(c+dx)+a+2b}{1-\cosh^2(c+dx)}d\cosh(c+dx)}{d}$$

$$\Big\downarrow \text{25}$$

$$\frac{\frac{1}{2}\int \frac{-2b\cosh^2(c+dx)+a+2b}{1-\cosh^2(c+dx)}d\cosh(c+dx)+\frac{a\cosh(c+dx)}{2\left(1-\cosh^2(c+dx)\right)}}{d}$$

$$\Big\downarrow \text{299}$$

$$\frac{\frac{1}{2}\left(a\int \frac{1}{1-\cosh^2(c+dx)}d\cosh(c+dx)+2b\cosh(c+dx)\right)+\frac{a\cosh(c+dx)}{2(1-\cosh^2(c+dx))}}{d}$$

$$\Big\downarrow \text{219}$$

$$\frac{\frac{1}{2}(a\text{arctanh}(\cosh(c+dx))+2b\cosh(c+dx))+\frac{a\cosh(c+dx)}{2(1-\cosh^2(c+dx))}}{d}$$

input
```
Int[Csch[c + d*x]^3*(a + b*Sinh[c + d*x]^4),x]
```

output
```
((a*ArcTanh[Cosh[c + d*x]] + 2*b*Cosh[c + d*x])/2 + (a*Cosh[c + d*x])/(2*(
1 - Cosh[c + d*x]^2)))/d
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 299
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2), x_Symbol] :> Simp[d*x
*((a + b*x^2)^(p + 1)/(b*(2*p + 3))), x] - Simp[(a*d - b*c*(2*p + 3))/(b*(2
*p + 3))   Int[(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
 a*d, 0] && NeQ[2*p + 3, 0]
```

rule 1471
```
Int[((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.),
x_Symbol] :> With[{Qx = PolynomialQuotient[(a + b*x^2 + c*x^4)^p, d + e*x^2
, x], R = Coeff[PolynomialRemainder[(a + b*x^2 + c*x^4)^p, d + e*x^2, x], x
, 0]}, Simp[(-R)*x*((d + e*x^2)^(q + 1)/(2*d*(q + 1))), x] + Simp[1/(2*d*(q
 + 1))   Int[(d + e*x^2)^(q + 1)*ExpandToSum[2*d*(q + 1)*Qx + R*(2*q + 3),
x], x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^
2 - b*d*e + a*e^2, 0] && IGtQ[p, 0] && LtQ[q, -1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3694
```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^4)^
(p_.), x_Symbol] :> With[{ff = FreeFactors[Cos[e + f*x], x]}, Simp[-ff/f
Subst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a + b - 2*b*ff^2*x^2 + b*ff^4*x^4)^p,
 x], x, Cos[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m -
 1)/2]
```

Maple [A] (verified)

Time = 0.55 (sec) , antiderivative size = 38, normalized size of antiderivative = 0.81

method	result	size
derivativedivides	$\frac{a\left(-\frac{\operatorname{csch}(dx+c)\coth(dx+c)}{2}+\operatorname{arctanh}\left(e^{dx+c}\right)\right)+b\cosh(dx+c)}{d}$	38
default	$\frac{a\left(-\frac{\operatorname{csch}(dx+c)\coth(dx+c)}{2}+\operatorname{arctanh}\left(e^{dx+c}\right)\right)+b\cosh(dx+c)}{d}$	38
parallelrisch	$\frac{8a\ln\left(\frac{1}{\sqrt{\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)}}\right)+a\left(\operatorname{sech}\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}-3\right)\operatorname{csch}\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}+\coth\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}a+8b(1+\cosh(dx+c))}{8d}$	72
risch	$\frac{e^{dx+c}b}{2d}+\frac{e^{-dx-c}b}{2d}-\frac{a\,e^{dx+c}\left(e^{2dx+2c}+1\right)}{d\left(e^{2dx+2c}-1\right)^{2}}+\frac{a\ln\left(e^{dx+c}+1\right)}{2d}-\frac{a\ln\left(e^{dx+c}-1\right)}{2d}$	95

input
```
int(csch(d*x+c)^3*(a+b*sinh(d*x+c)^4),x,method=_RETURNVERBOSE)
```

output
```
1/d*(a*(-1/2*csch(d*x+c)*coth(d*x+c)+arctanh(exp(d*x+c)))+b*cosh(d*x+c))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 690 vs. $2(43) = 86$.

Time = 0.10 (sec) , antiderivative size = 690, normalized size of antiderivative = 14.68

$$\int \operatorname{csch}^3(c+dx)\left(a+b\sinh^4(c+dx)\right)\,dx = \text{Too large to display}$$

input
```
integrate(csch(d*x+c)^3*(a+b*sinh(d*x+c)^4),x, algorithm="fricas")
```

output
```
1/2*(b*cosh(d*x + c)^6 + 6*b*cosh(d*x + c)*sinh(d*x + c)^5 + b*sinh(d*x +
c)^6 - (2*a + b)*cosh(d*x + c)^4 + (15*b*cosh(d*x + c)^2 - 2*a - b)*sinh(d
*x + c)^4 + 4*(5*b*cosh(d*x + c)^3 - (2*a + b)*cosh(d*x + c))*sinh(d*x + c
)^3 - (2*a + b)*cosh(d*x + c)^2 + (15*b*cosh(d*x + c)^4 - 6*(2*a + b)*cosh
(d*x + c)^2 - 2*a - b)*sinh(d*x + c)^2 + (a*cosh(d*x + c)^5 + 5*a*cosh(d*x
 + c)*sinh(d*x + c)^4 + a*sinh(d*x + c)^5 - 2*a*cosh(d*x + c)^3 + 2*(5*a*c
osh(d*x + c)^2 - a)*sinh(d*x + c)^3 + 2*(5*a*cosh(d*x + c)^3 - 3*a*cosh(d*
x + c))*sinh(d*x + c)^2 + a*cosh(d*x + c) + (5*a*cosh(d*x + c)^4 - 6*a*cos
h(d*x + c)^2 + a)*sinh(d*x + c))*log(cosh(d*x + c) + sinh(d*x + c) + 1) -
(a*cosh(d*x + c)^5 + 5*a*cosh(d*x + c)*sinh(d*x + c)^4 + a*sinh(d*x + c)^5
 - 2*a*cosh(d*x + c)^3 + 2*(5*a*cosh(d*x + c)^2 - a)*sinh(d*x + c)^3 + 2*(
5*a*cosh(d*x + c)^3 - 3*a*cosh(d*x + c))*sinh(d*x + c)^2 + a*cosh(d*x + c)
 + (5*a*cosh(d*x + c)^4 - 6*a*cosh(d*x + c)^2 + a)*sinh(d*x + c))*log(cosh
(d*x + c) + sinh(d*x + c) - 1) + 2*(3*b*cosh(d*x + c)^5 - 2*(2*a + b)*cosh
(d*x + c)^3 - (2*a + b)*cosh(d*x + c))*sinh(d*x + c) + b)/(d*cosh(d*x + c)
^5 + 5*d*cosh(d*x + c)*sinh(d*x + c)^4 + d*sinh(d*x + c)^5 - 2*d*cosh(d*x
+ c)^3 + 2*(5*d*cosh(d*x + c)^2 - d)*sinh(d*x + c)^3 + 2*(5*d*cosh(d*x + c
)^3 - 3*d*cosh(d*x + c))*sinh(d*x + c)^2 + d*cosh(d*x + c) + (5*d*cosh(d*x
 + c)^4 - 6*d*cosh(d*x + c)^2 + d)*sinh(d*x + c))
```

Sympy [F(-1)]

Timed out.

$$\int \operatorname{csch}^3(c+dx)\left(a+b\sinh^4(c+dx)\right)\,dx = \text{Timed out}$$

input
```
integrate(csch(d*x+c)**3*(a+b*sinh(d*x+c)**4),x)
```

output
```
Timed out
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 115 vs. 2(43) = 86.

Time = 0.07 (sec) , antiderivative size = 115, normalized size of antiderivative = 2.45

$$\int \operatorname{csch}^3(c+dx)\left(a+b\sinh^4(c+dx)\right)\,dx$$
$$=\frac{1}{2}\,b\left(\frac{e^{(dx+c)}}{d}+\frac{e^{(-dx-c)}}{d}\right)$$
$$+\frac{1}{2}\,a\left(\frac{\log\left(e^{(-dx-c)}+1\right)}{d}-\frac{\log\left(e^{(-dx-c)}-1\right)}{d}+\frac{2\left(e^{(-dx-c)}+e^{(-3\,dx-3\,c)}\right)}{d(2\,e^{(-2\,dx-2\,c)}-e^{(-4\,dx-4\,c)}-1)}\right)$$

input
```
integrate(csch(d*x+c)^3*(a+b*sinh(d*x+c)^4),x, algorithm="maxima")
```

output
```
1/2*b*(e^(d*x + c)/d + e^(-d*x - c)/d) + 1/2*a*(log(e^(-d*x - c) + 1)/d -
log(e^(-d*x - c) - 1)/d + 2*(e^(-d*x - c) + e^(-3*d*x - 3*c))/(d*(2*e^(-2*
d*x - 2*c) - e^(-4*d*x - 4*c) - 1)))
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 107 vs. 2(43) = 86.

Time = 0.14 (sec) , antiderivative size = 107, normalized size of antiderivative = 2.28

$$\int \operatorname{csch}^3(c+dx)\left(a+b\sinh^4(c+dx)\right)\,dx$$
$$=\frac{2\,b\left(e^{(dx+c)}+e^{(-dx-c)}\right)+a\log\left(e^{(dx+c)}+e^{(-dx-c)}+2\right)-a\log\left(e^{(dx+c)}+e^{(-dx-c)}-2\right)-\frac{4\,a\left(e^{(dx+c)}+e^{(-dx-}\right.}{\left(e^{(dx+c)}+e^{(-dx-c)}\right)^2}}{4\,d}$$

input
```
integrate(csch(d*x+c)^3*(a+b*sinh(d*x+c)^4),x, algorithm="giac")
```

output
```
1/4*(2*b*(e^(d*x + c) + e^(-d*x - c)) + a*log(e^(d*x + c) + e^(-d*x - c) +
 2) - a*log(e^(d*x + c) + e^(-d*x - c) - 2) - 4*a*(e^(d*x + c) + e^(-d*x -
 c))/((e^(d*x + c) + e^(-d*x - c))^2 - 4))/d
```

Mupad [B] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 126, normalized size of antiderivative = 2.68

$$\int \operatorname{csch}^3(c+dx)\left(a+b\sinh^4(c+dx)\right)\,dx=\frac{b\,\mathrm{e}^{-c-d\,x}}{2\,d}+\frac{b\,\mathrm{e}^{c+d\,x}}{2\,d}+\frac{\mathrm{atan}\left(\frac{a\,\mathrm{e}^{d\,x}\,\mathrm{e}^{c}\,\sqrt{-d^2}}{d\,\sqrt{a^2}}\right)\,\sqrt{a^2}}{\sqrt{-d^2}}-\frac{a\,\mathrm{e}^{c+d\,x}}{d\,\left(\mathrm{e}^{2\,c+2\,d\,x}-1\right)}-\frac{2\,a\,\mathrm{e}^{c+d\,x}}{d\,\left(\mathrm{e}^{4\,c+4\,d\,x}-2\,\mathrm{e}^{2\,c+2\,d\,x}+1\right)}$$

input
```
int((a + b*sinh(c + d*x)^4)/sinh(c + d*x)^3,x)
```

output
```
(b*exp(- c - d*x))/(2*d) + (b*exp(c + d*x))/(2*d) + (atan((a*exp(d*x)*exp(
c)*(-d^2)^(1/2))/(d*(a^2)^(1/2)))*(a^2)^(1/2))/(-d^2)^(1/2) - (a*exp(c + d
*x))/(d*(exp(2*c + 2*d*x) - 1)) - (2*a*exp(c + d*x))/(d*(exp(4*c + 4*d*x)
- 2*exp(2*c + 2*d*x) + 1))
```

Reduce [B] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 236, normalized size of antiderivative = 5.02

$$\int \operatorname{csch}^3(c+dx)\left(a+b\sinh^4(c+dx)\right)\,dx =\frac{e^{6dx+6c}b-e^{5dx+5c}\log\left(e^{dx+c}-1\right)a+e^{5dx+5c}\log\left(e^{dx+c}+1\right)a-2e^{4dx+4c}a-e^{4dx+4c}b+2e^{3dx+3c}\log\left(e^{dx+c}\right.}{2e^{dx+c}d\left(e^{4dx+\right.}}$$

input
```
int(csch(d*x+c)^3*(a+b*sinh(d*x+c)^4),x)
```

output
```
(e**(6*c + 6*d*x)*b - e**(5*c + 5*d*x)*log(e**(c + d*x) - 1)*a + e**(5*c +
 5*d*x)*log(e**(c + d*x) + 1)*a - 2*e**(4*c + 4*d*x)*a - e**(4*c + 4*d*x)*
b + 2*e**(3*c + 3*d*x)*log(e**(c + d*x) - 1)*a - 2*e**(3*c + 3*d*x)*log(e*
*(c + d*x) + 1)*a - 2*e**(2*c + 2*d*x)*a - e**(2*c + 2*d*x)*b - e**(c + d*
x)*log(e**(c + d*x) - 1)*a + e**(c + d*x)*log(e**(c + d*x) + 1)*a + b)/(2*
e**(c + d*x)*d*(e**(4*c + 4*d*x) - 2*e**(2*c + 2*d*x) + 1))
```

3.167 $\int \operatorname{csch}^4(c+dx) \left(a+b\sinh^4(c+dx)\right) \, dx$

Optimal result

Integrand size = 21, antiderivative size = 31

$$\int \operatorname{csch}^4(c+dx) \left(a+b\sinh^4(c+dx)\right) \, dx = bx+\frac{a\coth(c+dx)}{d}-\frac{a\coth^3(c+dx)}{3d}$$

output
```
b*x+a*coth(d*x+c)/d-1/3*a*coth(d*x+c)^3/d
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 40, normalized size of antiderivative = 1.29

$$\int \operatorname{csch}^4(c+dx) \left(a+b\sinh^4(c+dx)\right) \, dx = bx+\frac{2a\coth(c+dx)}{3d} - \frac{a\coth(c+dx)\operatorname{csch}^2(c+dx)}{3d}$$

input
```
Integrate[Csch[c + d*x]^4*(a + b*Sinh[c + d*x]^4),x]
```

output
```
b*x + (2*a*Coth[c + d*x])/(3*d) - (a*Coth[c + d*x]*Csch[c + d*x]^2)/(3*d)
```

Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 35, normalized size of antiderivative = 1.13, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.190$, Rules used = {3042, 3696, 1584, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \operatorname{csch}^4(c+dx)\left(a+b\sinh^4(c+dx)\right)\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{a+b\sin(ic+idx)^4}{\sin(ic+idx)^4}dx$$

$$\Big\downarrow \text{3696}$$

$$\frac{\int \frac{\coth^4(c+dx)\left((a+b)\tanh^4(c+dx)-2a\tanh^2(c+dx)+a\right)}{1-\tanh^2(c+dx)}d\tanh(c+dx)}{d}$$

$$\Big\downarrow \text{1584}$$

$$\frac{\int \left(a\coth^4(c+dx)-a\coth^2(c+dx)-\frac{b}{\tanh^2(c+dx)-1}\right)d\tanh(c+dx)}{d}$$

$$\Big\downarrow \text{2009}$$

$$\frac{-\frac{1}{3}a\coth^3(c+dx)+a\coth(c+dx)+b\text{arctanh}(\tanh(c+dx))}{d}$$

input
```
Int[Csch[c + d*x]^4*(a + b*Sinh[c + d*x]^4),x]
```

output
```
(b*ArcTanh[Tanh[c + d*x]] + a*Coth[c + d*x] - (a*Coth[c + d*x]^3)/3)/d
```

Defintions of rubi rules used

rule 1584
```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (
c_.)*(x_)^4)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(f*x)^m*(d + e*x^2)^q*
(a + b*x^2 + c*x^4)^p, x], x] /; FreeQ[{a, b, c, d, e, f, m, q}, x] && NeQ[
b^2 - 4*a*c, 0] && IGtQ[p, 0] && IGtQ[q, -2]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3696
```
Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^4)^(
p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Simp[ff^(m + 1
)/f   Subst[Int[x^m*((a + 2*a*ff^2*x^2 + (a + b)*ff^4*x^4)^p/(1 + ff^2*x^2)
^(m/2 + 2*p + 1)), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f}, x] &
& IntegerQ[m/2] && IntegerQ[p]
```

Maple [A] (verified)

Time = 0.49 (sec) , antiderivative size = 33, normalized size of antiderivative = 1.06

method	result	size
derivativedivides	$\frac{a\left(\frac{2}{3}-\frac{\operatorname{csch}(dx+c)^2}{3}\right)\coth(dx+c)+b(dx+c)}{d}$	33
default	$\frac{a\left(\frac{2}{3}-\frac{\operatorname{csch}(dx+c)^2}{3}\right)\coth(dx+c)+b(dx+c)}{d}$	33
risch	$bx-\frac{4a\left(3\,\mathrm{e}^{2dx+2c}-1\right)}{3d\left(\mathrm{e}^{2dx+2c}-1\right)^3}$	37
parallelrisch	$-\frac{-24dxb+a\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^3+\coth\left(\frac{dx}{2}+\frac{c}{2}\right)^3-9\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)-9\coth\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{24d}$	59

input
```
int(csch(d*x+c)^4*(a+b*sinh(d*x+c)^4),x,method=_RETURNVERBOSE)
```

output
```
1/d*(a*(2/3-1/3*csch(d*x+c)^2)*coth(d*x+c)+b*(d*x+c))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 129 vs. $2(29) = 58$.

Time = 0.09 (sec) , antiderivative size = 129, normalized size of antiderivative = 4.16

$$\int \operatorname{csch}^4(c+dx)\left(a+b\sinh^4(c+dx)\right)\,dx$$
$$= \frac{2\,a\cosh\left(dx+c\right)^3+6\,a\cosh\left(dx+c\right)\sinh\left(dx+c\right)^2+\left(3\,bdx-2\,a\right)\sinh\left(dx+c\right)^3-6\,a\cosh\left(dx+c\right)-3}{3\,\left(d\sinh\left(dx+c\right)^3+3\,\left(d\cosh\left(dx+c\right)^2-d\right)\sinh\left(dx\right.\right.}$$

input
```
integrate(csch(d*x+c)^4*(a+b*sinh(d*x+c)^4),x, algorithm="fricas")
```

output
```
1/3*(2*a*cosh(d*x + c)^3 + 6*a*cosh(d*x + c)*sinh(d*x + c)^2 + (3*b*d*x -
2*a)*sinh(d*x + c)^3 - 6*a*cosh(d*x + c) - 3*(3*b*d*x - (3*b*d*x - 2*a)*co
sh(d*x + c)^2 - 2*a)*sinh(d*x + c))/(d*sinh(d*x + c)^3 + 3*(d*cosh(d*x + c
)^2 - d)*sinh(d*x + c))
```

Sympy [F(-1)]

Timed out.

$$\int \operatorname{csch}^4(c+dx)\left(a+b\sinh^4(c+dx)\right)\,dx = \text{Timed out}$$

input
```
integrate(csch(d*x+c)**4*(a+b*sinh(d*x+c)**4),x)
```

output
```
Timed out
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 97 vs. $2(29) = 58$.

Time = 0.04 (sec) , antiderivative size = 97, normalized size of antiderivative = 3.13

$$\int \operatorname{csch}^4(c+dx)\left(a+b\sinh^4(c+dx)\right)\,dx = bx + \frac{4}{3}a\left(\frac{3e^{(-2dx-2c)}}{d(3e^{(-2dx-2c)}-3e^{(-4dx-4c)}+e^{(-6dx-6c)}-1)} - \frac{1}{d(3e^{(-2dx-2c)}-3e^{(-4dx-4c)}+e^{(-6dx-6c)}-1)}\right.$$

input
```
integrate(csch(d*x+c)^4*(a+b*sinh(d*x+c)^4),x, algorithm="maxima")
```

output
```
b*x + 4/3*a*(3*e^(-2*d*x - 2*c)/(d*(3*e^(-2*d*x - 2*c) - 3*e^(-4*d*x - 4*c
) + e^(-6*d*x - 6*c) - 1)) - 1/(d*(3*e^(-2*d*x - 2*c) - 3*e^(-4*d*x - 4*c)
 + e^(-6*d*x - 6*c) - 1)))
```

Giac [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 45, normalized size of antiderivative = 1.45

$$\int \operatorname{csch}^4(c+dx)\left(a+b\sinh^4(c+dx)\right)\,dx = \frac{3\,(dx+c)b - \frac{4\,(3\,ae^{(2\,dx+2\,c)}-a)}{\left(e^{(2\,dx+2\,c)}-1\right)^3}}{3\,d}$$

input
```
integrate(csch(d*x+c)^4*(a+b*sinh(d*x+c)^4),x, algorithm="giac")
```

output
```
1/3*(3*(d*x + c)*b - 4*(3*a*e^(2*d*x + 2*c) - a)/(e^(2*d*x + 2*c) - 1)^3)/
d
```

Mupad [B] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 81, normalized size of antiderivative = 2.61

$$\int \operatorname{csch}^4(c+dx) \left(a+b\sinh^4(c+dx)\right)\,dx = \frac{4\,a-12\,a\,\mathrm{e}^{2\,c+2\,d\,x}-3\,b\,d\,x+9\,b\,d\,x\,\mathrm{e}^{2\,c+2\,d\,x}-9\,b\,d\,x\,\mathrm{e}^{4\,c+4\,d\,x}+3\,b\,d\,x\,\mathrm{e}^{6\,c+6\,d\,x}}{3\,d\,{\left(\mathrm{e}^{2\,c+2\,d\,x}-1\right)}^3}$$

input
```
int((a + b*sinh(c + d*x)^4)/sinh(c + d*x)^4,x)
```

output
```
(4*a - 12*a*exp(2*c + 2*d*x) - 3*b*d*x + 9*b*d*x*exp(2*c + 2*d*x) - 9*b*d*
x*exp(4*c + 4*d*x) + 3*b*d*x*exp(6*c + 6*d*x))/(3*d*(exp(2*c + 2*d*x) - 1)
^3)
```

Reduce [B] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 110, normalized size of antiderivative = 3.55

$$\int \operatorname{csch}^4(c+dx) \left(a+b\sinh^4(c+dx)\right)\,dx = \frac{3e^{6dx+6c}bdx-9e^{4dx+4c}bdx-12e^{2dx+2c}a+9e^{2dx+2c}bdx+4a-3bdx}{3d\left(e^{6dx+6c}-3e^{4dx+4c}+3e^{2dx+2c}-1\right)}$$

input
```
int(csch(d*x+c)^4*(a+b*sinh(d*x+c)^4),x)
```

output
```
(3*e**(6*c + 6*d*x)*b*d*x - 9*e**(4*c + 4*d*x)*b*d*x - 12*e**(2*c + 2*d*x)
*a + 9*e**(2*c + 2*d*x)*b*d*x + 4*a - 3*b*d*x)/(3*d*(e**(6*c + 6*d*x) - 3*
e**(4*c + 4*d*x) + 3*e**(2*c + 2*d*x) - 1))
```

3.168 $\int \operatorname{csch}^5(c+dx)\left(a+b\sinh^4(c+dx)\right)\,dx$

Optimal result

Integrand size = 21, antiderivative size = 64

$$\int \operatorname{csch}^5(c+dx)\left(a+b\sinh^4(c+dx)\right)\,dx = -\frac{(3a+8b)\operatorname{arctanh}(\cosh(c+dx))}{8d} + \frac{3a\coth(c+dx)\operatorname{csch}(c+dx)}{8d} - \frac{a\coth(c+dx)\operatorname{csch}^3(c+dx)}{4d}$$

output
```
-1/8*(3*a+8*b)*arctanh(cosh(d*x+c))/d+3/8*a*coth(d*x+c)*csch(d*x+c)/d-1/4*
a*coth(d*x+c)*csch(d*x+c)^3/d
```

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 132 vs. 2(64) = 128.

Time = 0.07 (sec) , antiderivative size = 132, normalized size of antiderivative = 2.06

$$\int \operatorname{csch}^5(c+dx)\left(a+b\sinh^4(c+dx)\right)\,dx=-\frac{b\operatorname{arctanh}(\cosh(c+dx))}{d}+\frac{3a\operatorname{csch}^2\left(\frac{1}{2}(c+dx)\right)}{32d}-\frac{a\operatorname{csch}^4\left(\frac{1}{2}(c+dx)\right)}{64d}-\frac{3a\log\left(\cosh\left(\frac{1}{2}(c+dx)\right)\right)}{8d}+\frac{3a\log\left(\sinh\left(\frac{1}{2}(c+dx)\right)\right)}{8d}+\frac{3a\operatorname{sech}^2\left(\frac{1}{2}(c+dx)\right)}{32d}+\frac{a\operatorname{sech}^4\left(\frac{1}{2}(c+dx)\right)}{64d}$$

input
```
Integrate[Csch[c + d*x]^5*(a + b*Sinh[c + d*x]^4),x]
```

output
```
-((b*ArcTanh[Cosh[c + d*x]])/d) + (3*a*Csch[(c + d*x)/2]^2)/(32*d) - (a*Cs
ch[(c + d*x)/2]^4)/(64*d) - (3*a*Log[Cosh[(c + d*x)/2]])/(8*d) + (3*a*Log[
Sinh[(c + d*x)/2]])/(8*d) + (3*a*Sech[(c + d*x)/2]^2)/(32*d) + (a*Sech[(c
+ d*x)/2]^4)/(64*d)
```

Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 79, normalized size of antiderivative = 1.23, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {3042, 26, 3694, 1471, 25, 298, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \operatorname{csch}^5(c+dx)\left(a+b\sinh^4(c+dx)\right)\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{i(a+b\sin(ic+idx)^4)}{\sin(ic+idx)^5}dx$$

$$\Big\downarrow \ 26$$

$$i\int \frac{b\sin(ic+idx)^4+a}{\sin(ic+idx)^5}dx$$

$$\Big\downarrow \ 3694$$

$$-\frac{\int \frac{b\cosh^4(c+dx)-2b\cosh^2(c+dx)+a+b}{(1-\cosh^2(c+dx))^3}d\cosh(c+dx)}{d}$$

$$\Big\downarrow \ 1471$$

$$-\frac{\frac{a\cosh(c+dx)}{4(1-\cosh^2(c+dx))^2}-\frac{1}{4}\int -\frac{-4b\cosh^2(c+dx)+3a+4b}{(1-\cosh^2(c+dx))^2}d\cosh(c+dx)}{d}$$

$$\Big\downarrow \ 25$$

$$-\frac{\frac{1}{4}\int \frac{-4b\cosh^2(c+dx)+3a+4b}{(1-\cosh^2(c+dx))^2}d\cosh(c+dx)+\frac{a\cosh(c+dx)}{4(1-\cosh^2(c+dx))^2}}{d}$$

$$\Big\downarrow \ 298$$

$$-\frac{\frac{1}{4}\left(\frac{1}{2}(3a+8b)\int \frac{1}{1-\cosh^2(c+dx)}d\cosh(c+dx)+\frac{3a\cosh(c+dx)}{2(1-\cosh^2(c+dx))}\right)+\frac{a\cosh(c+dx)}{4(1-\cosh^2(c+dx))^2}}{d}$$

$$\Big\downarrow \ 219$$

$$-\frac{\frac{1}{4}\left(\frac{1}{2}(3a+8b)\text{arctanh}(\cosh(c+dx))+\frac{3a\cosh(c+dx)}{2(1-\cosh^2(c+dx))}\right)+\frac{a\cosh(c+dx)}{4(1-\cosh^2(c+dx))^2}}{d}$$

input
```
Int[Csch[c + d*x]^5*(a + b*Sinh[c + d*x]^4),x]
```

output
```
-(((a*Cosh[c + d*x])/(4*(1 - Cosh[c + d*x]^2)^2) + (((3*a + 8*b)*ArcTanh[C
osh[c + d*x]])/2 + (3*a*Cosh[c + d*x])/(2*(1 - Cosh[c + d*x]^2)))/4)/d)
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 298
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2), x_Symbol] :> Simp[(-(
b*c - a*d))*x*((a + b*x^2)^(p + 1)/(2*a*b*(p + 1))), x] - Simp[(a*d - b*c*(
2*p + 3))/(2*a*b*(p + 1))   Int[(a + b*x^2)^(p + 1), x], x] /; FreeQ[{a, b,
 c, d, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/2 + p, 0])
```

rule 1471
```
Int[((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.),
x_Symbol] :> With[{Qx = PolynomialQuotient[(a + b*x^2 + c*x^4)^p, d + e*x^2
, x], R = Coeff[PolynomialRemainder[(a + b*x^2 + c*x^4)^p, d + e*x^2, x], x
, 0]}, Simp[(-R)*x*((d + e*x^2)^(q + 1)/(2*d*(q + 1))), x] + Simp[1/(2*d*(q
 + 1))   Int[(d + e*x^2)^(q + 1)*ExpandToSum[2*d*(q + 1)*Qx + R*(2*q + 3),
x], x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^
2 - b*d*e + a*e^2, 0] && IGtQ[p, 0] && LtQ[q, -1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3694
```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^4)^
(p_.), x_Symbol] :> With[{ff = FreeFactors[Cos[e + f*x], x]}, Simp[-ff/f
Subst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a + b - 2*b*ff^2*x^2 + b*ff^4*x^4)^p,
 x], x, Cos[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m -
 1)/2]
```

Maple [A] (verified)

Time = 0.60 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.84

method	result
derivativedivides	$\frac{a\left(\left(-\frac{\operatorname{csch}(dx+c)^{3}}{4}+\frac{3\operatorname{csch}(dx+c)}{8}\right)\coth(dx+c)-\frac{3\operatorname{arctanh}\left(\mathrm{e}^{dx+c}\right)}{4}\right)-2b\operatorname{arctanh}\left(\mathrm{e}^{dx+c}\right)}{d}$
default	$\frac{a\left(\left(-\frac{\operatorname{csch}(dx+c)^{3}}{4}+\frac{3\operatorname{csch}(dx+c)}{8}\right)\coth(dx+c)-\frac{3\operatorname{arctanh}\left(\mathrm{e}^{dx+c}\right)}{4}\right)-2b\operatorname{arctanh}\left(\mathrm{e}^{dx+c}\right)}{d}$
parallelrisch	$\frac{(24a+64b)\ln\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-a\left(\coth\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}+\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}-8\right)\left(\coth\left(\frac{dx}{2}+\frac{c}{2}\right)-\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)\right)\left(\coth\left(\frac{dx}{2}+\frac{c}{2}\right.\right.}{64d}$
risch	$\frac{\mathrm{e}^{dx+c}a\left(3\,\mathrm{e}^{6dx+6c}-11\,\mathrm{e}^{4dx+4c}-11\,\mathrm{e}^{2dx+2c}+3\right)}{4d\left(\mathrm{e}^{2dx+2c}-1\right)^{4}}+\frac{3a\ln\left(\mathrm{e}^{dx+c}-1\right)}{8d}+\frac{\ln\left(\mathrm{e}^{dx+c}-1\right)b}{d}-\frac{3a\ln\left(\mathrm{e}^{dx+c}+1\right)}{8d}-\frac{\ln(\mathrm{e}^{d}}{}$

input
```
int(csch(d*x+c)^5*(a+b*sinh(d*x+c)^4),x,method=_RETURNVERBOSE)
```

output
```
1/d*(a*((-1/4*csch(d*x+c)^3+3/8*csch(d*x+c))*coth(d*x+c)-3/4*arctanh(exp(d
*x+c)))-2*b*arctanh(exp(d*x+c)))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1476 vs. 2(58) = 116.

Time = 0.10 (sec) , antiderivative size = 1476, normalized size of antiderivative = 23.06

$$\int \operatorname{csch}^5(c+dx)\left(a+b\sinh^4(c+dx)\right)\,dx = \text{Too large to display}$$

input
```
integrate(csch(d*x+c)^5*(a+b*sinh(d*x+c)^4),x, algorithm="fricas")
```

output
```
1/8*(6*a*cosh(d*x + c)^7 + 42*a*cosh(d*x + c)*sinh(d*x + c)^6 + 6*a*sinh(d
*x + c)^7 - 22*a*cosh(d*x + c)^5 + 2*(63*a*cosh(d*x + c)^2 - 11*a)*sinh(d*
x + c)^5 + 10*(21*a*cosh(d*x + c)^3 - 11*a*cosh(d*x + c))*sinh(d*x + c)^4
- 22*a*cosh(d*x + c)^3 + 2*(105*a*cosh(d*x + c)^4 - 110*a*cosh(d*x + c)^2
- 11*a)*sinh(d*x + c)^3 + 2*(63*a*cosh(d*x + c)^5 - 110*a*cosh(d*x + c)^3
- 33*a*cosh(d*x + c))*sinh(d*x + c)^2 + 6*a*cosh(d*x + c) - ((3*a + 8*b)*c
osh(d*x + c)^8 + 8*(3*a + 8*b)*cosh(d*x + c)*sinh(d*x + c)^7 + (3*a + 8*b)
*sinh(d*x + c)^8 - 4*(3*a + 8*b)*cosh(d*x + c)^6 + 4*(7*(3*a + 8*b)*cosh(d
*x + c)^2 - 3*a - 8*b)*sinh(d*x + c)^6 + 8*(7*(3*a + 8*b)*cosh(d*x + c)^3
- 3*(3*a + 8*b)*cosh(d*x + c))*sinh(d*x + c)^5 + 6*(3*a + 8*b)*cosh(d*x +
c)^4 + 2*(35*(3*a + 8*b)*cosh(d*x + c)^4 - 30*(3*a + 8*b)*cosh(d*x + c)^2
+ 9*a + 24*b)*sinh(d*x + c)^4 + 8*(7*(3*a + 8*b)*cosh(d*x + c)^5 - 10*(3*a
 + 8*b)*cosh(d*x + c)^3 + 3*(3*a + 8*b)*cosh(d*x + c))*sinh(d*x + c)^3 - 4
*(3*a + 8*b)*cosh(d*x + c)^2 + 4*(7*(3*a + 8*b)*cosh(d*x + c)^6 - 15*(3*a
+ 8*b)*cosh(d*x + c)^4 + 9*(3*a + 8*b)*cosh(d*x + c)^2 - 3*a - 8*b)*sinh(d
*x + c)^2 + 8*((3*a + 8*b)*cosh(d*x + c)^7 - 3*(3*a + 8*b)*cosh(d*x + c)^5
 + 3*(3*a + 8*b)*cosh(d*x + c)^3 - (3*a + 8*b)*cosh(d*x + c))*sinh(d*x + c
) + 3*a + 8*b)*log(cosh(d*x + c) + sinh(d*x + c) + 1) + ((3*a + 8*b)*cosh(
d*x + c)^8 + 8*(3*a + 8*b)*cosh(d*x + c)*sinh(d*x + c)^7 + (3*a + 8*b)*sin
h(d*x + c)^8 - 4*(3*a + 8*b)*cosh(d*x + c)^6 + 4*(7*(3*a + 8*b)*cosh(d*...
```

Sympy [F(-1)]

Timed out.

$$\int \operatorname{csch}^5(c+dx) \left(a+b\sinh^4(c+dx)\right) \, dx = \text{Timed out}$$

input
```
integrate(csch(d*x+c)**5*(a+b*sinh(d*x+c)**4),x)
```

output
```
Timed out
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 174 vs. $2(58) = 116$.

Time = 0.04 (sec) , antiderivative size = 174, normalized size of antiderivative = 2.72

$$\int \operatorname{csch}^5(c+dx)\left(a+b\sinh^4(c+dx)\right)\,dx = -\frac{1}{8}a\left(\frac{3\log\left(e^{(-dx-c)}+1\right)}{d}-\frac{3\log\left(e^{(-dx-c)}-1\right)}{d}+\frac{2\left(3e^{(-dx-c)}-11e^{(-3dx-3c)}-11e^{(-5dx-5c)}+3e\right.}{d(4e^{(-2dx-2c)}-6e^{(-4dx-4c)}+4e^{(-6dx-6c)}-e^{(-8}}\right. - b\left(\frac{\log\left(e^{(-dx-c)}+1\right)}{d}-\frac{\log\left(e^{(-dx-c)}-1\right)}{d}\right)$$

input
```
integrate(csch(d*x+c)^5*(a+b*sinh(d*x+c)^4),x, algorithm="maxima")
```

output
```
-1/8*a*(3*log(e^(-d*x - c) + 1)/d - 3*log(e^(-d*x - c) - 1)/d + 2*(3*e^(-d
*x - c) - 11*e^(-3*d*x - 3*c) - 11*e^(-5*d*x - 5*c) + 3*e^(-7*d*x - 7*c))/
(d*(4*e^(-2*d*x - 2*c) - 6*e^(-4*d*x - 4*c) + 4*e^(-6*d*x - 6*c) - e^(-8*d
*x - 8*c) - 1))) - b*(log(e^(-d*x - c) + 1)/d - log(e^(-d*x - c) - 1)/d)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 124 vs. $2(58) = 116$.

Time = 0.14 (sec) , antiderivative size = 124, normalized size of antiderivative = 1.94

$$\int \operatorname{csch}^5(c+dx)\left(a+b\sinh^4(c+dx)\right)\,dx = -\frac{(3a+8b)\log\left(e^{(dx+c)}+e^{(-dx-c)}+2\right)-(3a+8b)\log\left(e^{(dx+c)}+e^{(-dx-c)}-2\right)-\frac{4\left(3a\left(e^{(dx+c)}+e^{(-dx-c)}\right)^3\right.}{\left(\left(e^{(dx+c)}+e^{(-\right.\right.}}}{16d}$$

input
```
integrate(csch(d*x+c)^5*(a+b*sinh(d*x+c)^4),x, algorithm="giac")
```

output
```
-1/16*((3*a + 8*b)*log(e^(d*x + c) + e^(-d*x - c) + 2) - (3*a + 8*b)*log(e
^(d*x + c) + e^(-d*x - c) - 2) - 4*(3*a*(e^(d*x + c) + e^(-d*x - c))^3 - 2
0*a*(e^(d*x + c) + e^(-d*x - c)))/((e^(d*x + c) + e^(-d*x - c))^2 - 4)^2)/
d
```

Mupad [B] (verification not implemented)

Time = 1.67 (sec) , antiderivative size = 242, normalized size of antiderivative = 3.78

$$\int \operatorname{csch}^5(c+dx)\left(a+b\sinh^4(c+dx)\right)\,dx = \frac{3\,a\,e^{c+d\,x}}{4\,d\,\left(e^{2\,c+2\,d\,x}-1\right)}-\frac{\operatorname{atan}\left(\frac{e^{d\,x}\,e^c\,\left(3\,a\,\sqrt{-d^2}+8\,b\,\sqrt{-d^2}\right)}{d\,\sqrt{9\,a^2+48\,a\,b+64\,b^2}}\right)\,\sqrt{9\,a^2+48\,a\,b+64\,b^2}}{4\,\sqrt{-d^2}} - \frac{a\,e^{c+d\,x}}{2\,d\,\left(e^{4\,c+4\,d\,x}-2\,e^{2\,c+2\,d\,x}+1\right)}-\frac{6\,a\,e^{c+d\,x}}{d\,\left(3\,e^{2\,c+2\,d\,x}-3\,e^{4\,c+4\,d\,x}+e^{6\,c+6\,d\,x}-1\right)} - \frac{4\,a\,e^{c+d\,x}}{d\,\left(6\,e^{4\,c+4\,d\,x}-4\,e^{2\,c+2\,d\,x}-4\,e^{6\,c+6\,d\,x}+e^{8\,c+8\,d\,x}+1\right)}$$

input
```
int((a + b*sinh(c + d*x)^4)/sinh(c + d*x)^5,x)
```

output
```
(3*a*exp(c + d*x))/(4*d*(exp(2*c + 2*d*x) - 1)) - (atan((exp(d*x)*exp(c)*(
3*a*(-d^2)^(1/2) + 8*b*(-d^2)^(1/2)))/(d*(48*a*b + 9*a^2 + 64*b^2)^(1/2)))
*(48*a*b + 9*a^2 + 64*b^2)^(1/2))/(4*(-d^2)^(1/2)) - (a*exp(c + d*x))/(2*d
*(exp(4*c + 4*d*x) - 2*exp(2*c + 2*d*x) + 1)) - (6*a*exp(c + d*x))/(d*(3*e
xp(2*c + 2*d*x) - 3*exp(4*c + 4*d*x) + exp(6*c + 6*d*x) - 1)) - (4*a*exp(c
 + d*x))/(d*(6*exp(4*c + 4*d*x) - 4*exp(2*c + 2*d*x) - 4*exp(6*c + 6*d*x)
+ exp(8*c + 8*d*x) + 1))
```

Reduce [B] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 525, normalized size of antiderivative = 8.20

$$\int \operatorname{csch}^5(c+dx)\left(a+b\sinh^4(c+dx)\right)\,dx$$
$$= \frac{3e^{8dx+8c}\log\left(e^{dx+c}-1\right)a+8e^{8dx+8c}\log\left(e^{dx+c}-1\right)b-3e^{8dx+8c}\log\left(e^{dx+c}+1\right)a-8e^{8dx+8c}\log\left(e^{dx+c}+1\right)}{}$$

input
```
int(csch(d*x+c)^5*(a+b*sinh(d*x+c)^4),x)
```

output
```
(3*e**(8*c + 8*d*x)*log(e**(c + d*x) - 1)*a + 8*e**(8*c + 8*d*x)*log(e**(c
 + d*x) - 1)*b - 3*e**(8*c + 8*d*x)*log(e**(c + d*x) + 1)*a - 8*e**(8*c +
8*d*x)*log(e**(c + d*x) + 1)*b + 6*e**(7*c + 7*d*x)*a - 12*e**(6*c + 6*d*x
)*log(e**(c + d*x) - 1)*a - 32*e**(6*c + 6*d*x)*log(e**(c + d*x) - 1)*b +
12*e**(6*c + 6*d*x)*log(e**(c + d*x) + 1)*a + 32*e**(6*c + 6*d*x)*log(e**(
c + d*x) + 1)*b - 22*e**(5*c + 5*d*x)*a + 18*e**(4*c + 4*d*x)*log(e**(c +
d*x) - 1)*a + 48*e**(4*c + 4*d*x)*log(e**(c + d*x) - 1)*b - 18*e**(4*c + 4
*d*x)*log(e**(c + d*x) + 1)*a - 48*e**(4*c + 4*d*x)*log(e**(c + d*x) + 1)*
b - 22*e**(3*c + 3*d*x)*a - 12*e**(2*c + 2*d*x)*log(e**(c + d*x) - 1)*a -
32*e**(2*c + 2*d*x)*log(e**(c + d*x) - 1)*b + 12*e**(2*c + 2*d*x)*log(e**(
c + d*x) + 1)*a + 32*e**(2*c + 2*d*x)*log(e**(c + d*x) + 1)*b + 6*e**(c +
d*x)*a + 3*log(e**(c + d*x) - 1)*a + 8*log(e**(c + d*x) - 1)*b - 3*log(e**
(c + d*x) + 1)*a - 8*log(e**(c + d*x) + 1)*b)/(8*d*(e**(8*c + 8*d*x) - 4*e
**(6*c + 6*d*x) + 6*e**(4*c + 4*d*x) - 4*e**(2*c + 2*d*x) + 1))
```

3.169 $\int \text{csch}^6(c+dx)\left(a+b\sinh^4(c+dx)\right)\,dx$

Optimal result

Integrand size = 21, antiderivative size = 47

$$\int \text{csch}^6(c+dx)\left(a+b\sinh^4(c+dx)\right)\,dx=-\frac{(a+b)\coth(c+dx)}{d}+\frac{2a\coth^3(c+dx)}{3d}-\frac{a\coth^5(c+dx)}{5d}$$

output
```
-(a+b)*coth(d*x+c)/d+2/3*a*coth(d*x+c)^3/d-1/5*a*coth(d*x+c)^5/d
```

Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 71, normalized size of antiderivative = 1.51

$$\int \text{csch}^6(c+dx)\left(a+b\sinh^4(c+dx)\right)\,dx=-\frac{8a\coth(c+dx)}{15d}-\frac{b\coth(c+dx)}{d}+\frac{4a\coth(c+dx)\text{csch}^2(c+dx)}{15d}-\frac{a\coth(c+dx)\text{csch}^4(c+dx)}{5d}$$

input
```
Integrate[Csch[c + d*x]^6*(a + b*Sinh[c + d*x]^4),x]
```

output
```
(-8*a*Coth[c + d*x])/(15*d) - (b*Coth[c + d*x])/d + (4*a*Coth[c + d*x]*Csc
h[c + d*x]^2)/(15*d) - (a*Coth[c + d*x]*Csch[c + d*x]^4)/(5*d)
```

Rubi [A] (verified)

Time = 0.31 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.89, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.238$, Rules used = {3042, 25, 3696, 1433, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \operatorname{csch}^6(c+dx) \left(a+b\sinh^4(c+dx)\right) dx$$

$$\Big\downarrow \text{3042}$$

$$\int -\frac{a+b\sin(ic+idx)^4}{\sin(ic+idx)^6}dx$$

$$\Big\downarrow \text{25}$$

$$-\int \frac{b\sin(ic+idx)^4+a}{\sin(ic+idx)^6}dx$$

$$\Big\downarrow \text{3696}$$

$$\frac{\int \coth^6(c+dx)\left((a+b)\tanh^4(c+dx)-2a\tanh^2(c+dx)+a\right)d\tanh(c+dx)}{d}$$

$$\Big\downarrow \text{1433}$$

$$\frac{\int \left(a\coth^6(c+dx)-2a\coth^4(c+dx)+(a+b)\coth^2(c+dx)\right)d\tanh(c+dx)}{d}$$

$$\Big\downarrow \text{2009}$$

$$\frac{-(a+b)\coth(c+dx)-\frac{1}{5}a\coth^5(c+dx)+\frac{2}{3}a\coth^3(c+dx)}{d}$$

input
```
Int[Csch[c + d*x]^6*(a + b*Sinh[c + d*x]^4),x]
```

output
```
(-((a + b)*Coth[c + d*x]) + (2*a*Coth[c + d*x]^3)/3 - (a*Coth[c + d*x]^5)/
5)/d
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 1433
```
Int[((d_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol]
 :> Int[ExpandIntegrand[(d*x)^m*(a + b*x^2 + c*x^4)^p, x], x] /; FreeQ[{a,
b, c, d, m}, x] && IGtQ[p, 0] && (EqQ[p, 1] ||  !IntegerQ[(m + 1)/2])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3696
```
Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^4)^(
p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Simp[ff^(m + 1
)/f   Subst[Int[x^m*((a + 2*a*ff^2*x^2 + (a + b)*ff^4*x^4)^p/(1 + ff^2*x^2)
^(m/2 + 2*p + 1)), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f}, x] &
& IntegerQ[m/2] && IntegerQ[p]
```

Maple [A] (verified)

Time = 0.70 (sec) , antiderivative size = 45, normalized size of antiderivative = 0.96

method	result
derivativedivides	$\frac{a\left(-\frac{8}{15}-\frac{\operatorname{csch}(dx+c)^4}{5}+\frac{4\operatorname{csch}(dx+c)^2}{15}\right)\coth(dx+c)-b\coth(dx+c)}{d}$
default	$\frac{a\left(-\frac{8}{15}-\frac{\operatorname{csch}(dx+c)^4}{5}+\frac{4\operatorname{csch}(dx+c)^2}{15}\right)\coth(dx+c)-b\coth(dx+c)}{d}$
parallelrisch	$\frac{-3\coth\left(\frac{dx}{2}+\frac{c}{2}\right)^5a+25\coth\left(\frac{dx}{2}+\frac{c}{2}\right)^3a+(-150a-240b)\coth\left(\frac{dx}{2}+\frac{c}{2}\right)-3\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^4a-\frac{25\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^2a}{3}+50a\right.}{480d}$
risch	$-\frac{2\left(15\,e^{8dx+8c}b-60\,e^{6dx+6c}b+80\,e^{4dx+4c}a+90b\,e^{4dx+4c}-40\,e^{2dx+2c}a-60\,e^{2dx+2c}b+8a+15b\right)}{15d\left(e^{2dx+2c}-1\right)^5}$

input
```
int(csch(d*x+c)^6*(a+b*sinh(d*x+c)^4),x,method=_RETURNVERBOSE)
```

output
```
1/d*(a*(-8/15-1/5*csch(d*x+c)^4+4/15*csch(d*x+c)^2)*coth(d*x+c)-b*coth(d*x
+c))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 333 vs. $2(43) = 86$.

Time = 0.08 (sec) , antiderivative size = 333, normalized size of antiderivative = 7.09

$$\int \operatorname{csch}^6(c+dx)\left(a+b\sinh^4(c+dx)\right)\,dx =$$

$$-\frac{4\left((4\,a+15\,b)\cosh\left(dx+c\right)^4-16\,a\cosh\left(dx+\right.\right.}{15\left(d\cosh\left(dx+c\right)^6+6\,d\cosh\left(dx+c\right)\sinh\left(dx+c\right)^5+d\sinh\left(dx+c\right)^6-6\,d\cosh\left(dx+c\right)^4+3\left(5\,d\right.\right.}$$

input
```
integrate(csch(d*x+c)^6*(a+b*sinh(d*x+c)^4),x, algorithm="fricas")
```

output
```
-4/15*((4*a + 15*b)*cosh(d*x + c)^4 - 16*a*cosh(d*x + c)*sinh(d*x + c)^3 +
 (4*a + 15*b)*sinh(d*x + c)^4 - 20*(a + 3*b)*cosh(d*x + c)^2 + 2*(3*(4*a +
 15*b)*cosh(d*x + c)^2 - 10*a - 30*b)*sinh(d*x + c)^2 - 8*(2*a*cosh(d*x +
c)^3 - 5*a*cosh(d*x + c))*sinh(d*x + c) + 40*a + 45*b)/(d*cosh(d*x + c)^6
+ 6*d*cosh(d*x + c)*sinh(d*x + c)^5 + d*sinh(d*x + c)^6 - 6*d*cosh(d*x + c
)^4 + 3*(5*d*cosh(d*x + c)^2 - 2*d)*sinh(d*x + c)^4 + 4*(5*d*cosh(d*x + c)
^3 - 4*d*cosh(d*x + c))*sinh(d*x + c)^3 + 15*d*cosh(d*x + c)^2 + 3*(5*d*co
sh(d*x + c)^4 - 12*d*cosh(d*x + c)^2 + 5*d)*sinh(d*x + c)^2 + 2*(3*d*cosh(
d*x + c)^5 - 8*d*cosh(d*x + c)^3 + 5*d*cosh(d*x + c))*sinh(d*x + c) - 10*d
)
```

Sympy [F(-1)]

Timed out.

$$\int \operatorname{csch}^6(c+dx)\left(a+b\sinh^4(c+dx)\right)\,dx = \text{Timed out}$$

input
```
integrate(csch(d*x+c)**6*(a+b*sinh(d*x+c)**4),x)
```

output
```
Timed out
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 228 vs. $2(43) = 86$.

Time = 0.04 (sec) , antiderivative size = 228, normalized size of antiderivative = 4.85

$$\int \operatorname{csch}^6(c+dx)\left(a+b\sinh^4(c+dx)\right)\,dx =$$
$$-\frac{16}{15}a\left(\frac{5e^{(-2dx-2c)}}{d(5e^{(-2dx-2c)}-10e^{(-4dx-4c)}+10e^{(-6dx-6c)}-5e^{(-8dx-8c)}+e^{(-10dx-10c)}-1)}-\frac{}{d(5e^{(-2dx-2}}\right.$$
$$+\frac{2b}{d(e^{(-2dx-2c)}-1)}$$

input
```
integrate(csch(d*x+c)^6*(a+b*sinh(d*x+c)^4),x, algorithm="maxima")
```

output
```
-16/15*a*(5*e^(-2*d*x - 2*c)/(d*(5*e^(-2*d*x - 2*c) - 10*e^(-4*d*x - 4*c)
+ 10*e^(-6*d*x - 6*c) - 5*e^(-8*d*x - 8*c) + e^(-10*d*x - 10*c) - 1)) - 10
*e^(-4*d*x - 4*c)/(d*(5*e^(-2*d*x - 2*c) - 10*e^(-4*d*x - 4*c) + 10*e^(-6*
d*x - 6*c) - 5*e^(-8*d*x - 8*c) + e^(-10*d*x - 10*c) - 1)) - 1/(d*(5*e^(-2
*d*x - 2*c) - 10*e^(-4*d*x - 4*c) + 10*e^(-6*d*x - 6*c) - 5*e^(-8*d*x - 8*
c) + e^(-10*d*x - 10*c) - 1))) + 2*b/(d*(e^(-2*d*x - 2*c) - 1))
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 97 vs. $2(43) = 86$.

Time = 0.14 (sec) , antiderivative size = 97, normalized size of antiderivative = 2.06

$$\int \operatorname{csch}^6(c+dx)\left(a+b\sinh^4(c+dx)\right)\,dx = -\frac{2\left(15\,be^{(8\,dx+8\,c)}-60\,be^{(6\,dx+6\,c)}+80\,ae^{(4\,dx+4\,c)}+90\,be^{(4\,dx+4\,c)}-40\,ae^{(2\,dx+2\,c)}-60\,be^{(2\,dx+2\,c)}+8\,a+\right.}{15\,d\left(e^{(2\,dx+2\,c)}-1\right)^5}$$

input
```
integrate(csch(d*x+c)^6*(a+b*sinh(d*x+c)^4),x, algorithm="giac")
```

output
```
-2/15*(15*b*e^(8*d*x + 8*c) - 60*b*e^(6*d*x + 6*c) + 80*a*e^(4*d*x + 4*c)
+ 90*b*e^(4*d*x + 4*c) - 40*a*e^(2*d*x + 2*c) - 60*b*e^(2*d*x + 2*c) + 8*a
 + 15*b)/(d*(e^(2*d*x + 2*c) - 1)^5)
```

Mupad [B] (verification not implemented)

Time = 1.60 (sec) , antiderivative size = 337, normalized size of antiderivative = 7.17

$$\begin{aligned}
&\int \operatorname{csch}^6(c+dx)\left(a+b\sinh^4(c+dx)\right)\,dx \\
&= \frac{\frac{2\,b}{5\,d}+\frac{6\,b\,\mathrm{e}^{4\,c+4\,d\,x}}{5\,d}-\frac{2\,b\,\mathrm{e}^{6\,c+6\,d\,x}}{5\,d}-\frac{2\,\mathrm{e}^{2\,c+2\,d\,x}\,(8\,a+3\,b)}{5\,d}}{6\,\mathrm{e}^{4\,c+4\,d\,x}-4\,\mathrm{e}^{2\,c+2\,d\,x}-4\,\mathrm{e}^{6\,c+6\,d\,x}+\mathrm{e}^{8\,c+8\,d\,x}+1} \\
&\quad -\frac{\frac{2\,(8\,a+3\,b)}{15\,d}-\frac{4\,b\,\mathrm{e}^{2\,c+2\,d\,x}}{5\,d}+\frac{2\,b\,\mathrm{e}^{4\,c+4\,d\,x}}{5\,d}}{3\,\mathrm{e}^{2\,c+2\,d\,x}-3\,\mathrm{e}^{4\,c+4\,d\,x}+\mathrm{e}^{6\,c+6\,d\,x}-1} \\
&\quad -\frac{\frac{2\,b}{5\,d}-\frac{8\,b\,\mathrm{e}^{2\,c+2\,d\,x}}{5\,d}-\frac{8\,b\,\mathrm{e}^{6\,c+6\,d\,x}}{5\,d}+\frac{2\,b\,\mathrm{e}^{8\,c+8\,d\,x}}{5\,d}+\frac{4\,\mathrm{e}^{4\,c+4\,d\,x}\,(8\,a+3\,b)}{5\,d}}{5\,\mathrm{e}^{2\,c+2\,d\,x}-10\,\mathrm{e}^{4\,c+4\,d\,x}+10\,\mathrm{e}^{6\,c+6\,d\,x}-5\,\mathrm{e}^{8\,c+8\,d\,x}+\mathrm{e}^{10\,c+10\,d\,x}-1} \\
&\quad -\frac{4\,b}{5\,d\,\left(\mathrm{e}^{2\,c+2\,d\,x}-1\right)}
\end{aligned}$$

input
```
int((a + b*sinh(c + d*x)^4)/sinh(c + d*x)^6,x)
```

output
```
((2*b)/(5*d) + (6*b*exp(4*c + 4*d*x))/(5*d) - (2*b*exp(6*c + 6*d*x))/(5*d)
 - (2*exp(2*c + 2*d*x)*(8*a + 3*b))/(5*d))/(6*exp(4*c + 4*d*x) - 4*exp(2*c
 + 2*d*x) - 4*exp(6*c + 6*d*x) + exp(8*c + 8*d*x) + 1) - ((2*(8*a + 3*b))/
(15*d) - (4*b*exp(2*c + 2*d*x))/(5*d) + (2*b*exp(4*c + 4*d*x))/(5*d))/(3*e
xp(2*c + 2*d*x) - 3*exp(4*c + 4*d*x) + exp(6*c + 6*d*x) - 1) - ((2*b)/(5*d
) - (8*b*exp(2*c + 2*d*x))/(5*d) - (8*b*exp(6*c + 6*d*x))/(5*d) + (2*b*exp
(8*c + 8*d*x))/(5*d) + (4*exp(4*c + 4*d*x)*(8*a + 3*b))/(5*d))/(5*exp(2*c
+ 2*d*x) - 10*exp(4*c + 4*d*x) + 10*exp(6*c + 6*d*x) - 5*exp(8*c + 8*d*x)
+ exp(10*c + 10*d*x) - 1) - (4*b)/(5*d*(exp(2*c + 2*d*x) - 1))
```

Reduce [B] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 152, normalized size of antiderivative = 3.23

$$\int \operatorname{csch}^6(c+dx) \left(a+b\sinh^4(c+dx)\right) dx = \frac{-\frac{2e^{10dx+10c}b}{5}+4e^{6dx+6c}b-\frac{32e^{4dx+4c}a}{3}-8e^{4dx+4c}b+\frac{16e^{2dx+2c}a}{3}+6e^{2dx+2c}b-\frac{16a}{15}-\frac{8b}{5}}{d\left(e^{10dx+10c}-5e^{8dx+8c}+10e^{6dx+6c}-10e^{4dx+4c}+5e^{2dx+2c}-1\right)}$$

input
```
int(csch(d*x+c)^6*(a+b*sinh(d*x+c)^4),x)
```

output
```
(2*( - 3*e**(10*c + 10*d*x)*b + 30*e**(6*c + 6*d*x)*b - 80*e**(4*c + 4*d*x
)*a - 60*e**(4*c + 4*d*x)*b + 40*e**(2*c + 2*d*x)*a + 45*e**(2*c + 2*d*x)*
b - 8*a - 12*b))/(15*d*(e**(10*c + 10*d*x) - 5*e**(8*c + 8*d*x) + 10*e**(6
*c + 6*d*x) - 10*e**(4*c + 4*d*x) + 5*e**(2*c + 2*d*x) - 1))
```

3.170 $\int \operatorname{csch}^7(c+dx)\left(a+b\sinh^4(c+dx)\right)\,dx$

Optimal result

Integrand size = 21, antiderivative size = 92

$$\int \operatorname{csch}^7(c+dx)\left(a+b\sinh^4(c+dx)\right)\,dx=\frac{(5a+8b)\operatorname{arctanh}(\cosh(c+dx))}{16d}-\frac{(5a+8b)\coth(c+dx)\operatorname{csch}(c+dx)}{16d}+\frac{5a\coth(c+dx)\operatorname{csch}^3(c+dx)}{24d}-\frac{a\coth(c+dx)\operatorname{csch}^5(c+dx)}{6d}$$

output
```
1/16*(5*a+8*b)*arctanh(cosh(d*x+c))/d-1/16*(5*a+8*b)*coth(d*x+c)*csch(d*x+
c)/d+5/24*a*coth(d*x+c)*csch(d*x+c)^3/d-1/6*a*coth(d*x+c)*csch(d*x+c)^5/d
```

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 237 vs. 2(92) = 184.

Time = 0.12 (sec) , antiderivative size = 237, normalized size of antiderivative = 2.58

$$\int \operatorname{csch}^7(c+dx)\left(a+b\sinh^4(c+dx)\right)\,dx = -\frac{5a\operatorname{csch}^2\left(\frac{1}{2}(c+dx)\right)}{64d} - \frac{b\operatorname{csch}^2\left(\frac{1}{2}(c+dx)\right)}{8d} + \frac{a\operatorname{csch}^4\left(\frac{1}{2}(c+dx)\right)}{64d} - \frac{a\operatorname{csch}^6\left(\frac{1}{2}(c+dx)\right)}{384d} + \frac{5a\log\left(\cosh\left(\frac{1}{2}(c+dx)\right)\right)}{16d} + \frac{b\log\left(\cosh\left(\frac{1}{2}(c+dx)\right)\right)}{2d} - \frac{5a\log\left(\sinh\left(\frac{1}{2}(c+dx)\right)\right)}{16d} - \frac{b\log\left(\sinh\left(\frac{1}{2}(c+dx)\right)\right)}{2d} - \frac{5a\operatorname{sech}^2\left(\frac{1}{2}(c+dx)\right)}{64d} - \frac{b\operatorname{sech}^2\left(\frac{1}{2}(c+dx)\right)}{8d} - \frac{a\operatorname{sech}^4\left(\frac{1}{2}(c+dx)\right)}{64d} - \frac{a\operatorname{sech}^6\left(\frac{1}{2}(c+dx)\right)}{384d}$$

input
```
Integrate[Csch[c + d*x]^7*(a + b*Sinh[c + d*x]^4),x]
```

output
```
(-5*a*Csch[(c + d*x)/2]^2)/(64*d) - (b*Csch[(c + d*x)/2]^2)/(8*d) + (a*Csc
h[(c + d*x)/2]^4)/(64*d) - (a*Csch[(c + d*x)/2]^6)/(384*d) + (5*a*Log[Cosh
[(c + d*x)/2]])/(16*d) + (b*Log[Cosh[(c + d*x)/2]])/(2*d) - (5*a*Log[Sinh[
(c + d*x)/2]])/(16*d) - (b*Log[Sinh[(c + d*x)/2]])/(2*d) - (5*a*Sech[(c +
d*x)/2]^2)/(64*d) - (b*Sech[(c + d*x)/2]^2)/(8*d) - (a*Sech[(c + d*x)/2]^4
)/(64*d) - (a*Sech[(c + d*x)/2]^6)/(384*d)
```

Rubi [A] (verified)

Time = 0.31 (sec) , antiderivative size = 107, normalized size of antiderivative = 1.16, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.381$, Rules used = {3042, 26, 3694, 1471, 25, 298, 215, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \text{csch}^7(c+dx)\left(a+b\sinh^4(c+dx)\right)\,dx$$

$$\Big\downarrow \; 3042$$

$$\int -\frac{i\left(a+b\sin(ic+idx)^4\right)}{\sin(ic+idx)^7}dx$$

$$\Big\downarrow \; 26$$

$$-i\int \frac{b\sin(ic+idx)^4+a}{\sin(ic+idx)^7}dx$$

$$\Big\downarrow \; 3694$$

$$\frac{\int \frac{b\cosh^4(c+dx)-2b\cosh^2(c+dx)+a+b}{\left(1-\cosh^2(c+dx)\right)^4}d\cosh(c+dx)}{d}$$

$$\Big\downarrow \; 1471$$

$$\frac{\frac{a\cosh(c+dx)}{6\left(1-\cosh^2(c+dx)\right)^3}-\frac{1}{6}\int -\frac{-6b\cosh^2(c+dx)+5a+6b}{\left(1-\cosh^2(c+dx)\right)^3}d\cosh(c+dx)}{d}$$

$$\Big\downarrow \; 25$$

$$\frac{\frac{1}{6}\int \frac{-6b\cosh^2(c+dx)+5a+6b}{\left(1-\cosh^2(c+dx)\right)^3}d\cosh(c+dx)+\frac{a\cosh(c+dx)}{6\left(1-\cosh^2(c+dx)\right)^3}}{d}$$

$$\Big\downarrow \; 298$$

$$\frac{\frac{1}{6}\left(\frac{3}{4}(5a+8b)\int \frac{1}{\left(1-\cosh^2(c+dx)\right)^2}d\cosh(c+dx)+\frac{5a\cosh(c+dx)}{4\left(1-\cosh^2(c+dx)\right)^2}\right)+\frac{a\cosh(c+dx)}{6\left(1-\cosh^2(c+dx)\right)^3}}{d}$$

$$\Big\downarrow \; 215$$

$$\frac{\frac{1}{6}\left(\frac{3}{4}(5a+8b)\left(\frac{1}{2}\int \frac{1}{1-\cosh^2(c+dx)}d\cosh(c+dx)+\frac{\cosh(c+dx)}{2\left(1-\cosh^2(c+dx)\right)}\right)+\frac{5a\cosh(c+dx)}{4\left(1-\cosh^2(c+dx)\right)^2}\right)+\frac{a\cosh(c+dx)}{6\left(1-\cosh^2(c+dx)\right)^3}}{d}$$

$$\Big\downarrow \; 219$$

$$\frac{\frac{1}{6}\left(\frac{3}{4}(5a+8b)\left(\frac{1}{2}\text{arctanh}(\cosh(c+dx))+\frac{\cosh(c+dx)}{2\left(1-\cosh^2(c+dx)\right)}\right)+\frac{5a\cosh(c+dx)}{4\left(1-\cosh^2(c+dx)\right)^2}\right)+\frac{a\cosh(c+dx)}{6\left(1-\cosh^2(c+dx)\right)^3}}{d}$$

input
```
Int[Csch[c + d*x]^7*(a + b*Sinh[c + d*x]^4),x]
```

output
```
((a*Cosh[c + d*x])/(6*(1 - Cosh[c + d*x]^2)^3) + ((5*a*Cosh[c + d*x])/(4*(
1 - Cosh[c + d*x]^2)^2) + (3*(5*a + 8*b)*(ArcTanh[Cosh[c + d*x]]/2 + Cosh[
c + d*x]/(2*(1 - Cosh[c + d*x]^2))))/4)/6)/d
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 215
```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(-x)*((a + b*x^2)^(p + 1)
/(2*a*(p + 1))), x] + Simp[(2*p + 3)/(2*a*(p + 1))   Int[(a + b*x^2)^(p + 1
), x], x] /; FreeQ[{a, b}, x] && LtQ[p, -1] && (IntegerQ[4*p] || IntegerQ[6
*p])
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 298
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2), x_Symbol] :> Simp[(-(
b*c - a*d))*x*((a + b*x^2)^(p + 1)/(2*a*b*(p + 1))), x] - Simp[(a*d - b*c*(
2*p + 3))/(2*a*b*(p + 1))   Int[(a + b*x^2)^(p + 1), x], x] /; FreeQ[{a, b,
 c, d, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/2 + p, 0])
```

rule 1471
```
Int[((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.),
x_Symbol] :> With[{Qx = PolynomialQuotient[(a + b*x^2 + c*x^4)^p, d + e*x^2
, x], R = Coeff[PolynomialRemainder[(a + b*x^2 + c*x^4)^p, d + e*x^2, x], x
, 0]}, Simp[(-R)*x*((d + e*x^2)^(q + 1)/(2*d*(q + 1))), x] + Simp[1/(2*d*(q
 + 1))   Int[(d + e*x^2)^(q + 1)*ExpandToSum[2*d*(q + 1)*Qx + R*(2*q + 3),
x], x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^
2 - b*d*e + a*e^2, 0] && IGtQ[p, 0] && LtQ[q, -1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3694
```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^4)^
(p_.), x_Symbol] :> With[{ff = FreeFactors[Cos[e + f*x], x]}, Simp[-ff/f
Subst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a + b - 2*b*ff^2*x^2 + b*ff^4*x^4)^p,
 x], x, Cos[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m -
 1)/2]
```

Maple [A] (verified)

Time = 0.85 (sec) , antiderivative size = 78, normalized size of antiderivative = 0.85

method	result
derivativedivides	$\frac{a\left(\left(-\frac{\operatorname{csch}(dx+c)^5}{6}+\frac{5\operatorname{csch}(dx+c)^3}{24}-\frac{5\operatorname{csch}(dx+c)}{16}\right)\coth(dx+c)+\frac{5\operatorname{arctanh}\left(e^{dx+c}\right)}{8}\right)+b\left(-\frac{\operatorname{csch}(dx+c)\coth(dx+c)}{2}+\operatorname{arcta}\right.}{d}$
default	$\frac{a\left(\left(-\frac{\operatorname{csch}(dx+c)^5}{6}+\frac{5\operatorname{csch}(dx+c)^3}{24}-\frac{5\operatorname{csch}(dx+c)}{16}\right)\coth(dx+c)+\frac{5\operatorname{arctanh}\left(e^{dx+c}\right)}{8}\right)+b\left(-\frac{\operatorname{csch}(dx+c)\coth(dx+c)}{2}+\operatorname{arcta}\right.}{d}$
parallelrisch	$\frac{24(-5a-8b)\ln\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-\left(\coth\left(\frac{dx}{2}+\frac{c}{2}\right)^4a+a\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^2-9\right)\coth\left(\frac{dx}{2}+\frac{c}{2}\right)^2+\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^4a-9\tanh\right.}{384d}$
risch	$-\frac{e^{dx+c}\left(15e^{10dx+10c}a+24e^{10dx+10c}b-85e^{8dx+8c}a-72e^{8dx+8c}b+198e^{6dx+6c}a+48e^{6dx+6c}b+198e^{4dx+4c}a+48be^{4}\right.}{24d\left(e^{2dx+2c}-1\right)^6}$

input
```
int(csch(d*x+c)^7*(a+b*sinh(d*x+c)^4),x,method=_RETURNVERBOSE)
```

output
```
1/d*(a*((-1/6*csch(d*x+c)^5+5/24*csch(d*x+c)^3-5/16*csch(d*x+c))*coth(d*x+
c)+5/8*arctanh(exp(d*x+c)))+b*(-1/2*csch(d*x+c)*coth(d*x+c)+arctanh(exp(d*
x+c))))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 3115 vs. $2(84) = 168$.

Time = 0.11 (sec) , antiderivative size = 3115, normalized size of antiderivative = 33.86

$$\int \operatorname{csch}^7(c+dx)\left(a+b\sinh^4(c+dx)\right)\,dx = \text{Too large to display}$$

input
```
integrate(csch(d*x+c)^7*(a+b*sinh(d*x+c)^4),x, algorithm="fricas")
```

output
```
Too large to include
```

Sympy [F(-1)]

Timed out.

$$\int \operatorname{csch}^7(c+dx)\left(a+b\sinh^4(c+dx)\right)\,dx = \text{Timed out}$$

input
```
integrate(csch(d*x+c)**7*(a+b*sinh(d*x+c)**4),x)
```

output
```
Timed out
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 268 vs. $2(84) = 168$.

Time = 0.05 (sec) , antiderivative size = 268, normalized size of antiderivative = 2.91

$$\int \operatorname{csch}^7(c+dx)\left(a+b\sinh^4(c+dx)\right)\,dx$$
$$=\frac{1}{48}a\left(\frac{15\log\left(e^{(-dx-c)}+1\right)}{d}-\frac{15\log\left(e^{(-dx-c)}-1\right)}{d}+\frac{2\left(15\,e^{(-dx-c)}-85\,e^{(-3\,dx-3\,c)}+198\,e^{(-5\,dx-5\,c)}+\right.}{d(6\,e^{(-2\,dx-2\,c)}-15\,e^{(-4\,dx-4\,c)}+20\,e^{(-6\,dx-6\,c)}-1}\right.$$
$$+\frac{1}{2}b\left(\frac{\log\left(e^{(-dx-c)}+1\right)}{d}-\frac{\log\left(e^{(-dx-c)}-1\right)}{d}+\frac{2\left(e^{(-dx-c)}+e^{(-3\,dx-3\,c)}\right)}{d(2\,e^{(-2\,dx-2\,c)}-e^{(-4\,dx-4\,c)}-1)}\right)$$

input
```
integrate(csch(d*x+c)^7*(a+b*sinh(d*x+c)^4),x, algorithm="maxima")
```

output
```
1/48*a*(15*log(e^(-d*x - c) + 1)/d - 15*log(e^(-d*x - c) - 1)/d + 2*(15*e^
(-d*x - c) - 85*e^(-3*d*x - 3*c) + 198*e^(-5*d*x - 5*c) + 198*e^(-7*d*x -
7*c) - 85*e^(-9*d*x - 9*c) + 15*e^(-11*d*x - 11*c))/(d*(6*e^(-2*d*x - 2*c)
 - 15*e^(-4*d*x - 4*c) + 20*e^(-6*d*x - 6*c) - 15*e^(-8*d*x - 8*c) + 6*e^(
-10*d*x - 10*c) - e^(-12*d*x - 12*c) - 1))) + 1/2*b*(log(e^(-d*x - c) + 1)
/d - log(e^(-d*x - c) - 1)/d + 2*(e^(-d*x - c) + e^(-3*d*x - 3*c))/(d*(2*e
^(-2*d*x - 2*c) - e^(-4*d*x - 4*c) - 1)))
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 207 vs. $2(84) = 168$.

Time = 0.15 (sec) , antiderivative size = 207, normalized size of antiderivative = 2.25

$$\int \operatorname{csch}^7(c+dx)\left(a+b\sinh^4(c+dx)\right)\,dx$$
$$=\frac{3\,(5\,a+8\,b)\log\left(e^{(dx+c)}+e^{(-dx-c)}+2\right)-3\,(5\,a+8\,b)\log\left(e^{(dx+c)}+e^{(-dx-c)}-2\right)-\frac{4\left(15\,a\left(e^{(dx+c)}+e^{(-dx-c)}\right.\right.}{}}{96}$$

input
```
integrate(csch(d*x+c)^7*(a+b*sinh(d*x+c)^4),x, algorithm="giac")
```

output
```
1/96*(3*(5*a + 8*b)*log(e^(d*x + c) + e^(-d*x - c) + 2) - 3*(5*a + 8*b)*lo
g(e^(d*x + c) + e^(-d*x - c) - 2) - 4*(15*a*(e^(d*x + c) + e^(-d*x - c))^5
 + 24*b*(e^(d*x + c) + e^(-d*x - c))^5 - 160*a*(e^(d*x + c) + e^(-d*x - c)
)^3 - 192*b*(e^(d*x + c) + e^(-d*x - c))^3 + 528*a*(e^(d*x + c) + e^(-d*x
- c)) + 384*b*(e^(d*x + c) + e^(-d*x - c)))/((e^(d*x + c) + e^(-d*x - c))^
2 - 4)^3)/d
```

Mupad [B] (verification not implemented)

Time = 1.62 (sec) , antiderivative size = 472, normalized size of antiderivative = 5.13

$$\begin{aligned}
&\int \operatorname{csch}^7(c+dx)\left(a+b\sinh^4(c+dx)\right)\,dx\\
&=\frac{\operatorname{atan}\left(\frac{e^{dx}\,e^{c}\,\left(5\,a\,\sqrt{-d^2}+8\,b\,\sqrt{-d^2}\right)}{d\,\sqrt{25\,a^2+80\,a\,b+64\,b^2}}\right)\,\sqrt{25\,a^2+80\,a\,b+64\,b^2}}{8\,\sqrt{-d^2}}\\
&-\frac{\frac{2\,b\,e^{9\,c+9\,d\,x}}{3\,d}-\frac{8\,b\,e^{7\,c+7\,d\,x}}{3\,d}-\frac{8\,b\,e^{3\,c+3\,d\,x}}{3\,d}+\frac{4\,e^{5\,c+5\,d\,x}\,\left(8\,a+3\,b\right)}{3\,d}+\frac{2\,b\,e^{c+d\,x}}{3\,d}}{15\,e^{4\,c+4\,d\,x}-6\,e^{2\,c+2\,d\,x}-20\,e^{6\,c+6\,d\,x}+15\,e^{8\,c+8\,d\,x}-6\,e^{10\,c+10\,d\,x}+e^{12\,c+12\,d\,x}+1}\\
&+\frac{e^{c+d\,x}\,\left(5\,a-16\,b\right)}{12\,d\,\left(e^{4\,c+4\,d\,x}-2\,e^{2\,c+2\,d\,x}+1\right)}-\frac{a\,e^{c+d\,x}}{3\,d\,\left(3\,e^{2\,c+2\,d\,x}-3\,e^{4\,c+4\,d\,x}+e^{6\,c+6\,d\,x}-1\right)}\\
&-\frac{22\,a\,e^{c+d\,x}}{3\,d\,\left(6\,e^{4\,c+4\,d\,x}-4\,e^{2\,c+2\,d\,x}-4\,e^{6\,c+6\,d\,x}+e^{8\,c+8\,d\,x}+1\right)}-\frac{e^{c+d\,x}\,\left(5\,a+8\,b\right)}{8\,d\,\left(e^{2\,c+2\,d\,x}-1\right)}\\
&-\frac{16\,a\,e^{c+d\,x}}{3\,d\,\left(5\,e^{2\,c+2\,d\,x}-10\,e^{4\,c+4\,d\,x}+10\,e^{6\,c+6\,d\,x}-5\,e^{8\,c+8\,d\,x}+e^{10\,c+10\,d\,x}-1\right)}
\end{aligned}$$

input
```
int((a + b*sinh(c + d*x)^4)/sinh(c + d*x)^7,x)
```

output
```
(atan((exp(d*x)*exp(c)*(5*a*(-d^2)^(1/2) + 8*b*(-d^2)^(1/2)))/(d*(80*a*b +
 25*a^2 + 64*b^2)^(1/2)))*(80*a*b + 25*a^2 + 64*b^2)^(1/2))/(8*(-d^2)^(1/2
)) - ((2*b*exp(9*c + 9*d*x))/(3*d) - (8*b*exp(7*c + 7*d*x))/(3*d) - (8*b*e
xp(3*c + 3*d*x))/(3*d) + (4*exp(5*c + 5*d*x)*(8*a + 3*b))/(3*d) + (2*b*exp
(c + d*x))/(3*d))/(15*exp(4*c + 4*d*x) - 6*exp(2*c + 2*d*x) - 20*exp(6*c +
 6*d*x) + 15*exp(8*c + 8*d*x) - 6*exp(10*c + 10*d*x) + exp(12*c + 12*d*x)
+ 1) + (exp(c + d*x)*(5*a - 16*b))/(12*d*(exp(4*c + 4*d*x) - 2*exp(2*c + 2
*d*x) + 1)) - (a*exp(c + d*x))/(3*d*(3*exp(2*c + 2*d*x) - 3*exp(4*c + 4*d*
x) + exp(6*c + 6*d*x) - 1)) - (22*a*exp(c + d*x))/(3*d*(6*exp(4*c + 4*d*x)
 - 4*exp(2*c + 2*d*x) - 4*exp(6*c + 6*d*x) + exp(8*c + 8*d*x) + 1)) - (exp
(c + d*x)*(5*a + 8*b))/(8*d*(exp(2*c + 2*d*x) - 1)) - (16*a*exp(c + d*x))/
(3*d*(5*exp(2*c + 2*d*x) - 10*exp(4*c + 4*d*x) + 10*exp(6*c + 6*d*x) - 5*e
xp(8*c + 8*d*x) + exp(10*c + 10*d*x) - 1))
```

Reduce [B] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 834, normalized size of antiderivative = 9.07

$$\int \operatorname{csch}^7(c+dx)\left(a+b\sinh^4(c+dx)\right)\,dx = \text{Too large to display}$$

input
```
int(csch(d*x+c)^7*(a+b*sinh(d*x+c)^4),x)
```

output
```
( - 15*e**(12*c + 12*d*x)*log(e**(c + d*x) - 1)*a - 24*e**(12*c + 12*d*x)*
log(e**(c + d*x) - 1)*b + 15*e**(12*c + 12*d*x)*log(e**(c + d*x) + 1)*a +
24*e**(12*c + 12*d*x)*log(e**(c + d*x) + 1)*b - 30*e**(11*c + 11*d*x)*a -
48*e**(11*c + 11*d*x)*b + 90*e**(10*c + 10*d*x)*log(e**(c + d*x) - 1)*a +
144*e**(10*c + 10*d*x)*log(e**(c + d*x) - 1)*b - 90*e**(10*c + 10*d*x)*log
(e**(c + d*x) + 1)*a - 144*e**(10*c + 10*d*x)*log(e**(c + d*x) + 1)*b + 17
0*e**(9*c + 9*d*x)*a + 144*e**(9*c + 9*d*x)*b - 225*e**(8*c + 8*d*x)*log(e
**(c + d*x) - 1)*a - 360*e**(8*c + 8*d*x)*log(e**(c + d*x) - 1)*b + 225*e*
*(8*c + 8*d*x)*log(e**(c + d*x) + 1)*a + 360*e**(8*c + 8*d*x)*log(e**(c +
d*x) + 1)*b - 396*e**(7*c + 7*d*x)*a - 96*e**(7*c + 7*d*x)*b + 300*e**(6*c
 + 6*d*x)*log(e**(c + d*x) - 1)*a + 480*e**(6*c + 6*d*x)*log(e**(c + d*x)
- 1)*b - 300*e**(6*c + 6*d*x)*log(e**(c + d*x) + 1)*a - 480*e**(6*c + 6*d*
x)*log(e**(c + d*x) + 1)*b - 396*e**(5*c + 5*d*x)*a - 96*e**(5*c + 5*d*x)*
b - 225*e**(4*c + 4*d*x)*log(e**(c + d*x) - 1)*a - 360*e**(4*c + 4*d*x)*lo
g(e**(c + d*x) - 1)*b + 225*e**(4*c + 4*d*x)*log(e**(c + d*x) + 1)*a + 360
*e**(4*c + 4*d*x)*log(e**(c + d*x) + 1)*b + 170*e**(3*c + 3*d*x)*a + 144*e
**(3*c + 3*d*x)*b + 90*e**(2*c + 2*d*x)*log(e**(c + d*x) - 1)*a + 144*e**(
2*c + 2*d*x)*log(e**(c + d*x) - 1)*b - 90*e**(2*c + 2*d*x)*log(e**(c + d*x
) + 1)*a - 144*e**(2*c + 2*d*x)*log(e**(c + d*x) + 1)*b - 30*e**(c + d*x)*
a - 48*e**(c + d*x)*b - 15*log(e**(c + d*x) - 1)*a - 24*log(e**(c + d*x...
```

3.171 $\int \sinh^3(c+dx)\left(a+b\sinh^4(c+dx)\right)^2\,dx$

Optimal result

Integrand size = 23, antiderivative size = 120

$$\int \sinh^3(c+dx)\left(a+b\sinh^4(c+dx)\right)^2\,dx = -\frac{(a+b)^2\cosh(c+dx)}{d} + \frac{(a+b)(a+5b)\cosh^3(c+dx)}{3d} - \frac{2b(3a+5b)\cosh^5(c+dx)}{5d} + \frac{2b(a+5b)\cosh^7(c+dx)}{7d} - \frac{5b^2\cosh^9(c+dx)}{9d} + \frac{b^2\cosh^{11}(c+dx)}{11d}$$

output
```
-(a+b)^2*cosh(d*x+c)/d+1/3*(a+b)*(a+5*b)*cosh(d*x+c)^3/d-2/5*b*(3*a+5*b)*c
osh(d*x+c)^5/d+2/7*b*(a+5*b)*cosh(d*x+c)^7/d-5/9*b^2*cosh(d*x+c)^9/d+1/11*
b^2*cosh(d*x+c)^11/d
```

Mathematica [A] (verified)

Time = 0.22 (sec) , antiderivative size = 207, normalized size of antiderivative = 1.72

$$\int \sinh^3(c+dx)\left(a+b\sinh^4(c+dx)\right)^2\,dx$$
$$=-\frac{3a^2\cosh(c+dx)}{4d}-\frac{35ab\cosh(c+dx)}{32d}-\frac{231b^2\cosh(c+dx)}{512d}+\frac{a^2\cosh(3(c+dx))}{12d}+\frac{7ab\cosh(3(c+dx))}{32d}+\frac{55b^2\cosh(3(c+dx))}{512d}-\frac{7ab\cosh(5(c+dx))}{160d}-\frac{33b^2\cosh(5(c+dx))}{1024d}+\frac{ab\cosh(7(c+dx))}{224d}+\frac{55b^2\cosh(7(c+dx))}{7168d}-\frac{11b^2\cosh(9(c+dx))}{9216d}+\frac{b^2\cosh(11(c+dx))}{11264d}$$

input
```
Integrate[Sinh[c + d*x]^3*(a + b*Sinh[c + d*x]^4)^2,x]
```

output
```
(-3*a^2*Cosh[c + d*x])/(4*d) - (35*a*b*Cosh[c + d*x])/(32*d) - (231*b^2*Co
sh[c + d*x])/(512*d) + (a^2*Cosh[3*(c + d*x)])/(12*d) + (7*a*b*Cosh[3*(c +
 d*x)])/(32*d) + (55*b^2*Cosh[3*(c + d*x)])/(512*d) - (7*a*b*Cosh[5*(c + d
*x)])/(160*d) - (33*b^2*Cosh[5*(c + d*x)])/(1024*d) + (a*b*Cosh[7*(c + d*x
)])/(224*d) + (55*b^2*Cosh[7*(c + d*x)])/(7168*d) - (11*b^2*Cosh[9*(c + d*
x)])/(9216*d) + (b^2*Cosh[11*(c + d*x)])/(11264*d)
```

Rubi [A] (verified)

Time = 0.39 (sec) , antiderivative size = 106, normalized size of antiderivative = 0.88, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.217$, Rules used = {3042, 26, 3694, 1467, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sinh^3(c+dx)\left(a+b\sinh^4(c+dx)\right)^2\,dx$$
$$\Big\downarrow \text{3042}$$
$$\int i\sin(ic+idx)^3\left(a+b\sin(ic+idx)^4\right)^2dx$$

$$\Big\downarrow 26$$

$$i\int \sin(ic+idx)^3\left(b\sin(ic+idx)^4+a\right)^2dx$$

$$\Big\downarrow 3694$$

$$-\frac{\int \left(1-\cosh^2(c+dx)\right)\left(b\cosh^4(c+dx)-2b\cosh^2(c+dx)+a+b\right)^2d\cosh(c+dx)}{d}$$

$$\Big\downarrow 1467$$

$$-\frac{\int \left(-b^2\cosh^{10}(c+dx)+5b^2\cosh^8(c+dx)-2b(a+5b)\cosh^6(c+dx)+2b(3a+5b)\cosh^4(c+dx)+(-a-5b\right.}{d}$$

$$\Big\downarrow 2009$$

$$-\frac{-\frac{2}{7}b(a+5b)\cosh^7(c+dx)+\frac{2}{5}b(3a+5b)\cosh^5(c+dx)-\frac{1}{3}(a+b)(a+5b)\cosh^3(c+dx)+(a+b)^2\cosh(c+}{d}$$

input
```
Int[Sinh[c + d*x]^3*(a + b*Sinh[c + d*x]^4)^2,x]
```

output
```
-(((a + b)^2*Cosh[c + d*x] - ((a + b)*(a + 5*b)*Cosh[c + d*x]^3)/3 + (2*b*
(3*a + 5*b)*Cosh[c + d*x]^5)/5 - (2*b*(a + 5*b)*Cosh[c + d*x]^7)/7 + (5*b^
2*Cosh[c + d*x]^9)/9 - (b^2*Cosh[c + d*x]^11)/11)/d)
```

Defintions of rubi rules used

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 1467
```
Int[((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.),
 x_Symbol] :> Int[ExpandIntegrand[(d + e*x^2)^q*(a + b*x^2 + c*x^4)^p, x],
x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e
 + a*e^2, 0] && IGtQ[p, 0] && IGtQ[q, -2]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3694
```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^4)^
(p_.), x_Symbol] :> With[{ff = FreeFactors[Cos[e + f*x], x]}, Simp[-ff/f
Subst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a + b - 2*b*ff^2*x^2 + b*ff^4*x^4)^p,
 x], x, Cos[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m -
 1)/2]
```

Maple [A] (verified)

Time = 148.51 (sec) , antiderivative size = 130, normalized size of antiderivative = 1.08

method	result
parallelrisch	$\frac{(295680a^2+776160ab+381150b^2)\cosh(3dx+3c)-155232\left(a+\frac{165b}{224}\right)b\cosh(5dx+5c)+15840\left(a+\frac{55b}{32}\right)b\cosh(7dx+7c)+}{}$
derivativedivides	$\frac{a^2\left(-\frac{2}{3}+\frac{\sinh(dx+c)^2}{3}\right)\cosh(dx+c)+2ab\left(-\frac{16}{35}+\frac{\sinh(dx+c)^6}{7}-\frac{6\sinh(dx+c)^4}{35}+\frac{8\sinh(dx+c)^2}{35}\right)\cosh(dx+c)+b^2\left(-\frac{256}{693}+\right.}{d}$
default	$\frac{a^2\left(-\frac{2}{3}+\frac{\sinh(dx+c)^2}{3}\right)\cosh(dx+c)+2ab\left(-\frac{16}{35}+\frac{\sinh(dx+c)^6}{7}-\frac{6\sinh(dx+c)^4}{35}+\frac{8\sinh(dx+c)^2}{35}\right)\cosh(dx+c)+b^2\left(-\frac{256}{693}+\right.}{d}$
parts	$\frac{a^2\left(-\frac{2}{3}+\frac{\sinh(dx+c)^2}{3}\right)\cosh(dx+c)}{d}+\frac{b^2\left(-\frac{256}{693}+\frac{\sinh(dx+c)^{10}}{11}-\frac{10\sinh(dx+c)^8}{99}+\frac{80\sinh(dx+c)^6}{693}-\frac{32\sinh(dx+c)^4}{231}+128\right.}{d}$
risch	$\frac{b^2e^{11dx+11c}}{22528d}-\frac{11b^2e^{9dx+9c}}{18432d}+\frac{b\,e^{7dx+7c}a}{448d}+\frac{55b^2e^{7dx+7c}}{14336d}-\frac{7b\,e^{5dx+5c}a}{320d}-\frac{33b^2e^{5dx+5c}}{2048d}+\frac{e^{3dx+3c}a^2}{24d}+7$
orering	Expression too large to display

input
```
int(sinh(d*x+c)^3*(a+b*sinh(d*x+c)^4)^2,x,method=_RETURNVERBOSE)
```

output
```
1/3548160*((295680*a^2+776160*a*b+381150*b^2)*cosh(3*d*x+3*c)-155232*(a+16
5/224*b)*b*cosh(5*d*x+5*c)+15840*(a+55/32*b)*b*cosh(7*d*x+7*c)+315*b^2*cos
h(11*d*x+11*c)-4235*b^2*cosh(9*d*x+9*c)+(-2661120*a^2-3880800*a*b-1600830*
b^2)*cosh(d*x+c)-2365440*a^2-3244032*a*b-1310720*b^2)/d
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 404 vs. $2(110) = 220$.

Time = 0.10 (sec) , antiderivative size = 404, normalized size of antiderivative = 3.37

$$\int \sinh^3(c+dx)\left(a+b\sinh^4(c+dx)\right)^2\,dx$$
$$= \frac{315\,b^2\cosh(dx+c)^{11} + 3465\,b^2\cosh(dx+c)\sinh(dx+c)^{10} - 4235\,b^2\cosh(dx+c)^9 + 3465\,(15\,b^2\cosh}{}$$

input
```
integrate(sinh(d*x+c)^3*(a+b*sinh(d*x+c)^4)^2,x, algorithm="fricas")
```

output
```
1/3548160*(315*b^2*cosh(d*x + c)^11 + 3465*b^2*cosh(d*x + c)*sinh(d*x + c)
^10 - 4235*b^2*cosh(d*x + c)^9 + 3465*(15*b^2*cosh(d*x + c)^3 - 11*b^2*cos
h(d*x + c))*sinh(d*x + c)^8 + 495*(32*a*b + 55*b^2)*cosh(d*x + c)^7 + 1155
*(126*b^2*cosh(d*x + c)^5 - 308*b^2*cosh(d*x + c)^3 + 3*(32*a*b + 55*b^2)*
cosh(d*x + c))*sinh(d*x + c)^6 - 693*(224*a*b + 165*b^2)*cosh(d*x + c)^5 +
 3465*(30*b^2*cosh(d*x + c)^7 - 154*b^2*cosh(d*x + c)^5 + 5*(32*a*b + 55*b
^2)*cosh(d*x + c)^3 - (224*a*b + 165*b^2)*cosh(d*x + c))*sinh(d*x + c)^4 +
 2310*(128*a^2 + 336*a*b + 165*b^2)*cosh(d*x + c)^3 + 3465*(5*b^2*cosh(d*x
 + c)^9 - 44*b^2*cosh(d*x + c)^7 + 3*(32*a*b + 55*b^2)*cosh(d*x + c)^5 - 2
*(224*a*b + 165*b^2)*cosh(d*x + c)^3 + 2*(128*a^2 + 336*a*b + 165*b^2)*cos
h(d*x + c))*sinh(d*x + c)^2 - 6930*(384*a^2 + 560*a*b + 231*b^2)*cosh(d*x
+ c))/d
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 280 vs. $2(109) = 218$.

Time = 1.86 (sec) , antiderivative size = 280, normalized size of antiderivative = 2.33

$$\int \sinh^3(c+dx)\left(a+b\sinh^4(c+dx)\right)^2\,dx$$
$$= \begin{cases} \frac{a^2\sinh^2(c+dx)\cosh(c+dx)}{d} - \frac{2a^2\cosh^3(c+dx)}{3d} + \frac{2ab\sinh^6(c+dx)\cosh(c+dx)}{d} - \frac{4ab\sinh^4(c+dx)\cosh^3(c+dx)}{d} + \frac{16ab\sinh^2(c+d}{5} \\ x\left(a+b\sinh^4(c)\right)^2\sinh^3(c) \end{cases}$$

input
```
integrate(sinh(d*x+c)**3*(a+b*sinh(d*x+c)**4)**2,x)
```

output
```
Piecewise((a**2*sinh(c + d*x)**2*cosh(c + d*x)/d - 2*a**2*cosh(c + d*x)**3
/(3*d) + 2*a*b*sinh(c + d*x)**6*cosh(c + d*x)/d - 4*a*b*sinh(c + d*x)**4*c
osh(c + d*x)**3/d + 16*a*b*sinh(c + d*x)**2*cosh(c + d*x)**5/(5*d) - 32*a*
b*cosh(c + d*x)**7/(35*d) + b**2*sinh(c + d*x)**10*cosh(c + d*x)/d - 10*b*
*2*sinh(c + d*x)**8*cosh(c + d*x)**3/(3*d) + 16*b**2*sinh(c + d*x)**6*cosh
(c + d*x)**5/(3*d) - 32*b**2*sinh(c + d*x)**4*cosh(c + d*x)**7/(7*d) + 128
*b**2*sinh(c + d*x)**2*cosh(c + d*x)**9/(63*d) - 256*b**2*cosh(c + d*x)**1
1/(693*d), Ne(d, 0)), (x*(a + b*sinh(c)**4)**2*sinh(c)**3, True))
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 307 vs. $2(110) = 220$.

Time = 0.05 (sec) , antiderivative size = 307, normalized size of antiderivative = 2.56

$$\int \sinh^3(c+dx)\left(a+b\sinh^4(c+dx)\right)^2\,dx =$$

$$-\frac{1}{1419264}b^2\left(\frac{\left(847\,e^{(-2\,dx-2\,c)}-5445\,e^{(-4\,dx-4\,c)}+22869\,e^{(-6\,dx-6\,c)}-76230\,e^{(-8\,dx-8\,c)}+320166\,e^{(-10\,d}\right.}{d}\right.$$

$$-\frac{1}{2240}ab\left(\frac{\left(49\,e^{(-2\,dx-2\,c)}-245\,e^{(-4\,dx-4\,c)}+1225\,e^{(-6\,dx-6\,c)}-5\right)e^{(7\,dx+7\,c)}}{d}+\frac{1225\,e^{(-dx-c)}-245\,e^{(-3\,d}}{}\right.$$

$$+\frac{1}{24}a^2\left(\frac{e^{(3\,dx+3\,c)}}{d}-\frac{9\,e^{(dx+c)}}{d}-\frac{9\,e^{(-dx-c)}}{d}+\frac{e^{(-3\,dx-3\,c)}}{d}\right)$$

input
```
integrate(sinh(d*x+c)^3*(a+b*sinh(d*x+c)^4)^2,x, algorithm="maxima")
```

output
```
-1/1419264*b^2*((847*e^(-2*d*x - 2*c) - 5445*e^(-4*d*x - 4*c) + 22869*e^(-
6*d*x - 6*c) - 76230*e^(-8*d*x - 8*c) + 320166*e^(-10*d*x - 10*c) - 63)*e^
(11*d*x + 11*c)/d + (320166*e^(-d*x - c) - 76230*e^(-3*d*x - 3*c) + 22869*
e^(-5*d*x - 5*c) - 5445*e^(-7*d*x - 7*c) + 847*e^(-9*d*x - 9*c) - 63*e^(-1
1*d*x - 11*c))/d) - 1/2240*a*b*((49*e^(-2*d*x - 2*c) - 245*e^(-4*d*x - 4*c
) + 1225*e^(-6*d*x - 6*c) - 5)*e^(7*d*x + 7*c)/d + (1225*e^(-d*x - c) - 24
5*e^(-3*d*x - 3*c) + 49*e^(-5*d*x - 5*c) - 5*e^(-7*d*x - 7*c))/d) + 1/24*a
^2*(e^(3*d*x + 3*c)/d - 9*e^(d*x + c)/d - 9*e^(-d*x - c)/d + e^(-3*d*x - 3
*c)/d)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 278 vs. $2(110) = 220$.

Time = 0.16 (sec) , antiderivative size = 278, normalized size of antiderivative = 2.32

$$\int \sinh^3(c+dx)\left(a+b\sinh^4(c+dx)\right)^2\,dx = \frac{b^2e^{(11\,dx+11\,c)}}{22528\,d} - \frac{11\,b^2e^{(9\,dx+9\,c)}}{18432\,d} - \frac{11\,b^2e^{(-9\,dx-9\,c)}}{18432\,d} + \frac{b^2e^{(-11\,dx-11\,c)}}{22528\,d} + \frac{(32\,ab+55\,b^2)e^{(7\,dx+7\,c)}}{14336\,d} - \frac{(224\,ab+165\,b^2)e^{(5\,dx+5\,c)}}{10240\,d} + \frac{(128\,a^2+336\,ab+165\,b^2)e^{(3\,dx+3\,c)}}{3072\,d} - \frac{(384\,a^2+560\,ab+231\,b^2)e^{(dx+c)}}{1024\,d} - \frac{(384\,a^2+560\,ab+231\,b^2)e^{(-dx-c)}}{1024\,d} + \frac{(128\,a^2+336\,ab+165\,b^2)e^{(-3\,dx-3\,c)}}{3072\,d} - \frac{(224\,ab+165\,b^2)e^{(-5\,dx-5\,c)}}{10240\,d} + \frac{(32\,ab+55\,b^2)e^{(-7\,dx-7\,c)}}{14336\,d}$$

input
```
integrate(sinh(d*x+c)^3*(a+b*sinh(d*x+c)^4)^2,x, algorithm="giac")
```

output
```
1/22528*b^2*e^(11*d*x + 11*c)/d - 11/18432*b^2*e^(9*d*x + 9*c)/d - 11/1843
2*b^2*e^(-9*d*x - 9*c)/d + 1/22528*b^2*e^(-11*d*x - 11*c)/d + 1/14336*(32*
a*b + 55*b^2)*e^(7*d*x + 7*c)/d - 1/10240*(224*a*b + 165*b^2)*e^(5*d*x + 5
*c)/d + 1/3072*(128*a^2 + 336*a*b + 165*b^2)*e^(3*d*x + 3*c)/d - 1/1024*(3
84*a^2 + 560*a*b + 231*b^2)*e^(d*x + c)/d - 1/1024*(384*a^2 + 560*a*b + 23
1*b^2)*e^(-d*x - c)/d + 1/3072*(128*a^2 + 336*a*b + 165*b^2)*e^(-3*d*x - 3
*c)/d - 1/10240*(224*a*b + 165*b^2)*e^(-5*d*x - 5*c)/d + 1/14336*(32*a*b +
 55*b^2)*e^(-7*d*x - 7*c)/d
```

Mupad [B] (verification not implemented)

Time = 0.36 (sec) , antiderivative size = 150, normalized size of antiderivative = 1.25

$$\int \sinh^3(c+dx) \left(a+b\sinh^4(c+dx)\right)^2 \, dx =$$
$$-\frac{-\frac{a^2\cosh(c+dx)^3}{3}+a^2\cosh(c+dx)-\frac{2ab\cosh(c+dx)^7}{7}+\frac{6ab\cosh(c+dx)^5}{5}-2ab\cosh(c+dx)^3+2ab\cosh(c$$

input
```
int(sinh(c + d*x)^3*(a + b*sinh(c + d*x)^4)^2,x)
```

output
```
-(a^2*cosh(c + d*x) + b^2*cosh(c + d*x) - (a^2*cosh(c + d*x)^3)/3 - (5*b^2
*cosh(c + d*x)^3)/3 + 2*b^2*cosh(c + d*x)^5 - (10*b^2*cosh(c + d*x)^7)/7 +
 (5*b^2*cosh(c + d*x)^9)/9 - (b^2*cosh(c + d*x)^11)/11 + 2*a*b*cosh(c + d*
x) - 2*a*b*cosh(c + d*x)^3 + (6*a*b*cosh(c + d*x)^5)/5 - (2*a*b*cosh(c + d
*x)^7)/7)/d
```

Reduce [B] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 360, normalized size of antiderivative = 3.00

$$\int \sinh^3(c+dx) \left(a+b\sinh^4(c+dx)\right)^2 \, dx$$
$$= \frac{315e^{22dx+22c}b^2-4235e^{20dx+20c}b^2+15840e^{18dx+18c}ab+27225e^{18dx+18c}b^2-155232e^{16dx+16c}ab-114345e^{1}$$

input
```
int(sinh(d*x+c)^3*(a+b*sinh(d*x+c)^4)^2,x)
```

output

```
(315*e**(22*c + 22*d*x)*b**2 - 4235*e**(20*c + 20*d*x)*b**2 + 15840*e**(18
*c + 18*d*x)*a*b + 27225*e**(18*c + 18*d*x)*b**2 - 155232*e**(16*c + 16*d*
x)*a*b - 114345*e**(16*c + 16*d*x)*b**2 + 295680*e**(14*c + 14*d*x)*a**2 +
 776160*e**(14*c + 14*d*x)*a*b + 381150*e**(14*c + 14*d*x)*b**2 - 2661120*
e**(12*c + 12*d*x)*a**2 - 3880800*e**(12*c + 12*d*x)*a*b - 1600830*e**(12*
c + 12*d*x)*b**2 - 2661120*e**(10*c + 10*d*x)*a**2 - 3880800*e**(10*c + 10
*d*x)*a*b - 1600830*e**(10*c + 10*d*x)*b**2 + 295680*e**(8*c + 8*d*x)*a**2
 + 776160*e**(8*c + 8*d*x)*a*b + 381150*e**(8*c + 8*d*x)*b**2 - 155232*e**
(6*c + 6*d*x)*a*b - 114345*e**(6*c + 6*d*x)*b**2 + 15840*e**(4*c + 4*d*x)*
a*b + 27225*e**(4*c + 4*d*x)*b**2 - 4235*e**(2*c + 2*d*x)*b**2 + 315*b**2)
/(7096320*e**(11*c + 11*d*x)*d)
```

3.172 $\int \sinh^2(c+dx) \left(a+b\sinh^4(c+dx)\right)^2 dx$

Optimal result

Integrand size = 23, antiderivative size = 161

$$\begin{aligned}
&\int \sinh^2(c+dx) \left(a+b\sinh^4(c+dx)\right)^2 dx \\
&= -\frac{1}{256}\left(128a^2+160ab+63b^2\right)x + \frac{(128a^2+352ab+193b^2)\cosh(c+dx)\sinh(c+dx)}{256d} \\
&\quad - \frac{b(416a+447b)\cosh^3(c+dx)\sinh(c+dx)}{384d} \\
&\quad + \frac{b(160a+513b)\cosh^5(c+dx)\sinh(c+dx)}{480d} \\
&\quad - \frac{41b^2\cosh^7(c+dx)\sinh(c+dx)}{80d} + \frac{b^2\cosh^9(c+dx)\sinh(c+dx)}{10d}
\end{aligned}$$

output
```
-1/256*(128*a^2+160*a*b+63*b^2)*x+1/256*(128*a^2+352*a*b+193*b^2)*cosh(d*x
+c)*sinh(d*x+c)/d-1/384*b*(416*a+447*b)*cosh(d*x+c)^3*sinh(d*x+c)/d+1/480*
b*(160*a+513*b)*cosh(d*x+c)^5*sinh(d*x+c)/d-41/80*b^2*cosh(d*x+c)^7*sinh(d
*x+c)/d+1/10*b^2*cosh(d*x+c)^9*sinh(d*x+c)/d
```

Mathematica [A] (verified)

Time = 0.34 (sec) , antiderivative size = 139, normalized size of antiderivative = 0.86

$$\int \sinh^2(c+dx)\left(a+b\sinh^4(c+dx)\right)^2\,dx = -\frac{15360a^2c+19200abc+7560b^2c+15360a^2dx+19200abdx+7560b^2dx-60(128a^2+240ab+105b^2)}{}$$

input
```
Integrate[Sinh[c + d*x]^2*(a + b*Sinh[c + d*x]^4)^2,x]
```

output
```
-1/30720*(15360*a^2*c + 19200*a*b*c + 7560*b^2*c + 15360*a^2*d*x + 19200*a
*b*d*x + 7560*b^2*d*x - 60*(128*a^2 + 240*a*b + 105*b^2)*Sinh[2*(c + d*x)]
 + 360*b*(8*a + 5*b)*Sinh[4*(c + d*x)] - 320*a*b*Sinh[6*(c + d*x)] - 450*b
^2*Sinh[6*(c + d*x)] + 75*b^2*Sinh[8*(c + d*x)] - 6*b^2*Sinh[10*(c + d*x)]
)/d
```

Rubi [A] (verified)

Time = 0.78 (sec) , antiderivative size = 208, normalized size of antiderivative = 1.29, number of steps used = 13, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.522$, Rules used = {3042, 25, 3696, 1580, 25, 2345, 2345, 27, 1471, 27, 298, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sinh^2(c+dx)\left(a+b\sinh^4(c+dx)\right)^2\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int -\sin(ic+idx)^2\left(a+b\sin(ic+idx)^4\right)^2\,dx$$

$$\Big\downarrow \text{25}$$

$$-\int \sin(ic+idx)^2\left(b\sin(ic+idx)^4+a\right)^2\,dx$$

$$\Big\downarrow \text{3696}$$

$$\frac{\int \frac{\tanh^2(c+dx)\left((a+b)\tanh^4(c+dx)-2a\tanh^2(c+dx)+a\right)^2}{\left(1-\tanh^2(c+dx)\right)^6}d\tanh(c+dx)}{d}$$

$$\Big\downarrow \textbf{1580}$$

$$\frac{\frac{1}{10}\int -\frac{10(a+b)^2\tanh^8(c+dx)-10(3a-b)(a+b)\tanh^6(c+dx)+10\left(3a^2+b^2\right)\tanh^4(c+dx)-10\left(a^2-b^2\right)\tanh^2(c+dx)+b^2}{\left(1-\tanh^2(c+dx)\right)^5}d\tanh(c+dx)+\frac{}{1}}{d}$$

$$\Big\downarrow \textbf{25}$$

$$\frac{\frac{b^2\tanh(c+dx)}{10\left(1-\tanh^2(c+dx)\right)^5}-\frac{1}{10}\int \frac{10(a+b)^2\tanh^8(c+dx)-10(3a-b)(a+b)\tanh^6(c+dx)+10\left(3a^2+b^2\right)\tanh^4(c+dx)-10\left(a^2-b^2\right)\tanh^2(c+dx)+b^2}{\left(1-\tanh^2(c+dx)\right)^5}}{d}$$

$$\Big\downarrow \textbf{2345}$$

$$\frac{\frac{1}{10}\left(\frac{1}{8}\int \frac{80(a+b)^2\tanh^6(c+dx)-160\left(a^2-b^2\right)\tanh^4(c+dx)+80\left(a^2+3b^2\right)\tanh^2(c+dx)+33b^2}{\left(1-\tanh^2(c+dx)\right)^4}d\tanh(c+dx)-\frac{41b^2\tanh(c+dx)}{8\left(1-\tanh^2(c+dx)\right)^4}\right)+}{d}$$

$$\Big\downarrow \textbf{2345}$$

$$\frac{\frac{1}{10}\left(\frac{1}{8}\left(\frac{b(160a+513b)\tanh(c+dx)}{6\left(1-\tanh^2(c+dx)\right)^3}-\frac{1}{6}\int \frac{5\left(96(a+b)^2\tanh^4(c+dx)-96(a-3b)(a+b)\tanh^2(c+dx)+b(32a+63b)\right)}{\left(1-\tanh^2(c+dx)\right)^3}d\tanh(c+dx)\right)-\frac{4}{8(1}\right.}{d}$$

$$\Big\downarrow \textbf{27}$$

$$\frac{\frac{1}{10}\left(\frac{1}{8}\left(\frac{b(160a+513b)\tanh(c+dx)}{6\left(1-\tanh^2(c+dx)\right)^3}-\frac{5}{6}\int \frac{96(a+b)^2\tanh^4(c+dx)-96(a-3b)(a+b)\tanh^2(c+dx)+b(32a+63b)}{\left(1-\tanh^2(c+dx)\right)^3}d\tanh(c+dx)\right)-\frac{41b^2}{8(1-\text{ta}}\right.}{d}$$

$$\Big\downarrow \textbf{1471}$$

$$\frac{\frac{1}{10}\left(\frac{1}{8}\left(\frac{b(160a+513b)\tanh(c+dx)}{6\left(1-\tanh^2(c+dx)\right)^3}-\frac{5}{6}\left(\frac{b(416a+447b)\tanh(c+dx)}{4\left(1-\tanh^2(c+dx)\right)^2}-\frac{1}{4}\int \frac{3\left(128(a+b)^2\tanh^2(c+dx)+b(96a+65b)\right)}{\left(1-\tanh^2(c+dx)\right)^2}d\tanh(c+dx)\right)\right)\right.}{d}$$

$$\Big\downarrow \textbf{27}$$

$$\frac{\frac{1}{10}\left(\frac{1}{8}\left(\frac{b(160a+513b)\tanh(c+dx)}{6\left(1-\tanh^2(c+dx)\right)^3}-\frac{5}{6}\left(\frac{b(416a+447b)\tanh(c+dx)}{4\left(1-\tanh^2(c+dx)\right)^2}-\frac{3}{4}\int \frac{128(a+b)^2\tanh^2(c+dx)+b(96a+65b)}{\left(1-\tanh^2(c+dx)\right)^2}d\tanh(c+dx)\right)\right)-\right.}{d}$$

$$\Big\downarrow \textbf{298}$$

$$\frac{\frac{1}{10}\left(\frac{1}{8}\left(\frac{b(160a+513b)\tanh(c+dx)}{6(1-\tanh^2(c+dx))^3}-\frac{5}{6}\left(\frac{b(416a+447b)\tanh(c+dx)}{4(1-\tanh^2(c+dx))^2}-\frac{3}{4}\left(\frac{(128a^2+352ab+193b^2)\tanh(c+dx)}{2(1-\tanh^2(c+dx))}-\frac{1}{2}(128a^2+160ab+\right.\right.\right.\right.}{d}$$

$\downarrow$ 219

$$\frac{\frac{1}{10}\left(\frac{1}{8}\left(\frac{b(160a+513b)\tanh(c+dx)}{6(1-\tanh^2(c+dx))^3}-\frac{5}{6}\left(\frac{b(416a+447b)\tanh(c+dx)}{4(1-\tanh^2(c+dx))^2}-\frac{3}{4}\left(\frac{(128a^2+352ab+193b^2)\tanh(c+dx)}{2(1-\tanh^2(c+dx))}-\frac{1}{2}(128a^2+160ab+\right.\right.\right.\right.}{d}$$

input
```
Int[Sinh[c + d*x]^2*(a + b*Sinh[c + d*x]^4)^2,x]
```

output
```
((b^2*Tanh[c + d*x])/(10*(1 - Tanh[c + d*x]^2)^5) + ((-41*b^2*Tanh[c + d*x
])/(8*(1 - Tanh[c + d*x]^2)^4) + ((b*(160*a + 513*b)*Tanh[c + d*x])/(6*(1
- Tanh[c + d*x]^2)^3) - (5*((b*(416*a + 447*b)*Tanh[c + d*x])/(4*(1 - Tanh
[c + d*x]^2)^2) - (3*(-1/2*((128*a^2 + 160*a*b + 63*b^2)*ArcTanh[Tanh[c +
d*x]]) + ((128*a^2 + 352*a*b + 193*b^2)*Tanh[c + d*x])/(2*(1 - Tanh[c + d*
x]^2))))/4))/6)/8)/10)/d
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 298
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2), x_Symbol] :> Simp[(-(
b*c - a*d))*x*((a + b*x^2)^(p + 1)/(2*a*b*(p + 1))), x] - Simp[(a*d - b*c*(
2*p + 3))/(2*a*b*(p + 1))   Int[(a + b*x^2)^(p + 1), x], x] /; FreeQ[{a, b,
 c, d, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/2 + p, 0])
```

rule 1471
```
Int[((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.),
x_Symbol] :> With[{Qx = PolynomialQuotient[(a + b*x^2 + c*x^4)^p, d + e*x^2
, x], R = Coeff[PolynomialRemainder[(a + b*x^2 + c*x^4)^p, d + e*x^2, x], x
, 0]}, Simp[(-R)*x*((d + e*x^2)^(q + 1)/(2*d*(q + 1))), x] + Simp[1/(2*d*(q
 + 1))   Int[(d + e*x^2)^(q + 1)*ExpandToSum[2*d*(q + 1)*Qx + R*(2*q + 3),
x], x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^
2 - b*d*e + a*e^2, 0] && IGtQ[p, 0] && LtQ[q, -1]
```

rule 1580
```
Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)
^4)^(p_.), x_Symbol] :> Simp[(-d)^(m/2 - 1)*(c*d^2 - b*d*e + a*e^2)^p*x*((d
 + e*x^2)^(q + 1)/(2*e^(2*p + m/2)*(q + 1))), x] + Simp[1/(2*e^(2*p + m/2)*
(q + 1))   Int[(d + e*x^2)^(q + 1)*ExpandToSum[Together[(1/(d + e*x^2))*(2*
e^(2*p + m/2)*(q + 1)*x^m*(a + b*x^2 + c*x^4)^p - (-d)^(m/2 - 1)*(c*d^2 - b
*d*e + a*e^2)^p*(d + e*(2*q + 3)*x^2))], x], x], x] /; FreeQ[{a, b, c, d, e
}, x] && NeQ[b^2 - 4*a*c, 0] && IGtQ[p, 0] && ILtQ[q, -1] && IGtQ[m/2, 0]
```

rule 2345
```
Int[(Pq_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{Q = PolynomialQuot
ient[Pq, a + b*x^2, x], f = Coeff[PolynomialRemainder[Pq, a + b*x^2, x], x,
 0], g = Coeff[PolynomialRemainder[Pq, a + b*x^2, x], x, 1]}, Simp[(a*g - b
*f*x)*((a + b*x^2)^(p + 1)/(2*a*b*(p + 1))), x] + Simp[1/(2*a*(p + 1))   In
t[(a + b*x^2)^(p + 1)*ExpandToSum[2*a*(p + 1)*Q + f*(2*p + 3), x], x], x]]
/; FreeQ[{a, b}, x] && PolyQ[Pq, x] && LtQ[p, -1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3696
```
Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^4)^(
p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Simp[ff^(m + 1
)/f   Subst[Int[x^m*((a + 2*a*ff^2*x^2 + (a + b)*ff^4*x^4)^p/(1 + ff^2*x^2)
^(m/2 + 2*p + 1)), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f}, x] &
& IntegerQ[m/2] && IntegerQ[p]
```

Maple [A] (verified)

Time = 49.20 (sec) , antiderivative size = 114, normalized size of antiderivative = 0.71

method	result
parallelrisch	$\frac{(7680a^2+14400ab+6300b^2)\sinh(2dx+2c)-2880b\left(a+\frac{5b}{8}\right)\sinh(4dx+4c)+(320ab+450b^2)\sinh(6dx+6c)+6b^2\sinh(10}{30720d}$
derivativedivides	$\frac{a^2\left(\frac{\cosh(dx+c)\sinh(dx+c)}{2}-\frac{dx}{2}-\frac{c}{2}\right)+2ab\left(\left(\frac{\sinh(dx+c)^5}{6}-\frac{5\sinh(dx+c)^3}{24}+\frac{5\sinh(dx+c)}{16}\right)\cosh(dx+c)-\frac{5dx}{16}-\frac{5c}{16}\right)+b^2\left(\left(\right.\right.}{d}$
default	$\frac{a^2\left(\frac{\cosh(dx+c)\sinh(dx+c)}{2}-\frac{dx}{2}-\frac{c}{2}\right)+2ab\left(\left(\frac{\sinh(dx+c)^5}{6}-\frac{5\sinh(dx+c)^3}{24}+\frac{5\sinh(dx+c)}{16}\right)\cosh(dx+c)-\frac{5dx}{16}-\frac{5c}{16}\right)+b^2\left(\left(\right.\right.}{d}$
parts	$\frac{a^2\left(\frac{\cosh(dx+c)\sinh(dx+c)}{2}-\frac{dx}{2}-\frac{c}{2}\right)}{d}+\frac{b^2\left(\left(\frac{\sinh(dx+c)^9}{10}-\frac{9\sinh(dx+c)^7}{80}+\frac{21\sinh(dx+c)^5}{160}-\frac{21\sinh(dx+c)^3}{128}+\frac{63\sinh(dx+c}{256}\right.\right.}{d}$
risch	$-\frac{a^2x}{2}-\frac{5abx}{8}-\frac{63b^2x}{256}+\frac{b^2e^{10dx+10c}}{10240d}-\frac{5b^2e^{8dx+8c}}{4096d}+\frac{b\,e^{6dx+6c}a}{192d}+\frac{15b^2e^{6dx+6c}}{2048d}-\frac{3\,e^{4dx+4c}ab}{64d}-\frac{15\,e}{}$
orering	Expression too large to display

input
```
int(sinh(d*x+c)^2*(a+b*sinh(d*x+c)^4)^2,x,method=_RETURNVERBOSE)
```

output
```
1/30720*((7680*a^2+14400*a*b+6300*b^2)*sinh(2*d*x+2*c)-2880*b*(a+5/8*b)*si
nh(4*d*x+4*c)+(320*a*b+450*b^2)*sinh(6*d*x+6*c)+6*b^2*sinh(10*d*x+10*c)-75
*b^2*sinh(8*d*x+8*c)-15360*(a^2+5/4*a*b+63/128*b^2)*d*x)/d
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 305 vs. $2(149) = 298$.

Time = 0.09 (sec) , antiderivative size = 305, normalized size of antiderivative = 1.89

$$\int \sinh^2(c+dx)\left(a+b\sinh^4(c+dx)\right)^2\,dx$$
$$=\frac{15\,b^2\cosh(dx+c)\sinh(dx+c)^9+30\left(6\,b^2\cosh(dx+c)^3-5\,b^2\cosh(dx+c)\right)\sinh(dx+c)^7+3\left(126\,b\right.}{}$$

input
```
integrate(sinh(d*x+c)^2*(a+b*sinh(d*x+c)^4)^2,x, algorithm="fricas")
```

output
```
1/7680*(15*b^2*cosh(d*x + c)*sinh(d*x + c)^9 + 30*(6*b^2*cosh(d*x + c)^3 -
 5*b^2*cosh(d*x + c))*sinh(d*x + c)^7 + 3*(126*b^2*cosh(d*x + c)^5 - 350*b
^2*cosh(d*x + c)^3 + 5*(32*a*b + 45*b^2)*cosh(d*x + c))*sinh(d*x + c)^5 +
10*(18*b^2*cosh(d*x + c)^7 - 105*b^2*cosh(d*x + c)^5 + 5*(32*a*b + 45*b^2)
*cosh(d*x + c)^3 - 36*(8*a*b + 5*b^2)*cosh(d*x + c))*sinh(d*x + c)^3 - 30*
(128*a^2 + 160*a*b + 63*b^2)*d*x + 15*(b^2*cosh(d*x + c)^9 - 10*b^2*cosh(d
*x + c)^7 + (32*a*b + 45*b^2)*cosh(d*x + c)^5 - 24*(8*a*b + 5*b^2)*cosh(d*
x + c)^3 + 2*(128*a^2 + 240*a*b + 105*b^2)*cosh(d*x + c))*sinh(d*x + c))/d
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 484 vs. $2(155) = 310$.

Time = 1.35 (sec) , antiderivative size = 484, normalized size of antiderivative = 3.01

$$\int \sinh^2(c+dx)\left(a+b\sinh^4(c+dx)\right)^2\,dx$$

$$= \begin{cases} \frac{a^2x\sinh^2(c+dx)}{2} - \frac{a^2x\cosh^2(c+dx)}{2} + \frac{a^2\sinh(c+dx)\cosh(c+dx)}{2d} + \frac{5abx\sinh^6(c+dx)}{8} - \frac{15abx\sinh^4(c+dx)\cosh^2(c+dx)}{8} + \frac{15a\ldots}{} \\ x\left(a+b\sinh^4(c)\right)^2\sinh^2(c) \end{cases}$$

input
```
integrate(sinh(d*x+c)**2*(a+b*sinh(d*x+c)**4)**2,x)
```

output
```
Piecewise((a**2*x*sinh(c + d*x)**2/2 - a**2*x*cosh(c + d*x)**2/2 + a**2*si
nh(c + d*x)*cosh(c + d*x)/(2*d) + 5*a*b*x*sinh(c + d*x)**6/8 - 15*a*b*x*si
nh(c + d*x)**4*cosh(c + d*x)**2/8 + 15*a*b*x*sinh(c + d*x)**2*cosh(c + d*x
)**4/8 - 5*a*b*x*cosh(c + d*x)**6/8 + 11*a*b*sinh(c + d*x)**5*cosh(c + d*x
)/(8*d) - 5*a*b*sinh(c + d*x)**3*cosh(c + d*x)**3/(3*d) + 5*a*b*sinh(c + d
*x)*cosh(c + d*x)**5/(8*d) + 63*b**2*x*sinh(c + d*x)**10/256 - 315*b**2*x*
sinh(c + d*x)**8*cosh(c + d*x)**2/256 + 315*b**2*x*sinh(c + d*x)**6*cosh(c
 + d*x)**4/128 - 315*b**2*x*sinh(c + d*x)**4*cosh(c + d*x)**6/128 + 315*b*
*2*x*sinh(c + d*x)**2*cosh(c + d*x)**8/256 - 63*b**2*x*cosh(c + d*x)**10/2
56 + 193*b**2*sinh(c + d*x)**9*cosh(c + d*x)/(256*d) - 237*b**2*sinh(c + d
*x)**7*cosh(c + d*x)**3/(128*d) + 21*b**2*sinh(c + d*x)**5*cosh(c + d*x)**
5/(10*d) - 147*b**2*sinh(c + d*x)**3*cosh(c + d*x)**7/(128*d) + 63*b**2*si
nh(c + d*x)*cosh(c + d*x)**9/(256*d), Ne(d, 0)), (x*(a + b*sinh(c)**4)**2*
sinh(c)**2, True))
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 260, normalized size of antiderivative = 1.61

$$\int \sinh^2(c+dx)\left(a+b\sinh^4(c+dx)\right)^2\,dx = -\frac{1}{8}a^2\left(4x-\frac{e^{(2dx+2c)}}{d}+\frac{e^{(-2dx-2c)}}{d}\right)$$
$$-\frac{1}{20480}b^2\left(\frac{\left(25e^{(-2dx-2c)}-150e^{(-4dx-4c)}+600e^{(-6dx-6c)}-2100e^{(-8dx-8c)}-2\right)e^{(10dx+10c)}}{d}+\frac{5040\,(\right.$$
$$-\frac{1}{192}ab\left(\frac{\left(9e^{(-2dx-2c)}-45e^{(-4dx-4c)}-1\right)e^{(6dx+6c)}}{d}+\frac{120\,(dx+c)}{d}+\frac{45e^{(-2dx-2c)}-9e^{(-4dx-4c)}+e}{d}\right.$$

input
```
integrate(sinh(d*x+c)^2*(a+b*sinh(d*x+c)^4)^2,x, algorithm="maxima")
```

output
```
-1/8*a^2*(4*x - e^(2*d*x + 2*c)/d + e^(-2*d*x - 2*c)/d) - 1/20480*b^2*((25
*e^(-2*d*x - 2*c) - 150*e^(-4*d*x - 4*c) + 600*e^(-6*d*x - 6*c) - 2100*e^(
-8*d*x - 8*c) - 2)*e^(10*d*x + 10*c)/d + 5040*(d*x + c)/d + (2100*e^(-2*d*
x - 2*c) - 600*e^(-4*d*x - 4*c) + 150*e^(-6*d*x - 6*c) - 25*e^(-8*d*x - 8*
c) + 2*e^(-10*d*x - 10*c))/d) - 1/192*a*b*((9*e^(-2*d*x - 2*c) - 45*e^(-4*
d*x - 4*c) - 1)*e^(6*d*x + 6*c)/d + 120*(d*x + c)/d + (45*e^(-2*d*x - 2*c)
 - 9*e^(-4*d*x - 4*c) + e^(-6*d*x - 6*c))/d)
```

Giac [A] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 241, normalized size of antiderivative = 1.50

$$\int \sinh^2(c+dx)\left(a+b\sinh^4(c+dx)\right)^2\,dx$$
$$= -\frac{1}{256}\left(128a^2+160ab+63b^2\right)x+\frac{b^2e^{(10dx+10c)}}{10240d}-\frac{5b^2e^{(8dx+8c)}}{4096d}+\frac{5b^2e^{(-8dx-8c)}}{4096d}$$
$$-\frac{b^2e^{(-10dx-10c)}}{10240d}+\frac{(32ab+45b^2)e^{(6dx+6c)}}{6144d}-\frac{3(8ab+5b^2)e^{(4dx+4c)}}{512d}$$
$$+\frac{(128a^2+240ab+105b^2)e^{(2dx+2c)}}{1024d}-\frac{(128a^2+240ab+105b^2)e^{(-2dx-2c)}}{1024d}$$
$$+\frac{3(8ab+5b^2)e^{(-4dx-4c)}}{512d}-\frac{(32ab+45b^2)e^{(-6dx-6c)}}{6144d}$$

input
```
integrate(sinh(d*x+c)^2*(a+b*sinh(d*x+c)^4)^2,x, algorithm="giac")
```

output
```
-1/256*(128*a^2 + 160*a*b + 63*b^2)*x + 1/10240*b^2*e^(10*d*x + 10*c)/d -
5/4096*b^2*e^(8*d*x + 8*c)/d + 5/4096*b^2*e^(-8*d*x - 8*c)/d - 1/10240*b^2
*e^(-10*d*x - 10*c)/d + 1/6144*(32*a*b + 45*b^2)*e^(6*d*x + 6*c)/d - 3/512
*(8*a*b + 5*b^2)*e^(4*d*x + 4*c)/d + 1/1024*(128*a^2 + 240*a*b + 105*b^2)*
e^(2*d*x + 2*c)/d - 1/1024*(128*a^2 + 240*a*b + 105*b^2)*e^(-2*d*x - 2*c)/
d + 3/512*(8*a*b + 5*b^2)*e^(-4*d*x - 4*c)/d - 1/6144*(32*a*b + 45*b^2)*e^
(-6*d*x - 6*c)/d
```

Mupad [B] (verification not implemented)

Time = 0.41 (sec) , antiderivative size = 149, normalized size of antiderivative = 0.93

$$\int \sinh^2(c+dx)\left(a+b\sinh^4(c+dx)\right)^2\,dx = \frac{960\,a^2\sinh(2\,c+2\,d\,x)+\frac{1575\,b^2\sinh(2\,c+2\,d\,x)}{2}-225\,b^2\sinh(4\,c+4\,d\,x)+\frac{225\,b^2\sinh(6\,c+6\,d\,x)}{4}-\frac{75\,b^2\sinh(8\,c+8\,d\,x)}{8}$$

input
```
int(sinh(c + d*x)^2*(a + b*sinh(c + d*x)^4)^2,x)
```

output
```
(960*a^2*sinh(2*c + 2*d*x) + (1575*b^2*sinh(2*c + 2*d*x))/2 - 225*b^2*sinh
(4*c + 4*d*x) + (225*b^2*sinh(6*c + 6*d*x))/4 - (75*b^2*sinh(8*c + 8*d*x))
/8 + (3*b^2*sinh(10*c + 10*d*x))/4 + 1800*a*b*sinh(2*c + 2*d*x) - 360*a*b*
sinh(4*c + 4*d*x) + 40*a*b*sinh(6*c + 6*d*x) - 1920*a^2*d*x - 945*b^2*d*x
- 2400*a*b*d*x)/(3840*d)
```

Reduce [B] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 322, normalized size of antiderivative = 2.00

$$\int \sinh^2(c+dx)\left(a+b\sinh^4(c+dx)\right)^2\,dx = \frac{6e^{20dx+20c}b^2-75e^{18dx+18c}b^2+320e^{16dx+16c}ab+450e^{16dx+16c}b^2-2880e^{14dx+14c}ab-1800e^{14dx+14c}b^2+76}{}$$

input
```
int(sinh(d*x+c)^2*(a+b*sinh(d*x+c)^4)^2,x)
```

output
```
(6*e**(20*c + 20*d*x)*b**2 - 75*e**(18*c + 18*d*x)*b**2 + 320*e**(16*c + 1
6*d*x)*a*b + 450*e**(16*c + 16*d*x)*b**2 - 2880*e**(14*c + 14*d*x)*a*b - 1
800*e**(14*c + 14*d*x)*b**2 + 7680*e**(12*c + 12*d*x)*a**2 + 14400*e**(12*
c + 12*d*x)*a*b + 6300*e**(12*c + 12*d*x)*b**2 - 30720*e**(10*c + 10*d*x)*
a**2*d*x - 38400*e**(10*c + 10*d*x)*a*b*d*x - 15120*e**(10*c + 10*d*x)*b**
2*d*x - 7680*e**(8*c + 8*d*x)*a**2 - 14400*e**(8*c + 8*d*x)*a*b - 6300*e**
(8*c + 8*d*x)*b**2 + 2880*e**(6*c + 6*d*x)*a*b + 1800*e**(6*c + 6*d*x)*b**
2 - 320*e**(4*c + 4*d*x)*a*b - 450*e**(4*c + 4*d*x)*b**2 + 75*e**(2*c + 2*
d*x)*b**2 - 6*b**2)/(61440*e**(10*c + 10*d*x)*d)
```

3.173 $\int \sinh(c+dx)\left(a+b\sinh^4(c+dx)\right)^2\,dx$

Optimal result

Integrand size = 21, antiderivative size = 92

$$\int \sinh(c+dx)\left(a+b\sinh^4(c+dx)\right)^2\,dx = \frac{(a+b)^2\cosh(c+dx)}{d} - \frac{4b(a+b)\cosh^3(c+dx)}{3d} + \frac{2b(a+3b)\cosh^5(c+dx)}{5d} - \frac{4b^2\cosh^7(c+dx)}{7d} + \frac{b^2\cosh^9(c+dx)}{9d}$$

output
```
(a+b)^2*cosh(d*x+c)/d-4/3*b*(a+b)*cosh(d*x+c)^3/d+2/5*b*(a+3*b)*cosh(d*x+c
)^5/d-4/7*b^2*cosh(d*x+c)^7/d+1/9*b^2*cosh(d*x+c)^9/d
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 164, normalized size of antiderivative = 1.78

$$\int \sinh(c+dx)\left(a+b\sinh^4(c+dx)\right)^2\,dx=\frac{a^2\cosh(c)\cosh(dx)}{d}+\frac{5ab\cosh(c+dx)}{4d}+\frac{63b^2\cosh(c+dx)}{128d}-\frac{5ab\cosh(3(c+dx))}{24d}-\frac{7b^2\cosh(3(c+dx))}{64d}+\frac{ab\cosh(5(c+dx))}{40d}+\frac{9b^2\cosh(5(c+dx))}{320d}-\frac{9b^2\cosh(7(c+dx))}{1792d}+\frac{b^2\cosh(9(c+dx))}{2304d}+\frac{a^2\sinh(c)\sinh(dx)}{d}$$

input
```
Integrate[Sinh[c + d*x]*(a + b*Sinh[c + d*x]^4)^2,x]
```

output
```
(a^2*Cosh[c]*Cosh[d*x])/d + (5*a*b*Cosh[c + d*x])/(4*d) + (63*b^2*Cosh[c +
 d*x])/(128*d) - (5*a*b*Cosh[3*(c + d*x)])/(24*d) - (7*b^2*Cosh[3*(c + d*x
)])/(64*d) + (a*b*Cosh[5*(c + d*x)])/(40*d) + (9*b^2*Cosh[5*(c + d*x)])/(3
20*d) - (9*b^2*Cosh[7*(c + d*x)])/(1792*d) + (b^2*Cosh[9*(c + d*x)])/(2304
*d) + (a^2*Sinh[c]*Sinh[d*x])/d
```

Rubi [A] (verified)

Time = 0.31 (sec) , antiderivative size = 81, normalized size of antiderivative = 0.88, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.238, Rules used = {3042, 26, 3694, 1403, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sinh(c+dx)\left(a+b\sinh^4(c+dx)\right)^2\,dx$$

$$\Big\downarrow \ 3042$$

$$\int -i\sin(ic+idx)\left(a+b\sin(ic+idx)^4\right)^2dx$$

$$\Big\downarrow \ 26$$

$$-i\int \sin(ic+idx)\left(b\sin(ic+idx)^4+a\right)^2dx$$

$$\Big\downarrow \ 3694$$

$$\frac{\int \left(b\cosh^4(c+dx)-2b\cosh^2(c+dx)+a+b\right)^2d\cosh(c+dx)}{d}$$

$$\Big\downarrow \ 1403$$

$$\frac{\int \left(b^2\cosh^8(c+dx)-4b^2\cosh^6(c+dx)+2ab\left(\frac{3b}{a}+1\right)\cosh^4(c+dx)-4ab\left(\frac{b}{a}+1\right)\cosh^2(c+dx)+a^2\left(\frac{b(2a+b)}{a^2}+\right.\right.}{d}$$

$$\Big\downarrow \ 2009$$

$$\frac{\frac{2}{5}b(a+3b)\cosh^5(c+dx)-\frac{4}{3}b(a+b)\cosh^3(c+dx)+(a+b)^2\cosh(c+dx)+\frac{1}{9}b^2\cosh^9(c+dx)-\frac{4}{7}b^2\cosh^7(c+}{d}$$

input
```
Int[Sinh[c + d*x]*(a + b*Sinh[c + d*x]^4)^2,x]
```

output
```
((a + b)^2*Cosh[c + d*x] - (4*b*(a + b)*Cosh[c + d*x]^3)/3 + (2*b*(a + 3*b
)*Cosh[c + d*x]^5)/5 - (4*b^2*Cosh[c + d*x]^7)/7 + (b^2*Cosh[c + d*x]^9)/9
)/d
```

Defintions of rubi rules used

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 1403
```
Int[((a_.) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Int[ExpandInte
grand[(a + b*x^2 + c*x^4)^p, x], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a
*c, 0] && IGtQ[p, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3694
```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^4)^
(p_.), x_Symbol] :> With[{ff = FreeFactors[Cos[e + f*x], x]}, Simp[-ff/f
Subst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a + b - 2*b*ff^2*x^2 + b*ff^4*x^4)^p,
 x], x, Cos[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m -
 1)/2]
```

Maple [A] (verified)

Time = 18.25 (sec) , antiderivative size = 100, normalized size of antiderivative = 1.09

method	result
derivativedivides	$\frac{a^2\cosh(dx+c)+2ab\left(\frac{8}{15}+\frac{\sinh(dx+c)^4}{5}-\frac{4\sinh(dx+c)^2}{15}\right)\cosh(dx+c)+b^2\left(\frac{128}{315}+\frac{\sinh(dx+c)^8}{9}-\frac{8\sinh(dx+c)^6}{63}+\frac{16\sinh(dx+}{105}\right.}{d}$
default	$\frac{a^2\cosh(dx+c)+2ab\left(\frac{8}{15}+\frac{\sinh(dx+c)^4}{5}-\frac{4\sinh(dx+c)^2}{15}\right)\cosh(dx+c)+b^2\left(\frac{128}{315}+\frac{\sinh(dx+c)^8}{9}-\frac{8\sinh(dx+c)^6}{63}+\frac{16\sinh(dx+}{105}\right.}{d}$
parallelrisch	$\frac{-16800\left(a+\frac{21b}{40}\right)b\cosh(3dx+3c)+2016\left(a+\frac{9b}{8}\right)b\cosh(5dx+5c)-405b^2\cosh(7dx+7c)+35b^2\cosh(9dx+9c)+(80640a^2}{80640d}$
parts	$\frac{b^2\left(\frac{128}{315}+\frac{\sinh(dx+c)^8}{9}-\frac{8\sinh(dx+c)^6}{63}+\frac{16\sinh(dx+c)^4}{105}-\frac{64\sinh(dx+c)^2}{315}\right)\cosh(dx+c)}{d}+\frac{a^2\cosh(dx+c)}{d}+\frac{2ab\left(\frac{8}{15}+\right.}{}$
risch	$\frac{b^2e^{9dx+9c}}{4608d}-\frac{9b^2e^{7dx+7c}}{3584d}+\frac{b\,e^{5dx+5c}a}{80d}+\frac{9b^2e^{5dx+5c}}{640d}-\frac{5\,e^{3dx+3c}ab}{48d}-\frac{7\,e^{3dx+3c}b^2}{128d}+\frac{e^{dx+c}a^2}{2d}+\frac{5\,e^{dx+c}a}{8d}$

input
```
int(sinh(d*x+c)*(a+b*sinh(d*x+c)^4)^2,x,method=_RETURNVERBOSE)
```

output
```
1/d*(a^2*cosh(d*x+c)+2*a*b*(8/15+1/5*sinh(d*x+c)^4-4/15*sinh(d*x+c)^2)*cos
h(d*x+c)+b^2*(128/315+1/9*sinh(d*x+c)^8-8/63*sinh(d*x+c)^6+16/105*sinh(d*x
+c)^4-64/315*sinh(d*x+c)^2)*cosh(d*x+c))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 279 vs. $2(84) = 168$.

Time = 0.10 (sec) , antiderivative size = 279, normalized size of antiderivative = 3.03

$$\int \sinh(c+dx)\left(a+b\sinh^4(c+dx)\right)^2\,dx$$
$$=\frac{35\,b^2\cosh\left(dx+c\right)^9+315\,b^2\cosh\left(dx+c\right)\sinh\left(dx+c\right)^8-405\,b^2\cosh\left(dx+c\right)^7+105\,\left(28\,b^2\cosh\left(dx+\right.\right.}{}$$

input
```
integrate(sinh(d*x+c)*(a+b*sinh(d*x+c)^4)^2,x, algorithm="fricas")
```

output
```
1/80640*(35*b^2*cosh(d*x + c)^9 + 315*b^2*cosh(d*x + c)*sinh(d*x + c)^8 -
405*b^2*cosh(d*x + c)^7 + 105*(28*b^2*cosh(d*x + c)^3 - 27*b^2*cosh(d*x +
c))*sinh(d*x + c)^6 + 252*(8*a*b + 9*b^2)*cosh(d*x + c)^5 + 315*(14*b^2*co
sh(d*x + c)^5 - 45*b^2*cosh(d*x + c)^3 + 4*(8*a*b + 9*b^2)*cosh(d*x + c))*
sinh(d*x + c)^4 - 420*(40*a*b + 21*b^2)*cosh(d*x + c)^3 + 315*(4*b^2*cosh(
d*x + c)^7 - 27*b^2*cosh(d*x + c)^5 + 8*(8*a*b + 9*b^2)*cosh(d*x + c)^3 -
4*(40*a*b + 21*b^2)*cosh(d*x + c))*sinh(d*x + c)^2 + 630*(128*a^2 + 160*a*
b + 63*b^2)*cosh(d*x + c))/d
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 204 vs. $2(83) = 166$.

Time = 0.94 (sec) , antiderivative size = 204, normalized size of antiderivative = 2.22

$$\int \sinh(c+dx)\left(a+b\sinh^4(c+dx)\right)^2\,dx$$
$$=\begin{cases}\frac{a^2\cosh\left(c+dx\right)}{d}+\frac{2ab\sinh^4\left(c+dx\right)\cosh\left(c+dx\right)}{d}-\frac{8ab\sinh^2\left(c+dx\right)\cosh^3\left(c+dx\right)}{3d}+\frac{16ab\cosh^5\left(c+dx\right)}{15d}+\frac{b^2\sinh^8\left(c+dx\right)\cosh\left(c+dx\right)}{d} \\ x\left(a+b\sinh^4\left(c\right)\right)^2\sinh\left(c\right)\end{cases}$$

input
```
integrate(sinh(d*x+c)*(a+b*sinh(d*x+c)**4)**2,x)
```

output
```
Piecewise((a**2*cosh(c + d*x)/d + 2*a*b*sinh(c + d*x)**4*cosh(c + d*x)/d -
 8*a*b*sinh(c + d*x)**2*cosh(c + d*x)**3/(3*d) + 16*a*b*cosh(c + d*x)**5/(
15*d) + b**2*sinh(c + d*x)**8*cosh(c + d*x)/d - 8*b**2*sinh(c + d*x)**6*co
sh(c + d*x)**3/(3*d) + 16*b**2*sinh(c + d*x)**4*cosh(c + d*x)**5/(5*d) - 6
4*b**2*sinh(c + d*x)**2*cosh(c + d*x)**7/(35*d) + 128*b**2*cosh(c + d*x)**
9/(315*d), Ne(d, 0)), (x*(a + b*sinh(c)**4)**2*sinh(c), True))
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 226 vs. $2(84) = 168$.

Time = 0.04 (sec) , antiderivative size = 226, normalized size of antiderivative = 2.46

$$\int \sinh(c+dx)\left(a+b\sinh^4(c+dx)\right)^2 dx = -\frac{1}{161280}b^2\left(\frac{\left(405\,e^{(-2\,dx-2\,c)}-2268\,e^{(-4\,dx-4\,c)}+8820\,e^{(-6\,dx-6\,c)}-39690\,e^{(-8\,dx-8\,c)}-35\right)e^{(9\,dx+9\,c)}}{d}-\right. + \frac{1}{240}ab\left(\frac{3\,e^{(5\,dx+5\,c)}}{d}-\frac{25\,e^{(3\,dx+3\,c)}}{d}+\frac{150\,e^{(dx+c)}}{d}+\frac{150\,e^{(-dx-c)}}{d}-\frac{25\,e^{(-3\,dx-3\,c)}}{d}+\frac{3\,e^{(-5\,dx-5\,c)}}{d}\right) + \frac{a^2\cosh(dx+c)}{d}$$

input
```
integrate(sinh(d*x+c)*(a+b*sinh(d*x+c)^4)^2,x, algorithm="maxima")
```

output
```
-1/161280*b^2*((405*e^(-2*d*x - 2*c) - 2268*e^(-4*d*x - 4*c) + 8820*e^(-6*
d*x - 6*c) - 39690*e^(-8*d*x - 8*c) - 35)*e^(9*d*x + 9*c)/d - (39690*e^(-d
*x - c) - 8820*e^(-3*d*x - 3*c) + 2268*e^(-5*d*x - 5*c) - 405*e^(-7*d*x -
7*c) + 35*e^(-9*d*x - 9*c))/d) + 1/240*a*b*(3*e^(5*d*x + 5*c)/d - 25*e^(3*
d*x + 3*c)/d + 150*e^(d*x + c)/d + 150*e^(-d*x - c)/d - 25*e^(-3*d*x - 3*c
)/d + 3*e^(-5*d*x - 5*c)/d) + a^2*cosh(d*x + c)/d
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 220 vs. $2(84) = 168$.

Time = 0.17 (sec) , antiderivative size = 220, normalized size of antiderivative = 2.39

$$\int \sinh(c+dx)\left(a+b\sinh^4(c+dx)\right)^2\,dx$$
$$= \frac{b^2e^{(9\,dx+9\,c)}}{4608\,d} - \frac{9\,b^2e^{(7\,dx+7\,c)}}{3584\,d} - \frac{9\,b^2e^{(-7\,dx-7\,c)}}{3584\,d} + \frac{b^2e^{(-9\,dx-9\,c)}}{4608\,d} + \frac{(8\,ab+9\,b^2)e^{(5\,dx+5\,c)}}{640\,d} - \frac{(40\,ab+21\,b^2)e^{(3\,dx+3\,c)}}{384\,d} + \frac{(128\,a^2+160\,ab+63\,b^2)e^{(dx+c)}}{256\,d} + \frac{(128\,a^2+160\,ab+63\,b^2)e^{(-dx-c)}}{256\,d} - \frac{(40\,ab+21\,b^2)e^{(-3\,dx-3\,c)}}{384\,d} + \frac{(8\,ab+9\,b^2)e^{(-5\,dx-5\,c)}}{640\,d}$$

input
```
integrate(sinh(d*x+c)*(a+b*sinh(d*x+c)^4)^2,x, algorithm="giac")
```

output
```
1/4608*b^2*e^(9*d*x + 9*c)/d - 9/3584*b^2*e^(7*d*x + 7*c)/d - 9/3584*b^2*e
^(-7*d*x - 7*c)/d + 1/4608*b^2*e^(-9*d*x - 9*c)/d + 1/640*(8*a*b + 9*b^2)*
e^(5*d*x + 5*c)/d - 1/384*(40*a*b + 21*b^2)*e^(3*d*x + 3*c)/d + 1/256*(128
*a^2 + 160*a*b + 63*b^2)*e^(d*x + c)/d + 1/256*(128*a^2 + 160*a*b + 63*b^2
)*e^(-d*x - c)/d - 1/384*(40*a*b + 21*b^2)*e^(-3*d*x - 3*c)/d + 1/640*(8*a
*b + 9*b^2)*e^(-5*d*x - 5*c)/d
```

Mupad [B] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 111, normalized size of antiderivative = 1.21

$$\int \sinh(c+dx)\left(a+b\sinh^4(c+dx)\right)^2\,dx$$
$$= \frac{a^2\cosh(c+d\,x) + \frac{2\,a\,b\cosh(c+d\,x)^5}{5} - \frac{4\,a\,b\cosh(c+d\,x)^3}{3} + 2\,a\,b\cosh(c+d\,x) + \frac{b^2\cosh(c+d\,x)^9}{9} - \frac{4\,b^2\cosh(c+d\,x)^7}{7} + \cdots}{d}$$

input
```
int(sinh(c + d*x)*(a + b*sinh(c + d*x)^4)^2,x)
```

output
```
(a^2*cosh(c + d*x) + b^2*cosh(c + d*x) - (4*b^2*cosh(c + d*x)^3)/3 + (6*b^
2*cosh(c + d*x)^5)/5 - (4*b^2*cosh(c + d*x)^7)/7 + (b^2*cosh(c + d*x)^9)/9
 + 2*a*b*cosh(c + d*x) - (4*a*b*cosh(c + d*x)^3)/3 + (2*a*b*cosh(c + d*x)^
5)/5)/d
```

Reduce [B] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 272, normalized size of antiderivative = 2.96

$$\int \sinh(c+dx)\left(a+b\sinh^4(c+dx)\right)^2\,dx = \frac{35e^{18dx+18c}b^2-405e^{16dx+16c}b^2+2016e^{14dx+14c}ab+2268e^{14dx+14c}b^2-16800e^{12dx+12c}ab-8820e^{12dx+12c}b^2}{}$$

input
```
int(sinh(d*x+c)*(a+b*sinh(d*x+c)^4)^2,x)
```

output
```
(35*e**(18*c + 18*d*x)*b**2 - 405*e**(16*c + 16*d*x)*b**2 + 2016*e**(14*c
+ 14*d*x)*a*b + 2268*e**(14*c + 14*d*x)*b**2 - 16800*e**(12*c + 12*d*x)*a*
b - 8820*e**(12*c + 12*d*x)*b**2 + 80640*e**(10*c + 10*d*x)*a**2 + 100800*
e**(10*c + 10*d*x)*a*b + 39690*e**(10*c + 10*d*x)*b**2 + 80640*e**(8*c + 8
*d*x)*a**2 + 100800*e**(8*c + 8*d*x)*a*b + 39690*e**(8*c + 8*d*x)*b**2 - 1
6800*e**(6*c + 6*d*x)*a*b - 8820*e**(6*c + 6*d*x)*b**2 + 2016*e**(4*c + 4*
d*x)*a*b + 2268*e**(4*c + 4*d*x)*b**2 - 405*e**(2*c + 2*d*x)*b**2 + 35*b**
2)/(161280*e**(9*c + 9*d*x)*d)
```

3.174 $\int \left(a+b\sinh^4(c+dx)\right)^2\,dx$

Optimal result

Integrand size = 14, antiderivative size = 125

$$\begin{aligned}\int \left(a+b\sinh^4(c+dx)\right)^2\,dx=&\ \frac{1}{128}\left(128a^2+96ab+35b^2\right)x\\&-\frac{b(160a+93b)\cosh(c+dx)\sinh(c+dx)}{128d}\\&+\frac{b(96a+163b)\cosh^3(c+dx)\sinh(c+dx)}{192d}\\&-\frac{25b^2\cosh^5(c+dx)\sinh(c+dx)}{48d}\\&+\frac{b^2\cosh^7(c+dx)\sinh(c+dx)}{8d}\end{aligned}$$

output
```
1/128*(128*a^2+96*a*b+35*b^2)*x-1/128*b*(160*a+93*b)*cosh(d*x+c)*sinh(d*x+
c)/d+1/192*b*(96*a+163*b)*cosh(d*x+c)^3*sinh(d*x+c)/d-25/48*b^2*cosh(d*x+c
)^5*sinh(d*x+c)/d+1/8*b^2*cosh(d*x+c)^7*sinh(d*x+c)/d
```

Mathematica [A] (verified)

Time = 0.25 (sec) , antiderivative size = 92, normalized size of antiderivative = 0.74

$$\int \left(a+b\sinh^4(c+dx)\right)^2\,dx$$
$$=\frac{24(128a^2+96ab+35b^2)\,(c+dx)-96b(16a+7b)\sinh(2(c+dx))+24b(8a+7b)\sinh(4(c+dx))-32b}{3072d}$$

input
```
Integrate[(a + b*Sinh[c + d*x]^4)^2,x]
```

output
```
(24*(128*a^2 + 96*a*b + 35*b^2)*(c + d*x) - 96*b*(16*a + 7*b)*Sinh[2*(c +
d*x)] + 24*b*(8*a + 7*b)*Sinh[4*(c + d*x)] - 32*b^2*Sinh[6*(c + d*x)] + 3*
b^2*Sinh[8*(c + d*x)])/(3072*d)
```

Rubi [A] (verified)

Time = 0.49 (sec) , antiderivative size = 164, normalized size of antiderivative = 1.31, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.714$, Rules used = {3042, 3688, 1471, 25, 2345, 25, 1471, 27, 298, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(a+b\sinh^4(c+dx)\right)^2\,dx$$

$$\Big\downarrow \ 3042$$

$$\int \left(a+b\sin(ic+idx)^4\right)^2dx$$

$$\Big\downarrow \ 3688$$

$$\frac{\int \frac{\left((a+b)\tanh^4(c+dx)-2a\tanh^2(c+dx)+a\right)^2}{\left(1-\tanh^2(c+dx)\right)^5}d\tanh(c+dx)}{d}$$

$$\Big\downarrow \ 1471$$

$$\frac{\frac{b^2\tanh(c+dx)}{8(1-\tanh^2(c+dx))^4}-\frac{1}{8}\int -\frac{-8(a+b)^2\tanh^6(c+dx)+8(3a-b)(a+b)\tanh^4(c+dx)-8(3a^2+b^2)\tanh^2(c+dx)+8a^2-b^2}{(1-\tanh^2(c+dx))^4}d\tanh(c+dx)}{d}$$

$\downarrow$ 25

$$\frac{\frac{1}{8}\int \frac{-8(a+b)^2\tanh^6(c+dx)+8(3a-b)(a+b)\tanh^4(c+dx)-8(3a^2+b^2)\tanh^2(c+dx)+8a^2-b^2}{(1-\tanh^2(c+dx))^4}d\tanh(c+dx)+\frac{b^2\tanh(c+dx)}{8(1-\tanh^2(c+dx))^4}}{d}$$

$\downarrow$ 2345

$$\frac{\frac{1}{8}\left(-\frac{1}{6}\int -\frac{48(a+b)^2\tanh^4(c+dx)-96(a^2-b^2)\tanh^2(c+dx)+48a^2+19b^2}{(1-\tanh^2(c+dx))^3}d\tanh(c+dx)-\frac{25b^2\tanh(c+dx)}{6(1-\tanh^2(c+dx))^3}\right)+\frac{b^2\tanh(c+dx)}{8(1-\tanh^2(c+dx))^4}}{d}$$

$\downarrow$ 25

$$\frac{\frac{1}{8}\left(\frac{1}{6}\int \frac{48(a+b)^2\tanh^4(c+dx)-96(a^2-b^2)\tanh^2(c+dx)+48a^2+19b^2}{(1-\tanh^2(c+dx))^3}d\tanh(c+dx)-\frac{25b^2\tanh(c+dx)}{6(1-\tanh^2(c+dx))^3}\right)+\frac{b^2\tanh(c+dx)}{8(1-\tanh^2(c+dx))^4}}{d}$$

$\downarrow$ 1471

$$\frac{\frac{1}{8}\left(\frac{1}{6}\left(\frac{b(96a+163b)\tanh(c+dx)}{4(1-\tanh^2(c+dx))^2}-\frac{1}{4}\int -\frac{3(64a^2-32ba-29b^2-64(a+b)^2\tanh^2(c+dx))}{(1-\tanh^2(c+dx))^2}d\tanh(c+dx)\right)-\frac{25b^2\tanh(c+dx)}{6(1-\tanh^2(c+dx))^3}\right)+\frac{b^2\tanh(c+dx)}{8(1-\tanh^2(c+dx))^4}}{d}$$

$\downarrow$ 27

$$\frac{\frac{1}{8}\left(\frac{1}{6}\left(\frac{3}{4}\int \frac{64a^2-32ba-29b^2-64(a+b)^2\tanh^2(c+dx)}{(1-\tanh^2(c+dx))^2}d\tanh(c+dx)+\frac{b(96a+163b)\tanh(c+dx)}{4(1-\tanh^2(c+dx))^2}\right)-\frac{25b^2\tanh(c+dx)}{6(1-\tanh^2(c+dx))^3}\right)+\frac{b^2\tanh(c+dx)}{8(1-\tanh^2(c+dx))^4}}{d}$$

$\downarrow$ 298

$$\frac{\frac{1}{8}\left(\frac{1}{6}\left(\frac{3}{4}\left(\frac{1}{2}(128a^2+96ab+35b^2)\int \frac{1}{1-\tanh^2(c+dx)}d\tanh(c+dx)-\frac{b(160a+93b)\tanh(c+dx)}{2(1-\tanh^2(c+dx))}\right)+\frac{b(96a+163b)\tanh(c+dx)}{4(1-\tanh^2(c+dx))^2}\right)-\frac{25b^2\tanh(c+dx)}{6(1-\tanh^2(c+dx))^3}\right)+\frac{b^2\tanh(c+dx)}{8(1-\tanh^2(c+dx))^4}}{d}$$

$\downarrow$ 219

$$\frac{\frac{1}{8}\left(\frac{1}{6}\left(\frac{3}{4}\left(\frac{1}{2}(128a^2+96ab+35b^2)\text{arctanh}(\tanh(c+dx))-\frac{b(160a+93b)\tanh(c+dx)}{2(1-\tanh^2(c+dx))}\right)+\frac{b(96a+163b)\tanh(c+dx)}{4(1-\tanh^2(c+dx))^2}\right)-\frac{25b^2\tanh(c+dx)}{6(1-\tanh^2(c+dx))^3}\right)+\frac{b^2\tanh(c+dx)}{8(1-\tanh^2(c+dx))^4}}{d}$$

input
```
Int[(a + b*Sinh[c + d*x]^4)^2,x]
```

output
```
((b^2*Tanh[c + d*x])/(8*(1 - Tanh[c + d*x]^2)^4) + ((-25*b^2*Tanh[c + d*x]
)/(6*(1 - Tanh[c + d*x]^2)^3) + ((b*(96*a + 163*b)*Tanh[c + d*x])/(4*(1 -
Tanh[c + d*x]^2)^2) + (3*(((128*a^2 + 96*a*b + 35*b^2)*ArcTanh[Tanh[c + d*
x]])/2 - (b*(160*a + 93*b)*Tanh[c + d*x])/(2*(1 - Tanh[c + d*x]^2))))/4)/6
)/8)/d
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 298
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2), x_Symbol] :> Simp[(-(
b*c - a*d))*x*((a + b*x^2)^(p + 1)/(2*a*b*(p + 1))), x] - Simp[(a*d - b*c*(
2*p + 3))/(2*a*b*(p + 1))   Int[(a + b*x^2)^(p + 1), x], x] /; FreeQ[{a, b,
 c, d, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/2 + p, 0])
```

rule 1471
```
Int[((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.),
x_Symbol] :> With[{Qx = PolynomialQuotient[(a + b*x^2 + c*x^4)^p, d + e*x^2
, x], R = Coeff[PolynomialRemainder[(a + b*x^2 + c*x^4)^p, d + e*x^2, x], x
, 0]}, Simp[(-R)*x*((d + e*x^2)^(q + 1)/(2*d*(q + 1))), x] + Simp[1/(2*d*(q
 + 1))   Int[(d + e*x^2)^(q + 1)*ExpandToSum[2*d*(q + 1)*Qx + R*(2*q + 3),
x], x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^
2 - b*d*e + a*e^2, 0] && IGtQ[p, 0] && LtQ[q, -1]
```

rule 2345
```
Int[(Pq_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{Q = PolynomialQuot
ient[Pq, a + b*x^2, x], f = Coeff[PolynomialRemainder[Pq, a + b*x^2, x], x,
 0], g = Coeff[PolynomialRemainder[Pq, a + b*x^2, x], x, 1]}, Simp[(a*g - b
*f*x)*((a + b*x^2)^(p + 1)/(2*a*b*(p + 1))), x] + Simp[1/(2*a*(p + 1))   In
t[(a + b*x^2)^(p + 1)*ExpandToSum[2*a*(p + 1)*Q + f*(2*p + 3), x], x], x]]
/; FreeQ[{a, b}, x] && PolyQ[Pq, x] && LtQ[p, -1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3688
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^4)^(p_.), x_Symbol] :> With[{ff =
 FreeFactors[Tan[e + f*x], x]}, Simp[ff/f   Subst[Int[(a + 2*a*ff^2*x^2 + (
a + b)*ff^4*x^4)^p/(1 + ff^2*x^2)^(2*p + 1), x], x, Tan[e + f*x]/ff], x]] /
; FreeQ[{a, b, e, f}, x] && IntegerQ[p]
```

Maple [A] (verified)

Time = 0.19 (sec) , antiderivative size = 111, normalized size of antiderivative = 0.89

$$\frac{b^2\left(\left(\frac{\sinh(dx+c)^7}{8}-\frac{7\sinh(dx+c)^5}{48}+\frac{35\sinh(dx+c)^3}{192}-\frac{35\sinh(dx+c)}{128}\right)\cosh\left(dx+c\right)+\frac{35dx}{128}+\frac{35c}{128}\right)+2ab\left(\left(\frac{\sinh(dx+c)^3}{4}\right.\right.}{d}$$

input
```
int((a+b*sinh(d*x+c)^4)^2,x)
```

output
```
1/d*(b^2*((1/8*sinh(d*x+c)^7-7/48*sinh(d*x+c)^5+35/192*sinh(d*x+c)^3-35/12
8*sinh(d*x+c))*cosh(d*x+c)+35/128*d*x+35/128*c)+2*a*b*((1/4*sinh(d*x+c)^3-
3/8*sinh(d*x+c))*cosh(d*x+c)+3/8*d*x+3/8*c)+a^2*(d*x+c))
```

Fricas [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 205, normalized size of antiderivative = 1.64

$$\int \left(a+b\sinh^4(c+dx)\right)^2\,dx$$
$$=\frac{3\,b^2\cosh\left(dx+c\right)\sinh\left(dx+c\right)^7+3\left(7\,b^2\cosh\left(dx+c\right)^3-8\,b^2\cosh\left(dx+c\right)\right)\sinh\left(dx+c\right)^5+\left(21\,b^2\cosh\right.}{}$$

input
```
integrate((a+b*sinh(d*x+c)^4)^2,x, algorithm="fricas")
```

output
```
1/384*(3*b^2*cosh(d*x + c)*sinh(d*x + c)^7 + 3*(7*b^2*cosh(d*x + c)^3 - 8*
b^2*cosh(d*x + c))*sinh(d*x + c)^5 + (21*b^2*cosh(d*x + c)^5 - 80*b^2*cosh
(d*x + c)^3 + 12*(8*a*b + 7*b^2)*cosh(d*x + c))*sinh(d*x + c)^3 + 3*(128*a
^2 + 96*a*b + 35*b^2)*d*x + 3*(b^2*cosh(d*x + c)^7 - 8*b^2*cosh(d*x + c)^5
 + 4*(8*a*b + 7*b^2)*cosh(d*x + c)^3 - 8*(16*a*b + 7*b^2)*cosh(d*x + c))*s
inh(d*x + c))/d
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 332 vs. 2(116) = 232.

Time = 0.70 (sec) , antiderivative size = 332, normalized size of antiderivative = 2.66

$$\int \left(a+b\sinh^4(c+dx)\right)^2\,dx$$
$$=\begin{cases} a^2x+\frac{3abx\sinh^4(c+dx)}{4}-\frac{3abx\sinh^2(c+dx)\cosh^2(c+dx)}{2}+\frac{3abx\cosh^4(c+dx)}{4}+\frac{5ab\sinh^3(c+dx)\cosh(c+dx)}{4d}-\frac{3ab\sinh(c+dx}{4} \\ x\left(a+b\sinh^4(c)\right)^2 \end{cases}$$

input
```
integrate((a+b*sinh(d*x+c)**4)**2,x)
```

output
```
Piecewise((a**2*x + 3*a*b*x*sinh(c + d*x)**4/4 - 3*a*b*x*sinh(c + d*x)**2*
cosh(c + d*x)**2/2 + 3*a*b*x*cosh(c + d*x)**4/4 + 5*a*b*sinh(c + d*x)**3*c
osh(c + d*x)/(4*d) - 3*a*b*sinh(c + d*x)*cosh(c + d*x)**3/(4*d) + 35*b**2*
x*sinh(c + d*x)**8/128 - 35*b**2*x*sinh(c + d*x)**6*cosh(c + d*x)**2/32 +
105*b**2*x*sinh(c + d*x)**4*cosh(c + d*x)**4/64 - 35*b**2*x*sinh(c + d*x)*
*2*cosh(c + d*x)**6/32 + 35*b**2*x*cosh(c + d*x)**8/128 + 93*b**2*sinh(c +
 d*x)**7*cosh(c + d*x)/(128*d) - 511*b**2*sinh(c + d*x)**5*cosh(c + d*x)**
3/(384*d) + 385*b**2*sinh(c + d*x)**3*cosh(c + d*x)**5/(384*d) - 35*b**2*s
inh(c + d*x)*cosh(c + d*x)**7/(128*d), Ne(d, 0)), (x*(a + b*sinh(c)**4)**2
, True))
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 183, normalized size of antiderivative = 1.46

$$\int \left(a+b\sinh^4(c+dx)\right)^2\,dx$$
$$=\frac{1}{32}\,ab\left(24\,x+\frac{e^{(4\,dx+4\,c)}}{d}-\frac{8\,e^{(2\,dx+2\,c)}}{d}+\frac{8\,e^{(-2\,dx-2\,c)}}{d}-\frac{e^{(-4\,dx-4\,c)}}{d}\right)+a^2x$$
$$-\frac{1}{6144}\,b^2\left(\frac{\left(32\,e^{(-2\,dx-2\,c)}-168\,e^{(-4\,dx-4\,c)}+672\,e^{(-6\,dx-6\,c)}-3\right)e^{(8\,dx+8\,c)}}{d}-\frac{1680\,(dx+c)}{d}-\frac{672\,e^{(-2\,dx}}{}\right.$$

input
```
integrate((a+b*sinh(d*x+c)^4)^2,x, algorithm="maxima")
```

output
```
1/32*a*b*(24*x + e^(4*d*x + 4*c)/d - 8*e^(2*d*x + 2*c)/d + 8*e^(-2*d*x - 2
*c)/d - e^(-4*d*x - 4*c)/d) + a^2*x - 1/6144*b^2*((32*e^(-2*d*x - 2*c) - 1
68*e^(-4*d*x - 4*c) + 672*e^(-6*d*x - 6*c) - 3)*e^(8*d*x + 8*c)/d - 1680*(
d*x + c)/d - (672*e^(-2*d*x - 2*c) - 168*e^(-4*d*x - 4*c) + 32*e^(-6*d*x -
 6*c) - 3*e^(-8*d*x - 8*c))/d)
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 183, normalized size of antiderivative = 1.46

$$\int \left(a+b\sinh^4(c+dx)\right)^2\,dx=\frac{1}{128}\left(128\,a^2+96\,ab+35\,b^2\right)x+\frac{b^2e^{(8\,dx+8\,c)}}{2048\,d}-\frac{b^2e^{(6\,dx+6\,c)}}{192\,d}+\frac{b^2e^{(-6\,dx-6\,c)}}{192\,d}-\frac{b^2e^{(-8\,dx-8\,c)}}{2048\,d}+\frac{(8\,ab+7\,b^2)e^{(4\,dx+4\,c)}}{256\,d}-\frac{(16\,ab+7\,b^2)e^{(2\,dx+2\,c)}}{64\,d}+\frac{(16\,ab+7\,b^2)e^{(-2\,dx-2\,c)}}{64\,d}-\frac{(8\,ab+7\,b^2)e^{(-4\,dx-4\,c)}}{256\,d}$$

input
```
integrate((a+b*sinh(d*x+c)^4)^2,x, algorithm="giac")
```

output
```
1/128*(128*a^2 + 96*a*b + 35*b^2)*x + 1/2048*b^2*e^(8*d*x + 8*c)/d - 1/192
*b^2*e^(6*d*x + 6*c)/d + 1/192*b^2*e^(-6*d*x - 6*c)/d - 1/2048*b^2*e^(-8*d
*x - 8*c)/d + 1/256*(8*a*b + 7*b^2)*e^(4*d*x + 4*c)/d - 1/64*(16*a*b + 7*b
^2)*e^(2*d*x + 2*c)/d + 1/64*(16*a*b + 7*b^2)*e^(-2*d*x - 2*c)/d - 1/256*(
8*a*b + 7*b^2)*e^(-4*d*x - 4*c)/d
```

Mupad [B] (verification not implemented)

Time = 1.75 (sec) , antiderivative size = 108, normalized size of antiderivative = 0.86

$$\int \left(a+b\sinh^4(c+dx)\right)^2\,dx$$
$$=\frac{21\,b^2\sinh(4\,c+4\,d\,x)-84\,b^2\sinh(2\,c+2\,d\,x)-4\,b^2\sinh(6\,c+6\,d\,x)+\frac{3\,b^2\sinh(8\,c+8\,d\,x)}{8}-192\,a\,b\sinh(2\,c}{384\,d}$$

input
```
int((a + b*sinh(c + d*x)^4)^2,x)
```

output
```
(21*b^2*sinh(4*c + 4*d*x) - 84*b^2*sinh(2*c + 2*d*x) - 4*b^2*sinh(6*c + 6*
d*x) + (3*b^2*sinh(8*c + 8*d*x))/8 - 192*a*b*sinh(2*c + 2*d*x) + 24*a*b*si
nh(4*c + 4*d*x) + 384*a^2*d*x + 105*b^2*d*x + 288*a*b*d*x)/(384*d)
```

Reduce [B] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 234, normalized size of antiderivative = 1.87

$$\int \left(a+b\sinh^4(c+dx)\right)^2\,dx$$
$$=\frac{3e^{16dx+16c}b^2-32e^{14dx+14c}b^2+192e^{12dx+12c}ab+168e^{12dx+12c}b^2-1536e^{10dx+10c}ab-672e^{10dx+10c}b^2+614}{}$$

input
```
int((a+b*sinh(d*x+c)^4)^2,x)
```

output
```
(3*e**(16*c + 16*d*x)*b**2 - 32*e**(14*c + 14*d*x)*b**2 + 192*e**(12*c + 1
2*d*x)*a*b + 168*e**(12*c + 12*d*x)*b**2 - 1536*e**(10*c + 10*d*x)*a*b - 6
72*e**(10*c + 10*d*x)*b**2 + 6144*e**(8*c + 8*d*x)*a**2*d*x + 4608*e**(8*c
 + 8*d*x)*a*b*d*x + 1680*e**(8*c + 8*d*x)*b**2*d*x + 1536*e**(6*c + 6*d*x)
*a*b + 672*e**(6*c + 6*d*x)*b**2 - 192*e**(4*c + 4*d*x)*a*b - 168*e**(4*c
+ 4*d*x)*b**2 + 32*e**(2*c + 2*d*x)*b**2 - 3*b**2)/(6144*e**(8*c + 8*d*x)*
d)
```

3.175 $\int \operatorname{csch}(c+dx) \left(a+b\sinh^4(c+dx)\right)^2 dx$

Optimal result

Integrand size = 21, antiderivative size = 92

$$\int \operatorname{csch}(c+dx) \left(a+b\sinh^4(c+dx)\right)^2 dx = -\frac{a^2\operatorname{arctanh}(\cosh(c+dx))}{d} - \frac{b(2a+b)\cosh(c+dx)}{d} + \frac{b(2a+3b)\cosh^3(c+dx)}{3d} - \frac{3b^2\cosh^5(c+dx)}{5d} + \frac{b^2\cosh^7(c+dx)}{7d}$$

output
```
-a^2*arctanh(cosh(d*x+c))/d-b*(2*a+b)*cosh(d*x+c)/d+1/3*b*(2*a+3*b)*cosh(d
*x+c)^3/d-3/5*b^2*cosh(d*x+c)^5/d+1/7*b^2*cosh(d*x+c)^7/d
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 118, normalized size of antiderivative = 1.28

$$\int \operatorname{csch}(c+dx)\left(a+b\sinh^4(c+dx)\right)^2\,dx=-\frac{a^2\operatorname{arctanh}(\cosh(c+dx))}{d}-\frac{3ab\cosh(c+dx)}{2d}-\frac{35b^2\cosh(c+dx)}{64d}+\frac{ab\cosh(3(c+dx))}{6d}+\frac{7b^2\cosh(3(c+dx))}{64d}-\frac{7b^2\cosh(5(c+dx))}{320d}+\frac{b^2\cosh(7(c+dx))}{448d}$$

input
```
Integrate[Csch[c + d*x]*(a + b*Sinh[c + d*x]^4)^2,x]
```

output
```
-((a^2*ArcTanh[Cosh[c + d*x]])/d) - (3*a*b*Cosh[c + d*x])/(2*d) - (35*b^2*
Cosh[c + d*x])/(64*d) + (a*b*Cosh[3*(c + d*x)])/(6*d) + (7*b^2*Cosh[3*(c +
 d*x)])/(64*d) - (7*b^2*Cosh[5*(c + d*x)])/(320*d) + (b^2*Cosh[7*(c + d*x)
])/(448*d)
```

Rubi [A] (verified)

Time = 0.31 (sec) , antiderivative size = 80, normalized size of antiderivative = 0.87, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}=0.238$, Rules used = {3042, 26, 3694, 1467, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \operatorname{csch}(c+dx)\left(a+b\sinh^4(c+dx)\right)^2\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{i\left(a+b\sin(ic+idx)^4\right)^2}{\sin(ic+idx)}dx$$

$$\Big\downarrow \text{26}$$

$$i\int \frac{\left(b\sin(ic+idx)^4+a\right)^2}{\sin(ic+idx)}dx$$

$$\Big\downarrow \; 3694$$

$$-\frac{\int \frac{\left(b\cosh^4(c+dx)-2b\cosh^2(c+dx)+a+b\right)^2}{1-\cosh^2(c+dx)}d\cosh(c+dx)}{d}$$

$$\Big\downarrow \; 1467$$

$$-\frac{\int \left(-b^2\cosh^6(c+dx)+3b^2\cosh^4(c+dx)-b(2a+3b)\cosh^2(c+dx)+b(2a+b)+\frac{a^2}{1-\cosh^2(c+dx)}\right)d\cosh(c+ }{d}$$

$$\Big\downarrow \; 2009$$

$$-\frac{a^2\text{arctanh}(\cosh(c+dx))-\frac{1}{3}b(2a+3b)\cosh^3(c+dx)+b(2a+b)\cosh(c+dx)-\frac{1}{7}b^2\cosh^7(c+dx)+\frac{3}{5}b^2\cosh}{d}$$

input
```
Int[Csch[c + d*x]*(a + b*Sinh[c + d*x]^4)^2,x]
```

output
```
-((a^2*ArcTanh[Cosh[c + d*x]] + b*(2*a + b)*Cosh[c + d*x] - (b*(2*a + 3*b)
*Cosh[c + d*x]^3)/3 + (3*b^2*Cosh[c + d*x]^5)/5 - (b^2*Cosh[c + d*x]^7)/7)
/d)
```

Defintions of rubi rules used

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 1467
```
Int[((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.),
 x_Symbol] :> Int[ExpandIntegrand[(d + e*x^2)^q*(a + b*x^2 + c*x^4)^p, x],
x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e
 + a*e^2, 0] && IGtQ[p, 0] && IGtQ[q, -2]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3694
```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^4)^
(p_.), x_Symbol] :> With[{ff = FreeFactors[Cos[e + f*x], x]}, Simp[-ff/f
Subst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a + b - 2*b*ff^2*x^2 + b*ff^4*x^4)^p,
 x], x, Cos[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m -
 1)/2]
```

Maple [A] (verified)

Time = 2.44 (sec) , antiderivative size = 82, normalized size of antiderivative = 0.89

method	result
derivativedivides	$\frac{-2a^2 \operatorname{arctanh}\left(e^{dx+c}\right)+2ab\left(-\frac{2}{3}+\frac{\sinh(dx+c)^2}{3}\right)\cosh(dx+c)+b^2\left(-\frac{16}{35}+\frac{\sinh(dx+c)^6}{7}-\frac{6\sinh(dx+c)^4}{35}+\frac{8\sinh(dx+c)^2}{35}\right) c}{d}$
default	$\frac{-2a^2 \operatorname{arctanh}\left(e^{dx+c}\right)+2ab\left(-\frac{2}{3}+\frac{\sinh(dx+c)^2}{3}\right)\cosh(dx+c)+b^2\left(-\frac{16}{35}+\frac{\sinh(dx+c)^6}{7}-\frac{6\sinh(dx+c)^4}{35}+\frac{8\sinh(dx+c)^2}{35}\right) c}{d}$
parallelrisch	$\frac{6720a^2 \ln\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-10080\left(\left(-\frac{a}{9}-\frac{7b}{96}\right)\cosh(3dx+3c)+\frac{7b\cosh(5dx+5c)}{480}-\frac{b\cosh(7dx+7c)}{672}+\left(a+\frac{35b}{96}\right)\cosh(dx+c)\right.}{6720d}$
risch	$\frac{b^2 e^{7dx+7c}}{896d}-\frac{7b^2 e^{5dx+5c}}{640d}+\frac{e^{3dx+3c}ab}{12d}+\frac{7\,e^{3dx+3c}b^2}{128d}-\frac{3\,e^{dx+c}ab}{4d}-\frac{35\,e^{dx+c}b^2}{128d}-\frac{3\,e^{-dx-c}ab}{4d}-\frac{35\,e^{-dx-}}{128d}$

input
```
int(csch(d*x+c)*(a+b*sinh(d*x+c)^4)^2,x,method=_RETURNVERBOSE)
```

output
```
1/d*(-2*a^2*arctanh(exp(d*x+c))+2*a*b*(-2/3+1/3*sinh(d*x+c)^2)*cosh(d*x+c)
+b^2*(-16/35+1/7*sinh(d*x+c)^6-6/35*sinh(d*x+c)^4+8/35*sinh(d*x+c)^2)*cosh
(d*x+c))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1575 vs. $2(86) = 172$.

Time = 0.10 (sec) , antiderivative size = 1575, normalized size of antiderivative = 17.12

$$\int \operatorname{csch}(c+dx) \left(a+b\sinh^4(c+dx)\right)^2 \, dx = \text{Too large to display}$$

input
```
integrate(csch(d*x+c)*(a+b*sinh(d*x+c)^4)^2,x, algorithm="fricas")
```

output
```
1/13440*(15*b^2*cosh(d*x + c)^14 + 210*b^2*cosh(d*x + c)*sinh(d*x + c)^13
+ 15*b^2*sinh(d*x + c)^14 - 147*b^2*cosh(d*x + c)^12 + 21*(65*b^2*cosh(d*x
 + c)^2 - 7*b^2)*sinh(d*x + c)^12 + 84*(65*b^2*cosh(d*x + c)^3 - 21*b^2*co
sh(d*x + c))*sinh(d*x + c)^11 + 35*(32*a*b + 21*b^2)*cosh(d*x + c)^10 + 7*
(2145*b^2*cosh(d*x + c)^4 - 1386*b^2*cosh(d*x + c)^2 + 160*a*b + 105*b^2)*
sinh(d*x + c)^10 + 70*(429*b^2*cosh(d*x + c)^5 - 462*b^2*cosh(d*x + c)^3 +
 5*(32*a*b + 21*b^2)*cosh(d*x + c))*sinh(d*x + c)^9 - 105*(96*a*b + 35*b^2
)*cosh(d*x + c)^8 + 105*(429*b^2*cosh(d*x + c)^6 - 693*b^2*cosh(d*x + c)^4
 + 15*(32*a*b + 21*b^2)*cosh(d*x + c)^2 - 96*a*b - 35*b^2)*sinh(d*x + c)^8
 + 24*(2145*b^2*cosh(d*x + c)^7 - 4851*b^2*cosh(d*x + c)^5 + 175*(32*a*b +
 21*b^2)*cosh(d*x + c)^3 - 35*(96*a*b + 35*b^2)*cosh(d*x + c))*sinh(d*x +
c)^7 - 105*(96*a*b + 35*b^2)*cosh(d*x + c)^6 + 21*(2145*b^2*cosh(d*x + c)^
8 - 6468*b^2*cosh(d*x + c)^6 + 350*(32*a*b + 21*b^2)*cosh(d*x + c)^4 - 140
*(96*a*b + 35*b^2)*cosh(d*x + c)^2 - 480*a*b - 175*b^2)*sinh(d*x + c)^6 +
42*(715*b^2*cosh(d*x + c)^9 - 2772*b^2*cosh(d*x + c)^7 + 210*(32*a*b + 21*
b^2)*cosh(d*x + c)^5 - 140*(96*a*b + 35*b^2)*cosh(d*x + c)^3 - 15*(96*a*b
+ 35*b^2)*cosh(d*x + c))*sinh(d*x + c)^5 + 35*(32*a*b + 21*b^2)*cosh(d*x +
 c)^4 + 35*(429*b^2*cosh(d*x + c)^10 - 2079*b^2*cosh(d*x + c)^8 + 210*(32*
a*b + 21*b^2)*cosh(d*x + c)^6 - 210*(96*a*b + 35*b^2)*cosh(d*x + c)^4 - 45
*(96*a*b + 35*b^2)*cosh(d*x + c)^2 + 32*a*b + 21*b^2)*sinh(d*x + c)^4 -...
```

Sympy [F(-1)]

Timed out.

$$\int \operatorname{csch}(c+dx)\left(a+b\sinh^4(c+dx)\right)^2\,dx=\text{Timed out}$$

input
```
integrate(csch(d*x+c)*(a+b*sinh(d*x+c)**4)**2,x)
```

output
```
Timed out
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 177 vs. 2(86) = 172.

Time = 0.05 (sec) , antiderivative size = 177, normalized size of antiderivative = 1.92

$$\int \operatorname{csch}(c+dx)\left(a+b\sinh^4(c+dx)\right)^2\,dx=$$
$$-\frac{1}{4480}\,b^2\left(\frac{\left(49\,e^{(-2\,dx-2\,c)}-245\,e^{(-4\,dx-4\,c)}+1225\,e^{(-6\,dx-6\,c)}-5\right)e^{(7\,dx+7\,c)}}{d}+\frac{1225\,e^{(-dx-c)}-245\,e^{(-3\,d}}{}\right.$$
$$+\frac{1}{12}\,ab\left(\frac{e^{(3\,dx+3\,c)}}{d}-\frac{9\,e^{(dx+c)}}{d}-\frac{9\,e^{(-dx-c)}}{d}+\frac{e^{(-3\,dx-3\,c)}}{d}\right)+\frac{a^2\log\left(\tanh\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)}{d}$$

input
```
integrate(csch(d*x+c)*(a+b*sinh(d*x+c)^4)^2,x, algorithm="maxima")
```

output
```
-1/4480*b^2*((49*e^(-2*d*x - 2*c) - 245*e^(-4*d*x - 4*c) + 1225*e^(-6*d*x
 - 6*c) - 5)*e^(7*d*x + 7*c)/d + (1225*e^(-d*x - c) - 245*e^(-3*d*x - 3*c)
 + 49*e^(-5*d*x - 5*c) - 5*e^(-7*d*x - 7*c))/d) + 1/12*a*b*(e^(3*d*x + 3*c)
/d - 9*e^(d*x + c)/d - 9*e^(-d*x - c)/d + e^(-3*d*x - 3*c)/d) + a^2*log(ta
nh(1/2*d*x + 1/2*c))/d
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 196 vs. $2(86) = 172$.

Time = 0.17 (sec) , antiderivative size = 196, normalized size of antiderivative = 2.13

$$\int \operatorname{csch}(c+dx)\left(a+b\sinh^4(c+dx)\right)^2\,dx = \frac{15\,b^2e^{(7\,dx+7\,c)} - 147\,b^2e^{(5\,dx+5\,c)} + 1120\,abe^{(3\,dx+3\,c)} + 735\,b^2e^{(3\,dx+3\,c)} - 10080\,abe^{(dx+c)} - 3675\,b^2e^{(dx+c)} - \cdots}{\cdots}$$

input
```
integrate(csch(d*x+c)*(a+b*sinh(d*x+c)^4)^2,x, algorithm="giac")
```

output
```
1/13440*(15*b^2*e^(7*d*x + 7*c) - 147*b^2*e^(5*d*x + 5*c) + 1120*a*b*e^(3*
d*x + 3*c) + 735*b^2*e^(3*d*x + 3*c) - 10080*a*b*e^(d*x + c) - 3675*b^2*e^
(d*x + c) - 13440*a^2*log(e^(d*x + c) + 1) + 13440*a^2*log(abs(e^(d*x + c)
 - 1)) - (10080*a*b*e^(6*d*x + 6*c) + 3675*b^2*e^(6*d*x + 6*c) - 1120*a*b*
e^(4*d*x + 4*c) - 735*b^2*e^(4*d*x + 4*c) + 147*b^2*e^(2*d*x + 2*c) - 15*b
^2)*e^(-7*d*x - 7*c))/d
```

Mupad [B] (verification not implemented)

Time = 0.35 (sec) , antiderivative size = 198, normalized size of antiderivative = 2.15

$$\int \operatorname{csch}(c+dx)\left(a+b\sinh^4(c+dx)\right)^2\,dx = \frac{b^2\,\mathrm{e}^{-7\,c-7\,d\,x}}{896\,d} - \frac{2\,\mathrm{atan}\left(\frac{a^2\,\mathrm{e}^{d\,x}\,\mathrm{e}^{c}\,\sqrt{-d^2}}{d\,\sqrt{a^4}}\right)\,\sqrt{a^4}}{\sqrt{-d^2}} - \frac{\mathrm{e}^{-c-d\,x}\,(35\,b^2+96\,a\,b)}{128\,d} - \frac{7\,b^2\,\mathrm{e}^{-5\,c-5\,d\,x}}{640\,d} - \frac{7\,b^2\,\mathrm{e}^{5\,c+5\,d\,x}}{640\,d} - \frac{\mathrm{e}^{c+d\,x}\,(35\,b^2+96\,a\,b)}{128\,d} + \frac{b^2\,\mathrm{e}^{7\,c+7\,d\,x}}{896\,d} + \frac{b\,\mathrm{e}^{-3\,c-3\,d\,x}\,(32\,a+21\,b)}{384\,d} + \frac{b\,\mathrm{e}^{3\,c+3\,d\,x}\,(32\,a+21\,b)}{384\,d}$$

input
```
int((a + b*sinh(c + d*x)^4)^2/sinh(c + d*x),x)
```

output
```
(b^2*exp(- 7*c - 7*d*x))/(896*d) - (2*atan((a^2*exp(d*x)*exp(c)*(-d^2)^(1/
2))/(d*(a^4)^(1/2)))*(a^4)^(1/2))/(-d^2)^(1/2) - (exp(- c - d*x)*(96*a*b +
 35*b^2))/(128*d) - (7*b^2*exp(- 5*c - 5*d*x))/(640*d) - (7*b^2*exp(5*c +
5*d*x))/(640*d) - (exp(c + d*x)*(96*a*b + 35*b^2))/(128*d) + (b^2*exp(7*c
+ 7*d*x))/(896*d) + (b*exp(- 3*c - 3*d*x)*(32*a + 21*b))/(384*d) + (b*exp(
3*c + 3*d*x)*(32*a + 21*b))/(384*d)
```

Reduce [B] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 234, normalized size of antiderivative = 2.54

$$\int \operatorname{csch}(c+dx)\left(a+b\sinh^4(c+dx)\right)^2\,dx = \frac{15e^{14dx+14c}b^2-147e^{12dx+12c}b^2+1120e^{10dx+10c}ab+735e^{10dx+10c}b^2-10080e^{8dx+8c}ab-3675e^{8dx+8c}b^2+1}{}$$

input
```
int(csch(d*x+c)*(a+b*sinh(d*x+c)^4)^2,x)
```

output
```
(15*e**(14*c + 14*d*x)*b**2 - 147*e**(12*c + 12*d*x)*b**2 + 1120*e**(10*c
+ 10*d*x)*a*b + 735*e**(10*c + 10*d*x)*b**2 - 10080*e**(8*c + 8*d*x)*a*b -
 3675*e**(8*c + 8*d*x)*b**2 + 13440*e**(7*c + 7*d*x)*log(e**(c + d*x) - 1)
*a**2 - 13440*e**(7*c + 7*d*x)*log(e**(c + d*x) + 1)*a**2 - 10080*e**(6*c
+ 6*d*x)*a*b - 3675*e**(6*c + 6*d*x)*b**2 + 1120*e**(4*c + 4*d*x)*a*b + 73
5*e**(4*c + 4*d*x)*b**2 - 147*e**(2*c + 2*d*x)*b**2 + 15*b**2)/(13440*e**(
7*c + 7*d*x)*d)
```

3.176 $\int \operatorname{csch}^2(c+dx)\left(a+b\sinh^4(c+dx)\right)^2\,dx$

Optimal result

Integrand size = 23, antiderivative size = 103

$$\int \operatorname{csch}^2(c+dx)\left(a+b\sinh^4(c+dx)\right)^2\,dx = -\frac{1}{16}b(16a+5b)x-\frac{a^2\coth(c+dx)}{d}+\frac{b(16a+11b)\cosh(c+dx)\sinh(c+dx)}{16d}-\frac{13b^2\cosh^3(c+dx)\sinh(c+dx)}{24d}+\frac{b^2\cosh^5(c+dx)\sinh(c+dx)}{6d}$$

output
```
-1/16*b*(16*a+5*b)*x-a^2*coth(d*x+c)/d+1/16*b*(16*a+11*b)*cosh(d*x+c)*sinh
(d*x+c)/d-13/24*b^2*cosh(d*x+c)^3*sinh(d*x+c)/d+1/6*b^2*cosh(d*x+c)^5*sinh
(d*x+c)/d
```

Mathematica [A] (verified)

Time = 0.20 (sec) , antiderivative size = 77, normalized size of antiderivative = 0.75

$$\int \operatorname{csch}^2(c+dx)\left(a+b\sinh^4(c+dx)\right)^2 dx$$
$$= \frac{-192a^2\coth(c+dx)+b(-192ac-60bc-192adx-60bdx+(96a+45b)\sinh(2(c+dx))-9b\sinh(4(c+dx))+b\sinh(6(c+dx)))}{192d}$$

input
```
Integrate[Csch[c + d*x]^2*(a + b*Sinh[c + d*x]^4)^2,x]
```

output
```
(-192*a^2*Coth[c + d*x] + b*(-192*a*c - 60*b*c - 192*a*d*x - 60*b*d*x + (9
6*a + 45*b)*Sinh[2*(c + d*x)] - 9*b*Sinh[4*(c + d*x)] + b*Sinh[6*(c + d*x)
]))/(192*d)
```

Rubi [A] (verified)

Time = 0.53 (sec) , antiderivative size = 134, normalized size of antiderivative = 1.30, number of steps used = 12, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.478$, Rules used = {3042, 25, 3696, 1582, 25, 2336, 27, 1582, 25, 359, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \operatorname{csch}^2(c+dx)\left(a+b\sinh^4(c+dx)\right)^2 dx$$

$$\Big\downarrow \text{3042}$$

$$\int -\frac{\left(a+b\sin(ic+idx)^4\right)^2}{\sin(ic+idx)^2}dx$$

$$\Big\downarrow \text{25}$$

$$-\int \frac{\left(b\sin(ic+idx)^4+a\right)^2}{\sin(ic+idx)^2}dx$$

$$\Big\downarrow \text{3696}$$

$$\frac{\int \frac{\coth^2(c+dx)\left((a+b)\tanh^4(c+dx)-2a\tanh^2(c+dx)+a\right)^2}{\left(1-\tanh^2(c+dx)\right)^4}d\tanh(c+dx)}{d}$$

$$\Big\downarrow \text{1582}$$

$$\frac{\frac{b^2\tanh(c+dx)}{6(1-\tanh^2(c+dx))^3}-\frac{1}{6}\int -\frac{\coth^2(c+dx)\left(-6(a+b)^2\tanh^6(c+dx)+6(3a-b)(a+b)\tanh^4(c+dx)-\left(18a^2+b^2\right)\tanh^2(c+dx)+6a^2\right)}{(1-\tanh^2(c+dx))^3}d\tanh(c+}{d}$$

$$\Big\downarrow \text{25}$$

$$\frac{\frac{1}{6}\int \frac{\coth^2(c+dx)\left(-6(a+b)^2\tanh^6(c+dx)+6(3a-b)(a+b)\tanh^4(c+dx)-\left(18a^2+b^2\right)\tanh^2(c+dx)+6a^2\right)}{(1-\tanh^2(c+dx))^3}d\tanh(c+dx)+\frac{b^2\tanh(c+dx}{6(1-\tanh^2(c+d}}{d}$$

$$\Big\downarrow \text{2336}$$

$$\frac{\frac{1}{6}\left(-\frac{1}{4}\int -\frac{3\coth^2(c+dx)\left(8(a+b)^2\tanh^4(c+dx)-\left(16a^2-3b^2\right)\tanh^2(c+dx)+8a^2\right)}{(1-\tanh^2(c+dx))^2}d\tanh(c+dx)-\frac{13b^2\tanh(c+dx)}{4(1-\tanh^2(c+dx))^2}\right)+\frac{b^2\tan}{6(1-\tan}}{d}$$

$$\Big\downarrow \text{27}$$

$$\frac{\frac{1}{6}\left(\frac{3}{4}\int \frac{\coth^2(c+dx)\left(8(a+b)^2\tanh^4(c+dx)-\left(16a^2-3b^2\right)\tanh^2(c+dx)+8a^2\right)}{(1-\tanh^2(c+dx))^2}d\tanh(c+dx)-\frac{13b^2\tanh(c+dx)}{4(1-\tanh^2(c+dx))^2}\right)+\frac{b^2\tanh(c+a}{6(1-\tanh^2(c+}}{d}$$

$$\Big\downarrow \text{1582}$$

$$\frac{\frac{1}{6}\left(\frac{3}{4}\left(\frac{b(16a+11b)\tanh(c+dx)}{2(1-\tanh^2(c+dx))}-\frac{1}{2}\int -\frac{\coth^2(c+dx)\left(16a^2-\left(16a^2+16ba+5b^2\right)\tanh^2(c+dx)\right)}{1-\tanh^2(c+dx)}d\tanh(c+dx)\right)-\frac{13b^2\tanh(c+dx)}{4(1-\tanh^2(c+dx))^2}\right)}{d}$$

$$\Big\downarrow \text{25}$$

$$\frac{\frac{1}{6}\left(\frac{3}{4}\left(\frac{1}{2}\int \frac{\coth^2(c+dx)\left(16a^2-\left(16a^2+16ba+5b^2\right)\tanh^2(c+dx)\right)}{1-\tanh^2(c+dx)}d\tanh(c+dx)+\frac{b(16a+11b)\tanh(c+dx)}{2(1-\tanh^2(c+dx))}\right)-\frac{13b^2\tanh(c+dx)}{4(1-\tanh^2(c+dx))^2}\right)}{d}$$

$$\Big\downarrow \text{359}$$

$$\frac{\frac{1}{6}\left(\frac{3}{4}\left(\frac{1}{2}\left(-b(16a+5b)\int \frac{1}{1-\tanh^2(c+dx)}d\tanh(c+dx)-16a^2\coth(c+dx)\right)+\frac{b(16a+11b)\tanh(c+dx)}{2(1-\tanh^2(c+dx))}\right)-\frac{13b^2\tanh(c+}{4(1-\tanh^2(c+}}{d}$$

$$\Big\downarrow \text{219}$$

$$\frac{\frac{1}{6}\left(\frac{3}{4}\left(\frac{1}{2}\left(-16a^2\coth(c+dx)-b(16a+5b)\text{arctanh}(\tanh(c+dx))\right)+\frac{b(16a+11b)\tanh(c+dx)}{2(1-\tanh^2(c+dx))}\right)-\frac{13b^2\tanh(c+dx)}{4(1-\tanh^2(c+dx))^2}\right)}{d}$$

input
```
Int[Csch[c + d*x]^2*(a + b*Sinh[c + d*x]^4)^2,x]
```

output
```
((b^2*Tanh[c + d*x])/(6*(1 - Tanh[c + d*x]^2)^3) + ((-13*b^2*Tanh[c + d*x]
)/(4*(1 - Tanh[c + d*x]^2)^2) + (3*((-(b*(16*a + 5*b)*ArcTanh[Tanh[c + d*x
]]) - 16*a^2*Coth[c + d*x])/2 + (b*(16*a + 11*b)*Tanh[c + d*x])/(2*(1 - Ta
nh[c + d*x]^2))))/4)/6)/d
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 359
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2), x
_Symbol] :> Simp[c*(e*x)^(m + 1)*((a + b*x^2)^(p + 1)/(a*e*(m + 1))), x] +
Simp[(a*d*(m + 1) - b*c*(m + 2*p + 3))/(a*e^2*(m + 1))   Int[(e*x)^(m + 2)*
(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c - a*d, 0]
&& LtQ[m, -1] &&  !ILtQ[p, -1]
```

rule 1582
```
Int[(x_)^(m_)*((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^
4)^(p_.), x_Symbol] :> Simp[(-d)^(m/2 - 1)*(c*d^2 - b*d*e + a*e^2)^p*x*((d
+ e*x^2)^(q + 1)/(2*e^(2*p + m/2)*(q + 1))), x] + Simp[(-d)^(m/2 - 1)/(2*e^
(2*p)*(q + 1))   Int[x^m*(d + e*x^2)^(q + 1)*ExpandToSum[Together[(1/(d + e
*x^2))*(2*(-d)^(-m/2 + 1)*e^(2*p)*(q + 1)*(a + b*x^2 + c*x^4)^p - ((c*d^2 -
 b*d*e + a*e^2)^p/(e^(m/2)*x^m))*(d + e*(2*q + 3)*x^2))], x], x], x] /; Fre
eQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && IGtQ[p, 0] && ILtQ[q, -1]
&& ILtQ[m/2, 0]
```

rule 2336
```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[
{Q = PolynomialQuotient[(c*x)^m*Pq, a + b*x^2, x], f = Coeff[PolynomialRema
inder[(c*x)^m*Pq, a + b*x^2, x], x, 0], g = Coeff[PolynomialRemainder[(c*x)
^m*Pq, a + b*x^2, x], x, 1]}, Simp[(a*g - b*f*x)*((a + b*x^2)^(p + 1)/(2*a*
b*(p + 1))), x] + Simp[1/(2*a*(p + 1))   Int[(c*x)^m*(a + b*x^2)^(p + 1)*Ex
pandToSum[(2*a*(p + 1)*Q)/(c*x)^m + (f*(2*p + 3))/(c*x)^m, x], x], x]] /; F
reeQ[{a, b, c}, x] && PolyQ[Pq, x] && LtQ[p, -1] && ILtQ[m, 0]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3696
```
Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^4)^(
p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Simp[ff^(m + 1
)/f   Subst[Int[x^m*((a + 2*a*ff^2*x^2 + (a + b)*ff^4*x^4)^p/(1 + ff^2*x^2)
^(m/2 + 2*p + 1)), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f}, x] &
& IntegerQ[m/2] && IntegerQ[p]
```

Maple [A] (verified)

Time = 1.54 (sec) , antiderivative size = 91, normalized size of antiderivative = 0.88

method	result
derivativedivides	$\frac{-\coth(dx+c)a^2+2ab\left(\frac{\cosh(dx+c)\sinh(dx+c)}{2}-\frac{dx}{2}-\frac{c}{2}\right)+b^2\left(\left(\frac{\sinh(dx+c)^5}{6}-\frac{5\sinh(dx+c)^3}{24}+\frac{5\sinh(dx+c)}{16}\right)\cosh(dx+c)-\right.}{d}$
default	$\frac{-\coth(dx+c)a^2+2ab\left(\frac{\cosh(dx+c)\sinh(dx+c)}{2}-\frac{dx}{2}-\frac{c}{2}\right)+b^2\left(\left(\frac{\sinh(dx+c)^5}{6}-\frac{5\sinh(dx+c)^3}{24}+\frac{5\sinh(dx+c)}{16}\right)\cosh(dx+c)-\right.}{d}$
parallelrisch	$\frac{-192abdx-60b^2dx+45b^2\sinh(2dx+2c)-9b^2\sinh(4dx+4c)+b^2\sinh(6dx+6c)-192\coth\left(\frac{dx}{2}+\frac{c}{2}\right)a^2+96ab\sinh(2dx+2}{192d}$
risch	$-abx-\frac{5b^2x}{16}+\frac{b^2e^{6dx+6c}}{384d}-\frac{3e^{4dx+4c}b^2}{128d}+\frac{e^{2dx+2c}ab}{4d}+\frac{15e^{2dx+2c}b^2}{128d}-\frac{e^{-2dx-2c}ab}{4d}-\frac{15e^{-2dx-2c}b^2}{128d}$

input
```
int(csch(d*x+c)^2*(a+b*sinh(d*x+c)^4)^2,x,method=_RETURNVERBOSE)
```

output
```
1/d*(-coth(d*x+c)*a^2+2*a*b*(1/2*cosh(d*x+c)*sinh(d*x+c)-1/2*d*x-1/2*c)+b^
2*((1/6*sinh(d*x+c)^5-5/24*sinh(d*x+c)^3+5/16*sinh(d*x+c))*cosh(d*x+c)-5/1
6*d*x-5/16*c))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 217 vs. 2(95) = 190.

Time = 0.12 (sec) , antiderivative size = 217, normalized size of antiderivative = 2.11

$$\int \operatorname{csch}^2(c+dx)\left(a+b\sinh^4(c+dx)\right)^2\,dx$$
$$=\frac{b^2\cosh(dx+c)^7+7\,b^2\cosh(dx+c)\sinh(dx+c)^6-10\,b^2\cosh(dx+c)^5+5\left(7\,b^2\cosh(dx+c)^3-10\,b\right.}{}$$

input
```
integrate(csch(d*x+c)^2*(a+b*sinh(d*x+c)^4)^2,x, algorithm="fricas")
```

output
```
1/384*(b^2*cosh(d*x + c)^7 + 7*b^2*cosh(d*x + c)*sinh(d*x + c)^6 - 10*b^2*
cosh(d*x + c)^5 + 5*(7*b^2*cosh(d*x + c)^3 - 10*b^2*cosh(d*x + c))*sinh(d*
x + c)^4 + 6*(16*a*b + 9*b^2)*cosh(d*x + c)^3 + (21*b^2*cosh(d*x + c)^5 -
100*b^2*cosh(d*x + c)^3 + 18*(16*a*b + 9*b^2)*cosh(d*x + c))*sinh(d*x + c)
^2 - 3*(128*a^2 + 32*a*b + 15*b^2)*cosh(d*x + c) - 24*((16*a*b + 5*b^2)*d*
x - 16*a^2)*sinh(d*x + c))/(d*sinh(d*x + c))
```

Sympy [F(-1)]

Timed out.

$$\int \operatorname{csch}^2(c+dx)\left(a+b\sinh^4(c+dx)\right)^2\,dx=\text{Timed out}$$

input
```
integrate(csch(d*x+c)**2*(a+b*sinh(d*x+c)**4)**2,x)
```

output
```
Timed out
```

Maxima [A] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 146, normalized size of antiderivative = 1.42

$$\int \operatorname{csch}^2(c+dx)\left(a+b\sinh^4(c+dx)\right)^2\,dx=-\frac{1}{4}\,ab\left(4\,x-\frac{e^{(2\,dx+2\,c)}}{d}+\frac{e^{(-2\,dx-2\,c)}}{d}\right)$$
$$-\frac{1}{384}\,b^2\left(\frac{\left(9\,e^{(-2\,dx-2\,c)}-45\,e^{(-4\,dx-4\,c)}-1\right)e^{(6\,dx+6\,c)}}{d}+\frac{120\,(dx+c)}{d}+\frac{45\,e^{(-2\,dx-2\,c)}-9\,e^{(-4\,dx-4\,c)}+e}{d}\right.$$
$$+\frac{2\,a^2}{d\left(e^{(-2\,dx-2\,c)}-1\right)}$$

input
```
integrate(csch(d*x+c)^2*(a+b*sinh(d*x+c)^4)^2,x, algorithm="maxima")
```

output
```
-1/4*a*b*(4*x - e^(2*d*x + 2*c)/d + e^(-2*d*x - 2*c)/d) - 1/384*b^2*((9*e^
(-2*d*x - 2*c) - 45*e^(-4*d*x - 4*c) - 1)*e^(6*d*x + 6*c)/d + 120*(d*x + c
)/d + (45*e^(-2*d*x - 2*c) - 9*e^(-4*d*x - 4*c) + e^(-6*d*x - 6*c))/d) + 2
*a^2/(d*(e^(-2*d*x - 2*c) - 1))
```

Giac [A] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 179, normalized size of antiderivative = 1.74

$$\int \operatorname{csch}^2(c+dx) \left(a+b\sinh^4(c+dx)\right)^2 \, dx = \frac{b^2e^{(6\,dx+6\,c)} - 9\,b^2e^{(4\,dx+4\,c)} + 96\,abe^{(2\,dx+2\,c)} + 45\,b^2e^{(2\,dx+2\,c)} - 24\,(16\,ab+5\,b^2)(dx+c) + \left(352\,abe^{(6\,dx+6}\right.}{384\,d}$$

input
```
integrate(csch(d*x+c)^2*(a+b*sinh(d*x+c)^4)^2,x, algorithm="giac")
```

output
```
1/384*(b^2*e^(6*d*x + 6*c) - 9*b^2*e^(4*d*x + 4*c) + 96*a*b*e^(2*d*x + 2*c
) + 45*b^2*e^(2*d*x + 2*c) - 24*(16*a*b + 5*b^2)*(d*x + c) + (352*a*b*e^(6
*d*x + 6*c) + 110*b^2*e^(6*d*x + 6*c) - 96*a*b*e^(4*d*x + 4*c) - 45*b^2*e^
(4*d*x + 4*c) + 9*b^2*e^(2*d*x + 2*c) - b^2)*e^(-6*d*x - 6*c) - 768*a^2/(e
^(2*d*x + 2*c) - 1))/d
```

Mupad [B] (verification not implemented)

Time = 1.72 (sec) , antiderivative size = 148, normalized size of antiderivative = 1.44

$$\int \operatorname{csch}^2(c+dx) \left(a+b\sinh^4(c+dx)\right)^2 \, dx = \frac{3\,b^2\,\mathrm{e}^{-4\,c-4\,d\,x}}{128\,d} - \frac{2\,a^2}{d\,\left(\mathrm{e}^{2\,c+2\,d\,x}-1\right)} - \frac{\mathrm{e}^{-2\,c-2\,d\,x}\,\left(15\,b^2+32\,a\,b\right)}{128\,d} - x\,\left(\frac{5\,b^2}{16}+a\,b\right) - \frac{3\,b^2\,\mathrm{e}^{4\,c+4\,d\,x}}{128\,d} - \frac{b^2\,\mathrm{e}^{-6\,c-6\,d\,x}}{384\,d} + \frac{b^2\,\mathrm{e}^{6\,c+6\,d\,x}}{384\,d} + \frac{b\,\mathrm{e}^{2\,c+2\,d\,x}\,\left(32\,a+15\,b\right)}{128\,d}$$

input
```
int((a + b*sinh(c + d*x)^4)^2/sinh(c + d*x)^2,x)
```

output
```
(3*b^2*exp(- 4*c - 4*d*x))/(128*d) - (2*a^2)/(d*(exp(2*c + 2*d*x) - 1)) -
(exp(- 2*c - 2*d*x)*(32*a*b + 15*b^2))/(128*d) - x*(a*b + (5*b^2)/16) - (3
*b^2*exp(4*c + 4*d*x))/(128*d) - (b^2*exp(- 6*c - 6*d*x))/(384*d) + (b^2*e
xp(6*c + 6*d*x))/(384*d) + (b*exp(2*c + 2*d*x)*(32*a + 15*b))/(128*d)
```

Reduce [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 247, normalized size of antiderivative = 2.40

$$\int \operatorname{csch}^2(c+dx)\left(a+b\sinh^4(c+dx)\right)^2\,dx$$
$$= \frac{e^{14dx+14c}b^2 - 10e^{12dx+12c}b^2 + 96e^{10dx+10c}ab + 54e^{10dx+10c}b^2 - 768e^{8dx+8c}a^2 - 384e^{8dx+8c}abdx - 192e^{8dx+8}}{384e^{}}$$

input
```
int(csch(d*x+c)^2*(a+b*sinh(d*x+c)^4)^2,x)
```

output
```
(e**(14*c + 14*d*x)*b**2 - 10*e**(12*c + 12*d*x)*b**2 + 96*e**(10*c + 10*d
*x)*a*b + 54*e**(10*c + 10*d*x)*b**2 - 768*e**(8*c + 8*d*x)*a**2 - 384*e**
(8*c + 8*d*x)*a*b*d*x - 192*e**(8*c + 8*d*x)*a*b - 120*e**(8*c + 8*d*x)*b*
*2*d*x - 90*e**(8*c + 8*d*x)*b**2 + 384*e**(6*c + 6*d*x)*a*b*d*x + 120*e**
(6*c + 6*d*x)*b**2*d*x + 96*e**(4*c + 4*d*x)*a*b + 54*e**(4*c + 4*d*x)*b**
2 - 10*e**(2*c + 2*d*x)*b**2 + b**2)/(384*e**(6*c + 6*d*x)*d*(e**(2*c + 2*
d*x) - 1))
```

3.177 $\int \operatorname{csch}^3(c+dx) \left(a+b\sinh^4(c+dx)\right)^2 \, dx$

Optimal result

Integrand size = 23, antiderivative size = 92

$$\int \operatorname{csch}^3(c+dx) \left(a+b\sinh^4(c+dx)\right)^2 \, dx = \frac{a^2 \operatorname{arctanh}(\cosh(c+dx))}{2d} + \frac{b(2a+b)\cosh(c+dx)}{d} - \frac{2b^2\cosh^3(c+dx)}{3d} + \frac{b^2\cosh^5(c+dx)}{5d} - \frac{a^2\coth(c+dx)\operatorname{csch}(c+dx)}{2d}$$

output
```
1/2*a^2*arctanh(cosh(d*x+c))/d+b*(2*a+b)*cosh(d*x+c)/d-2/3*b^2*cosh(d*x+c)
^3/d+1/5*b^2*cosh(d*x+c)^5/d-1/2*a^2*coth(d*x+c)*csch(d*x+c)/d
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 165, normalized size of antiderivative = 1.79

$$\int \operatorname{csch}^3(c+dx)\left(a+b\sinh^4(c+dx)\right)^2\,dx$$
$$=\frac{2ab\cosh(c)\cosh(dx)}{d}+\frac{5b^2\cosh(c+dx)}{8d}-\frac{5b^2\cosh(3(c+dx))}{48d}+\frac{b^2\cosh(5(c+dx))}{80d}-\frac{a^2\operatorname{csch}^2\left(\frac{1}{2}(c+dx)\right)}{8d}+\frac{a^2\log\left(\cosh\left(\frac{1}{2}(c+dx)\right)\right)}{2d}-\frac{a^2\log\left(\sinh\left(\frac{1}{2}(c+dx)\right)\right)}{2d}-\frac{a^2\operatorname{sech}^2\left(\frac{1}{2}(c+dx)\right)}{8d}+\frac{2ab\sinh(c)\sinh(dx)}{d}$$

input
```
Integrate[Csch[c + d*x]^3*(a + b*Sinh[c + d*x]^4)^2,x]
```

output
```
(2*a*b*Cosh[c]*Cosh[d*x])/d + (5*b^2*Cosh[c + d*x])/(8*d) - (5*b^2*Cosh[3*
(c + d*x)])/(48*d) + (b^2*Cosh[5*(c + d*x)])/(80*d) - (a^2*Csch[(c + d*x)/
2]^2)/(8*d) + (a^2*Log[Cosh[(c + d*x)/2]])/(2*d) - (a^2*Log[Sinh[(c + d*x)
/2]])/(2*d) - (a^2*Sech[(c + d*x)/2]^2)/(8*d) + (2*a*b*Sinh[c]*Sinh[d*x])/
d
```

Rubi [A] (verified)

Time = 0.41 (sec) , antiderivative size = 92, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.304, Rules used = {3042, 26, 3694, 1471, 25, 2341, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \operatorname{csch}^3(c+dx)\left(a+b\sinh^4(c+dx)\right)^2\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int -\frac{i\left(a+b\sin(ic+idx)^4\right)^2}{\sin(ic+idx)^3}dx$$

$$\Big\downarrow \text{26}$$

$$-i\int \frac{\left(b\sin(ic+idx)^4+a\right)^2}{\sin(ic+idx)^3}dx$$

$$\Big\downarrow \text{ 3694}$$

$$\frac{\int \frac{\left(b\cosh^4(c+dx)-2b\cosh^2(c+dx)+a+b\right)^2}{\left(1-\cosh^2(c+dx)\right)^2}d\cosh(c+dx)}{d}$$

$$\Big\downarrow \text{ 1471}$$

$$\frac{\frac{a^2\cosh(c+dx)}{2(1-\cosh^2(c+dx))}-\frac{1}{2}\int -\frac{-2b^2\cosh^6(c+dx)+6b^2\cosh^4(c+dx)-2b(2a+3b)\cosh^2(c+dx)+a^2+2b^2+4ab}{1-\cosh^2(c+dx)}d\cosh(c+dx)}{d}$$

$$\Big\downarrow \text{ 25}$$

$$\frac{\frac{1}{2}\int \frac{-2b^2\cosh^6(c+dx)+6b^2\cosh^4(c+dx)-2b(2a+3b)\cosh^2(c+dx)+a^2+2b^2+4ab}{1-\cosh^2(c+dx)}d\cosh(c+dx)+\frac{a^2\cosh(c+dx)}{2(1-\cosh^2(c+dx))}}{d}$$

$$\Big\downarrow \text{ 2341}$$

$$\frac{\frac{1}{2}\int \left(2b^2\cosh^4(c+dx)-4b^2\cosh^2(c+dx)+2b(2a+b)+\frac{a^2}{1-\cosh^2(c+dx)}\right)d\cosh(c+dx)+\frac{a^2\cosh(c+dx)}{2(1-\cosh^2(c+dx))}}{d}$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{\frac{1}{2}\left(a^2\text{arctanh}(\cosh(c+dx))+2b(2a+b)\cosh(c+dx)+\frac{2}{5}b^2\cosh^5(c+dx)-\frac{4}{3}b^2\cosh^3(c+dx)\right)+\frac{a^2\cosh(c+dx)}{2(1-\cosh^2(c+dx))}}{d}$$

input

```
Int[Csch[c + d*x]^3*(a + b*Sinh[c + d*x]^4)^2,x]
```

output

```
((a^2*Cosh[c + d*x])/(2*(1 - Cosh[c + d*x]^2)) + (a^2*ArcTanh[Cosh[c + d*x
]] + 2*b*(2*a + b)*Cosh[c + d*x] - (4*b^2*Cosh[c + d*x]^3)/3 + (2*b^2*Cosh
[c + d*x]^5)/5)/2)/d
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 1471
```
Int[((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.),
x_Symbol] :> With[{Qx = PolynomialQuotient[(a + b*x^2 + c*x^4)^p, d + e*x^2
, x], R = Coeff[PolynomialRemainder[(a + b*x^2 + c*x^4)^p, d + e*x^2, x], x
, 0]}, Simp[(-R)*x*((d + e*x^2)^(q + 1)/(2*d*(q + 1))), x] + Simp[1/(2*d*(q
 + 1))   Int[(d + e*x^2)^(q + 1)*ExpandToSum[2*d*(q + 1)*Qx + R*(2*q + 3),
x], x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^
2 - b*d*e + a*e^2, 0] && IGtQ[p, 0] && LtQ[q, -1]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2341
```
Int[(Pq_)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[Pq*
(a + b*x^2)^p, x], x] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3694
```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^4)^
(p_.), x_Symbol] :> With[{ff = FreeFactors[Cos[e + f*x], x]}, Simp[-ff/f
Subst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a + b - 2*b*ff^2*x^2 + b*ff^4*x^4)^p,
 x], x, Cos[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m -
 1)/2]
```

Maple [A] (verified)

Time = 1.44 (sec) , antiderivative size = 74, normalized size of antiderivative = 0.80

method	result
derivativedivides	$\frac{a^2\left(-\frac{\operatorname{csch}(dx+c)\coth(dx+c)}{2}+\operatorname{arctanh}\left(e^{dx+c}\right)\right)+2ab\cosh(dx+c)+b^2\left(\frac{8}{15}+\frac{\sinh(dx+c)^4}{5}-\frac{4\sinh(dx+c)^2}{15}\right)\cosh(dx+c)}{d}$
default	$\frac{a^2\left(-\frac{\operatorname{csch}(dx+c)\coth(dx+c)}{2}+\operatorname{arctanh}\left(e^{dx+c}\right)\right)+2ab\cosh(dx+c)+b^2\left(\frac{8}{15}+\frac{\sinh(dx+c)^4}{5}-\frac{4\sinh(dx+c)^2}{15}\right)\cosh(dx+c)}{d}$
parallelrisch	$\frac{8a^2\ln\left(\frac{1}{\sqrt{\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)}}\right)+a^2\left(\operatorname{sech}\left(\frac{dx}{2}+\frac{c}{2}\right)^2-7\right)\operatorname{csch}\left(\frac{dx}{2}+\frac{c}{2}\right)^2+5\coth\left(\frac{dx}{2}+\frac{c}{2}\right)^2a^2+16\left(-\frac{5b\cosh(3dx+3c)}{96}+\frac{b\cosh(}{1}\right.}{8d}$
risch	$\frac{b^2e^{5dx+5c}}{160d}-\frac{5\,e^{3dx+3c}b^2}{96d}+\frac{e^{dx+c}ab}{d}+\frac{5\,e^{dx+c}b^2}{16d}+\frac{e^{-dx-c}ab}{d}+\frac{5\,e^{-dx-c}b^2}{16d}-\frac{5\,e^{-3dx-3c}b^2}{96d}+\frac{b^2e^{-5dx-5}}{160d}$

input
```
int(csch(d*x+c)^3*(a+b*sinh(d*x+c)^4)^2,x,method=_RETURNVERBOSE)
```

output
```
1/d*(a^2*(-1/2*csch(d*x+c)*coth(d*x+c)+arctanh(exp(d*x+c)))+2*a*b*cosh(d*x
+c)+b^2*(8/15+1/5*sinh(d*x+c)^4-4/15*sinh(d*x+c)^2)*cosh(d*x+c))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2272 vs. 2(84) = 168.

Time = 0.12 (sec) , antiderivative size = 2272, normalized size of antiderivative = 24.70

$$\int \operatorname{csch}^3(c+dx)\left(a+b\sinh^4(c+dx)\right)^2\,dx = \text{Too large to display}$$

input
```
integrate(csch(d*x+c)^3*(a+b*sinh(d*x+c)^4)^2,x, algorithm="fricas")
```

output
```
1/480*(3*b^2*cosh(d*x + c)^14 + 42*b^2*cosh(d*x + c)*sinh(d*x + c)^13 + 3*
b^2*sinh(d*x + c)^14 - 31*b^2*cosh(d*x + c)^12 + (273*b^2*cosh(d*x + c)^2
- 31*b^2)*sinh(d*x + c)^12 + 12*(91*b^2*cosh(d*x + c)^3 - 31*b^2*cosh(d*x
+ c))*sinh(d*x + c)^11 + (480*a*b + 203*b^2)*cosh(d*x + c)^10 + (3003*b^2*
cosh(d*x + c)^4 - 2046*b^2*cosh(d*x + c)^2 + 480*a*b + 203*b^2)*sinh(d*x +
 c)^10 + 2*(3003*b^2*cosh(d*x + c)^5 - 3410*b^2*cosh(d*x + c)^3 + 5*(480*a
*b + 203*b^2)*cosh(d*x + c))*sinh(d*x + c)^9 - 5*(96*a^2 + 96*a*b + 35*b^2
)*cosh(d*x + c)^8 + (9009*b^2*cosh(d*x + c)^6 - 15345*b^2*cosh(d*x + c)^4
+ 45*(480*a*b + 203*b^2)*cosh(d*x + c)^2 - 480*a^2 - 480*a*b - 175*b^2)*si
nh(d*x + c)^8 + 8*(1287*b^2*cosh(d*x + c)^7 - 3069*b^2*cosh(d*x + c)^5 + 1
5*(480*a*b + 203*b^2)*cosh(d*x + c)^3 - 5*(96*a^2 + 96*a*b + 35*b^2)*cosh(
d*x + c))*sinh(d*x + c)^7 - 5*(96*a^2 + 96*a*b + 35*b^2)*cosh(d*x + c)^6 +
 (9009*b^2*cosh(d*x + c)^8 - 28644*b^2*cosh(d*x + c)^6 + 210*(480*a*b + 20
3*b^2)*cosh(d*x + c)^4 - 140*(96*a^2 + 96*a*b + 35*b^2)*cosh(d*x + c)^2 -
480*a^2 - 480*a*b - 175*b^2)*sinh(d*x + c)^6 + 2*(3003*b^2*cosh(d*x + c)^9
 - 12276*b^2*cosh(d*x + c)^7 + 126*(480*a*b + 203*b^2)*cosh(d*x + c)^5 - 1
40*(96*a^2 + 96*a*b + 35*b^2)*cosh(d*x + c)^3 - 15*(96*a^2 + 96*a*b + 35*b
^2)*cosh(d*x + c))*sinh(d*x + c)^5 + (480*a*b + 203*b^2)*cosh(d*x + c)^4 +
 (3003*b^2*cosh(d*x + c)^10 - 15345*b^2*cosh(d*x + c)^8 + 210*(480*a*b + 2
03*b^2)*cosh(d*x + c)^6 - 350*(96*a^2 + 96*a*b + 35*b^2)*cosh(d*x + c)^...
```

Sympy [F(-1)]

Timed out.

$$\int \operatorname{csch}^3(c+dx)\left(a+b\sinh^4(c+dx)\right)^2\,dx=\text{Timed out}$$

input
```
integrate(csch(d*x+c)**3*(a+b*sinh(d*x+c)**4)**2,x)
```

output
```
Timed out
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 204 vs. $2(84) = 168$.

Time = 0.05 (sec) , antiderivative size = 204, normalized size of antiderivative = 2.22

$$\int \operatorname{csch}^3(c+dx)\left(a+b\sinh^4(c+dx)\right)^2\,dx$$
$$= \frac{1}{480}b^2\left(\frac{3e^{(5dx+5c)}}{d} - \frac{25e^{(3dx+3c)}}{d} + \frac{150e^{(dx+c)}}{d} + \frac{150e^{(-dx-c)}}{d} - \frac{25e^{(-3dx-3c)}}{d} + \frac{3e^{(-5dx-5c)}}{d}\right) + ab\left(\frac{e^{(dx+c)}}{d} + \frac{e^{(-dx-c)}}{d}\right) + \frac{1}{2}a^2\left(\frac{\log\left(e^{(-dx-c)}+1\right)}{d} - \frac{\log\left(e^{(-dx-c)}-1\right)}{d} + \frac{2\left(e^{(-dx-c)}+e^{(-3dx-3c)}\right)}{d(2e^{(-2dx-2c)} - e^{(-4dx-4c)} - 1)}\right)$$

input
```
integrate(csch(d*x+c)^3*(a+b*sinh(d*x+c)^4)^2,x, algorithm="maxima")
```

output
```
1/480*b^2*(3*e^(5*d*x + 5*c)/d - 25*e^(3*d*x + 3*c)/d + 150*e^(d*x + c)/d
+ 150*e^(-d*x - c)/d - 25*e^(-3*d*x - 3*c)/d + 3*e^(-5*d*x - 5*c)/d) + a*b
*(e^(d*x + c)/d + e^(-d*x - c)/d) + 1/2*a^2*(log(e^(-d*x - c) + 1)/d - log
(e^(-d*x - c) - 1)/d + 2*(e^(-d*x - c) + e^(-3*d*x - 3*c))/(d*(2*e^(-2*d*x
 - 2*c) - e^(-4*d*x - 4*c) - 1)))
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 182 vs. $2(84) = 168$.

Time = 0.19 (sec) , antiderivative size = 182, normalized size of antiderivative = 1.98

$$\int \operatorname{csch}^3(c+dx)\left(a+b\sinh^4(c+dx)\right)^2\,dx$$
$$= \frac{3b^2\left(e^{(dx+c)}+e^{(-dx-c)}\right)^5 - 40b^2\left(e^{(dx+c)}+e^{(-dx-c)}\right)^3 + 480ab\left(e^{(dx+c)}+e^{(-dx-c)}\right) + 240b^2\left(e^{(dx+c)}+e^{(-dx}\right.}{480\,}$$

input
```
integrate(csch(d*x+c)^3*(a+b*sinh(d*x+c)^4)^2,x, algorithm="giac")
```

output
```
1/480*(3*b^2*(e^(d*x + c) + e^(-d*x - c))^5 - 40*b^2*(e^(d*x + c) + e^(-d*
x - c))^3 + 480*a*b*(e^(d*x + c) + e^(-d*x - c)) + 240*b^2*(e^(d*x + c) +
e^(-d*x - c)) + 120*a^2*log(e^(d*x + c) + e^(-d*x - c) + 2) - 120*a^2*log(
e^(d*x + c) + e^(-d*x - c) - 2) - 480*a^2*(e^(d*x + c) + e^(-d*x - c))/((e
^(d*x + c) + e^(-d*x - c))^2 - 4))/d
```

Mupad [B] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 214, normalized size of antiderivative = 2.33

$$\int \operatorname{csch}^3(c+dx)\left(a+b\sinh^4(c+dx)\right)^2\,dx = \frac{\operatorname{atan}\left(\frac{a^2\,e^{d\,x}\,e^c\,\sqrt{-d^2}}{d\,\sqrt{a^4}}\right)\,\sqrt{a^4}}{\sqrt{-d^2}} - \frac{5\,b^2\,e^{-3\,c-3\,d\,x}}{96\,d} - \frac{5\,b^2\,e^{3\,c+3\,d\,x}}{96\,d} + \frac{b^2\,e^{-5\,c-5\,d\,x}}{160\,d} + \frac{b^2\,e^{5\,c+5\,d\,x}}{160\,d} + \frac{b\,e^{-c-d\,x}\,(16\,a+5\,b)}{16\,d} - \frac{a^2\,e^{c+d\,x}}{d\,\left(e^{2\,c+2\,d\,x}-1\right)} + \frac{b\,e^{c+d\,x}\,(16\,a+5\,b)}{16\,d} - \frac{2\,a^2\,e^{c+d\,x}}{d\,\left(e^{4\,c+4\,d\,x}-2\,e^{2\,c+2\,d\,x}+1\right)}$$

input
```
int((a + b*sinh(c + d*x)^4)^2/sinh(c + d*x)^3,x)
```

output
```
(atan((a^2*exp(d*x)*exp(c)*(-d^2)^(1/2))/(d*(a^4)^(1/2)))*(a^4)^(1/2))/(-d
^2)^(1/2) - (5*b^2*exp(- 3*c - 3*d*x))/(96*d) - (5*b^2*exp(3*c + 3*d*x))/(
96*d) + (b^2*exp(- 5*c - 5*d*x))/(160*d) + (b^2*exp(5*c + 5*d*x))/(160*d)
+ (b*exp(- c - d*x)*(16*a + 5*b))/(16*d) - (a^2*exp(c + d*x))/(d*(exp(2*c
+ 2*d*x) - 1)) + (b*exp(c + d*x)*(16*a + 5*b))/(16*d) - (2*a^2*exp(c + d*x
))/(d*(exp(4*c + 4*d*x) - 2*exp(2*c + 2*d*x) + 1))
```

Reduce [B] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 390, normalized size of antiderivative = 4.24

$$\int \operatorname{csch}^3(c+dx) \left(a+b\sinh^4(c+dx)\right)^2 dx$$
$$= \frac{3e^{14dx+14c}b^2 - 31e^{12dx+12c}b^2 + 480e^{10dx+10c}ab + 203e^{10dx+10c}b^2 - 240e^{9dx+9c}\log\left(e^{dx+c}-1\right)a^2 + 240e^{9dx+}}{}$$

input
```
int(csch(d*x+c)^3*(a+b*sinh(d*x+c)^4)^2,x)
```

output
```
(3*e**(14*c + 14*d*x)*b**2 - 31*e**(12*c + 12*d*x)*b**2 + 480*e**(10*c + 1
0*d*x)*a*b + 203*e**(10*c + 10*d*x)*b**2 - 240*e**(9*c + 9*d*x)*log(e**(c
+ d*x) - 1)*a**2 + 240*e**(9*c + 9*d*x)*log(e**(c + d*x) + 1)*a**2 - 480*e
**(8*c + 8*d*x)*a**2 - 480*e**(8*c + 8*d*x)*a*b - 175*e**(8*c + 8*d*x)*b**
2 + 480*e**(7*c + 7*d*x)*log(e**(c + d*x) - 1)*a**2 - 480*e**(7*c + 7*d*x)
*log(e**(c + d*x) + 1)*a**2 - 480*e**(6*c + 6*d*x)*a**2 - 480*e**(6*c + 6*
d*x)*a*b - 175*e**(6*c + 6*d*x)*b**2 - 240*e**(5*c + 5*d*x)*log(e**(c + d*
x) - 1)*a**2 + 240*e**(5*c + 5*d*x)*log(e**(c + d*x) + 1)*a**2 + 480*e**(4
*c + 4*d*x)*a*b + 203*e**(4*c + 4*d*x)*b**2 - 31*e**(2*c + 2*d*x)*b**2 + 3
*b**2)/(480*e**(5*c + 5*d*x)*d*(e**(4*c + 4*d*x) - 2*e**(2*c + 2*d*x) + 1)
)
```

3.178 $\int \mathrm{csch}^4(c+dx) \left(a+b\sinh^4(c+dx)\right)^2 \, dx$

Optimal result

Integrand size = 23, antiderivative size = 91

$$\int \mathrm{csch}^4(c+dx) \left(a+b\sinh^4(c+dx)\right)^2 \, dx = \frac{1}{8}b(16a+3b)x + \frac{a^2\coth(c+dx)}{d} - \frac{a^2\coth^3(c+dx)}{3d} - \frac{5b^2\cosh(c+dx)\sinh(c+dx)}{8d} + \frac{b^2\cosh^3(c+dx)\sinh(c+dx)}{4d}$$

output
```
1/8*b*(16*a+3*b)*x+a^2*coth(d*x+c)/d-1/3*a^2*coth(d*x+c)^3/d-5/8*b^2*cosh(
d*x+c)*sinh(d*x+c)/d+1/4*b^2*cosh(d*x+c)^3*sinh(d*x+c)/d
```

Mathematica [A] (verified)

Time = 0.22 (sec) , antiderivative size = 68, normalized size of antiderivative = 0.75

$$\int \mathrm{csch}^4(c+dx) \left(a+b\sinh^4(c+dx)\right)^2 \, dx = \frac{-32a^2\coth(c+dx)\left(-2+\mathrm{csch}^2(c+dx)\right) + 3b(12bc+64adx+12bdx-8b\sinh(2(c+dx))+b\sinh(4(c}{96d}$$

input
```
Integrate[Csch[c + d*x]^4*(a + b*Sinh[c + d*x]^4)^2,x]
```

output
```
(-32*a^2*Coth[c + d*x]*(-2 + Csch[c + d*x]^2) + 3*b*(12*b*c + 64*a*d*x + 1
2*b*d*x - 8*b*Sinh[2*(c + d*x)] + b*Sinh[4*(c + d*x)]))/(96*d)
```

Rubi [A] (verified)

Time = 0.47 (sec) , antiderivative size = 111, normalized size of antiderivative = 1.22, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.304$, Rules used = {3042, 3696, 1582, 2336, 25, 1584, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \operatorname{csch}^4(c+dx)\left(a+b\sinh^4(c+dx)\right)^2dx$$

$$\Big\downarrow \ 3042$$

$$\int \frac{\left(a+b\sin(ic+idx)^4\right)^2}{\sin(ic+idx)^4}dx$$

$$\Big\downarrow \ 3696$$

$$\frac{\int \frac{\coth^4(c+dx)\left((a+b)\tanh^4(c+dx)-2a\tanh^2(c+dx)+a\right)^2}{\left(1-\tanh^2(c+dx)\right)^3}d\tanh(c+dx)}{d}$$

$$\Big\downarrow \ 1582$$

$$\frac{\frac{1}{4}\int \frac{\coth^4(c+dx)\left(-4(a+b)^2\tanh^6(c+dx)+\left(12a^2+8ba-b^2\right)\tanh^4(c+dx)-12a^2\tanh^2(c+dx)+4a^2\right)}{\left(1-\tanh^2(c+dx)\right)^2}d\tanh(c+dx)+\frac{b^2\tanh(c+dx)}{4\left(1-\tanh^2(c+dx)\right)^2}}{d}$$

$$\Big\downarrow \ 2336$$

$$\frac{\frac{1}{4}\left(-\frac{1}{2}\int -\frac{\coth^4(c+dx)\left(\left(8a^2+16ba+3b^2\right)\tanh^4(c+dx)-16a^2\tanh^2(c+dx)+8a^2\right)}{1-\tanh^2(c+dx)}d\tanh(c+dx)-\frac{5b^2\tanh(c+dx)}{2\left(1-\tanh^2(c+dx)\right)}\right)+\frac{b^2\tanh(c+dx)}{4\left(1-\tanh^2(c+dx)\right)^2}}{d}$$

$$\Big\downarrow \ 25$$

$$\frac{\frac{1}{4}\left(\frac{1}{2}\int \frac{\coth^4(c+dx)\left(\left(8a^2+16ba+3b^2\right)\tanh^4(c+dx)-16a^2\tanh^2(c+dx)+8a^2\right)}{1-\tanh^2(c+dx)}d\tanh(c+dx)-\frac{5b^2\tanh(c+dx)}{2\left(1-\tanh^2(c+dx)\right)}\right)+\frac{b^2\tanh(c+d}{4(1-\tanh^2(c+}}{d}$$

$$\Big\downarrow \ 1584$$

$$\frac{\frac{1}{4}\left(\frac{1}{2}\int \left(8a^2\coth^4(c+dx)-8a^2\coth^2(c+dx)-\frac{b(16a+3b)}{\tanh^2(c+dx)-1}\right)d\tanh(c+dx)-\frac{5b^2\tanh(c+dx)}{2\left(1-\tanh^2(c+dx)\right)}\right)+\frac{b^2\tanh(c+}{4(1-\tanh^2(c}}{d}$$

$$\Big\downarrow \ 2009$$

$$\frac{\frac{1}{4}\left(\frac{1}{2}\left(-\frac{8}{3}a^2\coth^3(c+dx)+8a^2\coth(c+dx)+b(16a+3b)\text{arctanh}(\tanh(c+dx))\right)-\frac{5b^2\tanh(c+dx)}{2\left(1-\tanh^2(c+dx)\right)}\right)+\frac{b^2\tan}{4(1-\tan}}{d}$$

input
```
Int[Csch[c + d*x]^4*(a + b*Sinh[c + d*x]^4)^2,x]
```

output
```
((b^2*Tanh[c + d*x])/(4*(1 - Tanh[c + d*x]^2)^2) + ((b*(16*a + 3*b)*ArcTan
h[Tanh[c + d*x]] + 8*a^2*Coth[c + d*x] - (8*a^2*Coth[c + d*x]^3)/3)/2 - (5
*b^2*Tanh[c + d*x])/(2*(1 - Tanh[c + d*x]^2)))/4)/d
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 1582
```
Int[(x_)^(m_)*((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^
4)^(p_.), x_Symbol] :> Simp[(-d)^(m/2 - 1)*(c*d^2 - b*d*e + a*e^2)^p*x*((d
+ e*x^2)^(q + 1)/(2*e^(2*p + m/2)*(q + 1))), x] + Simp[(-d)^(m/2 - 1)/(2*e^
(2*p)*(q + 1))   Int[x^m*(d + e*x^2)^(q + 1)*ExpandToSum[Together[(1/(d + e
*x^2))*(2*(-d)^(-m/2 + 1)*e^(2*p)*(q + 1)*(a + b*x^2 + c*x^4)^p - ((c*d^2 -
 b*d*e + a*e^2)^p/(e^(m/2)*x^m))*(d + e*(2*q + 3)*x^2))], x], x], x] /; Fre
eQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && IGtQ[p, 0] && ILtQ[q, -1]
&& ILtQ[m/2, 0]
```

rule 1584
```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (
c_.)*(x_)^4)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(f*x)^m*(d + e*x^2)^q*
(a + b*x^2 + c*x^4)^p, x], x] /; FreeQ[{a, b, c, d, e, f, m, q}, x] && NeQ[
b^2 - 4*a*c, 0] && IGtQ[p, 0] && IGtQ[q, -2]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2336
```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[
{Q = PolynomialQuotient[(c*x)^m*Pq, a + b*x^2, x], f = Coeff[PolynomialRema
inder[(c*x)^m*Pq, a + b*x^2, x], x, 0], g = Coeff[PolynomialRemainder[(c*x)
^m*Pq, a + b*x^2, x], x, 1]}, Simp[(a*g - b*f*x)*((a + b*x^2)^(p + 1)/(2*a*
b*(p + 1))), x] + Simp[1/(2*a*(p + 1))   Int[(c*x)^m*(a + b*x^2)^(p + 1)*Ex
pandToSum[(2*a*(p + 1)*Q)/(c*x)^m + (f*(2*p + 3))/(c*x)^m, x], x], x]] /; F
reeQ[{a, b, c}, x] && PolyQ[Pq, x] && LtQ[p, -1] && ILtQ[m, 0]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3696
```
Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^4)^(
p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Simp[ff^(m + 1
)/f   Subst[Int[x^m*((a + 2*a*ff^2*x^2 + (a + b)*ff^4*x^4)^p/(1 + ff^2*x^2)
^(m/2 + 2*p + 1)), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f}, x] &
& IntegerQ[m/2] && IntegerQ[p]
```

Maple [A] (verified)

Time = 1.33 (sec) , antiderivative size = 75, normalized size of antiderivative = 0.82

method	result
derivativedivides	$\frac{a^2\left(\frac{2}{3}-\frac{\operatorname{csch}(dx+c)^2}{3}\right)\coth(dx+c)+2ab(dx+c)+b^2\left(\left(\frac{\sinh(dx+c)^3}{4}-\frac{3\sinh(dx+c)}{8}\right)\cosh(dx+c)+\frac{3dx}{8}+\frac{3c}{8}\right)}{d}$
default	$\frac{a^2\left(\frac{2}{3}-\frac{\operatorname{csch}(dx+c)^2}{3}\right)\coth(dx+c)+2ab(dx+c)+b^2\left(\left(\frac{\sinh(dx+c)^3}{4}-\frac{3\sinh(dx+c)}{8}\right)\cosh(dx+c)+\frac{3dx}{8}+\frac{3c}{8}\right)}{d}$
parallelrisch	$\frac{-\left(\cosh(dx+c)-\frac{\cosh(3dx+3c)}{3}\right)a^2\operatorname{sech}\left(\frac{dx}{2}+\frac{c}{2}\right)^3\operatorname{csch}\left(\frac{dx}{2}+\frac{c}{2}\right)^3+32\left(-\frac{b\sinh(2dx+2c)}{8}+\frac{b\sinh(4dx+4c)}{64}+dx\left(a+\frac{3b}{16}\right)\right)b}{16d}$
risch	$2abx+\frac{3b^2x}{8}+\frac{e^{4dx+4c}b^2}{64d}-\frac{e^{2dx+2c}b^2}{8d}+\frac{e^{-2dx-2c}b^2}{8d}-\frac{e^{-4dx-4c}b^2}{64d}-\frac{4a^2(3e^{2dx+2c}-1)}{3d(e^{2dx+2c}-1)^3}$

input
```
int(csch(d*x+c)^4*(a+b*sinh(d*x+c)^4)^2,x,method=_RETURNVERBOSE)
```

output
```
1/d*(a^2*(2/3-1/3*csch(d*x+c)^2)*coth(d*x+c)+2*a*b*(d*x+c)+b^2*((1/4*sinh(
d*x+c)^3-3/8*sinh(d*x+c))*cosh(d*x+c)+3/8*d*x+3/8*c))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 300 vs. $2(83) = 166$.

Time = 0.10 (sec) , antiderivative size = 300, normalized size of antiderivative = 3.30

$$\int \operatorname{csch}^4(c+dx)\left(a+b\sinh^4(c+dx)\right)^2\,dx$$
$$=\frac{3\,b^2\cosh(dx+c)^7+21\,b^2\cosh(dx+c)\sinh(dx+c)^6-33\,b^2\cosh(dx+c)^5+15\left(7\,b^2\cosh(dx+c)^3-\right.}{}$$

input
```
integrate(csch(d*x+c)^4*(a+b*sinh(d*x+c)^4)^2,x, algorithm="fricas")
```

output
```
1/192*(3*b^2*cosh(d*x + c)^7 + 21*b^2*cosh(d*x + c)*sinh(d*x + c)^6 - 33*b
^2*cosh(d*x + c)^5 + 15*(7*b^2*cosh(d*x + c)^3 - 11*b^2*cosh(d*x + c))*sin
h(d*x + c)^4 + (128*a^2 + 81*b^2)*cosh(d*x + c)^3 + 8*(3*(16*a*b + 3*b^2)*
d*x - 16*a^2)*sinh(d*x + c)^3 + 3*(21*b^2*cosh(d*x + c)^5 - 110*b^2*cosh(d
*x + c)^3 + (128*a^2 + 81*b^2)*cosh(d*x + c))*sinh(d*x + c)^2 - 3*(128*a^2
 + 17*b^2)*cosh(d*x + c) - 24*(3*(16*a*b + 3*b^2)*d*x - (3*(16*a*b + 3*b^2
)*d*x - 16*a^2)*cosh(d*x + c)^2 - 16*a^2)*sinh(d*x + c))/(d*sinh(d*x + c)^
3 + 3*(d*cosh(d*x + c)^2 - d)*sinh(d*x + c))
```

Sympy [F(-1)]

Timed out.

$$\int \operatorname{csch}^4(c+dx)\left(a+b\sinh^4(c+dx)\right)^2\,dx = \text{Timed out}$$

input
```
integrate(csch(d*x+c)**4*(a+b*sinh(d*x+c)**4)**2,x)
```

output
```
Timed out
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 165, normalized size of antiderivative = 1.81

$$\int \operatorname{csch}^4(c+dx)\left(a+b\sinh^4(c+dx)\right)^2\,dx$$
$$= \frac{1}{64}b^2\left(24\,x+\frac{e^{(4\,dx+4\,c)}}{d}-\frac{8\,e^{(2\,dx+2\,c)}}{d}+\frac{8\,e^{(-2\,dx-2\,c)}}{d}-\frac{e^{(-4\,dx-4\,c)}}{d}\right)+2\,abx$$
$$+\frac{4}{3}a^2\left(\frac{3\,e^{(-2\,dx-2\,c)}}{d(3\,e^{(-2\,dx-2\,c)}-3\,e^{(-4\,dx-4\,c)}+e^{(-6\,dx-6\,c)}-1)}-\frac{1}{d(3\,e^{(-2\,dx-2\,c)}-3\,e^{(-4\,dx-4\,c)}+e^{(-6\,dx-6\,c)}-1}\right.$$

input
```
integrate(csch(d*x+c)^4*(a+b*sinh(d*x+c)^4)^2,x, algorithm="maxima")
```

output
```
1/64*b^2*(24*x + e^(4*d*x + 4*c)/d - 8*e^(2*d*x + 2*c)/d + 8*e^(-2*d*x - 2
*c)/d - e^(-4*d*x - 4*c)/d) + 2*a*b*x + 4/3*a^2*(3*e^(-2*d*x - 2*c)/(d*(3*
e^(-2*d*x - 2*c) - 3*e^(-4*d*x - 4*c) + e^(-6*d*x - 6*c) - 1)) - 1/(d*(3*e
^(-2*d*x - 2*c) - 3*e^(-4*d*x - 4*c) + e^(-6*d*x - 6*c) - 1)))
```

Giac [A] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 142, normalized size of antiderivative = 1.56

$$\int \operatorname{csch}^4(c+dx)\left(a+b\sinh^4(c+dx)\right)^2\,dx$$
$$=\frac{3\,b^2e^{(4\,dx+4\,c)}-24\,b^2e^{(2\,dx+2\,c)}+24\,(16\,ab+3\,b^2)(dx+c)-3\,\left(96\,abe^{(4\,dx+4\,c)}+18\,b^2e^{(4\,dx+4\,c)}-8\,b^2e^{(2\,dx+}\right.}{192\,d}$$

input
```
integrate(csch(d*x+c)^4*(a+b*sinh(d*x+c)^4)^2,x, algorithm="giac")
```

output
```
1/192*(3*b^2*e^(4*d*x + 4*c) - 24*b^2*e^(2*d*x + 2*c) + 24*(16*a*b + 3*b^2
)*(d*x + c) - 3*(96*a*b*e^(4*d*x + 4*c) + 18*b^2*e^(4*d*x + 4*c) - 8*b^2*e
^(2*d*x + 2*c) + b^2)*e^(-4*d*x - 4*c) - 256*(3*a^2*e^(2*d*x + 2*c) - a^2)
/(e^(2*d*x + 2*c) - 1)^3)/d
```

Mupad [B] (verification not implemented)

Time = 1.64 (sec) , antiderivative size = 164, normalized size of antiderivative = 1.80

$$\int \operatorname{csch}^4(c+dx)\left(a+b\sinh^4(c+dx)\right)^2\,dx$$
$$=\frac{b\,x\,(16\,a+3\,b)}{8}-\frac{4\,a^2}{3\,d\,\left(\mathrm{e}^{4\,c+4\,d\,x}-2\,\mathrm{e}^{2\,c+2\,d\,x}+1\right)}+\frac{b^2\,\mathrm{e}^{-2\,c-2\,d\,x}}{8\,d}-\frac{b^2\,\mathrm{e}^{2\,c+2\,d\,x}}{8\,d}$$
$$-\frac{b^2\,\mathrm{e}^{-4\,c-4\,d\,x}}{64\,d}+\frac{b^2\,\mathrm{e}^{4\,c+4\,d\,x}}{64\,d}-\frac{8\,a^2\,\mathrm{e}^{2\,c+2\,d\,x}}{3\,d\,\left(3\,\mathrm{e}^{2\,c+2\,d\,x}-3\,\mathrm{e}^{4\,c+4\,d\,x}+\mathrm{e}^{6\,c+6\,d\,x}-1\right)}$$

input
```
int((a + b*sinh(c + d*x)^4)^2/sinh(c + d*x)^4,x)
```

output
```
(b*x*(16*a + 3*b))/8 - (4*a^2)/(3*d*(exp(4*c + 4*d*x) - 2*exp(2*c + 2*d*x)
 + 1)) + (b^2*exp(- 2*c - 2*d*x))/(8*d) - (b^2*exp(2*c + 2*d*x))/(8*d) - (
b^2*exp(- 4*c - 4*d*x))/(64*d) + (b^2*exp(4*c + 4*d*x))/(64*d) - (8*a^2*ex
p(2*c + 2*d*x))/(3*d*(3*exp(2*c + 2*d*x) - 3*exp(4*c + 4*d*x) + exp(6*c +
6*d*x) - 1))
```

Reduce [B] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 313, normalized size of antiderivative = 3.44

$$\int \operatorname{csch}^4(c+dx)\left(a+b\sinh^4(c+dx)\right)^2\,dx$$
$$= \frac{3e^{14dx+14c}b^2 - 33e^{12dx+12c}b^2 + 384e^{10dx+10c}abdx + 72e^{10dx+10c}b^2dx + 64e^{10dx+10c}b^2 - 1152e^{8dx+8c}abdx - }{}$$

input
```
int(csch(d*x+c)^4*(a+b*sinh(d*x+c)^4)^2,x)
```

output
```
(3*e**(14*c + 14*d*x)*b**2 - 33*e**(12*c + 12*d*x)*b**2 + 384*e**(10*c + 1
0*d*x)*a*b*d*x + 72*e**(10*c + 10*d*x)*b**2*d*x + 64*e**(10*c + 10*d*x)*b*
*2 - 1152*e**(8*c + 8*d*x)*a*b*d*x - 216*e**(8*c + 8*d*x)*b**2*d*x - 768*e
**(6*c + 6*d*x)*a**2 + 1152*e**(6*c + 6*d*x)*a*b*d*x + 216*e**(6*c + 6*d*x
)*b**2*d*x - 102*e**(6*c + 6*d*x)*b**2 + 256*e**(4*c + 4*d*x)*a**2 - 384*e
**(4*c + 4*d*x)*a*b*d*x - 72*e**(4*c + 4*d*x)*b**2*d*x + 98*e**(4*c + 4*d*
x)*b**2 - 33*e**(2*c + 2*d*x)*b**2 + 3*b**2)/(192*e**(4*c + 4*d*x)*d*(e**(
6*c + 6*d*x) - 3*e**(4*c + 4*d*x) + 3*e**(2*c + 2*d*x) - 1))
```

3.179 $\int \operatorname{csch}^5(c+dx)\left(a+b\sinh^4(c+dx)\right)^2\,dx$

Optimal result

Integrand size = 23, antiderivative size = 101

$$\int \operatorname{csch}^5(c+dx)\left(a+b\sinh^4(c+dx)\right)^2\,dx = -\frac{a(3a+16b)\operatorname{arctanh}(\cosh(c+dx))}{8d} - \frac{b^2\cosh(c+dx)}{d} + \frac{b^2\cosh^3(c+dx)}{3d} + \frac{3a^2\coth(c+dx)\operatorname{csch}(c+dx)}{8d} - \frac{a^2\coth(c+dx)\operatorname{csch}^3(c+dx)}{4d}$$

output
```
-1/8*a*(3*a+16*b)*arctanh(cosh(d*x+c))/d-b^2*cosh(d*x+c)/d+1/3*b^2*cosh(d*
x+c)^3/d+3/8*a^2*coth(d*x+c)*csch(d*x+c)/d-1/4*a^2*coth(d*x+c)*csch(d*x+c)
^3/d
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 179, normalized size of antiderivative = 1.77

$$\int \operatorname{csch}^5(c+dx)\left(a+b\sinh^4(c+dx)\right)^2\,dx$$
$$=-\frac{2ab\text{arctanh}(\cosh(c+dx))}{d}-\frac{3b^2\cosh(c+dx)}{4d}+\frac{b^2\cosh(3(c+dx))}{12d}+\frac{3a^2\text{csch}^2\left(\frac{1}{2}(c+dx)\right)}{32d}-\frac{a^2\text{csch}^4\left(\frac{1}{2}(c+dx)\right)}{64d}-\frac{3a^2\log\left(\cosh\left(\frac{1}{2}(c+dx)\right)\right)}{8d}+\frac{3a^2\log\left(\sinh\left(\frac{1}{2}(c+dx)\right)\right)}{8d}+\frac{3a^2\text{sech}^2\left(\frac{1}{2}(c+dx)\right)}{32d}+\frac{a^2\text{sech}^4\left(\frac{1}{2}(c+dx)\right)}{64d}$$

input
```
Integrate[Csch[c + d*x]^5*(a + b*Sinh[c + d*x]^4)^2,x]
```

output
```
(-2*a*b*ArcTanh[Cosh[c + d*x]])/d - (3*b^2*Cosh[c + d*x])/(4*d) + (b^2*Cos
h[3*(c + d*x)])/(12*d) + (3*a^2*Csch[(c + d*x)/2]^2)/(32*d) - (a^2*Csch[(c
 + d*x)/2]^4)/(64*d) - (3*a^2*Log[Cosh[(c + d*x)/2]])/(8*d) + (3*a^2*Log[S
inh[(c + d*x)/2]])/(8*d) + (3*a^2*Sech[(c + d*x)/2]^2)/(32*d) + (a^2*Sech[
(c + d*x)/2]^4)/(64*d)
```

Rubi [A] (verified)

Time = 0.46 (sec) , antiderivative size = 112, normalized size of antiderivative = 1.11, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.391, Rules used = {3042, 26, 3694, 1471, 25, 2345, 25, 1467, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \operatorname{csch}^5(c+dx)\left(a+b\sinh^4(c+dx)\right)^2\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{i\left(a+b\sin(ic+idx)^4\right)^2}{\sin(ic+idx)^5}dx$$

$$\Big\downarrow \text{26}$$

$$i\int \frac{\left(b\sin(ic+idx)^4+a\right)^2}{\sin(ic+idx)^5}dx$$

$$\Big\downarrow \text{3694}$$

$$-\frac{\int \frac{\left(b\cosh^4(c+dx)-2b\cosh^2(c+dx)+a+b\right)^2}{\left(1-\cosh^2(c+dx)\right)^3}d\cosh(c+dx)}{d}$$

$$\Big\downarrow \text{1471}$$

$$-\frac{\frac{a^2\cosh(c+dx)}{4\left(1-\cosh^2(c+dx)\right)^2}-\frac{1}{4}\int -\frac{-4b^2\cosh^6(c+dx)+12b^2\cosh^4(c+dx)-4b(2a+3b)\cosh^2(c+dx)+(a+2b)(3a+2b)}{\left(1-\cosh^2(c+dx)\right)^2}d\cosh(c+dx)}{d}$$

$$\Big\downarrow \text{25}$$

$$-\frac{\frac{1}{4}\int \frac{-4b^2\cosh^6(c+dx)+12b^2\cosh^4(c+dx)-4b(2a+3b)\cosh^2(c+dx)+(a+2b)(3a+2b)}{\left(1-\cosh^2(c+dx)\right)^2}d\cosh(c+dx)+\frac{a^2\cosh(c+dx)}{4\left(1-\cosh^2(c+dx)\right)^2}}{d}$$

$$\Big\downarrow \text{2345}$$

$$-\frac{\frac{1}{4}\left(\frac{3a^2\cosh(c+dx)}{2\left(1-\cosh^2(c+dx)\right)}-\frac{1}{2}\int -\frac{8b^2\cosh^4(c+dx)-16b^2\cosh^2(c+dx)+3a^2+8b^2+16ab}{1-\cosh^2(c+dx)}d\cosh(c+dx)\right)+\frac{a^2\cosh(c+dx)}{4\left(1-\cosh^2(c+dx)\right)^2}}{d}$$

$$\Big\downarrow \text{25}$$

$$-\frac{\frac{1}{4}\left(\frac{1}{2}\int \frac{8b^2\cosh^4(c+dx)-16b^2\cosh^2(c+dx)+3a^2+8b^2+16ab}{1-\cosh^2(c+dx)}d\cosh(c+dx)+\frac{3a^2\cosh(c+dx)}{2\left(1-\cosh^2(c+dx)\right)}\right)+\frac{a^2\cosh(c+dx)}{4\left(1-\cosh^2(c+dx)\right)^2}}{d}$$

$$\Big\downarrow \text{1467}$$

$$-\frac{\frac{1}{4}\left(\frac{1}{2}\int \left(-8\cosh^2(c+dx)b^2+8b^2+\frac{3a^2+16ba}{1-\cosh^2(c+dx)}\right)d\cosh(c+dx)+\frac{3a^2\cosh(c+dx)}{2\left(1-\cosh^2(c+dx)\right)}\right)+\frac{a^2\cosh(c+dx)}{4\left(1-\cosh^2(c+dx)\right)^2}}{d}$$

$$\Big\downarrow \text{2009}$$

$$-\frac{\frac{1}{4}\left(\frac{3a^2\cosh(c+dx)}{2\left(1-\cosh^2(c+dx)\right)}+\frac{1}{2}\left(a(3a+16b)\text{arctanh}(\cosh(c+dx))-\frac{8}{3}b^2\cosh^3(c+dx)+8b^2\cosh(c+dx)\right)\right)+\frac{a^2\cos\ldots}{4(1-\cos\ldots}}{d}$$

input
```
Int[Csch[c + d*x]^5*(a + b*Sinh[c + d*x]^4)^2,x]
```

output
```
-(((a^2*Cosh[c + d*x])/(4*(1 - Cosh[c + d*x]^2)^2) + ((3*a^2*Cosh[c + d*x]
)/(2*(1 - Cosh[c + d*x]^2)) + (a*(3*a + 16*b)*ArcTanh[Cosh[c + d*x]] + 8*b
^2*Cosh[c + d*x] - (8*b^2*Cosh[c + d*x]^3)/3)/2)/4)/d)
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 1467
```
Int[((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.),
 x_Symbol] :> Int[ExpandIntegrand[(d + e*x^2)^q*(a + b*x^2 + c*x^4)^p, x],
x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e
 + a*e^2, 0] && IGtQ[p, 0] && IGtQ[q, -2]
```

rule 1471
```
Int[((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.),
x_Symbol] :> With[{Qx = PolynomialQuotient[(a + b*x^2 + c*x^4)^p, d + e*x^2
, x], R = Coeff[PolynomialRemainder[(a + b*x^2 + c*x^4)^p, d + e*x^2, x], x
, 0]}, Simp[(-R)*x*((d + e*x^2)^(q + 1)/(2*d*(q + 1))), x] + Simp[1/(2*d*(q
 + 1))   Int[(d + e*x^2)^(q + 1)*ExpandToSum[2*d*(q + 1)*Qx + R*(2*q + 3),
x], x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^
2 - b*d*e + a*e^2, 0] && IGtQ[p, 0] && LtQ[q, -1]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2345
```
Int[(Pq_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{Q = PolynomialQuot
ient[Pq, a + b*x^2, x], f = Coeff[PolynomialRemainder[Pq, a + b*x^2, x], x,
 0], g = Coeff[PolynomialRemainder[Pq, a + b*x^2, x], x, 1]}, Simp[(a*g - b
*f*x)*((a + b*x^2)^(p + 1)/(2*a*b*(p + 1))), x] + Simp[1/(2*a*(p + 1))   In
t[(a + b*x^2)^(p + 1)*ExpandToSum[2*a*(p + 1)*Q + f*(2*p + 3), x], x], x]]
/; FreeQ[{a, b}, x] && PolyQ[Pq, x] && LtQ[p, -1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3694
```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^4)^
(p_.), x_Symbol] :> With[{ff = FreeFactors[Cos[e + f*x], x]}, Simp[-ff/f
Subst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a + b - 2*b*ff^2*x^2 + b*ff^4*x^4)^p,
 x], x, Cos[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m -
 1)/2]
```

Maple [A] (verified)

Time = 1.35 (sec) , antiderivative size = 79, normalized size of antiderivative = 0.78

method	result
derivativedivides	$\frac{a^2\left(\left(-\frac{\operatorname{csch}(dx+c)^3}{4}+\frac{3\operatorname{csch}(dx+c)}{8}\right)\coth(dx+c)-\frac{3\operatorname{arctanh}\left(e^{dx+c}\right)}{4}\right)-4ab\operatorname{arctanh}\left(e^{dx+c}\right)+b^2\left(-\frac{2}{3}+\frac{\sinh(dx+c)^2}{3}\right)\mathrm{c}}{d}$
default	$\frac{a^2\left(\left(-\frac{\operatorname{csch}(dx+c)^3}{4}+\frac{3\operatorname{csch}(dx+c)}{8}\right)\coth(dx+c)-\frac{3\operatorname{arctanh}\left(e^{dx+c}\right)}{4}\right)-4ab\operatorname{arctanh}\left(e^{dx+c}\right)+b^2\left(-\frac{2}{3}+\frac{\sinh(dx+c)^2}{3}\right)\mathrm{c}}{d}$
parallelrisch	$\frac{192a\left(a+\frac{16b}{3}\right)\ln\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-11\left(\cosh(dx+c)-\frac{27\cosh(2dx+2c)}{44}-\frac{3\cosh(3dx+3c)}{11}+\frac{27\cosh(4dx+4c)}{176}+\frac{81}{176}\right)a^2\operatorname{sech}(}{512d}$
risch	$\frac{e^{3dx+3c}b^2}{24d}-\frac{3e^{dx+c}b^2}{8d}-\frac{3e^{-dx-c}b^2}{8d}+\frac{e^{-3dx-3c}b^2}{24d}+\frac{e^{dx+c}a^2\left(3e^{6dx+6c}-11e^{4dx+4c}-11e^{2dx+2c}+3\right)}{4d\left(e^{2dx+2c}-1\right)^4}-3a$

input
```
int(csch(d*x+c)^5*(a+b*sinh(d*x+c)^4)^2,x,method=_RETURNVERBOSE)
```

output
```
1/d*(a^2*((-1/4*csch(d*x+c)^3+3/8*csch(d*x+c))*coth(d*x+c)-3/4*arctanh(exp
(d*x+c)))-4*a*b*arctanh(exp(d*x+c))+b^2*(-2/3+1/3*sinh(d*x+c)^2)*cosh(d*x+
c))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 3356 vs. 2(93) = 186.

Time = 0.14 (sec) , antiderivative size = 3356, normalized size of antiderivative = 33.23

$$\int \operatorname{csch}^5(c+dx)\left(a+b\sinh^4(c+dx)\right)^2\,dx = \text{Too large to display}$$

input
```
integrate(csch(d*x+c)^5*(a+b*sinh(d*x+c)^4)^2,x, algorithm="fricas")
```

output
```
Too large to include
```

Sympy [F(-1)]

Timed out.

$$\int \operatorname{csch}^5(c+dx)\left(a+b\sinh^4(c+dx)\right)^2\,dx = \text{Timed out}$$

input
```
integrate(csch(d*x+c)**5*(a+b*sinh(d*x+c)**4)**2,x)
```

output
```
Timed out
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 234 vs. 2(93) = 186.

Time = 0.04 (sec) , antiderivative size = 234, normalized size of antiderivative = 2.32

$$\int \operatorname{csch}^5(c+dx)\left(a+b\sinh^4(c+dx)\right)^2\,dx$$
$$= \frac{1}{24}b^2\left(\frac{e^{(3\,dx+3\,c)}}{d} - \frac{9\,e^{(dx+c)}}{d} - \frac{9\,e^{(-dx-c)}}{d} + \frac{e^{(-3\,dx-3\,c)}}{d}\right)$$
$$- \frac{1}{8}a^2\left(\frac{3\log\left(e^{(-dx-c)}+1\right)}{d} - \frac{3\log\left(e^{(-dx-c)}-1\right)}{d} + \frac{2\left(3\,e^{(-dx-c)} - 11\,e^{(-3\,dx-3\,c)} - 11\,e^{(-5\,dx-5\,c)} + 3\,e^{(-\right.}}{d(4\,e^{(-2\,dx-2\,c)} - 6\,e^{(-4\,dx-4\,c)} + 4\,e^{(-6\,dx-6\,c)} - e^{(-}}\right.$$
$$- 2\,ab\left(\frac{\log\left(e^{(-dx-c)}+1\right)}{d} - \frac{\log\left(e^{(-dx-c)}-1\right)}{d}\right)$$

input
```
integrate(csch(d*x+c)^5*(a+b*sinh(d*x+c)^4)^2,x, algorithm="maxima")
```

output
```
1/24*b^2*(e^(3*d*x + 3*c)/d - 9*e^(d*x + c)/d - 9*e^(-d*x - c)/d + e^(-3*d
*x - 3*c)/d) - 1/8*a^2*(3*log(e^(-d*x - c) + 1)/d - 3*log(e^(-d*x - c) - 1
)/d + 2*(3*e^(-d*x - c) - 11*e^(-3*d*x - 3*c) - 11*e^(-5*d*x - 5*c) + 3*e^
(-7*d*x - 7*c))/(d*(4*e^(-2*d*x - 2*c) - 6*e^(-4*d*x - 4*c) + 4*e^(-6*d*x
- 6*c) - e^(-8*d*x - 8*c) - 1))) - 2*a*b*(log(e^(-d*x - c) + 1)/d - log(e^
(-d*x - c) - 1)/d)
```

Giac [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 179, normalized size of antiderivative = 1.77

$$\int \operatorname{csch}^5(c+dx)\left(a+b\sinh^4(c+dx)\right)^2\,dx$$
$$= \frac{2\,b^2\left(e^{(dx+c)}+e^{(-dx-c)}\right)^3-24\,b^2\left(e^{(dx+c)}+e^{(-dx-c)}\right)-3\,(3\,a^2+16\,ab)\log\left(e^{(dx+c)}+e^{(-dx-c)}+2\right)+3\,(3\,a}{48\,d}$$

input
```
integrate(csch(d*x+c)^5*(a+b*sinh(d*x+c)^4)^2,x, algorithm="giac")
```

output
```
1/48*(2*b^2*(e^(d*x + c) + e^(-d*x - c))^3 - 24*b^2*(e^(d*x + c) + e^(-d*x
 - c)) - 3*(3*a^2 + 16*a*b)*log(e^(d*x + c) + e^(-d*x - c) + 2) + 3*(3*a^2
 + 16*a*b)*log(e^(d*x + c) + e^(-d*x - c) - 2) + 12*(3*a^2*(e^(d*x + c) +
e^(-d*x - c))^3 - 20*a^2*(e^(d*x + c) + e^(-d*x - c)))/((e^(d*x + c) + e^(
-d*x - c))^2 - 4)^2)/d
```

Mupad [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 328, normalized size of antiderivative = 3.25

$$\int \operatorname{csch}^5(c+dx)\left(a+b\sinh^4(c+dx)\right)^2\,dx$$
$$=\frac{b^2\,\mathrm{e}^{-3\,c-3\,d\,x}}{24\,d}-\frac{3\,b^2\,\mathrm{e}^{-c-d\,x}}{8\,d}-\frac{3\,b^2\,\mathrm{e}^{c+d\,x}}{8\,d}+\frac{b^2\,\mathrm{e}^{3\,c+3\,d\,x}}{24\,d}$$
$$-\frac{\operatorname{atan}\left(\frac{\mathrm{e}^{d\,x}\,\mathrm{e}^{c}\,\left(3\,a^2\,\sqrt{-d^2}+16\,a\,b\,\sqrt{-d^2}\right)}{d\,\sqrt{9\,a^4+96\,a^3\,b+256\,a^2\,b^2}}\right)\,\sqrt{9\,a^4+96\,a^3\,b+256\,a^2\,b^2}}{4\,\sqrt{-d^2}}$$
$$-\frac{6\,a^2\,\mathrm{e}^{c+d\,x}}{d\,\left(3\,\mathrm{e}^{2\,c+2\,d\,x}-3\,\mathrm{e}^{4\,c+4\,d\,x}+\mathrm{e}^{6\,c+6\,d\,x}-1\right)}$$
$$-\frac{4\,a^2\,\mathrm{e}^{c+d\,x}}{d\,\left(6\,\mathrm{e}^{4\,c+4\,d\,x}-4\,\mathrm{e}^{2\,c+2\,d\,x}-4\,\mathrm{e}^{6\,c+6\,d\,x}+\mathrm{e}^{8\,c+8\,d\,x}+1\right)}$$
$$+\frac{3\,a^2\,\mathrm{e}^{c+d\,x}}{4\,d\,\left(\mathrm{e}^{2\,c+2\,d\,x}-1\right)}-\frac{a^2\,\mathrm{e}^{c+d\,x}}{2\,d\,\left(\mathrm{e}^{4\,c+4\,d\,x}-2\,\mathrm{e}^{2\,c+2\,d\,x}+1\right)}$$

input
```
int((a + b*sinh(c + d*x)^4)^2/sinh(c + d*x)^5,x)
```

output
```
(b^2*exp(- 3*c - 3*d*x))/(24*d) - (3*b^2*exp(- c - d*x))/(8*d) - (3*b^2*ex
p(c + d*x))/(8*d) + (b^2*exp(3*c + 3*d*x))/(24*d) - (atan((exp(d*x)*exp(c)
*(3*a^2*(-d^2)^(1/2) + 16*a*b*(-d^2)^(1/2)))/(d*(96*a^3*b + 9*a^4 + 256*a^
2*b^2)^(1/2)))*(96*a^3*b + 9*a^4 + 256*a^2*b^2)^(1/2))/(4*(-d^2)^(1/2)) -
(6*a^2*exp(c + d*x))/(d*(3*exp(2*c + 2*d*x) - 3*exp(4*c + 4*d*x) + exp(6*c
 + 6*d*x) - 1)) - (4*a^2*exp(c + d*x))/(d*(6*exp(4*c + 4*d*x) - 4*exp(2*c
+ 2*d*x) - 4*exp(6*c + 6*d*x) + exp(8*c + 8*d*x) + 1)) + (3*a^2*exp(c + d*
x))/(4*d*(exp(2*c + 2*d*x) - 1)) - (a^2*exp(c + d*x))/(2*d*(exp(4*c + 4*d*
x) - 2*exp(2*c + 2*d*x) + 1))
```

Reduce [B] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 725, normalized size of antiderivative = 7.18

$$\int \operatorname{csch}^5(c+dx)\left(a+b\sinh^4(c+dx)\right)^2\,dx$$
$$= \frac{54e^{7dx+7c}\log\left(e^{dx+c}-1\right)a^2-54e^{7dx+7c}\log\left(e^{dx+c}+1\right)a^2+192e^{9dx+9c}\log\left(e^{dx+c}+1\right)ab+288e^{7dx+7c}\log\left(\right.}{}$$

input
```
int(csch(d*x+c)^5*(a+b*sinh(d*x+c)^4)^2,x)
```

output
```
(e**(14*c + 14*d*x)*b**2 - 13*e**(12*c + 12*d*x)*b**2 + 9*e**(11*c + 11*d*
x)*log(e**(c + d*x) - 1)*a**2 + 48*e**(11*c + 11*d*x)*log(e**(c + d*x) - 1
)*a*b - 9*e**(11*c + 11*d*x)*log(e**(c + d*x) + 1)*a**2 - 48*e**(11*c + 11
*d*x)*log(e**(c + d*x) + 1)*a*b + 18*e**(10*c + 10*d*x)*a**2 + 33*e**(10*c
 + 10*d*x)*b**2 - 36*e**(9*c + 9*d*x)*log(e**(c + d*x) - 1)*a**2 - 192*e**
(9*c + 9*d*x)*log(e**(c + d*x) - 1)*a*b + 36*e**(9*c + 9*d*x)*log(e**(c +
d*x) + 1)*a**2 + 192*e**(9*c + 9*d*x)*log(e**(c + d*x) + 1)*a*b - 66*e**(8
*c + 8*d*x)*a**2 - 21*e**(8*c + 8*d*x)*b**2 + 54*e**(7*c + 7*d*x)*log(e**(
c + d*x) - 1)*a**2 + 288*e**(7*c + 7*d*x)*log(e**(c + d*x) - 1)*a*b - 54*e
**(7*c + 7*d*x)*log(e**(c + d*x) + 1)*a**2 - 288*e**(7*c + 7*d*x)*log(e**(
c + d*x) + 1)*a*b - 66*e**(6*c + 6*d*x)*a**2 - 21*e**(6*c + 6*d*x)*b**2 -
36*e**(5*c + 5*d*x)*log(e**(c + d*x) - 1)*a**2 - 192*e**(5*c + 5*d*x)*log(
e**(c + d*x) - 1)*a*b + 36*e**(5*c + 5*d*x)*log(e**(c + d*x) + 1)*a**2 + 1
92*e**(5*c + 5*d*x)*log(e**(c + d*x) + 1)*a*b + 18*e**(4*c + 4*d*x)*a**2 +
 33*e**(4*c + 4*d*x)*b**2 + 9*e**(3*c + 3*d*x)*log(e**(c + d*x) - 1)*a**2
+ 48*e**(3*c + 3*d*x)*log(e**(c + d*x) - 1)*a*b - 9*e**(3*c + 3*d*x)*log(e
**(c + d*x) + 1)*a**2 - 48*e**(3*c + 3*d*x)*log(e**(c + d*x) + 1)*a*b - 13
*e**(2*c + 2*d*x)*b**2 + b**2)/(24*e**(3*c + 3*d*x)*d*(e**(8*c + 8*d*x) -
4*e**(6*c + 6*d*x) + 6*e**(4*c + 4*d*x) - 4*e**(2*c + 2*d*x) + 1))
```

3.180 $\int \operatorname{csch}^6(c+dx)\left(a+b\sinh^4(c+dx)\right)^2\,dx$

Optimal result

Integrand size = 23, antiderivative size = 84

$$\int \operatorname{csch}^6(c+dx)\left(a+b\sinh^4(c+dx)\right)^2\,dx=-\frac{b^2x}{2}-\frac{a(a+2b)\coth(c+dx)}{d}+\frac{2a^2\coth^3(c+dx)}{3d}-\frac{a^2\coth^5(c+dx)}{5d}+\frac{b^2\cosh(c+dx)\sinh(c+dx)}{2d}$$

output
```
-1/2*b^2*x-a*(a+2*b)*coth(d*x+c)/d+2/3*a^2*coth(d*x+c)^3/d-1/5*a^2*coth(d*
x+c)^5/d+1/2*b^2*cosh(d*x+c)*sinh(d*x+c)/d
```

Mathematica [A] (verified)

Time = 0.50 (sec) , antiderivative size = 67, normalized size of antiderivative = 0.80

$$\int \operatorname{csch}^6(c+dx)\left(a+b\sinh^4(c+dx)\right)^2\,dx=\frac{-4a\coth(c+dx)\left(8a+30b-4a\operatorname{csch}^2(c+dx)+3a\operatorname{csch}^4(c+dx)\right)+15b^2(-2(c+dx)+\sinh(2(c+dx)}{60d}$$

input
```
Integrate[Csch[c + d*x]^6*(a + b*Sinh[c + d*x]^4)^2,x]
```

output
```
(-4*a*Coth[c + d*x]*(8*a + 30*b - 4*a*Csch[c + d*x]^2 + 3*a*Csch[c + d*x]^
4) + 15*b^2*(-2*(c + d*x) + Sinh[2*(c + d*x)]))/(60*d)
```

Rubi [A] (verified)

Time = 0.46 (sec) , antiderivative size = 93, normalized size of antiderivative = 1.11, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.304, Rules used = {3042, 25, 3696, 1582, 25, 2333, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \operatorname{csch}^6(c+dx)\left(a+b\sinh^4(c+dx)\right)^2dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int -\frac{\left(a+b\sin(ic+idx)^4\right)^2}{\sin(ic+idx)^6}dx$$

$$\Big\downarrow \text{ 25}$$

$$-\int \frac{\left(b\sin(ic+idx)^4+a\right)^2}{\sin(ic+idx)^6}dx$$

$$\Big\downarrow \text{ 3696}$$

$$\frac{\int \frac{\coth^6(c+dx)\left((a+b)\tanh^4(c+dx)-2a\tanh^2(c+dx)+a\right)^2}{\left(1-\tanh^2(c+dx)\right)^2}d\tanh(c+dx)}{d}$$

$$\Big\downarrow \text{ 1582}$$

$$\frac{\frac{b^2\tanh(c+dx)}{2\left(1-\tanh^2(c+dx)\right)}-\frac{1}{2}\int -\frac{\coth^6(c+dx)\left(-\left(\left(2a^2+4ba+b^2\right)\tanh^6(c+dx)\right)+2a(3a+2b)\tanh^4(c+dx)-6a^2\tanh^2(c+dx)+2a^2\right)}{1-\tanh^2(c+dx)}d\tanh(c+dx)}{d}$$

$$\Big\downarrow \text{ 25}$$

$$\frac{\frac{1}{2}\int \frac{\coth^6(c+dx)\left(-\left(\left(2a^2+4ba+b^2\right)\tanh^6(c+dx)\right)+2a(3a+2b)\tanh^4(c+dx)-6a^2\tanh^2(c+dx)+2a^2\right)}{1-\tanh^2(c+dx)}d\tanh(c+dx)+\frac{b^2\tanh(c+dx)}{2\left(1-\tanh^2(c+dx)\right)}}{d}$$

$$\Big\downarrow \text{ 2333}$$

$$\frac{\frac{1}{2}\int \left(2a^2\coth^6(c+dx)-4a^2\coth^4(c+dx)+2a(a+2b)\coth^2(c+dx)+\frac{b^2}{\tanh^2(c+dx)-1}\right)d\tanh(c+dx)+\frac{b^2\tan}{2(1-\tan}}{d}$$

$$\Big\downarrow \text{2009}$$

$$\frac{\frac{1}{2}\left(-\frac{2}{5}a^2\coth^5(c+dx)+\frac{4}{3}a^2\coth^3(c+dx)-2a(a+2b)\coth(c+dx)-b^2\text{arctanh}(\tanh(c+dx))\right)+\frac{b^2\tanh(c+}{2(1-\tanh^2(c}}{d}$$

input
```
Int[Csch[c + d*x]^6*(a + b*Sinh[c + d*x]^4)^2,x]
```

output
```
((-(b^2*ArcTanh[Tanh[c + d*x]]) - 2*a*(a + 2*b)*Coth[c + d*x] + (4*a^2*Cot
h[c + d*x]^3)/3 - (2*a^2*Coth[c + d*x]^5)/5)/2 + (b^2*Tanh[c + d*x])/(2*(1
 - Tanh[c + d*x]^2)))/d
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 1582
```
Int[(x_)^(m_)*((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^
4)^(p_.), x_Symbol] :> Simp[(-d)^(m/2 - 1)*(c*d^2 - b*d*e + a*e^2)^p*x*((d
 + e*x^2)^(q + 1)/(2*e^(2*p + m/2)*(q + 1))), x] + Simp[(-d)^(m/2 - 1)/(2*e^
(2*p)*(q + 1))   Int[x^m*(d + e*x^2)^(q + 1)*ExpandToSum[Together[(1/(d + e
*x^2))*(2*(-d)^(-m/2 + 1)*e^(2*p)*(q + 1)*(a + b*x^2 + c*x^4)^p - ((c*d^2 -
 b*d*e + a*e^2)^p/(e^(m/2)*x^m))*(d + e*(2*q + 3)*x^2))], x], x], x] /; Fre
eQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && IGtQ[p, 0] && ILtQ[q, -1]
&& ILtQ[m/2, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2333
```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[
ExpandIntegrand[(c*x)^m*Pq*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x]
&& PolyQ[Pq, x] && IGtQ[p, -2]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3696
```
Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^4)^(
p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Simp[ff^(m + 1
)/f   Subst[Int[x^m*((a + 2*a*ff^2*x^2 + (a + b)*ff^4*x^4)^p/(1 + ff^2*x^2)
^(m/2 + 2*p + 1)), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f}, x] &
& IntegerQ[m/2] && IntegerQ[p]
```

Maple [A] (verified)

Time = 1.29 (sec) , antiderivative size = 74, normalized size of antiderivative = 0.88

method	result
derivativedivides	$\frac{a^2\left(-\frac{8}{15}-\frac{\operatorname{csch}(dx+c)^4}{5}+\frac{4\operatorname{csch}(dx+c)^2}{15}\right)\coth(dx+c)-2\coth(dx+c)ab+b^2\left(\frac{\cosh(dx+c)\sinh(dx+c)}{2}-\frac{dx}{2}-\frac{c}{2}\right)}{d}$
default	$\frac{a^2\left(-\frac{8}{15}-\frac{\operatorname{csch}(dx+c)^4}{5}+\frac{4\operatorname{csch}(dx+c)^2}{15}\right)\coth(dx+c)-2\coth(dx+c)ab+b^2\left(\frac{\cosh(dx+c)\sinh(dx+c)}{2}-\frac{dx}{2}-\frac{c}{2}\right)}{d}$
parallelrisch	$\frac{-\operatorname{sech}\left(\frac{dx}{2}+\frac{c}{2}\right)^5a^2\left(\cosh(dx+c)-\frac{\cosh(3dx+3c)}{2}+\frac{\cosh(5dx+5c)}{10}\right)\operatorname{csch}\left(\frac{dx}{2}+\frac{c}{2}\right)^5+96\operatorname{sech}\left(\frac{dx}{2}+\frac{c}{2}\right)\operatorname{csch}\left(\frac{dx}{2}+\frac{c}{2}\right)ab-19}{96d}$
risch	$-\frac{b^2x}{2}+\frac{e^{2dx+2c}b^2}{8d}-\frac{e^{-2dx-2c}b^2}{8d}-\frac{4a\left(15e^{8dx+8c}b-60e^{6dx+6c}b+40e^{4dx+4c}a+90b\,e^{4dx+4c}-20e^{2dx+2c}a-60e\right.}{15d\left(e^{2dx+2c}-1\right)^5}$

input
```
int(csch(d*x+c)^6*(a+b*sinh(d*x+c)^4)^2,x,method=_RETURNVERBOSE)
```

output
```
1/d*(a^2*(-8/15-1/5*csch(d*x+c)^4+4/15*csch(d*x+c)^2)*coth(d*x+c)-2*coth(d
*x+c)*a*b+b^2*(1/2*cosh(d*x+c)*sinh(d*x+c)-1/2*d*x-1/2*c))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 457 vs. $2(76) = 152$.

Time = 0.10 (sec) , antiderivative size = 457, normalized size of antiderivative = 5.44

$$\int \operatorname{csch}^6(c+dx)\left(a+b\sinh^4(c+dx)\right)^2\,dx = \frac{15\,b^2\cosh\left(dx+c\right)^7+105\,b^2\cosh\left(dx+c\right)\sinh\left(dx+c\right)^6-\left(64\,a^2+240\,ab+75\,b^2\right)\cosh\left(dx+c\right)^5-4\,(1}{}$$

input
```
integrate(csch(d*x+c)^6*(a+b*sinh(d*x+c)^4)^2,x, algorithm="fricas")
```

output
```
1/120*(15*b^2*cosh(d*x + c)^7 + 105*b^2*cosh(d*x + c)*sinh(d*x + c)^6 - (6
4*a^2 + 240*a*b + 75*b^2)*cosh(d*x + c)^5 - 4*(15*b^2*d*x - 16*a^2 - 60*a*
b)*sinh(d*x + c)^5 + 5*(105*b^2*cosh(d*x + c)^3 - (64*a^2 + 240*a*b + 75*b
^2)*cosh(d*x + c))*sinh(d*x + c)^4 + 5*(64*a^2 + 144*a*b + 27*b^2)*cosh(d*
x + c)^3 + 20*(15*b^2*d*x - 2*(15*b^2*d*x - 16*a^2 - 60*a*b)*cosh(d*x + c)
^2 - 16*a^2 - 60*a*b)*sinh(d*x + c)^3 + 5*(63*b^2*cosh(d*x + c)^5 - 2*(64*
a^2 + 240*a*b + 75*b^2)*cosh(d*x + c)^3 + 3*(64*a^2 + 144*a*b + 27*b^2)*co
sh(d*x + c))*sinh(d*x + c)^2 - 5*(128*a^2 + 96*a*b + 15*b^2)*cosh(d*x + c)
 - 20*((15*b^2*d*x - 16*a^2 - 60*a*b)*cosh(d*x + c)^4 + 30*b^2*d*x - 3*(15
*b^2*d*x - 16*a^2 - 60*a*b)*cosh(d*x + c)^2 - 32*a^2 - 120*a*b)*sinh(d*x +
 c))/(d*sinh(d*x + c)^5 + 5*(2*d*cosh(d*x + c)^2 - d)*sinh(d*x + c)^3 + 5*
(d*cosh(d*x + c)^4 - 3*d*cosh(d*x + c)^2 + 2*d)*sinh(d*x + c))
```

Sympy [F(-1)]

Timed out.

$$\int \operatorname{csch}^6(c+dx)\left(a+b\sinh^4(c+dx)\right)^2\,dx = \text{Timed out}$$

input
```
integrate(csch(d*x+c)**6*(a+b*sinh(d*x+c)**4)**2,x)
```

output
```
Timed out
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 267 vs. $2(76) = 152$.

Time = 0.05 (sec) , antiderivative size = 267, normalized size of antiderivative = 3.18

$$\int \operatorname{csch}^6(c+dx)\left(a+b\sinh^4(c+dx)\right)^2\,dx = -\frac{1}{8}b^2\left(4x-\frac{e^{(2dx+2c)}}{d}+\frac{e^{(-2dx-2c)}}{d}\right) - \frac{16}{15}a^2\left(\frac{5e^{(-2dx-2c)}}{d(5e^{(-2dx-2c)}-10e^{(-4dx-4c)}+10e^{(-6dx-6c)}-5e^{(-8dx-8c)}+e^{(-10dx-10c)}-1)} - \frac{}{d(5e^{(-2dx-}}\right. + \frac{4ab}{d(e^{(-2dx-2c)}-1)}$$

input
```
integrate(csch(d*x+c)^6*(a+b*sinh(d*x+c)^4)^2,x, algorithm="maxima")
```

output
```
-1/8*b^2*(4*x - e^(2*d*x + 2*c)/d + e^(-2*d*x - 2*c)/d) - 16/15*a^2*(5*e^(
-2*d*x - 2*c)/(d*(5*e^(-2*d*x - 2*c) - 10*e^(-4*d*x - 4*c) + 10*e^(-6*d*x
 - 6*c) - 5*e^(-8*d*x - 8*c) + e^(-10*d*x - 10*c) - 1)) - 10*e^(-4*d*x - 4*
c)/(d*(5*e^(-2*d*x - 2*c) - 10*e^(-4*d*x - 4*c) + 10*e^(-6*d*x - 6*c) - 5*
e^(-8*d*x - 8*c) + e^(-10*d*x - 10*c) - 1)) - 1/(d*(5*e^(-2*d*x - 2*c) - 1
0*e^(-4*d*x - 4*c) + 10*e^(-6*d*x - 6*c) - 5*e^(-8*d*x - 8*c) + e^(-10*d*x
 - 10*c) - 1))) + 4*a*b/(d*(e^(-2*d*x - 2*c) - 1))
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 166 vs. $2(76) = 152$.

Time = 0.19 (sec) , antiderivative size = 166, normalized size of antiderivative = 1.98

$$\int \operatorname{csch}^6(c+dx)\left(a+b\sinh^4(c+dx)\right)^2\,dx = -\frac{60(dx+c)b^2-15b^2e^{(2dx+2c)}-15\left(2b^2e^{(2dx+2c)}-b^2\right)e^{(-2dx-2c)}+\frac{32\left(15abe^{(8dx+8c)}-60abe^{(6dx+6c)}+40a^2e^{(4}\right.}{}}{120d}$$

input
```
integrate(csch(d*x+c)^6*(a+b*sinh(d*x+c)^4)^2,x, algorithm="giac")
```

output
```
-1/120*(60*(d*x + c)*b^2 - 15*b^2*e^(2*d*x + 2*c) - 15*(2*b^2*e^(2*d*x + 2
*c) - b^2)*e^(-2*d*x - 2*c) + 32*(15*a*b*e^(8*d*x + 8*c) - 60*a*b*e^(6*d*x
 + 6*c) + 40*a^2*e^(4*d*x + 4*c) + 90*a*b*e^(4*d*x + 4*c) - 20*a^2*e^(2*d*
x + 2*c) - 60*a*b*e^(2*d*x + 2*c) + 4*a^2 + 15*a*b)/(e^(2*d*x + 2*c) - 1)^
5)/d
```

Mupad [B] (verification not implemented)

Time = 1.62 (sec) , antiderivative size = 397, normalized size of antiderivative = 4.73

$$\int \operatorname{csch}^6(c+dx)\left(a+b\sinh^4(c+dx)\right)^2\,dx$$
$$= \frac{b^2 e^{2c+2dx}}{8d} - \frac{\frac{4e^{2c+2dx}(4a^2+3ba)}{5d} - \frac{4ab}{5d} - \frac{12abe^{4c+4dx}}{5d} + \frac{4abe^{6c+6dx}}{5d}}{6e^{4c+4dx} - 4e^{2c+2dx} - 4e^{6c+6dx} + e^{8c+8dx} + 1}$$
$$- \frac{b^2 x}{2} - \frac{\frac{4(4a^2+3ba)}{15d} - \frac{8abe^{2c+2dx}}{5d} + \frac{4abe^{4c+4dx}}{5d}}{3e^{2c+2dx} - 3e^{4c+4dx} + e^{6c+6dx} - 1} - \frac{b^2 e^{-2c-2dx}}{8d}$$
$$- \frac{\frac{8e^{4c+4dx}(4a^2+3ba)}{5d} + \frac{4ab}{5d} - \frac{16abe^{2c+2dx}}{5d} - \frac{16abe^{6c+6dx}}{5d} + \frac{4abe^{8c+8dx}}{5d}}{5e^{2c+2dx} - 10e^{4c+4dx} + 10e^{6c+6dx} - 5e^{8c+8dx} + e^{10c+10dx} - 1}$$
$$- \frac{8ab}{5d(e^{2c+2dx}-1)}$$

input
```
int((a + b*sinh(c + d*x)^4)^2/sinh(c + d*x)^6,x)
```

output
```
(b^2*exp(2*c + 2*d*x))/(8*d) - ((4*exp(2*c + 2*d*x)*(3*a*b + 4*a^2))/(5*d)
 - (4*a*b)/(5*d) - (12*a*b*exp(4*c + 4*d*x))/(5*d) + (4*a*b*exp(6*c + 6*d*
x))/(5*d))/(6*exp(4*c + 4*d*x) - 4*exp(2*c + 2*d*x) - 4*exp(6*c + 6*d*x) +
 exp(8*c + 8*d*x) + 1) - (b^2*x)/2 - ((4*(3*a*b + 4*a^2))/(15*d) - (8*a*b*
exp(2*c + 2*d*x))/(5*d) + (4*a*b*exp(4*c + 4*d*x))/(5*d))/(3*exp(2*c + 2*d
*x) - 3*exp(4*c + 4*d*x) + exp(6*c + 6*d*x) - 1) - (b^2*exp(- 2*c - 2*d*x)
)/(8*d) - ((8*exp(4*c + 4*d*x)*(3*a*b + 4*a^2))/(5*d) + (4*a*b)/(5*d) - (1
6*a*b*exp(2*c + 2*d*x))/(5*d) - (16*a*b*exp(6*c + 6*d*x))/(5*d) + (4*a*b*e
xp(8*c + 8*d*x))/(5*d))/(5*exp(2*c + 2*d*x) - 10*exp(4*c + 4*d*x) + 10*exp
(6*c + 6*d*x) - 5*exp(8*c + 8*d*x) + exp(10*c + 10*d*x) - 1) - (8*a*b)/(5*
d*(exp(2*c + 2*d*x) - 1))
```

Reduce [B] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 392, normalized size of antiderivative = 4.67

$$\int \operatorname{csch}^6(c+dx) \left(a+b\sinh^4(c+dx)\right)^2 \, dx$$
$$= \frac{15e^{14dx+14c}b^2 - 96e^{12dx+12c}ab - 60e^{12dx+12c}b^2dx - 48e^{12dx+12c}b^2 + 300e^{10dx+10c}b^2dx + 960e^{8dx+8c}ab - 60\cdots}{}$$

input
```
int(csch(d*x+c)^6*(a+b*sinh(d*x+c)^4)^2,x)
```

output
```
(15*e**(14*c + 14*d*x)*b**2 - 96*e**(12*c + 12*d*x)*a*b - 60*e**(12*c + 12
*d*x)*b**2*d*x - 48*e**(12*c + 12*d*x)*b**2 + 300*e**(10*c + 10*d*x)*b**2*
d*x + 960*e**(8*c + 8*d*x)*a*b - 600*e**(8*c + 8*d*x)*b**2*d*x + 195*e**(8
*c + 8*d*x)*b**2 - 1280*e**(6*c + 6*d*x)*a**2 - 1920*e**(6*c + 6*d*x)*a*b
+ 600*e**(6*c + 6*d*x)*b**2*d*x - 345*e**(6*c + 6*d*x)*b**2 + 640*e**(4*c
+ 4*d*x)*a**2 + 1440*e**(4*c + 4*d*x)*a*b - 300*e**(4*c + 4*d*x)*b**2*d*x
+ 270*e**(4*c + 4*d*x)*b**2 - 128*e**(2*c + 2*d*x)*a**2 - 384*e**(2*c + 2*
d*x)*a*b + 60*e**(2*c + 2*d*x)*b**2*d*x - 102*e**(2*c + 2*d*x)*b**2 + 15*b
**2)/(120*e**(2*c + 2*d*x)*d*(e**(10*c + 10*d*x) - 5*e**(8*c + 8*d*x) + 10
*e**(6*c + 6*d*x) - 10*e**(4*c + 4*d*x) + 5*e**(2*c + 2*d*x) - 1))
```

3.181 $\int \operatorname{csch}^7(c+dx)\left(a+b\sinh^4(c+dx)\right)^2\,dx$

Optimal result

Integrand size = 23, antiderivative size = 111

$$\int \operatorname{csch}^7(c+dx)\left(a+b\sinh^4(c+dx)\right)^2\,dx = \frac{a(5a+16b)\operatorname{arctanh}(\cosh(c+dx))}{16d} + \frac{b^2\cosh(c+dx)}{d} - \frac{a(5a+16b)\coth(c+dx)\operatorname{csch}(c+dx)}{16d} + \frac{5a^2\coth(c+dx)\operatorname{csch}^3(c+dx)}{24d} - \frac{a^2\coth(c+dx)\operatorname{csch}^5(c+dx)}{6d}$$

output
```
1/16*a*(5*a+16*b)*arctanh(cosh(d*x+c))/d+b^2*cosh(d*x+c)/d-1/16*a*(5*a+16*
b)*coth(d*x+c)*csch(d*x+c)/d+5/24*a^2*coth(d*x+c)*csch(d*x+c)^3/d-1/6*a^2*
coth(d*x+c)*csch(d*x+c)^5/d
```

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 278 vs. $2(111) = 222$.

Time = 0.09 (sec) , antiderivative size = 278, normalized size of antiderivative = 2.50

$$\int \operatorname{csch}^7(c+dx)\left(a+b\sinh^4(c+dx)\right)^2 dx = \frac{b^2\cosh(c)\cosh(dx)}{d} - \frac{5a^2\operatorname{csch}^2\left(\frac{1}{2}(c+dx)\right)}{64d} - \frac{ab\operatorname{csch}^2\left(\frac{1}{2}(c+dx)\right)}{4d} + \frac{a^2\operatorname{csch}^4\left(\frac{1}{2}(c+dx)\right)}{64d} - \frac{a^2\operatorname{csch}^6\left(\frac{1}{2}(c+dx)\right)}{384d} + \frac{5a^2\log\left(\cosh\left(\frac{1}{2}(c+dx)\right)\right)}{16d} + \frac{ab\log\left(\cosh\left(\frac{1}{2}(c+dx)\right)\right)}{d} - \frac{5a^2\log\left(\sinh\left(\frac{1}{2}(c+dx)\right)\right)}{16d} - \frac{ab\log\left(\sinh\left(\frac{1}{2}(c+dx)\right)\right)}{d} - \frac{5a^2\operatorname{sech}^2\left(\frac{1}{2}(c+dx)\right)}{64d} - \frac{ab\operatorname{sech}^2\left(\frac{1}{2}(c+dx)\right)}{4d} - \frac{a^2\operatorname{sech}^4\left(\frac{1}{2}(c+dx)\right)}{64d} - \frac{a^2\operatorname{sech}^6\left(\frac{1}{2}(c+dx)\right)}{384d} + \frac{b^2\sinh(c)\sinh(dx)}{d}$$

input
```
Integrate[Csch[c + d*x]^7*(a + b*Sinh[c + d*x]^4)^2,x]
```

output
```
(b^2*Cosh[c]*Cosh[d*x])/d - (5*a^2*Csch[(c + d*x)/2]^2)/(64*d) - (a*b*Csch
[(c + d*x)/2]^2)/(4*d) + (a^2*Csch[(c + d*x)/2]^4)/(64*d) - (a^2*Csch[(c +
 d*x)/2]^6)/(384*d) + (5*a^2*Log[Cosh[(c + d*x)/2]])/(16*d) + (a*b*Log[Cos
h[(c + d*x)/2]])/d - (5*a^2*Log[Sinh[(c + d*x)/2]])/(16*d) - (a*b*Log[Sinh
[(c + d*x)/2]])/d - (5*a^2*Sech[(c + d*x)/2]^2)/(64*d) - (a*b*Sech[(c + d*
x)/2]^2)/(4*d) - (a^2*Sech[(c + d*x)/2]^4)/(64*d) - (a^2*Sech[(c + d*x)/2]
^6)/(384*d) + (b^2*Sinh[c]*Sinh[d*x])/d
```

Rubi [A] (verified)

Time = 0.49 (sec) , antiderivative size = 133, normalized size of antiderivative = 1.20, number of steps used = 12, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.478$, Rules used = {3042, 26, 3694, 1471, 25, 2345, 27, 1471, 25, 299, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \operatorname{csch}^7(c+dx)\left(a+b\sinh^4(c+dx)\right)^2\,dx$$

$\downarrow$ 3042

$$\int -\frac{i\left(a+b\sin(ic+idx)^4\right)^2}{\sin(ic+idx)^7}dx$$

$\downarrow$ 26

$$-i\int \frac{\left(b\sin(ic+idx)^4+a\right)^2}{\sin(ic+idx)^7}dx$$

$\downarrow$ 3694

$$\frac{\int \frac{\left(b\cosh^4(c+dx)-2b\cosh^2(c+dx)+a+b\right)^2}{\left(1-\cosh^2(c+dx)\right)^4}d\cosh(c+dx)}{d}$$

$\downarrow$ 1471

$$\frac{\frac{a^2\cosh(c+dx)}{6\left(1-\cosh^2(c+dx)\right)^3}-\frac{1}{6}\int -\frac{-6b^2\cosh^6(c+dx)+18b^2\cosh^4(c+dx)-6b(2a+3b)\cosh^2(c+dx)+5a^2+6b^2+12ab}{\left(1-\cosh^2(c+dx)\right)^3}d\cosh(c+dx)}{d}$$

$\downarrow$ 25

$$\frac{\frac{1}{6}\int \frac{-6b^2\cosh^6(c+dx)+18b^2\cosh^4(c+dx)-6b(2a+3b)\cosh^2(c+dx)+5a^2+6b^2+12ab}{\left(1-\cosh^2(c+dx)\right)^3}d\cosh(c+dx)+\frac{a^2\cosh(c+dx)}{6\left(1-\cosh^2(c+dx)\right)^3}}{d}$$

$\downarrow$ 2345

$$\frac{\frac{1}{6}\left(\frac{5a^2\cosh(c+dx)}{4\left(1-\cosh^2(c+dx)\right)^2}-\frac{1}{4}\int -\frac{3\left(8b^2\cosh^4(c+dx)-16b^2\cosh^2(c+dx)+5a^2+8b^2+16ab\right)}{\left(1-\cosh^2(c+dx)\right)^2}d\cosh(c+dx)\right)+\frac{a^2\cosh(c+dx)}{6\left(1-\cosh^2(c+dx)\right)^3}}{d}$$

$\downarrow$ 27

$$\frac{\frac{1}{6}\left(\frac{3}{4}\int \frac{8b^2\cosh^4(c+dx)-16b^2\cosh^2(c+dx)+5a^2+8b^2+16ab}{\left(1-\cosh^2(c+dx)\right)^2}d\cosh(c+dx)+\frac{5a^2\cosh(c+dx)}{4\left(1-\cosh^2(c+dx)\right)^2}\right)+\frac{a^2\cosh(c+dx)}{6\left(1-\cosh^2(c+dx)\right)^3}}{d}$$

$\downarrow$ 1471

$$\frac{\frac{1}{6}\left(\frac{3}{4}\left(\frac{a(5a+16b)\cosh(c+dx)}{2\left(1-\cosh^2(c+dx)\right)}-\frac{1}{2}\int -\frac{5a^2+16ba+16b^2-16b^2\cosh^2(c+dx)}{1-\cosh^2(c+dx)}d\cosh(c+dx)\right)+\frac{5a^2\cosh(c+dx)}{4\left(1-\cosh^2(c+dx)\right)^2}\right)+\frac{a^2\cosh(c+}{6(1-\cosh^2(c+}}{d}$$

$\downarrow$ 25

$$\frac{\frac{1}{6}\left(\frac{3}{4}\left(\frac{1}{2}\int \frac{5a^2+16ba+16b^2-16b^2\cosh^2(c+dx)}{1-\cosh^2(c+dx)}d\cosh(c+dx)+\frac{a(5a+16b)\cosh(c+dx)}{2(1-\cosh^2(c+dx))}\right)+\frac{5a^2\cosh(c+dx)}{4(1-\cosh^2(c+dx))^2}\right)+\frac{a^2\cosh(c+dx)}{6(1-\cosh^2(c+dx}}{d}$$

$\downarrow$ 299

$$\frac{\frac{1}{6}\left(\frac{3}{4}\left(\frac{1}{2}\left(a(5a+16b)\int \frac{1}{1-\cosh^2(c+dx)}d\cosh(c+dx)+16b^2\cosh(c+dx)\right)+\frac{a(5a+16b)\cosh(c+dx)}{2(1-\cosh^2(c+dx))}\right)+\frac{5a^2\cosh(c+dx)}{4(1-\cosh^2(c+dx))}\right.}{d}$$

$\downarrow$ 219

$$\frac{\frac{1}{6}\left(\frac{5a^2\cosh(c+dx)}{4(1-\cosh^2(c+dx))^2}+\frac{3}{4}\left(\frac{1}{2}(a(5a+16b)\text{arctanh}(\cosh(c+dx))+16b^2\cosh(c+dx))+\frac{a(5a+16b)\cosh(c+dx)}{2(1-\cosh^2(c+dx))}\right)\right)+\frac{}{6}}{d}$$

input
```
Int[Csch[c + d*x]^7*(a + b*Sinh[c + d*x]^4)^2,x]
```

output
```
((a^2*Cosh[c + d*x])/(6*(1 - Cosh[c + d*x]^2)^3) + ((5*a^2*Cosh[c + d*x])/
(4*(1 - Cosh[c + d*x]^2)^2) + (3*((a*(5*a + 16*b)*ArcTanh[Cosh[c + d*x]] +
 16*b^2*Cosh[c + d*x])/2 + (a*(5*a + 16*b)*Cosh[c + d*x])/(2*(1 - Cosh[c +
 d*x]^2))))/4)/6)/d
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 299
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2), x_Symbol] :> Simp[d*x
*((a + b*x^2)^(p + 1)/(b*(2*p + 3))), x] - Simp[(a*d - b*c*(2*p + 3))/(b*(2
*p + 3))   Int[(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
 a*d, 0] && NeQ[2*p + 3, 0]
```

rule 1471
```
Int[((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.),
x_Symbol] :> With[{Qx = PolynomialQuotient[(a + b*x^2 + c*x^4)^p, d + e*x^2
, x], R = Coeff[PolynomialRemainder[(a + b*x^2 + c*x^4)^p, d + e*x^2, x], x
, 0]}, Simp[(-R)*x*((d + e*x^2)^(q + 1)/(2*d*(q + 1))), x] + Simp[1/(2*d*(q
 + 1))   Int[(d + e*x^2)^(q + 1)*ExpandToSum[2*d*(q + 1)*Qx + R*(2*q + 3),
x], x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^
2 - b*d*e + a*e^2, 0] && IGtQ[p, 0] && LtQ[q, -1]
```

rule 2345
```
Int[(Pq_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{Q = PolynomialQuot
ient[Pq, a + b*x^2, x], f = Coeff[PolynomialRemainder[Pq, a + b*x^2, x], x,
 0], g = Coeff[PolynomialRemainder[Pq, a + b*x^2, x], x, 1]}, Simp[(a*g - b
*f*x)*((a + b*x^2)^(p + 1)/(2*a*b*(p + 1))), x] + Simp[1/(2*a*(p + 1))   In
t[(a + b*x^2)^(p + 1)*ExpandToSum[2*a*(p + 1)*Q + f*(2*p + 3), x], x], x]]
/; FreeQ[{a, b}, x] && PolyQ[Pq, x] && LtQ[p, -1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3694
```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^4)^
(p_.), x_Symbol] :> With[{ff = FreeFactors[Cos[e + f*x], x]}, Simp[-ff/f
Subst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a + b - 2*b*ff^2*x^2 + b*ff^4*x^4)^p,
 x], x, Cos[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m -
 1)/2]
```

Maple [A] (verified)

Time = 1.32 (sec) , antiderivative size = 92, normalized size of antiderivative = 0.83

method	result
derivativedivides	$\frac{a^2\left(\left(-\frac{\operatorname{csch}(dx+c)^5}{6}+\frac{5\operatorname{csch}(dx+c)^3}{24}-\frac{5\operatorname{csch}(dx+c)}{16}\right)\coth(dx+c)+\frac{5\operatorname{arctanh}\left(e^{dx+c}\right)}{8}\right)+2ab\left(-\frac{\operatorname{csch}(dx+c)\coth(dx+c)}{2}+\text{ar}\right.}{d}$
default	$\frac{a^2\left(\left(-\frac{\operatorname{csch}(dx+c)^5}{6}+\frac{5\operatorname{csch}(dx+c)^3}{24}-\frac{5\operatorname{csch}(dx+c)}{16}\right)\coth(dx+c)+\frac{5\operatorname{arctanh}\left(e^{dx+c}\right)}{8}\right)+2ab\left(-\frac{\operatorname{csch}(dx+c)\coth(dx+c)}{2}+\text{ar}\right.}{d}$
parallelrisch	$\frac{-5120a\left(a+\frac{16b}{5}\right)\ln\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-5\left(\cosh(5dx+5c)-\frac{3\cosh(6dx+6c)}{16}+\frac{66\cosh(dx+c)}{5}-\frac{45\cosh(2dx+2c)}{16}-\frac{17\cosh(3dx+}{3}\right.}{}$
risch	$\frac{e^{dx+c}b^2}{2d}+\frac{e^{-dx-c}b^2}{2d}-\frac{e^{dx+c}a\left(15\,e^{10dx+10c}a+48\,e^{10dx+10c}b-85\,e^{8dx+8c}a-144\,e^{8dx+8c}b+198\,e^{6dx+6c}a+96\,e^{6dx}\right.}{24d\left(e^{2dx+2c}-1\right)^6}$

input
```
int(csch(d*x+c)^7*(a+b*sinh(d*x+c)^4)^2,x,method=_RETURNVERBOSE)
```

output
```
1/d*(a^2*((-1/6*csch(d*x+c)^5+5/24*csch(d*x+c)^3-5/16*csch(d*x+c))*coth(d*
x+c)+5/8*arctanh(exp(d*x+c)))+2*a*b*(-1/2*csch(d*x+c)*coth(d*x+c)+arctanh(
exp(d*x+c)))+b^2*cosh(d*x+c))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 4500 vs. $2(103) = 206$.

Time = 0.16 (sec) , antiderivative size = 4500, normalized size of antiderivative = 40.54

$$\int \operatorname{csch}^7(c+dx)\left(a+b\sinh^4(c+dx)\right)^2\,dx=\text{Too large to display}$$

input
```
integrate(csch(d*x+c)^7*(a+b*sinh(d*x+c)^4)^2,x, algorithm="fricas")
```

output
```
Too large to include
```

Sympy [F(-1)]

Timed out.

$$\int \operatorname{csch}^7(c+dx)\left(a+b\sinh^4(c+dx)\right)^2\,dx=\text{Timed out}$$

input
```
integrate(csch(d*x+c)**7*(a+b*sinh(d*x+c)**4)**2,x)
```

output
```
Timed out
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 299 vs. $2(103) = 206$.

Time = 0.04 (sec) , antiderivative size = 299, normalized size of antiderivative = 2.69

$$\int \operatorname{csch}^7(c+dx)\left(a+b\sinh^4(c+dx)\right)^2\,dx=\frac{1}{2}b^2\left(\frac{e^{(dx+c)}}{d}+\frac{e^{(-dx-c)}}{d}\right)+\frac{1}{48}a^2\left(\frac{15\log\left(e^{(-dx-c)}+1\right)}{d}-\frac{15\log\left(e^{(-dx-c)}-1\right)}{d}+\frac{2\left(15e^{(-dx-c)}-85e^{(-3dx-3c)}+198e^{(-5dx-5c)}\right.}{d(6e^{(-2dx-2c)}-15e^{(-4dx-4c)}+20e^{(-6dx-6c)}}\right.+ab\left(\frac{\log\left(e^{(-dx-c)}+1\right)}{d}-\frac{\log\left(e^{(-dx-c)}-1\right)}{d}+\frac{2\left(e^{(-dx-c)}+e^{(-3dx-3c)}\right)}{d(2e^{(-2dx-2c)}-e^{(-4dx-4c)}-1)}\right)$$

input
```
integrate(csch(d*x+c)^7*(a+b*sinh(d*x+c)^4)^2,x, algorithm="maxima")
```

output
```
1/2*b^2*(e^(d*x + c)/d + e^(-d*x - c)/d) + 1/48*a^2*(15*log(e^(-d*x - c) +
 1)/d - 15*log(e^(-d*x - c) - 1)/d + 2*(15*e^(-d*x - c) - 85*e^(-3*d*x - 3
*c) + 198*e^(-5*d*x - 5*c) + 198*e^(-7*d*x - 7*c) - 85*e^(-9*d*x - 9*c) +
15*e^(-11*d*x - 11*c))/(d*(6*e^(-2*d*x - 2*c) - 15*e^(-4*d*x - 4*c) + 20*e
^(-6*d*x - 6*c) - 15*e^(-8*d*x - 8*c) + 6*e^(-10*d*x - 10*c) - e^(-12*d*x
- 12*c) - 1))) + a*b*(log(e^(-d*x - c) + 1)/d - log(e^(-d*x - c) - 1)/d +
2*(e^(-d*x - c) + e^(-3*d*x - 3*c))/(d*(2*e^(-2*d*x - 2*c) - e^(-4*d*x - 4
*c) - 1)))
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 243 vs. $2(103) = 206$.

Time = 0.20 (sec) , antiderivative size = 243, normalized size of antiderivative = 2.19

$$\int \operatorname{csch}^7(c+dx)\left(a+b\sinh^4(c+dx)\right)^2\,dx$$

$$= \frac{48\,b^2\left(e^{(dx+c)}+e^{(-dx-c)}\right)+3\,(5\,a^2+16\,ab)\log\left(e^{(dx+c)}+e^{(-dx-c)}+2\right)-3\,(5\,a^2+16\,ab)\log\left(e^{(dx+c)}+e^{(}\right.}{}$$

input
```
integrate(csch(d*x+c)^7*(a+b*sinh(d*x+c)^4)^2,x, algorithm="giac")
```

output
```
1/96*(48*b^2*(e^(d*x + c) + e^(-d*x - c)) + 3*(5*a^2 + 16*a*b)*log(e^(d*x
+ c) + e^(-d*x - c) + 2) - 3*(5*a^2 + 16*a*b)*log(e^(d*x + c) + e^(-d*x -
c) - 2) - 4*(15*a^2*(e^(d*x + c) + e^(-d*x - c))^5 + 48*a*b*(e^(d*x + c) +
 e^(-d*x - c))^5 - 160*a^2*(e^(d*x + c) + e^(-d*x - c))^3 - 384*a*b*(e^(d*
x + c) + e^(-d*x - c))^3 + 528*a^2*(e^(d*x + c) + e^(-d*x - c)) + 768*a*b*
(e^(d*x + c) + e^(-d*x - c)))/((e^(d*x + c) + e^(-d*x - c))^2 - 4)^3)/d
```

Mupad [B] (verification not implemented)

Time = 1.66 (sec) , antiderivative size = 535, normalized size of antiderivative = 4.82

$$\int \operatorname{csch}^7(c+dx)\left(a+b\sinh^4(c+dx)\right)^2\,dx$$
$$= \frac{b^2\,\mathrm{e}^{c+d\,x}}{2\,d} - \frac{\frac{8\,\mathrm{e}^{5\,c+5\,d\,x}\,\left(4\,a^2+3\,b\,a\right)}{3\,d}+\frac{4\,a\,b\,\mathrm{e}^{c+d\,x}}{3\,d}-\frac{16\,a\,b\,\mathrm{e}^{3\,c+3\,d\,x}}{3\,d}-\frac{16\,a\,b\,\mathrm{e}^{7\,c+7\,d\,x}}{3\,d}+\frac{4\,a\,b\,\mathrm{e}^{9\,c+9\,d\,x}}{3\,d}}{15\,\mathrm{e}^{4\,c+4\,d\,x}-6\,\mathrm{e}^{2\,c+2\,d\,x}-20\,\mathrm{e}^{6\,c+6\,d\,x}+15\,\mathrm{e}^{8\,c+8\,d\,x}-6\,\mathrm{e}^{10\,c+10\,d\,x}+\mathrm{e}^{12\,c+12\,d\,x}+1}$$
$$+ \frac{b^2\,\mathrm{e}^{-c-d\,x}}{2\,d} + \frac{\mathrm{atan}\left(\frac{\mathrm{e}^{d\,x}\,\mathrm{e}^{c}\,\left(5\,a^2\,\sqrt{-d^2}+16\,a\,b\,\sqrt{-d^2}\right)}{d\,\sqrt{25\,a^4+160\,a^3\,b+256\,a^2\,b^2}}\right)\,\sqrt{25\,a^4+160\,a^3\,b+256\,a^2\,b^2}}{8\,\sqrt{-d^2}}$$
$$- \frac{a^2\,\mathrm{e}^{c+d\,x}}{3\,d\,\left(3\,\mathrm{e}^{2\,c+2\,d\,x}-3\,\mathrm{e}^{4\,c+4\,d\,x}+\mathrm{e}^{6\,c+6\,d\,x}-1\right)}$$
$$- \frac{22\,a^2\,\mathrm{e}^{c+d\,x}}{3\,d\,\left(6\,\mathrm{e}^{4\,c+4\,d\,x}-4\,\mathrm{e}^{2\,c+2\,d\,x}-4\,\mathrm{e}^{6\,c+6\,d\,x}+\mathrm{e}^{8\,c+8\,d\,x}+1\right)}$$
$$- \frac{16\,a^2\,\mathrm{e}^{c+d\,x}}{3\,d\,\left(5\,\mathrm{e}^{2\,c+2\,d\,x}-10\,\mathrm{e}^{4\,c+4\,d\,x}+10\,\mathrm{e}^{6\,c+6\,d\,x}-5\,\mathrm{e}^{8\,c+8\,d\,x}+\mathrm{e}^{10\,c+10\,d\,x}-1\right)}$$
$$- \frac{\mathrm{e}^{c+d\,x}\,\left(5\,a^2+16\,b\,a\right)}{8\,d\,\left(\mathrm{e}^{2\,c+2\,d\,x}-1\right)} - \frac{\mathrm{e}^{c+d\,x}\,\left(32\,a\,b-5\,a^2\right)}{12\,d\,\left(\mathrm{e}^{4\,c+4\,d\,x}-2\,\mathrm{e}^{2\,c+2\,d\,x}+1\right)}$$

input
```
int((a + b*sinh(c + d*x)^4)^2/sinh(c + d*x)^7,x)
```

output
```
(b^2*exp(c + d*x))/(2*d) - ((8*exp(5*c + 5*d*x)*(3*a*b + 4*a^2))/(3*d) + (
4*a*b*exp(c + d*x))/(3*d) - (16*a*b*exp(3*c + 3*d*x))/(3*d) - (16*a*b*exp(
7*c + 7*d*x))/(3*d) + (4*a*b*exp(9*c + 9*d*x))/(3*d))/(15*exp(4*c + 4*d*x)
 - 6*exp(2*c + 2*d*x) - 20*exp(6*c + 6*d*x) + 15*exp(8*c + 8*d*x) - 6*exp(
10*c + 10*d*x) + exp(12*c + 12*d*x) + 1) + (b^2*exp(- c - d*x))/(2*d) + (a
tan((exp(d*x)*exp(c)*(5*a^2*(-d^2)^(1/2) + 16*a*b*(-d^2)^(1/2)))/(d*(160*a
^3*b + 25*a^4 + 256*a^2*b^2)^(1/2)))*(160*a^3*b + 25*a^4 + 256*a^2*b^2)^(1
/2))/(8*(-d^2)^(1/2)) - (a^2*exp(c + d*x))/(3*d*(3*exp(2*c + 2*d*x) - 3*ex
p(4*c + 4*d*x) + exp(6*c + 6*d*x) - 1)) - (22*a^2*exp(c + d*x))/(3*d*(6*ex
p(4*c + 4*d*x) - 4*exp(2*c + 2*d*x) - 4*exp(6*c + 6*d*x) + exp(8*c + 8*d*x
) + 1)) - (16*a^2*exp(c + d*x))/(3*d*(5*exp(2*c + 2*d*x) - 10*exp(4*c + 4*
d*x) + 10*exp(6*c + 6*d*x) - 5*exp(8*c + 8*d*x) + exp(10*c + 10*d*x) - 1))
 - (exp(c + d*x)*(16*a*b + 5*a^2))/(8*d*(exp(2*c + 2*d*x) - 1)) - (exp(c +
 d*x)*(32*a*b - 5*a^2))/(12*d*(exp(4*c + 4*d*x) - 2*exp(2*c + 2*d*x) + 1))
```

Reduce [B] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 1047, normalized size of antiderivative = 9.43

$$\int \operatorname{csch}^7(c+dx) \left(a+b\sinh^4(c+dx)\right)^2 \, dx = \text{Too large to display}$$

input
```
int(csch(d*x+c)^7*(a+b*sinh(d*x+c)^4)^2,x)
```

output
```
(24*e**(14*c + 14*d*x)*b**2 - 15*e**(13*c + 13*d*x)*log(e**(c + d*x) - 1)*
a**2 - 48*e**(13*c + 13*d*x)*log(e**(c + d*x) - 1)*a*b + 15*e**(13*c + 13*
d*x)*log(e**(c + d*x) + 1)*a**2 + 48*e**(13*c + 13*d*x)*log(e**(c + d*x) +
 1)*a*b - 30*e**(12*c + 12*d*x)*a**2 - 96*e**(12*c + 12*d*x)*a*b - 120*e**
(12*c + 12*d*x)*b**2 + 90*e**(11*c + 11*d*x)*log(e**(c + d*x) - 1)*a**2 +
288*e**(11*c + 11*d*x)*log(e**(c + d*x) - 1)*a*b - 90*e**(11*c + 11*d*x)*l
og(e**(c + d*x) + 1)*a**2 - 288*e**(11*c + 11*d*x)*log(e**(c + d*x) + 1)*a
*b + 170*e**(10*c + 10*d*x)*a**2 + 288*e**(10*c + 10*d*x)*a*b + 216*e**(10
*c + 10*d*x)*b**2 - 225*e**(9*c + 9*d*x)*log(e**(c + d*x) - 1)*a**2 - 720*
e**(9*c + 9*d*x)*log(e**(c + d*x) - 1)*a*b + 225*e**(9*c + 9*d*x)*log(e**(
c + d*x) + 1)*a**2 + 720*e**(9*c + 9*d*x)*log(e**(c + d*x) + 1)*a*b - 396*
e**(8*c + 8*d*x)*a**2 - 192*e**(8*c + 8*d*x)*a*b - 120*e**(8*c + 8*d*x)*b*
*2 + 300*e**(7*c + 7*d*x)*log(e**(c + d*x) - 1)*a**2 + 960*e**(7*c + 7*d*x
)*log(e**(c + d*x) - 1)*a*b - 300*e**(7*c + 7*d*x)*log(e**(c + d*x) + 1)*a
**2 - 960*e**(7*c + 7*d*x)*log(e**(c + d*x) + 1)*a*b - 396*e**(6*c + 6*d*x
)*a**2 - 192*e**(6*c + 6*d*x)*a*b - 120*e**(6*c + 6*d*x)*b**2 - 225*e**(5*
c + 5*d*x)*log(e**(c + d*x) - 1)*a**2 - 720*e**(5*c + 5*d*x)*log(e**(c + d
*x) - 1)*a*b + 225*e**(5*c + 5*d*x)*log(e**(c + d*x) + 1)*a**2 + 720*e**(5
*c + 5*d*x)*log(e**(c + d*x) + 1)*a*b + 170*e**(4*c + 4*d*x)*a**2 + 288*e*
*(4*c + 4*d*x)*a*b + 216*e**(4*c + 4*d*x)*b**2 + 90*e**(3*c + 3*d*x)*lo...
```

3.182 $\int \sinh^5(c+dx) \left(a+b\sinh^4(c+dx)\right)^3 dx$

Optimal result

Integrand size = 23, antiderivative size = 220

$$\int \sinh^5(c+dx) \left(a+b\sinh^4(c+dx)\right)^3 dx = \frac{(a+b)^3\cosh(c+dx)}{d} - \frac{2(a+b)^2(a+4b)\cosh^3(c+dx)}{3d} + \frac{(a+b)\left(a^2+17ab+28b^2\right)\cosh^5(c+dx)}{5d} - \frac{4b(3a^2+15ab+14b^2)\cosh^7(c+dx)}{7d} + \frac{b(3a^2+45ab+70b^2)\cosh^9(c+dx)}{9d} - \frac{2b^2(9a+28b)\cosh^{11}(c+dx)}{11d} + \frac{b^2(3a+28b)\cosh^{13}(c+dx)}{13d} - \frac{8b^3\cosh^{15}(c+dx)}{15d} + \frac{b^3\cosh^{17}(c+dx)}{17d}$$

output
```
(a+b)^3*cosh(d*x+c)/d-2/3*(a+b)^2*(a+4*b)*cosh(d*x+c)^3/d+1/5*(a+b)*(a^2+1
7*a*b+28*b^2)*cosh(d*x+c)^5/d-4/7*b*(3*a^2+15*a*b+14*b^2)*cosh(d*x+c)^7/d+
1/9*b*(3*a^2+45*a*b+70*b^2)*cosh(d*x+c)^9/d-2/11*b^2*(9*a+28*b)*cosh(d*x+c
)^11/d+1/13*b^2*(3*a+28*b)*cosh(d*x+c)^13/d-8/15*b^3*cosh(d*x+c)^15/d+1/17
*b^3*cosh(d*x+c)^17/d
```

Mathematica [A] (verified)

Time = 11.13 (sec) , antiderivative size = 288, normalized size of antiderivative = 1.31

$$\int \sinh^5(c+dx) \left(a+b\sinh^4(c+dx)\right)^3 dx$$
$$= \frac{1531530(20480a^3+48384a^2b+41184ab^2+12155b^3)\cosh(c+dx)-2042040(2560a^3+8064a^2b+7722}{}$$

input
```
Integrate[Sinh[c + d*x]^5*(a + b*Sinh[c + d*x]^4)^3,x]
```

output
```
(1531530*(20480*a^3 + 48384*a^2*b + 41184*a*b^2 + 12155*b^3)*Cosh[c + d*x]
 - 2042040*(2560*a^3 + 8064*a^2*b + 7722*a*b^2 + 2431*b^3)*Cosh[3*(c + d*x
)] + 627314688*a^3*Cosh[5*(c + d*x)] + 4234374144*a^2*b*Cosh[5*(c + d*x)]
+ 5256210960*a*b^2*Cosh[5*(c + d*x)] + 1895421528*b^3*Cosh[5*(c + d*x)] -
756138240*a^2*b*Cosh[7*(c + d*x)] - 1501774560*a*b^2*Cosh[7*(c + d*x)] - 6
76936260*b^3*Cosh[7*(c + d*x)] + 65345280*a^2*b*Cosh[9*(c + d*x)] + 318558
240*a*b^2*Cosh[9*(c + d*x)] + 202502300*b^3*Cosh[9*(c + d*x)] - 43439760*a
*b^2*Cosh[11*(c + d*x)] - 47338200*b^3*Cosh[11*(c + d*x)] + 2827440*a*b^2*
Cosh[13*(c + d*x)] + 8011080*b^3*Cosh[13*(c + d*x)] - 867867*b^3*Cosh[15*(
c + d*x)] + 45045*b^3*Cosh[17*(c + d*x)])/(50185175040*d)
```

Rubi [A] (verified)

Time = 0.47 (sec) , antiderivative size = 197, normalized size of antiderivative = 0.90, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.217$, Rules used = {3042, 26, 3694, 1467, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sinh^5(c+dx) \left(a+b\sinh^4(c+dx)\right)^3 dx$$

$$\Big\downarrow \text{3042}$$

$$\int -i\sin(ic+idx)^5 \left(a+b\sin(ic+idx)^4\right)^3 dx$$

$$\Big\downarrow \text{26}$$

$$-i\int \sin(ic+idx)^5\left(b\sin(ic+idx)^4+a\right)^3dx$$

$$\Big\downarrow \text{3694}$$

$$\frac{\int \left(1-\cosh^2(c+dx)\right)^2\left(b\cosh^4(c+dx)-2b\cosh^2(c+dx)+a+b\right)^3d\cosh(c+dx)}{d}$$

$$\Big\downarrow \text{1467}$$

$$\frac{\int \left(b^3\cosh^{16}(c+dx)-8b^3\cosh^{14}(c+dx)+b^2(3a+28b)\cosh^{12}(c+dx)-2b^2(9a+28b)\cosh^{10}(c+dx)+b(3a^2$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{9}b\left(3a^2+45ab+70b^2\right)\cosh^9(c+dx)-\frac{4}{7}b\left(3a^2+15ab+14b^2\right)\cosh^7(c+dx)+\frac{1}{5}(a+b)\left(a^2+17ab+28b^2\right)\cosh$$

input
```
Int[Sinh[c + d*x]^5*(a + b*Sinh[c + d*x]^4)^3,x]
```

output
```
((a + b)^3*Cosh[c + d*x] - (2*(a + b)^2*(a + 4*b)*Cosh[c + d*x]^3)/3 + ((a
 + b)*(a^2 + 17*a*b + 28*b^2)*Cosh[c + d*x]^5)/5 - (4*b*(3*a^2 + 15*a*b +
14*b^2)*Cosh[c + d*x]^7)/7 + (b*(3*a^2 + 45*a*b + 70*b^2)*Cosh[c + d*x]^9)
/9 - (2*b^2*(9*a + 28*b)*Cosh[c + d*x]^11)/11 + (b^2*(3*a + 28*b)*Cosh[c +
 d*x]^13)/13 - (8*b^3*Cosh[c + d*x]^15)/15 + (b^3*Cosh[c + d*x]^17)/17)/d
```

Defintions of rubi rules used

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 1467
```
Int[((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.),
 x_Symbol] :> Int[ExpandIntegrand[(d + e*x^2)^q*(a + b*x^2 + c*x^4)^p, x],
x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e
 + a*e^2, 0] && IGtQ[p, 0] && IGtQ[q, -2]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3694
```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^4)^
(p_.), x_Symbol] :> With[{ff = FreeFactors[Cos[e + f*x], x]}, Simp[-ff/f
Subst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a + b - 2*b*ff^2*x^2 + b*ff^4*x^4)^p,
 x], x, Cos[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m -
 1)/2]
```

Maple [A] (verified)

Time = 0.35 (sec) , antiderivative size = 258, normalized size of antiderivative = 1.17

$$\frac{a^3\left(\frac{8}{15}+\frac{\sinh(dx+c)^4}{5}-\frac{4\sinh(dx+c)^2}{15}\right)\cosh\left(dx+c\right)+3a^2b\left(\frac{128}{315}+\frac{\sinh(dx+c)^8}{9}-\frac{8\sinh(dx+c)^6}{63}+\frac{16\sinh(dx+c)^4}{105}-\frac{64\,\mathrm{si}}{}\right.}{}$$

input
```
int(sinh(d*x+c)^5*(a+b*sinh(d*x+c)^4)^3,x)
```

output
```
1/d*(a^3*(8/15+1/5*sinh(d*x+c)^4-4/15*sinh(d*x+c)^2)*cosh(d*x+c)+3*a^2*b*(
128/315+1/9*sinh(d*x+c)^8-8/63*sinh(d*x+c)^6+16/105*sinh(d*x+c)^4-64/315*s
inh(d*x+c)^2)*cosh(d*x+c)+3*b^2*a*(1024/3003+1/13*sinh(d*x+c)^12-12/143*si
nh(d*x+c)^10+40/429*sinh(d*x+c)^8-320/3003*sinh(d*x+c)^6+128/1001*sinh(d*x
+c)^4-512/3003*sinh(d*x+c)^2)*cosh(d*x+c)+b^3*(32768/109395+1/17*sinh(d*x+
c)^16-16/255*sinh(d*x+c)^14+224/3315*sinh(d*x+c)^12-896/12155*sinh(d*x+c)^
10+1792/21879*sinh(d*x+c)^8-2048/21879*sinh(d*x+c)^6+4096/36465*sinh(d*x+c
)^4-16384/109395*sinh(d*x+c)^2)*cosh(d*x+c))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1030 vs. 2(204) = 408.

Time = 0.10 (sec) , antiderivative size = 1030, normalized size of antiderivative = 4.68

$$\int \sinh^5(c+dx)\left(a+b\sinh^4(c+dx)\right)^3\,dx=\text{Too large to display}$$

input
```
integrate(sinh(d*x+c)^5*(a+b*sinh(d*x+c)^4)^3,x, algorithm="fricas")
```

output
```
1/50185175040*(45045*b^3*cosh(d*x + c)^17 + 765765*b^3*cosh(d*x + c)*sinh(
d*x + c)^16 - 867867*b^3*cosh(d*x + c)^15 + 765765*(40*b^3*cosh(d*x + c)^3
 - 17*b^3*cosh(d*x + c))*sinh(d*x + c)^14 + 471240*(6*a*b^2 + 17*b^3)*cosh
(d*x + c)^13 + 255255*(1092*b^3*cosh(d*x + c)^5 - 1547*b^3*cosh(d*x + c)^3
 + 24*(6*a*b^2 + 17*b^3)*cosh(d*x + c))*sinh(d*x + c)^12 - 556920*(78*a*b^
2 + 85*b^3)*cosh(d*x + c)^11 + 153153*(5720*b^3*cosh(d*x + c)^7 - 17017*b^
3*cosh(d*x + c)^5 + 880*(6*a*b^2 + 17*b^3)*cosh(d*x + c)^3 - 40*(78*a*b^2
+ 85*b^3)*cosh(d*x + c))*sinh(d*x + c)^10 + 340340*(192*a^2*b + 936*a*b^2
+ 595*b^3)*cosh(d*x + c)^9 + 765765*(1430*b^3*cosh(d*x + c)^9 - 7293*b^3*c
osh(d*x + c)^7 + 792*(6*a*b^2 + 17*b^3)*cosh(d*x + c)^5 - 120*(78*a*b^2 +
85*b^3)*cosh(d*x + c)^3 + 4*(192*a^2*b + 936*a*b^2 + 595*b^3)*cosh(d*x + c
))*sinh(d*x + c)^8 - 437580*(1728*a^2*b + 3432*a*b^2 + 1547*b^3)*cosh(d*x
+ c)^7 + 255255*(2184*b^3*cosh(d*x + c)^11 - 17017*b^3*cosh(d*x + c)^9 + 3
168*(6*a*b^2 + 17*b^3)*cosh(d*x + c)^7 - 1008*(78*a*b^2 + 85*b^3)*cosh(d*x
 + c)^5 + 112*(192*a^2*b + 936*a*b^2 + 595*b^3)*cosh(d*x + c)^3 - 12*(1728
*a^2*b + 3432*a*b^2 + 1547*b^3)*cosh(d*x + c))*sinh(d*x + c)^6 + 1225224*(
512*a^3 + 3456*a^2*b + 4290*a*b^2 + 1547*b^3)*cosh(d*x + c)^5 + 765765*(14
0*b^3*cosh(d*x + c)^13 - 1547*b^3*cosh(d*x + c)^11 + 440*(6*a*b^2 + 17*b^3
)*cosh(d*x + c)^9 - 240*(78*a*b^2 + 85*b^3)*cosh(d*x + c)^7 + 56*(192*a^2*
b + 936*a*b^2 + 595*b^3)*cosh(d*x + c)^5 - 20*(1728*a^2*b + 3432*a*b^2 ...
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 592 vs. $2(204) = 408$.

Time = 11.24 (sec) , antiderivative size = 592, normalized size of antiderivative = 2.69

$$\int \sinh^5(c+dx) \left(a+b\sinh^4(c+dx)\right)^3 \, dx = \text{Too large to display}$$

input
```
integrate(sinh(d*x+c)**5*(a+b*sinh(d*x+c)**4)**3,x)
```

output

```
Piecewise((a**3*sinh(c + d*x)**4*cosh(c + d*x)/d - 4*a**3*sinh(c + d*x)**2
*cosh(c + d*x)**3/(3*d) + 8*a**3*cosh(c + d*x)**5/(15*d) + 3*a**2*b*sinh(c
 + d*x)**8*cosh(c + d*x)/d - 8*a**2*b*sinh(c + d*x)**6*cosh(c + d*x)**3/d
+ 48*a**2*b*sinh(c + d*x)**4*cosh(c + d*x)**5/(5*d) - 192*a**2*b*sinh(c +
d*x)**2*cosh(c + d*x)**7/(35*d) + 128*a**2*b*cosh(c + d*x)**9/(105*d) + 3*
a*b**2*sinh(c + d*x)**12*cosh(c + d*x)/d - 12*a*b**2*sinh(c + d*x)**10*cos
h(c + d*x)**3/d + 24*a*b**2*sinh(c + d*x)**8*cosh(c + d*x)**5/d - 192*a*b*
*2*sinh(c + d*x)**6*cosh(c + d*x)**7/(7*d) + 128*a*b**2*sinh(c + d*x)**4*c
osh(c + d*x)**9/(7*d) - 512*a*b**2*sinh(c + d*x)**2*cosh(c + d*x)**11/(77*
d) + 1024*a*b**2*cosh(c + d*x)**13/(1001*d) + b**3*sinh(c + d*x)**16*cosh(
c + d*x)/d - 16*b**3*sinh(c + d*x)**14*cosh(c + d*x)**3/(3*d) + 224*b**3*s
inh(c + d*x)**12*cosh(c + d*x)**5/(15*d) - 128*b**3*sinh(c + d*x)**10*cosh
(c + d*x)**7/(5*d) + 256*b**3*sinh(c + d*x)**8*cosh(c + d*x)**9/(9*d) - 20
48*b**3*sinh(c + d*x)**6*cosh(c + d*x)**11/(99*d) + 4096*b**3*sinh(c + d*x
)**4*cosh(c + d*x)**13/(429*d) - 16384*b**3*sinh(c + d*x)**2*cosh(c + d*x)
**15/(6435*d) + 32768*b**3*cosh(c + d*x)**17/(109395*d), Ne(d, 0)), (x*(a
+ b*sinh(c)**4)**3*sinh(c)**5, True))
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 600 vs. $2(204) = 408$.

Time = 0.06 (sec) , antiderivative size = 600, normalized size of antiderivative = 2.73

$$\int \sinh^5(c+dx)\left(a+b\sinh^4(c+dx)\right)^3\,dx = \text{Too large to display}$$

input

```
integrate(sinh(d*x+c)^5*(a+b*sinh(d*x+c)^4)^3,x, algorithm="maxima")
```

output
```
-1/14338621440*b^3*((123981*e^(-2*d*x - 2*c) - 1144440*e^(-4*d*x - 4*c) +
6762600*e^(-6*d*x - 6*c) - 28928900*e^(-8*d*x - 8*c) + 96705180*e^(-10*d*x
 - 10*c) - 270774504*e^(-12*d*x - 12*c) + 709171320*e^(-14*d*x - 14*c) - 2
659392450*e^(-16*d*x - 16*c) - 6435)*e^(17*d*x + 17*c)/d - (2659392450*e^(
-d*x - c) - 709171320*e^(-3*d*x - 3*c) + 270774504*e^(-5*d*x - 5*c) - 9670
5180*e^(-7*d*x - 7*c) + 28928900*e^(-9*d*x - 9*c) - 6762600*e^(-11*d*x - 1
1*c) + 1144440*e^(-13*d*x - 13*c) - 123981*e^(-15*d*x - 15*c) + 6435*e^(-1
7*d*x - 17*c))/d) - 1/8200192*a*b^2*((3549*e^(-2*d*x - 2*c) - 26026*e^(-4*
d*x - 4*c) + 122694*e^(-6*d*x - 6*c) - 429429*e^(-8*d*x - 8*c) + 1288287*e
^(-10*d*x - 10*c) - 5153148*e^(-12*d*x - 12*c) - 231)*e^(13*d*x + 13*c)/d
- (5153148*e^(-d*x - c) - 1288287*e^(-3*d*x - 3*c) + 429429*e^(-5*d*x - 5*
c) - 122694*e^(-7*d*x - 7*c) + 26026*e^(-9*d*x - 9*c) - 3549*e^(-11*d*x -
11*c) + 231*e^(-13*d*x - 13*c))/d) - 1/53760*a^2*b*((405*e^(-2*d*x - 2*c)
- 2268*e^(-4*d*x - 4*c) + 8820*e^(-6*d*x - 6*c) - 39690*e^(-8*d*x - 8*c) -
 35)*e^(9*d*x + 9*c)/d - (39690*e^(-d*x - c) - 8820*e^(-3*d*x - 3*c) + 226
8*e^(-5*d*x - 5*c) - 405*e^(-7*d*x - 7*c) + 35*e^(-9*d*x - 9*c))/d) + 1/48
0*a^3*(3*e^(5*d*x + 5*c)/d - 25*e^(3*d*x + 3*c)/d + 150*e^(d*x + c)/d + 15
0*e^(-d*x - c)/d - 25*e^(-3*d*x - 3*c)/d + 3*e^(-5*d*x - 5*c)/d)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 520 vs. $2(204) = 408$.

Time = 0.25 (sec) , antiderivative size = 520, normalized size of antiderivative = 2.36

$$\int \sinh^5(c+dx)\left(a+b\sinh^4(c+dx)\right)^3\,dx$$
$$= \frac{b^3e^{(17\,dx+17\,c)}}{2228224\,d} - \frac{17\,b^3e^{(15\,dx+15\,c)}}{1966080\,d} - \frac{17\,b^3e^{(-15\,dx-15\,c)}}{1966080\,d} + \frac{b^3e^{(-17\,dx-17\,c)}}{2228224\,d} + \frac{(6\,ab^2+17\,b^3)e^{(13\,dx+13\,c)}}{212992\,d} - \frac{(78\,ab^2+85\,b^3)e^{(11\,dx+11\,c)}}{180224\,d} + \frac{(192\,a^2b+936\,ab^2+595\,b^3)e^{(9\,dx+9\,c)}}{294912\,d} - \frac{(1728\,a^2b+3432\,ab^2+1547\,b^3)e^{(7\,dx+7\,c)}}{229376\,d} + \frac{(512\,a^3+3456\,a^2b+4290\,ab^2+1547\,b^3)e^{(5\,dx+5\,c)}}{81920\,d} - \frac{(2560\,a^3+8064\,a^2b+7722\,ab^2+2431\,b^3)e^{(3\,dx+3\,c)}}{49152\,d} + \frac{(20480\,a^3+48384\,a^2b+41184\,ab^2+12155\,b^3)e^{(dx+c)}}{65536\,d} + \frac{(20480\,a^3+48384\,a^2b+41184\,ab^2+12155\,b^3)e^{(-dx-c)}}{65536\,d} - \frac{(2560\,a^3+8064\,a^2b+7722\,ab^2+2431\,b^3)e^{(-3\,dx-3\,c)}}{49152\,d} + \frac{(512\,a^3+3456\,a^2b+4290\,ab^2+1547\,b^3)e^{(-5\,dx-5\,c)}}{81920\,d} - \frac{(1728\,a^2b+3432\,ab^2+1547\,b^3)e^{(-7\,dx-7\,c)}}{229376\,d} + \frac{(192\,a^2b+936\,ab^2+595\,b^3)e^{(-9\,dx-9\,c)}}{294912\,d} - \frac{(78\,ab^2+85\,b^3)e^{(-11\,dx-11\,c)}}{180224\,d} + \frac{(6\,ab^2+17\,b^3)e^{(-13\,dx-13\,c)}}{212992\,d}$$

input

```
integrate(sinh(d*x+c)^5*(a+b*sinh(d*x+c)^4)^3,x, algorithm="giac")
```

output
```
1/2228224*b^3*e^(17*d*x + 17*c)/d - 17/1966080*b^3*e^(15*d*x + 15*c)/d - 1
7/1966080*b^3*e^(-15*d*x - 15*c)/d + 1/2228224*b^3*e^(-17*d*x - 17*c)/d +
1/212992*(6*a*b^2 + 17*b^3)*e^(13*d*x + 13*c)/d - 1/180224*(78*a*b^2 + 85*
b^3)*e^(11*d*x + 11*c)/d + 1/294912*(192*a^2*b + 936*a*b^2 + 595*b^3)*e^(9
*d*x + 9*c)/d - 1/229376*(1728*a^2*b + 3432*a*b^2 + 1547*b^3)*e^(7*d*x + 7
*c)/d + 1/81920*(512*a^3 + 3456*a^2*b + 4290*a*b^2 + 1547*b^3)*e^(5*d*x +
5*c)/d - 1/49152*(2560*a^3 + 8064*a^2*b + 7722*a*b^2 + 2431*b^3)*e^(3*d*x
+ 3*c)/d + 1/65536*(20480*a^3 + 48384*a^2*b + 41184*a*b^2 + 12155*b^3)*e^(
d*x + c)/d + 1/65536*(20480*a^3 + 48384*a^2*b + 41184*a*b^2 + 12155*b^3)*e
^(-d*x - c)/d - 1/49152*(2560*a^3 + 8064*a^2*b + 7722*a*b^2 + 2431*b^3)*e^
(-3*d*x - 3*c)/d + 1/81920*(512*a^3 + 3456*a^2*b + 4290*a*b^2 + 1547*b^3)*
e^(-5*d*x - 5*c)/d - 1/229376*(1728*a^2*b + 3432*a*b^2 + 1547*b^3)*e^(-7*d
*x - 7*c)/d + 1/294912*(192*a^2*b + 936*a*b^2 + 595*b^3)*e^(-9*d*x - 9*c)/
d - 1/180224*(78*a*b^2 + 85*b^3)*e^(-11*d*x - 11*c)/d + 1/212992*(6*a*b^2
+ 17*b^3)*e^(-13*d*x - 13*c)/d
```

Mupad [B] (verification not implemented)

Time = 2.57 (sec) , antiderivative size = 319, normalized size of antiderivative = 1.45

$$\int \sinh^5(c+dx)\left(a+b\sinh^4(c+dx)\right)^3\,dx$$

$$= \frac{\frac{a^3\cosh(c+d\,x)^5}{5}-\frac{2\,a^3\cosh(c+d\,x)^3}{3}+a^3\cosh(c+d\,x)+\frac{a^2\,b\cosh(c+d\,x)^9}{3}-\frac{12\,a^2\,b\cosh(c+d\,x)^7}{7}+\frac{18\,a^2\,b\cosh(c+d\,x)^5}{5}-4}{}$$

input
```
int(sinh(c + d*x)^5*(a + b*sinh(c + d*x)^4)^3,x)
```

output
```
(a^3*cosh(c + d*x) + b^3*cosh(c + d*x) - (2*a^3*cosh(c + d*x)^3)/3 + (a^3*
cosh(c + d*x)^5)/5 - (8*b^3*cosh(c + d*x)^3)/3 + (28*b^3*cosh(c + d*x)^5)/
5 - 8*b^3*cosh(c + d*x)^7 + (70*b^3*cosh(c + d*x)^9)/9 - (56*b^3*cosh(c +
d*x)^11)/11 + (28*b^3*cosh(c + d*x)^13)/13 - (8*b^3*cosh(c + d*x)^15)/15 +
 (b^3*cosh(c + d*x)^17)/17 - 6*a*b^2*cosh(c + d*x)^3 - 4*a^2*b*cosh(c + d*
x)^3 + 9*a*b^2*cosh(c + d*x)^5 + (18*a^2*b*cosh(c + d*x)^5)/5 - (60*a*b^2*
cosh(c + d*x)^7)/7 - (12*a^2*b*cosh(c + d*x)^7)/7 + 5*a*b^2*cosh(c + d*x)^
9 + (a^2*b*cosh(c + d*x)^9)/3 - (18*a*b^2*cosh(c + d*x)^11)/11 + (3*a*b^2*
cosh(c + d*x)^13)/13 + 3*a*b^2*cosh(c + d*x) + 3*a^2*b*cosh(c + d*x))/d
```

Reduce [B] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 752, normalized size of antiderivative = 3.42

$$\int \sinh^5(c+dx)\left(a+b\sinh^4(c+dx)\right)^3\,dx$$
$$=\frac{2827440e^{30dx+30c}a\,b^2-43439760e^{28dx+28c}a\,b^2+65345280e^{26dx+26c}a^2b+318558240e^{26dx+26c}a\,b^2-75613}{}$$

input
```
int(sinh(d*x+c)^5*(a+b*sinh(d*x+c)^4)^3,x)
```

output
```
(45045*e**(34*c + 34*d*x)*b**3 - 867867*e**(32*c + 32*d*x)*b**3 + 2827440*
e**(30*c + 30*d*x)*a*b**2 + 8011080*e**(30*c + 30*d*x)*b**3 - 43439760*e**
(28*c + 28*d*x)*a*b**2 - 47338200*e**(28*c + 28*d*x)*b**3 + 65345280*e**(2
6*c + 26*d*x)*a**2*b + 318558240*e**(26*c + 26*d*x)*a*b**2 + 202502300*e**
(26*c + 26*d*x)*b**3 - 756138240*e**(24*c + 24*d*x)*a**2*b - 1501774560*e*
*(24*c + 24*d*x)*a*b**2 - 676936260*e**(24*c + 24*d*x)*b**3 + 627314688*e*
*(22*c + 22*d*x)*a**3 + 4234374144*e**(22*c + 22*d*x)*a**2*b + 5256210960*
e**(22*c + 22*d*x)*a*b**2 + 1895421528*e**(22*c + 22*d*x)*b**3 - 522762240
0*e**(20*c + 20*d*x)*a**3 - 16467010560*e**(20*c + 20*d*x)*a**2*b - 157686
32880*e**(20*c + 20*d*x)*a*b**2 - 4964199240*e**(20*c + 20*d*x)*b**3 + 313
65734400*e**(18*c + 18*d*x)*a**3 + 74101547520*e**(18*c + 18*d*x)*a**2*b +
 63074531520*e**(18*c + 18*d*x)*a*b**2 + 18615747150*e**(18*c + 18*d*x)*b*
*3 + 31365734400*e**(16*c + 16*d*x)*a**3 + 74101547520*e**(16*c + 16*d*x)*
a**2*b + 63074531520*e**(16*c + 16*d*x)*a*b**2 + 18615747150*e**(16*c + 16
*d*x)*b**3 - 5227622400*e**(14*c + 14*d*x)*a**3 - 16467010560*e**(14*c + 1
4*d*x)*a**2*b - 15768632880*e**(14*c + 14*d*x)*a*b**2 - 4964199240*e**(14*
c + 14*d*x)*b**3 + 627314688*e**(12*c + 12*d*x)*a**3 + 4234374144*e**(12*c
 + 12*d*x)*a**2*b + 5256210960*e**(12*c + 12*d*x)*a*b**2 + 1895421528*e**(
12*c + 12*d*x)*b**3 - 756138240*e**(10*c + 10*d*x)*a**2*b - 1501774560*e**
(10*c + 10*d*x)*a*b**2 - 676936260*e**(10*c + 10*d*x)*b**3 + 65345280*e...
```

3.183 $\int \sinh^3(c+dx) \left(a+b\sinh^4(c+dx)\right)^3 \, dx$

Optimal result

Integrand size = 23, antiderivative size = 183

$$\int \sinh^3(c+dx) \left(a+b\sinh^4(c+dx)\right)^3 \, dx = -\frac{(a+b)^3\cosh(c+dx)}{d} + \frac{(a+b)^2(a+7b)\cosh^3(c+dx)}{3d} - \frac{3b(a+b)(3a+7b)\cosh^5(c+dx)}{5d} + \frac{b(3a^2+30ab+35b^2)\cosh^7(c+dx)}{7d} - \frac{5b^2(3a+7b)\cosh^9(c+dx)}{9d} + \frac{3b^2(a+7b)\cosh^{11}(c+dx)}{11d} - \frac{7b^3\cosh^{13}(c+dx)}{13d} + \frac{b^3\cosh^{15}(c+dx)}{15d}$$

output
```
-(a+b)^3*cosh(d*x+c)/d+1/3*(a+b)^2*(a+7*b)*cosh(d*x+c)^3/d-3/5*b*(a+b)*(3*
a+7*b)*cosh(d*x+c)^5/d+1/7*b*(3*a^2+30*a*b+35*b^2)*cosh(d*x+c)^7/d-5/9*b^2
*(3*a+7*b)*cosh(d*x+c)^9/d+3/11*b^2*(a+7*b)*cosh(d*x+c)^11/d-7/13*b^3*cosh
(d*x+c)^13/d+1/15*b^3*cosh(d*x+c)^15/d
```

Mathematica [A] (verified)

Time = 11.49 (sec) , antiderivative size = 185, normalized size of antiderivative = 1.01

$$\int \sinh^3(c+dx)\left(a+b\sinh^4(c+dx)\right)^3\,dx$$
$$=\frac{-135135(4096a^3+8960a^2b+7392ab^2+2145b^3)\cosh(c+dx)+15015(4096a^3+16128a^2b+15840ab^2}{}$$

input
```
Integrate[Sinh[c + d*x]^3*(a + b*Sinh[c + d*x]^4)^3,x]
```

output
```
(-135135*(4096*a^3 + 8960*a^2*b + 7392*a*b^2 + 2145*b^3)*Cosh[c + d*x] + 1
5015*(4096*a^3 + 16128*a^2*b + 15840*a*b^2 + 5005*b^3)*Cosh[3*(c + d*x)] +
 b*(-27027*(1792*a^2 + 2640*a*b + 1001*b^2)*Cosh[5*(c + d*x)] + 19305*(256
*a^2 + 880*a*b + 455*b^2)*Cosh[7*(c + d*x)] - 7*b*(715*(528*a + 455*b)*Cos
h[9*(c + d*x)] - 1755*(16*a + 35*b)*Cosh[11*(c + d*x)] + 7425*b*Cosh[13*(c
 + d*x)] - 429*b*Cosh[15*(c + d*x)])))/(738017280*d)
```

Rubi [A] (verified)

Time = 0.42 (sec) , antiderivative size = 163, normalized size of antiderivative = 0.89, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.217$, Rules used = {3042, 26, 3694, 1467, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sinh^3(c+dx)\left(a+b\sinh^4(c+dx)\right)^3\,dx$$
$$\Big\downarrow \text{ 3042}$$
$$\int i\sin(ic+idx)^3\left(a+b\sin(ic+idx)^4\right)^3dx$$
$$\Big\downarrow \text{ 26}$$
$$i\int \sin(ic+idx)^3\left(b\sin(ic+idx)^4+a\right)^3dx$$
$$\Big\downarrow \text{ 3694}$$

$$-\frac{\int \left(1-\cosh^2(c+dx)\right)\left(b\cosh^4(c+dx)-2b\cosh^2(c+dx)+a+b\right)^3d\cosh(c+dx)}{d}$$

$\Big\downarrow$ 1467

$$-\frac{\int \left(-b^3\cosh^{14}(c+dx)+7b^3\cosh^{12}(c+dx)-3b^2(a+7b)\cosh^{10}(c+dx)+5b^2(3a+7b)\cosh^8(c+dx)-b(3a^2\right.}{}$$

$\Big\downarrow$ 2009

$$-\frac{-\frac{1}{7}b(3a^2+30ab+35b^2)\cosh^7(c+dx)-\frac{3}{11}b^2(a+7b)\cosh^{11}(c+dx)+\frac{5}{9}b^2(3a+7b)\cosh^9(c+dx)+\frac{3}{5}b(a+b}{}$$

input
```
Int[Sinh[c + d*x]^3*(a + b*Sinh[c + d*x]^4)^3,x]
```

output
```
-(((a + b)^3*Cosh[c + d*x] - ((a + b)^2*(a + 7*b)*Cosh[c + d*x]^3)/3 + (3*
b*(a + b)*(3*a + 7*b)*Cosh[c + d*x]^5)/5 - (b*(3*a^2 + 30*a*b + 35*b^2)*Co
sh[c + d*x]^7)/7 + (5*b^2*(3*a + 7*b)*Cosh[c + d*x]^9)/9 - (3*b^2*(a + 7*b
)*Cosh[c + d*x]^11)/11 + (7*b^3*Cosh[c + d*x]^13)/13 - (b^3*Cosh[c + d*x]^
15)/15)/d)
```

Defintions of rubi rules used

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 1467
```
Int[((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.),
 x_Symbol] :> Int[ExpandIntegrand[(d + e*x^2)^q*(a + b*x^2 + c*x^4)^p, x],
x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e
 + a*e^2, 0] && IGtQ[p, 0] && IGtQ[q, -2]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3694
```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^4)^
(p_.), x_Symbol] :> With[{ff = FreeFactors[Cos[e + f*x], x]}, Simp[-ff/f
Subst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a + b - 2*b*ff^2*x^2 + b*ff^4*x^4)^p,
 x], x, Cos[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m -
 1)/2]
```

Maple [A] (verified)

Time = 0.33 (sec) , antiderivative size = 218, normalized size of antiderivative = 1.19

$$a^3\left(-\frac{2}{3}+\frac{\sinh(dx+c)^2}{3}\right)\cosh\left(dx+c\right)+3a^2b\left(-\frac{16}{35}+\frac{\sinh(dx+c)^6}{7}-\frac{6\sinh(dx+c)^4}{35}+\frac{8\sinh(dx+c)^2}{35}\right)\cosh\left(dx+c\right)+3$$

input
```
int(sinh(d*x+c)^3*(a+b*sinh(d*x+c)^4)^3,x)
```

output
```
1/d*(a^3*(-2/3+1/3*sinh(d*x+c)^2)*cosh(d*x+c)+3*a^2*b*(-16/35+1/7*sinh(d*x
+c)^6-6/35*sinh(d*x+c)^4+8/35*sinh(d*x+c)^2)*cosh(d*x+c)+3*b^2*a*(-256/693
+1/11*sinh(d*x+c)^10-10/99*sinh(d*x+c)^8+80/693*sinh(d*x+c)^6-32/231*sinh(
d*x+c)^4+128/693*sinh(d*x+c)^2)*cosh(d*x+c)+b^3*(-2048/6435+1/15*sinh(d*x+
c)^14-14/195*sinh(d*x+c)^12+56/715*sinh(d*x+c)^10-112/1287*sinh(d*x+c)^8+1
28/1287*sinh(d*x+c)^6-256/2145*sinh(d*x+c)^4+1024/6435*sinh(d*x+c)^2)*cosh
(d*x+c))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 795 vs. 2(169) = 338.

Time = 0.10 (sec) , antiderivative size = 795, normalized size of antiderivative = 4.34

$$\int \sinh^3(c+dx)\left(a+b\sinh^4(c+dx)\right)^3\,dx = \text{Too large to display}$$

input
```
integrate(sinh(d*x+c)^3*(a+b*sinh(d*x+c)^4)^3,x, algorithm="fricas")
```

output
```
1/738017280*(3003*b^3*cosh(d*x + c)^15 + 45045*b^3*cosh(d*x + c)*sinh(d*x
+ c)^14 - 51975*b^3*cosh(d*x + c)^13 + 15015*(91*b^3*cosh(d*x + c)^3 - 45*
b^3*cosh(d*x + c))*sinh(d*x + c)^12 + 12285*(16*a*b^2 + 35*b^3)*cosh(d*x +
 c)^11 + 9009*(1001*b^3*cosh(d*x + c)^5 - 1650*b^3*cosh(d*x + c)^3 + 15*(1
6*a*b^2 + 35*b^3)*cosh(d*x + c))*sinh(d*x + c)^10 - 5005*(528*a*b^2 + 455*
b^3)*cosh(d*x + c)^9 + 45045*(429*b^3*cosh(d*x + c)^7 - 1485*b^3*cosh(d*x
+ c)^5 + 45*(16*a*b^2 + 35*b^3)*cosh(d*x + c)^3 - (528*a*b^2 + 455*b^3)*co
sh(d*x + c))*sinh(d*x + c)^8 + 19305*(256*a^2*b + 880*a*b^2 + 455*b^3)*cos
h(d*x + c)^7 + 15015*(1001*b^3*cosh(d*x + c)^9 - 5940*b^3*cosh(d*x + c)^7
+ 378*(16*a*b^2 + 35*b^3)*cosh(d*x + c)^5 - 28*(528*a*b^2 + 455*b^3)*cosh(
d*x + c)^3 + 9*(256*a^2*b + 880*a*b^2 + 455*b^3)*cosh(d*x + c))*sinh(d*x +
 c)^6 - 27027*(1792*a^2*b + 2640*a*b^2 + 1001*b^3)*cosh(d*x + c)^5 + 45045
*(91*b^3*cosh(d*x + c)^11 - 825*b^3*cosh(d*x + c)^9 + 90*(16*a*b^2 + 35*b^
3)*cosh(d*x + c)^7 - 14*(528*a*b^2 + 455*b^3)*cosh(d*x + c)^5 + 15*(256*a^
2*b + 880*a*b^2 + 455*b^3)*cosh(d*x + c)^3 - 3*(1792*a^2*b + 2640*a*b^2 +
1001*b^3)*cosh(d*x + c))*sinh(d*x + c)^4 + 15015*(4096*a^3 + 16128*a^2*b +
 15840*a*b^2 + 5005*b^3)*cosh(d*x + c)^3 + 45045*(7*b^3*cosh(d*x + c)^13 -
 90*b^3*cosh(d*x + c)^11 + 15*(16*a*b^2 + 35*b^3)*cosh(d*x + c)^9 - 4*(528
*a*b^2 + 455*b^3)*cosh(d*x + c)^7 + 9*(256*a^2*b + 880*a*b^2 + 455*b^3)*co
sh(d*x + c)^5 - 6*(1792*a^2*b + 2640*a*b^2 + 1001*b^3)*cosh(d*x + c)^3 ...
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 484 vs. $2(167) = 334$.

Time = 6.19 (sec) , antiderivative size = 484, normalized size of antiderivative = 2.64

$$\int \sinh^3(c+dx)\left(a+b\sinh^4(c+dx)\right)^3\,dx$$
$$= \begin{cases} \frac{a^3\sinh^2(c+dx)\cosh(c+dx)}{d} - \frac{2a^3\cosh^3(c+dx)}{3d} + \frac{3a^2b\sinh^6(c+dx)\cosh(c+dx)}{d} - \frac{6a^2b\sinh^4(c+dx)\cosh^3(c+dx)}{d} + \frac{24a^2b\sinh^2(c \\ x\left(a+b\sinh^4(c)\right)^3\sinh^3(c) \end{cases}$$

input
```
integrate(sinh(d*x+c)**3*(a+b*sinh(d*x+c)**4)**3,x)
```

output
```
Piecewise((a**3*sinh(c + d*x)**2*cosh(c + d*x)/d - 2*a**3*cosh(c + d*x)**3
/(3*d) + 3*a**2*b*sinh(c + d*x)**6*cosh(c + d*x)/d - 6*a**2*b*sinh(c + d*x
)**4*cosh(c + d*x)**3/d + 24*a**2*b*sinh(c + d*x)**2*cosh(c + d*x)**5/(5*d
) - 48*a**2*b*cosh(c + d*x)**7/(35*d) + 3*a*b**2*sinh(c + d*x)**10*cosh(c
+ d*x)/d - 10*a*b**2*sinh(c + d*x)**8*cosh(c + d*x)**3/d + 16*a*b**2*sinh(
c + d*x)**6*cosh(c + d*x)**5/d - 96*a*b**2*sinh(c + d*x)**4*cosh(c + d*x)*
*7/(7*d) + 128*a*b**2*sinh(c + d*x)**2*cosh(c + d*x)**9/(21*d) - 256*a*b**
2*cosh(c + d*x)**11/(231*d) + b**3*sinh(c + d*x)**14*cosh(c + d*x)/d - 14*
b**3*sinh(c + d*x)**12*cosh(c + d*x)**3/(3*d) + 56*b**3*sinh(c + d*x)**10*
cosh(c + d*x)**5/(5*d) - 16*b**3*sinh(c + d*x)**8*cosh(c + d*x)**7/d + 128
*b**3*sinh(c + d*x)**6*cosh(c + d*x)**9/(9*d) - 256*b**3*sinh(c + d*x)**4*
cosh(c + d*x)**11/(33*d) + 1024*b**3*sinh(c + d*x)**2*cosh(c + d*x)**13/(4
29*d) - 2048*b**3*cosh(c + d*x)**15/(6435*d), Ne(d, 0)), (x*(a + b*sinh(c)
**4)**3*sinh(c)**3, True))
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 501 vs. $2(169) = 338$.

Time = 0.04 (sec) , antiderivative size = 501, normalized size of antiderivative = 2.74

$$\int \sinh^3(c+dx) \left(a+b\sinh^4(c+dx)\right)^3 \, dx = \text{Too large to display}$$

input
```
integrate(sinh(d*x+c)^3*(a+b*sinh(d*x+c)^4)^3,x, algorithm="maxima")
```

output
```
-1/210862080*b^3*((7425*e^(-2*d*x - 2*c) - 61425*e^(-4*d*x - 4*c) + 325325
*e^(-6*d*x - 6*c) - 1254825*e^(-8*d*x - 8*c) + 3864861*e^(-10*d*x - 10*c)
- 10735725*e^(-12*d*x - 12*c) + 41409225*e^(-14*d*x - 14*c) - 429)*e^(15*d
*x + 15*c)/d + (41409225*e^(-d*x - c) - 10735725*e^(-3*d*x - 3*c) + 386486
1*e^(-5*d*x - 5*c) - 1254825*e^(-7*d*x - 7*c) + 325325*e^(-9*d*x - 9*c) -
61425*e^(-11*d*x - 11*c) + 7425*e^(-13*d*x - 13*c) - 429*e^(-15*d*x - 15*c
))/d) - 1/473088*a*b^2*((847*e^(-2*d*x - 2*c) - 5445*e^(-4*d*x - 4*c) + 22
869*e^(-6*d*x - 6*c) - 76230*e^(-8*d*x - 8*c) + 320166*e^(-10*d*x - 10*c)
- 63)*e^(11*d*x + 11*c)/d + (320166*e^(-d*x - c) - 76230*e^(-3*d*x - 3*c)
+ 22869*e^(-5*d*x - 5*c) - 5445*e^(-7*d*x - 7*c) + 847*e^(-9*d*x - 9*c) -
63*e^(-11*d*x - 11*c))/d) - 3/4480*a^2*b*((49*e^(-2*d*x - 2*c) - 245*e^(-4
*d*x - 4*c) + 1225*e^(-6*d*x - 6*c) - 5)*e^(7*d*x + 7*c)/d + (1225*e^(-d*x
 - c) - 245*e^(-3*d*x - 3*c) + 49*e^(-5*d*x - 5*c) - 5*e^(-7*d*x - 7*c))/d
) + 1/24*a^3*(e^(3*d*x + 3*c)/d - 9*e^(d*x + c)/d - 9*e^(-d*x - c)/d + e^(
-3*d*x - 3*c)/d)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 446 vs. $2(169) = 338$.

Time = 0.25 (sec) , antiderivative size = 446, normalized size of antiderivative = 2.44

$$\int \sinh^3(c+dx)\left(a+b\sinh^4(c+dx)\right)^3\,dx$$
$$= \frac{b^3e^{(15\,dx+15\,c)}}{491520\,d} - \frac{15\,b^3e^{(13\,dx+13\,c)}}{425984\,d} - \frac{15\,b^3e^{(-13\,dx-13\,c)}}{425984\,d} + \frac{b^3e^{(-15\,dx-15\,c)}}{491520\,d} + \frac{3\,(16\,ab^2+35\,b^3)e^{(11\,dx+11\,c)}}{360448\,d} - \frac{(528\,ab^2+455\,b^3)e^{(9\,dx+9\,c)}}{294912\,d} + \frac{3\,(256\,a^2b+880\,ab^2+455\,b^3)e^{(7\,dx+7\,c)}}{229376\,d} - \frac{3\,(1792\,a^2b+2640\,ab^2+1001\,b^3)e^{(5\,dx+5\,c)}}{163840\,d} + \frac{(4096\,a^3+16128\,a^2b+15840\,ab^2+5005\,b^3)e^{(3\,dx+3\,c)}}{98304\,d} - \frac{3\,(4096\,a^3+8960\,a^2b+7392\,ab^2+2145\,b^3)e^{(dx+c)}}{32768\,d} - \frac{3\,(4096\,a^3+8960\,a^2b+7392\,ab^2+2145\,b^3)e^{(-dx-c)}}{32768\,d} + \frac{(4096\,a^3+16128\,a^2b+15840\,ab^2+5005\,b^3)e^{(-3\,dx-3\,c)}}{98304\,d} - \frac{3\,(1792\,a^2b+2640\,ab^2+1001\,b^3)e^{(-5\,dx-5\,c)}}{163840\,d} + \frac{3\,(256\,a^2b+880\,ab^2+455\,b^3)e^{(-7\,dx-7\,c)}}{229376\,d} - \frac{(528\,ab^2+455\,b^3)e^{(-9\,dx-9\,c)}}{294912\,d} + \frac{3\,(16\,ab^2+35\,b^3)e^{(-11\,dx-11\,c)}}{360448\,d}$$

input
```
integrate(sinh(d*x+c)^3*(a+b*sinh(d*x+c)^4)^3,x, algorithm="giac")
```

output
```
1/491520*b^3*e^(15*d*x + 15*c)/d - 15/425984*b^3*e^(13*d*x + 13*c)/d - 15/
425984*b^3*e^(-13*d*x - 13*c)/d + 1/491520*b^3*e^(-15*d*x - 15*c)/d + 3/36
0448*(16*a*b^2 + 35*b^3)*e^(11*d*x + 11*c)/d - 1/294912*(528*a*b^2 + 455*b
^3)*e^(9*d*x + 9*c)/d + 3/229376*(256*a^2*b + 880*a*b^2 + 455*b^3)*e^(7*d*
x + 7*c)/d - 3/163840*(1792*a^2*b + 2640*a*b^2 + 1001*b^3)*e^(5*d*x + 5*c)
/d + 1/98304*(4096*a^3 + 16128*a^2*b + 15840*a*b^2 + 5005*b^3)*e^(3*d*x +
3*c)/d - 3/32768*(4096*a^3 + 8960*a^2*b + 7392*a*b^2 + 2145*b^3)*e^(d*x +
c)/d - 3/32768*(4096*a^3 + 8960*a^2*b + 7392*a*b^2 + 2145*b^3)*e^(-d*x - c
)/d + 1/98304*(4096*a^3 + 16128*a^2*b + 15840*a*b^2 + 5005*b^3)*e^(-3*d*x
- 3*c)/d - 3/163840*(1792*a^2*b + 2640*a*b^2 + 1001*b^3)*e^(-5*d*x - 5*c)/
d + 3/229376*(256*a^2*b + 880*a*b^2 + 455*b^3)*e^(-7*d*x - 7*c)/d - 1/2949
12*(528*a*b^2 + 455*b^3)*e^(-9*d*x - 9*c)/d + 3/360448*(16*a*b^2 + 35*b^3)
*e^(-11*d*x - 11*c)/d
```

Mupad [B] (verification not implemented)

Time = 2.19 (sec) , antiderivative size = 266, normalized size of antiderivative = 1.45

$$\int \sinh^3(c+dx)\left(a+b\sinh^4(c+dx)\right)^3\,dx = -\frac{-\frac{a^3\cosh(c+d\,x)^3}{3}+a^3\cosh(c+d\,x)-\frac{3\,a^2\,b\cosh(c+d\,x)^7}{7}+\frac{9\,a^2\,b\cosh(c+d\,x)^5}{5}-3\,a^2\,b\cosh(c+d\,x)^3+3\,a^2\,b\cos}{}$$

input
```
int(sinh(c + d*x)^3*(a + b*sinh(c + d*x)^4)^3,x)
```

output
```
-(a^3*cosh(c + d*x) + b^3*cosh(c + d*x) - (a^3*cosh(c + d*x)^3)/3 - (7*b^3
*cosh(c + d*x)^3)/3 + (21*b^3*cosh(c + d*x)^5)/5 - 5*b^3*cosh(c + d*x)^7 +
 (35*b^3*cosh(c + d*x)^9)/9 - (21*b^3*cosh(c + d*x)^11)/11 + (7*b^3*cosh(c
 + d*x)^13)/13 - (b^3*cosh(c + d*x)^15)/15 - 5*a*b^2*cosh(c + d*x)^3 - 3*a
^2*b*cosh(c + d*x)^3 + 6*a*b^2*cosh(c + d*x)^5 + (9*a^2*b*cosh(c + d*x)^5)
/5 - (30*a*b^2*cosh(c + d*x)^7)/7 - (3*a^2*b*cosh(c + d*x)^7)/7 + (5*a*b^2
*cosh(c + d*x)^9)/3 - (3*a*b^2*cosh(c + d*x)^11)/11 + 3*a*b^2*cosh(c + d*x
) + 3*a^2*b*cosh(c + d*x))/d
```

Reduce [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 628, normalized size of antiderivative = 3.43

$$\int \sinh^3(c+dx)\left(a+b\sinh^4(c+dx)\right)^3\,dx$$
$$=\frac{196560e^{26dx+26c}a\,b^2-2642640e^{24dx+24c}a\,b^2+4942080e^{22dx+22c}a^2b+16988400e^{22dx+22c}a\,b^2-48432384e^{}}{}$$

input
```
int(sinh(d*x+c)^3*(a+b*sinh(d*x+c)^4)^3,x)
```

output
```
(3003*e**(30*c + 30*d*x)*b**3 - 51975*e**(28*c + 28*d*x)*b**3 + 196560*e**
(26*c + 26*d*x)*a*b**2 + 429975*e**(26*c + 26*d*x)*b**3 - 2642640*e**(24*c
 + 24*d*x)*a*b**2 - 2277275*e**(24*c + 24*d*x)*b**3 + 4942080*e**(22*c + 2
2*d*x)*a**2*b + 16988400*e**(22*c + 22*d*x)*a*b**2 + 8783775*e**(22*c + 22
*d*x)*b**3 - 48432384*e**(20*c + 20*d*x)*a**2*b - 71351280*e**(20*c + 20*d
*x)*a*b**2 - 27054027*e**(20*c + 20*d*x)*b**3 + 61501440*e**(18*c + 18*d*x
)*a**3 + 242161920*e**(18*c + 18*d*x)*a**2*b + 237837600*e**(18*c + 18*d*x
)*a*b**2 + 75150075*e**(18*c + 18*d*x)*b**3 - 553512960*e**(16*c + 16*d*x)
*a**3 - 1210809600*e**(16*c + 16*d*x)*a**2*b - 998917920*e**(16*c + 16*d*x
)*a*b**2 - 289864575*e**(16*c + 16*d*x)*b**3 - 553512960*e**(14*c + 14*d*x
)*a**3 - 1210809600*e**(14*c + 14*d*x)*a**2*b - 998917920*e**(14*c + 14*d*
x)*a*b**2 - 289864575*e**(14*c + 14*d*x)*b**3 + 61501440*e**(12*c + 12*d*x
)*a**3 + 242161920*e**(12*c + 12*d*x)*a**2*b + 237837600*e**(12*c + 12*d*x
)*a*b**2 + 75150075*e**(12*c + 12*d*x)*b**3 - 48432384*e**(10*c + 10*d*x)*
a**2*b - 71351280*e**(10*c + 10*d*x)*a*b**2 - 27054027*e**(10*c + 10*d*x)*
b**3 + 4942080*e**(8*c + 8*d*x)*a**2*b + 16988400*e**(8*c + 8*d*x)*a*b**2
+ 8783775*e**(8*c + 8*d*x)*b**3 - 2642640*e**(6*c + 6*d*x)*a*b**2 - 227727
5*e**(6*c + 6*d*x)*b**3 + 196560*e**(4*c + 4*d*x)*a*b**2 + 429975*e**(4*c
+ 4*d*x)*b**3 - 51975*e**(2*c + 2*d*x)*b**3 + 3003*b**3)/(1476034560*e**(1
5*c + 15*d*x)*d)
```

3.184 $\int \sinh(c+dx) \left(a+b\sinh^4(c+dx)\right)^3 dx$

Optimal result

Integrand size = 21, antiderivative size = 143

$$\begin{aligned}
&\int \sinh(c+dx) \left(a+b\sinh^4(c+dx)\right)^3 dx \\
&= \frac{(a+b)^3\cosh(c+dx)}{d} - \frac{2b(a+b)^2\cosh^3(c+dx)}{d} \\
&\quad + \frac{3b(a+b)(a+5b)\cosh^5(c+dx)}{5d} - \frac{4b^2(3a+5b)\cosh^7(c+dx)}{7d} \\
&\quad + \frac{b^2(a+5b)\cosh^9(c+dx)}{3d} - \frac{6b^3\cosh^{11}(c+dx)}{11d} + \frac{b^3\cosh^{13}(c+dx)}{13d}
\end{aligned}$$

output
```
(a+b)^3*cosh(d*x+c)/d-2*b*(a+b)^2*cosh(d*x+c)^3/d+3/5*b*(a+b)*(a+5*b)*cosh
(d*x+c)^5/d-4/7*b^2*(3*a+5*b)*cosh(d*x+c)^7/d+1/3*b^2*(a+5*b)*cosh(d*x+c)^
9/d-6/11*b^3*cosh(d*x+c)^11/d+1/13*b^3*cosh(d*x+c)^13/d
```

Mathematica [A] (verified)

Time = 1.68 (sec) , antiderivative size = 157, normalized size of antiderivative = 1.10

$$\int \sinh(c+dx)\left(a+b\sinh^4(c+dx)\right)^3\,dx$$
$$=\frac{60060(1024a^3+1920a^2b+1512ab^2+429b^3)\cosh(c+dx)-15015b(1280a^2+1344ab+429b^2)\cosh(3(c$$

input
```
Integrate[Sinh[c + d*x]*(a + b*Sinh[c + d*x]^4)^3,x]
```

output
```
(60060*(1024*a^3 + 1920*a^2*b + 1512*a*b^2 + 429*b^3)*Cosh[c + d*x] - 1501
5*b*(1280*a^2 + 1344*a*b + 429*b^2)*Cosh[3*(c + d*x)] + 3003*b*(768*a^2 +
1728*a*b + 715*b^2)*Cosh[5*(c + d*x)] - 4290*b^2*(216*a + 143*b)*Cosh[7*(c
 + d*x)] + 10010*b^2*(8*a + 13*b)*Cosh[9*(c + d*x)] - 17745*b^3*Cosh[11*(c
 + d*x)] + 1155*b^3*Cosh[13*(c + d*x)])/(61501440*d)
```

Rubi [A] (verified)

Time = 0.38 (sec) , antiderivative size = 126, normalized size of antiderivative = 0.88, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.238, Rules used = {3042, 26, 3694, 1403, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sinh(c+dx)\left(a+b\sinh^4(c+dx)\right)^3\,dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int -i\sin(ic+idx)\left(a+b\sin(ic+idx)^4\right)^3dx$$

$$\Big\downarrow \text{ 26}$$

$$-i\int \sin(ic+idx)\left(b\sin(ic+idx)^4+a\right)^3dx$$

$$\Big\downarrow \text{ 3694}$$

$$\frac{\int \left(b\cosh^4(c+dx)-2b\cosh^2(c+dx)+a+b\right)^3 d\cosh(c+dx)}{d}$$

$\big\downarrow$ 1403

$$\frac{\int \left(b^3\cosh^{12}(c+dx)-6b^3\cosh^{10}(c+dx)+12b^3\left(\frac{a+b}{4b}+1\right)\cosh^8(c+dx)-8b^3\left(\frac{3(a+b)}{2b}+1\right)\cosh^6(c+dx)+12b\right.}{d}$$

$\big\downarrow$ 2009

$$\frac{\frac{1}{3}b^2(a+5b)\cosh^9(c+dx)-\frac{4}{7}b^2(3a+5b)\cosh^7(c+dx)+\frac{3}{5}b(a+b)(a+5b)\cosh^5(c+dx)-2b(a+b)^2\cosh^3(c+\right.}{d}$$

input
```
Int[Sinh[c + d*x]*(a + b*Sinh[c + d*x]^4)^3,x]
```

output
```
((a + b)^3*Cosh[c + d*x] - 2*b*(a + b)^2*Cosh[c + d*x]^3 + (3*b*(a + b)*(a
 + 5*b)*Cosh[c + d*x]^5)/5 - (4*b^2*(3*a + 5*b)*Cosh[c + d*x]^7)/7 + (b^2*
(a + 5*b)*Cosh[c + d*x]^9)/3 - (6*b^3*Cosh[c + d*x]^11)/11 + (b^3*Cosh[c +
 d*x]^13)/13)/d
```

Defintions of rubi rules used

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 1403
```
Int[((a_.) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Int[ExpandInte
grand[(a + b*x^2 + c*x^4)^p, x], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a
*c, 0] && IGtQ[p, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3694
```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^4)^
(p_.), x_Symbol] :> With[{ff = FreeFactors[Cos[e + f*x], x]}, Simp[-ff/f
Subst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a + b - 2*b*ff^2*x^2 + b*ff^4*x^4)^p,
 x], x, Cos[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m -
 1)/2]
```

Maple [A] (verified)

Time = 0.28 (sec) , antiderivative size = 176, normalized size of antiderivative = 1.23

$$\frac{a^3\cosh(dx+c)+3a^2b\left(\frac{8}{15}+\frac{\sinh(dx+c)^4}{5}-\frac{4\sinh(dx+c)^2}{15}\right)\cosh(dx+c)+3b^2a\left(\frac{128}{315}+\frac{\sinh(dx+c)^8}{9}-\frac{8\sinh(dx+c)^6}{63}\right.}{}$$

input
```
int(sinh(d*x+c)*(a+b*sinh(d*x+c)^4)^3,x)
```

output
```
1/d*(a^3*cosh(d*x+c)+3*a^2*b*(8/15+1/5*sinh(d*x+c)^4-4/15*sinh(d*x+c)^2)*c
osh(d*x+c)+3*b^2*a*(128/315+1/9*sinh(d*x+c)^8-8/63*sinh(d*x+c)^6+16/105*si
nh(d*x+c)^4-64/315*sinh(d*x+c)^2)*cosh(d*x+c)+b^3*(1024/3003+1/13*sinh(d*x
+c)^12-12/143*sinh(d*x+c)^10+40/429*sinh(d*x+c)^8-320/3003*sinh(d*x+c)^6+1
28/1001*sinh(d*x+c)^4-512/3003*sinh(d*x+c)^2)*cosh(d*x+c))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 594 vs. 2(133) = 266.

Time = 0.10 (sec) , antiderivative size = 594, normalized size of antiderivative = 4.15

$$\int \sinh(c+dx)\left(a+b\sinh^4(c+dx)\right)^3\,dx = \text{Too large to display}$$

input
```
integrate(sinh(d*x+c)*(a+b*sinh(d*x+c)^4)^3,x, algorithm="fricas")
```

output
```
1/61501440*(1155*b^3*cosh(d*x + c)^13 + 15015*b^3*cosh(d*x + c)*sinh(d*x +
 c)^12 - 17745*b^3*cosh(d*x + c)^11 + 15015*(22*b^3*cosh(d*x + c)^3 - 13*b
^3*cosh(d*x + c))*sinh(d*x + c)^10 + 10010*(8*a*b^2 + 13*b^3)*cosh(d*x + c
)^9 + 45045*(33*b^3*cosh(d*x + c)^5 - 65*b^3*cosh(d*x + c)^3 + 2*(8*a*b^2
+ 13*b^3)*cosh(d*x + c))*sinh(d*x + c)^8 - 4290*(216*a*b^2 + 143*b^3)*cosh
(d*x + c)^7 + 30030*(66*b^3*cosh(d*x + c)^7 - 273*b^3*cosh(d*x + c)^5 + 28
*(8*a*b^2 + 13*b^3)*cosh(d*x + c)^3 - (216*a*b^2 + 143*b^3)*cosh(d*x + c))
*sinh(d*x + c)^6 + 3003*(768*a^2*b + 1728*a*b^2 + 715*b^3)*cosh(d*x + c)^5
 + 15015*(55*b^3*cosh(d*x + c)^9 - 390*b^3*cosh(d*x + c)^7 + 84*(8*a*b^2 +
 13*b^3)*cosh(d*x + c)^5 - 10*(216*a*b^2 + 143*b^3)*cosh(d*x + c)^3 + (768
*a^2*b + 1728*a*b^2 + 715*b^3)*cosh(d*x + c))*sinh(d*x + c)^4 - 15015*(128
0*a^2*b + 1344*a*b^2 + 429*b^3)*cosh(d*x + c)^3 + 15015*(6*b^3*cosh(d*x +
c)^11 - 65*b^3*cosh(d*x + c)^9 + 24*(8*a*b^2 + 13*b^3)*cosh(d*x + c)^7 - 6
*(216*a*b^2 + 143*b^3)*cosh(d*x + c)^5 + 2*(768*a^2*b + 1728*a*b^2 + 715*b
^3)*cosh(d*x + c)^3 - 3*(1280*a^2*b + 1344*a*b^2 + 429*b^3)*cosh(d*x + c))
*sinh(d*x + c)^2 + 60060*(1024*a^3 + 1920*a^2*b + 1512*a*b^2 + 429*b^3)*co
sh(d*x + c))/d
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 377 vs. $2(131) = 262$.

Time = 3.48 (sec) , antiderivative size = 377, normalized size of antiderivative = 2.64

$$\int \sinh(c+dx)\left(a+b\sinh^4(c+dx)\right)^3\,dx$$
$$= \begin{cases} \frac{a^3\cosh(c+dx)}{d} + \frac{3a^2b\sinh^4(c+dx)\cosh(c+dx)}{d} - \frac{4a^2b\sinh^2(c+dx)\cosh^3(c+dx)}{d} + \frac{8a^2b\cosh^5(c+dx)}{5d} + \frac{3ab^2\sinh^8(c+dx)\cosh(c+dx)}{d} \\ x\left(a+b\sinh^4(c)\right)^3\sinh(c) \end{cases}$$

input
```
integrate(sinh(d*x+c)*(a+b*sinh(d*x+c)**4)**3,x)
```

output
```
Piecewise((a**3*cosh(c + d*x)/d + 3*a**2*b*sinh(c + d*x)**4*cosh(c + d*x)/
d - 4*a**2*b*sinh(c + d*x)**2*cosh(c + d*x)**3/d + 8*a**2*b*cosh(c + d*x)*
*5/(5*d) + 3*a*b**2*sinh(c + d*x)**8*cosh(c + d*x)/d - 8*a*b**2*sinh(c + d
*x)**6*cosh(c + d*x)**3/d + 48*a*b**2*sinh(c + d*x)**4*cosh(c + d*x)**5/(5
*d) - 192*a*b**2*sinh(c + d*x)**2*cosh(c + d*x)**7/(35*d) + 128*a*b**2*cos
h(c + d*x)**9/(105*d) + b**3*sinh(c + d*x)**12*cosh(c + d*x)/d - 4*b**3*si
nh(c + d*x)**10*cosh(c + d*x)**3/d + 8*b**3*sinh(c + d*x)**8*cosh(c + d*x)
**5/d - 64*b**3*sinh(c + d*x)**6*cosh(c + d*x)**7/(7*d) + 128*b**3*sinh(c
+ d*x)**4*cosh(c + d*x)**9/(21*d) - 512*b**3*sinh(c + d*x)**2*cosh(c + d*x
)**11/(231*d) + 1024*b**3*cosh(c + d*x)**13/(3003*d), Ne(d, 0)), (x*(a + b
*sinh(c)**4)**3*sinh(c), True))
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 399 vs. $2(133) = 266$.

Time = 0.05 (sec) , antiderivative size = 399, normalized size of antiderivative = 2.79

$$\int \sinh(c+dx)\left(a+b\sinh^4(c+dx)\right)^3\,dx = -\frac{1}{24600576}\,b^3\left(\frac{\left(3549\,e^{(-2\,dx-2\,c)}-26026\,e^{(-4\,dx-4\,c)}+122694\,e^{(-6\,dx-6\,c)}-429429\,e^{(-8\,dx-8\,c)}+1288287\right.}{d}\right. - \frac{1}{53760}\,ab^2\left(\frac{\left(405\,e^{(-2\,dx-2\,c)}-2268\,e^{(-4\,dx-4\,c)}+8820\,e^{(-6\,dx-6\,c)}-39690\,e^{(-8\,dx-8\,c)}-35\right)e^{(9\,dx+9\,c)}}{d}-\right. + \frac{1}{160}\,a^2b\left(\frac{3\,e^{(5\,dx+5\,c)}}{d}-\frac{25\,e^{(3\,dx+3\,c)}}{d}+\frac{150\,e^{(dx+c)}}{d}+\frac{150\,e^{(-dx-c)}}{d}-\frac{25\,e^{(-3\,dx-3\,c)}}{d}+\frac{3\,e^{(-5\,dx-5\,c)}}{d}\right) + \frac{a^3\cosh\left(dx+c\right)}{d}$$

input
```
integrate(sinh(d*x+c)*(a+b*sinh(d*x+c)^4)^3,x, algorithm="maxima")
```

output
```
-1/24600576*b^3*((3549*e^(-2*d*x - 2*c) - 26026*e^(-4*d*x - 4*c) + 122694*
e^(-6*d*x - 6*c) - 429429*e^(-8*d*x - 8*c) + 1288287*e^(-10*d*x - 10*c) -
5153148*e^(-12*d*x - 12*c) - 231)*e^(13*d*x + 13*c)/d - (5153148*e^(-d*x -
 c) - 1288287*e^(-3*d*x - 3*c) + 429429*e^(-5*d*x - 5*c) - 122694*e^(-7*d*
x - 7*c) + 26026*e^(-9*d*x - 9*c) - 3549*e^(-11*d*x - 11*c) + 231*e^(-13*d
*x - 13*c))/d) - 1/53760*a*b^2*((405*e^(-2*d*x - 2*c) - 2268*e^(-4*d*x - 4
*c) + 8820*e^(-6*d*x - 6*c) - 39690*e^(-8*d*x - 8*c) - 35)*e^(9*d*x + 9*c)
/d - (39690*e^(-d*x - c) - 8820*e^(-3*d*x - 3*c) + 2268*e^(-5*d*x - 5*c) -
 405*e^(-7*d*x - 7*c) + 35*e^(-9*d*x - 9*c))/d) + 1/160*a^2*b*(3*e^(5*d*x
+ 5*c)/d - 25*e^(3*d*x + 3*c)/d + 150*e^(d*x + c)/d + 150*e^(-d*x - c)/d -
 25*e^(-3*d*x - 3*c)/d + 3*e^(-5*d*x - 5*c)/d) + a^3*cosh(d*x + c)/d
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 372 vs. $2(133) = 266$.

Time = 0.23 (sec) , antiderivative size = 372, normalized size of antiderivative = 2.60

$$
\begin{aligned}
&\int \sinh(c+dx)\left(a+b\sinh^4(c+dx)\right)^3\,dx \\
&= \frac{b^3e^{(13\,dx+13\,c)}}{106496\,d} - \frac{13\,b^3e^{(11\,dx+11\,c)}}{90112\,d} - \frac{13\,b^3e^{(-11\,dx-11\,c)}}{90112\,d} + \frac{b^3e^{(-13\,dx-13\,c)}}{106496\,d} \\
&+ \frac{(8\,ab^2+13\,b^3)e^{(9\,dx+9\,c)}}{12288\,d} - \frac{(216\,ab^2+143\,b^3)e^{(7\,dx+7\,c)}}{28672\,d} \\
&+ \frac{(768\,a^2b+1728\,ab^2+715\,b^3)e^{(5\,dx+5\,c)}}{40960\,d} - \frac{(1280\,a^2b+1344\,ab^2+429\,b^3)e^{(3\,dx+3\,c)}}{8192\,d} \\
&+ \frac{(1024\,a^3+1920\,a^2b+1512\,ab^2+429\,b^3)e^{(dx+c)}}{2048\,d} \\
&+ \frac{(1024\,a^3+1920\,a^2b+1512\,ab^2+429\,b^3)e^{(-dx-c)}}{2048\,d} \\
&- \frac{(1280\,a^2b+1344\,ab^2+429\,b^3)e^{(-3\,dx-3\,c)}}{8192\,d} + \frac{(768\,a^2b+1728\,ab^2+715\,b^3)e^{(-5\,dx-5\,c)}}{40960\,d} \\
&- \frac{(216\,ab^2+143\,b^3)e^{(-7\,dx-7\,c)}}{28672\,d} + \frac{(8\,ab^2+13\,b^3)e^{(-9\,dx-9\,c)}}{12288\,d}
\end{aligned}
$$

input
```
integrate(sinh(d*x+c)*(a+b*sinh(d*x+c)^4)^3,x, algorithm="giac")
```

output
```
1/106496*b^3*e^(13*d*x + 13*c)/d - 13/90112*b^3*e^(11*d*x + 11*c)/d - 13/9
0112*b^3*e^(-11*d*x - 11*c)/d + 1/106496*b^3*e^(-13*d*x - 13*c)/d + 1/1228
8*(8*a*b^2 + 13*b^3)*e^(9*d*x + 9*c)/d - 1/28672*(216*a*b^2 + 143*b^3)*e^(
7*d*x + 7*c)/d + 1/40960*(768*a^2*b + 1728*a*b^2 + 715*b^3)*e^(5*d*x + 5*c
)/d - 1/8192*(1280*a^2*b + 1344*a*b^2 + 429*b^3)*e^(3*d*x + 3*c)/d + 1/204
8*(1024*a^3 + 1920*a^2*b + 1512*a*b^2 + 429*b^3)*e^(d*x + c)/d + 1/2048*(1
024*a^3 + 1920*a^2*b + 1512*a*b^2 + 429*b^3)*e^(-d*x - c)/d - 1/8192*(1280
*a^2*b + 1344*a*b^2 + 429*b^3)*e^(-3*d*x - 3*c)/d + 1/40960*(768*a^2*b + 1
728*a*b^2 + 715*b^3)*e^(-5*d*x - 5*c)/d - 1/28672*(216*a*b^2 + 143*b^3)*e^
(-7*d*x - 7*c)/d + 1/12288*(8*a*b^2 + 13*b^3)*e^(-9*d*x - 9*c)/d
```

Mupad [B] (verification not implemented)

Time = 0.47 (sec) , antiderivative size = 211, normalized size of antiderivative = 1.48

$$\int \sinh(c+dx)\left(a+b\sinh^4(c+dx)\right)^3\,dx$$
$$= \frac{a^3\cosh(c+d\,x)+\frac{3\,a^2\,b\cosh(c+d\,x)^5}{5}-2\,a^2\,b\cosh(c+d\,x)^3+3\,a^2\,b\cosh(c+d\,x)+\frac{a\,b^2\cosh(c+d\,x)^9}{3}-\frac{12\,a\,b^2\cos}{}}{}$$

input
```
int(sinh(c + d*x)*(a + b*sinh(c + d*x)^4)^3,x)
```

output
```
(a^3*cosh(c + d*x) + b^3*cosh(c + d*x) - 2*b^3*cosh(c + d*x)^3 + 3*b^3*cos
h(c + d*x)^5 - (20*b^3*cosh(c + d*x)^7)/7 + (5*b^3*cosh(c + d*x)^9)/3 - (6
*b^3*cosh(c + d*x)^11)/11 + (b^3*cosh(c + d*x)^13)/13 - 4*a*b^2*cosh(c + d
*x)^3 - 2*a^2*b*cosh(c + d*x)^3 + (18*a*b^2*cosh(c + d*x)^5)/5 + (3*a^2*b*
cosh(c + d*x)^5)/5 - (12*a*b^2*cosh(c + d*x)^7)/7 + (a*b^2*cosh(c + d*x)^9
)/3 + 3*a*b^2*cosh(c + d*x) + 3*a^2*b*cosh(c + d*x))/d
```

Reduce [B] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 504, normalized size of antiderivative = 3.52

$$\int \sinh(c+dx)\left(a+b\sinh^4(c+dx)\right)^3 dx$$
$$= \frac{80080e^{22dx+22c}a\,b^2 - 926640e^{20dx+20c}a\,b^2 + 2306304e^{18dx+18c}a^2b + 1155b^3 - 17745e^{24dx+24c}b^3 - 17745e^{2d}}{}$$

input
```
int(sinh(d*x+c)*(a+b*sinh(d*x+c)^4)^3,x)
```

output
```
(1155*e**(26*c + 26*d*x)*b**3 - 17745*e**(24*c + 24*d*x)*b**3 + 80080*e**(
22*c + 22*d*x)*a*b**2 + 130130*e**(22*c + 22*d*x)*b**3 - 926640*e**(20*c +
 20*d*x)*a*b**2 - 613470*e**(20*c + 20*d*x)*b**3 + 2306304*e**(18*c + 18*d
*x)*a**2*b + 5189184*e**(18*c + 18*d*x)*a*b**2 + 2147145*e**(18*c + 18*d*x
)*b**3 - 19219200*e**(16*c + 16*d*x)*a**2*b - 20180160*e**(16*c + 16*d*x)*
a*b**2 - 6441435*e**(16*c + 16*d*x)*b**3 + 61501440*e**(14*c + 14*d*x)*a**
3 + 115315200*e**(14*c + 14*d*x)*a**2*b + 90810720*e**(14*c + 14*d*x)*a*b*
*2 + 25765740*e**(14*c + 14*d*x)*b**3 + 61501440*e**(12*c + 12*d*x)*a**3 +
 115315200*e**(12*c + 12*d*x)*a**2*b + 90810720*e**(12*c + 12*d*x)*a*b**2
+ 25765740*e**(12*c + 12*d*x)*b**3 - 19219200*e**(10*c + 10*d*x)*a**2*b -
20180160*e**(10*c + 10*d*x)*a*b**2 - 6441435*e**(10*c + 10*d*x)*b**3 + 230
6304*e**(8*c + 8*d*x)*a**2*b + 5189184*e**(8*c + 8*d*x)*a*b**2 + 2147145*e
**(8*c + 8*d*x)*b**3 - 926640*e**(6*c + 6*d*x)*a*b**2 - 613470*e**(6*c + 6
*d*x)*b**3 + 80080*e**(4*c + 4*d*x)*a*b**2 + 130130*e**(4*c + 4*d*x)*b**3
- 17745*e**(2*c + 2*d*x)*b**3 + 1155*b**3)/(123002880*e**(13*c + 13*d*x)*d
)
```

3.185 $\int \operatorname{csch}(c+dx)\left(a+b\sinh^4(c+dx)\right)^3\,dx$

Optimal result

Integrand size = 21, antiderivative size = 158

$$\int \operatorname{csch}(c+dx)\left(a+b\sinh^4(c+dx)\right)^3\,dx = -\frac{a^3\operatorname{arctanh}(\cosh(c+dx))}{d} - \frac{b(3a^2+3ab+b^2)\cosh(c+dx)}{d} + \frac{b(3a^2+9ab+5b^2)\cosh^3(c+dx)}{3d} - \frac{b^2(9a+10b)\cosh^5(c+dx)}{5d} + \frac{b^2(3a+10b)\cosh^7(c+dx)}{7d} - \frac{5b^3\cosh^9(c+dx)}{9d} + \frac{b^3\cosh^{11}(c+dx)}{11d}$$

output
```
-a^3*arctanh(cosh(d*x+c))/d-b*(3*a^2+3*a*b+b^2)*cosh(d*x+c)/d+1/3*b*(3*a^2
+9*a*b+5*b^2)*cosh(d*x+c)^3/d-1/5*b^2*(9*a+10*b)*cosh(d*x+c)^5/d+1/7*b^2*(
3*a+10*b)*cosh(d*x+c)^7/d-5/9*b^3*cosh(d*x+c)^9/d+1/11*b^3*cosh(d*x+c)^11/
d
```

Mathematica [A] (verified)

Time = 5.71 (sec) , antiderivative size = 155, normalized size of antiderivative = 0.98

$$\int \operatorname{csch}(c+dx)\left(a+b\sinh^4(c+dx)\right)^3\,dx$$
$$=\frac{-20790b(384a^2+280ab+77b^2)\cosh(c+dx)+6930b(8a+5b)(16a+11b)\cosh(3(c+dx))-2079b^2(11$$

input
```
Integrate[Csch[c + d*x]*(a + b*Sinh[c + d*x]^4)^3,x]
```

output
```
(-20790*b*(384*a^2 + 280*a*b + 77*b^2)*Cosh[c + d*x] + 6930*b*(8*a + 5*b)*
(16*a + 11*b)*Cosh[3*(c + d*x)] - 2079*b^2*(112*a + 55*b)*Cosh[5*(c + d*x)
] + 495*b^2*(48*a + 55*b)*Cosh[7*(c + d*x)] - 4235*b^3*Cosh[9*(c + d*x)] +
 315*b^3*Cosh[11*(c + d*x)] - 3548160*a^3*Log[Cosh[(c + d*x)/2]] + 3548160
*a^3*Log[Sinh[(c + d*x)/2]])/(3548160*d)
```

Rubi [A] (verified)

Time = 0.39 (sec) , antiderivative size = 140, normalized size of antiderivative = 0.89, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.238, Rules used = {3042, 26, 3694, 1467, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \operatorname{csch}(c+dx)\left(a+b\sinh^4(c+dx)\right)^3\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{i\left(a+b\sin(ic+idx)^4\right)^3}{\sin(ic+idx)}dx$$

$$\Big\downarrow \text{26}$$

$$i\int \frac{\left(b\sin(ic+idx)^4+a\right)^3}{\sin(ic+idx)}dx$$

$$\Big\downarrow \text{3694}$$

$$-\frac{\int \frac{\left(b\cosh^4(c+dx)-2b\cosh^2(c+dx)+a+b\right)^3}{1-\cosh^2(c+dx)}d\cosh(c+dx)}{d}$$

$\downarrow$ 1467

$$-\frac{\int \left(-b^3\cosh^{10}(c+dx)+5b^3\cosh^8(c+dx)-b^2(3a+10b)\cosh^6(c+dx)+b^2(9a+10b)\cosh^4(c+dx)-b(3a^2\right.}{d}$$

$\downarrow$ 2009

$$-\frac{a^3\text{arctanh}(\cosh(c+dx))-\frac{1}{3}b\left(3a^2+9ab+5b^2\right)\cosh^3(c+dx)+b\left(3a^2+3ab+b^2\right)\cosh(c+dx)-\frac{1}{7}b^2(3a+1}{d}$$

input
```
Int[Csch[c + d*x]*(a + b*Sinh[c + d*x]^4)^3,x]
```

output
```
-((a^3*ArcTanh[Cosh[c + d*x]] + b*(3*a^2 + 3*a*b + b^2)*Cosh[c + d*x] - (b
*(3*a^2 + 9*a*b + 5*b^2)*Cosh[c + d*x]^3)/3 + (b^2*(9*a + 10*b)*Cosh[c + d
*x]^5)/5 - (b^2*(3*a + 10*b)*Cosh[c + d*x]^7)/7 + (5*b^3*Cosh[c + d*x]^9)/
9 - (b^3*Cosh[c + d*x]^11)/11)/d)
```

Defintions of rubi rules used

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 1467
```
Int[((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.),
 x_Symbol] :> Int[ExpandIntegrand[(d + e*x^2)^q*(a + b*x^2 + c*x^4)^p, x],
x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e
 + a*e^2, 0] && IGtQ[p, 0] && IGtQ[q, -2]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3694
```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^4)^
(p_.), x_Symbol] :> With[{ff = FreeFactors[Cos[e + f*x], x]}, Simp[-ff/f
Subst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a + b - 2*b*ff^2*x^2 + b*ff^4*x^4)^p,
 x], x, Cos[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m -
 1)/2]
```

Maple [A] (verified)

Time = 14.88 (sec) , antiderivative size = 143, normalized size of antiderivative = 0.91

method	result
parallelrisch	$\frac{4a^3\ln\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-9b\left(-\frac{\left(a+\frac{11b}{16}\right)\left(a+\frac{5b}{8}\right)\cosh(3dx+3c)}{9}+\frac{7\left(a+\frac{55b}{112}\right)b\cosh(5dx+5c)}{240}-\frac{\left(a+\frac{55b}{48}\right)b\cosh(7dx+7c)}{336}-\frac{b^2\cos}{}\right.}{4d}$
derivativedivides	$\frac{-2a^3\operatorname{arctanh}\left(e^{dx+c}\right)+3a^2b\left(-\frac{2}{3}+\frac{\sinh(dx+c)^2}{3}\right)\cosh(dx+c)+3b^2a\left(-\frac{16}{35}+\frac{\sinh(dx+c)^6}{7}-\frac{6\sinh(dx+c)^4}{35}+\frac{8\sinh(dx+c)^2}{35}\right.}{}$
default	$\frac{-2a^3\operatorname{arctanh}\left(e^{dx+c}\right)+3a^2b\left(-\frac{2}{3}+\frac{\sinh(dx+c)^2}{3}\right)\cosh(dx+c)+3b^2a\left(-\frac{16}{35}+\frac{\sinh(dx+c)^6}{7}-\frac{6\sinh(dx+c)^4}{35}+\frac{8\sinh(dx+c)^2}{35}\right.}{}$
risch	$-\frac{11b^3e^{9dx+9c}}{18432d}-\frac{11b^3e^{-9dx-9c}}{18432d}+\frac{b^3e^{-11dx-11c}}{22528d}+\frac{b^3e^{11dx+11c}}{22528d}-\frac{231\,e^{dx+c}b^3}{1024d}-\frac{231\,e^{-dx-c}b^3}{1024d}+\frac{55\,e^{-3dx}}{102}$

input
```
int(csch(d*x+c)*(a+b*sinh(d*x+c)^4)^3,x,method=_RETURNVERBOSE)
```

output
```
1/4*(4*a^3*ln(tanh(1/2*d*x+1/2*c))-9*b*(-1/9*(a+11/16*b)*(a+5/8*b)*cosh(3*
d*x+3*c)+7/240*(a+55/112*b)*b*cosh(5*d*x+5*c)-1/336*(a+55/48*b)*b*cosh(7*d
*x+7*c)-1/25344*b^2*cosh(11*d*x+11*c)+11/20736*b^2*cosh(9*d*x+9*c)+(a^2+35
/48*a*b+77/384*b^2)*cosh(d*x+c)+8/9*a^2+64/105*a*b+1024/6237*b^2))/d
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 3824 vs. 2(148) = 296.

Time = 0.13 (sec) , antiderivative size = 3824, normalized size of antiderivative = 24.20

$$\int \operatorname{csch}(c+dx)\left(a+b\sinh^4(c+dx)\right)^3\,dx = \text{Too large to display}$$

input
```
integrate(csch(d*x+c)*(a+b*sinh(d*x+c)^4)^3,x, algorithm="fricas")
```

output
```
Too large to include
```

Sympy [F(-1)]

Timed out.

$$\int \operatorname{csch}(c+dx)\left(a+b\sinh^4(c+dx)\right)^3\,dx=\text{Timed out}$$

input
```
integrate(csch(d*x+c)*(a+b*sinh(d*x+c)**4)**3,x)
```

output
```
Timed out
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 327 vs. 2(148) = 296.

Time = 0.04 (sec) , antiderivative size = 327, normalized size of antiderivative = 2.07

$$\int \operatorname{csch}(c+dx)\left(a+b\sinh^4(c+dx)\right)^3\,dx=$$
$$-\frac{1}{1419264}b^3\left(\frac{\left(847\,e^{(-2\,dx-2\,c)}-5445\,e^{(-4\,dx-4\,c)}+22869\,e^{(-6\,dx-6\,c)}-76230\,e^{(-8\,dx-8\,c)}+320166\,e^{(-10\,d}\right.}{d}\right.$$
$$-\frac{3}{4480}ab^2\left(\frac{\left(49\,e^{(-2\,dx-2\,c)}-245\,e^{(-4\,dx-4\,c)}+1225\,e^{(-6\,dx-6\,c)}-5\right)e^{(7\,dx+7\,c)}}{d}+\frac{1225\,e^{(-dx-c)}-245\,e^{(-3}}{}\right.$$
$$+\frac{1}{8}a^2b\left(\frac{e^{(3\,dx+3\,c)}}{d}-\frac{9\,e^{(dx+c)}}{d}-\frac{9\,e^{(-dx-c)}}{d}+\frac{e^{(-3\,dx-3\,c)}}{d}\right)+\frac{a^3\log\left(\tanh\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)}{d}$$

input
```
integrate(csch(d*x+c)*(a+b*sinh(d*x+c)^4)^3,x, algorithm="maxima")
```

output
```
-1/1419264*b^3*((847*e^(-2*d*x - 2*c) - 5445*e^(-4*d*x - 4*c) + 22869*e^(-
6*d*x - 6*c) - 76230*e^(-8*d*x - 8*c) + 320166*e^(-10*d*x - 10*c) - 63)*e^
(11*d*x + 11*c)/d + (320166*e^(-d*x - c) - 76230*e^(-3*d*x - 3*c) + 22869*
e^(-5*d*x - 5*c) - 5445*e^(-7*d*x - 7*c) + 847*e^(-9*d*x - 9*c) - 63*e^(-1
1*d*x - 11*c))/d) - 3/4480*a*b^2*((49*e^(-2*d*x - 2*c) - 245*e^(-4*d*x - 4
*c) + 1225*e^(-6*d*x - 6*c) - 5)*e^(7*d*x + 7*c)/d + (1225*e^(-d*x - c) -
245*e^(-3*d*x - 3*c) + 49*e^(-5*d*x - 5*c) - 5*e^(-7*d*x - 7*c))/d) + 1/8*
a^2*b*(e^(3*d*x + 3*c)/d - 9*e^(d*x + c)/d - 9*e^(-d*x - c)/d + e^(-3*d*x
- 3*c)/d) + a^3*log(tanh(1/2*d*x + 1/2*c))/d
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 377 vs. $2(148) = 296$.

Time = 0.27 (sec) , antiderivative size = 377, normalized size of antiderivative = 2.39

$$\int \operatorname{csch}(c+dx)\left(a+b\sinh^4(c+dx)\right)^3\,dx$$
$$= \frac{315\,b^3e^{(11\,dx+11\,c)} - 4235\,b^3e^{(9\,dx+9\,c)} + 23760\,ab^2e^{(7\,dx+7\,c)} + 27225\,b^3e^{(7\,dx+7\,c)} - 232848\,ab^2e^{(5\,dx+5\,c)} - 11}{}$$

input
```
integrate(csch(d*x+c)*(a+b*sinh(d*x+c)^4)^3,x, algorithm="giac")
```

output
```
1/7096320*(315*b^3*e^(11*d*x + 11*c) - 4235*b^3*e^(9*d*x + 9*c) + 23760*a*
b^2*e^(7*d*x + 7*c) + 27225*b^3*e^(7*d*x + 7*c) - 232848*a*b^2*e^(5*d*x +
5*c) - 114345*b^3*e^(5*d*x + 5*c) + 887040*a^2*b*e^(3*d*x + 3*c) + 1164240
*a*b^2*e^(3*d*x + 3*c) + 381150*b^3*e^(3*d*x + 3*c) - 7983360*a^2*b*e^(d*x
 + c) - 5821200*a*b^2*e^(d*x + c) - 1600830*b^3*e^(d*x + c) - 7096320*a^3*
log(e^(d*x + c) + 1) + 7096320*a^3*log(abs(e^(d*x + c) - 1)) - (7983360*a^
2*b*e^(10*d*x + 10*c) + 5821200*a*b^2*e^(10*d*x + 10*c) + 1600830*b^3*e^(1
0*d*x + 10*c) - 887040*a^2*b*e^(8*d*x + 8*c) - 1164240*a*b^2*e^(8*d*x + 8*
c) - 381150*b^3*e^(8*d*x + 8*c) + 232848*a*b^2*e^(6*d*x + 6*c) + 114345*b^
3*e^(6*d*x + 6*c) - 23760*a*b^2*e^(4*d*x + 4*c) - 27225*b^3*e^(4*d*x + 4*c
) + 4235*b^3*e^(2*d*x + 2*c) - 315*b^3)*e^(-11*d*x - 11*c))/d
```

Mupad [B] (verification not implemented)

Time = 0.65 (sec) , antiderivative size = 326, normalized size of antiderivative = 2.06

$$\int \operatorname{csch}(c+dx)\left(a+b\sinh^4(c+dx)\right)^3\,dx$$
$$= \frac{e^{-3c-3dx}\left(128a^2b+168ab^2+55b^3\right)}{1024d} - \frac{2\,\mathrm{atan}\left(\frac{a^3\,e^{dx}\,e^{c}\sqrt{-d^2}}{d\sqrt{a^6}}\right)\sqrt{a^6}}{\sqrt{-d^2}}$$
$$+ \frac{e^{3c+3dx}\left(128a^2b+168ab^2+55b^3\right)}{1024d} - \frac{11b^3e^{-9c-9dx}}{18432d} - \frac{11b^3e^{9c+9dx}}{18432d}$$
$$+ \frac{b^3e^{-11c-11dx}}{22528d} + \frac{b^3e^{11c+11dx}}{22528d} - \frac{3be^{-c-dx}\left(384a^2+280ab+77b^2\right)}{1024d}$$
$$+ \frac{b^2e^{-7c-7dx}(48a+55b)}{14336d} + \frac{b^2e^{7c+7dx}(48a+55b)}{14336d} - \frac{3b^2e^{-5c-5dx}(112a+55b)}{10240d}$$
$$- \frac{3b^2e^{5c+5dx}(112a+55b)}{10240d} - \frac{3be^{c+dx}\left(384a^2+280ab+77b^2\right)}{1024d}$$

input
```
int((a + b*sinh(c + d*x)^4)^3/sinh(c + d*x),x)
```

output
```
(exp(- 3*c - 3*d*x)*(168*a*b^2 + 128*a^2*b + 55*b^3))/(1024*d) - (2*atan((
a^3*exp(d*x)*exp(c)*(-d^2)^(1/2))/(d*(a^6)^(1/2)))*(a^6)^(1/2))/(-d^2)^(1/
2) + (exp(3*c + 3*d*x)*(168*a*b^2 + 128*a^2*b + 55*b^3))/(1024*d) - (11*b^
3*exp(- 9*c - 9*d*x))/(18432*d) - (11*b^3*exp(9*c + 9*d*x))/(18432*d) + (b
^3*exp(- 11*c - 11*d*x))/(22528*d) + (b^3*exp(11*c + 11*d*x))/(22528*d) -
(3*b*exp(- c - d*x)*(280*a*b + 384*a^2 + 77*b^2))/(1024*d) + (b^2*exp(- 7*
c - 7*d*x)*(48*a + 55*b))/(14336*d) + (b^2*exp(7*c + 7*d*x)*(48*a + 55*b))
/(14336*d) - (3*b^2*exp(- 5*c - 5*d*x)*(112*a + 55*b))/(10240*d) - (3*b^2*
exp(5*c + 5*d*x)*(112*a + 55*b))/(10240*d) - (3*b*exp(c + d*x)*(280*a*b +
384*a^2 + 77*b^2))/(1024*d)
```

Reduce [B] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 430, normalized size of antiderivative = 2.72

$$\int \operatorname{csch}(c+dx)\left(a+b\sinh^4(c+dx)\right)^3\,dx$$
$$=\frac{315b^3+7096320e^{11dx+11c}\log\left(e^{dx+c}-1\right)a^3-7096320e^{11dx+11c}\log\left(e^{dx+c}+1\right)a^3-4235e^{2dx+2c}b^3-16008}{}$$

input
```
int(csch(d*x+c)*(a+b*sinh(d*x+c)^4)^3,x)
```

output
```
(315*e**(22*c + 22*d*x)*b**3 - 4235*e**(20*c + 20*d*x)*b**3 + 23760*e**(18
*c + 18*d*x)*a*b**2 + 27225*e**(18*c + 18*d*x)*b**3 - 232848*e**(16*c + 16
*d*x)*a*b**2 - 114345*e**(16*c + 16*d*x)*b**3 + 887040*e**(14*c + 14*d*x)*
a**2*b + 1164240*e**(14*c + 14*d*x)*a*b**2 + 381150*e**(14*c + 14*d*x)*b**
3 - 7983360*e**(12*c + 12*d*x)*a**2*b - 5821200*e**(12*c + 12*d*x)*a*b**2
- 1600830*e**(12*c + 12*d*x)*b**3 + 7096320*e**(11*c + 11*d*x)*log(e**(c +
 d*x) - 1)*a**3 - 7096320*e**(11*c + 11*d*x)*log(e**(c + d*x) + 1)*a**3 -
7983360*e**(10*c + 10*d*x)*a**2*b - 5821200*e**(10*c + 10*d*x)*a*b**2 - 16
00830*e**(10*c + 10*d*x)*b**3 + 887040*e**(8*c + 8*d*x)*a**2*b + 1164240*e
**(8*c + 8*d*x)*a*b**2 + 381150*e**(8*c + 8*d*x)*b**3 - 232848*e**(6*c + 6
*d*x)*a*b**2 - 114345*e**(6*c + 6*d*x)*b**3 + 23760*e**(4*c + 4*d*x)*a*b**
2 + 27225*e**(4*c + 4*d*x)*b**3 - 4235*e**(2*c + 2*d*x)*b**3 + 315*b**3)/(
7096320*e**(11*c + 11*d*x)*d)
```

3.186 $\int \operatorname{csch}^3(c+dx) \left(a+b\sinh^4(c+dx)\right)^3 \, dx$

Optimal result

Integrand size = 23, antiderivative size = 148

$$\int \operatorname{csch}^3(c+dx) \left(a+b\sinh^4(c+dx)\right)^3 \, dx = \frac{a^3 \operatorname{arctanh}(\cosh(c+dx))}{2d} + \frac{b(3a^2+3ab+b^2)\cosh(c+dx)}{d} - \frac{2b^2(3a+2b)\cosh^3(c+dx)}{3d} + \frac{3b^2(a+2b)\cosh^5(c+dx)}{5d} - \frac{4b^3\cosh^7(c+dx)}{7d} + \frac{b^3\cosh^9(c+dx)}{9d} - \frac{a^3\coth(c+dx)\operatorname{csch}(c+dx)}{2d}$$

output
```
1/2*a^3*arctanh(cosh(d*x+c))/d+b*(3*a^2+3*a*b+b^2)*cosh(d*x+c)/d-2/3*b^2*(
3*a+2*b)*cosh(d*x+c)^3/d+3/5*b^2*(a+2*b)*cosh(d*x+c)^5/d-4/7*b^3*cosh(d*x+
c)^7/d+1/9*b^3*cosh(d*x+c)^9/d-1/2*a^3*coth(d*x+c)*csch(d*x+c)/d
```

Mathematica [A] (verified)

Time = 4.31 (sec) , antiderivative size = 171, normalized size of antiderivative = 1.16

$$\int \operatorname{csch}^3(c+dx)\left(a+b\sinh^4(c+dx)\right)^3\,dx$$
$$=\frac{1890b(128a^2+80ab+21b^2)\cosh(c+dx)-1260b^2(20a+7b)\cosh(3(c+dx))+3024ab^2\cosh(5(c+dx)}{}$$

input
```
Integrate[Csch[c + d*x]^3*(a + b*Sinh[c + d*x]^4)^3,x]
```

output
```
(1890*b*(128*a^2 + 80*a*b + 21*b^2)*Cosh[c + d*x] - 1260*b^2*(20*a + 7*b)*
Cosh[3*(c + d*x)] + 3024*a*b^2*Cosh[5*(c + d*x)] + 2268*b^3*Cosh[5*(c + d*
x)] - 405*b^3*Cosh[7*(c + d*x)] + 35*b^3*Cosh[9*(c + d*x)] - 10080*a^3*Csc
h[(c + d*x)/2]^2 + 40320*a^3*Log[Cosh[(c + d*x)/2]] - 40320*a^3*Log[Sinh[(
c + d*x)/2]] - 10080*a^3*Sech[(c + d*x)/2]^2)/(80640*d)
```

Rubi [A] (verified)

Time = 0.59 (sec) , antiderivative size = 142, normalized size of antiderivative = 0.96, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}=0.304$, Rules used = {3042, 26, 3694, 1471, 25, 2341, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \operatorname{csch}^3(c+dx)\left(a+b\sinh^4(c+dx)\right)^3\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int -\frac{i\left(a+b\sin(ic+idx)^4\right)^3}{\sin(ic+idx)^3}dx$$

$$\Big\downarrow \text{26}$$

$$-i\int \frac{\left(b\sin(ic+idx)^4+a\right)^3}{\sin(ic+idx)^3}dx$$

$$\Big\downarrow \text{3694}$$

$$\frac{\int \frac{\left(b\cosh^4(c+dx)-2b\cosh^2(c+dx)+a+b\right)^3}{\left(1-\cosh^2(c+dx)\right)^2}d\cosh(c+dx)}{d}$$

$\Big\downarrow$ 1471

$$\frac{\frac{a^3\cosh(c+dx)}{2\left(1-\cosh^2(c+dx)\right)}-\frac{1}{2}\int -\frac{-2b^3\cosh^{10}(c+dx)+10b^3\cosh^8(c+dx)-2b^2(3a+10b)\cosh^6(c+dx)+2b^2(9a+10b)\cosh^4(c+dx)-2b\left(3a^2+9ba+5b^2\right.}{1-\cosh^2(c+dx)}}{d}$$

$\Big\downarrow$ 25

$$\frac{\frac{1}{2}\int \frac{-2b^3\cosh^{10}(c+dx)+10b^3\cosh^8(c+dx)-2b^2(3a+10b)\cosh^6(c+dx)+2b^2(9a+10b)\cosh^4(c+dx)-2b\left(3a^2+9ba+5b^2\right)\cosh^2(c+dx)+a^3+2b^3+}{1-\cosh^2(c+dx)}}{d}$$

$\Big\downarrow$ 2341

$$\frac{\frac{1}{2}\int \Big(2b^3\cosh^8(c+dx)-8b^3\cosh^6(c+dx)+6b^2(a+2b)\cosh^4(c+dx)-4b^2(3a+2b)\cosh^2(c+dx)+2b(3a^2+}{d}$$

$\Big\downarrow$ 2009

$$\frac{\frac{a^3\cosh(c+dx)}{2\left(1-\cosh^2(c+dx)\right)}+\frac{1}{2}\left(a^3\text{arctanh}(\cosh(c+dx))+2b\left(3a^2+3ab+b^2\right)\cosh(c+dx)+\frac{6}{5}b^2(a+2b)\cosh^5(c+dx)-\right.}{d}$$

input

```
Int[Csch[c + d*x]^3*(a + b*Sinh[c + d*x]^4)^3,x]
```

output

```
((a^3*Cosh[c + d*x])/(2*(1 - Cosh[c + d*x]^2)) + (a^3*ArcTanh[Cosh[c + d*x
]] + 2*b*(3*a^2 + 3*a*b + b^2)*Cosh[c + d*x] - (4*b^2*(3*a + 2*b)*Cosh[c +
 d*x]^3)/3 + (6*b^2*(a + 2*b)*Cosh[c + d*x]^5)/5 - (8*b^3*Cosh[c + d*x]^7)
/7 + (2*b^3*Cosh[c + d*x]^9)/9)/2)/d
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 1471
```
Int[((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.),
x_Symbol] :> With[{Qx = PolynomialQuotient[(a + b*x^2 + c*x^4)^p, d + e*x^2
, x], R = Coeff[PolynomialRemainder[(a + b*x^2 + c*x^4)^p, d + e*x^2, x], x
, 0]}, Simp[(-R)*x*((d + e*x^2)^(q + 1)/(2*d*(q + 1))), x] + Simp[1/(2*d*(q
 + 1))   Int[(d + e*x^2)^(q + 1)*ExpandToSum[2*d*(q + 1)*Qx + R*(2*q + 3),
x], x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^
2 - b*d*e + a*e^2, 0] && IGtQ[p, 0] && LtQ[q, -1]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2341
```
Int[(Pq_)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[Pq*
(a + b*x^2)^p, x], x] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3694
```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^4)^
(p_.), x_Symbol] :> With[{ff = FreeFactors[Cos[e + f*x], x]}, Simp[-ff/f
Subst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a + b - 2*b*ff^2*x^2 + b*ff^4*x^4)^p,
 x], x, Cos[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m -
 1)/2]
```

Maple [A] (verified)

Time = 5.48 (sec) , antiderivative size = 130, normalized size of antiderivative = 0.88

method	result
derivativedivides	$\frac{a^3\left(-\frac{\operatorname{csch}(dx+c)\coth(dx+c)}{2}+\operatorname{arctanh}\left(e^{dx+c}\right)\right)+3a^2b\cosh(dx+c)+3b^2a\left(\frac{8}{15}+\frac{\sinh(dx+c)^4}{5}-\frac{4\sinh(dx+c)^2}{15}\right)\cosh(dx+c}{d}$
default	$\frac{a^3\left(-\frac{\operatorname{csch}(dx+c)\coth(dx+c)}{2}+\operatorname{arctanh}\left(e^{dx+c}\right)\right)+3a^2b\cosh(dx+c)+3b^2a\left(\frac{8}{15}+\frac{\sinh(dx+c)^4}{5}-\frac{4\sinh(dx+c)^2}{15}\right)\cosh(dx+c}{d}$
parallelrisch	$\frac{8a^3\ln\left(\frac{1}{\sqrt{\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)}}\right)+a^3\left(\operatorname{sech}\left(\frac{dx}{2}+\frac{c}{2}\right)^2-11\right)\operatorname{csch}\left(\frac{dx}{2}+\frac{c}{2}\right)^2+9\coth\left(\frac{dx}{2}+\frac{c}{2}\right)^2a^3+24b\left(-\frac{5\left(a+\frac{7b}{20}\right)b\cosh(3dx+3}{48}\right.}{8d}$
risch	$\frac{b^3e^{9dx+9c}}{4608d}-\frac{9b^3e^{7dx+7c}}{3584d}+\frac{3b^2e^{5dx+5c}a}{160d}+\frac{9b^3e^{5dx+5c}}{640d}-\frac{5e^{3dx+3c}b^2a}{32d}-\frac{7e^{3dx+3c}b^3}{128d}+\frac{3e^{dx+c}a^2b}{2d}+\frac{15e}{}$

input
```
int(csch(d*x+c)^3*(a+b*sinh(d*x+c)^4)^3,x,method=_RETURNVERBOSE)
```

output
```
1/d*(a^3*(-1/2*csch(d*x+c)*coth(d*x+c)+arctanh(exp(d*x+c)))+3*a^2*b*cosh(d
*x+c)+3*b^2*a*(8/15+1/5*sinh(d*x+c)^4-4/15*sinh(d*x+c)^2)*cosh(d*x+c)+b^3*
(128/315+1/9*sinh(d*x+c)^8-8/63*sinh(d*x+c)^6+16/105*sinh(d*x+c)^4-64/315*
sinh(d*x+c)^2)*cosh(d*x+c))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 4895 vs. 2(136) = 272.

Time = 0.16 (sec) , antiderivative size = 4895, normalized size of antiderivative = 33.07

$$\int \operatorname{csch}^3(c+dx)\left(a+b\sinh^4(c+dx)\right)^3\,dx = \text{Too large to display}$$

input
```
integrate(csch(d*x+c)^3*(a+b*sinh(d*x+c)^4)^3,x, algorithm="fricas")
```

output
```
Too large to include
```

Sympy [F(-1)]

Timed out.

$$\int \operatorname{csch}^3(c+dx)\left(a+b\sinh^4(c+dx)\right)^3\,dx=\text{Timed out}$$

input
```
integrate(csch(d*x+c)**3*(a+b*sinh(d*x+c)**4)**3,x)
```

output
```
Timed out
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 334 vs. $2(136) = 272$.

Time = 0.05 (sec) , antiderivative size = 334, normalized size of antiderivative = 2.26

$$\int \operatorname{csch}^3(c+dx)\left(a+b\sinh^4(c+dx)\right)^3\,dx = -\frac{1}{161280}\,b^3\left(\frac{\left(405\,e^{(-2\,dx-2\,c)}-2268\,e^{(-4\,dx-4\,c)}+8820\,e^{(-6\,dx-6\,c)}-39690\,e^{(-8\,dx-8\,c)}-35\right)e^{(9\,dx+9\,c)}}{d}-\right.$$
$$+\frac{1}{160}\,ab^2\left(\frac{3\,e^{(5\,dx+5\,c)}}{d}-\frac{25\,e^{(3\,dx+3\,c)}}{d}+\frac{150\,e^{(dx+c)}}{d}+\frac{150\,e^{(-dx-c)}}{d}-\frac{25\,e^{(-3\,dx-3\,c)}}{d}+\frac{3\,e^{(-5\,dx-5\,c)}}{d}\right)$$
$$+\frac{3}{2}\,a^2b\left(\frac{e^{(dx+c)}}{d}+\frac{e^{(-dx-c)}}{d}\right)$$
$$+\frac{1}{2}\,a^3\left(\frac{\log\left(e^{(-dx-c)}+1\right)}{d}-\frac{\log\left(e^{(-dx-c)}-1\right)}{d}+\frac{2\left(e^{(-dx-c)}+e^{(-3\,dx-3\,c)}\right)}{d(2\,e^{(-2\,dx-2\,c)}-e^{(-4\,dx-4\,c)}-1)}\right)$$

input
```
integrate(csch(d*x+c)^3*(a+b*sinh(d*x+c)^4)^3,x, algorithm="maxima")
```

output
```
-1/161280*b^3*((405*e^(-2*d*x - 2*c) - 2268*e^(-4*d*x - 4*c) + 8820*e^(-6*
d*x - 6*c) - 39690*e^(-8*d*x - 8*c) - 35)*e^(9*d*x + 9*c)/d - (39690*e^(-d
*x - c) - 8820*e^(-3*d*x - 3*c) + 2268*e^(-5*d*x - 5*c) - 405*e^(-7*d*x -
7*c) + 35*e^(-9*d*x - 9*c))/d) + 1/160*a*b^2*(3*e^(5*d*x + 5*c)/d - 25*e^(
3*d*x + 3*c)/d + 150*e^(d*x + c)/d + 150*e^(-d*x - c)/d - 25*e^(-3*d*x - 3
*c)/d + 3*e^(-5*d*x - 5*c)/d) + 3/2*a^2*b*(e^(d*x + c)/d + e^(-d*x - c)/d)
 + 1/2*a^3*(log(e^(-d*x - c) + 1)/d - log(e^(-d*x - c) - 1)/d + 2*(e^(-d*x
 - c) + e^(-3*d*x - 3*c))/(d*(2*e^(-2*d*x - 2*c) - e^(-4*d*x - 4*c) - 1)))
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 300 vs. $2(136) = 272$.

Time = 0.29 (sec) , antiderivative size = 300, normalized size of antiderivative = 2.03

$$\int \operatorname{csch}^3(c+dx)\left(a+b\sinh^4(c+dx)\right)^3\,dx$$
$$= \frac{35\,b^3\left(e^{(dx+c)}+e^{(-dx-c)}\right)^9-720\,b^3\left(e^{(dx+c)}+e^{(-dx-c)}\right)^7+3024\,ab^2\left(e^{(dx+c)}+e^{(-dx-c)}\right)^5+6048\,b^3\left(e^{(dx+c)}\right.}{}$$

input
```
integrate(csch(d*x+c)^3*(a+b*sinh(d*x+c)^4)^3,x, algorithm="giac")
```

output
```
1/161280*(35*b^3*(e^(d*x + c) + e^(-d*x - c))^9 - 720*b^3*(e^(d*x + c) + e
^(-d*x - c))^7 + 3024*a*b^2*(e^(d*x + c) + e^(-d*x - c))^5 + 6048*b^3*(e^(
d*x + c) + e^(-d*x - c))^5 - 40320*a*b^2*(e^(d*x + c) + e^(-d*x - c))^3 -
26880*b^3*(e^(d*x + c) + e^(-d*x - c))^3 + 241920*a^2*b*(e^(d*x + c) + e^(
-d*x - c)) + 241920*a*b^2*(e^(d*x + c) + e^(-d*x - c)) + 80640*b^3*(e^(d*x
 + c) + e^(-d*x - c)) + 40320*a^3*log(e^(d*x + c) + e^(-d*x - c) + 2) - 40
320*a^3*log(e^(d*x + c) + e^(-d*x - c) - 2) - 161280*a^3*(e^(d*x + c) + e^
(-d*x - c))/((e^(d*x + c) + e^(-d*x - c))^2 - 4))/d
```

Mupad [B] (verification not implemented)

Time = 2.23 (sec) , antiderivative size = 326, normalized size of antiderivative = 2.20

$$\int \operatorname{csch}^3(c+dx)\left(a+b\sinh^4(c+dx)\right)^3\,dx$$
$$=\frac{\operatorname{atan}\left(\frac{a^3\,\mathrm{e}^{d\,x}\,\mathrm{e}^{c}\,\sqrt{-d^2}}{d\,\sqrt{a^6}}\right)\,\sqrt{a^6}}{\sqrt{-d^2}}-\frac{9\,b^3\,\mathrm{e}^{-7\,c-7\,d\,x}}{3584\,d}-\frac{9\,b^3\,\mathrm{e}^{7\,c+7\,d\,x}}{3584\,d}+\frac{b^3\,\mathrm{e}^{-9\,c-9\,d\,x}}{4608\,d}+\frac{b^3\,\mathrm{e}^{9\,c+9\,d\,x}}{4608\,d}+\frac{3\,b\,\mathrm{e}^{-c-d\,x}\,\left(128\,a^2+80\,a\,b+21\,b^2\right)}{256\,d}+\frac{3\,b^2\,\mathrm{e}^{-5\,c-5\,d\,x}\,\left(4\,a+3\,b\right)}{640\,d}+\frac{3\,b^2\,\mathrm{e}^{5\,c+5\,d\,x}\,\left(4\,a+3\,b\right)}{640\,d}-\frac{b^2\,\mathrm{e}^{-3\,c-3\,d\,x}\,\left(20\,a+7\,b\right)}{128\,d}-\frac{b^2\,\mathrm{e}^{3\,c+3\,d\,x}\,\left(20\,a+7\,b\right)}{128\,d}+\frac{3\,b\,\mathrm{e}^{c+d\,x}\,\left(128\,a^2+80\,a\,b+21\,b^2\right)}{256\,d}-\frac{a^3\,\mathrm{e}^{c+d\,x}}{d\,\left(\mathrm{e}^{2\,c+2\,d\,x}-1\right)}-\frac{2\,a^3\,\mathrm{e}^{c+d\,x}}{d\,\left(\mathrm{e}^{4\,c+4\,d\,x}-2\,\mathrm{e}^{2\,c+2\,d\,x}+1\right)}$$

input
```
int((a + b*sinh(c + d*x)^4)^3/sinh(c + d*x)^3,x)
```

output
```
(atan((a^3*exp(d*x)*exp(c)*(-d^2)^(1/2))/(d*(a^6)^(1/2)))*(a^6)^(1/2))/(-d
^2)^(1/2) - (9*b^3*exp(- 7*c - 7*d*x))/(3584*d) - (9*b^3*exp(7*c + 7*d*x))
/(3584*d) + (b^3*exp(- 9*c - 9*d*x))/(4608*d) + (b^3*exp(9*c + 9*d*x))/(46
08*d) + (3*b*exp(- c - d*x)*(80*a*b + 128*a^2 + 21*b^2))/(256*d) + (3*b^2*
exp(- 5*c - 5*d*x)*(4*a + 3*b))/(640*d) + (3*b^2*exp(5*c + 5*d*x)*(4*a + 3
*b))/(640*d) - (b^2*exp(- 3*c - 3*d*x)*(20*a + 7*b))/(128*d) - (b^2*exp(3*
c + 3*d*x)*(20*a + 7*b))/(128*d) + (3*b*exp(c + d*x)*(80*a*b + 128*a^2 + 2
1*b^2))/(256*d) - (a^3*exp(c + d*x))/(d*(exp(2*c + 2*d*x) - 1)) - (2*a^3*e
xp(c + d*x))/(d*(exp(4*c + 4*d*x) - 2*exp(2*c + 2*d*x) + 1))
```

Reduce [B] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 586, normalized size of antiderivative = 3.96

$$\int \operatorname{csch}^3(c+dx)\left(a+b\sinh^4(c+dx)\right)^3\,dx$$
$$=\frac{35b^3+161280e^{11dx+11c}\log\left(e^{dx+c}-1\right)a^3-161280e^{11dx+11c}\log\left(e^{dx+c}+1\right)a^3-475e^{2dx+2c}b^3-161280e^{10}}{}$$

input
```
int(csch(d*x+c)^3*(a+b*sinh(d*x+c)^4)^3,x)
```

output

```
(35*e**(22*c + 22*d*x)*b**3 - 475*e**(20*c + 20*d*x)*b**3 + 3024*e**(18*c
+ 18*d*x)*a*b**2 + 3113*e**(18*c + 18*d*x)*b**3 - 31248*e**(16*c + 16*d*x)
*a*b**2 - 13761*e**(16*c + 16*d*x)*b**3 + 241920*e**(14*c + 14*d*x)*a**2*b
 + 204624*e**(14*c + 14*d*x)*a*b**2 + 59598*e**(14*c + 14*d*x)*b**3 - 8064
0*e**(13*c + 13*d*x)*log(e**(c + d*x) - 1)*a**3 + 80640*e**(13*c + 13*d*x)
*log(e**(c + d*x) + 1)*a**3 - 161280*e**(12*c + 12*d*x)*a**3 - 241920*e**(
12*c + 12*d*x)*a**2*b - 176400*e**(12*c + 12*d*x)*a*b**2 - 48510*e**(12*c
+ 12*d*x)*b**3 + 161280*e**(11*c + 11*d*x)*log(e**(c + d*x) - 1)*a**3 - 16
1280*e**(11*c + 11*d*x)*log(e**(c + d*x) + 1)*a**3 - 161280*e**(10*c + 10*
d*x)*a**3 - 241920*e**(10*c + 10*d*x)*a**2*b - 176400*e**(10*c + 10*d*x)*a
*b**2 - 48510*e**(10*c + 10*d*x)*b**3 - 80640*e**(9*c + 9*d*x)*log(e**(c +
 d*x) - 1)*a**3 + 80640*e**(9*c + 9*d*x)*log(e**(c + d*x) + 1)*a**3 + 2419
20*e**(8*c + 8*d*x)*a**2*b + 204624*e**(8*c + 8*d*x)*a*b**2 + 59598*e**(8*
c + 8*d*x)*b**3 - 31248*e**(6*c + 6*d*x)*a*b**2 - 13761*e**(6*c + 6*d*x)*b
**3 + 3024*e**(4*c + 4*d*x)*a*b**2 + 3113*e**(4*c + 4*d*x)*b**3 - 475*e**(
2*c + 2*d*x)*b**3 + 35*b**3)/(161280*e**(9*c + 9*d*x)*d*(e**(4*c + 4*d*x)
- 2*e**(2*c + 2*d*x) + 1))
```

3.187 $\int \operatorname{csch}^5(c+dx) \left(a+b\sinh^4(c+dx)\right)^3 \, dx$

Optimal result

Integrand size = 23, antiderivative size = 142

$$\int \operatorname{csch}^5(c+dx) \left(a+b\sinh^4(c+dx)\right)^3 \, dx = -\frac{3a^2(a+8b)\operatorname{arctanh}(\cosh(c+dx))}{8d} - \frac{b^2(3a+b)\cosh(c+dx)}{d} + \frac{b^2(a+b)\cosh^3(c+dx)}{d} - \frac{3b^3\cosh^5(c+dx)}{5d} + \frac{b^3\cosh^7(c+dx)}{7d} + \frac{3a^3\coth(c+dx)\operatorname{csch}(c+dx)}{8d} - \frac{a^3\coth(c+dx)\operatorname{csch}^3(c+dx)}{4d}$$

output
```
-3/8*a^2*(a+8*b)*arctanh(cosh(d*x+c))/d-b^2*(3*a+b)*cosh(d*x+c)/d+b^2*(a+b
)*cosh(d*x+c)^3/d-3/5*b^3*cosh(d*x+c)^5/d+1/7*b^3*cosh(d*x+c)^7/d+3/8*a^3*
coth(d*x+c)*csch(d*x+c)/d-1/4*a^3*coth(d*x+c)*csch(d*x+c)^3/d
```

Mathematica [A] (verified)

Time = 3.28 (sec) , antiderivative size = 206, normalized size of antiderivative = 1.45

$$\int \operatorname{csch}^5(c+dx)\left(a+b\sinh^4(c+dx)\right)^3\,dx$$
$$=\frac{-35b^2(144a+35b)\cosh(c+dx)+35b^2(16a+7b)\cosh(3(c+dx))-49b^3\cosh(5(c+dx))+5b^3\cosh(7(}{}$$

input
```
Integrate[Csch[c + d*x]^5*(a + b*Sinh[c + d*x]^4)^3,x]
```

output
```
(-35*b^2*(144*a + 35*b)*Cosh[c + d*x] + 35*b^2*(16*a + 7*b)*Cosh[3*(c + d*
x)] - 49*b^3*Cosh[5*(c + d*x)] + 5*b^3*Cosh[7*(c + d*x)] + 210*a^3*Csch[(c
 + d*x)/2]^2 - 35*a^3*Csch[(c + d*x)/2]^4 - 840*a^3*Log[Cosh[(c + d*x)/2]]
 - 6720*a^2*b*Log[Cosh[(c + d*x)/2]] + 840*a^3*Log[Sinh[(c + d*x)/2]] + 67
20*a^2*b*Log[Sinh[(c + d*x)/2]] + 210*a^3*Sech[(c + d*x)/2]^2 + 35*a^3*Sec
h[(c + d*x)/2]^4)/(2240*d)
```

Rubi [A] (verified)

Time = 0.73 (sec) , antiderivative size = 149, normalized size of antiderivative = 1.05, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.391, Rules used = {3042, 26, 3694, 1471, 25, 2345, 25, 2341, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \operatorname{csch}^5(c+dx)\left(a+b\sinh^4(c+dx)\right)^3\,dx$$

$$\Big\downarrow \; 3042$$

$$\int \frac{i\left(a+b\sin(ic+idx)^4\right)^3}{\sin(ic+idx)^5}dx$$

$$\Big\downarrow \; 26$$

$$i\int \frac{\left(b\sin(ic+idx)^4+a\right)^3}{\sin(ic+idx)^5}dx$$

$\downarrow$ 3694

$$-\frac{\int \frac{\left(b\cosh^4(c+dx)-2b\cosh^2(c+dx)+a+b\right)^3}{\left(1-\cosh^2(c+dx)\right)^3}d\cosh(c+dx)}{d}$$

$\downarrow$ 1471

$$-\frac{\frac{a^3\cosh(c+dx)}{4\left(1-\cosh^2(c+dx)\right)^2}-\frac{1}{4}\int -\frac{-4b^3\cosh^{10}(c+dx)+20b^3\cosh^8(c+dx)-4b^2(3a+10b)\cosh^6(c+dx)+4b^2(9a+10b)\cosh^4(c+dx)-4b\left(3a^2+9ba+\right.}{\left(1-\cosh^2(c+dx)\right)^2}}{d}$$

$\downarrow$ 25

$$-\frac{\frac{1}{4}\int \frac{-4b^3\cosh^{10}(c+dx)+20b^3\cosh^8(c+dx)-4b^2(3a+10b)\cosh^6(c+dx)+4b^2(9a+10b)\cosh^4(c+dx)-4b\left(3a^2+9ba+5b^2\right)\cosh^2(c+dx)+3a^3+4}{\left(1-\cosh^2(c+dx)\right)^2}}{d}$$

$\downarrow$ 2345

$$-\frac{\frac{1}{4}\left(\frac{3a^3\cosh(c+dx)}{2\left(1-\cosh^2(c+dx)\right)}-\frac{1}{2}\int -\frac{8b^3\cosh^8(c+dx)-32b^3\cosh^6(c+dx)+24b^2(a+2b)\cosh^4(c+dx)-16b^2(3a+2b)\cosh^2(c+dx)+3a^3+8b^3+24ab}{1-\cosh^2(c+dx)}\right.}{d}$$

$\downarrow$ 25

$$-\frac{\frac{1}{4}\left(\frac{1}{2}\int \frac{8b^3\cosh^8(c+dx)-32b^3\cosh^6(c+dx)+24b^2(a+2b)\cosh^4(c+dx)-16b^2(3a+2b)\cosh^2(c+dx)+3a^3+8b^3+24ab^2+24a^2b}{1-\cosh^2(c+dx)}d\cosh(c+d\right.}{d}$$

$\downarrow$ 2341

$$-\frac{\frac{1}{4}\left(\frac{1}{2}\int \left(-8b^3\cosh^6(c+dx)+24b^3\cosh^4(c+dx)-24b^2(a+b)\cosh^2(c+dx)+8b^2(3a+b)+\frac{3\left(a^3+8ba^2\right)}{1-\cosh^2(c+dx)}\right)d\right.}{d}$$

$\downarrow$ 2009

$$-\frac{\frac{a^3\cosh(c+dx)}{4\left(1-\cosh^2(c+dx)\right)^2}+\frac{1}{4}\left(\frac{3a^3\cosh(c+dx)}{2\left(1-\cosh^2(c+dx)\right)}+\frac{1}{2}\left(3a^2(a+8b)\text{arctanh}(\cosh(c+dx))-8b^2(a+b)\cosh^3(c+dx)+8b^2\right.\right.}{d}$$

input
```
Int[Csch[c + d*x]^5*(a + b*Sinh[c + d*x]^4)^3,x]
```

output
```
-(((a^3*Cosh[c + d*x])/(4*(1 - Cosh[c + d*x]^2)^2) + ((3*a^3*Cosh[c + d*x]
)/(2*(1 - Cosh[c + d*x]^2)) + (3*a^2*(a + 8*b)*ArcTanh[Cosh[c + d*x]] + 8*
b^2*(3*a + b)*Cosh[c + d*x] - 8*b^2*(a + b)*Cosh[c + d*x]^3 + (24*b^3*Cosh
[c + d*x]^5)/5 - (8*b^3*Cosh[c + d*x]^7)/7)/2)/4)/d)
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 1471
```
Int[((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.),
x_Symbol] :> With[{Qx = PolynomialQuotient[(a + b*x^2 + c*x^4)^p, d + e*x^2
, x], R = Coeff[PolynomialRemainder[(a + b*x^2 + c*x^4)^p, d + e*x^2, x], x
, 0]}, Simp[(-R)*x*((d + e*x^2)^(q + 1)/(2*d*(q + 1))), x] + Simp[1/(2*d*(q
 + 1))   Int[(d + e*x^2)^(q + 1)*ExpandToSum[2*d*(q + 1)*Qx + R*(2*q + 3),
x], x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^
2 - b*d*e + a*e^2, 0] && IGtQ[p, 0] && LtQ[q, -1]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2341
```
Int[(Pq_)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[Pq*
(a + b*x^2)^p, x], x] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

rule 2345
```
Int[(Pq_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{Q = PolynomialQuot
ient[Pq, a + b*x^2, x], f = Coeff[PolynomialRemainder[Pq, a + b*x^2, x], x,
 0], g = Coeff[PolynomialRemainder[Pq, a + b*x^2, x], x, 1]}, Simp[(a*g - b
*f*x)*((a + b*x^2)^(p + 1)/(2*a*b*(p + 1))), x] + Simp[1/(2*a*(p + 1))   In
t[(a + b*x^2)^(p + 1)*ExpandToSum[2*a*(p + 1)*Q + f*(2*p + 3), x], x], x]]
/; FreeQ[{a, b}, x] && PolyQ[Pq, x] && LtQ[p, -1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3694
```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^4)^
(p_.), x_Symbol] :> With[{ff = FreeFactors[Cos[e + f*x], x]}, Simp[-ff/f
Subst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a + b - 2*b*ff^2*x^2 + b*ff^4*x^4)^p,
 x], x, Cos[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m -
 1)/2]
```

Maple [A] (verified)

Time = 4.46 (sec) , antiderivative size = 125, normalized size of antiderivative = 0.88

method	result
derivativedivides	$\frac{a^3\left(\left(-\frac{\operatorname{csch}(dx+c)^3}{4}+\frac{3\operatorname{csch}(dx+c)}{8}\right)\coth(dx+c)-\frac{3\operatorname{arctanh}\left(e^{dx+c}\right)}{4}\right)-6a^2b\operatorname{arctanh}\left(e^{dx+c}\right)+3b^2a\left(-\frac{2}{3}+\frac{\sinh(dx+c)^2}{3}\right.}{d}$
default	$\frac{a^3\left(\left(-\frac{\operatorname{csch}(dx+c)^3}{4}+\frac{3\operatorname{csch}(dx+c)}{8}\right)\coth(dx+c)-\frac{3\operatorname{arctanh}\left(e^{dx+c}\right)}{4}\right)-6a^2b\operatorname{arctanh}\left(e^{dx+c}\right)+3b^2a\left(-\frac{2}{3}+\frac{\sinh(dx+c)^2}{3}\right.}{d}$
parallelrisch	$107520a^2(a+8b)\ln\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-6160\left(\cosh(dx+c)-\frac{7\cosh(2dx+2c)}{4}-\frac{3\cosh(3dx+3c)}{11}+\frac{7\cosh(4dx+4c)}{16}+\frac{21}{16}\right)\operatorname{sech}\left(\right.$
risch	$\frac{b^3e^{7dx+7c}}{896d}-\frac{7b^3e^{5dx+5c}}{640d}+\frac{e^{3dx+3c}b^2a}{8d}+\frac{7e^{3dx+3c}b^3}{128d}-\frac{9e^{dx+c}b^2a}{8d}-\frac{35e^{dx+c}b^3}{128d}-\frac{9e^{-dx-c}b^2a}{8d}-\frac{35e^{-}}{12}$

input
```
int(csch(d*x+c)^5*(a+b*sinh(d*x+c)^4)^3,x,method=_RETURNVERBOSE)
```

output
```
1/d*(a^3*((-1/4*csch(d*x+c)^3+3/8*csch(d*x+c))*coth(d*x+c)-3/4*arctanh(exp
(d*x+c)))-6*a^2*b*arctanh(exp(d*x+c))+3*b^2*a*(-2/3+1/3*sinh(d*x+c)^2)*cos
h(d*x+c)+b^3*(-16/35+1/7*sinh(d*x+c)^6-6/35*sinh(d*x+c)^4+8/35*sinh(d*x+c)
^2)*cosh(d*x+c))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 6441 vs. $2(132) = 264$.

Time = 0.17 (sec) , antiderivative size = 6441, normalized size of antiderivative = 45.36

$$\int \operatorname{csch}^5(c+dx) \left(a+b\sinh^4(c+dx)\right)^3 dx = \text{Too large to display}$$

input
```
integrate(csch(d*x+c)^5*(a+b*sinh(d*x+c)^4)^3,x, algorithm="fricas")
```

output
```
Too large to include
```

Sympy [F(-1)]

Timed out.

$$\int \operatorname{csch}^5(c+dx) \left(a+b\sinh^4(c+dx)\right)^3 dx = \text{Timed out}$$

input
```
integrate(csch(d*x+c)**5*(a+b*sinh(d*x+c)**4)**3,x)
```

output
```
Timed out
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 340 vs. $2(132) = 264$.

Time = 0.05 (sec) , antiderivative size = 340, normalized size of antiderivative = 2.39

$$\int \operatorname{csch}^5(c+dx)\left(a+b\sinh^4(c+dx)\right)^3\,dx = -\frac{1}{4480}b^3\left(\frac{\left(49\,e^{(-2\,dx-2\,c)}-245\,e^{(-4\,dx-4\,c)}+1225\,e^{(-6\,dx-6\,c)}-5\right)e^{(7\,dx+7\,c)}}{d}+\frac{1225\,e^{(-dx-c)}-245\,e^{(-3\,dx}}{}\right.$$
$$+\frac{1}{8}ab^2\left(\frac{e^{(3\,dx+3\,c)}}{d}-\frac{9\,e^{(dx+c)}}{d}-\frac{9\,e^{(-dx-c)}}{d}+\frac{e^{(-3\,dx-3\,c)}}{d}\right)$$
$$-\frac{1}{8}a^3\left(\frac{3\log\left(e^{(-dx-c)}+1\right)}{d}-\frac{3\log\left(e^{(-dx-c)}-1\right)}{d}+\frac{2\left(3\,e^{(-dx-c)}-11\,e^{(-3\,dx-3\,c)}-11\,e^{(-5\,dx-5\,c)}+3\,e\right.}{d(4\,e^{(-2\,dx-2\,c)}-6\,e^{(-4\,dx-4\,c)}+4\,e^{(-6\,dx-6\,c)}-e^{(-8}}\right.$$
$$-3\,a^2b\left(\frac{\log\left(e^{(-dx-c)}+1\right)}{d}-\frac{\log\left(e^{(-dx-c)}-1\right)}{d}\right)$$

input
```
integrate(csch(d*x+c)^5*(a+b*sinh(d*x+c)^4)^3,x, algorithm="maxima")
```

output
```
-1/4480*b^3*((49*e^(-2*d*x - 2*c) - 245*e^(-4*d*x - 4*c) + 1225*e^(-6*d*x
- 6*c) - 5)*e^(7*d*x + 7*c)/d + (1225*e^(-d*x - c) - 245*e^(-3*d*x - 3*c)
+ 49*e^(-5*d*x - 5*c) - 5*e^(-7*d*x - 7*c))/d) + 1/8*a*b^2*(e^(3*d*x + 3*c
)/d - 9*e^(d*x + c)/d - 9*e^(-d*x - c)/d + e^(-3*d*x - 3*c)/d) - 1/8*a^3*(
3*log(e^(-d*x - c) + 1)/d - 3*log(e^(-d*x - c) - 1)/d + 2*(3*e^(-d*x - c)
- 11*e^(-3*d*x - 3*c) - 11*e^(-5*d*x - 5*c) + 3*e^(-7*d*x - 7*c))/(d*(4*e^
(-2*d*x - 2*c) - 6*e^(-4*d*x - 4*c) + 4*e^(-6*d*x - 6*c) - e^(-8*d*x - 8*c
) - 1))) - 3*a^2*b*(log(e^(-d*x - c) + 1)/d - log(e^(-d*x - c) - 1)/d)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 271 vs. 2(132) = 264.

Time = 0.30 (sec) , antiderivative size = 271, normalized size of antiderivative = 1.91

$$\int \operatorname{csch}^5(c+dx)\left(a+b\sinh^4(c+dx)\right)^3\,dx$$
$$=\frac{5\,b^3\left(e^{(dx+c)}+e^{(-dx-c)}\right)^7-84\,b^3\left(e^{(dx+c)}+e^{(-dx-c)}\right)^5+560\,ab^2\left(e^{(dx+c)}+e^{(-dx-c)}\right)^3+560\,b^3\left(e^{(dx+c)}+e^{(-}\right.}{}$$

input
```
integrate(csch(d*x+c)^5*(a+b*sinh(d*x+c)^4)^3,x, algorithm="giac")
```

output
```
1/4480*(5*b^3*(e^(d*x + c) + e^(-d*x - c))^7 - 84*b^3*(e^(d*x + c) + e^(-d
*x - c))^5 + 560*a*b^2*(e^(d*x + c) + e^(-d*x - c))^3 + 560*b^3*(e^(d*x +
c) + e^(-d*x - c))^3 - 6720*a*b^2*(e^(d*x + c) + e^(-d*x - c)) - 2240*b^3*
(e^(d*x + c) + e^(-d*x - c)) - 840*(a^3 + 8*a^2*b)*log(e^(d*x + c) + e^(-d
*x - c) + 2) + 840*(a^3 + 8*a^2*b)*log(e^(d*x + c) + e^(-d*x - c) - 2) + 1
120*(3*a^3*(e^(d*x + c) + e^(-d*x - c))^3 - 20*a^3*(e^(d*x + c) + e^(-d*x
- c)))/((e^(d*x + c) + e^(-d*x - c))^2 - 4)^2)/d
```

Mupad [B] (verification not implemented)

Time = 2.69 (sec) , antiderivative size = 421, normalized size of antiderivative = 2.96

$$\begin{aligned}
&\int \operatorname{csch}^5(c+dx)\left(a+b\sinh^4(c+dx)\right)^3\,dx\\
&=\frac{b^3\,\mathrm{e}^{-7\,c-7\,d\,x}}{896\,d}-\frac{7\,b^3\,\mathrm{e}^{-5\,c-5\,d\,x}}{640\,d}-\frac{7\,b^3\,\mathrm{e}^{5\,c+5\,d\,x}}{640\,d}\\
&-\frac{3\,\mathrm{atan}\left(\frac{\mathrm{e}^{d\,x}\,\mathrm{e}^{c}\,\left(a^3\,\sqrt{-d^2}+8\,a^2\,b\,\sqrt{-d^2}\right)}{d\,\sqrt{a^6+16\,a^5\,b+64\,a^4\,b^2}}\right)\,\sqrt{a^6+16\,a^5\,b+64\,a^4\,b^2}}{4\,\sqrt{-d^2}}+\frac{b^3\,\mathrm{e}^{7\,c+7\,d\,x}}{896\,d}\\
&-\frac{6\,a^3\,\mathrm{e}^{c+d\,x}}{d\,\left(3\,\mathrm{e}^{2\,c+2\,d\,x}-3\,\mathrm{e}^{4\,c+4\,d\,x}+\mathrm{e}^{6\,c+6\,d\,x}-1\right)}-\frac{b^2\,\mathrm{e}^{c+d\,x}\,\left(144\,a+35\,b\right)}{128\,d}\\
&-\frac{4\,a^3\,\mathrm{e}^{c+d\,x}}{d\,\left(6\,\mathrm{e}^{4\,c+4\,d\,x}-4\,\mathrm{e}^{2\,c+2\,d\,x}-4\,\mathrm{e}^{6\,c+6\,d\,x}+\mathrm{e}^{8\,c+8\,d\,x}+1\right)}\\
&+\frac{b^2\,\mathrm{e}^{-3\,c-3\,d\,x}\,\left(16\,a+7\,b\right)}{128\,d}+\frac{b^2\,\mathrm{e}^{3\,c+3\,d\,x}\,\left(16\,a+7\,b\right)}{128\,d}-\frac{b^2\,\mathrm{e}^{-c-d\,x}\,\left(144\,a+35\,b\right)}{128\,d}\\
&+\frac{3\,a^3\,\mathrm{e}^{c+d\,x}}{4\,d\,\left(\mathrm{e}^{2\,c+2\,d\,x}-1\right)}-\frac{a^3\,\mathrm{e}^{c+d\,x}}{2\,d\,\left(\mathrm{e}^{4\,c+4\,d\,x}-2\,\mathrm{e}^{2\,c+2\,d\,x}+1\right)}
\end{aligned}$$

input
```
int((a + b*sinh(c + d*x)^4)^3/sinh(c + d*x)^5,x)
```

output
```
(b^3*exp(- 7*c - 7*d*x))/(896*d) - (7*b^3*exp(- 5*c - 5*d*x))/(640*d) - (7
*b^3*exp(5*c + 5*d*x))/(640*d) - (3*atan((exp(d*x)*exp(c)*(a^3*(-d^2)^(1/2
) + 8*a^2*b*(-d^2)^(1/2)))/(d*(16*a^5*b + a^6 + 64*a^4*b^2)^(1/2)))*(16*a^
5*b + a^6 + 64*a^4*b^2)^(1/2))/(4*(-d^2)^(1/2)) + (b^3*exp(7*c + 7*d*x))/(
896*d) - (6*a^3*exp(c + d*x))/(d*(3*exp(2*c + 2*d*x) - 3*exp(4*c + 4*d*x)
+ exp(6*c + 6*d*x) - 1)) - (b^2*exp(c + d*x)*(144*a + 35*b))/(128*d) - (4*
a^3*exp(c + d*x))/(d*(6*exp(4*c + 4*d*x) - 4*exp(2*c + 2*d*x) - 4*exp(6*c
+ 6*d*x) + exp(8*c + 8*d*x) + 1)) + (b^2*exp(- 3*c - 3*d*x)*(16*a + 7*b))/
(128*d) + (b^2*exp(3*c + 3*d*x)*(16*a + 7*b))/(128*d) - (b^2*exp(- c - d*x
)*(144*a + 35*b))/(128*d) + (3*a^3*exp(c + d*x))/(4*d*(exp(2*c + 2*d*x) -
1)) - (a^3*exp(c + d*x))/(2*d*(exp(4*c + 4*d*x) - 2*exp(2*c + 2*d*x) + 1))
```

Reduce [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 936, normalized size of antiderivative = 6.59

$$\int \operatorname{csch}^5(c+dx)\left(a+b\sinh^4(c+dx)\right)^3\,dx = \text{Too large to display}$$

input
```
int(csch(d*x+c)^5*(a+b*sinh(d*x+c)^4)^3,x)
```

output

```
(5*e**(22*c + 22*d*x)*b**3 - 69*e**(20*c + 20*d*x)*b**3 + 560*e**(18*c + 1
8*d*x)*a*b**2 + 471*e**(18*c + 18*d*x)*b**3 - 7280*e**(16*c + 16*d*x)*a*b*
*2 - 2519*e**(16*c + 16*d*x)*b**3 + 1680*e**(15*c + 15*d*x)*log(e**(c + d*
x) - 1)*a**3 + 13440*e**(15*c + 15*d*x)*log(e**(c + d*x) - 1)*a**2*b - 168
0*e**(15*c + 15*d*x)*log(e**(c + d*x) + 1)*a**3 - 13440*e**(15*c + 15*d*x)
*log(e**(c + d*x) + 1)*a**2*b + 3360*e**(14*c + 14*d*x)*a**3 + 18480*e**(1
4*c + 14*d*x)*a*b**2 + 5346*e**(14*c + 14*d*x)*b**3 - 6720*e**(13*c + 13*d
*x)*log(e**(c + d*x) - 1)*a**3 - 53760*e**(13*c + 13*d*x)*log(e**(c + d*x)
 - 1)*a**2*b + 6720*e**(13*c + 13*d*x)*log(e**(c + d*x) + 1)*a**3 + 53760*
e**(13*c + 13*d*x)*log(e**(c + d*x) + 1)*a**2*b - 12320*e**(12*c + 12*d*x)
*a**3 - 11760*e**(12*c + 12*d*x)*a*b**2 - 3234*e**(12*c + 12*d*x)*b**3 + 1
0080*e**(11*c + 11*d*x)*log(e**(c + d*x) - 1)*a**3 + 80640*e**(11*c + 11*d
*x)*log(e**(c + d*x) - 1)*a**2*b - 10080*e**(11*c + 11*d*x)*log(e**(c + d*
x) + 1)*a**3 - 80640*e**(11*c + 11*d*x)*log(e**(c + d*x) + 1)*a**2*b - 123
20*e**(10*c + 10*d*x)*a**3 - 11760*e**(10*c + 10*d*x)*a*b**2 - 3234*e**(10
*c + 10*d*x)*b**3 - 6720*e**(9*c + 9*d*x)*log(e**(c + d*x) - 1)*a**3 - 537
60*e**(9*c + 9*d*x)*log(e**(c + d*x) - 1)*a**2*b + 6720*e**(9*c + 9*d*x)*l
og(e**(c + d*x) + 1)*a**3 + 53760*e**(9*c + 9*d*x)*log(e**(c + d*x) + 1)*a
**2*b + 3360*e**(8*c + 8*d*x)*a**3 + 18480*e**(8*c + 8*d*x)*a*b**2 + 5346*
e**(8*c + 8*d*x)*b**3 + 1680*e**(7*c + 7*d*x)*log(e**(c + d*x) - 1)*a**...
```

3.188 $\int \operatorname{csch}^7(c+dx)\left(a+b\sinh^4(c+dx)\right)^3\,dx$

Optimal result

Integrand size = 23, antiderivative size = 156

$$\begin{aligned}\int \operatorname{csch}^7(c+dx)\left(a+b\sinh^4(c+dx)\right)^3\,dx &= \frac{a^2(5a+24b)\operatorname{arctanh}(\cosh(c+dx))}{16d}\\
&+\frac{b^2(3a+b)\cosh(c+dx)}{d}\\
&-\frac{2b^3\cosh^3(c+dx)}{3d}+\frac{b^3\cosh^5(c+dx)}{5d}\\
&-\frac{a^2(5a+24b)\coth(c+dx)\operatorname{csch}(c+dx)}{16d}\\
&+\frac{5a^3\coth(c+dx)\operatorname{csch}^3(c+dx)}{24d}\\
&-\frac{a^3\coth(c+dx)\operatorname{csch}^5(c+dx)}{6d}\end{aligned}$$

output
```
1/16*a^2*(5*a+24*b)*arctanh(cosh(d*x+c))/d+b^2*(3*a+b)*cosh(d*x+c)/d-2/3*b
^3*cosh(d*x+c)^3/d+1/5*b^3*cosh(d*x+c)^5/d-1/16*a^2*(5*a+24*b)*coth(d*x+c)
*csch(d*x+c)/d+5/24*a^3*coth(d*x+c)*csch(d*x+c)^3/d-1/6*a^3*coth(d*x+c)*cs
ch(d*x+c)^5/d
```

Mathematica [A] (verified)

Time = 3.17 (sec) , antiderivative size = 256, normalized size of antiderivative = 1.64

$$\int \operatorname{csch}^7(c+dx)\left(a+b\sinh^4(c+dx)\right)^3\,dx = -\frac{-240b^2(24a+5b)\cosh(c+dx)+200b^3\cosh(3(c+dx))-24b^3\cosh(5(c+dx))+150a^3\operatorname{csch}^2\left(\frac{1}{2}(c+\right.}{}$$

input
```
Integrate[Csch[c + d*x]^7*(a + b*Sinh[c + d*x]^4)^3,x]
```

output
```
-1/1920*(-240*b^2*(24*a + 5*b)*Cosh[c + d*x] + 200*b^3*Cosh[3*(c + d*x)] -
 24*b^3*Cosh[5*(c + d*x)] + 150*a^3*Csch[(c + d*x)/2]^2 + 720*a^2*b*Csch[(
c + d*x)/2]^2 - 30*a^3*Csch[(c + d*x)/2]^4 + 5*a^3*Csch[(c + d*x)/2]^6 - 6
00*a^3*Log[Cosh[(c + d*x)/2]] - 2880*a^2*b*Log[Cosh[(c + d*x)/2]] + 600*a^
3*Log[Sinh[(c + d*x)/2]] + 2880*a^2*b*Log[Sinh[(c + d*x)/2]] + 150*a^3*Sec
h[(c + d*x)/2]^2 + 720*a^2*b*Sech[(c + d*x)/2]^2 + 30*a^3*Sech[(c + d*x)/2
]^4 + 5*a^3*Sech[(c + d*x)/2]^6)/d
```

Rubi [A] (verified)

Time = 0.91 (sec) , antiderivative size = 172, normalized size of antiderivative = 1.10, number of steps used = 12, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.478$, Rules used = {3042, 26, 3694, 1471, 25, 2345, 27, 2345, 25, 2341, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \operatorname{csch}^7(c+dx)\left(a+b\sinh^4(c+dx)\right)^3\,dx$$

$$\Big\downarrow \; 3042$$

$$\int -\frac{i\left(a+b\sin(ic+idx)^4\right)^3}{\sin(ic+idx)^7}dx$$

$$\Big\downarrow \; 26$$

$$-i\int \frac{\left(b\sin(ic+idx)^4+a\right)^3}{\sin(ic+idx)^7}dx$$

$$\Big\downarrow \; 3694$$

$$\frac{\int \frac{\left(b\cosh^4(c+dx)-2b\cosh^2(c+dx)+a+b\right)^3}{\left(1-\cosh^2(c+dx)\right)^4}d\cosh(c+dx)}{d}$$

$$\Big\downarrow \; 1471$$

$$\frac{\frac{a^3\cosh(c+dx)}{6\left(1-\cosh^2(c+dx)\right)^3}-\frac{1}{6}\int -\frac{-6b^3\cosh^{10}(c+dx)+30b^3\cosh^8(c+dx)-6b^2(3a+10b)\cosh^6(c+dx)+6b^2(9a+10b)\cosh^4(c+dx)-6b\left(3a^2+9ba+5\cdots\right.}{\left(1-\cosh^2(c+dx)\right)^3}\cdots}{d}$$

$$\Big\downarrow \; 25$$

$$\frac{\frac{1}{6}\int \frac{-6b^3\cosh^{10}(c+dx)+30b^3\cosh^8(c+dx)-6b^2(3a+10b)\cosh^6(c+dx)+6b^2(9a+10b)\cosh^4(c+dx)-6b\left(3a^2+9ba+5b^2\right)\cosh^2(c+dx)+5a^3+6b^3\cdots}{\left(1-\cosh^2(c+dx)\right)^3}\cdots}{d}$$

$$\Big\downarrow \; 2345$$

$$\frac{\frac{1}{6}\left(\frac{5a^3\cosh(c+dx)}{4\left(1-\cosh^2(c+dx)\right)^2}-\frac{1}{4}\int -\frac{3\left(8b^3\cosh^8(c+dx)-32b^3\cosh^6(c+dx)+24b^2(a+2b)\cosh^4(c+dx)-16b^2(3a+2b)\cosh^2(c+dx)+5a^3+8b^3+24\cdots\right.}{\left(1-\cosh^2(c+dx)\right)^2}\cdots\right.}{d}$$

$$\Big\downarrow \; 27$$

$$\frac{\frac{1}{6}\left(\frac{3}{4}\int \frac{8b^3\cosh^8(c+dx)-32b^3\cosh^6(c+dx)+24b^2(a+2b)\cosh^4(c+dx)-16b^2(3a+2b)\cosh^2(c+dx)+5a^3+8b^3+24ab^2+24a^2b}{\left(1-\cosh^2(c+dx)\right)^2}d\cosh(c+dx\cdots\right.}{d}$$

$$\Big\downarrow \; 2345$$

$$\frac{\frac{1}{6}\left(\frac{3}{4}\left(\frac{a^2(5a+24b)\cosh(c+dx)}{2\left(1-\cosh^2(c+dx)\right)}-\frac{1}{2}\int -\frac{-16b^3\cosh^6(c+dx)+48b^3\cosh^4(c+dx)-48b^2(a+b)\cosh^2(c+dx)+5a^3+16b^3+48ab^2+24a^2b}{1-\cosh^2(c+dx)}d\cosh(\cdots\right.\right.}{d}$$

$$\Big\downarrow \; 25$$

$$\frac{\frac{1}{6}\left(\frac{3}{4}\left(\frac{1}{2}\int \frac{-16b^3\cosh^6(c+dx)+48b^3\cosh^4(c+dx)-48b^2(a+b)\cosh^2(c+dx)+5a^3+16b^3+48ab^2+24a^2b}{1-\cosh^2(c+dx)}d\cosh(c+dx)+\frac{a^2(5a+24b)\cosh\cdots}{2\left(1-\cosh^2(c+\cdots\right.}\right.\right.}{d}$$

$$\Big\downarrow \; 2341$$

$$\frac{\frac{1}{6}\left(\frac{3}{4}\left(\frac{1}{2}\int \left(16b^3\cosh^4(c+dx)-32b^3\cosh^2(c+dx)+16b^2(3a+b)+\frac{5a^3+24ba^2}{1-\cosh^2(c+dx)}\right)d\cosh(c+dx)+\frac{a^2(5a+24b)\,\mathrm{c}\cdots}{2\left(1-\cosh^2\cdots\right.}\right.\right.}{d}$$

$\Big\downarrow$ 2009

$$\frac{\frac{a^3\cosh(c+dx)}{6(1-\cosh^2(c+dx))^3}+\frac{1}{6}\left(\frac{5a^3\cosh(c+dx)}{4(1-\cosh^2(c+dx))^2}+\frac{3}{4}\left(\frac{1}{2}\left(a^2(5a+24b)\text{arctanh}(\cosh(c+dx))+16b^2(3a+b)\cosh(c+dx)-\right.\right.\right.}{d}$$

input
```
Int[Csch[c + d*x]^7*(a + b*Sinh[c + d*x]^4)^3,x]
```

output
```
((a^3*Cosh[c + d*x])/(6*(1 - Cosh[c + d*x]^2)^3) + ((5*a^3*Cosh[c + d*x])/
(4*(1 - Cosh[c + d*x]^2)^2) + (3*((a^2*(5*a + 24*b)*Cosh[c + d*x])/(2*(1 -
 Cosh[c + d*x]^2)) + (a^2*(5*a + 24*b)*ArcTanh[Cosh[c + d*x]] + 16*b^2*(3*
a + b)*Cosh[c + d*x] - (32*b^3*Cosh[c + d*x]^3)/3 + (16*b^3*Cosh[c + d*x]^
5)/5)/2))/4)/6)/d
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 1471
```
Int[((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.),
x_Symbol] :> With[{Qx = PolynomialQuotient[(a + b*x^2 + c*x^4)^p, d + e*x^2
, x], R = Coeff[PolynomialRemainder[(a + b*x^2 + c*x^4)^p, d + e*x^2, x], x
, 0]}, Simp[(-R)*x*((d + e*x^2)^(q + 1)/(2*d*(q + 1))), x] + Simp[1/(2*d*(q
 + 1))   Int[(d + e*x^2)^(q + 1)*ExpandToSum[2*d*(q + 1)*Qx + R*(2*q + 3),
x], x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^
2 - b*d*e + a*e^2, 0] && IGtQ[p, 0] && LtQ[q, -1]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2341
```
Int[(Pq_)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[Pq*
(a + b*x^2)^p, x], x] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

rule 2345
```
Int[(Pq_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{Q = PolynomialQuot
ient[Pq, a + b*x^2, x], f = Coeff[PolynomialRemainder[Pq, a + b*x^2, x], x,
 0], g = Coeff[PolynomialRemainder[Pq, a + b*x^2, x], x, 1]}, Simp[(a*g - b
*f*x)*((a + b*x^2)^(p + 1)/(2*a*b*(p + 1))), x] + Simp[1/(2*a*(p + 1))   In
t[(a + b*x^2)^(p + 1)*ExpandToSum[2*a*(p + 1)*Q + f*(2*p + 3), x], x], x]]
/; FreeQ[{a, b}, x] && PolyQ[Pq, x] && LtQ[p, -1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3694
```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^4)^
(p_.), x_Symbol] :> With[{ff = FreeFactors[Cos[e + f*x], x]}, Simp[-ff/f
Subst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a + b - 2*b*ff^2*x^2 + b*ff^4*x^4)^p,
 x], x, Cos[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m -
 1)/2]
```

Maple [A] (verified)

Time = 3.84 (sec) , antiderivative size = 128, normalized size of antiderivative = 0.82

method	result
derivativedivides	$\frac{a^3\left(\left(-\frac{\operatorname{csch}(dx+c)^5}{6}+\frac{5\operatorname{csch}(dx+c)^3}{24}-\frac{5\operatorname{csch}(dx+c)}{16}\right)\coth(dx+c)+\frac{5\operatorname{arctanh}\left(e^{dx+c}\right)}{8}\right)+3a^2b\left(-\frac{\operatorname{csch}(dx+c)\coth(dx+c)}{2}+a\right.}{d}$
default	$\frac{a^3\left(\left(-\frac{\operatorname{csch}(dx+c)^5}{6}+\frac{5\operatorname{csch}(dx+c)^3}{24}-\frac{5\operatorname{csch}(dx+c)}{16}\right)\coth(dx+c)+\frac{5\operatorname{arctanh}\left(e^{dx+c}\right)}{8}\right)+3a^2b\left(-\frac{\operatorname{csch}(dx+c)\coth(dx+c)}{2}+a\right.}{d}$
parallelrisch	$-5120a^2\left(a+\frac{24b}{5}\right)\ln\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-5a^3\operatorname{sech}\left(\frac{dx}{2}+\frac{c}{2}\right)^6\left(\cosh(5dx+5c)-\frac{65\cosh(6dx+6c)}{48}+\frac{66\cosh(dx+c)}{5}-\frac{325\cosh(2}{16}\right.$
risch	$\frac{b^3e^{5dx+5c}}{160d}-\frac{5e^{3dx+3c}b^3}{96d}+\frac{3e^{dx+c}b^2a}{2d}+\frac{5e^{dx+c}b^3}{16d}+\frac{3e^{-dx-c}b^2a}{2d}+\frac{5e^{-dx-c}b^3}{16d}-\frac{5e^{-3dx-3c}b^3}{96d}+\frac{b^3e^{-}}{1}$

input
```
int(csch(d*x+c)^7*(a+b*sinh(d*x+c)^4)^3,x,method=_RETURNVERBOSE)
```

output
```
1/d*(a^3*((-1/6*csch(d*x+c)^5+5/24*csch(d*x+c)^3-5/16*csch(d*x+c))*coth(d*
x+c)+5/8*arctanh(exp(d*x+c)))+3*a^2*b*(-1/2*csch(d*x+c)*coth(d*x+c)+arctan
h(exp(d*x+c)))+3*b^2*a*cosh(d*x+c)+b^3*(8/15+1/5*sinh(d*x+c)^4-4/15*sinh(d
*x+c)^2)*cosh(d*x+c))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 8547 vs. $2(144) = 288$.

Time = 0.20 (sec) , antiderivative size = 8547, normalized size of antiderivative = 54.79

$$\int \operatorname{csch}^7(c+dx) \left(a+b\sinh^4(c+dx)\right)^3 \, dx = \text{Too large to display}$$

input
```
integrate(csch(d*x+c)^7*(a+b*sinh(d*x+c)^4)^3,x, algorithm="fricas")
```

output
```
Too large to include
```

Sympy [F(-1)]

Timed out.

$$\int \operatorname{csch}^7(c+dx) \left(a+b\sinh^4(c+dx)\right)^3 \, dx = \text{Timed out}$$

input
```
integrate(csch(d*x+c)**7*(a+b*sinh(d*x+c)**4)**3,x)
```

output
```
Timed out
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 390 vs. $2(144) = 288$.

Time = 0.04 (sec) , antiderivative size = 390, normalized size of antiderivative = 2.50

$$\int \operatorname{csch}^7(c+dx) \left(a+b\sinh^4(c+dx)\right)^3 \, dx$$

$$= \frac{1}{480} b^3 \left(\frac{3 e^{(5 dx+5 c)}}{d} - \frac{25 e^{(3 dx+3 c)}}{d} + \frac{150 e^{(dx+c)}}{d} + \frac{150 e^{(-dx-c)}}{d} - \frac{25 e^{(-3 dx-3 c)}}{d} + \frac{3 e^{(-5 dx-5 c)}}{d} \right) + \frac{3}{2} ab^2 \left(\frac{e^{(dx+c)}}{d} + \frac{e^{(-dx-c)}}{d} \right) + \frac{1}{48} a^3 \left(\frac{15 \log\left(e^{(-dx-c)}+1\right)}{d} - \frac{15 \log\left(e^{(-dx-c)}-1\right)}{d} + \frac{2 \left(15 e^{(-dx-c)} - 85 e^{(-3 dx-3 c)} + 198 e^{(-5 dx-5 c)}\right.}{d(6 e^{(-2 dx-2 c)} - 15 e^{(-4 dx-4 c)} + 20 e^{(-6 dx-6 c)}} \right. + \frac{3}{2} a^2 b \left(\frac{\log\left(e^{(-dx-c)}+1\right)}{d} - \frac{\log\left(e^{(-dx-c)}-1\right)}{d} + \frac{2 \left(e^{(-dx-c)} + e^{(-3 dx-3 c)}\right)}{d(2 e^{(-2 dx-2 c)} - e^{(-4 dx-4 c)} - 1)} \right)$$

input
```
integrate(csch(d*x+c)^7*(a+b*sinh(d*x+c)^4)^3,x, algorithm="maxima")
```

output
```
1/480*b^3*(3*e^(5*d*x + 5*c)/d - 25*e^(3*d*x + 3*c)/d + 150*e^(d*x + c)/d
+ 150*e^(-d*x - c)/d - 25*e^(-3*d*x - 3*c)/d + 3*e^(-5*d*x - 5*c)/d) + 3/2
*a*b^2*(e^(d*x + c)/d + e^(-d*x - c)/d) + 1/48*a^3*(15*log(e^(-d*x - c) +
1)/d - 15*log(e^(-d*x - c) - 1)/d + 2*(15*e^(-d*x - c) - 85*e^(-3*d*x - 3*
c) + 198*e^(-5*d*x - 5*c) + 198*e^(-7*d*x - 7*c) - 85*e^(-9*d*x - 9*c) + 1
5*e^(-11*d*x - 11*c))/(d*(6*e^(-2*d*x - 2*c) - 15*e^(-4*d*x - 4*c) + 20*e^
(-6*d*x - 6*c) - 15*e^(-8*d*x - 8*c) + 6*e^(-10*d*x - 10*c) - e^(-12*d*x -
 12*c) - 1))) + 3/2*a^2*b*(log(e^(-d*x - c) + 1)/d - log(e^(-d*x - c) - 1)
/d + 2*(e^(-d*x - c) + e^(-3*d*x - 3*c))/(d*(2*e^(-2*d*x - 2*c) - e^(-4*d*
x - 4*c) - 1)))
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 321 vs. $2(144) = 288$.

Time = 0.32 (sec) , antiderivative size = 321, normalized size of antiderivative = 2.06

$$\int \operatorname{csch}^7(c+dx)\left(a+b\sinh^4(c+dx)\right)^3\,dx$$

$$= \frac{3\,b^3\left(e^{(dx+c)}+e^{(-dx-c)}\right)^5 - 40\,b^3\left(e^{(dx+c)}+e^{(-dx-c)}\right)^3 + 720\,ab^2\left(e^{(dx+c)}+e^{(-dx-c)}\right) + 240\,b^3\left(e^{(dx+c)}+e^{(-\cdots\right.}}{}$$

input
```
integrate(csch(d*x+c)^7*(a+b*sinh(d*x+c)^4)^3,x, algorithm="giac")
```

output
```
1/480*(3*b^3*(e^(d*x + c) + e^(-d*x - c))^5 - 40*b^3*(e^(d*x + c) + e^(-d*
x - c))^3 + 720*a*b^2*(e^(d*x + c) + e^(-d*x - c)) + 240*b^3*(e^(d*x + c)
+ e^(-d*x - c)) + 15*(5*a^3 + 24*a^2*b)*log(e^(d*x + c) + e^(-d*x - c) + 2
) - 15*(5*a^3 + 24*a^2*b)*log(e^(d*x + c) + e^(-d*x - c) - 2) - 20*(15*a^3
*(e^(d*x + c) + e^(-d*x - c))^5 + 72*a^2*b*(e^(d*x + c) + e^(-d*x - c))^5
- 160*a^3*(e^(d*x + c) + e^(-d*x - c))^3 - 576*a^2*b*(e^(d*x + c) + e^(-d*
x - c))^3 + 528*a^3*(e^(d*x + c) + e^(-d*x - c)) + 1152*a^2*b*(e^(d*x + c)
 + e^(-d*x - c)))/((e^(d*x + c) + e^(-d*x - c))^2 - 4)^3)/d
```

Mupad [B] (verification not implemented)

Time = 2.27 (sec) , antiderivative size = 633, normalized size of antiderivative = 4.06

$$\int \operatorname{csch}^7(c+dx)\left(a+b\sinh^4(c+dx)\right)^3\,dx$$
$$= \frac{\operatorname{atan}\left(\frac{e^{dx}\,e^{c}\left(5\,a^3\sqrt{-d^2}+24\,a^2\,b\sqrt{-d^2}\right)}{d\sqrt{25\,a^6+240\,a^5\,b+576\,a^4\,b^2}}\right)\sqrt{25\,a^6+240\,a^5\,b+576\,a^4\,b^2}}{8\sqrt{-d^2}}$$
$$-\frac{\frac{4\,e^{5\,c+5\,d\,x}\,(8\,a^3+9\,b\,a^2)}{3\,d}-\frac{8\,a^2\,b\,e^{3\,c+3\,d\,x}}{d}-\frac{8\,a^2\,b\,e^{7\,c+7\,d\,x}}{d}+\frac{2\,a^2\,b\,e^{9\,c+9\,d\,x}}{d}+\frac{2\,a^2\,b\,e^{c+d\,x}}{d}}{15\,e^{4\,c+4\,d\,x}-6\,e^{2\,c+2\,d\,x}-20\,e^{6\,c+6\,d\,x}+15\,e^{8\,c+8\,d\,x}-6\,e^{10\,c+10\,d\,x}+e^{12\,c+12\,d\,x}+1}$$
$$-\frac{5\,b^3\,e^{-3\,c-3\,d\,x}}{96\,d}-\frac{5\,b^3\,e^{3\,c+3\,d\,x}}{96\,d}+\frac{b^3\,e^{-5\,c-5\,d\,x}}{160\,d}+\frac{b^3\,e^{5\,c+5\,d\,x}}{160\,d}$$
$$-\frac{e^{c+d\,x}\,(5\,a^3+24\,b\,a^2)}{8\,d\,(e^{2\,c+2\,d\,x}-1)}-\frac{a^3\,e^{c+d\,x}}{3\,d\,(3\,e^{2\,c+2\,d\,x}-3\,e^{4\,c+4\,d\,x}+e^{6\,c+6\,d\,x}-1)}$$
$$+\frac{b^2\,e^{c+d\,x}\,(24\,a+5\,b)}{16\,d}-\frac{e^{c+d\,x}\,(48\,a^2\,b-5\,a^3)}{12\,d\,(e^{4\,c+4\,d\,x}-2\,e^{2\,c+2\,d\,x}+1)}$$
$$-\frac{22\,a^3\,e^{c+d\,x}}{3\,d\,(6\,e^{4\,c+4\,d\,x}-4\,e^{2\,c+2\,d\,x}-4\,e^{6\,c+6\,d\,x}+e^{8\,c+8\,d\,x}+1)}$$
$$-\frac{16\,a^3\,e^{c+d\,x}}{3\,d\,(5\,e^{2\,c+2\,d\,x}-10\,e^{4\,c+4\,d\,x}+10\,e^{6\,c+6\,d\,x}-5\,e^{8\,c+8\,d\,x}+e^{10\,c+10\,d\,x}-1)}$$
$$+\frac{b^2\,e^{-c-d\,x}\,(24\,a+5\,b)}{16\,d}$$

input
```
int((a + b*sinh(c + d*x)^4)^3/sinh(c + d*x)^7,x)
```

output
```
(atan((exp(d*x)*exp(c)*(5*a^3*(-d^2)^(1/2) + 24*a^2*b*(-d^2)^(1/2)))/(d*(2
40*a^5*b + 25*a^6 + 576*a^4*b^2)^(1/2)))*(240*a^5*b + 25*a^6 + 576*a^4*b^2
)^(1/2))/(8*(-d^2)^(1/2)) - ((4*exp(5*c + 5*d*x)*(9*a^2*b + 8*a^3))/(3*d)
- (8*a^2*b*exp(3*c + 3*d*x))/d - (8*a^2*b*exp(7*c + 7*d*x))/d + (2*a^2*b*e
xp(9*c + 9*d*x))/d + (2*a^2*b*exp(c + d*x))/d)/(15*exp(4*c + 4*d*x) - 6*ex
p(2*c + 2*d*x) - 20*exp(6*c + 6*d*x) + 15*exp(8*c + 8*d*x) - 6*exp(10*c +
10*d*x) + exp(12*c + 12*d*x) + 1) - (5*b^3*exp(- 3*c - 3*d*x))/(96*d) - (5
*b^3*exp(3*c + 3*d*x))/(96*d) + (b^3*exp(- 5*c - 5*d*x))/(160*d) + (b^3*ex
p(5*c + 5*d*x))/(160*d) - (exp(c + d*x)*(24*a^2*b + 5*a^3))/(8*d*(exp(2*c
+ 2*d*x) - 1)) - (a^3*exp(c + d*x))/(3*d*(3*exp(2*c + 2*d*x) - 3*exp(4*c +
 4*d*x) + exp(6*c + 6*d*x) - 1)) + (b^2*exp(c + d*x)*(24*a + 5*b))/(16*d)
- (exp(c + d*x)*(48*a^2*b - 5*a^3))/(12*d*(exp(4*c + 4*d*x) - 2*exp(2*c +
2*d*x) + 1)) - (22*a^3*exp(c + d*x))/(3*d*(6*exp(4*c + 4*d*x) - 4*exp(2*c
+ 2*d*x) - 4*exp(6*c + 6*d*x) + exp(8*c + 8*d*x) + 1)) - (16*a^3*exp(c + d
*x))/(3*d*(5*exp(2*c + 2*d*x) - 10*exp(4*c + 4*d*x) + 10*exp(6*c + 6*d*x)
- 5*exp(8*c + 8*d*x) + exp(10*c + 10*d*x) - 1)) + (b^2*exp(- c - d*x)*(24*
a + 5*b))/(16*d)
```

Reduce [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 1290, normalized size of antiderivative = 8.27

$$\int \operatorname{csch}^7(c+dx)\left(a+b\sinh^4(c+dx)\right)^3\,dx = \text{Too large to display}$$

input
```
int(csch(d*x+c)^7*(a+b*sinh(d*x+c)^4)^3,x)
```

output

```
(3*e**(22*c + 22*d*x)*b**3 - 43*e**(20*c + 20*d*x)*b**3 + 720*e**(18*c + 1
8*d*x)*a*b**2 + 345*e**(18*c + 18*d*x)*b**3 - 150*e**(17*c + 17*d*x)*log(e
**(c + d*x) - 1)*a**3 - 720*e**(17*c + 17*d*x)*log(e**(c + d*x) - 1)*a**2*
b + 150*e**(17*c + 17*d*x)*log(e**(c + d*x) + 1)*a**3 + 720*e**(17*c + 17*
d*x)*log(e**(c + d*x) + 1)*a**2*b - 300*e**(16*c + 16*d*x)*a**3 - 1440*e**
(16*c + 16*d*x)*a**2*b - 3600*e**(16*c + 16*d*x)*a*b**2 - 1185*e**(16*c +
16*d*x)*b**3 + 900*e**(15*c + 15*d*x)*log(e**(c + d*x) - 1)*a**3 + 4320*e*
*(15*c + 15*d*x)*log(e**(c + d*x) - 1)*a**2*b - 900*e**(15*c + 15*d*x)*log
(e**(c + d*x) + 1)*a**3 - 4320*e**(15*c + 15*d*x)*log(e**(c + d*x) + 1)*a*
*2*b + 1700*e**(14*c + 14*d*x)*a**3 + 4320*e**(14*c + 14*d*x)*a**2*b + 648
0*e**(14*c + 14*d*x)*a*b**2 + 1870*e**(14*c + 14*d*x)*b**3 - 2250*e**(13*c
 + 13*d*x)*log(e**(c + d*x) - 1)*a**3 - 10800*e**(13*c + 13*d*x)*log(e**(c
 + d*x) - 1)*a**2*b + 2250*e**(13*c + 13*d*x)*log(e**(c + d*x) + 1)*a**3 +
 10800*e**(13*c + 13*d*x)*log(e**(c + d*x) + 1)*a**2*b - 3960*e**(12*c + 1
2*d*x)*a**3 - 2880*e**(12*c + 12*d*x)*a**2*b - 3600*e**(12*c + 12*d*x)*a*b
**2 - 990*e**(12*c + 12*d*x)*b**3 + 3000*e**(11*c + 11*d*x)*log(e**(c + d*
x) - 1)*a**3 + 14400*e**(11*c + 11*d*x)*log(e**(c + d*x) - 1)*a**2*b - 300
0*e**(11*c + 11*d*x)*log(e**(c + d*x) + 1)*a**3 - 14400*e**(11*c + 11*d*x)
*log(e**(c + d*x) + 1)*a**2*b - 3960*e**(10*c + 10*d*x)*a**3 - 2880*e**(10
*c + 10*d*x)*a**2*b - 3600*e**(10*c + 10*d*x)*a*b**2 - 990*e**(10*c + 1...
```

3.189 $\int \operatorname{csch}^9(c+dx) \left(a+b\sinh^4(c+dx)\right)^3 \, dx$

Optimal result

Integrand size = 23, antiderivative size = 171

$$
\begin{aligned}
\int \operatorname{csch}^9(c+dx) \left(a+b\sinh^4(c+dx)\right)^3 \, dx
&= -\frac{a(35a^2+144ab+384b^2)\operatorname{arctanh}(\cosh(c+dx))}{128d} - \frac{b^3\cosh(c+dx)}{d} \\
&\quad + \frac{b^3\cosh^3(c+dx)}{3d} + \frac{a^2(35a+144b)\coth(c+dx)\operatorname{csch}(c+dx)}{128d} \\
&\quad - \frac{a^2(35a+144b)\coth(c+dx)\operatorname{csch}^3(c+dx)}{192d} \\
&\quad + \frac{7a^3\coth(c+dx)\operatorname{csch}^5(c+dx)}{48d} - \frac{a^3\coth(c+dx)\operatorname{csch}^7(c+dx)}{8d}
\end{aligned}
$$

output
```
-1/128*a*(35*a^2+144*a*b+384*b^2)*arctanh(cosh(d*x+c))/d-b^3*cosh(d*x+c)/d
+1/3*b^3*cosh(d*x+c)^3/d+1/128*a^2*(35*a+144*b)*coth(d*x+c)*csch(d*x+c)/d-
1/192*a^2*(35*a+144*b)*coth(d*x+c)*csch(d*x+c)^3/d+7/48*a^3*coth(d*x+c)*cs
ch(d*x+c)^5/d-1/8*a^3*coth(d*x+c)*csch(d*x+c)^7/d
```

Mathematica [A] (verified)

Time = 4.03 (sec) , antiderivative size = 233, normalized size of antiderivative = 1.36

$$\int \operatorname{csch}^9(c+dx)\left(a+b\sinh^4(c+dx)\right)^3\,dx$$
$$= \frac{-4608b^3\cosh(c+dx)+512b^3\cosh(3(c+dx))+a\left(12a(35a+144b)\operatorname{csch}^2\left(\frac{1}{2}(c+dx)\right)-18a(5a+16b)\mathrm{c}\right.}{}$$

input
```
Integrate[Csch[c + d*x]^9*(a + b*Sinh[c + d*x]^4)^3,x]
```

output
```
(-4608*b^3*Cosh[c + d*x] + 512*b^3*Cosh[3*(c + d*x)] + a*(12*a*(35*a + 144
*b)*Csch[(c + d*x)/2]^2 - 18*a*(5*a + 16*b)*Csch[(c + d*x)/2]^4 + 20*a^2*C
sch[(c + d*x)/2]^6 - 3*a^2*Csch[(c + d*x)/2]^8 - 48*(35*a^2 + 144*a*b + 38
4*b^2)*(Log[Cosh[(c + d*x)/2]] - Log[Sinh[(c + d*x)/2]]) + 12*a*(35*a + 14
4*b)*Sech[(c + d*x)/2]^2 + 18*a*(5*a + 16*b)*Sech[(c + d*x)/2]^4 + 20*a^2*
Sech[(c + d*x)/2]^6 + 3*a^2*Sech[(c + d*x)/2]^8))/(6144*d)
```

Rubi [A] (verified)

Time = 0.96 (sec) , antiderivative size = 198, normalized size of antiderivative = 1.16, number of steps used = 14, number of rules used = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.565$, Rules used = {3042, 26, 3694, 1471, 25, 2345, 25, 2345, 27, 2345, 25, 1467, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \operatorname{csch}^9(c+dx)\left(a+b\sinh^4(c+dx)\right)^3\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{i\left(a+b\sin(ic+idx)^4\right)^3}{\sin(ic+idx)^9}dx$$

$$\Big\downarrow \text{26}$$

$$i\int \frac{\left(b\sin(ic+idx)^4+a\right)^3}{\sin(ic+idx)^9}dx$$

$$\Big\downarrow \text{3694}$$

$$-\frac{\int \frac{\left(b\cosh^4(c+dx)-2b\cosh^2(c+dx)+a+b\right)^3}{\left(1-\cosh^2(c+dx)\right)^5}d\cosh(c+dx)}{d}$$

$$\Big\downarrow \text{1471}$$

$$-\frac{\frac{a^3\cosh(c+dx)}{8\left(1-\cosh^2(c+dx)\right)^4}-\frac{1}{8}\int -\frac{-8b^3\cosh^{10}(c+dx)+40b^3\cosh^8(c+dx)-8b^2(3a+10b)\cosh^6(c+dx)+8b^2(9a+10b)\cosh^4(c+dx)-8b\left(3a^2+9ba+\right.}{\left(1-\cosh^2(c+dx)\right)^4}}{d}$$

$$\Big\downarrow \text{25}$$

$$-\frac{\frac{1}{8}\int \frac{-8b^3\cosh^{10}(c+dx)+40b^3\cosh^8(c+dx)-8b^2(3a+10b)\cosh^6(c+dx)+8b^2(9a+10b)\cosh^4(c+dx)-8b\left(3a^2+9ba+5b^2\right)\cosh^2(c+dx)+(a+2b)}{\left(1-\cosh^2(c+dx)\right)^4}}{d}$$

$$\Big\downarrow \text{2345}$$

$$-\frac{\frac{1}{8}\left(\frac{7a^3\cosh(c+dx)}{6\left(1-\cosh^2(c+dx)\right)^3}-\frac{1}{6}\int -\frac{48b^3\cosh^8(c+dx)-192b^3\cosh^6(c+dx)+144b^2(a+2b)\cosh^4(c+dx)-96b^2(3a+2b)\cosh^2(c+dx)+35a^3+48b}{\left(1-\cosh^2(c+dx)\right)^3}\right.}{d}$$

$$\Big\downarrow \text{25}$$

$$-\frac{\frac{1}{8}\left(\frac{1}{6}\int \frac{48b^3\cosh^8(c+dx)-192b^3\cosh^6(c+dx)+144b^2(a+2b)\cosh^4(c+dx)-96b^2(3a+2b)\cosh^2(c+dx)+35a^3+48b^3+144ab^2+144a^2b}{\left(1-\cosh^2(c+dx)\right)^3}d\cosh\right.}{d}$$

$$\Big\downarrow \text{2345}$$

$$-\frac{\frac{1}{8}\left(\frac{1}{6}\left(\frac{a^2(35a+144b)\cosh(c+dx)}{4\left(1-\cosh^2(c+dx)\right)^2}-\frac{1}{4}\int -\frac{3\left(-64b^3\cosh^6(c+dx)+192b^3\cosh^4(c+dx)-192b^2(a+b)\cosh^2(c+dx)+35a^3+64b^3+192ab^2+14\right.}{\left(1-\cosh^2(c+dx)\right)^2}\right.\right.}{d}$$

$$\Big\downarrow \text{27}$$

$$-\frac{\frac{1}{8}\left(\frac{1}{6}\left(\frac{3}{4}\int \frac{-64b^3\cosh^6(c+dx)+192b^3\cosh^4(c+dx)-192b^2(a+b)\cosh^2(c+dx)+35a^3+64b^3+192ab^2+144a^2b}{\left(1-\cosh^2(c+dx)\right)^2}d\cosh(c+dx)+\frac{a^2(35a+}{4(1-}\right.\right.}{d}$$

$$\Big\downarrow \text{2345}$$

$$-\frac{\frac{1}{8}\left(\frac{1}{6}\left(\frac{3}{4}\left(\frac{a^2(35a+144b)\cosh(c+dx)}{2\left(1-\cosh^2(c+dx)\right)}-\frac{1}{2}\int -\frac{128b^3\cosh^4(c+dx)-256b^3\cosh^2(c+dx)+35a^3+128b^3+384ab^2+144a^2b}{1-\cosh^2(c+dx)}d\cosh(c+dx)\right)\right.\right.}{d}$$

$$\Big\downarrow \ 25$$

$$-\frac{\frac{1}{8}\left(\frac{1}{6}\left(\frac{3}{4}\left(\frac{1}{2}\int \frac{128b^3\cosh^4(c+dx)-256b^3\cosh^2(c+dx)+35a^3+128b^3+384ab^2+144a^2b}{1-\cosh^2(c+dx)}d\cosh(c+dx)+\frac{a^2(35a+144b)\cosh(c+dx)}{2(1-\cosh^2(c+dx))}\right)+\right.\right.}{d}$$

$$\Big\downarrow \ 1467$$

$$-\frac{\frac{1}{8}\left(\frac{1}{6}\left(\frac{3}{4}\left(\frac{1}{2}\int \left(-128\cosh^2(c+dx)b^3+128b^3+\frac{35a^3+144ba^2+384b^2a}{1-\cosh^2(c+dx)}\right)d\cosh(c+dx)+\frac{a^2(35a+144b)\cosh(c+dx)}{2(1-\cosh^2(c+dx))}\right)+\right.\right.}{d}$$

$$\Big\downarrow \ 2009$$

$$-\frac{\frac{a^3\cosh(c+dx)}{8(1-\cosh^2(c+dx))^4}+\frac{1}{8}\left(\frac{7a^3\cosh(c+dx)}{6(1-\cosh^2(c+dx))^3}+\frac{1}{6}\left(\frac{3}{4}\left(\frac{1}{2}\left(a(35a^2+144ab+384b^2)\operatorname{arctanh}(\cosh(c+dx))-\frac{128}{3}b^3\cosh\right.\right.\right.\right.}{d}$$

input
```
Int[Csch[c + d*x]^9*(a + b*Sinh[c + d*x]^4)^3,x]
```

output
```
-(((a^3*Cosh[c + d*x])/(8*(1 - Cosh[c + d*x]^2)^4) + ((7*a^3*Cosh[c + d*x]
)/(6*(1 - Cosh[c + d*x]^2)^3) + ((a^2*(35*a + 144*b)*Cosh[c + d*x])/(4*(1
- Cosh[c + d*x]^2)^2) + (3*((a^2*(35*a + 144*b)*Cosh[c + d*x])/(2*(1 - Cos
h[c + d*x]^2)) + (a*(35*a^2 + 144*a*b + 384*b^2)*ArcTanh[Cosh[c + d*x]] +
128*b^3*Cosh[c + d*x] - (128*b^3*Cosh[c + d*x]^3)/3)/2))/4)/6)/8)/d)
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 1467
```
Int[((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.),
 x_Symbol] :> Int[ExpandIntegrand[(d + e*x^2)^q*(a + b*x^2 + c*x^4)^p, x],
x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e
 + a*e^2, 0] && IGtQ[p, 0] && IGtQ[q, -2]
```

rule 1471
```
Int[((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.),
x_Symbol] :> With[{Qx = PolynomialQuotient[(a + b*x^2 + c*x^4)^p, d + e*x^2
, x], R = Coeff[PolynomialRemainder[(a + b*x^2 + c*x^4)^p, d + e*x^2, x], x
, 0]}, Simp[(-R)*x*((d + e*x^2)^(q + 1)/(2*d*(q + 1))), x] + Simp[1/(2*d*(q
 + 1))   Int[(d + e*x^2)^(q + 1)*ExpandToSum[2*d*(q + 1)*Qx + R*(2*q + 3),
x], x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^
2 - b*d*e + a*e^2, 0] && IGtQ[p, 0] && LtQ[q, -1]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2345
```
Int[(Pq_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{Q = PolynomialQuot
ient[Pq, a + b*x^2, x], f = Coeff[PolynomialRemainder[Pq, a + b*x^2, x], x,
 0], g = Coeff[PolynomialRemainder[Pq, a + b*x^2, x], x, 1]}, Simp[(a*g - b
*f*x)*((a + b*x^2)^(p + 1)/(2*a*b*(p + 1))), x] + Simp[1/(2*a*(p + 1))   In
t[(a + b*x^2)^(p + 1)*ExpandToSum[2*a*(p + 1)*Q + f*(2*p + 3), x], x], x]]
/; FreeQ[{a, b}, x] && PolyQ[Pq, x] && LtQ[p, -1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3694
```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^4)^
(p_.), x_Symbol] :> With[{ff = FreeFactors[Cos[e + f*x], x]}, Simp[-ff/f
Subst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a + b - 2*b*ff^2*x^2 + b*ff^4*x^4)^p,
 x], x, Cos[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m -
 1)/2]
```

Maple [A] (verified)

Time = 3.28 (sec) , antiderivative size = 143, normalized size of antiderivative = 0.84

method	result
derivativedivides	$\frac{a^3\left(\left(-\frac{\operatorname{csch}(dx+c)^7}{8}+\frac{7\operatorname{csch}(dx+c)^5}{48}-\frac{35\operatorname{csch}(dx+c)^3}{192}+\frac{35\operatorname{csch}(dx+c)}{128}\right)\coth(dx+c)-\frac{35\operatorname{arctanh}\left(e^{dx+c}\right)}{64}\right)+3a^2b\left(\left(-\operatorname{csc}\right.\right.}{}$
default	$\frac{a^3\left(\left(-\frac{\operatorname{csch}(dx+c)^7}{8}+\frac{7\operatorname{csch}(dx+c)^5}{48}-\frac{35\operatorname{csch}(dx+c)^3}{192}+\frac{35\operatorname{csch}(dx+c)}{128}\right)\coth(dx+c)-\frac{35\operatorname{arctanh}\left(e^{dx+c}\right)}{64}\right)+3a^2b\left(\left(-\operatorname{csc}\right.\right.}{}$
parallelrisch	$\frac{(6881280a^3+28311552a^2b+75497472b^2a)\ln\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-3220\left(\cosh(5dx+5c)-\frac{638\cosh(6dx+6c)}{805}-\frac{3\cosh(7dx+7c)}{23}\right.}{}$
risch	$\frac{e^{3dx+3c}b^3}{24d}-\frac{3\,e^{dx+c}b^3}{8d}-\frac{3\,e^{-dx-c}b^3}{8d}+\frac{e^{-3dx-3c}b^3}{24d}+\frac{e^{dx+c}a^2\left(105\,e^{14dx+14c}a+432\,e^{14dx+14c}b-805\,e^{12dx+12}\right.}{}$

input
```
int(csch(d*x+c)^9*(a+b*sinh(d*x+c)^4)^3,x,method=_RETURNVERBOSE)
```

output
```
1/d*(a^3*((-1/8*csch(d*x+c)^7+7/48*csch(d*x+c)^5-35/192*csch(d*x+c)^3+35/1
28*csch(d*x+c))*coth(d*x+c)-35/64*arctanh(exp(d*x+c)))+3*a^2*b*((-1/4*csch
(d*x+c)^3+3/8*csch(d*x+c))*coth(d*x+c)-3/4*arctanh(exp(d*x+c)))-6*b^2*a*ar
ctanh(exp(d*x+c))+b^3*(-2/3+1/3*sinh(d*x+c)^2)*cosh(d*x+c))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 10848 vs. $2(159) = 318$.

Time = 0.21 (sec) , antiderivative size = 10848, normalized size of antiderivative = 63.44

$$\int \operatorname{csch}^9(c+dx)\left(a+b\sinh^4(c+dx)\right)^3\,dx=\text{Too large to display}$$

input
```
integrate(csch(d*x+c)^9*(a+b*sinh(d*x+c)^4)^3,x, algorithm="fricas")
```

output
```
Too large to include
```

Sympy [F(-1)]

Timed out.

$$\int \operatorname{csch}^9(c+dx)\left(a+b\sinh^4(c+dx)\right)^3\,dx = \text{Timed out}$$

input
```
integrate(csch(d*x+c)**9*(a+b*sinh(d*x+c)**4)**3,x)
```

output
```
Timed out
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 463 vs. $2(159) = 318$.

Time = 0.06 (sec) , antiderivative size = 463, normalized size of antiderivative = 2.71

$$\int \operatorname{csch}^9(c+dx)\left(a+b\sinh^4(c+dx)\right)^3\,dx$$
$$= \frac{1}{24}b^3\left(\frac{e^{(3\,dx+3\,c)}}{d} - \frac{9\,e^{(dx+c)}}{d} - \frac{9\,e^{(-dx-c)}}{d} + \frac{e^{(-3\,dx-3\,c)}}{d}\right)$$
$$-\frac{1}{384}a^3\left(\frac{105\log\left(e^{(-dx-c)}+1\right)}{d} - \frac{105\log\left(e^{(-dx-c)}-1\right)}{d} + \frac{2\left(105\,e^{(-dx-c)} - 805\,e^{(-3\,dx-3\,c)} + 2681\,e^{(-}\right.}{d(8\,e^{(-2\,dx-2\,c)} - 28\,e^{(-4\,dx-4\,c)} + 56\,e}\right.$$
$$-\frac{3}{8}a^2b\left(\frac{3\log\left(e^{(-dx-c)}+1\right)}{d} - \frac{3\log\left(e^{(-dx-c)}-1\right)}{d} + \frac{2\left(3\,e^{(-dx-c)} - 11\,e^{(-3\,dx-3\,c)} - 11\,e^{(-5\,dx-5\,c)} + 3\right.}{d(4\,e^{(-2\,dx-2\,c)} - 6\,e^{(-4\,dx-4\,c)} + 4\,e^{(-6\,dx-6\,c)} - e^{(-}}\right.$$
$$-3\,ab^2\left(\frac{\log\left(e^{(-dx-c)}+1\right)}{d} - \frac{\log\left(e^{(-dx-c)}-1\right)}{d}\right)$$

input
```
integrate(csch(d*x+c)^9*(a+b*sinh(d*x+c)^4)^3,x, algorithm="maxima")
```

output
```
1/24*b^3*(e^(3*d*x + 3*c)/d - 9*e^(d*x + c)/d - 9*e^(-d*x - c)/d + e^(-3*d
*x - 3*c)/d) - 1/384*a^3*(105*log(e^(-d*x - c) + 1)/d - 105*log(e^(-d*x -
c) - 1)/d + 2*(105*e^(-d*x - c) - 805*e^(-3*d*x - 3*c) + 2681*e^(-5*d*x -
5*c) - 5053*e^(-7*d*x - 7*c) - 5053*e^(-9*d*x - 9*c) + 2681*e^(-11*d*x - 1
1*c) - 805*e^(-13*d*x - 13*c) + 105*e^(-15*d*x - 15*c))/(d*(8*e^(-2*d*x -
2*c) - 28*e^(-4*d*x - 4*c) + 56*e^(-6*d*x - 6*c) - 70*e^(-8*d*x - 8*c) + 5
6*e^(-10*d*x - 10*c) - 28*e^(-12*d*x - 12*c) + 8*e^(-14*d*x - 14*c) - e^(-
16*d*x - 16*c) - 1))) - 3/8*a^2*b*(3*log(e^(-d*x - c) + 1)/d - 3*log(e^(-d
*x - c) - 1)/d + 2*(3*e^(-d*x - c) - 11*e^(-3*d*x - 3*c) - 11*e^(-5*d*x -
5*c) + 3*e^(-7*d*x - 7*c))/(d*(4*e^(-2*d*x - 2*c) - 6*e^(-4*d*x - 4*c) + 4
*e^(-6*d*x - 6*c) - e^(-8*d*x - 8*c) - 1))) - 3*a*b^2*(log(e^(-d*x - c) +
1)/d - log(e^(-d*x - c) - 1)/d)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 335 vs. $2(159) = 318$.

Time = 0.33 (sec) , antiderivative size = 335, normalized size of antiderivative = 1.96

$$\int \operatorname{csch}^9(c+dx)\left(a+b\sinh^4(c+dx)\right)^3\,dx$$

$$= \frac{32\,b^3\left(e^{(dx+c)}+e^{(-dx-c)}\right)^3-384\,b^3\left(e^{(dx+c)}+e^{(-dx-c)}\right)-3\left(35\,a^3+144\,a^2b+384\,ab^2\right)\log\left(e^{(dx+c)}+e^{(-dx}\right.}{}$$

input
```
integrate(csch(d*x+c)^9*(a+b*sinh(d*x+c)^4)^3,x, algorithm="giac")
```

output
```
1/768*(32*b^3*(e^(d*x + c) + e^(-d*x - c))^3 - 384*b^3*(e^(d*x + c) + e^(-
d*x - c)) - 3*(35*a^3 + 144*a^2*b + 384*a*b^2)*log(e^(d*x + c) + e^(-d*x -
 c) + 2) + 3*(35*a^3 + 144*a^2*b + 384*a*b^2)*log(e^(d*x + c) + e^(-d*x -
c) - 2) + 4*(105*a^3*(e^(d*x + c) + e^(-d*x - c))^7 + 432*a^2*b*(e^(d*x +
c) + e^(-d*x - c))^7 - 1540*a^3*(e^(d*x + c) + e^(-d*x - c))^5 - 6336*a^2*
b*(e^(d*x + c) + e^(-d*x - c))^5 + 8176*a^3*(e^(d*x + c) + e^(-d*x - c))^3
 + 29952*a^2*b*(e^(d*x + c) + e^(-d*x - c))^3 - 17856*a^3*(e^(d*x + c) + e
^(-d*x - c)) - 46080*a^2*b*(e^(d*x + c) + e^(-d*x - c)))/((e^(d*x + c) + e
^(-d*x - c))^2 - 4)^4)/d
```

Mupad [B] (verification not implemented)

Time = 2.18 (sec) , antiderivative size = 759, normalized size of antiderivative = 4.44

$$\int \operatorname{csch}^9(c+dx) \left(a+b\sinh^4(c+dx)\right)^3 \, dx = \text{Too large to display}$$

input
```
int((a + b*sinh(c + d*x)^4)^3/sinh(c + d*x)^9,x)
```

output
```
(b^3*exp(- 3*c - 3*d*x))/(24*d) - (3*b^3*exp(- c - d*x))/(8*d) - (3*b^3*ex
p(c + d*x))/(8*d) + (b^3*exp(3*c + 3*d*x))/(24*d) - (atan((exp(d*x)*exp(c)
*(35*a^3*(-d^2)^(1/2) + 384*a*b^2*(-d^2)^(1/2) + 144*a^2*b*(-d^2)^(1/2)))/
(d*(10080*a^5*b + 1225*a^6 + 147456*a^2*b^4 + 110592*a^3*b^3 + 47616*a^4*b
^2)^(1/2)))*(10080*a^5*b + 1225*a^6 + 147456*a^2*b^4 + 110592*a^3*b^3 + 47
616*a^4*b^2)^(1/2))/(64*(-d^2)^(1/2)) + (exp(c + d*x)*(144*a^2*b + 35*a^3)
)/(64*d*(exp(2*c + 2*d*x) - 1)) - (exp(c + d*x)*(48*a^2*b + a^3))/(4*d*(6*
exp(4*c + 4*d*x) - 4*exp(2*c + 2*d*x) - 4*exp(6*c + 6*d*x) + exp(8*c + 8*d
*x) + 1)) - (exp(c + d*x)*(144*a^2*b + 35*a^3))/(96*d*(exp(4*c + 4*d*x) -
2*exp(2*c + 2*d*x) + 1)) - (exp(c + d*x)*(432*a^2*b - 7*a^3))/(24*d*(3*exp
(2*c + 2*d*x) - 3*exp(4*c + 4*d*x) + exp(6*c + 6*d*x) - 1)) - (170*a^3*exp
(c + d*x))/(3*d*(5*exp(2*c + 2*d*x) - 10*exp(4*c + 4*d*x) + 10*exp(6*c + 6
*d*x) - 5*exp(8*c + 8*d*x) + exp(10*c + 10*d*x) - 1)) - (404*a^3*exp(c + d
*x))/(3*d*(15*exp(4*c + 4*d*x) - 6*exp(2*c + 2*d*x) - 20*exp(6*c + 6*d*x)
+ 15*exp(8*c + 8*d*x) - 6*exp(10*c + 10*d*x) + exp(12*c + 12*d*x) + 1)) -
(112*a^3*exp(c + d*x))/(d*(7*exp(2*c + 2*d*x) - 21*exp(4*c + 4*d*x) + 35*e
xp(6*c + 6*d*x) - 35*exp(8*c + 8*d*x) + 21*exp(10*c + 10*d*x) - 7*exp(12*c
 + 12*d*x) + exp(14*c + 14*d*x) - 1)) - (32*a^3*exp(c + d*x))/(d*(28*exp(4
*c + 4*d*x) - 8*exp(2*c + 2*d*x) - 56*exp(6*c + 6*d*x) + 70*exp(8*c + 8*d*
x) - 56*exp(10*c + 10*d*x) + 28*exp(12*c + 12*d*x) - 8*exp(14*c + 14*d*...
```

Reduce [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 1920, normalized size of antiderivative = 11.23

$$\int \operatorname{csch}^9(c+dx) \left(a+b\sinh^4(c+dx)\right)^3 \, dx = \text{Too large to display}$$

input
```
int(csch(d*x+c)^9*(a+b*sinh(d*x+c)^4)^3,x)
```

output

```
(16*e**(22*c + 22*d*x)*b**3 - 272*e**(20*c + 20*d*x)*b**3 + 105*e**(19*c +
 19*d*x)*log(e**(c + d*x) - 1)*a**3 + 432*e**(19*c + 19*d*x)*log(e**(c + d
*x) - 1)*a**2*b + 1152*e**(19*c + 19*d*x)*log(e**(c + d*x) - 1)*a*b**2 - 1
05*e**(19*c + 19*d*x)*log(e**(c + d*x) + 1)*a**3 - 432*e**(19*c + 19*d*x)*
log(e**(c + d*x) + 1)*a**2*b - 1152*e**(19*c + 19*d*x)*log(e**(c + d*x) +
1)*a*b**2 + 210*e**(18*c + 18*d*x)*a**3 + 864*e**(18*c + 18*d*x)*a**2*b +
1456*e**(18*c + 18*d*x)*b**3 - 840*e**(17*c + 17*d*x)*log(e**(c + d*x) - 1
)*a**3 - 3456*e**(17*c + 17*d*x)*log(e**(c + d*x) - 1)*a**2*b - 9216*e**(1
7*c + 17*d*x)*log(e**(c + d*x) - 1)*a*b**2 + 840*e**(17*c + 17*d*x)*log(e*
*(c + d*x) + 1)*a**3 + 3456*e**(17*c + 17*d*x)*log(e**(c + d*x) + 1)*a**2*
b + 9216*e**(17*c + 17*d*x)*log(e**(c + d*x) + 1)*a*b**2 - 1610*e**(16*c +
 16*d*x)*a**3 - 6624*e**(16*c + 16*d*x)*a**2*b - 3760*e**(16*c + 16*d*x)*b
**3 + 2940*e**(15*c + 15*d*x)*log(e**(c + d*x) - 1)*a**3 + 12096*e**(15*c
+ 15*d*x)*log(e**(c + d*x) - 1)*a**2*b + 32256*e**(15*c + 15*d*x)*log(e**(
c + d*x) - 1)*a*b**2 - 2940*e**(15*c + 15*d*x)*log(e**(c + d*x) + 1)*a**3
- 12096*e**(15*c + 15*d*x)*log(e**(c + d*x) + 1)*a**2*b - 32256*e**(15*c +
 15*d*x)*log(e**(c + d*x) + 1)*a*b**2 + 5362*e**(14*c + 14*d*x)*a**3 + 146
88*e**(14*c + 14*d*x)*a**2*b + 5024*e**(14*c + 14*d*x)*b**3 - 5880*e**(13*
c + 13*d*x)*log(e**(c + d*x) - 1)*a**3 - 24192*e**(13*c + 13*d*x)*log(e**(
c + d*x) - 1)*a**2*b - 64512*e**(13*c + 13*d*x)*log(e**(c + d*x) - 1)*a...
```

3.190 $\int \operatorname{csch}^{11}(c+dx) \left(a+b\sinh^4(c+dx)\right)^3 dx$

Optimal result

Integrand size = 23, antiderivative size = 189

$$\begin{aligned}\int \operatorname{csch}^{11}(c+dx) \left(a+b\sinh^4(c+dx)\right)^3 dx &= \frac{3a(21a^2+80ab+128b^2)\operatorname{arctanh}(\cosh(c+dx))}{256d}+\frac{b^3\cosh(c+dx)}{d}\\ &\quad-\frac{3a(21a^2+80ab+128b^2)\coth(c+dx)\operatorname{csch}(c+dx)}{256d}\\ &\quad+\frac{a^2(21a+80b)\coth(c+dx)\operatorname{csch}^3(c+dx)}{128d}\\ &\quad-\frac{a^2(21a+80b)\coth(c+dx)\operatorname{csch}^5(c+dx)}{160d}\\ &\quad+\frac{9a^3\coth(c+dx)\operatorname{csch}^7(c+dx)}{80d}-\frac{a^3\coth(c+dx)\operatorname{csch}^9(c+dx)}{10d}\end{aligned}$$

output
```
3/256*a*(21*a^2+80*a*b+128*b^2)*arctanh(cosh(d*x+c))/d+b^3*cosh(d*x+c)/d-3
/256*a*(21*a^2+80*a*b+128*b^2)*coth(d*x+c)*csch(d*x+c)/d+1/128*a^2*(21*a+8
0*b)*coth(d*x+c)*csch(d*x+c)^3/d-1/160*a^2*(21*a+80*b)*coth(d*x+c)*csch(d*
x+c)^5/d+9/80*a^3*coth(d*x+c)*csch(d*x+c)^7/d-1/10*a^3*coth(d*x+c)*csch(d*
x+c)^9/d
```

Mathematica [A] (verified)

Time = 2.54 (sec) , antiderivative size = 279, normalized size of antiderivative = 1.48

$$\int \operatorname{csch}^{11}(c+dx)\left(a+b\sinh^4(c+dx)\right)^3\,dx=\frac{b^3\cosh(c+dx)}{d}$$
$$-\frac{a\left(60(21a^2+80ab+128b^2)\operatorname{csch}^2\left(\frac{1}{2}(c+dx)\right)-40a(7a+24b)\operatorname{csch}^4\left(\frac{1}{2}(c+dx)\right)+10a(7a+16b)\operatorname{csch}^6\right.}{}$$

input
```
Integrate[Csch[c + d*x]^11*(a + b*Sinh[c + d*x]^4)^3,x]
```

output
```
(b^3*Cosh[c + d*x])/d - (a*(60*(21*a^2 + 80*a*b + 128*b^2)*Csch[(c + d*x)/
2]^2 - 40*a*(7*a + 24*b)*Csch[(c + d*x)/2]^4 + 10*a*(7*a + 16*b)*Csch[(c +
 d*x)/2]^6 - 15*a^2*Csch[(c + d*x)/2]^8 + 2*a^2*Csch[(c + d*x)/2]^10 - 240
*(21*a^2 + 80*a*b + 128*b^2)*(Log[Cosh[(c + d*x)/2]] - Log[Sinh[(c + d*x)/
2]]) + 60*(21*a^2 + 80*a*b + 128*b^2)*Sech[(c + d*x)/2]^2 + 40*a*(7*a + 24
*b)*Sech[(c + d*x)/2]^4 + 10*a*(7*a + 16*b)*Sech[(c + d*x)/2]^6 + 15*a^2*S
ech[(c + d*x)/2]^8 + 2*a^2*Sech[(c + d*x)/2]^10))/(20480*d)
```

Rubi [A] (verified)

Time = 0.91 (sec) , antiderivative size = 228, normalized size of antiderivative = 1.21, number of steps used = 16, number of rules used = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.652, Rules used = {3042, 26, 3694, 1471, 25, 2345, 25, 2345, 27, 2345, 25, 1471, 25, 299, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \operatorname{csch}^{11}(c+dx)\left(a+b\sinh^4(c+dx)\right)^3\,dx$$
$$\Big\downarrow \ 3042$$
$$\int -\frac{i\left(a+b\sin(ic+idx)^4\right)^3}{\sin(ic+idx)^{11}}dx$$
$$\Big\downarrow \ 26$$
$$-i\int \frac{\left(b\sin(ic+idx)^4+a\right)^3}{\sin(ic+idx)^{11}}dx$$

$$\Big\downarrow \text{3694}$$

$$\frac{\int \frac{\left(b\cosh^4(c+dx)-2b\cosh^2(c+dx)+a+b\right)^3}{\left(1-\cosh^2(c+dx)\right)^6}d\cosh(c+dx)}{d}$$

$$\Big\downarrow \text{1471}$$

$$\frac{\frac{a^3\cosh(c+dx)}{10\left(1-\cosh^2(c+dx)\right)^5}-\frac{1}{10}\int -\frac{-10b^3\cosh^{10}(c+dx)+50b^3\cosh^8(c+dx)-10b^2(3a+10b)\cosh^6(c+dx)+10b^2(9a+10b)\cosh^4(c+dx)-10b(3a^2+}{\left(1-\cosh^2(c+dx)\right)^5}}{d}$$

$$\Big\downarrow \text{25}$$

$$\frac{\frac{1}{10}\int \frac{-10b^3\cosh^{10}(c+dx)+50b^3\cosh^8(c+dx)-10b^2(3a+10b)\cosh^6(c+dx)+10b^2(9a+10b)\cosh^4(c+dx)-10b(3a^2+9ba+5b^2)\cosh^2(c+dx)+9a^3}{\left(1-\cosh^2(c+dx)\right)^5}}{d}$$

$$\Big\downarrow \text{2345}$$

$$\frac{\frac{1}{10}\left(\frac{9a^3\cosh(c+dx)}{8\left(1-\cosh^2(c+dx)\right)^4}-\frac{1}{8}\int -\frac{80b^3\cosh^8(c+dx)-320b^3\cosh^6(c+dx)+240b^2(a+2b)\cosh^4(c+dx)-160b^2(3a+2b)\cosh^2(c+dx)+63a^3+80b}{\left(1-\cosh^2(c+dx)\right)^4}\right.}{d}$$

$$\Big\downarrow \text{25}$$

$$\frac{\frac{1}{10}\left(\frac{1}{8}\int \frac{80b^3\cosh^8(c+dx)-320b^3\cosh^6(c+dx)+240b^2(a+2b)\cosh^4(c+dx)-160b^2(3a+2b)\cosh^2(c+dx)+63a^3+80b^3+240ab^2+240a^2b}{\left(1-\cosh^2(c+dx)\right)^4}d\cosh\right.}{d}$$

$$\Big\downarrow \text{2345}$$

$$\frac{\frac{1}{10}\left(\frac{1}{8}\left(\frac{a^2(21a+80b)\cosh(c+dx)}{2\left(1-\cosh^2(c+dx)\right)^3}-\frac{1}{6}\int -\frac{15\left(-32b^3\cosh^6(c+dx)+96b^3\cosh^4(c+dx)-96b^2(a+b)\cosh^2(c+dx)+21a^3+32b^3+96ab^2+80a^2b\right)}{\left(1-\cosh^2(c+dx)\right)^3}\right.\right.}{d}$$

$$\Big\downarrow \text{27}$$

$$\frac{\frac{1}{10}\left(\frac{1}{8}\left(\frac{5}{2}\int \frac{-32b^3\cosh^6(c+dx)+96b^3\cosh^4(c+dx)-96b^2(a+b)\cosh^2(c+dx)+21a^3+32b^3+96ab^2+80a^2b}{\left(1-\cosh^2(c+dx)\right)^3}d\cosh(c+dx)+\frac{a^2(21a+80b)}{2\left(1-\cosh^2(\right.}\right.\right.}{d}$$

$$\Big\downarrow \text{2345}$$

$$\frac{\frac{1}{10}\left(\frac{1}{8}\left(\frac{5}{2}\left(\frac{a^2(21a+80b)\cosh(c+dx)}{4\left(1-\cosh^2(c+dx)\right)^2}-\frac{1}{4}\int -\frac{128b^3\cosh^4(c+dx)-256b^3\cosh^2(c+dx)+63a^3+128b^3+384ab^2+240a^2b}{\left(1-\cosh^2(c+dx)\right)^2}d\cosh(c+dx)\right)\right.\right.}{d}$$

$\downarrow$ 25

$$\frac{\frac{1}{10}\left(\frac{1}{8}\left(\frac{5}{2}\left(\frac{1}{4}\int\frac{128b^3\cosh^4(c+dx)-256b^3\cosh^2(c+dx)+63a^3+128b^3+384ab^2+240a^2b}{(1-\cosh^2(c+dx))^2}d\cosh(c+dx)+\frac{a^2(21a+80b)\cosh(c+dx)}{4(1-\cosh^2(c+dx))^2}\right)+\right.\right.\right.}{d}$$

$\downarrow$ 1471

$$\frac{\frac{1}{10}\left(\frac{1}{8}\left(\frac{5}{2}\left(\frac{1}{4}\left(\frac{3a(21a^2+80ab+128b^2)\cosh(c+dx)}{2(1-\cosh^2(c+dx))}-\frac{1}{2}\int-\frac{63a^3+240ba^2+384b^2a+256b^3-256b^3\cosh^2(c+dx)}{1-\cosh^2(c+dx)}d\cosh(c+dx)\right)+\frac{a^2(2}{4(}\right.\right.\right.\right.}{d}$$

$\downarrow$ 25

$$\frac{\frac{1}{10}\left(\frac{1}{8}\left(\frac{5}{2}\left(\frac{1}{4}\left(\frac{1}{2}\int\frac{63a^3+240ba^2+384b^2a+256b^3-256b^3\cosh^2(c+dx)}{1-\cosh^2(c+dx)}d\cosh(c+dx)+\frac{3a(21a^2+80ab+128b^2)\cosh(c+dx)}{2(1-\cosh^2(c+dx))}\right)+\frac{a^2(21a}{4(1-}\right.\right.\right.\right.}{d}$$

$\downarrow$ 299

$$\frac{\frac{1}{10}\left(\frac{1}{8}\left(\frac{5}{2}\left(\frac{1}{4}\left(\frac{1}{2}\left(3a(21a^2+80ab+128b^2)\int\frac{1}{1-\cosh^2(c+dx)}d\cosh(c+dx)+256b^3\cosh(c+dx)\right)+\frac{3a(21a^2+80ab+128}{2(1-\cosh^2(}\right.\right.\right.\right.\right.}{d}$$

$\downarrow$ 219

$$\frac{\frac{a^3\cosh(c+dx)}{10(1-\cosh^2(c+dx))^5}+\frac{1}{10}\left(\frac{9a^3\cosh(c+dx)}{8(1-\cosh^2(c+dx))^4}+\frac{1}{8}\left(\frac{5}{2}\left(\frac{1}{4}\left(\frac{1}{2}\left(3a(21a^2+80ab+128b^2)\operatorname{arctanh}(\cosh(c+dx))+256b^3\right.\right.\right.\right.\right.\right.}{d}$$

input
```
Int[Csch[c + d*x]^11*(a + b*Sinh[c + d*x]^4)^3,x]
```

output
```
((a^3*Cosh[c + d*x])/(10*(1 - Cosh[c + d*x]^2)^5) + ((9*a^3*Cosh[c + d*x])
/(8*(1 - Cosh[c + d*x]^2)^4) + ((a^2*(21*a + 80*b)*Cosh[c + d*x])/(2*(1 -
Cosh[c + d*x]^2)^3) + (5*((a^2*(21*a + 80*b)*Cosh[c + d*x])/(4*(1 - Cosh[c
 + d*x]^2)^2) + ((3*a*(21*a^2 + 80*a*b + 128*b^2)*ArcTanh[Cosh[c + d*x]] +
 256*b^3*Cosh[c + d*x])/2 + (3*a*(21*a^2 + 80*a*b + 128*b^2)*Cosh[c + d*x]
)/(2*(1 - Cosh[c + d*x]^2)))/4))/2)/8)/10)/d
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 299
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2), x_Symbol] :> Simp[d*x
*((a + b*x^2)^(p + 1)/(b*(2*p + 3))), x] - Simp[(a*d - b*c*(2*p + 3))/(b*(2
*p + 3))   Int[(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
 a*d, 0] && NeQ[2*p + 3, 0]
```

rule 1471
```
Int[((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.),
x_Symbol] :> With[{Qx = PolynomialQuotient[(a + b*x^2 + c*x^4)^p, d + e*x^2
, x], R = Coeff[PolynomialRemainder[(a + b*x^2 + c*x^4)^p, d + e*x^2, x], x
, 0]}, Simp[(-R)*x*((d + e*x^2)^(q + 1)/(2*d*(q + 1))), x] + Simp[1/(2*d*(q
 + 1))   Int[(d + e*x^2)^(q + 1)*ExpandToSum[2*d*(q + 1)*Qx + R*(2*q + 3),
x], x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^
2 - b*d*e + a*e^2, 0] && IGtQ[p, 0] && LtQ[q, -1]
```

rule 2345
```
Int[(Pq_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{Q = PolynomialQuot
ient[Pq, a + b*x^2, x], f = Coeff[PolynomialRemainder[Pq, a + b*x^2, x], x,
 0], g = Coeff[PolynomialRemainder[Pq, a + b*x^2, x], x, 1]}, Simp[(a*g - b
*f*x)*((a + b*x^2)^(p + 1)/(2*a*b*(p + 1))), x] + Simp[1/(2*a*(p + 1))   In
t[(a + b*x^2)^(p + 1)*ExpandToSum[2*a*(p + 1)*Q + f*(2*p + 3), x], x], x]]
/; FreeQ[{a, b}, x] && PolyQ[Pq, x] && LtQ[p, -1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3694
```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^4)^
(p_.), x_Symbol] :> With[{ff = FreeFactors[Cos[e + f*x], x]}, Simp[-ff/f
Subst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a + b - 2*b*ff^2*x^2 + b*ff^4*x^4)^p,
 x], x, Cos[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m -
 1)/2]
```

Maple [A] (verified)

Time = 3.21 (sec) , antiderivative size = 166, normalized size of antiderivative = 0.88

method	result
derivativedivides	$\frac{a^3\left(\left(-\frac{\operatorname{csch}(dx+c)^9}{10}+\frac{9\operatorname{csch}(dx+c)^7}{80}-\frac{21\operatorname{csch}(dx+c)^5}{160}+\frac{21\operatorname{csch}(dx+c)^3}{128}-\frac{63\operatorname{csch}(dx+c)}{256}\right)\coth(dx+c)+\frac{63\operatorname{arctanh}\left(e^{dx+c}\right)}{128}\right)}{}$
default	$\frac{a^3\left(\left(-\frac{\operatorname{csch}(dx+c)^9}{10}+\frac{9\operatorname{csch}(dx+c)^7}{80}-\frac{21\operatorname{csch}(dx+c)^5}{160}+\frac{21\operatorname{csch}(dx+c)^3}{128}-\frac{63\operatorname{csch}(dx+c)}{256}\right)\coth(dx+c)+\frac{63\operatorname{arctanh}\left(e^{dx+c}\right)}{128}\right)}{}$
parallelrisch	$\frac{-\frac{63a\left(a^2+\frac{80}{21}ab+\frac{128}{21}b^2\right)\ln\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{256}-\frac{3297\operatorname{sech}\left(\frac{dx}{2}+\frac{c}{2}\right)^{10}a^3\left(\cosh(5dx+5c)-\frac{1125\cosh(6dx+6c)}{5024}-\frac{145\cosh(7dx+7c)}{628}+1\right.}{}}{}$
risch	$\frac{e^{dx+c}b^3}{2d}+\frac{e^{-dx-c}b^3}{2d}-\frac{e^{dx+c}a\left(315a^2e^{18dx+18c}+1920b^2e^{18dx+18c}-3045a^2e^{16dx+16c}-13440b^2e^{16dx+16c}+13188a\right.}{}$

input
```
int(csch(d*x+c)^11*(a+b*sinh(d*x+c)^4)^3,x,method=_RETURNVERBOSE)
```

output
```
1/d*(a^3*((-1/10*csch(d*x+c)^9+9/80*csch(d*x+c)^7-21/160*csch(d*x+c)^5+21/
128*csch(d*x+c)^3-63/256*csch(d*x+c))*coth(d*x+c)+63/128*arctanh(exp(d*x+c
)))+3*a^2*b*((-1/6*csch(d*x+c)^5+5/24*csch(d*x+c)^3-5/16*csch(d*x+c))*coth
(d*x+c)+5/8*arctanh(exp(d*x+c)))+3*b^2*a*(-1/2*csch(d*x+c)*coth(d*x+c)+arc
tanh(exp(d*x+c)))+b^3*cosh(d*x+c))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 13503 vs. $2(177) = 354$.

Time = 0.22 (sec) , antiderivative size = 13503, normalized size of antiderivative = 71.44

$$\int \operatorname{csch}^{11}(c+dx) \left(a+b\sinh^4(c+dx)\right)^3 \, dx = \text{Too large to display}$$

input
```
integrate(csch(d*x+c)^11*(a+b*sinh(d*x+c)^4)^3,x, algorithm="fricas")
```

output
```
Too large to include
```

Sympy [F(-1)]

Timed out.

$$\int \operatorname{csch}^{11}(c+dx) \left(a+b\sinh^4(c+dx)\right)^3 \, dx = \text{Timed out}$$

input
```
integrate(csch(d*x+c)**11*(a+b*sinh(d*x+c)**4)**3,x)
```

output
```
Timed out
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 573 vs. $2(177) = 354$.

Time = 0.05 (sec) , antiderivative size = 573, normalized size of antiderivative = 3.03

$$\int \operatorname{csch}^{11}(c+dx) \left(a+b\sinh^4(c+dx)\right)^3 \, dx = \text{Too large to display}$$

input
```
integrate(csch(d*x+c)^11*(a+b*sinh(d*x+c)^4)^3,x, algorithm="maxima")
```

output
```
1/2*b^3*(e^(d*x + c)/d + e^(-d*x - c)/d) + 1/1280*a^3*(315*log(e^(-d*x - c
) + 1)/d - 315*log(e^(-d*x - c) - 1)/d + 2*(315*e^(-d*x - c) - 3045*e^(-3*
d*x - 3*c) + 13188*e^(-5*d*x - 5*c) - 33660*e^(-7*d*x - 7*c) + 55970*e^(-9
*d*x - 9*c) + 55970*e^(-11*d*x - 11*c) - 33660*e^(-13*d*x - 13*c) + 13188*
e^(-15*d*x - 15*c) - 3045*e^(-17*d*x - 17*c) + 315*e^(-19*d*x - 19*c))/(d*
(10*e^(-2*d*x - 2*c) - 45*e^(-4*d*x - 4*c) + 120*e^(-6*d*x - 6*c) - 210*e^
(-8*d*x - 8*c) + 252*e^(-10*d*x - 10*c) - 210*e^(-12*d*x - 12*c) + 120*e^(
-14*d*x - 14*c) - 45*e^(-16*d*x - 16*c) + 10*e^(-18*d*x - 18*c) - e^(-20*d
*x - 20*c) - 1))) + 1/16*a^2*b*(15*log(e^(-d*x - c) + 1)/d - 15*log(e^(-d*
x - c) - 1)/d + 2*(15*e^(-d*x - c) - 85*e^(-3*d*x - 3*c) + 198*e^(-5*d*x -
 5*c) + 198*e^(-7*d*x - 7*c) - 85*e^(-9*d*x - 9*c) + 15*e^(-11*d*x - 11*c)
)/(d*(6*e^(-2*d*x - 2*c) - 15*e^(-4*d*x - 4*c) + 20*e^(-6*d*x - 6*c) - 15*
e^(-8*d*x - 8*c) + 6*e^(-10*d*x - 10*c) - e^(-12*d*x - 12*c) - 1))) + 3/2*
a*b^2*(log(e^(-d*x - c) + 1)/d - log(e^(-d*x - c) - 1)/d + 2*(e^(-d*x - c)
 + e^(-3*d*x - 3*c))/(d*(2*e^(-2*d*x - 2*c) - e^(-4*d*x - 4*c) - 1)))
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 477 vs. $2(177) = 354$.

Time = 0.33 (sec) , antiderivative size = 477, normalized size of antiderivative = 2.52

$$\int \operatorname{csch}^{11}(c+dx)\left(a+b\sinh^4(c+dx)\right)^3\,dx$$

$$= \frac{1280\,b^3\left(e^{(dx+c)}+e^{(-dx-c)}\right)+15\left(21\,a^3+80\,a^2b+128\,ab^2\right)\log\left(e^{(dx+c)}+e^{(-dx-c)}+2\right)-15\left(21\,a^3+80\,a\right.}{}$$

input
```
integrate(csch(d*x+c)^11*(a+b*sinh(d*x+c)^4)^3,x, algorithm="giac")
```

output
```
1/2560*(1280*b^3*(e^(d*x + c) + e^(-d*x - c)) + 15*(21*a^3 + 80*a^2*b + 12
8*a*b^2)*log(e^(d*x + c) + e^(-d*x - c) + 2) - 15*(21*a^3 + 80*a^2*b + 128
*a*b^2)*log(e^(d*x + c) + e^(-d*x - c) - 2) - 4*(315*a^3*(e^(d*x + c) + e^
(-d*x - c))^9 + 1200*a^2*b*(e^(d*x + c) + e^(-d*x - c))^9 + 1920*a*b^2*(e^
(d*x + c) + e^(-d*x - c))^9 - 5880*a^3*(e^(d*x + c) + e^(-d*x - c))^7 - 22
400*a^2*b*(e^(d*x + c) + e^(-d*x - c))^7 - 30720*a*b^2*(e^(d*x + c) + e^(-
d*x - c))^7 + 43008*a^3*(e^(d*x + c) + e^(-d*x - c))^5 + 163840*a^2*b*(e^(
d*x + c) + e^(-d*x - c))^5 + 184320*a*b^2*(e^(d*x + c) + e^(-d*x - c))^5 -
 151680*a^3*(e^(d*x + c) + e^(-d*x - c))^3 - 542720*a^2*b*(e^(d*x + c) + e
^(-d*x - c))^3 - 491520*a*b^2*(e^(d*x + c) + e^(-d*x - c))^3 + 247040*a^3*
(e^(d*x + c) + e^(-d*x - c)) + 675840*a^2*b*(e^(d*x + c) + e^(-d*x - c)) +
 491520*a*b^2*(e^(d*x + c) + e^(-d*x - c)))/((e^(d*x + c) + e^(-d*x - c))^
2 - 4)^5)/d
```

Mupad [B] (verification not implemented)

Time = 2.19 (sec) , antiderivative size = 1194, normalized size of antiderivative = 6.32

$$\int \operatorname{csch}^{11}(c+dx) \left(a+b\sinh^4(c+dx)\right)^3 \, dx = \text{Too large to display}$$

input
```
int((a + b*sinh(c + d*x)^4)^3/sinh(c + d*x)^11,x)
```

output
```
(b^3*exp(c + d*x))/(2*d) - ((24*exp(5*c + 5*d*x)*(7*a*b^2 + 4*a^2*b))/(5*d
) - (48*exp(7*c + 7*d*x)*(7*a*b^2 + 8*a^2*b))/(5*d) - (48*exp(11*c + 11*d*
x)*(7*a*b^2 + 8*a^2*b))/(5*d) + (24*exp(13*c + 13*d*x)*(7*a*b^2 + 4*a^2*b)
)/(5*d) + (4*exp(9*c + 9*d*x)*(105*a*b^2 + 144*a^2*b + 128*a^3))/(5*d) - (
48*a*b^2*exp(3*c + 3*d*x))/(5*d) - (48*a*b^2*exp(15*c + 15*d*x))/(5*d) + (
6*a*b^2*exp(17*c + 17*d*x))/(5*d) + (6*a*b^2*exp(c + d*x))/(5*d))/(45*exp(
4*c + 4*d*x) - 10*exp(2*c + 2*d*x) - 120*exp(6*c + 6*d*x) + 210*exp(8*c +
8*d*x) - 252*exp(10*c + 10*d*x) + 210*exp(12*c + 12*d*x) - 120*exp(14*c +
14*d*x) + 45*exp(16*c + 16*d*x) - 10*exp(18*c + 18*d*x) + exp(20*c + 20*d*
x) + 1) + (b^3*exp(- c - d*x))/(2*d) + (3*atan((exp(d*x)*exp(c)*(21*a^3*(-
d^2)^(1/2) + 128*a*b^2*(-d^2)^(1/2) + 80*a^2*b*(-d^2)^(1/2)))/(d*(3360*a^5
*b + 441*a^6 + 16384*a^2*b^4 + 20480*a^3*b^3 + 11776*a^4*b^2)^(1/2)))*(336
0*a^5*b + 441*a^6 + 16384*a^2*b^4 + 20480*a^3*b^3 + 11776*a^4*b^2)^(1/2))/
(128*(-d^2)^(1/2)) - (exp(c + d*x)*(208*a^2*b + a^3))/(5*d*(5*exp(2*c + 2*
d*x) - 10*exp(4*c + 4*d*x) + 10*exp(6*c + 6*d*x) - 5*exp(8*c + 8*d*x) + ex
p(10*c + 10*d*x) - 1)) - (exp(c + d*x)*(80*a^2*b + 21*a^3))/(80*d*(3*exp(2
*c + 2*d*x) - 3*exp(4*c + 4*d*x) + exp(6*c + 6*d*x) - 1)) - (3*exp(c + d*x
)*(464*a^2*b - 3*a^3))/(40*d*(6*exp(4*c + 4*d*x) - 4*exp(2*c + 2*d*x) - 4*
exp(6*c + 6*d*x) + exp(8*c + 8*d*x) + 1)) - (3*exp(c + d*x)*(128*a*b^2 + 8
0*a^2*b + 21*a^3))/(128*d*(exp(2*c + 2*d*x) - 1)) - (1032*a^3*exp(c + d...
```

Reduce [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 2453, normalized size of antiderivative = 12.98

$$\int \operatorname{csch}^{11}(c+dx)\left(a+b\sinh^4(c+dx)\right)^3 dx = \text{Too large to display}$$

input
```
int(csch(d*x+c)^11*(a+b*sinh(d*x+c)^4)^3,x)
```

output
```
(640*e**(22*c + 22*d*x)*b**3 - 315*e**(21*c + 21*d*x)*log(e**(c + d*x) - 1
)*a**3 - 1200*e**(21*c + 21*d*x)*log(e**(c + d*x) - 1)*a**2*b - 1920*e**(2
1*c + 21*d*x)*log(e**(c + d*x) - 1)*a*b**2 + 315*e**(21*c + 21*d*x)*log(e*
*(c + d*x) + 1)*a**3 + 1200*e**(21*c + 21*d*x)*log(e**(c + d*x) + 1)*a**2*
b + 1920*e**(21*c + 21*d*x)*log(e**(c + d*x) + 1)*a*b**2 - 630*e**(20*c +
20*d*x)*a**3 - 2400*e**(20*c + 20*d*x)*a**2*b - 3840*e**(20*c + 20*d*x)*a*
b**2 - 5760*e**(20*c + 20*d*x)*b**3 + 3150*e**(19*c + 19*d*x)*log(e**(c +
d*x) - 1)*a**3 + 12000*e**(19*c + 19*d*x)*log(e**(c + d*x) - 1)*a**2*b + 1
9200*e**(19*c + 19*d*x)*log(e**(c + d*x) - 1)*a*b**2 - 3150*e**(19*c + 19*
d*x)*log(e**(c + d*x) + 1)*a**3 - 12000*e**(19*c + 19*d*x)*log(e**(c + d*x
) + 1)*a**2*b - 19200*e**(19*c + 19*d*x)*log(e**(c + d*x) + 1)*a*b**2 + 60
90*e**(18*c + 18*d*x)*a**3 + 23200*e**(18*c + 18*d*x)*a**2*b + 26880*e**(1
8*c + 18*d*x)*a*b**2 + 22400*e**(18*c + 18*d*x)*b**3 - 14175*e**(17*c + 17
*d*x)*log(e**(c + d*x) - 1)*a**3 - 54000*e**(17*c + 17*d*x)*log(e**(c + d*
x) - 1)*a**2*b - 86400*e**(17*c + 17*d*x)*log(e**(c + d*x) - 1)*a*b**2 + 1
4175*e**(17*c + 17*d*x)*log(e**(c + d*x) + 1)*a**3 + 54000*e**(17*c + 17*d
*x)*log(e**(c + d*x) + 1)*a**2*b + 86400*e**(17*c + 17*d*x)*log(e**(c + d*
x) + 1)*a*b**2 - 26376*e**(16*c + 16*d*x)*a**3 - 100480*e**(16*c + 16*d*x)
*a**2*b - 76800*e**(16*c + 16*d*x)*a*b**2 - 48000*e**(16*c + 16*d*x)*b**3
+ 37800*e**(15*c + 15*d*x)*log(e**(c + d*x) - 1)*a**3 + 144000*e**(15*c...
```

3.191 $\int \operatorname{csch}^{13}(c+dx)\left(a+b\sinh^4(c+dx)\right)^3 dx$

Optimal result

Integrand size = 23, antiderivative size = 220

$$\begin{aligned}
&\int \operatorname{csch}^{13}(c+dx)\left(a+b\sinh^4(c+dx)\right)^3 dx \\
&= -\frac{(231a^3+840a^2b+1152ab^2+1024b^3)\operatorname{arctanh}(\cosh(c+dx))}{1024d} \\
&+ \frac{3a(77a^2+280ab+384b^2)\coth(c+dx)\operatorname{csch}(c+dx)}{1024d} \\
&- \frac{a(77a^2+280ab+384b^2)\coth(c+dx)\operatorname{csch}^3(c+dx)}{512d} \\
&+ \frac{7a^2(11a+40b)\coth(c+dx)\operatorname{csch}^5(c+dx)}{640d} \\
&- \frac{3a^2(11a+40b)\coth(c+dx)\operatorname{csch}^7(c+dx)}{320d} \\
&+ \frac{11a^3\coth(c+dx)\operatorname{csch}^9(c+dx)}{120d} - \frac{a^3\coth(c+dx)\operatorname{csch}^{11}(c+dx)}{12d}
\end{aligned}$$

output

```
-1/1024*(231*a^3+840*a^2*b+1152*a*b^2+1024*b^3)*arctanh(cosh(d*x+c))/d+3/1
024*a*(77*a^2+280*a*b+384*b^2)*coth(d*x+c)*csch(d*x+c)/d-1/512*a*(77*a^2+2
80*a*b+384*b^2)*coth(d*x+c)*csch(d*x+c)^3/d+7/640*a^2*(11*a+40*b)*coth(d*x
+c)*csch(d*x+c)^5/d-3/320*a^2*(11*a+40*b)*coth(d*x+c)*csch(d*x+c)^7/d+11/1
20*a^3*coth(d*x+c)*csch(d*x+c)^9/d-1/12*a^3*coth(d*x+c)*csch(d*x+c)^11/d
```

Mathematica [A] (verified)

Time = 2.07 (sec) , antiderivative size = 260, normalized size of antiderivative = 1.18

$$\int \operatorname{csch}^{13}(c+dx)\left(a+b\sinh^4(c+dx)\right)^3\,dx$$
$$=\frac{-30a(76555a^2+75816ab+45696b^2)\coth(c+dx)\operatorname{csch}^{11}(c+dx)+2a(750629a^2+2074200ab+142272\ldots}{}$$

input
```
Integrate[Csch[c + d*x]^13*(a + b*Sinh[c + d*x]^4)^3,x]
```

output
```
(-30*a*(76555*a^2 + 75816*a*b + 45696*b^2)*Coth[c + d*x]*Csch[c + d*x]^11
+ 2*a*(750629*a^2 + 2074200*a*b + 1422720*b^2)*Cosh[3*(c + d*x)]*Csch[c +
d*x]^12 - 9*a*(77099*a^2 + 280360*a*b + 246400*b^2)*Cosh[5*(c + d*x)]*Csch
[c + d*x]^12 + 63*a*(3421*a^2 + 12440*a*b + 14720*b^2)*Cosh[7*(c + d*x)]*C
sch[c + d*x]^12 - 525*a*(77*a^2 + 280*a*b + 384*b^2)*Cosh[9*(c + d*x)]*Csc
h[c + d*x]^12 + 45*a*(77*a^2 + 280*a*b + 384*b^2)*Cosh[11*(c + d*x)]*Csch[
c + d*x]^12 - 15360*(231*a^3 + 840*a^2*b + 1152*a*b^2 + 1024*b^3)*(Log[Cos
h[(c + d*x)/2]] - Log[Sinh[(c + d*x)/2]]))/(15728640*d)
```

Rubi [A] (verified)

Time = 0.90 (sec) , antiderivative size = 267, normalized size of antiderivative = 1.21, number of steps used = 16, number of rules used = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.652, Rules used = {3042, 26, 3694, 1471, 25, 2345, 27, 2345, 25, 2345, 27, 1471, 25, 298, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \operatorname{csch}^{13}(c+dx)\left(a+b\sinh^4(c+dx)\right)^3\,dx$$

$$\Big\downarrow \; 3042$$

$$\int \frac{i\left(a+b\sin(ic+idx)^4\right)^3}{\sin(ic+idx)^{13}}dx$$

$$\Big\downarrow \; 26$$

$$i\int \frac{\left(b\sin(ic+idx)^4+a\right)^3}{\sin(ic+idx)^{13}}dx$$

$\Big\downarrow$ 3694

$$-\frac{\int \frac{\left(b\cosh^4(c+dx)-2b\cosh^2(c+dx)+a+b\right)^3}{\left(1-\cosh^2(c+dx)\right)^7}d\cosh(c+dx)}{d}$$

$\Big\downarrow$ 1471

$$-\frac{\frac{a^3\cosh(c+dx)}{12\left(1-\cosh^2(c+dx)\right)^6}-\frac{1}{12}\int -\frac{-12b^3\cosh^{10}(c+dx)+60b^3\cosh^8(c+dx)-12b^2(3a+10b)\cosh^6(c+dx)+12b^2(9a+10b)\cosh^4(c+dx)-12b(3a}{\left(1-\cosh^2(c+dx)\right)^6}}{d}$$

$\Big\downarrow$ 25

$$-\frac{\frac{1}{12}\int \frac{-12b^3\cosh^{10}(c+dx)+60b^3\cosh^8(c+dx)-12b^2(3a+10b)\cosh^6(c+dx)+12b^2(9a+10b)\cosh^4(c+dx)-12b\left(3a^2+9ba+5b^2\right)\cosh^2(c+dx)+1}{\left(1-\cosh^2(c+dx)\right)^6}}{d}$$

$\Big\downarrow$ 2345

$$-\frac{\frac{1}{12}\left(\frac{11a^3\cosh(c+dx)}{10\left(1-\cosh^2(c+dx)\right)^5}-\frac{1}{10}\int -\frac{3\left(40b^3\cosh^8(c+dx)-160b^3\cosh^6(c+dx)+120b^2(a+2b)\cosh^4(c+dx)-80b^2(3a+2b)\cosh^2(c+dx)+33a\right.}{\left(1-\cosh^2(c+dx)\right)^5}\right.}{d}$$

$\Big\downarrow$ 27

$$-\frac{\frac{1}{12}\left(\frac{3}{10}\int \frac{40b^3\cosh^8(c+dx)-160b^3\cosh^6(c+dx)+120b^2(a+2b)\cosh^4(c+dx)-80b^2(3a+2b)\cosh^2(c+dx)+33a^3+40b^3+120ab^2+120a^2b}{\left(1-\cosh^2(c+dx)\right)^5}d\cos\right.}{d}$$

$\Big\downarrow$ 2345

$$-\frac{\frac{1}{12}\left(\frac{3}{10}\left(\frac{3a^2(11a+40b)\cosh(c+dx)}{8\left(1-\cosh^2(c+dx)\right)^4}-\frac{1}{8}\int -\frac{-320b^3\cosh^6(c+dx)+960b^3\cosh^4(c+dx)-960b^2(a+b)\cosh^2(c+dx)+231a^3+320b^3+960ab^2-}{\left(1-\cosh^2(c+dx)\right)^4}\right.\right.}{d}$$

$\Big\downarrow$ 25

$$-\frac{\frac{1}{12}\left(\frac{3}{10}\left(\frac{1}{8}\int \frac{-320b^3\cosh^6(c+dx)+960b^3\cosh^4(c+dx)-960b^2(a+b)\cosh^2(c+dx)+231a^3+320b^3+960ab^2+840a^2b}{\left(1-\cosh^2(c+dx)\right)^4}d\cosh(c+dx)+\frac{3a}{}\right.\right.}{d}$$

$\Big\downarrow$ 2345

$$-\frac{\frac{1}{12}\left(\frac{3}{10}\left(\frac{1}{8}\left(\frac{7a^2(11a+40b)\cosh(c+dx)}{2(1-\cosh^2(c+dx))^3}-\frac{1}{6}\int -\frac{15(128b^3\cosh^4(c+dx)-256b^3\cosh^2(c+dx)+77a^3+128b^3+384ab^2+280a^2b)}{(1-\cosh^2(c+dx))^3}d\cosh(c+\right.\right.\right.}{d}$$

$$\Big\downarrow 27$$

$$-\frac{\frac{1}{12}\left(\frac{3}{10}\left(\frac{1}{8}\left(\frac{5}{2}\int \frac{128b^3\cosh^4(c+dx)-256b^3\cosh^2(c+dx)+77a^3+128b^3+384ab^2+280a^2b}{(1-\cosh^2(c+dx))^3}d\cosh(c+dx)+\frac{7a^2(11a+40b)\cosh(c+dx)}{2(1-\cosh^2(c+dx))^3}\right)\right.\right.}{d}$$

$$\Big\downarrow 1471$$

$$-\frac{\frac{1}{12}\left(\frac{3}{10}\left(\frac{1}{8}\left(\frac{5}{2}\left(\frac{a(77a^2+280ab+384b^2)\cosh(c+dx)}{4(1-\cosh^2(c+dx))^2}-\frac{1}{4}\int -\frac{231a^3+840ba^2+1152b^2a+512b^3-512b^3\cosh^2(c+dx)}{(1-\cosh^2(c+dx))^2}d\cosh(c+dx)\right)-\right.\right.\right.}{d}$$

$$\Big\downarrow 25$$

$$-\frac{\frac{1}{12}\left(\frac{3}{10}\left(\frac{1}{8}\left(\frac{5}{2}\left(\frac{1}{4}\int \frac{231a^3+840ba^2+1152b^2a+512b^3-512b^3\cosh^2(c+dx)}{(1-\cosh^2(c+dx))^2}d\cosh(c+dx)+\frac{a(77a^2+280ab+384b^2)\cosh(c+dx)}{4(1-\cosh^2(c+dx))^2}\right)+\right.\right.\right.}{d}$$

$$\Big\downarrow 298$$

$$-\frac{\frac{1}{12}\left(\frac{3}{10}\left(\frac{1}{8}\left(\frac{5}{2}\left(\frac{1}{4}\left(\frac{1}{2}(231a^3+840a^2b+1152ab^2+1024b^3)\int \frac{1}{1-\cosh^2(c+dx)}d\cosh(c+dx)+\frac{3a(77a^2+280ab+384b^2)\,\mathrm{c}}{2(1-\cosh^2(c+dx}\right.\right.\right.\right.\right.}{}$$

$$\Big\downarrow 219$$

$$-\frac{\frac{a^3\cosh(c+dx)}{12(1-\cosh^2(c+dx))^6}+\frac{1}{12}\left(\frac{11a^3\cosh(c+dx)}{10(1-\cosh^2(c+dx))^5}+\frac{3}{10}\left(\frac{3a^2(11a+40b)\cosh(c+dx)}{8(1-\cosh^2(c+dx))^4}+\frac{1}{8}\left(\frac{7a^2(11a+40b)\cosh(c+dx)}{2(1-\cosh^2(c+dx))^3}+\frac{5}{2}\left(\frac{a(77a^2+2}{4(}\right.\right.\right.\right.}{}$$

input
```
Int[Csch[c + d*x]^13*(a + b*Sinh[c + d*x]^4)^3,x]
```

output
```
-(((a^3*Cosh[c + d*x])/(12*(1 - Cosh[c + d*x]^2)^6) + ((11*a^3*Cosh[c + d*
x])/(10*(1 - Cosh[c + d*x]^2)^5) + (3*((3*a^2*(11*a + 40*b)*Cosh[c + d*x])
/(8*(1 - Cosh[c + d*x]^2)^4) + ((7*a^2*(11*a + 40*b)*Cosh[c + d*x])/(2*(1
- Cosh[c + d*x]^2)^3) + (5*((a*(77*a^2 + 280*a*b + 384*b^2)*Cosh[c + d*x])
/(4*(1 - Cosh[c + d*x]^2)^2) + (((231*a^3 + 840*a^2*b + 1152*a*b^2 + 1024*
b^3)*ArcTanh[Cosh[c + d*x]])/2 + (3*a*(77*a^2 + 280*a*b + 384*b^2)*Cosh[c
+ d*x])/(2*(1 - Cosh[c + d*x]^2)))/4))/2)/8))/10)/12)/d)
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 298
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2), x_Symbol] :> Simp[(-(
b*c - a*d))*x*((a + b*x^2)^(p + 1)/(2*a*b*(p + 1))), x] - Simp[(a*d - b*c*(
2*p + 3))/(2*a*b*(p + 1))   Int[(a + b*x^2)^(p + 1), x], x] /; FreeQ[{a, b,
 c, d, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/2 + p, 0])
```

rule 1471
```
Int[((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.),
x_Symbol] :> With[{Qx = PolynomialQuotient[(a + b*x^2 + c*x^4)^p, d + e*x^2
, x], R = Coeff[PolynomialRemainder[(a + b*x^2 + c*x^4)^p, d + e*x^2, x], x
, 0]}, Simp[(-R)*x*((d + e*x^2)^(q + 1)/(2*d*(q + 1))), x] + Simp[1/(2*d*(q
 + 1))   Int[(d + e*x^2)^(q + 1)*ExpandToSum[2*d*(q + 1)*Qx + R*(2*q + 3),
x], x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^
2 - b*d*e + a*e^2, 0] && IGtQ[p, 0] && LtQ[q, -1]
```

rule 2345
```
Int[(Pq_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{Q = PolynomialQuot
ient[Pq, a + b*x^2, x], f = Coeff[PolynomialRemainder[Pq, a + b*x^2, x], x,
 0], g = Coeff[PolynomialRemainder[Pq, a + b*x^2, x], x, 1]}, Simp[(a*g - b
*f*x)*((a + b*x^2)^(p + 1)/(2*a*b*(p + 1))), x] + Simp[1/(2*a*(p + 1))   In
t[(a + b*x^2)^(p + 1)*ExpandToSum[2*a*(p + 1)*Q + f*(2*p + 3), x], x], x]]
/; FreeQ[{a, b}, x] && PolyQ[Pq, x] && LtQ[p, -1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3694
```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^4)^
(p_.), x_Symbol] :> With[{ff = FreeFactors[Cos[e + f*x], x]}, Simp[-ff/f
Subst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a + b - 2*b*ff^2*x^2 + b*ff^4*x^4)^p,
 x], x, Cos[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m -
 1)/2]
```

Maple [A] (verified)

Time = 3.15 (sec) , antiderivative size = 202, normalized size of antiderivative = 0.92

method	result
derivativedivides	$\frac{a^3\left(\left(-\frac{\operatorname{csch}(dx+c)^{11}}{12}+\frac{11\operatorname{csch}(dx+c)^9}{120}-\frac{33\operatorname{csch}(dx+c)^7}{320}+\frac{77\operatorname{csch}(dx+c)^5}{640}-\frac{77\operatorname{csch}(dx+c)^3}{512}+\frac{231\operatorname{csch}(dx+c)}{1024}\right)\coth(dx+c)-\right.}{}$
default	$\frac{a^3\left(\left(-\frac{\operatorname{csch}(dx+c)^{11}}{12}+\frac{11\operatorname{csch}(dx+c)^9}{120}-\frac{33\operatorname{csch}(dx+c)^7}{320}+\frac{77\operatorname{csch}(dx+c)^5}{640}-\frac{77\operatorname{csch}(dx+c)^3}{512}+\frac{231\operatorname{csch}(dx+c)}{1024}\right)\coth(dx+c)-\right.}{}$
parallelrisch	$\frac{(14533263360a^3+52848230400a^2b+72477573120b^2a+64424509440b^3)\ln\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-40425\left(a^2\operatorname{sech}\left(\frac{dx}{2}+\frac{c}{2}\right)^8\right.}{}$
risch	$\frac{e^{dx+c}a(215523a^2e^{18dx+18c}+927360b^2e^{18dx+18c}-693891a^2e^{16dx+16c}-2217600b^2e^{16dx+16c}+1501258a^2e^{14dx+14c}+2}{}$

input
```
int(csch(d*x+c)^13*(a+b*sinh(d*x+c)^4)^3,x,method=_RETURNVERBOSE)
```

output
```
1/d*(a^3*((-1/12*csch(d*x+c)^11+11/120*csch(d*x+c)^9-33/320*csch(d*x+c)^7+
77/640*csch(d*x+c)^5-77/512*csch(d*x+c)^3+231/1024*csch(d*x+c))*coth(d*x+c
)-231/512*arctanh(exp(d*x+c)))+3*a^2*b*((-1/8*csch(d*x+c)^7+7/48*csch(d*x+
c)^5-35/192*csch(d*x+c)^3+35/128*csch(d*x+c))*coth(d*x+c)-35/64*arctanh(ex
p(d*x+c)))+3*b^2*a*((-1/4*csch(d*x+c)^3+3/8*csch(d*x+c))*coth(d*x+c)-3/4*a
rctanh(exp(d*x+c)))-2*b^3*arctanh(exp(d*x+c)))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 17811 vs. $2(206) = 412$.

Time = 0.29 (sec) , antiderivative size = 17811, normalized size of antiderivative = 80.96

$$\int \operatorname{csch}^{13}(c+dx)\left(a+b\sinh^4(c+dx)\right)^3\,dx = \text{Too large to display}$$

input
```
integrate(csch(d*x+c)^13*(a+b*sinh(d*x+c)^4)^3,x, algorithm="fricas")
```

output
```
Too large to include
```

Sympy [F(-1)]

Timed out.

$$\int \operatorname{csch}^{13}(c+dx) \left(a+b\sinh^4(c+dx)\right)^3 \, dx = \text{Timed out}$$

input
```
integrate(csch(d*x+c)**13*(a+b*sinh(d*x+c)**4)**3,x)
```

output
```
Timed out
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 720 vs. 2(206) = 412.

Time = 0.06 (sec) , antiderivative size = 720, normalized size of antiderivative = 3.27

$$\int \operatorname{csch}^{13}(c+dx) \left(a+b\sinh^4(c+dx)\right)^3 \, dx = \text{Too large to display}$$

input
```
integrate(csch(d*x+c)^13*(a+b*sinh(d*x+c)^4)^3,x, algorithm="maxima")
```

output
```
-1/15360*a^3*(3465*log(e^(-d*x - c) + 1)/d - 3465*log(e^(-d*x - c) - 1)/d
+ 2*(3465*e^(-d*x - c) - 40425*e^(-3*d*x - 3*c) + 215523*e^(-5*d*x - 5*c)
- 693891*e^(-7*d*x - 7*c) + 1501258*e^(-9*d*x - 9*c) - 2296650*e^(-11*d*x
- 11*c) - 2296650*e^(-13*d*x - 13*c) + 1501258*e^(-15*d*x - 15*c) - 693891
*e^(-17*d*x - 17*c) + 215523*e^(-19*d*x - 19*c) - 40425*e^(-21*d*x - 21*c)
 + 3465*e^(-23*d*x - 23*c))/(d*(12*e^(-2*d*x - 2*c) - 66*e^(-4*d*x - 4*c)
+ 220*e^(-6*d*x - 6*c) - 495*e^(-8*d*x - 8*c) + 792*e^(-10*d*x - 10*c) - 9
24*e^(-12*d*x - 12*c) + 792*e^(-14*d*x - 14*c) - 495*e^(-16*d*x - 16*c) +
220*e^(-18*d*x - 18*c) - 66*e^(-20*d*x - 20*c) + 12*e^(-22*d*x - 22*c) - e
^(-24*d*x - 24*c) - 1))) - 1/128*a^2*b*(105*log(e^(-d*x - c) + 1)/d - 105*
log(e^(-d*x - c) - 1)/d + 2*(105*e^(-d*x - c) - 805*e^(-3*d*x - 3*c) + 268
1*e^(-5*d*x - 5*c) - 5053*e^(-7*d*x - 7*c) - 5053*e^(-9*d*x - 9*c) + 2681*
e^(-11*d*x - 11*c) - 805*e^(-13*d*x - 13*c) + 105*e^(-15*d*x - 15*c))/(d*(
8*e^(-2*d*x - 2*c) - 28*e^(-4*d*x - 4*c) + 56*e^(-6*d*x - 6*c) - 70*e^(-8*
d*x - 8*c) + 56*e^(-10*d*x - 10*c) - 28*e^(-12*d*x - 12*c) + 8*e^(-14*d*x
- 14*c) - e^(-16*d*x - 16*c) - 1))) - 3/8*a*b^2*(3*log(e^(-d*x - c) + 1)/d
 - 3*log(e^(-d*x - c) - 1)/d + 2*(3*e^(-d*x - c) - 11*e^(-3*d*x - 3*c) - 1
1*e^(-5*d*x - 5*c) + 3*e^(-7*d*x - 7*c))/(d*(4*e^(-2*d*x - 2*c) - 6*e^(-4*
d*x - 4*c) + 4*e^(-6*d*x - 6*c) - e^(-8*d*x - 8*c) - 1))) - b^3*(log(e^(-d
*x - c) + 1)/d - log(e^(-d*x - c) - 1)/d)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 537 vs. $2(206) = 412$.

Time = 0.35 (sec) , antiderivative size = 537, normalized size of antiderivative = 2.44

$$\int \operatorname{csch}^{13}(c+dx)\left(a+b\sinh^4(c+dx)\right)^3\,dx = \text{Too large to display}$$

input
```
integrate(csch(d*x+c)^13*(a+b*sinh(d*x+c)^4)^3,x, algorithm="giac")
```

output
```
-1/30720*(15*(231*a^3 + 840*a^2*b + 1152*a*b^2 + 1024*b^3)*log(e^(d*x + c)
 + e^(-d*x - c) + 2) - 15*(231*a^3 + 840*a^2*b + 1152*a*b^2 + 1024*b^3)*lo
g(e^(d*x + c) + e^(-d*x - c) - 2) - 4*(3465*a^3*(e^(d*x + c) + e^(-d*x - c
))^11 + 12600*a^2*b*(e^(d*x + c) + e^(-d*x - c))^11 + 17280*a*b^2*(e^(d*x
+ c) + e^(-d*x - c))^11 - 78540*a^3*(e^(d*x + c) + e^(-d*x - c))^9 - 28560
0*a^2*b*(e^(d*x + c) + e^(-d*x - c))^9 - 391680*a*b^2*(e^(d*x + c) + e^(-d
*x - c))^9 + 731808*a^3*(e^(d*x + c) + e^(-d*x - c))^7 + 2661120*a^2*b*(e^
(d*x + c) + e^(-d*x - c))^7 + 3502080*a*b^2*(e^(d*x + c) + e^(-d*x - c))^7
 - 3560832*a^3*(e^(d*x + c) + e^(-d*x - c))^5 - 12948480*a^2*b*(e^(d*x + c
) + e^(-d*x - c))^5 - 15482880*a*b^2*(e^(d*x + c) + e^(-d*x - c))^5 + 9391
360*a^3*(e^(d*x + c) + e^(-d*x - c))^3 + 32839680*a^2*b*(e^(d*x + c) + e^(
-d*x - c))^3 + 33914880*a*b^2*(e^(d*x + c) + e^(-d*x - c))^3 - 12180480*a^
3*(e^(d*x + c) + e^(-d*x - c)) - 34283520*a^2*b*(e^(d*x + c) + e^(-d*x - c
)) - 29491200*a*b^2*(e^(d*x + c) + e^(-d*x - c)))/((e^(d*x + c) + e^(-d*x
- c))^2 - 4)^6)/d
```

Mupad [B] (verification not implemented)

Time = 2.01 (sec) , antiderivative size = 1314, normalized size of antiderivative = 5.97

$$\int \operatorname{csch}^{13}(c+dx) \left(a+b\sinh^4(c+dx)\right)^3 dx = \text{Too large to display}$$

input
```
int((a + b*sinh(c + d*x)^4)^3/sinh(c + d*x)^13,x)
```

output
```
(3*exp(c + d*x)*(384*a*b^2 + 280*a^2*b + 77*a^3))/(512*d*(exp(2*c + 2*d*x)
 - 1)) - (5632*a^3*exp(c + d*x))/(3*d*(11*exp(2*c + 2*d*x) - 55*exp(4*c +
4*d*x) + 165*exp(6*c + 6*d*x) - 330*exp(8*c + 8*d*x) + 462*exp(10*c + 10*d
*x) - 462*exp(12*c + 12*d*x) + 330*exp(14*c + 14*d*x) - 165*exp(16*c + 16*
d*x) + 55*exp(18*c + 18*d*x) - 11*exp(20*c + 20*d*x) + exp(22*c + 22*d*x)
- 1)) - (1024*a^3*exp(c + d*x))/(3*d*(66*exp(4*c + 4*d*x) - 12*exp(2*c + 2
*d*x) - 220*exp(6*c + 6*d*x) + 495*exp(8*c + 8*d*x) - 792*exp(10*c + 10*d*
x) + 924*exp(12*c + 12*d*x) - 792*exp(14*c + 14*d*x) + 495*exp(16*c + 16*d
*x) - 220*exp(18*c + 18*d*x) + 66*exp(20*c + 20*d*x) - 12*exp(22*c + 22*d*
x) + exp(24*c + 24*d*x) + 1)) - (exp(c + d*x)*(2424*a^2*b + a^3))/(6*d*(15
*exp(4*c + 4*d*x) - 6*exp(2*c + 2*d*x) - 20*exp(6*c + 6*d*x) + 15*exp(8*c
+ 8*d*x) - 6*exp(10*c + 10*d*x) + exp(12*c + 12*d*x) + 1)) - (atan((exp(d*
x)*exp(c)*(231*a^3*(-d^2)^(1/2) + 1024*b^3*(-d^2)^(1/2) + 1152*a*b^2*(-d^2
)^(1/2) + 840*a^2*b*(-d^2)^(1/2)))/(d*(2359296*a*b^5 + 388080*a^5*b + 5336
1*a^6 + 1048576*b^6 + 3047424*a^2*b^4 + 2408448*a^3*b^3 + 1237824*a^4*b^2)
^(1/2)))*(2359296*a*b^5 + 388080*a^5*b + 53361*a^6 + 1048576*b^6 + 3047424
*a^2*b^4 + 2408448*a^3*b^3 + 1237824*a^4*b^2)^(1/2))/(512*(-d^2)^(1/2)) -
(exp(c + d*x)*(10200*a^2*b - 11*a^3))/(60*d*(5*exp(2*c + 2*d*x) - 10*exp(4
*c + 4*d*x) + 10*exp(6*c + 6*d*x) - 5*exp(8*c + 8*d*x) + exp(10*c + 10*d*x
) - 1)) - (exp(c + d*x)*(384*a*b^2 + 280*a^2*b + 77*a^3))/(256*d*(exp(4...
```

Reduce [B] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 3279, normalized size of antiderivative = 14.90

$$\int \operatorname{csch}^{13}(c+dx)\left(a+b\sinh^4(c+dx)\right)^3\,dx = \text{Too large to display}$$

input
```
int(csch(d*x+c)^13*(a+b*sinh(d*x+c)^4)^3,x)
```

output

```
(3465*e**(24*c + 24*d*x)*log(e**(c + d*x) - 1)*a**3 + 12600*e**(24*c + 24*
d*x)*log(e**(c + d*x) - 1)*a**2*b + 17280*e**(24*c + 24*d*x)*log(e**(c + d
*x) - 1)*a*b**2 + 15360*e**(24*c + 24*d*x)*log(e**(c + d*x) - 1)*b**3 - 34
65*e**(24*c + 24*d*x)*log(e**(c + d*x) + 1)*a**3 - 12600*e**(24*c + 24*d*x
)*log(e**(c + d*x) + 1)*a**2*b - 17280*e**(24*c + 24*d*x)*log(e**(c + d*x)
 + 1)*a*b**2 - 15360*e**(24*c + 24*d*x)*log(e**(c + d*x) + 1)*b**3 + 6930*
e**(23*c + 23*d*x)*a**3 + 25200*e**(23*c + 23*d*x)*a**2*b + 34560*e**(23*c
 + 23*d*x)*a*b**2 - 41580*e**(22*c + 22*d*x)*log(e**(c + d*x) - 1)*a**3 -
151200*e**(22*c + 22*d*x)*log(e**(c + d*x) - 1)*a**2*b - 207360*e**(22*c +
 22*d*x)*log(e**(c + d*x) - 1)*a*b**2 - 184320*e**(22*c + 22*d*x)*log(e**(
c + d*x) - 1)*b**3 + 41580*e**(22*c + 22*d*x)*log(e**(c + d*x) + 1)*a**3 +
 151200*e**(22*c + 22*d*x)*log(e**(c + d*x) + 1)*a**2*b + 207360*e**(22*c
+ 22*d*x)*log(e**(c + d*x) + 1)*a*b**2 + 184320*e**(22*c + 22*d*x)*log(e**
(c + d*x) + 1)*b**3 - 80850*e**(21*c + 21*d*x)*a**3 - 294000*e**(21*c + 21
*d*x)*a**2*b - 403200*e**(21*c + 21*d*x)*a*b**2 + 228690*e**(20*c + 20*d*x
)*log(e**(c + d*x) - 1)*a**3 + 831600*e**(20*c + 20*d*x)*log(e**(c + d*x)
- 1)*a**2*b + 1140480*e**(20*c + 20*d*x)*log(e**(c + d*x) - 1)*a*b**2 + 10
13760*e**(20*c + 20*d*x)*log(e**(c + d*x) - 1)*b**3 - 228690*e**(20*c + 20
*d*x)*log(e**(c + d*x) + 1)*a**3 - 831600*e**(20*c + 20*d*x)*log(e**(c + d
*x) + 1)*a**2*b - 1140480*e**(20*c + 20*d*x)*log(e**(c + d*x) + 1)*a*b*...
```

3.192 $\int \sinh^2(c+dx) \left(a+b\sinh^4(c+dx)\right)^3 dx$

Optimal result

Integrand size = 23, antiderivative size = 255

$$\begin{aligned}
&\int \sinh^2(c+dx) \left(a+b\sinh^4(c+dx)\right)^3 dx \\
&= -\frac{(1024a^3+1920a^2b+1512ab^2+429b^3)\,x}{2048} \\
&\quad + \frac{(1024a^3+4224a^2b+4632ab^2+1619b^3)\cosh(c+dx)\sinh(c+dx)}{2048d} \\
&\quad - \frac{b(4992a^2+10728ab+5549b^2)\cosh^3(c+dx)\sinh(c+dx)}{3072d} \\
&\quad + \frac{b(1920a^2+12312ab+10579b^2)\cosh^5(c+dx)\sinh(c+dx)}{3840d} \\
&\quad - \frac{b^2(6888a+11821b)\cosh^7(c+dx)\sinh(c+dx)}{4480d} \\
&\quad + \frac{b^2(504a+2593b)\cosh^9(c+dx)\sinh(c+dx)}{1680d} \\
&\quad - \frac{85b^3\cosh^{11}(c+dx)\sinh(c+dx)}{168d} + \frac{b^3\cosh^{13}(c+dx)\sinh(c+dx)}{14d}
\end{aligned}$$

output
```
-1/2048*(1024*a^3+1920*a^2*b+1512*a*b^2+429*b^3)*x+1/2048*(1024*a^3+4224*a
^2*b+4632*a*b^2+1619*b^3)*cosh(d*x+c)*sinh(d*x+c)/d-1/3072*b*(4992*a^2+107
28*a*b+5549*b^2)*cosh(d*x+c)^3*sinh(d*x+c)/d+1/3840*b*(1920*a^2+12312*a*b+
10579*b^2)*cosh(d*x+c)^5*sinh(d*x+c)/d-1/4480*b^2*(6888*a+11821*b)*cosh(d*
x+c)^7*sinh(d*x+c)/d+1/1680*b^2*(504*a+2593*b)*cosh(d*x+c)^9*sinh(d*x+c)/d
-85/168*b^3*cosh(d*x+c)^11*sinh(d*x+c)/d+1/14*b^3*cosh(d*x+c)^13*sinh(d*x+
c)/d
```

Mathematica [A] (verified)

Time = 1.89 (sec) , antiderivative size = 189, normalized size of antiderivative = 0.74

$$\int \sinh^2(c+dx)\left(a+b\sinh^4(c+dx)\right)^3\,dx$$
$$=\frac{-840(1024a^3+1920a^2b+1512ab^2+429b^3)\,(c+dx)+105(4096a^3+11520a^2b+10080ab^2+3003b^3)\,\mathrm{si}}{}$$

input
```
Integrate[Sinh[c + d*x]^2*(a + b*Sinh[c + d*x]^4)^3,x]
```

output
```
(-840*(1024*a^3 + 1920*a^2*b + 1512*a*b^2 + 429*b^3)*(c + d*x) + 105*(4096
*a^3 + 11520*a^2*b + 10080*a*b^2 + 3003*b^3)*Sinh[2*(c + d*x)] - 105*b*(23
04*a^2 + 2880*a*b + 1001*b^2)*Sinh[4*(c + d*x)] + 35*b*(768*a^2 + 2160*a*b
 + 1001*b^2)*Sinh[6*(c + d*x)] - 105*b^2*(120*a + 91*b)*Sinh[8*(c + d*x)]
+ 21*b^2*(48*a + 91*b)*Sinh[10*(c + d*x)] - 245*b^3*Sinh[12*(c + d*x)] + 1
5*b^3*Sinh[14*(c + d*x)])/(1720320*d)
```

Rubi [A] (verified)

Time = 1.33 (sec) , antiderivative size = 318, normalized size of antiderivative = 1.25, number of steps used = 17, number of rules used = 16, $\frac{\text{number of rules}}{\text{integrand size}}=0.696$, Rules used = {3042, 25, 3696, 1580, 25, 2345, 2345, 27, 2345, 27, 2345, 27, 1471, 27, 298, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sinh^2(c+dx)\left(a+b\sinh^4(c+dx)\right)^3\,dx$$

$\big\downarrow$ 3042

$$\int -\sin(ic+idx)^2\left(a+b\sin(ic+idx)^4\right)^3\,dx$$

$\big\downarrow$ 25

$$-\int \sin(ic+idx)^2\left(b\sin(ic+idx)^4+a\right)^3\,dx$$

$\big\downarrow$ 3696

$$\frac{\int \frac{\tanh^2(c+dx)\left((a+b)\tanh^4(c+dx)-2a\tanh^2(c+dx)+a\right)^3}{\left(1-\tanh^2(c+dx)\right)^8}d\tanh(c+dx)}{d}$$

$\big\downarrow$ 1580

$$\frac{\frac{1}{14}\int -\frac{14(a+b)^3\tanh^{12}(c+dx)-14(5a-b)(a+b)^2\tanh^{10}(c+dx)+14\left(10a^3+9ba^2+b^3\right)\tanh^8(c+dx)-14\left(10a^3+3ba^2-b^3\right)\tanh^6(c+dx)+14\left(5a^3\right.}{\left(1-\tanh^2(c+dx)\right)^7}}{d}$$

$\big\downarrow$ 25

$$\frac{\frac{b^3\tanh(c+dx)}{14\left(1-\tanh^2(c+dx)\right)^7}-\frac{1}{14}\int \frac{14(a+b)^3\tanh^{12}(c+dx)-14(5a-b)(a+b)^2\tanh^{10}(c+dx)+14\left(10a^3+9ba^2+b^3\right)\tanh^8(c+dx)-14\left(10a^3+3ba^2-b^3\right.}{\left(1-\tanh^2(c+dx)\right)^7}}{d}$$

$\big\downarrow$ 2345

$$\frac{\frac{1}{14}\left(\frac{1}{12}\int \frac{168(a+b)^3\tanh^{10}(c+dx)-336(2a-b)(a+b)^2\tanh^8(c+dx)+504\left(2a^3+ba^2+b^3\right)\tanh^6(c+dx)-672\left(a^3-b^3\right)\tanh^4(c+dx)+168\left(a^3+5b^3\right.}{\left(1-\tanh^2(c+dx)\right)^6}\right.}{d}$$

$\big\downarrow$ 2345

$$\frac{\frac{1}{14}\left(\frac{1}{12}\left(\frac{b^2(504a+2593b)\tanh(c+dx)}{10\left(1-\tanh^2(c+dx)\right)^5}-\frac{1}{10}\int \frac{3\left(560(a+b)^3\tanh^8(c+dx)-1680(a-b)(a+b)^2\tanh^6(c+dx)+1680\left(a^3+b^2a+2b^3\right)\tanh^4(c+dx)\right.}{\left(1-\tanh^2(c+dx)\right)^5}\right.\right.}{d}$$

$\big\downarrow$ 27

$$\frac{\frac{1}{14}\left(\frac{1}{12}\left(\frac{b^2(504a+2593b)\tanh(c+dx)}{10\left(1-\tanh^2(c+dx)\right)^5}-\frac{3}{10}\int \frac{560(a+b)^3\tanh^8(c+dx)-1680(a-b)(a+b)^2\tanh^6(c+dx)+1680\left(a^3+b^2a+2b^3\right)\tanh^4(c+dx)-\right.}{\left(1-\tanh^2(c+dx)\right)^5}\right.\right.}{d}$$

$\Big\downarrow$ 2345

$$\frac{\frac{1}{14}\left(\frac{1}{12}\left(\frac{b^2(504a+2593b)\tanh(c+dx)}{10\left(1-\tanh^2(c+dx)\right)^5}-\frac{3}{10}\left(\frac{b^2(6888a+11821b)\tanh(c+dx)}{8\left(1-\tanh^2(c+dx)\right)^4}-\frac{1}{8}\int\frac{7\left(640(a+b)^3\tanh^6(c+dx)-1280(a-2b)(a+b)^2\tanh^4(c\cdots\right.}{(1-\tan\cdots}\right.\right.\right.}{d}$$

$\Big\downarrow$ 27

$$\frac{\frac{1}{14}\left(\frac{1}{12}\left(\frac{b^2(504a+2593b)\tanh(c+dx)}{10\left(1-\tanh^2(c+dx)\right)^5}-\frac{3}{10}\left(\frac{b^2(6888a+11821b)\tanh(c+dx)}{8\left(1-\tanh^2(c+dx)\right)^4}-\frac{7}{8}\int\frac{640(a+b)^3\tanh^6(c+dx)-1280(a-2b)(a+b)^2\tanh^4(c+\cdots}{(1-\tanh\cdots}\right.\right.\right.}{d}$$

$\Big\downarrow$ 2345

$$\frac{1}{14}\left(\frac{1}{12}\left(\frac{b^2(504a+2593b)\tanh(c+dx)}{10\left(1-\tanh^2(c+dx)\right)^5}-\frac{3}{10}\left(\frac{b^2(6888a+11821b)\tanh(c+dx)}{8\left(1-\tanh^2(c+dx)\right)^4}-\frac{7}{8}\left(\frac{b\left(1920a^2+12312ab+10579b^2\right)\tanh(c+dx)}{6\left(1-\tanh^2(c+dx)\right)^3}-\frac{1}{6}\int\frac{5(76\cdots}{\cdots}\right.\right.\right.\right.$$

$\Big\downarrow$ 27

$$\frac{1}{14}\left(\frac{1}{12}\left(\frac{b^2(504a+2593b)\tanh(c+dx)}{10\left(1-\tanh^2(c+dx)\right)^5}-\frac{3}{10}\left(\frac{b^2(6888a+11821b)\tanh(c+dx)}{8\left(1-\tanh^2(c+dx)\right)^4}-\frac{7}{8}\left(\frac{b\left(1920a^2+12312ab+10579b^2\right)\tanh(c+dx)}{6\left(1-\tanh^2(c+dx)\right)^3}-\frac{5}{6}\int\frac{768\cdots}{\cdots}\right.\right.\right.\right.$$

$\Big\downarrow$ 1471

$$\frac{1}{14}\left(\frac{1}{12}\left(\frac{b^2(504a+2593b)\tanh(c+dx)}{10\left(1-\tanh^2(c+dx)\right)^5}-\frac{3}{10}\left(\frac{b^2(6888a+11821b)\tanh(c+dx)}{8\left(1-\tanh^2(c+dx)\right)^4}-\frac{7}{8}\left(\frac{b\left(1920a^2+12312ab+10579b^2\right)\tanh(c+dx)}{6\left(1-\tanh^2(c+dx)\right)^3}-\frac{5}{6}\left(\frac{b(49\cdots}{\cdots}\right.\right.\right.\right.\right.$$

$\Big\downarrow$ 27

$$\frac{1}{14}\left(\frac{1}{12}\left(\frac{b^2(504a+2593b)\tanh(c+dx)}{10\left(1-\tanh^2(c+dx)\right)^5}-\frac{3}{10}\left(\frac{b^2(6888a+11821b)\tanh(c+dx)}{8\left(1-\tanh^2(c+dx)\right)^4}-\frac{7}{8}\left(\frac{b\left(1920a^2+12312ab+10579b^2\right)\tanh(c+dx)}{6\left(1-\tanh^2(c+dx)\right)^3}-\frac{5}{6}\left(\frac{b(49\cdots}{\cdots}\right.\right.\right.\right.\right.$$

$\Big\downarrow$ 298

$$\frac{1}{14}\left(\frac{1}{12}\left(\frac{b^2(504a+2593b)\tanh(c+dx)}{10\left(1-\tanh^2(c+dx)\right)^5}-\frac{3}{10}\left(\frac{b^2(6888a+11821b)\tanh(c+dx)}{8\left(1-\tanh^2(c+dx)\right)^4}-\frac{7}{8}\left(\frac{b\left(1920a^2+12312ab+10579b^2\right)\tanh(c+dx)}{6\left(1-\tanh^2(c+dx)\right)^3}-\frac{5}{6}\left(\frac{b(49\cdots}{\cdots}\right.\right.\right.\right.\right.$$

$\Big\downarrow$ 219

$$\frac{1}{14}\left(\frac{1}{12}\left(\frac{b^2(504a+2593b)\tanh(c+dx)}{10(1-\tanh^2(c+dx))^5}-\frac{3}{10}\left(\frac{b^2(6888a+11821b)\tanh(c+dx)}{8(1-\tanh^2(c+dx))^4}-\frac{7}{8}\left(\frac{b(1920a^2+12312ab+10579b^2)\tanh(c+dx)}{6(1-\tanh^2(c+dx))^3}-\frac{5}{6}\left(\frac{b(499\right.\right.\right.\right.\right.$$

input
```
Int[Sinh[c + d*x]^2*(a + b*Sinh[c + d*x]^4)^3,x]
```

output
```
((b^3*Tanh[c + d*x])/(14*(1 - Tanh[c + d*x]^2)^7) + ((-85*b^3*Tanh[c + d*x
])/(12*(1 - Tanh[c + d*x]^2)^6) + ((b^2*(504*a + 2593*b)*Tanh[c + d*x])/(1
0*(1 - Tanh[c + d*x]^2)^5) - (3*((b^2*(6888*a + 11821*b)*Tanh[c + d*x])/(8
*(1 - Tanh[c + d*x]^2)^4) - (7*((b*(1920*a^2 + 12312*a*b + 10579*b^2)*Tanh
[c + d*x])/(6*(1 - Tanh[c + d*x]^2)^3) - (5*((b*(4992*a^2 + 10728*a*b + 55
49*b^2)*Tanh[c + d*x])/(4*(1 - Tanh[c + d*x]^2)^2) - (3*(-1/2*((1024*a^3 +
 1920*a^2*b + 1512*a*b^2 + 429*b^3)*ArcTanh[Tanh[c + d*x]]) + ((1024*a^3 +
 4224*a^2*b + 4632*a*b^2 + 1619*b^3)*Tanh[c + d*x])/(2*(1 - Tanh[c + d*x]^
2))))/4))/6))/8))/10)/12)/14)/d
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 298
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2), x_Symbol] :> Simp[(-(
b*c - a*d))*x*((a + b*x^2)^(p + 1)/(2*a*b*(p + 1))), x] - Simp[(a*d - b*c*(
2*p + 3))/(2*a*b*(p + 1))   Int[(a + b*x^2)^(p + 1), x], x] /; FreeQ[{a, b,
 c, d, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/2 + p, 0])
```

rule 1471
```
Int[((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.),
x_Symbol] :> With[{Qx = PolynomialQuotient[(a + b*x^2 + c*x^4)^p, d + e*x^2
, x], R = Coeff[PolynomialRemainder[(a + b*x^2 + c*x^4)^p, d + e*x^2, x], x
, 0]}, Simp[(-R)*x*((d + e*x^2)^(q + 1)/(2*d*(q + 1))), x] + Simp[1/(2*d*(q
 + 1))   Int[(d + e*x^2)^(q + 1)*ExpandToSum[2*d*(q + 1)*Qx + R*(2*q + 3),
x], x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^
2 - b*d*e + a*e^2, 0] && IGtQ[p, 0] && LtQ[q, -1]
```

rule 1580
```
Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)
^4)^(p_.), x_Symbol] :> Simp[(-d)^(m/2 - 1)*(c*d^2 - b*d*e + a*e^2)^p*x*((d
 + e*x^2)^(q + 1)/(2*e^(2*p + m/2)*(q + 1))), x] + Simp[1/(2*e^(2*p + m/2)*
(q + 1))   Int[(d + e*x^2)^(q + 1)*ExpandToSum[Together[(1/(d + e*x^2))*(2*
e^(2*p + m/2)*(q + 1)*x^m*(a + b*x^2 + c*x^4)^p - (-d)^(m/2 - 1)*(c*d^2 - b
*d*e + a*e^2)^p*(d + e*(2*q + 3)*x^2))], x], x], x] /; FreeQ[{a, b, c, d, e
}, x] && NeQ[b^2 - 4*a*c, 0] && IGtQ[p, 0] && ILtQ[q, -1] && IGtQ[m/2, 0]
```

rule 2345
```
Int[(Pq_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{Q = PolynomialQuot
ient[Pq, a + b*x^2, x], f = Coeff[PolynomialRemainder[Pq, a + b*x^2, x], x,
 0], g = Coeff[PolynomialRemainder[Pq, a + b*x^2, x], x, 1]}, Simp[(a*g - b
*f*x)*((a + b*x^2)^(p + 1)/(2*a*b*(p + 1))), x] + Simp[1/(2*a*(p + 1))   In
t[(a + b*x^2)^(p + 1)*ExpandToSum[2*a*(p + 1)*Q + f*(2*p + 3), x], x], x]]
/; FreeQ[{a, b}, x] && PolyQ[Pq, x] && LtQ[p, -1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3696
```
Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^4)^(
p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Simp[ff^(m + 1
)/f   Subst[Int[x^m*((a + 2*a*ff^2*x^2 + (a + b)*ff^4*x^4)^p/(1 + ff^2*x^2)
^(m/2 + 2*p + 1)), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f}, x] &
& IntegerQ[m/2] && IntegerQ[p]
```

Maple [A] (verified)

Time = 0.25 (sec) , antiderivative size = 240, normalized size of antiderivative = 0.94

$$\frac{a^3\left(\frac{\cosh(dx+c)\sinh(dx+c)}{2}-\frac{dx}{2}-\frac{c}{2}\right)+3a^2b\left(\left(\frac{\sinh(dx+c)^5}{6}-\frac{5\sinh(dx+c)^3}{24}+\frac{5\sinh(dx+c)}{16}\right)\cosh\left(dx+c\right)-\frac{5dx}{16}-\frac{5c}{16}\right)}{}$$

input
```
int(sinh(d*x+c)^2*(a+b*sinh(d*x+c)^4)^3,x)
```

output
```
1/d*(a^3*(1/2*cosh(d*x+c)*sinh(d*x+c)-1/2*d*x-1/2*c)+3*a^2*b*((1/6*sinh(d*
x+c)^5-5/24*sinh(d*x+c)^3+5/16*sinh(d*x+c))*cosh(d*x+c)-5/16*d*x-5/16*c)+3
*b^2*a*((1/10*sinh(d*x+c)^9-9/80*sinh(d*x+c)^7+21/160*sinh(d*x+c)^5-21/128
*sinh(d*x+c)^3+63/256*sinh(d*x+c))*cosh(d*x+c)-63/256*d*x-63/256*c)+b^3*((
1/14*sinh(d*x+c)^13-13/168*sinh(d*x+c)^11+143/1680*sinh(d*x+c)^9-429/4480*
sinh(d*x+c)^7+143/1280*sinh(d*x+c)^5-143/1024*sinh(d*x+c)^3+429/2048*sinh(
d*x+c))*cosh(d*x+c)-429/2048*d*x-429/2048*c))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 627 vs. 2(239) = 478.

Time = 0.10 (sec) , antiderivative size = 627, normalized size of antiderivative = 2.46

$$\int \sinh^2(c+dx)\left(a+b\sinh^4(c+dx)\right)^3\,dx=\text{Too large to display}$$

input
```
integrate(sinh(d*x+c)^2*(a+b*sinh(d*x+c)^4)^3,x, algorithm="fricas")
```

output
```
1/860160*(105*b^3*cosh(d*x + c)*sinh(d*x + c)^13 + 210*(13*b^3*cosh(d*x +
c)^3 - 7*b^3*cosh(d*x + c))*sinh(d*x + c)^11 + 35*(429*b^3*cosh(d*x + c)^5
 - 770*b^3*cosh(d*x + c)^3 + 3*(48*a*b^2 + 91*b^3)*cosh(d*x + c))*sinh(d*x
 + c)^9 + 60*(429*b^3*cosh(d*x + c)^7 - 1617*b^3*cosh(d*x + c)^5 + 21*(48*
a*b^2 + 91*b^3)*cosh(d*x + c)^3 - 7*(120*a*b^2 + 91*b^3)*cosh(d*x + c))*si
nh(d*x + c)^7 + 21*(715*b^3*cosh(d*x + c)^9 - 4620*b^3*cosh(d*x + c)^7 + 1
26*(48*a*b^2 + 91*b^3)*cosh(d*x + c)^5 - 140*(120*a*b^2 + 91*b^3)*cosh(d*x
 + c)^3 + 5*(768*a^2*b + 2160*a*b^2 + 1001*b^3)*cosh(d*x + c))*sinh(d*x +
c)^5 + 70*(39*b^3*cosh(d*x + c)^11 - 385*b^3*cosh(d*x + c)^9 + 18*(48*a*b^
2 + 91*b^3)*cosh(d*x + c)^7 - 42*(120*a*b^2 + 91*b^3)*cosh(d*x + c)^5 + 5*
(768*a^2*b + 2160*a*b^2 + 1001*b^3)*cosh(d*x + c)^3 - 3*(2304*a^2*b + 2880
*a*b^2 + 1001*b^3)*cosh(d*x + c))*sinh(d*x + c)^3 - 420*(1024*a^3 + 1920*a
^2*b + 1512*a*b^2 + 429*b^3)*d*x + 105*(b^3*cosh(d*x + c)^13 - 14*b^3*cosh
(d*x + c)^11 + (48*a*b^2 + 91*b^3)*cosh(d*x + c)^9 - 4*(120*a*b^2 + 91*b^3
)*cosh(d*x + c)^7 + (768*a^2*b + 2160*a*b^2 + 1001*b^3)*cosh(d*x + c)^5 -
2*(2304*a^2*b + 2880*a*b^2 + 1001*b^3)*cosh(d*x + c)^3 + (4096*a^3 + 11520
*a^2*b + 10080*a*b^2 + 3003*b^3)*cosh(d*x + c))*sinh(d*x + c))/d
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 877 vs. $2(250) = 500$.

Time = 4.77 (sec) , antiderivative size = 877, normalized size of antiderivative = 3.44

$$\int \sinh^2(c+dx) \left(a+b\sinh^4(c+dx)\right)^3 \, dx = \text{Too large to display}$$

input
```
integrate(sinh(d*x+c)**2*(a+b*sinh(d*x+c)**4)**3,x)
```

output
```
Piecewise((a**3*x*sinh(c + d*x)**2/2 - a**3*x*cosh(c + d*x)**2/2 + a**3*si
nh(c + d*x)*cosh(c + d*x)/(2*d) + 15*a**2*b*x*sinh(c + d*x)**6/16 - 45*a**
2*b*x*sinh(c + d*x)**4*cosh(c + d*x)**2/16 + 45*a**2*b*x*sinh(c + d*x)**2*
cosh(c + d*x)**4/16 - 15*a**2*b*x*cosh(c + d*x)**6/16 + 33*a**2*b*sinh(c +
 d*x)**5*cosh(c + d*x)/(16*d) - 5*a**2*b*sinh(c + d*x)**3*cosh(c + d*x)**3
/(2*d) + 15*a**2*b*sinh(c + d*x)*cosh(c + d*x)**5/(16*d) + 189*a*b**2*x*si
nh(c + d*x)**10/256 - 945*a*b**2*x*sinh(c + d*x)**8*cosh(c + d*x)**2/256 +
 945*a*b**2*x*sinh(c + d*x)**6*cosh(c + d*x)**4/128 - 945*a*b**2*x*sinh(c
+ d*x)**4*cosh(c + d*x)**6/128 + 945*a*b**2*x*sinh(c + d*x)**2*cosh(c + d*
x)**8/256 - 189*a*b**2*x*cosh(c + d*x)**10/256 + 579*a*b**2*sinh(c + d*x)*
*9*cosh(c + d*x)/(256*d) - 711*a*b**2*sinh(c + d*x)**7*cosh(c + d*x)**3/(1
28*d) + 63*a*b**2*sinh(c + d*x)**5*cosh(c + d*x)**5/(10*d) - 441*a*b**2*si
nh(c + d*x)**3*cosh(c + d*x)**7/(128*d) + 189*a*b**2*sinh(c + d*x)*cosh(c
+ d*x)**9/(256*d) + 429*b**3*x*sinh(c + d*x)**14/2048 - 3003*b**3*x*sinh(c
 + d*x)**12*cosh(c + d*x)**2/2048 + 9009*b**3*x*sinh(c + d*x)**10*cosh(c +
 d*x)**4/2048 - 15015*b**3*x*sinh(c + d*x)**8*cosh(c + d*x)**6/2048 + 1501
5*b**3*x*sinh(c + d*x)**6*cosh(c + d*x)**8/2048 - 9009*b**3*x*sinh(c + d*x
)**4*cosh(c + d*x)**10/2048 + 3003*b**3*x*sinh(c + d*x)**2*cosh(c + d*x)**
12/2048 - 429*b**3*x*cosh(c + d*x)**14/2048 + 1619*b**3*sinh(c + d*x)**13*
cosh(c + d*x)/(2048*d) - 4511*b**3*sinh(c + d*x)**11*cosh(c + d*x)**3/(...
```

Maxima [A] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 442, normalized size of antiderivative = 1.73

$$\int \sinh^2(c+dx)\left(a+b\sinh^4(c+dx)\right)^3\,dx = -\frac{1}{8}\,a^3\left(4\,x-\frac{e^{(2\,dx+2\,c)}}{d}+\frac{e^{(-2\,dx-2\,c)}}{d}\right)$$
$$-\frac{1}{3440640}\,b^3\left(\frac{\left(245\,e^{(-2\,dx-2\,c)}-1911\,e^{(-4\,dx-4\,c)}+9555\,e^{(-6\,dx-6\,c)}-35035\,e^{(-8\,dx-8\,c)}+105105\,e^{(-10\,dx-}\right.}{d}\right.$$
$$-\frac{3}{20480}\,ab^2\left(\frac{\left(25\,e^{(-2\,dx-2\,c)}-150\,e^{(-4\,dx-4\,c)}+600\,e^{(-6\,dx-6\,c)}-2100\,e^{(-8\,dx-8\,c)}-2\right)e^{(10\,dx+10\,c)}}{d}+\frac{5040}{}\right.$$
$$-\frac{1}{128}\,a^2b\left(\frac{\left(9\,e^{(-2\,dx-2\,c)}-45\,e^{(-4\,dx-4\,c)}-1\right)e^{(6\,dx+6\,c)}}{d}+\frac{120\,(dx+c)}{d}+\frac{45\,e^{(-2\,dx-2\,c)}-9\,e^{(-4\,dx-4\,c)}+}{d}\right.$$

input
```
integrate(sinh(d*x+c)^2*(a+b*sinh(d*x+c)^4)^3,x, algorithm="maxima")
```

output
```
-1/8*a^3*(4*x - e^(2*d*x + 2*c)/d + e^(-2*d*x - 2*c)/d) - 1/3440640*b^3*((
245*e^(-2*d*x - 2*c) - 1911*e^(-4*d*x - 4*c) + 9555*e^(-6*d*x - 6*c) - 350
35*e^(-8*d*x - 8*c) + 105105*e^(-10*d*x - 10*c) - 315315*e^(-12*d*x - 12*c
) - 15)*e^(14*d*x + 14*c)/d + 720720*(d*x + c)/d + (315315*e^(-2*d*x - 2*c
) - 105105*e^(-4*d*x - 4*c) + 35035*e^(-6*d*x - 6*c) - 9555*e^(-8*d*x - 8*
c) + 1911*e^(-10*d*x - 10*c) - 245*e^(-12*d*x - 12*c) + 15*e^(-14*d*x - 14
*c))/d) - 3/20480*a*b^2*((25*e^(-2*d*x - 2*c) - 150*e^(-4*d*x - 4*c) + 600
*e^(-6*d*x - 6*c) - 2100*e^(-8*d*x - 8*c) - 2)*e^(10*d*x + 10*c)/d + 5040*
(d*x + c)/d + (2100*e^(-2*d*x - 2*c) - 600*e^(-4*d*x - 4*c) + 150*e^(-6*d*
x - 6*c) - 25*e^(-8*d*x - 8*c) + 2*e^(-10*d*x - 10*c))/d) - 1/128*a^2*b*((
9*e^(-2*d*x - 2*c) - 45*e^(-4*d*x - 4*c) - 1)*e^(6*d*x + 6*c)/d + 120*(d*x
 + c)/d + (45*e^(-2*d*x - 2*c) - 9*e^(-4*d*x - 4*c) + e^(-6*d*x - 6*c))/d)
```

Giac [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 401, normalized size of antiderivative = 1.57

$$\begin{aligned}
&\int \sinh^2(c+dx)\left(a+b\sinh^4(c+dx)\right)^3\,dx \\
&= \frac{b^3e^{(14\,dx+14\,c)}}{229376\,d} - \frac{7\,b^3e^{(12\,dx+12\,c)}}{98304\,d} + \frac{7\,b^3e^{(-12\,dx-12\,c)}}{98304\,d} - \frac{b^3e^{(-14\,dx-14\,c)}}{229376\,d} \\
&\quad - \frac{1}{2048}\left(1024\,a^3 + 1920\,a^2b + 1512\,ab^2 + 429\,b^3\right)x + \frac{(48\,ab^2 + 91\,b^3)e^{(10\,dx+10\,c)}}{163840\,d} \\
&\quad - \frac{(120\,ab^2 + 91\,b^3)e^{(8\,dx+8\,c)}}{32768\,d} + \frac{(768\,a^2b + 2160\,ab^2 + 1001\,b^3)e^{(6\,dx+6\,c)}}{98304\,d} \\
&\quad - \frac{(2304\,a^2b + 2880\,ab^2 + 1001\,b^3)e^{(4\,dx+4\,c)}}{32768\,d} \\
&\quad + \frac{(4096\,a^3 + 11520\,a^2b + 10080\,ab^2 + 3003\,b^3)e^{(2\,dx+2\,c)}}{32768\,d} \\
&\quad - \frac{(4096\,a^3 + 11520\,a^2b + 10080\,ab^2 + 3003\,b^3)e^{(-2\,dx-2\,c)}}{32768\,d} \\
&\quad + \frac{(2304\,a^2b + 2880\,ab^2 + 1001\,b^3)e^{(-4\,dx-4\,c)}}{32768\,d} \\
&\quad - \frac{(768\,a^2b + 2160\,ab^2 + 1001\,b^3)e^{(-6\,dx-6\,c)}}{98304\,d} \\
&\quad + \frac{(120\,ab^2 + 91\,b^3)e^{(-8\,dx-8\,c)}}{32768\,d} - \frac{(48\,ab^2 + 91\,b^3)e^{(-10\,dx-10\,c)}}{163840\,d}
\end{aligned}$$

input
```
integrate(sinh(d*x+c)^2*(a+b*sinh(d*x+c)^4)^3,x, algorithm="giac")
```

output
```
1/229376*b^3*e^(14*d*x + 14*c)/d - 7/98304*b^3*e^(12*d*x + 12*c)/d + 7/983
04*b^3*e^(-12*d*x - 12*c)/d - 1/229376*b^3*e^(-14*d*x - 14*c)/d - 1/2048*(
1024*a^3 + 1920*a^2*b + 1512*a*b^2 + 429*b^3)*x + 1/163840*(48*a*b^2 + 91*
b^3)*e^(10*d*x + 10*c)/d - 1/32768*(120*a*b^2 + 91*b^3)*e^(8*d*x + 8*c)/d
+ 1/98304*(768*a^2*b + 2160*a*b^2 + 1001*b^3)*e^(6*d*x + 6*c)/d - 1/32768*
(2304*a^2*b + 2880*a*b^2 + 1001*b^3)*e^(4*d*x + 4*c)/d + 1/32768*(4096*a^3
 + 11520*a^2*b + 10080*a*b^2 + 3003*b^3)*e^(2*d*x + 2*c)/d - 1/32768*(4096
*a^3 + 11520*a^2*b + 10080*a*b^2 + 3003*b^3)*e^(-2*d*x - 2*c)/d + 1/32768*
(2304*a^2*b + 2880*a*b^2 + 1001*b^3)*e^(-4*d*x - 4*c)/d - 1/98304*(768*a^2
*b + 2160*a*b^2 + 1001*b^3)*e^(-6*d*x - 6*c)/d + 1/32768*(120*a*b^2 + 91*b
^3)*e^(-8*d*x - 8*c)/d - 1/163840*(48*a*b^2 + 91*b^3)*e^(-10*d*x - 10*c)/d
```

Mupad [B] (verification not implemented)

Time = 0.81 (sec) , antiderivative size = 393, normalized size of antiderivative = 1.54

$$\int \sinh^2(c+dx)\left(a+b\sinh^4(c+dx)\right)^3\,dx = \frac{e^{6c+6dx}\,(768\,a^2\,b+2160\,a\,b^2+1001\,b^3)}{98304\,d} - \frac{e^{-6c-6dx}\,(768\,a^2\,b+2160\,a\,b^2+1001\,b^3)}{98304\,d} - x\left(\frac{a^3}{2}+\frac{15\,a^2\,b}{16}+\frac{189\,a\,b^2}{256}+\frac{429\,b^3}{2048}\right) + \frac{e^{-4c-4dx}\,(2304\,a^2\,b+2880\,a\,b^2+1001\,b^3)}{32768\,d} - \frac{e^{4c+4dx}\,(2304\,a^2\,b+2880\,a\,b^2+1001\,b^3)}{32768\,d} - \frac{e^{-2c-2dx}\,(4096\,a^3+11520\,a^2\,b+10080\,a\,b^2+3003\,b^3)}{32768\,d} + \frac{e^{2c+2dx}\,(4096\,a^3+11520\,a^2\,b+10080\,a\,b^2+3003\,b^3)}{32768\,d} + \frac{7\,b^3\,e^{-12c-12dx}}{98304\,d} - \frac{7\,b^3\,e^{12c+12dx}}{98304\,d} - \frac{b^3\,e^{-14c-14dx}}{229376\,d} + \frac{b^3\,e^{14c+14dx}}{229376\,d} - \frac{b^2\,e^{-10c-10dx}\,(48\,a+91\,b)}{163840\,d} + \frac{b^2\,e^{10c+10dx}\,(48\,a+91\,b)}{163840\,d} + \frac{b^2\,e^{-8c-8dx}\,(120\,a+91\,b)}{32768\,d} - \frac{b^2\,e^{8c+8dx}\,(120\,a+91\,b)}{32768\,d}$$

input
```
int(sinh(c + d*x)^2*(a + b*sinh(c + d*x)^4)^3,x)
```

output
```
(exp(6*c + 6*d*x)*(2160*a*b^2 + 768*a^2*b + 1001*b^3))/(98304*d) - (exp(-
6*c - 6*d*x)*(2160*a*b^2 + 768*a^2*b + 1001*b^3))/(98304*d) - x*((189*a*b^
2)/256 + (15*a^2*b)/16 + a^3/2 + (429*b^3)/2048) + (exp(- 4*c - 4*d*x)*(28
80*a*b^2 + 2304*a^2*b + 1001*b^3))/(32768*d) - (exp(4*c + 4*d*x)*(2880*a*b
^2 + 2304*a^2*b + 1001*b^3))/(32768*d) - (exp(- 2*c - 2*d*x)*(10080*a*b^2
+ 11520*a^2*b + 4096*a^3 + 3003*b^3))/(32768*d) + (exp(2*c + 2*d*x)*(10080
*a*b^2 + 11520*a^2*b + 4096*a^3 + 3003*b^3))/(32768*d) + (7*b^3*exp(- 12*c
 - 12*d*x))/(98304*d) - (7*b^3*exp(12*c + 12*d*x))/(98304*d) - (b^3*exp(-
14*c - 14*d*x))/(229376*d) + (b^3*exp(14*c + 14*d*x))/(229376*d) - (b^2*ex
p(- 10*c - 10*d*x)*(48*a + 91*b))/(163840*d) + (b^2*exp(10*c + 10*d*x)*(48
*a + 91*b))/(163840*d) + (b^2*exp(- 8*c - 8*d*x)*(120*a + 91*b))/(32768*d)
 - (b^2*exp(8*c + 8*d*x)*(120*a + 91*b))/(32768*d)
```

Reduce [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 574, normalized size of antiderivative = 2.25

$$\int \sinh^2(c+dx)\left(a+b\sinh^4(c+dx)\right)^3\,dx$$
$$= \frac{1008e^{24dx+24c}a\,b^2 - 12600e^{22dx+22c}a\,b^2 + 26880e^{20dx+20c}a^2b + 75600e^{20dx+20c}a\,b^2 - 241920e^{18dx+18c}a^2b - \cdots}{}$$

input
```
int(sinh(d*x+c)^2*(a+b*sinh(d*x+c)^4)^3,x)
```

output

```
(15*e**(28*c + 28*d*x)*b**3 - 245*e**(26*c + 26*d*x)*b**3 + 1008*e**(24*c
+ 24*d*x)*a*b**2 + 1911*e**(24*c + 24*d*x)*b**3 - 12600*e**(22*c + 22*d*x)
*a*b**2 - 9555*e**(22*c + 22*d*x)*b**3 + 26880*e**(20*c + 20*d*x)*a**2*b +
 75600*e**(20*c + 20*d*x)*a*b**2 + 35035*e**(20*c + 20*d*x)*b**3 - 241920*
e**(18*c + 18*d*x)*a**2*b - 302400*e**(18*c + 18*d*x)*a*b**2 - 105105*e**(
18*c + 18*d*x)*b**3 + 430080*e**(16*c + 16*d*x)*a**3 + 1209600*e**(16*c +
16*d*x)*a**2*b + 1058400*e**(16*c + 16*d*x)*a*b**2 + 315315*e**(16*c + 16*
d*x)*b**3 - 1720320*e**(14*c + 14*d*x)*a**3*d*x - 3225600*e**(14*c + 14*d*
x)*a**2*b*d*x - 2540160*e**(14*c + 14*d*x)*a*b**2*d*x - 720720*e**(14*c +
14*d*x)*b**3*d*x - 430080*e**(12*c + 12*d*x)*a**3 - 1209600*e**(12*c + 12*
d*x)*a**2*b - 1058400*e**(12*c + 12*d*x)*a*b**2 - 315315*e**(12*c + 12*d*x
)*b**3 + 241920*e**(10*c + 10*d*x)*a**2*b + 302400*e**(10*c + 10*d*x)*a*b*
*2 + 105105*e**(10*c + 10*d*x)*b**3 - 26880*e**(8*c + 8*d*x)*a**2*b - 7560
0*e**(8*c + 8*d*x)*a*b**2 - 35035*e**(8*c + 8*d*x)*b**3 + 12600*e**(6*c +
6*d*x)*a*b**2 + 9555*e**(6*c + 6*d*x)*b**3 - 1008*e**(4*c + 4*d*x)*a*b**2
- 1911*e**(4*c + 4*d*x)*b**3 + 245*e**(2*c + 2*d*x)*b**3 - 15*b**3)/(34406
40*e**(14*c + 14*d*x)*d)
```

3.193 $\int \left(a+b\sinh^4(c+dx)\right)^3\,dx$

Optimal result

Integrand size = 14, antiderivative size = 211

$$\begin{aligned}\int \left(a+b\sinh^4(c+dx)\right)^3\,dx &= \frac{(1024a^3+1152a^2b+840ab^2+231b^3)\,x}{1024}\\ &\quad-\frac{b(1920a^2+2232ab+793b^2)\cosh(c+dx)\sinh(c+dx)}{1024d}\\ &\quad+\frac{b(1152a^2+3912ab+2279b^2)\cosh^3(c+dx)\sinh(c+dx)}{1536d}\\ &\quad-\frac{b^2(3000a+3481b)\cosh^5(c+dx)\sinh(c+dx)}{1920d}\\ &\quad+\frac{3b^2(40a+139b)\cosh^7(c+dx)\sinh(c+dx)}{320d}\\ &\quad-\frac{61b^3\cosh^9(c+dx)\sinh(c+dx)}{120d}+\frac{b^3\cosh^{11}(c+dx)\sinh(c+dx)}{12d}\end{aligned}$$

output
```
1/1024*(1024*a^3+1152*a^2*b+840*a*b^2+231*b^3)*x-1/1024*b*(1920*a^2+2232*a
*b+793*b^2)*cosh(d*x+c)*sinh(d*x+c)/d+1/1536*b*(1152*a^2+3912*a*b+2279*b^2
)*cosh(d*x+c)^3*sinh(d*x+c)/d-1/1920*b^2*(3000*a+3481*b)*cosh(d*x+c)^5*sin
h(d*x+c)/d+3/320*b^2*(40*a+139*b)*cosh(d*x+c)^7*sinh(d*x+c)/d-61/120*b^3*c
osh(d*x+c)^9*sinh(d*x+c)/d+1/12*b^3*cosh(d*x+c)^11*sinh(d*x+c)/d
```

Mathematica [A] (verified)

Time = 0.60 (sec) , antiderivative size = 156, normalized size of antiderivative = 0.74

$$\int \left(a+b\sinh^4(c+dx)\right)^3\,dx$$
$$=\frac{120(1024a^3+1152a^2b+840ab^2+231b^3)\,(c+dx)-720b(128a^2+112ab+33b^2)\sinh(2(c+dx))+45b(}{}$$

input
```
Integrate[(a + b*Sinh[c + d*x]^4)^3,x]
```

output
```
(120*(1024*a^3 + 1152*a^2*b + 840*a*b^2 + 231*b^3)*(c + d*x) - 720*b*(128*
a^2 + 112*a*b + 33*b^2)*Sinh[2*(c + d*x)] + 45*b*(256*a^2 + 448*a*b + 165*
b^2)*Sinh[4*(c + d*x)] - 40*b^2*(96*a + 55*b)*Sinh[6*(c + d*x)] + 45*b^2*(
8*a + 11*b)*Sinh[8*(c + d*x)] - 72*b^3*Sinh[10*(c + d*x)] + 5*b^3*Sinh[12*
(c + d*x)])/(122880*d)
```

Rubi [A] (verified)

Time = 0.97 (sec) , antiderivative size = 266, normalized size of antiderivative = 1.26, number of steps used = 15, number of rules used = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 1.000, Rules used = {3042, 3688, 1471, 25, 2345, 27, 2345, 25, 2345, 27, 1471, 27, 298, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(a+b\sinh^4(c+dx)\right)^3\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \left(a+b\sin(ic+idx)^4\right)^3dx$$

$$\Big\downarrow \text{3688}$$

$$\frac{\int \frac{\left((a+b)\tanh^4(c+dx)-2a\tanh^2(c+dx)+a\right)^3}{\left(1-\tanh^2(c+dx)\right)^7}d\tanh(c+dx)}{d}$$

$$\Big\downarrow \text{1471}$$

$$\frac{\frac{b^3\tanh(c+dx)}{12\left(1-\tanh^2(c+dx)\right)^6}-\frac{1}{12}\int -\frac{-12(a+b)^3\tanh^{10}(c+dx)+12(5a-b)(a+b)^2\tanh^8(c+dx)-12\left(10a^3+9ba^2+b^3\right)\tanh^6(c+dx)+12\left(10a^3+3ba^2\right.}{\left(1-\tanh^2(c+dx)\right)^6}}{d}$$

$$\Big\downarrow \text{25}$$

$$\frac{\frac{1}{12}\int \frac{-12(a+b)^3\tanh^{10}(c+dx)+12(5a-b)(a+b)^2\tanh^8(c+dx)-12\left(10a^3+9ba^2+b^3\right)\tanh^6(c+dx)+12\left(10a^3+3ba^2-b^3\right)\tanh^4(c+dx)-12\left(5a^3+\right.}{\left(1-\tanh^2(c+dx)\right)^6}}{d}$$

$$\Big\downarrow \text{2345}$$

$$\frac{\frac{1}{12}\left(-\frac{1}{10}\int -\frac{3\left(40(a+b)^3\tanh^8(c+dx)-80(2a-b)(a+b)^2\tanh^6(c+dx)+120\left(2a^3+ba^2+b^3\right)\tanh^4(c+dx)-160\left(a^3-b^3\right)\tanh^2(c+dx)+40a^3+1\right.}{\left(1-\tanh^2(c+dx)\right)^5}\right.}{d}$$

$$\Big\downarrow \text{27}$$

$$\frac{\frac{1}{12}\left(\frac{3}{10}\int \frac{40(a+b)^3\tanh^8(c+dx)-80(2a-b)(a+b)^2\tanh^6(c+dx)+120\left(2a^3+ba^2+b^3\right)\tanh^4(c+dx)-160\left(a^3-b^3\right)\tanh^2(c+dx)+40a^3+17b^3}{\left(1-\tanh^2(c+dx)\right)^5}d\,t\right.}{d}$$

$$\Big\downarrow \text{2345}$$

$$\frac{\frac{1}{12}\left(\frac{3}{10}\left(\frac{3b^2(40a+139b)\tanh(c+dx)}{8\left(1-\tanh^2(c+dx)\right)^4}-\frac{1}{8}\int -\frac{-320(a+b)^3\tanh^6(c+dx)+960(a-b)(a+b)^2\tanh^4(c+dx)-960\left(a^3+b^2a+2b^3\right)\tanh^2(c+dx)+3}{\left(1-\tanh^2(c+dx)\right)^4}\right.\right.}{d}$$

$$\Big\downarrow \text{25}$$

$$\frac{\frac{1}{12}\left(\frac{3}{10}\left(\frac{1}{8}\int \frac{-320(a+b)^3\tanh^6(c+dx)+960(a-b)(a+b)^2\tanh^4(c+dx)-960\left(a^3+b^2a+2b^3\right)\tanh^2(c+dx)+320a^3-281b^3-120ab^2}{\left(1-\tanh^2(c+dx)\right)^4}d\tanh(c+\right.\right.}{d}$$

$$\Big\downarrow \text{2345}$$

$$\frac{\frac{1}{12}\left(\frac{3}{10}\left(\frac{1}{8}\left(-\frac{1}{6}\int -\frac{5\left(384(a+b)^3\tanh^4(c+dx)-768(a-2b)(a+b)^2\tanh^2(c+dx)+384a^3+359b^3+456ab^2\right)}{\left(1-\tanh^2(c+dx)\right)^3}d\tanh(c+dx)-\frac{b^2(3000a+3}{6(1-\mathrm{ta}}\right.\right.\right.}{d}$$

$$\Big\downarrow \text{27}$$

$$\frac{\frac{1}{12}\left(\frac{3}{10}\left(\frac{1}{8}\left(\frac{5}{6}\int \frac{384(a+b)^3\tanh^4(c+dx)-768(a-2b)(a+b)^2\tanh^2(c+dx)+384a^3+359b^3+456ab^2}{\left(1-\tanh^2(c+dx)\right)^3}d\tanh(c+dx)-\frac{b^2(3000a+3481b)\,\mathrm{ta}}{6(1-\tanh^2(c+}\right.\right.\right.}{d}$$

$\downarrow$ 1471

$$\frac{\frac{1}{12}\left(\frac{3}{10}\left(\frac{1}{8}\left(\frac{5}{6}\left(\frac{b(1152a^2+3912ab+2279b^2)\tanh(c+dx)}{4(1-\tanh^2(c+dx))^2}-\frac{1}{4}\int-\frac{3(512a^3-384ba^2-696b^2a-281b^3-512(a+b)^3\tanh^2(c+dx))}{(1-\tanh^2(c+dx))^2}d\tanh(c+\right.\right.\right.\right.}{d}$$

$\downarrow$ 27

$$\frac{\frac{1}{12}\left(\frac{3}{10}\left(\frac{1}{8}\left(\frac{5}{6}\left(\frac{3}{4}\int\frac{512a^3-384ba^2-696b^2a-281b^3-512(a+b)^3\tanh^2(c+dx)}{(1-\tanh^2(c+dx))^2}d\tanh(c+dx)+\frac{b(1152a^2+3912ab+2279b^2)\tanh(c+dx)}{4(1-\tanh^2(c+dx))^2}\right.\right.\right.\right.}{d}$$

$\downarrow$ 298

$$\frac{1}{12}\left(\frac{3}{10}\left(\frac{1}{8}\left(\frac{5}{6}\left(\frac{3}{4}\left(\frac{1}{2}\left(1024a^3+1152a^2b+840ab^2+231b^3\right)\int\frac{1}{1-\tanh^2(c+dx)}d\tanh(c+dx)-\frac{b(1920a^2+2232ab+793b^2)\tanh(c+dx)}{2(1-\tanh^2(c+dx))}\right.\right.\right.\right.\right.$$

$\downarrow$ 219

$$\frac{1}{12}\left(\frac{3}{10}\left(\frac{1}{8}\left(\frac{5}{6}\left(\frac{b(1152a^2+3912ab+2279b^2)\tanh(c+dx)}{4(1-\tanh^2(c+dx))^2}+\frac{3}{4}\left(\frac{1}{2}\left(1024a^3+1152a^2b+840ab^2+231b^3\right)\operatorname{arctanh}(\tanh(c+dx\right.\right.\right.\right.\right.$$

input
```
Int[(a + b*Sinh[c + d*x]^4)^3,x]
```

output
```
((b^3*Tanh[c + d*x])/(12*(1 - Tanh[c + d*x]^2)^6) + ((-61*b^3*Tanh[c + d*x
])/(10*(1 - Tanh[c + d*x]^2)^5) + (3*((3*b^2*(40*a + 139*b)*Tanh[c + d*x])
/(8*(1 - Tanh[c + d*x]^2)^4) + (-1/6*(b^2*(3000*a + 3481*b)*Tanh[c + d*x])
/(1 - Tanh[c + d*x]^2)^3 + (5*((b*(1152*a^2 + 3912*a*b + 2279*b^2)*Tanh[c
+ d*x])/(4*(1 - Tanh[c + d*x]^2)^2) + (3*(((1024*a^3 + 1152*a^2*b + 840*a*
b^2 + 231*b^3)*ArcTanh[Tanh[c + d*x]])/2 - (b*(1920*a^2 + 2232*a*b + 793*b
^2)*Tanh[c + d*x])/(2*(1 - Tanh[c + d*x]^2))))/4))/6)/8))/10)/12)/d
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 298
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2), x_Symbol] :> Simp[(-(
b*c - a*d))*x*((a + b*x^2)^(p + 1)/(2*a*b*(p + 1))), x] - Simp[(a*d - b*c*(
2*p + 3))/(2*a*b*(p + 1))   Int[(a + b*x^2)^(p + 1), x], x] /; FreeQ[{a, b,
 c, d, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/2 + p, 0])
```

rule 1471
```
Int[((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.),
x_Symbol] :> With[{Qx = PolynomialQuotient[(a + b*x^2 + c*x^4)^p, d + e*x^2
, x], R = Coeff[PolynomialRemainder[(a + b*x^2 + c*x^4)^p, d + e*x^2, x], x
, 0]}, Simp[(-R)*x*((d + e*x^2)^(q + 1)/(2*d*(q + 1))), x] + Simp[1/(2*d*(q
 + 1))   Int[(d + e*x^2)^(q + 1)*ExpandToSum[2*d*(q + 1)*Qx + R*(2*q + 3),
x], x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^
2 - b*d*e + a*e^2, 0] && IGtQ[p, 0] && LtQ[q, -1]
```

rule 2345
```
Int[(Pq_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{Q = PolynomialQuot
ient[Pq, a + b*x^2, x], f = Coeff[PolynomialRemainder[Pq, a + b*x^2, x], x,
 0], g = Coeff[PolynomialRemainder[Pq, a + b*x^2, x], x, 1]}, Simp[(a*g - b
*f*x)*((a + b*x^2)^(p + 1)/(2*a*b*(p + 1))), x] + Simp[1/(2*a*(p + 1))   In
t[(a + b*x^2)^(p + 1)*ExpandToSum[2*a*(p + 1)*Q + f*(2*p + 3), x], x], x]]
/; FreeQ[{a, b}, x] && PolyQ[Pq, x] && LtQ[p, -1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3688
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^4)^(p_.), x_Symbol] :> With[{ff =
 FreeFactors[Tan[e + f*x], x]}, Simp[ff/f   Subst[Int[(a + 2*a*ff^2*x^2 + (
a + b)*ff^4*x^4)^p/(1 + ff^2*x^2)^(2*p + 1), x], x, Tan[e + f*x]/ff], x]] /
; FreeQ[{a, b, e, f}, x] && IntegerQ[p]
```

Maple [A] (verified)

Time = 0.25 (sec) , antiderivative size = 193, normalized size of antiderivative = 0.91

$$\frac{b^3\left(\left(\frac{\sinh(dx+c)^{11}}{12}-\frac{11\sinh(dx+c)^9}{120}+\frac{33\sinh(dx+c)^7}{320}-\frac{77\sinh(dx+c)^5}{640}+\frac{77\sinh(dx+c)^3}{512}-\frac{231\sinh(dx+c)}{1024}\right)\cosh(dx+c)+\right.}{}$$

input
```
int((a+b*sinh(d*x+c)^4)^3,x)
```

output
```
1/d*(b^3*((1/12*sinh(d*x+c)^11-11/120*sinh(d*x+c)^9+33/320*sinh(d*x+c)^7-7
7/640*sinh(d*x+c)^5+77/512*sinh(d*x+c)^3-231/1024*sinh(d*x+c))*cosh(d*x+c)
+231/1024*d*x+231/1024*c)+3*b^2*a*((1/8*sinh(d*x+c)^7-7/48*sinh(d*x+c)^5+3
5/192*sinh(d*x+c)^3-35/128*sinh(d*x+c))*cosh(d*x+c)+35/128*d*x+35/128*c)+3
*a^2*b*((1/4*sinh(d*x+c)^3-3/8*sinh(d*x+c))*cosh(d*x+c)+3/8*d*x+3/8*c)+a^3
*(d*x+c))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 461 vs. 2(197) = 394.

Time = 0.09 (sec) , antiderivative size = 461, normalized size of antiderivative = 2.18

$$\int \left(a+b\sinh^4(c+dx)\right)^3\,dx$$
$$=\frac{15\,b^3\cosh(dx+c)\sinh(dx+c)^{11}+5\left(55\,b^3\cosh(dx+c)^3-36\,b^3\cosh(dx+c)\right)\sinh(dx+c)^9+90\,(1}{}$$

input
```
integrate((a+b*sinh(d*x+c)^4)^3,x, algorithm="fricas")
```

output
```
1/30720*(15*b^3*cosh(d*x + c)*sinh(d*x + c)^11 + 5*(55*b^3*cosh(d*x + c)^3
 - 36*b^3*cosh(d*x + c))*sinh(d*x + c)^9 + 90*(11*b^3*cosh(d*x + c)^5 - 24
*b^3*cosh(d*x + c)^3 + (8*a*b^2 + 11*b^3)*cosh(d*x + c))*sinh(d*x + c)^7 +
 6*(165*b^3*cosh(d*x + c)^7 - 756*b^3*cosh(d*x + c)^5 + 105*(8*a*b^2 + 11*
b^3)*cosh(d*x + c)^3 - 10*(96*a*b^2 + 55*b^3)*cosh(d*x + c))*sinh(d*x + c)
^5 + 5*(55*b^3*cosh(d*x + c)^9 - 432*b^3*cosh(d*x + c)^7 + 126*(8*a*b^2 +
11*b^3)*cosh(d*x + c)^5 - 40*(96*a*b^2 + 55*b^3)*cosh(d*x + c)^3 + 9*(256*
a^2*b + 448*a*b^2 + 165*b^3)*cosh(d*x + c))*sinh(d*x + c)^3 + 30*(1024*a^3
 + 1152*a^2*b + 840*a*b^2 + 231*b^3)*d*x + 15*(b^3*cosh(d*x + c)^11 - 12*b
^3*cosh(d*x + c)^9 + 6*(8*a*b^2 + 11*b^3)*cosh(d*x + c)^7 - 4*(96*a*b^2 +
55*b^3)*cosh(d*x + c)^5 + 3*(256*a^2*b + 448*a*b^2 + 165*b^3)*cosh(d*x + c
)^3 - 24*(128*a^2*b + 112*a*b^2 + 33*b^3)*cosh(d*x + c))*sinh(d*x + c))/d
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 666 vs. $2(202) = 404$.

Time = 2.62 (sec) , antiderivative size = 666, normalized size of antiderivative = 3.16

$$\int \left(a + b\sinh^4(c + dx)\right)^3 \, dx = \text{Too large to display}$$

input
```
integrate((a+b*sinh(d*x+c)**4)**3,x)
```

output
```
Piecewise((a**3*x + 9*a**2*b*x*sinh(c + d*x)**4/8 - 9*a**2*b*x*sinh(c + d*
x)**2*cosh(c + d*x)**2/4 + 9*a**2*b*x*cosh(c + d*x)**4/8 + 15*a**2*b*sinh(
c + d*x)**3*cosh(c + d*x)/(8*d) - 9*a**2*b*sinh(c + d*x)*cosh(c + d*x)**3/
(8*d) + 105*a*b**2*x*sinh(c + d*x)**8/128 - 105*a*b**2*x*sinh(c + d*x)**6*
cosh(c + d*x)**2/32 + 315*a*b**2*x*sinh(c + d*x)**4*cosh(c + d*x)**4/64 -
105*a*b**2*x*sinh(c + d*x)**2*cosh(c + d*x)**6/32 + 105*a*b**2*x*cosh(c +
d*x)**8/128 + 279*a*b**2*sinh(c + d*x)**7*cosh(c + d*x)/(128*d) - 511*a*b*
*2*sinh(c + d*x)**5*cosh(c + d*x)**3/(128*d) + 385*a*b**2*sinh(c + d*x)**3
*cosh(c + d*x)**5/(128*d) - 105*a*b**2*sinh(c + d*x)*cosh(c + d*x)**7/(128
*d) + 231*b**3*x*sinh(c + d*x)**12/1024 - 693*b**3*x*sinh(c + d*x)**10*cos
h(c + d*x)**2/512 + 3465*b**3*x*sinh(c + d*x)**8*cosh(c + d*x)**4/1024 - 1
155*b**3*x*sinh(c + d*x)**6*cosh(c + d*x)**6/256 + 3465*b**3*x*sinh(c + d*
x)**4*cosh(c + d*x)**8/1024 - 693*b**3*x*sinh(c + d*x)**2*cosh(c + d*x)**1
0/512 + 231*b**3*x*cosh(c + d*x)**12/1024 + 793*b**3*sinh(c + d*x)**11*cos
h(c + d*x)/(1024*d) - 7337*b**3*sinh(c + d*x)**9*cosh(c + d*x)**3/(3072*d)
 + 9273*b**3*sinh(c + d*x)**7*cosh(c + d*x)**5/(2560*d) - 7623*b**3*sinh(c
 + d*x)**5*cosh(c + d*x)**7/(2560*d) + 1309*b**3*sinh(c + d*x)**3*cosh(c +
 d*x)**9/(1024*d) - 231*b**3*sinh(c + d*x)*cosh(c + d*x)**11/(1024*d), Ne(
d, 0)), (x*(a + b*sinh(c)**4)**3, True))
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 344, normalized size of antiderivative = 1.63

$$\int \left(a+b\sinh^4(c+dx)\right)^3\,dx$$
$$= \frac{3}{64}\,a^2b\left(24\,x+\frac{e^{(4\,dx+4\,c)}}{d}-\frac{8\,e^{(2\,dx+2\,c)}}{d}+\frac{8\,e^{(-2\,dx-2\,c)}}{d}-\frac{e^{(-4\,dx-4\,c)}}{d}\right)+a^3x$$
$$-\frac{1}{245760}\,b^3\left(\frac{\left(72\,e^{(-2\,dx-2\,c)}-495\,e^{(-4\,dx-4\,c)}+2200\,e^{(-6\,dx-6\,c)}-7425\,e^{(-8\,dx-8\,c)}+23760\,e^{(-10\,dx-10\,c)}-\right.}{d}\right.$$
$$-\frac{1}{2048}\,ab^2\left(\frac{\left(32\,e^{(-2\,dx-2\,c)}-168\,e^{(-4\,dx-4\,c)}+672\,e^{(-6\,dx-6\,c)}-3\right)e^{(8\,dx+8\,c)}}{d}-\frac{1680\,(dx+c)}{d}-\frac{672\,e^{(-2\,c}}{}\right.$$

input
```
integrate((a+b*sinh(d*x+c)^4)^3,x, algorithm="maxima")
```

output
```
3/64*a^2*b*(24*x + e^(4*d*x + 4*c)/d - 8*e^(2*d*x + 2*c)/d + 8*e^(-2*d*x -
 2*c)/d - e^(-4*d*x - 4*c)/d) + a^3*x - 1/245760*b^3*((72*e^(-2*d*x - 2*c)
 - 495*e^(-4*d*x - 4*c) + 2200*e^(-6*d*x - 6*c) - 7425*e^(-8*d*x - 8*c) +
23760*e^(-10*d*x - 10*c) - 5)*e^(12*d*x + 12*c)/d - 55440*(d*x + c)/d - (2
3760*e^(-2*d*x - 2*c) - 7425*e^(-4*d*x - 4*c) + 2200*e^(-6*d*x - 6*c) - 49
5*e^(-8*d*x - 8*c) + 72*e^(-10*d*x - 10*c) - 5*e^(-12*d*x - 12*c))/d) - 1/
2048*a*b^2*((32*e^(-2*d*x - 2*c) - 168*e^(-4*d*x - 4*c) + 672*e^(-6*d*x -
6*c) - 3)*e^(8*d*x + 8*c)/d - 1680*(d*x + c)/d - (672*e^(-2*d*x - 2*c) - 1
68*e^(-4*d*x - 4*c) + 32*e^(-6*d*x - 6*c) - 3*e^(-8*d*x - 8*c))/d)
```

Giac [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 327, normalized size of antiderivative = 1.55

$$\begin{aligned}\int \left(a+b\sinh^4(c+dx)\right)^3\,dx &= \frac{b^3e^{(12\,dx+12\,c)}}{49152\,d}-\frac{3\,b^3e^{(10\,dx+10\,c)}}{10240\,d}\\&+\frac{3\,b^3e^{(-10\,dx-10\,c)}}{10240\,d}-\frac{b^3e^{(-12\,dx-12\,c)}}{49152\,d}\\&+\frac{1}{1024}\left(1024\,a^3+1152\,a^2b+840\,ab^2+231\,b^3\right)x\\&+\frac{3\,(8\,ab^2+11\,b^3)e^{(8\,dx+8\,c)}}{16384\,d}-\frac{(96\,ab^2+55\,b^3)e^{(6\,dx+6\,c)}}{6144\,d}\\&+\frac{3\,(256\,a^2b+448\,ab^2+165\,b^3)e^{(4\,dx+4\,c)}}{16384\,d}\\&-\frac{3\,(128\,a^2b+112\,ab^2+33\,b^3)e^{(2\,dx+2\,c)}}{1024\,d}\\&+\frac{3\,(128\,a^2b+112\,ab^2+33\,b^3)e^{(-2\,dx-2\,c)}}{1024\,d}\\&-\frac{3\,(256\,a^2b+448\,ab^2+165\,b^3)e^{(-4\,dx-4\,c)}}{16384\,d}\\&+\frac{(96\,ab^2+55\,b^3)e^{(-6\,dx-6\,c)}}{6144\,d}-\frac{3\,(8\,ab^2+11\,b^3)e^{(-8\,dx-8\,c)}}{16384\,d}\end{aligned}$$

input
```
integrate((a+b*sinh(d*x+c)^4)^3,x, algorithm="giac")
```

output
```
1/49152*b^3*e^(12*d*x + 12*c)/d - 3/10240*b^3*e^(10*d*x + 10*c)/d + 3/1024
0*b^3*e^(-10*d*x - 10*c)/d - 1/49152*b^3*e^(-12*d*x - 12*c)/d + 1/1024*(10
24*a^3 + 1152*a^2*b + 840*a*b^2 + 231*b^3)*x + 3/16384*(8*a*b^2 + 11*b^3)*
e^(8*d*x + 8*c)/d - 1/6144*(96*a*b^2 + 55*b^3)*e^(6*d*x + 6*c)/d + 3/16384
*(256*a^2*b + 448*a*b^2 + 165*b^3)*e^(4*d*x + 4*c)/d - 3/1024*(128*a^2*b +
 112*a*b^2 + 33*b^3)*e^(2*d*x + 2*c)/d + 3/1024*(128*a^2*b + 112*a*b^2 + 3
3*b^3)*e^(-2*d*x - 2*c)/d - 3/16384*(256*a^2*b + 448*a*b^2 + 165*b^3)*e^(-
4*d*x - 4*c)/d + 1/6144*(96*a*b^2 + 55*b^3)*e^(-6*d*x - 6*c)/d - 3/16384*(
8*a*b^2 + 11*b^3)*e^(-8*d*x - 8*c)/d
```

Mupad [B] (verification not implemented)

Time = 2.07 (sec) , antiderivative size = 210, normalized size of antiderivative = 1.00

$$\int \left(a+b\sinh^4(c+dx)\right)^3\,dx$$
$$=\frac{\frac{7425\,b^3\sinh(4\,c+4\,d\,x)}{8}-2970\,b^3\sinh(2\,c+2\,d\,x)-275\,b^3\sinh(6\,c+6\,d\,x)+\frac{495\,b^3\sinh(8\,c+8\,d\,x)}{8}-9\,b^3\sinh(10\,c$$

input
```
int((a + b*sinh(c + d*x)^4)^3,x)
```

output
```
((7425*b^3*sinh(4*c + 4*d*x))/8 - 2970*b^3*sinh(2*c + 2*d*x) - 275*b^3*sin
h(6*c + 6*d*x) + (495*b^3*sinh(8*c + 8*d*x))/8 - 9*b^3*sinh(10*c + 10*d*x)
 + (5*b^3*sinh(12*c + 12*d*x))/8 - 10080*a*b^2*sinh(2*c + 2*d*x) - 11520*a
^2*b*sinh(2*c + 2*d*x) + 2520*a*b^2*sinh(4*c + 4*d*x) + 1440*a^2*b*sinh(4*
c + 4*d*x) - 480*a*b^2*sinh(6*c + 6*d*x) + 45*a*b^2*sinh(8*c + 8*d*x) + 15
360*a^3*d*x + 3465*b^3*d*x + 12600*a*b^2*d*x + 17280*a^2*b*d*x)/(15360*d)
```

Reduce [B] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 450, normalized size of antiderivative = 2.13

$$\int \left(a+b\sinh^4(c+dx)\right)^3\,dx$$
$$=\frac{360e^{20dx+20c}a\,b^2-5b^3+5e^{24dx+24c}b^3+55440e^{12dx+12c}b^3dx+72e^{2dx+2c}b^3+23760e^{10dx+10c}b^3+245760e^{1}$$

input

```
int((a+b*sinh(d*x+c)^4)^3,x)
```

output

```
(5*e**(24*c + 24*d*x)*b**3 - 72*e**(22*c + 22*d*x)*b**3 + 360*e**(20*c + 2
0*d*x)*a*b**2 + 495*e**(20*c + 20*d*x)*b**3 - 3840*e**(18*c + 18*d*x)*a*b*
*2 - 2200*e**(18*c + 18*d*x)*b**3 + 11520*e**(16*c + 16*d*x)*a**2*b + 2016
0*e**(16*c + 16*d*x)*a*b**2 + 7425*e**(16*c + 16*d*x)*b**3 - 92160*e**(14*
c + 14*d*x)*a**2*b - 80640*e**(14*c + 14*d*x)*a*b**2 - 23760*e**(14*c + 14
*d*x)*b**3 + 245760*e**(12*c + 12*d*x)*a**3*d*x + 276480*e**(12*c + 12*d*x
)*a**2*b*d*x + 201600*e**(12*c + 12*d*x)*a*b**2*d*x + 55440*e**(12*c + 12*
d*x)*b**3*d*x + 92160*e**(10*c + 10*d*x)*a**2*b + 80640*e**(10*c + 10*d*x)
*a*b**2 + 23760*e**(10*c + 10*d*x)*b**3 - 11520*e**(8*c + 8*d*x)*a**2*b -
20160*e**(8*c + 8*d*x)*a*b**2 - 7425*e**(8*c + 8*d*x)*b**3 + 3840*e**(6*c
+ 6*d*x)*a*b**2 + 2200*e**(6*c + 6*d*x)*b**3 - 360*e**(4*c + 4*d*x)*a*b**2
 - 495*e**(4*c + 4*d*x)*b**3 + 72*e**(2*c + 2*d*x)*b**3 - 5*b**3)/(245760*
e**(12*c + 12*d*x)*d)
```

3.194 $\int \operatorname{csch}^2(c+dx) \left(a+b\sinh^4(c+dx)\right)^3 \, dx$

Optimal result

Integrand size = 23, antiderivative size = 181

$$\begin{aligned}
&\int \operatorname{csch}^2(c+dx) \left(a+b\sinh^4(c+dx)\right)^3 \, dx \\
&= -\frac{3}{256}b\left(128a^2+80ab+21b^2\right)x - \frac{a^3\coth(c+dx)}{d} \\
&\quad + \frac{b(384a^2+528ab+193b^2)\cosh(c+dx)\sinh(c+dx)}{256d} \\
&\quad - \frac{b^2(208a+149b)\cosh^3(c+dx)\sinh(c+dx)}{128d} \\
&\quad + \frac{b^2(80a+171b)\cosh^5(c+dx)\sinh(c+dx)}{160d} \\
&\quad - \frac{41b^3\cosh^7(c+dx)\sinh(c+dx)}{80d} + \frac{b^3\cosh^9(c+dx)\sinh(c+dx)}{10d}
\end{aligned}$$

output
```
-3/256*b*(128*a^2+80*a*b+21*b^2)*x-a^3*coth(d*x+c)/d+1/256*b*(384*a^2+528*
a*b+193*b^2)*cosh(d*x+c)*sinh(d*x+c)/d-1/128*b^2*(208*a+149*b)*cosh(d*x+c)
^3*sinh(d*x+c)/d+1/160*b^2*(80*a+171*b)*cosh(d*x+c)^5*sinh(d*x+c)/d-41/80*
b^3*cosh(d*x+c)^7*sinh(d*x+c)/d+1/10*b^3*cosh(d*x+c)^9*sinh(d*x+c)/d
```

Mathematica [A] (verified)

Time = 6.52 (sec) , antiderivative size = 134, normalized size of antiderivative = 0.74

$$\int \operatorname{csch}^2(c+dx) \left(a+b\sinh^4(c+dx)\right)^3 \, dx$$
$$= \frac{-120b(128a^2+80ab+21b^2)(c+dx) - 10240a^3\coth(c+dx) + 60b(128a^2+120ab+35b^2)\sinh(2(c+$$

input
```
Integrate[Csch[c + d*x]^2*(a + b*Sinh[c + d*x]^4)^3,x]
```

output
```
(-120*b*(128*a^2 + 80*a*b + 21*b^2)*(c + d*x) - 10240*a^3*Coth[c + d*x] +
60*b*(128*a^2 + 120*a*b + 35*b^2)*Sinh[2*(c + d*x)] - 120*b^2*(12*a + 5*b)
*Sinh[4*(c + d*x)] + 10*b^2*(16*a + 15*b)*Sinh[6*(c + d*x)] - 25*b^3*Sinh[
8*(c + d*x)] + 2*b^3*Sinh[10*(c + d*x)])/(10240*d)
```

Rubi [A] (verified)

Time = 0.97 (sec) , antiderivative size = 228, normalized size of antiderivative = 1.26, number of steps used = 16, number of rules used = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.652$, Rules used = {3042, 25, 3696, 1582, 25, 2336, 25, 2336, 27, 2336, 25, 1582, 25, 359, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \operatorname{csch}^2(c+dx) \left(a+b\sinh^4(c+dx)\right)^3 \, dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int -\frac{\left(a+b\sin(ic+idx)^4\right)^3}{\sin(ic+idx)^2}dx$$

$$\Big\downarrow \text{ 25}$$

$$-\int \frac{\left(b\sin(ic+idx)^4+a\right)^3}{\sin(ic+idx)^2}dx$$

$$\Big\downarrow \text{ 3696}$$

$$\frac{\int \frac{\coth^2(c+dx)\left((a+b)\tanh^4(c+dx)-2a\tanh^2(c+dx)+a\right)^3}{\left(1-\tanh^2(c+dx)\right)^6}d\tanh(c+dx)}{d}$$

$\big\downarrow$ 1582

$$\frac{\frac{b^3\tanh(c+dx)}{10\left(1-\tanh^2(c+dx)\right)^5}-\frac{1}{10}\int -\frac{\coth^2(c+dx)\left(-10(a+b)^3\tanh^{10}(c+dx)+10(5a-b)(a+b)^2\tanh^8(c+dx)-10\left(10a^3+9ba^2+b^3\right)\tanh^6(c+dx)+1}{\left(1-\tanh^2(c+dx)\right)^5}}{d}$$

$\big\downarrow$ 25

$$\frac{\frac{1}{10}\int \frac{\coth^2(c+dx)\left(-10(a+b)^3\tanh^{10}(c+dx)+10(5a-b)(a+b)^2\tanh^8(c+dx)-10\left(10a^3+9ba^2+b^3\right)\tanh^6(c+dx)+10\left(10a^3+3ba^2-b^3\right)\tanh^4(c+}{\left(1-\tanh^2(c+dx)\right)^5}}{d}$$

$\big\downarrow$ 2336

$$\frac{\frac{1}{10}\left(-\frac{1}{8}\int -\frac{\coth^2(c+dx)\left(80(a+b)^3\tanh^8(c+dx)-160(2a-b)(a+b)^2\tanh^6(c+dx)+240\left(2a^3+ba^2+b^3\right)\tanh^4(c+dx)-\left(320a^3-33b^3\right)\tanh^2(c+}{\left(1-\tanh^2(c+dx)\right)^4}\right.}{d}$$

$\big\downarrow$ 25

$$\frac{\frac{1}{10}\left(\frac{1}{8}\int \frac{\coth^2(c+dx)\left(80(a+b)^3\tanh^8(c+dx)-160(2a-b)(a+b)^2\tanh^6(c+dx)+240\left(2a^3+ba^2+b^3\right)\tanh^4(c+dx)-\left(320a^3-33b^3\right)\tanh^2(c+dx)-}{\left(1-\tanh^2(c+dx)\right)^4}\right.}{d}$$

$\big\downarrow$ 2336

$$\frac{\frac{1}{10}\left(\frac{1}{8}\left(\frac{b^2(80a+171b)\tanh(c+dx)}{2\left(1-\tanh^2(c+dx)\right)^3}-\frac{1}{6}\int -\frac{15\coth^2(c+dx)\left(-32(a+b)^3\tanh^6(c+dx)+96(a-b)(a+b)^2\tanh^4(c+dx)-\left(96a^3+16b^2a+21b^3\right)\text{ta}}{\left(1-\tanh^2(c+dx)\right)^3}\right.\right.}{d}$$

$\big\downarrow$ 27

$$\frac{\frac{1}{10}\left(\frac{1}{8}\left(\frac{5}{2}\int \frac{\coth^2(c+dx)\left(-32(a+b)^3\tanh^6(c+dx)+96(a-b)(a+b)^2\tanh^4(c+dx)-\left(96a^3+16b^2a+21b^3\right)\tanh^2(c+dx)+32a^3\right)}{\left(1-\tanh^2(c+dx)\right)^3}d\tanh(c+d\right.\right.}{d}$$

$\big\downarrow$ 2336

$$\frac{\frac{1}{10}\left(\frac{1}{8}\left(\frac{5}{2}\left(-\frac{1}{4}\int -\frac{\coth^2(c+dx)\left(128(a+b)^3\tanh^4(c+dx)-\left(256a^3-144b^2a-65b^3\right)\tanh^2(c+dx)+128a^3\right)}{\left(1-\tanh^2(c+dx)\right)^2}d\tanh(c+dx)-\frac{b^2(208a+14}{4(1-\tan}\right.\right.\right.}{d}$$

$\big\downarrow$ 25

$$\frac{\frac{1}{10}\left(\frac{1}{8}\left(\frac{5}{2}\left(\frac{1}{4}\int \frac{\coth^2(c+dx)\left(128(a+b)^3\tanh^4(c+dx)-\left(256a^3-144b^2a-65b^3\right)\tanh^2(c+dx)+128a^3\right)}{\left(1-\tanh^2(c+dx)\right)^2}d\tanh(c+dx)-\frac{b^2(208a+149b)\tanh}{4(1-\tanh^2(c}\right.\right.\right.}{d}$$

$\Big\downarrow$ 1582

$$\frac{\frac{1}{10}\left(\frac{1}{8}\left(\frac{5}{2}\left(\frac{1}{4}\left(\frac{b(384a^2+528ab+193b^2)\tanh(c+dx)}{2(1-\tanh^2(c+dx))}-\frac{1}{2}\int -\frac{\coth^2(c+dx)\left(256a^3-\left(256a^3+384ba^2+240b^2a+63b^3\right)\tanh^2(c+dx)\right)}{1-\tanh^2(c+dx)}d\tanh(c\right.\right.\right.\right.}{d}$$

$\Big\downarrow$ 25

$$\frac{\frac{1}{10}\left(\frac{1}{8}\left(\frac{5}{2}\left(\frac{1}{4}\left(\frac{1}{2}\int \frac{\coth^2(c+dx)\left(256a^3-\left(256a^3+384ba^2+240b^2a+63b^3\right)\tanh^2(c+dx)\right)}{1-\tanh^2(c+dx)}d\tanh(c+dx)+\frac{b(384a^2+528ab+193b^2)\tanh(c}{2(1-\tanh^2(c+dx))}\right.\right.\right.\right.}{d}$$

$\Big\downarrow$ 359

$$\frac{\frac{1}{10}\left(\frac{1}{8}\left(\frac{5}{2}\left(\frac{1}{4}\left(\frac{1}{2}\left(-3b\left(128a^2+80ab+21b^2\right)\int \frac{1}{1-\tanh^2(c+dx)}d\tanh(c+dx)-256a^3\coth(c+dx)\right)+\frac{b(384a^2+528ab-}{2(1-\tan}\right.\right.\right.\right.}{d}$$

$\Big\downarrow$ 219

$$\frac{\frac{1}{10}\left(\frac{1}{8}\left(\frac{5}{2}\left(\frac{1}{4}\left(\frac{b(384a^2+528ab+193b^2)\tanh(c+dx)}{2(1-\tanh^2(c+dx))}+\frac{1}{2}\left(-256a^3\coth(c+dx)-3b\left(128a^2+80ab+21b^2\right)\operatorname{arctanh}(\tanh(c\right.\right.\right.\right.\right.}{d}$$

input
```
Int[Csch[c + d*x]^2*(a + b*Sinh[c + d*x]^4)^3,x]
```

output
```
((b^3*Tanh[c + d*x])/(10*(1 - Tanh[c + d*x]^2)^5) + ((-41*b^3*Tanh[c + d*x
])/(8*(1 - Tanh[c + d*x]^2)^4) + ((b^2*(80*a + 171*b)*Tanh[c + d*x])/(2*(1
 - Tanh[c + d*x]^2)^3) + (5*(-1/4*(b^2*(208*a + 149*b)*Tanh[c + d*x])/(1 -
 Tanh[c + d*x]^2)^2 + ((-3*b*(128*a^2 + 80*a*b + 21*b^2)*ArcTanh[Tanh[c +
d*x]] - 256*a^3*Coth[c + d*x])/2 + (b*(384*a^2 + 528*a*b + 193*b^2)*Tanh[c
 + d*x])/(2*(1 - Tanh[c + d*x]^2)))/4))/2)/8)/10)/d
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 359
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2), x
_Symbol] :> Simp[c*(e*x)^(m + 1)*((a + b*x^2)^(p + 1)/(a*e*(m + 1))), x] +
Simp[(a*d*(m + 1) - b*c*(m + 2*p + 3))/(a*e^2*(m + 1))   Int[(e*x)^(m + 2)*
(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c - a*d, 0]
&& LtQ[m, -1] &&  !ILtQ[p, -1]
```

rule 1582
```
Int[(x_)^(m_)*((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^
4)^(p_.), x_Symbol] :> Simp[(-d)^(m/2 - 1)*(c*d^2 - b*d*e + a*e^2)^p*x*((d
+ e*x^2)^(q + 1)/(2*e^(2*p + m/2)*(q + 1))), x] + Simp[(-d)^(m/2 - 1)/(2*e^
(2*p)*(q + 1))   Int[x^m*(d + e*x^2)^(q + 1)*ExpandToSum[Together[(1/(d + e
*x^2))*(2*(-d)^(-m/2 + 1)*e^(2*p)*(q + 1)*(a + b*x^2 + c*x^4)^p - ((c*d^2 -
 b*d*e + a*e^2)^p/(e^(m/2)*x^m))*(d + e*(2*q + 3)*x^2))], x], x], x] /; Fre
eQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && IGtQ[p, 0] && ILtQ[q, -1]
&& ILtQ[m/2, 0]
```

rule 2336
```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[
{Q = PolynomialQuotient[(c*x)^m*Pq, a + b*x^2, x], f = Coeff[PolynomialRema
inder[(c*x)^m*Pq, a + b*x^2, x], x, 0], g = Coeff[PolynomialRemainder[(c*x)
^m*Pq, a + b*x^2, x], x, 1]}, Simp[(a*g - b*f*x)*((a + b*x^2)^(p + 1)/(2*a*
b*(p + 1))), x] + Simp[1/(2*a*(p + 1))   Int[(c*x)^m*(a + b*x^2)^(p + 1)*Ex
pandToSum[(2*a*(p + 1)*Q)/(c*x)^m + (f*(2*p + 3))/(c*x)^m, x], x], x]] /; F
reeQ[{a, b, c}, x] && PolyQ[Pq, x] && LtQ[p, -1] && ILtQ[m, 0]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3696
```
Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^4)^(
p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Simp[ff^(m + 1
)/f   Subst[Int[x^m*((a + 2*a*ff^2*x^2 + (a + b)*ff^4*x^4)^p/(1 + ff^2*x^2)
^(m/2 + 2*p + 1)), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f}, x] &
& IntegerQ[m/2] && IntegerQ[p]
```

Maple [A] (verified)

Time = 6.20 (sec) , antiderivative size = 151, normalized size of antiderivative = 0.83

method	result
parallelrisch	$\frac{-2\coth\left(\frac{dx}{2}+\frac{c}{2}\right)a^3+\operatorname{sech}\left(\frac{dx}{2}+\frac{c}{2}\right)\operatorname{csch}\left(\frac{dx}{2}+\frac{c}{2}\right)a^3-3\left(\frac{\left(-a^2-\frac{15}{16}ab-\frac{35}{128}b^2\right)\sinh(2dx+2c)}{2}+\frac{3b\left(a+\frac{5b}{12}\right)\sinh(4dx+4c)}{32}-\frac{(}{}\right.}{2d}$
derivativedivides	$\frac{-a^3\coth(dx+c)+3a^2b\left(\frac{\cosh(dx+c)\sinh(dx+c)}{2}-\frac{dx}{2}-\frac{c}{2}\right)+3b^2a\left(\left(\frac{\sinh(dx+c)^5}{6}-\frac{5\sinh(dx+c)^3}{24}+\frac{5\sinh(dx+c)}{16}\right)\cosh(dx+\right.}{}$
default	$\frac{-a^3\coth(dx+c)+3a^2b\left(\frac{\cosh(dx+c)\sinh(dx+c)}{2}-\frac{dx}{2}-\frac{c}{2}\right)+3b^2a\left(\left(\frac{\sinh(dx+c)^5}{6}-\frac{5\sinh(dx+c)^3}{24}+\frac{5\sinh(dx+c)}{16}\right)\cosh(dx+\right.}{}$
risch	$-\frac{3a^2bx}{2}-\frac{15a\,b^2x}{16}-\frac{63b^3x}{256}+\frac{b^3\mathrm{e}^{10dx+10c}}{10240d}-\frac{5b^3\mathrm{e}^{8dx+8c}}{4096d}+\frac{b^2\mathrm{e}^{6dx+6c}a}{128d}+\frac{15b^3\mathrm{e}^{6dx+6c}}{2048d}-\frac{9\,\mathrm{e}^{4dx+4c}b^2a}{128d}-$

input
```
int(csch(d*x+c)^2*(a+b*sinh(d*x+c)^4)^3,x,method=_RETURNVERBOSE)
```

output
```
1/2*(-2*coth(1/2*d*x+1/2*c)*a^3+sech(1/2*d*x+1/2*c)*csch(1/2*d*x+1/2*c)*a^
3-3*(1/2*(-a^2-15/16*a*b-35/128*b^2)*sinh(2*d*x+2*c)+3/32*b*(a+5/12*b)*sin
h(4*d*x+4*c)-1/96*(a+15/16*b)*b*sinh(6*d*x+6*c)-1/7680*b^2*sinh(10*d*x+10*
c)+5/3072*b^2*sinh(8*d*x+8*c)+d*x*(a^2+5/8*a*b+21/128*b^2))*b)/d
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 474 vs. $2(169) = 338$.

Time = 0.10 (sec) , antiderivative size = 474, normalized size of antiderivative = 2.62

$$\int \operatorname{csch}^2(c+dx)\left(a+b\sinh^4(c+dx)\right)^3\,dx = \frac{2\,b^3\cosh\left(dx+c\right)^{11}+22\,b^3\cosh\left(dx+c\right)\sinh\left(dx+c\right)^{10}-27\,b^3\cosh\left(dx+c\right)^9+3\left(110\,b^3\cosh\left(dx+c\right)^3\right.}{}$$

input
```
integrate(csch(d*x+c)^2*(a+b*sinh(d*x+c)^4)^3,x, algorithm="fricas")
```

output
```
1/20480*(2*b^3*cosh(d*x + c)^11 + 22*b^3*cosh(d*x + c)*sinh(d*x + c)^10 -
27*b^3*cosh(d*x + c)^9 + 3*(110*b^3*cosh(d*x + c)^3 - 81*b^3*cosh(d*x + c)
)*sinh(d*x + c)^8 + 5*(32*a*b^2 + 35*b^3)*cosh(d*x + c)^7 + 7*(132*b^3*cos
h(d*x + c)^5 - 324*b^3*cosh(d*x + c)^3 + 5*(32*a*b^2 + 35*b^3)*cosh(d*x +
c))*sinh(d*x + c)^6 - 50*(32*a*b^2 + 15*b^3)*cosh(d*x + c)^5 + (660*b^3*co
sh(d*x + c)^7 - 3402*b^3*cosh(d*x + c)^5 + 175*(32*a*b^2 + 35*b^3)*cosh(d*
x + c)^3 - 250*(32*a*b^2 + 15*b^3)*cosh(d*x + c))*sinh(d*x + c)^4 + 60*(12
8*a^2*b + 144*a*b^2 + 45*b^3)*cosh(d*x + c)^3 + (110*b^3*cosh(d*x + c)^9 -
 972*b^3*cosh(d*x + c)^7 + 105*(32*a*b^2 + 35*b^3)*cosh(d*x + c)^5 - 500*(
32*a*b^2 + 15*b^3)*cosh(d*x + c)^3 + 180*(128*a^2*b + 144*a*b^2 + 45*b^3)*
cosh(d*x + c))*sinh(d*x + c)^2 - 20*(1024*a^3 + 384*a^2*b + 360*a*b^2 + 10
5*b^3)*cosh(d*x + c) + 80*(256*a^3 - 3*(128*a^2*b + 80*a*b^2 + 21*b^3)*d*x
)*sinh(d*x + c))/(d*sinh(d*x + c))
```

Sympy [F(-1)]

Timed out.

$$\int \operatorname{csch}^2(c+dx)\left(a+b\sinh^4(c+dx)\right)^3\,dx = \text{Timed out}$$

input
```
integrate(csch(d*x+c)**2*(a+b*sinh(d*x+c)**4)**3,x)
```

output
```
Timed out
```

Maxima [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 284, normalized size of antiderivative = 1.57

$$\int \operatorname{csch}^2(c+dx)\left(a+b\sinh^4(c+dx)\right)^3\,dx = -\frac{3}{8}a^2b\left(4x-\frac{e^{(2dx+2c)}}{d}+\frac{e^{(-2dx-2c)}}{d}\right)$$
$$-\frac{1}{20480}b^3\left(\frac{\left(25e^{(-2dx-2c)}-150e^{(-4dx-4c)}+600e^{(-6dx-6c)}-2100e^{(-8dx-8c)}-2\right)e^{(10dx+10c)}}{d}+\frac{5040(\ldots}{}\right.$$
$$-\frac{1}{128}ab^2\left(\frac{\left(9e^{(-2dx-2c)}-45e^{(-4dx-4c)}-1\right)e^{(6dx+6c)}}{d}+\frac{120(dx+c)}{d}+\frac{45e^{(-2dx-2c)}-9e^{(-4dx-4c)}+\ldots}{d}\right.$$
$$+\frac{2a^3}{d(e^{(-2dx-2c)}-1)}$$

input
```
integrate(csch(d*x+c)^2*(a+b*sinh(d*x+c)^4)^3,x, algorithm="maxima")
```

output
```
-3/8*a^2*b*(4*x - e^(2*d*x + 2*c)/d + e^(-2*d*x - 2*c)/d) - 1/20480*b^3*((
25*e^(-2*d*x - 2*c) - 150*e^(-4*d*x - 4*c) + 600*e^(-6*d*x - 6*c) - 2100*e
^(-8*d*x - 8*c) - 2)*e^(10*d*x + 10*c)/d + 5040*(d*x + c)/d + (2100*e^(-2*
d*x - 2*c) - 600*e^(-4*d*x - 4*c) + 150*e^(-6*d*x - 6*c) - 25*e^(-8*d*x -
8*c) + 2*e^(-10*d*x - 10*c))/d) - 1/128*a*b^2*((9*e^(-2*d*x - 2*c) - 45*e^
(-4*d*x - 4*c) - 1)*e^(6*d*x + 6*c)/d + 120*(d*x + c)/d + (45*e^(-2*d*x -
2*c) - 9*e^(-4*d*x - 4*c) + e^(-6*d*x - 6*c))/d) + 2*a^3/(d*(e^(-2*d*x - 2
*c) - 1))
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 355 vs. $2(169) = 338$.

Time = 0.28 (sec) , antiderivative size = 355, normalized size of antiderivative = 1.96

$$\int \operatorname{csch}^2(c+dx)\left(a+b\sinh^4(c+dx)\right)^3\,dx$$
$$=\frac{2b^3e^{(10dx+10c)}-25b^3e^{(8dx+8c)}+160ab^2e^{(6dx+6c)}+150b^3e^{(6dx+6c)}-1440ab^2e^{(4dx+4c)}-600b^3e^{(4dx+4c)}\ldots}{\ldots}$$

input
```
integrate(csch(d*x+c)^2*(a+b*sinh(d*x+c)^4)^3,x, algorithm="giac")
```

output
```
1/20480*(2*b^3*e^(10*d*x + 10*c) - 25*b^3*e^(8*d*x + 8*c) + 160*a*b^2*e^(6
*d*x + 6*c) + 150*b^3*e^(6*d*x + 6*c) - 1440*a*b^2*e^(4*d*x + 4*c) - 600*b
^3*e^(4*d*x + 4*c) + 7680*a^2*b*e^(2*d*x + 2*c) + 7200*a*b^2*e^(2*d*x + 2*
c) + 2100*b^3*e^(2*d*x + 2*c) - 240*(128*a^2*b + 80*a*b^2 + 21*b^3)*(d*x +
 c) - 40960*a^3/(e^(2*d*x + 2*c) - 1) + (35072*a^2*b*e^(10*d*x + 10*c) + 2
1920*a*b^2*e^(10*d*x + 10*c) + 5754*b^3*e^(10*d*x + 10*c) - 7680*a^2*b*e^(
8*d*x + 8*c) - 7200*a*b^2*e^(8*d*x + 8*c) - 2100*b^3*e^(8*d*x + 8*c) + 144
0*a*b^2*e^(6*d*x + 6*c) + 600*b^3*e^(6*d*x + 6*c) - 160*a*b^2*e^(4*d*x + 4
*c) - 150*b^3*e^(4*d*x + 4*c) + 25*b^3*e^(2*d*x + 2*c) - 2*b^3)*e^(-10*d*x
 - 10*c))/d
```

Mupad [B] (verification not implemented)

Time = 2.01 (sec) , antiderivative size = 265, normalized size of antiderivative = 1.46

$$\int \operatorname{csch}^2(c+dx)\left(a+b\sinh^4(c+dx)\right)^3\,dx = \frac{5\,b^3\,\mathrm{e}^{-8\,c-8\,d\,x}}{4096\,d}-\frac{2\,a^3}{d\,\left(\mathrm{e}^{2\,c+2\,d\,x}-1\right)}-\frac{5\,b^3\,\mathrm{e}^{8\,c+8\,d\,x}}{4096\,d}-\frac{b^3\,\mathrm{e}^{-10\,c-10\,d\,x}}{10240\,d}+\frac{b^3\,\mathrm{e}^{10\,c+10\,d\,x}}{10240\,d}-\frac{3\,b\,x\,\left(128\,a^2+80\,a\,b+21\,b^2\right)}{256}-\frac{3\,b\,\mathrm{e}^{-2\,c-2\,d\,x}\,\left(128\,a^2+120\,a\,b+35\,b^2\right)}{1024\,d}+\frac{3\,b\,\mathrm{e}^{2\,c+2\,d\,x}\,\left(128\,a^2+120\,a\,b+35\,b^2\right)}{1024\,d}+\frac{3\,b^2\,\mathrm{e}^{-4\,c-4\,d\,x}\,\left(12\,a+5\,b\right)}{512\,d}-\frac{3\,b^2\,\mathrm{e}^{4\,c+4\,d\,x}\,\left(12\,a+5\,b\right)}{512\,d}-\frac{b^2\,\mathrm{e}^{-6\,c-6\,d\,x}\,\left(16\,a+15\,b\right)}{2048\,d}+\frac{b^2\,\mathrm{e}^{6\,c+6\,d\,x}\,\left(16\,a+15\,b\right)}{2048\,d}$$

input
```
int((a + b*sinh(c + d*x)^4)^3/sinh(c + d*x)^2,x)
```

output
```
(5*b^3*exp(- 8*c - 8*d*x))/(4096*d) - (2*a^3)/(d*(exp(2*c + 2*d*x) - 1)) -
 (5*b^3*exp(8*c + 8*d*x))/(4096*d) - (b^3*exp(- 10*c - 10*d*x))/(10240*d)
+ (b^3*exp(10*c + 10*d*x))/(10240*d) - (3*b*x*(80*a*b + 128*a^2 + 21*b^2))
/256 - (3*b*exp(- 2*c - 2*d*x)*(120*a*b + 128*a^2 + 35*b^2))/(1024*d) + (3
*b*exp(2*c + 2*d*x)*(120*a*b + 128*a^2 + 35*b^2))/(1024*d) + (3*b^2*exp(-
4*c - 4*d*x)*(12*a + 5*b))/(512*d) - (3*b^2*exp(4*c + 4*d*x)*(12*a + 5*b))
/(512*d) - (b^2*exp(- 6*c - 6*d*x)*(16*a + 15*b))/(2048*d) + (b^2*exp(6*c
+ 6*d*x)*(16*a + 15*b))/(2048*d)
```

Reduce [B] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 468, normalized size of antiderivative = 2.59

$$\int \operatorname{csch}^2(c+dx)\left(a+b\sinh^4(c+dx)\right)^3\,dx$$
$$=\frac{2b^3+30720e^{10dx+10c}a^2bdx+19200e^{10dx+10c}ab^2dx-5040e^{12dx+12c}b^3dx-27e^{2dx+2c}b^3+5040e^{10dx+10c}b^3d}{}$$

input
```
int(csch(d*x+c)^2*(a+b*sinh(d*x+c)^4)^3,x)
```

output
```
(2*e**(22*c + 22*d*x)*b**3 - 27*e**(20*c + 20*d*x)*b**3 + 160*e**(18*c + 1
8*d*x)*a*b**2 + 175*e**(18*c + 18*d*x)*b**3 - 1600*e**(16*c + 16*d*x)*a*b*
*2 - 750*e**(16*c + 16*d*x)*b**3 + 7680*e**(14*c + 14*d*x)*a**2*b + 8640*e
**(14*c + 14*d*x)*a*b**2 + 2700*e**(14*c + 14*d*x)*b**3 - 40960*e**(12*c +
 12*d*x)*a**3 - 30720*e**(12*c + 12*d*x)*a**2*b*d*x - 15360*e**(12*c + 12*
d*x)*a**2*b - 19200*e**(12*c + 12*d*x)*a*b**2*d*x - 14400*e**(12*c + 12*d*
x)*a*b**2 - 5040*e**(12*c + 12*d*x)*b**3*d*x - 4200*e**(12*c + 12*d*x)*b**
3 + 30720*e**(10*c + 10*d*x)*a**2*b*d*x + 19200*e**(10*c + 10*d*x)*a*b**2*
d*x + 5040*e**(10*c + 10*d*x)*b**3*d*x + 7680*e**(8*c + 8*d*x)*a**2*b + 86
40*e**(8*c + 8*d*x)*a*b**2 + 2700*e**(8*c + 8*d*x)*b**3 - 1600*e**(6*c + 6
*d*x)*a*b**2 - 750*e**(6*c + 6*d*x)*b**3 + 160*e**(4*c + 4*d*x)*a*b**2 + 1
75*e**(4*c + 4*d*x)*b**3 - 27*e**(2*c + 2*d*x)*b**3 + 2*b**3)/(20480*e**(1
0*c + 10*d*x)*d*(e**(2*c + 2*d*x) - 1))
```

3.195 $\int \text{csch}^4(c+dx)\left(a+b\sinh^4(c+dx)\right)^3\,dx$

Optimal result

Integrand size = 23, antiderivative size = 161

$$\begin{aligned}
&\int \text{csch}^4(c+dx)\left(a+b\sinh^4(c+dx)\right)^3\,dx\\
&=\frac{1}{128}b\left(384a^2+144ab+35b^2\right)x+\frac{a^3\coth(c+dx)}{d}\\
&\quad-\frac{a^3\coth^3(c+dx)}{3d}-\frac{3b^2(80a+31b)\cosh(c+dx)\sinh(c+dx)}{128d}\\
&\quad+\frac{b^2(144a+163b)\cosh^3(c+dx)\sinh(c+dx)}{192d}\\
&\quad-\frac{25b^3\cosh^5(c+dx)\sinh(c+dx)}{48d}+\frac{b^3\cosh^7(c+dx)\sinh(c+dx)}{8d}
\end{aligned}$$

output
```
1/128*b*(384*a^2+144*a*b+35*b^2)*x+a^3*coth(d*x+c)/d-1/3*a^3*coth(d*x+c)^3
/d-3/128*b^2*(80*a+31*b)*cosh(d*x+c)*sinh(d*x+c)/d+1/192*b^2*(144*a+163*b)
*cosh(d*x+c)^3*sinh(d*x+c)/d-25/48*b^3*cosh(d*x+c)^5*sinh(d*x+c)/d+1/8*b^3
*cosh(d*x+c)^7*sinh(d*x+c)/d
```

Mathematica [A] (verified)

Time = 2.39 (sec) , antiderivative size = 131, normalized size of antiderivative = 0.81

$$\int \operatorname{csch}^4(c+dx)\left(a+b\sinh^4(c+dx)\right)^3\,dx$$
$$=\frac{-1024a^3\coth(c+dx)\left(-2+\operatorname{csch}^2(c+dx)\right)+b(9216a^2c+3456abc+840b^2c+9216a^2dx+3456abdx+\cdots}{\cdots}$$

input
```
Integrate[Csch[c + d*x]^4*(a + b*Sinh[c + d*x]^4)^3,x]
```

output
```
(-1024*a^3*Coth[c + d*x]*(-2 + Csch[c + d*x]^2) + b*(9216*a^2*c + 3456*a*b
*c + 840*b^2*c + 9216*a^2*d*x + 3456*a*b*d*x + 840*b^2*d*x - 96*b*(24*a +
7*b)*Sinh[2*(c + d*x)] + 24*b*(12*a + 7*b)*Sinh[4*(c + d*x)] - 32*b^2*Sinh
[6*(c + d*x)] + 3*b^2*Sinh[8*(c + d*x)]))/(3072*d)
```

Rubi [A] (verified)

Time = 1.06 (sec) , antiderivative size = 197, normalized size of antiderivative = 1.22, number of steps used = 12, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.478, Rules used = {3042, 3696, 1582, 2336, 25, 2336, 27, 2336, 25, 1584, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \operatorname{csch}^4(c+dx)\left(a+b\sinh^4(c+dx)\right)^3\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\left(a+b\sin(ic+idx)^4\right)^3}{\sin(ic+idx)^4}dx$$

$$\Big\downarrow \text{3696}$$

$$\frac{\int \frac{\coth^4(c+dx)\left((a+b)\tanh^4(c+dx)-2a\tanh^2(c+dx)+a\right)^3}{\left(1-\tanh^2(c+dx)\right)^5}d\tanh(c+dx)}{d}$$

$$\Big\downarrow \text{1582}$$

$$\frac{\frac{1}{8}\int \frac{\coth^4(c+dx)\left(-8(a+b)^3\tanh^{10}(c+dx)+8(5a-b)(a+b)^2\tanh^8(c+dx)-8\left(10a^3+9ba^2+b^3\right)\tanh^6(c+dx)+\left(80a^3+24ba^2-b^3\right)\tanh^4(c+dx)-}{\left(1-\tanh^2(c+dx)\right)^4}}{d}$$

$$\Big\downarrow \; 2336$$

$$\frac{\frac{1}{8}\left(-\frac{1}{6}\int -\frac{\coth^4(c+dx)\left(48(a+b)^3\tanh^8(c+dx)-96(2a-b)(a+b)^2\tanh^6(c+dx)+\left(288a^3+144ba^2+19b^3\right)\tanh^4(c+dx)-192a^3\tanh^2(c+dx)+\right.}{\left(1-\tanh^2(c+dx)\right)^3}}{d}$$

$$\Big\downarrow \; 25$$

$$\frac{\frac{1}{8}\left(\frac{1}{6}\int \frac{\coth^4(c+dx)\left(48(a+b)^3\tanh^8(c+dx)-96(2a-b)(a+b)^2\tanh^6(c+dx)+\left(288a^3+144ba^2+19b^3\right)\tanh^4(c+dx)-192a^3\tanh^2(c+dx)+48a^3\right.}{\left(1-\tanh^2(c+dx)\right)^3}}{d}$$

$$\Big\downarrow \; 2336$$

$$\frac{\frac{1}{8}\left(\frac{1}{6}\left(\frac{b^2(144a+163b)\tanh(c+dx)}{4\left(1-\tanh^2(c+dx)\right)^2}-\frac{1}{4}\int -\frac{3\coth^4(c+dx)\left(-64(a+b)^3\tanh^6(c+dx)+\left(192a^3+192ba^2-48b^2a-29b^3\right)\tanh^4(c+dx)-192a^3\tanh\right.}{\left(1-\tanh^2(c+dx)\right)^2}}{d}$$

$$\Big\downarrow \; 27$$

$$\frac{\frac{1}{8}\left(\frac{1}{6}\left(\frac{3}{4}\int \frac{\coth^4(c+dx)\left(-64(a+b)^3\tanh^6(c+dx)+\left(192a^3+192ba^2-48b^2a-29b^3\right)\tanh^4(c+dx)-192a^3\tanh^2(c+dx)+64a^3\right)}{\left(1-\tanh^2(c+dx)\right)^2}d\tanh(c+dx\right.}{d}$$

$$\Big\downarrow \; 2336$$

$$\frac{\frac{1}{8}\left(\frac{1}{6}\left(\frac{3}{4}\left(-\frac{1}{2}\int -\frac{\coth^4(c+dx)\left(\left(128a^3+384ba^2+144b^2a+35b^3\right)\tanh^4(c+dx)-256a^3\tanh^2(c+dx)+128a^3\right)}{1-\tanh^2(c+dx)}d\tanh(c+dx)-\frac{3b^2(80a+}{2(1-t}\right.}{d}$$

$$\Big\downarrow \; 25$$

$$\frac{\frac{1}{8}\left(\frac{1}{6}\left(\frac{3}{4}\left(\frac{1}{2}\int \frac{\coth^4(c+dx)\left(\left(128a^3+384ba^2+144b^2a+35b^3\right)\tanh^4(c+dx)-256a^3\tanh^2(c+dx)+128a^3\right)}{1-\tanh^2(c+dx)}d\tanh(c+dx)-\frac{3b^2(80a+31b)}{2(1-\tanh^2}\right.}{d}$$

$$\Big\downarrow \; 1584$$

$$\frac{\frac{1}{8}\left(\frac{1}{6}\left(\frac{3}{4}\left(\frac{1}{2}\int \left(128a^3\coth^4(c+dx)-128a^3\coth^2(c+dx)-\frac{b\left(384a^2+144ba+35b^2\right)}{\tanh^2(c+dx)-1}\right)d\tanh(c+dx)-\frac{3b^2(80a+31b)\tan}{2(1-\tanh^2(c+}\right.}{d}$$

$\downarrow$ 2009

$$\frac{\frac{1}{8}\left(\frac{1}{6}\left(\frac{3}{4}\left(\frac{1}{2}\left(-\frac{128}{3}a^3\coth^3(c+dx)+128a^3\coth(c+dx)+b\left(384a^2+144ab+35b^2\right)\operatorname{arctanh}(\tanh(c+dx))\right)-\frac{3b}{}\right.\right.\right.\right.}{d}$$

input
```
Int[Csch[c + d*x]^4*(a + b*Sinh[c + d*x]^4)^3,x]
```

output
```
((b^3*Tanh[c + d*x])/(8*(1 - Tanh[c + d*x]^2)^4) + ((-25*b^3*Tanh[c + d*x]
)/(6*(1 - Tanh[c + d*x]^2)^3) + ((b^2*(144*a + 163*b)*Tanh[c + d*x])/(4*(1
 - Tanh[c + d*x]^2)^2) + (3*((b*(384*a^2 + 144*a*b + 35*b^2)*ArcTanh[Tanh[
c + d*x]] + 128*a^3*Coth[c + d*x] - (128*a^3*Coth[c + d*x]^3)/3)/2 - (3*b^
2*(80*a + 31*b)*Tanh[c + d*x])/(2*(1 - Tanh[c + d*x]^2))))/4)/6)/8)/d
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 1582
```
Int[(x_)^(m_)*((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^
4)^(p_.), x_Symbol] :> Simp[(-d)^(m/2 - 1)*(c*d^2 - b*d*e + a*e^2)^p*x*((d
+ e*x^2)^(q + 1)/(2*e^(2*p + m/2)*(q + 1))), x] + Simp[(-d)^(m/2 - 1)/(2*e^
(2*p)*(q + 1))   Int[x^m*(d + e*x^2)^(q + 1)*ExpandToSum[Together[(1/(d + e
*x^2))*(2*(-d)^(-m/2 + 1)*e^(2*p)*(q + 1)*(a + b*x^2 + c*x^4)^p - ((c*d^2 -
 b*d*e + a*e^2)^p/(e^(m/2)*x^m))*(d + e*(2*q + 3)*x^2))], x], x], x] /; Fre
eQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && IGtQ[p, 0] && ILtQ[q, -1]
&& ILtQ[m/2, 0]
```

rule 1584
```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (
c_.)*(x_)^4)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(f*x)^m*(d + e*x^2)^q*
(a + b*x^2 + c*x^4)^p, x], x] /; FreeQ[{a, b, c, d, e, f, m, q}, x] && NeQ[
b^2 - 4*a*c, 0] && IGtQ[p, 0] && IGtQ[q, -2]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2336
```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[
{Q = PolynomialQuotient[(c*x)^m*Pq, a + b*x^2, x], f = Coeff[PolynomialRema
inder[(c*x)^m*Pq, a + b*x^2, x], x, 0], g = Coeff[PolynomialRemainder[(c*x)
^m*Pq, a + b*x^2, x], x, 1]}, Simp[(a*g - b*f*x)*((a + b*x^2)^(p + 1)/(2*a*
b*(p + 1))), x] + Simp[1/(2*a*(p + 1))   Int[(c*x)^m*(a + b*x^2)^(p + 1)*Ex
pandToSum[(2*a*(p + 1)*Q)/(c*x)^m + (f*(2*p + 3))/(c*x)^m, x], x], x]] /; F
reeQ[{a, b, c}, x] && PolyQ[Pq, x] && LtQ[p, -1] && ILtQ[m, 0]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3696
```
Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^4)^(
p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Simp[ff^(m + 1
)/f   Subst[Int[x^m*((a + 2*a*ff^2*x^2 + (a + b)*ff^4*x^4)^p/(1 + ff^2*x^2)
^(m/2 + 2*p + 1)), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f}, x] &
& IntegerQ[m/2] && IntegerQ[p]
```

Maple [A] (verified)

Time = 5.06 (sec) , antiderivative size = 134, normalized size of antiderivative = 0.83

method	result
parallelrisch	$\frac{-\operatorname{sech}\left(\frac{dx}{2}+\frac{c}{2}\right)^{3} a^{3}\left(\cosh(dx+c)-\frac{\cosh(3dx+3c)}{3}\right)\operatorname{csch}\left(\frac{dx}{2}+\frac{c}{2}\right)^{3}+48\left(-\frac{b\left(a+\frac{7b}{24}\right)\sinh(2dx+2c)}{4}+\frac{\left(a+\frac{7b}{12}\right)b\sinh(4dx+4c)}{32}\right.}{16d}$
derivativedivides	$\frac{a^{3}\left(\frac{2}{3}-\frac{\operatorname{csch}(dx+c)^{2}}{3}\right)\coth(dx+c)+3a^{2}b(dx+c)+3b^{2}a\left(\left(\frac{\sinh(dx+c)^{3}}{4}-\frac{3\sinh(dx+c)}{8}\right)\cosh(dx+c)+\frac{3dx}{8}+\frac{3c}{8}\right)+b^{3}\left(\left(\right.\right.}{d}$
default	$\frac{a^{3}\left(\frac{2}{3}-\frac{\operatorname{csch}(dx+c)^{2}}{3}\right)\coth(dx+c)+3a^{2}b(dx+c)+3b^{2}a\left(\left(\frac{\sinh(dx+c)^{3}}{4}-\frac{3\sinh(dx+c)}{8}\right)\cosh(dx+c)+\frac{3dx}{8}+\frac{3c}{8}\right)+b^{3}\left(\left(\right.\right.}{d}$
risch	$3a^{2}bx+\frac{9a\,b^{2}x}{8}+\frac{35b^{3}x}{128}+\frac{b^{3}e^{8dx+8c}}{2048d}-\frac{b^{3}e^{6dx+6c}}{192d}+\frac{3\,e^{4dx+4c}b^{2}a}{64d}+\frac{7\,e^{4dx+4c}b^{3}}{256d}-\frac{3\,e^{2dx+2c}b^{2}a}{8d}-$

input
```
int(csch(d*x+c)^4*(a+b*sinh(d*x+c)^4)^3,x,method=_RETURNVERBOSE)
```

output
```
1/16*(-sech(1/2*d*x+1/2*c)^3*a^3*(cosh(d*x+c)-1/3*cosh(3*d*x+3*c))*csch(1/
2*d*x+1/2*c)^3+48*(-1/4*b*(a+7/24*b)*sinh(2*d*x+2*c)+1/32*(a+7/12*b)*b*sin
h(4*d*x+4*c)-1/288*b^2*sinh(6*d*x+6*c)+1/3072*b^2*sinh(8*d*x+8*c)+d*x*(a^2
+3/8*a*b+35/384*b^2))*b)/d
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 567 vs. $2(149) = 298$.

Time = 0.09 (sec) , antiderivative size = 567, normalized size of antiderivative = 3.52

$$\int \operatorname{csch}^4(c+dx)\left(a+b\sinh^4(c+dx)\right)^3\,dx = \text{Too large to display}$$

input
```
integrate(csch(d*x+c)^4*(a+b*sinh(d*x+c)^4)^3,x, algorithm="fricas")
```

output
```
1/6144*(3*b^3*cosh(d*x + c)^11 + 33*b^3*cosh(d*x + c)*sinh(d*x + c)^10 - 4
1*b^3*cosh(d*x + c)^9 + 9*(55*b^3*cosh(d*x + c)^3 - 41*b^3*cosh(d*x + c))*
sinh(d*x + c)^8 + 3*(96*a*b^2 + 91*b^3)*cosh(d*x + c)^7 + 21*(66*b^3*cosh(
d*x + c)^5 - 164*b^3*cosh(d*x + c)^3 + (96*a*b^2 + 91*b^3)*cosh(d*x + c))*
sinh(d*x + c)^6 - 3*(1056*a*b^2 + 425*b^3)*cosh(d*x + c)^5 + 3*(330*b^3*co
sh(d*x + c)^7 - 1722*b^3*cosh(d*x + c)^5 + 35*(96*a*b^2 + 91*b^3)*cosh(d*x
 + c)^3 - 5*(1056*a*b^2 + 425*b^3)*cosh(d*x + c))*sinh(d*x + c)^4 + 8*(512
*a^3 + 972*a*b^2 + 319*b^3)*cosh(d*x + c)^3 - 16*(256*a^3 - 3*(384*a^2*b +
 144*a*b^2 + 35*b^3)*d*x)*sinh(d*x + c)^3 + 3*(55*b^3*cosh(d*x + c)^9 - 49
2*b^3*cosh(d*x + c)^7 + 21*(96*a*b^2 + 91*b^3)*cosh(d*x + c)^5 - 10*(1056*
a*b^2 + 425*b^3)*cosh(d*x + c)^3 + 8*(512*a^3 + 972*a*b^2 + 319*b^3)*cosh(
d*x + c))*sinh(d*x + c)^2 - 24*(512*a^3 + 204*a*b^2 + 63*b^3)*cosh(d*x + c
) + 48*(256*a^3 - 3*(384*a^2*b + 144*a*b^2 + 35*b^3)*d*x - (256*a^3 - 3*(3
84*a^2*b + 144*a*b^2 + 35*b^3)*d*x)*cosh(d*x + c)^2)*sinh(d*x + c))/(d*sin
h(d*x + c)^3 + 3*(d*cosh(d*x + c)^2 - d)*sinh(d*x + c))
```

Sympy [F(-1)]

Timed out.

$$\int \operatorname{csch}^4(c+dx)\left(a+b\sinh^4(c+dx)\right)^3\,dx = \text{Timed out}$$

input
```
integrate(csch(d*x+c)**4*(a+b*sinh(d*x+c)**4)**3,x)
```

output
```
Timed out
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 282, normalized size of antiderivative = 1.75

$$\int \operatorname{csch}^4(c+dx)\left(a+b\sinh^4(c+dx)\right)^3\,dx = \frac{3}{64}ab^2\left(24x+\frac{e^{(4dx+4c)}}{d}-\frac{8e^{(2dx+2c)}}{d}+\frac{8e^{(-2dx-2c)}}{d}-\frac{e^{(-4dx-4c)}}{d}\right)+3a^2bx - \frac{1}{6144}b^3\left(\frac{(32e^{(-2dx-2c)}-168e^{(-4dx-4c)}+672e^{(-6dx-6c)}-3)e^{(8dx+8c)}}{d}-\frac{1680(dx+c)}{d}-\frac{672e^{(-2dx}}{}\right. + \frac{4}{3}a^3\left(\frac{3e^{(-2dx-2c)}}{d(3e^{(-2dx-2c)}-3e^{(-4dx-4c)}+e^{(-6dx-6c)}-1)}-\frac{1}{d(3e^{(-2dx-2c)}-3e^{(-4dx-4c)}+e^{(-6dx-6c)}-1}\right.$$

input
```
integrate(csch(d*x+c)^4*(a+b*sinh(d*x+c)^4)^3,x, algorithm="maxima")
```

output
```
3/64*a*b^2*(24*x + e^(4*d*x + 4*c)/d - 8*e^(2*d*x + 2*c)/d + 8*e^(-2*d*x -
 2*c)/d - e^(-4*d*x - 4*c)/d) + 3*a^2*b*x - 1/6144*b^3*((32*e^(-2*d*x - 2*
c) - 168*e^(-4*d*x - 4*c) + 672*e^(-6*d*x - 6*c) - 3)*e^(8*d*x + 8*c)/d -
1680*(d*x + c)/d - (672*e^(-2*d*x - 2*c) - 168*e^(-4*d*x - 4*c) + 32*e^(-6
*d*x - 6*c) - 3*e^(-8*d*x - 8*c))/d) + 4/3*a^3*(3*e^(-2*d*x - 2*c)/(d*(3*e
^(-2*d*x - 2*c) - 3*e^(-4*d*x - 4*c) + e^(-6*d*x - 6*c) - 1)) - 1/(d*(3*e^
(-2*d*x - 2*c) - 3*e^(-4*d*x - 4*c) + e^(-6*d*x - 6*c) - 1)))
```

Giac [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 285, normalized size of antiderivative = 1.77

$$\int \operatorname{csch}^4(c+dx)\left(a+b\sinh^4(c+dx)\right)^3\,dx$$
$$=\frac{3\,b^3e^{(8\,dx+8\,c)}-32\,b^3e^{(6\,dx+6\,c)}+288\,ab^2e^{(4\,dx+4\,c)}+168\,b^3e^{(4\,dx+4\,c)}-2304\,ab^2e^{(2\,dx+2\,c)}-672\,b^3e^{(2\,dx+2\,c)}-}{}$$

input
```
integrate(csch(d*x+c)^4*(a+b*sinh(d*x+c)^4)^3,x, algorithm="giac")
```

output
```
1/6144*(3*b^3*e^(8*d*x + 8*c) - 32*b^3*e^(6*d*x + 6*c) + 288*a*b^2*e^(4*d*
x + 4*c) + 168*b^3*e^(4*d*x + 4*c) - 2304*a*b^2*e^(2*d*x + 2*c) - 672*b^3*
e^(2*d*x + 2*c) + 48*(384*a^2*b + 144*a*b^2 + 35*b^3)*(d*x + c) - (19200*a
^2*b*e^(8*d*x + 8*c) + 7200*a*b^2*e^(8*d*x + 8*c) + 1750*b^3*e^(8*d*x + 8*
c) - 2304*a*b^2*e^(6*d*x + 6*c) - 672*b^3*e^(6*d*x + 6*c) + 288*a*b^2*e^(4
*d*x + 4*c) + 168*b^3*e^(4*d*x + 4*c) - 32*b^3*e^(2*d*x + 2*c) + 3*b^3)*e^
(-8*d*x - 8*c) - 8192*(3*a^3*e^(2*d*x + 2*c) - a^3)/(e^(2*d*x + 2*c) - 1)^
3)/d
```

Mupad [B] (verification not implemented)

Time = 0.49 (sec) , antiderivative size = 269, normalized size of antiderivative = 1.67

$$\int \operatorname{csch}^4(c+dx)\left(a+b\sinh^4(c+dx)\right)^3\,dx$$
$$=x\left(3\,a^2\,b+\frac{9\,a\,b^2}{8}+\frac{35\,b^3}{128}\right)-\frac{4\,a^3}{3\,d\,\left(\mathrm{e}^{4\,c+4\,d\,x}-2\,\mathrm{e}^{2\,c+2\,d\,x}+1\right)}$$
$$+\frac{b^3\,\mathrm{e}^{-6\,c-6\,d\,x}}{192\,d}-\frac{b^3\,\mathrm{e}^{6\,c+6\,d\,x}}{192\,d}-\frac{b^3\,\mathrm{e}^{-8\,c-8\,d\,x}}{2048\,d}+\frac{b^3\,\mathrm{e}^{8\,c+8\,d\,x}}{2048\,d}$$
$$-\frac{8\,a^3\,\mathrm{e}^{2\,c+2\,d\,x}}{3\,d\,\left(3\,\mathrm{e}^{2\,c+2\,d\,x}-3\,\mathrm{e}^{4\,c+4\,d\,x}+\mathrm{e}^{6\,c+6\,d\,x}-1\right)}-\frac{b^2\,\mathrm{e}^{-4\,c-4\,d\,x}\,\left(12\,a+7\,b\right)}{256\,d}$$
$$+\frac{b^2\,\mathrm{e}^{4\,c+4\,d\,x}\,\left(12\,a+7\,b\right)}{256\,d}+\frac{b^2\,\mathrm{e}^{-2\,c-2\,d\,x}\,\left(24\,a+7\,b\right)}{64\,d}-\frac{b^2\,\mathrm{e}^{2\,c+2\,d\,x}\,\left(24\,a+7\,b\right)}{64\,d}$$

input
```
int((a + b*sinh(c + d*x)^4)^3/sinh(c + d*x)^4,x)
```

output
```
x*((9*a*b^2)/8 + 3*a^2*b + (35*b^3)/128) - (4*a^3)/(3*d*(exp(4*c + 4*d*x)
- 2*exp(2*c + 2*d*x) + 1)) + (b^3*exp(- 6*c - 6*d*x))/(192*d) - (b^3*exp(6
*c + 6*d*x))/(192*d) - (b^3*exp(- 8*c - 8*d*x))/(2048*d) + (b^3*exp(8*c +
8*d*x))/(2048*d) - (8*a^3*exp(2*c + 2*d*x))/(3*d*(3*exp(2*c + 2*d*x) - 3*e
xp(4*c + 4*d*x) + exp(6*c + 6*d*x) - 1)) - (b^2*exp(- 4*c - 4*d*x)*(12*a +
 7*b))/(256*d) + (b^2*exp(4*c + 4*d*x)*(12*a + 7*b))/(256*d) + (b^2*exp(-
2*c - 2*d*x)*(24*a + 7*b))/(64*d) - (b^2*exp(2*c + 2*d*x)*(24*a + 7*b))/(6
4*d)
```

Reduce [B] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 565, normalized size of antiderivative = 3.51

$$\int \operatorname{csch}^4(c+dx) \left(a+b\sinh^4(c+dx)\right)^3 dx$$
$$= \frac{3b^3 - 1680e^{8dx+8c}b^3dx + 55296e^{10dx+10c}a^2bdx + 20736e^{10dx+10c}a\,b^2dx + 1680e^{14dx+14c}b^3dx - 5040e^{12dx+1}}{}$$

input
```
int(csch(d*x+c)^4*(a+b*sinh(d*x+c)^4)^3,x)
```

output
```
(3*e**(22*c + 22*d*x)*b**3 - 41*e**(20*c + 20*d*x)*b**3 + 288*e**(18*c + 1
8*d*x)*a*b**2 + 273*e**(18*c + 18*d*x)*b**3 - 3168*e**(16*c + 16*d*x)*a*b*
*2 - 1275*e**(16*c + 16*d*x)*b**3 + 18432*e**(14*c + 14*d*x)*a**2*b*d*x +
6912*e**(14*c + 14*d*x)*a*b**2*d*x + 6144*e**(14*c + 14*d*x)*a*b**2 + 1680
*e**(14*c + 14*d*x)*b**3*d*x + 2048*e**(14*c + 14*d*x)*b**3 - 55296*e**(12
*c + 12*d*x)*a**2*b*d*x - 20736*e**(12*c + 12*d*x)*a*b**2*d*x - 5040*e**(1
2*c + 12*d*x)*b**3*d*x - 24576*e**(10*c + 10*d*x)*a**3 + 55296*e**(10*c +
10*d*x)*a**2*b*d*x + 20736*e**(10*c + 10*d*x)*a*b**2*d*x - 9792*e**(10*c +
 10*d*x)*a*b**2 + 5040*e**(10*c + 10*d*x)*b**3*d*x - 3024*e**(10*c + 10*d*
x)*b**3 + 8192*e**(8*c + 8*d*x)*a**3 - 18432*e**(8*c + 8*d*x)*a**2*b*d*x -
 6912*e**(8*c + 8*d*x)*a*b**2*d*x + 9408*e**(8*c + 8*d*x)*a*b**2 - 1680*e*
*(8*c + 8*d*x)*b**3*d*x + 3056*e**(8*c + 8*d*x)*b**3 - 3168*e**(6*c + 6*d*
x)*a*b**2 - 1275*e**(6*c + 6*d*x)*b**3 + 288*e**(4*c + 4*d*x)*a*b**2 + 273
*e**(4*c + 4*d*x)*b**3 - 41*e**(2*c + 2*d*x)*b**3 + 3*b**3)/(6144*e**(8*c
+ 8*d*x)*d*(e**(6*c + 6*d*x) - 3*e**(4*c + 4*d*x) + 3*e**(2*c + 2*d*x) - 1
))
```

3.196 $\int \operatorname{csch}^6(c+dx) \left(a+b\sinh^4(c+dx)\right)^3 dx$

Optimal result

Integrand size = 23, antiderivative size = 148

$$\int \operatorname{csch}^6(c+dx) \left(a+b\sinh^4(c+dx)\right)^3 dx = -\frac{1}{16}b^2(24a+5b)x - \frac{a^2(a+3b)\coth(c+dx)}{d} + \frac{2a^3\coth^3(c+dx)}{3d} - \frac{a^3\coth^5(c+dx)}{5d} + \frac{b^2(24a+11b)\cosh(c+dx)\sinh(c+dx)}{16d} - \frac{13b^3\cosh^3(c+dx)\sinh(c+dx)}{24d} + \frac{b^3\cosh^5(c+dx)\sinh(c+dx)}{6d}$$

output
```
-1/16*b^2*(24*a+5*b)*x-a^2*(a+3*b)*coth(d*x+c)/d+2/3*a^3*coth(d*x+c)^3/d-1
/5*a^3*coth(d*x+c)^5/d+1/16*b^2*(24*a+11*b)*cosh(d*x+c)*sinh(d*x+c)/d-13/2
4*b^3*cosh(d*x+c)^3*sinh(d*x+c)/d+1/6*b^3*cosh(d*x+c)^5*sinh(d*x+c)/d
```

Mathematica [A] (verified)

Time = 3.82 (sec) , antiderivative size = 110, normalized size of antiderivative = 0.74

$$\int \operatorname{csch}^6(c+dx)\left(a+b\sinh^4(c+dx)\right)^3\,dx$$
$$=\frac{-64a^2\coth(c+dx)\left(8a+45b-4a\operatorname{csch}^2(c+dx)+3a\operatorname{csch}^4(c+dx)\right)+5b^2(-288ac-60bc-288adx-}{960d}$$

input
```
Integrate[Csch[c + d*x]^6*(a + b*Sinh[c + d*x]^4)^3,x]
```

output
```
(-64*a^2*Coth[c + d*x]*(8*a + 45*b - 4*a*Csch[c + d*x]^2 + 3*a*Csch[c + d*
x]^4) + 5*b^2*(-288*a*c - 60*b*c - 288*a*d*x - 60*b*d*x + 9*(16*a + 5*b)*S
inh[2*(c + d*x)] - 9*b*Sinh[4*(c + d*x)] + b*Sinh[6*(c + d*x)]))/(960*d)
```

Rubi [A] (verified)

Time = 1.01 (sec) , antiderivative size = 173, normalized size of antiderivative = 1.17, number of steps used = 12, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}}=0.478$, Rules used = {3042, 25, 3696, 1582, 25, 2336, 27, 2336, 25, 2333, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \operatorname{csch}^6(c+dx)\left(a+b\sinh^4(c+dx)\right)^3\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int -\frac{\left(a+b\sin(ic+idx)^4\right)^3}{\sin(ic+idx)^6}dx$$

$$\Big\downarrow \text{25}$$

$$-\int \frac{\left(b\sin(ic+idx)^4+a\right)^3}{\sin(ic+idx)^6}dx$$

$$\Big\downarrow \text{3696}$$

$$\frac{\int \frac{\coth^6(c+dx)\left((a+b)\tanh^4(c+dx)-2a\tanh^2(c+dx)+a\right)^3}{\left(1-\tanh^2(c+dx)\right)^4}d\tanh(c+dx)}{d}$$

$$\Big\downarrow \text{1582}$$

$$\frac{\frac{b^3\tanh(c+dx)}{6\left(1-\tanh^2(c+dx)\right)^3}-\frac{1}{6}\int -\frac{\coth^6(c+dx)\left(-6(a+b)^3\tanh^{10}(c+dx)+6(5a-b)(a+b)^2\tanh^8(c+dx)-\left(60a^3+54ba^2+b^3\right)\tanh^6(c+dx)+6a^2(10\right.}{\left(1-\tanh^2(c+dx)\right)^3}}{d}$$

$$\Big\downarrow \text{25}$$

$$\frac{\frac{1}{6}\int \frac{\coth^6(c+dx)\left(-6(a+b)^3\tanh^{10}(c+dx)+6(5a-b)(a+b)^2\tanh^8(c+dx)-\left(60a^3+54ba^2+b^3\right)\tanh^6(c+dx)+6a^2(10a+3b)\tanh^4(c+dx)-30a^3\right.}{\left(1-\tanh^2(c+dx)\right)^3}}{d}$$

$$\Big\downarrow \text{2336}$$

$$\frac{\frac{1}{6}\left(-\frac{1}{4}\int -\frac{3\coth^6(c+dx)\left(8(a+b)^3\tanh^8(c+dx)-\left(32a^3+48ba^2-3b^3\right)\tanh^6(c+dx)+24a^2(2a+b)\tanh^4(c+dx)-32a^3\tanh^2(c+dx)+8a^3\right)}{\left(1-\tanh^2(c+dx)\right)^2}d\,\mathrm{ta}\right.}{d}$$

$$\Big\downarrow \text{27}$$

$$\frac{\frac{1}{6}\left(\frac{3}{4}\int \frac{\coth^6(c+dx)\left(8(a+b)^3\tanh^8(c+dx)-\left(32a^3+48ba^2-3b^3\right)\tanh^6(c+dx)+24a^2(2a+b)\tanh^4(c+dx)-32a^3\tanh^2(c+dx)+8a^3\right)}{\left(1-\tanh^2(c+dx)\right)^2}d\tanh(c\right.}{d}$$

$$\Big\downarrow \text{2336}$$

$$\frac{\frac{1}{6}\left(\frac{3}{4}\left(\frac{b^2(24a+11b)\tanh(c+dx)}{2\left(1-\tanh^2(c+dx)\right)}-\frac{1}{2}\int -\frac{\coth^6(c+dx)\left(-\left(\left(16a^3+48ba^2+24b^2a+5b^3\right)\tanh^6(c+dx)\right)+48a^2(a+b)\tanh^4(c+dx)-48a^3\tanh^2(c\right.}{1-\tanh^2(c+dx)}\right.\right.}{d}$$

$$\Big\downarrow \text{25}$$

$$\frac{\frac{1}{6}\left(\frac{3}{4}\left(\frac{1}{2}\int \frac{\coth^6(c+dx)\left(-\left(\left(16a^3+48ba^2+24b^2a+5b^3\right)\tanh^6(c+dx)\right)+48a^2(a+b)\tanh^4(c+dx)-48a^3\tanh^2(c+dx)+16a^3\right)}{1-\tanh^2(c+dx)}d\tanh(c+dx)\right.\right.}{d}$$

$$\Big\downarrow \text{2333}$$

$$\frac{\frac{1}{6}\left(\frac{3}{4}\left(\frac{1}{2}\int \left(16a^3\coth^6(c+dx)-32a^3\coth^4(c+dx)+16a^2(a+3b)\coth^2(c+dx)+\frac{b^2(24a+5b)}{\tanh^2(c+dx)-1}\right)d\tanh(c+d\right.\right.}{d}$$

$$\Big\downarrow \text{2009}$$

$$\frac{\frac{1}{6}\left(\frac{3}{4}\left(\frac{1}{2}\left(-\frac{16}{5}a^3\coth^5(c+dx)+\frac{32}{3}a^3\coth^3(c+dx)-16a^2(a+3b)\coth(c+dx)-b^2(24a+5b)\text{arctanh}(\tanh(c+\right.\right.\right.}{d}$$

input
```
Int[Csch[c + d*x]^6*(a + b*Sinh[c + d*x]^4)^3,x]
```

output
```
((b^3*Tanh[c + d*x])/(6*(1 - Tanh[c + d*x]^2)^3) + ((-13*b^3*Tanh[c + d*x]
)/(4*(1 - Tanh[c + d*x]^2)^2) + (3*((-(b^2*(24*a + 5*b)*ArcTanh[Tanh[c + d
*x]]) - 16*a^2*(a + 3*b)*Coth[c + d*x] + (32*a^3*Coth[c + d*x]^3)/3 - (16*
a^3*Coth[c + d*x]^5)/5)/2 + (b^2*(24*a + 11*b)*Tanh[c + d*x])/(2*(1 - Tanh
[c + d*x]^2))))/4)/6)/d
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 1582
```
Int[(x_)^(m_)*((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^
4)^(p_.), x_Symbol] :> Simp[(-d)^(m/2 - 1)*(c*d^2 - b*d*e + a*e^2)^p*x*((d
+ e*x^2)^(q + 1)/(2*e^(2*p + m/2)*(q + 1))), x] + Simp[(-d)^(m/2 - 1)/(2*e^
(2*p)*(q + 1))   Int[x^m*(d + e*x^2)^(q + 1)*ExpandToSum[Together[(1/(d + e
*x^2))*(2*(-d)^(-m/2 + 1)*e^(2*p)*(q + 1)*(a + b*x^2 + c*x^4)^p - ((c*d^2 -
 b*d*e + a*e^2)^p/(e^(m/2)*x^m))*(d + e*(2*q + 3)*x^2))], x], x], x] /; Fre
eQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && IGtQ[p, 0] && ILtQ[q, -1]
&& ILtQ[m/2, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2333
```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[
ExpandIntegrand[(c*x)^m*Pq*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x]
&& PolyQ[Pq, x] && IGtQ[p, -2]
```

rule 2336
```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[
{Q = PolynomialQuotient[(c*x)^m*Pq, a + b*x^2, x], f = Coeff[PolynomialRema
inder[(c*x)^m*Pq, a + b*x^2, x], x, 0], g = Coeff[PolynomialRemainder[(c*x)
^m*Pq, a + b*x^2, x], x, 1]}, Simp[(a*g - b*f*x)*((a + b*x^2)^(p + 1)/(2*a*
b*(p + 1))), x] + Simp[1/(2*a*(p + 1))   Int[(c*x)^m*(a + b*x^2)^(p + 1)*Ex
pandToSum[(2*a*(p + 1)*Q)/(c*x)^m + (f*(2*p + 3))/(c*x)^m, x], x], x]] /; F
reeQ[{a, b, c}, x] && PolyQ[Pq, x] && LtQ[p, -1] && ILtQ[m, 0]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3696
```
Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^4)^(
p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Simp[ff^(m + 1
)/f   Subst[Int[x^m*((a + 2*a*ff^2*x^2 + (a + b)*ff^4*x^4)^p/(1 + ff^2*x^2)
^(m/2 + 2*p + 1)), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f}, x] &
& IntegerQ[m/2] && IntegerQ[p]
```

Maple [A] (verified)

Time = 4.14 (sec) , antiderivative size = 126, normalized size of antiderivative = 0.85

method	result
derivativedivides	$\frac{a^3\left(-\frac{8}{15}-\frac{\operatorname{csch}(dx+c)^4}{5}+\frac{4\operatorname{csch}(dx+c)^2}{15}\right)\coth(dx+c)-3a^2b\coth(dx+c)+3b^2a\left(\frac{\cosh(dx+c)\sinh(dx+c)}{2}-\frac{dx}{2}-\frac{c}{2}\right)+b^3\left(\left(\left(\frac{s}{}\right.\right.\right.}{d}$
default	$\frac{a^3\left(-\frac{8}{15}-\frac{\operatorname{csch}(dx+c)^4}{5}+\frac{4\operatorname{csch}(dx+c)^2}{15}\right)\coth(dx+c)-3a^2b\coth(dx+c)+3b^2a\left(\frac{\cosh(dx+c)\sinh(dx+c)}{2}-\frac{dx}{2}-\frac{c}{2}\right)+b^3\left(\left(\left(\frac{s}{}\right.\right.\right.}{d}$
parallelrisch	$\frac{-10\left(\cosh(dx+c)-\frac{\cosh(3dx+3c)}{2}+\frac{\cosh(5dx+5c)}{10}\right)a^3\operatorname{sech}\left(\frac{dx}{2}+\frac{c}{2}\right)^5\operatorname{csch}\left(\frac{dx}{2}+\frac{c}{2}\right)^5+1440\operatorname{sech}\left(\frac{dx}{2}+\frac{c}{2}\right)\operatorname{csch}\left(\frac{dx}{2}+\frac{c}{2}\right)a^2}{960d}$
risch	$-\frac{3ab^2x}{2}-\frac{5b^3x}{16}+\frac{b^3e^{6dx+6c}}{384d}-\frac{3e^{4dx+4c}b^3}{128d}+\frac{3e^{2dx+2c}b^2a}{8d}+\frac{15e^{2dx+2c}b^3}{128d}-\frac{3e^{-2dx-2c}b^2a}{8d}-\frac{15e^{-2dx}}{128}$

input
```
int(csch(d*x+c)^6*(a+b*sinh(d*x+c)^4)^3,x,method=_RETURNVERBOSE)
```

output
```
1/d*(a^3*(-8/15-1/5*csch(d*x+c)^4+4/15*csch(d*x+c)^2)*coth(d*x+c)-3*a^2*b*
coth(d*x+c)+3*b^2*a*(1/2*cosh(d*x+c)*sinh(d*x+c)-1/2*d*x-1/2*c)+b^3*((1/6*
sinh(d*x+c)^5-5/24*sinh(d*x+c)^3+5/16*sinh(d*x+c))*cosh(d*x+c)-5/16*d*x-5/
16*c))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 768 vs. $2(136) = 272$.

Time = 0.10 (sec) , antiderivative size = 768, normalized size of antiderivative = 5.19

$$\int \operatorname{csch}^6(c+dx)\left(a+b\sinh^4(c+dx)\right)^3\,dx = \text{Too large to display}$$

input
```
integrate(csch(d*x+c)^6*(a+b*sinh(d*x+c)^4)^3,x, algorithm="fricas")
```

output
```
1/1920*(5*b^3*cosh(d*x + c)^11 + 55*b^3*cosh(d*x + c)*sinh(d*x + c)^10 - 7
0*b^3*cosh(d*x + c)^9 + 15*(55*b^3*cosh(d*x + c)^3 - 42*b^3*cosh(d*x + c))
*sinh(d*x + c)^8 + 20*(36*a*b^2 + 25*b^3)*cosh(d*x + c)^7 + 70*(33*b^3*cos
h(d*x + c)^5 - 84*b^3*cosh(d*x + c)^3 + 2*(36*a*b^2 + 25*b^3)*cosh(d*x + c
))*sinh(d*x + c)^6 - (1024*a^3 + 5760*a^2*b + 3600*a*b^2 + 1625*b^3)*cosh(
d*x + c)^5 + 8*(128*a^3 + 720*a^2*b - 15*(24*a*b^2 + 5*b^3)*d*x)*sinh(d*x
+ c)^5 + 5*(330*b^3*cosh(d*x + c)^7 - 1764*b^3*cosh(d*x + c)^5 + 140*(36*a
*b^2 + 25*b^3)*cosh(d*x + c)^3 - (1024*a^3 + 5760*a^2*b + 3600*a*b^2 + 162
5*b^3)*cosh(d*x + c))*sinh(d*x + c)^4 + 20*(256*a^3 + 864*a^2*b + 324*a*b^
2 + 125*b^3)*cosh(d*x + c)^3 - 40*(128*a^3 + 720*a^2*b - 15*(24*a*b^2 + 5*
b^3)*d*x - 2*(128*a^3 + 720*a^2*b - 15*(24*a*b^2 + 5*b^3)*d*x)*cosh(d*x +
c)^2)*sinh(d*x + c)^3 + 5*(55*b^3*cosh(d*x + c)^9 - 504*b^3*cosh(d*x + c)^
7 + 84*(36*a*b^2 + 25*b^3)*cosh(d*x + c)^5 - 2*(1024*a^3 + 5760*a^2*b + 36
00*a*b^2 + 1625*b^3)*cosh(d*x + c)^3 + 12*(256*a^3 + 864*a^2*b + 324*a*b^2
 + 125*b^3)*cosh(d*x + c))*sinh(d*x + c)^2 - 10*(1024*a^3 + 1152*a^2*b + 3
60*a*b^2 + 131*b^3)*cosh(d*x + c) + 40*((128*a^3 + 720*a^2*b - 15*(24*a*b^
2 + 5*b^3)*d*x)*cosh(d*x + c)^4 + 256*a^3 + 1440*a^2*b - 30*(24*a*b^2 + 5*
b^3)*d*x - 3*(128*a^3 + 720*a^2*b - 15*(24*a*b^2 + 5*b^3)*d*x)*cosh(d*x +
c)^2)*sinh(d*x + c))/(d*sinh(d*x + c)^5 + 5*(2*d*cosh(d*x + c)^2 - d)*sinh
(d*x + c)^3 + 5*(d*cosh(d*x + c)^4 - 3*d*cosh(d*x + c)^2 + 2*d)*sinh(d*...
```

Sympy [F(-1)]

Timed out.

$$\int \operatorname{csch}^6(c+dx)\left(a+b\sinh^4(c+dx)\right)^3\,dx=\text{Timed out}$$

input
```
integrate(csch(d*x+c)**6*(a+b*sinh(d*x+c)**4)**3,x)
```

output
```
Timed out
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 359 vs. $2(136) = 272$.

Time = 0.05 (sec) , antiderivative size = 359, normalized size of antiderivative = 2.43

$$\int \operatorname{csch}^6(c+dx)\left(a+b\sinh^4(c+dx)\right)^3\,dx=-\frac{3}{8}ab^2\left(4x-\frac{e^{(2dx+2c)}}{d}+\frac{e^{(-2dx-2c)}}{d}\right)$$
$$-\frac{1}{384}b^3\left(\frac{\left(9e^{(-2dx-2c)}-45e^{(-4dx-4c)}-1\right)e^{(6dx+6c)}}{d}+\frac{120(dx+c)}{d}+\frac{45e^{(-2dx-2c)}-9e^{(-4dx-4c)}+e}{d}\right.$$
$$-\frac{16}{15}a^3\left(\frac{5e^{(-2dx-2c)}}{d\left(5e^{(-2dx-2c)}-10e^{(-4dx-4c)}+10e^{(-6dx-6c)}-5e^{(-8dx-8c)}+e^{(-10dx-10c)}-1\right)}-\frac{}{d(5e^{(-2dx-}}\right.$$
$$+\frac{6a^2b}{d\left(e^{(-2dx-2c)}-1\right)}$$

input
```
integrate(csch(d*x+c)^6*(a+b*sinh(d*x+c)^4)^3,x, algorithm="maxima")
```

output
```
-3/8*a*b^2*(4*x - e^(2*d*x + 2*c)/d + e^(-2*d*x - 2*c)/d) - 1/384*b^3*((9*
e^(-2*d*x - 2*c) - 45*e^(-4*d*x - 4*c) - 1)*e^(6*d*x + 6*c)/d + 120*(d*x +
 c)/d + (45*e^(-2*d*x - 2*c) - 9*e^(-4*d*x - 4*c) + e^(-6*d*x - 6*c))/d) -
 16/15*a^3*(5*e^(-2*d*x - 2*c)/(d*(5*e^(-2*d*x - 2*c) - 10*e^(-4*d*x - 4*c
) + 10*e^(-6*d*x - 6*c) - 5*e^(-8*d*x - 8*c) + e^(-10*d*x - 10*c) - 1)) -
10*e^(-4*d*x - 4*c)/(d*(5*e^(-2*d*x - 2*c) - 10*e^(-4*d*x - 4*c) + 10*e^(-
6*d*x - 6*c) - 5*e^(-8*d*x - 8*c) + e^(-10*d*x - 10*c) - 1)) - 1/(d*(5*e^(
-2*d*x - 2*c) - 10*e^(-4*d*x - 4*c) + 10*e^(-6*d*x - 6*c) - 5*e^(-8*d*x -
8*c) + e^(-10*d*x - 10*c) - 1))) + 6*a^2*b/(d*(e^(-2*d*x - 2*c) - 1))
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 286 vs. 2(136) = 272.

Time = 0.30 (sec) , antiderivative size = 286, normalized size of antiderivative = 1.93

$$\int \operatorname{csch}^6(c+dx)\left(a+b\sinh^4(c+dx)\right)^3\,dx$$
$$=\frac{5\,b^3e^{(6\,dx+6\,c)}-45\,b^3e^{(4\,dx+4\,c)}+720\,ab^2e^{(2\,dx+2\,c)}+225\,b^3e^{(2\,dx+2\,c)}-120\,(24\,ab^2+5\,b^3)(dx+c)+5\,(528}{}$$

input
```
integrate(csch(d*x+c)^6*(a+b*sinh(d*x+c)^4)^3,x, algorithm="giac")
```

output
```
1/1920*(5*b^3*e^(6*d*x + 6*c) - 45*b^3*e^(4*d*x + 4*c) + 720*a*b^2*e^(2*d*
x + 2*c) + 225*b^3*e^(2*d*x + 2*c) - 120*(24*a*b^2 + 5*b^3)*(d*x + c) + 5*
(528*a*b^2*e^(6*d*x + 6*c) + 110*b^3*e^(6*d*x + 6*c) - 144*a*b^2*e^(4*d*x
+ 4*c) - 45*b^3*e^(4*d*x + 4*c) + 9*b^3*e^(2*d*x + 2*c) - b^3)*e^(-6*d*x -
 6*c) - 256*(45*a^2*b*e^(8*d*x + 8*c) - 180*a^2*b*e^(6*d*x + 6*c) + 80*a^3
*e^(4*d*x + 4*c) + 270*a^2*b*e^(4*d*x + 4*c) - 40*a^3*e^(2*d*x + 2*c) - 18
0*a^2*b*e^(2*d*x + 2*c) + 8*a^3 + 45*a^2*b)/(e^(2*d*x + 2*c) - 1)^5)/d
```

Mupad [B] (verification not implemented)

Time = 1.97 (sec) , antiderivative size = 511, normalized size of antiderivative = 3.45

$$\int \operatorname{csch}^6(c+dx)\left(a+b\sinh^4(c+dx)\right)^3\,dx$$
$$=\frac{\frac{6\,a^2\,b}{5\,d}-\frac{2\,\mathrm{e}^{2\,c+2\,d\,x}\,\left(8\,a^3+9\,b\,a^2\right)}{5\,d}+\frac{18\,a^2\,b\,\mathrm{e}^{4\,c+4\,d\,x}}{5\,d}-\frac{6\,a^2\,b\,\mathrm{e}^{6\,c+6\,d\,x}}{5\,d}}{6\,\mathrm{e}^{4\,c+4\,d\,x}-4\,\mathrm{e}^{2\,c+2\,d\,x}-4\,\mathrm{e}^{6\,c+6\,d\,x}+\mathrm{e}^{8\,c+8\,d\,x}+1}$$
$$-\frac{\frac{2\,\left(8\,a^3+9\,b\,a^2\right)}{15\,d}-\frac{12\,a^2\,b\,\mathrm{e}^{2\,c+2\,d\,x}}{5\,d}+\frac{6\,a^2\,b\,\mathrm{e}^{4\,c+4\,d\,x}}{5\,d}}{3\,\mathrm{e}^{2\,c+2\,d\,x}-3\,\mathrm{e}^{4\,c+4\,d\,x}+\mathrm{e}^{6\,c+6\,d\,x}-1}$$
$$-\frac{\frac{6\,a^2\,b}{5\,d}+\frac{4\,\mathrm{e}^{4\,c+4\,d\,x}\,\left(8\,a^3+9\,b\,a^2\right)}{5\,d}-\frac{24\,a^2\,b\,\mathrm{e}^{2\,c+2\,d\,x}}{5\,d}-\frac{24\,a^2\,b\,\mathrm{e}^{6\,c+6\,d\,x}}{5\,d}+\frac{6\,a^2\,b\,\mathrm{e}^{8\,c+8\,d\,x}}{5\,d}}{5\,\mathrm{e}^{2\,c+2\,d\,x}-10\,\mathrm{e}^{4\,c+4\,d\,x}+10\,\mathrm{e}^{6\,c+6\,d\,x}-5\,\mathrm{e}^{8\,c+8\,d\,x}+\mathrm{e}^{10\,c+10\,d\,x}-1}$$
$$-\frac{b^2\,x\,\left(24\,a+5\,b\right)}{16}+\frac{3\,b^3\,\mathrm{e}^{-4\,c-4\,d\,x}}{128\,d}-\frac{3\,b^3\,\mathrm{e}^{4\,c+4\,d\,x}}{128\,d}-\frac{b^3\,\mathrm{e}^{-6\,c-6\,d\,x}}{384\,d}+\frac{b^3\,\mathrm{e}^{6\,c+6\,d\,x}}{384\,d}$$
$$-\frac{3\,b^2\,\mathrm{e}^{-2\,c-2\,d\,x}\,\left(16\,a+5\,b\right)}{128\,d}+\frac{3\,b^2\,\mathrm{e}^{2\,c+2\,d\,x}\,\left(16\,a+5\,b\right)}{128\,d}-\frac{12\,a^2\,b}{5\,d\,\left(\mathrm{e}^{2\,c+2\,d\,x}-1\right)}$$

input
```
int((a + b*sinh(c + d*x)^4)^3/sinh(c + d*x)^6,x)
```

output
```
((6*a^2*b)/(5*d) - (2*exp(2*c + 2*d*x)*(9*a^2*b + 8*a^3))/(5*d) + (18*a^2*
b*exp(4*c + 4*d*x))/(5*d) - (6*a^2*b*exp(6*c + 6*d*x))/(5*d))/(6*exp(4*c +
 4*d*x) - 4*exp(2*c + 2*d*x) - 4*exp(6*c + 6*d*x) + exp(8*c + 8*d*x) + 1)
- ((2*(9*a^2*b + 8*a^3))/(15*d) - (12*a^2*b*exp(2*c + 2*d*x))/(5*d) + (6*a
^2*b*exp(4*c + 4*d*x))/(5*d))/(3*exp(2*c + 2*d*x) - 3*exp(4*c + 4*d*x) + e
xp(6*c + 6*d*x) - 1) - ((6*a^2*b)/(5*d) + (4*exp(4*c + 4*d*x)*(9*a^2*b + 8
*a^3))/(5*d) - (24*a^2*b*exp(2*c + 2*d*x))/(5*d) - (24*a^2*b*exp(6*c + 6*d
*x))/(5*d) + (6*a^2*b*exp(8*c + 8*d*x))/(5*d))/(5*exp(2*c + 2*d*x) - 10*ex
p(4*c + 4*d*x) + 10*exp(6*c + 6*d*x) - 5*exp(8*c + 8*d*x) + exp(10*c + 10*
d*x) - 1) - (b^2*x*(24*a + 5*b))/16 + (3*b^3*exp(- 4*c - 4*d*x))/(128*d) -
 (3*b^3*exp(4*c + 4*d*x))/(128*d) - (b^3*exp(- 6*c - 6*d*x))/(384*d) + (b^
3*exp(6*c + 6*d*x))/(384*d) - (3*b^2*exp(- 2*c - 2*d*x)*(16*a + 5*b))/(128
*d) + (3*b^2*exp(2*c + 2*d*x)*(16*a + 5*b))/(128*d) - (12*a^2*b)/(5*d*(exp
(2*c + 2*d*x) - 1))
```

Reduce [B] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 682, normalized size of antiderivative = 4.61

$$\int \operatorname{csch}^6(c+dx)\left(a+b\sinh^4(c+dx)\right)^3\,dx$$
$$= \frac{5b^3 - 3000e^{8dx+8c}b^3dx + 28800e^{10dx+10c}a\,b^2dx + 3000e^{14dx+14c}b^3dx - 2880e^{16dx+16c}a\,b^2dx - 6000e^{12dx+12c}}{}$$

input
```
int(csch(d*x+c)^6*(a+b*sinh(d*x+c)^4)^3,x)
```

output

```
(5*e**(22*c + 22*d*x)*b**3 - 70*e**(20*c + 20*d*x)*b**3 + 720*e**(18*c + 1
8*d*x)*a*b**2 + 500*e**(18*c + 18*d*x)*b**3 - 2304*e**(16*c + 16*d*x)*a**2
*b - 2880*e**(16*c + 16*d*x)*a*b**2*d*x - 2304*e**(16*c + 16*d*x)*a*b**2 -
 600*e**(16*c + 16*d*x)*b**3*d*x - 1125*e**(16*c + 16*d*x)*b**3 + 14400*e*
*(14*c + 14*d*x)*a*b**2*d*x + 3000*e**(14*c + 14*d*x)*b**3*d*x + 23040*e**
(12*c + 12*d*x)*a**2*b - 28800*e**(12*c + 12*d*x)*a*b**2*d*x + 9360*e**(12
*c + 12*d*x)*a*b**2 - 6000*e**(12*c + 12*d*x)*b**3*d*x + 3690*e**(12*c + 1
2*d*x)*b**3 - 20480*e**(10*c + 10*d*x)*a**3 - 46080*e**(10*c + 10*d*x)*a**
2*b + 28800*e**(10*c + 10*d*x)*a*b**2*d*x - 16560*e**(10*c + 10*d*x)*a*b**
2 + 6000*e**(10*c + 10*d*x)*b**3*d*x - 6310*e**(10*c + 10*d*x)*b**3 + 1024
0*e**(8*c + 8*d*x)*a**3 + 34560*e**(8*c + 8*d*x)*a**2*b - 14400*e**(8*c +
8*d*x)*a*b**2*d*x + 12960*e**(8*c + 8*d*x)*a*b**2 - 3000*e**(8*c + 8*d*x)*
b**3*d*x + 5000*e**(8*c + 8*d*x)*b**3 - 2048*e**(6*c + 6*d*x)*a**3 - 9216*
e**(6*c + 6*d*x)*a**2*b + 2880*e**(6*c + 6*d*x)*a*b**2*d*x - 4896*e**(6*c
+ 6*d*x)*a*b**2 + 600*e**(6*c + 6*d*x)*b**3*d*x - 2125*e**(6*c + 6*d*x)*b*
*3 + 720*e**(4*c + 4*d*x)*a*b**2 + 500*e**(4*c + 4*d*x)*b**3 - 70*e**(2*c
+ 2*d*x)*b**3 + 5*b**3)/(1920*e**(6*c + 6*d*x)*d*(e**(10*c + 10*d*x) - 5*e
**(8*c + 8*d*x) + 10*e**(6*c + 6*d*x) - 10*e**(4*c + 4*d*x) + 5*e**(2*c +
2*d*x) - 1))
```

3.197 $\int \operatorname{csch}^8(c+dx)\left(a+b\sinh^4(c+dx)\right)^3\,dx$

Optimal result

Integrand size = 23, antiderivative size = 133

$$\int \operatorname{csch}^8(c+dx)\left(a+b\sinh^4(c+dx)\right)^3\,dx=\frac{3}{8}b^2(8a+b)x+\frac{a^2(a+3b)\coth(c+dx)}{d}-\frac{a^2(a+b)\coth^3(c+dx)}{d}+\frac{3a^3\coth^5(c+dx)}{5d}-\frac{a^3\coth^7(c+dx)}{7d}-\frac{5b^3\cosh(c+dx)\sinh(c+dx)}{8d}+\frac{b^3\cosh^3(c+dx)\sinh(c+dx)}{4d}$$

output
```
3/8*b^2*(8*a+b)*x+a^2*(a+3*b)*coth(d*x+c)/d-a^2*(a+b)*coth(d*x+c)^3/d+3/5*
a^3*coth(d*x+c)^5/d-1/7*a^3*coth(d*x+c)^7/d-5/8*b^3*cosh(d*x+c)*sinh(d*x+c
)/d+1/4*b^3*cosh(d*x+c)^3*sinh(d*x+c)/d
```

Mathematica [A] (verified)

Time = 1.74 (sec) , antiderivative size = 106, normalized size of antiderivative = 0.80

$$\int \operatorname{csch}^8(c+dx)\left(a+b\sinh^4(c+dx)\right)^3\,dx$$
$$=\frac{-32a^2\coth(c+dx)\left(-2(8a+35b)+(8a+35b)\operatorname{csch}^2(c+dx)-6a\operatorname{csch}^4(c+dx)+5a\operatorname{csch}^6(c+dx)\right)+}{1120d}$$

input
```
Integrate[Csch[c + d*x]^8*(a + b*Sinh[c + d*x]^4)^3,x]
```

output
```
(-32*a^2*Coth[c + d*x]*(-2*(8*a + 35*b) + (8*a + 35*b)*Csch[c + d*x]^2 - 6
*a*Csch[c + d*x]^4 + 5*a*Csch[c + d*x]^6) + 35*b^2*(12*(8*a + b)*(c + d*x)
 - 8*b*Sinh[2*(c + d*x)] + b*Sinh[4*(c + d*x)]))/(1120*d)
```

Rubi [A] (verified)

Time = 0.83 (sec) , antiderivative size = 148, normalized size of antiderivative = 1.11, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}=0.304$, Rules used = {3042, 3696, 1582, 2336, 25, 2333, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \operatorname{csch}^8(c+dx)\left(a+b\sinh^4(c+dx)\right)^3\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\left(a+b\sin(ic+idx)^4\right)^3}{\sin(ic+idx)^8}dx$$

$$\Big\downarrow \text{3696}$$

$$\frac{\int \frac{\coth^8(c+dx)\left((a+b)\tanh^4(c+dx)-2a\tanh^2(c+dx)+a\right)^3}{\left(1-\tanh^2(c+dx)\right)^3}d\tanh(c+dx)}{d}$$

$$\Big\downarrow \text{1582}$$

$$\frac{\frac{1}{4}\int \frac{\coth^8(c+dx)\left(-4(a+b)^3\tanh^{10}(c+dx)+\left(20a^3+36ba^2+12b^2a-b^3\right)\tanh^8(c+dx)-4a^2(10a+9b)\tanh^6(c+dx)+4a^2(10a+3b)\tanh^4(c+dx)-2}{\left(1-\tanh^2(c+dx)\right)^2}}{d}$$

$\Big\downarrow$ 2336

$$\frac{\frac{1}{4}\left(-\frac{1}{2}\int -\frac{\coth^8(c+dx)\left(\left(8a^3+24ba^2+24b^2a+3b^3\right)\tanh^8(c+dx)-16a^2(2a+3b)\tanh^6(c+dx)+24a^2(2a+b)\tanh^4(c+dx)-32a^3\tanh^2(c+dx)+\right.}{1-\tanh^2(c+dx)}}{d}$$

$\Big\downarrow$ 25

$$\frac{\frac{1}{4}\left(\frac{1}{2}\int \frac{\coth^8(c+dx)\left(\left(8a^3+24ba^2+24b^2a+3b^3\right)\tanh^8(c+dx)-16a^2(2a+3b)\tanh^6(c+dx)+24a^2(2a+b)\tanh^4(c+dx)-32a^3\tanh^2(c+dx)+8a^3\right)}{1-\tanh^2(c+dx)}}{d}$$

$\Big\downarrow$ 2333

$$\frac{\frac{1}{4}\left(\frac{1}{2}\int \left(8a^3\coth^8(c+dx)-24a^3\coth^6(c+dx)+24a^2(a+b)\coth^4(c+dx)-8a^2(a+3b)\coth^2(c+dx)-\frac{3b^2}{\tanh^2}\right.\right.}{d}$$

$\Big\downarrow$ 2009

$$\frac{\frac{1}{4}\left(\frac{1}{2}\left(-\frac{8}{7}a^3\coth^7(c+dx)+\frac{24}{5}a^3\coth^5(c+dx)-8a^2(a+b)\coth^3(c+dx)+8a^2(a+3b)\coth(c+dx)+3b^2(8a\right.\right.}{d}$$

input

```
Int[Csch[c + d*x]^8*(a + b*Sinh[c + d*x]^4)^3,x]
```

output

```
((b^3*Tanh[c + d*x])/(4*(1 - Tanh[c + d*x]^2)^2) + ((3*b^2*(8*a + b)*ArcTa
nh[Tanh[c + d*x]] + 8*a^2*(a + 3*b)*Coth[c + d*x] - 8*a^2*(a + b)*Coth[c +
 d*x]^3 + (24*a^3*Coth[c + d*x]^5)/5 - (8*a^3*Coth[c + d*x]^7)/7)/2 - (5*b
^3*Tanh[c + d*x])/(2*(1 - Tanh[c + d*x]^2)))/4)/d
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 1582
```
Int[(x_)^(m_)*((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^
4)^(p_.), x_Symbol] :> Simp[(-d)^(m/2 - 1)*(c*d^2 - b*d*e + a*e^2)^p*x*((d
+ e*x^2)^(q + 1)/(2*e^(2*p + m/2)*(q + 1))), x] + Simp[(-d)^(m/2 - 1)/(2*e^
(2*p)*(q + 1))   Int[x^m*(d + e*x^2)^(q + 1)*ExpandToSum[Together[(1/(d + e
*x^2))*(2*(-d)^(-m/2 + 1)*e^(2*p)*(q + 1)*(a + b*x^2 + c*x^4)^p - ((c*d^2 -
 b*d*e + a*e^2)^p/(e^(m/2)*x^m))*(d + e*(2*q + 3)*x^2))], x], x], x] /; Fre
eQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && IGtQ[p, 0] && ILtQ[q, -1]
&& ILtQ[m/2, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2333
```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[
ExpandIntegrand[(c*x)^m*Pq*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x]
&& PolyQ[Pq, x] && IGtQ[p, -2]
```

rule 2336
```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[
{Q = PolynomialQuotient[(c*x)^m*Pq, a + b*x^2, x], f = Coeff[PolynomialRema
inder[(c*x)^m*Pq, a + b*x^2, x], x, 0], g = Coeff[PolynomialRemainder[(c*x)
^m*Pq, a + b*x^2, x], x, 1]}, Simp[(a*g - b*f*x)*((a + b*x^2)^(p + 1)/(2*a*
b*(p + 1))), x] + Simp[1/(2*a*(p + 1))   Int[(c*x)^m*(a + b*x^2)^(p + 1)*Ex
pandToSum[(2*a*(p + 1)*Q)/(c*x)^m + (f*(2*p + 3))/(c*x)^m, x], x], x]] /; F
reeQ[{a, b, c}, x] && PolyQ[Pq, x] && LtQ[p, -1] && ILtQ[m, 0]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3696
```
Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^4)^(
p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Simp[ff^(m + 1
)/f   Subst[Int[x^m*((a + 2*a*ff^2*x^2 + (a + b)*ff^4*x^4)^p/(1 + ff^2*x^2)
^(m/2 + 2*p + 1)), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f}, x] &
& IntegerQ[m/2] && IntegerQ[p]
```

Maple [A] (verified)

Time = 3.48 (sec) , antiderivative size = 121, normalized size of antiderivative = 0.91

method	result
derivativedivides	$\frac{a^3\left(\frac{16}{35}-\frac{\operatorname{csch}(dx+c)^6}{7}+\frac{6\operatorname{csch}(dx+c)^4}{35}-\frac{8\operatorname{csch}(dx+c)^2}{35}\right)\coth(dx+c)+3a^2b\left(\frac{2}{3}-\frac{\operatorname{csch}(dx+c)^2}{3}\right)\coth(dx+c)+3b^2a(dx+c)+}{d}$
default	$\frac{a^3\left(\frac{16}{35}-\frac{\operatorname{csch}(dx+c)^6}{7}+\frac{6\operatorname{csch}(dx+c)^4}{35}-\frac{8\operatorname{csch}(dx+c)^2}{35}\right)\coth(dx+c)+3a^2b\left(\frac{2}{3}-\frac{\operatorname{csch}(dx+c)^2}{3}\right)\coth(dx+c)+3b^2a(dx+c)+}{d}$
parallelrisch	$\frac{-\left(\cosh(dx+c)-\frac{3\cosh(3dx+3c)}{5}+\frac{\cosh(5dx+5c)}{5}-\frac{\cosh(7dx+7c)}{35}\right)\operatorname{sech}\left(\frac{dx}{2}+\frac{c}{2}\right)^7a^3\operatorname{csch}\left(\frac{dx}{2}+\frac{c}{2}\right)^7-96b\,a^2\operatorname{sech}\left(\frac{dx}{2}+\frac{c}{2}\right)^3}{512d}$
risch	$3a\,b^2x+\frac{3b^3x}{8}+\frac{e^{4dx+4c}b^3}{64d}-\frac{e^{2dx+2c}b^3}{8d}+\frac{e^{-2dx-2c}b^3}{8d}-\frac{e^{-4dx-4c}b^3}{64d}-\frac{4a^2\left(105\,e^{10dx+10c}b-455\,e^{8dx+8c}\right.}{}$

input
```
int(csch(d*x+c)^8*(a+b*sinh(d*x+c)^4)^3,x,method=_RETURNVERBOSE)
```

output
```
1/d*(a^3*(16/35-1/7*csch(d*x+c)^6+6/35*csch(d*x+c)^4-8/35*csch(d*x+c)^2)*c
oth(d*x+c)+3*a^2*b*(2/3-1/3*csch(d*x+c)^2)*coth(d*x+c)+3*b^2*a*(d*x+c)+b^3
*((1/4*sinh(d*x+c)^3-3/8*sinh(d*x+c))*cosh(d*x+c)+3/8*d*x+3/8*c))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 928 vs. $2(123) = 246$.

Time = 0.09 (sec) , antiderivative size = 928, normalized size of antiderivative = 6.98

$$\int \operatorname{csch}^8(c+dx)\left(a+b\sinh^4(c+dx)\right)^3\,dx = \text{Too large to display}$$

input
```
integrate(csch(d*x+c)^8*(a+b*sinh(d*x+c)^4)^3,x, algorithm="fricas")
```

output
```
1/2240*(35*b^3*cosh(d*x + c)^11 + 385*b^3*cosh(d*x + c)*sinh(d*x + c)^10 -
 525*b^3*cosh(d*x + c)^9 + 525*(11*b^3*cosh(d*x + c)^3 - 9*b^3*cosh(d*x +
c))*sinh(d*x + c)^8 + (1024*a^3 + 4480*a^2*b + 2695*b^3)*cosh(d*x + c)^7 -
 8*(128*a^3 + 560*a^2*b - 105*(8*a*b^2 + b^3)*d*x)*sinh(d*x + c)^7 + 7*(23
10*b^3*cosh(d*x + c)^5 - 6300*b^3*cosh(d*x + c)^3 + (1024*a^3 + 4480*a^2*b
 + 2695*b^3)*cosh(d*x + c))*sinh(d*x + c)^6 - 7*(1024*a^3 + 4480*a^2*b + 9
75*b^3)*cosh(d*x + c)^5 + 56*(128*a^3 + 560*a^2*b - 105*(8*a*b^2 + b^3)*d*
x - 3*(128*a^3 + 560*a^2*b - 105*(8*a*b^2 + b^3)*d*x)*cosh(d*x + c)^2)*sin
h(d*x + c)^5 + 35*(330*b^3*cosh(d*x + c)^7 - 1890*b^3*cosh(d*x + c)^5 + (1
024*a^3 + 4480*a^2*b + 2695*b^3)*cosh(d*x + c)^3 - (1024*a^3 + 4480*a^2*b
+ 975*b^3)*cosh(d*x + c))*sinh(d*x + c)^4 + 42*(512*a^3 + 1600*a^2*b + 215
*b^3)*cosh(d*x + c)^3 - 56*(5*(128*a^3 + 560*a^2*b - 105*(8*a*b^2 + b^3)*d
*x)*cosh(d*x + c)^4 + 384*a^3 + 1680*a^2*b - 315*(8*a*b^2 + b^3)*d*x - 10*
(128*a^3 + 560*a^2*b - 105*(8*a*b^2 + b^3)*d*x)*cosh(d*x + c)^2)*sinh(d*x
+ c)^3 + 7*(275*b^3*cosh(d*x + c)^9 - 2700*b^3*cosh(d*x + c)^7 + 3*(1024*a
^3 + 4480*a^2*b + 2695*b^3)*cosh(d*x + c)^5 - 10*(1024*a^3 + 4480*a^2*b +
975*b^3)*cosh(d*x + c)^3 + 18*(512*a^3 + 1600*a^2*b + 215*b^3)*cosh(d*x +
c))*sinh(d*x + c)^2 - 70*(512*a^3 + 576*a^2*b + 63*b^3)*cosh(d*x + c) - 56
*((128*a^3 + 560*a^2*b - 105*(8*a*b^2 + b^3)*d*x)*cosh(d*x + c)^6 - 5*(128
*a^3 + 560*a^2*b - 105*(8*a*b^2 + b^3)*d*x)*cosh(d*x + c)^4 - 640*a^3 -...
```

Sympy [F(-1)]

Timed out.

$$\int \operatorname{csch}^8(c+dx)\left(a+b\sinh^4(c+dx)\right)^3\,dx=\text{Timed out}$$

input
```
integrate(csch(d*x+c)**8*(a+b*sinh(d*x+c)**4)**3,x)
```

output
```
Timed out
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 537 vs. $2(123) = 246$.

Time = 0.05 (sec) , antiderivative size = 537, normalized size of antiderivative = 4.04

$$\int \operatorname{csch}^8(c+dx)\left(a+b\sinh^4(c+dx)\right)^3\,dx = \text{Too large to display}$$

input
```
integrate(csch(d*x+c)^8*(a+b*sinh(d*x+c)^4)^3,x, algorithm="maxima")
```

output
```
1/64*b^3*(24*x + e^(4*d*x + 4*c)/d - 8*e^(2*d*x + 2*c)/d + 8*e^(-2*d*x - 2
*c)/d - e^(-4*d*x - 4*c)/d) + 3*a*b^2*x + 32/35*a^3*(7*e^(-2*d*x - 2*c)/(d
*(7*e^(-2*d*x - 2*c) - 21*e^(-4*d*x - 4*c) + 35*e^(-6*d*x - 6*c) - 35*e^(-
8*d*x - 8*c) + 21*e^(-10*d*x - 10*c) - 7*e^(-12*d*x - 12*c) + e^(-14*d*x -
 14*c) - 1)) - 21*e^(-4*d*x - 4*c)/(d*(7*e^(-2*d*x - 2*c) - 21*e^(-4*d*x -
 4*c) + 35*e^(-6*d*x - 6*c) - 35*e^(-8*d*x - 8*c) + 21*e^(-10*d*x - 10*c)
- 7*e^(-12*d*x - 12*c) + e^(-14*d*x - 14*c) - 1)) + 35*e^(-6*d*x - 6*c)/(d
*(7*e^(-2*d*x - 2*c) - 21*e^(-4*d*x - 4*c) + 35*e^(-6*d*x - 6*c) - 35*e^(-
8*d*x - 8*c) + 21*e^(-10*d*x - 10*c) - 7*e^(-12*d*x - 12*c) + e^(-14*d*x -
 14*c) - 1)) - 1/(d*(7*e^(-2*d*x - 2*c) - 21*e^(-4*d*x - 4*c) + 35*e^(-6*d
*x - 6*c) - 35*e^(-8*d*x - 8*c) + 21*e^(-10*d*x - 10*c) - 7*e^(-12*d*x - 1
2*c) + e^(-14*d*x - 14*c) - 1))) + 4*a^2*b*(3*e^(-2*d*x - 2*c)/(d*(3*e^(-2
*d*x - 2*c) - 3*e^(-4*d*x - 4*c) + e^(-6*d*x - 6*c) - 1)) - 1/(d*(3*e^(-2*
d*x - 2*c) - 3*e^(-4*d*x - 4*c) + e^(-6*d*x - 6*c) - 1)))
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 253 vs. $2(123) = 246$.

Time = 0.31 (sec) , antiderivative size = 253, normalized size of antiderivative = 1.90

$$\int \operatorname{csch}^8(c+dx)\left(a+b\sinh^4(c+dx)\right)^3\,dx$$
$$= \frac{35\,b^3e^{(4\,dx+4\,c)} - 280\,b^3e^{(2\,dx+2\,c)} + 840\,(8\,ab^2+b^3)(dx+c) - 35\,\left(144\,ab^2e^{(4\,dx+4\,c)} + 18\,b^3e^{(4\,dx+4\,c)} - 8\,b^3e\right.}{}$$

input
```
integrate(csch(d*x+c)^8*(a+b*sinh(d*x+c)^4)^3,x, algorithm="giac")
```

output
```
1/2240*(35*b^3*e^(4*d*x + 4*c) - 280*b^3*e^(2*d*x + 2*c) + 840*(8*a*b^2 +
b^3)*(d*x + c) - 35*(144*a*b^2*e^(4*d*x + 4*c) + 18*b^3*e^(4*d*x + 4*c) -
8*b^3*e^(2*d*x + 2*c) + b^3)*e^(-4*d*x - 4*c) - 256*(105*a^2*b*e^(10*d*x +
 10*c) - 455*a^2*b*e^(8*d*x + 8*c) + 280*a^3*e^(6*d*x + 6*c) + 770*a^2*b*e
^(6*d*x + 6*c) - 168*a^3*e^(4*d*x + 4*c) - 630*a^2*b*e^(4*d*x + 4*c) + 56*
a^3*e^(2*d*x + 2*c) + 245*a^2*b*e^(2*d*x + 2*c) - 8*a^3 - 35*a^2*b)/(e^(2*
d*x + 2*c) - 1)^7)/d
```

Mupad [B] (verification not implemented)

Time = 1.92 (sec) , antiderivative size = 749, normalized size of antiderivative = 5.63

$$\int \operatorname{csch}^8(c+dx)\left(a+b\sinh^4(c+dx)\right)^3\,dx = \text{Too large to display}$$

input
```
int((a + b*sinh(c + d*x)^4)^3/sinh(c + d*x)^8,x)
```

output
```
((32*a^2*b)/(35*d) - (16*exp(2*c + 2*d*x)*(9*a^2*b + 8*a^3))/(35*d) + (192
*a^2*b*exp(4*c + 4*d*x))/(35*d) - (16*a^2*b*exp(6*c + 6*d*x))/(7*d))/(5*ex
p(2*c + 2*d*x) - 10*exp(4*c + 4*d*x) + 10*exp(6*c + 6*d*x) - 5*exp(8*c + 8
*d*x) + exp(10*c + 10*d*x) - 1) - ((16*exp(6*c + 6*d*x)*(9*a^2*b + 8*a^3))
/(7*d) + (24*a^2*b*exp(2*c + 2*d*x))/(7*d) - (96*a^2*b*exp(4*c + 4*d*x))/(
7*d) - (96*a^2*b*exp(8*c + 8*d*x))/(7*d) + (24*a^2*b*exp(10*c + 10*d*x))/(
7*d))/(7*exp(2*c + 2*d*x) - 21*exp(4*c + 4*d*x) + 35*exp(6*c + 6*d*x) - 35
*exp(8*c + 8*d*x) + 21*exp(10*c + 10*d*x) - 7*exp(12*c + 12*d*x) + exp(14*
c + 14*d*x) - 1) - ((4*a^2*b)/(7*d) + (8*exp(4*c + 4*d*x)*(9*a^2*b + 8*a^3
))/(7*d) - (32*a^2*b*exp(2*c + 2*d*x))/(7*d) - (64*a^2*b*exp(6*c + 6*d*x))
/(7*d) + (20*a^2*b*exp(8*c + 8*d*x))/(7*d))/(15*exp(4*c + 4*d*x) - 6*exp(2
*c + 2*d*x) - 20*exp(6*c + 6*d*x) + 15*exp(8*c + 8*d*x) - 6*exp(10*c + 10*
d*x) + exp(12*c + 12*d*x) + 1) + ((32*a^2*b)/(35*d) - (8*a^2*b*exp(2*c + 2
*d*x))/(7*d))/(3*exp(2*c + 2*d*x) - 3*exp(4*c + 4*d*x) + exp(6*c + 6*d*x)
- 1) - ((4*(9*a^2*b + 8*a^3))/(35*d) - (96*a^2*b*exp(2*c + 2*d*x))/(35*d)
+ (12*a^2*b*exp(4*c + 4*d*x))/(7*d))/(6*exp(4*c + 4*d*x) - 4*exp(2*c + 2*d
*x) - 4*exp(6*c + 6*d*x) + exp(8*c + 8*d*x) + 1) + (3*b^2*x*(8*a + b))/8 +
 (b^3*exp(- 2*c - 2*d*x))/(8*d) - (b^3*exp(2*c + 2*d*x))/(8*d) - (b^3*exp(
- 4*c - 4*d*x))/(64*d) + (b^3*exp(4*c + 4*d*x))/(64*d) - (4*a^2*b)/(7*d*(e
xp(4*c + 4*d*x) - 2*exp(2*c + 2*d*x) + 1))
```

Reduce [B] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 695, normalized size of antiderivative = 5.23

$$\int \operatorname{csch}^8(c+dx)\left(a+b\sinh^4(c+dx)\right)^3\,dx$$
$$= \frac{35b^3 - 17640e^{8dx+8c}b^3dx + 235200e^{10dx+10c}a\,b^2dx + 17640e^{14dx+14c}b^3dx - 47040e^{16dx+16c}a\,b^2dx - 29400}{}$$

input
```
int(csch(d*x+c)^8*(a+b*sinh(d*x+c)^4)^3,x)
```

output
```
(35*e**(22*c + 22*d*x)*b**3 - 525*e**(20*c + 20*d*x)*b**3 + 6720*e**(18*c
+ 18*d*x)*a*b**2*d*x + 840*e**(18*c + 18*d*x)*b**3*d*x + 1720*e**(18*c + 1
8*d*x)*b**3 - 47040*e**(16*c + 16*d*x)*a*b**2*d*x - 5880*e**(16*c + 16*d*x
)*b**3*d*x - 26880*e**(14*c + 14*d*x)*a**2*b + 141120*e**(14*c + 14*d*x)*a
*b**2*d*x + 17640*e**(14*c + 14*d*x)*b**3*d*x - 11445*e**(14*c + 14*d*x)*b
**3 + 116480*e**(12*c + 12*d*x)*a**2*b - 235200*e**(12*c + 12*d*x)*a*b**2*
d*x - 29400*e**(12*c + 12*d*x)*b**3*d*x + 29715*e**(12*c + 12*d*x)*b**3 -
71680*e**(10*c + 10*d*x)*a**3 - 197120*e**(10*c + 10*d*x)*a**2*b + 235200*
e**(10*c + 10*d*x)*a*b**2*d*x + 29400*e**(10*c + 10*d*x)*b**3*d*x - 38535*
e**(10*c + 10*d*x)*b**3 + 43008*e**(8*c + 8*d*x)*a**3 + 161280*e**(8*c + 8
*d*x)*a**2*b - 141120*e**(8*c + 8*d*x)*a*b**2*d*x - 17640*e**(8*c + 8*d*x)
*b**3*d*x + 29505*e**(8*c + 8*d*x)*b**3 - 14336*e**(6*c + 6*d*x)*a**3 - 62
720*e**(6*c + 6*d*x)*a**2*b + 47040*e**(6*c + 6*d*x)*a*b**2*d*x + 5880*e**
(6*c + 6*d*x)*b**3*d*x - 13650*e**(6*c + 6*d*x)*b**3 + 2048*e**(4*c + 4*d*
x)*a**3 + 8960*e**(4*c + 4*d*x)*a**2*b - 6720*e**(4*c + 4*d*x)*a*b**2*d*x
- 840*e**(4*c + 4*d*x)*b**3*d*x + 3670*e**(4*c + 4*d*x)*b**3 - 525*e**(2*c
 + 2*d*x)*b**3 + 35*b**3)/(2240*e**(4*c + 4*d*x)*d*(e**(14*c + 14*d*x) - 7
*e**(12*c + 12*d*x) + 21*e**(10*c + 10*d*x) - 35*e**(8*c + 8*d*x) + 35*e**
(6*c + 6*d*x) - 21*e**(4*c + 4*d*x) + 7*e**(2*c + 2*d*x) - 1))
```

3.198 $\int \operatorname{csch}^{10}(c+dx) \left(a+b\sinh^4(c+dx)\right)^3 dx$

Optimal result

Integrand size = 23, antiderivative size = 140

$$\int \operatorname{csch}^{10}(c+dx) \left(a+b\sinh^4(c+dx)\right)^3 dx = -\frac{b^3x}{2} - \frac{a(a^2+3ab+3b^2)\coth(c+dx)}{d} + \frac{2a^2(2a+3b)\coth^3(c+dx)}{3d} - \frac{3a^2(2a+b)\coth^5(c+dx)}{5d} + \frac{4a^3\coth^7(c+dx)}{7d} - \frac{a^3\coth^9(c+dx)}{9d} + \frac{b^3\cosh(c+dx)\sinh(c+dx)}{2d}$$

output
```
-1/2*b^3*x-a*(a^2+3*a*b+3*b^2)*coth(d*x+c)/d+2/3*a^2*(2*a+3*b)*coth(d*x+c)
^3/d-3/5*a^2*(2*a+b)*coth(d*x+c)^5/d+4/7*a^3*coth(d*x+c)^7/d-1/9*a^3*coth(
d*x+c)^9/d+1/2*b^3*cosh(d*x+c)*sinh(d*x+c)/d
```

Mathematica [A] (verified)

Time = 2.39 (sec) , antiderivative size = 115, normalized size of antiderivative = 0.82

$$\int \operatorname{csch}^{10}(c+dx)\left(a+b\sinh^4(c+dx)\right)^3\,dx$$
$$=\frac{-4a\coth(c+dx)\left(128a^2+504ab+945b^2-4a(16a+63b)\operatorname{csch}^2(c+dx)+3a(16a+63b)\operatorname{csch}^4(c+dx)\right.}{1260d}$$

input
```
Integrate[Csch[c + d*x]^10*(a + b*Sinh[c + d*x]^4)^3,x]
```

output
```
(-4*a*Coth[c + d*x]*(128*a^2 + 504*a*b + 945*b^2 - 4*a*(16*a + 63*b)*Csch[
c + d*x]^2 + 3*a*(16*a + 63*b)*Csch[c + d*x]^4 - 40*a^2*Csch[c + d*x]^6 +
35*a^2*Csch[c + d*x]^8) + 315*b^3*(-2*(c + d*x) + Sinh[2*(c + d*x)]))/(126
0*d)
```

Rubi [A] (verified)

Time = 0.67 (sec) , antiderivative size = 143, normalized size of antiderivative = 1.02, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}=0.304$, Rules used = {3042, 25, 3696, 1582, 25, 2333, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \operatorname{csch}^{10}(c+dx)\left(a+b\sinh^4(c+dx)\right)^3\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int -\frac{\left(a+b\sin(ic+idx)^4\right)^3}{\sin(ic+idx)^{10}}dx$$

$$\Big\downarrow \text{25}$$

$$-\int \frac{\left(b\sin(ic+idx)^4+a\right)^3}{\sin(ic+idx)^{10}}dx$$

$$\Big\downarrow \text{3696}$$

$$\frac{\int \frac{\coth^{10}(c+dx)\left((a+b)\tanh^4(c+dx)-2a\tanh^2(c+dx)+a\right)^3}{\left(1-\tanh^2(c+dx)\right)^2}d\tanh(c+dx)}{d}$$

$\big\downarrow$ 1582

$$\frac{\frac{b^3\tanh(c+dx)}{2(1-\tanh^2(c+dx))}-\frac{1}{2}\int -\frac{\coth^{10}(c+dx)\left(-\left(\left(2a^3+6ba^2+6b^2a+b^3\right)\tanh^{10}(c+dx)\right)+2a\left(5a^2+9ba+3b^2\right)\tanh^8(c+dx)-2a^2(10a+9b)\tanh^6(c}{1-\tanh^2(c+dx)}}{d}$$

$\big\downarrow$ 25

$$\frac{\frac{1}{2}\int \frac{\coth^{10}(c+dx)\left(-\left(\left(2a^3+6ba^2+6b^2a+b^3\right)\tanh^{10}(c+dx)\right)+2a\left(5a^2+9ba+3b^2\right)\tanh^8(c+dx)-2a^2(10a+9b)\tanh^6(c+dx)+2a^2(10a+3b)\tanh^4}{1-\tanh^2(c+dx)}}{d}$$

$\big\downarrow$ 2333

$$\frac{\frac{1}{2}\int \Big(2a^3\coth^{10}(c+dx)-8a^3\coth^8(c+dx)+6a^2(2a+b)\coth^6(c+dx)-4a^2(2a+3b)\coth^4(c+dx)+2a\big(a^2}{d}$$

$\big\downarrow$ 2009

$$\frac{\frac{1}{2}\left(-\frac{2}{9}a^3\coth^9(c+dx)+\frac{8}{7}a^3\coth^7(c+dx)-2a\left(a^2+3ab+3b^2\right)\coth(c+dx)-\frac{6}{5}a^2(2a+b)\coth^5(c+dx)+\frac{4}{3}c\right.}{d}$$

input
```
Int[Csch[c + d*x]^10*(a + b*Sinh[c + d*x]^4)^3,x]
```

output
```
((-(b^3*ArcTanh[Tanh[c + d*x]]) - 2*a*(a^2 + 3*a*b + 3*b^2)*Coth[c + d*x]
+ (4*a^2*(2*a + 3*b)*Coth[c + d*x]^3)/3 - (6*a^2*(2*a + b)*Coth[c + d*x]^5
)/5 + (8*a^3*Coth[c + d*x]^7)/7 - (2*a^3*Coth[c + d*x]^9)/9)/2 + (b^3*Tanh
[c + d*x])/(2*(1 - Tanh[c + d*x]^2)))/d
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 1582
```
Int[(x_)^(m_)*((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^
4)^(p_.), x_Symbol] :> Simp[(-d)^(m/2 - 1)*(c*d^2 - b*d*e + a*e^2)^p*x*((d
+ e*x^2)^(q + 1)/(2*e^(2*p + m/2)*(q + 1))), x] + Simp[(-d)^(m/2 - 1)/(2*e^
(2*p)*(q + 1))   Int[x^m*(d + e*x^2)^(q + 1)*ExpandToSum[Together[(1/(d + e
*x^2))*(2*(-d)^(-m/2 + 1)*e^(2*p)*(q + 1)*(a + b*x^2 + c*x^4)^p - ((c*d^2 -
 b*d*e + a*e^2)^p/(e^(m/2)*x^m))*(d + e*(2*q + 3)*x^2))], x], x], x] /; Fre
eQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && IGtQ[p, 0] && ILtQ[q, -1]
&& ILtQ[m/2, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2333
```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[
ExpandIntegrand[(c*x)^m*Pq*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x]
&& PolyQ[Pq, x] && IGtQ[p, -2]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3696
```
Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^4)^(
p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Simp[ff^(m + 1
)/f   Subst[Int[x^m*((a + 2*a*ff^2*x^2 + (a + b)*ff^4*x^4)^p/(1 + ff^2*x^2)
^(m/2 + 2*p + 1)), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f}, x] &
& IntegerQ[m/2] && IntegerQ[p]
```

Maple [A] (verified)

Time = 2.98 (sec) , antiderivative size = 130, normalized size of antiderivative = 0.93

method	result
derivativedivides	$\frac{a^3\left(-\frac{128}{315}-\frac{\operatorname{csch}(dx+c)^8}{9}+\frac{8\operatorname{csch}(dx+c)^6}{63}-\frac{16\operatorname{csch}(dx+c)^4}{105}+\frac{64\operatorname{csch}(dx+c)^2}{315}\right)\coth(dx+c)+3a^2b\left(-\frac{8}{15}-\frac{\operatorname{csch}(dx+c)^4}{5}+\frac{4\operatorname{csc}}{}\right.}{d}$
default	$\frac{a^3\left(-\frac{128}{315}-\frac{\operatorname{csch}(dx+c)^8}{9}+\frac{8\operatorname{csch}(dx+c)^6}{63}-\frac{16\operatorname{csch}(dx+c)^4}{105}+\frac{64\operatorname{csch}(dx+c)^2}{315}\right)\coth(dx+c)+3a^2b\left(-\frac{8}{15}-\frac{\operatorname{csch}(dx+c)^4}{5}+\frac{4\operatorname{csc}}{}\right.}{d}$
parallelrisch	$\frac{-a^3\operatorname{sech}\left(\frac{dx}{2}+\frac{c}{2}\right)^9\operatorname{csch}\left(\frac{dx}{2}+\frac{c}{2}\right)^9(\cosh(9dx+9c)+126\cosh(dx+c)+36\cosh(5dx+5c)-84\cosh(3dx+3c)-9\cosh(7dx-}{}$
risch	$-\frac{b^3x}{2}+\frac{e^{2dx+2c}b^3}{8d}-\frac{e^{-2dx-2c}b^3}{8d}-\frac{2a(945b^2e^{16dx+16c}-7560b^2e^{14dx+14c}+5040\,e^{12dx+12c}ab+26460\,e^{12dx+12c}}{}$

input
```
int(csch(d*x+c)^10*(a+b*sinh(d*x+c)^4)^3,x,method=_RETURNVERBOSE)
```

output
```
1/d*(a^3*(-128/315-1/9*csch(d*x+c)^8+8/63*csch(d*x+c)^6-16/105*csch(d*x+c)
^4+64/315*csch(d*x+c)^2)*coth(d*x+c)+3*a^2*b*(-8/15-1/5*csch(d*x+c)^4+4/15
*csch(d*x+c)^2)*coth(d*x+c)-3*b^2*a*coth(d*x+c)+b^3*(1/2*cosh(d*x+c)*sinh(
d*x+c)-1/2*d*x-1/2*c))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1314 vs. 2(128) = 256.

Time = 0.11 (sec) , antiderivative size = 1314, normalized size of antiderivative = 9.39

$$\int \operatorname{csch}^{10}(c+dx)\left(a+b\sinh^4(c+dx)\right)^3\,dx = \text{Too large to display}$$

input
```
integrate(csch(d*x+c)^10*(a+b*sinh(d*x+c)^4)^3,x, algorithm="fricas")
```

output
```
1/2520*(315*b^3*cosh(d*x + c)^11 + 3465*b^3*cosh(d*x + c)*sinh(d*x + c)^10
 - (1024*a^3 + 4032*a^2*b + 7560*a*b^2 + 2835*b^3)*cosh(d*x + c)^9 - 4*(31
5*b^3*d*x - 256*a^3 - 1008*a^2*b - 1890*a*b^2)*sinh(d*x + c)^9 + 9*(5775*b
^3*cosh(d*x + c)^3 - (1024*a^3 + 4032*a^2*b + 7560*a*b^2 + 2835*b^3)*cosh(
d*x + c))*sinh(d*x + c)^8 + 9*(1024*a^3 + 4032*a^2*b + 5880*a*b^2 + 1225*b
^3)*cosh(d*x + c)^7 + 36*(315*b^3*d*x - 256*a^3 - 1008*a^2*b - 1890*a*b^2
- 4*(315*b^3*d*x - 256*a^3 - 1008*a^2*b - 1890*a*b^2)*cosh(d*x + c)^2)*sin
h(d*x + c)^7 + 21*(6930*b^3*cosh(d*x + c)^5 - 4*(1024*a^3 + 4032*a^2*b + 7
560*a*b^2 + 2835*b^3)*cosh(d*x + c)^3 + 3*(1024*a^3 + 4032*a^2*b + 5880*a*
b^2 + 1225*b^3)*cosh(d*x + c))*sinh(d*x + c)^6 - 9*(4096*a^3 + 16128*a^2*b
 + 16800*a*b^2 + 2625*b^3)*cosh(d*x + c)^5 - 36*(1260*b^3*d*x + 14*(315*b^
3*d*x - 256*a^3 - 1008*a^2*b - 1890*a*b^2)*cosh(d*x + c)^4 - 1024*a^3 - 40
32*a^2*b - 7560*a*b^2 - 21*(315*b^3*d*x - 256*a^3 - 1008*a^2*b - 1890*a*b^
2)*cosh(d*x + c)^2)*sinh(d*x + c)^5 + 9*(11550*b^3*cosh(d*x + c)^7 - 14*(1
024*a^3 + 4032*a^2*b + 7560*a*b^2 + 2835*b^3)*cosh(d*x + c)^5 + 35*(1024*a
^3 + 4032*a^2*b + 5880*a*b^2 + 1225*b^3)*cosh(d*x + c)^3 - 5*(4096*a^3 + 1
6128*a^2*b + 16800*a*b^2 + 2625*b^3)*cosh(d*x + c))*sinh(d*x + c)^4 + 42*(
2048*a^3 + 6144*a^2*b + 5040*a*b^2 + 675*b^3)*cosh(d*x + c)^3 - 12*(28*(31
5*b^3*d*x - 256*a^3 - 1008*a^2*b - 1890*a*b^2)*cosh(d*x + c)^6 - 8820*b^3*
d*x - 105*(315*b^3*d*x - 256*a^3 - 1008*a^2*b - 1890*a*b^2)*cosh(d*x + ...
```

Sympy [F(-1)]

Timed out.

$$\int \operatorname{csch}^{10}(c+dx)\left(a+b\sinh^4(c+dx)\right)^3\,dx=\text{Timed out}$$

input
```
integrate(csch(d*x+c)**10*(a+b*sinh(d*x+c)**4)**3,x)
```

output
```
Timed out
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 842 vs. $2(128) = 256$.

Time = 0.05 (sec) , antiderivative size = 842, normalized size of antiderivative = 6.01

$$\int \operatorname{csch}^{10}(c+dx)\left(a+b\sinh^4(c+dx)\right)^3\,dx = \text{Too large to display}$$

input
```
integrate(csch(d*x+c)^10*(a+b*sinh(d*x+c)^4)^3,x, algorithm="maxima")
```

output
```
-1/8*b^3*(4*x - e^(2*d*x + 2*c)/d + e^(-2*d*x - 2*c)/d) - 256/315*a^3*(9*e
^(-2*d*x - 2*c)/(d*(9*e^(-2*d*x - 2*c) - 36*e^(-4*d*x - 4*c) + 84*e^(-6*d*
x - 6*c) - 126*e^(-8*d*x - 8*c) + 126*e^(-10*d*x - 10*c) - 84*e^(-12*d*x -
 12*c) + 36*e^(-14*d*x - 14*c) - 9*e^(-16*d*x - 16*c) + e^(-18*d*x - 18*c)
 - 1)) - 36*e^(-4*d*x - 4*c)/(d*(9*e^(-2*d*x - 2*c) - 36*e^(-4*d*x - 4*c)
+ 84*e^(-6*d*x - 6*c) - 126*e^(-8*d*x - 8*c) + 126*e^(-10*d*x - 10*c) - 84
*e^(-12*d*x - 12*c) + 36*e^(-14*d*x - 14*c) - 9*e^(-16*d*x - 16*c) + e^(-1
8*d*x - 18*c) - 1)) + 84*e^(-6*d*x - 6*c)/(d*(9*e^(-2*d*x - 2*c) - 36*e^(-
4*d*x - 4*c) + 84*e^(-6*d*x - 6*c) - 126*e^(-8*d*x - 8*c) + 126*e^(-10*d*x
 - 10*c) - 84*e^(-12*d*x - 12*c) + 36*e^(-14*d*x - 14*c) - 9*e^(-16*d*x -
16*c) + e^(-18*d*x - 18*c) - 1)) - 126*e^(-8*d*x - 8*c)/(d*(9*e^(-2*d*x -
2*c) - 36*e^(-4*d*x - 4*c) + 84*e^(-6*d*x - 6*c) - 126*e^(-8*d*x - 8*c) +
126*e^(-10*d*x - 10*c) - 84*e^(-12*d*x - 12*c) + 36*e^(-14*d*x - 14*c) - 9
*e^(-16*d*x - 16*c) + e^(-18*d*x - 18*c) - 1)) - 1/(d*(9*e^(-2*d*x - 2*c)
- 36*e^(-4*d*x - 4*c) + 84*e^(-6*d*x - 6*c) - 126*e^(-8*d*x - 8*c) + 126*e
^(-10*d*x - 10*c) - 84*e^(-12*d*x - 12*c) + 36*e^(-14*d*x - 14*c) - 9*e^(-
16*d*x - 16*c) + e^(-18*d*x - 18*c) - 1))) - 16/5*a^2*b*(5*e^(-2*d*x - 2*c
)/(d*(5*e^(-2*d*x - 2*c) - 10*e^(-4*d*x - 4*c) + 10*e^(-6*d*x - 6*c) - 5*e
^(-8*d*x - 8*c) + e^(-10*d*x - 10*c) - 1)) - 10*e^(-4*d*x - 4*c)/(d*(5*e^(
-2*d*x - 2*c) - 10*e^(-4*d*x - 4*c) + 10*e^(-6*d*x - 6*c) - 5*e^(-8*d*x...
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 360 vs. $2(128) = 256$.

Time = 0.32 (sec) , antiderivative size = 360, normalized size of antiderivative = 2.57

$$\int \operatorname{csch}^{10}(c+dx)\left(a+b\sinh^4(c+dx)\right)^3 dx =$$
$$-\frac{1260\,(dx+c)b^3-315\,b^3e^{(2\,dx+2\,c)}-315\,\left(2\,b^3e^{(2\,dx+2\,c)}-b^3\right)e^{(-2\,dx-2\,c)}+\frac{16\,\left(945\,ab^2e^{(16\,dx+16\,c)}-7560\,ab^2e^{(14\,d\ldots}\right.}{\ldots}}{\ldots}$$

input
```
integrate(csch(d*x+c)^10*(a+b*sinh(d*x+c)^4)^3,x, algorithm="giac")
```

output
```
-1/2520*(1260*(d*x + c)*b^3 - 315*b^3*e^(2*d*x + 2*c) - 315*(2*b^3*e^(2*d*
x + 2*c) - b^3)*e^(-2*d*x - 2*c) + 16*(945*a*b^2*e^(16*d*x + 16*c) - 7560*
a*b^2*e^(14*d*x + 14*c) + 5040*a^2*b*e^(12*d*x + 12*c) + 26460*a*b^2*e^(12
*d*x + 12*c) - 22680*a^2*b*e^(10*d*x + 10*c) - 52920*a*b^2*e^(10*d*x + 10*
c) + 16128*a^3*e^(8*d*x + 8*c) + 40824*a^2*b*e^(8*d*x + 8*c) + 66150*a*b^2
*e^(8*d*x + 8*c) - 10752*a^3*e^(6*d*x + 6*c) - 37296*a^2*b*e^(6*d*x + 6*c)
 - 52920*a*b^2*e^(6*d*x + 6*c) + 4608*a^3*e^(4*d*x + 4*c) + 18144*a^2*b*e^
(4*d*x + 4*c) + 26460*a*b^2*e^(4*d*x + 4*c) - 1152*a^3*e^(2*d*x + 2*c) - 4
536*a^2*b*e^(2*d*x + 2*c) - 7560*a*b^2*e^(2*d*x + 2*c) + 128*a^3 + 504*a^2
*b + 945*a*b^2)/(e^(2*d*x + 2*c) - 1)^9)/d
```

Mupad [B] (verification not implemented)

Time = 1.90 (sec) , antiderivative size = 1500, normalized size of antiderivative = 10.71

$$\int \operatorname{csch}^{10}(c+dx)\left(a+b\sinh^4(c+dx)\right)^3 dx = \text{Too large to display}$$

input
```
int((a + b*sinh(c + d*x)^4)^3/sinh(c + d*x)^10,x)
```

output
```
((2*a*b^2)/(3*d) - (2*exp(2*c + 2*d*x)*(7*a*b^2 + 4*a^2*b))/(3*d) + (2*exp
(4*c + 4*d*x)*(7*a*b^2 + 8*a^2*b))/d + (10*exp(8*c + 8*d*x)*(7*a*b^2 + 8*a
^2*b))/(3*d) - (2*exp(10*c + 10*d*x)*(7*a*b^2 + 4*a^2*b))/d - (2*exp(6*c +
 6*d*x)*(105*a*b^2 + 144*a^2*b + 128*a^3))/(9*d) + (14*a*b^2*exp(12*c + 12
*d*x))/(3*d) - (2*a*b^2*exp(14*c + 14*d*x))/(3*d))/(28*exp(4*c + 4*d*x) -
8*exp(2*c + 2*d*x) - 56*exp(6*c + 6*d*x) + 70*exp(8*c + 8*d*x) - 56*exp(10
*c + 10*d*x) + 28*exp(12*c + 12*d*x) - 8*exp(14*c + 14*d*x) + exp(16*c + 1
6*d*x) + 1) - ((2*(105*a*b^2 + 144*a^2*b + 128*a^3))/(315*d) - (8*exp(2*c
+ 2*d*x)*(7*a*b^2 + 8*a^2*b))/(21*d) + (4*exp(4*c + 4*d*x)*(7*a*b^2 + 4*a^
2*b))/(7*d) - (8*a*b^2*exp(6*c + 6*d*x))/(3*d) + (2*a*b^2*exp(8*c + 8*d*x)
)/(3*d))/(5*exp(2*c + 2*d*x) - 10*exp(4*c + 4*d*x) + 10*exp(6*c + 6*d*x) -
 5*exp(8*c + 8*d*x) + exp(10*c + 10*d*x) - 1) - ((2*(7*a*b^2 + 4*a^2*b))/(
21*d) - (4*a*b^2*exp(2*c + 2*d*x))/(3*d) + (2*a*b^2*exp(4*c + 4*d*x))/(3*d
))/(3*exp(2*c + 2*d*x) - 3*exp(4*c + 4*d*x) + exp(6*c + 6*d*x) - 1) - ((2*
a*b^2)/(3*d) + (8*exp(4*c + 4*d*x)*(7*a*b^2 + 4*a^2*b))/(3*d) - (16*exp(6*
c + 6*d*x)*(7*a*b^2 + 8*a^2*b))/(3*d) - (16*exp(10*c + 10*d*x)*(7*a*b^2 +
8*a^2*b))/(3*d) + (8*exp(12*c + 12*d*x)*(7*a*b^2 + 4*a^2*b))/(3*d) + (4*ex
p(8*c + 8*d*x)*(105*a*b^2 + 144*a^2*b + 128*a^3))/(9*d) - (16*a*b^2*exp(2*
c + 2*d*x))/(3*d) - (16*a*b^2*exp(14*c + 14*d*x))/(3*d) + (2*a*b^2*exp(16*
c + 16*d*x))/(3*d))/(9*exp(2*c + 2*d*x) - 36*exp(4*c + 4*d*x) + 84*exp(...
```

Reduce [B] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 784, normalized size of antiderivative = 5.60

$$\int \operatorname{csch}^{10}(c+dx) \left(a+b\sinh^4(c+dx)\right)^3 \, dx$$
$$= \frac{-1680e^{20dx+20c}a\,b^2+315b^3-105840e^{8dx+8c}b^3dx+105840e^{14dx+14c}b^3dx-158760e^{12dx+12c}b^3dx-4060e^{\cdots}}{\cdots}$$

input
```
int(csch(d*x+c)^10*(a+b*sinh(d*x+c)^4)^3,x)
```

output

```
(315*e**(22*c + 22*d*x)*b**3 - 1680*e**(20*c + 20*d*x)*a*b**2 - 1260*e**(2
0*c + 20*d*x)*b**3*d*x - 1610*e**(20*c + 20*d*x)*b**3 + 11340*e**(18*c + 1
8*d*x)*b**3*d*x + 60480*e**(16*c + 16*d*x)*a*b**2 - 45360*e**(16*c + 16*d*
x)*b**3*d*x + 20475*e**(16*c + 16*d*x)*b**3 - 80640*e**(14*c + 14*d*x)*a**
2*b - 282240*e**(14*c + 14*d*x)*a*b**2 + 105840*e**(14*c + 14*d*x)*b**3*d*
x - 74550*e**(14*c + 14*d*x)*b**3 + 362880*e**(12*c + 12*d*x)*a**2*b + 635
040*e**(12*c + 12*d*x)*a*b**2 - 158760*e**(12*c + 12*d*x)*b**3*d*x + 14112
0*e**(12*c + 12*d*x)*b**3 - 258048*e**(10*c + 10*d*x)*a**3 - 653184*e**(10
*c + 10*d*x)*a**2*b - 846720*e**(10*c + 10*d*x)*a*b**2 + 158760*e**(10*c +
 10*d*x)*b**3*d*x - 167580*e**(10*c + 10*d*x)*b**3 + 172032*e**(8*c + 8*d*
x)*a**3 + 596736*e**(8*c + 8*d*x)*a**2*b + 705600*e**(8*c + 8*d*x)*a*b**2
- 105840*e**(8*c + 8*d*x)*b**3*d*x + 131250*e**(8*c + 8*d*x)*b**3 - 73728*
e**(6*c + 6*d*x)*a**3 - 290304*e**(6*c + 6*d*x)*a**2*b - 362880*e**(6*c +
6*d*x)*a*b**2 + 45360*e**(6*c + 6*d*x)*b**3*d*x - 67725*e**(6*c + 6*d*x)*b
**3 + 18432*e**(4*c + 4*d*x)*a**3 + 72576*e**(4*c + 4*d*x)*a**2*b + 105840
*e**(4*c + 4*d*x)*a*b**2 - 11340*e**(4*c + 4*d*x)*b**3*d*x + 22050*e**(4*c
 + 4*d*x)*b**3 - 2048*e**(2*c + 2*d*x)*a**3 - 8064*e**(2*c + 2*d*x)*a**2*b
 - 13440*e**(2*c + 2*d*x)*a*b**2 + 1260*e**(2*c + 2*d*x)*b**3*d*x - 4060*e
**(2*c + 2*d*x)*b**3 + 315*b**3)/(2520*e**(2*c + 2*d*x)*d*(e**(18*c + 18*d
*x) - 9*e**(16*c + 16*d*x) + 36*e**(14*c + 14*d*x) - 84*e**(12*c + 12*d...
```

3.199 $\int \operatorname{csch}^{12}(c+dx)\left(a+b\sinh^4(c+dx)\right)^3\,dx$

Optimal result

Integrand size = 23, antiderivative size = 147

$$\begin{aligned}\int \operatorname{csch}^{12}(c+dx)\left(a+b\sinh^4(c+dx)\right)^3\,dx = b^3x+\frac{a(a^2+3ab+3b^2)\coth(c+dx)}{d}\\ -\frac{a(5a^2+9ab+3b^2)\coth^3(c+dx)}{3d}\\ +\frac{a^2(10a+9b)\coth^5(c+dx)}{5d}\\ -\frac{a^2(10a+3b)\coth^7(c+dx)}{7d}\\ +\frac{5a^3\coth^9(c+dx)}{9d}-\frac{a^3\coth^{11}(c+dx)}{11d}\end{aligned}$$

output
```
b^3*x+a*(a^2+3*a*b+3*b^2)*coth(d*x+c)/d-1/3*a*(5*a^2+9*a*b+3*b^2)*coth(d*x
+c)^3/d+1/5*a^2*(10*a+9*b)*coth(d*x+c)^5/d-1/7*a^2*(10*a+3*b)*coth(d*x+c)^
7/d+5/9*a^3*coth(d*x+c)^9/d-1/11*a^3*coth(d*x+c)^11/d
```

Mathematica [A] (verified)

Time = 6.25 (sec) , antiderivative size = 239, normalized size of antiderivative = 1.63

$$\int \operatorname{csch}^{12}(c+dx)\left(a+b\sinh^4(c+dx)\right)^3 dx = \frac{b^3(c+dx)}{d} + \frac{2(640a^3\cosh(c+dx)+2376a^2b\cosh(c+dx)+3465ab^2\cosh(c+dx))\operatorname{csch}(c+dx)}{3465d} + \frac{(-640a^3\cosh(c+dx)-2376a^2b\cosh(c+dx)-3465ab^2\cosh(c+dx))\operatorname{csch}^3(c+dx)}{3465d} + \frac{2(80a^3\cosh(c+dx)+297a^2b\cosh(c+dx))\operatorname{csch}^5(c+dx)}{1155d} + \frac{(-80a^3\cosh(c+dx)-297a^2b\cosh(c+dx))\operatorname{csch}^7(c+dx)}{693d} + \frac{10a^3\coth(c+dx)\operatorname{csch}^8(c+dx)}{99d} - \frac{a^3\coth(c+dx)\operatorname{csch}^{10}(c+dx)}{11d}$$

input
```
Integrate[Csch[c + d*x]^12*(a + b*Sinh[c + d*x]^4)^3,x]
```

output
```
(b^3*(c + d*x))/d + (2*(640*a^3*Cosh[c + d*x] + 2376*a^2*b*Cosh[c + d*x] +
 3465*a*b^2*Cosh[c + d*x])*Csch[c + d*x])/(3465*d) + ((-640*a^3*Cosh[c + d
*x] - 2376*a^2*b*Cosh[c + d*x] - 3465*a*b^2*Cosh[c + d*x])*Csch[c + d*x]^3
)/(3465*d) + (2*(80*a^3*Cosh[c + d*x] + 297*a^2*b*Cosh[c + d*x])*Csch[c +
d*x]^5)/(1155*d) + ((-80*a^3*Cosh[c + d*x] - 297*a^2*b*Cosh[c + d*x])*Csch
[c + d*x]^7)/(693*d) + (10*a^3*Coth[c + d*x]*Csch[c + d*x]^8)/(99*d) - (a^
3*Coth[c + d*x]*Csch[c + d*x]^10)/(11*d)
```

Rubi [A] (verified)

Time = 0.42 (sec) , antiderivative size = 139, normalized size of antiderivative = 0.95, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.174$, Rules used = {3042, 3696, 1584, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \operatorname{csch}^{12}(c+dx)\left(a+b\sinh^4(c+dx)\right)^3 dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{(a+b\sin(ic+idx)^4)^3}{\sin(ic+idx)^{12}}dx$$

$$\Big\downarrow \text{3696}$$

$$\frac{\int \frac{\coth^{12}(c+dx)((a+b)\tanh^4(c+dx)-2a\tanh^2(c+dx)+a)^3}{1-\tanh^2(c+dx)}d\tanh(c+dx)}{d}$$

$$\Big\downarrow \text{1584}$$

$$\frac{\int \left(a^3\coth^{12}(c+dx)-5a^3\coth^{10}(c+dx)+a^2(10a+3b)\coth^8(c+dx)-a^2(10a+9b)\coth^6(c+dx)+a(5a^2+\right.}{d}$$

$$\Big\downarrow \text{2009}$$

$$\frac{-\frac{1}{11}a^3\coth^{11}(c+dx)+\frac{5}{9}a^3\coth^9(c+dx)-\frac{1}{3}a(5a^2+9ab+3b^2)\coth^3(c+dx)+a(a^2+3ab+3b^2)\coth(c+d}{d}$$

input
```
Int[Csch[c + d*x]^12*(a + b*Sinh[c + d*x]^4)^3,x]
```

output
```
(b^3*ArcTanh[Tanh[c + d*x]] + a*(a^2 + 3*a*b + 3*b^2)*Coth[c + d*x] - (a*(
5*a^2 + 9*a*b + 3*b^2)*Coth[c + d*x]^3)/3 + (a^2*(10*a + 9*b)*Coth[c + d*x
]^5)/5 - (a^2*(10*a + 3*b)*Coth[c + d*x]^7)/7 + (5*a^3*Coth[c + d*x]^9)/9
- (a^3*Coth[c + d*x]^11)/11)/d
```

Defintions of rubi rules used

rule 1584
```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (
c_.)*(x_)^4)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(f*x)^m*(d + e*x^2)^q*
(a + b*x^2 + c*x^4)^p, x], x] /; FreeQ[{a, b, c, d, e, f, m, q}, x] && NeQ[
b^2 - 4*a*c, 0] && IGtQ[p, 0] && IGtQ[q, -2]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3696
```
Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^4)^(
p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Simp[ff^(m + 1
)/f   Subst[Int[x^m*((a + 2*a*ff^2*x^2 + (a + b)*ff^4*x^4)^p/(1 + ff^2*x^2)
^(m/2 + 2*p + 1)), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f}, x] &
& IntegerQ[m/2] && IntegerQ[p]
```

Maple [A] (verified)

Time = 2.63 (sec) , antiderivative size = 145, normalized size of antiderivative = 0.99

method	result
derivativedivides	$\frac{a^3\left(\frac{256}{693}-\frac{\operatorname{csch}(dx+c)^{10}}{11}+\frac{10\operatorname{csch}(dx+c)^8}{99}-\frac{80\operatorname{csch}(dx+c)^6}{693}+\frac{32\operatorname{csch}(dx+c)^4}{231}-\frac{128\operatorname{csch}(dx+c)^2}{693}\right)\coth(dx+c)+3a^2b\left(\frac{16}{35}-\operatorname{csc}\right.}{d}$
default	$\frac{a^3\left(\frac{256}{693}-\frac{\operatorname{csch}(dx+c)^{10}}{11}+\frac{10\operatorname{csch}(dx+c)^8}{99}-\frac{80\operatorname{csch}(dx+c)^6}{693}+\frac{32\operatorname{csch}(dx+c)^4}{231}-\frac{128\operatorname{csch}(dx+c)^2}{693}\right)\coth(dx+c)+3a^2b\left(\frac{16}{35}-\operatorname{csc}\right.}{d}$
parallelrisch	$-\left(\cosh(9dx+9c)-\frac{\cosh(11dx+11c)}{11}+42\cosh(dx+c)-30\cosh(3dx+3c)+15\cosh(5dx+5c)-5\cosh(7dx+7c)\right)\operatorname{sech}\left(\frac{dx}{2}+\right.$
risch	$b^3x-\frac{4a\left(10395b^2e^{18dx+18c}-86625b^2e^{16dx+16c}+83160ab\,e^{14dx+14c}+318780b^2e^{14dx+14c}-382536\,e^{12dx+12c}ab-67\right.}{}$

input
```
int(csch(d*x+c)^12*(a+b*sinh(d*x+c)^4)^3,x,method=_RETURNVERBOSE)
```

output
```
1/d*(a^3*(256/693-1/11*csch(d*x+c)^10+10/99*csch(d*x+c)^8-80/693*csch(d*x+
c)^6+32/231*csch(d*x+c)^4-128/693*csch(d*x+c)^2)*coth(d*x+c)+3*a^2*b*(16/3
5-1/7*csch(d*x+c)^6+6/35*csch(d*x+c)^4-8/35*csch(d*x+c)^2)*coth(d*x+c)+3*b
^2*a*(2/3-1/3*csch(d*x+c)^2)*coth(d*x+c)+b^3*(d*x+c))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1607 vs. $2(137) = 274$.

Time = 0.11 (sec) , antiderivative size = 1607, normalized size of antiderivative = 10.93

$$\int \operatorname{csch}^{12}(c+dx) \left(a+b\sinh^4(c+dx)\right)^3 \, dx = \text{Too large to display}$$

input
```
integrate(csch(d*x+c)^12*(a+b*sinh(d*x+c)^4)^3,x, algorithm="fricas")
```

output
```
1/3465*(2*(640*a^3 + 2376*a^2*b + 3465*a*b^2)*cosh(d*x + c)^11 + 22*(640*a
^3 + 2376*a^2*b + 3465*a*b^2)*cosh(d*x + c)*sinh(d*x + c)^10 + (3465*b^3*d
*x - 1280*a^3 - 4752*a^2*b - 6930*a*b^2)*sinh(d*x + c)^11 - 22*(640*a^3 +
2376*a^2*b + 3465*a*b^2)*cosh(d*x + c)^9 - 11*(3465*b^3*d*x - 1280*a^3 - 4
752*a^2*b - 6930*a*b^2 - 5*(3465*b^3*d*x - 1280*a^3 - 4752*a^2*b - 6930*a*
b^2)*cosh(d*x + c)^2)*sinh(d*x + c)^9 + 66*(5*(640*a^3 + 2376*a^2*b + 3465
*a*b^2)*cosh(d*x + c)^3 - 3*(640*a^3 + 2376*a^2*b + 3465*a*b^2)*cosh(d*x +
 c))*sinh(d*x + c)^8 + 110*(640*a^3 + 2376*a^2*b + 3087*a*b^2)*cosh(d*x +
c)^7 + 11*(17325*b^3*d*x + 30*(3465*b^3*d*x - 1280*a^3 - 4752*a^2*b - 6930
*a*b^2)*cosh(d*x + c)^4 - 6400*a^3 - 23760*a^2*b - 34650*a*b^2 - 36*(3465*
b^3*d*x - 1280*a^3 - 4752*a^2*b - 6930*a*b^2)*cosh(d*x + c)^2)*sinh(d*x +
c)^7 + 154*(6*(640*a^3 + 2376*a^2*b + 3465*a*b^2)*cosh(d*x + c)^5 - 12*(64
0*a^3 + 2376*a^2*b + 3465*a*b^2)*cosh(d*x + c)^3 + 5*(640*a^3 + 2376*a^2*b
 + 3087*a*b^2)*cosh(d*x + c))*sinh(d*x + c)^6 - 330*(640*a^3 + 2376*a^2*b
+ 2415*a*b^2)*cosh(d*x + c)^5 + 33*(14*(3465*b^3*d*x - 1280*a^3 - 4752*a^2
*b - 6930*a*b^2)*cosh(d*x + c)^6 - 17325*b^3*d*x - 42*(3465*b^3*d*x - 1280
*a^3 - 4752*a^2*b - 6930*a*b^2)*cosh(d*x + c)^4 + 6400*a^3 + 23760*a^2*b +
 34650*a*b^2 + 35*(3465*b^3*d*x - 1280*a^3 - 4752*a^2*b - 6930*a*b^2)*cosh
(d*x + c)^2)*sinh(d*x + c)^5 + 22*(30*(640*a^3 + 2376*a^2*b + 3465*a*b^2)*
cosh(d*x + c)^7 - 126*(640*a^3 + 2376*a^2*b + 3465*a*b^2)*cosh(d*x + c)...
```

Sympy [F(-1)]

Timed out.

$$\int \operatorname{csch}^{12}(c+dx)\left(a+b\sinh^4(c+dx)\right)^3\,dx=\text{Timed out}$$

input
```
integrate(csch(d*x+c)**12*(a+b*sinh(d*x+c)**4)**3,x)
```

output
```
Timed out
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1291 vs. $2(137) = 274$.

Time = 0.06 (sec) , antiderivative size = 1291, normalized size of antiderivative = 8.78

$$\int \operatorname{csch}^{12}(c+dx)\left(a+b\sinh^4(c+dx)\right)^3\,dx=\text{Too large to display}$$

input
```
integrate(csch(d*x+c)^12*(a+b*sinh(d*x+c)^4)^3,x, algorithm="maxima")
```

output
```
b^3*x + 512/693*a^3*(11*e^(-2*d*x - 2*c)/(d*(11*e^(-2*d*x - 2*c) - 55*e^(-
4*d*x - 4*c) + 165*e^(-6*d*x - 6*c) - 330*e^(-8*d*x - 8*c) + 462*e^(-10*d*
x - 10*c) - 462*e^(-12*d*x - 12*c) + 330*e^(-14*d*x - 14*c) - 165*e^(-16*d
*x - 16*c) + 55*e^(-18*d*x - 18*c) - 11*e^(-20*d*x - 20*c) + e^(-22*d*x -
22*c) - 1)) - 55*e^(-4*d*x - 4*c)/(d*(11*e^(-2*d*x - 2*c) - 55*e^(-4*d*x -
 4*c) + 165*e^(-6*d*x - 6*c) - 330*e^(-8*d*x - 8*c) + 462*e^(-10*d*x - 10*
c) - 462*e^(-12*d*x - 12*c) + 330*e^(-14*d*x - 14*c) - 165*e^(-16*d*x - 16
*c) + 55*e^(-18*d*x - 18*c) - 11*e^(-20*d*x - 20*c) + e^(-22*d*x - 22*c) -
 1)) + 165*e^(-6*d*x - 6*c)/(d*(11*e^(-2*d*x - 2*c) - 55*e^(-4*d*x - 4*c)
+ 165*e^(-6*d*x - 6*c) - 330*e^(-8*d*x - 8*c) + 462*e^(-10*d*x - 10*c) - 4
62*e^(-12*d*x - 12*c) + 330*e^(-14*d*x - 14*c) - 165*e^(-16*d*x - 16*c) +
55*e^(-18*d*x - 18*c) - 11*e^(-20*d*x - 20*c) + e^(-22*d*x - 22*c) - 1)) -
 330*e^(-8*d*x - 8*c)/(d*(11*e^(-2*d*x - 2*c) - 55*e^(-4*d*x - 4*c) + 165*
e^(-6*d*x - 6*c) - 330*e^(-8*d*x - 8*c) + 462*e^(-10*d*x - 10*c) - 462*e^(
-12*d*x - 12*c) + 330*e^(-14*d*x - 14*c) - 165*e^(-16*d*x - 16*c) + 55*e^(
-18*d*x - 18*c) - 11*e^(-20*d*x - 20*c) + e^(-22*d*x - 22*c) - 1)) + 462*e
^(-10*d*x - 10*c)/(d*(11*e^(-2*d*x - 2*c) - 55*e^(-4*d*x - 4*c) + 165*e^(-
6*d*x - 6*c) - 330*e^(-8*d*x - 8*c) + 462*e^(-10*d*x - 10*c) - 462*e^(-12*
d*x - 12*c) + 330*e^(-14*d*x - 14*c) - 165*e^(-16*d*x - 16*c) + 55*e^(-18*
d*x - 18*c) - 11*e^(-20*d*x - 20*c) + e^(-22*d*x - 22*c) - 1)) - 1/(d*(...
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 359 vs. $2(137) = 274$.

Time = 0.31 (sec) , antiderivative size = 359, normalized size of antiderivative = 2.44

$$\int \operatorname{csch}^{12}(c+dx)\left(a+b\sinh^4(c+dx)\right)^3\,dx$$

$$= \frac{3465\,(dx+c)b^3 - \frac{4\left(10395\,ab^2e^{(18\,dx+18\,c)} - 86625\,ab^2e^{(16\,dx+16\,c)} + 83160\,a^2be^{(14\,dx+14\,c)} + 318780\,ab^2e^{(14\,dx+14\,c)} - 382536\,a^2be^{(12\,dx-}}{}}{}$$

input
```
integrate(csch(d*x+c)^12*(a+b*sinh(d*x+c)^4)^3,x, algorithm="giac")
```

output
```
1/3465*(3465*(d*x + c)*b^3 - 4*(10395*a*b^2*e^(18*d*x + 18*c) - 86625*a*b^
2*e^(16*d*x + 16*c) + 83160*a^2*b*e^(14*d*x + 14*c) + 318780*a*b^2*e^(14*d
*x + 14*c) - 382536*a^2*b*e^(12*d*x + 12*c) - 679140*a*b^2*e^(12*d*x + 12*
c) + 295680*a^3*e^(10*d*x + 10*c) + 715176*a^2*b*e^(10*d*x + 10*c) + 92169
0*a*b^2*e^(10*d*x + 10*c) - 211200*a^3*e^(8*d*x + 8*c) - 700920*a^2*b*e^(8
*d*x + 8*c) - 824670*a*b^2*e^(8*d*x + 8*c) + 105600*a^3*e^(6*d*x + 6*c) +
392040*a^2*b*e^(6*d*x + 6*c) + 485100*a*b^2*e^(6*d*x + 6*c) - 35200*a^3*e^
(4*d*x + 4*c) - 130680*a^2*b*e^(4*d*x + 4*c) - 180180*a*b^2*e^(4*d*x + 4*c
) + 7040*a^3*e^(2*d*x + 2*c) + 26136*a^2*b*e^(2*d*x + 2*c) + 38115*a*b^2*e
^(2*d*x + 2*c) - 640*a^3 - 2376*a^2*b - 3465*a*b^2)/(e^(2*d*x + 2*c) - 1)^
11)/d
```

Mupad [B] (verification not implemented)

Time = 1.82 (sec) , antiderivative size = 1955, normalized size of antiderivative = 13.30

$$\int \operatorname{csch}^{12}(c+dx) \left(a+b\sinh^4(c+dx)\right)^3 \, dx = \text{Too large to display}$$

input
```
int((a + b*sinh(c + d*x)^4)^3/sinh(c + d*x)^12,x)
```

output
```
((64*a*b^2)/(165*d) - (32*exp(2*c + 2*d*x)*(7*a*b^2 + 4*a^2*b))/(55*d) + (
128*exp(4*c + 4*d*x)*(7*a*b^2 + 8*a^2*b))/(55*d) + (64*exp(8*c + 8*d*x)*(7
*a*b^2 + 8*a^2*b))/(11*d) - (224*exp(10*c + 10*d*x)*(7*a*b^2 + 4*a^2*b))/(
55*d) - (32*exp(6*c + 6*d*x)*(105*a*b^2 + 144*a^2*b + 128*a^3))/(99*d) + (
1792*a*b^2*exp(12*c + 12*d*x))/(165*d) - (96*a*b^2*exp(14*c + 14*d*x))/(55
*d))/(9*exp(2*c + 2*d*x) - 36*exp(4*c + 4*d*x) + 84*exp(6*c + 6*d*x) - 126
*exp(8*c + 8*d*x) + 126*exp(10*c + 10*d*x) - 84*exp(12*c + 12*d*x) + 36*ex
p(14*c + 14*d*x) - 9*exp(16*c + 16*d*x) + exp(18*c + 18*d*x) - 1) - ((4*(1
05*a*b^2 + 144*a^2*b + 128*a^3))/(693*d) - (32*exp(2*c + 2*d*x)*(7*a*b^2 +
 8*a^2*b))/(77*d) + (8*exp(4*c + 4*d*x)*(7*a*b^2 + 4*a^2*b))/(11*d) - (128
*a*b^2*exp(6*c + 6*d*x))/(33*d) + (12*a*b^2*exp(8*c + 8*d*x))/(11*d))/(15*
exp(4*c + 4*d*x) - 6*exp(2*c + 2*d*x) - 20*exp(6*c + 6*d*x) + 15*exp(8*c +
 8*d*x) - 6*exp(10*c + 10*d*x) + exp(12*c + 12*d*x) + 1) - ((4*(7*a*b^2 +
4*a^2*b))/(55*d) - (32*exp(2*c + 2*d*x)*(7*a*b^2 + 8*a^2*b))/(55*d) - (32*
exp(6*c + 6*d*x)*(7*a*b^2 + 8*a^2*b))/(11*d) + (28*exp(8*c + 8*d*x)*(7*a*b
^2 + 4*a^2*b))/(11*d) + (4*exp(4*c + 4*d*x)*(105*a*b^2 + 144*a^2*b + 128*a
^3))/(33*d) - (448*a*b^2*exp(10*c + 10*d*x))/(55*d) + (84*a*b^2*exp(12*c +
 12*d*x))/(55*d))/(28*exp(4*c + 4*d*x) - 8*exp(2*c + 2*d*x) - 56*exp(6*c +
 6*d*x) + 70*exp(8*c + 8*d*x) - 56*exp(10*c + 10*d*x) + 28*exp(12*c + 12*d
*x) - 8*exp(14*c + 14*d*x) + exp(16*c + 16*d*x) + 1) - ((4*(7*a*b^2 + 4...
```

Reduce [B] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 682, normalized size of antiderivative = 4.64

$$\int \operatorname{csch}^{12}(c+dx)\left(a+b\sinh^4(c+dx)\right)^3\,dx$$
$$= \frac{-1143450e^{8dx+8c}b^3dx + 13860a\,b^2 + 1143450e^{14dx+14c}b^3dx - 1600830e^{12dx+12c}b^3dx + 3465e^{22dx+22c}b^3dx}{}$$

input
```
int(csch(d*x+c)^12*(a+b*sinh(d*x+c)^4)^3,x)
```

output
```
(3465*e**(22*c + 22*d*x)*b**3*d*x - 38115*e**(20*c + 20*d*x)*b**3*d*x - 41
580*e**(18*c + 18*d*x)*a*b**2 + 190575*e**(18*c + 18*d*x)*b**3*d*x + 34650
0*e**(16*c + 16*d*x)*a*b**2 - 571725*e**(16*c + 16*d*x)*b**3*d*x - 332640*
e**(14*c + 14*d*x)*a**2*b - 1275120*e**(14*c + 14*d*x)*a*b**2 + 1143450*e*
*(14*c + 14*d*x)*b**3*d*x + 1530144*e**(12*c + 12*d*x)*a**2*b + 2716560*e*
*(12*c + 12*d*x)*a*b**2 - 1600830*e**(12*c + 12*d*x)*b**3*d*x - 1182720*e*
*(10*c + 10*d*x)*a**3 - 2860704*e**(10*c + 10*d*x)*a**2*b - 3686760*e**(10
*c + 10*d*x)*a*b**2 + 1600830*e**(10*c + 10*d*x)*b**3*d*x + 844800*e**(8*c
 + 8*d*x)*a**3 + 2803680*e**(8*c + 8*d*x)*a**2*b + 3298680*e**(8*c + 8*d*x
)*a*b**2 - 1143450*e**(8*c + 8*d*x)*b**3*d*x - 422400*e**(6*c + 6*d*x)*a**
3 - 1568160*e**(6*c + 6*d*x)*a**2*b - 1940400*e**(6*c + 6*d*x)*a*b**2 + 57
1725*e**(6*c + 6*d*x)*b**3*d*x + 140800*e**(4*c + 4*d*x)*a**3 + 522720*e**
(4*c + 4*d*x)*a**2*b + 720720*e**(4*c + 4*d*x)*a*b**2 - 190575*e**(4*c + 4
*d*x)*b**3*d*x - 28160*e**(2*c + 2*d*x)*a**3 - 104544*e**(2*c + 2*d*x)*a**
2*b - 152460*e**(2*c + 2*d*x)*a*b**2 + 38115*e**(2*c + 2*d*x)*b**3*d*x + 2
560*a**3 + 9504*a**2*b + 13860*a*b**2 - 3465*b**3*d*x)/(3465*d*(e**(22*c +
 22*d*x) - 11*e**(20*c + 20*d*x) + 55*e**(18*c + 18*d*x) - 165*e**(16*c +
16*d*x) + 330*e**(14*c + 14*d*x) - 462*e**(12*c + 12*d*x) + 462*e**(10*c +
 10*d*x) - 330*e**(8*c + 8*d*x) + 165*e**(6*c + 6*d*x) - 55*e**(4*c + 4*d*
x) + 11*e**(2*c + 2*d*x) - 1))
```

3.200 $\int \operatorname{csch}^{14}(c+dx)\left(a+b\sinh^4(c+dx)\right)^3\,dx$

Optimal result

Integrand size = 23, antiderivative size = 144

$$\begin{aligned}\int \operatorname{csch}^{14}(c+dx)\left(a+b\sinh^4(c+dx)\right)^3\,dx &= -\frac{(a+b)^3\coth(c+dx)}{d}+\frac{2a(a+b)^2\coth^3(c+dx)}{d}\\ &\quad-\frac{3a(a+b)(5a+b)\coth^5(c+dx)}{5d}+\frac{4a^2(5a+3b)\coth^7(c+dx)}{7d}\\ &\quad-\frac{a^2(5a+b)\coth^9(c+dx)}{3d}+\frac{6a^3\coth^{11}(c+dx)}{11d}-\frac{a^3\coth^{13}(c+dx)}{13d}\end{aligned}$$

output
```
-(a+b)^3*coth(d*x+c)/d+2*a*(a+b)^2*coth(d*x+c)^3/d-3/5*a*(a+b)*(5*a+b)*cot
h(d*x+c)^5/d+4/7*a^2*(5*a+3*b)*coth(d*x+c)^7/d-1/3*a^2*(5*a+b)*coth(d*x+c)
^9/d+6/11*a^3*coth(d*x+c)^11/d-1/13*a^3*coth(d*x+c)^13/d
```

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 350 vs. 2(144) = 288.

Time = 3.34 (sec) , antiderivative size = 350, normalized size of antiderivative = 2.43

$$\int \operatorname{csch}^{14}(c+dx)\left(a+b\sinh^4(c+dx)\right)^3\,dx = -\frac{(8580(1024a^3+1152a^2b+840ab^2+231b^3)\cosh(c+dx)-6435(1024a^3+2944a^2b+2408ab^2+693b}{}$$

input
```
Integrate[Csch[c + d*x]^14*(a + b*Sinh[c + d*x]^4)^3,x]
```

output
```
-1/61501440*((8580*(1024*a^3 + 1152*a^2*b + 840*a*b^2 + 231*b^3)*Cosh[c +
d*x] - 6435*(1024*a^3 + 2944*a^2*b + 2408*a*b^2 + 693*b^3)*Cosh[3*(c + d*x
)] + 3660800*a^3*Cosh[5*(c + d*x)] + 13087360*a^2*b*Cosh[5*(c + d*x)] + 13
093080*a*b^2*Cosh[5*(c + d*x)] + 4129125*b^3*Cosh[5*(c + d*x)] - 1464320*a
^3*Cosh[7*(c + d*x)] - 5234944*a^2*b*Cosh[7*(c + d*x)] - 6390384*a*b^2*Cos
h[7*(c + d*x)] - 2312310*b^3*Cosh[7*(c + d*x)] + 399360*a^3*Cosh[9*(c + d*
x)] + 1427712*a^2*b*Cosh[9*(c + d*x)] + 1873872*a*b^2*Cosh[9*(c + d*x)] +
810810*b^3*Cosh[9*(c + d*x)] - 66560*a^3*Cosh[11*(c + d*x)] - 237952*a^2*b
*Cosh[11*(c + d*x)] - 312312*a*b^2*Cosh[11*(c + d*x)] - 165165*b^3*Cosh[11
*(c + d*x)] + 5120*a^3*Cosh[13*(c + d*x)] + 18304*a^2*b*Cosh[13*(c + d*x)]
 + 24024*a*b^2*Cosh[13*(c + d*x)] + 15015*b^3*Cosh[13*(c + d*x)])*Csch[c +
 d*x]^13)/d
```

Rubi [A] (verified)

Time = 0.36 (sec) , antiderivative size = 127, normalized size of antiderivative = 0.88, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.217, Rules used = {3042, 25, 3696, 1433, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \operatorname{csch}^{14}(c+dx)\left(a+b\sinh^4(c+dx)\right)^3\,dx$$

$$\Big\downarrow \; 3042$$

$$\int -\frac{\left(a+b\sin(ic+idx)^4\right)^3}{\sin(ic+idx)^{14}}dx$$

$$\Big\downarrow \ 25$$

$$-\int \frac{\left(b\sin(ic+idx)^4+a\right)^3}{\sin(ic+idx)^{14}}dx$$

$$\Big\downarrow \ 3696$$

$$\frac{\int \coth^{14}(c+dx)\left((a+b)\tanh^4(c+dx)-2a\tanh^2(c+dx)+a\right)^3 d\tanh(c+dx)}{d}$$

$$\Big\downarrow \ 1433$$

$$\frac{\int \left(a^3\coth^{14}(c+dx)-6a^3\coth^{12}(c+dx)+3a^2(5a+b)\coth^{10}(c+dx)-4a^2(5a+3b)\coth^8(c+dx)+3a(a+b\right.}{d}$$

$$\Big\downarrow \ 2009$$

$$\frac{-\frac{1}{13}a^3\coth^{13}(c+dx)+\frac{6}{11}a^3\coth^{11}(c+dx)-\frac{1}{3}a^2(5a+b)\coth^9(c+dx)+\frac{4}{7}a^2(5a+3b)\coth^7(c+dx)-\frac{3}{5}a(a-}{d}$$

input
```
Int[Csch[c + d*x]^14*(a + b*Sinh[c + d*x]^4)^3,x]
```

output
```
(-((a + b)^3*Coth[c + d*x]) + 2*a*(a + b)^2*Coth[c + d*x]^3 - (3*a*(a + b)
*(5*a + b)*Coth[c + d*x]^5)/5 + (4*a^2*(5*a + 3*b)*Coth[c + d*x]^7)/7 - (a
^2*(5*a + b)*Coth[c + d*x]^9)/3 + (6*a^3*Coth[c + d*x]^11)/11 - (a^3*Coth[
c + d*x]^13)/13)/d
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 1433
```
Int[((d_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol]
 :> Int[ExpandIntegrand[(d*x)^m*(a + b*x^2 + c*x^4)^p, x], x] /; FreeQ[{a,
b, c, d, m}, x] && IGtQ[p, 0] && (EqQ[p, 1] ||  !IntegerQ[(m + 1)/2])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3696
```
Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^4)^(
p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Simp[ff^(m + 1
)/f   Subst[Int[x^m*((a + 2*a*ff^2*x^2 + (a + b)*ff^4*x^4)^p/(1 + ff^2*x^2)
^(m/2 + 2*p + 1)), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f}, x] &
& IntegerQ[m/2] && IntegerQ[p]
```

Maple [A] (verified)

Time = 3.54 (sec) , antiderivative size = 177, normalized size of antiderivative = 1.23

method	result
derivativedivides	$\frac{a^3\left(-\frac{1024}{3003}-\frac{\operatorname{csch}(dx+c)^{12}}{13}+\frac{12\operatorname{csch}(dx+c)^{10}}{143}-\frac{40\operatorname{csch}(dx+c)^8}{429}+\frac{320\operatorname{csch}(dx+c)^6}{3003}-\frac{128\operatorname{csch}(dx+c)^4}{1001}+\frac{512\operatorname{csch}(dx+c)^2}{3003}\right)\coth(\ldots}{\ldots}$
default	$\frac{a^3\left(-\frac{1024}{3003}-\frac{\operatorname{csch}(dx+c)^{12}}{13}+\frac{12\operatorname{csch}(dx+c)^{10}}{143}-\frac{40\operatorname{csch}(dx+c)^8}{429}+\frac{320\operatorname{csch}(dx+c)^6}{3003}-\frac{128\operatorname{csch}(dx+c)^4}{1001}+\frac{512\operatorname{csch}(dx+c)^2}{3003}\right)\coth(\ldots}{\ldots}$
parallelrisch	$\frac{-1155\coth\left(\frac{dx}{2}+\frac{c}{2}\right)^{13}a^3+17745\coth\left(\frac{dx}{2}+\frac{c}{2}\right)^{11}a^3-130130\left(a+\frac{8b}{13}\right)a^2\coth\left(\frac{dx}{2}+\frac{c}{2}\right)^9+613470a^2\left(a+\frac{216b}{143}\right)\coth\left(\frac{dx}{2}\ldots\right.}{\ldots}$
risch	$-\frac{2\left(7432425b^3e^{8dx+8c}-3303300b^3e^{6dx+6c}+399360a^3e^{4dx+4c}+990990b^3e^{4dx+4c}-66560a^3e^{2dx+2c}+24024b^2a+5120\ldots\right.}{\ldots}$

input
```
int(csch(d*x+c)^14*(a+b*sinh(d*x+c)^4)^3,x,method=_RETURNVERBOSE)
```

output
```
1/d*(a^3*(-1024/3003-1/13*csch(d*x+c)^12+12/143*csch(d*x+c)^10-40/429*csch
(d*x+c)^8+320/3003*csch(d*x+c)^6-128/1001*csch(d*x+c)^4+512/3003*csch(d*x+
c)^2)*coth(d*x+c)+3*a^2*b*(-128/315-1/9*csch(d*x+c)^8+8/63*csch(d*x+c)^6-1
6/105*csch(d*x+c)^4+64/315*csch(d*x+c)^2)*coth(d*x+c)+3*b^2*a*(-8/15-1/5*c
sch(d*x+c)^4+4/15*csch(d*x+c)^2)*coth(d*x+c)-b^3*coth(d*x+c))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2323 vs. $2(134) = 268$.

Time = 0.11 (sec) , antiderivative size = 2323, normalized size of antiderivative = 16.13

$$\int \operatorname{csch}^{14}(c+dx) \left(a+b\sinh^4(c+dx)\right)^3 \, dx = \text{Too large to display}$$

input
```
integrate(csch(d*x+c)^14*(a+b*sinh(d*x+c)^4)^3,x, algorithm="fricas")
```

output
```
-4/15015*((2560*a^3 + 9152*a^2*b + 12012*a*b^2 + 15015*b^3)*cosh(d*x + c)^
12 - 48*(640*a^3 + 2288*a^2*b + 3003*a*b^2)*cosh(d*x + c)*sinh(d*x + c)^11
 + (2560*a^3 + 9152*a^2*b + 12012*a*b^2 + 15015*b^3)*sinh(d*x + c)^12 - 52
*(640*a^3 + 2288*a^2*b + 3003*a*b^2 + 3465*b^3)*cosh(d*x + c)^10 - 2*(1664
0*a^3 + 59488*a^2*b + 78078*a*b^2 + 90090*b^3 - 33*(2560*a^3 + 9152*a^2*b
+ 12012*a*b^2 + 15015*b^3)*cosh(d*x + c)^2)*sinh(d*x + c)^10 - 40*(22*(640
*a^3 + 2288*a^2*b + 3003*a*b^2)*cosh(d*x + c)^3 - 13*(640*a^3 + 2288*a^2*b
 + 3003*a*b^2)*cosh(d*x + c))*sinh(d*x + c)^9 + 78*(2560*a^3 + 9152*a^2*b
+ 13552*a*b^2 + 12705*b^3)*cosh(d*x + c)^8 + 3*(165*(2560*a^3 + 9152*a^2*b
 + 12012*a*b^2 + 15015*b^3)*cosh(d*x + c)^4 + 66560*a^3 + 237952*a^2*b + 3
52352*a*b^2 + 330330*b^3 - 780*(640*a^3 + 2288*a^2*b + 3003*a*b^2 + 3465*b
^3)*cosh(d*x + c)^2)*sinh(d*x + c)^8 - 96*(33*(640*a^3 + 2288*a^2*b + 3003
*a*b^2)*cosh(d*x + c)^5 - 65*(640*a^3 + 2288*a^2*b + 3003*a*b^2)*cosh(d*x
+ c)^3 + 52*(320*a^3 + 1144*a^2*b + 1309*a*b^2)*cosh(d*x + c))*sinh(d*x +
c)^7 - 572*(1280*a^3 + 4576*a^2*b + 7581*a*b^2 + 5775*b^3)*cosh(d*x + c)^6
 + 4*(231*(2560*a^3 + 9152*a^2*b + 12012*a*b^2 + 15015*b^3)*cosh(d*x + c)^
6 - 2730*(640*a^3 + 2288*a^2*b + 3003*a*b^2 + 3465*b^3)*cosh(d*x + c)^4 -
183040*a^3 - 654368*a^2*b - 1084083*a*b^2 - 825825*b^3 + 546*(2560*a^3 + 9
152*a^2*b + 13552*a*b^2 + 12705*b^3)*cosh(d*x + c)^2)*sinh(d*x + c)^6 - 24
*(132*(640*a^3 + 2288*a^2*b + 3003*a*b^2)*cosh(d*x + c)^7 - 546*(640*a^...
```

Sympy [F(-1)]

Timed out.

$$\int \operatorname{csch}^{14}(c+dx) \left(a+b\sinh^4(c+dx)\right)^3 \, dx = \text{Timed out}$$

input
```
integrate(csch(d*x+c)**14*(a+b*sinh(d*x+c)**4)**3,x)
```

output
```
Timed out
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1916 vs. $2(134) = 268$.

Time = 0.05 (sec) , antiderivative size = 1916, normalized size of antiderivative = 13.31

$$\int \operatorname{csch}^{14}(c+dx) \left(a+b\sinh^4(c+dx)\right)^3 \, dx = \text{Too large to display}$$

input
```
integrate(csch(d*x+c)^14*(a+b*sinh(d*x+c)^4)^3,x, algorithm="maxima")
```

output
```
-2048/3003*a^3*(13*e^(-2*d*x - 2*c)/(d*(13*e^(-2*d*x - 2*c) - 78*e^(-4*d*x
 - 4*c) + 286*e^(-6*d*x - 6*c) - 715*e^(-8*d*x - 8*c) + 1287*e^(-10*d*x -
10*c) - 1716*e^(-12*d*x - 12*c) + 1716*e^(-14*d*x - 14*c) - 1287*e^(-16*d*
x - 16*c) + 715*e^(-18*d*x - 18*c) - 286*e^(-20*d*x - 20*c) + 78*e^(-22*d*
x - 22*c) - 13*e^(-24*d*x - 24*c) + e^(-26*d*x - 26*c) - 1)) - 78*e^(-4*d*
x - 4*c)/(d*(13*e^(-2*d*x - 2*c) - 78*e^(-4*d*x - 4*c) + 286*e^(-6*d*x - 6
*c) - 715*e^(-8*d*x - 8*c) + 1287*e^(-10*d*x - 10*c) - 1716*e^(-12*d*x - 1
2*c) + 1716*e^(-14*d*x - 14*c) - 1287*e^(-16*d*x - 16*c) + 715*e^(-18*d*x
- 18*c) - 286*e^(-20*d*x - 20*c) + 78*e^(-22*d*x - 22*c) - 13*e^(-24*d*x -
 24*c) + e^(-26*d*x - 26*c) - 1)) + 286*e^(-6*d*x - 6*c)/(d*(13*e^(-2*d*x
- 2*c) - 78*e^(-4*d*x - 4*c) + 286*e^(-6*d*x - 6*c) - 715*e^(-8*d*x - 8*c)
 + 1287*e^(-10*d*x - 10*c) - 1716*e^(-12*d*x - 12*c) + 1716*e^(-14*d*x - 1
4*c) - 1287*e^(-16*d*x - 16*c) + 715*e^(-18*d*x - 18*c) - 286*e^(-20*d*x -
 20*c) + 78*e^(-22*d*x - 22*c) - 13*e^(-24*d*x - 24*c) + e^(-26*d*x - 26*c
) - 1)) - 715*e^(-8*d*x - 8*c)/(d*(13*e^(-2*d*x - 2*c) - 78*e^(-4*d*x - 4*
c) + 286*e^(-6*d*x - 6*c) - 715*e^(-8*d*x - 8*c) + 1287*e^(-10*d*x - 10*c)
 - 1716*e^(-12*d*x - 12*c) + 1716*e^(-14*d*x - 14*c) - 1287*e^(-16*d*x - 1
6*c) + 715*e^(-18*d*x - 18*c) - 286*e^(-20*d*x - 20*c) + 78*e^(-22*d*x - 2
2*c) - 13*e^(-24*d*x - 24*c) + e^(-26*d*x - 26*c) - 1)) + 1287*e^(-10*d*x
- 10*c)/(d*(13*e^(-2*d*x - 2*c) - 78*e^(-4*d*x - 4*c) + 286*e^(-6*d*x -...
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 563 vs. $2(134) = 268$.

Time = 0.33 (sec) , antiderivative size = 563, normalized size of antiderivative = 3.91

$$\int \operatorname{csch}^{14}(c+dx)\left(a+b\sinh^4(c+dx)\right)^3\,dx = \text{Too large to display}$$

input
```
integrate(csch(d*x+c)^14*(a+b*sinh(d*x+c)^4)^3,x, algorithm="giac")
```

output
```
-2/15015*(15015*b^3*e^(24*d*x + 24*c) - 180180*b^3*e^(22*d*x + 22*c) + 240
240*a*b^2*e^(20*d*x + 20*c) + 990990*b^3*e^(20*d*x + 20*c) - 2042040*a*b^2
*e^(18*d*x + 18*c) - 3303300*b^3*e^(18*d*x + 18*c) + 2306304*a^2*b*e^(16*d
*x + 16*c) + 7711704*a*b^2*e^(16*d*x + 16*c) + 7432425*b^3*e^(16*d*x + 16*
c) - 10762752*a^2*b*e^(14*d*x + 14*c) - 17008992*a*b^2*e^(14*d*x + 14*c) -
 11891880*b^3*e^(14*d*x + 14*c) + 8785920*a^3*e^(12*d*x + 12*c) + 20646912
*a^2*b*e^(12*d*x + 12*c) + 24216192*a*b^2*e^(12*d*x + 12*c) + 13873860*b^3
*e^(12*d*x + 12*c) - 6589440*a^3*e^(10*d*x + 10*c) - 21250944*a^2*b*e^(10*
d*x + 10*c) - 23207184*a*b^2*e^(10*d*x + 10*c) - 11891880*b^3*e^(10*d*x +
10*c) + 3660800*a^3*e^(8*d*x + 8*c) + 13087360*a^2*b*e^(8*d*x + 8*c) + 151
35120*a*b^2*e^(8*d*x + 8*c) + 7432425*b^3*e^(8*d*x + 8*c) - 1464320*a^3*e^
(6*d*x + 6*c) - 5234944*a^2*b*e^(6*d*x + 6*c) - 6630624*a*b^2*e^(6*d*x + 6
*c) - 3303300*b^3*e^(6*d*x + 6*c) + 399360*a^3*e^(4*d*x + 4*c) + 1427712*a
^2*b*e^(4*d*x + 4*c) + 1873872*a*b^2*e^(4*d*x + 4*c) + 990990*b^3*e^(4*d*x
 + 4*c) - 66560*a^3*e^(2*d*x + 2*c) - 237952*a^2*b*e^(2*d*x + 2*c) - 31231
2*a*b^2*e^(2*d*x + 2*c) - 180180*b^3*e^(2*d*x + 2*c) + 5120*a^3 + 18304*a^
2*b + 24024*a*b^2 + 15015*b^3)/(d*(e^(2*d*x + 2*c) - 1)^13)
```

Mupad [B] (verification not implemented)

Time = 1.97 (sec) , antiderivative size = 3138, normalized size of antiderivative = 21.79

$$\int \operatorname{csch}^{14}(c+dx)\left(a+b\sinh^4(c+dx)\right)^3\,dx = \text{Too large to display}$$

input
```
int((a + b*sinh(c + d*x)^4)^3/sinh(c + d*x)^14,x)
```

output
```
((6*b^3*exp(4*c + 4*d*x))/(13*d) - (2*b^3*exp(6*c + 6*d*x))/(13*d) + (2*b^
2*(96*a + 55*b))/(715*d) - (6*b^2*exp(2*c + 2*d*x)*(8*a + 11*b))/(143*d))/
(6*exp(4*c + 4*d*x) - 4*exp(2*c + 2*d*x) - 4*exp(6*c + 6*d*x) + exp(8*c +
8*d*x) + 1) - ((2*(840*a*b^2 + 1152*a^2*b + 1024*a^3 + 231*b^3))/(3003*d)
- (12*b^3*exp(10*c + 10*d*x))/(13*d) + (2*b^3*exp(12*c + 12*d*x))/(13*d) -
 (4*b*exp(2*c + 2*d*x)*(112*a*b + 128*a^2 + 33*b^2))/(143*d) + (2*b*exp(4*
c + 4*d*x)*(448*a*b + 256*a^2 + 165*b^2))/(143*d) + (30*b^2*exp(8*c + 8*d*
x)*(8*a + 11*b))/(143*d) - (8*b^2*exp(6*c + 6*d*x)*(96*a + 55*b))/(143*d))
/(7*exp(2*c + 2*d*x) - 21*exp(4*c + 4*d*x) + 35*exp(6*c + 6*d*x) - 35*exp(
8*c + 8*d*x) + 21*exp(10*c + 10*d*x) - 7*exp(12*c + 12*d*x) + exp(14*c + 1
4*d*x) - 1) - ((8*exp(6*c + 6*d*x)*(840*a*b^2 + 1152*a^2*b + 1024*a^3 + 23
1*b^3))/(143*d) - (18*b^3*exp(16*c + 16*d*x))/(13*d) + (2*b^3*exp(18*c + 1
8*d*x))/(13*d) - (2*b^2*(96*a + 55*b))/(715*d) - (24*b*exp(4*c + 4*d*x)*(1
12*a*b + 128*a^2 + 33*b^2))/(143*d) - (84*b*exp(8*c + 8*d*x)*(112*a*b + 12
8*a^2 + 33*b^2))/(143*d) + (6*b*exp(2*c + 2*d*x)*(448*a*b + 256*a^2 + 165*
b^2))/(715*d) + (84*b*exp(10*c + 10*d*x)*(448*a*b + 256*a^2 + 165*b^2))/(7
15*d) + (72*b^2*exp(14*c + 14*d*x)*(8*a + 11*b))/(143*d) - (168*b^2*exp(12
*c + 12*d*x)*(96*a + 55*b))/(715*d))/(45*exp(4*c + 4*d*x) - 10*exp(2*c + 2
*d*x) - 120*exp(6*c + 6*d*x) + 210*exp(8*c + 8*d*x) - 252*exp(10*c + 10*d*
x) + 210*exp(12*c + 12*d*x) - 120*exp(14*c + 14*d*x) + 45*exp(16*c + 16...
```

Reduce [B] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 744, normalized size of antiderivative = 5.17

$$\int \operatorname{csch}^{14}(c+dx)\left(a+b\sinh^4(c+dx)\right)^3\,dx$$
$$= \frac{-32e^{20dx+20c}a\,b^2-\frac{24b^3}{13}-\frac{16a\,b^2}{5}-\frac{2048a^3}{3003}+22e^{2dx+2c}b^3+\frac{6144e^{10dx+10c}a^3}{7}+1386e^{10dx+10c}b^3+\frac{3328e^{2dx+2c}a^2b}{105}\cdots}{}$$

input
```
int(csch(d*x+c)^14*(a+b*sinh(d*x+c)^4)^3,x)
```

output

```
(2*( - 1155*e**(26*c + 26*d*x)*b**3 + 90090*e**(22*c + 22*d*x)*b**3 - 2402
40*e**(20*c + 20*d*x)*a*b**2 - 660660*e**(20*c + 20*d*x)*b**3 + 2042040*e*
*(18*c + 18*d*x)*a*b**2 + 2477475*e**(18*c + 18*d*x)*b**3 - 2306304*e**(16
*c + 16*d*x)*a**2*b - 7711704*e**(16*c + 16*d*x)*a*b**2 - 5945940*e**(16*c
 + 16*d*x)*b**3 + 10762752*e**(14*c + 14*d*x)*a**2*b + 17008992*e**(14*c +
 14*d*x)*a*b**2 + 9909900*e**(14*c + 14*d*x)*b**3 - 8785920*e**(12*c + 12*
d*x)*a**3 - 20646912*e**(12*c + 12*d*x)*a**2*b - 24216192*e**(12*c + 12*d*
x)*a*b**2 - 11891880*e**(12*c + 12*d*x)*b**3 + 6589440*e**(10*c + 10*d*x)*
a**3 + 21250944*e**(10*c + 10*d*x)*a**2*b + 23207184*e**(10*c + 10*d*x)*a*
b**2 + 10405395*e**(10*c + 10*d*x)*b**3 - 3660800*e**(8*c + 8*d*x)*a**3 -
13087360*e**(8*c + 8*d*x)*a**2*b - 15135120*e**(8*c + 8*d*x)*a*b**2 - 6606
600*e**(8*c + 8*d*x)*b**3 + 1464320*e**(6*c + 6*d*x)*a**3 + 5234944*e**(6*
c + 6*d*x)*a**2*b + 6630624*e**(6*c + 6*d*x)*a*b**2 + 2972970*e**(6*c + 6*
d*x)*b**3 - 399360*e**(4*c + 4*d*x)*a**3 - 1427712*e**(4*c + 4*d*x)*a**2*b
 - 1873872*e**(4*c + 4*d*x)*a*b**2 - 900900*e**(4*c + 4*d*x)*b**3 + 66560*
e**(2*c + 2*d*x)*a**3 + 237952*e**(2*c + 2*d*x)*a**2*b + 312312*e**(2*c +
2*d*x)*a*b**2 + 165165*e**(2*c + 2*d*x)*b**3 - 5120*a**3 - 18304*a**2*b -
24024*a*b**2 - 13860*b**3))/(15015*d*(e**(26*c + 26*d*x) - 13*e**(24*c + 2
4*d*x) + 78*e**(22*c + 22*d*x) - 286*e**(20*c + 20*d*x) + 715*e**(18*c + 1
8*d*x) - 1287*e**(16*c + 16*d*x) + 1716*e**(14*c + 14*d*x) - 1716*e**(1...
```

3.201 $\int \operatorname{csch}^{16}(c+dx)\left(a+b\sinh^4(c+dx)\right)^3\,dx$

Optimal result

Integrand size = 23, antiderivative size = 182

$$\begin{aligned}\int \operatorname{csch}^{16}(c+dx)\left(a+b\sinh^4(c+dx)\right)^3\,dx &= \frac{(a+b)^3\coth(c+dx)}{d}-\frac{(a+b)^2(7a+b)\coth^3(c+dx)}{3d}\\&\quad+\frac{3a(a+b)(7a+3b)\coth^5(c+dx)}{5d}-\frac{a(35a^2+30ab+3b^2)\coth^7(c+dx)}{7d}\\&\quad+\frac{5a^2(7a+3b)\coth^9(c+dx)}{9d}-\frac{3a^2(7a+b)\coth^{11}(c+dx)}{11d}\\&\quad+\frac{7a^3\coth^{13}(c+dx)}{13d}-\frac{a^3\coth^{15}(c+dx)}{15d}\end{aligned}$$

output
```
(a+b)^3*coth(d*x+c)/d-1/3*(a+b)^2*(7*a+b)*coth(d*x+c)^3/d+3/5*a*(a+b)*(7*a
+3*b)*coth(d*x+c)^5/d-1/7*a*(35*a^2+30*a*b+3*b^2)*coth(d*x+c)^7/d+5/9*a^2*
(7*a+3*b)*coth(d*x+c)^9/d-3/11*a^2*(7*a+b)*coth(d*x+c)^11/d+7/13*a^3*coth(
d*x+c)^13/d-1/15*a^3*coth(d*x+c)^15/d
```

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 404 vs. $2(182) = 364$.

Time = 4.35 (sec) , antiderivative size = 404, normalized size of antiderivative = 2.22

$$\int \operatorname{csch}^{16}(c+dx)\left(a+b\sinh^4(c+dx)\right)^3 dx = -\frac{(45045(1024a^3+1152a^2b+840ab^2+231b^3)\cosh(c+dx)-5005(7168a^3+20352a^2b+16632ab^2+4$$

input
```
Integrate[Csch[c + d*x]^16*(a + b*Sinh[c + d*x]^4)^3,x]
```

output
```
-1/369008640*((45045*(1024*a^3 + 1152*a^2*b + 840*a*b^2 + 231*b^3)*Cosh[c
+ d*x] - 5005*(7168*a^3 + 20352*a^2*b + 16632*a*b^2 + 4785*b^3)*Cosh[3*(c
+ d*x)] + 21525504*a^3*Cosh[5*(c + d*x)] + 74954880*a^2*b*Cosh[5*(c + d*x)
] + 74162088*a*b^2*Cosh[5*(c + d*x)] + 23288265*b^3*Cosh[5*(c + d*x)] - 97
84320*a^3*Cosh[7*(c + d*x)] - 34070400*a^2*b*Cosh[7*(c + d*x)] - 39999960*
a*b^2*Cosh[7*(c + d*x)] - 14189175*b^3*Cosh[7*(c + d*x)] + 3261440*a^3*Cos
h[9*(c + d*x)] + 11356800*a^2*b*Cosh[9*(c + d*x)] + 14054040*a*b^2*Cosh[9*
(c + d*x)] + 5720715*b^3*Cosh[9*(c + d*x)] - 752640*a^3*Cosh[11*(c + d*x)]
 - 2620800*a^2*b*Cosh[11*(c + d*x)] - 3243240*a*b^2*Cosh[11*(c + d*x)] - 1
486485*b^3*Cosh[11*(c + d*x)] + 107520*a^3*Cosh[13*(c + d*x)] + 374400*a^2
*b*Cosh[13*(c + d*x)] + 463320*a*b^2*Cosh[13*(c + d*x)] + 225225*b^3*Cosh[
13*(c + d*x)] - 7168*a^3*Cosh[15*(c + d*x)] - 24960*a^2*b*Cosh[15*(c + d*x
)] - 30888*a*b^2*Cosh[15*(c + d*x)] - 15015*b^3*Cosh[15*(c + d*x)])*Csch[c
 + d*x]^15)/d
```

Rubi [A] (verified)

Time = 0.42 (sec) , antiderivative size = 162, normalized size of antiderivative = 0.89, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.174$, Rules used = {3042, 3696, 1584, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \operatorname{csch}^{16}(c+dx)\left(a+b\sinh^4(c+dx)\right)^3\,dx$$

$\Big\downarrow$ 3042

$$\int \frac{\left(a+b\sin(ic+idx)^4\right)^3}{\sin(ic+idx)^{16}}dx$$

$\Big\downarrow$ 3696

$$\frac{\int \coth^{16}(c+dx)\left(1-\tanh^2(c+dx)\right)\left((a+b)\tanh^4(c+dx)-2a\tanh^2(c+dx)+a\right)^3d\tanh(c+dx)}{d}$$

$\Big\downarrow$ 1584

$$\int \left(a^3\coth^{16}(c+dx)-7a^3\coth^{14}(c+dx)+3a^2(7a+b)\coth^{12}(c+dx)-5a^2(7a+3b)\coth^{10}(c+dx)+a(35a^2$$

$\Big\downarrow$ 2009

$$-\tfrac{1}{15}a^3\coth^{15}(c+dx)+\tfrac{7}{13}a^3\coth^{13}(c+dx)-\tfrac{1}{7}a(35a^2+30ab+3b^2)\coth^7(c+dx)-\tfrac{3}{11}a^2(7a+b)\coth^{11}(c+$$

input
```
Int[Csch[c + d*x]^16*(a + b*Sinh[c + d*x]^4)^3,x]
```

output
```
((a + b)^3*Coth[c + d*x] - ((a + b)^2*(7*a + b)*Coth[c + d*x]^3)/3 + (3*a*
(a + b)*(7*a + 3*b)*Coth[c + d*x]^5)/5 - (a*(35*a^2 + 30*a*b + 3*b^2)*Coth
[c + d*x]^7)/7 + (5*a^2*(7*a + 3*b)*Coth[c + d*x]^9)/9 - (3*a^2*(7*a + b)*
Coth[c + d*x]^11)/11 + (7*a^3*Coth[c + d*x]^13)/13 - (a^3*Coth[c + d*x]^15
)/15)/d
```

Defintions of rubi rules used

rule 1584
```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (
c_.)*(x_)^4)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(f*x)^m*(d + e*x^2)^q*
(a + b*x^2 + c*x^4)^p, x], x] /; FreeQ[{a, b, c, d, e, f, m, q}, x] && NeQ[
b^2 - 4*a*c, 0] && IGtQ[p, 0] && IGtQ[q, -2]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3696
```
Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^4)^(
p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Simp[ff^(m + 1
)/f   Subst[Int[x^m*((a + 2*a*ff^2*x^2 + (a + b)*ff^4*x^4)^p/(1 + ff^2*x^2)
^(m/2 + 2*p + 1)), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f}, x] &
& IntegerQ[m/2] && IntegerQ[p]
```

Maple [A] (verified)

Time = 5.51 (sec) , antiderivative size = 218, normalized size of antiderivative = 1.20

method	result
derivativedivides	$\frac{a^3\left(\frac{2048}{6435}-\frac{\operatorname{csch}(dx+c)^{14}}{15}+\frac{14\operatorname{csch}(dx+c)^{12}}{195}-\frac{56\operatorname{csch}(dx+c)^{10}}{715}+\frac{112\operatorname{csch}(dx+c)^{8}}{1287}-\frac{128\operatorname{csch}(dx+c)^{6}}{1287}+\frac{256\operatorname{csch}(dx+c)^{4}}{2145}-\frac{1024\operatorname{cs}}{}\right.}{}$
default	$\frac{a^3\left(\frac{2048}{6435}-\frac{\operatorname{csch}(dx+c)^{14}}{15}+\frac{14\operatorname{csch}(dx+c)^{12}}{195}-\frac{56\operatorname{csch}(dx+c)^{10}}{715}+\frac{112\operatorname{csch}(dx+c)^{8}}{1287}-\frac{128\operatorname{csch}(dx+c)^{6}}{1287}+\frac{256\operatorname{csch}(dx+c)^{4}}{2145}-\frac{1024\operatorname{cs}}{}\right.}{}$
parallelrisch	$-\frac{7\operatorname{sech}\left(\frac{dx}{2}+\frac{c}{2}\right)^{15}\left(\left(-\frac{3}{13}a^3-\frac{3267}{7168}b^3-\frac{891}{896}b^2a-\frac{45}{56}a^2b\right)\cosh(11dx+11c)+\left(\frac{3}{91}a^3+\frac{495}{7168}b^3+\frac{891}{6272}b^2a+\frac{45}{392}a^2b\right)\cosh(1\right.}{}$
risch	$-\frac{4(-17342325b^3e^{8dx+8c}+6276270b^3e^{6dx+6c}-752640a^3e^{4dx+4c}-1531530b^3e^{4dx+4c}+107520a^3e^{2dx+2c}-30888b^2a-}{}$

input
```
int(csch(d*x+c)^16*(a+b*sinh(d*x+c)^4)^3,x,method=_RETURNVERBOSE)
```

output
```
1/d*(a^3*(2048/6435-1/15*csch(d*x+c)^14+14/195*csch(d*x+c)^12-56/715*csch(
d*x+c)^10+112/1287*csch(d*x+c)^8-128/1287*csch(d*x+c)^6+256/2145*csch(d*x+
c)^4-1024/6435*csch(d*x+c)^2)*coth(d*x+c)+3*a^2*b*(256/693-1/11*csch(d*x+c
)^10+10/99*csch(d*x+c)^8-80/693*csch(d*x+c)^6+32/231*csch(d*x+c)^4-128/693
*csch(d*x+c)^2)*coth(d*x+c)+3*b^2*a*(16/35-1/7*csch(d*x+c)^6+6/35*csch(d*x
+c)^4-8/35*csch(d*x+c)^2)*coth(d*x+c)+b^3*(2/3-1/3*csch(d*x+c)^2)*coth(d*x
+c))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2967 vs. $2(168) = 336$.

Time = 0.11 (sec) , antiderivative size = 2967, normalized size of antiderivative = 16.30

$$\int \operatorname{csch}^{16}(c+dx) \left(a+b\sinh^4(c+dx)\right)^3 \, dx = \text{Too large to display}$$

input
```
integrate(csch(d*x+c)^16*(a+b*sinh(d*x+c)^4)^3,x, algorithm="fricas")
```

output
```
8/45045*((3584*a^3 + 12480*a^2*b + 15444*a*b^2 - 15015*b^3)*cosh(d*x + c)^
13 + 13*(3584*a^3 + 12480*a^2*b + 15444*a*b^2 - 15015*b^3)*cosh(d*x + c)*s
inh(d*x + c)^12 - 2*(1792*a^3 + 6240*a^2*b + 7722*a*b^2 + 15015*b^3)*sinh(
d*x + c)^13 - 15*(3584*a^3 + 12480*a^2*b + 15444*a*b^2 - 11011*b^3)*cosh(d
*x + c)^11 + 6*(8960*a^3 + 31200*a^2*b + 38610*a*b^2 + 65065*b^3 - 26*(179
2*a^3 + 6240*a^2*b + 7722*a*b^2 + 15015*b^3)*cosh(d*x + c)^2)*sinh(d*x + c
)^11 + 11*(26*(3584*a^3 + 12480*a^2*b + 15444*a*b^2 - 15015*b^3)*cosh(d*x
+ c)^3 - 15*(3584*a^3 + 12480*a^2*b + 15444*a*b^2 - 11011*b^3)*cosh(d*x +
c))*sinh(d*x + c)^10 + 210*(1792*a^3 + 6240*a^2*b + 5148*a*b^2 - 3861*b^3)
*cosh(d*x + c)^9 - 10*(143*(1792*a^3 + 6240*a^2*b + 7722*a*b^2 + 15015*b^3
)*cosh(d*x + c)^4 + 37632*a^3 + 131040*a^2*b + 216216*a*b^2 + 234234*b^3 -
 165*(1792*a^3 + 6240*a^2*b + 7722*a*b^2 + 13013*b^3)*cosh(d*x + c)^2)*sin
h(d*x + c)^9 + 9*(143*(3584*a^3 + 12480*a^2*b + 15444*a*b^2 - 15015*b^3)*c
osh(d*x + c)^5 - 275*(3584*a^3 + 12480*a^2*b + 15444*a*b^2 - 11011*b^3)*co
sh(d*x + c)^3 + 210*(1792*a^3 + 6240*a^2*b + 5148*a*b^2 - 3861*b^3)*cosh(d
*x + c))*sinh(d*x + c)^8 - 182*(8960*a^3 + 31200*a^2*b + 13068*a*b^2 - 127
05*b^3)*cosh(d*x + c)^7 - 4*(858*(1792*a^3 + 6240*a^2*b + 7722*a*b^2 + 150
15*b^3)*cosh(d*x + c)^6 - 2475*(1792*a^3 + 6240*a^2*b + 7722*a*b^2 + 13013
*b^3)*cosh(d*x + c)^4 - 407680*a^3 - 1419600*a^2*b - 2918916*a*b^2 - 21471
45*b^3 + 3780*(896*a^3 + 3120*a^2*b + 5148*a*b^2 + 5577*b^3)*cosh(d*x +...
```

Sympy [F(-1)]

Timed out.

$$\int \operatorname{csch}^{16}(c+dx)\left(a+b\sinh^4(c+dx)\right)^3\,dx=\text{Timed out}$$

input
```
integrate(csch(d*x+c)**16*(a+b*sinh(d*x+c)**4)**3,x)
```

output
```
Timed out
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2731 vs. $2(168) = 336$.

Time = 0.07 (sec) , antiderivative size = 2731, normalized size of antiderivative = 15.01

$$\int \operatorname{csch}^{16}(c+dx)\left(a+b\sinh^4(c+dx)\right)^3\,dx=\text{Too large to display}$$

input
```
integrate(csch(d*x+c)^16*(a+b*sinh(d*x+c)^4)^3,x, algorithm="maxima")
```

output
```
4096/6435*a^3*(15*e^(-2*d*x - 2*c)/(d*(15*e^(-2*d*x - 2*c) - 105*e^(-4*d*x
 - 4*c) + 455*e^(-6*d*x - 6*c) - 1365*e^(-8*d*x - 8*c) + 3003*e^(-10*d*x -
 10*c) - 5005*e^(-12*d*x - 12*c) + 6435*e^(-14*d*x - 14*c) - 6435*e^(-16*d
*x - 16*c) + 5005*e^(-18*d*x - 18*c) - 3003*e^(-20*d*x - 20*c) + 1365*e^(-
22*d*x - 22*c) - 455*e^(-24*d*x - 24*c) + 105*e^(-26*d*x - 26*c) - 15*e^(-
28*d*x - 28*c) + e^(-30*d*x - 30*c) - 1)) - 105*e^(-4*d*x - 4*c)/(d*(15*e^
(-2*d*x - 2*c) - 105*e^(-4*d*x - 4*c) + 455*e^(-6*d*x - 6*c) - 1365*e^(-8*
d*x - 8*c) + 3003*e^(-10*d*x - 10*c) - 5005*e^(-12*d*x - 12*c) + 6435*e^(-
14*d*x - 14*c) - 6435*e^(-16*d*x - 16*c) + 5005*e^(-18*d*x - 18*c) - 3003*
e^(-20*d*x - 20*c) + 1365*e^(-22*d*x - 22*c) - 455*e^(-24*d*x - 24*c) + 10
5*e^(-26*d*x - 26*c) - 15*e^(-28*d*x - 28*c) + e^(-30*d*x - 30*c) - 1)) +
455*e^(-6*d*x - 6*c)/(d*(15*e^(-2*d*x - 2*c) - 105*e^(-4*d*x - 4*c) + 455*
e^(-6*d*x - 6*c) - 1365*e^(-8*d*x - 8*c) + 3003*e^(-10*d*x - 10*c) - 5005*
e^(-12*d*x - 12*c) + 6435*e^(-14*d*x - 14*c) - 6435*e^(-16*d*x - 16*c) + 5
005*e^(-18*d*x - 18*c) - 3003*e^(-20*d*x - 20*c) + 1365*e^(-22*d*x - 22*c)
 - 455*e^(-24*d*x - 24*c) + 105*e^(-26*d*x - 26*c) - 15*e^(-28*d*x - 28*c)
 + e^(-30*d*x - 30*c) - 1)) - 1365*e^(-8*d*x - 8*c)/(d*(15*e^(-2*d*x - 2*c
) - 105*e^(-4*d*x - 4*c) + 455*e^(-6*d*x - 6*c) - 1365*e^(-8*d*x - 8*c) +
3003*e^(-10*d*x - 10*c) - 5005*e^(-12*d*x - 12*c) + 6435*e^(-14*d*x - 14*c
) - 6435*e^(-16*d*x - 16*c) + 5005*e^(-18*d*x - 18*c) - 3003*e^(-20*d*x...
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 621 vs. $2(168) = 336$.

Time = 0.35 (sec) , antiderivative size = 621, normalized size of antiderivative = 3.41

$$\int \operatorname{csch}^{16}(c+dx) \left(a+b\sinh^4(c+dx)\right)^3 dx = \text{Too large to display}$$

input
```
integrate(csch(d*x+c)^16*(a+b*sinh(d*x+c)^4)^3,x, algorithm="giac")
```

output
```
-4/45045*(45045*b^3*e^(26*d*x + 26*c) - 555555*b^3*e^(24*d*x + 24*c) + 108
1080*a*b^2*e^(22*d*x + 22*c) + 3153150*b^3*e^(22*d*x + 22*c) - 9297288*a*b
^2*e^(20*d*x + 20*c) - 10900890*b^3*e^(20*d*x + 20*c) + 11531520*a^2*b*e^(
18*d*x + 18*c) + 35675640*a*b^2*e^(18*d*x + 18*c) + 25600575*b^3*e^(18*d*x
 + 18*c) - 54362880*a^2*b*e^(16*d*x + 16*c) - 80463240*a*b^2*e^(16*d*x + 1
6*c) - 43108065*b^3*e^(16*d*x + 16*c) + 46126080*a^3*e^(14*d*x + 14*c) + 1
06254720*a^2*b*e^(14*d*x + 14*c) + 118301040*a*b^2*e^(14*d*x + 14*c) + 535
13460*b^3*e^(14*d*x + 14*c) - 35875840*a^3*e^(12*d*x + 12*c) - 113393280*a
^2*b*e^(12*d*x + 12*c) - 118918800*a*b^2*e^(12*d*x + 12*c) - 49549500*b^3*
e^(12*d*x + 12*c) + 21525504*a^3*e^(10*d*x + 10*c) + 74954880*a^2*b*e^(10*
d*x + 10*c) + 83459376*a*b^2*e^(10*d*x + 10*c) + 34189155*b^3*e^(10*d*x +
10*c) - 9784320*a^3*e^(8*d*x + 8*c) - 34070400*a^2*b*e^(8*d*x + 8*c) - 410
81040*a*b^2*e^(8*d*x + 8*c) - 17342325*b^3*e^(8*d*x + 8*c) + 3261440*a^3*e
^(6*d*x + 6*c) + 11356800*a^2*b*e^(6*d*x + 6*c) + 14054040*a*b^2*e^(6*d*x
+ 6*c) + 6276270*b^3*e^(6*d*x + 6*c) - 752640*a^3*e^(4*d*x + 4*c) - 262080
0*a^2*b*e^(4*d*x + 4*c) - 3243240*a*b^2*e^(4*d*x + 4*c) - 1531530*b^3*e^(4
*d*x + 4*c) + 107520*a^3*e^(2*d*x + 2*c) + 374400*a^2*b*e^(2*d*x + 2*c) +
463320*a*b^2*e^(2*d*x + 2*c) + 225225*b^3*e^(2*d*x + 2*c) - 7168*a^3 - 249
60*a^2*b - 30888*a*b^2 - 15015*b^3)/(d*(e^(2*d*x + 2*c) - 1)^15)
```

Mupad [B] (verification not implemented)

Time = 2.10 (sec) , antiderivative size = 3823, normalized size of antiderivative = 21.01

$$\int \operatorname{csch}^{16}(c+dx)\left(a+b\sinh^4(c+dx)\right)^3\,dx = \text{Too large to display}$$

input
```
int((a + b*sinh(c + d*x)^4)^3/sinh(c + d*x)^16,x)
```

output
```
((32*b^3)/(455*d) - (8*b^3*exp(2*c + 2*d*x))/(105*d))/(3*exp(2*c + 2*d*x)
- 3*exp(4*c + 4*d*x) + exp(6*c + 6*d*x) - 1) - ((8*exp(8*c + 8*d*x)*(840*a
*b^2 + 1152*a^2*b + 1024*a^3 + 231*b^3))/(39*d) - (352*b^3*exp(18*c + 18*d
*x))/(91*d) + (44*b^3*exp(20*c + 20*d*x))/(105*d) + (4*b^2*(8*a + 11*b))/(
455*d) - (64*b*exp(6*c + 6*d*x)*(112*a*b + 128*a^2 + 33*b^2))/(91*d) - (12
8*b*exp(10*c + 10*d*x)*(112*a*b + 128*a^2 + 33*b^2))/(65*d) + (4*b*exp(4*c
 + 4*d*x)*(448*a*b + 256*a^2 + 165*b^2))/(91*d) + (24*b*exp(12*c + 12*d*x)
*(448*a*b + 256*a^2 + 165*b^2))/(65*d) + (132*b^2*exp(16*c + 16*d*x)*(8*a
+ 11*b))/(91*d) - (32*b^2*exp(2*c + 2*d*x)*(96*a + 55*b))/(1365*d) - (64*b
^2*exp(14*c + 14*d*x)*(96*a + 55*b))/(91*d))/(66*exp(4*c + 4*d*x) - 12*exp
(2*c + 2*d*x) - 220*exp(6*c + 6*d*x) + 495*exp(8*c + 8*d*x) - 792*exp(10*c
 + 10*d*x) + 924*exp(12*c + 12*d*x) - 792*exp(14*c + 14*d*x) + 495*exp(16*
c + 16*d*x) - 220*exp(18*c + 18*d*x) + 66*exp(20*c + 20*d*x) - 12*exp(22*c
 + 22*d*x) + exp(24*c + 24*d*x) + 1) + ((192*b^3*exp(4*c + 4*d*x))/(455*d)
 - (16*b^3*exp(6*c + 6*d*x))/(105*d) + (32*b^2*(96*a + 55*b))/(15015*d) -
(16*b^2*exp(2*c + 2*d*x)*(8*a + 11*b))/(455*d))/(5*exp(2*c + 2*d*x) - 10*e
xp(4*c + 4*d*x) + 10*exp(6*c + 6*d*x) - 5*exp(8*c + 8*d*x) + exp(10*c + 10
*d*x) - 1) - ((4*(840*a*b^2 + 1152*a^2*b + 1024*a^3 + 231*b^3))/(6435*d) -
 (96*b^3*exp(10*c + 10*d*x))/(65*d) + (4*b^3*exp(12*c + 12*d*x))/(15*d) -
(64*b*exp(2*c + 2*d*x)*(112*a*b + 128*a^2 + 33*b^2))/(2145*d) + (12*b*e...
```

Reduce [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 830, normalized size of antiderivative = 4.56

$$\int \operatorname{csch}^{16}(c+dx)\left(a+b\sinh^4(c+dx)\right)^3\,dx$$
$$= \frac{-96e^{22dx+22c}a\,b^2 + \frac{4128e^{20dx+20c}a\,b^2}{5} - 1024e^{18dx+18c}a^2b + \frac{4b^3}{3} + \frac{148e^{24dx+24c}b^3}{3} + \frac{96a\,b^2}{35} + \frac{4096a^3}{6435} - 20e^{2dx+2c}b^3}{}$$

input
```
int(csch(d*x+c)^16*(a+b*sinh(d*x+c)^4)^3,x)
```

output

```
(4*( - 45045*e**(26*c + 26*d*x)*b**3 + 555555*e**(24*c + 24*d*x)*b**3 - 10
81080*e**(22*c + 22*d*x)*a*b**2 - 3153150*e**(22*c + 22*d*x)*b**3 + 929728
8*e**(20*c + 20*d*x)*a*b**2 + 10900890*e**(20*c + 20*d*x)*b**3 - 11531520*
e**(18*c + 18*d*x)*a**2*b - 35675640*e**(18*c + 18*d*x)*a*b**2 - 25600575*
e**(18*c + 18*d*x)*b**3 + 54362880*e**(16*c + 16*d*x)*a**2*b + 80463240*e*
*(16*c + 16*d*x)*a*b**2 + 43108065*e**(16*c + 16*d*x)*b**3 - 46126080*e**(
14*c + 14*d*x)*a**3 - 106254720*e**(14*c + 14*d*x)*a**2*b - 118301040*e**(
14*c + 14*d*x)*a*b**2 - 53513460*e**(14*c + 14*d*x)*b**3 + 35875840*e**(12
*c + 12*d*x)*a**3 + 113393280*e**(12*c + 12*d*x)*a**2*b + 118918800*e**(12
*c + 12*d*x)*a*b**2 + 49549500*e**(12*c + 12*d*x)*b**3 - 21525504*e**(10*c
 + 10*d*x)*a**3 - 74954880*e**(10*c + 10*d*x)*a**2*b - 83459376*e**(10*c +
 10*d*x)*a*b**2 - 34189155*e**(10*c + 10*d*x)*b**3 + 9784320*e**(8*c + 8*d
*x)*a**3 + 34070400*e**(8*c + 8*d*x)*a**2*b + 41081040*e**(8*c + 8*d*x)*a*
b**2 + 17342325*e**(8*c + 8*d*x)*b**3 - 3261440*e**(6*c + 6*d*x)*a**3 - 11
356800*e**(6*c + 6*d*x)*a**2*b - 14054040*e**(6*c + 6*d*x)*a*b**2 - 627627
0*e**(6*c + 6*d*x)*b**3 + 752640*e**(4*c + 4*d*x)*a**3 + 2620800*e**(4*c +
 4*d*x)*a**2*b + 3243240*e**(4*c + 4*d*x)*a*b**2 + 1531530*e**(4*c + 4*d*x
)*b**3 - 107520*e**(2*c + 2*d*x)*a**3 - 374400*e**(2*c + 2*d*x)*a**2*b - 4
63320*e**(2*c + 2*d*x)*a*b**2 - 225225*e**(2*c + 2*d*x)*b**3 + 7168*a**3 +
 24960*a**2*b + 30888*a*b**2 + 15015*b**3))/(45045*d*(e**(30*c + 30*d*x...
```

3.202 $\int \operatorname{csch}^{18}(c+dx) \left(a+b\sinh^4(c+dx)\right)^3 \, dx$

Optimal result

Integrand size = 23, antiderivative size = 221

$$\int \operatorname{csch}^{18}(c+dx) \left(a+b\sinh^4(c+dx)\right)^3 \, dx = -\frac{(a+b)^3\coth(c+dx)}{d} + \frac{2(a+b)^2(4a+b)\coth^3(c+dx)}{3d} - \frac{(a+b)\left(28a^2+17ab+b^2\right)\coth^5(c+dx)}{5d} + \frac{4a(14a^2+15ab+3b^2)\coth^7(c+dx)}{7d} - \frac{a(70a^2+45ab+3b^2)\coth^9(c+dx)}{9d} + \frac{2a^2(28a+9b)\coth^{11}(c+dx)}{11d} - \frac{a^2(28a+3b)\coth^{13}(c+dx)}{13d} + \frac{8a^3\coth^{15}(c+dx)}{15d} - \frac{a^3\coth^{17}(c+dx)}{17d}$$

output
```
-(a+b)^3*coth(d*x+c)/d+2/3*(a+b)^2*(4*a+b)*coth(d*x+c)^3/d-1/5*(a+b)*(28*a
^2+17*a*b+b^2)*coth(d*x+c)^5/d+4/7*a*(14*a^2+15*a*b+3*b^2)*coth(d*x+c)^7/d
-1/9*a*(70*a^2+45*a*b+3*b^2)*coth(d*x+c)^9/d+2/11*a^2*(28*a+9*b)*coth(d*x+
c)^11/d-1/13*a^2*(28*a+3*b)*coth(d*x+c)^13/d+8/15*a^3*coth(d*x+c)^15/d-1/1
7*a^3*coth(d*x+c)^17/d
```

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 458 vs. $2(221) = 442$.

Time = 5.67 (sec) , antiderivative size = 458, normalized size of antiderivative = 2.07

$$\int \operatorname{csch}^{18}(c+dx)\left(a+b\sinh^4(c+dx)\right)^3\,dx = -\frac{(680680(1024a^3+1152a^2b+840ab^2+231b^3)\cosh(c+dx)-272272(2048a^3+5760a^2b+4704ab^2+}{}$$

input
```
Integrate[Csch[c + d*x]^18*(a + b*Sinh[c + d*x]^4)^3,x]
```

output
```
-1/6273146880*((680680*(1024*a^3 + 1152*a^2*b + 840*a*b^2 + 231*b^3)*Cosh[
c + d*x] - 272272*(2048*a^3 + 5760*a^2*b + 4704*a*b^2 + 1353*b^3)*Cosh[3*(
c + d*x)] + 354844672*a^3*Cosh[5*(c + d*x)] + 1211857920*a^2*b*Cosh[5*(c +
 d*x)] + 1189284096*a*b^2*Cosh[5*(c + d*x)] + 372263892*b^3*Cosh[5*(c + d*
x)] - 177422336*a^3*Cosh[7*(c + d*x)] - 605928960*a^2*b*Cosh[7*(c + d*x)]
- 692659968*a*b^2*Cosh[7*(c + d*x)] - 242288046*b^3*Cosh[7*(c + d*x)] + 68
239360*a^3*Cosh[9*(c + d*x)] + 233049600*a^2*b*Cosh[9*(c + d*x)] + 2777174
40*a*b^2*Cosh[9*(c + d*x)] + 108738630*b^3*Cosh[9*(c + d*x)] - 19496960*a^
3*Cosh[11*(c + d*x)] - 66585600*a^2*b*Cosh[11*(c + d*x)] - 79347840*a*b^2*
Cosh[11*(c + d*x)] - 33693660*b^3*Cosh[11*(c + d*x)] + 3899392*a^3*Cosh[13
*(c + d*x)] + 13317120*a^2*b*Cosh[13*(c + d*x)] + 15869568*a*b^2*Cosh[13*(
c + d*x)] + 6942936*b^3*Cosh[13*(c + d*x)] - 487424*a^3*Cosh[15*(c + d*x)]
 - 1664640*a^2*b*Cosh[15*(c + d*x)] - 1983696*a*b^2*Cosh[15*(c + d*x)] - 8
67867*b^3*Cosh[15*(c + d*x)] + 28672*a^3*Cosh[17*(c + d*x)] + 97920*a^2*b*
Cosh[17*(c + d*x)] + 116688*a*b^2*Cosh[17*(c + d*x)] + 51051*b^3*Cosh[17*(
c + d*x)])*Csch[c + d*x]^17)/d
```

Rubi [A] (verified)

Time = 0.45 (sec) , antiderivative size = 198, normalized size of antiderivative = 0.90, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.217$, Rules used = {3042, 25, 3696, 1584, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \operatorname{csch}^{18}(c+dx)\left(a+b\sinh^4(c+dx)\right)^3\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int -\frac{\left(a+b\sin(ic+idx)^4\right)^3}{\sin(ic+idx)^{18}}dx$$

$$\Big\downarrow \text{25}$$

$$-\int \frac{\left(b\sin(ic+idx)^4+a\right)^3}{\sin(ic+idx)^{18}}dx$$

$$\Big\downarrow \text{3696}$$

$$\frac{\int \coth^{18}(c+dx)\left(1-\tanh^2(c+dx)\right)^2\left((a+b)\tanh^4(c+dx)-2a\tanh^2(c+dx)+a\right)^3d\tanh(c+dx)}{d}$$

$$\Big\downarrow \text{1584}$$

$$\frac{\int \left(a^3\coth^{18}(c+dx)-8a^3\coth^{16}(c+dx)+a^2(28a+3b)\coth^{14}(c+dx)-2a^2(28a+9b)\coth^{12}(c+dx)+a(70\right.}{}$$

$$\Big\downarrow \text{2009}$$

$$\frac{-\frac{1}{17}a^3\coth^{17}(c+dx)+\frac{8}{15}a^3\coth^{15}(c+dx)-\frac{1}{9}a\left(70a^2+45ab+3b^2\right)\coth^9(c+dx)+\frac{4}{7}a\left(14a^2+15ab+3b^2\right)\text{co}}{}$$

input
```
Int[Csch[c + d*x]^18*(a + b*Sinh[c + d*x]^4)^3,x]
```

output
```
(-((a + b)^3*Coth[c + d*x]) + (2*(a + b)^2*(4*a + b)*Coth[c + d*x]^3)/3 -
((a + b)*(28*a^2 + 17*a*b + b^2)*Coth[c + d*x]^5)/5 + (4*a*(14*a^2 + 15*a*
b + 3*b^2)*Coth[c + d*x]^7)/7 - (a*(70*a^2 + 45*a*b + 3*b^2)*Coth[c + d*x]
^9)/9 + (2*a^2*(28*a + 9*b)*Coth[c + d*x]^11)/11 - (a^2*(28*a + 3*b)*Coth[
c + d*x]^13)/13 + (8*a^3*Coth[c + d*x]^15)/15 - (a^3*Coth[c + d*x]^17)/17)
/d
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 1584
```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (
c_.)*(x_)^4)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(f*x)^m*(d + e*x^2)^q*
(a + b*x^2 + c*x^4)^p, x], x] /; FreeQ[{a, b, c, d, e, f, m, q}, x] && NeQ[
b^2 - 4*a*c, 0] && IGtQ[p, 0] && IGtQ[q, -2]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3696
```
Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^4)^(
p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Simp[ff^(m + 1
)/f   Subst[Int[x^m*((a + 2*a*ff^2*x^2 + (a + b)*ff^4*x^4)^p/(1 + ff^2*x^2)
^(m/2 + 2*p + 1)), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f}, x] &
& IntegerQ[m/2] && IntegerQ[p]
```

Maple [A] (verified)

Time = 6.61 (sec) , antiderivative size = 258, normalized size of antiderivative = 1.17

method	result
derivativedivides	$\frac{a^3\left(-\frac{32768}{109395}-\frac{\operatorname{csch}(dx+c)^{16}}{17}+\frac{16\operatorname{csch}(dx+c)^{14}}{255}-\frac{224\operatorname{csch}(dx+c)^{12}}{3315}+\frac{896\operatorname{csch}(dx+c)^{10}}{12155}-\frac{1792\operatorname{csch}(dx+c)^{8}}{21879}+\frac{2048\operatorname{csch}(dx+c)^{6}}{21879}\right.}{}$
default	$\frac{a^3\left(-\frac{32768}{109395}-\frac{\operatorname{csch}(dx+c)^{16}}{17}+\frac{16\operatorname{csch}(dx+c)^{14}}{255}-\frac{224\operatorname{csch}(dx+c)^{12}}{3315}+\frac{896\operatorname{csch}(dx+c)^{10}}{12155}-\frac{1792\operatorname{csch}(dx+c)^{8}}{21879}+\frac{2048\operatorname{csch}(dx+c)^{6}}{21879}\right.}{}$
parallelrisch	$\frac{-45045\coth\left(\frac{dx}{2}+\frac{c}{2}\right)^{17}a^3+867867\coth\left(\frac{dx}{2}+\frac{c}{2}\right)^{15}a^3-8011080\left(\frac{6b}{17}+a\right)a^2\coth\left(\frac{dx}{2}+\frac{c}{2}\right)^{13}+47338200a^2\left(\frac{78b}{85}+a\right)\text{co}}{}$
risch	$-\frac{16(115120005b^3e^{8dx+8c}-34204170b^3e^{6dx+6c}+3899392a^3e^{4dx+4c}+6942936b^3e^{4dx+4c}-487424a^3e^{2dx+2c}+147026}{}$

input
```
int(csch(d*x+c)^18*(a+b*sinh(d*x+c)^4)^3,x,method=_RETURNVERBOSE)
```

output
```
1/d*(a^3*(-32768/109395-1/17*csch(d*x+c)^16+16/255*csch(d*x+c)^14-224/3315
*csch(d*x+c)^12+896/12155*csch(d*x+c)^10-1792/21879*csch(d*x+c)^8+2048/218
79*csch(d*x+c)^6-4096/36465*csch(d*x+c)^4+16384/109395*csch(d*x+c)^2)*coth
(d*x+c)+3*a^2*b*(-1024/3003-1/13*csch(d*x+c)^12+12/143*csch(d*x+c)^10-40/4
29*csch(d*x+c)^8+320/3003*csch(d*x+c)^6-128/1001*csch(d*x+c)^4+512/3003*cs
ch(d*x+c)^2)*coth(d*x+c)+3*b^2*a*(-128/315-1/9*csch(d*x+c)^8+8/63*csch(d*x
+c)^6-16/105*csch(d*x+c)^4+64/315*csch(d*x+c)^2)*coth(d*x+c)+b^3*(-8/15-1/
5*csch(d*x+c)^4+4/15*csch(d*x+c)^2)*coth(d*x+c))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 3585 vs. $2(205) = 410$.

Time = 0.13 (sec) , antiderivative size = 3585, normalized size of antiderivative = 16.22

$$\int \operatorname{csch}^{18}(c+dx) \left(a+b\sinh^4(c+dx)\right)^3 dx = \text{Too large to display}$$

input
```
integrate(csch(d*x+c)^18*(a+b*sinh(d*x+c)^4)^3,x, algorithm="fricas")
```

output
```
Too large to include
```

Sympy [F(-1)]

Timed out.

$$\int \operatorname{csch}^{18}(c+dx) \left(a+b\sinh^4(c+dx)\right)^3 dx = \text{Timed out}$$

input
```
integrate(csch(d*x+c)**18*(a+b*sinh(d*x+c)**4)**3,x)
```

output
```
Timed out
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 3719 vs. $2(205) = 410$.

Time = 0.06 (sec) , antiderivative size = 3719, normalized size of antiderivative = 16.83

$$\int \operatorname{csch}^{18}(c+dx)\left(a+b\sinh^4(c+dx)\right)^3\,dx = \text{Too large to display}$$

input
```
integrate(csch(d*x+c)^18*(a+b*sinh(d*x+c)^4)^3,x, algorithm="maxima")
```

output
```
-65536/109395*a^3*(17*e^(-2*d*x - 2*c)/(d*(17*e^(-2*d*x - 2*c) - 136*e^(-4
*d*x - 4*c) + 680*e^(-6*d*x - 6*c) - 2380*e^(-8*d*x - 8*c) + 6188*e^(-10*d
*x - 10*c) - 12376*e^(-12*d*x - 12*c) + 19448*e^(-14*d*x - 14*c) - 24310*e
^(-16*d*x - 16*c) + 24310*e^(-18*d*x - 18*c) - 19448*e^(-20*d*x - 20*c) +
12376*e^(-22*d*x - 22*c) - 6188*e^(-24*d*x - 24*c) + 2380*e^(-26*d*x - 26*
c) - 680*e^(-28*d*x - 28*c) + 136*e^(-30*d*x - 30*c) - 17*e^(-32*d*x - 32*
c) + e^(-34*d*x - 34*c) - 1)) - 136*e^(-4*d*x - 4*c)/(d*(17*e^(-2*d*x - 2*
c) - 136*e^(-4*d*x - 4*c) + 680*e^(-6*d*x - 6*c) - 2380*e^(-8*d*x - 8*c) +
 6188*e^(-10*d*x - 10*c) - 12376*e^(-12*d*x - 12*c) + 19448*e^(-14*d*x - 1
4*c) - 24310*e^(-16*d*x - 16*c) + 24310*e^(-18*d*x - 18*c) - 19448*e^(-20*
d*x - 20*c) + 12376*e^(-22*d*x - 22*c) - 6188*e^(-24*d*x - 24*c) + 2380*e^
(-26*d*x - 26*c) - 680*e^(-28*d*x - 28*c) + 136*e^(-30*d*x - 30*c) - 17*e^
(-32*d*x - 32*c) + e^(-34*d*x - 34*c) - 1)) + 680*e^(-6*d*x - 6*c)/(d*(17*
e^(-2*d*x - 2*c) - 136*e^(-4*d*x - 4*c) + 680*e^(-6*d*x - 6*c) - 2380*e^(-
8*d*x - 8*c) + 6188*e^(-10*d*x - 10*c) - 12376*e^(-12*d*x - 12*c) + 19448*
e^(-14*d*x - 14*c) - 24310*e^(-16*d*x - 16*c) + 24310*e^(-18*d*x - 18*c) -
 19448*e^(-20*d*x - 20*c) + 12376*e^(-22*d*x - 22*c) - 6188*e^(-24*d*x - 2
4*c) + 2380*e^(-26*d*x - 26*c) - 680*e^(-28*d*x - 28*c) + 136*e^(-30*d*x -
 30*c) - 17*e^(-32*d*x - 32*c) + e^(-34*d*x - 34*c) - 1)) - 2380*e^(-8*d*x
 - 8*c)/(d*(17*e^(-2*d*x - 2*c) - 136*e^(-4*d*x - 4*c) + 680*e^(-6*d*x ...
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 679 vs. $2(205) = 410$.

Time = 0.35 (sec) , antiderivative size = 679, normalized size of antiderivative = 3.07

$$\int \operatorname{csch}^{18}(c+dx) \left(a+b\sinh^4(c+dx)\right)^3 \, dx = \text{Too large to display}$$

input
```
integrate(csch(d*x+c)^18*(a+b*sinh(d*x+c)^4)^3,x, algorithm="giac")
```

output
```
-16/765765*(510510*b^3*e^(28*d*x + 28*c) - 6381375*b^3*e^(26*d*x + 26*c) +
 14702688*a*b^2*e^(24*d*x + 24*c) + 36807771*b^3*e^(24*d*x + 24*c) - 12742
3296*a*b^2*e^(22*d*x + 22*c) - 129771642*b^3*e^(22*d*x + 22*c) + 168030720
*a^2*b*e^(20*d*x + 20*c) + 494290368*a*b^2*e^(20*d*x + 20*c) + 312227916*b
^3*e^(20*d*x + 20*c) - 798145920*a^2*b*e^(18*d*x + 18*c) - 1132457040*a*b^
2*e^(18*d*x + 18*c) - 541906365*b^3*e^(18*d*x + 18*c) + 697016320*a^3*e^(1
6*d*x + 16*c) + 1582289280*a^2*b*e^(16*d*x + 16*c) + 1704228240*a*b^2*e^(1
6*d*x + 16*c) + 699143445*b^3*e^(16*d*x + 16*c) - 557613056*a^3*e^(14*d*x
+ 14*c) - 1736317440*a^2*b*e^(14*d*x + 14*c) - 1775057856*a*b^2*e^(14*d*x
+ 14*c) - 680611932*b^3*e^(14*d*x + 14*c) + 354844672*a^3*e^(12*d*x + 12*c
) + 1211857920*a^2*b*e^(12*d*x + 12*c) + 1316707392*a*b^2*e^(12*d*x + 12*c
) + 502035534*b^3*e^(12*d*x + 12*c) - 177422336*a^3*e^(10*d*x + 10*c) - 60
5928960*a^2*b*e^(10*d*x + 10*c) - 707362656*a*b^2*e^(10*d*x + 10*c) - 2790
95817*b^3*e^(10*d*x + 10*c) + 68239360*a^3*e^(8*d*x + 8*c) + 233049600*a^2
*b*e^(8*d*x + 8*c) + 277717440*a*b^2*e^(8*d*x + 8*c) + 115120005*b^3*e^(8*
d*x + 8*c) - 19496960*a^3*e^(6*d*x + 6*c) - 66585600*a^2*b*e^(6*d*x + 6*c)
 - 79347840*a*b^2*e^(6*d*x + 6*c) - 34204170*b^3*e^(6*d*x + 6*c) + 3899392
*a^3*e^(4*d*x + 4*c) + 13317120*a^2*b*e^(4*d*x + 4*c) + 15869568*a*b^2*e^(
4*d*x + 4*c) + 6942936*b^3*e^(4*d*x + 4*c) - 487424*a^3*e^(2*d*x + 2*c) -
1664640*a^2*b*e^(2*d*x + 2*c) - 1983696*a*b^2*e^(2*d*x + 2*c) - 867867*...
```

Mupad [B] (verification not implemented)

Time = 2.20 (sec) , antiderivative size = 4490, normalized size of antiderivative = 20.32

$$\int \operatorname{csch}^{18}(c+dx) \left(a+b\sinh^4(c+dx)\right)^3 dx = \text{Too large to display}$$

input
```
int((a + b*sinh(c + d*x)^4)^3/sinh(c + d*x)^18,x)
```

output
```
((24*b^3)/(595*d) - (64*exp(10*c + 10*d*x)*(840*a*b^2 + 1152*a^2*b + 1024*
a^3 + 231*b^3))/(85*d) + (6864*b^3*exp(20*c + 20*d*x))/(595*d) - (104*b^3*
exp(22*c + 22*d*x))/(85*d) + (48*b*exp(8*c + 8*d*x)*(112*a*b + 128*a^2 + 3
3*b^2))/(17*d) + (576*b*exp(12*c + 12*d*x)*(112*a*b + 128*a^2 + 33*b^2))/(
85*d) - (24*b*exp(6*c + 6*d*x)*(448*a*b + 256*a^2 + 165*b^2))/(119*d) - (1
44*b*exp(14*c + 14*d*x)*(448*a*b + 256*a^2 + 165*b^2))/(119*d) - (48*b^2*e
xp(2*c + 2*d*x)*(8*a + 11*b))/(595*d) - (528*b^2*exp(18*c + 18*d*x)*(8*a +
 11*b))/(119*d) + (16*b^2*exp(4*c + 4*d*x)*(96*a + 55*b))/(119*d) + (264*b
^2*exp(16*c + 16*d*x)*(96*a + 55*b))/(119*d))/(91*exp(4*c + 4*d*x) - 14*ex
p(2*c + 2*d*x) - 364*exp(6*c + 6*d*x) + 1001*exp(8*c + 8*d*x) - 2002*exp(1
0*c + 10*d*x) + 3003*exp(12*c + 12*d*x) - 3432*exp(14*c + 14*d*x) + 3003*e
xp(16*c + 16*d*x) - 2002*exp(18*c + 18*d*x) + 1001*exp(20*c + 20*d*x) - 36
4*exp(22*c + 22*d*x) + 91*exp(24*c + 24*d*x) - 14*exp(26*c + 26*d*x) + exp
(28*c + 28*d*x) + 1) + ((24*b^3)/(595*d) - (4*b^3*exp(2*c + 2*d*x))/(85*d)
)/(6*exp(4*c + 4*d*x) - 4*exp(2*c + 2*d*x) - 4*exp(6*c + 6*d*x) + exp(8*c
+ 8*d*x) + 1) - ((64*exp(8*c + 8*d*x)*(840*a*b^2 + 1152*a^2*b + 1024*a^3 +
 231*b^3))/(221*d) - (1056*b^3*exp(18*c + 18*d*x))/(119*d) + (88*b^3*exp(2
0*c + 20*d*x))/(85*d) + (48*b^2*(8*a + 11*b))/(7735*d) - (192*b*exp(6*c +
6*d*x)*(112*a*b + 128*a^2 + 33*b^2))/(221*d) - (3456*b*exp(10*c + 10*d*x)*
(112*a*b + 128*a^2 + 33*b^2))/(1105*d) + (72*b*exp(4*c + 4*d*x)*(448*a*...
```

Reduce [B] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 916, normalized size of antiderivative = 4.14

$$\int \operatorname{csch}^{18}(c+dx) \left(a+b\sinh^4(c+dx)\right)^3 dx = \text{Too large to display}$$

input
```
int(csch(d*x+c)^18*(a+b*sinh(d*x+c)^4)^3,x)
```

output

```
(16*( - 510510*e**(28*c + 28*d*x)*b**3 + 6381375*e**(26*c + 26*d*x)*b**3 -
 14702688*e**(24*c + 24*d*x)*a*b**2 - 36807771*e**(24*c + 24*d*x)*b**3 + 1
27423296*e**(22*c + 22*d*x)*a*b**2 + 129771642*e**(22*c + 22*d*x)*b**3 - 1
68030720*e**(20*c + 20*d*x)*a**2*b - 494290368*e**(20*c + 20*d*x)*a*b**2 -
 312227916*e**(20*c + 20*d*x)*b**3 + 798145920*e**(18*c + 18*d*x)*a**2*b +
 1132457040*e**(18*c + 18*d*x)*a*b**2 + 541906365*e**(18*c + 18*d*x)*b**3
- 697016320*e**(16*c + 16*d*x)*a**3 - 1582289280*e**(16*c + 16*d*x)*a**2*b
 - 1704228240*e**(16*c + 16*d*x)*a*b**2 - 699143445*e**(16*c + 16*d*x)*b**
3 + 557613056*e**(14*c + 14*d*x)*a**3 + 1736317440*e**(14*c + 14*d*x)*a**2
*b + 1775057856*e**(14*c + 14*d*x)*a*b**2 + 680611932*e**(14*c + 14*d*x)*b
**3 - 354844672*e**(12*c + 12*d*x)*a**3 - 1211857920*e**(12*c + 12*d*x)*a*
*2*b - 1316707392*e**(12*c + 12*d*x)*a*b**2 - 502035534*e**(12*c + 12*d*x)
*b**3 + 177422336*e**(10*c + 10*d*x)*a**3 + 605928960*e**(10*c + 10*d*x)*a
**2*b + 707362656*e**(10*c + 10*d*x)*a*b**2 + 279095817*e**(10*c + 10*d*x)
*b**3 - 68239360*e**(8*c + 8*d*x)*a**3 - 233049600*e**(8*c + 8*d*x)*a**2*b
 - 277717440*e**(8*c + 8*d*x)*a*b**2 - 115120005*e**(8*c + 8*d*x)*b**3 + 1
9496960*e**(6*c + 6*d*x)*a**3 + 66585600*e**(6*c + 6*d*x)*a**2*b + 7934784
0*e**(6*c + 6*d*x)*a*b**2 + 34204170*e**(6*c + 6*d*x)*b**3 - 3899392*e**(4
*c + 4*d*x)*a**3 - 13317120*e**(4*c + 4*d*x)*a**2*b - 15869568*e**(4*c + 4
*d*x)*a*b**2 - 6942936*e**(4*c + 4*d*x)*b**3 + 487424*e**(2*c + 2*d*x)*...
```

3.203 $\int \operatorname{csch}^{20}(c+dx)\left(a+b\sinh^4(c+dx)\right)^3\,dx$

Optimal result

Integrand size = 23, antiderivative size = 248

$$\begin{aligned}\int \operatorname{csch}^{20}(c+dx)\left(a+b\sinh^4(c+dx)\right)^3\,dx &= \frac{(a+b)^3\coth(c+dx)}{d}-\frac{(a+b)^2(3a+b)\coth^3(c+dx)}{d}\\&\quad+\frac{3(a+b)\left(12a^2+9ab+b^2\right)\coth^5(c+dx)}{5d}\\&\quad-\frac{\left(84a^3+105a^2b+30ab^2+b^3\right)\coth^7(c+dx)}{7d}+\frac{a\left(42a^2+35ab+5b^2\right)\coth^9(c+dx)}{3d}\\&\quad-\frac{3a\left(42a^2+21ab+b^2\right)\coth^{11}(c+dx)}{11d}+\frac{21a^2(4a+b)\coth^{13}(c+dx)}{13d}\\&\quad-\frac{a^2(12a+b)\coth^{15}(c+dx)}{5d}+\frac{9a^3\coth^{17}(c+dx)}{17d}-\frac{a^3\coth^{19}(c+dx)}{19d}\end{aligned}$$

output
```
(a+b)^3*coth(d*x+c)/d-(a+b)^2*(3*a+b)*coth(d*x+c)^3/d+3/5*(a+b)*(12*a^2+9*
a*b+b^2)*coth(d*x+c)^5/d-1/7*(84*a^3+105*a^2*b+30*a*b^2+b^3)*coth(d*x+c)^7
/d+1/3*a*(42*a^2+35*a*b+5*b^2)*coth(d*x+c)^9/d-3/11*a*(42*a^2+21*a*b+b^2)*
coth(d*x+c)^11/d+21/13*a^2*(4*a+b)*coth(d*x+c)^13/d-1/5*a^2*(12*a+b)*coth(
d*x+c)^15/d+9/17*a^3*coth(d*x+c)^17/d-1/19*a^3*coth(d*x+c)^19/d
```

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 548 vs. $2(248) = 496$.

Time = 7.35 (sec) , antiderivative size = 548, normalized size of antiderivative = 2.21

$$\int \operatorname{csch}^{20}(c+dx)\left(a+b\sinh^4(c+dx)\right)^3\,dx = \frac{(-7945986048a^3\cosh(c+dx) - 8939234304a^2b\cosh(c+dx) - 6518191680ab^2\cosh(c+dx) - 1792502}{}$$

input
```
Integrate[Csch[c + d*x]^20*(a + b*Sinh[c + d*x]^4)^3,x]
```

output
```
((-7945986048*a^3*Cosh[c + d*x] - 8939234304*a^2*b*Cosh[c + d*x] - 6518191
680*a*b^2*Cosh[c + d*x] - 1792502712*b^3*Cosh[c + d*x] + 6501261312*a^3*Co
sh[3*(c + d*x)] + 18149354496*a^2*b*Cosh[3*(c + d*x)] + 14814072000*a*b^2*
Cosh[3*(c + d*x)] + 4260103848*b^3*Cosh[3*(c + d*x)] - 4334174208*a^3*Cosh
[5*(c + d*x)] - 14582690304*a^2*b*Cosh[5*(c + d*x)] - 14221509120*a*b^2*Co
sh[5*(c + d*x)] - 4440518082*b^3*Cosh[5*(c + d*x)] + 2333786112*a^3*Cosh[7
*(c + d*x)] + 7852217856*a^2*b*Cosh[7*(c + d*x)] + 8803791360*a*b^2*Cosh[7
*(c + d*x)] + 3047642598*b^3*Cosh[7*(c + d*x)] - 1000194048*a^3*Cosh[9*(c
+ d*x)] - 3365236224*a^2*b*Cosh[9*(c + d*x)] - 3906077760*a*b^2*Cosh[9*(c
+ d*x)] - 1489040982*b^3*Cosh[9*(c + d*x)] + 333398016*a^3*Cosh[11*(c + d*
x)] + 1121745408*a^2*b*Cosh[11*(c + d*x)] + 1302025920*a*b^2*Cosh[11*(c +
d*x)] + 527386002*b^3*Cosh[11*(c + d*x)] - 83349504*a^3*Cosh[13*(c + d*x)]
 - 280436352*a^2*b*Cosh[13*(c + d*x)] - 325506480*a*b^2*Cosh[13*(c + d*x)]
 - 134271423*b^3*Cosh[13*(c + d*x)] + 14708736*a^3*Cosh[15*(c + d*x)] + 49
488768*a^2*b*Cosh[15*(c + d*x)] + 57442320*a*b^2*Cosh[15*(c + d*x)] + 2369
4957*b^3*Cosh[15*(c + d*x)] - 1634304*a^3*Cosh[17*(c + d*x)] - 5498752*a^2
*b*Cosh[17*(c + d*x)] - 6382480*a*b^2*Cosh[17*(c + d*x)] - 2632773*b^3*Cos
h[17*(c + d*x)] + 86016*a^3*Cosh[19*(c + d*x)] + 289408*a^2*b*Cosh[19*(c +
 d*x)] + 335920*a*b^2*Cosh[19*(c + d*x)] + 138567*b^3*Cosh[19*(c + d*x)])*
Csch[c + d*x]^19)/(79459860480*d)
```

Rubi [A] (verified)

Time = 0.50 (sec) , antiderivative size = 222, normalized size of antiderivative = 0.90, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.174, Rules used = {3042, 3696, 1584, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \operatorname{csch}^{20}(c+dx)\left(a+b\sinh^4(c+dx)\right)^3\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\left(a+b\sin(ic+idx)^4\right)^3}{\sin(ic+idx)^{20}}dx$$

$$\Big\downarrow \text{3696}$$

$$\frac{\int \coth^{20}(c+dx)\left(1-\tanh^2(c+dx)\right)^3\left((a+b)\tanh^4(c+dx)-2a\tanh^2(c+dx)+a\right)^3d\tanh(c+dx)}{d}$$

$$\Big\downarrow \text{1584}$$

$$\frac{\int \left(a^3\coth^{20}(c+dx)-9a^3\coth^{18}(c+dx)+3a^2(12a+b)\coth^{16}(c+dx)-21a^2(4a+b)\coth^{14}(c+dx)+3a(42\right.}{}$$

$$\Big\downarrow \text{2009}$$

$$\frac{-\frac{1}{19}a^3\coth^{19}(c+dx)+\frac{9}{17}a^3\coth^{17}(c+dx)-\frac{3}{11}a\left(42a^2+21ab+b^2\right)\coth^{11}(c+dx)+\frac{1}{3}a\left(42a^2+35ab+5b^2\right)\text{c}}{}$$

input
```
Int[Csch[c + d*x]^20*(a + b*Sinh[c + d*x]^4)^3,x]
```

output
```
((a + b)^3*Coth[c + d*x] - (a + b)^2*(3*a + b)*Coth[c + d*x]^3 + (3*(a + b
)*(12*a^2 + 9*a*b + b^2)*Coth[c + d*x]^5)/5 - ((84*a^3 + 105*a^2*b + 30*a*
b^2 + b^3)*Coth[c + d*x]^7)/7 + (a*(42*a^2 + 35*a*b + 5*b^2)*Coth[c + d*x]
^9)/3 - (3*a*(42*a^2 + 21*a*b + b^2)*Coth[c + d*x]^11)/11 + (21*a^2*(4*a +
 b)*Coth[c + d*x]^13)/13 - (a^2*(12*a + b)*Coth[c + d*x]^15)/5 + (9*a^3*Co
th[c + d*x]^17)/17 - (a^3*Coth[c + d*x]^19)/19)/d
```

Defintions of rubi rules used

rule 1584
```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (
c_.)*(x_)^4)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(f*x)^m*(d + e*x^2)^q*
(a + b*x^2 + c*x^4)^p, x], x] /; FreeQ[{a, b, c, d, e, f, m, q}, x] && NeQ[
b^2 - 4*a*c, 0] && IGtQ[p, 0] && IGtQ[q, -2]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3696
```
Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^4)^(
p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Simp[ff^(m + 1
)/f   Subst[Int[x^m*((a + 2*a*ff^2*x^2 + (a + b)*ff^4*x^4)^p/(1 + ff^2*x^2)
^(m/2 + 2*p + 1)), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f}, x] &
& IntegerQ[m/2] && IntegerQ[p]
```

Maple [A] (verified)

Time = 9.52 (sec) , antiderivative size = 298, normalized size of antiderivative = 1.20

method	result
derivativedivides	$\frac{a^3\left(\frac{65536}{230945}-\frac{\operatorname{csch}(dx+c)^{18}}{19}+\frac{18\operatorname{csch}(dx+c)^{16}}{323}-\frac{96\operatorname{csch}(dx+c)^{14}}{1615}+\frac{1344\operatorname{csch}(dx+c)^{12}}{20995}-\frac{16128\operatorname{csch}(dx+c)^{10}}{230945}+\frac{3584\operatorname{csch}(dx+c)^{8}}{46189}\right.}{}$
default	$\frac{a^3\left(\frac{65536}{230945}-\frac{\operatorname{csch}(dx+c)^{18}}{19}+\frac{18\operatorname{csch}(dx+c)^{16}}{323}-\frac{96\operatorname{csch}(dx+c)^{14}}{1615}+\frac{1344\operatorname{csch}(dx+c)^{12}}{20995}-\frac{16128\operatorname{csch}(dx+c)^{10}}{230945}+\frac{3584\operatorname{csch}(dx+c)^{8}}{46189}\right.}{}$
parallelrisch	$-\frac{9\operatorname{csch}\left(\frac{dx}{2}+\frac{c}{2}\right)^7\left(\operatorname{sech}\left(\frac{dx}{2}+\frac{c}{2}\right)^{12}a^3\left(-\frac{\cosh(11dx+11c)}{3}+\cosh(9dx+9c)-\frac{7\cosh(7dx+7c)}{3}+\frac{13\cosh(5dx+5c)}{3}-\frac{13\cosh(3c}{2}\right.\right.}{}$
risch	$-\frac{32\left(-532235847b^3e^{8dx+8c}+134271423b^3e^{6dx+6c}-14708736a^3e^{4dx+4c}-23694957b^3e^{4dx+4c}+1634304a^3e^{2dx+2c}+1\right.}{}$

input
```
int(csch(d*x+c)^20*(a+b*sinh(d*x+c)^4)^3,x,method=_RETURNVERBOSE)
```

output
```
1/d*(a^3*(65536/230945-1/19*csch(d*x+c)^18+18/323*csch(d*x+c)^16-96/1615*c
sch(d*x+c)^14+1344/20995*csch(d*x+c)^12-16128/230945*csch(d*x+c)^10+3584/4
6189*csch(d*x+c)^8-4096/46189*csch(d*x+c)^6+24576/230945*csch(d*x+c)^4-327
68/230945*csch(d*x+c)^2)*coth(d*x+c)+3*a^2*b*(2048/6435-1/15*csch(d*x+c)^1
4+14/195*csch(d*x+c)^12-56/715*csch(d*x+c)^10+112/1287*csch(d*x+c)^8-128/1
287*csch(d*x+c)^6+256/2145*csch(d*x+c)^4-1024/6435*csch(d*x+c)^2)*coth(d*x
+c)+3*b^2*a*(256/693-1/11*csch(d*x+c)^10+10/99*csch(d*x+c)^8-80/693*csch(d
*x+c)^6+32/231*csch(d*x+c)^4-128/693*csch(d*x+c)^2)*coth(d*x+c)+b^3*(16/35
-1/7*csch(d*x+c)^6+6/35*csch(d*x+c)^4-8/35*csch(d*x+c)^2)*coth(d*x+c))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 4259 vs. $2(232) = 464$.

Time = 0.11 (sec) , antiderivative size = 4259, normalized size of antiderivative = 17.17

$$\int \operatorname{csch}^{20}(c+dx) \left(a+b\sinh^4(c+dx)\right)^3 \, dx = \text{Too large to display}$$

input
```
integrate(csch(d*x+c)^20*(a+b*sinh(d*x+c)^4)^3,x, algorithm="fricas")
```

output
```
Too large to include
```

Sympy [F(-1)]

Timed out.

$$\int \operatorname{csch}^{20}(c+dx) \left(a+b\sinh^4(c+dx)\right)^3 \, dx = \text{Timed out}$$

input
```
integrate(csch(d*x+c)**20*(a+b*sinh(d*x+c)**4)**3,x)
```

output
```
Timed out
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 4883 vs. $2(232) = 464$.

Time = 0.07 (sec) , antiderivative size = 4883, normalized size of antiderivative = 19.69

$$\int \operatorname{csch}^{20}(c+dx)\left(a+b\sinh^4(c+dx)\right)^3\,dx = \text{Too large to display}$$

input
```
integrate(csch(d*x+c)^20*(a+b*sinh(d*x+c)^4)^3,x, algorithm="maxima")
```

output
```
131072/230945*a^3*(19*e^(-2*d*x - 2*c)/(d*(19*e^(-2*d*x - 2*c) - 171*e^(-4
*d*x - 4*c) + 969*e^(-6*d*x - 6*c) - 3876*e^(-8*d*x - 8*c) + 11628*e^(-10*
d*x - 10*c) - 27132*e^(-12*d*x - 12*c) + 50388*e^(-14*d*x - 14*c) - 75582*
e^(-16*d*x - 16*c) + 92378*e^(-18*d*x - 18*c) - 92378*e^(-20*d*x - 20*c) +
 75582*e^(-22*d*x - 22*c) - 50388*e^(-24*d*x - 24*c) + 27132*e^(-26*d*x -
26*c) - 11628*e^(-28*d*x - 28*c) + 3876*e^(-30*d*x - 30*c) - 969*e^(-32*d*
x - 32*c) + 171*e^(-34*d*x - 34*c) - 19*e^(-36*d*x - 36*c) + e^(-38*d*x -
38*c) - 1)) - 171*e^(-4*d*x - 4*c)/(d*(19*e^(-2*d*x - 2*c) - 171*e^(-4*d*x
 - 4*c) + 969*e^(-6*d*x - 6*c) - 3876*e^(-8*d*x - 8*c) + 11628*e^(-10*d*x
- 10*c) - 27132*e^(-12*d*x - 12*c) + 50388*e^(-14*d*x - 14*c) - 75582*e^(-
16*d*x - 16*c) + 92378*e^(-18*d*x - 18*c) - 92378*e^(-20*d*x - 20*c) + 755
82*e^(-22*d*x - 22*c) - 50388*e^(-24*d*x - 24*c) + 27132*e^(-26*d*x - 26*c
) - 11628*e^(-28*d*x - 28*c) + 3876*e^(-30*d*x - 30*c) - 969*e^(-32*d*x -
32*c) + 171*e^(-34*d*x - 34*c) - 19*e^(-36*d*x - 36*c) + e^(-38*d*x - 38*c
) - 1)) + 969*e^(-6*d*x - 6*c)/(d*(19*e^(-2*d*x - 2*c) - 171*e^(-4*d*x - 4
*c) + 969*e^(-6*d*x - 6*c) - 3876*e^(-8*d*x - 8*c) + 11628*e^(-10*d*x - 10
*c) - 27132*e^(-12*d*x - 12*c) + 50388*e^(-14*d*x - 14*c) - 75582*e^(-16*d
*x - 16*c) + 92378*e^(-18*d*x - 18*c) - 92378*e^(-20*d*x - 20*c) + 75582*e
^(-22*d*x - 22*c) - 50388*e^(-24*d*x - 24*c) + 27132*e^(-26*d*x - 26*c) -
11628*e^(-28*d*x - 28*c) + 3876*e^(-30*d*x - 30*c) - 969*e^(-32*d*x - 3...
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 737 vs. $2(232) = 464$.

Time = 0.36 (sec) , antiderivative size = 737, normalized size of antiderivative = 2.97

$$\int \operatorname{csch}^{20}(c+dx)\left(a+b\sinh^4(c+dx)\right)^3\,dx = \text{Too large to display}$$

input
```
integrate(csch(d*x+c)^20*(a+b*sinh(d*x+c)^4)^3,x, algorithm="giac")
```

output
```
-32/4849845*(4849845*b^3*e^(30*d*x + 30*c) - 61108047*b^3*e^(28*d*x + 28*c
) + 155195040*a*b^2*e^(26*d*x + 26*c) + 355978623*b^3*e^(26*d*x + 26*c) -
1352413920*a*b^2*e^(24*d*x + 24*c) - 1270797957*b^3*e^(24*d*x + 24*c) + 18
62340480*a^2*b*e^(22*d*x + 22*c) + 5287716720*a*b^2*e^(22*d*x + 22*c) + 31
06533573*b^3*e^(22*d*x + 22*c) - 8897848960*a^2*b*e^(20*d*x + 20*c) - 1225
6713040*a*b^2*e^(20*d*x + 20*c) - 5504019807*b^3*e^(20*d*x + 20*c) + 79459
86048*a^3*e^(18*d*x + 18*c) + 17837083264*a^2*b*e^(18*d*x + 18*c) + 187749
04720*a*b^2*e^(18*d*x + 18*c) + 7296522519*b^3*e^(18*d*x + 18*c) - 6501261
312*a^3*e^(16*d*x + 16*c) - 20011694976*a^2*b*e^(16*d*x + 16*c) - 20101788
720*a*b^2*e^(16*d*x + 16*c) - 7366637421*b^3*e^(16*d*x + 16*c) + 433417420
8*a^3*e^(14*d*x + 14*c) + 14582690304*a^2*b*e^(14*d*x + 14*c) + 1557392304
0*a*b^2*e^(14*d*x + 14*c) + 5711316039*b^3*e^(14*d*x + 14*c) - 2333786112*
a^3*e^(12*d*x + 12*c) - 7852217856*a^2*b*e^(12*d*x + 12*c) - 8958986400*a*
b^2*e^(12*d*x + 12*c) - 3403621221*b^3*e^(12*d*x + 12*c) + 1000194048*a^3*
e^(10*d*x + 10*c) + 3365236224*a^2*b*e^(10*d*x + 10*c) + 3906077760*a*b^2*
e^(10*d*x + 10*c) + 1550149029*b^3*e^(10*d*x + 10*c) - 333398016*a^3*e^(8*
d*x + 8*c) - 1121745408*a^2*b*e^(8*d*x + 8*c) - 1302025920*a*b^2*e^(8*d*x
+ 8*c) - 532235847*b^3*e^(8*d*x + 8*c) + 83349504*a^3*e^(6*d*x + 6*c) + 28
0436352*a^2*b*e^(6*d*x + 6*c) + 325506480*a*b^2*e^(6*d*x + 6*c) + 13427142
3*b^3*e^(6*d*x + 6*c) - 14708736*a^3*e^(4*d*x + 4*c) - 49488768*a^2*b*e...
```

Mupad [B] (verification not implemented)

Time = 2.28 (sec) , antiderivative size = 5190, normalized size of antiderivative = 20.93

$$\int \operatorname{csch}^{20}(c+dx) \left(a+b\sinh^4(c+dx)\right)^3 \, dx = \text{Too large to display}$$

input
```
int((a + b*sinh(c + d*x)^4)^3/sinh(c + d*x)^20,x)
```

output
```
((512*b*(112*a*b + 128*a^2 + 33*b^2))/(138567*d) + (1792*b^3*exp(8*c + 8*d
*x))/(969*d) - (448*b^3*exp(10*c + 10*d*x))/(969*d) - (64*b*exp(2*c + 2*d*
x)*(448*a*b + 256*a^2 + 165*b^2))/(12597*d) - (256*b^2*exp(6*c + 6*d*x)*(8
*a + 11*b))/(969*d) + (512*b^2*exp(4*c + 4*d*x)*(96*a + 55*b))/(12597*d))/
(9*exp(2*c + 2*d*x) - 36*exp(4*c + 4*d*x) + 84*exp(6*c + 6*d*x) - 126*exp(
8*c + 8*d*x) + 126*exp(10*c + 10*d*x) - 84*exp(12*c + 12*d*x) + 36*exp(14*
c + 14*d*x) - 9*exp(16*c + 16*d*x) + exp(18*c + 18*d*x) - 1) - ((8*b*(448*
a*b + 256*a^2 + 165*b^2))/(12597*d) + (128*exp(4*c + 4*d*x)*(840*a*b^2 + 1
152*a^2*b + 1024*a^3 + 231*b^3))/(4199*d) - (2816*b^3*exp(14*c + 14*d*x))/
(323*d) + (440*b^3*exp(16*c + 16*d*x))/(323*d) - (512*b*exp(2*c + 2*d*x)*(
112*a*b + 128*a^2 + 33*b^2))/(12597*d) - (2560*b*exp(6*c + 6*d*x)*(112*a*b
 + 128*a^2 + 33*b^2))/(4199*d) + (880*b*exp(8*c + 8*d*x)*(448*a*b + 256*a^
2 + 165*b^2))/(4199*d) + (704*b^2*exp(12*c + 12*d*x)*(8*a + 11*b))/(323*d)
 - (2816*b^2*exp(10*c + 10*d*x)*(96*a + 55*b))/(4199*d))/(66*exp(4*c + 4*d
*x) - 12*exp(2*c + 2*d*x) - 220*exp(6*c + 6*d*x) + 495*exp(8*c + 8*d*x) -
792*exp(10*c + 10*d*x) + 924*exp(12*c + 12*d*x) - 792*exp(14*c + 14*d*x) +
 495*exp(16*c + 16*d*x) - 220*exp(18*c + 18*d*x) + 66*exp(20*c + 20*d*x) -
 12*exp(22*c + 22*d*x) + exp(24*c + 24*d*x) + 1) - ((512*exp(18*c + 18*d*x
)*(840*a*b^2 + 1152*a^2*b + 1024*a^3 + 231*b^3))/(19*d) + (128*b^3*exp(6*c
 + 6*d*x))/(19*d) - (1536*b^3*exp(8*c + 8*d*x))/(19*d) - (1536*b^3*exp(...
```

Reduce [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 1002, normalized size of antiderivative = 4.04

$$\int \operatorname{csch}^{20}(c+dx) \left(a+b\sinh^4(c+dx)\right)^3 \, dx = \text{Too large to display}$$

input
```
int(csch(d*x+c)^20*(a+b*sinh(d*x+c)^4)^3,x)
```

output

```
(32*( - 4849845*e**(30*c + 30*d*x)*b**3 + 61108047*e**(28*c + 28*d*x)*b**3
 - 155195040*e**(26*c + 26*d*x)*a*b**2 - 355978623*e**(26*c + 26*d*x)*b**3
 + 1352413920*e**(24*c + 24*d*x)*a*b**2 + 1270797957*e**(24*c + 24*d*x)*b*
*3 - 1862340480*e**(22*c + 22*d*x)*a**2*b - 5287716720*e**(22*c + 22*d*x)*
a*b**2 - 3106533573*e**(22*c + 22*d*x)*b**3 + 8897848960*e**(20*c + 20*d*x
)*a**2*b + 12256713040*e**(20*c + 20*d*x)*a*b**2 + 5504019807*e**(20*c + 2
0*d*x)*b**3 - 7945986048*e**(18*c + 18*d*x)*a**3 - 17837083264*e**(18*c +
18*d*x)*a**2*b - 18774904720*e**(18*c + 18*d*x)*a*b**2 - 7296522519*e**(18
*c + 18*d*x)*b**3 + 6501261312*e**(16*c + 16*d*x)*a**3 + 20011694976*e**(1
6*c + 16*d*x)*a**2*b + 20101788720*e**(16*c + 16*d*x)*a*b**2 + 7366637421*
e**(16*c + 16*d*x)*b**3 - 4334174208*e**(14*c + 14*d*x)*a**3 - 14582690304
*e**(14*c + 14*d*x)*a**2*b - 15573923040*e**(14*c + 14*d*x)*a*b**2 - 57113
16039*e**(14*c + 14*d*x)*b**3 + 2333786112*e**(12*c + 12*d*x)*a**3 + 78522
17856*e**(12*c + 12*d*x)*a**2*b + 8958986400*e**(12*c + 12*d*x)*a*b**2 + 3
403621221*e**(12*c + 12*d*x)*b**3 - 1000194048*e**(10*c + 10*d*x)*a**3 - 3
365236224*e**(10*c + 10*d*x)*a**2*b - 3906077760*e**(10*c + 10*d*x)*a*b**2
 - 1550149029*e**(10*c + 10*d*x)*b**3 + 333398016*e**(8*c + 8*d*x)*a**3 +
1121745408*e**(8*c + 8*d*x)*a**2*b + 1302025920*e**(8*c + 8*d*x)*a*b**2 +
532235847*e**(8*c + 8*d*x)*b**3 - 83349504*e**(6*c + 6*d*x)*a**3 - 2804363
52*e**(6*c + 6*d*x)*a**2*b - 325506480*e**(6*c + 6*d*x)*a*b**2 - 134271...
```

3.204 $\int \frac{\sinh^7(c+dx)}{a-b\sinh^4(c+dx)}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 148

$$\int \frac{\sinh^7(c+dx)}{a-b\sinh^4(c+dx)}\,dx = -\frac{a\arctan\left(\frac{\sqrt[4]{b}\cosh(c+dx)}{\sqrt{\sqrt{a}-\sqrt{b}}}\right)}{2\sqrt{\sqrt{a}-\sqrt{b}}\,b^{7/4}d} + \frac{a\text{arctanh}\left(\frac{\sqrt[4]{b}\cosh(c+dx)}{\sqrt{\sqrt{a}+\sqrt{b}}}\right)}{2\sqrt{\sqrt{a}+\sqrt{b}}\,b^{7/4}d} + \frac{\cosh(c+dx)}{bd} - \frac{\cosh^3(c+dx)}{3bd}$$

output
```
-1/2*a*arctan(b^(1/4)*cosh(d*x+c)/(a^(1/2)-b^(1/2))^(1/2))/(a^(1/2)-b^(1/2
))^(1/2)/b^(7/4)/d+1/2*a*arctanh(b^(1/4)*cosh(d*x+c)/(a^(1/2)+b^(1/2))^(1/
2))/(a^(1/2)+b^(1/2))^(1/2)/b^(7/4)/d+cosh(d*x+c)/b/d-1/3*cosh(d*x+c)^3/b/
d
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

Time = 1.58 (sec) , antiderivative size = 390, normalized size of antiderivative = 2.64

$$\int \frac{\sinh^7(c+dx)}{a-b\sinh^4(c+dx)}\,dx$$
$$= \frac{18\cosh(c+dx)-2\cosh(3(c+dx))-3a\text{RootSum}\left[b-4b\#1^2-16a\#1^4+6b\#1^4-4b\#1^6+b\#1^8\&,\right.}{}$$

input
```
Integrate[Sinh[c + d*x]^7/(a - b*Sinh[c + d*x]^4),x]
```

output
```
(18*Cosh[c + d*x] - 2*Cosh[3*(c + d*x)] - 3*a*RootSum[b - 4*b*#1^2 - 16*a*
#1^4 + 6*b*#1^4 - 4*b*#1^6 + b*#1^8 & , (-c - d*x - 2*Log[-Cosh[(c + d*x)/
2] - Sinh[(c + d*x)/2] + Cosh[(c + d*x)/2]*#1 - Sinh[(c + d*x)/2]*#1] + 3*
c*#1^2 + 3*d*x*#1^2 + 6*Log[-Cosh[(c + d*x)/2] - Sinh[(c + d*x)/2] + Cosh[
(c + d*x)/2]*#1 - Sinh[(c + d*x)/2]*#1]*#1^2 - 3*c*#1^4 - 3*d*x*#1^4 - 6*L
og[-Cosh[(c + d*x)/2] - Sinh[(c + d*x)/2] + Cosh[(c + d*x)/2]*#1 - Sinh[(c
 + d*x)/2]*#1]*#1^4 + c*#1^6 + d*x*#1^6 + 2*Log[-Cosh[(c + d*x)/2] - Sinh[
(c + d*x)/2] + Cosh[(c + d*x)/2]*#1 - Sinh[(c + d*x)/2]*#1]*#1^6)/(-(b*#1)
 - 8*a*#1^3 + 3*b*#1^3 - 3*b*#1^5 + b*#1^7) & ])/(24*b*d)
```

Rubi [A] (verified)

Time = 0.43 (sec) , antiderivative size = 142, normalized size of antiderivative = 0.96, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.208$, Rules used = {3042, 26, 3694, 1484, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sinh^7(c+dx)}{a-b\sinh^4(c+dx)}\,dx$$
$$\Big\downarrow \text{3042}$$
$$\int \frac{i\sin(ic+idx)^7}{a-b\sin(ic+idx)^4}dx$$
$$\Big\downarrow \text{26}$$

$$i\int \frac{\sin(ic+idx)^7}{a-b\sin(ic+idx)^4}dx$$

$\Big\downarrow$ 3694

$$-\frac{\int \frac{\left(1-\cosh^2(c+dx)\right)^3}{-b\cosh^4(c+dx)+2b\cosh^2(c+dx)+a-b}d\cosh(c+dx)}{d}$$

$\Big\downarrow$ 1484

$$-\frac{\int \left(\frac{\cosh^2(c+dx)}{b}-\frac{1}{b}+\frac{a-a\cosh^2(c+dx)}{b\left(-b\cosh^4(c+dx)+2b\cosh^2(c+dx)+a-b\right)}\right)d\cosh(c+dx)}{d}$$

$\Big\downarrow$ 2009

$$-\frac{\frac{a\arctan\left(\frac{\sqrt[4]{b}\cosh(c+dx)}{\sqrt{\sqrt{a}-\sqrt{b}}}\right)}{2b^{7/4}\sqrt{\sqrt{a}-\sqrt{b}}}-\frac{a\text{arctanh}\left(\frac{\sqrt[4]{b}\cosh(c+dx)}{\sqrt{\sqrt{a}+\sqrt{b}}}\right)}{2b^{7/4}\sqrt{\sqrt{a}+\sqrt{b}}}+\frac{\cosh^3(c+dx)}{3b}-\frac{\cosh(c+dx)}{b}}{d}$$

input
```
Int[Sinh[c + d*x]^7/(a - b*Sinh[c + d*x]^4),x]
```

output
```
-(((a*ArcTan[(b^(1/4)*Cosh[c + d*x])/Sqrt[Sqrt[a] - Sqrt[b]]])/(2*Sqrt[Sqr
t[a] - Sqrt[b]]*b^(7/4)) - (a*ArcTanh[(b^(1/4)*Cosh[c + d*x])/Sqrt[Sqrt[a]
 + Sqrt[b]]])/(2*Sqrt[Sqrt[a] + Sqrt[b]]*b^(7/4)) - Cosh[c + d*x]/b + Cosh
[c + d*x]^3/(3*b))/d)
```

Defintions of rubi rules used

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 1484
```
Int[((d_) + (e_.)*(x_)^2)^(q_)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symb
ol] :> Int[ExpandIntegrand[(d + e*x^2)^q/(a + b*x^2 + c*x^4), x], x] /; Fre
eQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2,
 0] && IntegerQ[q]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3694
```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^4)^
(p_.), x_Symbol] :> With[{ff = FreeFactors[Cos[e + f*x], x]}, Simp[-ff/f
Subst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a + b - 2*b*ff^2*x^2 + b*ff^4*x^4)^p,
 x], x, Cos[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m -
 1)/2]
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 3.90 (sec) , antiderivative size = 172, normalized size of antiderivative = 1.16

method	result
risch	$-\frac{e^{3dx+3c}}{24bd}+\frac{3\,e^{dx+c}}{8bd}+\frac{3\,e^{-dx-c}}{8bd}-\frac{e^{-3dx-3c}}{24bd}+\left(\sum_{\_R=\text{RootOf}\left(\left(256a\,b^{7}d^{4}-256b^{8}d^{4}\right)\_Z^{4}+32a^{2}b^{4}d^{2}\_Z^{2}-a^{4}\right.}\right.$
derivativedivides	$\frac{\frac{8a^{2}\left(-\frac{\sqrt{ab}\,\arctan\left(\frac{-2\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}a+4\sqrt{ab}+2a}{4\sqrt{-ab-\sqrt{ab}\,a}}\right)}{16ab\sqrt{-ab-\sqrt{ab}\,a}}-\frac{\sqrt{ab}\,\arctan\left(\frac{2\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}a+4\sqrt{ab}-2a}{4\sqrt{-ab+\sqrt{ab}\,a}}\right)}{16ab\sqrt{-ab+\sqrt{ab}\,a}}\right)}{b}+\frac{1}{3b\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)^{3}}}{d}$
default	$\frac{\frac{8a^{2}\left(-\frac{\sqrt{ab}\,\arctan\left(\frac{-2\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}a+4\sqrt{ab}+2a}{4\sqrt{-ab-\sqrt{ab}\,a}}\right)}{16ab\sqrt{-ab-\sqrt{ab}\,a}}-\frac{\sqrt{ab}\,\arctan\left(\frac{2\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}a+4\sqrt{ab}-2a}{4\sqrt{-ab+\sqrt{ab}\,a}}\right)}{16ab\sqrt{-ab+\sqrt{ab}\,a}}\right)}{b}+\frac{1}{3b\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)^{3}}}{d}$

input
```
int(sinh(d*x+c)^7/(a-b*sinh(d*x+c)^4),x,method=_RETURNVERBOSE)
```

output
```
-1/24/b/d*exp(3*d*x+3*c)+3/8/b/d*exp(d*x+c)+3/8/b/d*exp(-d*x-c)-1/24/b/d*e
xp(-3*d*x-3*c)+sum(_R*ln(exp(2*d*x+2*c)+((128/a^2*b^5*d^3-128/a^3*b^6*d^3)
*_R^3+16/a*d*b^2*_R)*exp(d*x+c)+1),_R=RootOf((256*a*b^7*d^4-256*b^8*d^4)*_
Z^4+32*a^2*b^4*d^2*_Z^2-a^4))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1617 vs. $2(110) = 220$.

Time = 0.13 (sec) , antiderivative size = 1617, normalized size of antiderivative = 10.93

$$\int \frac{\sinh^7(c+dx)}{a-b\sinh^4(c+dx)}\,dx = \text{Too large to display}$$

input
```
integrate(sinh(d*x+c)^7/(a-b*sinh(d*x+c)^4),x, algorithm="fricas")
```

output
```
-1/24*(cosh(d*x + c)^6 + 6*cosh(d*x + c)*sinh(d*x + c)^5 + sinh(d*x + c)^6
 + 3*(5*cosh(d*x + c)^2 - 3)*sinh(d*x + c)^4 - 9*cosh(d*x + c)^4 + 4*(5*co
sh(d*x + c)^3 - 9*cosh(d*x + c))*sinh(d*x + c)^3 + 3*(5*cosh(d*x + c)^4 -
18*cosh(d*x + c)^2 - 3)*sinh(d*x + c)^2 - 6*(b*d*cosh(d*x + c)^3 + 3*b*d*c
osh(d*x + c)^2*sinh(d*x + c) + 3*b*d*cosh(d*x + c)*sinh(d*x + c)^2 + b*d*s
inh(d*x + c)^3)*sqrt(-((a*b^3 - b^4)*d^2*sqrt(a^5/((a^2*b^7 - 2*a*b^8 + b^
9)*d^4)) + a^2)/((a*b^3 - b^4)*d^2))*log(a^3*cosh(d*x + c)^2 + 2*a^3*cosh(
d*x + c)*sinh(d*x + c) + a^3*sinh(d*x + c)^2 + a^3 + 2*(a^2*b^2*d*cosh(d*x
 + c) + a^2*b^2*d*sinh(d*x + c) - ((a*b^5 - b^6)*d^3*cosh(d*x + c) + (a*b^
5 - b^6)*d^3*sinh(d*x + c))*sqrt(a^5/((a^2*b^7 - 2*a*b^8 + b^9)*d^4)))*sqr
t(-((a*b^3 - b^4)*d^2*sqrt(a^5/((a^2*b^7 - 2*a*b^8 + b^9)*d^4)) + a^2)/((a
*b^3 - b^4)*d^2))) + 6*(b*d*cosh(d*x + c)^3 + 3*b*d*cosh(d*x + c)^2*sinh(d
*x + c) + 3*b*d*cosh(d*x + c)*sinh(d*x + c)^2 + b*d*sinh(d*x + c)^3)*sqrt(
-((a*b^3 - b^4)*d^2*sqrt(a^5/((a^2*b^7 - 2*a*b^8 + b^9)*d^4)) + a^2)/((a*b
^3 - b^4)*d^2))*log(a^3*cosh(d*x + c)^2 + 2*a^3*cosh(d*x + c)*sinh(d*x + c
) + a^3*sinh(d*x + c)^2 + a^3 - 2*(a^2*b^2*d*cosh(d*x + c) + a^2*b^2*d*sin
h(d*x + c) - ((a*b^5 - b^6)*d^3*cosh(d*x + c) + (a*b^5 - b^6)*d^3*sinh(d*x
 + c))*sqrt(a^5/((a^2*b^7 - 2*a*b^8 + b^9)*d^4)))*sqrt(-((a*b^3 - b^4)*d^2
*sqrt(a^5/((a^2*b^7 - 2*a*b^8 + b^9)*d^4)) + a^2)/((a*b^3 - b^4)*d^2))) -
6*(b*d*cosh(d*x + c)^3 + 3*b*d*cosh(d*x + c)^2*sinh(d*x + c) + 3*b*d*co...
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\sinh^7(c+dx)}{a-b\sinh^4(c+dx)}\,dx = \text{Timed out}$$

input
```
integrate(sinh(d*x+c)**7/(a-b*sinh(d*x+c)**4),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{\sinh^7(c+dx)}{a-b\sinh^4(c+dx)}\,dx = \int -\frac{\sinh\left(dx+c\right)^7}{b\sinh\left(dx+c\right)^4-a}\,dx$$

input
```
integrate(sinh(d*x+c)^7/(a-b*sinh(d*x+c)^4),x, algorithm="maxima")
```

output
```
-1/24*(e^(6*d*x + 6*c) - 9*e^(4*d*x + 4*c) - 9*e^(2*d*x + 2*c) + 1)*e^(-3*
d*x - 3*c)/(b*d) - 1/128*integrate(256*(a*e^(7*d*x + 7*c) - 3*a*e^(5*d*x +
 5*c) + 3*a*e^(3*d*x + 3*c) - a*e^(d*x + c))/(b^2*e^(8*d*x + 8*c) - 4*b^2*
e^(6*d*x + 6*c) - 4*b^2*e^(2*d*x + 2*c) + b^2 - 2*(8*a*b*e^(4*c) - 3*b^2*e
^(4*c))*e^(4*d*x)), x)
```

Giac [F]

$$\int \frac{\sinh^7(c+dx)}{a-b\sinh^4(c+dx)}\,dx = \int -\frac{\sinh\left(dx+c\right)^7}{b\sinh\left(dx+c\right)^4-a}\,dx$$

input
```
integrate(sinh(d*x+c)^7/(a-b*sinh(d*x+c)^4),x, algorithm="giac")
```

output
```
sage0*x
```

Mupad [B] (verification not implemented)

Time = 10.66 (sec) , antiderivative size = 1124, normalized size of antiderivative = 7.59

$$\int \frac{\sinh^7(c+dx)}{a-b\sinh^4(c+dx)}\,dx = \text{Too large to display}$$

input
```
int(sinh(c + d*x)^7/(a - b*sinh(c + d*x)^4),x)
```

output
```
log(((((4194304*a^8*d^2*(exp(2*c + 2*d*x) + 1)*(3*a + b))/(b^11*(a - b)^2
) - (8388608*a^7*d^3*exp(c + d*x)*(a + b)*(-((a^5*b^7)^(1/2) + a^2*b^4)/(b
^7*d^2*(a - b)))^(1/2))/(b^10*(a - b)))*(-((a^5*b^7)^(1/2) + a^2*b^4)/(b^7
*d^2*(a - b)))^(1/2))/4 + (2097152*a^9*d*exp(c + d*x))/(b^13*(a - b)))*(-(
(a^5*b^7)^(1/2) + a^2*b^4)/(b^7*d^2*(a - b)))^(1/2))/4 - (262144*a^10*(exp
(2*c + 2*d*x) + 1)*(a + b))/(b^15*(a - b)^2))*(((a^5*b^7)^(1/2) + a^2*b^4)
/(16*(b^8*d^2 - a*b^7*d^2)))^(1/2) - log(((((4194304*a^8*d^2*(exp(2*c + 2
*d*x) + 1)*(3*a + b))/(b^11*(a - b)^2) + (8388608*a^7*d^3*exp(c + d*x)*(a
+ b)*(-((a^5*b^7)^(1/2) + a^2*b^4)/(b^7*d^2*(a - b)))^(1/2))/(b^10*(a - b)
))*(-((a^5*b^7)^(1/2) + a^2*b^4)/(b^7*d^2*(a - b)))^(1/2))/4 - (2097152*a^
9*d*exp(c + d*x))/(b^13*(a - b)))*(-((a^5*b^7)^(1/2) + a^2*b^4)/(b^7*d^2*(
a - b)))^(1/2))/4 - (262144*a^10*(exp(2*c + 2*d*x) + 1)*(a + b))/(b^15*(a
- b)^2))*(((a^5*b^7)^(1/2) + a^2*b^4)/(16*(b^8*d^2 - a*b^7*d^2)))^(1/2) +
log(((((4194304*a^8*d^2*(exp(2*c + 2*d*x) + 1)*(3*a + b))/(b^11*(a - b)^2
) - (8388608*a^7*d^3*exp(c + d*x)*(a + b)*(((a^5*b^7)^(1/2) - a^2*b^4)/(b^
7*d^2*(a - b)))^(1/2))/(b^10*(a - b)))*(((a^5*b^7)^(1/2) - a^2*b^4)/(b^7*d
^2*(a - b)))^(1/2))/4 + (2097152*a^9*d*exp(c + d*x))/(b^13*(a - b)))*(((a^
5*b^7)^(1/2) - a^2*b^4)/(b^7*d^2*(a - b)))^(1/2))/4 - (262144*a^10*(exp(2*
c + 2*d*x) + 1)*(a + b))/(b^15*(a - b)^2))*(-((a^5*b^7)^(1/2) - a^2*b^4)/(
16*(b^8*d^2 - a*b^7*d^2)))^(1/2) - log(((((4194304*a^8*d^2*(exp(2*c + ...
```

Reduce [F]

$$\int \frac{\sinh^7(c+dx)}{a-b\sinh^4(c+dx)}\,dx = \frac{-e^{6dx+6c}b+9e^{4dx+4c}b-768e^{3dx+6c}\left(\int \frac{e^{3dx}}{e^{8dx+8c}b-4e^{6dx+6c}b-16e^{4dx+4c}a+6e^{4dx+4c}b-4e^{2dx+2c}b+b}dx\right)a^2d-48e^{3dx+6c}}{}$$

input
```
int(sinh(d*x+c)^7/(a-b*sinh(d*x+c)^4),x)
```

output
```
( - e**(6*c + 6*d*x)*b + 9*e**(4*c + 4*d*x)*b - 768*e**(6*c + 3*d*x)*int(e
**(3*d*x)/(e**(8*c + 8*d*x)*b - 4*e**(6*c + 6*d*x)*b - 16*e**(4*c + 4*d*x)
*a + 6*e**(4*c + 4*d*x)*b - 4*e**(2*c + 2*d*x)*b + b),x)*a**2*d - 48*e**(6
*c + 3*d*x)*int(e**(3*d*x)/(e**(8*c + 8*d*x)*b - 4*e**(6*c + 6*d*x)*b - 16
*e**(4*c + 4*d*x)*a + 6*e**(4*c + 4*d*x)*b - 4*e**(2*c + 2*d*x)*b + b),x)*
a*b*d - 768*e**(4*c + 3*d*x)*int(e**(d*x)/(e**(8*c + 8*d*x)*b - 4*e**(6*c
+ 6*d*x)*b - 16*e**(4*c + 4*d*x)*a + 6*e**(4*c + 4*d*x)*b - 4*e**(2*c + 2*
d*x)*b + b),x)*a**2*d + 144*e**(4*c + 3*d*x)*int(e**(d*x)/(e**(8*c + 8*d*x
)*b - 4*e**(6*c + 6*d*x)*b - 16*e**(4*c + 4*d*x)*a + 6*e**(4*c + 4*d*x)*b
- 4*e**(2*c + 2*d*x)*b + b),x)*a*b*d + 48*e**(3*c + 3*d*x)*int(1/(e**(11*c
 + 11*d*x)*b - 4*e**(9*c + 9*d*x)*b - 16*e**(7*c + 7*d*x)*a + 6*e**(7*c +
7*d*x)*b - 4*e**(5*c + 5*d*x)*b + e**(3*c + 3*d*x)*b),x)*a*b*d - 144*e**(2
*c + 3*d*x)*int(1/(e**(8*c + 9*d*x)*b - 4*e**(6*c + 7*d*x)*b - 16*e**(4*c
+ 5*d*x)*a + 6*e**(4*c + 5*d*x)*b - 4*e**(2*c + 3*d*x)*b + e**(d*x)*b),x)*
a*b*d + 48*e**(2*c + 2*d*x)*a + 9*e**(2*c + 2*d*x)*b + 16*a - b)/(24*e**(3
*c + 3*d*x)*b**2*d)
```

3.205 $\int \frac{\sinh^5(c+dx)}{a-b\sinh^4(c+dx)}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 139

$$\int \frac{\sinh^5(c+dx)}{a-b\sinh^4(c+dx)}\,dx = \frac{\sqrt{a}\arctan\left(\frac{\sqrt[4]{b}\cosh(c+dx)}{\sqrt{\sqrt{a}-\sqrt{b}}}\right)}{2\sqrt{\sqrt{a}-\sqrt{b}}\,b^{5/4}d} + \frac{\sqrt{a}\,\text{arctanh}\left(\frac{\sqrt[4]{b}\cosh(c+dx)}{\sqrt{\sqrt{a}+\sqrt{b}}}\right)}{2\sqrt{\sqrt{a}+\sqrt{b}}\,b^{5/4}d} - \frac{\cosh(c+dx)}{bd}$$

output
```
1/2*a^(1/2)*arctan(b^(1/4)*cosh(d*x+c)/(a^(1/2)-b^(1/2))^(1/2))/(a^(1/2)-b
^(1/2))^(1/2)/b^(5/4)/d+1/2*a^(1/2)*arctanh(b^(1/4)*cosh(d*x+c)/(a^(1/2)+b
^(1/2))^(1/2))/(a^(1/2)+b^(1/2))^(1/2)/b^(5/4)/d-cosh(d*x+c)/b/d
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

Time = 1.44 (sec) , antiderivative size = 235, normalized size of antiderivative = 1.69

$$\int \frac{\sinh^5(c+dx)}{a-b\sinh^4(c+dx)}\,dx =$$
$$-\frac{2\cosh(c+dx)+a\text{RootSum}\left[b-4b\#1^2-16a\#1^4+6b\#1^4-4b\#1^6+b\#1^8\&,\frac{-c\#1-dx\#1-2\log\left(-\cos\right.}{}\right.}{}$$

input
```
Integrate[Sinh[c + d*x]^5/(a - b*Sinh[c + d*x]^4),x]
```

output
```
-1/2*(2*Cosh[c + d*x] + a*RootSum[b - 4*b*#1^2 - 16*a*#1^4 + 6*b*#1^4 - 4*
b*#1^6 + b*#1^8 & , (-(c*#1) - d*x*#1 - 2*Log[-Cosh[(c + d*x)/2] - Sinh[(c
 + d*x)/2] + Cosh[(c + d*x)/2]*#1 - Sinh[(c + d*x)/2]*#1]*#1 + c*#1^3 + d*
x*#1^3 + 2*Log[-Cosh[(c + d*x)/2] - Sinh[(c + d*x)/2] + Cosh[(c + d*x)/2]*
#1 - Sinh[(c + d*x)/2]*#1]*#1^3)/(-b - 8*a*#1^2 + 3*b*#1^2 - 3*b*#1^4 + b*
#1^6) & ])/(b*d)
```

Rubi [A] (verified)

Time = 0.37 (sec) , antiderivative size = 134, normalized size of antiderivative = 0.96, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.208$, Rules used = {3042, 26, 3694, 1484, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sinh^5(c+dx)}{a-b\sinh^4(c+dx)}dx$$
$$\Big\downarrow \text{3042}$$
$$\int -\frac{i\sin(ic+idx)^5}{a-b\sin(ic+idx)^4}dx$$
$$\Big\downarrow \text{26}$$

$$-i\int \frac{\sin(ic+idx)^5}{a-b\sin(ic+idx)^4}dx$$

$$\Big\downarrow \text{3694}$$

$$\frac{\int \frac{\left(1-\cosh^2(c+dx)\right)^2}{-b\cosh^4(c+dx)+2b\cosh^2(c+dx)+a-b}d\cosh(c+dx)}{d}$$

$$\Big\downarrow \text{1484}$$

$$\frac{\int \left(\frac{a}{b\left(-b\cosh^4(c+dx)+2b\cosh^2(c+dx)+a-b\right)}-\frac{1}{b}\right)d\cosh(c+dx)}{d}$$

$$\Big\downarrow \text{2009}$$

$$\frac{\frac{\sqrt{a}\arctan\left(\frac{\sqrt[4]{b}\cosh(c+dx)}{\sqrt{\sqrt{a}-\sqrt{b}}}\right)}{2b^{5/4}\sqrt{\sqrt{a}-\sqrt{b}}}+\frac{\sqrt{a}\text{arctanh}\left(\frac{\sqrt[4]{b}\cosh(c+dx)}{\sqrt{\sqrt{a}+\sqrt{b}}}\right)}{2b^{5/4}\sqrt{\sqrt{a}+\sqrt{b}}}-\frac{\cosh(c+dx)}{b}}{d}$$

input
```
Int[Sinh[c + d*x]^5/(a - b*Sinh[c + d*x]^4),x]
```

output
```
((Sqrt[a]*ArcTan[(b^(1/4)*Cosh[c + d*x])/Sqrt[Sqrt[a] - Sqrt[b]]])/(2*Sqrt
[Sqrt[a] - Sqrt[b]]*b^(5/4)) + (Sqrt[a]*ArcTanh[(b^(1/4)*Cosh[c + d*x])/Sq
rt[Sqrt[a] + Sqrt[b]]])/(2*Sqrt[Sqrt[a] + Sqrt[b]]*b^(5/4)) - Cosh[c + d*x
]/b)/d
```

Defintions of rubi rules used

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 1484
```
Int[((d_) + (e_.)*(x_)^2)^(q_)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symb
ol] :> Int[ExpandIntegrand[(d + e*x^2)^q/(a + b*x^2 + c*x^4), x], x] /; Fre
eQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2,
 0] && IntegerQ[q]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3694
```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^4)^
(p_.), x_Symbol] :> With[{ff = FreeFactors[Cos[e + f*x], x]}, Simp[-ff/f
Subst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a + b - 2*b*ff^2*x^2 + b*ff^4*x^4)^p,
 x], x, Cos[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m -
 1)/2]
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 1.56 (sec) , antiderivative size = 142, normalized size of antiderivative = 1.02

method	result
risch	$-\frac{e^{dx+c}}{2bd}-\frac{e^{-dx-c}}{2bd}+\left(\sum_{\_R=\operatorname{RootOf}\left(\left(256a\,b^{5}d^{4}-256b^{6}d^{4}\right)\_Z^{4}+32a\,d^{2}\_Z^{2}b^{3}-a^{2}\right)}\_R\ln\left(e^{2dx+2c}+\left(\left(\right.\right.\right.\right.$
derivativedivides	$\frac{\frac{2a^{2}\left(\frac{\arctan\left(\frac{2\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}a+4\sqrt{ab}-2a}{4\sqrt{-ab+\sqrt{ab}\,a}}\right)}{4a\sqrt{-ab+\sqrt{ab}\,a}}-\frac{\arctan\left(\frac{-2\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}a+4\sqrt{ab}+2a}{4\sqrt{-ab-\sqrt{ab}\,a}}\right)}{4a\sqrt{-ab-\sqrt{ab}\,a}}\right)}{b}-\frac{1}{b\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)}+\frac{1}{b\left(\tanh\left(\frac{dx}{2}\right.\right.}}{d}$
default	$\frac{\frac{2a^{2}\left(\frac{\arctan\left(\frac{2\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}a+4\sqrt{ab}-2a}{4\sqrt{-ab+\sqrt{ab}\,a}}\right)}{4a\sqrt{-ab+\sqrt{ab}\,a}}-\frac{\arctan\left(\frac{-2\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}a+4\sqrt{ab}+2a}{4\sqrt{-ab-\sqrt{ab}\,a}}\right)}{4a\sqrt{-ab-\sqrt{ab}\,a}}\right)}{b}-\frac{1}{b\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)}+\frac{1}{b\left(\tanh\left(\frac{dx}{2}\right.\right.}}{d}$

input
```
int(sinh(d*x+c)^5/(a-b*sinh(d*x+c)^4),x,method=_RETURNVERBOSE)
```

output
```
-1/2/b/d*exp(d*x+c)-1/2/b/d*exp(-d*x-c)+sum(_R*ln(exp(2*d*x+2*c)+((128/a*b
^4*d^3-128/a^2*b^5*d^3)*_R^3+(8*b*d+8/a*b^2*d)*_R)*exp(d*x+c)+1),_R=RootOf
((256*a*b^5*d^4-256*b^6*d^4)*_Z^4+32*a*d^2*_Z^2*b^3-a^2))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1247 vs. $2(99) = 198$.

Time = 0.16 (sec) , antiderivative size = 1247, normalized size of antiderivative = 8.97

$$\int \frac{\sinh^5(c+dx)}{a-b\sinh^4(c+dx)}\,dx = \text{Too large to display}$$

input
```
integrate(sinh(d*x+c)^5/(a-b*sinh(d*x+c)^4),x, algorithm="fricas")
```

output
```
1/4*((b*d*cosh(d*x + c) + b*d*sinh(d*x + c))*sqrt(-((a*b^2 - b^3)*d^2*sqrt
(a^3/((a^2*b^5 - 2*a*b^6 + b^7)*d^4)) + a)/((a*b^2 - b^3)*d^2))*log(a^2*co
sh(d*x + c)^2 + 2*a^2*cosh(d*x + c)*sinh(d*x + c) + a^2*sinh(d*x + c)^2 +
a^2 + 2*(a^2*b*d*cosh(d*x + c) + a^2*b*d*sinh(d*x + c) - ((a*b^4 - b^5)*d^
3*cosh(d*x + c) + (a*b^4 - b^5)*d^3*sinh(d*x + c))*sqrt(a^3/((a^2*b^5 - 2*
a*b^6 + b^7)*d^4)))*sqrt(-((a*b^2 - b^3)*d^2*sqrt(a^3/((a^2*b^5 - 2*a*b^6
+ b^7)*d^4)) + a)/((a*b^2 - b^3)*d^2))) - (b*d*cosh(d*x + c) + b*d*sinh(d*
x + c))*sqrt(-((a*b^2 - b^3)*d^2*sqrt(a^3/((a^2*b^5 - 2*a*b^6 + b^7)*d^4))
 + a)/((a*b^2 - b^3)*d^2))*log(a^2*cosh(d*x + c)^2 + 2*a^2*cosh(d*x + c)*s
inh(d*x + c) + a^2*sinh(d*x + c)^2 + a^2 - 2*(a^2*b*d*cosh(d*x + c) + a^2*
b*d*sinh(d*x + c) - ((a*b^4 - b^5)*d^3*cosh(d*x + c) + (a*b^4 - b^5)*d^3*s
inh(d*x + c))*sqrt(a^3/((a^2*b^5 - 2*a*b^6 + b^7)*d^4)))*sqrt(-((a*b^2 - b
^3)*d^2*sqrt(a^3/((a^2*b^5 - 2*a*b^6 + b^7)*d^4)) + a)/((a*b^2 - b^3)*d^2)
)) + (b*d*cosh(d*x + c) + b*d*sinh(d*x + c))*sqrt(((a*b^2 - b^3)*d^2*sqrt(
a^3/((a^2*b^5 - 2*a*b^6 + b^7)*d^4)) - a)/((a*b^2 - b^3)*d^2))*log(a^2*cos
h(d*x + c)^2 + 2*a^2*cosh(d*x + c)*sinh(d*x + c) + a^2*sinh(d*x + c)^2 + a
^2 + 2*(a^2*b*d*cosh(d*x + c) + a^2*b*d*sinh(d*x + c) + ((a*b^4 - b^5)*d^3
*cosh(d*x + c) + (a*b^4 - b^5)*d^3*sinh(d*x + c))*sqrt(a^3/((a^2*b^5 - 2*a
*b^6 + b^7)*d^4)))*sqrt(((a*b^2 - b^3)*d^2*sqrt(a^3/((a^2*b^5 - 2*a*b^6 +
b^7)*d^4)) - a)/((a*b^2 - b^3)*d^2))) - (b*d*cosh(d*x + c) + b*d*sinh(d...
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\sinh^5(c+dx)}{a-b\sinh^4(c+dx)}\,dx = \text{Timed out}$$

input
```
integrate(sinh(d*x+c)**5/(a-b*sinh(d*x+c)**4),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{\sinh^5(c+dx)}{a-b\sinh^4(c+dx)}\,dx = \int -\frac{\sinh(dx+c)^5}{b\sinh(dx+c)^4-a}\,dx$$

input
```
integrate(sinh(d*x+c)^5/(a-b*sinh(d*x+c)^4),x, algorithm="maxima")
```

output
```
-1/2*(e^(2*d*x + 2*c) + 1)*e^(-d*x - c)/(b*d) - 1/32*integrate(256*(a*e^(5
*d*x + 5*c) - a*e^(3*d*x + 3*c))/(b^2*e^(8*d*x + 8*c) - 4*b^2*e^(6*d*x + 6
*c) - 4*b^2*e^(2*d*x + 2*c) + b^2 - 2*(8*a*b*e^(4*c) - 3*b^2*e^(4*c))*e^(4
*d*x)), x)
```

Giac [F]

$$\int \frac{\sinh^5(c+dx)}{a-b\sinh^4(c+dx)}\,dx = \int -\frac{\sinh(dx+c)^5}{b\sinh(dx+c)^4-a}\,dx$$

input
```
integrate(sinh(d*x+c)^5/(a-b*sinh(d*x+c)^4),x, algorithm="giac")
```

output
```
sage0*x
```

Mupad [B] (verification not implemented)

Time = 8.88 (sec) , antiderivative size = 1046, normalized size of antiderivative = 7.53

$$\int \frac{\sinh^5(c+dx)}{a-b\sinh^4(c+dx)}\,dx = \text{Too large to display}$$

input
```
int(sinh(c + d*x)^5/(a - b*sinh(c + d*x)^4),x)
```

output
```
log((((((4194304*a^6*d^2*(exp(2*c + 2*d*x) + 1)*(3*a + b))/(b^9*(a - b)^2)
 + (16777216*a^6*d^3*exp(c + d*x)*(-((a^3*b^5)^(1/2) + a*b^3)/(b^5*d^2*(a
- b)))^(1/2))/(b^8*(a - b)))*(-((a^3*b^5)^(1/2) + a*b^3)/(b^5*d^2*(a - b))
)^(1/2))/4 - (2097152*a^7*d*exp(c + d*x))/(b^11*(a - b)))*(-((a^3*b^5)^(1/
2) + a*b^3)/(b^5*d^2*(a - b)))^(1/2))/4 - (262144*a^7*(exp(2*c + 2*d*x) +
1)*(a + b))/(b^12*(a - b)^2))*(((a^3*b^5)^(1/2) + a*b^3)/(16*(b^6*d^2 - a*
b^5*d^2)))^(1/2) - log((((((4194304*a^6*d^2*(exp(2*c + 2*d*x) + 1)*(3*a +
b))/(b^9*(a - b)^2) - (16777216*a^6*d^3*exp(c + d*x)*(-((a^3*b^5)^(1/2) +
a*b^3)/(b^5*d^2*(a - b)))^(1/2))/(b^8*(a - b)))*(-((a^3*b^5)^(1/2) + a*b^3
)/(b^5*d^2*(a - b)))^(1/2))/4 + (2097152*a^7*d*exp(c + d*x))/(b^11*(a - b)
))*(-((a^3*b^5)^(1/2) + a*b^3)/(b^5*d^2*(a - b)))^(1/2))/4 - (262144*a^7*(
exp(2*c + 2*d*x) + 1)*(a + b))/(b^12*(a - b)^2))*(((a^3*b^5)^(1/2) + a*b^3
)/(16*(b^6*d^2 - a*b^5*d^2)))^(1/2) - log((((((4194304*a^6*d^2*(exp(2*c +
2*d*x) + 1)*(3*a + b))/(b^9*(a - b)^2) - (16777216*a^6*d^3*exp(c + d*x)*((
(a^3*b^5)^(1/2) - a*b^3)/(b^5*d^2*(a - b)))^(1/2))/(b^8*(a - b)))*(((a^3*b
^5)^(1/2) - a*b^3)/(b^5*d^2*(a - b)))^(1/2))/4 + (2097152*a^7*d*exp(c + d*
x))/(b^11*(a - b)))*(((a^3*b^5)^(1/2) - a*b^3)/(b^5*d^2*(a - b)))^(1/2))/4
 - (262144*a^7*(exp(2*c + 2*d*x) + 1)*(a + b))/(b^12*(a - b)^2))*(-((a^3*b
^5)^(1/2) - a*b^3)/(16*(b^6*d^2 - a*b^5*d^2)))^(1/2) + log(((((4194304*a^
6*d^2*(exp(2*c + 2*d*x) + 1)*(3*a + b))/(b^9*(a - b)^2) + (16777216*a^6...
```

Reduce [F]

$$\int \frac{\sinh^5(c+dx)}{a-b\sinh^4(c+dx)}\,dx = \frac{-\cosh(dx+c) - \left(\int \frac{\sinh(dx+c)}{\sinh(dx+c)^4 b-a}dx\right)ad}{bd}$$

input
```
int(sinh(d*x+c)^5/(a-b*sinh(d*x+c)^4),x)
```

output
```
( - (cosh(c + d*x) + int(sinh(c + d*x)/(sinh(c + d*x)**4*b - a),x)*a*d))/(
b*d)
```

3.206 $\int \frac{\sinh^3(c+dx)}{a-b\sinh^4(c+dx)}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 115

$$\int \frac{\sinh^3(c+dx)}{a-b\sinh^4(c+dx)}\,dx = -\frac{\arctan\left(\frac{\sqrt[4]{b}\cosh(c+dx)}{\sqrt{\sqrt{a}-\sqrt{b}}}\right)}{2\sqrt{\sqrt{a}-\sqrt{b}}\,b^{3/4}d} + \frac{\operatorname{arctanh}\left(\frac{\sqrt[4]{b}\cosh(c+dx)}{\sqrt{\sqrt{a}+\sqrt{b}}}\right)}{2\sqrt{\sqrt{a}+\sqrt{b}}\,b^{3/4}d}$$

output
```
-1/2*arctan(b^(1/4)*cosh(d*x+c)/(a^(1/2)-b^(1/2))^(1/2))/(a^(1/2)-b^(1/2))
^(1/2)/b^(3/4)/d+1/2*arctanh(b^(1/4)*cosh(d*x+c)/(a^(1/2)+b^(1/2))^(1/2))/
(a^(1/2)+b^(1/2))^(1/2)/b^(3/4)/d
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

Time = 6.58 (sec) , antiderivative size = 365, normalized size of antiderivative = 3.17

$$\int \frac{\sinh^3(c+dx)}{a-b\sinh^4(c+dx)}\,dx =$$

$$-\frac{\text{RootSum}\left[b-4b\#1^2-16a\#1^4+6b\#1^4-4b\#1^6+b\#1^8\&, \frac{-c-dx-2\log\left(-\cosh\left(\frac{1}{2}(c+dx)\right)-\sinh\left(\frac{1}{2}(c+dx)\right)+\cos\right.}{}\right.}{}$$

input
```
Integrate[Sinh[c + d*x]^3/(a - b*Sinh[c + d*x]^4),x]
```

output
```
-1/8*RootSum[b - 4*b*#1^2 - 16*a*#1^4 + 6*b*#1^4 - 4*b*#1^6 + b*#1^8 & , (
-c - d*x - 2*Log[-Cosh[(c + d*x)/2] - Sinh[(c + d*x)/2] + Cosh[(c + d*x)/2
]*#1 - Sinh[(c + d*x)/2]*#1] + 3*c*#1^2 + 3*d*x*#1^2 + 6*Log[-Cosh[(c + d*
x)/2] - Sinh[(c + d*x)/2] + Cosh[(c + d*x)/2]*#1 - Sinh[(c + d*x)/2]*#1]*#
1^2 - 3*c*#1^4 - 3*d*x*#1^4 - 6*Log[-Cosh[(c + d*x)/2] - Sinh[(c + d*x)/2]
 + Cosh[(c + d*x)/2]*#1 - Sinh[(c + d*x)/2]*#1]*#1^4 + c*#1^6 + d*x*#1^6 +
 2*Log[-Cosh[(c + d*x)/2] - Sinh[(c + d*x)/2] + Cosh[(c + d*x)/2]*#1 - Sin
h[(c + d*x)/2]*#1]*#1^6)/(-(b*#1) - 8*a*#1^3 + 3*b*#1^3 - 3*b*#1^5 + b*#1^
7) & ]/d
```

Rubi [A] (verified)

Time = 0.31 (sec) , antiderivative size = 114, normalized size of antiderivative = 0.99, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {3042, 26, 3694, 1480, 218, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sinh^3(c+dx)}{a-b\sinh^4(c+dx)}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{i\sin(ic+idx)^3}{a-b\sin(ic+idx)^4}dx$$

$$\Big\downarrow \text{26}$$

$$i\int \frac{\sin(ic+idx)^3}{a-b\sin(ic+idx)^4}dx$$

$$\Big\downarrow \text{3694}$$

$$-\frac{\int \frac{1-\cosh^2(c+dx)}{-b\cosh^4(c+dx)+2b\cosh^2(c+dx)+a-b}d\cosh(c+dx)}{d}$$

$$\Big\downarrow \text{1480}$$

$$-\frac{-\frac{1}{2}\int \frac{1}{-b\cosh^2(c+dx)-\left(\sqrt{a}-\sqrt{b}\right)\sqrt{b}}d\cosh(c+dx)-\frac{1}{2}\int \frac{1}{\left(\sqrt{a}+\sqrt{b}\right)\sqrt{b}-b\cosh^2(c+dx)}d\cosh(c+dx)}{d}$$

$$\Big\downarrow \; 218$$

$$-\frac{\frac{\arctan\left(\frac{\sqrt[4]{b}\cosh(c+dx)}{\sqrt{\sqrt{a}-\sqrt{b}}}\right)}{2b^{3/4}\sqrt{\sqrt{a}-\sqrt{b}}}-\frac{1}{2}\int \frac{1}{\left(\sqrt{a}+\sqrt{b}\right)\sqrt{b}-b\cosh^2(c+dx)}d\cosh(c+dx)}{d}$$

$$\Big\downarrow \; 221$$

$$-\frac{\frac{\arctan\left(\frac{\sqrt[4]{b}\cosh(c+dx)}{\sqrt{\sqrt{a}-\sqrt{b}}}\right)}{2b^{3/4}\sqrt{\sqrt{a}-\sqrt{b}}}-\frac{\text{arctanh}\left(\frac{\sqrt[4]{b}\cosh(c+dx)}{\sqrt{\sqrt{a}+\sqrt{b}}}\right)}{2b^{3/4}\sqrt{\sqrt{a}+\sqrt{b}}}}{d}$$

input
```
Int[Sinh[c + d*x]^3/(a - b*Sinh[c + d*x]^4),x]
```

output
```
-((ArcTan[(b^(1/4)*Cosh[c + d*x])/Sqrt[Sqrt[a] - Sqrt[b]]]/(2*Sqrt[Sqrt[a]
 - Sqrt[b]]*b^(3/4)) - ArcTanh[(b^(1/4)*Cosh[c + d*x])/Sqrt[Sqrt[a] + Sqrt
[b]]]/(2*Sqrt[Sqrt[a] + Sqrt[b]]*b^(3/4)))/d)
```

Defintions of rubi rules used

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 1480
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Simp[(e/2 + (2*c*d - b*e)/(2*q))   Int[1/(
b/2 - q/2 + c*x^2), x], x] + Simp[(e/2 - (2*c*d - b*e)/(2*q))   Int[1/(b/2
+ q/2 + c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0]
 && NeQ[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3694
```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^4)^
(p_.), x_Symbol] :> With[{ff = FreeFactors[Cos[e + f*x], x]}, Simp[-ff/f
Subst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a + b - 2*b*ff^2*x^2 + b*ff^4*x^4)^p,
 x], x, Cos[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m -
 1)/2]
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.84 (sec) , antiderivative size = 89, normalized size of antiderivative = 0.77

method	result
risch	$\sum_{\_R=\mathrm{RootOf}\left(-1+(256a\,b^3d^4-256b^4d^4)\_Z^4+32d^2\_Z^2b^2\right)}\_R\ln\left(\mathrm{e}^{2dx+2c}+\left((128a\,b^2d^3-128b^3d^3)\_\right.\right.$
derivativedivides	$\frac{8a\left(-\frac{\sqrt{ab}\,\arctan\left(\frac{-2\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^2a+4\sqrt{ab}+2a}{4\sqrt{-ab-\sqrt{ab}\,a}}\right)}{16ab\sqrt{-ab-\sqrt{ab}\,a}}-\frac{\sqrt{ab}\,\arctan\left(\frac{2\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^2a+4\sqrt{ab}-2a}{4\sqrt{-ab+\sqrt{ab}\,a}}\right)}{16ab\sqrt{-ab+\sqrt{ab}\,a}}\right)}{d}$
default	$\frac{8a\left(-\frac{\sqrt{ab}\,\arctan\left(\frac{-2\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^2a+4\sqrt{ab}+2a}{4\sqrt{-ab-\sqrt{ab}\,a}}\right)}{16ab\sqrt{-ab-\sqrt{ab}\,a}}-\frac{\sqrt{ab}\,\arctan\left(\frac{2\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^2a+4\sqrt{ab}-2a}{4\sqrt{-ab+\sqrt{ab}\,a}}\right)}{16ab\sqrt{-ab+\sqrt{ab}\,a}}\right)}{d}$

input
```
int(sinh(d*x+c)^3/(a-b*sinh(d*x+c)^4),x,method=_RETURNVERBOSE)
```

output
```
sum(_R*ln(exp(2*d*x+2*c)+((128*a*b^2*d^3-128*b^3*d^3)*_R^3+16*b*d*_R)*exp(
d*x+c)+1),_R=RootOf(-1+(256*a*b^3*d^4-256*b^4*d^4)*_Z^4+32*d^2*_Z^2*b^2))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 975 vs. $2(79) = 158$.

Time = 0.13 (sec) , antiderivative size = 975, normalized size of antiderivative = 8.48

$$\int \frac{\sinh^3(c+dx)}{a-b\sinh^4(c+dx)}\,dx = \text{Too large to display}$$

input
```
integrate(sinh(d*x+c)^3/(a-b*sinh(d*x+c)^4),x, algorithm="fricas")
```

output
```
1/4*sqrt(-((a*b - b^2)*d^2*sqrt(a/((a^2*b^3 - 2*a*b^4 + b^5)*d^4)) + 1)/((
a*b - b^2)*d^2))*log(cosh(d*x + c)^2 + 2*cosh(d*x + c)*sinh(d*x + c) + sin
h(d*x + c)^2 + 2*(b*d*cosh(d*x + c) + b*d*sinh(d*x + c) - ((a*b^2 - b^3)*d
^3*cosh(d*x + c) + (a*b^2 - b^3)*d^3*sinh(d*x + c))*sqrt(a/((a^2*b^3 - 2*a
*b^4 + b^5)*d^4)))*sqrt(-((a*b - b^2)*d^2*sqrt(a/((a^2*b^3 - 2*a*b^4 + b^5
)*d^4)) + 1)/((a*b - b^2)*d^2)) + 1) - 1/4*sqrt(-((a*b - b^2)*d^2*sqrt(a/(
(a^2*b^3 - 2*a*b^4 + b^5)*d^4)) + 1)/((a*b - b^2)*d^2))*log(cosh(d*x + c)^
2 + 2*cosh(d*x + c)*sinh(d*x + c) + sinh(d*x + c)^2 - 2*(b*d*cosh(d*x + c)
 + b*d*sinh(d*x + c) - ((a*b^2 - b^3)*d^3*cosh(d*x + c) + (a*b^2 - b^3)*d^
3*sinh(d*x + c))*sqrt(a/((a^2*b^3 - 2*a*b^4 + b^5)*d^4)))*sqrt(-((a*b - b^
2)*d^2*sqrt(a/((a^2*b^3 - 2*a*b^4 + b^5)*d^4)) + 1)/((a*b - b^2)*d^2)) + 1
) + 1/4*sqrt(((a*b - b^2)*d^2*sqrt(a/((a^2*b^3 - 2*a*b^4 + b^5)*d^4)) - 1)
/((a*b - b^2)*d^2))*log(cosh(d*x + c)^2 + 2*cosh(d*x + c)*sinh(d*x + c) +
sinh(d*x + c)^2 + 2*(b*d*cosh(d*x + c) + b*d*sinh(d*x + c) + ((a*b^2 - b^3
)*d^3*cosh(d*x + c) + (a*b^2 - b^3)*d^3*sinh(d*x + c))*sqrt(a/((a^2*b^3 -
2*a*b^4 + b^5)*d^4)))*sqrt(((a*b - b^2)*d^2*sqrt(a/((a^2*b^3 - 2*a*b^4 + b
^5)*d^4)) - 1)/((a*b - b^2)*d^2)) + 1) - 1/4*sqrt(((a*b - b^2)*d^2*sqrt(a/
((a^2*b^3 - 2*a*b^4 + b^5)*d^4)) - 1)/((a*b - b^2)*d^2))*log(cosh(d*x + c)
^2 + 2*cosh(d*x + c)*sinh(d*x + c) + sinh(d*x + c)^2 - 2*(b*d*cosh(d*x + c
) + b*d*sinh(d*x + c) + ((a*b^2 - b^3)*d^3*cosh(d*x + c) + (a*b^2 - b^3...
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\sinh^3(c+dx)}{a-b\sinh^4(c+dx)}\,dx = \text{Timed out}$$

input
```
integrate(sinh(d*x+c)**3/(a-b*sinh(d*x+c)**4),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{\sinh^3(c+dx)}{a-b\sinh^4(c+dx)}\,dx = \int -\frac{\sinh(dx+c)^3}{b\sinh(dx+c)^4-a}\,dx$$

input
```
integrate(sinh(d*x+c)^3/(a-b*sinh(d*x+c)^4),x, algorithm="maxima")
```

output
```
-integrate(sinh(d*x + c)^3/(b*sinh(d*x + c)^4 - a), x)
```

Giac [F]

$$\int \frac{\sinh^3(c+dx)}{a-b\sinh^4(c+dx)}\,dx = \int -\frac{\sinh(dx+c)^3}{b\sinh(dx+c)^4-a}\,dx$$

input
```
integrate(sinh(d*x+c)^3/(a-b*sinh(d*x+c)^4),x, algorithm="giac")
```

output
```
sage0*x
```

Mupad [B] (verification not implemented)

Time = 7.01 (sec) , antiderivative size = 975, normalized size of antiderivative = 8.48

$$\int \frac{\sinh^3(c+dx)}{a-b\sinh^4(c+dx)}\,dx = \text{Too large to display}$$

input
```
int(sinh(c + d*x)^3/(a - b*sinh(c + d*x)^4),x)
```

output
```
log((((((4194304*a^4*d^2*(exp(2*c + 2*d*x) + 1)*(3*a + b))/(b^7*(a - b)^2)
 - (8388608*a^4*d^3*exp(c + d*x)*(a + b)*(-(b^2 - (a*b^3)^(1/2))/(b^3*d^2*
(a - b)))^(1/2))/(b^7*(a - b)))*(-(b^2 - (a*b^3)^(1/2))/(b^3*d^2*(a - b)))
^(1/2))/4 + (2097152*a^4*d*exp(c + d*x))/(b^8*(a - b)))*(-(b^2 - (a*b^3)^(
1/2))/(b^3*d^2*(a - b)))^(1/2))/4 - (262144*a^4*(exp(2*c + 2*d*x) + 1)*(a
+ b))/(b^9*(a - b)^2))*((b^2 - (a*b^3)^(1/2))/(16*(b^4*d^2 - a*b^3*d^2)))^
(1/2) - log((((((4194304*a^4*d^2*(exp(2*c + 2*d*x) + 1)*(3*a + b))/(b^7*(a
 - b)^2) + (8388608*a^4*d^3*exp(c + d*x)*(a + b)*(-(b^2 - (a*b^3)^(1/2))/(
b^3*d^2*(a - b)))^(1/2))/(b^7*(a - b)))*(-(b^2 - (a*b^3)^(1/2))/(b^3*d^2*(
a - b)))^(1/2))/4 - (2097152*a^4*d*exp(c + d*x))/(b^8*(a - b)))*(-(b^2 - (
a*b^3)^(1/2))/(b^3*d^2*(a - b)))^(1/2))/4 - (262144*a^4*(exp(2*c + 2*d*x)
+ 1)*(a + b))/(b^9*(a - b)^2))*((b^2 - (a*b^3)^(1/2))/(16*(b^4*d^2 - a*b^3
*d^2)))^(1/2) + log((((((4194304*a^4*d^2*(exp(2*c + 2*d*x) + 1)*(3*a + b))
/(b^7*(a - b)^2) - (8388608*a^4*d^3*exp(c + d*x)*(a + b)*(-(b^2 + (a*b^3)^
(1/2))/(b^3*d^2*(a - b)))^(1/2))/(b^7*(a - b)))*(-(b^2 + (a*b^3)^(1/2))/(b
^3*d^2*(a - b)))^(1/2))/4 + (2097152*a^4*d*exp(c + d*x))/(b^8*(a - b)))*(-
(b^2 + (a*b^3)^(1/2))/(b^3*d^2*(a - b)))^(1/2))/4 - (262144*a^4*(exp(2*c +
 2*d*x) + 1)*(a + b))/(b^9*(a - b)^2))*((b^2 + (a*b^3)^(1/2))/(16*(b^4*d^2
 - a*b^3*d^2)))^(1/2) - log((((((4194304*a^4*d^2*(exp(2*c + 2*d*x) + 1)*(3
*a + b))/(b^7*(a - b)^2) + (8388608*a^4*d^3*exp(c + d*x)*(a + b)*(-(b^2...
```

Reduce [F]

$$\int \frac{\sinh^3(c+dx)}{a-b\sinh^4(c+dx)}\,dx = -\left(\int \frac{\sinh(dx+c)^3}{\sinh(dx+c)^4 b-a}dx\right)$$

input
```
int(sinh(d*x+c)^3/(a-b*sinh(d*x+c)^4),x)
```

output
```
- int(sinh(c + d*x)**3/(sinh(c + d*x)**4*b - a),x)
```

3.207 $\int \frac{\sinh(c+dx)}{a-b\sinh^4(c+dx)}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 125

$$\int \frac{\sinh(c+dx)}{a-b\sinh^4(c+dx)}\,dx = \frac{\arctan\left(\frac{\sqrt[4]{b}\cosh(c+dx)}{\sqrt{\sqrt{a}-\sqrt{b}}}\right)}{2\sqrt{a}\sqrt{\sqrt{a}-\sqrt{b}}\sqrt[4]{b}d}+\frac{\operatorname{arctanh}\left(\frac{\sqrt[4]{b}\cosh(c+dx)}{\sqrt{\sqrt{a}+\sqrt{b}}}\right)}{2\sqrt{a}\sqrt{\sqrt{a}+\sqrt{b}}\sqrt[4]{b}d}$$

output
```
1/2*arctan(b^(1/4)*cosh(d*x+c)/(a^(1/2)-b^(1/2))^(1/2))/a^(1/2)/(a^(1/2)-b
^(1/2))^(1/2)/b^(1/4)/d+1/2*arctanh(b^(1/4)*cosh(d*x+c)/(a^(1/2)+b^(1/2))^
(1/2))/a^(1/2)/(a^(1/2)+b^(1/2))^(1/2)/b^(1/4)/d
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

Time = 4.20 (sec) , antiderivative size = 221, normalized size of antiderivative = 1.77

$$\int \frac{\sinh(c+dx)}{a-b\sinh^4(c+dx)}\,dx =$$

$$-\frac{\text{RootSum}\left[b-4b\#1^2-16a\#1^4+6b\#1^4-4b\#1^6+b\#1^8\&,\frac{-c\#1-dx\#1-2\log\left(-\cosh\left(\frac{1}{2}(c+dx)\right)-\sinh\left(\frac{1}{2}(c+\ldots\right.\right.}{\ldots}\right.}{\ldots}$$

input
```
Integrate[Sinh[c + d*x]/(a - b*Sinh[c + d*x]^4),x]
```

output
```
-1/2*RootSum[b - 4*b*#1^2 - 16*a*#1^4 + 6*b*#1^4 - 4*b*#1^6 + b*#1^8 & , (
-(c*#1) - d*x*#1 - 2*Log[-Cosh[(c + d*x)/2] - Sinh[(c + d*x)/2] + Cosh[(c
+ d*x)/2]*#1 - Sinh[(c + d*x)/2]*#1]*#1 + c*#1^3 + d*x*#1^3 + 2*Log[-Cosh[
(c + d*x)/2] - Sinh[(c + d*x)/2] + Cosh[(c + d*x)/2]*#1 - Sinh[(c + d*x)/2
]*#1]*#1^3)/(-b - 8*a*#1^2 + 3*b*#1^2 - 3*b*#1^4 + b*#1^6) & ]/d
```

Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 123, normalized size of antiderivative = 0.98, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.273$, Rules used = {3042, 26, 3694, 1406, 218, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sinh(c+dx)}{a-b\sinh^4(c+dx)}dx$$

$$\Big\downarrow \text{3042}$$

$$\int -\frac{i\sin(ic+idx)}{a-b\sin(ic+idx)^4}dx$$

$$\Big\downarrow \text{26}$$

$$-i\int \frac{\sin(ic+idx)}{a-b\sin(ic+idx)^4}dx$$

$$\Big\downarrow \text{3694}$$

$$\frac{\int \frac{1}{-b\cosh^4(c+dx)+2b\cosh^2(c+dx)+a-b}d\cosh(c+dx)}{d}$$

$$\Big\downarrow \text{1406}$$

$$\frac{\frac{\sqrt{b}\int \frac{1}{\left(\sqrt{a}+\sqrt{b}\right)\sqrt{b}-b\cosh^2(c+dx)}d\cosh(c+dx)}{2\sqrt{a}}-\frac{\sqrt{b}\int \frac{1}{-b\cosh^2(c+dx)-\left(\sqrt{a}-\sqrt{b}\right)\sqrt{b}}d\cosh(c+dx)}{2\sqrt{a}}}{d}$$

$$\Big\downarrow \text{218}$$

$$\frac{\frac{\sqrt{b}\int \frac{1}{\left(\sqrt{a}+\sqrt{b}\right)\sqrt{b}-b\cosh^2(c+dx)}d\cosh(c+dx)}{2\sqrt{a}}+\frac{\arctan\left(\frac{\sqrt[4]{b}\cosh(c+dx)}{\sqrt{\sqrt{a}-\sqrt{b}}}\right)}{2\sqrt{a}\sqrt[4]{b}\sqrt{\sqrt{a}-\sqrt{b}}}}{d}$$

$$\Big\downarrow 221$$

$$\frac{\frac{\arctan\left(\frac{\sqrt[4]{b}\cosh(c+dx)}{\sqrt{\sqrt{a}-\sqrt{b}}}\right)}{2\sqrt{a}\sqrt[4]{b}\sqrt{\sqrt{a}-\sqrt{b}}}+\frac{\text{arctanh}\left(\frac{\sqrt[4]{b}\cosh(c+dx)}{\sqrt{\sqrt{a}+\sqrt{b}}}\right)}{2\sqrt{a}\sqrt[4]{b}\sqrt{\sqrt{a}+\sqrt{b}}}}{d}$$

input
```
Int[Sinh[c + d*x]/(a - b*Sinh[c + d*x]^4),x]
```

output
```
(ArcTan[(b^(1/4)*Cosh[c + d*x])/Sqrt[Sqrt[a] - Sqrt[b]]]/(2*Sqrt[a]*Sqrt[S
qrt[a] - Sqrt[b]]*b^(1/4)) + ArcTanh[(b^(1/4)*Cosh[c + d*x])/Sqrt[Sqrt[a]
+ Sqrt[b]]]/(2*Sqrt[a]*Sqrt[Sqrt[a] + Sqrt[b]]*b^(1/4)))/d
```

Defintions of rubi rules used

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 1406
```
Int[((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(-1), x_Symbol] :> With[{q = Rt[b^
2 - 4*a*c, 2]}, Simp[c/q   Int[1/(b/2 - q/2 + c*x^2), x], x] - Simp[c/q   I
nt[1/(b/2 + q/2 + c*x^2), x], x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c
, 0] && PosQ[b^2 - 4*a*c]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3694
```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^4)^
(p_.), x_Symbol] :> With[{ff = FreeFactors[Cos[e + f*x], x]}, Simp[-ff/f
Subst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a + b - 2*b*ff^2*x^2 + b*ff^4*x^4)^p,
 x], x, Cos[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m -
 1)/2]
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.79 (sec) , antiderivative size = 98, normalized size of antiderivative = 0.78

method	result
risch	$\sum_{\_R=\mathrm{RootOf}\left(-1+\left(256 b\,d^{4} a^{3}-256 a^{2} b^{2} d^{4}\right)\_Z^{4}+32 a\,d^{2}\_Z^{2} b\right)} \_R \ln\left(\mathrm{e}^{2 d x+2 c}+\left(\left(128 a^{2} b\,d^{3}-128 a\,b^{2} d\right.\right.\right.$
derivativedivides	$\frac{2 a \left(\frac{\arctan\left(\frac{2 \tanh\left(\frac{d x}{2}+\frac{c}{2}\right)^{2} a+4 \sqrt{a b}-2 a}{4 \sqrt{-a b+\sqrt{a b}\, a}}\right)}{4 a \sqrt{-a b+\sqrt{a b}\, a}}-\frac{\arctan\left(\frac{-2 \tanh\left(\frac{d x}{2}+\frac{c}{2}\right)^{2} a+4 \sqrt{a b}+2 a}{4 \sqrt{-a b-\sqrt{a b}\, a}}\right)}{4 a \sqrt{-a b-\sqrt{a b}\, a}}\right)}{d}$
default	$\frac{2 a \left(\frac{\arctan\left(\frac{2 \tanh\left(\frac{d x}{2}+\frac{c}{2}\right)^{2} a+4 \sqrt{a b}-2 a}{4 \sqrt{-a b+\sqrt{a b}\, a}}\right)}{4 a \sqrt{-a b+\sqrt{a b}\, a}}-\frac{\arctan\left(\frac{-2 \tanh\left(\frac{d x}{2}+\frac{c}{2}\right)^{2} a+4 \sqrt{a b}+2 a}{4 \sqrt{-a b-\sqrt{a b}\, a}}\right)}{4 a \sqrt{-a b-\sqrt{a b}\, a}}\right)}{d}$

input
```
int(sinh(d*x+c)/(a-b*sinh(d*x+c)^4),x,method=_RETURNVERBOSE)
```

output
```
sum(_R*ln(exp(2*d*x+2*c)+((128*a^2*b*d^3-128*a*b^2*d^3)*_R^3+(8*a*d+8*b*d)
*_R)*exp(d*x+c)+1),_R=RootOf(-1+(256*a^3*b*d^4-256*a^2*b^2*d^4)*_Z^4+32*a*
d^2*_Z^2*b))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 979 vs. $2(85) = 170$.

Time = 0.12 (sec) , antiderivative size = 979, normalized size of antiderivative = 7.83

$$\int \frac{\sinh(c+dx)}{a-b\sinh^4(c+dx)}\,dx = \text{Too large to display}$$

input
```
integrate(sinh(d*x+c)/(a-b*sinh(d*x+c)^4),x, algorithm="fricas")
```

output
```
1/4*sqrt(-((a^2 - a*b)*d^2*sqrt(1/((a^3*b - 2*a^2*b^2 + a*b^3)*d^4)) + 1)/
((a^2 - a*b)*d^2))*log(cosh(d*x + c)^2 + 2*cosh(d*x + c)*sinh(d*x + c) + s
inh(d*x + c)^2 + 2*(a*d*cosh(d*x + c) + a*d*sinh(d*x + c) - ((a^2*b - a*b^
2)*d^3*cosh(d*x + c) + (a^2*b - a*b^2)*d^3*sinh(d*x + c))*sqrt(1/((a^3*b -
 2*a^2*b^2 + a*b^3)*d^4)))*sqrt(-((a^2 - a*b)*d^2*sqrt(1/((a^3*b - 2*a^2*b
^2 + a*b^3)*d^4)) + 1)/((a^2 - a*b)*d^2)) + 1) - 1/4*sqrt(-((a^2 - a*b)*d^
2*sqrt(1/((a^3*b - 2*a^2*b^2 + a*b^3)*d^4)) + 1)/((a^2 - a*b)*d^2))*log(co
sh(d*x + c)^2 + 2*cosh(d*x + c)*sinh(d*x + c) + sinh(d*x + c)^2 - 2*(a*d*c
osh(d*x + c) + a*d*sinh(d*x + c) - ((a^2*b - a*b^2)*d^3*cosh(d*x + c) + (a
^2*b - a*b^2)*d^3*sinh(d*x + c))*sqrt(1/((a^3*b - 2*a^2*b^2 + a*b^3)*d^4))
)*sqrt(-((a^2 - a*b)*d^2*sqrt(1/((a^3*b - 2*a^2*b^2 + a*b^3)*d^4)) + 1)/((
a^2 - a*b)*d^2)) + 1) + 1/4*sqrt(((a^2 - a*b)*d^2*sqrt(1/((a^3*b - 2*a^2*b
^2 + a*b^3)*d^4)) - 1)/((a^2 - a*b)*d^2))*log(cosh(d*x + c)^2 + 2*cosh(d*x
 + c)*sinh(d*x + c) + sinh(d*x + c)^2 + 2*(a*d*cosh(d*x + c) + a*d*sinh(d*
x + c) + ((a^2*b - a*b^2)*d^3*cosh(d*x + c) + (a^2*b - a*b^2)*d^3*sinh(d*x
 + c))*sqrt(1/((a^3*b - 2*a^2*b^2 + a*b^3)*d^4)))*sqrt(((a^2 - a*b)*d^2*sq
rt(1/((a^3*b - 2*a^2*b^2 + a*b^3)*d^4)) - 1)/((a^2 - a*b)*d^2)) + 1) - 1/4
*sqrt(((a^2 - a*b)*d^2*sqrt(1/((a^3*b - 2*a^2*b^2 + a*b^3)*d^4)) - 1)/((a^
2 - a*b)*d^2))*log(cosh(d*x + c)^2 + 2*cosh(d*x + c)*sinh(d*x + c) + sinh(
d*x + c)^2 - 2*(a*d*cosh(d*x + c) + a*d*sinh(d*x + c) + ((a^2*b - a*b^2...
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\sinh(c+dx)}{a-b\sinh^4(c+dx)}\,dx = \text{Timed out}$$

input
```
integrate(sinh(d*x+c)/(a-b*sinh(d*x+c)**4),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{\sinh(c+dx)}{a-b\sinh^4(c+dx)}\,dx = \int -\frac{\sinh(dx+c)}{b\sinh(dx+c)^4-a}\,dx$$

input
```
integrate(sinh(d*x+c)/(a-b*sinh(d*x+c)^4),x, algorithm="maxima")
```

output
```
-integrate(sinh(d*x + c)/(b*sinh(d*x + c)^4 - a), x)
```

Giac [F]

$$\int \frac{\sinh(c+dx)}{a-b\sinh^4(c+dx)}\,dx = \int -\frac{\sinh(dx+c)}{b\sinh(dx+c)^4-a}\,dx$$

input
```
integrate(sinh(d*x+c)/(a-b*sinh(d*x+c)^4),x, algorithm="giac")
```

output
```
sage0*x
```

Mupad [B] (verification not implemented)

Time = 9.14 (sec) , antiderivative size = 1007, normalized size of antiderivative = 8.06

$$\int \frac{\sinh(c+dx)}{a-b\sinh^4(c+dx)}\,dx = \text{Too large to display}$$

input
```
int(sinh(c + d*x)/(a - b*sinh(c + d*x)^4),x)
```

output
```
log(((((4194304*a^2*d^2*(exp(2*c + 2*d*x) + 1)*(3*a + b))/(b^5*(a - b)^2)
 + (16777216*a^3*d^3*exp(c + d*x)*(-(a*b - (a^3*b)^(1/2))/(a^2*b*d^2*(a -
b)))^(1/2))/(b^5*(a - b)))*(-(a*b - (a^3*b)^(1/2))/(a^2*b*d^2*(a - b)))^(1
/2))/4 - (2097152*a^2*d*exp(c + d*x))/(b^6*(a - b)))*(-(a*b - (a^3*b)^(1/2
))/(a^2*b*d^2*(a - b)))^(1/2))/4 - (262144*a*(exp(2*c + 2*d*x) + 1)*(a + b
))/(b^6*(a - b)^2))*(-(a*b - (a^3*b)^(1/2))/(16*(a^3*b*d^2 - a^2*b^2*d^2))
)^(1/2) - log(((((4194304*a^2*d^2*(exp(2*c + 2*d*x) + 1)*(3*a + b))/(b^5*
(a - b)^2) - (16777216*a^3*d^3*exp(c + d*x)*(-(a*b - (a^3*b)^(1/2))/(a^2*b
*d^2*(a - b)))^(1/2))/(b^5*(a - b)))*(-(a*b - (a^3*b)^(1/2))/(a^2*b*d^2*(a
 - b)))^(1/2))/4 + (2097152*a^2*d*exp(c + d*x))/(b^6*(a - b)))*(-(a*b - (a
^3*b)^(1/2))/(a^2*b*d^2*(a - b)))^(1/2))/4 - (262144*a*(exp(2*c + 2*d*x) +
 1)*(a + b))/(b^6*(a - b)^2))*(-(a*b - (a^3*b)^(1/2))/(16*(a^3*b*d^2 - a^2
*b^2*d^2)))^(1/2) - log(((((4194304*a^2*d^2*(exp(2*c + 2*d*x) + 1)*(3*a +
 b))/(b^5*(a - b)^2) - (16777216*a^3*d^3*exp(c + d*x)*(-(a*b + (a^3*b)^(1/
2))/(a^2*b*d^2*(a - b)))^(1/2))/(b^5*(a - b)))*(-(a*b + (a^3*b)^(1/2))/(a^
2*b*d^2*(a - b)))^(1/2))/4 + (2097152*a^2*d*exp(c + d*x))/(b^6*(a - b)))*(
-(a*b + (a^3*b)^(1/2))/(a^2*b*d^2*(a - b)))^(1/2))/4 - (262144*a*(exp(2*c
+ 2*d*x) + 1)*(a + b))/(b^6*(a - b)^2))*(-(a*b + (a^3*b)^(1/2))/(16*(a^3*b
*d^2 - a^2*b^2*d^2)))^(1/2) + log(((((4194304*a^2*d^2*(exp(2*c + 2*d*x) +
 1)*(3*a + b))/(b^5*(a - b)^2) + (16777216*a^3*d^3*exp(c + d*x)*(-(a*b ...
```

Reduce [F]

$$\int \frac{\sinh(c+dx)}{a-b\sinh^4(c+dx)}\,dx = -\left(\int \frac{\sinh(dx+c)}{\sinh(dx+c)^4 b-a}dx\right)$$

input
```
int(sinh(d*x+c)/(a-b*sinh(d*x+c)^4),x)
```

output

```
- int(sinh(c + d*x)/(sinh(c + d*x)**4*b - a),x)
```

3.208 $\int \frac{\operatorname{csch}(c+dx)}{a-b\sinh^4(c+dx)}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 136

$$\int \frac{\operatorname{csch}(c+dx)}{a-b\sinh^4(c+dx)}\,dx=-\frac{\sqrt[4]{b}\arctan\left(\frac{\sqrt[4]{b}\cosh(c+dx)}{\sqrt{\sqrt{a}-\sqrt{b}}}\right)}{2a\sqrt{\sqrt{a}-\sqrt{b}}d}-\frac{\operatorname{arctanh}(\cosh(c+dx))}{ad}+\frac{\sqrt[4]{b}\operatorname{arctanh}\left(\frac{\sqrt[4]{b}\cosh(c+dx)}{\sqrt{\sqrt{a}+\sqrt{b}}}\right)}{2a\sqrt{\sqrt{a}+\sqrt{b}}d}$$

output
```
-1/2*b^(1/4)*arctan(b^(1/4)*cosh(d*x+c)/(a^(1/2)-b^(1/2))^(1/2))/a/(a^(1/2
)-b^(1/2))^(1/2)/d-arctanh(cosh(d*x+c))/a/d+1/2*b^(1/4)*arctanh(b^(1/4)*co
sh(d*x+c)/(a^(1/2)+b^(1/2))^(1/2))/a/(a^(1/2)+b^(1/2))^(1/2)/d
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

Time = 9.26 (sec) , antiderivative size = 397, normalized size of antiderivative = 2.92

$$\int \frac{\operatorname{csch}(c+dx)}{a-b\sinh^4(c+dx)}\,dx =$$
$$-\frac{8\log\left(\cosh\left(\frac{1}{2}(c+dx)\right)\right)-8\log\left(\sinh\left(\frac{1}{2}(c+dx)\right)\right)+b\text{RootSum}\left[b-4b\#1^2-16a\#1^4+6b\#1^4-4b\right.}{}$$

input
```
Integrate[Csch[c + d*x]/(a - b*Sinh[c + d*x]^4),x]
```

output
```
-1/8*(8*Log[Cosh[(c + d*x)/2]] - 8*Log[Sinh[(c + d*x)/2]] + b*RootSum[b -
4*b*#1^2 - 16*a*#1^4 + 6*b*#1^4 - 4*b*#1^6 + b*#1^8 & , (-c - d*x - 2*Log[
-Cosh[(c + d*x)/2] - Sinh[(c + d*x)/2] + Cosh[(c + d*x)/2]*#1 - Sinh[(c +
d*x)/2]*#1] + 3*c*#1^2 + 3*d*x*#1^2 + 6*Log[-Cosh[(c + d*x)/2] - Sinh[(c +
 d*x)/2] + Cosh[(c + d*x)/2]*#1 - Sinh[(c + d*x)/2]*#1]*#1^2 - 3*c*#1^4 -
3*d*x*#1^4 - 6*Log[-Cosh[(c + d*x)/2] - Sinh[(c + d*x)/2] + Cosh[(c + d*x)
/2]*#1 - Sinh[(c + d*x)/2]*#1]*#1^4 + c*#1^6 + d*x*#1^6 + 2*Log[-Cosh[(c +
 d*x)/2] - Sinh[(c + d*x)/2] + Cosh[(c + d*x)/2]*#1 - Sinh[(c + d*x)/2]*#1
]*#1^6)/(-(b*#1) - 8*a*#1^3 + 3*b*#1^3 - 3*b*#1^5 + b*#1^7) & ])/(a*d)
```

Rubi [A] (verified)

Time = 0.38 (sec) , antiderivative size = 131, normalized size of antiderivative = 0.96, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.227$, Rules used = {3042, 26, 3694, 1484, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\operatorname{csch}(c+dx)}{a-b\sinh^4(c+dx)}\,dx$$
$$\Big\downarrow \text{3042}$$

$$\int \frac{i}{\sin(ic+idx)\left(a-b\sin(ic+idx)^4\right)}dx$$

$\big\downarrow$ 26

$$i\int \frac{1}{\sin(ic+idx)\left(a-b\sin(ic+idx)^4\right)}dx$$

$\big\downarrow$ 3694

$$-\frac{\int \frac{1}{(1-\cosh^2(c+dx))\left(-b\cosh^4(c+dx)+2b\cosh^2(c+dx)+a-b\right)}d\cosh(c+dx)}{d}$$

$\big\downarrow$ 1484

$$-\frac{\int \left(\frac{b-b\cosh^2(c+dx)}{a\left(-b\cosh^4(c+dx)+2b\cosh^2(c+dx)+a-b\right)}-\frac{1}{a\left(\cosh^2(c+dx)-1\right)}\right)d\cosh(c+dx)}{d}$$

$\big\downarrow$ 2009

$$-\frac{\frac{\sqrt[4]{b}\arctan\left(\frac{\sqrt[4]{b}\cosh(c+dx)}{\sqrt{\sqrt{a}-\sqrt{b}}}\right)}{2a\sqrt{\sqrt{a}-\sqrt{b}}}-\frac{\sqrt[4]{b}\text{arctanh}\left(\frac{\sqrt[4]{b}\cosh(c+dx)}{\sqrt{\sqrt{a}+\sqrt{b}}}\right)}{2a\sqrt{\sqrt{a}+\sqrt{b}}}+\frac{\text{arctanh}(\cosh(c+dx))}{a}}{d}$$

input
```
Int[Csch[c + d*x]/(a - b*Sinh[c + d*x]^4),x]
```

output
```
-(((b^(1/4)*ArcTan[(b^(1/4)*Cosh[c + d*x])/Sqrt[Sqrt[a] - Sqrt[b]]])/(2*a*
Sqrt[Sqrt[a] - Sqrt[b]]) + ArcTanh[Cosh[c + d*x]]/a - (b^(1/4)*ArcTanh[(b^
(1/4)*Cosh[c + d*x])/Sqrt[Sqrt[a] + Sqrt[b]]])/(2*a*Sqrt[Sqrt[a] + Sqrt[b]
]))/d)
```

Defintions of rubi rules used

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 1484
```
Int[((d_) + (e_.)*(x_)^2)^(q_)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symb
ol] :> Int[ExpandIntegrand[(d + e*x^2)^q/(a + b*x^2 + c*x^4), x], x] /; Fre
eQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2,
 0] && IntegerQ[q]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3694
```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^4)^
(p_.), x_Symbol] :> With[{ff = FreeFactors[Cos[e + f*x], x]}, Simp[-ff/f
Subst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a + b - 2*b*ff^2*x^2 + b*ff^4*x^4)^p,
 x], x, Cos[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m -
 1)/2]
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 1.01 (sec) , antiderivative size = 130, normalized size of antiderivative = 0.96

method	result
risch	$-\frac{\ln\left(\mathrm{e}^{dx+c}+1\right)}{ad}+\frac{\ln\left(\mathrm{e}^{dx+c}-1\right)}{ad}+2\left(\sum_{\_R=\mathrm{RootOf}\left(\left(4096a^5d^4-4096a^4b\,d^4\right)\_Z^4+128a^2b\,d^2\_Z^2-b\right)}\_R\ln\left(\mathrm{e}\right.\right.$
derivativedivides	$\frac{8b\left(-\frac{\sqrt{ab}\,\arctan\left(\frac{-2\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^2a+4\sqrt{ab}+2a}{4\sqrt{-ab-\sqrt{ab}\,a}}\right)}{16ab\sqrt{-ab-\sqrt{ab}\,a}}-\frac{\sqrt{ab}\,\arctan\left(\frac{2\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^2a+4\sqrt{ab}-2a}{4\sqrt{-ab+\sqrt{ab}\,a}}\right)}{16ab\sqrt{-ab+\sqrt{ab}\,a}}\right)+\frac{\ln\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{a}}{d}$
default	$\frac{8b\left(-\frac{\sqrt{ab}\,\arctan\left(\frac{-2\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^2a+4\sqrt{ab}+2a}{4\sqrt{-ab-\sqrt{ab}\,a}}\right)}{16ab\sqrt{-ab-\sqrt{ab}\,a}}-\frac{\sqrt{ab}\,\arctan\left(\frac{2\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^2a+4\sqrt{ab}-2a}{4\sqrt{-ab+\sqrt{ab}\,a}}\right)}{16ab\sqrt{-ab+\sqrt{ab}\,a}}\right)+\frac{\ln\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{a}}{d}$

input
```
int(csch(d*x+c)/(a-b*sinh(d*x+c)^4),x,method=_RETURNVERBOSE)
```

output
```
-1/a/d*ln(exp(d*x+c)+1)+1/a/d*ln(exp(d*x+c)-1)+2*sum(_R*ln(exp(2*d*x+2*c)+
((1024/b*d^3*a^4-1024*a^3*d^3)*_R^3+32*a*d*_R)*exp(d*x+c)+1),_R=RootOf((40
96*a^5*d^4-4096*a^4*b*d^4)*_Z^4+128*a^2*b*d^2*_Z^2-b))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1067 vs. $2(100) = 200$.

Time = 0.17 (sec) , antiderivative size = 1067, normalized size of antiderivative = 7.85

$$\int \frac{\operatorname{csch}(c+dx)}{a-b\sinh^4(c+dx)}\,dx = \text{Too large to display}$$

input
```
integrate(csch(d*x+c)/(a-b*sinh(d*x+c)^4),x, algorithm="fricas")
```

output
```
1/4*(a*d*sqrt(-((a^3 - a^2*b)*d^2*sqrt(b/((a^5 - 2*a^4*b + a^3*b^2)*d^4))
+ b)/((a^3 - a^2*b)*d^2))*log(b*cosh(d*x + c)^2 + 2*b*cosh(d*x + c)*sinh(d
*x + c) + b*sinh(d*x + c)^2 + 2*(a*b*d*cosh(d*x + c) + a*b*d*sinh(d*x + c)
 - ((a^4 - a^3*b)*d^3*cosh(d*x + c) + (a^4 - a^3*b)*d^3*sinh(d*x + c))*sqr
t(b/((a^5 - 2*a^4*b + a^3*b^2)*d^4)))*sqrt(-((a^3 - a^2*b)*d^2*sqrt(b/((a^
5 - 2*a^4*b + a^3*b^2)*d^4)) + b)/((a^3 - a^2*b)*d^2)) + b) - a*d*sqrt(-((
a^3 - a^2*b)*d^2*sqrt(b/((a^5 - 2*a^4*b + a^3*b^2)*d^4)) + b)/((a^3 - a^2*
b)*d^2))*log(b*cosh(d*x + c)^2 + 2*b*cosh(d*x + c)*sinh(d*x + c) + b*sinh(
d*x + c)^2 - 2*(a*b*d*cosh(d*x + c) + a*b*d*sinh(d*x + c) - ((a^4 - a^3*b)
*d^3*cosh(d*x + c) + (a^4 - a^3*b)*d^3*sinh(d*x + c))*sqrt(b/((a^5 - 2*a^4
*b + a^3*b^2)*d^4)))*sqrt(-((a^3 - a^2*b)*d^2*sqrt(b/((a^5 - 2*a^4*b + a^3
*b^2)*d^4)) + b)/((a^3 - a^2*b)*d^2)) + b) + a*d*sqrt(((a^3 - a^2*b)*d^2*s
qrt(b/((a^5 - 2*a^4*b + a^3*b^2)*d^4)) - b)/((a^3 - a^2*b)*d^2))*log(b*cos
h(d*x + c)^2 + 2*b*cosh(d*x + c)*sinh(d*x + c) + b*sinh(d*x + c)^2 + 2*(a*
b*d*cosh(d*x + c) + a*b*d*sinh(d*x + c) + ((a^4 - a^3*b)*d^3*cosh(d*x + c)
 + (a^4 - a^3*b)*d^3*sinh(d*x + c))*sqrt(b/((a^5 - 2*a^4*b + a^3*b^2)*d^4)
))*sqrt(((a^3 - a^2*b)*d^2*sqrt(b/((a^5 - 2*a^4*b + a^3*b^2)*d^4)) - b)/((
a^3 - a^2*b)*d^2)) + b) - a*d*sqrt(((a^3 - a^2*b)*d^2*sqrt(b/((a^5 - 2*a^4
*b + a^3*b^2)*d^4)) - b)/((a^3 - a^2*b)*d^2))*log(b*cosh(d*x + c)^2 + 2*b*
cosh(d*x + c)*sinh(d*x + c) + b*sinh(d*x + c)^2 - 2*(a*b*d*cosh(d*x + c...
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\operatorname{csch}(c+dx)}{a-b\sinh^4(c+dx)}\,dx = \text{Timed out}$$

input
```
integrate(csch(d*x+c)/(a-b*sinh(d*x+c)**4),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{\operatorname{csch}(c+dx)}{a-b\sinh^4(c+dx)}\,dx = \int -\frac{\operatorname{csch}(dx+c)}{b\sinh(dx+c)^4-a}\,dx$$

input
```
integrate(csch(d*x+c)/(a-b*sinh(d*x+c)^4),x, algorithm="maxima")
```

output
```
-log((e^(d*x + c) + 1)*e^(-c))/(a*d) + log((e^(d*x + c) - 1)*e^(-c))/(a*d)
 - 2*integrate((b*e^(7*d*x + 7*c) - 3*b*e^(5*d*x + 5*c) + 3*b*e^(3*d*x + 3
*c) - b*e^(d*x + c))/(a*b*e^(8*d*x + 8*c) - 4*a*b*e^(6*d*x + 6*c) - 4*a*b*
e^(2*d*x + 2*c) + a*b - 2*(8*a^2*e^(4*c) - 3*a*b*e^(4*c))*e^(4*d*x)), x)
```

Giac [F]

$$\int \frac{\operatorname{csch}(c+dx)}{a-b\sinh^4(c+dx)}\,dx = \int -\frac{\operatorname{csch}(dx+c)}{b\sinh(dx+c)^4-a}\,dx$$

input
```
integrate(csch(d*x+c)/(a-b*sinh(d*x+c)^4),x, algorithm="giac")
```

output
```
sage0*x
```

Mupad [B] (verification not implemented)

Time = 10.03 (sec) , antiderivative size = 1243, normalized size of antiderivative = 9.14

$$\int \frac{\operatorname{csch}(c+dx)}{a-b\sinh^4(c+dx)}\,dx = \text{Too large to display}$$

input
```
int(1/(sinh(c + d*x)*(a - b*sinh(c + d*x)^4)),x)
```

output
```
log(((((4294967296*a*d^2*(exp(2*c + 2*d*x) + 1)*(26*a*b - 49*a^2 + 15*b^2
))/(b^6*(a - b)^3) + (8589934592*a^2*d^3*exp(c + d*x)*(3*a*b + 16*a^2 - 15
*b^2)*(-(a^2*b + (a^5*b)^(1/2))/(a^4*d^2*(a - b)))^(1/2))/(b^7*(a - b)^2))
*(-(a^2*b + (a^5*b)^(1/2))/(a^4*d^2*(a - b)))^(1/2))/4 - (2147483648*d*exp
(c + d*x)*(17*a - 15*b))/(b^6*(a - b)^2))*(-(a^2*b + (a^5*b)^(1/2))/(a^4*d
^2*(a - b)))^(1/2))/4 + (268435456*(exp(2*c + 2*d*x) + 1)*(3*a*b + 16*a^2
- 15*b^2))/(a*b^6*(a - b)^3))*(-(a^2*b + (a^5*b)^(1/2))/(16*(a^5*d^2 - a^4
*b*d^2)))^(1/2) - log(((((4294967296*a*d^2*(exp(2*c + 2*d*x) + 1)*(26*a*b
 - 49*a^2 + 15*b^2))/(b^6*(a - b)^3) - (8589934592*a^2*d^3*exp(c + d*x)*(3
*a*b + 16*a^2 - 15*b^2)*(-(a^2*b + (a^5*b)^(1/2))/(a^4*d^2*(a - b)))^(1/2)
)/(b^7*(a - b)^2))*(-(a^2*b + (a^5*b)^(1/2))/(a^4*d^2*(a - b)))^(1/2))/4 +
 (2147483648*d*exp(c + d*x)*(17*a - 15*b))/(b^6*(a - b)^2))*(-(a^2*b + (a^
5*b)^(1/2))/(a^4*d^2*(a - b)))^(1/2))/4 + (268435456*(exp(2*c + 2*d*x) + 1
)*(3*a*b + 16*a^2 - 15*b^2))/(a*b^6*(a - b)^3))*(-(a^2*b + (a^5*b)^(1/2))/
(16*(a^5*d^2 - a^4*b*d^2)))^(1/2) - (2*atan((exp(d*x)*exp(c)*(65536*a^2*(-
a^2*d^2)^(1/2) + 50625*b^2*(-a^2*d^2)^(1/2) - 115200*a*b*(-a^2*d^2)^(1/2))
)/(65536*a^3*d + 50625*a*b^2*d - 115200*a^2*b*d)))/(-a^2*d^2)^(1/2) - log(
((((4294967296*a*d^2*(exp(2*c + 2*d*x) + 1)*(26*a*b - 49*a^2 + 15*b^2))/(
b^6*(a - b)^3) - (8589934592*a^2*d^3*exp(c + d*x)*(3*a*b + 16*a^2 - 15*b^2
)*(-(a^2*b - (a^5*b)^(1/2))/(a^4*d^2*(a - b)))^(1/2))/(b^7*(a - b)^2))*...
```

Reduce [F]

$$\int \frac{\operatorname{csch}(c+dx)}{a-b\sinh^4(c+dx)}\,dx = -\left(\int \frac{\operatorname{csch}(dx+c)}{\sinh(dx+c)^4 b-a}dx\right)$$

input
```
int(csch(d*x+c)/(a-b*sinh(d*x+c)^4),x)
```

output
```
- int(csch(c + d*x)/(sinh(c + d*x)**4*b - a),x)
```

3.209 $\int \frac{\operatorname{csch}^3(c+dx)}{a-b\sinh^4(c+dx)}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 164

$$\int \frac{\operatorname{csch}^3(c+dx)}{a-b\sinh^4(c+dx)}\,dx = \frac{b^{3/4}\arctan\left(\frac{\sqrt[4]{b}\cosh(c+dx)}{\sqrt{\sqrt{a}-\sqrt{b}}}\right)}{2a^{3/2}\sqrt{\sqrt{a}-\sqrt{b}}d} + \frac{\operatorname{arctanh}(\cosh(c+dx))}{2ad} + \frac{b^{3/4}\operatorname{arctanh}\left(\frac{\sqrt[4]{b}\cosh(c+dx)}{\sqrt{\sqrt{a}+\sqrt{b}}}\right)}{2a^{3/2}\sqrt{\sqrt{a}+\sqrt{b}}d} - \frac{\coth(c+dx)\operatorname{csch}(c+dx)}{2ad}$$

output
```
1/2*b^(3/4)*arctan(b^(1/4)*cosh(d*x+c)/(a^(1/2)-b^(1/2))^(1/2))/a^(3/2)/(a
^(1/2)-b^(1/2))^(1/2)/d+1/2*arctanh(cosh(d*x+c))/a/d+1/2*b^(3/4)*arctanh(b
^(1/4)*cosh(d*x+c)/(a^(1/2)+b^(1/2))^(1/2))/a^(3/2)/(a^(1/2)+b^(1/2))^(1/2
)/d-1/2*coth(d*x+c)*csch(d*x+c)/a/d
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

Time = 1.63 (sec) , antiderivative size = 278, normalized size of antiderivative = 1.70

$$\int \frac{\operatorname{csch}^3(c+dx)}{a-b\sinh^4(c+dx)}\,dx = -\frac{\operatorname{csch}^2\left(\frac{1}{2}(c+dx)\right)-4\log\left(\cosh\left(\frac{1}{2}(c+dx)\right)\right)+4\log\left(\sinh\left(\frac{1}{2}(c+dx)\right)\right)+4b\text{RootSum}\left[b-4b\#1^2-1\right.}{}$$

input
```
Integrate[Csch[c + d*x]^3/(a - b*Sinh[c + d*x]^4),x]
```

output
```
-1/8*(Csch[(c + d*x)/2]^2 - 4*Log[Cosh[(c + d*x)/2]] + 4*Log[Sinh[(c + d*x
)/2]] + 4*b*RootSum[b - 4*b*#1^2 - 16*a*#1^4 + 6*b*#1^4 - 4*b*#1^6 + b*#1^
8 & , (-(c*#1) - d*x*#1 - 2*Log[-Cosh[(c + d*x)/2] - Sinh[(c + d*x)/2] + C
osh[(c + d*x)/2]*#1 - Sinh[(c + d*x)/2]*#1]*#1 + c*#1^3 + d*x*#1^3 + 2*Log
[-Cosh[(c + d*x)/2] - Sinh[(c + d*x)/2] + Cosh[(c + d*x)/2]*#1 - Sinh[(c +
 d*x)/2]*#1]*#1^3)/(-b - 8*a*#1^2 + 3*b*#1^2 - 3*b*#1^4 + b*#1^6) & ] + Se
ch[(c + d*x)/2]^2)/(a*d)
```

Rubi [A] (verified)

Time = 0.39 (sec) , antiderivative size = 173, normalized size of antiderivative = 1.05, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.208$, Rules used = {3042, 26, 3694, 1484, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\operatorname{csch}^3(c+dx)}{a-b\sinh^4(c+dx)}dx$$

$$\Big\downarrow \; 3042$$

$$\int -\frac{i}{\sin(ic+idx)^3\left(a-b\sin(ic+idx)^4\right)}dx$$

$$\Big\downarrow \; 26$$

$$-i\int \frac{1}{\sin(ic+idx)^3\left(a-b\sin(ic+idx)^4\right)}dx$$

$$\Big\downarrow \text{ 3694}$$

$$\frac{\int \frac{1}{\left(1-\cosh^2(c+dx)\right)^2\left(-b\cosh^4(c+dx)+2b\cosh^2(c+dx)+a-b\right)}d\cosh(c+dx)}{d}$$

$$\Big\downarrow \text{ 1484}$$

$$\frac{\int \left(\frac{b}{a\left(-b\cosh^4(c+dx)+2b\cosh^2(c+dx)+a-b\right)}-\frac{1}{2a\left(\cosh^2(c+dx)-1\right)}+\frac{1}{4a(\cosh(c+dx)-1)^2}+\frac{1}{4a(\cosh(c+dx)+1)^2}\right)d\cosh(c+dx)}{d}$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{\frac{b^{3/4}\arctan\left(\frac{\sqrt[4]{b}\cosh(c+dx)}{\sqrt{\sqrt{a}-\sqrt{b}}}\right)}{2a^{3/2}\sqrt{\sqrt{a}-\sqrt{b}}}+\frac{b^{3/4}\text{arctanh}\left(\frac{\sqrt[4]{b}\cosh(c+dx)}{\sqrt{\sqrt{a}+\sqrt{b}}}\right)}{2a^{3/2}\sqrt{\sqrt{a}+\sqrt{b}}}+\frac{\text{arctanh}(\cosh(c+dx))}{2a}+\frac{1}{4a(1-\cosh(c+dx))}-\frac{1}{4a(\cosh(c+dx)+1)}}{d}$$

input
```
Int[Csch[c + d*x]^3/(a - b*Sinh[c + d*x]^4),x]
```

output
```
((b^(3/4)*ArcTan[(b^(1/4)*Cosh[c + d*x])/Sqrt[Sqrt[a] - Sqrt[b]]])/(2*a^(3
/2)*Sqrt[Sqrt[a] - Sqrt[b]]) + ArcTanh[Cosh[c + d*x]]/(2*a) + (b^(3/4)*Arc
Tanh[(b^(1/4)*Cosh[c + d*x])/Sqrt[Sqrt[a] + Sqrt[b]]])/(2*a^(3/2)*Sqrt[Sqr
t[a] + Sqrt[b]]) + 1/(4*a*(1 - Cosh[c + d*x])) - 1/(4*a*(1 + Cosh[c + d*x]
)))/d
```

Defintions of rubi rules used

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 1484
```
Int[((d_) + (e_.)*(x_)^2)^(q_)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symb
ol] :> Int[ExpandIntegrand[(d + e*x^2)^q/(a + b*x^2 + c*x^4), x], x] /; Fre
eQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2,
 0] && IntegerQ[q]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3694
```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^4)^
(p_.), x_Symbol] :> With[{ff = FreeFactors[Cos[e + f*x], x]}, Simp[-ff/f
Subst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a + b - 2*b*ff^2*x^2 + b*ff^4*x^4)^p,
 x], x, Cos[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m -
 1)/2]
```

Maple [A] (verified)

Time = 1.69 (sec) , antiderivative size = 181, normalized size of antiderivative = 1.10

method	result
derivativedivides	$\frac{\frac{\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^2}{8a}-\frac{1}{8a\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^2}-\frac{\ln\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{2a}+2b\left(\frac{\arctan\left(\frac{2\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^2a+4\sqrt{ab}-2a}{4\sqrt{-ab+\sqrt{ab}\,a}}\right)}{4a\sqrt{-ab+\sqrt{ab}\,a}}-\frac{\arctan\left(\frac{-2\tanh\left(\frac{dx}{2}+\cdots\right.}{4\sqrt{-a\cdots}}\right.}{4a\sqrt{-ab-\cdots}}\right.}{d}$ …
default	$\frac{\frac{\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^2}{8a}-\frac{1}{8a\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^2}-\frac{\ln\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{2a}+2b\left(\frac{\arctan\left(\frac{2\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^2a+4\sqrt{ab}-2a}{4\sqrt{-ab+\sqrt{ab}\,a}}\right)}{4a\sqrt{-ab+\sqrt{ab}\,a}}-\frac{\arctan\left(\frac{-2\tanh\left(\frac{dx}{2}+\cdots\right.}{4\sqrt{-a\cdots}}\right.}{4a\sqrt{-ab-\cdots}}\right.}{d}$ …
risch	$-\frac{e^{dx+c}\left(e^{2dx+2c}+1\right)}{da\left(e^{2dx+2c}-1\right)^2}+\frac{\ln\left(e^{dx+c}+1\right)}{2ad}-\frac{\ln\left(e^{dx+c}-1\right)}{2ad}+8\left(\sum_{\_R=\operatorname{RootOf}\left(\left(1048576a^7d^4-1048576a^6b\,d^4\right)\_Z^4+\cdots\right.}\right.$ …

input
```
int(csch(d*x+c)^3/(a-b*sinh(d*x+c)^4),x,method=_RETURNVERBOSE)
```

output
```
1/d*(1/8*tanh(1/2*d*x+1/2*c)^2/a-1/8/a/tanh(1/2*d*x+1/2*c)^2-1/2/a*ln(tanh
(1/2*d*x+1/2*c))+2*b*(1/4/a/(-a*b+(a*b)^(1/2)*a)^(1/2)*arctan(1/4*(2*tanh(
1/2*d*x+1/2*c)^2*a+4*(a*b)^(1/2)-2*a)/(-a*b+(a*b)^(1/2)*a)^(1/2))-1/4/a/(-
a*b-(a*b)^(1/2)*a)^(1/2)*arctan(1/4*(-2*tanh(1/2*d*x+1/2*c)^2*a+4*(a*b)^(1
/2)+2*a)/(-a*b-(a*b)^(1/2)*a)^(1/2))))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1954 vs. $2(120) = 240$.

Time = 0.17 (sec) , antiderivative size = 1954, normalized size of antiderivative = 11.91

$$\int \frac{\operatorname{csch}^3(c+dx)}{a-b\sinh^4(c+dx)}\,dx = \text{Too large to display}$$

input
```
integrate(csch(d*x+c)^3/(a-b*sinh(d*x+c)^4),x, algorithm="fricas")
```

output
```
-1/4*(4*cosh(d*x + c)^3 + 12*cosh(d*x + c)*sinh(d*x + c)^2 + 4*sinh(d*x +
c)^3 - (a*d*cosh(d*x + c)^4 + 4*a*d*cosh(d*x + c)*sinh(d*x + c)^3 + a*d*si
nh(d*x + c)^4 - 2*a*d*cosh(d*x + c)^2 + 2*(3*a*d*cosh(d*x + c)^2 - a*d)*si
nh(d*x + c)^2 + a*d + 4*(a*d*cosh(d*x + c)^3 - a*d*cosh(d*x + c))*sinh(d*x
 + c))*sqrt(-((a^4 - a^3*b)*d^2*sqrt(b^3/((a^7 - 2*a^6*b + a^5*b^2)*d^4))
+ b^2)/((a^4 - a^3*b)*d^2))*log(b^2*cosh(d*x + c)^2 + 2*b^2*cosh(d*x + c)*
sinh(d*x + c) + b^2*sinh(d*x + c)^2 + b^2 + 2*(a^2*b*d*cosh(d*x + c) + a^2
*b*d*sinh(d*x + c) - ((a^5 - a^4*b)*d^3*cosh(d*x + c) + (a^5 - a^4*b)*d^3*
sinh(d*x + c))*sqrt(b^3/((a^7 - 2*a^6*b + a^5*b^2)*d^4)))*sqrt(-((a^4 - a^
3*b)*d^2*sqrt(b^3/((a^7 - 2*a^6*b + a^5*b^2)*d^4)) + b^2)/((a^4 - a^3*b)*d
^2))) + (a*d*cosh(d*x + c)^4 + 4*a*d*cosh(d*x + c)*sinh(d*x + c)^3 + a*d*s
inh(d*x + c)^4 - 2*a*d*cosh(d*x + c)^2 + 2*(3*a*d*cosh(d*x + c)^2 - a*d)*s
inh(d*x + c)^2 + a*d + 4*(a*d*cosh(d*x + c)^3 - a*d*cosh(d*x + c))*sinh(d*
x + c))*sqrt(-((a^4 - a^3*b)*d^2*sqrt(b^3/((a^7 - 2*a^6*b + a^5*b^2)*d^4))
 + b^2)/((a^4 - a^3*b)*d^2))*log(b^2*cosh(d*x + c)^2 + 2*b^2*cosh(d*x + c)
*sinh(d*x + c) + b^2*sinh(d*x + c)^2 + b^2 - 2*(a^2*b*d*cosh(d*x + c) + a^
2*b*d*sinh(d*x + c) - ((a^5 - a^4*b)*d^3*cosh(d*x + c) + (a^5 - a^4*b)*d^3
*sinh(d*x + c))*sqrt(b^3/((a^7 - 2*a^6*b + a^5*b^2)*d^4)))*sqrt(-((a^4 - a
^3*b)*d^2*sqrt(b^3/((a^7 - 2*a^6*b + a^5*b^2)*d^4)) + b^2)/((a^4 - a^3*b)*
d^2))) - (a*d*cosh(d*x + c)^4 + 4*a*d*cosh(d*x + c)*sinh(d*x + c)^3 + a...
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\operatorname{csch}^3(c+dx)}{a-b\sinh^4(c+dx)}\,dx=\text{Timed out}$$

input
```
integrate(csch(d*x+c)**3/(a-b*sinh(d*x+c)**4),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{\operatorname{csch}^3(c+dx)}{a-b\sinh^4(c+dx)}\,dx=\int -\frac{\operatorname{csch}\left(dx+c\right)^3}{b\sinh\left(dx+c\right)^4-a}\,dx$$

input
```
integrate(csch(d*x+c)^3/(a-b*sinh(d*x+c)^4),x, algorithm="maxima")
```

output
```
-(e^(3*d*x + 3*c) + e^(d*x + c))/(a*d*e^(4*d*x + 4*c) - 2*a*d*e^(2*d*x + 2
*c) + a*d) + 1/2*log((e^(d*x + c) + 1)*e^(-c))/(a*d) - 1/2*log((e^(d*x + c
) - 1)*e^(-c))/(a*d) - 8*integrate((b*e^(5*d*x + 5*c) - b*e^(3*d*x + 3*c))
/(a*b*e^(8*d*x + 8*c) - 4*a*b*e^(6*d*x + 6*c) - 4*a*b*e^(2*d*x + 2*c) + a*
b - 2*(8*a^2*e^(4*c) - 3*a*b*e^(4*c))*e^(4*d*x)), x)
```

Giac [F]

$$\int \frac{\operatorname{csch}^3(c+dx)}{a-b\sinh^4(c+dx)}\,dx=\int -\frac{\operatorname{csch}\left(dx+c\right)^3}{b\sinh\left(dx+c\right)^4-a}\,dx$$

input
```
integrate(csch(d*x+c)^3/(a-b*sinh(d*x+c)^4),x, algorithm="giac")
```

output
```
sage0*x
```

Mupad [B] (verification not implemented)

Time = 13.07 (sec) , antiderivative size = 1517, normalized size of antiderivative = 9.25

$$\int \frac{\operatorname{csch}^3(c+dx)}{a-b\sinh^4(c+dx)}\,dx = \text{Too large to display}$$

input
```
int(1/(sinh(c + d*x)^3*(a - b*sinh(c + d*x)^4)),x)
```

output
```
atan((exp(d*x)*exp(c)*(256*a^6*(-a^2*d^2)^(1/2) + b^6*(-a^2*d^2)^(1/2) + 9
6*a^2*b^4*(-a^2*d^2)^(1/2) - 288*a^3*b^3*(-a^2*d^2)^(1/2) + 512*a^4*b^2*(-
a^2*d^2)^(1/2) - 16*a*b^5*(-a^2*d^2)^(1/2) - 512*a^5*b*(-a^2*d^2)^(1/2)))/
(256*a^7*d - 16*a^2*b^5*d + 96*a^3*b^4*d - 288*a^4*b^3*d + 512*a^5*b^2*d +
 a*b^6*d - 512*a^6*b*d))/(-a^2*d^2)^(1/2) - log(((((8589934592*d^3*exp(c
+ d*x)*(8*a^2 - 7*a*b + 3*b^2)*(-((a^7*b^3)^(1/2) + a^3*b^2)/(a^6*d^2*(a -
 b)))^(1/2))/(b^5*(a - b)^2) - (4294967296*d^2*(exp(2*c + 2*d*x) + 1)*(2*a
*b^2 - 7*a^2*b + 12*a^3 + b^3))/(a^2*b^4*(a - b)^3))*(-((a^7*b^3)^(1/2) +
a^3*b^2)/(a^6*d^2*(a - b)))^(1/2))/4 - (4294967296*d*exp(c + d*x)*(2*a^2 -
 2*a*b + b^2))/(a^3*b^4*(a - b)^2))*(-((a^7*b^3)^(1/2) + a^3*b^2)/(a^6*d^2
*(a - b)))^(1/2))/4 + (268435456*(exp(2*c + 2*d*x) + 1)*(4*a^3 - a*b^2 + b
^3))/(a^5*b^3*(a - b)^3))*(-((a^7*b^3)^(1/2) + a^3*b^2)/(16*(a^7*d^2 - a^6
*b*d^2)))^(1/2) + log((268435456*(exp(2*c + 2*d*x) + 1)*(4*a^3 - a*b^2 + b
^3))/(a^5*b^3*(a - b)^3) - ((((8589934592*d^3*exp(c + d*x)*(8*a^2 - 7*a*b
 + 3*b^2)*(-((a^7*b^3)^(1/2) + a^3*b^2)/(a^6*d^2*(a - b)))^(1/2))/(b^5*(a
- b)^2) + (4294967296*d^2*(exp(2*c + 2*d*x) + 1)*(2*a*b^2 - 7*a^2*b + 12*a
^3 + b^3))/(a^2*b^4*(a - b)^3))*(-((a^7*b^3)^(1/2) + a^3*b^2)/(a^6*d^2*(a
- b)))^(1/2))/4 - (4294967296*d*exp(c + d*x)*(2*a^2 - 2*a*b + b^2))/(a^3*b
^4*(a - b)^2))*(-((a^7*b^3)^(1/2) + a^3*b^2)/(a^6*d^2*(a - b)))^(1/2))/4)*
(-((a^7*b^3)^(1/2) + a^3*b^2)/(16*(a^7*d^2 - a^6*b*d^2)))^(1/2) - log((...
```

Reduce [F]

$$\int \frac{\operatorname{csch}^3(c+dx)}{a-b\sinh^4(c+dx)}\,dx = -\left(\int \frac{\operatorname{csch}(dx+c)^3}{\sinh(dx+c)^4 b-a}dx\right)$$

input
```
int(csch(d*x+c)^3/(a-b*sinh(d*x+c)^4),x)
```

output

```
- int(csch(c + d*x)**3/(sinh(c + d*x)**4*b - a),x)
```

3.210 $\int \frac{\sinh^6(c+dx)}{a-b\sinh^4(c+dx)}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 155

$$\int \frac{\sinh^6(c+dx)}{a-b\sinh^4(c+dx)}\,dx=\frac{x}{2b}-\frac{a^{3/4}\text{arctanh}\left(\frac{\sqrt{\sqrt{a}-\sqrt{b}}\tanh(c+dx)}{\sqrt[4]{a}}\right)}{2\sqrt{\sqrt{a}-\sqrt{b}}b^{3/2}d}+\frac{a^{3/4}\text{arctanh}\left(\frac{\sqrt{\sqrt{a}+\sqrt{b}}\tanh(c+dx)}{\sqrt[4]{a}}\right)}{2\sqrt{\sqrt{a}+\sqrt{b}}b^{3/2}d}-\frac{\cosh(c+dx)\sinh(c+dx)}{2bd}$$

output
```
1/2*x/b-1/2*a^(3/4)*arctanh((a^(1/2)-b^(1/2))^(1/2)*tanh(d*x+c)/a^(1/4))/(
a^(1/2)-b^(1/2))^(1/2)/b^(3/2)/d+1/2*a^(3/4)*arctanh((a^(1/2)+b^(1/2))^(1/
2)*tanh(d*x+c)/a^(1/4))/(a^(1/2)+b^(1/2))^(1/2)/b^(3/2)/d-1/2*cosh(d*x+c)*
sinh(d*x+c)/b/d
```

Mathematica [A] (verified)

Time = 1.55 (sec) , antiderivative size = 158, normalized size of antiderivative = 1.02

$$\int \frac{\sinh^6(c+dx)}{a-b\sinh^4(c+dx)}\,dx = \frac{2\sqrt{b}(c+dx)+\frac{2a\arctan\left(\frac{(\sqrt{a}-\sqrt{b})\tanh(c+dx)}{\sqrt{-a+\sqrt{a}\sqrt{b}}}\right)}{\sqrt{-a+\sqrt{a}\sqrt{b}}}+\frac{2a\text{arctanh}\left(\frac{(\sqrt{a}+\sqrt{b})\tanh(c+dx)}{\sqrt{a+\sqrt{a}\sqrt{b}}}\right)}{\sqrt{a+\sqrt{a}\sqrt{b}}}-\sqrt{b}\sinh(2(c+dx))}{4b^{3/2}d}$$

input
```
Integrate[Sinh[c + d*x]^6/(a - b*Sinh[c + d*x]^4),x]
```

output
```
(2*Sqrt[b]*(c + d*x) + (2*a*ArcTan[((Sqrt[a] - Sqrt[b])*Tanh[c + d*x])/Sqr
t[-a + Sqrt[a]*Sqrt[b]]])/Sqrt[-a + Sqrt[a]*Sqrt[b]] + (2*a*ArcTanh[((Sqrt
[a] + Sqrt[b])*Tanh[c + d*x])/Sqrt[a + Sqrt[a]*Sqrt[b]]])/Sqrt[a + Sqrt[a]
*Sqrt[b]] - Sqrt[b]*Sinh[2*(c + d*x)])/(4*b^(3/2)*d)
```

Rubi [A] (verified)

Time = 0.46 (sec) , antiderivative size = 173, normalized size of antiderivative = 1.12, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.208$, Rules used = {3042, 25, 3696, 1610, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sinh^6(c+dx)}{a-b\sinh^4(c+dx)}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int -\frac{\sin(ic+idx)^6}{a-b\sin(ic+idx)^4}dx$$

$$\Big\downarrow \text{25}$$

$$-\int \frac{\sin(ic+idx)^6}{a-b\sin(ic+idx)^4}dx$$

$$\Big\downarrow \text{3696}$$

$$\frac{\int \frac{\tanh^6(c+dx)}{(1-\tanh^2(c+dx))^2((a-b)\tanh^4(c+dx)-2a\tanh^2(c+dx)+a)}d\tanh(c+dx)}{d}$$

$$\Big\downarrow \text{1610}$$

$$\frac{\int \left(\frac{a\tanh^2(c+dx)}{b((a-b)\tanh^4(c+dx)-2a\tanh^2(c+dx)+a)}-\frac{1}{2b(\tanh^2(c+dx)-1)}-\frac{1}{4b(\tanh(c+dx)-1)^2}-\frac{1}{4b(\tanh(c+dx)+1)^2}\right)d\tanh(c+dx)}{d}$$

$$\Big\downarrow \text{2009}$$

$$\frac{-\frac{a^{3/4}\text{arctanh}\left(\frac{\sqrt{\sqrt{a}-\sqrt{b}}\tanh(c+dx)}{\sqrt[4]{a}}\right)}{2b^{3/2}\sqrt{\sqrt{a}-\sqrt{b}}}+\frac{a^{3/4}\text{arctanh}\left(\frac{\sqrt{\sqrt{a}+\sqrt{b}}\tanh(c+dx)}{\sqrt[4]{a}}\right)}{2b^{3/2}\sqrt{\sqrt{a}+\sqrt{b}}}+\frac{\text{arctanh}(\tanh(c+dx))}{2b}-\frac{1}{4b(1-\tanh(c+dx))}+\frac{1}{4b(t}}{d}$$

input
```
Int[Sinh[c + d*x]^6/(a - b*Sinh[c + d*x]^4),x]
```

output
```
(ArcTanh[Tanh[c + d*x]]/(2*b) - (a^(3/4)*ArcTanh[(Sqrt[Sqrt[a] - Sqrt[b]]*
Tanh[c + d*x])/a^(1/4)])/(2*Sqrt[Sqrt[a] - Sqrt[b]]*b^(3/2)) + (a^(3/4)*Ar
cTanh[(Sqrt[Sqrt[a] + Sqrt[b]]*Tanh[c + d*x])/a^(1/4)])/(2*Sqrt[Sqrt[a] +
Sqrt[b]]*b^(3/2)) - 1/(4*b*(1 - Tanh[c + d*x])) + 1/(4*b*(1 + Tanh[c + d*x
])))/d
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 1610
```
Int[(((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.))/((a_) + (b_.)*(x_)^2 +
 (c_.)*(x_)^4), x_Symbol] :> Int[ExpandIntegrand[(f*x)^m*((d + e*x^2)^q/(a
+ b*x^2 + c*x^4)), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b^2 - 4
*a*c, 0] && IntegerQ[q] && IntegerQ[m]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3696
```
Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^4)^(
p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Simp[ff^(m + 1
)/f   Subst[Int[x^m*((a + 2*a*ff^2*x^2 + (a + b)*ff^4*x^4)^p/(1 + ff^2*x^2)
^(m/2 + 2*p + 1)), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f}, x] &
& IntegerQ[m/2] && IntegerQ[p]
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 2.37 (sec) , antiderivative size = 162, normalized size of antiderivative = 1.05

method	result
risch	$\frac{x}{2b}-\frac{\mathrm{e}^{2dx+2c}}{8bd}+\frac{\mathrm{e}^{-2dx-2c}}{8bd}+\left(\sum_{\_R=\mathrm{RootOf}\left(\left(256a\,b^{6}d^{4}-256b^{7}d^{4}\right)\_Z^{4}-32a^{2}b^{3}d^{2}\_Z^{2}+a^{3}\right)}\_R\ln\left(\mathrm{e}^{2dx+2c}\right.\right.$
derivativedivides	$\frac{1}{2b\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^{2}}-\frac{1}{2b\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)}+\frac{\ln\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)}{2b}-\frac{a\left(\sum_{\_R=\mathrm{RootOf}\left(a\_Z^{8}-4a\_Z^{6}+(6a-16b)\_Z^{4}-4a\_\right.}\right.}{}$
default	$\frac{1}{2b\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^{2}}-\frac{1}{2b\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)}+\frac{\ln\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)}{2b}-\frac{a\left(\sum_{\_R=\mathrm{RootOf}\left(a\_Z^{8}-4a\_Z^{6}+(6a-16b)\_Z^{4}-4a\_\right.}\right.}{}$

input
```
int(sinh(d*x+c)^6/(a-b*sinh(d*x+c)^4),x,method=_RETURNVERBOSE)
```

output
```
1/2*x/b-1/8/b/d*exp(2*d*x+2*c)+1/8/b/d*exp(-2*d*x-2*c)+sum(_R*ln(exp(2*d*x
+2*c)+(-128/a*b^4*d^3+128/a^2*b^5*d^3)*_R^3+(32*b^2*d^2-32/a*d^2*b^3)*_R^2
+16*b*d*_R-2/b*a-1),_R=RootOf((256*a*b^6*d^4-256*b^7*d^4)*_Z^4-32*a^2*b^3*
d^2*_Z^2+a^3))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1441 vs. $2(111) = 222$.

Time = 0.16 (sec) , antiderivative size = 1441, normalized size of antiderivative = 9.30

$$\int \frac{\sinh^6(c+dx)}{a-b\sinh^4(c+dx)}\,dx = \text{Too large to display}$$

input
```
integrate(sinh(d*x+c)^6/(a-b*sinh(d*x+c)^4),x, algorithm="fricas")
```

output
```
1/8*(4*d*x*cosh(d*x + c)^2 - cosh(d*x + c)^4 - 4*cosh(d*x + c)*sinh(d*x +
c)^3 - sinh(d*x + c)^4 + 2*(2*d*x - 3*cosh(d*x + c)^2)*sinh(d*x + c)^2 - 2
*(b*d*cosh(d*x + c)^2 + 2*b*d*cosh(d*x + c)*sinh(d*x + c) + b*d*sinh(d*x +
 c)^2)*sqrt(((a*b^3 - b^4)*d^2*sqrt(a^3/((a^2*b^5 - 2*a*b^6 + b^7)*d^4)) +
 a^2)/((a*b^3 - b^4)*d^2))*log(a^2*cosh(d*x + c)^2 + 2*a^2*cosh(d*x + c)*s
inh(d*x + c) + a^2*sinh(d*x + c)^2 + 2*(a^2*b^2 - a*b^3)*d^2*sqrt(a^3/((a^
2*b^5 - 2*a*b^6 + b^7)*d^4)) - a^2 + 2*((a*b^4 - b^5)*d^3*sqrt(a^3/((a^2*b
^5 - 2*a*b^6 + b^7)*d^4)) - a^2*b*d)*sqrt(((a*b^3 - b^4)*d^2*sqrt(a^3/((a^
2*b^5 - 2*a*b^6 + b^7)*d^4)) + a^2)/((a*b^3 - b^4)*d^2))) + 2*(b*d*cosh(d*
x + c)^2 + 2*b*d*cosh(d*x + c)*sinh(d*x + c) + b*d*sinh(d*x + c)^2)*sqrt((
(a*b^3 - b^4)*d^2*sqrt(a^3/((a^2*b^5 - 2*a*b^6 + b^7)*d^4)) + a^2)/((a*b^3
 - b^4)*d^2))*log(a^2*cosh(d*x + c)^2 + 2*a^2*cosh(d*x + c)*sinh(d*x + c)
+ a^2*sinh(d*x + c)^2 + 2*(a^2*b^2 - a*b^3)*d^2*sqrt(a^3/((a^2*b^5 - 2*a*b
^6 + b^7)*d^4)) - a^2 - 2*((a*b^4 - b^5)*d^3*sqrt(a^3/((a^2*b^5 - 2*a*b^6
+ b^7)*d^4)) - a^2*b*d)*sqrt(((a*b^3 - b^4)*d^2*sqrt(a^3/((a^2*b^5 - 2*a*b
^6 + b^7)*d^4)) + a^2)/((a*b^3 - b^4)*d^2))) + 2*(b*d*cosh(d*x + c)^2 + 2*
b*d*cosh(d*x + c)*sinh(d*x + c) + b*d*sinh(d*x + c)^2)*sqrt(-((a*b^3 - b^4
)*d^2*sqrt(a^3/((a^2*b^5 - 2*a*b^6 + b^7)*d^4)) - a^2)/((a*b^3 - b^4)*d^2)
)*log(a^2*cosh(d*x + c)^2 + 2*a^2*cosh(d*x + c)*sinh(d*x + c) + a^2*sinh(d
*x + c)^2 - 2*(a^2*b^2 - a*b^3)*d^2*sqrt(a^3/((a^2*b^5 - 2*a*b^6 + b^7)...
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\sinh^6(c+dx)}{a-b\sinh^4(c+dx)}\,dx = \text{Timed out}$$

input
```
integrate(sinh(d*x+c)**6/(a-b*sinh(d*x+c)**4),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{\sinh^6(c+dx)}{a-b\sinh^4(c+dx)}\,dx = \int -\frac{\sinh\left(dx+c\right)^6}{b\sinh\left(dx+c\right)^4-a}\,dx$$

input
```
integrate(sinh(d*x+c)^6/(a-b*sinh(d*x+c)^4),x, algorithm="maxima")
```

output
```
1/8*(4*d*x*e^(2*d*x + 2*c) - e^(4*d*x + 4*c) + 1)*e^(-2*d*x - 2*c)/(b*d) -
 1/64*integrate(256*(a*e^(6*d*x + 6*c) - 2*a*e^(4*d*x + 4*c) + a*e^(2*d*x
+ 2*c))/(b^2*e^(8*d*x + 8*c) - 4*b^2*e^(6*d*x + 6*c) - 4*b^2*e^(2*d*x + 2*
c) + b^2 - 2*(8*a*b*e^(4*c) - 3*b^2*e^(4*c))*e^(4*d*x)), x)
```

Giac [F]

$$\int \frac{\sinh^6(c+dx)}{a-b\sinh^4(c+dx)}\,dx = \int -\frac{\sinh\left(dx+c\right)^6}{b\sinh\left(dx+c\right)^4-a}\,dx$$

input
```
integrate(sinh(d*x+c)^6/(a-b*sinh(d*x+c)^4),x, algorithm="giac")
```

output
```
sage0*x
```

Mupad [B] (verification not implemented)

Time = 12.25 (sec) , antiderivative size = 2191, normalized size of antiderivative = 14.14

$$\int \frac{\sinh^6(c+dx)}{a-b\sinh^4(c+dx)}\,dx = \text{Too large to display}$$

input
```
int(sinh(c + d*x)^6/(a - b*sinh(c + d*x)^4),x)
```

output
```
log(((((4194304*a^6*d^2*(512*a^4 - 1184*a^3*b - 253*a*b^3 - b^4 + 930*a^2
*b^2 + b^4*exp(2*c + 2*d*x) + 627*a*b^3*exp(2*c + 2*d*x) + 768*a^3*b*exp(2
*c + 2*d*x) - 1392*a^2*b^2*exp(2*c + 2*d*x)))/(b^12*(a - b)^2) - (16777216
*a^6*d^3*(((a^3*b^7)^(1/2) + a^2*b^3)/(b^6*d^2*(a - b)))^(1/2)*(40*a*b^2 -
 35*b^3 + 512*a^3*exp(2*c + 2*d*x) + 64*b^3*exp(2*c + 2*d*x) + 326*a*b^2*e
xp(2*c + 2*d*x) - 896*a^2*b*exp(2*c + 2*d*x)))/(b^11*(a - b)))*(((a^3*b^7)
^(1/2) + a^2*b^3)/(b^6*d^2*(a - b)))^(1/2))/4 - (2097152*a^7*d*(256*a^2*b
- 256*a*b^2 - 5*b^3 - 1024*a^3*exp(2*c + 2*d*x) + 6*b^3*exp(2*c + 2*d*x) +
 756*a*b^2*exp(2*c + 2*d*x) + 256*a^2*b*exp(2*c + 2*d*x)))/(b^14*(a - b)))
*(((a^3*b^7)^(1/2) + a^2*b^3)/(b^6*d^2*(a - b)))^(1/2))/4 - (524288*a^8*(1
85*a*b^2 - 464*a^2*b + 256*a^3 + 24*b^3 - 1024*a^3*exp(2*c + 2*d*x) - 35*b
^3*exp(2*c + 2*d*x) - 988*a*b^2*exp(2*c + 2*d*x) + 2048*a^2*b*exp(2*c + 2*
d*x)))/(b^15*(a - b)^2))*(-((a^3*b^7)^(1/2) + a^2*b^3)/(16*(b^7*d^2 - a*b^
6*d^2)))^(1/2) - log(((((4194304*a^6*d^2*(512*a^4 - 1184*a^3*b - 253*a*b^
3 - b^4 + 930*a^2*b^2 + b^4*exp(2*c + 2*d*x) + 627*a*b^3*exp(2*c + 2*d*x)
+ 768*a^3*b*exp(2*c + 2*d*x) - 1392*a^2*b^2*exp(2*c + 2*d*x)))/(b^12*(a -
b)^2) + (16777216*a^6*d^3*(((a^3*b^7)^(1/2) + a^2*b^3)/(b^6*d^2*(a - b)))^
(1/2)*(40*a*b^2 - 35*b^3 + 512*a^3*exp(2*c + 2*d*x) + 64*b^3*exp(2*c + 2*d
*x) + 326*a*b^2*exp(2*c + 2*d*x) - 896*a^2*b*exp(2*c + 2*d*x)))/(b^11*(a -
 b)))*(((a^3*b^7)^(1/2) + a^2*b^3)/(b^6*d^2*(a - b)))^(1/2))/4 + (20971...
```

Reduce [F]

$$\int \frac{\sinh^6(c+dx)}{a-b\sinh^4(c+dx)}\,dx = \text{too large to display}$$

input
```
int(sinh(d*x+c)^6/(a-b*sinh(d*x+c)^4),x)
```

output
```
( - 8*e**(4*c + 4*d*x)*a*b - e**(4*c + 4*d*x)*b**2 - 32768*e**(4*c + 2*d*x
)*int(e**(2*d*x)/(8*e**(8*c + 8*d*x)*a*b + e**(8*c + 8*d*x)*b**2 - 32*e**(
6*c + 6*d*x)*a*b - 4*e**(6*c + 6*d*x)*b**2 - 128*e**(4*c + 4*d*x)*a**2 + 3
2*e**(4*c + 4*d*x)*a*b + 6*e**(4*c + 4*d*x)*b**2 - 32*e**(2*c + 2*d*x)*a*b
 - 4*e**(2*c + 2*d*x)*b**2 + 8*a*b + b**2),x)*a**4*d + 10240*e**(4*c + 2*d
*x)*int(e**(2*d*x)/(8*e**(8*c + 8*d*x)*a*b + e**(8*c + 8*d*x)*b**2 - 32*e*
*(6*c + 6*d*x)*a*b - 4*e**(6*c + 6*d*x)*b**2 - 128*e**(4*c + 4*d*x)*a**2 +
 32*e**(4*c + 4*d*x)*a*b + 6*e**(4*c + 4*d*x)*b**2 - 32*e**(2*c + 2*d*x)*a
*b - 4*e**(2*c + 2*d*x)*b**2 + 8*a*b + b**2),x)*a**3*b*d + 1536*e**(4*c +
2*d*x)*int(e**(2*d*x)/(8*e**(8*c + 8*d*x)*a*b + e**(8*c + 8*d*x)*b**2 - 32
*e**(6*c + 6*d*x)*a*b - 4*e**(6*c + 6*d*x)*b**2 - 128*e**(4*c + 4*d*x)*a**
2 + 32*e**(4*c + 4*d*x)*a*b + 6*e**(4*c + 4*d*x)*b**2 - 32*e**(2*c + 2*d*x
)*a*b - 4*e**(2*c + 2*d*x)*b**2 + 8*a*b + b**2),x)*a**2*b**2*d - 32*e**(4*
c + 2*d*x)*int(e**(2*d*x)/(8*e**(8*c + 8*d*x)*a*b + e**(8*c + 8*d*x)*b**2
- 32*e**(6*c + 6*d*x)*a*b - 4*e**(6*c + 6*d*x)*b**2 - 128*e**(4*c + 4*d*x)
*a**2 + 32*e**(4*c + 4*d*x)*a*b + 6*e**(4*c + 4*d*x)*b**2 - 32*e**(2*c + 2
*d*x)*a*b - 4*e**(2*c + 2*d*x)*b**2 + 8*a*b + b**2),x)*a*b**3*d + 2048*e**
(2*c + 2*d*x)*int(1/(8*e**(10*c + 10*d*x)*a*b + e**(10*c + 10*d*x)*b**2 -
32*e**(8*c + 8*d*x)*a*b - 4*e**(8*c + 8*d*x)*b**2 - 128*e**(6*c + 6*d*x)*a
**2 + 32*e**(6*c + 6*d*x)*a*b + 6*e**(6*c + 6*d*x)*b**2 - 32*e**(4*c + ...
```

3.211 $\int \frac{\sinh^4(c+dx)}{a-b\sinh^4(c+dx)}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 127

$$\int \frac{\sinh^4(c+dx)}{a-b\sinh^4(c+dx)}\,dx = -\frac{x}{b}+\frac{\sqrt[4]{a}\,\text{arctanh}\left(\frac{\sqrt{\sqrt{a}-\sqrt{b}}\tanh(c+dx)}{\sqrt[4]{a}}\right)}{2\sqrt{\sqrt{a}-\sqrt{b}}\,bd}+\frac{\sqrt[4]{a}\,\text{arctanh}\left(\frac{\sqrt{\sqrt{a}+\sqrt{b}}\tanh(c+dx)}{\sqrt[4]{a}}\right)}{2\sqrt{\sqrt{a}+\sqrt{b}}\,bd}$$

output
```
-x/b+1/2*a^(1/4)*arctanh((a^(1/2)-b^(1/2))^(1/2)*tanh(d*x+c)/a^(1/4))/(a^(
1/2)-b^(1/2))^(1/2)/b/d+1/2*a^(1/4)*arctanh((a^(1/2)+b^(1/2))^(1/2)*tanh(d
*x+c)/a^(1/4))/(a^(1/2)+b^(1/2))^(1/2)/b/d
```

Mathematica [A] (verified)

Time = 2.52 (sec) , antiderivative size = 143, normalized size of antiderivative = 1.13

$$\int \frac{\sinh^4(c+dx)}{a-b\sinh^4(c+dx)}\,dx = \frac{-2(c+dx)-\frac{\sqrt{a}\arctan\left(\frac{(\sqrt{a}-\sqrt{b})\tanh(c+dx)}{\sqrt{-a+\sqrt{a}\sqrt{b}}}\right)}{\sqrt{-a+\sqrt{a}\sqrt{b}}}+\frac{\sqrt{a}\text{arctanh}\left(\frac{(\sqrt{a}+\sqrt{b})\tanh(c+dx)}{\sqrt{a+\sqrt{a}\sqrt{b}}}\right)}{\sqrt{a+\sqrt{a}\sqrt{b}}}}{2bd}$$

input
```
Integrate[Sinh[c + d*x]^4/(a - b*Sinh[c + d*x]^4),x]
```

output
```
(-2*(c + d*x) - (Sqrt[a]*ArcTan[((Sqrt[a] - Sqrt[b])*Tanh[c + d*x])/Sqrt[-
a + Sqrt[a]*Sqrt[b]]])/Sqrt[-a + Sqrt[a]*Sqrt[b]] + (Sqrt[a]*ArcTanh[((Sqr
t[a] + Sqrt[b])*Tanh[c + d*x])/Sqrt[a + Sqrt[a]*Sqrt[b]]])/Sqrt[a + Sqrt[a
]*Sqrt[b]])/(2*b*d)
```

Rubi [A] (verified)

Time = 0.40 (sec) , antiderivative size = 131, normalized size of antiderivative = 1.03, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {3042, 3696, 1610, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sinh^4(c+dx)}{a-b\sinh^4(c+dx)}\,dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \frac{\sin(ic+idx)^4}{a-b\sin(ic+idx)^4}dx$$

$$\Big\downarrow \text{ 3696}$$

$$\frac{\int \frac{\tanh^4(c+dx)}{(1-\tanh^2(c+dx))\left((a-b)\tanh^4(c+dx)-2a\tanh^2(c+dx)+a\right)}d\tanh(c+dx)}{d}$$

$$\Big\downarrow \text{1610}$$

$$\frac{\int \left(\frac{a(1-\tanh^2(c+dx))}{b((a-b)\tanh^4(c+dx)-2a\tanh^2(c+dx)+a)}+\frac{1}{b(\tanh^2(c+dx)-1)}\right)d\tanh(c+dx)}{d}$$

$$\Big\downarrow \text{2009}$$

$$\frac{\frac{\sqrt[4]{a}\text{arctanh}\left(\frac{\sqrt{\sqrt{a}-\sqrt{b}}\tanh(c+dx)}{\sqrt[4]{a}}\right)}{2b\sqrt{\sqrt{a}-\sqrt{b}}}+\frac{\sqrt[4]{a}\text{arctanh}\left(\frac{\sqrt{\sqrt{a}+\sqrt{b}}\tanh(c+dx)}{\sqrt[4]{a}}\right)}{2b\sqrt{\sqrt{a}+\sqrt{b}}}-\frac{\text{arctanh}(\tanh(c+dx))}{b}}{d}$$

input
```
Int[Sinh[c + d*x]^4/(a - b*Sinh[c + d*x]^4),x]
```

output
```
(-(ArcTanh[Tanh[c + d*x]]/b) + (a^(1/4)*ArcTanh[(Sqrt[Sqrt[a] - Sqrt[b]]*T
anh[c + d*x])/a^(1/4)])/(2*Sqrt[Sqrt[a] - Sqrt[b]]*b) + (a^(1/4)*ArcTanh[(
Sqrt[Sqrt[a] + Sqrt[b]]*Tanh[c + d*x])/a^(1/4)])/(2*Sqrt[Sqrt[a] + Sqrt[b]
]*b))/d
```

Defintions of rubi rules used

rule 1610
```
Int[(((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.))/((a_) + (b_.)*(x_)^2 +
 (c_.)*(x_)^4), x_Symbol] :> Int[ExpandIntegrand[(f*x)^m*((d + e*x^2)^q/(a
+ b*x^2 + c*x^4)), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b^2 - 4
*a*c, 0] && IntegerQ[q] && IntegerQ[m]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3696
```
Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^4)^(
p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Simp[ff^(m + 1
)/f   Subst[Int[x^m*((a + 2*a*ff^2*x^2 + (a + b)*ff^4*x^4)^p/(1 + ff^2*x^2)
^(m/2 + 2*p + 1)), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f}, x] &
& IntegerQ[m/2] && IntegerQ[p]
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.87 (sec) , antiderivative size = 121, normalized size of antiderivative = 0.95

method	result
risch	$-\frac{x}{b}+\left(\sum_{\_R=\mathrm{RootOf}\left(\left(256 a\,b^4 d^4-256 b^5 d^4\right)\_Z^4-32 a\,b^2 d^2\,\_Z^2+a\right)}\_R\ln\left(\mathrm{e}^{2dx+2c}+\left(-128 a\,b^2 d^3+12\right.\right.\right.$
derivativedivides	$\frac{-\frac{a\left(\sum_{\_R=\mathrm{RootOf}\left(a\,\_Z^8-4a\,\_Z^6+(6a-16b)\,\_Z^4-4a\,\_Z^2+a\right)}\frac{\left(\_R^6-3\_R^4+3\_R^2-1\right)\ln\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)-\_R\right)}{\_R^7 a-3\_R^5 a+3\_R^3 a-8\_R^3 b-\_R a}\right)}{4b}-\frac{1}{}}{d}$
default	$\frac{-\frac{a\left(\sum_{\_R=\mathrm{RootOf}\left(a\,\_Z^8-4a\,\_Z^6+(6a-16b)\,\_Z^4-4a\,\_Z^2+a\right)}\frac{\left(\_R^6-3\_R^4+3\_R^2-1\right)\ln\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)-\_R\right)}{\_R^7 a-3\_R^5 a+3\_R^3 a-8\_R^3 b-\_R a}\right)}{4b}-\frac{1}{}}{d}$

input
```
int(sinh(d*x+c)^4/(a-b*sinh(d*x+c)^4),x,method=_RETURNVERBOSE)
```

output
```
-x/b+sum(_R*ln(exp(2*d*x+2*c)+(-128*a*b^2*d^3+128*b^3*d^3)*_R^3+(32*a*b*d^
2-32*b^2*d^2)*_R^2+(8*a*d+8*b*d)*_R-2/b*a-1),_R=RootOf((256*a*b^4*d^4-256*
b^5*d^4)*_Z^4-32*a*b^2*d^2*_Z^2+a))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1009 vs. $2(91) = 182$.

Time = 0.14 (sec) , antiderivative size = 1009, normalized size of antiderivative = 7.94

$$\int \frac{\sinh^4(c+dx)}{a-b\sinh^4(c+dx)}\,dx = \text{Too large to display}$$

input
```
integrate(sinh(d*x+c)^4/(a-b*sinh(d*x+c)^4),x, algorithm="fricas")
```

output
```
-1/4*(b*sqrt(((a*b^2 - b^3)*d^2*sqrt(a/((a^2*b^3 - 2*a*b^4 + b^5)*d^4)) +
a)/((a*b^2 - b^3)*d^2))*log(2*(a*b - b^2)*d^2*sqrt(a/((a^2*b^3 - 2*a*b^4 +
 b^5)*d^4)) + cosh(d*x + c)^2 + 2*cosh(d*x + c)*sinh(d*x + c) + sinh(d*x +
 c)^2 + 2*((a*b^2 - b^3)*d^3*sqrt(a/((a^2*b^3 - 2*a*b^4 + b^5)*d^4)) - b*d
)*sqrt(((a*b^2 - b^3)*d^2*sqrt(a/((a^2*b^3 - 2*a*b^4 + b^5)*d^4)) + a)/((a
*b^2 - b^3)*d^2)) - 1) - b*sqrt(((a*b^2 - b^3)*d^2*sqrt(a/((a^2*b^3 - 2*a*
b^4 + b^5)*d^4)) + a)/((a*b^2 - b^3)*d^2))*log(2*(a*b - b^2)*d^2*sqrt(a/((
a^2*b^3 - 2*a*b^4 + b^5)*d^4)) + cosh(d*x + c)^2 + 2*cosh(d*x + c)*sinh(d*
x + c) + sinh(d*x + c)^2 - 2*((a*b^2 - b^3)*d^3*sqrt(a/((a^2*b^3 - 2*a*b^4
 + b^5)*d^4)) - b*d)*sqrt(((a*b^2 - b^3)*d^2*sqrt(a/((a^2*b^3 - 2*a*b^4 +
b^5)*d^4)) + a)/((a*b^2 - b^3)*d^2)) - 1) - b*sqrt(-((a*b^2 - b^3)*d^2*sqr
t(a/((a^2*b^3 - 2*a*b^4 + b^5)*d^4)) - a)/((a*b^2 - b^3)*d^2))*log(-2*(a*b
 - b^2)*d^2*sqrt(a/((a^2*b^3 - 2*a*b^4 + b^5)*d^4)) + cosh(d*x + c)^2 + 2*
cosh(d*x + c)*sinh(d*x + c) + sinh(d*x + c)^2 + 2*((a*b^2 - b^3)*d^3*sqrt(
a/((a^2*b^3 - 2*a*b^4 + b^5)*d^4)) + b*d)*sqrt(-((a*b^2 - b^3)*d^2*sqrt(a/
((a^2*b^3 - 2*a*b^4 + b^5)*d^4)) - a)/((a*b^2 - b^3)*d^2)) - 1) + b*sqrt(-
((a*b^2 - b^3)*d^2*sqrt(a/((a^2*b^3 - 2*a*b^4 + b^5)*d^4)) - a)/((a*b^2 -
b^3)*d^2))*log(-2*(a*b - b^2)*d^2*sqrt(a/((a^2*b^3 - 2*a*b^4 + b^5)*d^4))
+ cosh(d*x + c)^2 + 2*cosh(d*x + c)*sinh(d*x + c) + sinh(d*x + c)^2 - 2*((
a*b^2 - b^3)*d^3*sqrt(a/((a^2*b^3 - 2*a*b^4 + b^5)*d^4)) + b*d)*sqrt(-(...
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\sinh^4(c+dx)}{a-b\sinh^4(c+dx)}\,dx = \text{Timed out}$$

input
```
integrate(sinh(d*x+c)**4/(a-b*sinh(d*x+c)**4),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{\sinh^4(c+dx)}{a-b\sinh^4(c+dx)}\,dx = \int -\frac{\sinh(dx+c)^4}{b\sinh(dx+c)^4-a}\,dx$$

input
```
integrate(sinh(d*x+c)^4/(a-b*sinh(d*x+c)^4),x, algorithm="maxima")
```

output
```
-16*a*integrate(e^(4*d*x + 4*c)/(b^2*e^(8*d*x + 8*c) - 4*b^2*e^(6*d*x + 6*
c) - 4*b^2*e^(2*d*x + 2*c) + b^2 - 2*(8*a*b*e^(4*c) - 3*b^2*e^(4*c))*e^(4*
d*x)), x) - x/b
```

Giac [F]

$$\int \frac{\sinh^4(c+dx)}{a-b\sinh^4(c+dx)}\,dx = \int -\frac{\sinh(dx+c)^4}{b\sinh(dx+c)^4-a}\,dx$$

input
```
integrate(sinh(d*x+c)^4/(a-b*sinh(d*x+c)^4),x, algorithm="giac")
```

output
```
sage0*x
```

Mupad [B] (verification not implemented)

Time = 11.12 (sec) , antiderivative size = 1861, normalized size of antiderivative = 14.65

$$\int \frac{\sinh^4(c+dx)}{a-b\sinh^4(c+dx)}\,dx = \text{Too large to display}$$

input
```
int(sinh(c + d*x)^4/(a - b*sinh(c + d*x)^4),x)
```

output
```
log(((((524288*a^3*d^2*(31*a*b^2 - 128*a^2*b + 128*a^3 - b^3 + 256*a^3*ex
p(2*c + 2*d*x) + b^3*exp(2*c + 2*d*x) + 21*a*b^2*exp(2*c + 2*d*x) - 240*a^
2*b*exp(2*c + 2*d*x)))/(b^8*(a - b)) + (1048576*a^3*d^3*((a*b^2 - (a*b^5)^
(1/2))/(b^4*d^2*(a - b)))^(1/2)*(45*a*b^2 - 104*a^2*b + 64*a^3 - 3*b^3 + 4
*b^3*exp(2*c + 2*d*x) - 50*a*b^2*exp(2*c + 2*d*x) + 48*a^2*b*exp(2*c + 2*d
*x)))/(b^7*(a - b)))*((a*b^2 - (a*b^5)^(1/2))/(b^4*d^2*(a - b)))^(1/2))/4
+ (262144*a^4*d*(72*a*b - 64*a^2 - 9*b^2 + 256*a^2*exp(2*c + 2*d*x) + 31*b
^2*exp(2*c + 2*d*x) - 288*a*b*exp(2*c + 2*d*x)))/(b^9*(a - b)))*((a*b^2 -
(a*b^5)^(1/2))/(b^4*d^2*(a - b)))^(1/2))/4 + (32768*a^4*(128*a*b - 128*a^2
 - 15*b^2 + 256*a^2*exp(2*c + 2*d*x) + 29*b^2*exp(2*c + 2*d*x) - 304*a*b*e
xp(2*c + 2*d*x)))/(b^10*(a - b)))*(-(a*b^2 - (a*b^5)^(1/2))/(16*(b^5*d^2 -
 a*b^4*d^2)))^(1/2) - log(((((524288*a^3*d^2*(31*a*b^2 - 128*a^2*b + 128*
a^3 - b^3 + 256*a^3*exp(2*c + 2*d*x) + b^3*exp(2*c + 2*d*x) + 21*a*b^2*exp
(2*c + 2*d*x) - 240*a^2*b*exp(2*c + 2*d*x)))/(b^8*(a - b)) - (1048576*a^3*
d^3*((a*b^2 - (a*b^5)^(1/2))/(b^4*d^2*(a - b)))^(1/2)*(45*a*b^2 - 104*a^2*
b + 64*a^3 - 3*b^3 + 4*b^3*exp(2*c + 2*d*x) - 50*a*b^2*exp(2*c + 2*d*x) +
48*a^2*b*exp(2*c + 2*d*x)))/(b^7*(a - b)))*((a*b^2 - (a*b^5)^(1/2))/(b^4*d
^2*(a - b)))^(1/2))/4 - (262144*a^4*d*(72*a*b - 64*a^2 - 9*b^2 + 256*a^2*e
xp(2*c + 2*d*x) + 31*b^2*exp(2*c + 2*d*x) - 288*a*b*exp(2*c + 2*d*x)))/(b^
9*(a - b)))*((a*b^2 - (a*b^5)^(1/2))/(b^4*d^2*(a - b)))^(1/2))/4 + (327...
```

Reduce [F]

$$\int \frac{\sinh^4(c+dx)}{a-b\sinh^4(c+dx)}\,dx = \frac{-\left(\int \frac{1}{\sinh(dx+c)^4 b-a}dx\right)a-x}{b}$$

input
```
int(sinh(d*x+c)^4/(a-b*sinh(d*x+c)^4),x)
```

output
```
( - (int(1/(sinh(c + d*x)**4*b - a),x)*a + x))/b
```

3.212 $\int \frac{\sinh^2(c+dx)}{a-b\sinh^4(c+dx)}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 125

$$\int \frac{\sinh^2(c+dx)}{a-b\sinh^4(c+dx)}\,dx = -\frac{\operatorname{arctanh}\left(\frac{\sqrt{\sqrt{a}-\sqrt{b}}\tanh(c+dx)}{\sqrt[4]{a}}\right)}{2\sqrt[4]{a}\sqrt{\sqrt{a}-\sqrt{b}}\sqrt{b}d}+\frac{\operatorname{arctanh}\left(\frac{\sqrt{\sqrt{a}+\sqrt{b}}\tanh(c+dx)}{\sqrt[4]{a}}\right)}{2\sqrt[4]{a}\sqrt{\sqrt{a}+\sqrt{b}}\sqrt{b}d}$$

output
```
-1/2*arctanh((a^(1/2)-b^(1/2))^(1/2)*tanh(d*x+c)/a^(1/4))/a^(1/4)/(a^(1/2)
-b^(1/2))^(1/2)/b^(1/2)/d+1/2*arctanh((a^(1/2)+b^(1/2))^(1/2)*tanh(d*x+c)/
a^(1/4))/a^(1/4)/(a^(1/2)+b^(1/2))^(1/2)/b^(1/2)/d
```

Mathematica [A] (verified)

Time = 4.41 (sec) , antiderivative size = 127, normalized size of antiderivative = 1.02

$$\int \frac{\sinh^2(c+dx)}{a-b\sinh^4(c+dx)}\,dx = \frac{\frac{\arctan\left(\frac{(\sqrt{a}-\sqrt{b})\tanh(c+dx)}{\sqrt{-a+\sqrt{a}\sqrt{b}}}\right)}{\sqrt{-a+\sqrt{a}\sqrt{b}}}+\frac{\operatorname{arctanh}\left(\frac{(\sqrt{a}+\sqrt{b})\tanh(c+dx)}{\sqrt{a+\sqrt{a}\sqrt{b}}}\right)}{\sqrt{a+\sqrt{a}\sqrt{b}}}}{2\sqrt{b}d}$$

input
```
Integrate[Sinh[c + d*x]^2/(a - b*Sinh[c + d*x]^4),x]
```

output
```
(ArcTan[((Sqrt[a] - Sqrt[b])*Tanh[c + d*x])/Sqrt[-a + Sqrt[a]*Sqrt[b]]]/Sq
rt[-a + Sqrt[a]*Sqrt[b]] + ArcTanh[((Sqrt[a] + Sqrt[b])*Tanh[c + d*x])/Sqr
t[a + Sqrt[a]*Sqrt[b]]]/Sqrt[a + Sqrt[a]*Sqrt[b]])/(2*Sqrt[b]*d)
```

Rubi [A] (verified)

Time = 0.34 (sec) , antiderivative size = 168, normalized size of antiderivative = 1.34, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.208$, Rules used = {3042, 25, 3696, 1450, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sinh^2(c+dx)}{a-b\sinh^4(c+dx)}dx$$

$$\Big\downarrow \text{3042}$$

$$\int -\frac{\sin(ic+idx)^2}{a-b\sin(ic+idx)^4}dx$$

$$\Big\downarrow \text{25}$$

$$-\int \frac{\sin(ic+idx)^2}{a-b\sin(ic+idx)^4}dx$$

$$\Big\downarrow \text{3696}$$

$$\frac{\int \frac{\tanh^2(c+dx)}{(a-b)\tanh^4(c+dx)-2a\tanh^2(c+dx)+a}d\tanh(c+dx)}{d}$$

$$\Big\downarrow \text{1450}$$

$$\frac{\frac{1}{2}\left(1-\frac{\sqrt{a}}{\sqrt{b}}\right)\int \frac{1}{(a-b)\tanh^2(c+dx)-\sqrt{a}\left(\sqrt{a}-\sqrt{b}\right)}d\tanh(c+dx)+\frac{1}{2}\left(\frac{\sqrt{a}}{\sqrt{b}}+1\right)\int \frac{1}{(a-b)\tanh^2(c+dx)-\sqrt{a}\left(\sqrt{a}+\sqrt{b}\right)}d\tanh(c+}{d}$$

$$\Big\downarrow \text{221}$$

$$\frac{-\frac{\left(\frac{\sqrt{a}}{\sqrt{b}}+1\right)\operatorname{arctanh}\left(\frac{\sqrt{\sqrt{a}-\sqrt{b}}\tanh(c+dx)}{\sqrt[4]{a}}\right)}{2\sqrt[4]{a}\sqrt{\sqrt{a}-\sqrt{b}}\left(\sqrt{a}+\sqrt{b}\right)}-\frac{\left(1-\frac{\sqrt{a}}{\sqrt{b}}\right)\operatorname{arctanh}\left(\frac{\sqrt{\sqrt{a}+\sqrt{b}}\tanh(c+dx)}{\sqrt[4]{a}}\right)}{2\sqrt[4]{a}\left(\sqrt{a}-\sqrt{b}\right)\sqrt{\sqrt{a}+\sqrt{b}}}}{d}$$

input
```
Int[Sinh[c + d*x]^2/(a - b*Sinh[c + d*x]^4),x]
```

output
```
(-1/2*((1 + Sqrt[a]/Sqrt[b])*ArcTanh[(Sqrt[Sqrt[a] - Sqrt[b]]*Tanh[c + d*x
])/a^(1/4)])/(a^(1/4)*Sqrt[Sqrt[a] - Sqrt[b]]*(Sqrt[a] + Sqrt[b])) - ((1 -
 Sqrt[a]/Sqrt[b])*ArcTanh[(Sqrt[Sqrt[a] + Sqrt[b]]*Tanh[c + d*x])/a^(1/4)]
)/(2*a^(1/4)*(Sqrt[a] - Sqrt[b])*Sqrt[Sqrt[a] + Sqrt[b]]))/d
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 1450
```
Int[((d_.)*(x_))^(m_)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :> Wi
th[{q = Rt[b^2 - 4*a*c, 2]}, Simp[(d^2/2)*(b/q + 1)   Int[(d*x)^(m - 2)/(b/
2 + q/2 + c*x^2), x], x] - Simp[(d^2/2)*(b/q - 1)   Int[(d*x)^(m - 2)/(b/2
- q/2 + c*x^2), x], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[b^2 - 4*a*c, 0] &&
 GeQ[m, 2]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3696
```
Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^4)^(
p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Simp[ff^(m + 1
)/f   Subst[Int[x^m*((a + 2*a*ff^2*x^2 + (a + b)*ff^4*x^4)^p/(1 + ff^2*x^2)
^(m/2 + 2*p + 1)), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f}, x] &
& IntegerQ[m/2] && IntegerQ[p]
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.75 (sec) , antiderivative size = 94, normalized size of antiderivative = 0.75

method	result
derivativedivides	$-\frac{\sum_{\_R=\mathrm{RootOf}\left(a\_Z^8-4a\_Z^6+(6a-16b)\_Z^4-4a\_Z^2+a\right)}\frac{\left(\_R^4-\_R^2\right)\ln\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)-\_R\right)}{\_R^7a-3\_R^5a+3\_R^3a-8\_R^3b-\_Ra}}{d}$
default	$-\frac{\sum_{\_R=\mathrm{RootOf}\left(a\_Z^8-4a\_Z^6+(6a-16b)\_Z^4-4a\_Z^2+a\right)}\frac{\left(\_R^4-\_R^2\right)\ln\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)-\_R\right)}{\_R^7a-3\_R^5a+3\_R^3a-8\_R^3b-\_Ra}}{d}$
risch	$\sum_{\_R=\mathrm{RootOf}\left(1+(256a^2b^2d^4-256a\,b^3d^4)\_Z^4-32a\,d^2\_Z^2b\right)}\_R\ln\left(\mathrm{e}^{2dx+2c}+\left(-128a^2b\,d^3+128a\,b^2d^3\right.\right.$

input
```
int(sinh(d*x+c)^2/(a-b*sinh(d*x+c)^4),x,method=_RETURNVERBOSE)
```

output
```
-1/d*sum((_R^4-_R^2)/(_R^7*a-3*_R^5*a+3*_R^3*a-8*_R^3*b-_R*a)*ln(tanh(1/2*
d*x+1/2*c)-_R),_R=RootOf(a*_Z^8-4*a*_Z^6+(6*a-16*b)*_Z^4-4*a*_Z^2+a))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 975 vs. $2(85) = 170$.

Time = 0.12 (sec) , antiderivative size = 975, normalized size of antiderivative = 7.80

$$\int \frac{\sinh^2(c+dx)}{a-b\sinh^4(c+dx)}\,dx = \text{Too large to display}$$

input
```
integrate(sinh(d*x+c)^2/(a-b*sinh(d*x+c)^4),x, algorithm="fricas")
```

output
```
-1/4*sqrt(((a*b - b^2)*d^2*sqrt(1/((a^3*b - 2*a^2*b^2 + a*b^3)*d^4)) + 1)/
((a*b - b^2)*d^2))*log(2*(a^2 - a*b)*d^2*sqrt(1/((a^3*b - 2*a^2*b^2 + a*b^
3)*d^4)) + cosh(d*x + c)^2 + 2*cosh(d*x + c)*sinh(d*x + c) + sinh(d*x + c)
^2 + 2*((a^2*b - a*b^2)*d^3*sqrt(1/((a^3*b - 2*a^2*b^2 + a*b^3)*d^4)) - a*
d)*sqrt(((a*b - b^2)*d^2*sqrt(1/((a^3*b - 2*a^2*b^2 + a*b^3)*d^4)) + 1)/((
a*b - b^2)*d^2)) - 1) + 1/4*sqrt(((a*b - b^2)*d^2*sqrt(1/((a^3*b - 2*a^2*b
^2 + a*b^3)*d^4)) + 1)/((a*b - b^2)*d^2))*log(2*(a^2 - a*b)*d^2*sqrt(1/((a
^3*b - 2*a^2*b^2 + a*b^3)*d^4)) + cosh(d*x + c)^2 + 2*cosh(d*x + c)*sinh(d
*x + c) + sinh(d*x + c)^2 - 2*((a^2*b - a*b^2)*d^3*sqrt(1/((a^3*b - 2*a^2*
b^2 + a*b^3)*d^4)) - a*d)*sqrt(((a*b - b^2)*d^2*sqrt(1/((a^3*b - 2*a^2*b^2
 + a*b^3)*d^4)) + 1)/((a*b - b^2)*d^2)) - 1) + 1/4*sqrt(-((a*b - b^2)*d^2*
sqrt(1/((a^3*b - 2*a^2*b^2 + a*b^3)*d^4)) - 1)/((a*b - b^2)*d^2))*log(-2*(
a^2 - a*b)*d^2*sqrt(1/((a^3*b - 2*a^2*b^2 + a*b^3)*d^4)) + cosh(d*x + c)^2
 + 2*cosh(d*x + c)*sinh(d*x + c) + sinh(d*x + c)^2 + 2*((a^2*b - a*b^2)*d^
3*sqrt(1/((a^3*b - 2*a^2*b^2 + a*b^3)*d^4)) + a*d)*sqrt(-((a*b - b^2)*d^2*
sqrt(1/((a^3*b - 2*a^2*b^2 + a*b^3)*d^4)) - 1)/((a*b - b^2)*d^2)) - 1) - 1
/4*sqrt(-((a*b - b^2)*d^2*sqrt(1/((a^3*b - 2*a^2*b^2 + a*b^3)*d^4)) - 1)/(
(a*b - b^2)*d^2))*log(-2*(a^2 - a*b)*d^2*sqrt(1/((a^3*b - 2*a^2*b^2 + a*b^
3)*d^4)) + cosh(d*x + c)^2 + 2*cosh(d*x + c)*sinh(d*x + c) + sinh(d*x + c)
^2 - 2*((a^2*b - a*b^2)*d^3*sqrt(1/((a^3*b - 2*a^2*b^2 + a*b^3)*d^4)) +...
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\sinh^2(c+dx)}{a-b\sinh^4(c+dx)}\,dx = \text{Timed out}$$

input
```
integrate(sinh(d*x+c)**2/(a-b*sinh(d*x+c)**4),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{\sinh^2(c+dx)}{a-b\sinh^4(c+dx)}\,dx = \int -\frac{\sinh\left(dx+c\right)^2}{b\sinh\left(dx+c\right)^4-a}\,dx$$

input
```
integrate(sinh(d*x+c)^2/(a-b*sinh(d*x+c)^4),x, algorithm="maxima")
```

output
```
-integrate(sinh(d*x + c)^2/(b*sinh(d*x + c)^4 - a), x)
```

Giac [F]

$$\int \frac{\sinh^2(c+dx)}{a-b\sinh^4(c+dx)}\,dx = \int -\frac{\sinh\left(dx+c\right)^2}{b\sinh\left(dx+c\right)^4-a}\,dx$$

input
```
integrate(sinh(d*x+c)^2/(a-b*sinh(d*x+c)^4),x, algorithm="giac")
```

output
```
sage0*x
```

Mupad [B] (verification not implemented)

Time = 13.66 (sec) , antiderivative size = 1859, normalized size of antiderivative = 14.87

$$\int \frac{\sinh^2(c+dx)}{a-b\sinh^4(c+dx)}\,dx = \text{Too large to display}$$

input
```
int(sinh(c + d*x)^2/(a - b*sinh(c + d*x)^4),x)
```

output
```
log(((((262144*a^2*d^2*(102*a*b - 128*a^2 - 22*b^2 - 272*a^2*exp(2*c + 2*
d*x) + 19*b^2*exp(2*c + 2*d*x) + 189*a*b*exp(2*c + 2*d*x)))/(b^6*(a - b))
- (131072*a^2*d^3*((a*b + (a*b^3)^(1/2))/(a*b^2*d^2*(a - b)))^(1/2)*(119*a
*b^2 - 136*a^2*b + b^3 - 1024*a^3*exp(2*c + 2*d*x) + 9*b^3*exp(2*c + 2*d*x
) - 809*a*b^2*exp(2*c + 2*d*x) + 1808*a^2*b*exp(2*c + 2*d*x)))/(b^6*(a - b
)))*((a*b + (a*b^3)^(1/2))/(a*b^2*d^2*(a - b)))^(1/2))/4 + (32768*a*d*(120
*a^2*b - 129*a*b^2 + b^3 - 1024*a^3*exp(2*c + 2*d*x) - b^3*exp(2*c + 2*d*x
) + 201*a*b^2*exp(2*c + 2*d*x) + 816*a^2*b*exp(2*c + 2*d*x)))/(b^7*(a - b)
))*((a*b + (a*b^3)^(1/2))/(a*b^2*d^2*(a - b)))^(1/2))/4 - (16384*a*(106*a*
b - 128*a^2 - 2*b^2 + 240*a^2*exp(2*c + 2*d*x) + 3*b^2*exp(2*c + 2*d*x) -
275*a*b*exp(2*c + 2*d*x)))/(b^7*(a - b)))*(-(a*b + (a*b^3)^(1/2))/(16*(a*b
^3*d^2 - a^2*b^2*d^2)))^(1/2) - log(((((262144*a^2*d^2*(102*a*b - 128*a^2
 - 22*b^2 - 272*a^2*exp(2*c + 2*d*x) + 19*b^2*exp(2*c + 2*d*x) + 189*a*b*e
xp(2*c + 2*d*x)))/(b^6*(a - b)) + (131072*a^2*d^3*((a*b + (a*b^3)^(1/2))/(
a*b^2*d^2*(a - b)))^(1/2)*(119*a*b^2 - 136*a^2*b + b^3 - 1024*a^3*exp(2*c
+ 2*d*x) + 9*b^3*exp(2*c + 2*d*x) - 809*a*b^2*exp(2*c + 2*d*x) + 1808*a^2*
b*exp(2*c + 2*d*x)))/(b^6*(a - b)))*((a*b + (a*b^3)^(1/2))/(a*b^2*d^2*(a -
 b)))^(1/2))/4 - (32768*a*d*(120*a^2*b - 129*a*b^2 + b^3 - 1024*a^3*exp(2*
c + 2*d*x) - b^3*exp(2*c + 2*d*x) + 201*a*b^2*exp(2*c + 2*d*x) + 816*a^2*b
*exp(2*c + 2*d*x)))/(b^7*(a - b)))*((a*b + (a*b^3)^(1/2))/(a*b^2*d^2*(a...
```

Reduce [F]

$$\int \frac{\sinh^2(c+dx)}{a-b\sinh^4(c+dx)}\,dx$$
$$= \frac{64e^{4c}\left(\int \frac{e^{4dx}}{e^{8dx+8c}b-4e^{6dx+6c}b-16e^{4dx+4c}a+6e^{4dx+4c}b-4e^{2dx+2c}b+b}dx\right)ad-8e^{4c}\left(\int \frac{e^{4dx}}{e^{8dx+8c}b-4e^{6dx+6c}b-16e^{4dx+4c}a+6e^{4dx+4c}\cdots}\right.}{}$$

input
```
int(sinh(d*x+c)^2/(a-b*sinh(d*x+c)^4),x)
```

output

```
(64*e**(4*c)*int(e**(4*d*x)/(e**(8*c + 8*d*x)*b - 4*e**(6*c + 6*d*x)*b - 1
6*e**(4*c + 4*d*x)*a + 6*e**(4*c + 4*d*x)*b - 4*e**(2*c + 2*d*x)*b + b),x)
*a*d - 8*e**(4*c)*int(e**(4*d*x)/(e**(8*c + 8*d*x)*b - 4*e**(6*c + 6*d*x)*
b - 16*e**(4*c + 4*d*x)*a + 6*e**(4*c + 4*d*x)*b - 4*e**(2*c + 2*d*x)*b +
b),x)*b*d + 16*e**(2*c)*int(e**(2*d*x)/(e**(8*c + 8*d*x)*b - 4*e**(6*c + 6
*d*x)*b - 16*e**(4*c + 4*d*x)*a + 6*e**(4*c + 4*d*x)*b - 4*e**(2*c + 2*d*x
)*b + b),x)*b*d - 8*int(1/(e**(8*c + 8*d*x)*b - 4*e**(6*c + 6*d*x)*b - 16*
e**(4*c + 4*d*x)*a + 6*e**(4*c + 4*d*x)*b - 4*e**(2*c + 2*d*x)*b + b),x)*b
*d - log(e**(8*c + 8*d*x)*b - 4*e**(6*c + 6*d*x)*b - 16*e**(4*c + 4*d*x)*a
 + 6*e**(4*c + 4*d*x)*b - 4*e**(2*c + 2*d*x)*b + b) + 8*d*x)/(2*b*d)
```

3.213 $\int \frac{1}{a-b\sinh^4(c+dx)}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 115

$$\int \frac{1}{a-b\sinh^4(c+dx)}\,dx=\frac{\operatorname{arctanh}\left(\frac{\sqrt{\sqrt{a}-\sqrt{b}}\tanh(c+dx)}{\sqrt[4]{a}}\right)}{2a^{3/4}\sqrt{\sqrt{a}-\sqrt{b}}d}+\frac{\operatorname{arctanh}\left(\frac{\sqrt{\sqrt{a}+\sqrt{b}}\tanh(c+dx)}{\sqrt[4]{a}}\right)}{2a^{3/4}\sqrt{\sqrt{a}+\sqrt{b}}d}$$

output
```
1/2*arctanh((a^(1/2)-b^(1/2))^(1/2)*tanh(d*x+c)/a^(1/4))/a^(3/4)/(a^(1/2)-
b^(1/2))^(1/2)/d+1/2*arctanh((a^(1/2)+b^(1/2))^(1/2)*tanh(d*x+c)/a^(1/4))/
a^(3/4)/(a^(1/2)+b^(1/2))^(1/2)/d
```

Mathematica [A] (verified)

Time = 1.12 (sec) , antiderivative size = 128, normalized size of antiderivative = 1.11

$$\int \frac{1}{a-b\sinh^4(c+dx)}\,dx=\frac{-\frac{\arctan\left(\frac{(\sqrt{a}-\sqrt{b})\tanh(c+dx)}{\sqrt{-a+\sqrt{a}\sqrt{b}}}\right)}{\sqrt{-a+\sqrt{a}\sqrt{b}}}+\frac{\operatorname{arctanh}\left(\frac{(\sqrt{a}+\sqrt{b})\tanh(c+dx)}{\sqrt{a+\sqrt{a}\sqrt{b}}}\right)}{\sqrt{a+\sqrt{a}\sqrt{b}}}}{2\sqrt{a}d}$$

input
```
Integrate[(a - b*Sinh[c + d*x]^4)^(-1),x]
```

output
```
(-(ArcTan[((Sqrt[a] - Sqrt[b])*Tanh[c + d*x])/Sqrt[-a + Sqrt[a]*Sqrt[b]]]/
Sqrt[-a + Sqrt[a]*Sqrt[b]]) + ArcTanh[((Sqrt[a] + Sqrt[b])*Tanh[c + d*x])/
Sqrt[a + Sqrt[a]*Sqrt[b]]]/Sqrt[a + Sqrt[a]*Sqrt[b]])/(2*Sqrt[a]*d)
```

Rubi [A] (verified)

Time = 0.34 (sec) , antiderivative size = 168, normalized size of antiderivative = 1.46, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.267$, Rules used = {3042, 3688, 1480, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{a-b\sinh^4(c+dx)}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{a-b\sin(ic+idx)^4}dx$$

$$\Big\downarrow \text{3688}$$

$$\frac{\int \frac{1-\tanh^2(c+dx)}{(a-b)\tanh^4(c+dx)-2a\tanh^2(c+dx)+a}d\tanh(c+dx)}{d}$$

$$\Big\downarrow \text{1480}$$

$$\frac{-\frac{1}{2}\left(1-\frac{\sqrt{b}}{\sqrt{a}}\right)\int \frac{1}{(a-b)\tanh^2(c+dx)-\sqrt{a}\left(\sqrt{a}-\sqrt{b}\right)}d\tanh(c+dx)-\frac{1}{2}\left(\frac{\sqrt{b}}{\sqrt{a}}+1\right)\int \frac{1}{(a-b)\tanh^2(c+dx)-\sqrt{a}\left(\sqrt{a}+\sqrt{b}\right)}d\tanh(c+dx)}{d}$$

$$\Big\downarrow \text{221}$$

$$\frac{\frac{\left(\frac{\sqrt{b}}{\sqrt{a}}+1\right)\text{arctanh}\left(\frac{\sqrt{\sqrt{a}-\sqrt{b}}\tanh(c+dx)}{\sqrt[4]{a}}\right)}{2\sqrt[4]{a}\sqrt{\sqrt{a}-\sqrt{b}}\left(\sqrt{a}+\sqrt{b}\right)}+\frac{\left(1-\frac{\sqrt{b}}{\sqrt{a}}\right)\text{arctanh}\left(\frac{\sqrt{\sqrt{a}+\sqrt{b}}\tanh(c+dx)}{\sqrt[4]{a}}\right)}{2\sqrt[4]{a}\left(\sqrt{a}-\sqrt{b}\right)\sqrt{\sqrt{a}+\sqrt{b}}}}{d}$$

input
```
Int[(a - b*Sinh[c + d*x]^4)^(-1),x]
```

output
```
(((1 + Sqrt[b]/Sqrt[a])*ArcTanh[(Sqrt[Sqrt[a] - Sqrt[b]]*Tanh[c + d*x])/a^
(1/4)])/(2*a^(1/4)*Sqrt[Sqrt[a] - Sqrt[b]]*(Sqrt[a] + Sqrt[b])) + ((1 - Sq
rt[b]/Sqrt[a])*ArcTanh[(Sqrt[Sqrt[a] + Sqrt[b]]*Tanh[c + d*x])/a^(1/4)])/(
2*a^(1/4)*(Sqrt[a] - Sqrt[b])*Sqrt[Sqrt[a] + Sqrt[b]]))/d
```

Defintions of rubi rules used

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 1480
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Simp[(e/2 + (2*c*d - b*e)/(2*q))   Int[1/(
b/2 - q/2 + c*x^2), x], x] + Simp[(e/2 - (2*c*d - b*e)/(2*q))   Int[1/(b/2
+ q/2 + c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0]
 && NeQ[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3688
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^4)^(p_.), x_Symbol] :> With[{ff =
 FreeFactors[Tan[e + f*x], x]}, Simp[ff/f   Subst[Int[(a + 2*a*ff^2*x^2 + (
a + b)*ff^4*x^4)^p/(1 + ff^2*x^2)^(2*p + 1), x], x, Tan[e + f*x]/ff], x]] /
; FreeQ[{a, b, e, f}, x] && IntegerQ[p]
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.68 (sec) , antiderivative size = 102, normalized size of antiderivative = 0.89

method	result
derivativedivides	$\frac{\sum_{\_R=\mathrm{RootOf}\left(a\_Z^8-4a\_Z^6+(6a-16b)\_Z^4-4a\_Z^2+a\right)}\frac{\left(-\_R^6+3\_R^4-3\_R^2+1\right)\ln\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)-\_R\right)}{\_R^7a-3\_R^5a+3\_R^3a-8\_R^3b-\_Ra}}{4d}$
default	$\frac{\sum_{\_R=\mathrm{RootOf}\left(a\_Z^8-4a\_Z^6+(6a-16b)\_Z^4-4a\_Z^2+a\right)}\frac{\left(-\_R^6+3\_R^4-3\_R^2+1\right)\ln\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)-\_R\right)}{\_R^7a-3\_R^5a+3\_R^3a-8\_R^3b-\_Ra}}{4d}$
risch	$\sum_{\_R=\mathrm{RootOf}\left(1+(256a^4d^4-256b\,d^4a^3)\_Z^4-32a^2d^2\_Z^2\right)}\_R\ln\left(\mathrm{e}^{2dx+2c}+\left(-\frac{128d^3a^4}{b}+128a^3d^3\right)\_R\right.$

input
```
int(1/(a-b*sinh(d*x+c)^4),x,method=_RETURNVERBOSE)
```

output
```
1/4/d*sum((-_R^6+3*_R^4-3*_R^2+1)/(_R^7*a-3*_R^5*a+3*_R^3*a-8*_R^3*b-_R*a)
*ln(tanh(1/2*d*x+1/2*c)-_R),_R=RootOf(a*_Z^8-4*a*_Z^6+(6*a-16*b)*_Z^4-4*a*
_Z^2+a))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 975 vs. 2(79) = 158.

Time = 0.17 (sec) , antiderivative size = 975, normalized size of antiderivative = 8.48

$$\int \frac{1}{a-b\sinh^4(c+dx)}\,dx = \text{Too large to display}$$

input
```
integrate(1/(a-b*sinh(d*x+c)^4),x, algorithm="fricas")
```

output
```
-1/4*sqrt(((a^2 - a*b)*d^2*sqrt(b/((a^5 - 2*a^4*b + a^3*b^2)*d^4)) + 1)/((
a^2 - a*b)*d^2))*log(2*(a^3 - a^2*b)*d^2*sqrt(b/((a^5 - 2*a^4*b + a^3*b^2)
*d^4)) + b*cosh(d*x + c)^2 + 2*b*cosh(d*x + c)*sinh(d*x + c) + b*sinh(d*x
+ c)^2 + 2*((a^4 - a^3*b)*d^3*sqrt(b/((a^5 - 2*a^4*b + a^3*b^2)*d^4)) - a*
b*d)*sqrt(((a^2 - a*b)*d^2*sqrt(b/((a^5 - 2*a^4*b + a^3*b^2)*d^4)) + 1)/((
a^2 - a*b)*d^2)) - b) + 1/4*sqrt(((a^2 - a*b)*d^2*sqrt(b/((a^5 - 2*a^4*b +
 a^3*b^2)*d^4)) + 1)/((a^2 - a*b)*d^2))*log(2*(a^3 - a^2*b)*d^2*sqrt(b/((a
^5 - 2*a^4*b + a^3*b^2)*d^4)) + b*cosh(d*x + c)^2 + 2*b*cosh(d*x + c)*sinh
(d*x + c) + b*sinh(d*x + c)^2 - 2*((a^4 - a^3*b)*d^3*sqrt(b/((a^5 - 2*a^4*
b + a^3*b^2)*d^4)) - a*b*d)*sqrt(((a^2 - a*b)*d^2*sqrt(b/((a^5 - 2*a^4*b +
 a^3*b^2)*d^4)) + 1)/((a^2 - a*b)*d^2)) - b) + 1/4*sqrt(-((a^2 - a*b)*d^2*
sqrt(b/((a^5 - 2*a^4*b + a^3*b^2)*d^4)) - 1)/((a^2 - a*b)*d^2))*log(-2*(a^
3 - a^2*b)*d^2*sqrt(b/((a^5 - 2*a^4*b + a^3*b^2)*d^4)) + b*cosh(d*x + c)^2
 + 2*b*cosh(d*x + c)*sinh(d*x + c) + b*sinh(d*x + c)^2 + 2*((a^4 - a^3*b)*
d^3*sqrt(b/((a^5 - 2*a^4*b + a^3*b^2)*d^4)) + a*b*d)*sqrt(-((a^2 - a*b)*d^
2*sqrt(b/((a^5 - 2*a^4*b + a^3*b^2)*d^4)) - 1)/((a^2 - a*b)*d^2)) - b) - 1
/4*sqrt(-((a^2 - a*b)*d^2*sqrt(b/((a^5 - 2*a^4*b + a^3*b^2)*d^4)) - 1)/((a
^2 - a*b)*d^2))*log(-2*(a^3 - a^2*b)*d^2*sqrt(b/((a^5 - 2*a^4*b + a^3*b^2)
*d^4)) + b*cosh(d*x + c)^2 + 2*b*cosh(d*x + c)*sinh(d*x + c) + b*sinh(d*x
+ c)^2 - 2*((a^4 - a^3*b)*d^3*sqrt(b/((a^5 - 2*a^4*b + a^3*b^2)*d^4)) +...
```

Sympy [F(-1)]

Timed out.

$$\int \frac{1}{a-b\sinh^4(c+dx)}\,dx=\text{Timed out}$$

input
```
integrate(1/(a-b*sinh(d*x+c)**4),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{1}{a - b\sinh^4(c + dx)}\,dx = \int -\frac{1}{b\sinh{(dx + c)}^4 - a}\,dx$$

input
```
integrate(1/(a-b*sinh(d*x+c)^4),x, algorithm="maxima")
```

output
```
-integrate(1/(b*sinh(d*x + c)^4 - a), x)
```

Giac [F]

$$\int \frac{1}{a - b\sinh^4(c + dx)}\,dx = \int -\frac{1}{b\sinh{(dx + c)}^4 - a}\,dx$$

input
```
integrate(1/(a-b*sinh(d*x+c)^4),x, algorithm="giac")
```

output
```
sage0*x
```

Mupad [B] (verification not implemented)

Time = 10.99 (sec) , antiderivative size = 1787, normalized size of antiderivative = 15.54

$$\int \frac{1}{a - b\sinh^4(c + dx)}\,dx = \text{Too large to display}$$

input
```
int(1/(a - b*sinh(c + d*x)^4),x)
```

output
```
log(((((524288*d^2*(31*a*b^2 - 128*a^2*b + 128*a^3 - b^3 + 256*a^3*exp(2*
c + 2*d*x) + b^3*exp(2*c + 2*d*x) + 21*a*b^2*exp(2*c + 2*d*x) - 240*a^2*b*
exp(2*c + 2*d*x)))/(b^5*(a - b)) + (1048576*a*d^3*((a^2 - (a^3*b)^(1/2))/(
a^3*d^2*(a - b)))^(1/2)*(45*a*b^2 - 104*a^2*b + 64*a^3 - 3*b^3 + 4*b^3*exp
(2*c + 2*d*x) - 50*a*b^2*exp(2*c + 2*d*x) + 48*a^2*b*exp(2*c + 2*d*x)))/(b
^5*(a - b)))*((a^2 - (a^3*b)^(1/2))/(a^3*d^2*(a - b)))^(1/2))/4 + (262144*
d*(72*a*b - 64*a^2 - 9*b^2 + 256*a^2*exp(2*c + 2*d*x) + 31*b^2*exp(2*c + 2
*d*x) - 288*a*b*exp(2*c + 2*d*x)))/(b^5*(a - b)))*((a^2 - (a^3*b)^(1/2))/(
a^3*d^2*(a - b)))^(1/2))/4 + (32768*(128*a*b - 128*a^2 - 15*b^2 + 256*a^2*
exp(2*c + 2*d*x) + 29*b^2*exp(2*c + 2*d*x) - 304*a*b*exp(2*c + 2*d*x)))/(a
*b^5*(a - b)))*((a^2 - (a^3*b)^(1/2))/(16*(a^4*d^2 - a^3*b*d^2)))^(1/2) -
log(((((524288*d^2*(31*a*b^2 - 128*a^2*b + 128*a^3 - b^3 + 256*a^3*exp(2*
c + 2*d*x) + b^3*exp(2*c + 2*d*x) + 21*a*b^2*exp(2*c + 2*d*x) - 240*a^2*b*
exp(2*c + 2*d*x)))/(b^5*(a - b)) - (1048576*a*d^3*((a^2 - (a^3*b)^(1/2))/(
a^3*d^2*(a - b)))^(1/2)*(45*a*b^2 - 104*a^2*b + 64*a^3 - 3*b^3 + 4*b^3*exp
(2*c + 2*d*x) - 50*a*b^2*exp(2*c + 2*d*x) + 48*a^2*b*exp(2*c + 2*d*x)))/(b
^5*(a - b)))*((a^2 - (a^3*b)^(1/2))/(a^3*d^2*(a - b)))^(1/2))/4 - (262144*
d*(72*a*b - 64*a^2 - 9*b^2 + 256*a^2*exp(2*c + 2*d*x) + 31*b^2*exp(2*c + 2
*d*x) - 288*a*b*exp(2*c + 2*d*x)))/(b^5*(a - b)))*((a^2 - (a^3*b)^(1/2))/(
a^3*d^2*(a - b)))^(1/2))/4 + (32768*(128*a*b - 128*a^2 - 15*b^2 + 256*a...
```

Reduce [F]

$$\int \frac{1}{a-b\sinh^4(c+dx)}\,dx=-\left(\int \frac{1}{\sinh{(dx+c)}^4\,b-a}dx\right)$$

input
```
int(1/(a-b*sinh(d*x+c)^4),x)
```

output
```
 - int(1/(sinh(c + d*x)**4*b - a),x)
```

3.214 $\int \frac{\operatorname{csch}^2(c+dx)}{a-b\sinh^4(c+dx)}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 139

$$\int \frac{\operatorname{csch}^2(c+dx)}{a-b\sinh^4(c+dx)}\,dx = -\frac{\sqrt{b}\,\operatorname{arctanh}\left(\frac{\sqrt{\sqrt{a}-\sqrt{b}}\tanh(c+dx)}{\sqrt[4]{a}}\right)}{2a^{5/4}\sqrt{\sqrt{a}-\sqrt{b}}\,d} + \frac{\sqrt{b}\,\operatorname{arctanh}\left(\frac{\sqrt{\sqrt{a}+\sqrt{b}}\tanh(c+dx)}{\sqrt[4]{a}}\right)}{2a^{5/4}\sqrt{\sqrt{a}+\sqrt{b}}\,d} - \frac{\coth(c+dx)}{ad}$$

output
```
-1/2*b^(1/2)*arctanh((a^(1/2)-b^(1/2))^(1/2)*tanh(d*x+c)/a^(1/4))/a^(5/4)/
(a^(1/2)-b^(1/2))^(1/2)/d+1/2*b^(1/2)*arctanh((a^(1/2)+b^(1/2))^(1/2)*tanh
(d*x+c)/a^(1/4))/a^(5/4)/(a^(1/2)+b^(1/2))^(1/2)/d-coth(d*x+c)/a/d
```

Mathematica [A] (verified)

Time = 1.23 (sec) , antiderivative size = 143, normalized size of antiderivative = 1.03

$$\int \frac{\operatorname{csch}^2(c+dx)}{a-b\sinh^4(c+dx)}\,dx = \frac{\frac{\sqrt{b}\arctan\left(\frac{(\sqrt{a}-\sqrt{b})\tanh(c+dx)}{\sqrt{-a+\sqrt{a}\sqrt{b}}}\right)}{\sqrt{-a+\sqrt{a}\sqrt{b}}}+\frac{\sqrt{b}\operatorname{arctanh}\left(\frac{(\sqrt{a}+\sqrt{b})\tanh(c+dx)}{\sqrt{a+\sqrt{a}\sqrt{b}}}\right)}{\sqrt{a+\sqrt{a}\sqrt{b}}}-2\coth(c+dx)}{2ad}$$

input
```
Integrate[Csch[c + d*x]^2/(a - b*Sinh[c + d*x]^4),x]
```

output
```
((Sqrt[b]*ArcTan[((Sqrt[a] - Sqrt[b])*Tanh[c + d*x])/Sqrt[-a + Sqrt[a]*Sqr
t[b]]])/Sqrt[-a + Sqrt[a]*Sqrt[b]] + (Sqrt[b]*ArcTanh[((Sqrt[a] + Sqrt[b])
*Tanh[c + d*x])/Sqrt[a + Sqrt[a]*Sqrt[b]]])/Sqrt[a + Sqrt[a]*Sqrt[b]] - 2*
Coth[c + d*x])/(2*a*d)
```

Rubi [A] (verified)

Time = 0.43 (sec) , antiderivative size = 134, normalized size of antiderivative = 0.96, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.208, Rules used = {3042, 25, 3696, 1610, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\operatorname{csch}^2(c+dx)}{a-b\sinh^4(c+dx)}dx$$

$$\Big\downarrow \text{3042}$$

$$\int -\frac{1}{\sin(ic+idx)^2\left(a-b\sin(ic+idx)^4\right)}dx$$

$$\Big\downarrow \text{25}$$

$$-\int \frac{1}{\sin(ic+idx)^2\left(a-b\sin(ic+idx)^4\right)}dx$$

$$\Big\downarrow \text{3696}$$

$$\frac{\int \frac{\coth^2(c+dx)\left(1-\tanh^2(c+dx)\right)^2}{(a-b)\tanh^4(c+dx)-2a\tanh^2(c+dx)+a}d\tanh(c+dx)}{d}$$

$$\Big\downarrow \text{ 1610}$$

$$\frac{\int \left(\frac{\coth^2(c+dx)}{a}+\frac{b\tanh^2(c+dx)}{a\left((a-b)\tanh^4(c+dx)-2a\tanh^2(c+dx)+a\right)}\right)d\tanh(c+dx)}{d}$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{-\frac{\sqrt{b}\text{arctanh}\left(\frac{\sqrt{\sqrt{a}-\sqrt{b}}\tanh(c+dx)}{\sqrt[4]{a}}\right)}{2a^{5/4}\sqrt{\sqrt{a}-\sqrt{b}}}+\frac{\sqrt{b}\text{arctanh}\left(\frac{\sqrt{\sqrt{a}+\sqrt{b}}\tanh(c+dx)}{\sqrt[4]{a}}\right)}{2a^{5/4}\sqrt{\sqrt{a}+\sqrt{b}}}-\frac{\coth(c+dx)}{a}}{d}$$

input
```
Int[Csch[c + d*x]^2/(a - b*Sinh[c + d*x]^4),x]
```

output
```
(-1/2*(Sqrt[b]*ArcTanh[(Sqrt[Sqrt[a] - Sqrt[b]]*Tanh[c + d*x])/a^(1/4)])/(
a^(5/4)*Sqrt[Sqrt[a] - Sqrt[b]]) + (Sqrt[b]*ArcTanh[(Sqrt[Sqrt[a] + Sqrt[b
]]*Tanh[c + d*x])/a^(1/4)])/(2*a^(5/4)*Sqrt[Sqrt[a] + Sqrt[b]]) - Coth[c +
 d*x]/a)/d
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 1610
```
Int[(((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.))/((a_) + (b_.)*(x_)^2 +
 (c_.)*(x_)^4), x_Symbol] :> Int[ExpandIntegrand[(f*x)^m*((d + e*x^2)^q/(a
+ b*x^2 + c*x^4)), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b^2 - 4
*a*c, 0] && IntegerQ[q] && IntegerQ[m]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3696
```
Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^4)^(
p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Simp[ff^(m + 1
)/f   Subst[Int[x^m*((a + 2*a*ff^2*x^2 + (a + b)*ff^4*x^4)^p/(1 + ff^2*x^2)
^(m/2 + 2*p + 1)), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f}, x] &
& IntegerQ[m/2] && IntegerQ[p]
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 1.19 (sec) , antiderivative size = 130, normalized size of antiderivative = 0.94

method	result
derivativedivides	$\frac{-\frac{\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)}{2a}-\frac{b\left(\sum_{\_R=\mathrm{RootOf}\left(a\_Z^8-4a\_Z^6+(6a-16b)\_Z^4-4a\_Z^2+a\right)}\frac{\left(\_R^4-\_R^2\right)\ln\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)-\_R\right)}{\_R^7a-3\_R^5a+3\_R^3a-8\_R^3b-\_Ra}\right.}{a}}{d}$
default	$\frac{-\frac{\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)}{2a}-\frac{b\left(\sum_{\_R=\mathrm{RootOf}\left(a\_Z^8-4a\_Z^6+(6a-16b)\_Z^4-4a\_Z^2+a\right)}\frac{\left(\_R^4-\_R^2\right)\ln\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)-\_R\right)}{\_R^7a-3\_R^5a+3\_R^3a-8\_R^3b-\_Ra}\right.}{a}}{d}$
risch	$-\frac{2}{ad\left(e^{2dx+2c}-1\right)}+4\left(\sum_{\_R=\mathrm{RootOf}\left(\left(65536a^6d^4-65536a^5b\,d^4\right)\_Z^4-512a^3d^2\_Z^2b+b^2\right)}\_R\ln\left(e^{2dx+2c}+\right.\right.$

input
```
int(csch(d*x+c)^2/(a-b*sinh(d*x+c)^4),x,method=_RETURNVERBOSE)
```

output
```
1/d*(-1/2/a*tanh(1/2*d*x+1/2*c)-b/a*sum((_R^4-_R^2)/(_R^7*a-3*_R^5*a+3*_R^
3*a-8*_R^3*b-_R*a)*ln(tanh(1/2*d*x+1/2*c)-_R),_R=RootOf(a*_Z^8-4*a*_Z^6+(6
*a-16*b)*_Z^4-4*a*_Z^2+a))-1/2/a/tanh(1/2*d*x+1/2*c))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1305 vs. $2(99) = 198$.

Time = 0.14 (sec) , antiderivative size = 1305, normalized size of antiderivative = 9.39

$$\int \frac{\operatorname{csch}^2(c+dx)}{a-b\sinh^4(c+dx)}\,dx = \text{Too large to display}$$

input
```
integrate(csch(d*x+c)^2/(a-b*sinh(d*x+c)^4),x, algorithm="fricas")
```

output
```
-1/4*((a*d*cosh(d*x + c)^2 + 2*a*d*cosh(d*x + c)*sinh(d*x + c) + a*d*sinh(
d*x + c)^2 - a*d)*sqrt(((a^3 - a^2*b)*d^2*sqrt(b^3/((a^7 - 2*a^6*b + a^5*b
^2)*d^4)) + b)/((a^3 - a^2*b)*d^2))*log(b^2*cosh(d*x + c)^2 + 2*b^2*cosh(d
*x + c)*sinh(d*x + c) + b^2*sinh(d*x + c)^2 + 2*(a^4 - a^3*b)*d^2*sqrt(b^3
/((a^7 - 2*a^6*b + a^5*b^2)*d^4)) - b^2 + 2*((a^5 - a^4*b)*d^3*sqrt(b^3/((
a^7 - 2*a^6*b + a^5*b^2)*d^4)) - a^2*b*d)*sqrt(((a^3 - a^2*b)*d^2*sqrt(b^3
/((a^7 - 2*a^6*b + a^5*b^2)*d^4)) + b)/((a^3 - a^2*b)*d^2))) - (a*d*cosh(d
*x + c)^2 + 2*a*d*cosh(d*x + c)*sinh(d*x + c) + a*d*sinh(d*x + c)^2 - a*d)
*sqrt(((a^3 - a^2*b)*d^2*sqrt(b^3/((a^7 - 2*a^6*b + a^5*b^2)*d^4)) + b)/((
a^3 - a^2*b)*d^2))*log(b^2*cosh(d*x + c)^2 + 2*b^2*cosh(d*x + c)*sinh(d*x
+ c) + b^2*sinh(d*x + c)^2 + 2*(a^4 - a^3*b)*d^2*sqrt(b^3/((a^7 - 2*a^6*b
+ a^5*b^2)*d^4)) - b^2 - 2*((a^5 - a^4*b)*d^3*sqrt(b^3/((a^7 - 2*a^6*b + a
^5*b^2)*d^4)) - a^2*b*d)*sqrt(((a^3 - a^2*b)*d^2*sqrt(b^3/((a^7 - 2*a^6*b
+ a^5*b^2)*d^4)) + b)/((a^3 - a^2*b)*d^2))) - (a*d*cosh(d*x + c)^2 + 2*a*d
*cosh(d*x + c)*sinh(d*x + c) + a*d*sinh(d*x + c)^2 - a*d)*sqrt(-((a^3 - a^
2*b)*d^2*sqrt(b^3/((a^7 - 2*a^6*b + a^5*b^2)*d^4)) - b)/((a^3 - a^2*b)*d^2
))*log(b^2*cosh(d*x + c)^2 + 2*b^2*cosh(d*x + c)*sinh(d*x + c) + b^2*sinh(
d*x + c)^2 - 2*(a^4 - a^3*b)*d^2*sqrt(b^3/((a^7 - 2*a^6*b + a^5*b^2)*d^4))
 - b^2 + 2*((a^5 - a^4*b)*d^3*sqrt(b^3/((a^7 - 2*a^6*b + a^5*b^2)*d^4)) +
a^2*b*d)*sqrt(-((a^3 - a^2*b)*d^2*sqrt(b^3/((a^7 - 2*a^6*b + a^5*b^2)*d...
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\operatorname{csch}^2(c+dx)}{a-b\sinh^4(c+dx)}\,dx = \text{Timed out}$$

input
```
integrate(csch(d*x+c)**2/(a-b*sinh(d*x+c)**4),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{\operatorname{csch}^2(c+dx)}{a-b\sinh^4(c+dx)}\,dx = \int -\frac{\operatorname{csch}(dx+c)^2}{b\sinh(dx+c)^4-a}\,dx$$

input
```
integrate(csch(d*x+c)^2/(a-b*sinh(d*x+c)^4),x, algorithm="maxima")
```

output
```
-2/(a*d*e^(2*d*x + 2*c) - a*d) - 4*integrate((b*e^(6*d*x + 6*c) - 2*b*e^(4
*d*x + 4*c) + b*e^(2*d*x + 2*c))/(a*b*e^(8*d*x + 8*c) - 4*a*b*e^(6*d*x + 6
*c) - 4*a*b*e^(2*d*x + 2*c) + a*b - 2*(8*a^2*e^(4*c) - 3*a*b*e^(4*c))*e^(4
*d*x)), x)
```

Giac [F]

$$\int \frac{\operatorname{csch}^2(c+dx)}{a-b\sinh^4(c+dx)}\,dx = \int -\frac{\operatorname{csch}(dx+c)^2}{b\sinh(dx+c)^4-a}\,dx$$

input
```
integrate(csch(d*x+c)^2/(a-b*sinh(d*x+c)^4),x, algorithm="giac")
```

output
```
sage0*x
```

Mupad [B] (verification not implemented)

Time = 12.43 (sec) , antiderivative size = 2128, normalized size of antiderivative = 15.31

$$\int \frac{\operatorname{csch}^2(c+dx)}{a-b\sinh^4(c+dx)}\,dx = \text{Too large to display}$$

input
```
int(1/(sinh(c + d*x)^2*(a - b*sinh(c + d*x)^4)),x)
```

output
```
log((((((4194304*d^2*(512*a^4 - 1184*a^3*b - 253*a*b^3 - b^4 + 930*a^2*b^2
 + b^4*exp(2*c + 2*d*x) + 627*a*b^3*exp(2*c + 2*d*x) + 768*a^3*b*exp(2*c +
 2*d*x) - 1392*a^2*b^2*exp(2*c + 2*d*x)))/(a^2*b^4*(a - b)^2) - (16777216*
d^3*(((a^5*b^3)^(1/2) + a^3*b)/(a^5*d^2*(a - b)))^(1/2)*(40*a*b^2 - 35*b^3
 + 512*a^3*exp(2*c + 2*d*x) + 64*b^3*exp(2*c + 2*d*x) + 326*a*b^2*exp(2*c
+ 2*d*x) - 896*a^2*b*exp(2*c + 2*d*x)))/(b^5*(a - b)))*(((a^5*b^3)^(1/2) +
 a^3*b)/(a^5*d^2*(a - b)))^(1/2))/4 - (2097152*d*(256*a^2*b - 256*a*b^2 -
5*b^3 - 1024*a^3*exp(2*c + 2*d*x) + 6*b^3*exp(2*c + 2*d*x) + 756*a*b^2*exp
(2*c + 2*d*x) + 256*a^2*b*exp(2*c + 2*d*x)))/(a^3*b^4*(a - b)))*(((a^5*b^3
)^(1/2) + a^3*b)/(a^5*d^2*(a - b)))^(1/2))/4 - (524288*(185*a*b^2 - 464*a^
2*b + 256*a^3 + 24*b^3 - 1024*a^3*exp(2*c + 2*d*x) - 35*b^3*exp(2*c + 2*d*
x) - 988*a*b^2*exp(2*c + 2*d*x) + 2048*a^2*b*exp(2*c + 2*d*x)))/(a^4*b^3*(
a - b)^2))*(((a^5*b^3)^(1/2) + a^3*b)/(16*(a^6*d^2 - a^5*b*d^2)))^(1/2) -
log((((((4194304*d^2*(512*a^4 - 1184*a^3*b - 253*a*b^3 - b^4 + 930*a^2*b^2
 + b^4*exp(2*c + 2*d*x) + 627*a*b^3*exp(2*c + 2*d*x) + 768*a^3*b*exp(2*c +
 2*d*x) - 1392*a^2*b^2*exp(2*c + 2*d*x)))/(a^2*b^4*(a - b)^2) + (16777216*
d^3*(((a^5*b^3)^(1/2) + a^3*b)/(a^5*d^2*(a - b)))^(1/2)*(40*a*b^2 - 35*b^3
 + 512*a^3*exp(2*c + 2*d*x) + 64*b^3*exp(2*c + 2*d*x) + 326*a*b^2*exp(2*c
+ 2*d*x) - 896*a^2*b*exp(2*c + 2*d*x)))/(b^5*(a - b)))*(((a^5*b^3)^(1/2) +
 a^3*b)/(a^5*d^2*(a - b)))^(1/2))/4 + (2097152*d*(256*a^2*b - 256*a*b^2...
```

Reduce [F]

$$\int \frac{\operatorname{csch}^2(c+dx)}{a-b\sinh^4(c+dx)}\,dx = \text{Too large to display}$$

input
```
int(csch(d*x+c)^2/(a-b*sinh(d*x+c)^4),x)
```

output

```
(1024*e**(6*c + 2*d*x)*int(e**(4*d*x)/(e**(12*c + 12*d*x)*b - 6*e**(10*c +
 10*d*x)*b - 16*e**(8*c + 8*d*x)*a + 15*e**(8*c + 8*d*x)*b + 32*e**(6*c +
6*d*x)*a - 20*e**(6*c + 6*d*x)*b - 16*e**(4*c + 4*d*x)*a + 15*e**(4*c + 4*
d*x)*b - 6*e**(2*c + 2*d*x)*b + b),x)*a**2*d - 448*e**(6*c + 2*d*x)*int(e*
*(4*d*x)/(e**(12*c + 12*d*x)*b - 6*e**(10*c + 10*d*x)*b - 16*e**(8*c + 8*d
*x)*a + 15*e**(8*c + 8*d*x)*b + 32*e**(6*c + 6*d*x)*a - 20*e**(6*c + 6*d*x
)*b - 16*e**(4*c + 4*d*x)*a + 15*e**(4*c + 4*d*x)*b - 6*e**(2*c + 2*d*x)*b
 + b),x)*a*b*d + 256*e**(4*c + 2*d*x)*int(e**(2*d*x)/(e**(12*c + 12*d*x)*b
 - 6*e**(10*c + 10*d*x)*b - 16*e**(8*c + 8*d*x)*a + 15*e**(8*c + 8*d*x)*b
+ 32*e**(6*c + 6*d*x)*a - 20*e**(6*c + 6*d*x)*b - 16*e**(4*c + 4*d*x)*a +
15*e**(4*c + 4*d*x)*b - 6*e**(2*c + 2*d*x)*b + b),x)*a*b*d - 64*e**(2*c +
2*d*x)*int(1/(e**(12*c + 12*d*x)*b - 6*e**(10*c + 10*d*x)*b - 16*e**(8*c +
 8*d*x)*a + 15*e**(8*c + 8*d*x)*b + 32*e**(6*c + 6*d*x)*a - 20*e**(6*c + 6
*d*x)*b - 16*e**(4*c + 4*d*x)*a + 15*e**(4*c + 4*d*x)*b - 6*e**(2*c + 2*d*
x)*b + b),x)*a*b*d - 32*e**(2*c + 2*d*x)*log(e**(c + d*x) - 1)*a + 4*e**(2
*c + 2*d*x)*log(e**(c + d*x) - 1)*b - 32*e**(2*c + 2*d*x)*log(e**(c + d*x)
 + 1)*a + 4*e**(2*c + 2*d*x)*log(e**(c + d*x) + 1)*b - e**(2*c + 2*d*x)*lo
g(e**(8*c + 8*d*x)*b - 4*e**(6*c + 6*d*x)*b - 16*e**(4*c + 4*d*x)*a + 6*e*
*(4*c + 4*d*x)*b - 4*e**(2*c + 2*d*x)*b + b)*b + 64*e**(2*c + 2*d*x)*a*d*x
 - 32*e**(2*c + 2*d*x)*a - 1024*e**(4*c)*int(e**(4*d*x)/(e**(12*c + 12*...
```

3.215 $\int \frac{\operatorname{csch}^4(c+dx)}{a-b\sinh^4(c+dx)}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 148

$$\int \frac{\operatorname{csch}^4(c+dx)}{a-b\sinh^4(c+dx)}\,dx = \frac{b\operatorname{arctanh}\left(\frac{\sqrt{\sqrt{a}-\sqrt{b}}\tanh(c+dx)}{\sqrt[4]{a}}\right)}{2a^{7/4}\sqrt{\sqrt{a}-\sqrt{b}}d} + \frac{b\operatorname{arctanh}\left(\frac{\sqrt{\sqrt{a}+\sqrt{b}}\tanh(c+dx)}{\sqrt[4]{a}}\right)}{2a^{7/4}\sqrt{\sqrt{a}+\sqrt{b}}d} + \frac{\coth(c+dx)}{ad} - \frac{\coth^3(c+dx)}{3ad}$$

output
```
1/2*b*arctanh((a^(1/2)-b^(1/2))^(1/2)*tanh(d*x+c)/a^(1/4))/a^(7/4)/(a^(1/2
)-b^(1/2))^(1/2)/d+1/2*b*arctanh((a^(1/2)+b^(1/2))^(1/2)*tanh(d*x+c)/a^(1/
4))/a^(7/4)/(a^(1/2)+b^(1/2))^(1/2)/d+coth(d*x+c)/a/d-1/3*coth(d*x+c)^3/a/
d
```

Mathematica [A] (verified)

Time = 2.31 (sec) , antiderivative size = 165, normalized size of antiderivative = 1.11

$$\int \frac{\operatorname{csch}^4(c+dx)}{a-b\sinh^4(c+dx)}\,dx$$
$$= \frac{-\frac{3b\arctan\left(\frac{(\sqrt{a}-\sqrt{b})\tanh(c+dx)}{\sqrt{-a+\sqrt{a}\sqrt{b}}}\right)}{\sqrt{-a+\sqrt{a}\sqrt{b}}}+\frac{3b\text{arctanh}\left(\frac{(\sqrt{a}+\sqrt{b})\tanh(c+dx)}{\sqrt{a+\sqrt{a}\sqrt{b}}}\right)}{\sqrt{a+\sqrt{a}\sqrt{b}}}+4\sqrt{a}\coth(c+dx)-2\sqrt{a}\coth(c+dx)\operatorname{csch}^2}{6a^{3/2}d}$$

input
```
Integrate[Csch[c + d*x]^4/(a - b*Sinh[c + d*x]^4),x]
```

output
```
((-3*b*ArcTan[((Sqrt[a] - Sqrt[b])*Tanh[c + d*x])/Sqrt[-a + Sqrt[a]*Sqrt[b
]]])/Sqrt[-a + Sqrt[a]*Sqrt[b]] + (3*b*ArcTanh[((Sqrt[a] + Sqrt[b])*Tanh[c
 + d*x])/Sqrt[a + Sqrt[a]*Sqrt[b]]])/Sqrt[a + Sqrt[a]*Sqrt[b]] + 4*Sqrt[a]
*Coth[c + d*x] - 2*Sqrt[a]*Coth[c + d*x]*Csch[c + d*x]^2)/(6*a^(3/2)*d)
```

Rubi [A] (verified)

Time = 0.46 (sec) , antiderivative size = 140, normalized size of antiderivative = 0.95, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {3042, 3696, 1610, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\operatorname{csch}^4(c+dx)}{a-b\sinh^4(c+dx)}\,dx$$
$$\Big\downarrow \; 3042$$
$$\int \frac{1}{\sin(ic+idx)^4\left(a-b\sin(ic+idx)^4\right)}dx$$
$$\Big\downarrow \; 3696$$
$$\frac{\int \frac{\coth^4(c+dx)\left(1-\tanh^2(c+dx)\right)^3}{(a-b)\tanh^4(c+dx)-2a\tanh^2(c+dx)+a}d\tanh(c+dx)}{d}$$

$$\Big\downarrow \text{1610}$$

$$\frac{\int \left(\frac{\coth^4(c+dx)}{a}-\frac{\coth^2(c+dx)}{a}+\frac{b-b\tanh^2(c+dx)}{a\left((a-b)\tanh^4(c+dx)-2a\tanh^2(c+dx)+a\right)}\right)d\tanh(c+dx)}{d}$$

$$\Big\downarrow \text{2009}$$

$$\frac{\frac{b\text{arctanh}\left(\frac{\sqrt{\sqrt{a}-\sqrt{b}}\tanh(c+dx)}{\sqrt[4]{a}}\right)}{2a^{7/4}\sqrt{\sqrt{a}-\sqrt{b}}}+\frac{b\text{arctanh}\left(\frac{\sqrt{\sqrt{a}+\sqrt{b}}\tanh(c+dx)}{\sqrt[4]{a}}\right)}{2a^{7/4}\sqrt{\sqrt{a}+\sqrt{b}}}-\frac{\coth^3(c+dx)}{3a}+\frac{\coth(c+dx)}{a}}{d}$$

input
```
Int[Csch[c + d*x]^4/(a - b*Sinh[c + d*x]^4),x]
```

output
```
((b*ArcTanh[(Sqrt[Sqrt[a] - Sqrt[b]]*Tanh[c + d*x])/a^(1/4)])/(2*a^(7/4)*S
qrt[Sqrt[a] - Sqrt[b]]) + (b*ArcTanh[(Sqrt[Sqrt[a] + Sqrt[b]]*Tanh[c + d*x
])/a^(1/4)])/(2*a^(7/4)*Sqrt[Sqrt[a] + Sqrt[b]]) + Coth[c + d*x]/a - Coth[
c + d*x]^3/(3*a))/d
```

Defintions of rubi rules used

rule 1610
```
Int[(((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.))/((a_) + (b_.)*(x_)^2 +
 (c_.)*(x_)^4), x_Symbol] :> Int[ExpandIntegrand[(f*x)^m*((d + e*x^2)^q/(a
+ b*x^2 + c*x^4)), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b^2 - 4
*a*c, 0] && IntegerQ[q] && IntegerQ[m]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3696
```
Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^4)^(
p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Simp[ff^(m + 1
)/f   Subst[Int[x^m*((a + 2*a*ff^2*x^2 + (a + b)*ff^4*x^4)^p/(1 + ff^2*x^2)
^(m/2 + 2*p + 1)), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f}, x] &
& IntegerQ[m/2] && IntegerQ[p]
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 2.01 (sec) , antiderivative size = 168, normalized size of antiderivative = 1.14

method	result
derivativedivides	$\frac{-\frac{\frac{\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^{3}}{3}-3\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)}{8a}-\frac{b\left(\sum_{\_R=\mathrm{RootOf}\left(a\_Z^{8}-4a\_Z^{6}+(6a-16b)\_Z^{4}-4a\_Z^{2}+a\right)}\frac{\left(\_R^{6}-3\_R^{4}+3\_R^{2}-\right.}{\_R^{7}a-3\_R^{5}a+3\_}\right.}{4a}}{d}$
default	$\frac{-\frac{\frac{\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^{3}}{3}-3\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)}{8a}-\frac{b\left(\sum_{\_R=\mathrm{RootOf}\left(a\_Z^{8}-4a\_Z^{6}+(6a-16b)\_Z^{4}-4a\_Z^{2}+a\right)}\frac{\left(\_R^{6}-3\_R^{4}+3\_R^{2}-\right.}{\_R^{7}a-3\_R^{5}a+3\_}\right.}{4a}}{d}$
risch	$-\frac{4\left(3\,\mathrm{e}^{2dx+2c}-1\right)}{3ad\left(\mathrm{e}^{2dx+2c}-1\right)^{3}}+16\left(\sum_{\_R=\mathrm{RootOf}\left(\left(16777216a^{8}d^{4}-16777216a^{7}b\,d^{4}\right)\_Z^{4}-8192a^{4}b^{2}d^{2}\_Z^{2}+b^{4}\right)}\_R\ln\left(\right.\right.$

input
```
int(csch(d*x+c)^4/(a-b*sinh(d*x+c)^4),x,method=_RETURNVERBOSE)
```

output
```
1/d*(-1/8/a*(1/3*tanh(1/2*d*x+1/2*c)^3-3*tanh(1/2*d*x+1/2*c))-1/4*b/a*sum(
(_R^6-3*_R^4+3*_R^2-1)/(_R^7*a-3*_R^5*a+3*_R^3*a-8*_R^3*b-_R*a)*ln(tanh(1/
2*d*x+1/2*c)-_R),_R=RootOf(a*_Z^8-4*a*_Z^6+(6*a-16*b)*_Z^4-4*a*_Z^2+a))-1/
24/a/tanh(1/2*d*x+1/2*c)^3+3/8/a/tanh(1/2*d*x+1/2*c))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2206 vs. $2(110) = 220$.

Time = 0.15 (sec) , antiderivative size = 2206, normalized size of antiderivative = 14.91

$$\int \frac{\operatorname{csch}^4(c+dx)}{a-b\sinh^4(c+dx)}\,dx = \text{Too large to display}$$

input
```
integrate(csch(d*x+c)^4/(a-b*sinh(d*x+c)^4),x, algorithm="fricas")
```

output
```
1/12*(3*(a*d*cosh(d*x + c)^6 + 6*a*d*cosh(d*x + c)*sinh(d*x + c)^5 + a*d*s
inh(d*x + c)^6 - 3*a*d*cosh(d*x + c)^4 + 3*(5*a*d*cosh(d*x + c)^2 - a*d)*s
inh(d*x + c)^4 + 3*a*d*cosh(d*x + c)^2 + 4*(5*a*d*cosh(d*x + c)^3 - 3*a*d*
cosh(d*x + c))*sinh(d*x + c)^3 + 3*(5*a*d*cosh(d*x + c)^4 - 6*a*d*cosh(d*x
 + c)^2 + a*d)*sinh(d*x + c)^2 - a*d + 6*(a*d*cosh(d*x + c)^5 - 2*a*d*cosh
(d*x + c)^3 + a*d*cosh(d*x + c))*sinh(d*x + c))*sqrt(((a^4 - a^3*b)*d^2*sq
rt(b^5/((a^9 - 2*a^8*b + a^7*b^2)*d^4)) + b^2)/((a^4 - a^3*b)*d^2))*log(b^
4*cosh(d*x + c)^2 + 2*b^4*cosh(d*x + c)*sinh(d*x + c) + b^4*sinh(d*x + c)^
2 - b^4 + 2*(a^5*b - a^4*b^2)*d^2*sqrt(b^5/((a^9 - 2*a^8*b + a^7*b^2)*d^4)
) + 2*(a^2*b^3*d - (a^7 - a^6*b)*d^3*sqrt(b^5/((a^9 - 2*a^8*b + a^7*b^2)*d
^4)))*sqrt(((a^4 - a^3*b)*d^2*sqrt(b^5/((a^9 - 2*a^8*b + a^7*b^2)*d^4)) +
b^2)/((a^4 - a^3*b)*d^2))) - 3*(a*d*cosh(d*x + c)^6 + 6*a*d*cosh(d*x + c)*
sinh(d*x + c)^5 + a*d*sinh(d*x + c)^6 - 3*a*d*cosh(d*x + c)^4 + 3*(5*a*d*c
osh(d*x + c)^2 - a*d)*sinh(d*x + c)^4 + 3*a*d*cosh(d*x + c)^2 + 4*(5*a*d*c
osh(d*x + c)^3 - 3*a*d*cosh(d*x + c))*sinh(d*x + c)^3 + 3*(5*a*d*cosh(d*x
+ c)^4 - 6*a*d*cosh(d*x + c)^2 + a*d)*sinh(d*x + c)^2 - a*d + 6*(a*d*cosh(
d*x + c)^5 - 2*a*d*cosh(d*x + c)^3 + a*d*cosh(d*x + c))*sinh(d*x + c))*sqr
t(((a^4 - a^3*b)*d^2*sqrt(b^5/((a^9 - 2*a^8*b + a^7*b^2)*d^4)) + b^2)/((a^
4 - a^3*b)*d^2))*log(b^4*cosh(d*x + c)^2 + 2*b^4*cosh(d*x + c)*sinh(d*x +
c) + b^4*sinh(d*x + c)^2 - b^4 + 2*(a^5*b - a^4*b^2)*d^2*sqrt(b^5/((a^9...
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\operatorname{csch}^4(c+dx)}{a-b\sinh^4(c+dx)}\,dx = \text{Timed out}$$

input
```
integrate(csch(d*x+c)**4/(a-b*sinh(d*x+c)**4),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{\operatorname{csch}^4(c+dx)}{a-b\sinh^4(c+dx)}\,dx = \int -\frac{\operatorname{csch}(dx+c)^4}{b\sinh(dx+c)^4-a}\,dx$$

input
```
integrate(csch(d*x+c)^4/(a-b*sinh(d*x+c)^4),x, algorithm="maxima")
```

output
```
-16*b*integrate(e^(4*d*x + 4*c)/(a*b*e^(8*d*x + 8*c) - 4*a*b*e^(6*d*x + 6*
c) - 4*a*b*e^(2*d*x + 2*c) + a*b - 2*(8*a^2*e^(4*c) - 3*a*b*e^(4*c))*e^(4*
d*x)), x) - 4/3*(3*e^(2*d*x + 2*c) - 1)/(a*d*e^(6*d*x + 6*c) - 3*a*d*e^(4*
d*x + 4*c) + 3*a*d*e^(2*d*x + 2*c) - a*d)
```

Giac [F]

$$\int \frac{\operatorname{csch}^4(c+dx)}{a-b\sinh^4(c+dx)}\,dx = \int -\frac{\operatorname{csch}(dx+c)^4}{b\sinh(dx+c)^4-a}\,dx$$

input
```
integrate(csch(d*x+c)^4/(a-b*sinh(d*x+c)^4),x, algorithm="giac")
```

output
```
sage0*x
```

Mupad [B] (verification not implemented)

Time = 13.50 (sec) , antiderivative size = 2178, normalized size of antiderivative = 14.72

$$\int \frac{\operatorname{csch}^4(c+dx)}{a-b\sinh^4(c+dx)}\,dx = \text{Too large to display}$$

input
```
int(1/(sinh(c + d*x)^4*(a - b*sinh(c + d*x)^4)),x)
```

output
```
log((((((4194304*d^2*(512*a^4 - 1184*a^3*b - 253*a*b^3 - b^4 + 930*a^2*b^2
 + b^4*exp(2*c + 2*d*x) + 627*a*b^3*exp(2*c + 2*d*x) + 768*a^3*b*exp(2*c +
 2*d*x) - 1392*a^2*b^2*exp(2*c + 2*d*x)))/(a^4*b^2*(a - b)^2) + (8388608*d
^3*(((a^7*b^5)^(1/2) + a^4*b^2)/(a^7*d^2*(a - b)))^(1/2)*(181*a*b^2 - 432*
a^2*b + 256*a^3 + 5*b^3 - 512*a^3*exp(2*c + 2*d*x) - 6*b^3*exp(2*c + 2*d*x
) - 622*a*b^2*exp(2*c + 2*d*x) + 1152*a^2*b*exp(2*c + 2*d*x)))/(a^2*b^3*(a
 - b)))*(((a^7*b^5)^(1/2) + a^4*b^2)/(a^7*d^2*(a - b)))^(1/2))/4 + (209715
2*d*(176*a*b - 256*a^2 + 75*b^2 + 1536*a^2*exp(2*c + 2*d*x) - 134*b^2*exp(
2*c + 2*d*x) - 1408*a*b*exp(2*c + 2*d*x)))/(a^5*b*(a - b)))*(((a^7*b^5)^(1
/2) + a^4*b^2)/(a^7*d^2*(a - b)))^(1/2))/4 - (524288*(185*a*b^2 - 464*a^2*
b + 256*a^3 + 24*b^3 - 1024*a^3*exp(2*c + 2*d*x) - 35*b^3*exp(2*c + 2*d*x)
 - 988*a*b^2*exp(2*c + 2*d*x) + 2048*a^2*b*exp(2*c + 2*d*x)))/(a^7*(a - b)
^2))*(((a^7*b^5)^(1/2) + a^4*b^2)/(16*(a^8*d^2 - a^7*b*d^2)))^(1/2) - log(
(((((4194304*d^2*(512*a^4 - 1184*a^3*b - 253*a*b^3 - b^4 + 930*a^2*b^2 + b
^4*exp(2*c + 2*d*x) + 627*a*b^3*exp(2*c + 2*d*x) + 768*a^3*b*exp(2*c + 2*d
*x) - 1392*a^2*b^2*exp(2*c + 2*d*x)))/(a^4*b^2*(a - b)^2) - (8388608*d^3*(
((a^7*b^5)^(1/2) + a^4*b^2)/(a^7*d^2*(a - b)))^(1/2)*(181*a*b^2 - 432*a^2*
b + 256*a^3 + 5*b^3 - 512*a^3*exp(2*c + 2*d*x) - 6*b^3*exp(2*c + 2*d*x) -
622*a*b^2*exp(2*c + 2*d*x) + 1152*a^2*b*exp(2*c + 2*d*x)))/(a^2*b^3*(a - b
)))*(((a^7*b^5)^(1/2) + a^4*b^2)/(a^7*d^2*(a - b)))^(1/2))/4 - (2097152...
```

Reduce [F]

$$\int \frac{\operatorname{csch}^4(c+dx)}{a-b\sinh^4(c+dx)}\,dx = \text{too large to display}$$

input
```
int(csch(d*x+c)^4/(a-b*sinh(d*x+c)^4),x)
```

output

```
(8*( - 1179648*e**(10*c + 6*d*x)*int(e**(4*d*x)/(48*e**(16*c + 16*d*x)*a*b
 + e**(16*c + 16*d*x)*b**2 - 384*e**(14*c + 14*d*x)*a*b - 8*e**(14*c + 14*
d*x)*b**2 - 768*e**(12*c + 12*d*x)*a**2 + 1328*e**(12*c + 12*d*x)*a*b + 28
*e**(12*c + 12*d*x)*b**2 + 3072*e**(10*c + 10*d*x)*a**2 - 2624*e**(10*c +
10*d*x)*a*b - 56*e**(10*c + 10*d*x)*b**2 - 4608*e**(8*c + 8*d*x)*a**2 + 32
64*e**(8*c + 8*d*x)*a*b + 70*e**(8*c + 8*d*x)*b**2 + 3072*e**(6*c + 6*d*x)
*a**2 - 2624*e**(6*c + 6*d*x)*a*b - 56*e**(6*c + 6*d*x)*b**2 - 768*e**(4*c
 + 4*d*x)*a**2 + 1328*e**(4*c + 4*d*x)*a*b + 28*e**(4*c + 4*d*x)*b**2 - 38
4*e**(2*c + 2*d*x)*a*b - 8*e**(2*c + 2*d*x)*b**2 + 48*a*b + b**2),x)*a**4*
d + 638976*e**(10*c + 6*d*x)*int(e**(4*d*x)/(48*e**(16*c + 16*d*x)*a*b + e
**(16*c + 16*d*x)*b**2 - 384*e**(14*c + 14*d*x)*a*b - 8*e**(14*c + 14*d*x)
*b**2 - 768*e**(12*c + 12*d*x)*a**2 + 1328*e**(12*c + 12*d*x)*a*b + 28*e**
(12*c + 12*d*x)*b**2 + 3072*e**(10*c + 10*d*x)*a**2 - 2624*e**(10*c + 10*d
*x)*a*b - 56*e**(10*c + 10*d*x)*b**2 - 4608*e**(8*c + 8*d*x)*a**2 + 3264*e
**(8*c + 8*d*x)*a*b + 70*e**(8*c + 8*d*x)*b**2 + 3072*e**(6*c + 6*d*x)*a**
2 - 2624*e**(6*c + 6*d*x)*a*b - 56*e**(6*c + 6*d*x)*b**2 - 768*e**(4*c + 4
*d*x)*a**2 + 1328*e**(4*c + 4*d*x)*a*b + 28*e**(4*c + 4*d*x)*b**2 - 384*e*
*(2*c + 2*d*x)*a*b - 8*e**(2*c + 2*d*x)*b**2 + 48*a*b + b**2),x)*a**3*b*d
 - 13824*e**(10*c + 6*d*x)*int(e**(4*d*x)/(48*e**(16*c + 16*d*x)*a*b + e**(
16*c + 16*d*x)*b**2 - 384*e**(14*c + 14*d*x)*a*b - 8*e**(14*c + 14*d*x)...
```

3.216 $\int \frac{\sinh^9(c+dx)}{(a-b\sinh^4(c+dx))^2}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 235

$$\begin{aligned}\int \frac{\sinh^9(c+dx)}{(a-b\sinh^4(c+dx))^2}\,dx &= -\frac{\sqrt{a}\left(5\sqrt{a}-6\sqrt{b}\right)\arctan\left(\frac{\sqrt[4]{b}\cosh(c+dx)}{\sqrt{\sqrt{a}-\sqrt{b}}}\right)}{8\left(\sqrt{a}-\sqrt{b}\right)^{3/2}b^{9/4}d}\\ &\quad-\frac{\sqrt{a}\left(5\sqrt{a}+6\sqrt{b}\right)\operatorname{arctanh}\left(\frac{\sqrt[4]{b}\cosh(c+dx)}{\sqrt{\sqrt{a}+\sqrt{b}}}\right)}{8\left(\sqrt{a}+\sqrt{b}\right)^{3/2}b^{9/4}d}+\frac{\cosh(c+dx)}{b^2d}\\ &\quad+\frac{a\cosh(c+dx)\left(a+b-b\cosh^2(c+dx)\right)}{4(a-b)b^2d\left(a-b+2b\cosh^2(c+dx)-b\cosh^4(c+dx)\right)}\end{aligned}$$

output
```
-1/8*a^(1/2)*(5*a^(1/2)-6*b^(1/2))*arctan(b^(1/4)*cosh(d*x+c)/(a^(1/2)-b^(
1/2))^(1/2))/(a^(1/2)-b^(1/2))^(3/2)/b^(9/4)/d-1/8*a^(1/2)*(5*a^(1/2)+6*b^
(1/2))*arctanh(b^(1/4)*cosh(d*x+c)/(a^(1/2)+b^(1/2))^(1/2))/(a^(1/2)+b^(1/
2))^(3/2)/b^(9/4)/d+cosh(d*x+c)/b^2/d+1/4*a*cosh(d*x+c)*(a+b-b*cosh(d*x+c)
^2)/(a-b)/b^2/d/(a-b+2*b*cosh(d*x+c)^2-b*cosh(d*x+c)^4)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

Time = 3.34 (sec) , antiderivative size = 615, normalized size of antiderivative = 2.62

$$\int \frac{\sinh^9(c+dx)}{\left(a-b\sinh^4(c+dx)\right)^2}\,dx$$

$$= \frac{32\cosh(c+dx)+\frac{32a\cosh(c+dx)(2a+b-b\cosh(2(c+dx)))}{(a-b)(8a-3b+4b\cosh(2(c+dx))-b\cosh(4(c+dx)))}+\frac{a\text{RootSum}\left[b-4b\#1^2-16a\#1^4+6b\#1^4-4b\#1^6+b\#\right.}{}}{}$$

input
```
Integrate[Sinh[c + d*x]^9/(a - b*Sinh[c + d*x]^4)^2,x]
```

output
```
(32*Cosh[c + d*x] + (32*a*Cosh[c + d*x]*(2*a + b - b*Cosh[2*(c + d*x)]))/(
(a - b)*(8*a - 3*b + 4*b*Cosh[2*(c + d*x)] - b*Cosh[4*(c + d*x)])) + (a*Ro
otSum[b - 4*b*#1^2 - 16*a*#1^4 + 6*b*#1^4 - 4*b*#1^6 + b*#1^8 & , (-(b*c)
- b*d*x - 2*b*Log[-Cosh[(c + d*x)/2] - Sinh[(c + d*x)/2] + Cosh[(c + d*x)/
2]*#1 - Sinh[(c + d*x)/2]*#1] - 20*a*c*#1^2 + 27*b*c*#1^2 - 20*a*d*x*#1^2
+ 27*b*d*x*#1^2 - 40*a*Log[-Cosh[(c + d*x)/2] - Sinh[(c + d*x)/2] + Cosh[(
c + d*x)/2]*#1 - Sinh[(c + d*x)/2]*#1]*#1^2 + 54*b*Log[-Cosh[(c + d*x)/2]
- Sinh[(c + d*x)/2] + Cosh[(c + d*x)/2]*#1 - Sinh[(c + d*x)/2]*#1]*#1^2 +
20*a*c*#1^4 - 27*b*c*#1^4 + 20*a*d*x*#1^4 - 27*b*d*x*#1^4 + 40*a*Log[-Cosh
[(c + d*x)/2] - Sinh[(c + d*x)/2] + Cosh[(c + d*x)/2]*#1 - Sinh[(c + d*x)/
2]*#1]*#1^4 - 54*b*Log[-Cosh[(c + d*x)/2] - Sinh[(c + d*x)/2] + Cosh[(c +
d*x)/2]*#1 - Sinh[(c + d*x)/2]*#1]*#1^4 + b*c*#1^6 + b*d*x*#1^6 + 2*b*Log[
-Cosh[(c + d*x)/2] - Sinh[(c + d*x)/2] + Cosh[(c + d*x)/2]*#1 - Sinh[(c +
d*x)/2]*#1]*#1^6)/(-(b*#1) - 8*a*#1^3 + 3*b*#1^3 - 3*b*#1^5 + b*#1^7) & ])
/(a - b))/(32*b^2*d)
```

Rubi [A] (verified)

Time = 0.72 (sec) , antiderivative size = 259, normalized size of antiderivative = 1.10, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.292$, Rules used = {3042, 26, 3694, 1517, 27, 2205, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sinh^9(c+dx)}{\left(a-b\sinh^4(c+dx)\right)^2}dx$$

$$\Big\downarrow \text{3042}$$

$$\int -\frac{i\sin(ic+idx)^9}{\left(a-b\sin(ic+idx)^4\right)^2}dx$$

$$\Big\downarrow \text{26}$$

$$-i\int \frac{\sin(ic+idx)^9}{\left(a-b\sin(ic+idx)^4\right)^2}dx$$

$$\Big\downarrow \text{3694}$$

$$\frac{\int \frac{\left(1-\cosh^2(c+dx)\right)^4}{\left(-b\cosh^4(c+dx)+2b\cosh^2(c+dx)+a-b\right)^2}d\cosh(c+dx)}{d}$$

$$\Big\downarrow \text{1517}$$

$$\frac{\frac{a\cosh(c+dx)\left(a-b\cosh^2(c+dx)+b\right)}{4b^2(a-b)\left(a-b\cosh^4(c+dx)+2b\cosh^2(c+dx)-b\right)}-\frac{\int \frac{2\left(4a(a-b)\cosh^4(c+dx)-a(7a-8b)\cosh^2(c+dx)+a\left(\frac{a^2}{b}+a-4b\right)\right)}{-b\cosh^4(c+dx)+2b\cosh^2(c+dx)+a-b}d\cosh(c+dx)}{8ab(a-b)}}{d}$$

$$\Big\downarrow \text{27}$$

$$\frac{\frac{a\cosh(c+dx)\left(a-b\cosh^2(c+dx)+b\right)}{4b^2(a-b)\left(a-b\cosh^4(c+dx)+2b\cosh^2(c+dx)-b\right)}-\frac{\int \frac{4a(a-b)\cosh^4(c+dx)-a(7a-8b)\cosh^2(c+dx)+a\left(\frac{a^2}{b}+a-4b\right)}{-b\cosh^4(c+dx)+2b\cosh^2(c+dx)+a-b}d\cosh(c+dx)}{4ab(a-b)}}{d}$$

$$\Big\downarrow \text{2205}$$

$$\frac{\frac{a\cosh(c+dx)\left(a-b\cosh^2(c+dx)+b\right)}{4b^2(a-b)\left(a-b\cosh^4(c+dx)+2b\cosh^2(c+dx)-b\right)}-\frac{\int \left(\frac{b\cosh^2(c+dx)a^2+(5a-7b)a^2}{b\left(-b\cosh^4(c+dx)+2b\cosh^2(c+dx)+a-b\right)}-\frac{4a(a-b)}{b}\right)d\cosh(c+dx)}{4ab(a-b)}}{d}$$

$\downarrow$ 2009

$$\frac{\frac{a\cosh(c+dx)\left(a-b\cosh^2(c+dx)+b\right)}{4b^2(a-b)\left(a-b\cosh^4(c+dx)+2b\cosh^2(c+dx)-b\right)}-\frac{\frac{a^{3/2}\left(-\sqrt{a}\sqrt{b}+5a-6b\right)\arctan\left(\frac{\sqrt[4]{b}\cosh(c+dx)}{\sqrt{\sqrt{a}-\sqrt{b}}}\right)}{2b^{5/4}\sqrt{\sqrt{a}-\sqrt{b}}}+\frac{a^{3/2}\left(\sqrt{a}\sqrt{b}+5a-6b\right)\text{arctanh}\left(\frac{\sqrt[4]{b}\cosh(c+dx)}{\sqrt{\sqrt{a}+\sqrt{b}}}\right)}{2b^{5/4}\sqrt{\sqrt{a}+\sqrt{b}}}}{4ab(a-b)}}{d}$$

input
```
Int[Sinh[c + d*x]^9/(a - b*Sinh[c + d*x]^4)^2,x]
```

output
```
(-1/4*((a^(3/2)*(5*a - Sqrt[a]*Sqrt[b] - 6*b)*ArcTan[(b^(1/4)*Cosh[c + d*x
])/Sqrt[Sqrt[a] - Sqrt[b]]])/(2*Sqrt[Sqrt[a] - Sqrt[b]]*b^(5/4)) + (a^(3/2
)*(5*a + Sqrt[a]*Sqrt[b] - 6*b)*ArcTanh[(b^(1/4)*Cosh[c + d*x])/Sqrt[Sqrt[
a] + Sqrt[b]]])/(2*Sqrt[Sqrt[a] + Sqrt[b]]*b^(5/4)) - (4*a*(a - b)*Cosh[c
+ d*x])/b)/(a*(a - b)*b) + (a*Cosh[c + d*x]*(a + b - b*Cosh[c + d*x]^2))/(
4*(a - b)*b^2*(a - b + 2*b*Cosh[c + d*x]^2 - b*Cosh[c + d*x]^4)))/d
```

Defintions of rubi rules used

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 1517
```
Int[((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x
_Symbol] :> With[{f = Coeff[PolynomialRemainder[(d + e*x^2)^q, a + b*x^2 +
c*x^4, x], x, 0], g = Coeff[PolynomialRemainder[(d + e*x^2)^q, a + b*x^2 +
c*x^4, x], x, 2]}, Simp[x*(a + b*x^2 + c*x^4)^(p + 1)*((a*b*g - f*(b^2 - 2*
a*c) - c*(b*f - 2*a*g)*x^2)/(2*a*(p + 1)*(b^2 - 4*a*c))), x] + Simp[1/(2*a*
(p + 1)*(b^2 - 4*a*c))   Int[(a + b*x^2 + c*x^4)^(p + 1)*ExpandToSum[2*a*(p
 + 1)*(b^2 - 4*a*c)*PolynomialQuotient[(d + e*x^2)^q, a + b*x^2 + c*x^4, x]
 + b^2*f*(2*p + 3) - 2*a*c*f*(4*p + 5) - a*b*g + c*(4*p + 7)*(b*f - 2*a*g)*
x^2, x], x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ
[c*d^2 - b*d*e + a*e^2, 0] && IGtQ[q, 1] && LtQ[p, -1]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2205
```
Int[(Px_)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :> Int[ExpandInte
grand[Px/(a + b*x^2 + c*x^4), x], x] /; FreeQ[{a, b, c}, x] && PolyQ[Px, x^
2] && Expon[Px, x^2] > 1
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3694
```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^4)^
(p_.), x_Symbol] :> With[{ff = FreeFactors[Cos[e + f*x], x]}, Simp[-ff/f
Subst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a + b - 2*b*ff^2*x^2 + b*ff^4*x^4)^p,
 x], x, Cos[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m -
 1)/2]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 372 vs. $2(185) = 370$.

Time = 12.63 (sec) , antiderivative size = 373, normalized size of antiderivative = 1.59

method	result
derivativedivides	$$\frac{\frac{1}{b^2\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)}-\frac{2a\left(\frac{\frac{(a-2b)\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^6}{4a-4b}-\frac{(3a-8b)\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^4}{4(a-b)}+\frac{(3a+2b)\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^2}{4a-4b}-\frac{a}{4(a-b)}}{\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^8a-4\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^6a+6\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^4a-16b\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^4-4\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^2a+a}+\right.}{}}{}$$
default	$$\frac{\frac{1}{b^2\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)}-\frac{2a\left(\frac{\frac{(a-2b)\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^6}{4a-4b}-\frac{(3a-8b)\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^4}{4(a-b)}+\frac{(3a+2b)\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^2}{4a-4b}-\frac{a}{4(a-b)}}{\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^8a-4\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^6a+6\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^4a-16b\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^4-4\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^2a+a}+\right.}{}}{}$$
risch	$$\frac{e^{dx+c}}{2b^2d}+\frac{e^{-dx-c}}{2b^2d}+\frac{e^{dx+c}a\left(-e^{6dx+6c}b+4\,e^{4dx+4c}a+b\,e^{4dx+4c}+4\,e^{2dx+2c}a+e^{2dx+2c}b-b\right)}{2b^2(a-b)d\left(-e^{8dx+8c}b+4\,e^{6dx+6c}b+16\,e^{4dx+4c}a-6b\,e^{4dx+4c}+4\,e^{2dx+2c}b-b\right)}+\left(\sum_{\_R=\mathrm{Root}}\right.$$

input
```
int(sinh(d*x+c)^9/(a-b*sinh(d*x+c)^4)^2,x,method=_RETURNVERBOSE)
```

output
```
1/d*(1/b^2/(tanh(1/2*d*x+1/2*c)+1)-2*a/b^2*((1/4*(a-2*b)/(a-b)*tanh(1/2*d*
x+1/2*c)^6-1/4*(3*a-8*b)/(a-b)*tanh(1/2*d*x+1/2*c)^4+1/4*(3*a+2*b)/(a-b)*t
anh(1/2*d*x+1/2*c)^2-1/4*a/(a-b))/(tanh(1/2*d*x+1/2*c)^8*a-4*tanh(1/2*d*x+
1/2*c)^6*a+6*tanh(1/2*d*x+1/2*c)^4*a-16*b*tanh(1/2*d*x+1/2*c)^4-4*tanh(1/2
*d*x+1/2*c)^2*a+a)+1/4/(a-b)*a*(-1/4*((a*b)^(1/2)+5*a-6*b)/a/(-a*b-(a*b)^(
1/2)*a)^(1/2)*arctan(1/4*(-2*tanh(1/2*d*x+1/2*c)^2*a+4*(a*b)^(1/2)+2*a)/(-
a*b-(a*b)^(1/2)*a)^(1/2))+1/4*(-(a*b)^(1/2)+5*a-6*b)/a/(-a*b+(a*b)^(1/2)*a
)^(1/2)*arctan(1/4*(2*tanh(1/2*d*x+1/2*c)^2*a+4*(a*b)^(1/2)-2*a)/(-a*b+(a*
b)^(1/2)*a)^(1/2))))-1/b^2/(tanh(1/2*d*x+1/2*c)-1))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 7664 vs. 2(187) = 374.

Time = 0.33 (sec) , antiderivative size = 7664, normalized size of antiderivative = 32.61

$$\int \frac{\sinh^9(c+dx)}{(a-b\sinh^4(c+dx))^2}\,dx = \text{Too large to display}$$

input
```
integrate(sinh(d*x+c)^9/(a-b*sinh(d*x+c)^4)^2,x, algorithm="fricas")
```

output
```
Too large to include
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\sinh^9(c+dx)}{(a-b\sinh^4(c+dx))^2}\,dx = \text{Timed out}$$

input
```
integrate(sinh(d*x+c)**9/(a-b*sinh(d*x+c)**4)**2,x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{\sinh^9(c+dx)}{(a-b\sinh^4(c+dx))^2}\,dx = \int \frac{\sinh(dx+c)^9}{(b\sinh(dx+c)^4-a)^2}\,dx$$

input
```
integrate(sinh(d*x+c)^9/(a-b*sinh(d*x+c)^4)^2,x, algorithm="maxima")
```

output
```
1/2*(a*b - b^2 + (a*b*e^(10*c) - b^2*e^(10*c))*e^(10*d*x) - (2*a*b*e^(8*c)
 - 3*b^2*e^(8*c))*e^(8*d*x) - (20*a^2*e^(6*c) - 17*a*b*e^(6*c) + 2*b^2*e^(
6*c))*e^(6*d*x) - (20*a^2*e^(4*c) - 17*a*b*e^(4*c) + 2*b^2*e^(4*c))*e^(4*d
*x) - (2*a*b*e^(2*c) - 3*b^2*e^(2*c))*e^(2*d*x))/((a*b^3*d*e^(9*c) - b^4*d
*e^(9*c))*e^(9*d*x) - 4*(a*b^3*d*e^(7*c) - b^4*d*e^(7*c))*e^(7*d*x) - 2*(8
*a^2*b^2*d*e^(5*c) - 11*a*b^3*d*e^(5*c) + 3*b^4*d*e^(5*c))*e^(5*d*x) - 4*(
a*b^3*d*e^(3*c) - b^4*d*e^(3*c))*e^(3*d*x) + (a*b^3*d*e^c - b^4*d*e^c)*e^(
d*x)) + 1/512*integrate(256*(a*b*e^(7*d*x + 7*c) - a*b*e^(d*x + c) + (20*a
^2*e^(5*c) - 27*a*b*e^(5*c))*e^(5*d*x) - (20*a^2*e^(3*c) - 27*a*b*e^(3*c))
*e^(3*d*x))/(a*b^3 - b^4 + (a*b^3*e^(8*c) - b^4*e^(8*c))*e^(8*d*x) - 4*(a*
b^3*e^(6*c) - b^4*e^(6*c))*e^(6*d*x) - 2*(8*a^2*b^2*e^(4*c) - 11*a*b^3*e^(
4*c) + 3*b^4*e^(4*c))*e^(4*d*x) - 4*(a*b^3*e^(2*c) - b^4*e^(2*c))*e^(2*d*x
)), x)
```

Giac [F]

$$\int \frac{\sinh^9(c+dx)}{\left(a-b\sinh^4(c+dx)\right)^2}\,dx=\int \frac{\sinh\left(dx+c\right)^9}{\left(b\sinh\left(dx+c\right)^4-a\right)^2}\,dx$$

input
```
integrate(sinh(d*x+c)^9/(a-b*sinh(d*x+c)^4)^2,x, algorithm="giac")
```

output
```
sage0*x
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sinh^9(c+dx)}{\left(a-b\sinh^4(c+dx)\right)^2}\,dx=\int \frac{\sinh(c+d\,x)^9}{\left(a-b\sinh\left(c+d\,x\right)^4\right)^2}\,dx$$

input
```
int(sinh(c + d*x)^9/(a - b*sinh(c + d*x)^4)^2,x)
```

output
```
int(sinh(c + d*x)^9/(a - b*sinh(c + d*x)^4)^2, x)
```

Reduce [F]

$$\int \frac{\sinh^9(c+dx)}{\left(a-b\sinh^4(c+dx)\right)^2}\,dx = \text{too large to display}$$

input
```
int(sinh(d*x+c)^9/(a-b*sinh(d*x+c)^4)^2,x)
```

output
```
(315*e**(10*c + 10*d*x)*b**2 - 921600*e**(14*c + 9*d*x)*int(e**(5*d*x)/(e*
*(16*c + 16*d*x)*b**2 - 8*e**(14*c + 14*d*x)*b**2 - 32*e**(12*c + 12*d*x)*
a*b + 28*e**(12*c + 12*d*x)*b**2 + 128*e**(10*c + 10*d*x)*a*b - 56*e**(10*
c + 10*d*x)*b**2 + 256*e**(8*c + 8*d*x)*a**2 - 192*e**(8*c + 8*d*x)*a*b +
70*e**(8*c + 8*d*x)*b**2 + 128*e**(6*c + 6*d*x)*a*b - 56*e**(6*c + 6*d*x)*
b**2 - 32*e**(4*c + 4*d*x)*a*b + 28*e**(4*c + 4*d*x)*b**2 - 8*e**(2*c + 2*
d*x)*b**2 + b**2),x)*a**3*b*d - 744960*e**(14*c + 9*d*x)*int(e**(5*d*x)/(e
**(16*c + 16*d*x)*b**2 - 8*e**(14*c + 14*d*x)*b**2 - 32*e**(12*c + 12*d*x)
*a*b + 28*e**(12*c + 12*d*x)*b**2 + 128*e**(10*c + 10*d*x)*a*b - 56*e**(10
*c + 10*d*x)*b**2 + 256*e**(8*c + 8*d*x)*a**2 - 192*e**(8*c + 8*d*x)*a*b +
 70*e**(8*c + 8*d*x)*b**2 + 128*e**(6*c + 6*d*x)*a*b - 56*e**(6*c + 6*d*x)
*b**2 - 32*e**(4*c + 4*d*x)*a*b + 28*e**(4*c + 4*d*x)*b**2 - 8*e**(2*c + 2
*d*x)*b**2 + b**2),x)*a**2*b**2*d - 512*e**(14*c + 9*d*x)*int(e**(5*d*x)/(
e**(16*c + 16*d*x)*b**2 - 8*e**(14*c + 14*d*x)*b**2 - 32*e**(12*c + 12*d*x
)*a*b + 28*e**(12*c + 12*d*x)*b**2 + 128*e**(10*c + 10*d*x)*a*b - 56*e**(1
0*c + 10*d*x)*b**2 + 256*e**(8*c + 8*d*x)*a**2 - 192*e**(8*c + 8*d*x)*a*b
+ 70*e**(8*c + 8*d*x)*b**2 + 128*e**(6*c + 6*d*x)*a*b - 56*e**(6*c + 6*d*x
)*b**2 - 32*e**(4*c + 4*d*x)*a*b + 28*e**(4*c + 4*d*x)*b**2 - 8*e**(2*c +
2*d*x)*b**2 + b**2),x)*a*b**3*d - 3747840*e**(12*c + 9*d*x)*int(e**(3*d*x)
/(e**(16*c + 16*d*x)*b**2 - 8*e**(14*c + 14*d*x)*b**2 - 32*e**(12*c + 1...
```

3.217 $\int \frac{\sinh^7(c+dx)}{(a-b\sinh^4(c+dx))^2}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 210

$$\int \frac{\sinh^7(c+dx)}{(a-b\sinh^4(c+dx))^2}\,dx = \frac{\left(3\sqrt{a}-4\sqrt{b}\right)\arctan\left(\frac{\sqrt[4]{b}\cosh(c+dx)}{\sqrt{\sqrt{a}-\sqrt{b}}}\right)}{8\left(\sqrt{a}-\sqrt{b}\right)^{3/2}b^{7/4}d} - \frac{\left(3\sqrt{a}+4\sqrt{b}\right)\text{arctanh}\left(\frac{\sqrt[4]{b}\cosh(c+dx)}{\sqrt{\sqrt{a}+\sqrt{b}}}\right)}{8\left(\sqrt{a}+\sqrt{b}\right)^{3/2}b^{7/4}d} - \frac{a\cosh(c+dx)\left(2-\cosh^2(c+dx)\right)}{4(a-b)bd\left(a-b+2b\cosh^2(c+dx)-b\cosh^4(c+dx)\right)}$$

output
```
1/8*(3*a^(1/2)-4*b^(1/2))*arctan(b^(1/4)*cosh(d*x+c)/(a^(1/2)-b^(1/2))^(1/
2))/(a^(1/2)-b^(1/2))^(3/2)/b^(7/4)/d-1/8*(3*a^(1/2)+4*b^(1/2))*arctanh(b^
(1/4)*cosh(d*x+c)/(a^(1/2)+b^(1/2))^(1/2))/(a^(1/2)+b^(1/2))^(3/2)/b^(7/4)
/d-1/4*a*cosh(d*x+c)*(2-cosh(d*x+c)^2)/(a-b)/b/d/(a-b+2*b*cosh(d*x+c)^2-b*
cosh(d*x+c)^4)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

Time = 4.14 (sec) , antiderivative size = 737, normalized size of antiderivative = 3.51

$$\int \frac{\sinh^7(c+dx)}{(a-b\sinh^4(c+dx))^2}\,dx = \text{Too large to display}$$

input
```
Integrate[Sinh[c + d*x]^7/(a - b*Sinh[c + d*x]^4)^2,x]
```

output
```
-1/32*((-16*a*(-5*Cosh[c + d*x] + Cosh[3*(c + d*x)]))/(8*a - 3*b + 4*b*Cos
h[2*(c + d*x)] - b*Cosh[4*(c + d*x)]) + RootSum[b - 4*b*#1^2 - 16*a*#1^4 +
 6*b*#1^4 - 4*b*#1^6 + b*#1^8 & , (3*a*c - 4*b*c + 3*a*d*x - 4*b*d*x + 6*a
*Log[-Cosh[(c + d*x)/2] - Sinh[(c + d*x)/2] + Cosh[(c + d*x)/2]*#1 - Sinh[
(c + d*x)/2]*#1] - 8*b*Log[-Cosh[(c + d*x)/2] - Sinh[(c + d*x)/2] + Cosh[(
c + d*x)/2]*#1 - Sinh[(c + d*x)/2]*#1] - 5*a*c*#1^2 + 12*b*c*#1^2 - 5*a*d*
x*#1^2 + 12*b*d*x*#1^2 - 10*a*Log[-Cosh[(c + d*x)/2] - Sinh[(c + d*x)/2] +
 Cosh[(c + d*x)/2]*#1 - Sinh[(c + d*x)/2]*#1]*#1^2 + 24*b*Log[-Cosh[(c + d
*x)/2] - Sinh[(c + d*x)/2] + Cosh[(c + d*x)/2]*#1 - Sinh[(c + d*x)/2]*#1]*
#1^2 + 5*a*c*#1^4 - 12*b*c*#1^4 + 5*a*d*x*#1^4 - 12*b*d*x*#1^4 + 10*a*Log[
-Cosh[(c + d*x)/2] - Sinh[(c + d*x)/2] + Cosh[(c + d*x)/2]*#1 - Sinh[(c +
d*x)/2]*#1]*#1^4 - 24*b*Log[-Cosh[(c + d*x)/2] - Sinh[(c + d*x)/2] + Cosh[
(c + d*x)/2]*#1 - Sinh[(c + d*x)/2]*#1]*#1^4 - 3*a*c*#1^6 + 4*b*c*#1^6 - 3
*a*d*x*#1^6 + 4*b*d*x*#1^6 - 6*a*Log[-Cosh[(c + d*x)/2] - Sinh[(c + d*x)/2
] + Cosh[(c + d*x)/2]*#1 - Sinh[(c + d*x)/2]*#1]*#1^6 + 8*b*Log[-Cosh[(c +
 d*x)/2] - Sinh[(c + d*x)/2] + Cosh[(c + d*x)/2]*#1 - Sinh[(c + d*x)/2]*#1
]*#1^6)/(-(b*#1) - 8*a*#1^3 + 3*b*#1^3 - 3*b*#1^5 + b*#1^7) & ])/((a - b)*
b*d)
```

Rubi [A] (verified)

Time = 0.55 (sec) , antiderivative size = 228, normalized size of antiderivative = 1.09, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {3042, 26, 3694, 1517, 27, 1480, 218, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sinh^7(c+dx)}{\left(a-b\sinh^4(c+dx)\right)^2}dx$$

$$\Big\downarrow \; 3042$$

$$\int \frac{i\sin(ic+idx)^7}{\left(a-b\sin(ic+idx)^4\right)^2}dx$$

$$\Big\downarrow \; 26$$

$$i\int \frac{\sin(ic+idx)^7}{\left(a-b\sin(ic+idx)^4\right)^2}dx$$

$$\Big\downarrow \; 3694$$

$$-\frac{\int \frac{\left(1-\cosh^2(c+dx)\right)^3}{\left(-b\cosh^4(c+dx)+2b\cosh^2(c+dx)+a-b\right)^2}d\cosh(c+dx)}{d}$$

$$\Big\downarrow \; 1517$$

$$-\frac{\frac{a\cosh(c+dx)\left(2-\cosh^2(c+dx)\right)}{4b(a-b)\left(a-b\cosh^4(c+dx)+2b\cosh^2(c+dx)-b\right)}-\frac{\int \frac{2a\left(2(a-2b)-(3a-4b)\cosh^2(c+dx)\right)}{-b\cosh^4(c+dx)+2b\cosh^2(c+dx)+a-b}d\cosh(c+dx)}{8ab(a-b)}}{d}$$

$$\Big\downarrow \; 27$$

$$-\frac{\frac{a\cosh(c+dx)\left(2-\cosh^2(c+dx)\right)}{4b(a-b)\left(a-b\cosh^4(c+dx)+2b\cosh^2(c+dx)-b\right)}-\frac{\int \frac{2(a-2b)-(3a-4b)\cosh^2(c+dx)}{-b\cosh^4(c+dx)+2b\cosh^2(c+dx)+a-b}d\cosh(c+dx)}{4b(a-b)}}{d}$$

$$\Big\downarrow \; 1480$$

$$-\frac{\frac{a\cosh(c+dx)\left(2-\cosh^2(c+dx)\right)}{4b(a-b)\left(a-b\cosh^4(c+dx)+2b\cosh^2(c+dx)-b\right)}-\frac{-\frac{1}{2}\left(-\sqrt{a}\sqrt{b}+3a-4b\right)\int \frac{1}{-b\cosh^2(c+dx)-\left(\sqrt{a}-\sqrt{b}\right)\sqrt{b}}d\cosh(c+dx)-\frac{1}{2}\left(\sqrt{a}\sqrt{b}+3a-4b\right)\int}{4b(a-b)}}{d}$$

$$\Big\downarrow \; 218$$

$$-\frac{\frac{a\cosh(c+dx)\left(2-\cosh^2(c+dx)\right)}{4b(a-b)\left(a-b\cosh^4(c+dx)+2b\cosh^2(c+dx)-b\right)}-\frac{\frac{\left(-\sqrt{a}\sqrt{b}+3a-4b\right)\arctan\left(\frac{\sqrt[4]{b}\cosh(c+dx)}{\sqrt{\sqrt{a}-\sqrt{b}}}\right)}{2b^{3/4}\sqrt{\sqrt{a}-\sqrt{b}}}-\frac{1}{2}\left(\sqrt{a}\sqrt{b}+3a-4b\right)\int \frac{1}{\left(\sqrt{a}+\sqrt{b}\right)\sqrt{b}-b\cosh^2(c+d}}{4b(a-b)}}{d}$$

$$\Big\downarrow \; 221$$

$$-\frac{\frac{a\cosh(c+dx)(2-\cosh^2(c+dx))}{4b(a-b)(a-b\cosh^4(c+dx)+2b\cosh^2(c+dx)-b)}-\frac{\frac{(-\sqrt{a}\sqrt{b}+3a-4b)\arctan\left(\frac{\sqrt[4]{b}\cosh(c+dx)}{\sqrt{\sqrt{a}-\sqrt{b}}}\right)}{2b^{3/4}\sqrt{\sqrt{a}-\sqrt{b}}}-\frac{(\sqrt{a}\sqrt{b}+3a-4b)\text{arctanh}\left(\frac{\sqrt[4]{b}\cosh(c+dx)}{\sqrt{\sqrt{a}+\sqrt{b}}}\right)}{2b^{3/4}\sqrt{\sqrt{a}+\sqrt{b}}}}{4b(a-b)}}{d}$$

input
```
Int[Sinh[c + d*x]^7/(a - b*Sinh[c + d*x]^4)^2,x]
```

output
```
-((-1/4*(((3*a - Sqrt[a]*Sqrt[b] - 4*b)*ArcTan[(b^(1/4)*Cosh[c + d*x])/Sqr
t[Sqrt[a] - Sqrt[b]]])/(2*Sqrt[Sqrt[a] - Sqrt[b]]*b^(3/4)) - ((3*a + Sqrt[
a]*Sqrt[b] - 4*b)*ArcTanh[(b^(1/4)*Cosh[c + d*x])/Sqrt[Sqrt[a] + Sqrt[b]]]
)/(2*Sqrt[Sqrt[a] + Sqrt[b]]*b^(3/4))))/((a - b)*b) + (a*Cosh[c + d*x]*(2 -
 Cosh[c + d*x]^2))/(4*(a - b)*b*(a - b + 2*b*Cosh[c + d*x]^2 - b*Cosh[c +
d*x]^4)))/d)
```

Defintions of rubi rules used

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 1480
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Simp[(e/2 + (2*c*d - b*e)/(2*q))   Int[1/(
b/2 - q/2 + c*x^2), x], x] + Simp[(e/2 - (2*c*d - b*e)/(2*q))   Int[1/(b/2
+ q/2 + c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0]
 && NeQ[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

rule 1517
```
Int[((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x
_Symbol] :> With[{f = Coeff[PolynomialRemainder[(d + e*x^2)^q, a + b*x^2 +
c*x^4, x], x, 0], g = Coeff[PolynomialRemainder[(d + e*x^2)^q, a + b*x^2 +
c*x^4, x], x, 2]}, Simp[x*(a + b*x^2 + c*x^4)^(p + 1)*((a*b*g - f*(b^2 - 2*
a*c) - c*(b*f - 2*a*g)*x^2)/(2*a*(p + 1)*(b^2 - 4*a*c))), x] + Simp[1/(2*a*
(p + 1)*(b^2 - 4*a*c))   Int[(a + b*x^2 + c*x^4)^(p + 1)*ExpandToSum[2*a*(p
 + 1)*(b^2 - 4*a*c)*PolynomialQuotient[(d + e*x^2)^q, a + b*x^2 + c*x^4, x]
 + b^2*f*(2*p + 3) - 2*a*c*f*(4*p + 5) - a*b*g + c*(4*p + 7)*(b*f - 2*a*g)*
x^2, x], x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ
[c*d^2 - b*d*e + a*e^2, 0] && IGtQ[q, 1] && LtQ[p, -1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3694
```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^4)^
(p_.), x_Symbol] :> With[{ff = FreeFactors[Cos[e + f*x], x]}, Simp[-ff/f
Subst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a + b - 2*b*ff^2*x^2 + b*ff^4*x^4)^p,
 x], x, Cos[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m -
 1)/2]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 433 vs. $2(164) = 328$.

Time = 8.11 (sec) , antiderivative size = 434, normalized size of antiderivative = 2.07

method	result
derivativedivides	$128a^2\left(\frac{\frac{-\frac{\left(ab-\sqrt{ab}\,a+2\sqrt{ab}\,b\right)\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^2}{2a^2(a-b)}-\frac{b+\sqrt{ab}}{2a(a-b)}}{\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^4-2\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^2+\frac{4\sqrt{ab}\,\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^2}{a}+1}+\frac{\left(3\sqrt{ab}\,a-4\sqrt{ab}\,b-ab\right)\arctan\left(\frac{2\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^2a+4\sqrt{ab}-}{4\sqrt{-ab+\sqrt{ab}\,a}}\right.}{4a(a-b)\sqrt{-ab+\sqrt{ab}\,a}}}{256a\,b^2}\right.$
default	$128a^2\left(\frac{\frac{-\frac{\left(ab-\sqrt{ab}\,a+2\sqrt{ab}\,b\right)\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^2}{2a^2(a-b)}-\frac{b+\sqrt{ab}}{2a(a-b)}}{\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^4-2\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^2+\frac{4\sqrt{ab}\,\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^2}{a}+1}+\frac{\left(3\sqrt{ab}\,a-4\sqrt{ab}\,b-ab\right)\arctan\left(\frac{2\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^2a+4\sqrt{ab}-}{4\sqrt{-ab+\sqrt{ab}\,a}}\right.}{4a(a-b)\sqrt{-ab+\sqrt{ab}\,a}}}{256a\,b^2}\right.$
risch	$\frac{e^{dx+c}a\left(e^{6dx+6c}-5\,e^{4dx+4c}-5\,e^{2dx+2c}+1\right)}{2bd(a-b)\left(-e^{8dx+8c}b+4\,e^{6dx+6c}b+16\,e^{4dx+4c}a-6b\,e^{4dx+4c}+4\,e^{2dx+2c}b-b\right)}+\left(\_R=\text{RootOf}\left(\left(65536a^3b^7d^4-1966\right.\right.\right.$

input
```
int(sinh(d*x+c)^7/(a-b*sinh(d*x+c)^4)^2,x,method=_RETURNVERBOSE)
```

output
```
128/d*a^2*(1/256/a/b^2*((-1/2*(a*b-(a*b)^(1/2)*a+2*(a*b)^(1/2)*b)/a^2/(a-b
)*tanh(1/2*d*x+1/2*c)^2-1/2*(b+(a*b)^(1/2))/a/(a-b))/(tanh(1/2*d*x+1/2*c)^
4-2*tanh(1/2*d*x+1/2*c)^2+4*(a*b)^(1/2)/a*tanh(1/2*d*x+1/2*c)^2+1)+1/4*(3*
(a*b)^(1/2)*a-4*(a*b)^(1/2)*b-a*b)/a/(a-b)/(-a*b+(a*b)^(1/2)*a)^(1/2)*arct
an(1/4*(2*tanh(1/2*d*x+1/2*c)^2*a+4*(a*b)^(1/2)-2*a)/(-a*b+(a*b)^(1/2)*a)^
(1/2)))-1/256/a/b^2*((1/2*((a*b)^(1/2)*a-2*(a*b)^(1/2)*b+a*b)/a^2/(a-b)*ta
nh(1/2*d*x+1/2*c)^2+1/2*(b-(a*b)^(1/2))/a/(a-b))/(tanh(1/2*d*x+1/2*c)^4-4*
(a*b)^(1/2)/a*tanh(1/2*d*x+1/2*c)^2-2*tanh(1/2*d*x+1/2*c)^2+1)-1/4*(3*(a*b
)^(1/2)*a-4*(a*b)^(1/2)*b+a*b)/a/(a-b)/(-a*b-(a*b)^(1/2)*a)^(1/2)*arctan(1
/4*(-2*tanh(1/2*d*x+1/2*c)^2*a+4*(a*b)^(1/2)+2*a)/(-a*b-(a*b)^(1/2)*a)^(1/
2))))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 6266 vs. $2(161) = 322$.

Time = 0.24 (sec) , antiderivative size = 6266, normalized size of antiderivative = 29.84

$$\int \frac{\sinh^7(c+dx)}{(a-b\sinh^4(c+dx))^2}\,dx = \text{Too large to display}$$

input
```
integrate(sinh(d*x+c)^7/(a-b*sinh(d*x+c)^4)^2,x, algorithm="fricas")
```

output
```
Too large to include
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\sinh^7(c+dx)}{(a-b\sinh^4(c+dx))^2}\,dx = \text{Timed out}$$

input
```
integrate(sinh(d*x+c)**7/(a-b*sinh(d*x+c)**4)**2,x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{\sinh^7(c+dx)}{(a-b\sinh^4(c+dx))^2}\,dx = \int \frac{\sinh(dx+c)^7}{(b\sinh(dx+c)^4-a)^2}\,dx$$

input
```
integrate(sinh(d*x+c)^7/(a-b*sinh(d*x+c)^4)^2,x, algorithm="maxima")
```

output
```
-1/2*(a*e^(7*d*x + 7*c) - 5*a*e^(5*d*x + 5*c) - 5*a*e^(3*d*x + 3*c) + a*e^
(d*x + c))/(a*b^2*d - b^3*d + (a*b^2*d*e^(8*c) - b^3*d*e^(8*c))*e^(8*d*x)
- 4*(a*b^2*d*e^(6*c) - b^3*d*e^(6*c))*e^(6*d*x) - 2*(8*a^2*b*d*e^(4*c) - 1
1*a*b^2*d*e^(4*c) + 3*b^3*d*e^(4*c))*e^(4*d*x) - 4*(a*b^2*d*e^(2*c) - b^3*
d*e^(2*c))*e^(2*d*x)) + 1/128*integrate(64*((3*a*e^(7*c) - 4*b*e^(7*c))*e^
(7*d*x) - (5*a*e^(5*c) - 12*b*e^(5*c))*e^(5*d*x) + (5*a*e^(3*c) - 12*b*e^(
3*c))*e^(3*d*x) - (3*a*e^c - 4*b*e^c)*e^(d*x))/(a*b^2 - b^3 + (a*b^2*e^(8*
c) - b^3*e^(8*c))*e^(8*d*x) - 4*(a*b^2*e^(6*c) - b^3*e^(6*c))*e^(6*d*x) -
2*(8*a^2*b*e^(4*c) - 11*a*b^2*e^(4*c) + 3*b^3*e^(4*c))*e^(4*d*x) - 4*(a*b^
2*e^(2*c) - b^3*e^(2*c))*e^(2*d*x)), x)
```

Giac [F]

$$\int \frac{\sinh^7(c+dx)}{\left(a-b\sinh^4(c+dx)\right)^2}\,dx=\int \frac{\sinh\left(dx+c\right)^7}{\left(b\sinh\left(dx+c\right)^4-a\right)^2}\,dx$$

input
```
integrate(sinh(d*x+c)^7/(a-b*sinh(d*x+c)^4)^2,x, algorithm="giac")
```

output
```
sage0*x
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sinh^7(c+dx)}{\left(a-b\sinh^4(c+dx)\right)^2}\,dx=\int \frac{\sinh(c+d\,x)^7}{\left(a-b\sinh\left(c+d\,x\right)^4\right)^2}\,dx$$

input
```
int(sinh(c + d*x)^7/(a - b*sinh(c + d*x)^4)^2,x)
```

output
```
int(sinh(c + d*x)^7/(a - b*sinh(c + d*x)^4)^2, x)
```

Reduce [F]

$$\int \frac{\sinh^7(c+dx)}{(a-b\sinh^4(c+dx))^2}\,dx = \text{too large to display}$$

input
```
int(sinh(d*x+c)^7/(a-b*sinh(d*x+c)^4)^2,x)
```

output
```
(2*e**c*( - 16128*e**(14*c + 8*d*x)*int(e**(7*d*x)/(e**(16*c + 16*d*x)*b**
2 - 8*e**(14*c + 14*d*x)*b**2 - 32*e**(12*c + 12*d*x)*a*b + 28*e**(12*c +
12*d*x)*b**2 + 128*e**(10*c + 10*d*x)*a*b - 56*e**(10*c + 10*d*x)*b**2 + 2
56*e**(8*c + 8*d*x)*a**2 - 192*e**(8*c + 8*d*x)*a*b + 70*e**(8*c + 8*d*x)*
b**2 + 128*e**(6*c + 6*d*x)*a*b - 56*e**(6*c + 6*d*x)*b**2 - 32*e**(4*c +
4*d*x)*a*b + 28*e**(4*c + 4*d*x)*b**2 - 8*e**(2*c + 2*d*x)*b**2 + b**2),x)
*a**2*b*d - 2736*e**(14*c + 8*d*x)*int(e**(7*d*x)/(e**(16*c + 16*d*x)*b**2
 - 8*e**(14*c + 14*d*x)*b**2 - 32*e**(12*c + 12*d*x)*a*b + 28*e**(12*c + 1
2*d*x)*b**2 + 128*e**(10*c + 10*d*x)*a*b - 56*e**(10*c + 10*d*x)*b**2 + 25
6*e**(8*c + 8*d*x)*a**2 - 192*e**(8*c + 8*d*x)*a*b + 70*e**(8*c + 8*d*x)*b
**2 + 128*e**(6*c + 6*d*x)*a*b - 56*e**(6*c + 6*d*x)*b**2 - 32*e**(4*c + 4
*d*x)*a*b + 28*e**(4*c + 4*d*x)*b**2 - 8*e**(2*c + 2*d*x)*b**2 + b**2),x)*
a*b**2*d + 384*e**(14*c + 8*d*x)*int(e**(7*d*x)/(e**(16*c + 16*d*x)*b**2 -
 8*e**(14*c + 14*d*x)*b**2 - 32*e**(12*c + 12*d*x)*a*b + 28*e**(12*c + 12*
d*x)*b**2 + 128*e**(10*c + 10*d*x)*a*b - 56*e**(10*c + 10*d*x)*b**2 + 256*
e**(8*c + 8*d*x)*a**2 - 192*e**(8*c + 8*d*x)*a*b + 70*e**(8*c + 8*d*x)*b**
2 + 128*e**(6*c + 6*d*x)*a*b - 56*e**(6*c + 6*d*x)*b**2 - 32*e**(4*c + 4*d
*x)*a*b + 28*e**(4*c + 4*d*x)*b**2 - 8*e**(2*c + 2*d*x)*b**2 + b**2),x)*b*
*3*d - 40704*e**(12*c + 8*d*x)*int(e**(5*d*x)/(e**(16*c + 16*d*x)*b**2 - 8
*e**(14*c + 14*d*x)*b**2 - 32*e**(12*c + 12*d*x)*a*b + 28*e**(12*c + 12...
```

3.218 $\int \frac{\sinh^5(c+dx)}{(a-b\sinh^4(c+dx))^2}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 217

$$\int \frac{\sinh^5(c+dx)}{(a-b\sinh^4(c+dx))^2}\,dx = -\frac{\left(\sqrt{a}-2\sqrt{b}\right)\arctan\left(\frac{\sqrt[4]{b}\cosh(c+dx)}{\sqrt{\sqrt{a}-\sqrt{b}}}\right)}{8\sqrt{a}\left(\sqrt{a}-\sqrt{b}\right)^{3/2}b^{5/4}d} - \frac{\left(\sqrt{a}+2\sqrt{b}\right)\operatorname{arctanh}\left(\frac{\sqrt[4]{b}\cosh(c+dx)}{\sqrt{\sqrt{a}+\sqrt{b}}}\right)}{8\sqrt{a}\left(\sqrt{a}+\sqrt{b}\right)^{3/2}b^{5/4}d} + \frac{\cosh(c+dx)\left(a+b-b\cosh^2(c+dx)\right)}{4(a-b)bd\left(a-b+2b\cosh^2(c+dx)-b\cosh^4(c+dx)\right)}$$

output
```
-1/8*(a^(1/2)-2*b^(1/2))*arctan(b^(1/4)*cosh(d*x+c)/(a^(1/2)-b^(1/2))^(1/2
))/a^(1/2)/(a^(1/2)-b^(1/2))^(3/2)/b^(5/4)/d-1/8*(a^(1/2)+2*b^(1/2))*arcta
nh(b^(1/4)*cosh(d*x+c)/(a^(1/2)+b^(1/2))^(1/2))/a^(1/2)/(a^(1/2)+b^(1/2))^
(3/2)/b^(5/4)/d+1/4*cosh(d*x+c)*(a+b-b*cosh(d*x+c)^2)/(a-b)/b/d/(a-b+2*b*c
osh(d*x+c)^2-b*cosh(d*x+c)^4)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

Time = 4.02 (sec) , antiderivative size = 597, normalized size of antiderivative = 2.75

$$\int \frac{\sinh^5(c+dx)}{(a-b\sinh^4(c+dx))^2}\,dx$$
$$= \frac{\frac{32\cosh(c+dx)(2a+b-b\cosh(2(c+dx)))}{8a-3b+4b\cosh(2(c+dx))-b\cosh(4(c+dx))} + \text{RootSum}\left[b-4b\#1^2-16a\#1^4+6b\#1^4-4b\#1^6+b\#1^8\&, \frac{-bc-bdx-}{}\right.}{}$$

input
```
Integrate[Sinh[c + d*x]^5/(a - b*Sinh[c + d*x]^4)^2,x]
```

output
```
((32*Cosh[c + d*x]*(2*a + b - b*Cosh[2*(c + d*x)]))/(8*a - 3*b + 4*b*Cosh[
2*(c + d*x)] - b*Cosh[4*(c + d*x)]) + RootSum[b - 4*b*#1^2 - 16*a*#1^4 + 6
*b*#1^4 - 4*b*#1^6 + b*#1^8 & , (-(b*c) - b*d*x - 2*b*Log[-Cosh[(c + d*x)/
2] - Sinh[(c + d*x)/2] + Cosh[(c + d*x)/2]*#1 - Sinh[(c + d*x)/2]*#1] - 4*
a*c*#1^2 + 11*b*c*#1^2 - 4*a*d*x*#1^2 + 11*b*d*x*#1^2 - 8*a*Log[-Cosh[(c +
 d*x)/2] - Sinh[(c + d*x)/2] + Cosh[(c + d*x)/2]*#1 - Sinh[(c + d*x)/2]*#1
]*#1^2 + 22*b*Log[-Cosh[(c + d*x)/2] - Sinh[(c + d*x)/2] + Cosh[(c + d*x)/
2]*#1 - Sinh[(c + d*x)/2]*#1]*#1^2 + 4*a*c*#1^4 - 11*b*c*#1^4 + 4*a*d*x*#1
^4 - 11*b*d*x*#1^4 + 8*a*Log[-Cosh[(c + d*x)/2] - Sinh[(c + d*x)/2] + Cosh
[(c + d*x)/2]*#1 - Sinh[(c + d*x)/2]*#1]*#1^4 - 22*b*Log[-Cosh[(c + d*x)/2
] - Sinh[(c + d*x)/2] + Cosh[(c + d*x)/2]*#1 - Sinh[(c + d*x)/2]*#1]*#1^4
+ b*c*#1^6 + b*d*x*#1^6 + 2*b*Log[-Cosh[(c + d*x)/2] - Sinh[(c + d*x)/2] +
 Cosh[(c + d*x)/2]*#1 - Sinh[(c + d*x)/2]*#1]*#1^6)/(-(b*#1) - 8*a*#1^3 +
3*b*#1^3 - 3*b*#1^5 + b*#1^7) & ])/(32*(a - b)*b*d)
```

Rubi [A] (verified)

Time = 0.52 (sec) , antiderivative size = 227, normalized size of antiderivative = 1.05, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {3042, 26, 3694, 1517, 27, 1480, 218, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sinh^5(c+dx)}{\left(a-b\sinh^4(c+dx)\right)^2}\,dx$$

$\Big\downarrow$ 3042

$$\int -\frac{i\sin(ic+idx)^5}{\left(a-b\sin(ic+idx)^4\right)^2}dx$$

$\Big\downarrow$ 26

$$-i\int \frac{\sin(ic+idx)^5}{\left(a-b\sin(ic+idx)^4\right)^2}dx$$

$\Big\downarrow$ 3694

$$\frac{\int \frac{\left(1-\cosh^2(c+dx)\right)^2}{\left(-b\cosh^4(c+dx)+2b\cosh^2(c+dx)+a-b\right)^2}d\cosh(c+dx)}{d}$$

$\Big\downarrow$ 1517

$$\frac{\frac{\cosh(c+dx)\left(a-b\cosh^2(c+dx)+b\right)}{4b(a-b)\left(a-b\cosh^4(c+dx)+2b\cosh^2(c+dx)-b\right)}-\frac{\int \frac{2a\left(b\cosh^2(c+dx)+a-3b\right)}{-b\cosh^4(c+dx)+2b\cosh^2(c+dx)+a-b}d\cosh(c+dx)}{8ab(a-b)}}{d}$$

$\Big\downarrow$ 27

$$\frac{\frac{\cosh(c+dx)\left(a-b\cosh^2(c+dx)+b\right)}{4b(a-b)\left(a-b\cosh^4(c+dx)+2b\cosh^2(c+dx)-b\right)}-\frac{\int \frac{b\cosh^2(c+dx)+a-3b}{-b\cosh^4(c+dx)+2b\cosh^2(c+dx)+a-b}d\cosh(c+dx)}{4b(a-b)}}{d}$$

$\Big\downarrow$ 1480

$$\frac{\frac{\cosh(c+dx)\left(a-b\cosh^2(c+dx)+b\right)}{4b(a-b)\left(a-b\cosh^4(c+dx)+2b\cosh^2(c+dx)-b\right)}-\frac{\frac{1}{2}\sqrt{b}\left(\frac{a-2b}{\sqrt{a}}+\sqrt{b}\right)\int \frac{1}{\left(\sqrt{a}+\sqrt{b}\right)\sqrt{b}-b\cosh^2(c+dx)}d\cosh(c+dx)-\frac{1}{2}\sqrt{b}\left(\frac{a-2b}{\sqrt{a}}-\sqrt{b}\right)\int \frac{\cdots}{-b\cosh^2(c\cdots}}{4b(a-b)}}{d}$$

$\Big\downarrow$ 218

$$\frac{\frac{\cosh(c+dx)\left(a-b\cosh^2(c+dx)+b\right)}{4b(a-b)\left(a-b\cosh^4(c+dx)+2b\cosh^2(c+dx)-b\right)}-\frac{\frac{1}{2}\sqrt{b}\left(\frac{a-2b}{\sqrt{a}}+\sqrt{b}\right)\int \frac{1}{\left(\sqrt{a}+\sqrt{b}\right)\sqrt{b}-b\cosh^2(c+dx)}d\cosh(c+dx)+\frac{\left(\frac{a-2b}{\sqrt{a}}-\sqrt{b}\right)\arctan\left(\frac{\sqrt[4]{b}\cosh(\cdots}{\sqrt{\sqrt{a}-\cdots}}\right.}{2\sqrt[4]{b}\sqrt{\sqrt{a}-\sqrt{b}}}\cdots}{4b(a-b)}}{d}$$

$\Big\downarrow$ 221

$$\frac{\frac{\cosh(c+dx)\left(a-b\cosh^2(c+dx)+b\right)}{4b(a-b)\left(a-b\cosh^4(c+dx)+2b\cosh^2(c+dx)-b\right)}-\frac{\frac{\left(\frac{a-2b}{\sqrt{a}}-\sqrt{b}\right)\arctan\left(\frac{\sqrt[4]{b}\cosh(c+dx)}{\sqrt{\sqrt{a}-\sqrt{b}}}\right)}{2\sqrt[4]{b}\sqrt{\sqrt{a}-\sqrt{b}}}+\frac{\left(\frac{a-2b}{\sqrt{a}}+\sqrt{b}\right)\text{arctanh}\left(\frac{\sqrt[4]{b}\cosh(c+dx)}{\sqrt{\sqrt{a}+\sqrt{b}}}\right)}{2\sqrt[4]{b}\sqrt{\sqrt{a}+\sqrt{b}}}}{4b(a-b)}}{d}$$

input
```
Int[Sinh[c + d*x]^5/(a - b*Sinh[c + d*x]^4)^2,x]
```

output
```
(-1/4*((((a - 2*b)/Sqrt[a] - Sqrt[b])*ArcTan[(b^(1/4)*Cosh[c + d*x])/Sqrt[
Sqrt[a] - Sqrt[b]]])/(2*Sqrt[Sqrt[a] - Sqrt[b]]*b^(1/4)) + (((a - 2*b)/Sqr
t[a] + Sqrt[b])*ArcTanh[(b^(1/4)*Cosh[c + d*x])/Sqrt[Sqrt[a] + Sqrt[b]]])/
(2*Sqrt[Sqrt[a] + Sqrt[b]]*b^(1/4)))/((a - b)*b) + (Cosh[c + d*x]*(a + b -
 b*Cosh[c + d*x]^2))/(4*(a - b)*b*(a - b + 2*b*Cosh[c + d*x]^2 - b*Cosh[c
+ d*x]^4)))/d
```

Defintions of rubi rules used

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 1480
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Simp[(e/2 + (2*c*d - b*e)/(2*q))   Int[1/(
b/2 - q/2 + c*x^2), x], x] + Simp[(e/2 - (2*c*d - b*e)/(2*q))   Int[1/(b/2
+ q/2 + c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0]
 && NeQ[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

rule 1517
```
Int[((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x
_Symbol] :> With[{f = Coeff[PolynomialRemainder[(d + e*x^2)^q, a + b*x^2 +
c*x^4, x], x, 0], g = Coeff[PolynomialRemainder[(d + e*x^2)^q, a + b*x^2 +
c*x^4, x], x, 2]}, Simp[x*(a + b*x^2 + c*x^4)^(p + 1)*((a*b*g - f*(b^2 - 2*
a*c) - c*(b*f - 2*a*g)*x^2)/(2*a*(p + 1)*(b^2 - 4*a*c))), x] + Simp[1/(2*a*
(p + 1)*(b^2 - 4*a*c))   Int[(a + b*x^2 + c*x^4)^(p + 1)*ExpandToSum[2*a*(p
 + 1)*(b^2 - 4*a*c)*PolynomialQuotient[(d + e*x^2)^q, a + b*x^2 + c*x^4, x]
 + b^2*f*(2*p + 3) - 2*a*c*f*(4*p + 5) - a*b*g + c*(4*p + 7)*(b*f - 2*a*g)*
x^2, x], x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ
[c*d^2 - b*d*e + a*e^2, 0] && IGtQ[q, 1] && LtQ[p, -1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3694
```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^4)^
(p_.), x_Symbol] :> With[{ff = FreeFactors[Cos[e + f*x], x]}, Simp[-ff/f
Subst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a + b - 2*b*ff^2*x^2 + b*ff^4*x^4)^p,
 x], x, Cos[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m -
 1)/2]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 346 vs. $2(167) = 334$.

Time = 7.22 (sec) , antiderivative size = 347, normalized size of antiderivative = 1.60

method	result
derivativedivides	$\frac{\frac{-\frac{(a-2b)\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^{6}}{2b(a-b)}+\frac{(3a-8b)\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^{4}}{2b(a-b)}-\frac{(3a+2b)\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}}{2b(a-b)}+\frac{a}{2b(a-b)}}{\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^{8}a-4\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^{6}a+6\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^{4}a-16b\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^{4}-4\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}a+a}+\frac{a\left(-\frac{\left(-\sqrt{ab}-a+2b\right)\arctan}{4a}\right.}{}}{d}$
default	$\frac{\frac{-\frac{(a-2b)\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^{6}}{2b(a-b)}+\frac{(3a-8b)\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^{4}}{2b(a-b)}-\frac{(3a+2b)\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}}{2b(a-b)}+\frac{a}{2b(a-b)}}{\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^{8}a-4\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^{6}a+6\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^{4}a-16b\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^{4}-4\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}a+a}+\frac{a\left(-\frac{\left(-\sqrt{ab}-a+2b\right)\arctan}{4a}\right.}{}}{d}$
risch	$\frac{e^{dx+c}\left(-e^{6dx+6c}b+4\,e^{4dx+4c}a+b\,e^{4dx+4c}+4\,e^{2dx+2c}a+e^{2dx+2c}b-b\right)}{2b(a-b)d\left(-e^{8dx+8c}b+4\,e^{6dx+6c}b+16\,e^{4dx+4c}a-6b\,e^{4dx+4c}+4\,e^{2dx+2c}b-b\right)}+\Bigg(\ \_R=\mathrm{RootOf}\left(\left(65536a^{5}b^{5}d^{4}-1966\right.\right.$

input
```
int(sinh(d*x+c)^5/(a-b*sinh(d*x+c)^4)^2,x,method=_RETURNVERBOSE)
```

output
```
1/d*(32*(-1/64*(a-2*b)/b/(a-b)*tanh(1/2*d*x+1/2*c)^6+1/64*(3*a-8*b)/b/(a-b
)*tanh(1/2*d*x+1/2*c)^4-1/64*(3*a+2*b)/b/(a-b)*tanh(1/2*d*x+1/2*c)^2+1/64/
b/(a-b)*a)/(tanh(1/2*d*x+1/2*c)^8*a-4*tanh(1/2*d*x+1/2*c)^6*a+6*tanh(1/2*d
*x+1/2*c)^4*a-16*b*tanh(1/2*d*x+1/2*c)^4-4*tanh(1/2*d*x+1/2*c)^2*a+a)+1/2/
b/(a-b)*a*(-1/4*(-(a*b)^(1/2)-a+2*b)/a/(-a*b-(a*b)^(1/2)*a)^(1/2)*arctan(1
/4*(-2*tanh(1/2*d*x+1/2*c)^2*a+4*(a*b)^(1/2)+2*a)/(-a*b-(a*b)^(1/2)*a)^(1/
2))+1/4*((a*b)^(1/2)-a+2*b)/a/(-a*b+(a*b)^(1/2)*a)^(1/2)*arctan(1/4*(2*tan
h(1/2*d*x+1/2*c)^2*a+4*(a*b)^(1/2)-2*a)/(-a*b+(a*b)^(1/2)*a)^(1/2))))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 6250 vs. $2(169) = 338$.

Time = 0.25 (sec) , antiderivative size = 6250, normalized size of antiderivative = 28.80

$$\int \frac{\sinh^5(c+dx)}{(a-b\sinh^4(c+dx))^2}\,dx = \text{Too large to display}$$

input
```
integrate(sinh(d*x+c)^5/(a-b*sinh(d*x+c)^4)^2,x, algorithm="fricas")
```

output
```
Too large to include
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\sinh^5(c+dx)}{(a-b\sinh^4(c+dx))^2}\,dx = \text{Timed out}$$

input
```
integrate(sinh(d*x+c)**5/(a-b*sinh(d*x+c)**4)**2,x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{\sinh^5(c+dx)}{(a-b\sinh^4(c+dx))^2}\,dx = \int \frac{\sinh(dx+c)^5}{(b\sinh(dx+c)^4-a)^2}\,dx$$

input
```
integrate(sinh(d*x+c)^5/(a-b*sinh(d*x+c)^4)^2,x, algorithm="maxima")
```

output
```
-1/2*((4*a*e^(5*c) + b*e^(5*c))*e^(5*d*x) + (4*a*e^(3*c) + b*e^(3*c))*e^(3
*d*x) - b*e^(7*d*x + 7*c) - b*e^(d*x + c))/(a*b^2*d - b^3*d + (a*b^2*d*e^(
8*c) - b^3*d*e^(8*c))*e^(8*d*x) - 4*(a*b^2*d*e^(6*c) - b^3*d*e^(6*c))*e^(6
*d*x) - 2*(8*a^2*b*d*e^(4*c) - 11*a*b^2*d*e^(4*c) + 3*b^3*d*e^(4*c))*e^(4*
d*x) - 4*(a*b^2*d*e^(2*c) - b^3*d*e^(2*c))*e^(2*d*x)) + 1/32*integrate(16*
((4*a*e^(5*c) - 11*b*e^(5*c))*e^(5*d*x) - (4*a*e^(3*c) - 11*b*e^(3*c))*e^(
3*d*x) + b*e^(7*d*x + 7*c) - b*e^(d*x + c))/(a*b^2 - b^3 + (a*b^2*e^(8*c)
- b^3*e^(8*c))*e^(8*d*x) - 4*(a*b^2*e^(6*c) - b^3*e^(6*c))*e^(6*d*x) - 2*(
8*a^2*b*e^(4*c) - 11*a*b^2*e^(4*c) + 3*b^3*e^(4*c))*e^(4*d*x) - 4*(a*b^2*e
^(2*c) - b^3*e^(2*c))*e^(2*d*x)), x)
```

Giac [F]

$$\int \frac{\sinh^5(c+dx)}{\left(a-b\sinh^4(c+dx)\right)^2}\,dx=\int \frac{\sinh\left(dx+c\right)^5}{\left(b\sinh\left(dx+c\right)^4-a\right)^2}\,dx$$

input
```
integrate(sinh(d*x+c)^5/(a-b*sinh(d*x+c)^4)^2,x, algorithm="giac")
```

output
```
sage0*x
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sinh^5(c+dx)}{\left(a-b\sinh^4(c+dx)\right)^2}\,dx=\int \frac{\sinh(c+d\,x)^5}{\left(a-b\sinh\left(c+d\,x\right)^4\right)^2}\,dx$$

input
```
int(sinh(c + d*x)^5/(a - b*sinh(c + d*x)^4)^2,x)
```

output
```
int(sinh(c + d*x)^5/(a - b*sinh(c + d*x)^4)^2, x)
```

Reduce [F]

$$\int \frac{\sinh^5(c+dx)}{\left(a-b\sinh^4(c+dx)\right)^2}\,dx = \text{too large to display}$$

input
```
int(sinh(d*x+c)^5/(a-b*sinh(d*x+c)^4)^2,x)
```

output
```
(8*e**c*( - 2832*e**(14*c + 8*d*x)*int(e**(7*d*x)/(e**(16*c + 16*d*x)*b**2
 - 8*e**(14*c + 14*d*x)*b**2 - 32*e**(12*c + 12*d*x)*a*b + 28*e**(12*c + 1
2*d*x)*b**2 + 128*e**(10*c + 10*d*x)*a*b - 56*e**(10*c + 10*d*x)*b**2 + 25
6*e**(8*c + 8*d*x)*a**2 - 192*e**(8*c + 8*d*x)*a*b + 70*e**(8*c + 8*d*x)*b
**2 + 128*e**(6*c + 6*d*x)*a*b - 56*e**(6*c + 6*d*x)*b**2 - 32*e**(4*c + 4
*d*x)*a*b + 28*e**(4*c + 4*d*x)*b**2 - 8*e**(2*c + 2*d*x)*b**2 + b**2),x)*
a*b**2*d - 48*e**(14*c + 8*d*x)*int(e**(7*d*x)/(e**(16*c + 16*d*x)*b**2 -
8*e**(14*c + 14*d*x)*b**2 - 32*e**(12*c + 12*d*x)*a*b + 28*e**(12*c + 12*d
*x)*b**2 + 128*e**(10*c + 10*d*x)*a*b - 56*e**(10*c + 10*d*x)*b**2 + 256*e
**(8*c + 8*d*x)*a**2 - 192*e**(8*c + 8*d*x)*a*b + 70*e**(8*c + 8*d*x)*b**2
 + 128*e**(6*c + 6*d*x)*a*b - 56*e**(6*c + 6*d*x)*b**2 - 32*e**(4*c + 4*d*
x)*a*b + 28*e**(4*c + 4*d*x)*b**2 - 8*e**(2*c + 2*d*x)*b**2 + b**2),x)*b**
3*d - 3840*e**(12*c + 8*d*x)*int(e**(5*d*x)/(e**(16*c + 16*d*x)*b**2 - 8*e
**(14*c + 14*d*x)*b**2 - 32*e**(12*c + 12*d*x)*a*b + 28*e**(12*c + 12*d*x)
*b**2 + 128*e**(10*c + 10*d*x)*a*b - 56*e**(10*c + 10*d*x)*b**2 + 256*e**(
8*c + 8*d*x)*a**2 - 192*e**(8*c + 8*d*x)*a*b + 70*e**(8*c + 8*d*x)*b**2 +
128*e**(6*c + 6*d*x)*a*b - 56*e**(6*c + 6*d*x)*b**2 - 32*e**(4*c + 4*d*x)*
a*b + 28*e**(4*c + 4*d*x)*b**2 - 8*e**(2*c + 2*d*x)*b**2 + b**2),x)*a**2*b
*d + 3744*e**(12*c + 8*d*x)*int(e**(5*d*x)/(e**(16*c + 16*d*x)*b**2 - 8*e*
*(14*c + 14*d*x)*b**2 - 32*e**(12*c + 12*d*x)*a*b + 28*e**(12*c + 12*d*...
```

3.219 $\int \frac{\sinh^3(c+dx)}{(a-b\sinh^4(c+dx))^2} \, dx$

Optimal result

Integrand size = 24, antiderivative size = 186

$$\int \frac{\sinh^3(c+dx)}{(a-b\sinh^4(c+dx))^2}\,dx = -\frac{\arctan\left(\frac{\sqrt[4]{b}\cosh(c+dx)}{\sqrt{\sqrt{a}-\sqrt{b}}}\right)}{8\sqrt{a}\left(\sqrt{a}-\sqrt{b}\right)^{3/2}b^{3/4}d} + \frac{\operatorname{arctanh}\left(\frac{\sqrt[4]{b}\cosh(c+dx)}{\sqrt{\sqrt{a}+\sqrt{b}}}\right)}{8\sqrt{a}\left(\sqrt{a}+\sqrt{b}\right)^{3/2}b^{3/4}d} - \frac{\cosh(c+dx)\left(2-\cosh^2(c+dx)\right)}{4(a-b)d\left(a-b+2b\cosh^2(c+dx)-b\cosh^4(c+dx)\right)}$$

output
```
-1/8*arctan(b^(1/4)*cosh(d*x+c)/(a^(1/2)-b^(1/2))^(1/2))/a^(1/2)/(a^(1/2)-
b^(1/2))^(3/2)/b^(3/4)/d+1/8*arctanh(b^(1/4)*cosh(d*x+c)/(a^(1/2)+b^(1/2))
^(1/2))/a^(1/2)/(a^(1/2)+b^(1/2))^(3/2)/b^(3/4)/d-1/4*cosh(d*x+c)*(2-cosh(
d*x+c)^2)/(a-b)/d/(a-b+2*b*cosh(d*x+c)^2-b*cosh(d*x+c)^4)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

Time = 0.68 (sec) , antiderivative size = 422, normalized size of antiderivative = 2.27

$$\int \frac{\sinh^3(c+dx)}{(a-b\sinh^4(c+dx))^2}\,dx =$$

$$-\frac{\frac{16(-5\cosh(c+dx)+\cosh(3(c+dx)))}{-8a+3b-4b\cosh(2(c+dx))+b\cosh(4(c+dx))}+\text{RootSum}\Big[b-4b\#1^2-16a\#1^4+6b\#1^4-4b\#1^6+b\#1^8\&,\frac{-c-}{}$$

input
```
Integrate[Sinh[c + d*x]^3/(a - b*Sinh[c + d*x]^4)^2,x]
```

output
```
-1/32*((16*(-5*Cosh[c + d*x] + Cosh[3*(c + d*x)]))/(-8*a + 3*b - 4*b*Cosh[
2*(c + d*x)] + b*Cosh[4*(c + d*x)]) + RootSum[b - 4*b*#1^2 - 16*a*#1^4 + 6
*b*#1^4 - 4*b*#1^6 + b*#1^8 & , (-c - d*x - 2*Log[-Cosh[(c + d*x)/2] - Sin
h[(c + d*x)/2] + Cosh[(c + d*x)/2]*#1 - Sinh[(c + d*x)/2]*#1] + 7*c*#1^2 +
 7*d*x*#1^2 + 14*Log[-Cosh[(c + d*x)/2] - Sinh[(c + d*x)/2] + Cosh[(c + d*
x)/2]*#1 - Sinh[(c + d*x)/2]*#1]*#1^2 - 7*c*#1^4 - 7*d*x*#1^4 - 14*Log[-Co
sh[(c + d*x)/2] - Sinh[(c + d*x)/2] + Cosh[(c + d*x)/2]*#1 - Sinh[(c + d*x
)/2]*#1]*#1^4 + c*#1^6 + d*x*#1^6 + 2*Log[-Cosh[(c + d*x)/2] - Sinh[(c + d
*x)/2] + Cosh[(c + d*x)/2]*#1 - Sinh[(c + d*x)/2]*#1]*#1^6)/(-(b*#1) - 8*a
*#1^3 + 3*b*#1^3 - 3*b*#1^5 + b*#1^7) & ])/((a - b)*d)
```

Rubi [A] (verified)

Time = 0.44 (sec) , antiderivative size = 211, normalized size of antiderivative = 1.13, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {3042, 26, 3694, 1492, 27, 1480, 218, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sinh^3(c+dx)}{(a-b\sinh^4(c+dx))^2}\,dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \frac{i\sin(ic+idx)^3}{\left(a-b\sin(ic+idx)^4\right)^2}dx$$

$$\Big\downarrow \text{26}$$

$$i\int \frac{\sin(ic+idx)^3}{\left(a-b\sin(ic+idx)^4\right)^2}dx$$

$$\Big\downarrow \text{3694}$$

$$-\frac{\int \frac{1-\cosh^2(c+dx)}{\left(-b\cosh^4(c+dx)+2b\cosh^2(c+dx)+a-b\right)^2}d\cosh(c+dx)}{d}$$

$$\Big\downarrow \text{1492}$$

$$-\frac{\frac{\cosh(c+dx)\left(2-\cosh^2(c+dx)\right)}{4(a-b)\left(a-b\cosh^4(c+dx)+2b\cosh^2(c+dx)-b\right)}-\frac{\int -\frac{2ab\left(2-\cosh^2(c+dx)\right)}{-b\cosh^4(c+dx)+2b\cosh^2(c+dx)+a-b}d\cosh(c+dx)}{8ab(a-b)}}{d}$$

$$\Big\downarrow \text{27}$$

$$-\frac{\frac{\int \frac{2-\cosh^2(c+dx)}{-b\cosh^4(c+dx)+2b\cosh^2(c+dx)+a-b}d\cosh(c+dx)}{4(a-b)}+\frac{\cosh(c+dx)\left(2-\cosh^2(c+dx)\right)}{4(a-b)\left(a-b\cosh^4(c+dx)+2b\cosh^2(c+dx)-b\right)}}{d}$$

$$\Big\downarrow \text{1480}$$

$$-\frac{\frac{-\frac{1}{2}\left(\frac{\sqrt{b}}{\sqrt{a}}+1\right)\int \frac{1}{-b\cosh^2(c+dx)-\left(\sqrt{a}-\sqrt{b}\right)\sqrt{b}}d\cosh(c+dx)-\frac{1}{2}\left(1-\frac{\sqrt{b}}{\sqrt{a}}\right)\int \frac{1}{\left(\sqrt{a}+\sqrt{b}\right)\sqrt{b}-b\cosh^2(c+dx)}d\cosh(c+dx)}{4(a-b)}+\frac{\cosh(c+dx)(2-c}{4(a-b)(a-b\cosh^4(c+dx)}}{d}$$

$$\Big\downarrow \text{218}$$

$$-\frac{\frac{\frac{\left(\frac{\sqrt{b}}{\sqrt{a}}+1\right)\arctan\left(\frac{\sqrt[4]{b}\cosh(c+dx)}{\sqrt{\sqrt{a}-\sqrt{b}}}\right)}{2b^{3/4}\sqrt{\sqrt{a}-\sqrt{b}}}-\frac{1}{2}\left(1-\frac{\sqrt{b}}{\sqrt{a}}\right)\int \frac{1}{\left(\sqrt{a}+\sqrt{b}\right)\sqrt{b}-b\cosh^2(c+dx)}d\cosh(c+dx)}{4(a-b)}+\frac{\cosh(c+dx)\left(2-\cosh^2(c+dx)\right)}{4(a-b)\left(a-b\cosh^4(c+dx)+2b\cosh^2(c+dx)-b\right)}}{d}$$

$$\Big\downarrow \text{221}$$

$$-\frac{\frac{\frac{\left(\frac{\sqrt{b}}{\sqrt{a}}+1\right)\arctan\left(\frac{\sqrt[4]{b}\cosh(c+dx)}{\sqrt{\sqrt{a}-\sqrt{b}}}\right)}{2b^{3/4}\sqrt{\sqrt{a}-\sqrt{b}}}-\frac{\left(1-\frac{\sqrt{b}}{\sqrt{a}}\right)\text{arctanh}\left(\frac{\sqrt[4]{b}\cosh(c+dx)}{\sqrt{\sqrt{a}+\sqrt{b}}}\right)}{2b^{3/4}\sqrt{\sqrt{a}+\sqrt{b}}}}{4(a-b)}+\frac{\cosh(c+dx)\left(2-\cosh^2(c+dx)\right)}{4(a-b)\left(a-b\cosh^4(c+dx)+2b\cosh^2(c+dx)-b\right)}}{d}$$

input
```
Int[Sinh[c + d*x]^3/(a - b*Sinh[c + d*x]^4)^2,x]
```

output
```
-(((((1 + Sqrt[b]/Sqrt[a])*ArcTan[(b^(1/4)*Cosh[c + d*x])/Sqrt[Sqrt[a] - S
qrt[b]]])/(2*Sqrt[Sqrt[a] - Sqrt[b]]*b^(3/4)) - ((1 - Sqrt[b]/Sqrt[a])*Arc
Tanh[(b^(1/4)*Cosh[c + d*x])/Sqrt[Sqrt[a] + Sqrt[b]]])/(2*Sqrt[Sqrt[a] + S
qrt[b]]*b^(3/4)))/(4*(a - b)) + (Cosh[c + d*x]*(2 - Cosh[c + d*x]^2))/(4*(
a - b)*(a - b + 2*b*Cosh[c + d*x]^2 - b*Cosh[c + d*x]^4)))/d)
```

Defintions of rubi rules used

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 1480
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Simp[(e/2 + (2*c*d - b*e)/(2*q))   Int[1/(
b/2 - q/2 + c*x^2), x], x] + Simp[(e/2 - (2*c*d - b*e)/(2*q))   Int[1/(b/2
+ q/2 + c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0]
 && NeQ[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

rule 1492
```
Int[((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symb
ol] :> Simp[x*(a*b*e - d*(b^2 - 2*a*c) - c*(b*d - 2*a*e)*x^2)*((a + b*x^2 +
 c*x^4)^(p + 1)/(2*a*(p + 1)*(b^2 - 4*a*c))), x] + Simp[1/(2*a*(p + 1)*(b^2
 - 4*a*c))   Int[Simp[(2*p + 3)*d*b^2 - a*b*e - 2*a*c*d*(4*p + 5) + (4*p +
7)*(d*b - 2*a*e)*c*x^2, x]*(a + b*x^2 + c*x^4)^(p + 1), x], x] /; FreeQ[{a,
 b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] &&
 LtQ[p, -1] && IntegerQ[2*p]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3694
```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^4)^
(p_.), x_Symbol] :> With[{ff = FreeFactors[Cos[e + f*x], x]}, Simp[-ff/f
Subst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a + b - 2*b*ff^2*x^2 + b*ff^4*x^4)^p,
 x], x, Cos[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m -
 1)/2]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 321 vs. $2(144) = 288$.

Time = 6.11 (sec) , antiderivative size = 322, normalized size of antiderivative = 1.73

method	result
derivativedivides	$\frac{\frac{-\frac{\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^{6}}{2(a-b)}-\frac{(3a-8b)\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^{4}}{2a(a-b)}+\frac{5\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}}{2(a-b)}-\frac{1}{2(a-b)}}{\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^{8}a-4\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^{6}a+6\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^{4}a-16b\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^{4}-4\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}a+a}+a\left(\frac{\left(-b-\sqrt{ab}\right)\arctan\left(\frac{2\tan}{}\right.}{4ab\sqrt{-}}\right.}{d}$
default	$\frac{\frac{-\frac{\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^{6}}{2(a-b)}-\frac{(3a-8b)\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^{4}}{2a(a-b)}+\frac{5\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}}{2(a-b)}-\frac{1}{2(a-b)}}{\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^{8}a-4\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^{6}a+6\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^{4}a-16b\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^{4}-4\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}a+a}+a\left(\frac{\left(-b-\sqrt{ab}\right)\arctan\left(\frac{2\tan}{}\right.}{4ab\sqrt{-}}\right.}{d}$
risch	$\frac{e^{dx+c}\left(e^{6dx+6c}-5\,e^{4dx+4c}-5\,e^{2dx+2c}+1\right)}{2(a-b)d\left(-e^{8dx+8c}b+4\,e^{6dx+6c}b+16\,e^{4dx+4c}a-6b\,e^{4dx+4c}+4\,e^{2dx+2c}b-b\right)}+\left(\_R=\operatorname{RootOf}\left(-1+(65536a^{5}b^{3}d^{4}-\right.\right.$

input
```
int(sinh(d*x+c)^3/(a-b*sinh(d*x+c)^4)^2,x,method=_RETURNVERBOSE)
```

output
```
1/d*(8*(-1/16/(a-b)*tanh(1/2*d*x+1/2*c)^6-1/16*(3*a-8*b)/a/(a-b)*tanh(1/2*
d*x+1/2*c)^4+5/16/(a-b)*tanh(1/2*d*x+1/2*c)^2-1/16/(a-b))/(tanh(1/2*d*x+1/
2*c)^8*a-4*tanh(1/2*d*x+1/2*c)^6*a+6*tanh(1/2*d*x+1/2*c)^4*a-16*b*tanh(1/2
*d*x+1/2*c)^4-4*tanh(1/2*d*x+1/2*c)^2*a+a)+1/2/(a-b)*a*(1/4*(-b-(a*b)^(1/2
))/a/b/(-a*b+(a*b)^(1/2)*a)^(1/2)*arctan(1/4*(2*tanh(1/2*d*x+1/2*c)^2*a+4*
(a*b)^(1/2)-2*a)/(-a*b+(a*b)^(1/2)*a)^(1/2))-1/4*(-b+(a*b)^(1/2))/a/b/(-a*
b-(a*b)^(1/2)*a)^(1/2)*arctan(1/4*(-2*tanh(1/2*d*x+1/2*c)^2*a+4*(a*b)^(1/2
)+2*a)/(-a*b-(a*b)^(1/2)*a)^(1/2))))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 5238 vs. $2(141) = 282$.

Time = 0.21 (sec) , antiderivative size = 5238, normalized size of antiderivative = 28.16

$$\int \frac{\sinh^3(c+dx)}{(a-b\sinh^4(c+dx))^2}\,dx = \text{Too large to display}$$

input
```
integrate(sinh(d*x+c)^3/(a-b*sinh(d*x+c)^4)^2,x, algorithm="fricas")
```

output
```
Too large to include
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\sinh^3(c+dx)}{(a-b\sinh^4(c+dx))^2}\,dx = \text{Timed out}$$

input
```
integrate(sinh(d*x+c)**3/(a-b*sinh(d*x+c)**4)**2,x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{\sinh^3(c+dx)}{\left(a-b\sinh^4(c+dx)\right)^2}\,dx = \int \frac{\sinh\left(dx+c\right)^3}{\left(b\sinh\left(dx+c\right)^4-a\right)^2}\,dx$$

input
```
integrate(sinh(d*x+c)^3/(a-b*sinh(d*x+c)^4)^2,x, algorithm="maxima")
```

output
```
-1/2*(e^(7*d*x + 7*c) - 5*e^(5*d*x + 5*c) - 5*e^(3*d*x + 3*c) + e^(d*x + c
))/(a*b*d - b^2*d + (a*b*d*e^(8*c) - b^2*d*e^(8*c))*e^(8*d*x) - 4*(a*b*d*e
^(6*c) - b^2*d*e^(6*c))*e^(6*d*x) - 2*(8*a^2*d*e^(4*c) - 11*a*b*d*e^(4*c)
+ 3*b^2*d*e^(4*c))*e^(4*d*x) - 4*(a*b*d*e^(2*c) - b^2*d*e^(2*c))*e^(2*d*x)
) - 1/8*integrate(4*(e^(7*d*x + 7*c) - 7*e^(5*d*x + 5*c) + 7*e^(3*d*x + 3*
c) - e^(d*x + c))/(a*b - b^2 + (a*b*e^(8*c) - b^2*e^(8*c))*e^(8*d*x) - 4*(
a*b*e^(6*c) - b^2*e^(6*c))*e^(6*d*x) - 2*(8*a^2*e^(4*c) - 11*a*b*e^(4*c) +
 3*b^2*e^(4*c))*e^(4*d*x) - 4*(a*b*e^(2*c) - b^2*e^(2*c))*e^(2*d*x)), x)
```

Giac [F]

$$\int \frac{\sinh^3(c+dx)}{\left(a-b\sinh^4(c+dx)\right)^2}\,dx = \int \frac{\sinh\left(dx+c\right)^3}{\left(b\sinh\left(dx+c\right)^4-a\right)^2}\,dx$$

input
```
integrate(sinh(d*x+c)^3/(a-b*sinh(d*x+c)^4)^2,x, algorithm="giac")
```

output
```
sage0*x
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sinh^3(c+dx)}{\left(a-b\sinh^4(c+dx)\right)^2}\,dx = \int \frac{\sinh(c+d\,x)^3}{\left(a-b\sinh\left(c+d\,x\right)^4\right)^2}\,dx$$

input
```
int(sinh(c + d*x)^3/(a - b*sinh(c + d*x)^4)^2,x)
```

output
```
int(sinh(c + d*x)^3/(a - b*sinh(c + d*x)^4)^2, x)
```

Reduce [F]

$$\int \frac{\sinh^3(c+dx)}{\left(a-b\sinh^4(c+dx)\right)^2}\,dx = \text{too large to display}$$

input
```
int(sinh(d*x+c)^3/(a-b*sinh(d*x+c)^4)^2,x)
```

output

```
(32*e**c*(112*e**(14*c + 8*d*x)*int(e**(7*d*x)/(e**(16*c + 16*d*x)*b**2 -
8*e**(14*c + 14*d*x)*b**2 - 32*e**(12*c + 12*d*x)*a*b + 28*e**(12*c + 12*d
*x)*b**2 + 128*e**(10*c + 10*d*x)*a*b - 56*e**(10*c + 10*d*x)*b**2 + 256*e
**(8*c + 8*d*x)*a**2 - 192*e**(8*c + 8*d*x)*a*b + 70*e**(8*c + 8*d*x)*b**2
 + 128*e**(6*c + 6*d*x)*a*b - 56*e**(6*c + 6*d*x)*b**2 - 32*e**(4*c + 4*d*
x)*a*b + 28*e**(4*c + 4*d*x)*b**2 - 8*e**(2*c + 2*d*x)*b**2 + b**2),x)*a*b
*d + 3*e**(14*c + 8*d*x)*int(e**(7*d*x)/(e**(16*c + 16*d*x)*b**2 - 8*e**(1
4*c + 14*d*x)*b**2 - 32*e**(12*c + 12*d*x)*a*b + 28*e**(12*c + 12*d*x)*b**
2 + 128*e**(10*c + 10*d*x)*a*b - 56*e**(10*c + 10*d*x)*b**2 + 256*e**(8*c
+ 8*d*x)*a**2 - 192*e**(8*c + 8*d*x)*a*b + 70*e**(8*c + 8*d*x)*b**2 + 128*
e**(6*c + 6*d*x)*a*b - 56*e**(6*c + 6*d*x)*b**2 - 32*e**(4*c + 4*d*x)*a*b
+ 28*e**(4*c + 4*d*x)*b**2 - 8*e**(2*c + 2*d*x)*b**2 + b**2),x)*b**2*d - 1
44*e**(12*c + 8*d*x)*int(e**(5*d*x)/(e**(16*c + 16*d*x)*b**2 - 8*e**(14*c
+ 14*d*x)*b**2 - 32*e**(12*c + 12*d*x)*a*b + 28*e**(12*c + 12*d*x)*b**2 +
128*e**(10*c + 10*d*x)*a*b - 56*e**(10*c + 10*d*x)*b**2 + 256*e**(8*c + 8*
d*x)*a**2 - 192*e**(8*c + 8*d*x)*a*b + 70*e**(8*c + 8*d*x)*b**2 + 128*e**(
6*c + 6*d*x)*a*b - 56*e**(6*c + 6*d*x)*b**2 - 32*e**(4*c + 4*d*x)*a*b + 28
*e**(4*c + 4*d*x)*b**2 - 8*e**(2*c + 2*d*x)*b**2 + b**2),x)*a*b*d - 9*e**(
12*c + 8*d*x)*int(e**(5*d*x)/(e**(16*c + 16*d*x)*b**2 - 8*e**(14*c + 14*d*
x)*b**2 - 32*e**(12*c + 12*d*x)*a*b + 28*e**(12*c + 12*d*x)*b**2 + 128*...
```

3.220 $\int \frac{\sinh(c+dx)}{(a-b\sinh^4(c+dx))^2}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 221

$$\int \frac{\sinh(c+dx)}{(a-b\sinh^4(c+dx))^2}\,dx = \frac{\left(3\sqrt{a}-2\sqrt{b}\right)\arctan\left(\frac{\sqrt[4]{b}\cosh(c+dx)}{\sqrt{\sqrt{a}-\sqrt{b}}}\right)}{8a^{3/2}\left(\sqrt{a}-\sqrt{b}\right)^{3/2}\sqrt[4]{b}d}+\frac{\left(3\sqrt{a}+2\sqrt{b}\right)\operatorname{arctanh}\left(\frac{\sqrt[4]{b}\cosh(c+dx)}{\sqrt{\sqrt{a}+\sqrt{b}}}\right)}{8a^{3/2}\left(\sqrt{a}+\sqrt{b}\right)^{3/2}\sqrt[4]{b}d}+\frac{\cosh(c+dx)\left(a+b-b\cosh^2(c+dx)\right)}{4a(a-b)d\left(a-b+2b\cosh^2(c+dx)-b\cosh^4(c+dx)\right)}$$

output
```
1/8*(3*a^(1/2)-2*b^(1/2))*arctan(b^(1/4)*cosh(d*x+c)/(a^(1/2)-b^(1/2))^(1/
2))/a^(3/2)/(a^(1/2)-b^(1/2))^(3/2)/b^(1/4)/d+1/8*(3*a^(1/2)+2*b^(1/2))*ar
ctanh(b^(1/4)*cosh(d*x+c)/(a^(1/2)+b^(1/2))^(1/2))/a^(3/2)/(a^(1/2)+b^(1/2
))^(3/2)/b^(1/4)/d+1/4*cosh(d*x+c)*(a+b-b*cosh(d*x+c)^2)/a/(a-b)/d/(a-b+2*
b*cosh(d*x+c)^2-b*cosh(d*x+c)^4)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

Time = 0.50 (sec) , antiderivative size = 597, normalized size of antiderivative = 2.70

$$\int \frac{\sinh(c+dx)}{\left(a-b\sinh^4(c+dx)\right)^2}\,dx$$
$$= \frac{\frac{32\cosh(c+dx)(2a+b-b\cosh(2(c+dx)))}{8a-3b+4b\cosh(2(c+dx))-b\cosh(4(c+dx))} + \text{RootSum}\left[b-4b\#1^2-16a\#1^4+6b\#1^4-4b\#1^6+b\#1^8\&, \frac{-bc-bdx-}{}\right.}{}$$

input
```
Integrate[Sinh[c + d*x]/(a - b*Sinh[c + d*x]^4)^2,x]
```

output
```
((32*Cosh[c + d*x]*(2*a + b - b*Cosh[2*(c + d*x)]))/(8*a - 3*b + 4*b*Cosh[
2*(c + d*x)] - b*Cosh[4*(c + d*x)]) + RootSum[b - 4*b*#1^2 - 16*a*#1^4 + 6
*b*#1^4 - 4*b*#1^6 + b*#1^8 & , (-(b*c) - b*d*x - 2*b*Log[-Cosh[(c + d*x)/
2] - Sinh[(c + d*x)/2] + Cosh[(c + d*x)/2]*#1 - Sinh[(c + d*x)/2]*#1] + 12
*a*c*#1^2 - 5*b*c*#1^2 + 12*a*d*x*#1^2 - 5*b*d*x*#1^2 + 24*a*Log[-Cosh[(c
+ d*x)/2] - Sinh[(c + d*x)/2] + Cosh[(c + d*x)/2]*#1 - Sinh[(c + d*x)/2]*#
1]*#1^2 - 10*b*Log[-Cosh[(c + d*x)/2] - Sinh[(c + d*x)/2] + Cosh[(c + d*x)
/2]*#1 - Sinh[(c + d*x)/2]*#1]*#1^2 - 12*a*c*#1^4 + 5*b*c*#1^4 - 12*a*d*x*
#1^4 + 5*b*d*x*#1^4 - 24*a*Log[-Cosh[(c + d*x)/2] - Sinh[(c + d*x)/2] + Co
sh[(c + d*x)/2]*#1 - Sinh[(c + d*x)/2]*#1]*#1^4 + 10*b*Log[-Cosh[(c + d*x)
/2] - Sinh[(c + d*x)/2] + Cosh[(c + d*x)/2]*#1 - Sinh[(c + d*x)/2]*#1]*#1^
4 + b*c*#1^6 + b*d*x*#1^6 + 2*b*Log[-Cosh[(c + d*x)/2] - Sinh[(c + d*x)/2]
 + Cosh[(c + d*x)/2]*#1 - Sinh[(c + d*x)/2]*#1]*#1^6)/(-(b*#1) - 8*a*#1^3
+ 3*b*#1^3 - 3*b*#1^5 + b*#1^7) & ])/(32*a*(a - b)*d)
```

Rubi [A] (verified)

Time = 0.51 (sec) , antiderivative size = 231, normalized size of antiderivative = 1.05, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.364$, Rules used = {3042, 26, 3694, 1405, 27, 1480, 218, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sinh(c+dx)}{\left(a-b\sinh^4(c+dx)\right)^2}dx$$

$\Big\downarrow$ 3042

$$\int -\frac{i\sin(ic+idx)}{\left(a-b\sin(ic+idx)^4\right)^2}dx$$

$\Big\downarrow$ 26

$$-i\int \frac{\sin(ic+idx)}{\left(a-b\sin(ic+idx)^4\right)^2}dx$$

$\Big\downarrow$ 3694

$$\frac{\int \frac{1}{\left(-b\cosh^4(c+dx)+2b\cosh^2(c+dx)+a-b\right)^2}d\cosh(c+dx)}{d}$$

$\Big\downarrow$ 1405

$$\frac{\frac{\cosh(c+dx)\left(a-b\cosh^2(c+dx)+b\right)}{4a(a-b)\left(a-b\cosh^4(c+dx)+2b\cosh^2(c+dx)-b\right)}-\frac{\int -\frac{2b\left(-b\cosh^2(c+dx)+3a-b\right)}{-b\cosh^4(c+dx)+2b\cosh^2(c+dx)+a-b}d\cosh(c+dx)}{8ab(a-b)}}{d}$$

$\Big\downarrow$ 27

$$\frac{\frac{\int \frac{-b\cosh^2(c+dx)+3a-b}{-b\cosh^4(c+dx)+2b\cosh^2(c+dx)+a-b}d\cosh(c+dx)}{4a(a-b)}+\frac{\cosh(c+dx)\left(a-b\cosh^2(c+dx)+b\right)}{4a(a-b)\left(a-b\cosh^4(c+dx)+2b\cosh^2(c+dx)-b\right)}}{d}$$

$\Big\downarrow$ 1480

$$\frac{\frac{\frac{1}{2}\sqrt{b}\left(\frac{3a-2b}{\sqrt{a}}-\sqrt{b}\right)\int \frac{1}{\left(\sqrt{a}+\sqrt{b}\right)\sqrt{b}-b\cosh^2(c+dx)}d\cosh(c+dx)-\frac{1}{2}\sqrt{b}\left(\frac{3a-2b}{\sqrt{a}}+\sqrt{b}\right)\int \frac{1}{-b\cosh^2(c+dx)-\left(\sqrt{a}-\sqrt{b}\right)\sqrt{b}}d\cosh(c+dx)}{4a(a-b)}+\frac{\cosh(c+\cdots}{4a(a-b)(a-b\cdots}}{d}$$

$\Big\downarrow$ 218

$$\frac{\frac{\frac{1}{2}\sqrt{b}\left(\frac{3a-2b}{\sqrt{a}}-\sqrt{b}\right)\int \frac{1}{\left(\sqrt{a}+\sqrt{b}\right)\sqrt{b}-b\cosh^2(c+dx)}d\cosh(c+dx)+\frac{\left(\frac{3a-2b}{\sqrt{a}}+\sqrt{b}\right)\arctan\left(\frac{\sqrt[4]{b}\cosh(c+dx)}{\sqrt{\sqrt{a}-\sqrt{b}}}\right)}{2\sqrt[4]{b}\sqrt{\sqrt{a}-\sqrt{b}}}}{4a(a-b)}+\frac{\cosh(c+dx)\left(a-b\cosh^2(c+dx)+\cdots\right.}{4a(a-b)\left(a-b\cosh^4(c+dx)+2b\cosh^2(\cdots\right.}}{d}$$

$\Big\downarrow$ 221

$$\frac{\frac{\frac{\left(\frac{3a-2b}{\sqrt{a}}+\sqrt{b}\right)\arctan\left(\frac{\sqrt[4]{b}\cosh(c+dx)}{\sqrt{\sqrt{a}-\sqrt{b}}}\right)}{2\sqrt[4]{b}\sqrt{\sqrt{a}-\sqrt{b}}}+\frac{\left(\frac{3a-2b}{\sqrt{a}}-\sqrt{b}\right)\text{arctanh}\left(\frac{\sqrt[4]{b}\cosh(c+dx)}{\sqrt{\sqrt{a}+\sqrt{b}}}\right)}{2\sqrt[4]{b}\sqrt{\sqrt{a}+\sqrt{b}}}}{4a(a-b)}+\frac{\cosh(c+dx)\left(a-b\cosh^2(c+dx)+b\right)}{4a(a-b)\left(a-b\cosh^4(c+dx)+2b\cosh^2(c+dx)-b\right)}}{d}$$

input

```
Int[Sinh[c + d*x]/(a - b*Sinh[c + d*x]^4)^2,x]
```

output

```
(((((3*a - 2*b)/Sqrt[a] + Sqrt[b])*ArcTan[(b^(1/4)*Cosh[c + d*x])/Sqrt[Sqr
t[a] - Sqrt[b]]])/(2*Sqrt[Sqrt[a] - Sqrt[b]]*b^(1/4)) + (((3*a - 2*b)/Sqrt
[a] - Sqrt[b])*ArcTanh[(b^(1/4)*Cosh[c + d*x])/Sqrt[Sqrt[a] + Sqrt[b]]])/(
2*Sqrt[Sqrt[a] + Sqrt[b]]*b^(1/4)))/(4*a*(a - b)) + (Cosh[c + d*x]*(a + b
- b*Cosh[c + d*x]^2))/(4*a*(a - b)*(a - b + 2*b*Cosh[c + d*x]^2 - b*Cosh[c
 + d*x]^4)))/d
```

Defintions of rubi rules used

rule 26

```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 27

```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 218

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 221

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 1405
```
Int[((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Simp[(-x)*(b^2
- 2*a*c + b*c*x^2)*((a + b*x^2 + c*x^4)^(p + 1)/(2*a*(p + 1)*(b^2 - 4*a*c))
), x] + Simp[1/(2*a*(p + 1)*(b^2 - 4*a*c))   Int[(b^2 - 2*a*c + 2*(p + 1)*(
b^2 - 4*a*c) + b*c*(4*p + 7)*x^2)*(a + b*x^2 + c*x^4)^(p + 1), x], x] /; Fr
eeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && IntegerQ[2*p]
```

rule 1480
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Simp[(e/2 + (2*c*d - b*e)/(2*q))   Int[1/(
b/2 - q/2 + c*x^2), x], x] + Simp[(e/2 - (2*c*d - b*e)/(2*q))   Int[1/(b/2
+ q/2 + c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0]
 && NeQ[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3694
```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^4)^
(p_.), x_Symbol] :> With[{ff = FreeFactors[Cos[e + f*x], x]}, Simp[-ff/f
Subst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a + b - 2*b*ff^2*x^2 + b*ff^4*x^4)^p,
 x], x, Cos[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m -
 1)/2]
```

Maple [A] (verified)

Time = 6.63 (sec) , antiderivative size = 339, normalized size of antiderivative = 1.53

method	result
derivativedivides	$\frac{\frac{-\frac{(a-2b)\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^{6}}{2a(a-b)}+\frac{(3a-8b)\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^{4}}{2a(a-b)}-\frac{(3a+2b)\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}}{2a(a-b)}+\frac{2}{4a-4b}}{\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^{8}a-4\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^{6}a+6\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^{4}a-16b\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^{4}-4\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}a+a}+ -\frac{\left(-\sqrt{ab}+3a-2b\right)\arctan\Bigg(}{4a\sqrt{}}}{d}$
default	$\frac{\frac{-\frac{(a-2b)\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^{6}}{2a(a-b)}+\frac{(3a-8b)\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^{4}}{2a(a-b)}-\frac{(3a+2b)\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}}{2a(a-b)}+\frac{2}{4a-4b}}{\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^{8}a-4\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^{6}a+6\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^{4}a-16b\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^{4}-4\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}a+a}+ -\frac{\left(-\sqrt{ab}+3a-2b\right)\arctan\Bigg(}{4a\sqrt{}}}{d}$
risch	$\frac{e^{dx+c}\left(-e^{6dx+6c}b+4\,e^{4dx+4c}a+b\,e^{4dx+4c}+4\,e^{2dx+2c}a+e^{2dx+2c}b-b\right)}{2ad(a-b)\left(-e^{8dx+8c}b+4\,e^{6dx+6c}b+16\,e^{4dx+4c}a-6b\,e^{4dx+4c}+4\,e^{2dx+2c}b-b\right)}+\Bigg(\Bigg(_{\_R=\mathrm{RootOf}\left((65536a^{9}b\,d^{4}-1966\right.}$

input
```
int(sinh(d*x+c)/(a-b*sinh(d*x+c)^4)^2,x,method=_RETURNVERBOSE)
```

output
```
1/d*(2*(-1/4*(a-2*b)/a/(a-b)*tanh(1/2*d*x+1/2*c)^6+1/4*(3*a-8*b)/a/(a-b)*t
anh(1/2*d*x+1/2*c)^4-1/4*(3*a+2*b)/a/(a-b)*tanh(1/2*d*x+1/2*c)^2+1/4/(a-b)
)/(tanh(1/2*d*x+1/2*c)^8*a-4*tanh(1/2*d*x+1/2*c)^6*a+6*tanh(1/2*d*x+1/2*c)
^4*a-16*b*tanh(1/2*d*x+1/2*c)^4-4*tanh(1/2*d*x+1/2*c)^2*a+a)+1/2/(a-b)*(-1
/4*(-(a*b)^(1/2)+3*a-2*b)/a/(-a*b-(a*b)^(1/2)*a)^(1/2)*arctan(1/4*(-2*tanh
(1/2*d*x+1/2*c)^2*a+4*(a*b)^(1/2)+2*a)/(-a*b-(a*b)^(1/2)*a)^(1/2))+1/4*((a
*b)^(1/2)+3*a-2*b)/a/(-a*b+(a*b)^(1/2)*a)^(1/2)*arctan(1/4*(2*tanh(1/2*d*x
+1/2*c)^2*a+4*(a*b)^(1/2)-2*a)/(-a*b+(a*b)^(1/2)*a)^(1/2))))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 6018 vs. $2(173) = 346$.

Time = 0.27 (sec) , antiderivative size = 6018, normalized size of antiderivative = 27.23

$$\int \frac{\sinh(c+dx)}{\left(a-b\sinh^4(c+dx)\right)^2}\,dx = \text{Too large to display}$$

input
```
integrate(sinh(d*x+c)/(a-b*sinh(d*x+c)^4)^2,x, algorithm="fricas")
```

output
```
Too large to include
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\sinh(c+dx)}{\left(a-b\sinh^4(c+dx)\right)^2}\,dx = \text{Timed out}$$

input
```
integrate(sinh(d*x+c)/(a-b*sinh(d*x+c)**4)**2,x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{\sinh(c+dx)}{\left(a-b\sinh^4(c+dx)\right)^2}\,dx = \int \frac{\sinh\left(dx+c\right)}{\left(b\sinh\left(dx+c\right)^4-a\right)^2}\,dx$$

input
```
integrate(sinh(d*x+c)/(a-b*sinh(d*x+c)^4)^2,x, algorithm="maxima")
```

output
```
-1/2*((4*a*e^(5*c) + b*e^(5*c))*e^(5*d*x) + (4*a*e^(3*c) + b*e^(3*c))*e^(3
*d*x) - b*e^(7*d*x + 7*c) - b*e^(d*x + c))/(a^2*b*d - a*b^2*d + (a^2*b*d*e
^(8*c) - a*b^2*d*e^(8*c))*e^(8*d*x) - 4*(a^2*b*d*e^(6*c) - a*b^2*d*e^(6*c)
)*e^(6*d*x) - 2*(8*a^3*d*e^(4*c) - 11*a^2*b*d*e^(4*c) + 3*a*b^2*d*e^(4*c))
*e^(4*d*x) - 4*(a^2*b*d*e^(2*c) - a*b^2*d*e^(2*c))*e^(2*d*x)) + 1/2*integr
ate(-((12*a*e^(5*c) - 5*b*e^(5*c))*e^(5*d*x) - (12*a*e^(3*c) - 5*b*e^(3*c)
)*e^(3*d*x) - b*e^(7*d*x + 7*c) + b*e^(d*x + c))/(a^2*b - a*b^2 + (a^2*b*e
^(8*c) - a*b^2*e^(8*c))*e^(8*d*x) - 4*(a^2*b*e^(6*c) - a*b^2*e^(6*c))*e^(6
*d*x) - 2*(8*a^3*e^(4*c) - 11*a^2*b*e^(4*c) + 3*a*b^2*e^(4*c))*e^(4*d*x) -
 4*(a^2*b*e^(2*c) - a*b^2*e^(2*c))*e^(2*d*x)), x)
```

Giac [F]

$$\int \frac{\sinh(c+dx)}{(a-b\sinh^4(c+dx))^2}\,dx=\int \frac{\sinh(dx+c)}{(b\sinh(dx+c)^4-a)^2}\,dx$$

input
```
integrate(sinh(d*x+c)/(a-b*sinh(d*x+c)^4)^2,x, algorithm="giac")
```

output
```
sage0*x
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sinh(c+dx)}{(a-b\sinh^4(c+dx))^2}\,dx=\int \frac{\sinh(c+d\,x)}{\left(a-b\sinh(c+d\,x)^4\right)^2}\,dx$$

input
```
int(sinh(c + d*x)/(a - b*sinh(c + d*x)^4)^2,x)
```

output
```
int(sinh(c + d*x)/(a - b*sinh(c + d*x)^4)^2, x)
```

Reduce [F]

$$\int \frac{\sinh(c+dx)}{(a-b\sinh^4(c+dx))^2}\,dx=\text{too large to display}$$

input
```
int(sinh(d*x+c)/(a-b*sinh(d*x+c)^4)^2,x)
```

output

```
(128*e**c*(13*e**(14*c + 8*d*x)*int(e**(7*d*x)/(e**(16*c + 16*d*x)*b**2 -
8*e**(14*c + 14*d*x)*b**2 - 32*e**(12*c + 12*d*x)*a*b + 28*e**(12*c + 12*d
*x)*b**2 + 128*e**(10*c + 10*d*x)*a*b - 56*e**(10*c + 10*d*x)*b**2 + 256*e
**(8*c + 8*d*x)*a**2 - 192*e**(8*c + 8*d*x)*a*b + 70*e**(8*c + 8*d*x)*b**2
 + 128*e**(6*c + 6*d*x)*a*b - 56*e**(6*c + 6*d*x)*b**2 - 32*e**(4*c + 4*d*
x)*a*b + 28*e**(4*c + 4*d*x)*b**2 - 8*e**(2*c + 2*d*x)*b**2 + b**2),x)*b**
2*d + 48*e**(12*c + 8*d*x)*int(e**(5*d*x)/(e**(16*c + 16*d*x)*b**2 - 8*e**
(14*c + 14*d*x)*b**2 - 32*e**(12*c + 12*d*x)*a*b + 28*e**(12*c + 12*d*x)*b
**2 + 128*e**(10*c + 10*d*x)*a*b - 56*e**(10*c + 10*d*x)*b**2 + 256*e**(8*
c + 8*d*x)*a**2 - 192*e**(8*c + 8*d*x)*a*b + 70*e**(8*c + 8*d*x)*b**2 + 12
8*e**(6*c + 6*d*x)*a*b - 56*e**(6*c + 6*d*x)*b**2 - 32*e**(4*c + 4*d*x)*a*
b + 28*e**(4*c + 4*d*x)*b**2 - 8*e**(2*c + 2*d*x)*b**2 + b**2),x)*a*b*d -
18*e**(12*c + 8*d*x)*int(e**(5*d*x)/(e**(16*c + 16*d*x)*b**2 - 8*e**(14*c
+ 14*d*x)*b**2 - 32*e**(12*c + 12*d*x)*a*b + 28*e**(12*c + 12*d*x)*b**2 +
128*e**(10*c + 10*d*x)*a*b - 56*e**(10*c + 10*d*x)*b**2 + 256*e**(8*c + 8*
d*x)*a**2 - 192*e**(8*c + 8*d*x)*a*b + 70*e**(8*c + 8*d*x)*b**2 + 128*e**(
6*c + 6*d*x)*a*b - 56*e**(6*c + 6*d*x)*b**2 - 32*e**(4*c + 4*d*x)*a*b + 28
*e**(4*c + 4*d*x)*b**2 - 8*e**(2*c + 2*d*x)*b**2 + b**2),x)*b**2*d + 4*e**
(10*c + 8*d*x)*int(e**(3*d*x)/(e**(16*c + 16*d*x)*b**2 - 8*e**(14*c + 14*d
*x)*b**2 - 32*e**(12*c + 12*d*x)*a*b + 28*e**(12*c + 12*d*x)*b**2 + 128...
```

3.221 $\int \frac{\operatorname{csch}(c+dx)}{\left(a-b\sinh^4(c+dx)\right)^2}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 231

$$\begin{aligned}\int \frac{\operatorname{csch}(c+dx)}{\left(a-b\sinh^4(c+dx)\right)^2}\,dx &= -\frac{\left(5\sqrt{a}-4\sqrt{b}\right)\sqrt[4]{b}\arctan\left(\frac{\sqrt[4]{b}\cosh(c+dx)}{\sqrt{\sqrt{a}-\sqrt{b}}}\right)}{8a^2\left(\sqrt{a}-\sqrt{b}\right)^{3/2}d}-\frac{\operatorname{arctanh}(\cosh(c+dx))}{a^2d}\\&\quad+\frac{\left(5\sqrt{a}+4\sqrt{b}\right)\sqrt[4]{b}\operatorname{arctanh}\left(\frac{\sqrt[4]{b}\cosh(c+dx)}{\sqrt{\sqrt{a}+\sqrt{b}}}\right)}{8a^2\left(\sqrt{a}+\sqrt{b}\right)^{3/2}d}\\&\quad-\frac{b\cosh(c+dx)\left(2-\cosh^2(c+dx)\right)}{4a(a-b)d\left(a-b+2b\cosh^2(c+dx)-b\cosh^4(c+dx)\right)}\end{aligned}$$

output
```
-1/8*(5*a^(1/2)-4*b^(1/2))*b^(1/4)*arctan(b^(1/4)*cosh(d*x+c)/(a^(1/2)-b^(
1/2))^(1/2))/a^2/(a^(1/2)-b^(1/2))^(3/2)/d-arctanh(cosh(d*x+c))/a^2/d+1/8*
(5*a^(1/2)+4*b^(1/2))*b^(1/4)*arctanh(b^(1/4)*cosh(d*x+c)/(a^(1/2)+b^(1/2)
)^(1/2))/a^2/(a^(1/2)+b^(1/2))^(3/2)/d-1/4*b*cosh(d*x+c)*(2-cosh(d*x+c)^2)
/a/(a-b)/d/(a-b+2*b*cosh(d*x+c)^2-b*cosh(d*x+c)^4)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

Time = 3.45 (sec) , antiderivative size = 774, normalized size of antiderivative = 3.35

$$\int \frac{\operatorname{csch}(c+dx)}{(a-b\sinh^4(c+dx))^2}\,dx = \text{Too large to display}$$

input
```
Integrate[Csch[c + d*x]/(a - b*Sinh[c + d*x]^4)^2,x]
```

output
```
((16*a*b*(-5*Cosh[c + d*x] + Cosh[3*(c + d*x)]))/((a - b)*(8*a - 3*b + 4*b
*Cosh[2*(c + d*x)] - b*Cosh[4*(c + d*x)])) - 32*Log[Cosh[(c + d*x)/2]] + 3
2*Log[Sinh[(c + d*x)/2]] - (b*RootSum[b - 4*b*#1^2 - 16*a*#1^4 + 6*b*#1^4
- 4*b*#1^6 + b*#1^8 & , (-5*a*c + 4*b*c - 5*a*d*x + 4*b*d*x - 10*a*Log[-Co
sh[(c + d*x)/2] - Sinh[(c + d*x)/2] + Cosh[(c + d*x)/2]*#1 - Sinh[(c + d*x
)/2]*#1] + 8*b*Log[-Cosh[(c + d*x)/2] - Sinh[(c + d*x)/2] + Cosh[(c + d*x)
/2]*#1 - Sinh[(c + d*x)/2]*#1] + 19*a*c*#1^2 - 12*b*c*#1^2 + 19*a*d*x*#1^2
 - 12*b*d*x*#1^2 + 38*a*Log[-Cosh[(c + d*x)/2] - Sinh[(c + d*x)/2] + Cosh[
(c + d*x)/2]*#1 - Sinh[(c + d*x)/2]*#1]*#1^2 - 24*b*Log[-Cosh[(c + d*x)/2]
 - Sinh[(c + d*x)/2] + Cosh[(c + d*x)/2]*#1 - Sinh[(c + d*x)/2]*#1]*#1^2 -
 19*a*c*#1^4 + 12*b*c*#1^4 - 19*a*d*x*#1^4 + 12*b*d*x*#1^4 - 38*a*Log[-Cos
h[(c + d*x)/2] - Sinh[(c + d*x)/2] + Cosh[(c + d*x)/2]*#1 - Sinh[(c + d*x)
/2]*#1]*#1^4 + 24*b*Log[-Cosh[(c + d*x)/2] - Sinh[(c + d*x)/2] + Cosh[(c +
 d*x)/2]*#1 - Sinh[(c + d*x)/2]*#1]*#1^4 + 5*a*c*#1^6 - 4*b*c*#1^6 + 5*a*d
*x*#1^6 - 4*b*d*x*#1^6 + 10*a*Log[-Cosh[(c + d*x)/2] - Sinh[(c + d*x)/2] +
 Cosh[(c + d*x)/2]*#1 - Sinh[(c + d*x)/2]*#1]*#1^6 - 8*b*Log[-Cosh[(c + d*
x)/2] - Sinh[(c + d*x)/2] + Cosh[(c + d*x)/2]*#1 - Sinh[(c + d*x)/2]*#1]*#
1^6)/(-(b*#1) - 8*a*#1^3 + 3*b*#1^3 - 3*b*#1^5 + b*#1^7) & ])/(a - b))/(32
*a^2*d)
```

Rubi [A] (verified)

Time = 0.58 (sec) , antiderivative size = 311, normalized size of antiderivative = 1.35, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.227, Rules used = {3042, 26, 3694, 1567, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\text{csch}(c+dx)}{\left(a-b\sinh^4(c+dx)\right)^2}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{i}{\sin(ic+idx)\left(a-b\sin(ic+idx)^4\right)^2}dx$$

$$\Big\downarrow \text{26}$$

$$i\int \frac{1}{\sin(ic+idx)\left(a-b\sin(ic+idx)^4\right)^2}dx$$

$$\Big\downarrow \text{3694}$$

$$-\frac{\int \frac{1}{\left(1-\cosh^2(c+dx)\right)\left(-b\cosh^4(c+dx)+2b\cosh^2(c+dx)+a-b\right)^2}d\cosh(c+dx)}{d}$$

$$\Big\downarrow \text{1567}$$

$$-\frac{\int \left(\frac{b-b\cosh^2(c+dx)}{a^2\left(-b\cosh^4(c+dx)+2b\cosh^2(c+dx)+a-b\right)}+\frac{b-b\cosh^2(c+dx)}{a\left(-b\cosh^4(c+dx)+2b\cosh^2(c+dx)+a-b\right)^2}-\frac{1}{a^2\left(\cosh^2(c+dx)-1\right)}\right)d\cosh(c+dx)}{d}$$

$$\Big\downarrow \text{2009}$$

$$-\frac{\frac{\sqrt[4]{b}\arctan\left(\frac{\sqrt[4]{b}\cosh(c+dx)}{\sqrt{\sqrt{a}-\sqrt{b}}}\right)}{8a^{3/2}\left(\sqrt{a}-\sqrt{b}\right)^{3/2}}-\frac{\sqrt[4]{b}\text{arctanh}\left(\frac{\sqrt[4]{b}\cosh(c+dx)}{\sqrt{\sqrt{a}+\sqrt{b}}}\right)}{8a^{3/2}\left(\sqrt{a}+\sqrt{b}\right)^{3/2}}+\frac{\sqrt[4]{b}\arctan\left(\frac{\sqrt[4]{b}\cosh(c+dx)}{\sqrt{\sqrt{a}-\sqrt{b}}}\right)}{2a^2\sqrt{\sqrt{a}-\sqrt{b}}}-\frac{\sqrt[4]{b}\text{arctanh}\left(\frac{\sqrt[4]{b}\cosh(c+dx)}{\sqrt{\sqrt{a}+\sqrt{b}}}\right)}{2a^2\sqrt{\sqrt{a}+\sqrt{b}}}}{d}$$

input
```
Int[Csch[c + d*x]/(a - b*Sinh[c + d*x]^4)^2,x]
```

output
```
-(((b^(1/4)*ArcTan[(b^(1/4)*Cosh[c + d*x])/Sqrt[Sqrt[a] - Sqrt[b]]])/(8*a^
(3/2)*(Sqrt[a] - Sqrt[b])^(3/2)) + (b^(1/4)*ArcTan[(b^(1/4)*Cosh[c + d*x])
/Sqrt[Sqrt[a] - Sqrt[b]]])/(2*a^2*Sqrt[Sqrt[a] - Sqrt[b]]) + ArcTanh[Cosh[
c + d*x]]/a^2 - (b^(1/4)*ArcTanh[(b^(1/4)*Cosh[c + d*x])/Sqrt[Sqrt[a] + Sq
rt[b]]])/(8*a^(3/2)*(Sqrt[a] + Sqrt[b])^(3/2)) - (b^(1/4)*ArcTanh[(b^(1/4)
*Cosh[c + d*x])/Sqrt[Sqrt[a] + Sqrt[b]]])/(2*a^2*Sqrt[Sqrt[a] + Sqrt[b]])
+ (b*Cosh[c + d*x]*(2 - Cosh[c + d*x]^2))/(4*a*(a - b)*(a - b + 2*b*Cosh[c
 + d*x]^2 - b*Cosh[c + d*x]^4)))/d)
```

Defintions of rubi rules used

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 1567
```
Int[((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x
_Symbol] :> Int[ExpandIntegrand[(d + e*x^2)^q*(a + b*x^2 + c*x^4)^p, x], x]
 /; FreeQ[{a, b, c, d, e, p, q}, x] && NeQ[b^2 - 4*a*c, 0] && ((IntegerQ[p]
 && IntegerQ[q]) || IGtQ[p, 0] || IGtQ[q, 0])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3694
```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^4)^
(p_.), x_Symbol] :> With[{ff = FreeFactors[Cos[e + f*x], x]}, Simp[-ff/f
Subst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a + b - 2*b*ff^2*x^2 + b*ff^4*x^4)^p,
 x], x, Cos[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m -
 1)/2]
```

Maple [A] (verified)

Time = 4.29 (sec) , antiderivative size = 364, normalized size of antiderivative = 1.58

method	result
derivativedivides	$\frac{\frac{\ln\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{a^2}+\frac{16b\left(\frac{-\frac{a\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^6}{32(a-b)}-\frac{(3a-8b)\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^4}{32(a-b)}+\frac{5a\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^2}{32(a-b)}-\frac{a}{32(a-b)}}{\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^8a-4\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^6a+6\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^4a-16b\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^4-4\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^2a+a}+\cdots\right.}{}}{}$
default	$\frac{\frac{\ln\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{a^2}+\frac{16b\left(\frac{-\frac{a\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^6}{32(a-b)}-\frac{(3a-8b)\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^4}{32(a-b)}+\frac{5a\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^2}{32(a-b)}-\frac{a}{32(a-b)}}{\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^8a-4\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^6a+6\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^4a-16b\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^4-4\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^2a+a}+\cdots\right.}{}}{}$
risch	$\frac{b\,e^{dx+c}\left(e^{6dx+6c}-5\,e^{4dx+4c}-5\,e^{2dx+2c}+1\right)}{2a(a-b)d\left(-e^{8dx+8c}b+4\,e^{6dx+6c}b+16\,e^{4dx+4c}a-6b\,e^{4dx+4c}+4\,e^{2dx+2c}b-b\right)}+\frac{\ln\left(e^{dx+c}-1\right)}{a^2d}-\frac{\ln\left(e^{dx+c}+1\right)}{a^2d}+2$

input
```
int(csch(d*x+c)/(a-b*sinh(d*x+c)^4)^2,x,method=_RETURNVERBOSE)
```

output
```
1/d*(1/a^2*ln(tanh(1/2*d*x+1/2*c))+16*b/a^2*((-1/32*a/(a-b)*tanh(1/2*d*x+1
/2*c)^6-1/32*(3*a-8*b)/(a-b)*tanh(1/2*d*x+1/2*c)^4+5/32*a/(a-b)*tanh(1/2*d
*x+1/2*c)^2-1/32*a/(a-b))/(tanh(1/2*d*x+1/2*c)^8*a-4*tanh(1/2*d*x+1/2*c)^6
*a+6*tanh(1/2*d*x+1/2*c)^4*a-16*b*tanh(1/2*d*x+1/2*c)^4-4*tanh(1/2*d*x+1/2
*c)^2*a+a)+1/32/(a-b)*a*(-1/4*(5*(a*b)^(1/2)*a-4*(a*b)^(1/2)*b-a*b)/a/b/(-
a*b-(a*b)^(1/2)*a)^(1/2)*arctan(1/4*(-2*tanh(1/2*d*x+1/2*c)^2*a+4*(a*b)^(1
/2)+2*a)/(-a*b-(a*b)^(1/2)*a)^(1/2))+1/4*(-5*(a*b)^(1/2)*a+4*(a*b)^(1/2)*b
-a*b)/a/b/(-a*b+(a*b)^(1/2)*a)^(1/2)*arctan(1/4*(2*tanh(1/2*d*x+1/2*c)^2*a
+4*(a*b)^(1/2)-2*a)/(-a*b+(a*b)^(1/2)*a)^(1/2)))))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 7793 vs. $2(182) = 364$.

Time = 0.46 (sec) , antiderivative size = 7793, normalized size of antiderivative = 33.74

$$\int \frac{\operatorname{csch}(c+dx)}{(a-b\sinh^4(c+dx))^2}\,dx = \text{Too large to display}$$

input
```
integrate(csch(d*x+c)/(a-b*sinh(d*x+c)^4)^2,x, algorithm="fricas")
```

output
```
Too large to include
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\operatorname{csch}(c+dx)}{(a-b\sinh^4(c+dx))^2}\,dx = \text{Timed out}$$

input
```
integrate(csch(d*x+c)/(a-b*sinh(d*x+c)**4)**2,x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{\operatorname{csch}(c+dx)}{(a-b\sinh^4(c+dx))^2}\,dx = \int \frac{\operatorname{csch}(dx+c)}{(b\sinh(dx+c)^4-a)^2}\,dx$$

input
```
integrate(csch(d*x+c)/(a-b*sinh(d*x+c)^4)^2,x, algorithm="maxima")
```

output
```
-1/2*(b*e^(7*d*x + 7*c) - 5*b*e^(5*d*x + 5*c) - 5*b*e^(3*d*x + 3*c) + b*e^
(d*x + c))/(a^2*b*d - a*b^2*d + (a^2*b*d*e^(8*c) - a*b^2*d*e^(8*c))*e^(8*d
*x) - 4*(a^2*b*d*e^(6*c) - a*b^2*d*e^(6*c))*e^(6*d*x) - 2*(8*a^3*d*e^(4*c)
 - 11*a^2*b*d*e^(4*c) + 3*a*b^2*d*e^(4*c))*e^(4*d*x) - 4*(a^2*b*d*e^(2*c)
- a*b^2*d*e^(2*c))*e^(2*d*x)) - log((e^(d*x + c) + 1)*e^(-c))/(a^2*d) + lo
g((e^(d*x + c) - 1)*e^(-c))/(a^2*d) - 2*integrate(1/4*((5*a*b*e^(7*c) - 4*
b^2*e^(7*c))*e^(7*d*x) - (19*a*b*e^(5*c) - 12*b^2*e^(5*c))*e^(5*d*x) + (19
*a*b*e^(3*c) - 12*b^2*e^(3*c))*e^(3*d*x) - (5*a*b*e^c - 4*b^2*e^c)*e^(d*x)
)/(a^3*b - a^2*b^2 + (a^3*b*e^(8*c) - a^2*b^2*e^(8*c))*e^(8*d*x) - 4*(a^3*
b*e^(6*c) - a^2*b^2*e^(6*c))*e^(6*d*x) - 2*(8*a^4*e^(4*c) - 11*a^3*b*e^(4*
c) + 3*a^2*b^2*e^(4*c))*e^(4*d*x) - 4*(a^3*b*e^(2*c) - a^2*b^2*e^(2*c))*e^
(2*d*x)), x)
```

Giac [F]

$$\int \frac{\operatorname{csch}(c+dx)}{\left(a-b\sinh^4(c+dx)\right)^2}\,dx=\int \frac{\operatorname{csch}\left(dx+c\right)}{\left(b\sinh\left(dx+c\right)^4-a\right)^2}\,dx$$

input
```
integrate(csch(d*x+c)/(a-b*sinh(d*x+c)^4)^2,x, algorithm="giac")
```

output
```
sage0*x
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\operatorname{csch}(c+dx)}{\left(a-b\sinh^4(c+dx)\right)^2}\,dx=\int \frac{1}{\sinh\left(c+d\,x\right)\,\left(a-b\sinh\left(c+d\,x\right)^4\right)^2}\,dx$$

input
```
int(1/(sinh(c + d*x)*(a - b*sinh(c + d*x)^4)^2),x)
```

output
```
int(1/(sinh(c + d*x)*(a - b*sinh(c + d*x)^4)^2), x)
```

Reduce [F]

$$\int \frac{\operatorname{csch}(c+dx)}{\left(a-b\sinh^4(c+dx)\right)^2}\,dx = \text{too large to display}$$

input
```
int(csch(d*x+c)/(a-b*sinh(d*x+c)^4)^2,x)
```

output
```
(16*(1233792*e**(15*c + 8*d*x)*int(e**(7*d*x)/(63*e**(18*c + 18*d*x)*a*b**
2 - 8*e**(18*c + 18*d*x)*b**3 - 567*e**(16*c + 16*d*x)*a*b**2 + 72*e**(16*
c + 16*d*x)*b**3 - 2016*e**(14*c + 14*d*x)*a**2*b + 2524*e**(14*c + 14*d*x
)*a*b**2 - 288*e**(14*c + 14*d*x)*b**3 + 10080*e**(12*c + 12*d*x)*a**2*b -
 6572*e**(12*c + 12*d*x)*a*b**2 + 672*e**(12*c + 12*d*x)*b**3 + 16128*e**(
10*c + 10*d*x)*a**3 - 22208*e**(10*c + 10*d*x)*a**2*b + 10498*e**(10*c + 1
0*d*x)*a*b**2 - 1008*e**(10*c + 10*d*x)*b**3 - 16128*e**(8*c + 8*d*x)*a**3
 + 22208*e**(8*c + 8*d*x)*a**2*b - 10498*e**(8*c + 8*d*x)*a*b**2 + 1008*e*
*(8*c + 8*d*x)*b**3 - 10080*e**(6*c + 6*d*x)*a**2*b + 6572*e**(6*c + 6*d*x
)*a*b**2 - 672*e**(6*c + 6*d*x)*b**3 + 2016*e**(4*c + 4*d*x)*a**2*b - 2524
*e**(4*c + 4*d*x)*a*b**2 + 288*e**(4*c + 4*d*x)*b**3 + 567*e**(2*c + 2*d*x
)*a*b**2 - 72*e**(2*c + 2*d*x)*b**3 - 63*a*b**2 + 8*b**3),x)*a**2*b**2*d -
 350208*e**(15*c + 8*d*x)*int(e**(7*d*x)/(63*e**(18*c + 18*d*x)*a*b**2 - 8
*e**(18*c + 18*d*x)*b**3 - 567*e**(16*c + 16*d*x)*a*b**2 + 72*e**(16*c + 1
6*d*x)*b**3 - 2016*e**(14*c + 14*d*x)*a**2*b + 2524*e**(14*c + 14*d*x)*a*b
**2 - 288*e**(14*c + 14*d*x)*b**3 + 10080*e**(12*c + 12*d*x)*a**2*b - 6572
*e**(12*c + 12*d*x)*a*b**2 + 672*e**(12*c + 12*d*x)*b**3 + 16128*e**(10*c
+ 10*d*x)*a**3 - 22208*e**(10*c + 10*d*x)*a**2*b + 10498*e**(10*c + 10*d*x
)*a*b**2 - 1008*e**(10*c + 10*d*x)*b**3 - 16128*e**(8*c + 8*d*x)*a**3 + 22
208*e**(8*c + 8*d*x)*a**2*b - 10498*e**(8*c + 8*d*x)*a*b**2 + 1008*e**(...
```

3.222 $\int \frac{\sinh^8(c+dx)}{\left(a-b\sinh^4(c+dx)\right)^2}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 221

$$\int \frac{\sinh^8(c+dx)}{\left(a-b\sinh^4(c+dx)\right)^2}\,dx = \frac{x}{b^2}-\frac{\sqrt[4]{a}\left(4\sqrt{a}-5\sqrt{b}\right)\operatorname{arctanh}\left(\frac{\sqrt{\sqrt{a}-\sqrt{b}}\tanh(c+dx)}{\sqrt[4]{a}}\right)}{8\left(\sqrt{a}-\sqrt{b}\right)^{3/2}b^2d}-\frac{\sqrt[4]{a}\left(4\sqrt{a}+5\sqrt{b}\right)\operatorname{arctanh}\left(\frac{\sqrt{\sqrt{a}+\sqrt{b}}\tanh(c+dx)}{\sqrt[4]{a}}\right)}{8\left(\sqrt{a}+\sqrt{b}\right)^{3/2}b^2d}-\frac{a\tanh(c+dx)\left(1-2\tanh^2(c+dx)\right)}{4(a-b)bd\left(a-2a\tanh^2(c+dx)+(a-b)\tanh^4(c+dx)\right)}$$

output
```
x/b^2-1/8*a^(1/4)*(4*a^(1/2)-5*b^(1/2))*arctanh((a^(1/2)-b^(1/2))^(1/2)*ta
nh(d*x+c)/a^(1/4))/(a^(1/2)-b^(1/2))^(3/2)/b^2/d-1/8*a^(1/4)*(4*a^(1/2)+5*
b^(1/2))*arctanh((a^(1/2)+b^(1/2))^(1/2)*tanh(d*x+c)/a^(1/4))/(a^(1/2)+b^(
1/2))^(3/2)/b^2/d-1/4*a*tanh(d*x+c)*(1-2*tanh(d*x+c)^2)/(a-b)/b/d/(a-2*a*t
anh(d*x+c)^2+(a-b)*tanh(d*x+c)^4)
```

Mathematica [A] (verified)

Time = 9.02 (sec) , antiderivative size = 262, normalized size of antiderivative = 1.19

$$\int \frac{\sinh^8(c+dx)}{\left(a-b\sinh^4(c+dx)\right)^2}\,dx$$
$$= \frac{8(c+dx)+\frac{\sqrt{a}\left(4\sqrt{a}-5\sqrt{b}\right)\arctan\left(\frac{\left(\sqrt{a}-\sqrt{b}\right)\tanh(c+dx)}{\sqrt{-a+\sqrt{a}\sqrt{b}}}\right)}{\left(\sqrt{a}-\sqrt{b}\right)\sqrt{-a+\sqrt{a}\sqrt{b}}}-\frac{\sqrt{a}\left(4\sqrt{a}+5\sqrt{b}\right)\text{arctanh}\left(\frac{\left(\sqrt{a}+\sqrt{b}\right)\tanh(c+dx)}{\sqrt{a+\sqrt{a}\sqrt{b}}}\right)}{\left(\sqrt{a}+\sqrt{b}\right)\sqrt{a+\sqrt{a}\sqrt{b}}}+\frac{2ab(-6\sin}{(a-b)(8a-3b+4}}{8b^2d}$$

input
```
Integrate[Sinh[c + d*x]^8/(a - b*Sinh[c + d*x]^4)^2,x]
```

output
```
(8*(c + d*x) + (Sqrt[a]*(4*Sqrt[a] - 5*Sqrt[b])*ArcTan[((Sqrt[a] - Sqrt[b]
)*Tanh[c + d*x])/Sqrt[-a + Sqrt[a]*Sqrt[b]]])/((Sqrt[a] - Sqrt[b])*Sqrt[-a
 + Sqrt[a]*Sqrt[b]]) - (Sqrt[a]*(4*Sqrt[a] + 5*Sqrt[b])*ArcTanh[((Sqrt[a]
+ Sqrt[b])*Tanh[c + d*x])/Sqrt[a + Sqrt[a]*Sqrt[b]]])/((Sqrt[a] + Sqrt[b])
*Sqrt[a + Sqrt[a]*Sqrt[b]]) + (2*a*b*(-6*Sinh[2*(c + d*x)] + Sinh[4*(c + d
*x)]))/((a - b)*(8*a - 3*b + 4*b*Cosh[2*(c + d*x)] - b*Cosh[4*(c + d*x)]))
)/(8*b^2*d)
```

Rubi [A] (verified)

Time = 0.83 (sec) , antiderivative size = 389, normalized size of antiderivative = 1.76, number of steps used = 13, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$, Rules used = {3042, 3696, 1650, 27, 1598, 27, 1442, 27, 1480, 221, 1610, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sinh^8(c+dx)}{\left(a-b\sinh^4(c+dx)\right)^2}\,dx$$
$$\Big\downarrow \ 3042$$
$$\int \frac{\sin(ic+idx)^8}{\left(a-b\sin(ic+idx)^4\right)^2}dx$$
$$\Big\downarrow \ 3696$$

$$\frac{\int \frac{\tanh^8(c+dx)}{(1-\tanh^2(c+dx))\left((a-b)\tanh^4(c+dx)-2a\tanh^2(c+dx)+a\right)^2}d\tanh(c+dx)}{d}$$

$$\Big\downarrow \text{ 1650}$$

$$\frac{\frac{\int \frac{a\tanh^4(c+dx)\left(1-\tanh^2(c+dx)\right)}{\left((a-b)\tanh^4(c+dx)-2a\tanh^2(c+dx)+a\right)^2}d\tanh(c+dx)}{b}-\frac{\int \frac{\tanh^4(c+dx)}{\left(1-\tanh^2(c+dx)\right)\left((a-b)\tanh^4(c+dx)-2a\tanh^2(c+dx)+a\right)}d\tanh(c+dx)}{b}}{d}$$

$$\Big\downarrow \text{ 27}$$

$$\frac{\frac{a\int \frac{\tanh^4(c+dx)\left(1-\tanh^2(c+dx)\right)}{\left((a-b)\tanh^4(c+dx)-2a\tanh^2(c+dx)+a\right)^2}d\tanh(c+dx)}{b}-\frac{\int \frac{\tanh^4(c+dx)}{\left(1-\tanh^2(c+dx)\right)\left((a-b)\tanh^4(c+dx)-2a\tanh^2(c+dx)+a\right)}d\tanh(c+dx)}{b}}{d}$$

$$\Big\downarrow \text{ 1598}$$

$$\frac{\frac{a\left(\frac{\int -\frac{2b\tanh^4(c+dx)}{(a-b)\tanh^4(c+dx)-2a\tanh^2(c+dx)+a}d\tanh(c+dx)}{8ab}+\frac{\tanh^5(c+dx)}{4a\left((a-b)\tanh^4(c+dx)-2a\tanh^2(c+dx)+a\right)}\right)}{b}-\frac{\int \frac{\tanh^4(c+dx)}{\left(1-\tanh^2(c+dx)\right)\left((a-b)\tanh^4(c+dx)-2a\right.}}{b}}{d}$$

$$\Big\downarrow \text{ 27}$$

$$\frac{\frac{a\left(\frac{\tanh^5(c+dx)}{4a\left((a-b)\tanh^4(c+dx)-2a\tanh^2(c+dx)+a\right)}-\frac{\int \frac{\tanh^4(c+dx)}{(a-b)\tanh^4(c+dx)-2a\tanh^2(c+dx)+a}d\tanh(c+dx)}{4a}\right)}{b}-\frac{\int \frac{\tanh^4(c+dx)}{\left(1-\tanh^2(c+dx)\right)\left((a-b)\tanh^4(c+dx)-2a\right.}}{b}}{d}$$

$$\Big\downarrow \text{ 1442}$$

$$\frac{\frac{a\left(\frac{\tanh^5(c+dx)}{4a\left((a-b)\tanh^4(c+dx)-2a\tanh^2(c+dx)+a\right)}-\frac{\frac{\tanh(c+dx)}{a-b}-\frac{\int \frac{a\left(1-2\tanh^2(c+dx)\right)}{(a-b)\tanh^4(c+dx)-2a\tanh^2(c+dx)+a}d\tanh(c+dx)}{a-b}}{4a}\right)}{b}-\frac{\int \frac{\tanh^4}{\left(1-\tanh^2(c+dx)\right)\left((a-b)\tan\right.}}{}}{d}$$

$$\Big\downarrow \text{ 27}$$

$$\frac{\frac{a\left(\frac{\tanh^5(c+dx)}{4a\left((a-b)\tanh^4(c+dx)-2a\tanh^2(c+dx)+a\right)}-\frac{\frac{\tanh(c+dx)}{a-b}-\frac{a\int \frac{1-2\tanh^2(c+dx)}{(a-b)\tanh^4(c+dx)-2a\tanh^2(c+dx)+a}d\tanh(c+dx)}{a-b}}{4a}\right)}{b}-\frac{\int \frac{\tanh}{\left(1-\tanh^2(c+dx)\right)\left((a-b)\tan\right.}}{}}{d}$$

$$\Big\downarrow \; 1480$$

$$\frac{a\left(\frac{\tanh^5(c+dx)}{4a\left((a-b)\tanh^4(c+dx)-2a\tanh^2(c+dx)+a\right)}-\frac{\frac{\tanh(c+dx)}{a-b}-\frac{a\left(-\frac{1}{2}\left(2-\frac{a+b}{\sqrt{a}\sqrt{b}}\right)\int\frac{1}{(a-b)\tanh^2(c+dx)-\sqrt{a}\left(\sqrt{a}-\sqrt{b}\right)}d\tanh(c+dx)-\frac{1}{2}\left(\frac{a+b}{\sqrt{a}\sqrt{b}}+2\right)\int\frac{}{(a-b)}\right.}{a-b}}{4a}\right.}{b}$$

$$d$$

$$\Big\downarrow \; 221$$

$$\frac{a\left(\frac{\tanh^5(c+dx)}{4a\left((a-b)\tanh^4(c+dx)-2a\tanh^2(c+dx)+a\right)}-\frac{\frac{\tanh(c+dx)}{a-b}-\frac{a\left(\frac{\left(\frac{a+b}{\sqrt{a}\sqrt{b}}+2\right)\operatorname{arctanh}\left(\frac{\sqrt{\sqrt{a}-\sqrt{b}}\tanh(c+dx)}{\sqrt[4]{a}}\right)}{2\sqrt[4]{a}\sqrt{\sqrt{a}-\sqrt{b}}\left(\sqrt{a}+\sqrt{b}\right)}+\frac{\left(2-\frac{a+b}{\sqrt{a}\sqrt{b}}\right)\operatorname{arctanh}\left(\frac{\sqrt{\sqrt{a}+\sqrt{b}}\tanh}{\sqrt[4]{a}}\right)}{2\sqrt[4]{a}\left(\sqrt{a}-\sqrt{b}\right)\sqrt{\sqrt{a}+\sqrt{b}}}\right.}{a-b}}{4a}\right.}{b}$$

$$d$$

$$\Big\downarrow \; 1610$$

$$\frac{a\left(\frac{\tanh^5(c+dx)}{4a\left((a-b)\tanh^4(c+dx)-2a\tanh^2(c+dx)+a\right)}-\frac{\frac{\tanh(c+dx)}{a-b}-\frac{a\left(\frac{\left(\frac{a+b}{\sqrt{a}\sqrt{b}}+2\right)\operatorname{arctanh}\left(\frac{\sqrt{\sqrt{a}-\sqrt{b}}\tanh(c+dx)}{\sqrt[4]{a}}\right)}{2\sqrt[4]{a}\sqrt{\sqrt{a}-\sqrt{b}}\left(\sqrt{a}+\sqrt{b}\right)}+\frac{\left(2-\frac{a+b}{\sqrt{a}\sqrt{b}}\right)\operatorname{arctanh}\left(\frac{\sqrt{\sqrt{a}+\sqrt{b}}\tanh}{\sqrt[4]{a}}\right)}{2\sqrt[4]{a}\left(\sqrt{a}-\sqrt{b}\right)\sqrt{\sqrt{a}+\sqrt{b}}}\right.}{a-b}}{4a}\right.}{b}$$

$$d$$

$$\Big\downarrow \; 2009$$

$$\frac{a\left(\frac{\tanh^5(c+dx)}{4a\left((a-b)\tanh^4(c+dx)-2a\tanh^2(c+dx)+a\right)}-\frac{\frac{\tanh(c+dx)}{a-b}-\frac{a\left(\frac{\left(\frac{a+b}{\sqrt{a}\sqrt{b}}+2\right)\operatorname{arctanh}\left(\frac{\sqrt{\sqrt{a}-\sqrt{b}}\tanh(c+dx)}{\sqrt[4]{a}}\right)}{2\sqrt[4]{a}\sqrt{\sqrt{a}-\sqrt{b}}\left(\sqrt{a}+\sqrt{b}\right)}+\frac{\left(2-\frac{a+b}{\sqrt{a}\sqrt{b}}\right)\operatorname{arctanh}\left(\frac{\sqrt{\sqrt{a}+\sqrt{b}}\tanh}{\sqrt[4]{a}}\right)}{2\sqrt[4]{a}\left(\sqrt{a}-\sqrt{b}\right)\sqrt{\sqrt{a}+\sqrt{b}}}\right.}{a-b}}{4a}\right.}{b}$$

$$d$$

input
```
Int[Sinh[c + d*x]^8/(a - b*Sinh[c + d*x]^4)^2,x]
```

output
```
(-((-(ArcTanh[Tanh[c + d*x]]/b) + (a^(1/4)*ArcTanh[(Sqrt[Sqrt[a] - Sqrt[b]
]*Tanh[c + d*x])/a^(1/4)])/(2*Sqrt[Sqrt[a] - Sqrt[b]]*b) + (a^(1/4)*ArcTan
h[(Sqrt[Sqrt[a] + Sqrt[b]]*Tanh[c + d*x])/a^(1/4)])/(2*Sqrt[Sqrt[a] + Sqrt
[b]]*b))/b) + (a*(-1/4*(-((a*(((2 + (a + b)/(Sqrt[a]*Sqrt[b]))*ArcTanh[(Sq
rt[Sqrt[a] - Sqrt[b]]*Tanh[c + d*x])/a^(1/4)])/(2*a^(1/4)*Sqrt[Sqrt[a] - S
qrt[b]]*(Sqrt[a] + Sqrt[b])) + ((2 - (a + b)/(Sqrt[a]*Sqrt[b]))*ArcTanh[(S
qrt[Sqrt[a] + Sqrt[b]]*Tanh[c + d*x])/a^(1/4)])/(2*a^(1/4)*(Sqrt[a] - Sqrt
[b])*Sqrt[Sqrt[a] + Sqrt[b]])))/(a - b)) + Tanh[c + d*x]/(a - b))/a + Tanh
[c + d*x]^5/(4*a*(a - 2*a*Tanh[c + d*x]^2 + (a - b)*Tanh[c + d*x]^4))))/b)
/d
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 1442
```
Int[((d_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
:> Simp[d^3*(d*x)^(m - 3)*((a + b*x^2 + c*x^4)^(p + 1)/(c*(m + 4*p + 1))),
x] - Simp[d^4/(c*(m + 4*p + 1))   Int[(d*x)^(m - 4)*Simp[a*(m - 3) + b*(m +
 2*p - 1)*x^2, x]*(a + b*x^2 + c*x^4)^p, x], x] /; FreeQ[{a, b, c, d, p}, x
] && NeQ[b^2 - 4*a*c, 0] && GtQ[m, 3] && NeQ[m + 4*p + 1, 0] && IntegerQ[2*
p] && (IntegerQ[p] || IntegerQ[m])
```

rule 1480
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Simp[(e/2 + (2*c*d - b*e)/(2*q))   Int[1/(
b/2 - q/2 + c*x^2), x], x] + Simp[(e/2 - (2*c*d - b*e)/(2*q))   Int[1/(b/2
+ q/2 + c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0]
 && NeQ[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

rule 1598
```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(
x_)^4)^(p_.), x_Symbol] :> Simp[f*(f*x)^(m - 1)*(a + b*x^2 + c*x^4)^(p + 1)
*((b*d - 2*a*e - (b*e - 2*c*d)*x^2)/(2*(p + 1)*(b^2 - 4*a*c))), x] - Simp[f
^2/(2*(p + 1)*(b^2 - 4*a*c))   Int[(f*x)^(m - 2)*(a + b*x^2 + c*x^4)^(p + 1
)*Simp[(m - 1)*(b*d - 2*a*e) - (4*p + 4 + m + 1)*(b*e - 2*c*d)*x^2, x], x],
 x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] &&
 GtQ[m, 1] && IntegerQ[2*p] && (IntegerQ[p] || IntegerQ[m])
```

rule 1610
```
Int[(((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.))/((a_) + (b_.)*(x_)^2 +
 (c_.)*(x_)^4), x_Symbol] :> Int[ExpandIntegrand[(f*x)^m*((d + e*x^2)^q/(a
+ b*x^2 + c*x^4)), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b^2 - 4
*a*c, 0] && IntegerQ[q] && IntegerQ[m]
```

rule 1650
```
Int[(((f_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_))/((d_) +
 (e_.)*(x_)^2), x_Symbol] :> Simp[-f^4/(c*d^2 - b*d*e + a*e^2)   Int[(f*x)^
(m - 4)*(a*d + (b*d - a*e)*x^2)*(a + b*x^2 + c*x^4)^p, x], x] + Simp[d^2*(f
^4/(c*d^2 - b*d*e + a*e^2))   Int[(f*x)^(m - 4)*((a + b*x^2 + c*x^4)^(p + 1
)/(d + e*x^2)), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b^2 - 4*a*c,
0] && LtQ[p, -1] && GtQ[m, 2]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3696
```
Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^4)^(
p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Simp[ff^(m + 1
)/f   Subst[Int[x^m*((a + 2*a*ff^2*x^2 + (a + b)*ff^4*x^4)^p/(1 + ff^2*x^2)
^(m/2 + 2*p + 1)), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f}, x] &
& IntegerQ[m/2] && IntegerQ[p]
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 5.10 (sec) , antiderivative size = 331, normalized size of antiderivative = 1.50

method	result
derivativedivides	$2a\left(\frac{-\frac{b\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^7}{4(a-b)}+\frac{5b\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^5}{4(a-b)}+\frac{5b\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^3}{4(a-b)}-\frac{b\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)}{4(a-b)}}{\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^8a-4\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^6a+6\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^4a-16b\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^4-4\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^2a+a}+\frac{\_R=\text{RootOf}\left(a\_Z^8-4\right.}{}\right.$
default	$2a\left(\frac{-\frac{b\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^7}{4(a-b)}+\frac{5b\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^5}{4(a-b)}+\frac{5b\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^3}{4(a-b)}-\frac{b\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)}{4(a-b)}}{\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^8a-4\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^6a+6\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^4a-16b\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^4-4\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^2a+a}+\frac{\_R=\text{RootOf}\left(a\_Z^8-4\right.}{}\right.$
risch	$\frac{x}{b^2}+\frac{a\left(-e^{6dx+6c}b+8\,e^{4dx+4c}a-3b\,e^{4dx+4c}+5\,e^{2dx+2c}b-b\right)}{2b^2(a-b)d\left(-e^{8dx+8c}b+4\,e^{6dx+6c}b+16\,e^{4dx+4c}a-6b\,e^{4dx+4c}+4\,e^{2dx+2c}b-b\right)}+\left(\sum_{\_R=\text{RootOf}\left((65536a^3b^8\right.}\right.$

input

```
int(sinh(d*x+c)^8/(a-b*sinh(d*x+c)^4)^2,x,method=_RETURNVERBOSE)
```

output

```
1/d*(2*a/b^2*((-1/4*b/(a-b)*tanh(1/2*d*x+1/2*c)^7+5/4*b/(a-b)*tanh(1/2*d*x
+1/2*c)^5+5/4*b/(a-b)*tanh(1/2*d*x+1/2*c)^3-1/4*b/(a-b)*tanh(1/2*d*x+1/2*c
))/(tanh(1/2*d*x+1/2*c)^8*a-4*tanh(1/2*d*x+1/2*c)^6*a+6*tanh(1/2*d*x+1/2*c
)^4*a-16*b*tanh(1/2*d*x+1/2*c)^4-4*tanh(1/2*d*x+1/2*c)^2*a+a)+1/32/(a-b)*s
um(((4*a-5*b)*_R^6+(-12*a+19*b)*_R^4+(12*a-19*b)*_R^2-4*a+5*b)/(_R^7*a-3*_
R^5*a+3*_R^3*a-8*_R^3*b-_R*a)*ln(tanh(1/2*d*x+1/2*c)-_R),_R=RootOf(a*_Z^8-
4*a*_Z^6+(6*a-16*b)*_Z^4-4*a*_Z^2+a))-1/b^2*ln(tanh(1/2*d*x+1/2*c)-1)+1/b
^2*ln(tanh(1/2*d*x+1/2*c)+1))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 6944 vs. $2(175) = 350$.

Time = 0.53 (sec) , antiderivative size = 6944, normalized size of antiderivative = 31.42

$$\int \frac{\sinh^8(c+dx)}{(a-b\sinh^4(c+dx))^2}\,dx = \text{Too large to display}$$

input
```
integrate(sinh(d*x+c)^8/(a-b*sinh(d*x+c)^4)^2,x, algorithm="fricas")
```

output
```
Too large to include
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\sinh^8(c+dx)}{(a-b\sinh^4(c+dx))^2}\,dx = \text{Timed out}$$

input
```
integrate(sinh(d*x+c)**8/(a-b*sinh(d*x+c)**4)**2,x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{\sinh^8(c+dx)}{(a-b\sinh^4(c+dx))^2}\,dx = \int \frac{\sinh(dx+c)^8}{(b\sinh(dx+c)^4-a)^2}\,dx$$

input
```
integrate(sinh(d*x+c)^8/(a-b*sinh(d*x+c)^4)^2,x, algorithm="maxima")
```

output
```
1/2*(2*(a*b*d*e^(8*c) - b^2*d*e^(8*c))*x*e^(8*d*x) + a*b + 2*(a*b*d - b^2*
d)*x + (a*b*e^(6*c) - 8*(a*b*d*e^(6*c) - b^2*d*e^(6*c))*x)*e^(6*d*x) - (8*
a^2*e^(4*c) - 3*a*b*e^(4*c) + 4*(8*a^2*d*e^(4*c) - 11*a*b*d*e^(4*c) + 3*b^
2*d*e^(4*c))*x)*e^(4*d*x) - (5*a*b*e^(2*c) + 8*(a*b*d*e^(2*c) - b^2*d*e^(2
*c))*x)*e^(2*d*x))/(a*b^3*d - b^4*d + (a*b^3*d*e^(8*c) - b^4*d*e^(8*c))*e^
(8*d*x) - 4*(a*b^3*d*e^(6*c) - b^4*d*e^(6*c))*e^(6*d*x) - 2*(8*a^2*b^2*d*e
^(4*c) - 11*a*b^3*d*e^(4*c) + 3*b^4*d*e^(4*c))*e^(4*d*x) - 4*(a*b^3*d*e^(2
*c) - b^4*d*e^(2*c))*e^(2*d*x)) + 1/256*integrate(256*(a*b*e^(6*d*x + 6*c)
 + a*b*e^(2*d*x + 2*c) + 2*(8*a^2*e^(4*c) - 11*a*b*e^(4*c))*e^(4*d*x))/(a*
b^3 - b^4 + (a*b^3*e^(8*c) - b^4*e^(8*c))*e^(8*d*x) - 4*(a*b^3*e^(6*c) - b
^4*e^(6*c))*e^(6*d*x) - 2*(8*a^2*b^2*e^(4*c) - 11*a*b^3*e^(4*c) + 3*b^4*e^
(4*c))*e^(4*d*x) - 4*(a*b^3*e^(2*c) - b^4*e^(2*c))*e^(2*d*x)), x)
```

Giac [F]

$$\int \frac{\sinh^8(c+dx)}{\left(a-b\sinh^4(c+dx)\right)^2}\,dx=\int \frac{\sinh\left(dx+c\right)^8}{\left(b\sinh\left(dx+c\right)^4-a\right)^2}\,dx$$

input
```
integrate(sinh(d*x+c)^8/(a-b*sinh(d*x+c)^4)^2,x, algorithm="giac")
```

output
```
sage0*x
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sinh^8(c+dx)}{\left(a-b\sinh^4(c+dx)\right)^2}\,dx=\int \frac{\sinh(c+d\,x)^8}{\left(a-b\sinh\left(c+d\,x\right)^4\right)^2}\,dx$$

input
```
int(sinh(c + d*x)^8/(a - b*sinh(c + d*x)^4)^2,x)
```

output
```
int(sinh(c + d*x)^8/(a - b*sinh(c + d*x)^4)^2, x)
```

Reduce [F]

$$\int \frac{\sinh^8(c+dx)}{(a-b\sinh^4(c+dx))^2}\,dx = \text{too large to display}$$

input
```
int(sinh(d*x+c)^8/(a-b*sinh(d*x+c)^4)^2,x)
```

output
```
( - 1006632960*e**(12*c + 8*d*x)*int(e**(4*d*x)/(1920*e**(16*c + 16*d*x)*a
**2*b**2 + 80*e**(16*c + 16*d*x)*a*b**3 - 3*e**(16*c + 16*d*x)*b**4 - 1536
0*e**(14*c + 14*d*x)*a**2*b**2 - 640*e**(14*c + 14*d*x)*a*b**3 + 24*e**(14
*c + 14*d*x)*b**4 - 61440*e**(12*c + 12*d*x)*a**3*b + 51200*e**(12*c + 12*
d*x)*a**2*b**2 + 2336*e**(12*c + 12*d*x)*a*b**3 - 84*e**(12*c + 12*d*x)*b*
*4 + 245760*e**(10*c + 10*d*x)*a**3*b - 97280*e**(10*c + 10*d*x)*a**2*b**2
 - 4864*e**(10*c + 10*d*x)*a*b**3 + 168*e**(10*c + 10*d*x)*b**4 + 491520*e
**(8*c + 8*d*x)*a**4 - 348160*e**(8*c + 8*d*x)*a**3*b + 118272*e**(8*c + 8
*d*x)*a**2*b**2 + 6176*e**(8*c + 8*d*x)*a*b**3 - 210*e**(8*c + 8*d*x)*b**4
 + 245760*e**(6*c + 6*d*x)*a**3*b - 97280*e**(6*c + 6*d*x)*a**2*b**2 - 486
4*e**(6*c + 6*d*x)*a*b**3 + 168*e**(6*c + 6*d*x)*b**4 - 61440*e**(4*c + 4*
d*x)*a**3*b + 51200*e**(4*c + 4*d*x)*a**2*b**2 + 2336*e**(4*c + 4*d*x)*a*b
**3 - 84*e**(4*c + 4*d*x)*b**4 - 15360*e**(2*c + 2*d*x)*a**2*b**2 - 640*e*
*(2*c + 2*d*x)*a*b**3 + 24*e**(2*c + 2*d*x)*b**4 + 1920*a**2*b**2 + 80*a*b
**3 - 3*b**4),x)*a**7*b*d + 2285895680*e**(12*c + 8*d*x)*int(e**(4*d*x)/(1
920*e**(16*c + 16*d*x)*a**2*b**2 + 80*e**(16*c + 16*d*x)*a*b**3 - 3*e**(16
*c + 16*d*x)*b**4 - 15360*e**(14*c + 14*d*x)*a**2*b**2 - 640*e**(14*c + 14
*d*x)*a*b**3 + 24*e**(14*c + 14*d*x)*b**4 - 61440*e**(12*c + 12*d*x)*a**3*
b + 51200*e**(12*c + 12*d*x)*a**2*b**2 + 2336*e**(12*c + 12*d*x)*a*b**3 -
84*e**(12*c + 12*d*x)*b**4 + 245760*e**(10*c + 10*d*x)*a**3*b - 97280*e...
```

3.223 $\int \frac{\sinh^6(c+dx)}{(a-b\sinh^4(c+dx))^2}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 222

$$\int \frac{\sinh^6(c+dx)}{(a-b\sinh^4(c+dx))^2}\,dx = \frac{\left(2\sqrt{a}-3\sqrt{b}\right)\operatorname{arctanh}\left(\frac{\sqrt{\sqrt{a}-\sqrt{b}}\tanh(c+dx)}{\sqrt[4]{a}}\right)}{8\sqrt[4]{a}\left(\sqrt{a}-\sqrt{b}\right)^{3/2}b^{3/2}d} - \frac{\left(2\sqrt{a}+3\sqrt{b}\right)\operatorname{arctanh}\left(\frac{\sqrt{\sqrt{a}+\sqrt{b}}\tanh(c+dx)}{\sqrt[4]{a}}\right)}{8\sqrt[4]{a}\left(\sqrt{a}+\sqrt{b}\right)^{3/2}b^{3/2}d} + \frac{\tanh(c+dx)\left(a-(a+b)\tanh^2(c+dx)\right)}{4(a-b)bd\left(a-2a\tanh^2(c+dx)+(a-b)\tanh^4(c+dx)\right)}$$

output
```
1/8*(2*a^(1/2)-3*b^(1/2))*arctanh((a^(1/2)-b^(1/2))^(1/2)*tanh(d*x+c)/a^(1
/4))/a^(1/4)/(a^(1/2)-b^(1/2))^(3/2)/b^(3/2)/d-1/8*(2*a^(1/2)+3*b^(1/2))*a
rctanh((a^(1/2)+b^(1/2))^(1/2)*tanh(d*x+c)/a^(1/4))/a^(1/4)/(a^(1/2)+b^(1/
2))^(3/2)/b^(3/2)/d+1/4*tanh(d*x+c)*(a-(a+b)*tanh(d*x+c)^2)/(a-b)/b/d/(a-2
*a*tanh(d*x+c)^2+(a-b)*tanh(d*x+c)^4)
```

Mathematica [A] (verified)

Time = 4.82 (sec) , antiderivative size = 238, normalized size of antiderivative = 1.07

$$\int \frac{\sinh^6(c+dx)}{(a-b\sinh^4(c+dx))^2}\,dx$$
$$= \frac{\frac{\sqrt{b}\left(-2a+\sqrt{a}\sqrt{b}+3b\right)\arctan\left(\frac{\left(\sqrt{a}-\sqrt{b}\right)\tanh(c+dx)}{\sqrt{-a+\sqrt{a}\sqrt{b}}}\right)}{\sqrt{-a+\sqrt{a}\sqrt{b}}}+\frac{\sqrt{b}\left(-2a-\sqrt{a}\sqrt{b}+3b\right)\operatorname{arctanh}\left(\frac{\left(\sqrt{a}+\sqrt{b}\right)\tanh(c+dx)}{\sqrt{a+\sqrt{a}\sqrt{b}}}\right)}{\sqrt{a+\sqrt{a}\sqrt{b}}}-\frac{4b(-2a-b+b\cosh(2(c-}{8a-3b+4b\cosh(2(c+d}}{8(a-b)b^2d}$$

input
```
Integrate[Sinh[c + d*x]^6/(a - b*Sinh[c + d*x]^4)^2,x]
```

output
```
((Sqrt[b]*(-2*a + Sqrt[a]*Sqrt[b] + 3*b)*ArcTan[((Sqrt[a] - Sqrt[b])*Tanh[
c + d*x])/Sqrt[-a + Sqrt[a]*Sqrt[b]]])/Sqrt[-a + Sqrt[a]*Sqrt[b]] + (Sqrt[
b]*(-2*a - Sqrt[a]*Sqrt[b] + 3*b)*ArcTanh[((Sqrt[a] + Sqrt[b])*Tanh[c + d*
x])/Sqrt[a + Sqrt[a]*Sqrt[b]]])/Sqrt[a + Sqrt[a]*Sqrt[b]] - (4*b*(-2*a - b
 + b*Cosh[2*(c + d*x)])*Sinh[2*(c + d*x)])/(8*a - 3*b + 4*b*Cosh[2*(c + d*
x)] - b*Cosh[4*(c + d*x)]))/(8*(a - b)*b^2*d)
```

Rubi [A] (verified)

Time = 0.52 (sec) , antiderivative size = 273, normalized size of antiderivative = 1.23, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.375$, Rules used = {3042, 25, 3696, 1440, 27, 1602, 25, 1480, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sinh^6(c+dx)}{(a-b\sinh^4(c+dx))^2}\,dx$$

$$\Big\downarrow \; 3042$$

$$\int -\frac{\sin(ic+idx)^6}{(a-b\sin(ic+idx)^4)^2}dx$$

$$\Big\downarrow \; 25$$

$$-\int \frac{\sin(ic+idx)^6}{\left(a-b\sin(ic+idx)^4\right)^2}dx$$

$$\Big\downarrow \ 3696$$

$$\frac{\int \frac{\tanh^6(c+dx)}{\left((a-b)\tanh^4(c+dx)-2a\tanh^2(c+dx)+a\right)^2}d\tanh(c+dx)}{d}$$

$$\Big\downarrow \ 1440$$

$$\frac{\frac{\tanh^3(c+dx)\left(1-\tanh^2(c+dx)\right)}{4b\left((a-b)\tanh^4(c+dx)-2a\tanh^2(c+dx)+a\right)}-\frac{\int \frac{2a\tanh^2(c+dx)\left(3-\tanh^2(c+dx)\right)}{(a-b)\tanh^4(c+dx)-2a\tanh^2(c+dx)+a}d\tanh(c+dx)}{8ab}}{d}$$

$$\Big\downarrow \ 27$$

$$\frac{\frac{\tanh^3(c+dx)\left(1-\tanh^2(c+dx)\right)}{4b\left((a-b)\tanh^4(c+dx)-2a\tanh^2(c+dx)+a\right)}-\frac{\int \frac{\tanh^2(c+dx)\left(3-\tanh^2(c+dx)\right)}{(a-b)\tanh^4(c+dx)-2a\tanh^2(c+dx)+a}d\tanh(c+dx)}{4b}}{d}$$

$$\Big\downarrow \ 1602$$

$$\frac{\frac{\tanh^3(c+dx)\left(1-\tanh^2(c+dx)\right)}{4b\left((a-b)\tanh^4(c+dx)-2a\tanh^2(c+dx)+a\right)}-\frac{-\frac{\int -\frac{(a-3b)\tanh^2(c+dx)+a}{(a-b)\tanh^4(c+dx)-2a\tanh^2(c+dx)+a}d\tanh(c+dx)}{a-b}-\frac{\tanh(c+dx)}{a-b}}{4b}}{d}$$

$$\Big\downarrow \ 25$$

$$\frac{\frac{\tanh^3(c+dx)\left(1-\tanh^2(c+dx)\right)}{4b\left((a-b)\tanh^4(c+dx)-2a\tanh^2(c+dx)+a\right)}-\frac{\frac{\int \frac{(a-3b)\tanh^2(c+dx)+a}{(a-b)\tanh^4(c+dx)-2a\tanh^2(c+dx)+a}d\tanh(c+dx)}{a-b}-\frac{\tanh(c+dx)}{a-b}}{4b}}{d}$$

$$\Big\downarrow \ 1480$$

$$\frac{\frac{\tanh^3(c+dx)\left(1-\tanh^2(c+dx)\right)}{4b\left((a-b)\tanh^4(c+dx)-2a\tanh^2(c+dx)+a\right)}-\frac{\frac{\frac{1}{2}\left(-\frac{2\sqrt{a}(a-2b)}{\sqrt{b}}+a-3b\right)\int \frac{1}{(a-b)\tanh^2(c+dx)-\sqrt{a}\left(\sqrt{a}-\sqrt{b}\right)}d\tanh(c+dx)+\frac{1}{2}\left(\frac{2\sqrt{a}(a-2b)}{\sqrt{b}}+a-3b\right)\int \frac{}{(}}{a-b}}{4b}}{d}$$

$$\Big\downarrow \ 221$$

$$\frac{\frac{\tanh^3(c+dx)\left(1-\tanh^2(c+dx)\right)}{4b\left((a-b)\tanh^4(c+dx)-2a\tanh^2(c+dx)+a\right)}-\frac{\frac{-\frac{\left(\frac{2\sqrt{a}(a-2b)}{\sqrt{b}}+a-3b\right)\operatorname{arctanh}\left(\frac{\sqrt{\sqrt{a}-\sqrt{b}}\tanh(c+dx)}{\sqrt[4]{a}}\right)}{2\sqrt[4]{a}\sqrt{\sqrt{a}-\sqrt{b}}\left(\sqrt{a}+\sqrt{b}\right)}-\frac{\left(-\frac{2\sqrt{a}(a-2b)}{\sqrt{b}}+a-3b\right)\operatorname{arctanh}\left(\frac{\sqrt{\sqrt{}}}{}\right)}{2\sqrt[4]{a}\left(\sqrt{a}-\sqrt{b}\right)\sqrt{\sqrt{a}+}}}{a-b}}{4b}}{d}$$

input
```
Int[Sinh[c + d*x]^6/(a - b*Sinh[c + d*x]^4)^2,x]
```

output
```
(-1/4*((-1/2*((a + (2*Sqrt[a]*(a - 2*b))/Sqrt[b] - 3*b)*ArcTanh[(Sqrt[Sqrt
[a] - Sqrt[b]]*Tanh[c + d*x])/a^(1/4)])/(a^(1/4)*Sqrt[Sqrt[a] - Sqrt[b]]*(
Sqrt[a] + Sqrt[b])) - ((a - (2*Sqrt[a]*(a - 2*b))/Sqrt[b] - 3*b)*ArcTanh[(
Sqrt[Sqrt[a] + Sqrt[b]]*Tanh[c + d*x])/a^(1/4)])/(2*a^(1/4)*(Sqrt[a] - Sqr
t[b])*Sqrt[Sqrt[a] + Sqrt[b]]))/(a - b) - Tanh[c + d*x]/(a - b))/b + (Tanh
[c + d*x]^3*(1 - Tanh[c + d*x]^2))/(4*b*(a - 2*a*Tanh[c + d*x]^2 + (a - b)
*Tanh[c + d*x]^4)))/d
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 1440
```
Int[((d_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
:> Simp[(-d^3)*(d*x)^(m - 3)*(2*a + b*x^2)*((a + b*x^2 + c*x^4)^(p + 1)/(2*
(p + 1)*(b^2 - 4*a*c))), x] + Simp[d^4/(2*(p + 1)*(b^2 - 4*a*c))   Int[(d*x
)^(m - 4)*(2*a*(m - 3) + b*(m + 4*p + 3)*x^2)*(a + b*x^2 + c*x^4)^(p + 1),
x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && Gt
Q[m, 3] && IntegerQ[2*p] && (IntegerQ[p] || IntegerQ[m])
```

rule 1480
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Simp[(e/2 + (2*c*d - b*e)/(2*q))   Int[1/(
b/2 - q/2 + c*x^2), x], x] + Simp[(e/2 - (2*c*d - b*e)/(2*q))   Int[1/(b/2
+ q/2 + c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0]
 && NeQ[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

rule 1602
```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(
x_)^4)^(p_), x_Symbol] :> Simp[e*f*(f*x)^(m - 1)*((a + b*x^2 + c*x^4)^(p +
1)/(c*(m + 4*p + 3))), x] - Simp[f^2/(c*(m + 4*p + 3))   Int[(f*x)^(m - 2)*
(a + b*x^2 + c*x^4)^p*Simp[a*e*(m - 1) + (b*e*(m + 2*p + 1) - c*d*(m + 4*p
+ 3))*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && NeQ[b^2 - 4*a*c
, 0] && GtQ[m, 1] && NeQ[m + 4*p + 3, 0] && IntegerQ[2*p] && (IntegerQ[p] |
| IntegerQ[m])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3696
```
Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^4)^(
p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Simp[ff^(m + 1
)/f   Subst[Int[x^m*((a + 2*a*ff^2*x^2 + (a + b)*ff^4*x^4)^p/(1 + ff^2*x^2)
^(m/2 + 2*p + 1)), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f}, x] &
& IntegerQ[m/2] && IntegerQ[p]
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 3.86 (sec) , antiderivative size = 305, normalized size of antiderivative = 1.37

method	result
derivativedivides	$\frac{-\frac{128\left(-\frac{a\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^{7}}{256b(a-b)}+\frac{(a+4b)\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^{5}}{256b(a-b)}+\frac{(a+4b)\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^{3}}{256b(a-b)}-\frac{a\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)}{256b(a-b)}\right)}{\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^{8}a-4\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^{6}a+6\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^{4}a-16b\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^{4}-4\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}a+a}-\frac{\_R=\mathrm{RootOf}\left(a\_Z^{8}-4a\_\right.}{}}{d}$
default	$\frac{-\frac{128\left(-\frac{a\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^{7}}{256b(a-b)}+\frac{(a+4b)\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^{5}}{256b(a-b)}+\frac{(a+4b)\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^{3}}{256b(a-b)}-\frac{a\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)}{256b(a-b)}\right)}{\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^{8}a-4\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^{6}a+6\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^{4}a-16b\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^{4}-4\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}a+a}-\frac{\_R=\mathrm{RootOf}\left(a\_Z^{8}-4a\_\right.}{}}{d}$
risch	$\frac{2\,e^{6dx+6c}a-e^{6dx+6c}b-8\,e^{4dx+4c}a+3b\,e^{4dx+4c}-2\,e^{2dx+2c}a-3\,e^{2dx+2c}b+b}{2bd(a-b)\left(-e^{8dx+8c}b+4\,e^{6dx+6c}b+16\,e^{4dx+4c}a-6b\,e^{4dx+4c}+4\,e^{2dx+2c}b-b\right)}+\left(\sum_{\_R=\mathrm{RootOf}\left((65536a^{4}b^{6}d^{4}-1966\right.}\right.$

input
```
int(sinh(d*x+c)^6/(a-b*sinh(d*x+c)^4)^2,x,method=_RETURNVERBOSE)
```

output
```
1/d*(-128*(-1/256/b/(a-b)*a*tanh(1/2*d*x+1/2*c)^7+1/256/b*(a+4*b)/(a-b)*ta
nh(1/2*d*x+1/2*c)^5+1/256/b*(a+4*b)/(a-b)*tanh(1/2*d*x+1/2*c)^3-1/256/b/(a
-b)*a*tanh(1/2*d*x+1/2*c))/(tanh(1/2*d*x+1/2*c)^8*a-4*tanh(1/2*d*x+1/2*c)^
6*a+6*tanh(1/2*d*x+1/2*c)^4*a-16*b*tanh(1/2*d*x+1/2*c)^4-4*tanh(1/2*d*x+1/
2*c)^2*a+a)-1/16/b/(a-b)*sum((-a*_R^6+(-5*a+12*b)*_R^4+(5*a-12*b)*_R^2+a)/
(_R^7*a-3*_R^5*a+3*_R^3*a-8*_R^3*b-_R*a)*ln(tanh(1/2*d*x+1/2*c)-_R),_R=Roo
tOf(a*_Z^8-4*a*_Z^6+(6*a-16*b)*_Z^4-4*a*_Z^2+a)))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 6045 vs. $2(173) = 346$.

Time = 0.41 (sec) , antiderivative size = 6045, normalized size of antiderivative = 27.23

$$\int \frac{\sinh^6(c+dx)}{(a-b\sinh^4(c+dx))^2}\,dx = \text{Too large to display}$$

input
```
integrate(sinh(d*x+c)^6/(a-b*sinh(d*x+c)^4)^2,x, algorithm="fricas")
```

output
```
Too large to include
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\sinh^6(c+dx)}{(a-b\sinh^4(c+dx))^2}\,dx = \text{Timed out}$$

input
```
integrate(sinh(d*x+c)**6/(a-b*sinh(d*x+c)**4)**2,x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{\sinh^6(c+dx)}{(a-b\sinh^4(c+dx))^2}\,dx = \int \frac{\sinh(dx+c)^6}{(b\sinh(dx+c)^4-a)^2}\,dx$$

input
```
integrate(sinh(d*x+c)^6/(a-b*sinh(d*x+c)^4)^2,x, algorithm="maxima")
```

output
```
-1/2*((2*a*e^(6*c) - b*e^(6*c))*e^(6*d*x) - (8*a*e^(4*c) - 3*b*e^(4*c))*e^
(4*d*x) - (2*a*e^(2*c) + 3*b*e^(2*c))*e^(2*d*x) + b)/(a*b^2*d - b^3*d + (a
*b^2*d*e^(8*c) - b^3*d*e^(8*c))*e^(8*d*x) - 4*(a*b^2*d*e^(6*c) - b^3*d*e^(
6*c))*e^(6*d*x) - 2*(8*a^2*b*d*e^(4*c) - 11*a*b^2*d*e^(4*c) + 3*b^3*d*e^(4
*c))*e^(4*d*x) - 4*(a*b^2*d*e^(2*c) - b^3*d*e^(2*c))*e^(2*d*x)) + 1/64*int
egrate(64*((2*a*e^(6*c) - 3*b*e^(6*c))*e^(6*d*x) + (2*a*e^(2*c) - 3*b*e^(2
*c))*e^(2*d*x) + 6*b*e^(4*d*x + 4*c))/(a*b^2 - b^3 + (a*b^2*e^(8*c) - b^3*
e^(8*c))*e^(8*d*x) - 4*(a*b^2*e^(6*c) - b^3*e^(6*c))*e^(6*d*x) - 2*(8*a^2*
b*e^(4*c) - 11*a*b^2*e^(4*c) + 3*b^3*e^(4*c))*e^(4*d*x) - 4*(a*b^2*e^(2*c)
 - b^3*e^(2*c))*e^(2*d*x)), x)
```

Giac [F]

$$\int \frac{\sinh^6(c+dx)}{(a-b\sinh^4(c+dx))^2}\,dx = \int \frac{\sinh(dx+c)^6}{(b\sinh(dx+c)^4-a)^2}\,dx$$

input
```
integrate(sinh(d*x+c)^6/(a-b*sinh(d*x+c)^4)^2,x, algorithm="giac")
```

output
```
sage0*x
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sinh^6(c+dx)}{\left(a-b\sinh^4(c+dx)\right)^2}\,dx=\int \frac{\sinh(c+d\,x)^6}{\left(a-b\sinh\left(c+d\,x\right)^4\right)^2}\,dx$$

input
```
int(sinh(c + d*x)^6/(a - b*sinh(c + d*x)^4)^2,x)
```

output
```
int(sinh(c + d*x)^6/(a - b*sinh(c + d*x)^4)^2, x)
```

Reduce [F]

$$\int \frac{\sinh^6(c+dx)}{\left(a-b\sinh^4(c+dx)\right)^2}\,dx=\text{too large to display}$$

input
```
int(sinh(d*x+c)^6/(a-b*sinh(d*x+c)^4)^2,x)
```

output

```
(4026531840*e**(12*c + 8*d*x)*int(e**(4*d*x)/(1920*e**(16*c + 16*d*x)*a**2
*b**2 + 80*e**(16*c + 16*d*x)*a*b**3 - 3*e**(16*c + 16*d*x)*b**4 - 15360*e
**(14*c + 14*d*x)*a**2*b**2 - 640*e**(14*c + 14*d*x)*a*b**3 + 24*e**(14*c
+ 14*d*x)*b**4 - 61440*e**(12*c + 12*d*x)*a**3*b + 51200*e**(12*c + 12*d*x
)*a**2*b**2 + 2336*e**(12*c + 12*d*x)*a*b**3 - 84*e**(12*c + 12*d*x)*b**4
+ 245760*e**(10*c + 10*d*x)*a**3*b - 97280*e**(10*c + 10*d*x)*a**2*b**2 -
4864*e**(10*c + 10*d*x)*a*b**3 + 168*e**(10*c + 10*d*x)*b**4 + 491520*e**(
8*c + 8*d*x)*a**4 - 348160*e**(8*c + 8*d*x)*a**3*b + 118272*e**(8*c + 8*d*
x)*a**2*b**2 + 6176*e**(8*c + 8*d*x)*a*b**3 - 210*e**(8*c + 8*d*x)*b**4 +
245760*e**(6*c + 6*d*x)*a**3*b - 97280*e**(6*c + 6*d*x)*a**2*b**2 - 4864*e
**(6*c + 6*d*x)*a*b**3 + 168*e**(6*c + 6*d*x)*b**4 - 61440*e**(4*c + 4*d*x
)*a**3*b + 51200*e**(4*c + 4*d*x)*a**2*b**2 + 2336*e**(4*c + 4*d*x)*a*b**3
 - 84*e**(4*c + 4*d*x)*b**4 - 15360*e**(2*c + 2*d*x)*a**2*b**2 - 640*e**(2
*c + 2*d*x)*a*b**3 + 24*e**(2*c + 2*d*x)*b**4 + 1920*a**2*b**2 + 80*a*b**3
 - 3*b**4),x)*a**7*b*d - 8136949760*e**(12*c + 8*d*x)*int(e**(4*d*x)/(1920
*e**(16*c + 16*d*x)*a**2*b**2 + 80*e**(16*c + 16*d*x)*a*b**3 - 3*e**(16*c
+ 16*d*x)*b**4 - 15360*e**(14*c + 14*d*x)*a**2*b**2 - 640*e**(14*c + 14*d*
x)*a*b**3 + 24*e**(14*c + 14*d*x)*b**4 - 61440*e**(12*c + 12*d*x)*a**3*b +
 51200*e**(12*c + 12*d*x)*a**2*b**2 + 2336*e**(12*c + 12*d*x)*a*b**3 - 84*
e**(12*c + 12*d*x)*b**4 + 245760*e**(10*c + 10*d*x)*a**3*b - 97280*e**(...
```

3.224 $\int \frac{\sinh^4(c+dx)}{(a-b\sinh^4(c+dx))^2}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 186

$$\int \frac{\sinh^4(c+dx)}{(a-b\sinh^4(c+dx))^2}\,dx = \frac{\operatorname{arctanh}\left(\frac{\sqrt{\sqrt{a}-\sqrt{b}}\tanh(c+dx)}{\sqrt[4]{a}}\right)}{8a^{3/4}\left(\sqrt{a}-\sqrt{b}\right)^{3/2}\sqrt{b}d} - \frac{\operatorname{arctanh}\left(\frac{\sqrt{\sqrt{a}+\sqrt{b}}\tanh(c+dx)}{\sqrt[4]{a}}\right)}{8a^{3/4}\left(\sqrt{a}+\sqrt{b}\right)^{3/2}\sqrt{b}d} - \frac{\tanh(c+dx)\left(1-2\tanh^2(c+dx)\right)}{4(a-b)d\left(a-2a\tanh^2(c+dx)+(a-b)\tanh^4(c+dx)\right)}$$

output
```
1/8*arctanh((a^(1/2)-b^(1/2))^(1/2)*tanh(d*x+c)/a^(1/4))/a^(3/4)/(a^(1/2)-
b^(1/2))^(3/2)/b^(1/2)/d-1/8*arctanh((a^(1/2)+b^(1/2))^(1/2)*tanh(d*x+c)/a
^(1/4))/a^(3/4)/(a^(1/2)+b^(1/2))^(3/2)/b^(1/2)/d-1/4*tanh(d*x+c)*(1-2*tan
h(d*x+c)^2)/(a-b)/d/(a-2*a*tanh(d*x+c)^2+(a-b)*tanh(d*x+c)^4)
```

Mathematica [A] (verified)

Time = 9.05 (sec) , antiderivative size = 225, normalized size of antiderivative = 1.21

$$\int \frac{\sinh^4(c+dx)}{\left(a-b\sinh^4(c+dx)\right)^2}\,dx = -\frac{\frac{\left(\sqrt{a}+\sqrt{b}\right)\arctan\left(\frac{\left(\sqrt{a}-\sqrt{b}\right)\tanh(c+dx)}{\sqrt{-a+\sqrt{a}\sqrt{b}}}\right)}{\sqrt{a}\sqrt{-a+\sqrt{a}\sqrt{b}}\sqrt{b}}+\frac{\left(\sqrt{a}-\sqrt{b}\right)\text{arctanh}\left(\frac{\left(\sqrt{a}+\sqrt{b}\right)\tanh(c+dx)}{\sqrt{a+\sqrt{a}\sqrt{b}}}\right)}{\sqrt{a}\sqrt{a+\sqrt{a}\sqrt{b}}\sqrt{b}}-\frac{2(-6\sinh(2(c+dx))+\sinh(4(c+dx)))}{8a-3b+4b\cosh(2(c+dx))-b\cosh(4(c+dx))}}{8(a-b)d}$$

input
```
Integrate[Sinh[c + d*x]^4/(a - b*Sinh[c + d*x]^4)^2,x]
```

output
```
-1/8*(((Sqrt[a] + Sqrt[b])*ArcTan[((Sqrt[a] - Sqrt[b])*Tanh[c + d*x])/Sqrt
[-a + Sqrt[a]*Sqrt[b]]])/(Sqrt[a]*Sqrt[-a + Sqrt[a]*Sqrt[b]]*Sqrt[b]) + ((
Sqrt[a] - Sqrt[b])*ArcTanh[((Sqrt[a] + Sqrt[b])*Tanh[c + d*x])/Sqrt[a + Sq
rt[a]*Sqrt[b]]])/(Sqrt[a]*Sqrt[a + Sqrt[a]*Sqrt[b]]*Sqrt[b]) - (2*(-6*Sinh
[2*(c + d*x)] + Sinh[4*(c + d*x)]))/(8*a - 3*b + 4*b*Cosh[2*(c + d*x)] - b
*Cosh[4*(c + d*x)]))/((a - b)*d)
```

Rubi [A] (verified)

Time = 0.48 (sec) , antiderivative size = 251, normalized size of antiderivative = 1.35, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {3042, 3696, 1598, 27, 1442, 27, 1480, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sinh^4(c+dx)}{\left(a-b\sinh^4(c+dx)\right)^2}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\sin(ic+idx)^4}{\left(a-b\sin(ic+idx)^4\right)^2}dx$$

$$\Big\downarrow \text{3696}$$

$$\frac{\int \frac{\tanh^4(c+dx)\left(1-\tanh^2(c+dx)\right)}{\left((a-b)\tanh^4(c+dx)-2a\tanh^2(c+dx)+a\right)^2}d\tanh(c+dx)}{d}$$

$$\Big\downarrow \; 1598$$

$$\frac{\frac{\int -\frac{2b\tanh^4(c+dx)}{(a-b)\tanh^4(c+dx)-2a\tanh^2(c+dx)+a}d\tanh(c+dx)}{8ab}+\frac{\tanh^5(c+dx)}{4a\left((a-b)\tanh^4(c+dx)-2a\tanh^2(c+dx)+a\right)}}{d}$$

$$\Big\downarrow \; 27$$

$$\frac{\frac{\tanh^5(c+dx)}{4a\left((a-b)\tanh^4(c+dx)-2a\tanh^2(c+dx)+a\right)}-\frac{\int \frac{\tanh^4(c+dx)}{(a-b)\tanh^4(c+dx)-2a\tanh^2(c+dx)+a}d\tanh(c+dx)}{4a}}{d}$$

$$\Big\downarrow \; 1442$$

$$\frac{\frac{\tanh^5(c+dx)}{4a\left((a-b)\tanh^4(c+dx)-2a\tanh^2(c+dx)+a\right)}-\frac{\frac{\tanh(c+dx)}{a-b}-\frac{\int \frac{a\left(1-2\tanh^2(c+dx)\right)}{(a-b)\tanh^4(c+dx)-2a\tanh^2(c+dx)+a}d\tanh(c+dx)}{a-b}}{4a}}{d}$$

$$\Big\downarrow \; 27$$

$$\frac{\frac{\tanh^5(c+dx)}{4a\left((a-b)\tanh^4(c+dx)-2a\tanh^2(c+dx)+a\right)}-\frac{\frac{\tanh(c+dx)}{a-b}-\frac{a\int \frac{1-2\tanh^2(c+dx)}{(a-b)\tanh^4(c+dx)-2a\tanh^2(c+dx)+a}d\tanh(c+dx)}{a-b}}{4a}}{d}$$

$$\Big\downarrow \; 1480$$

$$\frac{\frac{\tanh^5(c+dx)}{4a\left((a-b)\tanh^4(c+dx)-2a\tanh^2(c+dx)+a\right)}-\frac{\frac{\tanh(c+dx)}{a-b}-\frac{a\left(-\frac{1}{2}\left(2-\frac{a+b}{\sqrt{a}\sqrt{b}}\right)\int \frac{1}{(a-b)\tanh^2(c+dx)-\sqrt{a}\left(\sqrt{a}-\sqrt{b}\right)}d\tanh(c+dx)-\frac{1}{2}\left(\frac{a+b}{\sqrt{a}\sqrt{b}}+2\right)\int \frac{\cdots}{(a\cdots}\right.}{a-b}}{4a}}{d}$$

$$\Big\downarrow \; 221$$

$$\frac{\frac{\tanh^5(c+dx)}{4a\left((a-b)\tanh^4(c+dx)-2a\tanh^2(c+dx)+a\right)}-\frac{\frac{\tanh(c+dx)}{a-b}-\frac{a\left(\frac{\left(\frac{a+b}{\sqrt{a}\sqrt{b}}+2\right)\text{arctanh}\left(\frac{\sqrt{\sqrt{a}-\sqrt{b}}\tanh(c+dx)}{\sqrt[4]{a}}\right)}{2\sqrt[4]{a}\sqrt{\sqrt{a}-\sqrt{b}}\left(\sqrt{a}+\sqrt{b}\right)}+\frac{\left(2-\frac{a+b}{\sqrt{a}\sqrt{b}}\right)\text{arctanh}\left(\frac{\sqrt{\sqrt{a}+\sqrt{\cdots}}}{\cdots}\right.}{2\sqrt[4]{a}\left(\sqrt{a}-\sqrt{b}\right)\sqrt{\sqrt{a}+\cdots}}\right.}{a-b}}{4a}}{d}$$

input
```
Int[Sinh[c + d*x]^4/(a - b*Sinh[c + d*x]^4)^2,x]
```

output
```
(-1/4*(-((a*(((2 + (a + b)/(Sqrt[a]*Sqrt[b]))*ArcTanh[(Sqrt[Sqrt[a] - Sqrt
[b]]*Tanh[c + d*x])/a^(1/4)])/(2*a^(1/4)*Sqrt[Sqrt[a] - Sqrt[b]]*(Sqrt[a]
+ Sqrt[b])) + ((2 - (a + b)/(Sqrt[a]*Sqrt[b]))*ArcTanh[(Sqrt[Sqrt[a] + Sqr
t[b]]*Tanh[c + d*x])/a^(1/4)])/(2*a^(1/4)*(Sqrt[a] - Sqrt[b])*Sqrt[Sqrt[a]
 + Sqrt[b]])))/(a - b)) + Tanh[c + d*x]/(a - b))/a + Tanh[c + d*x]^5/(4*a*
(a - 2*a*Tanh[c + d*x]^2 + (a - b)*Tanh[c + d*x]^4)))/d
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 1442
```
Int[((d_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
:> Simp[d^3*(d*x)^(m - 3)*((a + b*x^2 + c*x^4)^(p + 1)/(c*(m + 4*p + 1))),
x] - Simp[d^4/(c*(m + 4*p + 1))   Int[(d*x)^(m - 4)*Simp[a*(m - 3) + b*(m +
 2*p - 1)*x^2, x]*(a + b*x^2 + c*x^4)^p, x], x] /; FreeQ[{a, b, c, d, p}, x
] && NeQ[b^2 - 4*a*c, 0] && GtQ[m, 3] && NeQ[m + 4*p + 1, 0] && IntegerQ[2*
p] && (IntegerQ[p] || IntegerQ[m])
```

rule 1480
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Simp[(e/2 + (2*c*d - b*e)/(2*q))   Int[1/(
b/2 - q/2 + c*x^2), x], x] + Simp[(e/2 - (2*c*d - b*e)/(2*q))   Int[1/(b/2
+ q/2 + c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0]
 && NeQ[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

rule 1598
```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(
x_)^4)^(p_.), x_Symbol] :> Simp[f*(f*x)^(m - 1)*(a + b*x^2 + c*x^4)^(p + 1)
*((b*d - 2*a*e - (b*e - 2*c*d)*x^2)/(2*(p + 1)*(b^2 - 4*a*c))), x] - Simp[f
^2/(2*(p + 1)*(b^2 - 4*a*c))   Int[(f*x)^(m - 2)*(a + b*x^2 + c*x^4)^(p + 1
)*Simp[(m - 1)*(b*d - 2*a*e) - (4*p + 4 + m + 1)*(b*e - 2*c*d)*x^2, x], x],
 x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] &&
 GtQ[m, 1] && IntegerQ[2*p] && (IntegerQ[p] || IntegerQ[m])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3696
```
Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^4)^(
p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Simp[ff^(m + 1
)/f   Subst[Int[x^m*((a + 2*a*ff^2*x^2 + (a + b)*ff^4*x^4)^p/(1 + ff^2*x^2)
^(m/2 + 2*p + 1)), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f}, x] &
& IntegerQ[m/2] && IntegerQ[p]
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 3.16 (sec) , antiderivative size = 263, normalized size of antiderivative = 1.41

method	result
derivativedivides	$\frac{-\frac{32\left(\frac{\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^{7}}{64a-64b}-\frac{5\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^{5}}{64(a-b)}-\frac{5\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^{3}}{64(a-b)}+\frac{\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)}{64a-64b}\right)}{\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^{8}a-4\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^{6}a+6\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^{4}a-16b\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^{4}-4\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}a+a}-\frac{\_R=\mathrm{RootOf}\left(a\_Z^{8}-4a\_\right.}{}}{d}$
default	$\frac{-\frac{32\left(\frac{\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^{7}}{64a-64b}-\frac{5\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^{5}}{64(a-b)}-\frac{5\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^{3}}{64(a-b)}+\frac{\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)}{64a-64b}\right)}{\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^{8}a-4\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^{6}a+6\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^{4}a-16b\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^{4}-4\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}a+a}-\frac{\_R=\mathrm{RootOf}\left(a\_Z^{8}-4a\_\right.}{}}{d}$
risch	$\frac{-e^{6dx+6c}b+8e^{4dx+4c}a-3be^{4dx+4c}+5e^{2dx+2c}b-b}{2b(a-b)d\left(-e^{8dx+8c}b+4e^{6dx+6c}b+16e^{4dx+4c}a-6be^{4dx+4c}+4e^{2dx+2c}b-b\right)}+\Bigg(\sum_{\_R=\mathrm{RootOf}(1+(65536a^{6}b^{2}d^{4}-1}$

input
```
int(sinh(d*x+c)^4/(a-b*sinh(d*x+c)^4)^2,x,method=_RETURNVERBOSE)
```

output
```
1/d*(-32*(1/64/(a-b)*tanh(1/2*d*x+1/2*c)^7-5/64/(a-b)*tanh(1/2*d*x+1/2*c)^
5-5/64/(a-b)*tanh(1/2*d*x+1/2*c)^3+1/64/(a-b)*tanh(1/2*d*x+1/2*c))/(tanh(1
/2*d*x+1/2*c)^8*a-4*tanh(1/2*d*x+1/2*c)^6*a+6*tanh(1/2*d*x+1/2*c)^4*a-16*b
*tanh(1/2*d*x+1/2*c)^4-4*tanh(1/2*d*x+1/2*c)^2*a+a)-1/16/(a-b)*sum((_R^6-7
*_R^4+7*_R^2-1)/(_R^7*a-3*_R^5*a+3*_R^3*a-8*_R^3*b-_R*a)*ln(tanh(1/2*d*x+1
/2*c)-_R),_R=RootOf(a*_Z^8-4*a*_Z^6+(6*a-16*b)*_Z^4-4*a*_Z^2+a)))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 5658 vs. $2(144) = 288$.

Time = 0.26 (sec) , antiderivative size = 5658, normalized size of antiderivative = 30.42

$$\int \frac{\sinh^4(c+dx)}{(a-b\sinh^4(c+dx))^2}\,dx = \text{Too large to display}$$

input
```
integrate(sinh(d*x+c)^4/(a-b*sinh(d*x+c)^4)^2,x, algorithm="fricas")
```

output
```
Too large to include
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\sinh^4(c+dx)}{(a-b\sinh^4(c+dx))^2}\,dx = \text{Timed out}$$

input
```
integrate(sinh(d*x+c)**4/(a-b*sinh(d*x+c)**4)**2,x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{\sinh^4(c+dx)}{(a-b\sinh^4(c+dx))^2}\,dx = \int \frac{\sinh(dx+c)^4}{(b\sinh(dx+c)^4-a)^2}\,dx$$

input
```
integrate(sinh(d*x+c)^4/(a-b*sinh(d*x+c)^4)^2,x, algorithm="maxima")
```

output
```
-1/2*((8*a*e^(4*c) - 3*b*e^(4*c))*e^(4*d*x) - b*e^(6*d*x + 6*c) + 5*b*e^(2
*d*x + 2*c) - b)/(a*b^2*d - b^3*d + (a*b^2*d*e^(8*c) - b^3*d*e^(8*c))*e^(8
*d*x) - 4*(a*b^2*d*e^(6*c) - b^3*d*e^(6*c))*e^(6*d*x) - 2*(8*a^2*b*d*e^(4*
c) - 11*a*b^2*d*e^(4*c) + 3*b^3*d*e^(4*c))*e^(4*d*x) - 4*(a*b^2*d*e^(2*c)
- b^3*d*e^(2*c))*e^(2*d*x)) + 1/16*integrate(16*(e^(6*d*x + 6*c) - 6*e^(4*
d*x + 4*c) + e^(2*d*x + 2*c))/(a*b - b^2 + (a*b*e^(8*c) - b^2*e^(8*c))*e^(
8*d*x) - 4*(a*b*e^(6*c) - b^2*e^(6*c))*e^(6*d*x) - 2*(8*a^2*e^(4*c) - 11*a
*b*e^(4*c) + 3*b^2*e^(4*c))*e^(4*d*x) - 4*(a*b*e^(2*c) - b^2*e^(2*c))*e^(2
*d*x)), x)
```

Giac [F]

$$\int \frac{\sinh^4(c+dx)}{(a-b\sinh^4(c+dx))^2}\,dx = \int \frac{\sinh(dx+c)^4}{(b\sinh(dx+c)^4-a)^2}\,dx$$

input
```
integrate(sinh(d*x+c)^4/(a-b*sinh(d*x+c)^4)^2,x, algorithm="giac")
```

output
```
sage0*x
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sinh^4(c+dx)}{\left(a-b\sinh^4(c+dx)\right)^2}\,dx=\int \frac{\sinh(c+d\,x)^4}{\left(a-b\sinh\left(c+d\,x\right)^4\right)^2}\,dx$$

input
```
int(sinh(c + d*x)^4/(a - b*sinh(c + d*x)^4)^2,x)
```

output
```
int(sinh(c + d*x)^4/(a - b*sinh(c + d*x)^4)^2, x)
```

Reduce [F]

$$\int \frac{\sinh^4(c+dx)}{\left(a-b\sinh^4(c+dx)\right)^2}\,dx=\text{too large to display}$$

input
```
int(sinh(d*x+c)^4/(a-b*sinh(d*x+c)^4)^2,x)
```

output
```
(4*(251658240*e**(12*c + 8*d*x)*int(e**(4*d*x)/(1920*e**(16*c + 16*d*x)*a*
*2*b**2 + 80*e**(16*c + 16*d*x)*a*b**3 - 3*e**(16*c + 16*d*x)*b**4 - 15360
*e**(14*c + 14*d*x)*a**2*b**2 - 640*e**(14*c + 14*d*x)*a*b**3 + 24*e**(14*
c + 14*d*x)*b**4 - 61440*e**(12*c + 12*d*x)*a**3*b + 51200*e**(12*c + 12*d
*x)*a**2*b**2 + 2336*e**(12*c + 12*d*x)*a*b**3 - 84*e**(12*c + 12*d*x)*b**
4 + 245760*e**(10*c + 10*d*x)*a**3*b - 97280*e**(10*c + 10*d*x)*a**2*b**2
- 4864*e**(10*c + 10*d*x)*a*b**3 + 168*e**(10*c + 10*d*x)*b**4 + 491520*e*
*(8*c + 8*d*x)*a**4 - 348160*e**(8*c + 8*d*x)*a**3*b + 118272*e**(8*c + 8*
d*x)*a**2*b**2 + 6176*e**(8*c + 8*d*x)*a*b**3 - 210*e**(8*c + 8*d*x)*b**4
+ 245760*e**(6*c + 6*d*x)*a**3*b - 97280*e**(6*c + 6*d*x)*a**2*b**2 - 4864
*e**(6*c + 6*d*x)*a*b**3 + 168*e**(6*c + 6*d*x)*b**4 - 61440*e**(4*c + 4*d
*x)*a**3*b + 51200*e**(4*c + 4*d*x)*a**2*b**2 + 2336*e**(4*c + 4*d*x)*a*b*
*3 - 84*e**(4*c + 4*d*x)*b**4 - 15360*e**(2*c + 2*d*x)*a**2*b**2 - 640*e**
(2*c + 2*d*x)*a*b**3 + 24*e**(2*c + 2*d*x)*b**4 + 1920*a**2*b**2 + 80*a*b*
*3 - 3*b**4),x)*a**6*b*d + 57671680*e**(12*c + 8*d*x)*int(e**(4*d*x)/(1920
*e**(16*c + 16*d*x)*a**2*b**2 + 80*e**(16*c + 16*d*x)*a*b**3 - 3*e**(16*c
+ 16*d*x)*b**4 - 15360*e**(14*c + 14*d*x)*a**2*b**2 - 640*e**(14*c + 14*d*
x)*a*b**3 + 24*e**(14*c + 14*d*x)*b**4 - 61440*e**(12*c + 12*d*x)*a**3*b +
 51200*e**(12*c + 12*d*x)*a**2*b**2 + 2336*e**(12*c + 12*d*x)*a*b**3 - 84*
e**(12*c + 12*d*x)*b**4 + 245760*e**(10*c + 10*d*x)*a**3*b - 97280*e**(...
```

3.225 $\int \frac{\sinh^2(c+dx)}{\left(a-b\sinh^4(c+dx)\right)^2}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 220

$$\int \frac{\sinh^2(c+dx)}{\left(a-b\sinh^4(c+dx)\right)^2}\,dx = -\frac{\left(2\sqrt{a}-\sqrt{b}\right)\operatorname{arctanh}\left(\frac{\sqrt{\sqrt{a}-\sqrt{b}}\tanh(c+dx)}{\sqrt[4]{a}}\right)}{8a^{5/4}\left(\sqrt{a}-\sqrt{b}\right)^{3/2}\sqrt{b}d} + \frac{\left(2\sqrt{a}+\sqrt{b}\right)\operatorname{arctanh}\left(\frac{\sqrt{\sqrt{a}+\sqrt{b}}\tanh(c+dx)}{\sqrt[4]{a}}\right)}{8a^{5/4}\left(\sqrt{a}+\sqrt{b}\right)^{3/2}\sqrt{b}d} + \frac{\tanh(c+dx)\left(a-(a+b)\tanh^2(c+dx)\right)}{4a(a-b)d\left(a-2a\tanh^2(c+dx)+(a-b)\tanh^4(c+dx)\right)}$$

output
```
-1/8*(2*a^(1/2)-b^(1/2))*arctanh((a^(1/2)-b^(1/2))^(1/2)*tanh(d*x+c)/a^(1/
4))/a^(5/4)/(a^(1/2)-b^(1/2))^(3/2)/b^(1/2)/d+1/8*(2*a^(1/2)+b^(1/2))*arct
anh((a^(1/2)+b^(1/2))^(1/2)*tanh(d*x+c)/a^(1/4))/a^(5/4)/(a^(1/2)+b^(1/2))
^(3/2)/b^(1/2)/d+1/4*tanh(d*x+c)*(a-(a+b)*tanh(d*x+c)^2)/a/(a-b)/d/(a-2*a*
tanh(d*x+c)^2+(a-b)*tanh(d*x+c)^4)
```

Mathematica [A] (verified)

Time = 1.81 (sec) , antiderivative size = 253, normalized size of antiderivative = 1.15

$$\int \frac{\sinh^2(c+dx)}{(a-b\sinh^4(c+dx))^2}\,dx$$
$$=\frac{\frac{\sqrt{a}\left(2a+\sqrt{a}\sqrt{b}-b\right)\arctan\left(\frac{\left(\sqrt{a}-\sqrt{b}\right)\tanh(c+dx)}{\sqrt{-a+\sqrt{a}\sqrt{b}}}\right)}{\sqrt{-a+\sqrt{a}\sqrt{b}}\sqrt{b}}+\frac{\sqrt{a}\left(2a-\sqrt{a}\sqrt{b}-b\right)\operatorname{arctanh}\left(\frac{\left(\sqrt{a}+\sqrt{b}\right)\tanh(c+dx)}{\sqrt{a+\sqrt{a}\sqrt{b}}}\right)}{\sqrt{a+\sqrt{a}\sqrt{b}}\sqrt{b}}+\frac{4\sqrt{a}(2a+b-b\cosh(2(c+dx))}{8a-3b+4b\cosh(2(c+dx))-}}{8a^{3/2}(a-b)d}$$

input
```
Integrate[Sinh[c + d*x]^2/(a - b*Sinh[c + d*x]^4)^2,x]
```

output
```
((Sqrt[a]*(2*a + Sqrt[a]*Sqrt[b] - b)*ArcTan[((Sqrt[a] - Sqrt[b])*Tanh[c +
 d*x])/Sqrt[-a + Sqrt[a]*Sqrt[b]]])/(Sqrt[-a + Sqrt[a]*Sqrt[b]]*Sqrt[b]) +
 (Sqrt[a]*(2*a - Sqrt[a]*Sqrt[b] - b)*ArcTanh[((Sqrt[a] + Sqrt[b])*Tanh[c
+ d*x])/Sqrt[a + Sqrt[a]*Sqrt[b]]])/(Sqrt[a + Sqrt[a]*Sqrt[b]]*Sqrt[b]) +
(4*Sqrt[a]*(2*a + b - b*Cosh[2*(c + d*x)])*Sinh[2*(c + d*x)])/(8*a - 3*b +
 4*b*Cosh[2*(c + d*x)] - b*Cosh[4*(c + d*x)]))/(8*a^(3/2)*(a - b)*d)
```

Rubi [A] (verified)

Time = 0.50 (sec) , antiderivative size = 258, normalized size of antiderivative = 1.17, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.292, Rules used = {3042, 25, 3696, 1672, 27, 1480, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sinh^2(c+dx)}{(a-b\sinh^4(c+dx))^2}\,dx$$

$$\Big\downarrow \; 3042$$

$$\int -\frac{\sin(ic+idx)^2}{(a-b\sin(ic+idx)^4)^2}dx$$

$$\Big\downarrow \; 25$$

$$-\int \frac{\sin(ic+idx)^2}{\left(a-b\sin(ic+idx)^4\right)^2}dx$$

$\Big\downarrow$ 3696

$$\frac{\int \frac{\tanh^2(c+dx)\left(1-\tanh^2(c+dx)\right)^2}{\left((a-b)\tanh^4(c+dx)-2a\tanh^2(c+dx)+a\right)^2}d\tanh(c+dx)}{d}$$

$\Big\downarrow$ 1672

$$\frac{\frac{\tanh(c+dx)\left(a-(a+b)\tanh^2(c+dx)\right)}{4a(a-b)\left((a-b)\tanh^4(c+dx)-2a\tanh^2(c+dx)+a\right)}-\frac{\int \frac{2ab\left(a-(3a-b)\tanh^2(c+dx)\right)}{(a-b)\left((a-b)\tanh^4(c+dx)-2a\tanh^2(c+dx)+a\right)}d\tanh(c+dx)}{8a^2b}}{d}$$

$\Big\downarrow$ 27

$$\frac{\frac{\tanh(c+dx)\left(a-(a+b)\tanh^2(c+dx)\right)}{4a(a-b)\left((a-b)\tanh^4(c+dx)-2a\tanh^2(c+dx)+a\right)}-\frac{\int \frac{a-(3a-b)\tanh^2(c+dx)}{(a-b)\tanh^4(c+dx)-2a\tanh^2(c+dx)+a}d\tanh(c+dx)}{4a(a-b)}}{d}$$

$\Big\downarrow$ 1480

$$\frac{\frac{\tanh(c+dx)\left(a-(a+b)\tanh^2(c+dx)\right)}{4a(a-b)\left((a-b)\tanh^4(c+dx)-2a\tanh^2(c+dx)+a\right)}-\frac{-\frac{1}{2}\left(-\frac{2a^{3/2}}{\sqrt{b}}+3a-b\right)\int \frac{1}{(a-b)\tanh^2(c+dx)-\sqrt{a}\left(\sqrt{a}-\sqrt{b}\right)}d\tanh(c+dx)-\frac{1}{2}\left(\frac{2a^{3/2}}{\sqrt{b}}+3a-b\right)}{4a(a-b)}}{d}$$

$\Big\downarrow$ 221

$$\frac{\frac{\tanh(c+dx)\left(a-(a+b)\tanh^2(c+dx)\right)}{4a(a-b)\left((a-b)\tanh^4(c+dx)-2a\tanh^2(c+dx)+a\right)}-\frac{\frac{\left(\frac{2a^{3/2}}{\sqrt{b}}+3a-b\right)\text{arctanh}\left(\frac{\sqrt{\sqrt{a}-\sqrt{b}}\tanh(c+dx)}{\sqrt[4]{a}}\right)}{2\sqrt[4]{a}\sqrt{\sqrt{a}-\sqrt{b}}\left(\sqrt{a}+\sqrt{b}\right)}+\frac{\left(-\frac{2a^{3/2}}{\sqrt{b}}+3a-b\right)\text{arctanh}\left(\frac{\sqrt{\sqrt{a}+\sqrt{b}}}{}\right.}{2\sqrt[4]{a}\left(\sqrt{a}-\sqrt{b}\right)\sqrt{\sqrt{a}+\sqrt{b}}}}{4a(a-b)}}{d}$$

input
```
Int[Sinh[c + d*x]^2/(a - b*Sinh[c + d*x]^4)^2,x]
```

output
```
(-1/4*(((3*a + (2*a^(3/2))/Sqrt[b] - b)*ArcTanh[(Sqrt[Sqrt[a] - Sqrt[b]]*T
anh[c + d*x])/a^(1/4)])/(2*a^(1/4)*Sqrt[Sqrt[a] - Sqrt[b]]*(Sqrt[a] + Sqrt
[b])) + ((3*a - (2*a^(3/2))/Sqrt[b] - b)*ArcTanh[(Sqrt[Sqrt[a] + Sqrt[b]]*
Tanh[c + d*x])/a^(1/4)])/(2*a^(1/4)*(Sqrt[a] - Sqrt[b])*Sqrt[Sqrt[a] + Sqr
t[b]]))/(a*(a - b)) + (Tanh[c + d*x]*(a - (a + b)*Tanh[c + d*x]^2))/(4*a*(
a - b)*(a - 2*a*Tanh[c + d*x]^2 + (a - b)*Tanh[c + d*x]^4)))/d
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 1480
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Simp[(e/2 + (2*c*d - b*e)/(2*q))   Int[1/(
b/2 - q/2 + c*x^2), x], x] + Simp[(e/2 - (2*c*d - b*e)/(2*q))   Int[1/(b/2
+ q/2 + c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0]
 && NeQ[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

rule 1672
```
Int[(x_)^(m_)*((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^
4)^(p_), x_Symbol] :> With[{f = Coeff[PolynomialRemainder[x^m*(d + e*x^2)^q
, a + b*x^2 + c*x^4, x], x, 0], g = Coeff[PolynomialRemainder[x^m*(d + e*x^
2)^q, a + b*x^2 + c*x^4, x], x, 2]}, Simp[x*(a + b*x^2 + c*x^4)^(p + 1)*((a
*b*g - f*(b^2 - 2*a*c) - c*(b*f - 2*a*g)*x^2)/(2*a*(p + 1)*(b^2 - 4*a*c))),
 x] + Simp[1/(2*a*(p + 1)*(b^2 - 4*a*c))   Int[(a + b*x^2 + c*x^4)^(p + 1)*
Simp[ExpandToSum[2*a*(p + 1)*(b^2 - 4*a*c)*PolynomialQuotient[x^m*(d + e*x^
2)^q, a + b*x^2 + c*x^4, x] + b^2*f*(2*p + 3) - 2*a*c*f*(4*p + 5) - a*b*g +
 c*(4*p + 7)*(b*f - 2*a*g)*x^2, x], x], x], x]] /; FreeQ[{a, b, c, d, e}, x
] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && IGtQ[q, 1] && IGtQ[m/2, 0]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3696
```
Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^4)^(
p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Simp[ff^(m + 1
)/f   Subst[Int[x^m*((a + 2*a*ff^2*x^2 + (a + b)*ff^4*x^4)^p/(1 + ff^2*x^2)
^(m/2 + 2*p + 1)), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f}, x] &
& IntegerQ[m/2] && IntegerQ[p]
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 3.38 (sec) , antiderivative size = 297, normalized size of antiderivative = 1.35

method	result
derivativedivides	$\frac{-\frac{8\left(-\frac{\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^7}{16(a-b)}+\frac{(a+4b)\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^5}{16a(a-b)}+\frac{(a+4b)\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^3}{16a(a-b)}-\frac{\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)}{16(a-b)}\right)}{\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^8a-4\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^6a+6\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^4a-16b\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^4-4\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^2a+a}-\frac{\_R=\text{RootOf}\left(a\_Z^8-4a\_\right.}{}}{d}$
default	$\frac{-\frac{8\left(-\frac{\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^7}{16(a-b)}+\frac{(a+4b)\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^5}{16a(a-b)}+\frac{(a+4b)\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^3}{16a(a-b)}-\frac{\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)}{16(a-b)}\right)}{\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^8a-4\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^6a+6\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^4a-16b\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^4-4\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^2a+a}-\frac{\_R=\text{RootOf}\left(a\_Z^8-4a\_\right.}{}}{d}$
risch	$\frac{2\,e^{6dx+6c}a-e^{6dx+6c}b-8\,e^{4dx+4c}a+3b\,e^{4dx+4c}-2\,e^{2dx+2c}a-3\,e^{2dx+2c}b+b}{2ad(a-b)\left(-e^{8dx+8c}b+4\,e^{6dx+6c}b+16\,e^{4dx+4c}a-6b\,e^{4dx+4c}+4\,e^{2dx+2c}b-b\right)}+\Bigg(\_R=\text{RootOf}\left((65536a^8b^2d^4-1966\right.$

input
```
int(sinh(d*x+c)^2/(a-b*sinh(d*x+c)^4)^2,x,method=_RETURNVERBOSE)
```

output
```
1/d*(-8*(-1/16/(a-b)*tanh(1/2*d*x+1/2*c)^7+1/16*(a+4*b)/a/(a-b)*tanh(1/2*d
*x+1/2*c)^5+1/16*(a+4*b)/a/(a-b)*tanh(1/2*d*x+1/2*c)^3-1/16/(a-b)*tanh(1/2
*d*x+1/2*c))/(tanh(1/2*d*x+1/2*c)^8*a-4*tanh(1/2*d*x+1/2*c)^6*a+6*tanh(1/2
*d*x+1/2*c)^4*a-16*b*tanh(1/2*d*x+1/2*c)^4-4*tanh(1/2*d*x+1/2*c)^2*a+a)-1/
16/a/(a-b)*sum((-a*_R^6+(11*a-4*b)*_R^4+(-11*a+4*b)*_R^2+a)/(_R^7*a-3*_R^5
*a+3*_R^3*a-8*_R^3*b-_R*a)*ln(tanh(1/2*d*x+1/2*c)-_R),_R=RootOf(a*_Z^8-4*a
*_Z^6+(6*a-16*b)*_Z^4-4*a*_Z^2+a))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 6525 vs. $2(171) = 342$.

Time = 0.48 (sec) , antiderivative size = 6525, normalized size of antiderivative = 29.66

$$\int \frac{\sinh^2(c+dx)}{(a-b\sinh^4(c+dx))^2}\,dx = \text{Too large to display}$$

input
```
integrate(sinh(d*x+c)^2/(a-b*sinh(d*x+c)^4)^2,x, algorithm="fricas")
```

output
```
Too large to include
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\sinh^2(c+dx)}{(a-b\sinh^4(c+dx))^2}\,dx = \text{Timed out}$$

input
```
integrate(sinh(d*x+c)**2/(a-b*sinh(d*x+c)**4)**2,x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{\sinh^2(c+dx)}{(a-b\sinh^4(c+dx))^2}\,dx = \int \frac{\sinh(dx+c)^2}{(b\sinh(dx+c)^4-a)^2}\,dx$$

input
```
integrate(sinh(d*x+c)^2/(a-b*sinh(d*x+c)^4)^2,x, algorithm="maxima")
```

output
```
-1/2*((2*a*e^(6*c) - b*e^(6*c))*e^(6*d*x) - (8*a*e^(4*c) - 3*b*e^(4*c))*e^
(4*d*x) - (2*a*e^(2*c) + 3*b*e^(2*c))*e^(2*d*x) + b)/(a^2*b*d - a*b^2*d +
(a^2*b*d*e^(8*c) - a*b^2*d*e^(8*c))*e^(8*d*x) - 4*(a^2*b*d*e^(6*c) - a*b^2
*d*e^(6*c))*e^(6*d*x) - 2*(8*a^3*d*e^(4*c) - 11*a^2*b*d*e^(4*c) + 3*a*b^2*
d*e^(4*c))*e^(4*d*x) - 4*(a^2*b*d*e^(2*c) - a*b^2*d*e^(2*c))*e^(2*d*x)) -
1/4*integrate(4*((2*a*e^(6*c) - b*e^(6*c))*e^(6*d*x) - 2*(4*a*e^(4*c) - b*
e^(4*c))*e^(4*d*x) + (2*a*e^(2*c) - b*e^(2*c))*e^(2*d*x))/(a^2*b - a*b^2 +
 (a^2*b*e^(8*c) - a*b^2*e^(8*c))*e^(8*d*x) - 4*(a^2*b*e^(6*c) - a*b^2*e^(6
*c))*e^(6*d*x) - 2*(8*a^3*e^(4*c) - 11*a^2*b*e^(4*c) + 3*a*b^2*e^(4*c))*e^
(4*d*x) - 4*(a^2*b*e^(2*c) - a*b^2*e^(2*c))*e^(2*d*x)), x)
```

Giac [F]

$$\int \frac{\sinh^2(c+dx)}{\left(a-b\sinh^4(c+dx)\right)^2}\,dx=\int \frac{\sinh\left(dx+c\right)^2}{\left(b\sinh\left(dx+c\right)^4-a\right)^2}\,dx$$

input
```
integrate(sinh(d*x+c)^2/(a-b*sinh(d*x+c)^4)^2,x, algorithm="giac")
```

output
```
sage0*x
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sinh^2(c+dx)}{\left(a-b\sinh^4(c+dx)\right)^2}\,dx=\int \frac{\sinh(c+d\,x)^2}{\left(a-b\,\sinh\left(c+d\,x\right)^4\right)^2}\,dx$$

input
```
int(sinh(c + d*x)^2/(a - b*sinh(c + d*x)^4)^2,x)
```

output
```
int(sinh(c + d*x)^2/(a - b*sinh(c + d*x)^4)^2, x)
```

Reduce [F]

$$\int \frac{\sinh^2(c+dx)}{\left(a-b\sinh^4(c+dx)\right)^2}\,dx = \text{too large to display}$$

input

```
int(sinh(d*x+c)^2/(a-b*sinh(d*x+c)^4)^2,x)
```

output

```
(8*( - 251658240*e**(12*c + 8*d*x)*int(e**(4*d*x)/(1920*e**(16*c + 16*d*x)
*a**2*b**2 + 80*e**(16*c + 16*d*x)*a*b**3 - 3*e**(16*c + 16*d*x)*b**4 - 15
360*e**(14*c + 14*d*x)*a**2*b**2 - 640*e**(14*c + 14*d*x)*a*b**3 + 24*e**(
14*c + 14*d*x)*b**4 - 61440*e**(12*c + 12*d*x)*a**3*b + 51200*e**(12*c + 1
2*d*x)*a**2*b**2 + 2336*e**(12*c + 12*d*x)*a*b**3 - 84*e**(12*c + 12*d*x)*
b**4 + 245760*e**(10*c + 10*d*x)*a**3*b - 97280*e**(10*c + 10*d*x)*a**2*b*
*2 - 4864*e**(10*c + 10*d*x)*a*b**3 + 168*e**(10*c + 10*d*x)*b**4 + 491520
*e**(8*c + 8*d*x)*a**4 - 348160*e**(8*c + 8*d*x)*a**3*b + 118272*e**(8*c +
 8*d*x)*a**2*b**2 + 6176*e**(8*c + 8*d*x)*a*b**3 - 210*e**(8*c + 8*d*x)*b*
*4 + 245760*e**(6*c + 6*d*x)*a**3*b - 97280*e**(6*c + 6*d*x)*a**2*b**2 - 4
864*e**(6*c + 6*d*x)*a*b**3 + 168*e**(6*c + 6*d*x)*b**4 - 61440*e**(4*c +
4*d*x)*a**3*b + 51200*e**(4*c + 4*d*x)*a**2*b**2 + 2336*e**(4*c + 4*d*x)*a
*b**3 - 84*e**(4*c + 4*d*x)*b**4 - 15360*e**(2*c + 2*d*x)*a**2*b**2 - 640*
e**(2*c + 2*d*x)*a*b**3 + 24*e**(2*c + 2*d*x)*b**4 + 1920*a**2*b**2 + 80*a
*b**3 - 3*b**4),x)*a**6*b*d + 131072000*e**(12*c + 8*d*x)*int(e**(4*d*x)/(
1920*e**(16*c + 16*d*x)*a**2*b**2 + 80*e**(16*c + 16*d*x)*a*b**3 - 3*e**(1
6*c + 16*d*x)*b**4 - 15360*e**(14*c + 14*d*x)*a**2*b**2 - 640*e**(14*c + 1
4*d*x)*a*b**3 + 24*e**(14*c + 14*d*x)*b**4 - 61440*e**(12*c + 12*d*x)*a**3
*b + 51200*e**(12*c + 12*d*x)*a**2*b**2 + 2336*e**(12*c + 12*d*x)*a*b**3 -
 84*e**(12*c + 12*d*x)*b**4 + 245760*e**(10*c + 10*d*x)*a**3*b - 97280*...
```

3.226 $\int \frac{1}{\left(a-b\sinh^4(c+dx)\right)^2}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 210

$$\int \frac{1}{\left(a-b\sinh^4(c+dx)\right)^2}\,dx = \frac{\left(4\sqrt{a}-3\sqrt{b}\right)\operatorname{arctanh}\left(\frac{\sqrt{\sqrt{a}-\sqrt{b}}\tanh(c+dx)}{\sqrt[4]{a}}\right)}{8a^{7/4}\left(\sqrt{a}-\sqrt{b}\right)^{3/2}d} + \frac{\left(4\sqrt{a}+3\sqrt{b}\right)\operatorname{arctanh}\left(\frac{\sqrt{\sqrt{a}+\sqrt{b}}\tanh(c+dx)}{\sqrt[4]{a}}\right)}{8a^{7/4}\left(\sqrt{a}+\sqrt{b}\right)^{3/2}d} - \frac{b\tanh(c+dx)\left(1-2\tanh^2(c+dx)\right)}{4a(a-b)d\left(a-2a\tanh^2(c+dx)+(a-b)\tanh^4(c+dx)\right)}$$

output
```
1/8*(4*a^(1/2)-3*b^(1/2))*arctanh((a^(1/2)-b^(1/2))^(1/2)*tanh(d*x+c)/a^(1
/4))/a^(7/4)/(a^(1/2)-b^(1/2))^(3/2)/d+1/8*(4*a^(1/2)+3*b^(1/2))*arctanh((
a^(1/2)+b^(1/2))^(1/2)*tanh(d*x+c)/a^(1/4))/a^(7/4)/(a^(1/2)+b^(1/2))^(3/2
)/d-1/4*b*tanh(d*x+c)*(1-2*tanh(d*x+c)^2)/a/(a-b)/d/(a-2*a*tanh(d*x+c)^2+(
a-b)*tanh(d*x+c)^4)
```

Mathematica [A] (verified)

Time = 7.02 (sec) , antiderivative size = 230, normalized size of antiderivative = 1.10

$$\int \frac{1}{(a-b\sinh^4(c+dx))^2}\,dx = \frac{-\frac{\left(4a+\sqrt{a}\sqrt{b}-3b\right)\arctan\left(\frac{\left(\sqrt{a}-\sqrt{b}\right)\tanh(c+dx)}{\sqrt{-a+\sqrt{a}\sqrt{b}}}\right)}{\sqrt{-a+\sqrt{a}\sqrt{b}}}+\frac{\left(4a-\sqrt{a}\sqrt{b}-3b\right)\operatorname{arctanh}\left(\frac{\left(\sqrt{a}+\sqrt{b}\right)\tanh(c+dx)}{\sqrt{a+\sqrt{a}\sqrt{b}}}\right)}{\sqrt{a+\sqrt{a}\sqrt{b}}}+\frac{2\sqrt{a}b(-6\sinh(2(c+dx))+\sin}{8a-3b+4b\cosh(2(c+dx))-b\,c}}{8a^{3/2}(a-b)d}$$

input
```
Integrate[(a - b*Sinh[c + d*x]^4)^(-2),x]
```

output
```
(-(((4*a + Sqrt[a]*Sqrt[b] - 3*b)*ArcTan[((Sqrt[a] - Sqrt[b])*Tanh[c + d*x
])/Sqrt[-a + Sqrt[a]*Sqrt[b]]])/Sqrt[-a + Sqrt[a]*Sqrt[b]]) + ((4*a - Sqrt
[a]*Sqrt[b] - 3*b)*ArcTanh[((Sqrt[a] + Sqrt[b])*Tanh[c + d*x])/Sqrt[a + Sq
rt[a]*Sqrt[b]]])/Sqrt[a + Sqrt[a]*Sqrt[b]] + (2*Sqrt[a]*b*(-6*Sinh[2*(c +
d*x)] + Sinh[4*(c + d*x)]))/(8*a - 3*b + 4*b*Cosh[2*(c + d*x)] - b*Cosh[4*
(c + d*x)]))/(8*a^(3/2)*(a - b)*d)
```

Rubi [A] (verified)

Time = 0.53 (sec) , antiderivative size = 261, normalized size of antiderivative = 1.24, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.400$, Rules used = {3042, 3688, 1517, 27, 1480, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(a-b\sinh^4(c+dx))^2}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{(a-b\sin(ic+idx)^4)^2}dx$$

$$\Big\downarrow \text{3688}$$

$$\frac{\int \frac{\left(1-\tanh^2(c+dx)\right)^3}{\left((a-b)\tanh^4(c+dx)-2a\tanh^2(c+dx)+a\right)^2}d\tanh(c+dx)}{d}$$

$\Big\downarrow$ 1517

$$\frac{-\frac{\int -\frac{2ab\left(-2(2a-b)\tanh^2(c+dx)+4a-3b\right)}{(a-b)\left((a-b)\tanh^4(c+dx)-2a\tanh^2(c+dx)+a\right)}d\tanh(c+dx)}{8a^2b}-\frac{b\tanh(c+dx)\left(1-2\tanh^2(c+dx)\right)}{4a(a-b)\left((a-b)\tanh^4(c+dx)-2a\tanh^2(c+dx)+a\right)}}{d}$$

$\Big\downarrow$ 27

$$\frac{\frac{\int \frac{-2(2a-b)\tanh^2(c+dx)+4a-3b}{(a-b)\tanh^4(c+dx)-2a\tanh^2(c+dx)+a}d\tanh(c+dx)}{4a(a-b)}-\frac{b\tanh(c+dx)\left(1-2\tanh^2(c+dx)\right)}{4a(a-b)\left((a-b)\tanh^4(c+dx)-2a\tanh^2(c+dx)+a\right)}}{d}$$

$\Big\downarrow$ 1480

$$\frac{\frac{-\frac{\left(4\sqrt{a}-3\sqrt{b}\right)\left(\sqrt{a}+\sqrt{b}\right)^2\int \frac{1}{(a-b)\tanh^2(c+dx)-\sqrt{a}\left(\sqrt{a}+\sqrt{b}\right)}d\tanh(c+dx)}{2\sqrt{a}}-\frac{\left(4\sqrt{a}+3\sqrt{b}\right)\left(-2\sqrt{a}\sqrt{b}+a+b\right)\int \frac{1}{(a-b)\tanh^2(c+dx)-\sqrt{a}\left(\sqrt{a}-\sqrt{b}\right)}d\tanh(c+dx)}{2\sqrt{a}}}{4a(a-b)}}{d}$$

$\Big\downarrow$ 221

$$\frac{\frac{\frac{\left(4\sqrt{a}-3\sqrt{b}\right)\left(\sqrt{a}+\sqrt{b}\right)\text{arctanh}\left(\frac{\sqrt{\sqrt{a}-\sqrt{b}}\tanh(c+dx)}{\sqrt[4]{a}}\right)}{2a^{3/4}\sqrt{\sqrt{a}-\sqrt{b}}}+\frac{\left(4\sqrt{a}+3\sqrt{b}\right)\left(-2\sqrt{a}\sqrt{b}+a+b\right)\text{arctanh}\left(\frac{\sqrt{\sqrt{a}+\sqrt{b}}\tanh(c+dx)}{\sqrt[4]{a}}\right)}{2a^{3/4}\left(\sqrt{a}-\sqrt{b}\right)\sqrt{\sqrt{a}+\sqrt{b}}}}{4a(a-b)}-\frac{b\tanh(c+dx)}{4a(a-b)\left((a-b)\tanh^4\right.}}{d}$$

input
```
Int[(a - b*Sinh[c + d*x]^4)^(-2),x]
```

output
```
((((4*Sqrt[a] - 3*Sqrt[b])*(Sqrt[a] + Sqrt[b])*ArcTanh[(Sqrt[Sqrt[a] - Sqr
t[b]]*Tanh[c + d*x])/a^(1/4)])/(2*a^(3/4)*Sqrt[Sqrt[a] - Sqrt[b]]) + ((4*S
qrt[a] + 3*Sqrt[b])*(a - 2*Sqrt[a]*Sqrt[b] + b)*ArcTanh[(Sqrt[Sqrt[a] + Sq
rt[b]]*Tanh[c + d*x])/a^(1/4)])/(2*a^(3/4)*(Sqrt[a] - Sqrt[b])*Sqrt[Sqrt[a
] + Sqrt[b]]))/(4*a*(a - b)) - (b*Tanh[c + d*x]*(1 - 2*Tanh[c + d*x]^2))/(
4*a*(a - b)*(a - 2*a*Tanh[c + d*x]^2 + (a - b)*Tanh[c + d*x]^4)))/d
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 1480
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Simp[(e/2 + (2*c*d - b*e)/(2*q))   Int[1/(
b/2 - q/2 + c*x^2), x], x] + Simp[(e/2 - (2*c*d - b*e)/(2*q))   Int[1/(b/2
+ q/2 + c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0]
 && NeQ[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

rule 1517
```
Int[((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x
_Symbol] :> With[{f = Coeff[PolynomialRemainder[(d + e*x^2)^q, a + b*x^2 +
c*x^4, x], x, 0], g = Coeff[PolynomialRemainder[(d + e*x^2)^q, a + b*x^2 +
c*x^4, x], x, 2]}, Simp[x*(a + b*x^2 + c*x^4)^(p + 1)*((a*b*g - f*(b^2 - 2*
a*c) - c*(b*f - 2*a*g)*x^2)/(2*a*(p + 1)*(b^2 - 4*a*c))), x] + Simp[1/(2*a*
(p + 1)*(b^2 - 4*a*c))   Int[(a + b*x^2 + c*x^4)^(p + 1)*ExpandToSum[2*a*(p
 + 1)*(b^2 - 4*a*c)*PolynomialQuotient[(d + e*x^2)^q, a + b*x^2 + c*x^4, x]
 + b^2*f*(2*p + 3) - 2*a*c*f*(4*p + 5) - a*b*g + c*(4*p + 7)*(b*f - 2*a*g)*
x^2, x], x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ
[c*d^2 - b*d*e + a*e^2, 0] && IGtQ[q, 1] && LtQ[p, -1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3688
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^4)^(p_.), x_Symbol] :> With[{ff =
 FreeFactors[Tan[e + f*x], x]}, Simp[ff/f   Subst[Int[(a + 2*a*ff^2*x^2 + (
a + b)*ff^4*x^4)^p/(1 + ff^2*x^2)^(2*p + 1), x], x, Tan[e + f*x]/ff], x]] /
; FreeQ[{a, b, e, f}, x] && IntegerQ[p]
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 3.15 (sec) , antiderivative size = 307, normalized size of antiderivative = 1.46

method	result
derivativedivides	$\frac{-\frac{2\left(\frac{b\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^{7}}{4a(a-b)}-\frac{5b\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^{5}}{4a(a-b)}-\frac{5b\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^{3}}{4a(a-b)}+\frac{b\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)}{4a(a-b)}\right)}{\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^{8}a-4\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^{6}a+6\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^{4}a-16b\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^{4}-4\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}a+a}-\frac{\_R=\mathrm{RootOf}\left(a\_Z^{8}-4a\_\right.}{}}{d}$
default	$\frac{-\frac{2\left(\frac{b\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^{7}}{4a(a-b)}-\frac{5b\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^{5}}{4a(a-b)}-\frac{5b\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^{3}}{4a(a-b)}+\frac{b\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)}{4a(a-b)}\right)}{\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^{8}a-4\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^{6}a+6\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^{4}a-16b\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^{4}-4\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}a+a}-\frac{\_R=\mathrm{RootOf}\left(a\_Z^{8}-4a\_\right.}{}}{d}$
risch	$\frac{-e^{6dx+6c}b+8\,e^{4dx+4c}a-3b\,e^{4dx+4c}+5\,e^{2dx+2c}b-b}{2ad(a-b)\left(-e^{8dx+8c}b+4\,e^{6dx+6c}b+16\,e^{4dx+4c}a-6b\,e^{4dx+4c}+4\,e^{2dx+2c}b-b\right)}+\left(\sum_{\_R=\mathrm{RootOf}\left(\left(65536a^{10}d^{4}-19660\right.\right.}\right.$

input
```
int(1/(a-b*sinh(d*x+c)^4)^2,x,method=_RETURNVERBOSE)
```

output
```
1/d*(-2*(1/4*b/a/(a-b)*tanh(1/2*d*x+1/2*c)^7-5/4*b/a/(a-b)*tanh(1/2*d*x+1/
2*c)^5-5/4*b/a/(a-b)*tanh(1/2*d*x+1/2*c)^3+1/4*b/a/(a-b)*tanh(1/2*d*x+1/2*
c))/(tanh(1/2*d*x+1/2*c)^8*a-4*tanh(1/2*d*x+1/2*c)^6*a+6*tanh(1/2*d*x+1/2*
c)^4*a-16*b*tanh(1/2*d*x+1/2*c)^4-4*tanh(1/2*d*x+1/2*c)^2*a+a)-1/16/a/(a-b
)*sum(((4*a-3*b)*_R^6+(-12*a+5*b)*_R^4+(12*a-5*b)*_R^2-4*a+3*b)/(_R^7*a-3*
_R^5*a+3*_R^3*a-8*_R^3*b-_R*a)*ln(tanh(1/2*d*x+1/2*c)-_R),_R=RootOf(a*_Z^8
-4*a*_Z^6+(6*a-16*b)*_Z^4-4*a*_Z^2+a)))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 6522 vs. 2(164) = 328.

Time = 0.40 (sec) , antiderivative size = 6522, normalized size of antiderivative = 31.06

$$\int \frac{1}{\left(a-b\sinh^4(c+dx)\right)^2}\,dx = \text{Too large to display}$$

input
```
integrate(1/(a-b*sinh(d*x+c)^4)^2,x, algorithm="fricas")
```

output
```
Too large to include
```

Sympy [F(-1)]

Timed out.

$$\int \frac{1}{\left(a-b\sinh^4(c+dx)\right)^2}\,dx=\text{Timed out}$$

input
```
integrate(1/(a-b*sinh(d*x+c)**4)**2,x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{1}{\left(a-b\sinh^4(c+dx)\right)^2}\,dx=\int \frac{1}{\left(b\sinh\left(dx+c\right)^4-a\right)^2}\,dx$$

input
```
integrate(1/(a-b*sinh(d*x+c)^4)^2,x, algorithm="maxima")
```

output
```
-1/2*((8*a*e^(4*c) - 3*b*e^(4*c))*e^(4*d*x) - b*e^(6*d*x + 6*c) + 5*b*e^(2
*d*x + 2*c) - b)/(a^2*b*d - a*b^2*d + (a^2*b*d*e^(8*c) - a*b^2*d*e^(8*c))*
e^(8*d*x) - 4*(a^2*b*d*e^(6*c) - a*b^2*d*e^(6*c))*e^(6*d*x) - 2*(8*a^3*d*e
^(4*c) - 11*a^2*b*d*e^(4*c) + 3*a*b^2*d*e^(4*c))*e^(4*d*x) - 4*(a^2*b*d*e^
(2*c) - a*b^2*d*e^(2*c))*e^(2*d*x)) + integrate(-(2*(8*a*e^(4*c) - 5*b*e^(
4*c))*e^(4*d*x) - b*e^(6*d*x + 6*c) - b*e^(2*d*x + 2*c))/(a^2*b - a*b^2 +
(a^2*b*e^(8*c) - a*b^2*e^(8*c))*e^(8*d*x) - 4*(a^2*b*e^(6*c) - a*b^2*e^(6*
c))*e^(6*d*x) - 2*(8*a^3*e^(4*c) - 11*a^2*b*e^(4*c) + 3*a*b^2*e^(4*c))*e^(
4*d*x) - 4*(a^2*b*e^(2*c) - a*b^2*e^(2*c))*e^(2*d*x)), x)
```

Giac [F]

$$\int \frac{1}{\left(a-b\sinh^4(c+dx)\right)^2}\,dx=\int \frac{1}{\left(b\sinh\left(dx+c\right)^4-a\right)^2}\,dx$$

input
```
integrate(1/(a-b*sinh(d*x+c)^4)^2,x, algorithm="giac")
```

output
```
sage0*x
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\left(a-b\sinh^4(c+dx)\right)^2}\,dx=\int \frac{1}{\left(a-b\sinh\left(c+d\,x\right)^4\right)^2}\,dx$$

input
```
int(1/(a - b*sinh(c + d*x)^4)^2,x)
```

output
```
int(1/(a - b*sinh(c + d*x)^4)^2, x)
```

Reduce [F]

$$\int \frac{1}{\left(a-b\sinh^4(c+dx)\right)^2}\,dx=\text{too large to display}$$

input
```
int(1/(a-b*sinh(d*x+c)^4)^2,x)
```

output
```
(32*(94371840*e**(12*c + 8*d*x)*int(e**(4*d*x)/(1920*e**(16*c + 16*d*x)*a*
*2*b**2 + 80*e**(16*c + 16*d*x)*a*b**3 - 3*e**(16*c + 16*d*x)*b**4 - 15360
*e**(14*c + 14*d*x)*a**2*b**2 - 640*e**(14*c + 14*d*x)*a*b**3 + 24*e**(14*
c + 14*d*x)*b**4 - 61440*e**(12*c + 12*d*x)*a**3*b + 51200*e**(12*c + 12*d
*x)*a**2*b**2 + 2336*e**(12*c + 12*d*x)*a*b**3 - 84*e**(12*c + 12*d*x)*b**
4 + 245760*e**(10*c + 10*d*x)*a**3*b - 97280*e**(10*c + 10*d*x)*a**2*b**2
- 4864*e**(10*c + 10*d*x)*a*b**3 + 168*e**(10*c + 10*d*x)*b**4 + 491520*e*
*(8*c + 8*d*x)*a**4 - 348160*e**(8*c + 8*d*x)*a**3*b + 118272*e**(8*c + 8*
d*x)*a**2*b**2 + 6176*e**(8*c + 8*d*x)*a*b**3 - 210*e**(8*c + 8*d*x)*b**4
+ 245760*e**(6*c + 6*d*x)*a**3*b - 97280*e**(6*c + 6*d*x)*a**2*b**2 - 4864
*e**(6*c + 6*d*x)*a*b**3 + 168*e**(6*c + 6*d*x)*b**4 - 61440*e**(4*c + 4*d
*x)*a**3*b + 51200*e**(4*c + 4*d*x)*a**2*b**2 + 2336*e**(4*c + 4*d*x)*a*b*
*3 - 84*e**(4*c + 4*d*x)*b**4 - 15360*e**(2*c + 2*d*x)*a**2*b**2 - 640*e**
(2*c + 2*d*x)*a*b**3 + 24*e**(2*c + 2*d*x)*b**4 + 1920*a**2*b**2 + 80*a*b*
*3 - 3*b**4),x)*a**5*b*d - 57016320*e**(12*c + 8*d*x)*int(e**(4*d*x)/(1920
*e**(16*c + 16*d*x)*a**2*b**2 + 80*e**(16*c + 16*d*x)*a*b**3 - 3*e**(16*c
+ 16*d*x)*b**4 - 15360*e**(14*c + 14*d*x)*a**2*b**2 - 640*e**(14*c + 14*d*
x)*a*b**3 + 24*e**(14*c + 14*d*x)*b**4 - 61440*e**(12*c + 12*d*x)*a**3*b +
 51200*e**(12*c + 12*d*x)*a**2*b**2 + 2336*e**(12*c + 12*d*x)*a*b**3 - 84*
e**(12*c + 12*d*x)*b**4 + 245760*e**(10*c + 10*d*x)*a**3*b - 97280*e**(...
```

3.227 $\int \frac{\operatorname{csch}^2(c+dx)}{(a-b\sinh^4(c+dx))^2}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 237

$$\begin{aligned}\int \frac{\operatorname{csch}^2(c+dx)}{(a-b\sinh^4(c+dx))^2}\,dx &= -\frac{\left(6\sqrt{a}-5\sqrt{b}\right)\sqrt{b}\operatorname{arctanh}\left(\frac{\sqrt{\sqrt{a}-\sqrt{b}}\tanh(c+dx)}{\sqrt[4]{a}}\right)}{8a^{9/4}\left(\sqrt{a}-\sqrt{b}\right)^{3/2}d}\\ &\quad+\frac{\left(6\sqrt{a}+5\sqrt{b}\right)\sqrt{b}\operatorname{arctanh}\left(\frac{\sqrt{\sqrt{a}+\sqrt{b}}\tanh(c+dx)}{\sqrt[4]{a}}\right)}{8a^{9/4}\left(\sqrt{a}+\sqrt{b}\right)^{3/2}d}-\frac{\coth(c+dx)}{a^2d}\\ &\quad+\frac{b\tanh(c+dx)\left(a-(a+b)\tanh^2(c+dx)\right)}{4a^2(a-b)d\left(a-2a\tanh^2(c+dx)+(a-b)\tanh^4(c+dx)\right)}\end{aligned}$$

output
```
-1/8*(6*a^(1/2)-5*b^(1/2))*b^(1/2)*arctanh((a^(1/2)-b^(1/2))^(1/2)*tanh(d*
x+c)/a^(1/4))/a^(9/4)/(a^(1/2)-b^(1/2))^(3/2)/d+1/8*(6*a^(1/2)+5*b^(1/2))*
b^(1/2)*arctanh((a^(1/2)+b^(1/2))^(1/2)*tanh(d*x+c)/a^(1/4))/a^(9/4)/(a^(1
/2)+b^(1/2))^(3/2)/d-coth(d*x+c)/a^2/d+1/4*b*tanh(d*x+c)*(a-(a+b)*tanh(d*x
+c)^2)/a^2/(a-b)/d/(a-2*a*tanh(d*x+c)^2+(a-b)*tanh(d*x+c)^4)
```

Mathematica [A] (verified)

Time = 2.78 (sec) , antiderivative size = 272, normalized size of antiderivative = 1.15

$$\int \frac{\operatorname{csch}^2(c+dx)}{\left(a-b\sinh^4(c+dx)\right)^2}\,dx$$
$$= \frac{\frac{\left(6a\sqrt{b}-5\sqrt{a}b\right)\arctan\left(\frac{\left(\sqrt{a}-\sqrt{b}\right)\tanh(c+dx)}{\sqrt{-a+\sqrt{a}\sqrt{b}}}\right)}{\left(\sqrt{a}-\sqrt{b}\right)\sqrt{-a+\sqrt{a}\sqrt{b}}}+\frac{\left(6a\sqrt{b}+5\sqrt{a}b\right)\operatorname{arctanh}\left(\frac{\left(\sqrt{a}+\sqrt{b}\right)\tanh(c+dx)}{\sqrt{a+\sqrt{a}\sqrt{b}}}\right)}{\left(\sqrt{a}+\sqrt{b}\right)\sqrt{a+\sqrt{a}\sqrt{b}}}-8\sqrt{a}\coth(c+dx)+\frac{4\sqrt{a}}{(a-b)(8}}{8a^{5/2}d}$$

input
```
Integrate[Csch[c + d*x]^2/(a - b*Sinh[c + d*x]^4)^2,x]
```

output
```
(((6*a*Sqrt[b] - 5*Sqrt[a]*b)*ArcTan[((Sqrt[a] - Sqrt[b])*Tanh[c + d*x])/S
qrt[-a + Sqrt[a]*Sqrt[b]]])/((Sqrt[a] - Sqrt[b])*Sqrt[-a + Sqrt[a]*Sqrt[b]
]) + ((6*a*Sqrt[b] + 5*Sqrt[a]*b)*ArcTanh[((Sqrt[a] + Sqrt[b])*Tanh[c + d*
x])/Sqrt[a + Sqrt[a]*Sqrt[b]]])/((Sqrt[a] + Sqrt[b])*Sqrt[a + Sqrt[a]*Sqrt
[b]]) - 8*Sqrt[a]*Coth[c + d*x] + (4*Sqrt[a]*b*(2*a + b - b*Cosh[2*(c + d*
x)])*Sinh[2*(c + d*x)])/((a - b)*(8*a - 3*b + 4*b*Cosh[2*(c + d*x)] - b*Co
sh[4*(c + d*x)])))/(8*a^(5/2)*d)
```

Rubi [A] (verified)

Time = 0.79 (sec) , antiderivative size = 238, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.292$, Rules used = {3042, 25, 3696, 1673, 27, 2195, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\operatorname{csch}^2(c+dx)}{\left(a-b\sinh^4(c+dx)\right)^2}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int -\frac{1}{\sin(ic+idx)^2\left(a-b\sin(ic+idx)^4\right)^2}dx$$

$$\Big\downarrow \text{25}$$

$$-\int \frac{1}{\sin(ic+idx)^2\left(a-b\sin(ic+idx)^4\right)^2}dx$$

$$\Big\downarrow \; 3696$$

$$\frac{\int \frac{\coth^2(c+dx)\left(1-\tanh^2(c+dx)\right)^4}{\left((a-b)\tanh^4(c+dx)-2a\tanh^2(c+dx)+a\right)^2}d\tanh(c+dx)}{d}$$

$$\Big\downarrow \; 1673$$

$$\frac{\frac{b\tanh(c+dx)\left(a-(a+b)\tanh^2(c+dx)\right)}{4a^2(a-b)\left((a-b)\tanh^4(c+dx)-2a\tanh^2(c+dx)+a\right)}-\frac{\int -\frac{2\coth^2(c+dx)\left(\frac{b\left(4a^2-ba-b^2\right)\tanh^4(c+dx)}{a-b}-\frac{a(8a-7b)b\tanh^2(c+dx)}{a-b}+4ab\right)}{(a-b)\tanh^4(c+dx)-2a\tanh^2(c+dx)+a}d\tanh(c+dx}{8a^2b}}{d}$$

$$\Big\downarrow \; 27$$

$$\frac{\frac{\int \frac{\coth^2(c+dx)\left(\frac{b\left(4a^2-ba-b^2\right)\tanh^4(c+dx)}{a-b}-\frac{a(8a-7b)b\tanh^2(c+dx)}{a-b}+4ab\right)}{(a-b)\tanh^4(c+dx)-2a\tanh^2(c+dx)+a}d\tanh(c+dx)}{4a^2b}+\frac{b\tanh(c+dx)\left(a-(a+b)\tanh^2(c+dx)\right)}{4a^2(a-b)\left((a-b)\tanh^4(c+dx)-2a\tanh^2(c+dx)+a\right)}}{d}$$

$$\Big\downarrow \; 2195$$

$$\frac{\frac{\int \left(\frac{\left((7a-5b)\tanh^2(c+dx)-a\right)b^2}{(a-b)\left((a-b)\tanh^4(c+dx)-2a\tanh^2(c+dx)+a\right)}+4\coth^2(c+dx)b\right)d\tanh(c+dx)}{4a^2b}+\frac{b\tanh(c+dx)\left(a-(a+b)\tanh^2(c+dx)\right)}{4a^2(a-b)\left((a-b)\tanh^4(c+dx)-2a\tanh^2(c+dx)+a\right)}}{d}$$

$$\Big\downarrow \; 2009$$

$$\frac{\frac{-\frac{b^{3/2}\left(6\sqrt{a}-5\sqrt{b}\right)\text{arctanh}\left(\frac{\sqrt{\sqrt{a}-\sqrt{b}}\tanh(c+dx)}{\sqrt[4]{a}}\right)}{2\sqrt[4]{a}\left(\sqrt{a}-\sqrt{b}\right)^{3/2}}+\frac{b^{3/2}\left(6\sqrt{a}+5\sqrt{b}\right)\text{arctanh}\left(\frac{\sqrt{\sqrt{a}+\sqrt{b}}\tanh(c+dx)}{\sqrt[4]{a}}\right)}{2\sqrt[4]{a}\left(\sqrt{a}+\sqrt{b}\right)^{3/2}}-4b\coth(c+dx)}{4a^2b}+\frac{b\tanh(c+dx)}{4a^2(a-b)\left((a-b)\tan\right.}}{d}$$

input
```
Int[Csch[c + d*x]^2/(a - b*Sinh[c + d*x]^4)^2,x]
```

output
```
((-1/2*((6*Sqrt[a] - 5*Sqrt[b])*b^(3/2)*ArcTanh[(Sqrt[Sqrt[a] - Sqrt[b]]*T
anh[c + d*x])/a^(1/4)])/(a^(1/4)*(Sqrt[a] - Sqrt[b])^(3/2)) + ((6*Sqrt[a]
 + 5*Sqrt[b])*b^(3/2)*ArcTanh[(Sqrt[Sqrt[a] + Sqrt[b]]*Tanh[c + d*x])/a^(1/
4)])/(2*a^(1/4)*(Sqrt[a] + Sqrt[b])^(3/2)) - 4*b*Coth[c + d*x])/(4*a^2*b)
 + (b*Tanh[c + d*x]*(a - (a + b)*Tanh[c + d*x]^2))/(4*a^2*(a - b)*(a - 2*a*
Tanh[c + d*x]^2 + (a - b)*Tanh[c + d*x]^4)))/d
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 1673
```
Int[(x_)^(m_)*((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^
4)^(p_), x_Symbol] :> With[{f = Coeff[PolynomialRemainder[x^m*(d + e*x^2)^q
, a + b*x^2 + c*x^4, x], x, 0], g = Coeff[PolynomialRemainder[x^m*(d + e*x^
2)^q, a + b*x^2 + c*x^4, x], x, 2]}, Simp[x*(a + b*x^2 + c*x^4)^(p + 1)*((a
*b*g - f*(b^2 - 2*a*c) - c*(b*f - 2*a*g)*x^2)/(2*a*(p + 1)*(b^2 - 4*a*c))),
 x] + Simp[1/(2*a*(p + 1)*(b^2 - 4*a*c))   Int[x^m*(a + b*x^2 + c*x^4)^(p +
 1)*Simp[ExpandToSum[(2*a*(p + 1)*(b^2 - 4*a*c)*PolynomialQuotient[x^m*(d +
 e*x^2)^q, a + b*x^2 + c*x^4, x])/x^m + (b^2*f*(2*p + 3) - 2*a*c*f*(4*p + 5
) - a*b*g)/x^m + c*(4*p + 7)*(b*f - 2*a*g)*x^(2 - m), x], x], x], x]] /; Fr
eeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && IGtQ[q, 1]
&& ILtQ[m/2, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2195
```
Int[(Pq_)*((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_
Symbol] :> Int[ExpandIntegrand[(d*x)^m*Pq*(a + b*x^2 + c*x^4)^p, x], x] /;
FreeQ[{a, b, c, d, m}, x] && PolyQ[Pq, x^2] && IGtQ[p, -2]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3696
```
Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^4)^(
p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Simp[ff^(m + 1
)/f   Subst[Int[x^m*((a + 2*a*ff^2*x^2 + (a + b)*ff^4*x^4)^p/(1 + ff^2*x^2)
^(m/2 + 2*p + 1)), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f}, x] &
& IntegerQ[m/2] && IntegerQ[p]
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 4.40 (sec) , antiderivative size = 326, normalized size of antiderivative = 1.38

method	result
derivativedivides	$-\frac{\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)}{2a^2}-\frac{16b\left(\frac{-\frac{a\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^7}{32(a-b)}+\frac{(a+4b)\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^5}{32a-32b}+\frac{(a+4b)\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^3}{32a-32b}-\frac{a\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)}{32(a-b)}}{\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^8a-4\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^6a+6\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^4a-16b\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^4-4\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^2a+a}+\frac{\_R\cdots}{\cdots}\right.}{\cdots}$
default	$-\frac{\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)}{2a^2}-\frac{16b\left(\frac{-\frac{a\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^7}{32(a-b)}+\frac{(a+4b)\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^5}{32a-32b}+\frac{(a+4b)\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^3}{32a-32b}-\frac{a\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)}{32(a-b)}}{\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^8a-4\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^6a+6\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^4a-16b\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^4-4\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^2a+a}+\frac{\_R\cdots}{\cdots}\right.}{\cdots}$
risch	Expression too large to display

input
```
int(csch(d*x+c)^2/(a-b*sinh(d*x+c)^4)^2,x,method=_RETURNVERBOSE)
```

output
```
1/d*(-1/2/a^2*tanh(1/2*d*x+1/2*c)-16*b/a^2*((-1/32*a/(a-b)*tanh(1/2*d*x+1/
2*c)^7+1/32*(a+4*b)/(a-b)*tanh(1/2*d*x+1/2*c)^5+1/32*(a+4*b)/(a-b)*tanh(1/
2*d*x+1/2*c)^3-1/32*a/(a-b)*tanh(1/2*d*x+1/2*c))/(tanh(1/2*d*x+1/2*c)^8*a-
4*tanh(1/2*d*x+1/2*c)^6*a+6*tanh(1/2*d*x+1/2*c)^4*a-16*b*tanh(1/2*d*x+1/2*
c)^4-4*tanh(1/2*d*x+1/2*c)^2*a+a)+1/256/(a-b)*sum((-a*_R^6+(27*a-20*b)*_R^
4+(-27*a+20*b)*_R^2+a)/(_R^7*a-3*_R^5*a+3*_R^3*a-8*_R^3*b-_R*a)*ln(tanh(1/
2*d*x+1/2*c)-_R),_R=RootOf(a*_Z^8-4*a*_Z^6+(6*a-16*b)*_Z^4-4*a*_Z^2+a)))-1
/2/a^2/tanh(1/2*d*x+1/2*c))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 8824 vs. $2(188) = 376$.

Time = 0.63 (sec) , antiderivative size = 8824, normalized size of antiderivative = 37.23

$$\int \frac{\operatorname{csch}^2(c+dx)}{(a-b\sinh^4(c+dx))^2}\,dx = \text{Too large to display}$$

input
```
integrate(csch(d*x+c)^2/(a-b*sinh(d*x+c)^4)^2,x, algorithm="fricas")
```

output
```
Too large to include
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\operatorname{csch}^2(c+dx)}{(a-b\sinh^4(c+dx))^2}\,dx = \text{Timed out}$$

input
```
integrate(csch(d*x+c)**2/(a-b*sinh(d*x+c)**4)**2,x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{\operatorname{csch}^2(c+dx)}{(a-b\sinh^4(c+dx))^2}\,dx = \int \frac{\operatorname{csch}(dx+c)^2}{(b\sinh(dx+c)^4-a)^2}\,dx$$

input
```
integrate(csch(d*x+c)^2/(a-b*sinh(d*x+c)^4)^2,x, algorithm="maxima")
```

output
```
1/2*(4*a*b - 5*b^2 + (6*a*b*e^(8*c) - 5*b^2*e^(8*c))*e^(8*d*x) - 2*(13*a*b
*e^(6*c) - 10*b^2*e^(6*c))*e^(6*d*x) - 2*(32*a^2*e^(4*c) - 47*a*b*e^(4*c)
+ 15*b^2*e^(4*c))*e^(4*d*x) - 2*(7*a*b*e^(2*c) - 10*b^2*e^(2*c))*e^(2*d*x)
)/(a^3*b*d - a^2*b^2*d - (a^3*b*d*e^(10*c) - a^2*b^2*d*e^(10*c))*e^(10*d*x
) + 5*(a^3*b*d*e^(8*c) - a^2*b^2*d*e^(8*c))*e^(8*d*x) + 2*(8*a^4*d*e^(6*c)
 - 13*a^3*b*d*e^(6*c) + 5*a^2*b^2*d*e^(6*c))*e^(6*d*x) - 2*(8*a^4*d*e^(4*c
) - 13*a^3*b*d*e^(4*c) + 5*a^2*b^2*d*e^(4*c))*e^(4*d*x) - 5*(a^3*b*d*e^(2*
c) - a^2*b^2*d*e^(2*c))*e^(2*d*x)) - 4*integrate(1/4*((6*a*b*e^(6*c) - 5*b
^2*e^(6*c))*e^(6*d*x) - 2*(8*a*b*e^(4*c) - 5*b^2*e^(4*c))*e^(4*d*x) + (6*a
*b*e^(2*c) - 5*b^2*e^(2*c))*e^(2*d*x))/(a^3*b - a^2*b^2 + (a^3*b*e^(8*c) -
 a^2*b^2*e^(8*c))*e^(8*d*x) - 4*(a^3*b*e^(6*c) - a^2*b^2*e^(6*c))*e^(6*d*x
) - 2*(8*a^4*e^(4*c) - 11*a^3*b*e^(4*c) + 3*a^2*b^2*e^(4*c))*e^(4*d*x) - 4
*(a^3*b*e^(2*c) - a^2*b^2*e^(2*c))*e^(2*d*x)), x)
```

Giac [F]

$$\int \frac{\operatorname{csch}^2(c+dx)}{(a-b\sinh^4(c+dx))^2}\,dx = \int \frac{\operatorname{csch}(dx+c)^2}{(b\sinh(dx+c)^4-a)^2}\,dx$$

input
```
integrate(csch(d*x+c)^2/(a-b*sinh(d*x+c)^4)^2,x, algorithm="giac")
```

output
```
sage0*x
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\operatorname{csch}^2(c+dx)}{(a-b\sinh^4(c+dx))^2}\,dx = \int \frac{1}{\sinh(c+d\,x)^2\,\left(a-b\sinh(c+d\,x)^4\right)^2}\,dx$$

input
```
int(1/(sinh(c + d*x)^2*(a - b*sinh(c + d*x)^4)^2),x)
```

output
```
int(1/(sinh(c + d*x)^2*(a - b*sinh(c + d*x)^4)^2), x)
```

Reduce [F]

$$\int \frac{\operatorname{csch}^2(c+dx)}{(a-b\sinh^4(c+dx))^2}\,dx = \text{too large to display}$$

input
```
int(csch(d*x+c)^2/(a-b*sinh(d*x+c)^4)^2,x)
```

output
```
( - 112742891520*e**(14*c + 10*d*x)*int(e**(4*d*x)/(448*e**(20*c + 20*d*x)
*a**2*b**2 + 24*e**(20*c + 20*d*x)*a*b**3 - 15*e**(20*c + 20*d*x)*b**4 - 4
480*e**(18*c + 18*d*x)*a**2*b**2 - 240*e**(18*c + 18*d*x)*a*b**3 + 150*e**
(18*c + 18*d*x)*b**4 - 14336*e**(16*c + 16*d*x)*a**3*b + 19392*e**(16*c +
16*d*x)*a**2*b**2 + 1560*e**(16*c + 16*d*x)*a*b**3 - 675*e**(16*c + 16*d*x
)*b**4 + 86016*e**(14*c + 14*d*x)*a**3*b - 49152*e**(14*c + 14*d*x)*a**2*b
**2 - 5760*e**(14*c + 14*d*x)*a*b**3 + 1800*e**(14*c + 14*d*x)*b**4 + 1146
88*e**(12*c + 12*d*x)*a**4 - 208896*e**(12*c + 12*d*x)*a**3*b + 78720*e**(
12*c + 12*d*x)*a**2*b**2 + 12240*e**(12*c + 12*d*x)*a*b**3 - 3150*e**(12*c
 + 12*d*x)*b**4 - 229376*e**(10*c + 10*d*x)*a**4 + 274432*e**(10*c + 10*d*
x)*a**3*b - 89856*e**(10*c + 10*d*x)*a**2*b**2 - 15648*e**(10*c + 10*d*x)*
a*b**3 + 3780*e**(10*c + 10*d*x)*b**4 + 114688*e**(8*c + 8*d*x)*a**4 - 208
896*e**(8*c + 8*d*x)*a**3*b + 78720*e**(8*c + 8*d*x)*a**2*b**2 + 12240*e**
(8*c + 8*d*x)*a*b**3 - 3150*e**(8*c + 8*d*x)*b**4 + 86016*e**(6*c + 6*d*x)
*a**3*b - 49152*e**(6*c + 6*d*x)*a**2*b**2 - 5760*e**(6*c + 6*d*x)*a*b**3
+ 1800*e**(6*c + 6*d*x)*b**4 - 14336*e**(4*c + 4*d*x)*a**3*b + 19392*e**(4
*c + 4*d*x)*a**2*b**2 + 1560*e**(4*c + 4*d*x)*a*b**3 - 675*e**(4*c + 4*d*x
)*b**4 - 4480*e**(2*c + 2*d*x)*a**2*b**2 - 240*e**(2*c + 2*d*x)*a*b**3 + 1
50*e**(2*c + 2*d*x)*b**4 + 448*a**2*b**2 + 24*a*b**3 - 15*b**4),x)*a**8*b*
d + 163074539520*e**(14*c + 10*d*x)*int(e**(4*d*x)/(448*e**(20*c + 20*d...
```

3.228 $\int \frac{\sinh^9(c+dx)}{(a-b\sinh^4(c+dx))^3}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 315

$$\begin{aligned}
\int \frac{\sinh^9(c+dx)}{(a-b\sinh^4(c+dx))^3}\,dx &= \frac{\left(5a-14\sqrt{a}\sqrt{b}+12b\right)\arctan\left(\frac{\sqrt[4]{b}\cosh(c+dx)}{\sqrt{\sqrt{a}-\sqrt{b}}}\right)}{64\sqrt{a}\left(\sqrt{a}-\sqrt{b}\right)^{5/2}b^{9/4}d} \\
&\quad+\frac{\left(5a+14\sqrt{a}\sqrt{b}+12b\right)\operatorname{arctanh}\left(\frac{\sqrt[4]{b}\cosh(c+dx)}{\sqrt{\sqrt{a}+\sqrt{b}}}\right)}{64\sqrt{a}\left(\sqrt{a}+\sqrt{b}\right)^{5/2}b^{9/4}d} \\
&\quad+\frac{a\cosh(c+dx)\left(a+b-b\cosh^2(c+dx)\right)}{8(a-b)b^2d\left(a-b+2b\cosh^2(c+dx)-b\cosh^4(c+dx)\right)^2} \\
&\quad-\frac{\cosh(c+dx)\left(9a^2-11ab-10b^2-2(2a-5b)b\cosh^2(c+dx)\right)}{32(a-b)^2b^2d\left(a-b+2b\cosh^2(c+dx)-b\cosh^4(c+dx)\right)}
\end{aligned}$$

output
```
1/64*(5*a-14*a^(1/2)*b^(1/2)+12*b)*arctan(b^(1/4)*cosh(d*x+c)/(a^(1/2)-b^(
1/2))^(1/2))/a^(1/2)/(a^(1/2)-b^(1/2))^(5/2)/b^(9/4)/d+1/64*(5*a+14*a^(1/2
)*b^(1/2)+12*b)*arctanh(b^(1/4)*cosh(d*x+c)/(a^(1/2)+b^(1/2))^(1/2))/a^(1/
2)/(a^(1/2)+b^(1/2))^(5/2)/b^(9/4)/d+1/8*a*cosh(d*x+c)*(a+b-b*cosh(d*x+c)^
2)/(a-b)/b^2/d/(a-b+2*b*cosh(d*x+c)^2-b*cosh(d*x+c)^4)^2-1/32*cosh(d*x+c)*
(9*a^2-11*a*b-10*b^2-2*(2*a-5*b)*b*cosh(d*x+c)^2)/(a-b)^2/b^2/d/(a-b+2*b*c
osh(d*x+c)^2-b*cosh(d*x+c)^4)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

Time = 7.36 (sec) , antiderivative size = 1021, normalized size of antiderivative = 3.24

$$\int \frac{\sinh^9(c+dx)}{\left(a-b\sinh^4(c+dx)\right)^3}\,dx = \text{Too large to display}$$

input
```
Integrate[Sinh[c + d*x]^9/(a - b*Sinh[c + d*x]^4)^3,x]
```

output
```
((32*Cosh[c + d*x]*(-9*a^2 + 13*a*b + 5*b^2 + (2*a - 5*b)*b*Cosh[2*(c + d*
x)]))/(8*a - 3*b + 4*b*Cosh[2*(c + d*x)] - b*Cosh[4*(c + d*x)]) + (512*a*(
a - b)*Cosh[c + d*x]*(2*a + b - b*Cosh[2*(c + d*x)]))/(-8*a + 3*b - 4*b*Co
sh[2*(c + d*x)] + b*Cosh[4*(c + d*x)])^2 - RootSum[b - 4*b*#1^2 - 16*a*#1^
4 + 6*b*#1^4 - 4*b*#1^6 + b*#1^8 & , (-2*a*b*c + 5*b^2*c - 2*a*b*d*x + 5*b
^2*d*x - 4*a*b*Log[-Cosh[(c + d*x)/2] - Sinh[(c + d*x)/2] + Cosh[(c + d*x)
/2]*#1 - Sinh[(c + d*x)/2]*#1] + 10*b^2*Log[-Cosh[(c + d*x)/2] - Sinh[(c +
 d*x)/2] + Cosh[(c + d*x)/2]*#1 - Sinh[(c + d*x)/2]*#1] - 10*a^2*c*#1^2 +
28*a*b*c*#1^2 - 39*b^2*c*#1^2 - 10*a^2*d*x*#1^2 + 28*a*b*d*x*#1^2 - 39*b^2
*d*x*#1^2 - 20*a^2*Log[-Cosh[(c + d*x)/2] - Sinh[(c + d*x)/2] + Cosh[(c +
d*x)/2]*#1 - Sinh[(c + d*x)/2]*#1]*#1^2 + 56*a*b*Log[-Cosh[(c + d*x)/2] -
Sinh[(c + d*x)/2] + Cosh[(c + d*x)/2]*#1 - Sinh[(c + d*x)/2]*#1]*#1^2 - 78
*b^2*Log[-Cosh[(c + d*x)/2] - Sinh[(c + d*x)/2] + Cosh[(c + d*x)/2]*#1 - S
inh[(c + d*x)/2]*#1]*#1^2 + 10*a^2*c*#1^4 - 28*a*b*c*#1^4 + 39*b^2*c*#1^4
+ 10*a^2*d*x*#1^4 - 28*a*b*d*x*#1^4 + 39*b^2*d*x*#1^4 + 20*a^2*Log[-Cosh[(
c + d*x)/2] - Sinh[(c + d*x)/2] + Cosh[(c + d*x)/2]*#1 - Sinh[(c + d*x)/2]
*#1]*#1^4 - 56*a*b*Log[-Cosh[(c + d*x)/2] - Sinh[(c + d*x)/2] + Cosh[(c +
d*x)/2]*#1 - Sinh[(c + d*x)/2]*#1]*#1^4 + 78*b^2*Log[-Cosh[(c + d*x)/2] -
Sinh[(c + d*x)/2] + Cosh[(c + d*x)/2]*#1 - Sinh[(c + d*x)/2]*#1]*#1^4 + 2*
a*b*c*#1^6 - 5*b^2*c*#1^6 + 2*a*b*d*x*#1^6 - 5*b^2*d*x*#1^6 + 4*a*b*Log...
```

Rubi [A] (verified)

Time = 0.75 (sec) , antiderivative size = 370, normalized size of antiderivative = 1.17, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.417$, Rules used = {3042, 26, 3694, 1517, 27, 2206, 27, 1480, 218, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sinh^9(c+dx)}{\left(a-b\sinh^4(c+dx)\right)^3}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int -\frac{i\sin(ic+idx)^9}{\left(a-b\sin(ic+idx)^4\right)^3}dx$$

$$\Big\downarrow \text{26}$$

$$-i\int \frac{\sin(ic+idx)^9}{\left(a-b\sin(ic+idx)^4\right)^3}dx$$

$$\Big\downarrow \; 3694$$

$$\frac{\int \frac{\left(1-\cosh^2(c+dx)\right)^4}{\left(-b\cosh^4(c+dx)+2b\cosh^2(c+dx)+a-b\right)^3}d\cosh(c+dx)}{d}$$

$$\Big\downarrow \; 1517$$

$$\frac{\frac{a\cosh(c+dx)\left(a-b\cosh^2(c+dx)+b\right)}{8b^2(a-b)\left(a-b\cosh^4(c+dx)+2b\cosh^2(c+dx)-b\right)^2}-\frac{\int \frac{2\left(8a(a-b)\cosh^4(c+dx)-a(11a-16b)\cosh^2(c+dx)+\frac{a\left(a^2+ba-8b^2\right)}{b}\right)}{\left(-b\cosh^4(c+dx)+2b\cosh^2(c+dx)+a-b\right)^2}d\cosh(c+dx)}{16ab(a-b)}}{d}$$

$$\Big\downarrow \; 27$$

$$\frac{\frac{a\cosh(c+dx)\left(a-b\cosh^2(c+dx)+b\right)}{8b^2(a-b)\left(a-b\cosh^4(c+dx)+2b\cosh^2(c+dx)-b\right)^2}-\frac{\int \frac{8a(a-b)\cosh^4(c+dx)-a(11a-16b)\cosh^2(c+dx)+a\left(\frac{a^2}{b}+a-8b\right)}{\left(-b\cosh^4(c+dx)+2b\cosh^2(c+dx)+a-b\right)^2}d\cosh(c+dx)}{8ab(a-b)}}{d}$$

$$\Big\downarrow \; 2206$$

$$\frac{\frac{a\cosh(c+dx)\left(a-b\cosh^2(c+dx)+b\right)}{8b^2(a-b)\left(a-b\cosh^4(c+dx)+2b\cosh^2(c+dx)-b\right)^2}-\frac{\frac{a\cosh(c+dx)\left(9a^2-2b(2a-5b)\cosh^2(c+dx)-11ab-10b^2\right)}{4b(a-b)\left(a-b\cosh^4(c+dx)+2b\cosh^2(c+dx)-b\right)}-\frac{\int \frac{2a^2\left(5a^2-15ba+22b^2+2(2a-5b)b\cosh \ldots\right.}{-b\cosh^4(c+dx)+2b\cosh^2(c+dx) \ldots}}{8ab(a-b)}}{8ab(a-b)}}{d}$$

$$\Big\downarrow \; 27$$

$$\frac{\frac{a\cosh(c+dx)\left(a-b\cosh^2(c+dx)+b\right)}{8b^2(a-b)\left(a-b\cosh^4(c+dx)+2b\cosh^2(c+dx)-b\right)^2}-\frac{\frac{a\cosh(c+dx)\left(9a^2-2b(2a-5b)\cosh^2(c+dx)-11ab-10b^2\right)}{4b(a-b)\left(a-b\cosh^4(c+dx)+2b\cosh^2(c+dx)-b\right)}-\frac{a\int \frac{5a^2-15ba+22b^2+2(2a-5b)b\cosh^2(c \ldots}{-b\cosh^4(c+dx)+2b\cosh^2(c+dx)+ \ldots}}{4b(a-b)}}{8ab(a-b)}}{d}$$

$$\Big\downarrow \; 1480$$

$$\frac{\frac{a\cosh(c+dx)\left(a-b\cosh^2(c+dx)+b\right)}{8b^2(a-b)\left(a-b\cosh^4(c+dx)+2b\cosh^2(c+dx)-b\right)^2}-\frac{\frac{a\cosh(c+dx)\left(9a^2-2b(2a-5b)\cosh^2(c+dx)-11ab-10b^2\right)}{4b(a-b)\left(a-b\cosh^4(c+dx)+2b\cosh^2(c+dx)-b\right)}-\frac{a\left(\frac{\sqrt{b}\left(-2\sqrt{a}\sqrt{b}+a+b\right)\left(14\sqrt{a}\sqrt{b}+5a+1 \ldots\right.}{\ldots}\right.}{\ldots}}{\ldots}}{d}$$

$\Big\downarrow$ 218

$$\frac{\frac{a\cosh(c+dx)\left(a-b\cosh^2(c+dx)+b\right)}{8b^2(a-b)\left(a-b\cosh^4(c+dx)+2b\cosh^2(c+dx)-b\right)^2}-\frac{\frac{a\cosh(c+dx)\left(9a^2-2b(2a-5b)\cosh^2(c+dx)-11ab-10b^2\right)}{4b(a-b)\left(a-b\cosh^4(c+dx)+2b\cosh^2(c+dx)-b\right)}-\frac{a\left(\frac{\sqrt{b}\left(-2\sqrt{a}\sqrt{b}+a+b\right)\left(14\sqrt{a}\sqrt{b}+5a+1\right.}{}\right.}{}}{}}{d}$$

$\Big\downarrow$ 221

$$\frac{\frac{a\cosh(c+dx)\left(a-b\cosh^2(c+dx)+b\right)}{8b^2(a-b)\left(a-b\cosh^4(c+dx)+2b\cosh^2(c+dx)-b\right)^2}-\frac{\frac{a\cosh(c+dx)\left(9a^2-2b(2a-5b)\cosh^2(c+dx)-11ab-10b^2\right)}{4b(a-b)\left(a-b\cosh^4(c+dx)+2b\cosh^2(c+dx)-b\right)}-\frac{a\left(\frac{\left(-14\sqrt{a}\sqrt{b}+5a+12b\right)\left(\sqrt{a}+\sqrt{b}\right)^2\text{arc}}{2\sqrt{a}\sqrt[4]{b}\sqrt{\sqrt{a}-}}\right.}{}}{8ab(}}{d}$$

input
```
Int[Sinh[c + d*x]^9/(a - b*Sinh[c + d*x]^4)^3,x]
```

output
```
((a*Cosh[c + d*x]*(a + b - b*Cosh[c + d*x]^2))/(8*(a - b)*b^2*(a - b + 2*b
*Cosh[c + d*x]^2 - b*Cosh[c + d*x]^4)^2) - (-1/4*(a*(((Sqrt[a] + Sqrt[b])^
2*(5*a - 14*Sqrt[a]*Sqrt[b] + 12*b)*ArcTan[(b^(1/4)*Cosh[c + d*x])/Sqrt[Sq
rt[a] - Sqrt[b]]])/(2*Sqrt[a]*Sqrt[Sqrt[a] - Sqrt[b]]*b^(1/4)) + ((a - 2*S
qrt[a]*Sqrt[b] + b)*(5*a + 14*Sqrt[a]*Sqrt[b] + 12*b)*ArcTanh[(b^(1/4)*Cos
h[c + d*x])/Sqrt[Sqrt[a] + Sqrt[b]]])/(2*Sqrt[a]*Sqrt[Sqrt[a] + Sqrt[b]]*b
^(1/4))))/((a - b)*b) + (a*Cosh[c + d*x]*(9*a^2 - 11*a*b - 10*b^2 - 2*(2*a
 - 5*b)*b*Cosh[c + d*x]^2))/(4*(a - b)*b*(a - b + 2*b*Cosh[c + d*x]^2 - b*
Cosh[c + d*x]^4)))/(8*a*(a - b)*b))/d
```

Defintions of rubi rules used

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 1480
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Simp[(e/2 + (2*c*d - b*e)/(2*q))   Int[1/(
b/2 - q/2 + c*x^2), x], x] + Simp[(e/2 - (2*c*d - b*e)/(2*q))   Int[1/(b/2
+ q/2 + c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0]
 && NeQ[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

rule 1517
```
Int[((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x
_Symbol] :> With[{f = Coeff[PolynomialRemainder[(d + e*x^2)^q, a + b*x^2 +
c*x^4, x], x, 0], g = Coeff[PolynomialRemainder[(d + e*x^2)^q, a + b*x^2 +
c*x^4, x], x, 2]}, Simp[x*(a + b*x^2 + c*x^4)^(p + 1)*((a*b*g - f*(b^2 - 2*
a*c) - c*(b*f - 2*a*g)*x^2)/(2*a*(p + 1)*(b^2 - 4*a*c))), x] + Simp[1/(2*a*
(p + 1)*(b^2 - 4*a*c))   Int[(a + b*x^2 + c*x^4)^(p + 1)*ExpandToSum[2*a*(p
 + 1)*(b^2 - 4*a*c)*PolynomialQuotient[(d + e*x^2)^q, a + b*x^2 + c*x^4, x]
 + b^2*f*(2*p + 3) - 2*a*c*f*(4*p + 5) - a*b*g + c*(4*p + 7)*(b*f - 2*a*g)*
x^2, x], x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ
[c*d^2 - b*d*e + a*e^2, 0] && IGtQ[q, 1] && LtQ[p, -1]
```

rule 2206
```
Int[(Px_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> With[{d =
 Coeff[PolynomialRemainder[Px, a + b*x^2 + c*x^4, x], x, 0], e = Coeff[Poly
nomialRemainder[Px, a + b*x^2 + c*x^4, x], x, 2]}, Simp[x*(a + b*x^2 + c*x^
4)^(p + 1)*((a*b*e - d*(b^2 - 2*a*c) - c*(b*d - 2*a*e)*x^2)/(2*a*(p + 1)*(b
^2 - 4*a*c))), x] + Simp[1/(2*a*(p + 1)*(b^2 - 4*a*c))   Int[(a + b*x^2 + c
*x^4)^(p + 1)*ExpandToSum[2*a*(p + 1)*(b^2 - 4*a*c)*PolynomialQuotient[Px,
a + b*x^2 + c*x^4, x] + b^2*d*(2*p + 3) - 2*a*c*d*(4*p + 5) - a*b*e + c*(4*
p + 7)*(b*d - 2*a*e)*x^2, x], x], x]] /; FreeQ[{a, b, c}, x] && PolyQ[Px, x
^2] && Expon[Px, x^2] > 1 && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3694
```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^4)^
(p_.), x_Symbol] :> With[{ff = FreeFactors[Cos[e + f*x], x]}, Simp[-ff/f
Subst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a + b - 2*b*ff^2*x^2 + b*ff^4*x^4)^p,
 x], x, Cos[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m -
 1)/2]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 651 vs. $2(263) = 526$.

Time = 23.33 (sec) , antiderivative size = 652, normalized size of antiderivative = 2.07

method	result
derivativedivides	$\frac{\frac{a\left(5a^2-11ab+12b^2\right)\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^{14}}{16b^2\left(a^2-2ab+b^2\right)}-\frac{a\left(35a^2-85ab+104b^2\right)\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^{12}}{16b^2\left(a^2-2ab+b^2\right)}+\frac{\left(105a^3-407a^2b+652b^2a-320b^3\right)\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^{1}}{16b^2\left(a^2-2ab+b^2\right)}}{\left(\tanh\left(\frac{dx}{2}+\right.\right.}$
default	$\frac{\frac{a\left(5a^2-11ab+12b^2\right)\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^{14}}{16b^2\left(a^2-2ab+b^2\right)}-\frac{a\left(35a^2-85ab+104b^2\right)\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^{12}}{16b^2\left(a^2-2ab+b^2\right)}+\frac{\left(105a^3-407a^2b+652b^2a-320b^3\right)\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^{1}}{16b^2\left(a^2-2ab+b^2\right)}}{\left(\tanh\left(\frac{dx}{2}+\right.\right.}$
risch	Expression too large to display

input
```
int(sinh(d*x+c)^9/(a-b*sinh(d*x+c)^4)^3,x,method=_RETURNVERBOSE)
```

output
```
1/d*(512*(1/8192*a*(5*a^2-11*a*b+12*b^2)/b^2/(a^2-2*a*b+b^2)*tanh(1/2*d*x+
1/2*c)^14-1/8192/b^2*a*(35*a^2-85*a*b+104*b^2)/(a^2-2*a*b+b^2)*tanh(1/2*d*
x+1/2*c)^12+1/8192/b^2*(105*a^3-407*a^2*b+652*a*b^2-320*b^3)/(a^2-2*a*b+b^
2)*tanh(1/2*d*x+1/2*c)^10-1/8192*(175*a^3-865*a^2*b+1696*a*b^2-1408*b^3)/b
^2/(a^2-2*a*b+b^2)*tanh(1/2*d*x+1/2*c)^8+1/8192*(175*a^3-849*a^2*b+756*a*b
^2+320*b^3)/b^2/(a^2-2*a*b+b^2)*tanh(1/2*d*x+1/2*c)^6-1/8192*a*(105*a^2-38
3*a*b+248*b^2)/b^2/(a^2-2*a*b+b^2)*tanh(1/2*d*x+1/2*c)^4+1/8192*(35*a^2-77
*a*b-12*b^2)*a/b^2/(a^2-2*a*b+b^2)*tanh(1/2*d*x+1/2*c)^2-1/8192*a^2*(5*a-1
1*b)/b^2/(a^2-2*a*b+b^2))/(tanh(1/2*d*x+1/2*c)^8*a-4*tanh(1/2*d*x+1/2*c)^6
*a+6*tanh(1/2*d*x+1/2*c)^4*a-16*b*tanh(1/2*d*x+1/2*c)^4-4*tanh(1/2*d*x+1/2
*c)^2*a+a)^2+1/16/b^2/(a^2-2*a*b+b^2)*a*(-1/4*(4*(a*b)^(1/2)*a-10*(a*b)^(1
/2)*b+5*a^2-11*a*b+12*b^2)/a/(-a*b-(a*b)^(1/2)*a)^(1/2)*arctan(1/4*(-2*tan
h(1/2*d*x+1/2*c)^2*a+4*(a*b)^(1/2)+2*a)/(-a*b-(a*b)^(1/2)*a)^(1/2))+1/4*(-
4*(a*b)^(1/2)*a+10*(a*b)^(1/2)*b+5*a^2-11*a*b+12*b^2)/a/(-a*b+(a*b)^(1/2)*
a)^(1/2)*arctan(1/4*(2*tanh(1/2*d*x+1/2*c)^2*a+4*(a*b)^(1/2)-2*a)/(-a*b+(a
*b)^(1/2)*a)^(1/2))))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 21541 vs. $2(264) = 528$.

Time = 0.62 (sec) , antiderivative size = 21541, normalized size of antiderivative = 68.38

$$\int \frac{\sinh^9(c+dx)}{\left(a-b\sinh^4(c+dx)\right)^3}\,dx = \text{Too large to display}$$

input
```
integrate(sinh(d*x+c)^9/(a-b*sinh(d*x+c)^4)^3,x, algorithm="fricas")
```

output
```
Too large to include
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\sinh^9(c+dx)}{\left(a-b\sinh^4(c+dx)\right)^3}\,dx = \text{Timed out}$$

input
```
integrate(sinh(d*x+c)**9/(a-b*sinh(d*x+c)**4)**3,x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{\sinh^9(c+dx)}{\left(a-b\sinh^4(c+dx)\right)^3}\,dx = \int -\frac{\sinh\left(dx+c\right)^9}{\left(b\sinh\left(dx+c\right)^4-a\right)^3}\,dx$$

input
```
integrate(sinh(d*x+c)^9/(a-b*sinh(d*x+c)^4)^3,x, algorithm="maxima")
```

output

```
-1/8*((2*a*b^2*e^(15*c) - 5*b^3*e^(15*c))*e^(15*d*x) - (18*a^2*b*e^(13*c)
- 20*a*b^2*e^(13*c) - 25*b^3*e^(13*c))*e^(13*d*x) + 3*(18*a^2*b*e^(11*c) -
 8*a*b^2*e^(11*c) - 15*b^3*e^(11*c))*e^(11*d*x) + (160*a^3*e^(9*c) - 388*a
^2*b*e^(9*c) + 2*a*b^2*e^(9*c) + 25*b^3*e^(9*c))*e^(9*d*x) + (160*a^3*e^(7
*c) - 388*a^2*b*e^(7*c) + 2*a*b^2*e^(7*c) + 25*b^3*e^(7*c))*e^(7*d*x) + 3*
(18*a^2*b*e^(5*c) - 8*a*b^2*e^(5*c) - 15*b^3*e^(5*c))*e^(5*d*x) - (18*a^2*
b*e^(3*c) - 20*a*b^2*e^(3*c) - 25*b^3*e^(3*c))*e^(3*d*x) + (2*a*b^2*e^c -
5*b^3*e^c)*e^(d*x))/(a^2*b^4*d - 2*a*b^5*d + b^6*d + (a^2*b^4*d*e^(16*c) -
 2*a*b^5*d*e^(16*c) + b^6*d*e^(16*c))*e^(16*d*x) - 8*(a^2*b^4*d*e^(14*c) -
 2*a*b^5*d*e^(14*c) + b^6*d*e^(14*c))*e^(14*d*x) - 4*(8*a^3*b^3*d*e^(12*c)
 - 23*a^2*b^4*d*e^(12*c) + 22*a*b^5*d*e^(12*c) - 7*b^6*d*e^(12*c))*e^(12*d
*x) + 8*(16*a^3*b^3*d*e^(10*c) - 39*a^2*b^4*d*e^(10*c) + 30*a*b^5*d*e^(10*
c) - 7*b^6*d*e^(10*c))*e^(10*d*x) + 2*(128*a^4*b^2*d*e^(8*c) - 352*a^3*b^3
*d*e^(8*c) + 355*a^2*b^4*d*e^(8*c) - 166*a*b^5*d*e^(8*c) + 35*b^6*d*e^(8*c
))*e^(8*d*x) + 8*(16*a^3*b^3*d*e^(6*c) - 39*a^2*b^4*d*e^(6*c) + 30*a*b^5*d
*e^(6*c) - 7*b^6*d*e^(6*c))*e^(6*d*x) - 4*(8*a^3*b^3*d*e^(4*c) - 23*a^2*b^
4*d*e^(4*c) + 22*a*b^5*d*e^(4*c) - 7*b^6*d*e^(4*c))*e^(4*d*x) - 8*(a^2*b^4
*d*e^(2*c) - 2*a*b^5*d*e^(2*c) + b^6*d*e^(2*c))*e^(2*d*x)) - 1/512*integra
te(64*((2*a*b*e^(7*c) - 5*b^2*e^(7*c))*e^(7*d*x) + (10*a^2*e^(5*c) - 28*a*
b*e^(5*c) + 39*b^2*e^(5*c))*e^(5*d*x) - (10*a^2*e^(3*c) - 28*a*b*e^(3*c...
```

Giac [F]

$$\int \frac{\sinh^9(c+dx)}{(a-b\sinh^4(c+dx))^3}\,dx = \int -\frac{\sinh(dx+c)^9}{(b\sinh(dx+c)^4-a)^3}\,dx$$

input

```
integrate(sinh(d*x+c)^9/(a-b*sinh(d*x+c)^4)^3,x, algorithm="giac")
```

output

```
sage0*x
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sinh^9(c+dx)}{(a-b\sinh^4(c+dx))^3}\,dx = \int \frac{\sinh(c+d\,x)^9}{\left(a-b\sinh\left(c+d\,x\right)^4\right)^3}\,dx$$

input
```
int(sinh(c + d*x)^9/(a - b*sinh(c + d*x)^4)^3,x)
```

output
```
int(sinh(c + d*x)^9/(a - b*sinh(c + d*x)^4)^3, x)
```

Reduce [F]

$$\int \frac{\sinh^9(c+dx)}{(a-b\sinh^4(c+dx))^3}\,dx = \text{too large to display}$$

input
```
int(sinh(d*x+c)^9/(a-b*sinh(d*x+c)^4)^3,x)
```

output

```
(8*e**c*( - 104509440*e**(22*c + 16*d*x)*int(e**(7*d*x)/(e**(24*c + 24*d*x
)*b**3 - 12*e**(22*c + 22*d*x)*b**3 - 48*e**(20*c + 20*d*x)*a*b**2 + 66*e*
*(20*c + 20*d*x)*b**3 + 384*e**(18*c + 18*d*x)*a*b**2 - 220*e**(18*c + 18*
d*x)*b**3 + 768*e**(16*c + 16*d*x)*a**2*b - 1344*e**(16*c + 16*d*x)*a*b**2
 + 495*e**(16*c + 16*d*x)*b**3 - 3072*e**(14*c + 14*d*x)*a**2*b + 2688*e**
(14*c + 14*d*x)*a*b**2 - 792*e**(14*c + 14*d*x)*b**3 - 4096*e**(12*c + 12*
d*x)*a**3 + 4608*e**(12*c + 12*d*x)*a**2*b - 3360*e**(12*c + 12*d*x)*a*b**
2 + 924*e**(12*c + 12*d*x)*b**3 - 3072*e**(10*c + 10*d*x)*a**2*b + 2688*e*
*(10*c + 10*d*x)*a*b**2 - 792*e**(10*c + 10*d*x)*b**3 + 768*e**(8*c + 8*d*
x)*a**2*b - 1344*e**(8*c + 8*d*x)*a*b**2 + 495*e**(8*c + 8*d*x)*b**3 + 384
*e**(6*c + 6*d*x)*a*b**2 - 220*e**(6*c + 6*d*x)*b**3 - 48*e**(4*c + 4*d*x)
*a*b**2 + 66*e**(4*c + 4*d*x)*b**3 - 12*e**(2*c + 2*d*x)*b**3 + b**3),x)*a
**3*b**3*d - 41349632*e**(22*c + 16*d*x)*int(e**(7*d*x)/(e**(24*c + 24*d*x
)*b**3 - 12*e**(22*c + 22*d*x)*b**3 - 48*e**(20*c + 20*d*x)*a*b**2 + 66*e*
*(20*c + 20*d*x)*b**3 + 384*e**(18*c + 18*d*x)*a*b**2 - 220*e**(18*c + 18*
d*x)*b**3 + 768*e**(16*c + 16*d*x)*a**2*b - 1344*e**(16*c + 16*d*x)*a*b**2
 + 495*e**(16*c + 16*d*x)*b**3 - 3072*e**(14*c + 14*d*x)*a**2*b + 2688*e**
(14*c + 14*d*x)*a*b**2 - 792*e**(14*c + 14*d*x)*b**3 - 4096*e**(12*c + 12*
d*x)*a**3 + 4608*e**(12*c + 12*d*x)*a**2*b - 3360*e**(12*c + 12*d*x)*a*b**
2 + 924*e**(12*c + 12*d*x)*b**3 - 3072*e**(10*c + 10*d*x)*a**2*b + 2688...
```

3.229 $\int \frac{\sinh^7(c+dx)}{\left(a-b\sinh^4(c+dx)\right)^3}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 290

$$
\begin{aligned}
\int \frac{\sinh^7(c+dx)}{\left(a-b\sinh^4(c+dx)\right)^3}\,dx
&= \frac{3\left(\sqrt{a}-2\sqrt{b}\right)\arctan\left(\frac{\sqrt[4]{b}\cosh(c+dx)}{\sqrt{\sqrt{a}-\sqrt{b}}}\right)}{64\sqrt{a}\left(\sqrt{a}-\sqrt{b}\right)^{5/2}b^{7/4}d} - \frac{3\left(\sqrt{a}+2\sqrt{b}\right)\operatorname{arctanh}\left(\frac{\sqrt[4]{b}\cosh(c+dx)}{\sqrt{\sqrt{a}+\sqrt{b}}}\right)}{64\sqrt{a}\left(\sqrt{a}+\sqrt{b}\right)^{5/2}b^{7/4}d} \\
&\quad - \frac{a\cosh(c+dx)\left(2-\cosh^2(c+dx)\right)}{8(a-b)bd\left(a-b+2b\cosh^2(c+dx)-b\cosh^4(c+dx)\right)^2} \\
&\quad + \frac{\cosh(c+dx)\left(5a-17b-3(a-3b)\cosh^2(c+dx)\right)}{32(a-b)^2bd\left(a-b+2b\cosh^2(c+dx)-b\cosh^4(c+dx)\right)}
\end{aligned}
$$

output

```
3/64*(a^(1/2)-2*b^(1/2))*arctan(b^(1/4)*cosh(d*x+c)/(a^(1/2)-b^(1/2))^(1/2
))/a^(1/2)/(a^(1/2)-b^(1/2))^(5/2)/b^(7/4)/d-3/64*(a^(1/2)+2*b^(1/2))*arct
anh(b^(1/4)*cosh(d*x+c)/(a^(1/2)+b^(1/2))^(1/2))/a^(1/2)/(a^(1/2)+b^(1/2))
^(5/2)/b^(7/4)/d-1/8*a*cosh(d*x+c)*(2-cosh(d*x+c)^2)/(a-b)/b/d/(a-b+2*b*co
sh(d*x+c)^2-b*cosh(d*x+c)^4)^2+1/32*cosh(d*x+c)*(5*a-17*b-3*(a-3*b)*cosh(d
*x+c)^2)/(a-b)^2/b/d/(a-b+2*b*cosh(d*x+c)^2-b*cosh(d*x+c)^4)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

Time = 6.46 (sec) , antiderivative size = 802, normalized size of antiderivative = 2.77

$$\int \frac{\sinh^7(c+dx)}{\left(a-b\sinh^4(c+dx)\right)^3}\,dx = \text{Too large to display}$$

input
```
Integrate[Sinh[c + d*x]^7/(a - b*Sinh[c + d*x]^4)^3,x]
```

output
```
((-32*Cosh[c + d*x]*(-7*a + 25*b + 3*(a - 3*b)*Cosh[2*(c + d*x)]))/(8*a -
3*b + 4*b*Cosh[2*(c + d*x)] - b*Cosh[4*(c + d*x)]) + (512*a*(a - b)*(-5*Co
sh[c + d*x] + Cosh[3*(c + d*x)]))/(-8*a + 3*b - 4*b*Cosh[2*(c + d*x)] + b*
Cosh[4*(c + d*x)])^2 - 3*RootSum[b - 4*b*#1^2 - 16*a*#1^4 + 6*b*#1^4 - 4*b
*#1^6 + b*#1^8 & , (a*c - 3*b*c + a*d*x - 3*b*d*x + 2*a*Log[-Cosh[(c + d*x
)/2] - Sinh[(c + d*x)/2] + Cosh[(c + d*x)/2]*#1 - Sinh[(c + d*x)/2]*#1] -
6*b*Log[-Cosh[(c + d*x)/2] - Sinh[(c + d*x)/2] + Cosh[(c + d*x)/2]*#1 - Si
nh[(c + d*x)/2]*#1] - 3*a*c*#1^2 + 17*b*c*#1^2 - 3*a*d*x*#1^2 + 17*b*d*x*#
1^2 - 6*a*Log[-Cosh[(c + d*x)/2] - Sinh[(c + d*x)/2] + Cosh[(c + d*x)/2]*#
1 - Sinh[(c + d*x)/2]*#1]*#1^2 + 34*b*Log[-Cosh[(c + d*x)/2] - Sinh[(c + d
*x)/2] + Cosh[(c + d*x)/2]*#1 - Sinh[(c + d*x)/2]*#1]*#1^2 + 3*a*c*#1^4 -
17*b*c*#1^4 + 3*a*d*x*#1^4 - 17*b*d*x*#1^4 + 6*a*Log[-Cosh[(c + d*x)/2] -
Sinh[(c + d*x)/2] + Cosh[(c + d*x)/2]*#1 - Sinh[(c + d*x)/2]*#1]*#1^4 - 34
*b*Log[-Cosh[(c + d*x)/2] - Sinh[(c + d*x)/2] + Cosh[(c + d*x)/2]*#1 - Sin
h[(c + d*x)/2]*#1]*#1^4 - a*c*#1^6 + 3*b*c*#1^6 - a*d*x*#1^6 + 3*b*d*x*#1^
6 - 2*a*Log[-Cosh[(c + d*x)/2] - Sinh[(c + d*x)/2] + Cosh[(c + d*x)/2]*#1
- Sinh[(c + d*x)/2]*#1]*#1^6 + 6*b*Log[-Cosh[(c + d*x)/2] - Sinh[(c + d*x)
/2] + Cosh[(c + d*x)/2]*#1 - Sinh[(c + d*x)/2]*#1]*#1^6)/(-(b*#1) - 8*a*#1
^3 + 3*b*#1^3 - 3*b*#1^5 + b*#1^7) & ])/(256*(a - b)^2*b*d)
```

Rubi [A] (verified)

Time = 0.64 (sec) , antiderivative size = 305, normalized size of antiderivative = 1.05, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.417, Rules used = {3042, 26, 3694, 1517, 27, 1492, 27, 1480, 218, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sinh^7(c+dx)}{\left(a-b\sinh^4(c+dx)\right)^3}dx$$

$$\Big\downarrow \; 3042$$

$$\int \frac{i\sin(ic+idx)^7}{(a-b\sin(ic+idx)^4)^3}dx$$

$$\Big\downarrow \; 26$$

$$i\int \frac{\sin(ic+idx)^7}{(a-b\sin(ic+idx)^4)^3}dx$$

$$\Big\downarrow \; 3694$$

$$-\frac{\int \frac{\left(1-\cosh^2(c+dx)\right)^3}{\left(-b\cosh^4(c+dx)+2b\cosh^2(c+dx)+a-b\right)^3}d\cosh(c+dx)}{d}$$

$$\Big\downarrow \; 1517$$

$$-\frac{\frac{a\cosh(c+dx)\left(2-\cosh^2(c+dx)\right)}{8b(a-b)\left(a-b\cosh^4(c+dx)+2b\cosh^2(c+dx)-b\right)^2}-\frac{\int \frac{2a\left(2(a-4b)-(3a-8b)\cosh^2(c+dx)\right)}{\left(-b\cosh^4(c+dx)+2b\cosh^2(c+dx)+a-b\right)^2}d\cosh(c+dx)}{16ab(a-b)}}{d}$$

$$\Big\downarrow \; 27$$

$$-\frac{\frac{a\cosh(c+dx)\left(2-\cosh^2(c+dx)\right)}{8b(a-b)\left(a-b\cosh^4(c+dx)+2b\cosh^2(c+dx)-b\right)^2}-\frac{\int \frac{2(a-4b)-(3a-8b)\cosh^2(c+dx)}{\left(-b\cosh^4(c+dx)+2b\cosh^2(c+dx)+a-b\right)^2}d\cosh(c+dx)}{8b(a-b)}}{d}$$

$$\Big\downarrow \; 1492$$

$$-\frac{\frac{a\cosh(c+dx)\left(2-\cosh^2(c+dx)\right)}{8b(a-b)\left(a-b\cosh^4(c+dx)+2b\cosh^2(c+dx)-b\right)^2}-\frac{\frac{\cosh(c+dx)\left(-3(a-3b)\cosh^2(c+dx)+5a-17b\right)}{4(a-b)\left(a-b\cosh^4(c+dx)+2b\cosh^2(c+dx)-b\right)}-\frac{\int -\frac{6ab\left(-\left((a-3b)\cosh^2(c+dx)\right)+a-5b\right)}{-b\cosh^4(c+dx)+2b\cosh^2(c+dx)+a-b}d\cosh}{8ab(a-b)}}{8b(a-b)}}{d}$$

$\Big\downarrow$ 27

$$-\frac{\frac{a\cosh(c+dx)\left(2-\cosh^2(c+dx)\right)}{8b(a-b)\left(a-b\cosh^4(c+dx)+2b\cosh^2(c+dx)-b\right)^2}-\frac{\frac{3\int\frac{-\left((a-3b)\cosh^2(c+dx)\right)+a-5b}{-b\cosh^4(c+dx)+2b\cosh^2(c+dx)+a-b}d\cosh(c+dx)}{4(a-b)}+\frac{\cosh(c+dx)\left(-3(a-3b)\cosh^2(c+dx)+5a\ldots\right.}{4(a-b)\left(a-b\cosh^4(c+dx)+2b\cosh^2(c+\ldots\right.}}{8b(a-b)}}{d}$$

$\Big\downarrow$ 1480

$$-\frac{\frac{a\cosh(c+dx)\left(2-\cosh^2(c+dx)\right)}{8b(a-b)\left(a-b\cosh^4(c+dx)+2b\cosh^2(c+dx)-b\right)^2}-\frac{\frac{3\left(-\frac{1}{2}\left(-\frac{2b^{3/2}}{\sqrt{a}}+a-3b\right)\int\frac{1}{-b\cosh^2(c+dx)-\left(\sqrt{a}-\sqrt{b}\right)\sqrt{b}}d\cosh(c+dx)-\frac{1}{2}\left(\frac{2b^{3/2}}{\sqrt{a}}+a-3b\right)\int\frac{\ldots}{\left(\sqrt{\ldots}\right.}\ldots\right.}{4(a-b)}\ldots}{8b(a-b)}}{d}$$

$\Big\downarrow$ 218

$$-\frac{\frac{a\cosh(c+dx)\left(2-\cosh^2(c+dx)\right)}{8b(a-b)\left(a-b\cosh^4(c+dx)+2b\cosh^2(c+dx)-b\right)^2}-\frac{\frac{3\left(\frac{\left(-\frac{2b^{3/2}}{\sqrt{a}}+a-3b\right)\arctan\left(\frac{\sqrt[4]{b}\cosh(c+dx)}{\sqrt{\sqrt{a}-\sqrt{b}}}\right)}{2b^{3/4}\sqrt{\sqrt{a}-\sqrt{b}}}-\frac{1}{2}\left(\frac{2b^{3/2}}{\sqrt{a}}+a-3b\right)\int\frac{1}{\left(\sqrt{a}+\sqrt{b}\right)\sqrt{b}-b\cosh^2(c+\ldots}\ldots\right.}{4(a-b)}\ldots}{8b(a-b)}}{d}$$

$\Big\downarrow$ 221

$$-\frac{\frac{a\cosh(c+dx)\left(2-\cosh^2(c+dx)\right)}{8b(a-b)\left(a-b\cosh^4(c+dx)+2b\cosh^2(c+dx)-b\right)^2}-\frac{\frac{3\left(\frac{\left(-\frac{2b^{3/2}}{\sqrt{a}}+a-3b\right)\arctan\left(\frac{\sqrt[4]{b}\cosh(c+dx)}{\sqrt{\sqrt{a}-\sqrt{b}}}\right)}{2b^{3/4}\sqrt{\sqrt{a}-\sqrt{b}}}-\frac{\left(\frac{2b^{3/2}}{\sqrt{a}}+a-3b\right)\text{arctanh}\left(\frac{\sqrt[4]{b}\cosh(c+dx)}{\sqrt{\sqrt{a}+\sqrt{b}}}\ldots\right.}{2b^{3/4}\sqrt{\sqrt{a}+\sqrt{b}}}\ldots\right.}{4(a-b)}\ldots}{8b(a-b)}}{d}$$

input

```
Int[Sinh[c + d*x]^7/(a - b*Sinh[c + d*x]^4)^3,x]
```

output
```
-(((a*Cosh[c + d*x]*(2 - Cosh[c + d*x]^2))/(8*(a - b)*b*(a - b + 2*b*Cosh[
c + d*x]^2 - b*Cosh[c + d*x]^4)^2) - ((3*(((a - 3*b - (2*b^(3/2))/Sqrt[a])
*ArcTan[(b^(1/4)*Cosh[c + d*x])/Sqrt[Sqrt[a] - Sqrt[b]]])/(2*Sqrt[Sqrt[a]
- Sqrt[b]]*b^(3/4)) - ((a - 3*b + (2*b^(3/2))/Sqrt[a])*ArcTanh[(b^(1/4)*Co
sh[c + d*x])/Sqrt[Sqrt[a] + Sqrt[b]]])/(2*Sqrt[Sqrt[a] + Sqrt[b]]*b^(3/4))
))/(4*(a - b)) + (Cosh[c + d*x]*(5*a - 17*b - 3*(a - 3*b)*Cosh[c + d*x]^2)
)/(4*(a - b)*(a - b + 2*b*Cosh[c + d*x]^2 - b*Cosh[c + d*x]^4)))/(8*(a - b
)*b))/d)
```

Defintions of rubi rules used

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 1480
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Simp[(e/2 + (2*c*d - b*e)/(2*q))   Int[1/(
b/2 - q/2 + c*x^2), x], x] + Simp[(e/2 - (2*c*d - b*e)/(2*q))   Int[1/(b/2
+ q/2 + c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0]
 && NeQ[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

rule 1492
```
Int[((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symb
ol] :> Simp[x*(a*b*e - d*(b^2 - 2*a*c) - c*(b*d - 2*a*e)*x^2)*((a + b*x^2 +
 c*x^4)^(p + 1)/(2*a*(p + 1)*(b^2 - 4*a*c))), x] + Simp[1/(2*a*(p + 1)*(b^2
 - 4*a*c))   Int[Simp[(2*p + 3)*d*b^2 - a*b*e - 2*a*c*d*(4*p + 5) + (4*p +
7)*(d*b - 2*a*e)*c*x^2, x]*(a + b*x^2 + c*x^4)^(p + 1), x], x] /; FreeQ[{a,
 b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] &&
 LtQ[p, -1] && IntegerQ[2*p]
```

rule 1517
```
Int[((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x
_Symbol] :> With[{f = Coeff[PolynomialRemainder[(d + e*x^2)^q, a + b*x^2 +
c*x^4, x], x, 0], g = Coeff[PolynomialRemainder[(d + e*x^2)^q, a + b*x^2 +
c*x^4, x], x, 2]}, Simp[x*(a + b*x^2 + c*x^4)^(p + 1)*((a*b*g - f*(b^2 - 2*
a*c) - c*(b*f - 2*a*g)*x^2)/(2*a*(p + 1)*(b^2 - 4*a*c))), x] + Simp[1/(2*a*
(p + 1)*(b^2 - 4*a*c))   Int[(a + b*x^2 + c*x^4)^(p + 1)*ExpandToSum[2*a*(p
 + 1)*(b^2 - 4*a*c)*PolynomialQuotient[(d + e*x^2)^q, a + b*x^2 + c*x^4, x]
 + b^2*f*(2*p + 3) - 2*a*c*f*(4*p + 5) - a*b*g + c*(4*p + 7)*(b*f - 2*a*g)*
x^2, x], x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ
[c*d^2 - b*d*e + a*e^2, 0] && IGtQ[q, 1] && LtQ[p, -1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3694
```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^4)^
(p_.), x_Symbol] :> With[{ff = FreeFactors[Cos[e + f*x], x]}, Simp[-ff/f
Subst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a + b - 2*b*ff^2*x^2 + b*ff^4*x^4)^p,
 x], x, Cos[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m -
 1)/2]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 584 vs. $2(238) = 476$.

Time = 21.82 (sec) , antiderivative size = 585, normalized size of antiderivative = 2.02

method	result
derivativedivides	$\frac{-\frac{3a\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^{14}}{8\left(a^2-2ab+b^2\right)}-\frac{3(a-10b)a\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^{12}}{8b\left(a^2-2ab+b^2\right)}+\frac{\left(16a^2-111ab+80b^2\right)\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^{10}}{8b\left(a^2-2ab+b^2\right)}-\frac{\left(35a^3-26a^2b-64b^2a+256b^3\right)\tanh\left(\right.}{8ab\left(a^2-2ab+b^2\right)}}{\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^8a-4\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^6a+6\tanh\left(\frac{dx}{2}\right.\right.}$
default	$\frac{-\frac{3a\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^{14}}{8\left(a^2-2ab+b^2\right)}-\frac{3(a-10b)a\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^{12}}{8b\left(a^2-2ab+b^2\right)}+\frac{\left(16a^2-111ab+80b^2\right)\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^{10}}{8b\left(a^2-2ab+b^2\right)}-\frac{\left(35a^3-26a^2b-64b^2a+256b^3\right)\tanh\left(\right.}{8ab\left(a^2-2ab+b^2\right)}}{\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^8a-4\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^6a+6\tanh\left(\frac{dx}{2}\right.\right.}$
risch	Expression too large to display

input
```
int(sinh(d*x+c)^7/(a-b*sinh(d*x+c)^4)^3,x,method=_RETURNVERBOSE)
```

output
```
1/d*(128*(-3/1024*a/(a^2-2*a*b+b^2)*tanh(1/2*d*x+1/2*c)^14-3/1024/b*(a-10*
b)*a/(a^2-2*a*b+b^2)*tanh(1/2*d*x+1/2*c)^12+1/1024/b*(16*a^2-111*a*b+80*b^
2)/(a^2-2*a*b+b^2)*tanh(1/2*d*x+1/2*c)^10-1/1024*(35*a^3-26*a^2*b-64*a*b^2
+256*b^3)/a/b/(a^2-2*a*b+b^2)*tanh(1/2*d*x+1/2*c)^8+1/1024*(40*a^2+95*a*b-
336*b^2)/b/(a^2-2*a*b+b^2)*tanh(1/2*d*x+1/2*c)^6-1/1024*(25*a^2+54*a*b-64*
b^2)/b/(a^2-2*a*b+b^2)*tanh(1/2*d*x+1/2*c)^4+1/1024*(8*a+19*b)*a/b/(a^2-2*
a*b+b^2)*tanh(1/2*d*x+1/2*c)^2-1/1024*a*(a+2*b)/b/(a^2-2*a*b+b^2))/(tanh(1
/2*d*x+1/2*c)^8*a-4*tanh(1/2*d*x+1/2*c)^6*a+6*tanh(1/2*d*x+1/2*c)^4*a-16*b
*tanh(1/2*d*x+1/2*c)^4-4*tanh(1/2*d*x+1/2*c)^2*a+a)^2+3/8/b/(a^2-2*a*b+b^2
)*a*(-1/8*(-(a*b)^(1/2)*a+3*(a*b)^(1/2)*b-2*b^2)/a/b/(-a*b-(a*b)^(1/2)*a)^
(1/2)*arctan(1/4*(-2*tanh(1/2*d*x+1/2*c)^2*a+4*(a*b)^(1/2)+2*a)/(-a*b-(a*b
)^(1/2)*a)^(1/2))+1/8*((a*b)^(1/2)*a-3*(a*b)^(1/2)*b-2*b^2)/a/b/(-a*b+(a*b
)^(1/2)*a)^(1/2)*arctan(1/4*(2*tanh(1/2*d*x+1/2*c)^2*a+4*(a*b)^(1/2)-2*a)/
(-a*b+(a*b)^(1/2)*a)^(1/2))))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 20362 vs. $2(234) = 468$.

Time $= 0.46$ (sec) , antiderivative size $= 20362$, normalized size of antiderivative $= 70.21$

$$\int \frac{\sinh^7(c+dx)}{(a-b\sinh^4(c+dx))^3}\,dx = \text{Too large to display}$$

input
```
integrate(sinh(d*x+c)^7/(a-b*sinh(d*x+c)^4)^3,x, algorithm="fricas")
```

output
```
Too large to include
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\sinh^7(c+dx)}{(a-b\sinh^4(c+dx))^3}\,dx = \text{Timed out}$$

input
```
integrate(sinh(d*x+c)**7/(a-b*sinh(d*x+c)**4)**3,x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{\sinh^7(c+dx)}{(a-b\sinh^4(c+dx))^3}\,dx = \int -\frac{\sinh(dx+c)^7}{(b\sinh(dx+c)^4-a)^3}\,dx$$

input
```
integrate(sinh(d*x+c)^7/(a-b*sinh(d*x+c)^4)^3,x, algorithm="maxima")
```

output
```
1/16*(3*(a*b*e^(15*c) - 3*b^2*e^(15*c))*e^(15*d*x) - (23*a*b*e^(13*c) - 77
*b^2*e^(13*c))*e^(13*d*x) + (16*a^2*e^(11*c) + 131*a*b*e^(11*c) - 177*b^2*
e^(11*c))*e^(11*d*x) - (144*a^2*e^(9*c) + 367*a*b*e^(9*c) - 109*b^2*e^(9*c
))*e^(9*d*x) - (144*a^2*e^(7*c) + 367*a*b*e^(7*c) - 109*b^2*e^(7*c))*e^(7*
d*x) + (16*a^2*e^(5*c) + 131*a*b*e^(5*c) - 177*b^2*e^(5*c))*e^(5*d*x) - (2
3*a*b*e^(3*c) - 77*b^2*e^(3*c))*e^(3*d*x) + 3*(a*b*e^c - 3*b^2*e^c)*e^(d*x
))/(a^2*b^3*d - 2*a*b^4*d + b^5*d + (a^2*b^3*d*e^(16*c) - 2*a*b^4*d*e^(16*
c) + b^5*d*e^(16*c))*e^(16*d*x) - 8*(a^2*b^3*d*e^(14*c) - 2*a*b^4*d*e^(14*
c) + b^5*d*e^(14*c))*e^(14*d*x) - 4*(8*a^3*b^2*d*e^(12*c) - 23*a^2*b^3*d*e
^(12*c) + 22*a*b^4*d*e^(12*c) - 7*b^5*d*e^(12*c))*e^(12*d*x) + 8*(16*a^3*b
^2*d*e^(10*c) - 39*a^2*b^3*d*e^(10*c) + 30*a*b^4*d*e^(10*c) - 7*b^5*d*e^(1
0*c))*e^(10*d*x) + 2*(128*a^4*b*d*e^(8*c) - 352*a^3*b^2*d*e^(8*c) + 355*a^
2*b^3*d*e^(8*c) - 166*a*b^4*d*e^(8*c) + 35*b^5*d*e^(8*c))*e^(8*d*x) + 8*(1
6*a^3*b^2*d*e^(6*c) - 39*a^2*b^3*d*e^(6*c) + 30*a*b^4*d*e^(6*c) - 7*b^5*d*
e^(6*c))*e^(6*d*x) - 4*(8*a^3*b^2*d*e^(4*c) - 23*a^2*b^3*d*e^(4*c) + 22*a*
b^4*d*e^(4*c) - 7*b^5*d*e^(4*c))*e^(4*d*x) - 8*(a^2*b^3*d*e^(2*c) - 2*a*b^
4*d*e^(2*c) + b^5*d*e^(2*c))*e^(2*d*x)) + 1/128*integrate(24*((a*e^(7*c) -
 3*b*e^(7*c))*e^(7*d*x) - (3*a*e^(5*c) - 17*b*e^(5*c))*e^(5*d*x) + (3*a*e^
(3*c) - 17*b*e^(3*c))*e^(3*d*x) - (a*e^c - 3*b*e^c)*e^(d*x))/(a^2*b^2 - 2*
a*b^3 + b^4 + (a^2*b^2*e^(8*c) - 2*a*b^3*e^(8*c) + b^4*e^(8*c))*e^(8*d*...
```

Giac [F]

$$\int \frac{\sinh^7(c+dx)}{\left(a-b\sinh^4(c+dx)\right)^3}\,dx=\int -\frac{\sinh\left(dx+c\right)^7}{\left(b\sinh\left(dx+c\right)^4-a\right)^3}\,dx$$

input
```
integrate(sinh(d*x+c)^7/(a-b*sinh(d*x+c)^4)^3,x, algorithm="giac")
```

output
```
sage0*x
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sinh^7(c+dx)}{\left(a-b\sinh^4(c+dx)\right)^3}\,dx = \int \frac{\sinh(c+d\,x)^7}{\left(a-b\sinh\left(c+d\,x\right)^4\right)^3}\,dx$$

input
```
int(sinh(c + d*x)^7/(a - b*sinh(c + d*x)^4)^3,x)
```

output
```
int(sinh(c + d*x)^7/(a - b*sinh(c + d*x)^4)^3, x)
```

Reduce [F]

$$\int \frac{\sinh^7(c+dx)}{\left(a-b\sinh^4(c+dx)\right)^3}\,dx = \text{too large to display}$$

input
```
int(sinh(d*x+c)^7/(a-b*sinh(d*x+c)^4)^3,x)
```

output
```
(32*e**c*(1576960*e**(22*c + 16*d*x)*int(e**(7*d*x)/(e**(24*c + 24*d*x)*b*
*3 - 12*e**(22*c + 22*d*x)*b**3 - 48*e**(20*c + 20*d*x)*a*b**2 + 66*e**(20
*c + 20*d*x)*b**3 + 384*e**(18*c + 18*d*x)*a*b**2 - 220*e**(18*c + 18*d*x)
*b**3 + 768*e**(16*c + 16*d*x)*a**2*b - 1344*e**(16*c + 16*d*x)*a*b**2 + 4
95*e**(16*c + 16*d*x)*b**3 - 3072*e**(14*c + 14*d*x)*a**2*b + 2688*e**(14*
c + 14*d*x)*a*b**2 - 792*e**(14*c + 14*d*x)*b**3 - 4096*e**(12*c + 12*d*x)
*a**3 + 4608*e**(12*c + 12*d*x)*a**2*b - 3360*e**(12*c + 12*d*x)*a*b**2 +
924*e**(12*c + 12*d*x)*b**3 - 3072*e**(10*c + 10*d*x)*a**2*b + 2688*e**(10
*c + 10*d*x)*a*b**2 - 792*e**(10*c + 10*d*x)*b**3 + 768*e**(8*c + 8*d*x)*a
**2*b - 1344*e**(8*c + 8*d*x)*a*b**2 + 495*e**(8*c + 8*d*x)*b**3 + 384*e**
(6*c + 6*d*x)*a*b**2 - 220*e**(6*c + 6*d*x)*b**3 - 48*e**(4*c + 4*d*x)*a*b
**2 + 66*e**(4*c + 4*d*x)*b**3 - 12*e**(2*c + 2*d*x)*b**3 + b**3),x)*a**3*
b**2*d + 1664768*e**(22*c + 16*d*x)*int(e**(7*d*x)/(e**(24*c + 24*d*x)*b**
3 - 12*e**(22*c + 22*d*x)*b**3 - 48*e**(20*c + 20*d*x)*a*b**2 + 66*e**(20*
c + 20*d*x)*b**3 + 384*e**(18*c + 18*d*x)*a*b**2 - 220*e**(18*c + 18*d*x)*
b**3 + 768*e**(16*c + 16*d*x)*a**2*b - 1344*e**(16*c + 16*d*x)*a*b**2 + 49
5*e**(16*c + 16*d*x)*b**3 - 3072*e**(14*c + 14*d*x)*a**2*b + 2688*e**(14*c
 + 14*d*x)*a*b**2 - 792*e**(14*c + 14*d*x)*b**3 - 4096*e**(12*c + 12*d*x)*
a**3 + 4608*e**(12*c + 12*d*x)*a**2*b - 3360*e**(12*c + 12*d*x)*a*b**2 + 9
24*e**(12*c + 12*d*x)*b**3 - 3072*e**(10*c + 10*d*x)*a**2*b + 2688*e**(...
```

3.230 $\int \frac{\sinh^5(c+dx)}{\left(a-b\sinh^4(c+dx)\right)^3}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 313

$$\begin{aligned}\int \frac{\sinh^5(c+dx)}{\left(a-b\sinh^4(c+dx)\right)^3}\,dx &= -\frac{\left(3a-10\sqrt{a}\sqrt{b}+4b\right)\arctan\left(\frac{\sqrt[4]{b}\cosh(c+dx)}{\sqrt{\sqrt{a}-\sqrt{b}}}\right)}{64a^{3/2}\left(\sqrt{a}-\sqrt{b}\right)^{5/2}b^{5/4}d} \\ &\quad - \frac{\left(3a+10\sqrt{a}\sqrt{b}+4b\right)\operatorname{arctanh}\left(\frac{\sqrt[4]{b}\cosh(c+dx)}{\sqrt{\sqrt{a}+\sqrt{b}}}\right)}{64a^{3/2}\left(\sqrt{a}+\sqrt{b}\right)^{5/2}b^{5/4}d} \\ &\quad + \frac{\cosh(c+dx)\left(a+b-b\cosh^2(c+dx)\right)}{8(a-b)bd\left(a-b+2b\cosh^2(c+dx)-b\cosh^4(c+dx)\right)^2} \\ &\quad - \frac{\cosh(c+dx)\left(a^2-11ab-2b^2+2b(2a+b)\cosh^2(c+dx)\right)}{32a(a-b)^2bd\left(a-b+2b\cosh^2(c+dx)-b\cosh^4(c+dx)\right)}\end{aligned}$$

output
```
-1/64*(3*a-10*a^(1/2)*b^(1/2)+4*b)*arctan(b^(1/4)*cosh(d*x+c)/(a^(1/2)-b^(
1/2))^(1/2))/a^(3/2)/(a^(1/2)-b^(1/2))^(5/2)/b^(5/4)/d-1/64*(3*a+10*a^(1/2
)*b^(1/2)+4*b)*arctanh(b^(1/4)*cosh(d*x+c)/(a^(1/2)+b^(1/2))^(1/2))/a^(3/2
)/(a^(1/2)+b^(1/2))^(5/2)/b^(5/4)/d+1/8*cosh(d*x+c)*(a+b-b*cosh(d*x+c)^2)/
(a-b)/b/d/(a-b+2*b*cosh(d*x+c)^2-b*cosh(d*x+c)^4)^2-1/32*cosh(d*x+c)*(a^2-
11*a*b-2*b^2+2*b*(2*a+b)*cosh(d*x+c)^2)/a/(a-b)^2/b/d/(a-b+2*b*cosh(d*x+c)
^2-b*cosh(d*x+c)^4)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

Time = 6.89 (sec) , antiderivative size = 1019, normalized size of antiderivative = 3.26

$$\int \frac{\sinh^5(c+dx)}{(a-b\sinh^4(c+dx))^3}\,dx = \text{Too large to display}$$

input
```
Integrate[Sinh[c + d*x]^5/(a - b*Sinh[c + d*x]^4)^3,x]
```

output
```
-1/128*((32*Cosh[c + d*x]*(a^2 - 9*a*b - b^2 + b*(2*a + b)*Cosh[2*(c + d*x
)]))/(a*(8*a - 3*b + 4*b*Cosh[2*(c + d*x)] - b*Cosh[4*(c + d*x)])) - (512*
(a - b)*Cosh[c + d*x]*(2*a + b - b*Cosh[2*(c + d*x)]))/(-8*a + 3*b - 4*b*C
osh[2*(c + d*x)] + b*Cosh[4*(c + d*x)])^2 + RootSum[b - 4*b*#1^2 - 16*a*#1
^4 + 6*b*#1^4 - 4*b*#1^6 + b*#1^8 & , (2*a*b*c + b^2*c + 2*a*b*d*x + b^2*d
*x + 4*a*b*Log[-Cosh[(c + d*x)/2] - Sinh[(c + d*x)/2] + Cosh[(c + d*x)/2]*
#1 - Sinh[(c + d*x)/2]*#1] + 2*b^2*Log[-Cosh[(c + d*x)/2] - Sinh[(c + d*x)
/2] + Cosh[(c + d*x)/2]*#1 - Sinh[(c + d*x)/2]*#1] + 6*a^2*c*#1^2 - 32*a*b
*c*#1^2 + 5*b^2*c*#1^2 + 6*a^2*d*x*#1^2 - 32*a*b*d*x*#1^2 + 5*b^2*d*x*#1^2
 + 12*a^2*Log[-Cosh[(c + d*x)/2] - Sinh[(c + d*x)/2] + Cosh[(c + d*x)/2]*#
1 - Sinh[(c + d*x)/2]*#1]*#1^2 - 64*a*b*Log[-Cosh[(c + d*x)/2] - Sinh[(c +
 d*x)/2] + Cosh[(c + d*x)/2]*#1 - Sinh[(c + d*x)/2]*#1]*#1^2 + 10*b^2*Log[
-Cosh[(c + d*x)/2] - Sinh[(c + d*x)/2] + Cosh[(c + d*x)/2]*#1 - Sinh[(c +
d*x)/2]*#1]*#1^2 - 6*a^2*c*#1^4 + 32*a*b*c*#1^4 - 5*b^2*c*#1^4 - 6*a^2*d*x
*#1^4 + 32*a*b*d*x*#1^4 - 5*b^2*d*x*#1^4 - 12*a^2*Log[-Cosh[(c + d*x)/2] -
 Sinh[(c + d*x)/2] + Cosh[(c + d*x)/2]*#1 - Sinh[(c + d*x)/2]*#1]*#1^4 + 6
4*a*b*Log[-Cosh[(c + d*x)/2] - Sinh[(c + d*x)/2] + Cosh[(c + d*x)/2]*#1 -
Sinh[(c + d*x)/2]*#1]*#1^4 - 10*b^2*Log[-Cosh[(c + d*x)/2] - Sinh[(c + d*x
)/2] + Cosh[(c + d*x)/2]*#1 - Sinh[(c + d*x)/2]*#1]*#1^4 - 2*a*b*c*#1^6 -
b^2*c*#1^6 - 2*a*b*d*x*#1^6 - b^2*d*x*#1^6 - 4*a*b*Log[-Cosh[(c + d*x)/...
```

Rubi [A] (verified)

Time = 0.66 (sec) , antiderivative size = 360, normalized size of antiderivative = 1.15, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.417$, Rules used = {3042, 26, 3694, 1517, 27, 1492, 27, 1480, 218, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sinh^5(c+dx)}{\left(a-b\sinh^4(c+dx)\right)^3}dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int -\frac{i\sin(ic+idx)^5}{(a-b\sin(ic+idx)^4)^3}dx$$

$$\Big\downarrow \text{ 26}$$

$$-i\int \frac{\sin(ic+idx)^5}{\left(a-b\sin(ic+idx)^4\right)^3}dx$$

$\downarrow$ 3694

$$\frac{\int \frac{\left(1-\cosh^2(c+dx)\right)^2}{\left(-b\cosh^4(c+dx)+2b\cosh^2(c+dx)+a-b\right)^3}d\cosh(c+dx)}{d}$$

$\downarrow$ 1517

$$\frac{\frac{\cosh(c+dx)\left(a-b\cosh^2(c+dx)+b\right)}{8b(a-b)\left(a-b\cosh^4(c+dx)+2b\cosh^2(c+dx)-b\right)^2}-\frac{\int \frac{2a\left(5b\cosh^2(c+dx)+a-7b\right)}{\left(-b\cosh^4(c+dx)+2b\cosh^2(c+dx)+a-b\right)^2}d\cosh(c+dx)}{16ab(a-b)}}{d}$$

$\downarrow$ 27

$$\frac{\frac{\cosh(c+dx)\left(a-b\cosh^2(c+dx)+b\right)}{8b(a-b)\left(a-b\cosh^4(c+dx)+2b\cosh^2(c+dx)-b\right)^2}-\frac{\int \frac{5b\cosh^2(c+dx)+a-7b}{\left(-b\cosh^4(c+dx)+2b\cosh^2(c+dx)+a-b\right)^2}d\cosh(c+dx)}{8b(a-b)}}{d}$$

$\downarrow$ 1492

$$\frac{\frac{\cosh(c+dx)\left(a-b\cosh^2(c+dx)+b\right)}{8b(a-b)\left(a-b\cosh^4(c+dx)+2b\cosh^2(c+dx)-b\right)^2}-\frac{\frac{\cosh(c+dx)\left(a^2+2b(2a+b)\cosh^2(c+dx)-11ab-2b^2\right)}{4a(a-b)\left(a-b\cosh^4(c+dx)+2b\cosh^2(c+dx)-b\right)}-\frac{\int -\frac{2b\left(3a^2-17ba+2b^2+2b(2a+b)\cosh^2(c+dx)\right.}{-b\cosh^4(c+dx)+2b\cosh^2(c+dx)+a-b}}{8ab(a-b)}}{8b(a-b)}}{d}$$

$\downarrow$ 27

$$\frac{\frac{\cosh(c+dx)\left(a-b\cosh^2(c+dx)+b\right)}{8b(a-b)\left(a-b\cosh^4(c+dx)+2b\cosh^2(c+dx)-b\right)^2}-\frac{\frac{\int \frac{3a^2-17ba+2b^2+2b(2a+b)\cosh^2(c+dx)}{-b\cosh^4(c+dx)+2b\cosh^2(c+dx)+a-b}d\cosh(c+dx)}{4a(a-b)}+\frac{\cosh(c+dx)\left(a^2+2b(2a+b)\cosh^2(c+dx)-11\right.}{4a(a-b)\left(a-b\cosh^4(c+dx)+2b\cosh^2(c+d\right.}}{8b(a-b)}}{d}$$

$\downarrow$ 1480

$$\frac{\frac{\cosh(c+dx)\left(a-b\cosh^2(c+dx)+b\right)}{8b(a-b)\left(a-b\cosh^4(c+dx)+2b\cosh^2(c+dx)-b\right)^2}-\frac{\frac{\frac{\sqrt{b}\left(-2\sqrt{a}\sqrt{b}+a+b\right)\left(10\sqrt{a}\sqrt{b}+3a+4b\right)\int \frac{1}{\left(\sqrt{a}+\sqrt{b}\right)\sqrt{b}-b\cosh^2(c+dx)}d\cosh(c+dx)}{2\sqrt{a}}-\frac{\sqrt{b}\left(\sqrt{a}+\sqrt{b}\right)^2}{}}{4a(a-b)}}{}}{d}$$

$\downarrow$ 218

$$\frac{\frac{\cosh(c+dx)\left(a-b\cosh^2(c+dx)+b\right)}{8b(a-b)\left(a-b\cosh^4(c+dx)+2b\cosh^2(c+dx)-b\right)^2}-\frac{\frac{\frac{\sqrt{b}\left(-2\sqrt{a}\sqrt{b}+a+b\right)\left(10\sqrt{a}\sqrt{b}+3a+4b\right)\int\frac{1}{\left(\sqrt{a}+\sqrt{b}\right)\sqrt{b}-b\cosh^2(c+dx)}d\cosh(c+dx)}{2\sqrt{a}}+\frac{\left(-10\sqrt{a}\sqrt{b}+3\right.}{}}{4a(a-b)}}{8b(a}}{d}$$

$$\Big\downarrow \text{221}$$

$$\frac{\frac{\cosh(c+dx)\left(a-b\cosh^2(c+dx)+b\right)}{8b(a-b)\left(a-b\cosh^4(c+dx)+2b\cosh^2(c+dx)-b\right)^2}-\frac{\frac{\cosh(c+dx)\left(a^2+2b(2a+b)\cosh^2(c+dx)-11ab-2b^2\right)}{4a(a-b)\left(a-b\cosh^4(c+dx)+2b\cosh^2(c+dx)-b\right)}+\frac{\left(-10\sqrt{a}\sqrt{b}+3a+4b\right)\left(\sqrt{a}+\sqrt{b}\right)^2\arctan\left(\frac{\sqrt[4]{b}}{\sqrt{}}\right.}{2\sqrt{a}\sqrt[4]{b}\sqrt{\sqrt{a}-\sqrt{b}}}}{8b(a-b)}}{d}$$

input
```
Int[Sinh[c + d*x]^5/(a - b*Sinh[c + d*x]^4)^3,x]
```

output
```
((Cosh[c + d*x]*(a + b - b*Cosh[c + d*x]^2))/(8*(a - b)*b*(a - b + 2*b*Cos
h[c + d*x]^2 - b*Cosh[c + d*x]^4)^2) - ((((Sqrt[a] + Sqrt[b])^2*(3*a - 10*
Sqrt[a]*Sqrt[b] + 4*b)*ArcTan[(b^(1/4)*Cosh[c + d*x])/Sqrt[Sqrt[a] - Sqrt[
b]]])/(2*Sqrt[a]*Sqrt[Sqrt[a] - Sqrt[b]]*b^(1/4)) + ((a - 2*Sqrt[a]*Sqrt[b
] + b)*(3*a + 10*Sqrt[a]*Sqrt[b] + 4*b)*ArcTanh[(b^(1/4)*Cosh[c + d*x])/Sq
rt[Sqrt[a] + Sqrt[b]]])/(2*Sqrt[a]*Sqrt[Sqrt[a] + Sqrt[b]]*b^(1/4)))/(4*a*
(a - b)) + (Cosh[c + d*x]*(a^2 - 11*a*b - 2*b^2 + 2*b*(2*a + b)*Cosh[c + d
*x]^2))/(4*a*(a - b)*(a - b + 2*b*Cosh[c + d*x]^2 - b*Cosh[c + d*x]^4)))/(
8*(a - b)*b))/d
```

Defintions of rubi rules used

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 1480
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Simp[(e/2 + (2*c*d - b*e)/(2*q))   Int[1/(
b/2 - q/2 + c*x^2), x], x] + Simp[(e/2 - (2*c*d - b*e)/(2*q))   Int[1/(b/2
+ q/2 + c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0]
 && NeQ[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

rule 1492
```
Int[((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symb
ol] :> Simp[x*(a*b*e - d*(b^2 - 2*a*c) - c*(b*d - 2*a*e)*x^2)*((a + b*x^2 +
 c*x^4)^(p + 1)/(2*a*(p + 1)*(b^2 - 4*a*c))), x] + Simp[1/(2*a*(p + 1)*(b^2
 - 4*a*c))   Int[Simp[(2*p + 3)*d*b^2 - a*b*e - 2*a*c*d*(4*p + 5) + (4*p +
7)*(d*b - 2*a*e)*c*x^2, x]*(a + b*x^2 + c*x^4)^(p + 1), x], x] /; FreeQ[{a,
 b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] &&
 LtQ[p, -1] && IntegerQ[2*p]
```

rule 1517
```
Int[((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x
_Symbol] :> With[{f = Coeff[PolynomialRemainder[(d + e*x^2)^q, a + b*x^2 +
c*x^4, x], x, 0], g = Coeff[PolynomialRemainder[(d + e*x^2)^q, a + b*x^2 +
c*x^4, x], x, 2]}, Simp[x*(a + b*x^2 + c*x^4)^(p + 1)*((a*b*g - f*(b^2 - 2*
a*c) - c*(b*f - 2*a*g)*x^2)/(2*a*(p + 1)*(b^2 - 4*a*c))), x] + Simp[1/(2*a*
(p + 1)*(b^2 - 4*a*c))   Int[(a + b*x^2 + c*x^4)^(p + 1)*ExpandToSum[2*a*(p
 + 1)*(b^2 - 4*a*c)*PolynomialQuotient[(d + e*x^2)^q, a + b*x^2 + c*x^4, x]
 + b^2*f*(2*p + 3) - 2*a*c*f*(4*p + 5) - a*b*g + c*(4*p + 7)*(b*f - 2*a*g)*
x^2, x], x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ
[c*d^2 - b*d*e + a*e^2, 0] && IGtQ[q, 1] && LtQ[p, -1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3694
```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^4)^
(p_.), x_Symbol] :> With[{ff = FreeFactors[Cos[e + f*x], x]}, Simp[-ff/f
Subst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a + b - 2*b*ff^2*x^2 + b*ff^4*x^4)^p,
 x], x, Cos[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m -
 1)/2]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 649 vs. $2(261) = 522$.

Time = 20.54 (sec) , antiderivative size = 650, normalized size of antiderivative = 2.08

method	result
derivativedivides	$\frac{-\frac{\left(3a^2-13ab+4b^2\right)\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^{14}}{16b\left(a^2-2ab+b^2\right)}+\frac{3\left(7a^2-33ab+8b^2\right)\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^{12}}{16b\left(a^2-2ab+b^2\right)}-\frac{\left(63a^3-225a^2b+68b^2a+64b^3\right)\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^{10}}{16b\left(a^2-2ab+b^2\right)a}+\frac{3(3}{}}{\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^{8}a-4\,\text{ta}\right.}$
default	$\frac{-\frac{\left(3a^2-13ab+4b^2\right)\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^{14}}{16b\left(a^2-2ab+b^2\right)}+\frac{3\left(7a^2-33ab+8b^2\right)\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^{12}}{16b\left(a^2-2ab+b^2\right)}-\frac{\left(63a^3-225a^2b+68b^2a+64b^3\right)\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^{10}}{16b\left(a^2-2ab+b^2\right)a}+\frac{3(3}{}}{\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^{8}a-4\,\text{ta}\right.}$
risch	Expression too large to display

input
```
int(sinh(d*x+c)^5/(a-b*sinh(d*x+c)^4)^3,x,method=_RETURNVERBOSE)
```

output
```
1/d*(32*(-1/512*(3*a^2-13*a*b+4*b^2)/b/(a^2-2*a*b+b^2)*tanh(1/2*d*x+1/2*c)
^14+3/512/b*(7*a^2-33*a*b+8*b^2)/(a^2-2*a*b+b^2)*tanh(1/2*d*x+1/2*c)^12-1/
512/b*(63*a^3-225*a^2*b+68*a*b^2+64*b^3)/(a^2-2*a*b+b^2)/a*tanh(1/2*d*x+1/
2*c)^10+3/512*(35*a^3-61*a^2*b+32*a*b^2+128*b^3)/a/b/(a^2-2*a*b+b^2)*tanh(
1/2*d*x+1/2*c)^8-1/512/a*(105*a^3+9*a^2*b-452*a*b^2-64*b^3)/b/(a^2-2*a*b+b
^2)*tanh(1/2*d*x+1/2*c)^6+3/512*(21*a^2+29*a*b-40*b^2)/b/(a^2-2*a*b+b^2)*t
anh(1/2*d*x+1/2*c)^4-1/512*(21*a^2+37*a*b-4*b^2)/b/(a^2-2*a*b+b^2)*tanh(1/
2*d*x+1/2*c)^2+3/512*a*(a+b)/b/(a^2-2*a*b+b^2))/(tanh(1/2*d*x+1/2*c)^8*a-4
*tanh(1/2*d*x+1/2*c)^6*a+6*tanh(1/2*d*x+1/2*c)^4*a-16*b*tanh(1/2*d*x+1/2*c
)^4-4*tanh(1/2*d*x+1/2*c)^2*a+a)^2+1/16/b/(a^2-2*a*b+b^2)*(-1/4*(-4*(a*b)^
(1/2)*a-2*(a*b)^(1/2)*b-3*a^2+13*a*b-4*b^2)/a/(-a*b-(a*b)^(1/2)*a)^(1/2)*a
rctan(1/4*(-2*tanh(1/2*d*x+1/2*c)^2*a+4*(a*b)^(1/2)+2*a)/(-a*b-(a*b)^(1/2)
*a)^(1/2))+1/4*(4*(a*b)^(1/2)*a+2*(a*b)^(1/2)*b-3*a^2+13*a*b-4*b^2)/a/(-a*
b+(a*b)^(1/2)*a)^(1/2)*arctan(1/4*(2*tanh(1/2*d*x+1/2*c)^2*a+4*(a*b)^(1/2)
-2*a)/(-a*b+(a*b)^(1/2)*a)^(1/2))))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 22506 vs. $2(262) = 524$.

Time = 0.61 (sec) , antiderivative size = 22506, normalized size of antiderivative = 71.90

$$\int \frac{\sinh^5(c+dx)}{\left(a-b\sinh^4(c+dx)\right)^3}\,dx = \text{Too large to display}$$

input
```
integrate(sinh(d*x+c)^5/(a-b*sinh(d*x+c)^4)^3,x, algorithm="fricas")
```

output
```
Too large to include
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\sinh^5(c+dx)}{\left(a-b\sinh^4(c+dx)\right)^3}\,dx = \text{Timed out}$$

input
```
integrate(sinh(d*x+c)**5/(a-b*sinh(d*x+c)**4)**3,x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{\sinh^5(c+dx)}{\left(a-b\sinh^4(c+dx)\right)^3}\,dx = \int -\frac{\sinh\left(dx+c\right)^5}{\left(b\sinh\left(dx+c\right)^4-a\right)^3}\,dx$$

input
```
integrate(sinh(d*x+c)^5/(a-b*sinh(d*x+c)^4)^3,x, algorithm="maxima")
```

output
```
1/8*((2*a*b^2*e^(15*c) + b^3*e^(15*c))*e^(15*d*x) + (2*a^2*b*e^(13*c) - 24
*a*b^2*e^(13*c) - 5*b^3*e^(13*c))*e^(13*d*x) - (70*a^2*b*e^(11*c) - 76*a*b
^2*e^(11*c) - 9*b^3*e^(11*c))*e^(11*d*x) + (96*a^3*e^(9*c) + 164*a^2*b*e^(
9*c) - 54*a*b^2*e^(9*c) - 5*b^3*e^(9*c))*e^(9*d*x) + (96*a^3*e^(7*c) + 164
*a^2*b*e^(7*c) - 54*a*b^2*e^(7*c) - 5*b^3*e^(7*c))*e^(7*d*x) - (70*a^2*b*e
^(5*c) - 76*a*b^2*e^(5*c) - 9*b^3*e^(5*c))*e^(5*d*x) + (2*a^2*b*e^(3*c) -
24*a*b^2*e^(3*c) - 5*b^3*e^(3*c))*e^(3*d*x) + (2*a*b^2*e^c + b^3*e^c)*e^(d
*x))/(a^3*b^3*d - 2*a^2*b^4*d + a*b^5*d + (a^3*b^3*d*e^(16*c) - 2*a^2*b^4*
d*e^(16*c) + a*b^5*d*e^(16*c))*e^(16*d*x) - 8*(a^3*b^3*d*e^(14*c) - 2*a^2*
b^4*d*e^(14*c) + a*b^5*d*e^(14*c))*e^(14*d*x) - 4*(8*a^4*b^2*d*e^(12*c) -
23*a^3*b^3*d*e^(12*c) + 22*a^2*b^4*d*e^(12*c) - 7*a*b^5*d*e^(12*c))*e^(12*
d*x) + 8*(16*a^4*b^2*d*e^(10*c) - 39*a^3*b^3*d*e^(10*c) + 30*a^2*b^4*d*e^(
10*c) - 7*a*b^5*d*e^(10*c))*e^(10*d*x) + 2*(128*a^5*b*d*e^(8*c) - 352*a^4*
b^2*d*e^(8*c) + 355*a^3*b^3*d*e^(8*c) - 166*a^2*b^4*d*e^(8*c) + 35*a*b^5*d
*e^(8*c))*e^(8*d*x) + 8*(16*a^4*b^2*d*e^(6*c) - 39*a^3*b^3*d*e^(6*c) + 30*
a^2*b^4*d*e^(6*c) - 7*a*b^5*d*e^(6*c))*e^(6*d*x) - 4*(8*a^4*b^2*d*e^(4*c)
- 23*a^3*b^3*d*e^(4*c) + 22*a^2*b^4*d*e^(4*c) - 7*a*b^5*d*e^(4*c))*e^(4*d*
x) - 8*(a^3*b^3*d*e^(2*c) - 2*a^2*b^4*d*e^(2*c) + a*b^5*d*e^(2*c))*e^(2*d*
x)) + 1/32*integrate(4*((2*a*b*e^(7*c) + b^2*e^(7*c))*e^(7*d*x) + (6*a^2*e
^(5*c) - 32*a*b*e^(5*c) + 5*b^2*e^(5*c))*e^(5*d*x) - (6*a^2*e^(3*c) - 3...
```

Giac [F]

$$\int \frac{\sinh^5(c+dx)}{\left(a-b\sinh^4(c+dx)\right)^3}\,dx=\int -\frac{\sinh\left(dx+c\right)^5}{\left(b\sinh\left(dx+c\right)^4-a\right)^3}\,dx$$

input
```
integrate(sinh(d*x+c)^5/(a-b*sinh(d*x+c)^4)^3,x, algorithm="giac")
```

output
```
sage0*x
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sinh^5(c+dx)}{(a-b\sinh^4(c+dx))^3}\,dx = \int \frac{\sinh(c+d\,x)^5}{\left(a-b\sinh\left(c+d\,x\right)^4\right)^3}\,dx$$

input
```
int(sinh(c + d*x)^5/(a - b*sinh(c + d*x)^4)^3,x)
```

output
```
int(sinh(c + d*x)^5/(a - b*sinh(c + d*x)^4)^3, x)
```

Reduce [F]

$$\int \frac{\sinh^5(c+dx)}{(a-b\sinh^4(c+dx))^3}\,dx = \text{too large to display}$$

input
```
int(sinh(d*x+c)^5/(a-b*sinh(d*x+c)^4)^3,x)
```

output

```
(128*e**c*(4549888*e**(22*c + 16*d*x)*int(e**(7*d*x)/(e**(24*c + 24*d*x)*b
**3 - 12*e**(22*c + 22*d*x)*b**3 - 48*e**(20*c + 20*d*x)*a*b**2 + 66*e**(2
0*c + 20*d*x)*b**3 + 384*e**(18*c + 18*d*x)*a*b**2 - 220*e**(18*c + 18*d*x
)*b**3 + 768*e**(16*c + 16*d*x)*a**2*b - 1344*e**(16*c + 16*d*x)*a*b**2 +
495*e**(16*c + 16*d*x)*b**3 - 3072*e**(14*c + 14*d*x)*a**2*b + 2688*e**(14
*c + 14*d*x)*a*b**2 - 792*e**(14*c + 14*d*x)*b**3 - 4096*e**(12*c + 12*d*x
)*a**3 + 4608*e**(12*c + 12*d*x)*a**2*b - 3360*e**(12*c + 12*d*x)*a*b**2 +
 924*e**(12*c + 12*d*x)*b**3 - 3072*e**(10*c + 10*d*x)*a**2*b + 2688*e**(1
0*c + 10*d*x)*a*b**2 - 792*e**(10*c + 10*d*x)*b**3 + 768*e**(8*c + 8*d*x)*
a**2*b - 1344*e**(8*c + 8*d*x)*a*b**2 + 495*e**(8*c + 8*d*x)*b**3 + 384*e*
*(6*c + 6*d*x)*a*b**2 - 220*e**(6*c + 6*d*x)*b**3 - 48*e**(4*c + 4*d*x)*a*
b**2 + 66*e**(4*c + 4*d*x)*b**3 - 12*e**(2*c + 2*d*x)*b**3 + b**3),x)*a**2
*b**3*d + 920544*e**(22*c + 16*d*x)*int(e**(7*d*x)/(e**(24*c + 24*d*x)*b**
3 - 12*e**(22*c + 22*d*x)*b**3 - 48*e**(20*c + 20*d*x)*a*b**2 + 66*e**(20*
c + 20*d*x)*b**3 + 384*e**(18*c + 18*d*x)*a*b**2 - 220*e**(18*c + 18*d*x)*
b**3 + 768*e**(16*c + 16*d*x)*a**2*b - 1344*e**(16*c + 16*d*x)*a*b**2 + 49
5*e**(16*c + 16*d*x)*b**3 - 3072*e**(14*c + 14*d*x)*a**2*b + 2688*e**(14*c
 + 14*d*x)*a*b**2 - 792*e**(14*c + 14*d*x)*b**3 - 4096*e**(12*c + 12*d*x)*
a**3 + 4608*e**(12*c + 12*d*x)*a**2*b - 3360*e**(12*c + 12*d*x)*a*b**2 + 9
24*e**(12*c + 12*d*x)*b**3 - 3072*e**(10*c + 10*d*x)*a**2*b + 2688*e**(...
```

3.231 $\int \frac{\sinh^3(c+dx)}{(a-b\sinh^4(c+dx))^3}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 288

$$\begin{aligned}\int \frac{\sinh^3(c+dx)}{(a-b\sinh^4(c+dx))^3}\,dx &= -\frac{\left(5\sqrt{a}-2\sqrt{b}\right)\arctan\left(\frac{\sqrt[4]{b}\cosh(c+dx)}{\sqrt{\sqrt{a}-\sqrt{b}}}\right)}{64a^{3/2}\left(\sqrt{a}-\sqrt{b}\right)^{5/2}b^{3/4}d}+\frac{\left(5\sqrt{a}+2\sqrt{b}\right)\operatorname{arctanh}\left(\frac{\sqrt[4]{b}\cosh(c+dx)}{\sqrt{\sqrt{a}+\sqrt{b}}}\right)}{64a^{3/2}\left(\sqrt{a}+\sqrt{b}\right)^{5/2}b^{3/4}d}\\&\quad-\frac{\cosh(c+dx)\left(2-\cosh^2(c+dx)\right)}{8(a-b)d\left(a-b+2b\cosh^2(c+dx)-b\cosh^4(c+dx)\right)^2}\\&\quad-\frac{\cosh(c+dx)\left(11a+b-(5a+b)\cosh^2(c+dx)\right)}{32a(a-b)^2d\left(a-b+2b\cosh^2(c+dx)-b\cosh^4(c+dx)\right)}\end{aligned}$$

output
```
-1/64*(5*a^(1/2)-2*b^(1/2))*arctan(b^(1/4)*cosh(d*x+c)/(a^(1/2)-b^(1/2))^(
1/2))/a^(3/2)/(a^(1/2)-b^(1/2))^(5/2)/b^(3/4)/d+1/64*(5*a^(1/2)+2*b^(1/2))
*arctanh(b^(1/4)*cosh(d*x+c)/(a^(1/2)+b^(1/2))^(1/2))/a^(3/2)/(a^(1/2)+b^(
1/2))^(5/2)/b^(3/4)/d-1/8*cosh(d*x+c)*(2-cosh(d*x+c)^2)/(a-b)/d/(a-b+2*b*c
osh(d*x+c)^2-b*cosh(d*x+c)^4)^2-1/32*cosh(d*x+c)*(11*a+b-(5*a+b)*cosh(d*x+
c)^2)/a/(a-b)^2/d/(a-b+2*b*cosh(d*x+c)^2-b*cosh(d*x+c)^4)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

Time = 1.40 (sec) , antiderivative size = 802, normalized size of antiderivative = 2.78

$$\int \frac{\sinh^3(c+dx)}{\left(a-b\sinh^4(c+dx)\right)^3}\,dx = \text{Too large to display}$$

input
```
Integrate[Sinh[c + d*x]^3/(a - b*Sinh[c + d*x]^4)^3,x]
```

output
```
((32*Cosh[c + d*x]*(-17*a - b + (5*a + b)*Cosh[2*(c + d*x)]))/(a*(8*a - 3*
b + 4*b*Cosh[2*(c + d*x)] - b*Cosh[4*(c + d*x)])) + (512*(a - b)*(-5*Cosh[
c + d*x] + Cosh[3*(c + d*x)]))/(-8*a + 3*b - 4*b*Cosh[2*(c + d*x)] + b*Cos
h[4*(c + d*x)])^2 + RootSum[b - 4*b*#1^2 - 16*a*#1^4 + 6*b*#1^4 - 4*b*#1^6
 + b*#1^8 & , (5*a*c + b*c + 5*a*d*x + b*d*x + 10*a*Log[-Cosh[(c + d*x)/2]
 - Sinh[(c + d*x)/2] + Cosh[(c + d*x)/2]*#1 - Sinh[(c + d*x)/2]*#1] + 2*b*
Log[-Cosh[(c + d*x)/2] - Sinh[(c + d*x)/2] + Cosh[(c + d*x)/2]*#1 - Sinh[(
c + d*x)/2]*#1] - 47*a*c*#1^2 + 5*b*c*#1^2 - 47*a*d*x*#1^2 + 5*b*d*x*#1^2
- 94*a*Log[-Cosh[(c + d*x)/2] - Sinh[(c + d*x)/2] + Cosh[(c + d*x)/2]*#1 -
 Sinh[(c + d*x)/2]*#1]*#1^2 + 10*b*Log[-Cosh[(c + d*x)/2] - Sinh[(c + d*x)
/2] + Cosh[(c + d*x)/2]*#1 - Sinh[(c + d*x)/2]*#1]*#1^2 + 47*a*c*#1^4 - 5*
b*c*#1^4 + 47*a*d*x*#1^4 - 5*b*d*x*#1^4 + 94*a*Log[-Cosh[(c + d*x)/2] - Si
nh[(c + d*x)/2] + Cosh[(c + d*x)/2]*#1 - Sinh[(c + d*x)/2]*#1]*#1^4 - 10*b
*Log[-Cosh[(c + d*x)/2] - Sinh[(c + d*x)/2] + Cosh[(c + d*x)/2]*#1 - Sinh[
(c + d*x)/2]*#1]*#1^4 - 5*a*c*#1^6 - b*c*#1^6 - 5*a*d*x*#1^6 - b*d*x*#1^6
- 10*a*Log[-Cosh[(c + d*x)/2] - Sinh[(c + d*x)/2] + Cosh[(c + d*x)/2]*#1 -
 Sinh[(c + d*x)/2]*#1]*#1^6 - 2*b*Log[-Cosh[(c + d*x)/2] - Sinh[(c + d*x)/
2] + Cosh[(c + d*x)/2]*#1 - Sinh[(c + d*x)/2]*#1]*#1^6)/(-(b*#1) - 8*a*#1^
3 + 3*b*#1^3 - 3*b*#1^5 + b*#1^7) & ]/a)/(256*(a - b)^2*d)
```

Rubi [A] (verified)

Time = 0.66 (sec) , antiderivative size = 316, normalized size of antiderivative = 1.10, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.417, Rules used = {3042, 26, 3694, 1492, 27, 1492, 27, 1480, 218, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sinh^3(c+dx)}{\left(a-b\sinh^4(c+dx)\right)^3}dx$$

$$\Big\downarrow \ 3042$$

$$\int \frac{i\sin(ic+idx)^3}{\left(a-b\sin(ic+idx)^4\right)^3}dx$$

$$\Big\downarrow \ 26$$

$$i\int \frac{\sin(ic+idx)^3}{\left(a-b\sin(ic+idx)^4\right)^3}dx$$

$$\Big\downarrow \ 3694$$

$$-\frac{\int \frac{1-\cosh^2(c+dx)}{\left(-b\cosh^4(c+dx)+2b\cosh^2(c+dx)+a-b\right)^3}d\cosh(c+dx)}{d}$$

$$\Big\downarrow \ 1492$$

$$-\frac{\frac{\cosh(c+dx)\left(2-\cosh^2(c+dx)\right)}{8(a-b)\left(a-b\cosh^4(c+dx)+2b\cosh^2(c+dx)-b\right)^2}-\frac{\int -\frac{2ab\left(6-5\cosh^2(c+dx)\right)}{\left(-b\cosh^4(c+dx)+2b\cosh^2(c+dx)+a-b\right)^2}d\cosh(c+dx)}{16ab(a-b)}}{d}$$

$$\Big\downarrow \ 27$$

$$-\frac{\frac{\int \frac{6-5\cosh^2(c+dx)}{\left(-b\cosh^4(c+dx)+2b\cosh^2(c+dx)+a-b\right)^2}d\cosh(c+dx)}{8(a-b)}+\frac{\cosh(c+dx)\left(2-\cosh^2(c+dx)\right)}{8(a-b)\left(a-b\cosh^4(c+dx)+2b\cosh^2(c+dx)-b\right)^2}}{d}$$

$$\Big\downarrow \ 1492$$

$$-\frac{\frac{\frac{\cosh(c+dx)\left(-\left((5a+b)\cosh^2(c+dx)\right)+11a+b\right)}{4a(a-b)\left(a-b\cosh^4(c+dx)+2b\cosh^2(c+dx)-b\right)}-\frac{\int -\frac{2b\left(-\left((5a+b)\cosh^2(c+dx)\right)+13a-b\right)}{-b\cosh^4(c+dx)+2b\cosh^2(c+dx)+a-b}d\cosh(c+dx)}{8ab(a-b)}}{8(a-b)}+\frac{\cosh(c+dx)\left(2-\cosh^2(c+dx)\right)}{8(a-b)\left(a-b\cosh^4(c+dx)+2b\cosh^2(c+da\right.}}{d}$$

$\Big\downarrow$ 27

$$-\frac{\frac{\frac{\int \frac{-\left((5a+b)\cosh^2(c+dx)\right)+13a-b}{-b\cosh^4(c+dx)+2b\cosh^2(c+dx)+a-b}d\cosh(c+dx)}{4a(a-b)}+\frac{\cosh(c+dx)\left(-\left((5a+b)\cosh^2(c+dx)\right)+11a+b\right)}{4a(a-b)\left(a-b\cosh^4(c+dx)+2b\cosh^2(c+dx)-b\right)}}{8(a-b)}+\frac{\cosh(c+dx)\left(2-\cosh^2(c+dx)\right)}{8(a-b)\left(a-b\cosh^4(c+dx)+2b\cosh^2(c+dx)\right.}}{d}$$

$\Big\downarrow$ 1480

$$-\frac{\frac{\frac{-\frac{1}{2}\left(\frac{2\sqrt{b}(4a-b)}{\sqrt{a}}+5a+b\right)\int\frac{1}{-b\cosh^2(c+dx)-\left(\sqrt{a}-\sqrt{b}\right)\sqrt{b}}d\cosh(c+dx)-\frac{1}{2}\left(-\frac{2\sqrt{b}(4a-b)}{\sqrt{a}}+5a+b\right)\int\frac{1}{\left(\sqrt{a}+\sqrt{b}\right)\sqrt{b}-b\cosh^2(c+dx)}d\cosh(c+dx)}{4a(a-b)}+\frac{\cosh(c+dx)}{4a(a-b)\left(a\right.}}{8(a-b)}}{d}$$

$\Big\downarrow$ 218

$$-\frac{\frac{\frac{\frac{\left(\frac{2\sqrt{b}(4a-b)}{\sqrt{a}}+5a+b\right)\arctan\left(\frac{\sqrt[4]{b}\cosh(c+dx)}{\sqrt{\sqrt{a}-\sqrt{b}}}\right)}{2b^{3/4}\sqrt{\sqrt{a}-\sqrt{b}}}-\frac{1}{2}\left(-\frac{2\sqrt{b}(4a-b)}{\sqrt{a}}+5a+b\right)\int\frac{1}{\left(\sqrt{a}+\sqrt{b}\right)\sqrt{b}-b\cosh^2(c+dx)}d\cosh(c+dx)}{4a(a-b)}+\frac{\cosh(c+dx)\left(-\left((5a+b)\cosh^2(c+\right.\right.}{4a(a-b)\left(a-b\cosh^4(c+dx)+2b\cosh\right.}}{8(a-b)}}{d}$$

$\Big\downarrow$ 221

$$-\frac{\frac{\frac{\frac{\left(\frac{2\sqrt{b}(4a-b)}{\sqrt{a}}+5a+b\right)\arctan\left(\frac{\sqrt[4]{b}\cosh(c+dx)}{\sqrt{\sqrt{a}-\sqrt{b}}}\right)}{2b^{3/4}\sqrt{\sqrt{a}-\sqrt{b}}}-\frac{\left(-\frac{2\sqrt{b}(4a-b)}{\sqrt{a}}+5a+b\right)\text{arctanh}\left(\frac{\sqrt[4]{b}\cosh(c+dx)}{\sqrt{\sqrt{a}+\sqrt{b}}}\right)}{2b^{3/4}\sqrt{\sqrt{a}+\sqrt{b}}}}{4a(a-b)}+\frac{\cosh(c+dx)\left(-\left((5a+b)\cosh^2(c+dx)\right)+11a+b\right)}{4a(a-b)\left(a-b\cosh^4(c+dx)+2b\cosh^2(c+dx)-b\right.}}{8(a-b)}}{d}$$

input
```
Int[Sinh[c + d*x]^3/(a - b*Sinh[c + d*x]^4)^3,x]
```

output
```
-(((Cosh[c + d*x]*(2 - Cosh[c + d*x]^2))/(8*(a - b)*(a - b + 2*b*Cosh[c +
d*x]^2 - b*Cosh[c + d*x]^4)^2) + ((((5*a + (2*(4*a - b)*Sqrt[b])/Sqrt[a] +
 b)*ArcTan[(b^(1/4)*Cosh[c + d*x])/Sqrt[Sqrt[a] - Sqrt[b]]])/(2*Sqrt[Sqrt[
a] - Sqrt[b]]*b^(3/4)) - ((5*a - (2*(4*a - b)*Sqrt[b])/Sqrt[a] + b)*ArcTan
h[(b^(1/4)*Cosh[c + d*x])/Sqrt[Sqrt[a] + Sqrt[b]]])/(2*Sqrt[Sqrt[a] + Sqrt
[b]]*b^(3/4)))/(4*a*(a - b)) + (Cosh[c + d*x]*(11*a + b - (5*a + b)*Cosh[c
 + d*x]^2))/(4*a*(a - b)*(a - b + 2*b*Cosh[c + d*x]^2 - b*Cosh[c + d*x]^4)
))/(8*(a - b)))/d)
```

Defintions of rubi rules used

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 1480
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Simp[(e/2 + (2*c*d - b*e)/(2*q))   Int[1/(
b/2 - q/2 + c*x^2), x], x] + Simp[(e/2 - (2*c*d - b*e)/(2*q))   Int[1/(b/2
+ q/2 + c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0]
 && NeQ[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

rule 1492
```
Int[((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symb
ol] :> Simp[x*(a*b*e - d*(b^2 - 2*a*c) - c*(b*d - 2*a*e)*x^2)*((a + b*x^2 +
 c*x^4)^(p + 1)/(2*a*(p + 1)*(b^2 - 4*a*c))), x] + Simp[1/(2*a*(p + 1)*(b^2
 - 4*a*c))   Int[Simp[(2*p + 3)*d*b^2 - a*b*e - 2*a*c*d*(4*p + 5) + (4*p +
7)*(d*b - 2*a*e)*c*x^2, x]*(a + b*x^2 + c*x^4)^(p + 1), x], x] /; FreeQ[{a,
 b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] &&
 LtQ[p, -1] && IntegerQ[2*p]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3694
```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^4)^
(p_.), x_Symbol] :> With[{ff = FreeFactors[Cos[e + f*x], x]}, Simp[-ff/f
Subst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a + b - 2*b*ff^2*x^2 + b*ff^4*x^4)^p,
 x], x, Cos[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m -
 1)/2]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 592 vs. $2(236) = 472$.

Time = 20.07 (sec) , antiderivative size = 593, normalized size of antiderivative = 2.06

method	result
derivativedivides	$\frac{-\frac{(4a-b)\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^{14}}{8\left(a^2-2ab+b^2\right)}+\frac{\left(a^2+58ab-32b^2\right)\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^{12}}{8a\left(a^2-2ab+b^2\right)}+\frac{3\left(20a^2-73ab+48b^2\right)\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^{10}}{8a\left(a^2-2ab+b^2\right)}-\frac{\left(175a^3-550a^2b+832b^2a\right.}{8a^2\left(a^2-\right.}}{\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^8a-4\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^6a+\right.}$
default	$\frac{-\frac{(4a-b)\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^{14}}{8\left(a^2-2ab+b^2\right)}+\frac{\left(a^2+58ab-32b^2\right)\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^{12}}{8a\left(a^2-2ab+b^2\right)}+\frac{3\left(20a^2-73ab+48b^2\right)\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^{10}}{8a\left(a^2-2ab+b^2\right)}-\frac{\left(175a^3-550a^2b+832b^2a\right.}{8a^2\left(a^2-\right.}}{\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^8a-4\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^6a+\right.}$
risch	$\frac{e^{dx+c}\left(-5ab\,e^{14dx+14c}-b^2e^{14dx+14c}+49\,e^{12dx+12c}ab+5\,e^{12dx+12c}b^2+144\,e^{10dx+10c}a^2-165\,e^{10dx+10c}ab-9\,e^{10dx+10\right.}{16a(a-b)}$

input
```
int(sinh(d*x+c)^3/(a-b*sinh(d*x+c)^4)^3,x,method=_RETURNVERBOSE)
```

output
```
1/d*(8*(-1/64*(4*a-b)/(a^2-2*a*b+b^2)*tanh(1/2*d*x+1/2*c)^14+1/64*(a^2+58*
a*b-32*b^2)/a/(a^2-2*a*b+b^2)*tanh(1/2*d*x+1/2*c)^12+3/64/a*(20*a^2-73*a*b
+48*b^2)/(a^2-2*a*b+b^2)*tanh(1/2*d*x+1/2*c)^10-1/64/a^2*(175*a^3-550*a^2*
b+832*a*b^2-256*b^3)/(a^2-2*a*b+b^2)*tanh(1/2*d*x+1/2*c)^8+1/64/a*(220*a^2
-533*a*b+112*b^2)/(a^2-2*a*b+b^2)*tanh(1/2*d*x+1/2*c)^6-1/64*(141*a^2-158*
a*b+32*b^2)/a/(a^2-2*a*b+b^2)*tanh(1/2*d*x+1/2*c)^4+1/64*(44*a-17*b)/(a^2-
2*a*b+b^2)*tanh(1/2*d*x+1/2*c)^2-1/64*(5*a-2*b)/(a^2-2*a*b+b^2))/(tanh(1/2
*d*x+1/2*c)^8*a-4*tanh(1/2*d*x+1/2*c)^6*a+6*tanh(1/2*d*x+1/2*c)^4*a-16*b*t
anh(1/2*d*x+1/2*c)^4-4*tanh(1/2*d*x+1/2*c)^2*a+a)^2+1/8/(a^2-2*a*b+b^2)*(1
/8*(-5*(a*b)^(1/2)*a-(a*b)^(1/2)*b-8*a*b+2*b^2)/a/b/(-a*b+(a*b)^(1/2)*a)^(
1/2)*arctan(1/4*(2*tanh(1/2*d*x+1/2*c)^2*a+4*(a*b)^(1/2)-2*a)/(-a*b+(a*b)^
(1/2)*a)^(1/2))-1/8*(5*(a*b)^(1/2)*a+(a*b)^(1/2)*b-8*a*b+2*b^2)/a/b/(-a*b-
(a*b)^(1/2)*a)^(1/2)*arctan(1/4*(-2*tanh(1/2*d*x+1/2*c)^2*a+4*(a*b)^(1/2)+
2*a)/(-a*b-(a*b)^(1/2)*a)^(1/2))))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 20961 vs. $2(233) = 466$.

Time = 0.48 (sec) , antiderivative size = 20961, normalized size of antiderivative = 72.78

$$\int \frac{\sinh^3(c+dx)}{\left(a-b\sinh^4(c+dx)\right)^3}\,dx = \text{Too large to display}$$

input
```
integrate(sinh(d*x+c)^3/(a-b*sinh(d*x+c)^4)^3,x, algorithm="fricas")
```

output
```
Too large to include
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\sinh^3(c+dx)}{\left(a-b\sinh^4(c+dx)\right)^3}\,dx = \text{Timed out}$$

input
```
integrate(sinh(d*x+c)**3/(a-b*sinh(d*x+c)**4)**3,x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{\sinh^3(c+dx)}{\left(a-b\sinh^4(c+dx)\right)^3}\,dx = \int -\frac{\sinh\left(dx+c\right)^3}{\left(b\sinh\left(dx+c\right)^4-a\right)^3}\,dx$$

input
```
integrate(sinh(d*x+c)^3/(a-b*sinh(d*x+c)^4)^3,x, algorithm="maxima")
```

output
```
-1/16*((5*a*b*e^(15*c) + b^2*e^(15*c))*e^(15*d*x) - (49*a*b*e^(13*c) + 5*b
^2*e^(13*c))*e^(13*d*x) - 3*(48*a^2*e^(11*c) - 55*a*b*e^(11*c) - 3*b^2*e^(
11*c))*e^(11*d*x) + (784*a^2*e^(9*c) - 377*a*b*e^(9*c) - 5*b^2*e^(9*c))*e^
(9*d*x) + (784*a^2*e^(7*c) - 377*a*b*e^(7*c) - 5*b^2*e^(7*c))*e^(7*d*x) -
3*(48*a^2*e^(5*c) - 55*a*b*e^(5*c) - 3*b^2*e^(5*c))*e^(5*d*x) - (49*a*b*e^
(3*c) + 5*b^2*e^(3*c))*e^(3*d*x) + (5*a*b*e^c + b^2*e^c)*e^(d*x))/(a^3*b^2
*d - 2*a^2*b^3*d + a*b^4*d + (a^3*b^2*d*e^(16*c) - 2*a^2*b^3*d*e^(16*c) +
a*b^4*d*e^(16*c))*e^(16*d*x) - 8*(a^3*b^2*d*e^(14*c) - 2*a^2*b^3*d*e^(14*c
) + a*b^4*d*e^(14*c))*e^(14*d*x) - 4*(8*a^4*b*d*e^(12*c) - 23*a^3*b^2*d*e^
(12*c) + 22*a^2*b^3*d*e^(12*c) - 7*a*b^4*d*e^(12*c))*e^(12*d*x) + 8*(16*a^
4*b*d*e^(10*c) - 39*a^3*b^2*d*e^(10*c) + 30*a^2*b^3*d*e^(10*c) - 7*a*b^4*d
*e^(10*c))*e^(10*d*x) + 2*(128*a^5*d*e^(8*c) - 352*a^4*b*d*e^(8*c) + 355*a
^3*b^2*d*e^(8*c) - 166*a^2*b^3*d*e^(8*c) + 35*a*b^4*d*e^(8*c))*e^(8*d*x) +
 8*(16*a^4*b*d*e^(6*c) - 39*a^3*b^2*d*e^(6*c) + 30*a^2*b^3*d*e^(6*c) - 7*a
*b^4*d*e^(6*c))*e^(6*d*x) - 4*(8*a^4*b*d*e^(4*c) - 23*a^3*b^2*d*e^(4*c) +
22*a^2*b^3*d*e^(4*c) - 7*a*b^4*d*e^(4*c))*e^(4*d*x) - 8*(a^3*b^2*d*e^(2*c)
 - 2*a^2*b^3*d*e^(2*c) + a*b^4*d*e^(2*c))*e^(2*d*x)) - 1/8*integrate(1/2*(
(5*a*e^(7*c) + b*e^(7*c))*e^(7*d*x) - (47*a*e^(5*c) - 5*b*e^(5*c))*e^(5*d*
x) + (47*a*e^(3*c) - 5*b*e^(3*c))*e^(3*d*x) - (5*a*e^c + b*e^c)*e^(d*x))/(
a^3*b - 2*a^2*b^2 + a*b^3 + (a^3*b*e^(8*c) - 2*a^2*b^2*e^(8*c) + a*b^3*...
```

Giac [F]

$$\int \frac{\sinh^3(c+dx)}{\left(a-b\sinh^4(c+dx)\right)^3}\,dx = \int -\frac{\sinh\left(dx+c\right)^3}{\left(b\sinh\left(dx+c\right)^4-a\right)^3}\,dx$$

input
```
integrate(sinh(d*x+c)^3/(a-b*sinh(d*x+c)^4)^3,x, algorithm="giac")
```

output
```
sage0*x
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sinh^3(c+dx)}{\left(a-b\sinh^4(c+dx)\right)^3}\,dx = \int \frac{\sinh(c+d\,x)^3}{\left(a-b\sinh\left(c+d\,x\right)^4\right)^3}\,dx$$

input
```
int(sinh(c + d*x)^3/(a - b*sinh(c + d*x)^4)^3,x)
```

output
```
int(sinh(c + d*x)^3/(a - b*sinh(c + d*x)^4)^3, x)
```

Reduce [F]

$$\int \frac{\sinh^3(c+dx)}{\left(a-b\sinh^4(c+dx)\right)^3}\,dx = \text{too large to display}$$

input
```
int(sinh(d*x+c)^3/(a-b*sinh(d*x+c)^4)^3,x)
```

output

```
(512*e**c*( - 70400*e**(22*c + 16*d*x)*int(e**(7*d*x)/(e**(24*c + 24*d*x)*
b**3 - 12*e**(22*c + 22*d*x)*b**3 - 48*e**(20*c + 20*d*x)*a*b**2 + 66*e**(
20*c + 20*d*x)*b**3 + 384*e**(18*c + 18*d*x)*a*b**2 - 220*e**(18*c + 18*d*
x)*b**3 + 768*e**(16*c + 16*d*x)*a**2*b - 1344*e**(16*c + 16*d*x)*a*b**2 +
 495*e**(16*c + 16*d*x)*b**3 - 3072*e**(14*c + 14*d*x)*a**2*b + 2688*e**(1
4*c + 14*d*x)*a*b**2 - 792*e**(14*c + 14*d*x)*b**3 - 4096*e**(12*c + 12*d*
x)*a**3 + 4608*e**(12*c + 12*d*x)*a**2*b - 3360*e**(12*c + 12*d*x)*a*b**2
+ 924*e**(12*c + 12*d*x)*b**3 - 3072*e**(10*c + 10*d*x)*a**2*b + 2688*e**(
10*c + 10*d*x)*a*b**2 - 792*e**(10*c + 10*d*x)*b**3 + 768*e**(8*c + 8*d*x)
*a**2*b - 1344*e**(8*c + 8*d*x)*a*b**2 + 495*e**(8*c + 8*d*x)*b**3 + 384*e
**(6*c + 6*d*x)*a*b**2 - 220*e**(6*c + 6*d*x)*b**3 - 48*e**(4*c + 4*d*x)*a
*b**2 + 66*e**(4*c + 4*d*x)*b**3 - 12*e**(2*c + 2*d*x)*b**3 + b**3),x)*a**
2*b**2*d - 16464*e**(22*c + 16*d*x)*int(e**(7*d*x)/(e**(24*c + 24*d*x)*b**
3 - 12*e**(22*c + 22*d*x)*b**3 - 48*e**(20*c + 20*d*x)*a*b**2 + 66*e**(20*
c + 20*d*x)*b**3 + 384*e**(18*c + 18*d*x)*a*b**2 - 220*e**(18*c + 18*d*x)*
b**3 + 768*e**(16*c + 16*d*x)*a**2*b - 1344*e**(16*c + 16*d*x)*a*b**2 + 49
5*e**(16*c + 16*d*x)*b**3 - 3072*e**(14*c + 14*d*x)*a**2*b + 2688*e**(14*c
 + 14*d*x)*a*b**2 - 792*e**(14*c + 14*d*x)*b**3 - 4096*e**(12*c + 12*d*x)*
a**3 + 4608*e**(12*c + 12*d*x)*a**2*b - 3360*e**(12*c + 12*d*x)*a*b**2 + 9
24*e**(12*c + 12*d*x)*b**3 - 3072*e**(10*c + 10*d*x)*a**2*b + 2688*e**(...
```

3.232 $\int \frac{\sinh(c+dx)}{\left(a-b\sinh^4(c+dx)\right)^3}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 313

$$\begin{aligned}\int \frac{\sinh(c+dx)}{\left(a-b\sinh^4(c+dx)\right)^3}\,dx &= \frac{3\left(7a-10\sqrt{a}\sqrt{b}+4b\right)\arctan\left(\frac{\sqrt[4]{b}\cosh(c+dx)}{\sqrt{\sqrt{a}-\sqrt{b}}}\right)}{64a^{5/2}\left(\sqrt{a}-\sqrt{b}\right)^{5/2}\sqrt[4]{b}d} \\ &\quad+\frac{3\left(7a+10\sqrt{a}\sqrt{b}+4b\right)\operatorname{arctanh}\left(\frac{\sqrt[4]{b}\cosh(c+dx)}{\sqrt{\sqrt{a}+\sqrt{b}}}\right)}{64a^{5/2}\left(\sqrt{a}+\sqrt{b}\right)^{5/2}\sqrt[4]{b}d} \\ &\quad+\frac{\cosh(c+dx)\left(a+b-b\cosh^2(c+dx)\right)}{8a(a-b)d\left(a-b+2b\cosh^2(c+dx)-b\cosh^4(c+dx)\right)^2} \\ &\quad+\frac{\cosh(c+dx)\left((7a-3b)(a+2b)-6(2a-b)b\cosh^2(c+dx)\right)}{32a^2(a-b)^2d\left(a-b+2b\cosh^2(c+dx)-b\cosh^4(c+dx)\right)}\end{aligned}$$

output
```
3/64*(7*a-10*a^(1/2)*b^(1/2)+4*b)*arctan(b^(1/4)*cosh(d*x+c)/(a^(1/2)-b^(1
/2))^(1/2))/a^(5/2)/(a^(1/2)-b^(1/2))^(5/2)/b^(1/4)/d+3/64*(7*a+10*a^(1/2)
*b^(1/2)+4*b)*arctanh(b^(1/4)*cosh(d*x+c)/(a^(1/2)+b^(1/2))^(1/2))/a^(5/2)
/(a^(1/2)+b^(1/2))^(5/2)/b^(1/4)/d+1/8*cosh(d*x+c)*(a+b-b*cosh(d*x+c)^2)/a
/(a-b)/d/(a-b+2*b*cosh(d*x+c)^2-b*cosh(d*x+c)^4)^2+1/32*cosh(d*x+c)*((7*a-
3*b)*(a+2*b)-6*(2*a-b)*b*cosh(d*x+c)^2)/a^2/(a-b)^2/d/(a-b+2*b*cosh(d*x+c)
^2-b*cosh(d*x+c)^4)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

Time = 1.27 (sec) , antiderivative size = 1018, normalized size of antiderivative = 3.25

$$\int \frac{\sinh(c+dx)}{\left(a-b\sinh^4(c+dx)\right)^3}\,dx = \text{Too large to display}$$

input
```
Integrate[Sinh[c + d*x]/(a - b*Sinh[c + d*x]^4)^3,x]
```

output
```
((32*Cosh[c + d*x]*(7*a^2 + 5*a*b - 3*b^2 + 3*b*(-2*a + b)*Cosh[2*(c + d*x
)]))/(8*a - 3*b + 4*b*Cosh[2*(c + d*x)] - b*Cosh[4*(c + d*x)]) + (512*a*(a
 - b)*Cosh[c + d*x]*(2*a + b - b*Cosh[2*(c + d*x)]))/(-8*a + 3*b - 4*b*Cos
h[2*(c + d*x)] + b*Cosh[4*(c + d*x)])^2 + 3*RootSum[b - 4*b*#1^2 - 16*a*#1
^4 + 6*b*#1^4 - 4*b*#1^6 + b*#1^8 & , (-2*a*b*c + b^2*c - 2*a*b*d*x + b^2*
d*x - 4*a*b*Log[-Cosh[(c + d*x)/2] - Sinh[(c + d*x)/2] + Cosh[(c + d*x)/2]
*#1 - Sinh[(c + d*x)/2]*#1] + 2*b^2*Log[-Cosh[(c + d*x)/2] - Sinh[(c + d*x
)/2] + Cosh[(c + d*x)/2]*#1 - Sinh[(c + d*x)/2]*#1] + 14*a^2*c*#1^2 - 12*a
*b*c*#1^2 + 5*b^2*c*#1^2 + 14*a^2*d*x*#1^2 - 12*a*b*d*x*#1^2 + 5*b^2*d*x*#
1^2 + 28*a^2*Log[-Cosh[(c + d*x)/2] - Sinh[(c + d*x)/2] + Cosh[(c + d*x)/2
]*#1 - Sinh[(c + d*x)/2]*#1]*#1^2 - 24*a*b*Log[-Cosh[(c + d*x)/2] - Sinh[(
c + d*x)/2] + Cosh[(c + d*x)/2]*#1 - Sinh[(c + d*x)/2]*#1]*#1^2 + 10*b^2*L
og[-Cosh[(c + d*x)/2] - Sinh[(c + d*x)/2] + Cosh[(c + d*x)/2]*#1 - Sinh[(c
 + d*x)/2]*#1]*#1^2 - 14*a^2*c*#1^4 + 12*a*b*c*#1^4 - 5*b^2*c*#1^4 - 14*a^
2*d*x*#1^4 + 12*a*b*d*x*#1^4 - 5*b^2*d*x*#1^4 - 28*a^2*Log[-Cosh[(c + d*x)
/2] - Sinh[(c + d*x)/2] + Cosh[(c + d*x)/2]*#1 - Sinh[(c + d*x)/2]*#1]*#1^
4 + 24*a*b*Log[-Cosh[(c + d*x)/2] - Sinh[(c + d*x)/2] + Cosh[(c + d*x)/2]*
#1 - Sinh[(c + d*x)/2]*#1]*#1^4 - 10*b^2*Log[-Cosh[(c + d*x)/2] - Sinh[(c
+ d*x)/2] + Cosh[(c + d*x)/2]*#1 - Sinh[(c + d*x)/2]*#1]*#1^4 + 2*a*b*c*#1
^6 - b^2*c*#1^6 + 2*a*b*d*x*#1^6 - b^2*d*x*#1^6 + 4*a*b*Log[-Cosh[(c + ...
```

Rubi [A] (verified)

Time = 0.67 (sec) , antiderivative size = 363, normalized size of antiderivative = 1.16, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.455$, Rules used = {3042, 26, 3694, 1405, 27, 1492, 27, 1480, 218, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sinh(c+dx)}{\left(a-b\sinh^4(c+dx)\right)^3}\,dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int -\frac{i\sin(ic+idx)}{\left(a-b\sin(ic+idx)^4\right)^3}dx$$

$$\Big\downarrow \text{ 26}$$

$$-i\int \frac{\sin(ic+idx)}{\left(a-b\sin(ic+idx)^4\right)^3}dx$$

$\downarrow$ 3694

$$\frac{\int \frac{1}{\left(-b\cosh^4(c+dx)+2b\cosh^2(c+dx)+a-b\right)^3}d\cosh(c+dx)}{d}$$

$\downarrow$ 1405

$$\frac{\frac{\cosh(c+dx)\left(a-b\cosh^2(c+dx)+b\right)}{8a(a-b)\left(a-b\cosh^4(c+dx)+2b\cosh^2(c+dx)-b\right)^2}-\frac{\int -\frac{2b\left(-5b\cosh^2(c+dx)+7a-b\right)}{\left(-b\cosh^4(c+dx)+2b\cosh^2(c+dx)+a-b\right)^2}d\cosh(c+dx)}{16ab(a-b)}}{d}$$

$\downarrow$ 27

$$\frac{\frac{\int \frac{-5b\cosh^2(c+dx)+7a-b}{\left(-b\cosh^4(c+dx)+2b\cosh^2(c+dx)+a-b\right)^2}d\cosh(c+dx)}{8a(a-b)}+\frac{\cosh(c+dx)\left(a-b\cosh^2(c+dx)+b\right)}{8a(a-b)\left(a-b\cosh^4(c+dx)+2b\cosh^2(c+dx)-b\right)^2}}{d}$$

$\downarrow$ 1492

$$\frac{\frac{\frac{\cosh(c+dx)\left((7a-3b)(a+2b)-6b(2a-b)\cosh^2(c+dx)\right)}{4a(a-b)\left(a-b\cosh^4(c+dx)+2b\cosh^2(c+dx)-b\right)}-\frac{\int -\frac{6b\left(7a^2-5ba+2b^2-2(2a-b)b\cosh^2(c+dx)\right)}{-b\cosh^4(c+dx)+2b\cosh^2(c+dx)+a-b}d\cosh(c+dx)}{8ab(a-b)}}{8a(a-b)}+\frac{\cosh(c+dx)\left(a-b\cosh^2(c+d\right.}{8a(a-b)\left(a-b\cosh^4(c+dx)+2b\cosh\right.}}{d}$$

$\downarrow$ 27

$$\frac{\frac{\frac{3\int \frac{7a^2-5ba+2b^2-2(2a-b)b\cosh^2(c+dx)}{-b\cosh^4(c+dx)+2b\cosh^2(c+dx)+a-b}d\cosh(c+dx)}{4a(a-b)}+\frac{\cosh(c+dx)\left((7a-3b)(a+2b)-6b(2a-b)\cosh^2(c+dx)\right)}{4a(a-b)\left(a-b\cosh^4(c+dx)+2b\cosh^2(c+dx)-b\right)}}{8a(a-b)}+\frac{\cosh(c+dx)\left(a-b\cosh^2(c+dx)+\right.}{8a(a-b)\left(a-b\cosh^4(c+dx)+2b\cosh^2(c\right.}}{d}$$

$\downarrow$ 1480

$$\frac{\frac{3\left(\frac{\sqrt{b}\left(-2\sqrt{a}\sqrt{b}+a+b\right)\left(10\sqrt{a}\sqrt{b}+7a+4b\right)\int \frac{1}{\left(\sqrt{a}+\sqrt{b}\right)\sqrt{b}-b\cosh^2(c+dx)}d\cosh(c+dx)}{2\sqrt{a}}-\frac{\sqrt{b}\left(\sqrt{a}+\sqrt{b}\right)^2\left(-10\sqrt{a}\sqrt{b}+7a+4b\right)\int \frac{1}{-b\cosh^2(c+dx)-\left(\sqrt{a}-\sqrt{b}\right)\sqrt{b}}d}{2\sqrt{a}}\right.}{4a(a-b)}}{8a(a-b)}}{d}$$

$\downarrow$ 218

$$\frac{\frac{3\left(\frac{\sqrt{b}\left(-2\sqrt{a}\sqrt{b}+a+b\right)\left(10\sqrt{a}\sqrt{b}+7a+4b\right)\int\frac{1}{\left(\sqrt{a}+\sqrt{b}\right)\sqrt{b}-b\cosh^2(c+dx)}d\cosh(c+dx)}{2\sqrt{a}}+\frac{\left(-10\sqrt{a}\sqrt{b}+7a+4b\right)\left(\sqrt{a}+\sqrt{b}\right)^2\arctan\left(\frac{\sqrt[4]{b}\cosh(c+dx)}{\sqrt{\sqrt{a}-\sqrt{b}}}\right)}{2\sqrt{a}\sqrt[4]{b}\sqrt{\sqrt{a}-\sqrt{b}}}\right)}{4a(a-b)}+\frac{\cosh(c}{4a(}}{8a(a-b)}}{d}$$

$\downarrow$ 221

$$\frac{\frac{3\left(\frac{\left(-10\sqrt{a}\sqrt{b}+7a+4b\right)\left(\sqrt{a}+\sqrt{b}\right)^2\arctan\left(\frac{\sqrt[4]{b}\cosh(c+dx)}{\sqrt{\sqrt{a}-\sqrt{b}}}\right)}{2\sqrt{a}\sqrt[4]{b}\sqrt{\sqrt{a}-\sqrt{b}}}+\frac{\left(-2\sqrt{a}\sqrt{b}+a+b\right)\left(10\sqrt{a}\sqrt{b}+7a+4b\right)\text{arctanh}\left(\frac{\sqrt[4]{b}\cosh(c+dx)}{\sqrt{\sqrt{a}+\sqrt{b}}}\right)}{2\sqrt{a}\sqrt[4]{b}\sqrt{\sqrt{a}+\sqrt{b}}}\right)}{4a(a-b)}+\frac{\cosh(c+dx)\left((7a-3b)(\right.}{4a(a-b)\left(a-b\cosh\right.}}{8a(a-b)}}{d}$$

input
```
Int[Sinh[c + d*x]/(a - b*Sinh[c + d*x]^4)^3,x]
```

output
```
((Cosh[c + d*x]*(a + b - b*Cosh[c + d*x]^2))/(8*a*(a - b)*(a - b + 2*b*Cos
h[c + d*x]^2 - b*Cosh[c + d*x]^4)^2) + ((3*(((Sqrt[a] + Sqrt[b])^2*(7*a -
10*Sqrt[a]*Sqrt[b] + 4*b)*ArcTan[(b^(1/4)*Cosh[c + d*x])/Sqrt[Sqrt[a] - Sq
rt[b]]])/(2*Sqrt[a]*Sqrt[Sqrt[a] - Sqrt[b]]*b^(1/4)) + ((a - 2*Sqrt[a]*Sqr
t[b] + b)*(7*a + 10*Sqrt[a]*Sqrt[b] + 4*b)*ArcTanh[(b^(1/4)*Cosh[c + d*x])
/Sqrt[Sqrt[a] + Sqrt[b]]])/(2*Sqrt[a]*Sqrt[Sqrt[a] + Sqrt[b]]*b^(1/4))))/(
4*a*(a - b)) + (Cosh[c + d*x]*((7*a - 3*b)*(a + 2*b) - 6*(2*a - b)*b*Cosh[
c + d*x]^2))/(4*a*(a - b)*(a - b + 2*b*Cosh[c + d*x]^2 - b*Cosh[c + d*x]^4
)))/(8*a*(a - b)))/d
```

Defintions of rubi rules used

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 1405
```
Int[((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Simp[(-x)*(b^2
- 2*a*c + b*c*x^2)*((a + b*x^2 + c*x^4)^(p + 1)/(2*a*(p + 1)*(b^2 - 4*a*c))
), x] + Simp[1/(2*a*(p + 1)*(b^2 - 4*a*c))   Int[(b^2 - 2*a*c + 2*(p + 1)*(
b^2 - 4*a*c) + b*c*(4*p + 7)*x^2)*(a + b*x^2 + c*x^4)^(p + 1), x], x] /; Fr
eeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && IntegerQ[2*p]
```

rule 1480
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Simp[(e/2 + (2*c*d - b*e)/(2*q))   Int[1/(
b/2 - q/2 + c*x^2), x], x] + Simp[(e/2 - (2*c*d - b*e)/(2*q))   Int[1/(b/2
+ q/2 + c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0]
 && NeQ[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

rule 1492
```
Int[((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symb
ol] :> Simp[x*(a*b*e - d*(b^2 - 2*a*c) - c*(b*d - 2*a*e)*x^2)*((a + b*x^2 +
 c*x^4)^(p + 1)/(2*a*(p + 1)*(b^2 - 4*a*c))), x] + Simp[1/(2*a*(p + 1)*(b^2
 - 4*a*c))   Int[Simp[(2*p + 3)*d*b^2 - a*b*e - 2*a*c*d*(4*p + 5) + (4*p +
7)*(d*b - 2*a*e)*c*x^2, x]*(a + b*x^2 + c*x^4)^(p + 1), x], x] /; FreeQ[{a,
 b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] &&
 LtQ[p, -1] && IntegerQ[2*p]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3694
```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^4)^
(p_.), x_Symbol] :> With[{ff = FreeFactors[Cos[e + f*x], x]}, Simp[-ff/f
Subst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a + b - 2*b*ff^2*x^2 + b*ff^4*x^4)^p,
 x], x, Cos[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m -
 1)/2]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 640 vs. 2(261) = 522.

Time = 19.66 (sec) , antiderivative size = 641, normalized size of antiderivative = 2.05

method	result
derivativedivides	$\frac{-\frac{\left(11a^2-37ab+20b^2\right)\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^{14}}{16a\left(a^2-2ab+b^2\right)}+\frac{\left(77a^2-283ab+152b^2\right)\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^{12}}{16a\left(a^2-2ab+b^2\right)}-\frac{\left(231a^3-857a^2b+788b^2a-192b^3\right)\tanh\left(\frac{dx}{2}+\frac{c}{2}\right.}{16a^2\left(a^2-2ab+b^2\right)}}{\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right.\right.}$
default	$\frac{-\frac{\left(11a^2-37ab+20b^2\right)\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^{14}}{16a\left(a^2-2ab+b^2\right)}+\frac{\left(77a^2-283ab+152b^2\right)\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^{12}}{16a\left(a^2-2ab+b^2\right)}-\frac{\left(231a^3-857a^2b+788b^2a-192b^3\right)\tanh\left(\frac{dx}{2}+\frac{c}{2}\right.}{16a^2\left(a^2-2ab+b^2\right)}}{\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right.\right.}$
risch	Expression too large to display

input

```
int(sinh(d*x+c)/(a-b*sinh(d*x+c)^4)^3,x,method=_RETURNVERBOSE)
```

output

```
1/d*(2*(-1/32*(11*a^2-37*a*b+20*b^2)/a/(a^2-2*a*b+b^2)*tanh(1/2*d*x+1/2*c)
^14+1/32/a*(77*a^2-283*a*b+152*b^2)/(a^2-2*a*b+b^2)*tanh(1/2*d*x+1/2*c)^12
-1/32/a^2*(231*a^3-857*a^2*b+788*a*b^2-192*b^3)/(a^2-2*a*b+b^2)*tanh(1/2*d
*x+1/2*c)^10+1/32/a^2*(385*a^3-1231*a^2*b+1888*a*b^2-640*b^3)/(a^2-2*a*b+b
^2)*tanh(1/2*d*x+1/2*c)^8-1/32/a^2*(385*a^3-831*a^2*b-148*a*b^2+192*b^3)/(
a^2-2*a*b+b^2)*tanh(1/2*d*x+1/2*c)^6+1/32/a*(231*a^2-209*a*b+8*b^2)/(a^2-2
*a*b+b^2)*tanh(1/2*d*x+1/2*c)^4-1/32*(77*a^2-3*a*b-20*b^2)/a/(a^2-2*a*b+b^
2)*tanh(1/2*d*x+1/2*c)^2+1/32*(11*a-5*b)/(a^2-2*a*b+b^2))/(tanh(1/2*d*x+1/
2*c)^8*a-4*tanh(1/2*d*x+1/2*c)^6*a+6*tanh(1/2*d*x+1/2*c)^4*a-16*b*tanh(1/2
*d*x+1/2*c)^4-4*tanh(1/2*d*x+1/2*c)^2*a+a)^2+3/16/a/(a^2-2*a*b+b^2)*(-1/4*
(-4*(a*b)^(1/2)*a+2*(a*b)^(1/2)*b+7*a^2-9*a*b+4*b^2)/a/(-a*b-(a*b)^(1/2)*a
)^(1/2)*arctan(1/4*(-2*tanh(1/2*d*x+1/2*c)^2*a+4*(a*b)^(1/2)+2*a)/(-a*b-(a
*b)^(1/2)*a)^(1/2))+1/4*(4*(a*b)^(1/2)*a-2*(a*b)^(1/2)*b+7*a^2-9*a*b+4*b^2
)/a/(-a*b+(a*b)^(1/2)*a)^(1/2)*arctan(1/4*(2*tanh(1/2*d*x+1/2*c)^2*a+4*(a*
b)^(1/2)-2*a)/(-a*b+(a*b)^(1/2)*a)^(1/2))))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 22332 vs. $2(263) = 526$.

Time = 0.61 (sec) , antiderivative size = 22332, normalized size of antiderivative = 71.35

$$\int \frac{\sinh(c+dx)}{(a-b\sinh^4(c+dx))^3}\,dx = \text{Too large to display}$$

input
```
integrate(sinh(d*x+c)/(a-b*sinh(d*x+c)^4)^3,x, algorithm="fricas")
```

output
```
Too large to include
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\sinh(c+dx)}{(a-b\sinh^4(c+dx))^3}\,dx = \text{Timed out}$$

input
```
integrate(sinh(d*x+c)/(a-b*sinh(d*x+c)**4)**3,x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{\sinh(c+dx)}{(a-b\sinh^4(c+dx))^3}\,dx = \int -\frac{\sinh(dx+c)}{(b\sinh(dx+c)^4-a)^3}\,dx$$

input
```
integrate(sinh(d*x+c)/(a-b*sinh(d*x+c)^4)^3,x, algorithm="maxima")
```

output
```
1/8*(3*(2*a*b^2*e^(15*c) - b^3*e^(15*c))*e^(15*d*x) - (14*a^2*b*e^(13*c) +
 28*a*b^2*e^(13*c) - 15*b^3*e^(13*c))*e^(13*d*x) - (86*a^2*b*e^(11*c) - 12
8*a*b^2*e^(11*c) + 27*b^3*e^(11*c))*e^(11*d*x) + (352*a^3*e^(9*c) - 60*a^2
*b*e^(9*c) - 106*a*b^2*e^(9*c) + 15*b^3*e^(9*c))*e^(9*d*x) + (352*a^3*e^(7
*c) - 60*a^2*b*e^(7*c) - 106*a*b^2*e^(7*c) + 15*b^3*e^(7*c))*e^(7*d*x) - (
86*a^2*b*e^(5*c) - 128*a*b^2*e^(5*c) + 27*b^3*e^(5*c))*e^(5*d*x) - (14*a^2
*b*e^(3*c) + 28*a*b^2*e^(3*c) - 15*b^3*e^(3*c))*e^(3*d*x) + 3*(2*a*b^2*e^c
 - b^3*e^c)*e^(d*x))/(a^4*b^2*d - 2*a^3*b^3*d + a^2*b^4*d + (a^4*b^2*d*e^(
16*c) - 2*a^3*b^3*d*e^(16*c) + a^2*b^4*d*e^(16*c))*e^(16*d*x) - 8*(a^4*b^2
*d*e^(14*c) - 2*a^3*b^3*d*e^(14*c) + a^2*b^4*d*e^(14*c))*e^(14*d*x) - 4*(8
*a^5*b*d*e^(12*c) - 23*a^4*b^2*d*e^(12*c) + 22*a^3*b^3*d*e^(12*c) - 7*a^2*
b^4*d*e^(12*c))*e^(12*d*x) + 8*(16*a^5*b*d*e^(10*c) - 39*a^4*b^2*d*e^(10*c
) + 30*a^3*b^3*d*e^(10*c) - 7*a^2*b^4*d*e^(10*c))*e^(10*d*x) + 2*(128*a^6*
d*e^(8*c) - 352*a^5*b*d*e^(8*c) + 355*a^4*b^2*d*e^(8*c) - 166*a^3*b^3*d*e^
(8*c) + 35*a^2*b^4*d*e^(8*c))*e^(8*d*x) + 8*(16*a^5*b*d*e^(6*c) - 39*a^4*b
^2*d*e^(6*c) + 30*a^3*b^3*d*e^(6*c) - 7*a^2*b^4*d*e^(6*c))*e^(6*d*x) - 4*(
8*a^5*b*d*e^(4*c) - 23*a^4*b^2*d*e^(4*c) + 22*a^3*b^3*d*e^(4*c) - 7*a^2*b^
4*d*e^(4*c))*e^(4*d*x) - 8*(a^4*b^2*d*e^(2*c) - 2*a^3*b^3*d*e^(2*c) + a^2*
b^4*d*e^(2*c))*e^(2*d*x)) + 1/2*integrate(3/4*((2*a*b*e^(7*c) - b^2*e^(7*c
))*e^(7*d*x) - (14*a^2*e^(5*c) - 12*a*b*e^(5*c) + 5*b^2*e^(5*c))*e^(5*d...
```

Giac [F]

$$\int \frac{\sinh(c+dx)}{\left(a-b\sinh^4(c+dx)\right)^3}\,dx=\int -\frac{\sinh\left(dx+c\right)}{\left(b\sinh\left(dx+c\right)^4-a\right)^3}\,dx$$

input
```
integrate(sinh(d*x+c)/(a-b*sinh(d*x+c)^4)^3,x, algorithm="giac")
```

output
```
sage0*x
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sinh(c+dx)}{\left(a-b\sinh^4(c+dx)\right)^3}\,dx=\int \frac{\sinh(c+d\,x)}{\left(a-b\sinh\left(c+d\,x\right)^4\right)^3}\,dx$$

input
```
int(sinh(c + d*x)/(a - b*sinh(c + d*x)^4)^3,x)
```

output
```
int(sinh(c + d*x)/(a - b*sinh(c + d*x)^4)^3, x)
```

Reduce [F]

$$\int \frac{\sinh(c+dx)}{\left(a-b\sinh^4(c+dx)\right)^3}\,dx=\text{too large to display}$$

input
```
int(sinh(d*x+c)/(a-b*sinh(d*x+c)^4)^3,x)
```

output

```
(2048*e**c*( - 15952*e**(22*c + 16*d*x)*int(e**(7*d*x)/(e**(24*c + 24*d*x)
*b**3 - 12*e**(22*c + 22*d*x)*b**3 - 48*e**(20*c + 20*d*x)*a*b**2 + 66*e**
(20*c + 20*d*x)*b**3 + 384*e**(18*c + 18*d*x)*a*b**2 - 220*e**(18*c + 18*d
*x)*b**3 + 768*e**(16*c + 16*d*x)*a**2*b - 1344*e**(16*c + 16*d*x)*a*b**2
+ 495*e**(16*c + 16*d*x)*b**3 - 3072*e**(14*c + 14*d*x)*a**2*b + 2688*e**(
14*c + 14*d*x)*a*b**2 - 792*e**(14*c + 14*d*x)*b**3 - 4096*e**(12*c + 12*d
*x)*a**3 + 4608*e**(12*c + 12*d*x)*a**2*b - 3360*e**(12*c + 12*d*x)*a*b**2
 + 924*e**(12*c + 12*d*x)*b**3 - 3072*e**(10*c + 10*d*x)*a**2*b + 2688*e**
(10*c + 10*d*x)*a*b**2 - 792*e**(10*c + 10*d*x)*b**3 + 768*e**(8*c + 8*d*x
)*a**2*b - 1344*e**(8*c + 8*d*x)*a*b**2 + 495*e**(8*c + 8*d*x)*b**3 + 384*
e**(6*c + 6*d*x)*a*b**2 - 220*e**(6*c + 6*d*x)*b**3 - 48*e**(4*c + 4*d*x)*
a*b**2 + 66*e**(4*c + 4*d*x)*b**3 - 12*e**(2*c + 2*d*x)*b**3 + b**3),x)*a*
b**3*d - 2734*e**(22*c + 16*d*x)*int(e**(7*d*x)/(e**(24*c + 24*d*x)*b**3 -
 12*e**(22*c + 22*d*x)*b**3 - 48*e**(20*c + 20*d*x)*a*b**2 + 66*e**(20*c +
 20*d*x)*b**3 + 384*e**(18*c + 18*d*x)*a*b**2 - 220*e**(18*c + 18*d*x)*b**
3 + 768*e**(16*c + 16*d*x)*a**2*b - 1344*e**(16*c + 16*d*x)*a*b**2 + 495*e
**(16*c + 16*d*x)*b**3 - 3072*e**(14*c + 14*d*x)*a**2*b + 2688*e**(14*c +
14*d*x)*a*b**2 - 792*e**(14*c + 14*d*x)*b**3 - 4096*e**(12*c + 12*d*x)*a**
3 + 4608*e**(12*c + 12*d*x)*a**2*b - 3360*e**(12*c + 12*d*x)*a*b**2 + 924*
e**(12*c + 12*d*x)*b**3 - 3072*e**(10*c + 10*d*x)*a**2*b + 2688*e**(10*...
```

3.233 $\int \frac{\operatorname{csch}(c+dx)}{\left(a-b\sinh^4(c+dx)\right)^3}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 319

$$\begin{aligned}\int \frac{\operatorname{csch}(c+dx)}{\left(a-b\sinh^4(c+dx)\right)^3}\,dx &= -\frac{\sqrt[4]{b}\left(45a-74\sqrt{a}\sqrt{b}+32b\right)\arctan\left(\frac{\sqrt[4]{b}\cosh(c+dx)}{\sqrt{\sqrt{a}-\sqrt{b}}}\right)}{64a^3\left(\sqrt{a}-\sqrt{b}\right)^{5/2}d} - \frac{\operatorname{arctanh}(\cosh(c+dx))}{a^3d} \\ &\quad + \frac{\sqrt[4]{b}\left(45a+74\sqrt{a}\sqrt{b}+32b\right)\operatorname{arctanh}\left(\frac{\sqrt[4]{b}\cosh(c+dx)}{\sqrt{\sqrt{a}+\sqrt{b}}}\right)}{64a^3\left(\sqrt{a}+\sqrt{b}\right)^{5/2}d} \\ &\quad - \frac{b\cosh(c+dx)\left(2-\cosh^2(c+dx)\right)}{8a(a-b)d\left(a-b+2b\cosh^2(c+dx)-b\cosh^4(c+dx)\right)^2} \\ &\quad - \frac{b\cosh(c+dx)\left(3(9a-5b)-(13a-7b)\cosh^2(c+dx)\right)}{32a^2(a-b)^2d\left(a-b+2b\cosh^2(c+dx)-b\cosh^4(c+dx)\right)}\end{aligned}$$

output
```
-1/64*b^(1/4)*(45*a-74*a^(1/2)*b^(1/2)+32*b)*arctan(b^(1/4)*cosh(d*x+c)/(a
^(1/2)-b^(1/2))^(1/2))/a^3/(a^(1/2)-b^(1/2))^(5/2)/d-arctanh(cosh(d*x+c))/
a^3/d+1/64*b^(1/4)*(45*a+74*a^(1/2)*b^(1/2)+32*b)*arctanh(b^(1/4)*cosh(d*x
+c)/(a^(1/2)+b^(1/2))^(1/2))/a^3/(a^(1/2)+b^(1/2))^(5/2)/d-1/8*b*cosh(d*x+
c)*(2-cosh(d*x+c)^2)/a/(a-b)/d/(a-b+2*b*cosh(d*x+c)^2-b*cosh(d*x+c)^4)^2-1
/32*b*cosh(d*x+c)*(27*a-15*b-(13*a-7*b)*cosh(d*x+c)^2)/a^2/(a-b)^2/d/(a-b+
2*b*cosh(d*x+c)^2-b*cosh(d*x+c)^4)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

Time = 11.93 (sec) , antiderivative size = 1202, normalized size of antiderivative = 3.77

$$\int \frac{\operatorname{csch}(c+dx)}{\left(a-b\sinh^4(c+dx)\right)^3}\,dx = \text{Too large to display}$$

input
```
Integrate[Csch[c + d*x]/(a - b*Sinh[c + d*x]^4)^3,x]
```

output
```
((32*a*b*Cosh[c + d*x]*(-41*a + 23*b + (13*a - 7*b)*Cosh[2*(c + d*x)]))/((
a - b)^2*(8*a - 3*b + 4*b*Cosh[2*(c + d*x)] - b*Cosh[4*(c + d*x)])) + (512
*a^2*b*(-5*Cosh[c + d*x] + Cosh[3*(c + d*x)]))/((a - b)*(-8*a + 3*b - 4*b*
Cosh[2*(c + d*x)] + b*Cosh[4*(c + d*x)])^2) - 256*Log[Cosh[(c + d*x)/2]] +
 256*Log[Sinh[(c + d*x)/2]] - (b*RootSum[b - 4*b*#1^2 - 16*a*#1^4 + 6*b*#1
^4 - 4*b*#1^6 + b*#1^8 & , (-45*a^2*c + 71*a*b*c - 32*b^2*c - 45*a^2*d*x +
 71*a*b*d*x - 32*b^2*d*x - 90*a^2*Log[-Cosh[(c + d*x)/2] - Sinh[(c + d*x)/
2] + Cosh[(c + d*x)/2]*#1 - Sinh[(c + d*x)/2]*#1] + 142*a*b*Log[-Cosh[(c +
 d*x)/2] - Sinh[(c + d*x)/2] + Cosh[(c + d*x)/2]*#1 - Sinh[(c + d*x)/2]*#1
] - 64*b^2*Log[-Cosh[(c + d*x)/2] - Sinh[(c + d*x)/2] + Cosh[(c + d*x)/2]*
#1 - Sinh[(c + d*x)/2]*#1] + 199*a^2*c*#1^2 - 253*a*b*c*#1^2 + 96*b^2*c*#1
^2 + 199*a^2*d*x*#1^2 - 253*a*b*d*x*#1^2 + 96*b^2*d*x*#1^2 + 398*a^2*Log[-
Cosh[(c + d*x)/2] - Sinh[(c + d*x)/2] + Cosh[(c + d*x)/2]*#1 - Sinh[(c + d
*x)/2]*#1]*#1^2 - 506*a*b*Log[-Cosh[(c + d*x)/2] - Sinh[(c + d*x)/2] + Cos
h[(c + d*x)/2]*#1 - Sinh[(c + d*x)/2]*#1]*#1^2 + 192*b^2*Log[-Cosh[(c + d*
x)/2] - Sinh[(c + d*x)/2] + Cosh[(c + d*x)/2]*#1 - Sinh[(c + d*x)/2]*#1]*#
1^2 - 199*a^2*c*#1^4 + 253*a*b*c*#1^4 - 96*b^2*c*#1^4 - 199*a^2*d*x*#1^4 +
 253*a*b*d*x*#1^4 - 96*b^2*d*x*#1^4 - 398*a^2*Log[-Cosh[(c + d*x)/2] - Sin
h[(c + d*x)/2] + Cosh[(c + d*x)/2]*#1 - Sinh[(c + d*x)/2]*#1]*#1^4 + 506*a
*b*Log[-Cosh[(c + d*x)/2] - Sinh[(c + d*x)/2] + Cosh[(c + d*x)/2]*#1 - ...
```

Rubi [A] (verified)

Time = 1.09 (sec) , antiderivative size = 591, normalized size of antiderivative = 1.85, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.227$, Rules used = {3042, 26, 3694, 1567, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\text{csch}(c+dx)}{\left(a-b\sinh^4(c+dx)\right)^3}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{i}{\sin(ic+idx)\left(a-b\sin(ic+idx)^4\right)^3}dx$$

$$\Big\downarrow \text{26}$$

$$i\int \frac{1}{\sin(ic+idx)\left(a-b\sin(ic+idx)^4\right)^3}dx$$

$\Big\downarrow$ 3694

$$-\frac{\int \frac{1}{\left(1-\cosh^2(c+dx)\right)\left(-b\cosh^4(c+dx)+2b\cosh^2(c+dx)+a-b\right)^3}d\cosh(c+dx)}{d}$$

$\Big\downarrow$ 1567

$$-\frac{\int \left(\frac{b-b\cosh^2(c+dx)}{a^3\left(-b\cosh^4(c+dx)+2b\cosh^2(c+dx)+a-b\right)}+\frac{b-b\cosh^2(c+dx)}{a^2\left(-b\cosh^4(c+dx)+2b\cosh^2(c+dx)+a-b\right)^2}+\frac{b-b\cosh^2(c+dx)}{a\left(-b\cosh^4(c+dx)+2b\cosh^2(c+dx)+a-\right.}\right.}{d}$$

$\Big\downarrow$ 2009

$$-\frac{\frac{\sqrt[4]{b}\arctan\left(\frac{\sqrt[4]{b}\cosh(c+dx)}{\sqrt{\sqrt{a}-\sqrt{b}}}\right)}{8a^{5/2}\left(\sqrt{a}-\sqrt{b}\right)^{3/2}}+\frac{\sqrt[4]{b}\left(5\sqrt{a}-2\sqrt{b}\right)\arctan\left(\frac{\sqrt[4]{b}\cosh(c+dx)}{\sqrt{\sqrt{a}-\sqrt{b}}}\right)}{64a^{5/2}\left(\sqrt{a}-\sqrt{b}\right)^{5/2}}-\frac{\sqrt[4]{b}\left(5\sqrt{a}+2\sqrt{b}\right)\text{arctanh}\left(\frac{\sqrt[4]{b}\cosh(c+dx)}{\sqrt{\sqrt{a}+\sqrt{b}}}\right)}{64a^{5/2}\left(\sqrt{a}+\sqrt{b}\right)^{5/2}}-\frac{\sqrt[4]{b}\text{arc}}{8}}{}$$

input
```
Int[Csch[c + d*x]/(a - b*Sinh[c + d*x]^4)^3,x]
```

output
```
-((((5*Sqrt[a] - 2*Sqrt[b])*b^(1/4)*ArcTan[(b^(1/4)*Cosh[c + d*x])/Sqrt[Sq
rt[a] - Sqrt[b]]])/(64*a^(5/2)*(Sqrt[a] - Sqrt[b])^(5/2)) + (b^(1/4)*ArcTa
n[(b^(1/4)*Cosh[c + d*x])/Sqrt[Sqrt[a] - Sqrt[b]]])/(8*a^(5/2)*(Sqrt[a] -
Sqrt[b])^(3/2)) + (b^(1/4)*ArcTan[(b^(1/4)*Cosh[c + d*x])/Sqrt[Sqrt[a] - S
qrt[b]]])/(2*a^3*Sqrt[Sqrt[a] - Sqrt[b]]) + ArcTanh[Cosh[c + d*x]]/a^3 - (
b^(1/4)*ArcTanh[(b^(1/4)*Cosh[c + d*x])/Sqrt[Sqrt[a] + Sqrt[b]]])/(8*a^(5/
2)*(Sqrt[a] + Sqrt[b])^(3/2)) - (b^(1/4)*ArcTanh[(b^(1/4)*Cosh[c + d*x])/S
qrt[Sqrt[a] + Sqrt[b]]])/(2*a^3*Sqrt[Sqrt[a] + Sqrt[b]]) - ((5*Sqrt[a] + 2
*Sqrt[b])*b^(1/4)*ArcTanh[(b^(1/4)*Cosh[c + d*x])/Sqrt[Sqrt[a] + Sqrt[b]]]
)/(64*a^(5/2)*(Sqrt[a] + Sqrt[b])^(5/2)) + (b*Cosh[c + d*x]*(2 - Cosh[c +
d*x]^2))/(8*a*(a - b)*(a - b + 2*b*Cosh[c + d*x]^2 - b*Cosh[c + d*x]^4)^2)
 + (b*Cosh[c + d*x]*(2 - Cosh[c + d*x]^2))/(4*a^2*(a - b)*(a - b + 2*b*Cos
h[c + d*x]^2 - b*Cosh[c + d*x]^4)) + (b*Cosh[c + d*x]*(11*a + b - (5*a + b
)*Cosh[c + d*x]^2))/(32*a^2*(a - b)^2*(a - b + 2*b*Cosh[c + d*x]^2 - b*Cos
h[c + d*x]^4)))/d)
```

Defintions of rubi rules used

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 1567
```
Int[((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x
_Symbol] :> Int[ExpandIntegrand[(d + e*x^2)^q*(a + b*x^2 + c*x^4)^p, x], x]
 /; FreeQ[{a, b, c, d, e, p, q}, x] && NeQ[b^2 - 4*a*c, 0] && ((IntegerQ[p]
 && IntegerQ[q]) || IGtQ[p, 0] || IGtQ[q, 0])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3694
```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^4)^
(p_.), x_Symbol] :> With[{ff = FreeFactors[Cos[e + f*x], x]}, Simp[-ff/f
Subst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a + b - 2*b*ff^2*x^2 + b*ff^4*x^4)^p,
 x], x, Cos[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m -
 1)/2]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 646 vs. $2(268) = 536$.

Time = 14.16 (sec) , antiderivative size = 647, normalized size of antiderivative = 2.03

method	result
derivativedivides	$\frac{8b\left(\frac{-\frac{a^2(8a-5b)\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^{14}}{64\left(a^2-2ab+b^2\right)}+\frac{a\left(5a^2+86ab-64b^2\right)\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^{12}}{64a^2-128ab+64b^2}+\frac{a\left(104a^2-327ab+208b^2\right)\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^{10}}{64a^2-128ab+64b^2}-\frac{3\left(105a^3-\right.}{}}{\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^8a-4\tanh\right.}\right.}{}$
default	$\frac{8b\left(\frac{-\frac{a^2(8a-5b)\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^{14}}{64\left(a^2-2ab+b^2\right)}+\frac{a\left(5a^2+86ab-64b^2\right)\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^{12}}{64a^2-128ab+64b^2}+\frac{a\left(104a^2-327ab+208b^2\right)\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^{10}}{64a^2-128ab+64b^2}-\frac{3\left(105a^3-\right.}{}}{\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^8a-4\tanh\right.}\right.}{}$
risch	Expression too large to display

input
```
int(csch(d*x+c)/(a-b*sinh(d*x+c)^4)^3,x,method=_RETURNVERBOSE)
```

output
```
1/d*(8*b/a^3*((-1/64*a^2*(8*a-5*b)/(a^2-2*a*b+b^2)*tanh(1/2*d*x+1/2*c)^14+
1/64*a*(5*a^2+86*a*b-64*b^2)/(a^2-2*a*b+b^2)*tanh(1/2*d*x+1/2*c)^12+1/64*a
*(104*a^2-327*a*b+208*b^2)/(a^2-2*a*b+b^2)*tanh(1/2*d*x+1/2*c)^10-3/64*(10
5*a^3-358*a^2*b+576*a*b^2-256*b^3)/(a^2-2*a*b+b^2)*tanh(1/2*d*x+1/2*c)^8+1
/64*a*(400*a^2-1161*a*b+560*b^2)/(a^2-2*a*b+b^2)*tanh(1/2*d*x+1/2*c)^6-1/6
4*a*(257*a^2-370*a*b+128*b^2)/(a^2-2*a*b+b^2)*tanh(1/2*d*x+1/2*c)^4+1/64*(
80*a-53*b)*a^2/(a^2-2*a*b+b^2)*tanh(1/2*d*x+1/2*c)^2-3/64*a^2*(3*a-2*b)/(a
^2-2*a*b+b^2))/(tanh(1/2*d*x+1/2*c)^8*a-4*tanh(1/2*d*x+1/2*c)^6*a+6*tanh(1
/2*d*x+1/2*c)^4*a-16*b*tanh(1/2*d*x+1/2*c)^4-4*tanh(1/2*d*x+1/2*c)^2*a+a)^
2+1/64/(a^2-2*a*b+b^2)*a*(-1/8*(45*(a*b)^(1/2)*a^2-71*a*b*(a*b)^(1/2)+32*(
a*b)^(1/2)*b^2-16*a^2*b+10*b^2*a)/a/b/(-a*b-(a*b)^(1/2)*a)^(1/2)*arctan(1/
4*(-2*tanh(1/2*d*x+1/2*c)^2*a+4*(a*b)^(1/2)+2*a)/(-a*b-(a*b)^(1/2)*a)^(1/2
))+1/8*(-45*(a*b)^(1/2)*a^2+71*a*b*(a*b)^(1/2)-32*(a*b)^(1/2)*b^2-16*a^2*b
+10*b^2*a)/a/b/(-a*b+(a*b)^(1/2)*a)^(1/2)*arctan(1/4*(2*tanh(1/2*d*x+1/2*c
)^2*a+4*(a*b)^(1/2)-2*a)/(-a*b+(a*b)^(1/2)*a)^(1/2))))+1/a^3*ln(tanh(1/2*d
*x+1/2*c)))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 28586 vs. 2(263) = 526.

Time = 1.28 (sec) , antiderivative size = 28586, normalized size of antiderivative = 89.61

$$\int \frac{\operatorname{csch}(c+dx)}{\left(a-b\sinh^4(c+dx)\right)^3}\,dx = \text{Too large to display}$$

input
```
integrate(csch(d*x+c)/(a-b*sinh(d*x+c)^4)^3,x, algorithm="fricas")
```

output
```
Too large to include
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\operatorname{csch}(c+dx)}{\left(a-b\sinh^4(c+dx)\right)^3}\,dx = \text{Timed out}$$

input
```
integrate(csch(d*x+c)/(a-b*sinh(d*x+c)**4)**3,x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{\operatorname{csch}(c+dx)}{\left(a-b\sinh^4(c+dx)\right)^3}\,dx = \int -\frac{\operatorname{csch}\left(dx+c\right)}{\left(b\sinh\left(dx+c\right)^4-a\right)^3}\,dx$$

input
```
integrate(csch(d*x+c)/(a-b*sinh(d*x+c)^4)^3,x, algorithm="maxima")
```

output
```
-1/16*((13*a*b^2*e^(15*c) - 7*b^3*e^(15*c))*e^(15*d*x) - (121*a*b^2*e^(13*
c) - 67*b^3*e^(13*c))*e^(13*d*x) - (272*a^2*b*e^(11*c) - 461*a*b^2*e^(11*c
) + 159*b^3*e^(11*c))*e^(11*d*x) + (1424*a^2*b*e^(9*c) - 1121*a*b^2*e^(9*c
) + 99*b^3*e^(9*c))*e^(9*d*x) + (1424*a^2*b*e^(7*c) - 1121*a*b^2*e^(7*c) +
 99*b^3*e^(7*c))*e^(7*d*x) - (272*a^2*b*e^(5*c) - 461*a*b^2*e^(5*c) + 159*
b^3*e^(5*c))*e^(5*d*x) - (121*a*b^2*e^(3*c) - 67*b^3*e^(3*c))*e^(3*d*x) +
(13*a*b^2*e^c - 7*b^3*e^c)*e^(d*x))/(a^4*b^2*d - 2*a^3*b^3*d + a^2*b^4*d +
 (a^4*b^2*d*e^(16*c) - 2*a^3*b^3*d*e^(16*c) + a^2*b^4*d*e^(16*c))*e^(16*d*
x) - 8*(a^4*b^2*d*e^(14*c) - 2*a^3*b^3*d*e^(14*c) + a^2*b^4*d*e^(14*c))*e^
(14*d*x) - 4*(8*a^5*b*d*e^(12*c) - 23*a^4*b^2*d*e^(12*c) + 22*a^3*b^3*d*e^
(12*c) - 7*a^2*b^4*d*e^(12*c))*e^(12*d*x) + 8*(16*a^5*b*d*e^(10*c) - 39*a^
4*b^2*d*e^(10*c) + 30*a^3*b^3*d*e^(10*c) - 7*a^2*b^4*d*e^(10*c))*e^(10*d*x
) + 2*(128*a^6*d*e^(8*c) - 352*a^5*b*d*e^(8*c) + 355*a^4*b^2*d*e^(8*c) - 1
66*a^3*b^3*d*e^(8*c) + 35*a^2*b^4*d*e^(8*c))*e^(8*d*x) + 8*(16*a^5*b*d*e^(
6*c) - 39*a^4*b^2*d*e^(6*c) + 30*a^3*b^3*d*e^(6*c) - 7*a^2*b^4*d*e^(6*c))*
e^(6*d*x) - 4*(8*a^5*b*d*e^(4*c) - 23*a^4*b^2*d*e^(4*c) + 22*a^3*b^3*d*e^(
4*c) - 7*a^2*b^4*d*e^(4*c))*e^(4*d*x) - 8*(a^4*b^2*d*e^(2*c) - 2*a^3*b^3*d
*e^(2*c) + a^2*b^4*d*e^(2*c))*e^(2*d*x)) - log((e^(d*x + c) + 1)*e^(-c))/(
a^3*d) + log((e^(d*x + c) - 1)*e^(-c))/(a^3*d) - 2*integrate(1/32*((45*a^2
*b*e^(7*c) - 71*a*b^2*e^(7*c) + 32*b^3*e^(7*c))*e^(7*d*x) - (199*a^2*b*...
```

Giac [F]

$$\int \frac{\operatorname{csch}(c+dx)}{\left(a-b\sinh^4(c+dx)\right)^3}\,dx=\int -\frac{\operatorname{csch}\left(dx+c\right)}{\left(b\sinh\left(dx+c\right)^4-a\right)^3}\,dx$$

input
```
integrate(csch(d*x+c)/(a-b*sinh(d*x+c)^4)^3,x, algorithm="giac")
```

output
```
sage0*x
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\operatorname{csch}(c+dx)}{\left(a-b\sinh^4(c+dx)\right)^3}\,dx=\int \frac{1}{\sinh\left(c+d\,x\right)\,\left(a-b\,\sinh\left(c+d\,x\right)^4\right)^3}\,dx$$

input
```
int(1/(sinh(c + d*x)*(a - b*sinh(c + d*x)^4)^3),x)
```

output
```
int(1/(sinh(c + d*x)*(a - b*sinh(c + d*x)^4)^3), x)
```

Reduce [F]

$$\int \frac{\operatorname{csch}(c+dx)}{\left(a-b\sinh^4(c+dx)\right)^3}\,dx=\text{too large to display}$$

input
```
int(csch(d*x+c)/(a-b*sinh(d*x+c)^4)^3,x)
```

output

```
(16*( - 2298453136179200*e**(23*c + 16*d*x)*int(e**(7*d*x)/(85085*e**(26*c
 + 26*d*x)*a**2*b**3 - 816*e**(26*c + 26*d*x)*a*b**4 + 896*e**(26*c + 26*d
*x)*b**5 - 1106105*e**(24*c + 24*d*x)*a**2*b**3 + 10608*e**(24*c + 24*d*x)
*a*b**4 - 11648*e**(24*c + 24*d*x)*b**5 - 4084080*e**(22*c + 22*d*x)*a**3*
b**2 + 6675798*e**(22*c + 22*d*x)*a**2*b**3 - 106656*e**(22*c + 22*d*x)*a*
b**4 + 69888*e**(22*c + 22*d*x)*b**5 + 36756720*e**(20*c + 20*d*x)*a**3*b*
*2 - 24686822*e**(20*c + 20*d*x)*a**2*b**3 + 620448*e**(20*c + 20*d*x)*a*b
**4 - 256256*e**(20*c + 20*d*x)*b**5 + 65345280*e**(18*c + 18*d*x)*a**4*b
- 147653568*e**(18*c + 18*d*x)*a**3*b**2 + 62933951*e**(18*c + 18*d*x)*a**
2*b**3 - 2131728*e**(18*c + 18*d*x)*a*b**4 + 640640*e**(18*c + 18*d*x)*b**
5 - 326726400*e**(16*c + 16*d*x)*a**4*b + 346196160*e**(16*c + 16*d*x)*a**
3*b**2 - 116235147*e**(16*c + 16*d*x)*a**2*b**3 + 4662864*e**(16*c + 16*d*
x)*a*b**4 - 1153152*e**(16*c + 16*d*x)*b**5 - 348508160*e**(14*c + 14*d*x)
*a**5 + 656795136*e**(14*c + 14*d*x)*a**4*b - 524530976*e**(14*c + 14*d*x)
*a**3*b**2 + 157822308*e**(14*c + 14*d*x)*a**2*b**3 - 6819264*e**(14*c + 1
4*d*x)*a*b**4 + 1537536*e**(14*c + 14*d*x)*b**5 + 348508160*e**(12*c + 12*
d*x)*a**5 - 656795136*e**(12*c + 12*d*x)*a**4*b + 524530976*e**(12*c + 12*
d*x)*a**3*b**2 - 157822308*e**(12*c + 12*d*x)*a**2*b**3 + 6819264*e**(12*c
 + 12*d*x)*a*b**4 - 1537536*e**(12*c + 12*d*x)*b**5 + 326726400*e**(10*c +
 10*d*x)*a**4*b - 346196160*e**(10*c + 10*d*x)*a**3*b**2 + 116235147*e*...
```

3.234 $\int \frac{\sinh^8(c+dx)}{(a-b\sinh^4(c+dx))^3}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 307

$$\begin{aligned}\int \frac{\sinh^8(c+dx)}{(a-b\sinh^4(c+dx))^3}\,dx &= -\frac{\left(2\sqrt{a}-5\sqrt{b}\right)\operatorname{arctanh}\left(\frac{\sqrt{\sqrt{a}-\sqrt{b}}\tanh(c+dx)}{\sqrt[4]{a}}\right)}{64a^{3/4}\left(\sqrt{a}-\sqrt{b}\right)^{5/2}b^{3/2}d}\\
&\quad+\frac{\left(2\sqrt{a}+5\sqrt{b}\right)\operatorname{arctanh}\left(\frac{\sqrt{\sqrt{a}+\sqrt{b}}\tanh(c+dx)}{\sqrt[4]{a}}\right)}{64a^{3/4}\left(\sqrt{a}+\sqrt{b}\right)^{5/2}b^{3/2}d}\\
&\quad-\frac{a\tanh(c+dx)\left(3a+b-4(a+b)\tanh^2(c+dx)\right)}{8(a-b)^3d\left(a-2a\tanh^2(c+dx)+(a-b)\tanh^4(c+dx)\right)^2}\\
&\quad-\frac{\tanh(c+dx)\left(\frac{(a-9b)(a+b)}{(a-b)^3}-\frac{(a+19b)\tanh^2(c+dx)}{(a-b)^2}\right)}{32bd\left(a-2a\tanh^2(c+dx)+(a-b)\tanh^4(c+dx)\right)}\end{aligned}$$

output
```
-1/64*(2*a^(1/2)-5*b^(1/2))*arctanh((a^(1/2)-b^(1/2))^(1/2)*tanh(d*x+c)/a^
(1/4))/a^(3/4)/(a^(1/2)-b^(1/2))^(5/2)/b^(3/2)/d+1/64*(2*a^(1/2)+5*b^(1/2)
)*arctanh((a^(1/2)+b^(1/2))^(1/2)*tanh(d*x+c)/a^(1/4))/a^(3/4)/(a^(1/2)+b^
(1/2))^(5/2)/b^(3/2)/d-1/8*a*tanh(d*x+c)*(3*a+b-4*(a+b)*tanh(d*x+c)^2)/(a-
b)^3/d/(a-2*a*tanh(d*x+c)^2+(a-b)*tanh(d*x+c)^4)^2-1/32*tanh(d*x+c)*((a-9*
b)*(a+b)/(a-b)^3-(a+19*b)*tanh(d*x+c)^2/(a-b)^2)/b/d/(a-2*a*tanh(d*x+c)^2+
(a-b)*tanh(d*x+c)^4)
```

Mathematica [A] (verified)

Time = 12.37 (sec) , antiderivative size = 331, normalized size of antiderivative = 1.08

$$\int \frac{\sinh^8(c+dx)}{\left(a-b\sinh^4(c+dx)\right)^3}\,dx = \frac{\frac{\left(2\sqrt{a}-5\sqrt{b}\right)\left(\sqrt{a}+\sqrt{b}\right)^2\sqrt{b}\arctan\left(\frac{\left(\sqrt{a}-\sqrt{b}\right)\tanh(c+dx)}{\sqrt{-a+\sqrt{a}\sqrt{b}}}\right)}{\sqrt{a}\sqrt{-a+\sqrt{a}\sqrt{b}}}+\frac{\left(2a^{3/2}\sqrt{b}+ab-8\sqrt{a}b^{3/2}+5b^2\right)\text{arctanh}\left(\frac{\left(\sqrt{a}+\sqrt{b}\right)\tanh(c+dx)}{\sqrt{a+\sqrt{a}\sqrt{b}}}\right)}{\sqrt{a}\sqrt{a+\sqrt{a}\sqrt{b}}}+\frac{8b(5a-}{8a}}{64(a-b)^2b^2d}$$

input
```
Integrate[Sinh[c + d*x]^8/(a - b*Sinh[c + d*x]^4)^3,x]
```

output
```
(((2*Sqrt[a] - 5*Sqrt[b])*(Sqrt[a] + Sqrt[b])^2*Sqrt[b]*ArcTan[((Sqrt[a] -
 Sqrt[b])*Tanh[c + d*x])/Sqrt[-a + Sqrt[a]*Sqrt[b]]])/(Sqrt[a]*Sqrt[-a + S
qrt[a]*Sqrt[b]]) + ((2*a^(3/2)*Sqrt[b] + a*b - 8*Sqrt[a]*b^(3/2) + 5*b^2)*
ArcTanh[((Sqrt[a] + Sqrt[b])*Tanh[c + d*x])/Sqrt[a + Sqrt[a]*Sqrt[b]]])/(S
qrt[a]*Sqrt[a + Sqrt[a]*Sqrt[b]]) + (8*b*(5*a - 14*b + (-2*a + 5*b)*Cosh[2
*(c + d*x)])*Sinh[2*(c + d*x)])/(8*a - 3*b + 4*b*Cosh[2*(c + d*x)] - b*Cos
h[4*(c + d*x)]) + (64*a*(a - b)*b*(-6*Sinh[2*(c + d*x)] + Sinh[4*(c + d*x)
]))/(-8*a + 3*b - 4*b*Cosh[2*(c + d*x)] + b*Cosh[4*(c + d*x)])^2)/(64*(a -
 b)^2*b^2*d)
```

Rubi [A] (verified)

Time = 0.77 (sec) , antiderivative size = 365, normalized size of antiderivative = 1.19, number of steps used = 14, number of rules used = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.542, Rules used = {3042, 3696, 1598, 27, 1440, 27, 1602, 27, 1602, 25, 27, 1480, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sinh^8(c+dx)}{\left(a-b\sinh^4(c+dx)\right)^3}dx$$

$$\Big\downarrow \ 3042$$

$$\int \frac{\sin(ic+idx)^8}{\left(a-b\sin(ic+idx)^4\right)^3}dx$$

$$\Big\downarrow \ 3696$$

$$\frac{\int \frac{\tanh^8(c+dx)\left(1-\tanh^2(c+dx)\right)}{\left((a-b)\tanh^4(c+dx)-2a\tanh^2(c+dx)+a\right)^3}d\tanh(c+dx)}{d}$$

$$\Big\downarrow \ 1598$$

$$\frac{\frac{\int -\frac{2b\tanh^8(c+dx)}{\left((a-b)\tanh^4(c+dx)-2a\tanh^2(c+dx)+a\right)^2}d\tanh(c+dx)}{16ab}+\frac{\tanh^9(c+dx)}{8a\left((a-b)\tanh^4(c+dx)-2a\tanh^2(c+dx)+a\right)^2}}{d}$$

$$\Big\downarrow \ 27$$

$$\frac{\frac{\tanh^9(c+dx)}{8a\left((a-b)\tanh^4(c+dx)-2a\tanh^2(c+dx)+a\right)^2}-\frac{\int \frac{\tanh^8(c+dx)}{\left((a-b)\tanh^4(c+dx)-2a\tanh^2(c+dx)+a\right)^2}d\tanh(c+dx)}{8a}}{d}$$

$$\Big\downarrow \ 1440$$

$$\frac{\frac{\tanh^9(c+dx)}{8a\left((a-b)\tanh^4(c+dx)-2a\tanh^2(c+dx)+a\right)^2}-\frac{\frac{\tanh^5(c+dx)\left(1-\tanh^2(c+dx)\right)}{4b\left((a-b)\tanh^4(c+dx)-2a\tanh^2(c+dx)+a\right)}-\frac{\int \frac{2a\tanh^4(c+dx)\left(5-3\tanh^2(c+dx)\right)}{(a-b)\tanh^4(c+dx)-2a\tanh^2(c+dx)+a}d\tanh(c+dx)}{8ab}}{8a}}{d}$$

$$\Big\downarrow \ 27$$

$$\frac{\frac{\tanh^9(c+dx)}{8a\left((a-b)\tanh^4(c+dx)-2a\tanh^2(c+dx)+a\right)^2}-\frac{\frac{\tanh^5(c+dx)\left(1-\tanh^2(c+dx)\right)}{4b\left((a-b)\tanh^4(c+dx)-2a\tanh^2(c+dx)+a\right)}-\frac{\int\frac{\tanh^4(c+dx)\left(5-3\tanh^2(c+dx)\right)}{(a-b)\tanh^4(c+dx)-2a\tanh^2(c+dx)+a}d\tanh(c+dx)}{4b}}{8a}}{d}$$

$\downarrow$ 1602

$$\frac{\frac{\tanh^9(c+dx)}{8a\left((a-b)\tanh^4(c+dx)-2a\tanh^2(c+dx)+a\right)^2}-\frac{\frac{\tanh^5(c+dx)\left(1-\tanh^2(c+dx)\right)}{4b\left((a-b)\tanh^4(c+dx)-2a\tanh^2(c+dx)+a\right)}-\frac{-\frac{\int-\frac{3\tanh^2(c+dx)\left(3a-(a+5b)\tanh^2(c+dx)\right)}{(a-b)\tanh^4(c+dx)-2a\tanh^2(c+dx)+a}d\tanh(c+\cdots}{3(a-b)}\cdots}{4b}}{8a}}{d}$$

$\downarrow$ 27

$$\frac{\frac{\tanh^9(c+dx)}{8a\left((a-b)\tanh^4(c+dx)-2a\tanh^2(c+dx)+a\right)^2}-\frac{\frac{\tanh^5(c+dx)\left(1-\tanh^2(c+dx)\right)}{4b\left((a-b)\tanh^4(c+dx)-2a\tanh^2(c+dx)+a\right)}-\frac{\frac{\int\frac{\tanh^2(c+dx)\left(3a-(a+5b)\tanh^2(c+dx)\right)}{(a-b)\tanh^4(c+dx)-2a\tanh^2(c+dx)+a}d\tanh(c+dx)}{a-b}-\cdots}{4b}}{8a}}{d}$$

$\downarrow$ 1602

$$\frac{\frac{\tanh^9(c+dx)}{8a\left((a-b)\tanh^4(c+dx)-2a\tanh^2(c+dx)+a\right)^2}-\frac{\frac{\tanh^5(c+dx)\left(1-\tanh^2(c+dx)\right)}{4b\left((a-b)\tanh^4(c+dx)-2a\tanh^2(c+dx)+a\right)}-\frac{\frac{-\frac{\int-\frac{a\left((a-13b)\tanh^2(c+dx)+a+5b\right)}{(a-b)\tanh^4(c+dx)-2a\tanh^2(c+dx)+a}d\tanh(c+\cdots}{a-b}\cdots}{a-b}\cdots}{4b}}{8a}}{d}$$

$\downarrow$ 25

$$\frac{\frac{\tanh^9(c+dx)}{8a\left((a-b)\tanh^4(c+dx)-2a\tanh^2(c+dx)+a\right)^2}-\frac{\frac{\tanh^5(c+dx)\left(1-\tanh^2(c+dx)\right)}{4b\left((a-b)\tanh^4(c+dx)-2a\tanh^2(c+dx)+a\right)}-\frac{\frac{\frac{\int\frac{a\left((a-13b)\tanh^2(c+dx)+a+5b\right)}{(a-b)\tanh^4(c+dx)-2a\tanh^2(c+dx)+a}d\tanh(c+dx)}{a-b}-\cdots}{a-b}\cdots}{4b}}{8a}}{d}$$

$\downarrow$ 27

$$\frac{\frac{\tanh^9(c+dx)}{8a\left((a-b)\tanh^4(c+dx)-2a\tanh^2(c+dx)+a\right)^2}-\frac{\frac{\tanh^5(c+dx)\left(1-\tanh^2(c+dx)\right)}{4b\left((a-b)\tanh^4(c+dx)-2a\tanh^2(c+dx)+a\right)}-\frac{\frac{\frac{a\int\frac{(a-13b)\tanh^2(c+dx)+a+5b}{(a-b)\tanh^4(c+dx)-2a\tanh^2(c+dx)+a}d\tanh(c+dx\cdots}{a-b}\cdots}{a-b}\cdots}{4b}}{8a}}{d}$$

$$\Big\downarrow \textcolor{blue}{1480}$$

$$\frac{\frac{\tanh^9(c+dx)}{8a\left((a-b)\tanh^4(c+dx)-2a\tanh^2(c+dx)+a\right)^2}-\frac{\frac{\tanh^5(c+dx)\left(1-\tanh^2(c+dx)\right)}{4b\left((a-b)\tanh^4(c+dx)-2a\tanh^2(c+dx)+a\right)}-\frac{a\left(\frac{1}{2}\left(-\frac{2a^2-9ab-5b^2}{\sqrt{a}\sqrt{b}}+a-13b\right)\int\frac{1}{(a-b)\tanh^2(c+dx)-\cdots}\cdots\right)}{\cdots}}{\cdots}}{d}$$

$$\Big\downarrow \textcolor{blue}{221}$$

$$\frac{\frac{\tanh^9(c+dx)}{8a\left((a-b)\tanh^4(c+dx)-2a\tanh^2(c+dx)+a\right)^2}-\frac{\frac{\tanh^5(c+dx)\left(1-\tanh^2(c+dx)\right)}{4b\left((a-b)\tanh^4(c+dx)-2a\tanh^2(c+dx)+a\right)}-\frac{a\left(-\frac{\left(2\sqrt{a}-5\sqrt{b}\right)\left(\sqrt{a}+\sqrt{b}\right)^2\text{arctanh}\left(\frac{\sqrt{\sqrt{a}-\sqrt{b}}\tan\cdots}{\sqrt[4]{a}}\right)}{2a^{3/4}\sqrt{b}\sqrt{\sqrt{a}-\sqrt{b}}}\cdots\right)}{\cdots}}{\cdots}}{d}$$

input

```
Int[Sinh[c + d*x]^8/(a - b*Sinh[c + d*x]^4)^3,x]
```

output

```
(Tanh[c + d*x]^9/(8*a*(a - 2*a*Tanh[c + d*x]^2 + (a - b)*Tanh[c + d*x]^4)^
2) - ((Tanh[c + d*x]^5*(1 - Tanh[c + d*x]^2))/(4*b*(a - 2*a*Tanh[c + d*x]^
2 + (a - b)*Tanh[c + d*x]^4)) - (-(Tanh[c + d*x]^3/(a - b)) + ((a*(-1/2*((
2*Sqrt[a] - 5*Sqrt[b])*(Sqrt[a] + Sqrt[b])^2*ArcTanh[(Sqrt[Sqrt[a] - Sqrt[
b]]*Tanh[c + d*x])/a^(1/4)])/(a^(3/4)*Sqrt[Sqrt[a] - Sqrt[b]]*Sqrt[b]) - (
(a - 13*b - (2*a^2 - 9*a*b - 5*b^2)/(Sqrt[a]*Sqrt[b]))*ArcTanh[(Sqrt[Sqrt[
a] + Sqrt[b]]*Tanh[c + d*x])/a^(1/4)])/(2*a^(1/4)*(Sqrt[a] - Sqrt[b])*Sqrt
[Sqrt[a] + Sqrt[b]])))/(a - b) - ((a + 5*b)*Tanh[c + d*x])/(a - b))/(a - b
))/(4*b))/(8*a))/d
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 1440
```
Int[((d_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
:> Simp[(-d^3)*(d*x)^(m - 3)*(2*a + b*x^2)*((a + b*x^2 + c*x^4)^(p + 1)/(2*
(p + 1)*(b^2 - 4*a*c))), x] + Simp[d^4/(2*(p + 1)*(b^2 - 4*a*c))   Int[(d*x
)^(m - 4)*(2*a*(m - 3) + b*(m + 4*p + 3)*x^2)*(a + b*x^2 + c*x^4)^(p + 1),
x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && Gt
Q[m, 3] && IntegerQ[2*p] && (IntegerQ[p] || IntegerQ[m])
```

rule 1480
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Simp[(e/2 + (2*c*d - b*e)/(2*q))   Int[1/(
b/2 - q/2 + c*x^2), x], x] + Simp[(e/2 - (2*c*d - b*e)/(2*q))   Int[1/(b/2
+ q/2 + c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0]
 && NeQ[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

rule 1598
```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(
x_)^4)^(p_.), x_Symbol] :> Simp[f*(f*x)^(m - 1)*(a + b*x^2 + c*x^4)^(p + 1)
*((b*d - 2*a*e - (b*e - 2*c*d)*x^2)/(2*(p + 1)*(b^2 - 4*a*c))), x] - Simp[f
^2/(2*(p + 1)*(b^2 - 4*a*c))   Int[(f*x)^(m - 2)*(a + b*x^2 + c*x^4)^(p + 1
)*Simp[(m - 1)*(b*d - 2*a*e) - (4*p + 4 + m + 1)*(b*e - 2*c*d)*x^2, x], x],
 x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] &&
 GtQ[m, 1] && IntegerQ[2*p] && (IntegerQ[p] || IntegerQ[m])
```

rule 1602
```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(
x_)^4)^(p_), x_Symbol] :> Simp[e*f*(f*x)^(m - 1)*((a + b*x^2 + c*x^4)^(p +
1)/(c*(m + 4*p + 3))), x] - Simp[f^2/(c*(m + 4*p + 3))   Int[(f*x)^(m - 2)*
(a + b*x^2 + c*x^4)^p*Simp[a*e*(m - 1) + (b*e*(m + 2*p + 1) - c*d*(m + 4*p
+ 3))*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && NeQ[b^2 - 4*a*c
, 0] && GtQ[m, 1] && NeQ[m + 4*p + 3, 0] && IntegerQ[2*p] && (IntegerQ[p] |
| IntegerQ[m])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3696
```
Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^4)^(
p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Simp[ff^(m + 1
)/f   Subst[Int[x^m*((a + 2*a*ff^2*x^2 + (a + b)*ff^4*x^4)^p/(1 + ff^2*x^2)
^(m/2 + 2*p + 1)), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f}, x] &
& IntegerQ[m/2] && IntegerQ[p]
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 11.69 (sec) , antiderivative size = 535, normalized size of antiderivative = 1.74

method	result
derivativedivides	$-\frac{512\left(\frac{a(a+5b)\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)}{8192b\left(a^2-2ab+b^2\right)}-\frac{(5a+49b)a\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^3}{8192b\left(a^2-2ab+b^2\right)}+\frac{3\left(3a^2+55ab-48b^2\right)\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^5}{8192b\left(a^2-2ab+b^2\right)}-\frac{\left(5a^2+377ab-784b^2\right)\tanh\left(\frac{dx}{2}\right.}{8192b\left(a^2-2ab+b^2\right)}\right.}{\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^8a-4\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^6a+6\tan\right.}$
default	$-\frac{512\left(\frac{a(a+5b)\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)}{8192b\left(a^2-2ab+b^2\right)}-\frac{(5a+49b)a\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^3}{8192b\left(a^2-2ab+b^2\right)}+\frac{3\left(3a^2+55ab-48b^2\right)\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^5}{8192b\left(a^2-2ab+b^2\right)}-\frac{\left(5a^2+377ab-784b^2\right)\tanh\left(\frac{dx}{2}\right.}{8192b\left(a^2-2ab+b^2\right)}\right.}{\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^8a-4\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^6a+6\tan\right.}$
risch	Expression too large to display

input
```
int(sinh(d*x+c)^8/(a-b*sinh(d*x+c)^4)^3,x,method=_RETURNVERBOSE)
```

output
```
1/d*(-512*(1/8192*a*(a+5*b)/b/(a^2-2*a*b+b^2)*tanh(1/2*d*x+1/2*c)-1/8192*(
5*a+49*b)*a/b/(a^2-2*a*b+b^2)*tanh(1/2*d*x+1/2*c)^3+3/8192/b*(3*a^2+55*a*b
-48*b^2)/(a^2-2*a*b+b^2)*tanh(1/2*d*x+1/2*c)^5-1/8192*(5*a^2+377*a*b-784*b
^2)/b/(a^2-2*a*b+b^2)*tanh(1/2*d*x+1/2*c)^7-1/8192*(5*a^2+377*a*b-784*b^2)
/b/(a^2-2*a*b+b^2)*tanh(1/2*d*x+1/2*c)^9+3/8192/b*(3*a^2+55*a*b-48*b^2)/(a
^2-2*a*b+b^2)*tanh(1/2*d*x+1/2*c)^11-1/8192*(5*a+49*b)*a/b/(a^2-2*a*b+b^2)
*tanh(1/2*d*x+1/2*c)^13+1/8192*a*(a+5*b)/b/(a^2-2*a*b+b^2)*tanh(1/2*d*x+1/
2*c)^15)/(tanh(1/2*d*x+1/2*c)^8*a-4*tanh(1/2*d*x+1/2*c)^6*a+6*tanh(1/2*d*x
+1/2*c)^4*a-16*b*tanh(1/2*d*x+1/2*c)^4-4*tanh(1/2*d*x+1/2*c)^2*a+a)^2-1/12
8/b/(a^2-2*a*b+b^2)*sum(((a+5*b)*_R^6+(5*a-47*b)*_R^4+(-5*a+47*b)*_R^2-a-5
*b)/(_R^7*a-3*_R^5*a+3*_R^3*a-8*_R^3*b-_R*a)*ln(tanh(1/2*d*x+1/2*c)-_R),_R
=RootOf(a*_Z^8-4*a*_Z^6+(6*a-16*b)*_Z^4-4*a*_Z^2+a)))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 20486 vs. 2(257) = 514.

Time = 0.73 (sec) , antiderivative size = 20486, normalized size of antiderivative = 66.73

$$\int \frac{\sinh^8(c+dx)}{(a-b\sinh^4(c+dx))^3}\,dx = \text{Too large to display}$$

input
```
integrate(sinh(d*x+c)^8/(a-b*sinh(d*x+c)^4)^3,x, algorithm="fricas")
```

output
```
Too large to include
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\sinh^8(c+dx)}{(a-b\sinh^4(c+dx))^3}\,dx = \text{Timed out}$$

input
```
integrate(sinh(d*x+c)**8/(a-b*sinh(d*x+c)**4)**3,x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{\sinh^8(c+dx)}{\left(a-b\sinh^4(c+dx)\right)^3}\,dx=\int -\frac{\sinh\left(dx+c\right)^8}{\left(b\sinh\left(dx+c\right)^4-a\right)^3}\,dx$$

input
```
integrate(sinh(d*x+c)^8/(a-b*sinh(d*x+c)^4)^3,x, algorithm="maxima")
```

output
```
-1/8*(2*a*b^2 - 5*b^3 + (a*b^2*e^(14*c) - 4*b^3*e^(14*c))*e^(14*d*x) - (32
*a^2*b*e^(12*c) - 58*a*b^2*e^(12*c) - b^3*e^(12*c))*e^(12*d*x) + 3*(48*a^2
*b*e^(10*c) - 73*a*b^2*e^(10*c) + 20*b^3*e^(10*c))*e^(10*d*x) + (256*a^3*e
^(8*c) - 832*a^2*b*e^(8*c) + 550*a*b^2*e^(8*c) - 175*b^3*e^(8*c))*e^(8*d*x
) + (112*a^2*b*e^(6*c) - 533*a*b^2*e^(6*c) + 220*b^3*e^(6*c))*e^(6*d*x) -
(32*a^2*b*e^(4*c) - 158*a*b^2*e^(4*c) + 141*b^3*e^(4*c))*e^(4*d*x) - (17*a
*b^2*e^(2*c) - 44*b^3*e^(2*c))*e^(2*d*x))/(a^2*b^4*d - 2*a*b^5*d + b^6*d +
 (a^2*b^4*d*e^(16*c) - 2*a*b^5*d*e^(16*c) + b^6*d*e^(16*c))*e^(16*d*x) - 8
*(a^2*b^4*d*e^(14*c) - 2*a*b^5*d*e^(14*c) + b^6*d*e^(14*c))*e^(14*d*x) - 4
*(8*a^3*b^3*d*e^(12*c) - 23*a^2*b^4*d*e^(12*c) + 22*a*b^5*d*e^(12*c) - 7*b
^6*d*e^(12*c))*e^(12*d*x) + 8*(16*a^3*b^3*d*e^(10*c) - 39*a^2*b^4*d*e^(10*
c) + 30*a*b^5*d*e^(10*c) - 7*b^6*d*e^(10*c))*e^(10*d*x) + 2*(128*a^4*b^2*d
*e^(8*c) - 352*a^3*b^3*d*e^(8*c) + 355*a^2*b^4*d*e^(8*c) - 166*a*b^5*d*e^(
8*c) + 35*b^6*d*e^(8*c))*e^(8*d*x) + 8*(16*a^3*b^3*d*e^(6*c) - 39*a^2*b^4*
d*e^(6*c) + 30*a*b^5*d*e^(6*c) - 7*b^6*d*e^(6*c))*e^(6*d*x) - 4*(8*a^3*b^3
*d*e^(4*c) - 23*a^2*b^4*d*e^(4*c) + 22*a*b^5*d*e^(4*c) - 7*b^6*d*e^(4*c))*
e^(4*d*x) - 8*(a^2*b^4*d*e^(2*c) - 2*a*b^5*d*e^(2*c) + b^6*d*e^(2*c))*e^(2
*d*x)) - 1/256*integrate(64*((a*e^(6*c) - 4*b*e^(6*c))*e^(6*d*x) + (a*e^(2
*c) - 4*b*e^(2*c))*e^(2*d*x) + 18*b*e^(4*d*x + 4*c))/(a^2*b^2 - 2*a*b^3 +
b^4 + (a^2*b^2*e^(8*c) - 2*a*b^3*e^(8*c) + b^4*e^(8*c))*e^(8*d*x) - 4*(...
```

Giac [F]

$$\int \frac{\sinh^8(c+dx)}{\left(a-b\sinh^4(c+dx)\right)^3}\,dx=\int -\frac{\sinh\left(dx+c\right)^8}{\left(b\sinh\left(dx+c\right)^4-a\right)^3}\,dx$$

input
```
integrate(sinh(d*x+c)^8/(a-b*sinh(d*x+c)^4)^3,x, algorithm="giac")
```

output
```
sage0*x
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sinh^8(c+dx)}{\left(a-b\sinh^4(c+dx)\right)^3}\,dx=\int \frac{\sinh(c+d\,x)^8}{\left(a-b\sinh\left(c+d\,x\right)^4\right)^3}\,dx$$

input
```
int(sinh(c + d*x)^8/(a - b*sinh(c + d*x)^4)^3,x)
```

output
```
int(sinh(c + d*x)^8/(a - b*sinh(c + d*x)^4)^3, x)
```

Reduce [F]

$$\int \frac{\sinh^8(c+dx)}{\left(a-b\sinh^4(c+dx)\right)^3}\,dx=\text{too large to display}$$

input
```
int(sinh(d*x+c)^8/(a-b*sinh(d*x+c)^4)^3,x)
```

output

```
(4*(29554872554618880*e**(20*c + 16*d*x)*int(e**(4*d*x)/(3440640*e**(24*c
+ 24*d*x)*a**4*b**3 + 2150400*e**(24*c + 24*d*x)*a**3*b**4 + 16128*e**(24*
c + 24*d*x)*a**2*b**5 - 96*e**(24*c + 24*d*x)*a*b**6 + 35*e**(24*c + 24*d*
x)*b**7 - 41287680*e**(22*c + 22*d*x)*a**4*b**3 - 25804800*e**(22*c + 22*d
*x)*a**3*b**4 - 193536*e**(22*c + 22*d*x)*a**2*b**5 + 1152*e**(22*c + 22*d
*x)*a*b**6 - 420*e**(22*c + 22*d*x)*b**7 - 165150720*e**(20*c + 20*d*x)*a*
*5*b**2 + 123863040*e**(20*c + 20*d*x)*a**4*b**3 + 141152256*e**(20*c + 20
*d*x)*a**3*b**4 + 1069056*e**(20*c + 20*d*x)*a**2*b**5 - 8016*e**(20*c + 2
0*d*x)*a*b**6 + 2310*e**(20*c + 20*d*x)*b**7 + 1321205760*e**(18*c + 18*d*
x)*a**5*b**2 + 68812800*e**(18*c + 18*d*x)*a**4*b**3 - 466894848*e**(18*c
+ 18*d*x)*a**3*b**4 - 3585024*e**(18*c + 18*d*x)*a**2*b**5 + 34560*e**(18*
c + 18*d*x)*a*b**6 - 7700*e**(18*c + 18*d*x)*b**7 + 2642411520*e**(16*c +
16*d*x)*a**6*b - 2972712960*e**(16*c + 16*d*x)*a**5*b**2 - 1174634496*e**(
16*c + 16*d*x)*a**4*b**3 + 1042698240*e**(16*c + 16*d*x)*a**3*b**4 + 81392
64*e**(16*c + 16*d*x)*a**2*b**5 - 94560*e**(16*c + 16*d*x)*a*b**6 + 17325*
e**(16*c + 16*d*x)*b**7 - 10569646080*e**(14*c + 14*d*x)*a**6*b + 26424115
20*e**(14*c + 14*d*x)*a**5*b**2 + 3005743104*e**(14*c + 14*d*x)*a**4*b**3
- 1659469824*e**(14*c + 14*d*x)*a**3*b**4 - 13138944*e**(14*c + 14*d*x)*a*
*2*b**5 + 170112*e**(14*c + 14*d*x)*a*b**6 - 27720*e**(14*c + 14*d*x)*b**7
 - 14092861440*e**(12*c + 12*d*x)*a**7 + 7046430720*e**(12*c + 12*d*x)*...
```

3.235 $\int \frac{\sinh^6(c+dx)}{(a-b\sinh^4(c+dx))^3}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 345

$$\begin{aligned}\int \frac{\sinh^6(c+dx)}{(a-b\sinh^4(c+dx))^3}\,dx &= \frac{\left(4a-10\sqrt{a}\sqrt{b}+3b\right)\operatorname{arctanh}\left(\frac{\sqrt{\sqrt{a}-\sqrt{b}}\tanh(c+dx)}{\sqrt[4]{a}}\right)}{64a^{5/4}\left(\sqrt{a}-\sqrt{b}\right)^{5/2}b^{3/2}d} \\ &\quad - \frac{\left(4a+10\sqrt{a}\sqrt{b}+3b\right)\operatorname{arctanh}\left(\frac{\sqrt{\sqrt{a}+\sqrt{b}}\tanh(c+dx)}{\sqrt[4]{a}}\right)}{64a^{5/4}\left(\sqrt{a}+\sqrt{b}\right)^{5/2}b^{3/2}d} \\ &\quad + \frac{\tanh(c+dx)\left(a(a+3b)-(a^2+6ab+b^2)\tanh^2(c+dx)\right)}{8(a-b)^3d\left(a-2a\tanh^2(c+dx)+(a-b)\tanh^4(c+dx)\right)^2} \\ &\quad + \frac{\tanh(c+dx)\left(\frac{2a(a^2-ab-8b^2)}{(a-b)^3}-\frac{(2a^2+15ab+3b^2)\tanh^2(c+dx)}{(a-b)^2}\right)}{32abd\left(a-2a\tanh^2(c+dx)+(a-b)\tanh^4(c+dx)\right)}\end{aligned}$$

output
```
1/64*(4*a-10*a^(1/2)*b^(1/2)+3*b)*arctanh((a^(1/2)-b^(1/2))^(1/2)*tanh(d*x
+c)/a^(1/4))/a^(5/4)/(a^(1/2)-b^(1/2))^(5/2)/b^(3/2)/d-1/64*(4*a+10*a^(1/2
)*b^(1/2)+3*b)*arctanh((a^(1/2)+b^(1/2))^(1/2)*tanh(d*x+c)/a^(1/4))/a^(5/4
)/(a^(1/2)+b^(1/2))^(5/2)/b^(3/2)/d+1/8*tanh(d*x+c)*(a*(a+3*b)-(a^2+6*a*b+
b^2)*tanh(d*x+c)^2)/(a-b)^3/d/(a-2*a*tanh(d*x+c)^2+(a-b)*tanh(d*x+c)^4)^2+
1/32*tanh(d*x+c)*(2*a*(a^2-a*b-8*b^2)/(a-b)^3-(2*a^2+15*a*b+3*b^2)*tanh(d*
x+c)^2/(a-b)^2)/a/b/d/(a-2*a*tanh(d*x+c)^2+(a-b)*tanh(d*x+c)^4)
```

Mathematica [A] (verified)

Time = 7.69 (sec) , antiderivative size = 351, normalized size of antiderivative = 1.02

$$\int \frac{\sinh^6(c+dx)}{\left(a-b\sinh^4(c+dx)\right)^3}\,dx = -\frac{\frac{\left(\sqrt{a}+\sqrt{b}\right)^2\left(4a-10\sqrt{a}\sqrt{b}+3b\right)\arctan\left(\frac{\left(\sqrt{a}-\sqrt{b}\right)\tanh(c+dx)}{\sqrt{-a+\sqrt{a}\sqrt{b}}}\right)}{a\sqrt{-a+\sqrt{a}\sqrt{b}}b^{3/2}}+\frac{\left(\sqrt{a}-\sqrt{b}\right)^2\left(4a+10\sqrt{a}\sqrt{b}+3b\right)\text{arctanh}\left(\frac{\left(\sqrt{a}+\sqrt{b}\right)\tanh(c+dx)}{\sqrt{a+\sqrt{a}\sqrt{b}}}\right)}{a\sqrt{a+\sqrt{a}\sqrt{b}}b^{3/2}}+}{64(a-b)^2d}$$

input
```
Integrate[Sinh[c + d*x]^6/(a - b*Sinh[c + d*x]^4)^3,x]
```

output
```
-1/64*(((Sqrt[a] + Sqrt[b])^2*(4*a - 10*Sqrt[a]*Sqrt[b] + 3*b)*ArcTan[((Sq
rt[a] - Sqrt[b])*Tanh[c + d*x])/Sqrt[-a + Sqrt[a]*Sqrt[b]]])/(a*Sqrt[-a +
Sqrt[a]*Sqrt[b]]*b^(3/2)) + ((Sqrt[a] - Sqrt[b])^2*(4*a + 10*Sqrt[a]*Sqrt[
b] + 3*b)*ArcTanh[((Sqrt[a] + Sqrt[b])*Tanh[c + d*x])/Sqrt[a + Sqrt[a]*Sqr
t[b]]])/(a*Sqrt[a + Sqrt[a]*Sqrt[b]]*b^(3/2)) + (4*(4*a^2 - 19*a*b - 3*b^2
 + 3*b*(a + b)*Cosh[2*(c + d*x)])*Sinh[2*(c + d*x)])/(a*b*(8*a - 3*b + 4*b
*Cosh[2*(c + d*x)] - b*Cosh[4*(c + d*x)])) - (128*(a - b)*(2*a + b - b*Cos
h[2*(c + d*x)])*Sinh[2*(c + d*x)])/(b*(-8*a + 3*b - 4*b*Cosh[2*(c + d*x)]
+ b*Cosh[4*(c + d*x)])^2))/((a - b)^2*d)
```

Rubi [A] (verified)

Time = 0.91 (sec) , antiderivative size = 384, normalized size of antiderivative = 1.11, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.375, Rules used = {3042, 25, 3696, 1672, 27, 2206, 27, 1480, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sinh^6(c+dx)}{\left(a-b\sinh^4(c+dx)\right)^3}dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int -\frac{\sin(ic+idx)^6}{\left(a-b\sin(ic+idx)^4\right)^3}dx$$

$$\Big\downarrow \text{ 25}$$

$$-\int \frac{\sin(ic+idx)^6}{\left(a-b\sin(ic+idx)^4\right)^3}dx$$

$$\Big\downarrow \text{ 3696}$$

$$\frac{\int \frac{\tanh^6(c+dx)\left(1-\tanh^2(c+dx)\right)^2}{\left((a-b)\tanh^4(c+dx)-2a\tanh^2(c+dx)+a\right)^3}d\tanh(c+dx)}{d}$$

$$\Big\downarrow \text{ 1672}$$

$$\frac{\frac{\tanh(c+dx)\left(a(a+3b)-\left(a^2+6ab+b^2\right)\tanh^2(c+dx)\right)}{8(a-b)^3\left((a-b)\tanh^4(c+dx)-2a\tanh^2(c+dx)+a\right)^2}-\frac{\int \frac{2\left(-\frac{8a^2b\tanh^6(c+dx)}{a-b}-\frac{16a^2b^2\tanh^4(c+dx)}{(a-b)^2}+\frac{a^2b\left(5a^2+6ba-3b^2\right)\tanh^2(c+dx)}{(a-b)^3}+\frac{a^3b(a+3b)}{(a-b)^3}\cdots\right.}{\left((a-b)\tanh^4(c+dx)-2a\tanh^2(c+dx)+a\right)^2}}{16a^2b}\cdots}{d}$$

$$\Big\downarrow \text{ 27}$$

$$\frac{\frac{\tanh(c+dx)\left(a(a+3b)-\left(a^2+6ab+b^2\right)\tanh^2(c+dx)\right)}{8(a-b)^3\left((a-b)\tanh^4(c+dx)-2a\tanh^2(c+dx)+a\right)^2}-\frac{\int \frac{-\frac{8a^2b\tanh^6(c+dx)}{a-b}-\frac{16a^2b^2\tanh^4(c+dx)}{(a-b)^2}+\frac{a^2b\left(5a^2+6ba-3b^2\right)\tanh^2(c+dx)}{(a-b)^3}+\frac{a^3b(a+3b)}{(a-b)^3}}{\left((a-b)\tanh^4(c+dx)-2a\tanh^2(c+dx)+a\right)^2}d\mathrm{t}\cdots}{8a^2b}\cdots}{d}$$

$$\Big\downarrow \text{ 2206}$$

$$\frac{\frac{\tanh(c+dx)\left(a(a+3b)-\left(a^2+6ab+b^2\right)\tanh^2(c+dx)\right)}{8(a-b)^3\left((a-b)\tanh^4(c+dx)-2a\tanh^2(c+dx)+a\right)^2}-\frac{-\frac{\int -\frac{2a^3b\left(\left(2a^2-17ba+3b^2\right)\tanh^2(c+dx)+2a(a+2b)\right)}{(a-b)^2\left((a-b)\tanh^4(c+dx)-2a\tanh^2(c+dx)+a\right)}d\tanh(c+dx)}{8a^2b}-\frac{a\tanh(c+dx)\left(\frac{2a\left(a^2-ab\right.}{(a-b)}\right.}{4\left((a-b)\tanh\right.}}{8a^2b}}{d}$$

$\downarrow$ 27

$$\frac{\frac{\tanh(c+dx)\left(a(a+3b)-\left(a^2+6ab+b^2\right)\tanh^2(c+dx)\right)}{8(a-b)^3\left((a-b)\tanh^4(c+dx)-2a\tanh^2(c+dx)+a\right)^2}-\frac{\frac{a\int \frac{\left(2a^2-17ba+3b^2\right)\tanh^2(c+dx)+2a(a+2b)}{(a-b)\tanh^4(c+dx)-2a\tanh^2(c+dx)+a}d\tanh(c+dx)}{4(a-b)^2}-\frac{a\tanh(c+dx)\left(\frac{2a\left(a^2-ab-8b^2\right)}{(a-b)^3}-\right.}{4\left((a-b)\tanh^4(c+dx)-\right.}}{8a^2b}}{d}$$

$\downarrow$ 1480

$$\frac{\frac{\tanh(c+dx)\left(a(a+3b)-\left(a^2+6ab+b^2\right)\tanh^2(c+dx)\right)}{8(a-b)^3\left((a-b)\tanh^4(c+dx)-2a\tanh^2(c+dx)+a\right)^2}-\frac{a\left(\frac{\left(\sqrt{a}+\sqrt{b}\right)^3\left(-10\sqrt{a}\sqrt{b}+4a+3b\right)\int \frac{1}{(a-b)\tanh^2(c+dx)-\sqrt{a}\left(\sqrt{a}+\sqrt{b}\right)}d\tanh(c+dx)}{2\sqrt{b}}-\frac{\left(\sqrt{a}-\sqrt{b}\right.}{}\right.}{4(a-b)^2}}{d}$$

$\downarrow$ 221

$$\frac{\frac{\tanh(c+dx)\left(a(a+3b)-\left(a^2+6ab+b^2\right)\tanh^2(c+dx)\right)}{8(a-b)^3\left((a-b)\tanh^4(c+dx)-2a\tanh^2(c+dx)+a\right)^2}-\frac{a\left(\frac{\left(\sqrt{a}-\sqrt{b}\right)^2\left(10\sqrt{a}\sqrt{b}+4a+3b\right)\text{arctanh}\left(\frac{\sqrt{\sqrt{a}+\sqrt{b}}\tanh(c+dx)}{\sqrt[4]{a}}\right)}{2\sqrt[4]{a}\sqrt{b}\sqrt{\sqrt{a}+\sqrt{b}}}-\frac{\left(\sqrt{a}+\sqrt{b}\right)^2\left(-10\sqrt{a}\sqrt{b}-\right.}{}\right.}{4(a-b)^2}}{d}$$

input
```
Int[Sinh[c + d*x]^6/(a - b*Sinh[c + d*x]^4)^3,x]
```

output
```
((Tanh[c + d*x]*(a*(a + 3*b) - (a^2 + 6*a*b + b^2)*Tanh[c + d*x]^2))/(8*(a
 - b)^3*(a - 2*a*Tanh[c + d*x]^2 + (a - b)*Tanh[c + d*x]^4)^2) - ((a*(-1/2
*((Sqrt[a] + Sqrt[b])^2*(4*a - 10*Sqrt[a]*Sqrt[b] + 3*b)*ArcTanh[(Sqrt[Sqr
t[a] - Sqrt[b]]*Tanh[c + d*x])/a^(1/4)])/(a^(1/4)*Sqrt[Sqrt[a] - Sqrt[b]]*
Sqrt[b]) + ((Sqrt[a] - Sqrt[b])^2*(4*a + 10*Sqrt[a]*Sqrt[b] + 3*b)*ArcTanh
[(Sqrt[Sqrt[a] + Sqrt[b]]*Tanh[c + d*x])/a^(1/4)])/(2*a^(1/4)*Sqrt[Sqrt[a]
 + Sqrt[b]]*Sqrt[b])))/(4*(a - b)^2) - (a*Tanh[c + d*x]*((2*a*(a^2 - a*b -
 8*b^2))/(a - b)^3 - ((2*a^2 + 15*a*b + 3*b^2)*Tanh[c + d*x]^2)/(a - b)^2)
)/(4*(a - 2*a*Tanh[c + d*x]^2 + (a - b)*Tanh[c + d*x]^4)))/(8*a^2*b))/d
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 1480
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Simp[(e/2 + (2*c*d - b*e)/(2*q))   Int[1/(
b/2 - q/2 + c*x^2), x], x] + Simp[(e/2 - (2*c*d - b*e)/(2*q))   Int[1/(b/2
+ q/2 + c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0]
 && NeQ[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

rule 1672
```
Int[(x_)^(m_)*((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^
4)^(p_), x_Symbol] :> With[{f = Coeff[PolynomialRemainder[x^m*(d + e*x^2)^q
, a + b*x^2 + c*x^4, x], x, 0], g = Coeff[PolynomialRemainder[x^m*(d + e*x^
2)^q, a + b*x^2 + c*x^4, x], x, 2]}, Simp[x*(a + b*x^2 + c*x^4)^(p + 1)*((a
*b*g - f*(b^2 - 2*a*c) - c*(b*f - 2*a*g)*x^2)/(2*a*(p + 1)*(b^2 - 4*a*c))),
 x] + Simp[1/(2*a*(p + 1)*(b^2 - 4*a*c))   Int[(a + b*x^2 + c*x^4)^(p + 1)*
Simp[ExpandToSum[2*a*(p + 1)*(b^2 - 4*a*c)*PolynomialQuotient[x^m*(d + e*x^
2)^q, a + b*x^2 + c*x^4, x] + b^2*f*(2*p + 3) - 2*a*c*f*(4*p + 5) - a*b*g +
 c*(4*p + 7)*(b*f - 2*a*g)*x^2, x], x], x], x]] /; FreeQ[{a, b, c, d, e}, x
] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && IGtQ[q, 1] && IGtQ[m/2, 0]
```

rule 2206
```
Int[(Px_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> With[{d =
 Coeff[PolynomialRemainder[Px, a + b*x^2 + c*x^4, x], x, 0], e = Coeff[Poly
nomialRemainder[Px, a + b*x^2 + c*x^4, x], x, 2]}, Simp[x*(a + b*x^2 + c*x^
4)^(p + 1)*((a*b*e - d*(b^2 - 2*a*c) - c*(b*d - 2*a*e)*x^2)/(2*a*(p + 1)*(b
^2 - 4*a*c))), x] + Simp[1/(2*a*(p + 1)*(b^2 - 4*a*c))   Int[(a + b*x^2 + c
*x^4)^(p + 1)*ExpandToSum[2*a*(p + 1)*(b^2 - 4*a*c)*PolynomialQuotient[Px,
a + b*x^2 + c*x^4, x] + b^2*d*(2*p + 3) - 2*a*c*d*(4*p + 5) - a*b*e + c*(4*
p + 7)*(b*d - 2*a*e)*x^2, x], x], x]] /; FreeQ[{a, b, c}, x] && PolyQ[Px, x
^2] && Expon[Px, x^2] > 1 && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3696
```
Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^4)^(
p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Simp[ff^(m + 1
)/f   Subst[Int[x^m*((a + 2*a*ff^2*x^2 + (a + b)*ff^4*x^4)^p/(1 + ff^2*x^2)
^(m/2 + 2*p + 1)), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f}, x] &
& IntegerQ[m/2] && IntegerQ[p]
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 10.86 (sec) , antiderivative size = 594, normalized size of antiderivative = 1.72

method	result
derivativedivides	$-\frac{128\left(-\frac{a(a+2b)\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)}{1024b\left(a^2-2ab+b^2\right)}+\frac{\left(5a^2+24ab-2b^2\right)\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^3}{1024b\left(a^2-2ab+b^2\right)}-\frac{\left(9a^2+76ab-70b^2\right)\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^5}{1024b\left(a^2-2ab+b^2\right)}+\frac{\left(5a^3+54a^2b-164b^2a-\cdots\right.}{1024ab\left(a^2\cdots\right.}\right.}{\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^8a-4\tanh\left(\frac{d\cdots}{2}\right.\right.}$
default	$-\frac{128\left(-\frac{a(a+2b)\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)}{1024b\left(a^2-2ab+b^2\right)}+\frac{\left(5a^2+24ab-2b^2\right)\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^3}{1024b\left(a^2-2ab+b^2\right)}-\frac{\left(9a^2+76ab-70b^2\right)\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^5}{1024b\left(a^2-2ab+b^2\right)}+\frac{\left(5a^3+54a^2b-164b^2a-\cdots\right.}{1024ab\left(a^2\cdots\right.}\right.}{\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^8a-4\tanh\left(\frac{d\cdots}{2}\right.\right.}$
risch	Expression too large to display

input
```
int(sinh(d*x+c)^6/(a-b*sinh(d*x+c)^4)^3,x,method=_RETURNVERBOSE)
```

output
```
1/d*(-128*(-1/1024*a*(a+2*b)/b/(a^2-2*a*b+b^2)*tanh(1/2*d*x+1/2*c)+1/1024*
(5*a^2+24*a*b-2*b^2)/b/(a^2-2*a*b+b^2)*tanh(1/2*d*x+1/2*c)^3-1/1024/b*(9*a
^2+76*a*b-70*b^2)/(a^2-2*a*b+b^2)*tanh(1/2*d*x+1/2*c)^5+1/1024*(5*a^3+54*a
^2*b-164*a*b^2-96*b^3)/a/b/(a^2-2*a*b+b^2)*tanh(1/2*d*x+1/2*c)^7+1/1024*(5
*a^3+54*a^2*b-164*a*b^2-96*b^3)/a/b/(a^2-2*a*b+b^2)*tanh(1/2*d*x+1/2*c)^9-
1/1024/b*(9*a^2+76*a*b-70*b^2)/(a^2-2*a*b+b^2)*tanh(1/2*d*x+1/2*c)^11+1/10
24*(5*a^2+24*a*b-2*b^2)/b/(a^2-2*a*b+b^2)*tanh(1/2*d*x+1/2*c)^13-1/1024*a*
(a+2*b)/b/(a^2-2*a*b+b^2)*tanh(1/2*d*x+1/2*c)^15)/(tanh(1/2*d*x+1/2*c)^8*a
-4*tanh(1/2*d*x+1/2*c)^6*a+6*tanh(1/2*d*x+1/2*c)^4*a-16*b*tanh(1/2*d*x+1/2
*c)^4-4*tanh(1/2*d*x+1/2*c)^2*a+a)^2-1/64/a/b/(a^2-2*a*b+b^2)*sum((a*(-a-2
*b)*_R^6+(-5*a^2+32*a*b-6*b^2)*_R^4+(5*a^2-32*a*b+6*b^2)*_R^2+a^2+2*a*b)/(
_R^7*a-3*_R^5*a+3*_R^3*a-8*_R^3*b-_R*a)*ln(tanh(1/2*d*x+1/2*c)-_R),_R=Root
Of(a*_Z^8-4*a*_Z^6+(6*a-16*b)*_Z^4-4*a*_Z^2+a))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 22729 vs. $2(292) = 584$.

Time = 1.01 (sec) , antiderivative size = 22729, normalized size of antiderivative = 65.88

$$\int \frac{\sinh^6(c+dx)}{(a-b\sinh^4(c+dx))^3}\,dx = \text{Too large to display}$$

input
```
integrate(sinh(d*x+c)^6/(a-b*sinh(d*x+c)^4)^3,x, algorithm="fricas")
```

output
```
Too large to include
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\sinh^6(c+dx)}{(a-b\sinh^4(c+dx))^3}\,dx = \text{Timed out}$$

input
```
integrate(sinh(d*x+c)**6/(a-b*sinh(d*x+c)**4)**3,x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{\sinh^6(c+dx)}{(a-b\sinh^4(c+dx))^3}\,dx = \int -\frac{\sinh(dx+c)^6}{(b\sinh(dx+c)^4-a)^3}\,dx$$

input
```
integrate(sinh(d*x+c)^6/(a-b*sinh(d*x+c)^4)^3,x, algorithm="maxima")
```

output
```
-1/16*(3*a*b^2 + 3*b^3 - (4*a^2*b*e^(14*c) - 13*a*b^2*e^(14*c) + 3*b^3*e^(
14*c))*e^(14*d*x) + 3*(8*a^2*b*e^(12*c) - 33*a*b^2*e^(12*c) + 7*b^3*e^(12*
c))*e^(12*d*x) - (64*a^3*e^(10*c) + 68*a^2*b*e^(10*c) - 225*a*b^2*e^(10*c)
 + 63*b^3*e^(10*c))*e^(10*d*x) + 3*(128*a^3*e^(8*c) + 32*a^2*b*e^(8*c) - 6
1*a*b^2*e^(8*c) + 35*b^3*e^(8*c))*e^(8*d*x) + (64*a^3*e^(6*c) + 452*a^2*b*
e^(6*c) - 9*a*b^2*e^(6*c) - 105*b^3*e^(6*c))*e^(6*d*x) - 3*(40*a^2*b*e^(4*
c) - 29*a*b^2*e^(4*c) - 21*b^3*e^(4*c))*e^(4*d*x) + (4*a^2*b*e^(2*c) - 37*
a*b^2*e^(2*c) - 21*b^3*e^(2*c))*e^(2*d*x))/(a^3*b^3*d - 2*a^2*b^4*d + a*b^
5*d + (a^3*b^3*d*e^(16*c) - 2*a^2*b^4*d*e^(16*c) + a*b^5*d*e^(16*c))*e^(16
*d*x) - 8*(a^3*b^3*d*e^(14*c) - 2*a^2*b^4*d*e^(14*c) + a*b^5*d*e^(14*c))*e
^(14*d*x) - 4*(8*a^4*b^2*d*e^(12*c) - 23*a^3*b^3*d*e^(12*c) + 22*a^2*b^4*d
*e^(12*c) - 7*a*b^5*d*e^(12*c))*e^(12*d*x) + 8*(16*a^4*b^2*d*e^(10*c) - 39
*a^3*b^3*d*e^(10*c) + 30*a^2*b^4*d*e^(10*c) - 7*a*b^5*d*e^(10*c))*e^(10*d*
x) + 2*(128*a^5*b*d*e^(8*c) - 352*a^4*b^2*d*e^(8*c) + 355*a^3*b^3*d*e^(8*c
) - 166*a^2*b^4*d*e^(8*c) + 35*a*b^5*d*e^(8*c))*e^(8*d*x) + 8*(16*a^4*b^2*
d*e^(6*c) - 39*a^3*b^3*d*e^(6*c) + 30*a^2*b^4*d*e^(6*c) - 7*a*b^5*d*e^(6*c
))*e^(6*d*x) - 4*(8*a^4*b^2*d*e^(4*c) - 23*a^3*b^3*d*e^(4*c) + 22*a^2*b^4*
d*e^(4*c) - 7*a*b^5*d*e^(4*c))*e^(4*d*x) - 8*(a^3*b^3*d*e^(2*c) - 2*a^2*b^
4*d*e^(2*c) + a*b^5*d*e^(2*c))*e^(2*d*x)) + 1/64*integrate(8*((4*a^2*e^(6*
c) - 13*a*b*e^(6*c) + 3*b^2*e^(6*c))*e^(6*d*x) + 6*(7*a*b*e^(4*c) - b^2...
```

Giac [F]

$$\int \frac{\sinh^6(c+dx)}{\left(a-b\sinh^4(c+dx)\right)^3}\,dx=\int -\frac{\sinh\left(dx+c\right)^6}{\left(b\sinh\left(dx+c\right)^4-a\right)^3}\,dx$$

input
```
integrate(sinh(d*x+c)^6/(a-b*sinh(d*x+c)^4)^3,x, algorithm="giac")
```

output
```
sage0*x
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sinh^6(c+dx)}{\left(a-b\sinh^4(c+dx)\right)^3}\,dx=\int \frac{{\mathrm{sinh}\left(c+d\,x\right)}^6}{{\left(a-b\,{\mathrm{sinh}\left(c+d\,x\right)}^4\right)}^3}\,dx$$

input
```
int(sinh(c + d*x)^6/(a - b*sinh(c + d*x)^4)^3,x)
```

output
```
int(sinh(c + d*x)^6/(a - b*sinh(c + d*x)^4)^3, x)
```

Reduce [F]

$$\int \frac{\sinh^6(c+dx)}{\left(a-b\sinh^4(c+dx)\right)^3}\,dx=\text{too large to display}$$

input
```
int(sinh(d*x+c)^6/(a-b*sinh(d*x+c)^4)^3,x)
```

output

```
(16*( - 14777436277309440*e**(20*c + 16*d*x)*int(e**(4*d*x)/(3440640*e**(2
4*c + 24*d*x)*a**4*b**3 + 2150400*e**(24*c + 24*d*x)*a**3*b**4 + 16128*e**
(24*c + 24*d*x)*a**2*b**5 - 96*e**(24*c + 24*d*x)*a*b**6 + 35*e**(24*c + 2
4*d*x)*b**7 - 41287680*e**(22*c + 22*d*x)*a**4*b**3 - 25804800*e**(22*c +
22*d*x)*a**3*b**4 - 193536*e**(22*c + 22*d*x)*a**2*b**5 + 1152*e**(22*c +
22*d*x)*a*b**6 - 420*e**(22*c + 22*d*x)*b**7 - 165150720*e**(20*c + 20*d*x
)*a**5*b**2 + 123863040*e**(20*c + 20*d*x)*a**4*b**3 + 141152256*e**(20*c
+ 20*d*x)*a**3*b**4 + 1069056*e**(20*c + 20*d*x)*a**2*b**5 - 8016*e**(20*c
 + 20*d*x)*a*b**6 + 2310*e**(20*c + 20*d*x)*b**7 + 1321205760*e**(18*c + 1
8*d*x)*a**5*b**2 + 68812800*e**(18*c + 18*d*x)*a**4*b**3 - 466894848*e**(1
8*c + 18*d*x)*a**3*b**4 - 3585024*e**(18*c + 18*d*x)*a**2*b**5 + 34560*e**
(18*c + 18*d*x)*a*b**6 - 7700*e**(18*c + 18*d*x)*b**7 + 2642411520*e**(16*
c + 16*d*x)*a**6*b - 2972712960*e**(16*c + 16*d*x)*a**5*b**2 - 1174634496*
e**(16*c + 16*d*x)*a**4*b**3 + 1042698240*e**(16*c + 16*d*x)*a**3*b**4 + 8
139264*e**(16*c + 16*d*x)*a**2*b**5 - 94560*e**(16*c + 16*d*x)*a*b**6 + 17
325*e**(16*c + 16*d*x)*b**7 - 10569646080*e**(14*c + 14*d*x)*a**6*b + 2642
411520*e**(14*c + 14*d*x)*a**5*b**2 + 3005743104*e**(14*c + 14*d*x)*a**4*b
**3 - 1659469824*e**(14*c + 14*d*x)*a**3*b**4 - 13138944*e**(14*c + 14*d*x
)*a**2*b**5 + 170112*e**(14*c + 14*d*x)*a*b**6 - 27720*e**(14*c + 14*d*x)*
b**7 - 14092861440*e**(12*c + 12*d*x)*a**7 + 7046430720*e**(12*c + 12*d...
```

3.236 $\int \frac{\sinh^4(c+dx)}{(a-b\sinh^4(c+dx))^3}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 314

$$\begin{aligned}\int \frac{\sinh^4(c+dx)}{(a-b\sinh^4(c+dx))^3}\,dx &= \frac{3\left(2\sqrt{a}-\sqrt{b}\right)\operatorname{arctanh}\left(\frac{\sqrt{\sqrt{a}-\sqrt{b}}\tanh(c+dx)}{\sqrt[4]{a}}\right)}{64a^{7/4}\left(\sqrt{a}-\sqrt{b}\right)^{5/2}\sqrt{b}d} \\ &\quad - \frac{3\left(2\sqrt{a}+\sqrt{b}\right)\operatorname{arctanh}\left(\frac{\sqrt{\sqrt{a}+\sqrt{b}}\tanh(c+dx)}{\sqrt[4]{a}}\right)}{64a^{7/4}\left(\sqrt{a}+\sqrt{b}\right)^{5/2}\sqrt{b}d} \\ &\quad - \frac{b\tanh(c+dx)\left(3a+b-4(a+b)\tanh^2(c+dx)\right)}{8(a-b)^3d\left(a-2a\tanh^2(c+dx)+(a-b)\tanh^4(c+dx)\right)^2} \\ &\quad - \frac{\tanh(c+dx)\left(\frac{9a^2-24ab-b^2}{(a-b)^3}-\frac{(17a+3b)\tanh^2(c+dx)}{(a-b)^2}\right)}{32ad\left(a-2a\tanh^2(c+dx)+(a-b)\tanh^4(c+dx)\right)}\end{aligned}$$

output
```
3/64*(2*a^(1/2)-b^(1/2))*arctanh((a^(1/2)-b^(1/2))^(1/2)*tanh(d*x+c)/a^(1/
4))/a^(7/4)/(a^(1/2)-b^(1/2))^(5/2)/b^(1/2)/d-3/64*(2*a^(1/2)+b^(1/2))*arc
tanh((a^(1/2)+b^(1/2))^(1/2)*tanh(d*x+c)/a^(1/4))/a^(7/4)/(a^(1/2)+b^(1/2)
)^(5/2)/b^(1/2)/d-1/8*b*tanh(d*x+c)*(3*a+b-4*(a+b)*tanh(d*x+c)^2)/(a-b)^3/
d/(a-2*a*tanh(d*x+c)^2+(a-b)*tanh(d*x+c)^4)^2-1/32*tanh(d*x+c)*((9*a^2-24*
a*b-b^2)/(a-b)^3-(17*a+3*b)*tanh(d*x+c)^2/(a-b)^2)/a/d/(a-2*a*tanh(d*x+c)^
2+(a-b)*tanh(d*x+c)^4)
```

Mathematica [A] (verified)

Time = 12.97 (sec) , antiderivative size = 316, normalized size of antiderivative = 1.01

$$\int \frac{\sinh^4(c+dx)}{\left(a-b\sinh^4(c+dx)\right)^3}\,dx = \frac{-\frac{3\left(2a^{3/2}+3a\sqrt{b}-b^{3/2}\right)\arctan\left(\frac{\left(\sqrt{a}-\sqrt{b}\right)\tanh(c+dx)}{\sqrt{-a+\sqrt{a}\sqrt{b}}}\right)}{a^{3/2}\sqrt{-a+\sqrt{a}\sqrt{b}}\sqrt{b}}-\frac{3\left(2a^{3/2}-3a\sqrt{b}+b^{3/2}\right)\text{arctanh}\left(\frac{\left(\sqrt{a}+\sqrt{b}\right)\tanh(c+dx)}{\sqrt{a+\sqrt{a}\sqrt{b}}}\right)}{a^{3/2}\sqrt{a+\sqrt{a}\sqrt{b}}\sqrt{b}}+\frac{8(-7a-2b+(2a+b}{a(8a-3b+4b\cos}}{64(a-b)^2d}$$

input
```
Integrate[Sinh[c + d*x]^4/(a - b*Sinh[c + d*x]^4)^3,x]
```

output
```
((-3*(2*a^(3/2) + 3*a*Sqrt[b] - b^(3/2))*ArcTan[((Sqrt[a] - Sqrt[b])*Tanh[
c + d*x])/Sqrt[-a + Sqrt[a]*Sqrt[b]]])/(a^(3/2)*Sqrt[-a + Sqrt[a]*Sqrt[b]]
*Sqrt[b]) - (3*(2*a^(3/2) - 3*a*Sqrt[b] + b^(3/2))*ArcTanh[((Sqrt[a] + Sqr
t[b])*Tanh[c + d*x])/Sqrt[a + Sqrt[a]*Sqrt[b]]])/(a^(3/2)*Sqrt[a + Sqrt[a]
*Sqrt[b]]*Sqrt[b]) + (8*(-7*a - 2*b + (2*a + b)*Cosh[2*(c + d*x)])*Sinh[2*
(c + d*x)])/(a*(8*a - 3*b + 4*b*Cosh[2*(c + d*x)] - b*Cosh[4*(c + d*x)]))
+ (64*(a - b)*(-6*Sinh[2*(c + d*x)] + Sinh[4*(c + d*x)]))/(-8*a + 3*b - 4*
b*Cosh[2*(c + d*x)] + b*Cosh[4*(c + d*x)])^2)/(64*(a - b)^2*d)
```

Rubi [A] (verified)

Time = 0.93 (sec) , antiderivative size = 371, normalized size of antiderivative = 1.18, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {3042, 3696, 1672, 27, 2206, 27, 1480, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sinh^4(c+dx)}{\left(a-b\sinh^4(c+dx)\right)^3}\,dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \frac{\sin(ic+idx)^4}{\left(a-b\sin(ic+idx)^4\right)^3}dx$$

$$\Big\downarrow \text{ 3696}$$

$$\frac{\int \frac{\tanh^4(c+dx)\left(1-\tanh^2(c+dx)\right)^3}{\left((a-b)\tanh^4(c+dx)-2a\tanh^2(c+dx)+a\right)^3}d\tanh(c+dx)}{d}$$

$$\Big\downarrow \text{ 1672}$$

$$\frac{-\frac{\int -\frac{2\left(-\frac{8a^2b\tanh^6(c+dx)}{a-b}+\frac{8a^2(a-3b)b\tanh^4(c+dx)}{(a-b)^2}+\frac{4a^2(3a-b)b^2\tanh^2(c+dx)}{(a-b)^3}+\frac{a^2b^2(3a+b)}{(a-b)^3}\right)}{\left((a-b)\tanh^4(c+dx)-2a\tanh^2(c+dx)+a\right)^2}d\tanh(c+dx)}{16a^2b}-\frac{b\tanh(c+dx)\left(-4(a+b)\tanh^2(\cdots\right)}{8(a-b)^3\left((a-b)\tanh^4(c+dx)-2a\cdots\right)}}{d}$$

$$\Big\downarrow \text{ 27}$$

$$\frac{\frac{\int \frac{-\frac{8a^2b\tanh^6(c+dx)}{a-b}+\frac{8a^2(a-3b)b\tanh^4(c+dx)}{(a-b)^2}+\frac{4a^2(3a-b)b^2\tanh^2(c+dx)}{(a-b)^3}+\frac{a^2b^2(3a+b)}{(a-b)^3}}{\left((a-b)\tanh^4(c+dx)-2a\tanh^2(c+dx)+a\right)^2}d\tanh(c+dx)}{8a^2b}-\frac{b\tanh(c+dx)\left(-4(a+b)\tanh^2(c+dx)+\cdots\right)}{8(a-b)^3\left((a-b)\tanh^4(c+dx)-2a\tanh^2(c\cdots\right)}}{d}$$

$$\Big\downarrow \text{ 2206}$$

$$\frac{\frac{-\frac{\int -\frac{6a^3b^2\left(-\left((5a-b)\tanh^2(c+dx)\right)+3a-b\right)}{(a-b)^2\left((a-b)\tanh^4(c+dx)-2a\tanh^2(c+dx)+a\right)}d\tanh(c+dx)}{8a^2b}-\frac{ab\tanh(c+dx)\left(\frac{9a^2-24ab-b^2}{(a-b)^3}-\frac{(17a+3b)\tanh^2(c+dx)}{(a-b)^2}\right)}{4\left((a-b)\tanh^4(c+dx)-2a\tanh^2(c+dx)+a\right)}}{8a^2b}-\frac{b\tanh(c+dx)\left(-4(a\cdots\right)}{8(a-b)^3\left((a-b)\tanh^4(c\cdots\right)}}{d}$$

$$\Big\downarrow \text{ 27}$$

$$\frac{\frac{\frac{3ab\int \frac{-\left((5a-b)\tanh^2(c+dx)\right)+3a-b}{(a-b)\tanh^4(c+dx)-2a\tanh^2(c+dx)+a}d\tanh(c+dx)}{4(a-b)^2}-\frac{ab\tanh(c+dx)\left(\frac{9a^2-24ab-b^2}{(a-b)^3}-\frac{(17a+3b)\tanh^2(c+dx)}{(a-b)^2}\right)}{4\left((a-b)\tanh^4(c+dx)-2a\tanh^2(c+dx)+a\right)}}{8a^2b}-\frac{b\tanh(c+dx)(-4(a+b)\tanh}{8(a-b)^3\left((a-b)\tanh^4(c+dx)-2\right.}}{d}$$

$$\Big\downarrow \text{ 1480}$$

$$\frac{\frac{3ab\left(\frac{\left(\sqrt{a}-\sqrt{b}\right)^3\left(2\sqrt{a}+\sqrt{b}\right)\int \frac{1}{(a-b)\tanh^2(c+dx)-\sqrt{a}\left(\sqrt{a}-\sqrt{b}\right)}d\tanh(c+dx)}{2\sqrt{a}\sqrt{b}}-\frac{1}{2}\left(\frac{2a^2+3ab-b^2}{\sqrt{a}\sqrt{b}}+5a-b\right)\int \frac{1}{(a-b)\tanh^2(c+dx)-\sqrt{a}\left(\sqrt{a}+\sqrt{b}\right)}d\tanh(c+dx)\right)}{4(a-b)^2}}{8a^2b}}{d}$$

$$\Big\downarrow \text{ 221}$$

$$\frac{\frac{\frac{3ab\left(\frac{\left(\frac{2a^2+3ab-b^2}{\sqrt{a}\sqrt{b}}+5a-b\right)\text{arctanh}\left(\frac{\sqrt{\sqrt{a}-\sqrt{b}}\tanh(c+dx)}{\sqrt[4]{a}}\right)}{2\sqrt[4]{a}\sqrt{\sqrt{a}-\sqrt{b}}\left(\sqrt{a}+\sqrt{b}\right)}-\frac{\left(\sqrt{a}-\sqrt{b}\right)^2\left(2\sqrt{a}+\sqrt{b}\right)\text{arctanh}\left(\frac{\sqrt{\sqrt{a}+\sqrt{b}}\tanh(c+dx)}{\sqrt[4]{a}}\right)}{2a^{3/4}\sqrt{b}\sqrt{\sqrt{a}+\sqrt{b}}}\right)}{4(a-b)^2}-\frac{ab\tanh(c+dx)\left(\frac{9a^2-24a}{(a-b)}\right.}{4\left((a-b)\tanh^4(c+\right.}}{8a^2b}}{d}$$

input
```
Int[Sinh[c + d*x]^4/(a - b*Sinh[c + d*x]^4)^3,x]
```

output
```
(-1/8*(b*Tanh[c + d*x]*(3*a + b - 4*(a + b)*Tanh[c + d*x]^2))/((a - b)^3*(
a - 2*a*Tanh[c + d*x]^2 + (a - b)*Tanh[c + d*x]^4)^2) + ((3*a*b*(((5*a - b
 + (2*a^2 + 3*a*b - b^2)/(Sqrt[a]*Sqrt[b]))*ArcTanh[(Sqrt[Sqrt[a] - Sqrt[b
]]*Tanh[c + d*x])/a^(1/4)])/(2*a^(1/4)*Sqrt[Sqrt[a] - Sqrt[b]]*(Sqrt[a] +
Sqrt[b])) - ((Sqrt[a] - Sqrt[b])^2*(2*Sqrt[a] + Sqrt[b])*ArcTanh[(Sqrt[Sqr
t[a] + Sqrt[b]]*Tanh[c + d*x])/a^(1/4)])/(2*a^(3/4)*Sqrt[Sqrt[a] + Sqrt[b]
]*Sqrt[b])))/(4*(a - b)^2) - (a*b*Tanh[c + d*x]*((9*a^2 - 24*a*b - b^2)/(a
 - b)^3 - ((17*a + 3*b)*Tanh[c + d*x]^2)/(a - b)^2))/(4*(a - 2*a*Tanh[c +
d*x]^2 + (a - b)*Tanh[c + d*x]^4)))/(8*a^2*b))/d
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 1480
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Simp[(e/2 + (2*c*d - b*e)/(2*q))   Int[1/(
b/2 - q/2 + c*x^2), x], x] + Simp[(e/2 - (2*c*d - b*e)/(2*q))   Int[1/(b/2
+ q/2 + c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0]
 && NeQ[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

rule 1672
```
Int[(x_)^(m_)*((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^
4)^(p_), x_Symbol] :> With[{f = Coeff[PolynomialRemainder[x^m*(d + e*x^2)^q
, a + b*x^2 + c*x^4, x], x, 0], g = Coeff[PolynomialRemainder[x^m*(d + e*x^
2)^q, a + b*x^2 + c*x^4, x], x, 2]}, Simp[x*(a + b*x^2 + c*x^4)^(p + 1)*((a
*b*g - f*(b^2 - 2*a*c) - c*(b*f - 2*a*g)*x^2)/(2*a*(p + 1)*(b^2 - 4*a*c))),
 x] + Simp[1/(2*a*(p + 1)*(b^2 - 4*a*c))   Int[(a + b*x^2 + c*x^4)^(p + 1)*
Simp[ExpandToSum[2*a*(p + 1)*(b^2 - 4*a*c)*PolynomialQuotient[x^m*(d + e*x^
2)^q, a + b*x^2 + c*x^4, x] + b^2*f*(2*p + 3) - 2*a*c*f*(4*p + 5) - a*b*g +
 c*(4*p + 7)*(b*f - 2*a*g)*x^2, x], x], x], x]] /; FreeQ[{a, b, c, d, e}, x
] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && IGtQ[q, 1] && IGtQ[m/2, 0]
```

rule 2206
```
Int[(Px_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> With[{d =
 Coeff[PolynomialRemainder[Px, a + b*x^2 + c*x^4, x], x, 0], e = Coeff[Poly
nomialRemainder[Px, a + b*x^2 + c*x^4, x], x, 2]}, Simp[x*(a + b*x^2 + c*x^
4)^(p + 1)*((a*b*e - d*(b^2 - 2*a*c) - c*(b*d - 2*a*e)*x^2)/(2*a*(p + 1)*(b
^2 - 4*a*c))), x] + Simp[1/(2*a*(p + 1)*(b^2 - 4*a*c))   Int[(a + b*x^2 + c
*x^4)^(p + 1)*ExpandToSum[2*a*(p + 1)*(b^2 - 4*a*c)*PolynomialQuotient[Px,
a + b*x^2 + c*x^4, x] + b^2*d*(2*p + 3) - 2*a*c*d*(4*p + 5) - a*b*e + c*(4*
p + 7)*(b*d - 2*a*e)*x^2, x], x], x]] /; FreeQ[{a, b, c}, x] && PolyQ[Px, x
^2] && Expon[Px, x^2] > 1 && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3696
```
Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^4)^(
p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Simp[ff^(m + 1
)/f   Subst[Int[x^m*((a + 2*a*ff^2*x^2 + (a + b)*ff^4*x^4)^p/(1 + ff^2*x^2)
^(m/2 + 2*p + 1)), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f}, x] &
& IntegerQ[m/2] && IntegerQ[p]
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 10.18 (sec) , antiderivative size = 523, normalized size of antiderivative = 1.67

method	result
derivativedivides	$-\frac{32\left(\frac{3(3a-b)\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)}{512\left(a^2-2ab+b^2\right)}-\frac{(77a-23b)\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^3}{512\left(a^2-2ab+b^2\right)}+\frac{\left(177a^2-131ab-16b^2\right)\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^5}{512\left(a^2-2ab+b^2\right)a}-\frac{\left(109a^2-367ab-144b^2\right)\tanh\left(\right.}{512a\left(a^2-2ab+b^2\right)}\right.}{\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^8a-4\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^6a+6\right.}$
default	$-\frac{32\left(\frac{3(3a-b)\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)}{512\left(a^2-2ab+b^2\right)}-\frac{(77a-23b)\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^3}{512\left(a^2-2ab+b^2\right)}+\frac{\left(177a^2-131ab-16b^2\right)\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^5}{512\left(a^2-2ab+b^2\right)a}-\frac{\left(109a^2-367ab-144b^2\right)\tanh\left(\right.}{512a\left(a^2-2ab+b^2\right)}\right.}{\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^8a-4\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^6a+6\right.}$
risch	Expression too large to display

input
```
int(sinh(d*x+c)^4/(a-b*sinh(d*x+c)^4)^3,x,method=_RETURNVERBOSE)
```

output
```
1/d*(-32*(3/512*(3*a-b)/(a^2-2*a*b+b^2)*tanh(1/2*d*x+1/2*c)-1/512*(77*a-23
*b)/(a^2-2*a*b+b^2)*tanh(1/2*d*x+1/2*c)^3+1/512*(177*a^2-131*a*b-16*b^2)/(
a^2-2*a*b+b^2)/a*tanh(1/2*d*x+1/2*c)^5-1/512*(109*a^2-367*a*b-144*b^2)/a/(
a^2-2*a*b+b^2)*tanh(1/2*d*x+1/2*c)^7-1/512*(109*a^2-367*a*b-144*b^2)/a/(a^
2-2*a*b+b^2)*tanh(1/2*d*x+1/2*c)^9+1/512*(177*a^2-131*a*b-16*b^2)/(a^2-2*a
*b+b^2)/a*tanh(1/2*d*x+1/2*c)^11-1/512*(77*a-23*b)/(a^2-2*a*b+b^2)*tanh(1/
2*d*x+1/2*c)^13+3/512*(3*a-b)/(a^2-2*a*b+b^2)*tanh(1/2*d*x+1/2*c)^15)/(tan
h(1/2*d*x+1/2*c)^8*a-4*tanh(1/2*d*x+1/2*c)^6*a+6*tanh(1/2*d*x+1/2*c)^4*a-1
6*b*tanh(1/2*d*x+1/2*c)^4-4*tanh(1/2*d*x+1/2*c)^2*a+a)^2-3/128/(a^2-2*a*b+
b^2)/a*sum(((3*a-b)*_R^6+(-17*a+3*b)*_R^4+(17*a-3*b)*_R^2-3*a+b)/(_R^7*a-3
*_R^5*a+3*_R^3*a-8*_R^3*b-_R*a)*ln(tanh(1/2*d*x+1/2*c)-_R),_R=RootOf(a*_Z^
8-4*a*_Z^6+(6*a-16*b)*_Z^4-4*a*_Z^2+a))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 21932 vs. $2(264) = 528$.

Time = 0.77 (sec) , antiderivative size = 21932, normalized size of antiderivative = 69.85

$$\int \frac{\sinh^4(c+dx)}{(a-b\sinh^4(c+dx))^3}\,dx = \text{Too large to display}$$

input
```
integrate(sinh(d*x+c)^4/(a-b*sinh(d*x+c)^4)^3,x, algorithm="fricas")
```

output
```
Too large to include
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\sinh^4(c+dx)}{(a-b\sinh^4(c+dx))^3}\,dx = \text{Timed out}$$

input
```
integrate(sinh(d*x+c)**4/(a-b*sinh(d*x+c)**4)**3,x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{\sinh^4(c+dx)}{(a-b\sinh^4(c+dx))^3}\,dx = \int -\frac{\sinh(dx+c)^4}{(b\sinh(dx+c)^4-a)^3}\,dx$$

input
```
integrate(sinh(d*x+c)^4/(a-b*sinh(d*x+c)^4)^3,x, algorithm="maxima")
```

output
```
1/8*(3*a*b^2*e^(14*d*x + 14*c) + 2*a*b^2 + b^3 - 3*(10*a*b^2*e^(12*c) - b^
3*e^(12*c))*e^(12*d*x) - (80*a^2*b*e^(10*c) - 111*a*b^2*e^(10*c) + 16*b^3*
e^(10*c))*e^(10*d*x) + (256*a^3*e^(8*c) - 64*a^2*b*e^(8*c) - 26*a*b^2*e^(8
*c) + 35*b^3*e^(8*c))*e^(8*d*x) + (336*a^2*b*e^(6*c) - 95*a*b^2*e^(6*c) -
40*b^3*e^(6*c))*e^(6*d*x) - (64*a^2*b*e^(4*c) - 54*a*b^2*e^(4*c) - 25*b^3*
e^(4*c))*e^(4*d*x) - (19*a*b^2*e^(2*c) + 8*b^3*e^(2*c))*e^(2*d*x))/(a^3*b^
3*d - 2*a^2*b^4*d + a*b^5*d + (a^3*b^3*d*e^(16*c) - 2*a^2*b^4*d*e^(16*c) +
 a*b^5*d*e^(16*c))*e^(16*d*x) - 8*(a^3*b^3*d*e^(14*c) - 2*a^2*b^4*d*e^(14*
c) + a*b^5*d*e^(14*c))*e^(14*d*x) - 4*(8*a^4*b^2*d*e^(12*c) - 23*a^3*b^3*d
*e^(12*c) + 22*a^2*b^4*d*e^(12*c) - 7*a*b^5*d*e^(12*c))*e^(12*d*x) + 8*(16
*a^4*b^2*d*e^(10*c) - 39*a^3*b^3*d*e^(10*c) + 30*a^2*b^4*d*e^(10*c) - 7*a*
b^5*d*e^(10*c))*e^(10*d*x) + 2*(128*a^5*b*d*e^(8*c) - 352*a^4*b^2*d*e^(8*c
) + 355*a^3*b^3*d*e^(8*c) - 166*a^2*b^4*d*e^(8*c) + 35*a*b^5*d*e^(8*c))*e^
(8*d*x) + 8*(16*a^4*b^2*d*e^(6*c) - 39*a^3*b^3*d*e^(6*c) + 30*a^2*b^4*d*e^
(6*c) - 7*a*b^5*d*e^(6*c))*e^(6*d*x) - 4*(8*a^4*b^2*d*e^(4*c) - 23*a^3*b^3
*d*e^(4*c) + 22*a^2*b^4*d*e^(4*c) - 7*a*b^5*d*e^(4*c))*e^(4*d*x) - 8*(a^3*
b^3*d*e^(2*c) - 2*a^2*b^4*d*e^(2*c) + a*b^5*d*e^(2*c))*e^(2*d*x)) + 1/16*i
ntegrate(-12*(2*(4*a*e^(4*c) - b*e^(4*c))*e^(4*d*x) - a*e^(6*d*x + 6*c) -
a*e^(2*d*x + 2*c))/(a^3*b - 2*a^2*b^2 + a*b^3 + (a^3*b*e^(8*c) - 2*a^2*b^2
*e^(8*c) + a*b^3*e^(8*c))*e^(8*d*x) - 4*(a^3*b*e^(6*c) - 2*a^2*b^2*e^(6...
```

Giac [F]

$$\int \frac{\sinh^4(c+dx)}{(a-b\sinh^4(c+dx))^3}\,dx = \int -\frac{\sinh(dx+c)^4}{(b\sinh(dx+c)^4-a)^3}\,dx$$

input
```
integrate(sinh(d*x+c)^4/(a-b*sinh(d*x+c)^4)^3,x, algorithm="giac")
```

output
```
sage0*x
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sinh^4(c+dx)}{(a-b\sinh^4(c+dx))^3}\,dx = \int \frac{\sinh(c+d\,x)^4}{\left(a-b\sinh\left(c+d\,x\right)^4\right)^3}\,dx$$

input
```
int(sinh(c + d*x)^4/(a - b*sinh(c + d*x)^4)^3,x)
```

output
```
int(sinh(c + d*x)^4/(a - b*sinh(c + d*x)^4)^3, x)
```

Reduce [F]

$$\int \frac{\sinh^4(c+dx)}{(a-b\sinh^4(c+dx))^3}\,dx = \text{too large to display}$$

input
```
int(sinh(d*x+c)^4/(a-b*sinh(d*x+c)^4)^3,x)
```

output

```
(32*( - 3694359069327360*e**(20*c + 16*d*x)*int(e**(4*d*x)/(3440640*e**(24
*c + 24*d*x)*a**4*b**3 + 2150400*e**(24*c + 24*d*x)*a**3*b**4 + 16128*e**(
24*c + 24*d*x)*a**2*b**5 - 96*e**(24*c + 24*d*x)*a*b**6 + 35*e**(24*c + 24
*d*x)*b**7 - 41287680*e**(22*c + 22*d*x)*a**4*b**3 - 25804800*e**(22*c + 2
2*d*x)*a**3*b**4 - 193536*e**(22*c + 22*d*x)*a**2*b**5 + 1152*e**(22*c + 2
2*d*x)*a*b**6 - 420*e**(22*c + 22*d*x)*b**7 - 165150720*e**(20*c + 20*d*x)
*a**5*b**2 + 123863040*e**(20*c + 20*d*x)*a**4*b**3 + 141152256*e**(20*c +
 20*d*x)*a**3*b**4 + 1069056*e**(20*c + 20*d*x)*a**2*b**5 - 8016*e**(20*c
+ 20*d*x)*a*b**6 + 2310*e**(20*c + 20*d*x)*b**7 + 1321205760*e**(18*c + 18
*d*x)*a**5*b**2 + 68812800*e**(18*c + 18*d*x)*a**4*b**3 - 466894848*e**(18
*c + 18*d*x)*a**3*b**4 - 3585024*e**(18*c + 18*d*x)*a**2*b**5 + 34560*e**(
18*c + 18*d*x)*a*b**6 - 7700*e**(18*c + 18*d*x)*b**7 + 2642411520*e**(16*c
 + 16*d*x)*a**6*b - 2972712960*e**(16*c + 16*d*x)*a**5*b**2 - 1174634496*e
**(16*c + 16*d*x)*a**4*b**3 + 1042698240*e**(16*c + 16*d*x)*a**3*b**4 + 81
39264*e**(16*c + 16*d*x)*a**2*b**5 - 94560*e**(16*c + 16*d*x)*a*b**6 + 173
25*e**(16*c + 16*d*x)*b**7 - 10569646080*e**(14*c + 14*d*x)*a**6*b + 26424
11520*e**(14*c + 14*d*x)*a**5*b**2 + 3005743104*e**(14*c + 14*d*x)*a**4*b*
*3 - 1659469824*e**(14*c + 14*d*x)*a**3*b**4 - 13138944*e**(14*c + 14*d*x)
*a**2*b**5 + 170112*e**(14*c + 14*d*x)*a*b**6 - 27720*e**(14*c + 14*d*x)*b
**7 - 14092861440*e**(12*c + 12*d*x)*a**7 + 7046430720*e**(12*c + 12*d*...
```

3.237 $\int \frac{\sinh^2(c+dx)}{(a-b\sinh^4(c+dx))^3}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 348

$$
\begin{aligned}
\int \frac{\sinh^2(c+dx)}{(a-b\sinh^4(c+dx))^3}\,dx
&= -\frac{\left(12a-14\sqrt{a}\sqrt{b}+5b\right)\operatorname{arctanh}\left(\frac{\sqrt{\sqrt{a}-\sqrt{b}}\tanh(c+dx)}{\sqrt[4]{a}}\right)}{64a^{9/4}\left(\sqrt{a}-\sqrt{b}\right)^{5/2}\sqrt{b}d} \\
&\quad+\frac{\left(12a+14\sqrt{a}\sqrt{b}+5b\right)\operatorname{arctanh}\left(\frac{\sqrt{\sqrt{a}+\sqrt{b}}\tanh(c+dx)}{\sqrt[4]{a}}\right)}{64a^{9/4}\left(\sqrt{a}+\sqrt{b}\right)^{5/2}\sqrt{b}d} \\
&\quad+\frac{b\tanh(c+dx)\left(a(a+3b)-(a^2+6ab+b^2)\tanh^2(c+dx)\right)}{8a(a-b)^3d\left(a-2a\tanh^2(c+dx)+(a-b)\tanh^4(c+dx)\right)^2} \\
&\quad+\frac{\tanh(c+dx)\left(\frac{2a(5a^2-9ab-4b^2)}{(a-b)^3}-\frac{5(2a^2+3ab-b^2)\tanh^2(c+dx)}{(a-b)^2}\right)}{32a^2d\left(a-2a\tanh^2(c+dx)+(a-b)\tanh^4(c+dx)\right)}
\end{aligned}
$$

output
```
-1/64*(12*a-14*a^(1/2)*b^(1/2)+5*b)*arctanh((a^(1/2)-b^(1/2))^(1/2)*tanh(d
*x+c)/a^(1/4))/a^(9/4)/(a^(1/2)-b^(1/2))^(5/2)/b^(1/2)/d+1/64*(12*a+14*a^(
1/2)*b^(1/2)+5*b)*arctanh((a^(1/2)+b^(1/2))^(1/2)*tanh(d*x+c)/a^(1/4))/a^(
9/4)/(a^(1/2)+b^(1/2))^(5/2)/b^(1/2)/d+1/8*b*tanh(d*x+c)*(a*(a+3*b)-(a^2+6
*a*b+b^2)*tanh(d*x+c)^2)/a/(a-b)^3/d/(a-2*a*tanh(d*x+c)^2+(a-b)*tanh(d*x+c
)^4)^2+1/32*tanh(d*x+c)*(2*a*(5*a^2-9*a*b-4*b^2)/(a-b)^3-5*(2*a^2+3*a*b-b^
2)*tanh(d*x+c)^2/(a-b)^2)/a^2/d/(a-2*a*tanh(d*x+c)^2+(a-b)*tanh(d*x+c)^4)
```

Mathematica [A] (verified)

Time = 3.88 (sec) , antiderivative size = 343, normalized size of antiderivative = 0.99

$$\int \frac{\sinh^2(c+dx)}{\left(a-b\sinh^4(c+dx)\right)^3}\,dx = \frac{\frac{\left(\sqrt{a}+\sqrt{b}\right)^2\left(12a-14\sqrt{a}\sqrt{b}+5b\right)\arctan\left(\frac{\left(\sqrt{a}-\sqrt{b}\right)\tanh(c+dx)}{\sqrt{-a+\sqrt{a}\sqrt{b}}}\right)}{\sqrt{-a+\sqrt{a}\sqrt{b}}\sqrt{b}}+\frac{\left(\sqrt{a}-\sqrt{b}\right)^2\left(12a+14\sqrt{a}\sqrt{b}+5b\right)\text{arctanh}\left(\frac{\left(\sqrt{a}+\sqrt{b}\right)\tanh(c+dx)}{\sqrt{a+\sqrt{a}\sqrt{b}}}\right)}{\sqrt{a+\sqrt{a}\sqrt{b}}\sqrt{b}}+ 4}{64a^2(a-b)^2d}$$

input
```
Integrate[Sinh[c + d*x]^2/(a - b*Sinh[c + d*x]^4)^3,x]
```

output
```
(((Sqrt[a] + Sqrt[b])^2*(12*a - 14*Sqrt[a]*Sqrt[b] + 5*b)*ArcTan[((Sqrt[a]
 - Sqrt[b])*Tanh[c + d*x])/Sqrt[-a + Sqrt[a]*Sqrt[b]]])/(Sqrt[-a + Sqrt[a]
*Sqrt[b]]*Sqrt[b]) + ((Sqrt[a] - Sqrt[b])^2*(12*a + 14*Sqrt[a]*Sqrt[b] + 5
*b)*ArcTanh[((Sqrt[a] + Sqrt[b])*Tanh[c + d*x])/Sqrt[a + Sqrt[a]*Sqrt[b]]]
)/(Sqrt[a + Sqrt[a]*Sqrt[b]]*Sqrt[b]) + (4*(12*a^2 + 11*a*b - 5*b^2 + b*(-
11*a + 5*b)*Cosh[2*(c + d*x)])*Sinh[2*(c + d*x)])/(8*a - 3*b + 4*b*Cosh[2*
(c + d*x)] - b*Cosh[4*(c + d*x)]) + (128*a*(a - b)*(2*a + b - b*Cosh[2*(c
+ d*x)])*Sinh[2*(c + d*x)])/(-8*a + 3*b - 4*b*Cosh[2*(c + d*x)] + b*Cosh[4
*(c + d*x)])^2)/(64*a^2*(a - b)^2*d)
```

Rubi [A] (verified)

Time = 0.94 (sec) , antiderivative size = 390, normalized size of antiderivative = 1.12, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.375, Rules used = {3042, 25, 3696, 1672, 27, 2206, 27, 1480, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sinh^2(c+dx)}{\left(a-b\sinh^4(c+dx)\right)^3}dx$$

$$\Big\downarrow \text{3042}$$

$$\int -\frac{\sin(ic+idx)^2}{\left(a-b\sin(ic+idx)^4\right)^3}dx$$

$$\Big\downarrow \text{25}$$

$$-\int \frac{\sin(ic+idx)^2}{\left(a-b\sin(ic+idx)^4\right)^3}dx$$

$$\Big\downarrow \text{3696}$$

$$\frac{\int \frac{\tanh^2(c+dx)\left(1-\tanh^2(c+dx)\right)^4}{\left((a-b)\tanh^4(c+dx)-2a\tanh^2(c+dx)+a\right)^3}d\tanh(c+dx)}{d}$$

$$\Big\downarrow \text{1672}$$

$$\frac{\frac{b\tanh(c+dx)\left(a(a+3b)-\left(a^2+6ab+b^2\right)\tanh^2(c+dx)\right)}{8a(a-b)^3\left((a-b)\tanh^4(c+dx)-2a\tanh^2(c+dx)+a\right)^2}-\frac{\int \frac{2\left(-\frac{8a^2b\tanh^6(c+dx)}{a-b}+\frac{16a^2(a-2b)b\tanh^4(c+dx)}{(a-b)^2}-\frac{ab\left(8a^3-29ba^2+18b^2a-5b^3\right)\tanh^2(c+d}{(a-b)^3}\right.}{\left((a-b)\tanh^4(c+dx)-2a\tanh^2(c+dx)+a\right)^2}}{16a^2b}}{d}$$

$$\Big\downarrow \text{27}$$

$$\frac{\frac{b\tanh(c+dx)\left(a(a+3b)-\left(a^2+6ab+b^2\right)\tanh^2(c+dx)\right)}{8a(a-b)^3\left((a-b)\tanh^4(c+dx)-2a\tanh^2(c+dx)+a\right)^2}-\frac{\int \frac{-\frac{8a^2b\tanh^6(c+dx)}{a-b}+\frac{16a^2(a-2b)b\tanh^4(c+dx)}{(a-b)^2}-\frac{ab\left(8a^3-29ba^2+18b^2a-5b^3\right)\tanh^2(c+dx)}{(a-b)^3}}{\left((a-b)\tanh^4(c+dx)-2a\tanh^2(c+dx)+a\right)^2}}{8a^2b}}{d}$$

$$\Big\downarrow \text{2206}$$

$$\frac{\frac{b\tanh(c+dx)\left(a(a+3b)-\left(a^2+6ab+b^2\right)\tanh^2(c+dx)\right)}{8a(a-b)^3\left((a-b)\tanh^4(c+dx)-2a\tanh^2(c+dx)+a\right)^2}-\frac{-\frac{\int -\frac{2a^2b^2\left(2a(5a-2b)-\left(22a^2-15ba+5b^2\right)\tanh^2(c+dx)\right)}{(a-b)^2\left((a-b)\tanh^4(c+dx)-2a\tanh^2(c+dx)+a\right)}d\tanh(c+dx)}{8a^2b}-\frac{b\tanh(c+dx)\left(\frac{2a\left(5a\right.}{}\right.}{4\left((a-b\right.}}{8a^2b}}{d}$$

$\downarrow$ 27

$$\frac{\frac{b\tanh(c+dx)\left(a(a+3b)-\left(a^2+6ab+b^2\right)\tanh^2(c+dx)\right)}{8a(a-b)^3\left((a-b)\tanh^4(c+dx)-2a\tanh^2(c+dx)+a\right)^2}-\frac{\frac{b\int \frac{2a(5a-2b)-\left(22a^2-15ba+5b^2\right)\tanh^2(c+dx)}{(a-b)\tanh^4(c+dx)-2a\tanh^2(c+dx)+a}d\tanh(c+dx)}{4(a-b)^2}-\frac{b\tanh(c+dx)\left(\frac{2a\left(5a^2-9ab-4b\right.}{(a-b)^3}\right.}{4\left((a-b)\tanh^4(c+a\right.}}{8a^2b}}{d}$$

$\downarrow$ 1480

$$\frac{\frac{b\tanh(c+dx)\left(a(a+3b)-\left(a^2+6ab+b^2\right)\tanh^2(c+dx)\right)}{8a(a-b)^3\left((a-b)\tanh^4(c+dx)-2a\tanh^2(c+dx)+a\right)^2}-\frac{b\left(\frac{\left(\sqrt{a}-\sqrt{b}\right)^3\left(14\sqrt{a}\sqrt{b}+12a+5b\right)\int \frac{1}{(a-b)\tanh^2(c+dx)-\sqrt{a}\left(\sqrt{a}-\sqrt{b}\right)}d\tanh(c+dx)}{2\sqrt{b}}-\frac{\left(\sqrt{a}+\sqrt{\right.}}{}\right.}{4(a-b)^2}}{d}$$

$\downarrow$ 221

$$\frac{\frac{b\tanh(c+dx)\left(a(a+3b)-\left(a^2+6ab+b^2\right)\tanh^2(c+dx)\right)}{8a(a-b)^3\left((a-b)\tanh^4(c+dx)-2a\tanh^2(c+dx)+a\right)^2}-\frac{b\left(\frac{\left(\sqrt{a}+\sqrt{b}\right)^2\left(-14\sqrt{a}\sqrt{b}+12a+5b\right)\text{arctanh}\left(\frac{\sqrt{\sqrt{a}-\sqrt{b}}\tanh(c+dx)}{\sqrt[4]{a}}\right)}{2\sqrt[4]{a}\sqrt{b}\sqrt{\sqrt{a}-\sqrt{b}}}-\frac{\left(\sqrt{a}-\sqrt{b}\right)^2\left(14\sqrt{a}\sqrt{\right.}}{}\right.}{4(a-b)^2}}{d}$$

input
```
Int[Sinh[c + d*x]^2/(a - b*Sinh[c + d*x]^4)^3,x]
```

output
```
((b*Tanh[c + d*x]*(a*(a + 3*b) - (a^2 + 6*a*b + b^2)*Tanh[c + d*x]^2))/(8*
a*(a - b)^3*(a - 2*a*Tanh[c + d*x]^2 + (a - b)*Tanh[c + d*x]^4)^2) - ((b*(
((Sqrt[a] + Sqrt[b])^2*(12*a - 14*Sqrt[a]*Sqrt[b] + 5*b)*ArcTanh[(Sqrt[Sqr
t[a] - Sqrt[b]]*Tanh[c + d*x])/a^(1/4)])/(2*a^(1/4)*Sqrt[Sqrt[a] - Sqrt[b]
]*Sqrt[b]) - ((Sqrt[a] - Sqrt[b])^2*(12*a + 14*Sqrt[a]*Sqrt[b] + 5*b)*ArcT
anh[(Sqrt[Sqrt[a] + Sqrt[b]]*Tanh[c + d*x])/a^(1/4)])/(2*a^(1/4)*Sqrt[Sqrt
[a] + Sqrt[b]]*Sqrt[b])))/(4*(a - b)^2) - (b*Tanh[c + d*x]*((2*a*(5*a^2 -
9*a*b - 4*b^2))/(a - b)^3 - (5*(2*a^2 + 3*a*b - b^2)*Tanh[c + d*x]^2)/(a -
 b)^2))/(4*(a - 2*a*Tanh[c + d*x]^2 + (a - b)*Tanh[c + d*x]^4)))/(8*a^2*b)
)/d
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 1480
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Simp[(e/2 + (2*c*d - b*e)/(2*q))   Int[1/(
b/2 - q/2 + c*x^2), x], x] + Simp[(e/2 - (2*c*d - b*e)/(2*q))   Int[1/(b/2
+ q/2 + c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0]
 && NeQ[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

rule 1672
```
Int[(x_)^(m_)*((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^
4)^(p_), x_Symbol] :> With[{f = Coeff[PolynomialRemainder[x^m*(d + e*x^2)^q
, a + b*x^2 + c*x^4, x], x, 0], g = Coeff[PolynomialRemainder[x^m*(d + e*x^
2)^q, a + b*x^2 + c*x^4, x], x, 2]}, Simp[x*(a + b*x^2 + c*x^4)^(p + 1)*((a
*b*g - f*(b^2 - 2*a*c) - c*(b*f - 2*a*g)*x^2)/(2*a*(p + 1)*(b^2 - 4*a*c))),
 x] + Simp[1/(2*a*(p + 1)*(b^2 - 4*a*c))   Int[(a + b*x^2 + c*x^4)^(p + 1)*
Simp[ExpandToSum[2*a*(p + 1)*(b^2 - 4*a*c)*PolynomialQuotient[x^m*(d + e*x^
2)^q, a + b*x^2 + c*x^4, x] + b^2*f*(2*p + 3) - 2*a*c*f*(4*p + 5) - a*b*g +
 c*(4*p + 7)*(b*f - 2*a*g)*x^2, x], x], x], x]] /; FreeQ[{a, b, c, d, e}, x
] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && IGtQ[q, 1] && IGtQ[m/2, 0]
```

rule 2206
```
Int[(Px_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> With[{d =
 Coeff[PolynomialRemainder[Px, a + b*x^2 + c*x^4, x], x, 0], e = Coeff[Poly
nomialRemainder[Px, a + b*x^2 + c*x^4, x], x, 2]}, Simp[x*(a + b*x^2 + c*x^
4)^(p + 1)*((a*b*e - d*(b^2 - 2*a*c) - c*(b*d - 2*a*e)*x^2)/(2*a*(p + 1)*(b
^2 - 4*a*c))), x] + Simp[1/(2*a*(p + 1)*(b^2 - 4*a*c))   Int[(a + b*x^2 + c
*x^4)^(p + 1)*ExpandToSum[2*a*(p + 1)*(b^2 - 4*a*c)*PolynomialQuotient[Px,
a + b*x^2 + c*x^4, x] + b^2*d*(2*p + 3) - 2*a*c*d*(4*p + 5) - a*b*e + c*(4*
p + 7)*(b*d - 2*a*e)*x^2, x], x], x]] /; FreeQ[{a, b, c}, x] && PolyQ[Px, x
^2] && Expon[Px, x^2] > 1 && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3696
```
Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^4)^(
p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Simp[ff^(m + 1
)/f   Subst[Int[x^m*((a + 2*a*ff^2*x^2 + (a + b)*ff^4*x^4)^p/(1 + ff^2*x^2)
^(m/2 + 2*p + 1)), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f}, x] &
& IntegerQ[m/2] && IntegerQ[p]
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 10.68 (sec) , antiderivative size = 583, normalized size of antiderivative = 1.68

method	result
derivativedivides	$-\frac{8\left(-\frac{(5a-2b)\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)}{64\left(a^2-2ab+b^2\right)}+\frac{\left(25a^2+20ab-18b^2\right)\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^3}{64a\left(a^2-2ab+b^2\right)}-\frac{3\left(15a^2+8ab-18b^2\right)\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^5}{64a\left(a^2-2ab+b^2\right)}+\frac{\left(25a^3+2a^2b-388b^2a\right.}{64a^2\left(a^2\right.}}{\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^8a-4\tanh\right.}$
default	$-\frac{8\left(-\frac{(5a-2b)\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)}{64\left(a^2-2ab+b^2\right)}+\frac{\left(25a^2+20ab-18b^2\right)\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^3}{64a\left(a^2-2ab+b^2\right)}-\frac{3\left(15a^2+8ab-18b^2\right)\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^5}{64a\left(a^2-2ab+b^2\right)}+\frac{\left(25a^3+2a^2b-388b^2a\right.}{64a^2\left(a^2\right.}}{\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^8a-4\tanh\right.}$
risch	Expression too large to display

input
```
int(sinh(d*x+c)^2/(a-b*sinh(d*x+c)^4)^3,x,method=_RETURNVERBOSE)
```

output
```
1/d*(-8*(-1/64*(5*a-2*b)/(a^2-2*a*b+b^2)*tanh(1/2*d*x+1/2*c)+1/64*(25*a^2+
20*a*b-18*b^2)/a/(a^2-2*a*b+b^2)*tanh(1/2*d*x+1/2*c)^3-3/64/a*(15*a^2+8*a*
b-18*b^2)/(a^2-2*a*b+b^2)*tanh(1/2*d*x+1/2*c)^5+1/64*(25*a^3+2*a^2*b-388*a
*b^2+160*b^3)/a^2/(a^2-2*a*b+b^2)*tanh(1/2*d*x+1/2*c)^7+1/64*(25*a^3+2*a^2
*b-388*a*b^2+160*b^3)/a^2/(a^2-2*a*b+b^2)*tanh(1/2*d*x+1/2*c)^9-3/64/a*(15
*a^2+8*a*b-18*b^2)/(a^2-2*a*b+b^2)*tanh(1/2*d*x+1/2*c)^11+1/64*(25*a^2+20*
a*b-18*b^2)/a/(a^2-2*a*b+b^2)*tanh(1/2*d*x+1/2*c)^13-1/64*(5*a-2*b)/(a^2-2
*a*b+b^2)*tanh(1/2*d*x+1/2*c)^15)/(tanh(1/2*d*x+1/2*c)^8*a-4*tanh(1/2*d*x+
1/2*c)^6*a+6*tanh(1/2*d*x+1/2*c)^4*a-16*b*tanh(1/2*d*x+1/2*c)^4-4*tanh(1/2
*d*x+1/2*c)^2*a+a)^2-1/64/a^2/(a^2-2*a*b+b^2)*sum((a*(-5*a+2*b)*_R^6+(39*a
^2-28*a*b+10*b^2)*_R^4+(-39*a^2+28*a*b-10*b^2)*_R^2+5*a^2-2*a*b)/(_R^7*a-3
*_R^5*a+3*_R^3*a-8*_R^3*b-_R*a)*ln(tanh(1/2*d*x+1/2*c)-_R),_R=RootOf(a*_Z^
8-4*a*_Z^6+(6*a-16*b)*_Z^4-4*a*_Z^2+a))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 23355 vs. 2(296) = 592.

Time = 1.20 (sec) , antiderivative size = 23355, normalized size of antiderivative = 67.11

$$\int \frac{\sinh^2(c+dx)}{(a-b\sinh^4(c+dx))^3}\,dx = \text{Too large to display}$$

input
```
integrate(sinh(d*x+c)^2/(a-b*sinh(d*x+c)^4)^3,x, algorithm="fricas")
```

output
```
Too large to include
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\sinh^2(c+dx)}{(a-b\sinh^4(c+dx))^3}\,dx = \text{Timed out}$$

input
```
integrate(sinh(d*x+c)**2/(a-b*sinh(d*x+c)**4)**3,x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{\sinh^2(c+dx)}{(a-b\sinh^4(c+dx))^3}\,dx = \int -\frac{\sinh(dx+c)^2}{(b\sinh(dx+c)^4-a)^3}\,dx$$

input
```
integrate(sinh(d*x+c)^2/(a-b*sinh(d*x+c)^4)^3,x, algorithm="maxima")
```

output
```
-1/16*(11*a*b^2 - 5*b^3 + (12*a^2*b*e^(14*c) - 11*a*b^2*e^(14*c) + 5*b^3*e
^(14*c))*e^(14*d*x) - (104*a^2*b*e^(12*c) - 85*a*b^2*e^(12*c) + 35*b^3*e^(
12*c))*e^(12*d*x) - (320*a^3*e^(10*c) - 652*a^2*b*e^(10*c) + 407*a*b^2*e^(
10*c) - 105*b^3*e^(10*c))*e^(10*d*x) + (1408*a^3*e^(8*c) - 1696*a^2*b*e^(8
*c) + 865*a*b^2*e^(8*c) - 175*b^3*e^(8*c))*e^(8*d*x) + (320*a^3*e^(6*c) +
756*a^2*b*e^(6*c) - 849*a*b^2*e^(6*c) + 175*b^3*e^(6*c))*e^(6*d*x) - (248*
a^2*b*e^(4*c) - 383*a*b^2*e^(4*c) + 105*b^3*e^(4*c))*e^(4*d*x) - (12*a^2*b
*e^(2*c) + 77*a*b^2*e^(2*c) - 35*b^3*e^(2*c))*e^(2*d*x))/(a^4*b^2*d - 2*a^
3*b^3*d + a^2*b^4*d + (a^4*b^2*d*e^(16*c) - 2*a^3*b^3*d*e^(16*c) + a^2*b^4
*d*e^(16*c))*e^(16*d*x) - 8*(a^4*b^2*d*e^(14*c) - 2*a^3*b^3*d*e^(14*c) + a
^2*b^4*d*e^(14*c))*e^(14*d*x) - 4*(8*a^5*b*d*e^(12*c) - 23*a^4*b^2*d*e^(12
*c) + 22*a^3*b^3*d*e^(12*c) - 7*a^2*b^4*d*e^(12*c))*e^(12*d*x) + 8*(16*a^5
*b*d*e^(10*c) - 39*a^4*b^2*d*e^(10*c) + 30*a^3*b^3*d*e^(10*c) - 7*a^2*b^4*
d*e^(10*c))*e^(10*d*x) + 2*(128*a^6*d*e^(8*c) - 352*a^5*b*d*e^(8*c) + 355*
a^4*b^2*d*e^(8*c) - 166*a^3*b^3*d*e^(8*c) + 35*a^2*b^4*d*e^(8*c))*e^(8*d*x
) + 8*(16*a^5*b*d*e^(6*c) - 39*a^4*b^2*d*e^(6*c) + 30*a^3*b^3*d*e^(6*c) -
7*a^2*b^4*d*e^(6*c))*e^(6*d*x) - 4*(8*a^5*b*d*e^(4*c) - 23*a^4*b^2*d*e^(4*
c) + 22*a^3*b^3*d*e^(4*c) - 7*a^2*b^4*d*e^(4*c))*e^(4*d*x) - 8*(a^4*b^2*d*
e^(2*c) - 2*a^3*b^3*d*e^(2*c) + a^2*b^4*d*e^(2*c))*e^(2*d*x)) - 1/4*integr
ate(1/2*((12*a^2*e^(6*c) - 11*a*b*e^(6*c) + 5*b^2*e^(6*c))*e^(6*d*x) - ...
```

Giac [F]

$$\int \frac{\sinh^2(c+dx)}{(a-b\sinh^4(c+dx))^3}\,dx = \int -\frac{\sinh(dx+c)^2}{(b\sinh(dx+c)^4-a)^3}\,dx$$

input
```
integrate(sinh(d*x+c)^2/(a-b*sinh(d*x+c)^4)^3,x, algorithm="giac")
```

output
```
sage0*x
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sinh^2(c+dx)}{(a-b\sinh^4(c+dx))^3}\,dx = \int \frac{\sinh(c+d\,x)^2}{\left(a-b\sinh\left(c+d\,x\right)^4\right)^3}\,dx$$

input
```
int(sinh(c + d*x)^2/(a - b*sinh(c + d*x)^4)^3,x)
```

output
```
int(sinh(c + d*x)^2/(a - b*sinh(c + d*x)^4)^3, x)
```

Reduce [F]

$$\int \frac{\sinh^2(c+dx)}{(a-b\sinh^4(c+dx))^3}\,dx = \text{too large to display}$$

input
```
int(sinh(d*x+c)^2/(a-b*sinh(d*x+c)^4)^3,x)
```

output

```
(64*(11083077207982080*e**(20*c + 16*d*x)*int(e**(4*d*x)/(3440640*e**(24*c
 + 24*d*x)*a**4*b**3 + 2150400*e**(24*c + 24*d*x)*a**3*b**4 + 16128*e**(24
*c + 24*d*x)*a**2*b**5 - 96*e**(24*c + 24*d*x)*a*b**6 + 35*e**(24*c + 24*d
*x)*b**7 - 41287680*e**(22*c + 22*d*x)*a**4*b**3 - 25804800*e**(22*c + 22*
d*x)*a**3*b**4 - 193536*e**(22*c + 22*d*x)*a**2*b**5 + 1152*e**(22*c + 22*
d*x)*a*b**6 - 420*e**(22*c + 22*d*x)*b**7 - 165150720*e**(20*c + 20*d*x)*a
**5*b**2 + 123863040*e**(20*c + 20*d*x)*a**4*b**3 + 141152256*e**(20*c + 2
0*d*x)*a**3*b**4 + 1069056*e**(20*c + 20*d*x)*a**2*b**5 - 8016*e**(20*c +
20*d*x)*a*b**6 + 2310*e**(20*c + 20*d*x)*b**7 + 1321205760*e**(18*c + 18*d
*x)*a**5*b**2 + 68812800*e**(18*c + 18*d*x)*a**4*b**3 - 466894848*e**(18*c
 + 18*d*x)*a**3*b**4 - 3585024*e**(18*c + 18*d*x)*a**2*b**5 + 34560*e**(18
*c + 18*d*x)*a*b**6 - 7700*e**(18*c + 18*d*x)*b**7 + 2642411520*e**(16*c +
 16*d*x)*a**6*b - 2972712960*e**(16*c + 16*d*x)*a**5*b**2 - 1174634496*e**
(16*c + 16*d*x)*a**4*b**3 + 1042698240*e**(16*c + 16*d*x)*a**3*b**4 + 8139
264*e**(16*c + 16*d*x)*a**2*b**5 - 94560*e**(16*c + 16*d*x)*a*b**6 + 17325
*e**(16*c + 16*d*x)*b**7 - 10569646080*e**(14*c + 14*d*x)*a**6*b + 2642411
520*e**(14*c + 14*d*x)*a**5*b**2 + 3005743104*e**(14*c + 14*d*x)*a**4*b**3
 - 1659469824*e**(14*c + 14*d*x)*a**3*b**4 - 13138944*e**(14*c + 14*d*x)*a
**2*b**5 + 170112*e**(14*c + 14*d*x)*a*b**6 - 27720*e**(14*c + 14*d*x)*b**
7 - 14092861440*e**(12*c + 12*d*x)*a**7 + 7046430720*e**(12*c + 12*d*x)...
```

3.238 $\int \frac{1}{(a-b\sinh^4(c+dx))^3}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 320

$$\int \frac{1}{(a-b\sinh^4(c+dx))^3}\,dx = \frac{\left(32a-50\sqrt{a}\sqrt{b}+21b\right)\operatorname{arctanh}\left(\frac{\sqrt{\sqrt{a}-\sqrt{b}}\tanh(c+dx)}{\sqrt[4]{a}}\right)}{64a^{11/4}\left(\sqrt{a}-\sqrt{b}\right)^{5/2}d} + \frac{\left(32a+50\sqrt{a}\sqrt{b}+21b\right)\operatorname{arctanh}\left(\frac{\sqrt{\sqrt{a}+\sqrt{b}}\tanh(c+dx)}{\sqrt[4]{a}}\right)}{64a^{11/4}\left(\sqrt{a}+\sqrt{b}\right)^{5/2}d} - \frac{b^2\tanh(c+dx)\left(3a+b-4(a+b)\tanh^2(c+dx)\right)}{8a(a-b)^3d\left(a-2a\tanh^2(c+dx)+(a-b)\tanh^4(c+dx)\right)^2} - \frac{b\tanh(c+dx)\left(\frac{17a^2-40ab+7b^2}{(a-b)^3}-\frac{(33a-13b)\tanh^2(c+dx)}{(a-b)^2}\right)}{32a^2d\left(a-2a\tanh^2(c+dx)+(a-b)\tanh^4(c+dx)\right)}$$

output
```
1/64*(32*a-50*a^(1/2)*b^(1/2)+21*b)*arctanh((a^(1/2)-b^(1/2))^(1/2)*tanh(d
*x+c)/a^(1/4))/a^(11/4)/(a^(1/2)-b^(1/2))^(5/2)/d+1/64*(32*a+50*a^(1/2)*b^
(1/2)+21*b)*arctanh((a^(1/2)+b^(1/2))^(1/2)*tanh(d*x+c)/a^(1/4))/a^(11/4)/
(a^(1/2)+b^(1/2))^(5/2)/d-1/8*b^2*tanh(d*x+c)*(3*a+b-4*(a+b)*tanh(d*x+c)^2
)/a/(a-b)^3/d/(a-2*a*tanh(d*x+c)^2+(a-b)*tanh(d*x+c)^4)^2-1/32*b*tanh(d*x+
c)*((17*a^2-40*a*b+7*b^2)/(a-b)^3-(33*a-13*b)*tanh(d*x+c)^2/(a-b)^2)/a^2/d
/(a-2*a*tanh(d*x+c)^2+(a-b)*tanh(d*x+c)^4)
```

Mathematica [A] (verified)

Time = 12.21 (sec) , antiderivative size = 333, normalized size of antiderivative = 1.04

$$\int \frac{1}{\left(a-b\sinh^4(c+dx)\right)^3}\,dx = \frac{-\frac{\left(\sqrt{a}+\sqrt{b}\right)^2\left(32a-50\sqrt{a}\sqrt{b}+21b\right)\arctan\left(\frac{\left(\sqrt{a}-\sqrt{b}\right)\tanh(c+dx)}{\sqrt{-a+\sqrt{a}\sqrt{b}}}\right)}{\sqrt{-a+\sqrt{a}\sqrt{b}}}+\frac{\left(\sqrt{a}-\sqrt{b}\right)^2\left(32a+50\sqrt{a}\sqrt{b}+21b\right)\text{arctanh}\left(\frac{\left(\sqrt{a}+\sqrt{b}\right)\tanh(c+dx)}{\sqrt{a+\sqrt{a}\sqrt{b}}}\right)}{\sqrt{a+\sqrt{a}\sqrt{b}}}}{64a^{5/2}(a-b)^2d}$$

input
```
Integrate[(a - b*Sinh[c + d*x]^4)^(-3),x]
```

output
```
(-(((Sqrt[a] + Sqrt[b])^2*(32*a - 50*Sqrt[a]*Sqrt[b] + 21*b)*ArcTan[((Sqrt
[a] - Sqrt[b])*Tanh[c + d*x])/Sqrt[-a + Sqrt[a]*Sqrt[b]]])/Sqrt[-a + Sqrt[
a]*Sqrt[b]]) + ((Sqrt[a] - Sqrt[b])^2*(32*a + 50*Sqrt[a]*Sqrt[b] + 21*b)*A
rcTanh[((Sqrt[a] + Sqrt[b])*Tanh[c + d*x])/Sqrt[a + Sqrt[a]*Sqrt[b]]])/Sqr
t[a + Sqrt[a]*Sqrt[b]] + (8*Sqrt[a]*b*(-19*a + 10*b + (6*a - 3*b)*Cosh[2*(
c + d*x)])*Sinh[2*(c + d*x)])/(8*a - 3*b + 4*b*Cosh[2*(c + d*x)] - b*Cosh[
4*(c + d*x)]) + (64*a^(3/2)*(a - b)*b*(-6*Sinh[2*(c + d*x)] + Sinh[4*(c +
d*x)]))/(-8*a + 3*b - 4*b*Cosh[2*(c + d*x)] + b*Cosh[4*(c + d*x)])^2)/(64*
a^(5/2)*(a - b)^2*d)
```

Rubi [A] (verified)

Time = 0.89 (sec) , antiderivative size = 363, normalized size of antiderivative = 1.13, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.533, Rules used = {3042, 3688, 1517, 27, 2206, 27, 1480, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\left(a-b\sinh^4(c+dx)\right)^3}dx$$

$$\Big\downarrow \ 3042$$

$$\int \frac{1}{\left(a-b\sin(ic+idx)^4\right)^3}dx$$

$$\Big\downarrow \ 3688$$

$$\frac{\int \frac{\left(1-\tanh^2(c+dx)\right)^5}{\left((a-b)\tanh^4(c+dx)-2a\tanh^2(c+dx)+a\right)^3}d\tanh(c+dx)}{d}$$

$$\Big\downarrow \ 1517$$

$$\frac{-\frac{\int -\frac{2\left(-\frac{8a^2b\tanh^6(c+dx)}{a-b}+\frac{8a^2(3a-5b)b\tanh^4(c+dx)}{(a-b)^2}-\frac{4ab\left(6a^3-18ba^2+15b^2a-5b^3\right)\tanh^2(c+dx)}{(a-b)^3}+\frac{ab\left(8a^3-24ba^2+27b^2a-7b^3\right)}{(a-b)^3}\right)}{\left((a-b)\tanh^4(c+dx)-2a\tanh^2(c+dx)+a\right)^2}d\tanh(c+dx)}{16a^2b}-\frac{\cdots}{8\cdots}}{d}$$

$$\Big\downarrow \ 27$$

$$\frac{\frac{\int \frac{-\frac{8a^2b\tanh^6(c+dx)}{a-b}+\frac{8a^2(3a-5b)b\tanh^4(c+dx)}{(a-b)^2}-\frac{4ab\left(6a^3-18ba^2+15b^2a-5b^3\right)\tanh^2(c+dx)}{(a-b)^3}+\frac{ab\left(8a^3-24ba^2+27b^2a-7b^3\right)}{(a-b)^3}}{\left((a-b)\tanh^4(c+dx)-2a\tanh^2(c+dx)+a\right)^2}d\tanh(c+dx)}{8a^2b}-\frac{b^2\tanh\cdots}{8a(a-b)^3\cdots}}{d}$$

$$\Big\downarrow \ 2206$$

$$\frac{\frac{-\frac{\int -\frac{2a^2b^2\left(32a^2-47ba+21b^2-\left(32a^2-33ba+13b^2\right)\tanh^2(c+dx)\right)}{(a-b)^2\left((a-b)\tanh^4(c+dx)-2a\tanh^2(c+dx)+a\right)}d\tanh(c+dx)}{8a^2b}-\frac{b^2\tanh(c+dx)\left(\frac{17a^2-40ab+7b^2}{(a-b)^3}-\frac{(33a-13b)\tanh^2(c+dx)}{(a-b)^2}\right)}{4\left((a-b)\tanh^4(c+dx)-2a\tanh^2(c+dx)+a\right)}}{8a^2b}-\frac{b^2\tan\cdots}{8a(a-b)^3\cdots}}{d}$$

$$\Big\downarrow \; 27$$

$$\frac{\frac{b\int \frac{32a^2-47ba+21b^2-\left(32a^2-33ba+13b^2\right)\tanh^2(c+dx)}{(a-b)\tanh^4(c+dx)-2a\tanh^2(c+dx)+a}d\tanh(c+dx)}{4(a-b)^2}-\frac{b^2\tanh(c+dx)\left(\frac{17a^2-40ab+7b^2}{(a-b)^3}-\frac{(33a-13b)\tanh^2(c+dx)}{(a-b)^2}\right)}{4\left((a-b)\tanh^4(c+dx)-2a\tanh^2(c+dx)+a\right)}}{8a^2b}-\frac{b^2\tanh(c+dx)(\cdots}{8a(a-b)^3((a-b)\tan\cdots}}{d}$$

$$\Big\downarrow \; 1480$$

$$\frac{\frac{b\left(-\frac{\left(50\sqrt{a}\sqrt{b}+32a+21b\right)\left(\sqrt{a}-\sqrt{b}\right)^3\int \frac{1}{(a-b)\tanh^2(c+dx)-\sqrt{a}\left(\sqrt{a}-\sqrt{b}\right)}d\tanh(c+dx)}{2\sqrt{a}}-\frac{\left(\sqrt{a}+\sqrt{b}\right)^3\left(-50\sqrt{a}\sqrt{b}+32a+21b\right)\int \frac{1}{(a-b)\tanh^2(c+dx)-\sqrt{a}\left(\sqrt{a}+\sqrt{\cdots}\right.}}{2\sqrt{a}}\cdots}{4(a-b)^2}\cdots}{8a^2b}\cdots}{d}$$

$$\Big\downarrow \; 221$$

$$\frac{\frac{b\left(\frac{\left(50\sqrt{a}\sqrt{b}+32a+21b\right)\left(\sqrt{a}-\sqrt{b}\right)^2\text{arctanh}\left(\frac{\sqrt{\sqrt{a}+\sqrt{b}}\tanh(c+dx)}{\sqrt[4]{a}}\right)}{2a^{3/4}\sqrt{\sqrt{a}+\sqrt{b}}}+\frac{\left(\sqrt{a}+\sqrt{b}\right)^2\left(-50\sqrt{a}\sqrt{b}+32a+21b\right)\text{arctanh}\left(\frac{\sqrt{\sqrt{a}-\sqrt{b}}\tanh(c+dx)}{\sqrt[4]{a}}\right)}{2a^{3/4}\sqrt{\sqrt{a}-\sqrt{b}}}\right)}{4(a-b)^2}-\frac{b^2\tanh\cdots}{\cdots}}{8a^2b}\cdots}{d}$$

input
```
Int[(a - b*Sinh[c + d*x]^4)^(-3),x]
```

output
```
(-1/8*(b^2*Tanh[c + d*x]*(3*a + b - 4*(a + b)*Tanh[c + d*x]^2))/(a*(a - b)
^3*(a - 2*a*Tanh[c + d*x]^2 + (a - b)*Tanh[c + d*x]^4)^2) + ((b*(((Sqrt[a]
 + Sqrt[b])^2*(32*a - 50*Sqrt[a]*Sqrt[b] + 21*b)*ArcTanh[(Sqrt[Sqrt[a] - S
qrt[b]]*Tanh[c + d*x])/a^(1/4)])/(2*a^(3/4)*Sqrt[Sqrt[a] - Sqrt[b]]) + ((S
qrt[a] - Sqrt[b])^2*(32*a + 50*Sqrt[a]*Sqrt[b] + 21*b)*ArcTanh[(Sqrt[Sqrt[
a] + Sqrt[b]]*Tanh[c + d*x])/a^(1/4)])/(2*a^(3/4)*Sqrt[Sqrt[a] + Sqrt[b]])
))/(4*(a - b)^2) - (b^2*Tanh[c + d*x]*((17*a^2 - 40*a*b + 7*b^2)/(a - b)^3
 - ((33*a - 13*b)*Tanh[c + d*x]^2)/(a - b)^2))/(4*(a - 2*a*Tanh[c + d*x]^2
 + (a - b)*Tanh[c + d*x]^4)))/(8*a^2*b))/d
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 1480
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Simp[(e/2 + (2*c*d - b*e)/(2*q))   Int[1/(
b/2 - q/2 + c*x^2), x], x] + Simp[(e/2 - (2*c*d - b*e)/(2*q))   Int[1/(b/2
+ q/2 + c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0]
 && NeQ[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

rule 1517
```
Int[((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x
_Symbol] :> With[{f = Coeff[PolynomialRemainder[(d + e*x^2)^q, a + b*x^2 +
c*x^4, x], x, 0], g = Coeff[PolynomialRemainder[(d + e*x^2)^q, a + b*x^2 +
c*x^4, x], x, 2]}, Simp[x*(a + b*x^2 + c*x^4)^(p + 1)*((a*b*g - f*(b^2 - 2*
a*c) - c*(b*f - 2*a*g)*x^2)/(2*a*(p + 1)*(b^2 - 4*a*c))), x] + Simp[1/(2*a*
(p + 1)*(b^2 - 4*a*c))   Int[(a + b*x^2 + c*x^4)^(p + 1)*ExpandToSum[2*a*(p
 + 1)*(b^2 - 4*a*c)*PolynomialQuotient[(d + e*x^2)^q, a + b*x^2 + c*x^4, x]
 + b^2*f*(2*p + 3) - 2*a*c*f*(4*p + 5) - a*b*g + c*(4*p + 7)*(b*f - 2*a*g)*
x^2, x], x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ
[c*d^2 - b*d*e + a*e^2, 0] && IGtQ[q, 1] && LtQ[p, -1]
```

rule 2206
```
Int[(Px_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> With[{d =
 Coeff[PolynomialRemainder[Px, a + b*x^2 + c*x^4, x], x, 0], e = Coeff[Poly
nomialRemainder[Px, a + b*x^2 + c*x^4, x], x, 2]}, Simp[x*(a + b*x^2 + c*x^
4)^(p + 1)*((a*b*e - d*(b^2 - 2*a*c) - c*(b*d - 2*a*e)*x^2)/(2*a*(p + 1)*(b
^2 - 4*a*c))), x] + Simp[1/(2*a*(p + 1)*(b^2 - 4*a*c))   Int[(a + b*x^2 + c
*x^4)^(p + 1)*ExpandToSum[2*a*(p + 1)*(b^2 - 4*a*c)*PolynomialQuotient[Px,
a + b*x^2 + c*x^4, x] + b^2*d*(2*p + 3) - 2*a*c*d*(4*p + 5) - a*b*e + c*(4*
p + 7)*(b*d - 2*a*e)*x^2, x], x], x]] /; FreeQ[{a, b, c}, x] && PolyQ[Px, x
^2] && Expon[Px, x^2] > 1 && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3688
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^4)^(p_.), x_Symbol] :> With[{ff =
 FreeFactors[Tan[e + f*x], x]}, Simp[ff/f   Subst[Int[(a + 2*a*ff^2*x^2 + (
a + b)*ff^4*x^4)^p/(1 + ff^2*x^2)^(2*p + 1), x], x, Tan[e + f*x]/ff], x]] /
; FreeQ[{a, b, e, f}, x] && IntegerQ[p]
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 10.47 (sec) , antiderivative size = 577, normalized size of antiderivative = 1.80

method	result
derivativedivides	$-\frac{2\left(\frac{b(17a-11b)\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)}{32a\left(a^2-2ab+b^2\right)}-\frac{(149a-95b)b\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^3}{32a\left(a^2-2ab+b^2\right)}+\frac{b\left(345a^2-427ab+112b^2\right)\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^5}{32a^2\left(a^2-2ab+b^2\right)}-\frac{\left(213a^2-1111ab+496b^2\right.}{32a^2\left(a^2-2a\right.}\right.}{\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^8a-4\tanh\left(\frac{dx}{2}\right.\right.}$
default	$-\frac{2\left(\frac{b(17a-11b)\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)}{32a\left(a^2-2ab+b^2\right)}-\frac{(149a-95b)b\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^3}{32a\left(a^2-2ab+b^2\right)}+\frac{b\left(345a^2-427ab+112b^2\right)\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^5}{32a^2\left(a^2-2ab+b^2\right)}-\frac{\left(213a^2-1111ab+496b^2\right.}{32a^2\left(a^2-2a\right.}\right.}{\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^8a-4\tanh\left(\frac{dx}{2}\right.\right.}$
risch	Expression too large to display

input
```
int(1/(a-b*sinh(d*x+c)^4)^3,x,method=_RETURNVERBOSE)
```

output
```
1/d*(-2*(1/32*b*(17*a-11*b)/a/(a^2-2*a*b+b^2)*tanh(1/2*d*x+1/2*c)-1/32*(14
9*a-95*b)/a*b/(a^2-2*a*b+b^2)*tanh(1/2*d*x+1/2*c)^3+1/32/a^2*b*(345*a^2-42
7*a*b+112*b^2)/(a^2-2*a*b+b^2)*tanh(1/2*d*x+1/2*c)^5-1/32*(213*a^2-1111*a*
b+496*b^2)/a^2*b/(a^2-2*a*b+b^2)*tanh(1/2*d*x+1/2*c)^7-1/32*(213*a^2-1111*
a*b+496*b^2)/a^2*b/(a^2-2*a*b+b^2)*tanh(1/2*d*x+1/2*c)^9+1/32/a^2*b*(345*a
^2-427*a*b+112*b^2)/(a^2-2*a*b+b^2)*tanh(1/2*d*x+1/2*c)^11-1/32*(149*a-95*
b)/a*b/(a^2-2*a*b+b^2)*tanh(1/2*d*x+1/2*c)^13+1/32*b*(17*a-11*b)/a/(a^2-2*
a*b+b^2)*tanh(1/2*d*x+1/2*c)^15)/(tanh(1/2*d*x+1/2*c)^8*a-4*tanh(1/2*d*x+1
/2*c)^6*a+6*tanh(1/2*d*x+1/2*c)^4*a-16*b*tanh(1/2*d*x+1/2*c)^4-4*tanh(1/2*
d*x+1/2*c)^2*a+a)^2-1/128/(a^2-2*a*b+b^2)/a^2*sum(((32*a^2-47*a*b+21*b^2)*
_R^6+(-96*a^2+85*a*b-31*b^2)*_R^4+(96*a^2-85*a*b+31*b^2)*_R^2-32*a^2+47*a*
b-21*b^2)/(_R^7*a-3*_R^5*a+3*_R^3*a-8*_R^3*b-_R*a)*ln(tanh(1/2*d*x+1/2*c)-
_R),_R=RootOf(a*_Z^8-4*a*_Z^6+(6*a-16*b)*_Z^4-4*a*_Z^2+a))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 23125 vs. $2(274) = 548$.

Time = 1.25 (sec) , antiderivative size = 23125, normalized size of antiderivative = 72.27

$$\int \frac{1}{\left(a-b\sinh^4(c+dx)\right)^3}\,dx = \text{Too large to display}$$

input
```
integrate(1/(a-b*sinh(d*x+c)^4)^3,x, algorithm="fricas")
```

output
```
Too large to include
```

Sympy [F(-1)]

Timed out.

$$\int \frac{1}{\left(a-b\sinh^4(c+dx)\right)^3}\,dx = \text{Timed out}$$

input
```
integrate(1/(a-b*sinh(d*x+c)**4)**3,x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{1}{\left(a-b\sinh^4(c+dx)\right)^3}\,dx=\int -\frac{1}{\left(b\sinh\left(dx+c\right)^4-a\right)^3}\,dx$$

input
```
integrate(1/(a-b*sinh(d*x+c)^4)^3,x, algorithm="maxima")
```

output
```
1/8*(6*a*b^2 - 3*b^3 + (7*a*b^2*e^(14*c) - 4*b^3*e^(14*c))*e^(14*d*x) - (3
2*a^2*b*e^(12*c) + 2*a*b^2*e^(12*c) - 7*b^3*e^(12*c))*e^(12*d*x) - (16*a^2
*b*e^(10*c) - 3*a*b^2*e^(10*c) - 28*b^3*e^(10*c))*e^(10*d*x) + 3*(256*a^3*
e^(8*c) - 320*a^2*b*e^(8*c) + 166*a*b^2*e^(8*c) - 35*b^3*e^(8*c))*e^(8*d*x
) + (784*a^2*b*e^(6*c) - 723*a*b^2*e^(6*c) + 140*b^3*e^(6*c))*e^(6*d*x) -
(160*a^2*b*e^(4*c) - 266*a*b^2*e^(4*c) + 91*b^3*e^(4*c))*e^(4*d*x) - (55*a
*b^2*e^(2*c) - 28*b^3*e^(2*c))*e^(2*d*x))/(a^4*b^2*d - 2*a^3*b^3*d + a^2*b
^4*d + (a^4*b^2*d*e^(16*c) - 2*a^3*b^3*d*e^(16*c) + a^2*b^4*d*e^(16*c))*e^
(16*d*x) - 8*(a^4*b^2*d*e^(14*c) - 2*a^3*b^3*d*e^(14*c) + a^2*b^4*d*e^(14*
c))*e^(14*d*x) - 4*(8*a^5*b*d*e^(12*c) - 23*a^4*b^2*d*e^(12*c) + 22*a^3*b^
3*d*e^(12*c) - 7*a^2*b^4*d*e^(12*c))*e^(12*d*x) + 8*(16*a^5*b*d*e^(10*c) -
 39*a^4*b^2*d*e^(10*c) + 30*a^3*b^3*d*e^(10*c) - 7*a^2*b^4*d*e^(10*c))*e^(
10*d*x) + 2*(128*a^6*d*e^(8*c) - 352*a^5*b*d*e^(8*c) + 355*a^4*b^2*d*e^(8*
c) - 166*a^3*b^3*d*e^(8*c) + 35*a^2*b^4*d*e^(8*c))*e^(8*d*x) + 8*(16*a^5*b
*d*e^(6*c) - 39*a^4*b^2*d*e^(6*c) + 30*a^3*b^3*d*e^(6*c) - 7*a^2*b^4*d*e^(
6*c))*e^(6*d*x) - 4*(8*a^5*b*d*e^(4*c) - 23*a^4*b^2*d*e^(4*c) + 22*a^3*b^3
*d*e^(4*c) - 7*a^2*b^4*d*e^(4*c))*e^(4*d*x) - 8*(a^4*b^2*d*e^(2*c) - 2*a^3
*b^3*d*e^(2*c) + a^2*b^4*d*e^(2*c))*e^(2*d*x)) + integrate(1/4*((7*a*b*e^(
6*c) - 4*b^2*e^(6*c))*e^(6*d*x) - 2*(32*a^2*e^(4*c) - 40*a*b*e^(4*c) + 17*
b^2*e^(4*c))*e^(4*d*x) + (7*a*b*e^(2*c) - 4*b^2*e^(2*c))*e^(2*d*x))/(a^...
```

Giac [F]

$$\int \frac{1}{\left(a-b\sinh^4(c+dx)\right)^3}\,dx=\int -\frac{1}{\left(b\sinh\left(dx+c\right)^4-a\right)^3}\,dx$$

input
```
integrate(1/(a-b*sinh(d*x+c)^4)^3,x, algorithm="giac")
```

output
```
sage0*x
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\left(a-b\sinh^4(c+dx)\right)^3}\,dx=\int \frac{1}{\left(a-b\sinh\left(c+d\,x\right)^4\right)^3}\,dx$$

input
```
int(1/(a - b*sinh(c + d*x)^4)^3,x)
```

output
```
int(1/(a - b*sinh(c + d*x)^4)^3, x)
```

Reduce [F]

$$\int \frac{1}{\left(a-b\sinh^4(c+dx)\right)^3}\,dx=\text{too large to display}$$

input
```
int(1/(a-b*sinh(d*x+c)^4)^3,x)
```

output

```
(512*( - 3463461627494400*e**(20*c + 16*d*x)*int(e**(4*d*x)/(3440640*e**(2
4*c + 24*d*x)*a**4*b**3 + 2150400*e**(24*c + 24*d*x)*a**3*b**4 + 16128*e**
(24*c + 24*d*x)*a**2*b**5 - 96*e**(24*c + 24*d*x)*a*b**6 + 35*e**(24*c + 2
4*d*x)*b**7 - 41287680*e**(22*c + 22*d*x)*a**4*b**3 - 25804800*e**(22*c +
22*d*x)*a**3*b**4 - 193536*e**(22*c + 22*d*x)*a**2*b**5 + 1152*e**(22*c +
22*d*x)*a*b**6 - 420*e**(22*c + 22*d*x)*b**7 - 165150720*e**(20*c + 20*d*x
)*a**5*b**2 + 123863040*e**(20*c + 20*d*x)*a**4*b**3 + 141152256*e**(20*c
+ 20*d*x)*a**3*b**4 + 1069056*e**(20*c + 20*d*x)*a**2*b**5 - 8016*e**(20*c
 + 20*d*x)*a*b**6 + 2310*e**(20*c + 20*d*x)*b**7 + 1321205760*e**(18*c + 1
8*d*x)*a**5*b**2 + 68812800*e**(18*c + 18*d*x)*a**4*b**3 - 466894848*e**(1
8*c + 18*d*x)*a**3*b**4 - 3585024*e**(18*c + 18*d*x)*a**2*b**5 + 34560*e**
(18*c + 18*d*x)*a*b**6 - 7700*e**(18*c + 18*d*x)*b**7 + 2642411520*e**(16*
c + 16*d*x)*a**6*b - 2972712960*e**(16*c + 16*d*x)*a**5*b**2 - 1174634496*
e**(16*c + 16*d*x)*a**4*b**3 + 1042698240*e**(16*c + 16*d*x)*a**3*b**4 + 8
139264*e**(16*c + 16*d*x)*a**2*b**5 - 94560*e**(16*c + 16*d*x)*a*b**6 + 17
325*e**(16*c + 16*d*x)*b**7 - 10569646080*e**(14*c + 14*d*x)*a**6*b + 2642
411520*e**(14*c + 14*d*x)*a**5*b**2 + 3005743104*e**(14*c + 14*d*x)*a**4*b
**3 - 1659469824*e**(14*c + 14*d*x)*a**3*b**4 - 13138944*e**(14*c + 14*d*x
)*a**2*b**5 + 170112*e**(14*c + 14*d*x)*a*b**6 - 27720*e**(14*c + 14*d*x)*
b**7 - 14092861440*e**(12*c + 12*d*x)*a**7 + 7046430720*e**(12*c + 12*d...
```

3.239 $\int \frac{\operatorname{csch}^2(c+dx)}{\left(a-b\sinh^4(c+dx)\right)^3}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 357

$$\begin{aligned}\int \frac{\operatorname{csch}^2(c+dx)}{\left(a-b\sinh^4(c+dx)\right)^3}\,dx &= -\frac{3\sqrt{b}\left(20a-34\sqrt{a}\sqrt{b}+15b\right)\operatorname{arctanh}\left(\frac{\sqrt{\sqrt{a}-\sqrt{b}}\tanh(c+dx)}{\sqrt[4]{a}}\right)}{64a^{13/4}\left(\sqrt{a}-\sqrt{b}\right)^{5/2}d}\\
&\quad+\frac{3\sqrt{b}\left(20a+34\sqrt{a}\sqrt{b}+15b\right)\operatorname{arctanh}\left(\frac{\sqrt{\sqrt{a}+\sqrt{b}}\tanh(c+dx)}{\sqrt[4]{a}}\right)}{64a^{13/4}\left(\sqrt{a}+\sqrt{b}\right)^{5/2}d}-\frac{\coth(c+dx)}{a^3d}\\
&\quad+\frac{b^2\tanh(c+dx)\left(a(a+3b)-(a^2+6ab+b^2)\tanh^2(c+dx)\right)}{8a^2(a-b)^3d\left(a-2a\tanh^2(c+dx)+(a-b)\tanh^4(c+dx)\right)^2}\\
&\quad+\frac{b\tanh(c+dx)\left(2a^2(9a-17b)-(a-b)\left(18a^2+15ab-13b^2\right)\tanh^2(c+dx)\right)}{32a^3(a-b)^3d\left(a-2a\tanh^2(c+dx)+(a-b)\tanh^4(c+dx)\right)}\end{aligned}$$

output
```
-3/64*b^(1/2)*(20*a-34*a^(1/2)*b^(1/2)+15*b)*arctanh((a^(1/2)-b^(1/2))^(1/
2)*tanh(d*x+c)/a^(1/4))/a^(13/4)/(a^(1/2)-b^(1/2))^(5/2)/d+3/64*b^(1/2)*(2
0*a+34*a^(1/2)*b^(1/2)+15*b)*arctanh((a^(1/2)+b^(1/2))^(1/2)*tanh(d*x+c)/a
^(1/4))/a^(13/4)/(a^(1/2)+b^(1/2))^(5/2)/d-coth(d*x+c)/a^3/d+1/8*b^2*tanh(
d*x+c)*(a*(a+3*b)-(a^2+6*a*b+b^2)*tanh(d*x+c)^2)/a^2/(a-b)^3/d/(a-2*a*tanh
(d*x+c)^2+(a-b)*tanh(d*x+c)^4)^2+1/32*b*tanh(d*x+c)*(2*a^2*(9*a-17*b)-(a-b
)*(18*a^2+15*a*b-13*b^2)*tanh(d*x+c)^2)/a^3/(a-b)^3/d/(a-2*a*tanh(d*x+c)^2
+(a-b)*tanh(d*x+c)^4)
```

Mathematica [A] (verified)

Time = 7.93 (sec) , antiderivative size = 357, normalized size of antiderivative = 1.00

$$\int \frac{\operatorname{csch}^2(c+dx)}{\left(a-b\sinh^4(c+dx)\right)^3}\,dx$$

$$= \frac{\frac{3\sqrt{b}\left(20a-34\sqrt{a}\sqrt{b}+15b\right)\arctan\left(\frac{\left(\sqrt{a}-\sqrt{b}\right)\tanh(c+dx)}{\sqrt{-a+\sqrt{a}\sqrt{b}}}\right)}{\left(\sqrt{a}-\sqrt{b}\right)^2\sqrt{-a+\sqrt{a}\sqrt{b}}}+\frac{3\sqrt{b}\left(20a+34\sqrt{a}\sqrt{b}+15b\right)\operatorname{arctanh}\left(\frac{\left(\sqrt{a}+\sqrt{b}\right)\tanh(c+dx)}{\sqrt{a+\sqrt{a}\sqrt{b}}}\right)}{\left(\sqrt{a}+\sqrt{b}\right)^2\sqrt{a+\sqrt{a}\sqrt{b}}}-64\coth(c+}{64a^3 d}$$

input
```
Integrate[Csch[c + d*x]^2/(a - b*Sinh[c + d*x]^4)^3,x]
```

output
```
((3*Sqrt[b]*(20*a - 34*Sqrt[a]*Sqrt[b] + 15*b)*ArcTan[((Sqrt[a] - Sqrt[b])
*Tanh[c + d*x])/Sqrt[-a + Sqrt[a]*Sqrt[b]]])/((Sqrt[a] - Sqrt[b])^2*Sqrt[-
a + Sqrt[a]*Sqrt[b]]) + (3*Sqrt[b]*(20*a + 34*Sqrt[a]*Sqrt[b] + 15*b)*ArcT
anh[((Sqrt[a] + Sqrt[b])*Tanh[c + d*x])/Sqrt[a + Sqrt[a]*Sqrt[b]]])/((Sqrt
[a] + Sqrt[b])^2*Sqrt[a + Sqrt[a]*Sqrt[b]]) - 64*Coth[c + d*x] + (4*b*(28*
a^2 + 3*a*b - 13*b^2 + b*(-19*a + 13*b)*Cosh[2*(c + d*x)])*Sinh[2*(c + d*x
)])/((a - b)^2*(8*a - 3*b + 4*b*Cosh[2*(c + d*x)] - b*Cosh[4*(c + d*x)]))
+ (128*a*b*(2*a + b - b*Cosh[2*(c + d*x)])*Sinh[2*(c + d*x)])/((a - b)*(-8
*a + 3*b - 4*b*Cosh[2*(c + d*x)] + b*Cosh[4*(c + d*x)])^2))/(64*a^3*d)
```

Rubi [A] (verified)

Time = 1.68 (sec) , antiderivative size = 373, normalized size of antiderivative = 1.04, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.375$, Rules used = {3042, 25, 3696, 1673, 27, 2198, 27, 2195, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\operatorname{csch}^2(c+dx)}{\left(a-b\sinh^4(c+dx)\right)^3}dx$$

$$\Big\downarrow \text{3042}$$

$$\int -\frac{1}{\sin(ic+idx)^2\left(a-b\sin(ic+idx)^4\right)^3}dx$$

$$\Big\downarrow \text{25}$$

$$-\int \frac{1}{\sin(ic+idx)^2\left(a-b\sin(ic+idx)^4\right)^3}dx$$

$$\Big\downarrow \text{3696}$$

$$\frac{\int \frac{\coth^2(c+dx)\left(1-\tanh^2(c+dx)\right)^6}{\left((a-b)\tanh^4(c+dx)-2a\tanh^2(c+dx)+a\right)^3}d\tanh(c+dx)}{d}$$

$$\Big\downarrow \text{1673}$$

$$\frac{\frac{b^2\tanh(c+dx)\left(a(a+3b)-\left(a^2+6ab+b^2\right)\tanh^2(c+dx)\right)}{8a^2(a-b)^3\left((a-b)\tanh^4(c+dx)-2a\tanh^2(c+dx)+a\right)^2}-\frac{\int -\frac{2\coth^2(c+dx)\left(\frac{8a^2b\tanh^8(c+dx)}{a-b}-\frac{16a^2(2a-3b)b\tanh^6(c+dx)}{(a-b)^2}+\frac{b\left(48a^4-136ba^3+115b^2 a \cdots\right.}{(\cdots}\right.}{\left((a-b)\tanh^4(c+dx)-2a\cdots\right.}}{1\cdots}}{d}$$

$$\Big\downarrow \text{27}$$

$$\frac{\frac{\int \frac{\coth^2(c+dx)\left(\frac{8a^2b\tanh^8(c+dx)}{a-b}-\frac{16a^2(2a-3b)b\tanh^6(c+dx)}{(a-b)^2}+\frac{b\left(48a^4-136ba^3+115b^2a^2-30b^3a-5b^4\right)\tanh^4(c+dx)}{(a-b)^3}-\frac{ab\left(32a^3-96ba^2+97b^2a-29b^3\right)\tanh^2\cdots}{(a-b)^3}\right.}{\left((a-b)\tanh^4(c+dx)-2a\tanh^2(c+dx)+a\right)^2}}{8a^2b}\cdots}{d}$$

$$\Big\downarrow \text{2198}$$

$$\frac{\frac{\frac{b^2\tanh(c+dx)\left(\frac{2a^2(9a-17b)}{(a-b)^3}-\frac{(18a^2+15ab-13b^2)\tanh^2(c+dx)}{(a-b)^2}\right)}{4a\left((a-b)\tanh^4(c+dx)-2a\tanh^2(c+dx)+a\right)}-\frac{\int -\frac{2\coth^2(c+dx)\left(\frac{ab^2(32a^3-18ba^2-15b^2a+13b^3)\tanh^4(c+dx)}{(a-b)^2}-\frac{2a^2b^2(32a^2-55ba+2}{(a-b}\right.}{(a-b)\tanh^4(c+dx)-2a\tanh^2(c+dx)+a}}{8a^2b}}{8a^2b}}{d}$$

$\big\downarrow$ 27

$$\frac{\frac{\frac{\int\frac{\coth^2(c+dx)\left(\frac{ab^2(32a^3-18ba^2-15b^2a+13b^3)\tanh^4(c+dx)}{(a-b)^2}-\frac{2a^2b^2(32a^2-55ba+26b^2)\tanh^2(c+dx)}{(a-b)^2}+32a^2b^2\right)}{(a-b)\tanh^4(c+dx)-2a\tanh^2(c+dx)+a}d\tanh(c+dx)}{4a^2b}+\frac{b^2\tanh(c+dx)\left(\frac{2a^2(9a-17b}{(a-b)^3}\right.}{4a\left((a-b)\tanh^4(\right.}}{8a^2b}}{d}$$

$\big\downarrow$ 2195

$$\frac{\frac{\frac{\int\left(\frac{3a\left((26a^2-37ba+15b^2)\tanh^2(c+dx)-2a(3a-2b)\right)b^3}{(a-b)^2\left((a-b)\tanh^4(c+dx)-2a\tanh^2(c+dx)+a\right)}+32a\coth^2(c+dx)b^2\right)d\tanh(c+dx)}{4a^2b}+\frac{b^2\tanh(c+dx)\left(\frac{2a^2(9a-17b)}{(a-b)^3}-\frac{(18a^2+15ab-13b^2)\tanh^2(c+dx)}{(a-b)^2}\right.}{4a\left((a-b)\tanh^4(c+dx)-2a\tanh^2(c+dx)+a\right)}}{8a^2b}}{d}$$

$\big\downarrow$ 2009

$$\frac{\frac{b^2\tanh(c+dx)\left(a(a+3b)-(a^2+6ab+b^2)\tanh^2(c+dx)\right)}{8a^2(a-b)^3\left((a-b)\tanh^4(c+dx)-2a\tanh^2(c+dx)+a\right)^2}+\frac{\frac{b^2\tanh(c+dx)\left(\frac{2a^2(9a-17b)}{(a-b)^3}-\frac{(18a^2+15ab-13b^2)\tanh^2(c+dx)}{(a-b)^2}\right)}{4a\left((a-b)\tanh^4(c+dx)-2a\tanh^2(c+dx)+a\right)}+\frac{-\frac{3a^{3/4}b^{5/2}\left(-34\sqrt{a}\sqrt{b}\right.}{}}{}}{}}{d}$$

input
```
Int[Csch[c + d*x]^2/(a - b*Sinh[c + d*x]^4)^3,x]
```

output
```
((b^2*Tanh[c + d*x]*(a*(a + 3*b) - (a^2 + 6*a*b + b^2)*Tanh[c + d*x]^2))/(
8*a^2*(a - b)^3*(a - 2*a*Tanh[c + d*x]^2 + (a - b)*Tanh[c + d*x]^4)^2) + (
((-3*a^(3/4)*b^(5/2)*(20*a - 34*Sqrt[a]*Sqrt[b] + 15*b)*ArcTanh[(Sqrt[Sqrt
[a] - Sqrt[b]]*Tanh[c + d*x])/a^(1/4)])/(2*(Sqrt[a] - Sqrt[b])^(5/2)) + (3
*a^(3/4)*b^(5/2)*(20*a + 34*Sqrt[a]*Sqrt[b] + 15*b)*ArcTanh[(Sqrt[Sqrt[a]
+ Sqrt[b]]*Tanh[c + d*x])/a^(1/4)])/(2*(Sqrt[a] + Sqrt[b])^(5/2)) - 32*a*b
^2*Coth[c + d*x])/(4*a^2*b) + (b^2*Tanh[c + d*x]*((2*a^2*(9*a - 17*b))/(a
- b)^3 - ((18*a^2 + 15*a*b - 13*b^2)*Tanh[c + d*x]^2)/(a - b)^2))/(4*a*(a
- 2*a*Tanh[c + d*x]^2 + (a - b)*Tanh[c + d*x]^4)))/(8*a^2*b))/d
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 1673
```
Int[(x_)^(m_)*((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^
4)^(p_), x_Symbol] :> With[{f = Coeff[PolynomialRemainder[x^m*(d + e*x^2)^q
, a + b*x^2 + c*x^4, x], x, 0], g = Coeff[PolynomialRemainder[x^m*(d + e*x^
2)^q, a + b*x^2 + c*x^4, x], x, 2]}, Simp[x*(a + b*x^2 + c*x^4)^(p + 1)*((a
*b*g - f*(b^2 - 2*a*c) - c*(b*f - 2*a*g)*x^2)/(2*a*(p + 1)*(b^2 - 4*a*c))),
 x] + Simp[1/(2*a*(p + 1)*(b^2 - 4*a*c))   Int[x^m*(a + b*x^2 + c*x^4)^(p +
 1)*Simp[ExpandToSum[(2*a*(p + 1)*(b^2 - 4*a*c)*PolynomialQuotient[x^m*(d +
 e*x^2)^q, a + b*x^2 + c*x^4, x])/x^m + (b^2*f*(2*p + 3) - 2*a*c*f*(4*p + 5
) - a*b*g)/x^m + c*(4*p + 7)*(b*f - 2*a*g)*x^(2 - m), x], x], x], x]] /; Fr
eeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && IGtQ[q, 1]
&& ILtQ[m/2, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2195
```
Int[(Pq_)*((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_
Symbol] :> Int[ExpandIntegrand[(d*x)^m*Pq*(a + b*x^2 + c*x^4)^p, x], x] /;
FreeQ[{a, b, c, d, m}, x] && PolyQ[Pq, x^2] && IGtQ[p, -2]
```

rule 2198
```
Int[(Pq_)*(x_)^(m_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :>
 With[{Qx = PolynomialQuotient[x^m*Pq, a + b*x^2 + c*x^4, x], d = Coeff[Pol
ynomialRemainder[x^m*Pq, a + b*x^2 + c*x^4, x], x, 0], e = Coeff[Polynomial
Remainder[x^m*Pq, a + b*x^2 + c*x^4, x], x, 2]}, Simp[x*(a + b*x^2 + c*x^4)
^(p + 1)*((a*b*e - d*(b^2 - 2*a*c) - c*(b*d - 2*a*e)*x^2)/(2*a*(p + 1)*(b^2
 - 4*a*c))), x] + Simp[1/(2*a*(p + 1)*(b^2 - 4*a*c))   Int[x^m*(a + b*x^2 +
 c*x^4)^(p + 1)*ExpandToSum[(2*a*(p + 1)*(b^2 - 4*a*c)*Qx)/x^m + (b^2*d*(2*
p + 3) - 2*a*c*d*(4*p + 5) - a*b*e)/x^m + c*(4*p + 7)*(b*d - 2*a*e)*x^(2 -
m), x], x], x]] /; FreeQ[{a, b, c}, x] && PolyQ[Pq, x^2] && GtQ[Expon[Pq, x
^2], 1] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && ILtQ[m/2, 0]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3696
```
Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^4)^(
p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Simp[ff^(m + 1
)/f   Subst[Int[x^m*((a + 2*a*ff^2*x^2 + (a + b)*ff^4*x^4)^p/(1 + ff^2*x^2)
^(m/2 + 2*p + 1)), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f}, x] &
& IntegerQ[m/2] && IntegerQ[p]
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 13.99 (sec) , antiderivative size = 608, normalized size of antiderivative = 1.70

method	result
derivativedivides	$\dfrac{-\frac{\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)}{2a^3}-\frac{1}{2a^3\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)}-\frac{8b\left(-\frac{3a^2(3a-2b)\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)}{64\left(a^2-2ab+b^2\right)}+\frac{\left(45a^2+16ab-34b^2\right)a\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^3}{64a^2-128ab+64b^2}-\frac{a\left(81a^2-28ab-3\right.}{64\left(a^2\right.}\right.}{}}{}$
default	$\dfrac{-\frac{\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)}{2a^3}-\frac{1}{2a^3\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)}-\frac{8b\left(-\frac{3a^2(3a-2b)\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)}{64\left(a^2-2ab+b^2\right)}+\frac{\left(45a^2+16ab-34b^2\right)a\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^3}{64a^2-128ab+64b^2}-\frac{a\left(81a^2-28ab-3\right.}{64\left(a^2\right.}\right.}{}}{}$
risch	Expression too large to display

input

```
int(csch(d*x+c)^2/(a-b*sinh(d*x+c)^4)^3,x,method=_RETURNVERBOSE)
```

output

```
1/d*(-1/2/a^3*tanh(1/2*d*x+1/2*c)-1/2/a^3/tanh(1/2*d*x+1/2*c)-8*b/a^3*((-3
/64*a^2*(3*a-2*b)/(a^2-2*a*b+b^2)*tanh(1/2*d*x+1/2*c)+1/64*(45*a^2+16*a*b-
34*b^2)*a/(a^2-2*a*b+b^2)*tanh(1/2*d*x+1/2*c)^3-1/64*a*(81*a^2-28*a*b-38*b
^2)/(a^2-2*a*b+b^2)*tanh(1/2*d*x+1/2*c)^5+1/64*(45*a^3-50*a^2*b-612*a*b^2+
416*b^3)/(a^2-2*a*b+b^2)*tanh(1/2*d*x+1/2*c)^7+1/64*(45*a^3-50*a^2*b-612*a
*b^2+416*b^3)/(a^2-2*a*b+b^2)*tanh(1/2*d*x+1/2*c)^9-1/64*a*(81*a^2-28*a*b-
38*b^2)/(a^2-2*a*b+b^2)*tanh(1/2*d*x+1/2*c)^11+1/64*(45*a^2+16*a*b-34*b^2)
*a/(a^2-2*a*b+b^2)*tanh(1/2*d*x+1/2*c)^13-3/64*a^2*(3*a-2*b)/(a^2-2*a*b+b^
2)*tanh(1/2*d*x+1/2*c)^15)/(tanh(1/2*d*x+1/2*c)^8*a-4*tanh(1/2*d*x+1/2*c)^
6*a+6*tanh(1/2*d*x+1/2*c)^4*a-16*b*tanh(1/2*d*x+1/2*c)^4-4*tanh(1/2*d*x+1/
2*c)^2*a+a)^2+3/512/(a^2-2*a*b+b^2)*sum((a*(-3*a+2*b)*_R^6+(49*a^2-72*a*b+
30*b^2)*_R^4+(-49*a^2+72*a*b-30*b^2)*_R^2+3*a^2-2*a*b)/(_R^7*a-3*_R^5*a+3*
_R^3*a-8*_R^3*b-_R*a)*ln(tanh(1/2*d*x+1/2*c)-_R),_R=RootOf(a*_Z^8-4*a*_Z^6
+(6*a-16*b)*_Z^4-4*a*_Z^2+a))))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 28429 vs. $2(304) = 608$.

Time = 1.38 (sec) , antiderivative size = 28429, normalized size of antiderivative = 79.63

$$\int \frac{\operatorname{csch}^2(c+dx)}{(a-b\sinh^4(c+dx))^3}\,dx = \text{Too large to display}$$

input
```
integrate(csch(d*x+c)^2/(a-b*sinh(d*x+c)^4)^3,x, algorithm="fricas")
```

output
```
Too large to include
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\operatorname{csch}^2(c+dx)}{(a-b\sinh^4(c+dx))^3}\,dx = \text{Timed out}$$

input
```
integrate(csch(d*x+c)**2/(a-b*sinh(d*x+c)**4)**3,x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{\operatorname{csch}^2(c+dx)}{(a-b\sinh^4(c+dx))^3}\,dx = \int -\frac{\operatorname{csch}(dx+c)^2}{(b\sinh(dx+c)^4-a)^3}\,dx$$

input
```
integrate(csch(d*x+c)^2/(a-b*sinh(d*x+c)^4)^3,x, algorithm="maxima")
```

output
```
1/16*(32*a^2*b^2 - 83*a*b^3 + 45*b^4 + 3*(20*a^2*b^2*e^(16*c) - 33*a*b^3*e
^(16*c) + 15*b^4*e^(16*c))*e^(16*d*x) - 12*(43*a^2*b^2*e^(14*c) - 68*a*b^3
*e^(14*c) + 30*b^4*e^(14*c))*e^(14*d*x) - 4*(400*a^3*b*e^(12*c) - 1137*a^2
*b^2*e^(12*c) + 1031*a*b^3*e^(12*c) - 315*b^4*e^(12*c))*e^(12*d*x) + 12*(5
92*a^3*b*e^(10*c) - 1237*a^2*b^2*e^(10*c) + 886*a*b^3*e^(10*c) - 210*b^4*e
^(10*c))*e^(10*d*x) + 2*(4096*a^4*e^(8*c) - 12192*a^3*b*e^(8*c) + 13634*a^
2*b^2*e^(8*c) - 7113*a*b^3*e^(8*c) + 1575*b^4*e^(8*c))*e^(8*d*x) + 4*(880*
a^3*b*e^(6*c) - 2855*a^2*b^2*e^(6*c) + 2512*a*b^3*e^(6*c) - 630*b^4*e^(6*c
))*e^(6*d*x) - 4*(256*a^3*b*e^(4*c) - 823*a^2*b^2*e^(4*c) + 903*a*b^3*e^(4
*c) - 315*b^4*e^(4*c))*e^(4*d*x) - 12*(19*a^2*b^2*e^(2*c) - 54*a*b^3*e^(2*
c) + 30*b^4*e^(2*c))*e^(2*d*x))/(a^5*b^2*d - 2*a^4*b^3*d + a^3*b^4*d - (a^
5*b^2*d*e^(18*c) - 2*a^4*b^3*d*e^(18*c) + a^3*b^4*d*e^(18*c))*e^(18*d*x) +
 9*(a^5*b^2*d*e^(16*c) - 2*a^4*b^3*d*e^(16*c) + a^3*b^4*d*e^(16*c))*e^(16*
d*x) + 4*(8*a^6*b*d*e^(14*c) - 25*a^5*b^2*d*e^(14*c) + 26*a^4*b^3*d*e^(14*
c) - 9*a^3*b^4*d*e^(14*c))*e^(14*d*x) - 4*(40*a^6*b*d*e^(12*c) - 101*a^5*b
^2*d*e^(12*c) + 82*a^4*b^3*d*e^(12*c) - 21*a^3*b^4*d*e^(12*c))*e^(12*d*x)
- 2*(128*a^7*d*e^(10*c) - 416*a^6*b*d*e^(10*c) + 511*a^5*b^2*d*e^(10*c) -
286*a^4*b^3*d*e^(10*c) + 63*a^3*b^4*d*e^(10*c))*e^(10*d*x) + 2*(128*a^7*d*
e^(8*c) - 416*a^6*b*d*e^(8*c) + 511*a^5*b^2*d*e^(8*c) - 286*a^4*b^3*d*e^(8
*c) + 63*a^3*b^4*d*e^(8*c))*e^(8*d*x) + 4*(40*a^6*b*d*e^(6*c) - 101*a^5...
```

Giac [F]

$$\int \frac{\operatorname{csch}^2(c+dx)}{\left(a-b\sinh^4(c+dx)\right)^3}\,dx=\int -\frac{\operatorname{csch}\left(dx+c\right)^2}{\left(b\sinh\left(dx+c\right)^4-a\right)^3}\,dx$$

input
```
integrate(csch(d*x+c)^2/(a-b*sinh(d*x+c)^4)^3,x, algorithm="giac")
```

output
```
sage0*x
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\operatorname{csch}^2(c+dx)}{\left(a-b\sinh^4(c+dx)\right)^3}\,dx=\int \frac{1}{\sinh\left(c+d\,x\right)^2\,\left(a-b\sinh\left(c+d\,x\right)^4\right)^3}\,dx$$

input
```
int(1/(sinh(c + d*x)^2*(a - b*sinh(c + d*x)^4)^3),x)
```

output
```
int(1/(sinh(c + d*x)^2*(a - b*sinh(c + d*x)^4)^3), x)
```

Reduce [F]

$$\int \frac{\operatorname{csch}^2(c+dx)}{\left(a-b\sinh^4(c+dx)\right)^3}\,dx=\text{too large to display}$$

input
```
int(csch(d*x+c)^2/(a-b*sinh(d*x+c)^4)^3,x)
```

output

```
(96304974431690686464 0*e**(22*c + 18*d*x)*int(e**(4*d*x)/(9732096*e**(28*c
 + 28*d*x)*a**4*b**3 + 9236480*e**(28*c + 28*d*x)*a**3*b**4 + 254720*e**(2
8*c + 28*d*x)*a**2*b**5 + 127920*e**(28*c + 28*d*x)*a*b**6 + 8505*e**(28*c
 + 28*d*x)*b**7 - 136249344*e**(26*c + 26*d*x)*a**4*b**3 - 129310720*e**(2
6*c + 26*d*x)*a**3*b**4 - 3566080*e**(26*c + 26*d*x)*a**2*b**5 - 1790880*e
**(26*c + 26*d*x)*a*b**6 - 119070*e**(26*c + 26*d*x)*b**7 - 467140608*e**(
24*c + 24*d*x)*a**5*b**2 + 442269696*e**(24*c + 24*d*x)*a**4*b**3 + 828293
120*e**(24*c + 24*d*x)*a**3*b**4 + 17039360*e**(24*c + 24*d*x)*a**2*b**5 +
 11232480*e**(24*c + 24*d*x)*a*b**6 + 773955*e**(24*c + 24*d*x)*b**7 + 467
1406080*e**(22*c + 22*d*x)*a**5*b**2 + 891027456*e**(22*c + 22*d*x)*a**4*b
**3 - 3239813120*e**(22*c + 22*d*x)*a**3*b**4 - 31316480*e**(22*c + 22*d*x
)*a**2*b**5 - 42480480*e**(22*c + 22*d*x)*a*b**6 - 3095820*e**(22*c + 22*d
*x)*b**7 + 7474249728*e**(20*c + 20*d*x)*a**6*b - 13927710720*e**(20*c + 2
0*d*x)*a**5*b**2 - 10013343744*e**(20*c + 20*d*x)*a**4*b**3 + 8793763840*e
**(20*c + 20*d*x)*a**3*b**4 - 14800640*e**(20*c + 20*d*x)*a**2*b**5 + 1096
77120*e**(20*c + 20*d*x)*a*b**6 + 8513505*e**(20*c + 20*d*x)*b**7 - 448454
98368*e**(18*c + 18*d*x)*a**6*b + 13495173120*e**(18*c + 18*d*x)*a**5*b**2
 + 32544718848*e**(18*c + 18*d*x)*a**4*b**3 - 17613701120*e**(18*c + 18*d*
x)*a**3*b**4 + 187678720*e**(18*c + 18*d*x)*a**2*b**5 - 207107040*e**(18*c
 + 18*d*x)*a*b**6 - 17027010*e**(18*c + 18*d*x)*b**7 - 39862665216*e**(...
```

3.240 $\int \frac{\cosh^5(x)}{a+a\sinh^2(x)}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 18

$$\int \frac{\cosh^5(x)}{a+a\sinh^2(x)}\,dx=\frac{\sinh(x)}{a}+\frac{\sinh^3(x)}{3a}$$

output
```
sinh(x)/a+1/3*sinh(x)^3/a
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.83

$$\int \frac{\cosh^5(x)}{a+a\sinh^2(x)}\,dx=\frac{\sinh(x)+\frac{\sinh^3(x)}{3}}{a}$$

input
```
Integrate[Cosh[x]^5/(a + a*Sinh[x]^2),x]
```

output
```
(Sinh[x] + Sinh[x]^3/3)/a
```

Rubi [C] (verified)

Result contains complex when optimal does not.

Time = 0.27 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.33, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {3042, 3654, 3042, 3113, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cosh^5(x)}{a\sinh^2(x)+a}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\cos(ix)^5}{a-a\sin(ix)^2}dx$$

$$\Big\downarrow \text{3654}$$

$$\frac{\int \cosh^3(x)dx}{a}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\int \sin\left(ix+\frac{\pi}{2}\right)^3dx}{a}$$

$$\Big\downarrow \text{3113}$$

$$\frac{i\int \left(\sinh^2(x)+1\right)d(-i\sinh(x))}{a}$$

$$\Big\downarrow \text{2009}$$

$$\frac{i\left(-\frac{1}{3}i\sinh^3(x)-i\sinh(x)\right)}{a}$$

input
```
Int[Cosh[x]^5/(a + a*Sinh[x]^2),x]
```

output
```
(I*((-I)*Sinh[x] - (I/3)*Sinh[x]^3))/a
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3113
```
Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Simp[-d^(-1)   Subst[Int[Exp
and[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x]
 && IGtQ[(n - 1)/2, 0]
```

rule 3654
```
Int[(u_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> Simp[
a^p   Int[ActivateTrig[u*cos[e + f*x]^(2*p)], x], x] /; FreeQ[{a, b, e, f,
p}, x] && EqQ[a + b, 0] && IntegerQ[p]
```

Maple [A] (verified)

Time = 132.78 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.78

method	result	size
derivativedivides	$\frac{\frac{\sinh(x)^3}{3}+\sinh(x)}{a}$	14
default	$\frac{\frac{\sinh(x)^3}{3}+\sinh(x)}{a}$	14
risch	$\frac{e^{3x}}{24a}+\frac{3\,e^{x}}{8a}-\frac{3\,e^{-x}}{8a}-\frac{e^{-3x}}{24a}$	36

input
```
int(cosh(x)^5/(a+a*sinh(x)^2),x,method=_RETURNVERBOSE)
```

output
```
1/a*(1/3*sinh(x)^3+sinh(x))
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.11

$$\int \frac{\cosh^5(x)}{a+a\sinh^2(x)}\,dx = \frac{\sinh{(x)}^3 + 3\left(\cosh{(x)}^2+3\right)\sinh{(x)}}{12\,a}$$

input
```
integrate(cosh(x)^5/(a+a*sinh(x)^2),x, algorithm="fricas")
```

output
```
1/12*(sinh(x)^3 + 3*(cosh(x)^2 + 3)*sinh(x))/a
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 124 vs. 2(12) = 24.

Time = 2.28 (sec) , antiderivative size = 124, normalized size of antiderivative = 6.89

$$\int \frac{\cosh^5(x)}{a+a\sinh^2(x)}\,dx = -\frac{6\tanh^5\left(\frac{x}{2}\right)}{3a\tanh^6\left(\frac{x}{2}\right)-9a\tanh^4\left(\frac{x}{2}\right)+9a\tanh^2\left(\frac{x}{2}\right)-3a} + \frac{4\tanh^3\left(\frac{x}{2}\right)}{3a\tanh^6\left(\frac{x}{2}\right)-9a\tanh^4\left(\frac{x}{2}\right)+9a\tanh^2\left(\frac{x}{2}\right)-3a} - \frac{6\tanh\left(\frac{x}{2}\right)}{3a\tanh^6\left(\frac{x}{2}\right)-9a\tanh^4\left(\frac{x}{2}\right)+9a\tanh^2\left(\frac{x}{2}\right)-3a}$$

input
```
integrate(cosh(x)**5/(a+a*sinh(x)**2),x)
```

output
```
-6*tanh(x/2)**5/(3*a*tanh(x/2)**6 - 9*a*tanh(x/2)**4 + 9*a*tanh(x/2)**2 -
3*a) + 4*tanh(x/2)**3/(3*a*tanh(x/2)**6 - 9*a*tanh(x/2)**4 + 9*a*tanh(x/2)
**2 - 3*a) - 6*tanh(x/2)/(3*a*tanh(x/2)**6 - 9*a*tanh(x/2)**4 + 9*a*tanh(x
/2)**2 - 3*a)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 34 vs. $2(16) = 32$.

Time = 0.03 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.89

$$\int \frac{\cosh^5(x)}{a + a\sinh^2(x)}\,dx = \frac{\left(9\,e^{(-2\,x)} + 1\right)e^{(3\,x)}}{24\,a} - \frac{9\,e^{(-x)} + e^{(-3\,x)}}{24\,a}$$

input
```
integrate(cosh(x)^5/(a+a*sinh(x)^2),x, algorithm="maxima")
```

output
```
1/24*(9*e^(-2*x) + 1)*e^(3*x)/a - 1/24*(9*e^(-x) + e^(-3*x))/a
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.61

$$\int \frac{\cosh^5(x)}{a + a\sinh^2(x)}\,dx = -\frac{\left(9\,e^{(2\,x)} + 1\right)e^{(-3\,x)} - e^{(3\,x)} - 9\,e^{x}}{24\,a}$$

input
```
integrate(cosh(x)^5/(a+a*sinh(x)^2),x, algorithm="giac")
```

output
```
-1/24*((9*e^(2*x) + 1)*e^(-3*x) - e^(3*x) - 9*e^x)/a
```

Mupad [B] (verification not implemented)

Time = 1.95 (sec) , antiderivative size = 35, normalized size of antiderivative = 1.94

$$\int \frac{\cosh^5(x)}{a + a\sinh^2(x)}\,dx = \frac{e^{3\,x}}{24\,a} - \frac{e^{-3\,x}}{24\,a} - \frac{3\,e^{-x}}{8\,a} + \frac{3\,e^{x}}{8\,a}$$

input
```
int(cosh(x)^5/(a + a*sinh(x)^2),x)
```

output
```
exp(3*x)/(24*a) - exp(-3*x)/(24*a) - (3*exp(-x))/(8*a) + (3*exp(x))/(8*a)
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 33, normalized size of antiderivative = 1.83

$$\int \frac{\cosh^5(x)}{a + a\sinh^2(x)}\,dx = \frac{e^{6x} + 9e^{4x} - 9e^{2x} - 1}{24e^{3x}a}$$

input
```
int(cosh(x)^5/(a+a*sinh(x)^2),x)
```

output
```
(e**(6*x) + 9*e**(4*x) - 9*e**(2*x) - 1)/(24*e**(3*x)*a)
```

3.241 $\int \frac{\cosh^4(x)}{a+a\sinh^2(x)} dx$

Optimal result

Integrand size = 15, antiderivative size = 20

$$\int \frac{\cosh^4(x)}{a+a\sinh^2(x)} dx = \frac{x}{2a} + \frac{\cosh(x)\sinh(x)}{2a}$$

output
```
1/2*x/a+1/2*cosh(x)*sinh(x)/a
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.90

$$\int \frac{\cosh^4(x)}{a+a\sinh^2(x)} dx = \frac{\frac{x}{2} + \frac{1}{4}\sinh(2x)}{a}$$

input
```
Integrate[Cosh[x]^4/(a + a*Sinh[x]^2),x]
```

output
```
(x/2 + Sinh[2*x]/4)/a
```

Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.90, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {3042, 3654, 3042, 3115, 24}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cosh^4(x)}{a\sinh^2(x)+a}dx$$
$$\Big\downarrow \text{3042}$$
$$\int \frac{\cos(ix)^4}{a-a\sin(ix)^2}dx$$
$$\Big\downarrow \text{3654}$$
$$\frac{\int \cosh^2(x)dx}{a}$$
$$\Big\downarrow \text{3042}$$
$$\frac{\int \sin\left(ix+\frac{\pi}{2}\right)^2dx}{a}$$
$$\Big\downarrow \text{3115}$$
$$\frac{\frac{\int 1dx}{2}+\frac{1}{2}\sinh(x)\cosh(x)}{a}$$
$$\Big\downarrow \text{24}$$
$$\frac{\frac{x}{2}+\frac{1}{2}\sinh(x)\cosh(x)}{a}$$

input
```
Int[Cosh[x]^4/(a + a*Sinh[x]^2),x]
```

output
```
(x/2 + (Cosh[x]*Sinh[x])/2)/a
```

Defintions of rubi rules used

rule 24
```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3115
```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Simp[b^2*((n - 1)/n)   Int[(b*Sin
[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[
2*n]
```

rule 3654
```
Int[(u_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> Simp[
a^p   Int[ActivateTrig[u*cos[e + f*x]^(2*p)], x], x] /; FreeQ[{a, b, e, f,
p}, x] && EqQ[a + b, 0] && IntegerQ[p]
```

Maple [A] (verified)

Time = 52.29 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.30

method	result	size
risch	$\frac{x}{2a}+\frac{e^{2x}}{8a}-\frac{e^{-2x}}{8a}$	26
default	$\frac{-\frac{1}{2\left(1+\tanh\left(\frac{x}{2}\right)\right)^{2}}+\frac{2}{4+4\tanh\left(\frac{x}{2}\right)}+\frac{\ln\left(1+\tanh\left(\frac{x}{2}\right)\right)}{2}+\frac{1}{2\left(\tanh\left(\frac{x}{2}\right)-1\right)^{2}}+\frac{2}{4\tanh\left(\frac{x}{2}\right)-4}-\frac{\ln\left(\tanh\left(\frac{x}{2}\right)-1\right)}{2}}{a}$	65

input
```
int(cosh(x)^4/(a+a*sinh(x)^2),x,method=_RETURNVERBOSE)
```

output
```
1/2*x/a+1/8/a*exp(2*x)-1/8/a*exp(-2*x)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.60

$$\int \frac{\cosh^4(x)}{a + a\sinh^2(x)}\,dx = \frac{\cosh(x)\sinh(x) + x}{2\,a}$$

input
```
integrate(cosh(x)^4/(a+a*sinh(x)^2),x, algorithm="fricas")
```

output
```
1/2*(cosh(x)*sinh(x) + x)/a
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 153 vs. 2(14) = 28.

Time = 1.33 (sec) , antiderivative size = 153, normalized size of antiderivative = 7.65

$$\int \frac{\cosh^4(x)}{a + a\sinh^2(x)}\,dx = \frac{x\tanh^4\left(\frac{x}{2}\right)}{2a\tanh^4\left(\frac{x}{2}\right) - 4a\tanh^2\left(\frac{x}{2}\right) + 2a} - \frac{2x\tanh^2\left(\frac{x}{2}\right)}{2a\tanh^4\left(\frac{x}{2}\right) - 4a\tanh^2\left(\frac{x}{2}\right) + 2a} + \frac{x}{2a\tanh^4\left(\frac{x}{2}\right) - 4a\tanh^2\left(\frac{x}{2}\right) + 2a} + \frac{2\tanh^3\left(\frac{x}{2}\right)}{2a\tanh^4\left(\frac{x}{2}\right) - 4a\tanh^2\left(\frac{x}{2}\right) + 2a} + \frac{2\tanh\left(\frac{x}{2}\right)}{2a\tanh^4\left(\frac{x}{2}\right) - 4a\tanh^2\left(\frac{x}{2}\right) + 2a}$$

input
```
integrate(cosh(x)**4/(a+a*sinh(x)**2),x)
```

output
```
x*tanh(x/2)**4/(2*a*tanh(x/2)**4 - 4*a*tanh(x/2)**2 + 2*a) - 2*x*tanh(x/2)
**2/(2*a*tanh(x/2)**4 - 4*a*tanh(x/2)**2 + 2*a) + x/(2*a*tanh(x/2)**4 - 4*
a*tanh(x/2)**2 + 2*a) + 2*tanh(x/2)**3/(2*a*tanh(x/2)**4 - 4*a*tanh(x/2)**
2 + 2*a) + 2*tanh(x/2)/(2*a*tanh(x/2)**4 - 4*a*tanh(x/2)**2 + 2*a)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.25

$$\int \frac{\cosh^4(x)}{a + a\sinh^2(x)}\,dx = \frac{x}{2\,a} + \frac{e^{(2\,x)}}{8\,a} - \frac{e^{(-2\,x)}}{8\,a}$$

input
```
integrate(cosh(x)^4/(a+a*sinh(x)^2),x, algorithm="maxima")
```

output
```
1/2*x/a + 1/8*e^(2*x)/a - 1/8*e^(-2*x)/a
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.40

$$\int \frac{\cosh^4(x)}{a + a\sinh^2(x)}\,dx = -\frac{\left(2\,e^{(2\,x)} + 1\right)e^{(-2\,x)} - 4\,x - e^{(2\,x)}}{8\,a}$$

input
```
integrate(cosh(x)^4/(a+a*sinh(x)^2),x, algorithm="giac")
```

output
```
-1/8*((2*e^(2*x) + 1)*e^(-2*x) - 4*x - e^(2*x))/a
```

Mupad [B] (verification not implemented)

Time = 2.02 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.25

$$\int \frac{\cosh^4(x)}{a + a\sinh^2(x)}\,dx = \frac{e^{2\,x}}{8\,a} - \frac{e^{-2\,x}}{8\,a} + \frac{x}{2\,a}$$

input
```
int(cosh(x)^4/(a + a*sinh(x)^2),x)
```

output
```
exp(2*x)/(8*a) - exp(-2*x)/(8*a) + x/(2*a)
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.35

$$\int \frac{\cosh^4(x)}{a + a\sinh^2(x)}\, dx = \frac{e^{4x} + 4e^{2x}x - 1}{8e^{2x}a}$$

input
```
int(cosh(x)^4/(a+a*sinh(x)^2),x)
```

output
```
(e**(4*x) + 4*e**(2*x)*x - 1)/(8*e**(2*x)*a)
```

3.242 $\int \frac{\cosh^3(x)}{a+a\sinh^2(x)}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 6

$$\int \frac{\cosh^3(x)}{a+a\sinh^2(x)}\,dx=\frac{\sinh(x)}{a}$$

output
```
sinh(x)/a
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 6, normalized size of antiderivative = 1.00

$$\int \frac{\cosh^3(x)}{a+a\sinh^2(x)}\,dx=\frac{\sinh(x)}{a}$$

input
```
Integrate[Cosh[x]^3/(a + a*Sinh[x]^2),x]
```

output
```
Sinh[x]/a
```

Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 6, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.267$, Rules used = {3042, 3654, 3042, 3117}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cosh^3(x)}{a\sinh^2(x)+a}dx$$
$$\Big\downarrow \text{3042}$$
$$\int \frac{\cos(ix)^3}{a-a\sin(ix)^2}dx$$
$$\Big\downarrow \text{3654}$$
$$\frac{\int \cosh(x)dx}{a}$$
$$\Big\downarrow \text{3042}$$
$$\frac{\int \sin\left(ix+\frac{\pi}{2}\right)dx}{a}$$
$$\Big\downarrow \text{3117}$$
$$\frac{\sinh(x)}{a}$$

input

```
Int[Cosh[x]^3/(a + a*Sinh[x]^2),x]
```

output

```
Sinh[x]/a
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3117
```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

rule 3654
```
Int[(u_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> Simp[
a^p   Int[ActivateTrig[u*cos[e + f*x]^(2*p)], x], x] /; FreeQ[{a, b, e, f,
p}, x] && EqQ[a + b, 0] && IntegerQ[p]
```

Maple [A] (verified)

Time = 18.70 (sec) , antiderivative size = 7, normalized size of antiderivative = 1.17

method	result	size
derivativedivides	$\frac{\sinh(x)}{a}$	7
default	$\frac{\sinh(x)}{a}$	7
risch	$\frac{e^{x}}{2a}-\frac{e^{-x}}{2a}$	18

input
```
int(cosh(x)^3/(a+a*sinh(x)^2),x,method=_RETURNVERBOSE)
```

output
```
sinh(x)/a
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 6, normalized size of antiderivative = 1.00

$$\int \frac{\cosh^3(x)}{a+a\sinh^2(x)}\,dx=\frac{\sinh(x)}{a}$$

input
```
integrate(cosh(x)^3/(a+a*sinh(x)^2),x, algorithm="fricas")
```

output
```
sinh(x)/a
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 17 vs. $2(3) = 6$.

Time = 0.74 (sec) , antiderivative size = 17, normalized size of antiderivative = 2.83

$$\int \frac{\cosh^3(x)}{a + a\sinh^2(x)}\,dx = -\frac{2\tanh\left(\frac{x}{2}\right)}{a\tanh^2\left(\frac{x}{2}\right) - a}$$

input
```
integrate(cosh(x)**3/(a+a*sinh(x)**2),x)
```

output
```
-2*tanh(x/2)/(a*tanh(x/2)**2 - a)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 17 vs. $2(6) = 12$.

Time = 0.05 (sec) , antiderivative size = 17, normalized size of antiderivative = 2.83

$$\int \frac{\cosh^3(x)}{a + a\sinh^2(x)}\,dx = -\frac{e^{(-x)}}{2a} + \frac{e^x}{2a}$$

input
```
integrate(cosh(x)^3/(a+a*sinh(x)^2),x, algorithm="maxima")
```

output
```
-1/2*e^(-x)/a + 1/2*e^x/a
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 14 vs. $2(6) = 12$.

Time = 0.14 (sec) , antiderivative size = 14, normalized size of antiderivative = 2.33

$$\int \frac{\cosh^3(x)}{a + a\sinh^2(x)}\,dx = -\frac{e^{(-x)} - e^x}{2\,a}$$

input
```
integrate(cosh(x)^3/(a+a*sinh(x)^2),x, algorithm="giac")
```

output
```
-1/2*(e^(-x) - e^x)/a
```

Mupad [B] (verification not implemented)

Time = 1.97 (sec) , antiderivative size = 6, normalized size of antiderivative = 1.00

$$\int \frac{\cosh^3(x)}{a + a\sinh^2(x)}\,dx = \frac{\sinh(x)}{a}$$

input
```
int(cosh(x)^3/(a + a*sinh(x)^2),x)
```

output
```
sinh(x)/a
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 17, normalized size of antiderivative = 2.83

$$\int \frac{\cosh^3(x)}{a + a\sinh^2(x)}\,dx = \frac{e^{2x} - 1}{2e^x a}$$

input
```
int(cosh(x)^3/(a+a*sinh(x)^2),x)
```

output
```
(e**(2*x) - 1)/(2*e**x*a)
```

3.243 $\int \frac{\cosh^2(x)}{a+a\sinh^2(x)}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 5

$$\int \frac{\cosh^2(x)}{a+a\sinh^2(x)}\,dx = \frac{x}{a}$$

output
```
x/a
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 5, normalized size of antiderivative = 1.00

$$\int \frac{\cosh^2(x)}{a+a\sinh^2(x)}\,dx = \frac{x}{a}$$

input
```
Integrate[Cosh[x]^2/(a + a*Sinh[x]^2),x]
```

output
```
x/a
```

Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 5, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {3042, 3654, 24}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cosh^2(x)}{a \sinh^2(x)+a} \, dx$$
$$\Big\downarrow \text{3042}$$
$$\int \frac{\cos(ix)^2}{a-a\sin(ix)^2}dx$$
$$\Big\downarrow \text{3654}$$
$$\frac{\int 1dx}{a}$$
$$\Big\downarrow \text{24}$$
$$\frac{x}{a}$$

input
```
Int[Cosh[x]^2/(a + a*Sinh[x]^2),x]
```

output
```
x/a
```

Defintions of rubi rules used

rule 24
```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3654
```
Int[(u_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> Simp[
a^p   Int[ActivateTrig[u*cos[e + f*x]^(2*p)], x], x] /; FreeQ[{a, b, e, f,
p}, x] && EqQ[a + b, 0] && IntegerQ[p]
```

Maple [A] (verified)

Time = 4.56 (sec) , antiderivative size = 6, normalized size of antiderivative = 1.20

method	result	size
risch	$\frac{x}{a}$	6
default	$\frac{2\,\mathrm{arctanh}\left(\tanh\left(\frac{x}{2}\right)\right)}{a}$	11
orering	$\frac{x\cosh(x)^2}{a+a\sinh(x)^2}$	17

input
```
int(cosh(x)^2/(a+a*sinh(x)^2),x,method=_RETURNVERBOSE)
```

output
```
x/a
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 5, normalized size of antiderivative = 1.00

$$\int \frac{\cosh^2(x)}{a+a\sinh^2(x)}\,dx=\frac{x}{a}$$

input
```
integrate(cosh(x)^2/(a+a*sinh(x)^2),x, algorithm="fricas")
```

output
```
x/a
```

Sympy [A] (verification not implemented)

Time = 0.42 (sec) , antiderivative size = 2, normalized size of antiderivative = 0.40

$$\int \frac{\cosh^2(x)}{a + a\sinh^2(x)}\,dx = \frac{x}{a}$$

input
```
integrate(cosh(x)**2/(a+a*sinh(x)**2),x)
```

output
```
x/a
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 5, normalized size of antiderivative = 1.00

$$\int \frac{\cosh^2(x)}{a + a\sinh^2(x)}\,dx = \frac{x}{a}$$

input
```
integrate(cosh(x)^2/(a+a*sinh(x)^2),x, algorithm="maxima")
```

output
```
x/a
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 5, normalized size of antiderivative = 1.00

$$\int \frac{\cosh^2(x)}{a + a\sinh^2(x)}\,dx = \frac{x}{a}$$

input
```
integrate(cosh(x)^2/(a+a*sinh(x)^2),x, algorithm="giac")
```

output
```
x/a
```

Mupad [B] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 5, normalized size of antiderivative = 1.00

$$\int \frac{\cosh^2(x)}{a + a\sinh^2(x)}\,dx = \frac{x}{a}$$

input
```
int(cosh(x)^2/(a + a*sinh(x)^2),x)
```

output
```
x/a
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 5, normalized size of antiderivative = 1.00

$$\int \frac{\cosh^2(x)}{a + a\sinh^2(x)}\,dx = \frac{x}{a}$$

input
```
int(cosh(x)^2/(a+a*sinh(x)^2),x)
```

output
```
x/a
```

3.244 $\int \frac{\cosh(x)}{a+a\sinh^2(x)}\,dx$

Optimal result

Integrand size = 13, antiderivative size = 7

$$\int \frac{\cosh(x)}{a+a\sinh^2(x)}\,dx = \frac{\arctan(\sinh(x))}{a}$$

output
```
arctan(sinh(x))/a
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 8, normalized size of antiderivative = 1.14

$$\int \frac{\cosh(x)}{a+a\sinh^2(x)}\,dx = -\frac{\cot^{-1}(\sinh(x))}{a}$$

input
```
Integrate[Cosh[x]/(a + a*Sinh[x]^2),x]
```

output
```
-(ArcCot[Sinh[x]]/a)
```

Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 7, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.308$, Rules used = {3042, 3654, 3042, 4257}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cosh(x)}{a\sinh^2(x)+a}dx$$
$$\Big\downarrow \; 3042$$
$$\int \frac{\cos(ix)}{a-a\sin(ix)^2}dx$$
$$\Big\downarrow \; 3654$$
$$\frac{\int \operatorname{sech}(x)dx}{a}$$
$$\Big\downarrow \; 3042$$
$$\frac{\int \csc\left(ix+\frac{\pi}{2}\right)dx}{a}$$
$$\Big\downarrow \; 4257$$
$$\frac{\arctan(\sinh(x))}{a}$$

input
```
Int[Cosh[x]/(a + a*Sinh[x]^2),x]
```

output
```
ArcTan[Sinh[x]]/a
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3654
```
Int[(u_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> Simp[
a^p   Int[ActivateTrig[u*cos[e + f*x]^(2*p)], x], x] /; FreeQ[{a, b, e, f,
p}, x] && EqQ[a + b, 0] && IntegerQ[p]
```

rule 4257
```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Maple [A] (verified)

Time = 4.56 (sec) , antiderivative size = 8, normalized size of antiderivative = 1.14

method	result	size
derivativedivides	$\frac{\arctan(\sinh(x))}{a}$	8
default	$\frac{\arctan(\sinh(x))}{a}$	8
risch	$\frac{i\ln(e^{x}+i)}{a}-\frac{i\ln(e^{x}-i)}{a}$	26

input
```
int(cosh(x)/(a+a*sinh(x)^2),x,method=_RETURNVERBOSE)
```

output
```
arctan(sinh(x))/a
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 11, normalized size of antiderivative = 1.57

$$\int \frac{\cosh(x)}{a+a\sinh^2(x)}\,dx=\frac{2\,\arctan\left(\cosh\left(x\right)+\sinh\left(x\right)\right)}{a}$$

input
```
integrate(cosh(x)/(a+a*sinh(x)^2),x, algorithm="fricas")
```

output
```
2*arctan(cosh(x) + sinh(x))/a
```

Sympy [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 5, normalized size of antiderivative = 0.71

$$\int \frac{\cosh(x)}{a + a\sinh^2(x)}\,dx = \frac{\operatorname{atan}(\sinh(x))}{a}$$

input
```
integrate(cosh(x)/(a+a*sinh(x)**2),x)
```

output
```
atan(sinh(x))/a
```

Maxima [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 10, normalized size of antiderivative = 1.43

$$\int \frac{\cosh(x)}{a + a\sinh^2(x)}\,dx = -\frac{2\arctan\left(e^{(-x)}\right)}{a}$$

input
```
integrate(cosh(x)/(a+a*sinh(x)^2),x, algorithm="maxima")
```

output
```
-2*arctan(e^(-x))/a
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 8, normalized size of antiderivative = 1.14

$$\int \frac{\cosh(x)}{a + a\sinh^2(x)}\,dx = \frac{2\arctan(e^x)}{a}$$

input
```
integrate(cosh(x)/(a+a*sinh(x)^2),x, algorithm="giac")
```

output
```
2*arctan(e^x)/a
```

Mupad [B] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 7, normalized size of antiderivative = 1.00

$$\int \frac{\cosh(x)}{a + a\sinh^2(x)}\,dx = \frac{\text{atan}(\sinh(x))}{a}$$

input
```
int(cosh(x)/(a + a*sinh(x)^2),x)
```

output
```
atan(sinh(x))/a
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 7, normalized size of antiderivative = 1.00

$$\int \frac{\cosh(x)}{a + a\sinh^2(x)}\,dx = \frac{atan(\sinh(x))}{a}$$

input
```
int(cosh(x)/(a+a*sinh(x)^2),x)
```

output
```
atan(sinh(x))/a
```

3.245 $\int \frac{\text{sech}(x)}{a+a\sinh^2(x)}\,dx$

Optimal result

Integrand size = 13, antiderivative size = 22

$$\int \frac{\text{sech}(x)}{a+a\sinh^2(x)}\,dx = \frac{\arctan(\sinh(x))}{2a} + \frac{\text{sech}(x)\tanh(x)}{2a}$$

output
```
1/2*arctan(sinh(x))/a+1/2*sech(x)*tanh(x)/a
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.91

$$\int \frac{\text{sech}(x)}{a+a\sinh^2(x)}\,dx = \frac{\frac{1}{2}\arctan(\sinh(x)) + \frac{1}{2}\text{sech}(x)\tanh(x)}{a}$$

input
```
Integrate[Sech[x]/(a + a*Sinh[x]^2),x]
```

output
```
(ArcTan[Sinh[x]]/2 + (Sech[x]*Tanh[x])/2)/a
```

Rubi [A] (verified)

Time = 0.34 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.91, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.462, Rules used = {3042, 3654, 3042, 4255, 3042, 4257}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\text{sech}(x)}{a\sinh^2(x)+a}dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \frac{1}{\cos(ix)\left(a-a\sin(ix)^2\right)}dx$$

$$\Big\downarrow \text{ 3654}$$

$$\frac{\int \text{sech}^3(x)dx}{a}$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{\int \csc\left(ix+\frac{\pi}{2}\right)^3dx}{a}$$

$$\Big\downarrow \text{ 4255}$$

$$\frac{\frac{\int \text{sech}(x)dx}{2}+\frac{1}{2}\tanh(x)\text{sech}(x)}{a}$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{\frac{1}{2}\tanh(x)\text{sech}(x)+\frac{1}{2}\int \csc\left(ix+\frac{\pi}{2}\right)dx}{a}$$

$$\Big\downarrow \text{ 4257}$$

$$\frac{\frac{1}{2}\arctan(\sinh(x))+\frac{1}{2}\tanh(x)\text{sech}(x)}{a}$$

input
```
Int[Sech[x]/(a + a*Sinh[x]^2),x]
```

output
```
(ArcTan[Sinh[x]]/2 + (Sech[x]*Tanh[x])/2)/a
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3654
```
Int[(u_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> Simp[
a^p   Int[ActivateTrig[u*cos[e + f*x]^(2*p)], x], x] /; FreeQ[{a, b, e, f,
p}, x] && EqQ[a + b, 0] && IntegerQ[p]
```

rule 4255
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Csc[c + d*x])^(n - 1)/(d*(n - 1))), x] + Simp[b^2*((n - 2)/(n - 1))
  Int[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
&& IntegerQ[2*n]
```

rule 4257
```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 39 vs. $2(18) = 36$.

Time = 21.14 (sec) , antiderivative size = 40, normalized size of antiderivative = 1.82

method	result	size
default	$\frac{\frac{2\left(-\frac{\tanh\left(\frac{x}{2}\right)^{3}}{2}+\frac{\tanh\left(\frac{x}{2}\right)}{2}\right)}{\left(\tanh\left(\frac{x}{2}\right)^{2}+1\right)^{2}}+\arctan\left(\tanh\left(\frac{x}{2}\right)\right)}{a}$	40
risch	$\frac{e^{x}\left(e^{2x}-1\right)}{\left(e^{2x}+1\right)^{2}a}+\frac{i\ln\left(e^{x}+i\right)}{2a}-\frac{i\ln\left(e^{x}-i\right)}{2a}$	46

input
```
int(sech(x)/(a+a*sinh(x)^2),x,method=_RETURNVERBOSE)
```

output
```
2/a*((-1/2*tanh(1/2*x)^3+1/2*tanh(1/2*x))/(tanh(1/2*x)^2+1)^2+1/2*arctan(t
anh(1/2*x)))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 151 vs. $2(18) = 36$.

Time = 0.09 (sec) , antiderivative size = 151, normalized size of antiderivative = 6.86

$$\int \frac{\operatorname{sech}(x)}{a + a\sinh^2(x)}\,dx$$
$$= \frac{\cosh(x)^3 + 3\cosh(x)\sinh(x)^2 + \sinh(x)^3 + \left(\cosh(x)^4 + 4\cosh(x)\sinh(x)^3 + \sinh(x)^4 + 2\left(3\cosh(x\right.\right.}{a\cosh(x)^4 + 4a\cosh(x)\sinh(x)^3 + a\sinh(x)^4 + 2}$$

input
```
integrate(sech(x)/(a+a*sinh(x)^2),x, algorithm="fricas")
```

output
```
(cosh(x)^3 + 3*cosh(x)*sinh(x)^2 + sinh(x)^3 + (cosh(x)^4 + 4*cosh(x)*sinh
(x)^3 + sinh(x)^4 + 2*(3*cosh(x)^2 + 1)*sinh(x)^2 + 2*cosh(x)^2 + 4*(cosh(
x)^3 + cosh(x))*sinh(x) + 1)*arctan(cosh(x) + sinh(x)) + (3*cosh(x)^2 - 1)
*sinh(x) - cosh(x))/(a*cosh(x)^4 + 4*a*cosh(x)*sinh(x)^3 + a*sinh(x)^4 + 2
*a*cosh(x)^2 + 2*(3*a*cosh(x)^2 + a)*sinh(x)^2 + 4*(a*cosh(x)^3 + a*cosh(x
))*sinh(x) + a)
```

Sympy [F]

$$\int \frac{\operatorname{sech}(x)}{a + a\sinh^2(x)}\,dx = \frac{\int \frac{\operatorname{sech}(x)}{\sinh^2(x)+1}\,dx}{a}$$

input
```
integrate(sech(x)/(a+a*sinh(x)**2),x)
```

output
```
Integral(sech(x)/(sinh(x)**2 + 1), x)/a
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 40 vs. $2(18) = 36$.

Time = 0.15 (sec) , antiderivative size = 40, normalized size of antiderivative = 1.82

$$\int \frac{\operatorname{sech}(x)}{a + a\sinh^2(x)}\,dx = \frac{e^{(-x)} - e^{(-3x)}}{2\,ae^{(-2x)} + ae^{(-4x)} + a} - \frac{\arctan\left(e^{(-x)}\right)}{a}$$

input
```
integrate(sech(x)/(a+a*sinh(x)^2),x, algorithm="maxima")
```

output
```
(e^(-x) - e^(-3*x))/(2*a*e^(-2*x) + a*e^(-4*x) + a) - arctan(e^(-x))/a
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 52 vs. $2(18) = 36$.

Time = 0.12 (sec) , antiderivative size = 52, normalized size of antiderivative = 2.36

$$\int \frac{\operatorname{sech}(x)}{a + a\sinh^2(x)}\,dx = \frac{\pi + 2\arctan\left(\frac{1}{2}\left(e^{(2x)} - 1\right)e^{(-x)}\right)}{4\,a} - \frac{e^{(-x)} - e^{x}}{\left(\left(e^{(-x)} - e^{x}\right)^2 + 4\right)a}$$

input
```
integrate(sech(x)/(a+a*sinh(x)^2),x, algorithm="giac")
```

output
```
1/4*(pi + 2*arctan(1/2*(e^(2*x) - 1)*e^(-x)))/a - (e^(-x) - e^x)/(((e^(-x)
 - e^x)^2 + 4)*a)
```

Mupad [B] (verification not implemented)

Time = 1.99 (sec) , antiderivative size = 54, normalized size of antiderivative = 2.45

$$\int \frac{\operatorname{sech}(x)}{a + a\sinh^2(x)}\,dx = \frac{\operatorname{atan}\left(\frac{e^x\sqrt{a^2}}{a}\right)}{\sqrt{a^2}} - \frac{2\,e^x}{a\,\left(2\,e^{2x} + e^{4x} + 1\right)} + \frac{e^x}{a\,\left(e^{2x} + 1\right)}$$

input
```
int(1/(cosh(x)*(a + a*sinh(x)^2)),x)
```

output
```
atan((exp(x)*(a^2)^(1/2))/a)/(a^2)^(1/2) - (2*exp(x))/(a*(2*exp(2*x) + exp
(4*x) + 1)) + exp(x)/(a*(exp(2*x) + 1))
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 56, normalized size of antiderivative = 2.55

$$\int \frac{\operatorname{sech}(x)}{a+a\sinh^2(x)}\,dx = \frac{e^{4x}atan(e^x)+2e^{2x}atan(e^x)+atan(e^x)+e^{3x}-e^x}{a\,(e^{4x}+2e^{2x}+1)}$$

input
```
int(sech(x)/(a+a*sinh(x)^2),x)
```

output
```
(e**(4*x)*atan(e**x) + 2*e**(2*x)*atan(e**x) + atan(e**x) + e**(3*x) - e**
x)/(a*(e**(4*x) + 2*e**(2*x) + 1))
```

3.246 $\int \frac{\operatorname{sech}^3(x)}{a+a\sinh^2(x)}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 35

$$\int \frac{\operatorname{sech}^3(x)}{a+a\sinh^2(x)}\,dx=\frac{3\arctan(\sinh(x))}{8a}+\frac{3\operatorname{sech}(x)\tanh(x)}{8a}+\frac{\operatorname{sech}^3(x)\tanh(x)}{4a}$$

output
```
3/8*arctan(sinh(x))/a+3/8*sech(x)*tanh(x)/a+1/4*sech(x)^3*tanh(x)/a
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 30, normalized size of antiderivative = 0.86

$$\int \frac{\operatorname{sech}^3(x)}{a+a\sinh^2(x)}\,dx=\frac{\frac{3}{8}\arctan(\sinh(x))+\frac{3}{8}\operatorname{sech}(x)\tanh(x)+\frac{1}{4}\operatorname{sech}^3(x)\tanh(x)}{a}$$

input
```
Integrate[Sech[x]^3/(a + a*Sinh[x]^2),x]
```

output
```
((3*ArcTan[Sinh[x]])/8 + (3*Sech[x]*Tanh[x])/8 + (Sech[x]^3*Tanh[x])/4)/a
```

Rubi [A] (verified)

Time = 0.44 (sec) , antiderivative size = 35, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.533$, Rules used = {3042, 3654, 3042, 4255, 3042, 4255, 3042, 4257}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\text{sech}^3(x)}{a\sinh^2(x)+a}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{\cos(ix)^3\left(a-a\sin(ix)^2\right)}dx$$

$$\Big\downarrow \text{3654}$$

$$\frac{\int \text{sech}^5(x)dx}{a}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\int \csc\left(ix+\frac{\pi}{2}\right)^5dx}{a}$$

$$\Big\downarrow \text{4255}$$

$$\frac{\frac{3}{4}\int \text{sech}^3(x)dx+\frac{1}{4}\tanh(x)\text{sech}^3(x)}{a}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\frac{1}{4}\tanh(x)\text{sech}^3(x)+\frac{3}{4}\int \csc\left(ix+\frac{\pi}{2}\right)^3dx}{a}$$

$$\Big\downarrow \text{4255}$$

$$\frac{\frac{3}{4}\left(\frac{\int \text{sech}(x)dx}{2}+\frac{1}{2}\tanh(x)\text{sech}(x)\right)+\frac{1}{4}\tanh(x)\text{sech}^3(x)}{a}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\frac{1}{4}\tanh(x)\text{sech}^3(x)+\frac{3}{4}\left(\frac{1}{2}\tanh(x)\text{sech}(x)+\frac{1}{2}\int \csc\left(ix+\frac{\pi}{2}\right)dx\right)}{a}$$

$$\Big\downarrow \text{4257}$$

$$\frac{\frac{3}{4}\left(\frac{1}{2}\arctan(\sinh(x)) + \frac{1}{2}\tanh(x)\text{sech}(x)\right) + \frac{1}{4}\tanh(x)\text{sech}^3(x)}{a}$$

input
```
Int[Sech[x]^3/(a + a*Sinh[x]^2),x]
```

output
```
((Sech[x]^3*Tanh[x])/4 + (3*(ArcTan[Sinh[x]]/2 + (Sech[x]*Tanh[x])/2))/4)/
a
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3654
```
Int[(u_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> Simp[
a^p   Int[ActivateTrig[u*cos[e + f*x]^(2*p)], x], x] /; FreeQ[{a, b, e, f,
p}, x] && EqQ[a + b, 0] && IntegerQ[p]
```

rule 4255
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Csc[c + d*x])^(n - 1)/(d*(n - 1))), x] + Simp[b^2*((n - 2)/(n - 1))
  Int[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
&& IntegerQ[2*n]
```

rule 4257
```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Maple [A] (verified)

Time = 147.10 (sec) , antiderivative size = 56, normalized size of antiderivative = 1.60

method	result	size
default	$\frac{\frac{2\left(-\frac{5\tanh\left(\frac{x}{2}\right)^{7}}{8}+\frac{3\tanh\left(\frac{x}{2}\right)^{5}}{8}-\frac{3\tanh\left(\frac{x}{2}\right)^{3}}{8}+\frac{5\tanh\left(\frac{x}{2}\right)}{8}\right)}{\left(\tanh\left(\frac{x}{2}\right)^{2}+1\right)^{4}}+\frac{3\arctan\left(\tanh\left(\frac{x}{2}\right)\right)}{4}}{a}$	56
risch	$\frac{e^{x}\left(3\,e^{6x}+11\,e^{4x}-11\,e^{2x}-3\right)}{4\left(e^{2x}+1\right)^{4}a}+\frac{3i\ln\left(e^{x}+i\right)}{8a}-\frac{3i\ln\left(e^{x}-i\right)}{8a}$	61

input
```
int(sech(x)^3/(a+a*sinh(x)^2),x,method=_RETURNVERBOSE)
```

output
```
2/a*((-5/8*tanh(1/2*x)^7+3/8*tanh(1/2*x)^5-3/8*tanh(1/2*x)^3+5/8*tanh(1/2*
x))/(tanh(1/2*x)^2+1)^4+3/8*arctan(tanh(1/2*x)))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 488 vs. $2(29) = 58$.

Time = 0.09 (sec) , antiderivative size = 488, normalized size of antiderivative = 13.94

$$\int \frac{\operatorname{sech}^3(x)}{a+a\sinh^2(x)}\,dx = \text{Too large to display}$$

input
```
integrate(sech(x)^3/(a+a*sinh(x)^2),x, algorithm="fricas")
```

output
```
1/4*(3*cosh(x)^7 + 21*cosh(x)*sinh(x)^6 + 3*sinh(x)^7 + (63*cosh(x)^2 + 11
)*sinh(x)^5 + 11*cosh(x)^5 + 5*(21*cosh(x)^3 + 11*cosh(x))*sinh(x)^4 + (10
5*cosh(x)^4 + 110*cosh(x)^2 - 11)*sinh(x)^3 - 11*cosh(x)^3 + (63*cosh(x)^5
 + 110*cosh(x)^3 - 33*cosh(x))*sinh(x)^2 + 3*(cosh(x)^8 + 8*cosh(x)*sinh(x
)^7 + sinh(x)^8 + 4*(7*cosh(x)^2 + 1)*sinh(x)^6 + 4*cosh(x)^6 + 8*(7*cosh(
x)^3 + 3*cosh(x))*sinh(x)^5 + 2*(35*cosh(x)^4 + 30*cosh(x)^2 + 3)*sinh(x)^
4 + 6*cosh(x)^4 + 8*(7*cosh(x)^5 + 10*cosh(x)^3 + 3*cosh(x))*sinh(x)^3 + 4
*(7*cosh(x)^6 + 15*cosh(x)^4 + 9*cosh(x)^2 + 1)*sinh(x)^2 + 4*cosh(x)^2 +
8*(cosh(x)^7 + 3*cosh(x)^5 + 3*cosh(x)^3 + cosh(x))*sinh(x) + 1)*arctan(co
sh(x) + sinh(x)) + (21*cosh(x)^6 + 55*cosh(x)^4 - 33*cosh(x)^2 - 3)*sinh(x
) - 3*cosh(x))/(a*cosh(x)^8 + 8*a*cosh(x)*sinh(x)^7 + a*sinh(x)^8 + 4*a*co
sh(x)^6 + 4*(7*a*cosh(x)^2 + a)*sinh(x)^6 + 8*(7*a*cosh(x)^3 + 3*a*cosh(x)
)*sinh(x)^5 + 6*a*cosh(x)^4 + 2*(35*a*cosh(x)^4 + 30*a*cosh(x)^2 + 3*a)*si
nh(x)^4 + 8*(7*a*cosh(x)^5 + 10*a*cosh(x)^3 + 3*a*cosh(x))*sinh(x)^3 + 4*a
*cosh(x)^2 + 4*(7*a*cosh(x)^6 + 15*a*cosh(x)^4 + 9*a*cosh(x)^2 + a)*sinh(x
)^2 + 8*(a*cosh(x)^7 + 3*a*cosh(x)^5 + 3*a*cosh(x)^3 + a*cosh(x))*sinh(x)
+ a)
```

Sympy [F]

$$\int \frac{\operatorname{sech}^3(x)}{a + a\sinh^2(x)}\,dx = \frac{\int \frac{\operatorname{sech}^3(x)}{\sinh^2(x)+1}\,dx}{a}$$

input
```
integrate(sech(x)**3/(a+a*sinh(x)**2),x)
```

output
```
Integral(sech(x)**3/(sinh(x)**2 + 1), x)/a
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 69 vs. $2(29) = 58$.

Time = 0.12 (sec) , antiderivative size = 69, normalized size of antiderivative = 1.97

$$\int \frac{\operatorname{sech}^3(x)}{a + a\sinh^2(x)}\,dx = \frac{3\,e^{(-x)} + 11\,e^{(-3\,x)} - 11\,e^{(-5\,x)} - 3\,e^{(-7\,x)}}{4\,(4\,ae^{(-2\,x)} + 6\,ae^{(-4\,x)} + 4\,ae^{(-6\,x)} + ae^{(-8\,x)} + a)} - \frac{3\,\arctan\left(e^{(-x)}\right)}{4\,a}$$

input
```
integrate(sech(x)^3/(a+a*sinh(x)^2),x, algorithm="maxima")
```

output
```
1/4*(3*e^(-x) + 11*e^(-3*x) - 11*e^(-5*x) - 3*e^(-7*x))/(4*a*e^(-2*x) + 6*
a*e^(-4*x) + 4*a*e^(-6*x) + a*e^(-8*x) + a) - 3/4*arctan(e^(-x))/a
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 67 vs. $2(29) = 58$.

Time = 0.13 (sec) , antiderivative size = 67, normalized size of antiderivative = 1.91

$$\int \frac{\operatorname{sech}^3(x)}{a + a\sinh^2(x)}\,dx = \frac{3\left(\pi + 2\arctan\left(\frac{1}{2}\left(e^{(2x)} - 1\right)e^{(-x)}\right)\right)}{16a} - \frac{3\left(e^{(-x)} - e^x\right)^3 + 20e^{(-x)} - 20e^x}{4\left(\left(e^{(-x)} - e^x\right)^2 + 4\right)^2 a}$$

input
```
integrate(sech(x)^3/(a+a*sinh(x)^2),x, algorithm="giac")
```

output
```
3/16*(pi + 2*arctan(1/2*(e^(2*x) - 1)*e^(-x)))/a - 1/4*(3*(e^(-x) - e^x)^3
 + 20*e^(-x) - 20*e^x)/(((e^(-x) - e^x)^2 + 4)^2*a)
```

Mupad [B] (verification not implemented)

Time = 1.92 (sec) , antiderivative size = 118, normalized size of antiderivative = 3.37

$$\int \frac{\operatorname{sech}^3(x)}{a + a\sinh^2(x)}\,dx = \frac{3\operatorname{atan}\left(\frac{e^x\sqrt{a^2}}{a}\right)}{4\sqrt{a^2}} - \frac{4e^{3x}}{a\left(4e^{2x} + 6e^{4x} + 4e^{6x} + e^{8x} + 1\right)} - \frac{2e^x}{a\left(3e^{2x} + 3e^{4x} + e^{6x} + 1\right)} + \frac{e^x}{2a\left(2e^{2x} + e^{4x} + 1\right)} + \frac{3e^x}{4a\left(e^{2x} + 1\right)}$$

input
```
int(1/(cosh(x)^3*(a + a*sinh(x)^2)),x)
```

output
```
(3*atan((exp(x)*(a^2)^(1/2))/a))/(4*(a^2)^(1/2)) - (4*exp(3*x))/(a*(4*exp(
2*x) + 6*exp(4*x) + 4*exp(6*x) + exp(8*x) + 1)) - (2*exp(x))/(a*(3*exp(2*x
) + 3*exp(4*x) + exp(6*x) + 1)) + exp(x)/(2*a*(2*exp(2*x) + exp(4*x) + 1))
 + (3*exp(x))/(4*a*(exp(2*x) + 1))
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 112, normalized size of antiderivative = 3.20

$$\int \frac{\operatorname{sech}^3(x)}{a + a\sinh^2(x)}\,dx = \frac{3e^{8x}atan(e^x) + 12e^{6x}atan(e^x) + 18e^{4x}atan(e^x) + 12e^{2x}atan(e^x) + 3atan(e^x) + 3e^{7x} + 11e^{5x} - 11e^{3x} - 3}{4a\left(e^{8x} + 4e^{6x} + 6e^{4x} + 4e^{2x} + 1\right)}$$

input
```
int(sech(x)^3/(a+a*sinh(x)^2),x)
```

output
```
(3*e**(8*x)*atan(e**x) + 12*e**(6*x)*atan(e**x) + 18*e**(4*x)*atan(e**x) +
 12*e**(2*x)*atan(e**x) + 3*atan(e**x) + 3*e**(7*x) + 11*e**(5*x) - 11*e**
(3*x) - 3*e**x)/(4*a*(e**(8*x) + 4*e**(6*x) + 6*e**(4*x) + 4*e**(2*x) + 1)
)
```

3.247 $\int \cosh^4(c+dx)\left(a+b\sinh^2(c+dx)\right)\,dx$

Optimal result

Integrand size = 21, antiderivative size = 89

$$\int \cosh^4(c+dx)\left(a+b\sinh^2(c+dx)\right)\,dx = \frac{1}{16}(6a-b)x + \frac{(6a-b)\cosh(c+dx)\sinh(c+dx)}{16d} + \frac{(6a-b)\cosh^3(c+dx)\sinh(c+dx)}{24d} + \frac{b\cosh^5(c+dx)\sinh(c+dx)}{6d}$$

output
```
1/16*(6*a-b)*x+1/16*(6*a-b)*cosh(d*x+c)*sinh(d*x+c)/d+1/24*(6*a-b)*cosh(d*
x+c)^3*sinh(d*x+c)/d+1/6*b*cosh(d*x+c)^5*sinh(d*x+c)/d
```

Mathematica [A] (verified)

Time = 0.16 (sec) , antiderivative size = 63, normalized size of antiderivative = 0.71

$$\int \cosh^4(c+dx)\left(a+b\sinh^2(c+dx)\right)\,dx = \frac{72ac+72adx-12bdx+(48a-3b)\sinh(2(c+dx))+3(2a+b)\sinh(4(c+dx))+b\sinh(6(c+dx))}{192d}$$

input
```
Integrate[Cosh[c + d*x]^4*(a + b*Sinh[c + d*x]^2),x]
```

output
```
(72*a*c + 72*a*d*x - 12*b*d*x + (48*a - 3*b)*Sinh[2*(c + d*x)] + 3*(2*a +
b)*Sinh[4*(c + d*x)] + b*Sinh[6*(c + d*x)])/(192*d)
```

Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 106, normalized size of antiderivative = 1.19, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$, Rules used = {3042, 3670, 298, 215, 215, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cosh^4(c+dx)\left(a+b\sinh^2(c+dx)\right)\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \cos(ic+idx)^4\left(a-b\sin(ic+idx)^2\right)dx$$

$$\Big\downarrow \text{3670}$$

$$\frac{\int \frac{a-(a-b)\tanh^2(c+dx)}{\left(1-\tanh^2(c+dx)\right)^4}d\tanh(c+dx)}{d}$$

$$\Big\downarrow \text{298}$$

$$\frac{\frac{1}{6}(6a-b)\int \frac{1}{\left(1-\tanh^2(c+dx)\right)^3}d\tanh(c+dx)+\frac{b\tanh(c+dx)}{6\left(1-\tanh^2(c+dx)\right)^3}}{d}$$

$$\Big\downarrow \text{215}$$

$$\frac{\frac{1}{6}(6a-b)\left(\frac{3}{4}\int \frac{1}{\left(1-\tanh^2(c+dx)\right)^2}d\tanh(c+dx)+\frac{\tanh(c+dx)}{4\left(1-\tanh^2(c+dx)\right)^2}\right)+\frac{b\tanh(c+dx)}{6\left(1-\tanh^2(c+dx)\right)^3}}{d}$$

$$\Big\downarrow \text{215}$$

$$\frac{\frac{1}{6}(6a-b)\left(\frac{3}{4}\left(\frac{1}{2}\int \frac{1}{1-\tanh^2(c+dx)}d\tanh(c+dx)+\frac{\tanh(c+dx)}{2\left(1-\tanh^2(c+dx)\right)}\right)+\frac{\tanh(c+dx)}{4\left(1-\tanh^2(c+dx)\right)^2}\right)+\frac{b\tanh(c+dx)}{6\left(1-\tanh^2(c+dx)\right)^3}}{d}$$

$$\Big\downarrow \text{219}$$

$$\frac{\frac{1}{6}(6a-b)\left(\frac{3}{4}\left(\frac{1}{2}\text{arctanh}(\tanh(c+dx))+\frac{\tanh(c+dx)}{2(1-\tanh^2(c+dx))}\right)+\frac{\tanh(c+dx)}{4(1-\tanh^2(c+dx))^2}\right)+\frac{b\tanh(c+dx)}{6(1-\tanh^2(c+dx))^3}}{d}$$

input
```
Int[Cosh[c + d*x]^4*(a + b*Sinh[c + d*x]^2),x]
```

output
```
((b*Tanh[c + d*x])/(6*(1 - Tanh[c + d*x]^2)^3) + ((6*a - b)*(Tanh[c + d*x]
/(4*(1 - Tanh[c + d*x]^2)^2) + (3*(ArcTanh[Tanh[c + d*x]]/2 + Tanh[c + d*x
]/(2*(1 - Tanh[c + d*x]^2))))/4))/6)/d
```

Defintions of rubi rules used

rule 215
```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(-x)*((a + b*x^2)^(p + 1)
/(2*a*(p + 1))), x] + Simp[(2*p + 3)/(2*a*(p + 1))   Int[(a + b*x^2)^(p + 1
), x], x] /; FreeQ[{a, b}, x] && LtQ[p, -1] && (IntegerQ[4*p] || IntegerQ[6
*p])
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 298
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2), x_Symbol] :> Simp[(-(
b*c - a*d))*x*((a + b*x^2)^(p + 1)/(2*a*b*(p + 1))), x] - Simp[(a*d - b*c*(
2*p + 3))/(2*a*b*(p + 1))   Int[(a + b*x^2)^(p + 1), x], x] /; FreeQ[{a, b,
 c, d, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/2 + p, 0])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3670
```
Int[cos[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(
p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Simp[ff/f   Su
bst[Int[(a + (a + b)*ff^2*x^2)^p/(1 + ff^2*x^2)^(m/2 + p + 1), x], x, Tan[e
 + f*x]/ff], x]] /; FreeQ[{a, b, e, f}, x] && IntegerQ[m/2] && IntegerQ[p]
```

Maple [A] (verified)

Time = 67.62 (sec) , antiderivative size = 95, normalized size of antiderivative = 1.07

method	result
derivativedivides	$\frac{a\left(\left(\frac{\cosh(dx+c)^3}{4}+\frac{3\cosh(dx+c)}{8}\right)\sinh(dx+c)+\frac{3dx}{8}+\frac{3c}{8}\right)+b\left(\frac{\sinh(dx+c)\cosh(dx+c)^5}{6}-\frac{\left(\frac{\cosh(dx+c)^3}{4}+\frac{3\cosh(dx+c)}{8}\right)\sin}{6}\right.}{d}$
default	$\frac{a\left(\left(\frac{\cosh(dx+c)^3}{4}+\frac{3\cosh(dx+c)}{8}\right)\sinh(dx+c)+\frac{3dx}{8}+\frac{3c}{8}\right)+b\left(\frac{\sinh(dx+c)\cosh(dx+c)^5}{6}-\frac{\left(\frac{\cosh(dx+c)^3}{4}+\frac{3\cosh(dx+c)}{8}\right)\sin}{6}\right.}{d}$
risch	$\frac{3ax}{8}-\frac{bx}{16}+\frac{b\,e^{6dx+6c}}{384d}+\frac{e^{4dx+4c}a}{64d}+\frac{e^{4dx+4c}b}{128d}+\frac{e^{2dx+2c}a}{8d}-\frac{e^{2dx+2c}b}{128d}-\frac{e^{-2dx-2c}a}{8d}+\frac{e^{-2dx-2c}b}{128d}-e$
orering	$x\cosh\left(dx+c\right)^4\left(a+b\sinh\left(dx+c\right)^2\right)+\frac{\frac{49\cosh(dx+c)^3\left(a+b\sinh(dx+c)^2\right)d\sinh(dx+c)}{36}+\frac{49\cosh(dx+c)^5 b}{72}}{d^2}$

input
```
int(cosh(d*x+c)^4*(a+b*sinh(d*x+c)^2),x,method=_RETURNVERBOSE)
```

output
```
1/d*(a*((1/4*cosh(d*x+c)^3+3/8*cosh(d*x+c))*sinh(d*x+c)+3/8*d*x+3/8*c)+b*(
1/6*sinh(d*x+c)*cosh(d*x+c)^5-1/6*(1/4*cosh(d*x+c)^3+3/8*cosh(d*x+c))*sinh
(d*x+c)-1/16*d*x-1/16*c))
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 117, normalized size of antiderivative = 1.31

$$\int\cosh^4(c+dx)\left(a+b\sinh^2(c+dx)\right)\,dx$$
$$=\frac{3\,b\cosh\left(dx+c\right)\sinh\left(dx+c\right)^5+2\left(5\,b\cosh\left(dx+c\right)^3+3\left(2\,a+b\right)\cosh\left(dx+c\right)\right)\sinh\left(dx+c\right)^3+6\left(6\,a\right.}{96\,d}$$

input
```
integrate(cosh(d*x+c)^4*(a+b*sinh(d*x+c)^2),x, algorithm="fricas")
```

output
```
1/96*(3*b*cosh(d*x + c)*sinh(d*x + c)^5 + 2*(5*b*cosh(d*x + c)^3 + 3*(2*a
+ b)*cosh(d*x + c))*sinh(d*x + c)^3 + 6*(6*a - b)*d*x + 3*(b*cosh(d*x + c)
^5 + 2*(2*a + b)*cosh(d*x + c)^3 + (16*a - b)*cosh(d*x + c))*sinh(d*x + c)
)/d
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 250 vs. $2(76) = 152$.

Time = 0.36 (sec) , antiderivative size = 250, normalized size of antiderivative = 2.81

$$\int \cosh^4(c+dx)\left(a+b\sinh^2(c+dx)\right)\,dx$$
$$=\begin{cases} \frac{3ax\sinh^4(c+dx)}{8} - \frac{3ax\sinh^2(c+dx)\cosh^2(c+dx)}{4} + \frac{3ax\cosh^4(c+dx)}{8} - \frac{3a\sinh^3(c+dx)\cosh(c+dx)}{8d} + \frac{5a\sinh(c+dx)\cosh^3(c+dx)}{8d} \\ x\left(a+b\sinh^2(c)\right)\cosh^4(c) \end{cases}$$

input
```
integrate(cosh(d*x+c)**4*(a+b*sinh(d*x+c)**2),x)
```

output
```
Piecewise((3*a*x*sinh(c + d*x)**4/8 - 3*a*x*sinh(c + d*x)**2*cosh(c + d*x)
**2/4 + 3*a*x*cosh(c + d*x)**4/8 - 3*a*sinh(c + d*x)**3*cosh(c + d*x)/(8*d
) + 5*a*sinh(c + d*x)*cosh(c + d*x)**3/(8*d) + b*x*sinh(c + d*x)**6/16 - 3
*b*x*sinh(c + d*x)**4*cosh(c + d*x)**2/16 + 3*b*x*sinh(c + d*x)**2*cosh(c
+ d*x)**4/16 - b*x*cosh(c + d*x)**6/16 - b*sinh(c + d*x)**5*cosh(c + d*x)/
(16*d) + b*sinh(c + d*x)**3*cosh(c + d*x)**3/(6*d) + b*sinh(c + d*x)*cosh(
c + d*x)**5/(16*d), Ne(d, 0)), (x*(a + b*sinh(c)**2)*cosh(c)**4, True))
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 152, normalized size of antiderivative = 1.71

$$\int \cosh^4(c+dx)\left(a+b\sinh^2(c+dx)\right)\,dx$$
$$=\frac{1}{64}a\left(24x+\frac{e^{(4dx+4c)}}{d}+\frac{8e^{(2dx+2c)}}{d}-\frac{8e^{(-2dx-2c)}}{d}-\frac{e^{(-4dx-4c)}}{d}\right)$$
$$+\frac{1}{384}b\left(\frac{\left(3e^{(-2dx-2c)}-3e^{(-4dx-4c)}+1\right)e^{(6dx+6c)}}{d}-\frac{24(dx+c)}{d}+\frac{3e^{(-2dx-2c)}-3e^{(-4dx-4c)}-e^{(-6dx}}{d}\right.$$

input
```
integrate(cosh(d*x+c)^4*(a+b*sinh(d*x+c)^2),x, algorithm="maxima")
```

output
```
1/64*a*(24*x + e^(4*d*x + 4*c)/d + 8*e^(2*d*x + 2*c)/d - 8*e^(-2*d*x - 2*c
)/d - e^(-4*d*x - 4*c)/d) + 1/384*b*((3*e^(-2*d*x - 2*c) - 3*e^(-4*d*x - 4
*c) + 1)*e^(6*d*x + 6*c)/d - 24*(d*x + c)/d + (3*e^(-2*d*x - 2*c) - 3*e^(-
4*d*x - 4*c) - e^(-6*d*x - 6*c))/d)
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 121, normalized size of antiderivative = 1.36

$$\int \cosh^4(c+dx)\left(a+b\sinh^2(c+dx)\right)\,dx=\frac{1}{16}(6a-b)x+\frac{be^{(6dx+6c)}}{384d}+\frac{(2a+b)e^{(4dx+4c)}}{128d}+\frac{(16a-b)e^{(2dx+2c)}}{128d}-\frac{(16a-b)e^{(-2dx-2c)}}{128d}-\frac{(2a+b)e^{(-4dx-4c)}}{128d}-\frac{be^{(-6dx-6c)}}{384d}$$

input
```
integrate(cosh(d*x+c)^4*(a+b*sinh(d*x+c)^2),x, algorithm="giac")
```

output
```
1/16*(6*a - b)*x + 1/384*b*e^(6*d*x + 6*c)/d + 1/128*(2*a + b)*e^(4*d*x +
4*c)/d + 1/128*(16*a - b)*e^(2*d*x + 2*c)/d - 1/128*(16*a - b)*e^(-2*d*x -
 2*c)/d - 1/128*(2*a + b)*e^(-4*d*x - 4*c)/d - 1/384*b*e^(-6*d*x - 6*c)/d
```

Mupad [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 76, normalized size of antiderivative = 0.85

$$\int \cosh^4(c+dx)\left(a+b\sinh^2(c+dx)\right)\,dx$$
$$=\frac{12\,a\sinh(2\,c+2\,d\,x)+\frac{3\,a\sinh(4\,c+4\,d\,x)}{2}-\frac{3\,b\sinh(2\,c+2\,d\,x)}{4}+\frac{3\,b\sinh(4\,c+4\,d\,x)}{4}+\frac{b\sinh(6\,c+6\,d\,x)}{4}+18\,a\,d\,x-3\,b\,d\,x}{48\,d}$$

input
```
int(cosh(c + d*x)^4*(a + b*sinh(c + d*x)^2),x)
```

output
```
(12*a*sinh(2*c + 2*d*x) + (3*a*sinh(4*c + 4*d*x))/2 - (3*b*sinh(2*c + 2*d*
x))/4 + (3*b*sinh(4*c + 4*d*x))/4 + (b*sinh(6*c + 6*d*x))/4 + 18*a*d*x - 3
*b*d*x)/(48*d)
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 167, normalized size of antiderivative = 1.88

$$\int \cosh^4(c+dx)\left(a+b\sinh^2(c+dx)\right)\,dx$$
$$=\frac{e^{12dx+12c}b+6e^{10dx+10c}a+3e^{10dx+10c}b+48e^{8dx+8c}a-3e^{8dx+8c}b+144e^{6dx+6c}adx-24e^{6dx+6c}bdx-48e^{4dx}\cdots}{384e^{6dx+6c}d}$$

input
```
int(cosh(d*x+c)^4*(a+b*sinh(d*x+c)^2),x)
```

output
```
(e**(12*c + 12*d*x)*b + 6*e**(10*c + 10*d*x)*a + 3*e**(10*c + 10*d*x)*b +
48*e**(8*c + 8*d*x)*a - 3*e**(8*c + 8*d*x)*b + 144*e**(6*c + 6*d*x)*a*d*x
- 24*e**(6*c + 6*d*x)*b*d*x - 48*e**(4*c + 4*d*x)*a + 3*e**(4*c + 4*d*x)*b
 - 6*e**(2*c + 2*d*x)*a - 3*e**(2*c + 2*d*x)*b - b)/(384*e**(6*c + 6*d*x)*
d)
```

3.248 $\int \cosh^3(c+dx)\left(a+b\sinh^2(c+dx)\right)\,dx$

Optimal result

Integrand size = 21, antiderivative size = 46

$$\int \cosh^3(c+dx)\left(a+b\sinh^2(c+dx)\right)\,dx=\frac{a\sinh(c+dx)}{d}+\frac{(a+b)\sinh^3(c+dx)}{3d}+\frac{b\sinh^5(c+dx)}{5d}$$

output
```
a*sinh(d*x+c)/d+1/3*(a+b)*sinh(d*x+c)^3/d+1/5*b*sinh(d*x+c)^5/d
```

Mathematica [A] (verified)

Time = 0.10 (sec) , antiderivative size = 48, normalized size of antiderivative = 1.04

$$\int \cosh^3(c+dx)\left(a+b\sinh^2(c+dx)\right)\,dx=\frac{(100a-11b+4(5a+2b)\cosh(2(c+dx))+3b\cosh(4(c+dx)))\sinh(c+dx)}{120d}$$

input
```
Integrate[Cosh[c + d*x]^3*(a + b*Sinh[c + d*x]^2),x]
```

output
```
((100*a - 11*b + 4*(5*a + 2*b)*Cosh[2*(c + d*x)] + 3*b*Cosh[4*(c + d*x)])*
Sinh[c + d*x])/(120*d)
```

Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.89, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.190$, Rules used = {3042, 3669, 290, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cosh^3(c+dx)\left(a+b\sinh^2(c+dx)\right)\,dx$$

$$\Big\downarrow \; 3042$$

$$\int \cos(ic+idx)^3\left(a-b\sin(ic+idx)^2\right)dx$$

$$\Big\downarrow \; 3669$$

$$\frac{\int \left(\sinh^2(c+dx)+1\right)\left(b\sinh^2(c+dx)+a\right)d\sinh(c+dx)}{d}$$

$$\Big\downarrow \; 290$$

$$\frac{\int \left(b\sinh^4(c+dx)+(a+b)\sinh^2(c+dx)+a\right)d\sinh(c+dx)}{d}$$

$$\Big\downarrow \; 2009$$

$$\frac{\frac{1}{3}(a+b)\sinh^3(c+dx)+a\sinh(c+dx)+\frac{1}{5}b\sinh^5(c+dx)}{d}$$

input
```
Int[Cosh[c + d*x]^3*(a + b*Sinh[c + d*x]^2),x]
```

output
```
(a*Sinh[c + d*x] + ((a + b)*Sinh[c + d*x]^3)/3 + (b*Sinh[c + d*x]^5)/5)/d
```

Defintions of rubi rules used

rule 290
```
Int[((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.), x_Symbol] :> I
nt[ExpandIntegrand[(a + b*x^2)^p*(c + d*x^2)^q, x], x] /; FreeQ[{a, b, c, d
}, x] && NeQ[b*c - a*d, 0] && IGtQ[p, 0] && IGtQ[q, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3669
```
Int[cos[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^
(p_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Simp[ff/f   S
ubst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a + b*ff^2*x^2)^p, x], x, Sin[e + f*x]
/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m - 1)/2]
```

Maple [A] (verified)

Time = 27.52 (sec) , antiderivative size = 38, normalized size of antiderivative = 0.83

method	result
derivativedivides	$\frac{\frac{b\sinh(dx+c)^5}{5}+\frac{(a+b)\sinh(dx+c)^3}{3}+a\sinh(dx+c)}{d}$
default	$\frac{\frac{b\sinh(dx+c)^5}{5}+\frac{(a+b)\sinh(dx+c)^3}{3}+a\sinh(dx+c)}{d}$
risch	$\frac{b\,e^{5dx+5c}}{160d}+\frac{e^{3dx+3c}a}{24d}+\frac{e^{3dx+3c}b}{96d}+\frac{3\,e^{dx+c}a}{8d}-\frac{e^{dx+c}b}{16d}-\frac{3\,e^{-dx-c}a}{8d}+\frac{e^{-dx-c}b}{16d}-\frac{e^{-3dx-3c}a}{24d}-\frac{e^{-3dx-}}{96d}$
orering	$\frac{\frac{259\cosh(dx+c)^2\left(a+b\sinh(dx+c)^2\right)d\sinh(dx+c)}{75}+\frac{518\cosh(dx+c)^4b\sinh(dx+c)d}{225}}{d^2}-\frac{7\left(6d^3\sinh(dx+c)^3\left(a+b\sinh(dx+c)^2\right)\right.}{}$

input
```
int(cosh(d*x+c)^3*(a+b*sinh(d*x+c)^2),x,method=_RETURNVERBOSE)
```

output
```
1/d*(1/5*b*sinh(d*x+c)^5+1/3*(a+b)*sinh(d*x+c)^3+a*sinh(d*x+c))
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 82, normalized size of antiderivative = 1.78

$$\int \cosh^3(c+dx)\left(a+b\sinh^2(c+dx)\right)\,dx$$
$$= \frac{3\,b\sinh\left(dx+c\right)^5+5\left(6\,b\cosh\left(dx+c\right)^2+4\,a+b\right)\sinh\left(dx+c\right)^3+15\left(b\cosh\left(dx+c\right)^4+\left(4\,a+b\right)\cosh\right.}{240\,d}$$

input
```
integrate(cosh(d*x+c)^3*(a+b*sinh(d*x+c)^2),x, algorithm="fricas")
```

output
```
1/240*(3*b*sinh(d*x + c)^5 + 5*(6*b*cosh(d*x + c)^2 + 4*a + b)*sinh(d*x +
c)^3 + 15*(b*cosh(d*x + c)^4 + (4*a + b)*cosh(d*x + c)^2 + 12*a - 2*b)*sin
h(d*x + c))/d
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 85 vs. $2(39) = 78$.

Time = 0.22 (sec) , antiderivative size = 85, normalized size of antiderivative = 1.85

$$\int \cosh^3(c+dx)\left(a+b\sinh^2(c+dx)\right)\,dx$$
$$= \begin{cases} -\frac{2a\sinh^3(c+dx)}{3d}+\frac{a\sinh(c+dx)\cosh^2(c+dx)}{d}-\frac{2b\sinh^5(c+dx)}{15d}+\frac{b\sinh^3(c+dx)\cosh^2(c+dx)}{3d} & \text{for } d\neq 0\\ x\left(a+b\sinh^2(c)\right)\cosh^3(c) & \text{otherwise}\end{cases}$$

input
```
integrate(cosh(d*x+c)**3*(a+b*sinh(d*x+c)**2),x)
```

output
```
Piecewise((-2*a*sinh(c + d*x)**3/(3*d) + a*sinh(c + d*x)*cosh(c + d*x)**2/
d - 2*b*sinh(c + d*x)**5/(15*d) + b*sinh(c + d*x)**3*cosh(c + d*x)**2/(3*d
), Ne(d, 0)), (x*(a + b*sinh(c)**2)*cosh(c)**3, True))
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 136 vs. $2(42) = 84$.

Time = 0.04 (sec) , antiderivative size = 136, normalized size of antiderivative = 2.96

$$\int \cosh^3(c+dx)\left(a+b\sinh^2(c+dx)\right)\,dx = \frac{1}{480}\,b\left(\frac{\left(5\,e^{(-2\,dx-2\,c)}-30\,e^{(-4\,dx-4\,c)}+3\right)e^{(5\,dx+5\,c)}}{d}+\frac{30\,e^{(-dx-c)}-5\,e^{(-3\,dx-3\,c)}-3\,e^{(-5\,dx-5\,c)}}{d}\right) + \frac{1}{24}\,a\left(\frac{e^{(3\,dx+3\,c)}}{d}+\frac{9\,e^{(dx+c)}}{d}-\frac{9\,e^{(-dx-c)}}{d}-\frac{e^{(-3\,dx-3\,c)}}{d}\right)$$

input
```
integrate(cosh(d*x+c)^3*(a+b*sinh(d*x+c)^2),x, algorithm="maxima")
```

output
```
1/480*b*((5*e^(-2*d*x - 2*c) - 30*e^(-4*d*x - 4*c) + 3)*e^(5*d*x + 5*c)/d
+ (30*e^(-d*x - c) - 5*e^(-3*d*x - 3*c) - 3*e^(-5*d*x - 5*c))/d) + 1/24*a*
(e^(3*d*x + 3*c)/d + 9*e^(d*x + c)/d - 9*e^(-d*x - c)/d - e^(-3*d*x - 3*c)
/d)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 108 vs. $2(42) = 84$.

Time = 0.13 (sec) , antiderivative size = 108, normalized size of antiderivative = 2.35

$$\int \cosh^3(c+dx)\left(a+b\sinh^2(c+dx)\right)\,dx = \frac{be^{(5\,dx+5\,c)}}{160\,d}+\frac{(4\,a+b)e^{(3\,dx+3\,c)}}{96\,d}+\frac{(6\,a-b)e^{(dx+c)}}{16\,d}-\frac{(6\,a-b)e^{(-dx-c)}}{16\,d}-\frac{(4\,a+b)e^{(-3\,dx-3\,c)}}{96\,d}-\frac{be^{(-5\,dx-5\,c)}}{160\,d}$$

input
```
integrate(cosh(d*x+c)^3*(a+b*sinh(d*x+c)^2),x, algorithm="giac")
```

output
```
1/160*b*e^(5*d*x + 5*c)/d + 1/96*(4*a + b)*e^(3*d*x + 3*c)/d + 1/16*(6*a -
 b)*e^(d*x + c)/d - 1/16*(6*a - b)*e^(-d*x - c)/d - 1/96*(4*a + b)*e^(-3*d
*x - 3*c)/d - 1/160*b*e^(-5*d*x - 5*c)/d
```

Mupad [B] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 48, normalized size of antiderivative = 1.04

$$\int \cosh^3(c+dx) \left(a+b\sinh^2(c+dx)\right) \, dx = \frac{15\,a\sinh(c+d\,x)+5\,a\,{\sinh(c+d\,x)}^3+5\,b\,{\sinh(c+d\,x)}^3+3\,b\,{\sinh(c+d\,x)}^5}{15\,d}$$

input
```
int(cosh(c + d*x)^3*(a + b*sinh(c + d*x)^2),x)
```

output
```
(15*a*sinh(c + d*x) + 5*a*sinh(c + d*x)^3 + 5*b*sinh(c + d*x)^3 + 3*b*sinh
(c + d*x)^5)/(15*d)
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 138, normalized size of antiderivative = 3.00

$$\int \cosh^3(c+dx) \left(a+b\sinh^2(c+dx)\right) \, dx = \frac{3e^{10dx+10c}b+20e^{8dx+8c}a+5e^{8dx+8c}b+180e^{6dx+6c}a-30e^{6dx+6c}b-180e^{4dx+4c}a+30e^{4dx+4c}b-20e^{2dx+2c}}{480e^{5dx+5c}d}$$

input
```
int(cosh(d*x+c)^3*(a+b*sinh(d*x+c)^2),x)
```

output
```
(3*e**(10*c + 10*d*x)*b + 20*e**(8*c + 8*d*x)*a + 5*e**(8*c + 8*d*x)*b + 1
80*e**(6*c + 6*d*x)*a - 30*e**(6*c + 6*d*x)*b - 180*e**(4*c + 4*d*x)*a + 3
0*e**(4*c + 4*d*x)*b - 20*e**(2*c + 2*d*x)*a - 5*e**(2*c + 2*d*x)*b - 3*b)
/(480*e**(5*c + 5*d*x)*d)
```

3.249 $\int \cosh^2(c+dx)\left(a+b\sinh^2(c+dx)\right)\,dx$

Optimal result

Integrand size = 21, antiderivative size = 61

$$\int \cosh^2(c+dx)\left(a+b\sinh^2(c+dx)\right)\,dx = \frac{1}{8}(4a-b)x + \frac{(4a-b)\cosh(c+dx)\sinh(c+dx)}{8d} + \frac{b\cosh^3(c+dx)\sinh(c+dx)}{4d}$$

output
```
1/8*(4*a-b)*x+1/8*(4*a-b)*cosh(d*x+c)*sinh(d*x+c)/d+1/4*b*cosh(d*x+c)^3*si
nh(d*x+c)/d
```

Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 43, normalized size of antiderivative = 0.70

$$\int \cosh^2(c+dx)\left(a+b\sinh^2(c+dx)\right)\,dx = \frac{16ac+16adx-4bdx+8a\sinh(2(c+dx))+b\sinh(4(c+dx))}{32d}$$

input
```
Integrate[Cosh[c + d*x]^2*(a + b*Sinh[c + d*x]^2),x]
```

output
```
(16*a*c + 16*a*d*x - 4*b*d*x + 8*a*Sinh[2*(c + d*x)] + b*Sinh[4*(c + d*x)]
)/(32*d)
```

Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 77, normalized size of antiderivative = 1.26, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.238$, Rules used = {3042, 3670, 298, 215, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cosh^2(c+dx)\left(a+b\sinh^2(c+dx)\right)dx$$

$$\Big\downarrow \text{3042}$$

$$\int \cos(ic+idx)^2\left(a-b\sin(ic+idx)^2\right)dx$$

$$\Big\downarrow \text{3670}$$

$$\frac{\int \frac{a-(a-b)\tanh^2(c+dx)}{\left(1-\tanh^2(c+dx)\right)^3}d\tanh(c+dx)}{d}$$

$$\Big\downarrow \text{298}$$

$$\frac{\frac{1}{4}(4a-b)\int \frac{1}{\left(1-\tanh^2(c+dx)\right)^2}d\tanh(c+dx)+\frac{b\tanh(c+dx)}{4\left(1-\tanh^2(c+dx)\right)^2}}{d}$$

$$\Big\downarrow \text{215}$$

$$\frac{\frac{1}{4}(4a-b)\left(\frac{1}{2}\int \frac{1}{1-\tanh^2(c+dx)}d\tanh(c+dx)+\frac{\tanh(c+dx)}{2\left(1-\tanh^2(c+dx)\right)}\right)+\frac{b\tanh(c+dx)}{4\left(1-\tanh^2(c+dx)\right)^2}}{d}$$

$$\Big\downarrow \text{219}$$

$$\frac{\frac{1}{4}(4a-b)\left(\frac{1}{2}\text{arctanh}(\tanh(c+dx))+\frac{\tanh(c+dx)}{2\left(1-\tanh^2(c+dx)\right)}\right)+\frac{b\tanh(c+dx)}{4\left(1-\tanh^2(c+dx)\right)^2}}{d}$$

input
```
Int[Cosh[c + d*x]^2*(a + b*Sinh[c + d*x]^2),x]
```

output
```
((b*Tanh[c + d*x])/(4*(1 - Tanh[c + d*x]^2)^2) + ((4*a - b)*(ArcTanh[Tanh[
c + d*x]]/2 + Tanh[c + d*x]/(2*(1 - Tanh[c + d*x]^2))))/4)/d
```

Defintions of rubi rules used

rule 215
```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(-x)*((a + b*x^2)^(p + 1)
/(2*a*(p + 1))), x] + Simp[(2*p + 3)/(2*a*(p + 1))   Int[(a + b*x^2)^(p + 1
), x], x] /; FreeQ[{a, b}, x] && LtQ[p, -1] && (IntegerQ[4*p] || IntegerQ[6
*p])
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 298
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2), x_Symbol] :> Simp[(-(
b*c - a*d))*x*((a + b*x^2)^(p + 1)/(2*a*b*(p + 1))), x] - Simp[(a*d - b*c*(
2*p + 3))/(2*a*b*(p + 1))   Int[(a + b*x^2)^(p + 1), x], x] /; FreeQ[{a, b,
 c, d, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/2 + p, 0])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3670
```
Int[cos[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(
p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Simp[ff/f   Su
bst[Int[(a + (a + b)*ff^2*x^2)^p/(1 + ff^2*x^2)^(m/2 + p + 1), x], x, Tan[e
 + f*x]/ff], x]] /; FreeQ[{a, b, e, f}, x] && IntegerQ[m/2] && IntegerQ[p]
```

Maple [A] (verified)

Time = 10.33 (sec) , antiderivative size = 70, normalized size of antiderivative = 1.15

method	result
derivativedivides	$\frac{a\left(\frac{\cosh(dx+c)\sinh(dx+c)}{2}+\frac{dx}{2}+\frac{c}{2}\right)+b\left(\frac{\sinh(dx+c)\cosh(dx+c)^3}{4}-\frac{\cosh(dx+c)\sinh(dx+c)}{8}-\frac{dx}{8}-\frac{c}{8}\right)}{d}$
default	$\frac{a\left(\frac{\cosh(dx+c)\sinh(dx+c)}{2}+\frac{dx}{2}+\frac{c}{2}\right)+b\left(\frac{\sinh(dx+c)\cosh(dx+c)^3}{4}-\frac{\cosh(dx+c)\sinh(dx+c)}{8}-\frac{dx}{8}-\frac{c}{8}\right)}{d}$
risch	$\frac{ax}{2}-\frac{bx}{8}+\frac{e^{4dx+4c}b}{64d}+\frac{e^{2dx+2c}a}{8d}-\frac{e^{-2dx-2c}a}{8d}-\frac{e^{-4dx-4c}b}{64d}$
orering	$x\cosh(dx+c)^2\left(a+b\sinh(dx+c)^2\right)+\frac{\frac{5\cosh(dx+c)\left(a+b\sinh(dx+c)^2\right)d\sinh(dx+c)}{8}+\frac{5\cosh(dx+c)^3b\sinh}{8}}{d^2}$

input
```
int(cosh(d*x+c)^2*(a+b*sinh(d*x+c)^2),x,method=_RETURNVERBOSE)
```

output
```
1/d*(a*(1/2*cosh(d*x+c)*sinh(d*x+c)+1/2*d*x+1/2*c)+b*(1/4*sinh(d*x+c)*cosh
(d*x+c)^3-1/8*cosh(d*x+c)*sinh(d*x+c)-1/8*d*x-1/8*c))
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 59, normalized size of antiderivative = 0.97

$$\int \cosh^2(c+dx)\left(a+b\sinh^2(c+dx)\right)\,dx$$
$$=\frac{b\cosh(dx+c)\sinh(dx+c)^3+(4a-b)dx+\left(b\cosh(dx+c)^3+4a\cosh(dx+c)\right)\sinh(dx+c)}{8d}$$

input
```
integrate(cosh(d*x+c)^2*(a+b*sinh(d*x+c)^2),x, algorithm="fricas")
```

output
```
1/8*(b*cosh(d*x + c)*sinh(d*x + c)^3 + (4*a - b)*d*x + (b*cosh(d*x + c)^3
+ 4*a*cosh(d*x + c))*sinh(d*x + c))/d
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 150 vs. $2(49) = 98$.

Time = 0.17 (sec) , antiderivative size = 150, normalized size of antiderivative = 2.46

$$\int \cosh^2(c+dx)\left(a+b\sinh^2(c+dx)\right)\,dx$$
$$= \begin{cases} -\frac{ax\sinh^2(c+dx)}{2} + \frac{ax\cosh^2(c+dx)}{2} + \frac{a\sinh(c+dx)\cosh(c+dx)}{2d} - \frac{bx\sinh^4(c+dx)}{8} + \frac{bx\sinh^2(c+dx)\cosh^2(c+dx)}{4} - \frac{bx\cosh^4(c}{8} \\ x\left(a+b\sinh^2(c)\right)\cosh^2(c) \end{cases}$$

input
```
integrate(cosh(d*x+c)**2*(a+b*sinh(d*x+c)**2),x)
```

output
```
Piecewise((-a*x*sinh(c + d*x)**2/2 + a*x*cosh(c + d*x)**2/2 + a*sinh(c + d
*x)*cosh(c + d*x)/(2*d) - b*x*sinh(c + d*x)**4/8 + b*x*sinh(c + d*x)**2*co
sh(c + d*x)**2/4 - b*x*cosh(c + d*x)**4/8 + b*sinh(c + d*x)**3*cosh(c + d*
x)/(8*d) + b*sinh(c + d*x)*cosh(c + d*x)**3/(8*d), Ne(d, 0)), (x*(a + b*si
nh(c)**2)*cosh(c)**2, True))
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 76, normalized size of antiderivative = 1.25

$$\int \cosh^2(c+dx)\left(a+b\sinh^2(c+dx)\right)\,dx$$
$$= \frac{1}{8}a\left(4x + \frac{e^{(2dx+2c)}}{d} - \frac{e^{(-2dx-2c)}}{d}\right) - \frac{1}{64}b\left(\frac{8(dx+c)}{d} - \frac{e^{(4dx+4c)}}{d} + \frac{e^{(-4dx-4c)}}{d}\right)$$

input
```
integrate(cosh(d*x+c)^2*(a+b*sinh(d*x+c)^2),x, algorithm="maxima")
```

output
```
1/8*a*(4*x + e^(2*d*x + 2*c)/d - e^(-2*d*x - 2*c)/d) - 1/64*b*(8*(d*x + c)
/d - e^(4*d*x + 4*c)/d + e^(-4*d*x - 4*c)/d)
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 71, normalized size of antiderivative = 1.16

$$\int \cosh^2(c+dx) \left(a+b\sinh^2(c+dx)\right) \, dx = \frac{1}{8}(4a-b)x + \frac{be^{(4dx+4c)}}{64d} + \frac{ae^{(2dx+2c)}}{8d} - \frac{ae^{(-2dx-2c)}}{8d} - \frac{be^{(-4dx-4c)}}{64d}$$

input
```
integrate(cosh(d*x+c)^2*(a+b*sinh(d*x+c)^2),x, algorithm="giac")
```

output
```
1/8*(4*a - b)*x + 1/64*b*e^(4*d*x + 4*c)/d + 1/8*a*e^(2*d*x + 2*c)/d - 1/8
*a*e^(-2*d*x - 2*c)/d - 1/64*b*e^(-4*d*x - 4*c)/d
```

Mupad [B] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 38, normalized size of antiderivative = 0.62

$$\int \cosh^2(c+dx) \left(a+b\sinh^2(c+dx)\right) \, dx = \frac{ax}{2} - \frac{bx}{8} + \frac{\frac{a\sinh(2c+2dx)}{4} + \frac{b\sinh(4c+4dx)}{32}}{d}$$

input
```
int(cosh(c + d*x)^2*(a + b*sinh(c + d*x)^2),x)
```

output
```
(a*x)/2 - (b*x)/8 + ((a*sinh(2*c + 2*d*x))/4 + (b*sinh(4*c + 4*d*x))/32)/d
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 89, normalized size of antiderivative = 1.46

$$\int \cosh^2(c+dx) \left(a+b\sinh^2(c+dx)\right) \, dx = \frac{e^{8dx+8c}b + 8e^{6dx+6c}a + 32e^{4dx+4c}adx - 8e^{4dx+4c}bdx - 8e^{2dx+2c}a - b}{64e^{4dx+4c}d}$$

input
```
int(cosh(d*x+c)^2*(a+b*sinh(d*x+c)^2),x)
```

output
```
(e**(8*c + 8*d*x)*b + 8*e**(6*c + 6*d*x)*a + 32*e**(4*c + 4*d*x)*a*d*x - 8
*e**(4*c + 4*d*x)*b*d*x - 8*e**(2*c + 2*d*x)*a - b)/(64*e**(4*c + 4*d*x)*d
)
```

3.250 $\int \cosh(c+dx)\left(a+b\sinh^2(c+dx)\right)\,dx$

Optimal result

Integrand size = 19, antiderivative size = 28

$$\int \cosh(c+dx)\left(a+b\sinh^2(c+dx)\right)\,dx=\frac{a\sinh(c+dx)}{d}+\frac{b\sinh^3(c+dx)}{3d}$$

output
```
a*sinh(d*x+c)/d+1/3*b*sinh(d*x+c)^3/d
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 39, normalized size of antiderivative = 1.39

$$\int \cosh(c+dx)\left(a+b\sinh^2(c+dx)\right)\,dx=\frac{a\cosh(dx)\sinh(c)}{d}+\frac{a\cosh(c)\sinh(dx)}{d}+\frac{b\sinh^3(c+dx)}{3d}$$

input
```
Integrate[Cosh[c + d*x]*(a + b*Sinh[c + d*x]^2),x]
```

output
```
(a*Cosh[d*x]*Sinh[c])/d + (a*Cosh[c]*Sinh[d*x])/d + (b*Sinh[c + d*x]^3)/(3
*d)
```

Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.93, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.158$, Rules used = {3042, 3669, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cosh(c+dx)\left(a+b\sinh^2(c+dx)\right)\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \cos(ic+idx)\left(a-b\sin(ic+idx)^2\right)dx$$

$$\Big\downarrow \text{3669}$$

$$\frac{\int \left(b\sinh^2(c+dx)+a\right)d\sinh(c+dx)}{d}$$

$$\Big\downarrow \text{2009}$$

$$\frac{a\sinh(c+dx)+\frac{1}{3}b\sinh^3(c+dx)}{d}$$

input
```
Int[Cosh[c + d*x]*(a + b*Sinh[c + d*x]^2),x]
```

output
```
(a*Sinh[c + d*x] + (b*Sinh[c + d*x]^3)/3)/d
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3669
```
Int[cos[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^
(p_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Simp[ff/f   S
ubst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a + b*ff^2*x^2)^p, x], x, Sin[e + f*x]
/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m - 1)/2]
```

Maple [A] (verified)

Time = 3.69 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.89

method	result
derivativedivides	$\frac{\frac{b\sinh(dx+c)^3}{3}+a\sinh(dx+c)}{d}$
default	$\frac{\frac{b\sinh(dx+c)^3}{3}+a\sinh(dx+c)}{d}$
risch	$\frac{e^{3dx+3c}b}{24d}+\frac{e^{dx+c}a}{2d}-\frac{e^{dx+c}b}{8d}-\frac{e^{-dx-c}a}{2d}+\frac{e^{-dx-c}b}{8d}-\frac{e^{-3dx-3c}b}{24d}$
orering	$\frac{\frac{10d\sinh(dx+c)\left(a+b\sinh(dx+c)^2\right)}{9}+\frac{20b\sinh(dx+c)d\cosh(dx+c)^2}{9}}{d^2}-\frac{d^3\sinh(dx+c)\left(a+b\sinh(dx+c)^2\right)+20d^3\cosh(dx+c)}{9d^4}$

input
```
int(cosh(d*x+c)*(a+b*sinh(d*x+c)^2),x,method=_RETURNVERBOSE)
```

output
```
1/d*(1/3*b*sinh(d*x+c)^3+a*sinh(d*x+c))
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 41, normalized size of antiderivative = 1.46

$$\int\cosh(c+dx)\left(a+b\sinh^2(c+dx)\right)\,dx=\frac{b\sinh\left(dx+c\right)^3+3\left(b\cosh\left(dx+c\right)^2+4\,a-b\right)\sinh\left(dx+c\right)}{12\,d}$$

input
```
integrate(cosh(d*x+c)*(a+b*sinh(d*x+c)^2),x, algorithm="fricas")
```

output
```
1/12*(b*sinh(d*x + c)^3 + 3*(b*cosh(d*x + c)^2 + 4*a - b)*sinh(d*x + c))/d
```

Sympy [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.29

$$\int \cosh(c+dx)\left(a+b\sinh^2(c+dx)\right)\,dx = \begin{cases} \frac{a\sinh(c+dx)}{d} + \frac{b\sinh^3(c+dx)}{3d} & \text{for } d\neq 0 \\ x\left(a+b\sinh^2(c)\right)\cosh(c) & \text{otherwise} \end{cases}$$

input
```
integrate(cosh(d*x+c)*(a+b*sinh(d*x+c)**2),x)
```

output
```
Piecewise((a*sinh(c + d*x)/d + b*sinh(c + d*x)**3/(3*d), Ne(d, 0)), (x*(a
+ b*sinh(c)**2)*cosh(c), True))
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.93

$$\int \cosh(c+dx)\left(a+b\sinh^2(c+dx)\right)\,dx = \frac{b\sinh(dx+c)^3}{3\,d} + \frac{a\sinh(dx+c)}{d}$$

input
```
integrate(cosh(d*x+c)*(a+b*sinh(d*x+c)^2),x, algorithm="maxima")
```

output
```
1/3*b*sinh(d*x + c)^3/d + a*sinh(d*x + c)/d
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 70 vs. 2(26) = 52.

Time = 0.13 (sec) , antiderivative size = 70, normalized size of antiderivative = 2.50

$$\int \cosh(c+dx)\left(a+b\sinh^2(c+dx)\right)\,dx = \frac{be^{(3\,dx+3\,c)}}{24\,d} + \frac{(4\,a-b)e^{(dx+c)}}{8\,d} - \frac{(4\,a-b)e^{(-dx-c)}}{8\,d} - \frac{be^{(-3\,dx-3\,c)}}{24\,d}$$

input
```
integrate(cosh(d*x+c)*(a+b*sinh(d*x+c)^2),x, algorithm="giac")
```

output
```
1/24*b*e^(3*d*x + 3*c)/d + 1/8*(4*a - b)*e^(d*x + c)/d - 1/8*(4*a - b)*e^(
-d*x - c)/d - 1/24*b*e^(-3*d*x - 3*c)/d
```

Mupad [B] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.89

$$\int \cosh(c+dx)\left(a+b\sinh^2(c+dx)\right)\,dx=\frac{\sinh(c+d\,x)\,\left(b\sinh(c+d\,x)^2+3\,a\right)}{3\,d}$$

input
```
int(cosh(c + d*x)*(a + b*sinh(c + d*x)^2),x)
```

output
```
(sinh(c + d*x)*(3*a + b*sinh(c + d*x)^2))/(3*d)
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.89

$$\int \cosh(c+dx)\left(a+b\sinh^2(c+dx)\right)\,dx=\frac{\sinh(dx+c)\left(\sinh(dx+c)^2b+3a\right)}{3d}$$

input
```
int(cosh(d*x+c)*(a+b*sinh(d*x+c)^2),x)
```

output
```
(sinh(c + d*x)*(sinh(c + d*x)**2*b + 3*a))/(3*d)
```

3.251 $\int \operatorname{sech}(c+dx)\left(a+b\sinh^2(c+dx)\right)\,dx$

Optimal result

Integrand size = 19, antiderivative size = 28

$$\int \operatorname{sech}(c+dx)\left(a+b\sinh^2(c+dx)\right)\,dx=\frac{(a-b)\arctan(\sinh(c+dx))}{d}+\frac{b\sinh(c+dx)}{d}$$

output
```
(a-b)*arctan(sinh(d*x+c))/d+b*sinh(d*x+c)/d
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 38, normalized size of antiderivative = 1.36

$$\int \operatorname{sech}(c+dx)\left(a+b\sinh^2(c+dx)\right)\,dx=-\frac{a\cot^{-1}(\sinh(c+dx))}{d}-\frac{b\arctan(\sinh(c+dx))}{d}+\frac{b\sinh(c+dx)}{d}$$

input
```
Integrate[Sech[c + d*x]*(a + b*Sinh[c + d*x]^2),x]
```

output
```
-((a*ArcCot[Sinh[c + d*x]])/d) - (b*ArcTan[Sinh[c + d*x]])/d + (b*Sinh[c +
 d*x])/d
```

Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.93, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.211$, Rules used = {3042, 3669, 299, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \operatorname{sech}(c+dx)\left(a+b\sinh^2(c+dx)\right)\,dx$$

$$\Big\downarrow \ 3042$$

$$\int \frac{a-b\sin(ic+idx)^2}{\cos(ic+idx)}dx$$

$$\Big\downarrow \ 3669$$

$$\frac{\int \frac{b\sinh^2(c+dx)+a}{\sinh^2(c+dx)+1}d\sinh(c+dx)}{d}$$

$$\Big\downarrow \ 299$$

$$\frac{(a-b)\int \frac{1}{\sinh^2(c+dx)+1}d\sinh(c+dx)+b\sinh(c+dx)}{d}$$

$$\Big\downarrow \ 216$$

$$\frac{(a-b)\arctan(\sinh(c+dx))+b\sinh(c+dx)}{d}$$

input
```
Int[Sech[c + d*x]*(a + b*Sinh[c + d*x]^2),x]
```

output
```
((a - b)*ArcTan[Sinh[c + d*x]] + b*Sinh[c + d*x])/d
```

Defintions of rubi rules used

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 299
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2), x_Symbol] :> Simp[d*x
*((a + b*x^2)^(p + 1)/(b*(2*p + 3))), x] - Simp[(a*d - b*c*(2*p + 3))/(b*(2
*p + 3))   Int[(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
 a*d, 0] && NeQ[2*p + 3, 0]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3669
```
Int[cos[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^
(p_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Simp[ff/f   S
ubst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a + b*ff^2*x^2)^p, x], x, Sin[e + f*x]
/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m - 1)/2]
```

Maple [A] (verified)

Time = 2.88 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.21

method	result	size
derivativedivides	$\frac{2a\arctan\left(e^{dx+c}\right)+b\left(\sinh\left(dx+c\right)-2\arctan\left(e^{dx+c}\right)\right)}{d}$	34
default	$\frac{2a\arctan\left(e^{dx+c}\right)+b\left(\sinh\left(dx+c\right)-2\arctan\left(e^{dx+c}\right)\right)}{d}$	34
risch	$\frac{e^{dx+c}b}{2d}-\frac{e^{-dx-c}b}{2d}+\frac{i\ln\left(e^{dx+c}+i\right)a}{d}-\frac{i\ln\left(e^{dx+c}+i\right)b}{d}-\frac{i\ln\left(e^{dx+c}-i\right)a}{d}+\frac{i\ln\left(e^{dx+c}-i\right)b}{d}$	97

input
```
int(sech(d*x+c)*(a+b*sinh(d*x+c)^2),x,method=_RETURNVERBOSE)
```

output
```
1/d*(2*a*arctan(exp(d*x+c))+b*(sinh(d*x+c)-2*arctan(exp(d*x+c))))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 101 vs. $2(28) = 56$.

Time = 0.11 (sec) , antiderivative size = 101, normalized size of antiderivative = 3.61

$$\int \operatorname{sech}(c+dx)\left(a+b\sinh^2(c+dx)\right)\,dx = \frac{b\cosh(dx+c)^2 + 2\,b\cosh(dx+c)\sinh(dx+c) + b\sinh(dx+c)^2 + 4\left((a-b)\cosh(dx+c) + (a-b)\,\mathrm{s}\right.}{2\left(d\cosh(dx+c) + d\sinh(dx+c)\right)}$$

input
```
integrate(sech(d*x+c)*(a+b*sinh(d*x+c)^2),x, algorithm="fricas")
```

output
```
1/2*(b*cosh(d*x + c)^2 + 2*b*cosh(d*x + c)*sinh(d*x + c) + b*sinh(d*x + c)
^2 + 4*((a - b)*cosh(d*x + c) + (a - b)*sinh(d*x + c))*arctan(cosh(d*x + c
) + sinh(d*x + c)) - b)/(d*cosh(d*x + c) + d*sinh(d*x + c))
```

Sympy [F]

$$\int \operatorname{sech}(c+dx)\left(a+b\sinh^2(c+dx)\right)\,dx = \int \left(a+b\sinh^2(c+dx)\right)\operatorname{sech}(c+dx)\,dx$$

input
```
integrate(sech(d*x+c)*(a+b*sinh(d*x+c)**2),x)
```

output
```
Integral((a + b*sinh(c + d*x)**2)*sech(c + d*x), x)
```

Maxima [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 56, normalized size of antiderivative = 2.00

$$\int \operatorname{sech}(c+dx)\left(a+b\sinh^2(c+dx)\right)\,dx = \frac{1}{2}\,b\left(\frac{4\arctan\left(e^{(-dx-c)}\right)}{d} + \frac{e^{(dx+c)}}{d} - \frac{e^{(-dx-c)}}{d}\right) + \frac{a\arctan(\sinh(dx+c))}{d}$$

input
```
integrate(sech(d*x+c)*(a+b*sinh(d*x+c)^2),x, algorithm="maxima")
```

output
```
1/2*b*(4*arctan(e^(-d*x - c))/d + e^(d*x + c)/d - e^(-d*x - c)/d) + a*arct
an(sinh(d*x + c))/d
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 40, normalized size of antiderivative = 1.43

$$\int \operatorname{sech}(c+dx)\left(a+b\sinh^2(c+dx)\right)\,dx=\frac{4\,(a-b)\arctan\left(e^{(dx+c)}\right)+be^{(dx+c)}-be^{(-dx-c)}}{2\,d}$$

input
```
integrate(sech(d*x+c)*(a+b*sinh(d*x+c)^2),x, algorithm="giac")
```

output
```
1/2*(4*(a - b)*arctan(e^(d*x + c)) + b*e^(d*x + c) - b*e^(-d*x - c))/d
```

Mupad [B] (verification not implemented)

Time = 2.00 (sec) , antiderivative size = 88, normalized size of antiderivative = 3.14

$$\int \operatorname{sech}(c+dx)\left(a+b\sinh^2(c+dx)\right)\,dx=\frac{2\,\mathrm{atan}\left(\frac{e^{d\,x}\,e^{c}\,\left(a\,\sqrt{d^2}-b\,\sqrt{d^2}\right)}{d\,\sqrt{a^2-2\,a\,b+b^2}}\right)\,\sqrt{a^2-2\,a\,b+b^2}}{\sqrt{d^2}}-\frac{b\,e^{-c-d\,x}}{2\,d}+\frac{b\,e^{c+d\,x}}{2\,d}$$

input
```
int((a + b*sinh(c + d*x)^2)/cosh(c + d*x),x)
```

output
```
(2*atan((exp(d*x)*exp(c)*(a*(d^2)^(1/2) - b*(d^2)^(1/2)))/(d*(a^2 - 2*a*b
+ b^2)^(1/2)))*(a^2 - 2*a*b + b^2)^(1/2))/(d^2)^(1/2) - (b*exp(- c - d*x))
/(2*d) + (b*exp(c + d*x))/(2*d)
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 66, normalized size of antiderivative = 2.36

$$\int \operatorname{sech}(c+dx)\left(a+b\sinh^2(c+dx)\right)\,dx = \frac{4e^{dx+c}atan\left(e^{dx+c}\right)a-4e^{dx+c}atan\left(e^{dx+c}\right)b+e^{2dx+2c}b-b}{2e^{dx+c}d}$$

input
```
int(sech(d*x+c)*(a+b*sinh(d*x+c)^2),x)
```

output
```
(4*e**(c + d*x)*atan(e**(c + d*x))*a - 4*e**(c + d*x)*atan(e**(c + d*x))*b
 + e**(2*c + 2*d*x)*b - b)/(2*e**(c + d*x)*d)
```

3.252 $\int \operatorname{sech}^2(c+dx)\left(a+b\sinh^2(c+dx)\right)\,dx$

Optimal result

Integrand size = 21, antiderivative size = 19

$$\int \operatorname{sech}^2(c+dx)\left(a+b\sinh^2(c+dx)\right)\,dx=bx+\frac{(a-b)\tanh(c+dx)}{d}$$

output
```
b*x+(a-b)*tanh(d*x+c)/d
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.89

$$\int \operatorname{sech}^2(c+dx)\left(a+b\sinh^2(c+dx)\right)\,dx=\frac{b\operatorname{arctanh}(\tanh(c+dx))}{d}+\frac{a\tanh(c+dx)}{d}-\frac{b\tanh(c+dx)}{d}$$

input
```
Integrate[Sech[c + d*x]^2*(a + b*Sinh[c + d*x]^2),x]
```

output
```
(b*ArcTanh[Tanh[c + d*x]])/d + (a*Tanh[c + d*x])/d - (b*Tanh[c + d*x])/d
```

Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.37, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.190$, Rules used = {3042, 3670, 299, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \operatorname{sech}^2(c+dx) \left(a+b\sinh^2(c+dx)\right) \, dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \frac{a-b\sin(ic+idx)^2}{\cos(ic+idx)^2}dx$$

$$\Big\downarrow \text{ 3670}$$

$$\frac{\int \frac{a-(a-b)\tanh^2(c+dx)}{1-\tanh^2(c+dx)}d\tanh(c+dx)}{d}$$

$$\Big\downarrow \text{ 299}$$

$$\frac{b\int \frac{1}{1-\tanh^2(c+dx)}d\tanh(c+dx)+(a-b)\tanh(c+dx)}{d}$$

$$\Big\downarrow \text{ 219}$$

$$\frac{(a-b)\tanh(c+dx)+b\text{arctanh}(\tanh(c+dx))}{d}$$

input
```
Int[Sech[c + d*x]^2*(a + b*Sinh[c + d*x]^2),x]
```

output
```
(b*ArcTanh[Tanh[c + d*x]] + (a - b)*Tanh[c + d*x])/d
```

Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 299
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2), x_Symbol] :> Simp[d*x
*((a + b*x^2)^(p + 1)/(b*(2*p + 3))), x] - Simp[(a*d - b*c*(2*p + 3))/(b*(2
*p + 3))   Int[(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
 a*d, 0] && NeQ[2*p + 3, 0]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3670
```
Int[cos[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(
p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Simp[ff/f   Su
bst[Int[(a + (a + b)*ff^2*x^2)^p/(1 + ff^2*x^2)^(m/2 + p + 1), x], x, Tan[e
 + f*x]/ff], x]] /; FreeQ[{a, b, e, f}, x] && IntegerQ[m/2] && IntegerQ[p]
```

Maple [A] (verified)

Time = 1.10 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.53

method	result	size
derivativedivides	$\frac{\tanh(dx+c)a+b(dx+c-\tanh(dx+c))}{d}$	29
default	$\frac{\tanh(dx+c)a+b(dx+c-\tanh(dx+c))}{d}$	29
parallelrisch	$\frac{b(dx-\tanh(dx+c))\cosh(dx+c)+a\sinh(dx+c)}{d\cosh(dx+c)}$	42
risch	$bx-\frac{2a}{d(e^{2dx+2c}+1)}+\frac{2b}{d(e^{2dx+2c}+1)}$	43

input
```
int(sech(d*x+c)^2*(a+b*sinh(d*x+c)^2),x,method=_RETURNVERBOSE)
```

output
```
1/d*(tanh(d*x+c)*a+b*(d*x+c-tanh(d*x+c)))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 41 vs. $2(19) = 38$.

Time = 0.07 (sec) , antiderivative size = 41, normalized size of antiderivative = 2.16

$$\int \operatorname{sech}^2(c+dx) \left(a+b\sinh^2(c+dx)\right) \, dx = \frac{(bdx-a+b)\cosh(dx+c)+(a-b)\sinh(dx+c)}{d\cosh(dx+c)}$$

input
```
integrate(sech(d*x+c)^2*(a+b*sinh(d*x+c)^2),x, algorithm="fricas")
```

output
```
((b*d*x - a + b)*cosh(d*x + c) + (a - b)*sinh(d*x + c))/(d*cosh(d*x + c))
```

Sympy [F]

$$\int \operatorname{sech}^2(c+dx) \left(a+b\sinh^2(c+dx)\right) \, dx = \int \left(a+b\sinh^2(c+dx)\right)\operatorname{sech}^2(c+dx)\, dx$$

input
```
integrate(sech(d*x+c)**2*(a+b*sinh(d*x+c)**2),x)
```

output
```
Integral((a + b*sinh(c + d*x)**2)*sech(c + d*x)**2, x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 47 vs. $2(19) = 38$.

Time = 0.04 (sec) , antiderivative size = 47, normalized size of antiderivative = 2.47

$$\int \operatorname{sech}^2(c+dx) \left(a+b\sinh^2(c+dx)\right) \, dx = b\left(x+\frac{c}{d}-\frac{2}{d\left(e^{(-2\,dx-2\,c)}+1\right)}\right) + \frac{2\,a}{d\left(e^{(-2\,dx-2\,c)}+1\right)}$$

input
```
integrate(sech(d*x+c)^2*(a+b*sinh(d*x+c)^2),x, algorithm="maxima")
```

output
```
b*(x + c/d - 2/(d*(e^(-2*d*x - 2*c) + 1))) + 2*a/(d*(e^(-2*d*x - 2*c) + 1)
)
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.68

$$\int \operatorname{sech}^2(c+dx)\left(a+b\sinh^2(c+dx)\right)\,dx=\frac{(dx+c)b-\frac{2(a-b)}{e^{(2\,dx+2\,c)}+1}}{d}$$

input
```
integrate(sech(d*x+c)^2*(a+b*sinh(d*x+c)^2),x, algorithm="giac")
```

output
```
((d*x + c)*b - 2*(a - b)/(e^(2*d*x + 2*c) + 1))/d
```

Mupad [B] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.42

$$\int \operatorname{sech}^2(c+dx)\left(a+b\sinh^2(c+dx)\right)\,dx=b\,x-\frac{2\,(a-b)}{d\,\left(\mathrm{e}^{2\,c+2\,d\,x}+1\right)}$$

input
```
int((a + b*sinh(c + d*x)^2)/cosh(c + d*x)^2,x)
```

output
```
b*x - (2*(a - b))/(d*(exp(2*c + 2*d*x) + 1))
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 63, normalized size of antiderivative = 3.32

$$\int \operatorname{sech}^2(c+dx)\left(a+b\sinh^2(c+dx)\right)\,dx=\frac{2e^{2dx+2c}a+e^{2dx+2c}bdx-2e^{2dx+2c}b+bdx}{d\left(e^{2dx+2c}+1\right)}$$

input
```
int(sech(d*x+c)^2*(a+b*sinh(d*x+c)^2),x)
```

output
```
(2*e**(2*c + 2*d*x)*a + e**(2*c + 2*d*x)*b*d*x - 2*e**(2*c + 2*d*x)*b + b*
d*x)/(d*(e**(2*c + 2*d*x) + 1))
```

3.253 $\int \operatorname{sech}^3(c+dx)\left(a+b\sinh^2(c+dx)\right)\,dx$

Optimal result

Integrand size = 21, antiderivative size = 42

$$\int \operatorname{sech}^3(c+dx)\left(a+b\sinh^2(c+dx)\right)\,dx=\frac{(a+b)\arctan(\sinh(c+dx))}{2d}+\frac{(a-b)\operatorname{sech}(c+dx)\tanh(c+dx)}{2d}$$

output
```
1/2*(a+b)*arctan(sinh(d*x+c))/d+1/2*(a-b)*sech(d*x+c)*tanh(d*x+c)/d
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 71, normalized size of antiderivative = 1.69

$$\int \operatorname{sech}^3(c+dx)\left(a+b\sinh^2(c+dx)\right)\,dx=\frac{a\arctan(\sinh(c+dx))}{2d}+\frac{b\arctan(\sinh(c+dx))}{2d}+\frac{a\operatorname{sech}(c+dx)\tanh(c+dx)}{2d}-\frac{b\operatorname{sech}(c+dx)\tanh(c+dx)}{2d}$$

input
```
Integrate[Sech[c + d*x]^3*(a + b*Sinh[c + d*x]^2),x]
```

output
```
(a*ArcTan[Sinh[c + d*x]])/(2*d) + (b*ArcTan[Sinh[c + d*x]])/(2*d) + (a*Sec
h[c + d*x]*Tanh[c + d*x])/(2*d) - (b*Sech[c + d*x]*Tanh[c + d*x])/(2*d)
```

Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.10, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.190$, Rules used = {3042, 3669, 298, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \operatorname{sech}^3(c+dx) \left(a+b\sinh^2(c+dx)\right) \, dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{a-b\sin(ic+idx)^2}{\cos(ic+idx)^3}dx$$

$$\Big\downarrow \text{3669}$$

$$\frac{\int \frac{b\sinh^2(c+dx)+a}{\left(\sinh^2(c+dx)+1\right)^2}d\sinh(c+dx)}{d}$$

$$\Big\downarrow \text{298}$$

$$\frac{\frac{1}{2}(a+b)\int \frac{1}{\sinh^2(c+dx)+1}d\sinh(c+dx)+\frac{(a-b)\sinh(c+dx)}{2\left(\sinh^2(c+dx)+1\right)}}{d}$$

$$\Big\downarrow \text{216}$$

$$\frac{\frac{1}{2}(a+b)\arctan(\sinh(c+dx))+\frac{(a-b)\sinh(c+dx)}{2\left(\sinh^2(c+dx)+1\right)}}{d}$$

input
```
Int[Sech[c + d*x]^3*(a + b*Sinh[c + d*x]^2),x]
```

output
```
(((a + b)*ArcTan[Sinh[c + d*x]])/2 + ((a - b)*Sinh[c + d*x])/(2*(1 + Sinh[
c + d*x]^2)))/d
```

Defintions of rubi rules used

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 298
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2), x_Symbol] :> Simp[(-(
b*c - a*d))*x*((a + b*x^2)^(p + 1)/(2*a*b*(p + 1))), x] - Simp[(a*d - b*c*(
2*p + 3))/(2*a*b*(p + 1))   Int[(a + b*x^2)^(p + 1), x], x] /; FreeQ[{a, b,
 c, d, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/2 + p, 0])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3669
```
Int[cos[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^
(p_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Simp[ff/f   S
ubst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a + b*ff^2*x^2)^p, x], x, Sin[e + f*x]
/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m - 1)/2]
```

Maple [A] (verified)

Time = 6.71 (sec) , antiderivative size = 70, normalized size of antiderivative = 1.67

method	result	size
derivativedivides	$\frac{a\left(\frac{\operatorname{sech}(dx+c)\tanh(dx+c)}{2}+\arctan\left(e^{dx+c}\right)\right)+b\left(-\frac{\sinh(dx+c)}{\cosh(dx+c)^2}+\frac{\operatorname{sech}(dx+c)\tanh(dx+c)}{2}+\arctan\left(e^{dx+c}\right)\right)}{d}$	70
default	$\frac{a\left(\frac{\operatorname{sech}(dx+c)\tanh(dx+c)}{2}+\arctan\left(e^{dx+c}\right)\right)+b\left(-\frac{\sinh(dx+c)}{\cosh(dx+c)^2}+\frac{\operatorname{sech}(dx+c)\tanh(dx+c)}{2}+\arctan\left(e^{dx+c}\right)\right)}{d}$	70
risch	$\frac{e^{dx+c}(a-b)\left(e^{2dx+2c}-1\right)}{d\left(e^{2dx+2c}+1\right)^2}+\frac{i\ln\left(e^{dx+c}+i\right)a}{2d}+\frac{i\ln\left(e^{dx+c}+i\right)b}{2d}-\frac{i\ln\left(e^{dx+c}-i\right)a}{2d}-\frac{i\ln\left(e^{dx+c}-i\right)b}{2d}$	109

input
```
int(sech(d*x+c)^3*(a+b*sinh(d*x+c)^2),x,method=_RETURNVERBOSE)
```

output
```
1/d*(a*(1/2*sech(d*x+c)*tanh(d*x+c)+arctan(exp(d*x+c)))+b*(-sinh(d*x+c)/co
sh(d*x+c)^2+1/2*sech(d*x+c)*tanh(d*x+c)+arctan(exp(d*x+c))))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 324 vs. $2(38) = 76$.

Time = 0.10 (sec) , antiderivative size = 324, normalized size of antiderivative = 7.71

$$\int \operatorname{sech}^3(c+dx)\left(a+b\sinh^2(c+dx)\right)\,dx = \frac{(a-b)\cosh(dx+c)^3+3(a-b)\cosh(dx+c)\sinh(dx+c)^2+(a-b)\sinh(dx+c)^3+((a+b)\cosh(d}{}$$

input
```
integrate(sech(d*x+c)^3*(a+b*sinh(d*x+c)^2),x, algorithm="fricas")
```

output
```
((a - b)*cosh(d*x + c)^3 + 3*(a - b)*cosh(d*x + c)*sinh(d*x + c)^2 + (a -
b)*sinh(d*x + c)^3 + ((a + b)*cosh(d*x + c)^4 + 4*(a + b)*cosh(d*x + c)*si
nh(d*x + c)^3 + (a + b)*sinh(d*x + c)^4 + 2*(a + b)*cosh(d*x + c)^2 + 2*(3
*(a + b)*cosh(d*x + c)^2 + a + b)*sinh(d*x + c)^2 + 4*((a + b)*cosh(d*x +
c)^3 + (a + b)*cosh(d*x + c))*sinh(d*x + c) + a + b)*arctan(cosh(d*x + c)
+ sinh(d*x + c)) - (a - b)*cosh(d*x + c) + (3*(a - b)*cosh(d*x + c)^2 - a
+ b)*sinh(d*x + c))/(d*cosh(d*x + c)^4 + 4*d*cosh(d*x + c)*sinh(d*x + c)^3
 + d*sinh(d*x + c)^4 + 2*d*cosh(d*x + c)^2 + 2*(3*d*cosh(d*x + c)^2 + d)*s
inh(d*x + c)^2 + 4*(d*cosh(d*x + c)^3 + d*cosh(d*x + c))*sinh(d*x + c) + d
)
```

Sympy [F]

$$\int \operatorname{sech}^3(c+dx)\left(a+b\sinh^2(c+dx)\right)\,dx = \int \left(a+b\sinh^2(c+dx)\right)\operatorname{sech}^3(c+dx)\,dx$$

input
```
integrate(sech(d*x+c)**3*(a+b*sinh(d*x+c)**2),x)
```

output
```
Integral((a + b*sinh(c + d*x)**2)*sech(c + d*x)**3, x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 136 vs. $2(38) = 76$.

Time = 0.14 (sec) , antiderivative size = 136, normalized size of antiderivative = 3.24

$$\int \operatorname{sech}^3(c+dx)\left(a+b\sinh^2(c+dx)\right)\,dx = -b\left(\frac{\arctan\left(e^{(-dx-c)}\right)}{d}+\frac{e^{(-dx-c)}-e^{(-3\,dx-3\,c)}}{d(2\,e^{(-2\,dx-2\,c)}+e^{(-4\,dx-4\,c)}+1)}\right) - a\left(\frac{\arctan\left(e^{(-dx-c)}\right)}{d}-\frac{e^{(-dx-c)}-e^{(-3\,dx-3\,c)}}{d(2\,e^{(-2\,dx-2\,c)}+e^{(-4\,dx-4\,c)}+1)}\right)$$

input
```
integrate(sech(d*x+c)^3*(a+b*sinh(d*x+c)^2),x, algorithm="maxima")
```

output
```
-b*(arctan(e^(-d*x - c))/d + (e^(-d*x - c) - e^(-3*d*x - 3*c))/(d*(2*e^(-2
*d*x - 2*c) + e^(-4*d*x - 4*c) + 1))) - a*(arctan(e^(-d*x - c))/d - (e^(-d
*x - c) - e^(-3*d*x - 3*c))/(d*(2*e^(-2*d*x - 2*c) + e^(-4*d*x - 4*c) + 1)
))
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 105 vs. $2(38) = 76$.

Time = 0.14 (sec) , antiderivative size = 105, normalized size of antiderivative = 2.50

$$\int \operatorname{sech}^3(c+dx)\left(a+b\sinh^2(c+dx)\right)\,dx = \frac{\left(\pi+2\,\arctan\left(\frac{1}{2}\left(e^{(2\,dx+2\,c)}-1\right)e^{(-dx-c)}\right)\right)(a+b)+\frac{4\left(a\left(e^{(dx+c)}-e^{(-dx-c)}\right)-b\left(e^{(dx+c)}-e^{(-dx-c)}\right)\right)}{\left(e^{(dx+c)}-e^{(-dx-c)}\right)^2+4}}{4\,d}$$

input
```
integrate(sech(d*x+c)^3*(a+b*sinh(d*x+c)^2),x, algorithm="giac")
```

output
```
1/4*((pi + 2*arctan(1/2*(e^(2*d*x + 2*c) - 1)*e^(-d*x - c)))*(a + b) + 4*(
a*(e^(d*x + c) - e^(-d*x - c)) - b*(e^(d*x + c) - e^(-d*x - c)))/((e^(d*x
+ c) - e^(-d*x - c))^2 + 4))/d
```

Mupad [B] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 127, normalized size of antiderivative = 3.02

$$\int \operatorname{sech}^3(c+dx)\left(a+b\sinh^2(c+dx)\right)\,dx=\frac{\operatorname{atan}\left(\frac{\mathrm{e}^{d\,x}\,\mathrm{e}^c\,\left(a\,\sqrt{d^2}+b\,\sqrt{d^2}\right)}{d\,\sqrt{a^2+2\,a\,b+b^2}}\right)\,\sqrt{a^2+2\,a\,b+b^2}}{\sqrt{d^2}}+\frac{\mathrm{e}^{c+d\,x}\,\left(a-b\right)}{d\,\left(\mathrm{e}^{2\,c+2\,d\,x}+1\right)}-\frac{2\,\mathrm{e}^{c+d\,x}\,\left(a-b\right)}{d\,\left(2\,\mathrm{e}^{2\,c+2\,d\,x}+\mathrm{e}^{4\,c+4\,d\,x}+1\right)}$$

input
```
int((a + b*sinh(c + d*x)^2)/cosh(c + d*x)^3,x)
```

output
```
(atan((exp(d*x)*exp(c)*(a*(d^2)^(1/2) + b*(d^2)^(1/2)))/(d*(2*a*b + a^2 +
b^2)^(1/2)))*(2*a*b + a^2 + b^2)^(1/2))/(d^2)^(1/2) + (exp(c + d*x)*(a - b
))/(d*(exp(2*c + 2*d*x) + 1)) - (2*exp(c + d*x)*(a - b))/(d*(2*exp(2*c + 2
*d*x) + exp(4*c + 4*d*x) + 1))
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 177, normalized size of antiderivative = 4.21

$$\int \operatorname{sech}^3(c+dx)\left(a+b\sinh^2(c+dx)\right)\,dx=\frac{e^{4dx+4c}atan\left(e^{dx+c}\right)a+e^{4dx+4c}atan\left(e^{dx+c}\right)b+2e^{2dx+2c}atan\left(e^{dx+c}\right)a+2e^{2dx+2c}atan\left(e^{dx+c}\right)b+atan\left(e^{dx}\right.}{d\left(e^{4dx+4c}+2e^{2dx+2c}+1\right)}$$

input
```
int(sech(d*x+c)^3*(a+b*sinh(d*x+c)^2),x)
```

output
```
(e**(4*c + 4*d*x)*atan(e**(c + d*x))*a + e**(4*c + 4*d*x)*atan(e**(c + d*x
))*b + 2*e**(2*c + 2*d*x)*atan(e**(c + d*x))*a + 2*e**(2*c + 2*d*x)*atan(e
**(c + d*x))*b + atan(e**(c + d*x))*a + atan(e**(c + d*x))*b + e**(3*c + 3
*d*x)*a - e**(3*c + 3*d*x)*b - e**(c + d*x)*a + e**(c + d*x)*b)/(d*(e**(4*
c + 4*d*x) + 2*e**(2*c + 2*d*x) + 1))
```

3.254 $\int \operatorname{sech}^4(c+dx) \left(a+b\sinh^2(c+dx)\right) \, dx$

Optimal result

Integrand size = 21, antiderivative size = 32

$$\int \operatorname{sech}^4(c+dx) \left(a+b\sinh^2(c+dx)\right) \, dx = \frac{a\tanh(c+dx)}{d} - \frac{(a-b)\tanh^3(c+dx)}{3d}$$

output
```
a*tanh(d*x+c)/d-1/3*(a-b)*tanh(d*x+c)^3/d
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 44, normalized size of antiderivative = 1.38

$$\int \operatorname{sech}^4(c+dx) \left(a+b\sinh^2(c+dx)\right) \, dx = \frac{a\tanh(c+dx)}{d} - \frac{a\tanh^3(c+dx)}{3d} + \frac{b\tanh^3(c+dx)}{3d}$$

input
```
Integrate[Sech[c + d*x]^4*(a + b*Sinh[c + d*x]^2),x]
```

output
```
(a*Tanh[c + d*x])/d - (a*Tanh[c + d*x]^3)/(3*d) + (b*Tanh[c + d*x]^3)/(3*d
)
```

Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 30, normalized size of antiderivative = 0.94, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {3042, 3670, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \operatorname{sech}^4(c+dx)\left(a+b\sinh^2(c+dx)\right)dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{a-b\sin(ic+idx)^2}{\cos(ic+idx)^4}dx$$

$$\Big\downarrow \text{3670}$$

$$\frac{\int \left(a-(a-b)\tanh^2(c+dx)\right)d\tanh(c+dx)}{d}$$

$$\Big\downarrow \text{2009}$$

$$\frac{a\tanh(c+dx)-\frac{1}{3}(a-b)\tanh^3(c+dx)}{d}$$

input
```
Int[Sech[c + d*x]^4*(a + b*Sinh[c + d*x]^2),x]
```

output
```
(a*Tanh[c + d*x] - ((a - b)*Tanh[c + d*x]^3)/3)/d
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3670
```
Int[cos[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(
p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Simp[ff/f   Su
bst[Int[(a + (a + b)*ff^2*x^2)^p/(1 + ff^2*x^2)^(m/2 + p + 1), x], x, Tan[e
 + f*x]/ff], x]] /; FreeQ[{a, b, e, f}, x] && IntegerQ[m/2] && IntegerQ[p]
```

Maple [A] (verified)

Time = 17.01 (sec) , antiderivative size = 48, normalized size of antiderivative = 1.50

method	result	size
risch	$-\frac{2\left(3 b\,{\mathrm e}^{4 d x +4 c}+6\,{\mathrm e}^{2 d x +2 c} a +2 a +b \right)}{3 d \left({\mathrm e}^{2 d x +2 c}+1\right)^{3}}$	48
derivativedivides	$\frac{a \left(\frac{2}{3}+\frac{\operatorname{sech}\left(d x +c \right)^{2}}{3}\right) \tanh \left(d x +c \right)+b \left(-\frac{\sinh \left(d x +c \right)}{2 \cosh \left(d x +c \right)^{3}}+\frac{\left(\frac{2}{3}+\frac{\operatorname{sech}\left(d x +c \right)^{2}}{3}\right) \tanh \left(d x +c \right)}{2}\right)}{d}$	65
default	$\frac{a \left(\frac{2}{3}+\frac{\operatorname{sech}\left(d x +c \right)^{2}}{3}\right) \tanh \left(d x +c \right)+b \left(-\frac{\sinh \left(d x +c \right)}{2 \cosh \left(d x +c \right)^{3}}+\frac{\left(\frac{2}{3}+\frac{\operatorname{sech}\left(d x +c \right)^{2}}{3}\right) \tanh \left(d x +c \right)}{2}\right)}{d}$	65

input
```
int(sech(d*x+c)^4*(a+b*sinh(d*x+c)^2),x,method=_RETURNVERBOSE)
```

output
```
-2/3*(3*b*exp(4*d*x+4*c)+6*exp(2*d*x+2*c)*a+2*a+b)/d/(exp(2*d*x+2*c)+1)^3
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 159 vs. $2(30) = 60$.

Time = 0.07 (sec) , antiderivative size = 159, normalized size of antiderivative = 4.97

$$\int \operatorname{sech}^4(c+dx) \left(a+b\sinh^2(c+dx)\right) \, dx =$$

$$-\frac{4\left((a+2\,b)\cosh\left(dx+c\right)^2-2\,(a-b)\cosh\left(dx+c\right)s}{3\left(d\cosh\left(dx+c\right)^4+4\,d\cosh\left(dx+c\right)\sinh\left(dx+c\right)^3+d\sinh\left(dx+c\right)^4+4\,d\cosh\left(dx+c\right)^2+2\,(3\,d\,c}$$

input
```
integrate(sech(d*x+c)^4*(a+b*sinh(d*x+c)^2),x, algorithm="fricas")
```

output
```
-4/3*((a + 2*b)*cosh(d*x + c)^2 - 2*(a - b)*cosh(d*x + c)*sinh(d*x + c) +
(a + 2*b)*sinh(d*x + c)^2 + 3*a)/(d*cosh(d*x + c)^4 + 4*d*cosh(d*x + c)*si
nh(d*x + c)^3 + d*sinh(d*x + c)^4 + 4*d*cosh(d*x + c)^2 + 2*(3*d*cosh(d*x
+ c)^2 + 2*d)*sinh(d*x + c)^2 + 4*(d*cosh(d*x + c)^3 + d*cosh(d*x + c))*si
nh(d*x + c) + 3*d)
```

Sympy [F]

$$\int \operatorname{sech}^4(c+dx)\left(a+b\sinh^2(c+dx)\right)\,dx=\int \left(a+b\sinh^2{(c+dx)}\right)\operatorname{sech}^4{(c+dx)}\,dx$$

input
```
integrate(sech(d*x+c)**4*(a+b*sinh(d*x+c)**2),x)
```

output
```
Integral((a + b*sinh(c + d*x)**2)*sech(c + d*x)**4, x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 185 vs. $2(30) = 60$.

Time = 0.04 (sec) , antiderivative size = 185, normalized size of antiderivative = 5.78

$$\int \operatorname{sech}^4(c+dx)\left(a+b\sinh^2(c+dx)\right)\,dx$$
$$=\frac{4}{3}a\left(\frac{3e^{(-2dx-2c)}}{d(3e^{(-2dx-2c)}+3e^{(-4dx-4c)}+e^{(-6dx-6c)}+1)}+\frac{1}{d(3e^{(-2dx-2c)}+3e^{(-4dx-4c)}+e^{(-6dx-6c)}+1)}\right)$$
$$+\frac{2}{3}b\left(\frac{3e^{(-4dx-4c)}}{d(3e^{(-2dx-2c)}+3e^{(-4dx-4c)}+e^{(-6dx-6c)}+1)}+\frac{1}{d(3e^{(-2dx-2c)}+3e^{(-4dx-4c)}+e^{(-6dx-6c)}+1)}\right)$$

input
```
integrate(sech(d*x+c)^4*(a+b*sinh(d*x+c)^2),x, algorithm="maxima")
```

output
```
4/3*a*(3*e^(-2*d*x - 2*c)/(d*(3*e^(-2*d*x - 2*c) + 3*e^(-4*d*x - 4*c) + e^
(-6*d*x - 6*c) + 1)) + 1/(d*(3*e^(-2*d*x - 2*c) + 3*e^(-4*d*x - 4*c) + e^(
-6*d*x - 6*c) + 1))) + 2/3*b*(3*e^(-4*d*x - 4*c)/(d*(3*e^(-2*d*x - 2*c) +
3*e^(-4*d*x - 4*c) + e^(-6*d*x - 6*c) + 1)) + 1/(d*(3*e^(-2*d*x - 2*c) + 3
*e^(-4*d*x - 4*c) + e^(-6*d*x - 6*c) + 1)))
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 47, normalized size of antiderivative = 1.47

$$\int \operatorname{sech}^4(c+dx)\left(a+b\sinh^2(c+dx)\right)\,dx=-\frac{2\left(3\,be^{(4\,dx+4\,c)}+6\,ae^{(2\,dx+2\,c)}+2\,a+b\right)}{3\,d\left(e^{(2\,dx+2\,c)}+1\right)^3}$$

input
```
integrate(sech(d*x+c)^4*(a+b*sinh(d*x+c)^2),x, algorithm="giac")
```

output
```
-2/3*(3*b*e^(4*d*x + 4*c) + 6*a*e^(2*d*x + 2*c) + 2*a + b)/(d*(e^(2*d*x +
2*c) + 1)^3)
```

Mupad [B] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 47, normalized size of antiderivative = 1.47

$$\int \operatorname{sech}^4(c+dx)\left(a+b\sinh^2(c+dx)\right)\,dx=-\frac{2\left(2\,a+b+6\,a\,\mathrm{e}^{2\,c+2\,d\,x}+3\,b\,\mathrm{e}^{4\,c+4\,d\,x}\right)}{3\,d\,\left(\mathrm{e}^{2\,c+2\,d\,x}+1\right)^3}$$

input
```
int((a + b*sinh(c + d*x)^2)/cosh(c + d*x)^4,x)
```

output
```
-(2*(2*a + b + 6*a*exp(2*c + 2*d*x) + 3*b*exp(4*c + 4*d*x)))/(3*d*(exp(2*c
 + 2*d*x) + 1)^3)
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 85, normalized size of antiderivative = 2.66

$$\int \operatorname{sech}^4(c+dx)\left(a+b\sinh^2(c+dx)\right)\,dx=\frac{\frac{2e^{6dx+6c}b}{3}-4e^{2dx+2c}a+2e^{2dx+2c}b-\frac{4a}{3}}{d\left(e^{6dx+6c}+3e^{4dx+4c}+3e^{2dx+2c}+1\right)}$$

input
```
int(sech(d*x+c)^4*(a+b*sinh(d*x+c)^2),x)
```

output
```
(2*(e**(6*c + 6*d*x)*b - 6*e**(2*c + 2*d*x)*a + 3*e**(2*c + 2*d*x)*b - 2*a
))/(3*d*(e**(6*c + 6*d*x) + 3*e**(4*c + 4*d*x) + 3*e**(2*c + 2*d*x) + 1))
```

3.255 $\int \operatorname{sech}^5(c+dx) \left(a+b\sinh^2(c+dx)\right) dx$

Optimal result

Integrand size = 21, antiderivative size = 70

$$\int \operatorname{sech}^5(c+dx)\left(a+b\sinh^2(c+dx)\right)\,dx = \frac{(3a+b)\arctan(\sinh(c+dx))}{8d} + \frac{(3a+b)\operatorname{sech}(c+dx)\tanh(c+dx)}{8d} + \frac{(a-b)\operatorname{sech}^3(c+dx)\tanh(c+dx)}{4d}$$

output
```
1/8*(3*a+b)*arctan(sinh(d*x+c))/d+1/8*(3*a+b)*sech(d*x+c)*tanh(d*x+c)/d+1/
4*(a-b)*sech(d*x+c)^3*tanh(d*x+c)/d
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 115, normalized size of antiderivative = 1.64

$$\int \operatorname{sech}^5(c+dx) \left(a+b\sinh^2(c+dx)\right) dx = \frac{3a\arctan(\sinh(c+dx))}{8d} + \frac{b\arctan(\sinh(c+dx))}{8d} + \frac{3a\operatorname{sech}(c+dx)\tanh(c+dx)}{8d} + \frac{b\operatorname{sech}(c+dx)\tanh(c+dx)}{8d} + \frac{a\operatorname{sech}^3(c+dx)\tanh(c+dx)}{4d} - \frac{b\operatorname{sech}^3(c+dx)\tanh(c+dx)}{4d}$$

input
```
Integrate[Sech[c + d*x]^5*(a + b*Sinh[c + d*x]^2),x]
```

output
```
(3*a*ArcTan[Sinh[c + d*x]])/(8*d) + (b*ArcTan[Sinh[c + d*x]])/(8*d) + (3*a
*Sech[c + d*x]*Tanh[c + d*x])/(8*d) + (b*Sech[c + d*x]*Tanh[c + d*x])/(8*d
) + (a*Sech[c + d*x]^3*Tanh[c + d*x])/(4*d) - (b*Sech[c + d*x]^3*Tanh[c +
d*x])/(4*d)
```

Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 75, normalized size of antiderivative = 1.07, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.238$, Rules used = {3042, 3669, 298, 215, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \operatorname{sech}^5(c+dx) \left(a+b\sinh^2(c+dx)\right) dx$$

$$\Big\downarrow \; 3042$$

$$\int \frac{a-b\sin(ic+idx)^2}{\cos(ic+idx)^5}dx$$

$$\Big\downarrow \text{3669}$$

$$\frac{\int \frac{b\sinh^2(c+dx)+a}{\left(\sinh^2(c+dx)+1\right)^3}d\sinh(c+dx)}{d}$$

$$\Big\downarrow \text{298}$$

$$\frac{\frac{1}{4}(3a+b)\int \frac{1}{\left(\sinh^2(c+dx)+1\right)^2}d\sinh(c+dx)+\frac{(a-b)\sinh(c+dx)}{4\left(\sinh^2(c+dx)+1\right)^2}}{d}$$

$$\Big\downarrow \text{215}$$

$$\frac{\frac{1}{4}(3a+b)\left(\frac{1}{2}\int \frac{1}{\sinh^2(c+dx)+1}d\sinh(c+dx)+\frac{\sinh(c+dx)}{2\left(\sinh^2(c+dx)+1\right)}\right)+\frac{(a-b)\sinh(c+dx)}{4\left(\sinh^2(c+dx)+1\right)^2}}{d}$$

$$\Big\downarrow \text{216}$$

$$\frac{\frac{1}{4}(3a+b)\left(\frac{1}{2}\arctan(\sinh(c+dx))+\frac{\sinh(c+dx)}{2\left(\sinh^2(c+dx)+1\right)}\right)+\frac{(a-b)\sinh(c+dx)}{4\left(\sinh^2(c+dx)+1\right)^2}}{d}$$

input
```
Int[Sech[c + d*x]^5*(a + b*Sinh[c + d*x]^2),x]
```

output
```
(((a - b)*Sinh[c + d*x])/(4*(1 + Sinh[c + d*x]^2)^2) + ((3*a + b)*(ArcTan[
Sinh[c + d*x]]/2 + Sinh[c + d*x]/(2*(1 + Sinh[c + d*x]^2))))/4)/d
```

Defintions of rubi rules used

rule 215
```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(-x)*((a + b*x^2)^(p + 1)
/(2*a*(p + 1))), x] + Simp[(2*p + 3)/(2*a*(p + 1))   Int[(a + b*x^2)^(p + 1
), x], x] /; FreeQ[{a, b}, x] && LtQ[p, -1] && (IntegerQ[4*p] || IntegerQ[6
*p])
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 298
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2), x_Symbol] :> Simp[(-(
b*c - a*d))*x*((a + b*x^2)^(p + 1)/(2*a*b*(p + 1))), x] - Simp[(a*d - b*c*(
2*p + 3))/(2*a*b*(p + 1))   Int[(a + b*x^2)^(p + 1), x], x] /; FreeQ[{a, b,
 c, d, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/2 + p, 0])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3669
```
Int[cos[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^
(p_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Simp[ff/f   S
ubst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a + b*ff^2*x^2)^p, x], x, Sin[e + f*x]
/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m - 1)/2]
```

Maple [A] (verified)

Time = 33.96 (sec) , antiderivative size = 99, normalized size of antiderivative = 1.41

method	result
derivativedivides	$\frac{a\left(\left(\frac{\operatorname{sech}(dx+c)^3}{4}+\frac{3\operatorname{sech}(dx+c)}{8}\right)\tanh(dx+c)+\frac{3\arctan\left(e^{dx+c}\right)}{4}\right)+b\left(-\frac{\sinh(dx+c)}{3\cosh(dx+c)^4}+\frac{\left(\frac{\operatorname{sech}(dx+c)^3}{4}+\frac{3\operatorname{sech}(dx+c)}{8}\right)\mathrm{ta}}{3}\right.}{d}$
default	$\frac{a\left(\left(\frac{\operatorname{sech}(dx+c)^3}{4}+\frac{3\operatorname{sech}(dx+c)}{8}\right)\tanh(dx+c)+\frac{3\arctan\left(e^{dx+c}\right)}{4}\right)+b\left(-\frac{\sinh(dx+c)}{3\cosh(dx+c)^4}+\frac{\left(\frac{\operatorname{sech}(dx+c)^3}{4}+\frac{3\operatorname{sech}(dx+c)}{8}\right)\mathrm{ta}}{3}\right.}{d}$
risch	$\frac{e^{dx+c}\left(3e^{6dx+6c}a+e^{6dx+6c}b+11e^{4dx+4c}a-7be^{4dx+4c}-11e^{2dx+2c}a+7e^{2dx+2c}b-3a-b\right)}{4d\left(e^{2dx+2c}+1\right)^4}+\frac{3i\ln\left(e^{dx+c}+i\right)a}{8d}+\frac{i\ln}{}$

input
```
int(sech(d*x+c)^5*(a+b*sinh(d*x+c)^2),x,method=_RETURNVERBOSE)
```

output
```
1/d*(a*((1/4*sech(d*x+c)^3+3/8*sech(d*x+c))*tanh(d*x+c)+3/4*arctan(exp(d*x
+c)))+b*(-1/3*sinh(d*x+c)/cosh(d*x+c)^4+1/3*(1/4*sech(d*x+c)^3+3/8*sech(d*
x+c))*tanh(d*x+c)+1/4*arctan(exp(d*x+c))))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1046 vs. $2(64) = 128$.

Time = 0.12 (sec) , antiderivative size = 1046, normalized size of antiderivative = 14.94

$$\int \operatorname{sech}^5(c+dx)\left(a+b\sinh^2(c+dx)\right)\,dx = \text{Too large to display}$$

input
```
integrate(sech(d*x+c)^5*(a+b*sinh(d*x+c)^2),x, algorithm="fricas")
```

output
```
1/4*((3*a + b)*cosh(d*x + c)^7 + 7*(3*a + b)*cosh(d*x + c)*sinh(d*x + c)^6
 + (3*a + b)*sinh(d*x + c)^7 + (11*a - 7*b)*cosh(d*x + c)^5 + (21*(3*a + b
)*cosh(d*x + c)^2 + 11*a - 7*b)*sinh(d*x + c)^5 + 5*(7*(3*a + b)*cosh(d*x
+ c)^3 + (11*a - 7*b)*cosh(d*x + c))*sinh(d*x + c)^4 - (11*a - 7*b)*cosh(d
*x + c)^3 + (35*(3*a + b)*cosh(d*x + c)^4 + 10*(11*a - 7*b)*cosh(d*x + c)^
2 - 11*a + 7*b)*sinh(d*x + c)^3 + (21*(3*a + b)*cosh(d*x + c)^5 + 10*(11*a
 - 7*b)*cosh(d*x + c)^3 - 3*(11*a - 7*b)*cosh(d*x + c))*sinh(d*x + c)^2 +
((3*a + b)*cosh(d*x + c)^8 + 8*(3*a + b)*cosh(d*x + c)*sinh(d*x + c)^7 + (
3*a + b)*sinh(d*x + c)^8 + 4*(3*a + b)*cosh(d*x + c)^6 + 4*(7*(3*a + b)*co
sh(d*x + c)^2 + 3*a + b)*sinh(d*x + c)^6 + 8*(7*(3*a + b)*cosh(d*x + c)^3
+ 3*(3*a + b)*cosh(d*x + c))*sinh(d*x + c)^5 + 6*(3*a + b)*cosh(d*x + c)^4
 + 2*(35*(3*a + b)*cosh(d*x + c)^4 + 30*(3*a + b)*cosh(d*x + c)^2 + 9*a +
3*b)*sinh(d*x + c)^4 + 8*(7*(3*a + b)*cosh(d*x + c)^5 + 10*(3*a + b)*cosh(
d*x + c)^3 + 3*(3*a + b)*cosh(d*x + c))*sinh(d*x + c)^3 + 4*(3*a + b)*cosh
(d*x + c)^2 + 4*(7*(3*a + b)*cosh(d*x + c)^6 + 15*(3*a + b)*cosh(d*x + c)^
4 + 9*(3*a + b)*cosh(d*x + c)^2 + 3*a + b)*sinh(d*x + c)^2 + 8*((3*a + b)*
cosh(d*x + c)^7 + 3*(3*a + b)*cosh(d*x + c)^5 + 3*(3*a + b)*cosh(d*x + c)^
3 + (3*a + b)*cosh(d*x + c))*sinh(d*x + c) + 3*a + b)*arctan(cosh(d*x + c)
 + sinh(d*x + c)) - (3*a + b)*cosh(d*x + c) + (7*(3*a + b)*cosh(d*x + c)^6
 + 5*(11*a - 7*b)*cosh(d*x + c)^4 - 3*(11*a - 7*b)*cosh(d*x + c)^2 - 3*...
```

Sympy [F]

$$\int \operatorname{sech}^5(c+dx)\left(a+b\sinh^2(c+dx)\right)\,dx=\int \left(a+b\sinh^2{(c+dx)}\right)\operatorname{sech}^5{(c+dx)}\,dx$$

input
```
integrate(sech(d*x+c)**5*(a+b*sinh(d*x+c)**2),x)
```

output
```
Integral((a + b*sinh(c + d*x)**2)*sech(c + d*x)**5, x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 228 vs. $2(64) = 128$.

Time = 0.13 (sec) , antiderivative size = 228, normalized size of antiderivative = 3.26

$$\int \operatorname{sech}^5(c+dx)\left(a+b\sinh^2(c+dx)\right)\,dx=$$
$$-\frac{1}{4}\,a\left(\frac{3\,\arctan\left(e^{(-dx-c)}\right)}{d}-\frac{3\,e^{(-dx-c)}+11\,e^{(-3\,dx-3\,c)}-11\,e^{(-5\,dx-5\,c)}-3\,e^{(-7\,dx-7\,c)}}{d(4\,e^{(-2\,dx-2\,c)}+6\,e^{(-4\,dx-4\,c)}+4\,e^{(-6\,dx-6\,c)}+e^{(-8\,dx-8\,c)}+1)}\right)$$
$$-\frac{1}{4}\,b\left(\frac{\arctan\left(e^{(-dx-c)}\right)}{d}-\frac{e^{(-dx-c)}-7\,e^{(-3\,dx-3\,c)}+7\,e^{(-5\,dx-5\,c)}-e^{(-7\,dx-7\,c)}}{d(4\,e^{(-2\,dx-2\,c)}+6\,e^{(-4\,dx-4\,c)}+4\,e^{(-6\,dx-6\,c)}+e^{(-8\,dx-8\,c)}+1)}\right)$$

input
```
integrate(sech(d*x+c)^5*(a+b*sinh(d*x+c)^2),x, algorithm="maxima")
```

output
```
-1/4*a*(3*arctan(e^(-d*x - c))/d - (3*e^(-d*x - c) + 11*e^(-3*d*x - 3*c) -
 11*e^(-5*d*x - 5*c) - 3*e^(-7*d*x - 7*c))/(d*(4*e^(-2*d*x - 2*c) + 6*e^(-
4*d*x - 4*c) + 4*e^(-6*d*x - 6*c) + e^(-8*d*x - 8*c) + 1))) - 1/4*b*(arcta
n(e^(-d*x - c))/d - (e^(-d*x - c) - 7*e^(-3*d*x - 3*c) + 7*e^(-5*d*x - 5*c
) - e^(-7*d*x - 7*c))/(d*(4*e^(-2*d*x - 2*c) + 6*e^(-4*d*x - 4*c) + 4*e^(-
6*d*x - 6*c) + e^(-8*d*x - 8*c) + 1)))
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 153 vs. $2(64) = 128$.

Time = 0.14 (sec) , antiderivative size = 153, normalized size of antiderivative = 2.19

$$\int \operatorname{sech}^5(c+dx)\left(a+b\sinh^2(c+dx)\right)\,dx = \frac{\left(\pi + 2\arctan\left(\frac{1}{2}\left(e^{(2\,dx+2\,c)}-1\right)e^{(-dx-c)}\right)\right)(3\,a+b) + \frac{4\left(3\,a\left(e^{(dx+c)}-e^{(-dx-c)}\right)^3+b\left(e^{(dx+c)}-e^{(-dx-c)}\right)^3+20\,a\left(e^{(dx+c)}-\right.\right.}{\left(\left(e^{(dx+c)}-e^{(-dx-c)}\right)^2+4\right)^2}}{16\,d}$$

input
```
integrate(sech(d*x+c)^5*(a+b*sinh(d*x+c)^2),x, algorithm="giac")
```

output
```
1/16*((pi + 2*arctan(1/2*(e^(2*d*x + 2*c) - 1)*e^(-d*x - c)))*(3*a + b) +
4*(3*a*(e^(d*x + c) - e^(-d*x - c))^3 + b*(e^(d*x + c) - e^(-d*x - c))^3 +
 20*a*(e^(d*x + c) - e^(-d*x - c)) - 4*b*(e^(d*x + c) - e^(-d*x - c)))/((e
^(d*x + c) - e^(-d*x - c))^2 + 4)^2)/d
```

Mupad [B] (verification not implemented)

Time = 1.99 (sec) , antiderivative size = 280, normalized size of antiderivative = 4.00

$$\int \operatorname{sech}^5(c+dx)\left(a+b\sinh^2(c+dx)\right)\,dx = \frac{\operatorname{atan}\left(\frac{e^{d\,x}\,e^{c}\,\left(3\,a\,\sqrt{d^2}+b\,\sqrt{d^2}\right)}{d\,\sqrt{9\,a^2+6\,a\,b+b^2}}\right)\,\sqrt{9\,a^2+6\,a\,b+b^2}}{4\,\sqrt{d^2}} - \frac{\frac{b\,e^{5\,c+5\,d\,x}}{d}+\frac{2\,e^{3\,c+3\,d\,x}\,(2\,a-b)}{d}+\frac{b\,e^{c+d\,x}}{d}}{4\,e^{2\,c+2\,d\,x}+6\,e^{4\,c+4\,d\,x}+4\,e^{6\,c+6\,d\,x}+e^{8\,c+8\,d\,x}+1} + \frac{e^{c+d\,x}\,(3\,a+b)}{4\,d\,\left(e^{2\,c+2\,d\,x}+1\right)} + \frac{e^{c+d\,x}\,(a-3\,b)}{2\,d\,\left(2\,e^{2\,c+2\,d\,x}+e^{4\,c+4\,d\,x}+1\right)} - \frac{2\,e^{c+d\,x}\,(a-b)}{d\,\left(3\,e^{2\,c+2\,d\,x}+3\,e^{4\,c+4\,d\,x}+e^{6\,c+6\,d\,x}+1\right)}$$

input
```
int((a + b*sinh(c + d*x)^2)/cosh(c + d*x)^5,x)
```

output
```
(atan((exp(d*x)*exp(c)*(3*a*(d^2)^(1/2) + b*(d^2)^(1/2)))/(d*(6*a*b + 9*a^
2 + b^2)^(1/2)))*(6*a*b + 9*a^2 + b^2)^(1/2))/(4*(d^2)^(1/2)) - ((b*exp(5*
c + 5*d*x))/d + (2*exp(3*c + 3*d*x)*(2*a - b))/d + (b*exp(c + d*x))/d)/(4*
exp(2*c + 2*d*x) + 6*exp(4*c + 4*d*x) + 4*exp(6*c + 6*d*x) + exp(8*c + 8*d
*x) + 1) + (exp(c + d*x)*(3*a + b))/(4*d*(exp(2*c + 2*d*x) + 1)) + (exp(c
+ d*x)*(a - 3*b))/(2*d*(2*exp(2*c + 2*d*x) + exp(4*c + 4*d*x) + 1)) - (2*e
xp(c + d*x)*(a - b))/(d*(3*exp(2*c + 2*d*x) + 3*exp(4*c + 4*d*x) + exp(6*c
 + 6*d*x) + 1))
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 341, normalized size of antiderivative = 4.87

$$\int \operatorname{sech}^5(c+dx)\left(a+b\sinh^2(c+dx)\right)\,dx = \frac{3e^{8dx+8c}atan\left(e^{dx+c}\right)a+e^{8dx+8c}atan\left(e^{dx+c}\right)b+12e^{6dx+6c}atan\left(e^{dx+c}\right)a+4e^{6dx+6c}atan\left(e^{dx+c}\right)b+18e^{4dx}}{}$$

input
```
int(sech(d*x+c)^5*(a+b*sinh(d*x+c)^2),x)
```

output
```
(3*e**(8*c + 8*d*x)*atan(e**(c + d*x))*a + e**(8*c + 8*d*x)*atan(e**(c + d
*x))*b + 12*e**(6*c + 6*d*x)*atan(e**(c + d*x))*a + 4*e**(6*c + 6*d*x)*ata
n(e**(c + d*x))*b + 18*e**(4*c + 4*d*x)*atan(e**(c + d*x))*a + 6*e**(4*c +
 4*d*x)*atan(e**(c + d*x))*b + 12*e**(2*c + 2*d*x)*atan(e**(c + d*x))*a +
4*e**(2*c + 2*d*x)*atan(e**(c + d*x))*b + 3*atan(e**(c + d*x))*a + atan(e*
*(c + d*x))*b + 3*e**(7*c + 7*d*x)*a + e**(7*c + 7*d*x)*b + 11*e**(5*c + 5
*d*x)*a - 7*e**(5*c + 5*d*x)*b - 11*e**(3*c + 3*d*x)*a + 7*e**(3*c + 3*d*x
)*b - 3*e**(c + d*x)*a - e**(c + d*x)*b)/(4*d*(e**(8*c + 8*d*x) + 4*e**(6*
c + 6*d*x) + 6*e**(4*c + 4*d*x) + 4*e**(2*c + 2*d*x) + 1))
```

3.256 $\int \operatorname{sech}^6(c+dx) \left(a+b\sinh^2(c+dx)\right) \, dx$

Optimal result

Integrand size = 21, antiderivative size = 54

$$\int \operatorname{sech}^6(c+dx) \left(a+b\sinh^2(c+dx)\right) \, dx = \frac{a\tanh(c+dx)}{d} - \frac{(2a-b)\tanh^3(c+dx)}{3d} + \frac{(a-b)\tanh^5(c+dx)}{5d}$$

output
```
a*tanh(d*x+c)/d-1/3*(2*a-b)*tanh(d*x+c)^3/d+1/5*(a-b)*tanh(d*x+c)^5/d
```

Mathematica [A] (verified)

Time = 0.20 (sec) , antiderivative size = 102, normalized size of antiderivative = 1.89

$$\int \operatorname{sech}^6(c+dx) \left(a+b\sinh^2(c+dx)\right) \, dx = \frac{a\tanh(c+dx)}{d} + \frac{2b\tanh(c+dx)}{15d} + \frac{b\operatorname{sech}^2(c+dx)\tanh(c+dx)}{15d} - \frac{b\operatorname{sech}^4(c+dx)\tanh(c+dx)}{5d} - \frac{2a\tanh^3(c+dx)}{3d} + \frac{a\tanh^5(c+dx)}{5d}$$

input
```
Integrate[Sech[c + d*x]^6*(a + b*Sinh[c + d*x]^2),x]
```

output
```
(a*Tanh[c + d*x])/d + (2*b*Tanh[c + d*x])/(15*d) + (b*Sech[c + d*x]^2*Tanh
[c + d*x])/(15*d) - (b*Sech[c + d*x]^4*Tanh[c + d*x])/(5*d) - (2*a*Tanh[c
+ d*x]^3)/(3*d) + (a*Tanh[c + d*x]^5)/(5*d)
```

Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 49, normalized size of antiderivative = 0.91, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.190, Rules used = {3042, 3670, 290, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \operatorname{sech}^6(c+dx)\left(a+b\sinh^2(c+dx)\right)\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{a-b\sin(ic+idx)^2}{\cos(ic+idx)^6}dx$$

$$\Big\downarrow \text{3670}$$

$$\frac{\int \left(1-\tanh^2(c+dx)\right)\left(a-(a-b)\tanh^2(c+dx)\right)d\tanh(c+dx)}{d}$$

$$\Big\downarrow \text{290}$$

$$\frac{\int \left((a-b)\tanh^4(c+dx)-(2a-b)\tanh^2(c+dx)+a\right)d\tanh(c+dx)}{d}$$

$$\Big\downarrow \text{2009}$$

$$\frac{\frac{1}{5}(a-b)\tanh^5(c+dx)-\frac{1}{3}(2a-b)\tanh^3(c+dx)+a\tanh(c+dx)}{d}$$

input
```
Int[Sech[c + d*x]^6*(a + b*Sinh[c + d*x]^2),x]
```

output
```
(a*Tanh[c + d*x] - ((2*a - b)*Tanh[c + d*x]^3)/3 + ((a - b)*Tanh[c + d*x]^
5)/5)/d
```

Defintions of rubi rules used

rule 290
```
Int[((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.), x_Symbol] :> I
nt[ExpandIntegrand[(a + b*x^2)^p*(c + d*x^2)^q, x], x] /; FreeQ[{a, b, c, d
}, x] && NeQ[b*c - a*d, 0] && IGtQ[p, 0] && IGtQ[q, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3670
```
Int[cos[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(
p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Simp[ff/f   Su
bst[Int[(a + (a + b)*ff^2*x^2)^p/(1 + ff^2*x^2)^(m/2 + p + 1), x], x, Tan[e
 + f*x]/ff], x]] /; FreeQ[{a, b, e, f}, x] && IntegerQ[m/2] && IntegerQ[p]
```

Maple [A] (verified)

Time = 73.43 (sec) , antiderivative size = 84, normalized size of antiderivative = 1.56

method	result
risch	$-\frac{4\left(15\,{\mathrm e}^{6 d x +6 c} b +40\,{\mathrm e}^{4 d x +4 c} a -5 b\,{\mathrm e}^{4 d x +4 c}+20\,{\mathrm e}^{2 d x +2 c} a +5\,{\mathrm e}^{2 d x +2 c} b +4 a +b \right)}{15 d \left({\mathrm e}^{2 d x +2 c}+1\right)^{5}}$
derivativedivides	$\frac{a\left(\frac{8}{15}+\frac{\operatorname{sech}\left(d x +c \right)^{4}}{5}+\frac{4\,\operatorname{sech}\left(d x +c \right)^{2}}{15}\right)\tanh\left(d x +c \right)+b\left(-\frac{\sinh\left(d x +c \right)}{4 \cosh\left(d x +c \right)^{5}}+\frac{\left(\frac{8}{15}+\frac{\operatorname{sech}\left(d x +c \right)^{4}}{5}+\frac{4\,\operatorname{sech}\left(d x +c \right)^{2}}{15}\right)\tanh\left(d x +c \right)}{4}\right)}{d}$
default	$\frac{a\left(\frac{8}{15}+\frac{\operatorname{sech}\left(d x +c \right)^{4}}{5}+\frac{4\,\operatorname{sech}\left(d x +c \right)^{2}}{15}\right)\tanh\left(d x +c \right)+b\left(-\frac{\sinh\left(d x +c \right)}{4 \cosh\left(d x +c \right)^{5}}+\frac{\left(\frac{8}{15}+\frac{\operatorname{sech}\left(d x +c \right)^{4}}{5}+\frac{4\,\operatorname{sech}\left(d x +c \right)^{2}}{15}\right)\tanh\left(d x +c \right)}{4}\right)}{d}$

input
```
int(sech(d*x+c)^6*(a+b*sinh(d*x+c)^2),x,method=_RETURNVERBOSE)
```

output
```
-4/15*(15*exp(6*d*x+6*c)*b+40*exp(4*d*x+4*c)*a-5*b*exp(4*d*x+4*c)+20*exp(2
*d*x+2*c)*a+5*exp(2*d*x+2*c)*b+4*a+b)/d/(exp(2*d*x+2*c)+1)^5
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 343 vs. 2(50) = 100.

Time = 0.09 (sec) , antiderivative size = 343, normalized size of antiderivative = 6.35

$$\int \operatorname{sech}^6(c+dx)\left(a+b\sinh^2(c+dx)\right)\,dx =$$

$$-\frac{}{15\left(d\cosh\left(dx+c\right)^7+7\,d\cosh\left(dx+c\right)\sinh\left(dx+c\right)^6+d\sinh\left(dx+c\right)^7+5\,d\cosh\left(dx+c\right)^5+\left(21\,d\,c\right.\right.}$$

input
```
integrate(sech(d*x+c)^6*(a+b*sinh(d*x+c)^2),x, algorithm="fricas")
```

output
```
-8/15*(2*(a + 4*b)*cosh(d*x + c)^3 + 6*(a + 4*b)*cosh(d*x + c)*sinh(d*x +
c)^2 - (2*a - 7*b)*sinh(d*x + c)^3 + 30*a*cosh(d*x + c) - (3*(2*a - 7*b)*c
osh(d*x + c)^2 - 10*a + 5*b)*sinh(d*x + c))/(d*cosh(d*x + c)^7 + 7*d*cosh(
d*x + c)*sinh(d*x + c)^6 + d*sinh(d*x + c)^7 + 5*d*cosh(d*x + c)^5 + (21*d
*cosh(d*x + c)^2 + 5*d)*sinh(d*x + c)^5 + 5*(7*d*cosh(d*x + c)^3 + 5*d*cos
h(d*x + c))*sinh(d*x + c)^4 + 11*d*cosh(d*x + c)^3 + (35*d*cosh(d*x + c)^4
 + 50*d*cosh(d*x + c)^2 + 9*d)*sinh(d*x + c)^3 + (21*d*cosh(d*x + c)^5 + 5
0*d*cosh(d*x + c)^3 + 33*d*cosh(d*x + c))*sinh(d*x + c)^2 + 15*d*cosh(d*x
+ c) + (7*d*cosh(d*x + c)^6 + 25*d*cosh(d*x + c)^4 + 27*d*cosh(d*x + c)^2
+ 5*d)*sinh(d*x + c))
```

Sympy [F(-1)]

Timed out.

$$\int \operatorname{sech}^6(c+dx)\left(a+b\sinh^2(c+dx)\right)\,dx=\text{Timed out}$$

input
```
integrate(sech(d*x+c)**6*(a+b*sinh(d*x+c)**2),x)
```

output
```
Timed out
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 486 vs. $2(50) = 100$.

Time = 0.04 (sec) , antiderivative size = 486, normalized size of antiderivative = 9.00

$$\int \operatorname{sech}^6(c+dx)\left(a+b\sinh^2(c+dx)\right)\,dx$$
$$=\frac{16}{15}\,a\left(\frac{5\,e^{(-2\,dx-2\,c)}}{d(5\,e^{(-2\,dx-2\,c)}+10\,e^{(-4\,dx-4\,c)}+10\,e^{(-6\,dx-6\,c)}+5\,e^{(-8\,dx-8\,c)}+e^{(-10\,dx-10\,c)}+1)}+\frac{}{d(5\,e^{(-2\,dx-2\,c)}}\right.$$
$$+\frac{4}{15}\,b\left(\frac{5\,e^{(-2\,dx-2\,c)}}{d(5\,e^{(-2\,dx-2\,c)}+10\,e^{(-4\,dx-4\,c)}+10\,e^{(-6\,dx-6\,c)}+5\,e^{(-8\,dx-8\,c)}+e^{(-10\,dx-10\,c)}+1)}-\frac{}{d(5\,e^{(-2\,dx-2}}\right.$$

input
```
integrate(sech(d*x+c)^6*(a+b*sinh(d*x+c)^2),x, algorithm="maxima")
```

output
```
16/15*a*(5*e^(-2*d*x - 2*c)/(d*(5*e^(-2*d*x - 2*c) + 10*e^(-4*d*x - 4*c) +
 10*e^(-6*d*x - 6*c) + 5*e^(-8*d*x - 8*c) + e^(-10*d*x - 10*c) + 1)) + 10*
e^(-4*d*x - 4*c)/(d*(5*e^(-2*d*x - 2*c) + 10*e^(-4*d*x - 4*c) + 10*e^(-6*d
*x - 6*c) + 5*e^(-8*d*x - 8*c) + e^(-10*d*x - 10*c) + 1)) + 1/(d*(5*e^(-2*
d*x - 2*c) + 10*e^(-4*d*x - 4*c) + 10*e^(-6*d*x - 6*c) + 5*e^(-8*d*x - 8*c
) + e^(-10*d*x - 10*c) + 1))) + 4/15*b*(5*e^(-2*d*x - 2*c)/(d*(5*e^(-2*d*x
 - 2*c) + 10*e^(-4*d*x - 4*c) + 10*e^(-6*d*x - 6*c) + 5*e^(-8*d*x - 8*c) +
 e^(-10*d*x - 10*c) + 1)) - 5*e^(-4*d*x - 4*c)/(d*(5*e^(-2*d*x - 2*c) + 10
*e^(-4*d*x - 4*c) + 10*e^(-6*d*x - 6*c) + 5*e^(-8*d*x - 8*c) + e^(-10*d*x
- 10*c) + 1)) + 15*e^(-6*d*x - 6*c)/(d*(5*e^(-2*d*x - 2*c) + 10*e^(-4*d*x
- 4*c) + 10*e^(-6*d*x - 6*c) + 5*e^(-8*d*x - 8*c) + e^(-10*d*x - 10*c) + 1
)) + 1/(d*(5*e^(-2*d*x - 2*c) + 10*e^(-4*d*x - 4*c) + 10*e^(-6*d*x - 6*c)
+ 5*e^(-8*d*x - 8*c) + e^(-10*d*x - 10*c) + 1)))
```

Giac [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 83, normalized size of antiderivative = 1.54

$$\int \operatorname{sech}^6(c+dx)\left(a+b\sinh^2(c+dx)\right)\,dx = -\frac{4\left(15\,be^{(6\,dx+6\,c)}+40\,ae^{(4\,dx+4\,c)}-5\,be^{(4\,dx+4\,c)}+20\,ae^{(2\,dx+2\,c)}+5\,be^{(2\,dx+2\,c)}+4\,a+b\right)}{15\,d\left(e^{(2\,dx+2\,c)}+1\right)^5}$$

input
```
integrate(sech(d*x+c)^6*(a+b*sinh(d*x+c)^2),x, algorithm="giac")
```

output
```
-4/15*(15*b*e^(6*d*x + 6*c) + 40*a*e^(4*d*x + 4*c) - 5*b*e^(4*d*x + 4*c) +
 20*a*e^(2*d*x + 2*c) + 5*b*e^(2*d*x + 2*c) + 4*a + b)/(d*(e^(2*d*x + 2*c)
 + 1)^5)
```

Mupad [B] (verification not implemented)

Time = 1.99 (sec) , antiderivative size = 298, normalized size of antiderivative = 5.52

$$\int \operatorname{sech}^6(c+dx)\left(a+b\sinh^2(c+dx)\right)\,dx$$
$$= -\frac{\frac{8\,b\,e^{2\,c+2\,d\,x}}{5\,d}+\frac{8\,b\,e^{6\,c+6\,d\,x}}{5\,d}+\frac{16\,e^{4\,c+4\,d\,x}\,(2\,a-b)}{5\,d}}{5\,e^{2\,c+2\,d\,x}+10\,e^{4\,c+4\,d\,x}+10\,e^{6\,c+6\,d\,x}+5\,e^{8\,c+8\,d\,x}+e^{10\,c+10\,d\,x}+1}$$
$$-\frac{\frac{2\,b}{5\,d}+\frac{6\,b\,e^{4\,c+4\,d\,x}}{5\,d}+\frac{8\,e^{2\,c+2\,d\,x}\,(2\,a-b)}{5\,d}}{4\,e^{2\,c+2\,d\,x}+6\,e^{4\,c+4\,d\,x}+4\,e^{6\,c+6\,d\,x}+e^{8\,c+8\,d\,x}+1}$$
$$-\frac{\frac{8\,(2\,a-b)}{15\,d}+\frac{4\,b\,e^{2\,c+2\,d\,x}}{5\,d}}{3\,e^{2\,c+2\,d\,x}+3\,e^{4\,c+4\,d\,x}+e^{6\,c+6\,d\,x}+1}-\frac{2\,b}{5\,d\,\left(2\,e^{2\,c+2\,d\,x}+e^{4\,c+4\,d\,x}+1\right)}$$

input
```
int((a + b*sinh(c + d*x)^2)/cosh(c + d*x)^6,x)
```

output
```
- ((8*b*exp(2*c + 2*d*x))/(5*d) + (8*b*exp(6*c + 6*d*x))/(5*d) + (16*exp(4
*c + 4*d*x)*(2*a - b))/(5*d))/(5*exp(2*c + 2*d*x) + 10*exp(4*c + 4*d*x) +
10*exp(6*c + 6*d*x) + 5*exp(8*c + 8*d*x) + exp(10*c + 10*d*x) + 1) - ((2*b
)/(5*d) + (6*b*exp(4*c + 4*d*x))/(5*d) + (8*exp(2*c + 2*d*x)*(2*a - b))/(5
*d))/(4*exp(2*c + 2*d*x) + 6*exp(4*c + 4*d*x) + 4*exp(6*c + 6*d*x) + exp(8
*c + 8*d*x) + 1) - ((8*(2*a - b))/(15*d) + (4*b*exp(2*c + 2*d*x))/(5*d))/(
3*exp(2*c + 2*d*x) + 3*exp(4*c + 4*d*x) + exp(6*c + 6*d*x) + 1) - (2*b)/(5
*d*(2*exp(2*c + 2*d*x) + exp(4*c + 4*d*x) + 1))
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 139, normalized size of antiderivative = 2.57

$$\int \operatorname{sech}^6(c+dx)\left(a+b\sinh^2(c+dx)\right)\,dx$$
$$= \frac{-4e^{6dx+6c}b-\frac{32e^{4dx+4c}a}{3}+\frac{4e^{4dx+4c}b}{3}-\frac{16e^{2dx+2c}a}{3}-\frac{4e^{2dx+2c}b}{3}-\frac{16a}{15}-\frac{4b}{15}}{d\left(e^{10dx+10c}+5e^{8dx+8c}+10e^{6dx+6c}+10e^{4dx+4c}+5e^{2dx+2c}+1\right)}$$

input
```
int(sech(d*x+c)^6*(a+b*sinh(d*x+c)^2),x)
```

output

```
(4*( - 15*e**(6*c + 6*d*x)*b - 40*e**(4*c + 4*d*x)*a + 5*e**(4*c + 4*d*x)*
b - 20*e**(2*c + 2*d*x)*a - 5*e**(2*c + 2*d*x)*b - 4*a - b))/(15*d*(e**(10
*c + 10*d*x) + 5*e**(8*c + 8*d*x) + 10*e**(6*c + 6*d*x) + 10*e**(4*c + 4*d
*x) + 5*e**(2*c + 2*d*x) + 1))
```

3.257 $\int \cosh^4(c+dx) \left(a+b\sinh^2(c+dx)\right)^2 \, dx$

Optimal result

Integrand size = 23, antiderivative size = 144

$$\begin{aligned}
&\int \cosh^4(c+dx) \left(a+b\sinh^2(c+dx)\right)^2 \, dx \\
&= \frac{1}{128}\left(48a^2-16ab+3b^2\right)x + \frac{(48a^2-16ab+3b^2)\cosh(c+dx)\sinh(c+dx)}{128d} \\
&\quad + \frac{(48a^2-16ab+3b^2)\cosh^3(c+dx)\sinh(c+dx)}{192d} \\
&\quad + \frac{(16a-9b)b\cosh^5(c+dx)\sinh(c+dx)}{48d} + \frac{b^2\cosh^7(c+dx)\sinh(c+dx)}{8d}
\end{aligned}$$

output
```
1/128*(48*a^2-16*a*b+3*b^2)*x+1/128*(48*a^2-16*a*b+3*b^2)*cosh(d*x+c)*sinh
(d*x+c)/d+1/192*(48*a^2-16*a*b+3*b^2)*cosh(d*x+c)^3*sinh(d*x+c)/d+1/48*(16
*a-9*b)*b*cosh(d*x+c)^5*sinh(d*x+c)/d+1/8*b^2*cosh(d*x+c)^7*sinh(d*x+c)/d
```

Mathematica [A] (verified)

Time = 1.60 (sec) , antiderivative size = 98, normalized size of antiderivative = 0.68

$$\int \cosh^4(c+dx)\left(a+b\sinh^2(c+dx)\right)^2\,dx$$
$$= \frac{24(48a^2-16ab+3b^2)\,(c+dx)+96a(8a-b)\sinh(2(c+dx))+24(4a^2+4ab-b^2)\sinh(4(c+dx))+3}{3072d}$$

input
```
Integrate[Cosh[c + d*x]^4*(a + b*Sinh[c + d*x]^2)^2,x]
```

output
```
(24*(48*a^2 - 16*a*b + 3*b^2)*(c + d*x) + 96*a*(8*a - b)*Sinh[2*(c + d*x)]
 + 24*(4*a^2 + 4*a*b - b^2)*Sinh[4*(c + d*x)] + 32*a*b*Sinh[6*(c + d*x)] +
 3*b^2*Sinh[8*(c + d*x)])/(3072*d)
```

Rubi [A] (verified)

Time = 0.33 (sec) , antiderivative size = 168, normalized size of antiderivative = 1.17, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.348$, Rules used = {3042, 3670, 315, 25, 298, 215, 215, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cosh^4(c+dx)\left(a+b\sinh^2(c+dx)\right)^2\,dx$$
$$\Big\downarrow \; 3042$$
$$\int \cos(ic+idx)^4\left(a-b\sin(ic+idx)^2\right)^2dx$$
$$\Big\downarrow \; 3670$$
$$\frac{\int \frac{\left(a-(a-b)\tanh^2(c+dx)\right)^2}{\left(1-\tanh^2(c+dx)\right)^5}d\tanh(c+dx)}{d}$$
$$\Big\downarrow \; 315$$
$$\frac{\frac{b\tanh(c+dx)\left(a-(a-b)\tanh^2(c+dx)\right)}{8\left(1-\tanh^2(c+dx)\right)^4}-\frac{1}{8}\int -\frac{a(8a-b)-(8a-3b)(a-b)\tanh^2(c+dx)}{\left(1-\tanh^2(c+dx)\right)^4}d\tanh(c+dx)}{d}$$

$$\Big\downarrow \text{25}$$

$$\frac{\frac{1}{8}\int \frac{a(8a-b)-(8a-3b)(a-b)\tanh^2(c+dx)}{(1-\tanh^2(c+dx))^4}d\tanh(c+dx)+\frac{b\tanh(c+dx)(a-(a-b)\tanh^2(c+dx))}{8(1-\tanh^2(c+dx))^4}}{d}$$

$$\Big\downarrow \text{298}$$

$$\frac{\frac{1}{8}\left(\frac{1}{6}\left(48a^2-16ab+3b^2\right)\int \frac{1}{(1-\tanh^2(c+dx))^3}d\tanh(c+dx)+\frac{b(10a-3b)\tanh(c+dx)}{6(1-\tanh^2(c+dx))^3}\right)+\frac{b\tanh(c+dx)(a-(a-b)\tanh^2(c+dx)}{8(1-\tanh^2(c+dx))^4}}{d}$$

$$\Big\downarrow \text{215}$$

$$\frac{\frac{1}{8}\left(\frac{1}{6}\left(48a^2-16ab+3b^2\right)\left(\frac{3}{4}\int \frac{1}{(1-\tanh^2(c+dx))^2}d\tanh(c+dx)+\frac{\tanh(c+dx)}{4(1-\tanh^2(c+dx))^2}\right)+\frac{b(10a-3b)\tanh(c+dx)}{6(1-\tanh^2(c+dx))^3}\right)+\frac{b\tan}{}}{d}$$

$$\Big\downarrow \text{215}$$

$$\frac{\frac{1}{8}\left(\frac{1}{6}\left(48a^2-16ab+3b^2\right)\left(\frac{3}{4}\left(\frac{1}{2}\int \frac{1}{1-\tanh^2(c+dx)}d\tanh(c+dx)+\frac{\tanh(c+dx)}{2(1-\tanh^2(c+dx))}\right)+\frac{\tanh(c+dx)}{4(1-\tanh^2(c+dx))^2}\right)+\frac{b(10a-3}{6(1-\mathrm{ta}}}{d}$$

$$\Big\downarrow \text{219}$$

$$\frac{\frac{1}{8}\left(\frac{1}{6}\left(48a^2-16ab+3b^2\right)\left(\frac{3}{4}\left(\frac{1}{2}\text{arctanh}(\tanh(c+dx))+\frac{\tanh(c+dx)}{2(1-\tanh^2(c+dx))}\right)+\frac{\tanh(c+dx)}{4(1-\tanh^2(c+dx))^2}\right)+\frac{b(10a-3b)\tanh(c}{6(1-\tanh^2(c+dx}}{d}$$

input
```
Int[Cosh[c + d*x]^4*(a + b*Sinh[c + d*x]^2)^2,x]
```

output
```
((b*Tanh[c + d*x]*(a - (a - b)*Tanh[c + d*x]^2))/(8*(1 - Tanh[c + d*x]^2)^
4) + (((10*a - 3*b)*b*Tanh[c + d*x])/(6*(1 - Tanh[c + d*x]^2)^3) + ((48*a^
2 - 16*a*b + 3*b^2)*(Tanh[c + d*x]/(4*(1 - Tanh[c + d*x]^2)^2) + (3*(ArcTa
nh[Tanh[c + d*x]]/2 + Tanh[c + d*x]/(2*(1 - Tanh[c + d*x]^2))))/4))/6)/8)/
d
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 215
```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(-x)*((a + b*x^2)^(p + 1)
/(2*a*(p + 1))), x] + Simp[(2*p + 3)/(2*a*(p + 1))   Int[(a + b*x^2)^(p + 1
), x], x] /; FreeQ[{a, b}, x] && LtQ[p, -1] && (IntegerQ[4*p] || IntegerQ[6
*p])
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 298
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2), x_Symbol] :> Simp[(-(
b*c - a*d))*x*((a + b*x^2)^(p + 1)/(2*a*b*(p + 1))), x] - Simp[(a*d - b*c*(
2*p + 3))/(2*a*b*(p + 1))   Int[(a + b*x^2)^(p + 1), x], x] /; FreeQ[{a, b,
 c, d, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/2 + p, 0])
```

rule 315
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_), x_Symbol] :> Sim
p[(a*d - c*b)*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^(q - 1)/(2*a*b*(p + 1))),
x] - Simp[1/(2*a*b*(p + 1))   Int[(a + b*x^2)^(p + 1)*(c + d*x^2)^(q - 2)*S
imp[c*(a*d - c*b*(2*p + 3)) + d*(a*d*(2*(q - 1) + 1) - b*c*(2*(p + q) + 1))
*x^2, x], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -
1] && GtQ[q, 1] && IntBinomialQ[a, b, c, d, 2, p, q, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3670
```
Int[cos[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(
p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Simp[ff/f   Su
bst[Int[(a + (a + b)*ff^2*x^2)^p/(1 + ff^2*x^2)^(m/2 + p + 1), x], x, Tan[e
 + f*x]/ff], x]] /; FreeQ[{a, b, e, f}, x] && IntegerQ[m/2] && IntegerQ[p]
```

Maple [A] (verified)

Time = 0.14 (sec) , antiderivative size = 172, normalized size of antiderivative = 1.19

$$\frac{a^2\left(\left(\frac{\cosh(dx+c)^3}{4}+\frac{3\cosh(dx+c)}{8}\right)\sinh\left(dx+c\right)+\frac{3dx}{8}+\frac{3c}{8}\right)+2ab\left(\frac{\sinh(dx+c)\cosh(dx+c)^5}{6}-\frac{\left(\frac{\cosh(dx+c)^3}{4}+\frac{3\cosh(dx+c)}{8}\right)}{6}\right.}{}$$

input
```
int(cosh(d*x+c)^4*(a+b*sinh(d*x+c)^2)^2,x)
```

output
```
1/d*(a^2*((1/4*cosh(d*x+c)^3+3/8*cosh(d*x+c))*sinh(d*x+c)+3/8*d*x+3/8*c)+2
*a*b*(1/6*sinh(d*x+c)*cosh(d*x+c)^5-1/6*(1/4*cosh(d*x+c)^3+3/8*cosh(d*x+c)
)*sinh(d*x+c)-1/16*d*x-1/16*c)+b^2*(1/8*sinh(d*x+c)^3*cosh(d*x+c)^5-1/16*s
inh(d*x+c)*cosh(d*x+c)^5+1/16*(1/4*cosh(d*x+c)^3+3/8*cosh(d*x+c))*sinh(d*x
+c)+3/128*d*x+3/128*c))
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 212, normalized size of antiderivative = 1.47

$$\int \cosh^4(c+dx)\left(a+b\sinh^2(c+dx)\right)^2\,dx$$
$$=\frac{3\,b^2\cosh\left(dx+c\right)\sinh\left(dx+c\right)^7+3\left(7\,b^2\cosh\left(dx+c\right)^3+8\,ab\cosh\left(dx+c\right)\right)\sinh\left(dx+c\right)^5+\left(21\,b^2\cosh\right.}{}$$

input
```
integrate(cosh(d*x+c)^4*(a+b*sinh(d*x+c)^2)^2,x, algorithm="fricas")
```

output
```
1/384*(3*b^2*cosh(d*x + c)*sinh(d*x + c)^7 + 3*(7*b^2*cosh(d*x + c)^3 + 8*
a*b*cosh(d*x + c))*sinh(d*x + c)^5 + (21*b^2*cosh(d*x + c)^5 + 80*a*b*cosh
(d*x + c)^3 + 12*(4*a^2 + 4*a*b - b^2)*cosh(d*x + c))*sinh(d*x + c)^3 + 3*
(48*a^2 - 16*a*b + 3*b^2)*d*x + 3*(b^2*cosh(d*x + c)^7 + 8*a*b*cosh(d*x +
c)^5 + 4*(4*a^2 + 4*a*b - b^2)*cosh(d*x + c)^3 + 8*(8*a^2 - a*b)*cosh(d*x
+ c))*sinh(d*x + c))/d
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 481 vs. $2(134) = 268$.

Time = 0.70 (sec) , antiderivative size = 481, normalized size of antiderivative = 3.34

$$\int \cosh^4(c+dx)\left(a+b\sinh^2(c+dx)\right)^2\,dx$$

$$= \begin{cases} \frac{3a^2x\sinh^4(c+dx)}{8} - \frac{3a^2x\sinh^2(c+dx)\cosh^2(c+dx)}{4} + \frac{3a^2x\cosh^4(c+dx)}{8} - \frac{3a^2\sinh^3(c+dx)\cosh(c+dx)}{8d} + \frac{5a^2\sinh(c+dx)\cosh^3(}{8d} \\ x\left(a+b\sinh^2(c)\right)^2\cosh^4(c) \end{cases}$$

input
```
integrate(cosh(d*x+c)**4*(a+b*sinh(d*x+c)**2)**2,x)
```

output
```
Piecewise((3*a**2*x*sinh(c + d*x)**4/8 - 3*a**2*x*sinh(c + d*x)**2*cosh(c
+ d*x)**2/4 + 3*a**2*x*cosh(c + d*x)**4/8 - 3*a**2*sinh(c + d*x)**3*cosh(c
 + d*x)/(8*d) + 5*a**2*sinh(c + d*x)*cosh(c + d*x)**3/(8*d) + a*b*x*sinh(c
 + d*x)**6/8 - 3*a*b*x*sinh(c + d*x)**4*cosh(c + d*x)**2/8 + 3*a*b*x*sinh(
c + d*x)**2*cosh(c + d*x)**4/8 - a*b*x*cosh(c + d*x)**6/8 - a*b*sinh(c + d
*x)**5*cosh(c + d*x)/(8*d) + a*b*sinh(c + d*x)**3*cosh(c + d*x)**3/(3*d) +
 a*b*sinh(c + d*x)*cosh(c + d*x)**5/(8*d) + 3*b**2*x*sinh(c + d*x)**8/128
- 3*b**2*x*sinh(c + d*x)**6*cosh(c + d*x)**2/32 + 9*b**2*x*sinh(c + d*x)**
4*cosh(c + d*x)**4/64 - 3*b**2*x*sinh(c + d*x)**2*cosh(c + d*x)**6/32 + 3*
b**2*x*cosh(c + d*x)**8/128 - 3*b**2*sinh(c + d*x)**7*cosh(c + d*x)/(128*d
) + 11*b**2*sinh(c + d*x)**5*cosh(c + d*x)**3/(128*d) + 11*b**2*sinh(c + d
*x)**3*cosh(c + d*x)**5/(128*d) - 3*b**2*sinh(c + d*x)*cosh(c + d*x)**7/(1
28*d), Ne(d, 0)), (x*(a + b*sinh(c)**2)**2*cosh(c)**4, True))
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 225, normalized size of antiderivative = 1.56

$$\int \cosh^4(c+dx)\left(a+b\sinh^2(c+dx)\right)^2\,dx$$
$$=\frac{1}{64}a^2\left(24x+\frac{e^{(4dx+4c)}}{d}+\frac{8e^{(2dx+2c)}}{d}-\frac{8e^{(-2dx-2c)}}{d}-\frac{e^{(-4dx-4c)}}{d}\right)$$
$$-\frac{1}{2048}b^2\left(\frac{\left(8e^{(-4dx-4c)}-1\right)e^{(8dx+8c)}}{d}-\frac{48(dx+c)}{d}-\frac{8e^{(-4dx-4c)}-e^{(-8dx-8c)}}{d}\right)$$
$$+\frac{1}{192}ab\left(\frac{\left(3e^{(-2dx-2c)}-3e^{(-4dx-4c)}+1\right)e^{(6dx+6c)}}{d}-\frac{24(dx+c)}{d}+\frac{3e^{(-2dx-2c)}-3e^{(-4dx-4c)}-e^{(-6}}{d}\right.$$

input
```
integrate(cosh(d*x+c)^4*(a+b*sinh(d*x+c)^2)^2,x, algorithm="maxima")
```

output
```
1/64*a^2*(24*x + e^(4*d*x + 4*c)/d + 8*e^(2*d*x + 2*c)/d - 8*e^(-2*d*x - 2
*c)/d - e^(-4*d*x - 4*c)/d) - 1/2048*b^2*((8*e^(-4*d*x - 4*c) - 1)*e^(8*d*
x + 8*c)/d - 48*(d*x + c)/d - (8*e^(-4*d*x - 4*c) - e^(-8*d*x - 8*c))/d) +
 1/192*a*b*((3*e^(-2*d*x - 2*c) - 3*e^(-4*d*x - 4*c) + 1)*e^(6*d*x + 6*c)/
d - 24*(d*x + c)/d + (3*e^(-2*d*x - 2*c) - 3*e^(-4*d*x - 4*c) - e^(-6*d*x
- 6*c))/d)
```

Giac [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 191, normalized size of antiderivative = 1.33

$$\int \cosh^4(c+dx)\left(a+b\sinh^2(c+dx)\right)^2\,dx$$
$$=\frac{1}{128}\left(48a^2-16ab+3b^2\right)x+\frac{b^2e^{(8dx+8c)}}{2048d}+\frac{abe^{(6dx+6c)}}{192d}-\frac{abe^{(-6dx-6c)}}{192d}$$
$$-\frac{b^2e^{(-8dx-8c)}}{2048d}+\frac{(4a^2+4ab-b^2)e^{(4dx+4c)}}{256d}+\frac{(8a^2-ab)e^{(2dx+2c)}}{64d}$$
$$-\frac{(8a^2-ab)e^{(-2dx-2c)}}{64d}-\frac{(4a^2+4ab-b^2)e^{(-4dx-4c)}}{256d}$$

input
```
integrate(cosh(d*x+c)^4*(a+b*sinh(d*x+c)^2)^2,x, algorithm="giac")
```

output
```
1/128*(48*a^2 - 16*a*b + 3*b^2)*x + 1/2048*b^2*e^(8*d*x + 8*c)/d + 1/192*a
*b*e^(6*d*x + 6*c)/d - 1/192*a*b*e^(-6*d*x - 6*c)/d - 1/2048*b^2*e^(-8*d*x
 - 8*c)/d + 1/256*(4*a^2 + 4*a*b - b^2)*e^(4*d*x + 4*c)/d + 1/64*(8*a^2 -
a*b)*e^(2*d*x + 2*c)/d - 1/64*(8*a^2 - a*b)*e^(-2*d*x - 2*c)/d - 1/256*(4*
a^2 + 4*a*b - b^2)*e^(-4*d*x - 4*c)/d
```

Mupad [B] (verification not implemented)

Time = 0.38 (sec) , antiderivative size = 121, normalized size of antiderivative = 0.84

$$\int \cosh^4(c+dx) \left(a+b\sinh^2(c+dx)\right)^2 \, dx = \frac{96\,a^2\sinh(2\,c+2\,d\,x)+12\,a^2\sinh(4\,c+4\,d\,x)-3\,b^2\sinh(4\,c+4\,d\,x)+\frac{3\,b^2\sinh(8\,c+8\,d\,x)}{8}-12\,a\,b\sinh(2\,c+\ldots}{384\,d}$$

input
```
int(cosh(c + d*x)^4*(a + b*sinh(c + d*x)^2)^2,x)
```

output
```
(96*a^2*sinh(2*c + 2*d*x) + 12*a^2*sinh(4*c + 4*d*x) - 3*b^2*sinh(4*c + 4*
d*x) + (3*b^2*sinh(8*c + 8*d*x))/8 - 12*a*b*sinh(2*c + 2*d*x) + 12*a*b*sin
h(4*c + 4*d*x) + 4*a*b*sinh(6*c + 6*d*x) + 144*a^2*d*x + 9*b^2*d*x - 48*a*
b*d*x)/(384*d)
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 262, normalized size of antiderivative = 1.82

$$\int \cosh^4(c+dx) \left(a+b\sinh^2(c+dx)\right)^2 \, dx = \frac{3e^{16dx+16c}b^2+32e^{14dx+14c}ab+96e^{12dx+12c}a^2+96e^{12dx+12c}ab-24e^{12dx+12c}b^2+768e^{10dx+10c}a^2-96e^{10dx+\ldots}}{\ldots}$$

input
```
int(cosh(d*x+c)^4*(a+b*sinh(d*x+c)^2)^2,x)
```

output
```
(3*e**(16*c + 16*d*x)*b**2 + 32*e**(14*c + 14*d*x)*a*b + 96*e**(12*c + 12*
d*x)*a**2 + 96*e**(12*c + 12*d*x)*a*b - 24*e**(12*c + 12*d*x)*b**2 + 768*e
**(10*c + 10*d*x)*a**2 - 96*e**(10*c + 10*d*x)*a*b + 2304*e**(8*c + 8*d*x)
*a**2*d*x - 768*e**(8*c + 8*d*x)*a*b*d*x + 144*e**(8*c + 8*d*x)*b**2*d*x -
 768*e**(6*c + 6*d*x)*a**2 + 96*e**(6*c + 6*d*x)*a*b - 96*e**(4*c + 4*d*x)
*a**2 - 96*e**(4*c + 4*d*x)*a*b + 24*e**(4*c + 4*d*x)*b**2 - 32*e**(2*c +
2*d*x)*a*b - 3*b**2)/(6144*e**(8*c + 8*d*x)*d)
```

3.258 $\int \cosh^3(c+dx) \left(a+b\sinh^2(c+dx)\right)^2 \, dx$

Optimal result

Integrand size = 23, antiderivative size = 74

$$\int \cosh^3(c+dx) \left(a+b\sinh^2(c+dx)\right)^2 \, dx = \frac{a^2\sinh(c+dx)}{d} + \frac{a(a+2b)\sinh^3(c+dx)}{3d} + \frac{b(2a+b)\sinh^5(c+dx)}{5d} + \frac{b^2\sinh^7(c+dx)}{7d}$$

output
```
a^2*sinh(d*x+c)/d+1/3*a*(a+2*b)*sinh(d*x+c)^3/d+1/5*b*(2*a+b)*sinh(d*x+c)^
5/d+1/7*b^2*sinh(d*x+c)^7/d
```

Mathematica [A] (verified)

Time = 0.09 (sec) , antiderivative size = 64, normalized size of antiderivative = 0.86

$$\int \cosh^3(c+dx) \left(a+b\sinh^2(c+dx)\right)^2 \, dx = \frac{105a^2\sinh(c+dx) + 35a(a+2b)\sinh^3(c+dx) + 21b(2a+b)\sinh^5(c+dx) + 15b^2\sinh^7(c+dx)}{105d}$$

input
```
Integrate[Cosh[c + d*x]^3*(a + b*Sinh[c + d*x]^2)^2,x]
```

output
```
(105*a^2*Sinh[c + d*x] + 35*a*(a + 2*b)*Sinh[c + d*x]^3 + 21*b*(2*a + b)*S
inh[c + d*x]^5 + 15*b^2*Sinh[c + d*x]^7)/(105*d)
```

Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 66, normalized size of antiderivative = 0.89, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.174$, Rules used = {3042, 3669, 290, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cosh^3(c+dx)\left(a+b\sinh^2(c+dx)\right)^2dx$$

$$\Big\downarrow \text{3042}$$

$$\int \cos(ic+idx)^3\left(a-b\sin(ic+idx)^2\right)^2dx$$

$$\Big\downarrow \text{3669}$$

$$\frac{\int \left(\sinh^2(c+dx)+1\right)\left(b\sinh^2(c+dx)+a\right)^2d\sinh(c+dx)}{d}$$

$$\Big\downarrow \text{290}$$

$$\frac{\int \left(b^2\sinh^6(c+dx)+b(2a+b)\sinh^4(c+dx)+a(a+2b)\sinh^2(c+dx)+a^2\right)d\sinh(c+dx)}{d}$$

$$\Big\downarrow \text{2009}$$

$$\frac{a^2\sinh(c+dx)+\frac{1}{5}b(2a+b)\sinh^5(c+dx)+\frac{1}{3}a(a+2b)\sinh^3(c+dx)+\frac{1}{7}b^2\sinh^7(c+dx)}{d}$$

input
```
Int[Cosh[c + d*x]^3*(a + b*Sinh[c + d*x]^2)^2,x]
```

output
```
(a^2*Sinh[c + d*x] + (a*(a + 2*b)*Sinh[c + d*x]^3)/3 + (b*(2*a + b)*Sinh[c
 + d*x]^5)/5 + (b^2*Sinh[c + d*x]^7)/7)/d
```

Defintions of rubi rules used

rule 290
```
Int[((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.), x_Symbol] :> I
nt[ExpandIntegrand[(a + b*x^2)^p*(c + d*x^2)^q, x], x] /; FreeQ[{a, b, c, d
}, x] && NeQ[b*c - a*d, 0] && IGtQ[p, 0] && IGtQ[q, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3669
```
Int[cos[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^
(p_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Simp[ff/f   S
ubst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a + b*ff^2*x^2)^p, x], x, Sin[e + f*x]
/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m - 1)/2]
```

Maple [A] (verified)

Time = 0.12 (sec) , antiderivative size = 65, normalized size of antiderivative = 0.88

$$\frac{\frac{b^2 \sinh(dx+c)^7}{7} + \frac{(2ab+b^2) \sinh(dx+c)^5}{5} + \frac{(a^2+2ab) \sinh(dx+c)^3}{3} + \sinh(dx+c)\, a^2}{d}$$

input
```
int(cosh(d*x+c)^3*(a+b*sinh(d*x+c)^2)^2,x)
```

output
```
1/d*(1/7*b^2*sinh(d*x+c)^7+1/5*(2*a*b+b^2)*sinh(d*x+c)^5+1/3*(a^2+2*a*b)*s
inh(d*x+c)^3+sinh(d*x+c)*a^2)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 188 vs. $2(68) = 136$.

Time = 0.10 (sec) , antiderivative size = 188, normalized size of antiderivative = 2.54

$$\int \cosh^3(c+dx)\left(a+b\sinh^2(c+dx)\right)^2\,dx$$
$$= \frac{15\,b^2\sinh\left(dx+c\right)^7+21\left(15\,b^2\cosh\left(dx+c\right)^2+8\,ab-b^2\right)\sinh\left(dx+c\right)^5+35\left(15\,b^2\cosh\left(dx+c\right)^4+6\left(\right.\right.}{}$$

input
```
integrate(cosh(d*x+c)^3*(a+b*sinh(d*x+c)^2)^2,x, algorithm="fricas")
```

output
```
1/6720*(15*b^2*sinh(d*x + c)^7 + 21*(15*b^2*cosh(d*x + c)^2 + 8*a*b - b^2)
*sinh(d*x + c)^5 + 35*(15*b^2*cosh(d*x + c)^4 + 6*(8*a*b - b^2)*cosh(d*x +
 c)^2 + 16*a^2 + 8*a*b - 3*b^2)*sinh(d*x + c)^3 + 105*(b^2*cosh(d*x + c)^6
 + (8*a*b - b^2)*cosh(d*x + c)^4 + (16*a^2 + 8*a*b - 3*b^2)*cosh(d*x + c)^
2 + 48*a^2 - 16*a*b + 3*b^2)*sinh(d*x + c))/d
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 136 vs. $2(63) = 126$.

Time = 0.45 (sec) , antiderivative size = 136, normalized size of antiderivative = 1.84

$$\int \cosh^3(c+dx)\left(a+b\sinh^2(c+dx)\right)^2\,dx$$
$$= \begin{cases} -\frac{2a^2\sinh^3(c+dx)}{3d}+\frac{a^2\sinh(c+dx)\cosh^2(c+dx)}{d}-\frac{4ab\sinh^5(c+dx)}{15d}+\frac{2ab\sinh^3(c+dx)\cosh^2(c+dx)}{3d}-\frac{2b^2\sinh^7(c+dx)}{35d}+\frac{b^2\sin}{} \\ x\left(a+b\sinh^2(c)\right)^2\cosh^3(c) \end{cases}$$

input
```
integrate(cosh(d*x+c)**3*(a+b*sinh(d*x+c)**2)**2,x)
```

output
```
Piecewise((-2*a**2*sinh(c + d*x)**3/(3*d) + a**2*sinh(c + d*x)*cosh(c + d*
x)**2/d - 4*a*b*sinh(c + d*x)**5/(15*d) + 2*a*b*sinh(c + d*x)**3*cosh(c +
d*x)**2/(3*d) - 2*b**2*sinh(c + d*x)**7/(35*d) + b**2*sinh(c + d*x)**5*cos
h(c + d*x)**2/(5*d), Ne(d, 0)), (x*(a + b*sinh(c)**2)**2*cosh(c)**3, True)
)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 242 vs. $2(68) = 136$.

Time = 0.04 (sec) , antiderivative size = 242, normalized size of antiderivative = 3.27

$$\int \cosh^3(c+dx)\left(a+b\sinh^2(c+dx)\right)^2\,dx = -\frac{1}{4480}b^2\left(\frac{\left(7\,e^{(-2\,dx-2\,c)}+35\,e^{(-4\,dx-4\,c)}-105\,e^{(-6\,dx-6\,c)}-5\right)e^{(7\,dx+7\,c)}}{d}+\frac{105\,e^{(-dx-c)}-35\,e^{(-3\,dx-3\,c)}-}{d}\right.$$
$$+\frac{1}{240}ab\left(\frac{\left(5\,e^{(-2\,dx-2\,c)}-30\,e^{(-4\,dx-4\,c)}+3\right)e^{(5\,dx+5\,c)}}{d}+\frac{30\,e^{(-dx-c)}-5\,e^{(-3\,dx-3\,c)}-3\,e^{(-5\,dx-5\,c)}}{d}\right)$$
$$+\frac{1}{24}a^2\left(\frac{e^{(3\,dx+3\,c)}}{d}+\frac{9\,e^{(dx+c)}}{d}-\frac{9\,e^{(-dx-c)}}{d}-\frac{e^{(-3\,dx-3\,c)}}{d}\right)$$

input
```
integrate(cosh(d*x+c)^3*(a+b*sinh(d*x+c)^2)^2,x, algorithm="maxima")
```

output
```
-1/4480*b^2*((7*e^(-2*d*x - 2*c) + 35*e^(-4*d*x - 4*c) - 105*e^(-6*d*x - 6
*c) - 5)*e^(7*d*x + 7*c)/d + (105*e^(-d*x - c) - 35*e^(-3*d*x - 3*c) - 7*e
^(-5*d*x - 5*c) + 5*e^(-7*d*x - 7*c))/d) + 1/240*a*b*((5*e^(-2*d*x - 2*c)
- 30*e^(-4*d*x - 4*c) + 3)*e^(5*d*x + 5*c)/d + (30*e^(-d*x - c) - 5*e^(-3*
d*x - 3*c) - 3*e^(-5*d*x - 5*c))/d) + 1/24*a^2*(e^(3*d*x + 3*c)/d + 9*e^(d
*x + c)/d - 9*e^(-d*x - c)/d - e^(-3*d*x - 3*c)/d)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 196 vs. $2(68) = 136$.

Time = 0.14 (sec) , antiderivative size = 196, normalized size of antiderivative = 2.65

$$\int \cosh^3(c+dx)\left(a+b\sinh^2(c+dx)\right)^2\,dx$$
$$=\frac{b^2e^{(7\,dx+7\,c)}}{896\,d}-\frac{b^2e^{(-7\,dx-7\,c)}}{896\,d}+\frac{(8\,ab-b^2)e^{(5\,dx+5\,c)}}{640\,d}+\frac{(16\,a^2+8\,ab-3\,b^2)e^{(3\,dx+3\,c)}}{384\,d}$$
$$+\frac{(48\,a^2-16\,ab+3\,b^2)e^{(dx+c)}}{128\,d}-\frac{(48\,a^2-16\,ab+3\,b^2)e^{(-dx-c)}}{128\,d}$$
$$-\frac{(16\,a^2+8\,ab-3\,b^2)e^{(-3\,dx-3\,c)}}{384\,d}-\frac{(8\,ab-b^2)e^{(-5\,dx-5\,c)}}{640\,d}$$

input
```
integrate(cosh(d*x+c)^3*(a+b*sinh(d*x+c)^2)^2,x, algorithm="giac")
```

output
```
1/896*b^2*e^(7*d*x + 7*c)/d - 1/896*b^2*e^(-7*d*x - 7*c)/d + 1/640*(8*a*b
- b^2)*e^(5*d*x + 5*c)/d + 1/384*(16*a^2 + 8*a*b - 3*b^2)*e^(3*d*x + 3*c)/
d + 1/128*(48*a^2 - 16*a*b + 3*b^2)*e^(d*x + c)/d - 1/128*(48*a^2 - 16*a*b
 + 3*b^2)*e^(-d*x - c)/d - 1/384*(16*a^2 + 8*a*b - 3*b^2)*e^(-3*d*x - 3*c)
/d - 1/640*(8*a*b - b^2)*e^(-5*d*x - 5*c)/d
```

Mupad [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 80, normalized size of antiderivative = 1.08

$$\int \cosh^3(c+dx)\left(a+b\sinh^2(c+dx)\right)^2\,dx$$
$$= \frac{35\,a^2\sinh(c+d\,x)^3+105\,a^2\sinh(c+d\,x)+42\,a\,b\sinh(c+d\,x)^5+70\,a\,b\sinh(c+d\,x)^3+15\,b^2\sinh(c+}{105\,d}$$

input
```
int(cosh(c + d*x)^3*(a + b*sinh(c + d*x)^2)^2,x)
```

output
```
(105*a^2*sinh(c + d*x) + 35*a^2*sinh(c + d*x)^3 + 21*b^2*sinh(c + d*x)^5 +
 15*b^2*sinh(c + d*x)^7 + 70*a*b*sinh(c + d*x)^3 + 42*a*b*sinh(c + d*x)^5)
/(105*d)
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 272, normalized size of antiderivative = 3.68

$$\int \cosh^3(c+dx)\left(a+b\sinh^2(c+dx)\right)^2\,dx$$
$$= \frac{15e^{14dx+14c}b^2+168e^{12dx+12c}ab-21e^{12dx+12c}b^2+560e^{10dx+10c}a^2+280e^{10dx+10c}ab-105e^{10dx+10c}b^2+504}{}$$

input
```
int(cosh(d*x+c)^3*(a+b*sinh(d*x+c)^2)^2,x)
```

output

```
(15*e**(14*c + 14*d*x)*b**2 + 168*e**(12*c + 12*d*x)*a*b - 21*e**(12*c + 1
2*d*x)*b**2 + 560*e**(10*c + 10*d*x)*a**2 + 280*e**(10*c + 10*d*x)*a*b - 1
05*e**(10*c + 10*d*x)*b**2 + 5040*e**(8*c + 8*d*x)*a**2 - 1680*e**(8*c + 8
*d*x)*a*b + 315*e**(8*c + 8*d*x)*b**2 - 5040*e**(6*c + 6*d*x)*a**2 + 1680*
e**(6*c + 6*d*x)*a*b - 315*e**(6*c + 6*d*x)*b**2 - 560*e**(4*c + 4*d*x)*a*
*2 - 280*e**(4*c + 4*d*x)*a*b + 105*e**(4*c + 4*d*x)*b**2 - 168*e**(2*c +
2*d*x)*a*b + 21*e**(2*c + 2*d*x)*b**2 - 15*b**2)/(13440*e**(7*c + 7*d*x)*d
)
```

3.259 $\int \cosh^2(c+dx) \left(a+b\sinh^2(c+dx)\right)^2 \, dx$

Optimal result

Integrand size = 23, antiderivative size = 104

$$\int \cosh^2(c+dx) \left(a+b\sinh^2(c+dx)\right)^2 \, dx = \frac{1}{16}\left(8a^2-4ab+b^2\right)x+\frac{(8a^2-4ab+b^2)\cosh(c+dx)\sinh(c+dx)}{16d} + \frac{(12a-7b)b\cosh^3(c+dx)\sinh(c+dx)}{24d}+\frac{b^2\cosh^5(c+dx)\sinh(c+dx)}{6d}$$

output
```
1/16*(8*a^2-4*a*b+b^2)*x+1/16*(8*a^2-4*a*b+b^2)*cosh(d*x+c)*sinh(d*x+c)/d+
1/24*(12*a-7*b)*b*cosh(d*x+c)^3*sinh(d*x+c)/d+1/6*b^2*cosh(d*x+c)^5*sinh(d
*x+c)/d
```

Mathematica [A] (verified)

Time = 1.44 (sec) , antiderivative size = 79, normalized size of antiderivative = 0.76

$$\int \cosh^2(c+dx) \left(a+b\sinh^2(c+dx)\right)^2 \, dx = \frac{12(8a^2-4ab+b^2)(c+dx)+3(16a^2-b^2)\sinh(2(c+dx))+3(4a-b)b\sinh(4(c+dx))+b^2\sinh(6(c+}{192d}$$

input
```
Integrate[Cosh[c + d*x]^2*(a + b*Sinh[c + d*x]^2)^2,x]
```

output
```
(12*(8*a^2 - 4*a*b + b^2)*(c + d*x) + 3*(16*a^2 - b^2)*Sinh[2*(c + d*x)] +
 3*(4*a - b)*b*Sinh[4*(c + d*x)] + b^2*Sinh[6*(c + d*x)])/(192*d)
```

Rubi [A] (verified)

Time = 0.34 (sec) , antiderivative size = 137, normalized size of antiderivative = 1.32, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.304$, Rules used = {3042, 3670, 315, 25, 298, 215, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cosh^2(c+dx)\left(a+b\sinh^2(c+dx)\right)^2\,dx$$

$$\Big\downarrow \ 3042$$

$$\int \cos(ic+idx)^2\left(a-b\sin(ic+idx)^2\right)^2dx$$

$$\Big\downarrow \ 3670$$

$$\frac{\int \frac{\left(a-(a-b)\tanh^2(c+dx)\right)^2}{\left(1-\tanh^2(c+dx)\right)^4}d\tanh(c+dx)}{d}$$

$$\Big\downarrow \ 315$$

$$\frac{\frac{b\tanh(c+dx)\left(a-(a-b)\tanh^2(c+dx)\right)}{6\left(1-\tanh^2(c+dx)\right)^3}-\frac{1}{6}\int -\frac{a(6a-b)-3(a-b)(2a-b)\tanh^2(c+dx)}{\left(1-\tanh^2(c+dx)\right)^3}d\tanh(c+dx)}{d}$$

$$\Big\downarrow \ 25$$

$$\frac{\frac{1}{6}\int \frac{a(6a-b)-3(a-b)(2a-b)\tanh^2(c+dx)}{\left(1-\tanh^2(c+dx)\right)^3}d\tanh(c+dx)+\frac{b\tanh(c+dx)\left(a-(a-b)\tanh^2(c+dx)\right)}{6\left(1-\tanh^2(c+dx)\right)^3}}{d}$$

$$\Big\downarrow \ 298$$

$$\frac{\frac{1}{6}\left(\frac{3}{4}\left(8a^2-4ab+b^2\right)\int \frac{1}{\left(1-\tanh^2(c+dx)\right)^2}d\tanh(c+dx)+\frac{b(8a-3b)\tanh(c+dx)}{4\left(1-\tanh^2(c+dx)\right)^2}\right)+\frac{b\tanh(c+dx)\left(a-(a-b)\tanh^2(c+dx)\right)}{6\left(1-\tanh^2(c+dx)\right)^3}}{d}$$

$$\Big\downarrow \ 215$$

$$\frac{\frac{1}{6}\left(\frac{3}{4}\left(8a^2-4ab+b^2\right)\left(\frac{1}{2}\int\frac{1}{1-\tanh^2(c+dx)}d\tanh(c+dx)+\frac{\tanh(c+dx)}{2(1-\tanh^2(c+dx))}\right)+\frac{b(8a-3b)\tanh(c+dx)}{4(1-\tanh^2(c+dx))^2}\right)+\frac{b\tanh(c+dx)(a\cdots}{6(1-\tanh\cdots}}{d}$$

$$\Big\downarrow \text{ 219}$$

$$\frac{\frac{1}{6}\left(\frac{3}{4}\left(8a^2-4ab+b^2\right)\left(\frac{1}{2}\text{arctanh}(\tanh(c+dx))+\frac{\tanh(c+dx)}{2(1-\tanh^2(c+dx))}\right)+\frac{b(8a-3b)\tanh(c+dx)}{4(1-\tanh^2(c+dx))^2}\right)+\frac{b\tanh(c+dx)(a-(a-b)\tanh\cdots}{6(1-\tanh^2(c+d\cdots}}{d}$$

input
```
Int[Cosh[c + d*x]^2*(a + b*Sinh[c + d*x]^2)^2,x]
```

output
```
((b*Tanh[c + d*x]*(a - (a - b)*Tanh[c + d*x]^2))/(6*(1 - Tanh[c + d*x]^2)^
3) + (((8*a - 3*b)*b*Tanh[c + d*x])/(4*(1 - Tanh[c + d*x]^2)^2) + (3*(8*a^
2 - 4*a*b + b^2)*(ArcTanh[Tanh[c + d*x]]/2 + Tanh[c + d*x]/(2*(1 - Tanh[c
+ d*x]^2))))/4)/6)/d
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 215
```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(-x)*((a + b*x^2)^(p + 1)
/(2*a*(p + 1))), x] + Simp[(2*p + 3)/(2*a*(p + 1))   Int[(a + b*x^2)^(p + 1
), x], x] /; FreeQ[{a, b}, x] && LtQ[p, -1] && (IntegerQ[4*p] || IntegerQ[6
*p])
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 298
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2), x_Symbol] :> Simp[(-(
b*c - a*d))*x*((a + b*x^2)^(p + 1)/(2*a*b*(p + 1))), x] - Simp[(a*d - b*c*(
2*p + 3))/(2*a*b*(p + 1))   Int[(a + b*x^2)^(p + 1), x], x] /; FreeQ[{a, b,
 c, d, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/2 + p, 0])
```

rule 315
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_), x_Symbol] :> Sim
p[(a*d - c*b)*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^(q - 1)/(2*a*b*(p + 1))),
x] - Simp[1/(2*a*b*(p + 1))   Int[(a + b*x^2)^(p + 1)*(c + d*x^2)^(q - 2)*S
imp[c*(a*d - c*b*(2*p + 3)) + d*(a*d*(2*(q - 1) + 1) - b*c*(2*(p + q) + 1))
*x^2, x], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -
1] && GtQ[q, 1] && IntBinomialQ[a, b, c, d, 2, p, q, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3670
```
Int[cos[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(
p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Simp[ff/f   Su
bst[Int[(a + (a + b)*ff^2*x^2)^p/(1 + ff^2*x^2)^(m/2 + p + 1), x], x, Tan[e
 + f*x]/ff], x]] /; FreeQ[{a, b, e, f}, x] && IntegerQ[m/2] && IntegerQ[p]
```

Maple [A] (verified)

Time = 175.69 (sec) , antiderivative size = 134, normalized size of antiderivative = 1.29

method	result
derivativedivides	$\frac{a^2\left(\frac{\cosh(dx+c)\sinh(dx+c)}{2}+\frac{dx}{2}+\frac{c}{2}\right)+2ab\left(\frac{\sinh(dx+c)\cosh(dx+c)^3}{4}-\frac{\cosh(dx+c)\sinh(dx+c)}{8}-\frac{dx}{8}-\frac{c}{8}\right)+b^2\left(\frac{\sinh(dx+c)^3\cos}{6}\right.}{d}$
default	$\frac{a^2\left(\frac{\cosh(dx+c)\sinh(dx+c)}{2}+\frac{dx}{2}+\frac{c}{2}\right)+2ab\left(\frac{\sinh(dx+c)\cosh(dx+c)^3}{4}-\frac{\cosh(dx+c)\sinh(dx+c)}{8}-\frac{dx}{8}-\frac{c}{8}\right)+b^2\left(\frac{\sinh(dx+c)^3\cos}{6}\right.}{d}$
risch	$\frac{a^2x}{2}-\frac{abx}{4}+\frac{b^2x}{16}+\frac{b^2e^{6dx+6c}}{384d}+\frac{e^{4dx+4c}ab}{32d}-\frac{e^{4dx+4c}b^2}{128d}+\frac{e^{2dx+2c}a^2}{8d}-\frac{e^{2dx+2c}b^2}{128d}-\frac{e^{-2dx-2c}a^2}{8d}+\mathrm{e}$
orering	Expression too large to display

input
```
int(cosh(d*x+c)^2*(a+b*sinh(d*x+c)^2)^2,x,method=_RETURNVERBOSE)
```

output
```
1/d*(a^2*(1/2*cosh(d*x+c)*sinh(d*x+c)+1/2*d*x+1/2*c)+2*a*b*(1/4*sinh(d*x+c
)*cosh(d*x+c)^3-1/8*cosh(d*x+c)*sinh(d*x+c)-1/8*d*x-1/8*c)+b^2*(1/6*sinh(d
*x+c)^3*cosh(d*x+c)^3-1/8*sinh(d*x+c)*cosh(d*x+c)^3+1/16*cosh(d*x+c)*sinh(
d*x+c)+1/16*d*x+1/16*c))
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 143, normalized size of antiderivative = 1.38

$$\int \cosh^2(c+dx) \left(a+b\sinh^2(c+dx)\right)^2 dx$$
$$= \frac{3\,b^2\cosh\left(dx+c\right)\sinh\left(dx+c\right)^5+2\left(5\,b^2\cosh\left(dx+c\right)^3+3\left(4\,ab-b^2\right)\cosh\left(dx+c\right)\right)\sinh\left(dx+c\right)^3+6}{}$$

input
```
integrate(cosh(d*x+c)^2*(a+b*sinh(d*x+c)^2)^2,x, algorithm="fricas")
```

output
```
1/96*(3*b^2*cosh(d*x + c)*sinh(d*x + c)^5 + 2*(5*b^2*cosh(d*x + c)^3 + 3*(
4*a*b - b^2)*cosh(d*x + c))*sinh(d*x + c)^3 + 6*(8*a^2 - 4*a*b + b^2)*d*x
+ 3*(b^2*cosh(d*x + c)^5 + 2*(4*a*b - b^2)*cosh(d*x + c)^3 + (16*a^2 - b^2
)*cosh(d*x + c))*sinh(d*x + c))/d
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 314 vs. $2(95) = 190$.

Time = 0.35 (sec) , antiderivative size = 314, normalized size of antiderivative = 3.02

$$\int \cosh^2(c+dx) \left(a+b\sinh^2(c+dx)\right)^2 dx$$
$$= \begin{cases} -\frac{a^2x\sinh^2(c+dx)}{2}+\frac{a^2x\cosh^2(c+dx)}{2}+\frac{a^2\sinh(c+dx)\cosh(c+dx)}{2d}-\frac{abx\sinh^4(c+dx)}{4}+\frac{abx\sinh^2(c+dx)\cosh^2(c+dx)}{2}-\frac{abx\,c}{} \\ x\left(a+b\sinh^2(c)\right)^2\cosh^2(c) \end{cases}$$

input
```
integrate(cosh(d*x+c)**2*(a+b*sinh(d*x+c)**2)**2,x)
```

output
```
Piecewise((-a**2*x*sinh(c + d*x)**2/2 + a**2*x*cosh(c + d*x)**2/2 + a**2*s
inh(c + d*x)*cosh(c + d*x)/(2*d) - a*b*x*sinh(c + d*x)**4/4 + a*b*x*sinh(c
 + d*x)**2*cosh(c + d*x)**2/2 - a*b*x*cosh(c + d*x)**4/4 + a*b*sinh(c + d*
x)**3*cosh(c + d*x)/(4*d) + a*b*sinh(c + d*x)*cosh(c + d*x)**3/(4*d) - b**
2*x*sinh(c + d*x)**6/16 + 3*b**2*x*sinh(c + d*x)**4*cosh(c + d*x)**2/16 -
3*b**2*x*sinh(c + d*x)**2*cosh(c + d*x)**4/16 + b**2*x*cosh(c + d*x)**6/16
 + b**2*sinh(c + d*x)**5*cosh(c + d*x)/(16*d) + b**2*sinh(c + d*x)**3*cosh
(c + d*x)**3/(6*d) - b**2*sinh(c + d*x)*cosh(c + d*x)**5/(16*d), Ne(d, 0))
, (x*(a + b*sinh(c)**2)**2*cosh(c)**2, True))
```

Maxima [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 171, normalized size of antiderivative = 1.64

$$\int \cosh^2(c+dx)\left(a+b\sinh^2(c+dx)\right)^2\,dx = \frac{1}{8}\,a^2\left(4\,x+\frac{e^{(2\,dx+2\,c)}}{d}-\frac{e^{(-2\,dx-2\,c)}}{d}\right) - \frac{1}{384}\,b^2\left(\frac{\left(3\,e^{(-2\,dx-2\,c)}+3\,e^{(-4\,dx-4\,c)}-1\right)e^{(6\,dx+6\,c)}}{d}-\frac{24\,(dx+c)}{d}-\frac{3\,e^{(-2\,dx-2\,c)}+3\,e^{(-4\,dx-4\,c)}-e^{(-6\,c\cdots}}{d}\right. - \frac{1}{32}\,ab\left(\frac{8\,(dx+c)}{d}-\frac{e^{(4\,dx+4\,c)}}{d}+\frac{e^{(-4\,dx-4\,c)}}{d}\right)$$

input
```
integrate(cosh(d*x+c)^2*(a+b*sinh(d*x+c)^2)^2,x, algorithm="maxima")
```

output
```
1/8*a^2*(4*x + e^(2*d*x + 2*c)/d - e^(-2*d*x - 2*c)/d) - 1/384*b^2*((3*e^(
-2*d*x - 2*c) + 3*e^(-4*d*x - 4*c) - 1)*e^(6*d*x + 6*c)/d - 24*(d*x + c)/d
 - (3*e^(-2*d*x - 2*c) + 3*e^(-4*d*x - 4*c) - e^(-6*d*x - 6*c))/d) - 1/32*
a*b*(8*(d*x + c)/d - e^(4*d*x + 4*c)/d + e^(-4*d*x - 4*c)/d)
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 149, normalized size of antiderivative = 1.43

$$\int \cosh^2(c+dx)\left(a+b\sinh^2(c+dx)\right)^2\,dx = \frac{1}{16}\left(8\,a^2-4\,ab+b^2\right)x+\frac{b^2e^{(6\,dx+6\,c)}}{384\,d} - \frac{b^2e^{(-6\,dx-6\,c)}}{384\,d}+\frac{(4\,ab-b^2)e^{(4\,dx+4\,c)}}{128\,d} + \frac{(16\,a^2-b^2)e^{(2\,dx+2\,c)}}{128\,d} - \frac{(16\,a^2-b^2)e^{(-2\,dx-2\,c)}}{128\,d} - \frac{(4\,ab-b^2)e^{(-4\,dx-4\,c)}}{128\,d}$$

input
```
integrate(cosh(d*x+c)^2*(a+b*sinh(d*x+c)^2)^2,x, algorithm="giac")
```

output
```
1/16*(8*a^2 - 4*a*b + b^2)*x + 1/384*b^2*e^(6*d*x + 6*c)/d - 1/384*b^2*e^(
-6*d*x - 6*c)/d + 1/128*(4*a*b - b^2)*e^(4*d*x + 4*c)/d + 1/128*(16*a^2 -
b^2)*e^(2*d*x + 2*c)/d - 1/128*(16*a^2 - b^2)*e^(-2*d*x - 2*c)/d - 1/128*(
4*a*b - b^2)*e^(-4*d*x - 4*c)/d
```

Mupad [B] (verification not implemented)

Time = 2.19 (sec) , antiderivative size = 95, normalized size of antiderivative = 0.91

$$\int \cosh^2(c+dx)\left(a+b\sinh^2(c+dx)\right)^2\,dx = \frac{12\,a^2\sinh(2\,c+2\,d\,x)-\frac{3\,b^2\sinh(2\,c+2\,d\,x)}{4}-\frac{3\,b^2\sinh(4\,c+4\,d\,x)}{4}+\frac{b^2\sinh(6\,c+6\,d\,x)}{4}+3\,a\,b\sinh(4\,c+4\,d\,x)+24\,a^2}{48\,d}$$

input
```
int(cosh(c + d*x)^2*(a + b*sinh(c + d*x)^2)^2,x)
```

output
```
(12*a^2*sinh(2*c + 2*d*x) - (3*b^2*sinh(2*c + 2*d*x))/4 - (3*b^2*sinh(4*c
+ 4*d*x))/4 + (b^2*sinh(6*c + 6*d*x))/4 + 3*a*b*sinh(4*c + 4*d*x) + 24*a^2
*d*x + 3*b^2*d*x - 12*a*b*d*x)/(48*d)
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 205, normalized size of antiderivative = 1.97

$$\int \cosh^2(c+dx)\left(a+b\sinh^2(c+dx)\right)^2\,dx$$
$$= \frac{e^{12dx+12c}b^2+12e^{10dx+10c}ab-3e^{10dx+10c}b^2+48e^{8dx+8c}a^2-3e^{8dx+8c}b^2+192e^{6dx+6c}a^2dx-96e^{6dx+6c}abdx}{384e^{6dx+6c}d}$$

input
```
int(cosh(d*x+c)^2*(a+b*sinh(d*x+c)^2)^2,x)
```

output
```
(e**(12*c + 12*d*x)*b**2 + 12*e**(10*c + 10*d*x)*a*b - 3*e**(10*c + 10*d*x
)*b**2 + 48*e**(8*c + 8*d*x)*a**2 - 3*e**(8*c + 8*d*x)*b**2 + 192*e**(6*c
+ 6*d*x)*a**2*d*x - 96*e**(6*c + 6*d*x)*a*b*d*x + 24*e**(6*c + 6*d*x)*b**2
*d*x - 48*e**(4*c + 4*d*x)*a**2 + 3*e**(4*c + 4*d*x)*b**2 - 12*e**(2*c + 2
*d*x)*a*b + 3*e**(2*c + 2*d*x)*b**2 - b**2)/(384*e**(6*c + 6*d*x)*d)
```

3.260 $\int \cosh(c+dx)\left(a+b\sinh^2(c+dx)\right)^2\,dx$

Optimal result

Integrand size = 21, antiderivative size = 49

$$\int \cosh(c+dx)\left(a+b\sinh^2(c+dx)\right)^2\,dx=\frac{a^2\sinh(c+dx)}{d}+\frac{2ab\sinh^3(c+dx)}{3d}+\frac{b^2\sinh^5(c+dx)}{5d}$$

output
```
a^2*sinh(d*x+c)/d+2/3*a*b*sinh(d*x+c)^3/d+1/5*b^2*sinh(d*x+c)^5/d
```

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.90

$$\int \cosh(c+dx)\left(a+b\sinh^2(c+dx)\right)^2\,dx=\frac{a^2\sinh(c+dx)+\frac{2}{3}ab\sinh^3(c+dx)+\frac{1}{5}b^2\sinh^5(c+dx)}{d}$$

input
```
Integrate[Cosh[c + d*x]*(a + b*Sinh[c + d*x]^2)^2,x]
```

output
```
(a^2*Sinh[c + d*x] + (2*a*b*Sinh[c + d*x]^3)/3 + (b^2*Sinh[c + d*x]^5)/5)/
d
```

Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.90, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.190$, Rules used = {3042, 3669, 210, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cosh(c+dx)\left(a+b\sinh^2(c+dx)\right)^2 dx$$

$$\Big\downarrow \text{3042}$$

$$\int \cos(ic+idx)\left(a-b\sin(ic+idx)^2\right)^2 dx$$

$$\Big\downarrow \text{3669}$$

$$\frac{\int \left(b\sinh^2(c+dx)+a\right)^2 d\sinh(c+dx)}{d}$$

$$\Big\downarrow \text{210}$$

$$\frac{\int \left(b^2\sinh^4(c+dx)+2ab\sinh^2(c+dx)+a^2\right) d\sinh(c+dx)}{d}$$

$$\Big\downarrow \text{2009}$$

$$\frac{a^2\sinh(c+dx)+\frac{2}{3}ab\sinh^3(c+dx)+\frac{1}{5}b^2\sinh^5(c+dx)}{d}$$

input
```
Int[Cosh[c + d*x]*(a + b*Sinh[c + d*x]^2)^2,x]
```

output

```
(a^2*Sinh[c + d*x] + (2*a*b*Sinh[c + d*x]^3)/3 + (b^2*Sinh[c + d*x]^5)/5)/
d
```

Defintions of rubi rules used

rule 210
```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Int[ExpandIntegrand[(a + b*x^2
)^p, x], x] /; FreeQ[{a, b}, x] && IGtQ[p, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3669
```
Int[cos[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^
(p_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Simp[ff/f   S
ubst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a + b*ff^2*x^2)^p, x], x, Sin[e + f*x]
/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m - 1)/2]
```

Maple [A] (verified)

Time = 79.28 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.84

method	result
derivativedivides	$\frac{\frac{b^{2} \sinh(dx+c)^{5}}{5}+\frac{2ab \sinh(dx+c)^{3}}{3}+\sinh(dx+c) a^{2}}{d}$
default	$\frac{\frac{b^{2} \sinh(dx+c)^{5}}{5}+\frac{2ab \sinh(dx+c)^{3}}{3}+\sinh(dx+c) a^{2}}{d}$
risch	$\frac{b^{2} e^{5dx+5c}}{160d}+\frac{e^{3dx+3c} ab}{12d}-\frac{e^{3dx+3c} b^{2}}{32d}+\frac{e^{dx+c} a^{2}}{2d}-\frac{e^{dx+c} ab}{4d}+\frac{e^{dx+c} b^{2}}{16d}-\frac{e^{-dx-c} a^{2}}{2d}+\frac{e^{-dx-c} ab}{4d}-\frac{e^{-dx}}{16}$
orering	$\frac{\frac{259d \sinh(dx+c)\left(a+b \sinh(dx+c)^{2}\right)^{2}}{225}+\frac{1036 \cosh(dx+c)^{2}\left(a+b \sinh(dx+c)^{2}\right) b \sinh(dx+c) d}{225}}{d^{2}}-\frac{7\left(d^{3} \sinh(dx+c)\left(a+b \sinh(dx\right.\right.}{}$

input
```
int(cosh(d*x+c)*(a+b*sinh(d*x+c)^2)^2,x,method=_RETURNVERBOSE)
```

output
```
1/d*(1/5*b^2*sinh(d*x+c)^5+2/3*a*b*sinh(d*x+c)^3+sinh(d*x+c)*a^2)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 106 vs. $2(45) = 90$.

Time = 0.09 (sec) , antiderivative size = 106, normalized size of antiderivative = 2.16

$$\int \cosh(c+dx)\left(a+b\sinh^2(c+dx)\right)^2\,dx = \frac{3\,b^2\sinh\left(dx+c\right)^5+5\left(6\,b^2\cosh\left(dx+c\right)^2+8\,ab-3\,b^2\right)\sinh\left(dx+c\right)^3+15\left(b^2\cosh\left(dx+c\right)^4+(8\,ab-\right.}{240\,d}$$

input
```
integrate(cosh(d*x+c)*(a+b*sinh(d*x+c)^2)^2,x, algorithm="fricas")
```

output
```
1/240*(3*b^2*sinh(d*x + c)^5 + 5*(6*b^2*cosh(d*x + c)^2 + 8*a*b - 3*b^2)*s
inh(d*x + c)^3 + 15*(b^2*cosh(d*x + c)^4 + (8*a*b - 3*b^2)*cosh(d*x + c)^2
 + 16*a^2 - 8*a*b + 2*b^2)*sinh(d*x + c))/d
```

Sympy [A] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 58, normalized size of antiderivative = 1.18

$$\int \cosh(c+dx)\left(a+b\sinh^2(c+dx)\right)^2\,dx = \begin{cases} \frac{a^2\sinh\left(c+dx\right)}{d}+\frac{2ab\sinh^3\left(c+dx\right)}{3d}+\frac{b^2\sinh^5\left(c+dx\right)}{5d} & \text{for } d\neq 0 \\ x\left(a+b\sinh^2\left(c\right)\right)^2\cosh\left(c\right) & \text{otherwise} \end{cases}$$

input
```
integrate(cosh(d*x+c)*(a+b*sinh(d*x+c)**2)**2,x)
```

output
```
Piecewise((a**2*sinh(c + d*x)/d + 2*a*b*sinh(c + d*x)**3/(3*d) + b**2*sinh
(c + d*x)**5/(5*d), Ne(d, 0)), (x*(a + b*sinh(c)**2)**2*cosh(c), True))
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 45, normalized size of antiderivative = 0.92

$$\int \cosh(c+dx)\left(a+b\sinh^2(c+dx)\right)^2\,dx = \frac{b^2\sinh(dx+c)^5}{5\,d} + \frac{2\,ab\sinh(dx+c)^3}{3\,d} + \frac{a^2\sinh(dx+c)}{d}$$

input
```
integrate(cosh(d*x+c)*(a+b*sinh(d*x+c)^2)^2,x, algorithm="maxima")
```

output
```
1/5*b^2*sinh(d*x + c)^5/d + 2/3*a*b*sinh(d*x + c)^3/d + a^2*sinh(d*x + c)/
d
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 134 vs. 2(45) = 90.

Time = 0.14 (sec) , antiderivative size = 134, normalized size of antiderivative = 2.73

$$\int \cosh(c+dx)\left(a+b\sinh^2(c+dx)\right)^2\,dx = \frac{b^2e^{(5\,dx+5\,c)}}{160\,d} - \frac{b^2e^{(-5\,dx-5\,c)}}{160\,d} + \frac{(8\,ab-3\,b^2)e^{(3\,dx+3\,c)}}{96\,d} + \frac{(8\,a^2-4\,ab+b^2)e^{(dx+c)}}{16\,d} - \frac{(8\,a^2-4\,ab+b^2)e^{(-dx-c)}}{16\,d} - \frac{(8\,ab-3\,b^2)e^{(-3\,dx-3\,c)}}{96\,d}$$

input
```
integrate(cosh(d*x+c)*(a+b*sinh(d*x+c)^2)^2,x, algorithm="giac")
```

output
```
1/160*b^2*e^(5*d*x + 5*c)/d - 1/160*b^2*e^(-5*d*x - 5*c)/d + 1/96*(8*a*b -
 3*b^2)*e^(3*d*x + 3*c)/d + 1/16*(8*a^2 - 4*a*b + b^2)*e^(d*x + c)/d - 1/1
6*(8*a^2 - 4*a*b + b^2)*e^(-d*x - c)/d - 1/96*(8*a*b - 3*b^2)*e^(-3*d*x -
3*c)/d
```

Mupad [B] (verification not implemented)

Time = 2.03 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.86

$$\int \cosh(c+dx)\left(a+b\sinh^2(c+dx)\right)^2\,dx = \frac{\sinh(c+d\,x)\,\left(15\,a^2+10\,a\,b\sinh(c+d\,x)^2+3\,b^2\sinh(c+d\,x)^4\right)}{15\,d}$$

input
```
int(cosh(c + d*x)*(a + b*sinh(c + d*x)^2)^2,x)
```

output
```
(sinh(c + d*x)*(15*a^2 + 3*b^2*sinh(c + d*x)^4 + 10*a*b*sinh(c + d*x)^2))/
(15*d)
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.86

$$\int \cosh(c+dx)\left(a+b\sinh^2(c+dx)\right)^2\,dx = \frac{\sinh(dx+c)\left(3\sinh(dx+c)^4b^2+10\sinh(dx+c)^2ab+15a^2\right)}{15d}$$

input
```
int(cosh(d*x+c)*(a+b*sinh(d*x+c)^2)^2,x)
```

output
```
(sinh(c + d*x)*(3*sinh(c + d*x)**4*b**2 + 10*sinh(c + d*x)**2*a*b + 15*a**
2))/(15*d)
```

3.261 $\int \operatorname{sech}(c+dx) \left(a+b\sinh^2(c+dx)\right)^2 dx$

Optimal result

Integrand size = 21, antiderivative size = 55

$$\int \operatorname{sech}(c+dx) \left(a+b\sinh^2(c+dx)\right)^2 dx = \frac{(a-b)^2\arctan(\sinh(c+dx))}{d} + \frac{(2a-b)b\sinh(c+dx)}{d} + \frac{b^2\sinh^3(c+dx)}{3d}$$

output
```
(a-b)^2*arctan(sinh(d*x+c))/d+(2*a-b)*b*sinh(d*x+c)/d+1/3*b^2*sinh(d*x+c)^
3/d
```

Mathematica [A] (verified)

Time = 0.18 (sec) , antiderivative size = 70, normalized size of antiderivative = 1.27

$$\int \operatorname{sech}(c+dx) \left(a+b\sinh^2(c+dx)\right)^2 dx = \frac{\sinh(c+dx)\left(\frac{3(a-b)^2\operatorname{arctanh}\left(\sqrt{-\sinh^2(c+dx)}\right)}{\sqrt{-\sinh^2(c+dx)}} + b\left(6a+b\left(-3+\sinh^2(c+dx)\right)\right)\right)}{3d}$$

input
```
Integrate[Sech[c + d*x]*(a + b*Sinh[c + d*x]^2)^2,x]
```

output
```
(Sinh[c + d*x]*((3*(a - b)^2*ArcTanh[Sqrt[-Sinh[c + d*x]^2]])/Sqrt[-Sinh[c
 + d*x]^2] + b*(6*a + b*(-3 + Sinh[c + d*x]^2))))/(3*d)
```

Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 50, normalized size of antiderivative = 0.91, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.190$, Rules used = {3042, 3669, 300, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \operatorname{sech}(c+dx)\left(a+b\sinh^2(c+dx)\right)^2\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\left(a-b\sin(ic+idx)^2\right)^2}{\cos(ic+idx)}dx$$

$$\Big\downarrow \text{3669}$$

$$\frac{\int \frac{\left(b\sinh^2(c+dx)+a\right)^2}{\sinh^2(c+dx)+1}d\sinh(c+dx)}{d}$$

$$\Big\downarrow \text{300}$$

$$\frac{\int \left(\frac{(a-b)^2}{\sinh^2(c+dx)+1}+b^2\sinh^2(c+dx)+(2a-b)b\right)d\sinh(c+dx)}{d}$$

$$\Big\downarrow \text{2009}$$

$$\frac{(a-b)^2\arctan(\sinh(c+dx))+b(2a-b)\sinh(c+dx)+\frac{1}{3}b^2\sinh^3(c+dx)}{d}$$

input
```
Int[Sech[c + d*x]*(a + b*Sinh[c + d*x]^2)^2,x]
```

output
```
((a - b)^2*ArcTan[Sinh[c + d*x]] + (2*a - b)*b*Sinh[c + d*x] + (b^2*Sinh[c
 + d*x]^3)/3)/d
```

Defintions of rubi rules used

rule 300
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_), x_Symbol] :> Int
[PolynomialDivide[(a + b*x^2)^p, (c + d*x^2)^(-q), x], x] /; FreeQ[{a, b, c
, d}, x] && NeQ[b*c - a*d, 0] && IGtQ[p, 0] && ILtQ[q, 0] && GeQ[p, -q]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3669
```
Int[cos[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^
(p_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Simp[ff/f   S
ubst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a + b*ff^2*x^2)^p, x], x, Sin[e + f*x]
/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m - 1)/2]
```

Maple [A] (verified)

Time = 77.58 (sec) , antiderivative size = 70, normalized size of antiderivative = 1.27

method	result
derivativedivides	$\frac{2a^2\arctan(\mathrm{e}^{dx+c})+2ab(\sinh(dx+c)-2\arctan(\mathrm{e}^{dx+c}))+b^2\left(\frac{\sinh(dx+c)^3}{3}-\sinh(dx+c)+2\arctan(\mathrm{e}^{dx+c})\right)}{d}$
default	$\frac{2a^2\arctan(\mathrm{e}^{dx+c})+2ab(\sinh(dx+c)-2\arctan(\mathrm{e}^{dx+c}))+b^2\left(\frac{\sinh(dx+c)^3}{3}-\sinh(dx+c)+2\arctan(\mathrm{e}^{dx+c})\right)}{d}$
risch	$\frac{\mathrm{e}^{3dx+3c}b^2}{24d}+\frac{\mathrm{e}^{dx+c}ab}{d}-\frac{5\,\mathrm{e}^{dx+c}b^2}{8d}-\frac{\mathrm{e}^{-dx-c}ab}{d}+\frac{5\,\mathrm{e}^{-dx-c}b^2}{8d}-\frac{\mathrm{e}^{-3dx-3c}b^2}{24d}+\frac{i\ln(\mathrm{e}^{dx+c}+i)a^2}{d}-\frac{2i\ln(\mathrm{e}$

input
```
int(sech(d*x+c)*(a+b*sinh(d*x+c)^2)^2,x,method=_RETURNVERBOSE)
```

output
```
1/d*(2*a^2*arctan(exp(d*x+c))+2*a*b*(sinh(d*x+c)-2*arctan(exp(d*x+c)))+b^2
*(1/3*sinh(d*x+c)^3-sinh(d*x+c)+2*arctan(exp(d*x+c))))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 446 vs. $2(53) = 106$.

Time = 0.10 (sec) , antiderivative size = 446, normalized size of antiderivative = 8.11

$$\int \operatorname{sech}(c+dx)\left(a+b\sinh^2(c+dx)\right)^2\,dx = \frac{b^2\cosh(dx+c)^6+6\,b^2\cosh(dx+c)\sinh(dx+c)^5+b^2\sinh(dx+c)^6+3\,(8\,ab-5\,b^2)\cosh(dx+c)^4+}{}$$

input
```
integrate(sech(d*x+c)*(a+b*sinh(d*x+c)^2)^2,x, algorithm="fricas")
```

output
```
1/24*(b^2*cosh(d*x + c)^6 + 6*b^2*cosh(d*x + c)*sinh(d*x + c)^5 + b^2*sinh
(d*x + c)^6 + 3*(8*a*b - 5*b^2)*cosh(d*x + c)^4 + 3*(5*b^2*cosh(d*x + c)^2
 + 8*a*b - 5*b^2)*sinh(d*x + c)^4 + 4*(5*b^2*cosh(d*x + c)^3 + 3*(8*a*b -
5*b^2)*cosh(d*x + c))*sinh(d*x + c)^3 - 3*(8*a*b - 5*b^2)*cosh(d*x + c)^2
+ 3*(5*b^2*cosh(d*x + c)^4 + 6*(8*a*b - 5*b^2)*cosh(d*x + c)^2 - 8*a*b + 5
*b^2)*sinh(d*x + c)^2 - b^2 + 48*((a^2 - 2*a*b + b^2)*cosh(d*x + c)^3 + 3*
(a^2 - 2*a*b + b^2)*cosh(d*x + c)^2*sinh(d*x + c) + 3*(a^2 - 2*a*b + b^2)*
cosh(d*x + c)*sinh(d*x + c)^2 + (a^2 - 2*a*b + b^2)*sinh(d*x + c)^3)*arcta
n(cosh(d*x + c) + sinh(d*x + c)) + 6*(b^2*cosh(d*x + c)^5 + 2*(8*a*b - 5*b
^2)*cosh(d*x + c)^3 - (8*a*b - 5*b^2)*cosh(d*x + c))*sinh(d*x + c))/(d*cos
h(d*x + c)^3 + 3*d*cosh(d*x + c)^2*sinh(d*x + c) + 3*d*cosh(d*x + c)*sinh(
d*x + c)^2 + d*sinh(d*x + c)^3)
```

Sympy [F]

$$\int \operatorname{sech}(c+dx)\left(a+b\sinh^2(c+dx)\right)^2\,dx = \int \left(a+b\sinh^2(c+dx)\right)^2\operatorname{sech}(c+dx)\,dx$$

input
```
integrate(sech(d*x+c)*(a+b*sinh(d*x+c)**2)**2,x)
```

output
```
Integral((a + b*sinh(c + d*x)**2)**2*sech(c + d*x), x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 133 vs. $2(53) = 106$.

Time = 0.13 (sec) , antiderivative size = 133, normalized size of antiderivative = 2.42

$$\int \operatorname{sech}(c+dx)\left(a+b\sinh^2(c+dx)\right)^2\,dx = -\frac{1}{24}\,b^2\left(\frac{\left(15\,e^{(-2\,dx-2\,c)}-1\right)e^{(3\,dx+3\,c)}}{d}-\frac{15\,e^{(-dx-c)}-e^{(-3\,dx-3\,c)}}{d}+\frac{48\,\arctan\left(e^{(-dx-c)}\right)}{d}\right) + ab\left(\frac{4\,\arctan\left(e^{(-dx-c)}\right)}{d}+\frac{e^{(dx+c)}}{d}-\frac{e^{(-dx-c)}}{d}\right)+\frac{a^2\arctan\left(\sinh\left(dx+c\right)\right)}{d}$$

input
```
integrate(sech(d*x+c)*(a+b*sinh(d*x+c)^2)^2,x, algorithm="maxima")
```

output
```
-1/24*b^2*((15*e^(-2*d*x - 2*c) - 1)*e^(3*d*x + 3*c)/d - (15*e^(-d*x - c)
- e^(-3*d*x - 3*c))/d + 48*arctan(e^(-d*x - c))/d) + a*b*(4*arctan(e^(-d*x
 - c))/d + e^(d*x + c)/d - e^(-d*x - c)/d) + a^2*arctan(sinh(d*x + c))/d
```

Giac [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 102, normalized size of antiderivative = 1.85

$$\int \operatorname{sech}(c+dx)\left(a+b\sinh^2(c+dx)\right)^2\,dx = \frac{b^2e^{(3\,dx+3\,c)}+24\,abe^{(dx+c)}-15\,b^2e^{(dx+c)}+48\left(a^2-2\,ab+b^2\right)\arctan\left(e^{(dx+c)}\right)-\left(24\,abe^{(2\,dx+2\,c)}-15\,b^2e^{(}\right.}{24\,d}$$

input
```
integrate(sech(d*x+c)*(a+b*sinh(d*x+c)^2)^2,x, algorithm="giac")
```

output
```
1/24*(b^2*e^(3*d*x + 3*c) + 24*a*b*e^(d*x + c) - 15*b^2*e^(d*x + c) + 48*(
a^2 - 2*a*b + b^2)*arctan(e^(d*x + c)) - (24*a*b*e^(2*d*x + 2*c) - 15*b^2*
e^(2*d*x + 2*c) + b^2)*e^(-3*d*x - 3*c))/d
```

Mupad [B] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 182, normalized size of antiderivative = 3.31

$$\int \operatorname{sech}(c+dx)\left(a+b\sinh^2(c+dx)\right)^2\,dx$$
$$=\frac{b^2\,\mathrm{e}^{3\,c+3\,d\,x}}{24\,d}-\frac{b^2\,\mathrm{e}^{-3\,c-3\,d\,x}}{24\,d}-\frac{\mathrm{e}^{-c-d\,x}\,\left(8\,a\,b-5\,b^2\right)}{8\,d}+\frac{2\,\mathrm{atan}\left(\frac{\mathrm{e}^{d\,x}\,\mathrm{e}^{c}\,\left(a^2\,\sqrt{d^2}+b^2\,\sqrt{d^2}-2\,a\,b\,\sqrt{d^2}\right)}{d\,\sqrt{a^4-4\,a^3\,b+6\,a^2\,b^2-4\,a\,b^3+b^4}}\right)\,\sqrt{a^4-4\,a^3\,b+6\,a^2\,b^2-4\,a\,b^3+b^4}}{\sqrt{d^2}}+\frac{b\,\mathrm{e}^{c+d\,x}\,\left(8\,a-5\,b\right)}{8\,d}$$

input
```
int((a + b*sinh(c + d*x)^2)^2/cosh(c + d*x),x)
```

output
```
(b^2*exp(3*c + 3*d*x))/(24*d) - (b^2*exp(- 3*c - 3*d*x))/(24*d) - (exp(- c
 - d*x)*(8*a*b - 5*b^2))/(8*d) + (2*atan((exp(d*x)*exp(c)*(a^2*(d^2)^(1/2)
 + b^2*(d^2)^(1/2) - 2*a*b*(d^2)^(1/2)))/(d*(a^4 - 4*a^3*b - 4*a*b^3 + b^4
 + 6*a^2*b^2)^(1/2)))*(a^4 - 4*a^3*b - 4*a*b^3 + b^4 + 6*a^2*b^2)^(1/2))/(
d^2)^(1/2) + (b*exp(c + d*x)*(8*a - 5*b))/(8*d)
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 163, normalized size of antiderivative = 2.96

$$\int \operatorname{sech}(c+dx)\left(a+b\sinh^2(c+dx)\right)^2\,dx$$
$$=\frac{48e^{3dx+3c}atan\left(e^{dx+c}\right)a^2-96e^{3dx+3c}atan\left(e^{dx+c}\right)ab+48e^{3dx+3c}atan\left(e^{dx+c}\right)b^2+e^{6dx+6c}b^2+24e^{4dx+4c}ab}{24e^{3dx+3c}d}$$

input
```
int(sech(d*x+c)*(a+b*sinh(d*x+c)^2)^2,x)
```

output
```
(48*e**(3*c + 3*d*x)*atan(e**(c + d*x))*a**2 - 96*e**(3*c + 3*d*x)*atan(e*
*(c + d*x))*a*b + 48*e**(3*c + 3*d*x)*atan(e**(c + d*x))*b**2 + e**(6*c +
6*d*x)*b**2 + 24*e**(4*c + 4*d*x)*a*b - 15*e**(4*c + 4*d*x)*b**2 - 24*e**(
2*c + 2*d*x)*a*b + 15*e**(2*c + 2*d*x)*b**2 - b**2)/(24*e**(3*c + 3*d*x)*d
)
```

3.262 $\int \operatorname{sech}^2(c+dx) \left(a+b\sinh^2(c+dx)\right)^2 \, dx$

Optimal result

Integrand size = 23, antiderivative size = 53

$$\int \operatorname{sech}^2(c+dx) \left(a+b\sinh^2(c+dx)\right)^2 \, dx = \frac{1}{2}(4a-3b)bx + \frac{b^2\cosh(c+dx)\sinh(c+dx)}{2d} + \frac{(a-b)^2\tanh(c+dx)}{d}$$

output
```
1/2*(4*a-3*b)*b*x+1/2*b^2*cosh(d*x+c)*sinh(d*x+c)/d+(a-b)^2*tanh(d*x+c)/d
```

Mathematica [A] (verified)

Time = 0.63 (sec) , antiderivative size = 50, normalized size of antiderivative = 0.94

$$\int \operatorname{sech}^2(c+dx) \left(a+b\sinh^2(c+dx)\right)^2 \, dx = \frac{2(4a-3b)b(c+dx)+b^2\sinh(2(c+dx))+4(a-b)^2\tanh(c+dx)}{4d}$$

input
```
Integrate[Sech[c + d*x]^2*(a + b*Sinh[c + d*x]^2)^2,x]
```

output
```
(2*(4*a - 3*b)*b*(c + d*x) + b^2*Sinh[2*(c + d*x)] + 4*(a - b)^2*Tanh[c + 
d*x])/(4*d)
```

Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 65, normalized size of antiderivative = 1.23, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.174$, Rules used = {3042, 3670, 300, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \operatorname{sech}^2(c+dx)\left(a+b\sinh^2(c+dx)\right)^2\,dx$$

$$\Big\downarrow \ 3042$$

$$\int \frac{\left(a-b\sin(ic+idx)^2\right)^2}{\cos(ic+idx)^2}dx$$

$$\Big\downarrow \ 3670$$

$$\frac{\int \frac{\left(a-(a-b)\tanh^2(c+dx)\right)^2}{\left(1-\tanh^2(c+dx)\right)^2}d\tanh(c+dx)}{d}$$

$$\Big\downarrow \ 300$$

$$\frac{\int \left((a-b)^2+\frac{(2a-b)b-2(a-b)b\tanh^2(c+dx)}{\left(1-\tanh^2(c+dx)\right)^2}\right)d\tanh(c+dx)}{d}$$

$$\Big\downarrow \ 2009$$

$$\frac{\frac{1}{2}b(4a-3b)\text{arctanh}(\tanh(c+dx))+(a-b)^2\tanh(c+dx)+\frac{b^2\tanh(c+dx)}{2\left(1-\tanh^2(c+dx)\right)}}{d}$$

input
```
Int[Sech[c + d*x]^2*(a + b*Sinh[c + d*x]^2)^2,x]
```

output
```
(((4*a - 3*b)*b*ArcTanh[Tanh[c + d*x]])/2 + (a - b)^2*Tanh[c + d*x] + (b^2
*Tanh[c + d*x])/(2*(1 - Tanh[c + d*x]^2)))/d
```

Defintions of rubi rules used

rule 300
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_), x_Symbol] :> Int
[PolynomialDivide[(a + b*x^2)^p, (c + d*x^2)^(-q), x], x] /; FreeQ[{a, b, c
, d}, x] && NeQ[b*c - a*d, 0] && IGtQ[p, 0] && ILtQ[q, 0] && GeQ[p, -q]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3670
```
Int[cos[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(
p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Simp[ff/f   Su
bst[Int[(a + (a + b)*ff^2*x^2)^p/(1 + ff^2*x^2)^(m/2 + p + 1), x], x, Tan[e
 + f*x]/ff], x]] /; FreeQ[{a, b, e, f}, x] && IntegerQ[m/2] && IntegerQ[p]
```

Maple [A] (verified)

Time = 84.34 (sec) , antiderivative size = 71, normalized size of antiderivative = 1.34

method	result	size
derivativedivides	$\frac{\tanh(dx+c)a^2+2ab(dx+c-\tanh(dx+c))+b^2\left(\frac{\sinh(dx+c)^3}{2\cosh(dx+c)}-\frac{3dx}{2}-\frac{3c}{2}+\frac{3\tanh(dx+c)}{2}\right)}{d}$	71
default	$\frac{\tanh(dx+c)a^2+2ab(dx+c-\tanh(dx+c))+b^2\left(\frac{\sinh(dx+c)^3}{2\cosh(dx+c)}-\frac{3dx}{2}-\frac{3c}{2}+\frac{3\tanh(dx+c)}{2}\right)}{d}$	71
risch	$2abx-\frac{3b^2x}{2}+\frac{e^{2dx+2c}b^2}{8d}-\frac{e^{-2dx-2c}b^2}{8d}-\frac{2a^2}{d(e^{2dx+2c}+1)}+\frac{4ab}{d(e^{2dx+2c}+1)}-\frac{2b^2}{d(e^{2dx+2c}+1)}$	109

input
```
int(sech(d*x+c)^2*(a+b*sinh(d*x+c)^2)^2,x,method=_RETURNVERBOSE)
```

output
```
1/d*(tanh(d*x+c)*a^2+2*a*b*(d*x+c-tanh(d*x+c))+b^2*(1/2*sinh(d*x+c)^3/cosh
(d*x+c)-3/2*d*x-3/2*c+3/2*tanh(d*x+c)))
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 97, normalized size of antiderivative = 1.83

$$\int \operatorname{sech}^2(c+dx)\left(a+b\sinh^2(c+dx)\right)^2\,dx$$
$$=\frac{b^2\sinh\left(dx+c\right)^3+4\left(\left(4\,ab-3\,b^2\right)dx-2\,a^2+4\,ab-2\,b^2\right)\cosh\left(dx+c\right)+\left(3\,b^2\cosh\left(dx+c\right)^2+8\,a^2-1\right.}{8\,d\cosh\left(dx+c\right)}$$

input
```
integrate(sech(d*x+c)^2*(a+b*sinh(d*x+c)^2)^2,x, algorithm="fricas")
```

output
```
1/8*(b^2*sinh(d*x + c)^3 + 4*((4*a*b - 3*b^2)*d*x - 2*a^2 + 4*a*b - 2*b^2)
*cosh(d*x + c) + (3*b^2*cosh(d*x + c)^2 + 8*a^2 - 16*a*b + 9*b^2)*sinh(d*x
 + c))/(d*cosh(d*x + c))
```

Sympy [F]

$$\int \operatorname{sech}^2(c+dx)\left(a+b\sinh^2(c+dx)\right)^2\,dx=\int\left(a+b\sinh^2\left(c+dx\right)\right)^2\operatorname{sech}^2\left(c+dx\right)dx$$

input
```
integrate(sech(d*x+c)**2*(a+b*sinh(d*x+c)**2)**2,x)
```

output
```
Integral((a + b*sinh(c + d*x)**2)**2*sech(c + d*x)**2, x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 119 vs. 2(49) = 98.

Time = 0.04 (sec) , antiderivative size = 119, normalized size of antiderivative = 2.25

$$\int \operatorname{sech}^2(c+dx)\left(a+b\sinh^2(c+dx)\right)^2\,dx$$
$$=2\,ab\left(x+\frac{c}{d}-\frac{2}{d\left(e^{(-2\,dx-2\,c)}+1\right)}\right)$$
$$-\frac{1}{8}\,b^2\left(\frac{12\,(dx+c)}{d}+\frac{e^{(-2\,dx-2\,c)}}{d}-\frac{17\,e^{(-2\,dx-2\,c)}+1}{d\left(e^{(-2\,dx-2\,c)}+e^{(-4\,dx-4\,c)}\right)}\right)+\frac{2\,a^2}{d\left(e^{(-2\,dx-2\,c)}+1\right)}$$

input
```
integrate(sech(d*x+c)^2*(a+b*sinh(d*x+c)^2)^2,x, algorithm="maxima")
```

output
```
2*a*b*(x + c/d - 2/(d*(e^(-2*d*x - 2*c) + 1))) - 1/8*b^2*(12*(d*x + c)/d +
 e^(-2*d*x - 2*c)/d - (17*e^(-2*d*x - 2*c) + 1)/(d*(e^(-2*d*x - 2*c) + e^(
-4*d*x - 4*c)))) + 2*a^2/(d*(e^(-2*d*x - 2*c) + 1))
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 131 vs. $2(49) = 98$.

Time = 0.15 (sec) , antiderivative size = 131, normalized size of antiderivative = 2.47

$$\int \operatorname{sech}^2(c+dx)\left(a+b\sinh^2(c+dx)\right)^2\,dx = \frac{b^2e^{(2\,dx+2\,c)} + 4\,(4\,ab-3\,b^2)(dx+c) - \frac{4\,abe^{(4\,dx+4\,c)}-3\,b^2e^{(4\,dx+4\,c)}+16\,a^2e^{(2\,dx+2\,c)}-28\,abe^{(2\,dx+2\,c)}+14\,b^2e^{(2\,dx+2\,c)}+b^2}{e^{(4\,dx+4\,c)}+e^{(2\,dx+2\,c)}}}{8\,d}$$

input
```
integrate(sech(d*x+c)^2*(a+b*sinh(d*x+c)^2)^2,x, algorithm="giac")
```

output
```
1/8*(b^2*e^(2*d*x + 2*c) + 4*(4*a*b - 3*b^2)*(d*x + c) - (4*a*b*e^(4*d*x +
 4*c) - 3*b^2*e^(4*d*x + 4*c) + 16*a^2*e^(2*d*x + 2*c) - 28*a*b*e^(2*d*x +
 2*c) + 14*b^2*e^(2*d*x + 2*c) + b^2)/(e^(4*d*x + 4*c) + e^(2*d*x + 2*c)))
/d
```

Mupad [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 75, normalized size of antiderivative = 1.42

$$\int \operatorname{sech}^2(c+dx)\left(a+b\sinh^2(c+dx)\right)^2\,dx = \frac{b\,x\,(4\,a-3\,b)}{2} - \frac{b^2\,\mathrm{e}^{-2\,c-2\,d\,x}}{8\,d} + \frac{b^2\,\mathrm{e}^{2\,c+2\,d\,x}}{8\,d} - \frac{2\,(a^2-2\,a\,b+b^2)}{d\,\left(\mathrm{e}^{2\,c+2\,d\,x}+1\right)}$$

input
```
int((a + b*sinh(c + d*x)^2)^2/cosh(c + d*x)^2,x)
```

output
```
(b*x*(4*a - 3*b))/2 - (b^2*exp(- 2*c - 2*d*x))/(8*d) + (b^2*exp(2*c + 2*d*
x))/(8*d) - (2*(a^2 - 2*a*b + b^2))/(d*(exp(2*c + 2*d*x) + 1))
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 161, normalized size of antiderivative = 3.04

$$\int \operatorname{sech}^2(c+dx)\left(a+b\sinh^2(c+dx)\right)^2\,dx$$
$$=\frac{e^{6dx+6c}b^2+16e^{4dx+4c}a^2+16e^{4dx+4c}abdx-32e^{4dx+4c}ab-12e^{4dx+4c}b^2dx+18e^{4dx+4c}b^2+16e^{2dx+2c}abdx-}{8e^{2dx+2c}d\left(e^{2dx+2c}+1\right)}$$

input
```
int(sech(d*x+c)^2*(a+b*sinh(d*x+c)^2)^2,x)
```

output
```
(e**(6*c + 6*d*x)*b**2 + 16*e**(4*c + 4*d*x)*a**2 + 16*e**(4*c + 4*d*x)*a*
b*d*x - 32*e**(4*c + 4*d*x)*a*b - 12*e**(4*c + 4*d*x)*b**2*d*x + 18*e**(4*
c + 4*d*x)*b**2 + 16*e**(2*c + 2*d*x)*a*b*d*x - 12*e**(2*c + 2*d*x)*b**2*d
*x - b**2)/(8*e**(2*c + 2*d*x)*d*(e**(2*c + 2*d*x) + 1))
```

3.263 $\int \operatorname{sech}^3(c+dx) \left(a+b\sinh^2(c+dx)\right)^2 dx$

Optimal result

Integrand size = 23, antiderivative size = 64

$$\int \operatorname{sech}^3(c+dx) \left(a+b\sinh^2(c+dx)\right)^2 dx = \frac{(a-b)(a+3b)\arctan(\sinh(c+dx))}{2d} + \frac{b^2\sinh(c+dx)}{d} + \frac{(a-b)^2\operatorname{sech}(c+dx)\tanh(c+dx)}{2d}$$

output
```
1/2*(a-b)*(a+3*b)*arctan(sinh(d*x+c))/d+b^2*sinh(d*x+c)/d+1/2*(a-b)^2*sech
(d*x+c)*tanh(d*x+c)/d
```

Mathematica [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 3.02 (sec) , antiderivative size = 233, normalized size of antiderivative = 3.64

$$\int \operatorname{sech}^3(c+dx) \left(a+b\sinh^2(c+dx)\right)^2 dx$$

$$= \frac{\operatorname{csch}^3(c+dx) \left(-64\,{}_4F_3\left(\frac{3}{2},2,2,2;1,1,\frac{9}{2};-\sinh^2(c+dx)\right)\sinh^6(c+dx)\left(a+b\sinh^2(c+dx)\right)^2 - 35(375\right.}{}$$

input
```
Integrate[Sech[c + d*x]^3*(a + b*Sinh[c + d*x]^2)^2,x]
```

output
```
(Csch[c + d*x]^3*(-64*HypergeometricPFQ[{3/2, 2, 2, 2}, {1, 1, 9/2}, -Sinh
[c + d*x]^2]*Sinh[c + d*x]^6*(a + b*Sinh[c + d*x]^2)^2 - 35*(375*a^2 + a*(
37*a + 689*b + 61*b*Cosh[2*(c + d*x)])*Sinh[c + d*x]^2 + 303*b^2*Sinh[c +
d*x]^4 + 61*b^2*Sinh[c + d*x]^6) + (105*ArcTanh[Sqrt[-Sinh[c + d*x]^2]]*(1
25*a^2 + 2*a*(27*a + 125*b)*Sinh[c + d*x]^2 + (9*a^2 + 124*a*b + 101*b^2)*
Sinh[c + d*x]^4 + 2*b*(a + 27*b)*Sinh[c + d*x]^6 + b^2*Sinh[c + d*x]^8))/S
qrt[-Sinh[c + d*x]^2]))/(1680*d)
```

Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 65, normalized size of antiderivative = 1.02, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.174$, Rules used = {3042, 3669, 300, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \operatorname{sech}^3(c+dx)\left(a+b\sinh^2(c+dx)\right)^2\,dx$$

$$\Big\downarrow \; 3042$$

$$\int \frac{\left(a-b\sin(ic+idx)^2\right)^2}{\cos(ic+idx)^3}dx$$

$$\Big\downarrow \; 3669$$

$$\frac{\int \frac{\left(b\sinh^2(c+dx)+a\right)^2}{\left(\sinh^2(c+dx)+1\right)^2}d\sinh(c+dx)}{d}$$

$$\Big\downarrow \; 300$$

$$\frac{\int \left(b^2+\frac{a^2-b^2+2(a-b)b\sinh^2(c+dx)}{\left(\sinh^2(c+dx)+1\right)^2}\right)d\sinh(c+dx)}{d}$$

$$\Big\downarrow \; 2009$$

$$\frac{\frac{1}{2}(a+3b)(a-b)\arctan(\sinh(c+dx))+\frac{(a-b)^2\sinh(c+dx)}{2\left(\sinh^2(c+dx)+1\right)}+b^2\sinh(c+dx)}{d}$$

input
```
Int[Sech[c + d*x]^3*(a + b*Sinh[c + d*x]^2)^2,x]
```

output
```
(((a - b)*(a + 3*b)*ArcTan[Sinh[c + d*x]])/2 + b^2*Sinh[c + d*x] + ((a - b
)^2*Sinh[c + d*x])/(2*(1 + Sinh[c + d*x]^2)))/d
```

Defintions of rubi rules used

rule 300
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_), x_Symbol] :> Int
[PolynomialDivide[(a + b*x^2)^p, (c + d*x^2)^(-q), x], x] /; FreeQ[{a, b, c
, d}, x] && NeQ[b*c - a*d, 0] && IGtQ[p, 0] && ILtQ[q, 0] && GeQ[p, -q]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3669
```
Int[cos[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^
(p_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Simp[ff/f   S
ubst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a + b*ff^2*x^2)^p, x], x, Sin[e + f*x]
/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m - 1)/2]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 134 vs. 2(60) = 120.

Time = 70.12 (sec) , antiderivative size = 135, normalized size of antiderivative = 2.11

method	result
derivativedivides	$\frac{a^2\left(\frac{\operatorname{sech}(dx+c)\tanh(dx+c)}{2}+\arctan\left(\mathrm{e}^{dx+c}\right)\right)+2ab\left(-\frac{\sinh(dx+c)}{\cosh(dx+c)^2}+\frac{\operatorname{sech}(dx+c)\tanh(dx+c)}{2}+\arctan\left(\mathrm{e}^{dx+c}\right)\right)+b^2\left(\frac{\sinh(dx}{\cosh(dx}\right.}{d}$
default	$\frac{a^2\left(\frac{\operatorname{sech}(dx+c)\tanh(dx+c)}{2}+\arctan\left(\mathrm{e}^{dx+c}\right)\right)+2ab\left(-\frac{\sinh(dx+c)}{\cosh(dx+c)^2}+\frac{\operatorname{sech}(dx+c)\tanh(dx+c)}{2}+\arctan\left(\mathrm{e}^{dx+c}\right)\right)+b^2\left(\frac{\sinh(dx}{\cosh(dx}\right.}{d}$
risch	$\frac{\mathrm{e}^{dx+c}b^2}{2d}-\frac{\mathrm{e}^{-dx-c}b^2}{2d}+\frac{\mathrm{e}^{dx+c}(a^2-2ab+b^2)\left(\mathrm{e}^{2dx+2c}-1\right)}{d\left(\mathrm{e}^{2dx+2c}+1\right)^2}+\frac{i\ln\left(\mathrm{e}^{dx+c}+i\right)a^2}{2d}+\frac{i\ln\left(\mathrm{e}^{dx+c}+i\right)ab}{d}-\frac{3i\ln(\mathrm{e}^{dx+}}{2d}$

input
```
int(sech(d*x+c)^3*(a+b*sinh(d*x+c)^2)^2,x,method=_RETURNVERBOSE)
```

output
```
1/d*(a^2*(1/2*sech(d*x+c)*tanh(d*x+c)+arctan(exp(d*x+c)))+2*a*b*(-sinh(d*x
+c)/cosh(d*x+c)^2+1/2*sech(d*x+c)*tanh(d*x+c)+arctan(exp(d*x+c)))+b^2*(sin
h(d*x+c)^3/cosh(d*x+c)^2+3*sinh(d*x+c)/cosh(d*x+c)^2-3/2*sech(d*x+c)*tanh(
d*x+c)-3*arctan(exp(d*x+c))))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 759 vs. $2(60) = 120$.

Time = 0.10 (sec) , antiderivative size = 759, normalized size of antiderivative = 11.86

$$\int \operatorname{sech}^3(c+dx)\left(a+b\sinh^2(c+dx)\right)^2\,dx = \text{Too large to display}$$

input
```
integrate(sech(d*x+c)^3*(a+b*sinh(d*x+c)^2)^2,x, algorithm="fricas")
```

output
```
1/2*(b^2*cosh(d*x + c)^6 + 6*b^2*cosh(d*x + c)*sinh(d*x + c)^5 + b^2*sinh(
d*x + c)^6 + (2*a^2 - 4*a*b + 3*b^2)*cosh(d*x + c)^4 + (15*b^2*cosh(d*x +
c)^2 + 2*a^2 - 4*a*b + 3*b^2)*sinh(d*x + c)^4 + 4*(5*b^2*cosh(d*x + c)^3 +
 (2*a^2 - 4*a*b + 3*b^2)*cosh(d*x + c))*sinh(d*x + c)^3 - (2*a^2 - 4*a*b +
 3*b^2)*cosh(d*x + c)^2 + (15*b^2*cosh(d*x + c)^4 + 6*(2*a^2 - 4*a*b + 3*b
^2)*cosh(d*x + c)^2 - 2*a^2 + 4*a*b - 3*b^2)*sinh(d*x + c)^2 - b^2 + 2*((a
^2 + 2*a*b - 3*b^2)*cosh(d*x + c)^5 + 5*(a^2 + 2*a*b - 3*b^2)*cosh(d*x + c
)*sinh(d*x + c)^4 + (a^2 + 2*a*b - 3*b^2)*sinh(d*x + c)^5 + 2*(a^2 + 2*a*b
 - 3*b^2)*cosh(d*x + c)^3 + 2*(5*(a^2 + 2*a*b - 3*b^2)*cosh(d*x + c)^2 + a
^2 + 2*a*b - 3*b^2)*sinh(d*x + c)^3 + 2*(5*(a^2 + 2*a*b - 3*b^2)*cosh(d*x
+ c)^3 + 3*(a^2 + 2*a*b - 3*b^2)*cosh(d*x + c))*sinh(d*x + c)^2 + (a^2 + 2
*a*b - 3*b^2)*cosh(d*x + c) + (5*(a^2 + 2*a*b - 3*b^2)*cosh(d*x + c)^4 + 6
*(a^2 + 2*a*b - 3*b^2)*cosh(d*x + c)^2 + a^2 + 2*a*b - 3*b^2)*sinh(d*x + c
))*arctan(cosh(d*x + c) + sinh(d*x + c)) + 2*(3*b^2*cosh(d*x + c)^5 + 2*(2
*a^2 - 4*a*b + 3*b^2)*cosh(d*x + c)^3 - (2*a^2 - 4*a*b + 3*b^2)*cosh(d*x +
 c))*sinh(d*x + c))/(d*cosh(d*x + c)^5 + 5*d*cosh(d*x + c)*sinh(d*x + c)^4
 + d*sinh(d*x + c)^5 + 2*d*cosh(d*x + c)^3 + 2*(5*d*cosh(d*x + c)^2 + d)*s
inh(d*x + c)^3 + 2*(5*d*cosh(d*x + c)^3 + 3*d*cosh(d*x + c))*sinh(d*x + c)
^2 + d*cosh(d*x + c) + (5*d*cosh(d*x + c)^4 + 6*d*cosh(d*x + c)^2 + d)*sin
h(d*x + c))
```

Sympy [F]

$$\int \operatorname{sech}^3(c+dx)\left(a+b\sinh^2(c+dx)\right)^2\,dx=\int \left(a+b\sinh^2\left(c+dx\right)\right)^2\operatorname{sech}^3\left(c+dx\right)dx$$

input
```
integrate(sech(d*x+c)**3*(a+b*sinh(d*x+c)**2)**2,x)
```

output
```
Integral((a + b*sinh(c + d*x)**2)**2*sech(c + d*x)**3, x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 234 vs. $2(60) = 120$.

Time = 0.13 (sec) , antiderivative size = 234, normalized size of antiderivative = 3.66

$$\int \operatorname{sech}^3(c+dx)\left(a+b\sinh^2(c+dx)\right)^2\,dx$$
$$=\frac{1}{2}\,b^2\left(\frac{6\,\arctan\left(e^{(-dx-c)}\right)}{d}-\frac{e^{(-dx-c)}}{d}+\frac{4\,e^{(-2\,dx-2\,c)}-e^{(-4\,dx-4\,c)}+1}{d\left(e^{(-dx-c)}+2\,e^{(-3\,dx-3\,c)}+e^{(-5\,dx-5\,c)}\right)}\right)$$
$$-2\,ab\left(\frac{\arctan\left(e^{(-dx-c)}\right)}{d}+\frac{e^{(-dx-c)}-e^{(-3\,dx-3\,c)}}{d\left(2\,e^{(-2\,dx-2\,c)}+e^{(-4\,dx-4\,c)}+1\right)}\right)$$
$$-a^2\left(\frac{\arctan\left(e^{(-dx-c)}\right)}{d}-\frac{e^{(-dx-c)}-e^{(-3\,dx-3\,c)}}{d\left(2\,e^{(-2\,dx-2\,c)}+e^{(-4\,dx-4\,c)}+1\right)}\right)$$

input
```
integrate(sech(d*x+c)^3*(a+b*sinh(d*x+c)^2)^2,x, algorithm="maxima")
```

output
```
1/2*b^2*(6*arctan(e^(-d*x - c))/d - e^(-d*x - c)/d + (4*e^(-2*d*x - 2*c) -
 e^(-4*d*x - 4*c) + 1)/(d*(e^(-d*x - c) + 2*e^(-3*d*x - 3*c) + e^(-5*d*x -
 5*c)))) - 2*a*b*(arctan(e^(-d*x - c))/d + (e^(-d*x - c) - e^(-3*d*x - 3*c
))/(d*(2*e^(-2*d*x - 2*c) + e^(-4*d*x - 4*c) + 1))) - a^2*(arctan(e^(-d*x
- c))/d - (e^(-d*x - c) - e^(-3*d*x - 3*c))/(d*(2*e^(-2*d*x - 2*c) + e^(-4
*d*x - 4*c) + 1)))
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 163 vs. $2(60) = 120$.

Time = 0.15 (sec) , antiderivative size = 163, normalized size of antiderivative = 2.55

$$\int \operatorname{sech}^3(c+dx)\left(a+b\sinh^2(c+dx)\right)^2\,dx$$
$$=\frac{2\,b^2\left(e^{(dx+c)}-e^{(-dx-c)}\right)+\left(\pi+2\,\arctan\left(\frac{1}{2}\left(e^{(2\,dx+2\,c)}-1\right)e^{(-dx-c)}\right)\right)\left(a^2+2\,ab-3\,b^2\right)+\frac{4\left(a^2\left(e^{(dx+c)}-e^{(-dx}\right.\right.}{\cdots}}{4\,d}$$

input
```
integrate(sech(d*x+c)^3*(a+b*sinh(d*x+c)^2)^2,x, algorithm="giac")
```

output
```
1/4*(2*b^2*(e^(d*x + c) - e^(-d*x - c)) + (pi + 2*arctan(1/2*(e^(2*d*x + 2
*c) - 1)*e^(-d*x - c)))*(a^2 + 2*a*b - 3*b^2) + 4*(a^2*(e^(d*x + c) - e^(-
d*x - c)) - 2*a*b*(e^(d*x + c) - e^(-d*x - c)) + b^2*(e^(d*x + c) - e^(-d*
x - c)))/((e^(d*x + c) - e^(-d*x - c))^2 + 4))/d
```

Mupad [B] (verification not implemented)

Time = 2.09 (sec) , antiderivative size = 220, normalized size of antiderivative = 3.44

$$\int \operatorname{sech}^3(c+dx)\left(a+b\sinh^2(c+dx)\right)^2\,dx = \frac{b^2\,\mathrm{e}^{c+d\,x}}{2\,d}+\frac{\mathrm{atan}\left(\frac{\mathrm{e}^{d\,x}\,\mathrm{e}^{c}\,\left(a^2\,\sqrt{d^2}-3\,b^2\,\sqrt{d^2}+2\,a\,b\,\sqrt{d^2}\right)}{d\,\sqrt{a^4+4\,a^3\,b-2\,a^2\,b^2-12\,a\,b^3+9\,b^4}}\right)\,\sqrt{a^4+4\,a^3\,b-2\,a^2\,b^2-12\,a\,b^3+9\,b^4}}{\sqrt{d^2}} - \frac{b^2\,\mathrm{e}^{-c-d\,x}}{2\,d}+\frac{\mathrm{e}^{c+d\,x}\,\left(a^2-2\,a\,b+b^2\right)}{d\,\left(\mathrm{e}^{2\,c+2\,d\,x}+1\right)}-\frac{2\,\mathrm{e}^{c+d\,x}\,\left(a^2-2\,a\,b+b^2\right)}{d\,\left(2\,\mathrm{e}^{2\,c+2\,d\,x}+\mathrm{e}^{4\,c+4\,d\,x}+1\right)}$$

input
```
int((a + b*sinh(c + d*x)^2)^2/cosh(c + d*x)^3,x)
```

output
```
(b^2*exp(c + d*x))/(2*d) + (atan((exp(d*x)*exp(c)*(a^2*(d^2)^(1/2) - 3*b^2
*(d^2)^(1/2) + 2*a*b*(d^2)^(1/2)))/(d*(4*a^3*b - 12*a*b^3 + a^4 + 9*b^4 -
2*a^2*b^2)^(1/2)))*(4*a^3*b - 12*a*b^3 + a^4 + 9*b^4 - 2*a^2*b^2)^(1/2))/(
d^2)^(1/2) - (b^2*exp(- c - d*x))/(2*d) + (exp(c + d*x)*(a^2 - 2*a*b + b^2
))/(d*(exp(2*c + 2*d*x) + 1)) - (2*exp(c + d*x)*(a^2 - 2*a*b + b^2))/(d*(2
*exp(2*c + 2*d*x) + exp(4*c + 4*d*x) + 1))
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 343, normalized size of antiderivative = 5.36

$$\int \operatorname{sech}^3(c+dx)\left(a+b\sinh^2(c+dx)\right)^2\,dx = \frac{2e^{5dx+5c}atan\left(e^{dx+c}\right)a^2+4e^{5dx+5c}atan\left(e^{dx+c}\right)ab-6e^{5dx+5c}atan\left(e^{dx+c}\right)b^2+4e^{3dx+3c}atan\left(e^{dx+c}\right)a^2+8e}{}$$

input
```
int(sech(d*x+c)^3*(a+b*sinh(d*x+c)^2)^2,x)
```

output
```
(2*e**(5*c + 5*d*x)*atan(e**(c + d*x))*a**2 + 4*e**(5*c + 5*d*x)*atan(e**(
c + d*x))*a*b - 6*e**(5*c + 5*d*x)*atan(e**(c + d*x))*b**2 + 4*e**(3*c + 3
*d*x)*atan(e**(c + d*x))*a**2 + 8*e**(3*c + 3*d*x)*atan(e**(c + d*x))*a*b
- 12*e**(3*c + 3*d*x)*atan(e**(c + d*x))*b**2 + 2*e**(c + d*x)*atan(e**(c
+ d*x))*a**2 + 4*e**(c + d*x)*atan(e**(c + d*x))*a*b - 6*e**(c + d*x)*atan
(e**(c + d*x))*b**2 + e**(6*c + 6*d*x)*b**2 + 2*e**(4*c + 4*d*x)*a**2 - 4*
e**(4*c + 4*d*x)*a*b + 3*e**(4*c + 4*d*x)*b**2 - 2*e**(2*c + 2*d*x)*a**2 +
 4*e**(2*c + 2*d*x)*a*b - 3*e**(2*c + 2*d*x)*b**2 - b**2)/(2*e**(c + d*x)*
d*(e**(4*c + 4*d*x) + 2*e**(2*c + 2*d*x) + 1))
```

3.264 $\int \operatorname{sech}^4(c+dx) \left(a+b\sinh^2(c+dx)\right)^2 \, dx$

Optimal result

Integrand size = 23, antiderivative size = 47

$$\int \operatorname{sech}^4(c+dx) \left(a+b\sinh^2(c+dx)\right)^2 \, dx = b^2x+\frac{(a^2-b^2)\tanh(c+dx)}{d} - \frac{(a-b)^2\tanh^3(c+dx)}{3d}$$

output
```
b^2*x+(a^2-b^2)*tanh(d*x+c)/d-1/3*(a-b)^2*tanh(d*x+c)^3/d
```

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 95 vs. $2(47) = 94$.

Time = 0.08 (sec) , antiderivative size = 95, normalized size of antiderivative = 2.02

$$\int \operatorname{sech}^4(c+dx) \left(a+b\sinh^2(c+dx)\right)^2 \, dx = \frac{b^2\text{arctanh}(\tanh(c+dx))}{d}+\frac{a^2\tanh(c+dx)}{d} - \frac{b^2\tanh(c+dx)}{d}-\frac{a^2\tanh^3(c+dx)}{3d} + \frac{2ab\tanh^3(c+dx)}{3d}-\frac{b^2\tanh^3(c+dx)}{3d}$$

input
```
Integrate[Sech[c + d*x]^4*(a + b*Sinh[c + d*x]^2)^2,x]
```

output
```
(b^2*ArcTanh[Tanh[c + d*x]])/d + (a^2*Tanh[c + d*x])/d - (b^2*Tanh[c + d*x
])/d - (a^2*Tanh[c + d*x]^3)/(3*d) + (2*a*b*Tanh[c + d*x]^3)/(3*d) - (b^2*
Tanh[c + d*x]^3)/(3*d)
```

Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 51, normalized size of antiderivative = 1.09, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.174, Rules used = {3042, 3670, 300, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \operatorname{sech}^4(c+dx) \left(a+b\sinh^2(c+dx)\right)^2 \, dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\left(a-b\sin(ic+idx)^2\right)^2}{\cos(ic+idx)^4}dx$$

$$\Big\downarrow \text{3670}$$

$$\frac{\int \frac{\left(a-(a-b)\tanh^2(c+dx)\right)^2}{1-\tanh^2(c+dx)}d\tanh(c+dx)}{d}$$

$$\Big\downarrow \text{300}$$

$$\frac{\int \left(a^2-b^2-(a-b)^2\tanh^2(c+dx)+\frac{b^2}{1-\tanh^2(c+dx)}\right)d\tanh(c+dx)}{d}$$

$$\Big\downarrow \text{2009}$$

$$\frac{\left(a^2-b^2\right)\tanh(c+dx)-\frac{1}{3}(a-b)^2\tanh^3(c+dx)+b^2\text{arctanh}(\tanh(c+dx))}{d}$$

input
```
Int[Sech[c + d*x]^4*(a + b*Sinh[c + d*x]^2)^2,x]
```

output
```
(b^2*ArcTanh[Tanh[c + d*x]] + (a^2 - b^2)*Tanh[c + d*x] - ((a - b)^2*Tanh[
c + d*x]^3)/3)/d
```

Defintions of rubi rules used

rule 300
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_), x_Symbol] :> Int
[PolynomialDivide[(a + b*x^2)^p, (c + d*x^2)^(-q), x], x] /; FreeQ[{a, b, c
, d}, x] && NeQ[b*c - a*d, 0] && IGtQ[p, 0] && ILtQ[q, 0] && GeQ[p, -q]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3670
```
Int[cos[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(
p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Simp[ff/f   Su
bst[Int[(a + (a + b)*ff^2*x^2)^p/(1 + ff^2*x^2)^(m/2 + p + 1), x], x, Tan[e
 + f*x]/ff], x]] /; FreeQ[{a, b, e, f}, x] && IntegerQ[m/2] && IntegerQ[p]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 91 vs. 2(45) = 90.

Time = 54.50 (sec) , antiderivative size = 92, normalized size of antiderivative = 1.96

method	result
risch	$b^2x-\frac{4\left(3\,\mathrm{e}^{4dx+4c}ab-3\,\mathrm{e}^{4dx+4c}b^2+3\,\mathrm{e}^{2dx+2c}a^2-3b^2\mathrm{e}^{2dx+2c}+a^2+ab-2b^2\right)}{3d\left(\mathrm{e}^{2dx+2c}+1\right)^3}$
derivativedivides	$\frac{a^2\left(\frac{2}{3}+\frac{\mathrm{sech}(dx+c)^2}{3}\right)\tanh(dx+c)+2ab\left(-\frac{\sinh(dx+c)}{2\cosh(dx+c)^3}+\frac{\left(\frac{2}{3}+\frac{\mathrm{sech}(dx+c)^2}{3}\right)\tanh(dx+c)}{2}\right)+b^2\left(dx+c-\tanh(dx+c)-\right.}{d}$
default	$\frac{a^2\left(\frac{2}{3}+\frac{\mathrm{sech}(dx+c)^2}{3}\right)\tanh(dx+c)+2ab\left(-\frac{\sinh(dx+c)}{2\cosh(dx+c)^3}+\frac{\left(\frac{2}{3}+\frac{\mathrm{sech}(dx+c)^2}{3}\right)\tanh(dx+c)}{2}\right)+b^2\left(dx+c-\tanh(dx+c)-\right.}{d}$

input
```
int(sech(d*x+c)^4*(a+b*sinh(d*x+c)^2)^2,x,method=_RETURNVERBOSE)
```

output
```
b^2*x-4/3*(3*exp(4*d*x+4*c)*a*b-3*exp(4*d*x+4*c)*b^2+3*exp(2*d*x+2*c)*a^2-
3*b^2*exp(2*d*x+2*c)+a^2+a*b-2*b^2)/d/(exp(2*d*x+2*c)+1)^3
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 200 vs. $2(45) = 90$.

Time = 0.08 (sec) , antiderivative size = 200, normalized size of antiderivative = 4.26

$$\int \operatorname{sech}^4(c+dx)\left(a+b\sinh^2(c+dx)\right)^2\,dx = \frac{(3\,b^2dx-2\,a^2-2\,ab+4\,b^2)\cosh(dx+c)^3+3\,(3\,b^2dx-2\,a^2-2\,ab+4\,b^2)\cosh(dx+c)\sinh(dx+c)^2}{3\,\left(d\cosh(dx+c)^3\right.}$$

input
```
integrate(sech(d*x+c)^4*(a+b*sinh(d*x+c)^2)^2,x, algorithm="fricas")
```

output
```
1/3*((3*b^2*d*x - 2*a^2 - 2*a*b + 4*b^2)*cosh(d*x + c)^3 + 3*(3*b^2*d*x -
2*a^2 - 2*a*b + 4*b^2)*cosh(d*x + c)*sinh(d*x + c)^2 + 2*(a^2 + a*b - 2*b^
2)*sinh(d*x + c)^3 + 3*(3*b^2*d*x - 2*a^2 - 2*a*b + 4*b^2)*cosh(d*x + c) +
 6*((a^2 + a*b - 2*b^2)*cosh(d*x + c)^2 + a^2 - a*b)*sinh(d*x + c))/(d*cos
h(d*x + c)^3 + 3*d*cosh(d*x + c)*sinh(d*x + c)^2 + 3*d*cosh(d*x + c))
```

Sympy [F(-1)]

Timed out.

$$\int \operatorname{sech}^4(c+dx)\left(a+b\sinh^2(c+dx)\right)^2\,dx = \text{Timed out}$$

input
```
integrate(sech(d*x+c)**4*(a+b*sinh(d*x+c)**2)**2,x)
```

output
```
Timed out
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 267 vs. $2(45) = 90$.

Time = 0.05 (sec) , antiderivative size = 267, normalized size of antiderivative = 5.68

$$\int \operatorname{sech}^4(c+dx)\left(a+b\sinh^2(c+dx)\right)^2\,dx$$
$$=\frac{1}{3}b^2\left(3x+\frac{3c}{d}-\frac{4\left(3e^{(-2dx-2c)}+3e^{(-4dx-4c)}+2\right)}{d(3e^{(-2dx-2c)}+3e^{(-4dx-4c)}+e^{(-6dx-6c)}+1)}\right)$$
$$+\frac{4}{3}a^2\left(\frac{3e^{(-2dx-2c)}}{d(3e^{(-2dx-2c)}+3e^{(-4dx-4c)}+e^{(-6dx-6c)}+1)}+\frac{1}{d(3e^{(-2dx-2c)}+3e^{(-4dx-4c)}+e^{(-6dx-6c)}+1)}\right)$$
$$+\frac{4}{3}ab\left(\frac{3e^{(-4dx-4c)}}{d(3e^{(-2dx-2c)}+3e^{(-4dx-4c)}+e^{(-6dx-6c)}+1)}+\frac{1}{d(3e^{(-2dx-2c)}+3e^{(-4dx-4c)}+e^{(-6dx-6c)}+1)}\right)$$

input
```
integrate(sech(d*x+c)^4*(a+b*sinh(d*x+c)^2)^2,x, algorithm="maxima")
```

output
```
1/3*b^2*(3*x + 3*c/d - 4*(3*e^(-2*d*x - 2*c) + 3*e^(-4*d*x - 4*c) + 2)/(d*
(3*e^(-2*d*x - 2*c) + 3*e^(-4*d*x - 4*c) + e^(-6*d*x - 6*c) + 1))) + 4/3*a
^2*(3*e^(-2*d*x - 2*c)/(d*(3*e^(-2*d*x - 2*c) + 3*e^(-4*d*x - 4*c) + e^(-6
*d*x - 6*c) + 1)) + 1/(d*(3*e^(-2*d*x - 2*c) + 3*e^(-4*d*x - 4*c) + e^(-6*
d*x - 6*c) + 1))) + 4/3*a*b*(3*e^(-4*d*x - 4*c)/(d*(3*e^(-2*d*x - 2*c) + 3
*e^(-4*d*x - 4*c) + e^(-6*d*x - 6*c) + 1)) + 1/(d*(3*e^(-2*d*x - 2*c) + 3*
e^(-4*d*x - 4*c) + e^(-6*d*x - 6*c) + 1)))
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 98 vs. $2(45) = 90$.

Time = 0.15 (sec) , antiderivative size = 98, normalized size of antiderivative = 2.09

$$\int \operatorname{sech}^4(c+dx)\left(a+b\sinh^2(c+dx)\right)^2\,dx$$
$$=\frac{3(dx+c)b^2-\frac{4\left(3abe^{(4dx+4c)}-3b^2e^{(4dx+4c)}+3a^2e^{(2dx+2c)}-3b^2e^{(2dx+2c)}+a^2+ab-2b^2\right)}{\left(e^{(2dx+2c)}+1\right)^3}}{3d}$$

input
```
integrate(sech(d*x+c)^4*(a+b*sinh(d*x+c)^2)^2,x, algorithm="giac")
```

output
```
1/3*(3*(d*x + c)*b^2 - 4*(3*a*b*e^(4*d*x + 4*c) - 3*b^2*e^(4*d*x + 4*c) +
3*a^2*e^(2*d*x + 2*c) - 3*b^2*e^(2*d*x + 2*c) + a^2 + a*b - 2*b^2)/(e^(2*d
*x + 2*c) + 1)^3)/d
```

Mupad [B] (verification not implemented)

Time = 1.92 (sec) , antiderivative size = 194, normalized size of antiderivative = 4.13

$$\int \operatorname{sech}^4(c+dx) \left(a+b\sinh^2(c+dx)\right)^2 \, dx = b^2 x - \frac{\frac{4\,(a\,b-b^2)}{3\,d} - \frac{8\,e^{2\,c+2\,d\,x}\,(a\,b-a^2)}{3\,d} + \frac{4\,e^{4\,c+4\,d\,x}\,(a\,b-b^2)}{3\,d}}{3\,e^{2\,c+2\,d\,x}+3\,e^{4\,c+4\,d\,x}+e^{6\,c+6\,d\,x}+1} + \frac{\frac{4\,(a\,b-a^2)}{3\,d} - \frac{4\,e^{2\,c+2\,d\,x}\,(a\,b-b^2)}{3\,d}}{2\,e^{2\,c+2\,d\,x}+e^{4\,c+4\,d\,x}+1} - \frac{4\,(a\,b-b^2)}{3\,d\,\left(e^{2\,c+2\,d\,x}+1\right)}$$

input
```
int((a + b*sinh(c + d*x)^2)^2/cosh(c + d*x)^4,x)
```

output
```
b^2*x - ((4*(a*b - b^2))/(3*d) - (8*exp(2*c + 2*d*x)*(a*b - a^2))/(3*d) +
(4*exp(4*c + 4*d*x)*(a*b - b^2))/(3*d))/(3*exp(2*c + 2*d*x) + 3*exp(4*c +
4*d*x) + exp(6*c + 6*d*x) + 1) + ((4*(a*b - a^2))/(3*d) - (4*exp(2*c + 2*d
*x)*(a*b - b^2))/(3*d))/(2*exp(2*c + 2*d*x) + exp(4*c + 4*d*x) + 1) - (4*(
a*b - b^2))/(3*d*(exp(2*c + 2*d*x) + 1))
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 170, normalized size of antiderivative = 3.62

$$\int \operatorname{sech}^4(c+dx) \left(a+b\sinh^2(c+dx)\right)^2 \, dx = \frac{4e^{6dx+6c}ab + 3e^{6dx+6c}b^2dx - 4e^{6dx+6c}b^2 + 9e^{4dx+4c}b^2dx - 12e^{2dx+2c}a^2 + 12e^{2dx+2c}ab + 9e^{2dx+2c}b^2dx - 4a}{3d\left(e^{6dx+6c}+3e^{4dx+4c}+3e^{2dx+2c}+1\right)}$$

input
```
int(sech(d*x+c)^4*(a+b*sinh(d*x+c)^2)^2,x)
```

output
```
(4*e**(6*c + 6*d*x)*a*b + 3*e**(6*c + 6*d*x)*b**2*d*x - 4*e**(6*c + 6*d*x)
*b**2 + 9*e**(4*c + 4*d*x)*b**2*d*x - 12*e**(2*c + 2*d*x)*a**2 + 12*e**(2*
c + 2*d*x)*a*b + 9*e**(2*c + 2*d*x)*b**2*d*x - 4*a**2 + 3*b**2*d*x + 4*b**
2)/(3*d*(e**(6*c + 6*d*x) + 3*e**(4*c + 4*d*x) + 3*e**(2*c + 2*d*x) + 1))
```

3.265 $\int \mathrm{sech}^5(c+dx)\left(a+b\sinh^2(c+dx)\right)^2\,dx$

Optimal result

Integrand size = 23, antiderivative size = 89

$$\int \mathrm{sech}^5(c+dx)\left(a+b\sinh^2(c+dx)\right)^2\,dx = \frac{(3a^2+2ab+3b^2)\arctan(\sinh(c+dx))}{8d} + \frac{(a-b)(3a+5b)\mathrm{sech}(c+dx)\tanh(c+dx)}{8d} + \frac{(a-b)^2\mathrm{sech}^3(c+dx)\tanh(c+dx)}{4d}$$

output
```
1/8*(3*a^2+2*a*b+3*b^2)*arctan(sinh(d*x+c))/d+1/8*(a-b)*(3*a+5*b)*sech(d*x
+c)*tanh(d*x+c)/d+1/4*(a-b)^2*sech(d*x+c)^3*tanh(d*x+c)/d
```

Mathematica [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 6.38 (sec) , antiderivative size = 303, normalized size of antiderivative = 3.40

$$\int \mathrm{sech}^5(c+dx)\left(a+b\sinh^2(c+dx)\right)^2\,dx = -\frac{\mathrm{csch}^3(c+dx)\left(128\,{}_5F_4\left(\frac{3}{2},2,2,2,2;1,1,1,\frac{9}{2};-\sinh^2(c+dx)\right)\sinh^6(c+dx)\left(a+b\sinh^2(c+dx)\right)^2+\right.}{}$$

input
```
Integrate[Sech[c + d*x]^5*(a + b*Sinh[c + d*x]^2)^2,x]
```

output
```
-1/6720*(Csch[c + d*x]^3*(128*HypergeometricPFQ[{3/2, 2, 2, 2, 2}, {1, 1,
1, 9/2}, -Sinh[c + d*x]^2]*Sinh[c + d*x]^6*(a + b*Sinh[c + d*x]^2)^2 + 128
*HypergeometricPFQ[{3/2, 2, 2, 2}, {1, 1, 9/2}, -Sinh[c + d*x]^2]*Sinh[c +
 d*x]^6*(7*a^2 + 12*a*b*Sinh[c + d*x]^2 + 5*b^2*Sinh[c + d*x]^4) + 35*(337
5*a^2 + a*(657*a + 4643*b + 607*b*Cosh[2*(c + d*x)])*Sinh[c + d*x]^2 + 194
7*b^2*Sinh[c + d*x]^4 + 485*b^2*Sinh[c + d*x]^6) - (105*ArcTanh[Sqrt[-Sinh
[c + d*x]^2]]*(1125*a^2 + 2*a*(297*a + 875*b)*Sinh[c + d*x]^2 + (37*a^2 +
988*a*b + 649*b^2)*Sinh[c + d*x]^4 + 2*b*(11*a + 189*b)*Sinh[c + d*x]^6 +
9*b^2*Sinh[c + d*x]^8))/Sqrt[-Sinh[c + d*x]^2]))/d
```

Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 106, normalized size of antiderivative = 1.19, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.217$, Rules used = {3042, 3669, 315, 298, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \operatorname{sech}^5(c+dx)\left(a+b\sinh^2(c+dx)\right)^2\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\left(a-b\sin(ic+idx)^2\right)^2}{\cos(ic+idx)^5}dx$$

$$\Big\downarrow \text{3669}$$

$$\frac{\int \frac{\left(b\sinh^2(c+dx)+a\right)^2}{\left(\sinh^2(c+dx)+1\right)^3}d\sinh(c+dx)}{d}$$

$$\Big\downarrow \text{315}$$

$$\frac{\frac{1}{4}\int \frac{b(a+3b)\sinh^2(c+dx)+a(3a+b)}{\left(\sinh^2(c+dx)+1\right)^2}d\sinh(c+dx)+\frac{(a-b)\sinh(c+dx)\left(a+b\sinh^2(c+dx)\right)}{4\left(\sinh^2(c+dx)+1\right)^2}}{d}$$

$$\Big\downarrow \text{298}$$

$$\frac{\frac{1}{4}\left(\frac{1}{2}(3a^2+2ab+3b^2)\int\frac{1}{\sinh^2(c+dx)+1}d\sinh(c+dx)+\frac{3(a^2-b^2)\sinh(c+dx)}{2(\sinh^2(c+dx)+1)}\right)+\frac{(a-b)\sinh(c+dx)(a+b\sinh^2(c+dx))}{4(\sinh^2(c+dx)+1)^2}}{d}$$

$$\Big\downarrow \; 216$$

$$\frac{\frac{1}{4}\left(\frac{1}{2}(3a^2+2ab+3b^2)\arctan(\sinh(c+dx))+\frac{3(a^2-b^2)\sinh(c+dx)}{2(\sinh^2(c+dx)+1)}\right)+\frac{(a-b)\sinh(c+dx)(a+b\sinh^2(c+dx))}{4(\sinh^2(c+dx)+1)^2}}{d}$$

input
```
Int[Sech[c + d*x]^5*(a + b*Sinh[c + d*x]^2)^2,x]
```

output
```
(((a - b)*Sinh[c + d*x]*(a + b*Sinh[c + d*x]^2))/(4*(1 + Sinh[c + d*x]^2)^
2) + (((3*a^2 + 2*a*b + 3*b^2)*ArcTan[Sinh[c + d*x]])/2 + (3*(a^2 - b^2)*S
inh[c + d*x])/(2*(1 + Sinh[c + d*x]^2)))/4)/d
```

Defintions of rubi rules used

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 298
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2), x_Symbol] :> Simp[(-(
b*c - a*d))*x*((a + b*x^2)^(p + 1)/(2*a*b*(p + 1))), x] - Simp[(a*d - b*c*(
2*p + 3))/(2*a*b*(p + 1))   Int[(a + b*x^2)^(p + 1), x], x] /; FreeQ[{a, b,
 c, d, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/2 + p, 0])
```

rule 315
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_), x_Symbol] :> Sim
p[(a*d - c*b)*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^(q - 1)/(2*a*b*(p + 1))),
x] - Simp[1/(2*a*b*(p + 1))   Int[(a + b*x^2)^(p + 1)*(c + d*x^2)^(q - 2)*S
imp[c*(a*d - c*b*(2*p + 3)) + d*(a*d*(2*(q - 1) + 1) - b*c*(2*(p + q) + 1))
*x^2, x], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -
1] && GtQ[q, 1] && IntBinomialQ[a, b, c, d, 2, p, q, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3669
```
Int[cos[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^
(p_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Simp[ff/f   S
ubst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a + b*ff^2*x^2)^p, x], x, Sin[e + f*x]
/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m - 1)/2]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 176 vs. $2(83) = 166$.

Time = 100.40 (sec) , antiderivative size = 177, normalized size of antiderivative = 1.99

method	result
derivativedivides	$\frac{a^2\left(\left(\frac{\operatorname{sech}(dx+c)^3}{4}+\frac{3\operatorname{sech}(dx+c)}{8}\right)\tanh(dx+c)+\frac{3\arctan\left(e^{dx+c}\right)}{4}\right)+2ab\left(-\frac{\sinh(dx+c)}{3\cosh(dx+c)^4}+\frac{\left(\frac{\operatorname{sech}(dx+c)^3}{4}+\frac{3\operatorname{sech}(dx+c)}{8}\right.}{3}\right.}{}$
default	$\frac{a^2\left(\left(\frac{\operatorname{sech}(dx+c)^3}{4}+\frac{3\operatorname{sech}(dx+c)}{8}\right)\tanh(dx+c)+\frac{3\arctan\left(e^{dx+c}\right)}{4}\right)+2ab\left(-\frac{\sinh(dx+c)}{3\cosh(dx+c)^4}+\frac{\left(\frac{\operatorname{sech}(dx+c)^3}{4}+\frac{3\operatorname{sech}(dx+c)}{8}\right.}{3}\right.}{}$
risch	$\frac{e^{dx+c}\left(3e^{6dx+6c}a^2+2e^{6dx+6c}ab-5e^{6dx+6c}b^2+11e^{4dx+4c}a^2-14e^{4dx+4c}ab+3e^{4dx+4c}b^2-11e^{2dx+2c}a^2+14e^{2dx+2c}\right.}{4d\left(e^{2dx+2c}+1\right)^4}$

input
```
int(sech(d*x+c)^5*(a+b*sinh(d*x+c)^2)^2,x,method=_RETURNVERBOSE)
```

output
```
1/d*(a^2*((1/4*sech(d*x+c)^3+3/8*sech(d*x+c))*tanh(d*x+c)+3/4*arctan(exp(d
*x+c)))+2*a*b*(-1/3*sinh(d*x+c)/cosh(d*x+c)^4+1/3*(1/4*sech(d*x+c)^3+3/8*s
ech(d*x+c))*tanh(d*x+c)+1/4*arctan(exp(d*x+c)))+b^2*(-sinh(d*x+c)^3/cosh(d
*x+c)^4-sinh(d*x+c)/cosh(d*x+c)^4+(1/4*sech(d*x+c)^3+3/8*sech(d*x+c))*tanh
(d*x+c)+3/4*arctan(exp(d*x+c))))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1472 vs. $2(83) = 166$.

Time = 0.09 (sec) , antiderivative size = 1472, normalized size of antiderivative = 16.54

$$\int \operatorname{sech}^5(c+dx) \left(a+b\sinh^2(c+dx)\right)^2 \, dx = \text{Too large to display}$$

input
```
integrate(sech(d*x+c)^5*(a+b*sinh(d*x+c)^2)^2,x, algorithm="fricas")
```

output
```
1/4*((3*a^2 + 2*a*b - 5*b^2)*cosh(d*x + c)^7 + 7*(3*a^2 + 2*a*b - 5*b^2)*c
osh(d*x + c)*sinh(d*x + c)^6 + (3*a^2 + 2*a*b - 5*b^2)*sinh(d*x + c)^7 + (
11*a^2 - 14*a*b + 3*b^2)*cosh(d*x + c)^5 + (21*(3*a^2 + 2*a*b - 5*b^2)*cos
h(d*x + c)^2 + 11*a^2 - 14*a*b + 3*b^2)*sinh(d*x + c)^5 + 5*(7*(3*a^2 + 2*
a*b - 5*b^2)*cosh(d*x + c)^3 + (11*a^2 - 14*a*b + 3*b^2)*cosh(d*x + c))*si
nh(d*x + c)^4 - (11*a^2 - 14*a*b + 3*b^2)*cosh(d*x + c)^3 + (35*(3*a^2 + 2
*a*b - 5*b^2)*cosh(d*x + c)^4 + 10*(11*a^2 - 14*a*b + 3*b^2)*cosh(d*x + c)
^2 - 11*a^2 + 14*a*b - 3*b^2)*sinh(d*x + c)^3 + (21*(3*a^2 + 2*a*b - 5*b^2
)*cosh(d*x + c)^5 + 10*(11*a^2 - 14*a*b + 3*b^2)*cosh(d*x + c)^3 - 3*(11*a
^2 - 14*a*b + 3*b^2)*cosh(d*x + c))*sinh(d*x + c)^2 + ((3*a^2 + 2*a*b + 3*
b^2)*cosh(d*x + c)^8 + 8*(3*a^2 + 2*a*b + 3*b^2)*cosh(d*x + c)*sinh(d*x +
c)^7 + (3*a^2 + 2*a*b + 3*b^2)*sinh(d*x + c)^8 + 4*(3*a^2 + 2*a*b + 3*b^2)
*cosh(d*x + c)^6 + 4*(7*(3*a^2 + 2*a*b + 3*b^2)*cosh(d*x + c)^2 + 3*a^2 +
2*a*b + 3*b^2)*sinh(d*x + c)^6 + 8*(7*(3*a^2 + 2*a*b + 3*b^2)*cosh(d*x + c
)^3 + 3*(3*a^2 + 2*a*b + 3*b^2)*cosh(d*x + c))*sinh(d*x + c)^5 + 6*(3*a^2
+ 2*a*b + 3*b^2)*cosh(d*x + c)^4 + 2*(35*(3*a^2 + 2*a*b + 3*b^2)*cosh(d*x
+ c)^4 + 30*(3*a^2 + 2*a*b + 3*b^2)*cosh(d*x + c)^2 + 9*a^2 + 6*a*b + 9*b^
2)*sinh(d*x + c)^4 + 8*(7*(3*a^2 + 2*a*b + 3*b^2)*cosh(d*x + c)^5 + 10*(3*
a^2 + 2*a*b + 3*b^2)*cosh(d*x + c)^3 + 3*(3*a^2 + 2*a*b + 3*b^2)*cosh(d*x
+ c))*sinh(d*x + c)^3 + 4*(3*a^2 + 2*a*b + 3*b^2)*cosh(d*x + c)^2 + 4*(...
```

Sympy [F(-1)]

Timed out.

$$\int \operatorname{sech}^5(c+dx)\left(a+b\sinh^2(c+dx)\right)^2\,dx = \text{Timed out}$$

input
```
integrate(sech(d*x+c)**5*(a+b*sinh(d*x+c)**2)**2,x)
```

output
```
Timed out
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 347 vs. 2(83) = 166.

Time = 0.14 (sec) , antiderivative size = 347, normalized size of antiderivative = 3.90

$$\int \operatorname{sech}^5(c+dx)\left(a+b\sinh^2(c+dx)\right)^2\,dx =$$
$$-\frac{1}{4}b^2\left(\frac{3\arctan\left(e^{(-dx-c)}\right)}{d}+\frac{5e^{(-dx-c)}-3e^{(-3dx-3c)}+3e^{(-5dx-5c)}-5e^{(-7dx-7c)}}{d(4e^{(-2dx-2c)}+6e^{(-4dx-4c)}+4e^{(-6dx-6c)}+e^{(-8dx-8c)}+1)}\right)$$
$$-\frac{1}{4}a^2\left(\frac{3\arctan\left(e^{(-dx-c)}\right)}{d}-\frac{3e^{(-dx-c)}+11e^{(-3dx-3c)}-11e^{(-5dx-5c)}-3e^{(-7dx-7c)}}{d(4e^{(-2dx-2c)}+6e^{(-4dx-4c)}+4e^{(-6dx-6c)}+e^{(-8dx-8c)}+1)}\right)$$
$$-\frac{1}{2}ab\left(\frac{\arctan\left(e^{(-dx-c)}\right)}{d}-\frac{e^{(-dx-c)}-7e^{(-3dx-3c)}+7e^{(-5dx-5c)}-e^{(-7dx-7c)}}{d(4e^{(-2dx-2c)}+6e^{(-4dx-4c)}+4e^{(-6dx-6c)}+e^{(-8dx-8c)}+1)}\right)$$

input
```
integrate(sech(d*x+c)^5*(a+b*sinh(d*x+c)^2)^2,x, algorithm="maxima")
```

output
```
-1/4*b^2*(3*arctan(e^(-d*x - c))/d + (5*e^(-d*x - c) - 3*e^(-3*d*x - 3*c)
 + 3*e^(-5*d*x - 5*c) - 5*e^(-7*d*x - 7*c))/(d*(4*e^(-2*d*x - 2*c) + 6*e^(-
4*d*x - 4*c) + 4*e^(-6*d*x - 6*c) + e^(-8*d*x - 8*c) + 1))) - 1/4*a^2*(3*a
rctan(e^(-d*x - c))/d - (3*e^(-d*x - c) + 11*e^(-3*d*x - 3*c) - 11*e^(-5*d
*x - 5*c) - 3*e^(-7*d*x - 7*c))/(d*(4*e^(-2*d*x - 2*c) + 6*e^(-4*d*x - 4*c
) + 4*e^(-6*d*x - 6*c) + e^(-8*d*x - 8*c) + 1))) - 1/2*a*b*(arctan(e^(-d*x
 - c))/d - (e^(-d*x - c) - 7*e^(-3*d*x - 3*c) + 7*e^(-5*d*x - 5*c) - e^(-7
*d*x - 7*c))/(d*(4*e^(-2*d*x - 2*c) + 6*e^(-4*d*x - 4*c) + 4*e^(-6*d*x - 6
*c) + e^(-8*d*x - 8*c) + 1)))
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 218 vs. $2(83) = 166$.

Time = 0.15 (sec) , antiderivative size = 218, normalized size of antiderivative = 2.45

$$\int \operatorname{sech}^5(c+dx)\left(a+b\sinh^2(c+dx)\right)^2\,dx$$
$$= \frac{\left(\pi + 2\arctan\left(\frac{1}{2}\left(e^{(2\,dx+2\,c)}-1\right)e^{(-dx-c)}\right)\right)(3\,a^2+2\,ab+3\,b^2) + \frac{4\left(3\,a^2\left(e^{(dx+c)}-e^{(-dx-c)}\right)^3+2\,ab\left(e^{(dx+c)}-e^{(-dx-c)}\right.\right.}{}}{16\,d}$$

input
```
integrate(sech(d*x+c)^5*(a+b*sinh(d*x+c)^2)^2,x, algorithm="giac")
```

output
```
1/16*((pi + 2*arctan(1/2*(e^(2*d*x + 2*c) - 1)*e^(-d*x - c)))*(3*a^2 + 2*a
*b + 3*b^2) + 4*(3*a^2*(e^(d*x + c) - e^(-d*x - c))^3 + 2*a*b*(e^(d*x + c)
 - e^(-d*x - c))^3 - 5*b^2*(e^(d*x + c) - e^(-d*x - c))^3 + 20*a^2*(e^(d*x
 + c) - e^(-d*x - c)) - 8*a*b*(e^(d*x + c) - e^(-d*x - c)) - 12*b^2*(e^(d*
x + c) - e^(-d*x - c)))/((e^(d*x + c) - e^(-d*x - c))^2 + 4)^2)/d
```

Mupad [B] (verification not implemented)

Time = 1.92 (sec) , antiderivative size = 327, normalized size of antiderivative = 3.67

$$\int \operatorname{sech}^5(c+dx)\left(a+b\sinh^2(c+dx)\right)^2\,dx$$
$$= \frac{\operatorname{atan}\left(\frac{e^{d\,x}\,e^c\,\left(3\,a^2\,\sqrt{d^2}+3\,b^2\,\sqrt{d^2}+2\,a\,b\,\sqrt{d^2}\right)}{d\,\sqrt{9\,a^4+12\,a^3\,b+22\,a^2\,b^2+12\,a\,b^3+9\,b^4}}\right)\,\sqrt{9\,a^4+12\,a^3\,b+22\,a^2\,b^2+12\,a\,b^3+9\,b^4}}{4\,\sqrt{d^2}}$$
$$- \frac{6\,e^{c+d\,x}\,\left(a^2-2\,a\,b+b^2\right)}{d\,\left(3\,e^{2\,c+2\,d\,x}+3\,e^{4\,c+4\,d\,x}+e^{6\,c+6\,d\,x}+1\right)}$$
$$+ \frac{4\,e^{c+d\,x}\,\left(a^2-2\,a\,b+b^2\right)}{d\,\left(4\,e^{2\,c+2\,d\,x}+6\,e^{4\,c+4\,d\,x}+4\,e^{6\,c+6\,d\,x}+e^{8\,c+8\,d\,x}+1\right)}$$
$$+ \frac{e^{c+d\,x}\,\left(a^2-10\,a\,b+9\,b^2\right)}{2\,d\,\left(2\,e^{2\,c+2\,d\,x}+e^{4\,c+4\,d\,x}+1\right)} + \frac{e^{c+d\,x}\,\left(3\,a^2+2\,a\,b-5\,b^2\right)}{4\,d\,\left(e^{2\,c+2\,d\,x}+1\right)}$$

input
```
int((a + b*sinh(c + d*x)^2)^2/cosh(c + d*x)^5,x)
```

output
```
(atan((exp(d*x)*exp(c)*(3*a^2*(d^2)^(1/2) + 3*b^2*(d^2)^(1/2) + 2*a*b*(d^2
)^(1/2)))/(d*(12*a*b^3 + 12*a^3*b + 9*a^4 + 9*b^4 + 22*a^2*b^2)^(1/2)))*(1
2*a*b^3 + 12*a^3*b + 9*a^4 + 9*b^4 + 22*a^2*b^2)^(1/2))/(4*(d^2)^(1/2)) -
(6*exp(c + d*x)*(a^2 - 2*a*b + b^2))/(d*(3*exp(2*c + 2*d*x) + 3*exp(4*c +
4*d*x) + exp(6*c + 6*d*x) + 1)) + (4*exp(c + d*x)*(a^2 - 2*a*b + b^2))/(d*
(4*exp(2*c + 2*d*x) + 6*exp(4*c + 4*d*x) + 4*exp(6*c + 6*d*x) + exp(8*c +
8*d*x) + 1)) + (exp(c + d*x)*(a^2 - 10*a*b + 9*b^2))/(2*d*(2*exp(2*c + 2*d
*x) + exp(4*c + 4*d*x) + 1)) + (exp(c + d*x)*(2*a*b + 3*a^2 - 5*b^2))/(4*d
*(exp(2*c + 2*d*x) + 1))
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 533, normalized size of antiderivative = 5.99

$$\int \operatorname{sech}^5(c+dx)\left(a+b\sinh^2(c+dx)\right)^2 dx = \frac{2e^{8dx+8c}atan\left(e^{dx+c}\right)ab+8e^{6dx+6c}atan\left(e^{dx+c}\right)ab+12e^{4dx+4c}atan\left(e^{dx+c}\right)ab+8e^{2dx+2c}atan\left(e^{dx+c}\right)ab+3}{}$$

input
```
int(sech(d*x+c)^5*(a+b*sinh(d*x+c)^2)^2,x)
```

output
```
(3*e**(8*c + 8*d*x)*atan(e**(c + d*x))*a**2 + 2*e**(8*c + 8*d*x)*atan(e**(
c + d*x))*a*b + 3*e**(8*c + 8*d*x)*atan(e**(c + d*x))*b**2 + 12*e**(6*c +
6*d*x)*atan(e**(c + d*x))*a**2 + 8*e**(6*c + 6*d*x)*atan(e**(c + d*x))*a*b
 + 12*e**(6*c + 6*d*x)*atan(e**(c + d*x))*b**2 + 18*e**(4*c + 4*d*x)*atan(
e**(c + d*x))*a**2 + 12*e**(4*c + 4*d*x)*atan(e**(c + d*x))*a*b + 18*e**(4
*c + 4*d*x)*atan(e**(c + d*x))*b**2 + 12*e**(2*c + 2*d*x)*atan(e**(c + d*x
))*a**2 + 8*e**(2*c + 2*d*x)*atan(e**(c + d*x))*a*b + 12*e**(2*c + 2*d*x)*
atan(e**(c + d*x))*b**2 + 3*atan(e**(c + d*x))*a**2 + 2*atan(e**(c + d*x))
*a*b + 3*atan(e**(c + d*x))*b**2 + 3*e**(7*c + 7*d*x)*a**2 + 2*e**(7*c + 7
*d*x)*a*b - 5*e**(7*c + 7*d*x)*b**2 + 11*e**(5*c + 5*d*x)*a**2 - 14*e**(5*
c + 5*d*x)*a*b + 3*e**(5*c + 5*d*x)*b**2 - 11*e**(3*c + 3*d*x)*a**2 + 14*e
**(3*c + 3*d*x)*a*b - 3*e**(3*c + 3*d*x)*b**2 - 3*e**(c + d*x)*a**2 - 2*e*
*(c + d*x)*a*b + 5*e**(c + d*x)*b**2)/(4*d*(e**(8*c + 8*d*x) + 4*e**(6*c +
 6*d*x) + 6*e**(4*c + 4*d*x) + 4*e**(2*c + 2*d*x) + 1))
```

3.266 $\int \operatorname{sech}^6(c+dx) \left(a+b\sinh^2(c+dx)\right)^2 dx$

Optimal result

Integrand size = 23, antiderivative size = 57

$$\int \operatorname{sech}^6(c+dx) \left(a+b\sinh^2(c+dx)\right)^2 dx = \frac{a^2\tanh(c+dx)}{d} - \frac{2a(a-b)\tanh^3(c+dx)}{3d} + \frac{(a-b)^2\tanh^5(c+dx)}{5d}$$

output
```
a^2*tanh(d*x+c)/d-2/3*a*(a-b)*tanh(d*x+c)^3/d+1/5*(a-b)^2*tanh(d*x+c)^5/d
```

Mathematica [A] (verified)

Time = 0.98 (sec) , antiderivative size = 69, normalized size of antiderivative = 1.21

$$\int \operatorname{sech}^6(c+dx) \left(a+b\sinh^2(c+dx)\right)^2 dx = \frac{(8a^2+4ab+3b^2+2(2a^2+ab-3b^2)\operatorname{sech}^2(c+dx)+3(a-b)^2\operatorname{sech}^4(c+dx))\tanh(c+dx)}{15d}$$

input
```
Integrate[Sech[c + d*x]^6*(a + b*Sinh[c + d*x]^2)^2,x]
```

output
```
((8*a^2 + 4*a*b + 3*b^2 + 2*(2*a^2 + a*b - 3*b^2)*Sech[c + d*x]^2 + 3*(a -
 b)^2*Sech[c + d*x]^4)*Tanh[c + d*x])/(15*d)
```

Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 52, normalized size of antiderivative = 0.91, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.174$, Rules used = {3042, 3670, 210, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \operatorname{sech}^6(c+dx)\left(a+b\sinh^2(c+dx)\right)^2dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\left(a-b\sin(ic+idx)^2\right)^2}{\cos(ic+idx)^6}dx$$

$$\Big\downarrow \text{3670}$$

$$\frac{\int \left(a-(a-b)\tanh^2(c+dx)\right)^2d\tanh(c+dx)}{d}$$

$$\Big\downarrow \text{210}$$

$$\frac{\int \left((a-b)^2\tanh^4(c+dx)-2a(a-b)\tanh^2(c+dx)+a^2\right)d\tanh(c+dx)}{d}$$

$$\Big\downarrow \text{2009}$$

$$\frac{a^2\tanh(c+dx)+\frac{1}{5}(a-b)^2\tanh^5(c+dx)-\frac{2}{3}a(a-b)\tanh^3(c+dx)}{d}$$

input
```
Int[Sech[c + d*x]^6*(a + b*Sinh[c + d*x]^2)^2,x]
```

output
```
(a^2*Tanh[c + d*x] - (2*a*(a - b)*Tanh[c + d*x]^3)/3 + ((a - b)^2*Tanh[c +
 d*x]^5)/5)/d
```

Defintions of rubi rules used

rule 210
```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Int[ExpandIntegrand[(a + b*x^2
)^p, x], x] /; FreeQ[{a, b}, x] && IGtQ[p, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3670
```
Int[cos[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(
p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Simp[ff/f   Su
bst[Int[(a + (a + b)*ff^2*x^2)^p/(1 + ff^2*x^2)^(m/2 + p + 1), x], x, Tan[e
 + f*x]/ff], x]] /; FreeQ[{a, b, e, f}, x] && IntegerQ[m/2] && IntegerQ[p]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 128 vs. $2(53) = 106$.

Time = 190.42 (sec) , antiderivative size = 129, normalized size of antiderivative = 2.26

method	result
risch	$-\frac{2\left(15\,e^{8dx+8c}b^2+60\,e^{6dx+6c}ab+80\,e^{4dx+4c}a^2-20\,e^{4dx+4c}ab+30\,e^{4dx+4c}b^2+40\,e^{2dx+2c}a^2+20\,e^{2dx+2c}ba+8a^2+4ab\right.}{15d\left(e^{2dx+2c}+1\right)^5}$
derivativedivides	$\frac{a^2\left(\frac{8}{15}+\frac{\operatorname{sech}(dx+c)^4}{5}+\frac{4\operatorname{sech}(dx+c)^2}{15}\right)\tanh(dx+c)+2ab\left(-\frac{\sinh(dx+c)}{4\cosh(dx+c)^5}+\frac{\left(\frac{8}{15}+\frac{\operatorname{sech}(dx+c)^4}{5}+\frac{4\operatorname{sech}(dx+c)^2}{15}\right)\tanh(dx+}{4}\right.}{d}$
default	$\frac{a^2\left(\frac{8}{15}+\frac{\operatorname{sech}(dx+c)^4}{5}+\frac{4\operatorname{sech}(dx+c)^2}{15}\right)\tanh(dx+c)+2ab\left(-\frac{\sinh(dx+c)}{4\cosh(dx+c)^5}+\frac{\left(\frac{8}{15}+\frac{\operatorname{sech}(dx+c)^4}{5}+\frac{4\operatorname{sech}(dx+c)^2}{15}\right)\tanh(dx+}{4}\right.}{d}$

input
```
int(sech(d*x+c)^6*(a+b*sinh(d*x+c)^2)^2,x,method=_RETURNVERBOSE)
```

output
```
-2/15*(15*exp(8*d*x+8*c)*b^2+60*exp(6*d*x+6*c)*a*b+80*exp(4*d*x+4*c)*a^2-2
0*exp(4*d*x+4*c)*a*b+30*exp(4*d*x+4*c)*b^2+40*exp(2*d*x+2*c)*a^2+20*exp(2*
d*x+2*c)*b*a+8*a^2+4*a*b+3*b^2)/d/(exp(2*d*x+2*c)+1)^5
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 403 vs. $2(53) = 106$.

Time = 0.09 (sec) , antiderivative size = 403, normalized size of antiderivative = 7.07

$$\int \operatorname{sech}^6(c+dx)\left(a+b\sinh^2(c+dx)\right)^2\,dx = -\frac{4\left((4a^2+2ab+9b^2)\cosh(dx+c)^4-8(2a^2+ab-3b^2)\cosh(dx+c)\sinh(dx+\right.}{15\left(d\cosh(dx+c)^6+6d\cosh(dx+c)\sinh(dx+c)^5+d\sinh(dx+c)^6+6d\cosh(dx+c)^4+3(5d\right.}$$

input
```
integrate(sech(d*x+c)^6*(a+b*sinh(d*x+c)^2)^2,x, algorithm="fricas")
```

output
```
-4/15*((4*a^2 + 2*a*b + 9*b^2)*cosh(d*x + c)^4 - 8*(2*a^2 + a*b - 3*b^2)*c
osh(d*x + c)*sinh(d*x + c)^3 + (4*a^2 + 2*a*b + 9*b^2)*sinh(d*x + c)^4 + 2
0*(a^2 + 2*a*b)*cosh(d*x + c)^2 + 2*(3*(4*a^2 + 2*a*b + 9*b^2)*cosh(d*x +
c)^2 + 10*a^2 + 20*a*b)*sinh(d*x + c)^2 + 40*a^2 - 10*a*b + 15*b^2 - 8*((2
*a^2 + a*b - 3*b^2)*cosh(d*x + c)^3 + 5*(a^2 - a*b)*cosh(d*x + c))*sinh(d*
x + c))/(d*cosh(d*x + c)^6 + 6*d*cosh(d*x + c)*sinh(d*x + c)^5 + d*sinh(d*
x + c)^6 + 6*d*cosh(d*x + c)^4 + 3*(5*d*cosh(d*x + c)^2 + 2*d)*sinh(d*x +
c)^4 + 4*(5*d*cosh(d*x + c)^3 + 4*d*cosh(d*x + c))*sinh(d*x + c)^3 + 15*d*
cosh(d*x + c)^2 + 3*(5*d*cosh(d*x + c)^4 + 12*d*cosh(d*x + c)^2 + 5*d)*sin
h(d*x + c)^2 + 2*(3*d*cosh(d*x + c)^5 + 8*d*cosh(d*x + c)^3 + 5*d*cosh(d*x
 + c))*sinh(d*x + c) + 10*d)
```

Sympy [F(-1)]

Timed out.

$$\int \operatorname{sech}^6(c+dx) \left(a+b\sinh^2(c+dx)\right)^2 \, dx = \text{Timed out}$$

input
```
integrate(sech(d*x+c)**6*(a+b*sinh(d*x+c)**2)**2,x)
```

output
```
Timed out
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 698 vs. $2(53) = 106$.

Time = 0.04 (sec) , antiderivative size = 698, normalized size of antiderivative = 12.25

$$\int \operatorname{sech}^6(c+dx) \left(a+b\sinh^2(c+dx)\right)^2 \, dx = \text{Too large to display}$$

input
```
integrate(sech(d*x+c)^6*(a+b*sinh(d*x+c)^2)^2,x, algorithm="maxima")
```

output
```
16/15*a^2*(5*e^(-2*d*x - 2*c)/(d*(5*e^(-2*d*x - 2*c) + 10*e^(-4*d*x - 4*c)
 + 10*e^(-6*d*x - 6*c) + 5*e^(-8*d*x - 8*c) + e^(-10*d*x - 10*c) + 1)) + 1
0*e^(-4*d*x - 4*c)/(d*(5*e^(-2*d*x - 2*c) + 10*e^(-4*d*x - 4*c) + 10*e^(-6
*d*x - 6*c) + 5*e^(-8*d*x - 8*c) + e^(-10*d*x - 10*c) + 1)) + 1/(d*(5*e^(-
2*d*x - 2*c) + 10*e^(-4*d*x - 4*c) + 10*e^(-6*d*x - 6*c) + 5*e^(-8*d*x - 8
*c) + e^(-10*d*x - 10*c) + 1))) + 8/15*a*b*(5*e^(-2*d*x - 2*c)/(d*(5*e^(-2
*d*x - 2*c) + 10*e^(-4*d*x - 4*c) + 10*e^(-6*d*x - 6*c) + 5*e^(-8*d*x - 8*
c) + e^(-10*d*x - 10*c) + 1)) - 5*e^(-4*d*x - 4*c)/(d*(5*e^(-2*d*x - 2*c)
+ 10*e^(-4*d*x - 4*c) + 10*e^(-6*d*x - 6*c) + 5*e^(-8*d*x - 8*c) + e^(-10*
d*x - 10*c) + 1)) + 15*e^(-6*d*x - 6*c)/(d*(5*e^(-2*d*x - 2*c) + 10*e^(-4*
d*x - 4*c) + 10*e^(-6*d*x - 6*c) + 5*e^(-8*d*x - 8*c) + e^(-10*d*x - 10*c)
 + 1)) + 1/(d*(5*e^(-2*d*x - 2*c) + 10*e^(-4*d*x - 4*c) + 10*e^(-6*d*x - 6
*c) + 5*e^(-8*d*x - 8*c) + e^(-10*d*x - 10*c) + 1))) + 2/5*b^2*(10*e^(-4*d
*x - 4*c)/(d*(5*e^(-2*d*x - 2*c) + 10*e^(-4*d*x - 4*c) + 10*e^(-6*d*x - 6*
c) + 5*e^(-8*d*x - 8*c) + e^(-10*d*x - 10*c) + 1)) + 5*e^(-8*d*x - 8*c)/(d
*(5*e^(-2*d*x - 2*c) + 10*e^(-4*d*x - 4*c) + 10*e^(-6*d*x - 6*c) + 5*e^(-8
*d*x - 8*c) + e^(-10*d*x - 10*c) + 1)) + 1/(d*(5*e^(-2*d*x - 2*c) + 10*e^(
-4*d*x - 4*c) + 10*e^(-6*d*x - 6*c) + 5*e^(-8*d*x - 8*c) + e^(-10*d*x - 10
*c) + 1)))
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 128 vs. $2(53) = 106$.

Time = 0.15 (sec) , antiderivative size = 128, normalized size of antiderivative = 2.25

$$\int \operatorname{sech}^6(c+dx) \left(a+b\sinh^2(c+dx)\right)^2 dx = -\frac{2 \left(15 b^2 e^{(8 dx+8 c)} + 60 abe^{(6 dx+6 c)} + 80 a^2 e^{(4 dx+4 c)} - 20 abe^{(4 dx+4 c)} + 30 b^2 e^{(4 dx+4 c)} + 40 a^2 e^{(2 dx+2 c)} + \right.}{15 d\left(e^{(2 dx+2 c)} + 1\right)^5}$$

input
```
integrate(sech(d*x+c)^6*(a+b*sinh(d*x+c)^2)^2,x, algorithm="giac")
```

output
```
-2/15*(15*b^2*e^(8*d*x + 8*c) + 60*a*b*e^(6*d*x + 6*c) + 80*a^2*e^(4*d*x +
 4*c) - 20*a*b*e^(4*d*x + 4*c) + 30*b^2*e^(4*d*x + 4*c) + 40*a^2*e^(2*d*x
+ 2*c) + 20*a*b*e^(2*d*x + 2*c) + 8*a^2 + 4*a*b + 3*b^2)/(d*(e^(2*d*x + 2*
c) + 1)^5)
```

Mupad [B] (verification not implemented)

Time = 1.93 (sec) , antiderivative size = 464, normalized size of antiderivative = 8.14

$$\int \operatorname{sech}^6(c+dx)\left(a+b\sinh^2(c+dx)\right)^2\,dx$$
$$= -\frac{\frac{2(8a^2-8ab+3b^2)}{15d}+\frac{2b^2e^{4c+4dx}}{5d}+\frac{4be^{2c+2dx}(2a-b)}{5d}}{3e^{2c+2dx}+3e^{4c+4dx}+e^{6c+6dx}+1}$$
$$-\frac{\frac{2b^2}{5d}+\frac{2b^2e^{8c+8dx}}{5d}+\frac{4e^{4c+4dx}(8a^2-8ab+3b^2)}{5d}+\frac{8be^{2c+2dx}(2a-b)}{5d}+\frac{8be^{6c+6dx}(2a-b)}{5d}}{5e^{2c+2dx}+10e^{4c+4dx}+10e^{6c+6dx}+5e^{8c+8dx}+e^{10c+10dx}+1}$$
$$-\frac{\frac{2b(2a-b)}{5d}+\frac{2b^2e^{2c+2dx}}{5d}}{2e^{2c+2dx}+e^{4c+4dx}+1}$$
$$-\frac{\frac{2b(2a-b)}{5d}+\frac{2b^2e^{6c+6dx}}{5d}+\frac{2e^{2c+2dx}(8a^2-8ab+3b^2)}{5d}+\frac{6be^{4c+4dx}(2a-b)}{5d}}{4e^{2c+2dx}+6e^{4c+4dx}+4e^{6c+6dx}+e^{8c+8dx}+1}$$
$$-\frac{2b^2}{5d\left(e^{2c+2dx}+1\right)}$$

input
```
int((a + b*sinh(c + d*x)^2)^2/cosh(c + d*x)^6,x)
```

output
```
- ((2*(8*a^2 - 8*a*b + 3*b^2))/(15*d) + (2*b^2*exp(4*c + 4*d*x))/(5*d) + (
4*b*exp(2*c + 2*d*x)*(2*a - b))/(5*d))/(3*exp(2*c + 2*d*x) + 3*exp(4*c + 4
*d*x) + exp(6*c + 6*d*x) + 1) - ((2*b^2)/(5*d) + (2*b^2*exp(8*c + 8*d*x))/
(5*d) + (4*exp(4*c + 4*d*x)*(8*a^2 - 8*a*b + 3*b^2))/(5*d) + (8*b*exp(2*c
+ 2*d*x)*(2*a - b))/(5*d) + (8*b*exp(6*c + 6*d*x)*(2*a - b))/(5*d))/(5*exp
(2*c + 2*d*x) + 10*exp(4*c + 4*d*x) + 10*exp(6*c + 6*d*x) + 5*exp(8*c + 8*
d*x) + exp(10*c + 10*d*x) + 1) - ((2*b*(2*a - b))/(5*d) + (2*b^2*exp(2*c +
 2*d*x))/(5*d))/(2*exp(2*c + 2*d*x) + exp(4*c + 4*d*x) + 1) - ((2*b*(2*a -
 b))/(5*d) + (2*b^2*exp(6*c + 6*d*x))/(5*d) + (2*exp(2*c + 2*d*x)*(8*a^2 -
 8*a*b + 3*b^2))/(5*d) + (6*b*exp(4*c + 4*d*x)*(2*a - b))/(5*d))/(4*exp(2*
c + 2*d*x) + 6*exp(4*c + 4*d*x) + 4*exp(6*c + 6*d*x) + exp(8*c + 8*d*x) +
1) - (2*b^2)/(5*d*(exp(2*c + 2*d*x) + 1))
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 194, normalized size of antiderivative = 3.40

$$\int \operatorname{sech}^6(c+dx) \left(a+b\sinh^2(c+dx)\right)^2 \, dx$$
$$= \frac{\frac{2e^{10dx+10c}b^2}{5} - 8e^{6dx+6c}ab + 4e^{6dx+6c}b^2 - \frac{32e^{4dx+4c}a^2}{3} + \frac{8e^{4dx+4c}ab}{3} - \frac{16e^{2dx+2c}a^2}{3} - \frac{8e^{2dx+2c}ab}{3} + 2e^{2dx+2c}b^2 - \frac{16a}{15}}{d\left(e^{10dx+10c} + 5e^{8dx+8c} + 10e^{6dx+6c} + 10e^{4dx+4c} + 5e^{2dx+2c} + 1\right)}$$

input

```
int(sech(d*x+c)^6*(a+b*sinh(d*x+c)^2)^2,x)
```

output

```
(2*(3*e**(10*c + 10*d*x)*b**2 - 60*e**(6*c + 6*d*x)*a*b + 30*e**(6*c + 6*d
*x)*b**2 - 80*e**(4*c + 4*d*x)*a**2 + 20*e**(4*c + 4*d*x)*a*b - 40*e**(2*c
 + 2*d*x)*a**2 - 20*e**(2*c + 2*d*x)*a*b + 15*e**(2*c + 2*d*x)*b**2 - 8*a*
*2 - 4*a*b))/(15*d*(e**(10*c + 10*d*x) + 5*e**(8*c + 8*d*x) + 10*e**(6*c +
 6*d*x) + 10*e**(4*c + 4*d*x) + 5*e**(2*c + 2*d*x) + 1))
```

3.267 $\int \operatorname{sech}^7(c+dx) \left(a+b\sinh^2(c+dx)\right)^2 dx$

Optimal result

Integrand size = 23, antiderivative size = 121

$$\int \operatorname{sech}^7(c+dx) \left(a+b\sinh^2(c+dx)\right)^2 dx = \frac{(5a^2+2ab+b^2)\arctan(\sinh(c+dx))}{16d} + \frac{(5a^2+2ab+b^2)\operatorname{sech}(c+dx)\tanh(c+dx)}{16d} + \frac{(a-b)(5a+7b)\operatorname{sech}^3(c+dx)\tanh(c+dx)}{24d} + \frac{(a-b)^2\operatorname{sech}^5(c+dx)\tanh(c+dx)}{6d}$$

output
```
1/16*(5*a^2+2*a*b+b^2)*arctan(sinh(d*x+c))/d+1/16*(5*a^2+2*a*b+b^2)*sech(d
*x+c)*tanh(d*x+c)/d+1/24*(a-b)*(5*a+7*b)*sech(d*x+c)^3*tanh(d*x+c)/d+1/6*(
a-b)^2*sech(d*x+c)^5*tanh(d*x+c)/d
```

Mathematica [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 8.91 (sec) , antiderivative size = 715, normalized size of antiderivative = 5.91

$$\int \operatorname{sech}^7(c+dx) \left(a+b\sinh^2(c+dx)\right)^2 dx = \frac{\operatorname{csch}^3(c+dx)\left(65625a^2\operatorname{arctanh}\left(\sqrt{-\sinh^2(c+dx)}\right) + 36855a^2\operatorname{arctanh}\left(\sqrt{-\sinh^2(c+dx)}\right)\sinh^2(c+\right.}{}$$

input
```
Integrate[Sech[c + d*x]^7*(a + b*Sinh[c + d*x]^2)^2,x]
```

output
```
(Csch[c + d*x]^3*(65625*a^2*ArcTanh[Sqrt[-Sinh[c + d*x]^2]] + 36855*a^2*Ar
cTanh[Sqrt[-Sinh[c + d*x]^2]]*Sinh[c + d*x]^2 + 91875*a*b*ArcTanh[Sqrt[-Si
nh[c + d*x]^2]]*Sinh[c + d*x]^2 + 1680*a^2*ArcTanh[Sqrt[-Sinh[c + d*x]^2]]
*Sinh[c + d*x]^4 + 54180*a*b*ArcTanh[Sqrt[-Sinh[c + d*x]^2]]*Sinh[c + d*x]
^4 + 32970*b^2*ArcTanh[Sqrt[-Sinh[c + d*x]^2]]*Sinh[c + d*x]^4 + 1365*a*b*
ArcTanh[Sqrt[-Sinh[c + d*x]^2]]*Sinh[c + d*x]^6 + 19845*b^2*ArcTanh[Sqrt[-
Sinh[c + d*x]^2]]*Sinh[c + d*x]^6 + 525*b^2*ArcTanh[Sqrt[-Sinh[c + d*x]^2]
]*Sinh[c + d*x]^8 - 65625*a^2*Sqrt[-Sinh[c + d*x]^2] + 14980*a^2*(-Sinh[c
+ d*x]^2)^(3/2) + 91875*a*b*(-Sinh[c + d*x]^2)^(3/2) + 8855*b^2*Sinh[c + d
*x]^4*(-Sinh[c + d*x]^2)^(3/2) + 16*a^2*HypergeometricPFQ[{3/2, 2, 2, 2, 2
, 2}, {1, 1, 1, 1, 9/2}, -Sinh[c + d*x]^2]*Sinh[c + d*x]^4*(-Sinh[c + d*x]
^2)^(3/2) + 32*a*b*HypergeometricPFQ[{3/2, 2, 2, 2, 2, 2}, {1, 1, 1, 1, 9/
2}, -Sinh[c + d*x]^2]*Sinh[c + d*x]^6*(-Sinh[c + d*x]^2)^(3/2) + 16*b^2*Hy
pergeometricPFQ[{3/2, 2, 2, 2, 2, 2}, {1, 1, 1, 1, 9/2}, -Sinh[c + d*x]^2]
*Sinh[c + d*x]^8*(-Sinh[c + d*x]^2)^(3/2) - 23555*a*b*(-Sinh[c + d*x]^2)^(
5/2) - 32970*b^2*(-Sinh[c + d*x]^2)^(5/2) + 32*HypergeometricPFQ[{3/2, 2,
2, 2, 2}, {1, 1, 1, 9/2}, -Sinh[c + d*x]^2]*Sinh[c + d*x]^4*(-Sinh[c + d*x
]^2)^(3/2)*(5*a^2 + 9*a*b*Sinh[c + d*x]^2 + 4*b^2*Sinh[c + d*x]^4) + 4*Hyp
ergeometricPFQ[{3/2, 2, 2, 2}, {1, 1, 9/2}, -Sinh[c + d*x]^2]*Sinh[c + d*x
]^4*(-Sinh[c + d*x]^2)^(3/2)*(155*a^2 + 242*a*b*Sinh[c + d*x]^2 + 95*b^...
```

Rubi [A] (verified)

Time = 0.35 (sec) , antiderivative size = 134, normalized size of antiderivative = 1.11, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.261$, Rules used = {3042, 3669, 315, 298, 215, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \operatorname{sech}^7(c+dx) \left(a+b\sinh^2(c+dx)\right)^2 \, dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\left(a-b\sin(ic+idx)^2\right)^2}{\cos(ic+idx)^7}dx$$

$$\Big\downarrow \; 3669$$

$$\frac{\int \frac{\left(b\sinh^2(c+dx)+a\right)^2}{\left(\sinh^2(c+dx)+1\right)^4}d\sinh(c+dx)}{d}$$

$$\Big\downarrow \; 315$$

$$\frac{\frac{1}{6}\int \frac{3b(a+b)\sinh^2(c+dx)+a(5a+b)}{\left(\sinh^2(c+dx)+1\right)^3}d\sinh(c+dx)+\frac{(a-b)\sinh(c+dx)\left(a+b\sinh^2(c+dx)\right)}{6\left(\sinh^2(c+dx)+1\right)^3}}{d}$$

$$\Big\downarrow \; 298$$

$$\frac{\frac{1}{6}\left(\frac{3}{4}\left(5a^2+2ab+b^2\right)\int \frac{1}{\left(\sinh^2(c+dx)+1\right)^2}d\sinh(c+dx)+\frac{(a-b)(5a+3b)\sinh(c+dx)}{4\left(\sinh^2(c+dx)+1\right)^2}\right)+\frac{(a-b)\sinh(c+dx)\left(a+b\sinh^2(c+dx)\right)}{6\left(\sinh^2(c+dx)+1\right)^3}}{d}$$

$$\Big\downarrow \; 215$$

$$\frac{\frac{1}{6}\left(\frac{3}{4}\left(5a^2+2ab+b^2\right)\left(\frac{1}{2}\int \frac{1}{\sinh^2(c+dx)+1}d\sinh(c+dx)+\frac{\sinh(c+dx)}{2\left(\sinh^2(c+dx)+1\right)}\right)+\frac{(a-b)(5a+3b)\sinh(c+dx)}{4\left(\sinh^2(c+dx)+1\right)^2}\right)+\frac{(a-b)\sinh(c-}{6(\sin}}{d}$$

$$\Big\downarrow \; 216$$

$$\frac{\frac{1}{6}\left(\frac{3}{4}\left(5a^2+2ab+b^2\right)\left(\frac{1}{2}\arctan(\sinh(c+dx))+\frac{\sinh(c+dx)}{2\left(\sinh^2(c+dx)+1\right)}\right)+\frac{(a-b)(5a+3b)\sinh(c+dx)}{4\left(\sinh^2(c+dx)+1\right)^2}\right)+\frac{(a-b)\sinh(c+dx)(a+b}{6(\sinh^2(c+dx)}}{d}$$

input
```
Int[Sech[c + d*x]^7*(a + b*Sinh[c + d*x]^2)^2,x]
```

output
```
(((a - b)*Sinh[c + d*x]*(a + b*Sinh[c + d*x]^2))/(6*(1 + Sinh[c + d*x]^2)^
3) + (((a - b)*(5*a + 3*b)*Sinh[c + d*x])/(4*(1 + Sinh[c + d*x]^2)^2) + (3
*(5*a^2 + 2*a*b + b^2)*(ArcTan[Sinh[c + d*x]]/2 + Sinh[c + d*x]/(2*(1 + Si
nh[c + d*x]^2))))/4)/6)/d
```

Defintions of rubi rules used

rule 215
```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(-x)*((a + b*x^2)^(p + 1)
/(2*a*(p + 1))), x] + Simp[(2*p + 3)/(2*a*(p + 1))   Int[(a + b*x^2)^(p + 1
), x], x] /; FreeQ[{a, b}, x] && LtQ[p, -1] && (IntegerQ[4*p] || IntegerQ[6
*p])
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 298
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2), x_Symbol] :> Simp[(-(
b*c - a*d))*x*((a + b*x^2)^(p + 1)/(2*a*b*(p + 1))), x] - Simp[(a*d - b*c*(
2*p + 3))/(2*a*b*(p + 1))   Int[(a + b*x^2)^(p + 1), x], x] /; FreeQ[{a, b,
 c, d, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/2 + p, 0])
```

rule 315
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_), x_Symbol] :> Sim
p[(a*d - c*b)*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^(q - 1)/(2*a*b*(p + 1))),
x] - Simp[1/(2*a*b*(p + 1))   Int[(a + b*x^2)^(p + 1)*(c + d*x^2)^(q - 2)*S
imp[c*(a*d - c*b*(2*p + 3)) + d*(a*d*(2*(q - 1) + 1) - b*c*(2*(p + q) + 1))
*x^2, x], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -
1] && GtQ[q, 1] && IntBinomialQ[a, b, c, d, 2, p, q, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3669
```
Int[cos[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^
(p_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Simp[ff/f   S
ubst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a + b*ff^2*x^2)^p, x], x, Sin[e + f*x]
/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m - 1)/2]
```

Maple [A] (verified)

Time = 0.30 (sec) , antiderivative size = 208, normalized size of antiderivative = 1.72

$$a^2\left(\left(\frac{\operatorname{sech}(dx+c)^5}{6}+\frac{5\operatorname{sech}(dx+c)^3}{24}+\frac{5\operatorname{sech}(dx+c)}{16}\right)\tanh(dx+c)+\frac{5\arctan(e^{dx+c})}{8}\right)+2ab\left(-\frac{\sinh(dx+c)}{5\cosh(dx+c)^6}+\frac{\left(\frac{\operatorname{sech}(dx+c}{6}\right.}{}\right.$$

input
```
int(sech(d*x+c)^7*(a+b*sinh(d*x+c)^2)^2,x)
```

output
```
1/d*(a^2*((1/6*sech(d*x+c)^5+5/24*sech(d*x+c)^3+5/16*sech(d*x+c))*tanh(d*x
+c)+5/8*arctan(exp(d*x+c)))+2*a*b*(-1/5*sinh(d*x+c)/cosh(d*x+c)^6+1/5*(1/6
*sech(d*x+c)^5+5/24*sech(d*x+c)^3+5/16*sech(d*x+c))*tanh(d*x+c)+1/8*arctan
(exp(d*x+c)))+b^2*(-1/3*sinh(d*x+c)^3/cosh(d*x+c)^6-1/5*sinh(d*x+c)/cosh(d
*x+c)^6+1/5*(1/6*sech(d*x+c)^5+5/24*sech(d*x+c)^3+5/16*sech(d*x+c))*tanh(d
*x+c)+1/8*arctan(exp(d*x+c))))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2824 vs. 2(113) = 226.

Time = 0.11 (sec) , antiderivative size = 2824, normalized size of antiderivative = 23.34

$$\int \operatorname{sech}^7(c+dx)\left(a+b\sinh^2(c+dx)\right)^2\,dx=\text{Too large to display}$$

input
```
integrate(sech(d*x+c)^7*(a+b*sinh(d*x+c)^2)^2,x, algorithm="fricas")
```

output
```
1/24*(3*(5*a^2 + 2*a*b + b^2)*cosh(d*x + c)^11 + 33*(5*a^2 + 2*a*b + b^2)*
cosh(d*x + c)*sinh(d*x + c)^10 + 3*(5*a^2 + 2*a*b + b^2)*sinh(d*x + c)^11
+ (85*a^2 + 34*a*b - 47*b^2)*cosh(d*x + c)^9 + (165*(5*a^2 + 2*a*b + b^2)*
cosh(d*x + c)^2 + 85*a^2 + 34*a*b - 47*b^2)*sinh(d*x + c)^9 + 9*(55*(5*a^2
 + 2*a*b + b^2)*cosh(d*x + c)^3 + (85*a^2 + 34*a*b - 47*b^2)*cosh(d*x + c)
)*sinh(d*x + c)^8 + 6*(33*a^2 - 38*a*b + 13*b^2)*cosh(d*x + c)^7 + 6*(165*
(5*a^2 + 2*a*b + b^2)*cosh(d*x + c)^4 + 6*(85*a^2 + 34*a*b - 47*b^2)*cosh(
d*x + c)^2 + 33*a^2 - 38*a*b + 13*b^2)*sinh(d*x + c)^7 + 42*(33*(5*a^2 + 2
*a*b + b^2)*cosh(d*x + c)^5 + 2*(85*a^2 + 34*a*b - 47*b^2)*cosh(d*x + c)^3
 + (33*a^2 - 38*a*b + 13*b^2)*cosh(d*x + c))*sinh(d*x + c)^6 - 6*(33*a^2 -
 38*a*b + 13*b^2)*cosh(d*x + c)^5 + 6*(231*(5*a^2 + 2*a*b + b^2)*cosh(d*x
+ c)^6 + 21*(85*a^2 + 34*a*b - 47*b^2)*cosh(d*x + c)^4 + 21*(33*a^2 - 38*a
*b + 13*b^2)*cosh(d*x + c)^2 - 33*a^2 + 38*a*b - 13*b^2)*sinh(d*x + c)^5 +
 6*(165*(5*a^2 + 2*a*b + b^2)*cosh(d*x + c)^7 + 21*(85*a^2 + 34*a*b - 47*b
^2)*cosh(d*x + c)^5 + 35*(33*a^2 - 38*a*b + 13*b^2)*cosh(d*x + c)^3 - 5*(3
3*a^2 - 38*a*b + 13*b^2)*cosh(d*x + c))*sinh(d*x + c)^4 - (85*a^2 + 34*a*b
 - 47*b^2)*cosh(d*x + c)^3 + (495*(5*a^2 + 2*a*b + b^2)*cosh(d*x + c)^8 +
84*(85*a^2 + 34*a*b - 47*b^2)*cosh(d*x + c)^6 + 210*(33*a^2 - 38*a*b + 13*
b^2)*cosh(d*x + c)^4 - 60*(33*a^2 - 38*a*b + 13*b^2)*cosh(d*x + c)^2 - 85*
a^2 - 34*a*b + 47*b^2)*sinh(d*x + c)^3 + 3*(55*(5*a^2 + 2*a*b + b^2)*co...
```

Sympy [F(-1)]

Timed out.

$$\int \operatorname{sech}^7(c+dx)\left(a+b\sinh^2(c+dx)\right)^2\,dx=\text{Timed out}$$

input
```
integrate(sech(d*x+c)**7*(a+b*sinh(d*x+c)**2)**2,x)
```

output
```
Timed out
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 483 vs. $2(113) = 226$.

Time = 0.13 (sec) , antiderivative size = 483, normalized size of antiderivative = 3.99

$$\int \operatorname{sech}^7(c+dx)\left(a+b\sinh^2(c+dx)\right)^2\,dx =$$

$$-\frac{1}{24}a^2\left(\frac{15\arctan\left(e^{(-dx-c)}\right)}{d} - \frac{15\,e^{(-dx-c)}+85\,e^{(-3\,dx-3\,c)}+198\,e^{(-5\,dx-5\,c)}-198\,e^{(-7\,dx-7\,c)}-85\,e^{(-}}{d(6\,e^{(-2\,dx-2\,c)}+15\,e^{(-4\,dx-4\,c)}+20\,e^{(-6\,dx-6\,c)}+15\,e^{(-8\,dx-8\,c)}+6\,e^{(-10}}\right.$$

$$-\frac{1}{12}ab\left(\frac{3\arctan\left(e^{(-dx-c)}\right)}{d} - \frac{3\,e^{(-dx-c)}+17\,e^{(-3\,dx-3\,c)}-114\,e^{(-5\,dx-5\,c)}+114\,e^{(-7\,dx-7\,c)}-17\,e^{(-9}}{d(6\,e^{(-2\,dx-2\,c)}+15\,e^{(-4\,dx-4\,c)}+20\,e^{(-6\,dx-6\,c)}+15\,e^{(-8\,dx-8\,c)}+6\,e^{(-10\,d}}\right.$$

$$-\frac{1}{24}b^2\left(\frac{3\arctan\left(e^{(-dx-c)}\right)}{d} - \frac{3\,e^{(-dx-c)}-47\,e^{(-3\,dx-3\,c)}+78\,e^{(-5\,dx-5\,c)}-78\,e^{(-7\,dx-7\,c)}+47\,e^{(-9\,d}}{d(6\,e^{(-2\,dx-2\,c)}+15\,e^{(-4\,dx-4\,c)}+20\,e^{(-6\,dx-6\,c)}+15\,e^{(-8\,dx-8\,c)}+6\,e^{(-10\,d}}\right.$$

input
```
integrate(sech(d*x+c)^7*(a+b*sinh(d*x+c)^2)^2,x, algorithm="maxima")
```

output
```
-1/24*a^2*(15*arctan(e^(-d*x - c))/d - (15*e^(-d*x - c) + 85*e^(-3*d*x - 3
*c) + 198*e^(-5*d*x - 5*c) - 198*e^(-7*d*x - 7*c) - 85*e^(-9*d*x - 9*c) -
15*e^(-11*d*x - 11*c))/(d*(6*e^(-2*d*x - 2*c) + 15*e^(-4*d*x - 4*c) + 20*e
^(-6*d*x - 6*c) + 15*e^(-8*d*x - 8*c) + 6*e^(-10*d*x - 10*c) + e^(-12*d*x
- 12*c) + 1))) - 1/12*a*b*(3*arctan(e^(-d*x - c))/d - (3*e^(-d*x - c) + 17
*e^(-3*d*x - 3*c) - 114*e^(-5*d*x - 5*c) + 114*e^(-7*d*x - 7*c) - 17*e^(-9
*d*x - 9*c) - 3*e^(-11*d*x - 11*c))/(d*(6*e^(-2*d*x - 2*c) + 15*e^(-4*d*x
- 4*c) + 20*e^(-6*d*x - 6*c) + 15*e^(-8*d*x - 8*c) + 6*e^(-10*d*x - 10*c)
+ e^(-12*d*x - 12*c) + 1))) - 1/24*b^2*(3*arctan(e^(-d*x - c))/d - (3*e^(-
d*x - c) - 47*e^(-3*d*x - 3*c) + 78*e^(-5*d*x - 5*c) - 78*e^(-7*d*x - 7*c)
 + 47*e^(-9*d*x - 9*c) - 3*e^(-11*d*x - 11*c))/(d*(6*e^(-2*d*x - 2*c) + 15
*e^(-4*d*x - 4*c) + 20*e^(-6*d*x - 6*c) + 15*e^(-8*d*x - 8*c) + 6*e^(-10*d
*x - 10*c) + e^(-12*d*x - 12*c) + 1)))
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 291 vs. $2(113) = 226$.

Time = 0.16 (sec) , antiderivative size = 291, normalized size of antiderivative = 2.40

$$\int \operatorname{sech}^7(c+dx) \left(a+b\sinh^2(c+dx)\right)^2 \, dx$$

$$= \frac{3\left(\pi + 2\arctan\left(\frac{1}{2}\left(e^{(2\,dx+2\,c)}-1\right)e^{(-dx-c)}\right)\right)\left(5\,a^2+2\,ab+b^2\right) + \frac{4\left(15\,a^2\left(e^{(dx+c)}-e^{(-dx-c)}\right)^5+6\,ab\left(e^{(dx+c)}-e^{(-dx-c}\right.\right.}{}}{}$$

input
```
integrate(sech(d*x+c)^7*(a+b*sinh(d*x+c)^2)^2,x, algorithm="giac")
```

output
```
1/96*(3*(pi + 2*arctan(1/2*(e^(2*d*x + 2*c) - 1)*e^(-d*x - c)))*(5*a^2 + 2
*a*b + b^2) + 4*(15*a^2*(e^(d*x + c) - e^(-d*x - c))^5 + 6*a*b*(e^(d*x + c
) - e^(-d*x - c))^5 + 3*b^2*(e^(d*x + c) - e^(-d*x - c))^5 + 160*a^2*(e^(d
*x + c) - e^(-d*x - c))^3 + 64*a*b*(e^(d*x + c) - e^(-d*x - c))^3 - 32*b^2
*(e^(d*x + c) - e^(-d*x - c))^3 + 528*a^2*(e^(d*x + c) - e^(-d*x - c)) - 9
6*a*b*(e^(d*x + c) - e^(-d*x - c)) - 48*b^2*(e^(d*x + c) - e^(-d*x - c)))/
((e^(d*x + c) - e^(-d*x - c))^2 + 4)^3)/d
```

Mupad [B] (verification not implemented)

Time = 1.96 (sec) , antiderivative size = 582, normalized size of antiderivative = 4.81

$$\int \operatorname{sech}^7(c+dx) \left(a+b\sinh^2(c+dx)\right)^2 \, dx$$

$$= \frac{\operatorname{atan}\left(\frac{e^{d\,x}\,e^{c}\,\left(5\,a^2\,\sqrt{d^2}+b^2\,\sqrt{d^2}+2\,a\,b\,\sqrt{d^2}\right)}{d\,\sqrt{25\,a^4+20\,a^3\,b+14\,a^2\,b^2+4\,a\,b^3+b^4}}\right)\,\sqrt{25\,a^4+20\,a^3\,b+14\,a^2\,b^2+4\,a\,b^3+b^4}}{8\,\sqrt{d^2}}$$
$$- \frac{\frac{2\,b^2\,e^{c+d\,x}}{3\,d}+\frac{2\,b^2\,e^{9\,c+9\,d\,x}}{3\,d}+\frac{4\,e^{5\,c+5\,d\,x}\,\left(8\,a^2-8\,a\,b+3\,b^2\right)}{3\,d}+\frac{8\,b\,e^{3\,c+3\,d\,x}\,\left(2\,a-b\right)}{3\,d}+\frac{8\,b\,e^{7\,c+7\,d\,x}\,\left(2\,a-b\right)}{3\,d}}{6\,e^{2\,c+2\,d\,x}+15\,e^{4\,c+4\,d\,x}+20\,e^{6\,c+6\,d\,x}+15\,e^{8\,c+8\,d\,x}+6\,e^{10\,c+10\,d\,x}+e^{12\,c+12\,d\,x}+1}$$
$$- \frac{2\,e^{c+d\,x}\,\left(11\,a^2-26\,a\,b+15\,b^2\right)}{3\,d\,\left(4\,e^{2\,c+2\,d\,x}+6\,e^{4\,c+4\,d\,x}+4\,e^{6\,c+6\,d\,x}+e^{8\,c+8\,d\,x}+1\right)}+\frac{e^{c+d\,x}\,\left(5\,a^2+2\,a\,b+b^2\right)}{8\,d\,\left(e^{2\,c+2\,d\,x}+1\right)}$$
$$+ \frac{16\,e^{c+d\,x}\,\left(a^2-2\,a\,b+b^2\right)}{3\,d\,\left(5\,e^{2\,c+2\,d\,x}+10\,e^{4\,c+4\,d\,x}+10\,e^{6\,c+6\,d\,x}+5\,e^{8\,c+8\,d\,x}+e^{10\,c+10\,d\,x}+1\right)}$$
$$+ \frac{e^{c+d\,x}\,\left(a^2-22\,a\,b+21\,b^2\right)}{3\,d\,\left(3\,e^{2\,c+2\,d\,x}+3\,e^{4\,c+4\,d\,x}+e^{6\,c+6\,d\,x}+1\right)}+\frac{e^{c+d\,x}\,\left(5\,a^2+2\,a\,b-23\,b^2\right)}{12\,d\,\left(2\,e^{2\,c+2\,d\,x}+e^{4\,c+4\,d\,x}+1\right)}$$

input
```
int((a + b*sinh(c + d*x)^2)^2/cosh(c + d*x)^7,x)
```

output
```
(atan((exp(d*x)*exp(c)*(5*a^2*(d^2)^(1/2) + b^2*(d^2)^(1/2) + 2*a*b*(d^2)^
(1/2)))/(d*(4*a*b^3 + 20*a^3*b + 25*a^4 + b^4 + 14*a^2*b^2)^(1/2)))*(4*a*b
^3 + 20*a^3*b + 25*a^4 + b^4 + 14*a^2*b^2)^(1/2))/(8*(d^2)^(1/2)) - ((2*b^
2*exp(c + d*x))/(3*d) + (2*b^2*exp(9*c + 9*d*x))/(3*d) + (4*exp(5*c + 5*d*
x)*(8*a^2 - 8*a*b + 3*b^2))/(3*d) + (8*b*exp(3*c + 3*d*x)*(2*a - b))/(3*d)
 + (8*b*exp(7*c + 7*d*x)*(2*a - b))/(3*d))/(6*exp(2*c + 2*d*x) + 15*exp(4*
c + 4*d*x) + 20*exp(6*c + 6*d*x) + 15*exp(8*c + 8*d*x) + 6*exp(10*c + 10*d
*x) + exp(12*c + 12*d*x) + 1) - (2*exp(c + d*x)*(11*a^2 - 26*a*b + 15*b^2)
)/(3*d*(4*exp(2*c + 2*d*x) + 6*exp(4*c + 4*d*x) + 4*exp(6*c + 6*d*x) + exp
(8*c + 8*d*x) + 1)) + (exp(c + d*x)*(2*a*b + 5*a^2 + b^2))/(8*d*(exp(2*c +
 2*d*x) + 1)) + (16*exp(c + d*x)*(a^2 - 2*a*b + b^2))/(3*d*(5*exp(2*c + 2*
d*x) + 10*exp(4*c + 4*d*x) + 10*exp(6*c + 6*d*x) + 5*exp(8*c + 8*d*x) + ex
p(10*c + 10*d*x) + 1)) + (exp(c + d*x)*(a^2 - 22*a*b + 21*b^2))/(3*d*(3*ex
p(2*c + 2*d*x) + 3*exp(4*c + 4*d*x) + exp(6*c + 6*d*x) + 1)) + (exp(c + d*
x)*(2*a*b + 5*a^2 - 23*b^2))/(12*d*(2*exp(2*c + 2*d*x) + exp(4*c + 4*d*x)
+ 1))
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 781, normalized size of antiderivative = 6.45

$$\int \operatorname{sech}^7(c+dx) \left(a+b\sinh^2(c+dx)\right)^2 dx$$
$$= \frac{90e^{8dx+8c}atan\left(e^{dx+c}\right)ab+120e^{6dx+6c}atan\left(e^{dx+c}\right)ab+90e^{4dx+4c}atan\left(e^{dx+c}\right)ab+36e^{2dx+2c}atan\left(e^{dx+c}\right)a}{}$$

input
```
int(sech(d*x+c)^7*(a+b*sinh(d*x+c)^2)^2,x)
```

output

```
(15*e**(12*c + 12*d*x)*atan(e**(c + d*x))*a**2 + 6*e**(12*c + 12*d*x)*atan
(e**(c + d*x))*a*b + 3*e**(12*c + 12*d*x)*atan(e**(c + d*x))*b**2 + 90*e**
(10*c + 10*d*x)*atan(e**(c + d*x))*a**2 + 36*e**(10*c + 10*d*x)*atan(e**(c
 + d*x))*a*b + 18*e**(10*c + 10*d*x)*atan(e**(c + d*x))*b**2 + 225*e**(8*c
 + 8*d*x)*atan(e**(c + d*x))*a**2 + 90*e**(8*c + 8*d*x)*atan(e**(c + d*x))
*a*b + 45*e**(8*c + 8*d*x)*atan(e**(c + d*x))*b**2 + 300*e**(6*c + 6*d*x)*
atan(e**(c + d*x))*a**2 + 120*e**(6*c + 6*d*x)*atan(e**(c + d*x))*a*b + 60
*e**(6*c + 6*d*x)*atan(e**(c + d*x))*b**2 + 225*e**(4*c + 4*d*x)*atan(e**(
c + d*x))*a**2 + 90*e**(4*c + 4*d*x)*atan(e**(c + d*x))*a*b + 45*e**(4*c +
 4*d*x)*atan(e**(c + d*x))*b**2 + 90*e**(2*c + 2*d*x)*atan(e**(c + d*x))*a
**2 + 36*e**(2*c + 2*d*x)*atan(e**(c + d*x))*a*b + 18*e**(2*c + 2*d*x)*ata
n(e**(c + d*x))*b**2 + 15*atan(e**(c + d*x))*a**2 + 6*atan(e**(c + d*x))*a
*b + 3*atan(e**(c + d*x))*b**2 + 15*e**(11*c + 11*d*x)*a**2 + 6*e**(11*c +
 11*d*x)*a*b + 3*e**(11*c + 11*d*x)*b**2 + 85*e**(9*c + 9*d*x)*a**2 + 34*e
**(9*c + 9*d*x)*a*b - 47*e**(9*c + 9*d*x)*b**2 + 198*e**(7*c + 7*d*x)*a**2
 - 228*e**(7*c + 7*d*x)*a*b + 78*e**(7*c + 7*d*x)*b**2 - 198*e**(5*c + 5*d
*x)*a**2 + 228*e**(5*c + 5*d*x)*a*b - 78*e**(5*c + 5*d*x)*b**2 - 85*e**(3*
c + 3*d*x)*a**2 - 34*e**(3*c + 3*d*x)*a*b + 47*e**(3*c + 3*d*x)*b**2 - 15*
e**(c + d*x)*a**2 - 6*e**(c + d*x)*a*b - 3*e**(c + d*x)*b**2)/(24*d*(e**(1
2*c + 12*d*x) + 6*e**(10*c + 10*d*x) + 15*e**(8*c + 8*d*x) + 20*e**(6*c...
```

3.268 $\int \cosh^4(c+dx) \left(a+b\sinh^2(c+dx)\right)^3 \, dx$

Optimal result

Integrand size = 23, antiderivative size = 198

$$\begin{aligned}
&\int \cosh^4(c+dx) \left(a+b\sinh^2(c+dx)\right)^3 \, dx \\
&= \frac{3}{256}(4a-b)\left(8a^2-2ab+b^2\right)x+\frac{3(4a-b)\left(8a^2-2ab+b^2\right)\cosh(c+dx)\sinh(c+dx)}{256d} \\
&+ \frac{(4a-b)\left(8a^2-2ab+b^2\right)\cosh^3(c+dx)\sinh(c+dx)}{128d} \\
&+ \frac{b(80a^2-90ab+31b^2)\cosh^5(c+dx)\sinh(c+dx)}{160d} \\
&+ \frac{3(10a-7b)b^2\cosh^7(c+dx)\sinh(c+dx)}{80d}+\frac{b^3\cosh^9(c+dx)\sinh(c+dx)}{10d}
\end{aligned}$$

output

```
3/256*(4*a-b)*(8*a^2-2*a*b+b^2)*x+3/256*(4*a-b)*(8*a^2-2*a*b+b^2)*cosh(d*x
+c)*sinh(d*x+c)/d+1/128*(4*a-b)*(8*a^2-2*a*b+b^2)*cosh(d*x+c)^3*sinh(d*x+c
)/d+1/160*b*(80*a^2-90*a*b+31*b^2)*cosh(d*x+c)^5*sinh(d*x+c)/d+3/80*(10*a-
7*b)*b^2*cosh(d*x+c)^7*sinh(d*x+c)/d+1/10*b^3*cosh(d*x+c)^9*sinh(d*x+c)/d
```

Mathematica [A] (verified)

Time = 2.53 (sec) , antiderivative size = 144, normalized size of antiderivative = 0.73

$$\int \cosh^4(c+dx)\left(a+b\sinh^2(c+dx)\right)^3\,dx$$
$$= \frac{120(4a-b)\left(8a^2-2ab+b^2\right)(c+dx)+20(128a^3-24a^2b+b^3)\sinh(2(c+dx))+40(8a^3+12a^2b-6ab}{}$$

input
```
Integrate[Cosh[c + d*x]^4*(a + b*Sinh[c + d*x]^2)^3,x]
```

output
```
(120*(4*a - b)*(8*a^2 - 2*a*b + b^2)*(c + d*x) + 20*(128*a^3 - 24*a^2*b +
b^3)*Sinh[2*(c + d*x)] + 40*(8*a^3 + 12*a^2*b - 6*a*b^2 + b^3)*Sinh[4*(c +
 d*x)] - 10*b*(-16*a^2 + b^2)*Sinh[6*(c + d*x)] + 5*(6*a - b)*b^2*Sinh[8*(
c + d*x)] + 2*b^3*Sinh[10*(c + d*x)])/(10240*d)
```

Rubi [A] (verified)

Time = 0.50 (sec) , antiderivative size = 237, normalized size of antiderivative = 1.20, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.391$, Rules used = {3042, 3670, 315, 25, 401, 298, 215, 215, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cosh^4(c+dx)\left(a+b\sinh^2(c+dx)\right)^3\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \cos(ic+idx)^4\left(a-b\sin(ic+idx)^2\right)^3dx$$

$$\Big\downarrow \text{3670}$$

$$\frac{\int \frac{\left(a-(a-b)\tanh^2(c+dx)\right)^3}{\left(1-\tanh^2(c+dx)\right)^6}d\tanh(c+dx)}{d}$$

$$\Big\downarrow \text{315}$$

$$\frac{\frac{b\tanh(c+dx)\left(a-(a-b)\tanh^2(c+dx)\right)^2}{10\left(1-\tanh^2(c+dx)\right)^5}-\frac{1}{10}\int -\frac{\left(a-(a-b)\tanh^2(c+dx)\right)\left(a(10a-b)-5(a-b)(2a-b)\tanh^2(c+dx)\right)}{\left(1-\tanh^2(c+dx)\right)^5}d\tanh(c+dx)}{d}$$

$\Big\downarrow$ 25

$$\frac{\frac{1}{10}\int \frac{\left(a-(a-b)\tanh^2(c+dx)\right)\left(a(10a-b)-5(a-b)(2a-b)\tanh^2(c+dx)\right)}{\left(1-\tanh^2(c+dx)\right)^5}d\tanh(c+dx)+\frac{b\tanh(c+dx)\left(a-(a-b)\tanh^2(c+dx)\right)^2}{10\left(1-\tanh^2(c+dx)\right)^5}}{d}$$

$\Big\downarrow$ 401

$$\frac{\frac{1}{10}\left(\frac{1}{8}\int \frac{a\left(80a^2-22ba+5b^2\right)-5(a-b)\left(16a^2-10ba+3b^2\right)\tanh^2(c+dx)}{\left(1-\tanh^2(c+dx)\right)^4}d\tanh(c+dx)+\frac{b(14a-5b)\tanh(c+dx)\left(a-(a-b)\tanh^2(c+dx)\right)}{8\left(1-\tanh^2(c+dx)\right)^4}\right)}{d}$$

$\Big\downarrow$ 298

$$\frac{\frac{1}{10}\left(\frac{1}{8}\left(\frac{5}{2}(4a-b)\left(8a^2-2ab+b^2\right)\int \frac{1}{\left(1-\tanh^2(c+dx)\right)^3}d\tanh(c+dx)+\frac{b\left(36a^2-20ab+5b^2\right)\tanh(c+dx)}{2\left(1-\tanh^2(c+dx)\right)^3}\right)+\frac{b(14a-5b)\tanh(\ldots}{8(\ldots}\right.}{d}$$

$\Big\downarrow$ 215

$$\frac{\frac{1}{10}\left(\frac{1}{8}\left(\frac{5}{2}(4a-b)\left(8a^2-2ab+b^2\right)\left(\frac{3}{4}\int \frac{1}{\left(1-\tanh^2(c+dx)\right)^2}d\tanh(c+dx)+\frac{\tanh(c+dx)}{4\left(1-\tanh^2(c+dx)\right)^2}\right)+\frac{b\left(36a^2-20ab+5b^2\right)\tan\ldots}{2\left(1-\tanh^2(c+d\ldots\right.}\right.\right.}{d}$$

$\Big\downarrow$ 215

$$\frac{\frac{1}{10}\left(\frac{1}{8}\left(\frac{5}{2}(4a-b)\left(8a^2-2ab+b^2\right)\left(\frac{3}{4}\left(\frac{1}{2}\int \frac{1}{1-\tanh^2(c+dx)}d\tanh(c+dx)+\frac{\tanh(c+dx)}{2\left(1-\tanh^2(c+dx)\right)}\right)+\frac{\tanh(c+dx)}{4\left(1-\tanh^2(c+dx)\right)^2}\right)\right.\right.}{d}$$

$\Big\downarrow$ 219

$$\frac{\frac{1}{10}\left(\frac{1}{8}\left(\frac{5}{2}(4a-b)\left(8a^2-2ab+b^2\right)\left(\frac{3}{4}\left(\frac{1}{2}\text{arctanh}(\tanh(c+dx))+\frac{\tanh(c+dx)}{2\left(1-\tanh^2(c+dx)\right)}\right)+\frac{\tanh(c+dx)}{4\left(1-\tanh^2(c+dx)\right)^2}\right)+\frac{b(36a\ldots}{2\ldots}\right.\right.}{d}$$

input
```
Int[Cosh[c + d*x]^4*(a + b*Sinh[c + d*x]^2)^3,x]
```

output
```
((b*Tanh[c + d*x]*(a - (a - b)*Tanh[c + d*x]^2)^2)/(10*(1 - Tanh[c + d*x]^
2)^5) + (((14*a - 5*b)*b*Tanh[c + d*x]*(a - (a - b)*Tanh[c + d*x]^2))/(8*(
1 - Tanh[c + d*x]^2)^4) + ((b*(36*a^2 - 20*a*b + 5*b^2)*Tanh[c + d*x])/(2*
(1 - Tanh[c + d*x]^2)^3) + (5*(4*a - b)*(8*a^2 - 2*a*b + b^2)*(Tanh[c + d*
x]/(4*(1 - Tanh[c + d*x]^2)^2) + (3*(ArcTanh[Tanh[c + d*x]]/2 + Tanh[c + d
*x]/(2*(1 - Tanh[c + d*x]^2))))/4))/2)/8)/10)/d
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 215
```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(-x)*((a + b*x^2)^(p + 1)
/(2*a*(p + 1))), x] + Simp[(2*p + 3)/(2*a*(p + 1))   Int[(a + b*x^2)^(p + 1
), x], x] /; FreeQ[{a, b}, x] && LtQ[p, -1] && (IntegerQ[4*p] || IntegerQ[6
*p])
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 298
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2), x_Symbol] :> Simp[(-(
b*c - a*d))*x*((a + b*x^2)^(p + 1)/(2*a*b*(p + 1))), x] - Simp[(a*d - b*c*(
2*p + 3))/(2*a*b*(p + 1))   Int[(a + b*x^2)^(p + 1), x], x] /; FreeQ[{a, b,
 c, d, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/2 + p, 0])
```

rule 315
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_), x_Symbol] :> Sim
p[(a*d - c*b)*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^(q - 1)/(2*a*b*(p + 1))),
x] - Simp[1/(2*a*b*(p + 1))   Int[(a + b*x^2)^(p + 1)*(c + d*x^2)^(q - 2)*S
imp[c*(a*d - c*b*(2*p + 3)) + d*(a*d*(2*(q - 1) + 1) - b*c*(2*(p + q) + 1))
*x^2, x], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -
1] && GtQ[q, 1] && IntBinomialQ[a, b, c, d, 2, p, q, x]
```

rule 401
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_.)*((e_) + (f_.)*(x
_)^2), x_Symbol] :> Simp[(-(b*e - a*f))*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^
q/(a*b*2*(p + 1))), x] + Simp[1/(a*b*2*(p + 1))   Int[(a + b*x^2)^(p + 1)*(
c + d*x^2)^(q - 1)*Simp[c*(b*e*2*(p + 1) + b*e - a*f) + d*(b*e*2*(p + 1) +
(b*e - a*f)*(2*q + 1))*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && L
tQ[p, -1] && GtQ[q, 0]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3670
```
Int[cos[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(
p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Simp[ff/f   Su
bst[Int[(a + (a + b)*ff^2*x^2)^p/(1 + ff^2*x^2)^(m/2 + p + 1), x], x, Tan[e
 + f*x]/ff], x]] /; FreeQ[{a, b, e, f}, x] && IntegerQ[m/2] && IntegerQ[p]
```

Maple [A] (verified)

Time = 0.15 (sec) , antiderivative size = 267, normalized size of antiderivative = 1.35

$$a^3\left(\left(\frac{\cosh(dx+c)^3}{4}+\frac{3\cosh(dx+c)}{8}\right)\sinh\left(dx+c\right)+\frac{3dx}{8}+\frac{3c}{8}\right)+3a^2b\left(\frac{\sinh(dx+c)\cosh(dx+c)^5}{6}-\frac{\left(\frac{\cosh(dx+c)^3}{4}+\frac{3\cosh(dx+c)}{8}\right.}{6}\right.$$

input
```
int(cosh(d*x+c)^4*(a+b*sinh(d*x+c)^2)^3,x)
```

output
```
1/d*(a^3*((1/4*cosh(d*x+c)^3+3/8*cosh(d*x+c))*sinh(d*x+c)+3/8*d*x+3/8*c)+3
*a^2*b*(1/6*sinh(d*x+c)*cosh(d*x+c)^5-1/6*(1/4*cosh(d*x+c)^3+3/8*cosh(d*x+
c))*sinh(d*x+c)-1/16*d*x-1/16*c)+3*b^2*a*(1/8*sinh(d*x+c)^3*cosh(d*x+c)^5-
1/16*sinh(d*x+c)*cosh(d*x+c)^5+1/16*(1/4*cosh(d*x+c)^3+3/8*cosh(d*x+c))*si
nh(d*x+c)+3/128*d*x+3/128*c)+b^3*(1/10*sinh(d*x+c)^5*cosh(d*x+c)^5-1/16*si
nh(d*x+c)^3*cosh(d*x+c)^5+1/32*sinh(d*x+c)*cosh(d*x+c)^5-1/32*(1/4*cosh(d*
x+c)^3+3/8*cosh(d*x+c))*sinh(d*x+c)-3/256*d*x-3/256*c))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 376 vs. $2(186) = 372$.

Time = 0.09 (sec) , antiderivative size = 376, normalized size of antiderivative = 1.90

$$\int \cosh^4(c+dx) \left(a+b\sinh^2(c+dx)\right)^3 dx$$
$$= \frac{5 b^3 \cosh(dx+c) \sinh(dx+c)^9 + 10 \left(6 b^3 \cosh(dx+c)^3 + (6 ab^2 - b^3) \cosh(dx+c)\right) \sinh(dx+c)^7 + (}{}$$

input
```
integrate(cosh(d*x+c)^4*(a+b*sinh(d*x+c)^2)^3,x, algorithm="fricas")
```

output
```
1/2560*(5*b^3*cosh(d*x + c)*sinh(d*x + c)^9 + 10*(6*b^3*cosh(d*x + c)^3 +
(6*a*b^2 - b^3)*cosh(d*x + c))*sinh(d*x + c)^7 + (126*b^3*cosh(d*x + c)^5
+ 70*(6*a*b^2 - b^3)*cosh(d*x + c)^3 + 15*(16*a^2*b - b^3)*cosh(d*x + c))*
sinh(d*x + c)^5 + 10*(6*b^3*cosh(d*x + c)^7 + 7*(6*a*b^2 - b^3)*cosh(d*x +
 c)^5 + 5*(16*a^2*b - b^3)*cosh(d*x + c)^3 + 4*(8*a^3 + 12*a^2*b - 6*a*b^2
 + b^3)*cosh(d*x + c))*sinh(d*x + c)^3 + 30*(32*a^3 - 16*a^2*b + 6*a*b^2 -
 b^3)*d*x + 5*(b^3*cosh(d*x + c)^9 + 2*(6*a*b^2 - b^3)*cosh(d*x + c)^7 + 3
*(16*a^2*b - b^3)*cosh(d*x + c)^5 + 8*(8*a^3 + 12*a^2*b - 6*a*b^2 + b^3)*c
osh(d*x + c)^3 + 2*(128*a^3 - 24*a^2*b + b^3)*cosh(d*x + c))*sinh(d*x + c)
)/d
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 774 vs. $2(189) = 378$.

Time = 1.34 (sec) , antiderivative size = 774, normalized size of antiderivative = 3.91

$$\int \cosh^4(c+dx) \left(a+b\sinh^2(c+dx)\right)^3 dx = \text{Too large to display}$$

input
```
integrate(cosh(d*x+c)**4*(a+b*sinh(d*x+c)**2)**3,x)
```

output
```
Piecewise((3*a**3*x*sinh(c + d*x)**4/8 - 3*a**3*x*sinh(c + d*x)**2*cosh(c
+ d*x)**2/4 + 3*a**3*x*cosh(c + d*x)**4/8 - 3*a**3*sinh(c + d*x)**3*cosh(c
 + d*x)/(8*d) + 5*a**3*sinh(c + d*x)*cosh(c + d*x)**3/(8*d) + 3*a**2*b*x*s
inh(c + d*x)**6/16 - 9*a**2*b*x*sinh(c + d*x)**4*cosh(c + d*x)**2/16 + 9*a
**2*b*x*sinh(c + d*x)**2*cosh(c + d*x)**4/16 - 3*a**2*b*x*cosh(c + d*x)**6
/16 - 3*a**2*b*sinh(c + d*x)**5*cosh(c + d*x)/(16*d) + a**2*b*sinh(c + d*x
)**3*cosh(c + d*x)**3/(2*d) + 3*a**2*b*sinh(c + d*x)*cosh(c + d*x)**5/(16*
d) + 9*a*b**2*x*sinh(c + d*x)**8/128 - 9*a*b**2*x*sinh(c + d*x)**6*cosh(c
+ d*x)**2/32 + 27*a*b**2*x*sinh(c + d*x)**4*cosh(c + d*x)**4/64 - 9*a*b**2
*x*sinh(c + d*x)**2*cosh(c + d*x)**6/32 + 9*a*b**2*x*cosh(c + d*x)**8/128
- 9*a*b**2*sinh(c + d*x)**7*cosh(c + d*x)/(128*d) + 33*a*b**2*sinh(c + d*x
)**5*cosh(c + d*x)**3/(128*d) + 33*a*b**2*sinh(c + d*x)**3*cosh(c + d*x)**
5/(128*d) - 9*a*b**2*sinh(c + d*x)*cosh(c + d*x)**7/(128*d) + 3*b**3*x*sin
h(c + d*x)**10/256 - 15*b**3*x*sinh(c + d*x)**8*cosh(c + d*x)**2/256 + 15*
b**3*x*sinh(c + d*x)**6*cosh(c + d*x)**4/128 - 15*b**3*x*sinh(c + d*x)**4*
cosh(c + d*x)**6/128 + 15*b**3*x*sinh(c + d*x)**2*cosh(c + d*x)**8/256 - 3
*b**3*x*cosh(c + d*x)**10/256 - 3*b**3*sinh(c + d*x)**9*cosh(c + d*x)/(256
*d) + 7*b**3*sinh(c + d*x)**7*cosh(c + d*x)**3/(128*d) + b**3*sinh(c + d*x
)**5*cosh(c + d*x)**5/(10*d) - 7*b**3*sinh(c + d*x)**3*cosh(c + d*x)**7/(1
28*d) + 3*b**3*sinh(c + d*x)*cosh(c + d*x)**9/(256*d), Ne(d, 0)), (x*(a...
```

Maxima [A] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 363, normalized size of antiderivative = 1.83

$$\int \cosh^4(c+dx)\left(a+b\sinh^2(c+dx)\right)^3\,dx$$
$$= \frac{1}{64}a^3\left(24x+\frac{e^{(4dx+4c)}}{d}+\frac{8e^{(2dx+2c)}}{d}-\frac{8e^{(-2dx-2c)}}{d}-\frac{e^{(-4dx-4c)}}{d}\right)$$
$$-\frac{1}{20480}b^3\left(\frac{\left(5e^{(-2dx-2c)}+10e^{(-4dx-4c)}-40e^{(-6dx-6c)}-20e^{(-8dx-8c)}-2\right)e^{(10dx+10c)}}{d}+\frac{240(dx+c)}{d}\right.$$
$$-\frac{3}{2048}ab^2\left(\frac{\left(8e^{(-4dx-4c)}-1\right)e^{(8dx+8c)}}{d}-\frac{48(dx+c)}{d}-\frac{8e^{(-4dx-4c)}-e^{(-8dx-8c)}}{d}\right)$$
$$+\frac{1}{128}a^2b\left(\frac{\left(3e^{(-2dx-2c)}-3e^{(-4dx-4c)}+1\right)e^{(6dx+6c)}}{d}-\frac{24(dx+c)}{d}+\frac{3e^{(-2dx-2c)}-3e^{(-4dx-4c)}-e^{(-6}}{d}\right.$$

input
```
integrate(cosh(d*x+c)^4*(a+b*sinh(d*x+c)^2)^3,x, algorithm="maxima")
```

output
```
1/64*a^3*(24*x + e^(4*d*x + 4*c)/d + 8*e^(2*d*x + 2*c)/d - 8*e^(-2*d*x - 2
*c)/d - e^(-4*d*x - 4*c)/d) - 1/20480*b^3*((5*e^(-2*d*x - 2*c) + 10*e^(-4*
d*x - 4*c) - 40*e^(-6*d*x - 6*c) - 20*e^(-8*d*x - 8*c) - 2)*e^(10*d*x + 10
*c)/d + 240*(d*x + c)/d + (20*e^(-2*d*x - 2*c) + 40*e^(-4*d*x - 4*c) - 10*
e^(-6*d*x - 6*c) - 5*e^(-8*d*x - 8*c) + 2*e^(-10*d*x - 10*c))/d) - 3/2048*
a*b^2*((8*e^(-4*d*x - 4*c) - 1)*e^(8*d*x + 8*c)/d - 48*(d*x + c)/d - (8*e^
(-4*d*x - 4*c) - e^(-8*d*x - 8*c))/d) + 1/128*a^2*b*((3*e^(-2*d*x - 2*c) -
 3*e^(-4*d*x - 4*c) + 1)*e^(6*d*x + 6*c)/d - 24*(d*x + c)/d + (3*e^(-2*d*x
 - 2*c) - 3*e^(-4*d*x - 4*c) - e^(-6*d*x - 6*c))/d)
```

Giac [A] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 293, normalized size of antiderivative = 1.48

$$\begin{aligned}\int \cosh^4(c+dx)\left(a+b\sinh^2(c+dx)\right)^3\,dx &= \frac{b^3e^{(10\,dx+10\,c)}}{10240\,d}-\frac{b^3e^{(-10\,dx-10\,c)}}{10240\,d}+\frac{3}{256}\left(32\,a^3-16\,a^2b+6\,ab^2-b^3\right)x\\&\quad+\frac{(6\,ab^2-b^3)e^{(8\,dx+8\,c)}}{4096\,d}+\frac{(16\,a^2b-b^3)e^{(6\,dx+6\,c)}}{2048\,d}\\&\quad+\frac{(8\,a^3+12\,a^2b-6\,ab^2+b^3)e^{(4\,dx+4\,c)}}{512\,d}+\frac{(128\,a^3-24\,a^2b+b^3)e^{(2\,dx+2\,c)}}{1024\,d}\\&\quad-\frac{(128\,a^3-24\,a^2b+b^3)e^{(-2\,dx-2\,c)}}{1024\,d}-\frac{(8\,a^3+12\,a^2b-6\,ab^2+b^3)e^{(-4\,dx-4\,c)}}{512\,d}\\&\quad-\frac{(16\,a^2b-b^3)e^{(-6\,dx-6\,c)}}{2048\,d}-\frac{(6\,ab^2-b^3)e^{(-8\,dx-8\,c)}}{4096\,d}\end{aligned}$$

input
```
integrate(cosh(d*x+c)^4*(a+b*sinh(d*x+c)^2)^3,x, algorithm="giac")
```

output
```
1/10240*b^3*e^(10*d*x + 10*c)/d - 1/10240*b^3*e^(-10*d*x - 10*c)/d + 3/256
*(32*a^3 - 16*a^2*b + 6*a*b^2 - b^3)*x + 1/4096*(6*a*b^2 - b^3)*e^(8*d*x +
 8*c)/d + 1/2048*(16*a^2*b - b^3)*e^(6*d*x + 6*c)/d + 1/512*(8*a^3 + 12*a^
2*b - 6*a*b^2 + b^3)*e^(4*d*x + 4*c)/d + 1/1024*(128*a^3 - 24*a^2*b + b^3)
*e^(2*d*x + 2*c)/d - 1/1024*(128*a^3 - 24*a^2*b + b^3)*e^(-2*d*x - 2*c)/d
- 1/512*(8*a^3 + 12*a^2*b - 6*a*b^2 + b^3)*e^(-4*d*x - 4*c)/d - 1/2048*(16
*a^2*b - b^3)*e^(-6*d*x - 6*c)/d - 1/4096*(6*a*b^2 - b^3)*e^(-8*d*x - 8*c)
/d
```

Mupad [B] (verification not implemented)

Time = 0.60 (sec) , antiderivative size = 209, normalized size of antiderivative = 1.06

$$\int \cosh^4(c+dx)\left(a+b\sinh^2(c+dx)\right)^3\,dx$$
$$= \frac{320\,a^3\sinh(2\,c+2\,d\,x)+40\,a^3\sinh(4\,c+4\,d\,x)+\frac{5\,b^3\sinh(2\,c+2\,d\,x)}{2}+5\,b^3\sinh(4\,c+4\,d\,x)-\frac{5\,b^3\sinh(6\,c+6\,d\,x)}{4}}{}$$

input
```
int(cosh(c + d*x)^4*(a + b*sinh(c + d*x)^2)^3,x)
```

output
```
(320*a^3*sinh(2*c + 2*d*x) + 40*a^3*sinh(4*c + 4*d*x) + (5*b^3*sinh(2*c +
2*d*x))/2 + 5*b^3*sinh(4*c + 4*d*x) - (5*b^3*sinh(6*c + 6*d*x))/4 - (5*b^3
*sinh(8*c + 8*d*x))/8 + (b^3*sinh(10*c + 10*d*x))/4 - 60*a^2*b*sinh(2*c +
2*d*x) - 30*a*b^2*sinh(4*c + 4*d*x) + 60*a^2*b*sinh(4*c + 4*d*x) + 20*a^2*
b*sinh(6*c + 6*d*x) + (15*a*b^2*sinh(8*c + 8*d*x))/4 + 480*a^3*d*x - 15*b^
3*d*x + 90*a*b^2*d*x - 240*a^2*b*d*x)/(1280*d)
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 448, normalized size of antiderivative = 2.26

$$\int \cosh^4(c+dx)\left(a+b\sinh^2(c+dx)\right)^3\,dx$$
$$= \frac{-2b^3-3840e^{10dx+10c}a^2bdx+1440e^{10dx+10c}a\,b^2dx+5e^{2dx+2c}b^3+7680e^{10dx+10c}a^3dx-240e^{10dx+10c}b^3dx}{}$$

input
```
int(cosh(d*x+c)^4*(a+b*sinh(d*x+c)^2)^3,x)
```

output

```
(2*e**(20*c + 20*d*x)*b**3 + 30*e**(18*c + 18*d*x)*a*b**2 - 5*e**(18*c + 1
8*d*x)*b**3 + 160*e**(16*c + 16*d*x)*a**2*b - 10*e**(16*c + 16*d*x)*b**3 +
 320*e**(14*c + 14*d*x)*a**3 + 480*e**(14*c + 14*d*x)*a**2*b - 240*e**(14*
c + 14*d*x)*a*b**2 + 40*e**(14*c + 14*d*x)*b**3 + 2560*e**(12*c + 12*d*x)*
a**3 - 480*e**(12*c + 12*d*x)*a**2*b + 20*e**(12*c + 12*d*x)*b**3 + 7680*e
**(10*c + 10*d*x)*a**3*d*x - 3840*e**(10*c + 10*d*x)*a**2*b*d*x + 1440*e**
(10*c + 10*d*x)*a*b**2*d*x - 240*e**(10*c + 10*d*x)*b**3*d*x - 2560*e**(8*
c + 8*d*x)*a**3 + 480*e**(8*c + 8*d*x)*a**2*b - 20*e**(8*c + 8*d*x)*b**3 -
 320*e**(6*c + 6*d*x)*a**3 - 480*e**(6*c + 6*d*x)*a**2*b + 240*e**(6*c + 6
*d*x)*a*b**2 - 40*e**(6*c + 6*d*x)*b**3 - 160*e**(4*c + 4*d*x)*a**2*b + 10
*e**(4*c + 4*d*x)*b**3 - 30*e**(2*c + 2*d*x)*a*b**2 + 5*e**(2*c + 2*d*x)*b
**3 - 2*b**3)/(20480*e**(10*c + 10*d*x)*d)
```

3.269 $\int \cosh^3(c+dx) \left(a+b\sinh^2(c+dx)\right)^3 dx$

Optimal result

Integrand size = 23, antiderivative size = 98

$$\int \cosh^3(c+dx) \left(a+b\sinh^2(c+dx)\right)^3 dx = \frac{a^3\sinh(c+dx)}{d} + \frac{a^2(a+3b)\sinh^3(c+dx)}{3d} + \frac{3ab(a+b)\sinh^5(c+dx)}{5d} + \frac{b^2(3a+b)\sinh^7(c+dx)}{7d} + \frac{b^3\sinh^9(c+dx)}{9d}$$

output
```
a^3*sinh(d*x+c)/d+1/3*a^2*(a+3*b)*sinh(d*x+c)^3/d+3/5*a*b*(a+b)*sinh(d*x+c
)^5/d+1/7*b^2*(3*a+b)*sinh(d*x+c)^7/d+1/9*b^3*sinh(d*x+c)^9/d
```

Mathematica [A] (verified)

Time = 0.18 (sec) , antiderivative size = 83, normalized size of antiderivative = 0.85

$$\int \cosh^3(c+dx) \left(a+b\sinh^2(c+dx)\right)^3 dx = \frac{315a^3\sinh(c+dx) + 105a^2(a+3b)\sinh^3(c+dx) + 189ab(a+b)\sinh^5(c+dx) + 45b^2(3a+b)\sinh^7(c+}{315d}$$

input
```
Integrate[Cosh[c + d*x]^3*(a + b*Sinh[c + d*x]^2)^3,x]
```

output
```
(315*a^3*Sinh[c + d*x] + 105*a^2*(a + 3*b)*Sinh[c + d*x]^3 + 189*a*b*(a +
b)*Sinh[c + d*x]^5 + 45*b^2*(3*a + b)*Sinh[c + d*x]^7 + 35*b^3*Sinh[c + d*
x]^9)/(315*d)
```

Rubi [A] (verified)

Time = 0.31 (sec) , antiderivative size = 87, normalized size of antiderivative = 0.89, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.174$, Rules used = {3042, 3669, 290, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cosh^3(c+dx)\left(a+b\sinh^2(c+dx)\right)^3\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \cos(ic+idx)^3\left(a-b\sin(ic+idx)^2\right)^3dx$$

$$\Big\downarrow \text{3669}$$

$$\frac{\int \left(\sinh^2(c+dx)+1\right)\left(b\sinh^2(c+dx)+a\right)^3d\sinh(c+dx)}{d}$$

$$\Big\downarrow \text{290}$$

$$\frac{\int \left(b^3\sinh^8(c+dx)+b^2(3a+b)\sinh^6(c+dx)+3ab(a+b)\sinh^4(c+dx)+a^2(a+3b)\sinh^2(c+dx)+a^3\right)d\sin}{d}$$

$$\Big\downarrow \text{2009}$$

$$\frac{a^3\sinh(c+dx)+\frac{1}{3}a^2(a+3b)\sinh^3(c+dx)+\frac{1}{7}b^2(3a+b)\sinh^7(c+dx)+\frac{3}{5}ab(a+b)\sinh^5(c+dx)+\frac{1}{9}b^3\sinh^9}{d}$$

input
```
Int[Cosh[c + d*x]^3*(a + b*Sinh[c + d*x]^2)^3,x]
```

output
```
(a^3*Sinh[c + d*x] + (a^2*(a + 3*b)*Sinh[c + d*x]^3)/3 + (3*a*b*(a + b)*Si
nh[c + d*x]^5)/5 + (b^2*(3*a + b)*Sinh[c + d*x]^7)/7 + (b^3*Sinh[c + d*x]^
9)/9)/d
```

Defintions of rubi rules used

rule 290
```
Int[((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.), x_Symbol] :> I
nt[ExpandIntegrand[(a + b*x^2)^p*(c + d*x^2)^q, x], x] /; FreeQ[{a, b, c, d
}, x] && NeQ[b*c - a*d, 0] && IGtQ[p, 0] && IGtQ[q, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3669
```
Int[cos[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^
(p_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Simp[ff/f   S
ubst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a + b*ff^2*x^2)^p, x], x, Sin[e + f*x]
/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m - 1)/2]
```

Maple [A] (verified)

Time = 0.12 (sec) , antiderivative size = 92, normalized size of antiderivative = 0.94

$$\frac{\frac{b^3 \sinh(dx+c)^9}{9} + \frac{(3b^2a+b^3) \sinh(dx+c)^7}{7} + \frac{(3a^2b+3b^2a) \sinh(dx+c)^5}{5} + \frac{(a^3+3a^2b) \sinh(dx+c)^3}{3} + a^3 \sinh(dx+c)}{d}$$

input
```
int(cosh(d*x+c)^3*(a+b*sinh(d*x+c)^2)^3,x)
```

output
```
1/d*(1/9*b^3*sinh(d*x+c)^9+1/7*(3*a*b^2+b^3)*sinh(d*x+c)^7+1/5*(3*a^2*b+3*
a*b^2)*sinh(d*x+c)^5+1/3*(a^3+3*a^2*b)*sinh(d*x+c)^3+a^3*sinh(d*x+c))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 324 vs. $2(90) = 180$.

Time = 0.10 (sec) , antiderivative size = 324, normalized size of antiderivative = 3.31

$$\int \cosh^3(c+dx)\left(a+b\sinh^2(c+dx)\right)^3\,dx$$
$$= \frac{35\,b^3\sinh\left(dx+c\right)^9 + 45\left(28\,b^3\cosh\left(dx+c\right)^2 + 12\,ab^2 - 3\,b^3\right)\sinh\left(dx+c\right)^7 + 63\left(70\,b^3\cosh\left(dx+c\right)^4 + \right.}{}$$

input
```
integrate(cosh(d*x+c)^3*(a+b*sinh(d*x+c)^2)^3,x, algorithm="fricas")
```

output
```
1/80640*(35*b^3*sinh(d*x + c)^9 + 45*(28*b^3*cosh(d*x + c)^2 + 12*a*b^2 -
3*b^3)*sinh(d*x + c)^7 + 63*(70*b^3*cosh(d*x + c)^4 + 48*a^2*b - 12*a*b^2
+ 45*(4*a*b^2 - b^3)*cosh(d*x + c)^2)*sinh(d*x + c)^5 + 105*(28*b^3*cosh(d
*x + c)^6 + 45*(4*a*b^2 - b^3)*cosh(d*x + c)^4 + 64*a^3 + 48*a^2*b - 36*a*
b^2 + 8*b^3 + 72*(4*a^2*b - a*b^2)*cosh(d*x + c)^2)*sinh(d*x + c)^3 + 315*
(b^3*cosh(d*x + c)^8 + 3*(4*a*b^2 - b^3)*cosh(d*x + c)^6 + 12*(4*a^2*b - a
*b^2)*cosh(d*x + c)^4 + 192*a^3 - 96*a^2*b + 36*a*b^2 - 6*b^3 + 4*(16*a^3
+ 12*a^2*b - 9*a*b^2 + 2*b^3)*cosh(d*x + c)^2)*sinh(d*x + c))/d
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 182 vs. $2(87) = 174$.

Time = 0.92 (sec) , antiderivative size = 182, normalized size of antiderivative = 1.86

$$\int \cosh^3(c+dx)\left(a+b\sinh^2(c+dx)\right)^3\,dx$$
$$= \begin{cases} -\frac{2a^3\sinh^3(c+dx)}{3d} + \frac{a^3\sinh(c+dx)\cosh^2(c+dx)}{d} - \frac{2a^2b\sinh^5(c+dx)}{5d} + \frac{a^2b\sinh^3(c+dx)\cosh^2(c+dx)}{d} - \frac{6ab^2\sinh^7(c+dx)}{35d} + \frac{3a}{} \\ x\left(a+b\sinh^2(c)\right)^3\cosh^3(c) \end{cases}$$

input
```
integrate(cosh(d*x+c)**3*(a+b*sinh(d*x+c)**2)**3,x)
```

output
```
Piecewise((-2*a**3*sinh(c + d*x)**3/(3*d) + a**3*sinh(c + d*x)*cosh(c + d*
x)**2/d - 2*a**2*b*sinh(c + d*x)**5/(5*d) + a**2*b*sinh(c + d*x)**3*cosh(c
 + d*x)**2/d - 6*a*b**2*sinh(c + d*x)**7/(35*d) + 3*a*b**2*sinh(c + d*x)**
5*cosh(c + d*x)**2/(5*d) - 2*b**3*sinh(c + d*x)**9/(63*d) + b**3*sinh(c +
d*x)**7*cosh(c + d*x)**2/(7*d), Ne(d, 0)), (x*(a + b*sinh(c)**2)**3*cosh(c
)**3, True))
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 349 vs. $2(90) = 180$.

Time = 0.05 (sec) , antiderivative size = 349, normalized size of antiderivative = 3.56

$$\int \cosh^3(c+dx)\left(a+b\sinh^2(c+dx)\right)^3\,dx =$$

$$-\frac{1}{32256}b^3\left(\frac{\left(27\,e^{(-2\,dx-2\,c)}-168\,e^{(-6\,dx-6\,c)}+378\,e^{(-8\,dx-8\,c)}-7\right)e^{(9\,dx+9\,c)}}{d}-\frac{378\,e^{(-dx-c)}-168\,e^{(-3\,dx-}}{}\right.$$

$$-\frac{3}{4480}ab^2\left(\frac{\left(7\,e^{(-2\,dx-2\,c)}+35\,e^{(-4\,dx-4\,c)}-105\,e^{(-6\,dx-6\,c)}-5\right)e^{(7\,dx+7\,c)}}{d}+\frac{105\,e^{(-dx-c)}-35\,e^{(-3\,dx-3\,c)}}{}\right.$$

$$+\frac{1}{160}a^2b\left(\frac{\left(5\,e^{(-2\,dx-2\,c)}-30\,e^{(-4\,dx-4\,c)}+3\right)e^{(5\,dx+5\,c)}}{d}+\frac{30\,e^{(-dx-c)}-5\,e^{(-3\,dx-3\,c)}-3\,e^{(-5\,dx-5\,c)}}{d}\right)$$

$$+\frac{1}{24}a^3\left(\frac{e^{(3\,dx+3\,c)}}{d}+\frac{9\,e^{(dx+c)}}{d}-\frac{9\,e^{(-dx-c)}}{d}-\frac{e^{(-3\,dx-3\,c)}}{d}\right)$$

input
```
integrate(cosh(d*x+c)^3*(a+b*sinh(d*x+c)^2)^3,x, algorithm="maxima")
```

output
```
-1/32256*b^3*((27*e^(-2*d*x - 2*c) - 168*e^(-6*d*x - 6*c) + 378*e^(-8*d*x
- 8*c) - 7)*e^(9*d*x + 9*c)/d - (378*e^(-d*x - c) - 168*e^(-3*d*x - 3*c) +
 27*e^(-7*d*x - 7*c) - 7*e^(-9*d*x - 9*c))/d) - 3/4480*a*b^2*((7*e^(-2*d*x
 - 2*c) + 35*e^(-4*d*x - 4*c) - 105*e^(-6*d*x - 6*c) - 5)*e^(7*d*x + 7*c)/
d + (105*e^(-d*x - c) - 35*e^(-3*d*x - 3*c) - 7*e^(-5*d*x - 5*c) + 5*e^(-7
*d*x - 7*c))/d) + 1/160*a^2*b*((5*e^(-2*d*x - 2*c) - 30*e^(-4*d*x - 4*c) +
 3)*e^(5*d*x + 5*c)/d + (30*e^(-d*x - c) - 5*e^(-3*d*x - 3*c) - 3*e^(-5*d*
x - 5*c))/d) + 1/24*a^3*(e^(3*d*x + 3*c)/d + 9*e^(d*x + c)/d - 9*e^(-d*x -
 c)/d - e^(-3*d*x - 3*c)/d)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 286 vs. $2(90) = 180$.

Time = 0.16 (sec) , antiderivative size = 286, normalized size of antiderivative = 2.92

$$\int \cosh^3(c+dx)\left(a+b\sinh^2(c+dx)\right)^3\,dx = \frac{b^3e^{(9\,dx+9\,c)}}{4608\,d} - \frac{b^3e^{(-9\,dx-9\,c)}}{4608\,d} + \frac{3\,(4\,ab^2-b^3)e^{(7\,dx+7\,c)}}{3584\,d} + \frac{3\,(4\,a^2b-ab^2)e^{(5\,dx+5\,c)}}{640\,d} + \frac{(16\,a^3+12\,a^2b-9\,ab^2+2\,b^3)e^{(3\,dx+3\,c)}}{384\,d} + \frac{3\,(32\,a^3-16\,a^2b+6\,ab^2-b^3)e^{(dx+c)}}{256\,d} - \frac{3\,(32\,a^3-16\,a^2b+6\,ab^2-b^3)e^{(-dx-c)}}{256\,d} - \frac{(16\,a^3+12\,a^2b-9\,ab^2+2\,b^3)e^{(-3\,dx-3\,c)}}{384\,d} - \frac{3\,(4\,a^2b-ab^2)e^{(-5\,dx-5\,c)}}{640\,d} - \frac{3\,(4\,ab^2-b^3)e^{(-7\,dx-7\,c)}}{3584\,d}$$

input
```
integrate(cosh(d*x+c)^3*(a+b*sinh(d*x+c)^2)^3,x, algorithm="giac")
```

output
```
1/4608*b^3*e^(9*d*x + 9*c)/d - 1/4608*b^3*e^(-9*d*x - 9*c)/d + 3/3584*(4*a
*b^2 - b^3)*e^(7*d*x + 7*c)/d + 3/640*(4*a^2*b - a*b^2)*e^(5*d*x + 5*c)/d
+ 1/384*(16*a^3 + 12*a^2*b - 9*a*b^2 + 2*b^3)*e^(3*d*x + 3*c)/d + 3/256*(3
2*a^3 - 16*a^2*b + 6*a*b^2 - b^3)*e^(d*x + c)/d - 3/256*(32*a^3 - 16*a^2*b
 + 6*a*b^2 - b^3)*e^(-d*x - c)/d - 1/384*(16*a^3 + 12*a^2*b - 9*a*b^2 + 2*
b^3)*e^(-3*d*x - 3*c)/d - 3/640*(4*a^2*b - a*b^2)*e^(-5*d*x - 5*c)/d - 3/3
584*(4*a*b^2 - b^3)*e^(-7*d*x - 7*c)/d
```

Mupad [B] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 112, normalized size of antiderivative = 1.14

$$\int \cosh^3(c+dx)\left(a+b\sinh^2(c+dx)\right)^3\,dx = \frac{105\,a^3\sinh(c+d\,x)^3+315\,a^3\sinh(c+d\,x)+189\,a^2\,b\sinh(c+d\,x)^5+315\,a^2\,b\sinh(c+d\,x)^3+135\,a\,b^2\,\mathrm{s}}{315\,d}$$

input
```
int(cosh(c + d*x)^3*(a + b*sinh(c + d*x)^2)^3,x)
```

output
```
(315*a^3*sinh(c + d*x) + 105*a^3*sinh(c + d*x)^3 + 45*b^3*sinh(c + d*x)^7
+ 35*b^3*sinh(c + d*x)^9 + 315*a^2*b*sinh(c + d*x)^3 + 189*a*b^2*sinh(c +
d*x)^5 + 189*a^2*b*sinh(c + d*x)^5 + 135*a*b^2*sinh(c + d*x)^7)/(315*d)
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 412, normalized size of antiderivative = 4.20

$$\int \cosh^3(c+dx)\left(a+b\sinh^2(c+dx)\right)^3\,dx$$
$$= \frac{-35b^3 + 135e^{2dx+2c}b^3 + 60480e^{10dx+10c}a^3 - 1890e^{10dx+10c}b^3 + 540e^{16dx+16c}a\,b^2 + 3024e^{14dx+14c}a^2b - 756}{}$$

input
```
int(cosh(d*x+c)^3*(a+b*sinh(d*x+c)^2)^3,x)
```

output
```
(35*e**(18*c + 18*d*x)*b**3 + 540*e**(16*c + 16*d*x)*a*b**2 - 135*e**(16*c
 + 16*d*x)*b**3 + 3024*e**(14*c + 14*d*x)*a**2*b - 756*e**(14*c + 14*d*x)*
a*b**2 + 6720*e**(12*c + 12*d*x)*a**3 + 5040*e**(12*c + 12*d*x)*a**2*b - 3
780*e**(12*c + 12*d*x)*a*b**2 + 840*e**(12*c + 12*d*x)*b**3 + 60480*e**(10
*c + 10*d*x)*a**3 - 30240*e**(10*c + 10*d*x)*a**2*b + 11340*e**(10*c + 10*
d*x)*a*b**2 - 1890*e**(10*c + 10*d*x)*b**3 - 60480*e**(8*c + 8*d*x)*a**3 +
 30240*e**(8*c + 8*d*x)*a**2*b - 11340*e**(8*c + 8*d*x)*a*b**2 + 1890*e**(
8*c + 8*d*x)*b**3 - 6720*e**(6*c + 6*d*x)*a**3 - 5040*e**(6*c + 6*d*x)*a**
2*b + 3780*e**(6*c + 6*d*x)*a*b**2 - 840*e**(6*c + 6*d*x)*b**3 - 3024*e**(
4*c + 4*d*x)*a**2*b + 756*e**(4*c + 4*d*x)*a*b**2 - 540*e**(2*c + 2*d*x)*a
*b**2 + 135*e**(2*c + 2*d*x)*b**3 - 35*b**3)/(161280*e**(9*c + 9*d*x)*d)
```

3.270 $\int \cosh^2(c+dx) \left(a+b\sinh^2(c+dx)\right)^3 \, dx$

Optimal result

Integrand size = 23, antiderivative size = 163

$$\begin{aligned}
&\int \cosh^2(c+dx) \left(a+b\sinh^2(c+dx)\right)^3 \, dx \\
&= \frac{1}{128}\left(64a^3-48a^2b+24ab^2-5b^3\right) x \\
&\quad + \frac{(64a^3-48a^2b+24ab^2-5b^3)\cosh(c+dx)\sinh(c+dx)}{128d} \\
&\quad + \frac{b(144a^2-168ab+59b^2)\cosh^3(c+dx)\sinh(c+dx)}{192d} \\
&\quad + \frac{(24a-17b)b^2\cosh^5(c+dx)\sinh(c+dx)}{48d} + \frac{b^3\cosh^7(c+dx)\sinh(c+dx)}{8d}
\end{aligned}$$

output
```
1/128*(64*a^3-48*a^2*b+24*a*b^2-5*b^3)*x+1/128*(64*a^3-48*a^2*b+24*a*b^2-5
*b^3)*cosh(d*x+c)*sinh(d*x+c)/d+1/192*b*(144*a^2-168*a*b+59*b^2)*cosh(d*x+
c)^3*sinh(d*x+c)/d+1/48*(24*a-17*b)*b^2*cosh(d*x+c)^5*sinh(d*x+c)/d+1/8*b^
3*cosh(d*x+c)^7*sinh(d*x+c)/d
```

Mathematica [A] (verified)

Time = 2.22 (sec) , antiderivative size = 120, normalized size of antiderivative = 0.74

$$\int \cosh^2(c+dx)\left(a+b\sinh^2(c+dx)\right)^3\,dx = \frac{24(64a^3-48a^2b+24ab^2-5b^3)(c+dx)+48(16a^3-3ab^2+b^3)\sinh(2(c+dx))+24b(12a^2-6ab+b^2)\sinh(4(c+dx))+16(3a-b)b^2\sinh(6(c+dx))+3b^3\sinh(8(c+dx))}{3072d}$$

input
```
Integrate[Cosh[c + d*x]^2*(a + b*Sinh[c + d*x]^2)^3,x]
```

output
```
(24*(64*a^3 - 48*a^2*b + 24*a*b^2 - 5*b^3)*(c + d*x) + 48*(16*a^3 - 3*a*b^
2 + b^3)*Sinh[2*(c + d*x)] + 24*b*(12*a^2 - 6*a*b + b^2)*Sinh[4*(c + d*x)]
 + 16*(3*a - b)*b^2*Sinh[6*(c + d*x)] + 3*b^3*Sinh[8*(c + d*x)])/(3072*d)
```

Rubi [A] (verified)

Time = 0.47 (sec) , antiderivative size = 211, normalized size of antiderivative = 1.29, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.348$, Rules used = {3042, 3670, 315, 25, 401, 298, 215, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cosh^2(c+dx)\left(a+b\sinh^2(c+dx)\right)^3\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \cos(ic+idx)^2\left(a-b\sin(ic+idx)^2\right)^3dx$$

$$\Big\downarrow \text{3670}$$

$$\frac{\int \frac{\left(a-(a-b)\tanh^2(c+dx)\right)^3}{\left(1-\tanh^2(c+dx)\right)^5}d\tanh(c+dx)}{d}$$

$$\Big\downarrow \text{315}$$

$$\frac{\frac{b\tanh(c+dx)\left(a-(a-b)\tanh^2(c+dx)\right)^2}{8\left(1-\tanh^2(c+dx)\right)^4}-\frac{1}{8}\int -\frac{\left(a-(a-b)\tanh^2(c+dx)\right)\left(a(8a-b)-(8a-5b)(a-b)\tanh^2(c+dx)\right)}{\left(1-\tanh^2(c+dx)\right)^4}d\tanh(c+dx)}{d}$$

$\downarrow$ 25

$$\frac{\frac{1}{8}\int \frac{\left(a-(a-b)\tanh^2(c+dx)\right)\left(a(8a-b)-(8a-5b)(a-b)\tanh^2(c+dx)\right)}{\left(1-\tanh^2(c+dx)\right)^4}d\tanh(c+dx)+\frac{b\tanh(c+dx)\left(a-(a-b)\tanh^2(c+dx)\right)^2}{8\left(1-\tanh^2(c+dx)\right)^4}}{d}$$

$\downarrow$ 401

$$\frac{\frac{1}{8}\left(\frac{1}{6}\int \frac{a(48a^2-18ba+5b^2)-3(a-b)(16a^2-14ba+5b^2)\tanh^2(c+dx)}{\left(1-\tanh^2(c+dx)\right)^3}d\tanh(c+dx)+\frac{b(12a-5b)\tanh(c+dx)\left(a-(a-b)\tanh^2(c+dx)\right)}{6\left(1-\tanh^2(c+dx)\right)^3}\right)+}{d}$$

$\downarrow$ 298

$$\frac{\frac{1}{8}\left(\frac{1}{6}\left(\frac{3}{4}\left(64a^3-48a^2b+24ab^2-5b^3\right)\int \frac{1}{\left(1-\tanh^2(c+dx)\right)^2}d\tanh(c+dx)+\frac{b(72a^2-52ab+15b^2)\tanh(c+dx)}{4\left(1-\tanh^2(c+dx)\right)^2}\right)+\frac{b(12a-5b)}{}\right.}{d}$$

$\downarrow$ 215

$$\frac{\frac{1}{8}\left(\frac{1}{6}\left(\frac{3}{4}\left(64a^3-48a^2b+24ab^2-5b^3\right)\left(\frac{1}{2}\int \frac{1}{1-\tanh^2(c+dx)}d\tanh(c+dx)+\frac{\tanh(c+dx)}{2\left(1-\tanh^2(c+dx)\right)}\right)+\frac{b(72a^2-52ab+15b^2)\tan}{4\left(1-\tanh^2(c+dx\right.}\right.\right.}{d}$$

$\downarrow$ 219

$$\frac{\frac{1}{8}\left(\frac{1}{6}\left(\frac{b(72a^2-52ab+15b^2)\tanh(c+dx)}{4\left(1-\tanh^2(c+dx)\right)^2}+\frac{3}{4}\left(64a^3-48a^2b+24ab^2-5b^3\right)\left(\frac{1}{2}\text{arctanh}(\tanh(c+dx))+\frac{\tanh(c+dx)}{2\left(1-\tanh^2(c+dx)\right)}\right)\right.\right.}{d}$$

input
```
Int[Cosh[c + d*x]^2*(a + b*Sinh[c + d*x]^2)^3,x]
```

output
```
((b*Tanh[c + d*x]*(a - (a - b)*Tanh[c + d*x]^2)^2)/(8*(1 - Tanh[c + d*x]^2
)^4) + (((12*a - 5*b)*b*Tanh[c + d*x]*(a - (a - b)*Tanh[c + d*x]^2))/(6*(1
 - Tanh[c + d*x]^2)^3) + ((b*(72*a^2 - 52*a*b + 15*b^2)*Tanh[c + d*x])/(4*
(1 - Tanh[c + d*x]^2)^2) + (3*(64*a^3 - 48*a^2*b + 24*a*b^2 - 5*b^3)*(ArcT
anh[Tanh[c + d*x]]/2 + Tanh[c + d*x]/(2*(1 - Tanh[c + d*x]^2))))/4)/6)/8)/
d
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 215
```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(-x)*((a + b*x^2)^(p + 1)
/(2*a*(p + 1))), x] + Simp[(2*p + 3)/(2*a*(p + 1))   Int[(a + b*x^2)^(p + 1
), x], x] /; FreeQ[{a, b}, x] && LtQ[p, -1] && (IntegerQ[4*p] || IntegerQ[6
*p])
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 298
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2), x_Symbol] :> Simp[(-(
b*c - a*d))*x*((a + b*x^2)^(p + 1)/(2*a*b*(p + 1))), x] - Simp[(a*d - b*c*(
2*p + 3))/(2*a*b*(p + 1))   Int[(a + b*x^2)^(p + 1), x], x] /; FreeQ[{a, b,
 c, d, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/2 + p, 0])
```

rule 315
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_), x_Symbol] :> Sim
p[(a*d - c*b)*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^(q - 1)/(2*a*b*(p + 1))),
x] - Simp[1/(2*a*b*(p + 1))   Int[(a + b*x^2)^(p + 1)*(c + d*x^2)^(q - 2)*S
imp[c*(a*d - c*b*(2*p + 3)) + d*(a*d*(2*(q - 1) + 1) - b*c*(2*(p + q) + 1))
*x^2, x], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -
1] && GtQ[q, 1] && IntBinomialQ[a, b, c, d, 2, p, q, x]
```

rule 401
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_.)*((e_) + (f_.)*(x
_)^2), x_Symbol] :> Simp[(-(b*e - a*f))*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^
q/(a*b*2*(p + 1))), x] + Simp[1/(a*b*2*(p + 1))   Int[(a + b*x^2)^(p + 1)*(
c + d*x^2)^(q - 1)*Simp[c*(b*e*2*(p + 1) + b*e - a*f) + d*(b*e*2*(p + 1) +
(b*e - a*f)*(2*q + 1))*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && L
tQ[p, -1] && GtQ[q, 0]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3670
```
Int[cos[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(
p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Simp[ff/f   Su
bst[Int[(a + (a + b)*ff^2*x^2)^p/(1 + ff^2*x^2)^(m/2 + p + 1), x], x, Tan[e
 + f*x]/ff], x]] /; FreeQ[{a, b, e, f}, x] && IntegerQ[m/2] && IntegerQ[p]
```

Maple [A] (verified)

Time = 0.12 (sec) , antiderivative size = 216, normalized size of antiderivative = 1.33

$$a^3\left(\frac{\cosh(dx+c)\sinh(dx+c)}{2}+\frac{dx}{2}+\frac{c}{2}\right)+3a^2b\left(\frac{\sinh(dx+c)\cosh(dx+c)^3}{4}-\frac{\cosh(dx+c)\sinh(dx+c)}{8}-\frac{dx}{8}-\frac{c}{8}\right)+3b^2a\left(\frac{\sinh(dx+$$

input
```
int(cosh(d*x+c)^2*(a+b*sinh(d*x+c)^2)^3,x)
```

output
```
1/d*(a^3*(1/2*cosh(d*x+c)*sinh(d*x+c)+1/2*d*x+1/2*c)+3*a^2*b*(1/4*sinh(d*x
+c)*cosh(d*x+c)^3-1/8*cosh(d*x+c)*sinh(d*x+c)-1/8*d*x-1/8*c)+3*b^2*a*(1/6*
sinh(d*x+c)^3*cosh(d*x+c)^3-1/8*sinh(d*x+c)*cosh(d*x+c)^3+1/16*cosh(d*x+c)
*sinh(d*x+c)+1/16*d*x+1/16*c)+b^3*(1/8*sinh(d*x+c)^5*cosh(d*x+c)^3-5/48*si
nh(d*x+c)^3*cosh(d*x+c)^3+5/64*sinh(d*x+c)*cosh(d*x+c)^3-5/128*cosh(d*x+c)
*sinh(d*x+c)-5/128*d*x-5/128*c))
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 257, normalized size of antiderivative = 1.58

$$\int \cosh^2(c+dx)\left(a+b\sinh^2(c+dx)\right)^3\,dx$$
$$=\frac{3\,b^3\cosh\left(dx+c\right)\sinh\left(dx+c\right)^7+3\left(7\,b^3\cosh\left(dx+c\right)^3+4\left(3\,ab^2-b^3\right)\cosh\left(dx+c\right)\right)\sinh\left(dx+c\right)^5+\left(\right.}{}$$

input
```
integrate(cosh(d*x+c)^2*(a+b*sinh(d*x+c)^2)^3,x, algorithm="fricas")
```

output
```
1/384*(3*b^3*cosh(d*x + c)*sinh(d*x + c)^7 + 3*(7*b^3*cosh(d*x + c)^3 + 4*
(3*a*b^2 - b^3)*cosh(d*x + c))*sinh(d*x + c)^5 + (21*b^3*cosh(d*x + c)^5 +
 40*(3*a*b^2 - b^3)*cosh(d*x + c)^3 + 12*(12*a^2*b - 6*a*b^2 + b^3)*cosh(d
*x + c))*sinh(d*x + c)^3 + 3*(64*a^3 - 48*a^2*b + 24*a*b^2 - 5*b^3)*d*x +
3*(b^3*cosh(d*x + c)^7 + 4*(3*a*b^2 - b^3)*cosh(d*x + c)^5 + 4*(12*a^2*b -
 6*a*b^2 + b^3)*cosh(d*x + c)^3 + 4*(16*a^3 - 3*a*b^2 + b^3)*cosh(d*x + c)
)*sinh(d*x + c))/d
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 559 vs. 2(160) = 320.

Time = 0.72 (sec) , antiderivative size = 559, normalized size of antiderivative = 3.43

$$\int \cosh^2(c+dx)\left(a+b\sinh^2(c+dx)\right)^3\,dx=\text{Too large to display}$$

input
```
integrate(cosh(d*x+c)**2*(a+b*sinh(d*x+c)**2)**3,x)
```

output
```
Piecewise((-a**3*x*sinh(c + d*x)**2/2 + a**3*x*cosh(c + d*x)**2/2 + a**3*s
inh(c + d*x)*cosh(c + d*x)/(2*d) - 3*a**2*b*x*sinh(c + d*x)**4/8 + 3*a**2*
b*x*sinh(c + d*x)**2*cosh(c + d*x)**2/4 - 3*a**2*b*x*cosh(c + d*x)**4/8 +
3*a**2*b*sinh(c + d*x)**3*cosh(c + d*x)/(8*d) + 3*a**2*b*sinh(c + d*x)*cos
h(c + d*x)**3/(8*d) - 3*a*b**2*x*sinh(c + d*x)**6/16 + 9*a*b**2*x*sinh(c +
 d*x)**4*cosh(c + d*x)**2/16 - 9*a*b**2*x*sinh(c + d*x)**2*cosh(c + d*x)**
4/16 + 3*a*b**2*x*cosh(c + d*x)**6/16 + 3*a*b**2*sinh(c + d*x)**5*cosh(c +
 d*x)/(16*d) + a*b**2*sinh(c + d*x)**3*cosh(c + d*x)**3/(2*d) - 3*a*b**2*s
inh(c + d*x)*cosh(c + d*x)**5/(16*d) - 5*b**3*x*sinh(c + d*x)**8/128 + 5*b
**3*x*sinh(c + d*x)**6*cosh(c + d*x)**2/32 - 15*b**3*x*sinh(c + d*x)**4*co
sh(c + d*x)**4/64 + 5*b**3*x*sinh(c + d*x)**2*cosh(c + d*x)**6/32 - 5*b**3
*x*cosh(c + d*x)**8/128 + 5*b**3*sinh(c + d*x)**7*cosh(c + d*x)/(128*d) +
73*b**3*sinh(c + d*x)**5*cosh(c + d*x)**3/(384*d) - 55*b**3*sinh(c + d*x)*
*3*cosh(c + d*x)**5/(384*d) + 5*b**3*sinh(c + d*x)*cosh(c + d*x)**7/(128*d
), Ne(d, 0)), (x*(a + b*sinh(c)**2)**3*cosh(c)**2, True))
```

Maxima [A] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 287, normalized size of antiderivative = 1.76

$$\int \cosh^2(c+dx)\left(a+b\sinh^2(c+dx)\right)^3\,dx = \frac{1}{8}\,a^3\left(4\,x+\frac{e^{(2\,dx+2\,c)}}{d}-\frac{e^{(-2\,dx-2\,c)}}{d}\right)$$
$$-\frac{1}{6144}\,b^3\left(\frac{\left(16\,e^{(-2\,dx-2\,c)}-24\,e^{(-4\,dx-4\,c)}-48\,e^{(-6\,dx-6\,c)}-3\right)e^{(8\,dx+8\,c)}}{d}+\frac{240\,(dx+c)}{d}+\frac{48\,e^{(-2\,dx-2\,c)}}{}\right.$$
$$-\frac{1}{128}\,ab^2\left(\frac{\left(3\,e^{(-2\,dx-2\,c)}+3\,e^{(-4\,dx-4\,c)}-1\right)e^{(6\,dx+6\,c)}}{d}-\frac{24\,(dx+c)}{d}-\frac{3\,e^{(-2\,dx-2\,c)}+3\,e^{(-4\,dx-4\,c)}-e^{(-6}}{d}\right.$$
$$-\frac{3}{64}\,a^2b\left(\frac{8\,(dx+c)}{d}-\frac{e^{(4\,dx+4\,c)}}{d}+\frac{e^{(-4\,dx-4\,c)}}{d}\right)$$

input
```
integrate(cosh(d*x+c)^2*(a+b*sinh(d*x+c)^2)^3,x, algorithm="maxima")
```

output
```
1/8*a^3*(4*x + e^(2*d*x + 2*c)/d - e^(-2*d*x - 2*c)/d) - 1/6144*b^3*((16*e
^(-2*d*x - 2*c) - 24*e^(-4*d*x - 4*c) - 48*e^(-6*d*x - 6*c) - 3)*e^(8*d*x
+ 8*c)/d + 240*(d*x + c)/d + (48*e^(-2*d*x - 2*c) + 24*e^(-4*d*x - 4*c) -
16*e^(-6*d*x - 6*c) + 3*e^(-8*d*x - 8*c))/d) - 1/128*a*b^2*((3*e^(-2*d*x -
 2*c) + 3*e^(-4*d*x - 4*c) - 1)*e^(6*d*x + 6*c)/d - 24*(d*x + c)/d - (3*e^
(-2*d*x - 2*c) + 3*e^(-4*d*x - 4*c) - e^(-6*d*x - 6*c))/d) - 3/64*a^2*b*(8
*(d*x + c)/d - e^(4*d*x + 4*c)/d + e^(-4*d*x - 4*c)/d)
```

Giac [A] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 231, normalized size of antiderivative = 1.42

$$\int \cosh^2(c+dx)\left(a+b\sinh^2(c+dx)\right)^3\,dx = \frac{b^3e^{(8\,dx+8\,c)}}{2048\,d} - \frac{b^3e^{(-8\,dx-8\,c)}}{2048\,d} + \frac{1}{128}\left(64\,a^3-48\,a^2b+24\,ab^2-5\,b^3\right)x + \frac{(3\,ab^2-b^3)e^{(6\,dx+6\,c)}}{384\,d} + \frac{(12\,a^2b-6\,ab^2+b^3)e^{(4\,dx+4\,c)}}{256\,d} + \frac{(16\,a^3-3\,ab^2+b^3)e^{(2\,dx+2\,c)}}{128\,d} - \frac{(16\,a^3-3\,ab^2+b^3)e^{(-2\,dx-2\,c)}}{128\,d} - \frac{(12\,a^2b-6\,ab^2+b^3)e^{(-4\,dx-4\,c)}}{256\,d} - \frac{(3\,ab^2-b^3)e^{(-6\,dx-6\,c)}}{384\,d}$$

input
```
integrate(cosh(d*x+c)^2*(a+b*sinh(d*x+c)^2)^3,x, algorithm="giac")
```

output
```
1/2048*b^3*e^(8*d*x + 8*c)/d - 1/2048*b^3*e^(-8*d*x - 8*c)/d + 1/128*(64*a
^3 - 48*a^2*b + 24*a*b^2 - 5*b^3)*x + 1/384*(3*a*b^2 - b^3)*e^(6*d*x + 6*c
)/d + 1/256*(12*a^2*b - 6*a*b^2 + b^3)*e^(4*d*x + 4*c)/d + 1/128*(16*a^3 -
 3*a*b^2 + b^3)*e^(2*d*x + 2*c)/d - 1/128*(16*a^3 - 3*a*b^2 + b^3)*e^(-2*d
*x - 2*c)/d - 1/256*(12*a^2*b - 6*a*b^2 + b^3)*e^(-4*d*x - 4*c)/d - 1/384*
(3*a*b^2 - b^3)*e^(-6*d*x - 6*c)/d
```

Mupad [B] (verification not implemented)

Time = 0.43 (sec) , antiderivative size = 166, normalized size of antiderivative = 1.02

$$\int \cosh^2(c+dx)\left(a+b\sinh^2(c+dx)\right)^3\,dx$$
$$= \frac{96\,a^3\sinh(2\,c+2\,d\,x)+6\,b^3\sinh(2\,c+2\,d\,x)+3\,b^3\sinh(4\,c+4\,d\,x)-2\,b^3\sinh(6\,c+6\,d\,x)+\frac{3\,b^3\sinh(8\,c+}{8}}{}$$

input
```
int(cosh(c + d*x)^2*(a + b*sinh(c + d*x)^2)^3,x)
```

output
```
(96*a^3*sinh(2*c + 2*d*x) + 6*b^3*sinh(2*c + 2*d*x) + 3*b^3*sinh(4*c + 4*d
*x) - 2*b^3*sinh(6*c + 6*d*x) + (3*b^3*sinh(8*c + 8*d*x))/8 - 18*a*b^2*sin
h(2*c + 2*d*x) - 18*a*b^2*sinh(4*c + 4*d*x) + 36*a^2*b*sinh(4*c + 4*d*x) +
 6*a*b^2*sinh(6*c + 6*d*x) + 192*a^3*d*x - 15*b^3*d*x + 72*a*b^2*d*x - 144
*a^2*b*d*x)/(384*d)
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 356, normalized size of antiderivative = 2.18

$$\int \cosh^2(c+dx)\left(a+b\sinh^2(c+dx)\right)^3\,dx$$
$$= \frac{3e^{16dx+16c}b^3+48e^{14dx+14c}a\,b^2-16e^{14dx+14c}b^3+288e^{12dx+12c}a^2b-144e^{12dx+12c}a\,b^2+24e^{12dx+12c}b^3+768}{}$$

input
```
int(cosh(d*x+c)^2*(a+b*sinh(d*x+c)^2)^3,x)
```

output
```
(3*e**(16*c + 16*d*x)*b**3 + 48*e**(14*c + 14*d*x)*a*b**2 - 16*e**(14*c +
14*d*x)*b**3 + 288*e**(12*c + 12*d*x)*a**2*b - 144*e**(12*c + 12*d*x)*a*b*
*2 + 24*e**(12*c + 12*d*x)*b**3 + 768*e**(10*c + 10*d*x)*a**3 - 144*e**(10
*c + 10*d*x)*a*b**2 + 48*e**(10*c + 10*d*x)*b**3 + 3072*e**(8*c + 8*d*x)*a
**3*d*x - 2304*e**(8*c + 8*d*x)*a**2*b*d*x + 1152*e**(8*c + 8*d*x)*a*b**2*
d*x - 240*e**(8*c + 8*d*x)*b**3*d*x - 768*e**(6*c + 6*d*x)*a**3 + 144*e**(
6*c + 6*d*x)*a*b**2 - 48*e**(6*c + 6*d*x)*b**3 - 288*e**(4*c + 4*d*x)*a**2
*b + 144*e**(4*c + 4*d*x)*a*b**2 - 24*e**(4*c + 4*d*x)*b**3 - 48*e**(2*c +
 2*d*x)*a*b**2 + 16*e**(2*c + 2*d*x)*b**3 - 3*b**3)/(6144*e**(8*c + 8*d*x)
*d)
```

3.271 $\int \cosh(c+dx)\left(a+b\sinh^2(c+dx)\right)^3\,dx$

Optimal result

Integrand size = 21, antiderivative size = 67

$$\int \cosh(c+dx)\left(a+b\sinh^2(c+dx)\right)^3\,dx=\frac{a^3\sinh(c+dx)}{d}+\frac{a^2b\sinh^3(c+dx)}{d}+\frac{3ab^2\sinh^5(c+dx)}{5d}+\frac{b^3\sinh^7(c+dx)}{7d}$$

output
```
a^3*sinh(d*x+c)/d+a^2*b*sinh(d*x+c)^3/d+3/5*a*b^2*sinh(d*x+c)^5/d+1/7*b^3*
sinh(d*x+c)^7/d
```

Mathematica [A] (verified)

Time = 0.09 (sec) , antiderivative size = 59, normalized size of antiderivative = 0.88

$$\int \cosh(c+dx)\left(a+b\sinh^2(c+dx)\right)^3\,dx=\frac{a^3\sinh(c+dx)+a^2b\sinh^3(c+dx)+\frac{3}{5}ab^2\sinh^5(c+dx)+\frac{1}{7}b^3\sinh^7(c+dx)}{d}$$

input
```
Integrate[Cosh[c + d*x]*(a + b*Sinh[c + d*x]^2)^3,x]
```

output
```
(a^3*Sinh[c + d*x] + a^2*b*Sinh[c + d*x]^3 + (3*a*b^2*Sinh[c + d*x]^5)/5 +
 (b^3*Sinh[c + d*x]^7)/7)/d
```

Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 59, normalized size of antiderivative = 0.88, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.190, Rules used = {3042, 3669, 210, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cosh(c+dx)\left(a+b\sinh^2(c+dx)\right)^3 dx$$

$$\Big\downarrow \text{3042}$$

$$\int \cos(ic+idx)\left(a-b\sin(ic+idx)^2\right)^3 dx$$

$$\Big\downarrow \text{3669}$$

$$\frac{\int \left(b\sinh^2(c+dx)+a\right)^3 d\sinh(c+dx)}{d}$$

$$\Big\downarrow \text{210}$$

$$\frac{\int \left(b^3\sinh^6(c+dx)+3ab^2\sinh^4(c+dx)+3a^2b\sinh^2(c+dx)+a^3\right) d\sinh(c+dx)}{d}$$

$$\Big\downarrow \text{2009}$$

$$\frac{a^3\sinh(c+dx)+a^2b\sinh^3(c+dx)+\frac{3}{5}ab^2\sinh^5(c+dx)+\frac{1}{7}b^3\sinh^7(c+dx)}{d}$$

input
```
Int[Cosh[c + d*x]*(a + b*Sinh[c + d*x]^2)^3,x]
```

output
```
(a^3*Sinh[c + d*x] + a^2*b*Sinh[c + d*x]^3 + (3*a*b^2*Sinh[c + d*x]^5)/5 +
 (b^3*Sinh[c + d*x]^7)/7)/d
```

Defintions of rubi rules used

rule 210
```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Int[ExpandIntegrand[(a + b*x^2
)^p, x], x] /; FreeQ[{a, b}, x] && IGtQ[p, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3669
```
Int[cos[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^
(p_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Simp[ff/f   S
ubst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a + b*ff^2*x^2)^p, x], x, Sin[e + f*x]
/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m - 1)/2]
```

Maple [A] (verified)

Time = 0.05 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.84

$$\frac{\frac{b^3 \sinh(dx+c)^7}{7} + \frac{3b^2 a \sinh(dx+c)^5}{5} + a^2 b \sinh(dx+c)^3 + a^3 \sinh(dx+c)}{d}$$

input
```
int(cosh(d*x+c)*(a+b*sinh(d*x+c)^2)^3,x)
```

output
```
1/d*(1/7*b^3*sinh(d*x+c)^7+3/5*b^2*a*sinh(d*x+c)^5+a^2*b*sinh(d*x+c)^3+a^3
*sinh(d*x+c))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 209 vs. $2(63) = 126$.

Time = 0.09 (sec) , antiderivative size = 209, normalized size of antiderivative = 3.12

$$\int \cosh(c+dx)\left(a+b\sinh^2(c+dx)\right)^3\,dx$$
$$= \frac{5\,b^3\sinh\left(dx+c\right)^7+7\left(15\,b^3\cosh\left(dx+c\right)^2+12\,ab^2-5\,b^3\right)\sinh\left(dx+c\right)^5+35\left(5\,b^3\cosh\left(dx+c\right)^4+16\right.}{}$$

input
```
integrate(cosh(d*x+c)*(a+b*sinh(d*x+c)^2)^3,x, algorithm="fricas")
```

output
```
1/2240*(5*b^3*sinh(d*x + c)^7 + 7*(15*b^3*cosh(d*x + c)^2 + 12*a*b^2 - 5*b
^3)*sinh(d*x + c)^5 + 35*(5*b^3*cosh(d*x + c)^4 + 16*a^2*b - 12*a*b^2 + 3*
b^3 + 2*(12*a*b^2 - 5*b^3)*cosh(d*x + c)^2)*sinh(d*x + c)^3 + 35*(b^3*cosh
(d*x + c)^6 + (12*a*b^2 - 5*b^3)*cosh(d*x + c)^4 + 64*a^3 - 48*a^2*b + 24*
a*b^2 - 5*b^3 + 3*(16*a^2*b - 12*a*b^2 + 3*b^3)*cosh(d*x + c)^2)*sinh(d*x
+ c))/d
```

Sympy [A] (verification not implemented)

Time = 0.47 (sec) , antiderivative size = 75, normalized size of antiderivative = 1.12

$$\int \cosh(c+dx)\left(a+b\sinh^2(c+dx)\right)^3\,dx = \begin{cases} \frac{a^3\sinh\left(c+dx\right)}{d}+\frac{a^2b\sinh^3\left(c+dx\right)}{d}+\frac{3ab^2\sinh^5\left(c+dx\right)}{5d}+\frac{b^3\sinh^7\left(c+dx\right)}{7d} & \text{for } d\neq 0 \\ x\left(a+b\sinh^2\left(c\right)\right)^3\cosh\left(c\right) & \text{otherwise} \end{cases}$$

input
```
integrate(cosh(d*x+c)*(a+b*sinh(d*x+c)**2)**3,x)
```

output
```
Piecewise((a**3*sinh(c + d*x)/d + a**2*b*sinh(c + d*x)**3/d + 3*a*b**2*sin
h(c + d*x)**5/(5*d) + b**3*sinh(c + d*x)**7/(7*d), Ne(d, 0)), (x*(a + b*si
nh(c)**2)**3*cosh(c), True))
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 63, normalized size of antiderivative = 0.94

$$\int \cosh(c+dx)\left(a+b\sinh^2(c+dx)\right)^3\,dx = \frac{b^3\sinh\left(dx+c\right)^7}{7\,d}+\frac{3\,ab^2\sinh\left(dx+c\right)^5}{5\,d}+\frac{a^2b\sinh\left(dx+c\right)^3}{d}+\frac{a^3\sinh\left(dx+c\right)}{d}$$

input
```
integrate(cosh(d*x+c)*(a+b*sinh(d*x+c)^2)^3,x, algorithm="maxima")
```

output
```
1/7*b^3*sinh(d*x + c)^7/d + 3/5*a*b^2*sinh(d*x + c)^5/d + a^2*b*sinh(d*x +
 c)^3/d + a^3*sinh(d*x + c)/d
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 222 vs. 2(63) = 126.

Time = 0.46 (sec) , antiderivative size = 222, normalized size of antiderivative = 3.31

$$\int \cosh(c+dx)\left(a+b\sinh^2(c+dx)\right)^3\,dx = \frac{b^3e^{(7\,dx+7\,c)}}{896\,d}-\frac{b^3e^{(-7\,dx-7\,c)}}{896\,d}+\frac{(12\,ab^2-5\,b^3)e^{(5\,dx+5\,c)}}{640\,d}+\frac{(16\,a^2b-12\,ab^2+3\,b^3)e^{(3\,dx+3\,c)}}{128\,d}+\frac{(64\,a^3-48\,a^2b+24\,ab^2-5\,b^3)e^{(dx+c)}}{128\,d}-\frac{(64\,a^3-48\,a^2b+24\,ab^2-5\,b^3)e^{(-dx-c)}}{128\,d}-\frac{(16\,a^2b-12\,ab^2+3\,b^3)e^{(-3\,dx-3\,c)}}{128\,d}-\frac{(12\,ab^2-5\,b^3)e^{(-5\,dx-5\,c)}}{640\,d}$$

input
```
integrate(cosh(d*x+c)*(a+b*sinh(d*x+c)^2)^3,x, algorithm="giac")
```

output
```
1/896*b^3*e^(7*d*x + 7*c)/d - 1/896*b^3*e^(-7*d*x - 7*c)/d + 1/640*(12*a*b
^2 - 5*b^3)*e^(5*d*x + 5*c)/d + 1/128*(16*a^2*b - 12*a*b^2 + 3*b^3)*e^(3*d
*x + 3*c)/d + 1/128*(64*a^3 - 48*a^2*b + 24*a*b^2 - 5*b^3)*e^(d*x + c)/d -
 1/128*(64*a^3 - 48*a^2*b + 24*a*b^2 - 5*b^3)*e^(-d*x - c)/d - 1/128*(16*a
^2*b - 12*a*b^2 + 3*b^3)*e^(-3*d*x - 3*c)/d - 1/640*(12*a*b^2 - 5*b^3)*e^(
-5*d*x - 5*c)/d
```

Mupad [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 58, normalized size of antiderivative = 0.87

$$\int \cosh(c+dx)\left(a+b\sinh^2(c+dx)\right)^3\,dx$$
$$=\frac{\sinh(c+d\,x)\,\left(35\,a^3+35\,a^2\,b\sinh(c+d\,x)^2+21\,a\,b^2\sinh(c+d\,x)^4+5\,b^3\sinh(c+d\,x)^6\right)}{35\,d}$$

input
```
int(cosh(c + d*x)*(a + b*sinh(c + d*x)^2)^3,x)
```

output
```
(sinh(c + d*x)*(35*a^3 + 5*b^3*sinh(c + d*x)^6 + 35*a^2*b*sinh(c + d*x)^2
+ 21*a*b^2*sinh(c + d*x)^4))/(35*d)
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 58, normalized size of antiderivative = 0.87

$$\int \cosh(c+dx)\left(a+b\sinh^2(c+dx)\right)^3\,dx$$
$$=\frac{\sinh(dx+c)\left(5\sinh(dx+c)^6b^3+21\sinh(dx+c)^4a\,b^2+35\sinh(dx+c)^2a^2b+35a^3\right)}{35d}$$

input
```
int(cosh(d*x+c)*(a+b*sinh(d*x+c)^2)^3,x)
```

output
```
(sinh(c + d*x)*(5*sinh(c + d*x)**6*b**3 + 21*sinh(c + d*x)**4*a*b**2 + 35*
sinh(c + d*x)**2*a**2*b + 35*a**3))/(35*d)
```

3.272 $\int \operatorname{sech}(c+dx)\left(a+b\sinh^2(c+dx)\right)^3\,dx$

Optimal result

Integrand size = 21, antiderivative size = 86

$$\int \operatorname{sech}(c+dx)\left(a+b\sinh^2(c+dx)\right)^3\,dx = \frac{(a-b)^3\arctan(\sinh(c+dx))}{d} + \frac{b(3a^2-3ab+b^2)\sinh(c+dx)}{d} + \frac{(3a-b)b^2\sinh^3(c+dx)}{3d} + \frac{b^3\sinh^5(c+dx)}{5d}$$

output
```
(a-b)^3*arctan(sinh(d*x+c))/d+b*(3*a^2-3*a*b+b^2)*sinh(d*x+c)/d+1/3*(3*a-b
)*b^2*sinh(d*x+c)^3/d+1/5*b^3*sinh(d*x+c)^5/d
```

Mathematica [A] (verified)

Time = 0.39 (sec) , antiderivative size = 100, normalized size of antiderivative = 1.16

$$\int \operatorname{sech}(c+dx)\left(a+b\sinh^2(c+dx)\right)^3\,dx$$

$$= \frac{\sinh(c+dx)\left(\frac{15(a-b)^3\operatorname{arctanh}\left(\sqrt{-\sinh^2(c+dx)}\right)}{\sqrt{-\sinh^2(c+dx)}} + b\left(45a^2+15ab\left(-3+\sinh^2(c+dx)\right)+b^2\left(15-5\sinh^2(c\right.\right.\right.}{15d}$$

input
```
Integrate[Sech[c + d*x]*(a + b*Sinh[c + d*x]^2)^3,x]
```

output
```
(Sinh[c + d*x]*((15*(a - b)^3*ArcTanh[Sqrt[-Sinh[c + d*x]^2]])/Sqrt[-Sinh[
c + d*x]^2] + b*(45*a^2 + 15*a*b*(-3 + Sinh[c + d*x]^2) + b^2*(15 - 5*Sinh
[c + d*x]^2 + 3*Sinh[c + d*x]^4))))/(15*d)
```

Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 78, normalized size of antiderivative = 0.91, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.190$, Rules used = {3042, 3669, 300, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \text{sech}(c+dx)\left(a+b\sinh^2(c+dx)\right)^3 dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\left(a-b\sin(ic+idx)^2\right)^3}{\cos(ic+idx)}dx$$

$$\Big\downarrow \text{3669}$$

$$\frac{\int \frac{\left(b\sinh^2(c+dx)+a\right)^3}{\sinh^2(c+dx)+1}d\sinh(c+dx)}{d}$$

$$\Big\downarrow \text{300}$$

$$\frac{\int \left(b^3\sinh^4(c+dx)+(3a-b)b^2\sinh^2(c+dx)+b\left(3a^2-3ba+b^2\right)+\frac{(a-b)^3}{\sinh^2(c+dx)+1}\right)d\sinh(c+dx)}{d}$$

$$\Big\downarrow \text{2009}$$

$$\frac{b\left(3a^2-3ab+b^2\right)\sinh(c+dx)+(a-b)^3\arctan(\sinh(c+dx))+\frac{1}{3}b^2(3a-b)\sinh^3(c+dx)+\frac{1}{5}b^3\sinh^5(c+dx)}{d}$$

input
```
Int[Sech[c + d*x]*(a + b*Sinh[c + d*x]^2)^3,x]
```

output
```
((a - b)^3*ArcTan[Sinh[c + d*x]] + b*(3*a^2 - 3*a*b + b^2)*Sinh[c + d*x] +
 ((3*a - b)*b^2*Sinh[c + d*x]^3)/3 + (b^3*Sinh[c + d*x]^5)/5)/d
```

Defintions of rubi rules used

rule 300
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_), x_Symbol] :> Int
[PolynomialDivide[(a + b*x^2)^p, (c + d*x^2)^(-q), x], x] /; FreeQ[{a, b, c
, d}, x] && NeQ[b*c - a*d, 0] && IGtQ[p, 0] && ILtQ[q, 0] && GeQ[p, -q]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3669
```
Int[cos[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^
(p_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Simp[ff/f   S
ubst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a + b*ff^2*x^2)^p, x], x, Sin[e + f*x]
/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m - 1)/2]
```

Maple [A] (verified)

Time = 0.16 (sec) , antiderivative size = 114, normalized size of antiderivative = 1.33

$$\frac{2a^3\arctan\left(\mathrm{e}^{dx+c}\right)+3a^2b\left(\sinh\left(dx+c\right)-2\arctan\left(\mathrm{e}^{dx+c}\right)\right)+3b^2a\left(\frac{\sinh(dx+c)^3}{3}-\sinh\left(dx+c\right)+2\arctan\left(\right.\right.}{d}$$

input
```
int(sech(d*x+c)*(a+b*sinh(d*x+c)^2)^3,x)
```

output
```
1/d*(2*a^3*arctan(exp(d*x+c))+3*a^2*b*(sinh(d*x+c)-2*arctan(exp(d*x+c)))+3
*b^2*a*(1/3*sinh(d*x+c)^3-sinh(d*x+c)+2*arctan(exp(d*x+c)))+b^3*(1/5*sinh(
d*x+c)^5-1/3*sinh(d*x+c)^3+sinh(d*x+c)-2*arctan(exp(d*x+c))))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1114 vs. $2(82) = 164$.

Time = 0.10 (sec) , antiderivative size = 1114, normalized size of antiderivative = 12.95

$$\int \operatorname{sech}(c+dx)\left(a+b\sinh^2(c+dx)\right)^3\,dx = \text{Too large to display}$$

input
```
integrate(sech(d*x+c)*(a+b*sinh(d*x+c)^2)^3,x, algorithm="fricas")
```

output
```
1/480*(3*b^3*cosh(d*x + c)^10 + 30*b^3*cosh(d*x + c)*sinh(d*x + c)^9 + 3*b
^3*sinh(d*x + c)^10 + 5*(12*a*b^2 - 7*b^3)*cosh(d*x + c)^8 + 5*(27*b^3*cos
h(d*x + c)^2 + 12*a*b^2 - 7*b^3)*sinh(d*x + c)^8 + 40*(9*b^3*cosh(d*x + c)
^3 + (12*a*b^2 - 7*b^3)*cosh(d*x + c))*sinh(d*x + c)^7 + 30*(24*a^2*b - 30
*a*b^2 + 11*b^3)*cosh(d*x + c)^6 + 10*(63*b^3*cosh(d*x + c)^4 + 72*a^2*b -
 90*a*b^2 + 33*b^3 + 14*(12*a*b^2 - 7*b^3)*cosh(d*x + c)^2)*sinh(d*x + c)^
6 + 4*(189*b^3*cosh(d*x + c)^5 + 70*(12*a*b^2 - 7*b^3)*cosh(d*x + c)^3 + 4
5*(24*a^2*b - 30*a*b^2 + 11*b^3)*cosh(d*x + c))*sinh(d*x + c)^5 - 30*(24*a
^2*b - 30*a*b^2 + 11*b^3)*cosh(d*x + c)^4 + 10*(63*b^3*cosh(d*x + c)^6 + 3
5*(12*a*b^2 - 7*b^3)*cosh(d*x + c)^4 - 72*a^2*b + 90*a*b^2 - 33*b^3 + 45*(
24*a^2*b - 30*a*b^2 + 11*b^3)*cosh(d*x + c)^2)*sinh(d*x + c)^4 + 40*(9*b^3
*cosh(d*x + c)^7 + 7*(12*a*b^2 - 7*b^3)*cosh(d*x + c)^5 + 15*(24*a^2*b - 3
0*a*b^2 + 11*b^3)*cosh(d*x + c)^3 - 3*(24*a^2*b - 30*a*b^2 + 11*b^3)*cosh(
d*x + c))*sinh(d*x + c)^3 - 3*b^3 - 5*(12*a*b^2 - 7*b^3)*cosh(d*x + c)^2 +
 5*(27*b^3*cosh(d*x + c)^8 + 28*(12*a*b^2 - 7*b^3)*cosh(d*x + c)^6 + 90*(2
4*a^2*b - 30*a*b^2 + 11*b^3)*cosh(d*x + c)^4 - 12*a*b^2 + 7*b^3 - 36*(24*a
^2*b - 30*a*b^2 + 11*b^3)*cosh(d*x + c)^2)*sinh(d*x + c)^2 + 960*((a^3 - 3
*a^2*b + 3*a*b^2 - b^3)*cosh(d*x + c)^5 + 5*(a^3 - 3*a^2*b + 3*a*b^2 - b^3
)*cosh(d*x + c)^4*sinh(d*x + c) + 10*(a^3 - 3*a^2*b + 3*a*b^2 - b^3)*cosh(
d*x + c)^3*sinh(d*x + c)^2 + 10*(a^3 - 3*a^2*b + 3*a*b^2 - b^3)*cosh(d*...
```

Sympy [F]

$$\int \operatorname{sech}(c+dx)\left(a+b\sinh^2(c+dx)\right)^3\,dx=\int \left(a+b\sinh^2(c+dx)\right)^3\operatorname{sech}(c+dx)\,dx$$

input
```
integrate(sech(d*x+c)*(a+b*sinh(d*x+c)**2)**3,x)
```

output
```
Integral((a + b*sinh(c + d*x)**2)**3*sech(c + d*x), x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 233 vs. 2(82) = 164.

Time = 0.15 (sec) , antiderivative size = 233, normalized size of antiderivative = 2.71

$$\begin{aligned}\int \operatorname{sech}(c+dx)\left(a+b\sinh^2(c+dx)\right)^3\,dx=&\\ -\frac{1}{480}b^3&\left(\frac{\left(35\,e^{(-2\,dx-2\,c)}-330\,e^{(-4\,dx-4\,c)}-3\right)e^{(5\,dx+5\,c)}}{d}+\frac{330\,e^{(-dx-c)}-35\,e^{(-3\,dx-3\,c)}+3\,e^{(-5\,dx-5\,c)}}{d}-\right.\\ &-\frac{1}{8}ab^2\left(\frac{\left(15\,e^{(-2\,dx-2\,c)}-1\right)e^{(3\,dx+3\,c)}}{d}-\frac{15\,e^{(-dx-c)}-e^{(-3\,dx-3\,c)}}{d}+\frac{48\arctan\left(e^{(-dx-c)}\right)}{d}\right)\\ &+\frac{3}{2}a^2b\left(\frac{4\arctan\left(e^{(-dx-c)}\right)}{d}+\frac{e^{(dx+c)}}{d}-\frac{e^{(-dx-c)}}{d}\right)+\frac{a^3\arctan\left(\sinh\left(dx+c\right)\right)}{d}\end{aligned}$$

input
```
integrate(sech(d*x+c)*(a+b*sinh(d*x+c)^2)^3,x, algorithm="maxima")
```

output
```
-1/480*b^3*((35*e^(-2*d*x - 2*c) - 330*e^(-4*d*x - 4*c) - 3)*e^(5*d*x + 5*
c)/d + (330*e^(-d*x - c) - 35*e^(-3*d*x - 3*c) + 3*e^(-5*d*x - 5*c))/d - 9
60*arctan(e^(-d*x - c))/d) - 1/8*a*b^2*((15*e^(-2*d*x - 2*c) - 1)*e^(3*d*x
 + 3*c)/d - (15*e^(-d*x - c) - e^(-3*d*x - 3*c))/d + 48*arctan(e^(-d*x - c
))/d) + 3/2*a^2*b*(4*arctan(e^(-d*x - c))/d + e^(d*x + c)/d - e^(-d*x - c)
/d) + a^3*arctan(sinh(d*x + c))/d
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 204 vs. $2(82) = 164$.

Time = 0.15 (sec) , antiderivative size = 204, normalized size of antiderivative = 2.37

$$\int \operatorname{sech}(c+dx) \left(a+b\sinh^2(c+dx)\right)^3 dx = \frac{3\,b^3e^{(5\,dx+5\,c)} + 60\,ab^2e^{(3\,dx+3\,c)} - 35\,b^3e^{(3\,dx+3\,c)} + 720\,a^2be^{(dx+c)} - 900\,ab^2e^{(dx+c)} + 330\,b^3e^{(dx+c)} + 960\,(a}{}$$

input
```
integrate(sech(d*x+c)*(a+b*sinh(d*x+c)^2)^3,x, algorithm="giac")
```

output
```
1/480*(3*b^3*e^(5*d*x + 5*c) + 60*a*b^2*e^(3*d*x + 3*c) - 35*b^3*e^(3*d*x
+ 3*c) + 720*a^2*b*e^(d*x + c) - 900*a*b^2*e^(d*x + c) + 330*b^3*e^(d*x +
c) + 960*(a^3 - 3*a^2*b + 3*a*b^2 - b^3)*arctan(e^(d*x + c)) - (720*a^2*b*
e^(4*d*x + 4*c) - 900*a*b^2*e^(4*d*x + 4*c) + 330*b^3*e^(4*d*x + 4*c) + 60
*a*b^2*e^(2*d*x + 2*c) - 35*b^3*e^(2*d*x + 2*c) + 3*b^3)*e^(-5*d*x - 5*c))
/d
```

Mupad [B] (verification not implemented)

Time = 2.02 (sec) , antiderivative size = 294, normalized size of antiderivative = 3.42

$$\int \operatorname{sech}(c+dx) \left(a+b\sinh^2(c+dx)\right)^3 dx = \frac{e^{c+d\,x}\,(24\,a^2\,b-30\,a\,b^2+11\,b^3)}{16\,d} - \frac{e^{-c-d\,x}\,(24\,a^2\,b-30\,a\,b^2+11\,b^3)}{16\,d} + \frac{2\,\mathrm{atan}\left(\frac{e^{d\,x}\,e^{c}\,\left(a^3\,\sqrt{d^2}-b^3\,\sqrt{d^2}+3\,a\,b^2\,\sqrt{d^2}-3\,a^2\,b\,\sqrt{d^2}\right)}{d\,\sqrt{a^6-6\,a^5\,b+15\,a^4\,b^2-20\,a^3\,b^3+15\,a^2\,b^4-6\,a\,b^5+b^6}}\right)\,\sqrt{a^6-6\,a^5\,b+15\,a^4\,b^2-20\,a^3\,b^3+15\,a^2\,b^4-6\,a\,b^5}}{\sqrt{d^2}} - \frac{b^3\,e^{-5\,c-5\,d\,x}}{160\,d} + \frac{b^3\,e^{5\,c+5\,d\,x}}{160\,d} - \frac{b^2\,e^{-3\,c-3\,d\,x}\,(12\,a-7\,b)}{96\,d} + \frac{b^2\,e^{3\,c+3\,d\,x}\,(12\,a-7\,b)}{96\,d}$$

input
```
int((a + b*sinh(c + d*x)^2)^3/cosh(c + d*x),x)
```

output
```
(exp(c + d*x)*(24*a^2*b - 30*a*b^2 + 11*b^3))/(16*d) - (exp(- c - d*x)*(24
*a^2*b - 30*a*b^2 + 11*b^3))/(16*d) + (2*atan((exp(d*x)*exp(c)*(a^3*(d^2)^
(1/2) - b^3*(d^2)^(1/2) + 3*a*b^2*(d^2)^(1/2) - 3*a^2*b*(d^2)^(1/2)))/(d*(
a^6 - 6*a^5*b - 6*a*b^5 + b^6 + 15*a^2*b^4 - 20*a^3*b^3 + 15*a^4*b^2)^(1/2
)))*(a^6 - 6*a^5*b - 6*a*b^5 + b^6 + 15*a^2*b^4 - 20*a^3*b^3 + 15*a^4*b^2)
^(1/2))/(d^2)^(1/2) - (b^3*exp(- 5*c - 5*d*x))/(160*d) + (b^3*exp(5*c + 5*
d*x))/(160*d) - (b^2*exp(- 3*c - 3*d*x)*(12*a - 7*b))/(96*d) + (b^2*exp(3*
c + 3*d*x)*(12*a - 7*b))/(96*d)
```

Reduce [B] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 288, normalized size of antiderivative = 3.35

$$\int \operatorname{sech}(c+dx)\left(a+b\sinh^2(c+dx)\right)^3\,dx$$
$$= \frac{960e^{5dx+5c}atan\left(e^{dx+c}\right)a^3 - 2880e^{5dx+5c}atan\left(e^{dx+c}\right)a^2b + 2880e^{5dx+5c}atan\left(e^{dx+c}\right)ab^2 - 960e^{5dx+5c}atan}{}$$

input
```
int(sech(d*x+c)*(a+b*sinh(d*x+c)^2)^3,x)
```

output
```
(960*e**(5*c + 5*d*x)*atan(e**(c + d*x))*a**3 - 2880*e**(5*c + 5*d*x)*atan
(e**(c + d*x))*a**2*b + 2880*e**(5*c + 5*d*x)*atan(e**(c + d*x))*a*b**2 -
960*e**(5*c + 5*d*x)*atan(e**(c + d*x))*b**3 + 3*e**(10*c + 10*d*x)*b**3 +
 60*e**(8*c + 8*d*x)*a*b**2 - 35*e**(8*c + 8*d*x)*b**3 + 720*e**(6*c + 6*d
*x)*a**2*b - 900*e**(6*c + 6*d*x)*a*b**2 + 330*e**(6*c + 6*d*x)*b**3 - 720
*e**(4*c + 4*d*x)*a**2*b + 900*e**(4*c + 4*d*x)*a*b**2 - 330*e**(4*c + 4*d
*x)*b**3 - 60*e**(2*c + 2*d*x)*a*b**2 + 35*e**(2*c + 2*d*x)*b**3 - 3*b**3)
/(480*e**(5*c + 5*d*x)*d)
```

3.273 $\int \operatorname{sech}^2(c+dx)\left(a+b\sinh^2(c+dx)\right)^3\,dx$

Optimal result

Integrand size = 23, antiderivative size = 92

$$\int \operatorname{sech}^2(c+dx)\left(a+b\sinh^2(c+dx)\right)^3\,dx = \frac{3}{8}b\left(8a^2-12ab+5b^2\right)x + \frac{3(4a-3b)b^2\cosh(c+dx)\sinh(c+dx)}{8d} + \frac{b^3\cosh^3(c+dx)\sinh(c+dx)}{4d} + \frac{(a-b)^3\tanh(c+dx)}{d}$$

output
```
3/8*b*(8*a^2-12*a*b+5*b^2)*x+3/8*(4*a-3*b)*b^2*cosh(d*x+c)*sinh(d*x+c)/d+1
/4*b^3*cosh(d*x+c)^3*sinh(d*x+c)/d+(a-b)^3*tanh(d*x+c)/d
```

Mathematica [A] (verified)

Time = 1.62 (sec) , antiderivative size = 78, normalized size of antiderivative = 0.85

$$\int \operatorname{sech}^2(c+dx)\left(a+b\sinh^2(c+dx)\right)^3\,dx = \frac{12b(8a^2-12ab+5b^2)(c+dx)+8(3a-2b)b^2\sinh(2(c+dx))+b^3\sinh(4(c+dx))+32(a-b)^3\tanh(c}{32d}$$

input
```
Integrate[Sech[c + d*x]^2*(a + b*Sinh[c + d*x]^2)^3,x]
```

output
```
(12*b*(8*a^2 - 12*a*b + 5*b^2)*(c + d*x) + 8*(3*a - 2*b)*b^2*Sinh[2*(c + d
*x)] + b^3*Sinh[4*(c + d*x)] + 32*(a - b)^3*Tanh[c + d*x])/(32*d)
```

Rubi [A] (verified)

Time = 0.36 (sec) , antiderivative size = 107, normalized size of antiderivative = 1.16, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.174, Rules used = {3042, 3670, 300, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \operatorname{sech}^2(c+dx)\left(a+b\sinh^2(c+dx)\right)^3\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\left(a-b\sin(ic+idx)^2\right)^3}{\cos(ic+idx)^2}dx$$

$$\Big\downarrow \text{3670}$$

$$\frac{\int \frac{\left(a-(a-b)\tanh^2(c+dx)\right)^3}{\left(1-\tanh^2(c+dx)\right)^3}d\tanh(c+dx)}{d}$$

$$\Big\downarrow \text{300}$$

$$\frac{\int \left((a-b)^3+\frac{3(a-b)^2b\tanh^4(c+dx)-3(a-b)(2a-b)b\tanh^2(c+dx)+b\left(3a^2-3ba+b^2\right)}{\left(1-\tanh^2(c+dx)\right)^3}\right)d\tanh(c+dx)}{d}$$

$$\Big\downarrow \text{2009}$$

$$\frac{\frac{3}{8}b\left(8a^2-12ab+5b^2\right)\text{arctanh}(\tanh(c+dx))+\frac{3b^2(4a-3b)\tanh(c+dx)}{8\left(1-\tanh^2(c+dx)\right)}+(a-b)^3\tanh(c+dx)+\frac{b^3\tanh(c+dx)}{4\left(1-\tanh^2(c+dx)\right)^2}}{d}$$

input
```
Int[Sech[c + d*x]^2*(a + b*Sinh[c + d*x]^2)^3,x]
```

output
```
((3*b*(8*a^2 - 12*a*b + 5*b^2)*ArcTanh[Tanh[c + d*x]])/8 + (a - b)^3*Tanh[
c + d*x] + (b^3*Tanh[c + d*x])/(4*(1 - Tanh[c + d*x]^2)^2) + (3*(4*a - 3*b
)*b^2*Tanh[c + d*x])/(8*(1 - Tanh[c + d*x]^2)))/d
```

Defintions of rubi rules used

rule 300
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_), x_Symbol] :> Int
[PolynomialDivide[(a + b*x^2)^p, (c + d*x^2)^(-q), x], x] /; FreeQ[{a, b, c
, d}, x] && NeQ[b*c - a*d, 0] && IGtQ[p, 0] && ILtQ[q, 0] && GeQ[p, -q]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3670
```
Int[cos[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(
p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Simp[ff/f   Su
bst[Int[(a + (a + b)*ff^2*x^2)^p/(1 + ff^2*x^2)^(m/2 + p + 1), x], x, Tan[e
 + f*x]/ff], x]] /; FreeQ[{a, b, e, f}, x] && IntegerQ[m/2] && IntegerQ[p]
```

Maple [A] (verified)

Time = 0.16 (sec) , antiderivative size = 131, normalized size of antiderivative = 1.42

$$\frac{\tanh\left(dx+c\right)a^{3}+3a^{2}b\left(dx+c-\tanh\left(dx+c\right)\right)+3b^{2}a\left(\frac{\sinh(dx+c)^{3}}{2\cosh(dx+c)}-\frac{3dx}{2}-\frac{3c}{2}+\frac{3\tanh(dx+c)}{2}\right)+b^{3}\left(\frac{\sinh(dx+}{4\cosh(dx}\right.}{d}$$

input
```
int(sech(d*x+c)^2*(a+b*sinh(d*x+c)^2)^3,x)
```

output
```
1/d*(tanh(d*x+c)*a^3+3*a^2*b*(d*x+c-tanh(d*x+c))+3*b^2*a*(1/2*sinh(d*x+c)^
3/cosh(d*x+c)-3/2*d*x-3/2*c+3/2*tanh(d*x+c))+b^3*(1/4*sinh(d*x+c)^5/cosh(d
*x+c)-5/8*sinh(d*x+c)^3/cosh(d*x+c)+15/8*d*x+15/8*c-15/8*tanh(d*x+c)))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 178 vs. 2(86) = 172.

Time = 0.08 (sec) , antiderivative size = 178, normalized size of antiderivative = 1.93

$$\int \operatorname{sech}^2(c+dx) \left(a+b\sinh^2(c+dx)\right)^3 dx = \frac{b^3 \sinh(dx+c)^5 + \left(10\,b^3 \cosh(dx+c)^2 + 24\,ab^2 - 15\,b^3\right) \sinh(dx+c)^3 - 8\left(8\,a^3 - 24\,a^2b + 24\,ab^2 - 8\,b\right.}{}$$

input
```
integrate(sech(d*x+c)^2*(a+b*sinh(d*x+c)^2)^3,x, algorithm="fricas")
```

output
```
1/64*(b^3*sinh(d*x + c)^5 + (10*b^3*cosh(d*x + c)^2 + 24*a*b^2 - 15*b^3)*s
inh(d*x + c)^3 - 8*(8*a^3 - 24*a^2*b + 24*a*b^2 - 8*b^3 - 3*(8*a^2*b - 12*
a*b^2 + 5*b^3)*d*x)*cosh(d*x + c) + (5*b^3*cosh(d*x + c)^4 + 64*a^3 - 192*
a^2*b + 216*a*b^2 - 80*b^3 + 9*(8*a*b^2 - 5*b^3)*cosh(d*x + c)^2)*sinh(d*x
 + c))/(d*cosh(d*x + c))
```

Sympy [F(-1)]

Timed out.

$$\int \operatorname{sech}^2(c+dx) \left(a+b\sinh^2(c+dx)\right)^3 dx = \text{Timed out}$$

input
```
integrate(sech(d*x+c)**2*(a+b*sinh(d*x+c)**2)**3,x)
```

output
```
Timed out
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 215 vs. $2(86) = 172$.

Time = 0.08 (sec) , antiderivative size = 215, normalized size of antiderivative = 2.34

$$\int \operatorname{sech}^2(c+dx)\left(a+b\sinh^2(c+dx)\right)^3\,dx = 3\,a^2b\left(x+\frac{c}{d}-\frac{2}{d(e^{(-2\,dx-2\,c)}+1)}\right) + \frac{1}{64}\,b^3\left(\frac{120\,(dx+c)}{d}+\frac{16\,e^{(-2\,dx-2\,c)}-e^{(-4\,dx-4\,c)}}{d}-\frac{15\,e^{(-2\,dx-2\,c)}+144\,e^{(-4\,dx-4\,c)}-1}{d(e^{(-4\,dx-4\,c)}+e^{(-6\,dx-6\,c)})}\right) - \frac{3}{8}\,ab^2\left(\frac{12\,(dx+c)}{d}+\frac{e^{(-2\,dx-2\,c)}}{d}-\frac{17\,e^{(-2\,dx-2\,c)}+1}{d(e^{(-2\,dx-2\,c)}+e^{(-4\,dx-4\,c)})}\right)+\frac{2\,a^3}{d(e^{(-2\,dx-2\,c)}+1)}$$

input
```
integrate(sech(d*x+c)^2*(a+b*sinh(d*x+c)^2)^3,x, algorithm="maxima")
```

output
```
3*a^2*b*(x + c/d - 2/(d*(e^(-2*d*x - 2*c) + 1))) + 1/64*b^3*(120*(d*x + c)
/d + (16*e^(-2*d*x - 2*c) - e^(-4*d*x - 4*c))/d - (15*e^(-2*d*x - 2*c) + 1
44*e^(-4*d*x - 4*c) - 1)/(d*(e^(-4*d*x - 4*c) + e^(-6*d*x - 6*c)))) - 3/8*
a*b^2*(12*(d*x + c)/d + e^(-2*d*x - 2*c)/d - (17*e^(-2*d*x - 2*c) + 1)/(d*
(e^(-2*d*x - 2*c) + e^(-4*d*x - 4*c)))) + 2*a^3/(d*(e^(-2*d*x - 2*c) + 1))
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 197 vs. $2(86) = 172$.

Time = 0.18 (sec) , antiderivative size = 197, normalized size of antiderivative = 2.14

$$\int \operatorname{sech}^2(c+dx)\left(a+b\sinh^2(c+dx)\right)^3\,dx = \frac{b^3e^{(4\,dx+4\,c)}+24\,ab^2e^{(2\,dx+2\,c)}-16\,b^3e^{(2\,dx+2\,c)}+24\,(8\,a^2b-12\,ab^2+5\,b^3)(dx+c)-\left(144\,a^2be^{(4\,dx+4\,c)}-\right.}{64}$$

input
```
integrate(sech(d*x+c)^2*(a+b*sinh(d*x+c)^2)^3,x, algorithm="giac")
```

output
```
1/64*(b^3*e^(4*d*x + 4*c) + 24*a*b^2*e^(2*d*x + 2*c) - 16*b^3*e^(2*d*x + 2
*c) + 24*(8*a^2*b - 12*a*b^2 + 5*b^3)*(d*x + c) - (144*a^2*b*e^(4*d*x + 4*
c) - 216*a*b^2*e^(4*d*x + 4*c) + 90*b^3*e^(4*d*x + 4*c) + 24*a*b^2*e^(2*d*
x + 2*c) - 16*b^3*e^(2*d*x + 2*c) + b^3)*e^(-4*d*x - 4*c) - 128*(a^3 - 3*a
^2*b + 3*a*b^2 - b^3)/(e^(2*d*x + 2*c) + 1))/d
```

Mupad [B] (verification not implemented)

Time = 1.91 (sec) , antiderivative size = 141, normalized size of antiderivative = 1.53

$$\int \operatorname{sech}^2(c+dx) \left(a+b\sinh^2(c+dx)\right)^3 \, dx = \frac{b^3 \, e^{4\,c+4\,d\,x}}{64\,d} - \frac{b^3 \, e^{-4\,c-4\,d\,x}}{64\,d} - \frac{2\,(a^3-3\,a^2\,b+3\,a\,b^2-b^3)}{d\,\left(e^{2\,c+2\,d\,x}+1\right)} + \frac{3\,b\,x\,(8\,a^2-12\,a\,b+5\,b^2)}{8} - \frac{b^2\,e^{-2\,c-2\,d\,x}\,(3\,a-2\,b)}{8\,d} + \frac{b^2\,e^{2\,c+2\,d\,x}\,(3\,a-2\,b)}{8\,d}$$

input
```
int((a + b*sinh(c + d*x)^2)^3/cosh(c + d*x)^2,x)
```

output
```
(b^3*exp(4*c + 4*d*x))/(64*d) - (b^3*exp(- 4*c - 4*d*x))/(64*d) - (2*(3*a*
b^2 - 3*a^2*b + a^3 - b^3))/(d*(exp(2*c + 2*d*x) + 1)) + (3*b*x*(8*a^2 - 1
2*a*b + 5*b^2))/8 - (b^2*exp(- 2*c - 2*d*x)*(3*a - 2*b))/(8*d) + (b^2*exp(
2*c + 2*d*x)*(3*a - 2*b))/(8*d)
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 281, normalized size of antiderivative = 3.05

$$\int \operatorname{sech}^2(c+dx) \left(a+b\sinh^2(c+dx)\right)^3 \, dx = \frac{e^{10dx+10c}b^3+24e^{8dx+8c}a\,b^2-15e^{8dx+8c}b^3+128e^{6dx+6c}a^3+192e^{6dx+6c}a^2bdx-384e^{6dx+6c}a^2b-288e^{6dx+6}}{}$$

input
```
int(sech(d*x+c)^2*(a+b*sinh(d*x+c)^2)^3,x)
```

output
```
(e**(10*c + 10*d*x)*b**3 + 24*e**(8*c + 8*d*x)*a*b**2 - 15*e**(8*c + 8*d*x
)*b**3 + 128*e**(6*c + 6*d*x)*a**3 + 192*e**(6*c + 6*d*x)*a**2*b*d*x - 384
*e**(6*c + 6*d*x)*a**2*b - 288*e**(6*c + 6*d*x)*a*b**2*d*x + 432*e**(6*c +
 6*d*x)*a*b**2 + 120*e**(6*c + 6*d*x)*b**3*d*x - 160*e**(6*c + 6*d*x)*b**3
 + 192*e**(4*c + 4*d*x)*a**2*b*d*x - 288*e**(4*c + 4*d*x)*a*b**2*d*x + 120
*e**(4*c + 4*d*x)*b**3*d*x - 24*e**(2*c + 2*d*x)*a*b**2 + 15*e**(2*c + 2*d
*x)*b**3 - b**3)/(64*e**(4*c + 4*d*x)*d*(e**(2*c + 2*d*x) + 1))
```

3.274 $\int \text{sech}^3(c+dx)\left(a+b\sinh^2(c+dx)\right)^3\,dx$

Optimal result

Integrand size = 23, antiderivative size = 91

$$\int \text{sech}^3(c+dx)\left(a+b\sinh^2(c+dx)\right)^3\,dx = \frac{(a-b)^2(a+5b)\arctan(\sinh(c+dx))}{2d} + \frac{(3a-2b)b^2\sinh(c+dx)}{d} + \frac{b^3\sinh^3(c+dx)}{3d} + \frac{(a-b)^3\text{sech}(c+dx)\tanh(c+dx)}{2d}$$

output
```
1/2*(a-b)^2*(a+5*b)*arctan(sinh(d*x+c))/d+(3*a-2*b)*b^2*sinh(d*x+c)/d+1/3*
b^3*sinh(d*x+c)^3/d+1/2*(a-b)^3*sech(d*x+c)*tanh(d*x+c)/d
```

Mathematica [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 5.76 (sec) , antiderivative size = 347, normalized size of antiderivative = 3.81

$$\int \operatorname{sech}^3(c+dx)\left(a+b\sinh^2(c+dx)\right)^3\,dx$$

$$= \frac{\operatorname{csch}^5(c+dx)\left(-256\,{}_5F_4\left(\frac{3}{2},2,2,2,2;1,1,1,\frac{11}{2};-\sinh^2(c+dx)\right)\sinh^8(c+dx)\left(a+b\sinh^2(c+dx)\right)^3+\right.}{}$$

input
```
Integrate[Sech[c + d*x]^3*(a + b*Sinh[c + d*x]^2)^3,x]
```

output
```
(Csch[c + d*x]^5*(-256*HypergeometricPFQ[{3/2, 2, 2, 2, 2}, {1, 1, 1, 11/2
}, -Sinh[c + d*x]^2]*Sinh[c + d*x]^8*(a + b*Sinh[c + d*x]^2)^3 + 21*(36015
*a^3 + 5*a^2*(3224*a + 21609*b)*Sinh[c + d*x]^2 + 3*a*(491*a^2 + 16120*a*b
 + 36015*b^2)*Sinh[c + d*x]^4 + 3*b*(753*a^2 + 18280*a*b + 10805*b^2)*Sinh
[c + d*x]^6 + b^2*(2259*a + 17320*b)*Sinh[c + d*x]^8 + 753*b^3*Sinh[c + d*
x]^10) - (315*ArcTanh[Sqrt[-Sinh[c + d*x]^2]]*(2401*a^3 + 3*a^2*(625*a + 2
401*b)*Sinh[c + d*x]^2 + 3*a*(81*a^2 + 1875*a*b + 2401*b^2)*Sinh[c + d*x]^
4 + (-47*a^3 + 585*a^2*b + 6057*a*b^2 + 2161*b^3)*Sinh[c + d*x]^6 + 3*b*(a
^2 + 243*a*b + 625*b^2)*Sinh[c + d*x]^8 + 3*b^2*(a + 81*b)*Sinh[c + d*x]^1
0 + b^3*Sinh[c + d*x]^12))/Sqrt[-Sinh[c + d*x]^2]))/(30240*d)
```

Rubi [A] (verified)

Time = 0.34 (sec) , antiderivative size = 89, normalized size of antiderivative = 0.98, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.174$, Rules used = {3042, 3669, 300, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \operatorname{sech}^3(c+dx)\left(a+b\sinh^2(c+dx)\right)^3\,dx$$

$$\Big\downarrow \; 3042$$

$$\int \frac{\left(a-b\sin(ic+idx)^2\right)^3}{\cos(ic+idx)^3}dx$$

$$\Big\downarrow \text{3669}$$

$$\frac{\int \frac{\left(b\sinh^2(c+dx)+a\right)^3}{\left(\sinh^2(c+dx)+1\right)^2}d\sinh(c+dx)}{d}$$

$$\Big\downarrow \text{300}$$

$$\frac{\int \left(\sinh^2(c+dx)b^3+(3a-2b)b^2+\frac{3b\sinh^2(c+dx)(a-b)^2+(a+2b)(a-b)^2}{\left(\sinh^2(c+dx)+1\right)^2}\right)d\sinh(c+dx)}{d}$$

$$\Big\downarrow \text{2009}$$

$$\frac{\frac{1}{2}(a+5b)(a-b)^2\arctan(\sinh(c+dx))+b^2(3a-2b)\sinh(c+dx)+\frac{(a-b)^3\sinh(c+dx)}{2\left(\sinh^2(c+dx)+1\right)}+\frac{1}{3}b^3\sinh^3(c+dx)}{d}$$

input
```
Int[Sech[c + d*x]^3*(a + b*Sinh[c + d*x]^2)^3,x]
```

output
```
(((a - b)^2*(a + 5*b)*ArcTan[Sinh[c + d*x]])/2 + (3*a - 2*b)*b^2*Sinh[c +
d*x] + (b^3*Sinh[c + d*x]^3)/3 + ((a - b)^3*Sinh[c + d*x])/(2*(1 + Sinh[c
+ d*x]^2)))/d
```

Defintions of rubi rules used

rule 300
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_), x_Symbol] :> Int
[PolynomialDivide[(a + b*x^2)^p, (c + d*x^2)^(-q), x], x] /; FreeQ[{a, b, c
, d}, x] && NeQ[b*c - a*d, 0] && IGtQ[p, 0] && ILtQ[q, 0] && GeQ[p, -q]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3669
```
Int[cos[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^
(p_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Simp[ff/f   S
ubst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a + b*ff^2*x^2)^p, x], x, Sin[e + f*x]
/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m - 1)/2]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 218 vs. $2(85) = 170$.

Time = 0.17 (sec) , antiderivative size = 219, normalized size of antiderivative = 2.41

$$\frac{a^3\left(\frac{\operatorname{sech}(dx+c)\tanh(dx+c)}{2}+\arctan\left(e^{dx+c}\right)\right)+3a^2b\left(-\frac{\sinh(dx+c)}{\cosh(dx+c)^2}+\frac{\operatorname{sech}(dx+c)\tanh(dx+c)}{2}+\arctan\left(e^{dx+c}\right)\right)+3b^2\cdots}{}$$

input
```
int(sech(d*x+c)^3*(a+b*sinh(d*x+c)^2)^3,x)
```

output
```
1/d*(a^3*(1/2*sech(d*x+c)*tanh(d*x+c)+arctan(exp(d*x+c)))+3*a^2*b*(-sinh(d
*x+c)/cosh(d*x+c)^2+1/2*sech(d*x+c)*tanh(d*x+c)+arctan(exp(d*x+c)))+3*b^2*
a*(sinh(d*x+c)^3/cosh(d*x+c)^2+3*sinh(d*x+c)/cosh(d*x+c)^2-3/2*sech(d*x+c)
*tanh(d*x+c)-3*arctan(exp(d*x+c)))+b^3*(1/3*sinh(d*x+c)^5/cosh(d*x+c)^2-5/
3*sinh(d*x+c)^3/cosh(d*x+c)^2-5*sinh(d*x+c)/cosh(d*x+c)^2+5/2*sech(d*x+c)*
tanh(d*x+c)+5*arctan(exp(d*x+c))))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1679 vs. $2(85) = 170$.

Time = 0.10 (sec) , antiderivative size = 1679, normalized size of antiderivative = 18.45

$$\int \operatorname{sech}^3(c+dx)\left(a+b\sinh^2(c+dx)\right)^3\,dx = \text{Too large to display}$$

input
```
integrate(sech(d*x+c)^3*(a+b*sinh(d*x+c)^2)^3,x, algorithm="fricas")
```

output
```
1/24*(b^3*cosh(d*x + c)^10 + 10*b^3*cosh(d*x + c)*sinh(d*x + c)^9 + b^3*si
nh(d*x + c)^10 + (36*a*b^2 - 25*b^3)*cosh(d*x + c)^8 + (45*b^3*cosh(d*x +
c)^2 + 36*a*b^2 - 25*b^3)*sinh(d*x + c)^8 + 8*(15*b^3*cosh(d*x + c)^3 + (3
6*a*b^2 - 25*b^3)*cosh(d*x + c))*sinh(d*x + c)^7 + 2*(12*a^3 - 36*a^2*b +
54*a*b^2 - 25*b^3)*cosh(d*x + c)^6 + 2*(105*b^3*cosh(d*x + c)^4 + 12*a^3 -
 36*a^2*b + 54*a*b^2 - 25*b^3 + 14*(36*a*b^2 - 25*b^3)*cosh(d*x + c)^2)*si
nh(d*x + c)^6 + 4*(63*b^3*cosh(d*x + c)^5 + 14*(36*a*b^2 - 25*b^3)*cosh(d*
x + c)^3 + 3*(12*a^3 - 36*a^2*b + 54*a*b^2 - 25*b^3)*cosh(d*x + c))*sinh(d
*x + c)^5 - 2*(12*a^3 - 36*a^2*b + 54*a*b^2 - 25*b^3)*cosh(d*x + c)^4 + 2*
(105*b^3*cosh(d*x + c)^6 + 35*(36*a*b^2 - 25*b^3)*cosh(d*x + c)^4 - 12*a^3
 + 36*a^2*b - 54*a*b^2 + 25*b^3 + 15*(12*a^3 - 36*a^2*b + 54*a*b^2 - 25*b^
3)*cosh(d*x + c)^2)*sinh(d*x + c)^4 + 8*(15*b^3*cosh(d*x + c)^7 + 7*(36*a*
b^2 - 25*b^3)*cosh(d*x + c)^5 + 5*(12*a^3 - 36*a^2*b + 54*a*b^2 - 25*b^3)*
cosh(d*x + c)^3 - (12*a^3 - 36*a^2*b + 54*a*b^2 - 25*b^3)*cosh(d*x + c))*s
inh(d*x + c)^3 - b^3 - (36*a*b^2 - 25*b^3)*cosh(d*x + c)^2 + (45*b^3*cosh(
d*x + c)^8 + 28*(36*a*b^2 - 25*b^3)*cosh(d*x + c)^6 + 30*(12*a^3 - 36*a^2*
b + 54*a*b^2 - 25*b^3)*cosh(d*x + c)^4 - 36*a*b^2 + 25*b^3 - 12*(12*a^3 -
36*a^2*b + 54*a*b^2 - 25*b^3)*cosh(d*x + c)^2)*sinh(d*x + c)^2 + 24*((a^3
+ 3*a^2*b - 9*a*b^2 + 5*b^3)*cosh(d*x + c)^7 + 7*(a^3 + 3*a^2*b - 9*a*b^2
+ 5*b^3)*cosh(d*x + c)*sinh(d*x + c)^6 + (a^3 + 3*a^2*b - 9*a*b^2 + 5*b...
```

Sympy [F(-1)]

Timed out.

$$\int \operatorname{sech}^3(c+dx)\left(a+b\sinh^2(c+dx)\right)^3\,dx=\text{Timed out}$$

input
```
integrate(sech(d*x+c)**3*(a+b*sinh(d*x+c)**2)**3,x)
```

output
```
Timed out
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 357 vs. $2(85) = 170$.

Time = 0.12 (sec) , antiderivative size = 357, normalized size of antiderivative = 3.92

$$\int \operatorname{sech}^3(c+dx)\left(a+b\sinh^2(c+dx)\right)^3\,dx$$
$$=\frac{1}{24}b^3\left(\frac{27\,e^{(-dx-c)}-e^{(-3\,dx-3\,c)}}{d}-\frac{120\,\arctan\left(e^{(-dx-c)}\right)}{d}-\frac{25\,e^{(-2\,dx-2\,c)}+77\,e^{(-4\,dx-4\,c)}+3\,e^{(-6\,dx-6\,c)}-1}{d(e^{(-3\,dx-3\,c)}+2\,e^{(-5\,dx-5\,c)}+e^{(-7\,dx-7\,c)})}\right)$$
$$+\frac{3}{2}\,ab^2\left(\frac{6\,\arctan\left(e^{(-dx-c)}\right)}{d}-\frac{e^{(-dx-c)}}{d}+\frac{4\,e^{(-2\,dx-2\,c)}-e^{(-4\,dx-4\,c)}+1}{d(e^{(-dx-c)}+2\,e^{(-3\,dx-3\,c)}+e^{(-5\,dx-5\,c)})}\right)$$
$$-3\,a^2b\left(\frac{\arctan\left(e^{(-dx-c)}\right)}{d}+\frac{e^{(-dx-c)}-e^{(-3\,dx-3\,c)}}{d(2\,e^{(-2\,dx-2\,c)}+e^{(-4\,dx-4\,c)}+1)}\right)$$
$$-a^3\left(\frac{\arctan\left(e^{(-dx-c)}\right)}{d}-\frac{e^{(-dx-c)}-e^{(-3\,dx-3\,c)}}{d(2\,e^{(-2\,dx-2\,c)}+e^{(-4\,dx-4\,c)}+1)}\right)$$

input
```
integrate(sech(d*x+c)^3*(a+b*sinh(d*x+c)^2)^3,x, algorithm="maxima")
```

output
```
1/24*b^3*((27*e^(-d*x - c) - e^(-3*d*x - 3*c))/d - 120*arctan(e^(-d*x - c)
)/d - (25*e^(-2*d*x - 2*c) + 77*e^(-4*d*x - 4*c) + 3*e^(-6*d*x - 6*c) - 1)
/(d*(e^(-3*d*x - 3*c) + 2*e^(-5*d*x - 5*c) + e^(-7*d*x - 7*c)))) + 3/2*a*b
^2*(6*arctan(e^(-d*x - c))/d - e^(-d*x - c)/d + (4*e^(-2*d*x - 2*c) - e^(-
4*d*x - 4*c) + 1)/(d*(e^(-d*x - c) + 2*e^(-3*d*x - 3*c) + e^(-5*d*x - 5*c)
))) - 3*a^2*b*(arctan(e^(-d*x - c))/d + (e^(-d*x - c) - e^(-3*d*x - 3*c))/
(d*(2*e^(-2*d*x - 2*c) + e^(-4*d*x - 4*c) + 1))) - a^3*(arctan(e^(-d*x - c
))/d - (e^(-d*x - c) - e^(-3*d*x - 3*c))/(d*(2*e^(-2*d*x - 2*c) + e^(-4*d*
x - 4*c) + 1)))
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 247 vs. $2(85) = 170$.

Time = 0.17 (sec) , antiderivative size = 247, normalized size of antiderivative = 2.71

$$\int \operatorname{sech}^3(c+dx)\left(a+b\sinh^2(c+dx)\right)^3\,dx$$
$$= \frac{b^3\left(e^{(dx+c)}-e^{(-dx-c)}\right)^3+36\,ab^2\left(e^{(dx+c)}-e^{(-dx-c)}\right)-24\,b^3\left(e^{(dx+c)}-e^{(-dx-c)}\right)+6\left(\pi+2\arctan\left(\frac{1}{2}\left(e^{(2c}\right.\right.\right.}{}$$

input
```
integrate(sech(d*x+c)^3*(a+b*sinh(d*x+c)^2)^3,x, algorithm="giac")
```

output
```
1/24*(b^3*(e^(d*x + c) - e^(-d*x - c))^3 + 36*a*b^2*(e^(d*x + c) - e^(-d*x
 - c)) - 24*b^3*(e^(d*x + c) - e^(-d*x - c)) + 6*(pi + 2*arctan(1/2*(e^(2*
d*x + 2*c) - 1)*e^(-d*x - c)))*(a^3 + 3*a^2*b - 9*a*b^2 + 5*b^3) + 24*(a^3
*(e^(d*x + c) - e^(-d*x - c)) - 3*a^2*b*(e^(d*x + c) - e^(-d*x - c)) + 3*a
*b^2*(e^(d*x + c) - e^(-d*x - c)) - b^3*(e^(d*x + c) - e^(-d*x - c)))/((e^
(d*x + c) - e^(-d*x - c))^2 + 4))/d
```

Mupad [B] (verification not implemented)

Time = 3.25 (sec) , antiderivative size = 308, normalized size of antiderivative = 3.38

$$\int \operatorname{sech}^3(c+dx)\left(a+b\sinh^2(c+dx)\right)^3\,dx$$
$$= \frac{b^3\,e^{3\,c+3\,d\,x}}{24\,d} - \frac{b^3\,e^{-3\,c-3\,d\,x}}{24\,d} + \frac{3\,b^2\,e^{c+d\,x}\,(4\,a-3\,b)}{8\,d} + \frac{e^{c+d\,x}\,(a^3-3\,a^2\,b+3\,a\,b^2-b^3)}{d\,(e^{2\,c+2\,d\,x}+1)}$$
$$- \frac{2\,e^{c+d\,x}\,(a^3-3\,a^2\,b+3\,a\,b^2-b^3)}{d\,(2\,e^{2\,c+2\,d\,x}+e^{4\,c+4\,d\,x}+1)} - \frac{3\,b^2\,e^{-c-d\,x}\,(4\,a-3\,b)}{8\,d}$$
$$+ \frac{\ln\left(-e^{d\,x}\,e^{c}\,(a^3+3\,a^2\,b-9\,a\,b^2+5\,b^3)-(a-b)^2\,(a+5\,b)\,1i\right)\,(a-b)^2\,(a+5\,b)\,1i}{2\,d}$$
$$- \frac{\ln\left(-e^{d\,x}\,e^{c}\,(a^3+3\,a^2\,b-9\,a\,b^2+5\,b^3)+(a-b)^2\,(a+5\,b)\,1i\right)\,(a-b)^2\,(a+5\,b)\,1i}{2\,d}$$

input
```
int((a + b*sinh(c + d*x)^2)^3/cosh(c + d*x)^3,x)
```

output
```
(b^3*exp(3*c + 3*d*x))/(24*d) - (b^3*exp(- 3*c - 3*d*x))/(24*d) + (log(- (
a - b)^2*(a + 5*b)*1i - exp(d*x)*exp(c)*(3*a^2*b - 9*a*b^2 + a^3 + 5*b^3))
*(a - b)^2*(a + 5*b)*1i)/(2*d) - (log((a - b)^2*(a + 5*b)*1i - exp(d*x)*ex
p(c)*(3*a^2*b - 9*a*b^2 + a^3 + 5*b^3))*(a - b)^2*(a + 5*b)*1i)/(2*d) + (3
*b^2*exp(c + d*x)*(4*a - 3*b))/(8*d) + (exp(c + d*x)*(3*a*b^2 - 3*a^2*b +
a^3 - b^3))/(d*(exp(2*c + 2*d*x) + 1)) - (2*exp(c + d*x)*(3*a*b^2 - 3*a^2*
b + a^3 - b^3))/(d*(2*exp(2*c + 2*d*x) + exp(4*c + 4*d*x) + 1)) - (3*b^2*e
xp(- c - d*x)*(4*a - 3*b))/(8*d)
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 531, normalized size of antiderivative = 5.84

$$\int \operatorname{sech}^3(c+dx)\left(a+b\sinh^2(c+dx)\right)^3\,dx = \frac{-b^3+48e^{5dx+5c}atan\left(e^{dx+c}\right)a^3+240e^{5dx+5c}atan\left(e^{dx+c}\right)b^3+72e^{7dx+7c}atan\left(e^{dx+c}\right)a^2b-216e^{7dx+7c}atan}{}$$

input
```
int(sech(d*x+c)^3*(a+b*sinh(d*x+c)^2)^3,x)
```

output
```
(24*e**(7*c + 7*d*x)*atan(e**(c + d*x))*a**3 + 72*e**(7*c + 7*d*x)*atan(e*
*(c + d*x))*a**2*b - 216*e**(7*c + 7*d*x)*atan(e**(c + d*x))*a*b**2 + 120*
e**(7*c + 7*d*x)*atan(e**(c + d*x))*b**3 + 48*e**(5*c + 5*d*x)*atan(e**(c
+ d*x))*a**3 + 144*e**(5*c + 5*d*x)*atan(e**(c + d*x))*a**2*b - 432*e**(5*
c + 5*d*x)*atan(e**(c + d*x))*a*b**2 + 240*e**(5*c + 5*d*x)*atan(e**(c + d
*x))*b**3 + 24*e**(3*c + 3*d*x)*atan(e**(c + d*x))*a**3 + 72*e**(3*c + 3*d
*x)*atan(e**(c + d*x))*a**2*b - 216*e**(3*c + 3*d*x)*atan(e**(c + d*x))*a*
b**2 + 120*e**(3*c + 3*d*x)*atan(e**(c + d*x))*b**3 + e**(10*c + 10*d*x)*b
**3 + 36*e**(8*c + 8*d*x)*a*b**2 - 25*e**(8*c + 8*d*x)*b**3 + 24*e**(6*c +
 6*d*x)*a**3 - 72*e**(6*c + 6*d*x)*a**2*b + 108*e**(6*c + 6*d*x)*a*b**2 -
50*e**(6*c + 6*d*x)*b**3 - 24*e**(4*c + 4*d*x)*a**3 + 72*e**(4*c + 4*d*x)*
a**2*b - 108*e**(4*c + 4*d*x)*a*b**2 + 50*e**(4*c + 4*d*x)*b**3 - 36*e**(2
*c + 2*d*x)*a*b**2 + 25*e**(2*c + 2*d*x)*b**3 - b**3)/(24*e**(3*c + 3*d*x)
*d*(e**(4*c + 4*d*x) + 2*e**(2*c + 2*d*x) + 1))
```

3.275 $\int \operatorname{sech}^4(c+dx) \left(a+b\sinh^2(c+dx)\right)^3 \, dx$

Optimal result

Integrand size = 23, antiderivative size = 82

$$\int \operatorname{sech}^4(c+dx) \left(a+b\sinh^2(c+dx)\right)^3 \, dx = \frac{1}{2}(6a-5b)b^2x+\frac{b^3\cosh(c+dx)\sinh(c+dx)}{2d} + \frac{(a-b)^2(a+2b)\tanh(c+dx)}{d} - \frac{(a-b)^3\tanh^3(c+dx)}{3d}$$

output
```
1/2*(6*a-5*b)*b^2*x+1/2*b^3*cosh(d*x+c)*sinh(d*x+c)/d+(a-b)^2*(a+2*b)*tanh
(d*x+c)/d-1/3*(a-b)^3*tanh(d*x+c)^3/d
```

Mathematica [A] (verified)

Time = 1.93 (sec) , antiderivative size = 84, normalized size of antiderivative = 1.02

$$\int \operatorname{sech}^4(c+dx) \left(a+b\sinh^2(c+dx)\right)^3 \, dx = \frac{6(6a-5b)b^2(c+dx)+3b^3\sinh(2(c+dx))+2(a-b)^2(4a+5b+(2a+7b)\cosh(2(c+dx)))\operatorname{sech}^2(c+d}{12d}$$

input
```
Integrate[Sech[c + d*x]^4*(a + b*Sinh[c + d*x]^2)^3,x]
```

output
```
(6*(6*a - 5*b)*b^2*(c + d*x) + 3*b^3*Sinh[2*(c + d*x)] + 2*(a - b)^2*(4*a
+ 5*b + (2*a + 7*b)*Cosh[2*(c + d*x)])*Sech[c + d*x]^2*Tanh[c + d*x])/(12*
d)
```

Rubi [A] (verified)

Time = 0.33 (sec) , antiderivative size = 91, normalized size of antiderivative = 1.11, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.174$, Rules used = {3042, 3670, 300, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \operatorname{sech}^4(c+dx)\left(a+b\sinh^2(c+dx)\right)^3 dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\left(a-b\sin(ic+idx)^2\right)^3}{\cos(ic+idx)^4}dx$$

$$\Big\downarrow \text{3670}$$

$$\frac{\int \frac{\left(a-(a-b)\tanh^2(c+dx)\right)^3}{\left(1-\tanh^2(c+dx)\right)^2}d\tanh(c+dx)}{d}$$

$$\Big\downarrow \text{300}$$

$$\frac{\int \left(-\tanh^2(c+dx)(a-b)^3+(a+2b)(a-b)^2+\frac{(3a-2b)b^2-3(a-b)b^2\tanh^2(c+dx)}{\left(1-\tanh^2(c+dx)\right)^2}\right)d\tanh(c+dx)}{d}$$

$$\Big\downarrow \text{2009}$$

$$\frac{\frac{1}{2}b^2(6a-5b)\text{arctanh}(\tanh(c+dx))-\frac{1}{3}(a-b)^3\tanh^3(c+dx)+(a-b)^2(a+2b)\tanh(c+dx)+\frac{b^3\tanh(c+dx)}{2\left(1-\tanh^2(c+dx\right)}}{d}$$

input
```
Int[Sech[c + d*x]^4*(a + b*Sinh[c + d*x]^2)^3,x]
```

output
```
(((6*a - 5*b)*b^2*ArcTanh[Tanh[c + d*x]])/2 + (a - b)^2*(a + 2*b)*Tanh[c +
 d*x] - ((a - b)^3*Tanh[c + d*x]^3)/3 + (b^3*Tanh[c + d*x])/(2*(1 - Tanh[c
 + d*x]^2)))/d
```

Defintions of rubi rules used

rule 300
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_), x_Symbol] :> Int
[PolynomialDivide[(a + b*x^2)^p, (c + d*x^2)^(-q), x], x] /; FreeQ[{a, b, c
, d}, x] && NeQ[b*c - a*d, 0] && IGtQ[p, 0] && ILtQ[q, 0] && GeQ[p, -q]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3670
```
Int[cos[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(
p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Simp[ff/f   Su
bst[Int[(a + (a + b)*ff^2*x^2)^p/(1 + ff^2*x^2)^(m/2 + p + 1), x], x, Tan[e
 + f*x]/ff], x]] /; FreeQ[{a, b, e, f}, x] && IntegerQ[m/2] && IntegerQ[p]
```

Maple [A] (verified)

Time = 0.20 (sec) , antiderivative size = 148, normalized size of antiderivative = 1.80

$$\frac{a^3\left(\frac{2}{3}+\frac{\operatorname{sech}(dx+c)^2}{3}\right)\tanh\left(dx+c\right)+3a^2b\left(-\frac{\sinh(dx+c)}{2\cosh(dx+c)^3}+\frac{\left(\frac{2}{3}+\frac{\operatorname{sech}(dx+c)^2}{3}\right)\tanh(dx+c)}{2}\right)+3b^2a\left(dx+c-\tanh\right.}{d}$$

input
```
int(sech(d*x+c)^4*(a+b*sinh(d*x+c)^2)^3,x)
```

output
```
1/d*(a^3*(2/3+1/3*sech(d*x+c)^2)*tanh(d*x+c)+3*a^2*b*(-1/2*sinh(d*x+c)/cos
h(d*x+c)^3+1/2*(2/3+1/3*sech(d*x+c)^2)*tanh(d*x+c))+3*b^2*a*(d*x+c-tanh(d*
x+c)-1/3*tanh(d*x+c)^3)+b^3*(1/2*sinh(d*x+c)^5/cosh(d*x+c)^3-5/2*d*x-5/2*c
+5/2*tanh(d*x+c)+5/6*tanh(d*x+c)^3))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 321 vs. $2(76) = 152$.

Time = 0.11 (sec) , antiderivative size = 321, normalized size of antiderivative = 3.91

$$\int \operatorname{sech}^4(c+dx)\left(a+b\sinh^2(c+dx)\right)^3\,dx$$
$$= \frac{3\,b^3\sinh\left(dx+c\right)^5-4\left(4\,a^3+6\,a^2b-24\,ab^2+14\,b^3-3\left(6\,ab^2-5\,b^3\right)dx\right)\cosh\left(dx+c\right)^3-12\left(4\,a^3+6\,a^2\right.}{}$$

input
```
integrate(sech(d*x+c)^4*(a+b*sinh(d*x+c)^2)^3,x, algorithm="fricas")
```

output
```
1/24*(3*b^3*sinh(d*x + c)^5 - 4*(4*a^3 + 6*a^2*b - 24*a*b^2 + 14*b^3 - 3*(
6*a*b^2 - 5*b^3)*d*x)*cosh(d*x + c)^3 - 12*(4*a^3 + 6*a^2*b - 24*a*b^2 + 1
4*b^3 - 3*(6*a*b^2 - 5*b^3)*d*x)*cosh(d*x + c)*sinh(d*x + c)^2 + (30*b^3*c
osh(d*x + c)^2 + 16*a^3 + 24*a^2*b - 96*a*b^2 + 65*b^3)*sinh(d*x + c)^3 -
12*(4*a^3 + 6*a^2*b - 24*a*b^2 + 14*b^3 - 3*(6*a*b^2 - 5*b^3)*d*x)*cosh(d*
x + c) + 3*(5*b^3*cosh(d*x + c)^4 + 16*a^3 - 24*a^2*b + 10*b^3 + (16*a^3 +
 24*a^2*b - 96*a*b^2 + 65*b^3)*cosh(d*x + c)^2)*sinh(d*x + c))/(d*cosh(d*x
 + c)^3 + 3*d*cosh(d*x + c)*sinh(d*x + c)^2 + 3*d*cosh(d*x + c))
```

Sympy [F(-1)]

Timed out.

$$\int \operatorname{sech}^4(c+dx)\left(a+b\sinh^2(c+dx)\right)^3\,dx = \text{Timed out}$$

input
```
integrate(sech(d*x+c)**4*(a+b*sinh(d*x+c)**2)**3,x)
```

output
```
Timed out
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 382 vs. $2(76) = 152$.

Time = 0.06 (sec) , antiderivative size = 382, normalized size of antiderivative = 4.66

$$
\begin{aligned}
&\int \operatorname{sech}^4(c+dx)\left(a+b\sinh^2(c+dx)\right)^3\,dx \\
&= ab^2\left(3\,x+\frac{3\,c}{d}-\frac{4\left(3\,e^{(-2\,dx-2\,c)}+3\,e^{(-4\,dx-4\,c)}+2\right)}{d(3\,e^{(-2\,dx-2\,c)}+3\,e^{(-4\,dx-4\,c)}+e^{(-6\,dx-6\,c)}+1)}\right) \\
&\quad -\frac{1}{24}\,b^3\left(\frac{60\,(dx+c)}{d}+\frac{3\,e^{(-2\,dx-2\,c)}}{d}-\frac{121\,e^{(-2\,dx-2\,c)}+201\,e^{(-4\,dx-4\,c)}+147\,e^{(-6\,dx-6\,c)}+3}{d(e^{(-2\,dx-2\,c)}+3\,e^{(-4\,dx-4\,c)}+3\,e^{(-6\,dx-6\,c)}+e^{(-8\,dx-8\,c)})}\right) \\
&\quad +\frac{4}{3}\,a^3\left(\frac{3\,e^{(-2\,dx-2\,c)}}{d(3\,e^{(-2\,dx-2\,c)}+3\,e^{(-4\,dx-4\,c)}+e^{(-6\,dx-6\,c)}+1)}+\frac{1}{d(3\,e^{(-2\,dx-2\,c)}+3\,e^{(-4\,dx-4\,c)}+e^{(-6\,dx-6\,c)}+1)}\right) \\
&\quad +2\,a^2b\left(\frac{3\,e^{(-4\,dx-4\,c)}}{d(3\,e^{(-2\,dx-2\,c)}+3\,e^{(-4\,dx-4\,c)}+e^{(-6\,dx-6\,c)}+1)}+\frac{1}{d(3\,e^{(-2\,dx-2\,c)}+3\,e^{(-4\,dx-4\,c)}+e^{(-6\,dx-6\,c)}+1)}\right)
\end{aligned}
$$

input
```
integrate(sech(d*x+c)^4*(a+b*sinh(d*x+c)^2)^3,x, algorithm="maxima")
```

output
```
a*b^2*(3*x + 3*c/d - 4*(3*e^(-2*d*x - 2*c) + 3*e^(-4*d*x - 4*c) + 2)/(d*(3
*e^(-2*d*x - 2*c) + 3*e^(-4*d*x - 4*c) + e^(-6*d*x - 6*c) + 1))) - 1/24*b^
3*(60*(d*x + c)/d + 3*e^(-2*d*x - 2*c)/d - (121*e^(-2*d*x - 2*c) + 201*e^(
-4*d*x - 4*c) + 147*e^(-6*d*x - 6*c) + 3)/(d*(e^(-2*d*x - 2*c) + 3*e^(-4*d
*x - 4*c) + 3*e^(-6*d*x - 6*c) + e^(-8*d*x - 8*c)))) + 4/3*a^3*(3*e^(-2*d*
x - 2*c)/(d*(3*e^(-2*d*x - 2*c) + 3*e^(-4*d*x - 4*c) + e^(-6*d*x - 6*c) +
1)) + 1/(d*(3*e^(-2*d*x - 2*c) + 3*e^(-4*d*x - 4*c) + e^(-6*d*x - 6*c) + 1
))) + 2*a^2*b*(3*e^(-4*d*x - 4*c)/(d*(3*e^(-2*d*x - 2*c) + 3*e^(-4*d*x - 4
*c) + e^(-6*d*x - 6*c) + 1)) + 1/(d*(3*e^(-2*d*x - 2*c) + 3*e^(-4*d*x - 4*
c) + e^(-6*d*x - 6*c) + 1)))
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 208 vs. $2(76) = 152$.

Time = 0.18 (sec) , antiderivative size = 208, normalized size of antiderivative = 2.54

$$\int \operatorname{sech}^4(c+dx)\left(a+b\sinh^2(c+dx)\right)^3 dx$$
$$= \frac{3\,b^3e^{(2\,dx+2\,c)} + 12\,(6\,ab^2-5\,b^3)(dx+c) - 3\left(12\,ab^2e^{(2\,dx+2\,c)} - 10\,b^3e^{(2\,dx+2\,c)} + b^3\right)e^{(-2\,dx-2\,c)} - \frac{16\,(9\,a^2be^{(4}}{}}{24\,d}$$

input
```
integrate(sech(d*x+c)^4*(a+b*sinh(d*x+c)^2)^3,x, algorithm="giac")
```

output
```
1/24*(3*b^3*e^(2*d*x + 2*c) + 12*(6*a*b^2 - 5*b^3)*(d*x + c) - 3*(12*a*b^2
*e^(2*d*x + 2*c) - 10*b^3*e^(2*d*x + 2*c) + b^3)*e^(-2*d*x - 2*c) - 16*(9*
a^2*b*e^(4*d*x + 4*c) - 18*a*b^2*e^(4*d*x + 4*c) + 9*b^3*e^(4*d*x + 4*c) +
 6*a^3*e^(2*d*x + 2*c) - 18*a*b^2*e^(2*d*x + 2*c) + 12*b^3*e^(2*d*x + 2*c)
 + 2*a^3 + 3*a^2*b - 12*a*b^2 + 7*b^3)/(e^(2*d*x + 2*c) + 1)^3)/d
```

Mupad [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 273, normalized size of antiderivative = 3.33

$$\int \operatorname{sech}^4(c+dx)\left(a+b\sinh^2(c+dx)\right)^3 dx$$
$$= \frac{b^2\,x\,(6\,a-5\,b)}{2} - \frac{\frac{2\,(a^2\,b-2\,a\,b^2+b^3)}{d} + \frac{2\,\mathrm{e}^{4\,c+4\,d\,x}\,(a^2\,b-2\,a\,b^2+b^3)}{d} + \frac{4\,\mathrm{e}^{2\,c+2\,d\,x}\,(2\,a^3-3\,a^2\,b+b^3)}{3\,d}}{3\,\mathrm{e}^{2\,c+2\,d\,x}+3\,\mathrm{e}^{4\,c+4\,d\,x}+\mathrm{e}^{6\,c+6\,d\,x}+1}$$
$$- \frac{\frac{2\,(2\,a^3-3\,a^2\,b+b^3)}{3\,d} + \frac{2\,\mathrm{e}^{2\,c+2\,d\,x}\,(a^2\,b-2\,a\,b^2+b^3)}{d}}{2\,\mathrm{e}^{2\,c+2\,d\,x}+\mathrm{e}^{4\,c+4\,d\,x}+1}$$
$$- \frac{b^3\,\mathrm{e}^{-2\,c-2\,d\,x}}{8\,d} + \frac{b^3\,\mathrm{e}^{2\,c+2\,d\,x}}{8\,d} - \frac{2\,(a^2\,b-2\,a\,b^2+b^3)}{d\,(\mathrm{e}^{2\,c+2\,d\,x}+1)}$$

input
```
int((a + b*sinh(c + d*x)^2)^3/cosh(c + d*x)^4,x)
```

output
```
(b^2*x*(6*a - 5*b))/2 - ((2*(a^2*b - 2*a*b^2 + b^3))/d + (2*exp(4*c + 4*d*
x)*(a^2*b - 2*a*b^2 + b^3))/d + (4*exp(2*c + 2*d*x)*(2*a^3 - 3*a^2*b + b^3
))/(3*d))/(3*exp(2*c + 2*d*x) + 3*exp(4*c + 4*d*x) + exp(6*c + 6*d*x) + 1)
 - ((2*(2*a^3 - 3*a^2*b + b^3))/(3*d) + (2*exp(2*c + 2*d*x)*(a^2*b - 2*a*b
^2 + b^3))/d)/(2*exp(2*c + 2*d*x) + exp(4*c + 4*d*x) + 1) - (b^3*exp(- 2*c
 - 2*d*x))/(8*d) + (b^3*exp(2*c + 2*d*x))/(8*d) - (2*(a^2*b - 2*a*b^2 + b^
3))/(d*(exp(2*c + 2*d*x) + 1))
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 355, normalized size of antiderivative = 4.33

$$\int \operatorname{sech}^4(c+dx)\left(a+b\sinh^2(c+dx)\right)^3\,dx$$
$$= \frac{3e^{10dx+10c}b^3+48e^{8dx+8c}a^2b+72e^{8dx+8c}a\,b^2dx-96e^{8dx+8c}a\,b^2-60e^{8dx+8c}b^3dx+55e^{8dx+8c}b^3+216e^{6dx+6}}{}$$

input
```
int(sech(d*x+c)^4*(a+b*sinh(d*x+c)^2)^3,x)
```

output
```
(3*e**(10*c + 10*d*x)*b**3 + 48*e**(8*c + 8*d*x)*a**2*b + 72*e**(8*c + 8*d
*x)*a*b**2*d*x - 96*e**(8*c + 8*d*x)*a*b**2 - 60*e**(8*c + 8*d*x)*b**3*d*x
 + 55*e**(8*c + 8*d*x)*b**3 + 216*e**(6*c + 6*d*x)*a*b**2*d*x - 180*e**(6*
c + 6*d*x)*b**3*d*x - 96*e**(4*c + 4*d*x)*a**3 + 144*e**(4*c + 4*d*x)*a**2
*b + 216*e**(4*c + 4*d*x)*a*b**2*d*x - 180*e**(4*c + 4*d*x)*b**3*d*x - 60*
e**(4*c + 4*d*x)*b**3 - 32*e**(2*c + 2*d*x)*a**3 + 72*e**(2*c + 2*d*x)*a*b
**2*d*x + 96*e**(2*c + 2*d*x)*a*b**2 - 60*e**(2*c + 2*d*x)*b**3*d*x - 75*e
**(2*c + 2*d*x)*b**3 - 3*b**3)/(24*e**(2*c + 2*d*x)*d*(e**(6*c + 6*d*x) +
3*e**(4*c + 4*d*x) + 3*e**(2*c + 2*d*x) + 1))
```

3.276 $\int \operatorname{sech}^5(c+dx) \left(a+b\sinh^2(c+dx)\right)^3 dx$

Optimal result

Integrand size = 23, antiderivative size = 103

$$\int \operatorname{sech}^5(c+dx) \left(a+b\sinh^2(c+dx)\right)^3 dx = \frac{3(a-b)\left(4b^2+(a+b)^2\right)\arctan(\sinh(c+dx))}{8d} + \frac{b^3\sinh(c+dx)}{d} + \frac{3(a-b)^2(a+3b)\operatorname{sech}(c+dx)\tanh(c+dx)}{8d} + \frac{(a-b)^3\operatorname{sech}^3(c+dx)\tanh(c+dx)}{4d}$$

output
```
3/8*(a-b)*(4*b^2+(a+b)^2)*arctan(sinh(d*x+c))/d+b^3*sinh(d*x+c)/d+3/8*(a-b
)^2*(a+3*b)*sech(d*x+c)*tanh(d*x+c)/d+1/4*(a-b)^3*sech(d*x+c)^3*tanh(d*x+c
)/d
```

Mathematica [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 9.37 (sec) , antiderivative size = 472, normalized size of antiderivative = 4.58

$$\int \operatorname{sech}^5(c+dx) \left(a+b\sinh^2(c+dx)\right)^3 dx = -\frac{\operatorname{csch}^5(c+dx)\left(256\,{}_6F_5\left(\frac{3}{2},2,2,2,2,2;1,1,1,1,\frac{11}{2};-\sinh^2(c+dx)\right)\sinh^8(c+dx)\left(a+b\sinh^2(c+dx\right)\right.}{}$$

input
```
Integrate[Sech[c + d*x]^5*(a + b*Sinh[c + d*x]^2)^3,x]
```

output
```
-1/60480*(Csch[c + d*x]^5*(256*HypergeometricPFQ[{3/2, 2, 2, 2, 2, 2}, {1,
 1, 1, 1, 11/2}, -Sinh[c + d*x]^2]*Sinh[c + d*x]^8*(a + b*Sinh[c + d*x]^2)
^3 + 384*HypergeometricPFQ[{3/2, 2, 2, 2, 2}, {1, 1, 1, 11/2}, -Sinh[c + d
*x]^2]*Sinh[c + d*x]^8*(a + b*Sinh[c + d*x]^2)^2*(7*a + 5*b*Sinh[c + d*x]^
2) - 21*(15*a*b^2*Sinh[c + d*x]^4*(36015 + 21529*Sinh[c + d*x]^2 + 1128*Si
nh[c + d*x]^4) + 9*a^2*b*Sinh[c + d*x]^2*(72030 + 41615*Sinh[c + d*x]^2 +
2131*Sinh[c + d*x]^4) + b^3*Sinh[c + d*x]^6*(149460 + 90805*Sinh[c + d*x]^
2 + 4887*Sinh[c + d*x]^4) + a^3*(252105 + 140965*Sinh[c + d*x]^2 + 8226*Si
nh[c + d*x]^4)) + (315*ArcTanh[Sqrt[-Sinh[c + d*x]^2]]*(a^3*(16807 + 15000
*Sinh[c + d*x]^2 + 2187*Sinh[c + d*x]^4 - 62*Sinh[c + d*x]^6) + 9*a^2*b*Si
nh[c + d*x]^2*(4802 + 4375*Sinh[c + d*x]^2 + 640*Sinh[c + d*x]^4 + 3*Sinh[
c + d*x]^6) + b^3*Sinh[c + d*x]^6*(9964 + 9375*Sinh[c + d*x]^2 + 1458*Sinh
[c + d*x]^4 + 7*Sinh[c + d*x]^6) + 3*a*b^2*Sinh[c + d*x]^4*(12005 + 11178*
Sinh[c + d*x]^2 + 1701*Sinh[c + d*x]^4 + 8*Sinh[c + d*x]^6)))/Sqrt[-Sinh[c
 + d*x]^2]))/d
```

Rubi [A] (verified)

Time = 0.35 (sec) , antiderivative size = 105, normalized size of antiderivative = 1.02, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.174$, Rules used = {3042, 3669, 300, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \operatorname{sech}^5(c+dx)\left(a+b\sinh^2(c+dx)\right)^3\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\left(a-b\sin(ic+idx)^2\right)^3}{\cos(ic+idx)^5}dx$$

$$\Big\downarrow \text{3669}$$

$$\frac{\int \frac{\left(b\sinh^2(c+dx)+a\right)^3}{\left(\sinh^2(c+dx)+1\right)^3}d\sinh(c+dx)}{d}$$

$$\Big\downarrow \text{300}$$

$$\frac{\int \left(b^3+\frac{3(a-b)b^2\sinh^4(c+dx)+3b(a^2-b^2)\sinh^2(c+dx)+a^3-b^3}{\left(\sinh^2(c+dx)+1\right)^3}\right)d\sinh(c+dx)}{d}$$

$$\Big\downarrow \text{2009}$$

$$\frac{\frac{3}{8}\left((a+b)^2+4b^2\right)(a-b)\arctan(\sinh(c+dx))+\frac{(a-b)^3\sinh(c+dx)}{4\left(\sinh^2(c+dx)+1\right)^2}+\frac{3(a+3b)(a-b)^2\sinh(c+dx)}{8\left(\sinh^2(c+dx)+1\right)}+b^3\sinh(c+dx)}{d}$$

input
```
Int[Sech[c + d*x]^5*(a + b*Sinh[c + d*x]^2)^3,x]
```

output
```
((3*(a - b)*(4*b^2 + (a + b)^2)*ArcTan[Sinh[c + d*x]])/8 + b^3*Sinh[c + d*
x] + ((a - b)^3*Sinh[c + d*x])/(4*(1 + Sinh[c + d*x]^2)^2) + (3*(a - b)^2*
(a + 3*b)*Sinh[c + d*x])/(8*(1 + Sinh[c + d*x]^2)))/d
```

Defintions of rubi rules used

rule 300
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_), x_Symbol] :> Int
[PolynomialDivide[(a + b*x^2)^p, (c + d*x^2)^(-q), x], x] /; FreeQ[{a, b, c
, d}, x] && NeQ[b*c - a*d, 0] && IGtQ[p, 0] && ILtQ[q, 0] && GeQ[p, -q]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3669
```
Int[cos[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^
(p_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Simp[ff/f    S
ubst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a + b*ff^2*x^2)^p, x], x, Sin[e + f*x]
/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m - 1)/2]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 272 vs. $2(97) = 194$.

Time = 0.18 (sec) , antiderivative size = 273, normalized size of antiderivative = 2.65

$$a^3\left(\left(\frac{\operatorname{sech}(dx+c)^3}{4}+\frac{3\operatorname{sech}(dx+c)}{8}\right)\tanh(dx+c)+\frac{3\arctan\left(e^{dx+c}\right)}{4}\right)+3a^2b\left(-\frac{\sinh(dx+c)}{3\cosh(dx+c)^4}+\frac{\left(\frac{\operatorname{sech}(dx+c)^3}{4}+\frac{3\operatorname{sech}(dx+c)}{8}\right)}{3}\right.$$

input
```
int(sech(d*x+c)^5*(a+b*sinh(d*x+c)^2)^3,x)
```

output
```
1/d*(a^3*((1/4*sech(d*x+c)^3+3/8*sech(d*x+c))*tanh(d*x+c)+3/4*arctan(exp(d
*x+c)))+3*a^2*b*(-1/3*sinh(d*x+c)/cosh(d*x+c)^4+1/3*(1/4*sech(d*x+c)^3+3/8
*sech(d*x+c))*tanh(d*x+c)+1/4*arctan(exp(d*x+c))))+3*b^2*a*(-sinh(d*x+c)^3/
cosh(d*x+c)^4-sinh(d*x+c)/cosh(d*x+c)^4+(1/4*sech(d*x+c)^3+3/8*sech(d*x+c)
)*tanh(d*x+c)+3/4*arctan(exp(d*x+c)))+b^3*(sinh(d*x+c)^5/cosh(d*x+c)^4+5*s
inh(d*x+c)^3/cosh(d*x+c)^4+5*sinh(d*x+c)/cosh(d*x+c)^4-5*(1/4*sech(d*x+c)^
3+3/8*sech(d*x+c))*tanh(d*x+c)-15/4*arctan(exp(d*x+c))))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2245 vs. $2(97) = 194$.

Time = 0.11 (sec) , antiderivative size = 2245, normalized size of antiderivative = 21.80

$$\int \operatorname{sech}^5(c+dx)\left(a+b\sinh^2(c+dx)\right)^3\,dx=\text{Too large to display}$$

input
```
integrate(sech(d*x+c)^5*(a+b*sinh(d*x+c)^2)^3,x, algorithm="fricas")
```

output
```
1/4*(2*b^3*cosh(d*x + c)^10 + 20*b^3*cosh(d*x + c)*sinh(d*x + c)^9 + 2*b^3
*sinh(d*x + c)^10 + 3*(a^3 + a^2*b - 5*a*b^2 + 5*b^3)*cosh(d*x + c)^8 + 3*
(30*b^3*cosh(d*x + c)^2 + a^3 + a^2*b - 5*a*b^2 + 5*b^3)*sinh(d*x + c)^8 +
 24*(10*b^3*cosh(d*x + c)^3 + (a^3 + a^2*b - 5*a*b^2 + 5*b^3)*cosh(d*x + c
))*sinh(d*x + c)^7 + (11*a^3 - 21*a^2*b + 9*a*b^2 + 5*b^3)*cosh(d*x + c)^6
 + (420*b^3*cosh(d*x + c)^4 + 11*a^3 - 21*a^2*b + 9*a*b^2 + 5*b^3 + 84*(a^
3 + a^2*b - 5*a*b^2 + 5*b^3)*cosh(d*x + c)^2)*sinh(d*x + c)^6 + 6*(84*b^3*
cosh(d*x + c)^5 + 28*(a^3 + a^2*b - 5*a*b^2 + 5*b^3)*cosh(d*x + c)^3 + (11
*a^3 - 21*a^2*b + 9*a*b^2 + 5*b^3)*cosh(d*x + c))*sinh(d*x + c)^5 - (11*a^
3 - 21*a^2*b + 9*a*b^2 + 5*b^3)*cosh(d*x + c)^4 + (420*b^3*cosh(d*x + c)^6
 + 210*(a^3 + a^2*b - 5*a*b^2 + 5*b^3)*cosh(d*x + c)^4 - 11*a^3 + 21*a^2*b
 - 9*a*b^2 - 5*b^3 + 15*(11*a^3 - 21*a^2*b + 9*a*b^2 + 5*b^3)*cosh(d*x + c
)^2)*sinh(d*x + c)^4 + 4*(60*b^3*cosh(d*x + c)^7 + 42*(a^3 + a^2*b - 5*a*b
^2 + 5*b^3)*cosh(d*x + c)^5 + 5*(11*a^3 - 21*a^2*b + 9*a*b^2 + 5*b^3)*cosh
(d*x + c)^3 - (11*a^3 - 21*a^2*b + 9*a*b^2 + 5*b^3)*cosh(d*x + c))*sinh(d*
x + c)^3 - 2*b^3 - 3*(a^3 + a^2*b - 5*a*b^2 + 5*b^3)*cosh(d*x + c)^2 + 3*(
30*b^3*cosh(d*x + c)^8 + 28*(a^3 + a^2*b - 5*a*b^2 + 5*b^3)*cosh(d*x + c)^
6 + 5*(11*a^3 - 21*a^2*b + 9*a*b^2 + 5*b^3)*cosh(d*x + c)^4 - a^3 - a^2*b
+ 5*a*b^2 - 5*b^3 - 2*(11*a^3 - 21*a^2*b + 9*a*b^2 + 5*b^3)*cosh(d*x + c)^
2)*sinh(d*x + c)^2 + 3*((a^3 + a^2*b + 3*a*b^2 - 5*b^3)*cosh(d*x + c)^9...
```

Sympy [F(-1)]

Timed out.

$$\int \operatorname{sech}^5(c+dx) \left(a+b\sinh^2(c+dx)\right)^3 \, dx = \text{Timed out}$$

input
```
integrate(sech(d*x+c)**5*(a+b*sinh(d*x+c)**2)**3,x)
```

output
```
Timed out
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 489 vs. $2(97) = 194$.

Time = 0.14 (sec) , antiderivative size = 489, normalized size of antiderivative = 4.75

$$\int \operatorname{sech}^5(c+dx)\left(a+b\sinh^2(c+dx)\right)^3\,dx$$
$$= \frac{1}{4}b^3\left(\frac{15\arctan\left(e^{(-dx-c)}\right)}{d} - \frac{2e^{(-dx-c)}}{d} + \frac{17e^{(-2dx-2c)}+13e^{(-4dx-4c)}+7e^{(-6dx-6c)}-7e^{(-8dx-8c)}+2}{d\left(e^{(-dx-c)}+4e^{(-3dx-3c)}+6e^{(-5dx-5c)}+4e^{(-7dx-7c)}+e^{(-9dx-9c)}\right)}\right)$$
$$-\frac{3}{4}ab^2\left(\frac{3\arctan\left(e^{(-dx-c)}\right)}{d} + \frac{5e^{(-dx-c)}-3e^{(-3dx-3c)}+3e^{(-5dx-5c)}-5e^{(-7dx-7c)}}{d\left(4e^{(-2dx-2c)}+6e^{(-4dx-4c)}+4e^{(-6dx-6c)}+e^{(-8dx-8c)}+1\right)}\right)$$
$$-\frac{1}{4}a^3\left(\frac{3\arctan\left(e^{(-dx-c)}\right)}{d} - \frac{3e^{(-dx-c)}+11e^{(-3dx-3c)}-11e^{(-5dx-5c)}-3e^{(-7dx-7c)}}{d\left(4e^{(-2dx-2c)}+6e^{(-4dx-4c)}+4e^{(-6dx-6c)}+e^{(-8dx-8c)}+1\right)}\right)$$
$$-\frac{3}{4}a^2b\left(\frac{\arctan\left(e^{(-dx-c)}\right)}{d} - \frac{e^{(-dx-c)}-7e^{(-3dx-3c)}+7e^{(-5dx-5c)}-e^{(-7dx-7c)}}{d\left(4e^{(-2dx-2c)}+6e^{(-4dx-4c)}+4e^{(-6dx-6c)}+e^{(-8dx-8c)}+1\right)}\right)$$

input
```
integrate(sech(d*x+c)^5*(a+b*sinh(d*x+c)^2)^3,x, algorithm="maxima")
```

output
```
1/4*b^3*(15*arctan(e^(-d*x - c))/d - 2*e^(-d*x - c)/d + (17*e^(-2*d*x - 2*
c) + 13*e^(-4*d*x - 4*c) + 7*e^(-6*d*x - 6*c) - 7*e^(-8*d*x - 8*c) + 2)/(d
*(e^(-d*x - c) + 4*e^(-3*d*x - 3*c) + 6*e^(-5*d*x - 5*c) + 4*e^(-7*d*x - 7
*c) + e^(-9*d*x - 9*c)))) - 3/4*a*b^2*(3*arctan(e^(-d*x - c))/d + (5*e^(-d
*x - c) - 3*e^(-3*d*x - 3*c) + 3*e^(-5*d*x - 5*c) - 5*e^(-7*d*x - 7*c))/(d
*(4*e^(-2*d*x - 2*c) + 6*e^(-4*d*x - 4*c) + 4*e^(-6*d*x - 6*c) + e^(-8*d*x
 - 8*c) + 1))) - 1/4*a^3*(3*arctan(e^(-d*x - c))/d - (3*e^(-d*x - c) + 11*
e^(-3*d*x - 3*c) - 11*e^(-5*d*x - 5*c) - 3*e^(-7*d*x - 7*c))/(d*(4*e^(-2*d
*x - 2*c) + 6*e^(-4*d*x - 4*c) + 4*e^(-6*d*x - 6*c) + e^(-8*d*x - 8*c) + 1
))) - 3/4*a^2*b*(arctan(e^(-d*x - c))/d - (e^(-d*x - c) - 7*e^(-3*d*x - 3*
c) + 7*e^(-5*d*x - 5*c) - e^(-7*d*x - 7*c))/(d*(4*e^(-2*d*x - 2*c) + 6*e^(
-4*d*x - 4*c) + 4*e^(-6*d*x - 6*c) + e^(-8*d*x - 8*c) + 1)))
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 301 vs. $2(97) = 194$.

Time = 0.20 (sec) , antiderivative size = 301, normalized size of antiderivative = 2.92

$$\int \operatorname{sech}^5(c+dx)\left(a+b\sinh^2(c+dx)\right)^3\,dx$$
$$= \frac{8\,b^3\left(e^{(dx+c)}-e^{(-dx-c)}\right)+3\left(\pi+2\arctan\left(\frac{1}{2}\left(e^{(2\,dx+2\,c)}-1\right)e^{(-dx-c)}\right)\right)\left(a^3+a^2b+3\,ab^2-5\,b^3\right)+\frac{4\left(3\,a^3(e\right.}{}}{}$$

input
```
integrate(sech(d*x+c)^5*(a+b*sinh(d*x+c)^2)^3,x, algorithm="giac")
```

output
```
1/16*(8*b^3*(e^(d*x + c) - e^(-d*x - c)) + 3*(pi + 2*arctan(1/2*(e^(2*d*x
+ 2*c) - 1)*e^(-d*x - c)))*(a^3 + a^2*b + 3*a*b^2 - 5*b^3) + 4*(3*a^3*(e^(
d*x + c) - e^(-d*x - c))^3 + 3*a^2*b*(e^(d*x + c) - e^(-d*x - c))^3 - 15*a
*b^2*(e^(d*x + c) - e^(-d*x - c))^3 + 9*b^3*(e^(d*x + c) - e^(-d*x - c))^3
 + 20*a^3*(e^(d*x + c) - e^(-d*x - c)) - 12*a^2*b*(e^(d*x + c) - e^(-d*x -
 c)) - 36*a*b^2*(e^(d*x + c) - e^(-d*x - c)) + 28*b^3*(e^(d*x + c) - e^(-d
*x - c)))/((e^(d*x + c) - e^(-d*x - c))^2 + 4)^2)/d
```

Mupad [B] (verification not implemented)

Time = 1.82 (sec) , antiderivative size = 430, normalized size of antiderivative = 4.17

$$\int \operatorname{sech}^5(c+dx)\left(a+b\sinh^2(c+dx)\right)^3\,dx$$
$$= \frac{3\,\mathrm{atan}\left(\frac{e^{d\,x}\,e^{c}\,\left(a^3\,\sqrt{d^2}-5\,b^3\,\sqrt{d^2}+3\,a\,b^2\,\sqrt{d^2}+a^2\,b\,\sqrt{d^2}\right)}{d\,\sqrt{a^6+2\,a^5\,b+7\,a^4\,b^2-4\,a^3\,b^3-a^2\,b^4-30\,a\,b^5+25\,b^6}}\right)\,\sqrt{a^6+2\,a^5\,b+7\,a^4\,b^2-4\,a^3\,b^3-a^2\,b^4-30\,a\,b^5+25\,b^6}}{4\,\sqrt{d^2}}$$
$$+\frac{b^3\,e^{c+d\,x}}{2\,d}-\frac{b^3\,e^{-c-d\,x}}{2\,d}+\frac{3\,e^{c+d\,x}\,\left(a^3+a^2\,b-5\,a\,b^2+3\,b^3\right)}{4\,d\,\left(e^{2\,c+2\,d\,x}+1\right)}$$
$$+\frac{e^{c+d\,x}\,\left(a^3-15\,a^2\,b+27\,a\,b^2-13\,b^3\right)}{2\,d\,\left(2\,e^{2\,c+2\,d\,x}+e^{4\,c+4\,d\,x}+1\right)}-\frac{6\,e^{c+d\,x}\,\left(a^3-3\,a^2\,b+3\,a\,b^2-b^3\right)}{d\,\left(3\,e^{2\,c+2\,d\,x}+3\,e^{4\,c+4\,d\,x}+e^{6\,c+6\,d\,x}+1\right)}$$
$$+\frac{4\,e^{c+d\,x}\,\left(a^3-3\,a^2\,b+3\,a\,b^2-b^3\right)}{d\,\left(4\,e^{2\,c+2\,d\,x}+6\,e^{4\,c+4\,d\,x}+4\,e^{6\,c+6\,d\,x}+e^{8\,c+8\,d\,x}+1\right)}$$

input
```
int((a + b*sinh(c + d*x)^2)^3/cosh(c + d*x)^5,x)
```

output
```
(3*atan((exp(d*x)*exp(c)*(a^3*(d^2)^(1/2) - 5*b^3*(d^2)^(1/2) + 3*a*b^2*(d
^2)^(1/2) + a^2*b*(d^2)^(1/2)))/(d*(2*a^5*b - 30*a*b^5 + a^6 + 25*b^6 - a^
2*b^4 - 4*a^3*b^3 + 7*a^4*b^2)^(1/2)))*(2*a^5*b - 30*a*b^5 + a^6 + 25*b^6
- a^2*b^4 - 4*a^3*b^3 + 7*a^4*b^2)^(1/2))/(4*(d^2)^(1/2)) + (b^3*exp(c + d
*x))/(2*d) - (b^3*exp(- c - d*x))/(2*d) + (3*exp(c + d*x)*(a^2*b - 5*a*b^2
 + a^3 + 3*b^3))/(4*d*(exp(2*c + 2*d*x) + 1)) + (exp(c + d*x)*(27*a*b^2 -
15*a^2*b + a^3 - 13*b^3))/(2*d*(2*exp(2*c + 2*d*x) + exp(4*c + 4*d*x) + 1)
) - (6*exp(c + d*x)*(3*a*b^2 - 3*a^2*b + a^3 - b^3))/(d*(3*exp(2*c + 2*d*x
) + 3*exp(4*c + 4*d*x) + exp(6*c + 6*d*x) + 1)) + (4*exp(c + d*x)*(3*a*b^2
 - 3*a^2*b + a^3 - b^3))/(d*(4*exp(2*c + 2*d*x) + 6*exp(4*c + 4*d*x) + 4*e
xp(6*c + 6*d*x) + exp(8*c + 8*d*x) + 1))
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 791, normalized size of antiderivative = 7.68

$$\int \operatorname{sech}^5(c+dx)\left(a+b\sinh^2(c+dx)\right)^3\,dx = \frac{-2b^3+18e^{5dx+5c}atan\left(e^{dx+c}\right)a^3-90e^{5dx+5c}atan\left(e^{dx+c}\right)b^3+12e^{7dx+7c}atan\left(e^{dx+c}\right)a^2b+36e^{7dx+7c}atan\left(\right.}{}$$

input
```
int(sech(d*x+c)^5*(a+b*sinh(d*x+c)^2)^3,x)
```

output

```
(3*e**(9*c + 9*d*x)*atan(e**(c + d*x))*a**3 + 3*e**(9*c + 9*d*x)*atan(e**(
c + d*x))*a**2*b + 9*e**(9*c + 9*d*x)*atan(e**(c + d*x))*a*b**2 - 15*e**(9
*c + 9*d*x)*atan(e**(c + d*x))*b**3 + 12*e**(7*c + 7*d*x)*atan(e**(c + d*x
))*a**3 + 12*e**(7*c + 7*d*x)*atan(e**(c + d*x))*a**2*b + 36*e**(7*c + 7*d
*x)*atan(e**(c + d*x))*a*b**2 - 60*e**(7*c + 7*d*x)*atan(e**(c + d*x))*b**
3 + 18*e**(5*c + 5*d*x)*atan(e**(c + d*x))*a**3 + 18*e**(5*c + 5*d*x)*atan
(e**(c + d*x))*a**2*b + 54*e**(5*c + 5*d*x)*atan(e**(c + d*x))*a*b**2 - 90
*e**(5*c + 5*d*x)*atan(e**(c + d*x))*b**3 + 12*e**(3*c + 3*d*x)*atan(e**(c
 + d*x))*a**3 + 12*e**(3*c + 3*d*x)*atan(e**(c + d*x))*a**2*b + 36*e**(3*c
 + 3*d*x)*atan(e**(c + d*x))*a*b**2 - 60*e**(3*c + 3*d*x)*atan(e**(c + d*x
))*b**3 + 3*e**(c + d*x)*atan(e**(c + d*x))*a**3 + 3*e**(c + d*x)*atan(e**
(c + d*x))*a**2*b + 9*e**(c + d*x)*atan(e**(c + d*x))*a*b**2 - 15*e**(c +
d*x)*atan(e**(c + d*x))*b**3 + 2*e**(10*c + 10*d*x)*b**3 + 3*e**(8*c + 8*d
*x)*a**3 + 3*e**(8*c + 8*d*x)*a**2*b - 15*e**(8*c + 8*d*x)*a*b**2 + 15*e**
(8*c + 8*d*x)*b**3 + 11*e**(6*c + 6*d*x)*a**3 - 21*e**(6*c + 6*d*x)*a**2*b
 + 9*e**(6*c + 6*d*x)*a*b**2 + 5*e**(6*c + 6*d*x)*b**3 - 11*e**(4*c + 4*d*
x)*a**3 + 21*e**(4*c + 4*d*x)*a**2*b - 9*e**(4*c + 4*d*x)*a*b**2 - 5*e**(4
*c + 4*d*x)*b**3 - 3*e**(2*c + 2*d*x)*a**3 - 3*e**(2*c + 2*d*x)*a**2*b + 1
5*e**(2*c + 2*d*x)*a*b**2 - 15*e**(2*c + 2*d*x)*b**3 - 2*b**3)/(4*e**(c +
d*x)*d*(e**(8*c + 8*d*x) + 4*e**(6*c + 6*d*x) + 6*e**(4*c + 4*d*x) + 4*...
```

3.277 $\int \operatorname{sech}^6(c+dx)\left(a+b\sinh^2(c+dx)\right)^3\,dx$

Optimal result

Integrand size = 23, antiderivative size = 74

$$\int \operatorname{sech}^6(c+dx)\left(a+b\sinh^2(c+dx)\right)^3\,dx = b^3x+\frac{(a^3-b^3)\tanh(c+dx)}{d} - \frac{(a-b)^2(2a+b)\tanh^3(c+dx)}{3d} + \frac{(a-b)^3\tanh^5(c+dx)}{5d}$$

output
```
b^3*x+(a^3-b^3)*tanh(d*x+c)/d-1/3*(a-b)^2*(2*a+b)*tanh(d*x+c)^3/d+1/5*(a-b
)^3*tanh(d*x+c)^5/d
```

Mathematica [A] (verified)

Time = 1.30 (sec) , antiderivative size = 103, normalized size of antiderivative = 1.39

$$\int \operatorname{sech}^6(c+dx)\left(a+b\sinh^2(c+dx)\right)^3\,dx = \frac{b^3(c+dx)}{d} + \frac{(a-b)\left(64a^2+22ab+49b^2+12(4a^2+7ab+4b^2)\cosh(2(c+dx))+(8a^2+14ab+23b^2)\cosh(4(c+\right.}{120d}$$

input
```
Integrate[Sech[c + d*x]^6*(a + b*Sinh[c + d*x]^2)^3,x]
```

output
```
(b^3*(c + d*x))/d + ((a - b)*(64*a^2 + 22*a*b + 49*b^2 + 12*(4*a^2 + 7*a*b
 + 4*b^2)*Cosh[2*(c + d*x)] + (8*a^2 + 14*a*b + 23*b^2)*Cosh[4*(c + d*x)])
*Sech[c + d*x]^4*Tanh[c + d*x])/(120*d)
```

Rubi [A] (verified)

Time = 0.32 (sec) , antiderivative size = 75, normalized size of antiderivative = 1.01, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.174$, Rules used = {3042, 3670, 300, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \operatorname{sech}^6(c+dx)\left(a+b\sinh^2(c+dx)\right)^3dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\left(a-b\sin(ic+idx)^2\right)^3}{\cos(ic+idx)^6}dx$$

$$\Big\downarrow \text{3670}$$

$$\frac{\int \frac{\left(a-(a-b)\tanh^2(c+dx)\right)^3}{1-\tanh^2(c+dx)}d\tanh(c+dx)}{d}$$

$$\Big\downarrow \text{300}$$

$$\frac{\int \left((a-b)^3\tanh^4(c+dx)-(a-b)^2(2a+b)\tanh^2(c+dx)+a^3-b^3+\frac{b^3}{1-\tanh^2(c+dx)}\right)d\tanh(c+dx)}{d}$$

$$\Big\downarrow \text{2009}$$

$$\frac{\left(a^3-b^3\right)\tanh(c+dx)+\frac{1}{5}(a-b)^3\tanh^5(c+dx)-\frac{1}{3}(a-b)^2(2a+b)\tanh^3(c+dx)+b^3\text{arctanh}(\tanh(c+dx))}{d}$$

input
```
Int[Sech[c + d*x]^6*(a + b*Sinh[c + d*x]^2)^3,x]
```

output
```
(b^3*ArcTanh[Tanh[c + d*x]] + (a^3 - b^3)*Tanh[c + d*x] - ((a - b)^2*(2*a
+ b)*Tanh[c + d*x]^3)/3 + ((a - b)^3*Tanh[c + d*x]^5)/5)/d
```

Defintions of rubi rules used

rule 300
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_), x_Symbol] :> Int
[PolynomialDivide[(a + b*x^2)^p, (c + d*x^2)^(-q), x], x] /; FreeQ[{a, b, c
, d}, x] && NeQ[b*c - a*d, 0] && IGtQ[p, 0] && ILtQ[q, 0] && GeQ[p, -q]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3670
```
Int[cos[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(
p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Simp[ff/f   Su
bst[Int[(a + (a + b)*ff^2*x^2)^p/(1 + ff^2*x^2)^(m/2 + p + 1), x], x, Tan[e
 + f*x]/ff], x]] /; FreeQ[{a, b, e, f}, x] && IntegerQ[m/2] && IntegerQ[p]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 198 vs. $2(70) = 140$.

Time = 0.16 (sec) , antiderivative size = 199, normalized size of antiderivative = 2.69

$$a^3\left(\frac{8}{15}+\frac{\operatorname{sech}(dx+c)^4}{5}+\frac{4\operatorname{sech}(dx+c)^2}{15}\right)\tanh\left(dx+c\right)+3a^2b\left(-\frac{\sinh(dx+c)}{4\cosh(dx+c)^5}+\frac{\left(\frac{8}{15}+\frac{\operatorname{sech}(dx+c)^4}{5}+\frac{4\operatorname{sech}(dx+c)^2}{15}\right)\tanh(dx+c}{4}\right.$$

input
```
int(sech(d*x+c)^6*(a+b*sinh(d*x+c)^2)^3,x)
```

output
```
1/d*(a^3*(8/15+1/5*sech(d*x+c)^4+4/15*sech(d*x+c)^2)*tanh(d*x+c)+3*a^2*b*(
-1/4*sinh(d*x+c)/cosh(d*x+c)^5+1/4*(8/15+1/5*sech(d*x+c)^4+4/15*sech(d*x+c
)^2)*tanh(d*x+c))+3*b^2*a*(-1/2*sinh(d*x+c)^3/cosh(d*x+c)^5-3/8*sinh(d*x+c
)/cosh(d*x+c)^5+3/8*(8/15+1/5*sech(d*x+c)^4+4/15*sech(d*x+c)^2)*tanh(d*x+c
))+b^3*(d*x+c-tanh(d*x+c)-1/3*tanh(d*x+c)^3-1/5*tanh(d*x+c)^5))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 530 vs. $2(70) = 140$.

Time = 0.09 (sec) , antiderivative size = 530, normalized size of antiderivative = 7.16

$$\int \operatorname{sech}^6(c+dx)\left(a+b\sinh^2(c+dx)\right)^3\,dx$$
$$= \frac{(15\,b^3dx - 8\,a^3 - 6\,a^2b - 9\,ab^2 + 23\,b^3)\cosh\left(dx+c\right)^5 + 5\left(15\,b^3dx - 8\,a^3 - 6\,a^2b - 9\,ab^2 + 23\,b^3\right)\cosh\left(\right.}{}$$

input
```
integrate(sech(d*x+c)^6*(a+b*sinh(d*x+c)^2)^3,x, algorithm="fricas")
```

output
```
1/15*((15*b^3*d*x - 8*a^3 - 6*a^2*b - 9*a*b^2 + 23*b^3)*cosh(d*x + c)^5 +
5*(15*b^3*d*x - 8*a^3 - 6*a^2*b - 9*a*b^2 + 23*b^3)*cosh(d*x + c)*sinh(d*x
 + c)^4 + (8*a^3 + 6*a^2*b + 9*a*b^2 - 23*b^3)*sinh(d*x + c)^5 + 5*(15*b^3
*d*x - 8*a^3 - 6*a^2*b - 9*a*b^2 + 23*b^3)*cosh(d*x + c)^3 + 5*(8*a^3 + 6*
a^2*b - 9*a*b^2 - 5*b^3 + 2*(8*a^3 + 6*a^2*b + 9*a*b^2 - 23*b^3)*cosh(d*x
+ c)^2)*sinh(d*x + c)^3 + 5*(2*(15*b^3*d*x - 8*a^3 - 6*a^2*b - 9*a*b^2 + 2
3*b^3)*cosh(d*x + c)^3 + 3*(15*b^3*d*x - 8*a^3 - 6*a^2*b - 9*a*b^2 + 23*b^
3)*cosh(d*x + c))*sinh(d*x + c)^2 + 10*(15*b^3*d*x - 8*a^3 - 6*a^2*b - 9*a
*b^2 + 23*b^3)*cosh(d*x + c) + 5*((8*a^3 + 6*a^2*b + 9*a*b^2 - 23*b^3)*cos
h(d*x + c)^4 + 16*a^3 - 24*a^2*b + 18*a*b^2 - 10*b^3 + 3*(8*a^3 + 6*a^2*b
- 9*a*b^2 - 5*b^3)*cosh(d*x + c)^2)*sinh(d*x + c))/(d*cosh(d*x + c)^5 + 5*
d*cosh(d*x + c)*sinh(d*x + c)^4 + 5*d*cosh(d*x + c)^3 + 5*(2*d*cosh(d*x +
c)^3 + 3*d*cosh(d*x + c))*sinh(d*x + c)^2 + 10*d*cosh(d*x + c))
```

Sympy [F(-1)]

Timed out.

$$\int \operatorname{sech}^6(c+dx) \left(a+b\sinh^2(c+dx)\right)^3 \, dx = \text{Timed out}$$

input
```
integrate(sech(d*x+c)**6*(a+b*sinh(d*x+c)**2)**3,x)
```

output
```
Timed out
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 824 vs. $2(70) = 140$.

Time = 0.06 (sec) , antiderivative size = 824, normalized size of antiderivative = 11.14

$$\int \operatorname{sech}^6(c+dx) \left(a+b\sinh^2(c+dx)\right)^3 \, dx = \text{Too large to display}$$

input
```
integrate(sech(d*x+c)^6*(a+b*sinh(d*x+c)^2)^3,x, algorithm="maxima")
```

output
```
1/15*b^3*(15*x + 15*c/d - 2*(70*e^(-2*d*x - 2*c) + 140*e^(-4*d*x - 4*c) +
90*e^(-6*d*x - 6*c) + 45*e^(-8*d*x - 8*c) + 23)/(d*(5*e^(-2*d*x - 2*c) + 1
0*e^(-4*d*x - 4*c) + 10*e^(-6*d*x - 6*c) + 5*e^(-8*d*x - 8*c) + e^(-10*d*x
 - 10*c) + 1))) + 16/15*a^3*(5*e^(-2*d*x - 2*c)/(d*(5*e^(-2*d*x - 2*c) + 1
0*e^(-4*d*x - 4*c) + 10*e^(-6*d*x - 6*c) + 5*e^(-8*d*x - 8*c) + e^(-10*d*x
 - 10*c) + 1)) + 10*e^(-4*d*x - 4*c)/(d*(5*e^(-2*d*x - 2*c) + 10*e^(-4*d*x
 - 4*c) + 10*e^(-6*d*x - 6*c) + 5*e^(-8*d*x - 8*c) + e^(-10*d*x - 10*c) +
1)) + 1/(d*(5*e^(-2*d*x - 2*c) + 10*e^(-4*d*x - 4*c) + 10*e^(-6*d*x - 6*c)
 + 5*e^(-8*d*x - 8*c) + e^(-10*d*x - 10*c) + 1))) + 4/5*a^2*b*(5*e^(-2*d*x
 - 2*c)/(d*(5*e^(-2*d*x - 2*c) + 10*e^(-4*d*x - 4*c) + 10*e^(-6*d*x - 6*c)
 + 5*e^(-8*d*x - 8*c) + e^(-10*d*x - 10*c) + 1)) - 5*e^(-4*d*x - 4*c)/(d*(
5*e^(-2*d*x - 2*c) + 10*e^(-4*d*x - 4*c) + 10*e^(-6*d*x - 6*c) + 5*e^(-8*d
*x - 8*c) + e^(-10*d*x - 10*c) + 1)) + 15*e^(-6*d*x - 6*c)/(d*(5*e^(-2*d*x
 - 2*c) + 10*e^(-4*d*x - 4*c) + 10*e^(-6*d*x - 6*c) + 5*e^(-8*d*x - 8*c) +
 e^(-10*d*x - 10*c) + 1)) + 1/(d*(5*e^(-2*d*x - 2*c) + 10*e^(-4*d*x - 4*c)
 + 10*e^(-6*d*x - 6*c) + 5*e^(-8*d*x - 8*c) + e^(-10*d*x - 10*c) + 1))) +
6/5*a*b^2*(10*e^(-4*d*x - 4*c)/(d*(5*e^(-2*d*x - 2*c) + 10*e^(-4*d*x - 4*c
) + 10*e^(-6*d*x - 6*c) + 5*e^(-8*d*x - 8*c) + e^(-10*d*x - 10*c) + 1)) +
5*e^(-8*d*x - 8*c)/(d*(5*e^(-2*d*x - 2*c) + 10*e^(-4*d*x - 4*c) + 10*e^(-6
*d*x - 6*c) + 5*e^(-8*d*x - 8*c) + e^(-10*d*x - 10*c) + 1)) + 1/(d*(5*e...
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 213 vs. $2(70) = 140$.

Time = 0.18 (sec) , antiderivative size = 213, normalized size of antiderivative = 2.88

$$\int \operatorname{sech}^6(c+dx)\left(a+b\sinh^2(c+dx)\right)^3\,dx = \frac{15\,(dx+c)b^3 - \frac{2\,(45\,ab^2e^{(8\,dx+8\,c)}-45\,b^3e^{(8\,dx+8\,c)}+90\,a^2be^{(6\,dx+6\,c)}-90\,b^3e^{(6\,dx+6\,c)}+80\,a^3e^{(4\,dx+4\,c)}-30\,a^2be^{(4\,dx+4\,c)}+90\,ab^2e^{(4\,dx}}{\left(e^{(2\,dx+2\,c)}+1\right)^5}}{15\,d}$$

input
```
integrate(sech(d*x+c)^6*(a+b*sinh(d*x+c)^2)^3,x, algorithm="giac")
```

output
```
1/15*(15*(d*x + c)*b^3 - 2*(45*a*b^2*e^(8*d*x + 8*c) - 45*b^3*e^(8*d*x + 8
*c) + 90*a^2*b*e^(6*d*x + 6*c) - 90*b^3*e^(6*d*x + 6*c) + 80*a^3*e^(4*d*x
+ 4*c) - 30*a^2*b*e^(4*d*x + 4*c) + 90*a*b^2*e^(4*d*x + 4*c) - 140*b^3*e^(
4*d*x + 4*c) + 40*a^3*e^(2*d*x + 2*c) + 30*a^2*b*e^(2*d*x + 2*c) - 70*b^3*
e^(2*d*x + 2*c) + 8*a^3 + 6*a^2*b + 9*a*b^2 - 23*b^3)/(e^(2*d*x + 2*c) + 1
)^5)/d
```

Mupad [B] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 563, normalized size of antiderivative = 7.61

$$\int \operatorname{sech}^6(c+dx)\left(a+b\sinh^2(c+dx)\right)^3\,dx$$
$$= b^3x - \frac{\frac{2(8a^3-12a^2b+9ab^2-5b^3)}{15d} - \frac{12e^{2c+2dx}(ab^2-a^2b)}{5d} + \frac{6e^{4c+4dx}(ab^2-b^3)}{5d}}{3e^{2c+2dx}+3e^{4c+4dx}+e^{6c+6dx}+1}$$
$$+ \frac{\frac{6(ab^2-a^2b)}{5d} - \frac{6e^{2c+2dx}(ab^2-b^3)}{5d}}{2e^{2c+2dx}+e^{4c+4dx}+1}$$
$$+ \frac{\frac{6(ab^2-a^2b)}{5d} + \frac{18e^{4c+4dx}(ab^2-a^2b)}{5d} - \frac{2e^{2c+2dx}(8a^3-12a^2b+9ab^2-5b^3)}{5d} - \frac{6e^{6c+6dx}(ab^2-b^3)}{5d}}{4e^{2c+2dx}+6e^{4c+4dx}+4e^{6c+6dx}+e^{8c+8dx}+1}$$
$$- \frac{\frac{6(ab^2-b^3)}{5d} - \frac{24e^{2c+2dx}(ab^2-a^2b)}{5d} - \frac{24e^{6c+6dx}(ab^2-a^2b)}{5d} + \frac{4e^{4c+4dx}(8a^3-12a^2b+9ab^2-5b^3)}{5d} + \frac{6e^{8c+8dx}(ab^2-b^3)}{5d}}{5e^{2c+2dx}+10e^{4c+4dx}+10e^{6c+6dx}+5e^{8c+8dx}+e^{10c+10dx}+1}$$
$$- \frac{6(ab^2-b^3)}{5d\,(e^{2c+2dx}+1)}$$

input
```
int((a + b*sinh(c + d*x)^2)^3/cosh(c + d*x)^6,x)
```

output
```
b^3*x - ((2*(9*a*b^2 - 12*a^2*b + 8*a^3 - 5*b^3))/(15*d) - (12*exp(2*c + 2
*d*x)*(a*b^2 - a^2*b))/(5*d) + (6*exp(4*c + 4*d*x)*(a*b^2 - b^3))/(5*d))/(
3*exp(2*c + 2*d*x) + 3*exp(4*c + 4*d*x) + exp(6*c + 6*d*x) + 1) + ((6*(a*b
^2 - a^2*b))/(5*d) - (6*exp(2*c + 2*d*x)*(a*b^2 - b^3))/(5*d))/(2*exp(2*c
+ 2*d*x) + exp(4*c + 4*d*x) + 1) + ((6*(a*b^2 - a^2*b))/(5*d) + (18*exp(4*
c + 4*d*x)*(a*b^2 - a^2*b))/(5*d) - (2*exp(2*c + 2*d*x)*(9*a*b^2 - 12*a^2*
b + 8*a^3 - 5*b^3))/(5*d) - (6*exp(6*c + 6*d*x)*(a*b^2 - b^3))/(5*d))/(4*e
xp(2*c + 2*d*x) + 6*exp(4*c + 4*d*x) + 4*exp(6*c + 6*d*x) + exp(8*c + 8*d*
x) + 1) - ((6*(a*b^2 - b^3))/(5*d) - (24*exp(2*c + 2*d*x)*(a*b^2 - a^2*b))
/(5*d) - (24*exp(6*c + 6*d*x)*(a*b^2 - a^2*b))/(5*d) + (4*exp(4*c + 4*d*x)
*(9*a*b^2 - 12*a^2*b + 8*a^3 - 5*b^3))/(5*d) + (6*exp(8*c + 8*d*x)*(a*b^2
- b^3))/(5*d))/(5*exp(2*c + 2*d*x) + 10*exp(4*c + 4*d*x) + 10*exp(6*c + 6*
d*x) + 5*exp(8*c + 8*d*x) + exp(10*c + 10*d*x) + 1) - (6*(a*b^2 - b^3))/(5
*d*(exp(2*c + 2*d*x) + 1))
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 347, normalized size of antiderivative = 4.69

$$\int \operatorname{sech}^6(c+dx) \left(a+b\sinh^2(c+dx)\right)^3 dx = \frac{18e^{10dx+10c}a\,b^2+15e^{10dx+10c}b^3dx-18e^{10dx+10c}b^3+75e^{8dx+8c}b^3dx-180e^{6dx+6c}a^2b+180e^{6dx+6c}a\,b^2+15}{}$$

input
```
int(sech(d*x+c)^6*(a+b*sinh(d*x+c)^2)^3,x)
```

output
```
(18*e**(10*c + 10*d*x)*a*b**2 + 15*e**(10*c + 10*d*x)*b**3*d*x - 18*e**(10
*c + 10*d*x)*b**3 + 75*e**(8*c + 8*d*x)*b**3*d*x - 180*e**(6*c + 6*d*x)*a*
*2*b + 180*e**(6*c + 6*d*x)*a*b**2 + 150*e**(6*c + 6*d*x)*b**3*d*x - 160*e
**(4*c + 4*d*x)*a**3 + 60*e**(4*c + 4*d*x)*a**2*b + 150*e**(4*c + 4*d*x)*b
**3*d*x + 100*e**(4*c + 4*d*x)*b**3 - 80*e**(2*c + 2*d*x)*a**3 - 60*e**(2*
c + 2*d*x)*a**2*b + 90*e**(2*c + 2*d*x)*a*b**2 + 75*e**(2*c + 2*d*x)*b**3*
d*x + 50*e**(2*c + 2*d*x)*b**3 - 16*a**3 - 12*a**2*b + 15*b**3*d*x + 28*b*
*3)/(15*d*(e**(10*c + 10*d*x) + 5*e**(8*c + 8*d*x) + 10*e**(6*c + 6*d*x) +
 10*e**(4*c + 4*d*x) + 5*e**(2*c + 2*d*x) + 1))
```

3.278 $\int \operatorname{sech}^7(c+dx) \left(a+b\sinh^2(c+dx)\right)^3 dx$

Optimal result

Integrand size = 23, antiderivative size = 135

$$\int \operatorname{sech}^7(c+dx) \left(a+b\sinh^2(c+dx)\right)^3 dx = \frac{(a+b)(5a^2-2ab+5b^2)\arctan(\sinh(c+dx))}{16d} + \frac{(a-b)(5a^2+8ab+11b^2)\operatorname{sech}(c+dx)\tanh(c+dx)}{16d} + \frac{(a-b)^2(5a+13b)\operatorname{sech}^3(c+dx)\tanh(c+dx)}{24d} + \frac{(a-b)^3\operatorname{sech}^5(c+dx)\tanh(c+dx)}{6d}$$

output

```
1/16*(a+b)*(5*a^2-2*a*b+5*b^2)*arctan(sinh(d*x+c))/d+1/16*(a-b)*(5*a^2+8*a
*b+11*b^2)*sech(d*x+c)*tanh(d*x+c)/d+1/24*(a-b)^2*(5*a+13*b)*sech(d*x+c)^3
*tanh(d*x+c)/d+1/6*(a-b)^3*sech(d*x+c)^5*tanh(d*x+c)/d
```

Mathematica [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 10.97 (sec) , antiderivative size = 1192, normalized size of antiderivative = 8.83

$$\int \operatorname{sech}^7(c+dx) \left(a+b\sinh^2(c+dx)\right)^3 \, dx = \text{Too large to display}$$

input
```
Integrate[Sech[c + d*x]^7*(a + b*Sinh[c + d*x]^2)^3,x]
```

output
```
(Csch[c + d*x]^5*(-117228825*a^3*ArcTanh[Sqrt[-Sinh[c + d*x]^2]] - 1092656
25*a^3*ArcTanh[Sqrt[-Sinh[c + d*x]^2]]*Sinh[c + d*x]^2 - 274542345*a^2*b*A
rcTanh[Sqrt[-Sinh[c + d*x]^2]]*Sinh[c + d*x]^2 - 17069535*a^3*ArcTanh[Sqrt
[-Sinh[c + d*x]^2]]*Sinh[c + d*x]^4 - 260465625*a^2*b*ArcTanh[Sqrt[-Sinh[c
 + d*x]^2]]*Sinh[c + d*x]^4 - 215549775*a*b^2*ArcTanh[Sqrt[-Sinh[c + d*x]^
2]]*Sinh[c + d*x]^4 + 142065*a^3*ArcTanh[Sqrt[-Sinh[c + d*x]^2]]*Sinh[c +
d*x]^6 - 41427855*a^2*b*ArcTanh[Sqrt[-Sinh[c + d*x]^2]]*Sinh[c + d*x]^6 -
207173295*a*b^2*ArcTanh[Sqrt[-Sinh[c + d*x]^2]]*Sinh[c + d*x]^6 - 58009455
*b^3*ArcTanh[Sqrt[-Sinh[c + d*x]^2]]*Sinh[c + d*x]^6 - 210735*a^2*b*ArcTan
h[Sqrt[-Sinh[c + d*x]^2]]*Sinh[c + d*x]^8 - 33756345*a*b^2*ArcTanh[Sqrt[-S
inh[c + d*x]^2]]*Sinh[c + d*x]^8 - 56109375*b^3*ArcTanh[Sqrt[-Sinh[c + d*x
]^2]]*Sinh[c + d*x]^8 - 174825*a*b^2*ArcTanh[Sqrt[-Sinh[c + d*x]^2]]*Sinh[
c + d*x]^10 - 9261945*b^3*ArcTanh[Sqrt[-Sinh[c + d*x]^2]]*Sinh[c + d*x]^10
 - 48825*b^3*ArcTanh[Sqrt[-Sinh[c + d*x]^2]]*Sinh[c + d*x]^12 + 117228825*
a^3*Sqrt[-Sinh[c + d*x]^2] + 4093425*a^3*Sinh[c + d*x]^4*Sqrt[-Sinh[c + d*
x]^2] + 168951510*a^2*b*Sinh[c + d*x]^4*Sqrt[-Sinh[c + d*x]^2] + 215549775
*a*b^2*Sinh[c + d*x]^4*Sqrt[-Sinh[c + d*x]^2] + 9514449*a^2*b*Sinh[c + d*x
]^6*Sqrt[-Sinh[c + d*x]^2] + 135323370*a*b^2*Sinh[c + d*x]^6*Sqrt[-Sinh[c
+ d*x]^2] + 58009455*b^3*Sinh[c + d*x]^6*Sqrt[-Sinh[c + d*x]^2] + 7808535*
a*b^2*Sinh[c + d*x]^8*Sqrt[-Sinh[c + d*x]^2] + 36772890*b^3*Sinh[c + d*...
```

Rubi [A] (verified)

Time = 0.40 (sec) , antiderivative size = 170, normalized size of antiderivative = 1.26, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.304$, Rules used = {3042, 3669, 315, 401, 25, 298, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \text{sech}^7(c+dx)\left(a+b\sinh^2(c+dx)\right)^3dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\left(a-b\sin(ic+idx)^2\right)^3}{\cos(ic+idx)^7}dx$$

$$\Big\downarrow \text{3669}$$

$$\frac{\int \frac{\left(b\sinh^2(c+dx)+a\right)^3}{\left(\sinh^2(c+dx)+1\right)^4}d\sinh(c+dx)}{d}$$

$$\Big\downarrow \text{315}$$

$$\frac{\frac{1}{6}\int \frac{\left(b\sinh^2(c+dx)+a\right)\left(b(a+5b)\sinh^2(c+dx)+a(5a+b)\right)}{\left(\sinh^2(c+dx)+1\right)^3}d\sinh(c+dx)+\frac{(a-b)\sinh(c+dx)\left(a+b\sinh^2(c+dx)\right)^2}{6\left(\sinh^2(c+dx)+1\right)^3}}{d}$$

$$\Big\downarrow \text{401}$$

$$\frac{\frac{1}{6}\left(\frac{5(a^2-b^2)\sinh(c+dx)\left(a+b\sinh^2(c+dx)\right)}{4\left(\sinh^2(c+dx)+1\right)^2}-\frac{1}{4}\int -\frac{b(5a^2+4ba+15b^2)\sinh^2(c+dx)+a(15a^2+4ba+5b^2)}{\left(\sinh^2(c+dx)+1\right)^2}d\sinh(c+dx)\right)+\frac{(a-b)\sinh(c+dx)\left(a+b\sinh^2(c+dx)\right)^2}{6\left(\sinh^2(c+dx)+1\right)^3}}{d}$$

$$\Big\downarrow \text{25}$$

$$\frac{\frac{1}{6}\left(\frac{1}{4}\int \frac{b(5a^2+4ba+15b^2)\sinh^2(c+dx)+a(15a^2+4ba+5b^2)}{\left(\sinh^2(c+dx)+1\right)^2}d\sinh(c+dx)+\frac{5(a^2-b^2)\sinh(c+dx)\left(a+b\sinh^2(c+dx)\right)}{4\left(\sinh^2(c+dx)+1\right)^2}\right)+\frac{(a-b)\sinh(c+dx)\left(a+b\sinh^2(c+dx)\right)^2}{6\left(\sinh^2(c+dx)+1\right)^3}}{d}$$

$$\Big\downarrow \text{298}$$

$$\frac{\frac{1}{6}\left(\frac{1}{4}\left(\frac{3}{2}(a+b)\left(5a^2-2ab+5b^2\right)\int \frac{1}{\sinh^2(c+dx)+1}d\sinh(c+dx)+\frac{(a-b)(15a^2+14ab+15b^2)\sinh(c+dx)}{2\left(\sinh^2(c+dx)+1\right)}\right)+\frac{5(a^2-b^2)\sinh(c+dx)\left(a+b\sinh^2(c+dx)\right)}{4\left(\sinh^2(c+dx)+1\right)^2}\right)+\frac{(a-b)\sinh(c+dx)\left(a+b\sinh^2(c+dx)\right)^2}{6\left(\sinh^2(c+dx)+1\right)^3}}{d}$$

$$\downarrow 216$$

$$\frac{\frac{1}{6}\left(\frac{1}{4}\left(\frac{3}{2}(a+b)\left(5a^2-2ab+5b^2\right)\arctan(\sinh(c+dx))+\frac{(a-b)\left(15a^2+14ab+15b^2\right)\sinh(c+dx)}{2\left(\sinh^2(c+dx)+1\right)}\right)+\frac{5\left(a^2-b^2\right)\sinh(c+dx)(a+b}{4(\sinh^2(c+dx)+}\right.}{d}$$

input
```
Int[Sech[c + d*x]^7*(a + b*Sinh[c + d*x]^2)^3,x]
```

output
```
(((a - b)*Sinh[c + d*x]*(a + b*Sinh[c + d*x]^2)^2)/(6*(1 + Sinh[c + d*x]^2
)^3) + ((5*(a^2 - b^2)*Sinh[c + d*x]*(a + b*Sinh[c + d*x]^2))/(4*(1 + Sinh
[c + d*x]^2)^2) + ((3*(a + b)*(5*a^2 - 2*a*b + 5*b^2)*ArcTan[Sinh[c + d*x]
])/2 + ((a - b)*(15*a^2 + 14*a*b + 15*b^2)*Sinh[c + d*x])/(2*(1 + Sinh[c +
 d*x]^2)))/4)/6)/d
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 298
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2), x_Symbol] :> Simp[(-(
b*c - a*d))*x*((a + b*x^2)^(p + 1)/(2*a*b*(p + 1))), x] - Simp[(a*d - b*c*(
2*p + 3))/(2*a*b*(p + 1))   Int[(a + b*x^2)^(p + 1), x], x] /; FreeQ[{a, b,
 c, d, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/2 + p, 0])
```

rule 315
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_), x_Symbol] :> Sim
p[(a*d - c*b)*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^(q - 1)/(2*a*b*(p + 1))),
x] - Simp[1/(2*a*b*(p + 1))   Int[(a + b*x^2)^(p + 1)*(c + d*x^2)^(q - 2)*S
imp[c*(a*d - c*b*(2*p + 3)) + d*(a*d*(2*(q - 1) + 1) - b*c*(2*(p + q) + 1))
*x^2, x], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -
1] && GtQ[q, 1] && IntBinomialQ[a, b, c, d, 2, p, q, x]
```

rule 401
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_.)*((e_) + (f_.)*(x
_)^2), x_Symbol] :> Simp[(-(b*e - a*f))*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^
q/(a*b*2*(p + 1))), x] + Simp[1/(a*b*2*(p + 1))   Int[(a + b*x^2)^(p + 1)*(
c + d*x^2)^(q - 1)*Simp[c*(b*e*2*(p + 1) + b*e - a*f) + d*(b*e*2*(p + 1) +
(b*e - a*f)*(2*q + 1))*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && L
tQ[p, -1] && GtQ[q, 0]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3669
```
Int[cos[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^
(p_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Simp[ff/f   S
ubst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a + b*ff^2*x^2)^p, x], x, Sin[e + f*x]
/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m - 1)/2]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 313 vs. $2(127) = 254$.

Time = 0.27 (sec) , antiderivative size = 314, normalized size of antiderivative = 2.33

$$a^3\left(\left(\frac{\operatorname{sech}(dx+c)^5}{6}+\frac{5\operatorname{sech}(dx+c)^3}{24}+\frac{5\operatorname{sech}(dx+c)}{16}\right)\tanh\left(dx+c\right)+\frac{5\arctan\left(e^{dx+c}\right)}{8}\right)+3a^2b\left(-\frac{\sinh(dx+c)}{5\cosh(dx+c)^6}+\frac{\left(\frac{\operatorname{sech}(dx+c)^5}{6}\right.}{}\right.$$

input
```
int(sech(d*x+c)^7*(a+b*sinh(d*x+c)^2)^3,x)
```

output
```
1/d*(a^3*((1/6*sech(d*x+c)^5+5/24*sech(d*x+c)^3+5/16*sech(d*x+c))*tanh(d*x
+c)+5/8*arctan(exp(d*x+c)))+3*a^2*b*(-1/5*sinh(d*x+c)/cosh(d*x+c)^6+1/5*(1
/6*sech(d*x+c)^5+5/24*sech(d*x+c)^3+5/16*sech(d*x+c))*tanh(d*x+c)+1/8*arct
an(exp(d*x+c)))+3*b^2*a*(-1/3*sinh(d*x+c)^3/cosh(d*x+c)^6-1/5*sinh(d*x+c)/
cosh(d*x+c)^6+1/5*(1/6*sech(d*x+c)^5+5/24*sech(d*x+c)^3+5/16*sech(d*x+c))*
tanh(d*x+c)+1/8*arctan(exp(d*x+c)))+b^3*(-sinh(d*x+c)^5/cosh(d*x+c)^6-5/3*
sinh(d*x+c)^3/cosh(d*x+c)^6-sinh(d*x+c)/cosh(d*x+c)^6+(1/6*sech(d*x+c)^5+5
/24*sech(d*x+c)^3+5/16*sech(d*x+c))*tanh(d*x+c)+5/8*arctan(exp(d*x+c))))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 3675 vs. $2(127) = 254$.

Time = 0.12 (sec) , antiderivative size = 3675, normalized size of antiderivative = 27.22

$$\int \operatorname{sech}^7(c+dx) \left(a+b\sinh^2(c+dx)\right)^3 \, dx = \text{Too large to display}$$

input
```
integrate(sech(d*x+c)^7*(a+b*sinh(d*x+c)^2)^3,x, algorithm="fricas")
```

output
```
Too large to include
```

Sympy [F(-1)]

Timed out.

$$\int \operatorname{sech}^7(c+dx) \left(a+b\sinh^2(c+dx)\right)^3 \, dx = \text{Timed out}$$

input
```
integrate(sech(d*x+c)**7*(a+b*sinh(d*x+c)**2)**3,x)
```

output
```
Timed out
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 646 vs. $2(127) = 254$.

Time = 0.13 (sec) , antiderivative size = 646, normalized size of antiderivative = 4.79

$$\int \operatorname{sech}^7(c+dx) \left(a+b\sinh^2(c+dx)\right)^3 \, dx = \text{Too large to display}$$

input
```
integrate(sech(d*x+c)^7*(a+b*sinh(d*x+c)^2)^3,x, algorithm="maxima")
```

output
```
-1/24*b^3*(15*arctan(e^(-d*x - c))/d + (33*e^(-d*x - c) - 5*e^(-3*d*x - 3*
c) + 90*e^(-5*d*x - 5*c) - 90*e^(-7*d*x - 7*c) + 5*e^(-9*d*x - 9*c) - 33*e
^(-11*d*x - 11*c))/(d*(6*e^(-2*d*x - 2*c) + 15*e^(-4*d*x - 4*c) + 20*e^(-6
*d*x - 6*c) + 15*e^(-8*d*x - 8*c) + 6*e^(-10*d*x - 10*c) + e^(-12*d*x - 12
*c) + 1))) - 1/24*a^3*(15*arctan(e^(-d*x - c))/d - (15*e^(-d*x - c) + 85*e
^(-3*d*x - 3*c) + 198*e^(-5*d*x - 5*c) - 198*e^(-7*d*x - 7*c) - 85*e^(-9*d
*x - 9*c) - 15*e^(-11*d*x - 11*c))/(d*(6*e^(-2*d*x - 2*c) + 15*e^(-4*d*x -
 4*c) + 20*e^(-6*d*x - 6*c) + 15*e^(-8*d*x - 8*c) + 6*e^(-10*d*x - 10*c) +
 e^(-12*d*x - 12*c) + 1))) - 1/8*a^2*b*(3*arctan(e^(-d*x - c))/d - (3*e^(-
d*x - c) + 17*e^(-3*d*x - 3*c) - 114*e^(-5*d*x - 5*c) + 114*e^(-7*d*x - 7*
c) - 17*e^(-9*d*x - 9*c) - 3*e^(-11*d*x - 11*c))/(d*(6*e^(-2*d*x - 2*c) +
15*e^(-4*d*x - 4*c) + 20*e^(-6*d*x - 6*c) + 15*e^(-8*d*x - 8*c) + 6*e^(-10
*d*x - 10*c) + e^(-12*d*x - 12*c) + 1))) - 1/8*a*b^2*(3*arctan(e^(-d*x - c
))/d - (3*e^(-d*x - c) - 47*e^(-3*d*x - 3*c) + 78*e^(-5*d*x - 5*c) - 78*e^
(-7*d*x - 7*c) + 47*e^(-9*d*x - 9*c) - 3*e^(-11*d*x - 11*c))/(d*(6*e^(-2*d
*x - 2*c) + 15*e^(-4*d*x - 4*c) + 20*e^(-6*d*x - 6*c) + 15*e^(-8*d*x - 8*c
) + 6*e^(-10*d*x - 10*c) + e^(-12*d*x - 12*c) + 1)))
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 383 vs. $2(127) = 254$.

Time = 0.19 (sec) , antiderivative size = 383, normalized size of antiderivative = 2.84

$$\int \operatorname{sech}^7(c+dx)\left(a+b\sinh^2(c+dx)\right)^3\,dx$$

$$= \frac{3\left(\pi+2\arctan\left(\frac{1}{2}\left(e^{(2dx+2c)}-1\right)e^{(-dx-c)}\right)\right)\left(5a^3+3a^2b+3ab^2+5b^3\right)+\frac{4\left(15a^3\left(e^{(dx+c)}-e^{(-dx-c)}\right)^5+9a^2b(e^{(\cdots}}{\cdots}}{\cdots}$$

input
```
integrate(sech(d*x+c)^7*(a+b*sinh(d*x+c)^2)^3,x, algorithm="giac")
```

output
```
1/96*(3*(pi + 2*arctan(1/2*(e^(2*d*x + 2*c) - 1)*e^(-d*x - c)))*(5*a^3 + 3
*a^2*b + 3*a*b^2 + 5*b^3) + 4*(15*a^3*(e^(d*x + c) - e^(-d*x - c))^5 + 9*a
^2*b*(e^(d*x + c) - e^(-d*x - c))^5 + 9*a*b^2*(e^(d*x + c) - e^(-d*x - c))
^5 - 33*b^3*(e^(d*x + c) - e^(-d*x - c))^5 + 160*a^3*(e^(d*x + c) - e^(-d*
x - c))^3 + 96*a^2*b*(e^(d*x + c) - e^(-d*x - c))^3 - 96*a*b^2*(e^(d*x + c
) - e^(-d*x - c))^3 - 160*b^3*(e^(d*x + c) - e^(-d*x - c))^3 + 528*a^3*(e^
(d*x + c) - e^(-d*x - c)) - 144*a^2*b*(e^(d*x + c) - e^(-d*x - c)) - 144*a
*b^2*(e^(d*x + c) - e^(-d*x - c)) - 240*b^3*(e^(d*x + c) - e^(-d*x - c)))/
((e^(d*x + c) - e^(-d*x - c))^2 + 4)^3)/d
```

Mupad [B] (verification not implemented)

Time = 1.79 (sec) , antiderivative size = 601, normalized size of antiderivative = 4.45

$$\int \operatorname{sech}^7(c+dx)\left(a+b\sinh^2(c+dx)\right)^3\,dx$$

$$= \frac{\operatorname{atan}\left(\frac{e^{dx}\,e^{c}\left(5\,a^3\sqrt{d^2}+5\,b^3\sqrt{d^2}+3\,a\,b^2\sqrt{d^2}+3\,a^2\,b\sqrt{d^2}\right)}{d\sqrt{25\,a^6+30\,a^5\,b+39\,a^4\,b^2+68\,a^3\,b^3+39\,a^2\,b^4+30\,a\,b^5+25\,b^6}}\right)\sqrt{25\,a^6+30\,a^5\,b+39\,a^4\,b^2+68\,a^3\,b^3+39\,a^2\,b^4+}}{8\sqrt{d^2}}$$

$$-\frac{6\,e^{c+d\,x}\left(3\,a^3-11\,a^2\,b+13\,a\,b^2-5\,b^3\right)}{d\left(4\,e^{2\,c+2\,d\,x}+6\,e^{4\,c+4\,d\,x}+4\,e^{6\,c+6\,d\,x}+e^{8\,c+8\,d\,x}+1\right)}$$

$$+\frac{e^{c+d\,x}\left(a^3-57\,a^2\,b+111\,a\,b^2-55\,b^3\right)}{3\,d\left(3\,e^{2\,c+2\,d\,x}+3\,e^{4\,c+4\,d\,x}+e^{6\,c+6\,d\,x}+1\right)}$$

$$+\frac{e^{c+d\,x}\left(5\,a^3+3\,a^2\,b+3\,a\,b^2-11\,b^3\right)}{8\,d\left(e^{2\,c+2\,d\,x}+1\right)}+\frac{e^{c+d\,x}\left(5\,a^3+3\,a^2\,b-93\,a\,b^2+85\,b^3\right)}{12\,d\left(2\,e^{2\,c+2\,d\,x}+e^{4\,c+4\,d\,x}+1\right)}$$

$$+\frac{80\,e^{c+d\,x}\left(a^3-3\,a^2\,b+3\,a\,b^2-b^3\right)}{3\,d\left(5\,e^{2\,c+2\,d\,x}+10\,e^{4\,c+4\,d\,x}+10\,e^{6\,c+6\,d\,x}+5\,e^{8\,c+8\,d\,x}+e^{10\,c+10\,d\,x}+1\right)}$$

$$-\frac{32\,e^{c+d\,x}\left(a^3-3\,a^2\,b+3\,a\,b^2-b^3\right)}{3\,d\left(6\,e^{2\,c+2\,d\,x}+15\,e^{4\,c+4\,d\,x}+20\,e^{6\,c+6\,d\,x}+15\,e^{8\,c+8\,d\,x}+6\,e^{10\,c+10\,d\,x}+e^{12\,c+12\,d\,x}+1\right)}$$

input
```
int((a + b*sinh(c + d*x)^2)^3/cosh(c + d*x)^7,x)
```

output
```
(atan((exp(d*x)*exp(c)*(5*a^3*(d^2)^(1/2) + 5*b^3*(d^2)^(1/2) + 3*a*b^2*(d
^2)^(1/2) + 3*a^2*b*(d^2)^(1/2)))/(d*(30*a*b^5 + 30*a^5*b + 25*a^6 + 25*b^
6 + 39*a^2*b^4 + 68*a^3*b^3 + 39*a^4*b^2)^(1/2)))*(30*a*b^5 + 30*a^5*b + 2
5*a^6 + 25*b^6 + 39*a^2*b^4 + 68*a^3*b^3 + 39*a^4*b^2)^(1/2))/(8*(d^2)^(1/
2)) - (6*exp(c + d*x)*(13*a*b^2 - 11*a^2*b + 3*a^3 - 5*b^3))/(d*(4*exp(2*c
 + 2*d*x) + 6*exp(4*c + 4*d*x) + 4*exp(6*c + 6*d*x) + exp(8*c + 8*d*x) + 1
)) + (exp(c + d*x)*(111*a*b^2 - 57*a^2*b + a^3 - 55*b^3))/(3*d*(3*exp(2*c
+ 2*d*x) + 3*exp(4*c + 4*d*x) + exp(6*c + 6*d*x) + 1)) + (exp(c + d*x)*(3*
a*b^2 + 3*a^2*b + 5*a^3 - 11*b^3))/(8*d*(exp(2*c + 2*d*x) + 1)) + (exp(c +
 d*x)*(3*a^2*b - 93*a*b^2 + 5*a^3 + 85*b^3))/(12*d*(2*exp(2*c + 2*d*x) + e
xp(4*c + 4*d*x) + 1)) + (80*exp(c + d*x)*(3*a*b^2 - 3*a^2*b + a^3 - b^3))/
(3*d*(5*exp(2*c + 2*d*x) + 10*exp(4*c + 4*d*x) + 10*exp(6*c + 6*d*x) + 5*e
xp(8*c + 8*d*x) + exp(10*c + 10*d*x) + 1)) - (32*exp(c + d*x)*(3*a*b^2 - 3
*a^2*b + a^3 - b^3))/(3*d*(6*exp(2*c + 2*d*x) + 15*exp(4*c + 4*d*x) + 20*e
xp(6*c + 6*d*x) + 15*exp(8*c + 8*d*x) + 6*exp(10*c + 10*d*x) + exp(12*c +
12*d*x) + 1))
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 1058, normalized size of antiderivative = 7.84

$$\int \operatorname{sech}^7(c+dx) \left(a+b\sinh^2(c+dx)\right)^3 \, dx = \text{Too large to display}$$

input
```
int(sech(d*x+c)^7*(a+b*sinh(d*x+c)^2)^3,x)
```

output
```
(15*e**(12*c + 12*d*x)*atan(e**(c + d*x))*a**3 + 9*e**(12*c + 12*d*x)*atan
(e**(c + d*x))*a**2*b + 9*e**(12*c + 12*d*x)*atan(e**(c + d*x))*a*b**2 + 1
5*e**(12*c + 12*d*x)*atan(e**(c + d*x))*b**3 + 90*e**(10*c + 10*d*x)*atan(
e**(c + d*x))*a**3 + 54*e**(10*c + 10*d*x)*atan(e**(c + d*x))*a**2*b + 54*
e**(10*c + 10*d*x)*atan(e**(c + d*x))*a*b**2 + 90*e**(10*c + 10*d*x)*atan(
e**(c + d*x))*b**3 + 225*e**(8*c + 8*d*x)*atan(e**(c + d*x))*a**3 + 135*e*
*(8*c + 8*d*x)*atan(e**(c + d*x))*a**2*b + 135*e**(8*c + 8*d*x)*atan(e**(c
 + d*x))*a*b**2 + 225*e**(8*c + 8*d*x)*atan(e**(c + d*x))*b**3 + 300*e**(6
*c + 6*d*x)*atan(e**(c + d*x))*a**3 + 180*e**(6*c + 6*d*x)*atan(e**(c + d*
x))*a**2*b + 180*e**(6*c + 6*d*x)*atan(e**(c + d*x))*a*b**2 + 300*e**(6*c
+ 6*d*x)*atan(e**(c + d*x))*b**3 + 225*e**(4*c + 4*d*x)*atan(e**(c + d*x))
*a**3 + 135*e**(4*c + 4*d*x)*atan(e**(c + d*x))*a**2*b + 135*e**(4*c + 4*d
*x)*atan(e**(c + d*x))*a*b**2 + 225*e**(4*c + 4*d*x)*atan(e**(c + d*x))*b*
*3 + 90*e**(2*c + 2*d*x)*atan(e**(c + d*x))*a**3 + 54*e**(2*c + 2*d*x)*ata
n(e**(c + d*x))*a**2*b + 54*e**(2*c + 2*d*x)*atan(e**(c + d*x))*a*b**2 + 9
0*e**(2*c + 2*d*x)*atan(e**(c + d*x))*b**3 + 15*atan(e**(c + d*x))*a**3 +
9*atan(e**(c + d*x))*a**2*b + 9*atan(e**(c + d*x))*a*b**2 + 15*atan(e**(c
+ d*x))*b**3 + 15*e**(11*c + 11*d*x)*a**3 + 9*e**(11*c + 11*d*x)*a**2*b +
9*e**(11*c + 11*d*x)*a*b**2 - 33*e**(11*c + 11*d*x)*b**3 + 85*e**(9*c + 9*
d*x)*a**3 + 51*e**(9*c + 9*d*x)*a**2*b - 141*e**(9*c + 9*d*x)*a*b**2 + ...
```

3.279 $\int \operatorname{sech}^8(c+dx)\left(a+b\sinh^2(c+dx)\right)^3\,dx$

Optimal result

Integrand size = 23, antiderivative size = 80

$$\int \operatorname{sech}^8(c+dx)\left(a+b\sinh^2(c+dx)\right)^3\,dx=\frac{a^3\tanh(c+dx)}{d}-\frac{a^2(a-b)\tanh^3(c+dx)}{d}+\frac{3a(a-b)^2\tanh^5(c+dx)}{5d}-\frac{(a-b)^3\tanh^7(c+dx)}{7d}$$

output
```
a^3*tanh(d*x+c)/d-a^2*(a-b)*tanh(d*x+c)^3/d+3/5*a*(a-b)^2*tanh(d*x+c)^5/d-
1/7*(a-b)^3*tanh(d*x+c)^7/d
```

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 163 vs. 2(80) = 160.

Time = 2.41 (sec) , antiderivative size = 163, normalized size of antiderivative = 2.04

$$\int \operatorname{sech}^8(c+dx)\left(a+b\sinh^2(c+dx)\right)^3\,dx$$
$$=\frac{(512a^3-304a^2b+192ab^2-50b^3+(464a^3+232a^2b-246ab^2+75b^3)\cosh(2(c+dx))+2(64a^3+32a^2}{}$$

input
```
Integrate[Sech[c + d*x]^8*(a + b*Sinh[c + d*x]^2)^3,x]
```

output
```
((512*a^3 - 304*a^2*b + 192*a*b^2 - 50*b^3 + (464*a^3 + 232*a^2*b - 246*a*
b^2 + 75*b^3)*Cosh[2*(c + d*x)] + 2*(64*a^3 + 32*a^2*b + 24*a*b^2 - 15*b^3
)*Cosh[4*(c + d*x)] + 16*a^3*Cosh[6*(c + d*x)] + 8*a^2*b*Cosh[6*(c + d*x)]
 + 6*a*b^2*Cosh[6*(c + d*x)] + 5*b^3*Cosh[6*(c + d*x)])*Sech[c + d*x]^6*Ta
nh[c + d*x])/(1120*d)
```

Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 72, normalized size of antiderivative = 0.90, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.174, Rules used = {3042, 3670, 210, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \text{sech}^8(c+dx)\left(a+b\sinh^2(c+dx)\right)^3 dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\left(a-b\sin(ic+idx)^2\right)^3}{\cos(ic+idx)^8}dx$$

$$\Big\downarrow \text{3670}$$

$$\frac{\int \left(a-(a-b)\tanh^2(c+dx)\right)^3 d\tanh(c+dx)}{d}$$

$$\Big\downarrow \text{210}$$

$$\frac{\int \left(-(a-b)^3\tanh^6(c+dx)+3a(a-b)^2\tanh^4(c+dx)-3a^2(a-b)\tanh^2(c+dx)+a^3\right) d\tanh(c+dx)}{d}$$

$$\Big\downarrow \text{2009}$$

$$\frac{a^3\tanh(c+dx)-a^2(a-b)\tanh^3(c+dx)-\frac{1}{7}(a-b)^3\tanh^7(c+dx)+\frac{3}{5}a(a-b)^2\tanh^5(c+dx)}{d}$$

input
```
Int[Sech[c + d*x]^8*(a + b*Sinh[c + d*x]^2)^3,x]
```

output
```
(a^3*Tanh[c + d*x] - a^2*(a - b)*Tanh[c + d*x]^3 + (3*a*(a - b)^2*Tanh[c +
 d*x]^5)/5 - ((a - b)^3*Tanh[c + d*x]^7)/7)/d
```

Defintions of rubi rules used

rule 210
```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Int[ExpandIntegrand[(a + b*x^2
)^p, x], x] /; FreeQ[{a, b}, x] && IGtQ[p, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3670
```
Int[cos[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(
p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Simp[ff/f   Su
bst[Int[(a + (a + b)*ff^2*x^2)^p/(1 + ff^2*x^2)^(m/2 + p + 1), x], x, Tan[e
 + f*x]/ff], x]] /; FreeQ[{a, b, e, f}, x] && IntegerQ[m/2] && IntegerQ[p]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 288 vs. $2(76) = 152$.

Time = 0.30 (sec) , antiderivative size = 289, normalized size of antiderivative = 3.61

$$\frac{a^3\left(\frac{16}{35}+\frac{\operatorname{sech}(dx+c)^6}{7}+\frac{6\operatorname{sech}(dx+c)^4}{35}+\frac{8\operatorname{sech}(dx+c)^2}{35}\right)\tanh\left(dx+c\right)+3a^2b\left(-\frac{\sinh(dx+c)}{6\cosh(dx+c)^7}+\frac{\left(\frac{16}{35}+\frac{\operatorname{sech}(dx+c)^6}{7}+\frac{6\operatorname{sech}(d}{35}\right.}{}\right.}{}$$

input
```
int(sech(d*x+c)^8*(a+b*sinh(d*x+c)^2)^3,x)
```

output
```
1/d*(a^3*(16/35+1/7*sech(d*x+c)^6+6/35*sech(d*x+c)^4+8/35*sech(d*x+c)^2)*t
anh(d*x+c)+3*a^2*b*(-1/6*sinh(d*x+c)/cosh(d*x+c)^7+1/6*(16/35+1/7*sech(d*x
+c)^6+6/35*sech(d*x+c)^4+8/35*sech(d*x+c)^2)*tanh(d*x+c))+3*b^2*a*(-1/4*si
nh(d*x+c)^3/cosh(d*x+c)^7-1/8*sinh(d*x+c)/cosh(d*x+c)^7+1/8*(16/35+1/7*sec
h(d*x+c)^6+6/35*sech(d*x+c)^4+8/35*sech(d*x+c)^2)*tanh(d*x+c))+b^3*(-1/2*s
inh(d*x+c)^5/cosh(d*x+c)^7-5/8*sinh(d*x+c)^3/cosh(d*x+c)^7-5/16*sinh(d*x+c
)/cosh(d*x+c)^7+5/16*(16/35+1/7*sech(d*x+c)^6+6/35*sech(d*x+c)^4+8/35*sech
(d*x+c)^2)*tanh(d*x+c)))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 814 vs. $2(76) = 152$.

Time = 0.09 (sec) , antiderivative size = 814, normalized size of antiderivative = 10.18

$$\int \operatorname{sech}^8(c+dx) \left(a+b\sinh^2(c+dx)\right)^3 dx = \text{Too large to display}$$

input
```
integrate(sech(d*x+c)^8*(a+b*sinh(d*x+c)^2)^3,x, algorithm="fricas")
```

output
```
-4/35*((8*a^3 + 4*a^2*b + 3*a*b^2 + 20*b^3)*cosh(d*x + c)^6 - 6*(8*a^3 + 4
*a^2*b + 3*a*b^2 - 15*b^3)*cosh(d*x + c)*sinh(d*x + c)^5 + (8*a^3 + 4*a^2*
b + 3*a*b^2 + 20*b^3)*sinh(d*x + c)^6 + 14*(4*a^3 + 2*a^2*b + 9*a*b^2)*cos
h(d*x + c)^4 + (56*a^3 + 28*a^2*b + 126*a*b^2 + 15*(8*a^3 + 4*a^2*b + 3*a*
b^2 + 20*b^3)*cosh(d*x + c)^2)*sinh(d*x + c)^4 - 4*(5*(8*a^3 + 4*a^2*b + 3
*a*b^2 - 15*b^3)*cosh(d*x + c)^3 + 28*(2*a^3 + a^2*b - 3*a*b^2)*cosh(d*x +
 c))*sinh(d*x + c)^3 + 280*a^3 - 140*a^2*b + 210*a*b^2 + 7*(24*a^3 + 52*a^
2*b - 21*a*b^2 + 20*b^3)*cosh(d*x + c)^2 + (15*(8*a^3 + 4*a^2*b + 3*a*b^2
+ 20*b^3)*cosh(d*x + c)^4 + 168*a^3 + 364*a^2*b - 147*a*b^2 + 140*b^3 + 84
*(4*a^3 + 2*a^2*b + 9*a*b^2)*cosh(d*x + c)^2)*sinh(d*x + c)^2 - 2*(3*(8*a^
3 + 4*a^2*b + 3*a*b^2 - 15*b^3)*cosh(d*x + c)^5 + 56*(2*a^3 + a^2*b - 3*a*
b^2)*cosh(d*x + c)^3 + 7*(24*a^3 - 28*a^2*b + 9*a*b^2 - 5*b^3)*cosh(d*x +
c))*sinh(d*x + c))/(d*cosh(d*x + c)^8 + 8*d*cosh(d*x + c)*sinh(d*x + c)^7
+ d*sinh(d*x + c)^8 + 8*d*cosh(d*x + c)^6 + 4*(7*d*cosh(d*x + c)^2 + 2*d)*
sinh(d*x + c)^6 + 4*(14*d*cosh(d*x + c)^3 + 9*d*cosh(d*x + c))*sinh(d*x +
c)^5 + 28*d*cosh(d*x + c)^4 + 2*(35*d*cosh(d*x + c)^4 + 60*d*cosh(d*x + c)
^2 + 14*d)*sinh(d*x + c)^4 + 8*(7*d*cosh(d*x + c)^5 + 15*d*cosh(d*x + c)^3
 + 7*d*cosh(d*x + c))*sinh(d*x + c)^3 + 56*d*cosh(d*x + c)^2 + 4*(7*d*cosh
(d*x + c)^6 + 30*d*cosh(d*x + c)^4 + 42*d*cosh(d*x + c)^2 + 14*d)*sinh(d*x
 + c)^2 + 4*(2*d*cosh(d*x + c)^7 + 9*d*cosh(d*x + c)^5 + 14*d*cosh(d*x ...
```

Sympy [F(-1)]

Timed out.

$$\int \operatorname{sech}^8(c+dx) \left(a+b\sinh^2(c+dx)\right)^3 \, dx = \text{Timed out}$$

input
```
integrate(sech(d*x+c)**8*(a+b*sinh(d*x+c)**2)**3,x)
```

output
```
Timed out
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1754 vs. $2(76) = 152$.

Time = 0.06 (sec) , antiderivative size = 1754, normalized size of antiderivative = 21.92

$$\int \operatorname{sech}^8(c+dx)\left(a+b\sinh^2(c+dx)\right)^3\,dx = \text{Too large to display}$$

input
```
integrate(sech(d*x+c)^8*(a+b*sinh(d*x+c)^2)^3,x, algorithm="maxima")
```

output
```
32/35*a^3*(7*e^(-2*d*x - 2*c)/(d*(7*e^(-2*d*x - 2*c) + 21*e^(-4*d*x - 4*c)
 + 35*e^(-6*d*x - 6*c) + 35*e^(-8*d*x - 8*c) + 21*e^(-10*d*x - 10*c) + 7*e
^(-12*d*x - 12*c) + e^(-14*d*x - 14*c) + 1)) + 21*e^(-4*d*x - 4*c)/(d*(7*e
^(-2*d*x - 2*c) + 21*e^(-4*d*x - 4*c) + 35*e^(-6*d*x - 6*c) + 35*e^(-8*d*x
 - 8*c) + 21*e^(-10*d*x - 10*c) + 7*e^(-12*d*x - 12*c) + e^(-14*d*x - 14*c
) + 1)) + 35*e^(-6*d*x - 6*c)/(d*(7*e^(-2*d*x - 2*c) + 21*e^(-4*d*x - 4*c)
 + 35*e^(-6*d*x - 6*c) + 35*e^(-8*d*x - 8*c) + 21*e^(-10*d*x - 10*c) + 7*e
^(-12*d*x - 12*c) + e^(-14*d*x - 14*c) + 1)) + 1/(d*(7*e^(-2*d*x - 2*c) +
21*e^(-4*d*x - 4*c) + 35*e^(-6*d*x - 6*c) + 35*e^(-8*d*x - 8*c) + 21*e^(-1
0*d*x - 10*c) + 7*e^(-12*d*x - 12*c) + e^(-14*d*x - 14*c) + 1))) + 16/35*a
^2*b*(7*e^(-2*d*x - 2*c)/(d*(7*e^(-2*d*x - 2*c) + 21*e^(-4*d*x - 4*c) + 35
*e^(-6*d*x - 6*c) + 35*e^(-8*d*x - 8*c) + 21*e^(-10*d*x - 10*c) + 7*e^(-12
*d*x - 12*c) + e^(-14*d*x - 14*c) + 1)) + 21*e^(-4*d*x - 4*c)/(d*(7*e^(-2*
d*x - 2*c) + 21*e^(-4*d*x - 4*c) + 35*e^(-6*d*x - 6*c) + 35*e^(-8*d*x - 8*
c) + 21*e^(-10*d*x - 10*c) + 7*e^(-12*d*x - 12*c) + e^(-14*d*x - 14*c) + 1
)) - 35*e^(-6*d*x - 6*c)/(d*(7*e^(-2*d*x - 2*c) + 21*e^(-4*d*x - 4*c) + 35
*e^(-6*d*x - 6*c) + 35*e^(-8*d*x - 8*c) + 21*e^(-10*d*x - 10*c) + 7*e^(-12
*d*x - 12*c) + e^(-14*d*x - 14*c) + 1)) + 70*e^(-8*d*x - 8*c)/(d*(7*e^(-2*
d*x - 2*c) + 21*e^(-4*d*x - 4*c) + 35*e^(-6*d*x - 6*c) + 35*e^(-8*d*x - 8*
c) + 21*e^(-10*d*x - 10*c) + 7*e^(-12*d*x - 12*c) + e^(-14*d*x - 14*c) ...
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 260 vs. $2(76) = 152$.

Time = 0.19 (sec) , antiderivative size = 260, normalized size of antiderivative = 3.25

$$\int \operatorname{sech}^8(c+dx)\left(a+b\sinh^2(c+dx)\right)^3\,dx = -\frac{2\left(35\,b^3e^{(12\,dx+12\,c)}+210\,ab^2e^{(10\,dx+10\,c)}+560\,a^2be^{(8\,dx+8\,c)}-210\,ab^2e^{(8\,dx+8\,c)}+175\,b^3e^{(8\,dx+8\,c)}+560\,a\right.}{}$$

input
```
integrate(sech(d*x+c)^8*(a+b*sinh(d*x+c)^2)^3,x, algorithm="giac")
```

output
```
-2/35*(35*b^3*e^(12*d*x + 12*c) + 210*a*b^2*e^(10*d*x + 10*c) + 560*a^2*b*
e^(8*d*x + 8*c) - 210*a*b^2*e^(8*d*x + 8*c) + 175*b^3*e^(8*d*x + 8*c) + 56
0*a^3*e^(6*d*x + 6*c) - 280*a^2*b*e^(6*d*x + 6*c) + 420*a*b^2*e^(6*d*x + 6
*c) + 336*a^3*e^(4*d*x + 4*c) + 168*a^2*b*e^(4*d*x + 4*c) - 84*a*b^2*e^(4*
d*x + 4*c) + 105*b^3*e^(4*d*x + 4*c) + 112*a^3*e^(2*d*x + 2*c) + 56*a^2*b*
e^(2*d*x + 2*c) + 42*a*b^2*e^(2*d*x + 2*c) + 16*a^3 + 8*a^2*b + 6*a*b^2 +
5*b^3)/(d*(e^(2*d*x + 2*c) + 1)^7)
```

Mupad [B] (verification not implemented)

Time = 1.73 (sec) , antiderivative size = 994, normalized size of antiderivative = 12.42

$$\int \operatorname{sech}^8(c+dx)\left(a+b\sinh^2(c+dx)\right)^3\,dx = \text{Too large to display}$$

input
```
int((a + b*sinh(c + d*x)^2)^3/cosh(c + d*x)^8,x)
```

output
```
- ((2*b^3)/(7*d) + (8*exp(6*c + 6*d*x)*(18*a*b^2 - 24*a^2*b + 16*a^3 - 5*b
^3))/(7*d) + (2*b^3*exp(12*c + 12*d*x))/(7*d) + (6*b*exp(4*c + 4*d*x)*(16*
a^2 - 16*a*b + 5*b^2))/(7*d) + (6*b*exp(8*c + 8*d*x)*(16*a^2 - 16*a*b + 5*
b^2))/(7*d) + (12*b^2*exp(2*c + 2*d*x)*(2*a - b))/(7*d) + (12*b^2*exp(10*c
 + 10*d*x)*(2*a - b))/(7*d))/(7*exp(2*c + 2*d*x) + 21*exp(4*c + 4*d*x) + 3
5*exp(6*c + 6*d*x) + 35*exp(8*c + 8*d*x) + 21*exp(10*c + 10*d*x) + 7*exp(1
2*c + 12*d*x) + exp(14*c + 14*d*x) + 1) - ((4*exp(4*c + 4*d*x)*(18*a*b^2 -
 24*a^2*b + 16*a^3 - 5*b^3))/(7*d) + (2*b^3*exp(10*c + 10*d*x))/(7*d) + (2
*b^2*(2*a - b))/(7*d) + (2*b*exp(2*c + 2*d*x)*(16*a^2 - 16*a*b + 5*b^2))/(
7*d) + (4*b*exp(6*c + 6*d*x)*(16*a^2 - 16*a*b + 5*b^2))/(7*d) + (10*b^2*ex
p(8*c + 8*d*x)*(2*a - b))/(7*d))/(6*exp(2*c + 2*d*x) + 15*exp(4*c + 4*d*x)
 + 20*exp(6*c + 6*d*x) + 15*exp(8*c + 8*d*x) + 6*exp(10*c + 10*d*x) + exp(
12*c + 12*d*x) + 1) - ((2*(18*a*b^2 - 24*a^2*b + 16*a^3 - 5*b^3))/(35*d) +
 (2*b^3*exp(6*c + 6*d*x))/(7*d) + (6*b*exp(2*c + 2*d*x)*(16*a^2 - 16*a*b +
 5*b^2))/(35*d) + (6*b^2*exp(4*c + 4*d*x)*(2*a - b))/(7*d))/(4*exp(2*c + 2
*d*x) + 6*exp(4*c + 4*d*x) + 4*exp(6*c + 6*d*x) + exp(8*c + 8*d*x) + 1) -
((2*b^3*exp(2*c + 2*d*x))/(7*d) + (2*b^2*(2*a - b))/(7*d))/(2*exp(2*c + 2*
d*x) + exp(4*c + 4*d*x) + 1) - ((2*b*(16*a^2 - 16*a*b + 5*b^2))/(35*d) + (
8*exp(2*c + 2*d*x)*(18*a*b^2 - 24*a^2*b + 16*a^3 - 5*b^3))/(35*d) + (2*b^3
*exp(8*c + 8*d*x))/(7*d) + (12*b*exp(4*c + 4*d*x)*(16*a^2 - 16*a*b + 5*...
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 358, normalized size of antiderivative = 4.48

$$\int \operatorname{sech}^8(c+dx)\left(a+b\sinh^2(c+dx)\right)^3\,dx$$

$$= \frac{\frac{2e^{14dx+14c}b^3}{7} - 12e^{10dx+10c}a\,b^2 + 6e^{10dx+10c}b^3 - 32e^{8dx+8c}a^2b + 12e^{8dx+8c}a\,b^2 - 32e^{6dx+6c}a^3 + 16e^{6dx+6c}a^2b}{d\left(e^{14dx+14c} + 7e^{12dx+12c}\right.}$$

input
```
int(sech(d*x+c)^8*(a+b*sinh(d*x+c)^2)^3,x)
```

output
```
(2*(5*e**(14*c + 14*d*x)*b**3 - 210*e**(10*c + 10*d*x)*a*b**2 + 105*e**(10
*c + 10*d*x)*b**3 - 560*e**(8*c + 8*d*x)*a**2*b + 210*e**(8*c + 8*d*x)*a*b
**2 - 560*e**(6*c + 6*d*x)*a**3 + 280*e**(6*c + 6*d*x)*a**2*b - 420*e**(6*
c + 6*d*x)*a*b**2 + 175*e**(6*c + 6*d*x)*b**3 - 336*e**(4*c + 4*d*x)*a**3
- 168*e**(4*c + 4*d*x)*a**2*b + 84*e**(4*c + 4*d*x)*a*b**2 - 112*e**(2*c +
 2*d*x)*a**3 - 56*e**(2*c + 2*d*x)*a**2*b - 42*e**(2*c + 2*d*x)*a*b**2 + 3
5*e**(2*c + 2*d*x)*b**3 - 16*a**3 - 8*a**2*b - 6*a*b**2))/(35*d*(e**(14*c
+ 14*d*x) + 7*e**(12*c + 12*d*x) + 21*e**(10*c + 10*d*x) + 35*e**(8*c + 8*
d*x) + 35*e**(6*c + 6*d*x) + 21*e**(4*c + 4*d*x) + 7*e**(2*c + 2*d*x) + 1)
)
```

3.280 $\int \frac{\cosh^7(c+dx)}{a+b\sinh^2(c+dx)} dx$

Optimal result

Integrand size = 23, antiderivative size = 108

$$\int \frac{\cosh^7(c+dx)}{a+b\sinh^2(c+dx)} dx = -\frac{(a-b)^3 \arctan\left(\frac{\sqrt{b}\sinh(c+dx)}{\sqrt{a}}\right)}{\sqrt{a}b^{7/2}d} + \frac{(a^2-3ab+3b^2)\sinh(c+dx)}{b^3d} - \frac{(a-3b)\sinh^3(c+dx)}{3b^2d} + \frac{\sinh^5(c+dx)}{5bd}$$

output
```
-(a-b)^3*arctan(b^(1/2)*sinh(d*x+c)/a^(1/2))/a^(1/2)/b^(7/2)/d+(a^2-3*a*b+
3*b^2)*sinh(d*x+c)/b^3/d-1/3*(a-3*b)*sinh(d*x+c)^3/b^2/d+1/5*sinh(d*x+c)^5
/b/d
```

Mathematica [A] (verified)

Time = 0.36 (sec) , antiderivative size = 117, normalized size of antiderivative = 1.08

$$\int \frac{\cosh^7(c+dx)}{a+b\sinh^2(c+dx)} dx = \frac{30\sqrt{b}(8a^2-22ab+19b^2)\sinh(c+dx)+5b^{3/2}(-4a+9b)\sinh(3(c+dx))+\frac{3\left(80(a-b)^3\arctan\left(\frac{\sqrt{a}\operatorname{csch}(c+dx)}{\sqrt{b}}\right)\right.}{\sqrt{a}}}{240b^{7/2}d}$$

input
```
Integrate[Cosh[c + d*x]^7/(a + b*Sinh[c + d*x]^2),x]
```

output
```
(30*Sqrt[b]*(8*a^2 - 22*a*b + 19*b^2)*Sinh[c + d*x] + 5*b^(3/2)*(-4*a + 9*
b)*Sinh[3*(c + d*x)] + (3*(80*(a - b)^3*ArcTan[(Sqrt[a]*Csch[c + d*x])/Sqr
t[b]] + Sqrt[a]*b^(5/2)*Sinh[5*(c + d*x)]))/Sqrt[a])/(240*b^(7/2)*d)
```

Rubi [A] (verified)

Time = 0.33 (sec) , antiderivative size = 100, normalized size of antiderivative = 0.93, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.174$, Rules used = $\{3042, 3669, 300, 2009\}$

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cosh^7(c+dx)}{a+b\sinh^2(c+dx)}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\cos(ic+idx)^7}{a-b\sin(ic+idx)^2}dx$$

$$\Big\downarrow \text{3669}$$

$$\frac{\int \frac{\left(\sinh^2(c+dx)+1\right)^3}{b\sinh^2(c+dx)+a}d\sinh(c+dx)}{d}$$

$$\Big\downarrow \text{300}$$

$$\frac{\int \left(\frac{\sinh^4(c+dx)}{b}-\frac{(a-3b)\sinh^2(c+dx)}{b^2}+\frac{a^2-3ba+3b^2}{b^3}+\frac{-a^3+3ba^2-3b^2a+b^3}{b^3\left(b\sinh^2(c+dx)+a\right)}\right)d\sinh(c+dx)}{d}$$

$$\Big\downarrow \text{2009}$$

$$\frac{\frac{(a^2-3ab+3b^2)\sinh(c+dx)}{b^3}-\frac{(a-b)^3\arctan\left(\frac{\sqrt{b}\sinh(c+dx)}{\sqrt{a}}\right)}{\sqrt{a}b^{7/2}}-\frac{(a-3b)\sinh^3(c+dx)}{3b^2}+\frac{\sinh^5(c+dx)}{5b}}{d}$$

input
```
Int[Cosh[c + d*x]^7/(a + b*Sinh[c + d*x]^2),x]
```

output
```
(-(((a - b)^3*ArcTan[(Sqrt[b]*Sinh[c + d*x])/Sqrt[a]])/(Sqrt[a]*b^(7/2)))
+ ((a^2 - 3*a*b + 3*b^2)*Sinh[c + d*x])/b^3 - ((a - 3*b)*Sinh[c + d*x]^3)/
(3*b^2) + Sinh[c + d*x]^5/(5*b))/d
```

Defintions of rubi rules used

rule 300
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_), x_Symbol] :> Int
[PolynomialDivide[(a + b*x^2)^p, (c + d*x^2)^(-q), x], x] /; FreeQ[{a, b, c
, d}, x] && NeQ[b*c - a*d, 0] && IGtQ[p, 0] && ILtQ[q, 0] && GeQ[p, -q]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3669
```
Int[cos[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^
(p_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Simp[ff/f   S
ubst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a + b*ff^2*x^2)^p, x], x, Sin[e + f*x]
/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m - 1)/2]
```

Maple [A] (verified)

Time = 0.11 (sec) , antiderivative size = 123, normalized size of antiderivative = 1.14

$$\frac{\frac{\frac{b^2\sinh(dx+c)^5}{5}-\frac{ab\sinh(dx+c)^3}{3}+b^2\sinh(dx+c)^3+\sinh(dx+c)a^2-3\sinh(dx+c)ab+3\sinh(dx+c)b^2}{b^3}+\frac{(-a^3+3a^2b-3b^2a+b^3)\arctan\left(\frac{b\sinh(dx+c)}{\sqrt{ab}}\right)}{b^3\sqrt{ab}}}{d}$$

input
```
int(cosh(d*x+c)^7/(a+b*sinh(d*x+c)^2),x)
```

output
```
1/d*(1/b^3*(1/5*b^2*sinh(d*x+c)^5-1/3*a*b*sinh(d*x+c)^3+b^2*sinh(d*x+c)^3+
sinh(d*x+c)*a^2-3*sinh(d*x+c)*a*b+3*sinh(d*x+c)*b^2)+(-a^3+3*a^2*b-3*a*b^2
+b^3)/b^3/(a*b)^(1/2)*arctan(b*sinh(d*x+c)/(a*b)^(1/2)))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1551 vs. $2(96) = 192$.

Time = 0.13 (sec) , antiderivative size = 3069, normalized size of antiderivative = 28.42

$$\int \frac{\cosh^7(c+dx)}{a+b\sinh^2(c+dx)}\,dx = \text{Too large to display}$$

input
```
integrate(cosh(d*x+c)^7/(a+b*sinh(d*x+c)^2),x, algorithm="fricas")
```

output
```
Too large to include
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\cosh^7(c+dx)}{a+b\sinh^2(c+dx)}\,dx = \text{Timed out}$$

input
```
integrate(cosh(d*x+c)**7/(a+b*sinh(d*x+c)**2),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{\cosh^7(c+dx)}{a+b\sinh^2(c+dx)}\,dx = \int \frac{\cosh\left(dx+c\right)^7}{b\sinh\left(dx+c\right)^2+a}\,dx$$

input
```
integrate(cosh(d*x+c)^7/(a+b*sinh(d*x+c)^2),x, algorithm="maxima")
```

output
```
1/480*(3*b^2*e^(10*d*x + 10*c) - 3*b^2 - 5*(4*a*b*e^(8*c) - 9*b^2*e^(8*c))
*e^(8*d*x) + 30*(8*a^2*e^(6*c) - 22*a*b*e^(6*c) + 19*b^2*e^(6*c))*e^(6*d*x
) - 30*(8*a^2*e^(4*c) - 22*a*b*e^(4*c) + 19*b^2*e^(4*c))*e^(4*d*x) + 5*(4*
a*b*e^(2*c) - 9*b^2*e^(2*c))*e^(2*d*x))*e^(-5*d*x - 5*c)/(b^3*d) - 1/128*i
ntegrate(256*((a^3*e^(3*c) - 3*a^2*b*e^(3*c) + 3*a*b^2*e^(3*c) - b^3*e^(3*
c))*e^(3*d*x) + (a^3*e^c - 3*a^2*b*e^c + 3*a*b^2*e^c - b^3*e^c)*e^(d*x))/(
b^4*e^(4*d*x + 4*c) + b^4 + 2*(2*a*b^3*e^(2*c) - b^4*e^(2*c))*e^(2*d*x)),
x)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{\cosh^7(c+dx)}{a+b\sinh^2(c+dx)}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(cosh(d*x+c)^7/(a+b*sinh(d*x+c)^2),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Limit: Max order reached or unable
to make series expansion Error: Bad Argument Value
```

Mupad [B] (verification not implemented)

Time = 2.39 (sec) , antiderivative size = 954, normalized size of antiderivative = 8.83

$$\int \frac{\cosh^7(c+dx)}{a+b\sinh^2(c+dx)}\,dx = \text{Too large to display}$$

input
```
int(cosh(c + d*x)^7/(a + b*sinh(c + d*x)^2),x)
```

output
```
exp(5*c + 5*d*x)/(160*b*d) - exp(- 5*c - 5*d*x)/(160*b*d) - ((2*atan((exp(
d*x)*exp(c)*(a - b)^3*(a*b^7*d^2)^(1/2))/(2*a*b^3*d*((a - b)^6)^(1/2))) +
2*atan((a*b^8*exp(d*x)*exp(c)*((4*(12*a^3*b^5*d*(a^6 - 6*a^5*b - 6*a*b^5 +
 b^6 + 15*a^2*b^4 - 20*a^3*b^3 + 15*a^4*b^2)^(1/2) - 8*a^2*b^6*d*(a^6 - 6*
a^5*b - 6*a*b^5 + b^6 + 15*a^2*b^4 - 20*a^3*b^3 + 15*a^4*b^2)^(1/2) - 8*a^
4*b^4*d*(a^6 - 6*a^5*b - 6*a*b^5 + b^6 + 15*a^2*b^4 - 20*a^3*b^3 + 15*a^4*
b^2)^(1/2) + 2*a^5*b^3*d*(a^6 - 6*a^5*b - 6*a*b^5 + b^6 + 15*a^2*b^4 - 20*
a^3*b^3 + 15*a^4*b^2)^(1/2) + 2*a*b^7*d*(a^6 - 6*a^5*b - 6*a*b^5 + b^6 + 1
5*a^2*b^4 - 20*a^3*b^3 + 15*a^4*b^2)^(1/2)))/(a^2*b^15*d^2*(a - b)^3) - (2
*(a^7*(a*b^7*d^2)^(1/2) - b^7*(a*b^7*d^2)^(1/2) + 7*a*b^6*(a*b^7*d^2)^(1/2
) - 7*a^6*b*(a*b^7*d^2)^(1/2) - 21*a^2*b^5*(a*b^7*d^2)^(1/2) + 35*a^3*b^4*
(a*b^7*d^2)^(1/2) - 35*a^4*b^3*(a*b^7*d^2)^(1/2) + 21*a^5*b^2*(a*b^7*d^2)^
(1/2)))/(a^2*b^11*d*((a - b)^6)^(1/2)*(a*b^7*d^2)^(1/2)))*(a*b^7*d^2)^(1/2
))/(4*a^4 - 16*a^3*b - 16*a*b^3 + 4*b^4 + 24*a^2*b^2) + (2*exp(3*c)*exp(3*
d*x)*(a^7*(a*b^7*d^2)^(1/2) - b^7*(a*b^7*d^2)^(1/2) + 7*a*b^6*(a*b^7*d^2)^
(1/2) - 7*a^6*b*(a*b^7*d^2)^(1/2) - 21*a^2*b^5*(a*b^7*d^2)^(1/2) + 35*a^3*
b^4*(a*b^7*d^2)^(1/2) - 35*a^4*b^3*(a*b^7*d^2)^(1/2) + 21*a^5*b^2*(a*b^7*d
^2)^(1/2)))/(a*b^3*d*((a - b)^6)^(1/2)*(4*a^4 - 16*a^3*b - 16*a*b^3 + 4*b^
4 + 24*a^2*b^2))))*(a^6 - 6*a^5*b - 6*a*b^5 + b^6 + 15*a^2*b^4 - 20*a^3*b^
3 + 15*a^4*b^2)^(1/2))/(2*(a*b^7*d^2)^(1/2)) + (exp(c + d*x)*(8*a^2 - 2...
```

Reduce [B] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 1879, normalized size of antiderivative = 17.40

$$\int \frac{\cosh^7(c+dx)}{a+b\sinh^2(c+dx)}\,dx = \text{Too large to display}$$

input
```
int(cosh(d*x+c)^7/(a+b*sinh(d*x+c)^2),x)
```

output

```
(480*e**(5*c + 5*d*x)*sqrt(b)*sqrt(a)*sqrt(a - b)*sqrt(2*sqrt(a)*sqrt(a -
b) + 2*a - b)*atan((e**(c + d*x)*b)/(sqrt(b)*sqrt(2*sqrt(a)*sqrt(a - b) +
2*a - b)))*a**3 - 1440*e**(5*c + 5*d*x)*sqrt(b)*sqrt(a)*sqrt(a - b)*sqrt(2
*sqrt(a)*sqrt(a - b) + 2*a - b)*atan((e**(c + d*x)*b)/(sqrt(b)*sqrt(2*sqrt
(a)*sqrt(a - b) + 2*a - b)))*a**2*b + 1440*e**(5*c + 5*d*x)*sqrt(b)*sqrt(a
)*sqrt(a - b)*sqrt(2*sqrt(a)*sqrt(a - b) + 2*a - b)*atan((e**(c + d*x)*b)/
(sqrt(b)*sqrt(2*sqrt(a)*sqrt(a - b) + 2*a - b)))*a*b**2 - 480*e**(5*c + 5*
d*x)*sqrt(b)*sqrt(a)*sqrt(a - b)*sqrt(2*sqrt(a)*sqrt(a - b) + 2*a - b)*ata
n((e**(c + d*x)*b)/(sqrt(b)*sqrt(2*sqrt(a)*sqrt(a - b) + 2*a - b)))*b**3 -
 480*e**(5*c + 5*d*x)*sqrt(b)*sqrt(2*sqrt(a)*sqrt(a - b) + 2*a - b)*atan((
e**(c + d*x)*b)/(sqrt(b)*sqrt(2*sqrt(a)*sqrt(a - b) + 2*a - b)))*a**4 + 14
40*e**(5*c + 5*d*x)*sqrt(b)*sqrt(2*sqrt(a)*sqrt(a - b) + 2*a - b)*atan((e*
*(c + d*x)*b)/(sqrt(b)*sqrt(2*sqrt(a)*sqrt(a - b) + 2*a - b)))*a**3*b - 14
40*e**(5*c + 5*d*x)*sqrt(b)*sqrt(2*sqrt(a)*sqrt(a - b) + 2*a - b)*atan((e*
*(c + d*x)*b)/(sqrt(b)*sqrt(2*sqrt(a)*sqrt(a - b) + 2*a - b)))*a**2*b**2 +
 480*e**(5*c + 5*d*x)*sqrt(b)*sqrt(2*sqrt(a)*sqrt(a - b) + 2*a - b)*atan((
e**(c + d*x)*b)/(sqrt(b)*sqrt(2*sqrt(a)*sqrt(a - b) + 2*a - b)))*a*b**3 +
240*e**(5*c + 5*d*x)*sqrt(b)*sqrt(a)*sqrt(a - b)*sqrt(2*sqrt(a)*sqrt(a - b
) - 2*a + b)*log( - sqrt(2*sqrt(a)*sqrt(a - b) - 2*a + b) + e**(c + d*x)*s
qrt(b))*a**3 - 720*e**(5*c + 5*d*x)*sqrt(b)*sqrt(a)*sqrt(a - b)*sqrt(2*...
```

3.281 $\int \frac{\cosh^6(c+dx)}{a+b\sinh^2(c+dx)}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 121

$$\int \frac{\cosh^6(c+dx)}{a+b\sinh^2(c+dx)}\,dx = \frac{(8a^2-20ab+15b^2)\,x}{8b^3} - \frac{(a-b)^{5/2}\text{arctanh}\left(\frac{\sqrt{a-b}\tanh(c+dx)}{\sqrt{a}}\right)}{\sqrt{a}b^3d} - \frac{(4a-7b)\cosh(c+dx)\sinh(c+dx)}{8b^2d} + \frac{\cosh^3(c+dx)\sinh(c+dx)}{4bd}$$

output
```
1/8*(8*a^2-20*a*b+15*b^2)*x/b^3-(a-b)^(5/2)*arctanh((a-b)^(1/2)*tanh(d*x+c
)/a^(1/2))/a^(1/2)/b^3/d-1/8*(4*a-7*b)*cosh(d*x+c)*sinh(d*x+c)/b^2/d+1/4*c
osh(d*x+c)^3*sinh(d*x+c)/b/d
```

Mathematica [A] (verified)

Time = 0.41 (sec) , antiderivative size = 106, normalized size of antiderivative = 0.88

$$\int \frac{\cosh^6(c+dx)}{a+b\sinh^2(c+dx)}\,dx = \frac{-32(a-b)^{5/2}\text{arctanh}\left(\frac{\sqrt{a-b}\tanh(c+dx)}{\sqrt{a}}\right)+\sqrt{a}(4(8a^2-20ab+15b^2)\,(c+dx)-8(a-2b)b\sinh(2(c+dx)}{32\sqrt{a}b^3d}$$

input
```
Integrate[Cosh[c + d*x]^6/(a + b*Sinh[c + d*x]^2),x]
```

output
```
(-32*(a - b)^(5/2)*ArcTanh[(Sqrt[a - b]*Tanh[c + d*x])/Sqrt[a]] + Sqrt[a]*
(4*(8*a^2 - 20*a*b + 15*b^2)*(c + d*x) - 8*(a - 2*b)*b*Sinh[2*(c + d*x)] +
 b^2*Sinh[4*(c + d*x)]))/(32*Sqrt[a]*b^3*d)
```

Rubi [A] (verified)

Time = 0.43 (sec) , antiderivative size = 149, normalized size of antiderivative = 1.23, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.391$, Rules used = {3042, 3670, 316, 25, 402, 25, 397, 219, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cosh^6(c+dx)}{a+b\sinh^2(c+dx)}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\cos(ic+idx)^6}{a-b\sin(ic+idx)^2}dx$$

$$\Big\downarrow \text{3670}$$

$$\frac{\int \frac{1}{\left(1-\tanh^2(c+dx)\right)^3\left(a-(a-b)\tanh^2(c+dx)\right)}d\tanh(c+dx)}{d}$$

$$\Big\downarrow \text{316}$$

$$\frac{\frac{\int -\frac{3(a-b)\tanh^2(c+dx)+a-4b}{\left(1-\tanh^2(c+dx)\right)^2\left(a-(a-b)\tanh^2(c+dx)\right)}d\tanh(c+dx)}{4b}+\frac{\tanh(c+dx)}{4b\left(1-\tanh^2(c+dx)\right)^2}}{d}$$

$$\Big\downarrow \text{25}$$

$$\frac{\frac{\tanh(c+dx)}{4b\left(1-\tanh^2(c+dx)\right)^2}-\frac{\int \frac{3(a-b)\tanh^2(c+dx)+a-4b}{\left(1-\tanh^2(c+dx)\right)^2\left(a-(a-b)\tanh^2(c+dx)\right)}d\tanh(c+dx)}{4b}}{d}$$

$$\Big\downarrow \text{402}$$

$$\frac{\frac{\tanh(c+dx)}{4b(1-\tanh^2(c+dx))^2}-\frac{\frac{\int -\frac{4a^2-9ba+8b^2+(4a-7b)(a-b)\tanh^2(c+dx)}{\left(1-\tanh^2(c+dx)\right)\left(a-(a-b)\tanh^2(c+dx)\right)}d\tanh(c+dx)}{2b}+\frac{(4a-7b)\tanh(c+dx)}{2b\left(1-\tanh^2(c+dx)\right)}}{4b}}{d}$$

$\Big\downarrow$ 25

$$\frac{\frac{\tanh(c+dx)}{4b(1-\tanh^2(c+dx))^2}-\frac{\frac{(4a-7b)\tanh(c+dx)}{2b\left(1-\tanh^2(c+dx)\right)}-\frac{\int \frac{4a^2-9ba+8b^2+(4a-7b)(a-b)\tanh^2(c+dx)}{\left(1-\tanh^2(c+dx)\right)\left(a-(a-b)\tanh^2(c+dx)\right)}d\tanh(c+dx)}{2b}}{4b}}{d}$$

$\Big\downarrow$ 397

$$\frac{\frac{\tanh(c+dx)}{4b(1-\tanh^2(c+dx))^2}-\frac{\frac{(4a-7b)\tanh(c+dx)}{2b\left(1-\tanh^2(c+dx)\right)}-\frac{\frac{\left(8a^2-20ab+15b^2\right)\int\frac{1}{1-\tanh^2(c+dx)}d\tanh(c+dx)}{b}-\frac{8(a-b)^3\int\frac{1}{a-(a-b)\tanh^2(c+dx)}d\tanh(c+dx)}{b}}{2b}}{4b}}{d}$$

$\Big\downarrow$ 219

$$\frac{\frac{\tanh(c+dx)}{4b(1-\tanh^2(c+dx))^2}-\frac{\frac{(4a-7b)\tanh(c+dx)}{2b\left(1-\tanh^2(c+dx)\right)}-\frac{\frac{\left(8a^2-20ab+15b^2\right)\text{arctanh}(\tanh(c+dx))}{b}-\frac{8(a-b)^3\int\frac{1}{a-(a-b)\tanh^2(c+dx)}d\tanh(c+dx)}{b}}{2b}}{4b}}{d}$$

$\Big\downarrow$ 221

$$\frac{\frac{\tanh(c+dx)}{4b(1-\tanh^2(c+dx))^2}-\frac{\frac{(4a-7b)\tanh(c+dx)}{2b\left(1-\tanh^2(c+dx)\right)}-\frac{\frac{\left(8a^2-20ab+15b^2\right)\text{arctanh}(\tanh(c+dx))}{b}-\frac{8(a-b)^{5/2}\text{arctanh}\left(\frac{\sqrt{a-b}\tanh(c+dx)}{\sqrt{a}}\right)}{\sqrt{a}b}}{2b}}{4b}}{d}$$

input
```
Int[Cosh[c + d*x]^6/(a + b*Sinh[c + d*x]^2),x]
```

output
```
(Tanh[c + d*x]/(4*b*(1 - Tanh[c + d*x]^2)^2) - (-1/2*(((8*a^2 - 20*a*b + 1
5*b^2)*ArcTanh[Tanh[c + d*x]])/b - (8*(a - b)^(5/2)*ArcTanh[(Sqrt[a - b]*T
anh[c + d*x])/Sqrt[a]])/(Sqrt[a]*b))/b + ((4*a - 7*b)*Tanh[c + d*x])/(2*b*
(1 - Tanh[c + d*x]^2)))/(4*b))/d
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 316
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_), x_Symbol] :> Sim
p[(-b)*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^(q + 1)/(2*a*(p + 1)*(b*c - a*d))
), x] + Simp[1/(2*a*(p + 1)*(b*c - a*d))   Int[(a + b*x^2)^(p + 1)*(c + d*x
^2)^q*Simp[b*c + 2*(p + 1)*(b*c - a*d) + d*b*(2*(p + q + 2) + 1)*x^2, x], x
], x] /; FreeQ[{a, b, c, d, q}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] &&  !
( !IntegerQ[p] && IntegerQ[q] && LtQ[q, -1]) && IntBinomialQ[a, b, c, d, 2,
 p, q, x]
```

rule 397
```
Int[((e_) + (f_.)*(x_)^2)/(((a_) + (b_.)*(x_)^2)*((c_) + (d_.)*(x_)^2)), x_
Symbol] :> Simp[(b*e - a*f)/(b*c - a*d)   Int[1/(a + b*x^2), x], x] - Simp[
(d*e - c*f)/(b*c - a*d)   Int[1/(c + d*x^2), x], x] /; FreeQ[{a, b, c, d, e
, f}, x]
```

rule 402
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_.)*((e_) + (f_.)*(x
_)^2), x_Symbol] :> Simp[(-(b*e - a*f))*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^
(q + 1)/(a*2*(b*c - a*d)*(p + 1))), x] + Simp[1/(a*2*(b*c - a*d)*(p + 1))
 Int[(a + b*x^2)^(p + 1)*(c + d*x^2)^q*Simp[c*(b*e - a*f) + e*2*(b*c - a*d)
*(p + 1) + d*(b*e - a*f)*(2*(p + q + 2) + 1)*x^2, x], x], x] /; FreeQ[{a, b
, c, d, e, f, q}, x] && LtQ[p, -1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3670
```
Int[cos[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(
p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Simp[ff/f   Su
bst[Int[(a + (a + b)*ff^2*x^2)^p/(1 + ff^2*x^2)^(m/2 + p + 1), x], x, Tan[e
 + f*x]/ff], x]] /; FreeQ[{a, b, e, f}, x] && IntegerQ[m/2] && IntegerQ[p]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 430 vs. $2(107) = 214$.

Time = 177.88 (sec) , antiderivative size = 431, normalized size of antiderivative = 3.56

method	result
risch	$\frac{x\,a^2}{b^3}-\frac{5ax}{2b^2}+\frac{15x}{8b}+\frac{e^{4dx+4c}}{64bd}-\frac{e^{2dx+2c}a}{8b^2d}+\frac{e^{2dx+2c}}{4bd}+\frac{e^{-2dx-2c}a}{8b^2d}-\frac{e^{-2dx-2c}}{4bd}-\frac{e^{-4dx-4c}}{64bd}+\frac{a\sqrt{a(a-}}{}$
derivativedivides	$-\frac{1}{4b\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^4}+\frac{1}{2b\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^3}-\frac{-4a+11b}{8b^2\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^2}-\frac{-9b+4a}{8b^2\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)}+\frac{\left(8a^2-20ab+15b^2\right)\ln\left(\text{ta}\right.}{8b^3}$
default	$-\frac{1}{4b\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^4}+\frac{1}{2b\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^3}-\frac{-4a+11b}{8b^2\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^2}-\frac{-9b+4a}{8b^2\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)}+\frac{\left(8a^2-20ab+15b^2\right)\ln\left(\text{ta}\right.}{8b^3}$

input
```
int(cosh(d*x+c)^6/(a+b*sinh(d*x+c)^2),x,method=_RETURNVERBOSE)
```

output
```
x/b^3*a^2-5/2*a*x/b^2+15/8*x/b+1/64/b/d*exp(4*d*x+4*c)-1/8/b^2/d*exp(2*d*x
+2*c)*a+1/4/b/d*exp(2*d*x+2*c)+1/8/b^2/d*exp(-2*d*x-2*c)*a-1/4/b/d*exp(-2*
d*x-2*c)-1/64/b/d*exp(-4*d*x-4*c)+1/2*a*(a*(a-b))^(1/2)/d/b^3*ln(exp(2*d*x
+2*c)+(2*a+2*(a*(a-b))^(1/2)-b)/b)-(a*(a-b))^(1/2)/d/b^2*ln(exp(2*d*x+2*c)
+(2*a+2*(a*(a-b))^(1/2)-b)/b)+1/2/a*(a*(a-b))^(1/2)/d/b*ln(exp(2*d*x+2*c)+
(2*a+2*(a*(a-b))^(1/2)-b)/b)-1/2*a*(a*(a-b))^(1/2)/d/b^3*ln(exp(2*d*x+2*c)
-(-2*a+2*(a*(a-b))^(1/2)+b)/b)+(a*(a-b))^(1/2)/d/b^2*ln(exp(2*d*x+2*c)-(-2
*a+2*(a*(a-b))^(1/2)+b)/b)-1/2/a*(a*(a-b))^(1/2)/d/b*ln(exp(2*d*x+2*c)-(-2
*a+2*(a*(a-b))^(1/2)+b)/b)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 776 vs. $2(107) = 214$.

Time = 0.11 (sec) , antiderivative size = 1817, normalized size of antiderivative = 15.02

$$\int \frac{\cosh^6(c+dx)}{a+b\sinh^2(c+dx)}\,dx = \text{Too large to display}$$

input
```
integrate(cosh(d*x+c)^6/(a+b*sinh(d*x+c)^2),x, algorithm="fricas")
```

output
```
[1/64*(b^2*cosh(d*x + c)^8 + 8*b^2*cosh(d*x + c)*sinh(d*x + c)^7 + b^2*sin
h(d*x + c)^8 + 8*(8*a^2 - 20*a*b + 15*b^2)*d*x*cosh(d*x + c)^4 - 8*(a*b -
2*b^2)*cosh(d*x + c)^6 + 4*(7*b^2*cosh(d*x + c)^2 - 2*a*b + 4*b^2)*sinh(d*
x + c)^6 + 8*(7*b^2*cosh(d*x + c)^3 - 6*(a*b - 2*b^2)*cosh(d*x + c))*sinh(
d*x + c)^5 + 2*(35*b^2*cosh(d*x + c)^4 + 4*(8*a^2 - 20*a*b + 15*b^2)*d*x -
 60*(a*b - 2*b^2)*cosh(d*x + c)^2)*sinh(d*x + c)^4 + 8*(7*b^2*cosh(d*x + c
)^5 + 4*(8*a^2 - 20*a*b + 15*b^2)*d*x*cosh(d*x + c) - 20*(a*b - 2*b^2)*cos
h(d*x + c)^3)*sinh(d*x + c)^3 + 8*(a*b - 2*b^2)*cosh(d*x + c)^2 + 4*(7*b^2
*cosh(d*x + c)^6 + 12*(8*a^2 - 20*a*b + 15*b^2)*d*x*cosh(d*x + c)^2 - 30*(
a*b - 2*b^2)*cosh(d*x + c)^4 + 2*a*b - 4*b^2)*sinh(d*x + c)^2 + 32*((a^2 -
 2*a*b + b^2)*cosh(d*x + c)^4 + 4*(a^2 - 2*a*b + b^2)*cosh(d*x + c)^3*sinh
(d*x + c) + 6*(a^2 - 2*a*b + b^2)*cosh(d*x + c)^2*sinh(d*x + c)^2 + 4*(a^2
 - 2*a*b + b^2)*cosh(d*x + c)*sinh(d*x + c)^3 + (a^2 - 2*a*b + b^2)*sinh(d
*x + c)^4)*sqrt((a - b)/a)*log((b^2*cosh(d*x + c)^4 + 4*b^2*cosh(d*x + c)*
sinh(d*x + c)^3 + b^2*sinh(d*x + c)^4 + 2*(2*a*b - b^2)*cosh(d*x + c)^2 +
2*(3*b^2*cosh(d*x + c)^2 + 2*a*b - b^2)*sinh(d*x + c)^2 + 8*a^2 - 8*a*b +
b^2 + 4*(b^2*cosh(d*x + c)^3 + (2*a*b - b^2)*cosh(d*x + c))*sinh(d*x + c)
+ 4*(a*b*cosh(d*x + c)^2 + 2*a*b*cosh(d*x + c)*sinh(d*x + c) + a*b*sinh(d*
x + c)^2 + 2*a^2 - a*b)*sqrt((a - b)/a))/(b*cosh(d*x + c)^4 + 4*b*cosh(d*x
 + c)*sinh(d*x + c)^3 + b*sinh(d*x + c)^4 + 2*(2*a - b)*cosh(d*x + c)^2...
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\cosh^6(c+dx)}{a+b\sinh^2(c+dx)}\,dx = \text{Timed out}$$

input
```
integrate(cosh(d*x+c)**6/(a+b*sinh(d*x+c)**2),x)
```

output
```
Timed out
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{\cosh^6(c+dx)}{a+b\sinh^2(c+dx)}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate(cosh(d*x+c)^6/(a+b*sinh(d*x+c)^2),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(b-a>0)', see `assume?` for more
details)Is
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 226 vs. $2(107) = 214$.

Time = 0.72 (sec) , antiderivative size = 226, normalized size of antiderivative = 1.87

$$\int \frac{\cosh^6(c+dx)}{a+b\sinh^2(c+dx)}\,dx$$

$$= \frac{\frac{8\left(8a^2-20ab+15b^2\right)(dx+c)}{b^3} + \frac{be^{(4dx+4c)}-8ae^{(2dx+2c)}+16be^{(2dx+2c)}}{b^2} - \frac{\left(48a^2e^{(4dx+4c)}-120abe^{(4dx+4c)}+90b^2e^{(4dx+4c)}-8abe^{(2dx}\right.}{b^3}}{64d}$$

input
```
integrate(cosh(d*x+c)^6/(a+b*sinh(d*x+c)^2),x, algorithm="giac")
```

output
```
1/64*(8*(8*a^2 - 20*a*b + 15*b^2)*(d*x + c)/b^3 + (b*e^(4*d*x + 4*c) - 8*a
*e^(2*d*x + 2*c) + 16*b*e^(2*d*x + 2*c))/b^2 - (48*a^2*e^(4*d*x + 4*c) - 1
20*a*b*e^(4*d*x + 4*c) + 90*b^2*e^(4*d*x + 4*c) - 8*a*b*e^(2*d*x + 2*c) +
16*b^2*e^(2*d*x + 2*c) + b^2)*e^(-4*d*x - 4*c)/b^3 - 64*(a^3 - 3*a^2*b + 3
*a*b^2 - b^3)*arctan(1/2*(b*e^(2*d*x + 2*c) + 2*a - b)/sqrt(-a^2 + a*b))/(
sqrt(-a^2 + a*b)*b^3))/d
```

Mupad [B] (verification not implemented)

Time = 2.26 (sec) , antiderivative size = 264, normalized size of antiderivative = 2.18

$$\int \frac{\cosh^6(c+dx)}{a+b\sinh^2(c+dx)}\,dx = \frac{x\,(8\,a^2-20\,a\,b+15\,b^2)}{8\,b^3}-\frac{\mathrm{e}^{-4\,c-4\,d\,x}}{64\,b\,d}+\frac{\mathrm{e}^{4\,c+4\,d\,x}}{64\,b\,d}+\frac{\mathrm{e}^{-2\,c-2\,d\,x}\,(a-2\,b)}{8\,b^2\,d}-\frac{\mathrm{e}^{2\,c+2\,d\,x}\,(a-2\,b)}{8\,b^2\,d}+\frac{\ln\left(\frac{4\,\mathrm{e}^{2\,c+2\,d\,x}\,(a-b)^3}{b^4}-\frac{2\,(a-b)^{5/2}\,(b+2\,a\,\mathrm{e}^{2\,c+2\,d\,x}-b\,\mathrm{e}^{2\,c+2\,d\,x})}{\sqrt{a}\,b^4}\right)\,(a-b)^{5/2}}{2\,\sqrt{a}\,b^3\,d}-\frac{\ln\left(\frac{4\,\mathrm{e}^{2\,c+2\,d\,x}\,(a-b)^3}{b^4}+\frac{2\,(a-b)^{5/2}\,(b+2\,a\,\mathrm{e}^{2\,c+2\,d\,x}-b\,\mathrm{e}^{2\,c+2\,d\,x})}{\sqrt{a}\,b^4}\right)\,(a-b)^{5/2}}{2\,\sqrt{a}\,b^3\,d}$$

input
```
int(cosh(c + d*x)^6/(a + b*sinh(c + d*x)^2),x)
```

output
```
(x*(8*a^2 - 20*a*b + 15*b^2))/(8*b^3) - exp(- 4*c - 4*d*x)/(64*b*d) + exp(
4*c + 4*d*x)/(64*b*d) + (exp(- 2*c - 2*d*x)*(a - 2*b))/(8*b^2*d) - (exp(2*
c + 2*d*x)*(a - 2*b))/(8*b^2*d) + (log((4*exp(2*c + 2*d*x)*(a - b)^3)/b^4
- (2*(a - b)^(5/2)*(b + 2*a*exp(2*c + 2*d*x) - b*exp(2*c + 2*d*x)))/(a^(1/
2)*b^4))*(a - b)^(5/2))/(2*a^(1/2)*b^3*d) - (log((4*exp(2*c + 2*d*x)*(a -
b)^3)/b^4 + (2*(a - b)^(5/2)*(b + 2*a*exp(2*c + 2*d*x) - b*exp(2*c + 2*d*x
)))/(a^(1/2)*b^4))*(a - b)^(5/2))/(2*a^(1/2)*b^3*d)
```

Reduce [B] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 630, normalized size of antiderivative = 5.21

$$\int \frac{\cosh^6(c+dx)}{a+b\sinh^2(c+dx)}\,dx$$
$$= \frac{-32e^{4dx+4c}\sqrt{a}\,\sqrt{a-b}\log\left(-\sqrt{2\sqrt{a}\,\sqrt{a-b}-2a+b}+e^{dx+c}\sqrt{b}\right)a^2+64e^{4dx+4c}\sqrt{a}\,\sqrt{a-b}\log\left(-\sqrt{2\sqrt{}}\right.}{}$$

input
```
int(cosh(d*x+c)^6/(a+b*sinh(d*x+c)^2),x)
```

output
```
( - 32*e**(4*c + 4*d*x)*sqrt(a)*sqrt(a - b)*log( - sqrt(2*sqrt(a)*sqrt(a -
 b) - 2*a + b) + e**(c + d*x)*sqrt(b))*a**2 + 64*e**(4*c + 4*d*x)*sqrt(a)*
sqrt(a - b)*log( - sqrt(2*sqrt(a)*sqrt(a - b) - 2*a + b) + e**(c + d*x)*sq
rt(b))*a*b - 32*e**(4*c + 4*d*x)*sqrt(a)*sqrt(a - b)*log( - sqrt(2*sqrt(a)
*sqrt(a - b) - 2*a + b) + e**(c + d*x)*sqrt(b))*b**2 - 32*e**(4*c + 4*d*x)
*sqrt(a)*sqrt(a - b)*log(sqrt(2*sqrt(a)*sqrt(a - b) - 2*a + b) + e**(c + d
*x)*sqrt(b))*a**2 + 64*e**(4*c + 4*d*x)*sqrt(a)*sqrt(a - b)*log(sqrt(2*sqr
t(a)*sqrt(a - b) - 2*a + b) + e**(c + d*x)*sqrt(b))*a*b - 32*e**(4*c + 4*d
*x)*sqrt(a)*sqrt(a - b)*log(sqrt(2*sqrt(a)*sqrt(a - b) - 2*a + b) + e**(c
+ d*x)*sqrt(b))*b**2 + 32*e**(4*c + 4*d*x)*sqrt(a)*sqrt(a - b)*log(2*sqrt(
a)*sqrt(a - b) + e**(2*c + 2*d*x)*b + 2*a - b)*a**2 - 64*e**(4*c + 4*d*x)*
sqrt(a)*sqrt(a - b)*log(2*sqrt(a)*sqrt(a - b) + e**(2*c + 2*d*x)*b + 2*a -
 b)*a*b + 32*e**(4*c + 4*d*x)*sqrt(a)*sqrt(a - b)*log(2*sqrt(a)*sqrt(a - b
) + e**(2*c + 2*d*x)*b + 2*a - b)*b**2 + e**(8*c + 8*d*x)*a*b**2 - 8*e**(6
*c + 6*d*x)*a**2*b + 16*e**(6*c + 6*d*x)*a*b**2 + 64*e**(4*c + 4*d*x)*a**3
*d*x - 160*e**(4*c + 4*d*x)*a**2*b*d*x + 120*e**(4*c + 4*d*x)*a*b**2*d*x +
 8*e**(2*c + 2*d*x)*a**2*b - 16*e**(2*c + 2*d*x)*a*b**2 - a*b**2)/(64*e**(
4*c + 4*d*x)*a*b**3*d)
```

3.282 $\int \frac{\cosh^5(c+dx)}{a+b\sinh^2(c+dx)}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 77

$$\int \frac{\cosh^5(c+dx)}{a+b\sinh^2(c+dx)}\,dx = \frac{(a-b)^2\arctan\left(\frac{\sqrt{b}\sinh(c+dx)}{\sqrt{a}}\right)}{\sqrt{a}b^{5/2}d} - \frac{(a-2b)\sinh(c+dx)}{b^2d} + \frac{\sinh^3(c+dx)}{3bd}$$

output
```
(a-b)^2*arctan(b^(1/2)*sinh(d*x+c)/a^(1/2))/a^(1/2)/b^(5/2)/d-(a-2*b)*sinh
(d*x+c)/b^2/d+1/3*sinh(d*x+c)^3/b/d
```

Mathematica [A] (verified)

Time = 0.19 (sec) , antiderivative size = 79, normalized size of antiderivative = 1.03

$$\int \frac{\cosh^5(c+dx)}{a+b\sinh^2(c+dx)}\,dx = \frac{-\frac{12(a-b)^2\arctan\left(\frac{\sqrt{a}\operatorname{csch}(c+dx)}{\sqrt{b}}\right)}{\sqrt{a}} + 3\sqrt{b}(-4a+7b)\sinh(c+dx) + b^{3/2}\sinh(3(c+dx))}{12b^{5/2}d}$$

input
```
Integrate[Cosh[c + d*x]^5/(a + b*Sinh[c + d*x]^2),x]
```

output
```
((-12*(a - b)^2*ArcTan[(Sqrt[a]*Csch[c + d*x])/Sqrt[b]])/Sqrt[a] + 3*Sqrt[
b]*(-4*a + 7*b)*Sinh[c + d*x] + b^(3/2)*Sinh[3*(c + d*x)])/(12*b^(5/2)*d)
```

Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 72, normalized size of antiderivative = 0.94, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.174, Rules used = {3042, 3669, 300, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cosh^5(c+dx)}{a+b\sinh^2(c+dx)}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\cos(ic+idx)^5}{a-b\sin(ic+idx)^2}dx$$

$$\Big\downarrow \text{3669}$$

$$\frac{\int \frac{\left(\sinh^2(c+dx)+1\right)^2}{b\sinh^2(c+dx)+a}d\sinh(c+dx)}{d}$$

$$\Big\downarrow \text{300}$$

$$\frac{\int \left(\frac{\sinh^2(c+dx)}{b}+\frac{a^2-2ba+b^2}{b^2\left(b\sinh^2(c+dx)+a\right)}-\frac{a-2b}{b^2}\right)d\sinh(c+dx)}{d}$$

$$\Big\downarrow \text{2009}$$

$$\frac{\frac{(a-b)^2\arctan\left(\frac{\sqrt{b}\sinh(c+dx)}{\sqrt{a}}\right)}{\sqrt{a}b^{5/2}}-\frac{(a-2b)\sinh(c+dx)}{b^2}+\frac{\sinh^3(c+dx)}{3b}}{d}$$

input
```
Int[Cosh[c + d*x]^5/(a + b*Sinh[c + d*x]^2),x]
```

output
```
(((a - b)^2*ArcTan[(Sqrt[b]*Sinh[c + d*x])/Sqrt[a]])/(Sqrt[a]*b^(5/2)) - (
(a - 2*b)*Sinh[c + d*x])/b^2 + Sinh[c + d*x]^3/(3*b))/d
```

Defintions of rubi rules used

rule 300
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_), x_Symbol] :> Int
[PolynomialDivide[(a + b*x^2)^p, (c + d*x^2)^(-q), x], x] /; FreeQ[{a, b, c
, d}, x] && NeQ[b*c - a*d, 0] && IGtQ[p, 0] && ILtQ[q, 0] && GeQ[p, -q]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3669
```
Int[cos[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^
(p_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Simp[ff/f   S
ubst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a + b*ff^2*x^2)^p, x], x, Sin[e + f*x]
/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m - 1)/2]
```

Maple [A] (verified)

Time = 71.19 (sec) , antiderivative size = 74, normalized size of antiderivative = 0.96

method	result
derivativedivides	$\frac{-\frac{-\frac{b\sinh(dx+c)^3}{3}+a\sinh(dx+c)-2b\sinh(dx+c)}{b^2}+\frac{\left(a^2-2ab+b^2\right)\arctan\left(\frac{b\sinh(dx+c)}{\sqrt{ab}}\right)}{b^2\sqrt{ab}}}{d}$
default	$\frac{-\frac{-\frac{b\sinh(dx+c)^3}{3}+a\sinh(dx+c)-2b\sinh(dx+c)}{b^2}+\frac{\left(a^2-2ab+b^2\right)\arctan\left(\frac{b\sinh(dx+c)}{\sqrt{ab}}\right)}{b^2\sqrt{ab}}}{d}$
risch	$\frac{e^{3dx+3c}}{24bd}-\frac{e^{dx+c}a}{2b^2d}+\frac{7e^{dx+c}}{8bd}+\frac{e^{-dx-c}a}{2b^2d}-\frac{7e^{-dx-c}}{8bd}-\frac{e^{-3dx-3c}}{24bd}-\frac{\ln\left(e^{2dx+2c}-\frac{2a\,e^{dx+c}}{\sqrt{-ab}}-1\right)a^2}{2\sqrt{-ab}\,d\,b^2}+\ln\left(e^{2}\right.$

input
```
int(cosh(d*x+c)^5/(a+b*sinh(d*x+c)^2),x,method=_RETURNVERBOSE)
```

output
```
1/d*(-1/b^2*(-1/3*b*sinh(d*x+c)^3+a*sinh(d*x+c)-2*b*sinh(d*x+c))+(a^2-2*a*
b+b^2)/b^2/(a*b)^(1/2)*arctan(b*sinh(d*x+c)/(a*b)^(1/2)))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 704 vs. $2(67) = 134$.

Time = 0.12 (sec) , antiderivative size = 1493, normalized size of antiderivative = 19.39

$$\int \frac{\cosh^5(c+dx)}{a+b\sinh^2(c+dx)}\,dx = \text{Too large to display}$$

input
```
integrate(cosh(d*x+c)^5/(a+b*sinh(d*x+c)^2),x, algorithm="fricas")
```

output
```
[1/24*(a*b^2*cosh(d*x + c)^6 + 6*a*b^2*cosh(d*x + c)*sinh(d*x + c)^5 + a*b
^2*sinh(d*x + c)^6 - 3*(4*a^2*b - 7*a*b^2)*cosh(d*x + c)^4 + 3*(5*a*b^2*co
sh(d*x + c)^2 - 4*a^2*b + 7*a*b^2)*sinh(d*x + c)^4 + 4*(5*a*b^2*cosh(d*x +
 c)^3 - 3*(4*a^2*b - 7*a*b^2)*cosh(d*x + c))*sinh(d*x + c)^3 - a*b^2 + 3*(
4*a^2*b - 7*a*b^2)*cosh(d*x + c)^2 + 3*(5*a*b^2*cosh(d*x + c)^4 + 4*a^2*b
- 7*a*b^2 - 6*(4*a^2*b - 7*a*b^2)*cosh(d*x + c)^2)*sinh(d*x + c)^2 - 12*((
a^2 - 2*a*b + b^2)*cosh(d*x + c)^3 + 3*(a^2 - 2*a*b + b^2)*cosh(d*x + c)^2
*sinh(d*x + c) + 3*(a^2 - 2*a*b + b^2)*cosh(d*x + c)*sinh(d*x + c)^2 + (a^
2 - 2*a*b + b^2)*sinh(d*x + c)^3)*sqrt(-a*b)*log((b*cosh(d*x + c)^4 + 4*b*
cosh(d*x + c)*sinh(d*x + c)^3 + b*sinh(d*x + c)^4 - 2*(2*a + b)*cosh(d*x +
 c)^2 + 2*(3*b*cosh(d*x + c)^2 - 2*a - b)*sinh(d*x + c)^2 + 4*(b*cosh(d*x
+ c)^3 - (2*a + b)*cosh(d*x + c))*sinh(d*x + c) - 4*(cosh(d*x + c)^3 + 3*c
osh(d*x + c)*sinh(d*x + c)^2 + sinh(d*x + c)^3 + (3*cosh(d*x + c)^2 - 1)*s
inh(d*x + c) - cosh(d*x + c))*sqrt(-a*b) + b)/(b*cosh(d*x + c)^4 + 4*b*cos
h(d*x + c)*sinh(d*x + c)^3 + b*sinh(d*x + c)^4 + 2*(2*a - b)*cosh(d*x + c)
^2 + 2*(3*b*cosh(d*x + c)^2 + 2*a - b)*sinh(d*x + c)^2 + 4*(b*cosh(d*x + c
)^3 + (2*a - b)*cosh(d*x + c))*sinh(d*x + c) + b)) + 6*(a*b^2*cosh(d*x + c
)^5 - 2*(4*a^2*b - 7*a*b^2)*cosh(d*x + c)^3 + (4*a^2*b - 7*a*b^2)*cosh(d*x
 + c))*sinh(d*x + c))/(a*b^3*d*cosh(d*x + c)^3 + 3*a*b^3*d*cosh(d*x + c)^2
*sinh(d*x + c) + 3*a*b^3*d*cosh(d*x + c)*sinh(d*x + c)^2 + a*b^3*d*sinh...
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\cosh^5(c+dx)}{a+b\sinh^2(c+dx)}\,dx = \text{Timed out}$$

input
```
integrate(cosh(d*x+c)**5/(a+b*sinh(d*x+c)**2),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{\cosh^5(c+dx)}{a+b\sinh^2(c+dx)}\,dx = \int \frac{\cosh(dx+c)^5}{b\sinh(dx+c)^2+a}\,dx$$

input
```
integrate(cosh(d*x+c)^5/(a+b*sinh(d*x+c)^2),x, algorithm="maxima")
```

output
```
-1/24*(3*(4*a*e^(4*c) - 7*b*e^(4*c))*e^(4*d*x) - 3*(4*a*e^(2*c) - 7*b*e^(2
*c))*e^(2*d*x) - b*e^(6*d*x + 6*c) + b)*e^(-3*d*x - 3*c)/(b^2*d) + 1/32*in
tegrate(64*((a^2*e^(3*c) - 2*a*b*e^(3*c) + b^2*e^(3*c))*e^(3*d*x) + (a^2*e
^c - 2*a*b*e^c + b^2*e^c)*e^(d*x))/(b^3*e^(4*d*x + 4*c) + b^3 + 2*(2*a*b^2
*e^(2*c) - b^3*e^(2*c))*e^(2*d*x)), x)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{\cosh^5(c+dx)}{a+b\sinh^2(c+dx)}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(cosh(d*x+c)^5/(a+b*sinh(d*x+c)^2),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Limit: Max order reached or unable
to make series expansion Error: Bad Argument Value
```

Mupad [B] (verification not implemented)

Time = 2.17 (sec) , antiderivative size = 668, normalized size of antiderivative = 8.68

$$\int \frac{\cosh^5(c+dx)}{a+b\sinh^2(c+dx)}\,dx$$
$$= \frac{\left(2\,\mathrm{atan}\left(\frac{\mathrm{e}^{d\,x}\,\mathrm{e}^{c}\,(a-b)^2\,\sqrt{a\,b^5\,d^2}}{2\,a\,b^2\,d\,\sqrt{(a-b)^4}}\right)-2\,\mathrm{atan}\left(\frac{a\,b^6\,\mathrm{e}^{d\,x}\,\mathrm{e}^{c}\left(\frac{4\left(2\,a\,b^5\,d\,\sqrt{a^4-4\,a^3\,b+6\,a^2\,b^2-4\,a\,b^3+b^4}-6\,a^2\,b^4\,d\,\sqrt{a^4-4\,a^3\,b+6\,a^2\,b^2-4\,a\,b^3+b}\right.}{a^2\,b^{11}}\right.}{}\right.\right.}{}$$
$$-\frac{\mathrm{e}^{-3\,c-3\,d\,x}}{24\,b\,d}+\frac{\mathrm{e}^{3\,c+3\,d\,x}}{24\,b\,d}-\frac{\mathrm{e}^{c+d\,x}\,(4\,a-7\,b)}{8\,b^2\,d}+\frac{\mathrm{e}^{-c-d\,x}\,(4\,a-7\,b)}{8\,b^2\,d}$$

input
```
int(cosh(c + d*x)^5/(a + b*sinh(c + d*x)^2),x)
```

output
```
((2*atan((exp(d*x)*exp(c)*(a - b)^2*(a*b^5*d^2)^(1/2))/(2*a*b^2*d*((a - b)
^4)^(1/2))) - 2*atan((a*b^6*exp(d*x)*exp(c)*((4*(2*a*b^5*d*(a^4 - 4*a^3*b
- 4*a*b^3 + b^4 + 6*a^2*b^2)^(1/2) - 6*a^2*b^4*d*(a^4 - 4*a^3*b - 4*a*b^3
+ b^4 + 6*a^2*b^2)^(1/2) + 6*a^3*b^3*d*(a^4 - 4*a^3*b - 4*a*b^3 + b^4 + 6*
a^2*b^2)^(1/2) - 2*a^4*b^2*d*(a^4 - 4*a^3*b - 4*a*b^3 + b^4 + 6*a^2*b^2)^(
1/2)))/(a^2*b^11*d^2*(a - b)^2) + (2*(a^5*(a*b^5*d^2)^(1/2) - b^5*(a*b^5*d
^2)^(1/2) + 5*a*b^4*(a*b^5*d^2)^(1/2) - 5*a^4*b*(a*b^5*d^2)^(1/2) - 10*a^2
*b^3*(a*b^5*d^2)^(1/2) + 10*a^3*b^2*(a*b^5*d^2)^(1/2)))/(a^2*b^8*d*((a - b
)^4)^(1/2)*(a*b^5*d^2)^(1/2)))*(a*b^5*d^2)^(1/2))/(12*a*b^2 - 12*a^2*b + 4
*a^3 - 4*b^3) - (2*exp(3*c)*exp(3*d*x)*(a^5*(a*b^5*d^2)^(1/2) - b^5*(a*b^5
*d^2)^(1/2) + 5*a*b^4*(a*b^5*d^2)^(1/2) - 5*a^4*b*(a*b^5*d^2)^(1/2) - 10*a
^2*b^3*(a*b^5*d^2)^(1/2) + 10*a^3*b^2*(a*b^5*d^2)^(1/2)))/(a*b^2*d*((a - b
)^4)^(1/2)*(12*a*b^2 - 12*a^2*b + 4*a^3 - 4*b^3))))*(a^4 - 4*a^3*b - 4*a*b
^3 + b^4 + 6*a^2*b^2)^(1/2))/(2*(a*b^5*d^2)^(1/2)) - exp(- 3*c - 3*d*x)/(2
4*b*d) + exp(3*c + 3*d*x)/(24*b*d) - (exp(c + d*x)*(4*a - 7*b))/(8*b^2*d)
+ (exp(- c - d*x)*(4*a - 7*b))/(8*b^2*d)
```

Reduce [B] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 1346, normalized size of antiderivative = 17.48

$$\int \frac{\cosh^5(c+dx)}{a+b\sinh^2(c+dx)}\,dx = \text{Too large to display}$$

input
```
int(cosh(d*x+c)^5/(a+b*sinh(d*x+c)^2),x)
```

output
```
( - 24*e**(3*c + 3*d*x)*sqrt(b)*sqrt(a)*sqrt(a - b)*sqrt(2*sqrt(a)*sqrt(a
- b) + 2*a - b)*atan((e**(c + d*x)*b)/(sqrt(b)*sqrt(2*sqrt(a)*sqrt(a - b)
+ 2*a - b)))*a**2 + 48*e**(3*c + 3*d*x)*sqrt(b)*sqrt(a)*sqrt(a - b)*sqrt(2
*sqrt(a)*sqrt(a - b) + 2*a - b)*atan((e**(c + d*x)*b)/(sqrt(b)*sqrt(2*sqrt
(a)*sqrt(a - b) + 2*a - b)))*a*b - 24*e**(3*c + 3*d*x)*sqrt(b)*sqrt(a)*sqr
t(a - b)*sqrt(2*sqrt(a)*sqrt(a - b) + 2*a - b)*atan((e**(c + d*x)*b)/(sqrt
(b)*sqrt(2*sqrt(a)*sqrt(a - b) + 2*a - b)))*b**2 + 24*e**(3*c + 3*d*x)*sqr
t(b)*sqrt(2*sqrt(a)*sqrt(a - b) + 2*a - b)*atan((e**(c + d*x)*b)/(sqrt(b)*
sqrt(2*sqrt(a)*sqrt(a - b) + 2*a - b)))*a**3 - 48*e**(3*c + 3*d*x)*sqrt(b)
*sqrt(2*sqrt(a)*sqrt(a - b) + 2*a - b)*atan((e**(c + d*x)*b)/(sqrt(b)*sqrt
(2*sqrt(a)*sqrt(a - b) + 2*a - b)))*a**2*b + 24*e**(3*c + 3*d*x)*sqrt(b)*s
qrt(2*sqrt(a)*sqrt(a - b) + 2*a - b)*atan((e**(c + d*x)*b)/(sqrt(b)*sqrt(2
*sqrt(a)*sqrt(a - b) + 2*a - b)))*a*b**2 - 12*e**(3*c + 3*d*x)*sqrt(b)*sqr
t(a)*sqrt(a - b)*sqrt(2*sqrt(a)*sqrt(a - b) - 2*a + b)*log( - sqrt(2*sqrt(
a)*sqrt(a - b) - 2*a + b) + e**(c + d*x)*sqrt(b))*a**2 + 24*e**(3*c + 3*d*
x)*sqrt(b)*sqrt(a)*sqrt(a - b)*sqrt(2*sqrt(a)*sqrt(a - b) - 2*a + b)*log(
- sqrt(2*sqrt(a)*sqrt(a - b) - 2*a + b) + e**(c + d*x)*sqrt(b))*a*b - 12*e
**(3*c + 3*d*x)*sqrt(b)*sqrt(a)*sqrt(a - b)*sqrt(2*sqrt(a)*sqrt(a - b) - 2
*a + b)*log( - sqrt(2*sqrt(a)*sqrt(a - b) - 2*a + b) + e**(c + d*x)*sqrt(b
))*b**2 + 12*e**(3*c + 3*d*x)*sqrt(b)*sqrt(a)*sqrt(a - b)*sqrt(2*sqrt(a...
```

3.283 $\int \frac{\cosh^4(c+dx)}{a+b\sinh^2(c+dx)}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 81

$$\int \frac{\cosh^4(c+dx)}{a+b\sinh^2(c+dx)}\,dx=-\frac{(2a-3b)x}{2b^2}+\frac{(a-b)^{3/2}\operatorname{arctanh}\left(\frac{\sqrt{a-b}\tanh(c+dx)}{\sqrt{a}}\right)}{\sqrt{a}b^2d}+\frac{\cosh(c+dx)\sinh(c+dx)}{2bd}$$

output
```
-1/2*(2*a-3*b)*x/b^2+(a-b)^(3/2)*arctanh((a-b)^(1/2)*tanh(d*x+c)/a^(1/2))/
a^(1/2)/b^2/d+1/2*cosh(d*x+c)*sinh(d*x+c)/b/d
```

Mathematica [A] (verified)

Time = 0.25 (sec) , antiderivative size = 80, normalized size of antiderivative = 0.99

$$\int \frac{\cosh^4(c+dx)}{a+b\sinh^2(c+dx)}\,dx=\frac{4(a-b)^{3/2}\operatorname{arctanh}\left(\frac{\sqrt{a-b}\tanh(c+dx)}{\sqrt{a}}\right)+\sqrt{a}(-2(2a-3b)(c+dx)+b\sinh(2(c+dx)))}{4\sqrt{a}b^2d}$$

input
```
Integrate[Cosh[c + d*x]^4/(a + b*Sinh[c + d*x]^2),x]
```

output
```
(4*(a - b)^(3/2)*ArcTanh[(Sqrt[a - b]*Tanh[c + d*x])/Sqrt[a]] + Sqrt[a]*(-
2*(2*a - 3*b)*(c + d*x) + b*Sinh[2*(c + d*x)]))/(4*Sqrt[a]*b^2*d)
```

Rubi [A] (verified)

Time = 0.33 (sec) , antiderivative size = 99, normalized size of antiderivative = 1.22, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.304$, Rules used = {3042, 3670, 316, 25, 397, 219, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cosh^4(c+dx)}{a+b\sinh^2(c+dx)}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\cos(ic+idx)^4}{a-b\sin(ic+idx)^2}dx$$

$$\Big\downarrow \text{3670}$$

$$\frac{\int \frac{1}{\left(1-\tanh^2(c+dx)\right)^2\left(a-(a-b)\tanh^2(c+dx)\right)}d\tanh(c+dx)}{d}$$

$$\Big\downarrow \text{316}$$

$$\frac{\frac{\int -\frac{(a-b)\tanh^2(c+dx)+a-2b}{\left(1-\tanh^2(c+dx)\right)\left(a-(a-b)\tanh^2(c+dx)\right)}d\tanh(c+dx)}{2b}+\frac{\tanh(c+dx)}{2b(1-\tanh^2(c+dx))}}{d}$$

$$\Big\downarrow \text{25}$$

$$\frac{\frac{\tanh(c+dx)}{2b(1-\tanh^2(c+dx))}-\frac{\int \frac{(a-b)\tanh^2(c+dx)+a-2b}{\left(1-\tanh^2(c+dx)\right)\left(a-(a-b)\tanh^2(c+dx)\right)}d\tanh(c+dx)}{2b}}{d}$$

$$\Big\downarrow \text{397}$$

$$\frac{\frac{\tanh(c+dx)}{2b(1-\tanh^2(c+dx))}-\frac{\frac{(2a-3b)\int \frac{1}{1-\tanh^2(c+dx)}d\tanh(c+dx)}{b}-\frac{2(a-b)^2\int \frac{1}{a-(a-b)\tanh^2(c+dx)}d\tanh(c+dx)}{b}}{2b}}{d}$$

$$\Big\downarrow \text{219}$$

$$\frac{\frac{\tanh(c+dx)}{2b(1-\tanh^2(c+dx))}-\frac{\frac{(2a-3b)\text{arctanh}(\tanh(c+dx))}{b}-\frac{2(a-b)^2\int\frac{1}{a-(a-b)\tanh^2(c+dx)}d\tanh(c+dx)}{b}}{2b}}{d}$$

$$\Big\downarrow \text{221}$$

$$\frac{\frac{\tanh(c+dx)}{2b(1-\tanh^2(c+dx))}-\frac{\frac{(2a-3b)\text{arctanh}(\tanh(c+dx))}{b}-\frac{2(a-b)^{3/2}\text{arctanh}\left(\frac{\sqrt{a-b}\tanh(c+dx)}{\sqrt{a}}\right)}{\sqrt{a}b}}{2b}}{d}$$

input
```
Int[Cosh[c + d*x]^4/(a + b*Sinh[c + d*x]^2),x]
```

output
```
(-1/2*(((2*a - 3*b)*ArcTanh[Tanh[c + d*x]])/b - (2*(a - b)^(3/2)*ArcTanh[(
Sqrt[a - b]*Tanh[c + d*x])/Sqrt[a]])/(Sqrt[a]*b))/b + Tanh[c + d*x]/(2*b*(
1 - Tanh[c + d*x]^2)))/d
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 316
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_), x_Symbol] :> Sim
p[(-b)*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^(q + 1)/(2*a*(p + 1)*(b*c - a*d))
), x] + Simp[1/(2*a*(p + 1)*(b*c - a*d))   Int[(a + b*x^2)^(p + 1)*(c + d*x
^2)^q*Simp[b*c + 2*(p + 1)*(b*c - a*d) + d*b*(2*(p + q + 2) + 1)*x^2, x], x
], x] /; FreeQ[{a, b, c, d, q}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] &&  !
( !IntegerQ[p] && IntegerQ[q] && LtQ[q, -1]) && IntBinomialQ[a, b, c, d, 2,
 p, q, x]
```

rule 397
```
Int[((e_) + (f_.)*(x_)^2)/(((a_) + (b_.)*(x_)^2)*((c_) + (d_.)*(x_)^2)), x_
Symbol] :> Simp[(b*e - a*f)/(b*c - a*d)   Int[1/(a + b*x^2), x], x] - Simp[
(d*e - c*f)/(b*c - a*d)   Int[1/(c + d*x^2), x], x] /; FreeQ[{a, b, c, d, e
, f}, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3670
```
Int[cos[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(
p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Simp[ff/f   Su
bst[Int[(a + (a + b)*ff^2*x^2)^p/(1 + ff^2*x^2)^(m/2 + p + 1), x], x, Tan[e
 + f*x]/ff], x]] /; FreeQ[{a, b, e, f}, x] && IntegerQ[m/2] && IntegerQ[p]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 252 vs. $2(69) = 138$.

Time = 29.17 (sec) , antiderivative size = 253, normalized size of antiderivative = 3.12

method	result
risch	$-\frac{ax}{b^2}+\frac{3x}{2b}+\frac{e^{2dx+2c}}{8bd}-\frac{e^{-2dx-2c}}{8bd}+\frac{\sqrt{a(a-b)}\,\ln\left(e^{2dx+2c}-\frac{-2a+2\sqrt{a(a-b)}+b}{b}\right)}{2d\,b^2}-\frac{\sqrt{a(a-b)}\,\ln\left(e^{2dx+2c}-\right.}{2adb}$
derivativedivides	$\frac{2\left(a^2-2ab+b^2\right)a\left(\frac{\left(-\sqrt{-b(a-b)}-b\right)\arctan\left(\frac{\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)a}{\sqrt{\left(2\sqrt{-b(a-b)}-a+2b\right)a}}\right)}{2a\sqrt{-b(a-b)}\sqrt{\left(2\sqrt{-b(a-b)}-a+2b\right)a}}-\frac{\left(-\sqrt{-b(a-b)}+b\right)\operatorname{arctanh}\left(\frac{\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)a}{\sqrt{\left(2\sqrt{-b(a-b)}+a-2b\right)a}}\right)}{2a\sqrt{-b(a-b)}\sqrt{\left(2\sqrt{-b(a-b)}+a-2b\right)a}}\right.}{b^2}$
default	$\frac{2\left(a^2-2ab+b^2\right)a\left(\frac{\left(-\sqrt{-b(a-b)}-b\right)\arctan\left(\frac{\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)a}{\sqrt{\left(2\sqrt{-b(a-b)}-a+2b\right)a}}\right)}{2a\sqrt{-b(a-b)}\sqrt{\left(2\sqrt{-b(a-b)}-a+2b\right)a}}-\frac{\left(-\sqrt{-b(a-b)}+b\right)\operatorname{arctanh}\left(\frac{\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)a}{\sqrt{\left(2\sqrt{-b(a-b)}+a-2b\right)a}}\right)}{2a\sqrt{-b(a-b)}\sqrt{\left(2\sqrt{-b(a-b)}+a-2b\right)a}}\right.}{b^2}$

input
```
int(cosh(d*x+c)^4/(a+b*sinh(d*x+c)^2),x,method=_RETURNVERBOSE)
```

output
```
-a*x/b^2+3/2*x/b+1/8/b/d*exp(2*d*x+2*c)-1/8/b/d*exp(-2*d*x-2*c)+1/2*(a*(a-
b))^(1/2)/d/b^2*ln(exp(2*d*x+2*c)-(-2*a+2*(a*(a-b))^(1/2)+b)/b)-1/2/a*(a*(
a-b))^(1/2)/d/b*ln(exp(2*d*x+2*c)-(-2*a+2*(a*(a-b))^(1/2)+b)/b)-1/2*(a*(a-
b))^(1/2)/d/b^2*ln(exp(2*d*x+2*c)+(2*a+2*(a*(a-b))^(1/2)-b)/b)+1/2/a*(a*(a
-b))^(1/2)/d/b*ln(exp(2*d*x+2*c)+(2*a+2*(a*(a-b))^(1/2)-b)/b)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 305 vs. $2(69) = 138$.

Time = 0.11 (sec) , antiderivative size = 875, normalized size of antiderivative = 10.80

$$\int \frac{\cosh^4(c+dx)}{a+b\sinh^2(c+dx)}\,dx = \text{Too large to display}$$

input
```
integrate(cosh(d*x+c)^4/(a+b*sinh(d*x+c)^2),x, algorithm="fricas")
```

output
```
[-1/8*(4*(2*a - 3*b)*d*x*cosh(d*x + c)^2 - b*cosh(d*x + c)^4 - 4*b*cosh(d*
x + c)*sinh(d*x + c)^3 - b*sinh(d*x + c)^4 + 2*(2*(2*a - 3*b)*d*x - 3*b*co
sh(d*x + c)^2)*sinh(d*x + c)^2 + 4*((a - b)*cosh(d*x + c)^2 + 2*(a - b)*co
sh(d*x + c)*sinh(d*x + c) + (a - b)*sinh(d*x + c)^2)*sqrt((a - b)/a)*log((
b^2*cosh(d*x + c)^4 + 4*b^2*cosh(d*x + c)*sinh(d*x + c)^3 + b^2*sinh(d*x +
 c)^4 + 2*(2*a*b - b^2)*cosh(d*x + c)^2 + 2*(3*b^2*cosh(d*x + c)^2 + 2*a*b
 - b^2)*sinh(d*x + c)^2 + 8*a^2 - 8*a*b + b^2 + 4*(b^2*cosh(d*x + c)^3 + (
2*a*b - b^2)*cosh(d*x + c))*sinh(d*x + c) + 4*(a*b*cosh(d*x + c)^2 + 2*a*b
*cosh(d*x + c)*sinh(d*x + c) + a*b*sinh(d*x + c)^2 + 2*a^2 - a*b)*sqrt((a
- b)/a))/(b*cosh(d*x + c)^4 + 4*b*cosh(d*x + c)*sinh(d*x + c)^3 + b*sinh(d
*x + c)^4 + 2*(2*a - b)*cosh(d*x + c)^2 + 2*(3*b*cosh(d*x + c)^2 + 2*a - b
)*sinh(d*x + c)^2 + 4*(b*cosh(d*x + c)^3 + (2*a - b)*cosh(d*x + c))*sinh(d
*x + c) + b)) + 4*(2*(2*a - 3*b)*d*x*cosh(d*x + c) - b*cosh(d*x + c)^3)*si
nh(d*x + c) + b)/(b^2*d*cosh(d*x + c)^2 + 2*b^2*d*cosh(d*x + c)*sinh(d*x +
 c) + b^2*d*sinh(d*x + c)^2), -1/8*(4*(2*a - 3*b)*d*x*cosh(d*x + c)^2 - b*
cosh(d*x + c)^4 - 4*b*cosh(d*x + c)*sinh(d*x + c)^3 - b*sinh(d*x + c)^4 +
2*(2*(2*a - 3*b)*d*x - 3*b*cosh(d*x + c)^2)*sinh(d*x + c)^2 + 8*((a - b)*c
osh(d*x + c)^2 + 2*(a - b)*cosh(d*x + c)*sinh(d*x + c) + (a - b)*sinh(d*x
+ c)^2)*sqrt(-(a - b)/a)*arctan(-1/2*(b*cosh(d*x + c)^2 + 2*b*cosh(d*x + c
)*sinh(d*x + c) + b*sinh(d*x + c)^2 + 2*a - b)*sqrt(-(a - b)/a)/(a - b)...
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\cosh^4(c+dx)}{a+b\sinh^2(c+dx)}\,dx = \text{Timed out}$$

input
```
integrate(cosh(d*x+c)**4/(a+b*sinh(d*x+c)**2),x)
```

output
```
Timed out
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{\cosh^4(c+dx)}{a+b\sinh^2(c+dx)}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate(cosh(d*x+c)^4/(a+b*sinh(d*x+c)^2),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(b-a>0)', see `assume?` for more
details)Is
```

Giac [A] (verification not implemented)

Time = 0.55 (sec) , antiderivative size = 138, normalized size of antiderivative = 1.70

$$\int \frac{\cosh^4(c+dx)}{a+b\sinh^2(c+dx)}\,dx = -\frac{\frac{4\,(dx+c)(2\,a-3\,b)}{b^2} - \frac{e^{(2\,dx+2\,c)}}{b} - \frac{\left(4\,ae^{(2\,dx+2\,c)}-6\,be^{(2\,dx+2\,c)}-b\right)e^{(-2\,dx-2\,c)}}{b^2} - \frac{8\,(a^2-2\,ab+b^2)\arctan\left(\frac{be^{(2\,dx+2\,c)}+2\,a-b}{2\,\sqrt{-a^2+ab}}\right)}{\sqrt{-a^2+ab}b^2}}{8\,d}$$

input
```
integrate(cosh(d*x+c)^4/(a+b*sinh(d*x+c)^2),x, algorithm="giac")
```

output
```
-1/8*(4*(d*x + c)*(2*a - 3*b)/b^2 - e^(2*d*x + 2*c)/b - (4*a*e^(2*d*x + 2*
c) - 6*b*e^(2*d*x + 2*c) - b)*e^(-2*d*x - 2*c)/b^2 - 8*(a^2 - 2*a*b + b^2)
*arctan(1/2*(b*e^(2*d*x + 2*c) + 2*a - b)/sqrt(-a^2 + a*b))/(sqrt(-a^2 + a
*b)*b^2))/d
```

Mupad [B] (verification not implemented)

Time = 2.50 (sec) , antiderivative size = 300, normalized size of antiderivative = 3.70

$$\int \frac{\cosh^4(c+dx)}{a+b\sinh^2(c+dx)}\,dx = \frac{e^{2c+2dx}}{8bd} - \frac{e^{-2c-2dx}}{8bd} - \frac{x(2a-3b)}{2b^2} - \frac{\ln\left(\frac{4(a-b)^3\left(2ab-b^2+8a^2e^{2c+2dx}+b^2e^{2c+2dx}-8abe^{2c+2dx}\right)}{ab^6} - \frac{8(a-b)^{7/2}\left(b+4ae^{2c+2dx}-2be^{2c+2dx}\right)}{\sqrt{a}\,b^6}\right)(a-b)^{3/2}}{2\sqrt{a}\,b^2d} + \frac{\ln\left(\frac{4(a-b)^3\left(2ab-b^2+8a^2e^{2c+2dx}+b^2e^{2c+2dx}-8abe^{2c+2dx}\right)}{ab^6} + \frac{8(a-b)^{7/2}\left(b+4ae^{2c+2dx}-2be^{2c+2dx}\right)}{\sqrt{a}\,b^6}\right)(a-b)^{3/2}}{2\sqrt{a}\,b^2d}$$

input
```
int(cosh(c + d*x)^4/(a + b*sinh(c + d*x)^2),x)
```

output
```
exp(2*c + 2*d*x)/(8*b*d) - exp(- 2*c - 2*d*x)/(8*b*d) - (x*(2*a - 3*b))/(2
*b^2) - (log((4*(a - b)^3*(2*a*b - b^2 + 8*a^2*exp(2*c + 2*d*x) + b^2*exp(
2*c + 2*d*x) - 8*a*b*exp(2*c + 2*d*x)))/(a*b^6) - (8*(a - b)^(7/2)*(b + 4*
a*exp(2*c + 2*d*x) - 2*b*exp(2*c + 2*d*x)))/(a^(1/2)*b^6))*(a - b)^(3/2))/
(2*a^(1/2)*b^2*d) + (log((4*(a - b)^3*(2*a*b - b^2 + 8*a^2*exp(2*c + 2*d*x
) + b^2*exp(2*c + 2*d*x) - 8*a*b*exp(2*c + 2*d*x)))/(a*b^6) + (8*(a - b)^(
7/2)*(b + 4*a*exp(2*c + 2*d*x) - 2*b*exp(2*c + 2*d*x)))/(a^(1/2)*b^6))*(a
- b)^(3/2))/(2*a^(1/2)*b^2*d)
```

Reduce [B] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 376, normalized size of antiderivative = 4.64

$$\int \frac{\cosh^4(c+dx)}{a+b\sinh^2(c+dx)}\,dx$$
$$= \frac{4e^{2dx+2c}\sqrt{a}\,\sqrt{a-b}\log\left(-\sqrt{2\sqrt{a}\,\sqrt{a-b}-2a+b}+e^{dx+c}\sqrt{b}\right)a-4e^{2dx+2c}\sqrt{a}\,\sqrt{a-b}\log\left(-\sqrt{2\sqrt{a}\,\sqrt{a}\right.}{}$$

input
```
int(cosh(d*x+c)^4/(a+b*sinh(d*x+c)^2),x)
```

output
```
(4*e**(2*c + 2*d*x)*sqrt(a)*sqrt(a - b)*log( - sqrt(2*sqrt(a)*sqrt(a - b)
- 2*a + b) + e**(c + d*x)*sqrt(b))*a - 4*e**(2*c + 2*d*x)*sqrt(a)*sqrt(a -
 b)*log( - sqrt(2*sqrt(a)*sqrt(a - b) - 2*a + b) + e**(c + d*x)*sqrt(b))*b
 + 4*e**(2*c + 2*d*x)*sqrt(a)*sqrt(a - b)*log(sqrt(2*sqrt(a)*sqrt(a - b) -
 2*a + b) + e**(c + d*x)*sqrt(b))*a - 4*e**(2*c + 2*d*x)*sqrt(a)*sqrt(a -
b)*log(sqrt(2*sqrt(a)*sqrt(a - b) - 2*a + b) + e**(c + d*x)*sqrt(b))*b - 4
*e**(2*c + 2*d*x)*sqrt(a)*sqrt(a - b)*log(2*sqrt(a)*sqrt(a - b) + e**(2*c
+ 2*d*x)*b + 2*a - b)*a + 4*e**(2*c + 2*d*x)*sqrt(a)*sqrt(a - b)*log(2*sqr
t(a)*sqrt(a - b) + e**(2*c + 2*d*x)*b + 2*a - b)*b + e**(4*c + 4*d*x)*a*b
- 8*e**(2*c + 2*d*x)*a**2*d*x + 12*e**(2*c + 2*d*x)*a*b*d*x - a*b)/(8*e**(
2*c + 2*d*x)*a*b**2*d)
```

3.284 $\int \frac{\cosh^3(c+dx)}{a+b\sinh^2(c+dx)}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 52

$$\int \frac{\cosh^3(c+dx)}{a+b\sinh^2(c+dx)}\,dx=-\frac{(a-b)\arctan\left(\frac{\sqrt{b}\sinh(c+dx)}{\sqrt{a}}\right)}{\sqrt{a}b^{3/2}d}+\frac{\sinh(c+dx)}{bd}$$

output
```
-(a-b)*arctan(b^(1/2)*sinh(d*x+c)/a^(1/2))/a^(1/2)/b^(3/2)/d+sinh(d*x+c)/b
/d
```

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 50, normalized size of antiderivative = 0.96

$$\int \frac{\cosh^3(c+dx)}{a+b\sinh^2(c+dx)}\,dx=\frac{-\frac{(a-b)\arctan\left(\frac{\sqrt{b}\sinh(c+dx)}{\sqrt{a}}\right)}{\sqrt{a}b^{3/2}}+\frac{\sinh(c+dx)}{b}}{d}$$

input
```
Integrate[Cosh[c + d*x]^3/(a + b*Sinh[c + d*x]^2),x]
```

output
```
(-(((a - b)*ArcTan[(Sqrt[b]*Sinh[c + d*x])/Sqrt[a]])/(Sqrt[a]*b^(3/2))) +
Sinh[c + d*x]/b)/d
```

Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 50, normalized size of antiderivative = 0.96, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.174$, Rules used = {3042, 3669, 299, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cosh^3(c+dx)}{a+b\sinh^2(c+dx)}dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \frac{\cos(ic+idx)^3}{a-b\sin(ic+idx)^2}dx$$

$$\Big\downarrow \text{ 3669}$$

$$\frac{\int \frac{\sinh^2(c+dx)+1}{b\sinh^2(c+dx)+a}d\sinh(c+dx)}{d}$$

$$\Big\downarrow \text{ 299}$$

$$\frac{\frac{\sinh(c+dx)}{b}-\frac{(a-b)\int \frac{1}{b\sinh^2(c+dx)+a}d\sinh(c+dx)}{b}}{d}$$

$$\Big\downarrow \text{ 218}$$

$$\frac{\frac{\sinh(c+dx)}{b}-\frac{(a-b)\arctan\left(\frac{\sqrt{b}\sinh(c+dx)}{\sqrt{a}}\right)}{\sqrt{a}b^{3/2}}}{d}$$

input
```
Int[Cosh[c + d*x]^3/(a + b*Sinh[c + d*x]^2),x]
```

output
```
(-(((a - b)*ArcTan[(Sqrt[b]*Sinh[c + d*x])/Sqrt[a]])/(Sqrt[a]*b^(3/2))) +
Sinh[c + d*x]/b)/d
```

Defintions of rubi rules used

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 299
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2), x_Symbol] :> Simp[d*x
*((a + b*x^2)^(p + 1)/(b*(2*p + 3))), x] - Simp[(a*d - b*c*(2*p + 3))/(b*(2
*p + 3))   Int[(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
 a*d, 0] && NeQ[2*p + 3, 0]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3669
```
Int[cos[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^
(p_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Simp[ff/f   S
ubst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a + b*ff^2*x^2)^p, x], x, Sin[e + f*x]
/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m - 1)/2]
```

Maple [A] (verified)

Time = 10.22 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.85

method	result
derivativedivides	$\frac{\frac{\sinh(dx+c)}{b}+\frac{(-a+b)\arctan\left(\frac{b\sinh(dx+c)}{\sqrt{ab}}\right)}{b\sqrt{ab}}}{d}$
default	$\frac{\frac{\sinh(dx+c)}{b}+\frac{(-a+b)\arctan\left(\frac{b\sinh(dx+c)}{\sqrt{ab}}\right)}{b\sqrt{ab}}}{d}$
risch	$\frac{e^{dx+c}}{2bd}-\frac{e^{-dx-c}}{2bd}-\frac{\ln\left(e^{2dx+2c}+\frac{2a\,e^{dx+c}}{\sqrt{-ab}}-1\right)a}{2\sqrt{-ab}\,db}+\frac{\ln\left(e^{2dx+2c}+\frac{2a\,e^{dx+c}}{\sqrt{-ab}}-1\right)}{2\sqrt{-ab}\,d}+\frac{\ln\left(e^{2dx+2c}-\frac{2a\,e^{dx+c}}{\sqrt{-ab}}-1\right)a}{2\sqrt{-ab}\,db}-$

input
```
int(cosh(d*x+c)^3/(a+b*sinh(d*x+c)^2),x,method=_RETURNVERBOSE)
```

output
```
1/d*(1/b*sinh(d*x+c)+(-a+b)/b/(a*b)^(1/2)*arctan(b*sinh(d*x+c)/(a*b)^(1/2)
))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 254 vs. $2(44) = 88$.

Time = 0.14 (sec) , antiderivative size = 662, normalized size of antiderivative = 12.73

$$\int \frac{\cosh^3(c+dx)}{a+b\sinh^2(c+dx)}\,dx = \text{Too large to display}$$

input
```
integrate(cosh(d*x+c)^3/(a+b*sinh(d*x+c)^2),x, algorithm="fricas")
```

output
```
[1/2*(a*b*cosh(d*x + c)^2 + 2*a*b*cosh(d*x + c)*sinh(d*x + c) + a*b*sinh(d
*x + c)^2 + sqrt(-a*b)*((a - b)*cosh(d*x + c) + (a - b)*sinh(d*x + c))*log
((b*cosh(d*x + c)^4 + 4*b*cosh(d*x + c)*sinh(d*x + c)^3 + b*sinh(d*x + c)^
4 - 2*(2*a + b)*cosh(d*x + c)^2 + 2*(3*b*cosh(d*x + c)^2 - 2*a - b)*sinh(d
*x + c)^2 + 4*(b*cosh(d*x + c)^3 - (2*a + b)*cosh(d*x + c))*sinh(d*x + c)
 - 4*(cosh(d*x + c)^3 + 3*cosh(d*x + c)*sinh(d*x + c)^2 + sinh(d*x + c)^3 +
 (3*cosh(d*x + c)^2 - 1)*sinh(d*x + c) - cosh(d*x + c))*sqrt(-a*b) + b)/(b
*cosh(d*x + c)^4 + 4*b*cosh(d*x + c)*sinh(d*x + c)^3 + b*sinh(d*x + c)^4 +
 2*(2*a - b)*cosh(d*x + c)^2 + 2*(3*b*cosh(d*x + c)^2 + 2*a - b)*sinh(d*x
+ c)^2 + 4*(b*cosh(d*x + c)^3 + (2*a - b)*cosh(d*x + c))*sinh(d*x + c) + b
)) - a*b)/(a*b^2*d*cosh(d*x + c) + a*b^2*d*sinh(d*x + c)), 1/2*(a*b*cosh(d
*x + c)^2 + 2*a*b*cosh(d*x + c)*sinh(d*x + c) + a*b*sinh(d*x + c)^2 + 2*sq
rt(a*b)*((a - b)*cosh(d*x + c) + (a - b)*sinh(d*x + c))*arctan(2*sqrt(a*b)
/(b*cosh(d*x + c) + b*sinh(d*x + c))) - 2*sqrt(a*b)*((a - b)*cosh(d*x + c)
 + (a - b)*sinh(d*x + c))*arctan(1/2*(b*cosh(d*x + c)^3 + 3*b*cosh(d*x + c
)*sinh(d*x + c)^2 + b*sinh(d*x + c)^3 + (4*a - b)*cosh(d*x + c) + (3*b*cos
h(d*x + c)^2 + 4*a - b)*sinh(d*x + c))*sqrt(a*b)/(a*b)) - a*b)/(a*b^2*d*co
sh(d*x + c) + a*b^2*d*sinh(d*x + c))]
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\cosh^3(c+dx)}{a+b\sinh^2(c+dx)}\,dx = \text{Timed out}$$

input
```
integrate(cosh(d*x+c)**3/(a+b*sinh(d*x+c)**2),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{\cosh^3(c+dx)}{a+b\sinh^2(c+dx)}\,dx = \int \frac{\cosh(dx+c)^3}{b\sinh(dx+c)^2+a}\,dx$$

input
```
integrate(cosh(d*x+c)^3/(a+b*sinh(d*x+c)^2),x, algorithm="maxima")
```

output
```
1/2*(e^(2*d*x + 2*c) - 1)*e^(-d*x - c)/(b*d) - 1/8*integrate(16*((a*e^(3*c
) - b*e^(3*c))*e^(3*d*x) + (a*e^c - b*e^c)*e^(d*x))/(b^2*e^(4*d*x + 4*c) +
 b^2 + 2*(2*a*b*e^(2*c) - b^2*e^(2*c))*e^(2*d*x)), x)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{\cosh^3(c+dx)}{a+b\sinh^2(c+dx)}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(cosh(d*x+c)^3/(a+b*sinh(d*x+c)^2),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Limit: Max order reached or unable
to make series expansion Error: Bad Argument Value
```

Mupad [B] (verification not implemented)

Time = 2.01 (sec) , antiderivative size = 426, normalized size of antiderivative = 8.19

$$\int \frac{\cosh^3(c+dx)}{a+b\sinh^2(c+dx)}\,dx = \frac{e^{c+dx}}{2\,b\,d} - \frac{\left(2\,\mathrm{atan}\left(\frac{a\,b^4\,e^{d\,x}\,e^c\,\left(\frac{4\,\left(2\,a\,b^3\,d\,\sqrt{a^2-2\,a\,b+b^2}+2\,a^3\,b\,d\,\sqrt{a^2-2\,a\,b+b^2}-4\,a^2\,b^2\,d\,\sqrt{a^2-2\,a\,b+b^2}\right)}{a^2\,b^7\,d^2\,(a-b)}-\frac{2\,\left(a^3\,\sqrt{a\,b^3\,d^2}-b^3\,\sqrt{a\,b^3\,d^2}+3\,a\,b^2\,\sqrt{a\,b^3\,d^2}-\cdots\right.}{a^2\,b^5\,d\,\sqrt{(a-b)^2}\,\sqrt{a\,b^3\,d^2}}\cdots\right.}{4\,a^2-8\,a\,b+4\,b^2}\cdots\right.\right.}{\cdots} - \frac{e^{-c-d\,x}}{2\,b\,d}$$

input
```
int(cosh(c + d*x)^3/(a + b*sinh(c + d*x)^2),x)
```

output
```
exp(c + d*x)/(2*b*d) - ((2*atan((a*b^4*exp(d*x)*exp(c)*((4*(2*a*b^3*d*(a^2
 - 2*a*b + b^2)^(1/2) + 2*a^3*b*d*(a^2 - 2*a*b + b^2)^(1/2) - 4*a^2*b^2*d*
(a^2 - 2*a*b + b^2)^(1/2)))/(a^2*b^7*d^2*(a - b)) - (2*(a^3*(a*b^3*d^2)^(1
/2) - b^3*(a*b^3*d^2)^(1/2) + 3*a*b^2*(a*b^3*d^2)^(1/2) - 3*a^2*b*(a*b^3*d
^2)^(1/2)))/(a^2*b^5*d*((a - b)^2)^(1/2)*(a*b^3*d^2)^(1/2)))*(a*b^3*d^2)^(
1/2))/(4*a^2 - 8*a*b + 4*b^2) + (2*exp(3*c)*exp(3*d*x)*(a^3*(a*b^3*d^2)^(1
/2) - b^3*(a*b^3*d^2)^(1/2) + 3*a*b^2*(a*b^3*d^2)^(1/2) - 3*a^2*b*(a*b^3*d
^2)^(1/2)))/(a*b*d*((a - b)^2)^(1/2)*(4*a^2 - 8*a*b + 4*b^2))) + 2*atan((e
xp(d*x)*exp(c)*(a - b)*(a*b^3*d^2)^(1/2))/(2*a*b*d*((a - b)^2)^(1/2))))*(a
^2 - 2*a*b + b^2)^(1/2))/(2*(a*b^3*d^2)^(1/2)) - exp(- c - d*x)/(2*b*d)
```

Reduce [B] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 807, normalized size of antiderivative = 15.52

$$\int \frac{\cosh^3(c+dx)}{a+b\sinh^2(c+dx)}\,dx = \text{Too large to display}$$

input
```
int(cosh(d*x+c)^3/(a+b*sinh(d*x+c)^2),x)
```

output
```
(2*e**(c + d*x)*sqrt(b)*sqrt(a)*sqrt(a - b)*sqrt(2*sqrt(a)*sqrt(a - b) + 2
*a - b)*atan((e**(c + d*x)*b)/(sqrt(b)*sqrt(2*sqrt(a)*sqrt(a - b) + 2*a -
b)))*a - 2*e**(c + d*x)*sqrt(b)*sqrt(a)*sqrt(a - b)*sqrt(2*sqrt(a)*sqrt(a
- b) + 2*a - b)*atan((e**(c + d*x)*b)/(sqrt(b)*sqrt(2*sqrt(a)*sqrt(a - b)
+ 2*a - b)))*b - 2*e**(c + d*x)*sqrt(b)*sqrt(2*sqrt(a)*sqrt(a - b) + 2*a -
 b)*atan((e**(c + d*x)*b)/(sqrt(b)*sqrt(2*sqrt(a)*sqrt(a - b) + 2*a - b)))
*a**2 + 2*e**(c + d*x)*sqrt(b)*sqrt(2*sqrt(a)*sqrt(a - b) + 2*a - b)*atan(
(e**(c + d*x)*b)/(sqrt(b)*sqrt(2*sqrt(a)*sqrt(a - b) + 2*a - b)))*a*b + e*
*(c + d*x)*sqrt(b)*sqrt(a)*sqrt(a - b)*sqrt(2*sqrt(a)*sqrt(a - b) - 2*a +
b)*log( - sqrt(2*sqrt(a)*sqrt(a - b) - 2*a + b) + e**(c + d*x)*sqrt(b))*a
- e**(c + d*x)*sqrt(b)*sqrt(a)*sqrt(a - b)*sqrt(2*sqrt(a)*sqrt(a - b) - 2*
a + b)*log( - sqrt(2*sqrt(a)*sqrt(a - b) - 2*a + b) + e**(c + d*x)*sqrt(b)
)*b - e**(c + d*x)*sqrt(b)*sqrt(a)*sqrt(a - b)*sqrt(2*sqrt(a)*sqrt(a - b)
- 2*a + b)*log(sqrt(2*sqrt(a)*sqrt(a - b) - 2*a + b) + e**(c + d*x)*sqrt(b
))*a + e**(c + d*x)*sqrt(b)*sqrt(a)*sqrt(a - b)*sqrt(2*sqrt(a)*sqrt(a - b)
 - 2*a + b)*log(sqrt(2*sqrt(a)*sqrt(a - b) - 2*a + b) + e**(c + d*x)*sqrt(
b))*b + e**(c + d*x)*sqrt(b)*sqrt(2*sqrt(a)*sqrt(a - b) - 2*a + b)*log( -
sqrt(2*sqrt(a)*sqrt(a - b) - 2*a + b) + e**(c + d*x)*sqrt(b))*a**2 - e**(c
 + d*x)*sqrt(b)*sqrt(2*sqrt(a)*sqrt(a - b) - 2*a + b)*log( - sqrt(2*sqrt(a
)*sqrt(a - b) - 2*a + b) + e**(c + d*x)*sqrt(b))*a*b - e**(c + d*x)*sqr...
```

3.285 $\int \frac{\cosh^2(c+dx)}{a+b\sinh^2(c+dx)}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 50

$$\int \frac{\cosh^2(c+dx)}{a+b\sinh^2(c+dx)}\,dx=\frac{x}{b}-\frac{\sqrt{a-b}\,\mathrm{arctanh}\left(\frac{\sqrt{a-b}\tanh(c+dx)}{\sqrt{a}}\right)}{\sqrt{a}bd}$$

output
```
x/b-(a-b)^(1/2)*arctanh((a-b)^(1/2)*tanh(d*x+c)/a^(1/2))/a^(1/2)/b/d
```

Mathematica [A] (verified)

Time = 0.20 (sec) , antiderivative size = 50, normalized size of antiderivative = 1.00

$$\int \frac{\cosh^2(c+dx)}{a+b\sinh^2(c+dx)}\,dx=\frac{c+dx-\frac{\sqrt{a-b}\,\mathrm{arctanh}\left(\frac{\sqrt{a-b}\tanh(c+dx)}{\sqrt{a}}\right)}{\sqrt{a}}}{bd}$$

input
```
Integrate[Cosh[c + d*x]^2/(a + b*Sinh[c + d*x]^2),x]
```

output
```
(c + d*x - (Sqrt[a - b]*ArcTanh[(Sqrt[a - b]*Tanh[c + d*x])/Sqrt[a]])/Sqrt
[a])/(b*d)
```

Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 57, normalized size of antiderivative = 1.14, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.217, Rules used = {3042, 3670, 303, 219, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cosh^2(c+dx)}{a+b\sinh^2(c+dx)}dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \frac{\cos(ic+idx)^2}{a-b\sin(ic+idx)^2}dx$$

$$\Big\downarrow \text{ 3670}$$

$$\frac{\int \frac{1}{(1-\tanh^2(c+dx))(a-(a-b)\tanh^2(c+dx))}d\tanh(c+dx)}{d}$$

$$\Big\downarrow \text{ 303}$$

$$\frac{\frac{\int \frac{1}{1-\tanh^2(c+dx)}d\tanh(c+dx)}{b}-\frac{(a-b)\int \frac{1}{a-(a-b)\tanh^2(c+dx)}d\tanh(c+dx)}{b}}{d}$$

$$\Big\downarrow \text{ 219}$$

$$\frac{\frac{\text{arctanh}(\tanh(c+dx))}{b}-\frac{(a-b)\int \frac{1}{a-(a-b)\tanh^2(c+dx)}d\tanh(c+dx)}{b}}{d}$$

$$\Big\downarrow \text{ 221}$$

$$\frac{\frac{\text{arctanh}(\tanh(c+dx))}{b}-\frac{\sqrt{a-b}\,\text{arctanh}\left(\frac{\sqrt{a-b}\tanh(c+dx)}{\sqrt{a}}\right)}{\sqrt{a}b}}{d}$$

input
```
Int[Cosh[c + d*x]^2/(a + b*Sinh[c + d*x]^2),x]
```

output
```
(ArcTanh[Tanh[c + d*x]]/b - (Sqrt[a - b]*ArcTanh[(Sqrt[a - b]*Tanh[c + d*x
])/Sqrt[a]])/(Sqrt[a]*b))/d
```

Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 303
```
Int[1/(((a_) + (b_.)*(x_)^2)*((c_) + (d_.)*(x_)^2)), x_Symbol] :> Simp[b/(b
*c - a*d)   Int[1/(a + b*x^2), x], x] - Simp[d/(b*c - a*d)   Int[1/(c + d*x
^2), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3670
```
Int[cos[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(
p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Simp[ff/f   Su
bst[Int[(a + (a + b)*ff^2*x^2)^p/(1 + ff^2*x^2)^(m/2 + p + 1), x], x, Tan[e
 + f*x]/ff], x]] /; FreeQ[{a, b, e, f}, x] && IntegerQ[m/2] && IntegerQ[p]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 111 vs. $2(42) = 84$.

Time = 3.13 (sec) , antiderivative size = 112, normalized size of antiderivative = 2.24

method	result
risch	$\frac{x}{b}+\frac{\sqrt{a(a-b)}\,\ln\left(e^{2dx+2c}+\frac{2a+2\sqrt{a(a-b)}-b}{b}\right)}{2adb}-\frac{\sqrt{a(a-b)}\,\ln\left(e^{2dx+2c}-\frac{-2a+2\sqrt{a(a-b)}+b}{b}\right)}{2adb}$
derivativedivides	$\frac{-\frac{\ln\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}{b}+\frac{\ln\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)}{b}+\frac{2a(a-b)\left(-\frac{\left(\sqrt{-b(a-b)}-b\right)\operatorname{arctanh}\left(\frac{\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)a}{\sqrt{\left(2\sqrt{-b(a-b)}+a-2b\right)a}}\right)}{2a\sqrt{-b(a-b)}\sqrt{\left(2\sqrt{-b(a-b)}+a-2b\right)a}}+\frac{\left(\sqrt{-b(a-b)}\cdots\right.}{2a\sqrt{-\cdots}}\cdots\right.}{b}\cdots}{d}$
default	$\frac{-\frac{\ln\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}{b}+\frac{\ln\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)}{b}+\frac{2a(a-b)\left(-\frac{\left(\sqrt{-b(a-b)}-b\right)\operatorname{arctanh}\left(\frac{\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)a}{\sqrt{\left(2\sqrt{-b(a-b)}+a-2b\right)a}}\right)}{2a\sqrt{-b(a-b)}\sqrt{\left(2\sqrt{-b(a-b)}+a-2b\right)a}}+\frac{\left(\sqrt{-b(a-b)}\cdots\right.}{2a\sqrt{-\cdots}}\cdots\right.}{b}\cdots}{d}$

input
```
int(cosh(d*x+c)^2/(a+b*sinh(d*x+c)^2),x,method=_RETURNVERBOSE)
```

output
```
x/b+1/2/a*(a*(a-b))^(1/2)/d/b*ln(exp(2*d*x+2*c)+(2*a+2*(a*(a-b))^(1/2)-b)/
b)-1/2/a*(a*(a-b))^(1/2)/d/b*ln(exp(2*d*x+2*c)-(-2*a+2*(a*(a-b))^(1/2)+b)/
b)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 88 vs. $2(42) = 84$.

Time = 0.13 (sec) , antiderivative size = 443, normalized size of antiderivative = 8.86

$$\int \frac{\cosh^2(c+dx)}{a+b\sinh^2(c+dx)}\,dx$$

$$=\left[\frac{2\,dx+\sqrt{\frac{a-b}{a}}\log\left(\frac{b^2\cosh(dx+c)^4+4\,b^2\cosh(dx+c)\sinh(dx+c)^3+b^2\sinh(dx+c)^4+2\,(2\,ab-b^2)\cosh(dx+c)^2+2\left(3\,b^2\cosh(dx+c)^2+2\cdots\right.}{b\cosh(dx+c)^4+4\,b\cosh(dx+c)\sinh(dx+c)^3+b\sinh(dx+c)^4\cdots}\cdots\right.}{\cdots}\right.$$

input
```
integrate(cosh(d*x+c)^2/(a+b*sinh(d*x+c)^2),x, algorithm="fricas")
```

output
```
[1/2*(2*d*x + sqrt((a - b)/a)*log((b^2*cosh(d*x + c)^4 + 4*b^2*cosh(d*x +
c)*sinh(d*x + c)^3 + b^2*sinh(d*x + c)^4 + 2*(2*a*b - b^2)*cosh(d*x + c)^2
 + 2*(3*b^2*cosh(d*x + c)^2 + 2*a*b - b^2)*sinh(d*x + c)^2 + 8*a^2 - 8*a*b
 + b^2 + 4*(b^2*cosh(d*x + c)^3 + (2*a*b - b^2)*cosh(d*x + c))*sinh(d*x +
c) + 4*(a*b*cosh(d*x + c)^2 + 2*a*b*cosh(d*x + c)*sinh(d*x + c) + a*b*sinh
(d*x + c)^2 + 2*a^2 - a*b)*sqrt((a - b)/a))/(b*cosh(d*x + c)^4 + 4*b*cosh(
d*x + c)*sinh(d*x + c)^3 + b*sinh(d*x + c)^4 + 2*(2*a - b)*cosh(d*x + c)^2
 + 2*(3*b*cosh(d*x + c)^2 + 2*a - b)*sinh(d*x + c)^2 + 4*(b*cosh(d*x + c)^
3 + (2*a - b)*cosh(d*x + c))*sinh(d*x + c) + b)))/(b*d), (d*x + sqrt(-(a -
 b)/a)*arctan(-1/2*(b*cosh(d*x + c)^2 + 2*b*cosh(d*x + c)*sinh(d*x + c) +
b*sinh(d*x + c)^2 + 2*a - b)*sqrt(-(a - b)/a)/(a - b)))/(b*d)]
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\cosh^2(c+dx)}{a+b\sinh^2(c+dx)}\,dx = \text{Timed out}$$

input
```
integrate(cosh(d*x+c)**2/(a+b*sinh(d*x+c)**2),x)
```

output
```
Timed out
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{\cosh^2(c+dx)}{a+b\sinh^2(c+dx)}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate(cosh(d*x+c)^2/(a+b*sinh(d*x+c)^2),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(b-a>0)', see `assume?` for more
details)Is
```

Giac [A] (verification not implemented)

Time = 0.40 (sec) , antiderivative size = 68, normalized size of antiderivative = 1.36

$$\int \frac{\cosh^2(c+dx)}{a+b\sinh^2(c+dx)}\,dx = -\frac{\frac{(a-b)\arctan\left(\frac{be^{(2\,dx+2\,c)}+2\,a-b}{2\,\sqrt{-a^2+ab}}\right)}{\sqrt{-a^2+ab}b} - \frac{dx+c}{b}}{d}$$

input
```
integrate(cosh(d*x+c)^2/(a+b*sinh(d*x+c)^2),x, algorithm="giac")
```

output
```
-((a - b)*arctan(1/2*(b*e^(2*d*x + 2*c) + 2*a - b)/sqrt(-a^2 + a*b))/(sqrt
(-a^2 + a*b)*b) - (d*x + c)/b)/d
```

Mupad [B] (verification not implemented)

Time = 0.48 (sec) , antiderivative size = 166, normalized size of antiderivative = 3.32

$$\int \frac{\cosh^2(c+dx)}{a+b\sinh^2(c+dx)}\,dx = \frac{x}{b} + \frac{\ln\left(\frac{4\,\mathrm{e}^{2\,c+2\,d\,x}\,(a-b)}{b^2} - \frac{2\,\sqrt{a-b}\,\left(b+2\,a\,\mathrm{e}^{2\,c+2\,d\,x}-b\,\mathrm{e}^{2\,c+2\,d\,x}\right)}{\sqrt{a}\,b^2}\right)\sqrt{a-b}}{2\,\sqrt{a}\,b\,d} - \frac{\ln\left(\frac{4\,\mathrm{e}^{2\,c+2\,d\,x}\,(a-b)}{b^2} + \frac{2\,\sqrt{a-b}\,\left(b+2\,a\,\mathrm{e}^{2\,c+2\,d\,x}-b\,\mathrm{e}^{2\,c+2\,d\,x}\right)}{\sqrt{a}\,b^2}\right)\sqrt{a-b}}{2\,\sqrt{a}\,b\,d}$$

input
```
int(cosh(c + d*x)^2/(a + b*sinh(c + d*x)^2),x)
```

output
```
x/b + (log((4*exp(2*c + 2*d*x)*(a - b))/b^2 - (2*(a - b)^(1/2)*(b + 2*a*ex
p(2*c + 2*d*x) - b*exp(2*c + 2*d*x)))/(a^(1/2)*b^2))*(a - b)^(1/2))/(2*a^(
1/2)*b*d) - (log((4*exp(2*c + 2*d*x)*(a - b))/b^2 + (2*(a - b)^(1/2)*(b +
2*a*exp(2*c + 2*d*x) - b*exp(2*c + 2*d*x)))/(a^(1/2)*b^2))*(a - b)^(1/2))/
(2*a^(1/2)*b*d)
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 134, normalized size of antiderivative = 2.68

$$\int \frac{\cosh^2(c+dx)}{a+b\sinh^2(c+dx)}\,dx$$
$$= \frac{-\sqrt{a}\,\sqrt{a-b}\log\left(-\sqrt{2\sqrt{a}\,\sqrt{a-b}-2a+b}+e^{dx+c}\sqrt{b}\right)-\sqrt{a}\,\sqrt{a-b}\log\left(\sqrt{2\sqrt{a}\,\sqrt{a-b}-2a+b}+e^{c}\right.}{2abd}$$

input
```
int(cosh(d*x+c)^2/(a+b*sinh(d*x+c)^2),x)
```

output
```
( - sqrt(a)*sqrt(a - b)*log( - sqrt(2*sqrt(a)*sqrt(a - b) - 2*a + b) + e**
(c + d*x)*sqrt(b)) - sqrt(a)*sqrt(a - b)*log(sqrt(2*sqrt(a)*sqrt(a - b) -
2*a + b) + e**(c + d*x)*sqrt(b)) + sqrt(a)*sqrt(a - b)*log(2*sqrt(a)*sqrt(
a - b) + e**(2*c + 2*d*x)*b + 2*a - b) + 2*a*d*x)/(2*a*b*d)
```

3.286 $\int \frac{\cosh(c+dx)}{a+b\sinh^2(c+dx)}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 32

$$\int \frac{\cosh(c+dx)}{a+b\sinh^2(c+dx)}\,dx = \frac{\arctan\left(\frac{\sqrt{b}\sinh(c+dx)}{\sqrt{a}}\right)}{\sqrt{a}\sqrt{b}d}$$

output
```
arctan(b^(1/2)*sinh(d*x+c)/a^(1/2))/a^(1/2)/b^(1/2)/d
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.00

$$\int \frac{\cosh(c+dx)}{a+b\sinh^2(c+dx)}\,dx = \frac{\arctan\left(\frac{\sqrt{b}\sinh(c+dx)}{\sqrt{a}}\right)}{\sqrt{a}\sqrt{b}d}$$

input
```
Integrate[Cosh[c + d*x]/(a + b*Sinh[c + d*x]^2),x]
```

output
```
ArcTan[(Sqrt[b]*Sinh[c + d*x])/Sqrt[a]]/(Sqrt[a]*Sqrt[b]*d)
```

Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {3042, 3669, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cosh(c+dx)}{a+b\sinh^2(c+dx)}dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \frac{\cos(ic+idx)}{a-b\sin(ic+idx)^2}dx$$

$$\Big\downarrow \text{ 3669}$$

$$\frac{\int \frac{1}{b\sinh^2(c+dx)+a}d\sinh(c+dx)}{d}$$

$$\Big\downarrow \text{ 218}$$

$$\frac{\arctan\left(\frac{\sqrt{b}\sinh(c+dx)}{\sqrt{a}}\right)}{\sqrt{a}\sqrt{b}d}$$

input
```
Int[Cosh[c + d*x]/(a + b*Sinh[c + d*x]^2),x]
```

output
```
ArcTan[(Sqrt[b]*Sinh[c + d*x])/Sqrt[a]]/(Sqrt[a]*Sqrt[b]*d)
```

Defintions of rubi rules used

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3669
```
Int[cos[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^
(p_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Simp[ff/f   S
ubst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a + b*ff^2*x^2)^p, x], x, Sin[e + f*x]
/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m - 1)/2]
```

Maple [A] (verified)

Time = 2.77 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.75

method	result	size
derivativedivides	$\frac{\arctan\left(\frac{b\sinh(dx+c)}{\sqrt{ab}}\right)}{d\sqrt{ab}}$	24
default	$\frac{\arctan\left(\frac{b\sinh(dx+c)}{\sqrt{ab}}\right)}{d\sqrt{ab}}$	24
risch	$-\frac{\ln\left(e^{2dx+2c}-\frac{2a\,e^{dx+c}}{\sqrt{-ab}}-1\right)}{2\sqrt{-ab}\,d}+\frac{\ln\left(e^{2dx+2c}+\frac{2a\,e^{dx+c}}{\sqrt{-ab}}-1\right)}{2\sqrt{-ab}\,d}$	78

input
```
int(cosh(d*x+c)/(a+b*sinh(d*x+c)^2),x,method=_RETURNVERBOSE)
```

output
```
1/d/(a*b)^(1/2)*arctan(b*sinh(d*x+c)/(a*b)^(1/2))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 143 vs. 2(24) = 48.

Time = 0.09 (sec) , antiderivative size = 464, normalized size of antiderivative = 14.50

$$\int \frac{\cosh(c+dx)}{a+b\sinh^2(c+dx)}\,dx$$
$$= \left[-\frac{\sqrt{-ab}\log\left(\frac{b\cosh(dx+c)^4+4\,b\cosh(dx+c)\sinh(dx+c)^3+b\sinh(dx+c)^4-2\,(2\,a+b)\cosh(dx+c)^2+2\left(3\,b\cosh(dx+c)^2-2\,a-b\right)\sinh(dx\cdots}{b\cosh(dx+c)^4+4\,b\cosh(dx+c)\sinh(dx+c)^3+b\sinh(dx+c)^4+2\,(\cdots}\right.}{\cdots}\right.$$
$$-\frac{\sqrt{ab}\arctan\left(\frac{2\sqrt{ab}}{b\cosh(dx+c)+b\sinh(dx+c)}\right)-\sqrt{ab}\arctan\left(\frac{\left(b\cosh(dx+c)^3+3\,b\cosh(dx+c)\sinh(dx+c)^2+b\sinh(dx+c)^3+(4\,a\cdots}{2\,ab}\right.}{abd}$$

input
```
integrate(cosh(d*x+c)/(a+b*sinh(d*x+c)^2),x, algorithm="fricas")
```

output
```
[-1/2*sqrt(-a*b)*log((b*cosh(d*x + c)^4 + 4*b*cosh(d*x + c)*sinh(d*x + c)^
3 + b*sinh(d*x + c)^4 - 2*(2*a + b)*cosh(d*x + c)^2 + 2*(3*b*cosh(d*x + c)
^2 - 2*a - b)*sinh(d*x + c)^2 + 4*(b*cosh(d*x + c)^3 - (2*a + b)*cosh(d*x
+ c))*sinh(d*x + c) - 4*(cosh(d*x + c)^3 + 3*cosh(d*x + c)*sinh(d*x + c)^2
 + sinh(d*x + c)^3 + (3*cosh(d*x + c)^2 - 1)*sinh(d*x + c) - cosh(d*x + c)
)*sqrt(-a*b) + b)/(b*cosh(d*x + c)^4 + 4*b*cosh(d*x + c)*sinh(d*x + c)^3 +
 b*sinh(d*x + c)^4 + 2*(2*a - b)*cosh(d*x + c)^2 + 2*(3*b*cosh(d*x + c)^2
+ 2*a - b)*sinh(d*x + c)^2 + 4*(b*cosh(d*x + c)^3 + (2*a - b)*cosh(d*x + c
))*sinh(d*x + c) + b))/(a*b*d), -(sqrt(a*b)*arctan(2*sqrt(a*b)/(b*cosh(d*x
 + c) + b*sinh(d*x + c))) - sqrt(a*b)*arctan(1/2*(b*cosh(d*x + c)^3 + 3*b*
cosh(d*x + c)*sinh(d*x + c)^2 + b*sinh(d*x + c)^3 + (4*a - b)*cosh(d*x + c
) + (3*b*cosh(d*x + c)^2 + 4*a - b)*sinh(d*x + c))*sqrt(a*b)/(a*b)))/(a*b*
d)]
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 107 vs. $2(29) = 58$.

Time = 1.09 (sec) , antiderivative size = 107, normalized size of antiderivative = 3.34

$$\int \frac{\cosh(c+dx)}{a+b\sinh^2(c+dx)}\,dx = \begin{cases} \frac{\tilde{\infty}x\cosh(c)}{\sinh^2(c)} & \text{for } a=0 \wedge b=0 \wedge d=0 \\ \frac{\sinh(c+dx)}{ad} & \text{for } b=0 \\ \frac{x\cosh(c)}{a+b\sinh^2(c)} & \text{for } d=0 \\ -\frac{1}{bd\sinh(c+dx)} & \text{for } a=0 \\ \frac{\log\left(-\sqrt{-\frac{a}{b}}+\sinh(c+dx)\right)}{2bd\sqrt{-\frac{a}{b}}} - \frac{\log\left(\sqrt{-\frac{a}{b}}+\sinh(c+dx)\right)}{2bd\sqrt{-\frac{a}{b}}} & \text{otherwise} \end{cases}$$

input
```
integrate(cosh(d*x+c)/(a+b*sinh(d*x+c)**2),x)
```

output
```
Piecewise((zoo*x*cosh(c)/sinh(c)**2, Eq(a, 0) & Eq(b, 0) & Eq(d, 0)), (sin
h(c + d*x)/(a*d), Eq(b, 0)), (x*cosh(c)/(a + b*sinh(c)**2), Eq(d, 0)), (-1
/(b*d*sinh(c + d*x)), Eq(a, 0)), (log(-sqrt(-a/b) + sinh(c + d*x))/(2*b*d*
sqrt(-a/b)) - log(sqrt(-a/b) + sinh(c + d*x))/(2*b*d*sqrt(-a/b)), True))
```

Maxima [F]

$$\int \frac{\cosh(c+dx)}{a+b\sinh^2(c+dx)}\,dx = \int \frac{\cosh(dx+c)}{b\sinh(dx+c)^2+a}\,dx$$

input
```
integrate(cosh(d*x+c)/(a+b*sinh(d*x+c)^2),x, algorithm="maxima")
```

output
```
integrate(cosh(d*x + c)/(b*sinh(d*x + c)^2 + a), x)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{\cosh(c+dx)}{a+b\sinh^2(c+dx)}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(cosh(d*x+c)/(a+b*sinh(d*x+c)^2),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Limit: Max order reached or unable
to make series expansion Error: Bad Argument Value
```

Mupad [B] (verification not implemented)

Time = 1.74 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.72

$$\int \frac{\cosh(c+dx)}{a+b\sinh^2(c+dx)}\,dx = \frac{\operatorname{atan}\left(\frac{b\sinh(c+d\,x)}{\sqrt{a\,b}}\right)}{d\,\sqrt{a\,b}}$$

input
```
int(cosh(c + d*x)/(a + b*sinh(c + d*x)^2),x)
```

output
```
atan((b*sinh(c + d*x))/(a*b)^(1/2))/(d*(a*b)^(1/2))
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.97

$$\int \frac{\cosh(c+dx)}{a+b\sinh^2(c+dx)}\,dx = \frac{\sqrt{b}\,\sqrt{a}\,atan\left(\frac{\sinh(dx+c)b}{\sqrt{b}\,\sqrt{a}}\right)}{abd}$$

input
```
int(cosh(d*x+c)/(a+b*sinh(d*x+c)^2),x)
```

output
```
(sqrt(b)*sqrt(a)*atan((sinh(c + d*x)*b)/(sqrt(b)*sqrt(a))))/(a*b*d)
```

3.287 $\int \frac{\operatorname{sech}(c+dx)}{a+b\sinh^2(c+dx)}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 59

$$\int \frac{\operatorname{sech}(c+dx)}{a+b\sinh^2(c+dx)}\,dx=\frac{\arctan(\sinh(c+dx))}{(a-b)d}-\frac{\sqrt{b}\arctan\left(\frac{\sqrt{b}\sinh(c+dx)}{\sqrt{a}}\right)}{\sqrt{a}(a-b)d}$$

output
```
arctan(sinh(d*x+c))/(a-b)/d-b^(1/2)*arctan(b^(1/2)*sinh(d*x+c)/a^(1/2))/a^
(1/2)/(a-b)/d
```

Mathematica [A] (verified)

Time = 0.12 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.92

$$\int \frac{\operatorname{sech}(c+dx)}{a+b\sinh^2(c+dx)}\,dx=\frac{\frac{\sqrt{b}\arctan\left(\frac{\sqrt{a}\operatorname{csch}(c+dx)}{\sqrt{b}}\right)}{\sqrt{a}}+2\arctan\left(\tanh\left(\frac{1}{2}(c+dx)\right)\right)}{ad-bd}$$

input
```
Integrate[Sech[c + d*x]/(a + b*Sinh[c + d*x]^2),x]
```

output
```
((Sqrt[b]*ArcTan[(Sqrt[a]*Csch[c + d*x])/Sqrt[b]])/Sqrt[a] + 2*ArcTan[Tanh
[(c + d*x)/2]])/(a*d - b*d)
```

Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.97, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.238, Rules used = {3042, 3669, 303, 216, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\text{sech}(c+dx)}{a+b\sinh^2(c+dx)}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{\cos(ic+idx)\left(a-b\sin(ic+idx)^2\right)}dx$$

$$\Big\downarrow \text{3669}$$

$$\frac{\int \frac{1}{\left(\sinh^2(c+dx)+1\right)\left(b\sinh^2(c+dx)+a\right)}d\sinh(c+dx)}{d}$$

$$\Big\downarrow \text{303}$$

$$\frac{\frac{\int \frac{1}{\sinh^2(c+dx)+1}d\sinh(c+dx)}{a-b}-\frac{b\int \frac{1}{b\sinh^2(c+dx)+a}d\sinh(c+dx)}{a-b}}{d}$$

$$\Big\downarrow \text{216}$$

$$\frac{\frac{\arctan(\sinh(c+dx))}{a-b}-\frac{b\int \frac{1}{b\sinh^2(c+dx)+a}d\sinh(c+dx)}{a-b}}{d}$$

$$\Big\downarrow \text{218}$$

$$\frac{\frac{\arctan(\sinh(c+dx))}{a-b}-\frac{\sqrt{b}\arctan\left(\frac{\sqrt{b}\sinh(c+dx)}{\sqrt{a}}\right)}{\sqrt{a}(a-b)}}{d}$$

input
```
Int[Sech[c + d*x]/(a + b*Sinh[c + d*x]^2),x]
```

output
```
(ArcTan[Sinh[c + d*x]]/(a - b) - (Sqrt[b]*ArcTan[(Sqrt[b]*Sinh[c + d*x])/S
qrt[a]])/(Sqrt[a]*(a - b)))/d
```

Defintions of rubi rules used

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 303
```
Int[1/(((a_) + (b_.)*(x_)^2)*((c_) + (d_.)*(x_)^2)), x_Symbol] :> Simp[b/(b
*c - a*d)   Int[1/(a + b*x^2), x], x] - Simp[d/(b*c - a*d)   Int[1/(c + d*x
^2), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3669
```
Int[cos[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^
(p_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Simp[ff/f   S
ubst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a + b*ff^2*x^2)^p, x], x, Sin[e + f*x]
/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m - 1)/2]
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 21.90 (sec) , antiderivative size = 148, normalized size of antiderivative = 2.51

method	result
risch	$\frac{i\ln(e^{dx+c}+i)}{(a-b)d}-\frac{i\ln(e^{dx+c}-i)}{(a-b)d}+\frac{\sqrt{-ab}\,\ln\left(e^{2dx+2c}-\frac{2\sqrt{-ab}\,e^{dx+c}}{b}-1\right)}{2a(a-b)d}-\frac{\sqrt{-ab}\,\ln\left(e^{2dx+2c}+\frac{2\sqrt{-ab}\,e^{dx+c}}{b}-1\right.}{2a(a-b)d}$
derivativedivides	$\frac{-\frac{2ba\left(-\frac{\left(a+\sqrt{-b(a-b)}-b\right)\operatorname{arctanh}\left(\frac{\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)a}{\sqrt{\left(2\sqrt{-b(a-b)}+a-2b\right)a}}\right)}{2a\sqrt{-b(a-b)}\sqrt{\left(2\sqrt{-b(a-b)}+a-2b\right)a}}+\frac{\left(-a+\sqrt{-b(a-b)}+b\right)\arctan\left(\frac{\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)a}{\sqrt{\left(2\sqrt{-b(a-b)}-a+2b\right)a}}\right)}{2a\sqrt{-b(a-b)}\sqrt{\left(2\sqrt{-b(a-b)}-a+2b\right)a}}\right)}{a-b}+}{d}$
default	$\frac{-\frac{2ba\left(-\frac{\left(a+\sqrt{-b(a-b)}-b\right)\operatorname{arctanh}\left(\frac{\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)a}{\sqrt{\left(2\sqrt{-b(a-b)}+a-2b\right)a}}\right)}{2a\sqrt{-b(a-b)}\sqrt{\left(2\sqrt{-b(a-b)}+a-2b\right)a}}+\frac{\left(-a+\sqrt{-b(a-b)}+b\right)\arctan\left(\frac{\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)a}{\sqrt{\left(2\sqrt{-b(a-b)}-a+2b\right)a}}\right)}{2a\sqrt{-b(a-b)}\sqrt{\left(2\sqrt{-b(a-b)}-a+2b\right)a}}\right)}{a-b}+}{d}$

input
```
int(sech(d*x+c)/(a+b*sinh(d*x+c)^2),x,method=_RETURNVERBOSE)
```

output
```
I/(a-b)/d*ln(exp(d*x+c)+I)-I/(a-b)/d*ln(exp(d*x+c)-I)+1/2/a*(-a*b)^(1/2)/(
a-b)/d*ln(exp(2*d*x+2*c)-2*(-a*b)^(1/2)/b*exp(d*x+c)-1)-1/2/a*(-a*b)^(1/2)
/(a-b)/d*ln(exp(2*d*x+2*c)+2*(-a*b)^(1/2)/b*exp(d*x+c)-1)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 158 vs. $2(51) = 102$.

Time = 0.13 (sec) , antiderivative size = 511, normalized size of antiderivative = 8.66

$$\int \frac{\operatorname{sech}(c+dx)}{a+b\sinh^2(c+dx)}\,dx$$

$$= \left[-\frac{\sqrt{-\frac{b}{a}}\log\left(\frac{b\cosh(dx+c)^4+4\,b\cosh(dx+c)\sinh(dx+c)^3+b\sinh(dx+c)^4-2\,(2\,a+b)\cosh(dx+c)^2+2\left(3\,b\cosh(dx+c)^2-2\,a-b\right)\sinh(dx-}{b\cosh(dx+c)^4+4\,b\cosh(dx+c)\sinh(dx+c)^3+b\sinh(dx+c)^4-}\right.}{}\right.$$

$$-\frac{\sqrt{\frac{b}{a}}\arctan\left(\frac{1}{2}\sqrt{\frac{b}{a}}(\cosh(dx+c)+\sinh(dx+c))\right)+\sqrt{\frac{b}{a}}\arctan\left(\frac{\left(b\cosh(dx+c)^3+3\,b\cosh(dx+c)\sinh(dx+c)^2+\right.}{}\right.}{(a}$$

input
```
integrate(sech(d*x+c)/(a+b*sinh(d*x+c)^2),x, algorithm="fricas")
```

output
```
[-1/2*(sqrt(-b/a)*log((b*cosh(d*x + c)^4 + 4*b*cosh(d*x + c)*sinh(d*x + c)
^3 + b*sinh(d*x + c)^4 - 2*(2*a + b)*cosh(d*x + c)^2 + 2*(3*b*cosh(d*x + c
)^2 - 2*a - b)*sinh(d*x + c)^2 + 4*(b*cosh(d*x + c)^3 - (2*a + b)*cosh(d*x
 + c))*sinh(d*x + c) + 4*(a*cosh(d*x + c)^3 + 3*a*cosh(d*x + c)*sinh(d*x +
 c)^2 + a*sinh(d*x + c)^3 - a*cosh(d*x + c) + (3*a*cosh(d*x + c)^2 - a)*si
nh(d*x + c))*sqrt(-b/a) + b)/(b*cosh(d*x + c)^4 + 4*b*cosh(d*x + c)*sinh(d
*x + c)^3 + b*sinh(d*x + c)^4 + 2*(2*a - b)*cosh(d*x + c)^2 + 2*(3*b*cosh(
d*x + c)^2 + 2*a - b)*sinh(d*x + c)^2 + 4*(b*cosh(d*x + c)^3 + (2*a - b)*c
osh(d*x + c))*sinh(d*x + c) + b)) - 4*arctan(cosh(d*x + c) + sinh(d*x + c)
))/((a - b)*d), -(sqrt(b/a)*arctan(1/2*sqrt(b/a)*(cosh(d*x + c) + sinh(d*x
 + c))) + sqrt(b/a)*arctan(1/2*(b*cosh(d*x + c)^3 + 3*b*cosh(d*x + c)*sinh
(d*x + c)^2 + b*sinh(d*x + c)^3 + (4*a - b)*cosh(d*x + c) + (3*b*cosh(d*x
+ c)^2 + 4*a - b)*sinh(d*x + c))*sqrt(b/a)/b) - 2*arctan(cosh(d*x + c) + s
inh(d*x + c)))/((a - b)*d)]
```

Sympy [F]

$$\int \frac{\operatorname{sech}(c+dx)}{a+b\sinh^2(c+dx)}\,dx=\int \frac{\operatorname{sech}(c+dx)}{a+b\sinh^2(c+dx)}\,dx$$

input
```
integrate(sech(d*x+c)/(a+b*sinh(d*x+c)**2),x)
```

output
```
Integral(sech(c + d*x)/(a + b*sinh(c + d*x)**2), x)
```

Maxima [F]

$$\int \frac{\operatorname{sech}(c+dx)}{a+b\sinh^2(c+dx)}\,dx=\int \frac{\operatorname{sech}(dx+c)}{b\sinh(dx+c)^2+a}\,dx$$

input
```
integrate(sech(d*x+c)/(a+b*sinh(d*x+c)^2),x, algorithm="maxima")
```

output
```
2*arctan(e^(d*x + c))/(a*d - b*d) - 2*integrate((b*e^(3*d*x + 3*c) + b*e^(
d*x + c))/(a*b - b^2 + (a*b*e^(4*c) - b^2*e^(4*c))*e^(4*d*x) + 2*(2*a^2*e^
(2*c) - 3*a*b*e^(2*c) + b^2*e^(2*c))*e^(2*d*x)), x)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{\operatorname{sech}(c+dx)}{a+b\sinh^2(c+dx)}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(sech(d*x+c)/(a+b*sinh(d*x+c)^2),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Limit: Max order reached or unable
to make series expansion Error: Bad Argument Value
```

Mupad [B] (verification not implemented)

Time = 2.19 (sec) , antiderivative size = 648, normalized size of antiderivative = 10.98

$$\int \frac{\operatorname{sech}(c+dx)}{a+b\sinh^2(c+dx)}\,dx = \frac{2\,\mathrm{atan}\left(\frac{e^{d\,x}\,e^{c}\,\left(16\,a^2\,\sqrt{a^2\,d^2-2\,a\,b\,d^2+b^2\,d^2}+b^2\,\sqrt{a^2\,d^2-2\,a\,b\,d^2+b^2\,d^2}-8\,a\,b\,\sqrt{a^2\,d^2-2\,a\,b\,d^2+b^2\,d^2}\right)}{16\,d\,a^3-24\,d\,a^2\,b+9\,d\,a\,b^2-d\,b^3}\right)}{\sqrt{a^2\,d^2-2\,a\,b\,d^2+b^2\,d^2}} - \frac{\sqrt{b}\left(2\,\mathrm{atan}\left(\frac{\sqrt{b}\,e^{d\,x}\,e^{c}\,\sqrt{a\,d^2\,(a-b)^2}}{2\,a\,d\,(a-b)}\right)-2\,\mathrm{atan}\left(\frac{\left(a^3\,b^{5/2}\,\sqrt{a^3\,d^2-2\,a^2\,b\,d^2+a\,b^2\,d^2}-a^2\,b^{7/2}\,\sqrt{a^3\,d^2-2\,a^2\,b\,d^2+a\,b^2\,d^2}\right)\left(e^{d\,x}\,e^{c}\right.}{}\right.\right.}{}$$

input
```
int(1/(cosh(c + d*x)*(a + b*sinh(c + d*x)^2)),x)
```

output
```
(2*atan((exp(d*x)*exp(c)*(16*a^2*(a^2*d^2 + b^2*d^2 - 2*a*b*d^2)^(1/2) + b
^2*(a^2*d^2 + b^2*d^2 - 2*a*b*d^2)^(1/2) - 8*a*b*(a^2*d^2 + b^2*d^2 - 2*a*
b*d^2)^(1/2)))/(16*a^3*d - b^3*d + 9*a*b^2*d - 24*a^2*b*d)))/(a^2*d^2 + b^
2*d^2 - 2*a*b*d^2)^(1/2) - (b^(1/2)*(2*atan((b^(1/2)*exp(d*x)*exp(c)*(a*d^
2*(a - b)^2)^(1/2))/(2*a*d*(a - b))) - 2*atan(((a^3*b^(5/2)*(a^3*d^2 + a*b
^2*d^2 - 2*a^2*b*d^2)^(1/2) - a^2*b^(7/2)*(a^3*d^2 + a*b^2*d^2 - 2*a^2*b*d
^2)^(1/2))*(exp(d*x)*exp(c)*((64*(8*a^3*d + 2*a*b^2*d - 10*a^2*b*d))/(a*b^
3*(a*b - a^2)*(a*d^2*(a - b)^2)^(1/2)*(a^3*d^2 + a*b^2*d^2 - 2*a^2*b*d^2)^
(1/2)) + (32*(b^(3/2)*(a^3*d^2 + a*b^2*d^2 - 2*a^2*b*d^2)^(1/2) - 4*a*b^(1
/2)*(a^3*d^2 + a*b^2*d^2 - 2*a^2*b*d^2)^(1/2)))/(a^2*b^(5/2)*d*(a - b)*(a*
b - a^2)*(a^3*d^2 + a*b^2*d^2 - 2*a^2*b*d^2)^(1/2))) - (32*exp(3*c)*exp(3*
d*x)*(b^(3/2)*(a^3*d^2 + a*b^2*d^2 - 2*a^2*b*d^2)^(1/2) - 4*a*b^(1/2)*(a^3
*d^2 + a*b^2*d^2 - 2*a^2*b*d^2)^(1/2)))/(a^2*b^(5/2)*d*(a - b)*(a*b - a^2)
*(a^3*d^2 + a*b^2*d^2 - 2*a^2*b*d^2)^(1/2))))/(256*a - 64*b))))/(2*(a^3*d^
2 + a*b^2*d^2 - 2*a^2*b*d^2)^(1/2))
```

Reduce [B] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 364, normalized size of antiderivative = 6.17

$$\int \frac{\operatorname{sech}(c+dx)}{a+b\sinh^2(c+dx)}\,dx$$

$$= \frac{4atan\left(e^{dx+c}\right)ab+2\sqrt{b}\,\sqrt{a}\,\sqrt{a-b}\,\sqrt{2\sqrt{a}\,\sqrt{a-b}+2a-b}\,atan\left(\frac{e^{dx+c}b}{\sqrt{b}\,\sqrt{2\sqrt{a}\,\sqrt{a-b}+2a-b}}\right)-2\sqrt{b}\,\sqrt{2\sqrt{a}\,\sqrt{a-}}}{}$$

input
```
int(sech(d*x+c)/(a+b*sinh(d*x+c)^2),x)
```

output

```
(4*atan(e**(c + d*x))*a*b + 2*sqrt(b)*sqrt(a)*sqrt(a - b)*sqrt(2*sqrt(a)*s
qrt(a - b) + 2*a - b)*atan((e**(c + d*x)*b)/(sqrt(b)*sqrt(2*sqrt(a)*sqrt(a
 - b) + 2*a - b))) - 2*sqrt(b)*sqrt(2*sqrt(a)*sqrt(a - b) + 2*a - b)*atan(
(e**(c + d*x)*b)/(sqrt(b)*sqrt(2*sqrt(a)*sqrt(a - b) + 2*a - b)))*a + sqrt
(b)*sqrt(a)*sqrt(a - b)*sqrt(2*sqrt(a)*sqrt(a - b) - 2*a + b)*log( - sqrt(
2*sqrt(a)*sqrt(a - b) - 2*a + b) + e**(c + d*x)*sqrt(b)) - sqrt(b)*sqrt(a)
*sqrt(a - b)*sqrt(2*sqrt(a)*sqrt(a - b) - 2*a + b)*log(sqrt(2*sqrt(a)*sqrt
(a - b) - 2*a + b) + e**(c + d*x)*sqrt(b)) + sqrt(b)*sqrt(2*sqrt(a)*sqrt(a
 - b) - 2*a + b)*log( - sqrt(2*sqrt(a)*sqrt(a - b) - 2*a + b) + e**(c + d*
x)*sqrt(b))*a - sqrt(b)*sqrt(2*sqrt(a)*sqrt(a - b) - 2*a + b)*log(sqrt(2*s
qrt(a)*sqrt(a - b) - 2*a + b) + e**(c + d*x)*sqrt(b))*a)/(2*a*b*d*(a - b))
```

3.288 $\int \frac{\text{sech}^2(c+dx)}{a+b\sinh^2(c+dx)}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 60

$$\int \frac{\text{sech}^2(c+dx)}{a+b\sinh^2(c+dx)}\,dx = -\frac{b\,\text{arctanh}\left(\frac{\sqrt{a-b}\tanh(c+dx)}{\sqrt{a}}\right)}{\sqrt{a}(a-b)^{3/2}d}+\frac{\tanh(c+dx)}{(a-b)d}$$

output
```
-b*arctanh((a-b)^(1/2)*tanh(d*x+c)/a^(1/2))/a^(1/2)/(a-b)^(3/2)/d+tanh(d*x
+c)/(a-b)/d
```

Mathematica [A] (verified)

Time = 0.29 (sec) , antiderivative size = 60, normalized size of antiderivative = 1.00

$$\int \frac{\text{sech}^2(c+dx)}{a+b\sinh^2(c+dx)}\,dx = -\frac{b\,\text{arctanh}\left(\frac{\sqrt{a-b}\tanh(c+dx)}{\sqrt{a}}\right)}{\sqrt{a}(a-b)^{3/2}d}+\frac{\tanh(c+dx)}{(a-b)d}$$

input
```
Integrate[Sech[c + d*x]^2/(a + b*Sinh[c + d*x]^2),x]
```

output
```
-((b*ArcTanh[(Sqrt[a - b]*Tanh[c + d*x])/Sqrt[a]])/(Sqrt[a]*(a - b)^(3/2)*
d)) + Tanh[c + d*x]/((a - b)*d)
```

Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 58, normalized size of antiderivative = 0.97, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.174, Rules used = {3042, 3670, 299, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\operatorname{sech}^2(c+dx)}{a+b\sinh^2(c+dx)}\,dx$$

$$\Big\downarrow \; 3042$$

$$\int \frac{1}{\cos(ic+idx)^2\left(a-b\sin(ic+idx)^2\right)}dx$$

$$\Big\downarrow \; 3670$$

$$\frac{\int \frac{1-\tanh^2(c+dx)}{a-(a-b)\tanh^2(c+dx)}d\tanh(c+dx)}{d}$$

$$\Big\downarrow \; 299$$

$$\frac{\frac{\tanh(c+dx)}{a-b}-\frac{b\int \frac{1}{a-(a-b)\tanh^2(c+dx)}d\tanh(c+dx)}{a-b}}{d}$$

$$\Big\downarrow \; 221$$

$$\frac{\frac{\tanh(c+dx)}{a-b}-\frac{b\,\text{arctanh}\left(\frac{\sqrt{a-b}\tanh(c+dx)}{\sqrt{a}}\right)}{\sqrt{a}(a-b)^{3/2}}}{d}$$

input
```
Int[Sech[c + d*x]^2/(a + b*Sinh[c + d*x]^2),x]
```

output
```
(-((b*ArcTanh[(Sqrt[a - b]*Tanh[c + d*x])/Sqrt[a]])/(Sqrt[a]*(a - b)^(3/2)
)) + Tanh[c + d*x]/(a - b))/d
```

Defintions of rubi rules used

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 299
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2), x_Symbol] :> Simp[d*x
*((a + b*x^2)^(p + 1)/(b*(2*p + 3))), x] - Simp[(a*d - b*c*(2*p + 3))/(b*(2
*p + 3))   Int[(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
 a*d, 0] && NeQ[2*p + 3, 0]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3670
```
Int[cos[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(
p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Simp[ff/f   Su
bst[Int[(a + (a + b)*ff^2*x^2)^p/(1 + ff^2*x^2)^(m/2 + p + 1), x], x, Tan[e
 + f*x]/ff], x]] /; FreeQ[{a, b, e, f}, x] && IntegerQ[m/2] && IntegerQ[p]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 194 vs. $2(52) = 104$.

Time = 61.50 (sec) , antiderivative size = 195, normalized size of antiderivative = 3.25

method	result
risch	$-\frac{2}{d(a-b)\left(e^{2dx+2c}+1\right)}+\frac{b\ln\left(e^{2dx+2c}+\frac{2a\sqrt{a^2-ab}-b\sqrt{a^2-ab}+2a^2-2ab}{b\sqrt{a^2-ab}}\right)}{2\sqrt{a^2-ab}\,(a-b)d}-\frac{b\ln\left(e^{2dx+2c}+\frac{2a\sqrt{a^2-ab}-b\sqrt{a^2-ab}-2}{b\sqrt{a^2-ab}}\right)}{2\sqrt{a^2-ab}\,(a-b)d}$
derivativedivides	$\frac{\frac{2ba\left(-\frac{\left(\sqrt{-b(a-b)}-b\right)\operatorname{arctanh}\left(\frac{\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)a}{\sqrt{\left(2\sqrt{-b(a-b)}+a-2b\right)a}}\right)}{2a\sqrt{-b(a-b)}\sqrt{\left(2\sqrt{-b(a-b)}+a-2b\right)a}}+\frac{\left(\sqrt{-b(a-b)}+b\right)\arctan\left(\frac{\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)a}{\sqrt{\left(2\sqrt{-b(a-b)}-a+2b\right)a}}\right)}{2a\sqrt{-b(a-b)}\sqrt{\left(2\sqrt{-b(a-b)}-a+2b\right)a}}\right)}{a-b}+\frac{2\tan}{(a-b)\left(\tan\right.}}{d}$
default	$\frac{\frac{2ba\left(-\frac{\left(\sqrt{-b(a-b)}-b\right)\operatorname{arctanh}\left(\frac{\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)a}{\sqrt{\left(2\sqrt{-b(a-b)}+a-2b\right)a}}\right)}{2a\sqrt{-b(a-b)}\sqrt{\left(2\sqrt{-b(a-b)}+a-2b\right)a}}+\frac{\left(\sqrt{-b(a-b)}+b\right)\arctan\left(\frac{\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)a}{\sqrt{\left(2\sqrt{-b(a-b)}-a+2b\right)a}}\right)}{2a\sqrt{-b(a-b)}\sqrt{\left(2\sqrt{-b(a-b)}-a+2b\right)a}}\right)}{a-b}+\frac{2\tan}{(a-b)\left(\tan\right.}}{d}$

input
```
int(sech(d*x+c)^2/(a+b*sinh(d*x+c)^2),x,method=_RETURNVERBOSE)
```

output
```
-2/d/(a-b)/(exp(2*d*x+2*c)+1)+1/2/(a^2-a*b)^(1/2)*b/(a-b)/d*ln(exp(2*d*x+2
*c)+(2*a*(a^2-a*b)^(1/2)-b*(a^2-a*b)^(1/2)+2*a^2-2*a*b)/b/(a^2-a*b)^(1/2))
-1/2/(a^2-a*b)^(1/2)*b/(a-b)/d*ln(exp(2*d*x+2*c)+(2*a*(a^2-a*b)^(1/2)-b*(a
^2-a*b)^(1/2)-2*a^2+2*a*b)/b/(a^2-a*b)^(1/2))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 226 vs. $2(52) = 104$.

Time = 0.12 (sec) , antiderivative size = 709, normalized size of antiderivative = 11.82

$$\int \frac{\operatorname{sech}^2(c+dx)}{a+b\sinh^2(c+dx)}\,dx = \text{Too large to display}$$

input
```
integrate(sech(d*x+c)^2/(a+b*sinh(d*x+c)^2),x, algorithm="fricas")
```

output
```
[-1/2*((b*cosh(d*x + c)^2 + 2*b*cosh(d*x + c)*sinh(d*x + c) + b*sinh(d*x +
 c)^2 + b)*sqrt(a^2 - a*b)*log((b^2*cosh(d*x + c)^4 + 4*b^2*cosh(d*x + c)*
sinh(d*x + c)^3 + b^2*sinh(d*x + c)^4 + 2*(2*a*b - b^2)*cosh(d*x + c)^2 +
2*(3*b^2*cosh(d*x + c)^2 + 2*a*b - b^2)*sinh(d*x + c)^2 + 8*a^2 - 8*a*b +
b^2 + 4*(b^2*cosh(d*x + c)^3 + (2*a*b - b^2)*cosh(d*x + c))*sinh(d*x + c)
- 4*(b*cosh(d*x + c)^2 + 2*b*cosh(d*x + c)*sinh(d*x + c) + b*sinh(d*x + c)
^2 + 2*a - b)*sqrt(a^2 - a*b))/(b*cosh(d*x + c)^4 + 4*b*cosh(d*x + c)*sinh
(d*x + c)^3 + b*sinh(d*x + c)^4 + 2*(2*a - b)*cosh(d*x + c)^2 + 2*(3*b*cos
h(d*x + c)^2 + 2*a - b)*sinh(d*x + c)^2 + 4*(b*cosh(d*x + c)^3 + (2*a - b)
*cosh(d*x + c))*sinh(d*x + c) + b)) + 4*a^2 - 4*a*b)/((a^3 - 2*a^2*b + a*b
^2)*d*cosh(d*x + c)^2 + 2*(a^3 - 2*a^2*b + a*b^2)*d*cosh(d*x + c)*sinh(d*x
 + c) + (a^3 - 2*a^2*b + a*b^2)*d*sinh(d*x + c)^2 + (a^3 - 2*a^2*b + a*b^2
)*d), ((b*cosh(d*x + c)^2 + 2*b*cosh(d*x + c)*sinh(d*x + c) + b*sinh(d*x +
 c)^2 + b)*sqrt(-a^2 + a*b)*arctan(-1/2*(b*cosh(d*x + c)^2 + 2*b*cosh(d*x
+ c)*sinh(d*x + c) + b*sinh(d*x + c)^2 + 2*a - b)*sqrt(-a^2 + a*b)/(a^2 -
a*b)) - 2*a^2 + 2*a*b)/((a^3 - 2*a^2*b + a*b^2)*d*cosh(d*x + c)^2 + 2*(a^3
 - 2*a^2*b + a*b^2)*d*cosh(d*x + c)*sinh(d*x + c) + (a^3 - 2*a^2*b + a*b^2
)*d*sinh(d*x + c)^2 + (a^3 - 2*a^2*b + a*b^2)*d)]
```

Sympy [F]

$$\int \frac{\operatorname{sech}^2(c+dx)}{a+b\sinh^2(c+dx)}\,dx=\int \frac{\operatorname{sech}^2{(c+dx)}}{a+b\sinh^2{(c+dx)}}\,dx$$

input
```
integrate(sech(d*x+c)**2/(a+b*sinh(d*x+c)**2),x)
```

output
```
Integral(sech(c + d*x)**2/(a + b*sinh(c + d*x)**2), x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{\operatorname{sech}^2(c+dx)}{a+b\sinh^2(c+dx)}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate(sech(d*x+c)^2/(a+b*sinh(d*x+c)^2),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(b-a>0)', see `assume?` for more
details)Is
```

Giac [A] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 80, normalized size of antiderivative = 1.33

$$\int \frac{\operatorname{sech}^2(c+dx)}{a+b\sinh^2(c+dx)}\,dx = -\frac{\frac{b\arctan\left(\frac{be^{(2\,dx+2\,c)}+2\,a-b}{2\,\sqrt{-a^2+ab}}\right)}{\sqrt{-a^2+ab}(a-b)}+\frac{2}{(a-b)\left(e^{(2\,dx+2\,c)}+1\right)}}{d}$$

input
```
integrate(sech(d*x+c)^2/(a+b*sinh(d*x+c)^2),x, algorithm="giac")
```

output
```
-(b*arctan(1/2*(b*e^(2*d*x + 2*c) + 2*a - b)/sqrt(-a^2 + a*b))/(sqrt(-a^2
+ a*b)*(a - b)) + 2/((a - b)*(e^(2*d*x + 2*c) + 1)))/d
```

Mupad [B] (verification not implemented)

Time = 2.35 (sec) , antiderivative size = 265, normalized size of antiderivative = 4.42

$$\int \frac{\operatorname{sech}^2(c+dx)}{a+b\sinh^2(c+dx)}\,dx = \frac{b\ln\left(\frac{4\,\left(2\,a\,b-b^2+8\,a^2\,\mathrm{e}^{2\,c+2\,d\,x}+b^2\,\mathrm{e}^{2\,c+2\,d\,x}-8\,a\,b\,\mathrm{e}^{2\,c+2\,d\,x}\right)}{a\,{\left(a-b\right)}^3}-\frac{8\,b+32\,a\,\mathrm{e}^{2\,c+2\,d\,x}-16\,b\,\mathrm{e}^{2\,c+2\,d\,x}}{\sqrt{a}\,{\left(a-b\right)}^{5/2}}\right)}{2\,\sqrt{a}\,d\,{\left(a-b\right)}^{3/2}} - \frac{b\ln\left(\frac{8\,b+32\,a\,\mathrm{e}^{2\,c+2\,d\,x}-16\,b\,\mathrm{e}^{2\,c+2\,d\,x}}{\sqrt{a}\,{\left(a-b\right)}^{5/2}}+\frac{4\,\left(2\,a\,b-b^2+8\,a^2\,\mathrm{e}^{2\,c+2\,d\,x}+b^2\,\mathrm{e}^{2\,c+2\,d\,x}-8\,a\,b\,\mathrm{e}^{2\,c+2\,d\,x}\right)}{a\,{\left(a-b\right)}^3}\right)}{2\,\sqrt{a}\,d\,{\left(a-b\right)}^{3/2}} - \frac{2}{\left(\mathrm{e}^{2\,c+2\,d\,x}+1\right)\,\left(a\,d-b\,d\right)}$$

input
```
int(1/(cosh(c + d*x)^2*(a + b*sinh(c + d*x)^2)),x)
```

output
```
(b*log((4*(2*a*b - b^2 + 8*a^2*exp(2*c + 2*d*x) + b^2*exp(2*c + 2*d*x) - 8
*a*b*exp(2*c + 2*d*x)))/(a*(a - b)^3) - (8*b + 32*a*exp(2*c + 2*d*x) - 16*
b*exp(2*c + 2*d*x))/(a^(1/2)*(a - b)^(5/2))))/(2*a^(1/2)*d*(a - b)^(3/2))
- (b*log((8*b + 32*a*exp(2*c + 2*d*x) - 16*b*exp(2*c + 2*d*x))/(a^(1/2)*(a
 - b)^(5/2)) + (4*(2*a*b - b^2 + 8*a^2*exp(2*c + 2*d*x) + b^2*exp(2*c + 2*
d*x) - 8*a*b*exp(2*c + 2*d*x)))/(a*(a - b)^3)))/(2*a^(1/2)*d*(a - b)^(3/2)
) - 2/((exp(2*c + 2*d*x) + 1)*(a*d - b*d))
```

Reduce [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 363, normalized size of antiderivative = 6.05

$$\int \frac{\operatorname{sech}^2(c+dx)}{a+b\sinh^2(c+dx)}\,dx = \frac{-e^{2dx+2c}\sqrt{a}\,\sqrt{a-b}\log\left(-\sqrt{2\sqrt{a}\,\sqrt{a-b}-2a+b}+e^{dx+c}\sqrt{b}\right)b-e^{2dx+2c}\sqrt{a}\,\sqrt{a-b}\log\left(\sqrt{2\sqrt{a}\,\sqrt{a-}\right.}{}$$

input
```
int(sech(d*x+c)^2/(a+b*sinh(d*x+c)^2),x)
```

output

```
( - e**(2*c + 2*d*x)*sqrt(a)*sqrt(a - b)*log( - sqrt(2*sqrt(a)*sqrt(a - b)
 - 2*a + b) + e**(c + d*x)*sqrt(b))*b - e**(2*c + 2*d*x)*sqrt(a)*sqrt(a -
b)*log(sqrt(2*sqrt(a)*sqrt(a - b) - 2*a + b) + e**(c + d*x)*sqrt(b))*b + e
**(2*c + 2*d*x)*sqrt(a)*sqrt(a - b)*log(2*sqrt(a)*sqrt(a - b) + e**(2*c +
2*d*x)*b + 2*a - b)*b - sqrt(a)*sqrt(a - b)*log( - sqrt(2*sqrt(a)*sqrt(a -
 b) - 2*a + b) + e**(c + d*x)*sqrt(b))*b - sqrt(a)*sqrt(a - b)*log(sqrt(2*
sqrt(a)*sqrt(a - b) - 2*a + b) + e**(c + d*x)*sqrt(b))*b + sqrt(a)*sqrt(a
- b)*log(2*sqrt(a)*sqrt(a - b) + e**(2*c + 2*d*x)*b + 2*a - b)*b + 4*e**(2
*c + 2*d*x)*a**2 - 4*e**(2*c + 2*d*x)*a*b)/(2*a*d*(e**(2*c + 2*d*x)*a**2 -
 2*e**(2*c + 2*d*x)*a*b + e**(2*c + 2*d*x)*b**2 + a**2 - 2*a*b + b**2))
```

3.289 $\int \frac{\operatorname{sech}^3(c+dx)}{a+b\sinh^2(c+dx)}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 92

$$\int \frac{\operatorname{sech}^3(c+dx)}{a+b\sinh^2(c+dx)}\,dx = \frac{(a-3b)\arctan(\sinh(c+dx))}{2(a-b)^2d} + \frac{b^{3/2}\arctan\left(\frac{\sqrt{b}\sinh(c+dx)}{\sqrt{a}}\right)}{\sqrt{a}(a-b)^2d} + \frac{\operatorname{sech}(c+dx)\tanh(c+dx)}{2(a-b)d}$$

output
```
1/2*(a-3*b)*arctan(sinh(d*x+c))/(a-b)^2/d+b^(3/2)*arctan(b^(1/2)*sinh(d*x+
c)/a^(1/2))/a^(1/2)/(a-b)^2/d+1/2*sech(d*x+c)*tanh(d*x+c)/(a-b)/d
```

Mathematica [A] (verified)

Time = 0.20 (sec) , antiderivative size = 91, normalized size of antiderivative = 0.99

$$\int \frac{\operatorname{sech}^3(c+dx)}{a+b\sinh^2(c+dx)}\,dx = \frac{-2b^{3/2}\arctan\left(\frac{\sqrt{a}\operatorname{csch}(c+dx)}{\sqrt{b}}\right) + 2\sqrt{a}(a-3b)\arctan\left(\tanh\left(\frac{1}{2}(c+dx)\right)\right) + \sqrt{a}(a-b)\operatorname{sech}(c+dx)\tanh(}{2\sqrt{a}(a-b)^2d}$$

input
```
Integrate[Sech[c + d*x]^3/(a + b*Sinh[c + d*x]^2),x]
```

output
```
(-2*b^(3/2)*ArcTan[(Sqrt[a]*Csch[c + d*x])/Sqrt[b]] + 2*Sqrt[a]*(a - 3*b)*
ArcTan[Tanh[(c + d*x)/2]] + Sqrt[a]*(a - b)*Sech[c + d*x]*Tanh[c + d*x])/(
2*Sqrt[a]*(a - b)^2*d)
```

Rubi [A] (verified)

Time = 0.32 (sec) , antiderivative size = 103, normalized size of antiderivative = 1.12, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.304$, Rules used = {3042, 3669, 316, 25, 397, 216, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\text{sech}^3(c+dx)}{a+b\sinh^2(c+dx)}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{\cos(ic+idx)^3\left(a-b\sin(ic+idx)^2\right)}dx$$

$$\Big\downarrow \text{3669}$$

$$\frac{\int \frac{1}{\left(\sinh^2(c+dx)+1\right)^2\left(b\sinh^2(c+dx)+a\right)}d\sinh(c+dx)}{d}$$

$$\Big\downarrow \text{316}$$

$$\frac{\frac{\sinh(c+dx)}{2(a-b)\left(\sinh^2(c+dx)+1\right)}-\frac{\int -\frac{b\sinh^2(c+dx)+a-2b}{\left(\sinh^2(c+dx)+1\right)\left(b\sinh^2(c+dx)+a\right)}d\sinh(c+dx)}{2(a-b)}}{d}$$

$$\Big\downarrow \text{25}$$

$$\frac{\frac{\int \frac{b\sinh^2(c+dx)+a-2b}{\left(\sinh^2(c+dx)+1\right)\left(b\sinh^2(c+dx)+a\right)}d\sinh(c+dx)}{2(a-b)}+\frac{\sinh(c+dx)}{2(a-b)\left(\sinh^2(c+dx)+1\right)}}{d}$$

$$\Big\downarrow \text{397}$$

$$\frac{\frac{\frac{2b^2\int \frac{1}{b\sinh^2(c+dx)+a}d\sinh(c+dx)}{a-b}+\frac{(a-3b)\int \frac{1}{\sinh^2(c+dx)+1}d\sinh(c+dx)}{a-b}}{2(a-b)}+\frac{\sinh(c+dx)}{2(a-b)\left(\sinh^2(c+dx)+1\right)}}{d}$$

$$\Big\downarrow \text{216}$$

$$\frac{\frac{\frac{2b^2\int \frac{1}{b\sinh^2(c+dx)+a}d\sinh(c+dx)}{a-b}+\frac{(a-3b)\arctan(\sinh(c+dx))}{a-b}}{2(a-b)}+\frac{\sinh(c+dx)}{2(a-b)(\sinh^2(c+dx)+1)}}{d}$$

$$\Big\downarrow \text{218}$$

$$\frac{\frac{\frac{2b^{3/2}\arctan\left(\frac{\sqrt{b}\sinh(c+dx)}{\sqrt{a}}\right)}{\sqrt{a}(a-b)}+\frac{(a-3b)\arctan(\sinh(c+dx))}{a-b}}{2(a-b)}+\frac{\sinh(c+dx)}{2(a-b)(\sinh^2(c+dx)+1)}}{d}$$

input
```
Int[Sech[c + d*x]^3/(a + b*Sinh[c + d*x]^2),x]
```

output
```
((((a - 3*b)*ArcTan[Sinh[c + d*x]])/(a - b) + (2*b^(3/2)*ArcTan[(Sqrt[b]*S
inh[c + d*x])/Sqrt[a]])/(Sqrt[a]*(a - b)))/(2*(a - b)) + Sinh[c + d*x]/(2*
(a - b)*(1 + Sinh[c + d*x]^2)))/d
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 316
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_), x_Symbol] :> Sim
p[(-b)*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^(q + 1)/(2*a*(p + 1)*(b*c - a*d))
), x] + Simp[1/(2*a*(p + 1)*(b*c - a*d))   Int[(a + b*x^2)^(p + 1)*(c + d*x
^2)^q*Simp[b*c + 2*(p + 1)*(b*c - a*d) + d*b*(2*(p + q + 2) + 1)*x^2, x], x
], x] /; FreeQ[{a, b, c, d, q}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] &&  !
( !IntegerQ[p] && IntegerQ[q] && LtQ[q, -1]) && IntBinomialQ[a, b, c, d, 2,
 p, q, x]
```

rule 397
```
Int[((e_) + (f_.)*(x_)^2)/(((a_) + (b_.)*(x_)^2)*((c_) + (d_.)*(x_)^2)), x_
Symbol] :> Simp[(b*e - a*f)/(b*c - a*d)   Int[1/(a + b*x^2), x], x] - Simp[
(d*e - c*f)/(b*c - a*d)   Int[1/(c + d*x^2), x], x] /; FreeQ[{a, b, c, d, e
, f}, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3669
```
Int[cos[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^
(p_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Simp[ff/f   S
ubst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a + b*ff^2*x^2)^p, x], x, Sin[e + f*x]
/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m - 1)/2]
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 135.32 (sec) , antiderivative size = 241, normalized size of antiderivative = 2.62

method	result
risch	$\frac{e^{dx+c}\left(e^{2dx+2c}-1\right)}{d(a-b)\left(e^{2dx+2c}+1\right)^2}+\frac{i\ln\left(e^{dx+c}+i\right)a}{2(a-b)^2 d}-\frac{3i\ln\left(e^{dx+c}+i\right)b}{2(a-b)^2 d}-\frac{i\ln\left(e^{dx+c}-i\right)a}{2(a-b)^2 d}+\frac{3i\ln\left(e^{dx+c}-i\right)b}{2(a-b)^2 d}+\frac{\sqrt{-ab}\,b\ln\cdots}{\cdots}$
derivativedivides	$\frac{\frac{\frac{2\left(\left(-\frac{a}{2}+\frac{b}{2}\right)\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^3+\left(\frac{a}{2}-\frac{b}{2}\right)\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^2+1\right)^2}+(a-3b)\arctan\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{(a-b)^2}+2b^2a\left(-\frac{\left(a+\sqrt{-b(a-b)}-b\right)\operatorname{arctanh}\left(\cdots\right)}{2a\sqrt{-b(a-b)}\sqrt{\left(2\sqrt{-b\cdots}\right)}}\cdots\right)}{d}$
default	$\frac{\frac{\frac{2\left(\left(-\frac{a}{2}+\frac{b}{2}\right)\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^3+\left(\frac{a}{2}-\frac{b}{2}\right)\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^2+1\right)^2}+(a-3b)\arctan\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{(a-b)^2}+2b^2a\left(-\frac{\left(a+\sqrt{-b(a-b)}-b\right)\operatorname{arctanh}\left(\cdots\right)}{2a\sqrt{-b(a-b)}\sqrt{\left(2\sqrt{-b\cdots}\right)}}\cdots\right)}{d}$

input
```
int(sech(d*x+c)^3/(a+b*sinh(d*x+c)^2),x,method=_RETURNVERBOSE)
```

output
```
exp(d*x+c)*(exp(2*d*x+2*c)-1)/d/(a-b)/(exp(2*d*x+2*c)+1)^2+1/2*I/(a-b)^2/d
*ln(exp(d*x+c)+I)*a-3/2*I/(a-b)^2/d*ln(exp(d*x+c)+I)*b-1/2*I/(a-b)^2/d*ln(
exp(d*x+c)-I)*a+3/2*I/(a-b)^2/d*ln(exp(d*x+c)-I)*b+1/2/a*(-a*b)^(1/2)*b/(a
-b)^2/d*ln(exp(2*d*x+2*c)+2*(-a*b)^(1/2)/b*exp(d*x+c)-1)-1/2/a*(-a*b)^(1/2
)*b/(a-b)^2/d*ln(exp(2*d*x+2*c)-2*(-a*b)^(1/2)/b*exp(d*x+c)-1)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 772 vs. $2(80) = 160$.

Time = 0.15 (sec) , antiderivative size = 1644, normalized size of antiderivative = 17.87

$$\int \frac{\operatorname{sech}^3(c+dx)}{a+b\sinh^2(c+dx)}\,dx = \text{Too large to display}$$

input
```
integrate(sech(d*x+c)^3/(a+b*sinh(d*x+c)^2),x, algorithm="fricas")
```

output
```
[1/2*(2*(a - b)*cosh(d*x + c)^3 + 6*(a - b)*cosh(d*x + c)*sinh(d*x + c)^2
+ 2*(a - b)*sinh(d*x + c)^3 + (b*cosh(d*x + c)^4 + 4*b*cosh(d*x + c)*sinh(
d*x + c)^3 + b*sinh(d*x + c)^4 + 2*b*cosh(d*x + c)^2 + 2*(3*b*cosh(d*x + c
)^2 + b)*sinh(d*x + c)^2 + 4*(b*cosh(d*x + c)^3 + b*cosh(d*x + c))*sinh(d*
x + c) + b)*sqrt(-b/a)*log((b*cosh(d*x + c)^4 + 4*b*cosh(d*x + c)*sinh(d*x
 + c)^3 + b*sinh(d*x + c)^4 - 2*(2*a + b)*cosh(d*x + c)^2 + 2*(3*b*cosh(d*
x + c)^2 - 2*a - b)*sinh(d*x + c)^2 + 4*(b*cosh(d*x + c)^3 - (2*a + b)*cos
h(d*x + c))*sinh(d*x + c) + 4*(a*cosh(d*x + c)^3 + 3*a*cosh(d*x + c)*sinh(
d*x + c)^2 + a*sinh(d*x + c)^3 - a*cosh(d*x + c) + (3*a*cosh(d*x + c)^2 -
a)*sinh(d*x + c))*sqrt(-b/a) + b)/(b*cosh(d*x + c)^4 + 4*b*cosh(d*x + c)*s
inh(d*x + c)^3 + b*sinh(d*x + c)^4 + 2*(2*a - b)*cosh(d*x + c)^2 + 2*(3*b*
cosh(d*x + c)^2 + 2*a - b)*sinh(d*x + c)^2 + 4*(b*cosh(d*x + c)^3 + (2*a -
 b)*cosh(d*x + c))*sinh(d*x + c) + b)) + 2*((a - 3*b)*cosh(d*x + c)^4 + 4*
(a - 3*b)*cosh(d*x + c)*sinh(d*x + c)^3 + (a - 3*b)*sinh(d*x + c)^4 + 2*(a
 - 3*b)*cosh(d*x + c)^2 + 2*(3*(a - 3*b)*cosh(d*x + c)^2 + a - 3*b)*sinh(d
*x + c)^2 + 4*((a - 3*b)*cosh(d*x + c)^3 + (a - 3*b)*cosh(d*x + c))*sinh(d
*x + c) + a - 3*b)*arctan(cosh(d*x + c) + sinh(d*x + c)) - 2*(a - b)*cosh(
d*x + c) + 2*(3*(a - b)*cosh(d*x + c)^2 - a + b)*sinh(d*x + c))/((a^2 - 2*
a*b + b^2)*d*cosh(d*x + c)^4 + 4*(a^2 - 2*a*b + b^2)*d*cosh(d*x + c)*sinh(
d*x + c)^3 + (a^2 - 2*a*b + b^2)*d*sinh(d*x + c)^4 + 2*(a^2 - 2*a*b + b...
```

Sympy [F]

$$\int \frac{\operatorname{sech}^3(c+dx)}{a+b\sinh^2(c+dx)}\,dx = \int \frac{\operatorname{sech}^3(c+dx)}{a+b\sinh^2(c+dx)}\,dx$$

input
```
integrate(sech(d*x+c)**3/(a+b*sinh(d*x+c)**2),x)
```

output
```
Integral(sech(c + d*x)**3/(a + b*sinh(c + d*x)**2), x)
```

Maxima [F]

$$\int \frac{\operatorname{sech}^3(c+dx)}{a+b\sinh^2(c+dx)}\,dx = \int \frac{\operatorname{sech}(dx+c)^3}{b\sinh(dx+c)^2+a}\,dx$$

input
```
integrate(sech(d*x+c)^3/(a+b*sinh(d*x+c)^2),x, algorithm="maxima")
```

output
```
(a*e^c - 3*b*e^c)*arctan(e^(d*x + c))*e^(-c)/(a^2*d - 2*a*b*d + b^2*d) + (
e^(3*d*x + 3*c) - e^(d*x + c))/(a*d - b*d + (a*d*e^(4*c) - b*d*e^(4*c))*e^
(4*d*x) + 2*(a*d*e^(2*c) - b*d*e^(2*c))*e^(2*d*x)) + 8*integrate(1/4*(b^2*
e^(3*d*x + 3*c) + b^2*e^(d*x + c))/(a^2*b - 2*a*b^2 + b^3 + (a^2*b*e^(4*c)
 - 2*a*b^2*e^(4*c) + b^3*e^(4*c))*e^(4*d*x) + 2*(2*a^3*e^(2*c) - 5*a^2*b*e
^(2*c) + 4*a*b^2*e^(2*c) - b^3*e^(2*c))*e^(2*d*x)), x)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{\operatorname{sech}^3(c+dx)}{a+b\sinh^2(c+dx)}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(sech(d*x+c)^3/(a+b*sinh(d*x+c)^2),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Limit: Max order reached or unable
to make series expansion Error: Bad Argument Value
```

Mupad [B] (verification not implemented)

Time = 6.90 (sec) , antiderivative size = 2797, normalized size of antiderivative = 30.40

$$\int \frac{\operatorname{sech}^3(c+dx)}{a+b\sinh^2(c+dx)}\,dx = \text{Too large to display}$$

input
```
int(1/(cosh(c + d*x)^3*(a + b*sinh(c + d*x)^2)),x)
```

output
```
exp(c + d*x)/((exp(2*c + 2*d*x) + 1)*(a*d - b*d)) - (2*exp(c + d*x))/((a*d
 - b*d)*(2*exp(2*c + 2*d*x) + exp(4*c + 4*d*x) + 1)) + ((2*atan((b^2*exp(d
*x)*exp(c)*(a*d^2*(a - b)^4)^(1/2))/(2*a*d*(a - b)^2*(b^3)^(1/2))) - 2*ata
n((exp(d*x)*exp(c)*((64*(20*a^3*d*(b^3)^(5/2) - 232*a^6*d*(b^3)^(3/2) + 2*
a^9*d*(b^3)^(1/2) + 2*a*b^5*d*(b^3)^(3/2) + 10*a^5*b*d*(b^3)^(3/2) - 22*a^
8*b*d*(b^3)^(1/2) - 10*a^2*b^4*d*(b^3)^(3/2) - 20*a^4*b^2*d*(b^3)^(3/2) -
18*a^2*b^7*d*(b^3)^(1/2) + 102*a^3*b^6*d*(b^3)^(1/2) - 242*a^4*b^5*d*(b^3)
^(1/2) + 310*a^5*b^4*d*(b^3)^(1/2) + 98*a^7*b^2*d*(b^3)^(1/2)))/(a*b^4*(a
- b)^5*(a*b - a^2)*(a^2 - 2*a*b + b^2)*(a*d^2*(a - b)^4)^(1/2)*(3*a*b^2 -
3*a^2*b + a^3 - b^3)*(9*a*b^2 - 6*a^2*b + a^3 - b^3)*(a^5*d^2 + a*b^4*d^2
- 4*a^4*b*d^2 - 4*a^2*b^3*d^2 + 6*a^3*b^2*d^2)^(1/2)) - (32*(b^8*(a^5*d^2
+ a*b^4*d^2 - 4*a^4*b*d^2 - 4*a^2*b^3*d^2 + 6*a^3*b^2*d^2)^(1/2) + 36*a^2*
b^6*(a^5*d^2 + a*b^4*d^2 - 4*a^4*b*d^2 - 4*a^2*b^3*d^2 + 6*a^3*b^2*d^2)^(1
/2) - 47*a^3*b^5*(a^5*d^2 + a*b^4*d^2 - 4*a^4*b*d^2 - 4*a^2*b^3*d^2 + 6*a^
3*b^2*d^2)^(1/2) + 30*a^4*b^4*(a^5*d^2 + a*b^4*d^2 - 4*a^4*b*d^2 - 4*a^2*b
^3*d^2 + 6*a^3*b^2*d^2)^(1/2) - 9*a^5*b^3*(a^5*d^2 + a*b^4*d^2 - 4*a^4*b*d
^2 - 4*a^2*b^3*d^2 + 6*a^3*b^2*d^2)^(1/2) + a^6*b^2*(a^5*d^2 + a*b^4*d^2 -
 4*a^4*b*d^2 - 4*a^2*b^3*d^2 + 6*a^3*b^2*d^2)^(1/2) - 12*a*b^7*(a^5*d^2 +
a*b^4*d^2 - 4*a^4*b*d^2 - 4*a^2*b^3*d^2 + 6*a^3*b^2*d^2)^(1/2)))/(a^2*b^2*
d*(a - b)^7*(a*b - a^2)*(b^3)^(1/2)*(a^2 - 2*a*b + b^2)*(3*a*b^2 - 3*a^...
```

Reduce [B] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 1396, normalized size of antiderivative = 15.17

$$\int \frac{\operatorname{sech}^3(c+dx)}{a+b\sinh^2(c+dx)}\,dx = \text{Too large to display}$$

input
```
int(sech(d*x+c)^3/(a+b*sinh(d*x+c)^2),x)
```

output
```
(2*e**(4*c + 4*d*x)*atan(e**(c + d*x))*a**2 - 6*e**(4*c + 4*d*x)*atan(e**(
c + d*x))*a*b + 4*e**(2*c + 2*d*x)*atan(e**(c + d*x))*a**2 - 12*e**(2*c +
2*d*x)*atan(e**(c + d*x))*a*b + 2*atan(e**(c + d*x))*a**2 - 6*atan(e**(c +
 d*x))*a*b - 2*e**(4*c + 4*d*x)*sqrt(b)*sqrt(a)*sqrt(a - b)*sqrt(2*sqrt(a)
*sqrt(a - b) + 2*a - b)*atan((e**(c + d*x)*b)/(sqrt(b)*sqrt(2*sqrt(a)*sqrt
(a - b) + 2*a - b))) - 4*e**(2*c + 2*d*x)*sqrt(b)*sqrt(a)*sqrt(a - b)*sqrt
(2*sqrt(a)*sqrt(a - b) + 2*a - b)*atan((e**(c + d*x)*b)/(sqrt(b)*sqrt(2*sq
rt(a)*sqrt(a - b) + 2*a - b))) - 2*sqrt(b)*sqrt(a)*sqrt(a - b)*sqrt(2*sqrt
(a)*sqrt(a - b) + 2*a - b)*atan((e**(c + d*x)*b)/(sqrt(b)*sqrt(2*sqrt(a)*s
qrt(a - b) + 2*a - b))) + 2*e**(4*c + 4*d*x)*sqrt(b)*sqrt(2*sqrt(a)*sqrt(a
 - b) + 2*a - b)*atan((e**(c + d*x)*b)/(sqrt(b)*sqrt(2*sqrt(a)*sqrt(a - b)
 + 2*a - b)))*a + 4*e**(2*c + 2*d*x)*sqrt(b)*sqrt(2*sqrt(a)*sqrt(a - b) +
2*a - b)*atan((e**(c + d*x)*b)/(sqrt(b)*sqrt(2*sqrt(a)*sqrt(a - b) + 2*a -
 b)))*a + 2*sqrt(b)*sqrt(2*sqrt(a)*sqrt(a - b) + 2*a - b)*atan((e**(c + d*
x)*b)/(sqrt(b)*sqrt(2*sqrt(a)*sqrt(a - b) + 2*a - b)))*a - e**(4*c + 4*d*x
)*sqrt(b)*sqrt(a)*sqrt(a - b)*sqrt(2*sqrt(a)*sqrt(a - b) - 2*a + b)*log( -
 sqrt(2*sqrt(a)*sqrt(a - b) - 2*a + b) + e**(c + d*x)*sqrt(b)) + e**(4*c +
 4*d*x)*sqrt(b)*sqrt(a)*sqrt(a - b)*sqrt(2*sqrt(a)*sqrt(a - b) - 2*a + b)*
log(sqrt(2*sqrt(a)*sqrt(a - b) - 2*a + b) + e**(c + d*x)*sqrt(b)) - 2*e**(
2*c + 2*d*x)*sqrt(b)*sqrt(a)*sqrt(a - b)*sqrt(2*sqrt(a)*sqrt(a - b) - 2...
```

3.290 $\int \frac{\text{sech}^4(c+dx)}{a+b\sinh^2(c+dx)}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 88

$$\int \frac{\text{sech}^4(c+dx)}{a+b\sinh^2(c+dx)}\,dx = \frac{b^2\text{arctanh}\left(\frac{\sqrt{a-b}\tanh(c+dx)}{\sqrt{a}}\right)}{\sqrt{a}(a-b)^{5/2}d} + \frac{(a-2b)\tanh(c+dx)}{(a-b)^2d} - \frac{\tanh^3(c+dx)}{3(a-b)d}$$

output
```
b^2*arctanh((a-b)^(1/2)*tanh(d*x+c)/a^(1/2))/a^(1/2)/(a-b)^(5/2)/d+(a-2*b)
*tanh(d*x+c)/(a-b)^2/d-1/3*tanh(d*x+c)^3/(a-b)/d
```

Mathematica [A] (verified)

Time = 0.50 (sec) , antiderivative size = 84, normalized size of antiderivative = 0.95

$$\int \frac{\text{sech}^4(c+dx)}{a+b\sinh^2(c+dx)}\,dx = \frac{\frac{3b^2\text{arctanh}\left(\frac{\sqrt{a-b}\tanh(c+dx)}{\sqrt{a}}\right)}{\sqrt{a}(a-b)^{5/2}} + \frac{\left(2a-5b+(a-b)\text{sech}^2(c+dx)\right)\tanh(c+dx)}{(a-b)^2}}{3d}$$

input
```
Integrate[Sech[c + d*x]^4/(a + b*Sinh[c + d*x]^2),x]
```

output
```
((3*b^2*ArcTanh[(Sqrt[a - b]*Tanh[c + d*x])/Sqrt[a]])/(Sqrt[a]*(a - b)^(5/
2)) + ((2*a - 5*b + (a - b)*Sech[c + d*x]^2)*Tanh[c + d*x])/(a - b)^2)/(3*
d)
```

Rubi [A] (verified)

Time = 0.31 (sec) , antiderivative size = 83, normalized size of antiderivative = 0.94, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.174$, Rules used = {3042, 3670, 300, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\operatorname{sech}^4(c+dx)}{a+b\sinh^2(c+dx)}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{\cos(ic+idx)^4\left(a-b\sin(ic+idx)^2\right)}dx$$

$$\Big\downarrow \text{3670}$$

$$\frac{\int \frac{\left(1-\tanh^2(c+dx)\right)^2}{a-(a-b)\tanh^2(c+dx)}d\tanh(c+dx)}{d}$$

$$\Big\downarrow \text{300}$$

$$\frac{\int \left(\frac{b^2}{(a-b)^2\left(a-(a-b)\tanh^2(c+dx)\right)}-\frac{\tanh^2(c+dx)}{a-b}+\frac{a-2b}{(a-b)^2}\right)d\tanh(c+dx)}{d}$$

$$\Big\downarrow \text{2009}$$

$$\frac{\frac{b^2\text{arctanh}\left(\frac{\sqrt{a-b}\tanh(c+dx)}{\sqrt{a}}\right)}{\sqrt{a}(a-b)^{5/2}}-\frac{\tanh^3(c+dx)}{3(a-b)}+\frac{(a-2b)\tanh(c+dx)}{(a-b)^2}}{d}$$

input
```
Int[Sech[c + d*x]^4/(a + b*Sinh[c + d*x]^2),x]
```

output
```
((b^2*ArcTanh[(Sqrt[a - b]*Tanh[c + d*x])/Sqrt[a]])/(Sqrt[a]*(a - b)^(5/2)
) + ((a - 2*b)*Tanh[c + d*x])/(a - b)^2 - Tanh[c + d*x]^3/(3*(a - b)))/d
```

Defintions of rubi rules used

rule 300
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_), x_Symbol] :> Int
[PolynomialDivide[(a + b*x^2)^p, (c + d*x^2)^(-q), x], x] /; FreeQ[{a, b, c
, d}, x] && NeQ[b*c - a*d, 0] && IGtQ[p, 0] && ILtQ[q, 0] && GeQ[p, -q]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3670
```
Int[cos[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(
p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Simp[ff/f   Su
bst[Int[(a + (a + b)*ff^2*x^2)^p/(1 + ff^2*x^2)^(m/2 + p + 1), x], x, Tan[e
 + f*x]/ff], x]] /; FreeQ[{a, b, e, f}, x] && IntegerQ[m/2] && IntegerQ[p]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 241 vs. $2(78) = 156$.

Time = 283.41 (sec) , antiderivative size = 242, normalized size of antiderivative = 2.75

method	result
risch	$-\frac{2(-3b\,e^{4dx+4c}+6\,e^{2dx+2c}a-12\,e^{2dx+2c}b+2a-5b)}{3d(a-b)^2(e^{2dx+2c}+1)^3}+\frac{b^2\ln\left(e^{2dx+2c}+\frac{2a\sqrt{a^2-ab}-b\sqrt{a^2-ab}-2a^2+2ab}{b\sqrt{a^2-ab}}\right)}{2\sqrt{a^2-ab}\,(a-b)^2 d}-\frac{b^2\ln\Big(e}{}$
derivativedivides	$\frac{-\frac{2b^2a\left(-\frac{\left(\sqrt{-b(a-b)}-b\right)\operatorname{arctanh}\left(\frac{\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)a}{\sqrt{\left(2\sqrt{-b(a-b)}+a-2b\right)a}}\right)}{2a\sqrt{-b(a-b)}\sqrt{\left(2\sqrt{-b(a-b)}+a-2b\right)a}}+\frac{\left(\sqrt{-b(a-b)}+b\right)\arctan\left(\frac{\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)a}{\sqrt{\left(2\sqrt{-b(a-b)}-a+2b\right)a}}\right)}{2a\sqrt{-b(a-b)}\sqrt{\left(2\sqrt{-b(a-b)}-a+2b\right)a}}\right)}{(a-b)^2}-\frac{2\big((-a}{}}{d}$
default	$\frac{-\frac{2b^2a\left(-\frac{\left(\sqrt{-b(a-b)}-b\right)\operatorname{arctanh}\left(\frac{\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)a}{\sqrt{\left(2\sqrt{-b(a-b)}+a-2b\right)a}}\right)}{2a\sqrt{-b(a-b)}\sqrt{\left(2\sqrt{-b(a-b)}+a-2b\right)a}}+\frac{\left(\sqrt{-b(a-b)}+b\right)\arctan\left(\frac{\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)a}{\sqrt{\left(2\sqrt{-b(a-b)}-a+2b\right)a}}\right)}{2a\sqrt{-b(a-b)}\sqrt{\left(2\sqrt{-b(a-b)}-a+2b\right)a}}\right)}{(a-b)^2}-\frac{2\big((-a}{}}{d}$

input
```
int(sech(d*x+c)^4/(a+b*sinh(d*x+c)^2),x,method=_RETURNVERBOSE)
```

output
```
-2/3*(-3*b*exp(4*d*x+4*c)+6*exp(2*d*x+2*c)*a-12*exp(2*d*x+2*c)*b+2*a-5*b)/
d/(a-b)^2/(exp(2*d*x+2*c)+1)^3+1/2/(a^2-a*b)^(1/2)*b^2/(a-b)^2/d*ln(exp(2*
d*x+2*c)+(2*a*(a^2-a*b)^(1/2)-b*(a^2-a*b)^(1/2)-2*a^2+2*a*b)/b/(a^2-a*b)^(
1/2))-1/2/(a^2-a*b)^(1/2)*b^2/(a-b)^2/d*ln(exp(2*d*x+2*c)+(2*a*(a^2-a*b)^(
1/2)-b*(a^2-a*b)^(1/2)+2*a^2-2*a*b)/b/(a^2-a*b)^(1/2))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1094 vs. $2(78) = 156$.

Time = 0.12 (sec) , antiderivative size = 2444, normalized size of antiderivative = 27.77

$$\int \frac{\operatorname{sech}^4(c + dx)}{a + b\sinh^2(c + dx)}\, dx = \text{Too large to display}$$

input
```
integrate(sech(d*x+c)^4/(a+b*sinh(d*x+c)^2),x, algorithm="fricas")
```

output
```
[1/6*(12*(a^2*b - a*b^2)*cosh(d*x + c)^4 + 48*(a^2*b - a*b^2)*cosh(d*x + c
)*sinh(d*x + c)^3 + 12*(a^2*b - a*b^2)*sinh(d*x + c)^4 - 8*a^3 + 28*a^2*b
- 20*a*b^2 - 24*(a^3 - 3*a^2*b + 2*a*b^2)*cosh(d*x + c)^2 - 24*(a^3 - 3*a^
2*b + 2*a*b^2 - 3*(a^2*b - a*b^2)*cosh(d*x + c)^2)*sinh(d*x + c)^2 + 3*(b^
2*cosh(d*x + c)^6 + 6*b^2*cosh(d*x + c)*sinh(d*x + c)^5 + b^2*sinh(d*x + c
)^6 + 3*b^2*cosh(d*x + c)^4 + 3*(5*b^2*cosh(d*x + c)^2 + b^2)*sinh(d*x + c
)^4 + 3*b^2*cosh(d*x + c)^2 + 4*(5*b^2*cosh(d*x + c)^3 + 3*b^2*cosh(d*x +
c))*sinh(d*x + c)^3 + 3*(5*b^2*cosh(d*x + c)^4 + 6*b^2*cosh(d*x + c)^2 + b
^2)*sinh(d*x + c)^2 + b^2 + 6*(b^2*cosh(d*x + c)^5 + 2*b^2*cosh(d*x + c)^3
 + b^2*cosh(d*x + c))*sinh(d*x + c))*sqrt(a^2 - a*b)*log((b^2*cosh(d*x + c
)^4 + 4*b^2*cosh(d*x + c)*sinh(d*x + c)^3 + b^2*sinh(d*x + c)^4 + 2*(2*a*b
 - b^2)*cosh(d*x + c)^2 + 2*(3*b^2*cosh(d*x + c)^2 + 2*a*b - b^2)*sinh(d*x
 + c)^2 + 8*a^2 - 8*a*b + b^2 + 4*(b^2*cosh(d*x + c)^3 + (2*a*b - b^2)*cos
h(d*x + c))*sinh(d*x + c) - 4*(b*cosh(d*x + c)^2 + 2*b*cosh(d*x + c)*sinh(
d*x + c) + b*sinh(d*x + c)^2 + 2*a - b)*sqrt(a^2 - a*b))/(b*cosh(d*x + c)^
4 + 4*b*cosh(d*x + c)*sinh(d*x + c)^3 + b*sinh(d*x + c)^4 + 2*(2*a - b)*co
sh(d*x + c)^2 + 2*(3*b*cosh(d*x + c)^2 + 2*a - b)*sinh(d*x + c)^2 + 4*(b*c
osh(d*x + c)^3 + (2*a - b)*cosh(d*x + c))*sinh(d*x + c) + b)) + 48*((a^2*b
 - a*b^2)*cosh(d*x + c)^3 - (a^3 - 3*a^2*b + 2*a*b^2)*cosh(d*x + c))*sinh(
d*x + c))/((a^4 - 3*a^3*b + 3*a^2*b^2 - a*b^3)*d*cosh(d*x + c)^6 + 6*(a...
```

Sympy [F]

$$\int \frac{\operatorname{sech}^4(c+dx)}{a+b\sinh^2(c+dx)}\,dx = \int \frac{\operatorname{sech}^4(c+dx)}{a+b\sinh^2(c+dx)}\,dx$$

input
```
integrate(sech(d*x+c)**4/(a+b*sinh(d*x+c)**2),x)
```

output
```
Integral(sech(c + d*x)**4/(a + b*sinh(c + d*x)**2), x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{\operatorname{sech}^4(c+dx)}{a+b\sinh^2(c+dx)}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate(sech(d*x+c)^4/(a+b*sinh(d*x+c)^2),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(b-a>0)', see `assume?` for more
details)Is
```

Giac [A] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 138, normalized size of antiderivative = 1.57

$$\int \frac{\operatorname{sech}^4(c+dx)}{a+b\sinh^2(c+dx)}\,dx = \frac{\frac{3\,b^2\arctan\left(\frac{be^{(2\,dx+2\,c)}+2\,a-b}{2\,\sqrt{-a^2+ab}}\right)}{(a^2-2\,ab+b^2)\sqrt{-a^2+ab}} + \frac{2\,\left(3\,be^{(4\,dx+4\,c)}-6\,ae^{(2\,dx+2\,c)}+12\,be^{(2\,dx+2\,c)}-2\,a+5\,b\right)}{(a^2-2\,ab+b^2)\left(e^{(2\,dx+2\,c)}+1\right)^3}}{3\,d}$$

input
```
integrate(sech(d*x+c)^4/(a+b*sinh(d*x+c)^2),x, algorithm="giac")
```

output
```
1/3*(3*b^2*arctan(1/2*(b*e^(2*d*x + 2*c) + 2*a - b)/sqrt(-a^2 + a*b))/((a^
2 - 2*a*b + b^2)*sqrt(-a^2 + a*b)) + 2*(3*b*e^(4*d*x + 4*c) - 6*a*e^(2*d*x
 + 2*c) + 12*b*e^(2*d*x + 2*c) - 2*a + 5*b)/((a^2 - 2*a*b + b^2)*(e^(2*d*x
 + 2*c) + 1)^3))/d
```

Mupad [B] (verification not implemented)

Time = 3.26 (sec) , antiderivative size = 710, normalized size of antiderivative = 8.07

$$\int \frac{\operatorname{sech}^4(c+dx)}{a+b\sinh^2(c+dx)}\,dx = \frac{8}{3\,\left(a\,d-b\,d\right)\,\left(3\,{\mathrm{e}}^{2\,c+2\,d\,x}+3\,{\mathrm{e}}^{4\,c+4\,d\,x}+{\mathrm{e}}^{6\,c+6\,d\,x}+1\right)}$$
$$-\frac{4}{\left(a\,d-b\,d\right)\,\left(2\,{\mathrm{e}}^{2\,c+2\,d\,x}+{\mathrm{e}}^{4\,c+4\,d\,x}+1\right)}$$
$$+\frac{\mathrm{atan}\left(\left({\mathrm{e}}^{2\,c}\,{\mathrm{e}}^{2\,d\,x}\,\left(\frac{4}{d\,{\left(a-b\right)}^2\,\sqrt{b^4}\,\left(a^2-2\,a\,b+b^2\right)}+\frac{\left(2\,a-b\right)\,\left(2\,a^3\,d\,\sqrt{b^4}-b^3\,d\,\sqrt{b^4}+4\,a\,b^2\,d\,\sqrt{b^4}-5\,a^2\,b\,d\,\sqrt{b^4}\right)}{b^4\,\left(a^2-2\,a\,b+b^2\right)\,\sqrt{-a\,d^2\,{\left(a-b\right)}^5}\,\sqrt{-a^6\,d^2+5\,a^5\,b\,d^2-10\,a^4\,b^2\,d^2+10\,a^3\,b^3\,d^2-5\,a^{\cdots}}}\right.\right.\right.}{\cdots}$$
$$+\frac{2\,b}{\left({\mathrm{e}}^{2\,c+2\,d\,x}+1\right)\,\left(a-b\right)\,\left(a\,d-b\,d\right)}$$

input
```
int(1/(cosh(c + d*x)^4*(a + b*sinh(c + d*x)^2)),x)
```

output
```
8/(3*(a*d - b*d)*(3*exp(2*c + 2*d*x) + 3*exp(4*c + 4*d*x) + exp(6*c + 6*d*
x) + 1)) - 4/((a*d - b*d)*(2*exp(2*c + 2*d*x) + exp(4*c + 4*d*x) + 1)) + (
atan((exp(2*c)*exp(2*d*x)*(4/(d*(a - b)^2*(b^4)^(1/2)*(a^2 - 2*a*b + b^2))
 + ((2*a - b)*(2*a^3*d*(b^4)^(1/2) - b^3*d*(b^4)^(1/2) + 4*a*b^2*d*(b^4)^(
1/2) - 5*a^2*b*d*(b^4)^(1/2)))/(b^4*(a^2 - 2*a*b + b^2)*(-a*d^2*(a - b)^5)
^(1/2)*(a*b^5*d^2 - a^6*d^2 + 5*a^5*b*d^2 - 5*a^2*b^4*d^2 + 10*a^3*b^3*d^2
 - 10*a^4*b^2*d^2)^(1/2))) + ((2*a - b)*(b^3*d*(b^4)^(1/2) - 2*a*b^2*d*(b^
4)^(1/2) + a^2*b*d*(b^4)^(1/2)))/(b^4*(a^2 - 2*a*b + b^2)*(-a*d^2*(a - b)^
5)^(1/2)*(a*b^5*d^2 - a^6*d^2 + 5*a^5*b*d^2 - 5*a^2*b^4*d^2 + 10*a^3*b^3*d
^2 - 10*a^4*b^2*d^2)^(1/2)))*((b^3*(a*b^5*d^2 - a^6*d^2 + 5*a^5*b*d^2 - 5*
a^2*b^4*d^2 + 10*a^3*b^3*d^2 - 10*a^4*b^2*d^2)^(1/2))/2 - a*b^2*(a*b^5*d^2
 - a^6*d^2 + 5*a^5*b*d^2 - 5*a^2*b^4*d^2 + 10*a^3*b^3*d^2 - 10*a^4*b^2*d^2
)^(1/2) + (a^2*b*(a*b^5*d^2 - a^6*d^2 + 5*a^5*b*d^2 - 5*a^2*b^4*d^2 + 10*a
^3*b^3*d^2 - 10*a^4*b^2*d^2)^(1/2))/2))*(b^4)^(1/2))/(a*b^5*d^2 - a^6*d^2
+ 5*a^5*b*d^2 - 5*a^2*b^4*d^2 + 10*a^3*b^3*d^2 - 10*a^4*b^2*d^2)^(1/2) + (
2*b)/((exp(2*c + 2*d*x) + 1)*(a - b)*(a*d - b*d))
```

Reduce [B] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 911, normalized size of antiderivative = 10.35

$$\int \frac{\operatorname{sech}^4(c+dx)}{a+b\sinh^2(c+dx)}\,dx = \text{Too large to display}$$

input
```
int(sech(d*x+c)^4/(a+b*sinh(d*x+c)^2),x)
```

output
```
(3*e**(6*c + 6*d*x)*sqrt(a)*sqrt(a - b)*log( - sqrt(2*sqrt(a)*sqrt(a - b)
- 2*a + b) + e**(c + d*x)*sqrt(b))*b**2 + 3*e**(6*c + 6*d*x)*sqrt(a)*sqrt(
a - b)*log(sqrt(2*sqrt(a)*sqrt(a - b) - 2*a + b) + e**(c + d*x)*sqrt(b))*b
**2 - 3*e**(6*c + 6*d*x)*sqrt(a)*sqrt(a - b)*log(2*sqrt(a)*sqrt(a - b) + e
**(2*c + 2*d*x)*b + 2*a - b)*b**2 + 9*e**(4*c + 4*d*x)*sqrt(a)*sqrt(a - b)
*log( - sqrt(2*sqrt(a)*sqrt(a - b) - 2*a + b) + e**(c + d*x)*sqrt(b))*b**2
 + 9*e**(4*c + 4*d*x)*sqrt(a)*sqrt(a - b)*log(sqrt(2*sqrt(a)*sqrt(a - b) -
 2*a + b) + e**(c + d*x)*sqrt(b))*b**2 - 9*e**(4*c + 4*d*x)*sqrt(a)*sqrt(a
 - b)*log(2*sqrt(a)*sqrt(a - b) + e**(2*c + 2*d*x)*b + 2*a - b)*b**2 + 9*e
**(2*c + 2*d*x)*sqrt(a)*sqrt(a - b)*log( - sqrt(2*sqrt(a)*sqrt(a - b) - 2*
a + b) + e**(c + d*x)*sqrt(b))*b**2 + 9*e**(2*c + 2*d*x)*sqrt(a)*sqrt(a -
b)*log(sqrt(2*sqrt(a)*sqrt(a - b) - 2*a + b) + e**(c + d*x)*sqrt(b))*b**2
- 9*e**(2*c + 2*d*x)*sqrt(a)*sqrt(a - b)*log(2*sqrt(a)*sqrt(a - b) + e**(2
*c + 2*d*x)*b + 2*a - b)*b**2 + 3*sqrt(a)*sqrt(a - b)*log( - sqrt(2*sqrt(a
)*sqrt(a - b) - 2*a + b) + e**(c + d*x)*sqrt(b))*b**2 + 3*sqrt(a)*sqrt(a -
 b)*log(sqrt(2*sqrt(a)*sqrt(a - b) - 2*a + b) + e**(c + d*x)*sqrt(b))*b**2
 - 3*sqrt(a)*sqrt(a - b)*log(2*sqrt(a)*sqrt(a - b) + e**(2*c + 2*d*x)*b +
2*a - b)*b**2 - 4*e**(6*c + 6*d*x)*a**2*b + 4*e**(6*c + 6*d*x)*a*b**2 - 24
*e**(2*c + 2*d*x)*a**3 + 60*e**(2*c + 2*d*x)*a**2*b - 36*e**(2*c + 2*d*x)*
a*b**2 - 8*a**3 + 24*a**2*b - 16*a*b**2)/(6*a*d*(e**(6*c + 6*d*x)*a**3 ...
```

3.291 $\int \frac{\operatorname{sech}^5(c+dx)}{a+b\sinh^2(c+dx)}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 138

$$\int \frac{\operatorname{sech}^5(c+dx)}{a+b\sinh^2(c+dx)}\,dx = \frac{(3a^2-10ab+15b^2)\arctan(\sinh(c+dx))}{8(a-b)^3d} - \frac{b^{5/2}\arctan\left(\frac{\sqrt{b}\sinh(c+dx)}{\sqrt{a}}\right)}{\sqrt{a}(a-b)^3d} + \frac{(3a-7b)\operatorname{sech}(c+dx)\tanh(c+dx)}{8(a-b)^2d} + \frac{\operatorname{sech}^3(c+dx)\tanh(c+dx)}{4(a-b)d}$$

output
```
1/8*(3*a^2-10*a*b+15*b^2)*arctan(sinh(d*x+c))/(a-b)^3/d-b^(5/2)*arctan(b^(
1/2)*sinh(d*x+c)/a^(1/2))/a^(1/2)/(a-b)^3/d+1/8*(3*a-7*b)*sech(d*x+c)*tanh
(d*x+c)/(a-b)^2/d+1/4*sech(d*x+c)^3*tanh(d*x+c)/(a-b)/d
```

Mathematica [A] (verified)

Time = 0.38 (sec) , antiderivative size = 139, normalized size of antiderivative = 1.01

$$\int \frac{\operatorname{sech}^5(c+dx)}{a+b\sinh^2(c+dx)}\,dx$$
$$=\frac{8b^{5/2}\arctan\left(\frac{\sqrt{a}\operatorname{csch}(c+dx)}{\sqrt{b}}\right)+2\sqrt{a}(3a^2-10ab+15b^2)\arctan\left(\tanh\left(\frac{1}{2}(c+dx)\right)\right)+\sqrt{a}(3a^2-10ab+7}{8\sqrt{a}(a-b)^3d}$$

input
```
Integrate[Sech[c + d*x]^5/(a + b*Sinh[c + d*x]^2),x]
```

output
```
(8*b^(5/2)*ArcTan[(Sqrt[a]*Csch[c + d*x])/Sqrt[b]] + 2*Sqrt[a]*(3*a^2 - 10
*a*b + 15*b^2)*ArcTan[Tanh[(c + d*x)/2]] + Sqrt[a]*(3*a^2 - 10*a*b + 7*b^2
)*Sech[c + d*x]*Tanh[c + d*x] + 2*Sqrt[a]*(a - b)^2*Sech[c + d*x]^3*Tanh[c
 + d*x])/(8*Sqrt[a]*(a - b)^3*d)
```

Rubi [A] (verified)

Time = 0.39 (sec) , antiderivative size = 161, normalized size of antiderivative = 1.17, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.391, Rules used = {3042, 3669, 316, 25, 402, 25, 397, 216, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\operatorname{sech}^5(c+dx)}{a+b\sinh^2(c+dx)}\,dx$$
$$\Big\downarrow \text{ 3042}$$
$$\int \frac{1}{\cos(ic+idx)^5\left(a-b\sin(ic+idx)^2\right)}dx$$
$$\Big\downarrow \text{ 3669}$$
$$\frac{\int \frac{1}{\left(\sinh^2(c+dx)+1\right)^3\left(b\sinh^2(c+dx)+a\right)}d\sinh(c+dx)}{d}$$
$$\Big\downarrow \text{ 316}$$

$$\frac{\frac{\sinh(c+dx)}{4(a-b)\left(\sinh^2(c+dx)+1\right)^2}-\frac{\int -\frac{3b\sinh^2(c+dx)+3a-4b}{\left(\sinh^2(c+dx)+1\right)^2\left(b\sinh^2(c+dx)+a\right)}d\sinh(c+dx)}{4(a-b)}}{d}$$

$$\Big\downarrow \; 25$$

$$\frac{\frac{\int \frac{3b\sinh^2(c+dx)+3a-4b}{\left(\sinh^2(c+dx)+1\right)^2\left(b\sinh^2(c+dx)+a\right)}d\sinh(c+dx)}{4(a-b)}+\frac{\sinh(c+dx)}{4(a-b)\left(\sinh^2(c+dx)+1\right)^2}}{d}$$

$$\Big\downarrow \; 402$$

$$\frac{\frac{\frac{(3a-7b)\sinh(c+dx)}{2(a-b)\left(\sinh^2(c+dx)+1\right)}-\frac{\int -\frac{3a^2-7ba+8b^2+(3a-7b)b\sinh^2(c+dx)}{\left(\sinh^2(c+dx)+1\right)\left(b\sinh^2(c+dx)+a\right)}d\sinh(c+dx)}{2(a-b)}}{4(a-b)}+\frac{\sinh(c+dx)}{4(a-b)\left(\sinh^2(c+dx)+1\right)^2}}{d}$$

$$\Big\downarrow \; 25$$

$$\frac{\frac{\frac{\int \frac{3a^2-7ba+8b^2+(3a-7b)b\sinh^2(c+dx)}{\left(\sinh^2(c+dx)+1\right)\left(b\sinh^2(c+dx)+a\right)}d\sinh(c+dx)}{2(a-b)}+\frac{(3a-7b)\sinh(c+dx)}{2(a-b)\left(\sinh^2(c+dx)+1\right)}}{4(a-b)}+\frac{\sinh(c+dx)}{4(a-b)\left(\sinh^2(c+dx)+1\right)^2}}{d}$$

$$\Big\downarrow \; 397$$

$$\frac{\frac{\frac{\frac{\left(3a^2-10ab+15b^2\right)\int \frac{1}{\sinh^2(c+dx)+1}d\sinh(c+dx)}{a-b}-\frac{8b^3\int \frac{1}{b\sinh^2(c+dx)+a}d\sinh(c+dx)}{a-b}}{2(a-b)}+\frac{(3a-7b)\sinh(c+dx)}{2(a-b)\left(\sinh^2(c+dx)+1\right)}}{4(a-b)}+\frac{\sinh(c+dx)}{4(a-b)\left(\sinh^2(c+dx)+1\right)^2}}{d}$$

$$\Big\downarrow \; 216$$

$$\frac{\frac{\frac{\frac{\left(3a^2-10ab+15b^2\right)\arctan(\sinh(c+dx))}{a-b}-\frac{8b^3\int \frac{1}{b\sinh^2(c+dx)+a}d\sinh(c+dx)}{a-b}}{2(a-b)}+\frac{(3a-7b)\sinh(c+dx)}{2(a-b)\left(\sinh^2(c+dx)+1\right)}}{4(a-b)}+\frac{\sinh(c+dx)}{4(a-b)\left(\sinh^2(c+dx)+1\right)^2}}{d}$$

$$\Big\downarrow \; 218$$

$$\frac{\frac{\frac{\frac{\left(3a^2-10ab+15b^2\right)\arctan(\sinh(c+dx))}{a-b}-\frac{8b^{5/2}\arctan\left(\frac{\sqrt{b}\sinh(c+dx)}{\sqrt{a}}\right)}{\sqrt{a}(a-b)}}{2(a-b)}+\frac{(3a-7b)\sinh(c+dx)}{2(a-b)\left(\sinh^2(c+dx)+1\right)}}{4(a-b)}+\frac{\sinh(c+dx)}{4(a-b)\left(\sinh^2(c+dx)+1\right)^2}}{d}$$

input
```
Int[Sech[c + d*x]^5/(a + b*Sinh[c + d*x]^2),x]
```

output
```
(Sinh[c + d*x]/(4*(a - b)*(1 + Sinh[c + d*x]^2)^2) + ((((3*a^2 - 10*a*b +
15*b^2)*ArcTan[Sinh[c + d*x]])/(a - b) - (8*b^(5/2)*ArcTan[(Sqrt[b]*Sinh[c
 + d*x])/Sqrt[a]])/(Sqrt[a]*(a - b)))/(2*(a - b)) + ((3*a - 7*b)*Sinh[c +
d*x])/(2*(a - b)*(1 + Sinh[c + d*x]^2)))/(4*(a - b)))/d
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 316
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_), x_Symbol] :> Sim
p[(-b)*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^(q + 1)/(2*a*(p + 1)*(b*c - a*d))
), x] + Simp[1/(2*a*(p + 1)*(b*c - a*d))   Int[(a + b*x^2)^(p + 1)*(c + d*x
^2)^q*Simp[b*c + 2*(p + 1)*(b*c - a*d) + d*b*(2*(p + q + 2) + 1)*x^2, x], x
], x] /; FreeQ[{a, b, c, d, q}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] &&  !
( !IntegerQ[p] && IntegerQ[q] && LtQ[q, -1]) && IntBinomialQ[a, b, c, d, 2,
 p, q, x]
```

rule 397
```
Int[((e_) + (f_.)*(x_)^2)/(((a_) + (b_.)*(x_)^2)*((c_) + (d_.)*(x_)^2)), x_
Symbol] :> Simp[(b*e - a*f)/(b*c - a*d)   Int[1/(a + b*x^2), x], x] - Simp[
(d*e - c*f)/(b*c - a*d)   Int[1/(c + d*x^2), x], x] /; FreeQ[{a, b, c, d, e
, f}, x]
```

rule 402
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_.)*((e_) + (f_.)*(x
_)^2), x_Symbol] :> Simp[(-(b*e - a*f))*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^
(q + 1)/(a*2*(b*c - a*d)*(p + 1))), x] + Simp[1/(a*2*(b*c - a*d)*(p + 1))
 Int[(a + b*x^2)^(p + 1)*(c + d*x^2)^q*Simp[c*(b*e - a*f) + e*2*(b*c - a*d)
*(p + 1) + d*(b*e - a*f)*(2*(p + q + 2) + 1)*x^2, x], x], x] /; FreeQ[{a, b
, c, d, e, f, q}, x] && LtQ[p, -1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3669
```
Int[cos[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^
(p_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Simp[ff/f   S
ubst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a + b*ff^2*x^2)^p, x], x, Sin[e + f*x]
/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m - 1)/2]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 351 vs. $2(124) = 248$.

Time = 0.25 (sec) , antiderivative size = 352, normalized size of antiderivative = 2.55

$$\frac{\frac{2\left(\left(-\frac{5}{8}a^2+\frac{7}{4}ab-\frac{9}{8}b^2\right)\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^7+\left(\frac{3}{8}a^2-\frac{1}{4}ab-\frac{1}{8}b^2\right)\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^5+\left(-\frac{3}{8}a^2+\frac{1}{4}ab+\frac{1}{8}b^2\right)\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^3+\left(\frac{5}{8}a^2-\frac{7}{4}ab+\frac{9}{8}b^2\right)\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^2+1\right)^4}+\frac{\left(3a^2-10a\cdots\right.}{\cdots}}{(a-b)^3}$$

input
```
int(sech(d*x+c)^5/(a+b*sinh(d*x+c)^2),x)
```

output
```
1/d*(2/(a-b)^3*(((-5/8*a^2+7/4*a*b-9/8*b^2)*tanh(1/2*d*x+1/2*c)^7+(3/8*a^2
-1/4*a*b-1/8*b^2)*tanh(1/2*d*x+1/2*c)^5+(-3/8*a^2+1/4*a*b+1/8*b^2)*tanh(1/
2*d*x+1/2*c)^3+(5/8*a^2-7/4*a*b+9/8*b^2)*tanh(1/2*d*x+1/2*c))/(tanh(1/2*d*
x+1/2*c)^2+1)^4+1/8*(3*a^2-10*a*b+15*b^2)*arctan(tanh(1/2*d*x+1/2*c)))-2*b
^3/(a-b)^3*a*(-1/2*(a+(-b*(a-b))^(1/2)-b)/a/(-b*(a-b))^(1/2)/((2*(-b*(a-b)
)^(1/2)+a-2*b)*a)^(1/2)*arctanh(tanh(1/2*d*x+1/2*c)*a/((2*(-b*(a-b))^(1/2)
+a-2*b)*a)^(1/2))+1/2*(-a+(-b*(a-b))^(1/2)+b)/a/(-b*(a-b))^(1/2)/((2*(-b*(
a-b))^(1/2)-a+2*b)*a)^(1/2)*arctan(tanh(1/2*d*x+1/2*c)*a/((2*(-b*(a-b))^(1
/2)-a+2*b)*a)^(1/2))))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2827 vs. 2(124) = 248.

Time = 0.20 (sec) , antiderivative size = 5500, normalized size of antiderivative = 39.86

$$\int \frac{\operatorname{sech}^5(c+dx)}{a+b\sinh^2(c+dx)}\,dx = \text{Too large to display}$$

input
```
integrate(sech(d*x+c)^5/(a+b*sinh(d*x+c)^2),x, algorithm="fricas")
```

output
```
Too large to include
```

Sympy [F]

$$\int \frac{\operatorname{sech}^5(c+dx)}{a+b\sinh^2(c+dx)}\,dx = \int \frac{\operatorname{sech}^5(c+dx)}{a+b\sinh^2(c+dx)}\,dx$$

input
```
integrate(sech(d*x+c)**5/(a+b*sinh(d*x+c)**2),x)
```

output
```
Integral(sech(c + d*x)**5/(a + b*sinh(c + d*x)**2), x)
```

Maxima [F]

$$\int \frac{\operatorname{sech}^5(c+dx)}{a+b\sinh^2(c+dx)}\,dx = \int \frac{\operatorname{sech}(dx+c)^5}{b\sinh(dx+c)^2+a}\,dx$$

input
```
integrate(sech(d*x+c)^5/(a+b*sinh(d*x+c)^2),x, algorithm="maxima")
```

output
```
1/4*(3*a^2*e^c - 10*a*b*e^c + 15*b^2*e^c)*arctan(e^(d*x + c))*e^(-c)/(a^3*
d - 3*a^2*b*d + 3*a*b^2*d - b^3*d) + 1/4*((3*a*e^(7*c) - 7*b*e^(7*c))*e^(7
*d*x) + (11*a*e^(5*c) - 15*b*e^(5*c))*e^(5*d*x) - (11*a*e^(3*c) - 15*b*e^(
3*c))*e^(3*d*x) - (3*a*e^c - 7*b*e^c)*e^(d*x))/(a^2*d - 2*a*b*d + b^2*d +
(a^2*d*e^(8*c) - 2*a*b*d*e^(8*c) + b^2*d*e^(8*c))*e^(8*d*x) + 4*(a^2*d*e^(
6*c) - 2*a*b*d*e^(6*c) + b^2*d*e^(6*c))*e^(6*d*x) + 6*(a^2*d*e^(4*c) - 2*a
*b*d*e^(4*c) + b^2*d*e^(4*c))*e^(4*d*x) + 4*(a^2*d*e^(2*c) - 2*a*b*d*e^(2*
c) + b^2*d*e^(2*c))*e^(2*d*x)) - 32*integrate(1/16*(b^3*e^(3*d*x + 3*c) +
b^3*e^(d*x + c))/(a^3*b - 3*a^2*b^2 + 3*a*b^3 - b^4 + (a^3*b*e^(4*c) - 3*a
^2*b^2*e^(4*c) + 3*a*b^3*e^(4*c) - b^4*e^(4*c))*e^(4*d*x) + 2*(2*a^4*e^(2*
c) - 7*a^3*b*e^(2*c) + 9*a^2*b^2*e^(2*c) - 5*a*b^3*e^(2*c) + b^4*e^(2*c))*
e^(2*d*x)), x)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{\operatorname{sech}^5(c+dx)}{a+b\sinh^2(c+dx)}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(sech(d*x+c)^5/(a+b*sinh(d*x+c)^2),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Limit: Max order reached or unable
to make series expansion Error: Bad Argument Value
```

Mupad [B] (verification not implemented)

Time = 12.96 (sec) , antiderivative size = 6237, normalized size of antiderivative = 45.20

$$\int \frac{\operatorname{sech}^5(c+dx)}{a+b\sinh^2(c+dx)}\,dx = \text{Too large to display}$$

input
```
int(1/(cosh(c + d*x)^5*(a + b*sinh(c + d*x)^2)),x)
```

output
```
(4*exp(c + d*x))/((a*d - b*d)*(4*exp(2*c + 2*d*x) + 6*exp(4*c + 4*d*x) + 4
*exp(6*c + 6*d*x) + exp(8*c + 8*d*x) + 1)) - (6*exp(c + d*x))/((a*d - b*d)
*(3*exp(2*c + 2*d*x) + 3*exp(4*c + 4*d*x) + exp(6*c + 6*d*x) + 1)) + (atan
((exp(d*x)*exp(c)*(243*a^12*(a^6*d^2 + b^6*d^2 - 6*a*b^5*d^2 - 6*a^5*b*d^2
 + 15*a^2*b^4*d^2 - 20*a^3*b^3*d^2 + 15*a^4*b^2*d^2)^(1/2) + 3840*b^12*(a^
6*d^2 + b^6*d^2 - 6*a*b^5*d^2 - 6*a^5*b*d^2 + 15*a^2*b^4*d^2 - 20*a^3*b^3*
d^2 + 15*a^4*b^2*d^2)^(1/2) - 110560*a*b^11*(a^6*d^2 + b^6*d^2 - 6*a*b^5*d
^2 - 6*a^5*b*d^2 + 15*a^2*b^4*d^2 - 20*a^3*b^3*d^2 + 15*a^4*b^2*d^2)^(1/2)
 - 4050*a^11*b*(a^6*d^2 + b^6*d^2 - 6*a*b^5*d^2 - 6*a^5*b*d^2 + 15*a^2*b^4
*d^2 - 20*a^3*b^3*d^2 + 15*a^4*b^2*d^2)^(1/2) + 976143*a^2*b^10*(a^6*d^2 +
 b^6*d^2 - 6*a*b^5*d^2 - 6*a^5*b*d^2 + 15*a^2*b^4*d^2 - 20*a^3*b^3*d^2 + 1
5*a^4*b^2*d^2)^(1/2) - 2740050*a^3*b^9*(a^6*d^2 + b^6*d^2 - 6*a*b^5*d^2 -
6*a^5*b*d^2 + 15*a^2*b^4*d^2 - 20*a^3*b^3*d^2 + 15*a^4*b^2*d^2)^(1/2) + 42
52775*a^4*b^8*(a^6*d^2 + b^6*d^2 - 6*a*b^5*d^2 - 6*a^5*b*d^2 + 15*a^2*b^4*
d^2 - 20*a^3*b^3*d^2 + 15*a^4*b^2*d^2)^(1/2) - 4316760*a^5*b^7*(a^6*d^2 +
b^6*d^2 - 6*a*b^5*d^2 - 6*a^5*b*d^2 + 15*a^2*b^4*d^2 - 20*a^3*b^3*d^2 + 15
*a^4*b^2*d^2)^(1/2) + 3087390*a^6*b^6*(a^6*d^2 + b^6*d^2 - 6*a*b^5*d^2 - 6
*a^5*b*d^2 + 15*a^2*b^4*d^2 - 20*a^3*b^3*d^2 + 15*a^4*b^2*d^2)^(1/2) - 160
8364*a^7*b^5*(a^6*d^2 + b^6*d^2 - 6*a*b^5*d^2 - 6*a^5*b*d^2 + 15*a^2*b^4*d
^2 - 20*a^3*b^3*d^2 + 15*a^4*b^2*d^2)^(1/2) + 615750*a^8*b^4*(a^6*d^2 +...
```

Reduce [B] (verification not implemented)

Time = 0.47 (sec) , antiderivative size = 2728, normalized size of antiderivative = 19.77

$$\int \frac{\operatorname{sech}^5(c+dx)}{a+b\sinh^2(c+dx)}\,dx = \text{Too large to display}$$

input
```
int(sech(d*x+c)^5/(a+b*sinh(d*x+c)^2),x)
```

output

```
(3*e**(8*c + 8*d*x)*atan(e**(c + d*x))*a**3 - 10*e**(8*c + 8*d*x)*atan(e**
(c + d*x))*a**2*b + 15*e**(8*c + 8*d*x)*atan(e**(c + d*x))*a*b**2 + 12*e**
(6*c + 6*d*x)*atan(e**(c + d*x))*a**3 - 40*e**(6*c + 6*d*x)*atan(e**(c + d
*x))*a**2*b + 60*e**(6*c + 6*d*x)*atan(e**(c + d*x))*a*b**2 + 18*e**(4*c +
 4*d*x)*atan(e**(c + d*x))*a**3 - 60*e**(4*c + 4*d*x)*atan(e**(c + d*x))*a
**2*b + 90*e**(4*c + 4*d*x)*atan(e**(c + d*x))*a*b**2 + 12*e**(2*c + 2*d*x
)*atan(e**(c + d*x))*a**3 - 40*e**(2*c + 2*d*x)*atan(e**(c + d*x))*a**2*b
+ 60*e**(2*c + 2*d*x)*atan(e**(c + d*x))*a*b**2 + 3*atan(e**(c + d*x))*a**
3 - 10*atan(e**(c + d*x))*a**2*b + 15*atan(e**(c + d*x))*a*b**2 + 4*e**(8*
c + 8*d*x)*sqrt(b)*sqrt(a)*sqrt(a - b)*sqrt(2*sqrt(a)*sqrt(a - b) + 2*a -
b)*atan((e**(c + d*x)*b)/(sqrt(b)*sqrt(2*sqrt(a)*sqrt(a - b) + 2*a - b)))*
b + 16*e**(6*c + 6*d*x)*sqrt(b)*sqrt(a)*sqrt(a - b)*sqrt(2*sqrt(a)*sqrt(a
- b) + 2*a - b)*atan((e**(c + d*x)*b)/(sqrt(b)*sqrt(2*sqrt(a)*sqrt(a - b)
+ 2*a - b)))*b + 24*e**(4*c + 4*d*x)*sqrt(b)*sqrt(a)*sqrt(a - b)*sqrt(2*sq
rt(a)*sqrt(a - b) + 2*a - b)*atan((e**(c + d*x)*b)/(sqrt(b)*sqrt(2*sqrt(a)
*sqrt(a - b) + 2*a - b)))*b + 16*e**(2*c + 2*d*x)*sqrt(b)*sqrt(a)*sqrt(a -
 b)*sqrt(2*sqrt(a)*sqrt(a - b) + 2*a - b)*atan((e**(c + d*x)*b)/(sqrt(b)*s
qrt(2*sqrt(a)*sqrt(a - b) + 2*a - b)))*b + 4*sqrt(b)*sqrt(a)*sqrt(a - b)*s
qrt(2*sqrt(a)*sqrt(a - b) + 2*a - b)*atan((e**(c + d*x)*b)/(sqrt(b)*sqrt(2
*sqrt(a)*sqrt(a - b) + 2*a - b)))*b - 4*e**(8*c + 8*d*x)*sqrt(b)*sqrt(2...
```

3.292 $\int \frac{\text{sech}^6(c+dx)}{a+b\sinh^2(c+dx)}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 126

$$\int \frac{\text{sech}^6(c+dx)}{a+b\sinh^2(c+dx)}\,dx = -\frac{b^3\text{arctanh}\left(\frac{\sqrt{a-b}\tanh(c+dx)}{\sqrt{a}}\right)}{\sqrt{a}(a-b)^{7/2}d} + \frac{(a^2-3ab+3b^2)\tanh(c+dx)}{(a-b)^3d} - \frac{(2a-3b)\tanh^3(c+dx)}{3(a-b)^2d} + \frac{\tanh^5(c+dx)}{5(a-b)d}$$

output
```
-b^3*arctanh((a-b)^(1/2)*tanh(d*x+c)/a^(1/2))/a^(1/2)/(a-b)^(7/2)/d+(a^2-3
*a*b+3*b^2)*tanh(d*x+c)/(a-b)^3/d-1/3*(2*a-3*b)*tanh(d*x+c)^3/(a-b)^2/d+1/
5*tanh(d*x+c)^5/(a-b)/d
```

Mathematica [A] (verified)

Time = 0.76 (sec) , antiderivative size = 119, normalized size of antiderivative = 0.94

$$\int \frac{\operatorname{sech}^6(c+dx)}{a+b\sinh^2(c+dx)}\,dx = \frac{-\frac{15b^3\operatorname{arctanh}\left(\frac{\sqrt{a-b}\tanh(c+dx)}{\sqrt{a}}\right)}{\sqrt{a}(a-b)^{7/2}}+\frac{\left(8a^2-26ab+33b^2+(4a^2-13ab+9b^2)\operatorname{sech}^2(c+dx)+3(a-b)^2\operatorname{sech}^4(c+dx)\right)\tanh(c+dx)}{(a-b)^3}}{15d}$$

input
```
Integrate[Sech[c + d*x]^6/(a + b*Sinh[c + d*x]^2),x]
```

output
```
((-15*b^3*ArcTanh[(Sqrt[a - b]*Tanh[c + d*x])/Sqrt[a]])/(Sqrt[a]*(a - b)^(
7/2)) + ((8*a^2 - 26*a*b + 33*b^2 + (4*a^2 - 13*a*b + 9*b^2)*Sech[c + d*x]
^2 + 3*(a - b)^2*Sech[c + d*x]^4)*Tanh[c + d*x])/(a - b)^3)/(15*d)
```

Rubi [A] (verified)

Time = 0.37 (sec) , antiderivative size = 118, normalized size of antiderivative = 0.94, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.174, Rules used = {3042, 3670, 300, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\operatorname{sech}^6(c+dx)}{a+b\sinh^2(c+dx)}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{\cos(ic+idx)^6\left(a-b\sin(ic+idx)^2\right)}dx$$

$$\Big\downarrow \text{3670}$$

$$\frac{\int \frac{\left(1-\tanh^2(c+dx)\right)^3}{a-(a-b)\tanh^2(c+dx)}d\tanh(c+dx)}{d}$$

$$\Big\downarrow \text{300}$$

$$\frac{\int \left(\frac{\tanh^4(c+dx)}{a-b} - \frac{(2a-3b)\tanh^2(c+dx)}{(a-b)^2} + \frac{a^2-3ba+3b^2}{(a-b)^3} - \frac{b^3}{(a-b)^3(a-(a-b)\tanh^2(c+dx))}\right) d\tanh(c+dx)}{d}$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{\frac{(a^2-3ab+3b^2)\tanh(c+dx)}{(a-b)^3} - \frac{b^3\operatorname{arctanh}\left(\frac{\sqrt{a-b}\tanh(c+dx)}{\sqrt{a}}\right)}{\sqrt{a}(a-b)^{7/2}} + \frac{\tanh^5(c+dx)}{5(a-b)} - \frac{(2a-3b)\tanh^3(c+dx)}{3(a-b)^2}}{d}$$

input

```
Int[Sech[c + d*x]^6/(a + b*Sinh[c + d*x]^2),x]
```

output

```
(-((b^3*ArcTanh[(Sqrt[a - b]*Tanh[c + d*x])/Sqrt[a]])/(Sqrt[a]*(a - b)^(7/
2))) + ((a^2 - 3*a*b + 3*b^2)*Tanh[c + d*x])/(a - b)^3 - ((2*a - 3*b)*Tanh
[c + d*x]^3)/(3*(a - b)^2) + Tanh[c + d*x]^5/(5*(a - b)))/d
```

Defintions of rubi rules used

rule 300

```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_), x_Symbol] :> Int
[PolynomialDivide[(a + b*x^2)^p, (c + d*x^2)^(-q), x], x] /; FreeQ[{a, b, c
, d}, x] && NeQ[b*c - a*d, 0] && IGtQ[p, 0] && ILtQ[q, 0] && GeQ[p, -q]
```

rule 2009

```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042

```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3670

```
Int[cos[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(
p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Simp[ff/f   Su
bst[Int[(a + (a + b)*ff^2*x^2)^p/(1 + ff^2*x^2)^(m/2 + p + 1), x], x, Tan[e
 + f*x]/ff], x]] /; FreeQ[{a, b, e, f}, x] && IntegerQ[m/2] && IntegerQ[p]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 345 vs. $2(114) = 228$.

Time = 0.24 (sec) , antiderivative size = 346, normalized size of antiderivative = 2.75

$$\frac{2b^3a\left(-\frac{\left(\sqrt{-b(a-b)}-b\right)\operatorname{arctanh}\left(\frac{\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)a}{\sqrt{\left(2\sqrt{-b(a-b)}+a-2b\right)a}}\right)}{2a\sqrt{-b(a-b)}\sqrt{\left(2\sqrt{-b(a-b)}+a-2b\right)a}}+\frac{\left(\sqrt{-b(a-b)}+b\right)\arctan\left(\frac{\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)a}{\sqrt{\left(2\sqrt{-b(a-b)}-a+2b\right)a}}\right)}{2a\sqrt{-b(a-b)}\sqrt{\left(2\sqrt{-b(a-b)}-a+2b\right)a}}\right)}{(a-b)^3}-\frac{2\left(\left(-a^2+3ab-3b^2\right)\tanh\left(\frac{dx}{2}+\frac{c}{2}\right.\right.}{}$$

input
```
int(sech(d*x+c)^6/(a+b*sinh(d*x+c)^2),x)
```

output
```
1/d*(2*b^3/(a-b)^3*a*(-1/2*((-b*(a-b))^(1/2)-b)/a/(-b*(a-b))^(1/2)/((2*(-b
*(a-b))^(1/2)+a-2*b)*a)^(1/2)*arctanh(tanh(1/2*d*x+1/2*c)*a/((2*(-b*(a-b))
^(1/2)+a-2*b)*a)^(1/2))+1/2*((-b*(a-b))^(1/2)+b)/a/(-b*(a-b))^(1/2)/((2*(-
b*(a-b))^(1/2)-a+2*b)*a)^(1/2)*arctan(tanh(1/2*d*x+1/2*c)*a/((2*(-b*(a-b))
^(1/2)-a+2*b)*a)^(1/2)))-2/(a-b)^3*((-a^2+3*a*b-3*b^2)*tanh(1/2*d*x+1/2*c)
^9+(-4/3*a^2+16/3*a*b-8*b^2)*tanh(1/2*d*x+1/2*c)^7+(-58/15*a^2+166/15*a*b-
66/5*b^2)*tanh(1/2*d*x+1/2*c)^5+(-4/3*a^2+16/3*a*b-8*b^2)*tanh(1/2*d*x+1/2
*c)^3+(-a^2+3*a*b-3*b^2)*tanh(1/2*d*x+1/2*c))/(tanh(1/2*d*x+1/2*c)^2+1)^5)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2895 vs. $2(114) = 228$.

Time = 0.16 (sec) , antiderivative size = 6046, normalized size of antiderivative = 47.98

$$\int \frac{\operatorname{sech}^6(c+dx)}{a+b\sinh^2(c+dx)}\,dx = \text{Too large to display}$$

input
```
integrate(sech(d*x+c)^6/(a+b*sinh(d*x+c)^2),x, algorithm="fricas")
```

output
```
Too large to include
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\operatorname{sech}^6(c+dx)}{a+b\sinh^2(c+dx)}\,dx = \text{Timed out}$$

input
```
integrate(sech(d*x+c)**6/(a+b*sinh(d*x+c)**2),x)
```

output
```
Timed out
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{\operatorname{sech}^6(c+dx)}{a+b\sinh^2(c+dx)}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate(sech(d*x+c)^6/(a+b*sinh(d*x+c)^2),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(b-a>0)', see `assume?` for more
details)Is
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 253 vs. $2(114) = 228$.

Time = 0.23 (sec) , antiderivative size = 253, normalized size of antiderivative = 2.01

$$\int \frac{\operatorname{sech}^6(c+dx)}{a+b\sinh^2(c+dx)}\,dx = -\frac{\frac{15\,b^3\arctan\left(\frac{be^{(2\,dx+2\,c)}+2\,a-b}{2\,\sqrt{-a^2+ab}}\right)}{(a^3-3\,a^2b+3\,ab^2-b^3)\sqrt{-a^2+ab}} + \frac{2\,(15\,b^2e^{(8\,dx+8\,c)}-30\,abe^{(6\,dx+6\,c)}+90\,b^2e^{(6\,dx+6\,c)}+80\,a^2e^{(4\,dx+4\,c)}-230\,abe^{(4\,dx+4\,c)}+240\,b^2e^{(4\,dx}}{(a^3-3\,a^2b+3\,ab^2-b^3)\,(e^{(2\,dx+}}}{15\,d}$$

input
```
integrate(sech(d*x+c)^6/(a+b*sinh(d*x+c)^2),x, algorithm="giac")
```

output
```
-1/15*(15*b^3*arctan(1/2*(b*e^(2*d*x + 2*c) + 2*a - b)/sqrt(-a^2 + a*b))/(
(a^3 - 3*a^2*b + 3*a*b^2 - b^3)*sqrt(-a^2 + a*b)) + 2*(15*b^2*e^(8*d*x + 8
*c) - 30*a*b*e^(6*d*x + 6*c) + 90*b^2*e^(6*d*x + 6*c) + 80*a^2*e^(4*d*x +
4*c) - 230*a*b*e^(4*d*x + 4*c) + 240*b^2*e^(4*d*x + 4*c) + 40*a^2*e^(2*d*x
 + 2*c) - 130*a*b*e^(2*d*x + 2*c) + 150*b^2*e^(2*d*x + 2*c) + 8*a^2 - 26*a
*b + 33*b^2)/((a^3 - 3*a^2*b + 3*a*b^2 - b^3)*(e^(2*d*x + 2*c) + 1)^5))/d
```

Mupad [B] (verification not implemented)

Time = 3.75 (sec) , antiderivative size = 1152, normalized size of antiderivative = 9.14

$$\int \frac{\operatorname{sech}^6(c+dx)}{a+b\sinh^2(c+dx)}\,dx = \text{Too large to display}$$

input
```
int(1/(cosh(c + d*x)^6*(a + b*sinh(c + d*x)^2)),x)
```

output
```
16/((a*d - b*d)*(4*exp(2*c + 2*d*x) + 6*exp(4*c + 4*d*x) + 4*exp(6*c + 6*d
*x) + exp(8*c + 8*d*x) + 1)) - 32/(5*(a*d - b*d)*(5*exp(2*c + 2*d*x) + 10*
exp(4*c + 4*d*x) + 10*exp(6*c + 6*d*x) + 5*exp(8*c + 8*d*x) + exp(10*c + 1
0*d*x) + 1)) + (atan((exp(2*c)*exp(2*d*x)*((4*b)/(d*(a - b)^3*(b^6)^(1/2)*
(3*a*b^2 - 3*a^2*b + a^3 - b^3)) + ((2*a - b)*(2*a^4*d*(b^6)^(1/2) + b^4*d
*(b^6)^(1/2) - 5*a*b^3*d*(b^6)^(1/2) - 7*a^3*b*d*(b^6)^(1/2) + 9*a^2*b^2*d
*(b^6)^(1/2)))/(b^5*(-a*d^2*(a - b)^7)^(1/2)*(3*a*b^2 - 3*a^2*b + a^3 - b^
3)*(a*b^7*d^2 - a^8*d^2 + 7*a^7*b*d^2 - 7*a^2*b^6*d^2 + 21*a^3*b^5*d^2 - 3
5*a^4*b^4*d^2 + 35*a^5*b^3*d^2 - 21*a^6*b^2*d^2)^(1/2))) - ((2*a - b)*(b^4
*d*(b^6)^(1/2) - 3*a*b^3*d*(b^6)^(1/2) - a^3*b*d*(b^6)^(1/2) + 3*a^2*b^2*d
*(b^6)^(1/2)))/(b^5*(-a*d^2*(a - b)^7)^(1/2)*(3*a*b^2 - 3*a^2*b + a^3 - b^
3)*(a*b^7*d^2 - a^8*d^2 + 7*a^7*b*d^2 - 7*a^2*b^6*d^2 + 21*a^3*b^5*d^2 - 3
5*a^4*b^4*d^2 + 35*a^5*b^3*d^2 - 21*a^6*b^2*d^2)^(1/2)))*((b^4*(a*b^7*d^2
- a^8*d^2 + 7*a^7*b*d^2 - 7*a^2*b^6*d^2 + 21*a^3*b^5*d^2 - 35*a^4*b^4*d^2
+ 35*a^5*b^3*d^2 - 21*a^6*b^2*d^2)^(1/2))/2 + (3*a^2*b^2*(a*b^7*d^2 - a^8*
d^2 + 7*a^7*b*d^2 - 7*a^2*b^6*d^2 + 21*a^3*b^5*d^2 - 35*a^4*b^4*d^2 + 35*a
^5*b^3*d^2 - 21*a^6*b^2*d^2)^(1/2))/2 - (3*a*b^3*(a*b^7*d^2 - a^8*d^2 + 7*
a^7*b*d^2 - 7*a^2*b^6*d^2 + 21*a^3*b^5*d^2 - 35*a^4*b^4*d^2 + 35*a^5*b^3*d
^2 - 21*a^6*b^2*d^2)^(1/2))/2 - (a^3*b*(a*b^7*d^2 - a^8*d^2 + 7*a^7*b*d^2
- 7*a^2*b^6*d^2 + 21*a^3*b^5*d^2 - 35*a^4*b^4*d^2 + 35*a^5*b^3*d^2 - 21...
```

Reduce [B] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 1587, normalized size of antiderivative = 12.60

$$\int \frac{\operatorname{sech}^6(c+dx)}{a+b\sinh^2(c+dx)}\,dx = \text{Too large to display}$$

input
```
int(sech(d*x+c)^6/(a+b*sinh(d*x+c)^2),x)
```

output
```
( - 15*e**(10*c + 10*d*x)*sqrt(a)*sqrt(a - b)*log( - sqrt(2*sqrt(a)*sqrt(a
 - b) - 2*a + b) + e**(c + d*x)*sqrt(b))*b**3 - 15*e**(10*c + 10*d*x)*sqrt
(a)*sqrt(a - b)*log(sqrt(2*sqrt(a)*sqrt(a - b) - 2*a + b) + e**(c + d*x)*s
qrt(b))*b**3 + 15*e**(10*c + 10*d*x)*sqrt(a)*sqrt(a - b)*log(2*sqrt(a)*sqr
t(a - b) + e**(2*c + 2*d*x)*b + 2*a - b)*b**3 - 75*e**(8*c + 8*d*x)*sqrt(a
)*sqrt(a - b)*log( - sqrt(2*sqrt(a)*sqrt(a - b) - 2*a + b) + e**(c + d*x)*
sqrt(b))*b**3 - 75*e**(8*c + 8*d*x)*sqrt(a)*sqrt(a - b)*log(sqrt(2*sqrt(a)
*sqrt(a - b) - 2*a + b) + e**(c + d*x)*sqrt(b))*b**3 + 75*e**(8*c + 8*d*x)
*sqrt(a)*sqrt(a - b)*log(2*sqrt(a)*sqrt(a - b) + e**(2*c + 2*d*x)*b + 2*a
- b)*b**3 - 150*e**(6*c + 6*d*x)*sqrt(a)*sqrt(a - b)*log( - sqrt(2*sqrt(a)
*sqrt(a - b) - 2*a + b) + e**(c + d*x)*sqrt(b))*b**3 - 150*e**(6*c + 6*d*x
)*sqrt(a)*sqrt(a - b)*log(sqrt(2*sqrt(a)*sqrt(a - b) - 2*a + b) + e**(c +
d*x)*sqrt(b))*b**3 + 150*e**(6*c + 6*d*x)*sqrt(a)*sqrt(a - b)*log(2*sqrt(a
)*sqrt(a - b) + e**(2*c + 2*d*x)*b + 2*a - b)*b**3 - 150*e**(4*c + 4*d*x)*
sqrt(a)*sqrt(a - b)*log( - sqrt(2*sqrt(a)*sqrt(a - b) - 2*a + b) + e**(c +
 d*x)*sqrt(b))*b**3 - 150*e**(4*c + 4*d*x)*sqrt(a)*sqrt(a - b)*log(sqrt(2*
sqrt(a)*sqrt(a - b) - 2*a + b) + e**(c + d*x)*sqrt(b))*b**3 + 150*e**(4*c
+ 4*d*x)*sqrt(a)*sqrt(a - b)*log(2*sqrt(a)*sqrt(a - b) + e**(2*c + 2*d*x)*
b + 2*a - b)*b**3 - 75*e**(2*c + 2*d*x)*sqrt(a)*sqrt(a - b)*log( - sqrt(2*
sqrt(a)*sqrt(a - b) - 2*a + b) + e**(c + d*x)*sqrt(b))*b**3 - 75*e**(2*...
```

3.293 $\int \frac{\cosh^6(c+dx)}{(a+b\sinh^2(c+dx))^2}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 158

$$\int \frac{\cosh^6(c+dx)}{(a+b\sinh^2(c+dx))^2}\,dx = -\frac{(4a-5b)x}{2b^3} + \frac{(a-b)^{3/2}(4a+b)\operatorname{arctanh}\left(\frac{\sqrt{a-b}\tanh(c+dx)}{\sqrt{a}}\right)}{2a^{3/2}b^3d} + \frac{\cosh(c+dx)\sinh(c+dx)}{2bd\,(a-(a-b)\tanh^2(c+dx))} + \frac{(a-b)(2a-b)\tanh(c+dx)}{2ab^2d\,(a-(a-b)\tanh^2(c+dx))}$$

output
```
-1/2*(4*a-5*b)*x/b^3+1/2*(a-b)^(3/2)*(4*a+b)*arctanh((a-b)^(1/2)*tanh(d*x+
c)/a^(1/2))/a^(3/2)/b^3/d+1/2*cosh(d*x+c)*sinh(d*x+c)/b/d/(a-(a-b)*tanh(d*
x+c)^2)+1/2*(a-b)*(2*a-b)*tanh(d*x+c)/a/b^2/d/(a-(a-b)*tanh(d*x+c)^2)
```

Mathematica [A] (verified)

Time = 0.69 (sec) , antiderivative size = 118, normalized size of antiderivative = 0.75

$$\int \frac{\cosh^6(c+dx)}{\left(a+b\sinh^2(c+dx)\right)^2}\,dx = \frac{2(-4a+5b)(c+dx)+\frac{2(a-b)^{3/2}(4a+b)\text{arctanh}\left(\frac{\sqrt{a-b}\tanh(c+dx)}{\sqrt{a}}\right)}{a^{3/2}}+b\sinh(2(c+dx))+\frac{2(a-b)^2b\sinh(2(c+dx))}{a(2a-b+b\cosh(2(c+dx)))}}{4b^3d}$$

input
```
Integrate[Cosh[c + d*x]^6/(a + b*Sinh[c + d*x]^2)^2,x]
```

output
```
(2*(-4*a + 5*b)*(c + d*x) + (2*(a - b)^(3/2)*(4*a + b)*ArcTanh[(Sqrt[a - b
]*Tanh[c + d*x])/Sqrt[a]])/a^(3/2) + b*Sinh[2*(c + d*x)] + (2*(a - b)^2*b*
Sinh[2*(c + d*x)])/(a*(2*a - b + b*Cosh[2*(c + d*x)])))/(4*b^3*d)
```

Rubi [A] (verified)

Time = 0.48 (sec) , antiderivative size = 177, normalized size of antiderivative = 1.12, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.391$, Rules used = {3042, 3670, 316, 25, 402, 27, 397, 219, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cosh^6(c+dx)}{\left(a+b\sinh^2(c+dx)\right)^2}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\cos(ic+idx)^6}{\left(a-b\sin(ic+idx)^2\right)^2}dx$$

$$\Big\downarrow \text{3670}$$

$$\frac{\int \frac{1}{\left(1-\tanh^2(c+dx)\right)^2\left(a-(a-b)\tanh^2(c+dx)\right)^2}d\tanh(c+dx)}{d}$$

$$\Big\downarrow \text{316}$$

$$\frac{\frac{\int -\frac{3(a-b)\tanh^2(c+dx)+a-2b}{\left(1-\tanh^2(c+dx)\right)\left(a-(a-b)\tanh^2(c+dx)\right)^2}d\tanh(c+dx)}{2b}+\frac{\tanh(c+dx)}{2b\left(1-\tanh^2(c+dx)\right)\left(a-(a-b)\tanh^2(c+dx)\right)}}{d}$$

$$\Big\downarrow \; 25$$

$$\frac{\frac{\tanh(c+dx)}{2b\left(1-\tanh^2(c+dx)\right)\left(a-(a-b)\tanh^2(c+dx)\right)}-\frac{\int \frac{3(a-b)\tanh^2(c+dx)+a-2b}{\left(1-\tanh^2(c+dx)\right)\left(a-(a-b)\tanh^2(c+dx)\right)^2}d\tanh(c+dx)}{2b}}{d}$$

$$\Big\downarrow \; 402$$

$$\frac{\frac{\tanh(c+dx)}{2b\left(1-\tanh^2(c+dx)\right)\left(a-(a-b)\tanh^2(c+dx)\right)}-\frac{-\frac{\int -\frac{2\left(2a^2-2ba-b^2+(a-b)(2a-b)\tanh^2(c+dx)\right)}{\left(1-\tanh^2(c+dx)\right)\left(a-(a-b)\tanh^2(c+dx)\right)}d\tanh(c+dx)}{2ab}-\frac{(a-b)(2a-b)\tanh(c+dx)}{ab\left(a-(a-b)\tanh^2(c+dx)\right)}}{2b}}{d}$$

$$\Big\downarrow \; 27$$

$$\frac{\frac{\tanh(c+dx)}{2b\left(1-\tanh^2(c+dx)\right)\left(a-(a-b)\tanh^2(c+dx)\right)}-\frac{\frac{\int \frac{2a^2-2ba-b^2+(a-b)(2a-b)\tanh^2(c+dx)}{\left(1-\tanh^2(c+dx)\right)\left(a-(a-b)\tanh^2(c+dx)\right)}d\tanh(c+dx)}{ab}-\frac{(a-b)(2a-b)\tanh(c+dx)}{ab\left(a-(a-b)\tanh^2(c+dx)\right)}}{2b}}{d}$$

$$\Big\downarrow \; 397$$

$$\frac{\frac{\tanh(c+dx)}{2b\left(1-\tanh^2(c+dx)\right)\left(a-(a-b)\tanh^2(c+dx)\right)}-\frac{\frac{\frac{a(4a-5b)\int \frac{1}{1-\tanh^2(c+dx)}d\tanh(c+dx)}{b}-\frac{(a-b)^2(4a+b)\int \frac{1}{a-(a-b)\tanh^2(c+dx)}d\tanh(c+dx)}{b}}{ab}-\frac{(a-\cdots}{ab(a-\cdots}}{2b}}{d}$$

$$\Big\downarrow \; 219$$

$$\frac{\frac{\tanh(c+dx)}{2b\left(1-\tanh^2(c+dx)\right)\left(a-(a-b)\tanh^2(c+dx)\right)}-\frac{\frac{\frac{a(4a-5b)\text{arctanh}(\tanh(c+dx))}{b}-\frac{(a-b)^2(4a+b)\int \frac{1}{a-(a-b)\tanh^2(c+dx)}d\tanh(c+dx)}{b}}{ab}-\frac{(a-b)(2a-\cdots}{ab\left(a-(a-b)\cdots\right.}}{2b}}{d}$$

$$\Big\downarrow \; 221$$

$$\frac{\frac{\tanh(c+dx)}{2b\left(1-\tanh^2(c+dx)\right)\left(a-(a-b)\tanh^2(c+dx)\right)}-\frac{\frac{\frac{a(4a-5b)\text{arctanh}(\tanh(c+dx))}{b}-\frac{(a-b)^{3/2}(4a+b)\text{arctanh}\left(\frac{\sqrt{a-b}\tanh(c+dx)}{\sqrt{a}}\right)}{\sqrt{a}b}}{ab}-\frac{(a-b)(2a-b)\,\text{ta}\cdots}{ab\left(a-(a-b)\,\text{tan}\cdots\right.}}{2b}}{d}$$

input
```
Int[Cosh[c + d*x]^6/(a + b*Sinh[c + d*x]^2)^2,x]
```

output
```
(Tanh[c + d*x]/(2*b*(1 - Tanh[c + d*x]^2)*(a - (a - b)*Tanh[c + d*x]^2)) -
 (((a*(4*a - 5*b)*ArcTanh[Tanh[c + d*x]])/b - ((a - b)^(3/2)*(4*a + b)*Arc
Tanh[(Sqrt[a - b]*Tanh[c + d*x])/Sqrt[a]])/(Sqrt[a]*b))/(a*b) - ((a - b)*(
2*a - b)*Tanh[c + d*x])/(a*b*(a - (a - b)*Tanh[c + d*x]^2)))/(2*b))/d
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 316
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_), x_Symbol] :> Sim
p[(-b)*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^(q + 1)/(2*a*(p + 1)*(b*c - a*d))
), x] + Simp[1/(2*a*(p + 1)*(b*c - a*d))   Int[(a + b*x^2)^(p + 1)*(c + d*x
^2)^q*Simp[b*c + 2*(p + 1)*(b*c - a*d) + d*b*(2*(p + q + 2) + 1)*x^2, x], x
], x] /; FreeQ[{a, b, c, d, q}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] &&  !
( !IntegerQ[p] && IntegerQ[q] && LtQ[q, -1]) && IntBinomialQ[a, b, c, d, 2,
 p, q, x]
```

rule 397
```
Int[((e_) + (f_.)*(x_)^2)/(((a_) + (b_.)*(x_)^2)*((c_) + (d_.)*(x_)^2)), x_
Symbol] :> Simp[(b*e - a*f)/(b*c - a*d)   Int[1/(a + b*x^2), x], x] - Simp[
(d*e - c*f)/(b*c - a*d)   Int[1/(c + d*x^2), x], x] /; FreeQ[{a, b, c, d, e
, f}, x]
```

rule 402
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_.)*((e_) + (f_.)*(x
_)^2), x_Symbol] :> Simp[(-(b*e - a*f))*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^
(q + 1)/(a*2*(b*c - a*d)*(p + 1))), x] + Simp[1/(a*2*(b*c - a*d)*(p + 1))
 Int[(a + b*x^2)^(p + 1)*(c + d*x^2)^q*Simp[c*(b*e - a*f) + e*2*(b*c - a*d)
*(p + 1) + d*(b*e - a*f)*(2*(p + q + 2) + 1)*x^2, x], x], x] /; FreeQ[{a, b
, c, d, e, f, q}, x] && LtQ[p, -1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3670
```
Int[cos[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(
p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Simp[ff/f   Su
bst[Int[(a + (a + b)*ff^2*x^2)^p/(1 + ff^2*x^2)^(m/2 + p + 1), x], x, Tan[e
 + f*x]/ff], x]] /; FreeQ[{a, b, e, f}, x] && IntegerQ[m/2] && IntegerQ[p]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 424 vs. $2(142) = 284$.

Time = 0.24 (sec) , antiderivative size = 425, normalized size of antiderivative = 2.69

$$\frac{\frac{1}{2b^2\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)^2}+\frac{1}{2b^2\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}+\frac{(4a-5b)\ln\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}{2b^3}-\frac{2\left(\frac{-\frac{b\left(a^2-2ab+b^2\right)\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^3}{2a}-\frac{b\left(a^2-2ab+b^2\right)\tanh}{2a}}{\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^4a-2\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^2a+4b\tanh\left(\frac{dx}{2}+\frac{c}{2}\right.}\right.}{}}{}$$

input
```
int(cosh(d*x+c)^6/(a+b*sinh(d*x+c)^2)^2,x)
```

output
```
1/d*(1/2/b^2/(tanh(1/2*d*x+1/2*c)-1)^2+1/2/b^2/(tanh(1/2*d*x+1/2*c)-1)+1/2
*(4*a-5*b)/b^3*ln(tanh(1/2*d*x+1/2*c)-1)-2/b^3*((-1/2*b*(a^2-2*a*b+b^2)/a*
tanh(1/2*d*x+1/2*c)^3-1/2*b*(a^2-2*a*b+b^2)/a*tanh(1/2*d*x+1/2*c))/(tanh(1
/2*d*x+1/2*c)^4*a-2*tanh(1/2*d*x+1/2*c)^2*a+4*b*tanh(1/2*d*x+1/2*c)^2+a)+1
/2*(4*a^3-7*a^2*b+2*a*b^2+b^3)*(-1/2*((-b*(a-b))^(1/2)-b)/a/(-b*(a-b))^(1/
2)/((2*(-b*(a-b))^(1/2)+a-2*b)*a)^(1/2)*arctanh(tanh(1/2*d*x+1/2*c)*a/((2*
(-b*(a-b))^(1/2)+a-2*b)*a)^(1/2))+1/2*((-b*(a-b))^(1/2)+b)/a/(-b*(a-b))^(1
/2)/((2*(-b*(a-b))^(1/2)-a+2*b)*a)^(1/2)*arctan(tanh(1/2*d*x+1/2*c)*a/((2*
(-b*(a-b))^(1/2)-a+2*b)*a)^(1/2))))-1/2/b^2/(tanh(1/2*d*x+1/2*c)+1)^2+1/2/
b^2/(tanh(1/2*d*x+1/2*c)+1)+1/2/b^3*(-4*a+5*b)*ln(tanh(1/2*d*x+1/2*c)+1))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1682 vs. $2(144) = 288$.

Time = 0.14 (sec) , antiderivative size = 3629, normalized size of antiderivative = 22.97

$$\int \frac{\cosh^6(c+dx)}{\left(a+b\sinh^2(c+dx)\right)^2}\,dx = \text{Too large to display}$$

input
```
integrate(cosh(d*x+c)^6/(a+b*sinh(d*x+c)^2)^2,x, algorithm="fricas")
```

output
```
Too large to include
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\cosh^6(c+dx)}{\left(a+b\sinh^2(c+dx)\right)^2}\,dx = \text{Timed out}$$

input
```
integrate(cosh(d*x+c)**6/(a+b*sinh(d*x+c)**2)**2,x)
```

output
```
Timed out
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{\cosh^6(c+dx)}{\left(a+b\sinh^2(c+dx)\right)^2}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate(cosh(d*x+c)^6/(a+b*sinh(d*x+c)^2)^2,x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(b-a>0)', see `assume?` for more
details)Is
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 305 vs. $2(144) = 288$.

Time = 1.23 (sec) , antiderivative size = 305, normalized size of antiderivative = 1.93

$$\int \frac{\cosh^6(c+dx)}{\left(a+b\sinh^2(c+dx)\right)^2}\,dx =$$

$$-\frac{\frac{12\,(dx+c)(4\,a-5\,b)}{b^3}-\frac{3\,e^{(2\,dx+2\,c)}}{b^2}-\frac{12\,\left(4\,a^3-7\,a^2b+2\,ab^2+b^3\right)\arctan\left(\frac{be^{(2\,dx+2\,c)}+2\,a-b}{2\,\sqrt{-a^2+ab}}\right)}{\sqrt{-a^2+ab}ab^3}-\frac{8\,a^2be^{(6\,dx+6\,c)}-10\,ab^2e^{(6\,dx+6\,c)}-16\,c}{}}{24\,d}$$

input
```
integrate(cosh(d*x+c)^6/(a+b*sinh(d*x+c)^2)^2,x, algorithm="giac")
```

output
```
-1/24*(12*(d*x + c)*(4*a - 5*b)/b^3 - 3*e^(2*d*x + 2*c)/b^2 - 12*(4*a^3 -
7*a^2*b + 2*a*b^2 + b^3)*arctan(1/2*(b*e^(2*d*x + 2*c) + 2*a - b)/sqrt(-a^
2 + a*b))/(sqrt(-a^2 + a*b)*a*b^3) - (8*a^2*b*e^(6*d*x + 6*c) - 10*a*b^2*e
^(6*d*x + 6*c) - 16*a^3*e^(4*d*x + 4*c) + 64*a^2*b*e^(4*d*x + 4*c) - 79*a*
b^2*e^(4*d*x + 4*c) + 24*b^3*e^(4*d*x + 4*c) - 28*a^2*b*e^(2*d*x + 2*c) +
44*a*b^2*e^(2*d*x + 2*c) - 24*b^3*e^(2*d*x + 2*c) - 3*a*b^2)/((b*e^(6*d*x
+ 6*c) + 4*a*e^(4*d*x + 4*c) - 2*b*e^(4*d*x + 4*c) + b*e^(2*d*x + 2*c))*a*
b^3))/d
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\cosh^6(c+dx)}{\left(a+b\sinh^2(c+dx)\right)^2}\,dx = \int \frac{\cosh(c+d\,x)^6}{\left(b\sinh\left(c+d\,x\right)^2+a\right)^2}\,dx$$

input
```
int(cosh(c + d*x)^6/(a + b*sinh(c + d*x)^2)^2,x)
```

output
```
int(cosh(c + d*x)^6/(a + b*sinh(c + d*x)^2)^2, x)
```

Reduce [B] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 1910, normalized size of antiderivative = 12.09

$$\int \frac{\cosh^6(c+dx)}{\left(a+b\sinh^2(c+dx)\right)^2}\,dx = \text{Too large to display}$$

input
```
int(cosh(d*x+c)^6/(a+b*sinh(d*x+c)^2)^2,x)
```

output

```
(8*e**(6*c + 6*d*x)*sqrt(a)*sqrt(a - b)*log( - sqrt(2*sqrt(a)*sqrt(a - b)
- 2*a + b) + e**(c + d*x)*sqrt(b))*a**2*b - 6*e**(6*c + 6*d*x)*sqrt(a)*sqr
t(a - b)*log( - sqrt(2*sqrt(a)*sqrt(a - b) - 2*a + b) + e**(c + d*x)*sqrt(
b))*a*b**2 - 2*e**(6*c + 6*d*x)*sqrt(a)*sqrt(a - b)*log( - sqrt(2*sqrt(a)*
sqrt(a - b) - 2*a + b) + e**(c + d*x)*sqrt(b))*b**3 + 8*e**(6*c + 6*d*x)*s
qrt(a)*sqrt(a - b)*log(sqrt(2*sqrt(a)*sqrt(a - b) - 2*a + b) + e**(c + d*x
)*sqrt(b))*a**2*b - 6*e**(6*c + 6*d*x)*sqrt(a)*sqrt(a - b)*log(sqrt(2*sqrt
(a)*sqrt(a - b) - 2*a + b) + e**(c + d*x)*sqrt(b))*a*b**2 - 2*e**(6*c + 6*
d*x)*sqrt(a)*sqrt(a - b)*log(sqrt(2*sqrt(a)*sqrt(a - b) - 2*a + b) + e**(c
 + d*x)*sqrt(b))*b**3 - 8*e**(6*c + 6*d*x)*sqrt(a)*sqrt(a - b)*log(2*sqrt(
a)*sqrt(a - b) + e**(2*c + 2*d*x)*b + 2*a - b)*a**2*b + 6*e**(6*c + 6*d*x)
*sqrt(a)*sqrt(a - b)*log(2*sqrt(a)*sqrt(a - b) + e**(2*c + 2*d*x)*b + 2*a
- b)*a*b**2 + 2*e**(6*c + 6*d*x)*sqrt(a)*sqrt(a - b)*log(2*sqrt(a)*sqrt(a
- b) + e**(2*c + 2*d*x)*b + 2*a - b)*b**3 + 32*e**(4*c + 4*d*x)*sqrt(a)*sq
rt(a - b)*log( - sqrt(2*sqrt(a)*sqrt(a - b) - 2*a + b) + e**(c + d*x)*sqrt
(b))*a**3 - 40*e**(4*c + 4*d*x)*sqrt(a)*sqrt(a - b)*log( - sqrt(2*sqrt(a)*
sqrt(a - b) - 2*a + b) + e**(c + d*x)*sqrt(b))*a**2*b + 4*e**(4*c + 4*d*x)
*sqrt(a)*sqrt(a - b)*log( - sqrt(2*sqrt(a)*sqrt(a - b) - 2*a + b) + e**(c
+ d*x)*sqrt(b))*a*b**2 + 4*e**(4*c + 4*d*x)*sqrt(a)*sqrt(a - b)*log( - sqr
t(2*sqrt(a)*sqrt(a - b) - 2*a + b) + e**(c + d*x)*sqrt(b))*b**3 + 32*e*...
```

3.294 $\int \frac{\cosh^5(c+dx)}{\left(a+b\sinh^2(c+dx)\right)^2}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 104

$$\int \frac{\cosh^5(c+dx)}{\left(a+b\sinh^2(c+dx)\right)^2}\,dx = -\frac{(3a^2-2ab-b^2)\arctan\left(\frac{\sqrt{b}\sinh(c+dx)}{\sqrt{a}}\right)}{2a^{3/2}b^{5/2}d} + \frac{\sinh(c+dx)}{b^2d} + \frac{(a-b)^2\sinh(c+dx)}{2ab^2d\left(a+b\sinh^2(c+dx)\right)}$$

output
```
-1/2*(3*a^2-2*a*b-b^2)*arctan(b^(1/2)*sinh(d*x+c)/a^(1/2))/a^(3/2)/b^(5/2)
/d+sinh(d*x+c)/b^2/d+1/2*(a-b)^2*sinh(d*x+c)/a/b^2/d/(a+b*sinh(d*x+c)^2)
```

Mathematica [A] (verified)

Time = 0.23 (sec) , antiderivative size = 106, normalized size of antiderivative = 1.02

$$\int \frac{\cosh^5(c+dx)}{\left(a+b\sinh^2(c+dx)\right)^2}\,dx = \frac{-\frac{(-3a^2+2ab+b^2)\arctan\left(\frac{\sqrt{a}\operatorname{csch}(c+dx)}{\sqrt{b}}\right)}{a^{3/2}} + 2\sqrt{b}\sinh(c+dx) + \frac{2(a-b)^2\sqrt{b}\sinh(c+dx)}{a(2a-b+b\cosh(2(c+dx)))}}{2b^{5/2}d}$$

input
```
Integrate[Cosh[c + d*x]^5/(a + b*Sinh[c + d*x]^2)^2,x]
```

output
```
(-(((-3*a^2 + 2*a*b + b^2)*ArcTan[(Sqrt[a]*Csch[c + d*x])/Sqrt[b]])/a^(3/2
)) + 2*Sqrt[b]*Sinh[c + d*x] + (2*(a - b)^2*Sqrt[b]*Sinh[c + d*x])/(a*(2*a
 - b + b*Cosh[2*(c + d*x)])))/(2*b^(5/2)*d)
```

Rubi [A] (verified)

Time = 0.36 (sec) , antiderivative size = 99, normalized size of antiderivative = 0.95, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.174$, Rules used = {3042, 3669, 300, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cosh^5(c+dx)}{\left(a+b\sinh^2(c+dx)\right)^2}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\cos(ic+idx)^5}{(a-b\sin(ic+idx)^2)^2}dx$$

$$\Big\downarrow \text{3669}$$

$$\frac{\int \frac{\left(\sinh^2(c+dx)+1\right)^2}{\left(b\sinh^2(c+dx)+a\right)^2}d\sinh(c+dx)}{d}$$

$$\Big\downarrow \text{300}$$

$$\frac{\int \left(\frac{1}{b^2}-\frac{a^2-b^2+2(a-b)b\sinh^2(c+dx)}{b^2\left(b\sinh^2(c+dx)+a\right)^2}\right)d\sinh(c+dx)}{d}$$

$$\Big\downarrow \text{2009}$$

$$\frac{-\frac{(3a^2-2ab-b^2)\arctan\left(\frac{\sqrt{b}\sinh(c+dx)}{\sqrt{a}}\right)}{2a^{3/2}b^{5/2}}+\frac{(a-b)^2\sinh(c+dx)}{2ab^2(a+b\sinh^2(c+dx))}+\frac{\sinh(c+dx)}{b^2}}{d}$$

input
```
Int[Cosh[c + d*x]^5/(a + b*Sinh[c + d*x]^2)^2,x]
```

output
```
(-1/2*((3*a^2 - 2*a*b - b^2)*ArcTan[(Sqrt[b]*Sinh[c + d*x])/Sqrt[a]])/(a^(
3/2)*b^(5/2)) + Sinh[c + d*x]/b^2 + ((a - b)^2*Sinh[c + d*x])/(2*a*b^2*(a
+ b*Sinh[c + d*x]^2)))/d
```

Defintions of rubi rules used

rule 300
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_), x_Symbol] :> Int
[PolynomialDivide[(a + b*x^2)^p, (c + d*x^2)^(-q), x], x] /; FreeQ[{a, b, c
, d}, x] && NeQ[b*c - a*d, 0] && IGtQ[p, 0] && ILtQ[q, 0] && GeQ[p, -q]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3669
```
Int[cos[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^
(p_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Simp[ff/f   S
ubst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a + b*ff^2*x^2)^p, x], x, Sin[e + f*x]
/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m - 1)/2]
```

Maple [A] (verified)

Time = 143.92 (sec) , antiderivative size = 97, normalized size of antiderivative = 0.93

method	result
derivativedivides	$\frac{\frac{\sinh(dx+c)}{b^2}-\frac{-\frac{(a^2-2ab+b^2)\sinh(dx+c)}{2a\left(a+b\sinh(dx+c)^2\right)}+\frac{(3a^2-2ab-b^2)\arctan\left(\frac{b\sinh(dx+c)}{\sqrt{ab}}\right)}{2a\sqrt{ab}}}{b^2}}{d}$
default	$\frac{\frac{\sinh(dx+c)}{b^2}-\frac{-\frac{(a^2-2ab+b^2)\sinh(dx+c)}{2a\left(a+b\sinh(dx+c)^2\right)}+\frac{(3a^2-2ab-b^2)\arctan\left(\frac{b\sinh(dx+c)}{\sqrt{ab}}\right)}{2a\sqrt{ab}}}{b^2}}{d}$
risch	$\frac{e^{dx+c}}{2b^2d}-\frac{e^{-dx-c}}{2b^2d}+\frac{(a^2-2ab+b^2)e^{dx+c}(e^{2dx+2c}-1)}{d\,b^2a\left(b\,e^{4dx+4c}+4\,e^{2dx+2c}a-2\,e^{2dx+2c}b+b\right)}-\frac{3a\ln\left(e^{2dx+2c}+\frac{2a\,e^{dx+c}}{\sqrt{-ab}}-1\right)}{4\sqrt{-ab}\,d\,b^2}+\frac{\ln\left(e^{2dx+2c}\right.}{2\sqrt{}}$

input
```
int(cosh(d*x+c)^5/(a+b*sinh(d*x+c)^2)^2,x,method=_RETURNVERBOSE)
```

output
```
1/d*(1/b^2*sinh(d*x+c)-1/b^2*(-1/2*(a^2-2*a*b+b^2)/a*sinh(d*x+c)/(a+b*sinh
(d*x+c)^2)+1/2*(3*a^2-2*a*b-b^2)/a/(a*b)^(1/2)*arctan(b*sinh(d*x+c)/(a*b)^
(1/2))))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1444 vs. $2(92) = 184$.

Time $= 0.13$ (sec) , antiderivative size $= 2741$, normalized size of antiderivative $= 26.36$

$$\int \frac{\cosh^5(c+dx)}{\left(a+b\sinh^2(c+dx)\right)^2}\,dx = \text{Too large to display}$$

input
```
integrate(cosh(d*x+c)^5/(a+b*sinh(d*x+c)^2)^2,x, algorithm="fricas")
```

output
```
[1/4*(2*a^2*b^2*cosh(d*x + c)^6 + 12*a^2*b^2*cosh(d*x + c)*sinh(d*x + c)^5
 + 2*a^2*b^2*sinh(d*x + c)^6 + 2*(6*a^3*b - 7*a^2*b^2 + 2*a*b^3)*cosh(d*x
+ c)^4 + 2*(15*a^2*b^2*cosh(d*x + c)^2 + 6*a^3*b - 7*a^2*b^2 + 2*a*b^3)*si
nh(d*x + c)^4 - 2*a^2*b^2 + 8*(5*a^2*b^2*cosh(d*x + c)^3 + (6*a^3*b - 7*a^
2*b^2 + 2*a*b^3)*cosh(d*x + c))*sinh(d*x + c)^3 - 2*(6*a^3*b - 7*a^2*b^2 +
 2*a*b^3)*cosh(d*x + c)^2 + 2*(15*a^2*b^2*cosh(d*x + c)^4 - 6*a^3*b + 7*a^
2*b^2 - 2*a*b^3 + 6*(6*a^3*b - 7*a^2*b^2 + 2*a*b^3)*cosh(d*x + c)^2)*sinh(
d*x + c)^2 + ((3*a^2*b - 2*a*b^2 - b^3)*cosh(d*x + c)^5 + 5*(3*a^2*b - 2*a
*b^2 - b^3)*cosh(d*x + c)*sinh(d*x + c)^4 + (3*a^2*b - 2*a*b^2 - b^3)*sinh
(d*x + c)^5 + 2*(6*a^3 - 7*a^2*b + b^3)*cosh(d*x + c)^3 + 2*(6*a^3 - 7*a^2
*b + b^3 + 5*(3*a^2*b - 2*a*b^2 - b^3)*cosh(d*x + c)^2)*sinh(d*x + c)^3 +
2*(5*(3*a^2*b - 2*a*b^2 - b^3)*cosh(d*x + c)^3 + 3*(6*a^3 - 7*a^2*b + b^3)
*cosh(d*x + c))*sinh(d*x + c)^2 + (3*a^2*b - 2*a*b^2 - b^3)*cosh(d*x + c)
+ (5*(3*a^2*b - 2*a*b^2 - b^3)*cosh(d*x + c)^4 + 3*a^2*b - 2*a*b^2 - b^3 +
 6*(6*a^3 - 7*a^2*b + b^3)*cosh(d*x + c)^2)*sinh(d*x + c))*sqrt(-a*b)*log(
(b*cosh(d*x + c)^4 + 4*b*cosh(d*x + c)*sinh(d*x + c)^3 + b*sinh(d*x + c)^4
 - 2*(2*a + b)*cosh(d*x + c)^2 + 2*(3*b*cosh(d*x + c)^2 - 2*a - b)*sinh(d*
x + c)^2 + 4*(b*cosh(d*x + c)^3 - (2*a + b)*cosh(d*x + c))*sinh(d*x + c) -
 4*(cosh(d*x + c)^3 + 3*cosh(d*x + c)*sinh(d*x + c)^2 + sinh(d*x + c)^3 +
(3*cosh(d*x + c)^2 - 1)*sinh(d*x + c) - cosh(d*x + c))*sqrt(-a*b) + b)/...
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\cosh^5(c+dx)}{\left(a+b\sinh^2(c+dx)\right)^2}\,dx = \text{Timed out}$$

input
```
integrate(cosh(d*x+c)**5/(a+b*sinh(d*x+c)**2)**2,x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{\cosh^5(c+dx)}{\left(a+b\sinh^2(c+dx)\right)^2}\,dx = \int \frac{\cosh\left(dx+c\right)^5}{\left(b\sinh\left(dx+c\right)^2+a\right)^2}\,dx$$

input
```
integrate(cosh(d*x+c)^5/(a+b*sinh(d*x+c)^2)^2,x, algorithm="maxima")
```

output
```
1/2*(a*b*e^(6*d*x + 6*c) - a*b + (6*a^2*e^(4*c) - 7*a*b*e^(4*c) + 2*b^2*e^
(4*c))*e^(4*d*x) - (6*a^2*e^(2*c) - 7*a*b*e^(2*c) + 2*b^2*e^(2*c))*e^(2*d*
x))/(a*b^3*d*e^(5*d*x + 5*c) + a*b^3*d*e^(d*x + c) + 2*(2*a^2*b^2*d*e^(3*c
) - a*b^3*d*e^(3*c))*e^(3*d*x)) - 1/32*integrate(32*((3*a^2*e^(3*c) - 2*a*
b*e^(3*c) - b^2*e^(3*c))*e^(3*d*x) + (3*a^2*e^c - 2*a*b*e^c - b^2*e^c)*e^(
d*x))/(a*b^3*e^(4*d*x + 4*c) + a*b^3 + 2*(2*a^2*b^2*e^(2*c) - a*b^3*e^(2*c
))*e^(2*d*x)), x)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{\cosh^5(c+dx)}{\left(a+b\sinh^2(c+dx)\right)^2}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(cosh(d*x+c)^5/(a+b*sinh(d*x+c)^2)^2,x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Limit: Max order reached or unable
to make series expansion Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\cosh^5(c+dx)}{\left(a+b\sinh^2(c+dx)\right)^2}\,dx=\int \frac{\cosh(c+d\,x)^5}{\left(b\sinh\left(c+d\,x\right)^2+a\right)^2}\,dx$$

input
```
int(cosh(c + d*x)^5/(a + b*sinh(c + d*x)^2)^2,x)
```

output
```
int(cosh(c + d*x)^5/(a + b*sinh(c + d*x)^2)^2, x)
```

Reduce [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 3876, normalized size of antiderivative = 37.27

$$\int \frac{\cosh^5(c+dx)}{\left(a+b\sinh^2(c+dx)\right)^2}\,dx=\text{Too large to display}$$

input
```
int(cosh(d*x+c)^5/(a+b*sinh(d*x+c)^2)^2,x)
```

output

```
(6*e**(5*c + 5*d*x)*sqrt(b)*sqrt(a)*sqrt(a - b)*sqrt(2*sqrt(a)*sqrt(a - b)
 + 2*a - b)*atan((e**(c + d*x)*b)/(sqrt(b)*sqrt(2*sqrt(a)*sqrt(a - b) + 2*
a - b)))*a**2*b - 4*e**(5*c + 5*d*x)*sqrt(b)*sqrt(a)*sqrt(a - b)*sqrt(2*sq
rt(a)*sqrt(a - b) + 2*a - b)*atan((e**(c + d*x)*b)/(sqrt(b)*sqrt(2*sqrt(a)
*sqrt(a - b) + 2*a - b)))*a*b**2 - 2*e**(5*c + 5*d*x)*sqrt(b)*sqrt(a)*sqrt
(a - b)*sqrt(2*sqrt(a)*sqrt(a - b) + 2*a - b)*atan((e**(c + d*x)*b)/(sqrt(
b)*sqrt(2*sqrt(a)*sqrt(a - b) + 2*a - b)))*b**3 + 24*e**(3*c + 3*d*x)*sqrt
(b)*sqrt(a)*sqrt(a - b)*sqrt(2*sqrt(a)*sqrt(a - b) + 2*a - b)*atan((e**(c
+ d*x)*b)/(sqrt(b)*sqrt(2*sqrt(a)*sqrt(a - b) + 2*a - b)))*a**3 - 28*e**(3
*c + 3*d*x)*sqrt(b)*sqrt(a)*sqrt(a - b)*sqrt(2*sqrt(a)*sqrt(a - b) + 2*a -
 b)*atan((e**(c + d*x)*b)/(sqrt(b)*sqrt(2*sqrt(a)*sqrt(a - b) + 2*a - b)))
*a**2*b + 4*e**(3*c + 3*d*x)*sqrt(b)*sqrt(a)*sqrt(a - b)*sqrt(2*sqrt(a)*sq
rt(a - b) + 2*a - b)*atan((e**(c + d*x)*b)/(sqrt(b)*sqrt(2*sqrt(a)*sqrt(a
- b) + 2*a - b)))*b**3 + 6*e**(c + d*x)*sqrt(b)*sqrt(a)*sqrt(a - b)*sqrt(2
*sqrt(a)*sqrt(a - b) + 2*a - b)*atan((e**(c + d*x)*b)/(sqrt(b)*sqrt(2*sqrt
(a)*sqrt(a - b) + 2*a - b)))*a**2*b - 4*e**(c + d*x)*sqrt(b)*sqrt(a)*sqrt(
a - b)*sqrt(2*sqrt(a)*sqrt(a - b) + 2*a - b)*atan((e**(c + d*x)*b)/(sqrt(b
)*sqrt(2*sqrt(a)*sqrt(a - b) + 2*a - b)))*a*b**2 - 2*e**(c + d*x)*sqrt(b)*
sqrt(a)*sqrt(a - b)*sqrt(2*sqrt(a)*sqrt(a - b) + 2*a - b)*atan((e**(c + d*
x)*b)/(sqrt(b)*sqrt(2*sqrt(a)*sqrt(a - b) + 2*a - b)))*b**3 - 6*e**(5*c...
```

3.295 $\int \frac{\cosh^4(c+dx)}{\left(a+b\sinh^2(c+dx)\right)^2}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 100

$$\int \frac{\cosh^4(c+dx)}{\left(a+b\sinh^2(c+dx)\right)^2}\,dx = \frac{x}{b^2} - \frac{\sqrt{a-b}(2a+b)\text{arctanh}\left(\frac{\sqrt{a-b}\tanh(c+dx)}{\sqrt{a}}\right)}{2a^{3/2}b^2d} - \frac{(a-b)\tanh(c+dx)}{2abd\left(a-(a-b)\tanh^2(c+dx)\right)}$$

output
```
x/b^2-1/2*(a-b)^(1/2)*(2*a+b)*arctanh((a-b)^(1/2)*tanh(d*x+c)/a^(1/2))/a^(
3/2)/b^2/d-1/2*(a-b)*tanh(d*x+c)/a/b/d/(a-(a-b)*tanh(d*x+c)^2)
```

Mathematica [A] (verified)

Time = 0.66 (sec) , antiderivative size = 108, normalized size of antiderivative = 1.08

$$\int \frac{\cosh^4(c+dx)}{\left(a+b\sinh^2(c+dx)\right)^2}\,dx = \frac{2(c+dx) - \frac{(2a^2-ab-b^2)\text{arctanh}\left(\frac{\sqrt{a-b}\tanh(c+dx)}{\sqrt{a}}\right)}{a^{3/2}\sqrt{a-b}} + \frac{b(-a+b)\sinh(2(c+dx))}{a(2a-b+b\cosh(2(c+dx)))}}{2b^2d}$$

input
```
Integrate[Cosh[c + d*x]^4/(a + b*Sinh[c + d*x]^2)^2,x]
```

output
```
(2*(c + d*x) - ((2*a^2 - a*b - b^2)*ArcTanh[(Sqrt[a - b]*Tanh[c + d*x])/Sq
rt[a]])/(a^(3/2)*Sqrt[a - b]) + (b*(-a + b)*Sinh[2*(c + d*x)])/(a*(2*a - b
 + b*Cosh[2*(c + d*x)])))/(2*b^2*d)
```

Rubi [A] (verified)

Time = 0.38 (sec) , antiderivative size = 115, normalized size of antiderivative = 1.15, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.304$, Rules used = {3042, 3670, 316, 25, 397, 219, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cosh^4(c+dx)}{\left(a+b\sinh^2(c+dx)\right)^2}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\cos(ic+idx)^4}{\left(a-b\sin(ic+idx)^2\right)^2}dx$$

$$\Big\downarrow \text{3670}$$

$$\frac{\int \frac{1}{\left(1-\tanh^2(c+dx)\right)\left(a-(a-b)\tanh^2(c+dx)\right)^2}d\tanh(c+dx)}{d}$$

$$\Big\downarrow \text{316}$$

$$\frac{-\frac{\int -\frac{(a-b)\tanh^2(c+dx)+a+b}{\left(1-\tanh^2(c+dx)\right)\left(a-(a-b)\tanh^2(c+dx)\right)}d\tanh(c+dx)}{2ab}-\frac{(a-b)\tanh(c+dx)}{2ab\left(a-(a-b)\tanh^2(c+dx)\right)}}{d}$$

$$\Big\downarrow \text{25}$$

$$\frac{\frac{\int \frac{(a-b)\tanh^2(c+dx)+a+b}{\left(1-\tanh^2(c+dx)\right)\left(a-(a-b)\tanh^2(c+dx)\right)}d\tanh(c+dx)}{2ab}-\frac{(a-b)\tanh(c+dx)}{2ab\left(a-(a-b)\tanh^2(c+dx)\right)}}{d}$$

$$\Big\downarrow \text{397}$$

$$\frac{\frac{\frac{2a\int \frac{1}{1-\tanh^2(c+dx)}d\tanh(c+dx)}{b}-\frac{(a-b)(2a+b)\int \frac{1}{a-(a-b)\tanh^2(c+dx)}d\tanh(c+dx)}{b}}{2ab}-\frac{(a-b)\tanh(c+dx)}{2ab(a-(a-b)\tanh^2(c+dx))}}{d}$$

$$\Big\downarrow \; 219$$

$$\frac{\frac{\frac{2a\text{arctanh}(\tanh(c+dx))}{b}-\frac{(a-b)(2a+b)\int \frac{1}{a-(a-b)\tanh^2(c+dx)}d\tanh(c+dx)}{b}}{2ab}-\frac{(a-b)\tanh(c+dx)}{2ab(a-(a-b)\tanh^2(c+dx))}}{d}$$

$$\Big\downarrow \; 221$$

$$\frac{\frac{\frac{2a\text{arctanh}(\tanh(c+dx))}{b}-\frac{\sqrt{a-b}(2a+b)\text{arctanh}\left(\frac{\sqrt{a-b}\tanh(c+dx)}{\sqrt{a}}\right)}{\sqrt{a}b}}{2ab}-\frac{(a-b)\tanh(c+dx)}{2ab(a-(a-b)\tanh^2(c+dx))}}{d}$$

input
```
Int[Cosh[c + d*x]^4/(a + b*Sinh[c + d*x]^2)^2,x]
```

output
```
(((2*a*ArcTanh[Tanh[c + d*x]])/b - (Sqrt[a - b]*(2*a + b)*ArcTanh[(Sqrt[a
- b]*Tanh[c + d*x])/Sqrt[a]])/(Sqrt[a]*b))/(2*a*b) - ((a - b)*Tanh[c + d*x
])/(2*a*b*(a - (a - b)*Tanh[c + d*x]^2)))/d
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 316
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_), x_Symbol] :> Sim
p[(-b)*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^(q + 1)/(2*a*(p + 1)*(b*c - a*d))
), x] + Simp[1/(2*a*(p + 1)*(b*c - a*d))   Int[(a + b*x^2)^(p + 1)*(c + d*x
^2)^q*Simp[b*c + 2*(p + 1)*(b*c - a*d) + d*b*(2*(p + q + 2) + 1)*x^2, x], x
], x] /; FreeQ[{a, b, c, d, q}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] &&  !
( !IntegerQ[p] && IntegerQ[q] && LtQ[q, -1]) && IntBinomialQ[a, b, c, d, 2,
 p, q, x]
```

rule 397
```
Int[((e_) + (f_.)*(x_)^2)/(((a_) + (b_.)*(x_)^2)*((c_) + (d_.)*(x_)^2)), x_
Symbol] :> Simp[(b*e - a*f)/(b*c - a*d)   Int[1/(a + b*x^2), x], x] - Simp[
(d*e - c*f)/(b*c - a*d)   Int[1/(c + d*x^2), x], x] /; FreeQ[{a, b, c, d, e
, f}, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3670
```
Int[cos[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(
p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Simp[ff/f   Su
bst[Int[(a + (a + b)*ff^2*x^2)^p/(1 + ff^2*x^2)^(m/2 + p + 1), x], x, Tan[e
 + f*x]/ff], x]] /; FreeQ[{a, b, e, f}, x] && IntegerQ[m/2] && IntegerQ[p]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 314 vs. $2(88) = 176$.

Time $= 58.01$ (sec) , antiderivative size $= 315$, normalized size of antiderivative $= 3.15$

method	result
risch	$\frac{x}{b^2}+\frac{2\,\mathrm{e}^{2dx+2c}a^2-3\,\mathrm{e}^{2dx+2c}ba+b^2\mathrm{e}^{2dx+2c}+ab-b^2}{d\,b^2a\left(b\,\mathrm{e}^{4dx+4c}+4\,\mathrm{e}^{2dx+2c}a-2\,\mathrm{e}^{2dx+2c}b+b\right)}+\frac{\sqrt{a(a-b)}\,\ln\left(\mathrm{e}^{2dx+2c}+\frac{2a+2\sqrt{a(a-b)}-b}{b}\right)}{2ad\,b^2}+\frac{\sqrt{a(a-b)}\,\ln\left(\right.}{}$
derivativedivides	$\frac{\frac{2\left(-\frac{b(a-b)\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^3}{2a}-\frac{b(a-b)\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)}{2a}\right)}{\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^4a-2\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^2a+4b\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^2+a}+\left(2a^2-ab-b^2\right)\left(-\frac{\left(\sqrt{-b(a-b)}-b\right)\operatorname{arctanh}\left(\frac{\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)a}{\sqrt{\left(2\sqrt{-b(a-b)}+a-2b\right)a}}\right.}{2a\sqrt{-b(a-b)}\sqrt{\left(2\sqrt{-b(a-b)}+a-2b\right)a}}\right.}{b^2}}{d}$
default	$\frac{\frac{2\left(-\frac{b(a-b)\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^3}{2a}-\frac{b(a-b)\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)}{2a}\right)}{\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^4a-2\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^2a+4b\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^2+a}+\left(2a^2-ab-b^2\right)\left(-\frac{\left(\sqrt{-b(a-b)}-b\right)\operatorname{arctanh}\left(\frac{\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)a}{\sqrt{\left(2\sqrt{-b(a-b)}+a-2b\right)a}}\right.}{2a\sqrt{-b(a-b)}\sqrt{\left(2\sqrt{-b(a-b)}+a-2b\right)a}}\right.}{b^2}}{d}$

input
```
int(cosh(d*x+c)^4/(a+b*sinh(d*x+c)^2)^2,x,method=_RETURNVERBOSE)
```

output
```
x/b^2+(2*exp(2*d*x+2*c)*a^2-3*exp(2*d*x+2*c)*b*a+b^2*exp(2*d*x+2*c)+a*b-b^
2)/d/b^2/a/(b*exp(4*d*x+4*c)+4*exp(2*d*x+2*c)*a-2*exp(2*d*x+2*c)*b+b)+1/2/
a*(a*(a-b))^(1/2)/d/b^2*ln(exp(2*d*x+2*c)+(2*a+2*(a*(a-b))^(1/2)-b)/b)+1/4
/a^2*(a*(a-b))^(1/2)/d/b*ln(exp(2*d*x+2*c)+(2*a+2*(a*(a-b))^(1/2)-b)/b)-1/
2/a*(a*(a-b))^(1/2)/d/b^2*ln(exp(2*d*x+2*c)-(-2*a+2*(a*(a-b))^(1/2)+b)/b)-
1/4/a^2*(a*(a-b))^(1/2)/d/b*ln(exp(2*d*x+2*c)-(-2*a+2*(a*(a-b))^(1/2)+b)/b
)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 631 vs. $2(89) = 178$.

Time = 0.13 (sec) , antiderivative size = 1527, normalized size of antiderivative = 15.27

$$\int \frac{\cosh^4(c+dx)}{\left(a+b\sinh^2(c+dx)\right)^2}\,dx = \text{Too large to display}$$

input
```
integrate(cosh(d*x+c)^4/(a+b*sinh(d*x+c)^2)^2,x, algorithm="fricas")
```

output
```
[1/4*(4*a*b*d*x*cosh(d*x + c)^4 + 16*a*b*d*x*cosh(d*x + c)*sinh(d*x + c)^3
 + 4*a*b*d*x*sinh(d*x + c)^4 + 4*a*b*d*x + 4*(2*(2*a^2 - a*b)*d*x + 2*a^2
- 3*a*b + b^2)*cosh(d*x + c)^2 + 4*(6*a*b*d*x*cosh(d*x + c)^2 + 2*(2*a^2 -
 a*b)*d*x + 2*a^2 - 3*a*b + b^2)*sinh(d*x + c)^2 + ((2*a*b + b^2)*cosh(d*x
 + c)^4 + 4*(2*a*b + b^2)*cosh(d*x + c)*sinh(d*x + c)^3 + (2*a*b + b^2)*si
nh(d*x + c)^4 + 2*(4*a^2 - b^2)*cosh(d*x + c)^2 + 2*(3*(2*a*b + b^2)*cosh(
d*x + c)^2 + 4*a^2 - b^2)*sinh(d*x + c)^2 + 2*a*b + b^2 + 4*((2*a*b + b^2)
*cosh(d*x + c)^3 + (4*a^2 - b^2)*cosh(d*x + c))*sinh(d*x + c))*sqrt((a - b
)/a)*log((b^2*cosh(d*x + c)^4 + 4*b^2*cosh(d*x + c)*sinh(d*x + c)^3 + b^2*
sinh(d*x + c)^4 + 2*(2*a*b - b^2)*cosh(d*x + c)^2 + 2*(3*b^2*cosh(d*x + c)
^2 + 2*a*b - b^2)*sinh(d*x + c)^2 + 8*a^2 - 8*a*b + b^2 + 4*(b^2*cosh(d*x
+ c)^3 + (2*a*b - b^2)*cosh(d*x + c))*sinh(d*x + c) + 4*(a*b*cosh(d*x + c)
^2 + 2*a*b*cosh(d*x + c)*sinh(d*x + c) + a*b*sinh(d*x + c)^2 + 2*a^2 - a*b
)*sqrt((a - b)/a))/(b*cosh(d*x + c)^4 + 4*b*cosh(d*x + c)*sinh(d*x + c)^3
+ b*sinh(d*x + c)^4 + 2*(2*a - b)*cosh(d*x + c)^2 + 2*(3*b*cosh(d*x + c)^2
 + 2*a - b)*sinh(d*x + c)^2 + 4*(b*cosh(d*x + c)^3 + (2*a - b)*cosh(d*x +
c))*sinh(d*x + c) + b)) + 4*a*b - 4*b^2 + 8*(2*a*b*d*x*cosh(d*x + c)^3 + (
2*(2*a^2 - a*b)*d*x + 2*a^2 - 3*a*b + b^2)*cosh(d*x + c))*sinh(d*x + c))/(
a*b^3*d*cosh(d*x + c)^4 + 4*a*b^3*d*cosh(d*x + c)*sinh(d*x + c)^3 + a*b^3*
d*sinh(d*x + c)^4 + a*b^3*d + 2*(2*a^2*b^2 - a*b^3)*d*cosh(d*x + c)^2 +...
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\cosh^4(c+dx)}{\left(a+b\sinh^2(c+dx)\right)^2}\,dx = \text{Timed out}$$

input
```
integrate(cosh(d*x+c)**4/(a+b*sinh(d*x+c)**2)**2,x)
```

output
```
Timed out
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{\cosh^4(c+dx)}{\left(a+b\sinh^2(c+dx)\right)^2}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate(cosh(d*x+c)^4/(a+b*sinh(d*x+c)^2)^2,x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(b-a>0)', see `assume?` for more
details)Is
```

Giac [A] (verification not implemented)

Time = 0.94 (sec) , antiderivative size = 178, normalized size of antiderivative = 1.78

$$\int \frac{\cosh^4(c+dx)}{\left(a+b\sinh^2(c+dx)\right)^2}\,dx = \frac{\frac{2\,(dx+c)}{b^2} - \frac{\left(2\,a^2-ab-b^2\right)\arctan\left(\frac{be^{(2\,dx+2\,c)}+2\,a-b}{2\,\sqrt{-a^2+ab}}\right)}{\sqrt{-a^2+ab}ab^2} + \frac{2\,\left(2\,a^2e^{(2\,dx+2\,c)}-3\,abe^{(2\,dx+2\,c)}+b^2e^{(2\,dx+2\,c)}+ab-b^2\right)}{\left(be^{(4\,dx+4\,c)}+4\,ae^{(2\,dx+2\,c)}-2\,be^{(2\,dx+2\,c)}+b\right)ab^2}}{2\,d}$$

input
```
integrate(cosh(d*x+c)^4/(a+b*sinh(d*x+c)^2)^2,x, algorithm="giac")
```

output
```
1/2*(2*(d*x + c)/b^2 - (2*a^2 - a*b - b^2)*arctan(1/2*(b*e^(2*d*x + 2*c) +
 2*a - b)/sqrt(-a^2 + a*b))/(sqrt(-a^2 + a*b)*a*b^2) + 2*(2*a^2*e^(2*d*x +
 2*c) - 3*a*b*e^(2*d*x + 2*c) + b^2*e^(2*d*x + 2*c) + a*b - b^2)/((b*e^(4*
d*x + 4*c) + 4*a*e^(2*d*x + 2*c) - 2*b*e^(2*d*x + 2*c) + b)*a*b^2))/d
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\cosh^4(c+dx)}{(a+b\sinh^2(c+dx))^2}\,dx = \int \frac{\cosh(c+d\,x)^4}{\left(b\sinh\left(c+d\,x\right)^2+a\right)^2}\,dx$$

input
```
int(cosh(c + d*x)^4/(a + b*sinh(c + d*x)^2)^2,x)
```

output
```
int(cosh(c + d*x)^4/(a + b*sinh(c + d*x)^2)^2, x)
```

Reduce [B] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 1033, normalized size of antiderivative = 10.33

$$\int \frac{\cosh^4(c+dx)}{(a+b\sinh^2(c+dx))^2}\,dx = \text{Too large to display}$$

input
```
int(cosh(d*x+c)^4/(a+b*sinh(d*x+c)^2)^2,x)
```

output
```
( - 2*e**(4*c + 4*d*x)*sqrt(a)*sqrt(a - b)*log( - sqrt(2*sqrt(a)*sqrt(a -
b) - 2*a + b) + e**(c + d*x)*sqrt(b))*a*b - e**(4*c + 4*d*x)*sqrt(a)*sqrt(
a - b)*log( - sqrt(2*sqrt(a)*sqrt(a - b) - 2*a + b) + e**(c + d*x)*sqrt(b)
)*b**2 - 2*e**(4*c + 4*d*x)*sqrt(a)*sqrt(a - b)*log(sqrt(2*sqrt(a)*sqrt(a
- b) - 2*a + b) + e**(c + d*x)*sqrt(b))*a*b - e**(4*c + 4*d*x)*sqrt(a)*sqr
t(a - b)*log(sqrt(2*sqrt(a)*sqrt(a - b) - 2*a + b) + e**(c + d*x)*sqrt(b))
*b**2 + 2*e**(4*c + 4*d*x)*sqrt(a)*sqrt(a - b)*log(2*sqrt(a)*sqrt(a - b) +
 e**(2*c + 2*d*x)*b + 2*a - b)*a*b + e**(4*c + 4*d*x)*sqrt(a)*sqrt(a - b)*
log(2*sqrt(a)*sqrt(a - b) + e**(2*c + 2*d*x)*b + 2*a - b)*b**2 - 8*e**(2*c
 + 2*d*x)*sqrt(a)*sqrt(a - b)*log( - sqrt(2*sqrt(a)*sqrt(a - b) - 2*a + b)
 + e**(c + d*x)*sqrt(b))*a**2 + 2*e**(2*c + 2*d*x)*sqrt(a)*sqrt(a - b)*log
( - sqrt(2*sqrt(a)*sqrt(a - b) - 2*a + b) + e**(c + d*x)*sqrt(b))*b**2 - 8
*e**(2*c + 2*d*x)*sqrt(a)*sqrt(a - b)*log(sqrt(2*sqrt(a)*sqrt(a - b) - 2*a
 + b) + e**(c + d*x)*sqrt(b))*a**2 + 2*e**(2*c + 2*d*x)*sqrt(a)*sqrt(a - b
)*log(sqrt(2*sqrt(a)*sqrt(a - b) - 2*a + b) + e**(c + d*x)*sqrt(b))*b**2 +
 8*e**(2*c + 2*d*x)*sqrt(a)*sqrt(a - b)*log(2*sqrt(a)*sqrt(a - b) + e**(2*
c + 2*d*x)*b + 2*a - b)*a**2 - 2*e**(2*c + 2*d*x)*sqrt(a)*sqrt(a - b)*log(
2*sqrt(a)*sqrt(a - b) + e**(2*c + 2*d*x)*b + 2*a - b)*b**2 - 2*sqrt(a)*sqr
t(a - b)*log( - sqrt(2*sqrt(a)*sqrt(a - b) - 2*a + b) + e**(c + d*x)*sqrt(
b))*a*b - sqrt(a)*sqrt(a - b)*log( - sqrt(2*sqrt(a)*sqrt(a - b) - 2*a +...
```

3.296 $\int \frac{\cosh^3(c+dx)}{\left(a+b\sinh^2(c+dx)\right)^2}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 77

$$\int \frac{\cosh^3(c+dx)}{\left(a+b\sinh^2(c+dx)\right)^2}\,dx=\frac{(a+b)\arctan\left(\frac{\sqrt{b}\sinh(c+dx)}{\sqrt{a}}\right)}{2a^{3/2}b^{3/2}d}-\frac{(a-b)\sinh(c+dx)}{2abd\left(a+b\sinh^2(c+dx)\right)}$$

output
```
1/2*(a+b)*arctan(b^(1/2)*sinh(d*x+c)/a^(1/2))/a^(3/2)/b^(3/2)/d-1/2*(a-b)*
sinh(d*x+c)/a/b/d/(a+b*sinh(d*x+c)^2)
```

Mathematica [A] (verified)

Time = 0.24 (sec) , antiderivative size = 75, normalized size of antiderivative = 0.97

$$\int \frac{\cosh^3(c+dx)}{\left(a+b\sinh^2(c+dx)\right)^2}\,dx=\frac{\frac{(a+b)\arctan\left(\frac{\sqrt{b}\sinh(c+dx)}{\sqrt{a}}\right)}{2a^{3/2}b^{3/2}}-\frac{(a-b)\sinh(c+dx)}{2ab(a+b\sinh^2(c+dx))}}{d}$$

input
```
Integrate[Cosh[c + d*x]^3/(a + b*Sinh[c + d*x]^2)^2,x]
```

output
```
(((a + b)*ArcTan[(Sqrt[b]*Sinh[c + d*x])/Sqrt[a]])/(2*a^(3/2)*b^(3/2)) - (
(a - b)*Sinh[c + d*x])/(2*a*b*(a + b*Sinh[c + d*x]^2)))/d
```

Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 75, normalized size of antiderivative = 0.97, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.174$, Rules used = {3042, 3669, 298, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cosh^3(c+dx)}{\left(a+b\sinh^2(c+dx)\right)^2}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\cos(ic+idx)^3}{\left(a-b\sin(ic+idx)^2\right)^2}dx$$

$$\Big\downarrow \text{3669}$$

$$\frac{\int \frac{\sinh^2(c+dx)+1}{\left(b\sinh^2(c+dx)+a\right)^2}d\sinh(c+dx)}{d}$$

$$\Big\downarrow \text{298}$$

$$\frac{\frac{(a+b)\int \frac{1}{b\sinh^2(c+dx)+a}d\sinh(c+dx)}{2ab}-\frac{(a-b)\sinh(c+dx)}{2ab(a+b\sinh^2(c+dx))}}{d}$$

$$\Big\downarrow \text{218}$$

$$\frac{\frac{(a+b)\arctan\left(\frac{\sqrt{b}\sinh(c+dx)}{\sqrt{a}}\right)}{2a^{3/2}b^{3/2}}-\frac{(a-b)\sinh(c+dx)}{2ab(a+b\sinh^2(c+dx))}}{d}$$

input
```
Int[Cosh[c + d*x]^3/(a + b*Sinh[c + d*x]^2)^2,x]
```

output
```
(((a + b)*ArcTan[(Sqrt[b]*Sinh[c + d*x])/Sqrt[a]])/(2*a^(3/2)*b^(3/2)) - (
(a - b)*Sinh[c + d*x])/(2*a*b*(a + b*Sinh[c + d*x]^2)))/d
```

Defintions of rubi rules used

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 298
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2), x_Symbol] :> Simp[(-(
b*c - a*d))*x*((a + b*x^2)^(p + 1)/(2*a*b*(p + 1))), x] - Simp[(a*d - b*c*(
2*p + 3))/(2*a*b*(p + 1))   Int[(a + b*x^2)^(p + 1), x], x] /; FreeQ[{a, b,
 c, d, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/2 + p, 0])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3669
```
Int[cos[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^
(p_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Simp[ff/f   S
ubst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a + b*ff^2*x^2)^p, x], x, Sin[e + f*x]
/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m - 1)/2]
```

Maple [A] (verified)

Time = 58.04 (sec) , antiderivative size = 69, normalized size of antiderivative = 0.90

method	result
derivativedivides	$\frac{-\frac{(a-b)\sinh(dx+c)}{2ab\left(a+b\sinh(dx+c)^2\right)}+\frac{(a+b)\arctan\left(\frac{b\sinh(dx+c)}{\sqrt{ab}}\right)}{2ab\sqrt{ab}}}{d}$
default	$\frac{-\frac{(a-b)\sinh(dx+c)}{2ab\left(a+b\sinh(dx+c)^2\right)}+\frac{(a+b)\arctan\left(\frac{b\sinh(dx+c)}{\sqrt{ab}}\right)}{2ab\sqrt{ab}}}{d}$
risch	$-\frac{e^{dx+c}(a-b)\left(e^{2dx+2c}-1\right)}{abd\left(b\,e^{4dx+4c}+4\,e^{2dx+2c}a-2\,e^{2dx+2c}b+b\right)}-\frac{\ln\left(e^{2dx+2c}-\frac{2a\,e^{dx+c}}{\sqrt{-ab}}-1\right)}{4\sqrt{-ab}\,db}-\frac{\ln\left(e^{2dx+2c}-\frac{2a\,e^{dx+c}}{\sqrt{-ab}}-1\right)}{4\sqrt{-ab}\,da}+\frac{\ln\left(e\right.}{}$

input
```
int(cosh(d*x+c)^3/(a+b*sinh(d*x+c)^2)^2,x,method=_RETURNVERBOSE)
```

output
```
1/d*(-1/2*(a-b)/a/b*sinh(d*x+c)/(a+b*sinh(d*x+c)^2)+1/2*(a+b)/a/b/(a*b)^(1
/2)*arctan(b*sinh(d*x+c)/(a*b)^(1/2)))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 805 vs. $2(65) = 130$.

Time = 0.11 (sec) , antiderivative size = 1617, normalized size of antiderivative = 21.00

$$\int \frac{\cosh^3(c+dx)}{\left(a+b\sinh^2(c+dx)\right)^2}\,dx = \text{Too large to display}$$

input
```
integrate(cosh(d*x+c)^3/(a+b*sinh(d*x+c)^2)^2,x, algorithm="fricas")
```

output
```
[-1/4*(4*(a^2*b - a*b^2)*cosh(d*x + c)^3 + 12*(a^2*b - a*b^2)*cosh(d*x + c
)*sinh(d*x + c)^2 + 4*(a^2*b - a*b^2)*sinh(d*x + c)^3 + ((a*b + b^2)*cosh(
d*x + c)^4 + 4*(a*b + b^2)*cosh(d*x + c)*sinh(d*x + c)^3 + (a*b + b^2)*sin
h(d*x + c)^4 + 2*(2*a^2 + a*b - b^2)*cosh(d*x + c)^2 + 2*(3*(a*b + b^2)*co
sh(d*x + c)^2 + 2*a^2 + a*b - b^2)*sinh(d*x + c)^2 + a*b + b^2 + 4*((a*b +
 b^2)*cosh(d*x + c)^3 + (2*a^2 + a*b - b^2)*cosh(d*x + c))*sinh(d*x + c))*
sqrt(-a*b)*log((b*cosh(d*x + c)^4 + 4*b*cosh(d*x + c)*sinh(d*x + c)^3 + b*
sinh(d*x + c)^4 - 2*(2*a + b)*cosh(d*x + c)^2 + 2*(3*b*cosh(d*x + c)^2 - 2
*a - b)*sinh(d*x + c)^2 + 4*(b*cosh(d*x + c)^3 - (2*a + b)*cosh(d*x + c))*
sinh(d*x + c) - 4*(cosh(d*x + c)^3 + 3*cosh(d*x + c)*sinh(d*x + c)^2 + sin
h(d*x + c)^3 + (3*cosh(d*x + c)^2 - 1)*sinh(d*x + c) - cosh(d*x + c))*sqrt
(-a*b) + b)/(b*cosh(d*x + c)^4 + 4*b*cosh(d*x + c)*sinh(d*x + c)^3 + b*sin
h(d*x + c)^4 + 2*(2*a - b)*cosh(d*x + c)^2 + 2*(3*b*cosh(d*x + c)^2 + 2*a
- b)*sinh(d*x + c)^2 + 4*(b*cosh(d*x + c)^3 + (2*a - b)*cosh(d*x + c))*sin
h(d*x + c) + b)) - 4*(a^2*b - a*b^2)*cosh(d*x + c) - 4*(a^2*b - a*b^2 - 3*
(a^2*b - a*b^2)*cosh(d*x + c)^2)*sinh(d*x + c))/(a^2*b^3*d*cosh(d*x + c)^4
 + 4*a^2*b^3*d*cosh(d*x + c)*sinh(d*x + c)^3 + a^2*b^3*d*sinh(d*x + c)^4 +
 a^2*b^3*d + 2*(2*a^3*b^2 - a^2*b^3)*d*cosh(d*x + c)^2 + 2*(3*a^2*b^3*d*co
sh(d*x + c)^2 + (2*a^3*b^2 - a^2*b^3)*d)*sinh(d*x + c)^2 + 4*(a^2*b^3*d*co
sh(d*x + c)^3 + (2*a^3*b^2 - a^2*b^3)*d*cosh(d*x + c))*sinh(d*x + c)), ...
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\cosh^3(c+dx)}{\left(a+b\sinh^2(c+dx)\right)^2}\,dx = \text{Timed out}$$

input
```
integrate(cosh(d*x+c)**3/(a+b*sinh(d*x+c)**2)**2,x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{\cosh^3(c+dx)}{\left(a+b\sinh^2(c+dx)\right)^2}\,dx = \int \frac{\cosh\left(dx+c\right)^3}{\left(b\sinh\left(dx+c\right)^2+a\right)^2}\,dx$$

input
```
integrate(cosh(d*x+c)^3/(a+b*sinh(d*x+c)^2)^2,x, algorithm="maxima")
```

output
```
-((a*e^(3*c) - b*e^(3*c))*e^(3*d*x) - (a*e^c - b*e^c)*e^(d*x))/(a*b^2*d*e^
(4*d*x + 4*c) + a*b^2*d + 2*(2*a^2*b*d*e^(2*c) - a*b^2*d*e^(2*c))*e^(2*d*x
)) + 1/8*integrate(8*((a*e^(3*c) + b*e^(3*c))*e^(3*d*x) + (a*e^c + b*e^c)*
e^(d*x))/(a*b^2*e^(4*d*x + 4*c) + a*b^2 + 2*(2*a^2*b*e^(2*c) - a*b^2*e^(2*
c))*e^(2*d*x)), x)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{\cosh^3(c+dx)}{\left(a+b\sinh^2(c+dx)\right)^2}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(cosh(d*x+c)^3/(a+b*sinh(d*x+c)^2)^2,x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Limit: Max order reached or unable
to make series expansion Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\cosh^3(c+dx)}{(a+b\sinh^2(c+dx))^2}\,dx = \int \frac{\cosh(c+d\,x)^3}{\left(b\sinh(c+d\,x)^2+a\right)^2}\,dx$$

input
```
int(cosh(c + d*x)^3/(a + b*sinh(c + d*x)^2)^2,x)
```

output
```
int(cosh(c + d*x)^3/(a + b*sinh(c + d*x)^2)^2, x)
```

Reduce [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 2867, normalized size of antiderivative = 37.23

$$\int \frac{\cosh^3(c+dx)}{(a+b\sinh^2(c+dx))^2}\,dx = \text{Too large to display}$$

input
```
int(cosh(d*x+c)^3/(a+b*sinh(d*x+c)^2)^2,x)
```

output
```
( - 2*e**(4*c + 4*d*x)*sqrt(b)*sqrt(a)*sqrt(a - b)*sqrt(2*sqrt(a)*sqrt(a -
 b) + 2*a - b)*atan((e**(c + d*x)*b)/(sqrt(b)*sqrt(2*sqrt(a)*sqrt(a - b) +
 2*a - b)))*a*b - 2*e**(4*c + 4*d*x)*sqrt(b)*sqrt(a)*sqrt(a - b)*sqrt(2*sq
rt(a)*sqrt(a - b) + 2*a - b)*atan((e**(c + d*x)*b)/(sqrt(b)*sqrt(2*sqrt(a)
*sqrt(a - b) + 2*a - b)))*b**2 - 8*e**(2*c + 2*d*x)*sqrt(b)*sqrt(a)*sqrt(a
 - b)*sqrt(2*sqrt(a)*sqrt(a - b) + 2*a - b)*atan((e**(c + d*x)*b)/(sqrt(b)
*sqrt(2*sqrt(a)*sqrt(a - b) + 2*a - b)))*a**2 - 4*e**(2*c + 2*d*x)*sqrt(b)
*sqrt(a)*sqrt(a - b)*sqrt(2*sqrt(a)*sqrt(a - b) + 2*a - b)*atan((e**(c + d
*x)*b)/(sqrt(b)*sqrt(2*sqrt(a)*sqrt(a - b) + 2*a - b)))*a*b + 4*e**(2*c +
2*d*x)*sqrt(b)*sqrt(a)*sqrt(a - b)*sqrt(2*sqrt(a)*sqrt(a - b) + 2*a - b)*a
tan((e**(c + d*x)*b)/(sqrt(b)*sqrt(2*sqrt(a)*sqrt(a - b) + 2*a - b)))*b**2
 - 2*sqrt(b)*sqrt(a)*sqrt(a - b)*sqrt(2*sqrt(a)*sqrt(a - b) + 2*a - b)*ata
n((e**(c + d*x)*b)/(sqrt(b)*sqrt(2*sqrt(a)*sqrt(a - b) + 2*a - b)))*a*b -
2*sqrt(b)*sqrt(a)*sqrt(a - b)*sqrt(2*sqrt(a)*sqrt(a - b) + 2*a - b)*atan((
e**(c + d*x)*b)/(sqrt(b)*sqrt(2*sqrt(a)*sqrt(a - b) + 2*a - b)))*b**2 + 2*
e**(4*c + 4*d*x)*sqrt(b)*sqrt(2*sqrt(a)*sqrt(a - b) + 2*a - b)*atan((e**(c
 + d*x)*b)/(sqrt(b)*sqrt(2*sqrt(a)*sqrt(a - b) + 2*a - b)))*a**2*b + 2*e**
(4*c + 4*d*x)*sqrt(b)*sqrt(2*sqrt(a)*sqrt(a - b) + 2*a - b)*atan((e**(c +
d*x)*b)/(sqrt(b)*sqrt(2*sqrt(a)*sqrt(a - b) + 2*a - b)))*a*b**2 + 8*e**(2*
c + 2*d*x)*sqrt(b)*sqrt(2*sqrt(a)*sqrt(a - b) + 2*a - b)*atan((e**(c + ...
```

3.297 $\int \frac{\cosh^2(c+dx)}{(a+b\sinh^2(c+dx))^2}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 79

$$\int \frac{\cosh^2(c+dx)}{(a+b\sinh^2(c+dx))^2}\,dx = \frac{\operatorname{arctanh}\left(\frac{\sqrt{a-b}\tanh(c+dx)}{\sqrt{a}}\right)}{2a^{3/2}\sqrt{a-b}\,d} + \frac{\tanh(c+dx)}{2ad\left(a-(a-b)\tanh^2(c+dx)\right)}$$

output
```
1/2*arctanh((a-b)^(1/2)*tanh(d*x+c)/a^(1/2))/a^(3/2)/(a-b)^(1/2)/d+1/2*tan
h(d*x+c)/a/d/(a-(a-b)*tanh(d*x+c)^2)
```

Mathematica [A] (verified)

Time = 0.36 (sec) , antiderivative size = 78, normalized size of antiderivative = 0.99

$$\int \frac{\cosh^2(c+dx)}{(a+b\sinh^2(c+dx))^2}\,dx = \frac{\frac{\operatorname{arctanh}\left(\frac{\sqrt{a-b}\tanh(c+dx)}{\sqrt{a}}\right)}{\sqrt{a-b}} + \frac{\sqrt{a}\sinh(2(c+dx))}{2a-b+b\cosh(2(c+dx))}}{2a^{3/2}d}$$

input
```
Integrate[Cosh[c + d*x]^2/(a + b*Sinh[c + d*x]^2)^2,x]
```

output
```
(ArcTanh[(Sqrt[a - b]*Tanh[c + d*x])/Sqrt[a]]/Sqrt[a - b] + (Sqrt[a]*Sinh[
2*(c + d*x)])/(2*a - b + b*Cosh[2*(c + d*x)]))/(2*a^(3/2)*d)
```

Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 77, normalized size of antiderivative = 0.97, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.174$, Rules used = {3042, 3670, 215, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cosh^2(c+dx)}{(a+b\sinh^2(c+dx))^2}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\cos(ic+idx)^2}{(a-b\sin(ic+idx)^2)^2}dx$$

$$\Big\downarrow \text{3670}$$

$$\frac{\int \frac{1}{(a-(a-b)\tanh^2(c+dx))^2}d\tanh(c+dx)}{d}$$

$$\Big\downarrow \text{215}$$

$$\frac{\frac{\int \frac{1}{a-(a-b)\tanh^2(c+dx)}d\tanh(c+dx)}{2a}+\frac{\tanh(c+dx)}{2a(a-(a-b)\tanh^2(c+dx))}}{d}$$

$$\Big\downarrow \text{221}$$

$$\frac{\frac{\text{arctanh}\left(\frac{\sqrt{a-b}\tanh(c+dx)}{\sqrt{a}}\right)}{2a^{3/2}\sqrt{a-b}}+\frac{\tanh(c+dx)}{2a(a-(a-b)\tanh^2(c+dx))}}{d}$$

input
```
Int[Cosh[c + d*x]^2/(a + b*Sinh[c + d*x]^2)^2,x]
```

output
```
(ArcTanh[(Sqrt[a - b]*Tanh[c + d*x])/Sqrt[a]]/(2*a^(3/2)*Sqrt[a - b]) + Ta
nh[c + d*x]/(2*a*(a - (a - b)*Tanh[c + d*x]^2)))/d
```

Defintions of rubi rules used

rule 215
```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(-x)*((a + b*x^2)^(p + 1)
/(2*a*(p + 1))), x] + Simp[(2*p + 3)/(2*a*(p + 1))   Int[(a + b*x^2)^(p + 1
), x], x] /; FreeQ[{a, b}, x] && LtQ[p, -1] && (IntegerQ[4*p] || IntegerQ[6
*p])
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3670
```
Int[cos[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(
p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Simp[ff/f   Su
bst[Int[(a + (a + b)*ff^2*x^2)^p/(1 + ff^2*x^2)^(m/2 + p + 1), x], x, Tan[e
 + f*x]/ff], x]] /; FreeQ[{a, b, e, f}, x] && IntegerQ[m/2] && IntegerQ[p]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 235 vs. $2(67) = 134$.

Time = 57.58 (sec) , antiderivative size = 236, normalized size of antiderivative = 2.99

method	result
risch	$-\frac{2\,\mathrm{e}^{2dx+2c}a-\mathrm{e}^{2dx+2c}b+b}{abd\left(b\,\mathrm{e}^{4dx+4c}+4\,\mathrm{e}^{2dx+2c}a-2\,\mathrm{e}^{2dx+2c}b+b\right)}+\frac{\ln\left(\mathrm{e}^{2dx+2c}+\frac{2a\sqrt{a^2-ab}-b\sqrt{a^2-ab}-2a^2+2ab}{b\sqrt{a^2-ab}}\right)}{4\sqrt{a^2-ab}\,da}-$
derivativedivides	$\frac{-\frac{2\left(-\frac{\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^3}{2a}-\frac{\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)}{2a}\right)}{\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^4a-2\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^2a+4b\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^2+a}+\frac{\left(\sqrt{-b(a-b)}-b\right)\operatorname{arctanh}\left(\frac{\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)a}{\sqrt{\left(2\sqrt{-b(a-b)}+a-2b\right)a}}\right)}{2a\sqrt{-b(a-b)}\sqrt{\left(2\sqrt{-b(a-b)}+a-2b\right)a}}-}{d}$
default	$\frac{-\frac{2\left(-\frac{\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^3}{2a}-\frac{\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)}{2a}\right)}{\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^4a-2\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^2a+4b\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^2+a}+\frac{\left(\sqrt{-b(a-b)}-b\right)\operatorname{arctanh}\left(\frac{\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)a}{\sqrt{\left(2\sqrt{-b(a-b)}+a-2b\right)a}}\right)}{2a\sqrt{-b(a-b)}\sqrt{\left(2\sqrt{-b(a-b)}+a-2b\right)a}}-}{d}$

input
```
int(cosh(d*x+c)^2/(a+b*sinh(d*x+c)^2)^2,x,method=_RETURNVERBOSE)
```

output
```
-(2*exp(2*d*x+2*c)*a-exp(2*d*x+2*c)*b+b)/a/b/d/(b*exp(4*d*x+4*c)+4*exp(2*d
*x+2*c)*a-2*exp(2*d*x+2*c)*b+b)+1/4/(a^2-a*b)^(1/2)/d/a*ln(exp(2*d*x+2*c)+
(2*a*(a^2-a*b)^(1/2)-b*(a^2-a*b)^(1/2)-2*a^2+2*a*b)/b/(a^2-a*b)^(1/2))-1/4
/(a^2-a*b)^(1/2)/d/a*ln(exp(2*d*x+2*c)+(2*a*(a^2-a*b)^(1/2)-b*(a^2-a*b)^(1
/2)+2*a^2-2*a*b)/b/(a^2-a*b)^(1/2))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 582 vs. $2(68) = 136$.

Time = 0.16 (sec) , antiderivative size = 1421, normalized size of antiderivative = 17.99

$$\int \frac{\cosh^2(c+dx)}{\left(a+b\sinh^2(c+dx)\right)^2}\,dx = \text{Too large to display}$$

input
```
integrate(cosh(d*x+c)^2/(a+b*sinh(d*x+c)^2)^2,x, algorithm="fricas")
```

output
```
[-1/4*(4*a^2*b - 4*a*b^2 + 4*(2*a^3 - 3*a^2*b + a*b^2)*cosh(d*x + c)^2 + 8
*(2*a^3 - 3*a^2*b + a*b^2)*cosh(d*x + c)*sinh(d*x + c) + 4*(2*a^3 - 3*a^2*
b + a*b^2)*sinh(d*x + c)^2 - (b^2*cosh(d*x + c)^4 + 4*b^2*cosh(d*x + c)*si
nh(d*x + c)^3 + b^2*sinh(d*x + c)^4 + 2*(2*a*b - b^2)*cosh(d*x + c)^2 + 2*
(3*b^2*cosh(d*x + c)^2 + 2*a*b - b^2)*sinh(d*x + c)^2 + b^2 + 4*(b^2*cosh(
d*x + c)^3 + (2*a*b - b^2)*cosh(d*x + c))*sinh(d*x + c))*sqrt(a^2 - a*b)*l
og((b^2*cosh(d*x + c)^4 + 4*b^2*cosh(d*x + c)*sinh(d*x + c)^3 + b^2*sinh(d
*x + c)^4 + 2*(2*a*b - b^2)*cosh(d*x + c)^2 + 2*(3*b^2*cosh(d*x + c)^2 + 2
*a*b - b^2)*sinh(d*x + c)^2 + 8*a^2 - 8*a*b + b^2 + 4*(b^2*cosh(d*x + c)^3
 + (2*a*b - b^2)*cosh(d*x + c))*sinh(d*x + c) - 4*(b*cosh(d*x + c)^2 + 2*b
*cosh(d*x + c)*sinh(d*x + c) + b*sinh(d*x + c)^2 + 2*a - b)*sqrt(a^2 - a*b
))/(b*cosh(d*x + c)^4 + 4*b*cosh(d*x + c)*sinh(d*x + c)^3 + b*sinh(d*x + c
)^4 + 2*(2*a - b)*cosh(d*x + c)^2 + 2*(3*b*cosh(d*x + c)^2 + 2*a - b)*sinh
(d*x + c)^2 + 4*(b*cosh(d*x + c)^3 + (2*a - b)*cosh(d*x + c))*sinh(d*x + c
) + b)))/((a^3*b^2 - a^2*b^3)*d*cosh(d*x + c)^4 + 4*(a^3*b^2 - a^2*b^3)*d*
cosh(d*x + c)*sinh(d*x + c)^3 + (a^3*b^2 - a^2*b^3)*d*sinh(d*x + c)^4 + 2*
(2*a^4*b - 3*a^3*b^2 + a^2*b^3)*d*cosh(d*x + c)^2 + 2*(3*(a^3*b^2 - a^2*b^
3)*d*cosh(d*x + c)^2 + (2*a^4*b - 3*a^3*b^2 + a^2*b^3)*d)*sinh(d*x + c)^2
+ (a^3*b^2 - a^2*b^3)*d + 4*((a^3*b^2 - a^2*b^3)*d*cosh(d*x + c)^3 + (2*a^
4*b - 3*a^3*b^2 + a^2*b^3)*d*cosh(d*x + c))*sinh(d*x + c)), -1/2*(2*a^2...
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\cosh^2(c+dx)}{\left(a+b\sinh^2(c+dx)\right)^2}\,dx = \text{Timed out}$$

input
```
integrate(cosh(d*x+c)**2/(a+b*sinh(d*x+c)**2)**2,x)
```

output
```
Timed out
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{\cosh^2(c+dx)}{\left(a+b\sinh^2(c+dx)\right)^2}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate(cosh(d*x+c)^2/(a+b*sinh(d*x+c)^2)^2,x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(b-a>0)', see `assume?` for more
details)Is
```

Giac [A] (verification not implemented)

Time = 0.46 (sec) , antiderivative size = 126, normalized size of antiderivative = 1.59

$$\int \frac{\cosh^2(c+dx)}{\left(a+b\sinh^2(c+dx)\right)^2}\,dx = \frac{\frac{\arctan\left(\frac{be^{(2\,dx+2\,c)}+2\,a-b}{2\,\sqrt{-a^2+ab}}\right)}{\sqrt{-a^2+ab}a} - \frac{2\,\left(2\,ae^{(2\,dx+2\,c)}-be^{(2\,dx+2\,c)}+b\right)}{\left(be^{(4\,dx+4\,c)}+4\,ae^{(2\,dx+2\,c)}-2\,be^{(2\,dx+2\,c)}+b\right)ab}}{2\,d}$$

input
```
integrate(cosh(d*x+c)^2/(a+b*sinh(d*x+c)^2)^2,x, algorithm="giac")
```

output
```
1/2*(arctan(1/2*(b*e^(2*d*x + 2*c) + 2*a - b)/sqrt(-a^2 + a*b))/(sqrt(-a^2
 + a*b)*a) - 2*(2*a*e^(2*d*x + 2*c) - b*e^(2*d*x + 2*c) + b)/((b*e^(4*d*x
+ 4*c) + 4*a*e^(2*d*x + 2*c) - 2*b*e^(2*d*x + 2*c) + b)*a*b))/d
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\cosh^2(c+dx)}{\left(a+b\sinh^2(c+dx)\right)^2}\,dx = \int \frac{\cosh(c+d\,x)^2}{\left(b\sinh\left(c+d\,x\right)^2+a\right)^2}\,dx$$

input
```
int(cosh(c + d*x)^2/(a + b*sinh(c + d*x)^2)^2,x)
```

output
```
int(cosh(c + d*x)^2/(a + b*sinh(c + d*x)^2)^2, x)
```

Reduce [B] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 700, normalized size of antiderivative = 8.86

$$\int \frac{\cosh^2(c+dx)}{\left(a+b\sinh^2(c+dx)\right)^2}\,dx$$
$$= \frac{e^{4dx+4c}\sqrt{a}\,\sqrt{a-b}\,\log\left(-\sqrt{2\sqrt{a}\,\sqrt{a-b}-2a+b}+e^{dx+c}\sqrt{b}\right)b+e^{4dx+4c}\sqrt{a}\,\sqrt{a-b}\,\log\left(\sqrt{2\sqrt{a}\,\sqrt{a-b}\right.}{}$$

input
```
int(cosh(d*x+c)^2/(a+b*sinh(d*x+c)^2)^2,x)
```

output
```
(e**(4*c + 4*d*x)*sqrt(a)*sqrt(a - b)*log( - sqrt(2*sqrt(a)*sqrt(a - b) -
2*a + b) + e**(c + d*x)*sqrt(b))*b + e**(4*c + 4*d*x)*sqrt(a)*sqrt(a - b)*
log(sqrt(2*sqrt(a)*sqrt(a - b) - 2*a + b) + e**(c + d*x)*sqrt(b))*b - e**(
4*c + 4*d*x)*sqrt(a)*sqrt(a - b)*log(2*sqrt(a)*sqrt(a - b) + e**(2*c + 2*d
*x)*b + 2*a - b)*b + 4*e**(2*c + 2*d*x)*sqrt(a)*sqrt(a - b)*log( - sqrt(2*
sqrt(a)*sqrt(a - b) - 2*a + b) + e**(c + d*x)*sqrt(b))*a - 2*e**(2*c + 2*d
*x)*sqrt(a)*sqrt(a - b)*log( - sqrt(2*sqrt(a)*sqrt(a - b) - 2*a + b) + e**
(c + d*x)*sqrt(b))*b + 4*e**(2*c + 2*d*x)*sqrt(a)*sqrt(a - b)*log(sqrt(2*s
qrt(a)*sqrt(a - b) - 2*a + b) + e**(c + d*x)*sqrt(b))*a - 2*e**(2*c + 2*d*
x)*sqrt(a)*sqrt(a - b)*log(sqrt(2*sqrt(a)*sqrt(a - b) - 2*a + b) + e**(c +
 d*x)*sqrt(b))*b - 4*e**(2*c + 2*d*x)*sqrt(a)*sqrt(a - b)*log(2*sqrt(a)*sq
rt(a - b) + e**(2*c + 2*d*x)*b + 2*a - b)*a + 2*e**(2*c + 2*d*x)*sqrt(a)*s
qrt(a - b)*log(2*sqrt(a)*sqrt(a - b) + e**(2*c + 2*d*x)*b + 2*a - b)*b + s
qrt(a)*sqrt(a - b)*log( - sqrt(2*sqrt(a)*sqrt(a - b) - 2*a + b) + e**(c +
d*x)*sqrt(b))*b + sqrt(a)*sqrt(a - b)*log(sqrt(2*sqrt(a)*sqrt(a - b) - 2*a
 + b) + e**(c + d*x)*sqrt(b))*b - sqrt(a)*sqrt(a - b)*log(2*sqrt(a)*sqrt(a
 - b) + e**(2*c + 2*d*x)*b + 2*a - b)*b + 2*e**(4*c + 4*d*x)*a**2 - 2*e**(
4*c + 4*d*x)*a*b - 2*a**2 + 2*a*b)/(4*a**2*d*(e**(4*c + 4*d*x)*a*b - e**(4
*c + 4*d*x)*b**2 + 4*e**(2*c + 2*d*x)*a**2 - 6*e**(2*c + 2*d*x)*a*b + 2*e*
*(2*c + 2*d*x)*b**2 + a*b - b**2))
```

3.298 $\int \frac{\cosh(c+dx)}{\left(a+b\sinh^2(c+dx)\right)^2}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 66

$$\int \frac{\cosh(c+dx)}{\left(a+b\sinh^2(c+dx)\right)^2}\,dx=\frac{\arctan\left(\frac{\sqrt{b}\sinh(c+dx)}{\sqrt{a}}\right)}{2a^{3/2}\sqrt{b}d}+\frac{\sinh(c+dx)}{2ad\left(a+b\sinh^2(c+dx)\right)}$$

output
```
1/2*arctan(b^(1/2)*sinh(d*x+c)/a^(1/2))/a^(3/2)/b^(1/2)/d+1/2*sinh(d*x+c)/
a/d/(a+b*sinh(d*x+c)^2)
```

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 64, normalized size of antiderivative = 0.97

$$\int \frac{\cosh(c+dx)}{\left(a+b\sinh^2(c+dx)\right)^2}\,dx=\frac{\frac{\arctan\left(\frac{\sqrt{b}\sinh(c+dx)}{\sqrt{a}}\right)}{2a^{3/2}\sqrt{b}}+\frac{\sinh(c+dx)}{2a\left(a+b\sinh^2(c+dx)\right)}}{d}$$

input
```
Integrate[Cosh[c + d*x]/(a + b*Sinh[c + d*x]^2)^2,x]
```

output
```
(ArcTan[(Sqrt[b]*Sinh[c + d*x])/Sqrt[a]]/(2*a^(3/2)*Sqrt[b]) + Sinh[c + d*
x]/(2*a*(a + b*Sinh[c + d*x]^2)))/d
```

Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 64, normalized size of antiderivative = 0.97, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.190, Rules used = {3042, 3669, 215, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cosh(c+dx)}{\left(a+b\sinh^2(c+dx)\right)^2}dx$$

$$\Big\downarrow \; 3042$$

$$\int \frac{\cos(ic+idx)}{\left(a-b\sin(ic+idx)^2\right)^2}dx$$

$$\Big\downarrow \; 3669$$

$$\frac{\int \frac{1}{\left(b\sinh^2(c+dx)+a\right)^2}d\sinh(c+dx)}{d}$$

$$\Big\downarrow \; 215$$

$$\frac{\frac{\int \frac{1}{b\sinh^2(c+dx)+a}d\sinh(c+dx)}{2a}+\frac{\sinh(c+dx)}{2a\left(a+b\sinh^2(c+dx)\right)}}{d}$$

$$\Big\downarrow \; 218$$

$$\frac{\frac{\arctan\left(\frac{\sqrt{b}\sinh(c+dx)}{\sqrt{a}}\right)}{2a^{3/2}\sqrt{b}}+\frac{\sinh(c+dx)}{2a\left(a+b\sinh^2(c+dx)\right)}}{d}$$

input
```
Int[Cosh[c + d*x]/(a + b*Sinh[c + d*x]^2)^2,x]
```

output
```
(ArcTan[(Sqrt[b]*Sinh[c + d*x])/Sqrt[a]]/(2*a^(3/2)*Sqrt[b]) + Sinh[c + d*
x]/(2*a*(a + b*Sinh[c + d*x]^2)))/d
```

Defintions of rubi rules used

rule 215
```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(-x)*((a + b*x^2)^(p + 1)
/(2*a*(p + 1))), x] + Simp[(2*p + 3)/(2*a*(p + 1))   Int[(a + b*x^2)^(p + 1
), x], x] /; FreeQ[{a, b}, x] && LtQ[p, -1] && (IntegerQ[4*p] || IntegerQ[6
*p])
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3669
```
Int[cos[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^
(p_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Simp[ff/f   S
ubst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a + b*ff^2*x^2)^p, x], x, Sin[e + f*x]
/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m - 1)/2]
```

Maple [A] (verified)

Time = 59.60 (sec) , antiderivative size = 55, normalized size of antiderivative = 0.83

method	result	size
derivativedivides	$\frac{\frac{\sinh(dx+c)}{2a\left(a+b\sinh(dx+c)^2\right)}+\frac{\arctan\left(\frac{b\sinh(dx+c)}{\sqrt{ab}}\right)}{2a\sqrt{ab}}}{d}$	55
default	$\frac{\frac{\sinh(dx+c)}{2a\left(a+b\sinh(dx+c)^2\right)}+\frac{\arctan\left(\frac{b\sinh(dx+c)}{\sqrt{ab}}\right)}{2a\sqrt{ab}}}{d}$	55
risch	$\frac{e^{dx+c}\left(e^{2dx+2c}-1\right)}{ad\left(b\,e^{4dx+4c}+4\,e^{2dx+2c}a-2\,e^{2dx+2c}b+b\right)}-\frac{\ln\left(e^{2dx+2c}-\frac{2a\,e^{dx+c}}{\sqrt{-ab}}-1\right)}{4\sqrt{-ab}\,da}+\frac{\ln\left(e^{2dx+2c}+\frac{2a\,e^{dx+c}}{\sqrt{-ab}}-1\right)}{4\sqrt{-ab}\,da}$	147

input
```
int(cosh(d*x+c)/(a+b*sinh(d*x+c)^2)^2,x,method=_RETURNVERBOSE)
```

output
```
1/d*(1/2*sinh(d*x+c)/a/(a+b*sinh(d*x+c)^2)+1/2/a/(a*b)^(1/2)*arctan(b*sinh
(d*x+c)/(a*b)^(1/2)))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 630 vs. $2(54) = 108$.

Time = 0.10 (sec) , antiderivative size = 1324, normalized size of antiderivative = 20.06

$$\int \frac{\cosh(c+dx)}{\left(a+b\sinh^2(c+dx)\right)^2}\,dx = \text{Too large to display}$$

input
```
integrate(cosh(d*x+c)/(a+b*sinh(d*x+c)^2)^2,x, algorithm="fricas")
```

output
```
[1/4*(4*a*b*cosh(d*x + c)^3 + 12*a*b*cosh(d*x + c)*sinh(d*x + c)^2 + 4*a*b
*sinh(d*x + c)^3 - 4*a*b*cosh(d*x + c) - (b*cosh(d*x + c)^4 + 4*b*cosh(d*x
 + c)*sinh(d*x + c)^3 + b*sinh(d*x + c)^4 + 2*(2*a - b)*cosh(d*x + c)^2 +
2*(3*b*cosh(d*x + c)^2 + 2*a - b)*sinh(d*x + c)^2 + 4*(b*cosh(d*x + c)^3 +
 (2*a - b)*cosh(d*x + c))*sinh(d*x + c) + b)*sqrt(-a*b)*log((b*cosh(d*x +
c)^4 + 4*b*cosh(d*x + c)*sinh(d*x + c)^3 + b*sinh(d*x + c)^4 - 2*(2*a + b)
*cosh(d*x + c)^2 + 2*(3*b*cosh(d*x + c)^2 - 2*a - b)*sinh(d*x + c)^2 + 4*(
b*cosh(d*x + c)^3 - (2*a + b)*cosh(d*x + c))*sinh(d*x + c) - 4*(cosh(d*x +
 c)^3 + 3*cosh(d*x + c)*sinh(d*x + c)^2 + sinh(d*x + c)^3 + (3*cosh(d*x +
c)^2 - 1)*sinh(d*x + c) - cosh(d*x + c))*sqrt(-a*b) + b)/(b*cosh(d*x + c)^
4 + 4*b*cosh(d*x + c)*sinh(d*x + c)^3 + b*sinh(d*x + c)^4 + 2*(2*a - b)*co
sh(d*x + c)^2 + 2*(3*b*cosh(d*x + c)^2 + 2*a - b)*sinh(d*x + c)^2 + 4*(b*c
osh(d*x + c)^3 + (2*a - b)*cosh(d*x + c))*sinh(d*x + c) + b)) + 4*(3*a*b*c
osh(d*x + c)^2 - a*b)*sinh(d*x + c))/(a^2*b^2*d*cosh(d*x + c)^4 + 4*a^2*b^
2*d*cosh(d*x + c)*sinh(d*x + c)^3 + a^2*b^2*d*sinh(d*x + c)^4 + a^2*b^2*d
+ 2*(2*a^3*b - a^2*b^2)*d*cosh(d*x + c)^2 + 2*(3*a^2*b^2*d*cosh(d*x + c)^2
 + (2*a^3*b - a^2*b^2)*d)*sinh(d*x + c)^2 + 4*(a^2*b^2*d*cosh(d*x + c)^3 +
 (2*a^3*b - a^2*b^2)*d*cosh(d*x + c))*sinh(d*x + c)), 1/2*(2*a*b*cosh(d*x
+ c)^3 + 6*a*b*cosh(d*x + c)*sinh(d*x + c)^2 + 2*a*b*sinh(d*x + c)^3 - 2*a
*b*cosh(d*x + c) - (b*cosh(d*x + c)^4 + 4*b*cosh(d*x + c)*sinh(d*x + c)...
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 377 vs. $2(54) = 108$.

Time = 6.13 (sec) , antiderivative size = 377, normalized size of antiderivative = 5.71

$$\int \frac{\cosh(c+dx)}{\left(a+b\sinh^2(c+dx)\right)^2}\,dx$$
$$= \begin{cases} \frac{\tilde{\infty}x\cosh(c)}{\sinh^4(c)} \\ \frac{\sinh(c+dx)}{a^2d} \\ -\frac{1}{3b^2d\sinh^3(c+dx)} \\ \frac{x\cosh(c)}{\left(a+b\sinh^2(c)\right)^2} \\ \frac{a\log\left(-\sqrt{-\frac{a}{b}}+\sinh(c+dx)\right)}{4a^2bd\sqrt{-\frac{a}{b}}+4ab^2d\sqrt{-\frac{a}{b}}\sinh^2(c+dx)} - \frac{a\log\left(\sqrt{-\frac{a}{b}}+\sinh(c+dx)\right)}{4a^2bd\sqrt{-\frac{a}{b}}+4ab^2d\sqrt{-\frac{a}{b}}\sinh^2(c+dx)} + \frac{2b\sqrt{-\frac{a}{b}}\sinh(c+dx)}{4a^2bd\sqrt{-\frac{a}{b}}+4ab^2d\sqrt{-\frac{a}{b}}\sinh^2(c+dx)} + \frac{b\log\left(-\sqrt{}\right.}{4a^2bd\sqrt{}} \end{cases}$$

input
```
integrate(cosh(d*x+c)/(a+b*sinh(d*x+c)**2)**2,x)
```

output
```
Piecewise((zoo*x*cosh(c)/sinh(c)**4, Eq(a, 0) & Eq(b, 0) & Eq(d, 0)), (sin
h(c + d*x)/(a**2*d), Eq(b, 0)), (-1/(3*b**2*d*sinh(c + d*x)**3), Eq(a, 0))
, (x*cosh(c)/(a + b*sinh(c)**2)**2, Eq(d, 0)), (a*log(-sqrt(-a/b) + sinh(c
 + d*x))/(4*a**2*b*d*sqrt(-a/b) + 4*a*b**2*d*sqrt(-a/b)*sinh(c + d*x)**2)
- a*log(sqrt(-a/b) + sinh(c + d*x))/(4*a**2*b*d*sqrt(-a/b) + 4*a*b**2*d*sq
rt(-a/b)*sinh(c + d*x)**2) + 2*b*sqrt(-a/b)*sinh(c + d*x)/(4*a**2*b*d*sqrt
(-a/b) + 4*a*b**2*d*sqrt(-a/b)*sinh(c + d*x)**2) + b*log(-sqrt(-a/b) + sin
h(c + d*x))*sinh(c + d*x)**2/(4*a**2*b*d*sqrt(-a/b) + 4*a*b**2*d*sqrt(-a/b
)*sinh(c + d*x)**2) - b*log(sqrt(-a/b) + sinh(c + d*x))*sinh(c + d*x)**2/(
4*a**2*b*d*sqrt(-a/b) + 4*a*b**2*d*sqrt(-a/b)*sinh(c + d*x)**2), True))
```

Maxima [F]

$$\int \frac{\cosh(c+dx)}{\left(a+b\sinh^2(c+dx)\right)^2}\,dx = \int \frac{\cosh(dx+c)}{\left(b\sinh(dx+c)^2+a\right)^2}\,dx$$

input
```
integrate(cosh(d*x+c)/(a+b*sinh(d*x+c)^2)^2,x, algorithm="maxima")
```

output
```
(e^(3*d*x + 3*c) - e^(d*x + c))/(a*b*d*e^(4*d*x + 4*c) + a*b*d + 2*(2*a^2*
d*e^(2*c) - a*b*d*e^(2*c))*e^(2*d*x)) + 1/2*integrate(2*(e^(3*d*x + 3*c) +
 e^(d*x + c))/(a*b*e^(4*d*x + 4*c) + a*b + 2*(2*a^2*e^(2*c) - a*b*e^(2*c))
*e^(2*d*x)), x)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{\cosh(c+dx)}{\left(a+b\sinh^2(c+dx)\right)^2}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(cosh(d*x+c)/(a+b*sinh(d*x+c)^2)^2,x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Limit: Max order reached or unable
to make series expansion Error: Bad Argument Value
```

Mupad [B] (verification not implemented)

Time = 1.76 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.82

$$\int \frac{\cosh(c+dx)}{\left(a+b\sinh^2(c+dx)\right)^2}\,dx = \frac{\sinh(c+d\,x)}{2\,a\,\left(b\,d\sinh(c+d\,x)^2+a\,d\right)} + \frac{\operatorname{atan}\left(\frac{\sqrt{b}\sinh(c+d\,x)}{\sqrt{a}}\right)}{2\,a^{3/2}\,\sqrt{b}\,d}$$

input
```
int(cosh(c + d*x)/(a + b*sinh(c + d*x)^2)^2,x)
```

output
```
sinh(c + d*x)/(2*a*(a*d + b*d*sinh(c + d*x)^2)) + atan((b^(1/2)*sinh(c + d
*x))/a^(1/2))/(2*a^(3/2)*b^(1/2)*d)
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 89, normalized size of antiderivative = 1.35

$$\int \frac{\cosh(c+dx)}{\left(a+b\sinh^2(c+dx)\right)^2}\,dx = \frac{\sqrt{b}\,\sqrt{a}\,atan\left(\frac{\sinh(dx+c)b}{\sqrt{b}\,\sqrt{a}}\right)\sinh\left(dx+c\right)^2 b+\sqrt{b}\,\sqrt{a}\,atan\left(\frac{\sinh(dx+c)b}{\sqrt{b}\,\sqrt{a}}\right)a+\sinh\left(dx+c\right)ab}{2a^2bd\left(\sinh\left(dx+c\right)^2 b+a\right)}$$

input
```
int(cosh(d*x+c)/(a+b*sinh(d*x+c)^2)^2,x)
```

output
```
(sqrt(b)*sqrt(a)*atan((sinh(c + d*x)*b)/(sqrt(b)*sqrt(a)))*sinh(c + d*x)**
2*b + sqrt(b)*sqrt(a)*atan((sinh(c + d*x)*b)/(sqrt(b)*sqrt(a)))*a + sinh(c
 + d*x)*a*b)/(2*a**2*b*d*(sinh(c + d*x)**2*b + a))
```

3.299 $\int \frac{\text{sech}(c+dx)}{\left(a+b\sinh^2(c+dx)\right)^2}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 106

$$\int \frac{\text{sech}(c+dx)}{\left(a+b\sinh^2(c+dx)\right)^2}\,dx=\frac{\arctan(\sinh(c+dx))}{(a-b)^2d}-\frac{(3a-b)\sqrt{b}\arctan\left(\frac{\sqrt{b}\sinh(c+dx)}{\sqrt{a}}\right)}{2a^{3/2}(a-b)^2d}-\frac{b\sinh(c+dx)}{2a(a-b)d\left(a+b\sinh^2(c+dx)\right)}$$

output
```
arctan(sinh(d*x+c))/(a-b)^2/d-1/2*(3*a-b)*b^(1/2)*arctan(b^(1/2)*sinh(d*x+
c)/a^(1/2))/a^(3/2)/(a-b)^2/d-1/2*b*sinh(d*x+c)/a/(a-b)/d/(a+b*sinh(d*x+c)
^2)
```

Mathematica [A] (verified)

Time = 0.31 (sec) , antiderivative size = 174, normalized size of antiderivative = 1.64

$$\int \frac{\text{sech}(c+dx)}{\left(a+b\sinh^2(c+dx)\right)^2}\,dx$$

$$=\frac{(2a-b)\left(-\sqrt{b}(-3a+b)\arctan\left(\frac{\sqrt{a}\text{csch}(c+dx)}{\sqrt{b}}\right)+4a^{3/2}\arctan\left(\tanh\left(\frac{1}{2}(c+dx)\right)\right)\right)+\left(-b^{3/2}(-3a+b)\right.}{2a^{3/2}(a-b)^2d(2a-b}$$

input
```
Integrate[Sech[c + d*x]/(a + b*Sinh[c + d*x]^2)^2,x]
```

output
```
((2*a - b)*(-(Sqrt[b]*(-3*a + b)*ArcTan[(Sqrt[a]*Csch[c + d*x])/Sqrt[b]])
+ 4*a^(3/2)*ArcTan[Tanh[(c + d*x)/2]]) + (-(b^(3/2)*(-3*a + b)*ArcTan[(Sqr
t[a]*Csch[c + d*x])/Sqrt[b]]) + 4*a^(3/2)*b*ArcTan[Tanh[(c + d*x)/2]])*Cos
h[2*(c + d*x)] - 2*Sqrt[a]*(a - b)*b*Sinh[c + d*x])/(2*a^(3/2)*(a - b)^2*d
*(2*a - b + b*Cosh[2*(c + d*x)]))
```

Rubi [A] (verified)

Time = 0.33 (sec) , antiderivative size = 116, normalized size of antiderivative = 1.09, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {3042, 3669, 316, 397, 216, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\text{sech}(c+dx)}{\left(a+b\sinh^2(c+dx)\right)^2}dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \frac{1}{\cos(ic+idx)\left(a-b\sin(ic+idx)^2\right)^2}dx$$

$$\Big\downarrow \text{ 3669}$$

$$\frac{\int \frac{1}{\left(\sinh^2(c+dx)+1\right)\left(b\sinh^2(c+dx)+a\right)^2}d\sinh(c+dx)}{d}$$

$$\Big\downarrow \text{ 316}$$

$$\frac{\frac{\int \frac{-b\sinh^2(c+dx)+2a-b}{\left(\sinh^2(c+dx)+1\right)\left(b\sinh^2(c+dx)+a\right)}d\sinh(c+dx)}{2a(a-b)}-\frac{b\sinh(c+dx)}{2a(a-b)\left(a+b\sinh^2(c+dx)\right)}}{d}$$

$$\Big\downarrow \text{ 397}$$

$$\frac{\frac{\frac{2a\int \frac{1}{\sinh^2(c+dx)+1}d\sinh(c+dx)}{a-b}-\frac{b(3a-b)\int \frac{1}{b\sinh^2(c+dx)+a}d\sinh(c+dx)}{a-b}}{2a(a-b)}-\frac{b\sinh(c+dx)}{2a(a-b)\left(a+b\sinh^2(c+dx)\right)}}{d}$$

$$\Big\downarrow \text{216}$$

$$\frac{\frac{\frac{2a\arctan(\sinh(c+dx))}{a-b}-\frac{b(3a-b)\int\frac{1}{b\sinh^2(c+dx)+a}d\sinh(c+dx)}{a-b}}{2a(a-b)}-\frac{b\sinh(c+dx)}{2a(a-b)(a+b\sinh^2(c+dx))}}{d}$$

$$\Big\downarrow \text{218}$$

$$\frac{\frac{\frac{2a\arctan(\sinh(c+dx))}{a-b}-\frac{\sqrt{b}(3a-b)\arctan\left(\frac{\sqrt{b}\sinh(c+dx)}{\sqrt{a}}\right)}{\sqrt{a}(a-b)}}{2a(a-b)}-\frac{b\sinh(c+dx)}{2a(a-b)(a+b\sinh^2(c+dx))}}{d}$$

input
```
Int[Sech[c + d*x]/(a + b*Sinh[c + d*x]^2)^2,x]
```

output
```
(((2*a*ArcTan[Sinh[c + d*x]])/(a - b) - ((3*a - b)*Sqrt[b]*ArcTan[(Sqrt[b]
*Sinh[c + d*x])/Sqrt[a]])/(Sqrt[a]*(a - b)))/(2*a*(a - b)) - (b*Sinh[c + d
*x])/(2*a*(a - b)*(a + b*Sinh[c + d*x]^2)))/d
```

Defintions of rubi rules used

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 316
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_), x_Symbol] :> Sim
p[(-b)*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^(q + 1)/(2*a*(p + 1)*(b*c - a*d))
), x] + Simp[1/(2*a*(p + 1)*(b*c - a*d))   Int[(a + b*x^2)^(p + 1)*(c + d*x
^2)^q*Simp[b*c + 2*(p + 1)*(b*c - a*d) + d*b*(2*(p + q + 2) + 1)*x^2, x], x
], x] /; FreeQ[{a, b, c, d, q}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] &&  !
( !IntegerQ[p] && IntegerQ[q] && LtQ[q, -1]) && IntBinomialQ[a, b, c, d, 2,
 p, q, x]
```

rule 397
```
Int[((e_) + (f_.)*(x_)^2)/(((a_) + (b_.)*(x_)^2)*((c_) + (d_.)*(x_)^2)), x_
Symbol] :> Simp[(b*e - a*f)/(b*c - a*d)   Int[1/(a + b*x^2), x], x] - Simp[
(d*e - c*f)/(b*c - a*d)   Int[1/(c + d*x^2), x], x] /; FreeQ[{a, b, c, d, e
, f}, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3669
```
Int[cos[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^
(p_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Simp[ff/f   S
ubst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a + b*ff^2*x^2)^p, x], x, Sin[e + f*x]
/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m - 1)/2]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 304 vs. $2(94) = 188$.

Time = 0.24 (sec) , antiderivative size = 305, normalized size of antiderivative = 2.88

$$-\frac{\frac{2b\left(\frac{-\frac{(a-b)\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^3}{2a}+\frac{(a-b)\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)}{2a}}{\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^4 a-2\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^2 a+4b\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^2+a}+\frac{(3a-b)\left(-\frac{\left(a+\sqrt{-b(a-b)}-b\right)\operatorname{arctanh}\left(\frac{\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)a}{\sqrt{\left(2\sqrt{-b(a-b)}+a-2b\right)a}}\right)}{2a\sqrt{-b(a-b)}\sqrt{\left(2\sqrt{-b(a-b)}+a-2b\right)a}}+\frac{\left(-a+\sqrt{-b(a-b)}+b\right)\text{ar}\cdots}{2a\sqrt{-b(a-b)}\cdots}\cdots\right.}{2}\cdots\right.}{(a-b)^2}\cdots}{d}$$

input
```
int(sech(d*x+c)/(a+b*sinh(d*x+c)^2)^2,x)
```

output
```
1/d*(-2*b/(a-b)^2*((-1/2*(a-b)/a*tanh(1/2*d*x+1/2*c)^3+1/2*(a-b)/a*tanh(1/
2*d*x+1/2*c))/(tanh(1/2*d*x+1/2*c)^4*a-2*tanh(1/2*d*x+1/2*c)^2*a+4*b*tanh(
1/2*d*x+1/2*c)^2+a)+1/2*(3*a-b)*(-1/2*(a+(-b*(a-b))^(1/2)-b)/a/(-b*(a-b))^
(1/2)/((2*(-b*(a-b))^(1/2)+a-2*b)*a)^(1/2)*arctanh(tanh(1/2*d*x+1/2*c)*a/(
(2*(-b*(a-b))^(1/2)+a-2*b)*a)^(1/2))+1/2*(-a+(-b*(a-b))^(1/2)+b)/a/(-b*(a-
b))^(1/2)/((2*(-b*(a-b))^(1/2)-a+2*b)*a)^(1/2)*arctan(tanh(1/2*d*x+1/2*c)*
a/((2*(-b*(a-b))^(1/2)-a+2*b)*a)^(1/2))))+2/(a-b)^2*arctan(tanh(1/2*d*x+1/
2*c)))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1068 vs. $2(94) = 188$.

Time = 0.15 (sec) , antiderivative size = 2143, normalized size of antiderivative = 20.22

$$\int \frac{\operatorname{sech}(c+dx)}{\left(a+b\sinh^2(c+dx)\right)^2}\,dx = \text{Too large to display}$$

input
```
integrate(sech(d*x+c)/(a+b*sinh(d*x+c)^2)^2,x, algorithm="fricas")
```

output

```
[-1/4*(4*(a*b - b^2)*cosh(d*x + c)^3 + 12*(a*b - b^2)*cosh(d*x + c)*sinh(d
*x + c)^2 + 4*(a*b - b^2)*sinh(d*x + c)^3 + ((3*a*b - b^2)*cosh(d*x + c)^4
 + 4*(3*a*b - b^2)*cosh(d*x + c)*sinh(d*x + c)^3 + (3*a*b - b^2)*sinh(d*x
+ c)^4 + 2*(6*a^2 - 5*a*b + b^2)*cosh(d*x + c)^2 + 2*(3*(3*a*b - b^2)*cosh
(d*x + c)^2 + 6*a^2 - 5*a*b + b^2)*sinh(d*x + c)^2 + 3*a*b - b^2 + 4*((3*a
*b - b^2)*cosh(d*x + c)^3 + (6*a^2 - 5*a*b + b^2)*cosh(d*x + c))*sinh(d*x
+ c))*sqrt(-b/a)*log((b*cosh(d*x + c)^4 + 4*b*cosh(d*x + c)*sinh(d*x + c)^
3 + b*sinh(d*x + c)^4 - 2*(2*a + b)*cosh(d*x + c)^2 + 2*(3*b*cosh(d*x + c)
^2 - 2*a - b)*sinh(d*x + c)^2 + 4*(b*cosh(d*x + c)^3 - (2*a + b)*cosh(d*x
+ c))*sinh(d*x + c) + 4*(a*cosh(d*x + c)^3 + 3*a*cosh(d*x + c)*sinh(d*x +
c)^2 + a*sinh(d*x + c)^3 - a*cosh(d*x + c) + (3*a*cosh(d*x + c)^2 - a)*sin
h(d*x + c))*sqrt(-b/a) + b)/(b*cosh(d*x + c)^4 + 4*b*cosh(d*x + c)*sinh(d*
x + c)^3 + b*sinh(d*x + c)^4 + 2*(2*a - b)*cosh(d*x + c)^2 + 2*(3*b*cosh(d
*x + c)^2 + 2*a - b)*sinh(d*x + c)^2 + 4*(b*cosh(d*x + c)^3 + (2*a - b)*co
sh(d*x + c))*sinh(d*x + c) + b)) - 8*(a*b*cosh(d*x + c)^4 + 4*a*b*cosh(d*x
 + c)*sinh(d*x + c)^3 + a*b*sinh(d*x + c)^4 + 2*(2*a^2 - a*b)*cosh(d*x + c
)^2 + 2*(3*a*b*cosh(d*x + c)^2 + 2*a^2 - a*b)*sinh(d*x + c)^2 + a*b + 4*(a
*b*cosh(d*x + c)^3 + (2*a^2 - a*b)*cosh(d*x + c))*sinh(d*x + c))*arctan(co
sh(d*x + c) + sinh(d*x + c)) - 4*(a*b - b^2)*cosh(d*x + c) + 4*(3*(a*b - b
^2)*cosh(d*x + c)^2 - a*b + b^2)*sinh(d*x + c))/((a^3*b - 2*a^2*b^2 + a...
```

Sympy [F]

$$\int \frac{\operatorname{sech}(c+dx)}{\left(a+b\sinh^2(c+dx)\right)^2}\,dx = \int \frac{\operatorname{sech}\left(c+dx\right)}{\left(a+b\sinh^2\left(c+dx\right)\right)^2}\,dx$$

input

```
integrate(sech(d*x+c)/(a+b*sinh(d*x+c)**2)**2,x)
```

output

```
Integral(sech(c + d*x)/(a + b*sinh(c + d*x)**2)**2, x)
```

Maxima [F]

$$\int \frac{\operatorname{sech}(c+dx)}{\left(a+b\sinh^2(c+dx)\right)^2}\,dx = \int \frac{\operatorname{sech}(dx+c)}{\left(b\sinh(dx+c)^2+a\right)^2}\,dx$$

input
```
integrate(sech(d*x+c)/(a+b*sinh(d*x+c)^2)^2,x, algorithm="maxima")
```

output
```
-(b*e^(3*d*x + 3*c) - b*e^(d*x + c))/(a^2*b*d - a*b^2*d + (a^2*b*d*e^(4*c)
 - a*b^2*d*e^(4*c))*e^(4*d*x) + 2*(2*a^3*d*e^(2*c) - 3*a^2*b*d*e^(2*c) + a
*b^2*d*e^(2*c))*e^(2*d*x)) + 2*arctan(e^(d*x + c))/(a^2*d - 2*a*b*d + b^2*
d) - 2*integrate(1/2*((3*a*b*e^(3*c) - b^2*e^(3*c))*e^(3*d*x) + (3*a*b*e^c
 - b^2*e^c)*e^(d*x))/(a^3*b - 2*a^2*b^2 + a*b^3 + (a^3*b*e^(4*c) - 2*a^2*b
^2*e^(4*c) + a*b^3*e^(4*c))*e^(4*d*x) + 2*(2*a^4*e^(2*c) - 5*a^3*b*e^(2*c)
 + 4*a^2*b^2*e^(2*c) - a*b^3*e^(2*c))*e^(2*d*x)), x)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{\operatorname{sech}(c+dx)}{\left(a+b\sinh^2(c+dx)\right)^2}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(sech(d*x+c)/(a+b*sinh(d*x+c)^2)^2,x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Limit: Max order reached or unable
to make series expansion Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\operatorname{sech}(c+dx)}{\left(a+b\sinh^2(c+dx)\right)^2}\,dx=\int \frac{1}{\cosh\left(c+d\,x\right)\,\left(b\sinh\left(c+d\,x\right)^2+a\right)^2}\,dx$$

input
```
int(1/(cosh(c + d*x)*(a + b*sinh(c + d*x)^2)^2),x)
```

output
```
int(1/(cosh(c + d*x)*(a + b*sinh(c + d*x)^2)^2), x)
```

Reduce [B] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 3045, normalized size of antiderivative = 28.73

$$\int \frac{\operatorname{sech}(c+dx)}{\left(a+b\sinh^2(c+dx)\right)^2}\,dx=\text{Too large to display}$$

input
```
int(sech(d*x+c)/(a+b*sinh(d*x+c)^2)^2,x)
```

output
```
(8*e**(4*c + 4*d*x)*atan(e**(c + d*x))*a**2*b**2 + 32*e**(2*c + 2*d*x)*ata
n(e**(c + d*x))*a**3*b - 16*e**(2*c + 2*d*x)*atan(e**(c + d*x))*a**2*b**2
+ 8*atan(e**(c + d*x))*a**2*b**2 + 6*e**(4*c + 4*d*x)*sqrt(b)*sqrt(a)*sqrt
(a - b)*sqrt(2*sqrt(a)*sqrt(a - b) + 2*a - b)*atan((e**(c + d*x)*b)/(sqrt(
b)*sqrt(2*sqrt(a)*sqrt(a - b) + 2*a - b)))*a*b - 2*e**(4*c + 4*d*x)*sqrt(b
)*sqrt(a)*sqrt(a - b)*sqrt(2*sqrt(a)*sqrt(a - b) + 2*a - b)*atan((e**(c +
d*x)*b)/(sqrt(b)*sqrt(2*sqrt(a)*sqrt(a - b) + 2*a - b)))*b**2 + 24*e**(2*c
 + 2*d*x)*sqrt(b)*sqrt(a)*sqrt(a - b)*sqrt(2*sqrt(a)*sqrt(a - b) + 2*a - b
)*atan((e**(c + d*x)*b)/(sqrt(b)*sqrt(2*sqrt(a)*sqrt(a - b) + 2*a - b)))*a
**2 - 20*e**(2*c + 2*d*x)*sqrt(b)*sqrt(a)*sqrt(a - b)*sqrt(2*sqrt(a)*sqrt(
a - b) + 2*a - b)*atan((e**(c + d*x)*b)/(sqrt(b)*sqrt(2*sqrt(a)*sqrt(a - b
) + 2*a - b)))*a*b + 4*e**(2*c + 2*d*x)*sqrt(b)*sqrt(a)*sqrt(a - b)*sqrt(2
*sqrt(a)*sqrt(a - b) + 2*a - b)*atan((e**(c + d*x)*b)/(sqrt(b)*sqrt(2*sqrt
(a)*sqrt(a - b) + 2*a - b)))*b**2 + 6*sqrt(b)*sqrt(a)*sqrt(a - b)*sqrt(2*s
qrt(a)*sqrt(a - b) + 2*a - b)*atan((e**(c + d*x)*b)/(sqrt(b)*sqrt(2*sqrt(a
)*sqrt(a - b) + 2*a - b)))*a*b - 2*sqrt(b)*sqrt(a)*sqrt(a - b)*sqrt(2*sqrt
(a)*sqrt(a - b) + 2*a - b)*atan((e**(c + d*x)*b)/(sqrt(b)*sqrt(2*sqrt(a)*s
qrt(a - b) + 2*a - b)))*b**2 - 6*e**(4*c + 4*d*x)*sqrt(b)*sqrt(2*sqrt(a)*s
qrt(a - b) + 2*a - b)*atan((e**(c + d*x)*b)/(sqrt(b)*sqrt(2*sqrt(a)*sqrt(a
 - b) + 2*a - b)))*a**2*b + 2*e**(4*c + 4*d*x)*sqrt(b)*sqrt(2*sqrt(a)*s...
```

3.300 $\int \frac{\text{sech}^2(c+dx)}{\left(a+b\sinh^2(c+dx)\right)^2}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 114

$$\int \frac{\text{sech}^2(c+dx)}{\left(a+b\sinh^2(c+dx)\right)^2}\,dx = -\frac{(4a-b)b\,\text{arctanh}\left(\frac{\sqrt{a-b}\tanh(c+dx)}{\sqrt{a}}\right)}{2a^{3/2}(a-b)^{5/2}d} + \frac{\tanh(c+dx)}{(a-b)^2d} + \frac{b^2\tanh(c+dx)}{2a(a-b)^2d\left(a-(a-b)\tanh^2(c+dx)\right)}$$

output
```
-1/2*(4*a-b)*b*arctanh((a-b)^(1/2)*tanh(d*x+c)/a^(1/2))/a^(3/2)/(a-b)^(5/2
)/d+tanh(d*x+c)/(a-b)^2/d+1/2*b^2*tanh(d*x+c)/a/(a-b)^2/d/(a-(a-b)*tanh(d*
x+c)^2)
```

Mathematica [A] (verified)

Time = 0.84 (sec) , antiderivative size = 105, normalized size of antiderivative = 0.92

$$\int \frac{\text{sech}^2(c+dx)}{\left(a+b\sinh^2(c+dx)\right)^2}\,dx = \frac{-\frac{(4a-b)b\,\text{arctanh}\left(\frac{\sqrt{a-b}\tanh(c+dx)}{\sqrt{a}}\right)}{a^{3/2}(a-b)^{5/2}} + \frac{\frac{b^2\sinh(2(c+dx))}{a(2a-b+b\cosh(2(c+dx)))}+2\tanh(c+dx)}{(a-b)^2}}{2d}$$

input
```
Integrate[Sech[c + d*x]^2/(a + b*Sinh[c + d*x]^2)^2,x]
```

output
```
(-(((4*a - b)*b*ArcTanh[(Sqrt[a - b]*Tanh[c + d*x])/Sqrt[a]])/(a^(3/2)*(a
- b)^(5/2))) + ((b^2*Sinh[2*(c + d*x)])/(a*(2*a - b + b*Cosh[2*(c + d*x)])
) + 2*Tanh[c + d*x])/(a - b)^2)/(2*d)
```

Rubi [A] (verified)

Time = 0.42 (sec) , antiderivative size = 109, normalized size of antiderivative = 0.96, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.174$, Rules used = {3042, 3670, 300, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\operatorname{sech}^2(c+dx)}{\left(a+b\sinh^2(c+dx)\right)^2}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{\cos(ic+idx)^2\left(a-b\sin(ic+idx)^2\right)^2}dx$$

$$\Big\downarrow \text{3670}$$

$$\frac{\int \frac{\left(1-\tanh^2(c+dx)\right)^2}{\left(a-(a-b)\tanh^2(c+dx)\right)^2}d\tanh(c+dx)}{d}$$

$$\Big\downarrow \text{300}$$

$$\frac{\int \left(\frac{1}{(a-b)^2}-\frac{(2a-b)b-2(a-b)b\tanh^2(c+dx)}{(a-b)^2\left((b-a)\tanh^2(c+dx)+a\right)^2}\right)d\tanh(c+dx)}{d}$$

$$\Big\downarrow \text{2009}$$

$$\frac{-\frac{b(4a-b)\text{arctanh}\left(\frac{\sqrt{a-b}\tanh(c+dx)}{\sqrt{a}}\right)}{2a^{3/2}(a-b)^{5/2}}+\frac{b^2\tanh(c+dx)}{2a(a-b)^2\left(a-(a-b)\tanh^2(c+dx)\right)}+\frac{\tanh(c+dx)}{(a-b)^2}}{d}$$

input
```
Int[Sech[c + d*x]^2/(a + b*Sinh[c + d*x]^2)^2,x]
```

output
```
(-1/2*((4*a - b)*b*ArcTanh[(Sqrt[a - b]*Tanh[c + d*x])/Sqrt[a]])/(a^(3/2)*
(a - b)^(5/2)) + Tanh[c + d*x]/(a - b)^2 + (b^2*Tanh[c + d*x])/(2*a*(a - b
)^2*(a - (a - b)*Tanh[c + d*x]^2)))/d
```

Defintions of rubi rules used

rule 300
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_), x_Symbol] :> Int
[PolynomialDivide[(a + b*x^2)^p, (c + d*x^2)^(-q), x], x] /; FreeQ[{a, b, c
, d}, x] && NeQ[b*c - a*d, 0] && IGtQ[p, 0] && ILtQ[q, 0] && GeQ[p, -q]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3670
```
Int[cos[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(
p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Simp[ff/f   Su
bst[Int[(a + (a + b)*ff^2*x^2)^p/(1 + ff^2*x^2)^(m/2 + p + 1), x], x, Tan[e
 + f*x]/ff], x]] /; FreeQ[{a, b, e, f}, x] && IntegerQ[m/2] && IntegerQ[p]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 306 vs. $2(102) = 204$.

Time = 0.26 (sec) , antiderivative size = 307, normalized size of antiderivative = 2.69

$$\frac{\frac{2\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)}{(a-b)^2\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^2+1\right)}+\frac{2b\left(\frac{\frac{b\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^3}{2a}+\frac{b\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)}{2a}}{\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^4a-2\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^2a+4b\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^2+a}+\frac{(4a-b)\left(-\frac{\left(\sqrt{-b(a-b)}-b\right)\operatorname{arctanh}\left(\frac{\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)a}{\sqrt{\left(2\sqrt{-b(a-b)}+a-2\cdots\right.}}\right)}{2a\sqrt{-b(a-b)}\sqrt{\left(2\sqrt{-b(a-b)}+a-2b\right)a}}\cdots\right)}{\cdots}\right)}{(a-b)^2}}{d}$$

input
```
int(sech(d*x+c)^2/(a+b*sinh(d*x+c)^2)^2,x)
```

output
```
1/d*(2/(a-b)^2*tanh(1/2*d*x+1/2*c)/(tanh(1/2*d*x+1/2*c)^2+1)+2*b/(a-b)^2*(
(1/2*b/a*tanh(1/2*d*x+1/2*c)^3+1/2*b/a*tanh(1/2*d*x+1/2*c))/(tanh(1/2*d*x+
1/2*c)^4*a-2*tanh(1/2*d*x+1/2*c)^2*a+4*b*tanh(1/2*d*x+1/2*c)^2+a)+1/2*(4*a
-b)*(-1/2*((-b*(a-b))^(1/2)-b)/a/(-b*(a-b))^(1/2)/((2*(-b*(a-b))^(1/2)+a-2
*b)*a)^(1/2)*arctanh(tanh(1/2*d*x+1/2*c)*a/((2*(-b*(a-b))^(1/2)+a-2*b)*a)^
(1/2))+1/2*((-b*(a-b))^(1/2)+b)/a/(-b*(a-b))^(1/2)/((2*(-b*(a-b))^(1/2)-a+
2*b)*a)^(1/2)*arctan(tanh(1/2*d*x+1/2*c)*a/((2*(-b*(a-b))^(1/2)-a+2*b)*a)^
(1/2)))))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1446 vs. $2(103) = 206$.

Time = 0.14 (sec) , antiderivative size = 3147, normalized size of antiderivative = 27.61

$$\int \frac{\operatorname{sech}^2(c+dx)}{\left(a+b\sinh^2(c+dx)\right)^2}\,dx = \text{Too large to display}$$

input
```
integrate(sech(d*x+c)^2/(a+b*sinh(d*x+c)^2)^2,x, algorithm="fricas")
```

output
```
Too large to include
```

Sympy [F]

$$\int \frac{\operatorname{sech}^2(c+dx)}{\left(a+b\sinh^2(c+dx)\right)^2}\,dx = \int \frac{\operatorname{sech}^2(c+dx)}{\left(a+b\sinh^2(c+dx)\right)^2}\,dx$$

input
```
integrate(sech(d*x+c)**2/(a+b*sinh(d*x+c)**2)**2,x)
```

output
```
Integral(sech(c + d*x)**2/(a + b*sinh(c + d*x)**2)**2, x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{\operatorname{sech}^2(c+dx)}{(a+b\sinh^2(c+dx))^2}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate(sech(d*x+c)^2/(a+b*sinh(d*x+c)^2)^2,x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(b-a>0)', see `assume?` for more
details)Is
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 221 vs. $2(103) = 206$.

Time = 0.29 (sec) , antiderivative size = 221, normalized size of antiderivative = 1.94

$$\int \frac{\operatorname{sech}^2(c+dx)}{(a+b\sinh^2(c+dx))^2}\,dx = -\frac{\frac{(4\,ab-b^2)\arctan\left(\frac{be^{(2\,dx+2\,c)}+2\,a-b}{2\,\sqrt{-a^2+ab}}\right)}{(a^3-2\,a^2b+ab^2)\sqrt{-a^2+ab}}+\frac{2\,\left(4\,abe^{(4\,dx+4\,c)}-b^2e^{(4\,dx+4\,c)}+8\,a^2e^{(2\,dx+2\,c)}-2\,abe^{(2\,dx+2\,c)}+2\,ab+b^2\right)}{(a^3-2\,a^2b+ab^2)\left(be^{(6\,dx+6\,c)}+4\,ae^{(4\,dx+4\,c)}-be^{(4\,dx+4\,c)}+4\,ae^{(2\,dx+2\,c)}-be^{(2\,dx+2\,c)}+b\right)}}{2\,d}$$

input
```
integrate(sech(d*x+c)^2/(a+b*sinh(d*x+c)^2)^2,x, algorithm="giac")
```

output
```
-1/2*((4*a*b - b^2)*arctan(1/2*(b*e^(2*d*x + 2*c) + 2*a - b)/sqrt(-a^2 + a
*b))/((a^3 - 2*a^2*b + a*b^2)*sqrt(-a^2 + a*b)) + 2*(4*a*b*e^(4*d*x + 4*c)
 - b^2*e^(4*d*x + 4*c) + 8*a^2*e^(2*d*x + 2*c) - 2*a*b*e^(2*d*x + 2*c) + 2
*a*b + b^2)/((a^3 - 2*a^2*b + a*b^2)*(b*e^(6*d*x + 6*c) + 4*a*e^(4*d*x + 4
*c) - b*e^(4*d*x + 4*c) + 4*a*e^(2*d*x + 2*c) - b*e^(2*d*x + 2*c) + b)))/d
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\operatorname{sech}^2(c+dx)}{\left(a+b\sinh^2(c+dx)\right)^2}\,dx = \int \frac{1}{\cosh\left(c+d\,x\right)^2\,\left(b\sinh\left(c+d\,x\right)^2+a\right)^2}\,dx$$

input
```
int(1/(cosh(c + d*x)^2*(a + b*sinh(c + d*x)^2)^2),x)
```

output
```
int(1/(cosh(c + d*x)^2*(a + b*sinh(c + d*x)^2)^2), x)
```

Reduce [B] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 1893, normalized size of antiderivative = 16.61

$$\int \frac{\operatorname{sech}^2(c+dx)}{\left(a+b\sinh^2(c+dx)\right)^2}\,dx = \text{Too large to display}$$

input
```
int(sech(d*x+c)^2/(a+b*sinh(d*x+c)^2)^2,x)
```

output

```
( - 4*e**(6*c + 6*d*x)*sqrt(a)*sqrt(a - b)*log( - sqrt(2*sqrt(a)*sqrt(a -
b) - 2*a + b) + e**(c + d*x)*sqrt(b))*a*b**2 + e**(6*c + 6*d*x)*sqrt(a)*sq
rt(a - b)*log( - sqrt(2*sqrt(a)*sqrt(a - b) - 2*a + b) + e**(c + d*x)*sqrt
(b))*b**3 - 4*e**(6*c + 6*d*x)*sqrt(a)*sqrt(a - b)*log(sqrt(2*sqrt(a)*sqrt
(a - b) - 2*a + b) + e**(c + d*x)*sqrt(b))*a*b**2 + e**(6*c + 6*d*x)*sqrt(
a)*sqrt(a - b)*log(sqrt(2*sqrt(a)*sqrt(a - b) - 2*a + b) + e**(c + d*x)*sq
rt(b))*b**3 + 4*e**(6*c + 6*d*x)*sqrt(a)*sqrt(a - b)*log(2*sqrt(a)*sqrt(a
- b) + e**(2*c + 2*d*x)*b + 2*a - b)*a*b**2 - e**(6*c + 6*d*x)*sqrt(a)*sqr
t(a - b)*log(2*sqrt(a)*sqrt(a - b) + e**(2*c + 2*d*x)*b + 2*a - b)*b**3 -
16*e**(4*c + 4*d*x)*sqrt(a)*sqrt(a - b)*log( - sqrt(2*sqrt(a)*sqrt(a - b)
- 2*a + b) + e**(c + d*x)*sqrt(b))*a**2*b + 8*e**(4*c + 4*d*x)*sqrt(a)*sqr
t(a - b)*log( - sqrt(2*sqrt(a)*sqrt(a - b) - 2*a + b) + e**(c + d*x)*sqrt(
b))*a*b**2 - e**(4*c + 4*d*x)*sqrt(a)*sqrt(a - b)*log( - sqrt(2*sqrt(a)*sq
rt(a - b) - 2*a + b) + e**(c + d*x)*sqrt(b))*b**3 - 16*e**(4*c + 4*d*x)*sq
rt(a)*sqrt(a - b)*log(sqrt(2*sqrt(a)*sqrt(a - b) - 2*a + b) + e**(c + d*x)
*sqrt(b))*a**2*b + 8*e**(4*c + 4*d*x)*sqrt(a)*sqrt(a - b)*log(sqrt(2*sqrt(
a)*sqrt(a - b) - 2*a + b) + e**(c + d*x)*sqrt(b))*a*b**2 - e**(4*c + 4*d*x
)*sqrt(a)*sqrt(a - b)*log(sqrt(2*sqrt(a)*sqrt(a - b) - 2*a + b) + e**(c +
d*x)*sqrt(b))*b**3 + 16*e**(4*c + 4*d*x)*sqrt(a)*sqrt(a - b)*log(2*sqrt(a)
*sqrt(a - b) + e**(2*c + 2*d*x)*b + 2*a - b)*a**2*b - 8*e**(4*c + 4*d*x...
```

3.301 $\int \frac{\operatorname{sech}^3(c+dx)}{(a+b\sinh^2(c+dx))^2}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 157

$$\int \frac{\operatorname{sech}^3(c+dx)}{(a+b\sinh^2(c+dx))^2}\,dx = \frac{(a-5b)\arctan(\sinh(c+dx))}{2(a-b)^3d} + \frac{(5a-b)b^{3/2}\arctan\left(\frac{\sqrt{b}\sinh(c+dx)}{\sqrt{a}}\right)}{2a^{3/2}(a-b)^3d} + \frac{b(a+b)\sinh(c+dx)}{2a(a-b)^2d\,(a+b\sinh^2(c+dx))} + \frac{\operatorname{sech}(c+dx)\tanh(c+dx)}{2(a-b)d\,(a+b\sinh^2(c+dx))}$$

output
```
1/2*(a-5*b)*arctan(sinh(d*x+c))/(a-b)^3/d+1/2*(5*a-b)*b^(3/2)*arctan(b^(1/
2)*sinh(d*x+c)/a^(1/2))/a^(3/2)/(a-b)^3/d+1/2*b*(a+b)*sinh(d*x+c)/a/(a-b)^
2/d/(a+b*sinh(d*x+c)^2)+1/2*sech(d*x+c)*tanh(d*x+c)/(a-b)/d/(a+b*sinh(d*x+
c)^2)
```

Mathematica [A] (verified)

Time = 0.64 (sec) , antiderivative size = 230, normalized size of antiderivative = 1.46

$$\int \frac{\operatorname{sech}^3(c+dx)}{\left(a+b\sinh^2(c+dx)\right)^2}\,dx$$
$$= \frac{2\sqrt{a}(a-b)b^2\sinh(c+dx)+(2a-b)\left(b^{3/2}(-5a+b)\arctan\left(\frac{\sqrt{a}\operatorname{csch}(c+dx)}{\sqrt{b}}\right)+2a^{3/2}(a-5b)\arctan\left(\tan\right.\right.}{}$$

input
```
Integrate[Sech[c + d*x]^3/(a + b*Sinh[c + d*x]^2)^2,x]
```

output
```
(2*Sqrt[a]*(a - b)*b^2*Sinh[c + d*x] + (2*a - b)*(b^(3/2)*(-5*a + b)*ArcTa
n[(Sqrt[a]*Csch[c + d*x])/Sqrt[b]] + 2*a^(3/2)*(a - 5*b)*ArcTan[Tanh[(c +
d*x)/2]] + a^(3/2)*(a - b)*Sech[c + d*x]*Tanh[c + d*x]) + b*Cosh[2*(c + d*
x)]*(b^(3/2)*(-5*a + b)*ArcTan[(Sqrt[a]*Csch[c + d*x])/Sqrt[b]] + 2*a^(3/2
)*(a - 5*b)*ArcTan[Tanh[(c + d*x)/2]] + a^(3/2)*(a - b)*Sech[c + d*x]*Tanh
[c + d*x]))/(2*a^(3/2)*(a - b)^3*d*(2*a - b + b*Cosh[2*(c + d*x)]))
```

Rubi [A] (verified)

Time = 0.44 (sec) , antiderivative size = 171, normalized size of antiderivative = 1.09, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.391$, Rules used = {3042, 3669, 316, 25, 402, 27, 397, 216, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\operatorname{sech}^3(c+dx)}{\left(a+b\sinh^2(c+dx)\right)^2}\,dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \frac{1}{\cos(ic+idx)^3\left(a-b\sin(ic+idx)^2\right)^2}dx$$

$$\Big\downarrow \text{ 3669}$$

$$\frac{\int \frac{1}{\left(\sinh^2(c+dx)+1\right)^2\left(b\sinh^2(c+dx)+a\right)^2}d\sinh(c+dx)}{d}$$

$$\Big\downarrow \text{316}$$

$$\frac{\frac{\sinh(c+dx)}{2(a-b)\left(\sinh^2(c+dx)+1\right)\left(a+b\sinh^2(c+dx)\right)}-\frac{\int -\frac{3b\sinh^2(c+dx)+a-2b}{\left(\sinh^2(c+dx)+1\right)\left(b\sinh^2(c+dx)+a\right)^2}d\sinh(c+dx)}{2(a-b)}}{d}$$

$$\Big\downarrow \text{25}$$

$$\frac{\frac{\int \frac{3b\sinh^2(c+dx)+a-2b}{\left(\sinh^2(c+dx)+1\right)\left(b\sinh^2(c+dx)+a\right)^2}d\sinh(c+dx)}{2(a-b)}+\frac{\sinh(c+dx)}{2(a-b)\left(\sinh^2(c+dx)+1\right)\left(a+b\sinh^2(c+dx)\right)}}{d}$$

$$\Big\downarrow \text{402}$$

$$\frac{\frac{\frac{\int \frac{2\left(a^2-4ba+b^2+b(a+b)\sinh^2(c+dx)\right)}{\left(\sinh^2(c+dx)+1\right)\left(b\sinh^2(c+dx)+a\right)}d\sinh(c+dx)}{2a(a-b)}+\frac{b(a+b)\sinh(c+dx)}{a(a-b)\left(a+b\sinh^2(c+dx)\right)}}{2(a-b)}+\frac{\sinh(c+dx)}{2(a-b)\left(\sinh^2(c+dx)+1\right)\left(a+b\sinh^2(c+dx)\right)}}{d}$$

$$\Big\downarrow \text{27}$$

$$\frac{\frac{\frac{\int \frac{a^2-4ba+b^2+b(a+b)\sinh^2(c+dx)}{\left(\sinh^2(c+dx)+1\right)\left(b\sinh^2(c+dx)+a\right)}d\sinh(c+dx)}{a(a-b)}+\frac{b(a+b)\sinh(c+dx)}{a(a-b)\left(a+b\sinh^2(c+dx)\right)}}{2(a-b)}+\frac{\sinh(c+dx)}{2(a-b)\left(\sinh^2(c+dx)+1\right)\left(a+b\sinh^2(c+dx)\right)}}{d}$$

$$\Big\downarrow \text{397}$$

$$\frac{\frac{\frac{\frac{b^2(5a-b)\int \frac{1}{b\sinh^2(c+dx)+a}d\sinh(c+dx)}{a-b}+\frac{a(a-5b)\int \frac{1}{\sinh^2(c+dx)+1}d\sinh(c+dx)}{a-b}}{a(a-b)}+\frac{b(a+b)\sinh(c+dx)}{a(a-b)\left(a+b\sinh^2(c+dx)\right)}}{2(a-b)}+\frac{\sinh(c+dx)}{2(a-b)\left(\sinh^2(c+dx)+1\right)\left(a+b\sinh^2(c+dx)\right)}}{d}$$

$$\Big\downarrow \text{216}$$

$$\frac{\frac{\frac{\frac{b^2(5a-b)\int \frac{1}{b\sinh^2(c+dx)+a}d\sinh(c+dx)}{a-b}+\frac{a(a-5b)\arctan(\sinh(c+dx))}{a-b}}{a(a-b)}+\frac{b(a+b)\sinh(c+dx)}{a(a-b)\left(a+b\sinh^2(c+dx)\right)}}{2(a-b)}+\frac{\sinh(c+dx)}{2(a-b)\left(\sinh^2(c+dx)+1\right)\left(a+b\sinh^2(c+dx)\right)}}{d}$$

$$\Big\downarrow \text{218}$$

$$\frac{\frac{\frac{\frac{b^{3/2}(5a-b)\arctan\left(\frac{\sqrt{b}\sinh(c+dx)}{\sqrt{a}}\right)}{\sqrt{a}(a-b)}+\frac{a(a-5b)\arctan(\sinh(c+dx))}{a-b}}{a(a-b)}+\frac{b(a+b)\sinh(c+dx)}{a(a-b)\left(a+b\sinh^2(c+dx)\right)}}{2(a-b)}+\frac{\sinh(c+dx)}{2(a-b)\left(\sinh^2(c+dx)+1\right)\left(a+b\sinh^2(c+dx)\right)}}{d}$$

input
```
Int[Sech[c + d*x]^3/(a + b*Sinh[c + d*x]^2)^2,x]
```

output
```
(Sinh[c + d*x]/(2*(a - b)*(1 + Sinh[c + d*x]^2)*(a + b*Sinh[c + d*x]^2)) +
 (((a*(a - 5*b)*ArcTan[Sinh[c + d*x]])/(a - b) + ((5*a - b)*b^(3/2)*ArcTan
[(Sqrt[b]*Sinh[c + d*x])/Sqrt[a]])/(Sqrt[a]*(a - b)))/(a*(a - b)) + (b*(a
+ b)*Sinh[c + d*x])/(a*(a - b)*(a + b*Sinh[c + d*x]^2)))/(2*(a - b)))/d
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 316
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_), x_Symbol] :> Sim
p[(-b)*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^(q + 1)/(2*a*(p + 1)*(b*c - a*d))
), x] + Simp[1/(2*a*(p + 1)*(b*c - a*d))   Int[(a + b*x^2)^(p + 1)*(c + d*x
^2)^q*Simp[b*c + 2*(p + 1)*(b*c - a*d) + d*b*(2*(p + q + 2) + 1)*x^2, x], x
], x] /; FreeQ[{a, b, c, d, q}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] &&  !
( !IntegerQ[p] && IntegerQ[q] && LtQ[q, -1]) && IntBinomialQ[a, b, c, d, 2,
 p, q, x]
```

rule 397
```
Int[((e_) + (f_.)*(x_)^2)/(((a_) + (b_.)*(x_)^2)*((c_) + (d_.)*(x_)^2)), x_
Symbol] :> Simp[(b*e - a*f)/(b*c - a*d)   Int[1/(a + b*x^2), x], x] - Simp[
(d*e - c*f)/(b*c - a*d)   Int[1/(c + d*x^2), x], x] /; FreeQ[{a, b, c, d, e
, f}, x]
```

rule 402
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_.)*((e_) + (f_.)*(x
_)^2), x_Symbol] :> Simp[(-(b*e - a*f))*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^
(q + 1)/(a*2*(b*c - a*d)*(p + 1))), x] + Simp[1/(a*2*(b*c - a*d)*(p + 1))
 Int[(a + b*x^2)^(p + 1)*(c + d*x^2)^q*Simp[c*(b*e - a*f) + e*2*(b*c - a*d)
*(p + 1) + d*(b*e - a*f)*(2*(p + q + 2) + 1)*x^2, x], x], x] /; FreeQ[{a, b
, c, d, e, f, q}, x] && LtQ[p, -1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3669
```
Int[cos[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^
(p_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Simp[ff/f   S
ubst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a + b*ff^2*x^2)^p, x], x, Sin[e + f*x]
/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m - 1)/2]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 367 vs. $2(141) = 282$.

Time = 0.25 (sec) , antiderivative size = 368, normalized size of antiderivative = 2.34

$$\frac{\frac{\frac{2\left(\left(-\frac{a}{2}+\frac{b}{2}\right)\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^3+\left(\frac{a}{2}-\frac{b}{2}\right)\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^2+1\right)^2}+(a-5b)\arctan\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{(a-b)^3}+\frac{2b^2\left(\frac{-\frac{(a-b)\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^3}{2a}+\frac{(a-b)\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)}{2a}}{\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^4a-2\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^2a+4b\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^2+a} + \cdots\right.}{\cdots}}{d}$$

input
```
int(sech(d*x+c)^3/(a+b*sinh(d*x+c)^2)^2,x)
```

output
```
1/d*(2/(a-b)^3*(((-1/2*a+1/2*b)*tanh(1/2*d*x+1/2*c)^3+(1/2*a-1/2*b)*tanh(1
/2*d*x+1/2*c))/(tanh(1/2*d*x+1/2*c)^2+1)^2+1/2*(a-5*b)*arctan(tanh(1/2*d*x
+1/2*c)))+2*b^2/(a-b)^3*((-1/2*(a-b)/a*tanh(1/2*d*x+1/2*c)^3+1/2*(a-b)/a*t
anh(1/2*d*x+1/2*c))/(tanh(1/2*d*x+1/2*c)^4*a-2*tanh(1/2*d*x+1/2*c)^2*a+4*b
*tanh(1/2*d*x+1/2*c)^2+a)+1/2*(5*a-b)*(-1/2*(a+(-b*(a-b))^(1/2)-b)/a/(-b*(
a-b))^(1/2)/((2*(-b*(a-b))^(1/2)+a-2*b)*a)^(1/2)*arctanh(tanh(1/2*d*x+1/2*
c)*a/((2*(-b*(a-b))^(1/2)+a-2*b)*a)^(1/2))+1/2*(-a+(-b*(a-b))^(1/2)+b)/a/(
-b*(a-b))^(1/2)/((2*(-b*(a-b))^(1/2)-a+2*b)*a)^(1/2)*arctan(tanh(1/2*d*x+1
/2*c)*a/((2*(-b*(a-b))^(1/2)-a+2*b)*a)^(1/2)))))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 3474 vs. $2(141) = 282$.

Time = 0.21 (sec) , antiderivative size = 6548, normalized size of antiderivative = 41.71

$$\int \frac{\operatorname{sech}^3(c+dx)}{\left(a+b\sinh^2(c+dx)\right)^2}\,dx = \text{Too large to display}$$

input
```
integrate(sech(d*x+c)^3/(a+b*sinh(d*x+c)^2)^2,x, algorithm="fricas")
```

output
```
Too large to include
```

Sympy [F]

$$\int \frac{\operatorname{sech}^3(c+dx)}{\left(a+b\sinh^2(c+dx)\right)^2}\,dx=\int \frac{\operatorname{sech}^3\left(c+dx\right)}{\left(a+b\sinh^2\left(c+dx\right)\right)^2}\,dx$$

input
```
integrate(sech(d*x+c)**3/(a+b*sinh(d*x+c)**2)**2,x)
```

output
```
Integral(sech(c + d*x)**3/(a + b*sinh(c + d*x)**2)**2, x)
```

Maxima [F]

$$\int \frac{\operatorname{sech}^3(c+dx)}{\left(a+b\sinh^2(c+dx)\right)^2}\,dx=\int \frac{\operatorname{sech}\left(dx+c\right)^3}{\left(b\sinh\left(dx+c\right)^2+a\right)^2}\,dx$$

input
```
integrate(sech(d*x+c)^3/(a+b*sinh(d*x+c)^2)^2,x, algorithm="maxima")
```

output
```
(a*e^c - 5*b*e^c)*arctan(e^(d*x + c))*e^(-c)/(a^3*d - 3*a^2*b*d + 3*a*b^2*
d - b^3*d) + ((a*b*e^(7*c) + b^2*e^(7*c))*e^(7*d*x) + (4*a^2*e^(5*c) - 3*a
*b*e^(5*c) + b^2*e^(5*c))*e^(5*d*x) - (4*a^2*e^(3*c) - 3*a*b*e^(3*c) + b^2
*e^(3*c))*e^(3*d*x) - (a*b*e^c + b^2*e^c)*e^(d*x))/(a^3*b*d - 2*a^2*b^2*d
+ a*b^3*d + (a^3*b*d*e^(8*c) - 2*a^2*b^2*d*e^(8*c) + a*b^3*d*e^(8*c))*e^(8
*d*x) + 4*(a^4*d*e^(6*c) - 2*a^3*b*d*e^(6*c) + a^2*b^2*d*e^(6*c))*e^(6*d*x
) + 2*(4*a^4*d*e^(4*c) - 9*a^3*b*d*e^(4*c) + 6*a^2*b^2*d*e^(4*c) - a*b^3*d
*e^(4*c))*e^(4*d*x) + 4*(a^4*d*e^(2*c) - 2*a^3*b*d*e^(2*c) + a^2*b^2*d*e^(
2*c))*e^(2*d*x)) + 8*integrate(1/8*((5*a*b^2*e^(3*c) - b^3*e^(3*c))*e^(3*d
*x) + (5*a*b^2*e^c - b^3*e^c)*e^(d*x))/(a^4*b - 3*a^3*b^2 + 3*a^2*b^3 - a*
b^4 + (a^4*b*e^(4*c) - 3*a^3*b^2*e^(4*c) + 3*a^2*b^3*e^(4*c) - a*b^4*e^(4*
c))*e^(4*d*x) + 2*(2*a^5*e^(2*c) - 7*a^4*b*e^(2*c) + 9*a^3*b^2*e^(2*c) - 5
*a^2*b^3*e^(2*c) + a*b^4*e^(2*c))*e^(2*d*x)), x)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{\operatorname{sech}^3(c+dx)}{\left(a+b\sinh^2(c+dx)\right)^2}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(sech(d*x+c)^3/(a+b*sinh(d*x+c)^2)^2,x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Limit: Max order reached or unable
to make series expansion Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\operatorname{sech}^3(c+dx)}{\left(a+b\sinh^2(c+dx)\right)^2}\,dx = \int \frac{1}{\cosh\left(c+d\,x\right)^3\,\left(b\,\sinh\left(c+d\,x\right)^2+a\right)^2}\,dx$$

input
```
int(1/(cosh(c + d*x)^3*(a + b*sinh(c + d*x)^2)^2),x)
```

output
```
int(1/(cosh(c + d*x)^3*(a + b*sinh(c + d*x)^2)^2), x)
```

Reduce [B] (verification not implemented)

Time = 0.55 (sec) , antiderivative size = 5140, normalized size of antiderivative = 32.74

$$\int \frac{\operatorname{sech}^3(c+dx)}{\left(a+b\sinh^2(c+dx)\right)^2}\,dx = \text{Too large to display}$$

input
```
int(sech(d*x+c)^3/(a+b*sinh(d*x+c)^2)^2,x)
```

output

```
(4*e**(8*c + 8*d*x)*atan(e**(c + d*x))*a**3*b - 20*e**(8*c + 8*d*x)*atan(e
**(c + d*x))*a**2*b**2 + 16*e**(6*c + 6*d*x)*atan(e**(c + d*x))*a**4 - 80*
e**(6*c + 6*d*x)*atan(e**(c + d*x))*a**3*b + 32*e**(4*c + 4*d*x)*atan(e**(
c + d*x))*a**4 - 168*e**(4*c + 4*d*x)*atan(e**(c + d*x))*a**3*b + 40*e**(4
*c + 4*d*x)*atan(e**(c + d*x))*a**2*b**2 + 16*e**(2*c + 2*d*x)*atan(e**(c
+ d*x))*a**4 - 80*e**(2*c + 2*d*x)*atan(e**(c + d*x))*a**3*b + 4*atan(e**(
c + d*x))*a**3*b - 20*atan(e**(c + d*x))*a**2*b**2 - 10*e**(8*c + 8*d*x)*s
qrt(b)*sqrt(a)*sqrt(a - b)*sqrt(2*sqrt(a)*sqrt(a - b) + 2*a - b)*atan((e**
(c + d*x)*b)/(sqrt(b)*sqrt(2*sqrt(a)*sqrt(a - b) + 2*a - b)))*a*b + 2*e**(
8*c + 8*d*x)*sqrt(b)*sqrt(a)*sqrt(a - b)*sqrt(2*sqrt(a)*sqrt(a - b) + 2*a
- b)*atan((e**(c + d*x)*b)/(sqrt(b)*sqrt(2*sqrt(a)*sqrt(a - b) + 2*a - b))
)*b**2 - 40*e**(6*c + 6*d*x)*sqrt(b)*sqrt(a)*sqrt(a - b)*sqrt(2*sqrt(a)*sq
rt(a - b) + 2*a - b)*atan((e**(c + d*x)*b)/(sqrt(b)*sqrt(2*sqrt(a)*sqrt(a
- b) + 2*a - b)))*a**2 + 8*e**(6*c + 6*d*x)*sqrt(b)*sqrt(a)*sqrt(a - b)*sq
rt(2*sqrt(a)*sqrt(a - b) + 2*a - b)*atan((e**(c + d*x)*b)/(sqrt(b)*sqrt(2*
sqrt(a)*sqrt(a - b) + 2*a - b)))*a*b - 80*e**(4*c + 4*d*x)*sqrt(b)*sqrt(a)
*sqrt(a - b)*sqrt(2*sqrt(a)*sqrt(a - b) + 2*a - b)*atan((e**(c + d*x)*b)/(
sqrt(b)*sqrt(2*sqrt(a)*sqrt(a - b) + 2*a - b)))*a**2 + 36*e**(4*c + 4*d*x)
*sqrt(b)*sqrt(a)*sqrt(a - b)*sqrt(2*sqrt(a)*sqrt(a - b) + 2*a - b)*atan((e
**(c + d*x)*b)/(sqrt(b)*sqrt(2*sqrt(a)*sqrt(a - b) + 2*a - b)))*a*b - 4...
```

3.302 $\int \frac{\operatorname{sech}^4(c+dx)}{\left(a+b\sinh^2(c+dx)\right)^2}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 143

$$\int \frac{\operatorname{sech}^4(c+dx)}{\left(a+b\sinh^2(c+dx)\right)^2}\,dx = \frac{(6a-b)b^2\operatorname{arctanh}\left(\frac{\sqrt{a-b}\tanh(c+dx)}{\sqrt{a}}\right)}{2a^{3/2}(a-b)^{7/2}d} + \frac{(a-3b)\tanh(c+dx)}{(a-b)^3d} - \frac{\tanh^3(c+dx)}{3(a-b)^2d} - \frac{b^3\tanh(c+dx)}{2a(a-b)^3d\left(a-(a-b)\tanh^2(c+dx)\right)}$$

output
```
1/2*(6*a-b)*b^2*arctanh((a-b)^(1/2)*tanh(d*x+c)/a^(1/2))/a^(3/2)/(a-b)^(7/
2)/d+(a-3*b)*tanh(d*x+c)/(a-b)^3/d-1/3*tanh(d*x+c)^3/(a-b)^2/d-1/2*b^3*tan
h(d*x+c)/a/(a-b)^3/d/(a-(a-b)*tanh(d*x+c)^2)
```

Mathematica [A] (verified)

Time = 1.53 (sec) , antiderivative size = 130, normalized size of antiderivative = 0.91

$$\int \frac{\operatorname{sech}^4(c+dx)}{\left(a+b\sinh^2(c+dx)\right)^2}\,dx = \frac{\frac{3(6a-b)b^2\operatorname{arctanh}\left(\frac{\sqrt{a-b}\tanh(c+dx)}{\sqrt{a}}\right)}{a^{3/2}(a-b)^{7/2}}+\frac{-\frac{3b^3\sinh(2(c+dx))}{a(2a-b+b\cosh(2(c+dx)))}+2\left(2(a-4b)+(a-b)\operatorname{sech}^2(c+dx)\right)\tanh(c+dx)}{(a-b)^3}}{6d}$$

input
```
Integrate[Sech[c + d*x]^4/(a + b*Sinh[c + d*x]^2)^2,x]
```

output
```
((3*(6*a - b)*b^2*ArcTanh[(Sqrt[a - b]*Tanh[c + d*x])/Sqrt[a]])/(a^(3/2)*(
a - b)^(7/2)) + ((-3*b^3*Sinh[2*(c + d*x)])/(a*(2*a - b + b*Cosh[2*(c + d*
x)])) + 2*(2*(a - 4*b) + (a - b)*Sech[c + d*x]^2)*Tanh[c + d*x])/(a - b)^3
)/(6*d)
```

Rubi [A] (verified)

Time = 0.44 (sec) , antiderivative size = 135, normalized size of antiderivative = 0.94, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.174$, Rules used = {3042, 3670, 300, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\operatorname{sech}^4(c+dx)}{\left(a+b\sinh^2(c+dx)\right)^2}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{\cos(ic+idx)^4\left(a-b\sin(ic+idx)^2\right)^2}dx$$

$$\Big\downarrow \text{3670}$$

$$\frac{\int \frac{\left(1-\tanh^2(c+dx)\right)^3}{\left(a-(a-b)\tanh^2(c+dx)\right)^2}d\tanh(c+dx)}{d}$$

$$\Big\downarrow \text{300}$$

$$\frac{\int \left(-\frac{\tanh^2(c+dx)}{(a-b)^2} + \frac{(3a-b)b^2 - 3(a-b)b^2\tanh^2(c+dx)}{(a-b)^3\left((b-a)\tanh^2(c+dx)+a\right)^2} + \frac{a-3b}{(a-b)^3}\right) d\tanh(c+dx)}{d}$$

$$\Big\downarrow \text{2009}$$

$$\frac{\frac{b^2(6a-b)\operatorname{arctanh}\left(\frac{\sqrt{a-b}\tanh(c+dx)}{\sqrt{a}}\right)}{2a^{3/2}(a-b)^{7/2}} - \frac{b^3\tanh(c+dx)}{2a(a-b)^3\left(a-(a-b)\tanh^2(c+dx)\right)} - \frac{\tanh^3(c+dx)}{3(a-b)^2} + \frac{(a-3b)\tanh(c+dx)}{(a-b)^3}}{d}$$

input
```
Int[Sech[c + d*x]^4/(a + b*Sinh[c + d*x]^2)^2,x]
```

output
```
(((6*a - b)*b^2*ArcTanh[(Sqrt[a - b]*Tanh[c + d*x])/Sqrt[a]])/(2*a^(3/2)*(
a - b)^(7/2)) + ((a - 3*b)*Tanh[c + d*x])/(a - b)^3 - Tanh[c + d*x]^3/(3*(
a - b)^2) - (b^3*Tanh[c + d*x])/(2*a*(a - b)^3*(a - (a - b)*Tanh[c + d*x]^
2)))/d
```

Defintions of rubi rules used

rule 300
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_), x_Symbol] :> Int
[PolynomialDivide[(a + b*x^2)^p, (c + d*x^2)^(-q), x], x] /; FreeQ[{a, b, c
, d}, x] && NeQ[b*c - a*d, 0] && IGtQ[p, 0] && ILtQ[q, 0] && GeQ[p, -q]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3670
```
Int[cos[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(
p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Simp[ff/f   Su
bst[Int[(a + (a + b)*ff^2*x^2)^p/(1 + ff^2*x^2)^(m/2 + p + 1), x], x, Tan[e
 + f*x]/ff], x]] /; FreeQ[{a, b, e, f}, x] && IntegerQ[m/2] && IntegerQ[p]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 355 vs. 2(129) = 258.

Time = 0.27 (sec) , antiderivative size = 356, normalized size of antiderivative = 2.49

$$\frac{-\frac{2\left((-a+3b)\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^5+\left(-\frac{2a}{3}+\frac{14b}{3}\right)\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^3+(-a+3b)\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{(a-b)^3\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^2+1\right)^3}-\frac{2b^2\left(\frac{\frac{b\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^3}{2a}+\frac{b\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)}{2a}}{\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^4a-2\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^2a+4b\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)\right.}{}}{d}$$

input
```
int(sech(d*x+c)^4/(a+b*sinh(d*x+c)^2)^2,x)
```

output
```
1/d*(-2/(a-b)^3*((-a+3*b)*tanh(1/2*d*x+1/2*c)^5+(-2/3*a+14/3*b)*tanh(1/2*d
*x+1/2*c)^3+(-a+3*b)*tanh(1/2*d*x+1/2*c))/(tanh(1/2*d*x+1/2*c)^2+1)^3-2/(a
-b)^3*b^2*((1/2*b/a*tanh(1/2*d*x+1/2*c)^3+1/2*b/a*tanh(1/2*d*x+1/2*c))/(ta
nh(1/2*d*x+1/2*c)^4*a-2*tanh(1/2*d*x+1/2*c)^2*a+4*b*tanh(1/2*d*x+1/2*c)^2+
a)+1/2*(6*a-b)*(-1/2*((-b*(a-b))^(1/2)-b)/a/(-b*(a-b))^(1/2)/((2*(-b*(a-b)
)^(1/2)+a-2*b)*a)^(1/2)*arctanh(tanh(1/2*d*x+1/2*c)*a/((2*(-b*(a-b))^(1/2)
+a-2*b)*a)^(1/2))+1/2*((-b*(a-b))^(1/2)+b)/a/(-b*(a-b))^(1/2)/((2*(-b*(a-b
))^(1/2)-a+2*b)*a)^(1/2)*arctan(tanh(1/2*d*x+1/2*c)*a/((2*(-b*(a-b))^(1/2)
-a+2*b)*a)^(1/2)))))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 3819 vs. 2(130) = 260.

Time = 0.19 (sec) , antiderivative size = 7894, normalized size of antiderivative = 55.20

$$\int \frac{\operatorname{sech}^4(c+dx)}{\left(a+b\sinh^2(c+dx)\right)^2}\,dx=\text{Too large to display}$$

input
```
integrate(sech(d*x+c)^4/(a+b*sinh(d*x+c)^2)^2,x, algorithm="fricas")
```

output
```
Too large to include
```

Sympy [F]

$$\int \frac{\operatorname{sech}^4(c+dx)}{(a+b\sinh^2(c+dx))^2}\,dx = \int \frac{\operatorname{sech}^4{(c+dx)}}{\left(a+b\sinh^2{(c+dx)}\right)^2}\,dx$$

input
```
integrate(sech(d*x+c)**4/(a+b*sinh(d*x+c)**2)**2,x)
```

output
```
Integral(sech(c + d*x)**4/(a + b*sinh(c + d*x)**2)**2, x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{\operatorname{sech}^4(c+dx)}{(a+b\sinh^2(c+dx))^2}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate(sech(d*x+c)^4/(a+b*sinh(d*x+c)^2)^2,x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(b-a>0)', see `assume?` for more
details)Is
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 270 vs. $2(130) = 260$.

Time = 0.28 (sec) , antiderivative size = 270, normalized size of antiderivative = 1.89

$$\int \frac{\operatorname{sech}^4(c+dx)}{\left(a+b\sinh^2(c+dx)\right)^2}\,dx = \frac{\frac{3\,(6\,ab^2-b^3)\arctan\left(\frac{be^{(2\,dx+2\,c)}+2\,a-b}{2\,\sqrt{-a^2+ab}}\right)}{(a^4-3\,a^3b+3\,a^2b^2-ab^3)\sqrt{-a^2+ab}} + \frac{6\,\left(2\,ab^2e^{(2\,dx+2\,c)}-b^3e^{(2\,dx+2\,c)}+b^3\right)}{(a^4-3\,a^3b+3\,a^2b^2-ab^3)\left(be^{(4\,dx+4\,c)}+4\,ae^{(2\,dx+2\,c)}-2\,be^{(2\,dx+2\,c)}+b\right)} + \frac{8\,(3\,be^{(4\,dx+4\,c)}-3\,ae}{(a^3-3\,a^2b+3}}{6\,d}$$

input
```
integrate(sech(d*x+c)^4/(a+b*sinh(d*x+c)^2)^2,x, algorithm="giac")
```

output
```
1/6*(3*(6*a*b^2 - b^3)*arctan(1/2*(b*e^(2*d*x + 2*c) + 2*a - b)/sqrt(-a^2
+ a*b))/((a^4 - 3*a^3*b + 3*a^2*b^2 - a*b^3)*sqrt(-a^2 + a*b)) + 6*(2*a*b^
2*e^(2*d*x + 2*c) - b^3*e^(2*d*x + 2*c) + b^3)/((a^4 - 3*a^3*b + 3*a^2*b^2
 - a*b^3)*(b*e^(4*d*x + 4*c) + 4*a*e^(2*d*x + 2*c) - 2*b*e^(2*d*x + 2*c) +
 b)) + 8*(3*b*e^(4*d*x + 4*c) - 3*a*e^(2*d*x + 2*c) + 9*b*e^(2*d*x + 2*c)
- a + 4*b)/((a^3 - 3*a^2*b + 3*a*b^2 - b^3)*(e^(2*d*x + 2*c) + 1)^3))/d
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\operatorname{sech}^4(c+dx)}{\left(a+b\sinh^2(c+dx)\right)^2}\,dx = \int \frac{1}{\cosh\left(c+d\,x\right)^4\,\left(b\,\sinh\left(c+d\,x\right)^2+a\right)^2}\,dx$$

input
```
int(1/(cosh(c + d*x)^4*(a + b*sinh(c + d*x)^2)^2),x)
```

output
```
int(1/(cosh(c + d*x)^4*(a + b*sinh(c + d*x)^2)^2), x)
```

Reduce [B] (verification not implemented)

Time = 0.39 (sec) , antiderivative size = 4421, normalized size of antiderivative = 30.92

$$\int \frac{\operatorname{sech}^4(c+dx)}{\left(a+b\sinh^2(c+dx)\right)^2}\,dx = \text{Too large to display}$$

input
```
int(sech(d*x+c)^4/(a+b*sinh(d*x+c)^2)^2,x)
```

output
```
(72*e**(10*c + 10*d*x)*sqrt(a)*sqrt(a - b)*log( - sqrt(2*sqrt(a)*sqrt(a -
b) - 2*a + b) + e**(c + d*x)*sqrt(b))*a**2*b**3 + 6*e**(10*c + 10*d*x)*sqr
t(a)*sqrt(a - b)*log( - sqrt(2*sqrt(a)*sqrt(a - b) - 2*a + b) + e**(c + d*
x)*sqrt(b))*a*b**4 - 3*e**(10*c + 10*d*x)*sqrt(a)*sqrt(a - b)*log( - sqrt(
2*sqrt(a)*sqrt(a - b) - 2*a + b) + e**(c + d*x)*sqrt(b))*b**5 + 72*e**(10*
c + 10*d*x)*sqrt(a)*sqrt(a - b)*log(sqrt(2*sqrt(a)*sqrt(a - b) - 2*a + b)
+ e**(c + d*x)*sqrt(b))*a**2*b**3 + 6*e**(10*c + 10*d*x)*sqrt(a)*sqrt(a -
b)*log(sqrt(2*sqrt(a)*sqrt(a - b) - 2*a + b) + e**(c + d*x)*sqrt(b))*a*b**
4 - 3*e**(10*c + 10*d*x)*sqrt(a)*sqrt(a - b)*log(sqrt(2*sqrt(a)*sqrt(a - b
) - 2*a + b) + e**(c + d*x)*sqrt(b))*b**5 - 72*e**(10*c + 10*d*x)*sqrt(a)*
sqrt(a - b)*log(2*sqrt(a)*sqrt(a - b) + e**(2*c + 2*d*x)*b + 2*a - b)*a**2
*b**3 - 6*e**(10*c + 10*d*x)*sqrt(a)*sqrt(a - b)*log(2*sqrt(a)*sqrt(a - b)
 + e**(2*c + 2*d*x)*b + 2*a - b)*a*b**4 + 3*e**(10*c + 10*d*x)*sqrt(a)*sqr
t(a - b)*log(2*sqrt(a)*sqrt(a - b) + e**(2*c + 2*d*x)*b + 2*a - b)*b**5 +
288*e**(8*c + 8*d*x)*sqrt(a)*sqrt(a - b)*log( - sqrt(2*sqrt(a)*sqrt(a - b)
 - 2*a + b) + e**(c + d*x)*sqrt(b))*a**3*b**2 + 96*e**(8*c + 8*d*x)*sqrt(a
)*sqrt(a - b)*log( - sqrt(2*sqrt(a)*sqrt(a - b) - 2*a + b) + e**(c + d*x)*
sqrt(b))*a**2*b**3 - 6*e**(8*c + 8*d*x)*sqrt(a)*sqrt(a - b)*log( - sqrt(2*
sqrt(a)*sqrt(a - b) - 2*a + b) + e**(c + d*x)*sqrt(b))*a*b**4 - 3*e**(8*c
+ 8*d*x)*sqrt(a)*sqrt(a - b)*log( - sqrt(2*sqrt(a)*sqrt(a - b) - 2*a + ...
```

3.303 $\int \frac{\cosh^6(c+dx)}{\left(a+b\sinh^2(c+dx)\right)^3}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 160

$$\int \frac{\cosh^6(c+dx)}{\left(a+b\sinh^2(c+dx)\right)^3}\,dx = \frac{x}{b^3} - \frac{\sqrt{a-b}(8a^2+4ab+3b^2)\,\text{arctanh}\left(\frac{\sqrt{a-b}\tanh(c+dx)}{\sqrt{a}}\right)}{8a^{5/2}b^3d} - \frac{(a-b)\tanh(c+dx)}{4abd\left(a-(a-b)\tanh^2(c+dx)\right)^2} - \frac{(a-b)(4a+3b)\tanh(c+dx)}{8a^2b^2d\left(a-(a-b)\tanh^2(c+dx)\right)}$$

output
```
x/b^3-1/8*(a-b)^(1/2)*(8*a^2+4*a*b+3*b^2)*arctanh((a-b)^(1/2)*tanh(d*x+c)/
a^(1/2))/a^(5/2)/b^3/d-1/4*(a-b)*tanh(d*x+c)/a/b/d/(a-(a-b)*tanh(d*x+c)^2)
^2-1/8*(a-b)*(4*a+3*b)*tanh(d*x+c)/a^2/b^2/d/(a-(a-b)*tanh(d*x+c)^2)
```

Mathematica [A] (verified)

Time = 1.24 (sec) , antiderivative size = 164, normalized size of antiderivative = 1.02

$$\int \frac{\cosh^6(c+dx)}{\left(a+b\sinh^2(c+dx)\right)^3}\,dx$$
$$= \frac{8(c+dx)-\frac{(8a^3-4a^2b-ab^2-3b^3)\operatorname{arctanh}\left(\frac{\sqrt{a-b}\tanh(c+dx)}{\sqrt{a}}\right)}{a^{5/2}\sqrt{a-b}}+\frac{4(a-b)^2b\sinh(2(c+dx))}{a(2a-b+b\cosh(2(c+dx)))^2}+\frac{3b(-2a^2+ab+b^2)\sinh(2(c+dx))}{a^2(2a-b+b\cosh(2(c+dx)))}}{8b^3d}$$

input
```
Integrate[Cosh[c + d*x]^6/(a + b*Sinh[c + d*x]^2)^3,x]
```

output
```
(8*(c + d*x) - ((8*a^3 - 4*a^2*b - a*b^2 - 3*b^3)*ArcTanh[(Sqrt[a - b]*Tan
h[c + d*x])/Sqrt[a]])/(a^(5/2)*Sqrt[a - b]) + (4*(a - b)^2*b*Sinh[2*(c + d
*x)])/(a*(2*a - b + b*Cosh[2*(c + d*x)])^2) + (3*b*(-2*a^2 + a*b + b^2)*Si
nh[2*(c + d*x)])/(a^2*(2*a - b + b*Cosh[2*(c + d*x)])))/(8*b^3*d)
```

Rubi [A] (verified)

Time = 0.43 (sec) , antiderivative size = 181, normalized size of antiderivative = 1.13, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.391, Rules used = {3042, 3670, 316, 25, 402, 25, 397, 219, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cosh^6(c+dx)}{\left(a+b\sinh^2(c+dx)\right)^3}\,dx$$
$$\Big\downarrow \text{ 3042}$$
$$\int \frac{\cos(ic+idx)^6}{(a-b\sin(ic+idx)^2)^3}dx$$
$$\Big\downarrow \text{ 3670}$$
$$\frac{\int \frac{1}{(1-\tanh^2(c+dx))\left(a-(a-b)\tanh^2(c+dx)\right)^3}d\tanh(c+dx)}{d}$$
$$\Big\downarrow \text{ 316}$$

$$\frac{-\frac{\int -\frac{3(a-b)\tanh^2(c+dx)+a+3b}{\left(1-\tanh^2(c+dx)\right)\left(a-(a-b)\tanh^2(c+dx)\right)^2}d\tanh(c+dx)}{4ab}-\frac{(a-b)\tanh(c+dx)}{4ab\left(a-(a-b)\tanh^2(c+dx)\right)^2}}{d}$$

$$\Big\downarrow \ 25$$

$$\frac{\frac{\int \frac{3(a-b)\tanh^2(c+dx)+a+3b}{\left(1-\tanh^2(c+dx)\right)\left(a-(a-b)\tanh^2(c+dx)\right)^2}d\tanh(c+dx)}{4ab}-\frac{(a-b)\tanh(c+dx)}{4ab\left(a-(a-b)\tanh^2(c+dx)\right)^2}}{d}$$

$$\Big\downarrow \ 402$$

$$\frac{\frac{\frac{\left(-\frac{4a}{b}+\frac{3b}{a}+1\right)\tanh(c+dx)}{2\left(a-(a-b)\tanh^2(c+dx)\right)}-\frac{\int -\frac{4a^2+ba+3b^2+(a-b)(4a+3b)\tanh^2(c+dx)}{\left(1-\tanh^2(c+dx)\right)\left(a-(a-b)\tanh^2(c+dx)\right)}d\tanh(c+dx)}{2ab}}{4ab}-\frac{(a-b)\tanh(c+dx)}{4ab\left(a-(a-b)\tanh^2(c+dx)\right)^2}}{d}$$

$$\Big\downarrow \ 25$$

$$\frac{\frac{\frac{\int \frac{4a^2+ba+3b^2+(a-b)(4a+3b)\tanh^2(c+dx)}{\left(1-\tanh^2(c+dx)\right)\left(a-(a-b)\tanh^2(c+dx)\right)}d\tanh(c+dx)}{2ab}+\frac{\left(-\frac{4a}{b}+\frac{3b}{a}+1\right)\tanh(c+dx)}{2\left(a-(a-b)\tanh^2(c+dx)\right)}}{4ab}-\frac{(a-b)\tanh(c+dx)}{4ab\left(a-(a-b)\tanh^2(c+dx)\right)^2}}{d}$$

$$\Big\downarrow \ 397$$

$$\frac{\frac{\frac{\frac{8a^2\int \frac{1}{1-\tanh^2(c+dx)}d\tanh(c+dx)}{b}-\frac{(a-b)\left(8a^2+4ab+3b^2\right)\int \frac{1}{a-(a-b)\tanh^2(c+dx)}d\tanh(c+dx)}{b}}{2ab}+\frac{\left(-\frac{4a}{b}+\frac{3b}{a}+1\right)\tanh(c+dx)}{2\left(a-(a-b)\tanh^2(c+dx)\right)}}{4ab}-\frac{(a-b)\tanh(c+dx)}{4ab\left(a-(a-b)\tanh^2(c+dx)\right)^2}}{d}$$

$$\Big\downarrow \ 219$$

$$\frac{\frac{\frac{\frac{8a^2\text{arctanh}(\tanh(c+dx))}{b}-\frac{(a-b)\left(8a^2+4ab+3b^2\right)\int \frac{1}{a-(a-b)\tanh^2(c+dx)}d\tanh(c+dx)}{b}}{2ab}+\frac{\left(-\frac{4a}{b}+\frac{3b}{a}+1\right)\tanh(c+dx)}{2\left(a-(a-b)\tanh^2(c+dx)\right)}}{4ab}-\frac{(a-b)\tanh(c+dx)}{4ab\left(a-(a-b)\tanh^2(c+dx)\right)^2}}{d}$$

$$\Big\downarrow \ 221$$

$$\frac{\frac{\frac{\frac{8a^2\text{arctanh}(\tanh(c+dx))}{b}-\frac{\sqrt{a-b}\left(8a^2+4ab+3b^2\right)\text{arctanh}\left(\frac{\sqrt{a-b}\tanh(c+dx)}{\sqrt{a}}\right)}{\sqrt{ab}}}{2ab}+\frac{\left(-\frac{4a}{b}+\frac{3b}{a}+1\right)\tanh(c+dx)}{2\left(a-(a-b)\tanh^2(c+dx)\right)}}{4ab}-\frac{(a-b)\tanh(c+dx)}{4ab\left(a-(a-b)\tanh^2(c+dx)\right)^2}}{d}$$

input
```
Int[Cosh[c + d*x]^6/(a + b*Sinh[c + d*x]^2)^3,x]
```

output
```
(-1/4*((a - b)*Tanh[c + d*x])/(a*b*(a - (a - b)*Tanh[c + d*x]^2)^2) + (((8
*a^2*ArcTanh[Tanh[c + d*x]])/b - (Sqrt[a - b]*(8*a^2 + 4*a*b + 3*b^2)*ArcT
anh[(Sqrt[a - b]*Tanh[c + d*x])/Sqrt[a]])/(Sqrt[a]*b))/(2*a*b) + ((1 - (4*
a)/b + (3*b)/a)*Tanh[c + d*x])/(2*(a - (a - b)*Tanh[c + d*x]^2)))/(4*a*b))
/d
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 316
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_), x_Symbol] :> Sim
p[(-b)*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^(q + 1)/(2*a*(p + 1)*(b*c - a*d))
), x] + Simp[1/(2*a*(p + 1)*(b*c - a*d))   Int[(a + b*x^2)^(p + 1)*(c + d*x
^2)^q*Simp[b*c + 2*(p + 1)*(b*c - a*d) + d*b*(2*(p + q + 2) + 1)*x^2, x], x
], x] /; FreeQ[{a, b, c, d, q}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] &&  !
( !IntegerQ[p] && IntegerQ[q] && LtQ[q, -1]) && IntBinomialQ[a, b, c, d, 2,
 p, q, x]
```

rule 397
```
Int[((e_) + (f_.)*(x_)^2)/(((a_) + (b_.)*(x_)^2)*((c_) + (d_.)*(x_)^2)), x_
Symbol] :> Simp[(b*e - a*f)/(b*c - a*d)   Int[1/(a + b*x^2), x], x] - Simp[
(d*e - c*f)/(b*c - a*d)   Int[1/(c + d*x^2), x], x] /; FreeQ[{a, b, c, d, e
, f}, x]
```

rule 402
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_.)*((e_) + (f_.)*(x
_)^2), x_Symbol] :> Simp[(-(b*e - a*f))*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^
(q + 1)/(a*2*(b*c - a*d)*(p + 1))), x] + Simp[1/(a*2*(b*c - a*d)*(p + 1))
 Int[(a + b*x^2)^(p + 1)*(c + d*x^2)^q*Simp[c*(b*e - a*f) + e*2*(b*c - a*d)
*(p + 1) + d*(b*e - a*f)*(2*(p + q + 2) + 1)*x^2, x], x], x] /; FreeQ[{a, b
, c, d, e, f, q}, x] && LtQ[p, -1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3670
```
Int[cos[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(
p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Simp[ff/f   Su
bst[Int[(a + (a + b)*ff^2*x^2)^p/(1 + ff^2*x^2)^(m/2 + p + 1), x], x, Tan[e
 + f*x]/ff], x]] /; FreeQ[{a, b, e, f}, x] && IntegerQ[m/2] && IntegerQ[p]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 428 vs. $2(146) = 292$.

Time = 0.25 (sec) , antiderivative size = 429, normalized size of antiderivative = 2.68

$$\frac{\frac{\ln\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)}{b^3}-\frac{\ln\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}{b^3}+\frac{2\left(-\frac{b\left(4a^2+ab-5b^2\right)\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^7}{8a}+\frac{\left(4a^3-23a^2b+7b^2a+12b^3\right)b\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^5}{8a^2}+\frac{\left(4a^3-23a^2b+7b^2 \cdots\right.}{\cdots}\right.}{\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^4a-2\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^2a+4b\tanh\left(\frac{dx}{2}+\cdots\right.\right.}}{\cdots}$$

input
```
int(cosh(d*x+c)^6/(a+b*sinh(d*x+c)^2)^3,x)
```

output
```
1/d*(1/b^3*ln(tanh(1/2*d*x+1/2*c)+1)-1/b^3*ln(tanh(1/2*d*x+1/2*c)-1)+2/b^3
*((-1/8*b*(4*a^2+a*b-5*b^2)/a*tanh(1/2*d*x+1/2*c)^7+1/8*(4*a^3-23*a^2*b+7*
a*b^2+12*b^3)/a^2*b*tanh(1/2*d*x+1/2*c)^5+1/8*(4*a^3-23*a^2*b+7*a*b^2+12*b
^3)/a^2*b*tanh(1/2*d*x+1/2*c)^3-1/8*b*(4*a^2+a*b-5*b^2)/a*tanh(1/2*d*x+1/2
*c))/(tanh(1/2*d*x+1/2*c)^4*a-2*tanh(1/2*d*x+1/2*c)^2*a+4*b*tanh(1/2*d*x+1
/2*c)^2+a)^2+1/8/a*(8*a^3-4*a^2*b-a*b^2-3*b^3)*(-1/2*((-b*(a-b))^(1/2)-b)/
a/(-b*(a-b))^(1/2)/((2*(-b*(a-b))^(1/2)+a-2*b)*a)^(1/2)*arctanh(tanh(1/2*d
*x+1/2*c)*a/((2*(-b*(a-b))^(1/2)+a-2*b)*a)^(1/2))+1/2*((-b*(a-b))^(1/2)+b)
/a/(-b*(a-b))^(1/2)/((2*(-b*(a-b))^(1/2)-a+2*b)*a)^(1/2)*arctan(tanh(1/2*d
*x+1/2*c)*a/((2*(-b*(a-b))^(1/2)-a+2*b)*a)^(1/2)))))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2623 vs. $2(148) = 296$.

Time = 0.17 (sec) , antiderivative size = 5511, normalized size of antiderivative = 34.44

$$\int \frac{\cosh^6(c+dx)}{\left(a+b\sinh^2(c+dx)\right)^3}\,dx = \text{Too large to display}$$

input
```
integrate(cosh(d*x+c)^6/(a+b*sinh(d*x+c)^2)^3,x, algorithm="fricas")
```

output
```
Too large to include
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\cosh^6(c+dx)}{\left(a+b\sinh^2(c+dx)\right)^3}\,dx = \text{Timed out}$$

input
```
integrate(cosh(d*x+c)**6/(a+b*sinh(d*x+c)**2)**3,x)
```

output
```
Timed out
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{\cosh^6(c+dx)}{\left(a+b\sinh^2(c+dx)\right)^3}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate(cosh(d*x+c)^6/(a+b*sinh(d*x+c)^2)^3,x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(b-a>0)', see `assume?` for more
details)Is
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 353 vs. 2(148) = 296.

Time = 0.97 (sec) , antiderivative size = 353, normalized size of antiderivative = 2.21

$$\int \frac{\cosh^6(c+dx)}{\left(a+b\sinh^2(c+dx)\right)^3}\,dx = \frac{\frac{8\,(dx+c)}{b^3} - \frac{(8\,a^3-4\,a^2b-ab^2-3\,b^3)\arctan\left(\frac{be^{(2\,dx+2\,c)}+2\,a-b}{2\,\sqrt{-a^2+ab}}\right)}{\sqrt{-a^2+ab}a^2b^3} + \frac{2\,(16\,a^3be^{(6\,dx+6\,c)}-20\,a^2b^2e^{(6\,dx+6\,c)}+ab^3e^{(6\,dx+6\,c)}+3\,b^4e^{(6\,dx+6\,c)}+}{}}{}$$

input
```
integrate(cosh(d*x+c)^6/(a+b*sinh(d*x+c)^2)^3,x, algorithm="giac")
```

output
```
1/8*(8*(d*x + c)/b^3 - (8*a^3 - 4*a^2*b - a*b^2 - 3*b^3)*arctan(1/2*(b*e^(
2*d*x + 2*c) + 2*a - b)/sqrt(-a^2 + a*b))/(sqrt(-a^2 + a*b)*a^2*b^3) + 2*(
16*a^3*b*e^(6*d*x + 6*c) - 20*a^2*b^2*e^(6*d*x + 6*c) + a*b^3*e^(6*d*x + 6
*c) + 3*b^4*e^(6*d*x + 6*c) + 48*a^4*e^(4*d*x + 4*c) - 72*a^3*b*e^(4*d*x +
 4*c) + 18*a^2*b^2*e^(4*d*x + 4*c) + 15*a*b^3*e^(4*d*x + 4*c) - 9*b^4*e^(4
*d*x + 4*c) + 32*a^3*b*e^(2*d*x + 2*c) - 28*a^2*b^2*e^(2*d*x + 2*c) - 13*a
*b^3*e^(2*d*x + 2*c) + 9*b^4*e^(2*d*x + 2*c) + 6*a^2*b^2 - 3*a*b^3 - 3*b^4
)/((b*e^(4*d*x + 4*c) + 4*a*e^(2*d*x + 2*c) - 2*b*e^(2*d*x + 2*c) + b)^2*a
^2*b^3))/d
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\cosh^6(c+dx)}{\left(a+b\sinh^2(c+dx)\right)^3}\,dx = \int \frac{\cosh(c+d\,x)^6}{\left(b\sinh\left(c+d\,x\right)^2+a\right)^3}\,dx$$

input
```
int(cosh(c + d*x)^6/(a + b*sinh(c + d*x)^2)^3,x)
```

output
```
int(cosh(c + d*x)^6/(a + b*sinh(c + d*x)^2)^3, x)
```

Reduce [B] (verification not implemented)

Time = 0.44 (sec) , antiderivative size = 20217, normalized size of antiderivative = 126.36

$$\int \frac{\cosh^6(c+dx)}{\left(a+b\sinh^2(c+dx)\right)^3}\,dx = \text{Too large to display}$$

input
```
int(cosh(d*x+c)^6/(a+b*sinh(d*x+c)^2)^3,x)
```

output

```
(192*e**(10*c + 10*d*x)*cosh(c + d*x)**5*sinh(c + d*x)*a**3*b**5 - 96*e**(
10*c + 10*d*x)*cosh(c + d*x)**5*sinh(c + d*x)*a**2*b**6 + 1536*e**(8*c + 8
*d*x)*cosh(c + d*x)**5*sinh(c + d*x)*a**4*b**4 - 1536*e**(8*c + 8*d*x)*cos
h(c + d*x)**5*sinh(c + d*x)*a**3*b**5 + 384*e**(8*c + 8*d*x)*cosh(c + d*x)
**5*sinh(c + d*x)*a**2*b**6 + 3072*e**(6*c + 6*d*x)*cosh(c + d*x)**5*sinh(
c + d*x)*a**5*b**3 - 4608*e**(6*c + 6*d*x)*cosh(c + d*x)**5*sinh(c + d*x)*
a**4*b**4 + 2688*e**(6*c + 6*d*x)*cosh(c + d*x)**5*sinh(c + d*x)*a**3*b**5
 - 576*e**(6*c + 6*d*x)*cosh(c + d*x)**5*sinh(c + d*x)*a**2*b**6 + 1536*e*
*(4*c + 4*d*x)*cosh(c + d*x)**5*sinh(c + d*x)*a**4*b**4 - 1536*e**(4*c + 4
*d*x)*cosh(c + d*x)**5*sinh(c + d*x)*a**3*b**5 + 384*e**(4*c + 4*d*x)*cosh
(c + d*x)**5*sinh(c + d*x)*a**2*b**6 + 192*e**(2*c + 2*d*x)*cosh(c + d*x)*
*5*sinh(c + d*x)*a**3*b**5 - 96*e**(2*c + 2*d*x)*cosh(c + d*x)**5*sinh(c +
 d*x)*a**2*b**6 + 960*e**(10*c + 10*d*x)*cosh(c + d*x)**3*sinh(c + d*x)**3
*a**3*b**5 - 480*e**(10*c + 10*d*x)*cosh(c + d*x)**3*sinh(c + d*x)**3*a**2
*b**6 + 1280*e**(10*c + 10*d*x)*cosh(c + d*x)**3*sinh(c + d*x)*a**4*b**4 -
 640*e**(10*c + 10*d*x)*cosh(c + d*x)**3*sinh(c + d*x)*a**3*b**5 + 7680*e*
*(8*c + 8*d*x)*cosh(c + d*x)**3*sinh(c + d*x)**3*a**4*b**4 - 7680*e**(8*c
+ 8*d*x)*cosh(c + d*x)**3*sinh(c + d*x)**3*a**3*b**5 + 1920*e**(8*c + 8*d*
x)*cosh(c + d*x)**3*sinh(c + d*x)**3*a**2*b**6 + 10240*e**(8*c + 8*d*x)*co
sh(c + d*x)**3*sinh(c + d*x)*a**5*b**3 - 10240*e**(8*c + 8*d*x)*cosh(c ...
```

3.304 $\int \frac{\cosh^5(c+dx)}{\left(a+b\sinh^2(c+dx)\right)^3}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 136

$$\int \frac{\cosh^5(c+dx)}{\left(a+b\sinh^2(c+dx)\right)^3}\,dx = \frac{(3a^2+2ab+3b^2)\arctan\left(\frac{\sqrt{b}\sinh(c+dx)}{\sqrt{a}}\right)}{8a^{5/2}b^{5/2}d} + \frac{(a-b)^2\sinh(c+dx)}{4ab^2d\left(a+b\sinh^2(c+dx)\right)^2} - \frac{(a-b)(5a+3b)\sinh(c+dx)}{8a^2b^2d\left(a+b\sinh^2(c+dx)\right)}$$

output
```
1/8*(3*a^2+2*a*b+3*b^2)*arctan(b^(1/2)*sinh(d*x+c)/a^(1/2))/a^(5/2)/b^(5/2
)/d+1/4*(a-b)^2*sinh(d*x+c)/a/b^2/d/(a+b*sinh(d*x+c)^2)^2-1/8*(a-b)*(5*a+3
*b)*sinh(d*x+c)/a^2/b^2/d/(a+b*sinh(d*x+c)^2)
```

Mathematica [A] (verified)

Time = 0.28 (sec) , antiderivative size = 149, normalized size of antiderivative = 1.10

$$\int \frac{\cosh^5(c+dx)}{\left(a+b\sinh^2(c+dx)\right)^3}\,dx$$
$$= \frac{-\left((3a^2+2ab+3b^2)\arctan\left(\frac{\sqrt{a}\operatorname{csch}(c+dx)}{\sqrt{b}}\right)\right)+\frac{8a^{3/2}(a-b)^2\sqrt{b}\sinh(c+dx)}{(2a-b+b\cosh(2(c+dx)))^2}-\frac{2\sqrt{a}\sqrt{b}(5a^2-2ab-3b^2)\sinh(c+dx)}{2a-b+b\cosh(2(c+dx))}}{8a^{5/2}b^{5/2}d}$$

input
```
Integrate[Cosh[c + d*x]^5/(a + b*Sinh[c + d*x]^2)^3,x]
```

output
```
(-((3*a^2 + 2*a*b + 3*b^2)*ArcTan[(Sqrt[a]*Csch[c + d*x])/Sqrt[b]]) + (8*a
^(3/2)*(a - b)^2*Sqrt[b]*Sinh[c + d*x])/(2*a - b + b*Cosh[2*(c + d*x)])^2
- (2*Sqrt[a]*Sqrt[b]*(5*a^2 - 2*a*b - 3*b^2)*Sinh[c + d*x])/(2*a - b + b*C
osh[2*(c + d*x)]))/(8*a^(5/2)*b^(5/2)*d)
```

Rubi [A] (verified)

Time = 0.33 (sec) , antiderivative size = 143, normalized size of antiderivative = 1.05, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.217$, Rules used = {3042, 3669, 315, 298, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cosh^5(c+dx)}{\left(a+b\sinh^2(c+dx)\right)^3}\,dx$$
$$\Big\downarrow \text{3042}$$
$$\int \frac{\cos(ic+idx)^5}{\left(a-b\sin(ic+idx)^2\right)^3}dx$$
$$\Big\downarrow \text{3669}$$
$$\frac{\int \frac{\left(\sinh^2(c+dx)+1\right)^2}{\left(b\sinh^2(c+dx)+a\right)^3}d\sinh(c+dx)}{d}$$

$$\Big\downarrow \text{315}$$

$$\frac{\frac{\int \frac{(3a+b)\sinh^2(c+dx)+a+3b}{\left(b\sinh^2(c+dx)+a\right)^2}d\sinh(c+dx)}{4ab}-\frac{(a-b)\sinh(c+dx)\left(\sinh^2(c+dx)+1\right)}{4ab\left(a+b\sinh^2(c+dx)\right)^2}}{d}$$

$$\Big\downarrow \text{298}$$

$$\frac{\frac{\frac{1}{2}\left(\frac{3a}{b}+\frac{3b}{a}+2\right)\int \frac{1}{b\sinh^2(c+dx)+a}d\sinh(c+dx)-\frac{3\left(\frac{a}{b}-\frac{b}{a}\right)\sinh(c+dx)}{2\left(a+b\sinh^2(c+dx)\right)}}{4ab}-\frac{(a-b)\sinh(c+dx)\left(\sinh^2(c+dx)+1\right)}{4ab\left(a+b\sinh^2(c+dx)\right)^2}}{d}$$

$$\Big\downarrow \text{218}$$

$$\frac{\frac{\frac{\left(\frac{3a}{b}+\frac{3b}{a}+2\right)\arctan\left(\frac{\sqrt{b}\sinh(c+dx)}{\sqrt{a}}\right)}{2\sqrt{a}\sqrt{b}}-\frac{3\left(\frac{a}{b}-\frac{b}{a}\right)\sinh(c+dx)}{2\left(a+b\sinh^2(c+dx)\right)}}{4ab}-\frac{(a-b)\sinh(c+dx)\left(\sinh^2(c+dx)+1\right)}{4ab\left(a+b\sinh^2(c+dx)\right)^2}}{d}$$

input
```
Int[Cosh[c + d*x]^5/(a + b*Sinh[c + d*x]^2)^3,x]
```

output
```
(-1/4*((a - b)*Sinh[c + d*x]*(1 + Sinh[c + d*x]^2))/(a*b*(a + b*Sinh[c + d
*x]^2)^2) + (((2 + (3*a)/b + (3*b)/a)*ArcTan[(Sqrt[b]*Sinh[c + d*x])/Sqrt[
a]])/(2*Sqrt[a]*Sqrt[b]) - (3*(a/b - b/a)*Sinh[c + d*x])/(2*(a + b*Sinh[c
+ d*x]^2)))/(4*a*b))/d
```

Defintions of rubi rules used

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 298
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2), x_Symbol] :> Simp[(-(
b*c - a*d))*x*((a + b*x^2)^(p + 1)/(2*a*b*(p + 1))), x] - Simp[(a*d - b*c*(
2*p + 3))/(2*a*b*(p + 1))   Int[(a + b*x^2)^(p + 1), x], x] /; FreeQ[{a, b,
 c, d, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/2 + p, 0])
```

rule 315
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_), x_Symbol] :> Sim
p[(a*d - c*b)*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^(q - 1)/(2*a*b*(p + 1))),
x] - Simp[1/(2*a*b*(p + 1))   Int[(a + b*x^2)^(p + 1)*(c + d*x^2)^(q - 2)*S
imp[c*(a*d - c*b*(2*p + 3)) + d*(a*d*(2*(q - 1) + 1) - b*c*(2*(p + q) + 1))
*x^2, x], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -
1] && GtQ[q, 1] && IntBinomialQ[a, b, c, d, 2, p, q, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3669
```
Int[cos[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^
(p_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Simp[ff/f   S
ubst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a + b*ff^2*x^2)^p, x], x, Sin[e + f*x]
/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m - 1)/2]
```

Maple [A] (verified)

Time = 0.11 (sec) , antiderivative size = 124, normalized size of antiderivative = 0.91

$$\frac{\frac{-\frac{\left(5a^2-2ab-3b^2\right)\sinh(dx+c)^3}{8a^2b}-\frac{\left(3a^2+2ab-5b^2\right)\sinh(dx+c)}{8b^2a}}{\left(a+b\sinh(dx+c)^2\right)^2}+\frac{\left(3a^2+2ab+3b^2\right)\arctan\left(\frac{b\sinh(dx+c)}{\sqrt{ab}}\right)}{8a^2b^2\sqrt{ab}}}{d}$$

input
```
int(cosh(d*x+c)^5/(a+b*sinh(d*x+c)^2)^3,x)
```

output
```
1/d*((-1/8*(5*a^2-2*a*b-3*b^2)/a^2/b*sinh(d*x+c)^3-1/8*(3*a^2+2*a*b-5*b^2)
/b^2/a*sinh(d*x+c))/(a+b*sinh(d*x+c)^2)^2+1/8*(3*a^2+2*a*b+3*b^2)/a^2/b^2/
(a*b)^(1/2)*arctan(b*sinh(d*x+c)/(a*b)^(1/2)))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 3268 vs. $2(122) = 244$.

Time = 0.16 (sec) , antiderivative size = 5846, normalized size of antiderivative = 42.99

$$\int \frac{\cosh^5(c+dx)}{(a+b\sinh^2(c+dx))^3}\,dx = \text{Too large to display}$$

input
```
integrate(cosh(d*x+c)^5/(a+b*sinh(d*x+c)^2)^3,x, algorithm="fricas")
```

output
```
Too large to include
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\cosh^5(c+dx)}{(a+b\sinh^2(c+dx))^3}\,dx = \text{Timed out}$$

input
```
integrate(cosh(d*x+c)**5/(a+b*sinh(d*x+c)**2)**3,x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{\cosh^5(c+dx)}{(a+b\sinh^2(c+dx))^3}\,dx = \int \frac{\cosh(dx+c)^5}{(b\sinh(dx+c)^2+a)^3}\,dx$$

input
```
integrate(cosh(d*x+c)^5/(a+b*sinh(d*x+c)^2)^3,x, algorithm="maxima")
```

output
```
-1/4*((5*a^2*b*e^(7*c) - 2*a*b^2*e^(7*c) - 3*b^3*e^(7*c))*e^(7*d*x) + (12*
a^3*e^(5*c) - 7*a^2*b*e^(5*c) - 14*a*b^2*e^(5*c) + 9*b^3*e^(5*c))*e^(5*d*x
) - (12*a^3*e^(3*c) - 7*a^2*b*e^(3*c) - 14*a*b^2*e^(3*c) + 9*b^3*e^(3*c))*
e^(3*d*x) - (5*a^2*b*e^c - 2*a*b^2*e^c - 3*b^3*e^c)*e^(d*x))/(a^2*b^4*d*e^
(8*d*x + 8*c) + a^2*b^4*d + 4*(2*a^3*b^3*d*e^(6*c) - a^2*b^4*d*e^(6*c))*e^
(6*d*x) + 2*(8*a^4*b^2*d*e^(4*c) - 8*a^3*b^3*d*e^(4*c) + 3*a^2*b^4*d*e^(4*
c))*e^(4*d*x) + 4*(2*a^3*b^3*d*e^(2*c) - a^2*b^4*d*e^(2*c))*e^(2*d*x)) + 1
/32*integrate(8*((3*a^2*e^(3*c) + 2*a*b*e^(3*c) + 3*b^2*e^(3*c))*e^(3*d*x)
 + (3*a^2*e^c + 2*a*b*e^c + 3*b^2*e^c)*e^(d*x))/(a^2*b^3*e^(4*d*x + 4*c) +
 a^2*b^3 + 2*(2*a^3*b^2*e^(2*c) - a^2*b^3*e^(2*c))*e^(2*d*x)), x)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{\cosh^5(c+dx)}{(a+b\sinh^2(c+dx))^3}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(cosh(d*x+c)^5/(a+b*sinh(d*x+c)^2)^3,x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Limit: Max order reached or unable
to make series expansion Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\cosh^5(c+dx)}{(a+b\sinh^2(c+dx))^3}\,dx = \int \frac{\cosh(c+d\,x)^5}{\left(b\,\sinh\left(c+d\,x\right)^2+a\right)^3}\,dx$$

input
```
int(cosh(c + d*x)^5/(a + b*sinh(c + d*x)^2)^3,x)
```

output
```
int(cosh(c + d*x)^5/(a + b*sinh(c + d*x)^2)^3, x)
```

Reduce [B] (verification not implemented)

Time = 0.32 (sec) , antiderivative size = 30251, normalized size of antiderivative = 222.43

$$\int \frac{\cosh^5(c+dx)}{\left(a+b\sinh^2(c+dx)\right)^3}\,dx = \text{Too large to display}$$

input
```
int(cosh(d*x+c)^5/(a+b*sinh(d*x+c)^2)^3,x)
```

output
```
(302*e**(9*c + 9*d*x)*sqrt(b)*sqrt(a)*sqrt(a - b)*sqrt(2*sqrt(a)*sqrt(a -
b) + 2*a - b)*atan((e**(c + d*x)*b)/(sqrt(b)*sqrt(2*sqrt(a)*sqrt(a - b) +
2*a - b)))*sinh(c + d*x)**4*a**2*b**4 - 12*e**(9*c + 9*d*x)*sqrt(b)*sqrt(a
)*sqrt(a - b)*sqrt(2*sqrt(a)*sqrt(a - b) + 2*a - b)*atan((e**(c + d*x)*b)/
(sqrt(b)*sqrt(2*sqrt(a)*sqrt(a - b) + 2*a - b)))*sinh(c + d*x)**4*a*b**5 -
 18*e**(9*c + 9*d*x)*sqrt(b)*sqrt(a)*sqrt(a - b)*sqrt(2*sqrt(a)*sqrt(a - b
) + 2*a - b)*atan((e**(c + d*x)*b)/(sqrt(b)*sqrt(2*sqrt(a)*sqrt(a - b) + 2
*a - b)))*sinh(c + d*x)**4*b**6 + 604*e**(9*c + 9*d*x)*sqrt(b)*sqrt(a)*sqr
t(a - b)*sqrt(2*sqrt(a)*sqrt(a - b) + 2*a - b)*atan((e**(c + d*x)*b)/(sqrt
(b)*sqrt(2*sqrt(a)*sqrt(a - b) + 2*a - b)))*sinh(c + d*x)**2*a**3*b**3 - 2
4*e**(9*c + 9*d*x)*sqrt(b)*sqrt(a)*sqrt(a - b)*sqrt(2*sqrt(a)*sqrt(a - b)
+ 2*a - b)*atan((e**(c + d*x)*b)/(sqrt(b)*sqrt(2*sqrt(a)*sqrt(a - b) + 2*a
 - b)))*sinh(c + d*x)**2*a**2*b**4 - 36*e**(9*c + 9*d*x)*sqrt(b)*sqrt(a)*s
qrt(a - b)*sqrt(2*sqrt(a)*sqrt(a - b) + 2*a - b)*atan((e**(c + d*x)*b)/(sq
rt(b)*sqrt(2*sqrt(a)*sqrt(a - b) + 2*a - b)))*sinh(c + d*x)**2*a*b**5 + 30
2*e**(9*c + 9*d*x)*sqrt(b)*sqrt(a)*sqrt(a - b)*sqrt(2*sqrt(a)*sqrt(a - b)
+ 2*a - b)*atan((e**(c + d*x)*b)/(sqrt(b)*sqrt(2*sqrt(a)*sqrt(a - b) + 2*a
 - b)))*a**4*b**2 - 12*e**(9*c + 9*d*x)*sqrt(b)*sqrt(a)*sqrt(a - b)*sqrt(2
*sqrt(a)*sqrt(a - b) + 2*a - b)*atan((e**(c + d*x)*b)/(sqrt(b)*sqrt(2*sqrt
(a)*sqrt(a - b) + 2*a - b)))*a**3*b**3 - 18*e**(9*c + 9*d*x)*sqrt(b)*sq...
```

3.305 $\int \frac{\cosh^4(c+dx)}{(a+b\sinh^2(c+dx))^3}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 114

$$\int \frac{\cosh^4(c+dx)}{(a+b\sinh^2(c+dx))^3}\,dx = \frac{3\operatorname{arctanh}\left(\frac{\sqrt{a-b}\tanh(c+dx)}{\sqrt{a}}\right)}{8a^{5/2}\sqrt{a-b}d} + \frac{\tanh(c+dx)}{4ad\,(a-(a-b)\tanh^2(c+dx))^2} + \frac{3\tanh(c+dx)}{8a^2d\,(a-(a-b)\tanh^2(c+dx))}$$

output
```
3/8*arctanh((a-b)^(1/2)*tanh(d*x+c)/a^(1/2))/a^(5/2)/(a-b)^(1/2)/d+1/4*tan
h(d*x+c)/a/d/(a-(a-b)*tanh(d*x+c)^2)^2+3/8*tanh(d*x+c)/a^2/d/(a-(a-b)*tanh
(d*x+c)^2)
```

Mathematica [A] (verified)

Time = 0.81 (sec) , antiderivative size = 102, normalized size of antiderivative = 0.89

$$\int \frac{\cosh^4(c+dx)}{\left(a+b\sinh^2(c+dx)\right)^3}\,dx = \frac{\frac{3\operatorname{arctanh}\left(\frac{\sqrt{a-b}\tanh(c+dx)}{\sqrt{a}}\right)}{\sqrt{a-b}}+\frac{\sqrt{a}(8a-3b+(2a+3b)\cosh(2(c+dx)))\sinh(2(c+dx))}{(2a-b+b\cosh(2(c+dx)))^2}}{8a^{5/2}d}$$

input
```
Integrate[Cosh[c + d*x]^4/(a + b*Sinh[c + d*x]^2)^3,x]
```

output
```
((3*ArcTanh[(Sqrt[a - b]*Tanh[c + d*x])/Sqrt[a]])/Sqrt[a - b] + (Sqrt[a]*(
8*a - 3*b + (2*a + 3*b)*Cosh[2*(c + d*x)])*Sinh[2*(c + d*x)])/(2*a - b + b
*Cosh[2*(c + d*x)])^2)/(8*a^(5/2)*d)
```

Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 117, normalized size of antiderivative = 1.03, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.217$, Rules used = {3042, 3670, 215, 215, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cosh^4(c+dx)}{\left(a+b\sinh^2(c+dx)\right)^3}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\cos(ic+idx)^4}{(a-b\sin(ic+idx)^2)^3}dx$$

$$\Big\downarrow \text{3670}$$

$$\frac{\int \frac{1}{\left(a-(a-b)\tanh^2(c+dx)\right)^3}d\tanh(c+dx)}{d}$$

$$\Big\downarrow \text{215}$$

$$\frac{\frac{3\int \frac{1}{\left(a-(a-b)\tanh^2(c+dx)\right)^2}d\tanh(c+dx)}{4a}+\frac{\tanh(c+dx)}{4a\left(a-(a-b)\tanh^2(c+dx)\right)^2}}{d}$$

$$\Big\downarrow \text{215}$$

$$\frac{\frac{3\left(\frac{\int \frac{1}{a-(a-b)\tanh^2(c+dx)}d\tanh(c+dx)}{2a}+\frac{\tanh(c+dx)}{2a\left(a-(a-b)\tanh^2(c+dx)\right)}\right)}{4a}+\frac{\tanh(c+dx)}{4a\left(a-(a-b)\tanh^2(c+dx)\right)^2}}{d}$$

$$\Big\downarrow \text{221}$$

$$\frac{\frac{3\left(\frac{\operatorname{arctanh}\left(\frac{\sqrt{a-b}\tanh(c+dx)}{\sqrt{a}}\right)}{2a^{3/2}\sqrt{a-b}}+\frac{\tanh(c+dx)}{2a\left(a-(a-b)\tanh^2(c+dx)\right)}\right)}{4a}+\frac{\tanh(c+dx)}{4a\left(a-(a-b)\tanh^2(c+dx)\right)^2}}{d}$$

input

```
Int[Cosh[c + d*x]^4/(a + b*Sinh[c + d*x]^2)^3,x]
```

output

```
(Tanh[c + d*x]/(4*a*(a - (a - b)*Tanh[c + d*x]^2)^2) + (3*(ArcTanh[(Sqrt[a
 - b]*Tanh[c + d*x])/Sqrt[a]]/(2*a^(3/2)*Sqrt[a - b]) + Tanh[c + d*x]/(2*a
*(a - (a - b)*Tanh[c + d*x]^2))))/(4*a))/d
```

Defintions of rubi rules used

rule 215

```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(-x)*((a + b*x^2)^(p + 1)
/(2*a*(p + 1))), x] + Simp[(2*p + 3)/(2*a*(p + 1))   Int[(a + b*x^2)^(p + 1
), x], x] /; FreeQ[{a, b}, x] && LtQ[p, -1] && (IntegerQ[4*p] || IntegerQ[6
*p])
```

rule 221

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 3042

```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3670
```
Int[cos[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(
p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Simp[ff/f   Su
bst[Int[(a + (a + b)*ff^2*x^2)^p/(1 + ff^2*x^2)^(m/2 + p + 1), x], x, Tan[e
 + f*x]/ff], x]] /; FreeQ[{a, b, e, f}, x] && IntegerQ[m/2] && IntegerQ[p]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 299 vs. $2(100) = 200$.

Time = 0.24 (sec) , antiderivative size = 300, normalized size of antiderivative = 2.63

$$\frac{-\frac{2\left(-\frac{5\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^7}{8a}-\frac{3(a+4b)\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^5}{8a^2}-\frac{3(a+4b)\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^3}{8a^2}-\frac{5\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)}{8a}\right)}{\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^4a-2\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^2a+4b\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^2+a\right)^2}-\frac{3\left(-\frac{\left(\sqrt{-b(a-b)}-b\right)\operatorname{arctanh}\left(\frac{\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)a}{\sqrt{\left(2\sqrt{-b(a-b)}+a-2b\right)a}}\right)}{2a\sqrt{-b(a-b)}\sqrt{\left(2\sqrt{-b(a-b)}+a-2b\right)a}}\cdots\right.}{4\cdots}}{d}$$

input
```
int(cosh(d*x+c)^4/(a+b*sinh(d*x+c)^2)^3,x)
```

output
```
1/d*(-2*(-5/8/a*tanh(1/2*d*x+1/2*c)^7-3/8*(a+4*b)/a^2*tanh(1/2*d*x+1/2*c)^
5-3/8*(a+4*b)/a^2*tanh(1/2*d*x+1/2*c)^3-5/8/a*tanh(1/2*d*x+1/2*c))/(tanh(1
/2*d*x+1/2*c)^4*a-2*tanh(1/2*d*x+1/2*c)^2*a+4*b*tanh(1/2*d*x+1/2*c)^2+a)^2
-3/4/a*(-1/2*((-b*(a-b))^(1/2)-b)/a/(-b*(a-b))^(1/2)/((2*(-b*(a-b))^(1/2)+
a-2*b)*a)^(1/2)*arctanh(tanh(1/2*d*x+1/2*c)*a/((2*(-b*(a-b))^(1/2)+a-2*b)*
a)^(1/2))+1/2*((-b*(a-b))^(1/2)+b)/a/(-b*(a-b))^(1/2)/((2*(-b*(a-b))^(1/2)
-a+2*b)*a)^(1/2)*arctan(tanh(1/2*d*x+1/2*c)*a/((2*(-b*(a-b))^(1/2)-a+2*b)*
a)^(1/2))))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2115 vs. $2(102) = 204$.

Time = 0.13 (sec) , antiderivative size = 4486, normalized size of antiderivative = 39.35

$$\int \frac{\cosh^4(c+dx)}{\left(a+b\sinh^2(c+dx)\right)^3}\,dx = \text{Too large to display}$$

input
```
integrate(cosh(d*x+c)^4/(a+b*sinh(d*x+c)^2)^3,x, algorithm="fricas")
```

output
```
Too large to include
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\cosh^4(c+dx)}{\left(a+b\sinh^2(c+dx)\right)^3}\,dx = \text{Timed out}$$

input
```
integrate(cosh(d*x+c)**4/(a+b*sinh(d*x+c)**2)**3,x)
```

output
```
Timed out
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{\cosh^4(c+dx)}{\left(a+b\sinh^2(c+dx)\right)^3}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate(cosh(d*x+c)^4/(a+b*sinh(d*x+c)^2)^3,x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(b-a>0)', see `assume?` for more
details)Is
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 244 vs. $2(102) = 204$.

Time = 1.03 (sec) , antiderivative size = 244, normalized size of antiderivative = 2.14

$$\int \frac{\cosh^4(c+dx)}{\left(a+b\sinh^2(c+dx)\right)^3}\,dx$$
$$= \frac{\frac{3\arctan\left(\frac{be^{(2\,dx+2\,c)}+2\,a-b}{2\sqrt{-a^2+ab}}\right)}{\sqrt{-a^2+ab}a^2} - \frac{2\,(8\,a^2be^{(6\,dx+6\,c)}-3\,b^3e^{(6\,dx+6\,c)}+16\,a^3e^{(4\,dx+4\,c)}+8\,a^2be^{(4\,dx+4\,c)}-18\,ab^2e^{(4\,dx+4\,c)}+9\,b^3e^{(4\,dx+4\,c)}+8\,a^2}{(be^{(4\,dx+4\,c)}+4\,ae^{(2\,dx+2\,c)}-2\,be^{(2\,dx+2\,c)}+b)^2a^2}}{8\,d}$$

input
```
integrate(cosh(d*x+c)^4/(a+b*sinh(d*x+c)^2)^3,x, algorithm="giac")
```

output
```
1/8*(3*arctan(1/2*(b*e^(2*d*x + 2*c) + 2*a - b)/sqrt(-a^2 + a*b))/(sqrt(-a
^2 + a*b)*a^2) - 2*(8*a^2*b*e^(6*d*x + 6*c) - 3*b^3*e^(6*d*x + 6*c) + 16*a
^3*e^(4*d*x + 4*c) + 8*a^2*b*e^(4*d*x + 4*c) - 18*a*b^2*e^(4*d*x + 4*c) +
9*b^3*e^(4*d*x + 4*c) + 8*a^2*b*e^(2*d*x + 2*c) + 16*a*b^2*e^(2*d*x + 2*c)
 - 9*b^3*e^(2*d*x + 2*c) + 2*a*b^2 + 3*b^3)/((b*e^(4*d*x + 4*c) + 4*a*e^(2
*d*x + 2*c) - 2*b*e^(2*d*x + 2*c) + b)^2*a^2*b^2))/d
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\cosh^4(c+dx)}{\left(a+b\sinh^2(c+dx)\right)^3}\,dx = \int \frac{\cosh(c+d\,x)^4}{\left(b\sinh\left(c+d\,x\right)^2+a\right)^3}\,dx$$

input
```
int(cosh(c + d*x)^4/(a + b*sinh(c + d*x)^2)^3,x)
```

output
```
int(cosh(c + d*x)^4/(a + b*sinh(c + d*x)^2)^3, x)
```

Reduce [B] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 11451, normalized size of antiderivative = 100.45

$$\int \frac{\cosh^4(c+dx)}{\left(a+b\sinh^2(c+dx)\right)^3}\,dx = \text{Too large to display}$$

input
```
int(cosh(d*x+c)^4/(a+b*sinh(d*x+c)^2)^3,x)
```

output
```
(32*e**(8*c + 8*d*x)*cosh(c + d*x)**3*sinh(c + d*x)*a**4*b**4 - 48*e**(8*c
 + 8*d*x)*cosh(c + d*x)**3*sinh(c + d*x)*a**3*b**5 + 16*e**(8*c + 8*d*x)*c
osh(c + d*x)**3*sinh(c + d*x)*a**2*b**6 + 256*e**(6*c + 6*d*x)*cosh(c + d*
x)**3*sinh(c + d*x)*a**5*b**3 - 512*e**(6*c + 6*d*x)*cosh(c + d*x)**3*sinh
(c + d*x)*a**4*b**4 + 320*e**(6*c + 6*d*x)*cosh(c + d*x)**3*sinh(c + d*x)*
a**3*b**5 - 64*e**(6*c + 6*d*x)*cosh(c + d*x)**3*sinh(c + d*x)*a**2*b**6 +
 512*e**(4*c + 4*d*x)*cosh(c + d*x)**3*sinh(c + d*x)*a**6*b**2 - 1280*e**(
4*c + 4*d*x)*cosh(c + d*x)**3*sinh(c + d*x)*a**5*b**3 + 1216*e**(4*c + 4*d
*x)*cosh(c + d*x)**3*sinh(c + d*x)*a**4*b**4 - 544*e**(4*c + 4*d*x)*cosh(c
 + d*x)**3*sinh(c + d*x)*a**3*b**5 + 96*e**(4*c + 4*d*x)*cosh(c + d*x)**3*
sinh(c + d*x)*a**2*b**6 + 256*e**(2*c + 2*d*x)*cosh(c + d*x)**3*sinh(c + d
*x)*a**5*b**3 - 512*e**(2*c + 2*d*x)*cosh(c + d*x)**3*sinh(c + d*x)*a**4*b
**4 + 320*e**(2*c + 2*d*x)*cosh(c + d*x)**3*sinh(c + d*x)*a**3*b**5 - 64*e
**(2*c + 2*d*x)*cosh(c + d*x)**3*sinh(c + d*x)*a**2*b**6 + 32*cosh(c + d*x
)**3*sinh(c + d*x)*a**4*b**4 - 48*cosh(c + d*x)**3*sinh(c + d*x)*a**3*b**5
 + 16*cosh(c + d*x)**3*sinh(c + d*x)*a**2*b**6 + 96*e**(8*c + 8*d*x)*cosh(
c + d*x)*sinh(c + d*x)**3*a**4*b**4 - 144*e**(8*c + 8*d*x)*cosh(c + d*x)*s
inh(c + d*x)**3*a**3*b**5 + 48*e**(8*c + 8*d*x)*cosh(c + d*x)*sinh(c + d*x
)**3*a**2*b**6 + 128*e**(8*c + 8*d*x)*cosh(c + d*x)*sinh(c + d*x)*a**5*b**
3 - 192*e**(8*c + 8*d*x)*cosh(c + d*x)*sinh(c + d*x)*a**4*b**4 + 64*e**...
```

3.306 $\int \frac{\cosh^3(c+dx)}{\left(a+b\sinh^2(c+dx)\right)^3}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 117

$$\int \frac{\cosh^3(c+dx)}{\left(a+b\sinh^2(c+dx)\right)^3}\,dx=\frac{(a+3b)\arctan\left(\frac{\sqrt{b}\sinh(c+dx)}{\sqrt{a}}\right)}{8a^{5/2}b^{3/2}d}-\frac{(a-b)\sinh(c+dx)}{4abd\left(a+b\sinh^2(c+dx)\right)^2}+\frac{(a+3b)\sinh(c+dx)}{8a^2bd\left(a+b\sinh^2(c+dx)\right)}$$

output
```
1/8*(a+3*b)*arctan(b^(1/2)*sinh(d*x+c)/a^(1/2))/a^(5/2)/b^(3/2)/d-1/4*(a-b
)*sinh(d*x+c)/a/b/d/(a+b*sinh(d*x+c)^2)^2+1/8*(a+3*b)*sinh(d*x+c)/a^2/b/d/
(a+b*sinh(d*x+c)^2)
```

Mathematica [A] (verified)

Time = 0.41 (sec) , antiderivative size = 114, normalized size of antiderivative = 0.97

$$\int \frac{\cosh^3(c+dx)}{\left(a+b\sinh^2(c+dx)\right)^3}\,dx = \frac{-\frac{\sinh(c+dx)}{\left(a+b\sinh^2(c+dx)\right)^2}+(a+3b)\left(\frac{3\arctan\left(\frac{\sqrt{b}\sinh(c+dx)}{\sqrt{a}}\right)}{8a^{5/2}\sqrt{b}}+\frac{\sinh(c+dx)\left(5a+3b\sinh^2(c+dx)\right)}{8a^2\left(a+b\sinh^2(c+dx)\right)^2}\right)}{3bd}$$

input
```
Integrate[Cosh[c + d*x]^3/(a + b*Sinh[c + d*x]^2)^3,x]
```

output
```
(-(Sinh[c + d*x]/(a + b*Sinh[c + d*x]^2)^2) + (a + 3*b)*((3*ArcTan[(Sqrt[b
]*Sinh[c + d*x])/Sqrt[a]])/(8*a^(5/2)*Sqrt[b]) + (Sinh[c + d*x]*(5*a + 3*b
*Sinh[c + d*x]^2))/(8*a^2*(a + b*Sinh[c + d*x]^2)^2)))/(3*b*d)
```

Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 115, normalized size of antiderivative = 0.98, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.217, Rules used = {3042, 3669, 298, 215, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cosh^3(c+dx)}{\left(a+b\sinh^2(c+dx)\right)^3}\,dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \frac{\cos(ic+idx)^3}{\left(a-b\sin(ic+idx)^2\right)^3}dx$$

$$\Big\downarrow \text{ 3669}$$

$$\frac{\int \frac{\sinh^2(c+dx)+1}{\left(b\sinh^2(c+dx)+a\right)^3}d\sinh(c+dx)}{d}$$

$$\Big\downarrow \text{ 298}$$

$$\frac{\frac{(a+3b)\int \frac{1}{\left(b\sinh^2(c+dx)+a\right)^2}d\sinh(c+dx)}{4ab}-\frac{(a-b)\sinh(c+dx)}{4ab\left(a+b\sinh^2(c+dx)\right)^2}}{d}$$

$$\Big\downarrow \text{215}$$

$$\frac{\frac{(a+3b)\left(\frac{\int \frac{1}{b\sinh^2(c+dx)+a}d\sinh(c+dx)}{2a}+\frac{\sinh(c+dx)}{2a\left(a+b\sinh^2(c+dx)\right)}\right)}{4ab}-\frac{(a-b)\sinh(c+dx)}{4ab\left(a+b\sinh^2(c+dx)\right)^2}}{d}$$

$$\Big\downarrow \text{218}$$

$$\frac{\frac{(a+3b)\left(\frac{\arctan\left(\frac{\sqrt{b}\sinh(c+dx)}{\sqrt{a}}\right)}{2a^{3/2}\sqrt{b}}+\frac{\sinh(c+dx)}{2a\left(a+b\sinh^2(c+dx)\right)}\right)}{4ab}-\frac{(a-b)\sinh(c+dx)}{4ab\left(a+b\sinh^2(c+dx)\right)^2}}{d}$$

input
```
Int[Cosh[c + d*x]^3/(a + b*Sinh[c + d*x]^2)^3,x]
```

output
```
(-1/4*((a - b)*Sinh[c + d*x])/(a*b*(a + b*Sinh[c + d*x]^2)^2) + ((a + 3*b)
*(ArcTan[(Sqrt[b]*Sinh[c + d*x])/Sqrt[a]]/(2*a^(3/2)*Sqrt[b]) + Sinh[c + d
*x]/(2*a*(a + b*Sinh[c + d*x]^2))))/(4*a*b))/d
```

Defintions of rubi rules used

rule 215
```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(-x)*((a + b*x^2)^(p + 1)
/(2*a*(p + 1))), x] + Simp[(2*p + 3)/(2*a*(p + 1))   Int[(a + b*x^2)^(p + 1
), x], x] /; FreeQ[{a, b}, x] && LtQ[p, -1] && (IntegerQ[4*p] || IntegerQ[6
*p])
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 298
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2), x_Symbol] :> Simp[(-(
b*c - a*d))*x*((a + b*x^2)^(p + 1)/(2*a*b*(p + 1))), x] - Simp[(a*d - b*c*(
2*p + 3))/(2*a*b*(p + 1))   Int[(a + b*x^2)^(p + 1), x], x] /; FreeQ[{a, b,
 c, d, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/2 + p, 0])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3669
```
Int[cos[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^
(p_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Simp[ff/f   S
ubst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a + b*ff^2*x^2)^p, x], x, Sin[e + f*x]
/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m - 1)/2]
```

Maple [A] (verified)

Time = 0.11 (sec) , antiderivative size = 91, normalized size of antiderivative = 0.78

$$\frac{\frac{\frac{(a+3b)\sinh(dx+c)^3}{8a^2}-\frac{(a-5b)\sinh(dx+c)}{8ab}}{\left(a+b\sinh(dx+c)^2\right)^2}+\frac{(a+3b)\arctan\left(\frac{b\sinh(dx+c)}{\sqrt{ab}}\right)}{8a^2b\sqrt{ab}}}{d}$$

input
```
int(cosh(d*x+c)^3/(a+b*sinh(d*x+c)^2)^3,x)
```

output
```
1/d*((1/8*(a+3*b)/a^2*sinh(d*x+c)^3-1/8*(a-5*b)/a/b*sinh(d*x+c))/(a+b*sinh
(d*x+c)^2)^2+1/8*(a+3*b)/a^2/b/(a*b)^(1/2)*arctan(b*sinh(d*x+c)/(a*b)^(1/2
)))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2700 vs. 2(103) = 206.

Time = 0.16 (sec) , antiderivative size = 4911, normalized size of antiderivative = 41.97

$$\int \frac{\cosh^3(c+dx)}{\left(a+b\sinh^2(c+dx)\right)^3}\,dx=\text{Too large to display}$$

input
```
integrate(cosh(d*x+c)^3/(a+b*sinh(d*x+c)^2)^3,x, algorithm="fricas")
```

output
```
Too large to include
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\cosh^3(c+dx)}{\left(a+b\sinh^2(c+dx)\right)^3}\,dx = \text{Timed out}$$

input
```
integrate(cosh(d*x+c)**3/(a+b*sinh(d*x+c)**2)**3,x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{\cosh^3(c+dx)}{\left(a+b\sinh^2(c+dx)\right)^3}\,dx = \int \frac{\cosh\left(dx+c\right)^3}{\left(b\sinh\left(dx+c\right)^2+a\right)^3}\,dx$$

input
```
integrate(cosh(d*x+c)^3/(a+b*sinh(d*x+c)^2)^3,x, algorithm="maxima")
```

output
```
1/4*((a*b*e^(7*c) + 3*b^2*e^(7*c))*e^(7*d*x) - (4*a^2*e^(5*c) - 17*a*b*e^(
5*c) + 9*b^2*e^(5*c))*e^(5*d*x) + (4*a^2*e^(3*c) - 17*a*b*e^(3*c) + 9*b^2*
e^(3*c))*e^(3*d*x) - (a*b*e^c + 3*b^2*e^c)*e^(d*x))/(a^2*b^3*d*e^(8*d*x +
8*c) + a^2*b^3*d + 4*(2*a^3*b^2*d*e^(6*c) - a^2*b^3*d*e^(6*c))*e^(6*d*x) +
 2*(8*a^4*b*d*e^(4*c) - 8*a^3*b^2*d*e^(4*c) + 3*a^2*b^3*d*e^(4*c))*e^(4*d*
x) + 4*(2*a^3*b^2*d*e^(2*c) - a^2*b^3*d*e^(2*c))*e^(2*d*x)) + 1/8*integrat
e(2*((a*e^(3*c) + 3*b*e^(3*c))*e^(3*d*x) + (a*e^c + 3*b*e^c)*e^(d*x))/(a^2
*b^2*e^(4*d*x + 4*c) + a^2*b^2 + 2*(2*a^3*b*e^(2*c) - a^2*b^2*e^(2*c))*e^(
2*d*x)), x)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{\cosh^3(c+dx)}{(a+b\sinh^2(c+dx))^3}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(cosh(d*x+c)^3/(a+b*sinh(d*x+c)^2)^3,x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Limit: Max order reached or unable
to make series expansion Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\cosh^3(c+dx)}{(a+b\sinh^2(c+dx))^3}\,dx = \int \frac{\cosh(c+d\,x)^3}{\left(b\sinh\left(c+d\,x\right)^2+a\right)^3}\,dx$$

input
```
int(cosh(c + d*x)^3/(a + b*sinh(c + d*x)^2)^3,x)
```

output
```
int(cosh(c + d*x)^3/(a + b*sinh(c + d*x)^2)^3, x)
```

Reduce [B] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 21865, normalized size of antiderivative = 186.88

$$\int \frac{\cosh^3(c+dx)}{(a+b\sinh^2(c+dx))^3}\,dx = \text{Too large to display}$$

input
```
int(cosh(d*x+c)^3/(a+b*sinh(d*x+c)^2)^3,x)
```

output
```
( - 18*e**(8*c + 8*d*x)*sqrt(b)*sqrt(a)*sqrt(a - b)*sqrt(2*sqrt(a)*sqrt(a
- b) + 2*a - b)*atan((e**(c + d*x)*b)/(sqrt(b)*sqrt(2*sqrt(a)*sqrt(a - b)
+ 2*a - b)))*sinh(c + d*x)**4*a*b**4 - 6*e**(8*c + 8*d*x)*sqrt(b)*sqrt(a)*
sqrt(a - b)*sqrt(2*sqrt(a)*sqrt(a - b) + 2*a - b)*atan((e**(c + d*x)*b)/(s
qrt(b)*sqrt(2*sqrt(a)*sqrt(a - b) + 2*a - b)))*sinh(c + d*x)**4*b**5 - 36*
e**(8*c + 8*d*x)*sqrt(b)*sqrt(a)*sqrt(a - b)*sqrt(2*sqrt(a)*sqrt(a - b) +
2*a - b)*atan((e**(c + d*x)*b)/(sqrt(b)*sqrt(2*sqrt(a)*sqrt(a - b) + 2*a -
 b)))*sinh(c + d*x)**2*a**2*b**3 - 12*e**(8*c + 8*d*x)*sqrt(b)*sqrt(a)*sqr
t(a - b)*sqrt(2*sqrt(a)*sqrt(a - b) + 2*a - b)*atan((e**(c + d*x)*b)/(sqrt
(b)*sqrt(2*sqrt(a)*sqrt(a - b) + 2*a - b)))*sinh(c + d*x)**2*a*b**4 - 18*e
**(8*c + 8*d*x)*sqrt(b)*sqrt(a)*sqrt(a - b)*sqrt(2*sqrt(a)*sqrt(a - b) + 2
*a - b)*atan((e**(c + d*x)*b)/(sqrt(b)*sqrt(2*sqrt(a)*sqrt(a - b) + 2*a -
b)))*a**3*b**2 - 6*e**(8*c + 8*d*x)*sqrt(b)*sqrt(a)*sqrt(a - b)*sqrt(2*sqr
t(a)*sqrt(a - b) + 2*a - b)*atan((e**(c + d*x)*b)/(sqrt(b)*sqrt(2*sqrt(a)*
sqrt(a - b) + 2*a - b)))*a**2*b**3 - 144*e**(6*c + 6*d*x)*sqrt(b)*sqrt(a)*
sqrt(a - b)*sqrt(2*sqrt(a)*sqrt(a - b) + 2*a - b)*atan((e**(c + d*x)*b)/(s
qrt(b)*sqrt(2*sqrt(a)*sqrt(a - b) + 2*a - b)))*sinh(c + d*x)**4*a**2*b**3
+ 24*e**(6*c + 6*d*x)*sqrt(b)*sqrt(a)*sqrt(a - b)*sqrt(2*sqrt(a)*sqrt(a -
b) + 2*a - b)*atan((e**(c + d*x)*b)/(sqrt(b)*sqrt(2*sqrt(a)*sqrt(a - b) +
2*a - b)))*sinh(c + d*x)**4*a*b**4 + 24*e**(6*c + 6*d*x)*sqrt(b)*sqrt(a...
```

3.307 $\int \frac{\cosh^2(c+dx)}{\left(a+b\sinh^2(c+dx)\right)^3}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 143

$$\int \frac{\cosh^2(c+dx)}{\left(a+b\sinh^2(c+dx)\right)^3}\,dx = \frac{(4a-3b)\text{arctanh}\left(\frac{\sqrt{a-b}\tanh(c+dx)}{\sqrt{a}}\right)}{8a^{5/2}(a-b)^{3/2}d} - \frac{b\tanh(c+dx)}{4a(a-b)d\left(a-(a-b)\tanh^2(c+dx)\right)^2} + \frac{(4a-3b)\tanh(c+dx)}{8a^2(a-b)d\left(a-(a-b)\tanh^2(c+dx)\right)}$$

output

```
1/8*(4*a-3*b)*arctanh((a-b)^(1/2)*tanh(d*x+c)/a^(1/2))/a^(5/2)/(a-b)^(3/2)
/d-1/4*b*tanh(d*x+c)/a/(a-b)/d/(a-(a-b)*tanh(d*x+c)^2)^2+1/8*(4*a-3*b)*tan
h(d*x+c)/a^2/(a-b)/d/(a-(a-b)*tanh(d*x+c)^2)
```

Mathematica [A] (verified)

Time = 0.99 (sec) , antiderivative size = 124, normalized size of antiderivative = 0.87

$$\int \frac{\cosh^2(c+dx)}{(a+b\sinh^2(c+dx))^3}\,dx = \frac{\frac{(4a-3b)\operatorname{arctanh}\left(\frac{\sqrt{a-b}\tanh(c+dx)}{\sqrt{a}}\right)}{(a-b)^{3/2}}+\frac{\sqrt{a}(8a^2-12ab+3b^2+(2a-3b)b\cosh(2(c+dx)))\sinh(2(c+dx))}{(a-b)(2a-b+b\cosh(2(c+dx)))^2}}{8a^{5/2}d}$$

input
```
Integrate[Cosh[c + d*x]^2/(a + b*Sinh[c + d*x]^2)^3,x]
```

output
```
(((4*a - 3*b)*ArcTanh[(Sqrt[a - b]*Tanh[c + d*x])/Sqrt[a]])/(a - b)^(3/2)
+ (Sqrt[a]*(8*a^2 - 12*a*b + 3*b^2 + (2*a - 3*b)*b*Cosh[2*(c + d*x)])*Sinh
[2*(c + d*x)])/((a - b)*(2*a - b + b*Cosh[2*(c + d*x)])^2))/(8*a^(5/2)*d)
```

Rubi [A] (verified)

Time = 0.31 (sec) , antiderivative size = 139, normalized size of antiderivative = 0.97, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.217, Rules used = {3042, 3670, 298, 215, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cosh^2(c+dx)}{(a+b\sinh^2(c+dx))^3}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\cos(ic+idx)^2}{(a-b\sin(ic+idx)^2)^3}dx$$

$$\Big\downarrow \text{3670}$$

$$\frac{\int \frac{1-\tanh^2(c+dx)}{(a-(a-b)\tanh^2(c+dx))^3}d\tanh(c+dx)}{d}$$

$$\Big\downarrow \text{298}$$

$$\frac{\frac{(4a-3b)\int \frac{1}{\left(a-(a-b)\tanh^2(c+dx)\right)^2}d\tanh(c+dx)}{4a(a-b)}-\frac{b\tanh(c+dx)}{4a(a-b)\left(a-(a-b)\tanh^2(c+dx)\right)^2}}{d}$$

$$\Big\downarrow \text{215}$$

$$\frac{\frac{(4a-3b)\left(\frac{\int \frac{1}{a-(a-b)\tanh^2(c+dx)}d\tanh(c+dx)}{2a}+\frac{\tanh(c+dx)}{2a\left(a-(a-b)\tanh^2(c+dx)\right)}\right)}{4a(a-b)}-\frac{b\tanh(c+dx)}{4a(a-b)\left(a-(a-b)\tanh^2(c+dx)\right)^2}}{d}$$

$$\Big\downarrow \text{221}$$

$$\frac{\frac{(4a-3b)\left(\frac{\operatorname{arctanh}\left(\frac{\sqrt{a-b}\tanh(c+dx)}{\sqrt{a}}\right)}{2a^{3/2}\sqrt{a-b}}+\frac{\tanh(c+dx)}{2a\left(a-(a-b)\tanh^2(c+dx)\right)}\right)}{4a(a-b)}-\frac{b\tanh(c+dx)}{4a(a-b)\left(a-(a-b)\tanh^2(c+dx)\right)^2}}{d}$$

input
```
Int[Cosh[c + d*x]^2/(a + b*Sinh[c + d*x]^2)^3,x]
```

output
```
(-1/4*(b*Tanh[c + d*x])/(a*(a - b)*(a - (a - b)*Tanh[c + d*x]^2)^2) + ((4*
a - 3*b)*(ArcTanh[(Sqrt[a - b]*Tanh[c + d*x])/Sqrt[a]]/(2*a^(3/2)*Sqrt[a -
 b]) + Tanh[c + d*x]/(2*a*(a - (a - b)*Tanh[c + d*x]^2))))/(4*a*(a - b)))/
d
```

Defintions of rubi rules used

rule 215
```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(-x)*((a + b*x^2)^(p + 1)
/(2*a*(p + 1))), x] + Simp[(2*p + 3)/(2*a*(p + 1))   Int[(a + b*x^2)^(p + 1
), x], x] /; FreeQ[{a, b}, x] && LtQ[p, -1] && (IntegerQ[4*p] || IntegerQ[6
*p])
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 298
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2), x_Symbol] :> Simp[(-(
b*c - a*d))*x*((a + b*x^2)^(p + 1)/(2*a*b*(p + 1))), x] - Simp[(a*d - b*c*(
2*p + 3))/(2*a*b*(p + 1))   Int[(a + b*x^2)^(p + 1), x], x] /; FreeQ[{a, b,
 c, d, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/2 + p, 0])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3670
```
Int[cos[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(
p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Simp[ff/f   Su
bst[Int[(a + (a + b)*ff^2*x^2)^p/(1 + ff^2*x^2)^(m/2 + p + 1), x], x, Tan[e
 + f*x]/ff], x]] /; FreeQ[{a, b, e, f}, x] && IntegerQ[m/2] && IntegerQ[p]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 375 vs. $2(129) = 258$.

Time = 0.19 (sec) , antiderivative size = 376, normalized size of antiderivative = 2.63

$$-\frac{\frac{2\left(-\frac{(4a-5b)\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^{7}}{8a(a-b)}+\frac{\left(4a^{2}-13ab+12b^{2}\right)\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^{5}}{8a^{2}(a-b)}+\frac{\left(4a^{2}-13ab+12b^{2}\right)\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^{3}}{8a^{2}(a-b)}-\frac{(4a-5b)\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)}{8a(a-b)}\right)}{\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^{4}a-2\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}a+4b\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}+a\right)^{2}}-\frac{(4a-3b)\left(-\frac{\left(\sqrt{-b(a}\right.}{2}\right.}{}}{d}$$

input
```
int(cosh(d*x+c)^2/(a+b*sinh(d*x+c)^2)^3,x)
```

output
```
1/d*(-2*(-1/8*(4*a-5*b)/a/(a-b)*tanh(1/2*d*x+1/2*c)^7+1/8*(4*a^2-13*a*b+12
*b^2)/a^2/(a-b)*tanh(1/2*d*x+1/2*c)^5+1/8*(4*a^2-13*a*b+12*b^2)/a^2/(a-b)*
tanh(1/2*d*x+1/2*c)^3-1/8*(4*a-5*b)/a/(a-b)*tanh(1/2*d*x+1/2*c))/(tanh(1/2
*d*x+1/2*c)^4*a-2*tanh(1/2*d*x+1/2*c)^2*a+4*b*tanh(1/2*d*x+1/2*c)^2+a)^2-1
/4/a*(4*a-3*b)/(a-b)*(-1/2*((-b*(a-b))^(1/2)-b)/a/(-b*(a-b))^(1/2)/((2*(-b
*(a-b))^(1/2)+a-2*b)*a)^(1/2)*arctanh(tanh(1/2*d*x+1/2*c)*a/((2*(-b*(a-b))
^(1/2)+a-2*b)*a)^(1/2))+1/2*((-b*(a-b))^(1/2)+b)/a/(-b*(a-b))^(1/2)/((2*(-
b*(a-b))^(1/2)-a+2*b)*a)^(1/2)*arctan(tanh(1/2*d*x+1/2*c)*a/((2*(-b*(a-b))
^(1/2)-a+2*b)*a)^(1/2))))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2464 vs. $2(131) = 262$.

Time = 0.17 (sec) , antiderivative size = 5183, normalized size of antiderivative = 36.24

$$\int \frac{\cosh^2(c+dx)}{\left(a+b\sinh^2(c+dx)\right)^3}\,dx = \text{Too large to display}$$

input
```
integrate(cosh(d*x+c)^2/(a+b*sinh(d*x+c)^2)^3,x, algorithm="fricas")
```

output
```
Too large to include
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\cosh^2(c+dx)}{\left(a+b\sinh^2(c+dx)\right)^3}\,dx = \text{Timed out}$$

input
```
integrate(cosh(d*x+c)**2/(a+b*sinh(d*x+c)**2)**3,x)
```

output
```
Timed out
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{\cosh^2(c+dx)}{\left(a+b\sinh^2(c+dx)\right)^3}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate(cosh(d*x+c)^2/(a+b*sinh(d*x+c)^2)^3,x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(b-a>0)', see `assume?` for more
details)Is
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 269 vs. 2(131) = 262.

Time = 0.64 (sec) , antiderivative size = 269, normalized size of antiderivative = 1.88

$$\int \frac{\cosh^2(c+dx)}{\left(a+b\sinh^2(c+dx)\right)^3}\,dx = \frac{\frac{(4a-3b)\arctan\left(\frac{be^{(2dx+2c)}+2a-b}{2\sqrt{-a^2+ab}}\right)}{(a^3-a^2b)\sqrt{-a^2+ab}} + \frac{2\left(4ab^2e^{(6dx+6c)}-3b^3e^{(6dx+6c)}-16a^3e^{(4dx+4c)}+40a^2be^{(4dx+4c)}-30ab^2e^{(4dx+4c)}+9b^3e^{(4dx+}\right.}{(a^3b-a^2b^2)\left(be^{(4dx+4c)}+4ae^{(2dx+2c)}-2be^{\cdot}\right.}}{8d}$$

input
```
integrate(cosh(d*x+c)^2/(a+b*sinh(d*x+c)^2)^3,x, algorithm="giac")
```

output
```
1/8*((4*a - 3*b)*arctan(1/2*(b*e^(2*d*x + 2*c) + 2*a - b)/sqrt(-a^2 + a*b)
)/((a^3 - a^2*b)*sqrt(-a^2 + a*b)) + 2*(4*a*b^2*e^(6*d*x + 6*c) - 3*b^3*e^
(6*d*x + 6*c) - 16*a^3*e^(4*d*x + 4*c) + 40*a^2*b*e^(4*d*x + 4*c) - 30*a*b
^2*e^(4*d*x + 4*c) + 9*b^3*e^(4*d*x + 4*c) - 16*a^2*b*e^(2*d*x + 2*c) + 28
*a*b^2*e^(2*d*x + 2*c) - 9*b^3*e^(2*d*x + 2*c) - 2*a*b^2 + 3*b^3)/((a^3*b
- a^2*b^2)*(b*e^(4*d*x + 4*c) + 4*a*e^(2*d*x + 2*c) - 2*b*e^(2*d*x + 2*c)
+ b)^2))/d
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\cosh^2(c+dx)}{\left(a+b\sinh^2(c+dx)\right)^3}\,dx = \int \frac{\cosh(c+d\,x)^2}{\left(b\sinh\left(c+d\,x\right)^2+a\right)^3}\,dx$$

input
```
int(cosh(c + d*x)^2/(a + b*sinh(c + d*x)^2)^3,x)
```

output
```
int(cosh(c + d*x)^2/(a + b*sinh(c + d*x)^2)^3, x)
```

Reduce [B] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 13353, normalized size of antiderivative = 93.38

$$\int \frac{\cosh^2(c+dx)}{\left(a+b\sinh^2(c+dx)\right)^3}\,dx = \text{Too large to display}$$

input
```
int(cosh(d*x+c)^2/(a+b*sinh(d*x+c)^2)^3,x)
```

output

```
(32*e**(8*c + 8*d*x)*cosh(c + d*x)*sinh(c + d*x)*a**5*b**3 - 80*e**(8*c +
8*d*x)*cosh(c + d*x)*sinh(c + d*x)*a**4*b**4 + 64*e**(8*c + 8*d*x)*cosh(c
+ d*x)*sinh(c + d*x)*a**3*b**5 - 16*e**(8*c + 8*d*x)*cosh(c + d*x)*sinh(c
+ d*x)*a**2*b**6 + 256*e**(6*c + 6*d*x)*cosh(c + d*x)*sinh(c + d*x)*a**6*b
**2 - 768*e**(6*c + 6*d*x)*cosh(c + d*x)*sinh(c + d*x)*a**5*b**3 + 832*e**
(6*c + 6*d*x)*cosh(c + d*x)*sinh(c + d*x)*a**4*b**4 - 384*e**(6*c + 6*d*x)
*cosh(c + d*x)*sinh(c + d*x)*a**3*b**5 + 64*e**(6*c + 6*d*x)*cosh(c + d*x)
*sinh(c + d*x)*a**2*b**6 + 512*e**(4*c + 4*d*x)*cosh(c + d*x)*sinh(c + d*x
)*a**7*b - 1792*e**(4*c + 4*d*x)*cosh(c + d*x)*sinh(c + d*x)*a**6*b**2 + 2
496*e**(4*c + 4*d*x)*cosh(c + d*x)*sinh(c + d*x)*a**5*b**3 - 1760*e**(4*c
+ 4*d*x)*cosh(c + d*x)*sinh(c + d*x)*a**4*b**4 + 640*e**(4*c + 4*d*x)*cosh
(c + d*x)*sinh(c + d*x)*a**3*b**5 - 96*e**(4*c + 4*d*x)*cosh(c + d*x)*sinh
(c + d*x)*a**2*b**6 + 256*e**(2*c + 2*d*x)*cosh(c + d*x)*sinh(c + d*x)*a**
6*b**2 - 768*e**(2*c + 2*d*x)*cosh(c + d*x)*sinh(c + d*x)*a**5*b**3 + 832*
e**(2*c + 2*d*x)*cosh(c + d*x)*sinh(c + d*x)*a**4*b**4 - 384*e**(2*c + 2*d
*x)*cosh(c + d*x)*sinh(c + d*x)*a**3*b**5 + 64*e**(2*c + 2*d*x)*cosh(c + d
*x)*sinh(c + d*x)*a**2*b**6 + 32*cosh(c + d*x)*sinh(c + d*x)*a**5*b**3 - 8
0*cosh(c + d*x)*sinh(c + d*x)*a**4*b**4 + 64*cosh(c + d*x)*sinh(c + d*x)*a
**3*b**5 - 16*cosh(c + d*x)*sinh(c + d*x)*a**2*b**6 + 8*e**(8*c + 8*d*x)*s
qrt(a)*sqrt(a - b)*log( - sqrt(2*sqrt(a)*sqrt(a - b) - 2*a + b) + e**(c...
```

3.308 $\int \frac{\cosh(c+dx)}{\left(a+b\sinh^2(c+dx)\right)^3} dx$

Optimal result

Integrand size = 21, antiderivative size = 96

$$\int \frac{\cosh(c+dx)}{\left(a+b\sinh^2(c+dx)\right)^3} dx = \frac{3\arctan\left(\frac{\sqrt{b}\sinh(c+dx)}{\sqrt{a}}\right)}{8a^{5/2}\sqrt{b}d} + \frac{\sinh(c+dx)}{4ad\left(a+b\sinh^2(c+dx)\right)^2} + \frac{3\sinh(c+dx)}{8a^2d\left(a+b\sinh^2(c+dx)\right)}$$

output
```
3/8*arctan(b^(1/2)*sinh(d*x+c)/a^(1/2))/a^(5/2)/b^(1/2)/d+1/4*sinh(d*x+c)/
a/d/(a+b*sinh(d*x+c)^2)^2+3/8*sinh(d*x+c)/a^2/d/(a+b*sinh(d*x+c)^2)
```

Mathematica [A] (verified)

Time = 0.20 (sec) , antiderivative size = 79, normalized size of antiderivative = 0.82

$$\int \frac{\cosh(c+dx)}{\left(a+b\sinh^2(c+dx)\right)^3} dx = \frac{\frac{3\arctan\left(\frac{\sqrt{b}\sinh(c+dx)}{\sqrt{a}}\right)}{8a^{5/2}\sqrt{b}} + \frac{\sinh(c+dx)\left(5a+3b\sinh^2(c+dx)\right)}{8a^2\left(a+b\sinh^2(c+dx)\right)^2}}{d}$$

input
```
Integrate[Cosh[c + d*x]/(a + b*Sinh[c + d*x]^2)^3,x]
```

output
```
((3*ArcTan[(Sqrt[b]*Sinh[c + d*x])/Sqrt[a]])/(8*a^(5/2)*Sqrt[b]) + (Sinh[c
 + d*x]*(5*a + 3*b*Sinh[c + d*x]^2))/(8*a^2*(a + b*Sinh[c + d*x]^2)^2))/d
```

Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 99, normalized size of antiderivative = 1.03, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.238$, Rules used = {3042, 3669, 215, 215, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cosh(c+dx)}{\left(a+b\sinh^2(c+dx)\right)^3}dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \frac{\cos(ic+idx)}{\left(a-b\sin(ic+idx)^2\right)^3}dx$$

$$\Big\downarrow \text{ 3669}$$

$$\frac{\int \frac{1}{\left(b\sinh^2(c+dx)+a\right)^3}d\sinh(c+dx)}{d}$$

$$\Big\downarrow \text{ 215}$$

$$\frac{\frac{3\int \frac{1}{\left(b\sinh^2(c+dx)+a\right)^2}d\sinh(c+dx)}{4a}+\frac{\sinh(c+dx)}{4a\left(a+b\sinh^2(c+dx)\right)^2}}{d}$$

$$\Big\downarrow \text{ 215}$$

$$\frac{\frac{3\left(\frac{\int \frac{1}{b\sinh^2(c+dx)+a}d\sinh(c+dx)}{2a}+\frac{\sinh(c+dx)}{2a\left(a+b\sinh^2(c+dx)\right)}\right)}{4a}+\frac{\sinh(c+dx)}{4a\left(a+b\sinh^2(c+dx)\right)^2}}{d}$$

$$\Big\downarrow \text{ 218}$$

$$\frac{\frac{3\left(\frac{\arctan\left(\frac{\sqrt{b}\sinh(c+dx)}{\sqrt{a}}\right)}{2a^{3/2}\sqrt{b}}+\frac{\sinh(c+dx)}{2a\left(a+b\sinh^2(c+dx)\right)}\right)}{4a}+\frac{\sinh(c+dx)}{4a\left(a+b\sinh^2(c+dx)\right)^2}}{d}$$

input
```
Int[Cosh[c + d*x]/(a + b*Sinh[c + d*x]^2)^3,x]
```

output
```
(Sinh[c + d*x]/(4*a*(a + b*Sinh[c + d*x]^2)^2) + (3*(ArcTan[(Sqrt[b]*Sinh[
c + d*x])/Sqrt[a]]/(2*a^(3/2)*Sqrt[b]) + Sinh[c + d*x]/(2*a*(a + b*Sinh[c
+ d*x]^2))))/(4*a))/d
```

Defintions of rubi rules used

rule 215
```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(-x)*((a + b*x^2)^(p + 1)
/(2*a*(p + 1))), x] + Simp[(2*p + 3)/(2*a*(p + 1))   Int[(a + b*x^2)^(p + 1
), x], x] /; FreeQ[{a, b}, x] && LtQ[p, -1] && (IntegerQ[4*p] || IntegerQ[6
*p])
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3669
```
Int[cos[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^
(p_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Simp[ff/f   S
ubst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a + b*ff^2*x^2)^p, x], x, Sin[e + f*x]
/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m - 1)/2]
```

Maple [A] (verified)

Time = 0.05 (sec) , antiderivative size = 85, normalized size of antiderivative = 0.89

$$\frac{\sinh\left(dx+c\right)}{4ad\left(a+b\sinh\left(dx+c\right)^{2}\right)^{2}}+\frac{3\sinh\left(dx+c\right)}{8a^{2}d\left(a+b\sinh\left(dx+c\right)^{2}\right)}+\frac{3\arctan\left(\frac{b\sinh(dx+c)}{\sqrt{ab}}\right)}{8d\,a^{2}\sqrt{ab}}$$

input
```
int(cosh(d*x+c)/(a+b*sinh(d*x+c)^2)^3,x)
```

output
```
1/4*sinh(d*x+c)/a/d/(a+b*sinh(d*x+c)^2)^2+3/8*sinh(d*x+c)/a^2/d/(a+b*sinh(
d*x+c)^2)+3/8/d/a^2/(a*b)^(1/2)*arctan(b*sinh(d*x+c)/(a*b)^(1/2))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2114 vs. $2(82) = 164$.

Time = 0.14 (sec) , antiderivative size = 3937, normalized size of antiderivative = 41.01

$$\int \frac{\cosh(c+dx)}{\left(a+b\sinh^2(c+dx)\right)^3}\,dx = \text{Too large to display}$$

input
```
integrate(cosh(d*x+c)/(a+b*sinh(d*x+c)^2)^3,x, algorithm="fricas")
```

output
```
Too large to include
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 835 vs. $2(85) = 170$.

Time = 23.92 (sec) , antiderivative size = 835, normalized size of antiderivative = 8.70

$$\int \frac{\cosh(c+dx)}{\left(a+b\sinh^2(c+dx)\right)^3}\,dx = \text{Too large to display}$$

input
```
integrate(cosh(d*x+c)/(a+b*sinh(d*x+c)**2)**3,x)
```

output
```
Piecewise((zoo*x*cosh(c)/sinh(c)**6, Eq(a, 0) & Eq(b, 0) & Eq(d, 0)), (sin
h(c + d*x)/(a**3*d), Eq(b, 0)), (-1/(5*b**3*d*sinh(c + d*x)**5), Eq(a, 0))
, (x*cosh(c)/(a + b*sinh(c)**2)**3, Eq(d, 0)), (3*a**2*log(-sqrt(-a/b) + s
inh(c + d*x))/(16*a**4*b*d*sqrt(-a/b) + 32*a**3*b**2*d*sqrt(-a/b)*sinh(c +
 d*x)**2 + 16*a**2*b**3*d*sqrt(-a/b)*sinh(c + d*x)**4) - 3*a**2*log(sqrt(-
a/b) + sinh(c + d*x))/(16*a**4*b*d*sqrt(-a/b) + 32*a**3*b**2*d*sqrt(-a/b)*
sinh(c + d*x)**2 + 16*a**2*b**3*d*sqrt(-a/b)*sinh(c + d*x)**4) + 10*a*b*sq
rt(-a/b)*sinh(c + d*x)/(16*a**4*b*d*sqrt(-a/b) + 32*a**3*b**2*d*sqrt(-a/b)
*sinh(c + d*x)**2 + 16*a**2*b**3*d*sqrt(-a/b)*sinh(c + d*x)**4) + 6*a*b*lo
g(-sqrt(-a/b) + sinh(c + d*x))*sinh(c + d*x)**2/(16*a**4*b*d*sqrt(-a/b) +
32*a**3*b**2*d*sqrt(-a/b)*sinh(c + d*x)**2 + 16*a**2*b**3*d*sqrt(-a/b)*sin
h(c + d*x)**4) - 6*a*b*log(sqrt(-a/b) + sinh(c + d*x))*sinh(c + d*x)**2/(1
6*a**4*b*d*sqrt(-a/b) + 32*a**3*b**2*d*sqrt(-a/b)*sinh(c + d*x)**2 + 16*a*
*2*b**3*d*sqrt(-a/b)*sinh(c + d*x)**4) + 6*b**2*sqrt(-a/b)*sinh(c + d*x)**
3/(16*a**4*b*d*sqrt(-a/b) + 32*a**3*b**2*d*sqrt(-a/b)*sinh(c + d*x)**2 + 1
6*a**2*b**3*d*sqrt(-a/b)*sinh(c + d*x)**4) + 3*b**2*log(-sqrt(-a/b) + sinh
(c + d*x))*sinh(c + d*x)**4/(16*a**4*b*d*sqrt(-a/b) + 32*a**3*b**2*d*sqrt(
-a/b)*sinh(c + d*x)**2 + 16*a**2*b**3*d*sqrt(-a/b)*sinh(c + d*x)**4) - 3*b
**2*log(sqrt(-a/b) + sinh(c + d*x))*sinh(c + d*x)**4/(16*a**4*b*d*sqrt(-a/
b) + 32*a**3*b**2*d*sqrt(-a/b)*sinh(c + d*x)**2 + 16*a**2*b**3*d*sqrt(-...
```

Maxima [F]

$$\int \frac{\cosh(c+dx)}{\left(a+b\sinh^2(c+dx)\right)^3}\,dx=\int \frac{\cosh\left(dx+c\right)}{\left(b\sinh\left(dx+c\right)^2+a\right)^3}\,dx$$

input
```
integrate(cosh(d*x+c)/(a+b*sinh(d*x+c)^2)^3,x, algorithm="maxima")
```

output
```
1/4*((20*a*e^(5*c) - 9*b*e^(5*c))*e^(5*d*x) - (20*a*e^(3*c) - 9*b*e^(3*c))
*e^(3*d*x) + 3*b*e^(7*d*x + 7*c) - 3*b*e^(d*x + c))/(a^2*b^2*d*e^(8*d*x +
8*c) + a^2*b^2*d + 4*(2*a^3*b*d*e^(6*c) - a^2*b^2*d*e^(6*c))*e^(6*d*x) + 2
*(8*a^4*d*e^(4*c) - 8*a^3*b*d*e^(4*c) + 3*a^2*b^2*d*e^(4*c))*e^(4*d*x) + 4
*(2*a^3*b*d*e^(2*c) - a^2*b^2*d*e^(2*c))*e^(2*d*x)) + 1/2*integrate(3/2*(e
^(3*d*x + 3*c) + e^(d*x + c))/(a^2*b*e^(4*d*x + 4*c) + a^2*b + 2*(2*a^3*e^
(2*c) - a^2*b*e^(2*c))*e^(2*d*x)), x)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{\cosh(c+dx)}{\left(a+b\sinh^2(c+dx)\right)^3}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(cosh(d*x+c)/(a+b*sinh(d*x+c)^2)^3,x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Limit: Max order reached or unable
to make series expansion Error: Bad Argument Value
```

Mupad [B] (verification not implemented)

Time = 1.83 (sec) , antiderivative size = 87, normalized size of antiderivative = 0.91

$$\int \frac{\cosh(c+dx)}{\left(a+b\sinh^2(c+dx)\right)^3}\,dx = \frac{\frac{5\sinh(c+d\,x)}{8\,a}+\frac{3\,b\sinh(c+d\,x)^3}{8\,a^2}}{d\,a^2+2\,d\,a\,b\sinh\left(c+d\,x\right)^2+d\,b^2\sinh\left(c+d\,x\right)^4} + \frac{3\,\mathrm{atan}\left(\frac{\sqrt{b}\sinh(c+d\,x)}{\sqrt{a}}\right)}{8\,a^{5/2}\sqrt{b}\,d}$$

input
```
int(cosh(c + d*x)/(a + b*sinh(c + d*x)^2)^3,x)
```

output
```
((5*sinh(c + d*x))/(8*a) + (3*b*sinh(c + d*x)^3)/(8*a^2))/(a^2*d + b^2*d*s
inh(c + d*x)^4 + 2*a*b*d*sinh(c + d*x)^2) + (3*atan((b^(1/2)*sinh(c + d*x)
)/a^(1/2)))/(8*a^(5/2)*b^(1/2)*d)
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 161, normalized size of antiderivative = 1.68

$$\int \frac{\cosh(c+dx)}{(a+b\sinh^2(c+dx))^3}\,dx$$
$$= \frac{3\sqrt{b}\,\sqrt{a}\,atan\left(\frac{\sinh(dx+c)b}{\sqrt{b}\,\sqrt{a}}\right)\sinh(dx+c)^4 b^2 + 6\sqrt{b}\,\sqrt{a}\,atan\left(\frac{\sinh(dx+c)b}{\sqrt{b}\,\sqrt{a}}\right)\sinh(dx+c)^2 ab + 3\sqrt{b}\,\sqrt{a}\,atan(}{8a^3bd\left(\sinh(dx+c)^4 b^2 + 2\sinh(dx+c)^2 ab + a\right.}$$

input
```
int(cosh(d*x+c)/(a+b*sinh(d*x+c)^2)^3,x)
```

output
```
(3*sqrt(b)*sqrt(a)*atan((sinh(c + d*x)*b)/(sqrt(b)*sqrt(a)))*sinh(c + d*x)
**4*b**2 + 6*sqrt(b)*sqrt(a)*atan((sinh(c + d*x)*b)/(sqrt(b)*sqrt(a)))*sin
h(c + d*x)**2*a*b + 3*sqrt(b)*sqrt(a)*atan((sinh(c + d*x)*b)/(sqrt(b)*sqrt
(a)))*a**2 + 3*sinh(c + d*x)**3*a*b**2 + 5*sinh(c + d*x)*a**2*b)/(8*a**3*b
*d*(sinh(c + d*x)**4*b**2 + 2*sinh(c + d*x)**2*a*b + a**2))
```

3.309 $\int \frac{\operatorname{sech}(c+dx)}{(a+b\sinh^2(c+dx))^3}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 159

$$\int \frac{\operatorname{sech}(c+dx)}{(a+b\sinh^2(c+dx))^3}\,dx = \frac{\arctan(\sinh(c+dx))}{(a-b)^3d} - \frac{\sqrt{b}(15a^2-10ab+3b^2)\arctan\left(\frac{\sqrt{b}\sinh(c+dx)}{\sqrt{a}}\right)}{8a^{5/2}(a-b)^3d} - \frac{b\sinh(c+dx)}{4a(a-b)d\left(a+b\sinh^2(c+dx)\right)^2} - \frac{(7a-3b)b\sinh(c+dx)}{8a^2(a-b)^2d\left(a+b\sinh^2(c+dx)\right)}$$

output
```
arctan(sinh(d*x+c))/(a-b)^3/d-1/8*b^(1/2)*(15*a^2-10*a*b+3*b^2)*arctan(b^(
1/2)*sinh(d*x+c)/a^(1/2))/a^(5/2)/(a-b)^3/d-1/4*b*sinh(d*x+c)/a/(a-b)/d/(a
+b*sinh(d*x+c)^2)^2-1/8*(7*a-3*b)*b*sinh(d*x+c)/a^2/(a-b)^2/d/(a+b*sinh(d*
x+c)^2)
```

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 321 vs. $2(159) = 318$.

Time = 0.55 (sec) , antiderivative size = 321, normalized size of antiderivative = 2.02

$$\int \frac{\operatorname{sech}(c+dx)}{(a+b\sinh^2(c+dx))^3}\,dx = \frac{(-2a+b)^2\left(\sqrt{b}(15a^2-10ab+3b^2)\arctan\left(\frac{\sqrt{a}\operatorname{csch}(c+dx)}{\sqrt{b}}\right)+16a^{5/2}\arctan\left(\tanh\left(\frac{1}{2}(c+dx)\right)\right)\right)+\left(b^{5/2}\right.}{}$$

input
```
Integrate[Sech[c + d*x]/(a + b*Sinh[c + d*x]^2)^3,x]
```

output
```
((-2*a + b)^2*(Sqrt[b]*(15*a^2 - 10*a*b + 3*b^2)*ArcTan[(Sqrt[a]*Csch[c +
d*x])/Sqrt[b]] + 16*a^(5/2)*ArcTan[Tanh[(c + d*x)/2]]) + (b^(5/2)*(15*a^2
- 10*a*b + 3*b^2)*ArcTan[(Sqrt[a]*Csch[c + d*x])/Sqrt[b]] + 16*a^(5/2)*b^2
*ArcTan[Tanh[(c + d*x)/2]])*Cosh[2*(c + d*x)]^2 - 2*Sqrt[a]*b*(18*a^3 - 35
*a^2*b + 20*a*b^2 - 3*b^3)*Sinh[c + d*x] - 2*b*Cosh[2*(c + d*x)]*(-((2*a -
 b)*(Sqrt[b]*(15*a^2 - 10*a*b + 3*b^2)*ArcTan[(Sqrt[a]*Csch[c + d*x])/Sqrt
[b]] + 16*a^(5/2)*ArcTan[Tanh[(c + d*x)/2]])) + Sqrt[a]*b*(7*a^2 - 10*a*b
+ 3*b^2)*Sinh[c + d*x]))/(8*a^(5/2)*(a - b)^3*d*(2*a - b + b*Cosh[2*(c + d
*x)])^2)
```

Rubi [A] (verified)

Time = 0.38 (sec) , antiderivative size = 183, normalized size of antiderivative = 1.15, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {3042, 3669, 316, 402, 397, 216, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\operatorname{sech}(c+dx)}{(a+b\sinh^2(c+dx))^3}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{\cos(ic+idx)\,(a-b\sin(ic+idx)^2)^3}dx$$

$$\Big\downarrow \text{ 3669}$$

$$\frac{\int \frac{1}{(\sinh^2(c+dx)+1)(b\sinh^2(c+dx)+a)^3}d\sinh(c+dx)}{d}$$

$$\Big\downarrow \text{ 316}$$

$$\frac{\frac{\int \frac{-3b\sinh^2(c+dx)+4a-3b}{\left(\sinh^2(c+dx)+1\right)\left(b\sinh^2(c+dx)+a\right)^2}d\sinh(c+dx)}{4a(a-b)}-\frac{b\sinh(c+dx)}{4a(a-b)\left(a+b\sinh^2(c+dx)\right)^2}}{d}$$

$$\Big\downarrow \text{ 402}$$

$$\frac{\frac{\frac{\int \frac{8a^2-7ba+3b^2-(7a-3b)b\sinh^2(c+dx)}{\left(\sinh^2(c+dx)+1\right)\left(b\sinh^2(c+dx)+a\right)}d\sinh(c+dx)}{2a(a-b)}-\frac{b(7a-3b)\sinh(c+dx)}{2a(a-b)\left(a+b\sinh^2(c+dx)\right)}}{4a(a-b)}-\frac{b\sinh(c+dx)}{4a(a-b)\left(a+b\sinh^2(c+dx)\right)^2}}{d}$$

$$\Big\downarrow \text{ 397}$$

$$\frac{\frac{\frac{\frac{8a^2\int \frac{1}{\sinh^2(c+dx)+1}d\sinh(c+dx)}{a-b}-\frac{b\left(15a^2-10ab+3b^2\right)\int \frac{1}{b\sinh^2(c+dx)+a}d\sinh(c+dx)}{a-b}}{2a(a-b)}-\frac{b(7a-3b)\sinh(c+dx)}{2a(a-b)\left(a+b\sinh^2(c+dx)\right)}}{4a(a-b)}-\frac{b\sinh(c+dx)}{4a(a-b)\left(a+b\sinh^2(c+dx)\right)^2}}{d}$$

$$\Big\downarrow \text{ 216}$$

$$\frac{\frac{\frac{\frac{8a^2\arctan(\sinh(c+dx))}{a-b}-\frac{b\left(15a^2-10ab+3b^2\right)\int \frac{1}{b\sinh^2(c+dx)+a}d\sinh(c+dx)}{a-b}}{2a(a-b)}-\frac{b(7a-3b)\sinh(c+dx)}{2a(a-b)\left(a+b\sinh^2(c+dx)\right)}}{4a(a-b)}-\frac{b\sinh(c+dx)}{4a(a-b)\left(a+b\sinh^2(c+dx)\right)^2}}{d}$$

$$\Big\downarrow \text{ 218}$$

$$\frac{\frac{\frac{\frac{8a^2\arctan(\sinh(c+dx))}{a-b}-\frac{\sqrt{b}\left(15a^2-10ab+3b^2\right)\arctan\left(\frac{\sqrt{b}\sinh(c+dx)}{\sqrt{a}}\right)}{\sqrt{a}(a-b)}}{2a(a-b)}-\frac{b(7a-3b)\sinh(c+dx)}{2a(a-b)\left(a+b\sinh^2(c+dx)\right)}}{4a(a-b)}-\frac{b\sinh(c+dx)}{4a(a-b)\left(a+b\sinh^2(c+dx)\right)^2}}{d}$$

input
```
Int[Sech[c + d*x]/(a + b*Sinh[c + d*x]^2)^3,x]
```

output
```
(-1/4*(b*Sinh[c + d*x])/(a*(a - b)*(a + b*Sinh[c + d*x]^2)^2) + (((8*a^2*A
rcTan[Sinh[c + d*x]])/(a - b) - (Sqrt[b]*(15*a^2 - 10*a*b + 3*b^2)*ArcTan[
(Sqrt[b]*Sinh[c + d*x])/Sqrt[a]])/(Sqrt[a]*(a - b)))/(2*a*(a - b)) - ((7*a
 - 3*b)*b*Sinh[c + d*x])/(2*a*(a - b)*(a + b*Sinh[c + d*x]^2)))/(4*a*(a -
b)))/d
```

Defintions of rubi rules used

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 316
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_), x_Symbol] :> Sim
p[(-b)*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^(q + 1)/(2*a*(p + 1)*(b*c - a*d))
), x] + Simp[1/(2*a*(p + 1)*(b*c - a*d))   Int[(a + b*x^2)^(p + 1)*(c + d*x
^2)^q*Simp[b*c + 2*(p + 1)*(b*c - a*d) + d*b*(2*(p + q + 2) + 1)*x^2, x], x
], x] /; FreeQ[{a, b, c, d, q}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] &&  !
( !IntegerQ[p] && IntegerQ[q] && LtQ[q, -1]) && IntBinomialQ[a, b, c, d, 2,
 p, q, x]
```

rule 397
```
Int[((e_) + (f_.)*(x_)^2)/(((a_) + (b_.)*(x_)^2)*((c_) + (d_.)*(x_)^2)), x_
Symbol] :> Simp[(b*e - a*f)/(b*c - a*d)   Int[1/(a + b*x^2), x], x] - Simp[
(d*e - c*f)/(b*c - a*d)   Int[1/(c + d*x^2), x], x] /; FreeQ[{a, b, c, d, e
, f}, x]
```

rule 402
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_.)*((e_) + (f_.)*(x
_)^2), x_Symbol] :> Simp[(-(b*e - a*f))*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^
(q + 1)/(a*2*(b*c - a*d)*(p + 1))), x] + Simp[1/(a*2*(b*c - a*d)*(p + 1))
 Int[(a + b*x^2)^(p + 1)*(c + d*x^2)^q*Simp[c*(b*e - a*f) + e*2*(b*c - a*d)
*(p + 1) + d*(b*e - a*f)*(2*(p + q + 2) + 1)*x^2, x], x], x] /; FreeQ[{a, b
, c, d, e, f, q}, x] && LtQ[p, -1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3669
```
Int[cos[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^
(p_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Simp[ff/f   S
ubst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a + b*ff^2*x^2)^p, x], x, Sin[e + f*x]
/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m - 1)/2]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 413 vs. $2(145) = 290$.

Time = 0.25 (sec) , antiderivative size = 414, normalized size of antiderivative = 2.60

$$-\frac{2b\left(\frac{-\frac{\left(9a^2-14ab+5b^2\right)\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^7}{8a}+\frac{\left(27a^3-70a^2b+55b^2a-12b^3\right)\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^5}{8a^2}-\frac{\left(27a^3-70a^2b+55b^2a-12b^3\right)\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^3}{8a^2}+\frac{\left(9a^2-14ab+5b^2\right)\tanh}{8a}}{\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^4a-2\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^2a+4b\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^2+a\right)^2}\right.}{(a-b}$$

input
```
int(sech(d*x+c)/(a+b*sinh(d*x+c)^2)^3,x)
```

output
```
1/d*(-2*b/(a-b)^3*((-1/8*(9*a^2-14*a*b+5*b^2)/a*tanh(1/2*d*x+1/2*c)^7+1/8*
(27*a^3-70*a^2*b+55*a*b^2-12*b^3)/a^2*tanh(1/2*d*x+1/2*c)^5-1/8*(27*a^3-70
*a^2*b+55*a*b^2-12*b^3)/a^2*tanh(1/2*d*x+1/2*c)^3+1/8*(9*a^2-14*a*b+5*b^2)
/a*tanh(1/2*d*x+1/2*c))/(tanh(1/2*d*x+1/2*c)^4*a-2*tanh(1/2*d*x+1/2*c)^2*a
+4*b*tanh(1/2*d*x+1/2*c)^2+a)^2+1/8/a*(15*a^2-10*a*b+3*b^2)*(-1/2*(a+(-b*(
a-b))^(1/2)-b)/a/(-b*(a-b))^(1/2)/((2*(-b*(a-b))^(1/2)+a-2*b)*a)^(1/2)*arc
tanh(tanh(1/2*d*x+1/2*c)*a/((2*(-b*(a-b))^(1/2)+a-2*b)*a)^(1/2))+1/2*(-a+(
-b*(a-b))^(1/2)+b)/a/(-b*(a-b))^(1/2)/((2*(-b*(a-b))^(1/2)-a+2*b)*a)^(1/2)
*arctan(tanh(1/2*d*x+1/2*c)*a/((2*(-b*(a-b))^(1/2)-a+2*b)*a)^(1/2))))+2/(a
-b)^3*arctan(tanh(1/2*d*x+1/2*c)))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 4384 vs. $2(145) = 290$.

Time = 0.24 (sec) , antiderivative size = 8083, normalized size of antiderivative = 50.84

$$\int \frac{\operatorname{sech}(c+dx)}{\left(a+b\sinh^2(c+dx)\right)^3}\,dx = \text{Too large to display}$$

input
```
integrate(sech(d*x+c)/(a+b*sinh(d*x+c)^2)^3,x, algorithm="fricas")
```

output
```
Too large to include
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\operatorname{sech}(c+dx)}{\left(a+b\sinh^2(c+dx)\right)^3}\,dx = \text{Timed out}$$

input
```
integrate(sech(d*x+c)/(a+b*sinh(d*x+c)**2)**3,x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{\operatorname{sech}(c+dx)}{\left(a+b\sinh^2(c+dx)\right)^3}\,dx = \int \frac{\operatorname{sech}\left(dx+c\right)}{\left(b\sinh\left(dx+c\right)^2+a\right)^3}\,dx$$

input
```
integrate(sech(d*x+c)/(a+b*sinh(d*x+c)^2)^3,x, algorithm="maxima")
```

output
```
-1/4*((7*a*b^2*e^(7*c) - 3*b^3*e^(7*c))*e^(7*d*x) + (36*a^2*b*e^(5*c) - 41
*a*b^2*e^(5*c) + 9*b^3*e^(5*c))*e^(5*d*x) - (36*a^2*b*e^(3*c) - 41*a*b^2*e
^(3*c) + 9*b^3*e^(3*c))*e^(3*d*x) - (7*a*b^2*e^c - 3*b^3*e^c)*e^(d*x))/(a^
4*b^2*d - 2*a^3*b^3*d + a^2*b^4*d + (a^4*b^2*d*e^(8*c) - 2*a^3*b^3*d*e^(8*
c) + a^2*b^4*d*e^(8*c))*e^(8*d*x) + 4*(2*a^5*b*d*e^(6*c) - 5*a^4*b^2*d*e^(
6*c) + 4*a^3*b^3*d*e^(6*c) - a^2*b^4*d*e^(6*c))*e^(6*d*x) + 2*(8*a^6*d*e^(
4*c) - 24*a^5*b*d*e^(4*c) + 27*a^4*b^2*d*e^(4*c) - 14*a^3*b^3*d*e^(4*c) +
3*a^2*b^4*d*e^(4*c))*e^(4*d*x) + 4*(2*a^5*b*d*e^(2*c) - 5*a^4*b^2*d*e^(2*c
) + 4*a^3*b^3*d*e^(2*c) - a^2*b^4*d*e^(2*c))*e^(2*d*x)) + 2*arctan(e^(d*x
+ c))/(a^3*d - 3*a^2*b*d + 3*a*b^2*d - b^3*d) - 2*integrate(1/8*((15*a^2*b
*e^(3*c) - 10*a*b^2*e^(3*c) + 3*b^3*e^(3*c))*e^(3*d*x) + (15*a^2*b*e^c - 1
0*a*b^2*e^c + 3*b^3*e^c)*e^(d*x))/(a^5*b - 3*a^4*b^2 + 3*a^3*b^3 - a^2*b^4
 + (a^5*b*e^(4*c) - 3*a^4*b^2*e^(4*c) + 3*a^3*b^3*e^(4*c) - a^2*b^4*e^(4*c
))*e^(4*d*x) + 2*(2*a^6*e^(2*c) - 7*a^5*b*e^(2*c) + 9*a^4*b^2*e^(2*c) - 5*
a^3*b^3*e^(2*c) + a^2*b^4*e^(2*c))*e^(2*d*x)), x)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{\operatorname{sech}(c+dx)}{\left(a+b\sinh^2(c+dx)\right)^3}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(sech(d*x+c)/(a+b*sinh(d*x+c)^2)^3,x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Limit: Max order reached or unable
to make series expansion Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\operatorname{sech}(c+dx)}{\left(a+b\sinh^2(c+dx)\right)^3}\,dx=\int \frac{1}{\cosh\left(c+d\,x\right)\,\left(b\sinh\left(c+d\,x\right)^2+a\right)^3}\,dx$$

input
```
int(1/(cosh(c + d*x)*(a + b*sinh(c + d*x)^2)^3),x)
```

output
```
int(1/(cosh(c + d*x)*(a + b*sinh(c + d*x)^2)^3), x)
```

Reduce [B] (verification not implemented)

Time = 0.39 (sec) , antiderivative size = 8621, normalized size of antiderivative = 54.22

$$\int \frac{\operatorname{sech}(c+dx)}{\left(a+b\sinh^2(c+dx)\right)^3}\,dx=\text{Too large to display}$$

input
```
int(sech(d*x+c)/(a+b*sinh(d*x+c)^2)^3,x)
```

output
```
(32*e**(8*c + 8*d*x)*atan(e**(c + d*x))*a**3*b**3 + 256*e**(6*c + 6*d*x)*a
tan(e**(c + d*x))*a**4*b**2 - 128*e**(6*c + 6*d*x)*atan(e**(c + d*x))*a**3
*b**3 + 512*e**(4*c + 4*d*x)*atan(e**(c + d*x))*a**5*b - 512*e**(4*c + 4*d
*x)*atan(e**(c + d*x))*a**4*b**2 + 192*e**(4*c + 4*d*x)*atan(e**(c + d*x))
*a**3*b**3 + 256*e**(2*c + 2*d*x)*atan(e**(c + d*x))*a**4*b**2 - 128*e**(2
*c + 2*d*x)*atan(e**(c + d*x))*a**3*b**3 + 32*atan(e**(c + d*x))*a**3*b**3
 + 30*e**(8*c + 8*d*x)*sqrt(b)*sqrt(a)*sqrt(a - b)*sqrt(2*sqrt(a)*sqrt(a -
 b) + 2*a - b)*atan((e**(c + d*x)*b)/(sqrt(b)*sqrt(2*sqrt(a)*sqrt(a - b) +
 2*a - b)))*a**2*b**2 - 20*e**(8*c + 8*d*x)*sqrt(b)*sqrt(a)*sqrt(a - b)*sq
rt(2*sqrt(a)*sqrt(a - b) + 2*a - b)*atan((e**(c + d*x)*b)/(sqrt(b)*sqrt(2*
sqrt(a)*sqrt(a - b) + 2*a - b)))*a*b**3 + 6*e**(8*c + 8*d*x)*sqrt(b)*sqrt(
a)*sqrt(a - b)*sqrt(2*sqrt(a)*sqrt(a - b) + 2*a - b)*atan((e**(c + d*x)*b)
/(sqrt(b)*sqrt(2*sqrt(a)*sqrt(a - b) + 2*a - b)))*b**4 + 240*e**(6*c + 6*d
*x)*sqrt(b)*sqrt(a)*sqrt(a - b)*sqrt(2*sqrt(a)*sqrt(a - b) + 2*a - b)*atan
((e**(c + d*x)*b)/(sqrt(b)*sqrt(2*sqrt(a)*sqrt(a - b) + 2*a - b)))*a**3*b
- 280*e**(6*c + 6*d*x)*sqrt(b)*sqrt(a)*sqrt(a - b)*sqrt(2*sqrt(a)*sqrt(a -
 b) + 2*a - b)*atan((e**(c + d*x)*b)/(sqrt(b)*sqrt(2*sqrt(a)*sqrt(a - b) +
 2*a - b)))*a**2*b**2 + 128*e**(6*c + 6*d*x)*sqrt(b)*sqrt(a)*sqrt(a - b)*s
qrt(2*sqrt(a)*sqrt(a - b) + 2*a - b)*atan((e**(c + d*x)*b)/(sqrt(b)*sqrt(2
*sqrt(a)*sqrt(a - b) + 2*a - b)))*a*b**3 - 24*e**(6*c + 6*d*x)*sqrt(b)*...
```

3.310 $\int \frac{\operatorname{sech}^2(c+dx)}{(a+b\sinh^2(c+dx))^3}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 172

$$\begin{aligned}\int \frac{\operatorname{sech}^2(c+dx)}{(a+b\sinh^2(c+dx))^3}\,dx =& -\frac{3b(8a^2-4ab+b^2)\operatorname{arctanh}\left(\frac{\sqrt{a-b}\tanh(c+dx)}{\sqrt{a}}\right)}{8a^{5/2}(a-b)^{7/2}d} \\ &+ \frac{\tanh(c+dx)}{(a-b)^3d} \\ &- \frac{b^3\tanh(c+dx)}{4a(a-b)^3d\left(a-(a-b)\tanh^2(c+dx)\right)^2} \\ &+ \frac{3(4a-b)b^2\tanh(c+dx)}{8a^2(a-b)^3d\left(a-(a-b)\tanh^2(c+dx)\right)}\end{aligned}$$

output

```
-3/8*b*(8*a^2-4*a*b+b^2)*arctanh((a-b)^(1/2)*tanh(d*x+c)/a^(1/2))/a^(5/2)/
(a-b)^(7/2)/d+tanh(d*x+c)/(a-b)^3/d-1/4*b^3*tanh(d*x+c)/a/(a-b)^3/d/(a-(a-
b)*tanh(d*x+c)^2)^2+3/8*(4*a-b)*b^2*tanh(d*x+c)/a^2/(a-b)^3/d/(a-(a-b)*tan
h(d*x+c)^2)
```

Mathematica [A] (verified)

Time = 1.20 (sec) , antiderivative size = 165, normalized size of antiderivative = 0.96

$$\int \frac{\operatorname{sech}^2(c+dx)}{\left(a+b\sinh^2(c+dx)\right)^3}\,dx$$
$$= \frac{-\frac{3b(8a^2-4ab+b^2)\operatorname{arctanh}\left(\frac{\sqrt{a-b}\tanh(c+dx)}{\sqrt{a}}\right)}{a^{5/2}(a-b)^{7/2}} + \frac{4b^2\sinh(2(c+dx))}{a(a-b)^2(2a-b+b\cosh(2(c+dx)))^2} + \frac{(10a-3b)b^2\sinh(2(c+dx))}{a^2(a-b)^3(2a-b+b\cosh(2(c+dx)))} + \frac{8\tanh(c+}{(a-b)}}{8d}$$

input
```
Integrate[Sech[c + d*x]^2/(a + b*Sinh[c + d*x]^2)^3,x]
```

output
```
((-3*b*(8*a^2 - 4*a*b + b^2)*ArcTanh[(Sqrt[a - b]*Tanh[c + d*x])/Sqrt[a]])
/(a^(5/2)*(a - b)^(7/2)) + (4*b^2*Sinh[2*(c + d*x)])/(a*(a - b)^2*(2*a - b
 + b*Cosh[2*(c + d*x)])^2) + ((10*a - 3*b)*b^2*Sinh[2*(c + d*x)])/(a^2*(a
- b)^3*(2*a - b + b*Cosh[2*(c + d*x)])) + (8*Tanh[c + d*x])/(a - b)^3)/(8*
d)
```

Rubi [A] (verified)

Time = 0.48 (sec) , antiderivative size = 164, normalized size of antiderivative = 0.95, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.174$, Rules used = {3042, 3670, 300, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\operatorname{sech}^2(c+dx)}{\left(a+b\sinh^2(c+dx)\right)^3}\,dx$$
$$\Big\downarrow \text{3042}$$
$$\int \frac{1}{\cos(ic+idx)^2\left(a-b\sin(ic+idx)^2\right)^3}dx$$
$$\Big\downarrow \text{3670}$$
$$\frac{\int \frac{\left(1-\tanh^2(c+dx)\right)^3}{\left(a-(a-b)\tanh^2(c+dx)\right)^3}d\tanh(c+dx)}{d}$$

$$\Big\downarrow \text{300}$$

$$\frac{\int \left(\frac{1}{(a-b)^3}-\frac{3(a-b)^2b\tanh^4(c+dx)-3(a-b)(2a-b)b\tanh^2(c+dx)+b(3a^2-3ba+b^2)}{(a-b)^3\left((b-a)\tanh^2(c+dx)+a\right)^3}\right)d\tanh(c+dx)}{d}$$

$$\Big\downarrow \text{2009}$$

$$\frac{\frac{3b^2(4a-b)\tanh(c+dx)}{8a^2(a-b)^3(a-(a-b)\tanh^2(c+dx))}-\frac{3b(8a^2-4ab+b^2)\text{arctanh}\left(\frac{\sqrt{a-b}\tanh(c+dx)}{\sqrt{a}}\right)}{8a^{5/2}(a-b)^{7/2}}-\frac{b^3\tanh(c+dx)}{4a(a-b)^3(a-(a-b)\tanh^2(c+dx))^2}+\frac{\tanh(c+dx)}{(a-b)^3}}{d}$$

input
```
Int[Sech[c + d*x]^2/(a + b*Sinh[c + d*x]^2)^3,x]
```

output
```
((-3*b*(8*a^2 - 4*a*b + b^2)*ArcTanh[(Sqrt[a - b]*Tanh[c + d*x])/Sqrt[a]])
/(8*a^(5/2)*(a - b)^(7/2)) + Tanh[c + d*x]/(a - b)^3 - (b^3*Tanh[c + d*x])
/(4*a*(a - b)^3*(a - (a - b)*Tanh[c + d*x]^2)^2) + (3*(4*a - b)*b^2*Tanh[c
 + d*x])/(8*a^2*(a - b)^3*(a - (a - b)*Tanh[c + d*x]^2)))/d
```

Defintions of rubi rules used

rule 300
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_), x_Symbol] :> Int
[PolynomialDivide[(a + b*x^2)^p, (c + d*x^2)^(-q), x], x] /; FreeQ[{a, b, c
, d}, x] && NeQ[b*c - a*d, 0] && IGtQ[p, 0] && ILtQ[q, 0] && GeQ[p, -q]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3670
```
Int[cos[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(
p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Simp[ff/f    Su
bst[Int[(a + (a + b)*ff^2*x^2)^p/(1 + ff^2*x^2)^(m/2 + p + 1), x], x, Tan[e
 + f*x]/ff], x]] /; FreeQ[{a, b, e, f}, x] && IntegerQ[m/2] && IntegerQ[p]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 393 vs. $2(158) = 316$.

Time = 0.25 (sec) , antiderivative size = 394, normalized size of antiderivative = 2.29

$$\frac{\frac{2b\left(\frac{\frac{b(12a-5b)\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^{7}}{8a}-\frac{3\left(4a^{2}-15ab+4b^{2}\right)b\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^{5}}{8a^{2}}-\frac{3\left(4a^{2}-15ab+4b^{2}\right)b\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^{3}}{8a^{2}}+\frac{b(12a-5b)\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)}{8a}}{\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^{4}a-2\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}a+4b\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}+a\right)^{2}}+\frac{3\left(8a^{2}-4ab+b^{2}\right)\left(-\frac{(\sqrt{\cdots}}{\cdots}\right.}{\cdots}\right.}{(a-b)^{3}}}{d}$$

input
```
int(sech(d*x+c)^2/(a+b*sinh(d*x+c)^2)^3,x)
```

output
```
1/d*(2*b/(a-b)^3*((1/8*b*(12*a-5*b)/a*tanh(1/2*d*x+1/2*c)^7-3/8*(4*a^2-15*
a*b+4*b^2)/a^2*b*tanh(1/2*d*x+1/2*c)^5-3/8*(4*a^2-15*a*b+4*b^2)/a^2*b*tanh
(1/2*d*x+1/2*c)^3+1/8*b*(12*a-5*b)/a*tanh(1/2*d*x+1/2*c))/(tanh(1/2*d*x+1/
2*c)^4*a-2*tanh(1/2*d*x+1/2*c)^2*a+4*b*tanh(1/2*d*x+1/2*c)^2+a)^2+3/8/a*(8
*a^2-4*a*b+b^2)*(-1/2*((-b*(a-b))^(1/2)-b)/a/(-b*(a-b))^(1/2)/((2*(-b*(a-b
))^(1/2)+a-2*b)*a)^(1/2)*arctanh(tanh(1/2*d*x+1/2*c)*a/((2*(-b*(a-b))^(1/2
)+a-2*b)*a)^(1/2))+1/2*((-b*(a-b))^(1/2)+b)/a/(-b*(a-b))^(1/2)/((2*(-b*(a-
b))^(1/2)-a+2*b)*a)^(1/2)*arctan(tanh(1/2*d*x+1/2*c)*a/((2*(-b*(a-b))^(1/2
)-a+2*b)*a)^(1/2))))+2/(a-b)^3*tanh(1/2*d*x+1/2*c)/(tanh(1/2*d*x+1/2*c)^2+
1))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 4593 vs. $2(160) = 320$.

Time = 0.21 (sec) , antiderivative size = 9442, normalized size of antiderivative = 54.90

$$\int \frac{\operatorname{sech}^2(c+dx)}{\left(a+b\sinh^2(c+dx)\right)^3}\,dx = \text{Too large to display}$$

input
```
integrate(sech(d*x+c)^2/(a+b*sinh(d*x+c)^2)^3,x, algorithm="fricas")
```

output
```
Too large to include
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\operatorname{sech}^2(c+dx)}{\left(a+b\sinh^2(c+dx)\right)^3}\,dx = \text{Timed out}$$

input
```
integrate(sech(d*x+c)**2/(a+b*sinh(d*x+c)**2)**3,x)
```

output
```
Timed out
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{\operatorname{sech}^2(c+dx)}{\left(a+b\sinh^2(c+dx)\right)^3}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate(sech(d*x+c)^2/(a+b*sinh(d*x+c)^2)^3,x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(b-a>0)', see `assume?` for more
details)Is
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 367 vs. $2(160) = 320$.

Time = 0.30 (sec) , antiderivative size = 367, normalized size of antiderivative = 2.13

$$\int \frac{\operatorname{sech}^2(c+dx)}{\left(a+b\sinh^2(c+dx)\right)^3}\,dx =$$

$$-\frac{\frac{3\,(8\,a^2b-4\,ab^2+b^3)\arctan\left(\frac{be^{(2\,dx+2\,c)}+2\,a-b}{2\,\sqrt{-a^2+ab}}\right)}{(a^5-3\,a^4b+3\,a^3b^2-a^2b^3)\sqrt{-a^2+ab}}+\frac{2\,(16\,a^2b^2e^{(6\,dx+6\,c)}-12\,ab^3e^{(6\,dx+6\,c)}+3\,b^4e^{(6\,dx+6\,c)}+80\,a^3be^{(4\,dx+4\,c)}-104\,a^2b^2e^{(4}}{(a^5-3\,a^4b+3\,a^3b^2-a^2b^3}}{}$$

input
```
integrate(sech(d*x+c)^2/(a+b*sinh(d*x+c)^2)^3,x, algorithm="giac")
```

output
```
-1/8*(3*(8*a^2*b - 4*a*b^2 + b^3)*arctan(1/2*(b*e^(2*d*x + 2*c) + 2*a - b)
/sqrt(-a^2 + a*b))/((a^5 - 3*a^4*b + 3*a^3*b^2 - a^2*b^3)*sqrt(-a^2 + a*b)
) + 2*(16*a^2*b^2*e^(6*d*x + 6*c) - 12*a*b^3*e^(6*d*x + 6*c) + 3*b^4*e^(6*
d*x + 6*c) + 80*a^3*b*e^(4*d*x + 4*c) - 104*a^2*b^2*e^(4*d*x + 4*c) + 54*a
*b^3*e^(4*d*x + 4*c) - 9*b^4*e^(4*d*x + 4*c) + 64*a^2*b^2*e^(2*d*x + 2*c)
- 52*a*b^3*e^(2*d*x + 2*c) + 9*b^4*e^(2*d*x + 2*c) + 10*a*b^3 - 3*b^4)/((a
^5 - 3*a^4*b + 3*a^3*b^2 - a^2*b^3)*(b*e^(4*d*x + 4*c) + 4*a*e^(2*d*x + 2*
c) - 2*b*e^(2*d*x + 2*c) + b)^2) + 16/((a^3 - 3*a^2*b + 3*a*b^2 - b^3)*(e^
(2*d*x + 2*c) + 1)))/d
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\operatorname{sech}^2(c+dx)}{\left(a+b\sinh^2(c+dx)\right)^3}\,dx = \int \frac{1}{\cosh\left(c+d\,x\right)^2\,\left(b\sinh\left(c+d\,x\right)^2+a\right)^3}\,dx$$

input
```
int(1/(cosh(c + d*x)^2*(a + b*sinh(c + d*x)^2)^3),x)
```

output
```
int(1/(cosh(c + d*x)^2*(a + b*sinh(c + d*x)^2)^3), x)
```

Reduce [B] (verification not implemented)

Time = 0.32 (sec) , antiderivative size = 5857, normalized size of antiderivative = 34.05

$$\int \frac{\operatorname{sech}^2(c+dx)}{\left(a+b\sinh^2(c+dx)\right)^3}\,dx = \text{Too large to display}$$

input
```
int(sech(d*x+c)^2/(a+b*sinh(d*x+c)^2)^3,x)
```

output
```
( - 192*e**(10*c + 10*d*x)*sqrt(a)*sqrt(a - b)*log( - sqrt(2*sqrt(a)*sqrt(
a - b) - 2*a + b) + e**(c + d*x)*sqrt(b))*a**3*b**3 + 168*e**(10*c + 10*d*
x)*sqrt(a)*sqrt(a - b)*log( - sqrt(2*sqrt(a)*sqrt(a - b) - 2*a + b) + e**(
c + d*x)*sqrt(b))*a**2*b**4 - 60*e**(10*c + 10*d*x)*sqrt(a)*sqrt(a - b)*lo
g( - sqrt(2*sqrt(a)*sqrt(a - b) - 2*a + b) + e**(c + d*x)*sqrt(b))*a*b**5
+ 9*e**(10*c + 10*d*x)*sqrt(a)*sqrt(a - b)*log( - sqrt(2*sqrt(a)*sqrt(a -
b) - 2*a + b) + e**(c + d*x)*sqrt(b))*b**6 - 192*e**(10*c + 10*d*x)*sqrt(a
)*sqrt(a - b)*log(sqrt(2*sqrt(a)*sqrt(a - b) - 2*a + b) + e**(c + d*x)*sqr
t(b))*a**3*b**3 + 168*e**(10*c + 10*d*x)*sqrt(a)*sqrt(a - b)*log(sqrt(2*sq
rt(a)*sqrt(a - b) - 2*a + b) + e**(c + d*x)*sqrt(b))*a**2*b**4 - 60*e**(10
*c + 10*d*x)*sqrt(a)*sqrt(a - b)*log(sqrt(2*sqrt(a)*sqrt(a - b) - 2*a + b)
 + e**(c + d*x)*sqrt(b))*a*b**5 + 9*e**(10*c + 10*d*x)*sqrt(a)*sqrt(a - b)
*log(sqrt(2*sqrt(a)*sqrt(a - b) - 2*a + b) + e**(c + d*x)*sqrt(b))*b**6 +
192*e**(10*c + 10*d*x)*sqrt(a)*sqrt(a - b)*log(2*sqrt(a)*sqrt(a - b) + e**
(2*c + 2*d*x)*b + 2*a - b)*a**3*b**3 - 168*e**(10*c + 10*d*x)*sqrt(a)*sqrt
(a - b)*log(2*sqrt(a)*sqrt(a - b) + e**(2*c + 2*d*x)*b + 2*a - b)*a**2*b**
4 + 60*e**(10*c + 10*d*x)*sqrt(a)*sqrt(a - b)*log(2*sqrt(a)*sqrt(a - b) +
e**(2*c + 2*d*x)*b + 2*a - b)*a*b**5 - 9*e**(10*c + 10*d*x)*sqrt(a)*sqrt(a
 - b)*log(2*sqrt(a)*sqrt(a - b) + e**(2*c + 2*d*x)*b + 2*a - b)*b**6 - 153
6*e**(8*c + 8*d*x)*sqrt(a)*sqrt(a - b)*log( - sqrt(2*sqrt(a)*sqrt(a - b...
```

3.311 $\int \frac{\operatorname{sech}^3(c+dx)}{(a+b\sinh^2(c+dx))^3}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 217

$$\begin{aligned}\int \frac{\operatorname{sech}^3(c+dx)}{(a+b\sinh^2(c+dx))^3}\,dx &= \frac{(a-7b)\arctan(\sinh(c+dx))}{2(a-b)^4 d} \\ &+ \frac{b^{3/2}(35a^2-14ab+3b^2)\arctan\left(\frac{\sqrt{b}\sinh(c+dx)}{\sqrt{a}}\right)}{8a^{5/2}(a-b)^4 d} \\ &+ \frac{b(2a+b)\sinh(c+dx)}{4a(a-b)^2 d\,(a+b\sinh^2(c+dx))^2} \\ &+ \frac{(4a-b)b(a+3b)\sinh(c+dx)}{8a^2(a-b)^3 d\,(a+b\sinh^2(c+dx))} \\ &+ \frac{\operatorname{sech}(c+dx)\tanh(c+dx)}{2(a-b)d\,(a+b\sinh^2(c+dx))^2}\end{aligned}$$

output
```
1/2*(a-7*b)*arctan(sinh(d*x+c))/(a-b)^4/d+1/8*b^(3/2)*(35*a^2-14*a*b+3*b^2
)*arctan(b^(1/2)*sinh(d*x+c)/a^(1/2))/a^(5/2)/(a-b)^4/d+1/4*b*(2*a+b)*sinh
(d*x+c)/a/(a-b)^2/d/(a+b*sinh(d*x+c)^2)^2+1/8*(4*a-b)*b*(a+3*b)*sinh(d*x+c
)/a^2/(a-b)^3/d/(a+b*sinh(d*x+c)^2)+1/2*sech(d*x+c)*tanh(d*x+c)/(a-b)/d/(a
+b*sinh(d*x+c)^2)^2
```

Mathematica [A] (verified)

Time = 1.04 (sec) , antiderivative size = 232, normalized size of antiderivative = 1.07

$$\int \frac{\operatorname{sech}^3(c+dx)}{\left(a+b\sinh^2(c+dx)\right)^3}\,dx$$
$$= \frac{-35a^2b^{3/2}\arctan\left(\frac{\sqrt{a}\operatorname{csch}(c+dx)}{\sqrt{b}}\right)+14ab^{5/2}\arctan\left(\frac{\sqrt{a}\operatorname{csch}(c+dx)}{\sqrt{b}}\right)-3b^{7/2}\arctan\left(\frac{\sqrt{a}\operatorname{csch}(c+dx)}{\sqrt{b}}\right)+8a^{7/2}}{}$$

input
```
Integrate[Sech[c + d*x]^3/(a + b*Sinh[c + d*x]^2)^3,x]
```

output
```
(-35*a^2*b^(3/2)*ArcTan[(Sqrt[a]*Csch[c + d*x])/Sqrt[b]] + 14*a*b^(5/2)*Ar
cTan[(Sqrt[a]*Csch[c + d*x])/Sqrt[b]] - 3*b^(7/2)*ArcTan[(Sqrt[a]*Csch[c +
 d*x])/Sqrt[b]] + 8*a^(7/2)*ArcTan[Tanh[(c + d*x)/2]] - 56*a^(5/2)*b*ArcTa
n[Tanh[(c + d*x)/2]] + (2*Sqrt[a]*(a - b)*b^2*(26*a^2 - 21*a*b + 3*b^2 + (
11*a - 3*b)*b*Cosh[2*(c + d*x)])*Sinh[c + d*x])/(2*a - b + b*Cosh[2*(c + d
*x)])^2 + 4*a^(5/2)*(a - b)*Sech[c + d*x]*Tanh[c + d*x])/(8*a^(5/2)*(a - b
)^4*d)
```

Rubi [A] (verified)

Time = 0.48 (sec) , antiderivative size = 252, normalized size of antiderivative = 1.16, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.435$, Rules used = {3042, 3669, 316, 25, 402, 27, 402, 397, 216, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\operatorname{sech}^3(c+dx)}{\left(a+b\sinh^2(c+dx)\right)^3}\,dx$$

$$\Big\downarrow \; 3042$$

$$\int \frac{1}{\cos(ic+idx)^3\left(a-b\sin(ic+idx)^2\right)^3}dx$$

$$\Big\downarrow \; 3669$$

$$\frac{\int \frac{1}{\left(\sinh^2(c+dx)+1\right)^2\left(b\sinh^2(c+dx)+a\right)^3}d\sinh(c+dx)}{d}$$

$$\Big\downarrow \; 316$$

$$\frac{\frac{\sinh(c+dx)}{2(a-b)\left(\sinh^2(c+dx)+1\right)\left(a+b\sinh^2(c+dx)\right)^2}-\frac{\int -\frac{5b\sinh^2(c+dx)+a-2b}{\left(\sinh^2(c+dx)+1\right)\left(b\sinh^2(c+dx)+a\right)^3}d\sinh(c+dx)}{2(a-b)}}{d}$$

$$\Big\downarrow \; 25$$

$$\frac{\frac{\int \frac{5b\sinh^2(c+dx)+a-2b}{\left(\sinh^2(c+dx)+1\right)\left(b\sinh^2(c+dx)+a\right)^3}d\sinh(c+dx)}{2(a-b)}+\frac{\sinh(c+dx)}{2(a-b)\left(\sinh^2(c+dx)+1\right)\left(a+b\sinh^2(c+dx)\right)^2}}{d}$$

$$\Big\downarrow \; 402$$

$$\frac{\frac{\frac{\int \frac{2\left(2a^2-8ba+3b^2+3b(2a+b)\sinh^2(c+dx)\right)}{\left(\sinh^2(c+dx)+1\right)\left(b\sinh^2(c+dx)+a\right)^2}d\sinh(c+dx)}{4a(a-b)}+\frac{b(2a+b)\sinh(c+dx)}{2a(a-b)\left(a+b\sinh^2(c+dx)\right)^2}}{2(a-b)}+\frac{\sinh(c+dx)}{2(a-b)\left(\sinh^2(c+dx)+1\right)\left(a+b\sinh^2(c+dx)\right)^2}}{d}$$

$$\Big\downarrow \; 27$$

$$\frac{\frac{\frac{\int \frac{2a^2-8ba+3b^2+3b(2a+b)\sinh^2(c+dx)}{\left(\sinh^2(c+dx)+1\right)\left(b\sinh^2(c+dx)+a\right)^2}d\sinh(c+dx)}{2a(a-b)}+\frac{b(2a+b)\sinh(c+dx)}{2a(a-b)\left(a+b\sinh^2(c+dx)\right)^2}}{2(a-b)}+\frac{\sinh(c+dx)}{2(a-b)\left(\sinh^2(c+dx)+1\right)\left(a+b\sinh^2(c+dx)\right)^2}}{d}$$

$$\Big\downarrow \; 402$$

$$\frac{\frac{\frac{\frac{\int \frac{4a^3-24ba^2+11b^2a-3b^3+(4a-b)b(a+3b)\sinh^2(c+dx)}{\left(\sinh^2(c+dx)+1\right)\left(b\sinh^2(c+dx)+a\right)}d\sinh(c+dx)}{2a(a-b)}+\frac{b(4a-b)(a+3b)\sinh(c+dx)}{2a(a-b)\left(a+b\sinh^2(c+dx)\right)}}{2a(a-b)}+\frac{b(2a+b)\sinh(c+dx)}{2a(a-b)\left(a+b\sinh^2(c+dx)\right)^2}}{2(a-b)}+\frac{\sinh}{2(a-b)\left(\sinh^2(c+dx)\cdots\right.}}{d}$$

$$\Big\downarrow \; 397$$

$$\frac{\frac{\frac{\frac{\frac{b^2\left(35a^2-14ab+3b^2\right)\int \frac{1}{b\sinh^2(c+dx)+a}d\sinh(c+dx)}{a-b}+\frac{4a^2(a-7b)\int \frac{1}{\sinh^2(c+dx)+1}d\sinh(c+dx)}{a-b}}{2a(a-b)}+\frac{b(4a-b)(a+3b)\sinh(c+dx)}{2a(a-b)\left(a+b\sinh^2(c+dx)\right)}}{2a(a-b)}+\frac{b(2a+b)\sinh(c+dx)}{2a(a-b)\left(a+b\sinh^2(c+dx)\right)^2}\cdots}{2(a-b)}}{d}$$

$$\Big\downarrow \text{216}$$

$$\frac{\frac{\frac{\frac{\frac{b^2(35a^2-14ab+3b^2)\int \frac{1}{b\sinh^2(c+dx)+a}d\sinh(c+dx)}{a-b}+\frac{4a^2(a-7b)\arctan(\sinh(c+dx))}{a-b}}{2a(a-b)}+\frac{b(4a-b)(a+3b)\sinh(c+dx)}{2a(a-b)\left(a+b\sinh^2(c+dx)\right)}}{2a(a-b)}+\frac{b(2a+b)\sinh(c+dx)}{2a(a-b)\left(a+b\sinh^2(c+dx)\right)^2}}{2(a-b)}+\frac{\ldots}{2(a-\ldots}}{d}$$

$$\Big\downarrow \text{218}$$

$$\frac{\frac{\frac{\frac{\frac{b^{3/2}(35a^2-14ab+3b^2)\arctan\left(\frac{\sqrt{b}\sinh(c+dx)}{\sqrt{a}}\right)}{\sqrt{a}(a-b)}+\frac{4a^2(a-7b)\arctan(\sinh(c+dx))}{a-b}}{2a(a-b)}+\frac{b(4a-b)(a+3b)\sinh(c+dx)}{2a(a-b)\left(a+b\sinh^2(c+dx)\right)}}{2a(a-b)}+\frac{b(2a+b)\sinh(c+dx)}{2a(a-b)\left(a+b\sinh^2(c+dx)\right)^2}}{2(a-b)}+\frac{\ldots}{2(a-b)(\mathrm{si}\ldots}}{d}$$

input

```
Int[Sech[c + d*x]^3/(a + b*Sinh[c + d*x]^2)^3,x]
```

output

```
(Sinh[c + d*x]/(2*(a - b)*(1 + Sinh[c + d*x]^2)*(a + b*Sinh[c + d*x]^2)^2)
 + ((b*(2*a + b)*Sinh[c + d*x])/(2*a*(a - b)*(a + b*Sinh[c + d*x]^2)^2) +
(((4*a^2*(a - 7*b)*ArcTan[Sinh[c + d*x]])/(a - b) + (b^(3/2)*(35*a^2 - 14*
a*b + 3*b^2)*ArcTan[(Sqrt[b]*Sinh[c + d*x])/Sqrt[a]])/(Sqrt[a]*(a - b)))/(
2*a*(a - b)) + ((4*a - b)*b*(a + 3*b)*Sinh[c + d*x])/(2*a*(a - b)*(a + b*S
inh[c + d*x]^2)))/(2*a*(a - b)))/(2*(a - b)))/d
```

Defintions of rubi rules used

rule 25

```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27

```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 216

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 316
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_), x_Symbol] :> Sim
p[(-b)*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^(q + 1)/(2*a*(p + 1)*(b*c - a*d))
), x] + Simp[1/(2*a*(p + 1)*(b*c - a*d))   Int[(a + b*x^2)^(p + 1)*(c + d*x
^2)^q*Simp[b*c + 2*(p + 1)*(b*c - a*d) + d*b*(2*(p + q + 2) + 1)*x^2, x], x
], x] /; FreeQ[{a, b, c, d, q}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] &&  !
( !IntegerQ[p] && IntegerQ[q] && LtQ[q, -1]) && IntBinomialQ[a, b, c, d, 2,
 p, q, x]
```

rule 397
```
Int[((e_) + (f_.)*(x_)^2)/(((a_) + (b_.)*(x_)^2)*((c_) + (d_.)*(x_)^2)), x_
Symbol] :> Simp[(b*e - a*f)/(b*c - a*d)   Int[1/(a + b*x^2), x], x] - Simp[
(d*e - c*f)/(b*c - a*d)   Int[1/(c + d*x^2), x], x] /; FreeQ[{a, b, c, d, e
, f}, x]
```

rule 402
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_.)*((e_) + (f_.)*(x
_)^2), x_Symbol] :> Simp[(-(b*e - a*f))*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^
(q + 1)/(a*2*(b*c - a*d)*(p + 1))), x] + Simp[1/(a*2*(b*c - a*d)*(p + 1))
 Int[(a + b*x^2)^(p + 1)*(c + d*x^2)^q*Simp[c*(b*e - a*f) + e*2*(b*c - a*d)
*(p + 1) + d*(b*e - a*f)*(2*(p + q + 2) + 1)*x^2, x], x], x] /; FreeQ[{a, b
, c, d, e, f, q}, x] && LtQ[p, -1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3669
```
Int[cos[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^
(p_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Simp[ff/f   S
ubst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a + b*ff^2*x^2)^p, x], x, Sin[e + f*x]
/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m - 1)/2]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 476 vs. $2(199) = 398$.

Time = 0.25 (sec) , antiderivative size = 477, normalized size of antiderivative = 2.20

$$\frac{\frac{2\left(\left(-\frac{a}{2}+\frac{b}{2}\right)\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^{3}+\left(\frac{a}{2}-\frac{b}{2}\right)\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}+1\right)^{2}}+(a-7b)\arctan\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{(a-b)^{4}}+\frac{2b^{2}\left(\frac{-\frac{\left(13a^{2}-18ab+5b^{2}\right)\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^{7}}{8a}+\frac{\left(39a^{3}-98a^{2}b+71b^{2}\cdots\right.}{\cdots}}{\left(\tanh\left(\cdots\right.\right.}\right.}{\cdots}$$

input
```
int(sech(d*x+c)^3/(a+b*sinh(d*x+c)^2)^3,x)
```

output
```
1/d*(2/(a-b)^4*(((-1/2*a+1/2*b)*tanh(1/2*d*x+1/2*c)^3+(1/2*a-1/2*b)*tanh(1
/2*d*x+1/2*c))/(tanh(1/2*d*x+1/2*c)^2+1)^2+1/2*(a-7*b)*arctan(tanh(1/2*d*x
+1/2*c)))+2*b^2/(a-b)^4*((-1/8*(13*a^2-18*a*b+5*b^2)/a*tanh(1/2*d*x+1/2*c)
^7+1/8*(39*a^3-98*a^2*b+71*a*b^2-12*b^3)/a^2*tanh(1/2*d*x+1/2*c)^5-1/8*(39
*a^3-98*a^2*b+71*a*b^2-12*b^3)/a^2*tanh(1/2*d*x+1/2*c)^3+1/8*(13*a^2-18*a*
b+5*b^2)/a*tanh(1/2*d*x+1/2*c))/(tanh(1/2*d*x+1/2*c)^4*a-2*tanh(1/2*d*x+1/
2*c)^2*a+4*b*tanh(1/2*d*x+1/2*c)^2+a)^2+1/8/a*(35*a^2-14*a*b+3*b^2)*(-1/2*
(a+(-b*(a-b))^(1/2)-b)/a/(-b*(a-b))^(1/2)/((2*(-b*(a-b))^(1/2)+a-2*b)*a)^(
1/2)*arctanh(tanh(1/2*d*x+1/2*c)*a/((2*(-b*(a-b))^(1/2)+a-2*b)*a)^(1/2))+1
/2*(-a+(-b*(a-b))^(1/2)+b)/a/(-b*(a-b))^(1/2)/((2*(-b*(a-b))^(1/2)-a+2*b)*
a)^(1/2)*arctan(tanh(1/2*d*x+1/2*c)*a/((2*(-b*(a-b))^(1/2)-a+2*b)*a)^(1/2)
))))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 10237 vs. 2(199) = 398.

Time = 0.46 (sec) , antiderivative size = 18765, normalized size of antiderivative = 86.47

$$\int \frac{\operatorname{sech}^3(c+dx)}{\left(a+b\sinh^2(c+dx)\right)^3}\,dx = \text{Too large to display}$$

input
```
integrate(sech(d*x+c)^3/(a+b*sinh(d*x+c)^2)^3,x, algorithm="fricas")
```

output
```
Too large to include
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\operatorname{sech}^3(c+dx)}{\left(a+b\sinh^2(c+dx)\right)^3}\,dx = \text{Timed out}$$

input
```
integrate(sech(d*x+c)**3/(a+b*sinh(d*x+c)**2)**3,x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{\operatorname{sech}^3(c+dx)}{\left(a+b\sinh^2(c+dx)\right)^3}\,dx = \int \frac{\operatorname{sech}\left(dx+c\right)^3}{\left(b\sinh\left(dx+c\right)^2+a\right)^3}\,dx$$

input
```
integrate(sech(d*x+c)^3/(a+b*sinh(d*x+c)^2)^3,x, algorithm="maxima")
```

output
```
(a*e^c - 7*b*e^c)*arctan(e^(d*x + c))*e^(-c)/(a^4*d - 4*a^3*b*d + 6*a^2*b^
2*d - 4*a*b^3*d + b^4*d) + 1/4*((4*a^2*b^2*e^(11*c) + 11*a*b^3*e^(11*c) -
3*b^4*e^(11*c))*e^(11*d*x) + (32*a^3*b*e^(9*c) + 32*a^2*b^2*e^(9*c) - 31*a
*b^3*e^(9*c) + 3*b^4*e^(9*c))*e^(9*d*x) + 2*(32*a^4*e^(7*c) - 48*a^3*b*e^(
7*c) + 46*a^2*b^2*e^(7*c) - 21*a*b^3*e^(7*c) + 3*b^4*e^(7*c))*e^(7*d*x) -
2*(32*a^4*e^(5*c) - 48*a^3*b*e^(5*c) + 46*a^2*b^2*e^(5*c) - 21*a*b^3*e^(5*
c) + 3*b^4*e^(5*c))*e^(5*d*x) - (32*a^3*b*e^(3*c) + 32*a^2*b^2*e^(3*c) - 3
1*a*b^3*e^(3*c) + 3*b^4*e^(3*c))*e^(3*d*x) - (4*a^2*b^2*e^c + 11*a*b^3*e^c
 - 3*b^4*e^c)*e^(d*x))/(a^5*b^2*d - 3*a^4*b^3*d + 3*a^3*b^4*d - a^2*b^5*d
+ (a^5*b^2*d*e^(12*c) - 3*a^4*b^3*d*e^(12*c) + 3*a^3*b^4*d*e^(12*c) - a^2*
b^5*d*e^(12*c))*e^(12*d*x) + 2*(4*a^6*b*d*e^(10*c) - 13*a^5*b^2*d*e^(10*c)
 + 15*a^4*b^3*d*e^(10*c) - 7*a^3*b^4*d*e^(10*c) + a^2*b^5*d*e^(10*c))*e^(1
0*d*x) + (16*a^7*d*e^(8*c) - 48*a^6*b*d*e^(8*c) + 47*a^5*b^2*d*e^(8*c) - 1
3*a^4*b^3*d*e^(8*c) - 3*a^3*b^4*d*e^(8*c) + a^2*b^5*d*e^(8*c))*e^(8*d*x) +
 4*(8*a^7*d*e^(6*c) - 28*a^6*b*d*e^(6*c) + 37*a^5*b^2*d*e^(6*c) - 23*a^4*b
^3*d*e^(6*c) + 7*a^3*b^4*d*e^(6*c) - a^2*b^5*d*e^(6*c))*e^(6*d*x) + (16*a^
7*d*e^(4*c) - 48*a^6*b*d*e^(4*c) + 47*a^5*b^2*d*e^(4*c) - 13*a^4*b^3*d*e^(
4*c) - 3*a^3*b^4*d*e^(4*c) + a^2*b^5*d*e^(4*c))*e^(4*d*x) + 2*(4*a^6*b*d*e
^(2*c) - 13*a^5*b^2*d*e^(2*c) + 15*a^4*b^3*d*e^(2*c) - 7*a^3*b^4*d*e^(2*c)
 + a^2*b^5*d*e^(2*c))*e^(2*d*x)) + 8*integrate(1/32*((35*a^2*b^2*e^(3*c...
```

Giac [F(-2)]

Exception generated.

$$\int \frac{\operatorname{sech}^3(c+dx)}{\left(a+b\sinh^2(c+dx)\right)^3}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(sech(d*x+c)^3/(a+b*sinh(d*x+c)^2)^3,x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Limit: Max order reached or unable
to make series expansion Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\operatorname{sech}^3(c+dx)}{\left(a+b\sinh^2(c+dx)\right)^3}\,dx = \int \frac{1}{\cosh\left(c+d\,x\right)^3\,\left(b\sinh\left(c+d\,x\right)^2+a\right)^3}\,dx$$

input
```
int(1/(cosh(c + d*x)^3*(a + b*sinh(c + d*x)^2)^3),x)
```

output
```
int(1/(cosh(c + d*x)^3*(a + b*sinh(c + d*x)^2)^3), x)
```

Reduce [B] (verification not implemented)

Time = 1.71 (sec) , antiderivative size = 13647, normalized size of antiderivative = 62.89

$$\int \frac{\operatorname{sech}^3(c+dx)}{\left(a+b\sinh^2(c+dx)\right)^3}\,dx = \text{Too large to display}$$

input
```
int(sech(d*x+c)^3/(a+b*sinh(d*x+c)^2)^3,x)
```

output

```
(16*e**(12*c + 12*d*x)*atan(e**(c + d*x))*a**4*b**2 - 112*e**(12*c + 12*d*
x)*atan(e**(c + d*x))*a**3*b**3 + 128*e**(10*c + 10*d*x)*atan(e**(c + d*x)
)*a**5*b - 928*e**(10*c + 10*d*x)*atan(e**(c + d*x))*a**4*b**2 + 224*e**(1
0*c + 10*d*x)*atan(e**(c + d*x))*a**3*b**3 + 256*e**(8*c + 8*d*x)*atan(e**
(c + d*x))*a**6 - 1792*e**(8*c + 8*d*x)*atan(e**(c + d*x))*a**5*b - 16*e**
(8*c + 8*d*x)*atan(e**(c + d*x))*a**4*b**2 + 112*e**(8*c + 8*d*x)*atan(e**
(c + d*x))*a**3*b**3 + 512*e**(6*c + 6*d*x)*atan(e**(c + d*x))*a**6 - 3840
*e**(6*c + 6*d*x)*atan(e**(c + d*x))*a**5*b + 1856*e**(6*c + 6*d*x)*atan(e
**(c + d*x))*a**4*b**2 - 448*e**(6*c + 6*d*x)*atan(e**(c + d*x))*a**3*b**3
 + 256*e**(4*c + 4*d*x)*atan(e**(c + d*x))*a**6 - 1792*e**(4*c + 4*d*x)*at
an(e**(c + d*x))*a**5*b - 16*e**(4*c + 4*d*x)*atan(e**(c + d*x))*a**4*b**2
 + 112*e**(4*c + 4*d*x)*atan(e**(c + d*x))*a**3*b**3 + 128*e**(2*c + 2*d*x
)*atan(e**(c + d*x))*a**5*b - 928*e**(2*c + 2*d*x)*atan(e**(c + d*x))*a**4
*b**2 + 224*e**(2*c + 2*d*x)*atan(e**(c + d*x))*a**3*b**3 + 16*atan(e**(c
+ d*x))*a**4*b**2 - 112*atan(e**(c + d*x))*a**3*b**3 - 70*e**(12*c + 12*d*
x)*sqrt(b)*sqrt(a)*sqrt(a - b)*sqrt(2*sqrt(a)*sqrt(a - b) + 2*a - b)*atan(
(e**(c + d*x)*b)/(sqrt(b)*sqrt(2*sqrt(a)*sqrt(a - b) + 2*a - b)))*a**2*b**
2 + 28*e**(12*c + 12*d*x)*sqrt(b)*sqrt(a)*sqrt(a - b)*sqrt(2*sqrt(a)*sqrt(
a - b) + 2*a - b)*atan((e**(c + d*x)*b)/(sqrt(b)*sqrt(2*sqrt(a)*sqrt(a - b
) + 2*a - b)))*a*b**3 - 6*e**(12*c + 12*d*x)*sqrt(b)*sqrt(a)*sqrt(a - b...
```

3.312 $\int \frac{\operatorname{sech}^4(c+dx)}{\left(a+b\sinh^2(c+dx)\right)^3}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 203

$$\int \frac{\operatorname{sech}^4(c+dx)}{\left(a+b\sinh^2(c+dx)\right)^3}\,dx = \frac{b^2(48a^2-16ab+3b^2)\operatorname{arctanh}\left(\frac{\sqrt{a-b}\tanh(c+dx)}{\sqrt{a}}\right)}{8a^{5/2}(a-b)^{9/2}d} + \frac{(a-4b)\tanh(c+dx)}{(a-b)^4d} - \frac{\tanh^3(c+dx)}{3(a-b)^3d} + \frac{b^4\tanh(c+dx)}{4a(a-b)^4d\left(a-(a-b)\tanh^2(c+dx)\right)^2} - \frac{(16a-3b)b^3\tanh(c+dx)}{8a^2(a-b)^4d\left(a-(a-b)\tanh^2(c+dx)\right)}$$

output
```
1/8*b^2*(48*a^2-16*a*b+3*b^2)*arctanh((a-b)^(1/2)*tanh(d*x+c)/a^(1/2))/a^(
5/2)/(a-b)^(9/2)/d+(a-4*b)*tanh(d*x+c)/(a-b)^4/d-1/3*tanh(d*x+c)^3/(a-b)^3
/d+1/4*b^4*tanh(d*x+c)/a/(a-b)^4/d/(a-(a-b)*tanh(d*x+c)^2)^2-1/8*(16*a-3*b
)*b^3*tanh(d*x+c)/a^2/(a-b)^4/d/(a-(a-b)*tanh(d*x+c)^2)
```

Mathematica [A] (verified)

Time = 1.73 (sec) , antiderivative size = 169, normalized size of antiderivative = 0.83

$$\int \frac{\operatorname{sech}^4(c+dx)}{\left(a+b\sinh^2(c+dx)\right)^3}\,dx$$
$$= \frac{\frac{3b^2(48a^2-16ab+3b^2)\operatorname{arctanh}\left(\frac{\sqrt{a-b}\tanh(c+dx)}{\sqrt{a}}\right)}{a^{5/2}(a-b)^{9/2}} + \frac{\frac{3b^3\left(-32a^2+24ab-3b^2+b(-14a+3b)\cosh(2(c+dx))\right)\sinh(2(c+dx))}{a^2(2a-b+b\cosh(2(c+dx)))^2}+8\left(2a-11b+(a-b)\operatorname{sech}\right.}{(a-b)^4}}{24d}$$

input
```
Integrate[Sech[c + d*x]^4/(a + b*Sinh[c + d*x]^2)^3,x]
```

output
```
((3*b^2*(48*a^2 - 16*a*b + 3*b^2)*ArcTanh[(Sqrt[a - b]*Tanh[c + d*x])/Sqrt
[a]])/(a^(5/2)*(a - b)^(9/2)) + ((3*b^3*(-32*a^2 + 24*a*b - 3*b^2 + b*(-14
*a + 3*b)*Cosh[2*(c + d*x)])*Sinh[2*(c + d*x)])/(a^2*(2*a - b + b*Cosh[2*(
c + d*x)])^2) + 8*(2*a - 11*b + (a - b)*Sech[c + d*x]^2)*Tanh[c + d*x])/(a
 - b)^4)/(24*d)
```

Rubi [A] (verified)

Time = 0.52 (sec) , antiderivative size = 192, normalized size of antiderivative = 0.95, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.174$, Rules used = {3042, 3670, 300, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\operatorname{sech}^4(c+dx)}{\left(a+b\sinh^2(c+dx)\right)^3}\,dx$$
$$\Big\downarrow \text{3042}$$
$$\int \frac{1}{\cos(ic+idx)^4\left(a-b\sin(ic+idx)^2\right)^3}dx$$
$$\Big\downarrow \text{3670}$$
$$\frac{\int \frac{\left(1-\tanh^2(c+dx)\right)^4}{\left(a-(a-b)\tanh^2(c+dx)\right)^3}d\tanh(c+dx)}{d}$$

$$\Big\downarrow \text{300}$$

$$\frac{\int \left(-\frac{\tanh^2(c+dx)}{(a-b)^3}+\frac{6(a-b)^2b^2\tanh^4(c+dx)-4(a-b)(3a-b)b^2\tanh^2(c+dx)+b^2\left(6a^2-4ba+b^2\right)}{(a-b)^4\left((b-a)\tanh^2(c+dx)+a\right)^3}+\frac{a-4b}{(a-b)^4}\right)d\tanh(c+dx)}{d}$$

$$\Big\downarrow \text{2009}$$

$$\frac{-\frac{b^3(16a-3b)\tanh(c+dx)}{8a^2(a-b)^4\left(a-(a-b)\tanh^2(c+dx)\right)}+\frac{b^2\left(48a^2-16ab+3b^2\right)\text{arctanh}\left(\frac{\sqrt{a-b}\tanh(c+dx)}{\sqrt{a}}\right)}{8a^{5/2}(a-b)^{9/2}}+\frac{b^4\tanh(c+dx)}{4a(a-b)^4\left(a-(a-b)\tanh^2(c+dx)\right)^2}-\frac{\tanh^3(c}{3(a-}}{d}$$

input
```
Int[Sech[c + d*x]^4/(a + b*Sinh[c + d*x]^2)^3,x]
```

output
```
((b^2*(48*a^2 - 16*a*b + 3*b^2)*ArcTanh[(Sqrt[a - b]*Tanh[c + d*x])/Sqrt[a
]])/(8*a^(5/2)*(a - b)^(9/2)) + ((a - 4*b)*Tanh[c + d*x])/(a - b)^4 - Tanh
[c + d*x]^3/(3*(a - b)^3) + (b^4*Tanh[c + d*x])/(4*a*(a - b)^4*(a - (a - b
)*Tanh[c + d*x]^2)^2) - ((16*a - 3*b)*b^3*Tanh[c + d*x])/(8*a^2*(a - b)^4*
(a - (a - b)*Tanh[c + d*x]^2)))/d
```

Defintions of rubi rules used

rule 300
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_), x_Symbol] :> Int
[PolynomialDivide[(a + b*x^2)^p, (c + d*x^2)^(-q), x], x] /; FreeQ[{a, b, c
, d}, x] && NeQ[b*c - a*d, 0] && IGtQ[p, 0] && ILtQ[q, 0] && GeQ[p, -q]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3670
```
Int[cos[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(
p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Simp[ff/f   Su
bst[Int[(a + (a + b)*ff^2*x^2)^p/(1 + ff^2*x^2)^(m/2 + p + 1), x], x, Tan[e
 + f*x]/ff], x]] /; FreeQ[{a, b, e, f}, x] && IntegerQ[m/2] && IntegerQ[p]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 444 vs. $2(187) = 374$.

Time = 0.27 (sec) , antiderivative size = 445, normalized size of antiderivative = 2.19

$$-\frac{2b^2\left(\frac{\frac{b(16a-5b)\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^7}{8a}-\frac{\left(16a^2-61ab+12b^2\right)b\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^5}{8a^2}-\frac{\left(16a^2-61ab+12b^2\right)b\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^3}{8a^2}+\frac{b(16a-5b)\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)}{8a}}{\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^4a-2\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^2a+4b\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^2+a\right)^2}+\frac{\left(48a^2-16ab+3b^2\right)\cdots}{\cdots}\right.}{(a-b)^4}$$

input
```
int(sech(d*x+c)^4/(a+b*sinh(d*x+c)^2)^3,x)
```

output
```
1/d*(-2/(a-b)^4*b^2*((1/8*b*(16*a-5*b)/a*tanh(1/2*d*x+1/2*c)^7-1/8*(16*a^2
-61*a*b+12*b^2)/a^2*b*tanh(1/2*d*x+1/2*c)^5-1/8*(16*a^2-61*a*b+12*b^2)/a^2
*b*tanh(1/2*d*x+1/2*c)^3+1/8*b*(16*a-5*b)/a*tanh(1/2*d*x+1/2*c))/(tanh(1/2
*d*x+1/2*c)^4*a-2*tanh(1/2*d*x+1/2*c)^2*a+4*b*tanh(1/2*d*x+1/2*c)^2+a)^2+1
/8/a*(48*a^2-16*a*b+3*b^2)*(-1/2*((-b*(a-b))^(1/2)-b)/a/(-b*(a-b))^(1/2)/(
(2*(-b*(a-b))^(1/2)+a-2*b)*a)^(1/2)*arctanh(tanh(1/2*d*x+1/2*c)*a/((2*(-b*
(a-b))^(1/2)+a-2*b)*a)^(1/2))+1/2*((-b*(a-b))^(1/2)+b)/a/(-b*(a-b))^(1/2)/
((2*(-b*(a-b))^(1/2)-a+2*b)*a)^(1/2)*arctan(tanh(1/2*d*x+1/2*c)*a/((2*(-b*
(a-b))^(1/2)-a+2*b)*a)^(1/2))))-2/(a-b)^4*((-a+4*b)*tanh(1/2*d*x+1/2*c)^5+
(-2/3*a+20/3*b)*tanh(1/2*d*x+1/2*c)^3+(-a+4*b)*tanh(1/2*d*x+1/2*c))/(tanh(
1/2*d*x+1/2*c)^2+1)^3)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 9388 vs. $2(189) = 378$.

Time = 0.30 (sec) , antiderivative size = 19032, normalized size of antiderivative = 93.75

$$\int \frac{\operatorname{sech}^4(c+dx)}{(a+b\sinh^2(c+dx))^3}\,dx = \text{Too large to display}$$

input
```
integrate(sech(d*x+c)^4/(a+b*sinh(d*x+c)^2)^3,x, algorithm="fricas")
```

output
```
Too large to include
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\operatorname{sech}^4(c+dx)}{(a+b\sinh^2(c+dx))^3}\,dx = \text{Timed out}$$

input
```
integrate(sech(d*x+c)**4/(a+b*sinh(d*x+c)**2)**3,x)
```

output
```
Timed out
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{\operatorname{sech}^4(c+dx)}{(a+b\sinh^2(c+dx))^3}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate(sech(d*x+c)^4/(a+b*sinh(d*x+c)^2)^3,x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(b-a>0)', see `assume?` for more
details)Is
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 436 vs. $2(189) = 378$.

Time = 0.38 (sec) , antiderivative size = 436, normalized size of antiderivative = 2.15

$$\int \frac{\operatorname{sech}^4(c+dx)}{(a+b\sinh^2(c+dx))^3}\,dx$$
$$= \frac{\frac{3\,(48\,a^2b^2-16\,ab^3+3\,b^4)\arctan\left(\frac{be^{(2\,dx+2\,c)}+2\,a-b}{2\,\sqrt{-a^2+ab}}\right)}{(a^6-4\,a^5b+6\,a^4b^2-4\,a^3b^3+a^2b^4)\sqrt{-a^2+ab}}+\frac{6\,(24\,a^2b^3e^{(6\,dx+6\,c)}-16\,ab^4e^{(6\,dx+6\,c)}+3\,b^5e^{(6\,dx+6\,c)}+112\,a^3b^2e^{(4\,dx+4\,c)}-136\,a^2b^3\cdots}{(a^6-4\,a^5b+6\,a^4b^2-4\,a^3b^3\cdots}}{\cdots}$$

input
```
integrate(sech(d*x+c)^4/(a+b*sinh(d*x+c)^2)^3,x, algorithm="giac")
```

output
```
1/24*(3*(48*a^2*b^2 - 16*a*b^3 + 3*b^4)*arctan(1/2*(b*e^(2*d*x + 2*c) + 2*
a - b)/sqrt(-a^2 + a*b))/((a^6 - 4*a^5*b + 6*a^4*b^2 - 4*a^3*b^3 + a^2*b^4
)*sqrt(-a^2 + a*b)) + 6*(24*a^2*b^3*e^(6*d*x + 6*c) - 16*a*b^4*e^(6*d*x +
6*c) + 3*b^5*e^(6*d*x + 6*c) + 112*a^3*b^2*e^(4*d*x + 4*c) - 136*a^2*b^3*e
^(4*d*x + 4*c) + 66*a*b^4*e^(4*d*x + 4*c) - 9*b^5*e^(4*d*x + 4*c) + 88*a^2
*b^3*e^(2*d*x + 2*c) - 64*a*b^4*e^(2*d*x + 2*c) + 9*b^5*e^(2*d*x + 2*c) +
14*a*b^4 - 3*b^5)/((a^6 - 4*a^5*b + 6*a^4*b^2 - 4*a^3*b^3 + a^2*b^4)*(b*e^
(4*d*x + 4*c) + 4*a*e^(2*d*x + 2*c) - 2*b*e^(2*d*x + 2*c) + b)^2) + 16*(9*
b*e^(4*d*x + 4*c) - 6*a*e^(2*d*x + 2*c) + 24*b*e^(2*d*x + 2*c) - 2*a + 11*
b)/((a^4 - 4*a^3*b + 6*a^2*b^2 - 4*a*b^3 + b^4)*(e^(2*d*x + 2*c) + 1)^3))/
d
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\operatorname{sech}^4(c+dx)}{\left(a+b\sinh^2(c+dx)\right)^3}\,dx=\int \frac{1}{\cosh\left(c+d\,x\right)^4\,{\left(b\,\sinh\left(c+d\,x\right)^2+a\right)}^3}\,dx$$

input
```
int(1/(cosh(c + d*x)^4*(a + b*sinh(c + d*x)^2)^3),x)
```

output
```
int(1/(cosh(c + d*x)^4*(a + b*sinh(c + d*x)^2)^3), x)
```

Reduce [B] (verification not implemented)

Time = 0.92 (sec) , antiderivative size = 8497, normalized size of antiderivative = 41.86

$$\int \frac{\operatorname{sech}^4(c+dx)}{\left(a+b\sinh^2(c+dx)\right)^3}\,dx=\text{Too large to display}$$

input
```
int(sech(d*x+c)^4/(a+b*sinh(d*x+c)^2)^3,x)
```

output

```
(1152*e**(14*c + 14*d*x)*sqrt(a)*sqrt(a - b)*log( - sqrt(2*sqrt(a)*sqrt(a
- b) - 2*a + b) + e**(c + d*x)*sqrt(b))*a**3*b**4 - 528*e**(14*c + 14*d*x)
*sqrt(a)*sqrt(a - b)*log( - sqrt(2*sqrt(a)*sqrt(a - b) - 2*a + b) + e**(c
+ d*x)*sqrt(b))*a**2*b**5 + 120*e**(14*c + 14*d*x)*sqrt(a)*sqrt(a - b)*log
( - sqrt(2*sqrt(a)*sqrt(a - b) - 2*a + b) + e**(c + d*x)*sqrt(b))*a*b**6 -
 9*e**(14*c + 14*d*x)*sqrt(a)*sqrt(a - b)*log( - sqrt(2*sqrt(a)*sqrt(a - b
) - 2*a + b) + e**(c + d*x)*sqrt(b))*b**7 + 1152*e**(14*c + 14*d*x)*sqrt(a
)*sqrt(a - b)*log(sqrt(2*sqrt(a)*sqrt(a - b) - 2*a + b) + e**(c + d*x)*sqr
t(b))*a**3*b**4 - 528*e**(14*c + 14*d*x)*sqrt(a)*sqrt(a - b)*log(sqrt(2*sq
rt(a)*sqrt(a - b) - 2*a + b) + e**(c + d*x)*sqrt(b))*a**2*b**5 + 120*e**(1
4*c + 14*d*x)*sqrt(a)*sqrt(a - b)*log(sqrt(2*sqrt(a)*sqrt(a - b) - 2*a + b
) + e**(c + d*x)*sqrt(b))*a*b**6 - 9*e**(14*c + 14*d*x)*sqrt(a)*sqrt(a - b
)*log(sqrt(2*sqrt(a)*sqrt(a - b) - 2*a + b) + e**(c + d*x)*sqrt(b))*b**7 -
 1152*e**(14*c + 14*d*x)*sqrt(a)*sqrt(a - b)*log(2*sqrt(a)*sqrt(a - b) + e
**(2*c + 2*d*x)*b + 2*a - b)*a**3*b**4 + 528*e**(14*c + 14*d*x)*sqrt(a)*sq
rt(a - b)*log(2*sqrt(a)*sqrt(a - b) + e**(2*c + 2*d*x)*b + 2*a - b)*a**2*b
**5 - 120*e**(14*c + 14*d*x)*sqrt(a)*sqrt(a - b)*log(2*sqrt(a)*sqrt(a - b)
 + e**(2*c + 2*d*x)*b + 2*a - b)*a*b**6 + 9*e**(14*c + 14*d*x)*sqrt(a)*sqr
t(a - b)*log(2*sqrt(a)*sqrt(a - b) + e**(2*c + 2*d*x)*b + 2*a - b)*b**7 +
9216*e**(12*c + 12*d*x)*sqrt(a)*sqrt(a - b)*log( - sqrt(2*sqrt(a)*sqrt(...
```

3.313 $\int \frac{\cosh^2(x)}{1-\sinh^2(x)} dx$

Optimal result

Integrand size = 15, antiderivative size = 19

$$\int \frac{\cosh^2(x)}{1-\sinh^2(x)} dx = -x+\sqrt{2}\text{arctanh}\left(\sqrt{2}\tanh(x)\right)$$

output
```
-x+arctanh(2^(1/2)*tanh(x))*2^(1/2)
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.26

$$\int \frac{\cosh^2(x)}{1-\sinh^2(x)} dx = -2\left(\frac{x}{2}-\frac{\text{arctanh}\left(\sqrt{2}\tanh(x)\right)}{\sqrt{2}}\right)$$

input
```
Integrate[Cosh[x]^2/(1 - Sinh[x]^2),x]
```

output
```
-2*(x/2 - ArcTanh[Sqrt[2]*Tanh[x]]/Sqrt[2])
```

Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.11, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.267, Rules used = {3042, 3670, 303, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cosh^2(x)}{1-\sinh^2(x)} \, dx$$

$$\Big\downarrow \ 3042$$

$$\int \frac{\cos(ix)^2}{1+\sin(ix)^2}dx$$

$$\Big\downarrow \ 3670$$

$$\int \frac{1}{\left(1-2\tanh^2(x)\right)\left(1-\tanh^2(x)\right)}d\tanh(x)$$

$$\Big\downarrow \ 303$$

$$2\int \frac{1}{1-2\tanh^2(x)}d\tanh(x)-\int \frac{1}{1-\tanh^2(x)}d\tanh(x)$$

$$\Big\downarrow \ 219$$

$$\sqrt{2}\text{arctanh}\left(\sqrt{2}\tanh(x)\right)-\text{arctanh}(\tanh(x))$$

input
```
Int[Cosh[x]^2/(1 - Sinh[x]^2),x]
```

output
```
-ArcTanh[Tanh[x]] + Sqrt[2]*ArcTanh[Sqrt[2]*Tanh[x]]
```

Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 303
```
Int[1/(((a_) + (b_.)*(x_)^2)*((c_) + (d_.)*(x_)^2)), x_Symbol] :> Simp[b/(b
*c - a*d)   Int[1/(a + b*x^2), x], x] - Simp[d/(b*c - a*d)   Int[1/(c + d*x
^2), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3670
```
Int[cos[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(
p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Simp[ff/f   Su
bst[Int[(a + (a + b)*ff^2*x^2)^p/(1 + ff^2*x^2)^(m/2 + p + 1), x], x, Tan[e
 + f*x]/ff], x]] /; FreeQ[{a, b, e, f}, x] && IntegerQ[m/2] && IntegerQ[p]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 38 vs. $2(15) = 30$.

Time = 0.91 (sec) , antiderivative size = 39, normalized size of antiderivative = 2.05

method	result
risch	$-x+\frac{\sqrt{2}\,\ln\left(\mathrm{e}^{2x}-3+2\sqrt{2}\right)}{2}-\frac{\sqrt{2}\,\ln\left(\mathrm{e}^{2x}-3-2\sqrt{2}\right)}{2}$
default	$\sqrt{2}\,\operatorname{arctanh}\left(\frac{\left(2\tanh\left(\frac{x}{2}\right)+2\right)\sqrt{2}}{4}\right)+\sqrt{2}\,\operatorname{arctanh}\left(\frac{\left(2\tanh\left(\frac{x}{2}\right)-2\right)\sqrt{2}}{4}\right)+\ln\left(\tanh\left(\frac{x}{2}\right)-1\right)-\ln\left(1+\mathrm{ta}\right.$

input
```
int(cosh(x)^2/(1-sinh(x)^2),x,method=_RETURNVERBOSE)
```

output
```
-x+1/2*2^(1/2)*ln(exp(2*x)-3+2*2^(1/2))-1/2*2^(1/2)*ln(exp(2*x)-3-2*2^(1/2
))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 70 vs. $2(15) = 30$.

Time = 0.13 (sec) , antiderivative size = 70, normalized size of antiderivative = 3.68

$$\int \frac{\cosh^2(x)}{1-\sinh^2(x)}\,dx$$
$$=\frac{1}{2}\sqrt{2}\log\left(-\frac{3\left(2\sqrt{2}-3\right)\cosh\left(x\right)^2-4\left(3\sqrt{2}-4\right)\cosh\left(x\right)\sinh\left(x\right)+3\left(2\sqrt{2}-3\right)\sinh\left(x\right)^2-2\sqrt{2}+3}{\cosh\left(x\right)^2+\sinh\left(x\right)^2-3}\right)$$
$$-x$$

input
```
integrate(cosh(x)^2/(1-sinh(x)^2),x, algorithm="fricas")
```

output
```
1/2*sqrt(2)*log(-(3*(2*sqrt(2) - 3)*cosh(x)^2 - 4*(3*sqrt(2) - 4)*cosh(x)*
sinh(x) + 3*(2*sqrt(2) - 3)*sinh(x)^2 - 2*sqrt(2) + 3)/(cosh(x)^2 + sinh(x
)^2 - 3)) - x
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 238 vs. $2(15) = 30$.

Time = 2.89 (sec) , antiderivative size = 238, normalized size of antiderivative = 12.53

$$\int \frac{\cosh^2(x)}{1-\sinh^2(x)}\,dx = -\frac{1331714x}{941664\sqrt{2}+1331714} - \frac{941664\sqrt{2}x}{941664\sqrt{2}+1331714} + \frac{941664\log\left(\tanh\left(\frac{x}{2}\right)-1+\sqrt{2}\right)}{941664\sqrt{2}+1331714} + \frac{665857\sqrt{2}\log\left(\tanh\left(\frac{x}{2}\right)-1+\sqrt{2}\right)}{941664\sqrt{2}+1331714} + \frac{941664\log\left(\tanh\left(\frac{x}{2}\right)+1+\sqrt{2}\right)}{941664\sqrt{2}+1331714} + \frac{665857\sqrt{2}\log\left(\tanh\left(\frac{x}{2}\right)+1+\sqrt{2}\right)}{941664\sqrt{2}+1331714} - \frac{665857\sqrt{2}\log\left(\tanh\left(\frac{x}{2}\right)-\sqrt{2}-1\right)}{941664\sqrt{2}+1331714} - \frac{941664\log\left(\tanh\left(\frac{x}{2}\right)-\sqrt{2}-1\right)}{941664\sqrt{2}+1331714} - \frac{665857\sqrt{2}\log\left(\tanh\left(\frac{x}{2}\right)-\sqrt{2}+1\right)}{941664\sqrt{2}+1331714} - \frac{941664\log\left(\tanh\left(\frac{x}{2}\right)-\sqrt{2}+1\right)}{941664\sqrt{2}+1331714}$$

input

```
integrate(cosh(x)**2/(1-sinh(x)**2),x)
```

output

```
-1331714*x/(941664*sqrt(2) + 1331714) - 941664*sqrt(2)*x/(941664*sqrt(2) +
 1331714) + 941664*log(tanh(x/2) - 1 + sqrt(2))/(941664*sqrt(2) + 1331714)
 + 665857*sqrt(2)*log(tanh(x/2) - 1 + sqrt(2))/(941664*sqrt(2) + 1331714)
+ 941664*log(tanh(x/2) + 1 + sqrt(2))/(941664*sqrt(2) + 1331714) + 665857*
sqrt(2)*log(tanh(x/2) + 1 + sqrt(2))/(941664*sqrt(2) + 1331714) - 665857*s
qrt(2)*log(tanh(x/2) - sqrt(2) - 1)/(941664*sqrt(2) + 1331714) - 941664*lo
g(tanh(x/2) - sqrt(2) - 1)/(941664*sqrt(2) + 1331714) - 665857*sqrt(2)*log
(tanh(x/2) - sqrt(2) + 1)/(941664*sqrt(2) + 1331714) - 941664*log(tanh(x/2
) - sqrt(2) + 1)/(941664*sqrt(2) + 1331714)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 64 vs. $2(15) = 30$.

Time = 0.12 (sec) , antiderivative size = 64, normalized size of antiderivative = 3.37

$$\int \frac{\cosh^2(x)}{1-\sinh^2(x)}\,dx = \frac{1}{2}\sqrt{2}\log\left(-\frac{\sqrt{2}-e^{(-x)}+1}{\sqrt{2}+e^{(-x)}-1}\right) - \frac{1}{2}\sqrt{2}\log\left(-\frac{\sqrt{2}-e^{(-x)}-1}{\sqrt{2}+e^{(-x)}+1}\right) - x$$

input
```
integrate(cosh(x)^2/(1-sinh(x)^2),x, algorithm="maxima")
```

output
```
1/2*sqrt(2)*log(-(sqrt(2) - e^(-x) + 1)/(sqrt(2) + e^(-x) - 1)) - 1/2*sqrt
(2)*log(-(sqrt(2) - e^(-x) - 1)/(sqrt(2) + e^(-x) + 1)) - x
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 41 vs. $2(15) = 30$.

Time = 0.12 (sec) , antiderivative size = 41, normalized size of antiderivative = 2.16

$$\int \frac{\cosh^2(x)}{1-\sinh^2(x)}\,dx = -\frac{1}{2}\sqrt{2}\log\left(\frac{\left|-4\sqrt{2}+2e^{(2x)}-6\right|}{\left|4\sqrt{2}+2e^{(2x)}-6\right|}\right) - x$$

input
```
integrate(cosh(x)^2/(1-sinh(x)^2),x, algorithm="giac")
```

output
```
-1/2*sqrt(2)*log(abs(-4*sqrt(2) + 2*e^(2*x) - 6)/abs(4*sqrt(2) + 2*e^(2*x)
 - 6)) - x
```

Mupad [B] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 56, normalized size of antiderivative = 2.95

$$\int \frac{\cosh^2(x)}{1-\sinh^2(x)}\,dx = \frac{\sqrt{2}\,\ln\left(8\,e^{2\,x}+\frac{\sqrt{2}\,(12\,e^{2\,x}-4)}{2}\right)}{2} - \frac{\sqrt{2}\,\ln\left(8\,e^{2\,x}-\frac{\sqrt{2}\,(12\,e^{2\,x}-4)}{2}\right)}{2} - x$$

input
```
int(-cosh(x)^2/(sinh(x)^2 - 1),x)
```

output
```
(2^(1/2)*log(8*exp(2*x) + (2^(1/2)*(12*exp(2*x) - 4))/2))/2 - (2^(1/2)*log
(8*exp(2*x) - (2^(1/2)*(12*exp(2*x) - 4))/2))/2 - x
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 56, normalized size of antiderivative = 2.95

$$\int \frac{\cosh^2(x)}{1-\sinh^2(x)}\,dx = -\frac{\sqrt{2}\log\left(e^x-\sqrt{2}-1\right)}{2} + \frac{\sqrt{2}\log\left(e^x-\sqrt{2}+1\right)}{2} + \frac{\sqrt{2}\log\left(e^x+\sqrt{2}-1\right)}{2} - \frac{\sqrt{2}\log\left(e^x+\sqrt{2}+1\right)}{2} - x$$

input
```
int(cosh(x)^2/(1-sinh(x)^2),x)
```

output
```
( - sqrt(2)*log(e**x - sqrt(2) - 1) + sqrt(2)*log(e**x - sqrt(2) + 1) + sq
rt(2)*log(e**x + sqrt(2) - 1) - sqrt(2)*log(e**x + sqrt(2) + 1) - 2*x)/2
```

3.314 $\int \frac{\cosh^3(x)}{1-\sinh^2(x)} dx$

Optimal result

Integrand size = 15, antiderivative size = 10

$$\int \frac{\cosh^3(x)}{1-\sinh^2(x)} dx = 2\text{arctanh}(\sinh(x)) - \sinh(x)$$

output
```
2*arctanh(sinh(x))-sinh(x)
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.40

$$\int \frac{\cosh^3(x)}{1-\sinh^2(x)} dx = -2\left(-\text{arctanh}(\sinh(x)) + \frac{\sinh(x)}{2}\right)$$

input
```
Integrate[Cosh[x]^3/(1 - Sinh[x]^2),x]
```

output
```
-2*(-ArcTanh[Sinh[x]] + Sinh[x]/2)
```

Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 10, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.267$, Rules used = {3042, 3669, 299, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cosh^3(x)}{1-\sinh^2(x)} \, dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\cos(ix)^3}{1+\sin(ix)^2}dx$$

$$\Big\downarrow \text{3669}$$

$$\int \frac{\sinh^2(x)+1}{1-\sinh^2(x)}d\sinh(x)$$

$$\Big\downarrow \text{299}$$

$$2\int \frac{1}{1-\sinh^2(x)}d\sinh(x)-\sinh(x)$$

$$\Big\downarrow \text{219}$$

$$2\text{arctanh}(\sinh(x))-\sinh(x)$$

input
```
Int[Cosh[x]^3/(1 - Sinh[x]^2),x]
```

output
```
2*ArcTanh[Sinh[x]] - Sinh[x]
```

Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 299
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2), x_Symbol] :> Simp[d*x
*((a + b*x^2)^(p + 1)/(b*(2*p + 3))), x] - Simp[(a*d - b*c*(2*p + 3))/(b*(2
*p + 3))   Int[(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
 a*d, 0] && NeQ[2*p + 3, 0]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3669
```
Int[cos[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^
(p_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Simp[ff/f   S
ubst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a + b*ff^2*x^2)^p, x], x, Sin[e + f*x]
/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m - 1)/2]
```

Maple [A] (verified)

Time = 3.80 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.80

method	result	size
derivativedivides	$-\sinh \left(x \right)-\ln \left(-1+\sinh \left(x \right)\right)+\ln \left(1+\sinh \left(x \right)\right)$	18
default	$-\sinh \left(x \right)-\ln \left(-1+\sinh \left(x \right)\right)+\ln \left(1+\sinh \left(x \right)\right)$	18
risch	$-\frac{{\mathrm e}^{x}}{2}+\frac{{\mathrm e}^{-x}}{2}+\ln \left({\mathrm e}^{2 x}+2 \,{\mathrm e}^{x}-1\right)-\ln \left({\mathrm e}^{2 x}-2 \,{\mathrm e}^{x}-1\right)$	36

input
```
int(cosh(x)^3/(1-sinh(x)^2),x,method=_RETURNVERBOSE)
```

output
```
-sinh(x)-ln(-1+sinh(x))+ln(1+sinh(x))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 71 vs. $2(10) = 20$.

Time = 0.10 (sec) , antiderivative size = 71, normalized size of antiderivative = 7.10

$$\int \frac{\cosh^3(x)}{1-\sinh^2(x)}\,dx = -\frac{\cosh(x)^2 - 2(\cosh(x)+\sinh(x))\log\left(\frac{2(\sinh(x)+1)}{\cosh(x)-\sinh(x)}\right) + 2(\cosh(x)+\sinh(x))\log\left(\frac{2(\sinh(x)-1)}{\cosh(x)-\sinh(x)}\right) +}{2(\cosh(x)+\sinh(x))}$$

input
```
integrate(cosh(x)^3/(1-sinh(x)^2),x, algorithm="fricas")
```

output
```
-1/2*(cosh(x)^2 - 2*(cosh(x) + sinh(x))*log(2*(sinh(x) + 1)/(cosh(x) - sin
h(x))) + 2*(cosh(x) + sinh(x))*log(2*(sinh(x) - 1)/(cosh(x) - sinh(x))) +
2*cosh(x)*sinh(x) + sinh(x)^2 - 1)/(cosh(x) + sinh(x))
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 129 vs. $2(8) = 16$.

Time = 0.56 (sec) , antiderivative size = 129, normalized size of antiderivative = 12.90

$$\begin{aligned}\int \frac{\cosh^3(x)}{1-\sinh^2(x)}\,dx &= \frac{\log\left(\tanh^2\left(\frac{x}{2}\right) - 2\tanh\left(\frac{x}{2}\right) - 1\right)\tanh^2\left(\frac{x}{2}\right)}{\tanh^2\left(\frac{x}{2}\right) - 1} \\ &\quad - \frac{\log\left(\tanh^2\left(\frac{x}{2}\right) - 2\tanh\left(\frac{x}{2}\right) - 1\right)}{\tanh^2\left(\frac{x}{2}\right) - 1} \\ &\quad - \frac{\log\left(\tanh^2\left(\frac{x}{2}\right) + 2\tanh\left(\frac{x}{2}\right) - 1\right)\tanh^2\left(\frac{x}{2}\right)}{\tanh^2\left(\frac{x}{2}\right) - 1} \\ &\quad + \frac{\log\left(\tanh^2\left(\frac{x}{2}\right) + 2\tanh\left(\frac{x}{2}\right) - 1\right)}{\tanh^2\left(\frac{x}{2}\right) - 1} + \frac{2\tanh\left(\frac{x}{2}\right)}{\tanh^2\left(\frac{x}{2}\right) - 1}\end{aligned}$$

input
```
integrate(cosh(x)**3/(1-sinh(x)**2),x)
```

output
```
log(tanh(x/2)**2 - 2*tanh(x/2) - 1)*tanh(x/2)**2/(tanh(x/2)**2 - 1) - log(
tanh(x/2)**2 - 2*tanh(x/2) - 1)/(tanh(x/2)**2 - 1) - log(tanh(x/2)**2 + 2*
tanh(x/2) - 1)*tanh(x/2)**2/(tanh(x/2)**2 - 1) + log(tanh(x/2)**2 + 2*tanh
(x/2) - 1)/(tanh(x/2)**2 - 1) + 2*tanh(x/2)/(tanh(x/2)**2 - 1)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 39 vs. $2(10) = 20$.

Time = 0.04 (sec) , antiderivative size = 39, normalized size of antiderivative = 3.90

$$\int \frac{\cosh^3(x)}{1-\sinh^2(x)}\,dx = \frac{1}{2}\,e^{(-x)} - \frac{1}{2}\,e^{x} - \log\left(2\,e^{(-x)} + e^{(-2\,x)} - 1\right) + \log\left(-2\,e^{(-x)} + e^{(-2\,x)} - 1\right)$$

input
```
integrate(cosh(x)^3/(1-sinh(x)^2),x, algorithm="maxima")
```

output
```
1/2*e^(-x) - 1/2*e^x - log(2*e^(-x) + e^(-2*x) - 1) + log(-2*e^(-x) + e^(-
2*x) - 1)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 37 vs. $2(10) = 20$.

Time = 0.13 (sec) , antiderivative size = 37, normalized size of antiderivative = 3.70

$$\int \frac{\cosh^3(x)}{1-\sinh^2(x)}\,dx = \frac{1}{2}\,e^{(-x)} - \frac{1}{2}\,e^{x} + \log\left(\left|-e^{(-x)} + e^{x} + 2\right|\right) - \log\left(\left|-e^{(-x)} + e^{x} - 2\right|\right)$$

input
```
integrate(cosh(x)^3/(1-sinh(x)^2),x, algorithm="giac")
```

output
```
1/2*e^(-x) - 1/2*e^x + log(abs(-e^(-x) + e^x + 2)) - log(abs(-e^(-x) + e^x
 - 2))
```

Mupad [B] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 39, normalized size of antiderivative = 3.90

$$\int \frac{\cosh^3(x)}{1-\sinh^2(x)}\,dx = \frac{e^{-x}}{2} - \ln\left(32\,e^{2\,x} - 64\,e^{x} - 32\right) + \ln\left(32\,e^{2\,x} + 64\,e^{x} - 32\right) - \frac{e^{x}}{2}$$

input
```
int(-cosh(x)^3/(sinh(x)^2 - 1),x)
```

output
```
exp(-x)/2 - log(32*exp(2*x) - 64*exp(x) - 32) + log(32*exp(2*x) + 64*exp(x
) - 32) - exp(x)/2
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 72, normalized size of antiderivative = 7.20

$$\int \frac{\cosh^3(x)}{1-\sinh^2(x)}\,dx$$
$$= \frac{-e^{2x} - 2e^{x}\log\left(e^{x}-\sqrt{2}-1\right) + 2e^{x}\log\left(e^{x}-\sqrt{2}+1\right) - 2e^{x}\log\left(e^{x}+\sqrt{2}-1\right) + 2e^{x}\log\left(e^{x}+\sqrt{2}+1\right) +}{2e^{x}}$$

input
```
int(cosh(x)^3/(1-sinh(x)^2),x)
```

output
```
( - e**(2*x) - 2*e**x*log(e**x - sqrt(2) - 1) + 2*e**x*log(e**x - sqrt(2)
+ 1) - 2*e**x*log(e**x + sqrt(2) - 1) + 2*e**x*log(e**x + sqrt(2) + 1) + 1
)/(2*e**x)
```

3.315 $\int \frac{\cosh^4(x)}{1-\sinh^2(x)} dx$

Optimal result

Integrand size = 15, antiderivative size = 30

$$\int \frac{\cosh^4(x)}{1-\sinh^2(x)} dx = -\frac{5x}{2} + 2\sqrt{2}\text{arctanh}\left(\sqrt{2}\tanh(x)\right) - \frac{1}{2}\cosh(x)\sinh(x)$$

output
```
-5/2*x+2*arctanh(2^(1/2)*tanh(x))*2^(1/2)-1/2*cosh(x)*sinh(x)
```

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.07

$$\int \frac{\cosh^4(x)}{1-\sinh^2(x)} dx = -2\left(\frac{5x}{4} - \sqrt{2}\text{arctanh}\left(\sqrt{2}\tanh(x)\right) + \frac{1}{8}\sinh(2x)\right)$$

input
```
Integrate[Cosh[x]^4/(1 - Sinh[x]^2),x]
```

output
```
-2*((5*x)/4 - Sqrt[2]*ArcTanh[Sqrt[2]*Tanh[x]] + Sinh[2*x]/8)
```

Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 43, normalized size of antiderivative = 1.43, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.400, Rules used = {3042, 3670, 316, 25, 397, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cosh^4(x)}{1-\sinh^2(x)}dx$$
$$\Big\downarrow \ 3042$$
$$\int \frac{\cos(ix)^4}{1+\sin(ix)^2}dx$$
$$\Big\downarrow \ 3670$$
$$\int \frac{1}{\left(1-2\tanh^2(x)\right)\left(1-\tanh^2(x)\right)^2}d\tanh(x)$$
$$\Big\downarrow \ 316$$
$$-\frac{1}{2}\int -\frac{2\tanh^2(x)+3}{\left(1-2\tanh^2(x)\right)\left(1-\tanh^2(x)\right)}d\tanh(x)-\frac{\tanh(x)}{2\left(1-\tanh^2(x)\right)}$$
$$\Big\downarrow \ 25$$
$$\frac{1}{2}\int \frac{2\tanh^2(x)+3}{\left(1-2\tanh^2(x)\right)\left(1-\tanh^2(x)\right)}d\tanh(x)-\frac{\tanh(x)}{2\left(1-\tanh^2(x)\right)}$$
$$\Big\downarrow \ 397$$
$$\frac{1}{2}\left(8\int \frac{1}{1-2\tanh^2(x)}d\tanh(x)-5\int \frac{1}{1-\tanh^2(x)}d\tanh(x)\right)-\frac{\tanh(x)}{2\left(1-\tanh^2(x)\right)}$$
$$\Big\downarrow \ 219$$
$$\frac{1}{2}\left(4\sqrt{2}\text{arctanh}\left(\sqrt{2}\tanh(x)\right)-5\text{arctanh}(\tanh(x))\right)-\frac{\tanh(x)}{2\left(1-\tanh^2(x)\right)}$$

input
```
Int[Cosh[x]^4/(1 - Sinh[x]^2),x]
```

output
```
(-5*ArcTanh[Tanh[x]] + 4*Sqrt[2]*ArcTanh[Sqrt[2]*Tanh[x]])/2 - Tanh[x]/(2*
(1 - Tanh[x]^2))
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 316
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_), x_Symbol] :> Sim
p[(-b)*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^(q + 1)/(2*a*(p + 1)*(b*c - a*d))
), x] + Simp[1/(2*a*(p + 1)*(b*c - a*d))   Int[(a + b*x^2)^(p + 1)*(c + d*x
^2)^q*Simp[b*c + 2*(p + 1)*(b*c - a*d) + d*b*(2*(p + q + 2) + 1)*x^2, x], x
], x] /; FreeQ[{a, b, c, d, q}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] &&  !
( !IntegerQ[p] && IntegerQ[q] && LtQ[q, -1]) && IntBinomialQ[a, b, c, d, 2,
 p, q, x]
```

rule 397
```
Int[((e_) + (f_.)*(x_)^2)/(((a_) + (b_.)*(x_)^2)*((c_) + (d_.)*(x_)^2)), x_
Symbol] :> Simp[(b*e - a*f)/(b*c - a*d)   Int[1/(a + b*x^2), x], x] - Simp[
(d*e - c*f)/(b*c - a*d)   Int[1/(c + d*x^2), x], x] /; FreeQ[{a, b, c, d, e
, f}, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3670
```
Int[cos[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(
p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Simp[ff/f   Su
bst[Int[(a + (a + b)*ff^2*x^2)^p/(1 + ff^2*x^2)^(m/2 + p + 1), x], x, Tan[e
 + f*x]/ff], x]] /; FreeQ[{a, b, e, f}, x] && IntegerQ[m/2] && IntegerQ[p]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 49 vs. 2(22) = 44.

Time = 11.24 (sec) , antiderivative size = 50, normalized size of antiderivative = 1.67

method	result
risch	$-\frac{5x}{2}-\frac{e^{2x}}{8}+\frac{e^{-2x}}{8}+\sqrt{2}\,\ln\left(e^{2x}-3+2\sqrt{2}\right)-\sqrt{2}\,\ln\left(e^{2x}-3-2\sqrt{2}\right)$
default	$-\frac{1}{2\left(\tanh\left(\frac{x}{2}\right)-1\right)^{2}}-\frac{1}{2\left(\tanh\left(\frac{x}{2}\right)-1\right)}+\frac{5\ln\left(\tanh\left(\frac{x}{2}\right)-1\right)}{2}+\frac{1}{2\left(1+\tanh\left(\frac{x}{2}\right)\right)^{2}}-\frac{1}{2\left(1+\tanh\left(\frac{x}{2}\right)\right)}-\frac{5\ln\left(1+\tanh\left(\frac{x}{2}\right)\right)}{2}+2$

input
```
int(cosh(x)^4/(1-sinh(x)^2),x,method=_RETURNVERBOSE)
```

output
```
-5/2*x-1/8*exp(2*x)+1/8*exp(-2*x)+2^(1/2)*ln(exp(2*x)-3+2*2^(1/2))-2^(1/2)
*ln(exp(2*x)-3-2*2^(1/2))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 163 vs. 2(22) = 44.

Time = 0.11 (sec) , antiderivative size = 163, normalized size of antiderivative = 5.43

$$\int \frac{\cosh^4(x)}{1-\sinh^2(x)}\,dx =$$

$$-\frac{\cosh\left(x\right)^{4}+4\,\cosh\left(x\right)\sinh\left(x\right)^{3}+\sinh\left(x\right)^{4}+20\,x\cosh\left(x\right)^{2}+2\,\left(3\,\cosh\left(x\right)^{2}+10\,x\right)\sinh\left(x\right)^{2}-8\,\left(\sqrt{\right.}}{}$$

input
```
integrate(cosh(x)^4/(1-sinh(x)^2),x, algorithm="fricas")
```

output
```
-1/8*(cosh(x)^4 + 4*cosh(x)*sinh(x)^3 + sinh(x)^4 + 20*x*cosh(x)^2 + 2*(3*
cosh(x)^2 + 10*x)*sinh(x)^2 - 8*(sqrt(2)*cosh(x)^2 + 2*sqrt(2)*cosh(x)*sin
h(x) + sqrt(2)*sinh(x)^2)*log(-(3*(2*sqrt(2) - 3)*cosh(x)^2 - 4*(3*sqrt(2)
 - 4)*cosh(x)*sinh(x) + 3*(2*sqrt(2) - 3)*sinh(x)^2 - 2*sqrt(2) + 3)/(cosh
(x)^2 + sinh(x)^2 - 3)) + 4*(cosh(x)^3 + 10*x*cosh(x))*sinh(x) - 1)/(cosh(
x)^2 + 2*cosh(x)*sinh(x) + sinh(x)^2)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2431 vs. $2(29) = 58$.

Time = 6.89 (sec) , antiderivative size = 2431, normalized size of antiderivative = 81.03

$$\int \frac{\cosh^4(x)}{1-\sinh^2(x)}\,dx = \text{Too large to display}$$

input
```
integrate(cosh(x)**4/(1-sinh(x)**2),x)
```

output
```
-2716698600*sqrt(2)*x*tanh(x/2)**4/(1536796802*tanh(x/2)**4 + 1086679440*s
qrt(2)*tanh(x/2)**4 - 3073593604*tanh(x/2)**2 - 2173358880*sqrt(2)*tanh(x/
2)**2 + 1536796802 + 1086679440*sqrt(2)) - 3841992005*x*tanh(x/2)**4/(1536
796802*tanh(x/2)**4 + 1086679440*sqrt(2)*tanh(x/2)**4 - 3073593604*tanh(x/
2)**2 - 2173358880*sqrt(2)*tanh(x/2)**2 + 1536796802 + 1086679440*sqrt(2))
 + 7683984010*x*tanh(x/2)**2/(1536796802*tanh(x/2)**4 + 1086679440*sqrt(2)
*tanh(x/2)**4 - 3073593604*tanh(x/2)**2 - 2173358880*sqrt(2)*tanh(x/2)**2
+ 1536796802 + 1086679440*sqrt(2)) + 5433397200*sqrt(2)*x*tanh(x/2)**2/(15
36796802*tanh(x/2)**4 + 1086679440*sqrt(2)*tanh(x/2)**4 - 3073593604*tanh(
x/2)**2 - 2173358880*sqrt(2)*tanh(x/2)**2 + 1536796802 + 1086679440*sqrt(2
)) - 2716698600*sqrt(2)*x/(1536796802*tanh(x/2)**4 + 1086679440*sqrt(2)*ta
nh(x/2)**4 - 3073593604*tanh(x/2)**2 - 2173358880*sqrt(2)*tanh(x/2)**2 + 1
536796802 + 1086679440*sqrt(2)) - 3841992005*x/(1536796802*tanh(x/2)**4 +
1086679440*sqrt(2)*tanh(x/2)**4 - 3073593604*tanh(x/2)**2 - 2173358880*sqr
t(2)*tanh(x/2)**2 + 1536796802 + 1086679440*sqrt(2)) + 2173358880*log(tanh
(x/2) - 1 + sqrt(2))*tanh(x/2)**4/(1536796802*tanh(x/2)**4 + 1086679440*sq
rt(2)*tanh(x/2)**4 - 3073593604*tanh(x/2)**2 - 2173358880*sqrt(2)*tanh(x/2
)**2 + 1536796802 + 1086679440*sqrt(2)) + 1536796802*sqrt(2)*log(tanh(x/2)
 - 1 + sqrt(2))*tanh(x/2)**4/(1536796802*tanh(x/2)**4 + 1086679440*sqrt(2)
*tanh(x/2)**4 - 3073593604*tanh(x/2)**2 - 2173358880*sqrt(2)*tanh(x/2)*...
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 75 vs. $2(22) = 44$.

Time = 0.11 (sec) , antiderivative size = 75, normalized size of antiderivative = 2.50

$$\int \frac{\cosh^4(x)}{1-\sinh^2(x)}\,dx = \sqrt{2}\log\left(-\frac{\sqrt{2}-e^{(-x)}+1}{\sqrt{2}+e^{(-x)}-1}\right) - \sqrt{2}\log\left(-\frac{\sqrt{2}-e^{(-x)}-1}{\sqrt{2}+e^{(-x)}+1}\right) - \frac{5}{2}\,x - \frac{1}{8}\,e^{(2\,x)} + \frac{1}{8}\,e^{(-2\,x)}$$

input
```
integrate(cosh(x)^4/(1-sinh(x)^2),x, algorithm="maxima")
```

output
```
sqrt(2)*log(-(sqrt(2) - e^(-x) + 1)/(sqrt(2) + e^(-x) - 1)) - sqrt(2)*log(
-(sqrt(2) - e^(-x) - 1)/(sqrt(2) + e^(-x) + 1)) - 5/2*x - 1/8*e^(2*x) + 1/
8*e^(-2*x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 61 vs. $2(22) = 44$.

Time = 0.13 (sec) , antiderivative size = 61, normalized size of antiderivative = 2.03

$$\int \frac{\cosh^4(x)}{1-\sinh^2(x)}\,dx = \frac{1}{8}\left(10\,e^{(2\,x)}+1\right)e^{(-2\,x)} - \sqrt{2}\log\left(\frac{\left|-4\,\sqrt{2}+2\,e^{(2\,x)}-6\right|}{\left|4\,\sqrt{2}+2\,e^{(2\,x)}-6\right|}\right) - \frac{5}{2}\,x - \frac{1}{8}\,e^{(2\,x)}$$

input
```
integrate(cosh(x)^4/(1-sinh(x)^2),x, algorithm="giac")
```

output
```
1/8*(10*e^(2*x) + 1)*e^(-2*x) - sqrt(2)*log(abs(-4*sqrt(2) + 2*e^(2*x) - 6
)/abs(4*sqrt(2) + 2*e^(2*x) - 6)) - 5/2*x - 1/8*e^(2*x)
```

Mupad [B] (verification not implemented)

Time = 1.70 (sec) , antiderivative size = 66, normalized size of antiderivative = 2.20

$$\int \frac{\cosh^4(x)}{1-\sinh^2(x)}\,dx = \frac{e^{-2x}}{8} - \frac{5x}{2} - \frac{e^{2x}}{8} + \sqrt{2}\ln\left(16\,e^{2x} + \sqrt{2}\left(12\,e^{2x}-4\right)\right) - \sqrt{2}\ln\left(16\,e^{2x} - \sqrt{2}\left(12\,e^{2x}-4\right)\right)$$

input
```
int(-cosh(x)^4/(sinh(x)^2 - 1),x)
```

output
```
exp(-2*x)/8 - (5*x)/2 - exp(2*x)/8 + 2^(1/2)*log(16*exp(2*x) + 2^(1/2)*(12
*exp(2*x) - 4)) - 2^(1/2)*log(16*exp(2*x) - 2^(1/2)*(12*exp(2*x) - 4))
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 98, normalized size of antiderivative = 3.27

$$\int \frac{\cosh^4(x)}{1-\sinh^2(x)}\,dx = \frac{-e^{4x} - 8e^{2x}\sqrt{2}\log\left(e^x-\sqrt{2}-1\right) + 8e^{2x}\sqrt{2}\log\left(e^x-\sqrt{2}+1\right) + 8e^{2x}\sqrt{2}\log\left(e^x+\sqrt{2}-1\right) - 8e^{2x}\sqrt{2}\log}{8e^{2x}}$$

input
```
int(cosh(x)^4/(1-sinh(x)^2),x)
```

output
```
( - e**(4*x) - 8*e**(2*x)*sqrt(2)*log(e**x - sqrt(2) - 1) + 8*e**(2*x)*sqr
t(2)*log(e**x - sqrt(2) + 1) + 8*e**(2*x)*sqrt(2)*log(e**x + sqrt(2) - 1)
- 8*e**(2*x)*sqrt(2)*log(e**x + sqrt(2) + 1) - 20*e**(2*x)*x + 1)/(8*e**(2
*x))
```

3.316 $\int \cosh^3(e+fx)\sqrt{a+b\sinh^2(e+fx)}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 117

$$\int \cosh^3(e+fx)\sqrt{a+b\sinh^2(e+fx)}\,dx = -\frac{a(a-4b)\text{arctanh}\left(\frac{\sqrt{b}\sinh(e+fx)}{\sqrt{a+b\sinh^2(e+fx)}}\right)}{8b^{3/2}f} - \frac{(a-4b)\sinh(e+fx)\sqrt{a+b\sinh^2(e+fx)}}{8bf} + \frac{\sinh(e+fx)\left(a+b\sinh^2(e+fx)\right)^{3/2}}{4bf}$$

output
```
-1/8*a*(a-4*b)*arctanh(b^(1/2)*sinh(f*x+e)/(a+b*sinh(f*x+e)^2)^(1/2))/b^(3
/2)/f-1/8*(a-4*b)*sinh(f*x+e)*(a+b*sinh(f*x+e)^2)^(1/2)/b/f+1/4*sinh(f*x+e
)*(a+b*sinh(f*x+e)^2)^(3/2)/b/f
```

Mathematica [A] (verified)

Time = 0.46 (sec) , antiderivative size = 124, normalized size of antiderivative = 1.06

$$\int \cosh^3(e+fx)\sqrt{a+b\sinh^2(e+fx)}\,dx$$
$$=\frac{\sqrt{a+b\sinh^2(e+fx)}\left(-\sqrt{a}(a-4b)\text{arcsinh}\left(\frac{\sqrt{b}\sinh(e+fx)}{\sqrt{a}}\right)+\sqrt{b}(a+3b+b\cosh(2(e+fx)))\sinh(e+f\right.}{8b^{3/2}f\sqrt{1+\frac{b\sinh^2(e+fx)}{a}}}$$

input
```
Integrate[Cosh[e + f*x]^3*Sqrt[a + b*Sinh[e + f*x]^2],x]
```

output
```
(Sqrt[a + b*Sinh[e + f*x]^2]*(-(Sqrt[a]*(a - 4*b)*ArcSinh[(Sqrt[b]*Sinh[e
+ f*x])/Sqrt[a]]) + Sqrt[b]*(a + 3*b + b*Cosh[2*(e + f*x)])*Sinh[e + f*x]*
Sqrt[1 + (b*Sinh[e + f*x]^2)/a]))/(8*b^(3/2)*f*Sqrt[1 + (b*Sinh[e + f*x]^2
)/a])
```

Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 112, normalized size of antiderivative = 0.96, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.240, Rules used = {3042, 3669, 299, 211, 224, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cosh^3(e+fx)\sqrt{a+b\sinh^2(e+fx)}\,dx$$
$$\Big\downarrow \text{3042}$$
$$\int \cos(ie+ifx)^3\sqrt{a-b\sin(ie+ifx)^2}dx$$
$$\Big\downarrow \text{3669}$$
$$\frac{\int \left(\sinh^2(e+fx)+1\right)\sqrt{b\sinh^2(e+fx)+a}d\sinh(e+fx)}{f}$$

$$\Big\downarrow \text{299}$$

$$\frac{\frac{\sinh(e+fx)\left(a+b\sinh^2(e+fx)\right)^{3/2}}{4b}-\frac{(a-4b)\int\sqrt{b\sinh^2(e+fx)+a}d\sinh(e+fx)}{4b}}{f}$$

$$\Big\downarrow \text{211}$$

$$\frac{\frac{\sinh(e+fx)\left(a+b\sinh^2(e+fx)\right)^{3/2}}{4b}-\frac{(a-4b)\left(\frac{1}{2}a\int\frac{1}{\sqrt{b\sinh^2(e+fx)+a}}d\sinh(e+fx)+\frac{1}{2}\sinh(e+fx)\sqrt{a+b\sinh^2(e+fx)}\right)}{4b}}{f}$$

$$\Big\downarrow \text{224}$$

$$\frac{\frac{\sinh(e+fx)\left(a+b\sinh^2(e+fx)\right)^{3/2}}{4b}-\frac{(a-4b)\left(\frac{1}{2}a\int\frac{1}{1-\frac{b\sinh^2(e+fx)}{b\sinh^2(e+fx)+a}}d\frac{\sinh(e+fx)}{\sqrt{b\sinh^2(e+fx)+a}}+\frac{1}{2}\sinh(e+fx)\sqrt{a+b\sinh^2(e+fx)}\right)}{4b}}{f}$$

$$\Big\downarrow \text{219}$$

$$\frac{\frac{\sinh(e+fx)\left(a+b\sinh^2(e+fx)\right)^{3/2}}{4b}-\frac{(a-4b)\left(\frac{a\,\text{arctanh}\left(\frac{\sqrt{b}\sinh(e+fx)}{\sqrt{a+b\sinh^2(e+fx)}}\right)}{2\sqrt{b}}+\frac{1}{2}\sinh(e+fx)\sqrt{a+b\sinh^2(e+fx)}\right)}{4b}}{f}$$

input
```
Int[Cosh[e + f*x]^3*Sqrt[a + b*Sinh[e + f*x]^2],x]
```

output
```
((Sinh[e + f*x]*(a + b*Sinh[e + f*x]^2)^(3/2))/(4*b) - ((a - 4*b)*((a*ArcT
anh[(Sqrt[b]*Sinh[e + f*x])/Sqrt[a + b*Sinh[e + f*x]^2]])/(2*Sqrt[b]) + (S
inh[e + f*x]*Sqrt[a + b*Sinh[e + f*x]^2])/2))/(4*b))/f
```

Defintions of rubi rules used

rule 211
```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[x*((a + b*x^2)^p/(2*p + 1
)), x] + Simp[2*a*(p/(2*p + 1))   Int[(a + b*x^2)^(p - 1), x], x] /; FreeQ[
{a, b}, x] && GtQ[p, 0] && (IntegerQ[4*p] || IntegerQ[6*p])
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

rule 299
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2), x_Symbol] :> Simp[d*x
*((a + b*x^2)^(p + 1)/(b*(2*p + 3))), x] - Simp[(a*d - b*c*(2*p + 3))/(b*(2
*p + 3))   Int[(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
 a*d, 0] && NeQ[2*p + 3, 0]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3669
```
Int[cos[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^
(p_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Simp[ff/f   S
ubst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a + b*ff^2*x^2)^p, x], x, Sin[e + f*x]
/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m - 1)/2]
```

Maple [A] (verified)

Time = 0.41 (sec) , antiderivative size = 146, normalized size of antiderivative = 1.25

method	result
derivativedivides	$\frac{\frac{\sinh(fx+e)\sqrt{a+b\sinh(fx+e)^2}}{2}+\frac{a\ln\left(\sqrt{b}\,\sinh(fx+e)+\sqrt{a+b\sinh(fx+e)^2}\right)}{2\sqrt{b}}+\frac{\sinh(fx+e)\left(a+b\sinh(fx+e)^2\right)^{\frac{3}{2}}}{4b}-\frac{a\left(\frac{\sinh(fx+e)\sqrt{}}{}\right.}{}}{f}$
default	$\frac{\frac{\sinh(fx+e)\sqrt{a+b\sinh(fx+e)^2}}{2}+\frac{a\ln\left(\sqrt{b}\,\sinh(fx+e)+\sqrt{a+b\sinh(fx+e)^2}\right)}{2\sqrt{b}}+\frac{\sinh(fx+e)\left(a+b\sinh(fx+e)^2\right)^{\frac{3}{2}}}{4b}-\frac{a\left(\frac{\sinh(fx+e)\sqrt{}}{}\right.}{}}{f}$

input
```
int(cosh(f*x+e)^3*(a+b*sinh(f*x+e)^2)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/f*(1/2*sinh(f*x+e)*(a+b*sinh(f*x+e)^2)^(1/2)+1/2*a/b^(1/2)*ln(b^(1/2)*si
nh(f*x+e)+(a+b*sinh(f*x+e)^2)^(1/2))+1/4*sinh(f*x+e)*(a+b*sinh(f*x+e)^2)^(
3/2)/b-1/4*a/b*(1/2*sinh(f*x+e)*(a+b*sinh(f*x+e)^2)^(1/2)+1/2*a/b^(1/2)*ln
(b^(1/2)*sinh(f*x+e)+(a+b*sinh(f*x+e)^2)^(1/2))))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1073 vs. $2(101) = 202$.

Time = 0.19 (sec) , antiderivative size = 3169, normalized size of antiderivative = 27.09

$$\int \cosh^3(e+fx)\sqrt{a+b\sinh^2(e+fx)}\,dx = \text{Too large to display}$$

input
```
integrate(cosh(f*x+e)^3*(a+b*sinh(f*x+e)^2)^(1/2),x, algorithm="fricas")
```

output
```
Too large to include
```

Sympy [F(-1)]

Timed out.

$$\int \cosh^3(e+fx)\sqrt{a+b\sinh^2(e+fx)}\,dx = \text{Timed out}$$

input
```
integrate(cosh(f*x+e)**3*(a+b*sinh(f*x+e)**2)**(1/2),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \cosh^3(e+fx)\sqrt{a+b\sinh^2(e+fx)}\,dx = \int \sqrt{b\sinh{(fx+e)}^2+a}\cosh{(fx+e)}^3\,dx$$

input
```
integrate(cosh(f*x+e)^3*(a+b*sinh(f*x+e)^2)^(1/2),x, algorithm="maxima")
```

output
```
integrate(sqrt(b*sinh(f*x + e)^2 + a)*cosh(f*x + e)^3, x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 871 vs. $2(101) = 202$.

Time = 0.39 (sec) , antiderivative size = 871, normalized size of antiderivative = 7.44

$$\int \cosh^3(e+fx)\sqrt{a+b\sinh^2(e+fx)}\,dx = \text{Too large to display}$$

input
```
integrate(cosh(f*x+e)^3*(a+b*sinh(f*x+e)^2)^(1/2),x, algorithm="giac")
```

output
```
1/64*(sqrt(b*e^(4*f*x + 4*e) + 4*a*e^(2*f*x + 2*e) - 2*b*e^(2*f*x + 2*e) +
 b)*((2*a*e^(6*e) + 5*b*e^(6*e))*e^(-2*e)/b + e^(2*f*x + 6*e)) - 8*(a^2*e^
(4*e) - 4*a*b*e^(4*e))*arctan(-(sqrt(b)*e^(2*f*x + 2*e) - sqrt(b*e^(4*f*x
+ 4*e) + 4*a*e^(2*f*x + 2*e) - 2*b*e^(2*f*x + 2*e) + b))/sqrt(-b))/(sqrt(-
b)*b) + 4*(a^2*sqrt(b)*e^(4*e) - 4*a*b^(3/2)*e^(4*e))*log(abs(-(sqrt(b)*e^
(2*f*x + 2*e) - sqrt(b*e^(4*f*x + 4*e) + 4*a*e^(2*f*x + 2*e) - 2*b*e^(2*f*
x + 2*e) + b))*b - 2*a*sqrt(b) + b^(3/2)))/b^2 + 4*(2*(sqrt(b)*e^(2*f*x +
2*e) - sqrt(b*e^(4*f*x + 4*e) + 4*a*e^(2*f*x + 2*e) - 2*b*e^(2*f*x + 2*e)
+ b))^3*a^2*e^(4*e) + 4*(sqrt(b)*e^(2*f*x + 2*e) - sqrt(b*e^(4*f*x + 4*e)
+ 4*a*e^(2*f*x + 2*e) - 2*b*e^(2*f*x + 2*e) + b))^3*a*b*e^(4*e) - 2*(sqrt(
b)*e^(2*f*x + 2*e) - sqrt(b*e^(4*f*x + 4*e) + 4*a*e^(2*f*x + 2*e) - 2*b*e^
(2*f*x + 2*e) + b))^3*b^2*e^(4*e) + 4*(sqrt(b)*e^(2*f*x + 2*e) - sqrt(b*e^
(4*f*x + 4*e) + 4*a*e^(2*f*x + 2*e) - 2*b*e^(2*f*x + 2*e) + b))^2*a*b^(3/2
)*e^(4*e) + (sqrt(b)*e^(2*f*x + 2*e) - sqrt(b*e^(4*f*x + 4*e) + 4*a*e^(2*f
*x + 2*e) - 2*b*e^(2*f*x + 2*e) + b))^2*b^(5/2)*e^(4*e) + 2*(sqrt(b)*e^(2*
f*x + 2*e) - sqrt(b*e^(4*f*x + 4*e) + 4*a*e^(2*f*x + 2*e) - 2*b*e^(2*f*x +
 2*e) + b))*a^2*b*e^(4*e) - 8*(sqrt(b)*e^(2*f*x + 2*e) - sqrt(b*e^(4*f*x +
 4*e) + 4*a*e^(2*f*x + 2*e) - 2*b*e^(2*f*x + 2*e) + b))*a*b^2*e^(4*e) + 4*
(sqrt(b)*e^(2*f*x + 2*e) - sqrt(b*e^(4*f*x + 4*e) + 4*a*e^(2*f*x + 2*e) -
2*b*e^(2*f*x + 2*e) + b))*b^3*e^(4*e) - 3*b^(7/2)*e^(4*e))/(((sqrt(b)*e...
```

Mupad [F(-1)]

Timed out.

$$\int \cosh^3(e+fx)\sqrt{a+b\sinh^2(e+fx)}\,dx = \int \cosh(e+f\,x)^3\,\sqrt{b\sinh\left(e+f\,x\right)^2+a}\,dx$$

input
```
int(cosh(e + f*x)^3*(a + b*sinh(e + f*x)^2)^(1/2),x)
```

output
```
int(cosh(e + f*x)^3*(a + b*sinh(e + f*x)^2)^(1/2), x)
```

Reduce [F]

$$\int \cosh^3(e+fx)\sqrt{a+b\sinh^2(e+fx)}\,dx=\int \sqrt{\sinh(fx+e)^2\,b+a}\,\cosh(fx+e)^3\,dx$$

input
```
int(cosh(f*x+e)^3*(a+b*sinh(f*x+e)^2)^(1/2),x)
```

output
```
int(sqrt(sinh(e + f*x)**2*b + a)*cosh(e + f*x)**3,x)
```

3.317 $\int \cosh(e+fx)\sqrt{a+b\sinh^2(e+fx)}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 72

$$\int \cosh(e+fx)\sqrt{a+b\sinh^2(e+fx)}\,dx = \frac{a\text{arctanh}\left(\frac{\sqrt{b}\sinh(e+fx)}{\sqrt{a+b\sinh^2(e+fx)}}\right)}{2\sqrt{b}f} + \frac{\sinh(e+fx)\sqrt{a+b\sinh^2(e+fx)}}{2f}$$

output
```
1/2*a*arctanh(b^(1/2)*sinh(f*x+e)/(a+b*sinh(f*x+e)^2)^(1/2))/b^(1/2)/f+1/2
*sinh(f*x+e)*(a+b*sinh(f*x+e)^2)^(1/2)/f
```

Mathematica [A] (verified)

Time = 0.17 (sec) , antiderivative size = 96, normalized size of antiderivative = 1.33

$$\int \cosh(e+fx)\sqrt{a+b\sinh^2(e+fx)}\,dx = \frac{\sqrt{b}\sinh(e+fx)\left(a+b\sinh^2(e+fx)\right)+a^{3/2}\text{arcsinh}\left(\frac{\sqrt{b}\sinh(e+fx)}{\sqrt{a}}\right)\sqrt{1+\frac{b\sinh^2(e+fx)}{a}}}{2\sqrt{b}f\sqrt{a+b\sinh^2(e+fx)}}$$

input
```
Integrate[Cosh[e + f*x]*Sqrt[a + b*Sinh[e + f*x]^2],x]
```

output
```
(Sqrt[b]*Sinh[e + f*x]*(a + b*Sinh[e + f*x]^2) + a^(3/2)*ArcSinh[(Sqrt[b]*
Sinh[e + f*x])/Sqrt[a]]*Sqrt[1 + (b*Sinh[e + f*x]^2)/a])/(2*Sqrt[b]*f*Sqrt
[a + b*Sinh[e + f*x]^2])
```

Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 70, normalized size of antiderivative = 0.97, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.217, Rules used = {3042, 3669, 211, 224, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cosh(e+fx)\sqrt{a+b\sinh^2(e+fx)}\,dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \cos(ie+ifx)\sqrt{a-b\sin(ie+ifx)^2}dx$$

$$\Big\downarrow \text{ 3669}$$

$$\frac{\int \sqrt{b\sinh^2(e+fx)+a}d\sinh(e+fx)}{f}$$

$$\Big\downarrow \text{ 211}$$

$$\frac{\frac{1}{2}a\int \frac{1}{\sqrt{b\sinh^2(e+fx)+a}}d\sinh(e+fx)+\frac{1}{2}\sinh(e+fx)\sqrt{a+b\sinh^2(e+fx)}}{f}$$

$$\Big\downarrow \text{ 224}$$

$$\frac{\frac{1}{2}a\int \frac{1}{1-\frac{b\sinh^2(e+fx)}{b\sinh^2(e+fx)+a}}d\frac{\sinh(e+fx)}{\sqrt{b\sinh^2(e+fx)+a}}+\frac{1}{2}\sinh(e+fx)\sqrt{a+b\sinh^2(e+fx)}}{f}$$

$$\Big\downarrow \text{ 219}$$

$$\frac{\frac{a\,\text{arctanh}\left(\frac{\sqrt{b}\sinh(e+fx)}{\sqrt{a+b\sinh^2(e+fx)}}\right)}{2\sqrt{b}}+\frac{1}{2}\sinh(e+fx)\sqrt{a+b\sinh^2(e+fx)}}{f}$$

input
```
Int[Cosh[e + f*x]*Sqrt[a + b*Sinh[e + f*x]^2],x]
```

output
```
((a*ArcTanh[(Sqrt[b]*Sinh[e + f*x])/Sqrt[a + b*Sinh[e + f*x]^2]])/(2*Sqrt[
b]) + (Sinh[e + f*x]*Sqrt[a + b*Sinh[e + f*x]^2])/2)/f
```

Defintions of rubi rules used

rule 211
```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[x*((a + b*x^2)^p/(2*p + 1
)), x] + Simp[2*a*(p/(2*p + 1))   Int[(a + b*x^2)^(p - 1), x], x] /; FreeQ[
{a, b}, x] && GtQ[p, 0] && (IntegerQ[4*p] || IntegerQ[6*p])
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3669
```
Int[cos[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^
(p_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Simp[ff/f   S
ubst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a + b*ff^2*x^2)^p, x], x, Sin[e + f*x]
/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m - 1)/2]
```

Maple [A] (verified)

Time = 0.18 (sec) , antiderivative size = 60, normalized size of antiderivative = 0.83

method	result	size
derivativedivides	$\frac{\frac{\sinh(fx+e)\sqrt{a+b\sinh(fx+e)^2}}{2}+\frac{a\ln\left(\sqrt{b}\,\sinh(fx+e)+\sqrt{a+b\sinh(fx+e)^2}\right)}{2\sqrt{b}}}{f}$	60
default	$\frac{\frac{\sinh(fx+e)\sqrt{a+b\sinh(fx+e)^2}}{2}+\frac{a\ln\left(\sqrt{b}\,\sinh(fx+e)+\sqrt{a+b\sinh(fx+e)^2}\right)}{2\sqrt{b}}}{f}$	60

input
```
int(cosh(f*x+e)*(a+b*sinh(f*x+e)^2)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/f*(1/2*sinh(f*x+e)*(a+b*sinh(f*x+e)^2)^(1/2)+1/2*a/b^(1/2)*ln(b^(1/2)*si
nh(f*x+e)+(a+b*sinh(f*x+e)^2)^(1/2)))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 644 vs. 2(60) = 120.

Time = 0.15 (sec) , antiderivative size = 2307, normalized size of antiderivative = 32.04

$$\int \cosh(e+fx)\sqrt{a+b\sinh^2(e+fx)}\,dx = \text{Too large to display}$$

input
```
integrate(cosh(f*x+e)*(a+b*sinh(f*x+e)^2)^(1/2),x, algorithm="fricas")
```

output
```
[1/8*((a*cosh(f*x + e)^2 + 2*a*cosh(f*x + e)*sinh(f*x + e) + a*sinh(f*x +
e)^2)*sqrt(b)*log(-((a^2*b - 2*a*b^2 + b^3)*cosh(f*x + e)^8 + 8*(a^2*b - 2
*a*b^2 + b^3)*cosh(f*x + e)*sinh(f*x + e)^7 + (a^2*b - 2*a*b^2 + b^3)*sinh
(f*x + e)^8 + 2*(a^3 - 4*a^2*b + 5*a*b^2 - 2*b^3)*cosh(f*x + e)^6 + 2*(a^3
 - 4*a^2*b + 5*a*b^2 - 2*b^3 + 14*(a^2*b - 2*a*b^2 + b^3)*cosh(f*x + e)^2)
*sinh(f*x + e)^6 + 4*(14*(a^2*b - 2*a*b^2 + b^3)*cosh(f*x + e)^3 + 3*(a^3
- 4*a^2*b + 5*a*b^2 - 2*b^3)*cosh(f*x + e))*sinh(f*x + e)^5 + (9*a^2*b - 1
4*a*b^2 + 6*b^3)*cosh(f*x + e)^4 + (70*(a^2*b - 2*a*b^2 + b^3)*cosh(f*x +
e)^4 + 9*a^2*b - 14*a*b^2 + 6*b^3 + 30*(a^3 - 4*a^2*b + 5*a*b^2 - 2*b^3)*c
osh(f*x + e)^2)*sinh(f*x + e)^4 + 4*(14*(a^2*b - 2*a*b^2 + b^3)*cosh(f*x +
 e)^5 + 10*(a^3 - 4*a^2*b + 5*a*b^2 - 2*b^3)*cosh(f*x + e)^3 + (9*a^2*b -
14*a*b^2 + 6*b^3)*cosh(f*x + e))*sinh(f*x + e)^3 + b^3 + 2*(3*a*b^2 - 2*b^
3)*cosh(f*x + e)^2 + 2*(14*(a^2*b - 2*a*b^2 + b^3)*cosh(f*x + e)^6 + 15*(a
^3 - 4*a^2*b + 5*a*b^2 - 2*b^3)*cosh(f*x + e)^4 + 3*a*b^2 - 2*b^3 + 3*(9*a
^2*b - 14*a*b^2 + 6*b^3)*cosh(f*x + e)^2)*sinh(f*x + e)^2 + sqrt(2)*((a^2
- 2*a*b + b^2)*cosh(f*x + e)^6 + 6*(a^2 - 2*a*b + b^2)*cosh(f*x + e)*sinh(
f*x + e)^5 + (a^2 - 2*a*b + b^2)*sinh(f*x + e)^6 - 3*(a^2 - 2*a*b + b^2)*c
osh(f*x + e)^4 + 3*(5*(a^2 - 2*a*b + b^2)*cosh(f*x + e)^2 - a^2 + 2*a*b -
b^2)*sinh(f*x + e)^4 + 4*(5*(a^2 - 2*a*b + b^2)*cosh(f*x + e)^3 - 3*(a^2 -
 2*a*b + b^2)*cosh(f*x + e))*sinh(f*x + e)^3 - (4*a*b - 3*b^2)*cosh(f*x...
```

Sympy [F]

$$\int \cosh(e+fx)\sqrt{a+b\sinh^2(e+fx)}\,dx = \int \sqrt{a+b\sinh^2{(e+fx)}}\cosh{(e+fx)}\,dx$$

input
```
integrate(cosh(f*x+e)*(a+b*sinh(f*x+e)**2)**(1/2),x)
```

output
```
Integral(sqrt(a + b*sinh(e + f*x)**2)*cosh(e + f*x), x)
```

Maxima [F]

$$\int \cosh(e+fx)\sqrt{a+b\sinh^2(e+fx)}\,dx=\int \sqrt{b\sinh{(fx+e)}^2+a}\cosh{(fx+e)}\;dx$$

input
```
integrate(cosh(f*x+e)*(a+b*sinh(f*x+e)^2)^(1/2),x, algorithm="maxima")
```

output
```
integrate(sqrt(b*sinh(f*x + e)^2 + a)*cosh(f*x + e), x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 407 vs. $2(60) = 120$.

Time = 0.26 (sec) , antiderivative size = 407, normalized size of antiderivative = 5.65

$$\int \cosh(e+fx)\sqrt{a+b\sinh^2(e+fx)}\,dx = \frac{\left(\frac{4\,a\arctan\left(-\frac{\sqrt{b}e^{(2\,fx+2\,e)}-\sqrt{be^{(4\,fx+4\,e)}+4\,ae^{(2\,fx+2\,e)}-2\,be^{(2\,fx+2\,e)}+b}}{\sqrt{-b}}\right)e^{(2\,e)}}{\sqrt{-b}}-\frac{2\,ae^{(2\,e)}\log\left(\left|-\left(\sqrt{b}e^{(2\,fx+2\,e)}-\sqrt{be^{(4\,fx+4\,e)}+4\,ae^{(2\,f}\right.\right.\right.}{\sqrt{b}}\right.}{}$$

input
```
integrate(cosh(f*x+e)*(a+b*sinh(f*x+e)^2)^(1/2),x, algorithm="giac")
```

output
```
1/8*(4*a*arctan(-(sqrt(b)*e^(2*f*x + 2*e) - sqrt(b*e^(4*f*x + 4*e) + 4*a*e
^(2*f*x + 2*e) - 2*b*e^(2*f*x + 2*e) + b))/sqrt(-b))*e^(2*e)/sqrt(-b) - 2*
a*e^(2*e)*log(abs(-(sqrt(b)*e^(2*f*x + 2*e) - sqrt(b*e^(4*f*x + 4*e) + 4*a
*e^(2*f*x + 2*e) - 2*b*e^(2*f*x + 2*e) + b))*b - 2*a*sqrt(b) + b^(3/2)))/s
qrt(b) + sqrt(b*e^(4*f*x + 4*e) + 4*a*e^(2*f*x + 2*e) - 2*b*e^(2*f*x + 2*e
) + b)*e^(2*e) + 2*(2*(sqrt(b)*e^(2*f*x + 2*e) - sqrt(b*e^(4*f*x + 4*e) +
4*a*e^(2*f*x + 2*e) - 2*b*e^(2*f*x + 2*e) + b))*a*e^(2*e) - (sqrt(b)*e^(2*
f*x + 2*e) - sqrt(b*e^(4*f*x + 4*e) + 4*a*e^(2*f*x + 2*e) - 2*b*e^(2*f*x +
 2*e) + b))*b*e^(2*e) + b^(3/2)*e^(2*e))/((sqrt(b)*e^(2*f*x + 2*e) - sqrt(
b*e^(4*f*x + 4*e) + 4*a*e^(2*f*x + 2*e) - 2*b*e^(2*f*x + 2*e) + b))^2 - b)
)*e^(-2*e)/f
```

Mupad [B] (verification not implemented)

Time = 1.84 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.85

$$\int \cosh(e+fx)\sqrt{a+b\sinh^2(e+fx)}\,dx = \frac{\sinh(e+f\,x)\,\sqrt{b\sinh\left(e+f\,x\right)^2+a}}{2\,f} + \frac{a\,\ln\left(\sqrt{b}\sinh(e+f\,x)+\sqrt{b\sinh\left(e+f\,x\right)^2+a}\right)}{2\,\sqrt{b}\,f}$$

input
```
int(cosh(e + f*x)*(a + b*sinh(e + f*x)^2)^(1/2),x)
```

output
```
(sinh(e + f*x)*(a + b*sinh(e + f*x)^2)^(1/2))/(2*f) + (a*log(b^(1/2)*sinh(
e + f*x) + (a + b*sinh(e + f*x)^2)^(1/2)))/(2*b^(1/2)*f)
```

Reduce [F]

$$\int \cosh(e+fx)\sqrt{a+b\sinh^2(e+fx)}\,dx = \frac{\sqrt{\sinh\left(fx+e\right)^2b+a}\,\sinh\left(fx+e\right)+\left(\int\frac{\sqrt{\sinh(fx+e)^2b+a}\,\cosh(fx+e)}{\sinh(fx+e)^2b+a}dx\right)af}{2f}$$

input
```
int(cosh(f*x+e)*(a+b*sinh(f*x+e)^2)^(1/2),x)
```

output
```
(sqrt(sinh(e + f*x)**2*b + a)*sinh(e + f*x) + int((sqrt(sinh(e + f*x)**2*b
 + a)*cosh(e + f*x))/(sinh(e + f*x)**2*b + a),x)*a*f)/(2*f)
```

3.318 $\int \text{sech}(e+fx)\sqrt{a+b\sinh^2(e+fx)}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 85

$$\int \text{sech}(e+fx)\sqrt{a+b\sinh^2(e+fx)}\,dx = \frac{\sqrt{a-b}\arctan\left(\frac{\sqrt{a-b}\sinh(e+fx)}{\sqrt{a+b\sinh^2(e+fx)}}\right)}{f} + \frac{\sqrt{b}\,\text{arctanh}\left(\frac{\sqrt{b}\sinh(e+fx)}{\sqrt{a+b\sinh^2(e+fx)}}\right)}{f}$$

output
```
(a-b)^(1/2)*arctan((a-b)^(1/2)*sinh(f*x+e)/(a+b*sinh(f*x+e)^2)^(1/2))/f+b^
(1/2)*arctanh(b^(1/2)*sinh(f*x+e)/(a+b*sinh(f*x+e)^2)^(1/2))/f
```

Mathematica [A] (verified)

Time = 0.51 (sec) , antiderivative size = 130, normalized size of antiderivative = 1.53

$$\int \text{sech}(e+fx)\sqrt{a+b\sinh^2(e+fx)}\,dx = \frac{\sqrt{a-b}\arctan\left(\frac{\sqrt{2a-2b}\sinh(e+fx)}{\sqrt{2a-b+b\cosh(2(e+fx))}}\right) + \frac{\sqrt{a}\sqrt{b}\,\text{arcsinh}\left(\frac{\sqrt{b}\sinh(e+fx)}{\sqrt{a}}\right)\sqrt{\frac{2a-b+b\cosh(2(e+fx))}{a}}}{\sqrt{2a-b+b\cosh(2(e+fx))}}}{f}$$

input
```
Integrate[Sech[e + f*x]*Sqrt[a + b*Sinh[e + f*x]^2],x]
```

output
```
(Sqrt[a - b]*ArcTan[(Sqrt[2*a - 2*b]*Sinh[e + f*x])/Sqrt[2*a - b + b*Cosh[
2*(e + f*x)]]] + (Sqrt[a]*Sqrt[b]*ArcSinh[(Sqrt[b]*Sinh[e + f*x])/Sqrt[a]]
*Sqrt[(2*a - b + b*Cosh[2*(e + f*x)])/a])/Sqrt[2*a - b + b*Cosh[2*(e + f*x
)]])/f
```

Rubi [A] (verified)

Time = 0.32 (sec) , antiderivative size = 83, normalized size of antiderivative = 0.98, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.304$, Rules used = {3042, 3669, 301, 224, 219, 291, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \operatorname{sech}(e+fx)\sqrt{a+b\sinh^2(e+fx)}\,dx$$

$$\Big\downarrow \; 3042$$

$$\int \frac{\sqrt{a-b\sin(ie+ifx)^2}}{\cos(ie+ifx)}dx$$

$$\Big\downarrow \; 3669$$

$$\frac{\int \frac{\sqrt{b\sinh^2(e+fx)+a}}{\sinh^2(e+fx)+1}d\sinh(e+fx)}{f}$$

$$\Big\downarrow \; 301$$

$$\frac{b\int \frac{1}{\sqrt{b\sinh^2(e+fx)+a}}d\sinh(e+fx)+(a-b)\int \frac{1}{\left(\sinh^2(e+fx)+1\right)\sqrt{b\sinh^2(e+fx)+a}}d\sinh(e+fx)}{f}$$

$$\Big\downarrow \; 224$$

$$\frac{(a-b)\int \frac{1}{\left(\sinh^2(e+fx)+1\right)\sqrt{b\sinh^2(e+fx)+a}}d\sinh(e+fx)+b\int \frac{1}{1-\frac{b\sinh^2(e+fx)}{b\sinh^2(e+fx)+a}}d\frac{\sinh(e+fx)}{\sqrt{b\sinh^2(e+fx)+a}}}{f}$$

$$\Big\downarrow \; 219$$

$$\frac{(a-b)\int \frac{1}{\left(\sinh^2(e+fx)+1\right)\sqrt{b\sinh^2(e+fx)+a}}d\sinh(e+fx)+\sqrt{b}\text{arctanh}\left(\frac{\sqrt{b}\sinh(e+fx)}{\sqrt{a+b\sinh^2(e+fx)}}\right)}{f}$$

$$\Big\downarrow \text{ 291}$$

$$\frac{(a-b)\int \frac{1}{1-\frac{(b-a)\sinh^2(e+fx)}{b\sinh^2(e+fx)+a}}d\frac{\sinh(e+fx)}{\sqrt{b\sinh^2(e+fx)+a}}+\sqrt{b}\text{arctanh}\left(\frac{\sqrt{b}\sinh(e+fx)}{\sqrt{a+b\sinh^2(e+fx)}}\right)}{f}$$

$$\Big\downarrow \text{ 216}$$

$$\frac{\sqrt{a-b}\arctan\left(\frac{\sqrt{a-b}\sinh(e+fx)}{\sqrt{a+b\sinh^2(e+fx)}}\right)+\sqrt{b}\text{arctanh}\left(\frac{\sqrt{b}\sinh(e+fx)}{\sqrt{a+b\sinh^2(e+fx)}}\right)}{f}$$

input
```
Int[Sech[e + f*x]*Sqrt[a + b*Sinh[e + f*x]^2],x]
```

output
```
(Sqrt[a - b]*ArcTan[(Sqrt[a - b]*Sinh[e + f*x])/Sqrt[a + b*Sinh[e + f*x]^2
]] + Sqrt[b]*ArcTanh[(Sqrt[b]*Sinh[e + f*x])/Sqrt[a + b*Sinh[e + f*x]^2]])
/f
```

Defintions of rubi rules used

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

rule 291
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*((c_) + (d_.)*(x_)^2)), x_Symbol] :> Subst
[Int[1/(c - (b*c - a*d)*x^2), x], x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b, c,
d}, x] && NeQ[b*c - a*d, 0]
```

rule 301
```
Int[((a_) + (b_.)*(x_)^2)^(p_.)/((c_) + (d_.)*(x_)^2), x_Symbol] :> Simp[b/
d   Int[(a + b*x^2)^(p - 1), x], x] - Simp[(b*c - a*d)/d   Int[(a + b*x^2)^
(p - 1)/(c + d*x^2), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0]
&& GtQ[p, 0] && (EqQ[p, 1/2] || EqQ[Denominator[p], 4] || (EqQ[p, 2/3] && E
qQ[b*c + 3*a*d, 0]))
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3669
```
Int[cos[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^
(p_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Simp[ff/f   S
ubst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a + b*ff^2*x^2)^p, x], x, Sin[e + f*x]
/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m - 1)/2]
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.28 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.60

method	result	size
default	$\frac{\text{`int/indef0`}\left(-\frac{-a-b\sinh(fx+e)^2}{\cosh(fx+e)^2\sqrt{a+b\sinh(fx+e)^2}},\sinh(fx+e)\right)}{f}$	51

input
```
int(sech(f*x+e)*(a+b*sinh(f*x+e)^2)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
`int/indef0`(-(-a-b*sinh(f*x+e)^2)/cosh(f*x+e)^2/(a+b*sinh(f*x+e)^2)^(1/2)
,sinh(f*x+e))/f
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 662 vs. $2(73) = 146$.

Time = 0.21 (sec) , antiderivative size = 4915, normalized size of antiderivative = 57.82

$$\int \operatorname{sech}(e+fx)\sqrt{a+b\sinh^2(e+fx)}\,dx = \text{Too large to display}$$

input
```
integrate(sech(f*x+e)*(a+b*sinh(f*x+e)^2)^(1/2),x, algorithm="fricas")
```

output
```
Too large to include
```

Sympy [F]

$$\int \operatorname{sech}(e+fx)\sqrt{a+b\sinh^2(e+fx)}\,dx = \int \sqrt{a+b\sinh^2(e+fx)}\operatorname{sech}(e+fx)\,dx$$

input
```
integrate(sech(f*x+e)*(a+b*sinh(f*x+e)**2)**(1/2),x)
```

output
```
Integral(sqrt(a + b*sinh(e + f*x)**2)*sech(e + f*x), x)
```

Maxima [F]

$$\int \operatorname{sech}(e+fx)\sqrt{a+b\sinh^2(e+fx)}\,dx = \int \sqrt{b\sinh(fx+e)^2+a}\operatorname{sech}(fx+e)\,dx$$

input
```
integrate(sech(f*x+e)*(a+b*sinh(f*x+e)^2)^(1/2),x, algorithm="maxima")
```

output
```
integrate(sqrt(b*sinh(f*x + e)^2 + a)*sech(f*x + e), x)
```

Giac [F(-2)]

Exception generated.

$$\int \operatorname{sech}(e+fx)\sqrt{a+b\sinh^2(e+fx)}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(sech(f*x+e)*(a+b*sinh(f*x+e)^2)^(1/2),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Degree mismatch inside factorisatio
n over extensionNot implemented, e.g. for multivariate mod/approx polynomi
alsError:
```

Mupad [F(-1)]

Timed out.

$$\int \operatorname{sech}(e+fx)\sqrt{a+b\sinh^2(e+fx)}\,dx = \int \frac{\sqrt{b\sinh\left(e+f\,x\right)^2+a}}{\cosh\left(e+f\,x\right)}\,dx$$

input
```
int((a + b*sinh(e + f*x)^2)^(1/2)/cosh(e + f*x),x)
```

output
```
int((a + b*sinh(e + f*x)^2)^(1/2)/cosh(e + f*x), x)
```

Reduce [F]

$$\int \operatorname{sech}(e+fx)\sqrt{a+b\sinh^2(e+fx)}\,dx = \int \sqrt{\sinh\left(fx+e\right)^2 b+a}\,\operatorname{sech}(fx+e)\,dx$$

input
```
int(sech(f*x+e)*(a+b*sinh(f*x+e)^2)^(1/2),x)
```

output
```
int(sqrt(sinh(e + f*x)**2*b + a)*sech(e + f*x),x)
```

3.319 $\int \operatorname{sech}^3(e+fx)\sqrt{a+b\sinh^2(e+fx)}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 86

$$\int \operatorname{sech}^3(e+fx)\sqrt{a+b\sinh^2(e+fx)}\,dx = \frac{a\arctan\left(\frac{\sqrt{a-b}\sinh(e+fx)}{\sqrt{a+b\sinh^2(e+fx)}}\right)}{2\sqrt{a-b}f} + \frac{\operatorname{sech}(e+fx)\sqrt{a+b\sinh^2(e+fx)}\tanh(e+fx)}{2f}$$

output
```
1/2*a*arctan((a-b)^(1/2)*sinh(f*x+e)/(a+b*sinh(f*x+e)^2)^(1/2))/(a-b)^(1/2
)/f+1/2*sech(f*x+e)*(a+b*sinh(f*x+e)^2)^(1/2)*tanh(f*x+e)/f
```

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 175 vs. 2(86) = 172.

Time = 1.05 (sec) , antiderivative size = 175, normalized size of antiderivative = 2.03

$$\int \operatorname{sech}^3(e+fx)\sqrt{a+b\sinh^2(e+fx)}\,dx = \frac{\sinh(e+fx)\left(\sqrt{2}a\operatorname{arctanh}\left(\frac{\sqrt{-\frac{(a-b)\sinh^2(e+fx)}{a}}}{\sqrt{1+\frac{b\sinh^2(e+fx)}{a}}}\right)\sqrt{\frac{2a-b+b\cosh(2(e+fx))}{a}} + (2a-b+b\cosh(2(e+fx)))\operatorname{sech}^2}{4f\sqrt{-\frac{(a-b)\sinh^2(e+fx)}{a}}\sqrt{a+b\sinh^2(e+fx)}}$$

input
```
Integrate[Sech[e + f*x]^3*Sqrt[a + b*Sinh[e + f*x]^2],x]
```

output
```
(Sinh[e + f*x]*(Sqrt[2]*a*ArcTanh[Sqrt[-(((a - b)*Sinh[e + f*x]^2)/a)]/Sqr
t[1 + (b*Sinh[e + f*x]^2)/a]]*Sqrt[(2*a - b + b*Cosh[2*(e + f*x)])/a] + (2
*a - b + b*Cosh[2*(e + f*x)])*Sech[e + f*x]^2*Sqrt[-(((a - b)*Sinh[e + f*x
]^2)/a)]))/(4*f*Sqrt[-(((a - b)*Sinh[e + f*x]^2)/a)]*Sqrt[a + b*Sinh[e + f
*x]^2])
```

Rubi [A] (verified)

Time = 0.32 (sec) , antiderivative size = 90, normalized size of antiderivative = 1.05, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {3042, 3669, 292, 291, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \operatorname{sech}^3(e+fx)\sqrt{a+b\sinh^2(e+fx)}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\sqrt{a-b\sin(ie+ifx)^2}}{\cos(ie+ifx)^3}dx$$

$$\Big\downarrow \text{3669}$$

$$\frac{\int \frac{\sqrt{b\sinh^2(e+fx)+a}}{\left(\sinh^2(e+fx)+1\right)^2}d\sinh(e+fx)}{f}$$

$$\Big\downarrow \text{292}$$

$$\frac{\frac{1}{2}a\int \frac{1}{\left(\sinh^2(e+fx)+1\right)\sqrt{b\sinh^2(e+fx)+a}}d\sinh(e+fx)+\frac{\sinh(e+fx)\sqrt{a+b\sinh^2(e+fx)}}{2\left(\sinh^2(e+fx)+1\right)}}{f}$$

$$\Big\downarrow \text{291}$$

$$\frac{\frac{1}{2}a\int \frac{1}{1-\frac{(b-a)\sinh^2(e+fx)}{b\sinh^2(e+fx)+a}}d\frac{\sinh(e+fx)}{\sqrt{b\sinh^2(e+fx)+a}}+\frac{\sinh(e+fx)\sqrt{a+b\sinh^2(e+fx)}}{2\left(\sinh^2(e+fx)+1\right)}}{f}$$

$$\Big\downarrow \text{216}$$

$$\frac{\frac{a\arctan\left(\frac{\sqrt{a-b}\sinh(e+fx)}{\sqrt{a+b\sinh^2(e+fx)}}\right)}{2\sqrt{a-b}}+\frac{\sinh(e+fx)\sqrt{a+b\sinh^2(e+fx)}}{2(\sinh^2(e+fx)+1)}}{f}$$

input
```
Int[Sech[e + f*x]^3*Sqrt[a + b*Sinh[e + f*x]^2],x]
```

output
```
((a*ArcTan[(Sqrt[a - b]*Sinh[e + f*x])/Sqrt[a + b*Sinh[e + f*x]^2]])/(2*Sq
rt[a - b]) + (Sinh[e + f*x]*Sqrt[a + b*Sinh[e + f*x]^2])/(2*(1 + Sinh[e +
f*x]^2)))/f
```

Defintions of rubi rules used

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 291
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*((c_) + (d_.)*(x_)^2)), x_Symbol] :> Subst
[Int[1/(c - (b*c - a*d)*x^2), x], x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b, c,
d}, x] && NeQ[b*c - a*d, 0]
```

rule 292
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_.), x_Symbol] :> Si
mp[(-x)*(a + b*x^2)^(p + 1)*((c + d*x^2)^q/(2*a*(p + 1))), x] - Simp[c*(q/(
a*(p + 1)))   Int[(a + b*x^2)^(p + 1)*(c + d*x^2)^(q - 1), x], x] /; FreeQ[
{a, b, c, d, p}, x] && NeQ[b*c - a*d, 0] && EqQ[2*(p + q + 1) + 1, 0] && Gt
Q[q, 0] && NeQ[p, -1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3669
```
Int[cos[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^
(p_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Simp[ff/f   S
ubst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a + b*ff^2*x^2)^p, x], x, Sin[e + f*x]
/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m - 1)/2]
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.40 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.41

method	result	size
default	$\frac{\text{`int/indef0`}\left(\frac{\sqrt{a+b\sinh(fx+e)^2}}{\cosh(fx+e)^4},\sinh(fx+e)\right)}{f}$	35

input
```
int(sech(f*x+e)^3*(a+b*sinh(f*x+e)^2)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
`int/indef0`(1/cosh(f*x+e)^4*(a+b*sinh(f*x+e)^2)^(1/2),sinh(f*x+e))/f
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 605 vs. 2(74) = 148.

Time = 0.16 (sec) , antiderivative size = 1327, normalized size of antiderivative = 15.43

$$\int \operatorname{sech}^3(e+fx)\sqrt{a+b\sinh^2(e+fx)}\,dx = \text{Too large to display}$$

input
```
integrate(sech(f*x+e)^3*(a+b*sinh(f*x+e)^2)^(1/2),x, algorithm="fricas")
```

output
```
[-1/4*((a*cosh(f*x + e)^4 + 4*a*cosh(f*x + e)*sinh(f*x + e)^3 + a*sinh(f*x
 + e)^4 + 2*a*cosh(f*x + e)^2 + 2*(3*a*cosh(f*x + e)^2 + a)*sinh(f*x + e)^
2 + 4*(a*cosh(f*x + e)^3 + a*cosh(f*x + e))*sinh(f*x + e) + a)*sqrt(-a + b
)*log(((a - 2*b)*cosh(f*x + e)^4 + 4*(a - 2*b)*cosh(f*x + e)*sinh(f*x + e)
^3 + (a - 2*b)*sinh(f*x + e)^4 - 2*(3*a - 2*b)*cosh(f*x + e)^2 + 2*(3*(a -
 2*b)*cosh(f*x + e)^2 - 3*a + 2*b)*sinh(f*x + e)^2 - 2*sqrt(2)*(cosh(f*x +
 e)^2 + 2*cosh(f*x + e)*sinh(f*x + e) + sinh(f*x + e)^2 - 1)*sqrt(-a + b)*
sqrt((b*cosh(f*x + e)^2 + b*sinh(f*x + e)^2 + 2*a - b)/(cosh(f*x + e)^2 -
2*cosh(f*x + e)*sinh(f*x + e) + sinh(f*x + e)^2)) + 4*((a - 2*b)*cosh(f*x
+ e)^3 - (3*a - 2*b)*cosh(f*x + e))*sinh(f*x + e) + a - 2*b)/(cosh(f*x + e
)^4 + 4*cosh(f*x + e)*sinh(f*x + e)^3 + sinh(f*x + e)^4 + 2*(3*cosh(f*x +
e)^2 + 1)*sinh(f*x + e)^2 + 2*cosh(f*x + e)^2 + 4*(cosh(f*x + e)^3 + cosh(
f*x + e))*sinh(f*x + e) + 1)) - 2*sqrt(2)*((a - b)*cosh(f*x + e)^2 + 2*(a
- b)*cosh(f*x + e)*sinh(f*x + e) + (a - b)*sinh(f*x + e)^2 - a + b)*sqrt((
b*cosh(f*x + e)^2 + b*sinh(f*x + e)^2 + 2*a - b)/(cosh(f*x + e)^2 - 2*cosh
(f*x + e)*sinh(f*x + e) + sinh(f*x + e)^2)))/((a - b)*f*cosh(f*x + e)^4 +
4*(a - b)*f*cosh(f*x + e)*sinh(f*x + e)^3 + (a - b)*f*sinh(f*x + e)^4 + 2*
(a - b)*f*cosh(f*x + e)^2 + 2*(3*(a - b)*f*cosh(f*x + e)^2 + (a - b)*f)*si
nh(f*x + e)^2 + (a - b)*f + 4*((a - b)*f*cosh(f*x + e)^3 + (a - b)*f*cosh(
f*x + e))*sinh(f*x + e)), 1/2*((a*cosh(f*x + e)^4 + 4*a*cosh(f*x + e)*s...
```

Sympy [F]

$$\int \operatorname{sech}^3(e+fx)\sqrt{a+b\sinh^2(e+fx)}\,dx = \int \sqrt{a+b\sinh^2(e+fx)}\operatorname{sech}^3(e+fx)\,dx$$

input
```
integrate(sech(f*x+e)**3*(a+b*sinh(f*x+e)**2)**(1/2),x)
```

output
```
Integral(sqrt(a + b*sinh(e + f*x)**2)*sech(e + f*x)**3, x)
```

Maxima [F]

$$\int \operatorname{sech}^3(e+fx)\sqrt{a+b\sinh^2(e+fx)}\,dx = \int \sqrt{b\sinh(fx+e)^2+a}\operatorname{sech}(fx+e)^3\,dx$$

input
```
integrate(sech(f*x+e)^3*(a+b*sinh(f*x+e)^2)^(1/2),x, algorithm="maxima")
```

output
```
integrate(sqrt(b*sinh(f*x + e)^2 + a)*sech(f*x + e)^3, x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 668 vs. $2(74) = 148$.

Time = 0.24 (sec) , antiderivative size = 668, normalized size of antiderivative = 7.77

$$\int \operatorname{sech}^3(e+fx)\sqrt{a+b\sinh^2(e+fx)}\,dx = \text{Too large to display}$$

input
```
integrate(sech(f*x+e)^3*(a+b*sinh(f*x+e)^2)^(1/2),x, algorithm="giac")
```

output
```
(a*arctan(-1/2*(sqrt(b)*e^(2*f*x + 2*e) - sqrt(b*e^(4*f*x + 4*e) + 4*a*e^(
2*f*x + 2*e) - 2*b*e^(2*f*x + 2*e) + b) + sqrt(b))/sqrt(a - b))*e^(-2*e)/(
sqrt(a - b)*f) - 2*((sqrt(b)*e^(2*f*x + 2*e) - sqrt(b*e^(4*f*x + 4*e) + 4*
a*e^(2*f*x + 2*e) - 2*b*e^(2*f*x + 2*e) + b))^3*a - 2*(sqrt(b)*e^(2*f*x +
2*e) - sqrt(b*e^(4*f*x + 4*e) + 4*a*e^(2*f*x + 2*e) - 2*b*e^(2*f*x + 2*e)
+ b))^3*b - 5*(sqrt(b)*e^(2*f*x + 2*e) - sqrt(b*e^(4*f*x + 4*e) + 4*a*e^(2
*f*x + 2*e) - 2*b*e^(2*f*x + 2*e) + b))^2*a*sqrt(b) + 2*(sqrt(b)*e^(2*f*x
+ 2*e) - sqrt(b*e^(4*f*x + 4*e) + 4*a*e^(2*f*x + 2*e) - 2*b*e^(2*f*x + 2*e
) + b))^2*b^(3/2) - 4*(sqrt(b)*e^(2*f*x + 2*e) - sqrt(b*e^(4*f*x + 4*e) +
4*a*e^(2*f*x + 2*e) - 2*b*e^(2*f*x + 2*e) + b))*a^2 - (sqrt(b)*e^(2*f*x +
2*e) - sqrt(b*e^(4*f*x + 4*e) + 4*a*e^(2*f*x + 2*e) - 2*b*e^(2*f*x + 2*e)
+ b))*a*b + 2*(sqrt(b)*e^(2*f*x + 2*e) - sqrt(b*e^(4*f*x + 4*e) + 4*a*e^(2
*f*x + 2*e) - 2*b*e^(2*f*x + 2*e) + b))*b^2 - 4*a^2*sqrt(b) + 5*a*b^(3/2)
- 2*b^(5/2))*e^(-2*e)/(((sqrt(b)*e^(2*f*x + 2*e) - sqrt(b*e^(4*f*x + 4*e)
+ 4*a*e^(2*f*x + 2*e) - 2*b*e^(2*f*x + 2*e) + b))^2 + 2*(sqrt(b)*e^(2*f*x
+ 2*e) - sqrt(b*e^(4*f*x + 4*e) + 4*a*e^(2*f*x + 2*e) - 2*b*e^(2*f*x + 2*e
) + b))*sqrt(b) + 4*a - 3*b)^2*f))*e^(2*e)
```

Mupad [F(-1)]

Timed out.

$$\int \operatorname{sech}^3(e+fx)\sqrt{a+b\sinh^2(e+fx)}\,dx=\int \frac{\sqrt{b\,{\sinh\left(e+f\,x\right)}^2+a}}{{\cosh\left(e+f\,x\right)}^3}\,dx$$

input
```
int((a + b*sinh(e + f*x)^2)^(1/2)/cosh(e + f*x)^3,x)
```

output
```
int((a + b*sinh(e + f*x)^2)^(1/2)/cosh(e + f*x)^3, x)
```

Reduce [F]

$$\int \operatorname{sech}^3(e+fx)\sqrt{a+b\sinh^2(e+fx)}\,dx=\int \sqrt{\sinh\left(fx+e\right)^2 b+a}\,\operatorname{sech}(fx+e)^3\,dx$$

input
```
int(sech(f*x+e)^3*(a+b*sinh(f*x+e)^2)^(1/2),x)
```

output
```
int(sqrt(sinh(e + f*x)**2*b + a)*sech(e + f*x)**3,x)
```

3.320 $\int \operatorname{sech}^5(e+fx)\sqrt{a+b\sinh^2(e+fx)}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 144

$$\int \operatorname{sech}^5(e+fx)\sqrt{a+b\sinh^2(e+fx)}\,dx = \frac{a(3a-4b)\arctan\left(\frac{\sqrt{a-b}\sinh(e+fx)}{\sqrt{a+b\sinh^2(e+fx)}}\right)}{8(a-b)^{3/2}f} + \frac{(3a-2b)\operatorname{sech}(e+fx)\sqrt{a+b\sinh^2(e+fx)}\tanh(e+fx)}{8(a-b)f} + \frac{\operatorname{sech}^3(e+fx)\sqrt{a+b\sinh^2(e+fx)}\tanh(e+fx)}{4f}$$

output

```
1/8*a*(3*a-4*b)*arctan((a-b)^(1/2)*sinh(f*x+e)/(a+b*sinh(f*x+e)^2)^(1/2))/
(a-b)^(3/2)/f+1/8*(3*a-2*b)*sech(f*x+e)*(a+b*sinh(f*x+e)^2)^(1/2)*tanh(f*x
+e)/(a-b)/f+1/4*sech(f*x+e)^3*(a+b*sinh(f*x+e)^2)^(1/2)*tanh(f*x+e)/f
```

Mathematica [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 10.05 (sec) , antiderivative size = 684, normalized size of antiderivative = 4.75

$$\int \operatorname{sech}^5(e+fx)\sqrt{a+b\sinh^2(e+fx)}\,dx =$$

$$-\frac{\operatorname{sech}^3(e+fx)\left(1+\frac{b\sinh^2(e+fx)}{a}\right)\tanh(e+fx)\left(-15a\arcsin\left(\sqrt{\frac{(a-b)\tanh^2(e+fx)}{a}}\right)-10b\arcsin\left(\sqrt{\frac{(a-}{}}\right.\right.}{}$$

input
```
Integrate[Sech[e + f*x]^5*Sqrt[a + b*Sinh[e + f*x]^2],x]
```

output
```
-1/40*(Sech[e + f*x]^3*(1 + (b*Sinh[e + f*x]^2)/a)*Tanh[e + f*x]*(-15*a*Ar
cSin[Sqrt[((a - b)*Tanh[e + f*x]^2)/a]] - 10*b*ArcSin[Sqrt[((a - b)*Tanh[e
 + f*x]^2)/a]]*Sinh[e + f*x]^2 - 30*a*Sqrt[(Sech[e + f*x]^2*(a + b*Sinh[e
+ f*x]^2))/a]*(((a - b)*Tanh[e + f*x]^2)/a)^(3/2) - 20*b*Sinh[e + f*x]^2*S
qrt[(Sech[e + f*x]^2*(a + b*Sinh[e + f*x]^2))/a]*(((a - b)*Tanh[e + f*x]^2
)/a)^(3/2) - 32*a*Hypergeometric2F1[2, 4, 7/2, ((a - b)*Tanh[e + f*x]^2)/a
]*Sqrt[(Sech[e + f*x]^2*(a + b*Sinh[e + f*x]^2))/a]*(((a - b)*Tanh[e + f*x
]^2)/a)^(5/2) - 32*b*Hypergeometric2F1[2, 4, 7/2, ((a - b)*Tanh[e + f*x]^2
)/a]*Sinh[e + f*x]^2*Sqrt[(Sech[e + f*x]^2*(a + b*Sinh[e + f*x]^2))/a]*(((
a - b)*Tanh[e + f*x]^2)/a)^(5/2) + 32*a*Hypergeometric2F1[2, 4, 7/2, ((a -
 b)*Tanh[e + f*x]^2)/a]*Sqrt[(Sech[e + f*x]^2*(a + b*Sinh[e + f*x]^2))/a]*
(((a - b)*Tanh[e + f*x]^2)/a)^(7/2) + 32*b*Hypergeometric2F1[2, 4, 7/2, ((
a - b)*Tanh[e + f*x]^2)/a]*Sinh[e + f*x]^2*Sqrt[(Sech[e + f*x]^2*(a + b*Si
nh[e + f*x]^2))/a]*(((a - b)*Tanh[e + f*x]^2)/a)^(7/2) + 15*a*Sqrt[((a - b
)*Sech[e + f*x]^2*(a + b*Sinh[e + f*x]^2)*Tanh[e + f*x]^2)/a^2] + 10*b*Sin
h[e + f*x]^2*Sqrt[((a - b)*Sech[e + f*x]^2*(a + b*Sinh[e + f*x]^2)*Tanh[e
+ f*x]^2)/a^2]))/(f*Sqrt[a + b*Sinh[e + f*x]^2]*Sqrt[(Sech[e + f*x]^2*(a +
 b*Sinh[e + f*x]^2))/a]*(((a - b)*Tanh[e + f*x]^2)/a)^(3/2))
```

Rubi [A] (verified)

Time = 0.36 (sec) , antiderivative size = 154, normalized size of antiderivative = 1.07, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.240$, Rules used = {3042, 3669, 296, 292, 291, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \text{sech}^5(e+fx)\sqrt{a+b\sinh^2(e+fx)}\,dx$$

$$\Big\downarrow \; 3042$$

$$\int \frac{\sqrt{a-b\sin(ie+ifx)^2}}{\cos(ie+ifx)^5}dx$$

$$\Big\downarrow \; 3669$$

$$\frac{\int \frac{\sqrt{b\sinh^2(e+fx)+a}}{\left(\sinh^2(e+fx)+1\right)^3}d\sinh(e+fx)}{f}$$

$$\Big\downarrow \; 296$$

$$\frac{\frac{(3a-4b)\int \frac{\sqrt{b\sinh^2(e+fx)+a}}{\left(\sinh^2(e+fx)+1\right)^2}d\sinh(e+fx)}{4(a-b)}+\frac{\sinh(e+fx)\left(a+b\sinh^2(e+fx)\right)^{3/2}}{4(a-b)\left(\sinh^2(e+fx)+1\right)^2}}{f}$$

$$\Big\downarrow \; 292$$

$$\frac{\frac{(3a-4b)\left(\frac{1}{2}a\int \frac{1}{\left(\sinh^2(e+fx)+1\right)\sqrt{b\sinh^2(e+fx)+a}}d\sinh(e+fx)+\frac{\sinh(e+fx)\sqrt{a+b\sinh^2(e+fx)}}{2\left(\sinh^2(e+fx)+1\right)}\right)}{4(a-b)}+\frac{\sinh(e+fx)\left(a+b\sinh^2(e+fx)\right)^{3/2}}{4(a-b)\left(\sinh^2(e+fx)+1\right)^2}}{f}$$

$$\Big\downarrow \; 291$$

$$\frac{\frac{(3a-4b)\left(\frac{1}{2}a\int \frac{1}{1-\frac{(b-a)\sinh^2(e+fx)}{b\sinh^2(e+fx)+a}}d\frac{\sinh(e+fx)}{\sqrt{b\sinh^2(e+fx)+a}}+\frac{\sinh(e+fx)\sqrt{a+b\sinh^2(e+fx)}}{2\left(\sinh^2(e+fx)+1\right)}\right)}{4(a-b)}+\frac{\sinh(e+fx)\left(a+b\sinh^2(e+fx)\right)^{3/2}}{4(a-b)\left(\sinh^2(e+fx)+1\right)^2}}{f}$$

$$\Big\downarrow \; 216$$

$$\frac{\frac{(3a-4b)\left(\frac{a\arctan\left(\frac{\sqrt{a-b}\sinh(e+fx)}{\sqrt{a+b\sinh^2(e+fx)}}\right)}{2\sqrt{a-b}}+\frac{\sinh(e+fx)\sqrt{a+b\sinh^2(e+fx)}}{2\left(\sinh^2(e+fx)+1\right)}\right)}{4(a-b)}+\frac{\sinh(e+fx)\left(a+b\sinh^2(e+fx)\right)^{3/2}}{4(a-b)\left(\sinh^2(e+fx)+1\right)^2}}{f}$$

input
```
Int[Sech[e + f*x]^5*Sqrt[a + b*Sinh[e + f*x]^2],x]
```

output
```
((Sinh[e + f*x]*(a + b*Sinh[e + f*x]^2)^(3/2))/(4*(a - b)*(1 + Sinh[e + f*
x]^2)^2) + ((3*a - 4*b)*((a*ArcTan[(Sqrt[a - b]*Sinh[e + f*x])/Sqrt[a + b*
Sinh[e + f*x]^2]])/(2*Sqrt[a - b]) + (Sinh[e + f*x]*Sqrt[a + b*Sinh[e + f*
x]^2])/(2*(1 + Sinh[e + f*x]^2))))/(4*(a - b)))/f
```

Defintions of rubi rules used

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 291
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*((c_) + (d_.)*(x_)^2)), x_Symbol] :> Subst
[Int[1/(c - (b*c - a*d)*x^2), x], x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b, c,
d}, x] && NeQ[b*c - a*d, 0]
```

rule 292
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_.), x_Symbol] :> Si
mp[(-x)*(a + b*x^2)^(p + 1)*((c + d*x^2)^q/(2*a*(p + 1))), x] - Simp[c*(q/(
a*(p + 1)))   Int[(a + b*x^2)^(p + 1)*(c + d*x^2)^(q - 1), x], x] /; FreeQ[
{a, b, c, d, p}, x] && NeQ[b*c - a*d, 0] && EqQ[2*(p + q + 1) + 1, 0] && Gt
Q[q, 0] && NeQ[p, -1]
```

rule 296
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_), x_Symbol] :> Sim
p[(-b)*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^(q + 1)/(2*a*(p + 1)*(b*c - a*d))
), x] + Simp[(b*c + 2*(p + 1)*(b*c - a*d))/(2*a*(p + 1)*(b*c - a*d))   Int[
(a + b*x^2)^(p + 1)*(c + d*x^2)^q, x], x] /; FreeQ[{a, b, c, d, q}, x] && N
eQ[b*c - a*d, 0] && EqQ[2*(p + q + 2) + 1, 0] && (LtQ[p, -1] ||  !LtQ[q, -1
]) && NeQ[p, -1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3669
```
Int[cos[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^
(p_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Simp[ff/f   S
ubst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a + b*ff^2*x^2)^p, x], x, Sin[e + f*x]
/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m - 1)/2]
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.19 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.24

$$\frac{\text{`int/indef0`}\left(\frac{\sqrt{a+b\sinh(fx+e)^2}}{\cosh(fx+e)^6}, \sinh\left(fx+e\right)\right)}{f}$$

input
```
int(sech(f*x+e)^5*(a+b*sinh(f*x+e)^2)^(1/2),x)
```

output
```
`int/indef0`(1/cosh(f*x+e)^6*(a+b*sinh(f*x+e)^2)^(1/2),sinh(f*x+e))/f
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1805 vs. $2(128) = 256$.

Time = 0.28 (sec) , antiderivative size = 3727, normalized size of antiderivative = 25.88

$$\int \operatorname{sech}^5(e+fx)\sqrt{a+b\sinh^2(e+fx)}\,dx = \text{Too large to display}$$

input
```
integrate(sech(f*x+e)^5*(a+b*sinh(f*x+e)^2)^(1/2),x, algorithm="fricas")
```

output
```
Too large to include
```

Sympy [F]

$$\int \operatorname{sech}^5(e+fx)\sqrt{a+b\sinh^2(e+fx)}\,dx = \int \sqrt{a+b\sinh^2{(e+fx)}}\operatorname{sech}^5{(e+fx)}\,dx$$

input
```
integrate(sech(f*x+e)**5*(a+b*sinh(f*x+e)**2)**(1/2),x)
```

output
```
Integral(sqrt(a + b*sinh(e + f*x)**2)*sech(e + f*x)**5, x)
```

Maxima [F]

$$\int \operatorname{sech}^5(e+fx)\sqrt{a+b\sinh^2(e+fx)}\,dx = \int \sqrt{b\sinh{(fx+e)}^2+a}\operatorname{sech}{(fx+e)}^5\,dx$$

input
```
integrate(sech(f*x+e)^5*(a+b*sinh(f*x+e)^2)^(1/2),x, algorithm="maxima")
```

output
```
integrate(sqrt(b*sinh(f*x + e)^2 + a)*sech(f*x + e)^5, x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2122 vs. $2(128) = 256$.

Time = 0.46 (sec) , antiderivative size = 2122, normalized size of antiderivative = 14.74

$$\int \operatorname{sech}^5(e+fx)\sqrt{a+b\sinh^2(e+fx)}\,dx = \text{Too large to display}$$

input
```
integrate(sech(f*x+e)^5*(a+b*sinh(f*x+e)^2)^(1/2),x, algorithm="giac")
```

output
```
1/4*((3*a^2 - 4*a*b)*arctan(-1/2*(sqrt(b)*e^(2*f*x + 2*e) - sqrt(b*e^(4*f*
x + 4*e) + 4*a*e^(2*f*x + 2*e) - 2*b*e^(2*f*x + 2*e) + b) + sqrt(b))/sqrt(
a - b))/((a*f*e^(4*e) - b*f*e^(4*e))*sqrt(a - b)) - 2*(3*(sqrt(b)*e^(2*f*x
 + 2*e) - sqrt(b*e^(4*f*x + 4*e) + 4*a*e^(2*f*x + 2*e) - 2*b*e^(2*f*x + 2*
e) + b))^7*a^2 - 4*(sqrt(b)*e^(2*f*x + 2*e) - sqrt(b*e^(4*f*x + 4*e) + 4*a
*e^(2*f*x + 2*e) - 2*b*e^(2*f*x + 2*e) + b))^7*a*b + 21*(sqrt(b)*e^(2*f*x
+ 2*e) - sqrt(b*e^(4*f*x + 4*e) + 4*a*e^(2*f*x + 2*e) - 2*b*e^(2*f*x + 2*e
) + b))^6*a^2*sqrt(b) - 60*(sqrt(b)*e^(2*f*x + 2*e) - sqrt(b*e^(4*f*x + 4*
e) + 4*a*e^(2*f*x + 2*e) - 2*b*e^(2*f*x + 2*e) + b))^6*a*b^(3/2) + 32*(sqr
t(b)*e^(2*f*x + 2*e) - sqrt(b*e^(4*f*x + 4*e) + 4*a*e^(2*f*x + 2*e) - 2*b*
e^(2*f*x + 2*e) + b))^6*b^(5/2) + 44*(sqrt(b)*e^(2*f*x + 2*e) - sqrt(b*e^(
4*f*x + 4*e) + 4*a*e^(2*f*x + 2*e) - 2*b*e^(2*f*x + 2*e) + b))^5*a^3 - 253
*(sqrt(b)*e^(2*f*x + 2*e) - sqrt(b*e^(4*f*x + 4*e) + 4*a*e^(2*f*x + 2*e) -
 2*b*e^(2*f*x + 2*e) + b))^5*a^2*b + 252*(sqrt(b)*e^(2*f*x + 2*e) - sqrt(b
*e^(4*f*x + 4*e) + 4*a*e^(2*f*x + 2*e) - 2*b*e^(2*f*x + 2*e) + b))^5*a*b^2
 - 64*(sqrt(b)*e^(2*f*x + 2*e) - sqrt(b*e^(4*f*x + 4*e) + 4*a*e^(2*f*x + 2
*e) - 2*b*e^(2*f*x + 2*e) + b))^5*b^3 - 292*(sqrt(b)*e^(2*f*x + 2*e) - sqr
t(b*e^(4*f*x + 4*e) + 4*a*e^(2*f*x + 2*e) - 2*b*e^(2*f*x + 2*e) + b))^4*a^
3*sqrt(b) + 317*(sqrt(b)*e^(2*f*x + 2*e) - sqrt(b*e^(4*f*x + 4*e) + 4*a*e^
(2*f*x + 2*e) - 2*b*e^(2*f*x + 2*e) + b))^4*a^2*b^(3/2) - 28*(sqrt(b)*e...
```

Mupad [F(-1)]

Timed out.

$$\int \operatorname{sech}^5(e+fx)\sqrt{a+b\sinh^2(e+fx)}\,dx=\int \frac{\sqrt{b\sinh\left(e+f\,x\right)^2+a}}{\cosh\left(e+f\,x\right)^5}\,dx$$

input
```
int((a + b*sinh(e + f*x)^2)^(1/2)/cosh(e + f*x)^5,x)
```

output
```
int((a + b*sinh(e + f*x)^2)^(1/2)/cosh(e + f*x)^5, x)
```

Reduce [F]

$$\int \operatorname{sech}^5(e+fx)\sqrt{a+b\sinh^2(e+fx)}\,dx=\int \sqrt{\sinh\left(fx+e\right)^2b+a}\,\operatorname{sech}(fx+e)^5\,dx$$

input
```
int(sech(f*x+e)^5*(a+b*sinh(f*x+e)^2)^(1/2),x)
```

output
```
int(sqrt(sinh(e + f*x)**2*b + a)*sech(e + f*x)**5,x)
```

3.321 $\int \cosh^4(e+fx)\sqrt{a+b\sinh^2(e+fx)}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 300

$$\int \cosh^4(e+fx)\sqrt{a+b\sinh^2(e+fx)}\,dx = \frac{(a+3b)\cosh(e+fx)\sinh(e+fx)\sqrt{a+b\sinh^2(e+fx)}}{15bf} + \frac{\cosh^3(e+fx)\sinh(e+fx)\sqrt{a+b\sinh^2(e+fx)}}{5f} + \frac{(2a^2-7ab-3b^2)\,E\left(\arctan(\sinh(e+fx))\,\middle|\,1-\frac{b}{a}\right)\operatorname{sech}(e+fx)\sqrt{a+b\sinh^2(e+fx)}}{15b^2f\sqrt{\frac{\operatorname{sech}^2(e+fx)(a+b\sinh^2(e+fx))}{a}}} - \frac{(a-9b)\operatorname{EllipticF}\left(\arctan(\sinh(e+fx)),1-\frac{b}{a}\right)\operatorname{sech}(e+fx)\sqrt{a+b\sinh^2(e+fx)}}{15bf\sqrt{\frac{\operatorname{sech}^2(e+fx)(a+b\sinh^2(e+fx))}{a}}} - \frac{(2a^2-7ab-3b^2)\sqrt{a+b\sinh^2(e+fx)}\tanh(e+fx)}{15b^2f}$$

output
```
1/15*(a+3*b)*cosh(f*x+e)*sinh(f*x+e)*(a+b*sinh(f*x+e)^2)^(1/2)/b/f+1/5*cos
h(f*x+e)^3*sinh(f*x+e)*(a+b*sinh(f*x+e)^2)^(1/2)/f+1/15*(2*a^2-7*a*b-3*b^2
)*EllipticE(sinh(f*x+e)/(1+sinh(f*x+e)^2)^(1/2),(1-b/a)^(1/2))*sech(f*x+e)
*(a+b*sinh(f*x+e)^2)^(1/2)/b^2/f/(sech(f*x+e)^2*(a+b*sinh(f*x+e)^2)/a)^(1/
2)-1/15*(a-9*b)*InverseJacobiAM(arctan(sinh(f*x+e)),(1-b/a)^(1/2))*sech(f*
x+e)*(a+b*sinh(f*x+e)^2)^(1/2)/b/f/(sech(f*x+e)^2*(a+b*sinh(f*x+e)^2)/a)^(
1/2)-1/15*(2*a^2-7*a*b-3*b^2)*(a+b*sinh(f*x+e)^2)^(1/2)*tanh(f*x+e)/b^2/f
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 1.21 (sec) , antiderivative size = 211, normalized size of antiderivative = 0.70

$$\int \cosh^4(e+fx)\sqrt{a+b\sinh^2(e+fx)}\,dx$$
$$= \frac{16ia(2a^2-7ab-3b^2)\sqrt{\frac{2a-b+b\cosh(2(e+fx))}{a}}E\left(i(e+fx)\,\middle|\,\frac{b}{a}\right)-32ia(a^2-4ab+3b^2)\sqrt{\frac{2a-b+b\cosh(2(e+fx))}{a}}\,\mathrm{I}}{240b^2f\sqrt{2}}$$

input
```
Integrate[Cosh[e + f*x]^4*Sqrt[a + b*Sinh[e + f*x]^2],x]
```

output
```
((16*I)*a*(2*a^2 - 7*a*b - 3*b^2)*Sqrt[(2*a - b + b*Cosh[2*(e + f*x)])/a]*
EllipticE[I*(e + f*x), b/a] - (32*I)*a*(a^2 - 4*a*b + 3*b^2)*Sqrt[(2*a - b
 + b*Cosh[2*(e + f*x)])/a]*EllipticF[I*(e + f*x), b/a] + Sqrt[2]*b*(8*a^2
+ 32*a*b - 15*b^2 + 4*b*(4*a + 3*b)*Cosh[2*(e + f*x)] + 3*b^2*Cosh[4*(e +
f*x)])*Sinh[2*(e + f*x)])/(240*b^2*f*Sqrt[2*a - b + b*Cosh[2*(e + f*x)]])
```

Rubi [A] (verified)

Time = 0.59 (sec) , antiderivative size = 348, normalized size of antiderivative = 1.16, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.360$, Rules used = {3042, 3671, 318, 25, 403, 406, 320, 388, 313}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cosh^4(e+fx)\sqrt{a+b\sinh^2(e+fx)}\,dx$$

$\downarrow$ 3042

$$\int \cos(ie+ifx)^4\sqrt{a-b\sin(ie+ifx)^2}dx$$

$\downarrow$ 3671

$$\frac{\sqrt{\cosh^2(e+fx)}\text{sech}(e+fx)\int \left(\sinh^2(e+fx)+1\right)^{3/2}\sqrt{b\sinh^2(e+fx)+a}d\sinh(e+fx)}{f}$$

$\downarrow$ 318

$$\frac{\sqrt{\cosh^2(e+fx)}\text{sech}(e+fx)\left(\frac{\int -\frac{\left(2(a-3b)\sinh^2(e+fx)+a-5b\right)\sqrt{b\sinh^2(e+fx)+a}}{\sqrt{\sinh^2(e+fx)+1}}d\sinh(e+fx)}{5b}+\frac{\sinh(e+fx)\sqrt{\sinh^2(e+fx)+1}(a+b\sinh}{5b}\right.}{f}$$

$\downarrow$ 25

$$\frac{\sqrt{\cosh^2(e+fx)}\text{sech}(e+fx)\left(\frac{\sinh(e+fx)\sqrt{\sinh^2(e+fx)+1}\left(a+b\sinh^2(e+fx)\right)^{3/2}}{5b}-\frac{\int \frac{\left(2(a-3b)\sinh^2(e+fx)+a-5b\right)\sqrt{b\sinh^2(e+fx)+a}}{\sqrt{\sinh^2(e+fx)+1}}}{5b}\right.}{f}$$

$\downarrow$ 403

$$\frac{\sqrt{\cosh^2(e+fx)}\text{sech}(e+fx)\left(\frac{\sinh(e+fx)\sqrt{\sinh^2(e+fx)+1}\left(a+b\sinh^2(e+fx)\right)^{3/2}}{5b}-\frac{\frac{1}{3}\int \frac{\left(2a^2-7ba-3b^2\right)\sinh^2(e+fx)+a(a-9b)}{\sqrt{\sinh^2(e+fx)+1}\sqrt{b\sinh^2(e+fx)+a}}d\sinh}{}\right.}{f}$$

$\downarrow$ 406

$$\sqrt{\cosh^2(e+fx)}\text{sech}(e+fx)\left(\frac{\sinh(e+fx)\sqrt{\sinh^2(e+fx)+1}\left(a+b\sinh^2(e+fx)\right)^{3/2}}{5b}-\frac{\frac{1}{3}\left(\left(2a^2-7ab-3b^2\right)\int \frac{\sinh^2(e+fx)}{\sqrt{\sinh^2(e+fx)+1}\sqrt{b\sinh^2}}\right.}{}\right.$$

$\downarrow$ 320

$$\sqrt{\cosh^2(e+fx)}\operatorname{sech}(e+fx)\left(\frac{\sinh(e+fx)\sqrt{\sinh^2(e+fx)+1}\left(a+b\sinh^2(e+fx)\right)^{3/2}}{5b}-\frac{\frac{1}{3}\left(\left(2a^2-7ab-3b^2\right)\int\frac{\sinh^2(e+fx)}{\sqrt{\sinh^2(e+fx)+1}\sqrt{b\sinh^2}}\right.}{}\right.$$

$\downarrow$ 388

$$\sqrt{\cosh^2(e+fx)}\operatorname{sech}(e+fx)\left(\frac{\sinh(e+fx)\sqrt{\sinh^2(e+fx)+1}\left(a+b\sinh^2(e+fx)\right)^{3/2}}{5b}-\frac{\frac{1}{3}\left(\left(2a^2-7ab-3b^2\right)\left(\frac{\sinh(e+fx)\sqrt{a+b\sinh^2(e+}}{b\sqrt{\sinh^2(e+fx)+1}}\right.\right.}{}\right.$$

$\downarrow$ 313

$$\sqrt{\cosh^2(e+fx)}\operatorname{sech}(e+fx)\left(\frac{\sinh(e+fx)\sqrt{\sinh^2(e+fx)+1}\left(a+b\sinh^2(e+fx)\right)^{3/2}}{5b}-\frac{\frac{1}{3}\left(\left(2a^2-7ab-3b^2\right)\left(\frac{\sinh(e+fx)\sqrt{a+b\sinh^2(e+}}{b\sqrt{\sinh^2(e+fx)+1}}\right.\right.}{}\right.$$

input
```
Int[Cosh[e + f*x]^4*Sqrt[a + b*Sinh[e + f*x]^2],x]
```

output
```
(Sqrt[Cosh[e + f*x]^2]*Sech[e + f*x]*((Sinh[e + f*x]*Sqrt[1 + Sinh[e + f*x
]^2]*(a + b*Sinh[e + f*x]^2)^(3/2))/(5*b) - ((2*(a - 3*b)*Sinh[e + f*x]*Sq
rt[1 + Sinh[e + f*x]^2]*Sqrt[a + b*Sinh[e + f*x]^2])/3 + (((a - 9*b)*Ellip
ticF[ArcTan[Sinh[e + f*x]], 1 - b/a]*Sqrt[a + b*Sinh[e + f*x]^2])/(Sqrt[1
+ Sinh[e + f*x]^2]*Sqrt[(a + b*Sinh[e + f*x]^2)/(a*(1 + Sinh[e + f*x]^2))]
) + (2*a^2 - 7*a*b - 3*b^2)*((Sinh[e + f*x]*Sqrt[a + b*Sinh[e + f*x]^2])/(
b*Sqrt[1 + Sinh[e + f*x]^2]) - (EllipticE[ArcTan[Sinh[e + f*x]], 1 - b/a]*
Sqrt[a + b*Sinh[e + f*x]^2])/(b*Sqrt[1 + Sinh[e + f*x]^2]*Sqrt[(a + b*Sinh
[e + f*x]^2)/(a*(1 + Sinh[e + f*x]^2))])))/3)/(5*b)))/f
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 313
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/((c_) + (d_.)*(x_)^2)^(3/2), x_Symbol] :> Sim
p[(Sqrt[a + b*x^2]/(c*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(c
+ d*x^2)))]))*EllipticE[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; FreeQ
[{a, b, c, d}, x] && PosQ[b/a] && PosQ[d/c]
```

rule 318
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_), x_Symbol] :> Sim
p[d*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^(q - 1)/(b*(2*(p + q) + 1))), x] + S
imp[1/(b*(2*(p + q) + 1))   Int[(a + b*x^2)^p*(c + d*x^2)^(q - 2)*Simp[c*(b
*c*(2*(p + q) + 1) - a*d) + d*(b*c*(2*(p + 2*q - 1) + 1) - a*d*(2*(q - 1) +
 1))*x^2, x], x], x] /; FreeQ[{a, b, c, d, p}, x] && NeQ[b*c - a*d, 0] && G
tQ[q, 1] && NeQ[2*(p + q) + 1, 0] &&  !IGtQ[p, 1] && IntBinomialQ[a, b, c,
d, 2, p, q, x]
```

rule 320
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(Sqrt[a + b*x^2]/(a*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(
c + d*x^2)))]))*EllipticF[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; Fre
eQ[{a, b, c, d}, x] && PosQ[d/c] && PosQ[b/a] &&  !SimplerSqrtQ[b/a, d/c]
```

rule 388
```
Int[(x_)^2/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol]
 :> Simp[x*(Sqrt[a + b*x^2]/(b*Sqrt[c + d*x^2])), x] - Simp[c/b   Int[Sqrt[
a + b*x^2]/(c + d*x^2)^(3/2), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
 a*d, 0] && PosQ[b/a] && PosQ[d/c] &&  !SimplerSqrtQ[b/a, d/c]
```

rule 403
```
Int[((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.)*((e_) + (f_.)*(
x_)^2), x_Symbol] :> Simp[f*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^q/(b*(2*(p +
 q + 1) + 1))), x] + Simp[1/(b*(2*(p + q + 1) + 1))   Int[(a + b*x^2)^p*(c
+ d*x^2)^(q - 1)*Simp[c*(b*e - a*f + b*e*2*(p + q + 1)) + (d*(b*e - a*f) +
f*2*q*(b*c - a*d) + b*d*e*2*(p + q + 1))*x^2, x], x], x] /; FreeQ[{a, b, c,
 d, e, f, p}, x] && GtQ[q, 0] && NeQ[2*(p + q + 1) + 1, 0]
```

rule 406
```
Int[((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.)*((e_) + (f_.)*(
x_)^2), x_Symbol] :> Simp[e   Int[(a + b*x^2)^p*(c + d*x^2)^q, x], x] + Sim
p[f   Int[x^2*(a + b*x^2)^p*(c + d*x^2)^q, x], x] /; FreeQ[{a, b, c, d, e,
f, p, q}, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3671
```
Int[cos[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(
p_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Simp[ff*(Sqrt[
Cos[e + f*x]^2]/(f*Cos[e + f*x]))   Subst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a
 + b*ff^2*x^2)^p, x], x, Sin[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x]
 && IntegerQ[m/2] &&  !IntegerQ[p]
```

Maple [A] (verified)

Time = 6.22 (sec) , antiderivative size = 521, normalized size of antiderivative = 1.74

method	result
default	$3\sqrt{-\frac{b}{a}}\, b^2\cosh(fx+e)^6\sinh(fx+e)+4\sqrt{-\frac{b}{a}}\, ab\cosh(fx+e)^4\sinh(fx+e)+\left(\sqrt{-\frac{b}{a}}\, a^2+2\sqrt{-\frac{b}{a}}\, ab-3\sqrt{-\frac{b}{a}}\, b^2\right)\cosh(fx+e)^2\sinh(f$

input
```
int(cosh(f*x+e)^4*(a+b*sinh(f*x+e)^2)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/15*(3*(-b/a)^(1/2)*b^2*cosh(f*x+e)^6*sinh(f*x+e)+4*(-b/a)^(1/2)*a*b*cosh
(f*x+e)^4*sinh(f*x+e)+((-b/a)^(1/2)*a^2+2*(-b/a)^(1/2)*a*b-3*(-b/a)^(1/2)*
b^2)*cosh(f*x+e)^2*sinh(f*x+e)+a^2*(b/a*cosh(f*x+e)^2+(a-b)/a)^(1/2)*(cosh
(f*x+e)^2)^(1/2)*EllipticF(sinh(f*x+e)*(-b/a)^(1/2),(1/b*a)^(1/2))+2*a*(b/
a*cosh(f*x+e)^2+(a-b)/a)^(1/2)*(cosh(f*x+e)^2)^(1/2)*EllipticF(sinh(f*x+e)
*(-b/a)^(1/2),(1/b*a)^(1/2))*b-3*(b/a*cosh(f*x+e)^2+(a-b)/a)^(1/2)*(cosh(f
*x+e)^2)^(1/2)*EllipticF(sinh(f*x+e)*(-b/a)^(1/2),(1/b*a)^(1/2))*b^2-2*(b/
a*cosh(f*x+e)^2+(a-b)/a)^(1/2)*(cosh(f*x+e)^2)^(1/2)*EllipticE(sinh(f*x+e)
*(-b/a)^(1/2),(1/b*a)^(1/2))*a^2+7*(b/a*cosh(f*x+e)^2+(a-b)/a)^(1/2)*(cosh
(f*x+e)^2)^(1/2)*EllipticE(sinh(f*x+e)*(-b/a)^(1/2),(1/b*a)^(1/2))*a*b+3*(
b/a*cosh(f*x+e)^2+(a-b)/a)^(1/2)*(cosh(f*x+e)^2)^(1/2)*EllipticE(sinh(f*x+
e)*(-b/a)^(1/2),(1/b*a)^(1/2))*b^2)/b/(-b/a)^(1/2)/cosh(f*x+e)/(a+b*sinh(f
*x+e)^2)^(1/2)/f
```

Fricas [F]

$$\int \cosh^4(e+fx)\sqrt{a+b\sinh^2(e+fx)}\,dx = \int \sqrt{b\sinh\left(fx+e\right)^2+a}\cosh\left(fx+e\right)^4\,dx$$

input
```
integrate(cosh(f*x+e)^4*(a+b*sinh(f*x+e)^2)^(1/2),x, algorithm="fricas")
```

output
```
integral(sqrt(b*sinh(f*x + e)^2 + a)*cosh(f*x + e)^4, x)
```

Sympy [F(-1)]

Timed out.

$$\int \cosh^4(e+fx)\sqrt{a+b\sinh^2(e+fx)}\,dx = \text{Timed out}$$

input
```
integrate(cosh(f*x+e)**4*(a+b*sinh(f*x+e)**2)**(1/2),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \cosh^4(e+fx)\sqrt{a+b\sinh^2(e+fx)}\,dx=\int \sqrt{b\sinh{(fx+e)}^2+a}\cosh{(fx+e)}^4\,dx$$

input
```
integrate(cosh(f*x+e)^4*(a+b*sinh(f*x+e)^2)^(1/2),x, algorithm="maxima")
```

output
```
integrate(sqrt(b*sinh(f*x + e)^2 + a)*cosh(f*x + e)^4, x)
```

Giac [F]

$$\int \cosh^4(e+fx)\sqrt{a+b\sinh^2(e+fx)}\,dx=\int \sqrt{b\sinh{(fx+e)}^2+a}\cosh{(fx+e)}^4\,dx$$

input
```
integrate(cosh(f*x+e)^4*(a+b*sinh(f*x+e)^2)^(1/2),x, algorithm="giac")
```

output
```
integrate(sqrt(b*sinh(f*x + e)^2 + a)*cosh(f*x + e)^4, x)
```

Mupad [F(-1)]

Timed out.

$$\int \cosh^4(e+fx)\sqrt{a+b\sinh^2(e+fx)}\,dx=\int {\cosh(e+f\,x)}^4\,\sqrt{b\,{\sinh\left(e+f\,x\right)}^2+a}\,dx$$

input
```
int(cosh(e + f*x)^4*(a + b*sinh(e + f*x)^2)^(1/2),x)
```

output
```
int(cosh(e + f*x)^4*(a + b*sinh(e + f*x)^2)^(1/2), x)
```

Reduce [F]

$$\int \cosh^4(e+fx)\sqrt{a+b\sinh^2(e+fx)}\,dx = \int \sqrt{\sinh(fx+e)^2 b+a}\,\cosh(fx+e)^4\,dx$$

input
```
int(cosh(f*x+e)^4*(a+b*sinh(f*x+e)^2)^(1/2),x)
```

output
```
int(sqrt(sinh(e + f*x)**2*b + a)*cosh(e + f*x)**4,x)
```

3.322 $\int \cosh^2(e+fx)\sqrt{a+b\sinh^2(e+fx)}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 223

$$\begin{aligned}
&\int \cosh^2(e+fx)\sqrt{a+b\sinh^2(e+fx)}\,dx \\
&= \frac{\cosh(e+fx)\sinh(e+fx)\sqrt{a+b\sinh^2(e+fx)}}{3f} \\
&\quad - \frac{(a+b)E\left(\arctan(\sinh(e+fx))\,\middle|\,1-\frac{b}{a}\right)\operatorname{sech}(e+fx)\sqrt{a+b\sinh^2(e+fx)}}{3bf\sqrt{\frac{\operatorname{sech}^2(e+fx)(a+b\sinh^2(e+fx))}{a}}} \\
&\quad + \frac{2\,\text{EllipticF}\left(\arctan(\sinh(e+fx)),1-\frac{b}{a}\right)\operatorname{sech}(e+fx)\sqrt{a+b\sinh^2(e+fx)}}{3f\sqrt{\frac{\operatorname{sech}^2(e+fx)(a+b\sinh^2(e+fx))}{a}}} \\
&\quad + \frac{(a+b)\sqrt{a+b\sinh^2(e+fx)}\tanh(e+fx)}{3bf}
\end{aligned}$$

output
```
1/3*cosh(f*x+e)*sinh(f*x+e)*(a+b*sinh(f*x+e)^2)^(1/2)/f-1/3*(a+b)*Elliptic
E(sinh(f*x+e)/(1+sinh(f*x+e)^2)^(1/2),(1-b/a)^(1/2))*sech(f*x+e)*(a+b*sinh
(f*x+e)^2)^(1/2)/b/f/(sech(f*x+e)^2*(a+b*sinh(f*x+e)^2)/a)^(1/2)+2/3*Inver
seJacobiAM(arctan(sinh(f*x+e)),(1-b/a)^(1/2))*sech(f*x+e)*(a+b*sinh(f*x+e)
^2)^(1/2)/f/(sech(f*x+e)^2*(a+b*sinh(f*x+e)^2)/a)^(1/2)+1/3*(a+b)*(a+b*sin
h(f*x+e)^2)^(1/2)*tanh(f*x+e)/b/f
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.73 (sec) , antiderivative size = 168, normalized size of antiderivative = 0.75

$$\int \cosh^2(e+fx)\sqrt{a+b\sinh^2(e+fx)}\,dx$$
$$= \frac{-2i\sqrt{2}a(a+b)\sqrt{\frac{2a-b+b\cosh(2(e+fx))}{a}}E\left(i(e+fx)\,\middle|\,\frac{b}{a}\right)+2i\sqrt{2}a(a-b)\sqrt{\frac{2a-b+b\cosh(2(e+fx))}{a}}\,\text{EllipticF}\left(i(e+\right.}{6bf\sqrt{4a-2b+2b\cosh(2(e+fx))}}$$

input
```
Integrate[Cosh[e + f*x]^2*Sqrt[a + b*Sinh[e + f*x]^2],x]
```

output
```
((-2*I)*Sqrt[2]*a*(a + b)*Sqrt[(2*a - b + b*Cosh[2*(e + f*x)])/a]*Elliptic
E[I*(e + f*x), b/a] + (2*I)*Sqrt[2]*a*(a - b)*Sqrt[(2*a - b + b*Cosh[2*(e
+ f*x)])/a]*EllipticF[I*(e + f*x), b/a] + b*(2*a - b + b*Cosh[2*(e + f*x)]
)*Sinh[2*(e + f*x)])/(6*b*f*Sqrt[4*a - 2*b + 2*b*Cosh[2*(e + f*x)]])
```

Rubi [A] (verified)

Time = 0.48 (sec) , antiderivative size = 276, normalized size of antiderivative = 1.24, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.320$, Rules used = {3042, 3671, 319, 27, 406, 320, 388, 313}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cosh^2(e+fx)\sqrt{a+b\sinh^2(e+fx)}\,dx$$
$$\Big\downarrow \text{3042}$$
$$\int \cos(ie+ifx)^2\sqrt{a-b\sin(ie+ifx)^2}dx$$
$$\Big\downarrow \text{3671}$$
$$\frac{\sqrt{\cosh^2(e+fx)}\text{sech}(e+fx)\int\sqrt{\sinh^2(e+fx)+1}\sqrt{b\sinh^2(e+fx)+a}d\sinh(e+fx)}{f}$$

$$\Big\downarrow \; 319$$

$$\frac{\sqrt{\cosh^2(e+fx)}\operatorname{sech}(e+fx)\left(\frac{2}{3}\int \frac{(a+b)\sinh^2(e+fx)+2a}{2\sqrt{\sinh^2(e+fx)+1}\sqrt{b\sinh^2(e+fx)+a}}d\sinh(e+fx)+\frac{1}{3}\sqrt{\sinh^2(e+fx)+1}\sinh(e\right.}{f}$$

$$\Big\downarrow \; 27$$

$$\frac{\sqrt{\cosh^2(e+fx)}\operatorname{sech}(e+fx)\left(\frac{1}{3}\int \frac{(a+b)\sinh^2(e+fx)+2a}{\sqrt{\sinh^2(e+fx)+1}\sqrt{b\sinh^2(e+fx)+a}}d\sinh(e+fx)+\frac{1}{3}\sqrt{\sinh^2(e+fx)+1}\sinh(e\right.}{f}$$

$$\Big\downarrow \; 406$$

$$\frac{\sqrt{\cosh^2(e+fx)}\operatorname{sech}(e+fx)\left(\frac{1}{3}\left(2a\int \frac{1}{\sqrt{\sinh^2(e+fx)+1}\sqrt{b\sinh^2(e+fx)+a}}d\sinh(e+fx)+(a+b)\int \frac{\sinh^2(e}{\sqrt{\sinh^2(e+fx)+1}\sqrt{}}\right.\right.}{f}$$

$$\Big\downarrow \; 320$$

$$\frac{\sqrt{\cosh^2(e+fx)}\operatorname{sech}(e+fx)\left(\frac{1}{3}\left((a+b)\int \frac{\sinh^2(e+fx)}{\sqrt{\sinh^2(e+fx)+1}\sqrt{b\sinh^2(e+fx)+a}}d\sinh(e+fx)+\frac{2\sqrt{a+b\sinh^2(e+fx)}\,\text{Ellip}}{\sqrt{\sinh^2(e+fx)}}\right.\right.}{f}$$

$$\Big\downarrow \; 388$$

$$\frac{\sqrt{\cosh^2(e+fx)}\operatorname{sech}(e+fx)\left(\frac{1}{3}\left((a+b)\left(\frac{\sinh(e+fx)\sqrt{a+b\sinh^2(e+fx)}}{b\sqrt{\sinh^2(e+fx)+1}}-\frac{\int \frac{\sqrt{b\sinh^2(e+fx)+a}}{\left(\sinh^2(e+fx)+1\right)^{3/2}}d\sinh(e+fx)}{b}\right)+\frac{2\sqrt{a+b\sinh}}{}\right.\right.}{f}$$

$$\Big\downarrow \; 313$$

$$\sqrt{\cosh^2(e+fx)}\operatorname{sech}(e+fx)\left(\frac{1}{3}\left(\frac{2\sqrt{a+b\sinh^2(e+fx)}\,\text{EllipticF}\left(\arctan(\sinh(e+fx)),1-\frac{b}{a}\right)}{\sqrt{\sinh^2(e+fx)+1}\sqrt{\frac{a+b\sinh^2(e+fx)}{a\left(\sinh^2(e+fx)+1\right)}}}+(a+b)\left(\frac{\sinh(e+fx)\sqrt{a+b\sinh^2}}{b\sqrt{\sinh^2(e+fx)+}}\right.\right.\right.$$

input
```
Int[Cosh[e + f*x]^2*Sqrt[a + b*Sinh[e + f*x]^2],x]
```

output
```
(Sqrt[Cosh[e + f*x]^2*Sech[e + f*x]*((Sinh[e + f*x]*Sqrt[1 + Sinh[e + f*x
]^2]*Sqrt[a + b*Sinh[e + f*x]^2])/3 + ((2*EllipticF[ArcTan[Sinh[e + f*x]],
 1 - b/a]*Sqrt[a + b*Sinh[e + f*x]^2])/(Sqrt[1 + Sinh[e + f*x]^2]*Sqrt[(a
+ b*Sinh[e + f*x]^2)/(a*(1 + Sinh[e + f*x]^2))]) + (a + b)*((Sinh[e + f*x]
*Sqrt[a + b*Sinh[e + f*x]^2])/(b*Sqrt[1 + Sinh[e + f*x]^2]) - (EllipticE[A
rcTan[Sinh[e + f*x]], 1 - b/a]*Sqrt[a + b*Sinh[e + f*x]^2])/(b*Sqrt[1 + Si
nh[e + f*x]^2]*Sqrt[(a + b*Sinh[e + f*x]^2)/(a*(1 + Sinh[e + f*x]^2))])))/
3))/f
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 313
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/((c_) + (d_.)*(x_)^2)^(3/2), x_Symbol] :> Sim
p[(Sqrt[a + b*x^2]/(c*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(c
+ d*x^2)))]))*EllipticE[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; FreeQ
[{a, b, c, d}, x] && PosQ[b/a] && PosQ[d/c]
```

rule 319
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_), x_Symbol] :> Sim
p[x*(a + b*x^2)^p*((c + d*x^2)^q/(2*(p + q) + 1)), x] + Simp[2/(2*(p + q) +
 1)   Int[(a + b*x^2)^(p - 1)*(c + d*x^2)^(q - 1)*Simp[a*c*(p + q) + (q*(b*
c - a*d) + a*d*(p + q))*x^2, x], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*
c - a*d, 0] && GtQ[q, 0] && GtQ[p, 0] && IntBinomialQ[a, b, c, d, 2, p, q,
x]
```

rule 320
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(Sqrt[a + b*x^2]/(a*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(
c + d*x^2)))]))*EllipticF[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; Fre
eQ[{a, b, c, d}, x] && PosQ[d/c] && PosQ[b/a] &&  !SimplerSqrtQ[b/a, d/c]
```

rule 388
```
Int[(x_)^2/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol]
 :> Simp[x*(Sqrt[a + b*x^2]/(b*Sqrt[c + d*x^2])), x] - Simp[c/b   Int[Sqrt[
a + b*x^2]/(c + d*x^2)^(3/2), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
 a*d, 0] && PosQ[b/a] && PosQ[d/c] &&  !SimplerSqrtQ[b/a, d/c]
```

rule 406
```
Int[((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.)*((e_) + (f_.)*(
x_)^2), x_Symbol] :> Simp[e   Int[(a + b*x^2)^p*(c + d*x^2)^q, x], x] + Sim
p[f   Int[x^2*(a + b*x^2)^p*(c + d*x^2)^q, x], x] /; FreeQ[{a, b, c, d, e,
f, p, q}, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3671
```
Int[cos[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(
p_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Simp[ff*(Sqrt[
Cos[e + f*x]^2]/(f*Cos[e + f*x]))   Subst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a
 + b*ff^2*x^2)^p, x], x, Sin[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x]
 && IntegerQ[m/2] &&  !IntegerQ[p]
```

Maple [A] (verified)

Time = 3.96 (sec) , antiderivative size = 351, normalized size of antiderivative = 1.57

method	result
default	$\frac{\sqrt{-\frac{b}{a}}\, b\cosh(fx+e)^{4}\sinh(fx+e)+\sqrt{-\frac{b}{a}}\, a\cosh(fx+e)^{2}\sinh(fx+e)-\sqrt{-\frac{b}{a}}\, b\cosh(fx+e)^{2}\sinh(fx+e)+a\sqrt{\frac{b\cosh(fx+e)^{2}}{a}+\frac{a-b}{a}}\,\sqrt{}}{}$

input
```
int(cosh(f*x+e)^2*(a+b*sinh(f*x+e)^2)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/3*((-b/a)^(1/2)*b*cosh(f*x+e)^4*sinh(f*x+e)+(-b/a)^(1/2)*a*cosh(f*x+e)^2
*sinh(f*x+e)-(-b/a)^(1/2)*b*cosh(f*x+e)^2*sinh(f*x+e)+a*(b/a*cosh(f*x+e)^2
+(a-b)/a)^(1/2)*(cosh(f*x+e)^2)^(1/2)*EllipticF(sinh(f*x+e)*(-b/a)^(1/2),(
1/b*a)^(1/2))-(b/a*cosh(f*x+e)^2+(a-b)/a)^(1/2)*(cosh(f*x+e)^2)^(1/2)*Elli
pticF(sinh(f*x+e)*(-b/a)^(1/2),(1/b*a)^(1/2))*b+(b/a*cosh(f*x+e)^2+(a-b)/a
)^(1/2)*(cosh(f*x+e)^2)^(1/2)*EllipticE(sinh(f*x+e)*(-b/a)^(1/2),(1/b*a)^(
1/2))*a+(b/a*cosh(f*x+e)^2+(a-b)/a)^(1/2)*(cosh(f*x+e)^2)^(1/2)*EllipticE(
sinh(f*x+e)*(-b/a)^(1/2),(1/b*a)^(1/2))*b)/(-b/a)^(1/2)/cosh(f*x+e)/(a+b*s
inh(f*x+e)^2)^(1/2)/f
```

Fricas [F]

$$\int \cosh^2(e+fx)\sqrt{a+b\sinh^2(e+fx)}\,dx = \int \sqrt{b\sinh\left(fx+e\right)^2+a}\cosh\left(fx+e\right)^2\,dx$$

input
```
integrate(cosh(f*x+e)^2*(a+b*sinh(f*x+e)^2)^(1/2),x, algorithm="fricas")
```

output
```
integral(sqrt(b*sinh(f*x + e)^2 + a)*cosh(f*x + e)^2, x)
```

Sympy [F]

$$\int \cosh^2(e+fx)\sqrt{a+b\sinh^2(e+fx)}\,dx = \int \sqrt{a+b\sinh^2\left(e+fx\right)}\cosh^2\left(e+fx\right)dx$$

input
```
integrate(cosh(f*x+e)**2*(a+b*sinh(f*x+e)**2)**(1/2),x)
```

output
```
Integral(sqrt(a + b*sinh(e + f*x)**2)*cosh(e + f*x)**2, x)
```

Maxima [F]

$$\int \cosh^2(e+fx)\sqrt{a+b\sinh^2(e+fx)}\,dx = \int \sqrt{b\sinh(fx+e)^2+a}\cosh(fx+e)^2\,dx$$

input
```
integrate(cosh(f*x+e)^2*(a+b*sinh(f*x+e)^2)^(1/2),x, algorithm="maxima")
```

output
```
integrate(sqrt(b*sinh(f*x + e)^2 + a)*cosh(f*x + e)^2, x)
```

Giac [F]

$$\int \cosh^2(e+fx)\sqrt{a+b\sinh^2(e+fx)}\,dx = \int \sqrt{b\sinh(fx+e)^2+a}\cosh(fx+e)^2\,dx$$

input
```
integrate(cosh(f*x+e)^2*(a+b*sinh(f*x+e)^2)^(1/2),x, algorithm="giac")
```

output
```
integrate(sqrt(b*sinh(f*x + e)^2 + a)*cosh(f*x + e)^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int \cosh^2(e+fx)\sqrt{a+b\sinh^2(e+fx)}\,dx = \int \cosh(e+f\,x)^2\,\sqrt{b\sinh(e+f\,x)^2+a}\,dx$$

input
```
int(cosh(e + f*x)^2*(a + b*sinh(e + f*x)^2)^(1/2),x)
```

output
```
int(cosh(e + f*x)^2*(a + b*sinh(e + f*x)^2)^(1/2), x)
```

Reduce [F]

$$\int \cosh^2(e+fx)\sqrt{a+b\sinh^2(e+fx)}\,dx=\int \sqrt{\sinh{(fx+e)}^2\,b+a}\,\cosh{(fx+e)}^2\,dx$$

input
```
int(cosh(f*x+e)^2*(a+b*sinh(f*x+e)^2)^(1/2),x)
```

output
```
int(sqrt(sinh(e + f*x)**2*b + a)*cosh(e + f*x)**2,x)
```

3.323 $\int \sqrt{a+b\sinh^2(e+fx)}\,dx$

Optimal result

Integrand size = 16, antiderivative size = 61

$$\int \sqrt{a+b\sinh^2(e+fx)}\,dx = -\frac{iE\left(ie+ifx\middle|\frac{b}{a}\right)\sqrt{a+b\sinh^2(e+fx)}}{f\sqrt{\frac{a+b\sinh^2(e+fx)}{a}}}$$

output
```
-I*EllipticE(sin(I*e+I*f*x),(b/a)^(1/2))*(a+b*sinh(f*x+e)^2)^(1/2)/f/((a+b
*sinh(f*x+e)^2)/a)^(1/2)
```

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 69, normalized size of antiderivative = 1.13

$$\int \sqrt{a+b\sinh^2(e+fx)}\,dx = -\frac{ia\sqrt{\frac{2a-b+b\cosh(2(e+fx))}{a}}E\left(i(e+fx)\middle|\frac{b}{a}\right)}{f\sqrt{2a-b+b\cosh(2(e+fx))}}$$

input
```
Integrate[Sqrt[a + b*Sinh[e + f*x]^2],x]
```

output
```
((-I)*a*Sqrt[(2*a - b + b*Cosh[2*(e + f*x)])/a]*EllipticE[I*(e + f*x), b/a
])/(f*Sqrt[2*a - b + b*Cosh[2*(e + f*x)]])
```

Rubi [A] (verified)

Time = 0.34 (sec) , antiderivative size = 60, normalized size of antiderivative = 0.98, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {3042, 3657, 3042, 3656}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{a+b\sinh^2(e+fx)}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \sqrt{a-b\sin(ie+ifx)^2}dx$$

$$\Big\downarrow \text{3657}$$

$$\frac{\sqrt{a+b\sinh^2(e+fx)}\int \sqrt{\frac{b\sinh^2(e+fx)}{a}+1}dx}{\sqrt{\frac{b\sinh^2(e+fx)}{a}+1}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\sqrt{a+b\sinh^2(e+fx)}\int \sqrt{1-\frac{b\sin(ie+ifx)^2}{a}}dx}{\sqrt{\frac{b\sinh^2(e+fx)}{a}+1}}$$

$$\Big\downarrow \text{3656}$$

$$-\frac{i\sqrt{a+b\sinh^2(e+fx)}E\left(ie+ifx\middle|\frac{b}{a}\right)}{f\sqrt{\frac{b\sinh^2(e+fx)}{a}+1}}$$

input
```
Int[Sqrt[a + b*Sinh[e + f*x]^2],x]
```

output
```
((-I)*EllipticE[I*e + I*f*x, b/a]*Sqrt[a + b*Sinh[e + f*x]^2])/(f*Sqrt[1 +
 (b*Sinh[e + f*x]^2)/a])
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3656
```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2], x_Symbol] :> Simp[(Sqrt[a
]/f)*EllipticE[e + f*x, -b/a], x] /; FreeQ[{a, b, e, f}, x] && GtQ[a, 0]
```

rule 3657
```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2], x_Symbol] :> Simp[Sqrt[a
+ b*Sin[e + f*x]^2]/Sqrt[1 + b*(Sin[e + f*x]^2/a)]   Int[Sqrt[1 + (b*Sin[e
+ f*x]^2)/a], x], x] /; FreeQ[{a, b, e, f}, x] &&  !GtQ[a, 0]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 139 vs. $2(57) = 114$.

Time = 2.05 (sec) , antiderivative size = 140, normalized size of antiderivative = 2.30

method	result
default	$\frac{\sqrt{\frac{a+b\sinh(fx+e)^2}{a}}\,\sqrt{\frac{\cosh(2fx+2e)}{2}+\frac{1}{2}}\,\left(a\,\text{EllipticF}\left(\sinh(fx+e)\sqrt{-\frac{b}{a}},\sqrt{\frac{a}{b}}\right)-b\,\text{EllipticF}\left(\sinh(fx+e)\sqrt{-\frac{b}{a}},\sqrt{\frac{a}{b}}\right)+b\,\text{EllipticE}\left(\text{si}\right.\right.}{\sqrt{-\frac{b}{a}}\,\cosh(fx+e)\sqrt{a+b\sinh(fx+e)^2}\,f}$

input
```
int((a+b*sinh(f*x+e)^2)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
((a+b*sinh(f*x+e)^2)/a)^(1/2)*(cosh(f*x+e)^2)^(1/2)*(a*EllipticF(sinh(f*x+
e)*(-b/a)^(1/2),(1/b*a)^(1/2))-b*EllipticF(sinh(f*x+e)*(-b/a)^(1/2),(1/b*a
)^(1/2))+b*EllipticE(sinh(f*x+e)*(-b/a)^(1/2),(1/b*a)^(1/2)))/(-b/a)^(1/2)
/cosh(f*x+e)/(a+b*sinh(f*x+e)^2)^(1/2)/f
```

Fricas [F]

$$\int \sqrt{a+b\sinh^2(e+fx)}\,dx=\int \sqrt{b\sinh{(fx+e)}^2+a}\,dx$$

input
```
integrate((a+b*sinh(f*x+e)^2)^(1/2),x, algorithm="fricas")
```

output
```
integral(sqrt(b*sinh(f*x + e)^2 + a), x)
```

Sympy [F]

$$\int \sqrt{a+b\sinh^2(e+fx)}\,dx=\int \sqrt{a+b\sinh^2{(e+fx)}}\,dx$$

input
```
integrate((a+b*sinh(f*x+e)**2)**(1/2),x)
```

output
```
Integral(sqrt(a + b*sinh(e + f*x)**2), x)
```

Maxima [F]

$$\int \sqrt{a+b\sinh^2(e+fx)}\,dx=\int \sqrt{b\sinh{(fx+e)}^2+a}\,dx$$

input
```
integrate((a+b*sinh(f*x+e)^2)^(1/2),x, algorithm="maxima")
```

output
```
integrate(sqrt(b*sinh(f*x + e)^2 + a), x)
```

Giac [F]

$$\int \sqrt{a+b\sinh^2(e+fx)}\,dx=\int \sqrt{b\sinh\left(fx+e\right)^2+a}\,dx$$

input
```
integrate((a+b*sinh(f*x+e)^2)^(1/2),x, algorithm="giac")
```

output
```
integrate(sqrt(b*sinh(f*x + e)^2 + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int \sqrt{a+b\sinh^2(e+fx)}\,dx=\int \sqrt{b\sinh\left(e+f\,x\right)^2+a}\,dx$$

input
```
int((a + b*sinh(e + f*x)^2)^(1/2),x)
```

output
```
int((a + b*sinh(e + f*x)^2)^(1/2), x)
```

Reduce [F]

$$\int \sqrt{a+b\sinh^2(e+fx)}\,dx=\int \sqrt{\sinh\left(fx+e\right)^2b+a}dx$$

input
```
int((a+b*sinh(f*x+e)^2)^(1/2),x)
```

output
```
int(sqrt(sinh(e + f*x)**2*b + a),x)
```

3.324 $\int \operatorname{sech}^2(e+fx)\sqrt{a+b\sinh^2(e+fx)}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 70

$$\int \operatorname{sech}^2(e+fx)\sqrt{a+b\sinh^2(e+fx)}\,dx = \frac{E\left(\arctan(\sinh(e+fx))\,\middle|\,1-\frac{b}{a}\right)\operatorname{sech}(e+fx)\sqrt{a+b\sinh^2(e+fx)}}{f\sqrt{\frac{\operatorname{sech}^2(e+fx)(a+b\sinh^2(e+fx))}{a}}}$$

output
```
EllipticE(sinh(f*x+e)/(1+sinh(f*x+e)^2)^(1/2),(1-b/a)^(1/2))*sech(f*x+e)*(
a+b*sinh(f*x+e)^2)^(1/2)/f/(sech(f*x+e)^2*(a+b*sinh(f*x+e)^2)/a)^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.55 (sec) , antiderivative size = 148, normalized size of antiderivative = 2.11

$$\int \operatorname{sech}^2(e+fx)\sqrt{a+b\sinh^2(e+fx)}\,dx = \frac{2ia\sqrt{\frac{2a-b+b\cosh(2(e+fx))}{a}}E\left(i(e+fx)\,\middle|\,\frac{b}{a}\right)-2ia\sqrt{\frac{2a-b+b\cosh(2(e+fx))}{a}}\operatorname{EllipticF}\left(i(e+fx),\frac{b}{a}\right)+\sqrt{2}(2a-b}{2f\sqrt{2a-b+b\cosh(2(e+fx))}}$$

input
```
Integrate[Sech[e + f*x]^2*Sqrt[a + b*Sinh[e + f*x]^2],x]
```

output
```
((2*I)*a*Sqrt[(2*a - b + b*Cosh[2*(e + f*x)])/a]*EllipticE[I*(e + f*x), b/
a] - (2*I)*a*Sqrt[(2*a - b + b*Cosh[2*(e + f*x)])/a]*EllipticF[I*(e + f*x)
, b/a] + Sqrt[2]*(2*a - b + b*Cosh[2*(e + f*x)])*Tanh[e + f*x])/(2*f*Sqrt[
2*a - b + b*Cosh[2*(e + f*x)]])
```

Rubi [A] (verified)

Time = 0.34 (sec) , antiderivative size = 100, normalized size of antiderivative = 1.43, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.120$, Rules used = {3042, 3671, 313}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \operatorname{sech}^2(e+fx)\sqrt{a+b\sinh^2(e+fx)}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\sqrt{a-b\sin(ie+ifx)^2}}{\cos(ie+ifx)^2}dx$$

$$\Big\downarrow \text{3671}$$

$$\frac{\sqrt{\cosh^2(e+fx)}\operatorname{sech}(e+fx)\int \frac{\sqrt{b\sinh^2(e+fx)+a}}{\left(\sinh^2(e+fx)+1\right)^{3/2}}d\sinh(e+fx)}{f}$$

$$\Big\downarrow \text{313}$$

$$\frac{\sqrt{\cosh^2(e+fx)}\operatorname{sech}(e+fx)\sqrt{a+b\sinh^2(e+fx)}E\left(\arctan(\sinh(e+fx))\left|1-\frac{b}{a}\right.\right)}{f\sqrt{\sinh^2(e+fx)+1}\sqrt{\frac{a+b\sinh^2(e+fx)}{a\left(\sinh^2(e+fx)+1\right)}}}$$

input
```
Int[Sech[e + f*x]^2*Sqrt[a + b*Sinh[e + f*x]^2],x]
```

output
```
(Sqrt[Cosh[e + f*x]^2]*EllipticE[ArcTan[Sinh[e + f*x]], 1 - b/a]*Sech[e +
f*x]*Sqrt[a + b*Sinh[e + f*x]^2])/(f*Sqrt[1 + Sinh[e + f*x]^2]*Sqrt[(a + b
*Sinh[e + f*x]^2)/(a*(1 + Sinh[e + f*x]^2))])
```

Defintions of rubi rules used

rule 313
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/((c_) + (d_.)*(x_)^2)^(3/2), x_Symbol] :> Sim
p[(Sqrt[a + b*x^2]/(c*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(c
+ d*x^2)))]))*EllipticE[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; FreeQ
[{a, b, c, d}, x] && PosQ[b/a] && PosQ[d/c]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3671
```
Int[cos[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(
p_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Simp[ff*(Sqrt[
Cos[e + f*x]^2]/(f*Cos[e + f*x]))   Subst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a
 + b*ff^2*x^2)^p, x], x, Sin[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x]
 && IntegerQ[m/2] &&  !IntegerQ[p]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 176 vs. $2(80) = 160$.

Time = 2.54 (sec) , antiderivative size = 177, normalized size of antiderivative = 2.53

method	result
default	$\frac{\sqrt{-\frac{b}{a}}\, b \sinh(fx+e)^3+b\sqrt{\frac{a+b\sinh(fx+e)^2}{a}}\,\sqrt{\frac{\cosh(2fx+2e)}{2}+\frac{1}{2}}\ \text{EllipticF}\left(\sinh(fx+e)\sqrt{-\frac{b}{a}}, \sqrt{\frac{a}{b}}\right)-b\sqrt{\frac{a+b\sinh(fx+e)^2}{a}}\,\sqrt{\frac{\cosh(2fx+}{2}}}{\sqrt{-\frac{b}{a}}\,\cosh(fx+e)\sqrt{a+b\sinh(fx+e)^2}\, f}$

input
```
int(sech(f*x+e)^2*(a+b*sinh(f*x+e)^2)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
((-b/a)^(1/2)*b*sinh(f*x+e)^3+b*((a+b*sinh(f*x+e)^2)/a)^(1/2)*(cosh(f*x+e)
^2)^(1/2)*EllipticF(sinh(f*x+e)*(-b/a)^(1/2),(1/b*a)^(1/2))-b*((a+b*sinh(f
*x+e)^2)/a)^(1/2)*(cosh(f*x+e)^2)^(1/2)*EllipticE(sinh(f*x+e)*(-b/a)^(1/2)
,(1/b*a)^(1/2))+(-b/a)^(1/2)*a*sinh(f*x+e))/(-b/a)^(1/2)/cosh(f*x+e)/(a+b*
sinh(f*x+e)^2)^(1/2)/f
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 535 vs. $2(80) = 160$.

Time = 0.11 (sec) , antiderivative size = 535, normalized size of antiderivative = 7.64

$$\int \operatorname{sech}^2(e+fx)\sqrt{a+b\sinh^2(e+fx)}\,dx =$$

$$-\frac{4\left(b\cosh\left(fx+e\right)^2+2\,b\cosh\left(fx+e\right)\sinh\left(fx+e\right)+b\sinh\left(fx+e\right)^2+b\right)\sqrt{b}\sqrt{\frac{2\,b\sqrt{\frac{a^2-ab}{b^2}}-2\,a+b}{b}}\sqrt{\frac{a^2-a}{b^2}}}{}$$

input
```
integrate(sech(f*x+e)^2*(a+b*sinh(f*x+e)^2)^(1/2),x, algorithm="fricas")
```

output
```
-(4*(b*cosh(f*x + e)^2 + 2*b*cosh(f*x + e)*sinh(f*x + e) + b*sinh(f*x + e)
^2 + b)*sqrt(b)*sqrt((2*b*sqrt((a^2 - a*b)/b^2) - 2*a + b)/b)*sqrt((a^2 -
a*b)/b^2)*elliptic_f(arcsin(sqrt((2*b*sqrt((a^2 - a*b)/b^2) - 2*a + b)/b)*
(cosh(f*x + e) + sinh(f*x + e))), (8*a^2 - 8*a*b + b^2 + 4*(2*a*b - b^2)*s
qrt((a^2 - a*b)/b^2))/b^2) + ((2*a - b)*cosh(f*x + e)^2 + 2*(2*a - b)*cosh
(f*x + e)*sinh(f*x + e) + (2*a - b)*sinh(f*x + e)^2 - 2*(b*cosh(f*x + e)^2
 + 2*b*cosh(f*x + e)*sinh(f*x + e) + b*sinh(f*x + e)^2 + b)*sqrt((a^2 - a*
b)/b^2) + 2*a - b)*sqrt(b)*sqrt((2*b*sqrt((a^2 - a*b)/b^2) - 2*a + b)/b)*e
lliptic_e(arcsin(sqrt((2*b*sqrt((a^2 - a*b)/b^2) - 2*a + b)/b)*(cosh(f*x +
 e) + sinh(f*x + e))), (8*a^2 - 8*a*b + b^2 + 4*(2*a*b - b^2)*sqrt((a^2 -
a*b)/b^2))/b^2) - sqrt(2)*(b*cosh(f*x + e) + b*sinh(f*x + e))*sqrt((b*cosh
(f*x + e)^2 + b*sinh(f*x + e)^2 + 2*a - b)/(cosh(f*x + e)^2 - 2*cosh(f*x +
 e)*sinh(f*x + e) + sinh(f*x + e)^2)))/(b*f*cosh(f*x + e)^2 + 2*b*f*cosh(f
*x + e)*sinh(f*x + e) + b*f*sinh(f*x + e)^2 + b*f)
```

Sympy [F]

$$\int \operatorname{sech}^2(e+fx)\sqrt{a+b\sinh^2(e+fx)}\,dx = \int \sqrt{a+b\sinh^2{(e+fx)}}\operatorname{sech}^2{(e+fx)}\,dx$$

input
```
integrate(sech(f*x+e)**2*(a+b*sinh(f*x+e)**2)**(1/2),x)
```

output
```
Integral(sqrt(a + b*sinh(e + f*x)**2)*sech(e + f*x)**2, x)
```

Maxima [F]

$$\int \operatorname{sech}^2(e+fx)\sqrt{a+b\sinh^2(e+fx)}\,dx = \int \sqrt{b\sinh{(fx+e)}^2+a}\operatorname{sech}{(fx+e)}^2\,dx$$

input
```
integrate(sech(f*x+e)^2*(a+b*sinh(f*x+e)^2)^(1/2),x, algorithm="maxima")
```

output
```
integrate(sqrt(b*sinh(f*x + e)^2 + a)*sech(f*x + e)^2, x)
```

Giac [F]

$$\int \operatorname{sech}^2(e+fx)\sqrt{a+b\sinh^2(e+fx)}\,dx = \int \sqrt{b\sinh{(fx+e)}^2+a}\operatorname{sech}{(fx+e)}^2\,dx$$

input
```
integrate(sech(f*x+e)^2*(a+b*sinh(f*x+e)^2)^(1/2),x, algorithm="giac")
```

output
```
integrate(sqrt(b*sinh(f*x + e)^2 + a)*sech(f*x + e)^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int \operatorname{sech}^2(e+fx)\sqrt{a+b\sinh^2(e+fx)}\,dx=\int \frac{\sqrt{b\sinh\left(e+f\,x\right)^2+a}}{\cosh\left(e+f\,x\right)^2}\,dx$$

input
```
int((a + b*sinh(e + f*x)^2)^(1/2)/cosh(e + f*x)^2,x)
```

output
```
int((a + b*sinh(e + f*x)^2)^(1/2)/cosh(e + f*x)^2, x)
```

Reduce [F]

$$\int \operatorname{sech}^2(e+fx)\sqrt{a+b\sinh^2(e+fx)}\,dx=\int \sqrt{\sinh\left(fx+e\right)^2b+a}\,\operatorname{sech}(fx+e)^2\,dx$$

input
```
int(sech(f*x+e)^2*(a+b*sinh(f*x+e)^2)^(1/2),x)
```

output
```
int(sqrt(sinh(e + f*x)**2*b + a)*sech(e + f*x)**2,x)
```

3.325 $\int \operatorname{sech}^4(e+fx)\sqrt{a+b\sinh^2(e+fx)}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 206

$$\begin{aligned}\int \operatorname{sech}^4(e+fx)\sqrt{a+b\sinh^2(e+fx)}\,dx &= \frac{(2a-b)E\left(\arctan(\sinh(e+fx))\left|1-\frac{b}{a}\right.\right)\operatorname{sech}(e+fx)\sqrt{a+b\sinh^2(e+fx)}}{3(a-b)f\sqrt{\frac{\operatorname{sech}^2(e+fx)(a+b\sinh^2(e+fx))}{a}}}\\ &\quad-\frac{b\,\text{EllipticF}\left(\arctan(\sinh(e+fx)),1-\frac{b}{a}\right)\operatorname{sech}(e+fx)\sqrt{a+b\sinh^2(e+fx)}}{3(a-b)f\sqrt{\frac{\operatorname{sech}^2(e+fx)(a+b\sinh^2(e+fx))}{a}}}\\ &\quad+\frac{\operatorname{sech}^2(e+fx)\sqrt{a+b\sinh^2(e+fx)}\tanh(e+fx)}{3f}\end{aligned}$$

output
```
1/3*(2*a-b)*EllipticE(sinh(f*x+e)/(1+sinh(f*x+e)^2)^(1/2),(1-b/a)^(1/2))*s
ech(f*x+e)*(a+b*sinh(f*x+e)^2)^(1/2)/(a-b)/f/(sech(f*x+e)^2*(a+b*sinh(f*x+
e)^2)/a)^(1/2)-1/3*b*InverseJacobiAM(arctan(sinh(f*x+e)),(1-b/a)^(1/2))*se
ch(f*x+e)*(a+b*sinh(f*x+e)^2)^(1/2)/(a-b)/f/(sech(f*x+e)^2*(a+b*sinh(f*x+e
)^2)/a)^(1/2)+1/3*sech(f*x+e)^2*(a+b*sinh(f*x+e)^2)^(1/2)*tanh(f*x+e)/f
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 2.14 (sec) , antiderivative size = 204, normalized size of antiderivative = 0.99

$$\int \operatorname{sech}^4(e+fx)\sqrt{a+b\sinh^2(e+fx)}\,dx$$
$$= \frac{8ia(2a-b)\sqrt{\frac{2a-b+b\cosh(2(e+fx))}{a}}E\left(i(e+fx)\left|\frac{b}{a}\right.\right)-16ia(a-b)\sqrt{\frac{2a-b+b\cosh(2(e+fx))}{a}}\operatorname{EllipticF}\left(i(e+fx),\right.}{24(a-b)f\sqrt{2a-}}$$

input
```
Integrate[Sech[e + f*x]^4*Sqrt[a + b*Sinh[e + f*x]^2],x]
```

output
```
((8*I)*a*(2*a - b)*Sqrt[(2*a - b + b*Cosh[2*(e + f*x)])/a]*EllipticE[I*(e
+ f*x), b/a] - (16*I)*a*(a - b)*Sqrt[(2*a - b + b*Cosh[2*(e + f*x)])/a]*El
lipticF[I*(e + f*x), b/a] + Sqrt[2]*((8*a^2 - 4*b^2)*Cosh[2*(e + f*x)] + (
2*a - b)*(8*a - 5*b + b*Cosh[4*(e + f*x)]))*Sech[e + f*x]^2*Tanh[e + f*x])
/(24*(a - b)*f*Sqrt[2*a - b + b*Cosh[2*(e + f*x)]])
```

Rubi [A] (verified)

Time = 0.45 (sec) , antiderivative size = 249, normalized size of antiderivative = 1.21, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.280$, Rules used = {3042, 3671, 314, 25, 400, 313, 320}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \operatorname{sech}^4(e+fx)\sqrt{a+b\sinh^2(e+fx)}\,dx$$
$$\Big\downarrow \; 3042$$
$$\int \frac{\sqrt{a-b\sin(ie+ifx)^2}}{\cos(ie+ifx)^4}dx$$
$$\Big\downarrow \; 3671$$

$$\frac{\sqrt{\cosh^2(e+fx)}\operatorname{sech}(e+fx)\int\frac{\sqrt{b\sinh^2(e+fx)+a}}{(\sinh^2(e+fx)+1)^{5/2}}d\sinh(e+fx)}{f}$$

$\big\downarrow$ 314

$$\frac{\sqrt{\cosh^2(e+fx)}\operatorname{sech}(e+fx)\left(\frac{\sinh(e+fx)\sqrt{a+b\sinh^2(e+fx)}}{3(\sinh^2(e+fx)+1)^{3/2}}-\frac{1}{3}\int-\frac{b\sinh^2(e+fx)+2a}{(\sinh^2(e+fx)+1)^{3/2}\sqrt{b\sinh^2(e+fx)+a}}d\sinh(e+fx)\right)}{f}$$

$\big\downarrow$ 25

$$\frac{\sqrt{\cosh^2(e+fx)}\operatorname{sech}(e+fx)\left(\frac{1}{3}\int\frac{b\sinh^2(e+fx)+2a}{(\sinh^2(e+fx)+1)^{3/2}\sqrt{b\sinh^2(e+fx)+a}}d\sinh(e+fx)+\frac{\sinh(e+fx)\sqrt{a+b\sinh^2(e+fx)}}{3(\sinh^2(e+fx)+1)^{3/2}}\right)}{f}$$

$\big\downarrow$ 400

$$\frac{\sqrt{\cosh^2(e+fx)}\operatorname{sech}(e+fx)\left(\frac{1}{3}\left(\frac{(2a-b)\int\frac{\sqrt{b\sinh^2(e+fx)+a}}{(\sinh^2(e+fx)+1)^{3/2}}d\sinh(e+fx)}{a-b}-\frac{ab\int\frac{1}{\sqrt{\sinh^2(e+fx)+1}\sqrt{b\sinh^2(e+fx)+a}}d\sinh(e+fx)}{a-b}\right.\right.}{f}$$

$\big\downarrow$ 313

$$\frac{\sqrt{\cosh^2(e+fx)}\operatorname{sech}(e+fx)\left(\frac{1}{3}\left(\frac{(2a-b)\sqrt{a+b\sinh^2(e+fx)}E\left(\arctan(\sinh(e+fx))\middle|1-\frac{b}{a}\right)}{(a-b)\sqrt{\sinh^2(e+fx)+1}\sqrt{\frac{a+b\sinh^2(e+fx)}{a\left(\sinh^2(e+fx)+1\right)}}}-\frac{ab\int\frac{1}{\sqrt{\sinh^2(e+fx)+1}\sqrt{b\sinh^2(e+fx)+}}}{a-b}\right.\right.}{f}$$

$\big\downarrow$ 320

$$\frac{\sqrt{\cosh^2(e+fx)}\operatorname{sech}(e+fx)\left(\frac{1}{3}\left(\frac{(2a-b)\sqrt{a+b\sinh^2(e+fx)}E\left(\arctan(\sinh(e+fx))\middle|1-\frac{b}{a}\right)}{(a-b)\sqrt{\sinh^2(e+fx)+1}\sqrt{\frac{a+b\sinh^2(e+fx)}{a\left(\sinh^2(e+fx)+1\right)}}}-\frac{b\sqrt{a+b\sinh^2(e+fx)}\,\text{EllipticF}\left(\text{arcta}\right.}{(a-b)\sqrt{\sinh^2(e+fx)+1}\sqrt{\frac{a+}{a(s}}}\right.\right.}{f}$$

input
```
Int[Sech[e + f*x]^4*Sqrt[a + b*Sinh[e + f*x]^2],x]
```

output
```
(Sqrt[Cosh[e + f*x]^2]*Sech[e + f*x]*((Sinh[e + f*x]*Sqrt[a + b*Sinh[e + f
*x]^2])/(3*(1 + Sinh[e + f*x]^2)^(3/2)) + (((2*a - b)*EllipticE[ArcTan[Sin
h[e + f*x]], 1 - b/a]*Sqrt[a + b*Sinh[e + f*x]^2])/((a - b)*Sqrt[1 + Sinh[
e + f*x]^2]*Sqrt[(a + b*Sinh[e + f*x]^2)/(a*(1 + Sinh[e + f*x]^2))]) - (b*
EllipticF[ArcTan[Sinh[e + f*x]], 1 - b/a]*Sqrt[a + b*Sinh[e + f*x]^2])/((a
 - b)*Sqrt[1 + Sinh[e + f*x]^2]*Sqrt[(a + b*Sinh[e + f*x]^2)/(a*(1 + Sinh[
e + f*x]^2))]))/3))/f
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 313
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/((c_) + (d_.)*(x_)^2)^(3/2), x_Symbol] :> Sim
p[(Sqrt[a + b*x^2]/(c*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(c
+ d*x^2)))]))*EllipticE[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; FreeQ
[{a, b, c, d}, x] && PosQ[b/a] && PosQ[d/c]
```

rule 314
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_), x_Symbol] :> Sim
p[(-x)*(a + b*x^2)^(p + 1)*((c + d*x^2)^q/(2*a*(p + 1))), x] + Simp[1/(2*a*
(p + 1))   Int[(a + b*x^2)^(p + 1)*(c + d*x^2)^(q - 1)*Simp[c*(2*p + 3) + d
*(2*(p + q + 1) + 1)*x^2, x], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
 a*d, 0] && LtQ[p, -1] && LtQ[0, q, 1] && IntBinomialQ[a, b, c, d, 2, p, q,
 x]
```

rule 320
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(Sqrt[a + b*x^2]/(a*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(
c + d*x^2)))]))*EllipticF[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; Fre
eQ[{a, b, c, d}, x] && PosQ[d/c] && PosQ[b/a] &&  !SimplerSqrtQ[b/a, d/c]
```

rule 400
```
Int[((e_) + (f_.)*(x_)^2)/(Sqrt[(a_) + (b_.)*(x_)^2]*((c_) + (d_.)*(x_)^2)^
(3/2)), x_Symbol] :> Simp[(b*e - a*f)/(b*c - a*d)   Int[1/(Sqrt[a + b*x^2]*
Sqrt[c + d*x^2]), x], x] - Simp[(d*e - c*f)/(b*c - a*d)   Int[Sqrt[a + b*x^
2]/(c + d*x^2)^(3/2), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && PosQ[b/a] &
& PosQ[d/c]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3671
```
Int[cos[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(
p_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Simp[ff*(Sqrt[
Cos[e + f*x]^2]/(f*Cos[e + f*x]))   Subst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a
 + b*ff^2*x^2)^p, x], x, Sin[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x]
 && IntegerQ[m/2] &&  !IntegerQ[p]
```

Maple [A] (verified)

Time = 6.08 (sec) , antiderivative size = 318, normalized size of antiderivative = 1.54

method	result
default	$\frac{\left(2\sqrt{-\frac{b}{a}}\, ab-\sqrt{-\frac{b}{a}}\, b^2\right)\cosh(fx+e)^4\sinh(fx+e)+\left(2\sqrt{-\frac{b}{a}}\, a^2-2\sqrt{-\frac{b}{a}}\, ab\right)\cosh(fx+e)^2\sinh(fx+e)+\sqrt{\frac{b\cosh(fx+e)^2}{a}+\frac{a-b}{a}}\, \sqrt{\mathrm{co}}}{}$
risch	Expression too large to display

input
```
int(sech(f*x+e)^4*(a+b*sinh(f*x+e)^2)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/3*((2*(-b/a)^(1/2)*a*b-(-b/a)^(1/2)*b^2)*cosh(f*x+e)^4*sinh(f*x+e)+(2*(-
b/a)^(1/2)*a^2-2*(-b/a)^(1/2)*a*b)*cosh(f*x+e)^2*sinh(f*x+e)+(b/a*cosh(f*x
+e)^2+(a-b)/a)^(1/2)*(cosh(f*x+e)^2)^(1/2)*b*(a*EllipticF(sinh(f*x+e)*(-b/
a)^(1/2),(1/b*a)^(1/2))-b*EllipticF(sinh(f*x+e)*(-b/a)^(1/2),(1/b*a)^(1/2)
)-2*EllipticE(sinh(f*x+e)*(-b/a)^(1/2),(1/b*a)^(1/2))*a+b*EllipticE(sinh(f
*x+e)*(-b/a)^(1/2),(1/b*a)^(1/2)))*cosh(f*x+e)^2+((-b/a)^(1/2)*a^2-2*(-b/a
)^(1/2)*a*b+(-b/a)^(1/2)*b^2)*sinh(f*x+e))/cosh(f*x+e)^3/(a-b)/(-b/a)^(1/2
)/(a+b*sinh(f*x+e)^2)^(1/2)/f
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2257 vs. $2(206) = 412$.

Time = 0.12 (sec) , antiderivative size = 2257, normalized size of antiderivative = 10.96

$$\int \operatorname{sech}^4(e+fx)\sqrt{a+b\sinh^2(e+fx)}\,dx = \text{Too large to display}$$

input
```
integrate(sech(f*x+e)^4*(a+b*sinh(f*x+e)^2)^(1/2),x, algorithm="fricas")
```

output
```
-1/3*(((4*a^2 - 4*a*b + b^2)*cosh(f*x + e)^6 + 6*(4*a^2 - 4*a*b + b^2)*cos
h(f*x + e)*sinh(f*x + e)^5 + (4*a^2 - 4*a*b + b^2)*sinh(f*x + e)^6 + 3*(4*
a^2 - 4*a*b + b^2)*cosh(f*x + e)^4 + 3*(5*(4*a^2 - 4*a*b + b^2)*cosh(f*x +
 e)^2 + 4*a^2 - 4*a*b + b^2)*sinh(f*x + e)^4 + 4*(5*(4*a^2 - 4*a*b + b^2)*
cosh(f*x + e)^3 + 3*(4*a^2 - 4*a*b + b^2)*cosh(f*x + e))*sinh(f*x + e)^3 +
 3*(4*a^2 - 4*a*b + b^2)*cosh(f*x + e)^2 + 3*(5*(4*a^2 - 4*a*b + b^2)*cosh
(f*x + e)^4 + 6*(4*a^2 - 4*a*b + b^2)*cosh(f*x + e)^2 + 4*a^2 - 4*a*b + b^
2)*sinh(f*x + e)^2 + 4*a^2 - 4*a*b + b^2 + 6*((4*a^2 - 4*a*b + b^2)*cosh(f
*x + e)^5 + 2*(4*a^2 - 4*a*b + b^2)*cosh(f*x + e)^3 + (4*a^2 - 4*a*b + b^2
)*cosh(f*x + e))*sinh(f*x + e) - 2*((2*a*b - b^2)*cosh(f*x + e)^6 + 6*(2*a
*b - b^2)*cosh(f*x + e)*sinh(f*x + e)^5 + (2*a*b - b^2)*sinh(f*x + e)^6 +
3*(2*a*b - b^2)*cosh(f*x + e)^4 + 3*(5*(2*a*b - b^2)*cosh(f*x + e)^2 + 2*a
*b - b^2)*sinh(f*x + e)^4 + 4*(5*(2*a*b - b^2)*cosh(f*x + e)^3 + 3*(2*a*b
- b^2)*cosh(f*x + e))*sinh(f*x + e)^3 + 3*(2*a*b - b^2)*cosh(f*x + e)^2 +
3*(5*(2*a*b - b^2)*cosh(f*x + e)^4 + 6*(2*a*b - b^2)*cosh(f*x + e)^2 + 2*a
*b - b^2)*sinh(f*x + e)^2 + 2*a*b - b^2 + 6*((2*a*b - b^2)*cosh(f*x + e)^5
 + 2*(2*a*b - b^2)*cosh(f*x + e)^3 + (2*a*b - b^2)*cosh(f*x + e))*sinh(f*x
 + e))*sqrt((a^2 - a*b)/b^2))*sqrt(b)*sqrt((2*b*sqrt((a^2 - a*b)/b^2) - 2*
a + b)/b)*elliptic_e(arcsin(sqrt((2*b*sqrt((a^2 - a*b)/b^2) - 2*a + b)/b)*
(cosh(f*x + e) + sinh(f*x + e))), (8*a^2 - 8*a*b + b^2 + 4*(2*a*b - b^2...
```

Sympy [F]

$$\int \operatorname{sech}^4(e+fx)\sqrt{a+b\sinh^2(e+fx)}\,dx = \int \sqrt{a+b\sinh^2(e+fx)}\operatorname{sech}^4(e+fx)\,dx$$

input
```
integrate(sech(f*x+e)**4*(a+b*sinh(f*x+e)**2)**(1/2),x)
```

output
```
Integral(sqrt(a + b*sinh(e + f*x)**2)*sech(e + f*x)**4, x)
```

Maxima [F]

$$\int \operatorname{sech}^4(e+fx)\sqrt{a+b\sinh^2(e+fx)}\,dx = \int \sqrt{b\sinh(fx+e)^2+a}\operatorname{sech}(fx+e)^4\,dx$$

input
```
integrate(sech(f*x+e)^4*(a+b*sinh(f*x+e)^2)^(1/2),x, algorithm="maxima")
```

output
```
integrate(sqrt(b*sinh(f*x + e)^2 + a)*sech(f*x + e)^4, x)
```

Giac [F]

$$\int \operatorname{sech}^4(e+fx)\sqrt{a+b\sinh^2(e+fx)}\,dx = \int \sqrt{b\sinh(fx+e)^2+a}\operatorname{sech}(fx+e)^4\,dx$$

input
```
integrate(sech(f*x+e)^4*(a+b*sinh(f*x+e)^2)^(1/2),x, algorithm="giac")
```

output
```
integrate(sqrt(b*sinh(f*x + e)^2 + a)*sech(f*x + e)^4, x)
```

Mupad [F(-1)]

Timed out.

$$\int \operatorname{sech}^4(e+fx)\sqrt{a+b\sinh^2(e+fx)}\,dx=\int \frac{\sqrt{b\sinh\left(e+f\,x\right)^2+a}}{\cosh\left(e+f\,x\right)^4}\,dx$$

input
```
int((a + b*sinh(e + f*x)^2)^(1/2)/cosh(e + f*x)^4,x)
```

output
```
int((a + b*sinh(e + f*x)^2)^(1/2)/cosh(e + f*x)^4, x)
```

Reduce [F]

$$\int \operatorname{sech}^4(e+fx)\sqrt{a+b\sinh^2(e+fx)}\,dx=\int \sqrt{\sinh\left(fx+e\right)^2b+a}\,\operatorname{sech}(fx+e)^4\,dx$$

input
```
int(sech(f*x+e)^4*(a+b*sinh(f*x+e)^2)^(1/2),x)
```

output
```
int(sqrt(sinh(e + f*x)**2*b + a)*sech(e + f*x)**4,x)
```

3.326 $\int \cosh^3(e+fx)\left(a+b\sinh^2(e+fx)\right)^{3/2}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 157

$$\int \cosh^3(e+fx)\left(a+b\sinh^2(e+fx)\right)^{3/2}\,dx = -\frac{a^2(a-6b)\text{arctanh}\left(\frac{\sqrt{b}\sinh(e+fx)}{\sqrt{a+b\sinh^2(e+fx)}}\right)}{16b^{3/2}f} - \frac{a(a-6b)\sinh(e+fx)\sqrt{a+b\sinh^2(e+fx)}}{16bf} - \frac{(a-6b)\sinh(e+fx)\left(a+b\sinh^2(e+fx)\right)^{3/2}}{24bf} + \frac{\sinh(e+fx)\left(a+b\sinh^2(e+fx)\right)^{5/2}}{6bf}$$

output
```
-1/16*a^2*(a-6*b)*arctanh(b^(1/2)*sinh(f*x+e)/(a+b*sinh(f*x+e)^2)^(1/2))/b
^(3/2)/f-1/16*a*(a-6*b)*sinh(f*x+e)*(a+b*sinh(f*x+e)^2)^(1/2)/b/f-1/24*(a-
6*b)*sinh(f*x+e)*(a+b*sinh(f*x+e)^2)^(3/2)/b/f+1/6*sinh(f*x+e)*(a+b*sinh(f
*x+e)^2)^(5/2)/b/f
```

Mathematica [A] (verified)

Time = 0.86 (sec) , antiderivative size = 149, normalized size of antiderivative = 0.95

$$\int \cosh^3(e+fx)\left(a+b\sinh^2(e+fx)\right)^{3/2}dx=\frac{\sqrt{a+b\sinh^2(e+fx)}\left(-3a^{3/2}(a-6b)\text{arcsinh}\left(\frac{\sqrt{b}\sinh(e+fx)}{\sqrt{a}}\right)+\sqrt{b}\sinh(e+fx)\sqrt{1+\frac{b\sinh^2}{}}\right.}{48b^{3/2}f\sqrt{1+\frac{b\sinh^2(e+f}{a}}}$$

input
```
Integrate[Cosh[e + f*x]^3*(a + b*Sinh[e + f*x]^2)^(3/2),x]
```

output
```
(Sqrt[a + b*Sinh[e + f*x]^2]*(-3*a^(3/2)*(a - 6*b)*ArcSinh[(Sqrt[b]*Sinh[e
 + f*x])/Sqrt[a]] + Sqrt[b]*Sinh[e + f*x]*Sqrt[1 + (b*Sinh[e + f*x]^2)/a]*
(3*a*(a + 10*b) + 2*b*(7*a + 6*b)*Sinh[e + f*x]^2 + 8*b^2*Sinh[e + f*x]^4)
))/(48*b^(3/2)*f*Sqrt[1 + (b*Sinh[e + f*x]^2)/a])
```

Rubi [A] (verified)

Time = 0.34 (sec) , antiderivative size = 144, normalized size of antiderivative = 0.92, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.280, Rules used = {3042, 3669, 299, 211, 211, 224, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cosh^3(e+fx)\left(a+b\sinh^2(e+fx)\right)^{3/2}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \cos(ie+ifx)^3\left(a-b\sin(ie+ifx)^2\right)^{3/2}dx$$

$$\Big\downarrow \text{3669}$$

$$\frac{\int \left(\sinh^2(e+fx)+1\right)\left(b\sinh^2(e+fx)+a\right)^{3/2}d\sinh(e+fx)}{f}$$

$$\Big\downarrow \text{299}$$

$$\frac{\frac{\sinh(e+fx)\left(a+b\sinh^2(e+fx)\right)^{5/2}}{6b}-\frac{(a-6b)\int \left(b\sinh^2(e+fx)+a\right)^{3/2}d\sinh(e+fx)}{6b}}{f}$$

$\downarrow$ 211

$$\frac{\frac{\sinh(e+fx)\left(a+b\sinh^2(e+fx)\right)^{5/2}}{6b}-\frac{(a-6b)\left(\frac{3}{4}a\int \sqrt{b\sinh^2(e+fx)+a}d\sinh(e+fx)+\frac{1}{4}\sinh(e+fx)\left(a+b\sinh^2(e+fx)\right)^{3/2}\right)}{6b}}{f}$$

$\downarrow$ 211

$$\frac{\frac{\sinh(e+fx)\left(a+b\sinh^2(e+fx)\right)^{5/2}}{6b}-\frac{(a-6b)\left(\frac{3}{4}a\left(\frac{1}{2}a\int \frac{1}{\sqrt{b\sinh^2(e+fx)+a}}d\sinh(e+fx)+\frac{1}{2}\sinh(e+fx)\sqrt{a+b\sinh^2(e+fx)}\right)+\frac{1}{4}\sinh(e+fx)(a}{6b}}{f}$$

$\downarrow$ 224

$$\frac{\frac{\sinh(e+fx)\left(a+b\sinh^2(e+fx)\right)^{5/2}}{6b}-\frac{(a-6b)\left(\frac{3}{4}a\left(\frac{1}{2}a\int \frac{1}{1-\frac{b\sinh^2(e+fx)}{b\sinh^2(e+fx)+a}}d\frac{\sinh(e+fx)}{\sqrt{b\sinh^2(e+fx)+a}}+\frac{1}{2}\sinh(e+fx)\sqrt{a+b\sinh^2(e+fx)}\right)+\frac{1}{4}\sinh(e}{6b}}{f}$$

$\downarrow$ 219

$$\frac{\frac{\sinh(e+fx)\left(a+b\sinh^2(e+fx)\right)^{5/2}}{6b}-\frac{(a-6b)\left(\frac{3}{4}a\left(\frac{a\,\text{arctanh}\left(\frac{\sqrt{b}\sinh(e+fx)}{\sqrt{a+b\sinh^2(e+fx)}}\right)}{2\sqrt{b}}+\frac{1}{2}\sinh(e+fx)\sqrt{a+b\sinh^2(e+fx)}\right)+\frac{1}{4}\sinh(e+fx)(a+b}{6b}}{f}$$

input
```
Int[Cosh[e + f*x]^3*(a + b*Sinh[e + f*x]^2)^(3/2),x]
```

output
```
((Sinh[e + f*x]*(a + b*Sinh[e + f*x]^2)^(5/2))/(6*b) - ((a - 6*b)*((Sinh[e
 + f*x]*(a + b*Sinh[e + f*x]^2)^(3/2))/4 + (3*a*((a*ArcTanh[(Sqrt[b]*Sinh[
e + f*x])/Sqrt[a + b*Sinh[e + f*x]^2]])/(2*Sqrt[b]) + (Sinh[e + f*x]*Sqrt[
a + b*Sinh[e + f*x]^2])/2))/4))/(6*b))/f
```

Defintions of rubi rules used

rule 211
```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[x*((a + b*x^2)^p/(2*p + 1
)), x] + Simp[2*a*(p/(2*p + 1))   Int[(a + b*x^2)^(p - 1), x], x] /; FreeQ[
{a, b}, x] && GtQ[p, 0] && (IntegerQ[4*p] || IntegerQ[6*p])
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

rule 299
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2), x_Symbol] :> Simp[d*x
*((a + b*x^2)^(p + 1)/(b*(2*p + 3))), x] - Simp[(a*d - b*c*(2*p + 3))/(b*(2
*p + 3))   Int[(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
 a*d, 0] && NeQ[2*p + 3, 0]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3669
```
Int[cos[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^
(p_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Simp[ff/f   S
ubst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a + b*ff^2*x^2)^p, x], x, Sin[e + f*x]
/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m - 1)/2]
```

Maple [A] (verified)

Time = 0.32 (sec) , antiderivative size = 215, normalized size of antiderivative = 1.37

method	result
derivativedivides	$\frac{\sinh(fx+e)\left(a+b\sinh(fx+e)^2\right)^{\frac{3}{2}}}{4f}+\frac{3a\sinh(fx+e)\sqrt{a+b\sinh(fx+e)^2}}{8f}+\frac{3a^2\ln\left(\sqrt{b}\,\sinh(fx+e)+\sqrt{a+b\sinh(fx}\right.}{8f\sqrt{b}}$
default	$\frac{\sinh(fx+e)\left(a+b\sinh(fx+e)^2\right)^{\frac{3}{2}}}{4f}+\frac{3a\sinh(fx+e)\sqrt{a+b\sinh(fx+e)^2}}{8f}+\frac{3a^2\ln\left(\sqrt{b}\,\sinh(fx+e)+\sqrt{a+b\sinh(fx}\right.}{8f\sqrt{b}}$

input
```
int(cosh(f*x+e)^3*(a+b*sinh(f*x+e)^2)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
1/4*sinh(f*x+e)*(a+b*sinh(f*x+e)^2)^(3/2)/f+3/8*a*sinh(f*x+e)*(a+b*sinh(f*
x+e)^2)^(1/2)/f+3/8/f*a^2/b^(1/2)*ln(b^(1/2)*sinh(f*x+e)+(a+b*sinh(f*x+e)^
2)^(1/2))+1/6*sinh(f*x+e)*(a+b*sinh(f*x+e)^2)^(5/2)/b/f-1/24/f/b*a*sinh(f*
x+e)*(a+b*sinh(f*x+e)^2)^(3/2)-1/16/f/b*a^2*sinh(f*x+e)*(a+b*sinh(f*x+e)^2
)^(1/2)-1/16/f/b^(3/2)*a^3*ln(b^(1/2)*sinh(f*x+e)+(a+b*sinh(f*x+e)^2)^(1/2
))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1734 vs. 2(137) = 274.

Time = 0.26 (sec) , antiderivative size = 4491, normalized size of antiderivative = 28.61

$$\int \cosh^3(e+fx)\left(a+b\sinh^2(e+fx)\right)^{3/2}\,dx=\text{Too large to display}$$

input
```
integrate(cosh(f*x+e)^3*(a+b*sinh(f*x+e)^2)^(3/2),x, algorithm="fricas")
```

output
```
Too large to include
```

Sympy [F(-1)]

Timed out.

$$\int \cosh^3(e+fx)\left(a+b\sinh^2(e+fx)\right)^{3/2}\,dx=\text{Timed out}$$

input
```
integrate(cosh(f*x+e)**3*(a+b*sinh(f*x+e)**2)**(3/2),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \cosh^3(e+fx)\left(a+b\sinh^2(e+fx)\right)^{3/2}\,dx=\int \left(b\sinh\left(fx+e\right)^2+a\right)^{\frac{3}{2}}\cosh\left(fx+e\right)^3\,dx$$

input
```
integrate(cosh(f*x+e)^3*(a+b*sinh(f*x+e)^2)^(3/2),x, algorithm="maxima")
```

output
```
integrate((b*sinh(f*x + e)^2 + a)^(3/2)*cosh(f*x + e)^3, x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1546 vs. $2(137) = 274$.

Time = 0.78 (sec) , antiderivative size = 1546, normalized size of antiderivative = 9.85

$$\int \cosh^3(e+fx)\left(a+b\sinh^2(e+fx)\right)^{3/2}\,dx=\text{Too large to display}$$

input
```
integrate(cosh(f*x+e)^3*(a+b*sinh(f*x+e)^2)^(3/2),x, algorithm="giac")
```

output
```
1/384*(((b*e^(2*f*x + 10*e) + (7*a*b^2*e^(14*e) + b^3*e^(14*e))*e^(-6*e)/b
^2)*e^(2*f*x) + (6*a^2*b*e^(12*e) + 39*a*b^2*e^(12*e) - 8*b^3*e^(12*e))*e^
(-6*e)/b^2)*sqrt(b*e^(4*f*x + 4*e) + 4*a*e^(2*f*x + 2*e) - 2*b*e^(2*f*x +
2*e) + b) - 24*(a^3*e^(6*e) - 6*a^2*b*e^(6*e))*arctan(-(sqrt(b)*e^(2*f*x +
 2*e) - sqrt(b*e^(4*f*x + 4*e) + 4*a*e^(2*f*x + 2*e) - 2*b*e^(2*f*x + 2*e)
 + b))/sqrt(-b))/(sqrt(-b)*b) + 12*(a^3*sqrt(b)*e^(6*e) - 6*a^2*b^(3/2)*e^
(6*e))*log(abs(-(sqrt(b)*e^(2*f*x + 2*e) - sqrt(b*e^(4*f*x + 4*e) + 4*a*e^
(2*f*x + 2*e) - 2*b*e^(2*f*x + 2*e) + b))*b - 2*a*sqrt(b) + b^(3/2)))/b^2
+ 2*(12*(sqrt(b)*e^(2*f*x + 2*e) - sqrt(b*e^(4*f*x + 4*e) + 4*a*e^(2*f*x +
 2*e) - 2*b*e^(2*f*x + 2*e) + b))^5*a^3*e^(6*e) + 72*(sqrt(b)*e^(2*f*x + 2
*e) - sqrt(b*e^(4*f*x + 4*e) + 4*a*e^(2*f*x + 2*e) - 2*b*e^(2*f*x + 2*e) +
 b))^5*a^2*b*e^(6*e) - 48*(sqrt(b)*e^(2*f*x + 2*e) - sqrt(b*e^(4*f*x + 4*e
) + 4*a*e^(2*f*x + 2*e) - 2*b*e^(2*f*x + 2*e) + b))^5*a*b^2*e^(6*e) + 9*(s
qrt(b)*e^(2*f*x + 2*e) - sqrt(b*e^(4*f*x + 4*e) + 4*a*e^(2*f*x + 2*e) - 2*
b*e^(2*f*x + 2*e) + b))^5*b^3*e^(6*e) + 48*(sqrt(b)*e^(2*f*x + 2*e) - sqrt
(b*e^(4*f*x + 4*e) + 4*a*e^(2*f*x + 2*e) - 2*b*e^(2*f*x + 2*e) + b))^4*a^2
*b^(3/2)*e^(6*e) + 24*(sqrt(b)*e^(2*f*x + 2*e) - sqrt(b*e^(4*f*x + 4*e) +
4*a*e^(2*f*x + 2*e) - 2*b*e^(2*f*x + 2*e) + b))^4*a*b^(5/2)*e^(6*e) - 9*(s
qrt(b)*e^(2*f*x + 2*e) - sqrt(b*e^(4*f*x + 4*e) + 4*a*e^(2*f*x + 2*e) - 2*
b*e^(2*f*x + 2*e) + b))^4*b^(7/2)*e^(6*e) + 32*(sqrt(b)*e^(2*f*x + 2*e)...
```

Mupad [F(-1)]

Timed out.

$$\int \cosh^3(e+fx) \left(a + b\sinh^2(e+fx)\right)^{3/2} dx = \int \cosh(e+f\,x)^3 \left(b\sinh(e+f\,x)^2+a\right)^{3/2} dx$$

input
```
int(cosh(e + f*x)^3*(a + b*sinh(e + f*x)^2)^(3/2),x)
```

output
```
int(cosh(e + f*x)^3*(a + b*sinh(e + f*x)^2)^(3/2), x)
```

Reduce [F]

$$\int \cosh^3(e+fx) \left(a+b\sinh^2(e+fx)\right)^{3/2} dx = \left(\int \sqrt{\sinh(fx+e)^2 b+a}\, \cosh(fx+e)^3 \sinh(fx+e)^2 dx\right) b + \left(\int \sqrt{\sinh(fx+e)^2 b+a}\, \cosh(fx+e)^3 dx\right) a$$

input
```
int(cosh(f*x+e)^3*(a+b*sinh(f*x+e)^2)^(3/2),x)
```

output
```
int(sqrt(sinh(e + f*x)**2*b + a)*cosh(e + f*x)**3*sinh(e + f*x)**2,x)*b +
int(sqrt(sinh(e + f*x)**2*b + a)*cosh(e + f*x)**3,x)*a
```

3.327 $\int \cosh(e+fx)\left(a+b\sinh^2(e+fx)\right)^{3/2}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 104

$$\int \cosh(e+fx)\left(a+b\sinh^2(e+fx)\right)^{3/2}\,dx = \frac{3a^2\operatorname{arctanh}\left(\frac{\sqrt{b}\sinh(e+fx)}{\sqrt{a+b\sinh^2(e+fx)}}\right)}{8\sqrt{b}f} + \frac{3a\sinh(e+fx)\sqrt{a+b\sinh^2(e+fx)}}{8f} + \frac{\sinh(e+fx)\left(a+b\sinh^2(e+fx)\right)^{3/2}}{4f}$$

output
```
3/8*a^2*arctanh(b^(1/2)*sinh(f*x+e)/(a+b*sinh(f*x+e)^2)^(1/2))/b^(1/2)/f+3
/8*a*sinh(f*x+e)*(a+b*sinh(f*x+e)^2)^(1/2)/f+1/4*sinh(f*x+e)*(a+b*sinh(f*x
+e)^2)^(3/2)/f
```

Mathematica [A] (verified)

Time = 0.32 (sec) , antiderivative size = 93, normalized size of antiderivative = 0.89

$$\int \cosh(e+fx)\left(a+b\sinh^2(e+fx)\right)^{3/2}\,dx = \frac{\sqrt{a+b\sinh^2(e+fx)}\left(5a\sinh(e+fx)+2b\sinh^3(e+fx)+\frac{3a^{3/2}\operatorname{arcsinh}\left(\frac{\sqrt{b}\sinh(e+fx)}{\sqrt{a}}\right)}{\sqrt{b}\sqrt{1+\frac{b\sinh^2(e+fx)}{a}}}\right)}{8f}$$

input
```
Integrate[Cosh[e + f*x]*(a + b*Sinh[e + f*x]^2)^(3/2),x]
```

output
```
(Sqrt[a + b*Sinh[e + f*x]^2]*(5*a*Sinh[e + f*x] + 2*b*Sinh[e + f*x]^3 + (3
*a^(3/2)*ArcSinh[(Sqrt[b]*Sinh[e + f*x])/Sqrt[a]])/(Sqrt[b]*Sqrt[1 + (b*Si
nh[e + f*x]^2)/a])))/(8*f)
```

Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 102, normalized size of antiderivative = 0.98, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.261$, Rules used = {3042, 3669, 211, 211, 224, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cosh(e+fx)\left(a+b\sinh^2(e+fx)\right)^{3/2}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \cos(ie+ifx)\left(a-b\sin(ie+ifx)^2\right)^{3/2}dx$$

$$\Big\downarrow \text{3669}$$

$$\frac{\int \left(b\sinh^2(e+fx)+a\right)^{3/2}d\sinh(e+fx)}{f}$$

$$\Big\downarrow \text{211}$$

$$\frac{\frac{3}{4}a\int \sqrt{b\sinh^2(e+fx)+a}d\sinh(e+fx)+\frac{1}{4}\sinh(e+fx)\left(a+b\sinh^2(e+fx)\right)^{3/2}}{f}$$

$$\Big\downarrow \text{211}$$

$$\frac{\frac{3}{4}a\left(\frac{1}{2}a\int \frac{1}{\sqrt{b\sinh^2(e+fx)+a}}d\sinh(e+fx)+\frac{1}{2}\sinh(e+fx)\sqrt{a+b\sinh^2(e+fx)}\right)+\frac{1}{4}\sinh(e+fx)\left(a+b\sinh^2(e\right.}{f}$$

$$\Big\downarrow \text{224}$$

$$\frac{\frac{3}{4}a\left(\frac{1}{2}a\int \frac{1}{1-\frac{b\sinh^2(e+fx)}{b\sinh^2(e+fx)+a}}d\frac{\sinh(e+fx)}{\sqrt{b\sinh^2(e+fx)+a}}+\frac{1}{2}\sinh(e+fx)\sqrt{a+b\sinh^2(e+fx)}\right)+\frac{1}{4}\sinh(e+fx)\left(a+b\sinh^2\right.}{f}$$

$$\Big\downarrow \text{219}$$

$$\frac{\frac{3}{4}a\left(\frac{a\text{arctanh}\left(\frac{\sqrt{b}\sinh(e+fx)}{\sqrt{a+b\sinh^2(e+fx)}}\right)}{2\sqrt{b}}+\frac{1}{2}\sinh(e+fx)\sqrt{a+b\sinh^2(e+fx)}\right)+\frac{1}{4}\sinh(e+fx)\left(a+b\sinh^2(e+fx)\right)^3}{f}$$

input
```
Int[Cosh[e + f*x]*(a + b*Sinh[e + f*x]^2)^(3/2),x]
```

output
```
((Sinh[e + f*x]*(a + b*Sinh[e + f*x]^2)^(3/2))/4 + (3*a*((a*ArcTanh[(Sqrt[
b]*Sinh[e + f*x])/Sqrt[a + b*Sinh[e + f*x]^2]])/(2*Sqrt[b]) + (Sinh[e + f*
x]*Sqrt[a + b*Sinh[e + f*x]^2])/2))/4)/f
```

Defintions of rubi rules used

rule 211
```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[x*((a + b*x^2)^p/(2*p + 1
)), x] + Simp[2*a*(p/(2*p + 1))   Int[(a + b*x^2)^(p - 1), x], x] /; FreeQ[
{a, b}, x] && GtQ[p, 0] && (IntegerQ[4*p] || IntegerQ[6*p])
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3669
```
Int[cos[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^
(p_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Simp[ff/f   S
ubst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a + b*ff^2*x^2)^p, x], x, Sin[e + f*x]
/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m - 1)/2]
```

Maple [A] (verified)

Time = 0.18 (sec) , antiderivative size = 90, normalized size of antiderivative = 0.87

method	result
derivativedivides	$\frac{\sinh(fx+e)\left(a+b\sinh(fx+e)^2\right)^{\frac{3}{2}}}{4f}+\frac{3a\sinh(fx+e)\sqrt{a+b\sinh(fx+e)^2}}{8f}+\frac{3a^2\ln\left(\sqrt{b}\,\sinh(fx+e)+\sqrt{a+b\sinh(fx}\right.}{8f\sqrt{b}}$
default	$\frac{\sinh(fx+e)\left(a+b\sinh(fx+e)^2\right)^{\frac{3}{2}}}{4f}+\frac{3a\sinh(fx+e)\sqrt{a+b\sinh(fx+e)^2}}{8f}+\frac{3a^2\ln\left(\sqrt{b}\,\sinh(fx+e)+\sqrt{a+b\sinh(fx}\right.}{8f\sqrt{b}}$

input
```
int(cosh(f*x+e)*(a+b*sinh(f*x+e)^2)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
1/4*sinh(f*x+e)*(a+b*sinh(f*x+e)^2)^(3/2)/f+3/8*a*sinh(f*x+e)*(a+b*sinh(f*
x+e)^2)^(1/2)/f+3/8/f*a^2/b^(1/2)*ln(b^(1/2)*sinh(f*x+e)+(a+b*sinh(f*x+e)^
2)^(1/2))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1014 vs. 2(88) = 176.

Time = 0.20 (sec) , antiderivative size = 3049, normalized size of antiderivative = 29.32

$$\int \cosh(e+fx)\left(a+b\sinh^2(e+fx)\right)^{3/2}\,dx=\text{Too large to display}$$

input
```
integrate(cosh(f*x+e)*(a+b*sinh(f*x+e)^2)^(3/2),x, algorithm="fricas")
```

output
```
Too large to include
```

Sympy [F(-1)]

Timed out.

$$\int \cosh(e+fx)\left(a+b\sinh^2(e+fx)\right)^{3/2}\,dx=\text{Timed out}$$

input
```
integrate(cosh(f*x+e)*(a+b*sinh(f*x+e)**2)**(3/2),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \cosh(e+fx)\left(a+b\sinh^2(e+fx)\right)^{3/2}\,dx=\int \left(b\sinh\left(fx+e\right)^2+a\right)^{\frac{3}{2}}\cosh\left(fx+e\right)\,dx$$

input
```
integrate(cosh(f*x+e)*(a+b*sinh(f*x+e)^2)^(3/2),x, algorithm="maxima")
```

output
```
integrate((b*sinh(f*x + e)^2 + a)^(3/2)*cosh(f*x + e), x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 791 vs. $2(88)=176$.

Time $=0.50$ (sec) , antiderivative size $=791$, normalized size of antiderivative $=7.61$

$$\int \cosh(e+fx)\left(a+b\sinh^2(e+fx)\right)^{3/2}\,dx=\text{Too large to display}$$

input
```
integrate(cosh(f*x+e)*(a+b*sinh(f*x+e)^2)^(3/2),x, algorithm="giac")
```

output
```
1/64*(24*a^2*arctan(-(sqrt(b)*e^(2*f*x + 2*e) - sqrt(b*e^(4*f*x + 4*e) + 4
*a*e^(2*f*x + 2*e) - 2*b*e^(2*f*x + 2*e) + b))/sqrt(-b))*e^(4*e)/sqrt(-b)
- 12*a^2*e^(4*e)*log(abs(-(sqrt(b)*e^(2*f*x + 2*e) - sqrt(b*e^(4*f*x + 4*e
) + 4*a*e^(2*f*x + 2*e) - 2*b*e^(2*f*x + 2*e) + b))*b - 2*a*sqrt(b) + b^(3
/2)))/sqrt(b) + sqrt(b*e^(4*f*x + 4*e) + 4*a*e^(2*f*x + 2*e) - 2*b*e^(2*f*
x + 2*e) + b)*(b*e^(2*f*x + 6*e) + (10*a*b*e^(6*e) - 3*b^2*e^(6*e))*e^(-2*
e)/b) + 4*(10*(sqrt(b)*e^(2*f*x + 2*e) - sqrt(b*e^(4*f*x + 4*e) + 4*a*e^(2
*f*x + 2*e) - 2*b*e^(2*f*x + 2*e) + b))^3*a^2*e^(4*e) - 8*(sqrt(b)*e^(2*f*
x + 2*e) - sqrt(b*e^(4*f*x + 4*e) + 4*a*e^(2*f*x + 2*e) - 2*b*e^(2*f*x + 2
*e) + b))^3*a*b*e^(4*e) + 2*(sqrt(b)*e^(2*f*x + 2*e) - sqrt(b*e^(4*f*x + 4
*e) + 4*a*e^(2*f*x + 2*e) - 2*b*e^(2*f*x + 2*e) + b))^3*b^2*e^(4*e) + 8*(s
qrt(b)*e^(2*f*x + 2*e) - sqrt(b*e^(4*f*x + 4*e) + 4*a*e^(2*f*x + 2*e) - 2*
b*e^(2*f*x + 2*e) + b))^2*a*b^(3/2)*e^(4*e) - 3*(sqrt(b)*e^(2*f*x + 2*e) -
 sqrt(b*e^(4*f*x + 4*e) + 4*a*e^(2*f*x + 2*e) - 2*b*e^(2*f*x + 2*e) + b))^
2*b^(5/2)*e^(4*e) - 6*(sqrt(b)*e^(2*f*x + 2*e) - sqrt(b*e^(4*f*x + 4*e) +
4*a*e^(2*f*x + 2*e) - 2*b*e^(2*f*x + 2*e) + b))*a^2*b*e^(4*e) + 4*(sqrt(b)
*e^(2*f*x + 2*e) - sqrt(b*e^(4*f*x + 4*e) + 4*a*e^(2*f*x + 2*e) - 2*b*e^(2
*f*x + 2*e) + b))*a*b^2*e^(4*e) - 4*a*b^(5/2)*e^(4*e) + b^(7/2)*e^(4*e))/(
(sqrt(b)*e^(2*f*x + 2*e) - sqrt(b*e^(4*f*x + 4*e) + 4*a*e^(2*f*x + 2*e) -
2*b*e^(2*f*x + 2*e) + b))^2 - b)^2)*e^(-4*e)/f
```

Mupad [B] (verification not implemented)

Time = 1.86 (sec) , antiderivative size = 60, normalized size of antiderivative = 0.58

$$\int \cosh(e+fx)\left(a+b\sinh^2(e+fx)\right)^{3/2}\,dx=\frac{\sinh(e+f\,x)\,\left(b\sinh(e+f\,x)^2+a\right)^{3/2}\,{}_2\mathrm{F}_1\left(-\frac{3}{2},\frac{1}{2};\ \frac{3}{2};\ -\frac{b\sinh(e+f\,x)^2}{a}\right)}{f\left(\frac{b\sinh(e+f\,x)^2}{a}+1\right)^{3/2}}$$

input
```
int(cosh(e + f*x)*(a + b*sinh(e + f*x)^2)^(3/2),x)
```

output
```
(sinh(e + f*x)*(a + b*sinh(e + f*x)^2)^(3/2)*hypergeom([-3/2, 1/2], 3/2, -
(b*sinh(e + f*x)^2)/a))/(f*((b*sinh(e + f*x)^2)/a + 1)^(3/2))
```

Reduce [F]

$$\int \cosh(e+fx)\left(a+b\sinh^2(e+fx)\right)^{3/2}\,dx=\frac{2\sqrt{\sinh(fx+e)^2b+a}\,\sinh(fx+e)^3b+5\sqrt{\sinh(fx+e)^2b+a}\,\sinh(fx+e)\,a+3\left(\int \frac{\sqrt{}}{}\right.}{8f}$$

input
```
int(cosh(f*x+e)*(a+b*sinh(f*x+e)^2)^(3/2),x)
```

output
```
(2*sqrt(sinh(e + f*x)**2*b + a)*sinh(e + f*x)**3*b + 5*sqrt(sinh(e + f*x)*
*2*b + a)*sinh(e + f*x)*a + 3*int((sqrt(sinh(e + f*x)**2*b + a)*cosh(e + f
*x))/(sinh(e + f*x)**2*b + a),x)*a**2*f)/(8*f)
```

3.328 $\int \operatorname{sech}(e+fx)\left(a+b\sinh^2(e+fx)\right)^{3/2}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 125

$$\int \operatorname{sech}(e+fx)\left(a+b\sinh^2(e+fx)\right)^{3/2}\,dx = \frac{(a-b)^{3/2}\arctan\left(\frac{\sqrt{a-b}\sinh(e+fx)}{\sqrt{a+b\sinh^2(e+fx)}}\right)}{f} + \frac{(3a-2b)\sqrt{b}\operatorname{arctanh}\left(\frac{\sqrt{b}\sinh(e+fx)}{\sqrt{a+b\sinh^2(e+fx)}}\right)}{2f} + \frac{b\sinh(e+fx)\sqrt{a+b\sinh^2(e+fx)}}{2f}$$

output
```
(a-b)^(3/2)*arctan((a-b)^(1/2)*sinh(f*x+e)/(a+b*sinh(f*x+e)^2)^(1/2))/f+1/
2*(3*a-2*b)*b^(1/2)*arctanh(b^(1/2)*sinh(f*x+e)/(a+b*sinh(f*x+e)^2)^(1/2))
/f+1/2*b*sinh(f*x+e)*(a+b*sinh(f*x+e)^2)^(1/2)/f
```

Mathematica [A] (verified)

Time = 0.34 (sec) , antiderivative size = 142, normalized size of antiderivative = 1.14

$$\int \operatorname{sech}(e+fx)\left(a+b\sinh^2(e+fx)\right)^{3/2}\,dx = \frac{4(a-b)^{3/2}\arctan\left(\frac{\sqrt{2a-2b}\sinh(e+fx)}{\sqrt{2a-b+b\cosh(2(e+fx))}}\right) + 2(3a-2b)\sqrt{b}\operatorname{arctanh}\left(\frac{\sqrt{2}\sqrt{b}\sinh(e+fx)}{\sqrt{2a-b+b\cosh(2(e+fx))}}\right) + b}{4f}$$

input
```
Integrate[Sech[e + f*x]*(a + b*Sinh[e + f*x]^2)^(3/2),x]
```

output
```
(4*(a - b)^(3/2)*ArcTan[(Sqrt[2*a - 2*b]*Sinh[e + f*x])/Sqrt[2*a - b + b*C
osh[2*(e + f*x)]]] + 2*(3*a - 2*b)*Sqrt[b]*ArcTanh[(Sqrt[2]*Sqrt[b]*Sinh[e
 + f*x])/Sqrt[2*a - b + b*Cosh[2*(e + f*x)]]] + b*Sqrt[4*a - 2*b + 2*b*Cos
h[2*(e + f*x)]]*Sinh[e + f*x])/(4*f)
```

Rubi [A] (verified)

Time = 0.35 (sec) , antiderivative size = 123, normalized size of antiderivative = 0.98, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.348$, Rules used = {3042, 3669, 318, 398, 224, 219, 291, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \operatorname{sech}(e+fx)\left(a+b\sinh^2(e+fx)\right)^{3/2}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\left(a-b\sin(ie+ifx)^2\right)^{3/2}}{\cos(ie+ifx)}dx$$

$$\Big\downarrow \text{3669}$$

$$\frac{\int \frac{\left(b\sinh^2(e+fx)+a\right)^{3/2}}{\sinh^2(e+fx)+1}d\sinh(e+fx)}{f}$$

$$\Big\downarrow \text{318}$$

$$\frac{\frac{1}{2}\int \frac{(3a-2b)b\sinh^2(e+fx)+a(2a-b)}{\left(\sinh^2(e+fx)+1\right)\sqrt{b\sinh^2(e+fx)+a}}d\sinh(e+fx)+\frac{1}{2}b\sinh(e+fx)\sqrt{a+b\sinh^2(e+fx)}}{f}$$

$$\Big\downarrow \text{398}$$

$$\frac{\frac{1}{2}\left(2(a-b)^2\int \frac{1}{\left(\sinh^2(e+fx)+1\right)\sqrt{b\sinh^2(e+fx)+a}}d\sinh(e+fx)+b(3a-2b)\int \frac{1}{\sqrt{b\sinh^2(e+fx)+a}}d\sinh(e+fx)\right)+\frac{1}{2}b}{f}$$

$$\Big\downarrow \text{224}$$

$$\frac{\frac{1}{2}\left(2(a-b)^2\int \frac{1}{(\sinh^2(e+fx)+1)\sqrt{b\sinh^2(e+fx)+a}}d\sinh(e+fx)+b(3a-2b)\int \frac{1}{1-\frac{b\sinh^2(e+fx)}{b\sinh^2(e+fx)+a}}d\frac{\sinh(e+fx)}{\sqrt{b\sinh^2(e+fx)+a}}\right)+\frac{1}{2}$$

$$\Big\downarrow \text{219}$$

$$\frac{\frac{1}{2}\left(2(a-b)^2\int \frac{1}{(\sinh^2(e+fx)+1)\sqrt{b\sinh^2(e+fx)+a}}d\sinh(e+fx)+\sqrt{b}(3a-2b)\text{arctanh}\left(\frac{\sqrt{b}\sinh(e+fx)}{\sqrt{a+b\sinh^2(e+fx)}}\right)\right)+\frac{1}{2}b\sin}{f}$$

$$\Big\downarrow \text{291}$$

$$\frac{\frac{1}{2}\left(2(a-b)^2\int \frac{1}{1-\frac{(b-a)\sinh^2(e+fx)}{b\sinh^2(e+fx)+a}}d\frac{\sinh(e+fx)}{\sqrt{b\sinh^2(e+fx)+a}}+\sqrt{b}(3a-2b)\text{arctanh}\left(\frac{\sqrt{b}\sinh(e+fx)}{\sqrt{a+b\sinh^2(e+fx)}}\right)\right)+\frac{1}{2}b\sinh(e+fx)\sqrt{}}{f}$$

$$\Big\downarrow \text{216}$$

$$\frac{\frac{1}{2}\left(2(a-b)^{3/2}\arctan\left(\frac{\sqrt{a-b}\sinh(e+fx)}{\sqrt{a+b\sinh^2(e+fx)}}\right)+\sqrt{b}(3a-2b)\text{arctanh}\left(\frac{\sqrt{b}\sinh(e+fx)}{\sqrt{a+b\sinh^2(e+fx)}}\right)\right)+\frac{1}{2}b\sinh(e+fx)\sqrt{a+b}}{f}$$

input
```
Int[Sech[e + f*x]*(a + b*Sinh[e + f*x]^2)^(3/2),x]
```

output
```
((2*(a - b)^(3/2)*ArcTan[(Sqrt[a - b]*Sinh[e + f*x])/Sqrt[a + b*Sinh[e + f
*x]^2]] + (3*a - 2*b)*Sqrt[b]*ArcTanh[(Sqrt[b]*Sinh[e + f*x])/Sqrt[a + b*S
inh[e + f*x]^2]])/2 + (b*Sinh[e + f*x]*Sqrt[a + b*Sinh[e + f*x]^2])/2)/f
```

Defintions of rubi rules used

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

rule 291
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*((c_) + (d_.)*(x_)^2)), x_Symbol] :> Subst
[Int[1/(c - (b*c - a*d)*x^2), x], x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b, c,
d}, x] && NeQ[b*c - a*d, 0]
```

rule 318
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_), x_Symbol] :> Sim
p[d*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^(q - 1)/(b*(2*(p + q) + 1))), x] + S
imp[1/(b*(2*(p + q) + 1))   Int[(a + b*x^2)^p*(c + d*x^2)^(q - 2)*Simp[c*(b
*c*(2*(p + q) + 1) - a*d) + d*(b*c*(2*(p + 2*q - 1) + 1) - a*d*(2*(q - 1) +
 1))*x^2, x], x], x] /; FreeQ[{a, b, c, d, p}, x] && NeQ[b*c - a*d, 0] && G
tQ[q, 1] && NeQ[2*(p + q) + 1, 0] &&  !IGtQ[p, 1] && IntBinomialQ[a, b, c,
d, 2, p, q, x]
```

rule 398
```
Int[((e_) + (f_.)*(x_)^2)/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2])
, x_Symbol] :> Simp[f/b   Int[1/Sqrt[c + d*x^2], x], x] + Simp[(b*e - a*f)/
b   Int[1/((a + b*x^2)*Sqrt[c + d*x^2]), x], x] /; FreeQ[{a, b, c, d, e, f}
, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3669
```
Int[cos[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^
(p_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Simp[ff/f   S
ubst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a + b*ff^2*x^2)^p, x], x, Sin[e + f*x]
/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m - 1)/2]
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.28 (sec) , antiderivative size = 63, normalized size of antiderivative = 0.50

method	result	size
default	$\frac{\text{`int/indef0`}\left(\frac{b^{2} \sinh(fx+e)^{4}+2\sinh(fx+e)^{2} ab+a^{2}}{\cosh(fx+e)^{2} \sqrt{a+b\sinh(fx+e)^{2}}}, \sinh(fx+e)\right)}{f}$	63

input
```
int(sech(f*x+e)*(a+b*sinh(f*x+e)^2)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
`int/indef0`((b^2*sinh(f*x+e)^4+2*sinh(f*x+e)^2*a*b+a^2)/cosh(f*x+e)^2/(a+
b*sinh(f*x+e)^2)^(1/2),sinh(f*x+e))/f
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 959 vs. 2(107) = 214.

Time = 0.36 (sec) , antiderivative size = 6113, normalized size of antiderivative = 48.90

$$\int \operatorname{sech}(e+fx) \left(a+b\sinh^2(e+fx)\right)^{3/2} dx = \text{Too large to display}$$

input
```
integrate(sech(f*x+e)*(a+b*sinh(f*x+e)^2)^(3/2),x, algorithm="fricas")
```

output
```
Too large to include
```

Sympy [F]

$$\int \operatorname{sech}(e+fx)\left(a+b\sinh^2(e+fx)\right)^{3/2}\,dx=\int \left(a+b\sinh^2{(e+fx)}\right)^{\frac{3}{2}}\operatorname{sech}{(e+fx)}\,dx$$

input
```
integrate(sech(f*x+e)*(a+b*sinh(f*x+e)**2)**(3/2),x)
```

output
```
Integral((a + b*sinh(e + f*x)**2)**(3/2)*sech(e + f*x), x)
```

Maxima [F]

$$\int \operatorname{sech}(e+fx)\left(a+b\sinh^2(e+fx)\right)^{3/2}\,dx=\int \left(b\sinh\left(fx+e\right)^2+a\right)^{\frac{3}{2}}\operatorname{sech}\left(fx+e\right)\,dx$$

input
```
integrate(sech(f*x+e)*(a+b*sinh(f*x+e)^2)^(3/2),x, algorithm="maxima")
```

output
```
integrate((b*sinh(f*x + e)^2 + a)^(3/2)*sech(f*x + e), x)
```

Giac [F(-2)]

Exception generated.

$$\int \operatorname{sech}(e+fx)\left(a+b\sinh^2(e+fx)\right)^{3/2}\,dx=\text{Exception raised: TypeError}$$

input
```
integrate(sech(f*x+e)*(a+b*sinh(f*x+e)^2)^(3/2),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Error: Bad Argument Type
```

Mupad [F(-1)]

Timed out.

$$\int \operatorname{sech}(e+fx)\left(a+b\sinh^2(e+fx)\right)^{3/2}\,dx=\int \frac{\left(b\sinh(e+f\,x)^2+a\right)^{3/2}}{\cosh\left(e+f\,x\right)}\,dx$$

input
```
int((a + b*sinh(e + f*x)^2)^(3/2)/cosh(e + f*x),x)
```

output
```
int((a + b*sinh(e + f*x)^2)^(3/2)/cosh(e + f*x), x)
```

Reduce [F]

$$\int \operatorname{sech}(e+fx)\left(a+b\sinh^2(e+fx)\right)^{3/2}\,dx=\left(\int \sqrt{\sinh\left(fx+e\right)^2b+a}\,\operatorname{sech}(fx+e)\sinh\left(fx+e\right)^2dx\right)b+\left(\int \sqrt{\sinh\left(fx+e\right)^2b+a}\,\operatorname{sech}(fx+e)\,dx\right)a$$

input
```
int(sech(f*x+e)*(a+b*sinh(f*x+e)^2)^(3/2),x)
```

output
```
int(sqrt(sinh(e + f*x)**2*b + a)*sech(e + f*x)*sinh(e + f*x)**2,x)*b + int
(sqrt(sinh(e + f*x)**2*b + a)*sech(e + f*x),x)*a
```

3.329 $\int \operatorname{sech}^3(e+fx)\left(a+b\sinh^2(e+fx)\right)^{3/2}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 133

$$\int \operatorname{sech}^3(e+fx)\left(a+b\sinh^2(e+fx)\right)^{3/2}\,dx=\frac{\sqrt{a-b}(a+2b)\arctan\left(\frac{\sqrt{a-b}\sinh(e+fx)}{\sqrt{a+b\sinh^2(e+fx)}}\right)}{2f}+\frac{b^{3/2}\text{arctanh}\left(\frac{\sqrt{b}\sinh(e+fx)}{\sqrt{a+b\sinh^2(e+fx)}}\right)}{f}+\frac{(a-b)\operatorname{sech}(e+fx)\sqrt{a+b\sinh^2(e+fx)}\tanh(e+fx)}{2f}$$

output

```
1/2*(a-b)^(1/2)*(a+2*b)*arctan((a-b)^(1/2)*sinh(f*x+e)/(a+b*sinh(f*x+e)^2)
^(1/2))/f+b^(3/2)*arctanh(b^(1/2)*sinh(f*x+e)/(a+b*sinh(f*x+e)^2)^(1/2))/f
+1/2*(a-b)*sech(f*x+e)*(a+b*sinh(f*x+e)^2)^(1/2)*tanh(f*x+e)/f
```

Mathematica [A] (verified)

Time = 0.52 (sec) , antiderivative size = 150, normalized size of antiderivative = 1.13

$$\int \operatorname{sech}^3(e+fx)\left(a+b\sinh^2(e+fx)\right)^{3/2}\,dx=\frac{2\sqrt{a-b}(a+2b)\arctan\left(\frac{\sqrt{2a-2b}\sinh(e+fx)}{\sqrt{2a-b+b\cosh(2(e+fx))}}\right)+4b^{3/2}\text{arctanh}\left(\frac{\sqrt{2}\sqrt{b}\sinh(e+fx)}{\sqrt{2a-b+b\cosh(2(e+fx))}}\right)+(a-}{4f}$$

input
```
Integrate[Sech[e + f*x]^3*(a + b*Sinh[e + f*x]^2)^(3/2),x]
```

output
```
(2*Sqrt[a - b]*(a + 2*b)*ArcTan[(Sqrt[2*a - 2*b]*Sinh[e + f*x])/Sqrt[2*a -
 b + b*Cosh[2*(e + f*x)]]] + 4*b^(3/2)*ArcTanh[(Sqrt[2]*Sqrt[b]*Sinh[e + f
*x])/Sqrt[2*a - b + b*Cosh[2*(e + f*x)]]] + (a - b)*Sqrt[4*a - 2*b + 2*b*C
osh[2*(e + f*x)]]*Sech[e + f*x]*Tanh[e + f*x])/(4*f)
```

Rubi [A] (verified)

Time = 0.38 (sec) , antiderivative size = 145, normalized size of antiderivative = 1.09, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.320, Rules used = {3042, 3669, 315, 398, 224, 219, 291, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \operatorname{sech}^3(e+fx)\left(a+b\sinh^2(e+fx)\right)^{3/2}\,dx$$

$$\Big\downarrow \; 3042$$

$$\int \frac{\left(a-b\sin(ie+ifx)^2\right)^{3/2}}{\cos(ie+ifx)^3}dx$$

$$\Big\downarrow \; 3669$$

$$\frac{\int \frac{\left(b\sinh^2(e+fx)+a\right)^{3/2}}{\left(\sinh^2(e+fx)+1\right)^2}d\sinh(e+fx)}{f}$$

$$\Big\downarrow \; 315$$

$$\frac{\frac{1}{2}\int \frac{2b^2\sinh^2(e+fx)+a(a+b)}{\left(\sinh^2(e+fx)+1\right)\sqrt{b\sinh^2(e+fx)+a}}d\sinh(e+fx)+\frac{(a-b)\sinh(e+fx)\sqrt{a+b\sinh^2(e+fx)}}{2\left(\sinh^2(e+fx)+1\right)}}{f}$$

$\downarrow$ 398

$$\frac{\frac{1}{2}\left(2b^2\int \frac{1}{\sqrt{b\sinh^2(e+fx)+a}}d\sinh(e+fx)-\left(2b^2-a(a+b)\right)\int \frac{1}{\left(\sinh^2(e+fx)+1\right)\sqrt{b\sinh^2(e+fx)+a}}d\sinh(e+fx)\right)+\frac{(a-}{}}{f}$$

$\downarrow$ 224

$$\frac{\frac{1}{2}\left(2b^2\int \frac{1}{1-\frac{b\sinh^2(e+fx)}{b\sinh^2(e+fx)+a}}d\frac{\sinh(e+fx)}{\sqrt{b\sinh^2(e+fx)+a}}-\left(2b^2-a(a+b)\right)\int \frac{1}{\left(\sinh^2(e+fx)+1\right)\sqrt{b\sinh^2(e+fx)+a}}d\sinh(e+fx)\right)+}{f}$$

$\downarrow$ 219

$$\frac{\frac{1}{2}\left(2b^{3/2}\text{arctanh}\left(\frac{\sqrt{b}\sinh(e+fx)}{\sqrt{a+b\sinh^2(e+fx)}}\right)-\left(2b^2-a(a+b)\right)\int \frac{1}{\left(\sinh^2(e+fx)+1\right)\sqrt{b\sinh^2(e+fx)+a}}d\sinh(e+fx)\right)+\frac{(a-b)\,s}{}}{f}$$

$\downarrow$ 291

$$\frac{\frac{1}{2}\left(2b^{3/2}\text{arctanh}\left(\frac{\sqrt{b}\sinh(e+fx)}{\sqrt{a+b\sinh^2(e+fx)}}\right)-\left(2b^2-a(a+b)\right)\int \frac{1}{1-\frac{(b-a)\sinh^2(e+fx)}{b\sinh^2(e+fx)+a}}d\frac{\sinh(e+fx)}{\sqrt{b\sinh^2(e+fx)+a}}\right)+\frac{(a-b)\sinh(e+fx)\sqrt{a}}{2\left(\sinh^2(e+\right.}}{f}$$

$\downarrow$ 216

$$\frac{\frac{1}{2}\left(2b^{3/2}\text{arctanh}\left(\frac{\sqrt{b}\sinh(e+fx)}{\sqrt{a+b\sinh^2(e+fx)}}\right)-\frac{\left(2b^2-a(a+b)\right)\arctan\left(\frac{\sqrt{a-b}\sinh(e+fx)}{\sqrt{a+b\sinh^2(e+fx)}}\right)}{\sqrt{a-b}}\right)+\frac{(a-b)\sinh(e+fx)\sqrt{a+b\sinh^2(e+fx)}}{2\left(\sinh^2(e+fx)+1\right)}}{f}$$

input
```
Int[Sech[e + f*x]^3*(a + b*Sinh[e + f*x]^2)^(3/2),x]
```

output
```
((-(((2*b^2 - a*(a + b))*ArcTan[(Sqrt[a - b]*Sinh[e + f*x])/Sqrt[a + b*Sin
h[e + f*x]^2]])/Sqrt[a - b]) + 2*b^(3/2)*ArcTanh[(Sqrt[b]*Sinh[e + f*x])/S
qrt[a + b*Sinh[e + f*x]^2]])/2 + ((a - b)*Sinh[e + f*x]*Sqrt[a + b*Sinh[e
+ f*x]^2])/(2*(1 + Sinh[e + f*x]^2)))/f
```

Defintions of rubi rules used

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

rule 291
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*((c_) + (d_.)*(x_)^2)), x_Symbol] :> Subst
[Int[1/(c - (b*c - a*d)*x^2), x], x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b, c,
d}, x] && NeQ[b*c - a*d, 0]
```

rule 315
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_), x_Symbol] :> Sim
p[(a*d - c*b)*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^(q - 1)/(2*a*b*(p + 1))),
x] - Simp[1/(2*a*b*(p + 1))   Int[(a + b*x^2)^(p + 1)*(c + d*x^2)^(q - 2)*S
imp[c*(a*d - c*b*(2*p + 3)) + d*(a*d*(2*(q - 1) + 1) - b*c*(2*(p + q) + 1))
*x^2, x], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -
1] && GtQ[q, 1] && IntBinomialQ[a, b, c, d, 2, p, q, x]
```

rule 398
```
Int[((e_) + (f_.)*(x_)^2)/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2])
, x_Symbol] :> Simp[f/b   Int[1/Sqrt[c + d*x^2], x], x] + Simp[(b*e - a*f)/
b   Int[1/((a + b*x^2)*Sqrt[c + d*x^2]), x], x] /; FreeQ[{a, b, c, d, e, f}
, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3669
```
Int[cos[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^
(p_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Simp[ff/f   S
ubst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a + b*ff^2*x^2)^p, x], x, Sin[e + f*x]
/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m - 1)/2]
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.37 (sec) , antiderivative size = 63, normalized size of antiderivative = 0.47

method	result	size
default	$\frac{\text{`int/indef0`}\left(\frac{b^{2} \sinh \left(f x +e \right)^{4}+2 \sinh \left(f x +e \right)^{2} a b +a^{2}}{\cosh \left(f x +e \right)^{4} \sqrt{a +b \sinh \left(f x +e \right)^{2}}}, \sinh \left(f x +e \right)\right)}{f}$	63

input
```
int(sech(f*x+e)^3*(a+b*sinh(f*x+e)^2)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
`int/indef0`((b^2*sinh(f*x+e)^4+2*sinh(f*x+e)^2*a*b+a^2)/cosh(f*x+e)^4/(a+
b*sinh(f*x+e)^2)^(1/2),sinh(f*x+e))/f
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1213 vs. 2(115) = 230.

Time = 0.33 (sec) , antiderivative size = 7126, normalized size of antiderivative = 53.58

$$\int \operatorname{sech}^3(e+fx) \left(a+b\sinh^2(e+fx)\right)^{3/2} dx = \text{Too large to display}$$

input
```
integrate(sech(f*x+e)^3*(a+b*sinh(f*x+e)^2)^(3/2),x, algorithm="fricas")
```

output
```
Too large to include
```

Sympy [F(-1)]

Timed out.

$$\int \operatorname{sech}^3(e+fx)\left(a+b\sinh^2(e+fx)\right)^{3/2}\,dx = \text{Timed out}$$

input
```
integrate(sech(f*x+e)**3*(a+b*sinh(f*x+e)**2)**(3/2),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \operatorname{sech}^3(e+fx)\left(a+b\sinh^2(e+fx)\right)^{3/2}\,dx = \int \left(b\sinh\left(fx+e\right)^2+a\right)^{\frac{3}{2}}\operatorname{sech}\left(fx+e\right)^3\,dx$$

input
```
integrate(sech(f*x+e)^3*(a+b*sinh(f*x+e)^2)^(3/2),x, algorithm="maxima")
```

output
```
integrate((b*sinh(f*x + e)^2 + a)^(3/2)*sech(f*x + e)^3, x)
```

Giac [F(-2)]

Exception generated.

$$\int \operatorname{sech}^3(e+fx)\left(a+b\sinh^2(e+fx)\right)^{3/2}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(sech(f*x+e)^3*(a+b*sinh(f*x+e)^2)^(3/2),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Error: Bad Argument Type
```

Mupad [F(-1)]

Timed out.

$$\int \operatorname{sech}^3(e+fx)\left(a+b\sinh^2(e+fx)\right)^{3/2}\,dx=\int \frac{\left(b\sinh(e+f\,x)^2+a\right)^{3/2}}{\cosh\left(e+f\,x\right)^3}\,dx$$

input
```
int((a + b*sinh(e + f*x)^2)^(3/2)/cosh(e + f*x)^3,x)
```

output
```
int((a + b*sinh(e + f*x)^2)^(3/2)/cosh(e + f*x)^3, x)
```

Reduce [F]

$$\int \operatorname{sech}^3(e+fx)\left(a+b\sinh^2(e+fx)\right)^{3/2}\,dx=\left(\int \sqrt{\sinh\left(fx+e\right)^2b+a}\,\operatorname{sech}(fx+e)^3\sinh\left(fx+e\right)^2dx\right)b+\left(\int \sqrt{\sinh\left(fx+e\right)^2b+a}\,\operatorname{sech}(fx+e)^3dx\right)a$$

input
```
int(sech(f*x+e)^3*(a+b*sinh(f*x+e)^2)^(3/2),x)
```

output
```
int(sqrt(sinh(e + f*x)**2*b + a)*sech(e + f*x)**3*sinh(e + f*x)**2,x)*b +
int(sqrt(sinh(e + f*x)**2*b + a)*sech(e + f*x)**3,x)*a
```

3.330 $\int \operatorname{sech}^5(e+fx)\left(a+b\sinh^2(e+fx)\right)^{3/2}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 126

$$\int \operatorname{sech}^5(e+fx)\left(a+b\sinh^2(e+fx)\right)^{3/2}\,dx=\frac{3a^2\arctan\left(\frac{\sqrt{a-b}\sinh(e+fx)}{\sqrt{a+b\sinh^2(e+fx)}}\right)}{8\sqrt{a-b}f}+\frac{3a\operatorname{sech}(e+fx)\sqrt{a+b\sinh^2(e+fx)}\tanh(e+fx)}{8f}+\frac{\operatorname{sech}^3(e+fx)\left(a+b\sinh^2(e+fx)\right)^{3/2}\tanh(e+fx)}{4f}$$

output
```
3/8*a^2*arctan((a-b)^(1/2)*sinh(f*x+e)/(a+b*sinh(f*x+e)^2)^(1/2))/(a-b)^(1
/2)/f+3/8*a*sech(f*x+e)*(a+b*sinh(f*x+e)^2)^(1/2)*tanh(f*x+e)/f+1/4*sech(f
*x+e)^3*(a+b*sinh(f*x+e)^2)^(3/2)*tanh(f*x+e)/f
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 0.09 (sec) , antiderivative size = 66, normalized size of antiderivative = 0.52

$$\int \operatorname{sech}^5(e+fx) \left(a+b\sinh^2(e+fx)\right)^{3/2} dx = \frac{a^2 \text{Hypergeometric2F1}\left(\frac{1}{2},3,\frac{3}{2},-\frac{(a-b)\sinh^2(e+fx)}{a+b\sinh^2(e+fx)}\right)\sinh(e+fx)}{f\sqrt{a+b\sinh^2(e+fx)}}$$

input
```
Integrate[Sech[e + f*x]^5*(a + b*Sinh[e + f*x]^2)^(3/2),x]
```

output
```
(a^2*Hypergeometric2F1[1/2, 3, 3/2, -(((a - b)*Sinh[e + f*x]^2)/(a + b*Sin
h[e + f*x]^2))]*Sinh[e + f*x])/(f*Sqrt[a + b*Sinh[e + f*x]^2])
```

Rubi [A] (verified)

Time = 0.32 (sec) , antiderivative size = 134, normalized size of antiderivative = 1.06, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.240$, Rules used = {3042, 3669, 292, 292, 291, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \operatorname{sech}^5(e+fx) \left(a+b\sinh^2(e+fx)\right)^{3/2} dx$$

$$\Big\downarrow \; 3042$$

$$\int \frac{\left(a-b\sin(ie+ifx)^2\right)^{3/2}}{\cos(ie+ifx)^5}dx$$

$$\Big\downarrow \; 3669$$

$$\frac{\int \frac{\left(b\sinh^2(e+fx)+a\right)^{3/2}}{\left(\sinh^2(e+fx)+1\right)^3}d\sinh(e+fx)}{f}$$

$$\Big\downarrow \; 292$$

$$\frac{\frac{3}{4}a\int \frac{\sqrt{b\sinh^2(e+fx)+a}}{\left(\sinh^2(e+fx)+1\right)^2}d\sinh(e+fx)+\frac{\sinh(e+fx)\left(a+b\sinh^2(e+fx)\right)^{3/2}}{4\left(\sinh^2(e+fx)+1\right)^2}}{f}$$

$$\Big\downarrow \text{292}$$

$$\frac{\frac{3}{4}a\left(\frac{1}{2}a\int \frac{1}{\left(\sinh^2(e+fx)+1\right)\sqrt{b\sinh^2(e+fx)+a}}d\sinh(e+fx)+\frac{\sinh(e+fx)\sqrt{a+b\sinh^2(e+fx)}}{2\left(\sinh^2(e+fx)+1\right)}\right)+\frac{\sinh(e+fx)\left(a+b\sinh^2(e+fx)\right)^{3/2}}{4\left(\sinh^2(e+fx)+1\right)^2}}{f}$$

$$\Big\downarrow \text{291}$$

$$\frac{\frac{3}{4}a\left(\frac{1}{2}a\int \frac{1}{1-\frac{(b-a)\sinh^2(e+fx)}{b\sinh^2(e+fx)+a}}d\frac{\sinh(e+fx)}{\sqrt{b\sinh^2(e+fx)+a}}+\frac{\sinh(e+fx)\sqrt{a+b\sinh^2(e+fx)}}{2\left(\sinh^2(e+fx)+1\right)}\right)+\frac{\sinh(e+fx)\left(a+b\sinh^2(e+fx)\right)^{3/2}}{4\left(\sinh^2(e+fx)+1\right)^2}}{f}$$

$$\Big\downarrow \text{216}$$

$$\frac{\frac{3}{4}a\left(\frac{a\arctan\left(\frac{\sqrt{a-b}\sinh(e+fx)}{\sqrt{a+b\sinh^2(e+fx)}}\right)}{2\sqrt{a-b}}+\frac{\sinh(e+fx)\sqrt{a+b\sinh^2(e+fx)}}{2\left(\sinh^2(e+fx)+1\right)}\right)+\frac{\sinh(e+fx)\left(a+b\sinh^2(e+fx)\right)^{3/2}}{4\left(\sinh^2(e+fx)+1\right)^2}}{f}$$

input
```
Int[Sech[e + f*x]^5*(a + b*Sinh[e + f*x]^2)^(3/2),x]
```

output
```
((Sinh[e + f*x]*(a + b*Sinh[e + f*x]^2)^(3/2))/(4*(1 + Sinh[e + f*x]^2)^2)
 + (3*a*((a*ArcTan[(Sqrt[a - b]*Sinh[e + f*x])/Sqrt[a + b*Sinh[e + f*x]^2]
])/(2*Sqrt[a - b]) + (Sinh[e + f*x]*Sqrt[a + b*Sinh[e + f*x]^2])/(2*(1 + S
inh[e + f*x]^2))))/4)/f
```

Defintions of rubi rules used

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 291
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*((c_) + (d_.)*(x_)^2)), x_Symbol] :> Subst
[Int[1/(c - (b*c - a*d)*x^2), x], x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b, c,
d}, x] && NeQ[b*c - a*d, 0]
```

rule 292
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_.), x_Symbol] :> Si
mp[(-x)*(a + b*x^2)^(p + 1)*((c + d*x^2)^q/(2*a*(p + 1))), x] - Simp[c*(q/(
a*(p + 1)))   Int[(a + b*x^2)^(p + 1)*(c + d*x^2)^(q - 1), x], x] /; FreeQ[
{a, b, c, d, p}, x] && NeQ[b*c - a*d, 0] && EqQ[2*(p + q + 1) + 1, 0] && Gt
Q[q, 0] && NeQ[p, -1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3669
```
Int[cos[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^
(p_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Simp[ff/f   S
ubst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a + b*ff^2*x^2)^p, x], x, Sin[e + f*x]
/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m - 1)/2]
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.51 (sec) , antiderivative size = 63, normalized size of antiderivative = 0.50

method	result	size
default	$\frac{\text{`int/indef0`}\left(\frac{b^{2} \sinh \left(f x+e\right)^{4}+2 \sinh \left(f x+e\right)^{2} a b+a^{2}}{\cosh \left(f x+e\right)^{6} \sqrt{a+b \sinh \left(f x+e\right)^{2}}}, \sinh \left(f x+e\right)\right)}{f}$	63

input
```
int(sech(f*x+e)^5*(a+b*sinh(f*x+e)^2)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
`int/indef0`((b^2*sinh(f*x+e)^4+2*sinh(f*x+e)^2*a*b+a^2)/cosh(f*x+e)^6/(a+
b*sinh(f*x+e)^2)^(1/2),sinh(f*x+e))/f
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1486 vs. 2(110) = 220.

Time = 0.28 (sec) , antiderivative size = 3089, normalized size of antiderivative = 24.52

$$\int \operatorname{sech}^5(e+fx) \left(a+b\sinh^2(e+fx)\right)^{3/2} \, dx = \text{Too large to display}$$

input
```
integrate(sech(f*x+e)^5*(a+b*sinh(f*x+e)^2)^(3/2),x, algorithm="fricas")
```

output
```
Too large to include
```

Sympy [F(-1)]

Timed out.

$$\int \operatorname{sech}^5(e+fx) \left(a+b\sinh^2(e+fx)\right)^{3/2} \, dx = \text{Timed out}$$

input
```
integrate(sech(f*x+e)**5*(a+b*sinh(f*x+e)**2)**(3/2),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \operatorname{sech}^5(e+fx) \left(a+b\sinh^2(e+fx)\right)^{3/2} \, dx = \int \left(b\sinh\left(fx+e\right)^2+a\right)^{\frac{3}{2}} \operatorname{sech}\left(fx+e\right)^5 \, dx$$

input
```
integrate(sech(f*x+e)^5*(a+b*sinh(f*x+e)^2)^(3/2),x, algorithm="maxima")
```

output
```
integrate((b*sinh(f*x + e)^2 + a)^(3/2)*sech(f*x + e)^5, x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2018 vs. $2(110) = 220$.

Time = 0.69 (sec) , antiderivative size = 2018, normalized size of antiderivative = 16.02

$$\int \operatorname{sech}^5(e+fx)\left(a+b\sinh^2(e+fx)\right)^{3/2}\,dx=\text{Too large to display}$$

input

```
integrate(sech(f*x+e)^5*(a+b*sinh(f*x+e)^2)^(3/2),x, algorithm="giac")
```

output

```
3/4*a^2*arctan(-1/2*(sqrt(b)*e^(2*f*x + 2*e) - sqrt(b*e^(4*f*x + 4*e) + 4*
a*e^(2*f*x + 2*e) - 2*b*e^(2*f*x + 2*e) + b) + sqrt(b))/sqrt(a - b))/(sqrt
(a - b)*f) - 1/2*(3*(sqrt(b)*e^(2*f*x + 2*e) - sqrt(b*e^(4*f*x + 4*e) + 4*
a*e^(2*f*x + 2*e) - 2*b*e^(2*f*x + 2*e) + b))^7*a^2 - 8*(sqrt(b)*e^(2*f*x
+ 2*e) - sqrt(b*e^(4*f*x + 4*e) + 4*a*e^(2*f*x + 2*e) - 2*b*e^(2*f*x + 2*e
) + b))^7*b^2 + 21*(sqrt(b)*e^(2*f*x + 2*e) - sqrt(b*e^(4*f*x + 4*e) + 4*a
*e^(2*f*x + 2*e) - 2*b*e^(2*f*x + 2*e) + b))^6*a^2*sqrt(b) - 64*(sqrt(b)*e
^(2*f*x + 2*e) - sqrt(b*e^(4*f*x + 4*e) + 4*a*e^(2*f*x + 2*e) - 2*b*e^(2*f
*x + 2*e) + b))^6*a*b^(3/2) + 8*(sqrt(b)*e^(2*f*x + 2*e) - sqrt(b*e^(4*f*x
 + 4*e) + 4*a*e^(2*f*x + 2*e) - 2*b*e^(2*f*x + 2*e) + b))^6*b^(5/2) + 44*(
sqrt(b)*e^(2*f*x + 2*e) - sqrt(b*e^(4*f*x + 4*e) + 4*a*e^(2*f*x + 2*e) - 2
*b*e^(2*f*x + 2*e) + b))^5*a^3 - 237*(sqrt(b)*e^(2*f*x + 2*e) - sqrt(b*e^(
4*f*x + 4*e) + 4*a*e^(2*f*x + 2*e) - 2*b*e^(2*f*x + 2*e) + b))^5*a^2*b + 9
6*(sqrt(b)*e^(2*f*x + 2*e) - sqrt(b*e^(4*f*x + 4*e) + 4*a*e^(2*f*x + 2*e)
- 2*b*e^(2*f*x + 2*e) + b))^5*a*b^2 - 8*(sqrt(b)*e^(2*f*x + 2*e) - sqrt(b*
e^(4*f*x + 4*e) + 4*a*e^(2*f*x + 2*e) - 2*b*e^(2*f*x + 2*e) + b))^5*b^3 -
292*(sqrt(b)*e^(2*f*x + 2*e) - sqrt(b*e^(4*f*x + 4*e) + 4*a*e^(2*f*x + 2*e
) - 2*b*e^(2*f*x + 2*e) + b))^4*a^3*sqrt(b) + 141*(sqrt(b)*e^(2*f*x + 2*e)
 - sqrt(b*e^(4*f*x + 4*e) + 4*a*e^(2*f*x + 2*e) - 2*b*e^(2*f*x + 2*e) + b)
)^4*a^2*b^(3/2) - 96*(sqrt(b)*e^(2*f*x + 2*e) - sqrt(b*e^(4*f*x + 4*e) ...
```

Mupad [F(-1)]

Timed out.

$$\int \operatorname{sech}^5(e+fx)\left(a+b\sinh^2(e+fx)\right)^{3/2}\,dx=\int \frac{\left(b\sinh(e+f\,x)^2+a\right)^{3/2}}{\cosh\left(e+f\,x\right)^5}\,dx$$

input
```
int((a + b*sinh(e + f*x)^2)^(3/2)/cosh(e + f*x)^5,x)
```

output
```
int((a + b*sinh(e + f*x)^2)^(3/2)/cosh(e + f*x)^5, x)
```

Reduce [F]

$$\int \operatorname{sech}^5(e+fx)\left(a+b\sinh^2(e+fx)\right)^{3/2}\,dx=\left(\int \sqrt{\sinh\left(fx+e\right)^2 b+a}\,\operatorname{sech}(fx+e)^5\sinh\left(fx+e\right)^2 dx\right)b+\left(\int \sqrt{\sinh\left(fx+e\right)^2 b+a}\,\operatorname{sech}(fx+e)^5\,dx\right)a$$

input
```
int(sech(f*x+e)^5*(a+b*sinh(f*x+e)^2)^(3/2),x)
```

output
```
int(sqrt(sinh(e + f*x)**2*b + a)*sech(e + f*x)**5*sinh(e + f*x)**2,x)*b +
int(sqrt(sinh(e + f*x)**2*b + a)*sech(e + f*x)**5,x)*a
```

3.331 $\int \operatorname{sech}^7(e+fx)\left(a+b\sinh^2(e+fx)\right)^{3/2}dx$

Optimal result

Integrand size = 25, antiderivative size = 205

$$\int \operatorname{sech}^7(e+fx)\left(a+b\sinh^2(e+fx)\right)^{3/2}dx=\frac{a^2(5a-6b)\arctan\left(\frac{\sqrt{a-b}\sinh(e+fx)}{\sqrt{a+b\sinh^2(e+fx)}}\right)}{16(a-b)^{3/2}f}+\frac{a(5a-6b)\operatorname{sech}(e+fx)\sqrt{a+b\sinh^2(e+fx)}\tanh(e+fx)}{16(a-b)f}+\frac{(5a-6b)\operatorname{sech}^3(e+fx)\left(a+b\sinh^2(e+fx)\right)^{3/2}\tanh(e+fx)}{24(a-b)f}+\frac{\operatorname{sech}^5(e+fx)\left(a+b\sinh^2(e+fx)\right)^{5/2}\tanh(e+fx)}{6(a-b)f}$$

output
```
1/16*a^2*(5*a-6*b)*arctan((a-b)^(1/2)*sinh(f*x+e)/(a+b*sinh(f*x+e)^2)^(1/2
))/(a-b)^(3/2)/f+1/16*a*(5*a-6*b)*sech(f*x+e)*(a+b*sinh(f*x+e)^2)^(1/2)*ta
nh(f*x+e)/(a-b)/f+1/24*(5*a-6*b)*sech(f*x+e)^3*(a+b*sinh(f*x+e)^2)^(3/2)*t
anh(f*x+e)/(a-b)/f+1/6*sech(f*x+e)^5*(a+b*sinh(f*x+e)^2)^(5/2)*tanh(f*x+e)
/(a-b)/f
```

Mathematica [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 10.91 (sec) , antiderivative size = 959, normalized size of antiderivative = 4.68

$$\int \operatorname{sech}^7(e+fx) \left(a+b\sinh^2(e+fx)\right)^{3/2} dx = \text{Too large to display}$$

input
```
Integrate[Sech[e + f*x]^7*(a + b*Sinh[e + f*x]^2)^(3/2),x]
```

output
```
(a^2*Sech[e + f*x]^3*(1 + (b*Sinh[e + f*x]^2)/a)^2*Tanh[e + f*x]*(45*a*Arc
Sin[Sqrt[((a - b)*Tanh[e + f*x]^2)/a]] + 30*b*ArcSin[Sqrt[((a - b)*Tanh[e
+ f*x]^2)/a]]*Sinh[e + f*x]^2 + 210*a*Sqrt[(Sech[e + f*x]^2*(a + b*Sinh[e
+ f*x]^2))/a]*(((a - b)*Tanh[e + f*x]^2)/a)^(3/2) + 140*b*Sinh[e + f*x]^2*
Sqrt[(Sech[e + f*x]^2*(a + b*Sinh[e + f*x]^2))/a]*(((a - b)*Tanh[e + f*x]^
2)/a)^(3/2) - 120*a*Sqrt[(Sech[e + f*x]^2*(a + b*Sinh[e + f*x]^2))/a]*(((a
 - b)*Tanh[e + f*x]^2)/a)^(5/2) + 256*a*Hypergeometric2F1[2, 5, 7/2, ((a -
 b)*Tanh[e + f*x]^2)/a]*Sqrt[(Sech[e + f*x]^2*(a + b*Sinh[e + f*x]^2))/a]*
(((a - b)*Tanh[e + f*x]^2)/a)^(5/2) - 80*b*Sinh[e + f*x]^2*Sqrt[(Sech[e +
f*x]^2*(a + b*Sinh[e + f*x]^2))/a]*(((a - b)*Tanh[e + f*x]^2)/a)^(5/2) + 2
56*b*Hypergeometric2F1[2, 5, 7/2, ((a - b)*Tanh[e + f*x]^2)/a]*Sinh[e + f*
x]^2*Sqrt[(Sech[e + f*x]^2*(a + b*Sinh[e + f*x]^2))/a]*(((a - b)*Tanh[e +
f*x]^2)/a)^(5/2) - 512*a*Hypergeometric2F1[2, 5, 7/2, ((a - b)*Tanh[e + f*
x]^2)/a]*Sqrt[(Sech[e + f*x]^2*(a + b*Sinh[e + f*x]^2))/a]*(((a - b)*Tanh[
e + f*x]^2)/a)^(7/2) - 512*b*Hypergeometric2F1[2, 5, 7/2, ((a - b)*Tanh[e
+ f*x]^2)/a]*Sinh[e + f*x]^2*Sqrt[(Sech[e + f*x]^2*(a + b*Sinh[e + f*x]^2)
)/a]*(((a - b)*Tanh[e + f*x]^2)/a)^(7/2) + 256*a*Hypergeometric2F1[2, 5, 7
/2, ((a - b)*Tanh[e + f*x]^2)/a]*Sqrt[(Sech[e + f*x]^2*(a + b*Sinh[e + f*x
]^2))/a]*(((a - b)*Tanh[e + f*x]^2)/a)^(9/2) + 256*b*Hypergeometric2F1[2,
5, 7/2, ((a - b)*Tanh[e + f*x]^2)/a]*Sinh[e + f*x]^2*Sqrt[(Sech[e + f*x...
```

Rubi [A] (verified)

Time = 0.37 (sec) , antiderivative size = 198, normalized size of antiderivative = 0.97, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.280$, Rules used = {3042, 3669, 296, 292, 292, 291, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \operatorname{sech}^7(e+fx)\left(a+b\sinh^2(e+fx)\right)^{3/2}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\left(a-b\sin(ie+ifx)^2\right)^{3/2}}{\cos(ie+ifx)^7}dx$$

$$\Big\downarrow \text{3669}$$

$$\frac{\int \frac{\left(b\sinh^2(e+fx)+a\right)^{3/2}}{\left(\sinh^2(e+fx)+1\right)^4}d\sinh(e+fx)}{f}$$

$$\Big\downarrow \text{296}$$

$$\frac{\frac{(5a-6b)\int \frac{\left(b\sinh^2(e+fx)+a\right)^{3/2}}{\left(\sinh^2(e+fx)+1\right)^3}d\sinh(e+fx)}{6(a-b)}+\frac{\sinh(e+fx)\left(a+b\sinh^2(e+fx)\right)^{5/2}}{6(a-b)\left(\sinh^2(e+fx)+1\right)^3}}{f}$$

$$\Big\downarrow \text{292}$$

$$\frac{\frac{(5a-6b)\left(\frac{3}{4}a\int \frac{\sqrt{b\sinh^2(e+fx)+a}}{\left(\sinh^2(e+fx)+1\right)^2}d\sinh(e+fx)+\frac{\sinh(e+fx)\left(a+b\sinh^2(e+fx)\right)^{3/2}}{4\left(\sinh^2(e+fx)+1\right)^2}\right)}{6(a-b)}+\frac{\sinh(e+fx)\left(a+b\sinh^2(e+fx)\right)^{5/2}}{6(a-b)\left(\sinh^2(e+fx)+1\right)^3}}{f}$$

$$\Big\downarrow \text{292}$$

$$\frac{\frac{(5a-6b)\left(\frac{3}{4}a\left(\frac{1}{2}a\int \frac{1}{\left(\sinh^2(e+fx)+1\right)\sqrt{b\sinh^2(e+fx)+a}}d\sinh(e+fx)+\frac{\sinh(e+fx)\sqrt{a+b\sinh^2(e+fx)}}{2\left(\sinh^2(e+fx)+1\right)}\right)+\frac{\sinh(e+fx)\left(a+b\sinh^2(e+fx)\right)^{3/2}}{4\left(\sinh^2(e+fx)+1\right)^2}\right)}{6(a-b)}+\cdots}{f}$$

$$\Big\downarrow \text{291}$$

$$\frac{\frac{(5a-6b)\left(\frac{3}{4}a\left(\frac{1}{2}a\int \frac{1}{1-\frac{(b-a)\sinh^2(e+fx)}{b\sinh^2(e+fx)+a}}d\frac{\sinh(e+fx)}{\sqrt{b\sinh^2(e+fx)+a}}+\frac{\sinh(e+fx)\sqrt{a+b\sinh^2(e+fx)}}{2\left(\sinh^2(e+fx)+1\right)}\right)+\frac{\sinh(e+fx)\left(a+b\sinh^2(e+fx)\right)^{3/2}}{4\left(\sinh^2(e+fx)+1\right)^2}\right)}{6(a-b)}+\frac{\sinh(e+fx)}{6(a-b)(}}{f}$$

$$\Big\downarrow \ 216$$

$$\frac{\frac{(5a-6b)\left(\frac{3}{4}a\left(\frac{a\arctan\left(\frac{\sqrt{a-b}\sinh(e+fx)}{\sqrt{a+b\sinh^2(e+fx)}}\right)}{2\sqrt{a-b}}+\frac{\sinh(e+fx)\sqrt{a+b\sinh^2(e+fx)}}{2\left(\sinh^2(e+fx)+1\right)}\right)+\frac{\sinh(e+fx)\left(a+b\sinh^2(e+fx)\right)^{3/2}}{4\left(\sinh^2(e+fx)+1\right)^2}\right)}{6(a-b)}+\frac{\sinh(e+fx)(a+b\sinh^2(e+f}{6(a-b)(\sinh^2(e+fx)+1}}{f}$$

input
```
Int[Sech[e + f*x]^7*(a + b*Sinh[e + f*x]^2)^(3/2),x]
```

output
```
((Sinh[e + f*x]*(a + b*Sinh[e + f*x]^2)^(5/2))/(6*(a - b)*(1 + Sinh[e + f*
x]^2)^3) + ((5*a - 6*b)*((Sinh[e + f*x]*(a + b*Sinh[e + f*x]^2)^(3/2))/(4*
(1 + Sinh[e + f*x]^2)^2) + (3*a*((a*ArcTan[(Sqrt[a - b]*Sinh[e + f*x])/Sqr
t[a + b*Sinh[e + f*x]^2]])/(2*Sqrt[a - b]) + (Sinh[e + f*x]*Sqrt[a + b*Sin
h[e + f*x]^2])/(2*(1 + Sinh[e + f*x]^2))))/4))/(6*(a - b)))/f
```

Defintions of rubi rules used

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 291
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*((c_) + (d_.)*(x_)^2)), x_Symbol] :> Subst
[Int[1/(c - (b*c - a*d)*x^2), x], x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b, c,
d}, x] && NeqQ[b*c - a*d, 0]
```

rule 292
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_.), x_Symbol] :> Si
mp[(-x)*(a + b*x^2)^(p + 1)*((c + d*x^2)^q/(2*a*(p + 1))), x] - Simp[c*(q/(
a*(p + 1)))   Int[(a + b*x^2)^(p + 1)*(c + d*x^2)^(q - 1), x], x] /; FreeQ[
{a, b, c, d, p}, x] && NeQ[b*c - a*d, 0] && EqQ[2*(p + q + 1) + 1, 0] && Gt
Q[q, 0] && NeQ[p, -1]
```

rule 296
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_), x_Symbol] :> Sim
p[(-b)*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^(q + 1)/(2*a*(p + 1)*(b*c - a*d))
), x] + Simp[(b*c + 2*(p + 1)*(b*c - a*d))/(2*a*(p + 1)*(b*c - a*d))   Int[
(a + b*x^2)^(p + 1)*(c + d*x^2)^q, x], x] /; FreeQ[{a, b, c, d, q}, x] && N
eQ[b*c - a*d, 0] && EqQ[2*(p + q + 2) + 1, 0] && (LtQ[p, -1] ||  !LtQ[q, -1
]) && NeQ[p, -1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3669
```
Int[cos[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^
(p_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Simp[ff/f   S
ubst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a + b*ff^2*x^2)^p, x], x, Sin[e + f*x]
/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m - 1)/2]
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.20 (sec) , antiderivative size = 63, normalized size of antiderivative = 0.31

$$\frac{\text{`int/indef0`}\left(\frac{b^{2} \sinh(fx+e)^{4}+2 \sinh(fx+e)^{2} a b+a^{2}}{\cosh(fx+e)^{8} \sqrt{a+b \sinh(fx+e)^{2}}}, \sinh\left(fx+e\right)\right)}{f}$$

input
```
int(sech(f*x+e)^7*(a+b*sinh(f*x+e)^2)^(3/2),x)
```

output
```
`int/indef0`((b^2*sinh(f*x+e)^4+2*sinh(f*x+e)^2*a*b+a^2)/cosh(f*x+e)^8/(a+
b*sinh(f*x+e)^2)^(1/2),sinh(f*x+e))/f
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 3758 vs. 2(185) = 370.

Time = 0.78 (sec) , antiderivative size = 7633, normalized size of antiderivative = 37.23

$$\int \operatorname{sech}^7(e+fx) \left(a+b\sinh^2(e+fx)\right)^{3/2} dx = \text{Too large to display}$$

input
```
integrate(sech(f*x+e)^7*(a+b*sinh(f*x+e)^2)^(3/2),x, algorithm="fricas")
```

output
```
Too large to include
```

Sympy [F(-1)]

Timed out.

$$\int \operatorname{sech}^7(e+fx) \left(a+b\sinh^2(e+fx)\right)^{3/2} dx = \text{Timed out}$$

input
```
integrate(sech(f*x+e)**7*(a+b*sinh(f*x+e)**2)**(3/2),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \operatorname{sech}^7(e+fx) \left(a+b\sinh^2(e+fx)\right)^{3/2} dx = \int \left(b\sinh\left(fx+e\right)^2+a\right)^{\frac{3}{2}} \operatorname{sech}\left(fx+e\right)^7 dx$$

input
```
integrate(sech(f*x+e)^7*(a+b*sinh(f*x+e)^2)^(3/2),x, algorithm="maxima")
```

output
```
integrate((b*sinh(f*x + e)^2 + a)^(3/2)*sech(f*x + e)^7, x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 4563 vs. $2(185) = 370$.

Time = 1.58 (sec) , antiderivative size = 4563, normalized size of antiderivative = 22.26

$$\int \operatorname{sech}^7(e+fx)\left(a+b\sinh^2(e+fx)\right)^{3/2}\,dx = \text{Too large to display}$$

input
```
integrate(sech(f*x+e)^7*(a+b*sinh(f*x+e)^2)^(3/2),x, algorithm="giac")
```

output
```
1/8*(5*a^3 - 6*a^2*b)*arctan(-1/2*(sqrt(b)*e^(2*f*x + 2*e) - sqrt(b*e^(4*f
*x + 4*e) + 4*a*e^(2*f*x + 2*e) - 2*b*e^(2*f*x + 2*e) + b) + sqrt(b))/sqrt
(a - b))/((a*f - b*f)*sqrt(a - b)) - 1/12*(15*(sqrt(b)*e^(2*f*x + 2*e) - s
qrt(b*e^(4*f*x + 4*e) + 4*a*e^(2*f*x + 2*e) - 2*b*e^(2*f*x + 2*e) + b))^11
*a^3 - 18*(sqrt(b)*e^(2*f*x + 2*e) - sqrt(b*e^(4*f*x + 4*e) + 4*a*e^(2*f*x
 + 2*e) - 2*b*e^(2*f*x + 2*e) + b))^11*a^2*b + 165*(sqrt(b)*e^(2*f*x + 2*e
) - sqrt(b*e^(4*f*x + 4*e) + 4*a*e^(2*f*x + 2*e) - 2*b*e^(2*f*x + 2*e) + b
))^10*a^3*sqrt(b) - 198*(sqrt(b)*e^(2*f*x + 2*e) - sqrt(b*e^(4*f*x + 4*e)
+ 4*a*e^(2*f*x + 2*e) - 2*b*e^(2*f*x + 2*e) + b))^10*a^2*b^(3/2) - 192*(sq
rt(b)*e^(2*f*x + 2*e) - sqrt(b*e^(4*f*x + 4*e) + 4*a*e^(2*f*x + 2*e) - 2*b
*e^(2*f*x + 2*e) + b))^10*a*b^(5/2) + 192*(sqrt(b)*e^(2*f*x + 2*e) - sqrt(
b*e^(4*f*x + 4*e) + 4*a*e^(2*f*x + 2*e) - 2*b*e^(2*f*x + 2*e) + b))^10*b^(
7/2) + 340*(sqrt(b)*e^(2*f*x + 2*e) - sqrt(b*e^(4*f*x + 4*e) + 4*a*e^(2*f*
x + 2*e) - 2*b*e^(2*f*x + 2*e) + b))^9*a^4 + 77*(sqrt(b)*e^(2*f*x + 2*e) -
 sqrt(b*e^(4*f*x + 4*e) + 4*a*e^(2*f*x + 2*e) - 2*b*e^(2*f*x + 2*e) + b))^
9*a^3*b - 2886*(sqrt(b)*e^(2*f*x + 2*e) - sqrt(b*e^(4*f*x + 4*e) + 4*a*e^(
2*f*x + 2*e) - 2*b*e^(2*f*x + 2*e) + b))^9*a^2*b^2 + 2944*(sqrt(b)*e^(2*f*
x + 2*e) - sqrt(b*e^(4*f*x + 4*e) + 4*a*e^(2*f*x + 2*e) - 2*b*e^(2*f*x + 2
*e) + b))^9*a*b^3 - 640*(sqrt(b)*e^(2*f*x + 2*e) - sqrt(b*e^(4*f*x + 4*e)
+ 4*a*e^(2*f*x + 2*e) - 2*b*e^(2*f*x + 2*e) + b))^9*b^4 + 3060*(sqrt(b)...
```

Mupad [F(-1)]

Timed out.

$$\int \operatorname{sech}^7(e+fx)\left(a+b\sinh^2(e+fx)\right)^{3/2}\,dx=\int \frac{\left(b\sinh(e+f\,x)^2+a\right)^{3/2}}{\cosh\left(e+f\,x\right)^7}\,dx$$

input
```
int((a + b*sinh(e + f*x)^2)^(3/2)/cosh(e + f*x)^7,x)
```

output
```
int((a + b*sinh(e + f*x)^2)^(3/2)/cosh(e + f*x)^7, x)
```

Reduce [F]

$$\int \operatorname{sech}^7(e+fx)\left(a+b\sinh^2(e+fx)\right)^{3/2}\,dx=\left(\int \sqrt{\sinh\left(fx+e\right)^2b+a}\,\operatorname{sech}(fx+e)^7\sinh\left(fx+e\right)^2dx\right)b+\left(\int \sqrt{\sinh\left(fx+e\right)^2b+a}\,\operatorname{sech}(fx+e)^7\,dx\right)a$$

input
```
int(sech(f*x+e)^7*(a+b*sinh(f*x+e)^2)^(3/2),x)
```

output
```
int(sqrt(sinh(e + f*x)**2*b + a)*sech(e + f*x)**7*sinh(e + f*x)**2,x)*b +
int(sqrt(sinh(e + f*x)**2*b + a)*sech(e + f*x)**7,x)*a
```

3.332 $\int \cosh^4(e+fx) \left(a+b\sinh^2(e+fx)\right)^{3/2} dx$

Optimal result

Integrand size = 25, antiderivative size = 355

$$\int \cosh^4(e+fx) \left(a+b\sinh^2(e+fx)\right)^{3/2} dx = -\frac{(2a^2-9ab-b^2)\cosh(e+fx)\sinh(e+fx)\sqrt{a+b\sinh^2(e+fx)}}{35bf} + \frac{3(a+b)\cosh(e+fx)\sinh(e+fx)\left(a+b\sinh^2(e+fx)\right)^{3/2}}{35bf} + \frac{\cosh^3(e+fx)\sinh(e+fx)\left(a+b\sinh^2(e+fx)\right)^{3/2}}{7f} + \frac{2(a+b)(a^2-6ab+b^2)E\left(\arctan(\sinh(e+fx))\middle|1-\frac{b}{a}\right)\operatorname{sech}(e+fx)\sqrt{a+b\sinh^2(e+fx)}}{35b^2f\sqrt{\frac{\operatorname{sech}^2(e+fx)(a+b\sinh^2(e+fx))}{a}}} - \frac{(a^2-18ab+b^2)\operatorname{EllipticF}\left(\arctan(\sinh(e+fx)),1-\frac{b}{a}\right)\operatorname{sech}(e+fx)\sqrt{a+b\sinh^2(e+fx)}}{35bf\sqrt{\frac{\operatorname{sech}^2(e+fx)(a+b\sinh^2(e+fx))}{a}}} - \frac{2(a+b)(a^2-6ab+b^2)\sqrt{a+b\sinh^2(e+fx)}\tanh(e+fx)}{35b^2f}$$

output
```
-1/35*(2*a^2-9*a*b-b^2)*cosh(f*x+e)*sinh(f*x+e)*(a+b*sinh(f*x+e)^2)^(1/2)/
b/f+3/35*(a+b)*cosh(f*x+e)*sinh(f*x+e)*(a+b*sinh(f*x+e)^2)^(3/2)/b/f+1/7*c
osh(f*x+e)^3*sinh(f*x+e)*(a+b*sinh(f*x+e)^2)^(3/2)/f+2/35*(a+b)*(a^2-6*a*b
+b^2)*EllipticE(sinh(f*x+e)/(1+sinh(f*x+e)^2)^(1/2),(1-b/a)^(1/2))*sech(f*
x+e)*(a+b*sinh(f*x+e)^2)^(1/2)/b^2/f/(sech(f*x+e)^2*(a+b*sinh(f*x+e)^2)/a)
^(1/2)-1/35*(a^2-18*a*b+b^2)*InverseJacobiAM(arctan(sinh(f*x+e)),(1-b/a)^(
1/2))*sech(f*x+e)*(a+b*sinh(f*x+e)^2)^(1/2)/b/f/(sech(f*x+e)^2*(a+b*sinh(f
*x+e)^2)/a)^(1/2)-2/35*(a+b)*(a^2-6*a*b+b^2)*(a+b*sinh(f*x+e)^2)^(1/2)*tan
h(f*x+e)/b^2/f
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 2.01 (sec) , antiderivative size = 256, normalized size of antiderivative = 0.72

$$\int \cosh^4(e+fx) \left(a+b\sinh^2(e+fx)\right)^{3/2} dx = \frac{128ia(a^3-5a^2b-5ab^2+b^3)\sqrt{\frac{2a-b+b\cosh(2(e+fx))}{a}}E\left(i(e+fx)\,\middle|\,\frac{b}{a}\right)-64ia(2a^3-11a^2b+8a}{}$$

input
```
Integrate[Cosh[e + f*x]^4*(a + b*Sinh[e + f*x]^2)^(3/2),x]
```

output
```
((128*I)*a*(a^3 - 5*a^2*b - 5*a*b^2 + b^3)*Sqrt[(2*a - b + b*Cosh[2*(e + f
*x)])/a]*EllipticE[I*(e + f*x), b/a] - (64*I)*a*(2*a^3 - 11*a^2*b + 8*a*b^
2 + b^3)*Sqrt[(2*a - b + b*Cosh[2*(e + f*x)])/a]*EllipticF[I*(e + f*x), b/
a] + Sqrt[2]*b*(32*a^3 + 400*a^2*b - 212*a*b^2 + 30*b^3 + b*(144*a^2 + 192
*a*b - 37*b^2)*Cosh[2*(e + f*x)] + 2*b^2*(26*a + b)*Cosh[4*(e + f*x)] + 5*
b^3*Cosh[6*(e + f*x)])*Sinh[2*(e + f*x)])/(2240*b^2*f*Sqrt[2*a - b + b*Cos
h[2*(e + f*x)]])
```

Rubi [A] (verified)

Time = 0.59 (sec) , antiderivative size = 415, normalized size of antiderivative = 1.17, number of steps used = 12, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.440, Rules used = {3042, 3671, 318, 403, 27, 403, 25, 406, 320, 388, 313}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cosh^4(e+fx)\left(a+b\sinh^2(e+fx)\right)^{3/2}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \cos(ie+ifx)^4\left(a-b\sin(ie+ifx)^2\right)^{3/2}dx$$

$$\Big\downarrow \text{3671}$$

$$\frac{\sqrt{\cosh^2(e+fx)}\text{sech}(e+fx)\int\left(\sinh^2(e+fx)+1\right)^{3/2}\left(b\sinh^2(e+fx)+a\right)^{3/2}d\sinh(e+fx)}{f}$$

$$\Big\downarrow \text{318}$$

$$\frac{\sqrt{\cosh^2(e+fx)}\text{sech}(e+fx)\left(\frac{1}{7}\int\frac{\left(\sinh^2(e+fx)+1\right)^{3/2}\left(2(4a-b)b\sinh^2(e+fx)+a(7a-b)\right)}{\sqrt{b\sinh^2(e+fx)+a}}d\sinh(e+fx)+\frac{1}{7}b\sinh(e+fx)\right.}{f}$$

$$\Big\downarrow \text{403}$$

$$\frac{\sqrt{\cosh^2(e+fx)}\text{sech}(e+fx)\left(\frac{1}{7}\left(\frac{\int\frac{3b\sqrt{\sinh^2(e+fx)+1}\left(\left(a^2+9ba-2b^2\right)\sinh^2(e+fx)+a(9a-b)\right)}{\sqrt{b\sinh^2(e+fx)+a}}d\sinh(e+fx)}{5b}+\frac{2}{5}(4a-b)\sinh(e+\right.\right.}{}$$

$$\Big\downarrow \text{27}$$

$$\frac{\sqrt{\cosh^2(e+fx)}\text{sech}(e+fx)\left(\frac{1}{7}\left(\frac{3}{5}\int\frac{\sqrt{\sinh^2(e+fx)+1}\left(\left(a^2+9ba-2b^2\right)\sinh^2(e+fx)+a(9a-b)\right)}{\sqrt{b\sinh^2(e+fx)+a}}d\sinh(e+fx)+\frac{2}{5}(4a-b)\right.\right.}{}$$

$$\Big\downarrow \text{403}$$

$$\sqrt{\cosh^2(e+fx)}\text{sech}(e+fx)\left(\frac{1}{7}\left(\frac{3}{5}\left(\frac{\int -\frac{2(a+b)\left(a^2-6ba+b^2\right)\sinh^2(e+fx)+a\left(a^2-18ba+b^2\right)}{\sqrt{\sinh^2(e+fx)+1}\sqrt{b\sinh^2(e+fx)+a}}d\sinh(e+fx)}{3b}+\frac{(a^2+9ab-2b^2)\sqrt{\sinh^2(e\right.\right.\right.$$

$\downarrow$ 25

$$\sqrt{\cosh^2(e+fx)}\text{sech}(e+fx)\left(\frac{1}{7}\left(\frac{3}{5}\left(\frac{(a^2+9ab-2b^2)\sinh(e+fx)\sqrt{\sinh^2(e+fx)+1}\sqrt{a+b\sinh^2(e+fx)}}{3b}-\frac{\int \frac{2(a+b)\left(a^2-6ba+b^2\right)\sinh}{\sqrt{\sinh^2(e+fx)+1}}\right.\right.\right.$$

$\downarrow$ 406

$$\sqrt{\cosh^2(e+fx)}\text{sech}(e+fx)\left(\frac{1}{7}\left(\frac{3}{5}\left(\frac{(a^2+9ab-2b^2)\sinh(e+fx)\sqrt{\sinh^2(e+fx)+1}\sqrt{a+b\sinh^2(e+fx)}}{3b}-\frac{a\left(a^2-18ab+b^2\right)\int \frac{}{\sqrt{\sinh^2}}\right.\right.\right.$$

$\downarrow$ 320

$$\sqrt{\cosh^2(e+fx)}\text{sech}(e+fx)\left(\frac{1}{7}\left(\frac{3}{5}\left(\frac{(a^2+9ab-2b^2)\sinh(e+fx)\sqrt{\sinh^2(e+fx)+1}\sqrt{a+b\sinh^2(e+fx)}}{3b}-\frac{2(a+b)(a^2-6ab+b^2)\int -}{}\right.\right.\right.$$

$\downarrow$ 388

$$\sqrt{\cosh^2(e+fx)}\text{sech}(e+fx)\left(\frac{1}{7}\left(\frac{3}{5}\left(\frac{(a^2+9ab-2b^2)\sinh(e+fx)\sqrt{\sinh^2(e+fx)+1}\sqrt{a+b\sinh^2(e+fx)}}{3b}-\frac{2(a+b)(a^2-6ab+b^2)\left(\frac{\text{si}}{}\right.}{}\right.\right.\right.$$

$\downarrow$ 313

$$\sqrt{\cosh^2(e+fx)}\operatorname{sech}(e+fx)\left(\frac{1}{7}\left(\frac{3}{5}\left(\frac{(a^2+9ab-2b^2)\sinh(e+fx)\sqrt{\sinh^2(e+fx)+1}\sqrt{a+b\sinh^2(e+fx)}}{3b}-\frac{\frac{(a^2-18ab+b^2)\sqrt{a+b\sinh^2}}{\sqrt{\sinh^2(e+}}}{}\right.\right.\right.$$

input
```
Int[Cosh[e + f*x]^4*(a + b*Sinh[e + f*x]^2)^(3/2),x]
```

output
```
(Sqrt[Cosh[e + f*x]^2]*Sech[e + f*x]*((b*Sinh[e + f*x]*(1 + Sinh[e + f*x]^
2)^(5/2)*Sqrt[a + b*Sinh[e + f*x]^2])/7 + ((2*(4*a - b)*Sinh[e + f*x]*(1 +
 Sinh[e + f*x]^2)^(3/2)*Sqrt[a + b*Sinh[e + f*x]^2])/5 + (3*(((a^2 + 9*a*b
 - 2*b^2)*Sinh[e + f*x]*Sqrt[1 + Sinh[e + f*x]^2]*Sqrt[a + b*Sinh[e + f*x]
^2])/(3*b) - (((a^2 - 18*a*b + b^2)*EllipticF[ArcTan[Sinh[e + f*x]], 1 - b
/a]*Sqrt[a + b*Sinh[e + f*x]^2])/(Sqrt[1 + Sinh[e + f*x]^2]*Sqrt[(a + b*Si
nh[e + f*x]^2)/(a*(1 + Sinh[e + f*x]^2))]) + 2*(a + b)*(a^2 - 6*a*b + b^2)
*((Sinh[e + f*x]*Sqrt[a + b*Sinh[e + f*x]^2])/(b*Sqrt[1 + Sinh[e + f*x]^2]
) - (EllipticE[ArcTan[Sinh[e + f*x]], 1 - b/a]*Sqrt[a + b*Sinh[e + f*x]^2]
)/(b*Sqrt[1 + Sinh[e + f*x]^2]*Sqrt[(a + b*Sinh[e + f*x]^2)/(a*(1 + Sinh[e
 + f*x]^2))])))/(3*b)))/5)/7))/f
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 313
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/((c_) + (d_.)*(x_)^2)^(3/2), x_Symbol] :> Sim
p[(Sqrt[a + b*x^2]/(c*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(c
+ d*x^2)))]))*EllipticE[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; FreeQ
[{a, b, c, d}, x] && PosQ[b/a] && PosQ[d/c]
```

rule 318
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_), x_Symbol] :> Sim
p[d*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^(q - 1)/(b*(2*(p + q) + 1))), x] + S
imp[1/(b*(2*(p + q) + 1))   Int[(a + b*x^2)^p*(c + d*x^2)^(q - 2)*Simp[c*(b
*c*(2*(p + q) + 1) - a*d) + d*(b*c*(2*(p + 2*q - 1) + 1) - a*d*(2*(q - 1) +
 1))*x^2, x], x], x] /; FreeQ[{a, b, c, d, p}, x] && NeQ[b*c - a*d, 0] && G
tQ[q, 1] && NeQ[2*(p + q) + 1, 0] &&  !IGtQ[p, 1] && IntBinomialQ[a, b, c,
d, 2, p, q, x]
```

rule 320
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(Sqrt[a + b*x^2]/(a*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(
c + d*x^2)))]))*EllipticF[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; Fre
eQ[{a, b, c, d}, x] && PosQ[d/c] && PosQ[b/a] &&  !SimplerSqrtQ[b/a, d/c]
```

rule 388
```
Int[(x_)^2/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol]
 :> Simp[x*(Sqrt[a + b*x^2]/(b*Sqrt[c + d*x^2])), x] - Simp[c/b   Int[Sqrt[
a + b*x^2]/(c + d*x^2)^(3/2), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
 a*d, 0] && PosQ[b/a] && PosQ[d/c] &&  !SimplerSqrtQ[b/a, d/c]
```

rule 403
```
Int[((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.)*((e_) + (f_.)*(
x_)^2), x_Symbol] :> Simp[f*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^q/(b*(2*(p +
 q + 1) + 1))), x] + Simp[1/(b*(2*(p + q + 1) + 1))   Int[(a + b*x^2)^p*(c
+ d*x^2)^(q - 1)*Simp[c*(b*e - a*f + b*e*2*(p + q + 1)) + (d*(b*e - a*f) +
f*2*q*(b*c - a*d) + b*d*e*2*(p + q + 1))*x^2, x], x], x] /; FreeQ[{a, b, c,
 d, e, f, p}, x] && GtQ[q, 0] && NeQ[2*(p + q + 1) + 1, 0]
```

rule 406
```
Int[((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.)*((e_) + (f_.)*(
x_)^2), x_Symbol] :> Simp[e   Int[(a + b*x^2)^p*(c + d*x^2)^q, x], x] + Sim
p[f   Int[x^2*(a + b*x^2)^p*(c + d*x^2)^q, x], x] /; FreeQ[{a, b, c, d, e,
f, p, q}, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3671
```
Int[cos[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(
p_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Simp[ff*(Sqrt[
Cos[e + f*x]^2]/(f*Cos[e + f*x]))   Subst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a
 + b*ff^2*x^2)^p, x], x, Sin[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x]
 && IntegerQ[m/2] &&  !IntegerQ[p]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 729 vs. $2(343) = 686$.

Time = 7.96 (sec) , antiderivative size = 730, normalized size of antiderivative = 2.06

method	result
default	$\frac{5\sqrt{-\frac{b}{a}}\, b^3 \cosh(fx+e)^8 \sinh(fx+e)+\left(13\sqrt{-\frac{b}{a}}\, a\, b^2-7\sqrt{-\frac{b}{a}}\, b^3\right) \cosh(fx+e)^6 \sinh(fx+e)+\left(9\sqrt{-\frac{b}{a}}\, a^2 b-\sqrt{-\frac{b}{a}}\, a\, b^2\right) \cosh(fx+e}{}$

input
```
int(cosh(f*x+e)^4*(a+b*sinh(f*x+e)^2)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
1/35*(5*(-b/a)^(1/2)*b^3*cosh(f*x+e)^8*sinh(f*x+e)+(13*(-b/a)^(1/2)*a*b^2-
7*(-b/a)^(1/2)*b^3)*cosh(f*x+e)^6*sinh(f*x+e)+(9*(-b/a)^(1/2)*a^2*b-(-b/a)
^(1/2)*a*b^2)*cosh(f*x+e)^4*sinh(f*x+e)+((-b/a)^(1/2)*a^3+8*(-b/a)^(1/2)*a
^2*b-11*(-b/a)^(1/2)*a*b^2+2*(-b/a)^(1/2)*b^3)*cosh(f*x+e)^2*sinh(f*x+e)+(
b/a*cosh(f*x+e)^2+(a-b)/a)^(1/2)*(cosh(f*x+e)^2)^(1/2)*EllipticF(sinh(f*x+
e)*(-b/a)^(1/2),(1/b*a)^(1/2))*a^3+8*(b/a*cosh(f*x+e)^2+(a-b)/a)^(1/2)*(co
sh(f*x+e)^2)^(1/2)*EllipticF(sinh(f*x+e)*(-b/a)^(1/2),(1/b*a)^(1/2))*a^2*b
-11*(b/a*cosh(f*x+e)^2+(a-b)/a)^(1/2)*(cosh(f*x+e)^2)^(1/2)*EllipticF(sinh
(f*x+e)*(-b/a)^(1/2),(1/b*a)^(1/2))*a*b^2+2*(b/a*cosh(f*x+e)^2+(a-b)/a)^(1
/2)*(cosh(f*x+e)^2)^(1/2)*EllipticF(sinh(f*x+e)*(-b/a)^(1/2),(1/b*a)^(1/2)
)*b^3-2*(b/a*cosh(f*x+e)^2+(a-b)/a)^(1/2)*(cosh(f*x+e)^2)^(1/2)*EllipticE(
sinh(f*x+e)*(-b/a)^(1/2),(1/b*a)^(1/2))*a^3+10*(b/a*cosh(f*x+e)^2+(a-b)/a)
^(1/2)*(cosh(f*x+e)^2)^(1/2)*EllipticE(sinh(f*x+e)*(-b/a)^(1/2),(1/b*a)^(1
/2))*a^2*b+10*(b/a*cosh(f*x+e)^2+(a-b)/a)^(1/2)*(cosh(f*x+e)^2)^(1/2)*Elli
pticE(sinh(f*x+e)*(-b/a)^(1/2),(1/b*a)^(1/2))*a*b^2-2*(b/a*cosh(f*x+e)^2+(
a-b)/a)^(1/2)*(cosh(f*x+e)^2)^(1/2)*EllipticE(sinh(f*x+e)*(-b/a)^(1/2),(1/
b*a)^(1/2))*b^3)/b/(-b/a)^(1/2)/cosh(f*x+e)/(a+b*sinh(f*x+e)^2)^(1/2)/f
```

Fricas [F]

$$\int \cosh^4(e+fx)\left(a+b\sinh^2(e+fx)\right)^{3/2}\,dx=\int \left(b\sinh\left(fx+e\right)^2+a\right)^{\frac{3}{2}}\cosh\left(fx+e\right)^4\,dx$$

input
```
integrate(cosh(f*x+e)^4*(a+b*sinh(f*x+e)^2)^(3/2),x, algorithm="fricas")
```

output
```
integral((b*cosh(f*x + e)^4*sinh(f*x + e)^2 + a*cosh(f*x + e)^4)*sqrt(b*si
nh(f*x + e)^2 + a), x)
```

Sympy [F(-1)]

Timed out.

$$\int \cosh^4(e+fx)\left(a+b\sinh^2(e+fx)\right)^{3/2}\,dx=\text{Timed out}$$

input
```
integrate(cosh(f*x+e)**4*(a+b*sinh(f*x+e)**2)**(3/2),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \cosh^4(e+fx)\left(a+b\sinh^2(e+fx)\right)^{3/2}\,dx=\int \left(b\sinh\left(fx+e\right)^2+a\right)^{\frac{3}{2}}\cosh\left(fx+e\right)^4\,dx$$

input
```
integrate(cosh(f*x+e)^4*(a+b*sinh(f*x+e)^2)^(3/2),x, algorithm="maxima")
```

output
```
integrate((b*sinh(f*x + e)^2 + a)^(3/2)*cosh(f*x + e)^4, x)
```

Giac [F]

$$\int \cosh^4(e+fx)\left(a+b\sinh^2(e+fx)\right)^{3/2}\,dx=\int \left(b\sinh\left(fx+e\right)^2+a\right)^{\frac{3}{2}}\cosh\left(fx+e\right)^4\,dx$$

input
```
integrate(cosh(f*x+e)^4*(a+b*sinh(f*x+e)^2)^(3/2),x, algorithm="giac")
```

output
```
integrate((b*sinh(f*x + e)^2 + a)^(3/2)*cosh(f*x + e)^4, x)
```

Mupad [F(-1)]

Timed out.

$$\int \cosh^4(e+fx)\left(a+b\sinh^2(e+fx)\right)^{3/2}\,dx=\int \cosh(e+f\,x)^4\left(b\sinh(e+f\,x)^2+a\right)^{3/2}\,dx$$

input
```
int(cosh(e + f*x)^4*(a + b*sinh(e + f*x)^2)^(3/2),x)
```

output
```
int(cosh(e + f*x)^4*(a + b*sinh(e + f*x)^2)^(3/2), x)
```

Reduce [F]

$$\int \cosh^4(e+fx)\left(a+b\sinh^2(e+fx)\right)^{3/2}\,dx=\left(\int \sqrt{\sinh\left(fx+e\right)^2b+a}\,\cosh\left(fx+e\right)^4\sinh\left(fx+e\right)^2dx\right)b+\left(\int \sqrt{\sinh\left(fx+e\right)^2b+a}\,\cosh\left(fx+e\right)^4dx\right)a$$

input
```
int(cosh(f*x+e)^4*(a+b*sinh(f*x+e)^2)^(3/2),x)
```

output

```
int(sqrt(sinh(e + f*x)**2*b + a)*cosh(e + f*x)**4*sinh(e + f*x)**2,x)*b +
int(sqrt(sinh(e + f*x)**2*b + a)*cosh(e + f*x)**4,x)*a
```

3.333 $\int \cosh^2(e+fx)\left(a+b\sinh^2(e+fx)\right)^{3/2}dx$

Optimal result

Integrand size = 25, antiderivative size = 294

$$\int \cosh^2(e+fx)\left(a+b\sinh^2(e+fx)\right)^{3/2}dx=\frac{(3a+b)\cosh(e+fx)\sinh(e+fx)\sqrt{a+b\sinh^2(e+fx)}}{15f}+\frac{\cosh(e+fx)\sinh(e+fx)\left(a+b\sinh^2(e+fx)\right)^{3/2}}{5f}-\frac{(3a^2+7ab-2b^2)\,E\left(\arctan(\sinh(e+fx))\left|1-\frac{b}{a}\right.\right)\operatorname{sech}(e+fx)\sqrt{a+b\sinh^2(e+fx)}}{15bf\sqrt{\frac{\operatorname{sech}^2(e+fx)(a+b\sinh^2(e+fx))}{a}}}+\frac{(9a-b)\operatorname{EllipticF}\left(\arctan(\sinh(e+fx)),1-\frac{b}{a}\right)\operatorname{sech}(e+fx)\sqrt{a+b\sinh^2(e+fx)}}{15f\sqrt{\frac{\operatorname{sech}^2(e+fx)(a+b\sinh^2(e+fx))}{a}}}+\frac{(3a^2+7ab-2b^2)\sqrt{a+b\sinh^2(e+fx)}\tanh(e+fx)}{15bf}$$

output
```
1/15*(3*a+b)*cosh(f*x+e)*sinh(f*x+e)*(a+b*sinh(f*x+e)^2)^(1/2)/f+1/5*cosh(
f*x+e)*sinh(f*x+e)*(a+b*sinh(f*x+e)^2)^(3/2)/f-1/15*(3*a^2+7*a*b-2*b^2)*El
lipticE(sinh(f*x+e)/(1+sinh(f*x+e)^2)^(1/2),(1-b/a)^(1/2))*sech(f*x+e)*(a+
b*sinh(f*x+e)^2)^(1/2)/b/f/(sech(f*x+e)^2*(a+b*sinh(f*x+e)^2)/a)^(1/2)+1/1
5*(9*a-b)*InverseJacobiAM(arctan(sinh(f*x+e)),(1-b/a)^(1/2))*sech(f*x+e)*(
a+b*sinh(f*x+e)^2)^(1/2)/f/(sech(f*x+e)^2*(a+b*sinh(f*x+e)^2)/a)^(1/2)+1/1
5*(3*a^2+7*a*b-2*b^2)*(a+b*sinh(f*x+e)^2)^(1/2)*tanh(f*x+e)/b/f
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 1.15 (sec) , antiderivative size = 213, normalized size of antiderivative = 0.72

$$\int \cosh^2(e+fx) \left(a+b\sinh^2(e+fx)\right)^{3/2} dx = \frac{-16ia(3a^2+7ab-2b^2)\sqrt{\frac{2a-b+b\cosh(2(e+fx))}{a}}E\left(i(e+fx)\,\middle|\,\frac{b}{a}\right)+16ia(3a^2-2ab-b^2)\sqrt{\frac{2a-b}{}}}{}$$

input
```
Integrate[Cosh[e + f*x]^2*(a + b*Sinh[e + f*x]^2)^(3/2),x]
```

output
```
((-16*I)*a*(3*a^2 + 7*a*b - 2*b^2)*Sqrt[(2*a - b + b*Cosh[2*(e + f*x)])/a]
*EllipticE[I*(e + f*x), b/a] + (16*I)*a*(3*a^2 - 2*a*b - b^2)*Sqrt[(2*a -
b + b*Cosh[2*(e + f*x)])/a]*EllipticF[I*(e + f*x), b/a] + Sqrt[2]*b*(48*a^
2 - 28*a*b + 5*b^2 + 4*(9*a - 2*b)*b*Cosh[2*(e + f*x)] + 3*b^2*Cosh[4*(e +
 f*x)])*Sinh[2*(e + f*x)])/(240*b*f*Sqrt[2*a - b + b*Cosh[2*(e + f*x)]])
```

Rubi [A] (verified)

Time = 0.51 (sec) , antiderivative size = 347, normalized size of antiderivative = 1.18, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.360$, Rules used = {3042, 3671, 318, 403, 27, 406, 320, 388, 313}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cosh^2(e+fx)\left(a+b\sinh^2(e+fx)\right)^{3/2}dx$$

$\Big\downarrow$ 3042

$$\int \cos(ie+ifx)^2\left(a-b\sin(ie+ifx)^2\right)^{3/2}dx$$

$\Big\downarrow$ 3671

$$\frac{\sqrt{\cosh^2(e+fx)}\text{sech}(e+fx)\int \sqrt{\sinh^2(e+fx)+1}\left(b\sinh^2(e+fx)+a\right)^{3/2}d\sinh(e+fx)}{f}$$

$\Big\downarrow$ 318

$$\frac{\sqrt{\cosh^2(e+fx)}\text{sech}(e+fx)\left(\frac{1}{5}\int \frac{\sqrt{\sinh^2(e+fx)+1}\left(2(3a-b)b\sinh^2(e+fx)+a(5a-b)\right)}{\sqrt{b\sinh^2(e+fx)+a}}d\sinh(e+fx)+\frac{1}{5}b\sinh(e+fx)\left(s\right.}{f}$$

$\Big\downarrow$ 403

$$\frac{\sqrt{\cosh^2(e+fx)}\text{sech}(e+fx)\left(\frac{1}{5}\left(\frac{\int \frac{b\left(\left(3a^2+7ba-2b^2\right)\sinh^2(e+fx)+a(9a-b)\right)}{\sqrt{\sinh^2(e+fx)+1}\sqrt{b\sinh^2(e+fx)+a}}d\sinh(e+fx)}{3b}+\frac{2}{3}(3a-b)\sqrt{\sinh^2(e+fx)+1}\,s\right.\right.}{f}$$

$\Big\downarrow$ 27

$$\frac{\sqrt{\cosh^2(e+fx)}\text{sech}(e+fx)\left(\frac{1}{5}\left(\frac{1}{3}\int \frac{\left(3a^2+7ba-2b^2\right)\sinh^2(e+fx)+a(9a-b)}{\sqrt{\sinh^2(e+fx)+1}\sqrt{b\sinh^2(e+fx)+a}}d\sinh(e+fx)+\frac{2}{3}(3a-b)\sqrt{\sinh^2(e+fx\right.\right.}{f}$$

$\Big\downarrow$ 406

$$\frac{\sqrt{\cosh^2(e+fx)}\text{sech}(e+fx)\left(\frac{1}{5}\left(\frac{1}{3}\left(\left(3a^2+7ab-2b^2\right)\int \frac{\sinh^2(e+fx)}{\sqrt{\sinh^2(e+fx)+1}\sqrt{b\sinh^2(e+fx)+a}}d\sinh(e+fx)+a(9a-\right.\right.\right.}{}$$

$\Big\downarrow$ 320

$$\sqrt{\cosh^2(e+fx)}\text{sech}(e+fx)\left(\frac{1}{5}\left(\frac{1}{3}\left(\left(3a^2+7ab-2b^2\right)\int\frac{\sinh^2(e+fx)}{\sqrt{\sinh^2(e+fx)+1}\sqrt{b\sinh^2(e+fx)+a}}d\sinh(e+fx)+\frac{(9a-b)}{}\right.\right.\right.$$

$$\Big\downarrow \text{388}$$

$$\sqrt{\cosh^2(e+fx)}\text{sech}(e+fx)\left(\frac{1}{5}\left(\frac{1}{3}\left(\left(3a^2+7ab-2b^2\right)\left(\frac{\sinh(e+fx)\sqrt{a+b\sinh^2(e+fx)}}{b\sqrt{\sinh^2(e+fx)+1}}-\frac{\int\frac{\sqrt{b\sinh^2(e+fx)+a}}{\left(\sinh^2(e+fx)+1\right)^{3/2}}d\sinh(e+}{b}\right.\right.\right.\right.$$

$$\Big\downarrow \text{313}$$

$$\sqrt{\cosh^2(e+fx)}\text{sech}(e+fx)\left(\frac{1}{5}\left(\frac{1}{3}\left(\left(3a^2+7ab-2b^2\right)\left(\frac{\sinh(e+fx)\sqrt{a+b\sinh^2(e+fx)}}{b\sqrt{\sinh^2(e+fx)+1}}-\frac{\sqrt{a+b\sinh^2(e+fx)}E\left(\arctan(\text{si}\right.}{b\sqrt{\sinh^2(e+fx)+1}\sqrt{\frac{a+b}{a\left(\sin\right.}}}\right.\right.\right.\right.$$

input
```
Int[Cosh[e + f*x]^2*(a + b*Sinh[e + f*x]^2)^(3/2),x]
```

output
```
(Sqrt[Cosh[e + f*x]^2]*Sech[e + f*x]*((b*Sinh[e + f*x]*(1 + Sinh[e + f*x]^
2)^(3/2)*Sqrt[a + b*Sinh[e + f*x]^2])/5 + ((2*(3*a - b)*Sinh[e + f*x]*Sqrt
[1 + Sinh[e + f*x]^2]*Sqrt[a + b*Sinh[e + f*x]^2])/3 + (((9*a - b)*Ellipti
cF[ArcTan[Sinh[e + f*x]], 1 - b/a]*Sqrt[a + b*Sinh[e + f*x]^2])/(Sqrt[1 +
Sinh[e + f*x]^2]*Sqrt[(a + b*Sinh[e + f*x]^2)/(a*(1 + Sinh[e + f*x]^2))])
+ (3*a^2 + 7*a*b - 2*b^2)*((Sinh[e + f*x]*Sqrt[a + b*Sinh[e + f*x]^2])/(b*
Sqrt[1 + Sinh[e + f*x]^2]) - (EllipticE[ArcTan[Sinh[e + f*x]], 1 - b/a]*Sq
rt[a + b*Sinh[e + f*x]^2])/(b*Sqrt[1 + Sinh[e + f*x]^2]*Sqrt[(a + b*Sinh[e
 + f*x]^2)/(a*(1 + Sinh[e + f*x]^2))]))))/3)/5))/f
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 313
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/((c_) + (d_.)*(x_)^2)^(3/2), x_Symbol] :> Sim
p[(Sqrt[a + b*x^2]/(c*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(c
+ d*x^2)))]))*EllipticE[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; FreeQ
[{a, b, c, d}, x] && PosQ[b/a] && PosQ[d/c]
```

rule 318
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_), x_Symbol] :> Sim
p[d*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^(q - 1)/(b*(2*(p + q) + 1))), x] + S
imp[1/(b*(2*(p + q) + 1))   Int[(a + b*x^2)^p*(c + d*x^2)^(q - 2)*Simp[c*(b
*c*(2*(p + q) + 1) - a*d) + d*(b*c*(2*(p + 2*q - 1) + 1) - a*d*(2*(q - 1) +
 1))*x^2, x], x], x] /; FreeQ[{a, b, c, d, p}, x] && NeQ[b*c - a*d, 0] && G
tQ[q, 1] && NeQ[2*(p + q) + 1, 0] &&  !IGtQ[p, 1] && IntBinomialQ[a, b, c,
d, 2, p, q, x]
```

rule 320
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(Sqrt[a + b*x^2]/(a*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(
c + d*x^2)))]))*EllipticF[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; Fre
eQ[{a, b, c, d}, x] && PosQ[d/c] && PosQ[b/a] &&  !SimplerSqrtQ[b/a, d/c]
```

rule 388
```
Int[(x_)^2/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol]
 :> Simp[x*(Sqrt[a + b*x^2]/(b*Sqrt[c + d*x^2])), x] - Simp[c/b   Int[Sqrt[
a + b*x^2]/(c + d*x^2)^(3/2), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
 a*d, 0] && PosQ[b/a] && PosQ[d/c] &&  !SimplerSqrtQ[b/a, d/c]
```

rule 403
```
Int[((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.)*((e_) + (f_.)*(
x_)^2), x_Symbol] :> Simp[f*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^q/(b*(2*(p +
 q + 1) + 1))), x] + Simp[1/(b*(2*(p + q + 1) + 1))   Int[(a + b*x^2)^p*(c
+ d*x^2)^(q - 1)*Simp[c*(b*e - a*f + b*e*2*(p + q + 1)) + (d*(b*e - a*f) +
f*2*q*(b*c - a*d) + b*d*e*2*(p + q + 1))*x^2, x], x], x] /; FreeQ[{a, b, c,
 d, e, f, p}, x] && GtQ[q, 0] && NeQ[2*(p + q + 1) + 1, 0]
```

rule 406
```
Int[((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.)*((e_) + (f_.)*(
x_)^2), x_Symbol] :> Simp[e   Int[(a + b*x^2)^p*(c + d*x^2)^q, x], x] + Sim
p[f   Int[x^2*(a + b*x^2)^p*(c + d*x^2)^q, x], x] /; FreeQ[{a, b, c, d, e,
f, p, q}, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3671
```
Int[cos[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(
p_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Simp[ff*(Sqrt[
Cos[e + f*x]^2]/(f*Cos[e + f*x]))   Subst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a
 + b*ff^2*x^2)^p, x], x, Sin[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x]
 && IntegerQ[m/2] &&  !IntegerQ[p]
```

Maple [A] (verified)

Time = 5.87 (sec) , antiderivative size = 535, normalized size of antiderivative = 1.82

method	result
default	$3\sqrt{-\frac{b}{a}}\, b^2 \cosh(fx+e)^6 \sinh(fx+e)+\left(9\sqrt{-\frac{b}{a}}\, ab-5\sqrt{-\frac{b}{a}}\, b^2\right)\cosh(fx+e)^4 \sinh(fx+e)+\left(6\sqrt{-\frac{b}{a}}\, a^2-8\sqrt{-\frac{b}{a}}\, ab+2\sqrt{-\frac{b}{a}}\, b^2\right)\cosh$

input
```
int(cosh(f*x+e)^2*(a+b*sinh(f*x+e)^2)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
1/15*(3*(-b/a)^(1/2)*b^2*cosh(f*x+e)^6*sinh(f*x+e)+(9*(-b/a)^(1/2)*a*b-5*(
-b/a)^(1/2)*b^2)*cosh(f*x+e)^4*sinh(f*x+e)+(6*(-b/a)^(1/2)*a^2-8*(-b/a)^(1
/2)*a*b+2*(-b/a)^(1/2)*b^2)*cosh(f*x+e)^2*sinh(f*x+e)+6*a^2*(b/a*cosh(f*x+
e)^2+(a-b)/a)^(1/2)*(cosh(f*x+e)^2)^(1/2)*EllipticF(sinh(f*x+e)*(-b/a)^(1/
2),(1/b*a)^(1/2))-8*a*(b/a*cosh(f*x+e)^2+(a-b)/a)^(1/2)*(cosh(f*x+e)^2)^(1
/2)*EllipticF(sinh(f*x+e)*(-b/a)^(1/2),(1/b*a)^(1/2))*b+2*(b/a*cosh(f*x+e)
^2+(a-b)/a)^(1/2)*(cosh(f*x+e)^2)^(1/2)*EllipticF(sinh(f*x+e)*(-b/a)^(1/2)
,(1/b*a)^(1/2))*b^2+3*(b/a*cosh(f*x+e)^2+(a-b)/a)^(1/2)*(cosh(f*x+e)^2)^(1
/2)*EllipticE(sinh(f*x+e)*(-b/a)^(1/2),(1/b*a)^(1/2))*a^2+7*(b/a*cosh(f*x+
e)^2+(a-b)/a)^(1/2)*(cosh(f*x+e)^2)^(1/2)*EllipticE(sinh(f*x+e)*(-b/a)^(1/
2),(1/b*a)^(1/2))*a*b-2*(b/a*cosh(f*x+e)^2+(a-b)/a)^(1/2)*(cosh(f*x+e)^2)^
(1/2)*EllipticE(sinh(f*x+e)*(-b/a)^(1/2),(1/b*a)^(1/2))*b^2)/(-b/a)^(1/2)/
cosh(f*x+e)/(a+b*sinh(f*x+e)^2)^(1/2)/f
```

Fricas [F]

$$\int \cosh^2(e+fx)\left(a+b\sinh^2(e+fx)\right)^{3/2}\,dx=\int \left(b\sinh\left(fx+e\right)^2+a\right)^{\frac{3}{2}}\cosh\left(fx+e\right)^2\,dx$$

input
```
integrate(cosh(f*x+e)^2*(a+b*sinh(f*x+e)^2)^(3/2),x, algorithm="fricas")
```

output
```
integral((b*cosh(f*x + e)^2*sinh(f*x + e)^2 + a*cosh(f*x + e)^2)*sqrt(b*si
nh(f*x + e)^2 + a), x)
```

Sympy [F(-1)]

Timed out.

$$\int \cosh^2(e+fx)\left(a+b\sinh^2(e+fx)\right)^{3/2}\,dx=\text{Timed out}$$

input
```
integrate(cosh(f*x+e)**2*(a+b*sinh(f*x+e)**2)**(3/2),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \cosh^2(e+fx)\left(a+b\sinh^2(e+fx)\right)^{3/2}\,dx=\int \left(b\sinh\left(fx+e\right)^2+a\right)^{\frac{3}{2}}\cosh\left(fx+e\right)^2\,dx$$

input
```
integrate(cosh(f*x+e)^2*(a+b*sinh(f*x+e)^2)^(3/2),x, algorithm="maxima")
```

output
```
integrate((b*sinh(f*x + e)^2 + a)^(3/2)*cosh(f*x + e)^2, x)
```

Giac [F]

$$\int \cosh^2(e+fx)\left(a+b\sinh^2(e+fx)\right)^{3/2}\,dx=\int \left(b\sinh\left(fx+e\right)^2+a\right)^{\frac{3}{2}}\cosh\left(fx+e\right)^2\,dx$$

input
```
integrate(cosh(f*x+e)^2*(a+b*sinh(f*x+e)^2)^(3/2),x, algorithm="giac")
```

output
```
integrate((b*sinh(f*x + e)^2 + a)^(3/2)*cosh(f*x + e)^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int \cosh^2(e+fx)\left(a+b\sinh^2(e+fx)\right)^{3/2}\,dx=\int \cosh(e+f\,x)^2\,\left(b\sinh(e+f\,x)^2+a\right)^{3/2}\,dx$$

input
```
int(cosh(e + f*x)^2*(a + b*sinh(e + f*x)^2)^(3/2),x)
```

output
```
int(cosh(e + f*x)^2*(a + b*sinh(e + f*x)^2)^(3/2), x)
```

Reduce [F]

$$\int \cosh^2(e+fx)\left(a+b\sinh^2(e+fx)\right)^{3/2}\,dx=\left(\int \sqrt{\sinh(fx+e)^2 b+a}\,\cosh(fx+e)^2\sinh(fx+e)^2\,dx\right)b+\left(\int \sqrt{\sinh(fx+e)^2 b+a}\,\cosh(fx+e)^2\,dx\right)a$$

input
```
int(cosh(f*x+e)^2*(a+b*sinh(f*x+e)^2)^(3/2),x)
```

output
```
int(sqrt(sinh(e + f*x)**2*b + a)*cosh(e + f*x)**2*sinh(e + f*x)**2,x)*b +
int(sqrt(sinh(e + f*x)**2*b + a)*cosh(e + f*x)**2,x)*a
```

3.334 $\int \left(a+b\sinh^2(e+fx)\right)^{3/2}\,dx$

Optimal result

Integrand size = 16, antiderivative size = 176

$$\int \left(a+b\sinh^2(e+fx)\right)^{3/2}\,dx = \frac{b\cosh(e+fx)\sinh(e+fx)\sqrt{a+b\sinh^2(e+fx)}}{3f} - \frac{2i(2a-b)E\left(ie+ifx\middle|\frac{b}{a}\right)\sqrt{a+b\sinh^2(e+fx)}}{3f\sqrt{\frac{a+b\sinh^2(e+fx)}{a}}} + \frac{ia(a-b)\,\text{EllipticF}\left(ie+ifx,\frac{b}{a}\right)\sqrt{\frac{a+b\sinh^2(e+fx)}{a}}}{3f\sqrt{a+b\sinh^2(e+fx)}}$$

output
```
1/3*b*cosh(f*x+e)*sinh(f*x+e)*(a+b*sinh(f*x+e)^2)^(1/2)/f-2/3*I*(2*a-b)*El
lipticE(sin(I*e+I*f*x),(b/a)^(1/2))*(a+b*sinh(f*x+e)^2)^(1/2)/f/((a+b*sinh
(f*x+e)^2)/a)^(1/2)+1/3*I*a*(a-b)*InverseJacobiAM(I*e+I*f*x,(b/a)^(1/2))*(
(a+b*sinh(f*x+e)^2)/a)^(1/2)/f/(a+b*sinh(f*x+e)^2)^(1/2)
```

Mathematica [A] (verified)

Time = 0.52 (sec) , antiderivative size = 169, normalized size of antiderivative = 0.96

$$\int \left(a+b\sinh^2(e+fx)\right)^{3/2}\,dx = \frac{-4i\sqrt{2}a(2a-b)\sqrt{\frac{2a-b+b\cosh(2(e+fx))}{a}}E\left(i(e+fx)\,\middle|\,\frac{b}{a}\right)+2i\sqrt{2}a(a-b)\sqrt{\frac{2a-b+b\cosh(2(e+fx))}{a}}\,\mathrm{E}}{6f\sqrt{4a-2b+2b\cosh(2(e}}$$

input
```
Integrate[(a + b*Sinh[e + f*x]^2)^(3/2),x]
```

output
```
((-4*I)*Sqrt[2]*a*(2*a - b)*Sqrt[(2*a - b + b*Cosh[2*(e + f*x)])/a]*Ellipt
icE[I*(e + f*x), b/a] + (2*I)*Sqrt[2]*a*(a - b)*Sqrt[(2*a - b + b*Cosh[2*(
e + f*x)])/a]*EllipticF[I*(e + f*x), b/a] + b*(2*a - b + b*Cosh[2*(e + f*x
)])*Sinh[2*(e + f*x)])/(6*f*Sqrt[4*a - 2*b + 2*b*Cosh[2*(e + f*x)]])
```

Rubi [A] (verified)

Time = 0.92 (sec) , antiderivative size = 175, normalized size of antiderivative = 0.99, number of steps used = 11, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.688$, Rules used = {3042, 3659, 3042, 3651, 3042, 3657, 3042, 3656, 3662, 3042, 3661}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(a+b\sinh^2(e+fx)\right)^{3/2}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \left(a-b\sin(ie+ifx)^2\right)^{3/2}dx$$

$$\Big\downarrow \text{3659}$$

$$\frac{1}{3}\int \frac{2(2a-b)b\sinh^2(e+fx)+a(3a-b)}{\sqrt{b\sinh^2(e+fx)+a}}dx+\frac{b\sinh(e+fx)\cosh(e+fx)\sqrt{a+b\sinh^2(e+fx)}}{3f}$$

$$\Big\downarrow \text{3042}$$

$$\frac{b\sinh(e+fx)\cosh(e+fx)\sqrt{a+b\sinh^2(e+fx)}}{3f}+\frac{1}{3}\int\frac{a(3a-b)-2(2a-b)b\sin(ie+ifx)^2}{\sqrt{a-b\sin(ie+ifx)^2}}dx$$

$$\Big\downarrow \text{3651}$$

$$\frac{1}{3}\left(2(2a-b)\int\sqrt{b\sinh^2(e+fx)+a}dx-a(a-b)\int\frac{1}{\sqrt{b\sinh^2(e+fx)+a}}dx\right)+\frac{b\sinh(e+fx)\cosh(e+fx)\sqrt{a+b\sinh^2(e+fx)}}{3f}$$

$$\Big\downarrow \text{3042}$$

$$\frac{b\sinh(e+fx)\cosh(e+fx)\sqrt{a+b\sinh^2(e+fx)}}{3f}+\frac{1}{3}\left(2(2a-b)\int\sqrt{a-b\sin(ie+ifx)^2}dx-a(a-b)\int\frac{1}{\sqrt{a-b\sin(ie+ifx)^2}}dx\right)$$

$$\Big\downarrow \text{3657}$$

$$\frac{b\sinh(e+fx)\cosh(e+fx)\sqrt{a+b\sinh^2(e+fx)}}{3f}+\frac{1}{3}\left(\frac{2(2a-b)\sqrt{a+b\sinh^2(e+fx)}\int\sqrt{\frac{b\sinh^2(e+fx)}{a}+1}dx}{\sqrt{\frac{b\sinh^2(e+fx)}{a}+1}}-a(a-b)\int\frac{1}{\sqrt{a-b\sin(ie+ifx)^2}}dx\right)$$

$$\Big\downarrow \text{3042}$$

$$\frac{b\sinh(e+fx)\cosh(e+fx)\sqrt{a+b\sinh^2(e+fx)}}{3f}+\frac{1}{3}\left(\frac{2(2a-b)\sqrt{a+b\sinh^2(e+fx)}\int\sqrt{1-\frac{b\sin(ie+ifx)^2}{a}}dx}{\sqrt{\frac{b\sinh^2(e+fx)}{a}+1}}-a(a-b)\int\frac{1}{\sqrt{a-b\sin(ie+ifx)^2}}dx\right)$$

$$\Big\downarrow \text{3656}$$

$$\frac{b\sinh(e+fx)\cosh(e+fx)\sqrt{a+b\sinh^2(e+fx)}}{3f}+\frac{1}{3}\left(-a(a-b)\int\frac{1}{\sqrt{a-b\sin(ie+ifx)^2}}dx-\frac{2i(2a-b)\sqrt{a+b\sinh^2(e+fx)}E\left(ie+ifx|\frac{b}{a}\right)}{f\sqrt{\frac{b\sinh^2(e+fx)}{a}+1}}\right)$$

$\Big\downarrow$ 3662

$$\frac{b\sinh(e+fx)\cosh(e+fx)\sqrt{a+b\sinh^2(e+fx)}}{3f}+\frac{1}{3}\left(-\frac{a(a-b)\sqrt{\frac{b\sinh^2(e+fx)}{a}+1}\int\frac{1}{\sqrt{\frac{b\sinh^2(e+fx)}{a}+1}}dx}{\sqrt{a+b\sinh^2(e+fx)}}-\frac{2i(2a-b)\sqrt{a+b\sinh^2(e+fx)}E\left(ie+ifx|\frac{b}{a}\right)}{f\sqrt{\frac{b\sinh^2(e+fx)}{a}+1}}\right)$$

$\Big\downarrow$ 3042

$$\frac{b\sinh(e+fx)\cosh(e+fx)\sqrt{a+b\sinh^2(e+fx)}}{3f}+\frac{1}{3}\left(-\frac{a(a-b)\sqrt{\frac{b\sinh^2(e+fx)}{a}+1}\int\frac{1}{\sqrt{1-\frac{b\sin(ie+ifx)^2}{a}}}dx}{\sqrt{a+b\sinh^2(e+fx)}}-\frac{2i(2a-b)\sqrt{a+b\sinh^2(e+fx)}E\left(ie+ifx|\frac{b}{a}\right)}{f\sqrt{\frac{b\sinh^2(e+fx)}{a}+1}}\right)$$

$\Big\downarrow$ 3661

$$\frac{b\sinh(e+fx)\cosh(e+fx)\sqrt{a+b\sinh^2(e+fx)}}{3f}+\frac{1}{3}\left(\frac{ia(a-b)\sqrt{\frac{b\sinh^2(e+fx)}{a}+1}\,\text{EllipticF}\left(ie+ifx,\frac{b}{a}\right)}{f\sqrt{a+b\sinh^2(e+fx)}}-\frac{2i(2a-b)\sqrt{a+b\sinh^2(e+fx)}E\left(ie+ifx|\frac{b}{a}\right)}{f\sqrt{\frac{b\sinh^2(e+fx)}{a}+1}}\right)$$

input
```
Int[(a + b*Sinh[e + f*x]^2)^(3/2),x]
```

output
```
(b*Cosh[e + f*x]*Sinh[e + f*x]*Sqrt[a + b*Sinh[e + f*x]^2])/(3*f) + (((-2*
I)*(2*a - b)*EllipticE[I*e + I*f*x, b/a]*Sqrt[a + b*Sinh[e + f*x]^2])/(f*S
qrt[1 + (b*Sinh[e + f*x]^2)/a]) + (I*a*(a - b)*EllipticF[I*e + I*f*x, b/a]
*Sqrt[1 + (b*Sinh[e + f*x]^2)/a])/(f*Sqrt[a + b*Sinh[e + f*x]^2]))/3
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3651
```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)]^2)/Sqrt[(a_) + (b_.)*sin[(e_.) +
 (f_.)*(x_)]^2], x_Symbol] :> Simp[B/b   Int[Sqrt[a + b*Sin[e + f*x]^2], x]
, x] + Simp[(A*b - a*B)/b   Int[1/Sqrt[a + b*Sin[e + f*x]^2], x], x] /; Fre
eQ[{a, b, e, f, A, B}, x]
```

rule 3656
```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2], x_Symbol] :> Simp[(Sqrt[a
]/f)*EllipticE[e + f*x, -b/a], x] /; FreeQ[{a, b, e, f}, x] && GtQ[a, 0]
```

rule 3657
```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2], x_Symbol] :> Simp[Sqrt[a
+ b*Sin[e + f*x]^2]/Sqrt[1 + b*(Sin[e + f*x]^2/a)]   Int[Sqrt[1 + (b*Sin[e
+ f*x]^2)/a], x], x] /; FreeQ[{a, b, e, f}, x] &&  !GtQ[a, 0]
```

rule 3659
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> Simp[(-b)*C
os[e + f*x]*Sin[e + f*x]*((a + b*Sin[e + f*x]^2)^(p - 1)/(2*f*p)), x] + Sim
p[1/(2*p)   Int[(a + b*Sin[e + f*x]^2)^(p - 2)*Simp[a*(b + 2*a*p) + b*(2*a
+ b)*(2*p - 1)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, e, f}, x] && NeQ[
a + b, 0] && GtQ[p, 1]
```

rule 3661
```
Int[1/Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2], x_Symbol] :> Simp[(1/(S
qrt[a]*f))*EllipticF[e + f*x, -b/a], x] /; FreeQ[{a, b, e, f}, x] && GtQ[a,
 0]
```

rule 3662
```
Int[1/Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2], x_Symbol] :> Simp[Sqrt[
1 + b*(Sin[e + f*x]^2/a)]/Sqrt[a + b*Sin[e + f*x]^2]   Int[1/Sqrt[1 + (b*Si
n[e + f*x]^2)/a], x], x] /; FreeQ[{a, b, e, f}, x] &&  !GtQ[a, 0]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 427 vs. $2(159) = 318$.

Time = 3.86 (sec) , antiderivative size = 428, normalized size of antiderivative = 2.43

method	result
default	$\frac{\sqrt{-\frac{b}{a}}\, b^2 \cosh(fx+e)^4 \sinh(fx+e)+\sqrt{-\frac{b}{a}}\, ab \cosh(fx+e)^2 \sinh(fx+e)-\sqrt{-\frac{b}{a}}\, b^2 \cosh(fx+e)^2 \sinh(fx+e)+3a^2\sqrt{\frac{b\cosh(fx+e)^2}{a}+}}{}$

input
```
int((a+b*sinh(f*x+e)^2)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
1/3*((-b/a)^(1/2)*b^2*cosh(f*x+e)^4*sinh(f*x+e)+(-b/a)^(1/2)*a*b*cosh(f*x+
e)^2*sinh(f*x+e)-(-b/a)^(1/2)*b^2*cosh(f*x+e)^2*sinh(f*x+e)+3*a^2*(b/a*cos
h(f*x+e)^2+(a-b)/a)^(1/2)*(cosh(f*x+e)^2)^(1/2)*EllipticF(sinh(f*x+e)*(-b/
a)^(1/2),(1/b*a)^(1/2))-5*a*(b/a*cosh(f*x+e)^2+(a-b)/a)^(1/2)*(cosh(f*x+e)
^2)^(1/2)*EllipticF(sinh(f*x+e)*(-b/a)^(1/2),(1/b*a)^(1/2))*b+2*(b/a*cosh(
f*x+e)^2+(a-b)/a)^(1/2)*(cosh(f*x+e)^2)^(1/2)*EllipticF(sinh(f*x+e)*(-b/a)
^(1/2),(1/b*a)^(1/2))*b^2+4*(b/a*cosh(f*x+e)^2+(a-b)/a)^(1/2)*(cosh(f*x+e)
^2)^(1/2)*EllipticE(sinh(f*x+e)*(-b/a)^(1/2),(1/b*a)^(1/2))*a*b-2*(b/a*cos
h(f*x+e)^2+(a-b)/a)^(1/2)*(cosh(f*x+e)^2)^(1/2)*EllipticE(sinh(f*x+e)*(-b/
a)^(1/2),(1/b*a)^(1/2))*b^2)/(-b/a)^(1/2)/cosh(f*x+e)/(a+b*sinh(f*x+e)^2)^
(1/2)/f
```

Fricas [F]

$$\int \left(a+b\sinh^2(e+fx)\right)^{3/2}\,dx=\int \left(b\sinh\left(fx+e\right)^2+a\right)^{\frac{3}{2}}\,dx$$

input
```
integrate((a+b*sinh(f*x+e)^2)^(3/2),x, algorithm="fricas")
```

output
```
integral((b*sinh(f*x + e)^2 + a)^(3/2), x)
```

Sympy [F]

$$\int \left(a+b\sinh^2(e+fx)\right)^{3/2}\,dx=\int \left(a+b\sinh^2\left(e+fx\right)\right)^{\frac{3}{2}}\,dx$$

input
```
integrate((a+b*sinh(f*x+e)**2)**(3/2),x)
```

output
```
Integral((a + b*sinh(e + f*x)**2)**(3/2), x)
```

Maxima [F]

$$\int \left(a+b\sinh^2(e+fx)\right)^{3/2}\,dx=\int \left(b\sinh\left(fx+e\right)^2+a\right)^{\frac{3}{2}}\,dx$$

input
```
integrate((a+b*sinh(f*x+e)^2)^(3/2),x, algorithm="maxima")
```

output
```
integrate((b*sinh(f*x + e)^2 + a)^(3/2), x)
```

Giac [F]

$$\int \left(a+b\sinh^2(e+fx)\right)^{3/2}\,dx=\int \left(b\sinh\left(fx+e\right)^2+a\right)^{\frac{3}{2}}\,dx$$

input
```
integrate((a+b*sinh(f*x+e)^2)^(3/2),x, algorithm="giac")
```

output
```
integrate((b*sinh(f*x + e)^2 + a)^(3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \left(a+b\sinh^2(e+fx)\right)^{3/2}\,dx=\int \left(b\sinh(e+f\,x)^2+a\right)^{3/2}\,dx$$

input
```
int((a + b*sinh(e + f*x)^2)^(3/2),x)
```

output
```
int((a + b*sinh(e + f*x)^2)^(3/2), x)
```

Reduce [F]

$$\int \left(a+b\sinh^2(e+fx)\right)^{3/2}\,dx=\left(\int \sqrt{\sinh\left(fx+e\right)^2b+a}dx\right)a+\left(\int \sqrt{\sinh\left(fx+e\right)^2b+a}\,\sinh\left(fx+e\right)^2dx\right)b$$

input
```
int((a+b*sinh(f*x+e)^2)^(3/2),x)
```

output
```
int(sqrt(sinh(e + f*x)**2*b + a),x)*a + int(sqrt(sinh(e + f*x)**2*b + a)*s
inh(e + f*x)**2,x)*b
```

3.335 $\int \operatorname{sech}^2(e+fx)\left(a+b\sinh^2(e+fx)\right)^{3/2}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 174

$$\int \operatorname{sech}^2(e+fx)\left(a+b\sinh^2(e+fx)\right)^{3/2}dx=\frac{(a-2b)E\left(\arctan(\sinh(e+fx))\,\middle|\,1-\frac{b}{a}\right)\operatorname{sech}(e+fx)\sqrt{a+b\sinh^2(e+fx)}}{f\sqrt{\frac{\operatorname{sech}^2(e+fx)(a+b\sinh^2(e+fx))}{a}}}+\frac{b\,\text{EllipticF}\left(\arctan(\sinh(e+fx)),1-\frac{b}{a}\right)\operatorname{sech}(e+fx)\sqrt{a+b\sinh^2(e+fx)}}{f\sqrt{\frac{\operatorname{sech}^2(e+fx)(a+b\sinh^2(e+fx))}{a}}}+\frac{b\sqrt{a+b\sinh^2(e+fx)}\tanh(e+fx)}{f}$$

output

```
(a-2*b)*EllipticE(sinh(f*x+e)/(1+sinh(f*x+e)^2)^(1/2),(1-b/a)^(1/2))*sech(
f*x+e)*(a+b*sinh(f*x+e)^2)^(1/2)/f/(sech(f*x+e)^2*(a+b*sinh(f*x+e)^2)/a)^(
1/2)+b*InverseJacobiAM(arctan(sinh(f*x+e)),(1-b/a)^(1/2))*sech(f*x+e)*(a+b
*sinh(f*x+e)^2)^(1/2)/f/(sech(f*x+e)^2*(a+b*sinh(f*x+e)^2)/a)^(1/2)+b*(a+b
*sinh(f*x+e)^2)^(1/2)*tanh(f*x+e)/f
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.93 (sec) , antiderivative size = 160, normalized size of antiderivative = 0.92

$$\int \operatorname{sech}^2(e+fx)\left(a+b\sinh^2(e+fx)\right)^{3/2}\,dx=\frac{2ia(a-2b)\sqrt{\frac{2a-b+b\cosh(2(e+fx))}{a}}E\left(i(e+fx)\left|\frac{b}{a}\right.\right)+(a-b)\left(-2ia\sqrt{\frac{2a-b+b\cosh(2(e+fx))}{a}}\,\text{Ellipt}\right.}{2f\sqrt{2a-b+b\cosh(2(e+}}$$

input
```
Integrate[Sech[e + f*x]^2*(a + b*Sinh[e + f*x]^2)^(3/2),x]
```

output
```
((2*I)*a*(a - 2*b)*Sqrt[(2*a - b + b*Cosh[2*(e + f*x)])/a]*EllipticE[I*(e
+ f*x), b/a] + (a - b)*((-2*I)*a*Sqrt[(2*a - b + b*Cosh[2*(e + f*x)])/a]*E
llipticF[I*(e + f*x), b/a] + Sqrt[2]*(2*a - b + b*Cosh[2*(e + f*x)])*Tanh[
e + f*x]))/(2*f*Sqrt[2*a - b + b*Cosh[2*(e + f*x)]])
```

Rubi [A] (verified)

Time = 0.44 (sec) , antiderivative size = 278, normalized size of antiderivative = 1.60, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.320, Rules used = {3042, 3671, 315, 27, 406, 320, 388, 313}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \operatorname{sech}^2(e+fx)\left(a+b\sinh^2(e+fx)\right)^{3/2}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\left(a-b\sin(ie+ifx)^2\right)^{3/2}}{\cos(ie+ifx)^2}dx$$

$$\Big\downarrow \text{3671}$$

$$\frac{\sqrt{\cosh^2(e+fx)}\text{sech}(e+fx)\int \frac{\left(b\sinh^2(e+fx)+a\right)^{3/2}}{\left(\sinh^2(e+fx)+1\right)^{3/2}}d\sinh(e+fx)}{f}$$

$$\Big\downarrow \text{315}$$

$$\frac{\sqrt{\cosh^2(e+fx)}\text{sech}(e+fx)\left(\int \frac{b\left(a-(a-2b)\sinh^2(e+fx)\right)}{\sqrt{\sinh^2(e+fx)+1}\sqrt{b\sinh^2(e+fx)+a}}d\sinh(e+fx)+\frac{(a-b)\sinh(e+fx)\sqrt{a+b\sinh^2(e+fx)}}{\sqrt{\sinh^2(e+fx)+1}}\right)}{f}$$

$$\Big\downarrow \text{27}$$

$$\frac{\sqrt{\cosh^2(e+fx)}\text{sech}(e+fx)\left(b\int \frac{a-(a-2b)\sinh^2(e+fx)}{\sqrt{\sinh^2(e+fx)+1}\sqrt{b\sinh^2(e+fx)+a}}d\sinh(e+fx)+\frac{(a-b)\sinh(e+fx)\sqrt{a+b\sinh^2(e+fx)}}{\sqrt{\sinh^2(e+fx)+1}}\right.}{f}$$

$$\Big\downarrow \text{406}$$

$$\frac{\sqrt{\cosh^2(e+fx)}\text{sech}(e+fx)\left(b\left(a\int \frac{1}{\sqrt{\sinh^2(e+fx)+1}\sqrt{b\sinh^2(e+fx)+a}}d\sinh(e+fx)-(a-2b)\int \frac{\sinh^2(e\cdots}{\sqrt{\sinh^2(e+fx)+1}\sqrt{\cdots}}\right.\right.}{f}$$

$$\Big\downarrow \text{320}$$

$$\frac{\sqrt{\cosh^2(e+fx)}\text{sech}(e+fx)\left(b\left(\frac{\sqrt{a+b\sinh^2(e+fx)}\text{EllipticF}\left(\arctan(\sinh(e+fx)),1-\frac{b}{a}\right)}{\sqrt{\sinh^2(e+fx)+1}\sqrt{\frac{a+b\sinh^2(e+fx)}{a\left(\sinh^2(e+fx)+1\right)}}}-(a-2b)\int \frac{\sinh^2(e+fx\cdots}{\sqrt{\sinh^2(e+fx)+1}\sqrt{b\sin\cdots}}\right.\right.}{f}$$

$$\Big\downarrow \text{388}$$

$$\frac{\sqrt{\cosh^2(e+fx)}\text{sech}(e+fx)\left(b\left(\frac{\sqrt{a+b\sinh^2(e+fx)}\text{EllipticF}\left(\arctan(\sinh(e+fx)),1-\frac{b}{a}\right)}{\sqrt{\sinh^2(e+fx)+1}\sqrt{\frac{a+b\sinh^2(e+fx)}{a\left(\sinh^2(e+fx)+1\right)}}}-(a-2b)\left(\frac{\sinh(e+fx)\sqrt{a+b\sinh^2\cdots}}{b\sqrt{\sinh^2(e+fx)+\cdots}}\right.\right.\right.}{f}$$

$$\Big\downarrow \text{313}$$

$$\frac{\sqrt{\cosh^2(e+fx)}\operatorname{sech}(e+fx)\left(b\left(\frac{\sqrt{a+b\sinh^2(e+fx)}\operatorname{EllipticF}\left(\arctan(\sinh(e+fx)),1-\frac{b}{a}\right)}{\sqrt{\sinh^2(e+fx)+1}\sqrt{\frac{a+b\sinh^2(e+fx)}{a\left(\sinh^2(e+fx)+1\right)}}}-(a-2b)\left(\frac{\sinh(e+fx)\sqrt{a+b\sinh^2}}{b\sqrt{\sinh^2(e+fx)+}}\right.\right.\right.}{f}$$

input
```
Int[Sech[e + f*x]^2*(a + b*Sinh[e + f*x]^2)^(3/2),x]
```

output
```
(Sqrt[Cosh[e + f*x]^2]*Sech[e + f*x]*(((a - b)*Sinh[e + f*x]*Sqrt[a + b*Si
nh[e + f*x]^2])/Sqrt[1 + Sinh[e + f*x]^2] + b*((EllipticF[ArcTan[Sinh[e +
f*x]], 1 - b/a]*Sqrt[a + b*Sinh[e + f*x]^2])/(Sqrt[1 + Sinh[e + f*x]^2]*Sq
rt[(a + b*Sinh[e + f*x]^2)/(a*(1 + Sinh[e + f*x]^2))]) - (a - 2*b)*((Sinh[
e + f*x]*Sqrt[a + b*Sinh[e + f*x]^2])/(b*Sqrt[1 + Sinh[e + f*x]^2]) - (Ell
ipticE[ArcTan[Sinh[e + f*x]], 1 - b/a]*Sqrt[a + b*Sinh[e + f*x]^2])/(b*Sqr
t[1 + Sinh[e + f*x]^2]*Sqrt[(a + b*Sinh[e + f*x]^2)/(a*(1 + Sinh[e + f*x]^
2))])))))/f
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 313
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/((c_) + (d_.)*(x_)^2)^(3/2), x_Symbol] :> Sim
p[(Sqrt[a + b*x^2]/(c*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(c
+ d*x^2)))]))*EllipticE[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; FreeQ
[{a, b, c, d}, x] && PosQ[b/a] && PosQ[d/c]
```

rule 315
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_), x_Symbol] :> Sim
p[(a*d - c*b)*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^(q - 1)/(2*a*b*(p + 1))),
x] - Simp[1/(2*a*b*(p + 1))   Int[(a + b*x^2)^(p + 1)*(c + d*x^2)^(q - 2)*S
imp[c*(a*d - c*b*(2*p + 3)) + d*(a*d*(2*(q - 1) + 1) - b*c*(2*(p + q) + 1))
*x^2, x], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -
1] && GtQ[q, 1] && IntBinomialQ[a, b, c, d, 2, p, q, x]
```

rule 320
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(Sqrt[a + b*x^2]/(a*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(
c + d*x^2)))]))*EllipticF[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; Fre
eQ[{a, b, c, d}, x] && PosQ[d/c] && PosQ[b/a] &&  !SimplerSqrtQ[b/a, d/c]
```

rule 388
```
Int[(x_)^2/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol]
 :> Simp[x*(Sqrt[a + b*x^2]/(b*Sqrt[c + d*x^2])), x] - Simp[c/b   Int[Sqrt[
a + b*x^2]/(c + d*x^2)^(3/2), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
 a*d, 0] && PosQ[b/a] && PosQ[d/c] &&  !SimplerSqrtQ[b/a, d/c]
```

rule 406
```
Int[((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.)*((e_) + (f_.)*(
x_)^2), x_Symbol] :> Simp[e   Int[(a + b*x^2)^p*(c + d*x^2)^q, x], x] + Sim
p[f   Int[x^2*(a + b*x^2)^p*(c + d*x^2)^q, x], x] /; FreeQ[{a, b, c, d, e,
f, p, q}, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3671
```
Int[cos[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(
p_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Simp[ff*(Sqrt[
Cos[e + f*x]^2]/(f*Cos[e + f*x]))   Subst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a
 + b*ff^2*x^2)^p, x], x, Sin[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x]
 && IntegerQ[m/2] &&  !IntegerQ[p]
```

Maple [A] (verified)

Time = 3.85 (sec) , antiderivative size = 334, normalized size of antiderivative = 1.92

method	result
default	$\frac{\sqrt{-\frac{b}{a}}\, ab \sinh(fx+e)^3-\sqrt{-\frac{b}{a}}\, b^2 \sinh(fx+e)^3+2a\sqrt{\frac{a+b\sinh(fx+e)^2}{a}}\,\sqrt{\frac{\cosh(2fx+2e)}{2}+\frac{1}{2}}\ \mathrm{EllipticF}\left(\sinh(fx+e)\sqrt{-\frac{b}{a}}, \sqrt{\frac{a}{b}}\right)b-2\sqrt{}}{}$

input
```
int(sech(f*x+e)^2*(a+b*sinh(f*x+e)^2)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
((-b/a)^(1/2)*a*b*sinh(f*x+e)^3-(-b/a)^(1/2)*b^2*sinh(f*x+e)^3+2*a*((a+b*s
inh(f*x+e)^2)/a)^(1/2)*(cosh(f*x+e)^2)^(1/2)*EllipticF(sinh(f*x+e)*(-b/a)^
(1/2),(1/b*a)^(1/2))*b-2*((a+b*sinh(f*x+e)^2)/a)^(1/2)*(cosh(f*x+e)^2)^(1/
2)*EllipticF(sinh(f*x+e)*(-b/a)^(1/2),(1/b*a)^(1/2))*b^2-((a+b*sinh(f*x+e)
^2)/a)^(1/2)*(cosh(f*x+e)^2)^(1/2)*EllipticE(sinh(f*x+e)*(-b/a)^(1/2),(1/b
*a)^(1/2))*a*b+2*((a+b*sinh(f*x+e)^2)/a)^(1/2)*(cosh(f*x+e)^2)^(1/2)*Ellip
ticE(sinh(f*x+e)*(-b/a)^(1/2),(1/b*a)^(1/2))*b^2+(-b/a)^(1/2)*a^2*sinh(f*x
+e)-(-b/a)^(1/2)*a*b*sinh(f*x+e))/(-b/a)^(1/2)/cosh(f*x+e)/(a+b*sinh(f*x+e
)^2)^(1/2)/f
```

Fricas [F]

$$\int \operatorname{sech}^2(e+fx)\left(a+b\sinh^2(e+fx)\right)^{3/2}\,dx=\int\left(b\sinh\left(fx+e\right)^2+a\right)^{\frac{3}{2}}\operatorname{sech}\left(fx+e\right)^2\,dx$$

input
```
integrate(sech(f*x+e)^2*(a+b*sinh(f*x+e)^2)^(3/2),x, algorithm="fricas")
```

output
```
integral((b*sech(f*x + e)^2*sinh(f*x + e)^2 + a*sech(f*x + e)^2)*sqrt(b*si
nh(f*x + e)^2 + a), x)
```

Sympy [F(-1)]

Timed out.

$$\int \operatorname{sech}^2(e+fx)\left(a+b\sinh^2(e+fx)\right)^{3/2}\,dx=\text{Timed out}$$

input
```
integrate(sech(f*x+e)**2*(a+b*sinh(f*x+e)**2)**(3/2),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \operatorname{sech}^2(e+fx)\left(a+b\sinh^2(e+fx)\right)^{3/2}\,dx=\int \left(b\sinh\left(fx+e\right)^2+a\right)^{\frac{3}{2}}\operatorname{sech}\left(fx+e\right)^2\,dx$$

input
```
integrate(sech(f*x+e)^2*(a+b*sinh(f*x+e)^2)^(3/2),x, algorithm="maxima")
```

output
```
integrate((b*sinh(f*x + e)^2 + a)^(3/2)*sech(f*x + e)^2, x)
```

Giac [F]

$$\int \operatorname{sech}^2(e+fx)\left(a+b\sinh^2(e+fx)\right)^{3/2}\,dx=\int \left(b\sinh\left(fx+e\right)^2+a\right)^{\frac{3}{2}}\operatorname{sech}\left(fx+e\right)^2\,dx$$

input
```
integrate(sech(f*x+e)^2*(a+b*sinh(f*x+e)^2)^(3/2),x, algorithm="giac")
```

output
```
integrate((b*sinh(f*x + e)^2 + a)^(3/2)*sech(f*x + e)^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int \operatorname{sech}^2(e+fx)\left(a+b\sinh^2(e+fx)\right)^{3/2}\,dx=\int \frac{\left(b\sinh(e+f\,x)^2+a\right)^{3/2}}{\cosh\left(e+f\,x\right)^2}\,dx$$

input
```
int((a + b*sinh(e + f*x)^2)^(3/2)/cosh(e + f*x)^2,x)
```

output
```
int((a + b*sinh(e + f*x)^2)^(3/2)/cosh(e + f*x)^2, x)
```

Reduce [F]

$$\int \operatorname{sech}^2(e+fx) \left(a+b\sinh^2(e+fx)\right)^{3/2} dx = \left(\int \sqrt{\sinh(fx+e)^2 b+a}\,\operatorname{sech}(fx+e)^2 \sinh(fx+e)^2 dx\right) b + \left(\int \sqrt{\sinh(fx+e)^2 b+a}\,\operatorname{sech}(fx+e)^2 dx\right) a$$

input

```
int(sech(f*x+e)^2*(a+b*sinh(f*x+e)^2)^(3/2),x)
```

output

```
int(sqrt(sinh(e + f*x)**2*b + a)*sech(e + f*x)**2*sinh(e + f*x)**2,x)*b +
int(sqrt(sinh(e + f*x)**2*b + a)*sech(e + f*x)**2,x)*a
```

3.336 $\int \operatorname{sech}^4(e+fx) \left(a+b\sinh^2(e+fx)\right)^{3/2} dx$

Optimal result

Integrand size = 25, antiderivative size = 193

$$\int \operatorname{sech}^4(e+fx) \left(a+b\sinh^2(e+fx)\right)^{3/2} dx = \frac{2(a+b)E\left(\arctan(\sinh(e+fx))\middle|1-\frac{b}{a}\right)\operatorname{sech}(e+fx)\sqrt{a+b\sinh^2(e+fx)}}{3f\sqrt{\frac{\operatorname{sech}^2(e+fx)\left(a+b\sinh^2(e+fx)\right)}{a}}} - \frac{b\operatorname{EllipticF}\left(\arctan(\sinh(e+fx)),1-\frac{b}{a}\right)\operatorname{sech}(e+fx)\sqrt{a+b\sinh^2(e+fx)}}{3f\sqrt{\frac{\operatorname{sech}^2(e+fx)\left(a+b\sinh^2(e+fx)\right)}{a}}} + \frac{(a-b)\operatorname{sech}^2(e+fx)\sqrt{a+b\sinh^2(e+fx)}\tanh(e+fx)}{3f}$$

output
```
2/3*(a+b)*EllipticE(sinh(f*x+e)/(1+sinh(f*x+e)^2)^(1/2),(1-b/a)^(1/2))*sec
h(f*x+e)*(a+b*sinh(f*x+e)^2)^(1/2)/f/(sech(f*x+e)^2*(a+b*sinh(f*x+e)^2)/a)
^(1/2)-1/3*b*InverseJacobiAM(arctan(sinh(f*x+e)),(1-b/a)^(1/2))*sech(f*x+e
)*(a+b*sinh(f*x+e)^2)^(1/2)/f/(sech(f*x+e)^2*(a+b*sinh(f*x+e)^2)/a)^(1/2)+
1/3*(a-b)*sech(f*x+e)^2*(a+b*sinh(f*x+e)^2)^(1/2)*tanh(f*x+e)/f
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 1.60 (sec) , antiderivative size = 197, normalized size of antiderivative = 1.02

$$\int \operatorname{sech}^4(e+fx)\left(a+b\sinh^2(e+fx)\right)^{3/2}\,dx=\frac{4ia(a+b)\sqrt{\frac{2a-b+b\cosh(2(e+fx))}{a}}E\left(i(e+fx)\,\middle|\,\frac{b}{a}\right)-2ia(2a+b)\sqrt{\frac{2a-b+b\cosh(2(e+fx))}{a}}\,\text{EllipticF}}{6f\sqrt{2a-b+b\,c}}$$

input
```
Integrate[Sech[e + f*x]^4*(a + b*Sinh[e + f*x]^2)^(3/2),x]
```

output
```
((4*I)*a*(a + b)*Sqrt[(2*a - b + b*Cosh[2*(e + f*x)])/a]*EllipticE[I*(e +
f*x), b/a] - (2*I)*a*(2*a + b)*Sqrt[(2*a - b + b*Cosh[2*(e + f*x)])/a]*Ell
ipticF[I*(e + f*x), b/a] + ((8*a^2 - 3*a*b + b^2 + (4*a^2 + 6*a*b - 2*b^2)
*Cosh[2*(e + f*x)] + b*(a + b)*Cosh[4*(e + f*x)])*Sech[e + f*x]^2*Tanh[e +
 f*x])/Sqrt[2])/(6*f*Sqrt[2*a - b + b*Cosh[2*(e + f*x)]])
```

Rubi [A] (verified)

Time = 0.41 (sec) , antiderivative size = 237, normalized size of antiderivative = 1.23, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.240$, Rules used = {3042, 3671, 315, 400, 313, 320}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \operatorname{sech}^4(e+fx)\left(a+b\sinh^2(e+fx)\right)^{3/2}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\left(a-b\sin(ie+ifx)^2\right)^{3/2}}{\cos(ie+ifx)^4}dx$$

$$\Big\downarrow \text{3671}$$

$$\frac{\sqrt{\cosh^2(e+fx)}\operatorname{sech}(e+fx)\int \frac{\left(b\sinh^2(e+fx)+a\right)^{3/2}}{\left(\sinh^2(e+fx)+1\right)^{5/2}}d\sinh(e+fx)}{f}$$

$\Big\downarrow$ 315

$$\frac{\sqrt{\cosh^2(e+fx)}\operatorname{sech}(e+fx)\left(\frac{1}{3}\int \frac{b(a+2b)\sinh^2(e+fx)+a(2a+b)}{\left(\sinh^2(e+fx)+1\right)^{3/2}\sqrt{b\sinh^2(e+fx)+a}}d\sinh(e+fx)+\frac{(a-b)\sinh(e+fx)\sqrt{a+b\sinh^2(e+}}{3\left(\sinh^2(e+fx)+1\right)^{3/2}}\right.}{f}$$

$\Big\downarrow$ 400

$$\frac{\sqrt{\cosh^2(e+fx)}\operatorname{sech}(e+fx)\left(\frac{1}{3}\left(2(a+b)\int \frac{\sqrt{b\sinh^2(e+fx)+a}}{\left(\sinh^2(e+fx)+1\right)^{3/2}}d\sinh(e+fx)-ab\int \frac{1}{\sqrt{\sinh^2(e+fx)+1}\sqrt{b\sinh^2(e+fx)-}}\right.\right.}{f}$$

$\Big\downarrow$ 313

$$\frac{\sqrt{\cosh^2(e+fx)}\operatorname{sech}(e+fx)\left(\frac{1}{3}\left(\frac{2(a+b)\sqrt{a+b\sinh^2(e+fx)}E\left(\arctan(\sinh(e+fx))\middle|1-\frac{b}{a}\right)}{\sqrt{\sinh^2(e+fx)+1}\sqrt{\frac{a+b\sinh^2(e+fx)}{a\left(\sinh^2(e+fx)+1\right)}}}-ab\int \frac{1}{\sqrt{\sinh^2(e+fx)+1}\sqrt{b\sinh^2(e+}}\right.\right.}{f}$$

$\Big\downarrow$ 320

$$\frac{\sqrt{\cosh^2(e+fx)}\operatorname{sech}(e+fx)\left(\frac{1}{3}\left(\frac{2(a+b)\sqrt{a+b\sinh^2(e+fx)}E\left(\arctan(\sinh(e+fx))\middle|1-\frac{b}{a}\right)}{\sqrt{\sinh^2(e+fx)+1}\sqrt{\frac{a+b\sinh^2(e+fx)}{a\left(\sinh^2(e+fx)+1\right)}}}-\frac{b\sqrt{a+b\sinh^2(e+fx)}\operatorname{EllipticF}\left(\operatorname{arcta}\right.}{\sqrt{\sinh^2(e+fx)+1}\sqrt{\frac{a+b\operatorname{si}}{a\left(\operatorname{sinh}\right.}}}\right.\right.}{f}$$

input
```
Int[Sech[e + f*x]^4*(a + b*Sinh[e + f*x]^2)^(3/2),x]
```

output
```
(Sqrt[Cosh[e + f*x]^2]*Sech[e + f*x]*(((a - b)*Sinh[e + f*x]*Sqrt[a + b*Si
nh[e + f*x]^2])/(3*(1 + Sinh[e + f*x]^2)^(3/2)) + ((2*(a + b)*EllipticE[Ar
cTan[Sinh[e + f*x]], 1 - b/a]*Sqrt[a + b*Sinh[e + f*x]^2])/(Sqrt[1 + Sinh[
e + f*x]^2]*Sqrt[(a + b*Sinh[e + f*x]^2)/(a*(1 + Sinh[e + f*x]^2))]) - (b*
EllipticF[ArcTan[Sinh[e + f*x]], 1 - b/a]*Sqrt[a + b*Sinh[e + f*x]^2])/(Sq
rt[1 + Sinh[e + f*x]^2]*Sqrt[(a + b*Sinh[e + f*x]^2)/(a*(1 + Sinh[e + f*x]
^2))]))/3))/f
```

Defintions of rubi rules used

rule 313
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/((c_) + (d_.)*(x_)^2)^(3/2), x_Symbol] :> Sim
p[(Sqrt[a + b*x^2]/(c*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(c
+ d*x^2)))]))*EllipticE[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; FreeQ
[{a, b, c, d}, x] && PosQ[b/a] && PosQ[d/c]
```

rule 315
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_), x_Symbol] :> Sim
p[(a*d - c*b)*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^(q - 1)/(2*a*b*(p + 1))),
x] - Simp[1/(2*a*b*(p + 1))   Int[(a + b*x^2)^(p + 1)*(c + d*x^2)^(q - 2)*S
imp[c*(a*d - c*b*(2*p + 3)) + d*(a*d*(2*(q - 1) + 1) - b*c*(2*(p + q) + 1))
*x^2, x], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -
1] && GtQ[q, 1] && IntBinomialQ[a, b, c, d, 2, p, q, x]
```

rule 320
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(Sqrt[a + b*x^2]/(a*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(
c + d*x^2)))]))*EllipticF[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; Fre
eQ[{a, b, c, d}, x] && PosQ[d/c] && PosQ[b/a] &&  !SimplerSqrtQ[b/a, d/c]
```

rule 400
```
Int[((e_) + (f_.)*(x_)^2)/(Sqrt[(a_) + (b_.)*(x_)^2]*((c_) + (d_.)*(x_)^2)^
(3/2)), x_Symbol] :> Simp[(b*e - a*f)/(b*c - a*d)   Int[1/(Sqrt[a + b*x^2]*
Sqrt[c + d*x^2]), x], x] - Simp[(d*e - c*f)/(b*c - a*d)   Int[Sqrt[a + b*x^
2]/(c + d*x^2)^(3/2), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && PosQ[b/a] &
& PosQ[d/c]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3671
```
Int[cos[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(
p_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Simp[ff*(Sqrt[
Cos[e + f*x]^2]/(f*Cos[e + f*x]))   Subst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a
 + b*ff^2*x^2)^p, x], x, Sin[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x]
 && IntegerQ[m/2] &&  !IntegerQ[p]
```

Maple [A] (verified)

Time = 5.04 (sec) , antiderivative size = 324, normalized size of antiderivative = 1.68

method	result
default	$\frac{\left(2\sqrt{-\frac{b}{a}}\,ab+2\sqrt{-\frac{b}{a}}\,b^{2}\right)\cosh(fx+e)^{4}\sinh(fx+e)+\left(2\sqrt{-\frac{b}{a}}\,a^{2}+\sqrt{-\frac{b}{a}}\,ab-3\sqrt{-\frac{b}{a}}\,b^{2}\right)\cosh(fx+e)^{2}\sinh(fx+e)+\sqrt{\frac{b\cosh(fx+e)^{2}}{a}}}{}$

input
```
int(sech(f*x+e)^4*(a+b*sinh(f*x+e)^2)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
1/3*((2*(-b/a)^(1/2)*a*b+2*(-b/a)^(1/2)*b^2)*cosh(f*x+e)^4*sinh(f*x+e)+(2*
(-b/a)^(1/2)*a^2+(-b/a)^(1/2)*a*b-3*(-b/a)^(1/2)*b^2)*cosh(f*x+e)^2*sinh(f
*x+e)+(b/a*cosh(f*x+e)^2+(a-b)/a)^(1/2)*(cosh(f*x+e)^2)^(1/2)*b*(a*Ellipti
cF(sinh(f*x+e)*(-b/a)^(1/2),(1/b*a)^(1/2))+2*b*EllipticF(sinh(f*x+e)*(-b/a
)^(1/2),(1/b*a)^(1/2))-2*EllipticE(sinh(f*x+e)*(-b/a)^(1/2),(1/b*a)^(1/2))
*a-2*b*EllipticE(sinh(f*x+e)*(-b/a)^(1/2),(1/b*a)^(1/2)))*cosh(f*x+e)^2+((
-b/a)^(1/2)*a^2-2*(-b/a)^(1/2)*a*b+(-b/a)^(1/2)*b^2)*sinh(f*x+e))/cosh(f*x
+e)^3/(-b/a)^(1/2)/(a+b*sinh(f*x+e)^2)^(1/2)/f
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2071 vs. 2(193) = 386.

Time = 0.12 (sec) , antiderivative size = 2071, normalized size of antiderivative = 10.73

$$\int \operatorname{sech}^4(e+fx)\left(a+b\sinh^2(e+fx)\right)^{3/2}\,dx=\text{Too large to display}$$

input
```
integrate(sech(f*x+e)^4*(a+b*sinh(f*x+e)^2)^(3/2),x, algorithm="fricas")
```

output
```
-2/3*(((2*a^2 + a*b - b^2)*cosh(f*x + e)^6 + 6*(2*a^2 + a*b - b^2)*cosh(f*
x + e)*sinh(f*x + e)^5 + (2*a^2 + a*b - b^2)*sinh(f*x + e)^6 + 3*(2*a^2 +
a*b - b^2)*cosh(f*x + e)^4 + 3*(5*(2*a^2 + a*b - b^2)*cosh(f*x + e)^2 + 2*
a^2 + a*b - b^2)*sinh(f*x + e)^4 + 4*(5*(2*a^2 + a*b - b^2)*cosh(f*x + e)^
3 + 3*(2*a^2 + a*b - b^2)*cosh(f*x + e))*sinh(f*x + e)^3 + 3*(2*a^2 + a*b
- b^2)*cosh(f*x + e)^2 + 3*(5*(2*a^2 + a*b - b^2)*cosh(f*x + e)^4 + 6*(2*a
^2 + a*b - b^2)*cosh(f*x + e)^2 + 2*a^2 + a*b - b^2)*sinh(f*x + e)^2 + 2*a
^2 + a*b - b^2 + 6*((2*a^2 + a*b - b^2)*cosh(f*x + e)^5 + 2*(2*a^2 + a*b -
 b^2)*cosh(f*x + e)^3 + (2*a^2 + a*b - b^2)*cosh(f*x + e))*sinh(f*x + e) -
 2*((a*b + b^2)*cosh(f*x + e)^6 + 6*(a*b + b^2)*cosh(f*x + e)*sinh(f*x + e
)^5 + (a*b + b^2)*sinh(f*x + e)^6 + 3*(a*b + b^2)*cosh(f*x + e)^4 + 3*(5*(
a*b + b^2)*cosh(f*x + e)^2 + a*b + b^2)*sinh(f*x + e)^4 + 4*(5*(a*b + b^2)
*cosh(f*x + e)^3 + 3*(a*b + b^2)*cosh(f*x + e))*sinh(f*x + e)^3 + 3*(a*b +
 b^2)*cosh(f*x + e)^2 + 3*(5*(a*b + b^2)*cosh(f*x + e)^4 + 6*(a*b + b^2)*c
osh(f*x + e)^2 + a*b + b^2)*sinh(f*x + e)^2 + a*b + b^2 + 6*((a*b + b^2)*c
osh(f*x + e)^5 + 2*(a*b + b^2)*cosh(f*x + e)^3 + (a*b + b^2)*cosh(f*x + e)
)*sinh(f*x + e))*sqrt((a^2 - a*b)/b^2))*sqrt(b)*sqrt((2*b*sqrt((a^2 - a*b)
/b^2) - 2*a + b)/b)*elliptic_e(arcsin(sqrt((2*b*sqrt((a^2 - a*b)/b^2) - 2*
a + b)/b)*(cosh(f*x + e) + sinh(f*x + e))), (8*a^2 - 8*a*b + b^2 + 4*(2*a*
b - b^2)*sqrt((a^2 - a*b)/b^2))/b^2) - ((2*a^2 - a*b)*cosh(f*x + e)^6 +...
```

Sympy [F(-1)]

Timed out.

$$\int \operatorname{sech}^4(e+fx)\left(a+b\sinh^2(e+fx)\right)^{3/2}\,dx=\text{Timed out}$$

input
```
integrate(sech(f*x+e)**4*(a+b*sinh(f*x+e)**2)**(3/2),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \operatorname{sech}^4(e+fx)\left(a+b\sinh^2(e+fx)\right)^{3/2}\,dx=\int \left(b\sinh\left(fx+e\right)^2+a\right)^{\frac{3}{2}}\operatorname{sech}\left(fx+e\right)^4\,dx$$

input
```
integrate(sech(f*x+e)^4*(a+b*sinh(f*x+e)^2)^(3/2),x, algorithm="maxima")
```

output
```
integrate((b*sinh(f*x + e)^2 + a)^(3/2)*sech(f*x + e)^4, x)
```

Giac [F]

$$\int \operatorname{sech}^4(e+fx)\left(a+b\sinh^2(e+fx)\right)^{3/2}\,dx=\int \left(b\sinh\left(fx+e\right)^2+a\right)^{\frac{3}{2}}\operatorname{sech}\left(fx+e\right)^4\,dx$$

input
```
integrate(sech(f*x+e)^4*(a+b*sinh(f*x+e)^2)^(3/2),x, algorithm="giac")
```

output
```
integrate((b*sinh(f*x + e)^2 + a)^(3/2)*sech(f*x + e)^4, x)
```

Mupad [F(-1)]

Timed out.

$$\int \operatorname{sech}^4(e+fx)\left(a+b\sinh^2(e+fx)\right)^{3/2}\,dx=\int \frac{\left(b\sinh(e+f\,x)^2+a\right)^{3/2}}{\cosh\left(e+f\,x\right)^4}\,dx$$

input
```
int((a + b*sinh(e + f*x)^2)^(3/2)/cosh(e + f*x)^4,x)
```

output
```
int((a + b*sinh(e + f*x)^2)^(3/2)/cosh(e + f*x)^4, x)
```

Reduce [F]

$$\int \operatorname{sech}^4(e+fx)\left(a+b\sinh^2(e+fx)\right)^{3/2}\,dx=\left(\int \sqrt{\sinh(fx+e)^2 b+a}\,\operatorname{sech}(fx+e)^4\sinh(fx+e)^2\,dx\right)b+\left(\int \sqrt{\sinh(fx+e)^2 b+a}\,\operatorname{sech}(fx+e)^4\,dx\right)a$$

input
```
int(sech(f*x+e)^4*(a+b*sinh(f*x+e)^2)^(3/2),x)
```

output
```
int(sqrt(sinh(e + f*x)**2*b + a)*sech(e + f*x)**4*sinh(e + f*x)**2,x)*b +
int(sqrt(sinh(e + f*x)**2*b + a)*sech(e + f*x)**4,x)*a
```

3.337 $\int \frac{\cosh^3(e+fx)}{\sqrt{a+b\sinh^2(e+fx)}}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 79

$$\int \frac{\cosh^3(e+fx)}{\sqrt{a+b\sinh^2(e+fx)}}\,dx = -\frac{(a-2b)\operatorname{arctanh}\left(\frac{\sqrt{b}\sinh(e+fx)}{\sqrt{a+b\sinh^2(e+fx)}}\right)}{2b^{3/2}f} + \frac{\sinh(e+fx)\sqrt{a+b\sinh^2(e+fx)}}{2bf}$$

output
```
-1/2*(a-2*b)*arctanh(b^(1/2)*sinh(f*x+e)/(a+b*sinh(f*x+e)^2)^(1/2))/b^(3/2
)/f+1/2*sinh(f*x+e)*(a+b*sinh(f*x+e)^2)^(1/2)/b/f
```

Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 77, normalized size of antiderivative = 0.97

$$\int \frac{\cosh^3(e+fx)}{\sqrt{a+b\sinh^2(e+fx)}}\,dx = \frac{-\frac{(a-2b)\operatorname{arctanh}\left(\frac{\sqrt{b}\sinh(e+fx)}{\sqrt{a+b\sinh^2(e+fx)}}\right)}{2b^{3/2}} + \frac{\sinh(e+fx)\sqrt{a+b\sinh^2(e+fx)}}{2b}}{f}$$

input
```
Integrate[Cosh[e + f*x]^3/Sqrt[a + b*Sinh[e + f*x]^2],x]
```

output
```
(-1/2*((a - 2*b)*ArcTanh[(Sqrt[b]*Sinh[e + f*x])/Sqrt[a + b*Sinh[e + f*x]^
2]])/b^(3/2) + (Sinh[e + f*x]*Sqrt[a + b*Sinh[e + f*x]^2])/(2*b))/f
```

Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 77, normalized size of antiderivative = 0.97, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {3042, 3669, 299, 224, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cosh^3(e+fx)}{\sqrt{a+b\sinh^2(e+fx)}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\cos(ie+ifx)^3}{\sqrt{a-b\sin(ie+ifx)^2}}dx$$

$$\Big\downarrow \text{3669}$$

$$\frac{\int \frac{\sinh^2(e+fx)+1}{\sqrt{b\sinh^2(e+fx)+a}}d\sinh(e+fx)}{f}$$

$$\Big\downarrow \text{299}$$

$$\frac{\frac{\sinh(e+fx)\sqrt{a+b\sinh^2(e+fx)}}{2b}-\frac{(a-2b)\int \frac{1}{\sqrt{b\sinh^2(e+fx)+a}}d\sinh(e+fx)}{2b}}{f}$$

$$\Big\downarrow \text{224}$$

$$\frac{\frac{\sinh(e+fx)\sqrt{a+b\sinh^2(e+fx)}}{2b}-\frac{(a-2b)\int \frac{1}{1-\frac{b\sinh^2(e+fx)}{b\sinh^2(e+fx)+a}}d\frac{\sinh(e+fx)}{\sqrt{b\sinh^2(e+fx)+a}}}{2b}}{f}$$

$$\Big\downarrow \text{219}$$

$$\frac{\frac{\sinh(e+fx)\sqrt{a+b\sinh^2(e+fx)}}{2b}-\frac{(a-2b)\text{arctanh}\left(\frac{\sqrt{b}\sinh(e+fx)}{\sqrt{a+b\sinh^2(e+fx)}}\right)}{2b^{3/2}}}{f}$$

input
```
Int[Cosh[e + f*x]^3/Sqrt[a + b*Sinh[e + f*x]^2],x]
```

output
```
(-1/2*((a - 2*b)*ArcTanh[(Sqrt[b]*Sinh[e + f*x])/Sqrt[a + b*Sinh[e + f*x]^
2]])/b^(3/2) + (Sinh[e + f*x]*Sqrt[a + b*Sinh[e + f*x]^2])/(2*b))/f
```

Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

rule 299
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2), x_Symbol] :> Simp[d*x
*((a + b*x^2)^(p + 1)/(b*(2*p + 3))), x] - Simp[(a*d - b*c*(2*p + 3))/(b*(2
*p + 3))   Int[(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
 a*d, 0] && NeQ[2*p + 3, 0]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3669
```
Int[cos[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^
(p_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Simp[ff/f   S
ubst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a + b*ff^2*x^2)^p, x], x, Sin[e + f*x]
/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m - 1)/2]
```

Maple [A] (verified)

Time = 0.31 (sec) , antiderivative size = 93, normalized size of antiderivative = 1.18

method	result	size
derivativedivides	$\frac{\frac{\ln\left(\sqrt{b}\,\sinh(fx+e)+\sqrt{a+b\sinh(fx+e)^2}\right)}{\sqrt{b}}+\frac{\sinh(fx+e)\sqrt{a+b\sinh(fx+e)^2}}{2b}-\frac{a\ln\left(\sqrt{b}\,\sinh(fx+e)+\sqrt{a+b\sinh(fx+e)^2}\right)}{2b^{\frac{3}{2}}}}{f}$	93
default	$\frac{\frac{\ln\left(\sqrt{b}\,\sinh(fx+e)+\sqrt{a+b\sinh(fx+e)^2}\right)}{\sqrt{b}}+\frac{\sinh(fx+e)\sqrt{a+b\sinh(fx+e)^2}}{2b}-\frac{a\ln\left(\sqrt{b}\,\sinh(fx+e)+\sqrt{a+b\sinh(fx+e)^2}\right)}{2b^{\frac{3}{2}}}}{f}$	93

input
```
int(cosh(f*x+e)^3/(a+b*sinh(f*x+e)^2)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/f*(ln(b^(1/2)*sinh(f*x+e)+(a+b*sinh(f*x+e)^2)^(1/2))/b^(1/2)+1/2*sinh(f*
x+e)/b*(a+b*sinh(f*x+e)^2)^(1/2)-1/2*a/b^(3/2)*ln(b^(1/2)*sinh(f*x+e)+(a+b
*sinh(f*x+e)^2)^(1/2)))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 673 vs. 2(67) = 134.

Time = 0.15 (sec) , antiderivative size = 2367, normalized size of antiderivative = 29.96

$$\int \frac{\cosh^3(e+fx)}{\sqrt{a+b\sinh^2(e+fx)}}\,dx = \text{Too large to display}$$

input
```
integrate(cosh(f*x+e)^3/(a+b*sinh(f*x+e)^2)^(1/2),x, algorithm="fricas")
```

output
```
[-1/8*(((a - 2*b)*cosh(f*x + e)^2 + 2*(a - 2*b)*cosh(f*x + e)*sinh(f*x + e
) + (a - 2*b)*sinh(f*x + e)^2)*sqrt(b)*log(-((a^2*b - 2*a*b^2 + b^3)*cosh(
f*x + e)^8 + 8*(a^2*b - 2*a*b^2 + b^3)*cosh(f*x + e)*sinh(f*x + e)^7 + (a^
2*b - 2*a*b^2 + b^3)*sinh(f*x + e)^8 + 2*(a^3 - 4*a^2*b + 5*a*b^2 - 2*b^3)
*cosh(f*x + e)^6 + 2*(a^3 - 4*a^2*b + 5*a*b^2 - 2*b^3 + 14*(a^2*b - 2*a*b^
2 + b^3)*cosh(f*x + e)^2)*sinh(f*x + e)^6 + 4*(14*(a^2*b - 2*a*b^2 + b^3)*
cosh(f*x + e)^3 + 3*(a^3 - 4*a^2*b + 5*a*b^2 - 2*b^3)*cosh(f*x + e))*sinh(
f*x + e)^5 + (9*a^2*b - 14*a*b^2 + 6*b^3)*cosh(f*x + e)^4 + (70*(a^2*b - 2
*a*b^2 + b^3)*cosh(f*x + e)^4 + 9*a^2*b - 14*a*b^2 + 6*b^3 + 30*(a^3 - 4*a
^2*b + 5*a*b^2 - 2*b^3)*cosh(f*x + e)^2)*sinh(f*x + e)^4 + 4*(14*(a^2*b -
2*a*b^2 + b^3)*cosh(f*x + e)^5 + 10*(a^3 - 4*a^2*b + 5*a*b^2 - 2*b^3)*cosh
(f*x + e)^3 + (9*a^2*b - 14*a*b^2 + 6*b^3)*cosh(f*x + e))*sinh(f*x + e)^3
+ b^3 + 2*(3*a*b^2 - 2*b^3)*cosh(f*x + e)^2 + 2*(14*(a^2*b - 2*a*b^2 + b^3
)*cosh(f*x + e)^6 + 15*(a^3 - 4*a^2*b + 5*a*b^2 - 2*b^3)*cosh(f*x + e)^4 +
 3*a*b^2 - 2*b^3 + 3*(9*a^2*b - 14*a*b^2 + 6*b^3)*cosh(f*x + e)^2)*sinh(f*
x + e)^2 + sqrt(2)*((a^2 - 2*a*b + b^2)*cosh(f*x + e)^6 + 6*(a^2 - 2*a*b +
 b^2)*cosh(f*x + e)*sinh(f*x + e)^5 + (a^2 - 2*a*b + b^2)*sinh(f*x + e)^6
- 3*(a^2 - 2*a*b + b^2)*cosh(f*x + e)^4 + 3*(5*(a^2 - 2*a*b + b^2)*cosh(f*
x + e)^2 - a^2 + 2*a*b - b^2)*sinh(f*x + e)^4 + 4*(5*(a^2 - 2*a*b + b^2)*c
osh(f*x + e)^3 - 3*(a^2 - 2*a*b + b^2)*cosh(f*x + e))*sinh(f*x + e)^3 -...
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\cosh^3(e+fx)}{\sqrt{a+b\sinh^2(e+fx)}}\,dx = \text{Timed out}$$

input
```
integrate(cosh(f*x+e)**3/(a+b*sinh(f*x+e)**2)**(1/2),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{\cosh^3(e+fx)}{\sqrt{a+b\sinh^2(e+fx)}}\,dx = \int \frac{\cosh(fx+e)^3}{\sqrt{b\sinh(fx+e)^2+a}}\,dx$$

input
```
integrate(cosh(f*x+e)^3/(a+b*sinh(f*x+e)^2)^(1/2),x, algorithm="maxima")
```

output
```
integrate(cosh(f*x + e)^3/sqrt(b*sinh(f*x + e)^2 + a), x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 433 vs. $2(67) = 134$.

Time = 0.22 (sec) , antiderivative size = 433, normalized size of antiderivative = 5.48

$$\int \frac{\cosh^3(e+fx)}{\sqrt{a+b\sinh^2(e+fx)}}\,dx = -\frac{\left(\frac{4\left(ae^{(2e)}-2be^{(2e)}\right)\arctan\left(-\frac{\sqrt{b}e^{(2fx+2e)}-\sqrt{be^{(4fx+4e)}+4ae^{(2fx+2e)}-2be^{(2fx+2e)}+b}}{\sqrt{-b}}\right)}{\sqrt{-b}b}-\frac{\sqrt{be^{(4fx+4e)}+4ae^{(2fx+2e)}-2be^{(2fx+\cdots}}}{b}\cdots\right.}{\cdots}$$

input
```
integrate(cosh(f*x+e)^3/(a+b*sinh(f*x+e)^2)^(1/2),x, algorithm="giac")
```

output
```
-1/8*(4*(a*e^(2*e) - 2*b*e^(2*e))*arctan(-(sqrt(b)*e^(2*f*x + 2*e) - sqrt(
b*e^(4*f*x + 4*e) + 4*a*e^(2*f*x + 2*e) - 2*b*e^(2*f*x + 2*e) + b))/sqrt(-
b))/(sqrt(-b)*b) - sqrt(b*e^(4*f*x + 4*e) + 4*a*e^(2*f*x + 2*e) - 2*b*e^(2
*f*x + 2*e) + b)*e^(2*e)/b - 2*(a*e^(2*e) - 2*b*e^(2*e))*log(abs((sqrt(b)*
e^(2*f*x + 2*e) - sqrt(b*e^(4*f*x + 4*e) + 4*a*e^(2*f*x + 2*e) - 2*b*e^(2*
f*x + 2*e) + b))*sqrt(b) + 2*a - b))/b^(3/2) - 2*(2*(sqrt(b)*e^(2*f*x + 2*
e) - sqrt(b*e^(4*f*x + 4*e) + 4*a*e^(2*f*x + 2*e) - 2*b*e^(2*f*x + 2*e) +
b))*a*e^(2*e) - (sqrt(b)*e^(2*f*x + 2*e) - sqrt(b*e^(4*f*x + 4*e) + 4*a*e^
(2*f*x + 2*e) - 2*b*e^(2*f*x + 2*e) + b))*b*e^(2*e) + b^(3/2)*e^(2*e))/(((
sqrt(b)*e^(2*f*x + 2*e) - sqrt(b*e^(4*f*x + 4*e) + 4*a*e^(2*f*x + 2*e) - 2
*b*e^(2*f*x + 2*e) + b))^2 - b)*b))*e^(-2*e)/f
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\cosh^3(e+fx)}{\sqrt{a+b\sinh^2(e+fx)}}\,dx = \int \frac{\cosh(e+f\,x)^3}{\sqrt{b\sinh\left(e+f\,x\right)^2+a}}\,dx$$

input
```
int(cosh(e + f*x)^3/(a + b*sinh(e + f*x)^2)^(1/2),x)
```

output
```
int(cosh(e + f*x)^3/(a + b*sinh(e + f*x)^2)^(1/2), x)
```

Reduce [F]

$$\int \frac{\cosh^3(e+fx)}{\sqrt{a+b\sinh^2(e+fx)}}\,dx = \int \frac{\sqrt{\sinh\left(fx+e\right)^2b+a}\,\cosh\left(fx+e\right)^3}{\sinh\left(fx+e\right)^2b+a}dx$$

input
```
int(cosh(f*x+e)^3/(a+b*sinh(f*x+e)^2)^(1/2),x)
```

output
```
int((sqrt(sinh(e + f*x)**2*b + a)*cosh(e + f*x)**3)/(sinh(e + f*x)**2*b +
a),x)
```

3.338 $\int \frac{\cosh(e+fx)}{\sqrt{a+b\sinh^2(e+fx)}}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 38

$$\int \frac{\cosh(e+fx)}{\sqrt{a+b\sinh^2(e+fx)}}\,dx = \frac{\operatorname{arctanh}\left(\frac{\sqrt{b}\sinh(e+fx)}{\sqrt{a+b\sinh^2(e+fx)}}\right)}{\sqrt{b}f}$$

output
```
arctanh(b^(1/2)*sinh(f*x+e)/(a+b*sinh(f*x+e)^2)^(1/2))/b^(1/2)/f
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 38, normalized size of antiderivative = 1.00

$$\int \frac{\cosh(e+fx)}{\sqrt{a+b\sinh^2(e+fx)}}\,dx = \frac{\operatorname{arctanh}\left(\frac{\sqrt{b}\sinh(e+fx)}{\sqrt{a+b\sinh^2(e+fx)}}\right)}{\sqrt{b}f}$$

input
```
Integrate[Cosh[e + f*x]/Sqrt[a + b*Sinh[e + f*x]^2],x]
```

output
```
ArcTanh[(Sqrt[b]*Sinh[e + f*x])/Sqrt[a + b*Sinh[e + f*x]^2]]/(Sqrt[b]*f)
```

Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 38, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.174, Rules used = {3042, 3669, 224, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cosh(e+fx)}{\sqrt{a+b\sinh^2(e+fx)}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\cos(ie+ifx)}{\sqrt{a-b\sin(ie+ifx)^2}}dx$$

$$\Big\downarrow \text{3669}$$

$$\frac{\int \frac{1}{\sqrt{b\sinh^2(e+fx)+a}}d\sinh(e+fx)}{f}$$

$$\Big\downarrow \text{224}$$

$$\frac{\int \frac{1}{1-\frac{b\sinh^2(e+fx)}{b\sinh^2(e+fx)+a}}d\frac{\sinh(e+fx)}{\sqrt{b\sinh^2(e+fx)+a}}}{f}$$

$$\Big\downarrow \text{219}$$

$$\frac{\text{arctanh}\left(\frac{\sqrt{b}\sinh(e+fx)}{\sqrt{a+b\sinh^2(e+fx)}}\right)}{\sqrt{b}f}$$

input
```
Int[Cosh[e + f*x]/Sqrt[a + b*Sinh[e + f*x]^2],x]
```

output
```
ArcTanh[(Sqrt[b]*Sinh[e + f*x])/Sqrt[a + b*Sinh[e + f*x]^2]]/(Sqrt[b]*f)
```

Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3669
```
Int[cos[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^
(p_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Simp[ff/f   S
ubst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a + b*ff^2*x^2)^p, x], x, Sin[e + f*x]
/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m - 1)/2]
```

Maple [A] (verified)

Time = 0.17 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.89

method	result	size
derivativedivides	$\frac{\ln\left(\sqrt{b}\,\sinh(fx+e)+\sqrt{a+b\sinh(fx+e)^2}\right)}{f\sqrt{b}}$	34
default	$\frac{\ln\left(\sqrt{b}\,\sinh(fx+e)+\sqrt{a+b\sinh(fx+e)^2}\right)}{f\sqrt{b}}$	34

input
```
int(cosh(f*x+e)/(a+b*sinh(f*x+e)^2)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/f*ln(b^(1/2)*sinh(f*x+e)+(a+b*sinh(f*x+e)^2)^(1/2))/b^(1/2)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 428 vs. $2(32) = 64$.

Time = 0.14 (sec) , antiderivative size = 1878, normalized size of antiderivative = 49.42

$$\int \frac{\cosh(e+fx)}{\sqrt{a+b\sinh^2(e+fx)}}\,dx = \text{Too large to display}$$

input
```
integrate(cosh(f*x+e)/(a+b*sinh(f*x+e)^2)^(1/2),x, algorithm="fricas")
```

output
```
[1/4*(sqrt(b)*log(-((a^2*b - 2*a*b^2 + b^3)*cosh(f*x + e)^8 + 8*(a^2*b - 2
*a*b^2 + b^3)*cosh(f*x + e)*sinh(f*x + e)^7 + (a^2*b - 2*a*b^2 + b^3)*sinh
(f*x + e)^8 + 2*(a^3 - 4*a^2*b + 5*a*b^2 - 2*b^3)*cosh(f*x + e)^6 + 2*(a^3
 - 4*a^2*b + 5*a*b^2 - 2*b^3 + 14*(a^2*b - 2*a*b^2 + b^3)*cosh(f*x + e)^2)
*sinh(f*x + e)^6 + 4*(14*(a^2*b - 2*a*b^2 + b^3)*cosh(f*x + e)^3 + 3*(a^3
- 4*a^2*b + 5*a*b^2 - 2*b^3)*cosh(f*x + e))*sinh(f*x + e)^5 + (9*a^2*b - 1
4*a*b^2 + 6*b^3)*cosh(f*x + e)^4 + (70*(a^2*b - 2*a*b^2 + b^3)*cosh(f*x +
e)^4 + 9*a^2*b - 14*a*b^2 + 6*b^3 + 30*(a^3 - 4*a^2*b + 5*a*b^2 - 2*b^3)*c
osh(f*x + e)^2)*sinh(f*x + e)^4 + 4*(14*(a^2*b - 2*a*b^2 + b^3)*cosh(f*x +
 e)^5 + 10*(a^3 - 4*a^2*b + 5*a*b^2 - 2*b^3)*cosh(f*x + e)^3 + (9*a^2*b -
14*a*b^2 + 6*b^3)*cosh(f*x + e))*sinh(f*x + e)^3 + b^3 + 2*(3*a*b^2 - 2*b^
3)*cosh(f*x + e)^2 + 2*(14*(a^2*b - 2*a*b^2 + b^3)*cosh(f*x + e)^6 + 15*(a
^3 - 4*a^2*b + 5*a*b^2 - 2*b^3)*cosh(f*x + e)^4 + 3*a*b^2 - 2*b^3 + 3*(9*a
^2*b - 14*a*b^2 + 6*b^3)*cosh(f*x + e)^2)*sinh(f*x + e)^2 + sqrt(2)*((a^2
- 2*a*b + b^2)*cosh(f*x + e)^6 + 6*(a^2 - 2*a*b + b^2)*cosh(f*x + e)*sinh(
f*x + e)^5 + (a^2 - 2*a*b + b^2)*sinh(f*x + e)^6 - 3*(a^2 - 2*a*b + b^2)*c
osh(f*x + e)^4 + 3*(5*(a^2 - 2*a*b + b^2)*cosh(f*x + e)^2 - a^2 + 2*a*b -
b^2)*sinh(f*x + e)^4 + 4*(5*(a^2 - 2*a*b + b^2)*cosh(f*x + e)^3 - 3*(a^2 -
 2*a*b + b^2)*cosh(f*x + e))*sinh(f*x + e)^3 - (4*a*b - 3*b^2)*cosh(f*x +
e)^2 + (15*(a^2 - 2*a*b + b^2)*cosh(f*x + e)^4 - 18*(a^2 - 2*a*b + b^2)...
```

Sympy [F]

$$\int \frac{\cosh(e+fx)}{\sqrt{a+b\sinh^2(e+fx)}}\,dx = \int \frac{\cosh\left(e+fx\right)}{\sqrt{a+b\sinh^2\left(e+fx\right)}}\,dx$$

input
```
integrate(cosh(f*x+e)/(a+b*sinh(f*x+e)**2)**(1/2),x)
```

output
```
Integral(cosh(e + f*x)/sqrt(a + b*sinh(e + f*x)**2), x)
```

Maxima [F]

$$\int \frac{\cosh(e+fx)}{\sqrt{a+b\sinh^2(e+fx)}}\,dx = \int \frac{\cosh\left(fx+e\right)}{\sqrt{b\sinh\left(fx+e\right)^2+a}}\,dx$$

input
```
integrate(cosh(f*x+e)/(a+b*sinh(f*x+e)^2)^(1/2),x, algorithm="maxima")
```

output
```
integrate(cosh(f*x + e)/sqrt(b*sinh(f*x + e)^2 + a), x)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{\cosh(e+fx)}{\sqrt{a+b\sinh^2(e+fx)}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(cosh(f*x+e)/(a+b*sinh(f*x+e)^2)^(1/2),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Error: Bad Argument Type
```

Mupad [B] (verification not implemented)

Time = 1.96 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.87

$$\int \frac{\cosh(e+fx)}{\sqrt{a+b\sinh^2(e+fx)}}\,dx = \frac{\ln\left(\sqrt{b}\sinh(e+f\,x)+\sqrt{b\sinh\left(e+f\,x\right)^2+a}\right)}{\sqrt{b}\,f}$$

input
```
int(cosh(e + f*x)/(a + b*sinh(e + f*x)^2)^(1/2),x)
```

output
```
log(b^(1/2)*sinh(e + f*x) + (a + b*sinh(e + f*x)^2)^(1/2))/(b^(1/2)*f)
```

Reduce [F]

$$\int \frac{\cosh(e+fx)}{\sqrt{a+b\sinh^2(e+fx)}}\,dx = \int \frac{\sqrt{\sinh\left(fx+e\right)^2 b+a}\,\cosh\left(fx+e\right)}{\sinh\left(fx+e\right)^2 b+a}dx$$

input
```
int(cosh(f*x+e)/(a+b*sinh(f*x+e)^2)^(1/2),x)
```

output
```
int((sqrt(sinh(e + f*x)**2*b + a)*cosh(e + f*x))/(sinh(e + f*x)**2*b + a),
x)
```

3.339 $\int \frac{\text{sech}(e+fx)}{\sqrt{a+b\sinh^2(e+fx)}}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 46

$$\int \frac{\text{sech}(e+fx)}{\sqrt{a+b\sinh^2(e+fx)}}\,dx=\frac{\arctan\left(\frac{\sqrt{a-b}\sinh(e+fx)}{\sqrt{a+b\sinh^2(e+fx)}}\right)}{\sqrt{a-b}f}$$

output
```
arctan((a-b)^(1/2)*sinh(f*x+e)/(a+b*sinh(f*x+e)^2)^(1/2))/(a-b)^(1/2)/f
```

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.00

$$\int \frac{\text{sech}(e+fx)}{\sqrt{a+b\sinh^2(e+fx)}}\,dx=\frac{\arctan\left(\frac{\sqrt{a-b}\sinh(e+fx)}{\sqrt{a+b\sinh^2(e+fx)}}\right)}{\sqrt{a-b}f}$$

input
```
Integrate[Sech[e + f*x]/Sqrt[a + b*Sinh[e + f*x]^2],x]
```

output
```
ArcTan[(Sqrt[a - b]*Sinh[e + f*x])/Sqrt[a + b*Sinh[e + f*x]^2]]/(Sqrt[a -
b]*f)
```

Rubi [A] (verified)

Time = 0.31 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.174, Rules used = {3042, 3669, 291, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\text{sech}(e+fx)}{\sqrt{a+b\sinh^2(e+fx)}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{\cos(ie+ifx)\sqrt{a-b\sin(ie+ifx)^2}}dx$$

$$\Big\downarrow \text{3669}$$

$$\frac{\int \frac{1}{\left(\sinh^2(e+fx)+1\right)\sqrt{b\sinh^2(e+fx)+a}}d\sinh(e+fx)}{f}$$

$$\Big\downarrow \text{291}$$

$$\frac{\int \frac{1}{1-\frac{(b-a)\sinh^2(e+fx)}{b\sinh^2(e+fx)+a}}d\frac{\sinh(e+fx)}{\sqrt{b\sinh^2(e+fx)+a}}}{f}$$

$$\Big\downarrow \text{216}$$

$$\frac{\arctan\left(\frac{\sqrt{a-b}\sinh(e+fx)}{\sqrt{a+b\sinh^2(e+fx)}}\right)}{f\sqrt{a-b}}$$

input
```
Int[Sech[e + f*x]/Sqrt[a + b*Sinh[e + f*x]^2],x]
```

output
```
ArcTan[(Sqrt[a - b]*Sinh[e + f*x])/Sqrt[a + b*Sinh[e + f*x]^2]]/(Sqrt[a -
b]*f)
```

Defintions of rubi rules used

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 291
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*((c_) + (d_.)*(x_)^2)), x_Symbol] :> Subst
[Int[1/(c - (b*c - a*d)*x^2), x], x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b, c,
d}, x] && NeQ[b*c - a*d, 0]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3669
```
Int[cos[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^
(p_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Simp[ff/f   S
ubst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a + b*ff^2*x^2)^p, x], x, Sin[e + f*x]
/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m - 1)/2]
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.31 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.76

method	result	size
default	$\frac{\text{`int/indef0`}\left(\frac{1}{\cosh(fx+e)^2\sqrt{a+b\sinh(fx+e)^2}},\sinh(fx+e)\right)}{f}$	35

input
```
int(sech(f*x+e)/(a+b*sinh(f*x+e)^2)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
`int/indef0`(1/cosh(f*x+e)^2/(a+b*sinh(f*x+e)^2)^(1/2),sinh(f*x+e))/f
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 237 vs. $2(40) = 80$.

Time = 0.11 (sec) , antiderivative size = 598, normalized size of antiderivative = 13.00

$$\int \frac{\operatorname{sech}(e+fx)}{\sqrt{a+b\sinh^2(e+fx)}}\,dx = \text{Too large to display}$$

input
```
integrate(sech(f*x+e)/(a+b*sinh(f*x+e)^2)^(1/2),x, algorithm="fricas")
```

output
```
[-1/2*sqrt(-a + b)*log(((a - 2*b)*cosh(f*x + e)^4 + 4*(a - 2*b)*cosh(f*x +
 e)*sinh(f*x + e)^3 + (a - 2*b)*sinh(f*x + e)^4 - 2*(3*a - 2*b)*cosh(f*x +
 e)^2 + 2*(3*(a - 2*b)*cosh(f*x + e)^2 - 3*a + 2*b)*sinh(f*x + e)^2 - 2*sq
rt(2)*(cosh(f*x + e)^2 + 2*cosh(f*x + e)*sinh(f*x + e) + sinh(f*x + e)^2 -
 1)*sqrt(-a + b)*sqrt((b*cosh(f*x + e)^2 + b*sinh(f*x + e)^2 + 2*a - b)/(c
osh(f*x + e)^2 - 2*cosh(f*x + e)*sinh(f*x + e) + sinh(f*x + e)^2)) + 4*((a
 - 2*b)*cosh(f*x + e)^3 - (3*a - 2*b)*cosh(f*x + e))*sinh(f*x + e) + a - 2
*b)/(cosh(f*x + e)^4 + 4*cosh(f*x + e)*sinh(f*x + e)^3 + sinh(f*x + e)^4 +
 2*(3*cosh(f*x + e)^2 + 1)*sinh(f*x + e)^2 + 2*cosh(f*x + e)^2 + 4*(cosh(f
*x + e)^3 + cosh(f*x + e))*sinh(f*x + e) + 1))/((a - b)*f), arctan(sqrt(2)
*(cosh(f*x + e)^2 + 2*cosh(f*x + e)*sinh(f*x + e) + sinh(f*x + e)^2 - 1)*s
qrt(a - b)*sqrt((b*cosh(f*x + e)^2 + b*sinh(f*x + e)^2 + 2*a - b)/(cosh(f*
x + e)^2 - 2*cosh(f*x + e)*sinh(f*x + e) + sinh(f*x + e)^2))/(b*cosh(f*x +
 e)^4 + 4*b*cosh(f*x + e)*sinh(f*x + e)^3 + b*sinh(f*x + e)^4 + 2*(2*a - b
)*cosh(f*x + e)^2 + 2*(3*b*cosh(f*x + e)^2 + 2*a - b)*sinh(f*x + e)^2 + 4*
(b*cosh(f*x + e)^3 + (2*a - b)*cosh(f*x + e))*sinh(f*x + e) + b))/(sqrt(a
- b)*f)]
```

Sympy [F]

$$\int \frac{\operatorname{sech}(e+fx)}{\sqrt{a+b\sinh^2(e+fx)}}\,dx = \int \frac{\operatorname{sech}(e+fx)}{\sqrt{a+b\sinh^2(e+fx)}}\,dx$$

input
```
integrate(sech(f*x+e)/(a+b*sinh(f*x+e)**2)**(1/2),x)
```

output
```
Integral(sech(e + f*x)/sqrt(a + b*sinh(e + f*x)**2), x)
```

Maxima [F]

$$\int \frac{\operatorname{sech}(e+fx)}{\sqrt{a+b\sinh^2(e+fx)}}\,dx = \int \frac{\operatorname{sech}(fx+e)}{\sqrt{b\sinh(fx+e)^2+a}}\,dx$$

input
```
integrate(sech(f*x+e)/(a+b*sinh(f*x+e)^2)^(1/2),x, algorithm="maxima")
```

output
```
integrate(sech(f*x + e)/sqrt(b*sinh(f*x + e)^2 + a), x)
```

Giac [A] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 80, normalized size of antiderivative = 1.74

$$\int \frac{\operatorname{sech}(e+fx)}{\sqrt{a+b\sinh^2(e+fx)}}\,dx = \frac{2\arctan\left(-\frac{\sqrt{b}e^{(2fx+2e)}-\sqrt{be^{(4fx+4e)}+4ae^{(2fx+2e)}-2be^{(2fx+2e)}+b}+\sqrt{b}}{2\sqrt{a-b}}\right)}{\sqrt{a-b}f}$$

input
```
integrate(sech(f*x+e)/(a+b*sinh(f*x+e)^2)^(1/2),x, algorithm="giac")
```

output
```
2*arctan(-1/2*(sqrt(b)*e^(2*f*x + 2*e) - sqrt(b*e^(4*f*x + 4*e) + 4*a*e^(2
*f*x + 2*e) - 2*b*e^(2*f*x + 2*e) + b) + sqrt(b))/sqrt(a - b))/(sqrt(a - b
)*f)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\operatorname{sech}(e+fx)}{\sqrt{a+b\sinh^2(e+fx)}}\,dx=\int \frac{1}{\cosh\left(e+f\,x\right)\,\sqrt{b\,{\sinh\left(e+f\,x\right)}^2+a}}\,dx$$

input
```
int(1/(cosh(e + f*x)*(a + b*sinh(e + f*x)^2)^(1/2)),x)
```

output
```
int(1/(cosh(e + f*x)*(a + b*sinh(e + f*x)^2)^(1/2)), x)
```

Reduce [F]

$$\int \frac{\operatorname{sech}(e+fx)}{\sqrt{a+b\sinh^2(e+fx)}}\,dx=\int \frac{\sqrt{\sinh\left(fx+e\right)^2 b+a}\,\operatorname{sech}(fx+e)}{\sinh\left(fx+e\right)^2 b+a}dx$$

input
```
int(sech(f*x+e)/(a+b*sinh(f*x+e)^2)^(1/2),x)
```

output
```
int((sqrt(sinh(e + f*x)**2*b + a)*sech(e + f*x))/(sinh(e + f*x)**2*b + a),
x)
```

3.340 $\int \frac{\text{sech}^3(e+fx)}{\sqrt{a+b\sinh^2(e+fx)}}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 97

$$\int \frac{\text{sech}^3(e+fx)}{\sqrt{a+b\sinh^2(e+fx)}}\,dx = \frac{(a-2b)\arctan\left(\frac{\sqrt{a-b}\sinh(e+fx)}{\sqrt{a+b\sinh^2(e+fx)}}\right)}{2(a-b)^{3/2}f} + \frac{\text{sech}(e+fx)\sqrt{a+b\sinh^2(e+fx)}\tanh(e+fx)}{2(a-b)f}$$

output
```
1/2*(a-2*b)*arctan((a-b)^(1/2)*sinh(f*x+e)/(a+b*sinh(f*x+e)^2)^(1/2))/(a-b
)^(3/2)/f+1/2*sech(f*x+e)*(a+b*sinh(f*x+e)^2)^(1/2)*tanh(f*x+e)/(a-b)/f
```

Mathematica [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 8.64 (sec) , antiderivative size = 417, normalized size of antiderivative = 4.30

$$\int \frac{\operatorname{sech}^3(e+fx)}{\sqrt{a+b\sinh^2(e+fx)}}\,dx$$

$$= \frac{\operatorname{sech}^3(e+fx)\left(1+\frac{b\sinh^2(e+fx)}{a}\right)\tanh(e+fx)\left(45a\arcsin\left(\sqrt{\frac{(a-b)\tanh^2(e+fx)}{a}}\right)+30b\arcsin\left(\sqrt{\frac{(a-b)\tan}{}}\right.\right.}{}$$

input
```
Integrate[Sech[e + f*x]^3/Sqrt[a + b*Sinh[e + f*x]^2],x]
```

output
```
(Sech[e + f*x]^3*(1 + (b*Sinh[e + f*x]^2)/a)*Tanh[e + f*x]*(45*a*ArcSin[Sq
rt[((a - b)*Tanh[e + f*x]^2)/a]] + 30*b*ArcSin[Sqrt[((a - b)*Tanh[e + f*x]
^2)/a]]*Sinh[e + f*x]^2 + 16*a*Hypergeometric2F1[2, 3, 7/2, ((a - b)*Tanh[
e + f*x]^2)/a]*Sqrt[(Sech[e + f*x]^2*(a + b*Sinh[e + f*x]^2))/a]*(((a - b)
*Tanh[e + f*x]^2)/a)^(5/2) + 16*b*Hypergeometric2F1[2, 3, 7/2, ((a - b)*Ta
nh[e + f*x]^2)/a]*Sinh[e + f*x]^2*Sqrt[(Sech[e + f*x]^2*(a + b*Sinh[e + f*
x]^2))/a]*(((a - b)*Tanh[e + f*x]^2)/a)^(5/2) - 45*a*Sqrt[((a - b)*Sech[e
+ f*x]^2*(a + b*Sinh[e + f*x]^2)*Tanh[e + f*x]^2)/a^2] - 30*b*Sinh[e + f*x
]^2*Sqrt[((a - b)*Sech[e + f*x]^2*(a + b*Sinh[e + f*x]^2)*Tanh[e + f*x]^2)
/a^2]))/(30*a*f*Sqrt[a + b*Sinh[e + f*x]^2]*Sqrt[(Sech[e + f*x]^2*(a + b*S
inh[e + f*x]^2))/a]*(((a - b)*Tanh[e + f*x]^2)/a)^(3/2))
```

Rubi [A] (verified)

Time = 0.36 (sec) , antiderivative size = 101, normalized size of antiderivative = 1.04, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {3042, 3669, 296, 291, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\operatorname{sech}^3(e+fx)}{\sqrt{a+b\sinh^2(e+fx)}}\,dx$$

$$\Big\downarrow \; 3042$$

$$\int \frac{1}{\cos(ie+ifx)^3\sqrt{a-b\sin(ie+ifx)^2}}dx$$

$$\Big\downarrow \text{3669}$$

$$\frac{\int \frac{1}{(\sinh^2(e+fx)+1)^2\sqrt{b\sinh^2(e+fx)+a}}d\sinh(e+fx)}{f}$$

$$\Big\downarrow \text{296}$$

$$\frac{\frac{(a-2b)\int \frac{1}{\left(\sinh^2(e+fx)+1\right)\sqrt{b\sinh^2(e+fx)+a}}d\sinh(e+fx)}{2(a-b)}+\frac{\sinh(e+fx)\sqrt{a+b\sinh^2(e+fx)}}{2(a-b)\left(\sinh^2(e+fx)+1\right)}}{f}$$

$$\Big\downarrow \text{291}$$

$$\frac{\frac{(a-2b)\int \frac{1}{1-\frac{(b-a)\sinh^2(e+fx)}{b\sinh^2(e+fx)+a}}d\frac{\sinh(e+fx)}{\sqrt{b\sinh^2(e+fx)+a}}}{2(a-b)}+\frac{\sinh(e+fx)\sqrt{a+b\sinh^2(e+fx)}}{2(a-b)\left(\sinh^2(e+fx)+1\right)}}{f}$$

$$\Big\downarrow \text{216}$$

$$\frac{\frac{(a-2b)\arctan\left(\frac{\sqrt{a-b}\sinh(e+fx)}{\sqrt{a+b\sinh^2(e+fx)}}\right)}{2(a-b)^{3/2}}+\frac{\sinh(e+fx)\sqrt{a+b\sinh^2(e+fx)}}{2(a-b)\left(\sinh^2(e+fx)+1\right)}}{f}$$

input
```
Int[Sech[e + f*x]^3/Sqrt[a + b*Sinh[e + f*x]^2],x]
```

output
```
(((a - 2*b)*ArcTan[(Sqrt[a - b]*Sinh[e + f*x])/Sqrt[a + b*Sinh[e + f*x]^2]
])/(2*(a - b)^(3/2)) + (Sinh[e + f*x]*Sqrt[a + b*Sinh[e + f*x]^2])/(2*(a -
 b)*(1 + Sinh[e + f*x]^2)))/f
```

Defintions of rubi rules used

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 291
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*((c_) + (d_.)*(x_)^2)), x_Symbol] :> Subst
[Int[1/(c - (b*c - a*d)*x^2), x], x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b, c,
d}, x] && NeQ[b*c - a*d, 0]
```

rule 296
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_), x_Symbol] :> Sim
p[(-b)*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^(q + 1)/(2*a*(p + 1)*(b*c - a*d))
), x] + Simp[(b*c + 2*(p + 1)*(b*c - a*d))/(2*a*(p + 1)*(b*c - a*d))   Int[
(a + b*x^2)^(p + 1)*(c + d*x^2)^q, x], x] /; FreeQ[{a, b, c, d, q}, x] && N
eQ[b*c - a*d, 0] && EqQ[2*(p + q + 2) + 1, 0] && (LtQ[p, -1] ||  !LtQ[q, -1
]) && NeQ[p, -1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3669
```
Int[cos[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^
(p_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Simp[ff/f   S
ubst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a + b*ff^2*x^2)^p, x], x, Sin[e + f*x]
/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m - 1)/2]
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 2.23 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.36

method	result	size
default	$\frac{\text{`int/indef0`}\left(\frac{1}{\cosh(fx+e)^4\sqrt{a+b\sinh(fx+e)^2}},\sinh(fx+e)\right)}{f}$	35
risch	Expression too large to display	377564

input
```
int(sech(f*x+e)^3/(a+b*sinh(f*x+e)^2)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
`int/indef0`(1/cosh(f*x+e)^4/(a+b*sinh(f*x+e)^2)^(1/2),sinh(f*x+e))/f
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 693 vs. $2(85) = 170$.

Time = 0.16 (sec) , antiderivative size = 1503, normalized size of antiderivative = 15.49

$$\int \frac{\operatorname{sech}^3(e+fx)}{\sqrt{a+b\sinh^2(e+fx)}}\,dx = \text{Too large to display}$$

input
```
integrate(sech(f*x+e)^3/(a+b*sinh(f*x+e)^2)^(1/2),x, algorithm="fricas")
```

output
```
[-1/4*(((a - 2*b)*cosh(f*x + e)^4 + 4*(a - 2*b)*cosh(f*x + e)*sinh(f*x + e
)^3 + (a - 2*b)*sinh(f*x + e)^4 + 2*(a - 2*b)*cosh(f*x + e)^2 + 2*(3*(a -
2*b)*cosh(f*x + e)^2 + a - 2*b)*sinh(f*x + e)^2 + 4*((a - 2*b)*cosh(f*x +
e)^3 + (a - 2*b)*cosh(f*x + e))*sinh(f*x + e) + a - 2*b)*sqrt(-a + b)*log(
((a - 2*b)*cosh(f*x + e)^4 + 4*(a - 2*b)*cosh(f*x + e)*sinh(f*x + e)^3 + (
a - 2*b)*sinh(f*x + e)^4 - 2*(3*a - 2*b)*cosh(f*x + e)^2 + 2*(3*(a - 2*b)*
cosh(f*x + e)^2 - 3*a + 2*b)*sinh(f*x + e)^2 - 2*sqrt(2)*(cosh(f*x + e)^2
+ 2*cosh(f*x + e)*sinh(f*x + e) + sinh(f*x + e)^2 - 1)*sqrt(-a + b)*sqrt((
b*cosh(f*x + e)^2 + b*sinh(f*x + e)^2 + 2*a - b)/(cosh(f*x + e)^2 - 2*cosh
(f*x + e)*sinh(f*x + e) + sinh(f*x + e)^2)) + 4*((a - 2*b)*cosh(f*x + e)^3
 - (3*a - 2*b)*cosh(f*x + e))*sinh(f*x + e) + a - 2*b)/(cosh(f*x + e)^4 +
4*cosh(f*x + e)*sinh(f*x + e)^3 + sinh(f*x + e)^4 + 2*(3*cosh(f*x + e)^2 +
 1)*sinh(f*x + e)^2 + 2*cosh(f*x + e)^2 + 4*(cosh(f*x + e)^3 + cosh(f*x +
e))*sinh(f*x + e) + 1)) - 2*sqrt(2)*((a - b)*cosh(f*x + e)^2 + 2*(a - b)*c
osh(f*x + e)*sinh(f*x + e) + (a - b)*sinh(f*x + e)^2 - a + b)*sqrt((b*cosh
(f*x + e)^2 + b*sinh(f*x + e)^2 + 2*a - b)/(cosh(f*x + e)^2 - 2*cosh(f*x +
 e)*sinh(f*x + e) + sinh(f*x + e)^2)))/((a^2 - 2*a*b + b^2)*f*cosh(f*x + e
)^4 + 4*(a^2 - 2*a*b + b^2)*f*cosh(f*x + e)*sinh(f*x + e)^3 + (a^2 - 2*a*b
 + b^2)*f*sinh(f*x + e)^4 + 2*(a^2 - 2*a*b + b^2)*f*cosh(f*x + e)^2 + 2*(3
*(a^2 - 2*a*b + b^2)*f*cosh(f*x + e)^2 + (a^2 - 2*a*b + b^2)*f)*sinh(f*...
```

Sympy [F]

$$\int \frac{\operatorname{sech}^3(e+fx)}{\sqrt{a+b\sinh^2(e+fx)}}\,dx = \int \frac{\operatorname{sech}^3(e+fx)}{\sqrt{a+b\sinh^2(e+fx)}}\,dx$$

input
```
integrate(sech(f*x+e)**3/(a+b*sinh(f*x+e)**2)**(1/2),x)
```

output
```
Integral(sech(e + f*x)**3/sqrt(a + b*sinh(e + f*x)**2), x)
```

Maxima [F]

$$\int \frac{\operatorname{sech}^3(e+fx)}{\sqrt{a+b\sinh^2(e+fx)}}\,dx = \int \frac{\operatorname{sech}(fx+e)^3}{\sqrt{b\sinh(fx+e)^2+a}}\,dx$$

input
```
integrate(sech(f*x+e)^3/(a+b*sinh(f*x+e)^2)^(1/2),x, algorithm="maxima")
```

output
```
integrate(sech(f*x + e)^3/sqrt(b*sinh(f*x + e)^2 + a), x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 694 vs. $2(85) = 170$.

Time = 0.27 (sec) , antiderivative size = 694, normalized size of antiderivative = 7.15

$$\int \frac{\operatorname{sech}^3(e+fx)}{\sqrt{a+b\sinh^2(e+fx)}}\,dx = \text{Too large to display}$$

input
```
integrate(sech(f*x+e)^3/(a+b*sinh(f*x+e)^2)^(1/2),x, algorithm="giac")
```

output
```
((a - 2*b)*arctan(-1/2*(sqrt(b)*e^(2*f*x + 2*e) - sqrt(b*e^(4*f*x + 4*e) +
 4*a*e^(2*f*x + 2*e) - 2*b*e^(2*f*x + 2*e) + b) + sqrt(b))/sqrt(a - b))/((
a*f*e^(4*e) - b*f*e^(4*e))*sqrt(a - b)) - 2*((sqrt(b)*e^(2*f*x + 2*e) - sq
rt(b*e^(4*f*x + 4*e) + 4*a*e^(2*f*x + 2*e) - 2*b*e^(2*f*x + 2*e) + b))^3*a
 - 2*(sqrt(b)*e^(2*f*x + 2*e) - sqrt(b*e^(4*f*x + 4*e) + 4*a*e^(2*f*x + 2*
e) - 2*b*e^(2*f*x + 2*e) + b))^3*b - 5*(sqrt(b)*e^(2*f*x + 2*e) - sqrt(b*e
^(4*f*x + 4*e) + 4*a*e^(2*f*x + 2*e) - 2*b*e^(2*f*x + 2*e) + b))^2*a*sqrt(
b) + 2*(sqrt(b)*e^(2*f*x + 2*e) - sqrt(b*e^(4*f*x + 4*e) + 4*a*e^(2*f*x +
2*e) - 2*b*e^(2*f*x + 2*e) + b))^2*b^(3/2) - 4*(sqrt(b)*e^(2*f*x + 2*e) -
sqrt(b*e^(4*f*x + 4*e) + 4*a*e^(2*f*x + 2*e) - 2*b*e^(2*f*x + 2*e) + b))*a
^2 - (sqrt(b)*e^(2*f*x + 2*e) - sqrt(b*e^(4*f*x + 4*e) + 4*a*e^(2*f*x + 2*
e) - 2*b*e^(2*f*x + 2*e) + b))*a*b + 2*(sqrt(b)*e^(2*f*x + 2*e) - sqrt(b*e
^(4*f*x + 4*e) + 4*a*e^(2*f*x + 2*e) - 2*b*e^(2*f*x + 2*e) + b))*b^2 - 4*a
^2*sqrt(b) + 5*a*b^(3/2) - 2*b^(5/2))/((a*f*e^(4*e) - b*f*e^(4*e))*((sqrt(
b)*e^(2*f*x + 2*e) - sqrt(b*e^(4*f*x + 4*e) + 4*a*e^(2*f*x + 2*e) - 2*b*e^
(2*f*x + 2*e) + b))^2 + 2*(sqrt(b)*e^(2*f*x + 2*e) - sqrt(b*e^(4*f*x + 4*e
) + 4*a*e^(2*f*x + 2*e) - 2*b*e^(2*f*x + 2*e) + b))*sqrt(b) + 4*a - 3*b)^2
))*e^(4*e)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\operatorname{sech}^3(e+fx)}{\sqrt{a+b\sinh^2(e+fx)}}\,dx = \int \frac{1}{\cosh\left(e+f\,x\right)^3\,\sqrt{b\sinh\left(e+f\,x\right)^2+a}}\,dx$$

input
```
int(1/(cosh(e + f*x)^3*(a + b*sinh(e + f*x)^2)^(1/2)),x)
```

output
```
int(1/(cosh(e + f*x)^3*(a + b*sinh(e + f*x)^2)^(1/2)), x)
```

Reduce [F]

$$\int \frac{\operatorname{sech}^3(e+fx)}{\sqrt{a+b\sinh^2(e+fx)}}\,dx = \int \frac{\sqrt{\sinh(fx+e)^2 b+a}\,\operatorname{sech}(fx+e)^3}{\sinh(fx+e)^2 b+a}dx$$

input
```
int(sech(f*x+e)^3/(a+b*sinh(f*x+e)^2)^(1/2),x)
```

output
```
int((sqrt(sinh(e + f*x)**2*b + a)*sech(e + f*x)**3)/(sinh(e + f*x)**2*b +
a),x)
```

3.341 $\int \frac{\cosh^4(e+fx)}{\sqrt{a+b\sinh^2(e+fx)}}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 241

$$\begin{aligned}\int \frac{\cosh^4(e+fx)}{\sqrt{a+b\sinh^2(e+fx)}}\,dx &= \frac{\cosh(e+fx)\sinh(e+fx)\sqrt{a+b\sinh^2(e+fx)}}{3bf}\\&+\frac{2(a-2b)E\left(\arctan(\sinh(e+fx))\,\middle|\,1-\frac{b}{a}\right)\operatorname{sech}(e+fx)\sqrt{a+b\sinh^2(e+fx)}}{3b^2f\sqrt{\frac{\operatorname{sech}^2(e+fx)(a+b\sinh^2(e+fx))}{a}}}\\&-\frac{(a-3b)\operatorname{EllipticF}\left(\arctan(\sinh(e+fx)),1-\frac{b}{a}\right)\operatorname{sech}(e+fx)\sqrt{a+b\sinh^2(e+fx)}}{3abf\sqrt{\frac{\operatorname{sech}^2(e+fx)(a+b\sinh^2(e+fx))}{a}}}\\&-\frac{2(a-2b)\sqrt{a+b\sinh^2(e+fx)}\tanh(e+fx)}{3b^2f}\end{aligned}$$

output
```
1/3*cosh(f*x+e)*sinh(f*x+e)*(a+b*sinh(f*x+e)^2)^(1/2)/b/f+2/3*(a-2*b)*Elli
pticE(sinh(f*x+e)/(1+sinh(f*x+e)^2)^(1/2),(1-b/a)^(1/2))*sech(f*x+e)*(a+b*
sinh(f*x+e)^2)^(1/2)/b^2/f/(sech(f*x+e)^2*(a+b*sinh(f*x+e)^2)/a)^(1/2)-1/3
*(a-3*b)*InverseJacobiAM(arctan(sinh(f*x+e)),(1-b/a)^(1/2))*sech(f*x+e)*(a
+b*sinh(f*x+e)^2)^(1/2)/a/b/f/(sech(f*x+e)^2*(a+b*sinh(f*x+e)^2)/a)^(1/2)-
2/3*(a-2*b)*(a+b*sinh(f*x+e)^2)^(1/2)*tanh(f*x+e)/b^2/f
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.84 (sec) , antiderivative size = 179, normalized size of antiderivative = 0.74

$$\int \frac{\cosh^4(e+fx)}{\sqrt{a+b\sinh^2(e+fx)}}\,dx$$
$$= \frac{4i\sqrt{2}a(a-2b)\sqrt{\frac{2a-b+b\cosh(2(e+fx))}{a}}E\left(i(e+fx)\,\middle|\,\frac{b}{a}\right)-2i\sqrt{2}(2a^2-5ab+3b^2)\sqrt{\frac{2a-b+b\cosh(2(e+fx))}{a}}\,\text{Ellipti}}{6b^2f\sqrt{4a-2b+2b\cosh(2(e+fx))}}$$

input
```
Integrate[Cosh[e + f*x]^4/Sqrt[a + b*Sinh[e + f*x]^2],x]
```

output
```
((4*I)*Sqrt[2]*a*(a - 2*b)*Sqrt[(2*a - b + b*Cosh[2*(e + f*x)])/a]*Ellipti
cE[I*(e + f*x), b/a] - (2*I)*Sqrt[2]*(2*a^2 - 5*a*b + 3*b^2)*Sqrt[(2*a - b
 + b*Cosh[2*(e + f*x)])/a]*EllipticF[I*(e + f*x), b/a] + b*(2*a - b + b*Co
sh[2*(e + f*x)])*Sinh[2*(e + f*x)])/(6*b^2*f*Sqrt[4*a - 2*b + 2*b*Cosh[2*(
e + f*x)]])
```

Rubi [A] (verified)

Time = 0.49 (sec) , antiderivative size = 292, normalized size of antiderivative = 1.21, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.320$, Rules used = {3042, 3671, 318, 25, 406, 320, 388, 313}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cosh^4(e+fx)}{\sqrt{a+b\sinh^2(e+fx)}}\,dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \frac{\cos(ie+ifx)^4}{\sqrt{a-b\sin(ie+ifx)^2}}dx$$

$$\Big\downarrow \text{ 3671}$$

$$\frac{\sqrt{\cosh^2(e+fx)}\text{sech}(e+fx)\int\frac{(\sinh^2(e+fx)+1)^{3/2}}{\sqrt{b\sinh^2(e+fx)+a}}d\sinh(e+fx)}{f}$$

$\downarrow$ 318

$$\frac{\sqrt{\cosh^2(e+fx)}\text{sech}(e+fx)\left(\frac{\int-\frac{2(a-2b)\sinh^2(e+fx)+a-3b}{\sqrt{\sinh^2(e+fx)+1}\sqrt{b\sinh^2(e+fx)+a}}d\sinh(e+fx)}{3b}+\frac{\sqrt{\sinh^2(e+fx)+1}\sinh(e+fx)\sqrt{a+b\sinh^2(e+fx)}}{3b}\right.}{f}$$

$\downarrow$ 25

$$\frac{\sqrt{\cosh^2(e+fx)}\text{sech}(e+fx)\left(\frac{\sinh(e+fx)\sqrt{\sinh^2(e+fx)+1}\sqrt{a+b\sinh^2(e+fx)}}{3b}-\frac{\int\frac{2(a-2b)\sinh^2(e+fx)+a-3b}{\sqrt{\sinh^2(e+fx)+1}\sqrt{b\sinh^2(e+fx)+a}}d\sinh(e+fx)}{3b}\right.}{f}$$

$\downarrow$ 406

$$\frac{\sqrt{\cosh^2(e+fx)}\text{sech}(e+fx)\left(\frac{\sinh(e+fx)\sqrt{\sinh^2(e+fx)+1}\sqrt{a+b\sinh^2(e+fx)}}{3b}-\frac{(a-3b)\int\frac{1}{\sqrt{\sinh^2(e+fx)+1}\sqrt{b\sinh^2(e+fx)+a}}d\sinh(\right.}{f}$$

$\downarrow$ 320

$$\frac{\sqrt{\cosh^2(e+fx)}\text{sech}(e+fx)\left(\frac{\sinh(e+fx)\sqrt{\sinh^2(e+fx)+1}\sqrt{a+b\sinh^2(e+fx)}}{3b}-\frac{2(a-2b)\int\frac{\sinh^2(e+fx)}{\sqrt{\sinh^2(e+fx)+1}\sqrt{b\sinh^2(e+fx)+a}}d\sinh}{}\right.}{f}$$

$\downarrow$ 388

$$\frac{\sqrt{\cosh^2(e+fx)}\text{sech}(e+fx)\left(\frac{\sinh(e+fx)\sqrt{\sinh^2(e+fx)+1}\sqrt{a+b\sinh^2(e+fx)}}{3b}-\frac{2(a-2b)\left(\frac{\sinh(e+fx)\sqrt{a+b\sinh^2(e+fx)}}{b\sqrt{\sinh^2(e+fx)+1}}-\frac{\int\frac{\sqrt{b\sin}}{(\sinh^2(}}{}\right.}{}\right.}{f}$$

$$\Big\downarrow \text{313}$$

$$\frac{\sqrt{\cosh^2(e+fx)}\operatorname{sech}(e+fx)\left(\frac{\sinh(e+fx)\sqrt{\sinh^2(e+fx)+1}\sqrt{a+b\sinh^2(e+fx)}}{3b}-\frac{\frac{(a-3b)\sqrt{a+b\sinh^2(e+fx)}\operatorname{EllipticF}\left(\arctan(\sinh(e+fx)\right.}{a\sqrt{\sinh^2(e+fx)+1}\sqrt{\frac{a+b\sinh^2(e+fx)}{a\left(\sinh^2(e+fx)+1\right)}}}}{\quad}\right.}{f}$$

input
```
Int[Cosh[e + f*x]^4/Sqrt[a + b*Sinh[e + f*x]^2],x]
```

output
```
(Sqrt[Cosh[e + f*x]^2]*Sech[e + f*x]*((Sinh[e + f*x]*Sqrt[1 + Sinh[e + f*x
]^2]*Sqrt[a + b*Sinh[e + f*x]^2])/(3*b) - (((a - 3*b)*EllipticF[ArcTan[Sin
h[e + f*x]], 1 - b/a]*Sqrt[a + b*Sinh[e + f*x]^2])/(a*Sqrt[1 + Sinh[e + f*
x]^2]*Sqrt[(a + b*Sinh[e + f*x]^2)/(a*(1 + Sinh[e + f*x]^2))]) + 2*(a - 2*
b)*((Sinh[e + f*x]*Sqrt[a + b*Sinh[e + f*x]^2])/(b*Sqrt[1 + Sinh[e + f*x]^
2]) - (EllipticE[ArcTan[Sinh[e + f*x]], 1 - b/a]*Sqrt[a + b*Sinh[e + f*x]^
2])/(b*Sqrt[1 + Sinh[e + f*x]^2]*Sqrt[(a + b*Sinh[e + f*x]^2)/(a*(1 + Sinh
[e + f*x]^2))])))/(3*b)))/f
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 313
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/((c_) + (d_.)*(x_)^2)^(3/2), x_Symbol] :> Sim
p[(Sqrt[a + b*x^2]/(c*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(c
 + d*x^2)))]))*EllipticE[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; FreeQ
[{a, b, c, d}, x] && PosQ[b/a] && PosQ[d/c]
```

rule 318
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_), x_Symbol] :> Sim
p[d*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^(q - 1)/(b*(2*(p + q) + 1))), x] + S
imp[1/(b*(2*(p + q) + 1))   Int[(a + b*x^2)^p*(c + d*x^2)^(q - 2)*Simp[c*(b
*c*(2*(p + q) + 1) - a*d) + d*(b*c*(2*(p + 2*q - 1) + 1) - a*d*(2*(q - 1) +
 1))*x^2, x], x], x] /; FreeQ[{a, b, c, d, p}, x] && NeQ[b*c - a*d, 0] && G
tQ[q, 1] && NeQ[2*(p + q) + 1, 0] &&  !IGtQ[p, 1] && IntBinomialQ[a, b, c,
d, 2, p, q, x]
```

rule 320
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(Sqrt[a + b*x^2]/(a*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(
c + d*x^2)))]))*EllipticF[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; Fre
eQ[{a, b, c, d}, x] && PosQ[d/c] && PosQ[b/a] &&  !SimplerSqrtQ[b/a, d/c]
```

rule 388
```
Int[(x_)^2/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol]
 :> Simp[x*(Sqrt[a + b*x^2]/(b*Sqrt[c + d*x^2])), x] - Simp[c/b   Int[Sqrt[
a + b*x^2]/(c + d*x^2)^(3/2), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
 a*d, 0] && PosQ[b/a] && PosQ[d/c] &&  !SimplerSqrtQ[b/a, d/c]
```

rule 406
```
Int[((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.)*((e_) + (f_.)*(
x_)^2), x_Symbol] :> Simp[e   Int[(a + b*x^2)^p*(c + d*x^2)^q, x], x] + Sim
p[f   Int[x^2*(a + b*x^2)^p*(c + d*x^2)^q, x], x] /; FreeQ[{a, b, c, d, e,
f, p, q}, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3671
```
Int[cos[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(
p_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Simp[ff*(Sqrt[
Cos[e + f*x]^2]/(f*Cos[e + f*x]))   Subst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a
 + b*ff^2*x^2)^p, x], x, Sin[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x]
 && IntegerQ[m/2] &&  !IntegerQ[p]
```

Maple [A] (verified)

Time = 3.12 (sec) , antiderivative size = 356, normalized size of antiderivative = 1.48

method	result
default	$\frac{\sqrt{-\frac{b}{a}}\, b\cosh(fx+e)^4\sinh(fx+e)+\sqrt{-\frac{b}{a}}\, a\cosh(fx+e)^2\sinh(fx+e)-\sqrt{-\frac{b}{a}}\, b\cosh(fx+e)^2\sinh(fx+e)+a\sqrt{\frac{b\cosh(fx+e)^2}{a}+\frac{a-b}{a}}\, \sqrt{}$

input
```
int(cosh(f*x+e)^4/(a+b*sinh(f*x+e)^2)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/3*((-b/a)^(1/2)*b*cosh(f*x+e)^4*sinh(f*x+e)+(-b/a)^(1/2)*a*cosh(f*x+e)^2
*sinh(f*x+e)-(-b/a)^(1/2)*b*cosh(f*x+e)^2*sinh(f*x+e)+a*(b/a*cosh(f*x+e)^2
+(a-b)/a)^(1/2)*(cosh(f*x+e)^2)^(1/2)*EllipticF(sinh(f*x+e)*(-b/a)^(1/2),(
1/b*a)^(1/2))-(b/a*cosh(f*x+e)^2+(a-b)/a)^(1/2)*(cosh(f*x+e)^2)^(1/2)*Elli
pticF(sinh(f*x+e)*(-b/a)^(1/2),(1/b*a)^(1/2))*b-2*(b/a*cosh(f*x+e)^2+(a-b)
/a)^(1/2)*(cosh(f*x+e)^2)^(1/2)*EllipticE(sinh(f*x+e)*(-b/a)^(1/2),(1/b*a)
^(1/2))*a+4*(b/a*cosh(f*x+e)^2+(a-b)/a)^(1/2)*(cosh(f*x+e)^2)^(1/2)*Ellipt
icE(sinh(f*x+e)*(-b/a)^(1/2),(1/b*a)^(1/2))*b)/b/(-b/a)^(1/2)/cosh(f*x+e)/
(a+b*sinh(f*x+e)^2)^(1/2)/f
```

Fricas [F]

$$\int \frac{\cosh^4(e+fx)}{\sqrt{a+b\sinh^2(e+fx)}}\,dx=\int \frac{\cosh\left(fx+e\right)^4}{\sqrt{b\sinh\left(fx+e\right)^2+a}}\,dx$$

input
```
integrate(cosh(f*x+e)^4/(a+b*sinh(f*x+e)^2)^(1/2),x, algorithm="fricas")
```

output
```
integral(cosh(f*x + e)^4/sqrt(b*sinh(f*x + e)^2 + a), x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\cosh^4(e+fx)}{\sqrt{a+b\sinh^2(e+fx)}}\,dx = \text{Timed out}$$

input
```
integrate(cosh(f*x+e)**4/(a+b*sinh(f*x+e)**2)**(1/2),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{\cosh^4(e+fx)}{\sqrt{a+b\sinh^2(e+fx)}}\,dx = \int \frac{\cosh\left(fx+e\right)^4}{\sqrt{b\sinh\left(fx+e\right)^2+a}}\,dx$$

input
```
integrate(cosh(f*x+e)^4/(a+b*sinh(f*x+e)^2)^(1/2),x, algorithm="maxima")
```

output
```
integrate(cosh(f*x + e)^4/sqrt(b*sinh(f*x + e)^2 + a), x)
```

Giac [F]

$$\int \frac{\cosh^4(e+fx)}{\sqrt{a+b\sinh^2(e+fx)}}\,dx = \int \frac{\cosh\left(fx+e\right)^4}{\sqrt{b\sinh\left(fx+e\right)^2+a}}\,dx$$

input
```
integrate(cosh(f*x+e)^4/(a+b*sinh(f*x+e)^2)^(1/2),x, algorithm="giac")
```

output
```
integrate(cosh(f*x + e)^4/sqrt(b*sinh(f*x + e)^2 + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\cosh^4(e+fx)}{\sqrt{a+b\sinh^2(e+fx)}}\,dx = \int \frac{\cosh(e+f\,x)^4}{\sqrt{b\sinh\left(e+f\,x\right)^2+a}}\,dx$$

input
```
int(cosh(e + f*x)^4/(a + b*sinh(e + f*x)^2)^(1/2),x)
```

output
```
int(cosh(e + f*x)^4/(a + b*sinh(e + f*x)^2)^(1/2), x)
```

Reduce [F]

$$\int \frac{\cosh^4(e+fx)}{\sqrt{a+b\sinh^2(e+fx)}}\,dx = \int \frac{\sqrt{\sinh\left(fx+e\right)^2 b+a}\,\cosh\left(fx+e\right)^4}{\sinh\left(fx+e\right)^2 b+a}dx$$

input
```
int(cosh(f*x+e)^4/(a+b*sinh(f*x+e)^2)^(1/2),x)
```

output
```
int((sqrt(sinh(e + f*x)**2*b + a)*cosh(e + f*x)**4)/(sinh(e + f*x)**2*b +
a),x)
```

3.342 $\int \frac{\cosh^2(e+fx)}{\sqrt{a+b\sinh^2(e+fx)}}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 177

$$\int \frac{\cosh^2(e+fx)}{\sqrt{a+b\sinh^2(e+fx)}}\,dx = -\frac{E\left(\arctan(\sinh(e+fx))\left|1-\frac{b}{a}\right.\right)\operatorname{sech}(e+fx)\sqrt{a+b\sinh^2(e+fx)}}{bf\sqrt{\frac{\operatorname{sech}^2(e+fx)(a+b\sinh^2(e+fx))}{a}}} + \frac{\operatorname{EllipticF}\left(\arctan(\sinh(e+fx)),1-\frac{b}{a}\right)\operatorname{sech}(e+fx)\sqrt{a+b\sinh^2(e+fx)}}{af\sqrt{\frac{\operatorname{sech}^2(e+fx)(a+b\sinh^2(e+fx))}{a}}} + \frac{\sqrt{a+b\sinh^2(e+fx)}\tanh(e+fx)}{bf}$$

output
```
-EllipticE(sinh(f*x+e)/(1+sinh(f*x+e)^2)^(1/2),(1-b/a)^(1/2))*sech(f*x+e)*
(a+b*sinh(f*x+e)^2)^(1/2)/b/f/(sech(f*x+e)^2*(a+b*sinh(f*x+e)^2)/a)^(1/2)+
InverseJacobiAM(arctan(sinh(f*x+e)),(1-b/a)^(1/2))*sech(f*x+e)*(a+b*sinh(f
*x+e)^2)^(1/2)/a/f/(sech(f*x+e)^2*(a+b*sinh(f*x+e)^2)/a)^(1/2)+(a+b*sinh(f
*x+e)^2)^(1/2)*tanh(f*x+e)/b/f
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.34 (sec) , antiderivative size = 95, normalized size of antiderivative = 0.54

$$\int \frac{\cosh^2(e+fx)}{\sqrt{a+b\sinh^2(e+fx)}}\,dx = -\frac{i\sqrt{\frac{2a-b+b\cosh(2(e+fx))}{a}}\left(aE\left(i(e+fx)\,\middle|\,\frac{b}{a}\right)+(-a+b)\,\text{EllipticF}\left(i(e+fx),\frac{b}{a}\right)\right)}{bf\sqrt{2a-b+b\cosh(2(e+fx))}}$$

input
```
Integrate[Cosh[e + f*x]^2/Sqrt[a + b*Sinh[e + f*x]^2],x]
```

output
```
((-I)*Sqrt[(2*a - b + b*Cosh[2*(e + f*x)])/a]*(a*EllipticE[I*(e + f*x), b/
a] + (-a + b)*EllipticF[I*(e + f*x), b/a]))/(b*f*Sqrt[2*a - b + b*Cosh[2*(
e + f*x)]])
```

Rubi [A] (verified)

Time = 0.43 (sec) , antiderivative size = 228, normalized size of antiderivative = 1.29, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.240$, Rules used = {3042, 3671, 324, 320, 388, 313}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cosh^2(e+fx)}{\sqrt{a+b\sinh^2(e+fx)}}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\cos(ie+ifx)^2}{\sqrt{a-b\sin(ie+ifx)^2}}dx$$

$$\Big\downarrow \text{3671}$$

$$\frac{\sqrt{\cosh^2(e+fx)}\text{sech}(e+fx)\int \frac{\sqrt{\sinh^2(e+fx)+1}}{\sqrt{b\sinh^2(e+fx)+a}}d\sinh(e+fx)}{f}$$

$\Big\downarrow$ 324

$$\frac{\sqrt{\cosh^2(e+fx)}\text{sech}(e+fx)\left(\int \frac{1}{\sqrt{\sinh^2(e+fx)+1}\sqrt{b\sinh^2(e+fx)+a}}d\sinh(e+fx)+\int \frac{\sinh^2(e+fx)}{\sqrt{\sinh^2(e+fx)+1}\sqrt{b\sinh^2(e+fx)+a}}\right.}{f}$$

$\Big\downarrow$ 320

$$\frac{\sqrt{\cosh^2(e+fx)}\text{sech}(e+fx)\left(\int \frac{\sinh^2(e+fx)}{\sqrt{\sinh^2(e+fx)+1}\sqrt{b\sinh^2(e+fx)+a}}d\sinh(e+fx)+\frac{\sqrt{a+b\sinh^2(e+fx)}\text{EllipticF}\left(\arctan(\sin\right.}{a\sqrt{\sinh^2(e+fx)+1}\sqrt{\frac{a+b\sinh^2}{a\left(\sinh^2(e\right.}}}\right.}{f}$$

$\Big\downarrow$ 388

$$\frac{\sqrt{\cosh^2(e+fx)}\text{sech}(e+fx)\left(-\frac{\int \frac{\sqrt{b\sinh^2(e+fx)+a}}{\left(\sinh^2(e+fx)+1\right)^{3/2}}d\sinh(e+fx)}{b}+\frac{\sqrt{a+b\sinh^2(e+fx)}\text{EllipticF}\left(\arctan(\sinh(e+fx)),1-\frac{b}{a}\right)}{a\sqrt{\sinh^2(e+fx)+1}\sqrt{\frac{a+b\sinh^2(e+fx)}{a\left(\sinh^2(e+fx)+1\right)}}}+\right.}{f}$$

$\Big\downarrow$ 313

$$\frac{\sqrt{\cosh^2(e+fx)}\text{sech}(e+fx)\left(\frac{\sqrt{a+b\sinh^2(e+fx)}\text{EllipticF}\left(\arctan(\sinh(e+fx)),1-\frac{b}{a}\right)}{a\sqrt{\sinh^2(e+fx)+1}\sqrt{\frac{a+b\sinh^2(e+fx)}{a\left(\sinh^2(e+fx)+1\right)}}}-\frac{\sqrt{a+b\sinh^2(e+fx)}E\left(\arctan(\sinh(e+fx\right.}{b\sqrt{\sinh^2(e+fx)+1}\sqrt{\frac{a+b\sinh^2(e+f}{a\left(\sinh^2(e+fx)\right.}}}\right.}{f}$$

input
```
Int[Cosh[e + f*x]^2/Sqrt[a + b*Sinh[e + f*x]^2],x]
```

output
```
(Sqrt[Cosh[e + f*x]^2]*Sech[e + f*x]*((Sinh[e + f*x]*Sqrt[a + b*Sinh[e + f
*x]^2])/(b*Sqrt[1 + Sinh[e + f*x]^2]) - (EllipticE[ArcTan[Sinh[e + f*x]],
1 - b/a]*Sqrt[a + b*Sinh[e + f*x]^2])/(b*Sqrt[1 + Sinh[e + f*x]^2]*Sqrt[(a
 + b*Sinh[e + f*x]^2)/(a*(1 + Sinh[e + f*x]^2))]) + (EllipticF[ArcTan[Sinh
[e + f*x]], 1 - b/a]*Sqrt[a + b*Sinh[e + f*x]^2])/(a*Sqrt[1 + Sinh[e + f*x
]^2]*Sqrt[(a + b*Sinh[e + f*x]^2)/(a*(1 + Sinh[e + f*x]^2))])))/f
```

Defintions of rubi rules used

rule 313
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/((c_) + (d_.)*(x_)^2)^(3/2), x_Symbol] :> Sim
p[(Sqrt[a + b*x^2]/(c*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(c
+ d*x^2)))]))*EllipticE[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; FreeQ
[{a, b, c, d}, x] && PosQ[b/a] && PosQ[d/c]
```

rule 320
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(Sqrt[a + b*x^2]/(a*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(
c + d*x^2)))]))*EllipticF[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; Fre
eQ[{a, b, c, d}, x] && PosQ[d/c] && PosQ[b/a] &&  !SimplerSqrtQ[b/a, d/c]
```

rule 324
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
a   Int[1/(Sqrt[a + b*x^2]*Sqrt[c + d*x^2]), x], x] + Simp[b   Int[x^2/(Sqr
t[a + b*x^2]*Sqrt[c + d*x^2]), x], x] /; FreeQ[{a, b, c, d}, x] && PosQ[d/c
] && PosQ[b/a]
```

rule 388
```
Int[(x_)^2/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol]
 :> Simp[x*(Sqrt[a + b*x^2]/(b*Sqrt[c + d*x^2])), x] - Simp[c/b   Int[Sqrt[
a + b*x^2]/(c + d*x^2)^(3/2), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
 a*d, 0] && PosQ[b/a] && PosQ[d/c] &&  !SimplerSqrtQ[b/a, d/c]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3671
```
Int[cos[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(
p_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Simp[ff*(Sqrt[
Cos[e + f*x]^2]/(f*Cos[e + f*x]))   Subst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a
 + b*ff^2*x^2)^p, x], x, Sin[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x]
 && IntegerQ[m/2] &&  !IntegerQ[p]
```

Maple [A] (verified)

Time = 1.52 (sec) , antiderivative size = 86, normalized size of antiderivative = 0.49

method	result	size
default	$\frac{\sqrt{\frac{a+b\sinh(fx+e)^2}{a}}\,\sqrt{\frac{\cosh(2fx+2e)}{2}+\frac{1}{2}}\ \operatorname{EllipticE}\left(\sinh(fx+e)\sqrt{-\frac{b}{a}},\sqrt{\frac{a}{b}}\right)}{\sqrt{-\frac{b}{a}}\ \cosh(fx+e)\sqrt{a+b\sinh(fx+e)^2}\,f}$	86

input
```
int(cosh(f*x+e)^2/(a+b*sinh(f*x+e)^2)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
((a+b*sinh(f*x+e)^2)/a)^(1/2)*(cosh(f*x+e)^2)^(1/2)*EllipticE(sinh(f*x+e)*
(-b/a)^(1/2),(1/b*a)^(1/2))/(-b/a)^(1/2)/cosh(f*x+e)/(a+b*sinh(f*x+e)^2)^(
1/2)/f
```

Fricas [F]

$$\int \frac{\cosh^2(e+fx)}{\sqrt{a+b\sinh^2(e+fx)}}\,dx=\int \frac{\cosh\left(fx+e\right)^2}{\sqrt{b\sinh\left(fx+e\right)^2+a}}\,dx$$

input
```
integrate(cosh(f*x+e)^2/(a+b*sinh(f*x+e)^2)^(1/2),x, algorithm="fricas")
```

output
```
integral(cosh(f*x + e)^2/sqrt(b*sinh(f*x + e)^2 + a), x)
```

Sympy [F]

$$\int \frac{\cosh^2(e+fx)}{\sqrt{a+b\sinh^2(e+fx)}}\,dx=\int \frac{\cosh^2{(e+fx)}}{\sqrt{a+b\sinh^2{(e+fx)}}}\,dx$$

input
```
integrate(cosh(f*x+e)**2/(a+b*sinh(f*x+e)**2)**(1/2),x)
```

output
```
Integral(cosh(e + f*x)**2/sqrt(a + b*sinh(e + f*x)**2), x)
```

Maxima [F]

$$\int \frac{\cosh^2(e+fx)}{\sqrt{a+b\sinh^2(e+fx)}}\,dx=\int \frac{\cosh{(fx+e)}^2}{\sqrt{b\sinh{(fx+e)}^2+a}}\,dx$$

input
```
integrate(cosh(f*x+e)^2/(a+b*sinh(f*x+e)^2)^(1/2),x, algorithm="maxima")
```

output
```
integrate(cosh(f*x + e)^2/sqrt(b*sinh(f*x + e)^2 + a), x)
```

Giac [F]

$$\int \frac{\cosh^2(e+fx)}{\sqrt{a+b\sinh^2(e+fx)}}\,dx=\int \frac{\cosh{(fx+e)}^2}{\sqrt{b\sinh{(fx+e)}^2+a}}\,dx$$

input
```
integrate(cosh(f*x+e)^2/(a+b*sinh(f*x+e)^2)^(1/2),x, algorithm="giac")
```

output
```
integrate(cosh(f*x + e)^2/sqrt(b*sinh(f*x + e)^2 + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\cosh^2(e+fx)}{\sqrt{a+b\sinh^2(e+fx)}}\,dx = \int \frac{\cosh(e+f\,x)^2}{\sqrt{b\sinh\left(e+f\,x\right)^2+a}}\,dx$$

input
```
int(cosh(e + f*x)^2/(a + b*sinh(e + f*x)^2)^(1/2),x)
```

output
```
int(cosh(e + f*x)^2/(a + b*sinh(e + f*x)^2)^(1/2), x)
```

Reduce [F]

$$\int \frac{\cosh^2(e+fx)}{\sqrt{a+b\sinh^2(e+fx)}}\,dx = \int \frac{\sqrt{\sinh\left(fx+e\right)^2b+a}\,\cosh\left(fx+e\right)^2}{\sinh\left(fx+e\right)^2b+a}dx$$

input
```
int(cosh(f*x+e)^2/(a+b*sinh(f*x+e)^2)^(1/2),x)
```

output
```
int((sqrt(sinh(e + f*x)**2*b + a)*cosh(e + f*x)**2)/(sinh(e + f*x)**2*b +
a),x)
```

3.343 $\int \frac{1}{\sqrt{a+b\sinh^2(e+fx)}}\,dx$

Optimal result

Integrand size = 16, antiderivative size = 61

$$\int \frac{1}{\sqrt{a+b\sinh^2(e+fx)}}\,dx=-\frac{i\,\text{EllipticF}\left(ie+ifx,\frac{b}{a}\right)\sqrt{\frac{a+b\sinh^2(e+fx)}{a}}}{f\sqrt{a+b\sinh^2(e+fx)}}$$

output
```
-I*InverseJacobiAM(I*e+I*f*x,(b/a)^(1/2))*((a+b*sinh(f*x+e)^2)/a)^(1/2)/f/
(a+b*sinh(f*x+e)^2)^(1/2)
```

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 68, normalized size of antiderivative = 1.11

$$\int \frac{1}{\sqrt{a+b\sinh^2(e+fx)}}\,dx=-\frac{i\sqrt{\frac{2a-b+b\cosh(2(e+fx))}{a}}\,\text{EllipticF}\left(i(e+fx),\frac{b}{a}\right)}{f\sqrt{2a-b+b\cosh(2(e+fx))}}$$

input
```
Integrate[1/Sqrt[a + b*Sinh[e + f*x]^2],x]
```

output
```
((-I)*Sqrt[(2*a - b + b*Cosh[2*(e + f*x)])/a]*EllipticF[I*(e + f*x), b/a])
/(f*Sqrt[2*a - b + b*Cosh[2*(e + f*x)]])
```

Rubi [A] (verified)

Time = 0.34 (sec) , antiderivative size = 60, normalized size of antiderivative = 0.98, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {3042, 3662, 3042, 3661}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt{a+b\sinh^2(e+fx)}}dx$$
$$\Big\downarrow \text{3042}$$
$$\int \frac{1}{\sqrt{a-b\sin(ie+ifx)^2}}dx$$
$$\Big\downarrow \text{3662}$$
$$\frac{\sqrt{\frac{b\sinh^2(e+fx)}{a}+1}\int \frac{1}{\sqrt{\frac{b\sinh^2(e+fx)}{a}+1}}dx}{\sqrt{a+b\sinh^2(e+fx)}}$$
$$\Big\downarrow \text{3042}$$
$$\frac{\sqrt{\frac{b\sinh^2(e+fx)}{a}+1}\int \frac{1}{\sqrt{1-\frac{b\sin(ie+ifx)^2}{a}}}dx}{\sqrt{a+b\sinh^2(e+fx)}}$$
$$\Big\downarrow \text{3661}$$
$$-\frac{i\sqrt{\frac{b\sinh^2(e+fx)}{a}+1}\,\text{EllipticF}\left(ie+ifx,\frac{b}{a}\right)}{f\sqrt{a+b\sinh^2(e+fx)}}$$

input
```
Int[1/Sqrt[a + b*Sinh[e + f*x]^2],x]
```

output
```
((-I)*EllipticF[I*e + I*f*x, b/a]*Sqrt[1 + (b*Sinh[e + f*x]^2)/a])/(f*Sqrt
[a + b*Sinh[e + f*x]^2])
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3661
```
Int[1/Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2], x_Symbol] :> Simp[(1/(S
qrt[a]*f))*EllipticF[e + f*x, -b/a], x] /; FreeQ[{a, b, e, f}, x] && GtQ[a,
 0]
```

rule 3662
```
Int[1/Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2], x_Symbol] :> Simp[Sqrt[
1 + b*(Sin[e + f*x]^2/a)]/Sqrt[a + b*Sin[e + f*x]^2]   Int[1/Sqrt[1 + (b*Si
n[e + f*x]^2)/a], x], x] /; FreeQ[{a, b, e, f}, x] &&  !GtQ[a, 0]
```

Maple [A] (verified)

Time = 0.59 (sec) , antiderivative size = 86, normalized size of antiderivative = 1.41

method	result	size
default	$\frac{\sqrt{\frac{a+b\sinh(fx+e)^2}{a}}\,\sqrt{\frac{\cosh(2fx+2e)}{2}+\frac{1}{2}}\,\operatorname{EllipticF}\left(\sinh(fx+e)\sqrt{-\frac{b}{a}},\sqrt{\frac{a}{b}}\right)}{\sqrt{-\frac{b}{a}}\,\cosh(fx+e)\sqrt{a+b\sinh(fx+e)^2}\,f}$	86

input
```
int(1/(a+b*sinh(f*x+e)^2)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/(-b/a)^(1/2)*((a+b*sinh(f*x+e)^2)/a)^(1/2)*(cosh(f*x+e)^2)^(1/2)*Ellipti
cF(sinh(f*x+e)*(-b/a)^(1/2),(1/b*a)^(1/2))/cosh(f*x+e)/(a+b*sinh(f*x+e)^2)
^(1/2)/f
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 147 vs. $2(53) = 106$.

Time = 0.11 (sec) , antiderivative size = 147, normalized size of antiderivative = 2.41

$$\int \frac{1}{\sqrt{a + b\sinh^2(e + fx)}}\,dx = -\frac{2\left(2\,b\sqrt{\frac{a^2-ab}{b^2}} + 2\,a - b\right)\sqrt{\frac{2\,b\sqrt{\frac{a^2-ab}{b^2}}-2\,a+b}{b}}F\left(\arcsin\left(\sqrt{\frac{2\,b\sqrt{\frac{a^2-ab}{b^2}}-2\,a+b}{b}}(\cosh\left(fx+e\right) + \sinh\left(fx+e\right)\right)\right.}{b^{\frac{3}{2}}f}$$

input
```
integrate(1/(a+b*sinh(f*x+e)^2)^(1/2),x, algorithm="fricas")
```

output
```
-2*(2*b*sqrt((a^2 - a*b)/b^2) + 2*a - b)*sqrt((2*b*sqrt((a^2 - a*b)/b^2) -
 2*a + b)/b)*elliptic_f(arcsin(sqrt((2*b*sqrt((a^2 - a*b)/b^2) - 2*a + b)/
b)*(cosh(f*x + e) + sinh(f*x + e))), (8*a^2 - 8*a*b + b^2 + 4*(2*a*b - b^2
)*sqrt((a^2 - a*b)/b^2))/b^2)/(b^(3/2)*f)
```

Sympy [F]

$$\int \frac{1}{\sqrt{a + b\sinh^2(e + fx)}}\,dx = \int \frac{1}{\sqrt{a + b\sinh^2\left(e + fx\right)}}\,dx$$

input
```
integrate(1/(a+b*sinh(f*x+e)**2)**(1/2),x)
```

output
```
Integral(1/sqrt(a + b*sinh(e + f*x)**2), x)
```

Maxima [F]

$$\int \frac{1}{\sqrt{a+b\sinh^2(e+fx)}}\,dx = \int \frac{1}{\sqrt{b\sinh\left(fx+e\right)^2+a}}\,dx$$

input
```
integrate(1/(a+b*sinh(f*x+e)^2)^(1/2),x, algorithm="maxima")
```

output
```
integrate(1/sqrt(b*sinh(f*x + e)^2 + a), x)
```

Giac [F]

$$\int \frac{1}{\sqrt{a+b\sinh^2(e+fx)}}\,dx = \int \frac{1}{\sqrt{b\sinh\left(fx+e\right)^2+a}}\,dx$$

input
```
integrate(1/(a+b*sinh(f*x+e)^2)^(1/2),x, algorithm="giac")
```

output
```
integrate(1/sqrt(b*sinh(f*x + e)^2 + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt{a+b\sinh^2(e+fx)}}\,dx = \int \frac{1}{\sqrt{b\sinh\left(e+f\,x\right)^2+a}}\,dx$$

input
```
int(1/(a + b*sinh(e + f*x)^2)^(1/2),x)
```

output
```
int(1/(a + b*sinh(e + f*x)^2)^(1/2), x)
```

Reduce [F]

$$\int \frac{1}{\sqrt{a+b\sinh^2(e+fx)}}\,dx=\int \frac{\sqrt{\sinh\left(fx+e\right)^2 b+a}}{\sinh\left(fx+e\right)^2 b+a}dx$$

input
```
int(1/(a+b*sinh(f*x+e)^2)^(1/2),x)
```

output
```
int(sqrt(sinh(e + f*x)**2*b + a)/(sinh(e + f*x)**2*b + a),x)
```

3.344 $\int \frac{\text{sech}^2(e+fx)}{\sqrt{a+b\sinh^2(e+fx)}}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 160

$$\int \frac{\text{sech}^2(e+fx)}{\sqrt{a+b\sinh^2(e+fx)}}\,dx = \frac{E\left(\arctan(\sinh(e+fx))\left|1-\frac{b}{a}\right.\right)\text{sech}(e+fx)\sqrt{a+b\sinh^2(e+fx)}}{(a-b)f\sqrt{\frac{\text{sech}^2(e+fx)(a+b\sinh^2(e+fx))}{a}}} - \frac{b\,\text{EllipticF}\left(\arctan(\sinh(e+fx)),1-\frac{b}{a}\right)\text{sech}(e+fx)\sqrt{a+b\sinh^2(e+fx)}}{a(a-b)f\sqrt{\frac{\text{sech}^2(e+fx)(a+b\sinh^2(e+fx))}{a}}}$$

output
```
EllipticE(sinh(f*x+e)/(1+sinh(f*x+e)^2)^(1/2),(1-b/a)^(1/2))*sech(f*x+e)*(
a+b*sinh(f*x+e)^2)^(1/2)/(a-b)/f/(sech(f*x+e)^2*(a+b*sinh(f*x+e)^2)/a)^(1/
2)-b*InverseJacobiAM(arctan(sinh(f*x+e)),(1-b/a)^(1/2))*sech(f*x+e)*(a+b*s
inh(f*x+e)^2)^(1/2)/a/(a-b)/f/(sech(f*x+e)^2*(a+b*sinh(f*x+e)^2)/a)^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.67 (sec) , antiderivative size = 159, normalized size of antiderivative = 0.99

$$\int \frac{\operatorname{sech}^2(e+fx)}{\sqrt{a+b\sinh^2(e+fx)}}\,dx = \frac{2ia\sqrt{\frac{2a-b+b\cosh(2(e+fx))}{a}}E\left(i(e+fx)\,\middle|\,\frac{b}{a}\right)-2i(a-b)\sqrt{\frac{2a-b+b\cosh(2(e+fx))}{a}}\operatorname{EllipticF}\left(i(e+fx),\frac{b}{a}\right)+\sqrt{2}(2\cdots}{2(a-b)f\sqrt{2a-b+b\cosh(2(e+fx))}}$$

input
```
Integrate[Sech[e + f*x]^2/Sqrt[a + b*Sinh[e + f*x]^2],x]
```

output
```
((2*I)*a*Sqrt[(2*a - b + b*Cosh[2*(e + f*x)])/a]*EllipticE[I*(e + f*x), b/
a] - (2*I)*(a - b)*Sqrt[(2*a - b + b*Cosh[2*(e + f*x)])/a]*EllipticF[I*(e
+ f*x), b/a] + Sqrt[2]*(2*a - b + b*Cosh[2*(e + f*x)])*Tanh[e + f*x])/(2*(
a - b)*f*Sqrt[2*a - b + b*Cosh[2*(e + f*x)]])
```

Rubi [A] (verified)

Time = 0.47 (sec) , antiderivative size = 283, normalized size of antiderivative = 1.77, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.320, Rules used = {3042, 3671, 316, 27, 324, 320, 388, 313}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\operatorname{sech}^2(e+fx)}{\sqrt{a+b\sinh^2(e+fx)}}dx$$

$$\Big\downarrow \ 3042$$

$$\int \frac{1}{\cos(ie+ifx)^2\sqrt{a-b\sin(ie+ifx)^2}}dx$$

$$\Big\downarrow \ 3671$$

$$\frac{\sqrt{\cosh^2(e+fx)}\text{sech}(e+fx)\int \frac{1}{\left(\sinh^2(e+fx)+1\right)^{3/2}\sqrt{b\sinh^2(e+fx)+a}}d\sinh(e+fx)}{f}$$

$\Big\downarrow$ 316

$$\frac{\sqrt{\cosh^2(e+fx)}\text{sech}(e+fx)\left(\frac{\sinh(e+fx)\sqrt{a+b\sinh^2(e+fx)}}{(a-b)\sqrt{\sinh^2(e+fx)+1}}-\frac{\int \frac{b\sqrt{\sinh^2(e+fx)+1}}{\sqrt{b\sinh^2(e+fx)+a}}d\sinh(e+fx)}{a-b}\right)}{f}$$

$\Big\downarrow$ 27

$$\frac{\sqrt{\cosh^2(e+fx)}\text{sech}(e+fx)\left(\frac{\sinh(e+fx)\sqrt{a+b\sinh^2(e+fx)}}{(a-b)\sqrt{\sinh^2(e+fx)+1}}-\frac{b\int \frac{\sqrt{\sinh^2(e+fx)+1}}{\sqrt{b\sinh^2(e+fx)+a}}d\sinh(e+fx)}{a-b}\right)}{f}$$

$\Big\downarrow$ 324

$$\frac{\sqrt{\cosh^2(e+fx)}\text{sech}(e+fx)\left(\frac{\sinh(e+fx)\sqrt{a+b\sinh^2(e+fx)}}{(a-b)\sqrt{\sinh^2(e+fx)+1}}-\frac{b\left(\int \frac{1}{\sqrt{\sinh^2(e+fx)+1}\sqrt{b\sinh^2(e+fx)+a}}d\sinh(e+fx)+\int \frac{\text{s}}{\sqrt{\sinh^2(e+f}}\right.}{a-b}\right.}{f}$$

$\Big\downarrow$ 320

$$\frac{\sqrt{\cosh^2(e+fx)}\text{sech}(e+fx)\left(\frac{\sinh(e+fx)\sqrt{a+b\sinh^2(e+fx)}}{(a-b)\sqrt{\sinh^2(e+fx)+1}}-\frac{b\left(\int \frac{\sinh^2(e+fx)}{\sqrt{\sinh^2(e+fx)+1}\sqrt{b\sinh^2(e+fx)+a}}d\sinh(e+fx)+\frac{\sqrt{a+b\sinh^2(e+}}{a\sqrt{\sinh}}\right.}{a-b}\right.}{f}$$

$\Big\downarrow$ 388

$$\frac{\sqrt{\cosh^2(e+fx)}\operatorname{sech}(e+fx)\left(\frac{\sinh(e+fx)\sqrt{a+b\sinh^2(e+fx)}}{(a-b)\sqrt{\sinh^2(e+fx)+1}}-\frac{b\left(-\frac{\int\frac{\sqrt{b\sinh^2(e+fx)+a}}{\left(\sinh^2(e+fx)+1\right)^{3/2}}d\sinh(e+fx)}{b}+\frac{\sqrt{a+b\sinh^2(e+fx)}\text{EllipticF}\left(\cdots\right.}{a\sqrt{\sinh^2(e+fx)+1}\sqrt{\cdots}}\right.}{a-b}\right.}{f}$$

$$\Big\downarrow \; 313$$

$$\frac{\sqrt{\cosh^2(e+fx)}\operatorname{sech}(e+fx)\left(\frac{\sinh(e+fx)\sqrt{a+b\sinh^2(e+fx)}}{(a-b)\sqrt{\sinh^2(e+fx)+1}}-\frac{b\left(\frac{\sqrt{a+b\sinh^2(e+fx)}\text{EllipticF}\left(\arctan(\sinh(e+fx)),1-\frac{b}{a}\right)}{a\sqrt{\sinh^2(e+fx)+1}\sqrt{\frac{a+b\sinh^2(e+fx)}{a\left(\sinh^2(e+fx)+1\right)}}}-\frac{\sqrt{a+b\sinh^2(\cdots}}{b\sqrt{\sinh^2(\cdots}}\right.}{a-\cdots}\right.}{f}$$

input
```
Int[Sech[e + f*x]^2/Sqrt[a + b*Sinh[e + f*x]^2],x]
```

output
```
(Sqrt[Cosh[e + f*x]^2]*Sech[e + f*x]*((Sinh[e + f*x]*Sqrt[a + b*Sinh[e + f
*x]^2])/((a - b)*Sqrt[1 + Sinh[e + f*x]^2]) - (b*((Sinh[e + f*x]*Sqrt[a +
b*Sinh[e + f*x]^2])/(b*Sqrt[1 + Sinh[e + f*x]^2]) - (EllipticE[ArcTan[Sinh
[e + f*x]], 1 - b/a]*Sqrt[a + b*Sinh[e + f*x]^2])/(b*Sqrt[1 + Sinh[e + f*x
]^2]*Sqrt[(a + b*Sinh[e + f*x]^2)/(a*(1 + Sinh[e + f*x]^2))]) + (EllipticF
[ArcTan[Sinh[e + f*x]], 1 - b/a]*Sqrt[a + b*Sinh[e + f*x]^2])/(a*Sqrt[1 +
Sinh[e + f*x]^2]*Sqrt[(a + b*Sinh[e + f*x]^2)/(a*(1 + Sinh[e + f*x]^2))]))
)/(a - b)))/f
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 313
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/((c_) + (d_.)*(x_)^2)^(3/2), x_Symbol] :> Sim
p[(Sqrt[a + b*x^2]/(c*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(c
 + d*x^2)))]))*EllipticE[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; FreeQ
[{a, b, c, d}, x] && PosQ[b/a] && PosQ[d/c]
```

rule 316
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_), x_Symbol] :> Sim
p[(-b)*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^(q + 1)/(2*a*(p + 1)*(b*c - a*d))
), x] + Simp[1/(2*a*(p + 1)*(b*c - a*d))   Int[(a + b*x^2)^(p + 1)*(c + d*x
^2)^q*Simp[b*c + 2*(p + 1)*(b*c - a*d) + d*b*(2*(p + q + 2) + 1)*x^2, x], x
], x] /; FreeQ[{a, b, c, d, q}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] &&  !
( !IntegerQ[p] && IntegerQ[q] && LtQ[q, -1]) && IntBinomialQ[a, b, c, d, 2,
 p, q, x]
```

rule 320
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(Sqrt[a + b*x^2]/(a*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(
c + d*x^2)))]))*EllipticF[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; Fre
eQ[{a, b, c, d}, x] && PosQ[d/c] && PosQ[b/a] &&  !SimplerSqrtQ[b/a, d/c]
```

rule 324
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
a   Int[1/(Sqrt[a + b*x^2]*Sqrt[c + d*x^2]), x], x] + Simp[b   Int[x^2/(Sqr
t[a + b*x^2]*Sqrt[c + d*x^2]), x], x] /; FreeQ[{a, b, c, d}, x] && PosQ[d/c
] && PosQ[b/a]
```

rule 388
```
Int[(x_)^2/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol]
 :> Simp[x*(Sqrt[a + b*x^2]/(b*Sqrt[c + d*x^2])), x] - Simp[c/b   Int[Sqrt[
a + b*x^2]/(c + d*x^2)^(3/2), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
 a*d, 0] && PosQ[b/a] && PosQ[d/c] &&  !SimplerSqrtQ[b/a, d/c]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3671
```
Int[cos[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(
p_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Simp[ff*(Sqrt[
Cos[e + f*x]^2]/(f*Cos[e + f*x]))   Subst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a
 + b*ff^2*x^2)^p, x], x, Sin[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x]
 && IntegerQ[m/2] &&  !IntegerQ[p]
```

Maple [A] (verified)

Time = 1.45 (sec) , antiderivative size = 133, normalized size of antiderivative = 0.83

method	result	siz
default	$-\frac{-\sqrt{-\frac{b}{a}}\, b \sinh(fx+e)^3+b\sqrt{\frac{a+b\sinh(fx+e)^2}{a}}\,\sqrt{\frac{\cosh(2fx+2e)}{2}+\frac{1}{2}}\,\operatorname{EllipticE}\left(\sinh(fx+e)\sqrt{-\frac{b}{a}},\sqrt{\frac{a}{b}}\right)-\sqrt{-\frac{b}{a}}\, a\sinh(fx+e)}{(a-b)\sqrt{-\frac{b}{a}}\,\cosh(fx+e)\sqrt{a+b\sinh(fx+e)^2}\, f}$	13
risch	Expression too large to display	49

input
```
int(sech(f*x+e)^2/(a+b*sinh(f*x+e)^2)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-(-(-b/a)^(1/2)*b*sinh(f*x+e)^3+b*((a+b*sinh(f*x+e)^2)/a)^(1/2)*(cosh(f*x+
e)^2)^(1/2)*EllipticE(sinh(f*x+e)*(-b/a)^(1/2),(1/b*a)^(1/2))-(-b/a)^(1/2)
*a*sinh(f*x+e))/(a-b)/(-b/a)^(1/2)/cosh(f*x+e)/(a+b*sinh(f*x+e)^2)^(1/2)/f
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 576 vs. 2(168) = 336.

Time = 0.10 (sec) , antiderivative size = 576, normalized size of antiderivative = 3.60

$$\int \frac{\operatorname{sech}^2(e+fx)}{\sqrt{a+b\sinh^2(e+fx)}}\,dx =$$

$$-\frac{\left((2\,a-b)\cosh\left(fx+e\right)^2+2\,(2\,a-b)\cosh\left(fx+e\right)\sinh\left(fx+e\right)+(2\,a-b)\sinh\left(fx+e\right)^2-2\,(b\,\mathrm{co}\right.}{}$$

input
```
integrate(sech(f*x+e)^2/(a+b*sinh(f*x+e)^2)^(1/2),x, algorithm="fricas")
```

output
```
-(((2*a - b)*cosh(f*x + e)^2 + 2*(2*a - b)*cosh(f*x + e)*sinh(f*x + e) + (
2*a - b)*sinh(f*x + e)^2 - 2*(b*cosh(f*x + e)^2 + 2*b*cosh(f*x + e)*sinh(f
*x + e) + b*sinh(f*x + e)^2 + b)*sqrt((a^2 - a*b)/b^2) + 2*a - b)*sqrt(b)*
sqrt((2*b*sqrt((a^2 - a*b)/b^2) - 2*a + b)/b)*elliptic_e(arcsin(sqrt((2*b*
sqrt((a^2 - a*b)/b^2) - 2*a + b)/b)*(cosh(f*x + e) + sinh(f*x + e))), (8*a
^2 - 8*a*b + b^2 + 4*(2*a*b - b^2)*sqrt((a^2 - a*b)/b^2))/b^2) - 2*((2*a -
 b)*cosh(f*x + e)^2 + 2*(2*a - b)*cosh(f*x + e)*sinh(f*x + e) + (2*a - b)*
sinh(f*x + e)^2 + 2*a - b)*sqrt(b)*sqrt((2*b*sqrt((a^2 - a*b)/b^2) - 2*a +
 b)/b)*elliptic_f(arcsin(sqrt((2*b*sqrt((a^2 - a*b)/b^2) - 2*a + b)/b)*(co
sh(f*x + e) + sinh(f*x + e))), (8*a^2 - 8*a*b + b^2 + 4*(2*a*b - b^2)*sqrt
((a^2 - a*b)/b^2))/b^2) - sqrt(2)*(b*cosh(f*x + e) + b*sinh(f*x + e))*sqrt
((b*cosh(f*x + e)^2 + b*sinh(f*x + e)^2 + 2*a - b)/(cosh(f*x + e)^2 - 2*co
sh(f*x + e)*sinh(f*x + e) + sinh(f*x + e)^2)))/((a*b - b^2)*f*cosh(f*x + e
)^2 + 2*(a*b - b^2)*f*cosh(f*x + e)*sinh(f*x + e) + (a*b - b^2)*f*sinh(f*x
 + e)^2 + (a*b - b^2)*f)
```

Sympy [F]

$$\int \frac{\operatorname{sech}^2(e+fx)}{\sqrt{a+b\sinh^2(e+fx)}}\,dx=\int \frac{\operatorname{sech}^2\left(e+fx\right)}{\sqrt{a+b\sinh^2\left(e+fx\right)}}\,dx$$

input
```
integrate(sech(f*x+e)**2/(a+b*sinh(f*x+e)**2)**(1/2),x)
```

output
```
Integral(sech(e + f*x)**2/sqrt(a + b*sinh(e + f*x)**2), x)
```

Maxima [F]

$$\int \frac{\operatorname{sech}^2(e+fx)}{\sqrt{a+b\sinh^2(e+fx)}}\,dx=\int \frac{\operatorname{sech}\left(fx+e\right)^2}{\sqrt{b\sinh\left(fx+e\right)^2+a}}\,dx$$

input
```
integrate(sech(f*x+e)^2/(a+b*sinh(f*x+e)^2)^(1/2),x, algorithm="maxima")
```

output
```
integrate(sech(f*x + e)^2/sqrt(b*sinh(f*x + e)^2 + a), x)
```

Giac [F]

$$\int \frac{\operatorname{sech}^2(e+fx)}{\sqrt{a+b\sinh^2(e+fx)}}\,dx = \int \frac{\operatorname{sech}\left(fx+e\right)^2}{\sqrt{b\sinh\left(fx+e\right)^2+a}}\,dx$$

input
```
integrate(sech(f*x+e)^2/(a+b*sinh(f*x+e)^2)^(1/2),x, algorithm="giac")
```

output
```
integrate(sech(f*x + e)^2/sqrt(b*sinh(f*x + e)^2 + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\operatorname{sech}^2(e+fx)}{\sqrt{a+b\sinh^2(e+fx)}}\,dx = \int \frac{1}{\cosh\left(e+f\,x\right)^2\,\sqrt{b\sinh\left(e+f\,x\right)^2+a}}\,dx$$

input
```
int(1/(cosh(e + f*x)^2*(a + b*sinh(e + f*x)^2)^(1/2)),x)
```

output
```
int(1/(cosh(e + f*x)^2*(a + b*sinh(e + f*x)^2)^(1/2)), x)
```

Reduce [F]

$$\int \frac{\operatorname{sech}^2(e+fx)}{\sqrt{a+b\sinh^2(e+fx)}}\,dx = \int \frac{\sqrt{\sinh\left(fx+e\right)^2b+a}\,\operatorname{sech}(fx+e)^2}{\sinh\left(fx+e\right)^2b+a}dx$$

input
```
int(sech(f*x+e)^2/(a+b*sinh(f*x+e)^2)^(1/2),x)
```

output
```
int((sqrt(sinh(e + f*x)**2*b + a)*sech(e + f*x)**2)/(sinh(e + f*x)**2*b +
a),x)
```

3.345 $\int \frac{\operatorname{sech}^4(e+fx)}{\sqrt{a+b\sinh^2(e+fx)}}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 219

$$\begin{aligned}\int \frac{\operatorname{sech}^4(e+fx)}{\sqrt{a+b\sinh^2(e+fx)}}\,dx &= \frac{2(a-2b)E\left(\arctan(\sinh(e+fx))\,\middle|\,1-\frac{b}{a}\right)\operatorname{sech}(e+fx)\sqrt{a+b\sinh^2(e+fx)}}{3(a-b)^2f\sqrt{\frac{\operatorname{sech}^2(e+fx)(a+b\sinh^2(e+fx))}{a}}}\\ &\quad-\frac{(a-3b)b\operatorname{EllipticF}\left(\arctan(\sinh(e+fx)),1-\frac{b}{a}\right)\operatorname{sech}(e+fx)\sqrt{a+b\sinh^2(e+fx)}}{3a(a-b)^2f\sqrt{\frac{\operatorname{sech}^2(e+fx)(a+b\sinh^2(e+fx))}{a}}}\\ &\quad+\frac{\operatorname{sech}^2(e+fx)\sqrt{a+b\sinh^2(e+fx)}\tanh(e+fx)}{3(a-b)f}\end{aligned}$$

output
```
2/3*(a-2*b)*EllipticE(sinh(f*x+e)/(1+sinh(f*x+e)^2)^(1/2),(1-b/a)^(1/2))*s
ech(f*x+e)*(a+b*sinh(f*x+e)^2)^(1/2)/(a-b)^2/f/(sech(f*x+e)^2*(a+b*sinh(f*
x+e)^2)/a)^(1/2)-1/3*(a-3*b)*b*InverseJacobiAM(arctan(sinh(f*x+e)),(1-b/a)
^(1/2))*sech(f*x+e)*(a+b*sinh(f*x+e)^2)^(1/2)/a/(a-b)^2/f/(sech(f*x+e)^2*(
a+b*sinh(f*x+e)^2)/a)^(1/2)+1/3*sech(f*x+e)^2*(a+b*sinh(f*x+e)^2)^(1/2)*ta
nh(f*x+e)/(a-b)/f
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 1.74 (sec) , antiderivative size = 219, normalized size of antiderivative = 1.00

$$\int \frac{\operatorname{sech}^4(e+fx)}{\sqrt{a+b\sinh^2(e+fx)}}\,dx$$
$$= \frac{4ia(a-2b)\sqrt{\frac{2a-b+b\cosh(2(e+fx))}{a}}E\left(i(e+fx)\left|\frac{b}{a}\right.\right)-2i(2a^2-5ab+3b^2)\sqrt{\frac{2a-b+b\cosh(2(e+fx))}{a}}\operatorname{EllipticF}\left(i(e\right.}{6(a-b)^2f\sqrt{2a-b+b\cosh}}$$

input
```
Integrate[Sech[e + f*x]^4/Sqrt[a + b*Sinh[e + f*x]^2],x]
```

output
```
((4*I)*a*(a - 2*b)*Sqrt[(2*a - b + b*Cosh[2*(e + f*x)])/a]*EllipticE[I*(e
+ f*x), b/a] - (2*I)*(2*a^2 - 5*a*b + 3*b^2)*Sqrt[(2*a - b + b*Cosh[2*(e +
 f*x)])/a]*EllipticF[I*(e + f*x), b/a] + ((8*a^2 - 15*a*b + 4*b^2 + (4*a^2
 - 6*a*b - 2*b^2)*Cosh[2*(e + f*x)] + (a - 2*b)*b*Cosh[4*(e + f*x)])*Sech[
e + f*x]^2*Tanh[e + f*x])/Sqrt[2])/(6*(a - b)^2*f*Sqrt[2*a - b + b*Cosh[2*
(e + f*x)]])
```

Rubi [A] (verified)

Time = 0.48 (sec) , antiderivative size = 270, normalized size of antiderivative = 1.23, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.280$, Rules used = {3042, 3671, 316, 25, 400, 313, 320}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\operatorname{sech}^4(e+fx)}{\sqrt{a+b\sinh^2(e+fx)}}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{\cos(ie+ifx)^4\sqrt{a-b\sin(ie+ifx)^2}}dx$$

$$\Big\downarrow \text{3671}$$

$$\frac{\sqrt{\cosh^2(e+fx)}\text{sech}(e+fx)\int \frac{1}{\left(\sinh^2(e+fx)+1\right)^{5/2}\sqrt{b\sinh^2(e+fx)+a}}d\sinh(e+fx)}{f}$$

$$\Big\downarrow \text{316}$$

$$\frac{\sqrt{\cosh^2(e+fx)}\text{sech}(e+fx)\left(\frac{\sinh(e+fx)\sqrt{a+b\sinh^2(e+fx)}}{3(a-b)\left(\sinh^2(e+fx)+1\right)^{3/2}}-\frac{\int -\frac{b\sinh^2(e+fx)+2a-3b}{\left(\sinh^2(e+fx)+1\right)^{3/2}\sqrt{b\sinh^2(e+fx)+a}}d\sinh(e+fx)}{3(a-b)}\right)}{f}$$

$$\Big\downarrow \text{25}$$

$$\frac{\sqrt{\cosh^2(e+fx)}\text{sech}(e+fx)\left(\frac{\int \frac{b\sinh^2(e+fx)+2a-3b}{\left(\sinh^2(e+fx)+1\right)^{3/2}\sqrt{b\sinh^2(e+fx)+a}}d\sinh(e+fx)}{3(a-b)}+\frac{\sinh(e+fx)\sqrt{a+b\sinh^2(e+fx)}}{3(a-b)\left(\sinh^2(e+fx)+1\right)^{3/2}}\right)}{f}$$

$$\Big\downarrow \text{400}$$

$$\frac{\sqrt{\cosh^2(e+fx)}\text{sech}(e+fx)\left(\frac{\frac{2(a-2b)\int \frac{\sqrt{b\sinh^2(e+fx)+a}}{\left(\sinh^2(e+fx)+1\right)^{3/2}}d\sinh(e+fx)}{a-b}-\frac{b(a-3b)\int \frac{1}{\sqrt{\sinh^2(e+fx)+1}\sqrt{b\sinh^2(e+fx)+a}}d\sinh(e+fx)}{a-b}}{3(a-b)}+\ldots\right.}{f}$$

$$\Big\downarrow \text{313}$$

$$\frac{\sqrt{\cosh^2(e+fx)}\text{sech}(e+fx)\left(\frac{\frac{2(a-2b)\sqrt{a+b\sinh^2(e+fx)}E\left(\arctan(\sinh(e+fx))\middle|1-\frac{b}{a}\right)}{(a-b)\sqrt{\sinh^2(e+fx)+1}\sqrt{\frac{a+b\sinh^2(e+fx)}{a\left(\sinh^2(e+fx)+1\right)}}}-\frac{b(a-3b)\int \frac{1}{\sqrt{\sinh^2(e+fx)+1}\sqrt{b\sinh^2(e+fx)+a}}d\sinh(e\ldots}{a-b}}{3(a-b)}\ldots\right.}{f}$$

$$\Big\downarrow \text{320}$$

$$\frac{\sqrt{\cosh^2(e+fx)}\operatorname{sech}(e+fx)\left(\frac{\frac{2(a-2b)\sqrt{a+b\sinh^2(e+fx)}E\left(\arctan(\sinh(e+fx))\middle|1-\frac{b}{a}\right)}{(a-b)\sqrt{\sinh^2(e+fx)+1}\sqrt{\frac{a+b\sinh^2(e+fx)}{a\left(\sinh^2(e+fx)+1\right)}}}-\frac{b(a-3b)\sqrt{a+b\sinh^2(e+fx)}\,\text{EllipticF}\left(\arctan(\sinh(e+fx\right.}{a(a-b)\sqrt{\sinh^2(e+fx)+1}\sqrt{\frac{a+b\sinh^2(e+fx)}{a\left(\sinh^2(e+fx)+1\right)}}}}{3(a-b)}\right.}{f}$$

input
```
Int[Sech[e + f*x]^4/Sqrt[a + b*Sinh[e + f*x]^2],x]
```

output
```
(Sqrt[Cosh[e + f*x]^2]*Sech[e + f*x]*((Sinh[e + f*x]*Sqrt[a + b*Sinh[e + f
*x]^2])/(3*(a - b)*(1 + Sinh[e + f*x]^2)^(3/2)) + ((2*(a - 2*b)*EllipticE[
ArcTan[Sinh[e + f*x]], 1 - b/a]*Sqrt[a + b*Sinh[e + f*x]^2])/((a - b)*Sqrt
[1 + Sinh[e + f*x]^2]*Sqrt[(a + b*Sinh[e + f*x]^2)/(a*(1 + Sinh[e + f*x]^2
))]) - ((a - 3*b)*b*EllipticF[ArcTan[Sinh[e + f*x]], 1 - b/a]*Sqrt[a + b*S
inh[e + f*x]^2])/(a*(a - b)*Sqrt[1 + Sinh[e + f*x]^2]*Sqrt[(a + b*Sinh[e +
 f*x]^2)/(a*(1 + Sinh[e + f*x]^2))]))/(3*(a - b))))/f
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 313
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/((c_) + (d_.)*(x_)^2)^(3/2), x_Symbol] :> Sim
p[(Sqrt[a + b*x^2]/(c*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(c
+ d*x^2)))]))*EllipticE[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; FreeQ
[{a, b, c, d}, x] && PosQ[b/a] && PosQ[d/c]
```

rule 316
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_), x_Symbol] :> Sim
p[(-b)*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^(q + 1)/(2*a*(p + 1)*(b*c - a*d))
), x] + Simp[1/(2*a*(p + 1)*(b*c - a*d))   Int[(a + b*x^2)^(p + 1)*(c + d*x
^2)^q*Simp[b*c + 2*(p + 1)*(b*c - a*d) + d*b*(2*(p + q + 2) + 1)*x^2, x], x
], x] /; FreeQ[{a, b, c, d, q}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] &&  !
( !IntegerQ[p] && IntegerQ[q] && LtQ[q, -1]) && IntBinomialQ[a, b, c, d, 2,
 p, q, x]
```

rule 320
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(Sqrt[a + b*x^2]/(a*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(
c + d*x^2)))]))*EllipticF[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; Fre
eQ[{a, b, c, d}, x] && PosQ[d/c] && PosQ[b/a] && !SimplerSqrtQ[b/a, d/c]
```

rule 400
```
Int[((e_) + (f_.)*(x_)^2)/(Sqrt[(a_) + (b_.)*(x_)^2]*((c_) + (d_.)*(x_)^2)^
(3/2)), x_Symbol] :> Simp[(b*e - a*f)/(b*c - a*d)   Int[1/(Sqrt[a + b*x^2]*
Sqrt[c + d*x^2]), x], x] - Simp[(d*e - c*f)/(b*c - a*d)   Int[Sqrt[a + b*x^
2]/(c + d*x^2)^(3/2), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && PosQ[b/a] &
& PosQ[d/c]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3671
```
Int[cos[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(
p_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Simp[ff*(Sqrt[
Cos[e + f*x]^2]/(f*Cos[e + f*x]))   Subst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a
 + b*ff^2*x^2)^p, x], x, Sin[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x]
 && IntegerQ[m/2] && !IntegerQ[p]
```

Maple [A] (verified)

Time = 2.04 (sec) , antiderivative size = 343, normalized size of antiderivative = 1.57

method	result
default	$\frac{\sqrt{\left(a+b\sinh(fx+e)^2\right)\cosh(fx+e)^2}\left(2\cosh(fx+e)^4\sqrt{-\frac{b}{a}}\,b(a-2b)\sinh(fx+e)+\cosh(fx+e)^2\sqrt{-\frac{b}{a}}\left(2a^2-5ab+3b^2\right)\sinh(fx+e)+\right.}{}$
risch	Expression too large to display

input
```
int(sech(f*x+e)^4/(a+b*sinh(f*x+e)^2)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/3*((a+b*sinh(f*x+e)^2)*cosh(f*x+e)^2)^(1/2)/cosh(f*x+e)^3/(-b/a)^(1/2)/(
b*cosh(f*x+e)^4+(a-b)*cosh(f*x+e)^2)^(1/2)/(a^2-2*a*b+b^2)*(2*cosh(f*x+e)^
4*(-b/a)^(1/2)*b*(a-2*b)*sinh(f*x+e)+cosh(f*x+e)^2*(-b/a)^(1/2)*(2*a^2-5*a
*b+3*b^2)*sinh(f*x+e)+(-b/a)^(1/2)*(a^2-2*a*b+b^2)*sinh(f*x+e)+(cosh(f*x+e
)^2)^(1/2)*(b/a*cosh(f*x+e)^2+(a-b)/a)^(1/2)*b*(a*EllipticF(sinh(f*x+e)*(-
b/a)^(1/2),(1/b*a)^(1/2))-b*EllipticF(sinh(f*x+e)*(-b/a)^(1/2),(1/b*a)^(1/
2))-2*EllipticE(sinh(f*x+e)*(-b/a)^(1/2),(1/b*a)^(1/2))*a+4*b*EllipticE(si
nh(f*x+e)*(-b/a)^(1/2),(1/b*a)^(1/2)))*cosh(f*x+e)^2)/(a+b*sinh(f*x+e)^2)^
(1/2)/f
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2443 vs. $2(219) = 438$.

Time = 0.15 (sec) , antiderivative size = 2443, normalized size of antiderivative = 11.16

$$\int \frac{\operatorname{sech}^4(e+fx)}{\sqrt{a+b\sinh^2(e+fx)}}\,dx = \text{Too large to display}$$

input
```
integrate(sech(f*x+e)^4/(a+b*sinh(f*x+e)^2)^(1/2),x, algorithm="fricas")
```

output
```
-2/3*(((2*a^2 - 5*a*b + 2*b^2)*cosh(f*x + e)^6 + 6*(2*a^2 - 5*a*b + 2*b^2)
*cosh(f*x + e)*sinh(f*x + e)^5 + (2*a^2 - 5*a*b + 2*b^2)*sinh(f*x + e)^6 +
 3*(2*a^2 - 5*a*b + 2*b^2)*cosh(f*x + e)^4 + 3*(5*(2*a^2 - 5*a*b + 2*b^2)*
cosh(f*x + e)^2 + 2*a^2 - 5*a*b + 2*b^2)*sinh(f*x + e)^4 + 4*(5*(2*a^2 - 5
*a*b + 2*b^2)*cosh(f*x + e)^3 + 3*(2*a^2 - 5*a*b + 2*b^2)*cosh(f*x + e))*s
inh(f*x + e)^3 + 3*(2*a^2 - 5*a*b + 2*b^2)*cosh(f*x + e)^2 + 3*(5*(2*a^2 -
 5*a*b + 2*b^2)*cosh(f*x + e)^4 + 6*(2*a^2 - 5*a*b + 2*b^2)*cosh(f*x + e)^
2 + 2*a^2 - 5*a*b + 2*b^2)*sinh(f*x + e)^2 + 2*a^2 - 5*a*b + 2*b^2 + 6*((2
*a^2 - 5*a*b + 2*b^2)*cosh(f*x + e)^5 + 2*(2*a^2 - 5*a*b + 2*b^2)*cosh(f*x
 + e)^3 + (2*a^2 - 5*a*b + 2*b^2)*cosh(f*x + e))*sinh(f*x + e) - 2*((a*b -
 2*b^2)*cosh(f*x + e)^6 + 6*(a*b - 2*b^2)*cosh(f*x + e)*sinh(f*x + e)^5 +
(a*b - 2*b^2)*sinh(f*x + e)^6 + 3*(a*b - 2*b^2)*cosh(f*x + e)^4 + 3*(5*(a*
b - 2*b^2)*cosh(f*x + e)^2 + a*b - 2*b^2)*sinh(f*x + e)^4 + 4*(5*(a*b - 2*
b^2)*cosh(f*x + e)^3 + 3*(a*b - 2*b^2)*cosh(f*x + e))*sinh(f*x + e)^3 + 3*
(a*b - 2*b^2)*cosh(f*x + e)^2 + 3*(5*(a*b - 2*b^2)*cosh(f*x + e)^4 + 6*(a*
b - 2*b^2)*cosh(f*x + e)^2 + a*b - 2*b^2)*sinh(f*x + e)^2 + a*b - 2*b^2 +
6*((a*b - 2*b^2)*cosh(f*x + e)^5 + 2*(a*b - 2*b^2)*cosh(f*x + e)^3 + (a*b
- 2*b^2)*cosh(f*x + e))*sinh(f*x + e))*sqrt((a^2 - a*b)/b^2))*sqrt(b)*sqrt
((2*b*sqrt((a^2 - a*b)/b^2) - 2*a + b)/b)*elliptic_e(arcsin(sqrt((2*b*sqrt
((a^2 - a*b)/b^2) - 2*a + b)/b)*(cosh(f*x + e) + sinh(f*x + e))), (8*a^...
```

Sympy [F]

$$\int \frac{\operatorname{sech}^4(e+fx)}{\sqrt{a+b\sinh^2(e+fx)}}\,dx=\int \frac{\operatorname{sech}^4(e+fx)}{\sqrt{a+b\sinh^2(e+fx)}}\,dx$$

input
```
integrate(sech(f*x+e)**4/(a+b*sinh(f*x+e)**2)**(1/2),x)
```

output
```
Integral(sech(e + f*x)**4/sqrt(a + b*sinh(e + f*x)**2), x)
```

Maxima [F]

$$\int \frac{\operatorname{sech}^4(e+fx)}{\sqrt{a+b\sinh^2(e+fx)}}\,dx = \int \frac{\operatorname{sech}(fx+e)^4}{\sqrt{b\sinh(fx+e)^2+a}}\,dx$$

input
```
integrate(sech(f*x+e)^4/(a+b*sinh(f*x+e)^2)^(1/2),x, algorithm="maxima")
```

output
```
integrate(sech(f*x + e)^4/sqrt(b*sinh(f*x + e)^2 + a), x)
```

Giac [F]

$$\int \frac{\operatorname{sech}^4(e+fx)}{\sqrt{a+b\sinh^2(e+fx)}}\,dx = \int \frac{\operatorname{sech}(fx+e)^4}{\sqrt{b\sinh(fx+e)^2+a}}\,dx$$

input
```
integrate(sech(f*x+e)^4/(a+b*sinh(f*x+e)^2)^(1/2),x, algorithm="giac")
```

output
```
integrate(sech(f*x + e)^4/sqrt(b*sinh(f*x + e)^2 + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\operatorname{sech}^4(e+fx)}{\sqrt{a+b\sinh^2(e+fx)}}\,dx = \int \frac{1}{\cosh(e+f\,x)^4\,\sqrt{b\sinh(e+f\,x)^2+a}}\,dx$$

input
```
int(1/(cosh(e + f*x)^4*(a + b*sinh(e + f*x)^2)^(1/2)),x)
```

output
```
int(1/(cosh(e + f*x)^4*(a + b*sinh(e + f*x)^2)^(1/2)), x)
```

Reduce [F]

$$\int \frac{\operatorname{sech}^4(e+fx)}{\sqrt{a+b\sinh^2(e+fx)}}\,dx = \int \frac{\sqrt{\sinh(fx+e)^2 b+a}\,\operatorname{sech}(fx+e)^4}{\sinh(fx+e)^2 b+a}dx$$

input
```
int(sech(f*x+e)^4/(a+b*sinh(f*x+e)^2)^(1/2),x)
```

output
```
int((sqrt(sinh(e + f*x)**2*b + a)*sech(e + f*x)**4)/(sinh(e + f*x)**2*b +
a),x)
```

3.346 $\int \frac{\cosh^3(e+fx)}{\left(a+b\sinh^2(e+fx)\right)^{3/2}}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 77

$$\int \frac{\cosh^3(e+fx)}{\left(a+b\sinh^2(e+fx)\right)^{3/2}}\,dx=\frac{\operatorname{arctanh}\left(\frac{\sqrt{b}\sinh(e+fx)}{\sqrt{a+b\sinh^2(e+fx)}}\right)}{b^{3/2}f}-\frac{(a-b)\sinh(e+fx)}{abf\sqrt{a+b\sinh^2(e+fx)}}$$

output
```
arctanh(b^(1/2)*sinh(f*x+e)/(a+b*sinh(f*x+e)^2)^(1/2))/b^(3/2)/f-(a-b)*sin
h(f*x+e)/a/b/f/(a+b*sinh(f*x+e)^2)^(1/2)
```

Mathematica [A] (verified)

Time = 0.12 (sec) , antiderivative size = 89, normalized size of antiderivative = 1.16

$$\int \frac{\cosh^3(e+fx)}{\left(a+b\sinh^2(e+fx)\right)^{3/2}}\,dx=\frac{\sqrt{b}(-a+b)\sinh(e+fx)+a^{3/2}\operatorname{arcsinh}\left(\frac{\sqrt{b}\sinh(e+fx)}{\sqrt{a}}\right)\sqrt{1+\frac{b\sinh^2(e+fx)}{a}}}{ab^{3/2}f\sqrt{a+b\sinh^2(e+fx)}}$$

input
```
Integrate[Cosh[e + f*x]^3/(a + b*Sinh[e + f*x]^2)^(3/2),x]
```

output
```
(Sqrt[b]*(-a + b)*Sinh[e + f*x] + a^(3/2)*ArcSinh[(Sqrt[b]*Sinh[e + f*x])/
Sqrt[a]]*Sqrt[1 + (b*Sinh[e + f*x]^2)/a])/(a*b^(3/2)*f*Sqrt[a + b*Sinh[e +
 f*x]^2])
```

Rubi [A] (verified)

Time = 0.33 (sec) , antiderivative size = 75, normalized size of antiderivative = 0.97, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {3042, 3669, 298, 224, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cosh^3(e+fx)}{\left(a+b\sinh^2(e+fx)\right)^{3/2}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\cos(ie+ifx)^3}{\left(a-b\sin(ie+ifx)^2\right)^{3/2}}dx$$

$$\Big\downarrow \text{3669}$$

$$\frac{\int \frac{\sinh^2(e+fx)+1}{\left(b\sinh^2(e+fx)+a\right)^{3/2}}d\sinh(e+fx)}{f}$$

$$\Big\downarrow \text{298}$$

$$\frac{\frac{\int \frac{1}{\sqrt{b\sinh^2(e+fx)+a}}d\sinh(e+fx)}{b}-\frac{(a-b)\sinh(e+fx)}{ab\sqrt{a+b\sinh^2(e+fx)}}}{f}$$

$$\Big\downarrow \text{224}$$

$$\frac{\frac{\int \frac{1}{1-\frac{b\sinh^2(e+fx)}{b\sinh^2(e+fx)+a}}d\frac{\sinh(e+fx)}{\sqrt{b\sinh^2(e+fx)+a}}}{b}-\frac{(a-b)\sinh(e+fx)}{ab\sqrt{a+b\sinh^2(e+fx)}}}{f}$$

$$\Big\downarrow \text{219}$$

$$\frac{\frac{\operatorname{arctanh}\left(\frac{\sqrt{b}\sinh(e+fx)}{\sqrt{a+b\sinh^2(e+fx)}}\right)}{b^{3/2}}-\frac{(a-b)\sinh(e+fx)}{ab\sqrt{a+b\sinh^2(e+fx)}}}{f}$$

input
```
Int[Cosh[e + f*x]^3/(a + b*Sinh[e + f*x]^2)^(3/2),x]
```

output
```
(ArcTanh[(Sqrt[b]*Sinh[e + f*x])/Sqrt[a + b*Sinh[e + f*x]^2]]/b^(3/2) - ((
a - b)*Sinh[e + f*x])/(a*b*Sqrt[a + b*Sinh[e + f*x]^2]))/f
```

Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

rule 298
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2), x_Symbol] :> Simp[(-(
b*c - a*d))*x*((a + b*x^2)^(p + 1)/(2*a*b*(p + 1))), x] - Simp[(a*d - b*c*(
2*p + 3))/(2*a*b*(p + 1))   Int[(a + b*x^2)^(p + 1), x], x] /; FreeQ[{a, b,
 c, d, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/2 + p, 0])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3669
```
Int[cos[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^
(p_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Simp[ff/f   S
ubst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a + b*ff^2*x^2)^p, x], x, Sin[e + f*x]
/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m - 1)/2]
```

Maple [A] (verified)

Time = 0.32 (sec) , antiderivative size = 85, normalized size of antiderivative = 1.10

method	result	size
derivativedivides	$\frac{\frac{\sinh(fx+e)}{a\sqrt{a+b\sinh(fx+e)^2}}-\frac{\sinh(fx+e)}{b\sqrt{a+b\sinh(fx+e)^2}}+\frac{\ln\left(\sqrt{b}\,\sinh(fx+e)+\sqrt{a+b\sinh(fx+e)^2}\right)}{b^{\frac{3}{2}}}}{f}$	85
default	$\frac{\frac{\sinh(fx+e)}{a\sqrt{a+b\sinh(fx+e)^2}}-\frac{\sinh(fx+e)}{b\sqrt{a+b\sinh(fx+e)^2}}+\frac{\ln\left(\sqrt{b}\,\sinh(fx+e)+\sqrt{a+b\sinh(fx+e)^2}\right)}{b^{\frac{3}{2}}}}{f}$	85

input
```
int(cosh(f*x+e)^3/(a+b*sinh(f*x+e)^2)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
1/f*(sinh(f*x+e)/a/(a+b*sinh(f*x+e)^2)^(1/2)-sinh(f*x+e)/b/(a+b*sinh(f*x+e
)^2)^(1/2)+1/b^(3/2)*ln(b^(1/2)*sinh(f*x+e)+(a+b*sinh(f*x+e)^2)^(1/2)))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 996 vs. 2(69) = 138.

Time = 0.21 (sec) , antiderivative size = 3014, normalized size of antiderivative = 39.14

$$\int \frac{\cosh^3(e+fx)}{\left(a+b\sinh^2(e+fx)\right)^{3/2}}\,dx = \text{Too large to display}$$

input
```
integrate(cosh(f*x+e)^3/(a+b*sinh(f*x+e)^2)^(3/2),x, algorithm="fricas")
```

output
```
Too large to include
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\cosh^3(e+fx)}{\left(a+b\sinh^2(e+fx)\right)^{3/2}}\,dx=\text{Timed out}$$

input
```
integrate(cosh(f*x+e)**3/(a+b*sinh(f*x+e)**2)**(3/2),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{\cosh^3(e+fx)}{\left(a+b\sinh^2(e+fx)\right)^{3/2}}\,dx=\int \frac{\cosh{(fx+e)}^3}{\left(b\sinh{(fx+e)}^2+a\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate(cosh(f*x+e)^3/(a+b*sinh(f*x+e)^2)^(3/2),x, algorithm="maxima")
```

output
```
integrate(cosh(f*x + e)^3/(b*sinh(f*x + e)^2 + a)^(3/2), x)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{\cosh^3(e+fx)}{\left(a+b\sinh^2(e+fx)\right)^{3/2}}\,dx=\text{Exception raised: TypeError}$$

input
```
integrate(cosh(f*x+e)^3/(a+b*sinh(f*x+e)^2)^(3/2),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Degree mismatch inside factorisatio
n over extensionNot implemented, e.g. for multivariate mod/approx polynomi
alsError:
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\cosh^3(e+fx)}{\left(a+b\sinh^2(e+fx)\right)^{3/2}}\,dx = \int \frac{\cosh(e+f\,x)^3}{\left(b\sinh\left(e+f\,x\right)^2+a\right)^{3/2}}\,dx$$

input
```
int(cosh(e + f*x)^3/(a + b*sinh(e + f*x)^2)^(3/2),x)
```

output
```
int(cosh(e + f*x)^3/(a + b*sinh(e + f*x)^2)^(3/2), x)
```

Reduce [F]

$$\int \frac{\cosh^3(e+fx)}{\left(a+b\sinh^2(e+fx)\right)^{3/2}}\,dx = \int \frac{\sqrt{\sinh\left(fx+e\right)^2 b+a}\,\cosh\left(fx+e\right)^3}{\sinh\left(fx+e\right)^4 b^2+2\sinh\left(fx+e\right)^2 ab+a^2}dx$$

input
```
int(cosh(f*x+e)^3/(a+b*sinh(f*x+e)^2)^(3/2),x)
```

output
```
int((sqrt(sinh(e + f*x)**2*b + a)*cosh(e + f*x)**3)/(sinh(e + f*x)**4*b**2
 + 2*sinh(e + f*x)**2*a*b + a**2),x)
```

3.347 $\int \frac{\cosh(e+fx)}{\left(a+b\sinh^2(e+fx)\right)^{3/2}}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 29

$$\int \frac{\cosh(e+fx)}{\left(a+b\sinh^2(e+fx)\right)^{3/2}}\,dx=\frac{\sinh(e+fx)}{af\sqrt{a+b\sinh^2(e+fx)}}$$

output
```
sinh(f*x+e)/a/f/(a+b*sinh(f*x+e)^2)^(1/2)
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.00

$$\int \frac{\cosh(e+fx)}{\left(a+b\sinh^2(e+fx)\right)^{3/2}}\,dx=\frac{\sinh(e+fx)}{af\sqrt{a+b\sinh^2(e+fx)}}$$

input
```
Integrate[Cosh[e + f*x]/(a + b*Sinh[e + f*x]^2)^(3/2),x]
```

output
```
Sinh[e + f*x]/(a*f*Sqrt[a + b*Sinh[e + f*x]^2])
```

Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.130, Rules used = {3042, 3669, 208}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cosh(e+fx)}{\left(a+b\sinh^2(e+fx)\right)^{3/2}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\cos(ie+ifx)}{\left(a-b\sin(ie+ifx)^2\right)^{3/2}}dx$$

$$\Big\downarrow \text{3669}$$

$$\frac{\int \frac{1}{\left(b\sinh^2(e+fx)+a\right)^{3/2}}d\sinh(e+fx)}{f}$$

$$\Big\downarrow \text{208}$$

$$\frac{\sinh(e+fx)}{af\sqrt{a+b\sinh^2(e+fx)}}$$

input
```
Int[Cosh[e + f*x]/(a + b*Sinh[e + f*x]^2)^(3/2),x]
```

output
```
Sinh[e + f*x]/(a*f*Sqrt[a + b*Sinh[e + f*x]^2])
```

Defintions of rubi rules used

rule 208
```
Int[((a_) + (b_.)*(x_)^2)^(-3/2), x_Symbol] :> Simp[x/(a*Sqrt[a + b*x^2]),
x] /; FreeQ[{a, b}, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3669
```
Int[cos[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^
(p_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Simp[ff/f   S
ubst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a + b*ff^2*x^2)^p, x], x, Sin[e + f*x]
/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m - 1)/2]
```

Maple [A] (verified)

Time = 0.23 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.97

method	result	size
derivativedivides	$\frac{\sinh(fx+e)}{af\sqrt{a+b\sinh(fx+e)^2}}$	28
default	$\frac{\sinh(fx+e)}{af\sqrt{a+b\sinh(fx+e)^2}}$	28

input
```
int(cosh(f*x+e)/(a+b*sinh(f*x+e)^2)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
sinh(f*x+e)/a/f/(a+b*sinh(f*x+e)^2)^(1/2)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 245 vs. 2(27) = 54.

Time = 0.12 (sec) , antiderivative size = 245, normalized size of antiderivative = 8.45

$$\int \frac{\cosh(e+fx)}{\left(a+b\sinh^2(e+fx)\right)^{3/2}}\,dx = \frac{\sqrt{2}(\cosh(f}{abf\cosh(fx+e)^4+4\,abf\cosh(fx+e)\sinh(fx+e)^3+abf\sinh(fx+}$$

input
```
integrate(cosh(f*x+e)/(a+b*sinh(f*x+e)^2)^(3/2),x, algorithm="fricas")
```

output
```
sqrt(2)*(cosh(f*x + e)^2 + 2*cosh(f*x + e)*sinh(f*x + e) + sinh(f*x + e)^2
 - 1)*sqrt((b*cosh(f*x + e)^2 + b*sinh(f*x + e)^2 + 2*a - b)/(cosh(f*x + e
)^2 - 2*cosh(f*x + e)*sinh(f*x + e) + sinh(f*x + e)^2))/(a*b*f*cosh(f*x +
e)^4 + 4*a*b*f*cosh(f*x + e)*sinh(f*x + e)^3 + a*b*f*sinh(f*x + e)^4 + 2*(
2*a^2 - a*b)*f*cosh(f*x + e)^2 + a*b*f + 2*(3*a*b*f*cosh(f*x + e)^2 + (2*a
^2 - a*b)*f)*sinh(f*x + e)^2 + 4*(a*b*f*cosh(f*x + e)^3 + (2*a^2 - a*b)*f*
cosh(f*x + e))*sinh(f*x + e))
```

Sympy [F]

$$\int \frac{\cosh(e+fx)}{(a+b\sinh^2(e+fx))^{3/2}}\,dx = \int \frac{\cosh(e+fx)}{(a+b\sinh^2(e+fx))^{\frac{3}{2}}}\,dx$$

input
```
integrate(cosh(f*x+e)/(a+b*sinh(f*x+e)**2)**(3/2),x)
```

output
```
Integral(cosh(e + f*x)/(a + b*sinh(e + f*x)**2)**(3/2), x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 236 vs. $2(27) = 54$.

Time = 0.15 (sec) , antiderivative size = 236, normalized size of antiderivative = 8.14

$$\int \frac{\cosh(e+fx)}{(a+b\sinh^2(e+fx))^{3/2}}\,dx = \frac{b^2e^{(-6\,fx-6\,e)} + 2\,ab - b^2 + (8\,a^2 - 8\,ab + 3\,b^2)e^{(-2\,fx-2\,e)} + 3\,(2\,ab - b^2)e^{(}}{2\,(a^2 - ab)(2\,(2\,a - b)e^{(-2\,fx-2\,e)} + be^{(-4\,fx-4\,e)} + b)^{\frac{3}{2}}f} - \frac{b^2 + 3\,(2\,ab - b^2)e^{(-2\,fx-2\,e)} + (8\,a^2 - 8\,ab + 3\,b^2)e^{(-4\,fx-4\,e)} + (2\,ab - b^2)e^{(-6\,fx-6\,e)}}{2\,(a^2 - ab)(2\,(2\,a - b)e^{(-2\,fx-2\,e)} + be^{(-4\,fx-4\,e)} + b)^{\frac{3}{2}}f}$$

input
```
integrate(cosh(f*x+e)/(a+b*sinh(f*x+e)^2)^(3/2),x, algorithm="maxima")
```

output
```
1/2*(b^2*e^(-6*f*x - 6*e) + 2*a*b - b^2 + (8*a^2 - 8*a*b + 3*b^2)*e^(-2*f*
x - 2*e) + 3*(2*a*b - b^2)*e^(-4*f*x - 4*e))/((a^2 - a*b)*(2*(2*a - b)*e^(
-2*f*x - 2*e) + b*e^(-4*f*x - 4*e) + b)^(3/2)*f) - 1/2*(b^2 + 3*(2*a*b - b
^2)*e^(-2*f*x - 2*e) + (8*a^2 - 8*a*b + 3*b^2)*e^(-4*f*x - 4*e) + (2*a*b -
 b^2)*e^(-6*f*x - 6*e))/((a^2 - a*b)*(2*(2*a - b)*e^(-2*f*x - 2*e) + b*e^(
-4*f*x - 4*e) + b)^(3/2)*f)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 131 vs. $2(27) = 54$.

Time = 0.22 (sec) , antiderivative size = 131, normalized size of antiderivative = 4.52

$$\int \frac{\cosh(e+fx)}{(a+b\sinh^2(e+fx))^{3/2}}\,dx = \frac{\frac{(afe^{(4e)}-bfe^{(4e)})e^{(2fx)}}{a^2f^2e^{(2e)}-abf^2e^{(2e)}} - \frac{afe^{(2e)}-bfe^{(2e)}}{a^2f^2e^{(2e)}-abf^2e^{(2e)}}}{\sqrt{be^{(4fx+4e)}+4ae^{(2fx+2e)}-2be^{(2fx+2e)}+b}}$$

input
```
integrate(cosh(f*x+e)/(a+b*sinh(f*x+e)^2)^(3/2),x, algorithm="giac")
```

output
```
((a*f*e^(4*e) - b*f*e^(4*e))*e^(2*f*x)/(a^2*f^2*e^(2*e) - a*b*f^2*e^(2*e))
 - (a*f*e^(2*e) - b*f*e^(2*e))/(a^2*f^2*e^(2*e) - a*b*f^2*e^(2*e)))/sqrt(b
*e^(4*f*x + 4*e) + 4*a*e^(2*f*x + 2*e) - 2*b*e^(2*f*x + 2*e) + b)
```

Mupad [B] (verification not implemented)

Time = 2.11 (sec) , antiderivative size = 191, normalized size of antiderivative = 6.59

$$\int \frac{\cosh(e+fx)}{(a+b\sinh^2(e+fx))^{3/2}}\,dx = -\frac{e^{e+fx}\sqrt{b\sinh(e+fx)^2+a}\left(\frac{2\cosh(e+fx)\,e^{e+fx}\,(b\,(2a-b)-b\,(4a-2b))}{f\,(a\,b^2-a^2\,b)} - \frac{2\,b^2\,e^{e+fx}\sinh(e+fx)}{f\,(a\,b^2-a^2\,b)} + \frac{b\,e^{2e+2fx}\,(4a-2b)}{f\,(a\,b^2-a^2\,b)}\right)}{4\,a\,e^{2e+2fx}-2\,b\,e^{2e+2fx}+2\,b\,e^{2e+2fx}\cosh(2e+2fx)}$$

input
```
int(cosh(e + f*x)/(a + b*sinh(e + f*x)^2)^(3/2),x)
```

output
```
-(exp(e + f*x)*(a + b*sinh(e + f*x)^2)^(1/2)*((2*cosh(e + f*x)*exp(e + f*x
)*(b*(2*a - b) - b*(4*a - 2*b)))/(f*(a*b^2 - a^2*b)) - (2*b^2*exp(e + f*x)
*sinh(e + f*x))/(f*(a*b^2 - a^2*b)) + (b*exp(2*e + 2*f*x)*(4*a - 2*b))/(f*
(a*b^2 - a^2*b))))/(4*a*exp(2*e + 2*f*x) - 2*b*exp(2*e + 2*f*x) + 2*b*exp(
2*e + 2*f*x)*cosh(2*e + 2*f*x))
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 40, normalized size of antiderivative = 1.38

$$\int \frac{\cosh(e+fx)}{\left(a+b\sinh^2(e+fx)\right)^{3/2}}\,dx = \frac{\sqrt{\sinh\left(fx+e\right)^2b+a}\,\sinh\left(fx+e\right)}{af\left(\sinh\left(fx+e\right)^2b+a\right)}$$

input
```
int(cosh(f*x+e)/(a+b*sinh(f*x+e)^2)^(3/2),x)
```

output
```
(sqrt(sinh(e + f*x)**2*b + a)*sinh(e + f*x))/(a*f*(sinh(e + f*x)**2*b + a)
)
```

3.348 $\int \frac{\text{sech}(e+fx)}{\left(a+b\sinh^2(e+fx)\right)^{3/2}}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 85

$$\int \frac{\text{sech}(e+fx)}{\left(a+b\sinh^2(e+fx)\right)^{3/2}}\,dx = \frac{\arctan\left(\frac{\sqrt{a-b}\sinh(e+fx)}{\sqrt{a+b\sinh^2(e+fx)}}\right)}{(a-b)^{3/2}f} - \frac{b\sinh(e+fx)}{a(a-b)f\sqrt{a+b\sinh^2(e+fx)}}$$

output
```
arctan((a-b)^(1/2)*sinh(f*x+e)/(a+b*sinh(f*x+e)^2)^(1/2))/(a-b)^(3/2)/f-b*
sinh(f*x+e)/a/(a-b)/f/(a+b*sinh(f*x+e)^2)^(1/2)
```

Mathematica [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 3.42 (sec) , antiderivative size = 254, normalized size of antiderivative = 2.99

$$\int \frac{\operatorname{sech}(e+fx)}{\left(a+b\sinh^2(e+fx)\right)^{3/2}}\,dx = \frac{\operatorname{sech}(e+fx)\tanh(e+fx)\left(\frac{4(a-b)\,\text{Hypergeometric2F1}\left(2,2,\frac{7}{2},\frac{(a-b)\tanh^2(e+fx)}{a}\right)(a+b}{15a^2}\right.}{}$$

input
```
Integrate[Sech[e + f*x]/(a + b*Sinh[e + f*x]^2)^(3/2),x]
```

output
```
(Sech[e + f*x]*Tanh[e + f*x]*((4*(a - b)*Hypergeometric2F1[2, 2, 7/2, ((a
- b)*Tanh[e + f*x]^2)/a]*(a + b*Sinh[e + f*x]^2)*Tanh[e + f*x]^2)/(15*a^2)
 + (Csch[e + f*x]^2*(3*a + 2*b*Sinh[e + f*x]^2)*(-(ArcSin[Sqrt[((a - b)*Ta
nh[e + f*x]^2)/a]]*(a + b*Sinh[e + f*x]^2)) + a*Cosh[e + f*x]^2*Sqrt[((a -
 b)*Sech[e + f*x]^2*(a + b*Sinh[e + f*x]^2)*Tanh[e + f*x]^2)/a^2]))/(a*(a
- b)*Sqrt[((a - b)*Sech[e + f*x]^2*(a + b*Sinh[e + f*x]^2)*Tanh[e + f*x]^2
)/a^2])))/(a*f*Sqrt[a + b*Sinh[e + f*x]^2])
```

Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 83, normalized size of antiderivative = 0.98, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.217$, Rules used = {3042, 3669, 296, 291, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\operatorname{sech}(e+fx)}{\left(a+b\sinh^2(e+fx)\right)^{3/2}}dx$$

$$\Big\downarrow \; 3042$$

$$\int \frac{1}{\cos(ie+ifx)\left(a-b\sin(ie+ifx)^2\right)^{3/2}}dx$$

$$\Big\downarrow \; 3669$$

$$\frac{\int \frac{1}{(\sinh^2(e+fx)+1)(b\sinh^2(e+fx)+a)^{3/2}}d\sinh(e+fx)}{f}$$

$$\Big\downarrow \text{ 296}$$

$$\frac{\frac{\int \frac{1}{(\sinh^2(e+fx)+1)\sqrt{b\sinh^2(e+fx)+a}}d\sinh(e+fx)}{a-b}-\frac{b\sinh(e+fx)}{a(a-b)\sqrt{a+b\sinh^2(e+fx)}}}{f}$$

$$\Big\downarrow \text{ 291}$$

$$\frac{\frac{\int \frac{1}{1-\frac{(b-a)\sinh^2(e+fx)}{b\sinh^2(e+fx)+a}}d\frac{\sinh(e+fx)}{\sqrt{b\sinh^2(e+fx)+a}}}{a-b}-\frac{b\sinh(e+fx)}{a(a-b)\sqrt{a+b\sinh^2(e+fx)}}}{f}$$

$$\Big\downarrow \text{ 216}$$

$$\frac{\frac{\arctan\left(\frac{\sqrt{a-b}\sinh(e+fx)}{\sqrt{a+b\sinh^2(e+fx)}}\right)}{(a-b)^{3/2}}-\frac{b\sinh(e+fx)}{a(a-b)\sqrt{a+b\sinh^2(e+fx)}}}{f}$$

input
```
Int[Sech[e + f*x]/(a + b*Sinh[e + f*x]^2)^(3/2),x]
```

output
```
(ArcTan[(Sqrt[a - b]*Sinh[e + f*x])/Sqrt[a + b*Sinh[e + f*x]^2]]/(a - b)^(
3/2) - (b*Sinh[e + f*x])/(a*(a - b)*Sqrt[a + b*Sinh[e + f*x]^2]))/f
```

Defintions of rubi rules used

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 291
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*((c_) + (d_.)*(x_)^2)), x_Symbol] :> Subst
[Int[1/(c - (b*c - a*d)*x^2), x], x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b, c,
d}, x] && NeQ[b*c - a*d, 0]
```

rule 296
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_), x_Symbol] :> Sim
p[(-b)*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^(q + 1)/(2*a*(p + 1)*(b*c - a*d))
), x] + Simp[(b*c + 2*(p + 1)*(b*c - a*d))/(2*a*(p + 1)*(b*c - a*d))   Int[
(a + b*x^2)^(p + 1)*(c + d*x^2)^q, x], x] /; FreeQ[{a, b, c, d, q}, x] && N
eQ[b*c - a*d, 0] && EqQ[2*(p + q + 2) + 1, 0] && (LtQ[p, -1] ||  !LtQ[q, -1
]) && NeQ[p, -1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3669
```
Int[cos[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^
(p_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Simp[ff/f   S
ubst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a + b*ff^2*x^2)^p, x], x, Sin[e + f*x]
/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m - 1)/2]
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.33 (sec) , antiderivative size = 101, normalized size of antiderivative = 1.19

method	result	size
default	$\frac{\text{`int/indef0`}\left(\frac{-a-b\sinh(fx+e)^2}{\left(-b^2\sinh(fx+e)^6+\left(-2ab-b^2\right)\sinh(fx+e)^4+\left(-a^2-2ab\right)\sinh(fx+e)^2-a^2\right)\sqrt{a+b\sinh(fx+e)^2}},\sinh(fx+e)\right)}{f}$	101

input
```
int(sech(f*x+e)/(a+b*sinh(f*x+e)^2)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
`int/indef0`((-a-b*sinh(f*x+e)^2)/(-b^2*sinh(f*x+e)^6+(-2*a*b-b^2)*sinh(f*
x+e)^4+(-a^2-2*a*b)*sinh(f*x+e)^2-a^2)/(a+b*sinh(f*x+e)^2)^(1/2),sinh(f*x+
e))/f
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 800 vs. $2(77) = 154$.

Time = 0.19 (sec) , antiderivative size = 1717, normalized size of antiderivative = 20.20

$$\int \frac{\operatorname{sech}(e+fx)}{\left(a+b\sinh^2(e+fx)\right)^{3/2}}\,dx = \text{Too large to display}$$

input
```
integrate(sech(f*x+e)/(a+b*sinh(f*x+e)^2)^(3/2),x, algorithm="fricas")
```

output
```
[1/2*((a*b*cosh(f*x + e)^4 + 4*a*b*cosh(f*x + e)*sinh(f*x + e)^3 + a*b*sin
h(f*x + e)^4 + 2*(2*a^2 - a*b)*cosh(f*x + e)^2 + 2*(3*a*b*cosh(f*x + e)^2
+ 2*a^2 - a*b)*sinh(f*x + e)^2 + a*b + 4*(a*b*cosh(f*x + e)^3 + (2*a^2 - a
*b)*cosh(f*x + e))*sinh(f*x + e))*sqrt(-a + b)*log(((a - 2*b)*cosh(f*x + e
)^4 + 4*(a - 2*b)*cosh(f*x + e)*sinh(f*x + e)^3 + (a - 2*b)*sinh(f*x + e)^
4 - 2*(3*a - 2*b)*cosh(f*x + e)^2 + 2*(3*(a - 2*b)*cosh(f*x + e)^2 - 3*a +
 2*b)*sinh(f*x + e)^2 + 2*sqrt(2)*(cosh(f*x + e)^2 + 2*cosh(f*x + e)*sinh(
f*x + e) + sinh(f*x + e)^2 - 1)*sqrt(-a + b)*sqrt((b*cosh(f*x + e)^2 + b*s
inh(f*x + e)^2 + 2*a - b)/(cosh(f*x + e)^2 - 2*cosh(f*x + e)*sinh(f*x + e)
 + sinh(f*x + e)^2)) + 4*((a - 2*b)*cosh(f*x + e)^3 - (3*a - 2*b)*cosh(f*x
 + e))*sinh(f*x + e) + a - 2*b)/(cosh(f*x + e)^4 + 4*cosh(f*x + e)*sinh(f*
x + e)^3 + sinh(f*x + e)^4 + 2*(3*cosh(f*x + e)^2 + 1)*sinh(f*x + e)^2 + 2
*cosh(f*x + e)^2 + 4*(cosh(f*x + e)^3 + cosh(f*x + e))*sinh(f*x + e) + 1))
 - 2*sqrt(2)*((a*b - b^2)*cosh(f*x + e)^2 + 2*(a*b - b^2)*cosh(f*x + e)*si
nh(f*x + e) + (a*b - b^2)*sinh(f*x + e)^2 - a*b + b^2)*sqrt((b*cosh(f*x +
e)^2 + b*sinh(f*x + e)^2 + 2*a - b)/(cosh(f*x + e)^2 - 2*cosh(f*x + e)*sin
h(f*x + e) + sinh(f*x + e)^2)))/((a^3*b - 2*a^2*b^2 + a*b^3)*f*cosh(f*x +
e)^4 + 4*(a^3*b - 2*a^2*b^2 + a*b^3)*f*cosh(f*x + e)*sinh(f*x + e)^3 + (a^
3*b - 2*a^2*b^2 + a*b^3)*f*sinh(f*x + e)^4 + 2*(2*a^4 - 5*a^3*b + 4*a^2*b^
2 - a*b^3)*f*cosh(f*x + e)^2 + 2*(3*(a^3*b - 2*a^2*b^2 + a*b^3)*f*cosh(...
```

Sympy [F]

$$\int \frac{\operatorname{sech}(e+fx)}{\left(a+b\sinh^2(e+fx)\right)^{3/2}}\,dx = \int \frac{\operatorname{sech}\left(e+fx\right)}{\left(a+b\sinh^{2}\left(e+fx\right)\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate(sech(f*x+e)/(a+b*sinh(f*x+e)**2)**(3/2),x)
```

output
```
Integral(sech(e + f*x)/(a + b*sinh(e + f*x)**2)**(3/2), x)
```

Maxima [F]

$$\int \frac{\operatorname{sech}(e+fx)}{\left(a+b\sinh^2(e+fx)\right)^{3/2}}\,dx = \int \frac{\operatorname{sech}\left(fx+e\right)}{\left(b\sinh\left(fx+e\right)^{2}+a\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate(sech(f*x+e)/(a+b*sinh(f*x+e)^2)^(3/2),x, algorithm="maxima")
```

output
```
integrate(sech(f*x + e)/(b*sinh(f*x + e)^2 + a)^(3/2), x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 323 vs. $2(77) = 154$.

Time = 0.26 (sec) , antiderivative size = 323, normalized size of antiderivative = 3.80

$$\int \frac{\operatorname{sech}(e+fx)}{\left(a+b\sinh^2(e+fx)\right)^{3/2}}\,dx = -\left(\frac{\frac{\left(a^2bfe^{(4e)}-2ab^2fe^{(4e)}+b^3fe^{(4e)}\right)e^{(2fx)}}{a^4f^2e^{(6e)}-3a^3bf^2e^{(6e)}+3a^2b^2f^2e^{(6e)}-ab^3f^2e^{(6e)}} - \frac{a^2bfe^{(2e)}-2ab^2fe^{(2e)}+b^3fe^{(2e)}}{a^4f^2e^{(6e)}-3a^3bf^2e^{(6e)}+3a^2b^2f^2e^{(6e)}-ab^3f^2e^{(6e)}}}{\sqrt{be^{(4fx+4e)}+4ae^{(2fx+2e)}-2be^{(2fx+2e)}+b}} - \frac{2\arctan\left(-\frac{\sqrt{b}e^{(2}}{}\right.}{}\right.$$

input
```
integrate(sech(f*x+e)/(a+b*sinh(f*x+e)^2)^(3/2),x, algorithm="giac")
```

output
```
-(((a^2*b*f*e^(4*e) - 2*a*b^2*f*e^(4*e) + b^3*f*e^(4*e))*e^(2*f*x)/(a^4*f^
2*e^(6*e) - 3*a^3*b*f^2*e^(6*e) + 3*a^2*b^2*f^2*e^(6*e) - a*b^3*f^2*e^(6*e
)) - (a^2*b*f*e^(2*e) - 2*a*b^2*f*e^(2*e) + b^3*f*e^(2*e))/(a^4*f^2*e^(6*e
) - 3*a^3*b*f^2*e^(6*e) + 3*a^2*b^2*f^2*e^(6*e) - a*b^3*f^2*e^(6*e)))/sqrt
(b*e^(4*f*x + 4*e) + 4*a*e^(2*f*x + 2*e) - 2*b*e^(2*f*x + 2*e) + b) - 2*ar
ctan(-1/2*(sqrt(b)*e^(2*f*x + 2*e) - sqrt(b*e^(4*f*x + 4*e) + 4*a*e^(2*f*x
 + 2*e) - 2*b*e^(2*f*x + 2*e) + b) + sqrt(b))/sqrt(a - b))/((a*f*e^(4*e) -
 b*f*e^(4*e))*sqrt(a - b)))*e^(4*e)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\operatorname{sech}(e+fx)}{\left(a+b\sinh^2(e+fx)\right)^{3/2}}\,dx=\int \frac{1}{\cosh\left(e+f\,x\right)\,\left(b\,{\sinh\left(e+f\,x\right)}^2+a\right)^{3/2}}\,dx$$

input
```
int(1/(cosh(e + f*x)*(a + b*sinh(e + f*x)^2)^(3/2)),x)
```

output
```
int(1/(cosh(e + f*x)*(a + b*sinh(e + f*x)^2)^(3/2)), x)
```

Reduce [F]

$$\int \frac{\operatorname{sech}(e+fx)}{\left(a+b\sinh^2(e+fx)\right)^{3/2}}\,dx=\int \frac{\sqrt{\sinh\left(fx+e\right)^2 b+a}\,\operatorname{sech}(fx+e)}{\sinh\left(fx+e\right)^4 b^2+2\sinh\left(fx+e\right)^2 ab+a^2}dx$$

input
```
int(sech(f*x+e)/(a+b*sinh(f*x+e)^2)^(3/2),x)
```

output
```
int((sqrt(sinh(e + f*x)**2*b + a)*sech(e + f*x))/(sinh(e + f*x)**4*b**2 +
2*sinh(e + f*x)**2*a*b + a**2),x)
```

3.349 $\int \frac{\operatorname{sech}^3(e+fx)}{\left(a+b\sinh^2(e+fx)\right)^{3/2}}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 142

$$\int \frac{\operatorname{sech}^3(e+fx)}{\left(a+b\sinh^2(e+fx)\right)^{3/2}}\,dx = \frac{(a-4b)\arctan\left(\frac{\sqrt{a-b}\sinh(e+fx)}{\sqrt{a+b\sinh^2(e+fx)}}\right)}{2(a-b)^{5/2}f} + \frac{b(a+2b)\sinh(e+fx)}{2a(a-b)^2f\sqrt{a+b\sinh^2(e+fx)}} + \frac{\operatorname{sech}(e+fx)\tanh(e+fx)}{2(a-b)f\sqrt{a+b\sinh^2(e+fx)}}$$

output
```
1/2*(a-4*b)*arctan((a-b)^(1/2)*sinh(f*x+e)/(a+b*sinh(f*x+e)^2)^(1/2))/(a-b
)^(5/2)/f+1/2*b*(a+2*b)*sinh(f*x+e)/a/(a-b)^2/f/(a+b*sinh(f*x+e)^2)^(1/2)+
1/2*sech(f*x+e)*tanh(f*x+e)/(a-b)/f/(a+b*sinh(f*x+e)^2)^(1/2)
```

Mathematica [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 3.84 (sec) , antiderivative size = 231, normalized size of antiderivative = 1.63

$$\int \frac{\operatorname{sech}^3(e+fx)}{\left(a+b\sinh^2(e+fx)\right)^{3/2}}\,dx = \frac{\operatorname{sech}^5(e+fx)\left(16(a-b)\,{}_3F_2\left(2,2,3;1,\frac{9}{2};\frac{(a-b)\tanh^2(e+fx)}{a}\right)\sinh^2(e+fx)\right.}{}$$

input
```
Integrate[Sech[e + f*x]^3/(a + b*Sinh[e + f*x]^2)^(3/2),x]
```

output
```
(Sech[e + f*x]^5*(16*(a - b)*HypergeometricPFQ[{2, 2, 3}, {1, 9/2}, ((a -
b)*Tanh[e + f*x]^2)/a]*Sinh[e + f*x]^2*(a + b*Sinh[e + f*x]^2)^2 + 16*(a -
 b)*Hypergeometric2F1[2, 3, 9/2, ((a - b)*Tanh[e + f*x]^2)/a]*Sinh[e + f*x
]^2*(4*a^2 + 7*a*b*Sinh[e + f*x]^2 + 3*b^2*Sinh[e + f*x]^4) + 7*a*Cosh[e +
 f*x]^2*Hypergeometric2F1[1, 2, 7/2, ((a - b)*Tanh[e + f*x]^2)/a]*(15*a^2
+ 20*a*b*Sinh[e + f*x]^2 + 8*b^2*Sinh[e + f*x]^4))*Tanh[e + f*x])/(105*a^4
*f*Sqrt[a + b*Sinh[e + f*x]^2])
```

Rubi [A] (verified)

Time = 0.38 (sec) , antiderivative size = 149, normalized size of antiderivative = 1.05, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.320$, Rules used = {3042, 3669, 316, 25, 402, 27, 291, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\operatorname{sech}^3(e+fx)}{\left(a+b\sinh^2(e+fx)\right)^{3/2}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{\cos(ie+ifx)^3\left(a-b\sin(ie+ifx)^2\right)^{3/2}}dx$$

$$\Big\downarrow \text{3669}$$

$$\frac{\int \frac{1}{\left(\sinh^2(e+fx)+1\right)^2\left(b\sinh^2(e+fx)+a\right)^{3/2}}d\sinh(e+fx)}{f}$$

$$\Big\downarrow \text{316}$$

$$\frac{\frac{\sinh(e+fx)}{2(a-b)\left(\sinh^2(e+fx)+1\right)\sqrt{a+b\sinh^2(e+fx)}}-\frac{\int -\frac{2b\sinh^2(e+fx)+a-2b}{\left(\sinh^2(e+fx)+1\right)\left(b\sinh^2(e+fx)+a\right)^{3/2}}d\sinh(e+fx)}{2(a-b)}}{f}$$

$$\Big\downarrow \text{25}$$

$$\frac{\frac{\int \frac{2b\sinh^2(e+fx)+a-2b}{\left(\sinh^2(e+fx)+1\right)\left(b\sinh^2(e+fx)+a\right)^{3/2}}d\sinh(e+fx)}{2(a-b)}+\frac{\sinh(e+fx)}{2(a-b)\left(\sinh^2(e+fx)+1\right)\sqrt{a+b\sinh^2(e+fx)}}}{f}$$

$\Big\downarrow$ 402

$$\frac{\frac{\frac{\int \frac{a(a-4b)}{\left(\sinh^2(e+fx)+1\right)\sqrt{b\sinh^2(e+fx)+a}}d\sinh(e+fx)}{a(a-b)}+\frac{b(a+2b)\sinh(e+fx)}{a(a-b)\sqrt{a+b\sinh^2(e+fx)}}}{2(a-b)}+\frac{\sinh(e+fx)}{2(a-b)\left(\sinh^2(e+fx)+1\right)\sqrt{a+b\sinh^2(e+fx)}}}{f}$$

$\Big\downarrow$ 27

$$\frac{\frac{\frac{(a-4b)\int \frac{1}{\left(\sinh^2(e+fx)+1\right)\sqrt{b\sinh^2(e+fx)+a}}d\sinh(e+fx)}{a-b}+\frac{b(a+2b)\sinh(e+fx)}{a(a-b)\sqrt{a+b\sinh^2(e+fx)}}}{2(a-b)}+\frac{\sinh(e+fx)}{2(a-b)\left(\sinh^2(e+fx)+1\right)\sqrt{a+b\sinh^2(e+fx)}}}{f}$$

$\Big\downarrow$ 291

$$\frac{\frac{\frac{(a-4b)\int \frac{1}{1-\frac{(b-a)\sinh^2(e+fx)}{b\sinh^2(e+fx)+a}}d\frac{\sinh(e+fx)}{\sqrt{b\sinh^2(e+fx)+a}}}{a-b}+\frac{b(a+2b)\sinh(e+fx)}{a(a-b)\sqrt{a+b\sinh^2(e+fx)}}}{2(a-b)}+\frac{\sinh(e+fx)}{2(a-b)\left(\sinh^2(e+fx)+1\right)\sqrt{a+b\sinh^2(e+fx)}}}{f}$$

$\Big\downarrow$ 216

$$\frac{\frac{\frac{(a-4b)\arctan\left(\frac{\sqrt{a-b}\sinh(e+fx)}{\sqrt{a+b\sinh^2(e+fx)}}\right)}{(a-b)^{3/2}}+\frac{b(a+2b)\sinh(e+fx)}{a(a-b)\sqrt{a+b\sinh^2(e+fx)}}}{2(a-b)}+\frac{\sinh(e+fx)}{2(a-b)\left(\sinh^2(e+fx)+1\right)\sqrt{a+b\sinh^2(e+fx)}}}{f}$$

input
```
Int[Sech[e + f*x]^3/(a + b*Sinh[e + f*x]^2)^(3/2),x]
```

output
```
(Sinh[e + f*x]/(2*(a - b)*(1 + Sinh[e + f*x]^2)*Sqrt[a + b*Sinh[e + f*x]^2
]) + (((a - 4*b)*ArcTan[(Sqrt[a - b]*Sinh[e + f*x])/Sqrt[a + b*Sinh[e + f*
x]^2]])/(a - b)^(3/2) + (b*(a + 2*b)*Sinh[e + f*x])/(a*(a - b)*Sqrt[a + b*
Sinh[e + f*x]^2]))/(2*(a - b)))/f
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 291
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*((c_) + (d_.)*(x_)^2)), x_Symbol] :> Subst
[Int[1/(c - (b*c - a*d)*x^2), x], x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b, c,
d}, x] && NeQ[b*c - a*d, 0]
```

rule 316
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_), x_Symbol] :> Sim
p[(-b)*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^(q + 1)/(2*a*(p + 1)*(b*c - a*d))
), x] + Simp[1/(2*a*(p + 1)*(b*c - a*d))   Int[(a + b*x^2)^(p + 1)*(c + d*x
^2)^q*Simp[b*c + 2*(p + 1)*(b*c - a*d) + d*b*(2*(p + q + 2) + 1)*x^2, x], x
], x] /; FreeQ[{a, b, c, d, q}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] &&  !
( !IntegerQ[p] && IntegerQ[q] && LtQ[q, -1]) && IntBinomialQ[a, b, c, d, 2,
 p, q, x]
```

rule 402
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_.)*((e_) + (f_.)*(x
_)^2), x_Symbol] :> Simp[(-(b*e - a*f))*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^
(q + 1)/(a*2*(b*c - a*d)*(p + 1))), x] + Simp[1/(a*2*(b*c - a*d)*(p + 1))
 Int[(a + b*x^2)^(p + 1)*(c + d*x^2)^q*Simp[c*(b*e - a*f) + e*2*(b*c - a*d)
*(p + 1) + d*(b*e - a*f)*(2*(p + q + 2) + 1)*x^2, x], x], x] /; FreeQ[{a, b
, c, d, e, f, q}, x] && LtQ[p, -1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3669
```
Int[cos[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^
(p_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Simp[ff/f   S
ubst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a + b*ff^2*x^2)^p, x], x, Sin[e + f*x]
/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m - 1)/2]
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.20 (sec) , antiderivative size = 95, normalized size of antiderivative = 0.67

$$\frac{\text{` int/indef0`}\left(-\frac{\sqrt{a+b\sinh(fx+e)^2}\,\cosh(fx+e)^2}{-b^2\cosh(fx+e)^{10}+(-2ab+2b^2)\cosh(fx+e)^8+(-a^2+2ab-b^2)\cosh(fx+e)^6},\sinh(fx+e)\right)}{f}$$

input
```
int(sech(f*x+e)^3/(a+b*sinh(f*x+e)^2)^(3/2),x)
```

output
```
`int/indef0`(-(a+b*sinh(f*x+e)^2)^(1/2)*cosh(f*x+e)^2/(-b^2*cosh(f*x+e)^10
+(-2*a*b+2*b^2)*cosh(f*x+e)^8+(-a^2+2*a*b-b^2)*cosh(f*x+e)^6),sinh(f*x+e))
/f
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2364 vs. 2(126) = 252.

Time = 0.37 (sec) , antiderivative size = 4845, normalized size of antiderivative = 34.12

$$\int \frac{\operatorname{sech}^3(e+fx)}{\left(a+b\sinh^2(e+fx)\right)^{3/2}}\,dx=\text{Too large to display}$$

input
```
integrate(sech(f*x+e)^3/(a+b*sinh(f*x+e)^2)^(3/2),x, algorithm="fricas")
```

output
```
Too large to include
```

Sympy [F]

$$\int \frac{\operatorname{sech}^3(e+fx)}{\left(a+b\sinh^2(e+fx)\right)^{3/2}}\,dx=\int \frac{\operatorname{sech}^3{\left(e+fx\right)}}{\left(a+b\sinh^2{\left(e+fx\right)}\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate(sech(f*x+e)**3/(a+b*sinh(f*x+e)**2)**(3/2),x)
```

output
```
Integral(sech(e + f*x)**3/(a + b*sinh(e + f*x)**2)**(3/2), x)
```

Maxima [F]

$$\int \frac{\operatorname{sech}^3(e+fx)}{\left(a+b\sinh^2(e+fx)\right)^{3/2}}\,dx=\int \frac{\operatorname{sech}\left(fx+e\right)^3}{\left(b\sinh\left(fx+e\right)^2+a\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate(sech(f*x+e)^3/(a+b*sinh(f*x+e)^2)^(3/2),x, algorithm="maxima")
```

output
```
integrate(sech(f*x + e)^3/(b*sinh(f*x + e)^2 + a)^(3/2), x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1031 vs. 2(126) = 252.

Time = 0.51 (sec) , antiderivative size = 1031, normalized size of antiderivative = 7.26

$$\int \frac{\operatorname{sech}^3(e+fx)}{\left(a+b\sinh^2(e+fx)\right)^{3/2}}\,dx=\text{Too large to display}$$

input
```
integrate(sech(f*x+e)^3/(a+b*sinh(f*x+e)^2)^(3/2),x, algorithm="giac")
```

output
```
(((a^3*b^2*f*e^(6*e) - 3*a^2*b^3*f*e^(6*e) + 3*a*b^4*f*e^(6*e) - b^5*f*e^(
6*e))*e^(2*f*x)/(a^6*f^2*e^(10*e) - 5*a^5*b*f^2*e^(10*e) + 10*a^4*b^2*f^2*
e^(10*e) - 10*a^3*b^3*f^2*e^(10*e) + 5*a^2*b^4*f^2*e^(10*e) - a*b^5*f^2*e^
(10*e)) - (a^3*b^2*f*e^(4*e) - 3*a^2*b^3*f*e^(4*e) + 3*a*b^4*f*e^(4*e) - b
^5*f*e^(4*e))/(a^6*f^2*e^(10*e) - 5*a^5*b*f^2*e^(10*e) + 10*a^4*b^2*f^2*e^
(10*e) - 10*a^3*b^3*f^2*e^(10*e) + 5*a^2*b^4*f^2*e^(10*e) - a*b^5*f^2*e^(1
0*e)))/sqrt(b*e^(4*f*x + 4*e) + 4*a*e^(2*f*x + 2*e) - 2*b*e^(2*f*x + 2*e)
+ b) + (a - 4*b)*arctan(-1/2*(sqrt(b)*e^(2*f*x + 2*e) - sqrt(b*e^(4*f*x +
4*e) + 4*a*e^(2*f*x + 2*e) - 2*b*e^(2*f*x + 2*e) + b) + sqrt(b))/sqrt(a -
b))/((a^2*f*e^(6*e) - 2*a*b*f*e^(6*e) + b^2*f*e^(6*e))*sqrt(a - b)) - 2*((
sqrt(b)*e^(2*f*x + 2*e) - sqrt(b*e^(4*f*x + 4*e) + 4*a*e^(2*f*x + 2*e) - 2
*b*e^(2*f*x + 2*e) + b))^3*a - 2*(sqrt(b)*e^(2*f*x + 2*e) - sqrt(b*e^(4*f*
x + 4*e) + 4*a*e^(2*f*x + 2*e) - 2*b*e^(2*f*x + 2*e) + b))^3*b - 5*(sqrt(b
)*e^(2*f*x + 2*e) - sqrt(b*e^(4*f*x + 4*e) + 4*a*e^(2*f*x + 2*e) - 2*b*e^(
2*f*x + 2*e) + b))^2*a*sqrt(b) + 2*(sqrt(b)*e^(2*f*x + 2*e) - sqrt(b*e^(4*
f*x + 4*e) + 4*a*e^(2*f*x + 2*e) - 2*b*e^(2*f*x + 2*e) + b))^2*b^(3/2) - 4
*(sqrt(b)*e^(2*f*x + 2*e) - sqrt(b*e^(4*f*x + 4*e) + 4*a*e^(2*f*x + 2*e) -
 2*b*e^(2*f*x + 2*e) + b))*a^2 - (sqrt(b)*e^(2*f*x + 2*e) - sqrt(b*e^(4*f*
x + 4*e) + 4*a*e^(2*f*x + 2*e) - 2*b*e^(2*f*x + 2*e) + b))*a*b + 2*(sqrt(b
)*e^(2*f*x + 2*e) - sqrt(b*e^(4*f*x + 4*e) + 4*a*e^(2*f*x + 2*e) - 2*b*...
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\operatorname{sech}^3(e+fx)}{\left(a+b\sinh^2(e+fx)\right)^{3/2}}\,dx=\int \frac{1}{\cosh\left(e+f\,x\right)^3\,\left(b\,\sinh\left(e+f\,x\right)^2+a\right)^{3/2}}\,dx$$

input
```
int(1/(cosh(e + f*x)^3*(a + b*sinh(e + f*x)^2)^(3/2)),x)
```

output
```
int(1/(cosh(e + f*x)^3*(a + b*sinh(e + f*x)^2)^(3/2)), x)
```

Reduce [F]

$$\int \frac{\operatorname{sech}^3(e+fx)}{(a+b\sinh^2(e+fx))^{3/2}}\,dx = \int \frac{\sqrt{\sinh(fx+e)^2 b + a}\,\operatorname{sech}(fx+e)^3}{\sinh(fx+e)^4 b^2 + 2\sinh(fx+e)^2 ab + a^2}dx$$

input
```
int(sech(f*x+e)^3/(a+b*sinh(f*x+e)^2)^(3/2),x)
```

output
```
int((sqrt(sinh(e + f*x)**2*b + a)*sech(e + f*x)**3)/(sinh(e + f*x)**4*b**2
 + 2*sinh(e + f*x)**2*a*b + a**2),x)
```

3.350 $\int \frac{\cosh^6(e+fx)}{\left(a+b\sinh^2(e+fx)\right)^{3/2}}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 296

$$\begin{aligned}\int \frac{\cosh^6(e+fx)}{\left(a+b\sinh^2(e+fx)\right)^{3/2}}\,dx =&-\frac{2(2a-3b)\cosh(e+fx)\sinh(e+fx)}{3b^2f\sqrt{a+b\sinh^2(e+fx)}}+\frac{\cosh^3(e+fx)\sinh(e+fx)}{3bf\sqrt{a+b\sinh^2(e+fx)}}\\&+\frac{(8a^2-13ab+3b^2)\cosh(e+fx)E\left(\arctan\left(\frac{\sqrt{b}\sinh(e+fx)}{\sqrt{a}}\right)|1-\frac{a}{b}\right)}{3\sqrt{a}b^{5/2}f\sqrt{\frac{a\cosh^2(e+fx)}{a+b\sinh^2(e+fx)}}\sqrt{a+b\sinh^2(e+fx)}}\\&-\frac{2\sqrt{a}(2a-3b)\cosh(e+fx)\operatorname{EllipticF}\left(\arctan\left(\frac{\sqrt{b}\sinh(e+fx)}{\sqrt{a}}\right),1-\frac{a}{b}\right)}{3b^{5/2}f\sqrt{\frac{a\cosh^2(e+fx)}{a+b\sinh^2(e+fx)}}\sqrt{a+b\sinh^2(e+fx)}}\end{aligned}$$

output
```
-2/3*(2*a-3*b)*cosh(f*x+e)*sinh(f*x+e)/b^2/f/(a+b*sinh(f*x+e)^2)^(1/2)+1/3
*cosh(f*x+e)^3*sinh(f*x+e)/b/f/(a+b*sinh(f*x+e)^2)^(1/2)+1/3*(8*a^2-13*a*b
+3*b^2)*cosh(f*x+e)*EllipticE(b^(1/2)*sinh(f*x+e)/a^(1/2)/(1+b*sinh(f*x+e)
^2/a)^(1/2),(1-a/b)^(1/2))/a^(1/2)/b^(5/2)/f/(a*cosh(f*x+e)^2/(a+b*sinh(f*
x+e)^2))^(1/2)/(a+b*sinh(f*x+e)^2)^(1/2)-2/3*a^(1/2)*(2*a-3*b)*cosh(f*x+e)
*InverseJacobiAM(arctan(b^(1/2)*sinh(f*x+e)/a^(1/2)),(1-a/b)^(1/2))/b^(5/2
)/f/(a*cosh(f*x+e)^2/(a+b*sinh(f*x+e)^2))^(1/2)/(a+b*sinh(f*x+e)^2)^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 1.09 (sec) , antiderivative size = 196, normalized size of antiderivative = 0.66

$$\int \frac{\cosh^6(e+fx)}{\left(a+b\sinh^2(e+fx)\right)^{3/2}}\,dx = \frac{4ia(8a^2-13ab+3b^2)\sqrt{\frac{2a-b+b\cosh(2(e+fx))}{a}}E\left(i(e+fx)\,\middle|\,\frac{b}{a}\right)-4ia(8a^2-1$$

input
```
Integrate[Cosh[e + f*x]^6/(a + b*Sinh[e + f*x]^2)^(3/2),x]
```

output
```
((4*I)*a*(8*a^2 - 13*a*b + 3*b^2)*Sqrt[(2*a - b + b*Cosh[2*(e + f*x)])/a]*
EllipticE[I*(e + f*x), b/a] - (4*I)*a*(8*a^2 - 17*a*b + 9*b^2)*Sqrt[(2*a -
 b + b*Cosh[2*(e + f*x)])/a]*EllipticF[I*(e + f*x), b/a] + Sqrt[2]*b*(8*a^
2 - 13*a*b + 6*b^2 + a*b*Cosh[2*(e + f*x)])*Sinh[2*(e + f*x)])/(12*a*b^3*f
*Sqrt[2*a - b + b*Cosh[2*(e + f*x)]])
```

Rubi [A] (verified)

Time = 0.53 (sec) , antiderivative size = 365, normalized size of antiderivative = 1.23, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.360$, Rules used = {3042, 3671, 315, 403, 25, 406, 320, 388, 313}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cosh^6(e+fx)}{\left(a+b\sinh^2(e+fx)\right)^{3/2}}dx$$

$$\Big\downarrow \; 3042$$

$$\int \frac{\cos(ie+ifx)^6}{\left(a-b\sin(ie+ifx)^2\right)^{3/2}}dx$$

$$\Big\downarrow \; 3671$$

$$\frac{\sqrt{\cosh^2(e+fx)}\text{sech}(e+fx)\int \frac{\left(\sinh^2(e+fx)+1\right)^{5/2}}{\left(b\sinh^2(e+fx)+a\right)^{3/2}}d\sinh(e+fx)}{f}$$

$$\Big\downarrow \; 315$$

$$\frac{\sqrt{\cosh^2(e+fx)}\text{sech}(e+fx)\left(\frac{\int \frac{\sqrt{\sinh^2(e+fx)+1}\left((4a-3b)\sinh^2(e+fx)+a\right)}{\sqrt{b\sinh^2(e+fx)+a}}d\sinh(e+fx)}{ab}-\frac{(a-b)\sinh(e+fx)\left(\sinh^2(e+fx)+1\right)^{3/2}}{ab\sqrt{a+b\sinh^2(e+fx)}}\right)}{f}$$

$$\Big\downarrow \; 403$$

$$\frac{\sqrt{\cosh^2(e+fx)}\text{sech}(e+fx)\left(\frac{\frac{\int -\frac{\left(8a^2-13ba+3b^2\right)\sinh^2(e+fx)+2a(2a-3b)}{\sqrt{\sinh^2(e+fx)+1}\sqrt{b\sinh^2(e+fx)+a}}d\sinh(e+fx)}{3b}+\frac{(4a-3b)\sqrt{\sinh^2(e+fx)+1}\sinh(e+fx)\sqrt{a+b\sinh^2(e\cdots}}{3b}}{ab}\cdots\right)}{f}$$

$$\Big\downarrow \; 25$$

$$\frac{\sqrt{\cosh^2(e+fx)}\text{sech}(e+fx)\left(\frac{\frac{(4a-3b)\sinh(e+fx)\sqrt{\sinh^2(e+fx)+1}\sqrt{a+b\sinh^2(e+fx)}}{3b}-\frac{\int \frac{\left(8a^2-13ba+3b^2\right)\sinh^2(e+fx)+2a(2a-3b)}{\sqrt{\sinh^2(e+fx)+1}\sqrt{b\sinh^2(e+fx)+a}}d\sinh(e+\cdots}{3b}}{ab}\cdots\right)}{f}$$

$$\Big\downarrow \; 406$$

$$\frac{\sqrt{\cosh^2(e+fx)}\text{sech}(e+fx)\left(\frac{\frac{(4a-3b)\sinh(e+fx)\sqrt{\sinh^2(e+fx)+1}\sqrt{a+b\sinh^2(e+fx)}}{3b}-\frac{\left(8a^2-13ab+3b^2\right)\int \frac{\sinh^2(e+fx)}{\sqrt{\sinh^2(e+fx)+1}\sqrt{b\sinh^2(e+fx)+\cdots}}\cdots}{\cdots}}{ab}\cdots\right)}{f}$$

$$\Big\downarrow \; 320$$

$$\frac{\sqrt{\cosh^2(e+fx)}\text{sech}(e+fx)\left(\frac{\frac{(4a-3b)\sinh(e+fx)\sqrt{\sinh^2(e+fx)+1}\sqrt{a+b\sinh^2(e+fx)}}{3b}-\frac{\left(8a^2-13ab+3b^2\right)\int \frac{\sinh^2(e+fx)}{\sqrt{\sinh^2(e+fx)+1}\sqrt{b\sinh^2(e+fx)+\cdots}}\cdots}{\cdots}}{ab}\cdots\right)}{f}$$

$$\Big\downarrow \; 388$$

$$\frac{\sqrt{\cosh^2(e+fx)}\operatorname{sech}(e+fx)\left(\frac{\frac{(4a-3b)\sinh(e+fx)\sqrt{\sinh^2(e+fx)+1}\sqrt{a+b\sinh^2(e+fx)}}{3b}-\frac{\left(8a^2-13ab+3b^2\right)\left(\frac{\sinh(e+fx)\sqrt{a+b\sinh^2(e+fx)}}{b\sqrt{\sinh^2(e+fx)+1}}-\frac{\int \cdots}{\cdots}\right.}{\cdots}}{ab}\right.}{f}$$

$\downarrow$ 313

$$\frac{\sqrt{\cosh^2(e+fx)}\operatorname{sech}(e+fx)\left(\frac{\frac{(4a-3b)\sinh(e+fx)\sqrt{\sinh^2(e+fx)+1}\sqrt{a+b\sinh^2(e+fx)}}{3b}-\frac{\left(8a^2-13ab+3b^2\right)\left(\frac{\sinh(e+fx)\sqrt{a+b\sinh^2(e+fx)}}{b\sqrt{\sinh^2(e+fx)+1}}-\frac{\sqrt{\cdots}}{\cdots}\right.}{\cdots}}{\cdots}\right.}{\cdots}$$

input

```
Int[Cosh[e + f*x]^6/(a + b*Sinh[e + f*x]^2)^(3/2),x]
```

output

```
(Sqrt[Cosh[e + f*x]^2]*Sech[e + f*x]*(-(((a - b)*Sinh[e + f*x]*(1 + Sinh[e
 + f*x]^2)^(3/2))/(a*b*Sqrt[a + b*Sinh[e + f*x]^2])) + (((4*a - 3*b)*Sinh[
e + f*x]*Sqrt[1 + Sinh[e + f*x]^2]*Sqrt[a + b*Sinh[e + f*x]^2])/(3*b) - ((
2*(2*a - 3*b)*EllipticF[ArcTan[Sinh[e + f*x]], 1 - b/a]*Sqrt[a + b*Sinh[e
+ f*x]^2])/(Sqrt[1 + Sinh[e + f*x]^2]*Sqrt[(a + b*Sinh[e + f*x]^2)/(a*(1 +
 Sinh[e + f*x]^2))]) + (8*a^2 - 13*a*b + 3*b^2)*((Sinh[e + f*x]*Sqrt[a + b
*Sinh[e + f*x]^2])/(b*Sqrt[1 + Sinh[e + f*x]^2]) - (EllipticE[ArcTan[Sinh[
e + f*x]], 1 - b/a]*Sqrt[a + b*Sinh[e + f*x]^2])/(b*Sqrt[1 + Sinh[e + f*x]
^2]*Sqrt[(a + b*Sinh[e + f*x]^2)/(a*(1 + Sinh[e + f*x]^2))])))/(3*b))/(a*b
)))/f
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 313
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/((c_) + (d_.)*(x_)^2)^(3/2), x_Symbol] :> Sim
p[(Sqrt[a + b*x^2]/(c*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(c
+ d*x^2)))]))*EllipticE[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; FreeQ
[{a, b, c, d}, x] && PosQ[b/a] && PosQ[d/c]
```

rule 315
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_), x_Symbol] :> Sim
p[(a*d - c*b)*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^(q - 1)/(2*a*b*(p + 1))),
x] - Simp[1/(2*a*b*(p + 1))   Int[(a + b*x^2)^(p + 1)*(c + d*x^2)^(q - 2)*S
imp[c*(a*d - c*b*(2*p + 3)) + d*(a*d*(2*(q - 1) + 1) - b*c*(2*(p + q) + 1))
*x^2, x], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -
1] && GtQ[q, 1] && IntBinomialQ[a, b, c, d, 2, p, q, x]
```

rule 320
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(Sqrt[a + b*x^2]/(a*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(
c + d*x^2)))]))*EllipticF[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; Fre
eQ[{a, b, c, d}, x] && PosQ[d/c] && PosQ[b/a] &&  !SimplerSqrtQ[b/a, d/c]
```

rule 388
```
Int[(x_)^2/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol]
 :> Simp[x*(Sqrt[a + b*x^2]/(b*Sqrt[c + d*x^2])), x] - Simp[c/b   Int[Sqrt[
a + b*x^2]/(c + d*x^2)^(3/2), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
 a*d, 0] && PosQ[b/a] && PosQ[d/c] &&  !SimplerSqrtQ[b/a, d/c]
```

rule 403
```
Int[((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.)*((e_) + (f_.)*(
x_)^2), x_Symbol] :> Simp[f*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^q/(b*(2*(p +
 q + 1) + 1))), x] + Simp[1/(b*(2*(p + q + 1) + 1))   Int[(a + b*x^2)^p*(c
+ d*x^2)^(q - 1)*Simp[c*(b*e - a*f + b*e*2*(p + q + 1)) + (d*(b*e - a*f) +
f*2*q*(b*c - a*d) + b*d*e*2*(p + q + 1))*x^2, x], x], x] /; FreeQ[{a, b, c,
 d, e, f, p}, x] && GtQ[q, 0] && NeQ[2*(p + q + 1) + 1, 0]
```

rule 406
```
Int[((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.)*((e_) + (f_.)*(
x_)^2), x_Symbol] :> Simp[e   Int[(a + b*x^2)^p*(c + d*x^2)^q, x], x] + Sim
p[f   Int[x^2*(a + b*x^2)^p*(c + d*x^2)^q, x], x] /; FreeQ[{a, b, c, d, e,
f, p, q}, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3671
```
Int[cos[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(
p_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Simp[ff*(Sqrt[
Cos[e + f*x]^2]/(f*Cos[e + f*x]))   Subst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a
 + b*ff^2*x^2)^p, x], x, Sin[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x]
 && IntegerQ[m/2] &&  !IntegerQ[p]
```

Maple [A] (verified)

Time = 5.70 (sec) , antiderivative size = 498, normalized size of antiderivative = 1.68

method	result
default	$\frac{\sqrt{-\frac{b}{a}}\, ab \cosh(fx+e)^4 \sinh(fx+e)+\left(4\sqrt{-\frac{b}{a}}\, a^2-7\sqrt{-\frac{b}{a}}\, ab+3\sqrt{-\frac{b}{a}}\, b^2\right)\cosh(fx+e)^2 \sinh(fx+e)+4a^2\sqrt{\frac{b\cosh(fx+e)^2}{a}+\frac{a-b}{a}}\,\sqrt{\cdots}}{}$

input
```
int(cosh(f*x+e)^6/(a+b*sinh(f*x+e)^2)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
1/3*((-b/a)^(1/2)*a*b*cosh(f*x+e)^4*sinh(f*x+e)+(4*(-b/a)^(1/2)*a^2-7*(-b/
a)^(1/2)*a*b+3*(-b/a)^(1/2)*b^2)*cosh(f*x+e)^2*sinh(f*x+e)+4*a^2*(b/a*cosh
(f*x+e)^2+(a-b)/a)^(1/2)*(cosh(f*x+e)^2)^(1/2)*EllipticF(sinh(f*x+e)*(-b/a
)^(1/2),(1/b*a)^(1/2))-7*a*(b/a*cosh(f*x+e)^2+(a-b)/a)^(1/2)*(cosh(f*x+e)^
2)^(1/2)*EllipticF(sinh(f*x+e)*(-b/a)^(1/2),(1/b*a)^(1/2))*b+3*(b/a*cosh(f
*x+e)^2+(a-b)/a)^(1/2)*(cosh(f*x+e)^2)^(1/2)*EllipticF(sinh(f*x+e)*(-b/a)^
(1/2),(1/b*a)^(1/2))*b^2-8*(b/a*cosh(f*x+e)^2+(a-b)/a)^(1/2)*(cosh(f*x+e)^
2)^(1/2)*EllipticE(sinh(f*x+e)*(-b/a)^(1/2),(1/b*a)^(1/2))*a^2+13*(b/a*cos
h(f*x+e)^2+(a-b)/a)^(1/2)*(cosh(f*x+e)^2)^(1/2)*EllipticE(sinh(f*x+e)*(-b/
a)^(1/2),(1/b*a)^(1/2))*a*b-3*(b/a*cosh(f*x+e)^2+(a-b)/a)^(1/2)*(cosh(f*x+
e)^2)^(1/2)*EllipticE(sinh(f*x+e)*(-b/a)^(1/2),(1/b*a)^(1/2))*b^2)/b^2/(-b
/a)^(1/2)/a/cosh(f*x+e)/(a+b*sinh(f*x+e)^2)^(1/2)/f
```

Fricas [F]

$$\int \frac{\cosh^6(e+fx)}{\left(a+b\sinh^2(e+fx)\right)^{3/2}}\,dx=\int \frac{\cosh\left(fx+e\right)^6}{\left(b\sinh\left(fx+e\right)^2+a\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate(cosh(f*x+e)^6/(a+b*sinh(f*x+e)^2)^(3/2),x, algorithm="fricas")
```

output
```
integral(sqrt(b*sinh(f*x + e)^2 + a)*cosh(f*x + e)^6/(b^2*sinh(f*x + e)^4
+ 2*a*b*sinh(f*x + e)^2 + a^2), x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\cosh^6(e+fx)}{\left(a+b\sinh^2(e+fx)\right)^{3/2}}\,dx=\text{Timed out}$$

input
```
integrate(cosh(f*x+e)**6/(a+b*sinh(f*x+e)**2)**(3/2),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{\cosh^6(e+fx)}{\left(a+b\sinh^2(e+fx)\right)^{3/2}}\,dx = \int \frac{\cosh\left(fx+e\right)^6}{\left(b\sinh\left(fx+e\right)^2+a\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate(cosh(f*x+e)^6/(a+b*sinh(f*x+e)^2)^(3/2),x, algorithm="maxima")
```

output
```
integrate(cosh(f*x + e)^6/(b*sinh(f*x + e)^2 + a)^(3/2), x)
```

Giac [F]

$$\int \frac{\cosh^6(e+fx)}{\left(a+b\sinh^2(e+fx)\right)^{3/2}}\,dx = \int \frac{\cosh\left(fx+e\right)^6}{\left(b\sinh\left(fx+e\right)^2+a\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate(cosh(f*x+e)^6/(a+b*sinh(f*x+e)^2)^(3/2),x, algorithm="giac")
```

output
```
integrate(cosh(f*x + e)^6/(b*sinh(f*x + e)^2 + a)^(3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\cosh^6(e+fx)}{\left(a+b\sinh^2(e+fx)\right)^{3/2}}\,dx = \int \frac{{\mathrm{cosh}\left(e+f\,x\right)}^6}{{\left(b\,{\mathrm{sinh}\left(e+f\,x\right)}^2+a\right)}^{3/2}}\,dx$$

input
```
int(cosh(e + f*x)^6/(a + b*sinh(e + f*x)^2)^(3/2),x)
```

output
```
int(cosh(e + f*x)^6/(a + b*sinh(e + f*x)^2)^(3/2), x)
```

Reduce [F]

$$\int \frac{\cosh^6(e+fx)}{\left(a+b\sinh^2(e+fx)\right)^{3/2}}\,dx = \int \frac{\sqrt{\sinh\left(fx+e\right)^2 b+a}\,\cosh\left(fx+e\right)^6}{\sinh\left(fx+e\right)^4 b^2+2\sinh\left(fx+e\right)^2 ab+a^2}dx$$

input
```
int(cosh(f*x+e)^6/(a+b*sinh(f*x+e)^2)^(3/2),x)
```

output
```
int((sqrt(sinh(e + f*x)**2*b + a)*cosh(e + f*x)**6)/(sinh(e + f*x)**4*b**2
 + 2*sinh(e + f*x)**2*a*b + a**2),x)
```

3.351 $\int \frac{\cosh^4(e+fx)}{(a+b\sinh^2(e+fx))^{3/2}}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 226

$$\int \frac{\cosh^4(e+fx)}{(a+b\sinh^2(e+fx))^{3/2}}\,dx = \frac{\cosh(e+fx)\sinh(e+fx)}{bf\sqrt{a+b\sinh^2(e+fx)}} - \frac{(2a-b)\cosh(e+fx)E\left(\arctan\left(\frac{\sqrt{b}\sinh(e+fx)}{\sqrt{a}}\right)|1-\frac{a}{b}\right)}{\sqrt{a}b^{3/2}f\sqrt{\frac{a\cosh^2(e+fx)}{a+b\sinh^2(e+fx)}}\sqrt{a+b\sinh^2(e+fx)}} + \frac{\sqrt{a}\cosh(e+fx)\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{b}\sinh(e+fx)}{\sqrt{a}}\right),1-\frac{a}{b}\right)}{b^{3/2}f\sqrt{\frac{a\cosh^2(e+fx)}{a+b\sinh^2(e+fx)}}\sqrt{a+b\sinh^2(e+fx)}}$$

output
```
cosh(f*x+e)*sinh(f*x+e)/b/f/(a+b*sinh(f*x+e)^2)^(1/2)-(2*a-b)*cosh(f*x+e)*
EllipticE(b^(1/2)*sinh(f*x+e)/a^(1/2)/(1+b*sinh(f*x+e)^2/a)^(1/2),(1-a/b)^
(1/2))/a^(1/2)/b^(3/2)/f/(a*cosh(f*x+e)^2/(a+b*sinh(f*x+e)^2))^(1/2)/(a+b*
sinh(f*x+e)^2)^(1/2)+a^(1/2)*cosh(f*x+e)*InverseJacobiAM(arctan(b^(1/2)*si
nh(f*x+e)/a^(1/2)),(1-a/b)^(1/2))/b^(3/2)/f/(a*cosh(f*x+e)^2/(a+b*sinh(f*x
+e)^2))^(1/2)/(a+b*sinh(f*x+e)^2)^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.65 (sec) , antiderivative size = 155, normalized size of antiderivative = 0.69

$$\int \frac{\cosh^4(e+fx)}{\left(a+b\sinh^2(e+fx)\right)^{3/2}}\,dx = \frac{-2ia(2a-b)\sqrt{\frac{2a-b+b\cosh(2(e+fx))}{a}}E\left(i(e+fx)\,\middle|\,\frac{b}{a}\right)+(a-b)\left(4ia\sqrt{\frac{2a-b+}{}}\right.}{2ab^2f\sqrt{2a-b+b\cosh}}$$

input
```
Integrate[Cosh[e + f*x]^4/(a + b*Sinh[e + f*x]^2)^(3/2),x]
```

output
```
((-2*I)*a*(2*a - b)*Sqrt[(2*a - b + b*Cosh[2*(e + f*x)])/a]*EllipticE[I*(e
 + f*x), b/a] + (a - b)*((4*I)*a*Sqrt[(2*a - b + b*Cosh[2*(e + f*x)])/a]*E
llipticF[I*(e + f*x), b/a] - Sqrt[2]*b*Sinh[2*(e + f*x)]))/(2*a*b^2*f*Sqrt
[2*a - b + b*Cosh[2*(e + f*x)]])
```

Rubi [A] (verified)

Time = 0.45 (sec) , antiderivative size = 291, normalized size of antiderivative = 1.29, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.280, Rules used = {3042, 3671, 315, 406, 320, 388, 313}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cosh^4(e+fx)}{\left(a+b\sinh^2(e+fx)\right)^{3/2}}dx$$

$$\Big\downarrow \; 3042$$

$$\int \frac{\cos(ie+ifx)^4}{(a-b\sin(ie+ifx)^2)^{3/2}}dx$$

$$\Big\downarrow \; 3671$$

$$\frac{\sqrt{\cosh^2(e+fx)}\text{sech}(e+fx)\int \frac{\left(\sinh^2(e+fx)+1\right)^{3/2}}{\left(b\sinh^2(e+fx)+a\right)^{3/2}}d\sinh(e+fx)}{f}$$

$\Big\downarrow$ 315

$$\frac{\sqrt{\cosh^2(e+fx)}\operatorname{sech}(e+fx)\left(\frac{\int \frac{(2a-b)\sinh^2(e+fx)+a}{\sqrt{\sinh^2(e+fx)+1}\sqrt{b\sinh^2(e+fx)+a}}d\sinh(e+fx)}{ab}-\frac{(a-b)\sinh(e+fx)\sqrt{\sinh^2(e+fx)+1}}{ab\sqrt{a+b\sinh^2(e+fx)}}\right)}{f}$$

$\Big\downarrow$ 406

$$\frac{\sqrt{\cosh^2(e+fx)}\operatorname{sech}(e+fx)\left(\frac{a\int \frac{1}{\sqrt{\sinh^2(e+fx)+1}\sqrt{b\sinh^2(e+fx)+a}}d\sinh(e+fx)+(2a-b)\int \frac{\sinh^2(e+fx)}{\sqrt{\sinh^2(e+fx)+1}\sqrt{b\sinh^2(e+fx)+a}}d\sinh(e$$

$\Big\downarrow$ 320

$$\frac{\sqrt{\cosh^2(e+fx)}\operatorname{sech}(e+fx)\left(\frac{(2a-b)\int \frac{\sinh^2(e+fx)}{\sqrt{\sinh^2(e+fx)+1}\sqrt{b\sinh^2(e+fx)+a}}d\sinh(e+fx)+\frac{\sqrt{a+b\sinh^2(e+fx)}\operatorname{EllipticF}\left(\arctan(\sinh(e+fx)),\right.}{\sqrt{\sinh^2(e+fx)+1}\sqrt{\frac{a+b\sinh^2(e+fx)}{a\left(\sinh^2(e+fx)+1\right)}}}}{ab}$$

$\Big\downarrow$ 388

$$\frac{\sqrt{\cosh^2(e+fx)}\operatorname{sech}(e+fx)\left(\frac{(2a-b)\left(\frac{\sinh(e+fx)\sqrt{a+b\sinh^2(e+fx)}}{b\sqrt{\sinh^2(e+fx)+1}}-\frac{\int \frac{\sqrt{b\sinh^2(e+fx)+a}}{\left(\sinh^2(e+fx)+1\right)^{3/2}}d\sinh(e+fx)}{b}\right)+\frac{\sqrt{a+b\sinh^2(e+fx)}\operatorname{EllipticF}\left(\right.}{\sqrt{\sinh^2(e+fx)+1}\sqrt{\frac{}{a}}}}{ab}$$

$\Big\downarrow$ 313

$$\frac{\sqrt{\cosh^2(e+fx)}\operatorname{sech}(e+fx)\left(\frac{\frac{\sqrt{a+b\sinh^2(e+fx)}\operatorname{EllipticF}\left(\arctan(\sinh(e+fx)),1-\frac{b}{a}\right)}{\sqrt{\sinh^2(e+fx)+1}\sqrt{\frac{a+b\sinh^2(e+fx)}{a\left(\sinh^2(e+fx)+1\right)}}}+(2a-b)\left(\frac{\sinh(e+fx)\sqrt{a+b\sinh^2(e+fx)}}{b\sqrt{\sinh^2(e+fx)+1}}-\frac{\sqrt{a+b\sinh^2(}{b\sqrt{\sinh^2}}\right.}{ab}$$

input
```
Int[Cosh[e + f*x]^4/(a + b*Sinh[e + f*x]^2)^(3/2),x]
```

output
```
(Sqrt[Cosh[e + f*x]^2]*Sech[e + f*x]*(-(((a - b)*Sinh[e + f*x]*Sqrt[1 + Si
nh[e + f*x]^2])/(a*b*Sqrt[a + b*Sinh[e + f*x]^2])) + ((EllipticF[ArcTan[Si
nh[e + f*x]], 1 - b/a]*Sqrt[a + b*Sinh[e + f*x]^2])/(Sqrt[1 + Sinh[e + f*x
]^2]*Sqrt[(a + b*Sinh[e + f*x]^2)/(a*(1 + Sinh[e + f*x]^2))]) + (2*a - b)*
((Sinh[e + f*x]*Sqrt[a + b*Sinh[e + f*x]^2])/(b*Sqrt[1 + Sinh[e + f*x]^2])
 - (EllipticE[ArcTan[Sinh[e + f*x]], 1 - b/a]*Sqrt[a + b*Sinh[e + f*x]^2])
/(b*Sqrt[1 + Sinh[e + f*x]^2]*Sqrt[(a + b*Sinh[e + f*x]^2)/(a*(1 + Sinh[e
+ f*x]^2))])))/(a*b)))/f
```

Defintions of rubi rules used

rule 313
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/((c_) + (d_.)*(x_)^2)^(3/2), x_Symbol] :> Sim
p[(Sqrt[a + b*x^2]/(c*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(c
+ d*x^2)))]))*EllipticE[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; FreeQ
[{a, b, c, d}, x] && PosQ[b/a] && PosQ[d/c]
```

rule 315
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_), x_Symbol] :> Sim
p[(a*d - c*b)*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^(q - 1)/(2*a*b*(p + 1))),
x] - Simp[1/(2*a*b*(p + 1))   Int[(a + b*x^2)^(p + 1)*(c + d*x^2)^(q - 2)*S
imp[c*(a*d - c*b*(2*p + 3)) + d*(a*d*(2*(q - 1) + 1) - b*c*(2*(p + q) + 1))
*x^2, x], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -
1] && GtQ[q, 1] && IntBinomialQ[a, b, c, d, 2, p, q, x]
```

rule 320
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(Sqrt[a + b*x^2]/(a*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(
c + d*x^2)))]))*EllipticF[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; Fre
eQ[{a, b, c, d}, x] && PosQ[d/c] && PosQ[b/a] &&  !SimplerSqrtQ[b/a, d/c]
```

rule 388
```
Int[(x_)^2/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol]
 :> Simp[x*(Sqrt[a + b*x^2]/(b*Sqrt[c + d*x^2])), x] - Simp[c/b   Int[Sqrt[
a + b*x^2]/(c + d*x^2)^(3/2), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
 a*d, 0] && PosQ[b/a] && PosQ[d/c] &&  !SimplerSqrtQ[b/a, d/c]
```

rule 406
```
Int[((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.)*((e_) + (f_.)*(
x_)^2), x_Symbol] :> Simp[e   Int[(a + b*x^2)^p*(c + d*x^2)^q, x], x] + Sim
p[f   Int[x^2*(a + b*x^2)^p*(c + d*x^2)^q, x], x] /; FreeQ[{a, b, c, d, e,
f, p, q}, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3671
```
Int[cos[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(
p_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Simp[ff*(Sqrt[
Cos[e + f*x]^2]/(f*Cos[e + f*x]))   Subst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a
 + b*ff^2*x^2)^p, x], x, Sin[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x]
 && IntegerQ[m/2] &&  !IntegerQ[p]
```

Maple [A] (verified)

Time = 5.44 (sec) , antiderivative size = 334, normalized size of antiderivative = 1.48

method	result
default	$-\frac{\sqrt{-\frac{b}{a}}\, a \cosh(fx+e)^{2} \sinh(fx+e)-\sqrt{-\frac{b}{a}}\, b \cosh(fx+e)^{2} \sinh(fx+e)+a \sqrt{\frac{b \cosh(fx+e)^{2}}{a}+\frac{a-b}{a}}\, \sqrt{\frac{\cosh(2fx+2e)}{2}+\frac{1}{2}}\, \operatorname{EllipticF}\left(\sinh \right.}{}$

input
```
int(cosh(f*x+e)^4/(a+b*sinh(f*x+e)^2)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-((-b/a)^(1/2)*a*cosh(f*x+e)^2*sinh(f*x+e)-(-b/a)^(1/2)*b*cosh(f*x+e)^2*si
nh(f*x+e)+a*(b/a*cosh(f*x+e)^2+(a-b)/a)^(1/2)*(cosh(f*x+e)^2)^(1/2)*Ellipt
icF(sinh(f*x+e)*(-b/a)^(1/2),(1/b*a)^(1/2))-(b/a*cosh(f*x+e)^2+(a-b)/a)^(1
/2)*(cosh(f*x+e)^2)^(1/2)*EllipticF(sinh(f*x+e)*(-b/a)^(1/2),(1/b*a)^(1/2)
)*b-2*(b/a*cosh(f*x+e)^2+(a-b)/a)^(1/2)*(cosh(f*x+e)^2)^(1/2)*EllipticE(si
nh(f*x+e)*(-b/a)^(1/2),(1/b*a)^(1/2))*a+(b/a*cosh(f*x+e)^2+(a-b)/a)^(1/2)*
(cosh(f*x+e)^2)^(1/2)*EllipticE(sinh(f*x+e)*(-b/a)^(1/2),(1/b*a)^(1/2))*b)
/b/(-b/a)^(1/2)/a/cosh(f*x+e)/(a+b*sinh(f*x+e)^2)^(1/2)/f
```

Fricas [F]

$$\int \frac{\cosh^4(e+fx)}{(a+b\sinh^2(e+fx))^{3/2}}\,dx = \int \frac{\cosh(fx+e)^4}{(b\sinh(fx+e)^2+a)^{\frac{3}{2}}}\,dx$$

input
```
integrate(cosh(f*x+e)^4/(a+b*sinh(f*x+e)^2)^(3/2),x, algorithm="fricas")
```

output
```
integral(sqrt(b*sinh(f*x + e)^2 + a)*cosh(f*x + e)^4/(b^2*sinh(f*x + e)^4
+ 2*a*b*sinh(f*x + e)^2 + a^2), x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\cosh^4(e+fx)}{(a+b\sinh^2(e+fx))^{3/2}}\,dx = \text{Timed out}$$

input
```
integrate(cosh(f*x+e)**4/(a+b*sinh(f*x+e)**2)**(3/2),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{\cosh^4(e+fx)}{(a+b\sinh^2(e+fx))^{3/2}}\,dx = \int \frac{\cosh(fx+e)^4}{(b\sinh(fx+e)^2+a)^{\frac{3}{2}}}\,dx$$

input
```
integrate(cosh(f*x+e)^4/(a+b*sinh(f*x+e)^2)^(3/2),x, algorithm="maxima")
```

output
```
integrate(cosh(f*x + e)^4/(b*sinh(f*x + e)^2 + a)^(3/2), x)
```

Giac [F]

$$\int \frac{\cosh^4(e+fx)}{\left(a+b\sinh^2(e+fx)\right)^{3/2}}\,dx=\int \frac{\cosh\left(fx+e\right)^4}{\left(b\sinh\left(fx+e\right)^2+a\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate(cosh(f*x+e)^4/(a+b*sinh(f*x+e)^2)^(3/2),x, algorithm="giac")
```

output
```
integrate(cosh(f*x + e)^4/(b*sinh(f*x + e)^2 + a)^(3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\cosh^4(e+fx)}{\left(a+b\sinh^2(e+fx)\right)^{3/2}}\,dx=\int \frac{\cosh(e+f\,x)^4}{\left(b\sinh\left(e+f\,x\right)^2+a\right)^{3/2}}\,dx$$

input
```
int(cosh(e + f*x)^4/(a + b*sinh(e + f*x)^2)^(3/2),x)
```

output
```
int(cosh(e + f*x)^4/(a + b*sinh(e + f*x)^2)^(3/2), x)
```

Reduce [F]

$$\int \frac{\cosh^4(e+fx)}{\left(a+b\sinh^2(e+fx)\right)^{3/2}}\,dx=\int \frac{\sqrt{\sinh\left(fx+e\right)^2b+a}\,\cosh\left(fx+e\right)^4}{\sinh\left(fx+e\right)^4b^2+2\sinh\left(fx+e\right)^2ab+a^2}dx$$

input
```
int(cosh(f*x+e)^4/(a+b*sinh(f*x+e)^2)^(3/2),x)
```

output
```
int((sqrt(sinh(e + f*x)**2*b + a)*cosh(e + f*x)**4)/(sinh(e + f*x)**4*b**2
 + 2*sinh(e + f*x)**2*a*b + a**2),x)
```

3.352 $\int \frac{\cosh^2(e+fx)}{(a+b\sinh^2(e+fx))^{3/2}}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 91

$$\int \frac{\cosh^2(e+fx)}{(a+b\sinh^2(e+fx))^{3/2}}\,dx = \frac{\cosh(e+fx)E\left(\arctan\left(\frac{\sqrt{b}\sinh(e+fx)}{\sqrt{a}}\right)|1-\frac{a}{b}\right)}{\sqrt{a}\sqrt{b}f\sqrt{\frac{a\cosh^2(e+fx)}{a+b\sinh^2(e+fx)}}\sqrt{a+b\sinh^2(e+fx)}}$$

output
```
cosh(f*x+e)*EllipticE(b^(1/2)*sinh(f*x+e)/a^(1/2)/(1+b*sinh(f*x+e)^2/a)^(1
/2),(1-a/b)^(1/2))/a^(1/2)/b^(1/2)/f/(a*cosh(f*x+e)^2/(a+b*sinh(f*x+e)^2))
^(1/2)/(a+b*sinh(f*x+e)^2)^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.42 (sec) , antiderivative size = 143, normalized size of antiderivative = 1.57

$$\int \frac{\cosh^2(e+fx)}{(a+b\sinh^2(e+fx))^{3/2}}\,dx = \frac{i\sqrt{2}a\sqrt{\frac{2a-b+b\cosh(2(e+fx))}{a}}E\left(i(e+fx)\,|\,\frac{b}{a}\right) - i\sqrt{2}a\sqrt{\frac{2a-b+b\cosh(2(e+fx))}{a}}\text{Ell}}{abf\sqrt{4a-2b+2b\cosh(2(e+fx}}$$

input
```
Integrate[Cosh[e + f*x]^2/(a + b*Sinh[e + f*x]^2)^(3/2),x]
```

output
```
(I*Sqrt[2]*a*Sqrt[(2*a - b + b*Cosh[2*(e + f*x)])/a]*EllipticE[I*(e + f*x)
, b/a] - I*Sqrt[2]*a*Sqrt[(2*a - b + b*Cosh[2*(e + f*x)])/a]*EllipticF[I*(
e + f*x), b/a] + b*Sinh[2*(e + f*x)])/(a*b*f*Sqrt[4*a - 2*b + 2*b*Cosh[2*(
e + f*x)]])
```

Rubi [A] (verified)

Time = 0.31 (sec) , antiderivative size = 119, normalized size of antiderivative = 1.31, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.120$, Rules used = {3042, 3671, 313}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cosh^2(e+fx)}{\left(a+b\sinh^2(e+fx)\right)^{3/2}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\cos(ie+ifx)^2}{\left(a-b\sin(ie+ifx)^2\right)^{3/2}}dx$$

$$\Big\downarrow \text{3671}$$

$$\frac{\sqrt{\cosh^2(e+fx)}\text{sech}(e+fx)\int \frac{\sqrt{\sinh^2(e+fx)+1}}{\left(b\sinh^2(e+fx)+a\right)^{3/2}}d\sinh(e+fx)}{f}$$

$$\Big\downarrow \text{313}$$

$$\frac{\sqrt{\sinh^2(e+fx)+1}\sqrt{\cosh^2(e+fx)}\text{sech}(e+fx)E\left(\arctan\left(\frac{\sqrt{b}\sinh(e+fx)}{\sqrt{a}}\right)|1-\frac{a}{b}\right)}{\sqrt{a}\sqrt{b}f\sqrt{\frac{a\left(\sinh^2(e+fx)+1\right)}{a+b\sinh^2(e+fx)}}\sqrt{a+b\sinh^2(e+fx)}}$$

input
```
Int[Cosh[e + f*x]^2/(a + b*Sinh[e + f*x]^2)^(3/2),x]
```

output
```
(Sqrt[Cosh[e + f*x]^2]*EllipticE[ArcTan[(Sqrt[b]*Sinh[e + f*x])/Sqrt[a]],
1 - a/b]*Sech[e + f*x]*Sqrt[1 + Sinh[e + f*x]^2])/(Sqrt[a]*Sqrt[b]*f*Sqrt[
(a*(1 + Sinh[e + f*x]^2))/(a + b*Sinh[e + f*x]^2)]*Sqrt[a + b*Sinh[e + f*x
]^2])
```

Defintions of rubi rules used

rule 313
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/((c_) + (d_.)*(x_)^2)^(3/2), x_Symbol] :> Sim
p[(Sqrt[a + b*x^2]/(c*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(c
+ d*x^2)))]))*EllipticE[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; FreeQ
[{a, b, c, d}, x] && PosQ[b/a] && PosQ[d/c]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3671
```
Int[cos[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(
p_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Simp[ff*(Sqrt[
Cos[e + f*x]^2]/(f*Cos[e + f*x]))   Subst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a
 + b*ff^2*x^2)^p, x], x, Sin[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x]
 && IntegerQ[m/2] &&  !IntegerQ[p]
```

Maple [A] (verified)

Time = 2.88 (sec) , antiderivative size = 181, normalized size of antiderivative = 1.99

method	result
default	$\frac{\sqrt{-\frac{b}{a}}\,\cosh(fx+e)^2\sinh(fx+e)+\sqrt{\frac{b\cosh(fx+e)^2}{a}+\frac{a-b}{a}}\,\sqrt{\frac{\cosh(2fx+2e)}{2}+\frac{1}{2}}\,\operatorname{EllipticF}\left(\sinh(fx+e)\sqrt{-\frac{b}{a}},\sqrt{\frac{a}{b}}\right)-\sqrt{\frac{b\cosh(fx+e)^2}{a}+}}{\sqrt{-\frac{b}{a}}\,a\cosh(fx+e)\sqrt{a+b\sinh(fx+e)^2}\,f}$

input
```
int(cosh(f*x+e)^2/(a+b*sinh(f*x+e)^2)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
((-b/a)^(1/2)*cosh(f*x+e)^2*sinh(f*x+e)+(b/a*cosh(f*x+e)^2+(a-b)/a)^(1/2)*
(cosh(f*x+e)^2)^(1/2)*EllipticF(sinh(f*x+e)*(-b/a)^(1/2),(1/b*a)^(1/2))-(b
/a*cosh(f*x+e)^2+(a-b)/a)^(1/2)*(cosh(f*x+e)^2)^(1/2)*EllipticE(sinh(f*x+e
)*(-b/a)^(1/2),(1/b*a)^(1/2)))/(-b/a)^(1/2)/a/cosh(f*x+e)/(a+b*sinh(f*x+e)
^2)^(1/2)/f
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1068 vs. $2(97) = 194$.

Time = 0.11 (sec) , antiderivative size = 1068, normalized size of antiderivative = 11.74

$$\int \frac{\cosh^2(e+fx)}{\left(a+b\sinh^2(e+fx)\right)^{3/2}}\,dx = \text{Too large to display}$$

input
```
integrate(cosh(f*x+e)^2/(a+b*sinh(f*x+e)^2)^(3/2),x, algorithm="fricas")
```

output
```
-(4*(b^2*cosh(f*x + e)^4 + 4*b^2*cosh(f*x + e)*sinh(f*x + e)^3 + b^2*sinh(
f*x + e)^4 + 2*(2*a*b - b^2)*cosh(f*x + e)^2 + 2*(3*b^2*cosh(f*x + e)^2 +
2*a*b - b^2)*sinh(f*x + e)^2 + b^2 + 4*(b^2*cosh(f*x + e)^3 + (2*a*b - b^2
)*cosh(f*x + e))*sinh(f*x + e))*sqrt(b)*sqrt((2*b*sqrt((a^2 - a*b)/b^2) -
2*a + b)/b)*sqrt((a^2 - a*b)/b^2)*elliptic_f(arcsin(sqrt((2*b*sqrt((a^2 -
a*b)/b^2) - 2*a + b)/b)*(cosh(f*x + e) + sinh(f*x + e))), (8*a^2 - 8*a*b +
 b^2 + 4*(2*a*b - b^2)*sqrt((a^2 - a*b)/b^2))/b^2) + ((2*a*b - b^2)*cosh(f
*x + e)^4 + 4*(2*a*b - b^2)*cosh(f*x + e)*sinh(f*x + e)^3 + (2*a*b - b^2)*
sinh(f*x + e)^4 + 2*(4*a^2 - 4*a*b + b^2)*cosh(f*x + e)^2 + 2*(3*(2*a*b -
b^2)*cosh(f*x + e)^2 + 4*a^2 - 4*a*b + b^2)*sinh(f*x + e)^2 + 2*a*b - b^2
+ 4*((2*a*b - b^2)*cosh(f*x + e)^3 + (4*a^2 - 4*a*b + b^2)*cosh(f*x + e))*
sinh(f*x + e) - 2*(b^2*cosh(f*x + e)^4 + 4*b^2*cosh(f*x + e)*sinh(f*x + e)
^3 + b^2*sinh(f*x + e)^4 + 2*(2*a*b - b^2)*cosh(f*x + e)^2 + 2*(3*b^2*cosh
(f*x + e)^2 + 2*a*b - b^2)*sinh(f*x + e)^2 + b^2 + 4*(b^2*cosh(f*x + e)^3
+ (2*a*b - b^2)*cosh(f*x + e))*sinh(f*x + e))*sqrt((a^2 - a*b)/b^2))*sqrt(
b)*sqrt((2*b*sqrt((a^2 - a*b)/b^2) - 2*a + b)/b)*elliptic_e(arcsin(sqrt((2
*b*sqrt((a^2 - a*b)/b^2) - 2*a + b)/b)*(cosh(f*x + e) + sinh(f*x + e))), (
8*a^2 - 8*a*b + b^2 + 4*(2*a*b - b^2)*sqrt((a^2 - a*b)/b^2))/b^2) - sqrt(2
)*(b^2*cosh(f*x + e)^3 + 3*b^2*cosh(f*x + e)*sinh(f*x + e)^2 + b^2*sinh(f*
x + e)^3 + (2*a*b - b^2)*cosh(f*x + e) + (3*b^2*cosh(f*x + e)^2 + 2*a*b...
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\cosh^2(e+fx)}{\left(a+b\sinh^2(e+fx)\right)^{3/2}}\,dx = \text{Timed out}$$

input
```
integrate(cosh(f*x+e)**2/(a+b*sinh(f*x+e)**2)**(3/2),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{\cosh^2(e+fx)}{\left(a+b\sinh^2(e+fx)\right)^{3/2}}\,dx = \int \frac{\cosh(fx+e)^2}{\left(b\sinh(fx+e)^2+a\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate(cosh(f*x+e)^2/(a+b*sinh(f*x+e)^2)^(3/2),x, algorithm="maxima")
```

output
```
integrate(cosh(f*x + e)^2/(b*sinh(f*x + e)^2 + a)^(3/2), x)
```

Giac [F]

$$\int \frac{\cosh^2(e+fx)}{\left(a+b\sinh^2(e+fx)\right)^{3/2}}\,dx = \int \frac{\cosh(fx+e)^2}{\left(b\sinh(fx+e)^2+a\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate(cosh(f*x+e)^2/(a+b*sinh(f*x+e)^2)^(3/2),x, algorithm="giac")
```

output
```
integrate(cosh(f*x + e)^2/(b*sinh(f*x + e)^2 + a)^(3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\cosh^2(e+fx)}{\left(a+b\sinh^2(e+fx)\right)^{3/2}}\,dx = \int \frac{\cosh(e+f\,x)^2}{\left(b\sinh\left(e+f\,x\right)^2+a\right)^{3/2}}\,dx$$

input
```
int(cosh(e + f*x)^2/(a + b*sinh(e + f*x)^2)^(3/2),x)
```

output
```
int(cosh(e + f*x)^2/(a + b*sinh(e + f*x)^2)^(3/2), x)
```

Reduce [F]

$$\int \frac{\cosh^2(e+fx)}{\left(a+b\sinh^2(e+fx)\right)^{3/2}}\,dx = \int \frac{\sqrt{\sinh\left(fx+e\right)^2 b+a}\,\cosh\left(fx+e\right)^2}{\sinh\left(fx+e\right)^4 b^2+2\sinh\left(fx+e\right)^2 ab+a^2}dx$$

input
```
int(cosh(f*x+e)^2/(a+b*sinh(f*x+e)^2)^(3/2),x)
```

output
```
int((sqrt(sinh(e + f*x)**2*b + a)*cosh(e + f*x)**2)/(sinh(e + f*x)**4*b**2
 + 2*sinh(e + f*x)**2*a*b + a**2),x)
```

3.353 $\int \frac{1}{\left(a+b\sinh^2(e+fx)\right)^{3/2}}\,dx$

Optimal result

Integrand size = 16, antiderivative size = 116

$$\int \frac{1}{\left(a+b\sinh^2(e+fx)\right)^{3/2}}\,dx = -\frac{b\cosh(e+fx)\sinh(e+fx)}{a(a-b)f\sqrt{a+b\sinh^2(e+fx)}} - \frac{iE\left(ie+ifx\middle|\frac{b}{a}\right)\sqrt{a+b\sinh^2(e+fx)}}{a(a-b)f\sqrt{\frac{a+b\sinh^2(e+fx)}{a}}}$$

output
```
-b*cosh(f*x+e)*sinh(f*x+e)/a/(a-b)/f/(a+b*sinh(f*x+e)^2)^(1/2)-I*EllipticE
(sin(I*e+I*f*x),(b/a)^(1/2))*(a+b*sinh(f*x+e)^2)^(1/2)/a/(a-b)/f/((a+b*sin
h(f*x+e)^2)/a)^(1/2)
```

Mathematica [A] (verified)

Time = 0.10 (sec) , antiderivative size = 100, normalized size of antiderivative = 0.86

$$\int \frac{1}{\left(a+b\sinh^2(e+fx)\right)^{3/2}}\,dx = \frac{-2ia\sqrt{\frac{2a-b+b\cosh(2(e+fx))}{a}}E\left(i(e+fx)\middle|\frac{b}{a}\right)-\sqrt{2}b\sinh(2(e+fx))}{2a(a-b)f\sqrt{2a-b+b\cosh(2(e+fx))}}$$

input
```
Integrate[(a + b*Sinh[e + f*x]^2)^(-3/2),x]
```

output
```
((-2*I)*a*Sqrt[(2*a - b + b*Cosh[2*(e + f*x)])/a]*EllipticE[I*(e + f*x), b
/a] - Sqrt[2]*b*Sinh[2*(e + f*x)])/(2*a*(a - b)*f*Sqrt[2*a - b + b*Cosh[2*
(e + f*x)]])
```

Rubi [A] (verified)

Time = 0.47 (sec) , antiderivative size = 115, normalized size of antiderivative = 0.99, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.438$, Rules used = {3042, 3663, 25, 3042, 3657, 3042, 3656}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\left(a+b\sinh^2(e+fx)\right)^{3/2}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{\left(a-b\sin(ie+ifx)^2\right)^{3/2}}dx$$

$$\Big\downarrow \text{3663}$$

$$-\frac{\int -\sqrt{b\sinh^2(e+fx)+a}dx}{a(a-b)}-\frac{b\sinh(e+fx)\cosh(e+fx)}{af(a-b)\sqrt{a+b\sinh^2(e+fx)}}$$

$$\Big\downarrow \text{25}$$

$$\frac{\int \sqrt{b\sinh^2(e+fx)+a}dx}{a(a-b)}-\frac{b\sinh(e+fx)\cosh(e+fx)}{af(a-b)\sqrt{a+b\sinh^2(e+fx)}}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{b\sinh(e+fx)\cosh(e+fx)}{af(a-b)\sqrt{a+b\sinh^2(e+fx)}}+\frac{\int \sqrt{a-b\sin(ie+ifx)^2}dx}{a(a-b)}$$

$$\Big\downarrow \text{3657}$$

$$\frac{\sqrt{a+b\sinh^2(e+fx)}\int \sqrt{\frac{b\sinh^2(e+fx)}{a}+1}dx}{a(a-b)\sqrt{\frac{b\sinh^2(e+fx)}{a}+1}}-\frac{b\sinh(e+fx)\cosh(e+fx)}{af(a-b)\sqrt{a+b\sinh^2(e+fx)}}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{b\sinh(e+fx)\cosh(e+fx)}{af(a-b)\sqrt{a+b\sinh^2(e+fx)}}+\frac{\sqrt{a+b\sinh^2(e+fx)}\int\sqrt{1-\frac{b\sin(ie+ifx)^2}{a}}dx}{a(a-b)\sqrt{\frac{b\sinh^2(e+fx)}{a}+1}}$$

$$\Big\downarrow \text{3656}$$

$$-\frac{b\sinh(e+fx)\cosh(e+fx)}{af(a-b)\sqrt{a+b\sinh^2(e+fx)}}-\frac{i\sqrt{a+b\sinh^2(e+fx)}E\left(ie+ifx|\frac{b}{a}\right)}{af(a-b)\sqrt{\frac{b\sinh^2(e+fx)}{a}+1}}$$

input
```
Int[(a + b*Sinh[e + f*x]^2)^(-3/2),x]
```

output
```
-((b*Cosh[e + f*x]*Sinh[e + f*x])/(a*(a - b)*f*Sqrt[a + b*Sinh[e + f*x]^2]
)) - (I*EllipticE[I*e + I*f*x, b/a]*Sqrt[a + b*Sinh[e + f*x]^2])/(a*(a - b
)*f*Sqrt[1 + (b*Sinh[e + f*x]^2)/a])
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3656
```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2], x_Symbol] :> Simp[(Sqrt[a
]/f)*EllipticE[e + f*x, -b/a], x] /; FreeQ[{a, b, e, f}, x] && GtQ[a, 0]
```

rule 3657
```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2], x_Symbol] :> Simp[Sqrt[a
+ b*Sin[e + f*x]^2]/Sqrt[1 + b*(Sin[e + f*x]^2/a)]   Int[Sqrt[1 + (b*Sin[e
+ f*x]^2)/a], x], x] /; FreeQ[{a, b, e, f}, x] &&  !GtQ[a, 0]
```

rule 3663
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> Simp[(-b)*C
os[e + f*x]*Sin[e + f*x]*((a + b*Sin[e + f*x]^2)^(p + 1)/(2*a*f*(p + 1)*(a
+ b))), x] + Simp[1/(2*a*(p + 1)*(a + b))   Int[(a + b*Sin[e + f*x]^2)^(p +
 1)*Simp[2*a*(p + 1) + b*(2*p + 3) - 2*b*(p + 2)*Sin[e + f*x]^2, x], x], x]
 /; FreeQ[{a, b, e, f}, x] && NeQ[a + b, 0] && LtQ[p, -1]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 251 vs. $2(110) = 220$.

Time = 1.03 (sec) , antiderivative size = 252, normalized size of antiderivative = 2.17

method	result
default	$\frac{-\sqrt{-\frac{b}{a}}\, b\cosh(fx+e)^2\sinh(fx+e)+a\sqrt{\frac{b\cosh(fx+e)^2}{a}+\frac{a-b}{a}}\,\sqrt{\frac{\cosh(2fx+2e)}{2}+\frac{1}{2}}\,\text{EllipticF}\left(\sinh(fx+e)\sqrt{-\frac{b}{a}},\sqrt{\frac{a}{b}}\right)-\sqrt{\frac{b\cosh(fx+e}{a}}}{a(a-b)\sqrt{-\frac{b}{a}}\,\cos}$

input
```
int(1/(a+b*sinh(f*x+e)^2)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
(-(-b/a)^(1/2)*b*cosh(f*x+e)^2*sinh(f*x+e)+a*(b/a*cosh(f*x+e)^2+(a-b)/a)^(
1/2)*(cosh(f*x+e)^2)^(1/2)*EllipticF(sinh(f*x+e)*(-b/a)^(1/2),(1/b*a)^(1/2
))-(b/a*cosh(f*x+e)^2+(a-b)/a)^(1/2)*(cosh(f*x+e)^2)^(1/2)*EllipticF(sinh(
f*x+e)*(-b/a)^(1/2),(1/b*a)^(1/2))*b+(b/a*cosh(f*x+e)^2+(a-b)/a)^(1/2)*(co
sh(f*x+e)^2)^(1/2)*EllipticE(sinh(f*x+e)*(-b/a)^(1/2),(1/b*a)^(1/2))*b)/a/
(a-b)/(-b/a)^(1/2)/cosh(f*x+e)/(a+b*sinh(f*x+e)^2)^(1/2)/f
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1464 vs. $2(106) = 212$.

Time = 0.14 (sec) , antiderivative size = 1464, normalized size of antiderivative = 12.62

$$\int \frac{1}{\left(a+b\sinh^2(e+fx)\right)^{3/2}}\,dx = \text{Too large to display}$$

input
```
integrate(1/(a+b*sinh(f*x+e)^2)^(3/2),x, algorithm="fricas")
```

output
```
(((2*a*b^2 - b^3)*cosh(f*x + e)^4 + 4*(2*a*b^2 - b^3)*cosh(f*x + e)*sinh(f
*x + e)^3 + (2*a*b^2 - b^3)*sinh(f*x + e)^4 + 2*a*b^2 - b^3 + 2*(4*a^2*b -
 4*a*b^2 + b^3)*cosh(f*x + e)^2 + 2*(4*a^2*b - 4*a*b^2 + b^3 + 3*(2*a*b^2
- b^3)*cosh(f*x + e)^2)*sinh(f*x + e)^2 + 4*((2*a*b^2 - b^3)*cosh(f*x + e)
^3 + (4*a^2*b - 4*a*b^2 + b^3)*cosh(f*x + e))*sinh(f*x + e) - 2*(b^3*cosh(
f*x + e)^4 + 4*b^3*cosh(f*x + e)*sinh(f*x + e)^3 + b^3*sinh(f*x + e)^4 + b
^3 + 2*(2*a*b^2 - b^3)*cosh(f*x + e)^2 + 2*(3*b^3*cosh(f*x + e)^2 + 2*a*b^
2 - b^3)*sinh(f*x + e)^2 + 4*(b^3*cosh(f*x + e)^3 + (2*a*b^2 - b^3)*cosh(f
*x + e))*sinh(f*x + e))*sqrt((a^2 - a*b)/b^2))*sqrt(b)*sqrt((2*b*sqrt((a^2
 - a*b)/b^2) - 2*a + b)/b)*elliptic_e(arcsin(sqrt((2*b*sqrt((a^2 - a*b)/b^
2) - 2*a + b)/b)*(cosh(f*x + e) + sinh(f*x + e))), (8*a^2 - 8*a*b + b^2 +
4*(2*a*b - b^2)*sqrt((a^2 - a*b)/b^2))/b^2) - 2*((2*a^2*b - a*b^2)*cosh(f*
x + e)^4 + 4*(2*a^2*b - a*b^2)*cosh(f*x + e)*sinh(f*x + e)^3 + (2*a^2*b -
a*b^2)*sinh(f*x + e)^4 + 2*a^2*b - a*b^2 + 2*(4*a^3 - 4*a^2*b + a*b^2)*cos
h(f*x + e)^2 + 2*(4*a^3 - 4*a^2*b + a*b^2 + 3*(2*a^2*b - a*b^2)*cosh(f*x +
 e)^2)*sinh(f*x + e)^2 + 4*((2*a^2*b - a*b^2)*cosh(f*x + e)^3 + (4*a^3 - 4
*a^2*b + a*b^2)*cosh(f*x + e))*sinh(f*x + e) + 2*((a*b^2 - b^3)*cosh(f*x +
 e)^4 + 4*(a*b^2 - b^3)*cosh(f*x + e)*sinh(f*x + e)^3 + (a*b^2 - b^3)*sinh
(f*x + e)^4 + a*b^2 - b^3 + 2*(2*a^2*b - 3*a*b^2 + b^3)*cosh(f*x + e)^2 +
2*(2*a^2*b - 3*a*b^2 + b^3 + 3*(a*b^2 - b^3)*cosh(f*x + e)^2)*sinh(f*x ...
```

Sympy [F]

$$\int \frac{1}{\left(a+b\sinh^2(e+fx)\right)^{3/2}}\,dx=\int \frac{1}{\left(a+b\sinh^2{(e+fx)}\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/(a+b*sinh(f*x+e)**2)**(3/2),x)
```

output
```
Integral((a + b*sinh(e + f*x)**2)**(-3/2), x)
```

Maxima [F]

$$\int \frac{1}{\left(a+b\sinh^2(e+fx)\right)^{3/2}}\,dx = \int \frac{1}{\left(b\sinh\left(fx+e\right)^2+a\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/(a+b*sinh(f*x+e)^2)^(3/2),x, algorithm="maxima")
```

output
```
integrate((b*sinh(f*x + e)^2 + a)^(-3/2), x)
```

Giac [F]

$$\int \frac{1}{\left(a+b\sinh^2(e+fx)\right)^{3/2}}\,dx = \int \frac{1}{\left(b\sinh\left(fx+e\right)^2+a\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/(a+b*sinh(f*x+e)^2)^(3/2),x, algorithm="giac")
```

output
```
integrate((b*sinh(f*x + e)^2 + a)^(-3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\left(a+b\sinh^2(e+fx)\right)^{3/2}}\,dx = \int \frac{1}{{\left(b\,{\mathrm{sinh}\left(e+f\,x\right)}^2+a\right)}^{3/2}}\,dx$$

input
```
int(1/(a + b*sinh(e + f*x)^2)^(3/2),x)
```

output
```
int(1/(a + b*sinh(e + f*x)^2)^(3/2), x)
```

Reduce [F]

$$\int \frac{1}{\left(a + b\sinh^2(e + fx)\right)^{3/2}}\,dx = \int \frac{\sqrt{\sinh\left(fx+e\right)^2 b + a}}{\sinh\left(fx+e\right)^4 b^2 + 2\sinh\left(fx+e\right)^2 ab + a^2}dx$$

input
```
int(1/(a+b*sinh(f*x+e)^2)^(3/2),x)
```

output
```
int(sqrt(sinh(e + f*x)**2*b + a)/(sinh(e + f*x)**4*b**2 + 2*sinh(e + f*x)*
*2*a*b + a**2),x)
```

3.354 $\int \frac{\operatorname{sech}^2(e+fx)}{(a+b\sinh^2(e+fx))^{3/2}}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 234

$$\int \frac{\operatorname{sech}^2(e+fx)}{(a+b\sinh^2(e+fx))^{3/2}}\,dx = \frac{\sqrt{b}(a+b)\cosh(e+fx)E\left(\arctan\left(\frac{\sqrt{b}\sinh(e+fx)}{\sqrt{a}}\right)|1-\frac{a}{b}\right)}{\sqrt{a}(a-b)^2f\sqrt{\frac{a\cosh^2(e+fx)}{a+b\sinh^2(e+fx)}}\sqrt{a+b\sinh^2(e+fx)}} - \frac{2\sqrt{a}\sqrt{b}\cosh(e+fx)\operatorname{EllipticF}\left(\arctan\left(\frac{\sqrt{b}\sinh(e+fx)}{\sqrt{a}}\right),1-\frac{a}{b}\right)}{(a-b)^2f\sqrt{\frac{a\cosh^2(e+fx)}{a+b\sinh^2(e+fx)}}\sqrt{a+b\sinh^2(e+fx)}} + \frac{\tanh(e+fx)}{(a-b)f\sqrt{a+b\sinh^2(e+fx)}}$$

output
```
b^(1/2)*(a+b)*cosh(f*x+e)*EllipticE(b^(1/2)*sinh(f*x+e)/a^(1/2)/(1+b*sinh(
f*x+e)^2/a)^(1/2),(1-a/b)^(1/2))/a^(1/2)/(a-b)^2/f/(a*cosh(f*x+e)^2/(a+b*s
inh(f*x+e)^2))^(1/2)/(a+b*sinh(f*x+e)^2)^(1/2)-2*a^(1/2)*b^(1/2)*cosh(f*x+
e)*InverseJacobiAM(arctan(b^(1/2)*sinh(f*x+e)/a^(1/2)),(1-a/b)^(1/2))/(a-b
)^2/f/(a*cosh(f*x+e)^2/(a+b*sinh(f*x+e)^2))^(1/2)/(a+b*sinh(f*x+e)^2)^(1/2
)+tanh(f*x+e)/(a-b)/f/(a+b*sinh(f*x+e)^2)^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 1.08 (sec) , antiderivative size = 178, normalized size of antiderivative = 0.76

$$\int \frac{\operatorname{sech}^2(e+fx)}{\left(a+b\sinh^2(e+fx)\right)^{3/2}}\,dx = \frac{i\sqrt{2}a(a+b)\sqrt{\frac{2a-b+b\cosh(2(e+fx))}{a}}E\left(i(e+fx)\,|\,\frac{b}{a}\right)-i\sqrt{2}a(a-b)\sqrt{\frac{2a-b+b}{}}}{a(a-b)^2f}$$

input
```
Integrate[Sech[e + f*x]^2/(a + b*Sinh[e + f*x]^2)^(3/2),x]
```

output
```
(I*Sqrt[2]*a*(a + b)*Sqrt[(2*a - b + b*Cosh[2*(e + f*x)])/a]*EllipticE[I*(
e + f*x), b/a] - I*Sqrt[2]*a*(a - b)*Sqrt[(2*a - b + b*Cosh[2*(e + f*x)])/
a]*EllipticF[I*(e + f*x), b/a] + (2*a^2 - a*b + b^2 + b*(a + b)*Cosh[2*(e
+ f*x)])*Tanh[e + f*x])/(a*(a - b)^2*f*Sqrt[4*a - 2*b + 2*b*Cosh[2*(e + f*
x)]])
```

Rubi [A] (verified)

Time = 0.42 (sec) , antiderivative size = 277, normalized size of antiderivative = 1.18, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.280$, Rules used = {3042, 3671, 316, 27, 400, 313, 320}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\operatorname{sech}^2(e+fx)}{\left(a+b\sinh^2(e+fx)\right)^{3/2}}dx$$

$$\Big\downarrow \; 3042$$

$$\int \frac{1}{\cos(ie+ifx)^2\left(a-b\sin(ie+ifx)^2\right)^{3/2}}dx$$

$$\Big\downarrow \; 3671$$

$$\frac{\sqrt{\cosh^2(e+fx)}\operatorname{sech}(e+fx)\int \frac{1}{\left(\sinh^2(e+fx)+1\right)^{3/2}\left(b\sinh^2(e+fx)+a\right)^{3/2}}d\sinh(e+fx)}{f}$$

$$\Big\downarrow \; 316$$

$$\frac{\sqrt{\cosh^2(e+fx)}\operatorname{sech}(e+fx)\left(\frac{\sinh(e+fx)}{(a-b)\sqrt{\sinh^2(e+fx)+1}\sqrt{a+b\sinh^2(e+fx)}}-\frac{\int \frac{b\left(1-\sinh^2(e+fx)\right)}{\sqrt{\sinh^2(e+fx)+1}\left(b\sinh^2(e+fx)+a\right)^{3/2}}d\sinh(e+fx)}{a-b}\right)}{f}$$

$$\Big\downarrow \; 27$$

$$\frac{\sqrt{\cosh^2(e+fx)}\operatorname{sech}(e+fx)\left(\frac{\sinh(e+fx)}{(a-b)\sqrt{\sinh^2(e+fx)+1}\sqrt{a+b\sinh^2(e+fx)}}-\frac{b\int \frac{1-\sinh^2(e+fx)}{\sqrt{\sinh^2(e+fx)+1}\left(b\sinh^2(e+fx)+a\right)^{3/2}}d\sinh(e+fx)}{a-b}\right)}{f}$$

$$\Big\downarrow \; 400$$

$$\frac{\sqrt{\cosh^2(e+fx)}\operatorname{sech}(e+fx)\left(\frac{\sinh(e+fx)}{(a-b)\sqrt{\sinh^2(e+fx)+1}\sqrt{a+b\sinh^2(e+fx)}}-\frac{b\left(\frac{2\int \frac{1}{\sqrt{\sinh^2(e+fx)+1}\sqrt{b\sinh^2(e+fx)+a}}d\sinh(e+fx)}{a-b}-\cdots\right)}{a-b}\right)}{f}$$

$$\Big\downarrow \; 313$$

$$\frac{\sqrt{\cosh^2(e+fx)}\operatorname{sech}(e+fx)\left(\frac{\sinh(e+fx)}{(a-b)\sqrt{\sinh^2(e+fx)+1}\sqrt{a+b\sinh^2(e+fx)}}-\frac{b\left(\frac{2\int \frac{1}{\sqrt{\sinh^2(e+fx)+1}\sqrt{b\sinh^2(e+fx)+a}}d\sinh(e+fx)}{a-b}-\cdots\right)}{a-\cdots}\right)}{f}$$

$$\Big\downarrow \; 320$$

$$\frac{\sqrt{\cosh^2(e+fx)}\operatorname{sech}(e+fx)\left(\frac{\sinh(e+fx)}{(a-b)\sqrt{\sinh^2(e+fx)+1}\sqrt{a+b\sinh^2(e+fx)}}-\frac{b\left(\frac{2\sqrt{a+b\sinh^2(e+fx)}\,\text{EllipticF}\left(\arctan(\sinh(e+fx)),1-\frac{b}{a}\right)}{a(a-b)\sqrt{\sinh^2(e+fx)+1}\sqrt{\frac{a+b\sinh^2(e+fx)}{a\left(\sinh^2(e+fx)+1\right)}}}\right.}{a}\right.}{f}$$

input
```
Int[Sech[e + f*x]^2/(a + b*Sinh[e + f*x]^2)^(3/2),x]
```

output
```
(Sqrt[Cosh[e + f*x]^2]*Sech[e + f*x]*(Sinh[e + f*x]/((a - b)*Sqrt[1 + Sinh
[e + f*x]^2]*Sqrt[a + b*Sinh[e + f*x]^2]) - (b*(-(((a + b)*EllipticE[ArcTa
n[(Sqrt[b]*Sinh[e + f*x])/Sqrt[a]], 1 - a/b]*Sqrt[1 + Sinh[e + f*x]^2])/(S
qrt[a]*(a - b)*Sqrt[b]*Sqrt[(a*(1 + Sinh[e + f*x]^2))/(a + b*Sinh[e + f*x]
^2)]*Sqrt[a + b*Sinh[e + f*x]^2])) + (2*EllipticF[ArcTan[Sinh[e + f*x]], 1
 - b/a]*Sqrt[a + b*Sinh[e + f*x]^2])/(a*(a - b)*Sqrt[1 + Sinh[e + f*x]^2]*
Sqrt[(a + b*Sinh[e + f*x]^2)/(a*(1 + Sinh[e + f*x]^2))])))/(a - b)))/f
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 313
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/((c_) + (d_.)*(x_)^2)^(3/2), x_Symbol] :> Sim
p[(Sqrt[a + b*x^2]/(c*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(c
 + d*x^2)))]))*EllipticE[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; FreeQ
[{a, b, c, d}, x] && PosQ[b/a] && PosQ[d/c]
```

rule 316
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_), x_Symbol] :> Sim
p[(-b)*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^(q + 1)/(2*a*(p + 1)*(b*c - a*d))
), x] + Simp[1/(2*a*(p + 1)*(b*c - a*d))   Int[(a + b*x^2)^(p + 1)*(c + d*x
^2)^q*Simp[b*c + 2*(p + 1)*(b*c - a*d) + d*b*(2*(p + q + 2) + 1)*x^2, x], x
], x] /; FreeQ[{a, b, c, d, q}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] &&  !
( !IntegerQ[p] && IntegerQ[q] && LtQ[q, -1]) && IntBinomialQ[a, b, c, d, 2,
 p, q, x]
```

rule 320
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(Sqrt[a + b*x^2]/(a*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(
c + d*x^2)))]))*EllipticF[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; Fre
eQ[{a, b, c, d}, x] && PosQ[d/c] && PosQ[b/a] &&  !SimplerSqrtQ[b/a, d/c]
```

rule 400
```
Int[((e_) + (f_.)*(x_)^2)/(Sqrt[(a_) + (b_.)*(x_)^2]*((c_) + (d_.)*(x_)^2)^
(3/2)), x_Symbol] :> Simp[(b*e - a*f)/(b*c - a*d)   Int[1/(Sqrt[a + b*x^2]*
Sqrt[c + d*x^2]), x], x] - Simp[(d*e - c*f)/(b*c - a*d)   Int[Sqrt[a + b*x^
2]/(c + d*x^2)^(3/2), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && PosQ[b/a] &
& PosQ[d/c]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3671
```
Int[cos[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(
p_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Simp[ff*(Sqrt[
Cos[e + f*x]^2]/(f*Cos[e + f*x]))   Subst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a
 + b*ff^2*x^2)^p, x], x, Sin[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x]
 && IntegerQ[m/2] &&  !IntegerQ[p]
```

Maple [A] (verified)

Time = 4.54 (sec) , antiderivative size = 345, normalized size of antiderivative = 1.47

method	result
default	$-\frac{-\sqrt{-\frac{b}{a}}\, ab\sinh(fx+e)^3-\sqrt{-\frac{b}{a}}\, b^2\sinh(fx+e)^3+a\sqrt{\frac{a+b\sinh(fx+e)^2}{a}}\,\sqrt{\frac{\cosh(2fx+2e)}{2}+\frac{1}{2}}\,\mathrm{EllipticF}\left(\sinh(fx+e)\sqrt{-\frac{b}{a}},\sqrt{\frac{a}{b}}\right)b-}{}$
risch	Expression too large to display

input
```
int(sech(f*x+e)^2/(a+b*sinh(f*x+e)^2)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-(-(-b/a)^(1/2)*a*b*sinh(f*x+e)^3-(-b/a)^(1/2)*b^2*sinh(f*x+e)^3+a*((a+b*s
inh(f*x+e)^2)/a)^(1/2)*(cosh(f*x+e)^2)^(1/2)*EllipticF(sinh(f*x+e)*(-b/a)^
(1/2),(1/b*a)^(1/2))*b-((a+b*sinh(f*x+e)^2)/a)^(1/2)*(cosh(f*x+e)^2)^(1/2)
*EllipticF(sinh(f*x+e)*(-b/a)^(1/2),(1/b*a)^(1/2))*b^2+((a+b*sinh(f*x+e)^2
)/a)^(1/2)*(cosh(f*x+e)^2)^(1/2)*EllipticE(sinh(f*x+e)*(-b/a)^(1/2),(1/b*a
)^(1/2))*a*b+((a+b*sinh(f*x+e)^2)/a)^(1/2)*(cosh(f*x+e)^2)^(1/2)*EllipticE
(sinh(f*x+e)*(-b/a)^(1/2),(1/b*a)^(1/2))*b^2-(-b/a)^(1/2)*a^2*sinh(f*x+e)-
(-b/a)^(1/2)*b^2*sinh(f*x+e))/(a-b)^2/a/(-b/a)^(1/2)/cosh(f*x+e)/(a+b*sinh
(f*x+e)^2)^(1/2)/f
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2786 vs. 2(228) = 456.

Time = 0.15 (sec) , antiderivative size = 2786, normalized size of antiderivative = 11.91

$$\int \frac{\mathrm{sech}^2(e+fx)}{\left(a+b\sinh^2(e+fx)\right)^{3/2}}\,dx = \text{Too large to display}$$

input
```
integrate(sech(f*x+e)^2/(a+b*sinh(f*x+e)^2)^(3/2),x, algorithm="fricas")
```

output
```
-(((2*a^2*b + a*b^2 - b^3)*cosh(f*x + e)^6 + 6*(2*a^2*b + a*b^2 - b^3)*cos
h(f*x + e)*sinh(f*x + e)^5 + (2*a^2*b + a*b^2 - b^3)*sinh(f*x + e)^6 + (8*
a^3 + 2*a^2*b - 5*a*b^2 + b^3)*cosh(f*x + e)^4 + (8*a^3 + 2*a^2*b - 5*a*b^
2 + b^3 + 15*(2*a^2*b + a*b^2 - b^3)*cosh(f*x + e)^2)*sinh(f*x + e)^4 + 4*
(5*(2*a^2*b + a*b^2 - b^3)*cosh(f*x + e)^3 + (8*a^3 + 2*a^2*b - 5*a*b^2 +
b^3)*cosh(f*x + e))*sinh(f*x + e)^3 + 2*a^2*b + a*b^2 - b^3 + (8*a^3 + 2*a
^2*b - 5*a*b^2 + b^3)*cosh(f*x + e)^2 + (15*(2*a^2*b + a*b^2 - b^3)*cosh(f
*x + e)^4 + 8*a^3 + 2*a^2*b - 5*a*b^2 + b^3 + 6*(8*a^3 + 2*a^2*b - 5*a*b^2
 + b^3)*cosh(f*x + e)^2)*sinh(f*x + e)^2 + 2*(3*(2*a^2*b + a*b^2 - b^3)*co
sh(f*x + e)^5 + 2*(8*a^3 + 2*a^2*b - 5*a*b^2 + b^3)*cosh(f*x + e)^3 + (8*a
^3 + 2*a^2*b - 5*a*b^2 + b^3)*cosh(f*x + e))*sinh(f*x + e) - 2*((a*b^2 + b
^3)*cosh(f*x + e)^6 + 6*(a*b^2 + b^3)*cosh(f*x + e)*sinh(f*x + e)^5 + (a*b
^2 + b^3)*sinh(f*x + e)^6 + (4*a^2*b + 3*a*b^2 - b^3)*cosh(f*x + e)^4 + (4
*a^2*b + 3*a*b^2 - b^3 + 15*(a*b^2 + b^3)*cosh(f*x + e)^2)*sinh(f*x + e)^4
 + 4*(5*(a*b^2 + b^3)*cosh(f*x + e)^3 + (4*a^2*b + 3*a*b^2 - b^3)*cosh(f*x
 + e))*sinh(f*x + e)^3 + a*b^2 + b^3 + (4*a^2*b + 3*a*b^2 - b^3)*cosh(f*x
+ e)^2 + (15*(a*b^2 + b^3)*cosh(f*x + e)^4 + 4*a^2*b + 3*a*b^2 - b^3 + 6*(
4*a^2*b + 3*a*b^2 - b^3)*cosh(f*x + e)^2)*sinh(f*x + e)^2 + 2*(3*(a*b^2 +
b^3)*cosh(f*x + e)^5 + 2*(4*a^2*b + 3*a*b^2 - b^3)*cosh(f*x + e)^3 + (4*a^
2*b + 3*a*b^2 - b^3)*cosh(f*x + e))*sinh(f*x + e))*sqrt((a^2 - a*b)/b^2...
```

Sympy [F]

$$\int \frac{\operatorname{sech}^2(e+fx)}{\left(a+b\sinh^2(e+fx)\right)^{3/2}}\,dx=\int \frac{\operatorname{sech}^2\left(e+fx\right)}{\left(a+b\sinh^2\left(e+fx\right)\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate(sech(f*x+e)**2/(a+b*sinh(f*x+e)**2)**(3/2),x)
```

output
```
Integral(sech(e + f*x)**2/(a + b*sinh(e + f*x)**2)**(3/2), x)
```

Maxima [F]

$$\int \frac{\operatorname{sech}^2(e+fx)}{\left(a+b\sinh^2(e+fx)\right)^{3/2}}\,dx = \int \frac{\operatorname{sech}\left(fx+e\right)^2}{\left(b\sinh\left(fx+e\right)^2+a\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate(sech(f*x+e)^2/(a+b*sinh(f*x+e)^2)^(3/2),x, algorithm="maxima")
```

output
```
integrate(sech(f*x + e)^2/(b*sinh(f*x + e)^2 + a)^(3/2), x)
```

Giac [F]

$$\int \frac{\operatorname{sech}^2(e+fx)}{\left(a+b\sinh^2(e+fx)\right)^{3/2}}\,dx = \int \frac{\operatorname{sech}\left(fx+e\right)^2}{\left(b\sinh\left(fx+e\right)^2+a\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate(sech(f*x+e)^2/(a+b*sinh(f*x+e)^2)^(3/2),x, algorithm="giac")
```

output
```
integrate(sech(f*x + e)^2/(b*sinh(f*x + e)^2 + a)^(3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\operatorname{sech}^2(e+fx)}{\left(a+b\sinh^2(e+fx)\right)^{3/2}}\,dx = \int \frac{1}{\cosh\left(e+f\,x\right)^2\,\left(b\sinh\left(e+f\,x\right)^2+a\right)^{3/2}}\,dx$$

input
```
int(1/(cosh(e + f*x)^2*(a + b*sinh(e + f*x)^2)^(3/2)),x)
```

output
```
int(1/(cosh(e + f*x)^2*(a + b*sinh(e + f*x)^2)^(3/2)), x)
```

Reduce [F]

$$\int \frac{\operatorname{sech}^2(e+fx)}{\left(a+b\sinh^2(e+fx)\right)^{3/2}}\,dx = \int \frac{\sqrt{\sinh\left(fx+e\right)^2 b+a}\,\operatorname{sech}(fx+e)^2}{\sinh\left(fx+e\right)^4 b^2+2\sinh\left(fx+e\right)^2 ab+a^2}dx$$

input
```
int(sech(f*x+e)^2/(a+b*sinh(f*x+e)^2)^(3/2),x)
```

output
```
int((sqrt(sinh(e + f*x)**2*b + a)*sech(e + f*x)**2)/(sinh(e + f*x)**4*b**2
 + 2*sinh(e + f*x)**2*a*b + a**2),x)
```

3.355 $\int \frac{\cosh^5(e+fx)}{\left(a+b\sinh^2(e+fx)\right)^{5/2}}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 126

$$\int \frac{\cosh^5(e+fx)}{\left(a+b\sinh^2(e+fx)\right)^{5/2}}\,dx=\frac{\operatorname{arctanh}\left(\frac{\sqrt{b}\sinh(e+fx)}{\sqrt{a+b\sinh^2(e+fx)}}\right)}{b^{5/2}f}+\frac{(a-b)^2\sinh(e+fx)}{3ab^2f\left(a+b\sinh^2(e+fx)\right)^{3/2}}-\frac{2(a-b)(2a+b)\sinh(e+fx)}{3a^2b^2f\sqrt{a+b\sinh^2(e+fx)}}$$

output
```
arctanh(b^(1/2)*sinh(f*x+e)/(a+b*sinh(f*x+e)^2)^(1/2))/b^(5/2)/f+1/3*(a-b)
^2*sinh(f*x+e)/a/b^2/f/(a+b*sinh(f*x+e)^2)^(3/2)-2/3*(a-b)*(2*a+b)*sinh(f*
x+e)/a^2/b^2/f/(a+b*sinh(f*x+e)^2)^(1/2)
```

Mathematica [A] (verified)

Time = 0.62 (sec) , antiderivative size = 126, normalized size of antiderivative = 1.00

$$\int \frac{\cosh^5(e+fx)}{\left(a+b\sinh^2(e+fx)\right)^{5/2}}\,dx=\frac{\frac{\operatorname{arctanh}\left(\frac{\sqrt{2}\sqrt{b}\sinh(e+fx)}{\sqrt{2a-b+b\cosh(2(e+fx))}}\right)}{b^{5/2}}+\frac{2\sqrt{2}(-a+b)\left(3a^2+ab-b^2+b(2a+b)\cosh(2(e+fx))\right)\sinh(e}{3a^2b^2(2a-b+b\cosh(2(e+fx)))^{3/2}}}{f}$$

input
```
Integrate[Cosh[e + f*x]^5/(a + b*Sinh[e + f*x]^2)^(5/2),x]
```

output
```
(ArcTanh[(Sqrt[2]*Sqrt[b]*Sinh[e + f*x])/Sqrt[2*a - b + b*Cosh[2*(e + f*x)
]]]/b^(5/2) + (2*Sqrt[2]*(-a + b)*(3*a^2 + a*b - b^2 + b*(2*a + b)*Cosh[2*
(e + f*x)])*Sinh[e + f*x])/(3*a^2*b^2*(2*a - b + b*Cosh[2*(e + f*x)])^(3/2
)))/f
```

Rubi [A] (verified)

Time = 0.34 (sec) , antiderivative size = 137, normalized size of antiderivative = 1.09, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.240$, Rules used = {3042, 3669, 315, 298, 224, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cosh^5(e+fx)}{\left(a+b\sinh^2(e+fx)\right)^{5/2}}dx$$

$$\Big\downarrow \; 3042$$

$$\int \frac{\cos(ie+ifx)^5}{\left(a-b\sin(ie+ifx)^2\right)^{5/2}}dx$$

$$\Big\downarrow \; 3669$$

$$\frac{\int \frac{\left(\sinh^2(e+fx)+1\right)^2}{\left(b\sinh^2(e+fx)+a\right)^{5/2}}d\sinh(e+fx)}{f}$$

$$\Big\downarrow \; 315$$

$$\frac{\frac{\int \frac{3a\sinh^2(e+fx)+a+2b}{\left(b\sinh^2(e+fx)+a\right)^{3/2}}d\sinh(e+fx)}{3ab}-\frac{(a-b)\sinh(e+fx)\left(\sinh^2(e+fx)+1\right)}{3ab\left(a+b\sinh^2(e+fx)\right)^{3/2}}}{f}$$

$$\Big\downarrow \; 298$$

$$\frac{\frac{\frac{3a\int \frac{1}{\sqrt{b\sinh^2(e+fx)+a}}d\sinh(e+fx)}{b}+\frac{\left(-\frac{3a}{b}+\frac{2b}{a}+1\right)\sinh(e+fx)}{\sqrt{a+b\sinh^2(e+fx)}}}{3ab}-\frac{(a-b)\sinh(e+fx)\left(\sinh^2(e+fx)+1\right)}{3ab\left(a+b\sinh^2(e+fx)\right)^{3/2}}}{f}$$

$$\Big\downarrow \; 224$$

$$\frac{\frac{\frac{3a\int \frac{1}{1-\frac{b\sinh^2(e+fx)}{b\sinh^2(e+fx)+a}}d\frac{\sinh(e+fx)}{\sqrt{b\sinh^2(e+fx)+a}}}{b}+\frac{\left(-\frac{3a}{b}+\frac{2b}{a}+1\right)\sinh(e+fx)}{\sqrt{a+b\sinh^2(e+fx)}}}{3ab}-\frac{(a-b)\sinh(e+fx)\left(\sinh^2(e+fx)+1\right)}{3ab\left(a+b\sinh^2(e+fx)\right)^{3/2}}}{f}$$

$$\Big\downarrow \; 219$$

$$\frac{\frac{\frac{3a\text{arctanh}\left(\frac{\sqrt{b}\sinh(e+fx)}{\sqrt{a+b\sinh^2(e+fx)}}\right)}{b^{3/2}}+\frac{\left(-\frac{3a}{b}+\frac{2b}{a}+1\right)\sinh(e+fx)}{\sqrt{a+b\sinh^2(e+fx)}}}{3ab}-\frac{(a-b)\sinh(e+fx)\left(\sinh^2(e+fx)+1\right)}{3ab\left(a+b\sinh^2(e+fx)\right)^{3/2}}}{f}$$

input
```
Int[Cosh[e + f*x]^5/(a + b*Sinh[e + f*x]^2)^(5/2),x]
```

output
```
(-1/3*((a - b)*Sinh[e + f*x]*(1 + Sinh[e + f*x]^2))/(a*b*(a + b*Sinh[e + f
*x]^2)^(3/2)) + ((3*a*ArcTanh[(Sqrt[b]*Sinh[e + f*x])/Sqrt[a + b*Sinh[e +
f*x]^2]])/b^(3/2) + ((1 - (3*a)/b + (2*b)/a)*Sinh[e + f*x])/Sqrt[a + b*Sin
h[e + f*x]^2])/(3*a*b))/f
```

Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

rule 298
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2), x_Symbol] :> Simp[(-(
b*c - a*d))*x*((a + b*x^2)^(p + 1)/(2*a*b*(p + 1))), x] - Simp[(a*d - b*c*(
2*p + 3))/(2*a*b*(p + 1))   Int[(a + b*x^2)^(p + 1), x], x] /; FreeQ[{a, b,
 c, d, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/2 + p, 0])
```

rule 315
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_), x_Symbol] :> Sim
p[(a*d - c*b)*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^(q - 1)/(2*a*b*(p + 1))),
x] - Simp[1/(2*a*b*(p + 1))   Int[(a + b*x^2)^(p + 1)*(c + d*x^2)^(q - 2)*S
imp[c*(a*d - c*b*(2*p + 3)) + d*(a*d*(2*(q - 1) + 1) - b*c*(2*(p + q) + 1))
*x^2, x], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -
1] && GtQ[q, 1] && IntBinomialQ[a, b, c, d, 2, p, q, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3669
```
Int[cos[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^
(p_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Simp[ff/f   S
ubst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a + b*ff^2*x^2)^p, x], x, Sin[e + f*x]
/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m - 1)/2]
```

Maple [A] (verified)

Time = 0.74 (sec) , antiderivative size = 208, normalized size of antiderivative = 1.65

method	result
derivativedivides	$\frac{\sinh(fx+e)}{3af\left(a+b\sinh(fx+e)^2\right)^{\frac{3}{2}}}+\frac{2\sinh(fx+e)}{3a^2f\sqrt{a+b\sinh(fx+e)^2}}-\frac{\sinh(fx+e)^3}{3fb\left(a+b\sinh(fx+e)^2\right)^{\frac{3}{2}}}-\frac{\sinh(fx+e)}{f\,b^2\sqrt{a+b\sinh(fx+e)^2}}+$
default	$\frac{\sinh(fx+e)}{3af\left(a+b\sinh(fx+e)^2\right)^{\frac{3}{2}}}+\frac{2\sinh(fx+e)}{3a^2f\sqrt{a+b\sinh(fx+e)^2}}-\frac{\sinh(fx+e)^3}{3fb\left(a+b\sinh(fx+e)^2\right)^{\frac{3}{2}}}-\frac{\sinh(fx+e)}{f\,b^2\sqrt{a+b\sinh(fx+e)^2}}+$

input
```
int(cosh(f*x+e)^5/(a+b*sinh(f*x+e)^2)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
1/3*sinh(f*x+e)/a/f/(a+b*sinh(f*x+e)^2)^(3/2)+2/3*sinh(f*x+e)/a^2/f/(a+b*s
inh(f*x+e)^2)^(1/2)-1/3/f*sinh(f*x+e)^3/b/(a+b*sinh(f*x+e)^2)^(3/2)-1/f/b^
2*sinh(f*x+e)/(a+b*sinh(f*x+e)^2)^(1/2)+1/f/b^(5/2)*ln(b^(1/2)*sinh(f*x+e)
+(a+b*sinh(f*x+e)^2)^(1/2))-2/3/f/b*sinh(f*x+e)/(a+b*sinh(f*x+e)^2)^(3/2)+
2/3/f/b/a*sinh(f*x+e)/(a+b*sinh(f*x+e)^2)^(1/2)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2820 vs. $2(112) = 224$.

Time = 0.48 (sec) , antiderivative size = 6662, normalized size of antiderivative = 52.87

$$\int \frac{\cosh^5(e+fx)}{(a+b\sinh^2(e+fx))^{5/2}}\,dx = \text{Too large to display}$$

input
```
integrate(cosh(f*x+e)^5/(a+b*sinh(f*x+e)^2)^(5/2),x, algorithm="fricas")
```

output
```
Too large to include
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\cosh^5(e+fx)}{(a+b\sinh^2(e+fx))^{5/2}}\,dx = \text{Timed out}$$

input
```
integrate(cosh(f*x+e)**5/(a+b*sinh(f*x+e)**2)**(5/2),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{\cosh^5(e+fx)}{(a+b\sinh^2(e+fx))^{5/2}}\,dx = \int \frac{\cosh(fx+e)^5}{(b\sinh(fx+e)^2+a)^{\frac{5}{2}}}\,dx$$

input
```
integrate(cosh(f*x+e)^5/(a+b*sinh(f*x+e)^2)^(5/2),x, algorithm="maxima")
```

output
```
integrate(cosh(f*x + e)^5/(b*sinh(f*x + e)^2 + a)^(5/2), x)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{\cosh^5(e+fx)}{\left(a+b\sinh^2(e+fx)\right)^{5/2}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(cosh(f*x+e)^5/(a+b*sinh(f*x+e)^2)^(5/2),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Degree mismatch inside factorisatio
n over extensionNot implemented, e.g. for multivariate mod/approx polynomi
alsError:
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\cosh^5(e+fx)}{\left(a+b\sinh^2(e+fx)\right)^{5/2}}\,dx = \int \frac{\cosh(e+f\,x)^5}{\left(b\sinh\left(e+f\,x\right)^2+a\right)^{5/2}}\,dx$$

input
```
int(cosh(e + f*x)^5/(a + b*sinh(e + f*x)^2)^(5/2),x)
```

output
```
int(cosh(e + f*x)^5/(a + b*sinh(e + f*x)^2)^(5/2), x)
```

Reduce [F]

$$\int \frac{\cosh^5(e+fx)}{\left(a+b\sinh^2(e+fx)\right)^{5/2}}\,dx = \int \frac{\sqrt{\sinh\left(fx+e\right)^2 b+a}\,\cosh\left(fx+e\right)^5}{\sinh\left(fx+e\right)^6 b^3+3\sinh\left(fx+e\right)^4 a\,b^2+3\sinh\left(fx+e\right)^2 a^2b+a^3}\,dx$$

input
```
int(cosh(f*x+e)^5/(a+b*sinh(f*x+e)^2)^(5/2),x)
```

output
```
int((sqrt(sinh(e + f*x)**2*b + a)*cosh(e + f*x)**5)/(sinh(e + f*x)**6*b**3
 + 3*sinh(e + f*x)**4*a*b**2 + 3*sinh(e + f*x)**2*a**2*b + a**3),x)
```

3.356 $\int \frac{\cosh^3(e+fx)}{\left(a+b\sinh^2(e+fx)\right)^{5/2}}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 81

$$\int \frac{\cosh^3(e+fx)}{\left(a+b\sinh^2(e+fx)\right)^{5/2}}\,dx = -\frac{(a-b)\sinh(e+fx)}{3abf\left(a+b\sinh^2(e+fx)\right)^{3/2}}+\frac{(a+2b)\sinh(e+fx)}{3a^2bf\sqrt{a+b\sinh^2(e+fx)}}$$

output
```
-1/3*(a-b)*sinh(f*x+e)/a/b/f/(a+b*sinh(f*x+e)^2)^(3/2)+1/3*(a+2*b)*sinh(f*
x+e)/a^2/b/f/(a+b*sinh(f*x+e)^2)^(1/2)
```

Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 50, normalized size of antiderivative = 0.62

$$\int \frac{\cosh^3(e+fx)}{\left(a+b\sinh^2(e+fx)\right)^{5/2}}\,dx = \frac{3a\sinh(e+fx)+(a+2b)\sinh^3(e+fx)}{3a^2f\left(a+b\sinh^2(e+fx)\right)^{3/2}}$$

input
```
Integrate[Cosh[e + f*x]^3/(a + b*Sinh[e + f*x]^2)^(5/2),x]
```

output
```
(3*a*Sinh[e + f*x] + (a + 2*b)*Sinh[e + f*x]^3)/(3*a^2*f*(a + b*Sinh[e + f
*x]^2)^(3/2))
```

Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 73, normalized size of antiderivative = 0.90, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.160, Rules used = {3042, 3669, 292, 208}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cosh^3(e+fx)}{\left(a+b\sinh^2(e+fx)\right)^{5/2}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\cos(ie+ifx)^3}{\left(a-b\sin(ie+ifx)^2\right)^{5/2}}dx$$

$$\Big\downarrow \text{3669}$$

$$\frac{\int \frac{\sinh^2(e+fx)+1}{\left(b\sinh^2(e+fx)+a\right)^{5/2}}d\sinh(e+fx)}{f}$$

$$\Big\downarrow \text{292}$$

$$\frac{\frac{2\int \frac{1}{\left(b\sinh^2(e+fx)+a\right)^{3/2}}d\sinh(e+fx)}{3a}+\frac{\sinh(e+fx)\left(\sinh^2(e+fx)+1\right)}{3a\left(a+b\sinh^2(e+fx)\right)^{3/2}}}{f}$$

$$\Big\downarrow \text{208}$$

$$\frac{\frac{2\sinh(e+fx)}{3a^2\sqrt{a+b\sinh^2(e+fx)}}+\frac{\left(\sinh^2(e+fx)+1\right)\sinh(e+fx)}{3a\left(a+b\sinh^2(e+fx)\right)^{3/2}}}{f}$$

input
```
Int[Cosh[e + f*x]^3/(a + b*Sinh[e + f*x]^2)^(5/2),x]
```

output
```
((Sinh[e + f*x]*(1 + Sinh[e + f*x]^2))/(3*a*(a + b*Sinh[e + f*x]^2)^(3/2))
 + (2*Sinh[e + f*x])/(3*a^2*Sqrt[a + b*Sinh[e + f*x]^2]))/f
```

Defintions of rubi rules used

rule 208
```
Int[((a_) + (b_.)*(x_)^2)^(-3/2), x_Symbol] :> Simp[x/(a*Sqrt[a + b*x^2]),
x] /; FreeQ[{a, b}, x]
```

rule 292
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_.), x_Symbol] :> Si
mp[(-x)*(a + b*x^2)^(p + 1)*((c + d*x^2)^q/(2*a*(p + 1))), x] - Simp[c*(q/(
a*(p + 1)))   Int[(a + b*x^2)^(p + 1)*(c + d*x^2)^(q - 1), x], x] /; FreeQ[
{a, b, c, d, p}, x] && NeQ[b*c - a*d, 0] && EqQ[2*(p + q + 1) + 1, 0] && Gt
Q[q, 0] && NeQ[p, -1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3669
```
Int[cos[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^
(p_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Simp[ff/f   S
ubst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a + b*ff^2*x^2)^p, x], x, Sin[e + f*x]
/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m - 1)/2]
```

Maple [A] (verified)

Time = 0.44 (sec) , antiderivative size = 138, normalized size of antiderivative = 1.70

method	result
derivativedivides	$\frac{\frac{\sinh(fx+e)}{3a\left(a+b\sinh(fx+e)^2\right)^{\frac{3}{2}}}+\frac{2\sinh(fx+e)}{3a^2\sqrt{a+b\sinh(fx+e)^2}}-\frac{\sinh(fx+e)}{2b\left(a+b\sinh(fx+e)^2\right)^{\frac{3}{2}}}+\frac{a\left(\frac{\sinh(fx+e)}{3a\left(a+b\sinh(fx+e)^2\right)^{\frac{3}{2}}}+\frac{2\sinh(fx+e)}{3a^2\sqrt{a+b\sinh(fx+e)^2}}\right)}{2b}}{f}$
default	$\frac{\frac{\sinh(fx+e)}{3a\left(a+b\sinh(fx+e)^2\right)^{\frac{3}{2}}}+\frac{2\sinh(fx+e)}{3a^2\sqrt{a+b\sinh(fx+e)^2}}-\frac{\sinh(fx+e)}{2b\left(a+b\sinh(fx+e)^2\right)^{\frac{3}{2}}}+\frac{a\left(\frac{\sinh(fx+e)}{3a\left(a+b\sinh(fx+e)^2\right)^{\frac{3}{2}}}+\frac{2\sinh(fx+e)}{3a^2\sqrt{a+b\sinh(fx+e)^2}}\right)}{2b}}{f}$

input
```
int(cosh(f*x+e)^3/(a+b*sinh(f*x+e)^2)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
1/f*(1/3*sinh(f*x+e)/a/(a+b*sinh(f*x+e)^2)^(3/2)+2/3/a^2*sinh(f*x+e)/(a+b*
sinh(f*x+e)^2)^(1/2)-1/2*sinh(f*x+e)/b/(a+b*sinh(f*x+e)^2)^(3/2)+1/2*a/b*(
1/3*sinh(f*x+e)/a/(a+b*sinh(f*x+e)^2)^(3/2)+2/3/a^2*sinh(f*x+e)/(a+b*sinh(
f*x+e)^2)^(1/2)))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 945 vs. $2(73) = 146$.

Time = 0.21 (sec) , antiderivative size = 945, normalized size of antiderivative = 11.67

$$\int \frac{\cosh^3(e+fx)}{\left(a+b\sinh^2(e+fx)\right)^{5/2}}\,dx = \text{Too large to display}$$

input
```
integrate(cosh(f*x+e)^3/(a+b*sinh(f*x+e)^2)^(5/2),x, algorithm="fricas")
```

output
```
1/3*sqrt(2)*((a + 2*b)*cosh(f*x + e)^6 + 6*(a + 2*b)*cosh(f*x + e)*sinh(f*
x + e)^5 + (a + 2*b)*sinh(f*x + e)^6 + 3*(3*a - 2*b)*cosh(f*x + e)^4 + 3*(
5*(a + 2*b)*cosh(f*x + e)^2 + 3*a - 2*b)*sinh(f*x + e)^4 + 4*(5*(a + 2*b)*
cosh(f*x + e)^3 + 3*(3*a - 2*b)*cosh(f*x + e))*sinh(f*x + e)^3 - 3*(3*a -
2*b)*cosh(f*x + e)^2 + 3*(5*(a + 2*b)*cosh(f*x + e)^4 + 6*(3*a - 2*b)*cosh
(f*x + e)^2 - 3*a + 2*b)*sinh(f*x + e)^2 + 6*((a + 2*b)*cosh(f*x + e)^5 +
2*(3*a - 2*b)*cosh(f*x + e)^3 - (3*a - 2*b)*cosh(f*x + e))*sinh(f*x + e) -
 a - 2*b)*sqrt((b*cosh(f*x + e)^2 + b*sinh(f*x + e)^2 + 2*a - b)/(cosh(f*x
 + e)^2 - 2*cosh(f*x + e)*sinh(f*x + e) + sinh(f*x + e)^2))/(a^2*b^2*f*cos
h(f*x + e)^8 + 8*a^2*b^2*f*cosh(f*x + e)*sinh(f*x + e)^7 + a^2*b^2*f*sinh(
f*x + e)^8 + 4*(2*a^3*b - a^2*b^2)*f*cosh(f*x + e)^6 + 4*(7*a^2*b^2*f*cosh
(f*x + e)^2 + (2*a^3*b - a^2*b^2)*f)*sinh(f*x + e)^6 + 2*(8*a^4 - 8*a^3*b
+ 3*a^2*b^2)*f*cosh(f*x + e)^4 + 8*(7*a^2*b^2*f*cosh(f*x + e)^3 + 3*(2*a^3
*b - a^2*b^2)*f*cosh(f*x + e))*sinh(f*x + e)^5 + a^2*b^2*f + 2*(35*a^2*b^2
*f*cosh(f*x + e)^4 + 30*(2*a^3*b - a^2*b^2)*f*cosh(f*x + e)^2 + (8*a^4 - 8
*a^3*b + 3*a^2*b^2)*f)*sinh(f*x + e)^4 + 4*(2*a^3*b - a^2*b^2)*f*cosh(f*x
+ e)^2 + 8*(7*a^2*b^2*f*cosh(f*x + e)^5 + 10*(2*a^3*b - a^2*b^2)*f*cosh(f*
x + e)^3 + (8*a^4 - 8*a^3*b + 3*a^2*b^2)*f*cosh(f*x + e))*sinh(f*x + e)^3
+ 4*(7*a^2*b^2*f*cosh(f*x + e)^6 + 15*(2*a^3*b - a^2*b^2)*f*cosh(f*x + e)^
4 + 3*(8*a^4 - 8*a^3*b + 3*a^2*b^2)*f*cosh(f*x + e)^2 + (2*a^3*b - a^2*...
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\cosh^3(e+fx)}{\left(a+b\sinh^2(e+fx)\right)^{5/2}}\,dx = \text{Timed out}$$

input
```
integrate(cosh(f*x+e)**3/(a+b*sinh(f*x+e)**2)**(5/2),x)
```

output
```
Timed out
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 927 vs. $2(73) = 146$.

Time = 0.17 (sec) , antiderivative size = 927, normalized size of antiderivative = 11.44

$$\int \frac{\cosh^3(e+fx)}{\left(a+b\sinh^2(e+fx)\right)^{5/2}}\,dx = \text{Too large to display}$$

input
```
integrate(cosh(f*x+e)^3/(a+b*sinh(f*x+e)^2)^(5/2),x, algorithm="maxima")
```

output
```
-1/12*(b^4*e^(-10*f*x - 10*e) - 4*a^3*b + 6*a^2*b^2 - b^4 - (16*a^4 - 32*a
^3*b + 6*a^2*b^2 + 10*a*b^3 - 5*b^4)*e^(-2*f*x - 2*e) + 10*(2*a^3*b - 3*a^
2*b^2 + 3*a*b^3 - b^4)*e^(-4*f*x - 4*e) + 10*(3*a^2*b^2 - 3*a*b^3 + b^4)*e
^(-6*f*x - 6*e) + 5*(2*a*b^3 - b^4)*e^(-8*f*x - 8*e))/((a^4 - 2*a^3*b + a^
2*b^2)*(2*(2*a - b)*e^(-2*f*x - 2*e) + b*e^(-4*f*x - 4*e) + b)^(5/2)*f) +
1/4*(2*a^2*b^2 - 2*a*b^3 + b^4 + 5*(4*a^3*b - 6*a^2*b^2 + 4*a*b^3 - b^4)*e
^(-2*f*x - 2*e) + 2*(24*a^4 - 48*a^3*b + 49*a^2*b^2 - 25*a*b^3 + 5*b^4)*e^
(-4*f*x - 4*e) + 10*(6*a^3*b - 9*a^2*b^2 + 5*a*b^3 - b^4)*e^(-6*f*x - 6*e)
 + 5*(4*a^2*b^2 - 4*a*b^3 + b^4)*e^(-8*f*x - 8*e) + (2*a*b^3 - b^4)*e^(-10
*f*x - 10*e))/((a^4 - 2*a^3*b + a^2*b^2)*(2*(2*a - b)*e^(-2*f*x - 2*e) + b
*e^(-4*f*x - 4*e) + b)^(5/2)*f) - 1/4*(2*a*b^3 - b^4 + 5*(4*a^2*b^2 - 4*a*
b^3 + b^4)*e^(-2*f*x - 2*e) + 10*(6*a^3*b - 9*a^2*b^2 + 5*a*b^3 - b^4)*e^(
-4*f*x - 4*e) + 2*(24*a^4 - 48*a^3*b + 49*a^2*b^2 - 25*a*b^3 + 5*b^4)*e^(-
6*f*x - 6*e) + 5*(4*a^3*b - 6*a^2*b^2 + 4*a*b^3 - b^4)*e^(-8*f*x - 8*e) +
(2*a^2*b^2 - 2*a*b^3 + b^4)*e^(-10*f*x - 10*e))/((a^4 - 2*a^3*b + a^2*b^2)
*(2*(2*a - b)*e^(-2*f*x - 2*e) + b*e^(-4*f*x - 4*e) + b)^(5/2)*f) + 1/12*(
b^4 + 5*(2*a*b^3 - b^4)*e^(-2*f*x - 2*e) + 10*(3*a^2*b^2 - 3*a*b^3 + b^4)*
e^(-4*f*x - 4*e) + 10*(2*a^3*b - 3*a^2*b^2 + 3*a*b^3 - b^4)*e^(-6*f*x - 6*
e) - (16*a^4 - 32*a^3*b + 6*a^2*b^2 + 10*a*b^3 - 5*b^4)*e^(-8*f*x - 8*e) -
 (4*a^3*b - 6*a^2*b^2 + b^4)*e^(-10*f*x - 10*e))/((a^4 - 2*a^3*b + a^2*...
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 408 vs. $2(73) = 146$.

Time = 0.35 (sec) , antiderivative size = 408, normalized size of antiderivative = 5.04

$$\int \frac{\cosh^3(e+fx)}{\left(a+b\sinh^2(e+fx)\right)^{5/2}}\,dx = \frac{\left(\left(\frac{\left(a^3f^3e^{(12\,e)}-3\,ab^2f^3e^{(12\,e)}+2\,b^3f^3e^{(12\,e)}\right)e^{(2\,fx)}}{a^4f^4e^{(6\,e)}-2\,a^3bf^4e^{(6\,e)}+a^2b^2f^4e^{(6\,e)}}+\frac{3\left(3\,a^3f^3e^{(10\,e)}-8\,a^2bf^3e^{(10\,e)}+7\,ab^2f^3e\cdots\right.}{a^4f^4e^{(6\,e)}-2\,a^3bf^4e^{(6\,e)}+a^2b\cdots}\cdots\right.\right.}{3\left(be^{(4\,f\cdots}\right.}$$

input
```
integrate(cosh(f*x+e)^3/(a+b*sinh(f*x+e)^2)^(5/2),x, algorithm="giac")
```

output
```
1/3*((((a^3*f^3*e^(12*e) - 3*a*b^2*f^3*e^(12*e) + 2*b^3*f^3*e^(12*e))*e^(2
*f*x)/(a^4*f^4*e^(6*e) - 2*a^3*b*f^4*e^(6*e) + a^2*b^2*f^4*e^(6*e)) + 3*(3
*a^3*f^3*e^(10*e) - 8*a^2*b*f^3*e^(10*e) + 7*a*b^2*f^3*e^(10*e) - 2*b^3*f^
3*e^(10*e))/(a^4*f^4*e^(6*e) - 2*a^3*b*f^4*e^(6*e) + a^2*b^2*f^4*e^(6*e)))
*e^(2*f*x) - 3*(3*a^3*f^3*e^(8*e) - 8*a^2*b*f^3*e^(8*e) + 7*a*b^2*f^3*e^(8
*e) - 2*b^3*f^3*e^(8*e))/(a^4*f^4*e^(6*e) - 2*a^3*b*f^4*e^(6*e) + a^2*b^2*
f^4*e^(6*e)))*e^(2*f*x) - (a^3*f^3*e^(6*e) - 3*a*b^2*f^3*e^(6*e) + 2*b^3*f
^3*e^(6*e))/(a^4*f^4*e^(6*e) - 2*a^3*b*f^4*e^(6*e) + a^2*b^2*f^4*e^(6*e)))
/(b*e^(4*f*x + 4*e) + 4*a*e^(2*f*x + 2*e) - 2*b*e^(2*f*x + 2*e) + b)^(3/2)
```

Mupad [B] (verification not implemented)

Time = 2.75 (sec) , antiderivative size = 144, normalized size of antiderivative = 1.78

$$\int \frac{\cosh^3(e+fx)}{\left(a+b\sinh^2(e+fx)\right)^{5/2}}\,dx = \frac{2\,e^{e+f\,x}\left(e^{2\,e+2\,f\,x}-1\right)\sqrt{a+b\left(\frac{e^{e+f\,x}}{2}-\frac{e^{-e-f\,x}}{2}\right)^2}\left(a+2\,b+10\,a\,e^{2\,e+2\,f\,x}\cdots\right.}{3\,a^2\,f\left(b+4\,a\,e^{2\,e+2\,f\,x}-2\,b\,e^{2\,e+2\,f\,x}+b\,e\cdots\right.}$$

input
```
int(cosh(e + f*x)^3/(a + b*sinh(e + f*x)^2)^(5/2),x)
```

output
```
(2*exp(e + f*x)*(exp(2*e + 2*f*x) - 1)*(a + b*(exp(e + f*x)/2 - exp(- e -
f*x)/2)^2)^(1/2)*(a + 2*b + 10*a*exp(2*e + 2*f*x) + a*exp(4*e + 4*f*x) - 4
*b*exp(2*e + 2*f*x) + 2*b*exp(4*e + 4*f*x)))/(3*a^2*f*(b + 4*a*exp(2*e + 2
*f*x) - 2*b*exp(2*e + 2*f*x) + b*exp(4*e + 4*f*x))^2)
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 110, normalized size of antiderivative = 1.36

$$\int \frac{\cosh^3(e+fx)}{\left(a+b\sinh^2(e+fx)\right)^{5/2}}\,dx = \frac{\sqrt{\sinh\left(fx+e\right)^2 b+a}\,\sinh\left(fx+e\right)\left(2\cosh\left(fx+e\right)^2\sinh\left(fx+e\right)^2 b+\right.}{3a^2 f\left(\sinh\left(fx+e\right)^4 b^2+2\sinh\right.}$$

input
```
int(cosh(f*x+e)^3/(a+b*sinh(f*x+e)^2)^(5/2),x)
```

output
```
(sqrt(sinh(e + f*x)**2*b + a)*sinh(e + f*x)*(2*cosh(e + f*x)**2*sinh(e + f
*x)**2*b + 3*cosh(e + f*x)**2*a - 2*sinh(e + f*x)**4*b - 2*sinh(e + f*x)**
2*a))/(3*a**2*f*(sinh(e + f*x)**4*b**2 + 2*sinh(e + f*x)**2*a*b + a**2))
```

3.357 $\int \frac{\cosh(e+fx)}{\left(a+b\sinh^2(e+fx)\right)^{5/2}}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 65

$$\int \frac{\cosh(e+fx)}{\left(a+b\sinh^2(e+fx)\right)^{5/2}}\,dx=\frac{\sinh(e+fx)}{3af\left(a+b\sinh^2(e+fx)\right)^{3/2}}+\frac{2\sinh(e+fx)}{3a^2f\sqrt{a+b\sinh^2(e+fx)}}$$

output
```
1/3*sinh(f*x+e)/a/f/(a+b*sinh(f*x+e)^2)^(3/2)+2/3*sinh(f*x+e)/a^2/f/(a+b*s
inh(f*x+e)^2)^(1/2)
```

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.72

$$\int \frac{\cosh(e+fx)}{\left(a+b\sinh^2(e+fx)\right)^{5/2}}\,dx=\frac{\sinh(e+fx)\left(3a+2b\sinh^2(e+fx)\right)}{3a^2f\left(a+b\sinh^2(e+fx)\right)^{3/2}}$$

input
```
Integrate[Cosh[e + f*x]/(a + b*Sinh[e + f*x]^2)^(5/2),x]
```

output
```
(Sinh[e + f*x]*(3*a + 2*b*Sinh[e + f*x]^2))/(3*a^2*f*(a + b*Sinh[e + f*x]^
2)^(3/2))
```

Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 63, normalized size of antiderivative = 0.97, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.174$, Rules used = {3042, 3669, 209, 208}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cosh(e+fx)}{\left(a+b\sinh^2(e+fx)\right)^{5/2}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\cos(ie+ifx)}{(a-b\sin(ie+ifx)^2)^{5/2}}dx$$

$$\Big\downarrow \text{3669}$$

$$\frac{\int \frac{1}{\left(b\sinh^2(e+fx)+a\right)^{5/2}}d\sinh(e+fx)}{f}$$

$$\Big\downarrow \text{209}$$

$$\frac{\frac{2\int \frac{1}{\left(b\sinh^2(e+fx)+a\right)^{3/2}}d\sinh(e+fx)}{3a}+\frac{\sinh(e+fx)}{3a\left(a+b\sinh^2(e+fx)\right)^{3/2}}}{f}$$

$$\Big\downarrow \text{208}$$

$$\frac{\frac{2\sinh(e+fx)}{3a^2\sqrt{a+b\sinh^2(e+fx)}}+\frac{\sinh(e+fx)}{3a\left(a+b\sinh^2(e+fx)\right)^{3/2}}}{f}$$

input
```
Int[Cosh[e + f*x]/(a + b*Sinh[e + f*x]^2)^(5/2),x]
```

output
```
(Sinh[e + f*x]/(3*a*(a + b*Sinh[e + f*x]^2)^(3/2)) + (2*Sinh[e + f*x])/(3*
a^2*Sqrt[a + b*Sinh[e + f*x]^2]))/f
```

Defintions of rubi rules used

rule 208
```
Int[((a_) + (b_.)*(x_)^2)^(-3/2), x_Symbol] :> Simp[x/(a*Sqrt[a + b*x^2]),
x] /; FreeQ[{a, b}, x]
```

rule 209
```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(-x)*((a + b*x^2)^(p + 1)
/(2*a*(p + 1))), x] + Simp[(2*p + 3)/(2*a*(p + 1))   Int[(a + b*x^2)^(p + 1
), x], x] /; FreeQ[{a, b}, x] && ILtQ[p + 3/2, 0]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3669
```
Int[cos[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^
(p_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Simp[ff/f   S
ubst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a + b*ff^2*x^2)^p, x], x, Sin[e + f*x]
/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m - 1)/2]
```

Maple [A] (verified)

Time = 2.11 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.86

method	result	size
derivativedivides	$\frac{\frac{\sinh(fx+e)}{3a\left(a+b\sinh(fx+e)^2\right)^{\frac{3}{2}}}+\frac{2\sinh(fx+e)}{3a^2\sqrt{a+b\sinh(fx+e)^2}}}{f}$	56
default	$\frac{\frac{\sinh(fx+e)}{3a\left(a+b\sinh(fx+e)^2\right)^{\frac{3}{2}}}+\frac{2\sinh(fx+e)}{3a^2\sqrt{a+b\sinh(fx+e)^2}}}{f}$	56
risch	Expression too large to display	405990

input
```
int(cosh(f*x+e)/(a+b*sinh(f*x+e)^2)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
1/f*(1/3*sinh(f*x+e)/a/(a+b*sinh(f*x+e)^2)^(3/2)+2/3/a^2*sinh(f*x+e)/(a+b*
sinh(f*x+e)^2)^(1/2))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 912 vs. $2(57) = 114$.

Time = 0.20 (sec) , antiderivative size = 912, normalized size of antiderivative = 14.03

$$\int \frac{\cosh(e + fx)}{\left(a + b\sinh^2(e + fx)\right)^{5/2}}\,dx = \text{Too large to display}$$

input
```
integrate(cosh(f*x+e)/(a+b*sinh(f*x+e)^2)^(5/2),x, algorithm="fricas")
```

output
```
2/3*sqrt(2)*(b*cosh(f*x + e)^6 + 6*b*cosh(f*x + e)*sinh(f*x + e)^5 + b*sin
h(f*x + e)^6 + 3*(2*a - b)*cosh(f*x + e)^4 + 3*(5*b*cosh(f*x + e)^2 + 2*a
- b)*sinh(f*x + e)^4 + 4*(5*b*cosh(f*x + e)^3 + 3*(2*a - b)*cosh(f*x + e))
*sinh(f*x + e)^3 - 3*(2*a - b)*cosh(f*x + e)^2 + 3*(5*b*cosh(f*x + e)^4 +
6*(2*a - b)*cosh(f*x + e)^2 - 2*a + b)*sinh(f*x + e)^2 + 6*(b*cosh(f*x + e
)^5 + 2*(2*a - b)*cosh(f*x + e)^3 - (2*a - b)*cosh(f*x + e))*sinh(f*x + e)
 - b)*sqrt((b*cosh(f*x + e)^2 + b*sinh(f*x + e)^2 + 2*a - b)/(cosh(f*x + e
)^2 - 2*cosh(f*x + e)*sinh(f*x + e) + sinh(f*x + e)^2))/(a^2*b^2*f*cosh(f*
x + e)^8 + 8*a^2*b^2*f*cosh(f*x + e)*sinh(f*x + e)^7 + a^2*b^2*f*sinh(f*x
+ e)^8 + 4*(2*a^3*b - a^2*b^2)*f*cosh(f*x + e)^6 + 4*(7*a^2*b^2*f*cosh(f*x
 + e)^2 + (2*a^3*b - a^2*b^2)*f)*sinh(f*x + e)^6 + 2*(8*a^4 - 8*a^3*b + 3*
a^2*b^2)*f*cosh(f*x + e)^4 + 8*(7*a^2*b^2*f*cosh(f*x + e)^3 + 3*(2*a^3*b -
 a^2*b^2)*f*cosh(f*x + e))*sinh(f*x + e)^5 + a^2*b^2*f + 2*(35*a^2*b^2*f*c
osh(f*x + e)^4 + 30*(2*a^3*b - a^2*b^2)*f*cosh(f*x + e)^2 + (8*a^4 - 8*a^3
*b + 3*a^2*b^2)*f)*sinh(f*x + e)^4 + 4*(2*a^3*b - a^2*b^2)*f*cosh(f*x + e)
^2 + 8*(7*a^2*b^2*f*cosh(f*x + e)^5 + 10*(2*a^3*b - a^2*b^2)*f*cosh(f*x +
e)^3 + (8*a^4 - 8*a^3*b + 3*a^2*b^2)*f*cosh(f*x + e))*sinh(f*x + e)^3 + 4*
(7*a^2*b^2*f*cosh(f*x + e)^6 + 15*(2*a^3*b - a^2*b^2)*f*cosh(f*x + e)^4 +
3*(8*a^4 - 8*a^3*b + 3*a^2*b^2)*f*cosh(f*x + e)^2 + (2*a^3*b - a^2*b^2)*f)
*sinh(f*x + e)^2 + 8*(a^2*b^2*f*cosh(f*x + e)^7 + 3*(2*a^3*b - a^2*b^2)...
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\cosh(e+fx)}{\left(a+b\sinh^2(e+fx)\right)^{5/2}}\,dx = \text{Timed out}$$

input
```
integrate(cosh(f*x+e)/(a+b*sinh(f*x+e)**2)**(5/2),x)
```

output
```
Timed out
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 485 vs. 2(57) = 114.

Time = 0.14 (sec) , antiderivative size = 485, normalized size of antiderivative = 7.46

$$\int \frac{\cosh(e+fx)}{\left(a+b\sinh^2(e+fx)\right)^{5/2}}\,dx = \frac{2\,a^2b^2 - 2\,ab^3 + b^4 + 5\,(4\,a^3b - 6\,a^2b^2 + 4\,ab^3 - b^4)e^{(-2\,fx-2\,e)} + 2\,(24\,a^4 -}{}$$
$$-\frac{2\,ab^3 - b^4 + 5\,(4\,a^2b^2 - 4\,ab^3 + b^4)e^{(-2\,fx-2\,e)} + 10\,(6\,a^3b - 9\,a^2b^2 + 5\,ab^3 - b^4)e^{(-4\,fx-4\,e)} + 2\,(24\,a^4 - 48}{3\,(a^4 - 2\,a^3b + a^2b^2)(2\,(2}$$

input
```
integrate(cosh(f*x+e)/(a+b*sinh(f*x+e)^2)^(5/2),x, algorithm="maxima")
```

output
```
1/3*(2*a^2*b^2 - 2*a*b^3 + b^4 + 5*(4*a^3*b - 6*a^2*b^2 + 4*a*b^3 - b^4)*e
^(-2*f*x - 2*e) + 2*(24*a^4 - 48*a^3*b + 49*a^2*b^2 - 25*a*b^3 + 5*b^4)*e^
(-4*f*x - 4*e) + 10*(6*a^3*b - 9*a^2*b^2 + 5*a*b^3 - b^4)*e^(-6*f*x - 6*e)
 + 5*(4*a^2*b^2 - 4*a*b^3 + b^4)*e^(-8*f*x - 8*e) + (2*a*b^3 - b^4)*e^(-10
*f*x - 10*e))/((a^4 - 2*a^3*b + a^2*b^2)*(2*(2*a - b)*e^(-2*f*x - 2*e) + b
*e^(-4*f*x - 4*e) + b)^(5/2)*f) - 1/3*(2*a*b^3 - b^4 + 5*(4*a^2*b^2 - 4*a*
b^3 + b^4)*e^(-2*f*x - 2*e) + 10*(6*a^3*b - 9*a^2*b^2 + 5*a*b^3 - b^4)*e^(
-4*f*x - 4*e) + 2*(24*a^4 - 48*a^3*b + 49*a^2*b^2 - 25*a*b^3 + 5*b^4)*e^(-
6*f*x - 6*e) + 5*(4*a^3*b - 6*a^2*b^2 + 4*a*b^3 - b^4)*e^(-8*f*x - 8*e) +
(2*a^2*b^2 - 2*a*b^3 + b^4)*e^(-10*f*x - 10*e))/((a^4 - 2*a^3*b + a^2*b^2)
*(2*(2*a - b)*e^(-2*f*x - 2*e) + b*e^(-4*f*x - 4*e) + b)^(5/2)*f)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 380 vs. $2(57) = 114$.

Time = 0.31 (sec) , antiderivative size = 380, normalized size of antiderivative = 5.85

$$\int \frac{\cosh(e+fx)}{\left(a+b\sinh^2(e+fx)\right)^{5/2}}\,dx = \frac{2\left(\left(\left(\frac{(a^2bfe^{(12\,e)}-2\,ab^2fe^{(12\,e)}+b^3fe^{(12\,e)})e^{(2\,fx)}}{a^4f^2e^{(6\,e)}-2\,a^3bf^2e^{(6\,e)}+a^2b^2f^2e^{(6\,e)}} + \frac{3\,(2\,a^3fe^{(10\,e)}-5\,a^2bfe^{(10\,e)}+4\,ab^2fe^{(10\,e}}{a^4f^2e^{(6\,e)}-2\,a^3bf^2e^{(6\,e)}+a^2b^2f^2}\right.\right.\right.}{3\,(be^{(4\,fx+4}}$$

input
```
integrate(cosh(f*x+e)/(a+b*sinh(f*x+e)^2)^(5/2),x, algorithm="giac")
```

output
```
2/3*((((a^2*b*f*e^(12*e) - 2*a*b^2*f*e^(12*e) + b^3*f*e^(12*e))*e^(2*f*x)/
(a^4*f^2*e^(6*e) - 2*a^3*b*f^2*e^(6*e) + a^2*b^2*f^2*e^(6*e)) + 3*(2*a^3*f
*e^(10*e) - 5*a^2*b*f*e^(10*e) + 4*a*b^2*f*e^(10*e) - b^3*f*e^(10*e))/(a^4
*f^2*e^(6*e) - 2*a^3*b*f^2*e^(6*e) + a^2*b^2*f^2*e^(6*e)))*e^(2*f*x) - 3*(
2*a^3*f*e^(8*e) - 5*a^2*b*f*e^(8*e) + 4*a*b^2*f*e^(8*e) - b^3*f*e^(8*e))/(
a^4*f^2*e^(6*e) - 2*a^3*b*f^2*e^(6*e) + a^2*b^2*f^2*e^(6*e)))*e^(2*f*x) -
(a^2*b*f*e^(6*e) - 2*a*b^2*f*e^(6*e) + b^3*f*e^(6*e))/(a^4*f^2*e^(6*e) - 2
*a^3*b*f^2*e^(6*e) + a^2*b^2*f^2*e^(6*e)))/(b*e^(4*f*x + 4*e) + 4*a*e^(2*f
*x + 2*e) - 2*b*e^(2*f*x + 2*e) + b)^(3/2)
```

Mupad [B] (verification not implemented)

Time = 2.45 (sec) , antiderivative size = 129, normalized size of antiderivative = 1.98

$$\int \frac{\cosh(e+fx)}{\left(a+b\sinh^2(e+fx)\right)^{5/2}}\,dx = \frac{4\,e^{e+f\,x}\,\left(e^{2\,e+2\,f\,x}-1\right)\,\sqrt{a+b\,\left(\frac{e^{e+f\,x}}{2}-\frac{e^{-e-f\,x}}{2}\right)^2}\,\left(b+6\,a\,e^{2\,e+2\,f\,x}-2\,b\,e^{2}\right.}{3\,a^2\,f\,\left(b+4\,a\,e^{2\,e+2\,f\,x}-2\,b\,e^{2\,e+2\,f\,x}+b\,e^{4\,e+4\,f\,x}\right)^2}$$

input
```
int(cosh(e + f*x)/(a + b*sinh(e + f*x)^2)^(5/2),x)
```

output
```
(4*exp(e + f*x)*(exp(2*e + 2*f*x) - 1)*(a + b*(exp(e + f*x)/2 - exp(- e -
f*x)/2)^2)^(1/2)*(b + 6*a*exp(2*e + 2*f*x) - 2*b*exp(2*e + 2*f*x) + b*exp(
4*e + 4*f*x)))/(3*a^2*f*(b + 4*a*exp(2*e + 2*f*x) - 2*b*exp(2*e + 2*f*x) +
 b*exp(4*e + 4*f*x))^2)
```

Reduce [F]

$$\int \frac{\cosh(e+fx)}{\left(a+b\sinh^2(e+fx)\right)^{5/2}}\,dx = \int \frac{\sqrt{\sinh\left(fx+e\right)^2 b+a}\,\cosh\left(fx+e\right)}{\sinh\left(fx+e\right)^6 b^3+3\sinh\left(fx+e\right)^4 a\,b^2+3\sinh\left(fx+e\right)^2 a^2 b+a^3}dx$$

input
```
int(cosh(f*x+e)/(a+b*sinh(f*x+e)^2)^(5/2),x)
```

output
```
int((sqrt(sinh(e + f*x)**2*b + a)*cosh(e + f*x))/(sinh(e + f*x)**6*b**3 +
3*sinh(e + f*x)**4*a*b**2 + 3*sinh(e + f*x)**2*a**2*b + a**3),x)
```

3.358 $\int \frac{\operatorname{sech}(e+fx)}{\left(a+b\sinh^2(e+fx)\right)^{5/2}}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 134

$$\int \frac{\operatorname{sech}(e+fx)}{\left(a+b\sinh^2(e+fx)\right)^{5/2}}\,dx = \frac{\arctan\left(\frac{\sqrt{a-b}\sinh(e+fx)}{\sqrt{a+b\sinh^2(e+fx)}}\right)}{(a-b)^{5/2}f} - \frac{b\sinh(e+fx)}{3a(a-b)f\left(a+b\sinh^2(e+fx)\right)^{3/2}} - \frac{(5a-2b)b\sinh(e+fx)}{3a^2(a-b)^2f\sqrt{a+b\sinh^2(e+fx)}}$$

output
```
arctan((a-b)^(1/2)*sinh(f*x+e)/(a+b*sinh(f*x+e)^2)^(1/2))/(a-b)^(5/2)/f-1/
3*b*sinh(f*x+e)/a/(a-b)/f/(a+b*sinh(f*x+e)^2)^(3/2)-1/3*(5*a-2*b)*b*sinh(f
*x+e)/a^2/(a-b)^2/f/(a+b*sinh(f*x+e)^2)^(1/2)
```

Mathematica [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 8.35 (sec) , antiderivative size = 1331, normalized size of antiderivative = 9.93

$$\int \frac{\operatorname{sech}(e+fx)}{\left(a+b\sinh^2(e+fx)\right)^{5/2}}\,dx = \text{Too large to display}$$

input
```
Integrate[Sech[e + f*x]/(a + b*Sinh[e + f*x]^2)^(5/2),x]
```

output
```
(Sech[e + f*x]*Tanh[e + f*x]*(1575*ArcSin[Sqrt[((a - b)*Tanh[e + f*x]^2)/a
]] + (2100*b*ArcSin[Sqrt[((a - b)*Tanh[e + f*x]^2)/a]]*Sinh[e + f*x]^2)/a
+ (840*b^2*ArcSin[Sqrt[((a - b)*Tanh[e + f*x]^2)/a]]*Sinh[e + f*x]^4)/a^2
- (3150*(a - b)*ArcSin[Sqrt[((a - b)*Tanh[e + f*x]^2)/a]]*Tanh[e + f*x]^2)
/a - (4200*(a - b)*b*ArcSin[Sqrt[((a - b)*Tanh[e + f*x]^2)/a]]*Sinh[e + f*
x]^2*Tanh[e + f*x]^2)/a^2 - (1680*(a - b)*b^2*ArcSin[Sqrt[((a - b)*Tanh[e
+ f*x]^2)/a]]*Sinh[e + f*x]^4*Tanh[e + f*x]^2)/a^3 + (1575*(a - b)^2*ArcSi
n[Sqrt[((a - b)*Tanh[e + f*x]^2)/a]]*Tanh[e + f*x]^4)/a^2 + (2100*(a - b)^
2*b*ArcSin[Sqrt[((a - b)*Tanh[e + f*x]^2)/a]]*Sinh[e + f*x]^2*Tanh[e + f*x
]^4)/a^3 + (840*(a - b)^2*b^2*ArcSin[Sqrt[((a - b)*Tanh[e + f*x]^2)/a]]*Si
nh[e + f*x]^4*Tanh[e + f*x]^4)/a^4 + 2100*Sqrt[(Sech[e + f*x]^2*(a + b*Sin
h[e + f*x]^2))/a]*(((a - b)*Tanh[e + f*x]^2)/a)^(3/2) + (2800*b*Sinh[e + f
*x]^2*Sqrt[(Sech[e + f*x]^2*(a + b*Sinh[e + f*x]^2))/a]*(((a - b)*Tanh[e +
 f*x]^2)/a)^(3/2))/a + (1120*b^2*Sinh[e + f*x]^4*Sqrt[(Sech[e + f*x]^2*(a
+ b*Sinh[e + f*x]^2))/a]*(((a - b)*Tanh[e + f*x]^2)/a)^(3/2))/a^2 + 96*Hyp
ergeometric2F1[2, 2, 9/2, ((a - b)*Tanh[e + f*x]^2)/a]*Sqrt[(Sech[e + f*x]
^2*(a + b*Sinh[e + f*x]^2))/a]*(((a - b)*Tanh[e + f*x]^2)/a)^(7/2) + 24*Hy
pergeometricPFQ[{2, 2, 2}, {1, 9/2}, ((a - b)*Tanh[e + f*x]^2)/a]*Sqrt[(Se
ch[e + f*x]^2*(a + b*Sinh[e + f*x]^2))/a]*(((a - b)*Tanh[e + f*x]^2)/a)^(7
/2) + (168*b*Hypergeometric2F1[2, 2, 9/2, ((a - b)*Tanh[e + f*x]^2)/a]*...
```

Rubi [A] (verified)

Time = 0.36 (sec) , antiderivative size = 144, normalized size of antiderivative = 1.07, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.304$, Rules used = {3042, 3669, 316, 402, 27, 291, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\text{sech}(e+fx)}{\left(a+b\sinh^2(e+fx)\right)^{5/2}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{\cos(ie+ifx)\left(a-b\sin(ie+ifx)^2\right)^{5/2}}dx$$

$$\Big\downarrow \text{3669}$$

$$\frac{\int \frac{1}{(\sinh^2(e+fx)+1)\left(b\sinh^2(e+fx)+a\right)^{5/2}}d\sinh(e+fx)}{f}$$

$$\Big\downarrow \text{316}$$

$$\frac{\frac{\int \frac{-2b\sinh^2(e+fx)+3a-2b}{\left(\sinh^2(e+fx)+1\right)\left(b\sinh^2(e+fx)+a\right)^{3/2}}d\sinh(e+fx)}{3a(a-b)}-\frac{b\sinh(e+fx)}{3a(a-b)\left(a+b\sinh^2(e+fx)\right)^{3/2}}}{f}$$

$$\Big\downarrow \text{402}$$

$$\frac{\frac{\frac{\int \frac{3a^2}{\left(\sinh^2(e+fx)+1\right)\sqrt{b\sinh^2(e+fx)+a}}d\sinh(e+fx)}{a(a-b)}-\frac{b(5a-2b)\sinh(e+fx)}{a(a-b)\sqrt{a+b\sinh^2(e+fx)}}}{3a(a-b)}-\frac{b\sinh(e+fx)}{3a(a-b)\left(a+b\sinh^2(e+fx)\right)^{3/2}}}{f}$$

$$\Big\downarrow \text{27}$$

$$\frac{\frac{\frac{3a\int \frac{1}{\left(\sinh^2(e+fx)+1\right)\sqrt{b\sinh^2(e+fx)+a}}d\sinh(e+fx)}{a-b}-\frac{b(5a-2b)\sinh(e+fx)}{a(a-b)\sqrt{a+b\sinh^2(e+fx)}}}{3a(a-b)}-\frac{b\sinh(e+fx)}{3a(a-b)\left(a+b\sinh^2(e+fx)\right)^{3/2}}}{f}$$

$$\Big\downarrow \text{291}$$

$$\frac{\frac{\frac{3a\int \frac{1}{1-\frac{(b-a)\sinh^2(e+fx)}{b\sinh^2(e+fx)+a}}d\frac{\sinh(e+fx)}{\sqrt{b\sinh^2(e+fx)+a}}}{a-b}-\frac{b(5a-2b)\sinh(e+fx)}{a(a-b)\sqrt{a+b\sinh^2(e+fx)}}}{3a(a-b)}-\frac{b\sinh(e+fx)}{3a(a-b)\left(a+b\sinh^2(e+fx)\right)^{3/2}}}{f}$$

$$\Big\downarrow \text{216}$$

$$\frac{\frac{\frac{3a\arctan\left(\frac{\sqrt{a-b}\sinh(e+fx)}{\sqrt{a+b\sinh^2(e+fx)}}\right)}{(a-b)^{3/2}}-\frac{b(5a-2b)\sinh(e+fx)}{a(a-b)\sqrt{a+b\sinh^2(e+fx)}}}{3a(a-b)}-\frac{b\sinh(e+fx)}{3a(a-b)\left(a+b\sinh^2(e+fx)\right)^{3/2}}}{f}$$

input

```
Int[Sech[e + f*x]/(a + b*Sinh[e + f*x]^2)^(5/2),x]
```

output
```
(-1/3*(b*Sinh[e + f*x])/(a*(a - b)*(a + b*Sinh[e + f*x]^2)^(3/2)) + ((3*a*
ArcTan[(Sqrt[a - b]*Sinh[e + f*x])/Sqrt[a + b*Sinh[e + f*x]^2]])/(a - b)^(
3/2) - ((5*a - 2*b)*b*Sinh[e + f*x])/(a*(a - b)*Sqrt[a + b*Sinh[e + f*x]^2
]))/(3*a*(a - b)))/f
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 291
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*((c_) + (d_.)*(x_)^2)), x_Symbol] :> Subst
[Int[1/(c - (b*c - a*d)*x^2), x], x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b, c,
d}, x] && NeQ[b*c - a*d, 0]
```

rule 316
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_), x_Symbol] :> Sim
p[(-b)*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^(q + 1)/(2*a*(p + 1)*(b*c - a*d))
), x] + Simp[1/(2*a*(p + 1)*(b*c - a*d))   Int[(a + b*x^2)^(p + 1)*(c + d*x
^2)^q*Simp[b*c + 2*(p + 1)*(b*c - a*d) + d*b*(2*(p + q + 2) + 1)*x^2, x], x
], x] /; FreeQ[{a, b, c, d, q}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] &&  !
( !IntegerQ[p] && IntegerQ[q] && LtQ[q, -1]) && IntBinomialQ[a, b, c, d, 2,
 p, q, x]
```

rule 402
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_.)*((e_) + (f_.)*(x
_)^2), x_Symbol] :> Simp[(-(b*e - a*f))*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^
(q + 1)/(a*2*(b*c - a*d)*(p + 1))), x] + Simp[1/(a*2*(b*c - a*d)*(p + 1))
 Int[(a + b*x^2)^(p + 1)*(c + d*x^2)^q*Simp[c*(b*e - a*f) + e*2*(b*c - a*d)
*(p + 1) + d*(b*e - a*f)*(2*(p + q + 2) + 1)*x^2, x], x], x] /; FreeQ[{a, b
, c, d, e, f, q}, x] && LtQ[p, -1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3669
```
Int[cos[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^
(p_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Simp[ff/f   S
ubst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a + b*ff^2*x^2)^p, x], x, Sin[e + f*x]
/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m - 1)/2]
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 3.69 (sec) , antiderivative size = 169, normalized size of antiderivative = 1.26

method	result
default	$\frac{\text{`int/indef0`}\left(\frac{-b^2 \sinh(fx+e)^4-2\sinh(fx+e)^2 ab-a^2}{\left(-b^4 \sinh(fx+e)^{10}+\left(-4a\,b^3-b^4\right)\sinh(fx+e)^8+\left(-6a^2b^2-4a\,b^3\right)\sinh(fx+e)^6+\left(-4a^3b-6a^2b^2\right)\sinh(fx+e)^4+\left(-a^4-4a^3b\right)\sin\right.}\right.}{f}$
risch	Expression too large to display

input
```
int(sech(f*x+e)/(a+b*sinh(f*x+e)^2)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
`int/indef0`((-b^2*sinh(f*x+e)^4-2*sinh(f*x+e)^2*a*b-a^2)/(-b^4*sinh(f*x+e
)^10+(-4*a*b^3-b^4)*sinh(f*x+e)^8+(-6*a^2*b^2-4*a*b^3)*sinh(f*x+e)^6+(-4*a
^3*b-6*a^2*b^2)*sinh(f*x+e)^4+(-a^4-4*a^3*b)*sinh(f*x+e)^2-a^4)/(a+b*sinh(
f*x+e)^2)^(1/2),sinh(f*x+e))/f
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2640 vs. 2(120) = 240.

Time = 0.39 (sec) , antiderivative size = 5396, normalized size of antiderivative = 40.27

$$\int \frac{\operatorname{sech}(e+fx)}{\left(a+b\sinh^2(e+fx)\right)^{5/2}}\,dx = \text{Too large to display}$$

input
```
integrate(sech(f*x+e)/(a+b*sinh(f*x+e)^2)^(5/2),x, algorithm="fricas")
```

output
```
Too large to include
```

Sympy [F]

$$\int \frac{\operatorname{sech}(e+fx)}{\left(a+b\sinh^2(e+fx)\right)^{5/2}}\,dx=\int \frac{\operatorname{sech}\left(e+fx\right)}{\left(a+b\sinh^2\left(e+fx\right)\right)^{\frac{5}{2}}}\,dx$$

input
```
integrate(sech(f*x+e)/(a+b*sinh(f*x+e)**2)**(5/2),x)
```

output
```
Integral(sech(e + f*x)/(a + b*sinh(e + f*x)**2)**(5/2), x)
```

Maxima [F]

$$\int \frac{\operatorname{sech}(e+fx)}{\left(a+b\sinh^2(e+fx)\right)^{5/2}}\,dx=\int \frac{\operatorname{sech}\left(fx+e\right)}{\left(b\sinh\left(fx+e\right)^2+a\right)^{\frac{5}{2}}}\,dx$$

input
```
integrate(sech(f*x+e)/(a+b*sinh(f*x+e)^2)^(5/2),x, algorithm="maxima")
```

output
```
integrate(sech(f*x + e)/(b*sinh(f*x + e)^2 + a)^(5/2), x)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{\operatorname{sech}(e+fx)}{\left(a+b\sinh^2(e+fx)\right)^{5/2}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(sech(f*x+e)/(a+b*sinh(f*x+e)^2)^(5/2),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Error: Bad Argument Type
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\operatorname{sech}(e+fx)}{\left(a+b\sinh^2(e+fx)\right)^{5/2}}\,dx = \int \frac{1}{\cosh\left(e+f\,x\right)\,\left(b\sinh\left(e+f\,x\right)^2+a\right)^{5/2}}\,dx$$

input
```
int(1/(cosh(e + f*x)*(a + b*sinh(e + f*x)^2)^(5/2)),x)
```

output
```
int(1/(cosh(e + f*x)*(a + b*sinh(e + f*x)^2)^(5/2)), x)
```

Reduce [F]

$$\int \frac{\operatorname{sech}(e+fx)}{\left(a+b\sinh^2(e+fx)\right)^{5/2}}\,dx = \int \frac{\sqrt{\sinh\left(fx+e\right)^2 b+a}\,\operatorname{sech}(fx+e)}{\sinh\left(fx+e\right)^6 b^3+3\sinh\left(fx+e\right)^4 a\,b^2+3\sinh\left(fx+e\right)^2 a^2b+a^3}\,dx$$

input
```
int(sech(f*x+e)/(a+b*sinh(f*x+e)^2)^(5/2),x)
```

output
```
int((sqrt(sinh(e + f*x)**2*b + a)*sech(e + f*x))/(sinh(e + f*x)**6*b**3 +
3*sinh(e + f*x)**4*a*b**2 + 3*sinh(e + f*x)**2*a**2*b + a**3),x)
```

3.359 $\int \frac{\cosh^6(e+fx)}{(a+b\sinh^2(e+fx))^{5/2}}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 307

$$\int \frac{\cosh^6(e+fx)}{(a+b\sinh^2(e+fx))^{5/2}}\,dx = -\frac{(a-b)\cosh^3(e+fx)\sinh(e+fx)}{3abf\,(a+b\sinh^2(e+fx))^{3/2}} + \frac{(4a-b)\cosh(e+fx)\sinh(e+fx)}{3ab^2f\sqrt{a+b\sinh^2(e+fx)}} - \frac{(8a^2-3ab-2b^2)\cosh(e+fx)E\left(\arctan\left(\frac{\sqrt{b}\sinh(e+fx)}{\sqrt{a}}\right)|1-\frac{a}{b}\right)}{3a^{3/2}b^{5/2}f\sqrt{\frac{a\cosh^2(e+fx)}{a+b\sinh^2(e+fx)}}\sqrt{a+b\sinh^2(e+fx)}} + \frac{(4a-b)\cosh(e+fx)\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{b}\sinh(e+fx)}{\sqrt{a}}\right),1-\frac{a}{b}\right)}{3\sqrt{a}b^{5/2}f\sqrt{\frac{a\cosh^2(e+fx)}{a+b\sinh^2(e+fx)}}\sqrt{a+b\sinh^2(e+fx)}}$$

output
```
-1/3*(a-b)*cosh(f*x+e)^3*sinh(f*x+e)/a/b/f/(a+b*sinh(f*x+e)^2)^(3/2)+1/3*(
4*a-b)*cosh(f*x+e)*sinh(f*x+e)/a/b^2/f/(a+b*sinh(f*x+e)^2)^(1/2)-1/3*(8*a^
2-3*a*b-2*b^2)*cosh(f*x+e)*EllipticE(b^(1/2)*sinh(f*x+e)/a^(1/2)/(1+b*sinh
(f*x+e)^2/a)^(1/2),(1-a/b)^(1/2))/a^(3/2)/b^(5/2)/f/(a*cosh(f*x+e)^2/(a+b*
sinh(f*x+e)^2))^(1/2)/(a+b*sinh(f*x+e)^2)^(1/2)+1/3*(4*a-b)*cosh(f*x+e)*In
verseJacobiAM(arctan(b^(1/2)*sinh(f*x+e)/a^(1/2)),(1-a/b)^(1/2))/a^(1/2)/b
^(5/2)/f/(a*cosh(f*x+e)^2/(a+b*sinh(f*x+e)^2))^(1/2)/(a+b*sinh(f*x+e)^2)^(
1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 1.95 (sec) , antiderivative size = 206, normalized size of antiderivative = 0.67

$$\int \frac{\cosh^6(e+fx)}{(a+b\sinh^2(e+fx))^{5/2}}\,dx = \frac{-2ia^2(8a^2-3ab-2b^2)\left(\frac{2a-b+b\cosh(2(e+fx))}{a}\right)^{3/2} E\left(i(e+fx)\,\middle|\,\frac{b}{a}\right)+\frac{1}{2}(a-$$

input
```
Integrate[Cosh[e + f*x]^6/(a + b*Sinh[e + f*x]^2)^(5/2),x]
```

output
```
((-2*I)*a^2*(8*a^2 - 3*a*b - 2*b^2)*((2*a - b + b*Cosh[2*(e + f*x)])/a)^(3
/2)*EllipticE[I*(e + f*x), b/a] + ((a - b)*((4*I)*a^2*(8*a + b)*((2*a - b
+ b*Cosh[2*(e + f*x)])/a)^(3/2)*EllipticF[I*(e + f*x), b/a] - 2*Sqrt[2]*b*
(8*a^2 + a*b - 2*b^2 + b*(5*a + 2*b)*Cosh[2*(e + f*x)])*Sinh[2*(e + f*x)])
)/2)/(6*a^2*b^3*f*(2*a - b + b*Cosh[2*(e + f*x)])^(3/2))
```

Rubi [A] (verified)

Time = 0.55 (sec) , antiderivative size = 370, normalized size of antiderivative = 1.21, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.360$, Rules used = {3042, 3671, 315, 401, 25, 406, 320, 388, 313}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cosh^6(e+fx)}{(a+b\sinh^2(e+fx))^{5/2}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\cos(ie+ifx)^6}{(a-b\sin(ie+ifx)^2)^{5/2}}dx$$

$$\Big\downarrow \text{3671}$$

$$\frac{\sqrt{\cosh^2(e+fx)}\text{sech}(e+fx)\int \frac{(\sinh^2(e+fx)+1)^{5/2}}{(b\sinh^2(e+fx)+a)^{5/2}}d\sinh(e+fx)}{f}$$

$\downarrow$ 315

$$\frac{\sqrt{\cosh^2(e+fx)}\operatorname{sech}(e+fx)\left(\frac{\int \frac{\sqrt{\sinh^2(e+fx)+1}\left((4a-b)\sinh^2(e+fx)+a+2b\right)}{\left(b\sinh^2(e+fx)+a\right)^{3/2}}d\sinh(e+fx)}{3ab}-\frac{(a-b)\sinh(e+fx)\left(\sinh^2(e+fx)+1\right)^{3/2}}{3ab\left(a+b\sinh^2(e+fx)\right)^{3/2}}\right.}{f}$$

$\downarrow$ 401

$$\frac{\sqrt{\cosh^2(e+fx)}\operatorname{sech}(e+fx)\left(\frac{\frac{2\left(-\frac{2a}{b}+\frac{b}{a}+1\right)\sinh(e+fx)\sqrt{\sinh^2(e+fx)+1}}{\sqrt{a+b\sinh^2(e+fx)}}-\frac{\int -\frac{\left(8a^2-3ba-2b^2\right)\sinh^2(e+fx)+a(4a-b)}{\sqrt{\sinh^2(e+fx)+1}\sqrt{b\sinh^2(e+fx)+a}}d\sinh(e+fx)}{ab}}{3ab}-\frac{(a-b}{}\right.}{f}$$

$\downarrow$ 25

$$\frac{\sqrt{\cosh^2(e+fx)}\operatorname{sech}(e+fx)\left(\frac{\frac{\int \frac{\left(8a^2-3ba-2b^2\right)\sinh^2(e+fx)+a(4a-b)}{\sqrt{\sinh^2(e+fx)+1}\sqrt{b\sinh^2(e+fx)+a}}d\sinh(e+fx)}{ab}+\frac{2\left(-\frac{2a}{b}+\frac{b}{a}+1\right)\sqrt{\sinh^2(e+fx)+1}\sinh(e+fx)}{\sqrt{a+b\sinh^2(e+fx)}}}{3ab}-\frac{(a-b)}{}\right.}{f}$$

$\downarrow$ 406

$$\frac{\sqrt{\cosh^2(e+fx)}\operatorname{sech}(e+fx)\left(\frac{\frac{\left(8a^2-3ab-2b^2\right)\int \frac{\sinh^2(e+fx)}{\sqrt{\sinh^2(e+fx)+1}\sqrt{b\sinh^2(e+fx)+a}}d\sinh(e+fx)+a(4a-b)\int \frac{1}{\sqrt{\sinh^2(e+fx)+1}\sqrt{b\sinh^2(e+fx}}}{ab}}{3ab}\right.}{f}$$

$\downarrow$ 320

$$\frac{\sqrt{\cosh^2(e+fx)}\operatorname{sech}(e+fx)\left(\frac{\frac{\left(8a^2-3ab-2b^2\right)\int\frac{\sinh^2(e+fx)}{\sqrt{\sinh^2(e+fx)+1}\sqrt{b\sinh^2(e+fx)+a}}d\sinh(e+fx)+\frac{(4a-b)\sqrt{a+b\sinh^2(e+fx)}\,\text{EllipticF}\left(\arctan(\right.}{\sqrt{\sinh^2(e+fx)+1}\sqrt{\frac{a+b\sinh^2(}{a\left(\sinh^2(e+\right.}}}}{ab}}{3ab}\right.}{f}$$

$\downarrow$ 388

$$\frac{\sqrt{\cosh^2(e+fx)}\operatorname{sech}(e+fx)\left(\frac{\frac{\left(8a^2-3ab-2b^2\right)\left(\frac{\sinh(e+fx)\sqrt{a+b\sinh^2(e+fx)}}{b\sqrt{\sinh^2(e+fx)+1}}-\frac{\int\frac{\sqrt{b\sinh^2(e+fx)+a}}{\left(\sinh^2(e+fx)+1\right)^{3/2}}d\sinh(e+fx)}{b}\right)+\frac{(4a-b)\sqrt{a+b\sinh^2(e-}}{\sqrt{\sinh^2(e-}}}{ab}}{3ab}\right.}{f}$$

$\downarrow$ 313

$$\frac{\sqrt{\cosh^2(e+fx)}\operatorname{sech}(e+fx)\left(\frac{\frac{\left(8a^2-3ab-2b^2\right)\left(\frac{\sinh(e+fx)\sqrt{a+b\sinh^2(e+fx)}}{b\sqrt{\sinh^2(e+fx)+1}}-\frac{\sqrt{a+b\sinh^2(e+fx)}E\left(\arctan(\sinh(e+fx))\middle|1-\frac{b}{a}\right)}{b\sqrt{\sinh^2(e+fx)+1}\sqrt{\frac{a+b\sinh^2(e+fx)}{a\left(\sinh^2(e+fx)+1\right)}}}\right)+\frac{(4a-b)\sqrt{c}}{}}{ab}}{3ab}\right.}{f}$$

input
```
Int[Cosh[e + f*x]^6/(a + b*Sinh[e + f*x]^2)^(5/2),x]
```

output
```
(Sqrt[Cosh[e + f*x]^2]*Sech[e + f*x]*(-1/3*((a - b)*Sinh[e + f*x]*(1 + Sin
h[e + f*x]^2)^(3/2))/(a*b*(a + b*Sinh[e + f*x]^2)^(3/2)) + ((2*(1 - (2*a)/
b + b/a)*Sinh[e + f*x]*Sqrt[1 + Sinh[e + f*x]^2])/Sqrt[a + b*Sinh[e + f*x]
^2] + (((4*a - b)*EllipticF[ArcTan[Sinh[e + f*x]], 1 - b/a]*Sqrt[a + b*Sin
h[e + f*x]^2])/(Sqrt[1 + Sinh[e + f*x]^2]*Sqrt[(a + b*Sinh[e + f*x]^2)/(a*
(1 + Sinh[e + f*x]^2))]) + (8*a^2 - 3*a*b - 2*b^2)*((Sinh[e + f*x]*Sqrt[a
+ b*Sinh[e + f*x]^2])/(b*Sqrt[1 + Sinh[e + f*x]^2]) - (EllipticE[ArcTan[Si
nh[e + f*x]], 1 - b/a]*Sqrt[a + b*Sinh[e + f*x]^2])/(b*Sqrt[1 + Sinh[e + f
*x]^2]*Sqrt[(a + b*Sinh[e + f*x]^2)/(a*(1 + Sinh[e + f*x]^2))])))/(a*b))/(
3*a*b)))/f
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 313
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/((c_) + (d_.)*(x_)^2)^(3/2), x_Symbol] :> Sim
p[(Sqrt[a + b*x^2]/(c*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(c
+ d*x^2)))]))*EllipticE[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; FreeQ
[{a, b, c, d}, x] && PosQ[b/a] && PosQ[d/c]
```

rule 315
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_), x_Symbol] :> Sim
p[(a*d - c*b)*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^(q - 1)/(2*a*b*(p + 1))),
x] - Simp[1/(2*a*b*(p + 1))   Int[(a + b*x^2)^(p + 1)*(c + d*x^2)^(q - 2)*S
imp[c*(a*d - c*b*(2*p + 3)) + d*(a*d*(2*(q - 1) + 1) - b*c*(2*(p + q) + 1))
*x^2, x], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -
1] && GtQ[q, 1] && IntBinomialQ[a, b, c, d, 2, p, q, x]
```

rule 320
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(Sqrt[a + b*x^2]/(a*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(
c + d*x^2)))]))*EllipticF[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; Fre
eQ[{a, b, c, d}, x] && PosQ[d/c] && PosQ[b/a] && !SimplerSqrtQ[b/a, d/c]
```

rule 388
```
Int[(x_)^2/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol]
 :> Simp[x*(Sqrt[a + b*x^2]/(b*Sqrt[c + d*x^2])), x] - Simp[c/b   Int[Sqrt[
a + b*x^2]/(c + d*x^2)^(3/2), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
 a*d, 0] && PosQ[b/a] && PosQ[d/c] &&  !SimplerSqrtQ[b/a, d/c]
```

rule 401
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_.)*((e_) + (f_.)*(x
_)^2), x_Symbol] :> Simp[(-(b*e - a*f))*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^
q/(a*b*2*(p + 1))), x] + Simp[1/(a*b*2*(p + 1))   Int[(a + b*x^2)^(p + 1)*(
c + d*x^2)^(q - 1)*Simp[c*(b*e*2*(p + 1) + b*e - a*f) + d*(b*e*2*(p + 1) +
(b*e - a*f)*(2*q + 1))*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && L
tQ[p, -1] && GtQ[q, 0]
```

rule 406
```
Int[((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.)*((e_) + (f_.)*(
x_)^2), x_Symbol] :> Simp[e   Int[(a + b*x^2)^p*(c + d*x^2)^q, x], x] + Sim
p[f   Int[x^2*(a + b*x^2)^p*(c + d*x^2)^q, x], x] /; FreeQ[{a, b, c, d, e,
f, p, q}, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3671
```
Int[cos[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(
p_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Simp[ff*(Sqrt[
Cos[e + f*x]^2]/(f*Cos[e + f*x]))   Subst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a
 + b*ff^2*x^2)^p, x], x, Sin[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x]
 && IntegerQ[m/2] &&  !IntegerQ[p]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 811 vs. $2(291) = 582$.

Time = 8.08 (sec) , antiderivative size = 812, normalized size of antiderivative = 2.64

method	result
default	$-\frac{\left(5\sqrt{-\frac{b}{a}}\,a^{2}b-3\sqrt{-\frac{b}{a}}\,a\,b^{2}-2\sqrt{-\frac{b}{a}}\,b^{3}\right)\cosh(fx+e)^{4}\sinh(fx+e)+\left(4\sqrt{-\frac{b}{a}}\,a^{3}-6\sqrt{-\frac{b}{a}}\,a^{2}b+2\sqrt{-\frac{b}{a}}\,b^{3}\right)\cosh(fx+e)^{2}\sinh(fx+}{}$

input
```
int(cosh(f*x+e)^6/(a+b*sinh(f*x+e)^2)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
-1/3*((5*(-b/a)^(1/2)*a^2*b-3*(-b/a)^(1/2)*a*b^2-2*(-b/a)^(1/2)*b^3)*cosh(
f*x+e)^4*sinh(f*x+e)+(4*(-b/a)^(1/2)*a^3-6*(-b/a)^(1/2)*a^2*b+2*(-b/a)^(1/
2)*b^3)*cosh(f*x+e)^2*sinh(f*x+e)+(b/a*cosh(f*x+e)^2+(a-b)/a)^(1/2)*(cosh(
f*x+e)^2)^(1/2)*b*(4*EllipticF(sinh(f*x+e)*(-b/a)^(1/2),(1/b*a)^(1/2))*a^2
-2*EllipticF(sinh(f*x+e)*(-b/a)^(1/2),(1/b*a)^(1/2))*a*b-2*EllipticF(sinh(
f*x+e)*(-b/a)^(1/2),(1/b*a)^(1/2))*b^2-8*EllipticE(sinh(f*x+e)*(-b/a)^(1/2
),(1/b*a)^(1/2))*a^2+3*EllipticE(sinh(f*x+e)*(-b/a)^(1/2),(1/b*a)^(1/2))*a
*b+2*EllipticE(sinh(f*x+e)*(-b/a)^(1/2),(1/b*a)^(1/2))*b^2)*cosh(f*x+e)^2+
4*(b/a*cosh(f*x+e)^2+(a-b)/a)^(1/2)*(cosh(f*x+e)^2)^(1/2)*EllipticF(sinh(f
*x+e)*(-b/a)^(1/2),(1/b*a)^(1/2))*a^3-6*(b/a*cosh(f*x+e)^2+(a-b)/a)^(1/2)*
(cosh(f*x+e)^2)^(1/2)*EllipticF(sinh(f*x+e)*(-b/a)^(1/2),(1/b*a)^(1/2))*a^
2*b+2*(b/a*cosh(f*x+e)^2+(a-b)/a)^(1/2)*(cosh(f*x+e)^2)^(1/2)*EllipticF(si
nh(f*x+e)*(-b/a)^(1/2),(1/b*a)^(1/2))*b^3-8*(b/a*cosh(f*x+e)^2+(a-b)/a)^(1
/2)*(cosh(f*x+e)^2)^(1/2)*EllipticE(sinh(f*x+e)*(-b/a)^(1/2),(1/b*a)^(1/2)
)*a^3+11*(b/a*cosh(f*x+e)^2+(a-b)/a)^(1/2)*(cosh(f*x+e)^2)^(1/2)*EllipticE
(sinh(f*x+e)*(-b/a)^(1/2),(1/b*a)^(1/2))*a^2*b-(b/a*cosh(f*x+e)^2+(a-b)/a)
^(1/2)*(cosh(f*x+e)^2)^(1/2)*EllipticE(sinh(f*x+e)*(-b/a)^(1/2),(1/b*a)^(1
/2))*a*b^2-2*(b/a*cosh(f*x+e)^2+(a-b)/a)^(1/2)*(cosh(f*x+e)^2)^(1/2)*Ellip
ticE(sinh(f*x+e)*(-b/a)^(1/2),(1/b*a)^(1/2))*b^3)/a^2/(a+b*sinh(f*x+e)^2)^
(3/2)/(-b/a)^(1/2)/b^2/cosh(f*x+e)/f
```

Fricas [F]

$$\int \frac{\cosh^6(e+fx)}{\left(a+b\sinh^2(e+fx)\right)^{5/2}}\,dx=\int \frac{\cosh\left(fx+e\right)^6}{\left(b\sinh\left(fx+e\right)^2+a\right)^{\frac{5}{2}}}\,dx$$

input
```
integrate(cosh(f*x+e)^6/(a+b*sinh(f*x+e)^2)^(5/2),x, algorithm="fricas")
```

output
```
integral(sqrt(b*sinh(f*x + e)^2 + a)*cosh(f*x + e)^6/(b^3*sinh(f*x + e)^6
+ 3*a*b^2*sinh(f*x + e)^4 + 3*a^2*b*sinh(f*x + e)^2 + a^3), x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\cosh^6(e+fx)}{\left(a+b\sinh^2(e+fx)\right)^{5/2}}\,dx = \text{Timed out}$$

input
```
integrate(cosh(f*x+e)**6/(a+b*sinh(f*x+e)**2)**(5/2),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{\cosh^6(e+fx)}{\left(a+b\sinh^2(e+fx)\right)^{5/2}}\,dx = \int \frac{\cosh\left(fx+e\right)^6}{\left(b\sinh\left(fx+e\right)^2+a\right)^{\frac{5}{2}}}\,dx$$

input
```
integrate(cosh(f*x+e)^6/(a+b*sinh(f*x+e)^2)^(5/2),x, algorithm="maxima")
```

output
```
integrate(cosh(f*x + e)^6/(b*sinh(f*x + e)^2 + a)^(5/2), x)
```

Giac [F]

$$\int \frac{\cosh^6(e+fx)}{\left(a+b\sinh^2(e+fx)\right)^{5/2}}\,dx = \int \frac{\cosh\left(fx+e\right)^6}{\left(b\sinh\left(fx+e\right)^2+a\right)^{\frac{5}{2}}}\,dx$$

input
```
integrate(cosh(f*x+e)^6/(a+b*sinh(f*x+e)^2)^(5/2),x, algorithm="giac")
```

output
```
integrate(cosh(f*x + e)^6/(b*sinh(f*x + e)^2 + a)^(5/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\cosh^6(e+fx)}{\left(a+b\sinh^2(e+fx)\right)^{5/2}}\,dx=\int \frac{\cosh(e+f\,x)^6}{\left(b\sinh\left(e+f\,x\right)^2+a\right)^{5/2}}\,dx$$

input
```
int(cosh(e + f*x)^6/(a + b*sinh(e + f*x)^2)^(5/2),x)
```

output
```
int(cosh(e + f*x)^6/(a + b*sinh(e + f*x)^2)^(5/2), x)
```

Reduce [F]

$$\int \frac{\cosh^6(e+fx)}{\left(a+b\sinh^2(e+fx)\right)^{5/2}}\,dx=\int \frac{\sqrt{\sinh\left(fx+e\right)^2 b+a}\,\cosh\left(fx+e\right)^6}{\sinh\left(fx+e\right)^6 b^3+3\sinh\left(fx+e\right)^4 a\,b^2+3\sinh\left(fx+e\right)^2 a^2b+a^3}dx$$

input
```
int(cosh(f*x+e)^6/(a+b*sinh(f*x+e)^2)^(5/2),x)
```

output
```
int((sqrt(sinh(e + f*x)**2*b + a)*cosh(e + f*x)**6)/(sinh(e + f*x)**6*b**3
 + 3*sinh(e + f*x)**4*a*b**2 + 3*sinh(e + f*x)**2*a**2*b + a**3),x)
```

3.360 $\int \frac{\cosh^4(e+fx)}{(a+b\sinh^2(e+fx))^{5/2}}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 238

$$\int \frac{\cosh^4(e+fx)}{(a+b\sinh^2(e+fx))^{5/2}}\,dx = -\frac{(a-b)\cosh(e+fx)\sinh(e+fx)}{3abf\,(a+b\sinh^2(e+fx))^{3/2}} + \frac{2(a+b)\cosh(e+fx)E\left(\arctan\left(\frac{\sqrt{b}\sinh(e+fx)}{\sqrt{a}}\right)|1-\frac{a}{b}\right)}{3a^{3/2}b^{3/2}f\sqrt{\frac{a\cosh^2(e+fx)}{a+b\sinh^2(e+fx)}}\sqrt{a+b\sinh^2(e+fx)}} - \frac{\cosh(e+fx)\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{b}\sinh(e+fx)}{\sqrt{a}}\right),1-\frac{a}{b}\right)}{3\sqrt{a}b^{3/2}f\sqrt{\frac{a\cosh^2(e+fx)}{a+b\sinh^2(e+fx)}}\sqrt{a+b\sinh^2(e+fx)}}$$

output

```
-1/3*(a-b)*cosh(f*x+e)*sinh(f*x+e)/a/b/f/(a+b*sinh(f*x+e)^2)^(3/2)+2/3*(a+
b)*cosh(f*x+e)*EllipticE(b^(1/2)*sinh(f*x+e)/a^(1/2)/(1+b*sinh(f*x+e)^2/a)
^(1/2),(1-a/b)^(1/2))/a^(3/2)/b^(3/2)/f/(a*cosh(f*x+e)^2/(a+b*sinh(f*x+e)^
2))^(1/2)/(a+b*sinh(f*x+e)^2)^(1/2)-1/3*cosh(f*x+e)*InverseJacobiAM(arctan
(b^(1/2)*sinh(f*x+e)/a^(1/2)),(1-a/b)^(1/2))/a^(1/2)/b^(3/2)/f/(a*cosh(f*x
+e)^2/(a+b*sinh(f*x+e)^2))^(1/2)/(a+b*sinh(f*x+e)^2)^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 1.26 (sec) , antiderivative size = 178, normalized size of antiderivative = 0.75

$$\int \frac{\cosh^4(e+fx)}{\left(a+b\sinh^2(e+fx)\right)^{5/2}}\,dx = \frac{2ia^2(a+b)\left(\frac{2a-b+b\cosh(2(e+fx))}{a}\right)^{3/2}E\left(i(e+fx)\,|\,\frac{b}{a}\right)-ia^2(2a+b)\left(\frac{2a-b+}{}\right.}{3a}$$

input
```
Integrate[Cosh[e + f*x]^4/(a + b*Sinh[e + f*x]^2)^(5/2),x]
```

output
```
((2*I)*a^2*(a + b)*((2*a - b + b*Cosh[2*(e + f*x)])/a)^(3/2)*EllipticE[I*(
e + f*x), b/a] - I*a^2*(2*a + b)*((2*a - b + b*Cosh[2*(e + f*x)])/a)^(3/2)
*EllipticF[I*(e + f*x), b/a] + Sqrt[2]*b*(a^2 + 2*a*b - b^2 + b*(a + b)*Co
sh[2*(e + f*x)])*Sinh[2*(e + f*x)])/(3*a^2*b^2*f*(2*a - b + b*Cosh[2*(e +
f*x)])^(3/2))
```

Rubi [A] (verified)

Time = 0.43 (sec) , antiderivative size = 270, normalized size of antiderivative = 1.13, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.240$, Rules used = {3042, 3671, 315, 400, 313, 320}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cosh^4(e+fx)}{\left(a+b\sinh^2(e+fx)\right)^{5/2}}dx$$

$$\Big\downarrow \; 3042$$

$$\int \frac{\cos(ie+ifx)^4}{\left(a-b\sin(ie+ifx)^2\right)^{5/2}}dx$$

$$\Big\downarrow \; 3671$$

$$\frac{\sqrt{\cosh^2(e+fx)}\text{sech}(e+fx)\int \frac{\left(\sinh^2(e+fx)+1\right)^{3/2}}{\left(b\sinh^2(e+fx)+a\right)^{5/2}}d\sinh(e+fx)}{f}$$

$\big\downarrow$ 315

$$\frac{\sqrt{\cosh^2(e+fx)}\text{sech}(e+fx)\left(\frac{\int\frac{(2a+b)\sinh^2(e+fx)+a+2b}{\sqrt{\sinh^2(e+fx)+1}\left(b\sinh^2(e+fx)+a\right)^{3/2}}d\sinh(e+fx)}{3ab}-\frac{(a-b)\sinh(e+fx)\sqrt{\sinh^2(e+fx)+1}}{3ab(a+b\sinh^2(e+fx))^{3/2}}\right)}{f}$$

$\big\downarrow$ 400

$$\frac{\sqrt{\cosh^2(e+fx)}\text{sech}(e+fx)\left(\frac{2(a+b)\int\frac{\sqrt{\sinh^2(e+fx)+1}}{\left(b\sinh^2(e+fx)+a\right)^{3/2}}d\sinh(e+fx)-\int\frac{1}{\sqrt{\sinh^2(e+fx)+1}\sqrt{b\sinh^2(e+fx)+a}}d\sinh(e+fx)}{3ab}-(a-\right.}{f}$$

$\big\downarrow$ 313

$$\frac{\sqrt{\cosh^2(e+fx)}\text{sech}(e+fx)\left(\frac{\frac{2(a+b)\sqrt{\sinh^2(e+fx)+1}E\left(\arctan\left(\frac{\sqrt{b}\sinh(e+fx)}{\sqrt{a}}\right)|1-\frac{a}{b}\right)}{\sqrt{a}\sqrt{b}\sqrt{\frac{a\left(\sinh^2(e+fx)+1\right)}{a+b\sinh^2(e+fx)}}\sqrt{a+b\sinh^2(e+fx)}}-\int\frac{1}{\sqrt{\sinh^2(e+fx)+1}\sqrt{b\sinh^2(e+fx)+a}}d\sinh(e+f}{3ab}\right.}{f}$$

$\big\downarrow$ 320

$$\frac{\sqrt{\cosh^2(e+fx)}\text{sech}(e+fx)\left(\frac{\frac{2(a+b)\sqrt{\sinh^2(e+fx)+1}E\left(\arctan\left(\frac{\sqrt{b}\sinh(e+fx)}{\sqrt{a}}\right)|1-\frac{a}{b}\right)}{\sqrt{a}\sqrt{b}\sqrt{\frac{a\left(\sinh^2(e+fx)+1\right)}{a+b\sinh^2(e+fx)}}\sqrt{a+b\sinh^2(e+fx)}}-\frac{\sqrt{a+b\sinh^2(e+fx)}\text{EllipticF}\left(\arctan(\sinh(e+fx)),1-\right.}{a\sqrt{\sinh^2(e+fx)+1}\sqrt{\frac{a+b\sinh^2(e+fx)}{a\left(\sinh^2(e+fx)+1\right)}}}}{3ab}\right.}{f}$$

input

```
Int[Cosh[e + f*x]^4/(a + b*Sinh[e + f*x]^2)^(5/2),x]
```

output
```
(Sqrt[Cosh[e + f*x]^2]*Sech[e + f*x]*(-1/3*((a - b)*Sinh[e + f*x]*Sqrt[1 +
 Sinh[e + f*x]^2])/(a*b*(a + b*Sinh[e + f*x]^2)^(3/2)) + ((2*(a + b)*Ellip
ticE[ArcTan[(Sqrt[b]*Sinh[e + f*x])/Sqrt[a]], 1 - a/b]*Sqrt[1 + Sinh[e + f
*x]^2])/(Sqrt[a]*Sqrt[b]*Sqrt[(a*(1 + Sinh[e + f*x]^2))/(a + b*Sinh[e + f*
x]^2)]*Sqrt[a + b*Sinh[e + f*x]^2]) - (EllipticF[ArcTan[Sinh[e + f*x]], 1
- b/a]*Sqrt[a + b*Sinh[e + f*x]^2])/(a*Sqrt[1 + Sinh[e + f*x]^2]*Sqrt[(a +
 b*Sinh[e + f*x]^2)/(a*(1 + Sinh[e + f*x]^2))]))/(3*a*b)))/f
```

Defintions of rubi rules used

rule 313
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/((c_) + (d_.)*(x_)^2)^(3/2), x_Symbol] :> Sim
p[(Sqrt[a + b*x^2]/(c*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(c
+ d*x^2)))]))*EllipticE[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; FreeQ
[{a, b, c, d}, x] && PosQ[b/a] && PosQ[d/c]
```

rule 315
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_), x_Symbol] :> Sim
p[(a*d - c*b)*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^(q - 1)/(2*a*b*(p + 1))),
x] - Simp[1/(2*a*b*(p + 1))   Int[(a + b*x^2)^(p + 1)*(c + d*x^2)^(q - 2)*S
imp[c*(a*d - c*b*(2*p + 3)) + d*(a*d*(2*(q - 1) + 1) - b*c*(2*(p + q) + 1))
*x^2, x], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -
1] && GtQ[q, 1] && IntBinomialQ[a, b, c, d, 2, p, q, x]
```

rule 320
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(Sqrt[a + b*x^2]/(a*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(
c + d*x^2)))]))*EllipticF[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; Fre
eQ[{a, b, c, d}, x] && PosQ[d/c] && PosQ[b/a] &&  !SimplerSqrtQ[b/a, d/c]
```

rule 400
```
Int[((e_) + (f_.)*(x_)^2)/(Sqrt[(a_) + (b_.)*(x_)^2]*((c_) + (d_.)*(x_)^2)^
(3/2)), x_Symbol] :> Simp[(b*e - a*f)/(b*c - a*d)   Int[1/(Sqrt[a + b*x^2]*
Sqrt[c + d*x^2]), x], x] - Simp[(d*e - c*f)/(b*c - a*d)   Int[Sqrt[a + b*x^
2]/(c + d*x^2)^(3/2), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && PosQ[b/a] &
& PosQ[d/c]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3671
```
Int[cos[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(
p_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Simp[ff*(Sqrt[
Cos[e + f*x]^2]/(f*Cos[e + f*x]))   Subst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a
 + b*ff^2*x^2)^p, x], x, Sin[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x]
 && IntegerQ[m/2] &&  !IntegerQ[p]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 596 vs. $2(226) = 452$.

Time = 5.80 (sec) , antiderivative size = 597, normalized size of antiderivative = 2.51

method	result
default	$\frac{\left(2\sqrt{-\frac{b}{a}}\, ab+2\sqrt{-\frac{b}{a}}\, b^{2}\right)\cosh(fx+e)^{4}\sinh(fx+e)+\left(\sqrt{-\frac{b}{a}}\, a^{2}+\sqrt{-\frac{b}{a}}\, ab-2\sqrt{-\frac{b}{a}}\, b^{2}\right)\cosh(fx+e)^{2}\sinh(fx+e)+\sqrt{\frac{b\cosh(fx+e)^{2}}{a}\cdots}}{\cdots}$

input
```
int(cosh(f*x+e)^4/(a+b*sinh(f*x+e)^2)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
1/3*((2*(-b/a)^(1/2)*a*b+2*(-b/a)^(1/2)*b^2)*cosh(f*x+e)^4*sinh(f*x+e)+((-
b/a)^(1/2)*a^2+(-b/a)^(1/2)*a*b-2*(-b/a)^(1/2)*b^2)*cosh(f*x+e)^2*sinh(f*x
+e)+(b/a*cosh(f*x+e)^2+(a-b)/a)^(1/2)*(cosh(f*x+e)^2)^(1/2)*b*(a*EllipticF
(sinh(f*x+e)*(-b/a)^(1/2),(1/b*a)^(1/2))+2*b*EllipticF(sinh(f*x+e)*(-b/a)^
(1/2),(1/b*a)^(1/2))-2*EllipticE(sinh(f*x+e)*(-b/a)^(1/2),(1/b*a)^(1/2))*a
-2*b*EllipticE(sinh(f*x+e)*(-b/a)^(1/2),(1/b*a)^(1/2)))*cosh(f*x+e)^2+a^2*
(b/a*cosh(f*x+e)^2+(a-b)/a)^(1/2)*(cosh(f*x+e)^2)^(1/2)*EllipticF(sinh(f*x
+e)*(-b/a)^(1/2),(1/b*a)^(1/2))+a*(b/a*cosh(f*x+e)^2+(a-b)/a)^(1/2)*(cosh(
f*x+e)^2)^(1/2)*EllipticF(sinh(f*x+e)*(-b/a)^(1/2),(1/b*a)^(1/2))*b-2*(b/a
*cosh(f*x+e)^2+(a-b)/a)^(1/2)*(cosh(f*x+e)^2)^(1/2)*EllipticF(sinh(f*x+e)*
(-b/a)^(1/2),(1/b*a)^(1/2))*b^2-2*(b/a*cosh(f*x+e)^2+(a-b)/a)^(1/2)*(cosh(
f*x+e)^2)^(1/2)*EllipticE(sinh(f*x+e)*(-b/a)^(1/2),(1/b*a)^(1/2))*a^2+2*(b
/a*cosh(f*x+e)^2+(a-b)/a)^(1/2)*(cosh(f*x+e)^2)^(1/2)*EllipticE(sinh(f*x+e
)*(-b/a)^(1/2),(1/b*a)^(1/2))*b^2)/a^2/(a+b*sinh(f*x+e)^2)^(3/2)/(-b/a)^(1
/2)/b/cosh(f*x+e)/f
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 4355 vs. $2(226) = 452$.

Time = 0.19 (sec) , antiderivative size = 4355, normalized size of antiderivative = 18.30

$$\int \frac{\cosh^4(e+fx)}{(a+b\sinh^2(e+fx))^{5/2}}\,dx = \text{Too large to display}$$

input
```
integrate(cosh(f*x+e)^4/(a+b*sinh(f*x+e)^2)^(5/2),x, algorithm="fricas")
```

output
```
Too large to include
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\cosh^4(e+fx)}{(a+b\sinh^2(e+fx))^{5/2}}\,dx = \text{Timed out}$$

input
```
integrate(cosh(f*x+e)**4/(a+b*sinh(f*x+e)**2)**(5/2),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{\cosh^4(e+fx)}{(a+b\sinh^2(e+fx))^{5/2}}\,dx = \int \frac{\cosh(fx+e)^4}{(b\sinh(fx+e)^2+a)^{\frac{5}{2}}}\,dx$$

input
```
integrate(cosh(f*x+e)^4/(a+b*sinh(f*x+e)^2)^(5/2),x, algorithm="maxima")
```

output
```
integrate(cosh(f*x + e)^4/(b*sinh(f*x + e)^2 + a)^(5/2), x)
```

Giac [F]

$$\int \frac{\cosh^4(e+fx)}{\left(a+b\sinh^2(e+fx)\right)^{5/2}}\,dx=\int \frac{\cosh\left(fx+e\right)^4}{\left(b\sinh\left(fx+e\right)^2+a\right)^{\frac{5}{2}}}\,dx$$

input
```
integrate(cosh(f*x+e)^4/(a+b*sinh(f*x+e)^2)^(5/2),x, algorithm="giac")
```

output
```
integrate(cosh(f*x + e)^4/(b*sinh(f*x + e)^2 + a)^(5/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\cosh^4(e+fx)}{\left(a+b\sinh^2(e+fx)\right)^{5/2}}\,dx=\int \frac{\cosh(e+f\,x)^4}{\left(b\sinh\left(e+f\,x\right)^2+a\right)^{5/2}}\,dx$$

input
```
int(cosh(e + f*x)^4/(a + b*sinh(e + f*x)^2)^(5/2),x)
```

output
```
int(cosh(e + f*x)^4/(a + b*sinh(e + f*x)^2)^(5/2), x)
```

Reduce [F]

$$\int \frac{\cosh^4(e+fx)}{\left(a+b\sinh^2(e+fx)\right)^{5/2}}\,dx=\int \frac{\sqrt{\sinh\left(fx+e\right)^2 b+a}\,\cosh\left(fx+e\right)^4}{\sinh\left(fx+e\right)^6 b^3+3\sinh\left(fx+e\right)^4 a\,b^2+3\sinh\left(fx+e\right)^2 a^2 b+a^3}dx$$

input
```
int(cosh(f*x+e)^4/(a+b*sinh(f*x+e)^2)^(5/2),x)
```

output
```
int((sqrt(sinh(e + f*x)**2*b + a)*cosh(e + f*x)**4)/(sinh(e + f*x)**6*b**3
 + 3*sinh(e + f*x)**4*a*b**2 + 3*sinh(e + f*x)**2*a**2*b + a**3),x)
```

3.361 $\int \frac{\cosh^2(e+fx)}{\left(a+b\sinh^2(e+fx)\right)^{5/2}}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 246

$$\int \frac{\cosh^2(e+fx)}{\left(a+b\sinh^2(e+fx)\right)^{5/2}}\,dx = \frac{\cosh(e+fx)\sinh(e+fx)}{3af\left(a+b\sinh^2(e+fx)\right)^{3/2}} + \frac{(a-2b)\cosh(e+fx)E\left(\arctan\left(\frac{\sqrt{b}\sinh(e+fx)}{\sqrt{a}}\right)|1-\frac{a}{b}\right)}{3a^{3/2}(a-b)\sqrt{b}f\sqrt{\frac{a\cosh^2(e+fx)}{a+b\sinh^2(e+fx)}}\sqrt{a+b\sinh^2(e+fx)}} + \frac{\cosh(e+fx)\operatorname{EllipticF}\left(\arctan\left(\frac{\sqrt{b}\sinh(e+fx)}{\sqrt{a}}\right),1-\frac{a}{b}\right)}{3\sqrt{a}(a-b)\sqrt{b}f\sqrt{\frac{a\cosh^2(e+fx)}{a+b\sinh^2(e+fx)}}\sqrt{a+b\sinh^2(e+fx)}}$$

output

```
1/3*cosh(f*x+e)*sinh(f*x+e)/a/f/(a+b*sinh(f*x+e)^2)^(3/2)+1/3*(a-2*b)*cosh
(f*x+e)*EllipticE(b^(1/2)*sinh(f*x+e)/a^(1/2)/(1+b*sinh(f*x+e)^2/a)^(1/2),
(1-a/b)^(1/2))/a^(3/2)/(a-b)/b^(1/2)/f/(a*cosh(f*x+e)^2/(a+b*sinh(f*x+e)^2
))^(1/2)/(a+b*sinh(f*x+e)^2)^(1/2)+1/3*cosh(f*x+e)*InverseJacobiAM(arctan(
b^(1/2)*sinh(f*x+e)/a^(1/2)),(1-a/b)^(1/2))/a^(1/2)/(a-b)/b^(1/2)/f/(a*cos
h(f*x+e)^2/(a+b*sinh(f*x+e)^2))^(1/2)/(a+b*sinh(f*x+e)^2)^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 1.20 (sec) , antiderivative size = 193, normalized size of antiderivative = 0.78

$$\int \frac{\cosh^2(e+fx)}{\left(a+b\sinh^2(e+fx)\right)^{5/2}}\,dx = \frac{2ia^2(a-2b)\left(\frac{2a-b+b\cosh(2(e+fx))}{a}\right)^{3/2}E\left(i(e+fx)\,\middle|\,\frac{b}{a}\right)-2ia^2(a-b)\left(\frac{2a-b}{\ldots}\right.}{6a\ldots}$$

input
```
Integrate[Cosh[e + f*x]^2/(a + b*Sinh[e + f*x]^2)^(5/2),x]
```

output
```
((2*I)*a^2*(a - 2*b)*((2*a - b + b*Cosh[2*(e + f*x)])/a)^(3/2)*EllipticE[I
*(e + f*x), b/a] - (2*I)*a^2*(a - b)*((2*a - b + b*Cosh[2*(e + f*x)])/a)^(
3/2)*EllipticF[I*(e + f*x), b/a] - Sqrt[2]*b*(-4*a^2 + 7*a*b - 2*b^2 - (a
- 2*b)*b*Cosh[2*(e + f*x)])*Sinh[2*(e + f*x)])/(6*a^2*(a - b)*b*f*(2*a - b
 + b*Cosh[2*(e + f*x)])^(3/2))
```

Rubi [A] (verified)

Time = 0.46 (sec) , antiderivative size = 273, normalized size of antiderivative = 1.11, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.280$, Rules used = {3042, 3671, 314, 25, 400, 313, 320}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cosh^2(e+fx)}{\left(a+b\sinh^2(e+fx)\right)^{5/2}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\cos(ie+ifx)^2}{\left(a-b\sin(ie+ifx)^2\right)^{5/2}}dx$$

$$\Big\downarrow \text{3671}$$

$$\frac{\sqrt{\cosh^2(e+fx)}\operatorname{sech}(e+fx)\int \frac{\sqrt{\sinh^2(e+fx)+1}}{\left(b\sinh^2(e+fx)+a\right)^{5/2}}d\sinh(e+fx)}{f}$$

$\Big\downarrow$ 314

$$\frac{\sqrt{\cosh^2(e+fx)}\text{sech}(e+fx)\left(\frac{\sinh(e+fx)\sqrt{\sinh^2(e+fx)+1}}{3a\left(a+b\sinh^2(e+fx)\right)^{3/2}}-\frac{\int -\frac{\sinh^2(e+fx)+2}{\sqrt{\sinh^2(e+fx)+1}\left(b\sinh^2(e+fx)+a\right)^{3/2}}d\sinh(e+fx)}{3a}\right)}{f}$$

$\Big\downarrow$ 25

$$\frac{\sqrt{\cosh^2(e+fx)}\text{sech}(e+fx)\left(\frac{\int \frac{\sinh^2(e+fx)+2}{\sqrt{\sinh^2(e+fx)+1}\left(b\sinh^2(e+fx)+a\right)^{3/2}}d\sinh(e+fx)}{3a}+\frac{\sqrt{\sinh^2(e+fx)+1}\sinh(e+fx)}{3a\left(a+b\sinh^2(e+fx)\right)^{3/2}}\right)}{f}$$

$\Big\downarrow$ 400

$$\frac{\sqrt{\cosh^2(e+fx)}\text{sech}(e+fx)\left(\frac{\frac{(a-2b)\int \frac{\sqrt{\sinh^2(e+fx)+1}}{\left(b\sinh^2(e+fx)+a\right)^{3/2}}d\sinh(e+fx)}{a-b}+\frac{\int \frac{1}{\sqrt{\sinh^2(e+fx)+1}\sqrt{b\sinh^2(e+fx)+a}}d\sinh(e+fx)}{a-b}}{3a}+\frac{\sqrt{\sinh^2}}{3a(a}\right.}{f}$$

$\Big\downarrow$ 313

$$\frac{\sqrt{\cosh^2(e+fx)}\text{sech}(e+fx)\left(\frac{\frac{\int \frac{1}{\sqrt{\sinh^2(e+fx)+1}\sqrt{b\sinh^2(e+fx)+a}}d\sinh(e+fx)}{a-b}+\frac{(a-2b)\sqrt{\sinh^2(e+fx)+1}E\left(\arctan\left(\frac{\sqrt{b}\sinh(e+fx)}{\sqrt{a}}\right)|1-\frac{a}{b}\right)}{\sqrt{a}\sqrt{b}(a-b)\sqrt{\frac{a\left(\sinh^2(e+fx)+1\right)}{a+b\sinh^2(e+fx)}}\sqrt{a+b\sinh^2(e+fx)}}}{3a}\right.}{f}$$

$\Big\downarrow$ 320

$$\frac{\sqrt{\cosh^2(e+fx)}\text{sech}(e+fx)\left(\frac{\frac{\sqrt{a+b\sinh^2(e+fx)}\text{EllipticF}\left(\arctan(\sinh(e+fx)),1-\frac{b}{a}\right)}{a(a-b)\sqrt{\sinh^2(e+fx)+1}\sqrt{\frac{a+b\sinh^2(e+fx)}{a\left(\sinh^2(e+fx)+1\right)}}}+\frac{(a-2b)\sqrt{\sinh^2(e+fx)+1}E\left(\arctan\left(\frac{\sqrt{b}\sinh(e+fx)}{\sqrt{a}}\right)|1-\right.}{\sqrt{a}\sqrt{b}(a-b)\sqrt{\frac{a\left(\sinh^2(e+fx)+1\right)}{a+b\sinh^2(e+fx)}}\sqrt{a+b\sinh^2(e+fx}}}{3a}\right.}{f}$$

input
```
Int[Cosh[e + f*x]^2/(a + b*Sinh[e + f*x]^2)^(5/2),x]
```

output
```
(Sqrt[Cosh[e + f*x]^2]*Sech[e + f*x]*((Sinh[e + f*x]*Sqrt[1 + Sinh[e + f*x
]^2])/(3*a*(a + b*Sinh[e + f*x]^2)^(3/2)) + (((a - 2*b)*EllipticE[ArcTan[(
Sqrt[b]*Sinh[e + f*x])/Sqrt[a]], 1 - a/b]*Sqrt[1 + Sinh[e + f*x]^2])/(Sqrt
[a]*(a - b)*Sqrt[b]*Sqrt[(a*(1 + Sinh[e + f*x]^2))/(a + b*Sinh[e + f*x]^2)
]*Sqrt[a + b*Sinh[e + f*x]^2]) + (EllipticF[ArcTan[Sinh[e + f*x]], 1 - b/a
]*Sqrt[a + b*Sinh[e + f*x]^2])/(a*(a - b)*Sqrt[1 + Sinh[e + f*x]^2]*Sqrt[(
a + b*Sinh[e + f*x]^2)/(a*(1 + Sinh[e + f*x]^2))]))/(3*a)))/f
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 313
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/((c_) + (d_.)*(x_)^2)^(3/2), x_Symbol] :> Sim
p[(Sqrt[a + b*x^2]/(c*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(c
+ d*x^2)))]))*EllipticE[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; FreeQ
[{a, b, c, d}, x] && PosQ[b/a] && PosQ[d/c]
```

rule 314
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_), x_Symbol] :> Sim
p[(-x)*(a + b*x^2)^(p + 1)*((c + d*x^2)^q/(2*a*(p + 1))), x] + Simp[1/(2*a*
(p + 1))   Int[(a + b*x^2)^(p + 1)*(c + d*x^2)^(q - 1)*Simp[c*(2*p + 3) + d
*(2*(p + q + 1) + 1)*x^2, x], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
 a*d, 0] && LtQ[p, -1] && LtQ[0, q, 1] && IntBinomialQ[a, b, c, d, 2, p, q,
 x]
```

rule 320
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(Sqrt[a + b*x^2]/(a*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(
c + d*x^2)))]))*EllipticF[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; Fre
eQ[{a, b, c, d}, x] && PosQ[d/c] && PosQ[b/a] &&  !SimplerSqrtQ[b/a, d/c]
```

rule 400
```
Int[((e_) + (f_.)*(x_)^2)/(Sqrt[(a_) + (b_.)*(x_)^2]*((c_) + (d_.)*(x_)^2)^
(3/2)), x_Symbol] :> Simp[(b*e - a*f)/(b*c - a*d)   Int[1/(Sqrt[a + b*x^2]*
Sqrt[c + d*x^2]), x], x] - Simp[(d*e - c*f)/(b*c - a*d)   Int[Sqrt[a + b*x^
2]/(c + d*x^2)^(3/2), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && PosQ[b/a] &
& PosQ[d/c]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3671
```
Int[cos[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(
p_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Simp[ff*(Sqrt[
Cos[e + f*x]^2]/(f*Cos[e + f*x]))   Subst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a
 + b*ff^2*x^2)^p, x], x, Sin[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x]
 && IntegerQ[m/2] &&  !IntegerQ[p]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 661 vs. $2(234) = 468$.

Time = 5.21 (sec) , antiderivative size = 662, normalized size of antiderivative = 2.69

method	result
default	$\frac{\sqrt{-\frac{b}{a}}\, ab \sinh(fx+e)^5 - 2\sqrt{-\frac{b}{a}}\, b^2 \sinh(fx+e)^5 + 2\sqrt{\frac{a+b\sinh(fx+e)^2}{a}}\, \sqrt{\frac{\cosh(2fx+2e)}{2}+\frac{1}{2}}\ \text{EllipticF}\left(\sinh(fx+e)\sqrt{-\frac{b}{a}}, \sqrt{\frac{a}{b}}\right) ab \sinh}{}$
risch	Expression too large to display

input
```
int(cosh(f*x+e)^2/(a+b*sinh(f*x+e)^2)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
1/3*((-b/a)^(1/2)*a*b*sinh(f*x+e)^5-2*(-b/a)^(1/2)*b^2*sinh(f*x+e)^5+2*((a
+b*sinh(f*x+e)^2)/a)^(1/2)*(cosh(f*x+e)^2)^(1/2)*EllipticF(sinh(f*x+e)*(-b
/a)^(1/2),(1/b*a)^(1/2))*a*b*sinh(f*x+e)^2-2*((a+b*sinh(f*x+e)^2)/a)^(1/2)
*(cosh(f*x+e)^2)^(1/2)*EllipticF(sinh(f*x+e)*(-b/a)^(1/2),(1/b*a)^(1/2))*b
^2*sinh(f*x+e)^2-((a+b*sinh(f*x+e)^2)/a)^(1/2)*(cosh(f*x+e)^2)^(1/2)*Ellip
ticE(sinh(f*x+e)*(-b/a)^(1/2),(1/b*a)^(1/2))*a*b*sinh(f*x+e)^2+2*((a+b*sin
h(f*x+e)^2)/a)^(1/2)*(cosh(f*x+e)^2)^(1/2)*EllipticE(sinh(f*x+e)*(-b/a)^(1
/2),(1/b*a)^(1/2))*b^2*sinh(f*x+e)^2+2*(-b/a)^(1/2)*a^2*sinh(f*x+e)^3-2*(-
b/a)^(1/2)*a*b*sinh(f*x+e)^3-2*(-b/a)^(1/2)*b^2*sinh(f*x+e)^3+2*a^2*((a+b*
sinh(f*x+e)^2)/a)^(1/2)*(cosh(f*x+e)^2)^(1/2)*EllipticF(sinh(f*x+e)*(-b/a)
^(1/2),(1/b*a)^(1/2))-2*a*((a+b*sinh(f*x+e)^2)/a)^(1/2)*(cosh(f*x+e)^2)^(1
/2)*EllipticF(sinh(f*x+e)*(-b/a)^(1/2),(1/b*a)^(1/2))*b-((a+b*sinh(f*x+e)^
2)/a)^(1/2)*(cosh(f*x+e)^2)^(1/2)*EllipticE(sinh(f*x+e)*(-b/a)^(1/2),(1/b*
a)^(1/2))*a^2+2*((a+b*sinh(f*x+e)^2)/a)^(1/2)*(cosh(f*x+e)^2)^(1/2)*Ellipt
icE(sinh(f*x+e)*(-b/a)^(1/2),(1/b*a)^(1/2))*a*b+2*(-b/a)^(1/2)*a^2*sinh(f*
x+e)-3*(-b/a)^(1/2)*a*b*sinh(f*x+e))/(a-b)/a^2/(a+b*sinh(f*x+e)^2)^(3/2)/(
-b/a)^(1/2)/cosh(f*x+e)/f
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 4770 vs. $2(234) = 468$.

Time = 0.22 (sec) , antiderivative size = 4770, normalized size of antiderivative = 19.39

$$\int \frac{\cosh^2(e+fx)}{\left(a+b\sinh^2(e+fx)\right)^{5/2}}\,dx = \text{Too large to display}$$

input
```
integrate(cosh(f*x+e)^2/(a+b*sinh(f*x+e)^2)^(5/2),x, algorithm="fricas")
```

output
```
Too large to include
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\cosh^2(e+fx)}{\left(a+b\sinh^2(e+fx)\right)^{5/2}}\,dx = \text{Timed out}$$

input
```
integrate(cosh(f*x+e)**2/(a+b*sinh(f*x+e)**2)**(5/2),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{\cosh^2(e+fx)}{\left(a+b\sinh^2(e+fx)\right)^{5/2}}\,dx = \int \frac{\cosh\left(fx+e\right)^2}{\left(b\sinh\left(fx+e\right)^2+a\right)^{\frac{5}{2}}}\,dx$$

input
```
integrate(cosh(f*x+e)^2/(a+b*sinh(f*x+e)^2)^(5/2),x, algorithm="maxima")
```

output
```
integrate(cosh(f*x + e)^2/(b*sinh(f*x + e)^2 + a)^(5/2), x)
```

Giac [F]

$$\int \frac{\cosh^2(e+fx)}{\left(a+b\sinh^2(e+fx)\right)^{5/2}}\,dx = \int \frac{\cosh\left(fx+e\right)^2}{\left(b\sinh\left(fx+e\right)^2+a\right)^{\frac{5}{2}}}\,dx$$

input
```
integrate(cosh(f*x+e)^2/(a+b*sinh(f*x+e)^2)^(5/2),x, algorithm="giac")
```

output
```
integrate(cosh(f*x + e)^2/(b*sinh(f*x + e)^2 + a)^(5/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\cosh^2(e+fx)}{\left(a+b\sinh^2(e+fx)\right)^{5/2}}\,dx=\int \frac{\cosh(e+f\,x)^2}{\left(b\sinh\left(e+f\,x\right)^2+a\right)^{5/2}}\,dx$$

input
```
int(cosh(e + f*x)^2/(a + b*sinh(e + f*x)^2)^(5/2),x)
```

output
```
int(cosh(e + f*x)^2/(a + b*sinh(e + f*x)^2)^(5/2), x)
```

Reduce [F]

$$\int \frac{\cosh^2(e+fx)}{\left(a+b\sinh^2(e+fx)\right)^{5/2}}\,dx=\int \frac{\sqrt{\sinh\left(fx+e\right)^2 b+a}\,\cosh\left(fx+e\right)^2}{\sinh\left(fx+e\right)^6 b^3+3\sinh\left(fx+e\right)^4 a\,b^2+3\sinh\left(fx+e\right)^2 a^2b+a^3}dx$$

input
```
int(cosh(f*x+e)^2/(a+b*sinh(f*x+e)^2)^(5/2),x)
```

output
```
int((sqrt(sinh(e + f*x)**2*b + a)*cosh(e + f*x)**2)/(sinh(e + f*x)**6*b**3
 + 3*sinh(e + f*x)**4*a*b**2 + 3*sinh(e + f*x)**2*a**2*b + a**3),x)
```

3.362 $\int \frac{1}{\left(a+b\sinh^2(e+fx)\right)^{5/2}}\,dx$

Optimal result

Integrand size = 16, antiderivative size = 253

$$\begin{aligned}\int \frac{1}{\left(a+b\sinh^2(e+fx)\right)^{5/2}}\,dx =& -\frac{b\cosh(e+fx)\sinh(e+fx)}{3a(a-b)f\left(a+b\sinh^2(e+fx)\right)^{3/2}} \\ &-\frac{2(2a-b)b\cosh(e+fx)\sinh(e+fx)}{3a^2(a-b)^2f\sqrt{a+b\sinh^2(e+fx)}} \\ &-\frac{2i(2a-b)E\left(ie+ifx\middle|\frac{b}{a}\right)\sqrt{a+b\sinh^2(e+fx)}}{3a^2(a-b)^2f\sqrt{\frac{a+b\sinh^2(e+fx)}{a}}} \\ &+\frac{i\,\text{EllipticF}\left(ie+ifx,\frac{b}{a}\right)\sqrt{\frac{a+b\sinh^2(e+fx)}{a}}}{3a(a-b)f\sqrt{a+b\sinh^2(e+fx)}}\end{aligned}$$

output
```
-1/3*b*cosh(f*x+e)*sinh(f*x+e)/a/(a-b)/f/(a+b*sinh(f*x+e)^2)^(3/2)-2/3*(2*
a-b)*b*cosh(f*x+e)*sinh(f*x+e)/a^2/(a-b)^2/f/(a+b*sinh(f*x+e)^2)^(1/2)-2/3
*I*(2*a-b)*EllipticE(sin(I*e+I*f*x),(b/a)^(1/2))*(a+b*sinh(f*x+e)^2)^(1/2)
/a^2/(a-b)^2/f/((a+b*sinh(f*x+e)^2)/a)^(1/2)+1/3*I*InverseJacobiAM(I*e+I*f
*x,(b/a)^(1/2))*((a+b*sinh(f*x+e)^2)/a)^(1/2)/a/(a-b)/f/(a+b*sinh(f*x+e)^2
)^(1/2)
```

Mathematica [A] (verified)

Time = 0.88 (sec) , antiderivative size = 190, normalized size of antiderivative = 0.75

$$\int \frac{1}{\left(a+b\sinh^2(e+fx)\right)^{5/2}}\,dx = \frac{-2ia^2(2a-b)\left(\frac{2a-b+b\cosh(2(e+fx))}{a}\right)^{3/2}E\left(i(e+fx)\,\middle|\,\frac{b}{a}\right)+ia^2(a-b)\left(\frac{2a-}{}\right.}{3}$$

input
```
Integrate[(a + b*Sinh[e + f*x]^2)^(-5/2),x]
```

output
```
((-2*I)*a^2*(2*a - b)*((2*a - b + b*Cosh[2*(e + f*x)])/a)^(3/2)*EllipticE[
I*(e + f*x), b/a] + I*a^2*(a - b)*((2*a - b + b*Cosh[2*(e + f*x)])/a)^(3/2
)*EllipticF[I*(e + f*x), b/a] + Sqrt[2]*b*(-5*a^2 + 5*a*b - b^2 + b*(-2*a
+ b)*Cosh[2*(e + f*x)])*Sinh[2*(e + f*x)])/(3*a^2*(a - b)^2*f*(2*a - b + b
*Cosh[2*(e + f*x)])^(3/2))
```

Rubi [A] (verified)

Time = 1.51 (sec) , antiderivative size = 258, normalized size of antiderivative = 1.02, number of steps used = 14, number of rules used = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.875$, Rules used = {3042, 3663, 25, 3042, 3652, 3042, 3651, 3042, 3657, 3042, 3656, 3662, 3042, 3661}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\left(a+b\sinh^2(e+fx)\right)^{5/2}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{(a-b\sin(ie+ifx)^2)^{5/2}}dx$$

$$\Big\downarrow \text{3663}$$

$$-\frac{\int -\frac{-b\sinh^2(e+fx)+3a-2b}{(b\sinh^2(e+fx)+a)^{3/2}}dx}{3a(a-b)}-\frac{b\sinh(e+fx)\cosh(e+fx)}{3af(a-b)\left(a+b\sinh^2(e+fx)\right)^{3/2}}$$

$$\Big\downarrow \text{25}$$

$$\frac{\int \frac{-b\sinh^2(e+fx)+3a-2b}{\left(b\sinh^2(e+fx)+a\right)^{3/2}}dx}{3a(a-b)}-\frac{b\sinh(e+fx)\cosh(e+fx)}{3af(a-b)\left(a+b\sinh^2(e+fx)\right)^{3/2}}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{b\sinh(e+fx)\cosh(e+fx)}{3af(a-b)\left(a+b\sinh^2(e+fx)\right)^{3/2}}+\frac{\int \frac{b\sin(ie+ifx)^2+3a-2b}{\left(a-b\sin(ie+ifx)^2\right)^{3/2}}dx}{3a(a-b)}$$

$$\Big\downarrow \text{3652}$$

$$\frac{\frac{\int \frac{2(2a-b)b\sinh^2(e+fx)+a(3a-b)}{\sqrt{b\sinh^2(e+fx)+a}}dx}{a(a-b)}-\frac{2b(2a-b)\sinh(e+fx)\cosh(e+fx)}{af(a-b)\sqrt{a+b\sinh^2(e+fx)}}}{3a(a-b)}-\frac{b\sinh(e+fx)\cosh(e+fx)}{3af(a-b)\left(a+b\sinh^2(e+fx)\right)^{3/2}}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{b\sinh(e+fx)\cosh(e+fx)}{3af(a-b)\left(a+b\sinh^2(e+fx)\right)^{3/2}}+\frac{-\frac{2b(2a-b)\sinh(e+fx)\cosh(e+fx)}{af(a-b)\sqrt{a+b\sinh^2(e+fx)}}+\frac{\int \frac{a(3a-b)-2(2a-b)b\sin(ie+ifx)^2}{\sqrt{a-b\sin(ie+ifx)^2}}dx}{a(a-b)}}{3a(a-b)}$$

$$\Big\downarrow \text{3651}$$

$$\frac{\frac{2(2a-b)\int \sqrt{b\sinh^2(e+fx)+a}dx-a(a-b)\int \frac{1}{\sqrt{b\sinh^2(e+fx)+a}}dx}{a(a-b)}-\frac{2b(2a-b)\sinh(e+fx)\cosh(e+fx)}{af(a-b)\sqrt{a+b\sinh^2(e+fx)}}}{3a(a-b)}-\frac{b\sinh(e+fx)\cosh(e+fx)}{3af(a-b)\left(a+b\sinh^2(e+fx)\right)^{3/2}}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{b\sinh(e+fx)\cosh(e+fx)}{3af(a-b)\left(a+b\sinh^2(e+fx)\right)^{3/2}}+\frac{-\frac{2b(2a-b)\sinh(e+fx)\cosh(e+fx)}{af(a-b)\sqrt{a+b\sinh^2(e+fx)}}+\frac{2(2a-b)\int \sqrt{a-b\sin(ie+ifx)^2}dx-a(a-b)\int \frac{1}{\sqrt{a-b\sin(ie+ifx)^2}}dx}{a(a-b)}}{3a(a-b)}$$

$$\Big\downarrow \text{3657}$$

$$\frac{-\frac{b\sinh(e+fx)\cosh(e+fx)}{3af(a-b)\left(a+b\sinh^2(e+fx)\right)^{3/2}}+}{}$$

$$\frac{-\frac{2b(2a-b)\sinh(e+fx)\cosh(e+fx)}{af(a-b)\sqrt{a+b\sinh^2(e+fx)}}+\frac{\frac{2(2a-b)\sqrt{a+b\sinh^2(e+fx)}\int\sqrt{\frac{b\sinh^2(e+fx)}{a}+1}dx}{\sqrt{\frac{b\sinh^2(e+fx)}{a}+1}}-a(a-b)\int\frac{1}{\sqrt{a-b\sin(ie+ifx)^2}}dx}{a(a-b)}}{3a(a-b)}$$

$\downarrow$ 3042

$$-\frac{b\sinh(e+fx)\cosh(e+fx)}{3af(a-b)\left(a+b\sinh^2(e+fx)\right)^{3/2}}+$$

$$\frac{-\frac{2b(2a-b)\sinh(e+fx)\cosh(e+fx)}{af(a-b)\sqrt{a+b\sinh^2(e+fx)}}+\frac{\frac{2(2a-b)\sqrt{a+b\sinh^2(e+fx)}\int\sqrt{1-\frac{b\sin(ie+ifx)^2}{a}}dx}{\sqrt{\frac{b\sinh^2(e+fx)}{a}+1}}-a(a-b)\int\frac{1}{\sqrt{a-b\sin(ie+ifx)^2}}dx}{a(a-b)}}{3a(a-b)}$$

$\downarrow$ 3656

$$-\frac{b\sinh(e+fx)\cosh(e+fx)}{3af(a-b)\left(a+b\sinh^2(e+fx)\right)^{3/2}}+$$

$$\frac{-\frac{2b(2a-b)\sinh(e+fx)\cosh(e+fx)}{af(a-b)\sqrt{a+b\sinh^2(e+fx)}}+\frac{-a(a-b)\int\frac{1}{\sqrt{a-b\sin(ie+ifx)^2}}dx-\frac{2i(2a-b)\sqrt{a+b\sinh^2(e+fx)}E\left(ie+ifx\middle|\frac{b}{a}\right)}{f\sqrt{\frac{b\sinh^2(e+fx)}{a}+1}}}{a(a-b)}}{3a(a-b)}$$

$\downarrow$ 3662

$$-\frac{b\sinh(e+fx)\cosh(e+fx)}{3af(a-b)\left(a+b\sinh^2(e+fx)\right)^{3/2}}+$$

$$\frac{-\frac{2b(2a-b)\sinh(e+fx)\cosh(e+fx)}{af(a-b)\sqrt{a+b\sinh^2(e+fx)}}+\frac{-\frac{a(a-b)\sqrt{\frac{b\sinh^2(e+fx)}{a}+1}\int\frac{1}{\sqrt{\frac{b\sinh^2(e+fx)}{a}+1}}dx}{\sqrt{a+b\sinh^2(e+fx)}}-\frac{2i(2a-b)\sqrt{a+b\sinh^2(e+fx)}E\left(ie+ifx\middle|\frac{b}{a}\right)}{f\sqrt{\frac{b\sinh^2(e+fx)}{a}+1}}}{a(a-b)}}{3a(a-b)}$$

$\downarrow$ 3042

$$-\frac{b\sinh(e+fx)\cosh(e+fx)}{3af(a-b)\left(a+b\sinh^2(e+fx)\right)^{3/2}}+$$

$$\frac{-\frac{2b(2a-b)\sinh(e+fx)\cosh(e+fx)}{af(a-b)\sqrt{a+b\sinh^2(e+fx)}}+\frac{-\frac{a(a-b)\sqrt{\frac{b\sinh^2(e+fx)}{a}+1}\int\frac{1}{\sqrt{1-\frac{b\sin(ie+ifx)^2}{a}}}dx}{\sqrt{a+b\sinh^2(e+fx)}}-\frac{2i(2a-b)\sqrt{a+b\sinh^2(e+fx)}E\left(ie+ifx\middle|\frac{b}{a}\right)}{f\sqrt{\frac{b\sinh^2(e+fx)}{a}+1}}}{a(a-b)}}{3a(a-b)}$$

$$\Big\downarrow \text{3661}$$

$$-\frac{b\sinh(e+fx)\cosh(e+fx)}{3af(a-b)\left(a+b\sinh^2(e+fx)\right)^{3/2}}+\frac{-\frac{2b(2a-b)\sinh(e+fx)\cosh(e+fx)}{af(a-b)\sqrt{a+b\sinh^2(e+fx)}}+\frac{\frac{ia(a-b)\sqrt{\frac{b\sinh^2(e+fx)}{a}+1}\,\text{EllipticF}\left(ie+ifx,\frac{b}{a}\right)}{f\sqrt{a+b\sinh^2(e+fx)}}-\frac{2i(2a-b)\sqrt{a+b\sinh^2(e+fx)}E\left(ie+ifx\middle|\frac{b}{a}\right)}{f\sqrt{\frac{b\sinh^2(e+fx)}{a}+1}}}{a(a-b)}}{3a(a-b)}$$

input
```
Int[(a + b*Sinh[e + f*x]^2)^(-5/2),x]
```

output
```
-1/3*(b*Cosh[e + f*x]*Sinh[e + f*x])/(a*(a - b)*f*(a + b*Sinh[e + f*x]^2)^
(3/2)) + ((-2*(2*a - b)*b*Cosh[e + f*x]*Sinh[e + f*x])/(a*(a - b)*f*Sqrt[a
 + b*Sinh[e + f*x]^2]) + (((-2*I)*(2*a - b)*EllipticE[I*e + I*f*x, b/a]*Sq
rt[a + b*Sinh[e + f*x]^2])/(f*Sqrt[1 + (b*Sinh[e + f*x]^2)/a]) + (I*a*(a -
 b)*EllipticF[I*e + I*f*x, b/a]*Sqrt[1 + (b*Sinh[e + f*x]^2)/a])/(f*Sqrt[a
 + b*Sinh[e + f*x]^2]))/(a*(a - b)))/(3*a*(a - b))
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3651
```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)]^2)/Sqrt[(a_) + (b_.)*sin[(e_.) +
 (f_.)*(x_)]^2], x_Symbol] :> Simp[B/b   Int[Sqrt[a + b*Sin[e + f*x]^2], x]
, x] + Simp[(A*b - a*B)/b   Int[1/Sqrt[a + b*Sin[e + f*x]^2], x], x] /; Fre
eQ[{a, b, e, f, A, B}, x]
```

rule 3652
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)]^2), x_Symbol] :> Simp[(-(A*b - a*B))*Cos[e + f*x]*Sin[e + f*x
]*((a + b*Sin[e + f*x]^2)^(p + 1)/(2*a*f*(a + b)*(p + 1))), x] - Simp[1/(2*
a*(a + b)*(p + 1))   Int[(a + b*Sin[e + f*x]^2)^(p + 1)*Simp[a*B - A*(2*a*(
p + 1) + b*(2*p + 3)) + 2*(A*b - a*B)*(p + 2)*Sin[e + f*x]^2, x], x], x] /;
 FreeQ[{a, b, e, f, A, B}, x] && LtQ[p, -1] && NeQ[a + b, 0]
```

rule 3656
```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2], x_Symbol] :> Simp[(Sqrt[a
]/f)*EllipticE[e + f*x, -b/a], x] /; FreeQ[{a, b, e, f}, x] && GtQ[a, 0]
```

rule 3657
```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2], x_Symbol] :> Simp[Sqrt[a
+ b*Sin[e + f*x]^2]/Sqrt[1 + b*(Sin[e + f*x]^2/a)]   Int[Sqrt[1 + (b*Sin[e
+ f*x]^2)/a], x], x] /; FreeQ[{a, b, e, f}, x] &&  !GtQ[a, 0]
```

rule 3661
```
Int[1/Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2], x_Symbol] :> Simp[(1/(S
qrt[a]*f))*EllipticF[e + f*x, -b/a], x] /; FreeQ[{a, b, e, f}, x] && GtQ[a,
 0]
```

rule 3662
```
Int[1/Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2], x_Symbol] :> Simp[Sqrt[
1 + b*(Sin[e + f*x]^2/a)]/Sqrt[a + b*Sin[e + f*x]^2]   Int[1/Sqrt[1 + (b*Si
n[e + f*x]^2)/a], x], x] /; FreeQ[{a, b, e, f}, x] &&  !GtQ[a, 0]
```

rule 3663
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> Simp[(-b)*C
os[e + f*x]*Sin[e + f*x]*((a + b*Sin[e + f*x]^2)^(p + 1)/(2*a*f*(p + 1)*(a
+ b))), x] + Simp[1/(2*a*(p + 1)*(a + b))   Int[(a + b*Sin[e + f*x]^2)^(p +
 1)*Simp[2*a*(p + 1) + b*(2*p + 3) - 2*b*(p + 2)*Sin[e + f*x]^2, x], x], x]
 /; FreeQ[{a, b, e, f}, x] && NeQ[a + b, 0] && LtQ[p, -1]
```

Maple [A] (verified)

Time = 1.45 (sec) , antiderivative size = 406, normalized size of antiderivative = 1.60

method	result
default	$\frac{\sqrt{\left(a+b\sinh(fx+e)^{2}\right)\cosh(fx+e)^{2}}\left(-\frac{\sinh(fx+e)\sqrt{\left(a+b\sinh(fx+e)^{2}\right)\cosh(fx+e)^{2}}}{3ba(a-b)\left(\sinh(fx+e)^{2}+\frac{a}{b}\right)^{2}}-\frac{2b\cosh(fx+e)^{2}\sinh(fx+e)(2a-b)}{3a^{2}(a-b)^{2}\sqrt{\left(a+b\sinh(fx+e)^{2}\right)\cosh(fx+e)^{2}}}+\frac{(3 \cdots}{}\right.}{}$
risch	Expression too large to display

input
```
int(1/(a+b*sinh(f*x+e)^2)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
((a+b*sinh(f*x+e)^2)*cosh(f*x+e)^2)^(1/2)*(-1/3/b/a/(a-b)*sinh(f*x+e)*((a+
b*sinh(f*x+e)^2)*cosh(f*x+e)^2)^(1/2)/(sinh(f*x+e)^2+1/b*a)^2-2/3*b*cosh(f
*x+e)^2/a^2/(a-b)^2*sinh(f*x+e)*(2*a-b)/((a+b*sinh(f*x+e)^2)*cosh(f*x+e)^2
)^(1/2)+(3*a-b)/(3*a^3-6*a^2*b+3*a*b^2)/(-b/a)^(1/2)*((a+b*sinh(f*x+e)^2)/
a)^(1/2)*(cosh(f*x+e)^2)^(1/2)/((a+b*sinh(f*x+e)^2)*cosh(f*x+e)^2)^(1/2)*E
llipticF(sinh(f*x+e)*(-b/a)^(1/2),(1/b*a)^(1/2))-2/3*b*(2*a-b)/a^2/(a-b)^2
/(-b/a)^(1/2)*((a+b*sinh(f*x+e)^2)/a)^(1/2)*(cosh(f*x+e)^2)^(1/2)/((a+b*si
nh(f*x+e)^2)*cosh(f*x+e)^2)^(1/2)*(EllipticF(sinh(f*x+e)*(-b/a)^(1/2),(1/b
*a)^(1/2))-EllipticE(sinh(f*x+e)*(-b/a)^(1/2),(1/b*a)^(1/2))))/cosh(f*x+e)
/(a+b*sinh(f*x+e)^2)^(1/2)/f
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 5442 vs. $2(225) = 450$.

Time = 0.25 (sec) , antiderivative size = 5442, normalized size of antiderivative = 21.51

$$\int \frac{1}{\left(a+b\sinh^2(e+fx)\right)^{5/2}}\,dx = \text{Too large to display}$$

input
```
integrate(1/(a+b*sinh(f*x+e)^2)^(5/2),x, algorithm="fricas")
```

output
```
Too large to include
```

Sympy [F]

$$\int \frac{1}{\left(a+b\sinh^2(e+fx)\right)^{5/2}}\,dx = \int \frac{1}{\left(a+b\sinh^2\left(e+fx\right)\right)^{\frac{5}{2}}}\,dx$$

input
```
integrate(1/(a+b*sinh(f*x+e)**2)**(5/2),x)
```

output
```
Integral((a + b*sinh(e + f*x)**2)**(-5/2), x)
```

Maxima [F]

$$\int \frac{1}{\left(a+b\sinh^2(e+fx)\right)^{5/2}}\,dx = \int \frac{1}{\left(b\sinh\left(fx+e\right)^2+a\right)^{\frac{5}{2}}}\,dx$$

input
```
integrate(1/(a+b*sinh(f*x+e)^2)^(5/2),x, algorithm="maxima")
```

output
```
integrate((b*sinh(f*x + e)^2 + a)^(-5/2), x)
```

Giac [F]

$$\int \frac{1}{\left(a+b\sinh^2(e+fx)\right)^{5/2}}\,dx = \int \frac{1}{\left(b\sinh\left(fx+e\right)^2+a\right)^{\frac{5}{2}}}\,dx$$

input
```
integrate(1/(a+b*sinh(f*x+e)^2)^(5/2),x, algorithm="giac")
```

output
```
integrate((b*sinh(f*x + e)^2 + a)^(-5/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\left(a+b\sinh^2(e+fx)\right)^{5/2}}\,dx=\int \frac{1}{{\left(b\,{\mathrm{sinh}\left(e+f\,x\right)}^2+a\right)}^{5/2}} \,d x$$

input
```
int(1/(a + b*sinh(e + f*x)^2)^(5/2),x)
```

output
```
int(1/(a + b*sinh(e + f*x)^2)^(5/2), x)
```

Reduce [F]

$$\int \frac{1}{\left(a+b\sinh^2(e+fx)\right)^{5/2}}\,dx=\int \frac{\sqrt{\sinh \left(f x +e \right)^{2} b +a}}{\sinh \left(f x +e \right)^{6} b^{3}+3 \sinh \left(f x +e \right)^{4} a \,b^{2}+3 \sinh \left(f x +e \right)^{2} a^{2} b +a^{3}}d x$$

input
```
int(1/(a+b*sinh(f*x+e)^2)^(5/2),x)
```

output
```
int(sqrt(sinh(e + f*x)**2*b + a)/(sinh(e + f*x)**6*b**3 + 3*sinh(e + f*x)*
*4*a*b**2 + 3*sinh(e + f*x)**2*a**2*b + a**3),x)
```

3.363 $\int \frac{\operatorname{sech}^2(e+fx)}{(a+b\sinh^2(e+fx))^{5/2}}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 309

$$\int \frac{\operatorname{sech}^2(e+fx)}{(a+b\sinh^2(e+fx))^{5/2}}\,dx = \frac{b(3a+b)\cosh(e+fx)\sinh(e+fx)}{3a(a-b)^2f\,(a+b\sinh^2(e+fx))^{3/2}} + \frac{\sqrt{b}(3a^2+7ab-2b^2)\cosh(e+fx)E\left(\arctan\left(\frac{\sqrt{b}\sinh(e+fx)}{\sqrt{a}}\right)|1-\frac{a}{b}\right)}{3a^{3/2}(a-b)^3f\sqrt{\frac{a\cosh^2(e+fx)}{a+b\sinh^2(e+fx)}}\sqrt{a+b\sinh^2(e+fx)}} - \frac{(9a-b)\sqrt{b}\cosh(e+fx)\operatorname{EllipticF}\left(\arctan\left(\frac{\sqrt{b}\sinh(e+fx)}{\sqrt{a}}\right),1-\frac{a}{b}\right)}{3\sqrt{a}(a-b)^3f\sqrt{\frac{a\cosh^2(e+fx)}{a+b\sinh^2(e+fx)}}\sqrt{a+b\sinh^2(e+fx)}} + \frac{\tanh(e+fx)}{(a-b)f\,(a+b\sinh^2(e+fx))^{3/2}}$$

output
```
1/3*b*(3*a+b)*cosh(f*x+e)*sinh(f*x+e)/a/(a-b)^2/f/(a+b*sinh(f*x+e)^2)^(3/2
)+1/3*b^(1/2)*(3*a^2+7*a*b-2*b^2)*cosh(f*x+e)*EllipticE(b^(1/2)*sinh(f*x+e
)/a^(1/2)/(1+b*sinh(f*x+e)^2/a)^(1/2),(1-a/b)^(1/2))/a^(3/2)/(a-b)^3/f/(a*
cosh(f*x+e)^2/(a+b*sinh(f*x+e)^2))^(1/2)/(a+b*sinh(f*x+e)^2)^(1/2)-1/3*(9*
a-b)*b^(1/2)*cosh(f*x+e)*InverseJacobiAM(arctan(b^(1/2)*sinh(f*x+e)/a^(1/2
)),(1-a/b)^(1/2))/a^(1/2)/(a-b)^3/f/(a*cosh(f*x+e)^2/(a+b*sinh(f*x+e)^2))^
(1/2)/(a+b*sinh(f*x+e)^2)^(1/2)+tanh(f*x+e)/(a-b)/f/(a+b*sinh(f*x+e)^2)^(3
/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 2.64 (sec) , antiderivative size = 260, normalized size of antiderivative = 0.84

$$\int \frac{\operatorname{sech}^2(e+fx)}{\left(a+b\sinh^2(e+fx)\right)^{5/2}}\,dx = \frac{2ia^2(3a^2+7ab-2b^2)\left(\frac{2a-b+b\cosh(2(e+fx))}{a}\right)^{3/2}E\left(i(e+fx)\,\middle|\,\frac{b}{a}\right)-2ia^2(3a^2}{}$$

input
```
Integrate[Sech[e + f*x]^2/(a + b*Sinh[e + f*x]^2)^(5/2),x]
```

output
```
((2*I)*a^2*(3*a^2 + 7*a*b - 2*b^2)*((2*a - b + b*Cosh[2*(e + f*x)])/a)^(3/
2)*EllipticE[I*(e + f*x), b/a] - (2*I)*a^2*(3*a^2 - 2*a*b - b^2)*((2*a - b
 + b*Cosh[2*(e + f*x)])/a)^(3/2)*EllipticF[I*(e + f*x), b/a] + ((24*a^4 -
24*a^3*b + 41*a^2*b^2 - 19*a*b^3 + 2*b^4 + 4*a*b*(6*a^2 + 5*a*b - 3*b^2)*C
osh[2*(e + f*x)] + b^2*(3*a^2 + 7*a*b - 2*b^2)*Cosh[4*(e + f*x)])*Tanh[e +
 f*x])/Sqrt[2])/(6*a^2*(a - b)^3*f*(2*a - b + b*Cosh[2*(e + f*x)])^(3/2))
```

Rubi [A] (verified)

Time = 0.57 (sec) , antiderivative size = 365, normalized size of antiderivative = 1.18, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.320, Rules used = {3042, 3671, 316, 27, 402, 400, 313, 320}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\operatorname{sech}^2(e+fx)}{\left(a+b\sinh^2(e+fx)\right)^{5/2}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{\cos(ie+ifx)^2\left(a-b\sin(ie+ifx)^2\right)^{5/2}}dx$$

$$\Big\downarrow \text{3671}$$

$$\frac{\sqrt{\cosh^2(e+fx)}\operatorname{sech}(e+fx)\int \frac{1}{\left(\sinh^2(e+fx)+1\right)^{3/2}\left(b\sinh^2(e+fx)+a\right)^{5/2}}d\sinh(e+fx)}{f}$$

$$\Big\downarrow \; 316$$

$$\frac{\sqrt{\cosh^2(e+fx)}\operatorname{sech}(e+fx)\left(\frac{\sinh(e+fx)}{(a-b)\sqrt{\sinh^2(e+fx)+1}\left(a+b\sinh^2(e+fx)\right)^{3/2}}-\frac{\int \frac{b\left(1-3\sinh^2(e+fx)\right)}{\sqrt{\sinh^2(e+fx)+1}\left(b\sinh^2(e+fx)+a\right)^{5/2}}d\sinh(e+fx)}{a-b}\right.}{f}$$

$$\Big\downarrow \; 27$$

$$\frac{\sqrt{\cosh^2(e+fx)}\operatorname{sech}(e+fx)\left(\frac{\sinh(e+fx)}{(a-b)\sqrt{\sinh^2(e+fx)+1}\left(a+b\sinh^2(e+fx)\right)^{3/2}}-\frac{b\int \frac{1-3\sinh^2(e+fx)}{\sqrt{\sinh^2(e+fx)+1}\left(b\sinh^2(e+fx)+a\right)^{5/2}}d\sinh(e+f}{a-b}\right.}{f}$$

$$\Big\downarrow \; 402$$

$$\frac{\sqrt{\cosh^2(e+fx)}\operatorname{sech}(e+fx)\left(\frac{\sinh(e+fx)}{(a-b)\sqrt{\sinh^2(e+fx)+1}\left(a+b\sinh^2(e+fx)\right)^{3/2}}-\frac{b\left(\frac{\int \frac{2(3a-b)-(3a+b)\sinh^2(e+fx)}{\sqrt{\sinh^2(e+fx)+1}\left(b\sinh^2(e+fx)+a\right)^{3/2}}d\sinh(e+f}{3a(a-b)}\right.}{a-b}\right.}{f}$$

$$\Big\downarrow \; 400$$

$$\frac{\sqrt{\cosh^2(e+fx)}\operatorname{sech}(e+fx)\left(\frac{\sinh(e+fx)}{(a-b)\sqrt{\sinh^2(e+fx)+1}\left(a+b\sinh^2(e+fx)\right)^{3/2}}-\frac{b\left(\frac{(9a-b)\int \frac{1}{\sqrt{\sinh^2(e+fx)+1}\sqrt{b\sinh^2(e+fx)+a}}d\sinh(e}{a-b}\right.}{}\right.}{f}$$

$$\Big\downarrow \; 313$$

$$\frac{\sqrt{\cosh^2(e+fx)}\text{sech}(e+fx)\left(\frac{\sinh(e+fx)}{(a-b)\sqrt{\sinh^2(e+fx)+1}\left(a+b\sinh^2(e+fx)\right)^{3/2}}-\frac{b\left(\frac{(9a-b)\int\frac{1}{\sqrt{\sinh^2(e+fx)+1}\sqrt{b\sinh^2(e+fx)+a}}d\sinh(e}{a-b}\right.}{}\right.}{f}$$

$\downarrow$ 320

$$\frac{\sqrt{\cosh^2(e+fx)}\text{sech}(e+fx)\left(\frac{\sinh(e+fx)}{(a-b)\sqrt{\sinh^2(e+fx)+1}\left(a+b\sinh^2(e+fx)\right)^{3/2}}-\frac{b\left(\frac{(9a-b)\sqrt{a+b\sinh^2(e+fx)}\,\text{EllipticF}\left(\arctan(\sinh(e+fx\right.}{a(a-b)\sqrt{\sinh^2(e+fx)+1}\sqrt{\frac{a+b\sinh^2(e+fx)}{a\left(\sinh^2(e+fx)+1\right.}}}\right.}{}\right.}{f}$$

input
```
Int[Sech[e + f*x]^2/(a + b*Sinh[e + f*x]^2)^(5/2),x]
```

output
```
(Sqrt[Cosh[e + f*x]^2]*Sech[e + f*x]*(Sinh[e + f*x]/((a - b)*Sqrt[1 + Sinh
[e + f*x]^2]*(a + b*Sinh[e + f*x]^2)^(3/2)) - (b*(-1/3*((3*a + b)*Sinh[e +
 f*x]*Sqrt[1 + Sinh[e + f*x]^2])/(a*(a - b)*(a + b*Sinh[e + f*x]^2)^(3/2))
 + (-(((3*a^2 + 7*a*b - 2*b^2)*EllipticE[ArcTan[(Sqrt[b]*Sinh[e + f*x])/Sq
rt[a]], 1 - a/b]*Sqrt[1 + Sinh[e + f*x]^2])/(Sqrt[a]*(a - b)*Sqrt[b]*Sqrt[
(a*(1 + Sinh[e + f*x]^2))/(a + b*Sinh[e + f*x]^2)]*Sqrt[a + b*Sinh[e + f*x
]^2])) + ((9*a - b)*EllipticF[ArcTan[Sinh[e + f*x]], 1 - b/a]*Sqrt[a + b*S
inh[e + f*x]^2])/(a*(a - b)*Sqrt[1 + Sinh[e + f*x]^2]*Sqrt[(a + b*Sinh[e +
 f*x]^2)/(a*(1 + Sinh[e + f*x]^2))]))/(3*a*(a - b))))/(a - b)))/f
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 313
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/((c_) + (d_.)*(x_)^2)^(3/2), x_Symbol] :> Sim
p[(Sqrt[a + b*x^2]/(c*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(c
+ d*x^2)))]))*EllipticE[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; FreeQ
[{a, b, c, d}, x] && PosQ[b/a] && PosQ[d/c]
```

rule 316
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_), x_Symbol] :> Sim
p[(-b)*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^(q + 1)/(2*a*(p + 1)*(b*c - a*d))
), x] + Simp[1/(2*a*(p + 1)*(b*c - a*d))   Int[(a + b*x^2)^(p + 1)*(c + d*x
^2)^q*Simp[b*c + 2*(p + 1)*(b*c - a*d) + d*b*(2*(p + q + 2) + 1)*x^2, x], x
], x] /; FreeQ[{a, b, c, d, q}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] &&  !
( !IntegerQ[p] && IntegerQ[q] && LtQ[q, -1]) && IntBinomialQ[a, b, c, d, 2,
 p, q, x]
```

rule 320
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(Sqrt[a + b*x^2]/(a*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(
c + d*x^2)))]))*EllipticF[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; Fre
eQ[{a, b, c, d}, x] && PosQ[d/c] && PosQ[b/a] &&  !SimplerSqrtQ[b/a, d/c]
```

rule 400
```
Int[((e_) + (f_.)*(x_)^2)/(Sqrt[(a_) + (b_.)*(x_)^2]*((c_) + (d_.)*(x_)^2)^
(3/2)), x_Symbol] :> Simp[(b*e - a*f)/(b*c - a*d)   Int[1/(Sqrt[a + b*x^2]*
Sqrt[c + d*x^2]), x], x] - Simp[(d*e - c*f)/(b*c - a*d)   Int[Sqrt[a + b*x^
2]/(c + d*x^2)^(3/2), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && PosQ[b/a] &
& PosQ[d/c]
```

rule 402
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_.)*((e_) + (f_.)*(x
_)^2), x_Symbol] :> Simp[(-(b*e - a*f))*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^
(q + 1)/(a*2*(b*c - a*d)*(p + 1))), x] + Simp[1/(a*2*(b*c - a*d)*(p + 1))
 Int[(a + b*x^2)^(p + 1)*(c + d*x^2)^q*Simp[c*(b*e - a*f) + e*2*(b*c - a*d)
*(p + 1) + d*(b*e - a*f)*(2*(p + q + 2) + 1)*x^2, x], x], x] /; FreeQ[{a, b
, c, d, e, f, q}, x] && LtQ[p, -1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3671
```
Int[cos[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(
p_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Simp[ff*(Sqrt[
Cos[e + f*x]^2]/(f*Cos[e + f*x]))   Subst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a
 + b*ff^2*x^2)^p, x], x, Sin[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x]
 && IntegerQ[m/2] &&  !IntegerQ[p]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 1001 vs. $2(295) = 590$.

Time = 6.72 (sec) , antiderivative size = 1002, normalized size of antiderivative = 3.24

method	result	size
default	Expression too large to display	1002
risch	Expression too large to display	88646

input
```
int(sech(f*x+e)^2/(a+b*sinh(f*x+e)^2)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
1/3*(3*(-b/a)^(1/2)*a^2*b^2*sinh(f*x+e)^5+7*(-b/a)^(1/2)*a*b^3*sinh(f*x+e)
^5-2*(-b/a)^(1/2)*b^4*sinh(f*x+e)^5-6*((a+b*sinh(f*x+e)^2)/a)^(1/2)*(cosh(
f*x+e)^2)^(1/2)*EllipticF(sinh(f*x+e)*(-b/a)^(1/2),(1/b*a)^(1/2))*a^2*b^2*
sinh(f*x+e)^2+8*((a+b*sinh(f*x+e)^2)/a)^(1/2)*(cosh(f*x+e)^2)^(1/2)*Ellipt
icF(sinh(f*x+e)*(-b/a)^(1/2),(1/b*a)^(1/2))*a*b^3*sinh(f*x+e)^2-2*((a+b*si
nh(f*x+e)^2)/a)^(1/2)*(cosh(f*x+e)^2)^(1/2)*EllipticF(sinh(f*x+e)*(-b/a)^(
1/2),(1/b*a)^(1/2))*b^4*sinh(f*x+e)^2-3*((a+b*sinh(f*x+e)^2)/a)^(1/2)*(cos
h(f*x+e)^2)^(1/2)*EllipticE(sinh(f*x+e)*(-b/a)^(1/2),(1/b*a)^(1/2))*a^2*b^
2*sinh(f*x+e)^2-7*((a+b*sinh(f*x+e)^2)/a)^(1/2)*(cosh(f*x+e)^2)^(1/2)*Elli
pticE(sinh(f*x+e)*(-b/a)^(1/2),(1/b*a)^(1/2))*a*b^3*sinh(f*x+e)^2+2*((a+b*
sinh(f*x+e)^2)/a)^(1/2)*(cosh(f*x+e)^2)^(1/2)*EllipticE(sinh(f*x+e)*(-b/a)
^(1/2),(1/b*a)^(1/2))*b^4*sinh(f*x+e)^2+6*(-b/a)^(1/2)*a^3*b*sinh(f*x+e)^3
+8*(-b/a)^(1/2)*a^2*b^2*sinh(f*x+e)^3+4*(-b/a)^(1/2)*a*b^3*sinh(f*x+e)^3-2
*(-b/a)^(1/2)*b^4*sinh(f*x+e)^3-6*((a+b*sinh(f*x+e)^2)/a)^(1/2)*(cosh(f*x+
e)^2)^(1/2)*EllipticF(sinh(f*x+e)*(-b/a)^(1/2),(1/b*a)^(1/2))*a^3*b+8*((a+
b*sinh(f*x+e)^2)/a)^(1/2)*(cosh(f*x+e)^2)^(1/2)*EllipticF(sinh(f*x+e)*(-b/
a)^(1/2),(1/b*a)^(1/2))*a^2*b^2-2*((a+b*sinh(f*x+e)^2)/a)^(1/2)*(cosh(f*x+
e)^2)^(1/2)*EllipticF(sinh(f*x+e)*(-b/a)^(1/2),(1/b*a)^(1/2))*a*b^3-3*((a+
b*sinh(f*x+e)^2)/a)^(1/2)*(cosh(f*x+e)^2)^(1/2)*EllipticE(sinh(f*x+e)*(-b/
a)^(1/2),(1/b*a)^(1/2))*a^3*b-7*((a+b*sinh(f*x+e)^2)/a)^(1/2)*(cosh(f*x...
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 8928 vs. $2(295) = 590$.

Time = 0.44 (sec) , antiderivative size = 8928, normalized size of antiderivative = 28.89

$$\int \frac{\operatorname{sech}^2(e+fx)}{\left(a+b\sinh^2(e+fx)\right)^{5/2}}\,dx = \text{Too large to display}$$

input
```
integrate(sech(f*x+e)^2/(a+b*sinh(f*x+e)^2)^(5/2),x, algorithm="fricas")
```

output
```
Too large to include
```

Sympy [F]

$$\int \frac{\operatorname{sech}^2(e+fx)}{(a+b\sinh^2(e+fx))^{5/2}}\,dx = \int \frac{\operatorname{sech}^2(e+fx)}{(a+b\sinh^2(e+fx))^{\frac{5}{2}}}\,dx$$

input
```
integrate(sech(f*x+e)**2/(a+b*sinh(f*x+e)**2)**(5/2),x)
```

output
```
Integral(sech(e + f*x)**2/(a + b*sinh(e + f*x)**2)**(5/2), x)
```

Maxima [F]

$$\int \frac{\operatorname{sech}^2(e+fx)}{(a+b\sinh^2(e+fx))^{5/2}}\,dx = \int \frac{\operatorname{sech}(fx+e)^2}{(b\sinh(fx+e)^2+a)^{\frac{5}{2}}}\,dx$$

input
```
integrate(sech(f*x+e)^2/(a+b*sinh(f*x+e)^2)^(5/2),x, algorithm="maxima")
```

output
```
integrate(sech(f*x + e)^2/(b*sinh(f*x + e)^2 + a)^(5/2), x)
```

Giac [F]

$$\int \frac{\operatorname{sech}^2(e+fx)}{(a+b\sinh^2(e+fx))^{5/2}}\,dx = \int \frac{\operatorname{sech}(fx+e)^2}{(b\sinh(fx+e)^2+a)^{\frac{5}{2}}}\,dx$$

input
```
integrate(sech(f*x+e)^2/(a+b*sinh(f*x+e)^2)^(5/2),x, algorithm="giac")
```

output
```
integrate(sech(f*x + e)^2/(b*sinh(f*x + e)^2 + a)^(5/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\operatorname{sech}^2(e+fx)}{\left(a+b\sinh^2(e+fx)\right)^{5/2}}\,dx=\int \frac{1}{\cosh\left(e+f\,x\right)^2\,\left(b\sinh\left(e+f\,x\right)^2+a\right)^{5/2}}\,dx$$

input
```
int(1/(cosh(e + f*x)^2*(a + b*sinh(e + f*x)^2)^(5/2)),x)
```

output
```
int(1/(cosh(e + f*x)^2*(a + b*sinh(e + f*x)^2)^(5/2)), x)
```

Reduce [F]

$$\int \frac{\operatorname{sech}^2(e+fx)}{\left(a+b\sinh^2(e+fx)\right)^{5/2}}\,dx=\int \frac{\sqrt{\sinh\left(fx+e\right)^2 b+a}\,\operatorname{sech}(fx+e)^2}{\sinh\left(fx+e\right)^6 b^3+3\sinh\left(fx+e\right)^4 a\,b^2+3\sinh\left(fx+e\right)^2 a^2 b+a^3}dx$$

input
```
int(sech(f*x+e)^2/(a+b*sinh(f*x+e)^2)^(5/2),x)
```

output
```
int((sqrt(sinh(e + f*x)**2*b + a)*sech(e + f*x)**2)/(sinh(e + f*x)**6*b**3
 + 3*sinh(e + f*x)**4*a*b**2 + 3*sinh(e + f*x)**2*a**2*b + a**3),x)
```

3.364 $\int (d\cosh(e+fx))^m \left(a+b\sinh^2(e+fx)\right)^p \, dx$

Optimal result

Integrand size = 25, antiderivative size = 117

$$\int (d\cosh(e+fx))^m \left(a+b\sinh^2(e+fx)\right)^p \, dx$$
$$= \frac{d \,\text{AppellF1}\left(\frac{1}{2}, \frac{1-m}{2}, -p, \frac{3}{2}, -\sinh^2(e+fx), -\frac{b\sinh^2(e+fx)}{a}\right) (d\cosh(e+fx))^{-1+m} \cosh^2(e+fx)^{\frac{1-m}{2}} \sinh}{f}$$

output
```
d*AppellF1(1/2,1/2-1/2*m,-p,3/2,-sinh(f*x+e)^2,-b*sinh(f*x+e)^2/a)*(d*cosh
(f*x+e))^(-1+m)*(cosh(f*x+e)^2)^(1/2-1/2*m)*sinh(f*x+e)*(a+b*sinh(f*x+e)^2
)^p/f/((1+b*sinh(f*x+e)^2/a)^p)
```

Mathematica [F]

$$\int (d\cosh(e+fx))^m \left(a+b\sinh^2(e+fx)\right)^p \, dx$$
$$= \int (d\cosh(e+fx))^m \left(a+b\sinh^2(e+fx)\right)^p \, dx$$

input
```
Integrate[(d*Cosh[e + f*x])^m*(a + b*Sinh[e + f*x]^2)^p,x]
```

output
```
Integrate[(d*Cosh[e + f*x])^m*(a + b*Sinh[e + f*x]^2)^p, x]
```

Rubi [A] (verified)

Time = 0.39 (sec) , antiderivative size = 117, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.160$, Rules used = {3042, 3672, 334, 333}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (d\cosh(e+fx))^m \left(a+b\sinh^2(e+fx)\right)^p dx$$

$$\Big\downarrow \ 3042$$

$$\int (d\cos(ie+ifx))^m \left(a-b\sin(ie+ifx)^2\right)^p dx$$

$$\Big\downarrow \ 3672$$

$$\frac{d\cosh^2(e+fx)^{\frac{1-m}{2}}(d\cosh(e+fx))^{m-1}\int \left(\sinh^2(e+fx)+1\right)^{\frac{m-1}{2}}\left(b\sinh^2(e+fx)+a\right)^p d\sinh(e+fx)}{f}$$

$$\Big\downarrow \ 334$$

$$\frac{d\cosh^2(e+fx)^{\frac{1-m}{2}}(d\cosh(e+fx))^{m-1}\left(a+b\sinh^2(e+fx)\right)^p\left(\frac{b\sinh^2(e+fx)}{a}+1\right)^{-p}\int \left(\sinh^2(e+fx)+1\right)^{\frac{m-1}{2}} (}{f}$$

$$\Big\downarrow \ 333$$

$$\frac{d\sinh(e+fx)\cosh^2(e+fx)^{\frac{1-m}{2}}(d\cosh(e+fx))^{m-1}\left(a+b\sinh^2(e+fx)\right)^p\left(\frac{b\sinh^2(e+fx)}{a}+1\right)^{-p}\text{AppellF1}\left(\frac{1}{2},\right.}{f}$$

input
```
Int[(d*Cosh[e + f*x])^m*(a + b*Sinh[e + f*x]^2)^p,x]
```

output
```
(d*AppellF1[1/2, (1 - m)/2, -p, 3/2, -Sinh[e + f*x]^2, -((b*Sinh[e + f*x]^
2)/a)]*(d*Cosh[e + f*x])^(-1 + m)*(Cosh[e + f*x]^2)^((1 - m)/2)*Sinh[e + f
*x]*(a + b*Sinh[e + f*x]^2)^p)/(f*(1 + (b*Sinh[e + f*x]^2)/a)^p)
```

Defintions of rubi rules used

rule 333
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_), x_Symbol] :> Sim
p[a^p*c^q*x*AppellF1[1/2, -p, -q, 3/2, (-b)*(x^2/a), (-d)*(x^2/c)], x] /; F
reeQ[{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && (IntegerQ[p] || GtQ[a,
0]) && (IntegerQ[q] || GtQ[c, 0])
```

rule 334
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_), x_Symbol] :> Sim
p[a^IntPart[p]*((a + b*x^2)^FracPart[p]/(1 + b*(x^2/a))^FracPart[p])   Int[
(1 + b*(x^2/a))^p*(c + d*x^2)^q, x], x] /; FreeQ[{a, b, c, d, p, q}, x] &&
NeQ[b*c - a*d, 0] &&  !(IntegerQ[p] || GtQ[a, 0])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3672
```
Int[(cos[(e_.) + (f_.)*(x_)]*(d_.))^(m_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)]^2)^(p_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Simp[f
f*d^(2*IntPart[(m - 1)/2] + 1)*((d*Cos[e + f*x])^(2*FracPart[(m - 1)/2])/(f
*(Cos[e + f*x]^2)^FracPart[(m - 1)/2]))   Subst[Int[(1 - ff^2*x^2)^((m - 1)
/2)*(a + b*ff^2*x^2)^p, x], x, Sin[e + f*x]/ff], x]] /; FreeQ[{a, b, d, e,
f, m, p}, x] &&  !IntegerQ[m]
```

Maple [F]

$$\int (d\cosh (fx+e))^m \left(a+b\sinh (fx+e)^2\right)^p dx$$

input
```
int((d*cosh(f*x+e))^m*(a+b*sinh(f*x+e)^2)^p,x)
```

output
```
int((d*cosh(f*x+e))^m*(a+b*sinh(f*x+e)^2)^p,x)
```

Fricas [F]

$$\int (d\cosh(e+fx))^m \left(a+b\sinh^2(e+fx)\right)^p \, dx = \int \left(b\sinh\left(fx+e\right)^2+a\right)^p (d\cosh\left(fx+e\right))^m \, dx$$

input
```
integrate((d*cosh(f*x+e))^m*(a+b*sinh(f*x+e)^2)^p,x, algorithm="fricas")
```

output
```
integral((b*sinh(f*x + e)^2 + a)^p*(d*cosh(f*x + e))^m, x)
```

Sympy [F(-1)]

Timed out.

$$\int (d\cosh(e+fx))^m \left(a+b\sinh^2(e+fx)\right)^p \, dx = \text{Timed out}$$

input
```
integrate((d*cosh(f*x+e))**m*(a+b*sinh(f*x+e)**2)**p,x)
```

output
```
Timed out
```

Maxima [F]

$$\int (d\cosh(e+fx))^m \left(a+b\sinh^2(e+fx)\right)^p \, dx = \int \left(b\sinh\left(fx+e\right)^2+a\right)^p (d\cosh\left(fx+e\right))^m \, dx$$

input
```
integrate((d*cosh(f*x+e))^m*(a+b*sinh(f*x+e)^2)^p,x, algorithm="maxima")
```

output
```
integrate((b*sinh(f*x + e)^2 + a)^p*(d*cosh(f*x + e))^m, x)
```

Giac [F]

$$\int (d\cosh(e+fx))^m \left(a+b\sinh^2(e+fx)\right)^p \, dx = \int \left(b\sinh\left(fx+e\right)^2+a\right)^p (d\cosh\left(fx+e\right))^m \, dx$$

input
```
integrate((d*cosh(f*x+e))^m*(a+b*sinh(f*x+e)^2)^p,x, algorithm="giac")
```

output
```
integrate((b*sinh(f*x + e)^2 + a)^p*(d*cosh(f*x + e))^m, x)
```

Mupad [F(-1)]

Timed out.

$$\int (d\cosh(e+fx))^m \left(a+b\sinh^2(e+fx)\right)^p \, dx = \int (d\cosh(e+f\,x))^m \left(b\sinh(e+f\,x)^2+a\right)^p \, dx$$

input
```
int((d*cosh(e + f*x))^m*(a + b*sinh(e + f*x)^2)^p,x)
```

output
```
int((d*cosh(e + f*x))^m*(a + b*sinh(e + f*x)^2)^p, x)
```

Reduce [F]

$$
\int (d\cosh(e+fx))^m \left(a+b\sinh^2(e+fx)\right)^p \, dx
$$
$$
= d^m\left(\int \left(\sinh\left(fx+e\right)^2 b+a\right)^p \cosh\left(fx+e\right)^m dx\right)
$$

input
```
int((d*cosh(f*x+e))^m*(a+b*sinh(f*x+e)^2)^p,x)
```

output
```
d**m*int((sinh(e + f*x)**2*b + a)**p*cosh(e + f*x)**m,x)
```

3.365 $\int \cosh^5(e+fx) \left(a+b\sinh^2(e+fx)\right)^p \, dx$

Optimal result

Integrand size = 23, antiderivative size = 208

$$\begin{aligned}\int \cosh^5(e+fx) \left(a+b\sinh^2(e+fx)\right)^p \, dx &= -\frac{(3a-2b(5+2p))\sinh(e+fx)\left(a+b\sinh^2(e+fx)\right)^{1+p}}{b^2f(3+2p)(5+2p)} \\ &+ \frac{\sinh^3(e+fx)\left(a+b\sinh^2(e+fx)\right)^{1+p}}{bf(5+2p)} \\ &+ \frac{(3a^2-2ab(5+2p)+b^2(15+16p+4p^2))\,\text{Hypergeometric2F1}\left(\frac{1}{2},-p,\frac{3}{2},-\frac{b\sinh^2(e+fx)}{a}\right)\sinh(e+fx)\,(}{b^2f(3+2p)(5+2p)}\end{aligned}$$

output
```
-(3*a-2*b*(5+2*p))*sinh(f*x+e)*(a+b*sinh(f*x+e)^2)^(p+1)/b^2/f/(3+2*p)/(5+
2*p)+sinh(f*x+e)^3*(a+b*sinh(f*x+e)^2)^(p+1)/b/f/(5+2*p)+(3*a^2-2*a*b*(5+2
*p)+b^2*(4*p^2+16*p+15))*hypergeom([1/2, -p],[3/2],-b*sinh(f*x+e)^2/a)*sin
h(f*x+e)*(a+b*sinh(f*x+e)^2)^p/b^2/f/(3+2*p)/(5+2*p)/((1+b*sinh(f*x+e)^2/a
)^p)
```

Mathematica [F]

$$\int \cosh^5(e+fx)\left(a+b\sinh^2(e+fx)\right)^p\,dx=\int \cosh^5(e+fx)\left(a+b\sinh^2(e+fx)\right)^p\,dx$$

input
```
Integrate[Cosh[e + f*x]^5*(a + b*Sinh[e + f*x]^2)^p,x]
```

output
```
Integrate[Cosh[e + f*x]^5*(a + b*Sinh[e + f*x]^2)^p, x]
```

Rubi [A] (verified)

Time = 0.44 (sec) , antiderivative size = 210, normalized size of antiderivative = 1.01, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.304, Rules used = {3042, 3669, 318, 25, 299, 238, 237}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cosh^5(e+fx)\left(a+b\sinh^2(e+fx)\right)^p\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \cos(ie+ifx)^5\left(a-b\sin(ie+ifx)^2\right)^p dx$$

$$\Big\downarrow \text{3669}$$

$$\frac{\int \left(\sinh^2(e+fx)+1\right)^2\left(b\sinh^2(e+fx)+a\right)^p d\sinh(e+fx)}{f}$$

$$\Big\downarrow \text{318}$$

$$\frac{\frac{\int -\left(b\sinh^2(e+fx)+a\right)^p\left((3a-b(2p+7))\sinh^2(e+fx)+a-b(2p+5)\right)d\sinh(e+fx)}{b(2p+5)}+\frac{\sinh(e+fx)\left(\sinh^2(e+fx)+1\right)\left(a+b\sinh^2(e+fx)\right)^{p+1}}{b(2p+5)}}{f}$$

$$\Big\downarrow \text{25}$$

$$\frac{\frac{\sinh(e+fx)\left(\sinh^2(e+fx)+1\right)\left(a+b\sinh^2(e+fx)\right)^{p+1}}{b(2p+5)}-\frac{\int\left(b\sinh^2(e+fx)+a\right)^p\left((3a-7b-2bp)\sinh^2(e+fx)+a-5b-2bp\right)d\sinh(e+fx)}{b(2p+5)}}{f}$$

$\downarrow$ 299

$$\frac{\frac{\sinh(e+fx)\left(\sinh^2(e+fx)+1\right)\left(a+b\sinh^2(e+fx)\right)^{p+1}}{b(2p+5)}-\frac{\frac{(3a-b(2p+7))\sinh(e+fx)\left(a+b\sinh^2(e+fx)\right)^{p+1}}{b(2p+3)}-\frac{\left(3a^2-2ab(2p+5)+b^2\left(4p^2+16p+15\right)\right)\int\left(b\sinh\right.}{b(2p+3)}}{b(2p+5)}}{f}$$

$\downarrow$ 238

$$\frac{\frac{\sinh(e+fx)\left(\sinh^2(e+fx)+1\right)\left(a+b\sinh^2(e+fx)\right)^{p+1}}{b(2p+5)}-\frac{\frac{(3a-b(2p+7))\sinh(e+fx)\left(a+b\sinh^2(e+fx)\right)^{p+1}}{b(2p+3)}-\frac{\left(3a^2-2ab(2p+5)+b^2\left(4p^2+16p+15\right)\right)\left(a+b\,\mathrm{s}\right.}{}}{b(2p+5)}}{f}$$

$\downarrow$ 237

$$\frac{\frac{\sinh(e+fx)\left(\sinh^2(e+fx)+1\right)\left(a+b\sinh^2(e+fx)\right)^{p+1}}{b(2p+5)}-\frac{\frac{(3a-b(2p+7))\sinh(e+fx)\left(a+b\sinh^2(e+fx)\right)^{p+1}}{b(2p+3)}-\frac{\left(3a^2-2ab(2p+5)+b^2\left(4p^2+16p+15\right)\right)\sinh(e}{}}{b(2}}{f}$$

input
```
Int[Cosh[e + f*x]^5*(a + b*Sinh[e + f*x]^2)^p,x]
```

output
```
((Sinh[e + f*x]*(1 + Sinh[e + f*x]^2)*(a + b*Sinh[e + f*x]^2)^(1 + p))/(b*
(5 + 2*p)) - (((3*a - b*(7 + 2*p))*Sinh[e + f*x]*(a + b*Sinh[e + f*x]^2)^(
1 + p))/(b*(3 + 2*p)) - ((3*a^2 - 2*a*b*(5 + 2*p) + b^2*(15 + 16*p + 4*p^2
))*Hypergeometric2F1[1/2, -p, 3/2, -((b*Sinh[e + f*x]^2)/a)]*Sinh[e + f*x]
*(a + b*Sinh[e + f*x]^2)^p)/(b*(3 + 2*p)*(1 + (b*Sinh[e + f*x]^2)/a)^p))/(
b*(5 + 2*p)))/f
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 237
```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[a^p*x*Hypergeometric2F1[-
p, 1/2, 1/2 + 1, (-b)*(x^2/a)], x] /; FreeQ[{a, b, p}, x] &&  !IntegerQ[2*p
] && GtQ[a, 0]
```

rule 238
```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[a^IntPart[p]*((a + b*x^2)
^FracPart[p]/(1 + b*(x^2/a))^FracPart[p])   Int[(1 + b*(x^2/a))^p, x], x] /
; FreeQ[{a, b, p}, x] &&  !IntegerQ[2*p] &&  !GtQ[a, 0]
```

rule 299
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2), x_Symbol] :> Simp[d*x
*((a + b*x^2)^(p + 1)/(b*(2*p + 3))), x] - Simp[(a*d - b*c*(2*p + 3))/(b*(2
*p + 3))   Int[(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
 a*d, 0] && NeQ[2*p + 3, 0]
```

rule 318
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_), x_Symbol] :> Sim
p[d*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^(q - 1)/(b*(2*(p + q) + 1))), x] + S
imp[1/(b*(2*(p + q) + 1))   Int[(a + b*x^2)^p*(c + d*x^2)^(q - 2)*Simp[c*(b
*c*(2*(p + q) + 1) - a*d) + d*(b*c*(2*(p + 2*q - 1) + 1) - a*d*(2*(q - 1) +
 1))*x^2, x], x], x] /; FreeQ[{a, b, c, d, p}, x] && NeQ[b*c - a*d, 0] && G
tQ[q, 1] && NeQ[2*(p + q) + 1, 0] &&  !IGtQ[p, 1] && IntBinomialQ[a, b, c,
d, 2, p, q, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3669
```
Int[cos[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^
(p_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Simp[ff/f   S
ubst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a + b*ff^2*x^2)^p, x], x, Sin[e + f*x]
/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m - 1)/2]
```

Maple [F]

$$\int \cosh(fx+e)^5 \left(a+b\sinh(fx+e)^2\right)^p dx$$

input
```
int(cosh(f*x+e)^5*(a+b*sinh(f*x+e)^2)^p,x)
```

output
```
int(cosh(f*x+e)^5*(a+b*sinh(f*x+e)^2)^p,x)
```

Fricas [F]

$$\int \cosh^5(e+fx)\left(a+b\sinh^2(e+fx)\right)^p dx = \int \left(b\sinh(fx+e)^2+a\right)^p \cosh(fx+e)^5 dx$$

input
```
integrate(cosh(f*x+e)^5*(a+b*sinh(f*x+e)^2)^p,x, algorithm="fricas")
```

output
```
integral((b*sinh(f*x + e)^2 + a)^p*cosh(f*x + e)^5, x)
```

Sympy [F(-1)]

Timed out.

$$\int \cosh^5(e+fx)\left(a+b\sinh^2(e+fx)\right)^p dx = \text{Timed out}$$

input
```
integrate(cosh(f*x+e)**5*(a+b*sinh(f*x+e)**2)**p,x)
```

output
```
Timed out
```

Maxima [F]

$$\int \cosh^5(e+fx) \left(a+b\sinh^2(e+fx)\right)^p \, dx = \int \left(b\sinh\left(fx+e\right)^2+a\right)^p \cosh\left(fx+e\right)^5 \, dx$$

input
```
integrate(cosh(f*x+e)^5*(a+b*sinh(f*x+e)^2)^p,x, algorithm="maxima")
```

output
```
integrate((b*sinh(f*x + e)^2 + a)^p*cosh(f*x + e)^5, x)
```

Giac [F]

$$\int \cosh^5(e+fx) \left(a+b\sinh^2(e+fx)\right)^p \, dx = \int \left(b\sinh\left(fx+e\right)^2+a\right)^p \cosh\left(fx+e\right)^5 \, dx$$

input
```
integrate(cosh(f*x+e)^5*(a+b*sinh(f*x+e)^2)^p,x, algorithm="giac")
```

output
```
integrate((b*sinh(f*x + e)^2 + a)^p*cosh(f*x + e)^5, x)
```

Mupad [F(-1)]

Timed out.

$$\int \cosh^5(e+fx) \left(a+b\sinh^2(e+fx)\right)^p \, dx = \int \cosh(e+f\,x)^5 \left(b\sinh(e+f\,x)^2+a\right)^p \, dx$$

input
```
int(cosh(e + f*x)^5*(a + b*sinh(e + f*x)^2)^p,x)
```

output
```
int(cosh(e + f*x)^5*(a + b*sinh(e + f*x)^2)^p, x)
```

Reduce [F]

$$\int \cosh^5(e+fx)\left(a+b\sinh^2(e+fx)\right)^p\,dx = \text{too large to display}$$

input
```
int(cosh(f*x+e)^5*(a+b*sinh(f*x+e)^2)^p,x)
```

output
```
(80*e**(10*e + 10*f*x)*(e**(4*e + 4*f*x)*b + 4*e**(2*e + 2*f*x)*a - 2*e**(
2*e + 2*f*x)*b + b)**p*b**4*p**4 - 200*e**(10*e + 10*f*x)*(e**(4*e + 4*f*x
)*b + 4*e**(2*e + 2*f*x)*a - 2*e**(2*e + 2*f*x)*b + b)**p*b**4*p**2 + 45*e
**(10*e + 10*f*x)*(e**(4*e + 4*f*x)*b + 4*e**(2*e + 2*f*x)*a - 2*e**(2*e +
 2*f*x)*b + b)**p*b**4 + 320*e**(8*e + 8*f*x)*(e**(4*e + 4*f*x)*b + 4*e**(
2*e + 2*f*x)*a - 2*e**(2*e + 2*f*x)*b + b)**p*a*b**3*p**4 - 480*e**(8*e +
8*f*x)*(e**(4*e + 4*f*x)*b + 4*e**(2*e + 2*f*x)*a - 2*e**(2*e + 2*f*x)*b +
 b)**p*a*b**3*p**3 - 80*e**(8*e + 8*f*x)*(e**(4*e + 4*f*x)*b + 4*e**(2*e +
 2*f*x)*a - 2*e**(2*e + 2*f*x)*b + b)**p*a*b**3*p**2 + 120*e**(8*e + 8*f*x
)*(e**(4*e + 4*f*x)*b + 4*e**(2*e + 2*f*x)*a - 2*e**(2*e + 2*f*x)*b + b)**
p*a*b**3*p + 240*e**(8*e + 8*f*x)*(e**(4*e + 4*f*x)*b + 4*e**(2*e + 2*f*x)
*a - 2*e**(2*e + 2*f*x)*b + b)**p*b**4*p**4 + 640*e**(8*e + 8*f*x)*(e**(4*
e + 4*f*x)*b + 4*e**(2*e + 2*f*x)*a - 2*e**(2*e + 2*f*x)*b + b)**p*b**4*p*
*3 - 1560*e**(8*e + 8*f*x)*(e**(4*e + 4*f*x)*b + 4*e**(2*e + 2*f*x)*a - 2*
e**(2*e + 2*f*x)*b + b)**p*b**4*p**2 - 160*e**(8*e + 8*f*x)*(e**(4*e + 4*f
*x)*b + 4*e**(2*e + 2*f*x)*a - 2*e**(2*e + 2*f*x)*b + b)**p*b**4*p + 375*e
**(8*e + 8*f*x)*(e**(4*e + 4*f*x)*b + 4*e**(2*e + 2*f*x)*a - 2*e**(2*e + 2
*f*x)*b + b)**p*b**4 - 1920*e**(6*e + 6*f*x)*(e**(4*e + 4*f*x)*b + 4*e**(2
*e + 2*f*x)*a - 2*e**(2*e + 2*f*x)*b + b)**p*a**2*b**2*p**3 + 3840*e**(6*e
 + 6*f*x)*(e**(4*e + 4*f*x)*b + 4*e**(2*e + 2*f*x)*a - 2*e**(2*e + 2*f*...
```

3.366 $\int \cosh^3(e+fx) \left(a+b\sinh^2(e+fx)\right)^p \, dx$

Optimal result

Integrand size = 23, antiderivative size = 125

$$\int \cosh^3(e+fx) \left(a+b\sinh^2(e+fx)\right)^p \, dx = \frac{\sinh(e+fx)\left(a+b\sinh^2(e+fx)\right)^{1+p}}{bf(3+2p)} - \frac{(a-b(3+2p))\,\text{Hypergeometric2F1}\left(\frac{1}{2},-p,\frac{3}{2},-\frac{b\sinh^2(e+fx)}{a}\right)\sinh(e+fx)\left(a+b\sinh^2(e+fx)\right)^p \left(1 -\right.}{bf(3+2p)}$$

output
```
sinh(f*x+e)*(a+b*sinh(f*x+e)^2)^(p+1)/b/f/(3+2*p)-(a-b*(3+2*p))*hypergeom(
[1/2, -p],[3/2],-b*sinh(f*x+e)^2/a)*sinh(f*x+e)*(a+b*sinh(f*x+e)^2)^p/b/f/
(3+2*p)/((1+b*sinh(f*x+e)^2/a)^p)
```

Mathematica [A] (verified)

Time = 0.15 (sec) , antiderivative size = 120, normalized size of antiderivative = 0.96

$$\int \cosh^3(e+fx) \left(a+b\sinh^2(e+fx)\right)^p \, dx = \frac{\sinh(e+fx)\left(a+b\sinh^2(e+fx)\right)^p \left(1+\frac{b\sinh^2(e+fx)}{a}\right)^{-p} \left((-a+b(3+2p))\,\text{Hypergeometric2F1}\left(\frac{1}{2},-p,\right.\right.}{bf(3+2p)}$$

input
```
Integrate[Cosh[e + f*x]^3*(a + b*Sinh[e + f*x]^2)^p,x]
```

output
```
(Sinh[e + f*x]*(a + b*Sinh[e + f*x]^2)^p*((-a + b*(3 + 2*p))*Hypergeometri
c2F1[1/2, -p, 3/2, -((b*Sinh[e + f*x]^2)/a)] + (a + b*Sinh[e + f*x]^2)*(1
+ (b*Sinh[e + f*x]^2)/a)^p))/(b*f*(3 + 2*p)*(1 + (b*Sinh[e + f*x]^2)/a)^p)
```

Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 117, normalized size of antiderivative = 0.94, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.217$, Rules used = {3042, 3669, 299, 238, 237}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cosh^3(e+fx)\left(a+b\sinh^2(e+fx)\right)^p dx$$

$$\Big\downarrow \text{3042}$$

$$\int \cos(ie+ifx)^3\left(a-b\sin(ie+ifx)^2\right)^p dx$$

$$\Big\downarrow \text{3669}$$

$$\frac{\int \left(\sinh^2(e+fx)+1\right)\left(b\sinh^2(e+fx)+a\right)^p d\sinh(e+fx)}{f}$$

$$\Big\downarrow \text{299}$$

$$\frac{\left(1-\frac{a}{2bp+3b}\right)\int \left(b\sinh^2(e+fx)+a\right)^p d\sinh(e+fx)+\frac{\sinh(e+fx)\left(a+b\sinh^2(e+fx)\right)^{p+1}}{b(2p+3)}}{f}$$

$$\Big\downarrow \text{238}$$

$$\frac{\left(1-\frac{a}{2bp+3b}\right)\left(a+b\sinh^2(e+fx)\right)^p\left(\frac{b\sinh^2(e+fx)}{a}+1\right)^{-p}\int \left(\frac{b\sinh^2(e+fx)}{a}+1\right)^p d\sinh(e+fx)+\frac{\sinh(e+fx)(a+b\sin}{b(2p+}}{f}$$

$$\Big\downarrow \text{237}$$

$$\frac{\left(1-\frac{a}{2bp+3b}\right)\sinh(e+fx)\left(a+b\sinh^2(e+fx)\right)^p\left(\frac{b\sinh^2(e+fx)}{a}+1\right)^{-p}\text{Hypergeometric2F1}\left(\frac{1}{2},-p,\frac{3}{2},-\frac{b\sinh^2(e+}{a}\right.}{f}$$

input
```
Int[Cosh[e + f*x]^3*(a + b*Sinh[e + f*x]^2)^p,x]
```

output
```
((Sinh[e + f*x]*(a + b*Sinh[e + f*x]^2)^(1 + p))/(b*(3 + 2*p)) + ((1 - a/(
3*b + 2*b*p))*Hypergeometric2F1[1/2, -p, 3/2, -((b*Sinh[e + f*x]^2)/a)]*Si
nh[e + f*x]*(a + b*Sinh[e + f*x]^2)^p)/(1 + (b*Sinh[e + f*x]^2)/a)^p)/f
```

Defintions of rubi rules used

rule 237
```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[a^p*x*Hypergeometric2F1[-
p, 1/2, 1/2 + 1, (-b)*(x^2/a)], x] /; FreeQ[{a, b, p}, x] &&  !IntegerQ[2*p
] && GtQ[a, 0]
```

rule 238
```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[a^IntPart[p]*((a + b*x^2)
^FracPart[p]/(1 + b*(x^2/a))^FracPart[p])   Int[(1 + b*(x^2/a))^p, x], x] /
; FreeQ[{a, b, p}, x] &&  !IntegerQ[2*p] &&  !GtQ[a, 0]
```

rule 299
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2), x_Symbol] :> Simp[d*x
*((a + b*x^2)^(p + 1)/(b*(2*p + 3))), x] - Simp[(a*d - b*c*(2*p + 3))/(b*(2
*p + 3))   Int[(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
 a*d, 0] && NeQ[2*p + 3, 0]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3669
```
Int[cos[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^
(p_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Simp[ff/f   S
ubst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a + b*ff^2*x^2)^p, x], x, Sin[e + f*x]
/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m - 1)/2]
```

Maple [F]

$$\int \cosh(fx+e)^3 \left(a+b\sinh(fx+e)^2\right)^p dx$$

input
```
int(cosh(f*x+e)^3*(a+b*sinh(f*x+e)^2)^p,x)
```

output
```
int(cosh(f*x+e)^3*(a+b*sinh(f*x+e)^2)^p,x)
```

Fricas [F]

$$\int \cosh^3(e+fx)\left(a+b\sinh^2(e+fx)\right)^p dx = \int \left(b\sinh(fx+e)^2+a\right)^p \cosh(fx+e)^3 dx$$

input
```
integrate(cosh(f*x+e)^3*(a+b*sinh(f*x+e)^2)^p,x, algorithm="fricas")
```

output
```
integral((b*sinh(f*x + e)^2 + a)^p*cosh(f*x + e)^3, x)
```

Sympy [F(-1)]

Timed out.

$$\int \cosh^3(e+fx)\left(a+b\sinh^2(e+fx)\right)^p dx = \text{Timed out}$$

input
```
integrate(cosh(f*x+e)**3*(a+b*sinh(f*x+e)**2)**p,x)
```

output
```
Timed out
```

Maxima [F]

$$\int \cosh^3(e+fx) \left(a+b\sinh^2(e+fx)\right)^p dx = \int \left(b\sinh\left(fx+e\right)^2+a\right)^p \cosh\left(fx+e\right)^3 dx$$

input
```
integrate(cosh(f*x+e)^3*(a+b*sinh(f*x+e)^2)^p,x, algorithm="maxima")
```

output
```
integrate((b*sinh(f*x + e)^2 + a)^p*cosh(f*x + e)^3, x)
```

Giac [F]

$$\int \cosh^3(e+fx) \left(a+b\sinh^2(e+fx)\right)^p dx = \int \left(b\sinh\left(fx+e\right)^2+a\right)^p \cosh\left(fx+e\right)^3 dx$$

input
```
integrate(cosh(f*x+e)^3*(a+b*sinh(f*x+e)^2)^p,x, algorithm="giac")
```

output
```
integrate((b*sinh(f*x + e)^2 + a)^p*cosh(f*x + e)^3, x)
```

Mupad [F(-1)]

Timed out.

$$\int \cosh^3(e+fx) \left(a+b\sinh^2(e+fx)\right)^p dx = \int \cosh(e+f\,x)^3 \left(b\sinh(e+f\,x)^2+a\right)^p dx$$

input
```
int(cosh(e + f*x)^3*(a + b*sinh(e + f*x)^2)^p,x)
```

output
```
int(cosh(e + f*x)^3*(a + b*sinh(e + f*x)^2)^p, x)
```

Reduce [F]

$$\int \cosh^3(e + fx) \left(a + b\sinh^2(e + fx)\right)^p \, dx = \text{too large to display}$$

input
```
int(cosh(f*x+e)^3*(a+b*sinh(f*x+e)^2)^p,x)
```

output
```
(12*e**(6*e + 6*f*x)*(e**(4*e + 4*f*x)*b + 4*e**(2*e + 2*f*x)*a - 2*e**(2*
e + 2*f*x)*b + b)**p*b**2*p**2 - 3*e**(6*e + 6*f*x)*(e**(4*e + 4*f*x)*b +
4*e**(2*e + 2*f*x)*a - 2*e**(2*e + 2*f*x)*b + b)**p*b**2 + 48*e**(4*e + 4*
f*x)*(e**(4*e + 4*f*x)*b + 4*e**(2*e + 2*f*x)*a - 2*e**(2*e + 2*f*x)*b + b
)**p*a*b*p**2 - 24*e**(4*e + 4*f*x)*(e**(4*e + 4*f*x)*b + 4*e**(2*e + 2*f*
x)*a - 2*e**(2*e + 2*f*x)*b + b)**p*a*b*p + 12*e**(4*e + 4*f*x)*(e**(4*e +
 4*f*x)*b + 4*e**(2*e + 2*f*x)*a - 2*e**(2*e + 2*f*x)*b + b)**p*b**2*p**2
+ 48*e**(4*e + 4*f*x)*(e**(4*e + 4*f*x)*b + 4*e**(2*e + 2*f*x)*a - 2*e**(2
*e + 2*f*x)*b + b)**p*b**2*p - 27*e**(4*e + 4*f*x)*(e**(4*e + 4*f*x)*b + 4
*e**(2*e + 2*f*x)*a - 2*e**(2*e + 2*f*x)*b + b)**p*b**2 - 96*e**(2*e + 2*f
*x)*(e**(4*e + 4*f*x)*b + 4*e**(2*e + 2*f*x)*a - 2*e**(2*e + 2*f*x)*b + b)
**p*a**2*p + 144*e**(2*e + 2*f*x)*(e**(4*e + 4*f*x)*b + 4*e**(2*e + 2*f*x)
*a - 2*e**(2*e + 2*f*x)*b + b)**p*a*b*p**2 + 312*e**(2*e + 2*f*x)*(e**(4*e
 + 4*f*x)*b + 4*e**(2*e + 2*f*x)*a - 2*e**(2*e + 2*f*x)*b + b)**p*a*b*p -
12*e**(2*e + 2*f*x)*(e**(4*e + 4*f*x)*b + 4*e**(2*e + 2*f*x)*a - 2*e**(2*e
 + 2*f*x)*b + b)**p*b**2*p**2 - 48*e**(2*e + 2*f*x)*(e**(4*e + 4*f*x)*b +
4*e**(2*e + 2*f*x)*a - 2*e**(2*e + 2*f*x)*b + b)**p*b**2*p + 27*e**(2*e +
2*f*x)*(e**(4*e + 4*f*x)*b + 4*e**(2*e + 2*f*x)*a - 2*e**(2*e + 2*f*x)*b +
 b)**p*b**2 - 12*(e**(4*e + 4*f*x)*b + 4*e**(2*e + 2*f*x)*a - 2*e**(2*e +
2*f*x)*b + b)**p*b**2*p**2 + 3*(e**(4*e + 4*f*x)*b + 4*e**(2*e + 2*f*x)...
```

3.367 $\int \cosh(e+fx)\left(a+b\sinh^2(e+fx)\right)^p\,dx$

Optimal result

Integrand size = 21, antiderivative size = 67

$$\int \cosh(e+fx)\left(a+b\sinh^2(e+fx)\right)^p\,dx = \frac{\text{Hypergeometric2F1}\left(\frac{1}{2},-p,\frac{3}{2},-\frac{b\sinh^2(e+fx)}{a}\right)\sinh(e+fx)\left(a+b\sinh^2(e+fx)\right)^p\left(1+\frac{b\sinh^2(e+fx)}{a}\right)^{-p}}{f}$$

output
```
hypergeom([1/2, -p],[3/2],-b*sinh(f*x+e)^2/a)*sinh(f*x+e)*(a+b*sinh(f*x+e)
^2)^p/f/((1+b*sinh(f*x+e)^2/a)^p)
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 67, normalized size of antiderivative = 1.00

$$\int \cosh(e+fx)\left(a+b\sinh^2(e+fx)\right)^p\,dx = \frac{\text{Hypergeometric2F1}\left(\frac{1}{2},-p,\frac{3}{2},-\frac{b\sinh^2(e+fx)}{a}\right)\sinh(e+fx)\left(a+b\sinh^2(e+fx)\right)^p\left(1+\frac{b\sinh^2(e+fx)}{a}\right)^{-p}}{f}$$

input
```
Integrate[Cosh[e + f*x]*(a + b*Sinh[e + f*x]^2)^p,x]
```

output
```
(Hypergeometric2F1[1/2, -p, 3/2, -((b*Sinh[e + f*x]^2)/a)]*Sinh[e + f*x]*(
a + b*Sinh[e + f*x]^2)^p)/(f*(1 + (b*Sinh[e + f*x]^2)/a)^p)
```

Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 67, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.190, Rules used = {3042, 3669, 238, 237}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cosh(e+fx)\left(a+b\sinh^2(e+fx)\right)^p dx$$

$$\Big\downarrow \text{3042}$$

$$\int \cos(ie+ifx)\left(a-b\sin(ie+ifx)^2\right)^p dx$$

$$\Big\downarrow \text{3669}$$

$$\frac{\int \left(b\sinh^2(e+fx)+a\right)^p d\sinh(e+fx)}{f}$$

$$\Big\downarrow \text{238}$$

$$\frac{\left(a+b\sinh^2(e+fx)\right)^p\left(\frac{b\sinh^2(e+fx)}{a}+1\right)^{-p}\int \left(\frac{b\sinh^2(e+fx)}{a}+1\right)^p d\sinh(e+fx)}{f}$$

$$\Big\downarrow \text{237}$$

$$\frac{\sinh(e+fx)\left(a+b\sinh^2(e+fx)\right)^p\left(\frac{b\sinh^2(e+fx)}{a}+1\right)^{-p}\text{Hypergeometric2F1}\left(\frac{1}{2},-p,\frac{3}{2},-\frac{b\sinh^2(e+fx)}{a}\right)}{f}$$

input
```
Int[Cosh[e + f*x]*(a + b*Sinh[e + f*x]^2)^p,x]
```

output
```
(Hypergeometric2F1[1/2, -p, 3/2, -((b*Sinh[e + f*x]^2)/a)]*Sinh[e + f*x]*(
a + b*Sinh[e + f*x]^2)^p)/(f*(1 + (b*Sinh[e + f*x]^2)/a)^p)
```

Defintions of rubi rules used

rule 237
```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[a^p*x*Hypergeometric2F1[-
p, 1/2, 1/2 + 1, (-b)*(x^2/a)], x] /; FreeQ[{a, b, p}, x] &&  !IntegerQ[2*p
] && GtQ[a, 0]
```

rule 238
```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[a^IntPart[p]*((a + b*x^2)
^FracPart[p]/(1 + b*(x^2/a))^FracPart[p])   Int[(1 + b*(x^2/a))^p, x], x] /
; FreeQ[{a, b, p}, x] &&  !IntegerQ[2*p] &&  !GtQ[a, 0]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3669
```
Int[cos[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^
(p_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Simp[ff/f   S
ubst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a + b*ff^2*x^2)^p, x], x, Sin[e + f*x]
/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m - 1)/2]
```

Maple [F]

$$\int \cosh \left(fx+e\right) \left(a+b \sinh \left(fx+e\right)^{2}\right)^{p}dx$$

input
```
int(cosh(f*x+e)*(a+b*sinh(f*x+e)^2)^p,x)
```

output
```
int(cosh(f*x+e)*(a+b*sinh(f*x+e)^2)^p,x)
```

Fricas [F]

$$\int \cosh(e+fx)\left(a+b\sinh^2(e+fx)\right)^p\,dx=\int \left(b\sinh\left(fx+e\right)^2+a\right)^p\cosh\left(fx+e\right)\,dx$$

input
```
integrate(cosh(f*x+e)*(a+b*sinh(f*x+e)^2)^p,x, algorithm="fricas")
```

output
```
integral((b*sinh(f*x + e)^2 + a)^p*cosh(f*x + e), x)
```

Sympy [F(-1)]

Timed out.

$$\int \cosh(e+fx)\left(a+b\sinh^2(e+fx)\right)^p\,dx=\text{Timed out}$$

input
```
integrate(cosh(f*x+e)*(a+b*sinh(f*x+e)**2)**p,x)
```

output
```
Timed out
```

Maxima [F]

$$\int \cosh(e+fx)\left(a+b\sinh^2(e+fx)\right)^p\,dx=\int \left(b\sinh\left(fx+e\right)^2+a\right)^p\cosh\left(fx+e\right)\,dx$$

input
```
integrate(cosh(f*x+e)*(a+b*sinh(f*x+e)^2)^p,x, algorithm="maxima")
```

output
```
integrate((b*sinh(f*x + e)^2 + a)^p*cosh(f*x + e), x)
```

Giac [F]

$$\int \cosh(e+fx)\left(a+b\sinh^2(e+fx)\right)^p\,dx=\int \left(b\sinh\left(fx+e\right)^2+a\right)^p\cosh\left(fx+e\right)\,dx$$

input
```
integrate(cosh(f*x+e)*(a+b*sinh(f*x+e)^2)^p,x, algorithm="giac")
```

output
```
integrate((b*sinh(f*x + e)^2 + a)^p*cosh(f*x + e), x)
```

Mupad [B] (verification not implemented)

Time = 2.46 (sec) , antiderivative size = 64, normalized size of antiderivative = 0.96

$$\int \cosh(e+fx)\left(a+b\sinh^2(e+fx)\right)^p\,dx=\frac{\sinh(e+f\,x)\,\left(b\sinh(e+f\,x)^2+a\right)^p\,{}_2\mathrm{F}_1\left(\frac{1}{2},-p;\ \frac{3}{2};\ -\frac{b\sinh(e+f\,x)^2}{a}\right)}{f\,\left(\frac{b\sinh(e+f\,x)^2}{a}+1\right)^p}$$

input
```
int(cosh(e + f*x)*(a + b*sinh(e + f*x)^2)^p,x)
```

output
```
(sinh(e + f*x)*(a + b*sinh(e + f*x)^2)^p*hypergeom([1/2, -p], 3/2, -(b*sin
h(e + f*x)^2)/a))/(f*((b*sinh(e + f*x)^2)/a + 1)^p)
```

Reduce [F]

$$\int \cosh(e+fx)\left(a+b\sinh^2(e+fx)\right)^p\,dx=\frac{\left(\sinh\left(fx+e\right)^2b+a\right)^p\sinh\left(fx+e\right)+4\left(\int\frac{\left(\sinh(fx+e)^2b+a\right)^p\cosh(fx+e)}{2\sinh(fx+e)^2bp+\sinh(fx+e)^2b+2ap+a}dx\right)af\,p^2+2\left(\int\frac{\left(\sinh(fx+e\right.}{2\sinh(fx+e)^2b}\right.}{f\left(2p+1\right)}$$

input
```
int(cosh(f*x+e)*(a+b*sinh(f*x+e)^2)^p,x)
```

output
```
((sinh(e + f*x)**2*b + a)**p*sinh(e + f*x) + 4*int(((sinh(e + f*x)**2*b +
a)**p*cosh(e + f*x))/(2*sinh(e + f*x)**2*b*p + sinh(e + f*x)**2*b + 2*a*p
+ a),x)*a*f*p**2 + 2*int(((sinh(e + f*x)**2*b + a)**p*cosh(e + f*x))/(2*si
nh(e + f*x)**2*b*p + sinh(e + f*x)**2*b + 2*a*p + a),x)*a*f*p)/(f*(2*p + 1
))
```

3.368 $\int \operatorname{sech}(e+fx) \left(a+b\sinh^2(e+fx)\right)^p \, dx$

Optimal result

Integrand size = 21, antiderivative size = 78

$$\int \operatorname{sech}(e+fx) \left(a+b\sinh^2(e+fx)\right)^p \, dx = \frac{\operatorname{AppellF1}\left(\frac{1}{2},1,-p,\frac{3}{2},-\sinh^2(e+fx),-\frac{b\sinh^2(e+fx)}{a}\right)\sinh(e+fx)\left(a+b\sinh^2(e+fx)\right)^p\left(1+\frac{b\sinh^2(e+}{a}\right.}{f}$$

output
```
AppellF1(1/2,1,-p,3/2,-sinh(f*x+e)^2,-b*sinh(f*x+e)^2/a)*sinh(f*x+e)*(a+b*
sinh(f*x+e)^2)^p/f/((1+b*sinh(f*x+e)^2/a)^p)
```

Mathematica [F]

$$\int \operatorname{sech}(e+fx) \left(a+b\sinh^2(e+fx)\right)^p \, dx = \int \operatorname{sech}(e+fx) \left(a+b\sinh^2(e+fx)\right)^p \, dx$$

input
```
Integrate[Sech[e + f*x]*(a + b*Sinh[e + f*x]^2)^p,x]
```

output
```
Integrate[Sech[e + f*x]*(a + b*Sinh[e + f*x]^2)^p, x]
```

Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 78, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.190$, Rules used = {3042, 3669, 334, 333}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \operatorname{sech}(e+fx)\left(a+b\sinh^2(e+fx)\right)^p\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\left(a-b\sin(ie+ifx)^2\right)^p}{\cos(ie+ifx)}dx$$

$$\Big\downarrow \text{3669}$$

$$\frac{\int \frac{\left(b\sinh^2(e+fx)+a\right)^p}{\sinh^2(e+fx)+1}d\sinh(e+fx)}{f}$$

$$\Big\downarrow \text{334}$$

$$\frac{\left(a+b\sinh^2(e+fx)\right)^p\left(\frac{b\sinh^2(e+fx)}{a}+1\right)^{-p}\int \frac{\left(\frac{b\sinh^2(e+fx)}{a}+1\right)^p}{\sinh^2(e+fx)+1}d\sinh(e+fx)}{f}$$

$$\Big\downarrow \text{333}$$

$$\frac{\sinh(e+fx)\left(a+b\sinh^2(e+fx)\right)^p\left(\frac{b\sinh^2(e+fx)}{a}+1\right)^{-p}\operatorname{AppellF1}\left(\frac{1}{2},1,-p,\frac{3}{2},-\sinh^2(e+fx),-\frac{b\sinh^2(e+fx)}{a}\right)}{f}$$

input
```
Int[Sech[e + f*x]*(a + b*Sinh[e + f*x]^2)^p,x]
```

output
```
(AppellF1[1/2, 1, -p, 3/2, -Sinh[e + f*x]^2, -((b*Sinh[e + f*x]^2)/a)]*Sin
h[e + f*x]*(a + b*Sinh[e + f*x]^2)^p)/(f*(1 + (b*Sinh[e + f*x]^2)/a)^p)
```

Defintions of rubi rules used

rule 333
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_), x_Symbol] :> Sim
p[a^p*c^q*x*AppellF1[1/2, -p, -q, 3/2, (-b)*(x^2/a), (-d)*(x^2/c)], x] /; F
reeQ[{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && (IntegerQ[p] || GtQ[a,
0]) && (IntegerQ[q] || GtQ[c, 0])
```

rule 334
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_), x_Symbol] :> Sim
p[a^IntPart[p]*((a + b*x^2)^FracPart[p]/(1 + b*(x^2/a))^FracPart[p])   Int[
(1 + b*(x^2/a))^p*(c + d*x^2)^q, x], x] /; FreeQ[{a, b, c, d, p, q}, x] &&
NeQ[b*c - a*d, 0] &&  !(IntegerQ[p] || GtQ[a, 0])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3669
```
Int[cos[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^
(p_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Simp[ff/f   S
ubst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a + b*ff^2*x^2)^p, x], x, Sin[e + f*x]
/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m - 1)/2]
```

Maple [F]

$$\int \operatorname{sech}(fx+e)\left(a+b\sinh(fx+e)^2\right)^p dx$$

input
```
int(sech(f*x+e)*(a+b*sinh(f*x+e)^2)^p,x)
```

output
```
int(sech(f*x+e)*(a+b*sinh(f*x+e)^2)^p,x)
```

Fricas [F]

$$\int \operatorname{sech}(e+fx)\left(a+b\sinh^2(e+fx)\right)^p\,dx=\int \left(b\sinh\left(fx+e\right)^2+a\right)^p\operatorname{sech}\left(fx+e\right)\,dx$$

input
```
integrate(sech(f*x+e)*(a+b*sinh(f*x+e)^2)^p,x, algorithm="fricas")
```

output
```
integral((b*sinh(f*x + e)^2 + a)^p*sech(f*x + e), x)
```

Sympy [F(-1)]

Timed out.

$$\int \operatorname{sech}(e+fx)\left(a+b\sinh^2(e+fx)\right)^p\,dx=\text{Timed out}$$

input
```
integrate(sech(f*x+e)*(a+b*sinh(f*x+e)**2)**p,x)
```

output
```
Timed out
```

Maxima [F]

$$\int \operatorname{sech}(e+fx)\left(a+b\sinh^2(e+fx)\right)^p\,dx=\int \left(b\sinh\left(fx+e\right)^2+a\right)^p\operatorname{sech}\left(fx+e\right)\,dx$$

input
```
integrate(sech(f*x+e)*(a+b*sinh(f*x+e)^2)^p,x, algorithm="maxima")
```

output
```
integrate((b*sinh(f*x + e)^2 + a)^p*sech(f*x + e), x)
```

Giac [F]

$$\int \operatorname{sech}(e+fx)\left(a+b\sinh^2(e+fx)\right)^p\,dx=\int \left(b\sinh\left(fx+e\right)^2+a\right)^p\operatorname{sech}\left(fx+e\right)\,dx$$

input
```
integrate(sech(f*x+e)*(a+b*sinh(f*x+e)^2)^p,x, algorithm="giac")
```

output
```
integrate((b*sinh(f*x + e)^2 + a)^p*sech(f*x + e), x)
```

Mupad [F(-1)]

Timed out.

$$\int \operatorname{sech}(e+fx)\left(a+b\sinh^2(e+fx)\right)^p\,dx=\int \frac{\left(b\sinh(e+f\,x)^2+a\right)^p}{\cosh\left(e+f\,x\right)}\,dx$$

input
```
int((a + b*sinh(e + f*x)^2)^p/cosh(e + f*x),x)
```

output
```
int((a + b*sinh(e + f*x)^2)^p/cosh(e + f*x), x)
```

Reduce [F]

$$\int \operatorname{sech}(e+fx)\left(a+b\sinh^2(e+fx)\right)^p\,dx=\int \left(\sinh\left(fx+e\right)^2b+a\right)^p\operatorname{sech}(fx+e)\,dx$$

input
```
int(sech(f*x+e)*(a+b*sinh(f*x+e)^2)^p,x)
```

output
```
int((sinh(e + f*x)**2*b + a)**p*sech(e + f*x),x)
```

3.369 $\int \operatorname{sech}^3(e+fx)\left(a+b\sinh^2(e+fx)\right)^p\,dx$

Optimal result

Integrand size = 23, antiderivative size = 78

$$\int \operatorname{sech}^3(e+fx)\left(a+b\sinh^2(e+fx)\right)^p\,dx = \frac{\operatorname{AppellF1}\left(\frac{1}{2},2,-p,\frac{3}{2},-\sinh^2(e+fx),-\frac{b\sinh^2(e+fx)}{a}\right)\sinh(e+fx)\left(a+b\sinh^2(e+fx)\right)^p\left(1+\frac{b\sinh^2(e+}{a}\right)}{f}$$

output
```
AppellF1(1/2,2,-p,3/2,-sinh(f*x+e)^2,-b*sinh(f*x+e)^2/a)*sinh(f*x+e)*(a+b*
sinh(f*x+e)^2)^p/f/((1+b*sinh(f*x+e)^2/a)^p)
```

Mathematica [F]

$$\int \operatorname{sech}^3(e+fx)\left(a+b\sinh^2(e+fx)\right)^p\,dx = \int \operatorname{sech}^3(e+fx)\left(a+b\sinh^2(e+fx)\right)^p\,dx$$

input
```
Integrate[Sech[e + f*x]^3*(a + b*Sinh[e + f*x]^2)^p,x]
```

output
```
Integrate[Sech[e + f*x]^3*(a + b*Sinh[e + f*x]^2)^p, x]
```

Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 78, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.174, Rules used = {3042, 3669, 334, 333}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \text{sech}^3(e+fx)\left(a+b\sinh^2(e+fx)\right)^p\,dx$$

$$\Big\downarrow \; 3042$$

$$\int \frac{\left(a-b\sin(ie+ifx)^2\right)^p}{\cos(ie+ifx)^3}dx$$

$$\Big\downarrow \; 3669$$

$$\frac{\int \frac{\left(b\sinh^2(e+fx)+a\right)^p}{\left(\sinh^2(e+fx)+1\right)^2}d\sinh(e+fx)}{f}$$

$$\Big\downarrow \; 334$$

$$\frac{\left(a+b\sinh^2(e+fx)\right)^p\left(\frac{b\sinh^2(e+fx)}{a}+1\right)^{-p}\int \frac{\left(\frac{b\sinh^2(e+fx)}{a}+1\right)^p}{\left(\sinh^2(e+fx)+1\right)^2}d\sinh(e+fx)}{f}$$

$$\Big\downarrow \; 333$$

$$\frac{\sinh(e+fx)\left(a+b\sinh^2(e+fx)\right)^p\left(\frac{b\sinh^2(e+fx)}{a}+1\right)^{-p}\text{AppellF1}\left(\frac{1}{2},2,-p,\frac{3}{2},-\sinh^2(e+fx),-\frac{b\sinh^2(e+fx)}{a}\right)}{f}$$

input
```
Int[Sech[e + f*x]^3*(a + b*Sinh[e + f*x]^2)^p,x]
```

output
```
(AppellF1[1/2, 2, -p, 3/2, -Sinh[e + f*x]^2, -((b*Sinh[e + f*x]^2)/a)]*Sin
h[e + f*x]*(a + b*Sinh[e + f*x]^2)^p)/(f*(1 + (b*Sinh[e + f*x]^2)/a)^p)
```

Defintions of rubi rules used

rule 333
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_), x_Symbol] :> Sim
p[a^p*c^q*x*AppellF1[1/2, -p, -q, 3/2, (-b)*(x^2/a), (-d)*(x^2/c)], x] /; F
reeQ[{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && (IntegerQ[p] || GtQ[a,
0]) && (IntegerQ[q] || GtQ[c, 0])
```

rule 334
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_), x_Symbol] :> Sim
p[a^IntPart[p]*((a + b*x^2)^FracPart[p]/(1 + b*(x^2/a))^FracPart[p])   Int[
(1 + b*(x^2/a))^p*(c + d*x^2)^q, x], x] /; FreeQ[{a, b, c, d, p, q}, x] &&
NeQ[b*c - a*d, 0] &&  !(IntegerQ[p] || GtQ[a, 0])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3669
```
Int[cos[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^
(p_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Simp[ff/f   S
ubst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a + b*ff^2*x^2)^p, x], x, Sin[e + f*x]
/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m - 1)/2]
```

Maple [F]

$$\int \operatorname{sech}(fx+e)^3 \left(a+b\sinh(fx+e)^2\right)^p dx$$

input
```
int(sech(f*x+e)^3*(a+b*sinh(f*x+e)^2)^p,x)
```

output
```
int(sech(f*x+e)^3*(a+b*sinh(f*x+e)^2)^p,x)
```

Fricas [F]

$$\int \operatorname{sech}^3(e+fx)\left(a+b\sinh^2(e+fx)\right)^p\,dx=\int \left(b\sinh\left(fx+e\right)^2+a\right)^p\operatorname{sech}\left(fx+e\right)^3\,dx$$

input
```
integrate(sech(f*x+e)^3*(a+b*sinh(f*x+e)^2)^p,x, algorithm="fricas")
```

output
```
integral((b*sinh(f*x + e)^2 + a)^p*sech(f*x + e)^3, x)
```

Sympy [F(-1)]

Timed out.

$$\int \operatorname{sech}^3(e+fx)\left(a+b\sinh^2(e+fx)\right)^p\,dx=\text{Timed out}$$

input
```
integrate(sech(f*x+e)**3*(a+b*sinh(f*x+e)**2)**p,x)
```

output
```
Timed out
```

Maxima [F]

$$\int \operatorname{sech}^3(e+fx)\left(a+b\sinh^2(e+fx)\right)^p\,dx=\int \left(b\sinh\left(fx+e\right)^2+a\right)^p\operatorname{sech}\left(fx+e\right)^3\,dx$$

input
```
integrate(sech(f*x+e)^3*(a+b*sinh(f*x+e)^2)^p,x, algorithm="maxima")
```

output
```
integrate((b*sinh(f*x + e)^2 + a)^p*sech(f*x + e)^3, x)
```

Giac [F]

$$\int \operatorname{sech}^3(e+fx)\left(a+b\sinh^2(e+fx)\right)^p\,dx=\int \left(b\sinh\left(fx+e\right)^2+a\right)^p\operatorname{sech}\left(fx+e\right)^3\,dx$$

input
```
integrate(sech(f*x+e)^3*(a+b*sinh(f*x+e)^2)^p,x, algorithm="giac")
```

output
```
integrate((b*sinh(f*x + e)^2 + a)^p*sech(f*x + e)^3, x)
```

Mupad [F(-1)]

Timed out.

$$\int \operatorname{sech}^3(e+fx)\left(a+b\sinh^2(e+fx)\right)^p\,dx=\int \frac{\left(b\sinh(e+f\,x)^2+a\right)^p}{\cosh\left(e+f\,x\right)^3}\,dx$$

input
```
int((a + b*sinh(e + f*x)^2)^p/cosh(e + f*x)^3,x)
```

output
```
int((a + b*sinh(e + f*x)^2)^p/cosh(e + f*x)^3, x)
```

Reduce [F]

$$\int \operatorname{sech}^3(e+fx)\left(a+b\sinh^2(e+fx)\right)^p\,dx=\int \operatorname{sech}(fx+e)^3\left(\sinh\left(fx+e\right)^2b+a\right)^p\,dx$$

input
```
int(sech(f*x+e)^3*(a+b*sinh(f*x+e)^2)^p,x)
```

output
```
int(sech(f*x+e)^3*(a+b*sinh(f*x+e)^2)^p,x)
```

3.370 $\int \cosh^4(e+fx)\left(a+b\sinh^2(e+fx)\right)^p\,dx$

Optimal result

Integrand size = 23, antiderivative size = 92

$$\int \cosh^4(e+fx)\left(a+b\sinh^2(e+fx)\right)^p\,dx$$
$$= \frac{\operatorname{AppellF1}\left(\frac{1}{2},-\frac{3}{2},-p,\frac{3}{2},-\sinh^2(e+fx),-\frac{b\sinh^2(e+fx)}{a}\right)\sqrt{\cosh^2(e+fx)}\left(a+b\sinh^2(e+fx)\right)^p\left(1+\frac{b\,\mathrm{si}}{}\right.}{f}$$

output
```
AppellF1(1/2,-3/2,-p,3/2,-sinh(f*x+e)^2,-b*sinh(f*x+e)^2/a)*(cosh(f*x+e)^2
)^(1/2)*(a+b*sinh(f*x+e)^2)^p*tanh(f*x+e)/f/((1+b*sinh(f*x+e)^2/a)^p)
```

Mathematica [F]

$$\int \cosh^4(e+fx)\left(a+b\sinh^2(e+fx)\right)^p\,dx = \int \cosh^4(e+fx)\left(a+b\sinh^2(e+fx)\right)^p\,dx$$

input
```
Integrate[Cosh[e + f*x]^4*(a + b*Sinh[e + f*x]^2)^p,x]
```

output
```
Integrate[Cosh[e + f*x]^4*(a + b*Sinh[e + f*x]^2)^p, x]
```

Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 92, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.174, Rules used = {3042, 3671, 334, 333}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cosh^4(e+fx)\left(a+b\sinh^2(e+fx)\right)^p dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \cos(ie+ifx)^4\left(a-b\sin(ie+ifx)^2\right)^p dx$$

$$\Big\downarrow \text{ 3671}$$

$$\frac{\sqrt{\cosh^2(e+fx)}\operatorname{sech}(e+fx)\int \left(\sinh^2(e+fx)+1\right)^{3/2}\left(b\sinh^2(e+fx)+a\right)^p d\sinh(e+fx)}{f}$$

$$\Big\downarrow \text{ 334}$$

$$\frac{\sqrt{\cosh^2(e+fx)}\operatorname{sech}(e+fx)\left(a+b\sinh^2(e+fx)\right)^p\left(\frac{b\sinh^2(e+fx)}{a}+1\right)^{-p}\int \left(\sinh^2(e+fx)+1\right)^{3/2}\left(\frac{b\sinh^2(e+fx)}{a}\right.}{f}$$

$$\Big\downarrow \text{ 333}$$

$$\frac{\sqrt{\cosh^2(e+fx)}\tanh(e+fx)\left(a+b\sinh^2(e+fx)\right)^p\left(\frac{b\sinh^2(e+fx)}{a}+1\right)^{-p}\operatorname{AppellF1}\left(\frac{1}{2},-\frac{3}{2},-p,\frac{3}{2},-\sinh^2(e+\right.}{f}$$

input
```
Int[Cosh[e + f*x]^4*(a + b*Sinh[e + f*x]^2)^p,x]
```

output
```
(AppellF1[1/2, -3/2, -p, 3/2, -Sinh[e + f*x]^2, -((b*Sinh[e + f*x]^2)/a)]*
Sqrt[Cosh[e + f*x]^2]*(a + b*Sinh[e + f*x]^2)^p*Tanh[e + f*x])/(f*(1 + (b*
Sinh[e + f*x]^2)/a)^p)
```

Defintions of rubi rules used

rule 333
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_), x_Symbol] :> Sim
p[a^p*c^q*x*AppellF1[1/2, -p, -q, 3/2, (-b)*(x^2/a), (-d)*(x^2/c)], x] /; F
reeQ[{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && (IntegerQ[p] || GtQ[a,
0]) && (IntegerQ[q] || GtQ[c, 0])
```

rule 334
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_), x_Symbol] :> Sim
p[a^IntPart[p]*((a + b*x^2)^FracPart[p]/(1 + b*(x^2/a))^FracPart[p])   Int[
(1 + b*(x^2/a))^p*(c + d*x^2)^q, x], x] /; FreeQ[{a, b, c, d, p, q}, x] &&
NeQ[b*c - a*d, 0] &&  !(IntegerQ[p] || GtQ[a, 0])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3671
```
Int[cos[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(
p_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Simp[ff*(Sqrt[
Cos[e + f*x]^2]/(f*Cos[e + f*x]))   Subst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a
 + b*ff^2*x^2)^p, x], x, Sin[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x]
 && IntegerQ[m/2] &&  !IntegerQ[p]
```

Maple [F]

$$\int \cosh (fx+e)^4 \left(a+b\sinh (fx+e)^2\right)^p dx$$

input
```
int(cosh(f*x+e)^4*(a+b*sinh(f*x+e)^2)^p,x)
```

output
```
int(cosh(f*x+e)^4*(a+b*sinh(f*x+e)^2)^p,x)
```

Fricas [F]

$$\int \cosh^4(e+fx)\left(a+b\sinh^2(e+fx)\right)^p\,dx = \int \left(b\sinh\left(fx+e\right)^2+a\right)^p\cosh\left(fx+e\right)^4\,dx$$

input
```
integrate(cosh(f*x+e)^4*(a+b*sinh(f*x+e)^2)^p,x, algorithm="fricas")
```

output
```
integral((b*sinh(f*x + e)^2 + a)^p*cosh(f*x + e)^4, x)
```

Sympy [F(-1)]

Timed out.

$$\int \cosh^4(e+fx)\left(a+b\sinh^2(e+fx)\right)^p\,dx = \text{Timed out}$$

input
```
integrate(cosh(f*x+e)**4*(a+b*sinh(f*x+e)**2)**p,x)
```

output
```
Timed out
```

Maxima [F]

$$\int \cosh^4(e+fx)\left(a+b\sinh^2(e+fx)\right)^p\,dx = \int \left(b\sinh\left(fx+e\right)^2+a\right)^p\cosh\left(fx+e\right)^4\,dx$$

input
```
integrate(cosh(f*x+e)^4*(a+b*sinh(f*x+e)^2)^p,x, algorithm="maxima")
```

output
```
integrate((b*sinh(f*x + e)^2 + a)^p*cosh(f*x + e)^4, x)
```

Giac [F]

$$\int \cosh^4(e+fx)\left(a+b\sinh^2(e+fx)\right)^p\,dx=\int \left(b\sinh\left(fx+e\right)^2+a\right)^p\cosh\left(fx+e\right)^4\,dx$$

input
```
integrate(cosh(f*x+e)^4*(a+b*sinh(f*x+e)^2)^p,x, algorithm="giac")
```

output
```
integrate((b*sinh(f*x + e)^2 + a)^p*cosh(f*x + e)^4, x)
```

Mupad [F(-1)]

Timed out.

$$\int \cosh^4(e+fx)\left(a+b\sinh^2(e+fx)\right)^p\,dx=\int \cosh(e+f\,x)^4\,\left(b\sinh(e+f\,x)^2+a\right)^p\,dx$$

input
```
int(cosh(e + f*x)^4*(a + b*sinh(e + f*x)^2)^p,x)
```

output
```
int(cosh(e + f*x)^4*(a + b*sinh(e + f*x)^2)^p, x)
```

Reduce [F]

$$\int \cosh^4(e+fx)\left(a+b\sinh^2(e+fx)\right)^p\,dx=\text{too large to display}$$

input
```
int(cosh(f*x+e)^4*(a+b*sinh(f*x+e)^2)^p,x)
```

output
```
(e**(8*e + 8*f*x)*(e**(4*e + 4*f*x)*b + 4*e**(2*e + 2*f*x)*a - 2*e**(2*e +
 2*f*x)*b + b)**p*b**2*p**3 - e**(8*e + 8*f*x)*(e**(4*e + 4*f*x)*b + 4*e**
(2*e + 2*f*x)*a - 2*e**(2*e + 2*f*x)*b + b)**p*b**2*p + 4*e**(6*e + 6*f*x)
*(e**(4*e + 4*f*x)*b + 4*e**(2*e + 2*f*x)*a - 2*e**(2*e + 2*f*x)*b + b)**p
*a*b*p**3 - 4*e**(6*e + 6*f*x)*(e**(4*e + 4*f*x)*b + 4*e**(2*e + 2*f*x)*a
- 2*e**(2*e + 2*f*x)*b + b)**p*a*b*p**2 + 2*e**(6*e + 6*f*x)*(e**(4*e + 4*
f*x)*b + 4*e**(2*e + 2*f*x)*a - 2*e**(2*e + 2*f*x)*b + b)**p*b**2*p**3 + 6
*e**(6*e + 6*f*x)*(e**(4*e + 4*f*x)*b + 4*e**(2*e + 2*f*x)*a - 2*e**(2*e +
 2*f*x)*b + b)**p*b**2*p**2 - 8*e**(6*e + 6*f*x)*(e**(4*e + 4*f*x)*b + 4*e
**(2*e + 2*f*x)*a - 2*e**(2*e + 2*f*x)*b + b)**p*b**2*p - 16*e**(4*e + 4*f
*x)*(e**(4*e + 4*f*x)*b + 4*e**(2*e + 2*f*x)*a - 2*e**(2*e + 2*f*x)*b + b)
**p*a**2*p**2 + 16*e**(4*e + 4*f*x)*(e**(4*e + 4*f*x)*b + 4*e**(2*e + 2*f*
x)*a - 2*e**(2*e + 2*f*x)*b + b)**p*a**2*p + 16*e**(4*e + 4*f*x)*(e**(4*e
+ 4*f*x)*b + 4*e**(2*e + 2*f*x)*a - 2*e**(2*e + 2*f*x)*b + b)**p*a*b*p**3
+ 32*e**(4*e + 4*f*x)*(e**(4*e + 4*f*x)*b + 4*e**(2*e + 2*f*x)*a - 2*e**(2
*e + 2*f*x)*b + b)**p*a*b*p**2 - 48*e**(4*e + 4*f*x)*(e**(4*e + 4*f*x)*b +
 4*e**(2*e + 2*f*x)*a - 2*e**(2*e + 2*f*x)*b + b)**p*a*b*p + 12*e**(4*e +
4*f*x)*(e**(4*e + 4*f*x)*b + 4*e**(2*e + 2*f*x)*a - 2*e**(2*e + 2*f*x)*b +
 b)**p*b**2*p - 12*e**(4*e + 4*f*x)*(e**(4*e + 4*f*x)*b + 4*e**(2*e + 2*f*
x)*a - 2*e**(2*e + 2*f*x)*b + b)**p*b**2 + 28*e**(2*e + 2*f*x)*(e**(4*e...
```

3.371 $\int \cosh^2(e+fx)\left(a+b\sinh^2(e+fx)\right)^p\,dx$

Optimal result

Integrand size = 23, antiderivative size = 92

$$\int \cosh^2(e+fx)\left(a+b\sinh^2(e+fx)\right)^p\,dx = \frac{\text{AppellF1}\left(\frac{1}{2},-\frac{1}{2},-p,\frac{3}{2},-\sinh^2(e+fx),-\frac{b\sinh^2(e+fx)}{a}\right)\sqrt{\cosh^2(e+fx)}\left(a+b\sinh^2(e+fx)\right)^p\left(1+\frac{b\,\text{si}}{}\right.}{f}$$

output
```
AppellF1(1/2,-1/2,-p,3/2,-sinh(f*x+e)^2,-b*sinh(f*x+e)^2/a)*(cosh(f*x+e)^2
)^(1/2)*(a+b*sinh(f*x+e)^2)^p*tanh(f*x+e)/f/((1+b*sinh(f*x+e)^2/a)^p)
```

Mathematica [F]

$$\int \cosh^2(e+fx)\left(a+b\sinh^2(e+fx)\right)^p\,dx = \int \cosh^2(e+fx)\left(a+b\sinh^2(e+fx)\right)^p\,dx$$

input
```
Integrate[Cosh[e + f*x]^2*(a + b*Sinh[e + f*x]^2)^p,x]
```

output
```
Integrate[Cosh[e + f*x]^2*(a + b*Sinh[e + f*x]^2)^p, x]
```

Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 92, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.174$, Rules used = {3042, 3671, 334, 333}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cosh^2(e+fx)\left(a+b\sinh^2(e+fx)\right)^p\,dx$$

$$\Big\downarrow \ 3042$$

$$\int \cos(ie+ifx)^2\left(a-b\sin(ie+ifx)^2\right)^p\,dx$$

$$\Big\downarrow \ 3671$$

$$\frac{\sqrt{\cosh^2(e+fx)}\text{sech}(e+fx)\int\sqrt{\sinh^2(e+fx)+1}\left(b\sinh^2(e+fx)+a\right)^p d\sinh(e+fx)}{f}$$

$$\Big\downarrow \ 334$$

$$\frac{\sqrt{\cosh^2(e+fx)}\text{sech}(e+fx)\left(a+b\sinh^2(e+fx)\right)^p\left(\frac{b\sinh^2(e+fx)}{a}+1\right)^{-p}\int\sqrt{\sinh^2(e+fx)+1}\left(\frac{b\sinh^2(e+fx)}{a}+\right.}{f}$$

$$\Big\downarrow \ 333$$

$$\frac{\sqrt{\cosh^2(e+fx)}\tanh(e+fx)\left(a+b\sinh^2(e+fx)\right)^p\left(\frac{b\sinh^2(e+fx)}{a}+1\right)^{-p}\text{AppellF1}\left(\frac{1}{2},-\frac{1}{2},-p,\frac{3}{2},-\sinh^2(e+\right.}{f}$$

input
```
Int[Cosh[e + f*x]^2*(a + b*Sinh[e + f*x]^2)^p,x]
```

output
```
(AppellF1[1/2, -1/2, -p, 3/2, -Sinh[e + f*x]^2, -((b*Sinh[e + f*x]^2)/a)]*
Sqrt[Cosh[e + f*x]^2]*(a + b*Sinh[e + f*x]^2)^p*Tanh[e + f*x])/(f*(1 + (b*
Sinh[e + f*x]^2)/a)^p)
```

Defintions of rubi rules used

rule 333
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_), x_Symbol] :> Sim
p[a^p*c^q*x*AppellF1[1/2, -p, -q, 3/2, (-b)*(x^2/a), (-d)*(x^2/c)], x] /; F
reeQ[{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && (IntegerQ[p] || GtQ[a,
0]) && (IntegerQ[q] || GtQ[c, 0])
```

rule 334
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_), x_Symbol] :> Sim
p[a^IntPart[p]*((a + b*x^2)^FracPart[p]/(1 + b*(x^2/a))^FracPart[p])   Int[
(1 + b*(x^2/a))^p*(c + d*x^2)^q, x], x] /; FreeQ[{a, b, c, d, p, q}, x] &&
NeQ[b*c - a*d, 0] &&  !(IntegerQ[p] || GtQ[a, 0])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3671
```
Int[cos[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(
p_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Simp[ff*(Sqrt[
Cos[e + f*x]^2]/(f*Cos[e + f*x]))   Subst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a
 + b*ff^2*x^2)^p, x], x, Sin[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x]
 && IntegerQ[m/2] &&  !IntegerQ[p]
```

Maple [F]

$$\int \cosh(fx+e)^2 \left(a+b\sinh(fx+e)^2\right)^p dx$$

input
```
int(cosh(f*x+e)^2*(a+b*sinh(f*x+e)^2)^p,x)
```

output
```
int(cosh(f*x+e)^2*(a+b*sinh(f*x+e)^2)^p,x)
```

Fricas [F]

$$\int \cosh^2(e+fx)\left(a+b\sinh^2(e+fx)\right)^p\,dx=\int \left(b\sinh\left(fx+e\right)^2+a\right)^p\cosh\left(fx+e\right)^2\,dx$$

input
```
integrate(cosh(f*x+e)^2*(a+b*sinh(f*x+e)^2)^p,x, algorithm="fricas")
```

output
```
integral((b*sinh(f*x + e)^2 + a)^p*cosh(f*x + e)^2, x)
```

Sympy [F(-1)]

Timed out.

$$\int \cosh^2(e+fx)\left(a+b\sinh^2(e+fx)\right)^p\,dx=\text{Timed out}$$

input
```
integrate(cosh(f*x+e)**2*(a+b*sinh(f*x+e)**2)**p,x)
```

output
```
Timed out
```

Maxima [F]

$$\int \cosh^2(e+fx)\left(a+b\sinh^2(e+fx)\right)^p\,dx=\int \left(b\sinh\left(fx+e\right)^2+a\right)^p\cosh\left(fx+e\right)^2\,dx$$

input
```
integrate(cosh(f*x+e)^2*(a+b*sinh(f*x+e)^2)^p,x, algorithm="maxima")
```

output
```
integrate((b*sinh(f*x + e)^2 + a)^p*cosh(f*x + e)^2, x)
```

Giac [F]

$$\int \cosh^2(e+fx)\left(a+b\sinh^2(e+fx)\right)^p\,dx=\int \left(b\sinh\left(fx+e\right)^2+a\right)^p\cosh\left(fx+e\right)^2\,dx$$

input
```
integrate(cosh(f*x+e)^2*(a+b*sinh(f*x+e)^2)^p,x, algorithm="giac")
```

output
```
integrate((b*sinh(f*x + e)^2 + a)^p*cosh(f*x + e)^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int \cosh^2(e+fx)\left(a+b\sinh^2(e+fx)\right)^p\,dx=\int \mathrm{cosh}(e+f\,x)^2\,\left(b\,\mathrm{sinh}(e+f\,x)^2+a\right)^p\,dx$$

input
```
int(cosh(e + f*x)^2*(a + b*sinh(e + f*x)^2)^p,x)
```

output
```
int(cosh(e + f*x)^2*(a + b*sinh(e + f*x)^2)^p, x)
```

Reduce [F]

$$\int \cosh^2(e+fx)\left(a+b\sinh^2(e+fx)\right)^p\,dx=\text{too large to display}$$

input
```
int(cosh(f*x+e)^2*(a+b*sinh(f*x+e)^2)^p,x)
```

output

```
(e**(4*e + 4*f*x)*(e**(4*e + 4*f*x)*b + 4*e**(2*e + 2*f*x)*a - 2*e**(2*e +
 2*f*x)*b + b)**p*b*p**2 - e**(4*e + 4*f*x)*(e**(4*e + 4*f*x)*b + 4*e**(2*
e + 2*f*x)*a - 2*e**(2*e + 2*f*x)*b + b)**p*b*p + 4*e**(2*e + 2*f*x)*(e**(
4*e + 4*f*x)*b + 4*e**(2*e + 2*f*x)*a - 2*e**(2*e + 2*f*x)*b + b)**p*a*p**
2 - 4*e**(2*e + 2*f*x)*(e**(4*e + 4*f*x)*b + 4*e**(2*e + 2*f*x)*a - 2*e**(
2*e + 2*f*x)*b + b)**p*a*p + 2*e**(2*e + 2*f*x)*(e**(4*e + 4*f*x)*b + 4*e*
*(2*e + 2*f*x)*a - 2*e**(2*e + 2*f*x)*b + b)**p*b*p - 2*e**(2*e + 2*f*x)*(
e**(4*e + 4*f*x)*b + 4*e**(2*e + 2*f*x)*a - 2*e**(2*e + 2*f*x)*b + b)**p*b
 + 8*(e**(4*e + 4*f*x)*b + 4*e**(2*e + 2*f*x)*a - 2*e**(2*e + 2*f*x)*b + b
)**p*a*p**2 + 8*(e**(4*e + 4*f*x)*b + 4*e**(2*e + 2*f*x)*a - 2*e**(2*e + 2
*f*x)*b + b)**p*a*p - (e**(4*e + 4*f*x)*b + 4*e**(2*e + 2*f*x)*a - 2*e**(2
*e + 2*f*x)*b + b)**p*b*p**2 - 3*(e**(4*e + 4*f*x)*b + 4*e**(2*e + 2*f*x)*
a - 2*e**(2*e + 2*f*x)*b + b)**p*b*p + 64*e**(2*e*p + 4*e + 2*f*p*x + 2*f*
x)*int((e**(4*e + 4*f*x)*b + 4*e**(2*e + 2*f*x)*a - 2*e**(2*e + 2*f*x)*b +
 b)**p/(e**(2*e*p + 6*e + 2*f*p*x + 4*f*x)*b*p - e**(2*e*p + 6*e + 2*f*p*x
 + 4*f*x)*b + 4*e**(2*e*p + 4*e + 2*f*p*x + 2*f*x)*a*p - 4*e**(2*e*p + 4*e
 + 2*f*p*x + 2*f*x)*a - 2*e**(2*e*p + 4*e + 2*f*p*x + 2*f*x)*b*p + 2*e**(2
*e*p + 4*e + 2*f*p*x + 2*f*x)*b + e**(2*e*p + 2*e + 2*f*p*x)*b*p - e**(2*e
*p + 2*e + 2*f*p*x)*b),x)*a**2*f*p**3 - 64*e**(2*e*p + 4*e + 2*f*p*x + 2*f
*x)*int((e**(4*e + 4*f*x)*b + 4*e**(2*e + 2*f*x)*a - 2*e**(2*e + 2*f*x)...
```

3.372 $\int (a+b\sinh^2(e+fx))^p\,dx$

Optimal result

Integrand size = 14, antiderivative size = 92

$$\int (a+b\sinh^2(e+fx))^p\,dx = \frac{\operatorname{AppellF1}\left(\frac{1}{2},\frac{1}{2},-p,\frac{3}{2},-\sinh^2(e+fx),-\frac{b\sinh^2(e+fx)}{a}\right)\sqrt{\cosh^2(e+fx)}\,(a+b\sinh^2(e+fx))^p\left(1+\frac{b\sinh}{}\right.}{f}$$

output
```
AppellF1(1/2,1/2,-p,3/2,-sinh(f*x+e)^2,-b*sinh(f*x+e)^2/a)*(cosh(f*x+e)^2)
^(1/2)*(a+b*sinh(f*x+e)^2)^p*tanh(f*x+e)/f/((1+b*sinh(f*x+e)^2/a)^p)
```

Mathematica [F]

$$\int (a+b\sinh^2(e+fx))^p\,dx = \int (a+b\sinh^2(e+fx))^p\,dx$$

input
```
Integrate[(a + b*Sinh[e + f*x]^2)^p,x]
```

output
```
Integrate[(a + b*Sinh[e + f*x]^2)^p, x]
```

Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 92, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {3042, 3664, 334, 333}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(a+b\sinh^2(e+fx)\right)^p\,dx$$

$$\Big\downarrow \; 3042$$

$$\int \left(a-b\sin(ie+ifx)^2\right)^p\,dx$$

$$\Big\downarrow \; 3664$$

$$\frac{\sqrt{\cosh^2(e+fx)}\text{sech}(e+fx)\int \frac{\left(b\sinh^2(e+fx)+a\right)^p}{\sqrt{\sinh^2(e+fx)+1}}d\sinh(e+fx)}{f}$$

$$\Big\downarrow \; 334$$

$$\frac{\sqrt{\cosh^2(e+fx)}\text{sech}(e+fx)\left(a+b\sinh^2(e+fx)\right)^p\left(\frac{b\sinh^2(e+fx)}{a}+1\right)^{-p}\int \frac{\left(\frac{b\sinh^2(e+fx)}{a}+1\right)^p}{\sqrt{\sinh^2(e+fx)+1}}d\sinh(e+fx)}{f}$$

$$\Big\downarrow \; 333$$

$$\frac{\sqrt{\cosh^2(e+fx)}\tanh(e+fx)\left(a+b\sinh^2(e+fx)\right)^p\left(\frac{b\sinh^2(e+fx)}{a}+1\right)^{-p}\text{AppellF1}\left(\frac{1}{2},\frac{1}{2},-p,\frac{3}{2},-\sinh^2(e+fx\right.}{f}$$

input
```
Int[(a + b*Sinh[e + f*x]^2)^p,x]
```

output
```
(AppellF1[1/2, 1/2, -p, 3/2, -Sinh[e + f*x]^2, -((b*Sinh[e + f*x]^2)/a)]*S
qrt[Cosh[e + f*x]^2]*(a + b*Sinh[e + f*x]^2)^p*Tanh[e + f*x])/(f*(1 + (b*S
inh[e + f*x]^2)/a)^p)
```

Defintions of rubi rules used

rule 333
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_), x_Symbol] :> Sim
p[a^p*c^q*x*AppellF1[1/2, -p, -q, 3/2, (-b)*(x^2/a), (-d)*(x^2/c)], x] /; F
reeQ[{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && (IntegerQ[p] || GtQ[a,
0]) && (IntegerQ[q] || GtQ[c, 0])
```

rule 334
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_), x_Symbol] :> Sim
p[a^IntPart[p]*((a + b*x^2)^FracPart[p]/(1 + b*(x^2/a))^FracPart[p])   Int[
(1 + b*(x^2/a))^p*(c + d*x^2)^q, x], x] /; FreeQ[{a, b, c, d, p, q}, x] &&
NeQ[b*c - a*d, 0] &&  !(IntegerQ[p] || GtQ[a, 0])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3664
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_.), x_Symbol] :> With[{ff =
 FreeFactors[Sin[e + f*x], x]}, Simp[ff*(Sqrt[Cos[e + f*x]^2]/(f*Cos[e + f*
x]))   Subst[Int[(a + b*ff^2*x^2)^p/Sqrt[1 - ff^2*x^2], x], x, Sin[e + f*x]
/ff], x]] /; FreeQ[{a, b, e, f, p}, x] &&  !IntegerQ[p]
```

Maple [F]

$$\int \left(a+b\sinh \left(fx+e\right)^{2}\right)^{p} dx$$

input
```
int((a+b*sinh(f*x+e)^2)^p,x)
```

output
```
int((a+b*sinh(f*x+e)^2)^p,x)
```

Fricas [F]

$$\int \left(a+b\sinh^2(e+fx)\right)^p\,dx=\int \left(b\sinh\left(fx+e\right)^2+a\right)^p\,dx$$

input
```
integrate((a+b*sinh(f*x+e)^2)^p,x, algorithm="fricas")
```

output
```
integral((b*sinh(f*x + e)^2 + a)^p, x)
```

Sympy [F(-1)]

Timed out.

$$\int \left(a+b\sinh^2(e+fx)\right)^p\,dx=\text{Timed out}$$

input
```
integrate((a+b*sinh(f*x+e)**2)**p,x)
```

output
```
Timed out
```

Maxima [F]

$$\int \left(a+b\sinh^2(e+fx)\right)^p\,dx=\int \left(b\sinh\left(fx+e\right)^2+a\right)^p\,dx$$

input
```
integrate((a+b*sinh(f*x+e)^2)^p,x, algorithm="maxima")
```

output
```
integrate((b*sinh(f*x + e)^2 + a)^p, x)
```

Giac [F]

$$\int (a+b\sinh^2(e+fx))^p\,dx=\int (b\sinh(fx+e)^2+a)^p\,dx$$

input
```
integrate((a+b*sinh(f*x+e)^2)^p,x, algorithm="giac")
```

output
```
integrate((b*sinh(f*x + e)^2 + a)^p, x)
```

Mupad [F(-1)]

Timed out.

$$\int (a+b\sinh^2(e+fx))^p\,dx=\int (b\sinh(e+f\,x)^2+a)^p\,dx$$

input
```
int((a + b*sinh(e + f*x)^2)^p,x)
```

output
```
int((a + b*sinh(e + f*x)^2)^p, x)
```

Reduce [F]

$$\int (a+b\sinh^2(e+fx))^p\,dx=\int (\sinh(fx+e)^2b+a)^p\,dx$$

input
```
int((a+b*sinh(f*x+e)^2)^p,x)
```

output
```
int((sinh(e + f*x)**2*b + a)**p,x)
```

3.373 $\int \operatorname{sech}^2(e+fx) \left(a+b\sinh^2(e+fx)\right)^p \, dx$

Optimal result

Integrand size = 23, antiderivative size = 92

$$\int \operatorname{sech}^2(e+fx) \left(a+b\sinh^2(e+fx)\right)^p \, dx$$
$$= \frac{\operatorname{AppellF1}\left(\frac{1}{2},\frac{3}{2},-p,\frac{3}{2},-\sinh^2(e+fx),-\frac{b\sinh^2(e+fx)}{a}\right)\sqrt{\cosh^2(e+fx)}\left(a+b\sinh^2(e+fx)\right)^p\left(1+\frac{b\sinh \cdots}{\cdots}\right.}{f}$$

output
```
AppellF1(1/2,3/2,-p,3/2,-sinh(f*x+e)^2,-b*sinh(f*x+e)^2/a)*(cosh(f*x+e)^2)
^(1/2)*(a+b*sinh(f*x+e)^2)^p*tanh(f*x+e)/f/((1+b*sinh(f*x+e)^2/a)^p)
```

Mathematica [F]

$$\int \operatorname{sech}^2(e+fx) \left(a+b\sinh^2(e+fx)\right)^p \, dx = \int \operatorname{sech}^2(e+fx) \left(a+b\sinh^2(e+fx)\right)^p \, dx$$

input
```
Integrate[Sech[e + f*x]^2*(a + b*Sinh[e + f*x]^2)^p,x]
```

output
```
Integrate[Sech[e + f*x]^2*(a + b*Sinh[e + f*x]^2)^p, x]
```

Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 92, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.174, Rules used = {3042, 3671, 334, 333}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \operatorname{sech}^2(e+fx) \left(a+b\sinh^2(e+fx)\right)^p \, dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \frac{\left(a-b\sin(ie+ifx)^2\right)^p}{\cos(ie+ifx)^2}dx$$

$$\Big\downarrow \text{ 3671}$$

$$\frac{\sqrt{\cosh^2(e+fx)}\operatorname{sech}(e+fx)\int \frac{\left(b\sinh^2(e+fx)+a\right)^p}{\left(\sinh^2(e+fx)+1\right)^{3/2}}d\sinh(e+fx)}{f}$$

$$\Big\downarrow \text{ 334}$$

$$\frac{\sqrt{\cosh^2(e+fx)}\operatorname{sech}(e+fx)\left(a+b\sinh^2(e+fx)\right)^p\left(\frac{b\sinh^2(e+fx)}{a}+1\right)^{-p}\int \frac{\left(\frac{b\sinh^2(e+fx)}{a}+1\right)^p}{\left(\sinh^2(e+fx)+1\right)^{3/2}}d\sinh(e+fx)}{f}$$

$$\Big\downarrow \text{ 333}$$

$$\frac{\sqrt{\cosh^2(e+fx)}\tanh(e+fx)\left(a+b\sinh^2(e+fx)\right)^p\left(\frac{b\sinh^2(e+fx)}{a}+1\right)^{-p}\operatorname{AppellF1}\left(\frac{1}{2},\frac{3}{2},-p,\frac{3}{2},-\sinh^2(e+fx)\right.}{f}$$

input
```
Int[Sech[e + f*x]^2*(a + b*Sinh[e + f*x]^2)^p,x]
```

output
```
(AppellF1[1/2, 3/2, -p, 3/2, -Sinh[e + f*x]^2, -((b*Sinh[e + f*x]^2)/a)]*S
qrt[Cosh[e + f*x]^2]*(a + b*Sinh[e + f*x]^2)^p*Tanh[e + f*x])/(f*(1 + (b*S
inh[e + f*x]^2)/a)^p)
```

Defintions of rubi rules used

rule 333
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_), x_Symbol] :> Sim
p[a^p*c^q*x*AppellF1[1/2, -p, -q, 3/2, (-b)*(x^2/a), (-d)*(x^2/c)], x] /; F
reeQ[{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && (IntegerQ[p] || GtQ[a,
0]) && (IntegerQ[q] || GtQ[c, 0])
```

rule 334
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_), x_Symbol] :> Sim
p[a^IntPart[p]*((a + b*x^2)^FracPart[p]/(1 + b*(x^2/a))^FracPart[p])   Int[
(1 + b*(x^2/a))^p*(c + d*x^2)^q, x], x] /; FreeQ[{a, b, c, d, p, q}, x] &&
NeQ[b*c - a*d, 0] &&  !(IntegerQ[p] || GtQ[a, 0])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3671
```
Int[cos[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(
p_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Simp[ff*(Sqrt[
Cos[e + f*x]^2]/(f*Cos[e + f*x]))   Subst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a
 + b*ff^2*x^2)^p, x], x, Sin[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x]
 && IntegerQ[m/2] &&  !IntegerQ[p]
```

Maple [F]

$$\int \operatorname{sech}(fx+e)^2 \left(a+b\sinh(fx+e)^2\right)^p dx$$

input
```
int(sech(f*x+e)^2*(a+b*sinh(f*x+e)^2)^p,x)
```

output
```
int(sech(f*x+e)^2*(a+b*sinh(f*x+e)^2)^p,x)
```

Fricas [F]

$$\int \operatorname{sech}^2(e+fx)\left(a+b\sinh^2(e+fx)\right)^p\,dx=\int \left(b\sinh\left(fx+e\right)^2+a\right)^p\operatorname{sech}\left(fx+e\right)^2\,dx$$

input
```
integrate(sech(f*x+e)^2*(a+b*sinh(f*x+e)^2)^p,x, algorithm="fricas")
```

output
```
integral((b*sinh(f*x + e)^2 + a)^p*sech(f*x + e)^2, x)
```

Sympy [F(-1)]

Timed out.

$$\int \operatorname{sech}^2(e+fx)\left(a+b\sinh^2(e+fx)\right)^p\,dx=\text{Timed out}$$

input
```
integrate(sech(f*x+e)**2*(a+b*sinh(f*x+e)**2)**p,x)
```

output
```
Timed out
```

Maxima [F]

$$\int \operatorname{sech}^2(e+fx)\left(a+b\sinh^2(e+fx)\right)^p\,dx=\int \left(b\sinh\left(fx+e\right)^2+a\right)^p\operatorname{sech}\left(fx+e\right)^2\,dx$$

input
```
integrate(sech(f*x+e)^2*(a+b*sinh(f*x+e)^2)^p,x, algorithm="maxima")
```

output
```
integrate((b*sinh(f*x + e)^2 + a)^p*sech(f*x + e)^2, x)
```

Giac [F]

$$\int \operatorname{sech}^2(e+fx) \left(a+b\sinh^2(e+fx)\right)^p \, dx = \int \left(b\sinh\left(fx+e\right)^2+a\right)^p \operatorname{sech}\left(fx+e\right)^2 \, dx$$

input
```
integrate(sech(f*x+e)^2*(a+b*sinh(f*x+e)^2)^p,x, algorithm="giac")
```

output
```
integrate((b*sinh(f*x + e)^2 + a)^p*sech(f*x + e)^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int \operatorname{sech}^2(e+fx) \left(a+b\sinh^2(e+fx)\right)^p \, dx = \int \frac{\left(b\sinh(e+f\,x)^2+a\right)^p}{\cosh\left(e+f\,x\right)^2} \, dx$$

input
```
int((a + b*sinh(e + f*x)^2)^p/cosh(e + f*x)^2,x)
```

output
```
int((a + b*sinh(e + f*x)^2)^p/cosh(e + f*x)^2, x)
```

Reduce [F]

$$\int \operatorname{sech}^2(e+fx) \left(a+b\sinh^2(e+fx)\right)^p \, dx = \int \operatorname{sech}(fx+e)^2 \left(\sinh\left(fx+e\right)^2 b+a\right)^p dx$$

input
```
int(sech(f*x+e)^2*(a+b*sinh(f*x+e)^2)^p,x)
```

output
```
int(sech(f*x+e)^2*(a+b*sinh(f*x+e)^2)^p,x)
```

3.374 $\int \operatorname{sech}^4(e+fx) \left(a+b\sinh^2(e+fx)\right)^p \, dx$

Optimal result

Integrand size = 23, antiderivative size = 92

$$\int \operatorname{sech}^4(e+fx) \left(a+b\sinh^2(e+fx)\right)^p \, dx = \frac{\operatorname{AppellF1}\left(\frac{1}{2},\frac{5}{2},-p,\frac{3}{2},-\sinh^2(e+fx),-\frac{b\sinh^2(e+fx)}{a}\right)\sqrt{\cosh^2(e+fx)}\left(a+b\sinh^2(e+fx)\right)^p\left(1+\frac{b\sinh}{}\right.}{f}$$

output
```
AppellF1(1/2,5/2,-p,3/2,-sinh(f*x+e)^2,-b*sinh(f*x+e)^2/a)*(cosh(f*x+e)^2)
^(1/2)*(a+b*sinh(f*x+e)^2)^p*tanh(f*x+e)/f/((1+b*sinh(f*x+e)^2/a)^p)
```

Mathematica [F]

$$\int \operatorname{sech}^4(e+fx) \left(a+b\sinh^2(e+fx)\right)^p \, dx = \int \operatorname{sech}^4(e+fx) \left(a+b\sinh^2(e+fx)\right)^p \, dx$$

input
```
Integrate[Sech[e + f*x]^4*(a + b*Sinh[e + f*x]^2)^p,x]
```

output
```
Integrate[Sech[e + f*x]^4*(a + b*Sinh[e + f*x]^2)^p, x]
```

Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 92, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.174, Rules used = {3042, 3671, 334, 333}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \operatorname{sech}^4(e+fx) \left(a+b \sinh^2(e+fx)\right)^p \, dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \frac{\left(a-b \sin(ie+ifx)^2\right)^p}{\cos(ie+ifx)^4}dx$$

$$\Big\downarrow \text{ 3671}$$

$$\frac{\sqrt{\cosh^2(e+fx)}\operatorname{sech}(e+fx) \int \frac{\left(b \sinh^2(e+fx)+a\right)^p}{\left(\sinh^2(e+fx)+1\right)^{5/2}}d\sinh(e+fx)}{f}$$

$$\Big\downarrow \text{ 334}$$

$$\frac{\sqrt{\cosh^2(e+fx)}\operatorname{sech}(e+fx) \left(a+b \sinh^2(e+fx)\right)^p \left(\frac{b \sinh^2(e+fx)}{a}+1\right)^{-p} \int \frac{\left(\frac{b \sinh^2(e+fx)}{a}+1\right)^p}{\left(\sinh^2(e+fx)+1\right)^{5/2}}d\sinh(e+fx)}{f}$$

$$\Big\downarrow \text{ 333}$$

$$\frac{\sqrt{\cosh^2(e+fx)} \tanh(e+fx) \left(a+b \sinh^2(e+fx)\right)^p \left(\frac{b \sinh^2(e+fx)}{a}+1\right)^{-p} \operatorname{AppellF1}\left(\frac{1}{2},\frac{5}{2},-p,\frac{3}{2},-\sinh^2(e+f}{f}$$

input
```
Int[Sech[e + f*x]^4*(a + b*Sinh[e + f*x]^2)^p,x]
```

output
```
(AppellF1[1/2, 5/2, -p, 3/2, -Sinh[e + f*x]^2, -((b*Sinh[e + f*x]^2)/a)]*S
qrt[Cosh[e + f*x]^2]*(a + b*Sinh[e + f*x]^2)^p*Tanh[e + f*x])/(f*(1 + (b*S
inh[e + f*x]^2)/a)^p)
```

Defintions of rubi rules used

rule 333
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_), x_Symbol] :> Sim
p[a^p*c^q*x*AppellF1[1/2, -p, -q, 3/2, (-b)*(x^2/a), (-d)*(x^2/c)], x] /; F
reeQ[{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && (IntegerQ[p] || GtQ[a,
0]) && (IntegerQ[q] || GtQ[c, 0])
```

rule 334
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_), x_Symbol] :> Sim
p[a^IntPart[p]*((a + b*x^2)^FracPart[p]/(1 + b*(x^2/a))^FracPart[p])   Int[
(1 + b*(x^2/a))^p*(c + d*x^2)^q, x], x] /; FreeQ[{a, b, c, d, p, q}, x] &&
NeQ[b*c - a*d, 0] &&  !(IntegerQ[p] || GtQ[a, 0])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3671
```
Int[cos[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(
p_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Simp[ff*(Sqrt[
Cos[e + f*x]^2]/(f*Cos[e + f*x]))   Subst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a
 + b*ff^2*x^2)^p, x], x, Sin[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x]
 && IntegerQ[m/2] &&  !IntegerQ[p]
```

Maple [F]

$$\int \operatorname{sech}(fx+e)^4 \left(a+b\sinh(fx+e)^2\right)^p dx$$

input
```
int(sech(f*x+e)^4*(a+b*sinh(f*x+e)^2)^p,x)
```

output
```
int(sech(f*x+e)^4*(a+b*sinh(f*x+e)^2)^p,x)
```

Fricas [F]

$$\int \operatorname{sech}^4(e+fx)\left(a+b\sinh^2(e+fx)\right)^p\,dx=\int \left(b\sinh\left(fx+e\right)^2+a\right)^p\operatorname{sech}\left(fx+e\right)^4\,dx$$

input
```
integrate(sech(f*x+e)^4*(a+b*sinh(f*x+e)^2)^p,x, algorithm="fricas")
```

output
```
integral((b*sinh(f*x + e)^2 + a)^p*sech(f*x + e)^4, x)
```

Sympy [F(-1)]

Timed out.

$$\int \operatorname{sech}^4(e+fx)\left(a+b\sinh^2(e+fx)\right)^p\,dx=\text{Timed out}$$

input
```
integrate(sech(f*x+e)**4*(a+b*sinh(f*x+e)**2)**p,x)
```

output
```
Timed out
```

Maxima [F]

$$\int \operatorname{sech}^4(e+fx)\left(a+b\sinh^2(e+fx)\right)^p\,dx=\int \left(b\sinh\left(fx+e\right)^2+a\right)^p\operatorname{sech}\left(fx+e\right)^4\,dx$$

input
```
integrate(sech(f*x+e)^4*(a+b*sinh(f*x+e)^2)^p,x, algorithm="maxima")
```

output
```
integrate((b*sinh(f*x + e)^2 + a)^p*sech(f*x + e)^4, x)
```

Giac [F]

$$\int \operatorname{sech}^4(e+fx)\left(a+b\sinh^2(e+fx)\right)^p\,dx=\int \left(b\sinh\left(fx+e\right)^2+a\right)^p\operatorname{sech}\left(fx+e\right)^4\,dx$$

input
```
integrate(sech(f*x+e)^4*(a+b*sinh(f*x+e)^2)^p,x, algorithm="giac")
```

output
```
integrate((b*sinh(f*x + e)^2 + a)^p*sech(f*x + e)^4, x)
```

Mupad [F(-1)]

Timed out.

$$\int \operatorname{sech}^4(e+fx)\left(a+b\sinh^2(e+fx)\right)^p\,dx=\int \frac{\left(b\sinh(e+f\,x)^2+a\right)^p}{\cosh\left(e+f\,x\right)^4}\,dx$$

input
```
int((a + b*sinh(e + f*x)^2)^p/cosh(e + f*x)^4,x)
```

output
```
int((a + b*sinh(e + f*x)^2)^p/cosh(e + f*x)^4, x)
```

Reduce [F]

$$\int \operatorname{sech}^4(e+fx)\left(a+b\sinh^2(e+fx)\right)^p\,dx=\int \operatorname{sech}(fx+e)^4\left(\sinh\left(fx+e\right)^2b+a\right)^p dx$$

input
```
int(sech(f*x+e)^4*(a+b*sinh(f*x+e)^2)^p,x)
```

output
```
int(sech(f*x+e)^4*(a+b*sinh(f*x+e)^2)^p,x)
```

3.375 $\int \frac{\cosh^5(c+dx)}{a+b\sqrt{\sinh(c+dx)}}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 259

$$\begin{aligned}\int \frac{\cosh^5(c+dx)}{a+b\sqrt{\sinh(c+dx)}}\,dx = &-\frac{2a(a^4+b^4)^2\log\left(a+b\sqrt{\sinh(c+dx)}\right)}{b^{10}d}\\ &+\frac{2(a^4+b^4)^2\sqrt{\sinh(c+dx)}}{b^9d}-\frac{a^3(a^4+2b^4)\sinh(c+dx)}{b^8d}\\ &+\frac{2a^2(a^4+2b^4)\sinh^{\frac{3}{2}}(c+dx)}{3b^7d}-\frac{a(a^4+2b^4)\sinh^2(c+dx)}{2b^6d}\\ &+\frac{2(a^4+2b^4)\sinh^{\frac{5}{2}}(c+dx)}{5b^5d}-\frac{a^3\sinh^3(c+dx)}{3b^4d}\\ &+\frac{2a^2\sinh^{\frac{7}{2}}(c+dx)}{7b^3d}-\frac{a\sinh^4(c+dx)}{4b^2d}+\frac{2\sinh^{\frac{9}{2}}(c+dx)}{9bd}\end{aligned}$$

output
```
-2*a*(a^4+b^4)^2*ln(a+b*sinh(d*x+c)^(1/2))/b^10/d+2*(a^4+b^4)^2*sinh(d*x+c
)^(1/2)/b^9/d-a^3*(a^4+2*b^4)*sinh(d*x+c)/b^8/d+2/3*a^2*(a^4+2*b^4)*sinh(d
*x+c)^(3/2)/b^7/d-1/2*a*(a^4+2*b^4)*sinh(d*x+c)^2/b^6/d+2/5*(a^4+2*b^4)*si
nh(d*x+c)^(5/2)/b^5/d-1/3*a^3*sinh(d*x+c)^3/b^4/d+2/7*a^2*sinh(d*x+c)^(7/2
)/b^3/d-1/4*a*sinh(d*x+c)^4/b^2/d+2/9*sinh(d*x+c)^(9/2)/b/d
```

Mathematica [A] (verified)

Time = 0.30 (sec) , antiderivative size = 220, normalized size of antiderivative = 0.85

$$\int \frac{\cosh^5(c+dx)}{a+b\sqrt{\sinh(c+dx)}}\,dx$$
$$= \frac{-2520a(a^4+b^4)^2\log\left(a+b\sqrt{\sinh(c+dx)}\right)+2520b(a^4+b^4)^2\sqrt{\sinh(c+dx)}-1260a^3b^2(a^4+2b^4)\sin}{}$$

input
```
Integrate[Cosh[c + d*x]^5/(a + b*Sqrt[Sinh[c + d*x]]),x]
```

output
```
(-2520*a*(a^4 + b^4)^2*Log[a + b*Sqrt[Sinh[c + d*x]]] + 2520*b*(a^4 + b^4)
^2*Sqrt[Sinh[c + d*x]] - 1260*a^3*b^2*(a^4 + 2*b^4)*Sinh[c + d*x] + 840*a^
2*b^3*(a^4 + 2*b^4)*Sinh[c + d*x]^(3/2) - 630*a*b^4*(a^4 + 2*b^4)*Sinh[c +
 d*x]^2 + 504*b^5*(a^4 + 2*b^4)*Sinh[c + d*x]^(5/2) - 420*a^3*b^6*Sinh[c +
 d*x]^3 + 360*a^2*b^7*Sinh[c + d*x]^(7/2) - 315*a*b^8*Sinh[c + d*x]^4 + 28
0*b^9*Sinh[c + d*x]^(9/2))/(1260*b^10*d)
```

Rubi [A] (verified)

Time = 0.58 (sec) , antiderivative size = 235, normalized size of antiderivative = 0.91, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {3042, 3702, 2429, 2123, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cosh^5(c+dx)}{a+b\sqrt{\sinh(c+dx)}}\,dx$$
$$\Big\downarrow \text{3042}$$
$$\int \frac{\cos(ic+idx)^5}{a+b\sqrt{-i\sin(ic+idx)}}dx$$
$$\Big\downarrow \text{3702}$$
$$\frac{\int \frac{\left(\sinh^2(c+dx)+1\right)^2}{a+b\sqrt{\sinh(c+dx)}}d\sinh(c+dx)}{d}$$

$$\Big\downarrow \text{2429}$$

$$\frac{2\int \frac{\sqrt{\sinh(c+dx)}\left(\sinh^2(c+dx)+1\right)^2}{a+b\sqrt{\sinh(c+dx)}}d\sqrt{\sinh(c+dx)}}{d}$$

$$\Big\downarrow \text{2123}$$

$$\frac{2\int \left(\frac{\sinh^4(c+dx)}{b}-\frac{a\sinh^{\frac{7}{2}}(c+dx)}{b^2}+\frac{a^2\sinh^3(c+dx)}{b^3}-\frac{a^3\sinh^{\frac{5}{2}}(c+dx)}{b^4}+\frac{(a^4+2b^4)\sinh^2(c+dx)}{b^5}-\frac{a(a^4+2b^4)\sinh^{\frac{3}{2}}(c+dx)}{b^6}+\frac{a^2(a}{}\right.}{d}$$

$$\Big\downarrow \text{2009}$$

$$\frac{2\left(-\frac{a(a^4+b^4)^2\log\left(a+b\sqrt{\sinh(c+dx)}\right)}{b^{10}}+\frac{(a^4+b^4)^2\sqrt{\sinh(c+dx)}}{b^9}-\frac{a(a^4+2b^4)\sinh^2(c+dx)}{4b^6}+\frac{(a^4+2b^4)\sinh^{\frac{5}{2}}(c+dx)}{5b^5}-\frac{a^3\sinh^3(c+dx}{6b^4}\right.}{d}$$

input
```
Int[Cosh[c + d*x]^5/(a + b*Sqrt[Sinh[c + d*x]]),x]
```

output
```
(2*(-((a*(a^4 + b^4)^2*Log[a + b*Sqrt[Sinh[c + d*x]]])/b^10) + ((a^4 + b^4
)^2*Sqrt[Sinh[c + d*x]])/b^9 - (a^3*(a^4 + 2*b^4)*Sinh[c + d*x])/(2*b^8) +
 (a^2*(a^4 + 2*b^4)*Sinh[c + d*x]^(3/2))/(3*b^7) - (a*(a^4 + 2*b^4)*Sinh[c
 + d*x]^2)/(4*b^6) + ((a^4 + 2*b^4)*Sinh[c + d*x]^(5/2))/(5*b^5) - (a^3*Si
nh[c + d*x]^3)/(6*b^4) + (a^2*Sinh[c + d*x]^(7/2))/(7*b^3) - (a*Sinh[c + d
*x]^4)/(8*b^2) + Sinh[c + d*x]^(9/2)/(9*b)))/d
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2123
```
Int[(Px_)*((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol]
:> Int[ExpandIntegrand[Px*(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c
, d, m, n}, x] && PolyQ[Px, x] && (IntegersQ[m, n] || IGtQ[m, -2])
```

rule 2429
```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{g = Denominator
[n]}, Simp[g   Subst[Int[x^(g - 1)*(Pq /. x -> x^g)*(a + b*x^(g*n))^p, x],
x, x^(1/g)], x]] /; FreeQ[{a, b, p}, x] && PolyQ[Pq, x] && FractionQ[n]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3702
```
Int[cos[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*((c_.)*sin[(e_.) + (f_.)*(x
_)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Si
mp[ff/f   Subst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a + b*(c*ff*x)^n)^p, x], x,
 Sin[e + f*x]/ff], x]] /; FreeQ[{a, b, c, e, f, n, p}, x] && IntegerQ[(m -
1)/2] && (EqQ[n, 4] || GtQ[m, 0] || IGtQ[p, 0] || IntegersQ[m, p])
```

Maple [A] (verified)

Time = 0.73 (sec) , antiderivative size = 262, normalized size of antiderivative = 1.01

method	result
derivativedivides	$\frac{\frac{2\sinh(dx+c)^{\frac{9}{2}}b^8}{9}-\frac{a\sinh(dx+c)^4b^7}{4}+\frac{2a^2\sinh(dx+c)^{\frac{7}{2}}b^6}{7}-\frac{a^3\sinh(dx+c)^3b^5}{3}+\frac{2a^4b^4\sinh(dx+c)^{\frac{5}{2}}}{5}+\frac{4b^8\sinh(dx+c)^{\frac{5}{2}}}{5}-\frac{a^5b^3\sin}{}}{}$
default	$\frac{\frac{2\sinh(dx+c)^{\frac{9}{2}}b^8}{9}-\frac{a\sinh(dx+c)^4b^7}{4}+\frac{2a^2\sinh(dx+c)^{\frac{7}{2}}b^6}{7}-\frac{a^3\sinh(dx+c)^3b^5}{3}+\frac{2a^4b^4\sinh(dx+c)^{\frac{5}{2}}}{5}+\frac{4b^8\sinh(dx+c)^{\frac{5}{2}}}{5}-\frac{a^5b^3\sin}{}}{}$

input
```
int(cosh(d*x+c)^5/(a+b*sinh(d*x+c)^(1/2)),x,method=_RETURNVERBOSE)
```

output
```
1/d*(2/b^9*(1/9*sinh(d*x+c)^(9/2)*b^8-1/8*a*sinh(d*x+c)^4*b^7+1/7*a^2*sinh
(d*x+c)^(7/2)*b^6-1/6*a^3*sinh(d*x+c)^3*b^5+1/5*a^4*b^4*sinh(d*x+c)^(5/2)+
2/5*b^8*sinh(d*x+c)^(5/2)-1/4*a^5*b^3*sinh(d*x+c)^2-1/2*a*b^7*sinh(d*x+c)^
2+1/3*a^6*b^2*sinh(d*x+c)^(3/2)+2/3*a^2*b^6*sinh(d*x+c)^(3/2)-1/2*a^7*b*si
nh(d*x+c)-a^3*b^5*sinh(d*x+c)+a^8*sinh(d*x+c)^(1/2)+2*a^4*b^4*sinh(d*x+c)^
(1/2)+b^8*sinh(d*x+c)^(1/2))-2*a*(a^8+2*a^4*b^4+b^8)/b^10*ln(a+b*sinh(d*x+
c)^(1/2)))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2595 vs. $2(233) = 466$.

Time = 0.77 (sec) , antiderivative size = 2595, normalized size of antiderivative = 10.02

$$\int \frac{\cosh^5(c+dx)}{a+b\sqrt{\sinh(c+dx)}}\,dx = \text{Too large to display}$$

input
```
integrate(cosh(d*x+c)^5/(a+b*sinh(d*x+c)^(1/2)),x, algorithm="fricas")
```

output
```
-1/20160*(315*a*b^8*cosh(d*x + c)^8 + 315*a*b^8*sinh(d*x + c)^8 + 840*a^3*
b^6*cosh(d*x + c)^7 - 840*a^3*b^6*cosh(d*x + c) + 315*a*b^8 + 840*(3*a*b^8
*cosh(d*x + c) + a^3*b^6)*sinh(d*x + c)^7 + 1260*(2*a^5*b^4 + 3*a*b^8)*cos
h(d*x + c)^6 + 420*(21*a*b^8*cosh(d*x + c)^2 + 14*a^3*b^6*cosh(d*x + c) +
6*a^5*b^4 + 9*a*b^8)*sinh(d*x + c)^6 + 2520*(4*a^7*b^2 + 7*a^3*b^6)*cosh(d
*x + c)^5 + 2520*(7*a*b^8*cosh(d*x + c)^3 + 7*a^3*b^6*cosh(d*x + c)^2 + 4*
a^7*b^2 + 7*a^3*b^6 + 3*(2*a^5*b^4 + 3*a*b^8)*cosh(d*x + c))*sinh(d*x + c)
^5 - 20160*((a^9 + 2*a^5*b^4 + a*b^8)*d*x + (a^9 + 2*a^5*b^4 + a*b^8)*c)*c
osh(d*x + c)^4 + 210*(105*a*b^8*cosh(d*x + c)^4 + 140*a^3*b^6*cosh(d*x + c
)^3 - 96*(a^9 + 2*a^5*b^4 + a*b^8)*d*x + 90*(2*a^5*b^4 + 3*a*b^8)*cosh(d*x
 + c)^2 - 96*(a^9 + 2*a^5*b^4 + a*b^8)*c + 60*(4*a^7*b^2 + 7*a^3*b^6)*cosh
(d*x + c))*sinh(d*x + c)^4 - 2520*(4*a^7*b^2 + 7*a^3*b^6)*cosh(d*x + c)^3
+ 840*(21*a*b^8*cosh(d*x + c)^5 + 35*a^3*b^6*cosh(d*x + c)^4 - 12*a^7*b^2
- 21*a^3*b^6 + 30*(2*a^5*b^4 + 3*a*b^8)*cosh(d*x + c)^3 + 30*(4*a^7*b^2 +
7*a^3*b^6)*cosh(d*x + c)^2 - 96*((a^9 + 2*a^5*b^4 + a*b^8)*d*x + (a^9 + 2*
a^5*b^4 + a*b^8)*c)*cosh(d*x + c))*sinh(d*x + c)^3 + 1260*(2*a^5*b^4 + 3*a
*b^8)*cosh(d*x + c)^2 + 1260*(7*a*b^8*cosh(d*x + c)^6 + 14*a^3*b^6*cosh(d*
x + c)^5 + 2*a^5*b^4 + 3*a*b^8 + 15*(2*a^5*b^4 + 3*a*b^8)*cosh(d*x + c)^4
+ 20*(4*a^7*b^2 + 7*a^3*b^6)*cosh(d*x + c)^3 - 96*((a^9 + 2*a^5*b^4 + a*b^
8)*d*x + (a^9 + 2*a^5*b^4 + a*b^8)*c)*cosh(d*x + c)^2 - 6*(4*a^7*b^2 + ...
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\cosh^5(c+dx)}{a+b\sqrt{\sinh(c+dx)}}\,dx = \text{Timed out}$$

input
```
integrate(cosh(d*x+c)**5/(a+b*sinh(d*x+c)**(1/2)),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{\cosh^5(c+dx)}{a+b\sqrt{\sinh(c+dx)}}\,dx = \int \frac{\cosh(dx+c)^5}{b\sqrt{\sinh(dx+c)}+a}\,dx$$

input
```
integrate(cosh(d*x+c)^5/(a+b*sinh(d*x+c)^(1/2)),x, algorithm="maxima")
```

output
```
integrate(cosh(d*x + c)^5/(b*sqrt(sinh(d*x + c)) + a), x)
```

Giac [F]

$$\int \frac{\cosh^5(c+dx)}{a+b\sqrt{\sinh(c+dx)}}\,dx = \int \frac{\cosh(dx+c)^5}{b\sqrt{\sinh(dx+c)}+a}\,dx$$

input
```
integrate(cosh(d*x+c)^5/(a+b*sinh(d*x+c)^(1/2)),x, algorithm="giac")
```

output
```
integrate(cosh(d*x + c)^5/(b*sqrt(sinh(d*x + c)) + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\cosh^5(c+dx)}{a+b\sqrt{\sinh(c+dx)}}\,dx = \int \frac{\cosh(c+d\,x)^5}{a+b\,\sqrt{\sinh(c+d\,x)}}\,dx$$

input
```
int(cosh(c + d*x)^5/(a + b*sinh(c + d*x)^(1/2)),x)
```

output
```
int(cosh(c + d*x)^5/(a + b*sinh(c + d*x)^(1/2)), x)
```

Reduce [F]

$$\int \frac{\cosh^5(c+dx)}{a+b\sqrt{\sinh(c+dx)}}\,dx = \frac{-3e^{8dx+8c}a\,b^8-8e^{7dx+7c}a^3b^6-24e^{6dx+6c}a^5b^4-36e^{6dx+6c}a\,b^8-96e^{5dx+5c}a^7b^2-168e^{5dx+5c}a^3b^6+192e^{4d}}{}$$

input
```
int(cosh(d*x+c)^5/(a+b*sinh(d*x+c)^(1/2)),x)
```

output
```
( - 3*e**(8*c + 8*d*x)*a*b**8 - 8*e**(7*c + 7*d*x)*a**3*b**6 - 24*e**(6*c
+ 6*d*x)*a**5*b**4 - 36*e**(6*c + 6*d*x)*a*b**8 - 96*e**(5*c + 5*d*x)*a**7
*b**2 - 168*e**(5*c + 5*d*x)*a**3*b**6 + 192*e**(4*c + 4*d*x)*int((sqrt(si
nh(c + d*x))*cosh(c + d*x)**5)/(sinh(c + d*x)*b**2 - a**2),x)*b**11*d - 19
2*e**(4*c + 4*d*x)*log(e**(2*c + 2*d*x)*b**2 - 2*e**(c + d*x)*a**2 - b**2)
*a**9 - 384*e**(4*c + 4*d*x)*log(e**(2*c + 2*d*x)*b**2 - 2*e**(c + d*x)*a*
*2 - b**2)*a**5*b**4 - 192*e**(4*c + 4*d*x)*log(e**(2*c + 2*d*x)*b**2 - 2*
e**(c + d*x)*a**2 - b**2)*a*b**8 + 192*e**(4*c + 4*d*x)*a**9*d*x + 384*e**
(4*c + 4*d*x)*a**5*b**4*d*x + 192*e**(4*c + 4*d*x)*a*b**8*d*x + 96*e**(3*c
 + 3*d*x)*a**7*b**2 + 168*e**(3*c + 3*d*x)*a**3*b**6 - 24*e**(2*c + 2*d*x)
*a**5*b**4 - 36*e**(2*c + 2*d*x)*a*b**8 + 8*e**(c + d*x)*a**3*b**6 - 3*a*b
**8)/(192*e**(4*c + 4*d*x)*b**10*d)
```

3.376 $\int \frac{\cosh^3(c+dx)}{a+b\sqrt{\sinh(c+dx)}}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 136

$$\int \frac{\cosh^3(c+dx)}{a+b\sqrt{\sinh(c+dx)}}\,dx = -\frac{2a(a^4+b^4)\log\left(a+b\sqrt{\sinh(c+dx)}\right)}{b^6d} + \frac{2(a^4+b^4)\sqrt{\sinh(c+dx)}}{b^5d} - \frac{a^3\sinh(c+dx)}{b^4d} + \frac{2a^2\sinh^{\frac{3}{2}}(c+dx)}{3b^3d} - \frac{a\sinh^2(c+dx)}{2b^2d} + \frac{2\sinh^{\frac{5}{2}}(c+dx)}{5bd}$$

output
```
-2*a*(a^4+b^4)*ln(a+b*sinh(d*x+c)^(1/2))/b^6/d+2*(a^4+b^4)*sinh(d*x+c)^(1/
2)/b^5/d-a^3*sinh(d*x+c)/b^4/d+2/3*a^2*sinh(d*x+c)^(3/2)/b^3/d-1/2*a*sinh(
d*x+c)^2/b^2/d+2/5*sinh(d*x+c)^(5/2)/b/d
```

Mathematica [A] (verified)

Time = 0.11 (sec) , antiderivative size = 117, normalized size of antiderivative = 0.86

$$\int \frac{\cosh^3(c+dx)}{a+b\sqrt{\sinh(c+dx)}}\,dx$$
$$= \frac{-60a(a^4+b^4)\log\left(a+b\sqrt{\sinh(c+dx)}\right)+60b(a^4+b^4)\sqrt{\sinh(c+dx)}-30a^3b^2\sinh(c+dx)+20a^2b^3}{30b^6d}$$

input
```
Integrate[Cosh[c + d*x]^3/(a + b*Sqrt[Sinh[c + d*x]]),x]
```

output
```
(-60*a*(a^4 + b^4)*Log[a + b*Sqrt[Sinh[c + d*x]]] + 60*b*(a^4 + b^4)*Sqrt[
Sinh[c + d*x]] - 30*a^3*b^2*Sinh[c + d*x] + 20*a^2*b^3*Sinh[c + d*x]^(3/2)
 - 15*a*b^4*Sinh[c + d*x]^2 + 12*b^5*Sinh[c + d*x]^(5/2))/(30*b^6*d)
```

Rubi [A] (verified)

Time = 0.45 (sec) , antiderivative size = 124, normalized size of antiderivative = 0.91, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {3042, 3702, 2429, 2123, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cosh^3(c+dx)}{a+b\sqrt{\sinh(c+dx)}}\,dx$$
$$\Big\downarrow \text{3042}$$
$$\int \frac{\cos(ic+idx)^3}{a+b\sqrt{-i\sin(ic+idx)}}dx$$
$$\Big\downarrow \text{3702}$$
$$\frac{\int \frac{\sinh^2(c+dx)+1}{a+b\sqrt{\sinh(c+dx)}}d\sinh(c+dx)}{d}$$
$$\Big\downarrow \text{2429}$$

$$\frac{2\int \frac{\sqrt{\sinh(c+dx)}(\sinh^2(c+dx)+1)}{a+b\sqrt{\sinh(c+dx)}}d\sqrt{\sinh(c+dx)}}{d}$$

$\Big\downarrow$ 2123

$$\frac{2\int \left(-\frac{\sqrt{\sinh(c+dx)}a^3}{b^4}+\frac{\sinh(c+dx)a^2}{b^3}-\frac{\sinh^{\frac{3}{2}}(c+dx)a}{b^2}-\frac{(a^4+b^4)a}{b^5\left(a+b\sqrt{\sinh(c+dx)}\right)}+\frac{\sinh^2(c+dx)}{b}+\frac{a^4+b^4}{b^5}\right)d\sqrt{\sinh(c+dx)}}{d}$$

$\Big\downarrow$ 2009

$$\frac{2\left(-\frac{a(a^4+b^4)\log\left(a+b\sqrt{\sinh(c+dx)}\right)}{b^6}+\frac{(a^4+b^4)\sqrt{\sinh(c+dx)}}{b^5}-\frac{a^3\sinh(c+dx)}{2b^4}+\frac{a^2\sinh^{\frac{3}{2}}(c+dx)}{3b^3}-\frac{a\sinh^2(c+dx)}{4b^2}+\frac{\sinh^{\frac{5}{2}}(c+dx)}{5b}\right)}{d}$$

input
```
Int[Cosh[c + d*x]^3/(a + b*Sqrt[Sinh[c + d*x]]),x]
```

output
```
(2*(-((a*(a^4 + b^4)*Log[a + b*Sqrt[Sinh[c + d*x]]])/b^6) + ((a^4 + b^4)*S
qrt[Sinh[c + d*x]])/b^5 - (a^3*Sinh[c + d*x])/(2*b^4) + (a^2*Sinh[c + d*x]
^(3/2))/(3*b^3) - (a*Sinh[c + d*x]^2)/(4*b^2) + Sinh[c + d*x]^(5/2)/(5*b))
)/d
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2123
```
Int[(Px_)*((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol]
:> Int[ExpandIntegrand[Px*(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c
, d, m, n}, x] && PolyQ[Px, x] && (IntegersQ[m, n] || IGtQ[m, -2])
```

rule 2429
```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{g = Denominator
[n]}, Simp[g   Subst[Int[x^(g - 1)*(Pq /. x -> x^g)*(a + b*x^(g*n))^p, x],
x, x^(1/g)], x]] /; FreeQ[{a, b, p}, x] && PolyQ[Pq, x] && FractionQ[n]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3702
```
Int[cos[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*((c_.)*sin[(e_.) + (f_.)*(x
_)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Si
mp[ff/f   Subst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a + b*(c*ff*x)^n)^p, x], x,
 Sin[e + f*x]/ff], x]] /; FreeQ[{a, b, c, e, f, n, p}, x] && IntegerQ[(m -
1)/2] && (EqQ[n, 4] || GtQ[m, 0] || IGtQ[p, 0] || IntegersQ[m, p])
```

Maple [A] (verified)

Time = 0.36 (sec) , antiderivative size = 117, normalized size of antiderivative = 0.86

method	result
derivativedivides	$\frac{\frac{\frac{2\sinh(dx+c)^{\frac{5}{2}}b^4}{5}-\frac{\sinh(dx+c)^2ab^3}{2}+\frac{2a^2\sinh(dx+c)^{\frac{3}{2}}b^2}{3}-a^3\sinh(dx+c)b+2a^4\sqrt{\sinh(dx+c)}+2b^4\sqrt{\sinh(dx+c)}}{b^5}-\frac{2a\left(a^4+b^4\right)\ln(}{}}{d}$
default	$\frac{\frac{\frac{2\sinh(dx+c)^{\frac{5}{2}}b^4}{5}-\frac{\sinh(dx+c)^2ab^3}{2}+\frac{2a^2\sinh(dx+c)^{\frac{3}{2}}b^2}{3}-a^3\sinh(dx+c)b+2a^4\sqrt{\sinh(dx+c)}+2b^4\sqrt{\sinh(dx+c)}}{b^5}-\frac{2a\left(a^4+b^4\right)\ln(}{}}{d}$

input
```
int(cosh(d*x+c)^3/(a+b*sinh(d*x+c)^(1/2)),x,method=_RETURNVERBOSE)
```

output
```
1/d*(2/b^5*(1/5*sinh(d*x+c)^(5/2)*b^4-1/4*sinh(d*x+c)^2*a*b^3+1/3*a^2*sinh
(d*x+c)^(3/2)*b^2-1/2*a^3*sinh(d*x+c)*b+a^4*sinh(d*x+c)^(1/2)+b^4*sinh(d*x
+c)^(1/2))-2*a*(a^4+b^4)/b^6*ln(a+b*sinh(d*x+c)^(1/2)))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 879 vs. 2(122) = 244.

Time = 0.68 (sec) , antiderivative size = 879, normalized size of antiderivative = 6.46

$$\int \frac{\cosh^3(c+dx)}{a+b\sqrt{\sinh(c+dx)}}\,dx = \text{Too large to display}$$

input
```
integrate(cosh(d*x+c)^3/(a+b*sinh(d*x+c)^(1/2)),x, algorithm="fricas")
```

output
```
-1/120*(15*a*b^4*cosh(d*x + c)^4 + 15*a*b^4*sinh(d*x + c)^4 + 60*a^3*b^2*c
osh(d*x + c)^3 - 60*a^3*b^2*cosh(d*x + c) + 15*a*b^4 + 60*(a*b^4*cosh(d*x
+ c) + a^3*b^2)*sinh(d*x + c)^3 - 120*((a^5 + a*b^4)*d*x + (a^5 + a*b^4)*c
)*cosh(d*x + c)^2 + 30*(3*a*b^4*cosh(d*x + c)^2 + 6*a^3*b^2*cosh(d*x + c)
- 4*(a^5 + a*b^4)*d*x - 4*(a^5 + a*b^4)*c)*sinh(d*x + c)^2 - 120*((a^5 + a
*b^4)*cosh(d*x + c)^2 + 2*(a^5 + a*b^4)*cosh(d*x + c)*sinh(d*x + c) + (a^5
 + a*b^4)*sinh(d*x + c)^2)*log((b^2*cosh(d*x + c)^2 + b^2*sinh(d*x + c)^2
+ 2*a^2*cosh(d*x + c) - b^2 + 2*(b^2*cosh(d*x + c) + a^2)*sinh(d*x + c) -
4*(a*b*cosh(d*x + c) + a*b*sinh(d*x + c))*sqrt(sinh(d*x + c)))/(b^2*cosh(d
*x + c)^2 + b^2*sinh(d*x + c)^2 - 2*a^2*cosh(d*x + c) - b^2 + 2*(b^2*cosh(
d*x + c) - a^2)*sinh(d*x + c))) + 120*((a^5 + a*b^4)*cosh(d*x + c)^2 + 2*(
a^5 + a*b^4)*cosh(d*x + c)*sinh(d*x + c) + (a^5 + a*b^4)*sinh(d*x + c)^2)*
log(2*(b^2*sinh(d*x + c) - a^2)/(cosh(d*x + c) - sinh(d*x + c))) + 60*(a*b
^4*cosh(d*x + c)^3 + 3*a^3*b^2*cosh(d*x + c)^2 - a^3*b^2 - 4*((a^5 + a*b^4
)*d*x + (a^5 + a*b^4)*c)*cosh(d*x + c))*sinh(d*x + c) - 4*(3*b^5*cosh(d*x
+ c)^4 + 3*b^5*sinh(d*x + c)^4 + 10*a^2*b^3*cosh(d*x + c)^3 - 10*a^2*b^3*c
osh(d*x + c) + 3*b^5 + 2*(6*b^5*cosh(d*x + c) + 5*a^2*b^3)*sinh(d*x + c)^3
 + 6*(10*a^4*b + 9*b^5)*cosh(d*x + c)^2 + 6*(3*b^5*cosh(d*x + c)^2 + 5*a^2
*b^3*cosh(d*x + c) + 10*a^4*b + 9*b^5)*sinh(d*x + c)^2 + 2*(6*b^5*cosh(d*x
 + c)^3 + 15*a^2*b^3*cosh(d*x + c)^2 - 5*a^2*b^3 + 6*(10*a^4*b + 9*b^5)...
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\cosh^3(c+dx)}{a+b\sqrt{\sinh(c+dx)}}\,dx = \text{Timed out}$$

input
```
integrate(cosh(d*x+c)**3/(a+b*sinh(d*x+c)**(1/2)),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{\cosh^3(c+dx)}{a+b\sqrt{\sinh(c+dx)}}\,dx = \int \frac{\cosh\left(dx+c\right)^3}{b\sqrt{\sinh\left(dx+c\right)}+a}\,dx$$

input
```
integrate(cosh(d*x+c)^3/(a+b*sinh(d*x+c)^(1/2)),x, algorithm="maxima")
```

output
```
integrate(cosh(d*x + c)^3/(b*sqrt(sinh(d*x + c)) + a), x)
```

Giac [F]

$$\int \frac{\cosh^3(c+dx)}{a+b\sqrt{\sinh(c+dx)}}\,dx = \int \frac{\cosh\left(dx+c\right)^3}{b\sqrt{\sinh\left(dx+c\right)}+a}\,dx$$

input
```
integrate(cosh(d*x+c)^3/(a+b*sinh(d*x+c)^(1/2)),x, algorithm="giac")
```

output
```
integrate(cosh(d*x + c)^3/(b*sqrt(sinh(d*x + c)) + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\cosh^3(c+dx)}{a+b\sqrt{\sinh(c+dx)}}\,dx = \int \frac{\cosh(c+d\,x)^3}{a+b\,\sqrt{\sinh\left(c+d\,x\right)}}\,dx$$

input
```
int(cosh(c + d*x)^3/(a + b*sinh(c + d*x)^(1/2)),x)
```

output
```
int(cosh(c + d*x)^3/(a + b*sinh(c + d*x)^(1/2)), x)
```

Reduce [F]

$$\int \frac{\cosh^3(c+dx)}{a+b\sqrt{\sinh(c+dx)}}\,dx$$
$$= \frac{-e^{4dx+4c}a\,b^4 - 4e^{3dx+3c}a^3b^2 + 8e^{2dx+2c}\left(\int \frac{\sqrt{\sinh(dx+c)}\,\cosh(dx+c)^3}{\sinh(dx+c)b^2-a^2}dx\right)b^7d - 8e^{2dx+2c}\log\left(e^{2dx+2c}b^2 - 2e^{dx+c}a\right.}{8e^{2c}}$$

input
```
int(cosh(d*x+c)^3/(a+b*sinh(d*x+c)^(1/2)),x)
```

output
```
( - e**(4*c + 4*d*x)*a*b**4 - 4*e**(3*c + 3*d*x)*a**3*b**2 + 8*e**(2*c + 2
*d*x)*int((sqrt(sinh(c + d*x))*cosh(c + d*x)**3)/(sinh(c + d*x)*b**2 - a**
2),x)*b**7*d - 8*e**(2*c + 2*d*x)*log(e**(2*c + 2*d*x)*b**2 - 2*e**(c + d*
x)*a**2 - b**2)*a**5 - 8*e**(2*c + 2*d*x)*log(e**(2*c + 2*d*x)*b**2 - 2*e*
*(c + d*x)*a**2 - b**2)*a*b**4 + 8*e**(2*c + 2*d*x)*a**5*d*x + 8*e**(2*c +
 2*d*x)*a*b**4*d*x + 4*e**(c + d*x)*a**3*b**2 - a*b**4)/(8*e**(2*c + 2*d*x
)*b**6*d)
```

3.377 $\int \frac{\cosh(c+dx)}{a+b\sqrt{\sinh(c+dx)}}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 43

$$\int \frac{\cosh(c+dx)}{a+b\sqrt{\sinh(c+dx)}}\,dx = -\frac{2a\log\left(a+b\sqrt{\sinh(c+dx)}\right)}{b^2d}+\frac{2\sqrt{\sinh(c+dx)}}{bd}$$

output
```
-2*a*ln(a+b*sinh(d*x+c)^(1/2))/b^2/d+2*sinh(d*x+c)^(1/2)/b/d
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.95

$$\int \frac{\cosh(c+dx)}{a+b\sqrt{\sinh(c+dx)}}\,dx = \frac{-\frac{2a\log\left(a+b\sqrt{\sinh(c+dx)}\right)}{b^2}+\frac{2\sqrt{\sinh(c+dx)}}{b}}{d}$$

input
```
Integrate[Cosh[c + d*x]/(a + b*Sqrt[Sinh[c + d*x]]),x]
```

output
```
((-2*a*Log[a + b*Sqrt[Sinh[c + d*x]]])/b^2 + (2*Sqrt[Sinh[c + d*x]])/b)/d
```

Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.95, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.217, Rules used = {3042, 3702, 774, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cosh(c+dx)}{a+b\sqrt{\sinh(c+dx)}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\cos(ic+idx)}{a+b\sqrt{-i\sin(ic+idx)}}dx$$

$$\Big\downarrow \text{3702}$$

$$\frac{\int \frac{1}{a+b\sqrt{\sinh(c+dx)}}d\sinh(c+dx)}{d}$$

$$\Big\downarrow \text{774}$$

$$\frac{2\int \frac{\sqrt{\sinh(c+dx)}}{a+b\sqrt{\sinh(c+dx)}}d\sqrt{\sinh(c+dx)}}{d}$$

$$\Big\downarrow \text{49}$$

$$\frac{2\int \left(\frac{1}{b}-\frac{a}{b\left(a+b\sqrt{\sinh(c+dx)}\right)}\right)d\sqrt{\sinh(c+dx)}}{d}$$

$$\Big\downarrow \text{2009}$$

$$\frac{2\left(\frac{\sqrt{\sinh(c+dx)}}{b}-\frac{a\log\left(a+b\sqrt{\sinh(c+dx)}\right)}{b^2}\right)}{d}$$

input
```
Int[Cosh[c + d*x]/(a + b*Sqrt[Sinh[c + d*x]]),x]
```

output
```
(2*(-((a*Log[a + b*Sqrt[Sinh[c + d*x]]])/b^2) + Sqrt[Sinh[c + d*x]]/b))/d
```

Defintions of rubi rules used

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 774
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = Denominator[n]},
Simp[k   Subst[Int[x^(k - 1)*(a + b*x^(k*n))^p, x], x, x^(1/k)], x]] /; Fre
eQ[{a, b, p}, x] && FractionQ[n]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3702
```
Int[cos[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*((c_.)*sin[(e_.) + (f_.)*(x
_)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Si
mp[ff/f   Subst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a + b*(c*ff*x)^n)^p, x], x,
 Sin[e + f*x]/ff], x]] /; FreeQ[{a, b, c, e, f, n, p}, x] && IntegerQ[(m -
1)/2] && (EqQ[n, 4] || GtQ[m, 0] || IGtQ[p, 0] || IntegersQ[m, p])
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 79 vs. $2(39) = 78$.

Time = 0.22 (sec) , antiderivative size = 80, normalized size of antiderivative = 1.86

method	result	size
derivativedivides	$\frac{\frac{2\sqrt{\sinh(dx+c)}}{b}-\frac{a\ln\left(a+b\sqrt{\sinh(dx+c)}\right)}{b^2}+\frac{a\ln\left(-b\sqrt{\sinh(dx+c)}+a\right)}{b^2}-\frac{a\ln\left(\sinh(dx+c)b^2-a^2\right)}{b^2}}{d}$	80
default	$\frac{\frac{2\sqrt{\sinh(dx+c)}}{b}-\frac{a\ln\left(a+b\sqrt{\sinh(dx+c)}\right)}{b^2}+\frac{a\ln\left(-b\sqrt{\sinh(dx+c)}+a\right)}{b^2}-\frac{a\ln\left(\sinh(dx+c)b^2-a^2\right)}{b^2}}{d}$	80

input
```
int(cosh(d*x+c)/(a+b*sinh(d*x+c)^(1/2)),x,method=_RETURNVERBOSE)
```

output
```
1/d*(2/b*sinh(d*x+c)^(1/2)-a/b^2*ln(a+b*sinh(d*x+c)^(1/2))+a/b^2*ln(-b*sin
h(d*x+c)^(1/2)+a)-a*ln(sinh(d*x+c)*b^2-a^2)/b^2)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 225 vs. $2(39) = 78$.

Time = 0.63 (sec) , antiderivative size = 225, normalized size of antiderivative = 5.23

$$\int \frac{\cosh(c+dx)}{a+b\sqrt{\sinh(c+dx)}}\,dx = \frac{adx + a\log\left(\frac{b^2\cosh(dx+c)^2+b^2\sinh(dx+c)^2+2\,a^2\cosh(dx+c)-b^2+2\,(b^2\cosh(dx+c)+a^2)\sinh(dx+c)-4\,(ab\cosh(dx+c)+ab\sinh(dx+c)}{b^2\cosh(dx+c)^2+b^2\sinh(dx+c)^2-2\,a^2\cosh(dx+c)-b^2+2\,(b^2\cosh(dx+c)-a^2)\sinh(dx+c)}\right.}{b^2d}$$

input
```
integrate(cosh(d*x+c)/(a+b*sinh(d*x+c)^(1/2)),x, algorithm="fricas")
```

output
```
(a*d*x + a*log((b^2*cosh(d*x + c)^2 + b^2*sinh(d*x + c)^2 + 2*a^2*cosh(d*x
 + c) - b^2 + 2*(b^2*cosh(d*x + c) + a^2)*sinh(d*x + c) - 4*(a*b*cosh(d*x
+ c) + a*b*sinh(d*x + c))*sqrt(sinh(d*x + c)))/(b^2*cosh(d*x + c)^2 + b^2*
sinh(d*x + c)^2 - 2*a^2*cosh(d*x + c) - b^2 + 2*(b^2*cosh(d*x + c) - a^2)*
sinh(d*x + c))) - a*log(2*(b^2*sinh(d*x + c) - a^2)/(cosh(d*x + c) - sinh(
d*x + c))) + 2*b*sqrt(sinh(d*x + c)))/(b^2*d)
```

Sympy [A] (verification not implemented)

Time = 1.23 (sec) , antiderivative size = 68, normalized size of antiderivative = 1.58

$$\int \frac{\cosh(c+dx)}{a+b\sqrt{\sinh(c+dx)}}\,dx = \begin{cases} \frac{x\cosh(c)}{a} & \text{for}\: b = 0 \wedge d = 0 \\ \frac{\sinh(c+dx)}{ad} & \text{for}\: b = 0 \\ \frac{x\cosh(c)}{a+b\sqrt{\sinh(c)}} & \text{for}\: d = 0 \\ -\frac{2a\log\left(\frac{a}{b}+\sqrt{\sinh(c+dx)}\right)}{b^2d} + \frac{2\sqrt{\sinh(c+dx)}}{bd} & \text{otherwise} \end{cases}$$

input
```
integrate(cosh(d*x+c)/(a+b*sinh(d*x+c)**(1/2)),x)
```

output
```
Piecewise((x*cosh(c)/a, Eq(b, 0) & Eq(d, 0)), (sinh(c + d*x)/(a*d), Eq(b,
0)), (x*cosh(c)/(a + b*sqrt(sinh(c))), Eq(d, 0)), (-2*a*log(a/b + sqrt(sin
h(c + d*x)))/(b**2*d) + 2*sqrt(sinh(c + d*x))/(b*d), True))
```

Maxima [F]

$$\int \frac{\cosh(c+dx)}{a+b\sqrt{\sinh(c+dx)}}\,dx = \int \frac{\cosh\left(dx+c\right)}{b\sqrt{\sinh\left(dx+c\right)}+a}\,dx$$

input
```
integrate(cosh(d*x+c)/(a+b*sinh(d*x+c)^(1/2)),x, algorithm="maxima")
```

output
```
integrate(cosh(d*x + c)/(b*sqrt(sinh(d*x + c)) + a), x)
```

Giac [F]

$$\int \frac{\cosh(c+dx)}{a+b\sqrt{\sinh(c+dx)}}\,dx = \int \frac{\cosh\left(dx+c\right)}{b\sqrt{\sinh\left(dx+c\right)}+a}\,dx$$

input
```
integrate(cosh(d*x+c)/(a+b*sinh(d*x+c)^(1/2)),x, algorithm="giac")
```

output
```
integrate(cosh(d*x + c)/(b*sqrt(sinh(d*x + c)) + a), x)
```

Mupad [B] (verification not implemented)

Time = 1.94 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.91

$$\int \frac{\cosh(c+dx)}{a+b\sqrt{\sinh(c+dx)}}\,dx = \frac{2\sqrt{\sinh\left(c+d\,x\right)}}{b\,d} - \frac{2\,a\,\ln\left(a+b\,\sqrt{\sinh\left(c+d\,x\right)}\right)}{b^2\,d}$$

input
```
int(cosh(c + d*x)/(a + b*sinh(c + d*x)^(1/2)),x)
```

output
```
(2*sinh(c + d*x)^(1/2))/(b*d) - (2*a*log(a + b*sinh(c + d*x)^(1/2)))/(b^2*
d)
```

Reduce [F]

$$\int \frac{\cosh(c+dx)}{a+b\sqrt{\sinh(c+dx)}}\,dx = \frac{2\sqrt{\sinh(dx+c)}\,b + \left(\int \frac{\sqrt{\sinh(dx+c)}\,\cosh(dx+c)}{\sinh(dx+c)^2 b^2 - \sinh(dx+c)a^2}dx\right)a^2bd - \log(\sinh(dx+c)\,b^2 - a^2)\,a}{b^2d}$$

input
```
int(cosh(d*x+c)/(a+b*sinh(d*x+c)^(1/2)),x)
```

output
```
(2*sqrt(sinh(c + d*x))*b + int((sqrt(sinh(c + d*x))*cosh(c + d*x))/(sinh(c
 + d*x)**2*b**2 - sinh(c + d*x)*a**2),x)*a**2*b*d - log(sinh(c + d*x)*b**2
 - a**2)*a)/(b**2*d)
```

3.378 $\int \frac{\text{sech}(c+dx)}{a+b\sqrt{\sinh(c+dx)}}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 230

$$\int \frac{\text{sech}(c+dx)}{a+b\sqrt{\sinh(c+dx)}}\,dx = \frac{b(a^2-b^2)\arctan\left(1-\sqrt{2}\sqrt{\sinh(c+dx)}\right)}{\sqrt{2}\,(a^4+b^4)\,d} - \frac{b(a^2-b^2)\arctan\left(1+\sqrt{2}\sqrt{\sinh(c+dx)}\right)}{\sqrt{2}\,(a^4+b^4)\,d} + \frac{a^3\arctan(\sinh(c+dx))}{(a^4+b^4)\,d} + \frac{b(a^2+b^2)\,\text{arctanh}\left(\frac{\sqrt{2}\sqrt{\sinh(c+dx)}}{1+\sinh(c+dx)}\right)}{\sqrt{2}\,(a^4+b^4)\,d} + \frac{ab^2\log(\cosh(c+dx))}{(a^4+b^4)\,d} - \frac{2ab^2\log\left(a+b\sqrt{\sinh(c+dx)}\right)}{(a^4+b^4)\,d}$$

output
```
-1/2*b*(a^2-b^2)*arctan(-1+2^(1/2)*sinh(d*x+c)^(1/2))*2^(1/2)/(a^4+b^4)/d-
1/2*b*(a^2-b^2)*arctan(1+2^(1/2)*sinh(d*x+c)^(1/2))*2^(1/2)/(a^4+b^4)/d+a^
3*arctan(sinh(d*x+c))/(a^4+b^4)/d+1/2*b*(a^2+b^2)*arctanh(2^(1/2)*sinh(d*x
+c)^(1/2)/(1+sinh(d*x+c)))*2^(1/2)/(a^4+b^4)/d+a*b^2*ln(cosh(d*x+c))/(a^4+
b^4)/d-2*a*b^2*ln(a+b*sinh(d*x+c)^(1/2))/(a^4+b^4)/d
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 0.19 (sec) , antiderivative size = 229, normalized size of antiderivative = 1.00

$$\int \frac{\operatorname{sech}(c+dx)}{a+b\sqrt{\sinh(c+dx)}}\,dx$$
$$= \frac{3\left(-2\sqrt{2}b^3\arctan\left(1-\sqrt{2}\sqrt{\sinh(c+dx)}\right)+2\sqrt{2}b^3\arctan\left(1+\sqrt{2}\sqrt{\sinh(c+dx)}\right)+4a^3\arctan(\sinh}{}$$

input
```
Integrate[Sech[c + d*x]/(a + b*Sqrt[Sinh[c + d*x]]),x]
```

output
```
(3*(-2*Sqrt[2]*b^3*ArcTan[1 - Sqrt[2]*Sqrt[Sinh[c + d*x]]] + 2*Sqrt[2]*b^3
*ArcTan[1 + Sqrt[2]*Sqrt[Sinh[c + d*x]]] + 4*a^3*ArcTan[Sinh[c + d*x]] + 4
*a*b^2*Log[Cosh[c + d*x]] - 8*a*b^2*Log[a + b*Sqrt[Sinh[c + d*x]]] - Sqrt[
2]*b^3*Log[1 - Sqrt[2]*Sqrt[Sinh[c + d*x]] + Sinh[c + d*x]] + Sqrt[2]*b^3*
Log[1 + Sqrt[2]*Sqrt[Sinh[c + d*x]] + Sinh[c + d*x]]) - 8*a^2*b*Hypergeome
tric2F1[3/4, 1, 7/4, -Sinh[c + d*x]^2]*Sinh[c + d*x]^(3/2))/(12*(a^4 + b^4
)*d)
```

Rubi [A] (verified)

Time = 0.83 (sec) , antiderivative size = 285, normalized size of antiderivative = 1.24, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.217$, Rules used = {3042, 3702, 7267, 7276, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\operatorname{sech}(c+dx)}{a+b\sqrt{\sinh(c+dx)}}\,dx$$
$$\Big\downarrow \; 3042$$
$$\int \frac{1}{\cos(ic+idx)\left(a+b\sqrt{-i\sin(ic+idx)}\right)}dx$$

$$\Big\downarrow \text{3702}$$

$$\frac{\int \frac{1}{\left(a+b\sqrt{\sinh(c+dx)}\right)(\sinh^2(c+dx)+1)}d\sinh(c+dx)}{d}$$

$$\Big\downarrow \text{7267}$$

$$\frac{2\int \frac{\sqrt{\sinh(c+dx)}}{\left(a+b\sqrt{\sinh(c+dx)}\right)(\sinh^2(c+dx)+1)}d\sqrt{\sinh(c+dx)}}{d}$$

$$\Big\downarrow \text{7276}$$

$$\frac{2\int \left(\frac{\sqrt{\sinh(c+dx)}a^3-b\sinh(c+dx)a^2+b^2\sinh^{\frac{3}{2}}(c+dx)a+b^3}{(a^4+b^4)(\sinh^2(c+dx)+1)}-\frac{ab^3}{(a^4+b^4)\left(a+b\sqrt{\sinh(c+dx)}\right)}\right)d\sqrt{\sinh(c+dx)}}{d}$$

$$\Big\downarrow \text{2009}$$

$$\frac{2\left(\frac{ab^2\log(\sinh^2(c+dx)+1)}{4(a^4+b^4)}-\frac{ab^2\log\left(a+b\sqrt{\sinh(c+dx)}\right)}{a^4+b^4}+\frac{a^3\arctan(\sinh(c+dx))}{2(a^4+b^4)}+\frac{b(a^2-b^2)\arctan\left(1-\sqrt{2}\sqrt{\sinh(c+dx)}\right)}{2\sqrt{2}(a^4+b^4)}-\frac{b(a^2-b^2}{}\right.}{d}$$

input
```
Int[Sech[c + d*x]/(a + b*Sqrt[Sinh[c + d*x]]),x]
```

output
```
(2*((b*(a^2 - b^2)*ArcTan[1 - Sqrt[2]*Sqrt[Sinh[c + d*x]]])/(2*Sqrt[2]*(a^
4 + b^4)) - (b*(a^2 - b^2)*ArcTan[1 + Sqrt[2]*Sqrt[Sinh[c + d*x]]])/(2*Sqr
t[2]*(a^4 + b^4)) + (a^3*ArcTan[Sinh[c + d*x]])/(2*(a^4 + b^4)) - (a*b^2*L
og[a + b*Sqrt[Sinh[c + d*x]]])/(a^4 + b^4) - (b*(a^2 + b^2)*Log[1 - Sqrt[2
]*Sqrt[Sinh[c + d*x]] + Sinh[c + d*x]])/(4*Sqrt[2]*(a^4 + b^4)) + (b*(a^2
+ b^2)*Log[1 + Sqrt[2]*Sqrt[Sinh[c + d*x]] + Sinh[c + d*x]])/(4*Sqrt[2]*(a
^4 + b^4)) + (a*b^2*Log[1 + Sinh[c + d*x]^2])/(4*(a^4 + b^4))))/d
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3702
```
Int[cos[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*((c_.)*sin[(e_.) + (f_.)*(x
_)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Si
mp[ff/f   Subst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a + b*(c*ff*x)^n)^p, x], x,
 Sin[e + f*x]/ff], x]] /; FreeQ[{a, b, c, e, f, n, p}, x] && IntegerQ[(m -
1)/2] && (EqQ[n, 4] || GtQ[m, 0] || IGtQ[p, 0] || IntegersQ[m, p])
```

rule 7267
```
Int[u_, x_Symbol] :> With[{lst = SubstForFractionalPowerOfLinear[u, x]}, Si
mp[lst[[2]]*lst[[4]]   Subst[Int[lst[[1]], x], x, lst[[3]]^(1/lst[[2]])], x
] /;  !FalseQ[lst] && SubstForFractionalPowerQ[u, lst[[3]], x]]
```

rule 7276
```
Int[(u_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{v = RationalFunctionE
xpand[u/(a + b*x^n), x]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && IGtQ
[n, 0]
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.50 (sec) , antiderivative size = 195, normalized size of antiderivative = 0.85

method	result
default	$\frac{a\left(-\frac{b^{2}\ln\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}a^{2}+2b^{2}\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)-a^{2}\right)}{a^{4}+b^{4}}+\frac{b^{2}\ln\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}+1\right)+2a^{2}\arctan\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{a^{4}+b^{4}}\right)}{d}+\frac{\text{`int/indef0`}\left(\frac{}{-b^{4}\cosh}\right.}{}$

input
```
int(sech(d*x+c)/(a+b*sinh(d*x+c)^(1/2)),x,method=_RETURNVERBOSE)
```

output
```
a/d*(-b^2/(a^4+b^4)*ln(tanh(1/2*d*x+1/2*c)^2*a^2+2*b^2*tanh(1/2*d*x+1/2*c)
-a^2)+2/(a^4+b^4)*(1/2*b^2*ln(tanh(1/2*d*x+1/2*c)^2+1)+a^2*arctan(tanh(1/2
*d*x+1/2*c))))+`int/indef0`(b*sinh(d*x+c)^(1/2)*(-sinh(d*x+c)*b^2+a^2)/(-b
^4*cosh(d*x+c)^4+2*a^2*b^2*cosh(d*x+c)^2*sinh(d*x+c)+(-a^4+b^4)*cosh(d*x+c
)^2),sinh(d*x+c))/d
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1811 vs. $2(211) = 422$.

Time = 0.96 (sec) , antiderivative size = 1811, normalized size of antiderivative = 7.87

$$\int \frac{\operatorname{sech}(c+dx)}{a+b\sqrt{\sinh(c+dx)}}\,dx = \text{Too large to display}$$

input
```
integrate(sech(d*x+c)/(a+b*sinh(d*x+c)^(1/2)),x, algorithm="fricas")
```

output
```
1/4*(4*sqrt(1/2)*(a^4 + b^4)*d*sqrt((a^4*b^2 - 2*a^2*b^4 + b^6)/((a^8 + 2*
a^4*b^4 + b^8)*d^2))*arctan(sqrt(1/2)*((a^4 + b^4)*d*cosh(d*x + c)^2 + (a^
4 + b^4)*d*sinh(d*x + c)^2 - 2*(a^4 + b^4)*d*cosh(d*x + c) - (a^4 + b^4)*d
 + 2*((a^4 + b^4)*d*cosh(d*x + c) - (a^4 + b^4)*d)*sinh(d*x + c))*sqrt((a^
4*b^2 - 2*a^2*b^4 + b^6)/((a^8 + 2*a^4*b^4 + b^8)*d^2))*sqrt(sinh(d*x + c)
)/(a^2*b - b^3 - (a^2*b - b^3)*cosh(d*x + c)^2 - 2*(a^2*b - b^3)*cosh(d*x
+ c)*sinh(d*x + c) - (a^2*b - b^3)*sinh(d*x + c)^2)) + 8*a^3*arctan(cosh(d
*x + c) + sinh(d*x + c)) + 4*a*b^2*log((b^2*cosh(d*x + c)^2 + b^2*sinh(d*x
 + c)^2 + 2*a^2*cosh(d*x + c) - b^2 + 2*(b^2*cosh(d*x + c) + a^2)*sinh(d*x
 + c) - 4*(a*b*cosh(d*x + c) + a*b*sinh(d*x + c))*sqrt(sinh(d*x + c)))/(b^
2*cosh(d*x + c)^2 + b^2*sinh(d*x + c)^2 - 2*a^2*cosh(d*x + c) - b^2 + 2*(b
^2*cosh(d*x + c) - a^2)*sinh(d*x + c))) + sqrt(1/2)*(a^4 + b^4)*d*sqrt((a^
4*b^2 + 2*a^2*b^4 + b^6)/((a^8 + 2*a^4*b^4 + b^8)*d^2))*log(((a^2*b + b^3)
*cosh(d*x + c)^4 + (a^2*b + b^3)*sinh(d*x + c)^4 + 8*(a^2*b + b^3)*cosh(d*
x + c)^3 + 4*(2*a^2*b + 2*b^3 + (a^2*b + b^3)*cosh(d*x + c))*sinh(d*x + c)
^3 + a^2*b + b^3 + 2*(a^2*b + b^3)*cosh(d*x + c)^2 + 2*(a^2*b + b^3 + 3*(a
^2*b + b^3)*cosh(d*x + c)^2 + 12*(a^2*b + b^3)*cosh(d*x + c))*sinh(d*x + c
)^2 + 8*sqrt(1/2)*((a^4 + b^4)*d*cosh(d*x + c)^3 + (a^4 + b^4)*d*sinh(d*x
+ c)^3 + 2*(a^4 + b^4)*d*cosh(d*x + c)^2 - (a^4 + b^4)*d*cosh(d*x + c) + (
3*(a^4 + b^4)*d*cosh(d*x + c) + 2*(a^4 + b^4)*d)*sinh(d*x + c)^2 + (3*(...
```

Sympy [F]

$$\int \frac{\operatorname{sech}(c+dx)}{a+b\sqrt{\sinh(c+dx)}}\,dx = \int \frac{\operatorname{sech}\left(c+dx\right)}{a+b\sqrt{\sinh\left(c+dx\right)}}\,dx$$

input
```
integrate(sech(d*x+c)/(a+b*sinh(d*x+c)**(1/2)),x)
```

output
```
Integral(sech(c + d*x)/(a + b*sqrt(sinh(c + d*x))), x)
```

Maxima [F]

$$\int \frac{\operatorname{sech}(c+dx)}{a+b\sqrt{\sinh(c+dx)}}\,dx = \int \frac{\operatorname{sech}\left(dx+c\right)}{b\sqrt{\sinh\left(dx+c\right)}+a}\,dx$$

input
```
integrate(sech(d*x+c)/(a+b*sinh(d*x+c)^(1/2)),x, algorithm="maxima")
```

output
```
integrate(sech(d*x + c)/(b*sqrt(sinh(d*x + c)) + a), x)
```

Giac [F]

$$\int \frac{\operatorname{sech}(c+dx)}{a+b\sqrt{\sinh(c+dx)}}\,dx = \int \frac{\operatorname{sech}\left(dx+c\right)}{b\sqrt{\sinh\left(dx+c\right)}+a}\,dx$$

input
```
integrate(sech(d*x+c)/(a+b*sinh(d*x+c)^(1/2)),x, algorithm="giac")
```

output
```
integrate(sech(d*x + c)/(b*sqrt(sinh(d*x + c)) + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\operatorname{sech}(c+dx)}{a+b\sqrt{\sinh(c+dx)}}\,dx=\int \frac{1}{\cosh\left(c+d\,x\right)\,\left(a+b\,\sqrt{\sinh\left(c+d\,x\right)}\right)}\,dx$$

input
```
int(1/(cosh(c + d*x)*(a + b*sinh(c + d*x)^(1/2))),x)
```

output
```
int(1/(cosh(c + d*x)*(a + b*sinh(c + d*x)^(1/2))), x)
```

Reduce [F]

$$\int \frac{\operatorname{sech}(c+dx)}{a+b\sqrt{\sinh(c+dx)}}\,dx$$
$$=\frac{2atan\left(e^{dx+c}\right)a^3+\left(\int \frac{\sqrt{\sinh(dx+c)}\,\operatorname{sech}(dx+c)}{\sinh(dx+c)b^2-a^2}dx\right)a^4bd+\left(\int \frac{\sqrt{\sinh(dx+c)}\,\operatorname{sech}(dx+c)}{\sinh(dx+c)b^2-a^2}dx\right)b^5d+\log\left(e^{2dx+2c}+1\right)a}{d\left(a^4+b^4\right)}$$

input
```
int(sech(d*x+c)/(a+b*sinh(d*x+c)^(1/2)),x)
```

output
```
(2*atan(e**(c + d*x))*a**3 + int((sqrt(sinh(c + d*x))*sech(c + d*x))/(sinh
(c + d*x)*b**2 - a**2),x)*a**4*b*d + int((sqrt(sinh(c + d*x))*sech(c + d*x
))/(sinh(c + d*x)*b**2 - a**2),x)*b**5*d + log(e**(2*c + 2*d*x) + 1)*a*b**
2 - log(e**(2*c + 2*d*x)*b**2 - 2*e**(c + d*x)*a**2 - b**2)*a*b**2)/(d*(a*
*4 + b**4))
```

3.379 $\int \frac{\cosh^5(c+dx)}{\left(a+b\sqrt{\sinh(c+dx)}\right)^2}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 270

$$\int \frac{\cosh^5(c+dx)}{\left(a+b\sqrt{\sinh(c+dx)}\right)^2}\,dx = \frac{2(a^4+b^4)(9a^4+b^4)\log\left(a+b\sqrt{\sinh(c+dx)}\right)}{b^{10}d} + \frac{2a(a^4+b^4)^2}{b^{10}d\left(a+b\sqrt{\sinh(c+dx)}\right)} - \frac{16a^3(a^4+b^4)\sqrt{\sinh(c+dx)}}{b^9d} + \frac{a^2(7a^4+6b^4)\sinh(c+dx)}{b^8d} - \frac{4a(3a^4+2b^4)\sinh^{\frac{3}{2}}(c+dx)}{3b^7d} + \frac{(5a^4+2b^4)\sinh^2(c+dx)}{2b^6d} - \frac{8a^3\sinh^{\frac{5}{2}}(c+dx)}{5b^5d} + \frac{a^2\sinh^3(c+dx)}{b^4d} - \frac{4a\sinh^{\frac{7}{2}}(c+dx)}{7b^3d} + \frac{\sinh^4(c+dx)}{4b^2d}$$

output
```
2*(a^4+b^4)*(9*a^4+b^4)*ln(a+b*sinh(d*x+c)^(1/2))/b^10/d+2*a*(a^4+b^4)^2/b
^10/d/(a+b*sinh(d*x+c)^(1/2))-16*a^3*(a^4+b^4)*sinh(d*x+c)^(1/2)/b^9/d+a^2
*(7*a^4+6*b^4)*sinh(d*x+c)/b^8/d-4/3*a*(3*a^4+2*b^4)*sinh(d*x+c)^(3/2)/b^7
/d+1/2*(5*a^4+2*b^4)*sinh(d*x+c)^2/b^6/d-8/5*a^3*sinh(d*x+c)^(5/2)/b^5/d+a
^2*sinh(d*x+c)^3/b^4/d-4/7*a*sinh(d*x+c)^(7/2)/b^3/d+1/4*sinh(d*x+c)^4/b^2
/d
```

Mathematica [A] (verified)

Time = 0.41 (sec) , antiderivative size = 288, normalized size of antiderivative = 1.07

$$\int \frac{\cosh^5(c+dx)}{\left(a+b\sqrt{\sinh(c+dx)}\right)^2}\,dx = \frac{840a(a^4+b^4)\left(a^4+b^4+(9a^4+b^4)\log\left(a+b\sqrt{\sinh(c+dx)}\right)\right)+840b(a^4+b^4)\left(-8a^4+(9a^4+b^4)\log\right.}{}$$

input
```
Integrate[Cosh[c + d*x]^5/(a + b*Sqrt[Sinh[c + d*x]])^2,x]
```

output
```
(840*a*(a^4 + b^4)*(a^4 + b^4 + (9*a^4 + b^4)*Log[a + b*Sqrt[Sinh[c + d*x]
]]) + 840*b*(a^4 + b^4)*(-8*a^4 + (9*a^4 + b^4)*Log[a + b*Sqrt[Sinh[c + d*
x]]])*Sqrt[Sinh[c + d*x]] - 420*a^3*b^2*(9*a^4 + 10*b^4)*Sinh[c + d*x] + 1
40*a^2*b^3*(9*a^4 + 10*b^4)*Sinh[c + d*x]^(3/2) - 70*a*b^4*(9*a^4 + 10*b^4
)*Sinh[c + d*x]^2 + 42*b^5*(9*a^4 + 10*b^4)*Sinh[c + d*x]^(5/2) - 252*a^3*
b^6*Sinh[c + d*x]^3 + 180*a^2*b^7*Sinh[c + d*x]^(7/2) - 135*a*b^8*Sinh[c +
 d*x]^4 + 105*b^9*Sinh[c + d*x]^(9/2))/(420*b^10*d*(a + b*Sqrt[Sinh[c + d*
x]]))
```

Rubi [A] (verified)

Time = 0.59 (sec) , antiderivative size = 249, normalized size of antiderivative = 0.92, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {3042, 3702, 2429, 2123, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cosh^5(c+dx)}{\left(a+b\sqrt{\sinh(c+dx)}\right)^2}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\cos(ic+idx)^5}{\left(a+b\sqrt{-i\sin(ic+idx)}\right)^2}dx$$

$$\Big\downarrow \text{3702}$$

$$\frac{\int \frac{\left(\sinh^2(c+dx)+1\right)^2}{\left(a+b\sqrt{\sinh(c+dx)}\right)^2}d\sinh(c+dx)}{d}$$

$$\Big\downarrow \text{2429}$$

$$\frac{2\int \frac{\sqrt{\sinh(c+dx)}\left(\sinh^2(c+dx)+1\right)^2}{\left(a+b\sqrt{\sinh(c+dx)}\right)^2}d\sqrt{\sinh(c+dx)}}{d}$$

$$\Big\downarrow \text{2123}$$

$$\frac{2\int \left(\frac{\sinh^{\frac{7}{2}}(c+dx)}{b^2}-\frac{2a\sinh^3(c+dx)}{b^3}+\frac{3a^2\sinh^{\frac{5}{2}}(c+dx)}{b^4}-\frac{4a^3\sinh^2(c+dx)}{b^5}+\frac{(5a^4+2b^4)\sinh^{\frac{3}{2}}(c+dx)}{b^6}-\frac{2a(3a^4+2b^4)\sinh(c+dx)}{b^7}+\right.}{d}$$

$$\Big\downarrow \text{2009}$$

$$\frac{2\left(\frac{a(a^4+b^4)^2}{b^{10}\left(a+b\sqrt{\sinh(c+dx)}\right)}+\frac{(a^4+b^4)(9a^4+b^4)\log\left(a+b\sqrt{\sinh(c+dx)}\right)}{b^{10}}-\frac{2a(3a^4+2b^4)\sinh^{\frac{3}{2}}(c+dx)}{3b^7}+\frac{(5a^4+2b^4)\sinh^2(c+dx)}{4b^6}-\frac{4a^3\text{si}}{}\right.}{d}$$

input
```
Int[Cosh[c + d*x]^5/(a + b*Sqrt[Sinh[c + d*x]])^2,x]
```

output
```
(2*(((a^4 + b^4)*(9*a^4 + b^4)*Log[a + b*Sqrt[Sinh[c + d*x]]])/b^10 + (a*(
a^4 + b^4)^2)/(b^10*(a + b*Sqrt[Sinh[c + d*x]])) - (8*a^3*(a^4 + b^4)*Sqrt
[Sinh[c + d*x]])/b^9 + (a^2*(7*a^4 + 6*b^4)*Sinh[c + d*x])/(2*b^8) - (2*a*
(3*a^4 + 2*b^4)*Sinh[c + d*x]^(3/2))/(3*b^7) + ((5*a^4 + 2*b^4)*Sinh[c + d
*x]^2)/(4*b^6) - (4*a^3*Sinh[c + d*x]^(5/2))/(5*b^5) + (a^2*Sinh[c + d*x]^
3)/(2*b^4) - (2*a*Sinh[c + d*x]^(7/2))/(7*b^3) + Sinh[c + d*x]^4/(8*b^2)))
/d
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2123
```
Int[(Px_)*((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol]
:> Int[ExpandIntegrand[Px*(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c
, d, m, n}, x] && PolyQ[Px, x] && (IntegersQ[m, n] || IGtQ[m, -2])
```

rule 2429
```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{g = Denominator
[n]}, Simp[g   Subst[Int[x^(g - 1)*(Pq /. x -> x^g)*(a + b*x^(g*n))^p, x],
x, x^(1/g)], x]] /; FreeQ[{a, b, p}, x] && PolyQ[Pq, x] && FractionQ[n]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3702
```
Int[cos[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*((c_.)*sin[(e_.) + (f_.)*(x
_)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Si
mp[ff/f   Subst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a + b*(c*ff*x)^n)^p, x], x,
 Sin[e + f*x]/ff], x]] /; FreeQ[{a, b, c, e, f, n, p}, x] && IntegerQ[(m -
1)/2] && (EqQ[n, 4] || GtQ[m, 0] || IGtQ[p, 0] || IntegersQ[m, p])
```

Maple [A] (verified)

Time = 0.69 (sec) , antiderivative size = 255, normalized size of antiderivative = 0.94

method	result
derivativedivides	$-\frac{\frac{2\left(-\frac{\sinh(dx+c)^4 b^7}{8}+\frac{2a\sinh(dx+c)^{\frac{7}{2}} b^6}{7}-\frac{a^2\sinh(dx+c)^3 b^5}{2}+\frac{4a^3\sinh(dx+c)^{\frac{5}{2}} b^4}{5}-\frac{5a^4 b^3\sinh(dx+c)^2}{4}-\frac{b^7\sinh(dx+c)^2}{2}+2a^5 b \cdots}{b^9}}{\cdots}$
default	$-\frac{\frac{2\left(-\frac{\sinh(dx+c)^4 b^7}{8}+\frac{2a\sinh(dx+c)^{\frac{7}{2}} b^6}{7}-\frac{a^2\sinh(dx+c)^3 b^5}{2}+\frac{4a^3\sinh(dx+c)^{\frac{5}{2}} b^4}{5}-\frac{5a^4 b^3\sinh(dx+c)^2}{4}-\frac{b^7\sinh(dx+c)^2}{2}+2a^5 b \cdots}{b^9}}{\cdots}$

input
```
int(cosh(d*x+c)^5/(a+b*sinh(d*x+c)^(1/2))^2,x,method=_RETURNVERBOSE)
```

output
```
1/d*(-2/b^9*(-1/8*sinh(d*x+c)^4*b^7+2/7*a*sinh(d*x+c)^(7/2)*b^6-1/2*a^2*si
nh(d*x+c)^3*b^5+4/5*a^3*sinh(d*x+c)^(5/2)*b^4-5/4*a^4*b^3*sinh(d*x+c)^2-1/
2*b^7*sinh(d*x+c)^2+2*a^5*b^2*sinh(d*x+c)^(3/2)+4/3*a*b^6*sinh(d*x+c)^(3/2
)-7/2*a^6*b*sinh(d*x+c)-3*a^2*b^5*sinh(d*x+c)+8*a^7*sinh(d*x+c)^(1/2)+8*a^
3*b^4*sinh(d*x+c)^(1/2))+2*a*(a^8+2*a^4*b^4+b^8)/b^10/(a+b*sinh(d*x+c)^(1/
2))+2/b^10*(9*a^8+10*a^4*b^4+b^8)*ln(a+b*sinh(d*x+c)^(1/2)))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 5181 vs. 2(248) = 496.

Time = 0.82 (sec) , antiderivative size = 5181, normalized size of antiderivative = 19.19

$$\int \frac{\cosh^5(c+dx)}{\left(a+b\sqrt{\sinh(c+dx)}\right)^2}\,dx = \text{Too large to display}$$

input
```
integrate(cosh(d*x+c)^5/(a+b*sinh(d*x+c)^(1/2))^2,x, algorithm="fricas")
```

output
```
Too large to include
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\cosh^5(c+dx)}{\left(a+b\sqrt{\sinh(c+dx)}\right)^2}\,dx = \text{Timed out}$$

input
```
integrate(cosh(d*x+c)**5/(a+b*sinh(d*x+c)**(1/2))**2,x)
```

output
```
Timed out
```

Maxima [F(-1)]

Timed out.

$$\int \frac{\cosh^5(c+dx)}{\left(a+b\sqrt{\sinh(c+dx)}\right)^2}\,dx = \text{Timed out}$$

input
```
integrate(cosh(d*x+c)^5/(a+b*sinh(d*x+c)^(1/2))^2,x, algorithm="maxima")
```

output
```
Timed out
```

Giac [F]

$$\int \frac{\cosh^5(c+dx)}{\left(a+b\sqrt{\sinh(c+dx)}\right)^2}\,dx = \int \frac{\cosh\left(dx+c\right)^5}{\left(b\sqrt{\sinh\left(dx+c\right)}+a\right)^2}\,dx$$

input
```
integrate(cosh(d*x+c)^5/(a+b*sinh(d*x+c)^(1/2))^2,x, algorithm="giac")
```

output
```
integrate(cosh(d*x + c)^5/(b*sqrt(sinh(d*x + c)) + a)^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\cosh^5(c+dx)}{\left(a+b\sqrt{\sinh(c+dx)}\right)^2}\,dx = \int \frac{\cosh(c+d\,x)^5}{\left(a+b\,\sqrt{\sinh\left(c+d\,x\right)}\right)^2}\,dx$$

input
```
int(cosh(c + d*x)^5/(a + b*sinh(c + d*x)^(1/2))^2,x)
```

output
```
int(cosh(c + d*x)^5/(a + b*sinh(c + d*x)^(1/2))^2, x)
```

Reduce [F]

$$\int \frac{\cosh^5(c+dx)}{\left(a+b\sqrt{\sinh(c+dx)}\right)^2}\,dx = \text{Too large to display}$$

input
```
int(cosh(d*x+c)^5/(a+b*sinh(d*x+c)^(1/2))^2,x)
```

output
```
(e**(10*c + 10*d*x)*b**10 + 6*e**(9*c + 9*d*x)*a**2*b**8 + 24*e**(8*c + 8*
d*x)*a**4*b**6 + 11*e**(8*c + 8*d*x)*b**10 + 144*e**(7*c + 7*d*x)*a**6*b**
4 + 136*e**(7*c + 7*d*x)*a**2*b**8 - 128*e**(6*c + 6*d*x)*int((sqrt(sinh(c
 + d*x))*cosh(c + d*x)**5)/(sinh(c + d*x)**2*b**4 - 2*sinh(c + d*x)*a**2*b
**2 + a**4),x)*a*b**13*d + 576*e**(6*c + 6*d*x)*log(e**(2*c + 2*d*x)*b**2
- 2*e**(c + d*x)*a**2 - b**2)*a**8*b**2 + 640*e**(6*c + 6*d*x)*log(e**(2*c
 + 2*d*x)*b**2 - 2*e**(c + d*x)*a**2 - b**2)*a**4*b**6 + 64*e**(6*c + 6*d*
x)*log(e**(2*c + 2*d*x)*b**2 - 2*e**(c + d*x)*a**2 - b**2)*b**10 - 576*e**
(6*c + 6*d*x)*a**8*b**2*d*x - 576*e**(6*c + 6*d*x)*a**8*b**2 - 640*e**(6*c
 + 6*d*x)*a**4*b**6*d*x - 856*e**(6*c + 6*d*x)*a**4*b**6 - 64*e**(6*c + 6*
d*x)*b**10*d*x - 308*e**(6*c + 6*d*x)*b**10 + 256*e**(5*c + 5*d*x)*int((sq
rt(sinh(c + d*x))*cosh(c + d*x)**5)/(sinh(c + d*x)**2*b**4 - 2*sinh(c + d*
x)*a**2*b**2 + a**4),x)*a**3*b**11*d - 1152*e**(5*c + 5*d*x)*log(e**(2*c +
 2*d*x)*b**2 - 2*e**(c + d*x)*a**2 - b**2)*a**10 - 1280*e**(5*c + 5*d*x)*l
og(e**(2*c + 2*d*x)*b**2 - 2*e**(c + d*x)*a**2 - b**2)*a**6*b**4 - 128*e**
(5*c + 5*d*x)*log(e**(2*c + 2*d*x)*b**2 - 2*e**(c + d*x)*a**2 - b**2)*a**2
*b**8 + 1152*e**(5*c + 5*d*x)*a**10*d*x + 1280*e**(5*c + 5*d*x)*a**6*b**4*
d*x + 128*e**(5*c + 5*d*x)*a**2*b**8*d*x + 128*e**(4*c + 4*d*x)*int((sqrt(
sinh(c + d*x))*cosh(c + d*x)**5)/(sinh(c + d*x)**2*b**4 - 2*sinh(c + d*x)*
a**2*b**2 + a**4),x)*a*b**13*d - 576*e**(4*c + 4*d*x)*log(e**(2*c + 2*d...
```

3.380 $\int \frac{\cosh^3(c+dx)}{\left(a+b\sqrt{\sinh(c+dx)}\right)^2} dx$

Optimal result

Integrand size = 25, antiderivative size = 142

$$\int \frac{\cosh^3(c+dx)}{\left(a+b\sqrt{\sinh(c+dx)}\right)^2} dx = \frac{2(5a^4+b^4)\log\left(a+b\sqrt{\sinh(c+dx)}\right)}{b^6d} + \frac{2a(a^4+b^4)}{b^6d\left(a+b\sqrt{\sinh(c+dx)}\right)} - \frac{8a^3\sqrt{\sinh(c+dx)}}{b^5d} + \frac{3a^2\sinh(c+dx)}{b^4d} - \frac{4a\sinh^{\frac{3}{2}}(c+dx)}{3b^3d} + \frac{\sinh^2(c+dx)}{2b^2d}$$

output
```
2*(5*a^4+b^4)*ln(a+b*sinh(d*x+c)^(1/2))/b^6/d+2*a*(a^4+b^4)/b^6/d/(a+b*sin
h(d*x+c)^(1/2))-8*a^3*sinh(d*x+c)^(1/2)/b^5/d+3*a^2*sinh(d*x+c)/b^4/d-4/3*
a*sinh(d*x+c)^(3/2)/b^3/d+1/2*sinh(d*x+c)^2/b^2/d
```

Mathematica [A] (verified)

Time = 0.31 (sec) , antiderivative size = 123, normalized size of antiderivative = 0.87

$$\int \frac{\cosh^3(c+dx)}{\left(a+b\sqrt{\sinh(c+dx)}\right)^2}\,dx$$
$$= \frac{12\left((5a^4+b^4)\log\left(a+b\sqrt{\sinh(c+dx)}\right)+\frac{a(a^4+b^4)}{a+b\sqrt{\sinh(c+dx)}}\right)-48a^3b\sqrt{\sinh(c+dx)}+18a^2b^2\sinh(c+dx}{6b^6d}$$

input
```
Integrate[Cosh[c + d*x]^3/(a + b*Sqrt[Sinh[c + d*x]])^2,x]
```

output
```
(12*((5*a^4 + b^4)*Log[a + b*Sqrt[Sinh[c + d*x]]] + (a*(a^4 + b^4))/(a + b
*Sqrt[Sinh[c + d*x]])) - 48*a^3*b*Sqrt[Sinh[c + d*x]] + 18*a^2*b^2*Sinh[c
+ d*x] - 8*a*b^3*Sinh[c + d*x]^(3/2) + 3*b^4*Sinh[c + d*x]^2)/(6*b^6*d)
```

Rubi [A] (verified)

Time = 0.48 (sec) , antiderivative size = 129, normalized size of antiderivative = 0.91, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {3042, 3702, 2429, 2123, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cosh^3(c+dx)}{\left(a+b\sqrt{\sinh(c+dx)}\right)^2}\,dx$$

$$\Big\downarrow \; 3042$$

$$\int \frac{\cos(ic+idx)^3}{\left(a+b\sqrt{-i\sin(ic+idx)}\right)^2}dx$$

$$\Big\downarrow \; 3702$$

$$\frac{\int \frac{\sinh^2(c+dx)+1}{\left(a+b\sqrt{\sinh(c+dx)}\right)^2}d\sinh(c+dx)}{d}$$

$$\Big\downarrow \text{2429}$$

$$\frac{2\int \frac{\sqrt{\sinh(c+dx)}(\sinh^2(c+dx)+1)}{\left(a+b\sqrt{\sinh(c+dx)}\right)^2}d\sqrt{\sinh(c+dx)}}{d}$$

$$\Big\downarrow \text{2123}$$

$$\frac{2\int \left(-\frac{4a^3}{b^5}+\frac{3\sqrt{\sinh(c+dx)}a^2}{b^4}-\frac{2\sinh(c+dx)a}{b^3}-\frac{(a^4+b^4)a}{b^5\left(a+b\sqrt{\sinh(c+dx)}\right)^2}+\frac{\sinh^{\frac{3}{2}}(c+dx)}{b^2}+\frac{5a^4+b^4}{b^5\left(a+b\sqrt{\sinh(c+dx)}\right)}\right)d\sqrt{\sinh(c+dx)}}{d}$$

$$\Big\downarrow \text{2009}$$

$$\frac{2\left(\frac{a(a^4+b^4)}{b^6\left(a+b\sqrt{\sinh(c+dx)}\right)}+\frac{(5a^4+b^4)\log\left(a+b\sqrt{\sinh(c+dx)}\right)}{b^6}-\frac{4a^3\sqrt{\sinh(c+dx)}}{b^5}+\frac{3a^2\sinh(c+dx)}{2b^4}-\frac{2a\sinh^{\frac{3}{2}}(c+dx)}{3b^3}+\frac{\sinh^2(c+dx)}{4b^2}\right)}{d}$$

input
```
Int[Cosh[c + d*x]^3/(a + b*Sqrt[Sinh[c + d*x]])^2,x]
```

output
```
(2*(((5*a^4 + b^4)*Log[a + b*Sqrt[Sinh[c + d*x]]])/b^6 + (a*(a^4 + b^4))/(
b^6*(a + b*Sqrt[Sinh[c + d*x]])) - (4*a^3*Sqrt[Sinh[c + d*x]])/b^5 + (3*a^
2*Sinh[c + d*x])/(2*b^4) - (2*a*Sinh[c + d*x]^(3/2))/(3*b^3) + Sinh[c + d*
x]^2/(4*b^2)))/d
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2123
```
Int[(Px_)*((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol]
:> Int[ExpandIntegrand[Px*(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c
, d, m, n}, x] && PolyQ[Px, x] && (IntegersQ[m, n] || IGtQ[m, -2])
```

rule 2429
```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{g = Denominator
[n]}, Simp[g   Subst[Int[x^(g - 1)*(Pq /. x -> x^g)*(a + b*x^(g*n))^p, x],
x, x^(1/g)], x]] /; FreeQ[{a, b, p}, x] && PolyQ[Pq, x] && FractionQ[n]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3702
```
Int[cos[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*((c_.)*sin[(e_.) + (f_.)*(x
_)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Si
mp[ff/f   Subst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a + b*(c*ff*x)^n)^p, x], x,
 Sin[e + f*x]/ff], x]] /; FreeQ[{a, b, c, e, f, n, p}, x] && IntegerQ[(m -
1)/2] && (EqQ[n, 4] || GtQ[m, 0] || IGtQ[p, 0] || IntegersQ[m, p])
```

Maple [A] (verified)

Time = 0.37 (sec) , antiderivative size = 118, normalized size of antiderivative = 0.83

method	result
derivativedivides	$\frac{-\frac{2\left(-\frac{b^3\sinh(dx+c)^2}{4}+\frac{2a\sinh(dx+c)^{\frac{3}{2}}b^2}{3}-\frac{3a^2b\sinh(dx+c)}{2}+4a^3\sqrt{\sinh(dx+c)}\right)}{b^5}+\frac{2a\left(a^4+b^4\right)}{b^6\left(a+b\sqrt{\sinh(dx+c)}\right)}+\frac{2\left(5a^4+b^4\right)\ln\left(a+b\right.}{b^6}}{d}$
default	$\frac{-\frac{2\left(-\frac{b^3\sinh(dx+c)^2}{4}+\frac{2a\sinh(dx+c)^{\frac{3}{2}}b^2}{3}-\frac{3a^2b\sinh(dx+c)}{2}+4a^3\sqrt{\sinh(dx+c)}\right)}{b^5}+\frac{2a\left(a^4+b^4\right)}{b^6\left(a+b\sqrt{\sinh(dx+c)}\right)}+\frac{2\left(5a^4+b^4\right)\ln\left(a+b\right.}{b^6}}{d}$

input
```
int(cosh(d*x+c)^3/(a+b*sinh(d*x+c)^(1/2))^2,x,method=_RETURNVERBOSE)
```

output
```
1/d*(-2/b^5*(-1/4*b^3*sinh(d*x+c)^2+2/3*a*sinh(d*x+c)^(3/2)*b^2-3/2*a^2*b*
sinh(d*x+c)+4*a^3*sinh(d*x+c)^(1/2))+2*a*(a^4+b^4)/b^6/(a+b*sinh(d*x+c)^(1
/2))+2/b^6*(5*a^4+b^4)*ln(a+b*sinh(d*x+c)^(1/2)))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2137 vs. $2(130) = 260$.

Time = 0.72 (sec) , antiderivative size = 2137, normalized size of antiderivative = 15.05

$$\int \frac{\cosh^3(c+dx)}{\left(a+b\sqrt{\sinh(c+dx)}\right)^2}\,dx = \text{Too large to display}$$

input
```
integrate(cosh(d*x+c)^3/(a+b*sinh(d*x+c)^(1/2))^2,x, algorithm="fricas")
```

output
```
1/24*(3*b^6*cosh(d*x + c)^6 + 3*b^6*sinh(d*x + c)^6 + 30*a^2*b^4*cosh(d*x
+ c)^5 + 30*a^2*b^4*cosh(d*x + c) - 3*b^6 + 6*(3*b^6*cosh(d*x + c) + 5*a^2
*b^4)*sinh(d*x + c)^5 - 3*(24*a^4*b^2 + b^6 + 8*(5*a^4*b^2 + b^6)*d*x + 8*
(5*a^4*b^2 + b^6)*c)*cosh(d*x + c)^4 + 3*(15*b^6*cosh(d*x + c)^2 + 50*a^2*
b^4*cosh(d*x + c) - 24*a^4*b^2 - b^6 - 8*(5*a^4*b^2 + b^6)*d*x - 8*(5*a^4*
b^2 + b^6)*c)*sinh(d*x + c)^4 - 24*(4*a^6 + 7*a^2*b^4 - 2*(5*a^6 + a^2*b^4
)*d*x - 2*(5*a^6 + a^2*b^4)*c)*cosh(d*x + c)^3 + 12*(5*b^6*cosh(d*x + c)^3
 + 25*a^2*b^4*cosh(d*x + c)^2 - 8*a^6 - 14*a^2*b^4 + 4*(5*a^6 + a^2*b^4)*d
*x + 4*(5*a^6 + a^2*b^4)*c - (24*a^4*b^2 + b^6 + 8*(5*a^4*b^2 + b^6)*d*x +
 8*(5*a^4*b^2 + b^6)*c)*cosh(d*x + c))*sinh(d*x + c)^3 + 3*(24*a^4*b^2 + b
^6 + 8*(5*a^4*b^2 + b^6)*d*x + 8*(5*a^4*b^2 + b^6)*c)*cosh(d*x + c)^2 + 3*
(15*b^6*cosh(d*x + c)^4 + 100*a^2*b^4*cosh(d*x + c)^3 + 24*a^4*b^2 + b^6 +
 8*(5*a^4*b^2 + b^6)*d*x - 6*(24*a^4*b^2 + b^6 + 8*(5*a^4*b^2 + b^6)*d*x +
 8*(5*a^4*b^2 + b^6)*c)*cosh(d*x + c)^2 + 8*(5*a^4*b^2 + b^6)*c - 24*(4*a^
6 + 7*a^2*b^4 - 2*(5*a^6 + a^2*b^4)*d*x - 2*(5*a^6 + a^2*b^4)*c)*cosh(d*x
+ c))*sinh(d*x + c)^2 + 24*((5*a^4*b^2 + b^6)*cosh(d*x + c)^4 + (5*a^4*b^2
 + b^6)*sinh(d*x + c)^4 - 2*(5*a^6 + a^2*b^4)*cosh(d*x + c)^3 - 2*(5*a^6 +
 a^2*b^4 - 2*(5*a^4*b^2 + b^6)*cosh(d*x + c))*sinh(d*x + c)^3 - (5*a^4*b^2
 + b^6)*cosh(d*x + c)^2 - (5*a^4*b^2 + b^6 - 6*(5*a^4*b^2 + b^6)*cosh(d*x
+ c)^2 + 6*(5*a^6 + a^2*b^4)*cosh(d*x + c))*sinh(d*x + c)^2 + 2*(2*(5*a...
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\cosh^3(c+dx)}{\left(a+b\sqrt{\sinh(c+dx)}\right)^2}\,dx = \text{Timed out}$$

input
```
integrate(cosh(d*x+c)**3/(a+b*sinh(d*x+c)**(1/2))**2,x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{\cosh^3(c+dx)}{\left(a+b\sqrt{\sinh(c+dx)}\right)^2} \, dx = \int \frac{\cosh(dx+c)^3}{\left(b\sqrt{\sinh(dx+c)}+a\right)^2} \, dx$$

input
```
integrate(cosh(d*x+c)^3/(a+b*sinh(d*x+c)^(1/2))^2,x, algorithm="maxima")
```

output
```
integrate(cosh(d*x + c)^3/(b*sqrt(sinh(d*x + c)) + a)^2, x)
```

Giac [F]

$$\int \frac{\cosh^3(c+dx)}{\left(a+b\sqrt{\sinh(c+dx)}\right)^2} \, dx = \int \frac{\cosh(dx+c)^3}{\left(b\sqrt{\sinh(dx+c)}+a\right)^2} \, dx$$

input
```
integrate(cosh(d*x+c)^3/(a+b*sinh(d*x+c)^(1/2))^2,x, algorithm="giac")
```

output
```
integrate(cosh(d*x + c)^3/(b*sqrt(sinh(d*x + c)) + a)^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\cosh^3(c+dx)}{\left(a+b\sqrt{\sinh(c+dx)}\right)^2} \, dx = \int \frac{\cosh(c+d\,x)^3}{\left(a+b\,\sqrt{\sinh(c+d\,x)}\right)^2} \, dx$$

input
```
int(cosh(c + d*x)^3/(a + b*sinh(c + d*x)^(1/2))^2,x)
```

output
```
int(cosh(c + d*x)^3/(a + b*sinh(c + d*x)^(1/2))^2, x)
```

Reduce [F]

$$\int \frac{\cosh^3(c+dx)}{\left(a+b\sqrt{\sinh(c+dx)}\right)^2}\,dx = \text{too large to display}$$

input
```
int(cosh(d*x+c)^3/(a+b*sinh(d*x+c)^(1/2))^2,x)
```

output
```
(96*e**(4*c + 4*d*x)*sqrt(sinh(c + d*x))*cosh(c + d*x)**2*a*b**7 - 192*e**
(3*c + 3*d*x)*sqrt(sinh(c + d*x))*cosh(c + d*x)**2*a**3*b**5 - 96*e**(2*c
+ 2*d*x)*sqrt(sinh(c + d*x))*cosh(c + d*x)**2*a*b**7 - 24*e**(4*c + 4*d*x)
*cosh(c + d*x)**2*a**2*b**6 + 48*e**(3*c + 3*d*x)*cosh(c + d*x)**2*a**4*b*
*4 + 24*e**(2*c + 2*d*x)*cosh(c + d*x)**2*a**2*b**6 - 128*e**(4*c + 4*d*x)
*sqrt(sinh(c + d*x))*sinh(c + d*x)**2*a*b**7 - 256*e**(4*c + 4*d*x)*sqrt(s
inh(c + d*x))*sinh(c + d*x)*a**3*b**5 + 768*e**(4*c + 4*d*x)*sqrt(sinh(c +
 d*x))*a**5*b**3 + 256*e**(3*c + 3*d*x)*sqrt(sinh(c + d*x))*sinh(c + d*x)*
*2*a**3*b**5 + 512*e**(3*c + 3*d*x)*sqrt(sinh(c + d*x))*sinh(c + d*x)*a**5
*b**3 - 1536*e**(3*c + 3*d*x)*sqrt(sinh(c + d*x))*a**7*b + 128*e**(2*c + 2
*d*x)*sqrt(sinh(c + d*x))*sinh(c + d*x)**2*a*b**7 + 256*e**(2*c + 2*d*x)*s
qrt(sinh(c + d*x))*sinh(c + d*x)*a**3*b**5 - 768*e**(2*c + 2*d*x)*sqrt(sin
h(c + d*x))*a**5*b**3 + 3*e**(6*c + 6*d*x)*sinh(c + d*x)*b**8 - 3*e**(6*c
+ 6*d*x)*a**2*b**6 + 18*e**(5*c + 5*d*x)*sinh(c + d*x)*a**2*b**6 - 18*e**(
5*c + 5*d*x)*a**4*b**4 + 384*e**(4*c + 4*d*x)*int((sqrt(sinh(c + d*x))*cos
h(c + d*x))/(sinh(c + d*x)**3*b**4 - 2*sinh(c + d*x)**2*a**2*b**2 + sinh(c
 + d*x)*a**4),x)*sinh(c + d*x)*a**7*b**5*d - 384*e**(4*c + 4*d*x)*int((sqr
t(sinh(c + d*x))*cosh(c + d*x))/(sinh(c + d*x)**3*b**4 - 2*sinh(c + d*x)**
2*a**2*b**2 + sinh(c + d*x)*a**4),x)*a**9*b**3*d + 48*e**(4*c + 4*d*x)*int
((sqrt(sinh(c + d*x))*cosh(c + d*x)**3)/(sinh(c + d*x)**3*b**4 - 2*sinh...
```

3.381 $\int \frac{\cosh(c+dx)}{\left(a+b\sqrt{\sinh(c+dx)}\right)^2}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 49

$$\int \frac{\cosh(c+dx)}{\left(a+b\sqrt{\sinh(c+dx)}\right)^2}\,dx = \frac{2\log\left(a+b\sqrt{\sinh(c+dx)}\right)}{b^2d}+\frac{2a}{b^2d\left(a+b\sqrt{\sinh(c+dx)}\right)}$$

output
```
2*ln(a+b*sinh(d*x+c)^(1/2))/b^2/d+2*a/b^2/d/(a+b*sinh(d*x+c)^(1/2))
```

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.86

$$\int \frac{\cosh(c+dx)}{\left(a+b\sqrt{\sinh(c+dx)}\right)^2}\,dx = \frac{2\left(\log\left(a+b\sqrt{\sinh(c+dx)}\right)+\frac{a}{a+b\sqrt{\sinh(c+dx)}}\right)}{b^2d}$$

input
```
Integrate[Cosh[c + d*x]/(a + b*Sqrt[Sinh[c + d*x]])^2,x]
```

output
```
(2*(Log[a + b*Sqrt[Sinh[c + d*x]]] + a/(a + b*Sqrt[Sinh[c + d*x]])))/(b^2*
d)
```

Rubi [A] (verified)

Time = 0.31 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.94, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.217$, Rules used = {3042, 3702, 774, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cosh(c+dx)}{\left(a+b\sqrt{\sinh(c+dx)}\right)^2}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\cos(ic+idx)}{\left(a+b\sqrt{-i\sin(ic+idx)}\right)^2}dx$$

$$\Big\downarrow \text{3702}$$

$$\frac{\int \frac{1}{\left(a+b\sqrt{\sinh(c+dx)}\right)^2}d\sinh(c+dx)}{d}$$

$$\Big\downarrow \text{774}$$

$$\frac{2\int \frac{\sqrt{\sinh(c+dx)}}{\left(a+b\sqrt{\sinh(c+dx)}\right)^2}d\sqrt{\sinh(c+dx)}}{d}$$

$$\Big\downarrow \text{49}$$

$$\frac{2\int \left(\frac{1}{b\left(a+b\sqrt{\sinh(c+dx)}\right)}-\frac{a}{b\left(a+b\sqrt{\sinh(c+dx)}\right)^2}\right)d\sqrt{\sinh(c+dx)}}{d}$$

$$\Big\downarrow \text{2009}$$

$$\frac{2\left(\frac{a}{b^2\left(a+b\sqrt{\sinh(c+dx)}\right)}+\frac{\log\left(a+b\sqrt{\sinh(c+dx)}\right)}{b^2}\right)}{d}$$

input
```
Int[Cosh[c + d*x]/(a + b*Sqrt[Sinh[c + d*x]])^2,x]
```

output
```
(2*(Log[a + b*Sqrt[Sinh[c + d*x]]]/b^2 + a/(b^2*(a + b*Sqrt[Sinh[c + d*x]]
))))/d
```

Defintions of rubi rules used

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 774
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = Denominator[n]},
Simp[k   Subst[Int[x^(k - 1)*(a + b*x^(k*n))^p, x], x, x^(1/k)], x]] /; Fre
eQ[{a, b, p}, x] && FractionQ[n]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3702
```
Int[cos[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*((c_.)*sin[(e_.) + (f_.)*(x
_)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Si
mp[ff/f   Subst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a + b*(c*ff*x)^n)^p, x], x,
 Sin[e + f*x]/ff], x]] /; FreeQ[{a, b, c, e, f, n, p}, x] && IntegerQ[(m -
1)/2] && (EqQ[n, 4] || GtQ[m, 0] || IGtQ[p, 0] || IntegersQ[m, p])
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 126 vs. 2(45) = 90.

Time = 0.24 (sec) , antiderivative size = 127, normalized size of antiderivative = 2.59

method	result
derivativedivides	$\frac{\frac{2a^2}{\left(-\sinh(dx+c)b^2+a^2\right)b^2}+\frac{\ln\left(-\sinh(dx+c)b^2+a^2\right)}{b^2}-\frac{a}{b^2\left(-b\sqrt{\sinh(dx+c)}+a\right)}-\frac{\ln\left(-b\sqrt{\sinh(dx+c)}+a\right)}{b^2}+\frac{a}{b^2\left(a+b\sqrt{\sinh(dx+c)}\right)}}{d}$
default	$\frac{\frac{2a^2}{\left(-\sinh(dx+c)b^2+a^2\right)b^2}+\frac{\ln\left(-\sinh(dx+c)b^2+a^2\right)}{b^2}-\frac{a}{b^2\left(-b\sqrt{\sinh(dx+c)}+a\right)}-\frac{\ln\left(-b\sqrt{\sinh(dx+c)}+a\right)}{b^2}+\frac{a}{b^2\left(a+b\sqrt{\sinh(dx+c)}\right)}}{d}$

input
```
int(cosh(d*x+c)/(a+b*sinh(d*x+c)^(1/2))^2,x,method=_RETURNVERBOSE)
```

output
```
1/d*(2*a^2/(-sinh(d*x+c)*b^2+a^2)/b^2+1/b^2*ln(-sinh(d*x+c)*b^2+a^2)-a/b^2
/(-b*sinh(d*x+c)^(1/2)+a)-1/b^2*ln(-b*sinh(d*x+c)^(1/2)+a)+a/b^2/(a+b*sinh
(d*x+c)^(1/2))+1/b^2*ln(a+b*sinh(d*x+c)^(1/2)))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 564 vs. 2(45) = 90.

Time = 0.13 (sec) , antiderivative size = 564, normalized size of antiderivative = 11.51

$$\int \frac{\cosh(c+dx)}{\left(a+b\sqrt{\sinh(c+dx)}\right)^2}\,dx$$
$$= \frac{b^2dx + b^2c - (b^2dx + b^2c)\cosh{(dx+c)}^2 - (b^2dx + b^2c)\sinh{(dx+c)}^2 + 2\left(a^2dx + a^2c - 2\,a^2\right)\cosh{(dx}}{}$$

input
```
integrate(cosh(d*x+c)/(a+b*sinh(d*x+c)^(1/2))^2,x, algorithm="fricas")
```

output
```
(b^2*d*x + b^2*c - (b^2*d*x + b^2*c)*cosh(d*x + c)^2 - (b^2*d*x + b^2*c)*s
inh(d*x + c)^2 + 2*(a^2*d*x + a^2*c - 2*a^2)*cosh(d*x + c) + (b^2*cosh(d*x
 + c)^2 + b^2*sinh(d*x + c)^2 - 2*a^2*cosh(d*x + c) - b^2 + 2*(b^2*cosh(d*
x + c) - a^2)*sinh(d*x + c))*log((b^2*cosh(d*x + c)^2 + b^2*sinh(d*x + c)^
2 + 2*a^2*cosh(d*x + c) - b^2 + 2*(b^2*cosh(d*x + c) + a^2)*sinh(d*x + c)
+ 4*(a*b*cosh(d*x + c) + a*b*sinh(d*x + c))*sqrt(sinh(d*x + c)))/(b^2*cosh
(d*x + c)^2 + b^2*sinh(d*x + c)^2 - 2*a^2*cosh(d*x + c) - b^2 + 2*(b^2*cos
h(d*x + c) - a^2)*sinh(d*x + c))) + (b^2*cosh(d*x + c)^2 + b^2*sinh(d*x +
c)^2 - 2*a^2*cosh(d*x + c) - b^2 + 2*(b^2*cosh(d*x + c) - a^2)*sinh(d*x +
c))*log(2*(b^2*sinh(d*x + c) - a^2)/(cosh(d*x + c) - sinh(d*x + c))) + 2*(
a^2*d*x + a^2*c - 2*a^2 - (b^2*d*x + b^2*c)*cosh(d*x + c))*sinh(d*x + c) +
 4*(a*b*cosh(d*x + c) + a*b*sinh(d*x + c))*sqrt(sinh(d*x + c)))/(b^4*d*cos
h(d*x + c)^2 + b^4*d*sinh(d*x + c)^2 - 2*a^2*b^2*d*cosh(d*x + c) - b^4*d +
 2*(b^4*d*cosh(d*x + c) - a^2*b^2*d)*sinh(d*x + c))
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 151 vs. $2(42) = 84$.

Time = 3.08 (sec) , antiderivative size = 151, normalized size of antiderivative = 3.08

$$\int \frac{\cosh(c+dx)}{\left(a+b\sqrt{\sinh(c+dx)}\right)^2}\,dx = \begin{cases} \frac{x\cosh(c)}{a^2} & \text{for } b=0 \wedge d=0 \\ \frac{\sinh(c+dx)}{a^2 d} & \text{for } b=0 \\ \frac{x\cosh(c)}{\left(a+b\sqrt{\sinh(c)}\right)^2} & \text{for } d=0 \\ \frac{2a\log\left(\frac{a}{b}+\sqrt{\sinh(c+dx)}\right)}{ab^2d+b^3d\sqrt{\sinh(c+dx)}} + \frac{2a}{ab^2d+b^3d\sqrt{\sinh(c+dx)}} + \frac{2b\log\left(\frac{a}{b}+\sqrt{\sinh(c+dx)}\right)\sqrt{\sinh(c+dx)}}{ab^2d+b^3d\sqrt{\sinh(c+dx)}} & \text{otherwise} \end{cases}$$

input
```
integrate(cosh(d*x+c)/(a+b*sinh(d*x+c)**(1/2))**2,x)
```

output
```
Piecewise((x*cosh(c)/a**2, Eq(b, 0) & Eq(d, 0)), (sinh(c + d*x)/(a**2*d),
Eq(b, 0)), (x*cosh(c)/(a + b*sqrt(sinh(c)))**2, Eq(d, 0)), (2*a*log(a/b +
sqrt(sinh(c + d*x)))/(a*b**2*d + b**3*d*sqrt(sinh(c + d*x))) + 2*a/(a*b**2
*d + b**3*d*sqrt(sinh(c + d*x))) + 2*b*log(a/b + sqrt(sinh(c + d*x)))*sqrt
(sinh(c + d*x))/(a*b**2*d + b**3*d*sqrt(sinh(c + d*x))), True))
```

Maxima [F]

$$\int \frac{\cosh(c+dx)}{\left(a+b\sqrt{\sinh(c+dx)}\right)^2}\,dx = \int \frac{\cosh\left(dx+c\right)}{\left(b\sqrt{\sinh\left(dx+c\right)}+a\right)^2}\,dx$$

input
```
integrate(cosh(d*x+c)/(a+b*sinh(d*x+c)^(1/2))^2,x, algorithm="maxima")
```

output
```
integrate(cosh(d*x + c)/(b*sqrt(sinh(d*x + c)) + a)^2, x)
```

Giac [F]

$$\int \frac{\cosh(c+dx)}{\left(a+b\sqrt{\sinh(c+dx)}\right)^2}\,dx = \int \frac{\cosh\left(dx+c\right)}{\left(b\sqrt{\sinh\left(dx+c\right)}+a\right)^2}\,dx$$

input
```
integrate(cosh(d*x+c)/(a+b*sinh(d*x+c)^(1/2))^2,x, algorithm="giac")
```

output
```
integrate(cosh(d*x + c)/(b*sqrt(sinh(d*x + c)) + a)^2, x)
```

Mupad [B] (verification not implemented)

Time = 2.35 (sec) , antiderivative size = 45, normalized size of antiderivative = 0.92

$$\int \frac{\cosh(c+dx)}{\left(a+b\sqrt{\sinh(c+dx)}\right)^2}\,dx = \frac{2\,a}{b^2\,\left(a\,d+b\,d\,\sqrt{\sinh\left(c+d\,x\right)}\right)} + \frac{2\,\ln\left(a+b\,\sqrt{\sinh\left(c+d\,x\right)}\right)}{b^2\,d}$$

input
```
int(cosh(c + d*x)/(a + b*sinh(c + d*x)^(1/2))^2,x)
```

output
```
(2*a)/(b^2*(a*d + b*d*sinh(c + d*x)^(1/2))) + (2*log(a + b*sinh(c + d*x)^(
1/2)))/(b^2*d)
```

Reduce [F]

$$\int \frac{\cosh(c+dx)}{\left(a+b\sqrt{\sinh(c+dx)}\right)^2}\,dx = \frac{4\sqrt{\sinh\left(dx+c\right)}\,ab+2\left(\int \frac{\sqrt{\sinh(dx+c)}\,\cosh(dx+c)}{\sinh(dx+c)^3b^4-2\sinh(dx+c)^2a^2b^2+\sinh(dx+c)a^4}dx\right)\sinh\left(dx+c\right)a^3b^3d-2\left(\int \frac{}{\sinh(dx+c)}\right.}{}$$

input
```
int(cosh(d*x+c)/(a+b*sinh(d*x+c)^(1/2))^2,x)
```

output
```
(4*sqrt(sinh(c + d*x))*a*b + 2*int((sqrt(sinh(c + d*x))*cosh(c + d*x))/(si
nh(c + d*x)**3*b**4 - 2*sinh(c + d*x)**2*a**2*b**2 + sinh(c + d*x)*a**4),x
)*sinh(c + d*x)*a**3*b**3*d - 2*int((sqrt(sinh(c + d*x))*cosh(c + d*x))/(s
inh(c + d*x)**3*b**4 - 2*sinh(c + d*x)**2*a**2*b**2 + sinh(c + d*x)*a**4),
x)*a**5*b*d + log(sinh(c + d*x)*b**2 - a**2)*sinh(c + d*x)*b**2 - log(sinh
(c + d*x)*b**2 - a**2)*a**2 - 2*sinh(c + d*x)*b**2)/(b**2*d*(sinh(c + d*x)
*b**2 - a**2))
```

3.382 $\int \frac{\operatorname{sech}(c+dx)}{\left(a+b\sqrt{\sinh(c+dx)}\right)^2}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 322

$$\begin{aligned}\int \frac{\operatorname{sech}(c+dx)}{\left(a+b\sqrt{\sinh(c+dx)}\right)^2}\,dx &= \frac{\sqrt{2}ab(a^4-2a^2b^2-b^4)\arctan\left(1-\sqrt{2}\sqrt{\sinh(c+dx)}\right)}{(a^4+b^4)^2d}\\
&\quad-\frac{\sqrt{2}ab(a^4-2a^2b^2-b^4)\arctan\left(1+\sqrt{2}\sqrt{\sinh(c+dx)}\right)}{(a^4+b^4)^2d}\\
&\quad+\frac{a^2(a^4-3b^4)\arctan(\sinh(c+dx))}{(a^4+b^4)^2d}\\
&\quad+\frac{\sqrt{2}ab(a^4+2a^2b^2-b^4)\operatorname{arctanh}\left(\frac{\sqrt{2}\sqrt{\sinh(c+dx)}}{1+\sinh(c+dx)}\right)}{(a^4+b^4)^2d}+\frac{b^2(3a^4-b^4)\log(\cosh(c+dx))}{(a^4+b^4)^2d}\\
&\quad-\frac{2b^2(3a^4-b^4)\log\left(a+b\sqrt{\sinh(c+dx)}\right)}{(a^4+b^4)^2d}+\frac{2ab^2}{(a^4+b^4)d\left(a+b\sqrt{\sinh(c+dx)}\right)}\end{aligned}$$

output
```
-2^(1/2)*a*b*(a^4-2*a^2*b^2-b^4)*arctan(-1+2^(1/2)*sinh(d*x+c)^(1/2))/(a^4
+b^4)^2/d-2^(1/2)*a*b*(a^4-2*a^2*b^2-b^4)*arctan(1+2^(1/2)*sinh(d*x+c)^(1/
2))/(a^4+b^4)^2/d+a^2*(a^4-3*b^4)*arctan(sinh(d*x+c))/(a^4+b^4)^2/d+2^(1/2
)*a*b*(a^4+2*a^2*b^2-b^4)*arctanh(2^(1/2)*sinh(d*x+c)^(1/2)/(1+sinh(d*x+c)
))/(a^4+b^4)^2/d+b^2*(3*a^4-b^4)*ln(cosh(d*x+c))/(a^4+b^4)^2/d-2*b^2*(3*a^
4-b^4)*ln(a+b*sinh(d*x+c)^(1/2))/(a^4+b^4)^2/d+2*a*b^2/(a^4+b^4)/d/(a+b*si
nh(d*x+c)^(1/2))
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 0.49 (sec) , antiderivative size = 300, normalized size of antiderivative = 0.93

$$\int \frac{\operatorname{sech}(c+dx)}{\left(a+b\sqrt{\sinh(c+dx)}\right)^2}\,dx$$

$$= \frac{-6\sqrt{2}a^3b^3\arctan\left(1-\sqrt{2}\sqrt{\sinh(c+dx)}\right)+6\sqrt{2}a^3b^3\arctan\left(1+\sqrt{2}\sqrt{\sinh(c+dx)}\right)+3a^2(a^4-3b^4)}{}$$

input
```
Integrate[Sech[c + d*x]/(a + b*Sqrt[Sinh[c + d*x]])^2,x]
```

output
```
(-6*Sqrt[2]*a^3*b^3*ArcTan[1 - Sqrt[2]*Sqrt[Sinh[c + d*x]]] + 6*Sqrt[2]*a^
3*b^3*ArcTan[1 + Sqrt[2]*Sqrt[Sinh[c + d*x]]] + 3*a^2*(a^4 - 3*b^4)*ArcTan
[Sinh[c + d*x]] - 3*b^2*(-3*a^4 + b^4)*Log[Cosh[c + d*x]] + 6*b^2*(-3*a^4
+ b^4)*Log[a + b*Sqrt[Sinh[c + d*x]]] - 3*Sqrt[2]*a^3*b^3*Log[1 - Sqrt[2]*
Sqrt[Sinh[c + d*x]] + Sinh[c + d*x]] + 3*Sqrt[2]*a^3*b^3*Log[1 + Sqrt[2]*S
qrt[Sinh[c + d*x]] + Sinh[c + d*x]] + (6*a*b^2*(a^4 + b^4))/(a + b*Sqrt[Si
nh[c + d*x]]) - 4*a*b*(a^4 - b^4)*Hypergeometric2F1[3/4, 1, 7/4, -Sinh[c +
 d*x]^2]*Sinh[c + d*x]^(3/2))/(3*(a^4 + b^4)^2*d)
```

Rubi [A] (verified)

Time = 1.07 (sec) , antiderivative size = 379, normalized size of antiderivative = 1.18, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.217, Rules used = {3042, 3702, 7267, 7276, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\text{sech}(c+dx)}{\left(a+b\sqrt{\sinh(c+dx)}\right)^2}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{\cos(ic+idx)\left(a+b\sqrt{-i\sin(ic+idx)}\right)^2}dx$$

$$\Big\downarrow \text{3702}$$

$$\frac{\int \frac{1}{\left(a+b\sqrt{\sinh(c+dx)}\right)^2(\sinh^2(c+dx)+1)}d\sinh(c+dx)}{d}$$

$$\Big\downarrow \text{7267}$$

$$\frac{2\int \frac{\sqrt{\sinh(c+dx)}}{\left(a+b\sqrt{\sinh(c+dx)}\right)^2(\sinh^2(c+dx)+1)}d\sqrt{\sinh(c+dx)}}{d}$$

$$\Big\downarrow \text{7276}$$

$$\frac{2\int \left(-\frac{ab^3}{(a^4+b^4)\left(a+b\sqrt{\sinh(c+dx)}\right)^2}+\frac{b^7-3a^4b^3}{(a^4+b^4)^2\left(a+b\sqrt{\sinh(c+dx)}\right)}+\frac{4a^3b^3+(3a^4-b^4)\sinh^{\frac{3}{2}}(c+dx)b^2-2a(a^4-b^4)\sinh(c+dx)b+a^2(a^{\cdots}}{(a^4+b^4)^2(\sinh^2(c+dx)+1)}\cdots}{d}$$

$$\Big\downarrow \text{2009}$$

$$\frac{2\left(\frac{ab^2}{(a^4+b^4)\left(a+b\sqrt{\sinh(c+dx)}\right)}+\frac{b^2(3a^4-b^4)\log(\sinh^2(c+dx)+1)}{4(a^4+b^4)^2}-\frac{b^2(3a^4-b^4)\log\left(a+b\sqrt{\sinh(c+dx)}\right)}{(a^4+b^4)^2}+\frac{a^2(a^4-3b^4)\arctan(\sinh(c+d\cdots}{2(a^4+b^4)^2}\cdots\right.}{\cdots}$$

input
```
Int[Sech[c + d*x]/(a + b*Sqrt[Sinh[c + d*x]])^2,x]
```

output
```
(2*((a*b*(a^4 - 2*a^2*b^2 - b^4)*ArcTan[1 - Sqrt[2]*Sqrt[Sinh[c + d*x]]])/
(Sqrt[2]*(a^4 + b^4)^2) - (a*b*(a^4 - 2*a^2*b^2 - b^4)*ArcTan[1 + Sqrt[2]*
Sqrt[Sinh[c + d*x]]])/(Sqrt[2]*(a^4 + b^4)^2) + (a^2*(a^4 - 3*b^4)*ArcTan[
Sinh[c + d*x]])/(2*(a^4 + b^4)^2) - (b^2*(3*a^4 - b^4)*Log[a + b*Sqrt[Sinh
[c + d*x]]])/(a^4 + b^4)^2 - (a*b*(a^4 + 2*a^2*b^2 - b^4)*Log[1 - Sqrt[2]*
Sqrt[Sinh[c + d*x]] + Sinh[c + d*x]])/(2*Sqrt[2]*(a^4 + b^4)^2) + (a*b*(a^
4 + 2*a^2*b^2 - b^4)*Log[1 + Sqrt[2]*Sqrt[Sinh[c + d*x]] + Sinh[c + d*x]])
/(2*Sqrt[2]*(a^4 + b^4)^2) + (b^2*(3*a^4 - b^4)*Log[1 + Sinh[c + d*x]^2])/
(4*(a^4 + b^4)^2) + (a*b^2)/((a^4 + b^4)*(a + b*Sqrt[Sinh[c + d*x]]))))/d
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3702
```
Int[cos[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*((c_.)*sin[(e_.) + (f_.)*(x
_)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Si
mp[ff/f   Subst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a + b*(c*ff*x)^n)^p, x], x,
 Sin[e + f*x]/ff], x]] /; FreeQ[{a, b, c, e, f, n, p}, x] && IntegerQ[(m -
1)/2] && (EqQ[n, 4] || GtQ[m, 0] || IGtQ[p, 0] || IntegersQ[m, p])
```

rule 7267
```
Int[u_, x_Symbol] :> With[{lst = SubstForFractionalPowerOfLinear[u, x]}, Si
mp[lst[[2]]*lst[[4]]   Subst[Int[lst[[1]], x], x, lst[[3]]^(1/lst[[2]])], x
] /;  !FalseQ[lst] && SubstForFractionalPowerQ[u, lst[[3]], x]]
```

rule 7276
```
Int[(u_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{v = RationalFunctionE
xpand[u/(a + b*x^n), x]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && IGtQ
[n, 0]
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.79 (sec) , antiderivative size = 376, normalized size of antiderivative = 1.17

method	result
default	$\frac{-\frac{2b^2\left(\frac{2(a^4+b^4)b^2\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)}{\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^2a^2+2b^2\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)-a^2}+\frac{(3a^4-b^4)\ln\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^2a^2+2b^2\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)-a^2\right)}{2}\right)}{(a^4+b^4)^2}+\frac{(3a^4b^2-b^6)\ln\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)\right.}{}}{d}$

input
```
int(sech(d*x+c)/(a+b*sinh(d*x+c)^(1/2))^2,x,method=_RETURNVERBOSE)
```

output
```
1/d*(-2*b^2/(a^4+b^4)^2*(2*(a^4+b^4)*b^2*tanh(1/2*d*x+1/2*c)/(tanh(1/2*d*x
+1/2*c)^2*a^2+2*b^2*tanh(1/2*d*x+1/2*c)-a^2)+1/2*(3*a^4-b^4)*ln(tanh(1/2*d
*x+1/2*c)^2*a^2+2*b^2*tanh(1/2*d*x+1/2*c)-a^2))+2/(a^8+2*a^4*b^4+b^8)*(1/2
*(3*a^4*b^2-b^6)*ln(tanh(1/2*d*x+1/2*c)^2+1)+(a^6-3*a^2*b^4)*arctan(tanh(1
/2*d*x+1/2*c))))+`int/indef0`(2*a*b*sinh(d*x+c)^(1/2)*(sinh(d*x+c)^2*b^4-2
*a^2*b^2*sinh(d*x+c)+a^4)/(-b^8*cosh(d*x+c)^6+4*a^2*b^6*cosh(d*x+c)^4*sinh
(d*x+c)+(-6*a^4*b^4+2*b^8)*cosh(d*x+c)^4+(4*a^6*b^2-4*a^2*b^6)*cosh(d*x+c)
^2*sinh(d*x+c)+(-a^8+6*a^4*b^4-b^8)*cosh(d*x+c)^2),sinh(d*x+c))/d
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 3834 vs. $2(300) = 600$.

Time = 5.17 (sec) , antiderivative size = 3834, normalized size of antiderivative = 11.91

$$\int \frac{\operatorname{sech}(c+dx)}{\left(a+b\sqrt{\sinh(c+dx)}\right)^2}\,dx = \text{Too large to display}$$

input
```
integrate(sech(d*x+c)/(a+b*sinh(d*x+c)^(1/2))^2,x, algorithm="fricas")
```

output
```
Too large to include
```

Sympy [F]

$$\int \frac{\operatorname{sech}(c+dx)}{\left(a+b\sqrt{\sinh(c+dx)}\right)^2}\,dx = \int \frac{\operatorname{sech}(c+dx)}{\left(a+b\sqrt{\sinh(c+dx)}\right)^2}\,dx$$

input
```
integrate(sech(d*x+c)/(a+b*sinh(d*x+c)**(1/2))**2,x)
```

output
```
Integral(sech(c + d*x)/(a + b*sqrt(sinh(c + d*x)))**2, x)
```

Maxima [F]

$$\int \frac{\operatorname{sech}(c+dx)}{\left(a+b\sqrt{\sinh(c+dx)}\right)^2}\,dx = \int \frac{\operatorname{sech}(dx+c)}{\left(b\sqrt{\sinh(dx+c)}+a\right)^2}\,dx$$

input
```
integrate(sech(d*x+c)/(a+b*sinh(d*x+c)^(1/2))^2,x, algorithm="maxima")
```

output
```
integrate(sech(d*x + c)/(b*sqrt(sinh(d*x + c)) + a)^2, x)
```

Giac [F]

$$\int \frac{\operatorname{sech}(c+dx)}{\left(a+b\sqrt{\sinh(c+dx)}\right)^2}\,dx = \int \frac{\operatorname{sech}(dx+c)}{\left(b\sqrt{\sinh(dx+c)}+a\right)^2}\,dx$$

input
```
integrate(sech(d*x+c)/(a+b*sinh(d*x+c)^(1/2))^2,x, algorithm="giac")
```

output
```
integrate(sech(d*x + c)/(b*sqrt(sinh(d*x + c)) + a)^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\mathrm{sech}(c+dx)}{\left(a+b\sqrt{\sinh(c+dx)}\right)^2}\,dx = \int \frac{1}{\cosh\left(c+d\,x\right)\,\left(a+b\,\sqrt{\sinh\left(c+d\,x\right)}\right)^2}\,dx$$

input
```
int(1/(cosh(c + d*x)*(a + b*sinh(c + d*x)^(1/2))^2),x)
```

output
```
int(1/(cosh(c + d*x)*(a + b*sinh(c + d*x)^(1/2))^2), x)
```

Reduce [F]

$$\int \frac{\mathrm{sech}(c+dx)}{\left(a+b\sqrt{\sinh(c+dx)}\right)^2}\,dx = \text{Too large to display}$$

input
```
int(sech(d*x+c)/(a+b*sinh(d*x+c)^(1/2))^2,x)
```

output

```
(2*e**(2*c + 2*d*x)*atan(e**(c + d*x))*a**6*b**2 - 6*e**(2*c + 2*d*x)*atan
(e**(c + d*x))*a**2*b**6 - 4*e**(c + d*x)*atan(e**(c + d*x))*a**8 + 12*e**
(c + d*x)*atan(e**(c + d*x))*a**4*b**4 - 2*atan(e**(c + d*x))*a**6*b**2 +
6*atan(e**(c + d*x))*a**2*b**6 - 2*e**(2*c + 2*d*x)*int((sqrt(sinh(c + d*x
))*sech(c + d*x))/(sinh(c + d*x)**2*b**4 - 2*sinh(c + d*x)*a**2*b**2 + a**
4),x)*a**9*b**3*d - 4*e**(2*c + 2*d*x)*int((sqrt(sinh(c + d*x))*sech(c + d
*x))/(sinh(c + d*x)**2*b**4 - 2*sinh(c + d*x)*a**2*b**2 + a**4),x)*a**5*b*
*7*d - 2*e**(2*c + 2*d*x)*int((sqrt(sinh(c + d*x))*sech(c + d*x))/(sinh(c
+ d*x)**2*b**4 - 2*sinh(c + d*x)*a**2*b**2 + a**4),x)*a*b**11*d + 3*e**(2*
c + 2*d*x)*log(e**(2*c + 2*d*x) + 1)*a**4*b**4 - e**(2*c + 2*d*x)*log(e**(
2*c + 2*d*x) + 1)*b**8 - 3*e**(2*c + 2*d*x)*log(e**(2*c + 2*d*x)*b**2 - 2*
e**(c + d*x)*a**2 - b**2)*a**4*b**4 + e**(2*c + 2*d*x)*log(e**(2*c + 2*d*x
)*b**2 - 2*e**(c + d*x)*a**2 - b**2)*b**8 - 2*e**(2*c + 2*d*x)*a**4*b**4 -
 2*e**(2*c + 2*d*x)*b**8 + 4*e**(c + d*x)*int((sqrt(sinh(c + d*x))*sech(c
+ d*x))/(sinh(c + d*x)**2*b**4 - 2*sinh(c + d*x)*a**2*b**2 + a**4),x)*a**1
1*b*d + 8*e**(c + d*x)*int((sqrt(sinh(c + d*x))*sech(c + d*x))/(sinh(c + d
*x)**2*b**4 - 2*sinh(c + d*x)*a**2*b**2 + a**4),x)*a**7*b**5*d + 4*e**(c +
 d*x)*int((sqrt(sinh(c + d*x))*sech(c + d*x))/(sinh(c + d*x)**2*b**4 - 2*s
inh(c + d*x)*a**2*b**2 + a**4),x)*a**3*b**9*d - 6*e**(c + d*x)*log(e**(2*c
 + 2*d*x) + 1)*a**6*b**2 + 2*e**(c + d*x)*log(e**(2*c + 2*d*x) + 1)*a**...
```

3.383 $\int \frac{\cosh^5(c+dx)}{a+b\sinh^n(c+dx)}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 130

$$\begin{aligned}\int \frac{\cosh^5(c+dx)}{a+b\sinh^n(c+dx)}\,dx &= \frac{\text{Hypergeometric2F1}\left(1,\frac{1}{n},1+\frac{1}{n},-\frac{b\sinh^n(c+dx)}{a}\right)\sinh(c+dx)}{ad} \\ &+ \frac{2\,\text{Hypergeometric2F1}\left(1,\frac{3}{n},\frac{3+n}{n},-\frac{b\sinh^n(c+dx)}{a}\right)\sinh^3(c+dx)}{3ad} \\ &+ \frac{\text{Hypergeometric2F1}\left(1,\frac{5}{n},\frac{5+n}{n},-\frac{b\sinh^n(c+dx)}{a}\right)\sinh^5(c+dx)}{5ad}\end{aligned}$$

output
```
hypergeom([1, 1/n],[1+1/n],-b*sinh(d*x+c)^n/a)*sinh(d*x+c)/a/d+2/3*hyperge
om([1, 3/n],[(3+n)/n],-b*sinh(d*x+c)^n/a)*sinh(d*x+c)^3/a/d+1/5*hypergeom(
[1, 5/n],[(5+n)/n],-b*sinh(d*x+c)^n/a)*sinh(d*x+c)^5/a/d
```

Mathematica [A] (verified)

Time = 0.11 (sec) , antiderivative size = 119, normalized size of antiderivative = 0.92

$$\int \frac{\cosh^5(c+dx)}{a+b\sinh^n(c+dx)}\,dx = \frac{15\,\text{Hypergeometric2F1}\left(1,\frac{1}{n},1+\frac{1}{n},-\frac{b\sinh^n(c+dx)}{a}\right)\sinh(c+dx)+10\,\text{Hypergeometric2F1}\left(1,\frac{3}{n},\frac{3+n}{n},-\frac{b}{}\right.}{15ad}$$

input
```
Integrate[Cosh[c + d*x]^5/(a + b*Sinh[c + d*x]^n),x]
```

output
```
(15*Hypergeometric2F1[1, n^(-1), 1 + n^(-1), -((b*Sinh[c + d*x]^n)/a)]*Sin
h[c + d*x] + 10*Hypergeometric2F1[1, 3/n, (3 + n)/n, -((b*Sinh[c + d*x]^n)
/a)]*Sinh[c + d*x]^3 + 3*Hypergeometric2F1[1, 5/n, (5 + n)/n, -((b*Sinh[c
+ d*x]^n)/a)]*Sinh[c + d*x]^5)/(15*a*d)
```

Rubi [A] (verified)

Time = 0.44 (sec) , antiderivative size = 125, normalized size of antiderivative = 0.96, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.174$, Rules used = {3042, 3702, 2432, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cosh^5(c+dx)}{a+b\sinh^n(c+dx)}dx$$

$$\Big\downarrow \; 3042$$

$$\int \frac{\cos(ic+idx)^5}{a+b(-i\sin(ic+idx))^n}dx$$

$$\Big\downarrow \; 3702$$

$$\frac{\int \frac{\left(\sinh^2(c+dx)+1\right)^2}{b\sinh^n(c+dx)+a}d\sinh(c+dx)}{d}$$

$$\Big\downarrow \; 2432$$

$$\frac{\int \left(\frac{\sinh^4(c+dx)}{b\sinh^n(c+dx)+a}+\frac{2\sinh^2(c+dx)}{b\sinh^n(c+dx)+a}+\frac{1}{b\sinh^n(c+dx)+a}\right)d\sinh(c+dx)}{d}$$

$$\Big\downarrow \text{2009}$$

$$\frac{\frac{\sinh(c+dx)\,\text{Hypergeometric2F1}\left(1,\frac{1}{n},1+\frac{1}{n},-\frac{b\sinh^n(c+dx)}{a}\right)}{a}+\frac{\sinh^5(c+dx)\,\text{Hypergeometric2F1}\left(1,\frac{5}{n},\frac{n+5}{n},-\frac{b\sinh^n(c+dx)}{a}\right)}{5a}+\frac{2\sinh^3(c+dx)}{}}{d}$$

input
```
Int[Cosh[c + d*x]^5/(a + b*Sinh[c + d*x]^n),x]
```

output
```
((Hypergeometric2F1[1, n^(-1), 1 + n^(-1), -((b*Sinh[c + d*x]^n)/a)]*Sinh[
c + d*x])/a + (2*Hypergeometric2F1[1, 3/n, (3 + n)/n, -((b*Sinh[c + d*x]^n
)/a)]*Sinh[c + d*x]^3)/(3*a) + (Hypergeometric2F1[1, 5/n, (5 + n)/n, -((b*
Sinh[c + d*x]^n)/a)]*Sinh[c + d*x]^5)/(5*a))/d
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2432
```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[
Pq*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, n, p}, x] && (PolyQ[Pq, x] || Poly
Q[Pq, x^n])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3702
```
Int[cos[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*((c_.)*sin[(e_.) + (f_.)*(x
_)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Si
mp[ff/f   Subst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a + b*(c*ff*x)^n)^p, x], x,
 Sin[e + f*x]/ff], x]] /; FreeQ[{a, b, c, e, f, n, p}, x] && IntegerQ[(m -
1)/2] && (EqQ[n, 4] || GtQ[m, 0] || IGtQ[p, 0] || IntegersQ[m, p])
```

Maple [F]

$$\int \frac{\cosh(dx+c)^5}{a+b\sinh(dx+c)^n}dx$$

input
```
int(cosh(d*x+c)^5/(a+b*sinh(d*x+c)^n),x)
```

output
```
int(cosh(d*x+c)^5/(a+b*sinh(d*x+c)^n),x)
```

Fricas [F]

$$\int \frac{\cosh^5(c+dx)}{a+b\sinh^n(c+dx)}\,dx = \int \frac{\cosh(dx+c)^5}{b\sinh(dx+c)^n+a}\,dx$$

input
```
integrate(cosh(d*x+c)^5/(a+b*sinh(d*x+c)^n),x, algorithm="fricas")
```

output
```
integral(cosh(d*x + c)^5/(b*sinh(d*x + c)^n + a), x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\cosh^5(c+dx)}{a+b\sinh^n(c+dx)}\,dx = \text{Timed out}$$

input
```
integrate(cosh(d*x+c)**5/(a+b*sinh(d*x+c)**n),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{\cosh^5(c+dx)}{a+b\sinh^n(c+dx)}\,dx = \int \frac{\cosh{(dx+c)}^5}{b\sinh{(dx+c)}^n+a}\,dx$$

input
```
integrate(cosh(d*x+c)^5/(a+b*sinh(d*x+c)^n),x, algorithm="maxima")
```

output
```
integrate(cosh(d*x + c)^5/(b*sinh(d*x + c)^n + a), x)
```

Giac [F]

$$\int \frac{\cosh^5(c+dx)}{a+b\sinh^n(c+dx)}\,dx = \int \frac{\cosh{(dx+c)}^5}{b\sinh{(dx+c)}^n+a}\,dx$$

input
```
integrate(cosh(d*x+c)^5/(a+b*sinh(d*x+c)^n),x, algorithm="giac")
```

output
```
integrate(cosh(d*x + c)^5/(b*sinh(d*x + c)^n + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\cosh^5(c+dx)}{a+b\sinh^n(c+dx)}\,dx = \int \frac{\cosh(c+d\,x)^5}{a+b\sinh{(c+d\,x)}^n}\,dx$$

input
```
int(cosh(c + d*x)^5/(a + b*sinh(c + d*x)^n),x)
```

output
```
int(cosh(c + d*x)^5/(a + b*sinh(c + d*x)^n), x)
```

Reduce [F]

$$\int \frac{\cosh^5(c+dx)}{a+b\sinh^n(c+dx)}\,dx$$
$$= \frac{e^{cn}2^n\left(e^{8c}\left(\int \frac{e^{dnx+5dx}}{(e^{2dx+2c}-1)^n b+e^{dnx+cn}2^n a}dx\right)+5e^{6c}\left(\int \frac{e^{dnx+3dx}}{(e^{2dx+2c}-1)^n b+e^{dnx+cn}2^n a}dx\right)+10e^{4c}\left(\int \frac{e^{dnx+dx}}{(e^{2dx+2c}-1)^n b+e^{dnx-}}\right.\right.}{}$$

input
```
int(cosh(d*x+c)^5/(a+b*sinh(d*x+c)^n),x)
```

output
```
(e**(c*n)*2**n*(e**(8*c)*int(e**(d*n*x + 5*d*x)/((e**(2*c + 2*d*x) - 1)**n
*b + e**(c*n + d*n*x)*2**n*a),x) + 5*e**(6*c)*int(e**(d*n*x + 3*d*x)/((e**
(2*c + 2*d*x) - 1)**n*b + e**(c*n + d*n*x)*2**n*a),x) + 10*e**(4*c)*int(e*
*(d*n*x + d*x)/((e**(2*c + 2*d*x) - 1)**n*b + e**(c*n + d*n*x)*2**n*a),x)
+ e**(3*c)*int(e**(d*n*x)/(e**(5*c + 5*d*x)*(e**(2*c + 2*d*x) - 1)**n*b +
e**(c*n + 5*c + d*n*x + 5*d*x)*2**n*a),x) + 10*e**(2*c)*int(e**(d*n*x)/(e*
*(d*x)*(e**(2*c + 2*d*x) - 1)**n*b + e**(c*n + d*n*x + d*x)*2**n*a),x) + 5
*int(e**(d*n*x)/(e**(3*d*x)*(e**(2*c + 2*d*x) - 1)**n*b + e**(c*n + d*n*x
+ 3*d*x)*2**n*a),x)))/(32*e**(3*c))
```

3.384 $\int \frac{\cosh^3(c+dx)}{a+b\sinh^n(c+dx)}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 84

$$\int \frac{\cosh^3(c+dx)}{a+b\sinh^n(c+dx)}\,dx = \frac{\text{Hypergeometric2F1}\left(1,\frac{1}{n},1+\frac{1}{n},-\frac{b\sinh^n(c+dx)}{a}\right)\sinh(c+dx)}{ad} + \frac{\text{Hypergeometric2F1}\left(1,\frac{3}{n},\frac{3+n}{n},-\frac{b\sinh^n(c+dx)}{a}\right)\sinh^3(c+dx)}{3ad}$$

output
```
hypergeom([1, 1/n],[1+1/n],-b*sinh(d*x+c)^n/a)*sinh(d*x+c)/a/d+1/3*hyperge
om([1, 3/n],[(3+n)/n],-b*sinh(d*x+c)^n/a)*sinh(d*x+c)^3/a/d
```

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 82, normalized size of antiderivative = 0.98

$$\int \frac{\cosh^3(c+dx)}{a+b\sinh^n(c+dx)}\,dx = \frac{\frac{\text{Hypergeometric2F1}\left(1,\frac{1}{n},1+\frac{1}{n},-\frac{b\sinh^n(c+dx)}{a}\right)\sinh(c+dx)}{a}+\frac{\text{Hypergeometric2F1}\left(1,\frac{3}{n},1+\frac{3}{n},-\frac{b\sinh^n(c+dx)}{a}\right)\sinh^3(c+dx)}{3a}}{d}$$

input
```
Integrate[Cosh[c + d*x]^3/(a + b*Sinh[c + d*x]^n),x]
```

output
```
((Hypergeometric2F1[1, n^(-1), 1 + n^(-1), -((b*Sinh[c + d*x]^n)/a)]*Sinh[
c + d*x])/a + (Hypergeometric2F1[1, 3/n, 1 + 3/n, -((b*Sinh[c + d*x]^n)/a)
]*Sinh[c + d*x]^3)/(3*a))/d
```

Rubi [A] (verified)

Time = 0.38 (sec) , antiderivative size = 82, normalized size of antiderivative = 0.98, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.174, Rules used = {3042, 3702, 2432, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cosh^3(c+dx)}{a+b\sinh^n(c+dx)}dx$$

$$\Big\downarrow \; 3042$$

$$\int \frac{\cos(ic+idx)^3}{a+b(-i\sin(ic+idx))^n}dx$$

$$\Big\downarrow \; 3702$$

$$\frac{\int \frac{\sinh^2(c+dx)+1}{b\sinh^n(c+dx)+a}d\sinh(c+dx)}{d}$$

$$\Big\downarrow \; 2432$$

$$\frac{\int \left(\frac{\sinh^2(c+dx)}{b\sinh^n(c+dx)+a}+\frac{1}{b\sinh^n(c+dx)+a}\right)d\sinh(c+dx)}{d}$$

$$\Big\downarrow \; 2009$$

$$\frac{\frac{\sinh(c+dx)\text{Hypergeometric2F1}\left(1,\frac{1}{n},1+\frac{1}{n},-\frac{b\sinh^n(c+dx)}{a}\right)}{a}+\frac{\sinh^3(c+dx)\text{Hypergeometric2F1}\left(1,\frac{3}{n},\frac{n+3}{n},-\frac{b\sinh^n(c+dx)}{a}\right)}{3a}}{d}$$

input
```
Int[Cosh[c + d*x]^3/(a + b*Sinh[c + d*x]^n),x]
```

output
```
((Hypergeometric2F1[1, n^(-1), 1 + n^(-1), -((b*Sinh[c + d*x]^n)/a)]*Sinh[
c + d*x])/a + (Hypergeometric2F1[1, 3/n, (3 + n)/n, -((b*Sinh[c + d*x]^n)/
a)]*Sinh[c + d*x]^3)/(3*a))/d
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2432
```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[
Pq*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, n, p}, x] && (PolyQ[Pq, x] || Poly
Q[Pq, x^n])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3702
```
Int[cos[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*((c_.)*sin[(e_.) + (f_.)*(x
_)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Si
mp[ff/f   Subst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a + b*(c*ff*x)^n)^p, x], x,
 Sin[e + f*x]/ff], x]] /; FreeQ[{a, b, c, e, f, n, p}, x] && IntegerQ[(m -
1)/2] && (EqQ[n, 4] || GtQ[m, 0] || IGtQ[p, 0] || IntegersQ[m, p])
```

Maple [F]

$$\int \frac{\cosh (dx + c)^3}{a + b \sinh (dx + c)^n} dx$$

input
```
int(cosh(d*x+c)^3/(a+b*sinh(d*x+c)^n),x)
```

output
```
int(cosh(d*x+c)^3/(a+b*sinh(d*x+c)^n),x)
```

Fricas [F]

$$\int \frac{\cosh^3(c+dx)}{a+b\sinh^n(c+dx)}\,dx = \int \frac{\cosh(dx+c)^3}{b\sinh(dx+c)^n+a}\,dx$$

input
```
integrate(cosh(d*x+c)^3/(a+b*sinh(d*x+c)^n),x, algorithm="fricas")
```

output
```
integral(cosh(d*x + c)^3/(b*sinh(d*x + c)^n + a), x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\cosh^3(c+dx)}{a+b\sinh^n(c+dx)}\,dx = \text{Timed out}$$

input
```
integrate(cosh(d*x+c)**3/(a+b*sinh(d*x+c)**n),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{\cosh^3(c+dx)}{a+b\sinh^n(c+dx)}\,dx = \int \frac{\cosh(dx+c)^3}{b\sinh(dx+c)^n+a}\,dx$$

input
```
integrate(cosh(d*x+c)^3/(a+b*sinh(d*x+c)^n),x, algorithm="maxima")
```

output
```
integrate(cosh(d*x + c)^3/(b*sinh(d*x + c)^n + a), x)
```

Giac [F]

$$\int \frac{\cosh^3(c+dx)}{a+b\sinh^n(c+dx)}\,dx = \int \frac{\cosh\left(dx+c\right)^3}{b\sinh\left(dx+c\right)^n+a}\,dx$$

input
```
integrate(cosh(d*x+c)^3/(a+b*sinh(d*x+c)^n),x, algorithm="giac")
```

output
```
integrate(cosh(d*x + c)^3/(b*sinh(d*x + c)^n + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\cosh^3(c+dx)}{a+b\sinh^n(c+dx)}\,dx = \int \frac{\cosh(c+d\,x)^3}{a+b\sinh\left(c+d\,x\right)^n}\,dx$$

input
```
int(cosh(c + d*x)^3/(a + b*sinh(c + d*x)^n),x)
```

output
```
int(cosh(c + d*x)^3/(a + b*sinh(c + d*x)^n), x)
```

Reduce [F]

$$\int \frac{\cosh^3(c+dx)}{a+b\sinh^n(c+dx)}\,dx = \frac{e^{cn}2^n\left(e^{4c}\left(\int \frac{e^{dnx+3dx}}{\left(e^{2dx+2c}-1\right)^n b+e^{dnx+cn}2^n a}dx\right)+3e^{2c}\left(\int \frac{e^{dnx+dx}}{\left(e^{2dx+2c}-1\right)^n b+e^{dnx+cn}2^n a}dx\right)+e^c\left(\int \frac{e^{dnx}}{e^{3dx+3c}\left(e^{2dx+2c}-1\right)^n b+e^{c}}\right.\right.}{8e^c}$$

input
```
int(cosh(d*x+c)^3/(a+b*sinh(d*x+c)^n),x)
```

output

```
(e**(c*n)*2**n*(e**(4*c)*int(e**(d*n*x + 3*d*x)/((e**(2*c + 2*d*x) - 1)**n
*b + e**(c*n + d*n*x)*2**n*a),x) + 3*e**(2*c)*int(e**(d*n*x + d*x)/((e**(2
*c + 2*d*x) - 1)**n*b + e**(c*n + d*n*x)*2**n*a),x) + e**c*int(e**(d*n*x)/
(e**(3*c + 3*d*x)*(e**(2*c + 2*d*x) - 1)**n*b + e**(c*n + 3*c + d*n*x + 3*
d*x)*2**n*a),x) + 3*int(e**(d*n*x)/(e**(d*x)*(e**(2*c + 2*d*x) - 1)**n*b +
 e**(c*n + d*n*x + d*x)*2**n*a),x)))/(8*e**c)
```

3.385 $\int \frac{\cosh(c+dx)}{a+b\sinh^n(c+dx)}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 37

$$\int \frac{\cosh(c+dx)}{a+b\sinh^n(c+dx)}\,dx=\frac{\text{Hypergeometric2F1}\left(1,\frac{1}{n},1+\frac{1}{n},-\frac{b\sinh^n(c+dx)}{a}\right)\sinh(c+dx)}{ad}$$

output
```
hypergeom([1, 1/n],[1+1/n],-b*sinh(d*x+c)^n/a)*sinh(d*x+c)/a/d
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.00

$$\int \frac{\cosh(c+dx)}{a+b\sinh^n(c+dx)}\,dx=\frac{\text{Hypergeometric2F1}\left(1,\frac{1}{n},1+\frac{1}{n},-\frac{b\sinh^n(c+dx)}{a}\right)\sinh(c+dx)}{ad}$$

input
```
Integrate[Cosh[c + d*x]/(a + b*Sinh[c + d*x]^n),x]
```

output
```
(Hypergeometric2F1[1, n^(-1), 1 + n^(-1), -((b*Sinh[c + d*x]^n)/a)]*Sinh[c
 + d*x])/(a*d)
```

Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {3042, 3702, 778}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cosh(c+dx)}{a+b\sinh^n(c+dx)}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\cos(ic+idx)}{a+b(-i\sin(ic+idx))^n}dx$$

$$\Big\downarrow \text{3702}$$

$$\frac{\int \frac{1}{b\sinh^n(c+dx)+a}d\sinh(c+dx)}{d}$$

$$\Big\downarrow \text{778}$$

$$\frac{\sinh(c+dx)\,\text{Hypergeometric2F1}\left(1,\frac{1}{n},1+\frac{1}{n},-\frac{b\sinh^n(c+dx)}{a}\right)}{ad}$$

input
```
Int[Cosh[c + d*x]/(a + b*Sinh[c + d*x]^n),x]
```

output
```
(Hypergeometric2F1[1, n^(-1), 1 + n^(-1), -((b*Sinh[c + d*x]^n)/a)]*Sinh[c
 + d*x])/(a*d)
```

Defintions of rubi rules used

rule 778
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p*x*Hypergeometric2F
1[-p, 1/n, 1/n + 1, (-b)*(x^n/a)], x] /; FreeQ[{a, b, n, p}, x] && !IGtQ[p
, 0] && !IntegerQ[1/n] && !ILtQ[Simplify[1/n + p], 0] && (IntegerQ[p] ||
GtQ[a, 0])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3702
```
Int[cos[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*((c_.)*sin[(e_.) + (f_.)*(x
_)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Si
mp[ff/f   Subst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a + b*(c*ff*x)^n)^p, x], x,
 Sin[e + f*x]/ff], x]] /; FreeQ[{a, b, c, e, f, n, p}, x] && IntegerQ[(m -
1)/2] && (EqQ[n, 4] || GtQ[m, 0] || IGtQ[p, 0] || IntegersQ[m, p])
```

Maple [F]

$$\int \frac{\cosh(dx+c)}{a+b\sinh(dx+c)^{n}}dx$$

input
```
int(cosh(d*x+c)/(a+b*sinh(d*x+c)^n),x)
```

output
```
int(cosh(d*x+c)/(a+b*sinh(d*x+c)^n),x)
```

Fricas [F]

$$\int \frac{\cosh(c+dx)}{a+b\sinh^n(c+dx)}\,dx = \int \frac{\cosh(dx+c)}{b\sinh(dx+c)^{n}+a}\,dx$$

input
```
integrate(cosh(d*x+c)/(a+b*sinh(d*x+c)^n),x, algorithm="fricas")
```

output
```
integral(cosh(d*x + c)/(b*sinh(d*x + c)^n + a), x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\cosh(c+dx)}{a+b\sinh^n(c+dx)}\,dx = \text{Timed out}$$

input
```
integrate(cosh(d*x+c)/(a+b*sinh(d*x+c)**n),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{\cosh(c+dx)}{a+b\sinh^n(c+dx)}\,dx = \int \frac{\cosh(dx+c)}{b\sinh(dx+c)^n+a}\,dx$$

input
```
integrate(cosh(d*x+c)/(a+b*sinh(d*x+c)^n),x, algorithm="maxima")
```

output
```
integrate(cosh(d*x + c)/(b*sinh(d*x + c)^n + a), x)
```

Giac [F]

$$\int \frac{\cosh(c+dx)}{a+b\sinh^n(c+dx)}\,dx = \int \frac{\cosh(dx+c)}{b\sinh(dx+c)^n+a}\,dx$$

input
```
integrate(cosh(d*x+c)/(a+b*sinh(d*x+c)^n),x, algorithm="giac")
```

output
```
integrate(cosh(d*x + c)/(b*sinh(d*x + c)^n + a), x)
```

Mupad [B] (verification not implemented)

Time = 1.87 (sec) , antiderivative size = 38, normalized size of antiderivative = 1.03

$$\int \frac{\cosh(c+dx)}{a+b\sinh^n(c+dx)}\,dx = \frac{\sinh(c+d\,x)\;{}_2\mathrm{F}_1\left(1,\frac{1}{n};\ \frac{1}{n}+1;\ -\frac{b\sinh(c+d\,x)^n}{a}\right)}{a\,d}$$

input
```
int(cosh(c + d*x)/(a + b*sinh(c + d*x)^n),x)
```

output
```
(sinh(c + d*x)*hypergeom([1, 1/n], 1/n + 1, -(b*sinh(c + d*x)^n)/a))/(a*d)
```

Reduce [F]

$$\int \frac{\cosh(c+dx)}{a+b\sinh^n(c+dx)}\,dx = \frac{e^{cn}2^n\left(e^{2c}\left(\int \frac{e^{dnx+dx}}{(e^{2dx+2c}-1)^n b+e^{dnx+cn}2^n a}dx\right)+\int \frac{e^{dnx}}{e^{dx}(e^{2dx+2c}-1)^n b+e^{dnx+cn+dx}2^n a}dx\right)}{2e^c}$$

input
```
int(cosh(d*x+c)/(a+b*sinh(d*x+c)^n),x)
```

output
```
(e**(c*n)*2**n*(e**(2*c)*int(e**(d*n*x + d*x)/((e**(2*c + 2*d*x) - 1)**n*b
 + e**(c*n + d*n*x)*2**n*a),x) + int(e**(d*n*x)/(e**(d*x)*(e**(2*c + 2*d*x
) - 1)**n*b + e**(c*n + d*n*x + d*x)*2**n*a),x)))/(2*e**c)
```

3.386 $\int \frac{\cosh^5(c+dx)}{(a+b\sinh^n(c+dx))^2}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 130

$$\int \frac{\cosh^5(c+dx)}{(a+b\sinh^n(c+dx))^2}\,dx = \frac{\text{Hypergeometric2F1}\left(2,\frac{1}{n},1+\frac{1}{n},-\frac{b\sinh^n(c+dx)}{a}\right)\sinh(c+dx)}{a^2d} + \frac{2\,\text{Hypergeometric2F1}\left(2,\frac{3}{n},\frac{3+n}{n},-\frac{b\sinh^n(c+dx)}{a}\right)\sinh^3(c+dx)}{3a^2d} + \frac{\text{Hypergeometric2F1}\left(2,\frac{5}{n},\frac{5+n}{n},-\frac{b\sinh^n(c+dx)}{a}\right)\sinh^5(c+dx)}{5a^2d}$$

output
```
hypergeom([2, 1/n],[1+1/n],-b*sinh(d*x+c)^n/a)*sinh(d*x+c)/a^2/d+2/3*hyper
geom([2, 3/n],[(3+n)/n],-b*sinh(d*x+c)^n/a)*sinh(d*x+c)^3/a^2/d+1/5*hyperg
eom([2, 5/n],[(5+n)/n],-b*sinh(d*x+c)^n/a)*sinh(d*x+c)^5/a^2/d
```

Mathematica [A] (verified)

Time = 0.09 (sec) , antiderivative size = 119, normalized size of antiderivative = 0.92

$$\int \frac{\cosh^5(c+dx)}{(a+b\sinh^n(c+dx))^2}\,dx$$
$$= \frac{15\,\text{Hypergeometric2F1}\left(2,\frac{1}{n},1+\frac{1}{n},-\frac{b\sinh^n(c+dx)}{a}\right)\sinh(c+dx)+10\,\text{Hypergeometric2F1}\left(2,\frac{3}{n},\frac{3+n}{n},-\frac{b}{}\right.}{15a^2d}$$

input
```
Integrate[Cosh[c + d*x]^5/(a + b*Sinh[c + d*x]^n)^2,x]
```

output
```
(15*Hypergeometric2F1[2, n^(-1), 1 + n^(-1), -((b*Sinh[c + d*x]^n)/a)]*Sin
h[c + d*x] + 10*Hypergeometric2F1[2, 3/n, (3 + n)/n, -((b*Sinh[c + d*x]^n)
/a)]*Sinh[c + d*x]^3 + 3*Hypergeometric2F1[2, 5/n, (5 + n)/n, -((b*Sinh[c
+ d*x]^n)/a)]*Sinh[c + d*x]^5)/(15*a^2*d)
```

Rubi [A] (verified)

Time = 0.45 (sec) , antiderivative size = 125, normalized size of antiderivative = 0.96, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.174, Rules used = {3042, 3702, 2432, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cosh^5(c+dx)}{(a+b\sinh^n(c+dx))^2}\,dx$$
$$\Big\downarrow \text{3042}$$
$$\int \frac{\cos(ic+idx)^5}{(a+b(-i\sin(ic+idx))^n)^2}dx$$
$$\Big\downarrow \text{3702}$$
$$\frac{\int \frac{(\sinh^2(c+dx)+1)^2}{(b\sinh^n(c+dx)+a)^2}d\sinh(c+dx)}{d}$$
$$\Big\downarrow \text{2432}$$

$$\frac{\int \left(\frac{\sinh^4(c+dx)}{(b\sinh^n(c+dx)+a)^2}+\frac{2\sinh^2(c+dx)}{(b\sinh^n(c+dx)+a)^2}+\frac{1}{(b\sinh^n(c+dx)+a)^2}\right)d\sinh(c+dx)}{d}$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{\frac{\sinh(c+dx)\,\text{Hypergeometric2F1}\left(2,\frac{1}{n},1+\frac{1}{n},-\frac{b\sinh^n(c+dx)}{a}\right)}{a^2}+\frac{\sinh^5(c+dx)\,\text{Hypergeometric2F1}\left(2,\frac{5}{n},\frac{n+5}{n},-\frac{b\sinh^n(c+dx)}{a}\right)}{5a^2}+\frac{2\sinh^3(c+dx)}{}}{d}$$

input
```
Int[Cosh[c + d*x]^5/(a + b*Sinh[c + d*x]^n)^2,x]
```

output
```
((Hypergeometric2F1[2, n^(-1), 1 + n^(-1), -((b*Sinh[c + d*x]^n)/a)]*Sinh[
c + d*x])/a^2 + (2*Hypergeometric2F1[2, 3/n, (3 + n)/n, -((b*Sinh[c + d*x]
^n)/a)]*Sinh[c + d*x]^3)/(3*a^2) + (Hypergeometric2F1[2, 5/n, (5 + n)/n, -
((b*Sinh[c + d*x]^n)/a)]*Sinh[c + d*x]^5)/(5*a^2))/d
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2432
```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[
Pq*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, n, p}, x] && (PolyQ[Pq, x] || Poly
Q[Pq, x^n])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3702
```
Int[cos[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*((c_.)*sin[(e_.) + (f_.)*(x
_)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Si
mp[ff/f   Subst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a + b*(c*ff*x)^n)^p, x], x,
 Sin[e + f*x]/ff], x]] /; FreeQ[{a, b, c, e, f, n, p}, x] && IntegerQ[(m -
1)/2] && (EqQ[n, 4] || GtQ[m, 0] || IGtQ[p, 0] || IntegersQ[m, p])
```

Maple [F]

$$\int \frac{\cosh(dx+c)^5}{(a+b\sinh(dx+c)^n)^2}dx$$

input
```
int(cosh(d*x+c)^5/(a+b*sinh(d*x+c)^n)^2,x)
```

output
```
int(cosh(d*x+c)^5/(a+b*sinh(d*x+c)^n)^2,x)
```

Fricas [F]

$$\int \frac{\cosh^5(c+dx)}{(a+b\sinh^n(c+dx))^2}\,dx = \int \frac{\cosh(dx+c)^5}{(b\sinh(dx+c)^n+a)^2}\,dx$$

input
```
integrate(cosh(d*x+c)^5/(a+b*sinh(d*x+c)^n)^2,x, algorithm="fricas")
```

output
```
integral(cosh(d*x + c)^5/(b^2*sinh(d*x + c)^(2*n) + 2*a*b*sinh(d*x + c)^n
+ a^2), x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\cosh^5(c+dx)}{(a+b\sinh^n(c+dx))^2}\,dx = \text{Timed out}$$

input
```
integrate(cosh(d*x+c)**5/(a+b*sinh(d*x+c)**n)**2,x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{\cosh^5(c+dx)}{(a+b\sinh^n(c+dx))^2}\,dx = \int \frac{\cosh(dx+c)^5}{(b\sinh(dx+c)^n+a)^2}\,dx$$

input
```
integrate(cosh(d*x+c)^5/(a+b*sinh(d*x+c)^n)^2,x, algorithm="maxima")
```

output
```
1/32*(2^n*e^(c*n + 10*d*x + 10*c) + 3*2^n*e^(c*n + 8*d*x + 8*c) + 2^(n + 1
)*e^(c*n + 6*d*x + 6*c) - 2^(n + 1)*e^(c*n + 4*d*x + 4*c) - 3*2^n*e^(c*n +
 2*d*x + 2*c) - 2^n*e^(c*n))*e^(d*n*x)/(2^n*a^2*d*n*e^(d*n*x + c*n + 5*d*x
 + 5*c) + a*b*d*n*e^(5*d*x + n*log(e^(d*x + c) + 1) + n*log(e^(d*x + c) -
1) + 5*c)) + 1/32*integrate((2^n*n*e^(c*n) - 5*2^n*e^(c*n) + (2^n*n*e^(c*n
) - 5*2^n*e^(c*n))*e^(10*d*x + 10*c) + (5*2^n*n*e^(c*n) - 9*2^n*e^(c*n))*e
^(8*d*x + 8*c) + (5*2^(n + 1)*n*e^(c*n) - 2^(n + 1)*e^(c*n))*e^(6*d*x + 6*
c) + (5*2^(n + 1)*n*e^(c*n) - 2^(n + 1)*e^(c*n))*e^(4*d*x + 4*c) + (5*2^n*
n*e^(c*n) - 9*2^n*e^(c*n))*e^(2*d*x + 2*c))*e^(d*n*x)/(2^n*a^2*n*e^(d*n*x
+ c*n + 5*d*x + 5*c) + a*b*n*e^(5*d*x + n*log(e^(d*x + c) + 1) + n*log(e^(
d*x + c) - 1) + 5*c)), x)
```

Giac [F]

$$\int \frac{\cosh^5(c+dx)}{(a+b\sinh^n(c+dx))^2}\,dx = \int \frac{\cosh(dx+c)^5}{(b\sinh(dx+c)^n+a)^2}\,dx$$

input
```
integrate(cosh(d*x+c)^5/(a+b*sinh(d*x+c)^n)^2,x, algorithm="giac")
```

output
```
integrate(cosh(d*x + c)^5/(b*sinh(d*x + c)^n + a)^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\cosh^5(c+dx)}{(a+b\sinh^n(c+dx))^2}\,dx = \int \frac{\cosh(c+d\,x)^5}{\left(a+b\sinh\left(c+d\,x\right)^n\right)^2}\,dx$$

input
```
int(cosh(c + d*x)^5/(a + b*sinh(c + d*x)^n)^2,x)
```

output
```
int(cosh(c + d*x)^5/(a + b*sinh(c + d*x)^n)^2, x)
```

Reduce [F]

$$\int \frac{\cosh^5(c+dx)}{(a+b\sinh^n(c+dx))^2}\,dx = \int \frac{\cosh\left(dx+c\right)^5}{\left(a+b\sinh\left(dx+c\right)^n\right)^2}dx$$

input
```
int(cosh(d*x+c)^5/(a+b*sinh(d*x+c)^n)^2,x)
```

output
```
int(cosh(d*x+c)^5/(a+b*sinh(d*x+c)^n)^2,x)
```

3.387 $\int \frac{\cosh^3(c+dx)}{(a+b\sinh^n(c+dx))^2}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 84

$$\int \frac{\cosh^3(c+dx)}{(a+b\sinh^n(c+dx))^2}\,dx = \frac{\text{Hypergeometric2F1}\left(2,\frac{1}{n},1+\frac{1}{n},-\frac{b\sinh^n(c+dx)}{a}\right)\sinh(c+dx)}{a^2d} + \frac{\text{Hypergeometric2F1}\left(2,\frac{3}{n},\frac{3+n}{n},-\frac{b\sinh^n(c+dx)}{a}\right)\sinh^3(c+dx)}{3a^2d}$$

output
```
hypergeom([2, 1/n],[1+1/n],-b*sinh(d*x+c)^n/a)*sinh(d*x+c)/a^2/d+1/3*hyper
geom([2, 3/n],[(3+n)/n],-b*sinh(d*x+c)^n/a)*sinh(d*x+c)^3/a^2/d
```

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 82, normalized size of antiderivative = 0.98

$$\int \frac{\cosh^3(c+dx)}{(a+b\sinh^n(c+dx))^2}\,dx = \frac{\frac{\text{Hypergeometric2F1}\left(2,\frac{1}{n},1+\frac{1}{n},-\frac{b\sinh^n(c+dx)}{a}\right)\sinh(c+dx)}{a^2} + \frac{\text{Hypergeometric2F1}\left(2,\frac{3}{n},1+\frac{3}{n},-\frac{b\sinh^n(c+dx)}{a}\right)\sinh^3(c+dx)}{3a^2}}{d}$$

input
```
Integrate[Cosh[c + d*x]^3/(a + b*Sinh[c + d*x]^n)^2,x]
```

output
```
((Hypergeometric2F1[2, n^(-1), 1 + n^(-1), -((b*Sinh[c + d*x]^n)/a)]*Sinh[
c + d*x])/a^2 + (Hypergeometric2F1[2, 3/n, 1 + 3/n, -((b*Sinh[c + d*x]^n)/
a)]*Sinh[c + d*x]^3)/(3*a^2))/d
```

Rubi [A] (verified)

Time = 0.37 (sec) , antiderivative size = 82, normalized size of antiderivative = 0.98, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.174, Rules used = {3042, 3702, 2432, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cosh^3(c+dx)}{(a+b\sinh^n(c+dx))^2}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\cos(ic+idx)^3}{(a+b(-i\sin(ic+idx))^n)^2}dx$$

$$\Big\downarrow \text{3702}$$

$$\frac{\int \frac{\sinh^2(c+dx)+1}{(b\sinh^n(c+dx)+a)^2}d\sinh(c+dx)}{d}$$

$$\Big\downarrow \text{2432}$$

$$\frac{\int \left(\frac{\sinh^2(c+dx)}{(b\sinh^n(c+dx)+a)^2}+\frac{1}{(b\sinh^n(c+dx)+a)^2}\right)d\sinh(c+dx)}{d}$$

$$\Big\downarrow \text{2009}$$

$$\frac{\frac{\sinh(c+dx)\,\text{Hypergeometric2F1}\left(2,\frac{1}{n},1+\frac{1}{n},-\frac{b\sinh^n(c+dx)}{a}\right)}{a^2}+\frac{\sinh^3(c+dx)\,\text{Hypergeometric2F1}\left(2,\frac{3}{n},\frac{n+3}{n},-\frac{b\sinh^n(c+dx)}{a}\right)}{3a^2}}{d}$$

input
```
Int[Cosh[c + d*x]^3/(a + b*Sinh[c + d*x]^n)^2,x]
```

output
```
((Hypergeometric2F1[2, n^(-1), 1 + n^(-1), -((b*Sinh[c + d*x]^n)/a)]*Sinh[
c + d*x])/a^2 + (Hypergeometric2F1[2, 3/n, (3 + n)/n, -((b*Sinh[c + d*x]^n
)/a)]*Sinh[c + d*x]^3)/(3*a^2))/d
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2432
```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[
Pq*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, n, p}, x] && (PolyQ[Pq, x] || Poly
Q[Pq, x^n])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3702
```
Int[cos[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*((c_.)*sin[(e_.) + (f_.)*(x
_)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Si
mp[ff/f   Subst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a + b*(c*ff*x)^n)^p, x], x,
 Sin[e + f*x]/ff], x]] /; FreeQ[{a, b, c, e, f, n, p}, x] && IntegerQ[(m -
1)/2] && (EqQ[n, 4] || GtQ[m, 0] || IGtQ[p, 0] || IntegersQ[m, p])
```

Maple [F]

$$\int \frac{\cosh(dx+c)^3}{(a+b\sinh(dx+c)^n)^2}dx$$

input
```
int(cosh(d*x+c)^3/(a+b*sinh(d*x+c)^n)^2,x)
```

output
```
int(cosh(d*x+c)^3/(a+b*sinh(d*x+c)^n)^2,x)
```

Fricas [F]

$$\int \frac{\cosh^3(c+dx)}{(a+b\sinh^n(c+dx))^2}\,dx = \int \frac{\cosh(dx+c)^3}{(b\sinh(dx+c)^n+a)^2}\,dx$$

input
```
integrate(cosh(d*x+c)^3/(a+b*sinh(d*x+c)^n)^2,x, algorithm="fricas")
```

output
```
integral(cosh(d*x + c)^3/(b^2*sinh(d*x + c)^(2*n) + 2*a*b*sinh(d*x + c)^n
+ a^2), x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\cosh^3(c+dx)}{(a+b\sinh^n(c+dx))^2}\,dx = \text{Timed out}$$

input
```
integrate(cosh(d*x+c)**3/(a+b*sinh(d*x+c)**n)**2,x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{\cosh^3(c+dx)}{(a+b\sinh^n(c+dx))^2}\,dx = \int \frac{\cosh(dx+c)^3}{(b\sinh(dx+c)^n+a)^2}\,dx$$

input
```
integrate(cosh(d*x+c)^3/(a+b*sinh(d*x+c)^n)^2,x, algorithm="maxima")
```

output
```
1/8*(2^n*e^(c*n + 6*d*x + 6*c) + 2^n*e^(c*n + 4*d*x + 4*c) - 2^n*e^(c*n +
2*d*x + 2*c) - 2^n*e^(c*n))*e^(d*n*x)/(2^n*a^2*d*n*e^(d*n*x + c*n + 3*d*x
+ 3*c) + a*b*d*n*e^(3*d*x + n*log(e^(d*x + c) + 1) + n*log(e^(d*x + c) - 1
) + 3*c)) + 1/8*integrate((2^n*n*e^(c*n) - 3*2^n*e^(c*n) + (2^n*n*e^(c*n)
- 3*2^n*e^(c*n))*e^(6*d*x + 6*c) + (3*2^n*n*e^(c*n) - 2^n*e^(c*n))*e^(4*d*
x + 4*c) + (3*2^n*n*e^(c*n) - 2^n*e^(c*n))*e^(2*d*x + 2*c))*e^(d*n*x)/(2^n
*a^2*n*e^(d*n*x + c*n + 3*d*x + 3*c) + a*b*n*e^(3*d*x + n*log(e^(d*x + c)
+ 1) + n*log(e^(d*x + c) - 1) + 3*c)), x)
```

Giac [F]

$$\int \frac{\cosh^3(c+dx)}{(a+b\sinh^n(c+dx))^2} \, dx = \int \frac{\cosh(dx+c)^3}{(b\sinh(dx+c)^n+a)^2} \, dx$$

input
```
integrate(cosh(d*x+c)^3/(a+b*sinh(d*x+c)^n)^2,x, algorithm="giac")
```

output
```
integrate(cosh(d*x + c)^3/(b*sinh(d*x + c)^n + a)^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\cosh^3(c+dx)}{(a+b\sinh^n(c+dx))^2} \, dx = \int \frac{\cosh(c+d\,x)^3}{(a+b\sinh(c+d\,x)^n)^2} \, dx$$

input
```
int(cosh(c + d*x)^3/(a + b*sinh(c + d*x)^n)^2,x)
```

output
```
int(cosh(c + d*x)^3/(a + b*sinh(c + d*x)^n)^2, x)
```

Reduce [F]

$$\int \frac{\cosh^3(c+dx)}{(a+b\sinh^n(c+dx))^2}\,dx = \text{too large to display}$$

input
```
int(cosh(d*x+c)^3/(a+b*sinh(d*x+c)^n)^2,x)
```

output
```
(3*e**(2*c*n + 6*c + 3*d*x)*(e**(2*c + 2*d*x) - 1)**n*2**(2*n)*int(e**(2*d
*n*x + 3*d*x)/((e**(2*c + 2*d*x) - 1)**(2*n)*b**2 + 2*e**(c*n + d*n*x)*(e*
*(2*c + 2*d*x) - 1)**n*2**n*a*b + e**(2*c*n + 2*d*n*x)*2**(2*n)*a**2),x)*a
**2*b*d*n**2 - 12*e**(2*c*n + 6*c + 3*d*x)*(e**(2*c + 2*d*x) - 1)**n*2**(2
*n)*int(e**(2*d*n*x + 3*d*x)/((e**(2*c + 2*d*x) - 1)**(2*n)*b**2 + 2*e**(c
*n + d*n*x)*(e**(2*c + 2*d*x) - 1)**n*2**n*a*b + e**(2*c*n + 2*d*n*x)*2**(
2*n)*a**2),x)*a**2*b*d*n + 9*e**(2*c*n + 6*c + 3*d*x)*(e**(2*c + 2*d*x) -
1)**n*2**(2*n)*int(e**(2*d*n*x + 3*d*x)/((e**(2*c + 2*d*x) - 1)**(2*n)*b**
2 + 2*e**(c*n + d*n*x)*(e**(2*c + 2*d*x) - 1)**n*2**n*a*b + e**(2*c*n + 2*
d*n*x)*2**(2*n)*a**2),x)*a**2*b*d + 9*e**(2*c*n + 4*c + 3*d*x)*(e**(2*c +
2*d*x) - 1)**n*2**(2*n)*int(e**(2*d*n*x + d*x)/((e**(2*c + 2*d*x) - 1)**(2
*n)*b**2 + 2*e**(c*n + d*n*x)*(e**(2*c + 2*d*x) - 1)**n*2**n*a*b + e**(2*c
*n + 2*d*n*x)*2**(2*n)*a**2),x)*a**2*b*d*n**2 - 36*e**(2*c*n + 4*c + 3*d*x
)*(e**(2*c + 2*d*x) - 1)**n*2**(2*n)*int(e**(2*d*n*x + d*x)/((e**(2*c + 2*
d*x) - 1)**(2*n)*b**2 + 2*e**(c*n + d*n*x)*(e**(2*c + 2*d*x) - 1)**n*2**n*
a*b + e**(2*c*n + 2*d*n*x)*2**(2*n)*a**2),x)*a**2*b*d*n + 27*e**(2*c*n + 4
*c + 3*d*x)*(e**(2*c + 2*d*x) - 1)**n*2**(2*n)*int(e**(2*d*n*x + d*x)/((e*
*(2*c + 2*d*x) - 1)**(2*n)*b**2 + 2*e**(c*n + d*n*x)*(e**(2*c + 2*d*x) - 1
)**n*2**n*a*b + e**(2*c*n + 2*d*n*x)*2**(2*n)*a**2),x)*a**2*b*d + 36*e**(c
*n + 2*c + 3*d*x)*(e**(2*c + 2*d*x) - 1)**n*2**n*int((e**(d*n*x)*(e**(2...
```

3.388 $\int \frac{\cosh(c+dx)}{(a+b\sinh^n(c+dx))^2}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 37

$$\int \frac{\cosh(c+dx)}{(a+b\sinh^n(c+dx))^2}\,dx = \frac{\text{Hypergeometric2F1}\left(2,\frac{1}{n},1+\frac{1}{n},-\frac{b\sinh^n(c+dx)}{a}\right)\sinh(c+dx)}{a^2d}$$

output
```
hypergeom([2, 1/n],[1+1/n],-b*sinh(d*x+c)^n/a)*sinh(d*x+c)/a^2/d
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.00

$$\int \frac{\cosh(c+dx)}{(a+b\sinh^n(c+dx))^2}\,dx = \frac{\text{Hypergeometric2F1}\left(2,\frac{1}{n},1+\frac{1}{n},-\frac{b\sinh^n(c+dx)}{a}\right)\sinh(c+dx)}{a^2d}$$

input
```
Integrate[Cosh[c + d*x]/(a + b*Sinh[c + d*x]^n)^2,x]
```

output
```
(Hypergeometric2F1[2, n^(-1), 1 + n^(-1), -((b*Sinh[c + d*x]^n)/a)]*Sinh[c
 + d*x])/(a^2*d)
```

Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {3042, 3702, 778}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cosh(c+dx)}{(a+b\sinh^n(c+dx))^2}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\cos(ic+idx)}{(a+b(-i\sin(ic+idx))^n)^2}dx$$

$$\Big\downarrow \text{3702}$$

$$\frac{\int \frac{1}{(b\sinh^n(c+dx)+a)^2}d\sinh(c+dx)}{d}$$

$$\Big\downarrow \text{778}$$

$$\frac{\sinh(c+dx)\,\text{Hypergeometric2F1}\left(2,\frac{1}{n},1+\frac{1}{n},-\frac{b\sinh^n(c+dx)}{a}\right)}{a^2d}$$

input
```
Int[Cosh[c + d*x]/(a + b*Sinh[c + d*x]^n)^2,x]
```

output
```
(Hypergeometric2F1[2, n^(-1), 1 + n^(-1), -((b*Sinh[c + d*x]^n)/a)]*Sinh[c
 + d*x])/(a^2*d)
```

Defintions of rubi rules used

rule 778
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p*x*Hypergeometric2F
1[-p, 1/n, 1/n + 1, (-b)*(x^n/a)], x] /; FreeQ[{a, b, n, p}, x] && !IGtQ[p
, 0] && !IntegerQ[1/n] && !ILtQ[Simplify[1/n + p], 0] && (IntegerQ[p] ||
GtQ[a, 0])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3702
```
Int[cos[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*((c_.)*sin[(e_.) + (f_.)*(x
_)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Si
mp[ff/f   Subst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a + b*(c*ff*x)^n)^p, x], x,
 Sin[e + f*x]/ff], x]] /; FreeQ[{a, b, c, e, f, n, p}, x] && IntegerQ[(m -
1)/2] && (EqQ[n, 4] || GtQ[m, 0] || IGtQ[p, 0] || IntegersQ[m, p])
```

Maple [F]

$$\int \frac{\cosh (dx+c)}{(a+b\sinh (dx+c)^{n})^{2}}dx$$

input
```
int(cosh(d*x+c)/(a+b*sinh(d*x+c)^n)^2,x)
```

output
```
int(cosh(d*x+c)/(a+b*sinh(d*x+c)^n)^2,x)
```

Fricas [F]

$$\int \frac{\cosh(c+dx)}{(a+b\sinh^{n}(c+dx))^{2}}\,dx=\int \frac{\cosh (dx+c)}{(b\sinh (dx+c)^{n}+a)^{2}}\,dx$$

input
```
integrate(cosh(d*x+c)/(a+b*sinh(d*x+c)^n)^2,x, algorithm="fricas")
```

output
```
integral(cosh(d*x + c)/(b^2*sinh(d*x + c)^(2*n) + 2*a*b*sinh(d*x + c)^n +
a^2), x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\cosh(c+dx)}{(a+b\sinh^n(c+dx))^2}\,dx = \text{Timed out}$$

input
```
integrate(cosh(d*x+c)/(a+b*sinh(d*x+c)**n)**2,x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{\cosh(c+dx)}{(a+b\sinh^n(c+dx))^2}\,dx = \int \frac{\cosh(dx+c)}{(b\sinh(dx+c)^n+a)^2}\,dx$$

input
```
integrate(cosh(d*x+c)/(a+b*sinh(d*x+c)^n)^2,x, algorithm="maxima")
```

output
```
1/2*(2^n*e^(c*n + 2*d*x + 2*c) - 2^n*e^(c*n))*e^(d*n*x)/(2^n*a^2*d*n*e^(d*
n*x + c*n + d*x + c) + a*b*d*n*e^(d*x + n*log(e^(d*x + c) + 1) + n*log(e^(
d*x + c) - 1) + c)) + 1/2*integrate((2^n*n*e^(c*n) - 2^n*e^(c*n) + (2^n*n*
e^(c*n) - 2^n*e^(c*n))*e^(2*d*x + 2*c))*e^(d*n*x)/(2^n*a^2*n*e^(d*n*x + c*
n + d*x + c) + a*b*n*e^(d*x + n*log(e^(d*x + c) + 1) + n*log(e^(d*x + c) -
 1) + c)), x)
```

Giac [F]

$$\int \frac{\cosh(c+dx)}{(a+b\sinh^n(c+dx))^2}\,dx = \int \frac{\cosh(dx+c)}{(b\sinh(dx+c)^n+a)^2}\,dx$$

input
```
integrate(cosh(d*x+c)/(a+b*sinh(d*x+c)^n)^2,x, algorithm="giac")
```

output
```
integrate(cosh(d*x + c)/(b*sinh(d*x + c)^n + a)^2, x)
```

Mupad [B] (verification not implemented)

Time = 1.93 (sec) , antiderivative size = 38, normalized size of antiderivative = 1.03

$$\int \frac{\cosh(c+dx)}{(a+b\sinh^n(c+dx))^2}\,dx = \frac{\sinh(c+d\,x)\ {}_2\mathrm{F}_1\left(2,\frac{1}{n};\ \frac{1}{n}+1;\ -\frac{b\sinh(c+d\,x)^n}{a}\right)}{a^2\,d}$$

input
```
int(cosh(c + d*x)/(a + b*sinh(c + d*x)^n)^2,x)
```

output
```
(sinh(c + d*x)*hypergeom([2, 1/n], 1/n + 1, -(b*sinh(c + d*x)^n)/a))/(a^2*
d)
```

Reduce [F]

$$\int \frac{\cosh(c+dx)}{(a+b\sinh^n(c+dx))^2}\,dx = \text{too large to display}$$

input
```
int(cosh(d*x+c)/(a+b*sinh(d*x+c)^n)^2,x)
```

output
```
(e**(2*c*n + 2*c + d*x)*sinh(c + d*x)**n*(e**(2*c + 2*d*x) - 1)**n*2**(2*n
)*int(e**(2*d*n*x + d*x)/((e**(2*c + 2*d*x) - 1)**(2*n)*b**2 + 2*e**(c*n +
 d*n*x)*(e**(2*c + 2*d*x) - 1)**n*2**n*a*b + e**(2*c*n + 2*d*n*x)*2**(2*n)
*a**2),x)*a**2*b**2*d*n - e**(2*c*n + 2*c + d*x)*sinh(c + d*x)**n*(e**(2*c
 + 2*d*x) - 1)**n*2**(2*n)*int(e**(2*d*n*x + d*x)/((e**(2*c + 2*d*x) - 1)*
*(2*n)*b**2 + 2*e**(c*n + d*n*x)*(e**(2*c + 2*d*x) - 1)**n*2**n*a*b + e**(
2*c*n + 2*d*n*x)*2**(2*n)*a**2),x)*a**2*b**2*d + 4*e**(c*n + d*x)*sinh(c +
 d*x)**n*(e**(2*c + 2*d*x) - 1)**n*2**n*int((e**(d*n*x)*(e**(2*c + 2*d*x)
- 1)**n)/(e**(2*c + 3*d*x)*(e**(2*c + 2*d*x) - 1)**(2*n)*b**2*n - e**(2*c
+ 3*d*x)*(e**(2*c + 2*d*x) - 1)**(2*n)*b**2 - e**(d*x)*(e**(2*c + 2*d*x) -
 1)**(2*n)*b**2*n + e**(d*x)*(e**(2*c + 2*d*x) - 1)**(2*n)*b**2 + 2*e**(c*
n + 2*c + d*n*x + 3*d*x)*(e**(2*c + 2*d*x) - 1)**n*2**n*a*b*n - 2*e**(c*n
+ 2*c + d*n*x + 3*d*x)*(e**(2*c + 2*d*x) - 1)**n*2**n*a*b - 2*e**(c*n + d*
n*x + d*x)*(e**(2*c + 2*d*x) - 1)**n*2**n*a*b*n + 2*e**(c*n + d*n*x + d*x)
*(e**(2*c + 2*d*x) - 1)**n*2**n*a*b + e**(2*c*n + 2*c + 2*d*n*x + 3*d*x)*2
**(2*n)*a**2*n - e**(2*c*n + 2*c + 2*d*n*x + 3*d*x)*2**(2*n)*a**2 - e**(2*
c*n + 2*d*n*x + d*x)*2**(2*n)*a**2*n + e**(2*c*n + 2*d*n*x + d*x)*2**(2*n)
*a**2),x)*a*b**3*d*n**2 - 4*e**(c*n + d*x)*sinh(c + d*x)**n*(e**(2*c + 2*d
*x) - 1)**n*2**n*int((e**(d*n*x)*(e**(2*c + 2*d*x) - 1)**n)/(e**(2*c + 3*d
*x)*(e**(2*c + 2*d*x) - 1)**(2*n)*b**2*n - e**(2*c + 3*d*x)*(e**(2*c + ...
```

3.389 $\int \frac{\coth(x)}{1-\sinh^2(x)} dx$

Optimal result

Integrand size = 13, antiderivative size = 17

$$\int \frac{\coth(x)}{1-\sinh^2(x)} dx = \log(\sinh(x)) - \frac{1}{2}\log\left(1-\sinh^2(x)\right)$$

output
```
ln(sinh(x))-1/2*ln(1-sinh(x)^2)
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.35

$$\int \frac{\coth(x)}{1-\sinh^2(x)} dx = -2\left(-\frac{1}{2}\log(\sinh(x)) + \frac{1}{4}\log\left(1-\sinh^2(x)\right)\right)$$

input
```
Integrate[Coth[x]/(1 - Sinh[x]^2),x]
```

output
```
-2*(-1/2*Log[Sinh[x]] + Log[1 - Sinh[x]^2]/4)
```

Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.24, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.462, Rules used = {3042, 26, 3673, 47, 14, 16}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\coth(x)}{1-\sinh^2(x)}dx$$
$$\Big\downarrow \text{3042}$$
$$\int \frac{i}{(1+\sin(ix)^2)\tan(ix)}dx$$
$$\Big\downarrow \text{26}$$
$$i\int \frac{1}{(\sin(ix)^2+1)\tan(ix)}dx$$
$$\Big\downarrow \text{3673}$$
$$\frac{1}{2}\int \frac{\text{csch}^2(x)}{1-\sinh^2(x)}d\sinh^2(x)$$
$$\Big\downarrow \text{47}$$
$$\frac{1}{2}\left(\int \frac{1}{1-\sinh^2(x)}d\sinh^2(x)+\int \text{csch}^2(x)d\sinh^2(x)\right)$$
$$\Big\downarrow \text{14}$$
$$\frac{1}{2}\left(\int \frac{1}{1-\sinh^2(x)}d\sinh^2(x)+\log\left(\sinh^2(x)\right)\right)$$
$$\Big\downarrow \text{16}$$
$$\frac{1}{2}\left(\log\left(\sinh^2(x)\right)-\log\left(1-\sinh^2(x)\right)\right)$$

input
```
Int[Coth[x]/(1 - Sinh[x]^2),x]
```

output
```
(Log[Sinh[x]^2] - Log[1 - Sinh[x]^2])/2
```

Defintions of rubi rules used

rule 14
```
Int[(a_.)/(x_), x_Symbol] :> Simp[a*Log[x], x] /; FreeQ[a, x]
```

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 47
```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] :> Simp[b/(b*c
 - a*d)   Int[1/(a + b*x), x], x] - Simp[d/(b*c - a*d)   Int[1/(c + d*x), x
], x] /; FreeQ[{a, b, c, d}, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3673
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_.)*tan[(e_.) + (f_.)*(x_)]^
(m_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x]^2, x]}, Simp[ff^((m
 + 1)/2)/(2*f)   Subst[Int[x^((m - 1)/2)*((a + b*ff*x)^p/(1 - ff*x)^((m + 1
)/2)), x], x, Sin[e + f*x]^2/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && Integ
erQ[(m - 1)/2]
```

Maple [A] (verified)

Time = 0.49 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.41

method	result	size
risch	$\ln\left(e^{2x}-1\right)-\frac{\ln\left(e^{4x}-6\,e^{2x}+1\right)}{2}$	24
default	$-\frac{\ln\left(\tanh\left(\frac{x}{2}\right)^{2}-2\tanh\left(\frac{x}{2}\right)-1\right)}{2}-\frac{\ln\left(\tanh\left(\frac{x}{2}\right)^{2}+2\tanh\left(\frac{x}{2}\right)-1\right)}{2}+\ln\left(\tanh\left(\frac{x}{2}\right)\right)$	41

input
```
int(coth(x)/(1-sinh(x)^2),x,method=_RETURNVERBOSE)
```

output
```
ln(exp(2*x)-1)-1/2*ln(exp(4*x)-6*exp(2*x)+1)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 47 vs. $2(15) = 30$.

Time = 0.09 (sec) , antiderivative size = 47, normalized size of antiderivative = 2.76

$$\int \frac{\coth(x)}{1-\sinh^2(x)}\,dx = -\frac{1}{2}\log\left(\frac{2\left(\cosh(x)^2+\sinh(x)^2-3\right)}{\cosh(x)^2-2\cosh(x)\sinh(x)+\sinh(x)^2}\right) + \log\left(\frac{2\sinh(x)}{\cosh(x)-\sinh(x)}\right)$$

input
```
integrate(coth(x)/(1-sinh(x)^2),x, algorithm="fricas")
```

output
```
-1/2*log(2*(cosh(x)^2 + sinh(x)^2 - 3)/(cosh(x)^2 - 2*cosh(x)*sinh(x) + si
nh(x)^2)) + log(2*sinh(x)/(cosh(x) - sinh(x)))
```

Sympy [F]

$$\int \frac{\coth(x)}{1-\sinh^2(x)}\,dx = -\int \frac{\coth(x)}{\sinh^2(x)-1}\,dx$$

input
```
integrate(coth(x)/(1-sinh(x)**2),x)
```

output
```
-Integral(coth(x)/(sinh(x)**2 - 1), x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 45 vs. $2(15) = 30$.

Time = 0.04 (sec) , antiderivative size = 45, normalized size of antiderivative = 2.65

$$\int \frac{\coth(x)}{1-\sinh^2(x)}\,dx = -\frac{1}{2}\log\left(2\,e^{(-x)}+e^{(-2\,x)}-1\right)+\log\left(e^{(-x)}+1\right)+\log\left(e^{(-x)}-1\right)-\frac{1}{2}\log\left(-2\,e^{(-x)}+e^{(-2\,x)}-1\right)$$

input
```
integrate(coth(x)/(1-sinh(x)^2),x, algorithm="maxima")
```

output
```
-1/2*log(2*e^(-x) + e^(-2*x) - 1) + log(e^(-x) + 1) + log(e^(-x) - 1) - 1/
2*log(-2*e^(-x) + e^(-2*x) - 1)
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.47

$$\int \frac{\coth(x)}{1-\sinh^2(x)}\,dx = -\frac{1}{2}\log\left(\left|e^{(4\,x)}-6\,e^{(2\,x)}+1\right|\right)+\log\left(\left|e^{(2\,x)}-1\right|\right)$$

input
```
integrate(coth(x)/(1-sinh(x)^2),x, algorithm="giac")
```

output
```
-1/2*log(abs(e^(4*x) - 6*e^(2*x) + 1)) + log(abs(e^(2*x) - 1))
```

Mupad [B] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.59

$$\int \frac{\coth(x)}{1-\sinh^2(x)}\,dx = \ln\left(5184\,\mathrm{e}^{2\,x}-5184\right)-\frac{\ln\left(9\,\mathrm{e}^{4\,x}-54\,\mathrm{e}^{2\,x}+9\right)}{2}$$

input
```
int(-coth(x)/(sinh(x)^2 - 1),x)
```

output
```
log(5184*exp(2*x) - 5184) - log(9*exp(4*x) - 54*exp(2*x) + 9)/2
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 57, normalized size of antiderivative = 3.35

$$\int \frac{\coth(x)}{1-\sinh^2(x)}\,dx = -\frac{\log\left(e^x-\sqrt{2}-1\right)}{2} - \frac{\log\left(e^x-\sqrt{2}+1\right)}{2} + \log(e^x-1) - \frac{\log\left(e^x+\sqrt{2}-1\right)}{2} - \frac{\log\left(e^x+\sqrt{2}+1\right)}{2} + \log(e^x+1)$$

input
```
int(coth(x)/(1-sinh(x)^2),x)
```

output
```
( - log(e**x - sqrt(2) - 1) - log(e**x - sqrt(2) + 1) + 2*log(e**x - 1) -
log(e**x + sqrt(2) - 1) - log(e**x + sqrt(2) + 1) + 2*log(e**x + 1))/2
```

3.390 $\int \sqrt{a+a\sinh^2(e+fx)}\tanh^5(e+fx)\,dx$

Optimal result

Integrand size = 25, antiderivative size = 63

$$\int \sqrt{a+a\sinh^2(e+fx)}\tanh^5(e+fx)\,dx = -\frac{a^2}{3f\left(a\cosh^2(e+fx)\right)^{3/2}}+\frac{2a}{f\sqrt{a\cosh^2(e+fx)}}+\frac{\sqrt{a\cosh^2(e+fx)}}{f}$$

output
```
-1/3*a^2/f/(a*cosh(f*x+e)^2)^(3/2)+2*a/f/(a*cosh(f*x+e)^2)^(1/2)+(a*cosh(f
*x+e)^2)^(1/2)/f
```

Mathematica [A] (verified)

Time = 0.20 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.70

$$\int \sqrt{a+a\sinh^2(e+fx)}\tanh^5(e+fx)\,dx = \frac{a(6+3\cosh^2(e+fx)-\operatorname{sech}^2(e+fx))}{3f\sqrt{a\cosh^2(e+fx)}}$$

input
```
Integrate[Sqrt[a + a*Sinh[e + f*x]^2]*Tanh[e + f*x]^5,x]
```

output
```
(a*(6 + 3*Cosh[e + f*x]^2 - Sech[e + f*x]^2))/(3*f*Sqrt[a*Cosh[e + f*x]^2]
)
```

Rubi [A] (verified)

Time = 0.40 (sec) , antiderivative size = 70, normalized size of antiderivative = 1.11, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.400, Rules used = {3042, 26, 3655, 26, 3042, 26, 3684, 8, 53, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \tanh^5(e+fx)\sqrt{a\sinh^2(e+fx)+a}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int -i\tan(ie+ifx)^5\sqrt{a-a\sin(ie+ifx)^2}dx$$

$$\Big\downarrow \text{26}$$

$$-i\int \sqrt{a-a\sin(ie+ifx)^2}\tan(ie+ifx)^5dx$$

$$\Big\downarrow \text{3655}$$

$$-i\int i\sqrt{a\cosh^2(e+fx)}\tanh^5(e+fx)dx$$

$$\Big\downarrow \text{26}$$

$$\int \tanh^5(e+fx)\sqrt{a\cosh^2(e+fx)}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{i\sqrt{a\sin\left(ie+ifx+\frac{\pi}{2}\right)^2}}{\tan\left(ie+ifx+\frac{\pi}{2}\right)^5}dx$$

$$\Big\downarrow \text{26}$$

$$i\int \frac{\sqrt{a\sin\left(\frac{1}{2}(2ie+\pi)+ifx\right)^2}}{\tan\left(\frac{1}{2}(2ie+\pi)+ifx\right)^5}dx$$

$$\Big\downarrow \text{3684}$$

$$\frac{\int \sqrt{a\cosh^2(e+fx)}\left(1-\cosh^2(e+fx)\right)^2\operatorname{sech}^6(e+fx)d\cosh^2(e+fx)}{2f}$$

$$\Big\downarrow \text{8}$$

$$\frac{a^3\int \frac{\left(1-\cosh^2(e+fx)\right)^2}{\left(a\cosh^2(e+fx)\right)^{5/2}}d\cosh^2(e+fx)}{2f}$$

$$\Big\downarrow \text{53}$$

$$\frac{a^3\int \left(\frac{1}{a^2\sqrt{a\cosh^2(e+fx)}}-\frac{2}{a\left(a\cosh^2(e+fx)\right)^{3/2}}+\frac{1}{\left(a\cosh^2(e+fx)\right)^{5/2}}\right)d\cosh^2(e+fx)}{2f}$$

$$\Big\downarrow \text{2009}$$

$$\frac{a^3\left(\frac{2\sqrt{a\cosh^2(e+fx)}}{a^3}+\frac{4}{a^2\sqrt{a\cosh^2(e+fx)}}-\frac{2}{3a\left(a\cosh^2(e+fx)\right)^{3/2}}\right)}{2f}$$

input
```
Int[Sqrt[a + a*Sinh[e + f*x]^2]*Tanh[e + f*x]^5,x]
```

output
```
(a^3*(-2/(3*a*(a*Cosh[e + f*x]^2)^(3/2)) + 4/(a^2*Sqrt[a*Cosh[e + f*x]^2])
 + (2*Sqrt[a*Cosh[e + f*x]^2])/a^3))/(2*f)
```

Defintions of rubi rules used

rule 8
```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_))^(p_), x_Symbol] :> Simp[1/a^m   Int[u*(a*
x)^(m + p), x], x] /; FreeQ[{a, m, p}, x] && IntegerQ[m]
```

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 53
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && LeQ[7*m + 4*n + 4, 0])
|| LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3655
```
Int[(u_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> Int[A
ctivateTrig[u*(a*cos[e + f*x]^2)^p], x] /; FreeQ[{a, b, e, f, p}, x] && EqQ
[a + b, 0]
```

rule 3684
```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*tan[(e_.) + (f_.)*(x_)]^(m_.
), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x]^2, x]}, Simp[ff^((m + 1
)/2)/(2*f)   Subst[Int[x^((m - 1)/2)*((b*ff^(n/2)*x^(n/2))^p/(1 - ff*x)^((m
 + 1)/2)), x], x, Sin[e + f*x]^2/ff], x]] /; FreeQ[{b, e, f, p}, x] && Inte
gerQ[(m - 1)/2] && IntegerQ[n/2]
```

Maple [A] (verified)

Time = 0.56 (sec) , antiderivative size = 49, normalized size of antiderivative = 0.78

method	result	size
default	$\frac{a\left(3\cosh(fx+e)^{4}+6\cosh(fx+e)^{2}-1\right)}{3\cosh(fx+e)^{2}\sqrt{a\cosh(fx+e)^{2}}\,f}$	49
risch	$\frac{\sqrt{\left(\mathrm{e}^{2fx+2e}+1\right)^{2}a\,\mathrm{e}^{-2fx-2e}}\,\left(3\,\mathrm{e}^{8fx+8e}+36\,\mathrm{e}^{6fx+6e}+50\,\mathrm{e}^{4fx+4e}+36\,\mathrm{e}^{2fx+2e}+3\right)}{6f\left(\mathrm{e}^{2fx+2e}+1\right)^{4}}$	91

input
```
int((a+a*sinh(f*x+e)^2)^(1/2)*tanh(f*x+e)^5,x,method=_RETURNVERBOSE)
```

output
```
1/3/cosh(f*x+e)^2*a*(3*cosh(f*x+e)^4+6*cosh(f*x+e)^2-1)/(a*cosh(f*x+e)^2)^
(1/2)/f
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 875 vs. $2(55) = 110$.

Time = 0.10 (sec) , antiderivative size = 875, normalized size of antiderivative = 13.89

$$\int \sqrt{a + a\sinh^2(e + fx)} \tanh^5(e + fx)\, dx = \text{Too large to display}$$

input
```
integrate((a+a*sinh(f*x+e)^2)^(1/2)*tanh(f*x+e)^5,x, algorithm="fricas")
```

output
```
1/6*(24*cosh(f*x + e)*e^(f*x + e)*sinh(f*x + e)^7 + 3*e^(f*x + e)*sinh(f*x
 + e)^8 + 12*(7*cosh(f*x + e)^2 + 3)*e^(f*x + e)*sinh(f*x + e)^6 + 24*(7*c
osh(f*x + e)^3 + 9*cosh(f*x + e))*e^(f*x + e)*sinh(f*x + e)^5 + 10*(21*cos
h(f*x + e)^4 + 54*cosh(f*x + e)^2 + 5)*e^(f*x + e)*sinh(f*x + e)^4 + 8*(21
*cosh(f*x + e)^5 + 90*cosh(f*x + e)^3 + 25*cosh(f*x + e))*e^(f*x + e)*sinh
(f*x + e)^3 + 12*(7*cosh(f*x + e)^6 + 45*cosh(f*x + e)^4 + 25*cosh(f*x + e
)^2 + 3)*e^(f*x + e)*sinh(f*x + e)^2 + 8*(3*cosh(f*x + e)^7 + 27*cosh(f*x
+ e)^5 + 25*cosh(f*x + e)^3 + 9*cosh(f*x + e))*e^(f*x + e)*sinh(f*x + e) +
 (3*cosh(f*x + e)^8 + 36*cosh(f*x + e)^6 + 50*cosh(f*x + e)^4 + 36*cosh(f*
x + e)^2 + 3)*e^(f*x + e))*sqrt(a*e^(4*f*x + 4*e) + 2*a*e^(2*f*x + 2*e) +
a)*e^(-f*x - e)/(f*cosh(f*x + e)^7 + (f*e^(2*f*x + 2*e) + f)*sinh(f*x + e)
^7 + 7*(f*cosh(f*x + e)*e^(2*f*x + 2*e) + f*cosh(f*x + e))*sinh(f*x + e)^6
 + 3*f*cosh(f*x + e)^5 + 3*(7*f*cosh(f*x + e)^2 + (7*f*cosh(f*x + e)^2 + f
)*e^(2*f*x + 2*e) + f)*sinh(f*x + e)^5 + 5*(7*f*cosh(f*x + e)^3 + 3*f*cosh
(f*x + e) + (7*f*cosh(f*x + e)^3 + 3*f*cosh(f*x + e))*e^(2*f*x + 2*e))*sin
h(f*x + e)^4 + 3*f*cosh(f*x + e)^3 + (35*f*cosh(f*x + e)^4 + 30*f*cosh(f*x
 + e)^2 + (35*f*cosh(f*x + e)^4 + 30*f*cosh(f*x + e)^2 + 3*f)*e^(2*f*x + 2
*e) + 3*f)*sinh(f*x + e)^3 + 3*(7*f*cosh(f*x + e)^5 + 10*f*cosh(f*x + e)^3
 + 3*f*cosh(f*x + e) + (7*f*cosh(f*x + e)^5 + 10*f*cosh(f*x + e)^3 + 3*f*c
osh(f*x + e))*e^(2*f*x + 2*e))*sinh(f*x + e)^2 + f*cosh(f*x + e) + (f*c...
```

Sympy [F]

$$\int \sqrt{a+a\sinh^2(e+fx)}\tanh^5(e+fx)\,dx = \int \sqrt{a\left(\sinh^2\left(e+fx\right)+1\right)}\tanh^5\left(e+fx\right)dx$$

input
```
integrate((a+a*sinh(f*x+e)**2)**(1/2)*tanh(f*x+e)**5,x)
```

output
```
Integral(sqrt(a*(sinh(e + f*x)**2 + 1))*tanh(e + f*x)**5, x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 292 vs. $2(55) = 110$.

Time = 0.21 (sec) , antiderivative size = 292, normalized size of antiderivative = 4.63

$$\begin{aligned}\int \sqrt{a+a\sinh^2(e+fx)}\tanh^5(e+fx)\,dx &= \frac{6\sqrt{a}e^{(-2fx-2e)}}{f\left(e^{(-fx-e)}+3e^{(-3fx-3e)}+3e^{(-5fx-5e)}+e^{(-7fx-7e)}\right)} \\ &+ \frac{25\sqrt{a}e^{(-4fx-4e)}}{3f\left(e^{(-fx-e)}+3e^{(-3fx-3e)}+3e^{(-5fx-5e)}+e^{(-7fx-7e)}\right)} \\ &+ \frac{6\sqrt{a}e^{(-6fx-6e)}}{f\left(e^{(-fx-e)}+3e^{(-3fx-3e)}+3e^{(-5fx-5e)}+e^{(-7fx-7e)}\right)} \\ &+ \frac{\sqrt{a}e^{(-8fx-8e)}}{2f\left(e^{(-fx-e)}+3e^{(-3fx-3e)}+3e^{(-5fx-5e)}+e^{(-7fx-7e)}\right)} \\ &+ \frac{\sqrt{a}}{2f\left(e^{(-fx-e)}+3e^{(-3fx-3e)}+3e^{(-5fx-5e)}+e^{(-7fx-7e)}\right)}\end{aligned}$$

input
```
integrate((a+a*sinh(f*x+e)^2)^(1/2)*tanh(f*x+e)^5,x, algorithm="maxima")
```

output
```
6*sqrt(a)*e^(-2*f*x - 2*e)/(f*(e^(-f*x - e) + 3*e^(-3*f*x - 3*e) + 3*e^(-5
*f*x - 5*e) + e^(-7*f*x - 7*e))) + 25/3*sqrt(a)*e^(-4*f*x - 4*e)/(f*(e^(-f
*x - e) + 3*e^(-3*f*x - 3*e) + 3*e^(-5*f*x - 5*e) + e^(-7*f*x - 7*e))) + 6
*sqrt(a)*e^(-6*f*x - 6*e)/(f*(e^(-f*x - e) + 3*e^(-3*f*x - 3*e) + 3*e^(-5*
f*x - 5*e) + e^(-7*f*x - 7*e))) + 1/2*sqrt(a)*e^(-8*f*x - 8*e)/(f*(e^(-f*x
 - e) + 3*e^(-3*f*x - 3*e) + 3*e^(-5*f*x - 5*e) + e^(-7*f*x - 7*e))) + 1/2
*sqrt(a)/(f*(e^(-f*x - e) + 3*e^(-3*f*x - 3*e) + 3*e^(-5*f*x - 5*e) + e^(-
7*f*x - 7*e)))
```

Giac [A] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 70, normalized size of antiderivative = 1.11

$$\int \sqrt{a + a\sinh^2(e + fx)}\tanh^5(e + fx)\,dx = \frac{\sqrt{a}\left(\frac{8\left(3\left(e^{(fx+e)} + e^{(-fx-e)}\right)^2 - 2\right)}{\left(e^{(fx+e)} + e^{(-fx-e)}\right)^3} + 3\,e^{(fx+e)} + 3\,e^{(-fx-e)}\right)}{6\,f}$$

input
```
integrate((a+a*sinh(f*x+e)^2)^(1/2)*tanh(f*x+e)^5,x, algorithm="giac")
```

output
```
1/6*sqrt(a)*(8*(3*(e^(f*x + e) + e^(-f*x - e))^2 - 2)/(e^(f*x + e) + e^(-f
*x - e))^3 + 3*e^(f*x + e) + 3*e^(-f*x - e))/f
```

Mupad [B] (verification not implemented)

Time = 1.92 (sec) , antiderivative size = 252, normalized size of antiderivative = 4.00

$$\int \sqrt{a+a\sinh^2(e+fx)}\tanh^5(e+fx)\,dx = \frac{\sqrt{a+a\left(\frac{e^{e+fx}}{2}-\frac{e^{-e-fx}}{2}\right)^2}}{f} + \frac{8\,e^{3e+3fx}\sqrt{a+a\left(\frac{e^{e+fx}}{2}-\frac{e^{-e-fx}}{2}\right)^2}}{f\,\left(e^{2e+2fx}+1\right)\left(e^{e+fx}+e^{3e+3fx}\right)} - \frac{16\,e^{3e+3fx}\sqrt{a+a\left(\frac{e^{e+fx}}{2}-\frac{e^{-e-fx}}{2}\right)^2}}{3f\,\left(e^{2e+2fx}+1\right)^2\left(e^{e+fx}+e^{3e+3fx}\right)} + \frac{16\,e^{3e+3fx}\sqrt{a+a\left(\frac{e^{e+fx}}{2}-\frac{e^{-e-fx}}{2}\right)^2}}{3f\,\left(e^{2e+2fx}+1\right)^3\left(e^{e+fx}+e^{3e+3fx}\right)}$$

input
```
int(tanh(e + f*x)^5*(a + a*sinh(e + f*x)^2)^(1/2),x)
```

output
```
(a + a*(exp(e + f*x)/2 - exp(- e - f*x)/2)^2)^(1/2)/f + (8*exp(3*e + 3*f*x
)*(a + a*(exp(e + f*x)/2 - exp(- e - f*x)/2)^2)^(1/2))/(f*(exp(2*e + 2*f*x
) + 1)*(exp(e + f*x) + exp(3*e + 3*f*x))) - (16*exp(3*e + 3*f*x)*(a + a*(e
xp(e + f*x)/2 - exp(- e - f*x)/2)^2)^(1/2))/(3*f*(exp(2*e + 2*f*x) + 1)^2*
(exp(e + f*x) + exp(3*e + 3*f*x))) + (16*exp(3*e + 3*f*x)*(a + a*(exp(e +
f*x)/2 - exp(- e - f*x)/2)^2)^(1/2))/(3*f*(exp(2*e + 2*f*x) + 1)^3*(exp(e
+ f*x) + exp(3*e + 3*f*x)))
```

Reduce [F]

$$\int \sqrt{a+a\sinh^2(e+fx)}\tanh^5(e+fx)\,dx = \sqrt{a}\left(\int \sqrt{\sinh(fx+e)^2+1}\,\tanh(fx+e)^5\,dx\right)$$

input
```
int((a+a*sinh(f*x+e)^2)^(1/2)*tanh(f*x+e)^5,x)
```

output
```
sqrt(a)*int(sqrt(sinh(e + f*x)**2 + 1)*tanh(e + f*x)**5,x)
```

3.391 $\int \sqrt{a+a\sinh^2(e+fx)}\tanh^3(e+fx)\,dx$

Optimal result

Integrand size = 25, antiderivative size = 38

$$\int \sqrt{a+a\sinh^2(e+fx)}\tanh^3(e+fx)\,dx=\frac{a}{f\sqrt{a\cosh^2(e+fx)}}+\frac{\sqrt{a\cosh^2(e+fx)}}{f}$$

output
```
a/f/(a*cosh(f*x+e)^2)^(1/2)+(a*cosh(f*x+e)^2)^(1/2)/f
```

Mathematica [A] (verified)

Time = 0.13 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.76

$$\int \sqrt{a+a\sinh^2(e+fx)}\tanh^3(e+fx)\,dx=\frac{a(1+\cosh^2(e+fx))}{f\sqrt{a\cosh^2(e+fx)}}$$

input
```
Integrate[Sqrt[a + a*Sinh[e + f*x]^2]*Tanh[e + f*x]^3,x]
```

output
```
(a*(1 + Cosh[e + f*x]^2))/(f*Sqrt[a*Cosh[e + f*x]^2])
```

Rubi [A] (verified)

Time = 0.37 (sec) , antiderivative size = 49, normalized size of antiderivative = 1.29, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.400, Rules used = {3042, 26, 3655, 26, 3042, 26, 3684, 8, 53, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \tanh^3(e+fx)\sqrt{a\sinh^2(e+fx)+a}dx$$

$$\Big\downarrow \text{3042}$$

$$\int i\tan(ie+ifx)^3\sqrt{a-a\sin(ie+ifx)^2}dx$$

$$\Big\downarrow \text{26}$$

$$i\int \sqrt{a-a\sin(ie+ifx)^2}\tan(ie+ifx)^3dx$$

$$\Big\downarrow \text{3655}$$

$$i\int -i\sqrt{a\cosh^2(e+fx)}\tanh^3(e+fx)dx$$

$$\Big\downarrow \text{26}$$

$$\int \tanh^3(e+fx)\sqrt{a\cosh^2(e+fx)}dx$$

$$\Big\downarrow \text{3042}$$

$$\int -\frac{i\sqrt{a\sin\left(ie+ifx+\frac{\pi}{2}\right)^2}}{\tan\left(ie+ifx+\frac{\pi}{2}\right)^3}dx$$

$$\Big\downarrow \text{26}$$

$$-i\int \frac{\sqrt{a\sin\left(\frac{1}{2}(2ie+\pi)+ifx\right)^2}}{\tan\left(\frac{1}{2}(2ie+\pi)+ifx\right)^3}dx$$

$$\Big\downarrow \text{3684}$$

$$-\frac{\int \sqrt{a\cosh^2(e+fx)}\left(1-\cosh^2(e+fx)\right)\text{sech}^4(e+fx)d\cosh^2(e+fx)}{2f}$$

$$\Big\downarrow \text{8}$$

$$-\frac{a^2\int \frac{1-\cosh^2(e+fx)}{\left(a\cosh^2(e+fx)\right)^{3/2}}d\cosh^2(e+fx)}{2f}$$

$$\Big\downarrow \; 53$$

$$-\frac{a^2\int \left(\frac{1}{\left(a\cosh^2(e+fx)\right)^{3/2}}-\frac{1}{a\sqrt{a\cosh^2(e+fx)}}\right)d\cosh^2(e+fx)}{2f}$$

$$\Big\downarrow \; 2009$$

$$-\frac{a^2\left(-\frac{2\sqrt{a\cosh^2(e+fx)}}{a^2}-\frac{2}{a\sqrt{a\cosh^2(e+fx)}}\right)}{2f}$$

input
```
Int[Sqrt[a + a*Sinh[e + f*x]^2]*Tanh[e + f*x]^3,x]
```

output
```
-1/2*(a^2*(-2/(a*Sqrt[a*Cosh[e + f*x]^2]) - (2*Sqrt[a*Cosh[e + f*x]^2])/a^
2))/f
```

Defintions of rubi rules used

rule 8
```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_))^(p_), x_Symbol] :> Simp[1/a^m   Int[u*(a*
x)^(m + p), x], x] /; FreeQ[{a, m, p}, x] && IntegerQ[m]
```

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 53
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && LeQ[7*m + 4*n + 4, 0])
|| LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3655
```
Int[(u_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> Int[A
ctivateTrig[u*(a*cos[e + f*x]^2)^p], x] /; FreeQ[{a, b, e, f, p}, x] && EqQ
[a + b, 0]
```

rule 3684
```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*tan[(e_.) + (f_.)*(x_)]^(m_.
), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x]^2, x]}, Simp[ff^((m + 1
)/2)/(2*f)   Subst[Int[x^((m - 1)/2)*((b*ff^(n/2)*x^(n/2))^p/(1 - ff*x)^((m
 + 1)/2)), x], x, Sin[e + f*x]^2/ff], x]] /; FreeQ[{b, e, f, p}, x] && Inte
gerQ[(m - 1)/2] && IntegerQ[n/2]
```

Maple [A] (verified)

Time = 0.47 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.74

method	result	size
default	$\frac{a\left(\cosh\left(fx+e\right)^{2}+1\right)}{\sqrt{a\cosh\left(fx+e\right)^{2}}\,f}$	28
risch	$\frac{\sqrt{\left(\mathrm{e}^{2fx+2e}+1\right)^{2}a\,\mathrm{e}^{-2fx-2e}}\,\left(\mathrm{e}^{4fx+4e}+6\,\mathrm{e}^{2fx+2e}+1\right)}{2f\left(\mathrm{e}^{2fx+2e}+1\right)^{2}}$	67

input
```
int((a+a*sinh(f*x+e)^2)^(1/2)*tanh(f*x+e)^3,x,method=_RETURNVERBOSE)
```

output
```
a*(cosh(f*x+e)^2+1)/(a*cosh(f*x+e)^2)^(1/2)/f
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 311 vs. 2(34) = 68.

Time = 0.09 (sec) , antiderivative size = 311, normalized size of antiderivative = 8.18

$$\int \sqrt{a+a\sinh^2(e+fx)}\tanh^3(e+fx)\,dx$$
$$= \frac{\left(4\cosh\left(fx+e\right)e^{(fx+e)}\sinh\left(fx+e\right)^3+e^{(fx+e)}\sinh\left(fx+e\right)^4+6\left(\cosh\left(fx+e\right)^2+1\right)e^{(fx+e)}\sinh\left(f\right.\right.}{2\left(f\cosh\left(fx+e\right)^3+\left(fe^{(2fx+2e)}+f\right)\sinh\left(fx+e\right)^3+3\left(f\cosh\left(fx+e\right)e^{(2fx+2e)}+f\cosh\left(fx+e\right)\right)s\right.}$$

input
```
integrate((a+a*sinh(f*x+e)^2)^(1/2)*tanh(f*x+e)^3,x, algorithm="fricas")
```

output
```
1/2*(4*cosh(f*x + e)*e^(f*x + e)*sinh(f*x + e)^3 + e^(f*x + e)*sinh(f*x +
e)^4 + 6*(cosh(f*x + e)^2 + 1)*e^(f*x + e)*sinh(f*x + e)^2 + 4*(cosh(f*x +
 e)^3 + 3*cosh(f*x + e))*e^(f*x + e)*sinh(f*x + e) + (cosh(f*x + e)^4 + 6*
cosh(f*x + e)^2 + 1)*e^(f*x + e))*sqrt(a*e^(4*f*x + 4*e) + 2*a*e^(2*f*x +
2*e) + a)*e^(-f*x - e)/(f*cosh(f*x + e)^3 + (f*e^(2*f*x + 2*e) + f)*sinh(f
*x + e)^3 + 3*(f*cosh(f*x + e)*e^(2*f*x + 2*e) + f*cosh(f*x + e))*sinh(f*x
 + e)^2 + f*cosh(f*x + e) + (f*cosh(f*x + e)^3 + f*cosh(f*x + e))*e^(2*f*x
 + 2*e) + (3*f*cosh(f*x + e)^2 + (3*f*cosh(f*x + e)^2 + f)*e^(2*f*x + 2*e)
 + f)*sinh(f*x + e))
```

Sympy [F]

$$\int \sqrt{a+a\sinh^2(e+fx)}\tanh^3(e+fx)\,dx$$
$$= \int \sqrt{a\left(\sinh^2\left(e+fx\right)+1\right)}\tanh^3\left(e+fx\right)\,dx$$

input
```
integrate((a+a*sinh(f*x+e)**2)**(1/2)*tanh(f*x+e)**3,x)
```

output
```
Integral(sqrt(a*(sinh(e + f*x)**2 + 1))*tanh(e + f*x)**3, x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 106 vs. $2(34) = 68$.

Time = 0.17 (sec) , antiderivative size = 106, normalized size of antiderivative = 2.79

$$\int \sqrt{a + a\sinh^2(e + fx)}\tanh^3(e + fx)\,dx = \frac{3\sqrt{a}e^{(-2\,fx-2\,e)}}{f(e^{(-fx-e)} + e^{(-3\,fx-3\,e)})} + \frac{\sqrt{a}e^{(-4\,fx-4\,e)}}{2\,f(e^{(-fx-e)} + e^{(-3\,fx-3\,e)})} + \frac{\sqrt{a}}{2\,f(e^{(-fx-e)} + e^{(-3\,fx-3\,e)})}$$

input
```
integrate((a+a*sinh(f*x+e)^2)^(1/2)*tanh(f*x+e)^3,x, algorithm="maxima")
```

output
```
3*sqrt(a)*e^(-2*f*x - 2*e)/(f*(e^(-f*x - e) + e^(-3*f*x - 3*e))) + 1/2*sqr
t(a)*e^(-4*f*x - 4*e)/(f*(e^(-f*x - e) + e^(-3*f*x - 3*e))) + 1/2*sqrt(a)/
(f*(e^(-f*x - e) + e^(-3*f*x - 3*e)))
```

Giac [A] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 44, normalized size of antiderivative = 1.16

$$\int \sqrt{a + a\sinh^2(e + fx)}\tanh^3(e + fx)\,dx = \frac{\sqrt{a}\left(\frac{4}{e^{(fx+e)}+e^{(-fx-e)}} + e^{(fx+e)} + e^{(-fx-e)}\right)}{2\,f}$$

input
```
integrate((a+a*sinh(f*x+e)^2)^(1/2)*tanh(f*x+e)^3,x, algorithm="giac")
```

output
```
1/2*sqrt(a)*(4/(e^(f*x + e) + e^(-f*x - e)) + e^(f*x + e) + e^(-f*x - e))/
f
```

Mupad [B] (verification not implemented)

Time = 1.89 (sec) , antiderivative size = 67, normalized size of antiderivative = 1.76

$$\int \sqrt{a+a\sinh^2(e+fx)}\tanh^3(e+fx)\,dx$$
$$=\frac{\sqrt{a+a\left(\frac{e^{e+f\,x}}{2}-\frac{e^{-e-f\,x}}{2}\right)^2}\,\left(6\,e^{2\,e+2\,f\,x}+e^{4\,e+4\,f\,x}+1\right)}{f\,\left(e^{2\,e+2\,f\,x}+1\right)^2}$$

input
```
int(tanh(e + f*x)^3*(a + a*sinh(e + f*x)^2)^(1/2),x)
```

output
```
((a + a*(exp(e + f*x)/2 - exp(- e - f*x)/2)^2)^(1/2)*(6*exp(2*e + 2*f*x) +
 exp(4*e + 4*f*x) + 1))/(f*(exp(2*e + 2*f*x) + 1)^2)
```

Reduce [F]

$$\int \sqrt{a+a\sinh^2(e+fx)}\tanh^3(e+fx)\,dx=\sqrt{a}\left(\int \sqrt{\sinh\left(fx+e\right)^2+1}\,\tanh\left(fx+e\right)^3dx\right)$$

input
```
int((a+a*sinh(f*x+e)^2)^(1/2)*tanh(f*x+e)^3,x)
```

output
```
sqrt(a)*int(sqrt(sinh(e + f*x)**2 + 1)*tanh(e + f*x)**3,x)
```

3.392 $\int \sqrt{a+a\sinh^2(e+fx)}\tanh(e+fx)\,dx$

Optimal result

Integrand size = 23, antiderivative size = 18

$$\int \sqrt{a+a\sinh^2(e+fx)}\tanh(e+fx)\,dx=\frac{\sqrt{a\cosh^2(e+fx)}}{f}$$

output
```
(a*cosh(f*x+e)^2)^(1/2)/f
```

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00

$$\int \sqrt{a+a\sinh^2(e+fx)}\tanh(e+fx)\,dx=\frac{\sqrt{a\cosh^2(e+fx)}}{f}$$

input
```
Integrate[Sqrt[a + a*Sinh[e + f*x]^2]*Tanh[e + f*x],x]
```

output
```
Sqrt[a*Cosh[e + f*x]^2]/f
```

Rubi [A] (verified)

Time = 0.32 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.391, Rules used = {3042, 26, 3655, 26, 3042, 26, 3684, 8, 17}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \tanh(e+fx)\sqrt{a\sinh^2(e+fx)+a}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int -i\tan(ie+ifx)\sqrt{a-a\sin(ie+ifx)^2}dx$$

$$\Big\downarrow \text{26}$$

$$-i\int \sqrt{a-a\sin(ie+ifx)^2}\tan(ie+ifx)dx$$

$$\Big\downarrow \text{3655}$$

$$-i\int i\sqrt{a\cosh^2(e+fx)}\tanh(e+fx)dx$$

$$\Big\downarrow \text{26}$$

$$\int \tanh(e+fx)\sqrt{a\cosh^2(e+fx)}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{i\sqrt{a\sin\left(ie+ifx+\frac{\pi}{2}\right)^2}}{\tan\left(ie+ifx+\frac{\pi}{2}\right)}dx$$

$$\Big\downarrow \text{26}$$

$$i\int \frac{\sqrt{a\sin\left(\frac{1}{2}(2ie+\pi)+ifx\right)^2}}{\tan\left(\frac{1}{2}(2ie+\pi)+ifx\right)}dx$$

$$\Big\downarrow \text{3684}$$

$$\frac{\int \sqrt{a\cosh^2(e+fx)}\text{sech}^2(e+fx)d\cosh^2(e+fx)}{2f}$$

$$\Big\downarrow \text{8}$$

$$\frac{a\int \frac{1}{\sqrt{a\cosh^2(e+fx)}}d\cosh^2(e+fx)}{2f}$$

$$\Big\downarrow \text{17}$$

$$\frac{\sqrt{a\cosh^2(e+fx)}}{f}$$

input
```
Int[Sqrt[a + a*Sinh[e + f*x]^2]*Tanh[e + f*x],x]
```

output
```
Sqrt[a*Cosh[e + f*x]^2]/f
```

Defintions of rubi rules used

rule 8
```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_))^(p_), x_Symbol] :> Simp[1/a^m   Int[u*(a*
x)^(m + p), x], x] /; FreeQ[{a, m, p}, x] && IntegerQ[m]
```

rule 17
```
Int[(c_.)*((a_.) + (b_.)*(x_))^(m_.), x_Symbol] :> Simp[c*((a + b*x)^(m + 1
)/(b*(m + 1))), x] /; FreeQ[{a, b, c, m}, x] && NeQ[m, -1]
```

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3655
```
Int[(u_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> Int[A
ctivateTrig[u*(a*cos[e + f*x]^2)^p], x] /; FreeQ[{a, b, e, f, p}, x] && EqQ
[a + b, 0]
```

rule 3684
```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*tan[(e_.) + (f_.)*(x_)]^(m_.
), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x]^2, x]}, Simp[ff^((m + 1
)/2)/(2*f)   Subst[Int[x^((m - 1)/2)*((b*ff^(n/2)*x^(n/2))^p/(1 - ff*x)^((m
 + 1)/2)), x], x, Sin[e + f*x]^2/ff], x]] /; FreeQ[{b, e, f, p}, x] && Inte
gerQ[(m - 1)/2] && IntegerQ[n/2]
```

Maple [A] (verified)

Time = 0.34 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.06

method	result	size
derivativedivides	$\frac{\sqrt{a+a\sinh(fx+e)^2}}{f}$	19
default	$\frac{\sqrt{a+a\sinh(fx+e)^2}}{f}$	19
risch	$\frac{\sqrt{\left(e^{2fx+2e}+1\right)^2 a\, e^{-2fx-2e}}\, e^{2fx+2e}}{2f\left(e^{2fx+2e}+1\right)}+\frac{\sqrt{\left(e^{2fx+2e}+1\right)^2 a\, e^{-2fx-2e}}}{2f\left(e^{2fx+2e}+1\right)}$	99

input
```
int((a+a*sinh(f*x+e)^2)^(1/2)*tanh(f*x+e),x,method=_RETURNVERBOSE)
```

output
```
(a+a*sinh(f*x+e)^2)^(1/2)/f
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 139 vs. 2(16) = 32.

Time = 0.09 (sec) , antiderivative size = 139, normalized size of antiderivative = 7.72

$$\int \sqrt{a+a\sinh^2(e+fx)}\tanh(e+fx)\,dx$$
$$=\frac{\left(2\cosh(fx+e)\,e^{(fx+e)}\sinh(fx+e)+e^{(fx+e)}\sinh(fx+e)^2+\left(\cosh(fx+e)^2+1\right)e^{(fx+e)}\right)\sqrt{ae^{(4fx+4e}}}{2\left(f\cosh(fx+e)\,e^{(2fx+2e)}+f\cosh(fx+e)+\left(fe^{(2fx+2e)}+f\right)\sinh(fx+e\right.}$$

input
```
integrate((a+a*sinh(f*x+e)^2)^(1/2)*tanh(f*x+e),x, algorithm="fricas")
```

output
```
1/2*(2*cosh(f*x + e)*e^(f*x + e)*sinh(f*x + e) + e^(f*x + e)*sinh(f*x + e)
^2 + (cosh(f*x + e)^2 + 1)*e^(f*x + e))*sqrt(a*e^(4*f*x + 4*e) + 2*a*e^(2*
f*x + 2*e) + a)*e^(-f*x - e)/(f*cosh(f*x + e)*e^(2*f*x + 2*e) + f*cosh(f*x
 + e) + (f*e^(2*f*x + 2*e) + f)*sinh(f*x + e))
```

Sympy [F]

$$\int \sqrt{a+a\sinh^2(e+fx)}\tanh(e+fx)\,dx=\int \sqrt{a\left(\sinh^2\left(e+fx\right)+1\right)}\tanh\left(e+fx\right)dx$$

input
```
integrate((a+a*sinh(f*x+e)**2)**(1/2)*tanh(f*x+e),x)
```

output
```
Integral(sqrt(a*(sinh(e + f*x)**2 + 1))*tanh(e + f*x), x)
```

Maxima [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.78

$$\int \sqrt{a+a\sinh^2(e+fx)}\tanh(e+fx)\,dx=\frac{\sqrt{a}e^{(fx+e)}}{2\,f}+\frac{\sqrt{a}e^{(-fx-e)}}{2\,f}$$

input
```
integrate((a+a*sinh(f*x+e)^2)^(1/2)*tanh(f*x+e),x, algorithm="maxima")
```

output
```
1/2*sqrt(a)*e^(f*x + e)/f + 1/2*sqrt(a)*e^(-f*x - e)/f
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.33

$$\int \sqrt{a+a\sinh^2(e+fx)}\tanh(e+fx)\,dx = \frac{\sqrt{a}\left(e^{(fx+e)}+e^{(-fx-e)}\right)}{2f}$$

input
```
integrate((a+a*sinh(f*x+e)^2)^(1/2)*tanh(f*x+e),x, algorithm="giac")
```

output
```
1/2*sqrt(a)*(e^(f*x + e) + e^(-f*x - e))/f
```

Mupad [B] (verification not implemented)

Time = 1.81 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00

$$\int \sqrt{a+a\sinh^2(e+fx)}\tanh(e+fx)\,dx = \frac{\sqrt{a\sinh\left(e+f\,x\right)^2+a}}{f}$$

input
```
int(tanh(e + f*x)*(a + a*sinh(e + f*x)^2)^(1/2),x)
```

output
```
(a + a*sinh(e + f*x)^2)^(1/2)/f
```

Reduce [F]

$$\int \sqrt{a+a\sinh^2(e+fx)}\tanh(e+fx)\,dx$$
$$= \sqrt{a}\left(\int \sqrt{\sinh\left(fx+e\right)^2+1}\,\tanh\left(fx+e\right)dx\right)$$

input
```
int((a+a*sinh(f*x+e)^2)^(1/2)*tanh(f*x+e),x)
```

output
```
sqrt(a)*int(sqrt(sinh(e + f*x)**2 + 1)*tanh(e + f*x),x)
```

3.393 $\int \coth(e+fx)\sqrt{a+a\sinh^2(e+fx)}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 50

$$\int \coth(e+fx)\sqrt{a+a\sinh^2(e+fx)}\,dx=-\frac{\sqrt{a}\,\mathrm{arctanh}\left(\frac{\sqrt{a\cosh^2(e+fx)}}{\sqrt{a}}\right)}{f}+\frac{\sqrt{a\cosh^2(e+fx)}}{f}$$

output
```
-a^(1/2)*arctanh((a*cosh(f*x+e)^2)^(1/2)/a^(1/2))/f+(a*cosh(f*x+e)^2)^(1/2
)/f
```

Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 49, normalized size of antiderivative = 0.98

$$\int \coth(e+fx)\sqrt{a+a\sinh^2(e+fx)}\,dx=-\frac{\sqrt{a}\,\mathrm{arctanh}\left(\frac{\sqrt{a\cosh^2(e+fx)}}{\sqrt{a}}\right)-\sqrt{a\cosh^2(e+fx)}}{f}$$

input
```
Integrate[Coth[e + f*x]*Sqrt[a + a*Sinh[e + f*x]^2],x]
```

output
```
-((Sqrt[a]*ArcTanh[Sqrt[a*Cosh[e + f*x]^2/Sqrt[a]] - Sqrt[a*Cosh[e + f*x]
^2])/f)
```

Rubi [A] (verified)

Time = 0.33 (sec) , antiderivative size = 52, normalized size of antiderivative = 1.04, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.435$, Rules used = {3042, 26, 3655, 26, 3042, 26, 3684, 60, 73, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \coth(e+fx)\sqrt{a\sinh^2(e+fx)+a}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{i\sqrt{a-a\sin(ie+ifx)^2}}{\tan(ie+ifx)}dx$$

$$\Big\downarrow \text{26}$$

$$i\int \frac{\sqrt{a-a\sin(ie+ifx)^2}}{\tan(ie+ifx)}dx$$

$$\Big\downarrow \text{3655}$$

$$i\int -i\sqrt{a\cosh^2(e+fx)}\coth(e+fx)dx$$

$$\Big\downarrow \text{26}$$

$$\int \coth(e+fx)\sqrt{a\cosh^2(e+fx)}dx$$

$$\Big\downarrow \text{3042}$$

$$\int -i\tan\left(ie+ifx+\frac{\pi}{2}\right)\sqrt{a\sin\left(ie+ifx+\frac{\pi}{2}\right)^2}dx$$

$$\Big\downarrow \text{26}$$

$$-i \int \sqrt{a \sin\left(\frac{1}{2}(2ie+\pi)+ifx\right)^2} \tan\left(\frac{1}{2}(2ie+\pi)+ifx\right) dx$$

$$\Big\downarrow \ 3684$$

$$-\frac{\int \frac{\sqrt{a\cosh^2(e+fx)}}{1-\cosh^2(e+fx)} d\cosh^2(e+fx)}{2f}$$

$$\Big\downarrow \ 60$$

$$-\frac{a\int \frac{1}{\sqrt{a\cosh^2(e+fx)}(1-\cosh^2(e+fx))} d\cosh^2(e+fx) - 2\sqrt{a\cosh^2(e+fx)}}{2f}$$

$$\Big\downarrow \ 73$$

$$-\frac{2\int \frac{1}{1-\frac{\cosh^4(e+fx)}{a}} d\sqrt{a\cosh^2(e+fx)} - 2\sqrt{a\cosh^2(e+fx)}}{2f}$$

$$\Big\downarrow \ 219$$

$$-\frac{2\sqrt{a}\text{arctanh}\left(\frac{\sqrt{a\cosh^2(e+fx)}}{\sqrt{a}}\right) - 2\sqrt{a\cosh^2(e+fx)}}{2f}$$

input
```
Int[Coth[e + f*x]*Sqrt[a + a*Sinh[e + f*x]^2],x]
```

output
```
-1/2*(2*Sqrt[a]*ArcTanh[Sqrt[a*Cosh[e + f*x]^2]/Sqrt[a]] - 2*Sqrt[a*Cosh[e
 + f*x]^2])/f
```

Defintions of rubi rules used

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 60
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Simp[n*((b*c - a*d)/(
b*(m + n + 1)))   Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
 c, d}, x] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ[m, 0] && ( !Integer
Q[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n + 2, 0] && IntLinear
Q[a, b, c, d, m, n, x]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3655
```
Int[(u_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> Int[A
ctivateTrig[u*(a*cos[e + f*x]^2)^p], x] /; FreeQ[{a, b, e, f, p}, x] && EqQ
[a + b, 0]
```

rule 3684
```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*tan[(e_.) + (f_.)*(x_)]^(m_.
), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x]^2, x]}, Simp[ff^((m + 1
)/2)/(2*f)   Subst[Int[x^((m - 1)/2)*((b*ff^(n/2)*x^(n/2))^p/(1 - ff*x)^((m
 + 1)/2)), x], x, Sin[e + f*x]^2/ff], x]] /; FreeQ[{b, e, f, p}, x] && Inte
gerQ[(m - 1)/2] && IntegerQ[n/2]
```

Maple [A] (verified)

Time = 0.46 (sec) , antiderivative size = 54, normalized size of antiderivative = 1.08

method	result
default	$\frac{a\cosh(fx+e)(2\cosh(fx+e)+\ln(\cosh(fx+e)-1)-\ln(\cosh(fx+e)+1))}{2\sqrt{a\cosh(fx+e)^2}\,f}$
risch	$\frac{\sqrt{(e^{2fx+2e}+1)^2 a\,e^{-2fx-2e}}\,e^{2fx+2e}}{2f(e^{2fx+2e}+1)}+\frac{\sqrt{(e^{2fx+2e}+1)^2 a\,e^{-2fx-2e}}}{2f(e^{2fx+2e}+1)}-\frac{\ln(e^{fx}+e^{-e})\sqrt{(e^{2fx+2e}+1)^2 a\,e^{-2fx-2e}}\,e^{fx+e}}{f(e^{2fx+2e}+1)}+\frac{\ln(}{}$

input
```
int(coth(f*x+e)*(a+a*sinh(f*x+e)^2)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/2*a*cosh(f*x+e)*(2*cosh(f*x+e)+ln(cosh(f*x+e)-1)-ln(cosh(f*x+e)+1))/(a*c
osh(f*x+e)^2)^(1/2)/f
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 200 vs. $2(42) = 84$.

Time = 0.10 (sec) , antiderivative size = 200, normalized size of antiderivative = 4.00

$$\int \coth(e+fx)\sqrt{a+a\sinh^2(e+fx)}\,dx$$
$$=\frac{\left(2\cosh\left(fx+e\right)e^{(fx+e)}\sinh\left(fx+e\right)+e^{(fx+e)}\sinh\left(fx+e\right)^2+\left(\cosh\left(fx+e\right)^2+1\right)e^{(fx+e)}+2\left(\cosh\left(\right.\right.\right.}{2\left(f\cosh\left(fx+e\right)e^{(2fx+2e)}+f\cosh\left(\right.\right.}$$

input
```
integrate(coth(f*x+e)*(a+a*sinh(f*x+e)^2)^(1/2),x, algorithm="fricas")
```

output
```
1/2*(2*cosh(f*x + e)*e^(f*x + e)*sinh(f*x + e) + e^(f*x + e)*sinh(f*x + e)
^2 + (cosh(f*x + e)^2 + 1)*e^(f*x + e) + 2*(cosh(f*x + e)*e^(f*x + e) + e^
(f*x + e)*sinh(f*x + e))*log((cosh(f*x + e) + sinh(f*x + e) - 1)/(cosh(f*x
 + e) + sinh(f*x + e) + 1)))*sqrt(a*e^(4*f*x + 4*e) + 2*a*e^(2*f*x + 2*e)
+ a)*e^(-f*x - e)/(f*cosh(f*x + e)*e^(2*f*x + 2*e) + f*cosh(f*x + e) + (f*
e^(2*f*x + 2*e) + f)*sinh(f*x + e))
```

Sympy [F]

$$\int \coth(e+fx)\sqrt{a+a\sinh^2(e+fx)}\,dx = \int \sqrt{a\left(\sinh^2\left(e+fx\right)+1\right)}\coth\left(e+fx\right)dx$$

input
```
integrate(coth(f*x+e)*(a+a*sinh(f*x+e)**2)**(1/2),x)
```

output
```
Integral(sqrt(a*(sinh(e + f*x)**2 + 1))*coth(e + f*x), x)
```

Maxima [A] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 68, normalized size of antiderivative = 1.36

$$\int \coth(e+fx)\sqrt{a+a\sinh^2(e+fx)}\,dx = \frac{\left(\sqrt{a}e^{(-2\,fx-2\,e)}+\sqrt{a}\right)e^{(fx+e)}}{2\,f} - \frac{\sqrt{a}\log\left(e^{(-fx-e)}+1\right)}{f} + \frac{\sqrt{a}\log\left(e^{(-fx-e)}-1\right)}{f}$$

input
```
integrate(coth(f*x+e)*(a+a*sinh(f*x+e)^2)^(1/2),x, algorithm="maxima")
```

output
```
1/2*(sqrt(a)*e^(-2*f*x - 2*e) + sqrt(a))*e^(f*x + e)/f - sqrt(a)*log(e^(-f
*x - e) + 1)/f + sqrt(a)*log(e^(-f*x - e) - 1)/f
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.94

$$\int \coth(e+fx)\sqrt{a+a\sinh^2(e+fx)}\,dx = \frac{\sqrt{a}\left(e^{(fx+e)}+e^{(-fx-e)}-2\,\log\left(e^{(fx+e)}+1\right)+2\,\log\left(\left|e^{(fx+e)}-1\right|\right)\right)}{2\,f}$$

input
```
integrate(coth(f*x+e)*(a+a*sinh(f*x+e)^2)^(1/2),x, algorithm="giac")
```

output
```
1/2*sqrt(a)*(e^(f*x + e) + e^(-f*x - e) - 2*log(e^(f*x + e) + 1) + 2*log(a
bs(e^(f*x + e) - 1)))/f
```

Mupad [F(-1)]

Timed out.

$$\int \coth(e+fx)\sqrt{a+a\sinh^2(e+fx)}\,dx = \int \coth(e+f\,x)\,\sqrt{a\sinh\left(e+f\,x\right)^2+a}\,dx$$

input
```
int(coth(e + f*x)*(a + a*sinh(e + f*x)^2)^(1/2),x)
```

output
```
int(coth(e + f*x)*(a + a*sinh(e + f*x)^2)^(1/2), x)
```

Reduce [F]

$$\int \coth(e+fx)\sqrt{a+a\sinh^2(e+fx)}\,dx = \sqrt{a}\left(\int \sqrt{\sinh\left(fx+e\right)^2+1}\,\coth\left(fx+e\right)dx\right)$$

input
```
int(coth(f*x+e)*(a+a*sinh(f*x+e)^2)^(1/2),x)
```

output
```
sqrt(a)*int(sqrt(sinh(e + f*x)**2 + 1)*coth(e + f*x),x)
```

3.394 $\int \coth^3(e+fx)\sqrt{a+a\sinh^2(e+fx)}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 87

$$\int \coth^3(e+fx)\sqrt{a+a\sinh^2(e+fx)}\,dx = -\frac{3\sqrt{a}\text{arctanh}\left(\frac{\sqrt{a\cosh^2(e+fx)}}{\sqrt{a}}\right)}{2f} + \frac{3\sqrt{a\cosh^2(e+fx)}}{2f} - \frac{\left(a\cosh^2(e+fx)\right)^{3/2}\text{csch}^2(e+fx)}{2af}$$

output
```
-3/2*a^(1/2)*arctanh((a*cosh(f*x+e)^2)^(1/2)/a^(1/2))/f+3/2*(a*cosh(f*x+e)
^2)^(1/2)/f-1/2*(a*cosh(f*x+e)^2)^(3/2)*csch(f*x+e)^2/a/f
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 0.11 (sec) , antiderivative size = 40, normalized size of antiderivative = 0.46

$$\int \coth^3(e+fx)\sqrt{a+a\sinh^2(e+fx)}\,dx = \frac{\left(a\cosh^2(e+fx)\right)^{5/2}\text{Hypergeometric2F1}\left(2,\frac{5}{2},\frac{7}{2},\cosh^2(e+fx)\right)}{5a^2f}$$

input
```
Integrate[Coth[e + f*x]^3*Sqrt[a + a*Sinh[e + f*x]^2],x]
```

output
```
((a*Cosh[e + f*x]^2)^(5/2)*Hypergeometric2F1[2, 5/2, 7/2, Cosh[e + f*x]^2]
)/(5*a^2*f)
```

Rubi [A] (verified)

Time = 0.36 (sec) , antiderivative size = 90, normalized size of antiderivative = 1.03, number of steps used = 13, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.480$, Rules used = {3042, 26, 3655, 26, 3042, 26, 3684, 8, 51, 60, 73, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \coth^3(e+fx)\sqrt{a\sinh^2(e+fx)+a}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int -\frac{i\sqrt{a-a\sin(ie+ifx)^2}}{\tan(ie+ifx)^3}dx$$

$$\Big\downarrow \text{26}$$

$$-i\int \frac{\sqrt{a-a\sin(ie+ifx)^2}}{\tan(ie+ifx)^3}dx$$

$$\Big\downarrow \text{3655}$$

$$-i\int i\sqrt{a\cosh^2(e+fx)}\coth^3(e+fx)dx$$

$$\Big\downarrow \text{26}$$

$$\int \coth^3(e+fx)\sqrt{a\cosh^2(e+fx)}dx$$

$$\Big\downarrow \text{3042}$$

$$\int i\tan\left(ie+ifx+\frac{\pi}{2}\right)^3\sqrt{a\sin\left(ie+ifx+\frac{\pi}{2}\right)^2}dx$$

$$\Big\downarrow \text{26}$$

$$i\int \sqrt{a\sin\left(\frac{1}{2}(2ie+\pi)+ifx\right)^2}\tan\left(\frac{1}{2}(2ie+\pi)+ifx\right)^3dx$$

$$\Big\downarrow \text{3684}$$

$$\frac{\int \frac{\cosh^2(e+fx)\sqrt{a\cosh^2(e+fx)}}{(1-\cosh^2(e+fx))^2}d\cosh^2(e+fx)}{2f}$$

$$\Big\downarrow \text{8}$$

$$\frac{\int \frac{(a\cosh^2(e+fx))^{3/2}}{(1-\cosh^2(e+fx))^2}d\cosh^2(e+fx)}{2af}$$

$$\Big\downarrow \text{51}$$

$$\frac{\frac{(a\cosh^2(e+fx))^{3/2}}{1-\cosh^2(e+fx)}-\frac{3}{2}a\int \frac{\sqrt{a\cosh^2(e+fx)}}{1-\cosh^2(e+fx)}d\cosh^2(e+fx)}{2af}$$

$$\Big\downarrow \text{60}$$

$$\frac{\frac{(a\cosh^2(e+fx))^{3/2}}{1-\cosh^2(e+fx)}-\frac{3}{2}a\left(a\int \frac{1}{\sqrt{a\cosh^2(e+fx)}(1-\cosh^2(e+fx))}d\cosh^2(e+fx)-2\sqrt{a\cosh^2(e+fx)}\right)}{2af}$$

$$\Big\downarrow \text{73}$$

$$\frac{\frac{(a\cosh^2(e+fx))^{3/2}}{1-\cosh^2(e+fx)}-\frac{3}{2}a\left(2\int \frac{1}{1-\frac{\cosh^4(e+fx)}{a}}d\sqrt{a\cosh^2(e+fx)}-2\sqrt{a\cosh^2(e+fx)}\right)}{2af}$$

$$\Big\downarrow \text{219}$$

$$\frac{\frac{(a\cosh^2(e+fx))^{3/2}}{1-\cosh^2(e+fx)}-\frac{3}{2}a\left(2\sqrt{a}\text{arctanh}\left(\frac{\sqrt{a\cosh^2(e+fx)}}{\sqrt{a}}\right)-2\sqrt{a\cosh^2(e+fx)}\right)}{2af}$$

input
```
Int[Coth[e + f*x]^3*Sqrt[a + a*Sinh[e + f*x]^2],x]
```

output
```
((a*Cosh[e + f*x]^2)^(3/2)/(1 - Cosh[e + f*x]^2) - (3*a*(2*Sqrt[a]*ArcTanh
[Sqrt[a*Cosh[e + f*x]^2]/Sqrt[a]] - 2*Sqrt[a*Cosh[e + f*x]^2]))/2)/(2*a*f)
```

Defintions of rubi rules used

rule 8
```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_))^(p_), x_Symbol] :> Simp[1/a^m   Int[u*(a*
x)^(m + p), x], x] /; FreeQ[{a, m, p}, x] && IntegerQ[m]
```

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 51
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + 1))), x] - Simp[d*(n/(b*(m + 1)))
Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d, n}, x
] && ILtQ[m, -1] && FractionQ[n] && GtQ[n, 0]
```

rule 60
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Simp[n*((b*c - a*d)/(
b*(m + n + 1)))   Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
 c, d}, x] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ[m, 0] && ( !Integer
Q[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n + 2, 0] && IntLinear
Q[a, b, c, d, m, n, x]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3655
```
Int[(u_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> Int[A
ctivateTrig[u*(a*cos[e + f*x]^2)^p], x] /; FreeQ[{a, b, e, f, p}, x] && EqQ
[a + b, 0]
```

rule 3684
```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*tan[(e_.) + (f_.)*(x_)]^(m_.
), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x]^2, x]}, Simp[ff^((m + 1
)/2)/(2*f)   Subst[Int[x^((m - 1)/2)*((b*ff^(n/2)*x^(n/2))^p/(1 - ff*x)^((m
 + 1)/2)), x], x, Sin[e + f*x]^2/ff], x]] /; FreeQ[{b, e, f, p}, x] && Inte
gerQ[(m - 1)/2] && IntegerQ[n/2]
```

Maple [A] (verified)

Time = 0.52 (sec) , antiderivative size = 102, normalized size of antiderivative = 1.17

method	result
default	$-\frac{a\cosh(fx+e)\left(-4\cosh(fx+e)\sinh(fx+e)^2+2\cosh(fx+e)+(3\ln(\cosh(fx+e)+1)-3\ln(\cosh(fx+e)-1))\sinh(fx+e)^2\right)}{4\sqrt{a\cosh(fx+e)^2}\,(\cosh(fx+e)-1)(\cosh(fx+e)+1)f}$
risch	$-\frac{\sqrt{\left(e^{2fx+2e}+1\right)^2 a\,e^{-2fx-2e}}\,\left(3\ln\left(e^{fx}+e^{-e}\right)e^{5fx+5e}-3\ln\left(e^{fx}-e^{-e}\right)e^{5fx+5e}-6\ln\left(e^{fx}+e^{-e}\right)e^{3fx+3e}+6\ln\left(e^{fx}-e^{-e}\right)e^{3fx+3e}\right.}{2f\left(e^{2fx+2e}+1\right)\left(e^{2fx+2e}-1\right)^2}$

input
```
int(coth(f*x+e)^3*(a+a*sinh(f*x+e)^2)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-1/4*a*cosh(f*x+e)*(-4*cosh(f*x+e)*sinh(f*x+e)^2+2*cosh(f*x+e)+(3*ln(cosh(
f*x+e)+1)-3*ln(cosh(f*x+e)-1))*sinh(f*x+e)^2)/(a*cosh(f*x+e)^2)^(1/2)/(cos
h(f*x+e)-1)/(cosh(f*x+e)+1)/f
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 764 vs. $2(71) = 142$.

Time = 0.11 (sec) , antiderivative size = 764, normalized size of antiderivative = 8.78

$$\int \coth^3(e+fx)\sqrt{a+a\sinh^2(e+fx)}\,dx = \text{Too large to display}$$

input
```
integrate(coth(f*x+e)^3*(a+a*sinh(f*x+e)^2)^(1/2),x, algorithm="fricas")
```

output
```
1/2*(6*cosh(f*x + e)*e^(f*x + e)*sinh(f*x + e)^5 + e^(f*x + e)*sinh(f*x +
e)^6 + 3*(5*cosh(f*x + e)^2 - 1)*e^(f*x + e)*sinh(f*x + e)^4 + 4*(5*cosh(f
*x + e)^3 - 3*cosh(f*x + e))*e^(f*x + e)*sinh(f*x + e)^3 + 3*(5*cosh(f*x +
 e)^4 - 6*cosh(f*x + e)^2 - 1)*e^(f*x + e)*sinh(f*x + e)^2 + 6*(cosh(f*x +
 e)^5 - 2*cosh(f*x + e)^3 - cosh(f*x + e))*e^(f*x + e)*sinh(f*x + e) + (co
sh(f*x + e)^6 - 3*cosh(f*x + e)^4 - 3*cosh(f*x + e)^2 + 1)*e^(f*x + e) + 3
*(5*cosh(f*x + e)*e^(f*x + e)*sinh(f*x + e)^4 + e^(f*x + e)*sinh(f*x + e)^
5 + 2*(5*cosh(f*x + e)^2 - 1)*e^(f*x + e)*sinh(f*x + e)^3 + 2*(5*cosh(f*x
+ e)^3 - 3*cosh(f*x + e))*e^(f*x + e)*sinh(f*x + e)^2 + (5*cosh(f*x + e)^4
 - 6*cosh(f*x + e)^2 + 1)*e^(f*x + e)*sinh(f*x + e) + (cosh(f*x + e)^5 - 2
*cosh(f*x + e)^3 + cosh(f*x + e))*e^(f*x + e))*log((cosh(f*x + e) + sinh(f
*x + e) - 1)/(cosh(f*x + e) + sinh(f*x + e) + 1)))*sqrt(a*e^(4*f*x + 4*e)
+ 2*a*e^(2*f*x + 2*e) + a)*e^(-f*x - e)/(f*cosh(f*x + e)^5 + (f*e^(2*f*x +
 2*e) + f)*sinh(f*x + e)^5 + 5*(f*cosh(f*x + e)*e^(2*f*x + 2*e) + f*cosh(f
*x + e))*sinh(f*x + e)^4 - 2*f*cosh(f*x + e)^3 + 2*(5*f*cosh(f*x + e)^2 +
(5*f*cosh(f*x + e)^2 - f)*e^(2*f*x + 2*e) - f)*sinh(f*x + e)^3 + 2*(5*f*co
sh(f*x + e)^3 - 3*f*cosh(f*x + e) + (5*f*cosh(f*x + e)^3 - 3*f*cosh(f*x +
e))*e^(2*f*x + 2*e))*sinh(f*x + e)^2 + f*cosh(f*x + e) + (f*cosh(f*x + e)^
5 - 2*f*cosh(f*x + e)^3 + f*cosh(f*x + e))*e^(2*f*x + 2*e) + (5*f*cosh(f*x
 + e)^4 - 6*f*cosh(f*x + e)^2 + (5*f*cosh(f*x + e)^4 - 6*f*cosh(f*x + e...
```

Sympy [F]

$$\int \coth^3(e+fx)\sqrt{a+a\sinh^2(e+fx)}\,dx = \int \sqrt{a\left(\sinh^2\left(e+fx\right)+1\right)}\coth^3\left(e+fx\right)dx$$

input
```
integrate(coth(f*x+e)**3*(a+a*sinh(f*x+e)**2)**(1/2),x)
```

output
```
Integral(sqrt(a*(sinh(e + f*x)**2 + 1))*coth(e + f*x)**3, x)
```

Maxima [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 126, normalized size of antiderivative = 1.45

$$\int \coth^3(e+fx)\sqrt{a+a\sinh^2(e+fx)}\,dx = -\frac{3\sqrt{a}\log\left(e^{(-fx-e)}+1\right)}{2f} + \frac{3\sqrt{a}\log\left(e^{(-fx-e)}-1\right)}{2f} - \frac{3\sqrt{a}e^{(-2fx-2e)} + 3\sqrt{a}e^{(-4fx-4e)} - \sqrt{a}e^{(-6fx-6e)} - \sqrt{a}}{2f\left(e^{(-fx-e)} - 2e^{(-3fx-3e)} + e^{(-5fx-5e)}\right)}$$

input
```
integrate(coth(f*x+e)^3*(a+a*sinh(f*x+e)^2)^(1/2),x, algorithm="maxima")
```

output
```
-3/2*sqrt(a)*log(e^(-f*x - e) + 1)/f + 3/2*sqrt(a)*log(e^(-f*x - e) - 1)/f
 - 1/2*(3*sqrt(a)*e^(-2*f*x - 2*e) + 3*sqrt(a)*e^(-4*f*x - 4*e) - sqrt(a)*
e^(-6*f*x - 6*e) - sqrt(a))/(f*(e^(-f*x - e) - 2*e^(-3*f*x - 3*e) + e^(-5*
f*x - 5*e)))
```

Giac [A] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 108, normalized size of antiderivative = 1.24

$$\int \coth^3(e+fx)\sqrt{a+a\sinh^2(e+fx)}\,dx = -\frac{\sqrt{a}\left(\frac{4\left(e^{(fx+e)}+e^{(-fx-e)}\right)}{\left(e^{(fx+e)}+e^{(-fx-e)}\right)^2-4} - 2e^{(fx+e)} - 2e^{(-fx-e)} + 3\log\left(e^{(fx+e)}+e^{(-fx-e)}+2\right) - 3\log\left(e^{(fx+e)}+e^{(-\right.\right.}{4f}$$

input
```
integrate(coth(f*x+e)^3*(a+a*sinh(f*x+e)^2)^(1/2),x, algorithm="giac")
```

output
```
-1/4*sqrt(a)*(4*(e^(f*x + e) + e^(-f*x - e))/((e^(f*x + e) + e^(-f*x - e))
^2 - 4) - 2*e^(f*x + e) - 2*e^(-f*x - e) + 3*log(e^(f*x + e) + e^(-f*x - e
) + 2) - 3*log(e^(f*x + e) + e^(-f*x - e) - 2))/f
```

Mupad [F(-1)]

Timed out.

$$\int \coth^3(e+fx)\sqrt{a+a\sinh^2(e+fx)}\,dx = \int \coth(e+f\,x)^3\,\sqrt{a\sinh\left(e+f\,x\right)^2+a}\,dx$$

input
```
int(coth(e + f*x)^3*(a + a*sinh(e + f*x)^2)^(1/2),x)
```

output
```
int(coth(e + f*x)^3*(a + a*sinh(e + f*x)^2)^(1/2), x)
```

Reduce [F]

$$\int \coth^3(e+fx)\sqrt{a+a\sinh^2(e+fx)}\,dx = \sqrt{a}\left(\int \sqrt{\sinh\left(fx+e\right)^2+1}\,\coth\left(fx+e\right)^3dx\right)$$

input
```
int(coth(f*x+e)^3*(a+a*sinh(f*x+e)^2)^(1/2),x)
```

output
```
sqrt(a)*int(sqrt(sinh(e + f*x)**2 + 1)*coth(e + f*x)**3,x)
```

3.395 $\int \sqrt{a+a\sinh^2(e+fx)}\tanh^6(e+fx)\,dx$

Optimal result

Integrand size = 25, antiderivative size = 120

$$\begin{aligned}\int \sqrt{a+a\sinh^2(e+fx)}\tanh^6(e+fx)\,dx &= -\frac{15\arctan(\sinh(e+fx))\sqrt{a\cosh^2(e+fx)}\operatorname{sech}(e+fx)}{8f}\\ &\quad+\frac{15\sqrt{a\cosh^2(e+fx)}\tanh(e+fx)}{8f}\\ &\quad-\frac{5\sqrt{a\cosh^2(e+fx)}\tanh^3(e+fx)}{8f}-\frac{\sqrt{a\cosh^2(e+fx)}\tanh^5(e+fx)}{4f}\end{aligned}$$

output
```
-15/8*arctan(sinh(f*x+e))*(a*cosh(f*x+e)^2)^(1/2)*sech(f*x+e)/f+15/8*(a*co
sh(f*x+e)^2)^(1/2)*tanh(f*x+e)/f-5/8*(a*cosh(f*x+e)^2)^(1/2)*tanh(f*x+e)^3
/f-1/4*(a*cosh(f*x+e)^2)^(1/2)*tanh(f*x+e)^5/f
```

Mathematica [A] (verified)

Time = 0.17 (sec) , antiderivative size = 75, normalized size of antiderivative = 0.62

$$\int \sqrt{a+a\sinh^2(e+fx)}\tanh^6(e+fx)\,dx =$$
$$-\frac{\sqrt{a\cosh^2(e+fx)}\mathrm{sech}^5(e+fx)\left(60\arctan(\sinh(e+fx))\cosh^4(e+fx)-5\sinh(e+fx)-15\sinh(3\right.}{32f}$$

input
```
Integrate[Sqrt[a + a*Sinh[e + f*x]^2]*Tanh[e + f*x]^6,x]
```

output
```
-1/32*(Sqrt[a*Cosh[e + f*x]^2]*Sech[e + f*x]^5*(60*ArcTan[Sinh[e + f*x]]*C
osh[e + f*x]^4 - 5*Sinh[e + f*x] - 15*Sinh[3*(e + f*x)] - 2*Sinh[5*(e + f*
x)]))/f
```

Rubi [A] (verified)

Time = 0.46 (sec) , antiderivative size = 99, normalized size of antiderivative = 0.82, number of steps used = 17, number of rules used = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.640, Rules used = {3042, 25, 3655, 25, 3042, 25, 3686, 25, 3042, 25, 3072, 25, 252, 252, 262, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \tanh^6(e+fx)\sqrt{a\sinh^2(e+fx)+a}\,dx$$
$$\Big\downarrow \text{3042}$$
$$\int \tan(ie+ifx)^6\left(-\sqrt{a-a\sin(ie+ifx)^2}\right)dx$$
$$\Big\downarrow \text{25}$$
$$-\int \sqrt{a-a\sin(ie+ifx)^2}\tan(ie+ifx)^6dx$$
$$\Big\downarrow \text{3655}$$
$$-\int -\sqrt{a\cosh^2(e+fx)}\tanh^6(e+fx)dx$$

$$\Big\downarrow \textcolor{blue}{25}$$

$$\int \tanh^6(e+fx)\sqrt{a\cosh^2(e+fx)}dx$$

$$\Big\downarrow \textcolor{blue}{3042}$$

$$\int -\frac{\sqrt{a\sin\left(ie+ifx+\frac{\pi}{2}\right)^2}}{\tan\left(ie+ifx+\frac{\pi}{2}\right)^6}dx$$

$$\Big\downarrow \textcolor{blue}{25}$$

$$-\int \frac{\sqrt{a\sin\left(\frac{1}{2}(2ie+\pi)+ifx\right)^2}}{\tan\left(\frac{1}{2}(2ie+\pi)+ifx\right)^6}dx$$

$$\Big\downarrow \textcolor{blue}{3686}$$

$$\text{sech}(e+fx)\left(-\sqrt{a\cosh^2(e+fx)}\right)\int -\sinh(e+fx)\tanh^5(e+fx)dx$$

$$\Big\downarrow \textcolor{blue}{25}$$

$$\text{sech}(e+fx)\sqrt{a\cosh^2(e+fx)}\int \sinh(e+fx)\tanh^5(e+fx)dx$$

$$\Big\downarrow \textcolor{blue}{3042}$$

$$\text{sech}(e+fx)\sqrt{a\cosh^2(e+fx)}\int -\sin(ie+ifx)\tan(ie+ifx)^5dx$$

$$\Big\downarrow \textcolor{blue}{25}$$

$$\text{sech}(e+fx)\left(-\sqrt{a\cosh^2(e+fx)}\right)\int \sin(ie+ifx)\tan(ie+ifx)^5dx$$

$$\Big\downarrow \textcolor{blue}{3072}$$

$$-\frac{\text{sech}(e+fx)\sqrt{a\cosh^2(e+fx)}\int -\frac{\sinh^6(e+fx)}{\left(\sinh^2(e+fx)+1\right)^3}d\sinh(e+fx)}{f}$$

$$\Big\downarrow \textcolor{blue}{25}$$

$$\frac{\text{sech}(e+fx)\sqrt{a\cosh^2(e+fx)}\int \frac{\sinh^6(e+fx)}{\left(\sinh^2(e+fx)+1\right)^3}d\sinh(e+fx)}{f}$$

$$\Big\downarrow \textcolor{blue}{252}$$

$$-\frac{\operatorname{sech}(e+fx)\sqrt{a\cosh^2(e+fx)}\left(\frac{\sinh^5(e+fx)}{4(\sinh^2(e+fx)+1)^2}-\frac{5}{4}\int\frac{\sinh^4(e+fx)}{(\sinh^2(e+fx)+1)^2}d\sinh(e+fx)\right)}{f}$$

$$\Big\downarrow \text{252}$$

$$-\frac{\operatorname{sech}(e+fx)\sqrt{a\cosh^2(e+fx)}\left(\frac{\sinh^5(e+fx)}{4(\sinh^2(e+fx)+1)^2}-\frac{5}{4}\left(\frac{3}{2}\int\frac{\sinh^2(e+fx)}{\sinh^2(e+fx)+1}d\sinh(e+fx)-\frac{\sinh^3(e+fx)}{2(\sinh^2(e+fx)+1)}\right)\right)}{f}$$

$$\Big\downarrow \text{262}$$

$$-\frac{\operatorname{sech}(e+fx)\sqrt{a\cosh^2(e+fx)}\left(\frac{\sinh^5(e+fx)}{4(\sinh^2(e+fx)+1)^2}-\frac{5}{4}\left(\frac{3}{2}\left(\sinh(e+fx)-\int\frac{1}{\sinh^2(e+fx)+1}d\sinh(e+fx)\right)-\frac{s}{2(si}\right.\right.}{f}$$

$$\Big\downarrow \text{216}$$

$$-\frac{\operatorname{sech}(e+fx)\sqrt{a\cosh^2(e+fx)}\left(\frac{\sinh^5(e+fx)}{4(\sinh^2(e+fx)+1)^2}-\frac{5}{4}\left(\frac{3}{2}(\sinh(e+fx)-\arctan(\sinh(e+fx)))-\frac{\sinh^3(e+fx)}{2(\sinh^2(e+fx)}\right.\right.}{f}$$

input
```
Int[Sqrt[a + a*Sinh[e + f*x]^2]*Tanh[e + f*x]^6,x]
```

output
```
-((Sqrt[a*Cosh[e + f*x]^2]*Sech[e + f*x]*(Sinh[e + f*x]^5/(4*(1 + Sinh[e +
 f*x]^2)^2) - (5*((3*(-ArcTan[Sinh[e + f*x]] + Sinh[e + f*x]))/2 - Sinh[e
+ f*x]^3/(2*(1 + Sinh[e + f*x]^2))))/4))/f)
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 252
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[c*(c*x
)^(m - 1)*((a + b*x^2)^(p + 1)/(2*b*(p + 1))), x] - Simp[c^2*((m - 1)/(2*b*
(p + 1)))   Int[(c*x)^(m - 2)*(a + b*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c
}, x] && LtQ[p, -1] && GtQ[m, 1] &&  !ILtQ[(m + 2*p + 3)/2, 0] && IntBinomi
alQ[a, b, c, 2, m, p, x]
```

rule 262
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[c*(c*x)
^(m - 1)*((a + b*x^2)^(p + 1)/(b*(m + 2*p + 1))), x] - Simp[a*c^2*((m - 1)/
(b*(m + 2*p + 1)))   Int[(c*x)^(m - 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b
, c, p}, x] && GtQ[m, 2 - 1] && NeQ[m + 2*p + 1, 0] && IntBinomialQ[a, b, c
, 2, m, p, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3072
```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*tan[(e_.) + (f_.)*(x_)]^(n_.), x_
Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Simp[ff/f   Subst[Int[
(ff*x)^(m + n)/(a^2 - ff^2*x^2)^((n + 1)/2), x], x, a*(Sin[e + f*x]/ff)], x
]] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n + 1)/2]
```

rule 3655
```
Int[(u_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> Int[A
ctivateTrig[u*(a*cos[e + f*x]^2)^p], x] /; FreeQ[{a, b, e, f, p}, x] && EqQ
[a + b, 0]
```

rule 3686
```
Int[(u_.)*((b_.)*sin[(e_.) + (f_.)*(x_)]^(n_))^(p_), x_Symbol] :> With[{ff
= FreeFactors[Sin[e + f*x], x]}, Simp[(b*ff^n)^IntPart[p]*((b*Sin[e + f*x]^
n)^FracPart[p]/(Sin[e + f*x]/ff)^(n*FracPart[p]))   Int[ActivateTrig[u]*(Si
n[e + f*x]/ff)^(n*p), x], x]] /; FreeQ[{b, e, f, n, p}, x] &&  !IntegerQ[p]
 && IntegerQ[n] && (EqQ[u, 1] || MatchQ[u, ((d_.)*(trig_)[e + f*x])^(m_.) /
; FreeQ[{d, m}, x] && MemberQ[{sin, cos, tan, cot, sec, csc}, trig]])
```

Maple [A] (verified)

Time = 0.60 (sec) , antiderivative size = 169, normalized size of antiderivative = 1.41

method	result
default	$\frac{\sqrt{a\sinh(fx+e)^2}\left(8\sqrt{a\sinh(fx+e)^2}\cosh(fx+e)^4\sqrt{-a}+15\ln\left(\frac{2\sqrt{-a}\sqrt{a\sinh(fx+e)^2}-2a}{\cosh(fx+e)}\right)a\cosh(fx+e)^4+9\sqrt{a\sinh(fx+e)^2}\cosh}{8\cosh(fx+e)^3\sqrt{-a}\sinh(fx+e)\sqrt{a\cosh(fx+e)^2}f}$
risch	$\frac{\sqrt{(e^{2fx+2e}+1)^2 a e^{-2fx-2e}}e^{2fx+2e}}{2f(e^{2fx+2e}+1)}-\frac{\sqrt{(e^{2fx+2e}+1)^2 a e^{-2fx-2e}}}{2f(e^{2fx+2e}+1)}+\frac{(9e^{6fx+6e}+e^{4fx+4e}-e^{2fx+2e}-9)\sqrt{(e^{2fx+2e}+1)^2 a e^{-}}}{4f(e^{2fx+2e}+1)^5}$

input
```
int((a+a*sinh(f*x+e)^2)^(1/2)*tanh(f*x+e)^6,x,method=_RETURNVERBOSE)
```

output
```
1/8/cosh(f*x+e)^3*(a*sinh(f*x+e)^2)^(1/2)*(8*(a*sinh(f*x+e)^2)^(1/2)*cosh(
f*x+e)^4*(-a)^(1/2)+15*ln(2/cosh(f*x+e)*((-a)^(1/2)*(a*sinh(f*x+e)^2)^(1/2
)-a))*a*cosh(f*x+e)^4+9*(a*sinh(f*x+e)^2)^(1/2)*cosh(f*x+e)^2*(-a)^(1/2)-2
*(-a)^(1/2)*(a*sinh(f*x+e)^2)^(1/2))/(-a)^(1/2)/sinh(f*x+e)/(a*cosh(f*x+e)
^2)^(1/2)/f
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1645 vs. 2(104) = 208.

Time = 0.13 (sec) , antiderivative size = 1645, normalized size of antiderivative = 13.71

$$\int\sqrt{a+a\sinh^2(e+fx)}\tanh^6(e+fx)\,dx=\text{Too large to display}$$

input
```
integrate((a+a*sinh(f*x+e)^2)^(1/2)*tanh(f*x+e)^6,x, algorithm="fricas")
```

output
```
1/4*(20*cosh(f*x + e)*e^(f*x + e)*sinh(f*x + e)^9 + 2*e^(f*x + e)*sinh(f*x
 + e)^10 + 15*(6*cosh(f*x + e)^2 + 1)*e^(f*x + e)*sinh(f*x + e)^8 + 120*(2
*cosh(f*x + e)^3 + cosh(f*x + e))*e^(f*x + e)*sinh(f*x + e)^7 + 5*(84*cosh
(f*x + e)^4 + 84*cosh(f*x + e)^2 + 1)*e^(f*x + e)*sinh(f*x + e)^6 + 6*(84*
cosh(f*x + e)^5 + 140*cosh(f*x + e)^3 + 5*cosh(f*x + e))*e^(f*x + e)*sinh(
f*x + e)^5 + 5*(84*cosh(f*x + e)^6 + 210*cosh(f*x + e)^4 + 15*cosh(f*x + e
)^2 - 1)*e^(f*x + e)*sinh(f*x + e)^4 + 20*(12*cosh(f*x + e)^7 + 42*cosh(f*
x + e)^5 + 5*cosh(f*x + e)^3 - cosh(f*x + e))*e^(f*x + e)*sinh(f*x + e)^3
+ 15*(6*cosh(f*x + e)^8 + 28*cosh(f*x + e)^6 + 5*cosh(f*x + e)^4 - 2*cosh(
f*x + e)^2 - 1)*e^(f*x + e)*sinh(f*x + e)^2 + 10*(2*cosh(f*x + e)^9 + 12*c
osh(f*x + e)^7 + 3*cosh(f*x + e)^5 - 2*cosh(f*x + e)^3 - 3*cosh(f*x + e))*
e^(f*x + e)*sinh(f*x + e) - 15*(9*cosh(f*x + e)*e^(f*x + e)*sinh(f*x + e)^
8 + e^(f*x + e)*sinh(f*x + e)^9 + 4*(9*cosh(f*x + e)^2 + 1)*e^(f*x + e)*si
nh(f*x + e)^7 + 28*(3*cosh(f*x + e)^3 + cosh(f*x + e))*e^(f*x + e)*sinh(f*
x + e)^6 + 6*(21*cosh(f*x + e)^4 + 14*cosh(f*x + e)^2 + 1)*e^(f*x + e)*sin
h(f*x + e)^5 + 2*(63*cosh(f*x + e)^5 + 70*cosh(f*x + e)^3 + 15*cosh(f*x +
e))*e^(f*x + e)*sinh(f*x + e)^4 + 4*(21*cosh(f*x + e)^6 + 35*cosh(f*x + e)
^4 + 15*cosh(f*x + e)^2 + 1)*e^(f*x + e)*sinh(f*x + e)^3 + 12*(3*cosh(f*x
+ e)^7 + 7*cosh(f*x + e)^5 + 5*cosh(f*x + e)^3 + cosh(f*x + e))*e^(f*x + e
)*sinh(f*x + e)^2 + (9*cosh(f*x + e)^8 + 28*cosh(f*x + e)^6 + 30*cosh(f...
```

Sympy [F]

$$\int \sqrt{a+a\sinh^2(e+fx)}\tanh^6(e+fx)\,dx = \int \sqrt{a\left(\sinh^2\left(e+fx\right)+1\right)}\tanh^6\left(e+fx\right)dx$$

input
```
integrate((a+a*sinh(f*x+e)**2)**(1/2)*tanh(f*x+e)**6,x)
```

output
```
Integral(sqrt(a*(sinh(e + f*x)**2 + 1))*tanh(e + f*x)**6, x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 891 vs. $2(104) = 208$.

Time = 0.18 (sec) , antiderivative size = 891, normalized size of antiderivative = 7.42

$$\int \sqrt{a+a\sinh^2(e+fx)}\tanh^6(e+fx)\,dx = \text{Too large to display}$$

input
```
integrate((a+a*sinh(f*x+e)^2)^(1/2)*tanh(f*x+e)^6,x, algorithm="maxima")
```

output
```
315/128*sqrt(a)*arctan(e^(-f*x - e))/f + 1/128*(105*sqrt(a)*arctan(e^(-f*x
 - e)) + (279*sqrt(a)*e^(-f*x - e) + 511*sqrt(a)*e^(-3*f*x - 3*e) + 385*sq
rt(a)*e^(-5*f*x - 5*e) + 105*sqrt(a)*e^(-7*f*x - 7*e))/(4*e^(-2*f*x - 2*e)
 + 6*e^(-4*f*x - 4*e) + 4*e^(-6*f*x - 6*e) + e^(-8*f*x - 8*e) + 1))/f + 1/
128*(105*sqrt(a)*arctan(e^(-f*x - e)) - (105*sqrt(a)*e^(-f*x - e) + 385*sq
rt(a)*e^(-3*f*x - 3*e) + 511*sqrt(a)*e^(-5*f*x - 5*e) + 279*sqrt(a)*e^(-7*
f*x - 7*e))/(4*e^(-2*f*x - 2*e) + 6*e^(-4*f*x - 4*e) + 4*e^(-6*f*x - 6*e)
+ e^(-8*f*x - 8*e) + 1))/f - 5/256*(15*sqrt(a)*arctan(e^(-f*x - e)) - (15*
sqrt(a)*e^(-f*x - e) + 55*sqrt(a)*e^(-3*f*x - 3*e) + 73*sqrt(a)*e^(-5*f*x
- 5*e) - 15*sqrt(a)*e^(-7*f*x - 7*e))/(4*e^(-2*f*x - 2*e) + 6*e^(-4*f*x -
4*e) + 4*e^(-6*f*x - 6*e) + e^(-8*f*x - 8*e) + 1))/f - 5/256*(15*sqrt(a)*a
rctan(e^(-f*x - e)) - (15*sqrt(a)*e^(-f*x - e) - 73*sqrt(a)*e^(-3*f*x - 3*
e) - 55*sqrt(a)*e^(-5*f*x - 5*e) - 15*sqrt(a)*e^(-7*f*x - 7*e))/(4*e^(-2*f
*x - 2*e) + 6*e^(-4*f*x - 4*e) + 4*e^(-6*f*x - 6*e) + e^(-8*f*x - 8*e) + 1
))/f + 5/64*(3*sqrt(a)*arctan(e^(-f*x - e)) - (3*sqrt(a)*e^(-f*x - e) + 11
*sqrt(a)*e^(-3*f*x - 3*e) - 11*sqrt(a)*e^(-5*f*x - 5*e) - 3*sqrt(a)*e^(-7*
f*x - 7*e))/(4*e^(-2*f*x - 2*e) + 6*e^(-4*f*x - 4*e) + 4*e^(-6*f*x - 6*e)
+ e^(-8*f*x - 8*e) + 1))/f + 1/256*(837*sqrt(a)*e^(-2*f*x - 2*e) + 1533*sq
rt(a)*e^(-4*f*x - 4*e) + 1155*sqrt(a)*e^(-6*f*x - 6*e) + 315*sqrt(a)*e^(-8
*f*x - 8*e) + 128*sqrt(a))/(f*(e^(-f*x - e) + 4*e^(-3*f*x - 3*e) + 6*e^...
```

Giac [A] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 124, normalized size of antiderivative = 1.03

$$\int \sqrt{a+a\sinh^2(e+fx)}\tanh^6(e+fx)\,dx = -\frac{\left(15\,\pi - \frac{4\left(9\left(e^{(fx+e)}-e^{(-fx-e)}\right)^3+28\,e^{(fx+e)}-28\,e^{(-fx-e)}\right)}{\left(\left(e^{(fx+e)}-e^{(-fx-e)}\right)^2+4\right)^2} + 30\arctan\left(\frac{1}{2}\left(e^{(2\,fx+2\,e)}-1\right)e^{(-fx-e)}\right) - 8\,e^{(fx+e)} \ldots}{16\,f}$$

input
```
integrate((a+a*sinh(f*x+e)^2)^(1/2)*tanh(f*x+e)^6,x, algorithm="giac")
```

output
```
-1/16*(15*pi - 4*(9*(e^(f*x + e) - e^(-f*x - e))^3 + 28*e^(f*x + e) - 28*e
^(-f*x - e))/((e^(f*x + e) - e^(-f*x - e))^2 + 4)^2 + 30*arctan(1/2*(e^(2*
f*x + 2*e) - 1)*e^(-f*x - e)) - 8*e^(f*x + e) + 8*e^(-f*x - e))*sqrt(a)/f
```

Mupad [F(-1)]

Timed out.

$$\int \sqrt{a+a\sinh^2(e+fx)}\tanh^6(e+fx)\,dx = \int \tanh(e+f\,x)^6\,\sqrt{a\sinh\left(e+f\,x\right)^2+a}\,dx$$

input
```
int(tanh(e + f*x)^6*(a + a*sinh(e + f*x)^2)^(1/2),x)
```

output
```
int(tanh(e + f*x)^6*(a + a*sinh(e + f*x)^2)^(1/2), x)
```

Reduce [F]

$$\int \sqrt{a+a\sinh^2(e+fx)}\tanh^6(e+fx)\,dx = \sqrt{a}\left(\int \sqrt{\sinh\left(fx+e\right)^2+1}\,\tanh\left(fx+e\right)^6dx\right)$$

input
```
int((a+a*sinh(f*x+e)^2)^(1/2)*tanh(f*x+e)^6,x)
```

output
```
sqrt(a)*int(sqrt(sinh(e + f*x)**2 + 1)*tanh(e + f*x)**6,x)
```

3.396 $\int \sqrt{a+a\sinh^2(e+fx)}\tanh^4(e+fx)\,dx$

Optimal result

Integrand size = 25, antiderivative size = 91

$$\int \sqrt{a+a\sinh^2(e+fx)}\tanh^4(e+fx)\,dx = -\frac{3\arctan(\sinh(e+fx))\sqrt{a\cosh^2(e+fx)}\operatorname{sech}(e+fx)}{2f} + \frac{3\sqrt{a\cosh^2(e+fx)}\tanh(e+fx)}{2f} - \frac{\sqrt{a\cosh^2(e+fx)}\tanh^3(e+fx)}{2f}$$

output
```
-3/2*arctan(sinh(f*x+e))*(a*cosh(f*x+e)^2)^(1/2)*sech(f*x+e)/f+3/2*(a*cosh
(f*x+e)^2)^(1/2)*tanh(f*x+e)/f-1/2*(a*cosh(f*x+e)^2)^(1/2)*tanh(f*x+e)^3/f
```

Mathematica [A] (verified)

Time = 0.14 (sec) , antiderivative size = 55, normalized size of antiderivative = 0.60

$$\int \sqrt{a+a\sinh^2(e+fx)}\tanh^4(e+fx)\,dx = \frac{a(-3\arctan(\sinh(e+fx))\cosh(e+fx)+(2+\cosh(2(e+fx)))\tanh(e+fx))}{2f\sqrt{a\cosh^2(e+fx)}}$$

input
```
Integrate[Sqrt[a + a*Sinh[e + f*x]^2]*Tanh[e + f*x]^4,x]
```

output
```
(a*(-3*ArcTan[Sinh[e + f*x]]*Cosh[e + f*x] + (2 + Cosh[2*(e + f*x)])*Tanh[
e + f*x]))/(2*f*Sqrt[a*Cosh[e + f*x]^2])
```

Rubi [A] (verified)

Time = 0.41 (sec) , antiderivative size = 69, normalized size of antiderivative = 0.76, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.360$, Rules used = {3042, 3655, 3042, 3686, 3042, 3072, 252, 262, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \tanh^4(e+fx)\sqrt{a\sinh^2(e+fx)+a}\,dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \tan(ie+ifx)^4\sqrt{a-a\sin(ie+ifx)^2}dx$$

$$\Big\downarrow \text{ 3655}$$

$$\int \tanh^4(e+fx)\sqrt{a\cosh^2(e+fx)}dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \frac{\sqrt{a\sin\left(ie+ifx+\frac{\pi}{2}\right)^2}}{\tan\left(ie+ifx+\frac{\pi}{2}\right)^4}dx$$

$$\Big\downarrow \text{ 3686}$$

$$\text{sech}(e+fx)\sqrt{a\cosh^2(e+fx)}\int \sinh(e+fx)\tanh^3(e+fx)dx$$

$$\Big\downarrow \text{ 3042}$$

$$\text{sech}(e+fx)\sqrt{a\cosh^2(e+fx)}\int \sin(ie+ifx)\tan(ie+ifx)^3dx$$

$$\Big\downarrow \text{ 3072}$$

$$\frac{\operatorname{sech}(e+fx)\sqrt{a\cosh^2(e+fx)}\int\frac{\sinh^4(e+fx)}{(\sinh^2(e+fx)+1)^2}d\sinh(e+fx)}{f}$$

$$\Big\downarrow \text{252}$$

$$\frac{\operatorname{sech}(e+fx)\sqrt{a\cosh^2(e+fx)}\left(\frac{3}{2}\int\frac{\sinh^2(e+fx)}{\sinh^2(e+fx)+1}d\sinh(e+fx)-\frac{\sinh^3(e+fx)}{2(\sinh^2(e+fx)+1)}\right)}{f}$$

$$\Big\downarrow \text{262}$$

$$\frac{\operatorname{sech}(e+fx)\sqrt{a\cosh^2(e+fx)}\left(\frac{3}{2}\left(\sinh(e+fx)-\int\frac{1}{\sinh^2(e+fx)+1}d\sinh(e+fx)\right)-\frac{\sinh^3(e+fx)}{2(\sinh^2(e+fx)+1)}\right)}{f}$$

$$\Big\downarrow \text{216}$$

$$\frac{\operatorname{sech}(e+fx)\sqrt{a\cosh^2(e+fx)}\left(\frac{3}{2}(\sinh(e+fx)-\arctan(\sinh(e+fx)))-\frac{\sinh^3(e+fx)}{2(\sinh^2(e+fx)+1)}\right)}{f}$$

input
```
Int[Sqrt[a + a*Sinh[e + f*x]^2]*Tanh[e + f*x]^4,x]
```

output
```
(Sqrt[a*Cosh[e + f*x]^2]*Sech[e + f*x]*((3*(-ArcTan[Sinh[e + f*x]] + Sinh[
e + f*x]))/2 - Sinh[e + f*x]^3/(2*(1 + Sinh[e + f*x]^2))))/f
```

Defintions of rubi rules used

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 252
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[c*(c*x
)^(m - 1)*((a + b*x^2)^(p + 1)/(2*b*(p + 1))), x] - Simp[c^2*((m - 1)/(2*b*
(p + 1)))   Int[(c*x)^(m - 2)*(a + b*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c
}, x] && LtQ[p, -1] && GtQ[m, 1] &&  !ILtQ[(m + 2*p + 3)/2, 0] && IntBinomi
alQ[a, b, c, 2, m, p, x]
```

rule 262
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[c*(c*x)
^(m - 1)*((a + b*x^2)^(p + 1)/(b*(m + 2*p + 1))), x] - Simp[a*c^2*((m - 1)/
(b*(m + 2*p + 1)))   Int[(c*x)^(m - 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b
, c, p}, x] && GtQ[m, 2 - 1] && NeQ[m + 2*p + 1, 0] && IntBinomialQ[a, b, c
, 2, m, p, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3072
```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*tan[(e_.) + (f_.)*(x_)]^(n_.), x_
Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Simp[ff/f   Subst[Int[
(ff*x)^(m + n)/(a^2 - ff^2*x^2)^((n + 1)/2), x], x, a*(Sin[e + f*x]/ff)], x
]] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n + 1)/2]
```

rule 3655
```
Int[(u_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> Int[A
ctivateTrig[u*(a*cos[e + f*x]^2)^p], x] /; FreeQ[{a, b, e, f, p}, x] && EqQ
[a + b, 0]
```

rule 3686
```
Int[(u_.)*((b_.)*sin[(e_.) + (f_.)*(x_)]^(n_))^(p_), x_Symbol] :> With[{ff
= FreeFactors[Sin[e + f*x], x]}, Simp[(b*ff^n)^IntPart[p]*((b*Sin[e + f*x]^
n)^FracPart[p]/(Sin[e + f*x]/ff)^(n*FracPart[p]))   Int[ActivateTrig[u]*(Si
n[e + f*x]/ff)^(n*p), x], x]] /; FreeQ[{b, e, f, n, p}, x] &&  !IntegerQ[p]
 && IntegerQ[n] && (EqQ[u, 1] || MatchQ[u, ((d_.)*(trig_)[e + f*x])^(m_.) /
; FreeQ[{d, m}, x] && MemberQ[{sin, cos, tan, cot, sec, csc}, trig]])
```

Maple [A] (verified)

Time = 0.52 (sec) , antiderivative size = 141, normalized size of antiderivative = 1.55

method	result
default	$\frac{\sqrt{a\sinh(fx+e)^2}\left(2\sqrt{a\sinh(fx+e)^2}\,\cosh(fx+e)^2\sqrt{-a}+3\ln\left(\frac{2\sqrt{-a}\,\sqrt{a\sinh(fx+e)^2-2a}}{\cosh(fx+e)}\right)\cosh(fx+e)^2 a+\sqrt{-a}\,\sqrt{a\sinh(fx+e)^2}\right)}{2\cosh(fx+e)\sqrt{-a}\,\sinh(fx+e)\sqrt{a\cosh(fx+e)^2}\,f}$
risch	$\frac{\sqrt{\left(e^{2fx+2e}+1\right)^2 a\,e^{-2fx-2e}}\,\left(3i\ln(e^{fx}-ie^{-e})e^{5fx+5e}-3i\ln(e^{fx}+ie^{-e})e^{5fx+5e}+6i\ln(e^{fx}-ie^{-e})e^{3fx+3e}-6i\ln(e^{fx}+ie^{-e})e^{3fx}\right.}{2f\left(e^{2fx+2e}+1\right)^3}$

input
```
int((a+a*sinh(f*x+e)^2)^(1/2)*tanh(f*x+e)^4,x,method=_RETURNVERBOSE)
```

output
```
1/2/cosh(f*x+e)*(a*sinh(f*x+e)^2)^(1/2)*(2*(a*sinh(f*x+e)^2)^(1/2)*cosh(f*
x+e)^2*(-a)^(1/2)+3*ln(2/cosh(f*x+e)*((-a)^(1/2)*(a*sinh(f*x+e)^2)^(1/2)-a
))*cosh(f*x+e)^2*a+(-a)^(1/2)*(a*sinh(f*x+e)^2)^(1/2))/(-a)^(1/2)/sinh(f*x
+e)/(a*cosh(f*x+e)^2)^(1/2)/f
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 742 vs. $2(79) = 158$.

Time = 0.11 (sec) , antiderivative size = 742, normalized size of antiderivative = 8.15

$$\int \sqrt{a + a\sinh^2(e + fx)}\tanh^4(e + fx)\,dx = \text{Too large to display}$$

input
```
integrate((a+a*sinh(f*x+e)^2)^(1/2)*tanh(f*x+e)^4,x, algorithm="fricas")
```

output
```
1/2*(6*cosh(f*x + e)*e^(f*x + e)*sinh(f*x + e)^5 + e^(f*x + e)*sinh(f*x +
e)^6 + 3*(5*cosh(f*x + e)^2 + 1)*e^(f*x + e)*sinh(f*x + e)^4 + 4*(5*cosh(f
*x + e)^3 + 3*cosh(f*x + e))*e^(f*x + e)*sinh(f*x + e)^3 + 3*(5*cosh(f*x +
 e)^4 + 6*cosh(f*x + e)^2 - 1)*e^(f*x + e)*sinh(f*x + e)^2 + 6*(cosh(f*x +
 e)^5 + 2*cosh(f*x + e)^3 - cosh(f*x + e))*e^(f*x + e)*sinh(f*x + e) - 6*(
5*cosh(f*x + e)*e^(f*x + e)*sinh(f*x + e)^4 + e^(f*x + e)*sinh(f*x + e)^5
+ 2*(5*cosh(f*x + e)^2 + 1)*e^(f*x + e)*sinh(f*x + e)^3 + 2*(5*cosh(f*x +
e)^3 + 3*cosh(f*x + e))*e^(f*x + e)*sinh(f*x + e)^2 + (5*cosh(f*x + e)^4 +
 6*cosh(f*x + e)^2 + 1)*e^(f*x + e)*sinh(f*x + e) + (cosh(f*x + e)^5 + 2*c
osh(f*x + e)^3 + cosh(f*x + e))*e^(f*x + e))*arctan(cosh(f*x + e) + sinh(f
*x + e)) + (cosh(f*x + e)^6 + 3*cosh(f*x + e)^4 - 3*cosh(f*x + e)^2 - 1)*e
^(f*x + e))*sqrt(a*e^(4*f*x + 4*e) + 2*a*e^(2*f*x + 2*e) + a)*e^(-f*x - e)
/(f*cosh(f*x + e)^5 + (f*e^(2*f*x + 2*e) + f)*sinh(f*x + e)^5 + 5*(f*cosh(
f*x + e)*e^(2*f*x + 2*e) + f*cosh(f*x + e))*sinh(f*x + e)^4 + 2*f*cosh(f*x
 + e)^3 + 2*(5*f*cosh(f*x + e)^2 + (5*f*cosh(f*x + e)^2 + f)*e^(2*f*x + 2*
e) + f)*sinh(f*x + e)^3 + 2*(5*f*cosh(f*x + e)^3 + 3*f*cosh(f*x + e) + (5*
f*cosh(f*x + e)^3 + 3*f*cosh(f*x + e))*e^(2*f*x + 2*e))*sinh(f*x + e)^2 +
f*cosh(f*x + e) + (f*cosh(f*x + e)^5 + 2*f*cosh(f*x + e)^3 + f*cosh(f*x +
e))*e^(2*f*x + 2*e) + (5*f*cosh(f*x + e)^4 + 6*f*cosh(f*x + e)^2 + (5*f*co
sh(f*x + e)^4 + 6*f*cosh(f*x + e)^2 + f)*e^(2*f*x + 2*e) + f)*sinh(f*x ...
```

Sympy [F]

$$\int \sqrt{a+a\sinh^2(e+fx)}\tanh^4(e+fx)\,dx = \int \sqrt{a\left(\sinh^2{(e+fx)}+1\right)}\tanh^4{(e+fx)}\,dx$$

input
```
integrate((a+a*sinh(f*x+e)**2)**(1/2)*tanh(f*x+e)**4,x)
```

output
```
Integral(sqrt(a*(sinh(e + f*x)**2 + 1))*tanh(e + f*x)**4, x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 387 vs. $2(79) = 158$.

Time = 0.16 (sec) , antiderivative size = 387, normalized size of antiderivative = 4.25

$$\int \sqrt{a + a\sinh^2(e + fx)}\tanh^4(e + fx)\,dx$$
$$= \frac{15\sqrt{a}\arctan\left(e^{(-fx-e)}\right)}{8f} + \frac{3\sqrt{a}\arctan\left(e^{(-fx-e)}\right) + \frac{5\sqrt{a}e^{(-fx-e)} + 3\sqrt{a}e^{(-3fx-3e)}}{2e^{(-2fx-2e)} + e^{(-4fx-4e)} + 1}}{4f}$$
$$+ \frac{3\sqrt{a}\arctan\left(e^{(-fx-e)}\right) - \frac{3\sqrt{a}e^{(-fx-e)} + 5\sqrt{a}e^{(-3fx-3e)}}{2e^{(-2fx-2e)} + e^{(-4fx-4e)} + 1}}{4f}$$
$$- \frac{3\left(\sqrt{a}\arctan\left(e^{(-fx-e)}\right) - \frac{\sqrt{a}e^{(-fx-e)} - \sqrt{a}e^{(-3fx-3e)}}{2e^{(-2fx-2e)} + e^{(-4fx-4e)} + 1}\right)}{8f}$$
$$+ \frac{25\sqrt{a}e^{(-2fx-2e)} + 15\sqrt{a}e^{(-4fx-4e)} + 8\sqrt{a}}{16f(e^{(-fx-e)} + 2e^{(-3fx-3e)} + e^{(-5fx-5e)})}$$
$$- \frac{15\sqrt{a}e^{(-fx-e)} + 25\sqrt{a}e^{(-3fx-3e)} + 8\sqrt{a}e^{(-5fx-5e)}}{16f(2e^{(-2fx-2e)} + e^{(-4fx-4e)} + 1)}$$

input
```
integrate((a+a*sinh(f*x+e)^2)^(1/2)*tanh(f*x+e)^4,x, algorithm="maxima")
```

output
```
15/8*sqrt(a)*arctan(e^(-f*x - e))/f + 1/4*(3*sqrt(a)*arctan(e^(-f*x - e))
+ (5*sqrt(a)*e^(-f*x - e) + 3*sqrt(a)*e^(-3*f*x - 3*e))/(2*e^(-2*f*x - 2*e
) + e^(-4*f*x - 4*e) + 1))/f + 1/4*(3*sqrt(a)*arctan(e^(-f*x - e)) - (3*sq
rt(a)*e^(-f*x - e) + 5*sqrt(a)*e^(-3*f*x - 3*e))/(2*e^(-2*f*x - 2*e) + e^(
-4*f*x - 4*e) + 1))/f - 3/8*(sqrt(a)*arctan(e^(-f*x - e)) - (sqrt(a)*e^(-f
*x - e) - sqrt(a)*e^(-3*f*x - 3*e))/(2*e^(-2*f*x - 2*e) + e^(-4*f*x - 4*e)
 + 1))/f + 1/16*(25*sqrt(a)*e^(-2*f*x - 2*e) + 15*sqrt(a)*e^(-4*f*x - 4*e)
 + 8*sqrt(a))/(f*(e^(-f*x - e) + 2*e^(-3*f*x - 3*e) + e^(-5*f*x - 5*e))) -
 1/16*(15*sqrt(a)*e^(-f*x - e) + 25*sqrt(a)*e^(-3*f*x - 3*e) + 8*sqrt(a)*e
^(-5*f*x - 5*e))/(f*(2*e^(-2*f*x - 2*e) + e^(-4*f*x - 4*e) + 1))
```

Giac [A] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 100, normalized size of antiderivative = 1.10

$$\int \sqrt{a+a\sinh^2(e+fx)}\tanh^4(e+fx)\,dx = -\frac{\left(3\,\pi - \frac{4\,\left(e^{(fx+e)}-e^{(-fx-e)}\right)}{\left(e^{(fx+e)}-e^{(-fx-e)}\right)^2+4} + 6\,\arctan\left(\frac{1}{2}\,\left(e^{(2\,fx+2\,e)}-1\right)e^{(-fx-e)}\right) - 2\,e^{(fx+e)} + 2\,e^{(-fx-e)}\right)\sqrt{a}}{4\,f}$$

input
```
integrate((a+a*sinh(f*x+e)^2)^(1/2)*tanh(f*x+e)^4,x, algorithm="giac")
```

output
```
-1/4*(3*pi - 4*(e^(f*x + e) - e^(-f*x - e))/((e^(f*x + e) - e^(-f*x - e))^
2 + 4) + 6*arctan(1/2*(e^(2*f*x + 2*e) - 1)*e^(-f*x - e)) - 2*e^(f*x + e)
+ 2*e^(-f*x - e))*sqrt(a)/f
```

Mupad [F(-1)]

Timed out.

$$\int \sqrt{a+a\sinh^2(e+fx)}\tanh^4(e+fx)\,dx = \int \tanh(e+f\,x)^4\,\sqrt{a\,\sinh\left(e+f\,x\right)^2+a}\,dx$$

input
```
int(tanh(e + f*x)^4*(a + a*sinh(e + f*x)^2)^(1/2),x)
```

output
```
int(tanh(e + f*x)^4*(a + a*sinh(e + f*x)^2)^(1/2), x)
```

Reduce [F]

$$\int \sqrt{a+a\sinh^2(e+fx)}\tanh^4(e+fx)\,dx = \sqrt{a}\left(\int \sqrt{\sinh\left(fx+e\right)^2+1}\,\tanh\left(fx+e\right)^4dx\right)$$

input
```
int((a+a*sinh(f*x+e)^2)^(1/2)*tanh(f*x+e)^4,x)
```

output
```
sqrt(a)*int(sqrt(sinh(e + f*x)**2 + 1)*tanh(e + f*x)**4,x)
```

3.397 $\int \sqrt{a+a\sinh^2(e+fx)}\tanh^2(e+fx)\,dx$

Optimal result

Integrand size = 25, antiderivative size = 57

$$\int \sqrt{a+a\sinh^2(e+fx)}\tanh^2(e+fx)\,dx = -\frac{\arctan(\sinh(e+fx))\sqrt{a\cosh^2(e+fx)}\operatorname{sech}(e+fx)}{f} + \frac{\sqrt{a\cosh^2(e+fx)}\tanh(e+fx)}{f}$$

output
```
-arctan(sinh(f*x+e))*(a*cosh(f*x+e)^2)^(1/2)*sech(f*x+e)/f+(a*cosh(f*x+e)^
2)^(1/2)*tanh(f*x+e)/f
```

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 40, normalized size of antiderivative = 0.70

$$\int \sqrt{a+a\sinh^2(e+fx)}\tanh^2(e+fx)\,dx = \frac{\sqrt{a\cosh^2(e+fx)}\operatorname{sech}(e+fx)(-\arctan(\sinh(e+fx))+\sinh(e+fx))}{f}$$

input
```
Integrate[Sqrt[a + a*Sinh[e + f*x]^2]*Tanh[e + f*x]^2,x]
```

output
```
(Sqrt[a*Cosh[e + f*x]^2]*Sech[e + f*x]*(-ArcTan[Sinh[e + f*x]] + Sinh[e +
f*x]))/f
```

Rubi [A] (verified)

Time = 0.43 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.72, number of steps used = 15, number of rules used = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.560$, Rules used = {3042, 25, 3655, 25, 3042, 25, 3686, 25, 3042, 25, 3072, 25, 262, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \tanh^2(e+fx)\sqrt{a\sinh^2(e+fx)+a}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \tan(ie+ifx)^2\left(-\sqrt{a-a\sin(ie+ifx)^2}\right)dx$$

$$\Big\downarrow \text{25}$$

$$-\int \sqrt{a-a\sin(ie+ifx)^2}\tan(ie+ifx)^2dx$$

$$\Big\downarrow \text{3655}$$

$$-\int -\sqrt{a\cosh^2(e+fx)}\tanh^2(e+fx)dx$$

$$\Big\downarrow \text{25}$$

$$\int \tanh^2(e+fx)\sqrt{a\cosh^2(e+fx)}dx$$

$$\Big\downarrow \text{3042}$$

$$\int -\frac{\sqrt{a\sin\left(ie+ifx+\frac{\pi}{2}\right)^2}}{\tan\left(ie+ifx+\frac{\pi}{2}\right)^2}dx$$

$$\Big\downarrow \text{25}$$

$$-\int \frac{\sqrt{a\sin\left(\frac{1}{2}(2ie+\pi)+ifx\right)^2}}{\tan\left(\frac{1}{2}(2ie+\pi)+ifx\right)^2}dx$$

$\Big\downarrow$ 3686

$$\operatorname{sech}(e+fx)\left(-\sqrt{a\cosh^2(e+fx)}\right)\int -\sinh(e+fx)\tanh(e+fx)dx$$

$\Big\downarrow$ 25

$$\operatorname{sech}(e+fx)\sqrt{a\cosh^2(e+fx)}\int \sinh(e+fx)\tanh(e+fx)dx$$

$\Big\downarrow$ 3042

$$\operatorname{sech}(e+fx)\sqrt{a\cosh^2(e+fx)}\int -\sin(ie+ifx)\tan(ie+ifx)dx$$

$\Big\downarrow$ 25

$$\operatorname{sech}(e+fx)\left(-\sqrt{a\cosh^2(e+fx)}\right)\int \sin(ie+ifx)\tan(ie+ifx)dx$$

$\Big\downarrow$ 3072

$$-\frac{\operatorname{sech}(e+fx)\sqrt{a\cosh^2(e+fx)}\int -\frac{\sinh^2(e+fx)}{\sinh^2(e+fx)+1}d\sinh(e+fx)}{f}$$

$\Big\downarrow$ 25

$$\frac{\operatorname{sech}(e+fx)\sqrt{a\cosh^2(e+fx)}\int \frac{\sinh^2(e+fx)}{\sinh^2(e+fx)+1}d\sinh(e+fx)}{f}$$

$\Big\downarrow$ 262

$$-\frac{\operatorname{sech}(e+fx)\sqrt{a\cosh^2(e+fx)}\left(\int \frac{1}{\sinh^2(e+fx)+1}d\sinh(e+fx)-\sinh(e+fx)\right)}{f}$$

$\Big\downarrow$ 216

$$-\frac{\operatorname{sech}(e+fx)\sqrt{a\cosh^2(e+fx)}(\arctan(\sinh(e+fx))-\sinh(e+fx))}{f}$$

input
```
Int[Sqrt[a + a*Sinh[e + f*x]^2]*Tanh[e + f*x]^2,x]
```

output
```
-((Sqrt[a*Cosh[e + f*x]^2]*Sech[e + f*x]*(ArcTan[Sinh[e + f*x]] - Sinh[e +
 f*x]))/f)
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 262
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[c*(c*x)
^(m - 1)*((a + b*x^2)^(p + 1)/(b*(m + 2*p + 1))), x] - Simp[a*c^2*((m - 1)/
(b*(m + 2*p + 1)))   Int[(c*x)^(m - 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b
, c, p}, x] && GtQ[m, 2 - 1] && NeQ[m + 2*p + 1, 0] && IntBinomialQ[a, b, c
, 2, m, p, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3072
```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*tan[(e_.) + (f_.)*(x_)]^(n_.), x_
Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Simp[ff/f   Subst[Int[
(ff*x)^(m + n)/(a^2 - ff^2*x^2)^((n + 1)/2), x], x, a*(Sin[e + f*x]/ff)], x
]] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n + 1)/2]
```

rule 3655
```
Int[(u_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> Int[A
ctivateTrig[u*(a*cos[e + f*x]^2)^p], x] /; FreeQ[{a, b, e, f, p}, x] && EqQ
[a + b, 0]
```

rule 3686
```
Int[(u_.)*((b_.)*sin[(e_.) + (f_.)*(x_)]^(n_))^(p_), x_Symbol] :> With[{ff
= FreeFactors[Sin[e + f*x], x]}, Simp[(b*ff^n)^IntPart[p]*((b*Sin[e + f*x]^
n)^FracPart[p]/(Sin[e + f*x]/ff)^(n*FracPart[p]))   Int[ActivateTrig[u]*(Si
n[e + f*x]/ff)^(n*p), x], x]] /; FreeQ[{b, e, f, n, p}, x] &&  !IntegerQ[p]
 && IntegerQ[n] && (EqQ[u, 1] || MatchQ[u, ((d_.)*(trig_)[e + f*x])^(m_.) /
; FreeQ[{d, m}, x] && MemberQ[{sin, cos, tan, cot, sec, csc}, trig]])
```

Maple [A] (verified)

Time = 0.48 (sec) , antiderivative size = 102, normalized size of antiderivative = 1.79

method	result
default	$\frac{\cosh(fx+e)\sqrt{a\sinh(fx+e)^2}\left(\sqrt{-a}\sqrt{a\sinh(fx+e)^2}+\ln\left(\frac{2\sqrt{-a}\sqrt{a\sinh(fx+e)^2}-2a}{\cosh(fx+e)}\right)a\right)}{\sqrt{-a}\sinh(fx+e)\sqrt{a\cosh(fx+e)^2}\,f}$
risch	$\frac{\sqrt{\left(e^{2fx+2e}+1\right)^2 a\,e^{-2fx-2e}}\,e^{2fx+2e}}{2f\left(e^{2fx+2e}+1\right)}-\frac{\sqrt{\left(e^{2fx+2e}+1\right)^2 a\,e^{-2fx-2e}}}{2f\left(e^{2fx+2e}+1\right)}+\frac{i\ln\left(e^{fx}-i e^{-e}\right)\sqrt{\left(e^{2fx+2e}+1\right)^2 a\,e^{-2fx-2e}}\,e^{fx+e}}{f\left(e^{2fx+2e}+1\right)}-\frac{i}{}$

input
```
int((a+a*sinh(f*x+e)^2)^(1/2)*tanh(f*x+e)^2,x,method=_RETURNVERBOSE)
```

output
```
cosh(f*x+e)*(a*sinh(f*x+e)^2)^(1/2)*((-a)^(1/2)*(a*sinh(f*x+e)^2)^(1/2)+ln
(2/cosh(f*x+e)*((-a)^(1/2)*(a*sinh(f*x+e)^2)^(1/2)-a))*a)/(-a)^(1/2)/sinh(
f*x+e)/(a*cosh(f*x+e)^2)^(1/2)/f
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 182 vs. $2(53) = 106$.

Time = 0.10 (sec) , antiderivative size = 182, normalized size of antiderivative = 3.19

$$\int\sqrt{a+a\sinh^2(e+fx)}\tanh^2(e+fx)\,dx$$
$$=\frac{\left(2\cosh\left(fx+e\right)e^{(fx+e)}\sinh\left(fx+e\right)+e^{(fx+e)}\sinh\left(fx+e\right)^2-4\left(\cosh\left(fx+e\right)e^{(fx+e)}+e^{(fx+e)}\sinh\left(j\right.\right.}{2\left(f\cosh\left(fx+e\right)e^{(2fx+2e)}+f\,c\right.}$$

input
```
integrate((a+a*sinh(f*x+e)^2)^(1/2)*tanh(f*x+e)^2,x, algorithm="fricas")
```

output
```
1/2*(2*cosh(f*x + e)*e^(f*x + e)*sinh(f*x + e) + e^(f*x + e)*sinh(f*x + e)
^2 - 4*(cosh(f*x + e)*e^(f*x + e) + e^(f*x + e)*sinh(f*x + e))*arctan(cosh
(f*x + e) + sinh(f*x + e)) + (cosh(f*x + e)^2 - 1)*e^(f*x + e))*sqrt(a*e^(
4*f*x + 4*e) + 2*a*e^(2*f*x + 2*e) + a)*e^(-f*x - e)/(f*cosh(f*x + e)*e^(2
*f*x + 2*e) + f*cosh(f*x + e) + (f*e^(2*f*x + 2*e) + f)*sinh(f*x + e))
```

Sympy [F]

$$\int \sqrt{a+a\sinh^2(e+fx)}\tanh^2(e+fx)\,dx = \int \sqrt{a\left(\sinh^2\left(e+fx\right)+1\right)}\tanh^2\left(e+fx\right)dx$$

input
```
integrate((a+a*sinh(f*x+e)**2)**(1/2)*tanh(f*x+e)**2,x)
```

output
```
Integral(sqrt(a*(sinh(e + f*x)**2 + 1))*tanh(e + f*x)**2, x)
```

Maxima [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 50, normalized size of antiderivative = 0.88

$$\int \sqrt{a+a\sinh^2(e+fx)}\tanh^2(e+fx)\,dx = \frac{2\sqrt{a}\arctan\left(e^{(-fx-e)}\right)}{f} + \frac{\sqrt{a}e^{(fx+e)}}{2f} - \frac{\sqrt{a}e^{(-fx-e)}}{2f}$$

input
```
integrate((a+a*sinh(f*x+e)^2)^(1/2)*tanh(f*x+e)^2,x, algorithm="maxima")
```

output
```
2*sqrt(a)*arctan(e^(-f*x - e))/f + 1/2*sqrt(a)*e^(f*x + e)/f - 1/2*sqrt(a)
*e^(-f*x - e)/f
```

Giac [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.61

$$\int \sqrt{a + a\sinh^2(e + fx)}\tanh^2(e + fx)\,dx = -\frac{\sqrt{a}\left(4\,\arctan\left(e^{(fx+e)}\right) - e^{(fx+e)} + e^{(-fx-e)}\right)}{2\,f}$$

input
```
integrate((a+a*sinh(f*x+e)^2)^(1/2)*tanh(f*x+e)^2,x, algorithm="giac")
```

output
```
-1/2*sqrt(a)*(4*arctan(e^(f*x + e)) - e^(f*x + e) + e^(-f*x - e))/f
```

Mupad [F(-1)]

Timed out.

$$\int \sqrt{a + a\sinh^2(e + fx)}\tanh^2(e + fx)\,dx = \int \tanh(e + f\,x)^2\,\sqrt{a\sinh\left(e + f\,x\right)^2 + a}\,dx$$

input
```
int(tanh(e + f*x)^2*(a + a*sinh(e + f*x)^2)^(1/2),x)
```

output
```
int(tanh(e + f*x)^2*(a + a*sinh(e + f*x)^2)^(1/2), x)
```

Reduce [F]

$$\int \sqrt{a + a\sinh^2(e + fx)}\tanh^2(e + fx)\,dx = \sqrt{a}\left(\int \sqrt{\sinh\left(fx + e\right)^2 + 1}\,\tanh\left(fx + e\right)^2 dx\right)$$

input
```
int((a+a*sinh(f*x+e)^2)^(1/2)*tanh(f*x+e)^2,x)
```

output
```
sqrt(a)*int(sqrt(sinh(e + f*x)**2 + 1)*tanh(e + f*x)**2,x)
```

3.398 $\int \coth^2(e+fx)\sqrt{a+a\sinh^2(e+fx)}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 56

$$\int \coth^2(e+fx)\sqrt{a+a\sinh^2(e+fx)}\,dx = -\frac{\sqrt{a\cosh^2(e+fx)}\text{csch}(e+fx)\text{sech}(e+fx)}{f}+\frac{\sqrt{a\cosh^2(e+fx)}\tanh(e+fx)}{f}$$

output
```
-(a*cosh(f*x+e)^2)^(1/2)*csch(f*x+e)*sech(f*x+e)/f+(a*cosh(f*x+e)^2)^(1/2)
*tanh(f*x+e)/f
```

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.62

$$\int \coth^2(e+fx)\sqrt{a+a\sinh^2(e+fx)}\,dx = -\frac{\sqrt{a\cosh^2(e+fx)}\left(-1+\text{csch}^2(e+fx)\right)\tanh(e+fx)}{f}$$

input
```
Integrate[Coth[e + f*x]^2*Sqrt[a + a*Sinh[e + f*x]^2],x]
```

output
```
-((Sqrt[a*Cosh[e + f*x]^2]*(-1 + Csch[e + f*x]^2)*Tanh[e + f*x])/f)
```

Rubi [C] (verified)

Result contains complex when optimal does not.

Time = 0.46 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.86, number of steps used = 14, number of rules used = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.520, Rules used = {3042, 25, 3655, 25, 3042, 25, 3686, 25, 3042, 25, 3070, 244, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \coth^2(e+fx)\sqrt{a\sinh^2(e+fx)+a}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int -\frac{\sqrt{a-a\sin(ie+ifx)^2}}{\tan(ie+ifx)^2}dx$$

$$\Big\downarrow \text{25}$$

$$-\int \frac{\sqrt{a-a\sin(ie+ifx)^2}}{\tan(ie+ifx)^2}dx$$

$$\Big\downarrow \text{3655}$$

$$-\int -\sqrt{a\cosh^2(e+fx)}\coth^2(e+fx)dx$$

$$\Big\downarrow \text{25}$$

$$\int \coth^2(e+fx)\sqrt{a\cosh^2(e+fx)}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \tan\left(ie+ifx+\frac{\pi}{2}\right)^2\left(-\sqrt{a\sin\left(ie+ifx+\frac{\pi}{2}\right)^2}\right)dx$$

$$\Big\downarrow \text{25}$$

$$-\int \sqrt{a\sin\left(\frac{1}{2}(2ie+\pi)+ifx\right)^2}\tan\left(\frac{1}{2}(2ie+\pi)+ifx\right)^2dx$$

$$\Big\downarrow \text{3686}$$

$$\operatorname{sech}(e+fx)\left(-\sqrt{a\cosh^2(e+fx)}\right)\int -\cosh(e+fx)\coth^2(e+fx)dx$$

$$\Big\downarrow \text{25}$$

$$\operatorname{sech}(e+fx)\sqrt{a\cosh^2(e+fx)}\int \cosh(e+fx)\coth^2(e+fx)dx$$

$$\Big\downarrow \text{3042}$$

$$\operatorname{sech}(e+fx)\sqrt{a\cosh^2(e+fx)}\int -\sin\left(ie+ifx+\frac{\pi}{2}\right)\tan\left(ie+ifx+\frac{\pi}{2}\right)^2dx$$

$$\Big\downarrow \text{25}$$

$$\operatorname{sech}(e+fx)\left(-\sqrt{a\cosh^2(e+fx)}\right)\int \sin\left(\frac{1}{2}(2ie+\pi)+ifx\right)\tan\left(\frac{1}{2}(2ie+\pi)+ifx\right)^2dx$$

$$\Big\downarrow \text{3070}$$

$$-\frac{i\operatorname{sech}(e+fx)\sqrt{a\cosh^2(e+fx)}\int -\operatorname{csch}^2(e+fx)\left(\sinh^2(e+fx)+1\right)d(-i\sinh(e+fx))}{f}$$

$$\Big\downarrow \text{244}$$

$$-\frac{i\operatorname{sech}(e+fx)\sqrt{a\cosh^2(e+fx)}\int \left(-\operatorname{csch}^2(e+fx)-1\right)d(-i\sinh(e+fx))}{f}$$

$$\Big\downarrow \text{2009}$$

$$-\frac{i\operatorname{sech}(e+fx)\sqrt{a\cosh^2(e+fx)}(i\sinh(e+fx)-i\operatorname{csch}(e+fx))}{f}$$

input
```
Int[Coth[e + f*x]^2*Sqrt[a + a*Sinh[e + f*x]^2],x]
```

output
```
((-I)*Sqrt[a*Cosh[e + f*x]^2]*Sech[e + f*x]*((-I)*Csch[e + f*x] + I*Sinh[e
 + f*x]))/f
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 244
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[Expand
Integrand[(c*x)^m*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x] && IGtQ[p
, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3070
```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*tan[(e_.) + (f_.)*(x_)]^(n_.), x_Symbol]
:> Simp[-f^(-1)   Subst[Int[(1 - x^2)^((m + n - 1)/2)/x^n, x], x, Cos[e + f
*x]], x] /; FreeQ[{e, f}, x] && IntegersQ[m, n, (m + n - 1)/2]
```

rule 3655
```
Int[(u_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> Int[A
ctivateTrig[u*(a*cos[e + f*x]^2)^p], x] /; FreeQ[{a, b, e, f, p}, x] && EqQ
[a + b, 0]
```

rule 3686
```
Int[(u_.)*((b_.)*sin[(e_.) + (f_.)*(x_)]^(n_))^(p_), x_Symbol] :> With[{ff
= FreeFactors[Sin[e + f*x], x]}, Simp[(b*ff^n)^IntPart[p]*((b*Sin[e + f*x]^
n)^FracPart[p]/(Sin[e + f*x]/ff)^(n*FracPart[p]))   Int[ActivateTrig[u]*(Si
n[e + f*x]/ff)^(n*p), x], x]] /; FreeQ[{b, e, f, n, p}, x] &&  !IntegerQ[p]
 && IntegerQ[n] && (EqQ[u, 1] || MatchQ[u, ((d_.)*(trig_)[e + f*x])^(m_.) /
; FreeQ[{d, m}, x] && MemberQ[{sin, cos, tan, cot, sec, csc}, trig]])
```

Maple [A] (verified)

Time = 0.45 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.75

method	result	size
default	$\frac{\cosh(fx+e)a\left(\cosh(fx+e)^2-2\right)}{\sinh(fx+e)\sqrt{a\cosh(fx+e)^2}\,f}$	42
risch	$\frac{\sqrt{\left(e^{2fx+2e}+1\right)^2 a\,e^{-2fx-2e}}\,\left(e^{4fx+4e}-6\,e^{2fx+2e}+1\right)}{2\left(e^{2fx+2e}-1\right)f\left(e^{2fx+2e}+1\right)}$	80

input
```
int(coth(f*x+e)^2*(a+a*sinh(f*x+e)^2)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
cosh(f*x+e)*a*(cosh(f*x+e)^2-2)/sinh(f*x+e)/(a*cosh(f*x+e)^2)^(1/2)/f
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 317 vs. 2(52) = 104.

Time = 0.10 (sec) , antiderivative size = 317, normalized size of antiderivative = 5.66

$$\int \coth^2(e+fx)\sqrt{a+a\sinh^2(e+fx)}\,dx$$
$$= \frac{\left(4\cosh\left(fx+e\right)e^{(fx+e)}\sinh\left(fx+e\right)^3+e^{(fx+e)}\sinh\left(fx+e\right)^4+6\left(\cosh\left(fx+e\right)^2-1\right)e^{(fx+e)}\sinh\left(f\right.\right.}{2\left(f\cosh\left(fx+e\right)^3+\left(fe^{(2fx+2e)}+f\right)\sinh\left(fx+e\right)^3+3\left(f\cosh\left(fx+e\right)e^{(2fx+2e)}+f\cosh\left(fx+e\right)\right)s\right.}$$

input
```
integrate(coth(f*x+e)^2*(a+a*sinh(f*x+e)^2)^(1/2),x, algorithm="fricas")
```

output
```
1/2*(4*cosh(f*x + e)*e^(f*x + e)*sinh(f*x + e)^3 + e^(f*x + e)*sinh(f*x +
e)^4 + 6*(cosh(f*x + e)^2 - 1)*e^(f*x + e)*sinh(f*x + e)^2 + 4*(cosh(f*x +
 e)^3 - 3*cosh(f*x + e))*e^(f*x + e)*sinh(f*x + e) + (cosh(f*x + e)^4 - 6*
cosh(f*x + e)^2 + 1)*e^(f*x + e))*sqrt(a*e^(4*f*x + 4*e) + 2*a*e^(2*f*x +
2*e) + a)*e^(-f*x - e)/(f*cosh(f*x + e)^3 + (f*e^(2*f*x + 2*e) + f)*sinh(f
*x + e)^3 + 3*(f*cosh(f*x + e)*e^(2*f*x + 2*e) + f*cosh(f*x + e))*sinh(f*x
 + e)^2 - f*cosh(f*x + e) + (f*cosh(f*x + e)^3 - f*cosh(f*x + e))*e^(2*f*x
 + 2*e) + (3*f*cosh(f*x + e)^2 + (3*f*cosh(f*x + e)^2 - f)*e^(2*f*x + 2*e)
 - f)*sinh(f*x + e))
```

Sympy [F]

$$\int \coth^2(e+fx)\sqrt{a+a\sinh^2(e+fx)}\,dx=\int \sqrt{a\left(\sinh^2\left(e+fx\right)+1\right)}\coth^2\left(e+fx\right)dx$$

input
```
integrate(coth(f*x+e)**2*(a+a*sinh(f*x+e)**2)**(1/2),x)
```

output
```
Integral(sqrt(a*(sinh(e + f*x)**2 + 1))*coth(e + f*x)**2, x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 125 vs. $2(52) = 104$.

Time = 0.14 (sec) , antiderivative size = 125, normalized size of antiderivative = 2.23

$$\int \coth^2(e+fx)\sqrt{a+a\sinh^2(e+fx)}\,dx=\frac{\sqrt{a}e^{(-fx-e)}}{f(e^{(-2\,fx-2\,e)}-1)}-\frac{2\,\sqrt{a}e^{(-2\,fx-2\,e)}-\sqrt{a}}{2\,f\left(e^{(-fx-e)}-e^{(-3\,fx-3\,e)}\right)}+\frac{2\,\sqrt{a}e^{(-fx-e)}-\sqrt{a}e^{(-3\,fx-3\,e)}}{2\,f\left(e^{(-2\,fx-2\,e)}-1\right)}$$

input
```
integrate(coth(f*x+e)^2*(a+a*sinh(f*x+e)^2)^(1/2),x, algorithm="maxima")
```

output
```
sqrt(a)*e^(-f*x - e)/(f*(e^(-2*f*x - 2*e) - 1)) - 1/2*(2*sqrt(a)*e^(-2*f*x
 - 2*e) - sqrt(a))/(f*(e^(-f*x - e) - e^(-3*f*x - 3*e))) + 1/2*(2*sqrt(a)*
e^(-f*x - e) - sqrt(a)*e^(-3*f*x - 3*e))/(f*(e^(-2*f*x - 2*e) - 1))
```

Giac [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.86

$$\int \coth^2(e+fx)\sqrt{a+a\sinh^2(e+fx)}\,dx = -\frac{\sqrt{a}\left(\frac{4}{e^{(fx+e)}-e^{(-fx-e)}}-e^{(fx+e)}+e^{(-fx-e)}\right)}{2\,f}$$

input
```
integrate(coth(f*x+e)^2*(a+a*sinh(f*x+e)^2)^(1/2),x, algorithm="giac")
```

output
```
-1/2*sqrt(a)*(4/(e^(f*x + e) - e^(-f*x - e)) - e^(f*x + e) + e^(-f*x - e))
/f
```

Mupad [B] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 67, normalized size of antiderivative = 1.20

$$\int \coth^2(e+fx)\sqrt{a+a\sinh^2(e+fx)}\,dx = \frac{\sqrt{a+a\left(\frac{e^{e+fx}}{2}-\frac{e^{-e-fx}}{2}\right)^2}\,\left(e^{4e+4fx}-6\,e^{2e+2fx}+1\right)}{f\,\left(e^{4e+4fx}-1\right)}$$

input
```
int(coth(e + f*x)^2*(a + a*sinh(e + f*x)^2)^(1/2),x)
```

output
```
((a + a*(exp(e + f*x)/2 - exp(- e - f*x)/2)^2)^(1/2)*(exp(4*e + 4*f*x) - 6
*exp(2*e + 2*f*x) + 1))/(f*(exp(4*e + 4*f*x) - 1))
```

Reduce [F]

$$\int \coth^2(e+fx)\sqrt{a+a\sinh^2(e+fx)}\,dx=\sqrt{a}\left(\int \sqrt{\sinh(fx+e)^2+1}\,\coth(fx+e)^2\,dx\right)$$

input
```
int(coth(f*x+e)^2*(a+a*sinh(f*x+e)^2)^(1/2),x)
```

output
```
sqrt(a)*int(sqrt(sinh(e + f*x)**2 + 1)*coth(e + f*x)**2,x)
```

3.399 $\int \coth^4(e+fx)\sqrt{a+a\sinh^2(e+fx)}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 91

$$\int \coth^4(e+fx)\sqrt{a+a\sinh^2(e+fx)}\,dx = -\frac{2\sqrt{a\cosh^2(e+fx)}\operatorname{csch}(e+fx)\operatorname{sech}(e+fx)}{f} - \frac{\sqrt{a\cosh^2(e+fx)}\operatorname{csch}^3(e+fx)\operatorname{sech}(e+fx)}{3f} + \frac{\sqrt{a\cosh^2(e+fx)}\tanh(e+fx)}{f}$$

output
```
-2*(a*cosh(f*x+e)^2)^(1/2)*csch(f*x+e)*sech(f*x+e)/f-1/3*(a*cosh(f*x+e)^2)
^(1/2)*csch(f*x+e)^3*sech(f*x+e)/f+(a*cosh(f*x+e)^2)^(1/2)*tanh(f*x+e)/f
```

Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.52

$$\int \coth^4(e+fx)\sqrt{a+a\sinh^2(e+fx)}\,dx = -\frac{\sqrt{a\cosh^2(e+fx)}\left(-3+6\operatorname{csch}^2(e+fx)+\operatorname{csch}^4(e+fx)\right)\tanh(e+fx)}{3f}$$

input
```
Integrate[Coth[e + f*x]^4*Sqrt[a + a*Sinh[e + f*x]^2],x]
```

output
```
-1/3*(Sqrt[a*Cosh[e + f*x]^2]*(-3 + 6*Csch[e + f*x]^2 + Csch[e + f*x]^4)*T
anh[e + f*x])/f
```

Rubi [C] (verified)

Result contains complex when optimal does not.

Time = 0.43 (sec) , antiderivative size = 62, normalized size of antiderivative = 0.68, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.320, Rules used = {3042, 3655, 3042, 3686, 3042, 3070, 244, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \coth^4(e+fx)\sqrt{a\sinh^2(e+fx)+a}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\sqrt{a-a\sin(ie+ifx)^2}}{\tan(ie+ifx)^4}dx$$

$$\Big\downarrow \text{3655}$$

$$\int \coth^4(e+fx)\sqrt{a\cosh^2(e+fx)}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \tan\left(ie+ifx+\frac{\pi}{2}\right)^4\sqrt{a\sin\left(ie+ifx+\frac{\pi}{2}\right)^2}dx$$

$$\Big\downarrow \text{3686}$$

$$\text{sech}(e+fx)\sqrt{a\cosh^2(e+fx)}\int \cosh(e+fx)\coth^4(e+fx)dx$$

$$\Big\downarrow \text{3042}$$

$$\text{sech}(e+fx)\sqrt{a\cosh^2(e+fx)}\int \sin\left(ie+ifx+\frac{\pi}{2}\right)\tan\left(ie+ifx+\frac{\pi}{2}\right)^4dx$$

$$\Big\downarrow \text{3070}$$

$$\frac{i\text{sech}(e+fx)\sqrt{a\cosh^2(e+fx)}\int \text{csch}^4(e+fx)\left(\sinh^2(e+fx)+1\right)^2d(-i\sinh(e+fx))}{f}$$

$$\Big\downarrow \text{ 244}$$

$$\frac{i\text{sech}(e+fx)\sqrt{a\cosh^2(e+fx)}\int \left(\text{csch}^4(e+fx)+2\text{csch}^2(e+fx)+1\right)d(-i\sinh(e+fx))}{f}$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{i\text{sech}(e+fx)\sqrt{a\cosh^2(e+fx)}\left(-i\sinh(e+fx)+\frac{1}{3}i\text{csch}^3(e+fx)+2i\text{csch}(e+fx)\right)}{f}$$

input
```
Int[Coth[e + f*x]^4*Sqrt[a + a*Sinh[e + f*x]^2],x]
```

output
```
(I*Sqrt[a*Cosh[e + f*x]^2]*Sech[e + f*x]*((2*I)*Csch[e + f*x] + (I/3)*Csch
[e + f*x]^3 - I*Sinh[e + f*x]))/f
```

Defintions of rubi rules used

rule 244
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[Expand
Integrand[(c*x)^m*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x] && IGtQ[p
, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3070
```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*tan[(e_.) + (f_.)*(x_)]^(n_.), x_Symbol]
:> Simp[-f^(-1)   Subst[Int[(1 - x^2)^((m + n - 1)/2)/x^n, x], x, Cos[e + f
*x]], x] /; FreeQ[{e, f}, x] && IntegersQ[m, n, (m + n - 1)/2]
```

rule 3655
```
Int[(u_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> Int[A
ctivateTrig[u*(a*cos[e + f*x]^2)^p], x] /; FreeQ[{a, b, e, f, p}, x] && EqQ
[a + b, 0]
```

rule 3686
```
Int[(u_.)*((b_.)*sin[(e_.) + (f_.)*(x_)]^(n_))^(p_), x_Symbol] :> With[{ff
= FreeFactors[Sin[e + f*x], x]}, Simp[(b*ff^n)^IntPart[p]*((b*Sin[e + f*x]^
n)^FracPart[p]/(Sin[e + f*x]/ff)^(n*FracPart[p]))   Int[ActivateTrig[u]*(Si
n[e + f*x]/ff)^(n*p), x], x]] /; FreeQ[{b, e, f, n, p}, x] &&  !IntegerQ[p]
 && IntegerQ[n] && (EqQ[u, 1] || MatchQ[u, ((d_.)*(trig_)[e + f*x])^(m_.) /
; FreeQ[{d, m}, x] && MemberQ[{sin, cos, tan, cot, sec, csc}, trig]])
```

Maple [A] (verified)

Time = 0.50 (sec) , antiderivative size = 75, normalized size of antiderivative = 0.82

method	result	size
default	$\frac{\cosh(fx+e)a\left(3\sinh(fx+e)^{4}-6\sinh(fx+e)^{2}-1\right)}{3(\cosh(fx+e)-1)(\cosh(fx+e)+1)\sinh(fx+e)\sqrt{a\cosh(fx+e)^{2}}\,f}$	75
risch	$-\frac{\sqrt{\left(\mathrm{e}^{2fx+2e}+1\right)^{2}a\,\mathrm{e}^{-2fx-2e}}\left(-3\,\mathrm{e}^{8fx+8e}+36\,\mathrm{e}^{6fx+6e}-50\,\mathrm{e}^{4fx+4e}+36\,\mathrm{e}^{2fx+2e}-3\right)}{6\left(\mathrm{e}^{2fx+2e}-1\right)^{3}f\left(\mathrm{e}^{2fx+2e}+1\right)}$	104

input
```
int(coth(f*x+e)^4*(a+a*sinh(f*x+e)^2)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/3*cosh(f*x+e)*a*(3*sinh(f*x+e)^4-6*sinh(f*x+e)^2-1)/(cosh(f*x+e)-1)/(cos
h(f*x+e)+1)/sinh(f*x+e)/(a*cosh(f*x+e)^2)^(1/2)/f
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 885 vs. 2(83) = 166.

Time = 0.13 (sec) , antiderivative size = 885, normalized size of antiderivative = 9.73

$$\int \coth^4(e+fx)\sqrt{a+a\sinh^2(e+fx)}\,dx = \text{Too large to display}$$

input
```
integrate(coth(f*x+e)^4*(a+a*sinh(f*x+e)^2)^(1/2),x, algorithm="fricas")
```

output
```
1/6*(24*cosh(f*x + e)*e^(f*x + e)*sinh(f*x + e)^7 + 3*e^(f*x + e)*sinh(f*x
 + e)^8 + 12*(7*cosh(f*x + e)^2 - 3)*e^(f*x + e)*sinh(f*x + e)^6 + 24*(7*c
osh(f*x + e)^3 - 9*cosh(f*x + e))*e^(f*x + e)*sinh(f*x + e)^5 + 10*(21*cos
h(f*x + e)^4 - 54*cosh(f*x + e)^2 + 5)*e^(f*x + e)*sinh(f*x + e)^4 + 8*(21
*cosh(f*x + e)^5 - 90*cosh(f*x + e)^3 + 25*cosh(f*x + e))*e^(f*x + e)*sinh
(f*x + e)^3 + 12*(7*cosh(f*x + e)^6 - 45*cosh(f*x + e)^4 + 25*cosh(f*x + e
)^2 - 3)*e^(f*x + e)*sinh(f*x + e)^2 + 8*(3*cosh(f*x + e)^7 - 27*cosh(f*x
+ e)^5 + 25*cosh(f*x + e)^3 - 9*cosh(f*x + e))*e^(f*x + e)*sinh(f*x + e) +
 (3*cosh(f*x + e)^8 - 36*cosh(f*x + e)^6 + 50*cosh(f*x + e)^4 - 36*cosh(f*
x + e)^2 + 3)*e^(f*x + e))*sqrt(a*e^(4*f*x + 4*e) + 2*a*e^(2*f*x + 2*e) +
a)*e^(-f*x - e)/(f*cosh(f*x + e)^7 + (f*e^(2*f*x + 2*e) + f)*sinh(f*x + e)
^7 + 7*(f*cosh(f*x + e)*e^(2*f*x + 2*e) + f*cosh(f*x + e))*sinh(f*x + e)^6
 - 3*f*cosh(f*x + e)^5 + 3*(7*f*cosh(f*x + e)^2 + (7*f*cosh(f*x + e)^2 - f
)*e^(2*f*x + 2*e) - f)*sinh(f*x + e)^5 + 5*(7*f*cosh(f*x + e)^3 - 3*f*cosh
(f*x + e) + (7*f*cosh(f*x + e)^3 - 3*f*cosh(f*x + e))*e^(2*f*x + 2*e))*sin
h(f*x + e)^4 + 3*f*cosh(f*x + e)^3 + (35*f*cosh(f*x + e)^4 - 30*f*cosh(f*x
 + e)^2 + (35*f*cosh(f*x + e)^4 - 30*f*cosh(f*x + e)^2 + 3*f)*e^(2*f*x + 2
*e) + 3*f)*sinh(f*x + e)^3 + 3*(7*f*cosh(f*x + e)^5 - 10*f*cosh(f*x + e)^3
 + 3*f*cosh(f*x + e) + (7*f*cosh(f*x + e)^5 - 10*f*cosh(f*x + e)^3 + 3*f*c
osh(f*x + e))*e^(2*f*x + 2*e))*sinh(f*x + e)^2 - f*cosh(f*x + e) + (f*c...
```

Sympy [F]

$$\int \coth^4(e+fx)\sqrt{a+a\sinh^2(e+fx)}\,dx=\int\sqrt{a\left(\sinh^2\left(e+fx\right)+1\right)}\coth^4\left(e+fx\right)dx$$

input
```
integrate(coth(f*x+e)**4*(a+a*sinh(f*x+e)**2)**(1/2),x)
```

output
```
Integral(sqrt(a*(sinh(e + f*x)**2 + 1))*coth(e + f*x)**4, x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 487 vs. $2(83) = 166$.

Time = 0.16 (sec) , antiderivative size = 487, normalized size of antiderivative = 5.35

$$\int \coth^4(e+fx)\sqrt{a+a\sinh^2(e+fx)}\,dx = -\frac{3\sqrt{a}\log\left(e^{(-fx-e)}+1\right)-3\sqrt{a}\log\left(e^{(-fx-e)}-1\right)-\frac{2\left(9\sqrt{a}e^{(-fx-e)}-8\sqrt{a}e^{(-3fx-3e)}+3\sqrt{a}e^{(-5fx-5e)}\right)}{3e^{(-2fx-2e)}-3e^{(-4fx-4e)}+e^{(-6fx-6e)}-1}}{12f} + \frac{3\sqrt{a}\log\left(e^{(-fx-e)}+1\right)-3\sqrt{a}\log\left(e^{(-fx-e)}-1\right)+\frac{2\left(3\sqrt{a}e^{(-fx-e)}-8\sqrt{a}e^{(-3fx-3e)}+9\sqrt{a}e^{(-5fx-5e)}\right)}{3e^{(-2fx-2e)}-3e^{(-4fx-4e)}+e^{(-6fx-6e)}-1}}{12f} + \frac{\sqrt{a}e^{(-3fx-3e)}}{f\left(3e^{(-2fx-2e)}-3e^{(-4fx-4e)}+e^{(-6fx-6e)}-1\right)} - \frac{33\sqrt{a}e^{(-2fx-2e)}-40\sqrt{a}e^{(-4fx-4e)}+15\sqrt{a}e^{(-6fx-6e)}-6\sqrt{a}}{12f\left(e^{(-fx-e)}-3e^{(-3fx-3e)}+3e^{(-5fx-5e)}-e^{(-7fx-7e)}\right)} + \frac{15\sqrt{a}e^{(-fx-e)}-40\sqrt{a}e^{(-3fx-3e)}+33\sqrt{a}e^{(-5fx-5e)}-6\sqrt{a}e^{(-7fx-7e)}}{12f\left(3e^{(-2fx-2e)}-3e^{(-4fx-4e)}+e^{(-6fx-6e)}-1\right)}$$

input
```
integrate(coth(f*x+e)^4*(a+a*sinh(f*x+e)^2)^(1/2),x, algorithm="maxima")
```

output
```
-1/12*(3*sqrt(a)*log(e^(-f*x - e) + 1) - 3*sqrt(a)*log(e^(-f*x - e) - 1) -
 2*(9*sqrt(a)*e^(-f*x - e) - 8*sqrt(a)*e^(-3*f*x - 3*e) + 3*sqrt(a)*e^(-5*
f*x - 5*e))/(3*e^(-2*f*x - 2*e) - 3*e^(-4*f*x - 4*e) + e^(-6*f*x - 6*e) -
1))/f + 1/12*(3*sqrt(a)*log(e^(-f*x - e) + 1) - 3*sqrt(a)*log(e^(-f*x - e)
 - 1) + 2*(3*sqrt(a)*e^(-f*x - e) - 8*sqrt(a)*e^(-3*f*x - 3*e) + 9*sqrt(a)
*e^(-5*f*x - 5*e))/(3*e^(-2*f*x - 2*e) - 3*e^(-4*f*x - 4*e) + e^(-6*f*x -
6*e) - 1))/f + sqrt(a)*e^(-3*f*x - 3*e)/(f*(3*e^(-2*f*x - 2*e) - 3*e^(-4*f
*x - 4*e) + e^(-6*f*x - 6*e) - 1)) - 1/12*(33*sqrt(a)*e^(-2*f*x - 2*e) - 4
0*sqrt(a)*e^(-4*f*x - 4*e) + 15*sqrt(a)*e^(-6*f*x - 6*e) - 6*sqrt(a))/(f*(
e^(-f*x - e) - 3*e^(-3*f*x - 3*e) + 3*e^(-5*f*x - 5*e) - e^(-7*f*x - 7*e))
) + 1/12*(15*sqrt(a)*e^(-f*x - e) - 40*sqrt(a)*e^(-3*f*x - 3*e) + 33*sqrt(
a)*e^(-5*f*x - 5*e) - 6*sqrt(a)*e^(-7*f*x - 7*e))/(f*(3*e^(-2*f*x - 2*e) -
 3*e^(-4*f*x - 4*e) + e^(-6*f*x - 6*e) - 1))
```

Giac [A] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 74, normalized size of antiderivative = 0.81

$$\int \coth^4(e+fx)\sqrt{a+a\sinh^2(e+fx)}\,dx$$
$$= -\frac{\sqrt{a}\left(\frac{8\left(3\left(e^{(fx+e)}-e^{(-fx-e)}\right)^2+2\right)}{\left(e^{(fx+e)}-e^{(-fx-e)}\right)^3}-3\,e^{(fx+e)}+3\,e^{(-fx-e)}\right)}{6\,f}$$

input
```
integrate(coth(f*x+e)^4*(a+a*sinh(f*x+e)^2)^(1/2),x, algorithm="giac")
```

output
```
-1/6*sqrt(a)*(8*(3*(e^(f*x + e) - e^(-f*x - e))^2 + 2)/(e^(f*x + e) - e^(-
f*x - e))^3 - 3*e^(f*x + e) + 3*e^(-f*x - e))/f
```

Mupad [B] (verification not implemented)

Time = 1.87 (sec) , antiderivative size = 281, normalized size of antiderivative = 3.09

$$\int \coth^4(e+fx)\sqrt{a+a\sinh^2(e+fx)}\,dx = -\frac{\left(\frac{1}{f}-\frac{e^{2\,e+2\,f\,x}}{f}\right)\sqrt{a+a\left(\frac{e^{e+f\,x}}{2}-\frac{e^{-e-f\,x}}{2}\right)^2}}{e^{2\,e+2\,f\,x}+1}$$
$$-\frac{8\,e^{3\,e+3\,f\,x}\sqrt{a+a\left(\frac{e^{e+f\,x}}{2}-\frac{e^{-e-f\,x}}{2}\right)^2}}{f\,\left(e^{2\,e+2\,f\,x}-1\right)\,\left(e^{e+f\,x}+e^{3\,e+3\,f\,x}\right)}$$
$$-\frac{16\,e^{3\,e+3\,f\,x}\sqrt{a+a\left(\frac{e^{e+f\,x}}{2}-\frac{e^{-e-f\,x}}{2}\right)^2}}{3\,f\,\left(e^{2\,e+2\,f\,x}-1\right)^2\,\left(e^{e+f\,x}+e^{3\,e+3\,f\,x}\right)}$$
$$-\frac{16\,e^{3\,e+3\,f\,x}\sqrt{a+a\left(\frac{e^{e+f\,x}}{2}-\frac{e^{-e-f\,x}}{2}\right)^2}}{3\,f\,\left(e^{2\,e+2\,f\,x}-1\right)^3\,\left(e^{e+f\,x}+e^{3\,e+3\,f\,x}\right)}$$

input
```
int(coth(e + f*x)^4*(a + a*sinh(e + f*x)^2)^(1/2),x)
```

output
```
- ((1/f - exp(2*e + 2*f*x)/f)*(a + a*(exp(e + f*x)/2 - exp(- e - f*x)/2)^2
)^(1/2))/(exp(2*e + 2*f*x) + 1) - (8*exp(3*e + 3*f*x)*(a + a*(exp(e + f*x)
/2 - exp(- e - f*x)/2)^2)^(1/2))/(f*(exp(2*e + 2*f*x) - 1)*(exp(e + f*x) +
 exp(3*e + 3*f*x))) - (16*exp(3*e + 3*f*x)*(a + a*(exp(e + f*x)/2 - exp(-
e - f*x)/2)^2)^(1/2))/(3*f*(exp(2*e + 2*f*x) - 1)^2*(exp(e + f*x) + exp(3*
e + 3*f*x))) - (16*exp(3*e + 3*f*x)*(a + a*(exp(e + f*x)/2 - exp(- e - f*x
)/2)^2)^(1/2))/(3*f*(exp(2*e + 2*f*x) - 1)^3*(exp(e + f*x) + exp(3*e + 3*f
*x)))
```

Reduce [F]

$$\int \coth^4(e+fx)\sqrt{a+a\sinh^2(e+fx)}\,dx=\sqrt{a}\left(\int\sqrt{\sinh(fx+e)^2+1}\coth(fx+e)^4\,dx\right)$$

input
```
int(coth(f*x+e)^4*(a+a*sinh(f*x+e)^2)^(1/2),x)
```

output
```
sqrt(a)*int(sqrt(sinh(e + f*x)**2 + 1)*coth(e + f*x)**4,x)
```

3.400 $\int \coth^6(e+fx)\sqrt{a+a\sinh^2(e+fx)}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 124

$$\int \coth^6(e+fx)\sqrt{a+a\sinh^2(e+fx)}\,dx = -\frac{3\sqrt{a\cosh^2(e+fx)}\text{csch}(e+fx)\text{sech}(e+fx)}{f} - \frac{\sqrt{a\cosh^2(e+fx)}\text{csch}^3(e+fx)\text{sech}(e+fx)}{f} - \frac{\sqrt{a\cosh^2(e+fx)}\text{csch}^5(e+fx)\text{sech}(e+fx)}{5f} + \frac{\sqrt{a\cosh^2(e+fx)}\tanh(e+fx)}{f}$$

output

```
-3*(a*cosh(f*x+e)^2)^(1/2)*csch(f*x+e)*sech(f*x+e)/f-(a*cosh(f*x+e)^2)^(1/
2)*csch(f*x+e)^3*sech(f*x+e)/f-1/5*(a*cosh(f*x+e)^2)^(1/2)*csch(f*x+e)^5*s
ech(f*x+e)/f+(a*cosh(f*x+e)^2)^(1/2)*tanh(f*x+e)/f
```

Mathematica [A] (verified)

Time = 0.13 (sec) , antiderivative size = 67, normalized size of antiderivative = 0.54

$$\int \coth^6(e+fx)\sqrt{a+a\sinh^2(e+fx)}\,dx$$
$$=\frac{\sqrt{a\cosh^2(e+fx)}(-182+235\cosh(2(e+fx))-90\cosh(4(e+fx))+5\cosh(6(e+fx)))\operatorname{csch}^5(e+fx}{160f}$$

input
```
Integrate[Coth[e + f*x]^6*Sqrt[a + a*Sinh[e + f*x]^2],x]
```

output
```
(Sqrt[a*Cosh[e + f*x]^2]*(-182 + 235*Cosh[2*(e + f*x)] - 90*Cosh[4*(e + f*
x)] + 5*Cosh[6*(e + f*x)])*Csch[e + f*x]^5*Sech[e + f*x])/(160*f)
```

Rubi [C] (verified)

Result contains complex when optimal does not.

Time = 0.46 (sec) , antiderivative size = 74, normalized size of antiderivative = 0.60, number of steps used = 14, number of rules used = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.520, Rules used = {3042, 25, 3655, 25, 3042, 25, 3686, 25, 3042, 25, 3070, 244, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \coth^6(e+fx)\sqrt{a\sinh^2(e+fx)+a}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int -\frac{\sqrt{a-a\sin(ie+ifx)^2}}{\tan(ie+ifx)^6}dx$$

$$\Big\downarrow \text{25}$$

$$-\int \frac{\sqrt{a-a\sin(ie+ifx)^2}}{\tan(ie+ifx)^6}dx$$

$$\Big\downarrow \text{3655}$$

$$-\int -\sqrt{a\cosh^2(e+fx)}\coth^6(e+fx)dx$$

$\Big\downarrow$ 25

$$\int \coth^6(e+fx)\sqrt{a\cosh^2(e+fx)}dx$$

$\Big\downarrow$ 3042

$$\int \tan\left(ie+ifx+\frac{\pi}{2}\right)^6\left(-\sqrt{a\sin\left(ie+ifx+\frac{\pi}{2}\right)^2}\right)dx$$

$\Big\downarrow$ 25

$$-\int \sqrt{a\sin\left(\frac{1}{2}(2ie+\pi)+ifx\right)^2}\tan\left(\frac{1}{2}(2ie+\pi)+ifx\right)^6dx$$

$\Big\downarrow$ 3686

$$\text{sech}(e+fx)\left(-\sqrt{a\cosh^2(e+fx)}\right)\int -\cosh(e+fx)\coth^6(e+fx)dx$$

$\Big\downarrow$ 25

$$\text{sech}(e+fx)\sqrt{a\cosh^2(e+fx)}\int \cosh(e+fx)\coth^6(e+fx)dx$$

$\Big\downarrow$ 3042

$$\text{sech}(e+fx)\sqrt{a\cosh^2(e+fx)}\int -\sin\left(ie+ifx+\frac{\pi}{2}\right)\tan\left(ie+ifx+\frac{\pi}{2}\right)^6dx$$

$\Big\downarrow$ 25

$$\text{sech}(e+fx)\left(-\sqrt{a\cosh^2(e+fx)}\right)\int \sin\left(\frac{1}{2}(2ie+\pi)+ifx\right)\tan\left(\frac{1}{2}(2ie+\pi)+ifx\right)^6dx$$

$\Big\downarrow$ 3070

$$-\frac{i\text{sech}(e+fx)\sqrt{a\cosh^2(e+fx)}\int -\text{csch}^6(e+fx)\left(\sinh^2(e+fx)+1\right)^3d(-i\sinh(e+fx))}{f}$$

$\Big\downarrow$ 244

$$-\frac{i\text{sech}(e+fx)\sqrt{a\cosh^2(e+fx)}\int \left(-\text{csch}^6(e+fx)-3\text{csch}^4(e+fx)-3\text{csch}^2(e+fx)-1\right)d(-i\sinh(e+fx))}{f}$$

$\Big\downarrow$ 2009

$$-\frac{i\operatorname{sech}(e+fx)\sqrt{a\cosh^2(e+fx)}\left(i\sinh(e+fx)-\frac{1}{5}i\operatorname{csch}^5(e+fx)-i\operatorname{csch}^3(e+fx)-3i\operatorname{csch}(e+fx)\right)}{f}$$

input
```
Int[Coth[e + f*x]^6*Sqrt[a + a*Sinh[e + f*x]^2],x]
```

output
```
((-I)*Sqrt[a*Cosh[e + f*x]^2]*Sech[e + f*x]*((-3*I)*Csch[e + f*x] - I*Csch
[e + f*x]^3 - (I/5)*Csch[e + f*x]^5 + I*Sinh[e + f*x]))/f
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 244
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[Expand
Integrand[(c*x)^m*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x] && IGtQ[p
, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3070
```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*tan[(e_.) + (f_.)*(x_)]^(n_.), x_Symbol]
:> Simp[-f^(-1)   Subst[Int[(1 - x^2)^((m + n - 1)/2)/x^n, x], x, Cos[e + f
*x]], x] /; FreeQ[{e, f}, x] && IntegersQ[m, n, (m + n - 1)/2]
```

rule 3655
```
Int[(u_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> Int[A
ctivateTrig[u*(a*cos[e + f*x]^2)^p], x] /; FreeQ[{a, b, e, f, p}, x] && EqQ
[a + b, 0]
```

rule 3686
```
Int[(u_.)*((b_.)*sin[(e_.) + (f_.)*(x_)]^(n_))^(p_), x_Symbol] :> With[{ff
= FreeFactors[Sin[e + f*x], x]}, Simp[(b*ff^n)^IntPart[p]*((b*Sin[e + f*x]^
n)^FracPart[p]/(Sin[e + f*x]/ff)^(n*FracPart[p]))   Int[ActivateTrig[u]*(Si
n[e + f*x]/ff)^(n*p), x], x]] /; FreeQ[{b, e, f, n, p}, x] &&  !IntegerQ[p]
 && IntegerQ[n] && (EqQ[u, 1] || MatchQ[u, ((d_.)*(trig_)[e + f*x])^(m_.) /
; FreeQ[{d, m}, x] && MemberQ[{sin, cos, tan, cot, sec, csc}, trig]])
```

Maple [A] (verified)

Time = 0.52 (sec) , antiderivative size = 85, normalized size of antiderivative = 0.69

method	result	size
default	$\frac{\cosh(fx+e)a\left(5\sinh(fx+e)^{6}-15\sinh(fx+e)^{4}-5\sinh(fx+e)^{2}-1\right)}{5(\cosh(fx+e)+1)^{2}(\cosh(fx+e)-1)^{2}\sinh(fx+e)\sqrt{a\cosh(fx+e)^{2}}\,f}$	85
risch	$-\frac{\sqrt{\left(e^{2fx+2e}+1\right)^{2}a\,e^{-2fx-2e}}\,\left(-5\,e^{12fx+12e}+90\,e^{10fx+10e}-235\,e^{8fx+8e}+364\,e^{6fx+6e}-235\,e^{4fx+4e}+90\,e^{2fx+2e}-5\right)}{10\left(e^{2fx+2e}-1\right)^{5}f\left(e^{2fx+2e}+1\right)}$	126

input
```
int(coth(f*x+e)^6*(a+a*sinh(f*x+e)^2)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/5*cosh(f*x+e)*a*(5*sinh(f*x+e)^6-15*sinh(f*x+e)^4-5*sinh(f*x+e)^2-1)/(co
sh(f*x+e)+1)^2/(cosh(f*x+e)-1)^2/sinh(f*x+e)/(a*cosh(f*x+e)^2)^(1/2)/f
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1696 vs. 2(114) = 228.

Time = 0.11 (sec) , antiderivative size = 1696, normalized size of antiderivative = 13.68

$$\int \coth^6(e+fx)\sqrt{a+a\sinh^2(e+fx)}\,dx = \text{Too large to display}$$

input
```
integrate(coth(f*x+e)^6*(a+a*sinh(f*x+e)^2)^(1/2),x, algorithm="fricas")
```

output
```
1/10*(60*cosh(f*x + e)*e^(f*x + e)*sinh(f*x + e)^11 + 5*e^(f*x + e)*sinh(f
*x + e)^12 + 30*(11*cosh(f*x + e)^2 - 3)*e^(f*x + e)*sinh(f*x + e)^10 + 10
0*(11*cosh(f*x + e)^3 - 9*cosh(f*x + e))*e^(f*x + e)*sinh(f*x + e)^9 + 5*(
495*cosh(f*x + e)^4 - 810*cosh(f*x + e)^2 + 47)*e^(f*x + e)*sinh(f*x + e)^
8 + 40*(99*cosh(f*x + e)^5 - 270*cosh(f*x + e)^3 + 47*cosh(f*x + e))*e^(f*
x + e)*sinh(f*x + e)^7 + 28*(165*cosh(f*x + e)^6 - 675*cosh(f*x + e)^4 + 2
35*cosh(f*x + e)^2 - 13)*e^(f*x + e)*sinh(f*x + e)^6 + 8*(495*cosh(f*x + e
)^7 - 2835*cosh(f*x + e)^5 + 1645*cosh(f*x + e)^3 - 273*cosh(f*x + e))*e^(
f*x + e)*sinh(f*x + e)^5 + 5*(495*cosh(f*x + e)^8 - 3780*cosh(f*x + e)^6 +
 3290*cosh(f*x + e)^4 - 1092*cosh(f*x + e)^2 + 47)*e^(f*x + e)*sinh(f*x +
e)^4 + 20*(55*cosh(f*x + e)^9 - 540*cosh(f*x + e)^7 + 658*cosh(f*x + e)^5
- 364*cosh(f*x + e)^3 + 47*cosh(f*x + e))*e^(f*x + e)*sinh(f*x + e)^3 + 10
*(33*cosh(f*x + e)^10 - 405*cosh(f*x + e)^8 + 658*cosh(f*x + e)^6 - 546*co
sh(f*x + e)^4 + 141*cosh(f*x + e)^2 - 9)*e^(f*x + e)*sinh(f*x + e)^2 + 4*(
15*cosh(f*x + e)^11 - 225*cosh(f*x + e)^9 + 470*cosh(f*x + e)^7 - 546*cosh
(f*x + e)^5 + 235*cosh(f*x + e)^3 - 45*cosh(f*x + e))*e^(f*x + e)*sinh(f*x
 + e) + (5*cosh(f*x + e)^12 - 90*cosh(f*x + e)^10 + 235*cosh(f*x + e)^8 -
364*cosh(f*x + e)^6 + 235*cosh(f*x + e)^4 - 90*cosh(f*x + e)^2 + 5)*e^(f*x
 + e))*sqrt(a*e^(4*f*x + 4*e) + 2*a*e^(2*f*x + 2*e) + a)*e^(-f*x - e)/(f*c
osh(f*x + e)^11 + (f*e^(2*f*x + 2*e) + f)*sinh(f*x + e)^11 + 11*(f*cosh...
```

Sympy [F(-1)]

Timed out.

$$\int \coth^6(e+fx)\sqrt{a+a\sinh^2(e+fx)}\,dx = \text{Timed out}$$

input
```
integrate(coth(f*x+e)**6*(a+a*sinh(f*x+e)**2)**(1/2),x)
```

output
```
Timed out
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1050 vs. $2(114) = 228$.

Time $= 0.19$ (sec) , antiderivative size $= 1050$, normalized size of antiderivative $= 8.47$

$$\int \coth^6(e+fx)\sqrt{a+a\sinh^2(e+fx)}\,dx = \text{Too large to display}$$

input
```
integrate(coth(f*x+e)^6*(a+a*sinh(f*x+e)^2)^(1/2),x, algorithm="maxima")
```

output
```
-1/320*(105*sqrt(a)*log(e^(-f*x - e) + 1) - 105*sqrt(a)*log(e^(-f*x - e) -
 1) - 2*(375*sqrt(a)*e^(-f*x - e) - 790*sqrt(a)*e^(-3*f*x - 3*e) + 896*sqr
t(a)*e^(-5*f*x - 5*e) - 490*sqrt(a)*e^(-7*f*x - 7*e) + 105*sqrt(a)*e^(-9*f
*x - 9*e))/(5*e^(-2*f*x - 2*e) - 10*e^(-4*f*x - 4*e) + 10*e^(-6*f*x - 6*e)
 - 5*e^(-8*f*x - 8*e) + e^(-10*f*x - 10*e) - 1))/f + 1/320*(105*sqrt(a)*lo
g(e^(-f*x - e) + 1) - 105*sqrt(a)*log(e^(-f*x - e) - 1) + 2*(105*sqrt(a)*e
^(-f*x - e) - 490*sqrt(a)*e^(-3*f*x - 3*e) + 896*sqrt(a)*e^(-5*f*x - 5*e)
- 790*sqrt(a)*e^(-7*f*x - 7*e) + 375*sqrt(a)*e^(-9*f*x - 9*e))/(5*e^(-2*f*
x - 2*e) - 10*e^(-4*f*x - 4*e) + 10*e^(-6*f*x - 6*e) - 5*e^(-8*f*x - 8*e)
+ e^(-10*f*x - 10*e) - 1))/f + 1/256*(15*sqrt(a)*log(e^(-f*x - e) + 1) - 1
5*sqrt(a)*log(e^(-f*x - e) - 1) + 2*(15*sqrt(a)*e^(-f*x - e) + 250*sqrt(a)
*e^(-3*f*x - 3*e) - 128*sqrt(a)*e^(-5*f*x - 5*e) + 70*sqrt(a)*e^(-7*f*x -
7*e) - 15*sqrt(a)*e^(-9*f*x - 9*e))/(5*e^(-2*f*x - 2*e) - 10*e^(-4*f*x - 4
*e) + 10*e^(-6*f*x - 6*e) - 5*e^(-8*f*x - 8*e) + e^(-10*f*x - 10*e) - 1))/
f - 1/256*(15*sqrt(a)*log(e^(-f*x - e) + 1) - 15*sqrt(a)*log(e^(-f*x - e)
- 1) + 2*(15*sqrt(a)*e^(-f*x - e) - 70*sqrt(a)*e^(-3*f*x - 3*e) + 128*sqrt
(a)*e^(-5*f*x - 5*e) - 250*sqrt(a)*e^(-7*f*x - 7*e) - 15*sqrt(a)*e^(-9*f*x
 - 9*e))/(5*e^(-2*f*x - 2*e) - 10*e^(-4*f*x - 4*e) + 10*e^(-6*f*x - 6*e) -
 5*e^(-8*f*x - 8*e) + e^(-10*f*x - 10*e) - 1))/f + 2*sqrt(a)*e^(-5*f*x - 5
*e)/(f*(5*e^(-2*f*x - 2*e) - 10*e^(-4*f*x - 4*e) + 10*e^(-6*f*x - 6*e) ...
```

Giac [A] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 96, normalized size of antiderivative = 0.77

$$\int \coth^6(e+fx)\sqrt{a+a\sinh^2(e+fx)}\,dx$$
$$= -\frac{\sqrt{a}\left(\frac{4\left(15\left(e^{(fx+e)}-e^{(-fx-e)}\right)^4+20\left(e^{(fx+e)}-e^{(-fx-e)}\right)^2+16\right)}{\left(e^{(fx+e)}-e^{(-fx-e)}\right)^5}-5e^{(fx+e)}+5e^{(-fx-e)}\right)}{10f}$$

input
```
integrate(coth(f*x+e)^6*(a+a*sinh(f*x+e)^2)^(1/2),x, algorithm="giac")
```

output
```
-1/10*sqrt(a)*(4*(15*(e^(f*x + e) - e^(-f*x - e))^4 + 20*(e^(f*x + e) - e^
(-f*x - e))^2 + 16)/(e^(f*x + e) - e^(-f*x - e))^5 - 5*e^(f*x + e) + 5*e^(
-f*x - e))/f
```

Mupad [B] (verification not implemented)

Time = 1.87 (sec) , antiderivative size = 427, normalized size of antiderivative = 3.44

$$\int \coth^6(e+fx)\sqrt{a+a\sinh^2(e+fx)}\,dx = -\frac{\left(\frac{1}{f}-\frac{e^{2e+2fx}}{f}\right)\sqrt{a+a\left(\frac{e^{e+fx}}{2}-\frac{e^{-e-fx}}{2}\right)^2}}{e^{2e+2fx}+1}$$
$$-\frac{12e^{3e+3fx}\sqrt{a+a\left(\frac{e^{e+fx}}{2}-\frac{e^{-e-fx}}{2}\right)^2}}{f\left(e^{2e+2fx}-1\right)\left(e^{e+fx}+e^{3e+3fx}\right)}$$
$$-\frac{16e^{3e+3fx}\sqrt{a+a\left(\frac{e^{e+fx}}{2}-\frac{e^{-e-fx}}{2}\right)^2}}{f\left(e^{2e+2fx}-1\right)^2\left(e^{e+fx}+e^{3e+3fx}\right)}$$
$$-\frac{144e^{3e+3fx}\sqrt{a+a\left(\frac{e^{e+fx}}{2}-\frac{e^{-e-fx}}{2}\right)^2}}{5f\left(e^{2e+2fx}-1\right)^3\left(e^{e+fx}+e^{3e+3fx}\right)}$$
$$-\frac{128e^{3e+3fx}\sqrt{a+a\left(\frac{e^{e+fx}}{2}-\frac{e^{-e-fx}}{2}\right)^2}}{5f\left(e^{2e+2fx}-1\right)^4\left(e^{e+fx}+e^{3e+3fx}\right)}$$
$$-\frac{64e^{3e+3fx}\sqrt{a+a\left(\frac{e^{e+fx}}{2}-\frac{e^{-e-fx}}{2}\right)^2}}{5f\left(e^{2e+2fx}-1\right)^5\left(e^{e+fx}+e^{3e+3fx}\right)}$$

input
```
int(coth(e + f*x)^6*(a + a*sinh(e + f*x)^2)^(1/2),x)
```

output
```
- ((1/f - exp(2*e + 2*f*x)/f)*(a + a*(exp(e + f*x)/2 - exp(- e - f*x)/2)^2
)^(1/2))/(exp(2*e + 2*f*x) + 1) - (12*exp(3*e + 3*f*x)*(a + a*(exp(e + f*x
)/2 - exp(- e - f*x)/2)^2)^(1/2))/(f*(exp(2*e + 2*f*x) - 1)*(exp(e + f*x)
+ exp(3*e + 3*f*x))) - (16*exp(3*e + 3*f*x)*(a + a*(exp(e + f*x)/2 - exp(-
 e - f*x)/2)^2)^(1/2))/(f*(exp(2*e + 2*f*x) - 1)^2*(exp(e + f*x) + exp(3*e
 + 3*f*x))) - (144*exp(3*e + 3*f*x)*(a + a*(exp(e + f*x)/2 - exp(- e - f*x
)/2)^2)^(1/2))/(5*f*(exp(2*e + 2*f*x) - 1)^3*(exp(e + f*x) + exp(3*e + 3*f
*x))) - (128*exp(3*e + 3*f*x)*(a + a*(exp(e + f*x)/2 - exp(- e - f*x)/2)^2
)^(1/2))/(5*f*(exp(2*e + 2*f*x) - 1)^4*(exp(e + f*x) + exp(3*e + 3*f*x)))
- (64*exp(3*e + 3*f*x)*(a + a*(exp(e + f*x)/2 - exp(- e - f*x)/2)^2)^(1/2)
)/(5*f*(exp(2*e + 2*f*x) - 1)^5*(exp(e + f*x) + exp(3*e + 3*f*x)))
```

Reduce [F]

$$\int \coth^6(e+fx)\sqrt{a+a\sinh^2(e+fx)}\,dx = \sqrt{a}\left(\int \sqrt{\sinh(fx+e)^2+1}\,\coth(fx+e)^6\,dx\right)$$

input
```
int(coth(f*x+e)^6*(a+a*sinh(f*x+e)^2)^(1/2),x)
```

output
```
sqrt(a)*int(sqrt(sinh(e + f*x)**2 + 1)*coth(e + f*x)**6,x)
```

3.401 $\int \frac{\tanh^5(e+fx)}{\sqrt{a+a\sinh^2(e+fx)}}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 66

$$\int \frac{\tanh^5(e+fx)}{\sqrt{a+a\sinh^2(e+fx)}}\,dx = -\frac{a^2}{5f\left(a\cosh^2(e+fx)\right)^{5/2}} + \frac{2a}{3f\left(a\cosh^2(e+fx)\right)^{3/2}} - \frac{1}{f\sqrt{a\cosh^2(e+fx)}}$$

output
```
-1/5*a^2/f/(a*cosh(f*x+e)^2)^(5/2)+2/3*a/f/(a*cosh(f*x+e)^2)^(3/2)-1/f/(a*
cosh(f*x+e)^2)^(1/2)
```

Mathematica [A] (verified)

Time = 0.16 (sec) , antiderivative size = 43, normalized size of antiderivative = 0.65

$$\int \frac{\tanh^5(e+fx)}{\sqrt{a+a\sinh^2(e+fx)}}\,dx = \frac{-15+10\operatorname{sech}^2(e+fx)-3\operatorname{sech}^4(e+fx)}{15f\sqrt{a\cosh^2(e+fx)}}$$

input
```
Integrate[Tanh[e + f*x]^5/Sqrt[a + a*Sinh[e + f*x]^2],x]
```

output
```
(-15 + 10*Sech[e + f*x]^2 - 3*Sech[e + f*x]^4)/(15*f*Sqrt[a*Cosh[e + f*x]^
2])
```

Rubi [A] (verified)

Time = 0.41 (sec) , antiderivative size = 72, normalized size of antiderivative = 1.09, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.400$, Rules used = {3042, 26, 3655, 26, 3042, 26, 3684, 8, 53, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\tanh^5(e+fx)}{\sqrt{a\sinh^2(e+fx)+a}}dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int -\frac{i\tan(ie+ifx)^5}{\sqrt{a-a\sin(ie+ifx)^2}}dx$$

$$\Big\downarrow \text{ 26}$$

$$-i\int \frac{\tan(ie+ifx)^5}{\sqrt{a-a\sin(ie+ifx)^2}}dx$$

$$\Big\downarrow \text{ 3655}$$

$$-i\int \frac{i\tanh^5(e+fx)}{\sqrt{a\cosh^2(e+fx)}}dx$$

$$\Big\downarrow \text{ 26}$$

$$\int \frac{\tanh^5(e+fx)}{\sqrt{a\cosh^2(e+fx)}}dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \frac{i}{\tan\left(ie+ifx+\frac{\pi}{2}\right)^5\sqrt{a\sin\left(ie+ifx+\frac{\pi}{2}\right)^2}}dx$$

$$\Big\downarrow \text{ 26}$$

$$i \int \frac{1}{\sqrt{a \sin \left(\frac{1}{2}(2ie+\pi)+ifx\right)^2} \tan \left(\frac{1}{2}(2ie+\pi)+ifx\right)^5}dx$$

$$\Big\downarrow \text{ 3684}$$

$$\frac{\int \frac{(1-\cosh^2(e+fx))^2 \text{sech}^6(e+fx)}{\sqrt{a\cosh^2(e+fx)}}d\cosh^2(e+fx)}{2f}$$

$$\Big\downarrow \text{ 8}$$

$$\frac{a^3 \int \frac{(1-\cosh^2(e+fx))^2}{(a\cosh^2(e+fx))^{7/2}}d\cosh^2(e+fx)}{2f}$$

$$\Big\downarrow \text{ 53}$$

$$\frac{a^3 \int \left(\frac{1}{(a\cosh^2(e+fx))^{7/2}}-\frac{2}{(a\cosh^2(e+fx))^{5/2}a}+\frac{1}{(a\cosh^2(e+fx))^{3/2}a^2}\right)d\cosh^2(e+fx)}{2f}$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{a^3 \left(-\frac{2}{a^3\sqrt{a\cosh^2(e+fx)}}+\frac{4}{3a^2(a\cosh^2(e+fx))^{3/2}}-\frac{2}{5a(a\cosh^2(e+fx))^{5/2}}\right)}{2f}$$

input
```
Int[Tanh[e + f*x]^5/Sqrt[a + a*Sinh[e + f*x]^2],x]
```

output
```
(a^3*(-2/(5*a*(a*Cosh[e + f*x]^2)^(5/2)) + 4/(3*a^2*(a*Cosh[e + f*x]^2)^(3
/2)) - 2/(a^3*Sqrt[a*Cosh[e + f*x]^2])))/(2*f)
```

Defintions of rubi rules used

rule 8
```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_))^(p_), x_Symbol] :> Simp[1/a^m   Int[u*(a*
x)^(m + p), x], x] /; FreeQ[{a, m, p}, x] && IntegerQ[m]
```

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 53
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && LeQ[7*m + 4*n + 4, 0])
|| LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3655
```
Int[(u_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> Int[A
ctivateTrig[u*(a*cos[e + f*x]^2)^p], x] /; FreeQ[{a, b, e, f, p}, x] && EqQ
[a + b, 0]
```

rule 3684
```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*tan[(e_.) + (f_.)*(x_)]^(m_.
), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x]^2, x]}, Simp[ff^((m + 1
)/2)/(2*f)   Subst[Int[x^((m - 1)/2)*((b*ff^(n/2)*x^(n/2))^p/(1 - ff*x)^((m
 + 1)/2)), x], x, Sin[e + f*x]^2/ff], x]] /; FreeQ[{b, e, f, p}, x] && Inte
gerQ[(m - 1)/2] && IntegerQ[n/2]
```

Maple [A] (verified)

Time = 0.42 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.73

method	result	size
default	$-\frac{15 \cosh(fx+e)^4-10 \cosh(fx+e)^2+3}{15 \cosh(fx+e)^4 \sqrt{a \cosh(fx+e)^2}\, f}$	48
risch	$-\frac{2\left(15\,{\rm e}^{8fx+8e}+20\,{\rm e}^{6fx+6e}+58\,{\rm e}^{4fx+4e}+20\,{\rm e}^{2fx+2e}+15\right)}{15\sqrt{\left({\rm e}^{2fx+2e}+1\right)^2 a\,{\rm e}^{-2fx-2e}}\,\left({\rm e}^{2fx+2e}+1\right)^4 f}$	91

input
```
int(tanh(f*x+e)^5/(a+a*sinh(f*x+e)^2)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-1/15/cosh(f*x+e)^4*(15*cosh(f*x+e)^4-10*cosh(f*x+e)^2+3)/(a*cosh(f*x+e)^2
)^(1/2)/f
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1387 vs. $2(56) = 112$.

Time = 0.11 (sec) , antiderivative size = 1387, normalized size of antiderivative = 21.02

$$\int \frac{\tanh^5(e+fx)}{\sqrt{a+a\sinh^2(e+fx)}}\,dx = \text{Too large to display}$$

input
```
integrate(tanh(f*x+e)^5/(a+a*sinh(f*x+e)^2)^(1/2),x, algorithm="fricas")
```

output
```
-2/15*(135*cosh(f*x + e)*e^(f*x + e)*sinh(f*x + e)^8 + 15*e^(f*x + e)*sinh
(f*x + e)^9 + 20*(27*cosh(f*x + e)^2 + 1)*e^(f*x + e)*sinh(f*x + e)^7 + 14
0*(9*cosh(f*x + e)^3 + cosh(f*x + e))*e^(f*x + e)*sinh(f*x + e)^6 + 2*(945
*cosh(f*x + e)^4 + 210*cosh(f*x + e)^2 + 29)*e^(f*x + e)*sinh(f*x + e)^5 +
 10*(189*cosh(f*x + e)^5 + 70*cosh(f*x + e)^3 + 29*cosh(f*x + e))*e^(f*x +
 e)*sinh(f*x + e)^4 + 20*(63*cosh(f*x + e)^6 + 35*cosh(f*x + e)^4 + 29*cos
h(f*x + e)^2 + 1)*e^(f*x + e)*sinh(f*x + e)^3 + 20*(27*cosh(f*x + e)^7 + 2
1*cosh(f*x + e)^5 + 29*cosh(f*x + e)^3 + 3*cosh(f*x + e))*e^(f*x + e)*sinh
(f*x + e)^2 + 5*(27*cosh(f*x + e)^8 + 28*cosh(f*x + e)^6 + 58*cosh(f*x + e
)^4 + 12*cosh(f*x + e)^2 + 3)*e^(f*x + e)*sinh(f*x + e) + (15*cosh(f*x + e
)^9 + 20*cosh(f*x + e)^7 + 58*cosh(f*x + e)^5 + 20*cosh(f*x + e)^3 + 15*co
sh(f*x + e))*e^(f*x + e))*sqrt(a*e^(4*f*x + 4*e) + 2*a*e^(2*f*x + 2*e) + a
)*e^(-f*x - e)/(a*f*cosh(f*x + e)^10 + (a*f*e^(2*f*x + 2*e) + a*f)*sinh(f*
x + e)^10 + 5*a*f*cosh(f*x + e)^8 + 10*(a*f*cosh(f*x + e)*e^(2*f*x + 2*e)
+ a*f*cosh(f*x + e))*sinh(f*x + e)^9 + 5*(9*a*f*cosh(f*x + e)^2 + a*f + (9
*a*f*cosh(f*x + e)^2 + a*f)*e^(2*f*x + 2*e))*sinh(f*x + e)^8 + 10*a*f*cosh
(f*x + e)^6 + 40*(3*a*f*cosh(f*x + e)^3 + a*f*cosh(f*x + e) + (3*a*f*cosh(
f*x + e)^3 + a*f*cosh(f*x + e))*e^(2*f*x + 2*e))*sinh(f*x + e)^7 + 10*(21*
a*f*cosh(f*x + e)^4 + 14*a*f*cosh(f*x + e)^2 + a*f + (21*a*f*cosh(f*x + e)
^4 + 14*a*f*cosh(f*x + e)^2 + a*f)*e^(2*f*x + 2*e))*sinh(f*x + e)^6 + 1...
```

Sympy [F]

$$\int \frac{\tanh^5(e+fx)}{\sqrt{a+a\sinh^2(e+fx)}}\,dx = \int \frac{\tanh^5(e+fx)}{\sqrt{a\left(\sinh^2(e+fx)+1\right)}}\,dx$$

input
```
integrate(tanh(f*x+e)**5/(a+a*sinh(f*x+e)**2)**(1/2),x)
```

output
```
Integral(tanh(e + f*x)**5/sqrt(a*(sinh(e + f*x)**2 + 1)), x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 446 vs. $2(56) = 112$.

Time = 0.20 (sec) , antiderivative size = 446, normalized size of antiderivative = 6.76

$$\int \frac{\tanh^5(e+fx)}{\sqrt{a+a\sinh^2(e+fx)}}\,dx = -\frac{2\,e^{(-fx-e)}}{\left(5\,\sqrt{a}e^{(-2\,fx-2\,e)}+10\,\sqrt{a}e^{(-4\,fx-4\,e)}+10\,\sqrt{a}e^{(-6\,fx-6\,e)}+5\,\sqrt{a}e^{(-8\,fx-8\,e)}+\sqrt{a}e^{(-10\,fx-10\,e)}+\sqrt{a}\right)f} - \frac{8\,e^{(-3\,fx-3\,e)}}{3\left(5\,\sqrt{a}e^{(-2\,fx-2\,e)}+10\,\sqrt{a}e^{(-4\,fx-4\,e)}+10\,\sqrt{a}e^{(-6\,fx-6\,e)}+5\,\sqrt{a}e^{(-8\,fx-8\,e)}+\sqrt{a}e^{(-10\,fx-10\,e)}+\sqrt{a}\right)f} - \frac{116\,e^{(-5\,fx-5\,e)}}{15\left(5\,\sqrt{a}e^{(-2\,fx-2\,e)}+10\,\sqrt{a}e^{(-4\,fx-4\,e)}+10\,\sqrt{a}e^{(-6\,fx-6\,e)}+5\,\sqrt{a}e^{(-8\,fx-8\,e)}+\sqrt{a}e^{(-10\,fx-10\,e)}+\sqrt{a}\right)f} - \frac{8\,e^{(-7\,fx-7\,e)}}{3\left(5\,\sqrt{a}e^{(-2\,fx-2\,e)}+10\,\sqrt{a}e^{(-4\,fx-4\,e)}+10\,\sqrt{a}e^{(-6\,fx-6\,e)}+5\,\sqrt{a}e^{(-8\,fx-8\,e)}+\sqrt{a}e^{(-10\,fx-10\,e)}+\sqrt{a}\right)f} - \frac{2\,e^{(-9\,fx-9\,e)}}{\left(5\,\sqrt{a}e^{(-2\,fx-2\,e)}+10\,\sqrt{a}e^{(-4\,fx-4\,e)}+10\,\sqrt{a}e^{(-6\,fx-6\,e)}+5\,\sqrt{a}e^{(-8\,fx-8\,e)}+\sqrt{a}e^{(-10\,fx-10\,e)}+\sqrt{a}\right)f}$$

input
```
integrate(tanh(f*x+e)^5/(a+a*sinh(f*x+e)^2)^(1/2),x, algorithm="maxima")
```

output
```
-2*e^(-f*x - e)/((5*sqrt(a)*e^(-2*f*x - 2*e) + 10*sqrt(a)*e^(-4*f*x - 4*e)
 + 10*sqrt(a)*e^(-6*f*x - 6*e) + 5*sqrt(a)*e^(-8*f*x - 8*e) + sqrt(a)*e^(-
10*f*x - 10*e) + sqrt(a))*f) - 8/3*e^(-3*f*x - 3*e)/((5*sqrt(a)*e^(-2*f*x
- 2*e) + 10*sqrt(a)*e^(-4*f*x - 4*e) + 10*sqrt(a)*e^(-6*f*x - 6*e) + 5*sqr
t(a)*e^(-8*f*x - 8*e) + sqrt(a)*e^(-10*f*x - 10*e) + sqrt(a))*f) - 116/15*
e^(-5*f*x - 5*e)/((5*sqrt(a)*e^(-2*f*x - 2*e) + 10*sqrt(a)*e^(-4*f*x - 4*e
) + 10*sqrt(a)*e^(-6*f*x - 6*e) + 5*sqrt(a)*e^(-8*f*x - 8*e) + sqrt(a)*e^(
-10*f*x - 10*e) + sqrt(a))*f) - 8/3*e^(-7*f*x - 7*e)/((5*sqrt(a)*e^(-2*f*x
 - 2*e) + 10*sqrt(a)*e^(-4*f*x - 4*e) + 10*sqrt(a)*e^(-6*f*x - 6*e) + 5*sq
rt(a)*e^(-8*f*x - 8*e) + sqrt(a)*e^(-10*f*x - 10*e) + sqrt(a))*f) - 2*e^(-
9*f*x - 9*e)/((5*sqrt(a)*e^(-2*f*x - 2*e) + 10*sqrt(a)*e^(-4*f*x - 4*e) +
10*sqrt(a)*e^(-6*f*x - 6*e) + 5*sqrt(a)*e^(-8*f*x - 8*e) + sqrt(a)*e^(-10*
f*x - 10*e) + sqrt(a))*f)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{\tanh^5(e+fx)}{\sqrt{a+a\sinh^2(e+fx)}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(tanh(f*x+e)^5/(a+a*sinh(f*x+e)^2)^(1/2),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [B] (verification not implemented)

Time = 1.88 (sec) , antiderivative size = 381, normalized size of antiderivative = 5.77

$$\int \frac{\tanh^5(e+fx)}{\sqrt{a+a\sinh^2(e+fx)}}\,dx = \frac{32\,e^{3\,e+3\,f\,x}\,\sqrt{a+a\,\left(\frac{e^{e+f\,x}}{2}-\frac{e^{-e-f\,x}}{2}\right)^2}}{3\,a\,f\,\left(e^{2\,e+2\,f\,x}+1\right)^2\,\left(e^{e+f\,x}+e^{3\,e+3\,f\,x}\right)} - \frac{4\,e^{3\,e+3\,f\,x}\,\sqrt{a+a\,\left(\frac{e^{e+f\,x}}{2}-\frac{e^{-e-f\,x}}{2}\right)^2}}{a\,f\,\left(e^{2\,e+2\,f\,x}+1\right)\,\left(e^{e+f\,x}+e^{3\,e+3\,f\,x}\right)} - \frac{352\,e^{3\,e+3\,f\,x}\,\sqrt{a+a\,\left(\frac{e^{e+f\,x}}{2}-\frac{e^{-e-f\,x}}{2}\right)^2}}{15\,a\,f\,\left(e^{2\,e+2\,f\,x}+1\right)^3\,\left(e^{e+f\,x}+e^{3\,e+3\,f\,x}\right)} + \frac{128\,e^{3\,e+3\,f\,x}\,\sqrt{a+a\,\left(\frac{e^{e+f\,x}}{2}-\frac{e^{-e-f\,x}}{2}\right)^2}}{5\,a\,f\,\left(e^{2\,e+2\,f\,x}+1\right)^4\,\left(e^{e+f\,x}+e^{3\,e+3\,f\,x}\right)} - \frac{64\,e^{3\,e+3\,f\,x}\,\sqrt{a+a\,\left(\frac{e^{e+f\,x}}{2}-\frac{e^{-e-f\,x}}{2}\right)^2}}{5\,a\,f\,\left(e^{2\,e+2\,f\,x}+1\right)^5\,\left(e^{e+f\,x}+e^{3\,e+3\,f\,x}\right)}$$

input
```
int(tanh(e + f*x)^5/(a + a*sinh(e + f*x)^2)^(1/2),x)
```

output
```
(32*exp(3*e + 3*f*x)*(a + a*(exp(e + f*x)/2 - exp(- e - f*x)/2)^2)^(1/2))/
(3*a*f*(exp(2*e + 2*f*x) + 1)^2*(exp(e + f*x) + exp(3*e + 3*f*x))) - (4*ex
p(3*e + 3*f*x)*(a + a*(exp(e + f*x)/2 - exp(- e - f*x)/2)^2)^(1/2))/(a*f*(
exp(2*e + 2*f*x) + 1)*(exp(e + f*x) + exp(3*e + 3*f*x))) - (352*exp(3*e +
3*f*x)*(a + a*(exp(e + f*x)/2 - exp(- e - f*x)/2)^2)^(1/2))/(15*a*f*(exp(2
*e + 2*f*x) + 1)^3*(exp(e + f*x) + exp(3*e + 3*f*x))) + (128*exp(3*e + 3*f
*x)*(a + a*(exp(e + f*x)/2 - exp(- e - f*x)/2)^2)^(1/2))/(5*a*f*(exp(2*e +
 2*f*x) + 1)^4*(exp(e + f*x) + exp(3*e + 3*f*x))) - (64*exp(3*e + 3*f*x)*(
a + a*(exp(e + f*x)/2 - exp(- e - f*x)/2)^2)^(1/2))/(5*a*f*(exp(2*e + 2*f*
x) + 1)^5*(exp(e + f*x) + exp(3*e + 3*f*x)))
```

Reduce [F]

$$\int \frac{\tanh^5(e+fx)}{\sqrt{a+a\sinh^2(e+fx)}}\,dx = \frac{\sqrt{a}\left(\int \frac{\sqrt{\sinh(fx+e)^2+1}\,\tanh(fx+e)^5}{\sinh(fx+e)^2+1}dx\right)}{a}$$

input
```
int(tanh(f*x+e)^5/(a+a*sinh(f*x+e)^2)^(1/2),x)
```

output
```
(sqrt(a)*int((sqrt(sinh(e + f*x)**2 + 1)*tanh(e + f*x)**5)/(sinh(e + f*x)*
*2 + 1),x))/a
```

3.402 $\int \frac{\tanh^3(e+fx)}{\sqrt{a+a\sinh^2(e+fx)}}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 42

$$\int \frac{\tanh^3(e+fx)}{\sqrt{a+a\sinh^2(e+fx)}}\,dx=\frac{a}{3f\left(a\cosh^2(e+fx)\right)^{3/2}}-\frac{1}{f\sqrt{a\cosh^2(e+fx)}}$$

output
```
1/3*a/f/(a*cosh(f*x+e)^2)^(3/2)-1/f/(a*cosh(f*x+e)^2)^(1/2)
```

Mathematica [A] (verified)

Time = 0.12 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.74

$$\int \frac{\tanh^3(e+fx)}{\sqrt{a+a\sinh^2(e+fx)}}\,dx=\frac{-3+\operatorname{sech}^2(e+fx)}{3f\sqrt{a\cosh^2(e+fx)}}$$

input
```
Integrate[Tanh[e + f*x]^3/Sqrt[a + a*Sinh[e + f*x]^2],x]
```

output
```
(-3 + Sech[e + f*x]^2)/(3*f*Sqrt[a*Cosh[e + f*x]^2])
```

Rubi [A] (verified)

Time = 0.37 (sec) , antiderivative size = 51, normalized size of antiderivative = 1.21, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.400, Rules used = {3042, 26, 3655, 26, 3042, 26, 3684, 8, 53, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\tanh^3(e+fx)}{\sqrt{a\sinh^2(e+fx)+a}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{i\tan(ie+ifx)^3}{\sqrt{a-a\sin(ie+ifx)^2}}dx$$

$$\Big\downarrow \text{26}$$

$$i\int \frac{\tan(ie+ifx)^3}{\sqrt{a-a\sin(ie+ifx)^2}}dx$$

$$\Big\downarrow \text{3655}$$

$$i\int -\frac{i\tanh^3(e+fx)}{\sqrt{a\cosh^2(e+fx)}}dx$$

$$\Big\downarrow \text{26}$$

$$\int \frac{\tanh^3(e+fx)}{\sqrt{a\cosh^2(e+fx)}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int -\frac{i}{\tan\left(ie+ifx+\frac{\pi}{2}\right)^3\sqrt{a\sin\left(ie+ifx+\frac{\pi}{2}\right)^2}}dx$$

$$\Big\downarrow \text{26}$$

$$-i\int \frac{1}{\sqrt{a\sin\left(\frac{1}{2}(2ie+\pi)+ifx\right)^2}\tan\left(\frac{1}{2}(2ie+\pi)+ifx\right)^3}dx$$

$$\Big\downarrow \text{3684}$$

$$-\frac{\int \frac{(1-\cosh^2(e+fx))\text{sech}^4(e+fx)}{\sqrt{a\cosh^2(e+fx)}}d\cosh^2(e+fx)}{2f}$$

$$\Big\downarrow \textbf{8}$$

$$-\frac{a^2\int \frac{1-\cosh^2(e+fx)}{(a\cosh^2(e+fx))^{5/2}}d\cosh^2(e+fx)}{2f}$$

$$\Big\downarrow \textbf{53}$$

$$-\frac{a^2\int \left(\frac{1}{(a\cosh^2(e+fx))^{5/2}}-\frac{1}{a(a\cosh^2(e+fx))^{3/2}}\right)d\cosh^2(e+fx)}{2f}$$

$$\Big\downarrow \textbf{2009}$$

$$-\frac{a^2\left(\frac{2}{a^2\sqrt{a\cosh^2(e+fx)}}-\frac{2}{3a(a\cosh^2(e+fx))^{3/2}}\right)}{2f}$$

input
```
Int[Tanh[e + f*x]^3/Sqrt[a + a*Sinh[e + f*x]^2],x]
```

output
```
-1/2*(a^2*(-2/(3*a*(a*Cosh[e + f*x]^2)^(3/2)) + 2/(a^2*Sqrt[a*Cosh[e + f*x
]^2])))/f
```

Defintions of rubi rules used

rule 8
```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_))^(p_), x_Symbol] :> Simp[1/a^m   Int[u*(a*
x)^(m + p), x], x] /; FreeQ[{a, m, p}, x] && IntegerQ[m]
```

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 53
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && LeQ[7*m + 4*n + 4, 0])
|| LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3655
```
Int[(u_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> Int[A
ctivateTrig[u*(a*cos[e + f*x]^2)^p], x] /; FreeQ[{a, b, e, f, p}, x] && EqQ
[a + b, 0]
```

rule 3684
```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*tan[(e_.) + (f_.)*(x_)]^(m_.
), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x]^2, x]}, Simp[ff^((m + 1
)/2)/(2*f)   Subst[Int[x^((m - 1)/2)*((b*ff^(n/2)*x^(n/2))^p/(1 - ff*x)^((m
 + 1)/2)), x], x, Sin[e + f*x]^2/ff], x]] /; FreeQ[{b, e, f, p}, x] && Inte
gerQ[(m - 1)/2] && IntegerQ[n/2]
```

Maple [A] (verified)

Time = 0.40 (sec) , antiderivative size = 38, normalized size of antiderivative = 0.90

method	result	size
default	$-\frac{3 \cosh\left(f x +e \right)^{2}-1}{3 \cosh\left(f x +e \right)^{2} \sqrt{a \cosh\left(f x +e \right)^{2}}\, f}$	38
risch	$-\frac{2 \left(3 \,{\mathrm e}^{4 f x +4 e}+2 \,{\mathrm e}^{2 f x +2 e}+3\right)}{3 \sqrt{\left({\mathrm e}^{2 f x +2 e}+1\right)^{2} a \,{\mathrm e}^{-2 f x -2 e}}\, \left({\mathrm e}^{2 f x +2 e}+1\right)^{2} f}$	69

input
```
int(tanh(f*x+e)^3/(a+a*sinh(f*x+e)^2)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-1/3/cosh(f*x+e)^2*(3*cosh(f*x+e)^2-1)/(a*cosh(f*x+e)^2)^(1/2)/f
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 641 vs. $2(36) = 72$.

Time = 0.09 (sec) , antiderivative size = 641, normalized size of antiderivative = 15.26

$$\int \frac{\tanh^3(e+fx)}{\sqrt{a+a\sinh^2(e+fx)}}\,dx = \text{Too large to display}$$

input
```
integrate(tanh(f*x+e)^3/(a+a*sinh(f*x+e)^2)^(1/2),x, algorithm="fricas")
```

output
```
-2/3*(15*cosh(f*x + e)*e^(f*x + e)*sinh(f*x + e)^4 + 3*e^(f*x + e)*sinh(f*
x + e)^5 + 2*(15*cosh(f*x + e)^2 + 1)*e^(f*x + e)*sinh(f*x + e)^3 + 6*(5*c
osh(f*x + e)^3 + cosh(f*x + e))*e^(f*x + e)*sinh(f*x + e)^2 + 3*(5*cosh(f*
x + e)^4 + 2*cosh(f*x + e)^2 + 1)*e^(f*x + e)*sinh(f*x + e) + (3*cosh(f*x
+ e)^5 + 2*cosh(f*x + e)^3 + 3*cosh(f*x + e))*e^(f*x + e))*sqrt(a*e^(4*f*x
 + 4*e) + 2*a*e^(2*f*x + 2*e) + a)*e^(-f*x - e)/(a*f*cosh(f*x + e)^6 + (a*
f*e^(2*f*x + 2*e) + a*f)*sinh(f*x + e)^6 + 3*a*f*cosh(f*x + e)^4 + 6*(a*f*
cosh(f*x + e)*e^(2*f*x + 2*e) + a*f*cosh(f*x + e))*sinh(f*x + e)^5 + 3*(5*
a*f*cosh(f*x + e)^2 + a*f + (5*a*f*cosh(f*x + e)^2 + a*f)*e^(2*f*x + 2*e))
*sinh(f*x + e)^4 + 3*a*f*cosh(f*x + e)^2 + 4*(5*a*f*cosh(f*x + e)^3 + 3*a*
f*cosh(f*x + e) + (5*a*f*cosh(f*x + e)^3 + 3*a*f*cosh(f*x + e))*e^(2*f*x +
 2*e))*sinh(f*x + e)^3 + 3*(5*a*f*cosh(f*x + e)^4 + 6*a*f*cosh(f*x + e)^2
+ a*f + (5*a*f*cosh(f*x + e)^4 + 6*a*f*cosh(f*x + e)^2 + a*f)*e^(2*f*x + 2
*e))*sinh(f*x + e)^2 + a*f + (a*f*cosh(f*x + e)^6 + 3*a*f*cosh(f*x + e)^4
+ 3*a*f*cosh(f*x + e)^2 + a*f)*e^(2*f*x + 2*e) + 6*(a*f*cosh(f*x + e)^5 +
2*a*f*cosh(f*x + e)^3 + a*f*cosh(f*x + e) + (a*f*cosh(f*x + e)^5 + 2*a*f*c
osh(f*x + e)^3 + a*f*cosh(f*x + e))*e^(2*f*x + 2*e))*sinh(f*x + e))
```

Sympy [F]

$$\int \frac{\tanh^3(e+fx)}{\sqrt{a+a\sinh^2(e+fx)}}\,dx = \int \frac{\tanh^3{(e+fx)}}{\sqrt{a\left(\sinh^2{(e+fx)}+1\right)}}\,dx$$

input
```
integrate(tanh(f*x+e)**3/(a+a*sinh(f*x+e)**2)**(1/2),x)
```

output
```
Integral(tanh(e + f*x)**3/sqrt(a*(sinh(e + f*x)**2 + 1)), x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 184 vs. 2(36) = 72.

Time = 0.18 (sec) , antiderivative size = 184, normalized size of antiderivative = 4.38

$$\int \frac{\tanh^3(e+fx)}{\sqrt{a+a\sinh^2(e+fx)}}\,dx = -\frac{2\,e^{(-fx-e)}}{\left(3\,\sqrt{a}e^{(-2\,fx-2\,e)}+3\,\sqrt{a}e^{(-4\,fx-4\,e)}+\sqrt{a}e^{(-6\,fx-6\,e)}+\sqrt{a}\right)f} - \frac{4\,e^{(-3\,fx-3\,e)}}{3\,\left(3\,\sqrt{a}e^{(-2\,fx-2\,e)}+3\,\sqrt{a}e^{(-4\,fx-4\,e)}+\sqrt{a}e^{(-6\,fx-6\,e)}+\sqrt{a}\right)f} - \frac{2\,e^{(-5\,fx-5\,e)}}{\left(3\,\sqrt{a}e^{(-2\,fx-2\,e)}+3\,\sqrt{a}e^{(-4\,fx-4\,e)}+\sqrt{a}e^{(-6\,fx-6\,e)}+\sqrt{a}\right)f}$$

input
```
integrate(tanh(f*x+e)^3/(a+a*sinh(f*x+e)^2)^(1/2),x, algorithm="maxima")
```

output
```
-2*e^(-f*x - e)/((3*sqrt(a)*e^(-2*f*x - 2*e) + 3*sqrt(a)*e^(-4*f*x - 4*e)
+ sqrt(a)*e^(-6*f*x - 6*e) + sqrt(a))*f) - 4/3*e^(-3*f*x - 3*e)/((3*sqrt(a
)*e^(-2*f*x - 2*e) + 3*sqrt(a)*e^(-4*f*x - 4*e) + sqrt(a)*e^(-6*f*x - 6*e)
 + sqrt(a))*f) - 2*e^(-5*f*x - 5*e)/((3*sqrt(a)*e^(-2*f*x - 2*e) + 3*sqrt(
a)*e^(-4*f*x - 4*e) + sqrt(a)*e^(-6*f*x - 6*e) + sqrt(a))*f)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{\tanh^3(e+fx)}{\sqrt{a+a\sinh^2(e+fx)}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(tanh(f*x+e)^3/(a+a*sinh(f*x+e)^2)^(1/2),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [B] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 82, normalized size of antiderivative = 1.95

$$\int \frac{\tanh^3(e+fx)}{\sqrt{a+a\sinh^2(e+fx)}}\,dx = -\frac{4\,e^{2\,e+2\,f\,x}\,\sqrt{a+a\,\left(\frac{e^{e+f\,x}}{2}-\frac{e^{-e-f\,x}}{2}\right)^2}\,\left(2\,e^{2\,e+2\,f\,x}+3\,e^{4\,e+4\,f\,x}+3\right)}{3\,a\,f\,\left(e^{2\,e+2\,f\,x}+1\right)^4}$$

input
```
int(tanh(e + f*x)^3/(a + a*sinh(e + f*x)^2)^(1/2),x)
```

output
```
-(4*exp(2*e + 2*f*x)*(a + a*(exp(e + f*x)/2 - exp(- e - f*x)/2)^2)^(1/2)*(
2*exp(2*e + 2*f*x) + 3*exp(4*e + 4*f*x) + 3))/(3*a*f*(exp(2*e + 2*f*x) + 1
)^4)
```

Reduce [F]

$$\int \frac{\tanh^3(e+fx)}{\sqrt{a+a\sinh^2(e+fx)}}\,dx = \frac{\sqrt{a}\left(\int \frac{\sqrt{\sinh(fx+e)^2+1}\,\tanh(fx+e)^3}{\sinh(fx+e)^2+1}dx\right)}{a}$$

input
```
int(tanh(f*x+e)^3/(a+a*sinh(f*x+e)^2)^(1/2),x)
```

output
```
(sqrt(a)*int((sqrt(sinh(e + f*x)**2 + 1)*tanh(e + f*x)**3)/(sinh(e + f*x)*
*2 + 1),x))/a
```

3.403 $\int \frac{\tanh(e+fx)}{\sqrt{a+a\sinh^2(e+fx)}}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 19

$$\int \frac{\tanh(e+fx)}{\sqrt{a+a\sinh^2(e+fx)}}\,dx = -\frac{1}{f\sqrt{a\cosh^2(e+fx)}}$$

output
```
-1/f/(a*cosh(f*x+e)^2)^(1/2)
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.00

$$\int \frac{\tanh(e+fx)}{\sqrt{a+a\sinh^2(e+fx)}}\,dx = -\frac{1}{f\sqrt{a\cosh^2(e+fx)}}$$

input
```
Integrate[Tanh[e + f*x]/Sqrt[a + a*Sinh[e + f*x]^2],x]
```

output
```
-(1/(f*Sqrt[a*Cosh[e + f*x]^2]))
```

Rubi [A] (verified)

Time = 0.34 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.391$, Rules used = {3042, 26, 3655, 26, 3042, 26, 3684, 8, 17}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\tanh(e+fx)}{\sqrt{a\sinh^2(e+fx)+a}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int -\frac{i\tan(ie+ifx)}{\sqrt{a-a\sin(ie+ifx)^2}}dx$$

$$\Big\downarrow \text{26}$$

$$-i\int \frac{\tan(ie+ifx)}{\sqrt{a-a\sin(ie+ifx)^2}}dx$$

$$\Big\downarrow \text{3655}$$

$$-i\int \frac{i\tanh(e+fx)}{\sqrt{a\cosh^2(e+fx)}}dx$$

$$\Big\downarrow \text{26}$$

$$\int \frac{\tanh(e+fx)}{\sqrt{a\cosh^2(e+fx)}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{i}{\tan\left(ie+ifx+\frac{\pi}{2}\right)\sqrt{a\sin\left(ie+ifx+\frac{\pi}{2}\right)^2}}dx$$

$$\Big\downarrow \text{26}$$

$$i\int \frac{1}{\sqrt{a\sin\left(\frac{1}{2}(2ie+\pi)+ifx\right)^2}\tan\left(\frac{1}{2}(2ie+\pi)+ifx\right)}dx$$

$$\Big\downarrow \text{3684}$$

$$\frac{\int \frac{\text{sech}^2(e+fx)}{\sqrt{a\cosh^2(e+fx)}}d\cosh^2(e+fx)}{2f}$$

$$\Big\downarrow \; 8$$

$$\frac{a\int \frac{1}{\left(a\cosh^2(e+fx)\right)^{3/2}}d\cosh^2(e+fx)}{2f}$$

$$\Big\downarrow \; 17$$

$$-\frac{1}{f\sqrt{a\cosh^2(e+fx)}}$$

input
```
Int[Tanh[e + f*x]/Sqrt[a + a*Sinh[e + f*x]^2],x]
```

output
```
-(1/(f*Sqrt[a*Cosh[e + f*x]^2]))
```

Defintions of rubi rules used

rule 8
```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_))^(p_), x_Symbol] :> Simp[1/a^m   Int[u*(a*
x)^(m + p), x], x] /; FreeQ[{a, m, p}, x] && IntegerQ[m]
```

rule 17
```
Int[(c_.)*((a_.) + (b_.)*(x_))^(m_.), x_Symbol] :> Simp[c*((a + b*x)^(m + 1
)/(b*(m + 1))), x] /; FreeQ[{a, b, c, m}, x] && NeQ[m, -1]
```

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3655
```
Int[(u_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> Int[A
ctivateTrig[u*(a*cos[e + f*x]^2)^p], x] /; FreeQ[{a, b, e, f, p}, x] && EqQ
[a + b, 0]
```

rule 3684
```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*tan[(e_.) + (f_.)*(x_)]^(m_.
), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x]^2, x]}, Simp[ff^((m + 1
)/2)/(2*f)   Subst[Int[x^((m - 1)/2)*((b*ff^(n/2)*x^(n/2))^p/(1 - ff*x)^((m
 + 1)/2)), x], x, Sin[e + f*x]^2/ff], x]] /; FreeQ[{b, e, f, p}, x] && Inte
gerQ[(m - 1)/2] && IntegerQ[n/2]
```

Maple [A] (verified)

Time = 0.29 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.05

method	result	size
derivativedivides	$-\frac{1}{f\sqrt{a+a\sinh(fx+e)^{2}}}$	20
default	$-\frac{1}{f\sqrt{a+a\sinh(fx+e)^{2}}}$	20
risch	$-\frac{2}{\sqrt{\left(\mathrm{e}^{2fx+2e}+1\right)^{2}a\,\mathrm{e}^{-2fx-2e}}\,f}$	32

input
```
int(tanh(f*x+e)/(a+a*sinh(f*x+e)^2)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-1/f/(a+a*sinh(f*x+e)^2)^(1/2)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 168 vs. $2(17) = 34$.

Time = 0.08 (sec) , antiderivative size = 168, normalized size of antiderivative = 8.84

$$\int \frac{\tanh(e+fx)}{\sqrt{a+a\sinh^2(e+fx)}}\,dx =$$

$$-\frac{2\sqrt{ae^{(4fx+4e)}+2ae^{(2fx+2e)}+a}\left(\cosh\left(fx+e\right)e^{(fx+e)}+e^{(fx+e)}\right.}{af\cosh\left(fx+e\right)^{2}+\left(afe^{(2fx+2e)}+af\right)\sinh\left(fx+e\right)^{2}+af+\left(af\cosh\left(fx+e\right)^{2}+af\right)e^{(2fx+2e)}+2}$$

input
```
integrate(tanh(f*x+e)/(a+a*sinh(f*x+e)^2)^(1/2),x, algorithm="fricas")
```

output
```
-2*sqrt(a*e^(4*f*x + 4*e) + 2*a*e^(2*f*x + 2*e) + a)*(cosh(f*x + e)*e^(f*x
 + e) + e^(f*x + e)*sinh(f*x + e))*e^(-f*x - e)/(a*f*cosh(f*x + e)^2 + (a*
f*e^(2*f*x + 2*e) + a*f)*sinh(f*x + e)^2 + a*f + (a*f*cosh(f*x + e)^2 + a*
f)*e^(2*f*x + 2*e) + 2*(a*f*cosh(f*x + e)*e^(2*f*x + 2*e) + a*f*cosh(f*x +
 e))*sinh(f*x + e))
```

Sympy [F]

$$\int \frac{\tanh(e+fx)}{\sqrt{a+a\sinh^2(e+fx)}}\,dx = \int \frac{\tanh(e+fx)}{\sqrt{a\left(\sinh^2(e+fx)+1\right)}}\,dx$$

input
```
integrate(tanh(f*x+e)/(a+a*sinh(f*x+e)**2)**(1/2),x)
```

output
```
Integral(tanh(e + f*x)/sqrt(a*(sinh(e + f*x)**2 + 1)), x)
```

Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 33, normalized size of antiderivative = 1.74

$$\int \frac{\tanh(e+fx)}{\sqrt{a+a\sinh^2(e+fx)}}\,dx = -\frac{2\,e^{(-fx-e)}}{\left(\sqrt{a}e^{(-2\,fx-2\,e)}+\sqrt{a}\right)f}$$

input
```
integrate(tanh(f*x+e)/(a+a*sinh(f*x+e)^2)^(1/2),x, algorithm="maxima")
```

output
```
-2*e^(-f*x - e)/((sqrt(a)*e^(-2*f*x - 2*e) + sqrt(a))*f)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{\tanh(e+fx)}{\sqrt{a+a\sinh^2(e+fx)}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(tanh(f*x+e)/(a+a*sinh(f*x+e)^2)^(1/2),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [B] (verification not implemented)

Time = 1.82 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.58

$$\int \frac{\tanh(e+fx)}{\sqrt{a+a\sinh^2(e+fx)}}\,dx = -\frac{\sqrt{a\sinh\left(e+f\,x\right)^2+a}}{a\,f\cosh\left(e+f\,x\right)^2}$$

input
```
int(tanh(e + f*x)/(a + a*sinh(e + f*x)^2)^(1/2),x)
```

output
```
-(a + a*sinh(e + f*x)^2)^(1/2)/(a*f*cosh(e + f*x)^2)
```

Reduce [F]

$$\int \frac{\tanh(e+fx)}{\sqrt{a+a\sinh^2(e+fx)}}\,dx = \frac{\sqrt{a}\left(\int \frac{\sqrt{\sinh(fx+e)^2+1}\,\tanh(fx+e)}{\sinh(fx+e)^2+1}dx\right)}{a}$$

input
```
int(tanh(f*x+e)/(a+a*sinh(f*x+e)^2)^(1/2),x)
```

output
```
(sqrt(a)*int((sqrt(sinh(e + f*x)**2 + 1)*tanh(e + f*x))/(sinh(e + f*x)**2
+ 1),x))/a
```

3.404 $\int \frac{\coth(e+fx)}{\sqrt{a+a\sinh^2(e+fx)}} dx$

Optimal result

Integrand size = 23, antiderivative size = 31

$$\int \frac{\coth(e+fx)}{\sqrt{a+a\sinh^2(e+fx)}} dx = -\frac{\operatorname{arctanh}\left(\frac{\sqrt{a\cosh^2(e+fx)}}{\sqrt{a}}\right)}{\sqrt{a}f}$$

output
```
-arctanh((a*cosh(f*x+e)^2)^(1/2)/a^(1/2))/a^(1/2)/f
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.03

$$\int \frac{\coth(e+fx)}{\sqrt{a+a\sinh^2(e+fx)}} dx = -\frac{\operatorname{arctanh}(\cosh(e+fx))\cosh(e+fx)}{f\sqrt{a\cosh^2(e+fx)}}$$

input
```
Integrate[Coth[e + f*x]/Sqrt[a + a*Sinh[e + f*x]^2],x]
```

output
```
-((ArcTanh[Cosh[e + f*x]]*Cosh[e + f*x])/(f*Sqrt[a*Cosh[e + f*x]^2]))
```

Rubi [A] (verified)

Time = 0.36 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.391$, Rules used = {3042, 26, 3655, 26, 3042, 26, 3684, 73, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\coth(e+fx)}{\sqrt{a\sinh^2(e+fx)+a}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{i}{\tan(ie+ifx)\sqrt{a-a\sin(ie+ifx)^2}}dx$$

$$\Big\downarrow \text{26}$$

$$i\int \frac{1}{\sqrt{a-a\sin(ie+ifx)^2}\tan(ie+ifx)}dx$$

$$\Big\downarrow \text{3655}$$

$$i\int -\frac{i\coth(e+fx)}{\sqrt{a\cosh^2(e+fx)}}dx$$

$$\Big\downarrow \text{26}$$

$$\int \frac{\coth(e+fx)}{\sqrt{a\cosh^2(e+fx)}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int -\frac{i\tan\left(ie+ifx+\frac{\pi}{2}\right)}{\sqrt{a\sin\left(ie+ifx+\frac{\pi}{2}\right)^2}}dx$$

$$\Big\downarrow \text{26}$$

$$-i\int \frac{\tan\left(\frac{1}{2}(2ie+\pi)+ifx\right)}{\sqrt{a\sin\left(\frac{1}{2}(2ie+\pi)+ifx\right)^2}}dx$$

$$\Big\downarrow \text{3684}$$

$$-\frac{\int \frac{1}{\sqrt{a\cosh^2(e+fx)}(1-\cosh^2(e+fx))}d\cosh^2(e+fx)}{2f}$$

$$\Big\downarrow \ 73$$

$$-\frac{\int \frac{1}{1-\frac{\cosh^4(e+fx)}{a}}d\sqrt{a\cosh^2(e+fx)}}{af}$$

$$\Big\downarrow \ 219$$

$$-\frac{\text{arctanh}\left(\frac{\sqrt{a\cosh^2(e+fx)}}{\sqrt{a}}\right)}{\sqrt{a}f}$$

input
```
Int[Coth[e + f*x]/Sqrt[a + a*Sinh[e + f*x]^2],x]
```

output
```
-(ArcTanh[Sqrt[a*Cosh[e + f*x]^2]/Sqrt[a]]/(Sqrt[a]*f))
```

Defintions of rubi rules used

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3655
```
Int[(u_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> Int[A
ctivateTrig[u*(a*cos[e + f*x]^2)^p], x] /; FreeQ[{a, b, e, f, p}, x] && EqQ
[a + b, 0]
```

rule 3684
```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*tan[(e_.) + (f_.)*(x_)]^(m_.
), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x]^2, x]}, Simp[ff^((m + 1
)/2)/(2*f)   Subst[Int[x^((m - 1)/2)*((b*ff^(n/2)*x^(n/2))^p/(1 - ff*x)^((m
 + 1)/2)), x], x, Sin[e + f*x]^2/ff], x]] /; FreeQ[{b, e, f, p}, x] && Inte
gerQ[(m - 1)/2] && IntegerQ[n/2]
```

Maple [A] (verified)

Time = 0.34 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.00

method	result	size
default	$-\frac{\cosh(fx+e)\,\operatorname{arctanh}(\cosh(fx+e))}{\sqrt{a\cosh(fx+e)^{2}}\,f}$	31
risch	$-\frac{\ln(\mathrm{e}^{fx}+\mathrm{e}^{-e})\,(\mathrm{e}^{2fx+2e}+1)\,\mathrm{e}^{-fx-e}}{f\sqrt{(\mathrm{e}^{2fx+2e}+1)^{2}a\,\mathrm{e}^{-2fx-2e}}}+\frac{\ln(\mathrm{e}^{fx}-\mathrm{e}^{-e})\,(\mathrm{e}^{2fx+2e}+1)\,\mathrm{e}^{-fx-e}}{f\sqrt{(\mathrm{e}^{2fx+2e}+1)^{2}a\,\mathrm{e}^{-2fx-2e}}}$	125

input
```
int(coth(f*x+e)/(a+a*sinh(f*x+e)^2)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-1/(a*cosh(f*x+e)^2)^(1/2)*cosh(f*x+e)*arctanh(cosh(f*x+e))/f
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 95 vs. $2(25) = 50$.

Time = 0.10 (sec) , antiderivative size = 174, normalized size of antiderivative = 5.61

$$\int \frac{\coth(e+fx)}{\sqrt{a+a\sinh^2(e+fx)}}\,dx$$
$$= \left[\frac{\sqrt{ae^{(4\,fx+4\,e)}+2\,ae^{(2\,fx+2\,e)}+a}\log\left(\frac{\cosh(fx+e)+\sinh(fx+e)-1}{\cosh(fx+e)+\sinh(fx+e)+1}\right)}{afe^{(2\,fx+2\,e)}+af}, \frac{2\sqrt{-a}\arctan\left(\frac{\sqrt{ae^{(4\,fx+4\,e)}+2\,ae}}{a\cosh(fx+e)e^{(2\,fx+2\,e)}+a\cosh(fx}\right.}{af}\right.$$

input
```
integrate(coth(f*x+e)/(a+a*sinh(f*x+e)^2)^(1/2),x, algorithm="fricas")
```

output
```
[sqrt(a*e^(4*f*x + 4*e) + 2*a*e^(2*f*x + 2*e) + a)*log((cosh(f*x + e) + si
nh(f*x + e) - 1)/(cosh(f*x + e) + sinh(f*x + e) + 1))/(a*f*e^(2*f*x + 2*e)
 + a*f), 2*sqrt(-a)*arctan(sqrt(a*e^(4*f*x + 4*e) + 2*a*e^(2*f*x + 2*e) +
a)*sqrt(-a)/(a*cosh(f*x + e)*e^(2*f*x + 2*e) + a*cosh(f*x + e) + (a*e^(2*f
*x + 2*e) + a)*sinh(f*x + e)))/(a*f)]
```

Sympy [F]

$$\int \frac{\coth(e+fx)}{\sqrt{a+a\sinh^2(e+fx)}}\,dx = \int \frac{\coth(e+fx)}{\sqrt{a\left(\sinh^2(e+fx)+1\right)}}\,dx$$

input
```
integrate(coth(f*x+e)/(a+a*sinh(f*x+e)**2)**(1/2),x)
```

output
```
Integral(coth(e + f*x)/sqrt(a*(sinh(e + f*x)**2 + 1)), x)
```

Maxima [A] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 40, normalized size of antiderivative = 1.29

$$\int \frac{\coth(e+fx)}{\sqrt{a+a\sinh^2(e+fx)}}\,dx = -\frac{\log\left(e^{(-fx-e)}+1\right)}{\sqrt{a}f} + \frac{\log\left(e^{(-fx-e)}-1\right)}{\sqrt{a}f}$$

input
```
integrate(coth(f*x+e)/(a+a*sinh(f*x+e)^2)^(1/2),x, algorithm="maxima")
```

output
```
-log(e^(-f*x - e) + 1)/(sqrt(a)*f) + log(e^(-f*x - e) - 1)/(sqrt(a)*f)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{\coth(e+fx)}{\sqrt{a+a\sinh^2(e+fx)}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(coth(f*x+e)/(a+a*sinh(f*x+e)^2)^(1/2),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\coth(e+fx)}{\sqrt{a+a\sinh^2(e+fx)}}\,dx = \int \frac{\coth(e+f\,x)}{\sqrt{a\sinh\left(e+f\,x\right)^2+a}}\,dx$$

input
```
int(coth(e + f*x)/(a + a*sinh(e + f*x)^2)^(1/2),x)
```

output
```
int(coth(e + f*x)/(a + a*sinh(e + f*x)^2)^(1/2), x)
```

Reduce [F]

$$\int \frac{\coth(e+fx)}{\sqrt{a+a\sinh^2(e+fx)}}\,dx = \frac{\sqrt{a}\left(\int \frac{\sqrt{\sinh(fx+e)^2+1}\,\coth(fx+e)}{\sinh(fx+e)^2+1}dx\right)}{a}$$

input
```
int(coth(f*x+e)/(a+a*sinh(f*x+e)^2)^(1/2),x)
```

output
```
(sqrt(a)*int((sqrt(sinh(e + f*x)**2 + 1)*coth(e + f*x))/(sinh(e + f*x)**2
+ 1),x))/a
```

3.405 $\int \frac{\coth^3(e+fx)}{\sqrt{a+a\sinh^2(e+fx)}}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 66

$$\int \frac{\coth^3(e+fx)}{\sqrt{a+a\sinh^2(e+fx)}}\,dx = -\frac{\operatorname{arctanh}\left(\frac{\sqrt{a\cosh^2(e+fx)}}{\sqrt{a}}\right)}{2\sqrt{a}f} - \frac{\sqrt{a\cosh^2(e+fx)}\operatorname{csch}^2(e+fx)}{2af}$$

output
```
-1/2*arctanh((a*cosh(f*x+e)^2)^(1/2)/a^(1/2))/a^(1/2)/f-1/2*(a*cosh(f*x+e)
^2)^(1/2)*csch(f*x+e)^2/a/f
```

Mathematica [A] (verified)

Time = 0.16 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.85

$$\int \frac{\coth^3(e+fx)}{\sqrt{a+a\sinh^2(e+fx)}}\,dx = -\frac{\operatorname{arctanh}\left(\sqrt{\cosh^2(e+fx)}\right)\sqrt{\cosh^2(e+fx)}+\coth^2(e+fx)}{2f\sqrt{a\cosh^2(e+fx)}}$$

input
```
Integrate[Coth[e + f*x]^3/Sqrt[a + a*Sinh[e + f*x]^2],x]
```

output
```
-1/2*(ArcTanh[Sqrt[Cosh[e + f*x]^2]]*Sqrt[Cosh[e + f*x]^2] + Coth[e + f*x]
^2)/(f*Sqrt[a*Cosh[e + f*x]^2])
```

Rubi [A] (verified)

Time = 0.41 (sec) , antiderivative size = 68, normalized size of antiderivative = 1.03, number of steps used = 12, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.440$, Rules used = {3042, 26, 3655, 26, 3042, 26, 3684, 8, 51, 73, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\coth^3(e+fx)}{\sqrt{a\sinh^2(e+fx)+a}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int -\frac{i}{\tan(ie+ifx)^3\sqrt{a-a\sin(ie+ifx)^2}}dx$$

$$\Big\downarrow \text{26}$$

$$-i\int \frac{1}{\sqrt{a-a\sin(ie+ifx)^2}\tan(ie+ifx)^3}dx$$

$$\Big\downarrow \text{3655}$$

$$-i\int \frac{i\coth^3(e+fx)}{\sqrt{a\cosh^2(e+fx)}}dx$$

$$\Big\downarrow \text{26}$$

$$\int \frac{\coth^3(e+fx)}{\sqrt{a\cosh^2(e+fx)}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{i\tan\left(ie+ifx+\frac{\pi}{2}\right)^3}{\sqrt{a\sin\left(ie+ifx+\frac{\pi}{2}\right)^2}}dx$$

$$\Big\downarrow \text{26}$$

$$i\int \frac{\tan\left(\frac{1}{2}(2ie+\pi)+ifx\right)^3}{\sqrt{a\sin\left(\frac{1}{2}(2ie+\pi)+ifx\right)^2}}dx$$

$$\Big\downarrow \text{3684}$$

$$\frac{\int \frac{\cosh^2(e+fx)}{\sqrt{a\cosh^2(e+fx)}(1-\cosh^2(e+fx))^2}d\cosh^2(e+fx)}{2f}$$

$$\Big\downarrow \text{8}$$

$$\frac{\int \frac{\sqrt{a\cosh^2(e+fx)}}{(1-\cosh^2(e+fx))^2}d\cosh^2(e+fx)}{2af}$$

$$\Big\downarrow \text{51}$$

$$\frac{\frac{\sqrt{a\cosh^2(e+fx)}}{1-\cosh^2(e+fx)}-\frac{1}{2}a\int \frac{1}{\sqrt{a\cosh^2(e+fx)}(1-\cosh^2(e+fx))}d\cosh^2(e+fx)}{2af}$$

$$\Big\downarrow \text{73}$$

$$\frac{\frac{\sqrt{a\cosh^2(e+fx)}}{1-\cosh^2(e+fx)}-\int \frac{1}{1-\frac{\cosh^4(e+fx)}{a}}d\sqrt{a\cosh^2(e+fx)}}{2af}$$

$$\Big\downarrow \text{219}$$

$$\frac{\frac{\sqrt{a\cosh^2(e+fx)}}{1-\cosh^2(e+fx)}-\sqrt{a}\text{arctanh}\left(\frac{\sqrt{a\cosh^2(e+fx)}}{\sqrt{a}}\right)}{2af}$$

input
```
Int[Coth[e + f*x]^3/Sqrt[a + a*Sinh[e + f*x]^2],x]
```

output
```
(-(Sqrt[a]*ArcTanh[Sqrt[a*Cosh[e + f*x]^2]/Sqrt[a]]) + Sqrt[a*Cosh[e + f*x
]^2]/(1 - Cosh[e + f*x]^2))/(2*a*f)
```

Defintions of rubi rules used

rule 8
```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_))^(p_), x_Symbol] :> Simp[1/a^m   Int[u*(a*
x)^(m + p), x], x] /; FreeQ[{a, m, p}, x] && IntegerQ[m]
```

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 51
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + 1))), x] - Simp[d*(n/(b*(m + 1)))
Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d, n}, x
] && ILtQ[m, -1] && FractionQ[n] && GtQ[n, 0]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3655
```
Int[(u_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> Int[A
ctivateTrig[u*(a*cos[e + f*x]^2)^p], x] /; FreeQ[{a, b, e, f, p}, x] && EqQ
[a + b, 0]
```

rule 3684
```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*tan[(e_.) + (f_.)*(x_)]^(m_.
), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x]^2, x]}, Simp[ff^((m + 1
)/2)/(2*f)   Subst[Int[x^((m - 1)/2)*((b*ff^(n/2)*x^(n/2))^p/(1 - ff*x)^((m
 + 1)/2)), x], x, Sin[e + f*x]^2/ff], x]] /; FreeQ[{b, e, f, p}, x] && Inte
gerQ[(m - 1)/2] && IntegerQ[n/2]
```

Maple [A] (verified)

Time = 0.42 (sec) , antiderivative size = 83, normalized size of antiderivative = 1.26

method	result	s
default	$-\frac{\cosh(fx+e)\left(2\cosh(fx+e)+(\ln(\cosh(fx+e)+1)-\ln(\cosh(fx+e)-1))\sinh(fx+e)^{2}\right)}{4\sqrt{a\cosh(fx+e)^{2}}\,(\cosh(fx+e)-1)(\cosh(fx+e)+1)f}$	8
risch	$-\frac{\left(e^{2fx+2e}+1\right)^{2}}{\sqrt{\left(e^{2fx+2e}+1\right)^{2}a\,e^{-2fx-2e}}\,f\left(e^{2fx+2e}-1\right)^{2}}-\frac{\ln\left(e^{fx}+e^{-e}\right)\left(e^{2fx+2e}+1\right)e^{-fx-e}}{2f\sqrt{\left(e^{2fx+2e}+1\right)^{2}a\,e^{-2fx-2e}}}+\frac{\ln\left(e^{fx}-e^{-e}\right)\left(e^{2fx+2e}+1\right)e^{-fx-e}}{2f\sqrt{\left(e^{2fx+2e}+1\right)^{2}a\,e^{-2fx-2e}}}$	1

input
```
int(coth(f*x+e)^3/(a+a*sinh(f*x+e)^2)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-1/4*cosh(f*x+e)*(2*cosh(f*x+e)+(ln(cosh(f*x+e)+1)-ln(cosh(f*x+e)-1))*sinh
(f*x+e)^2)/(a*cosh(f*x+e)^2)^(1/2)/(cosh(f*x+e)-1)/(cosh(f*x+e)+1)/f
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 529 vs. $2(54) = 108$.

Time = 0.10 (sec) , antiderivative size = 529, normalized size of antiderivative = 8.02

$$\int \frac{\coth^3(e+fx)}{\sqrt{a+a\sinh^2(e+fx)}}\,dx = \text{Too large to display}$$

input
```
integrate(coth(f*x+e)^3/(a+a*sinh(f*x+e)^2)^(1/2),x, algorithm="fricas")
```

output
```
-1/2*(6*cosh(f*x + e)*e^(f*x + e)*sinh(f*x + e)^2 + 2*e^(f*x + e)*sinh(f*x
 + e)^3 + 2*(3*cosh(f*x + e)^2 + 1)*e^(f*x + e)*sinh(f*x + e) + 2*(cosh(f*
x + e)^3 + cosh(f*x + e))*e^(f*x + e) - (4*cosh(f*x + e)*e^(f*x + e)*sinh(
f*x + e)^3 + e^(f*x + e)*sinh(f*x + e)^4 + 2*(3*cosh(f*x + e)^2 - 1)*e^(f*
x + e)*sinh(f*x + e)^2 + 4*(cosh(f*x + e)^3 - cosh(f*x + e))*e^(f*x + e)*s
inh(f*x + e) + (cosh(f*x + e)^4 - 2*cosh(f*x + e)^2 + 1)*e^(f*x + e))*log(
(cosh(f*x + e) + sinh(f*x + e) - 1)/(cosh(f*x + e) + sinh(f*x + e) + 1)))*
sqrt(a*e^(4*f*x + 4*e) + 2*a*e^(2*f*x + 2*e) + a)*e^(-f*x - e)/(a*f*cosh(f
*x + e)^4 + (a*f*e^(2*f*x + 2*e) + a*f)*sinh(f*x + e)^4 - 2*a*f*cosh(f*x +
 e)^2 + 4*(a*f*cosh(f*x + e)*e^(2*f*x + 2*e) + a*f*cosh(f*x + e))*sinh(f*x
 + e)^3 + 2*(3*a*f*cosh(f*x + e)^2 - a*f + (3*a*f*cosh(f*x + e)^2 - a*f)*e
^(2*f*x + 2*e))*sinh(f*x + e)^2 + a*f + (a*f*cosh(f*x + e)^4 - 2*a*f*cosh(
f*x + e)^2 + a*f)*e^(2*f*x + 2*e) + 4*(a*f*cosh(f*x + e)^3 - a*f*cosh(f*x
+ e) + (a*f*cosh(f*x + e)^3 - a*f*cosh(f*x + e))*e^(2*f*x + 2*e))*sinh(f*x
 + e))
```

Sympy [F]

$$\int \frac{\coth^3(e+fx)}{\sqrt{a+a\sinh^2(e+fx)}}\,dx=\int \frac{\coth^3{(e+fx)}}{\sqrt{a\left(\sinh^2{(e+fx)}+1\right)}}\,dx$$

input
```
integrate(coth(f*x+e)**3/(a+a*sinh(f*x+e)**2)**(1/2),x)
```

output
```
Integral(coth(e + f*x)**3/sqrt(a*(sinh(e + f*x)**2 + 1)), x)
```

Maxima [A] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 100, normalized size of antiderivative = 1.52

$$\int \frac{\coth^3(e+fx)}{\sqrt{a+a\sinh^2(e+fx)}}\,dx=-\frac{\log\left(e^{(-fx-e)}+1\right)}{2\sqrt{a}f}+\frac{\log\left(e^{(-fx-e)}-1\right)}{2\sqrt{a}f}+\frac{e^{(-fx-e)}+e^{(-3fx-3e)}}{\left(2\sqrt{a}e^{(-2fx-2e)}-\sqrt{a}e^{(-4fx-4e)}-\sqrt{a}\right)f}$$

input
```
integrate(coth(f*x+e)^3/(a+a*sinh(f*x+e)^2)^(1/2),x, algorithm="maxima")
```

output
```
-1/2*log(e^(-f*x - e) + 1)/(sqrt(a)*f) + 1/2*log(e^(-f*x - e) - 1)/(sqrt(a
)*f) + (e^(-f*x - e) + e^(-3*f*x - 3*e))/((2*sqrt(a)*e^(-2*f*x - 2*e) - sq
rt(a)*e^(-4*f*x - 4*e) - sqrt(a))*f)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{\coth^3(e+fx)}{\sqrt{a+a\sinh^2(e+fx)}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(coth(f*x+e)^3/(a+a*sinh(f*x+e)^2)^(1/2),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\coth^3(e+fx)}{\sqrt{a+a\sinh^2(e+fx)}}\,dx = \int \frac{\coth(e+f\,x)^3}{\sqrt{a\,\sinh\left(e+f\,x\right)^2+a}}\,dx$$

input
```
int(coth(e + f*x)^3/(a + a*sinh(e + f*x)^2)^(1/2),x)
```

output
```
int(coth(e + f*x)^3/(a + a*sinh(e + f*x)^2)^(1/2), x)
```

Reduce [F]

$$\int \frac{\coth^3(e+fx)}{\sqrt{a+a\sinh^2(e+fx)}}\,dx = \frac{\sqrt{a}\left(\int \frac{\sqrt{\sinh(fx+e)^2+1}\,\coth(fx+e)^3}{\sinh(fx+e)^2+1}dx\right)}{a}$$

input
```
int(coth(f*x+e)^3/(a+a*sinh(f*x+e)^2)^(1/2),x)
```

output
```
(sqrt(a)*int((sqrt(sinh(e + f*x)**2 + 1)*coth(e + f*x)**3)/(sinh(e + f*x)*
*2 + 1),x))/a
```

3.406 $\int \frac{\tanh^4(e+fx)}{\sqrt{a+a\sinh^2(e+fx)}}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 91

$$\int \frac{\tanh^4(e+fx)}{\sqrt{a+a\sinh^2(e+fx)}}\,dx = \frac{3\arctan(\sinh(e+fx))\cosh(e+fx)}{8f\sqrt{a\cosh^2(e+fx)}} - \frac{3\tanh(e+fx)}{8f\sqrt{a\cosh^2(e+fx)}} - \frac{\tanh^3(e+fx)}{4f\sqrt{a\cosh^2(e+fx)}}$$

output
```
3/8*arctan(sinh(f*x+e))*cosh(f*x+e)/f/(a*cosh(f*x+e)^2)^(1/2)-3/8*tanh(f*x
+e)/f/(a*cosh(f*x+e)^2)^(1/2)-1/4*tanh(f*x+e)^3/f/(a*cosh(f*x+e)^2)^(1/2)
```

Mathematica [A] (verified)

Time = 0.09 (sec) , antiderivative size = 66, normalized size of antiderivative = 0.73

$$\int \frac{\tanh^4(e+fx)}{\sqrt{a+a\sinh^2(e+fx)}}\,dx = \frac{3\arctan(\sinh(e+fx))\cosh(e+fx) + \tanh(e+fx)\left(3-6\text{sech}^2(e+fx)-8\tanh^2(e+fx)\right)}{8f\sqrt{a\cosh^2(e+fx)}}$$

input
```
Integrate[Tanh[e + f*x]^4/Sqrt[a + a*Sinh[e + f*x]^2],x]
```

output
```
(3*ArcTan[Sinh[e + f*x]]*Cosh[e + f*x] + Tanh[e + f*x]*(3 - 6*Sech[e + f*x
]^2 - 8*Tanh[e + f*x]^2))/(8*f*Sqrt[a*Cosh[e + f*x]^2])
```

Rubi [A] (verified)

Time = 0.65 (sec) , antiderivative size = 81, normalized size of antiderivative = 0.89, number of steps used = 12, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.480$, Rules used = {3042, 3655, 3042, 3686, 3042, 3091, 25, 3042, 25, 3091, 3042, 4257}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\tanh^4(e+fx)}{\sqrt{a\sinh^2(e+fx)+a}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\tan(ie+ifx)^4}{\sqrt{a-a\sin(ie+ifx)^2}}dx$$

$$\Big\downarrow \text{3655}$$

$$\int \frac{\tanh^4(e+fx)}{\sqrt{a\cosh^2(e+fx)}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{\tan\left(ie+ifx+\frac{\pi}{2}\right)^4\sqrt{a\sin\left(ie+ifx+\frac{\pi}{2}\right)^2}}dx$$

$$\Big\downarrow \text{3686}$$

$$\frac{\cosh(e+fx)\int \operatorname{sech}(e+fx)\tanh^4(e+fx)dx}{\sqrt{a\cosh^2(e+fx)}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\cosh(e+fx)\int \sec(ie+ifx)\tan(ie+ifx)^4dx}{\sqrt{a\cosh^2(e+fx)}}$$

$$\Big\downarrow \text{3091}$$

$$\frac{\cosh(e+fx)\left(-\frac{3}{4}\int -\text{sech}(e+fx)\tanh^2(e+fx)dx-\frac{\tanh^3(e+fx)\text{sech}(e+fx)}{4f}\right)}{\sqrt{a\cosh^2(e+fx)}}$$

$$\Big\downarrow \text{25}$$

$$\frac{\cosh(e+fx)\left(\frac{3}{4}\int \text{sech}(e+fx)\tanh^2(e+fx)dx-\frac{\tanh^3(e+fx)\text{sech}(e+fx)}{4f}\right)}{\sqrt{a\cosh^2(e+fx)}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\cosh(e+fx)\left(-\frac{\tanh^3(e+fx)\text{sech}(e+fx)}{4f}+\frac{3}{4}\int -\sec(ie+ifx)\tan(ie+ifx)^2dx\right)}{\sqrt{a\cosh^2(e+fx)}}$$

$$\Big\downarrow \text{25}$$

$$\frac{\cosh(e+fx)\left(-\frac{\tanh^3(e+fx)\text{sech}(e+fx)}{4f}-\frac{3}{4}\int \sec(ie+ifx)\tan(ie+ifx)^2dx\right)}{\sqrt{a\cosh^2(e+fx)}}$$

$$\Big\downarrow \text{3091}$$

$$\frac{\cosh(e+fx)\left(-\frac{3}{4}\left(\frac{\tanh(e+fx)\text{sech}(e+fx)}{2f}-\frac{1}{2}\int \text{sech}(e+fx)dx\right)-\frac{\tanh^3(e+fx)\text{sech}(e+fx)}{4f}\right)}{\sqrt{a\cosh^2(e+fx)}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\cosh(e+fx)\left(-\frac{\tanh^3(e+fx)\text{sech}(e+fx)}{4f}-\frac{3}{4}\left(\frac{\tanh(e+fx)\text{sech}(e+fx)}{2f}-\frac{1}{2}\int \csc\left(ie+ifx+\frac{\pi}{2}\right)dx\right)\right)}{\sqrt{a\cosh^2(e+fx)}}$$

$$\Big\downarrow \text{4257}$$

$$\frac{\cosh(e+fx)\left(-\frac{3}{4}\left(\frac{\tanh(e+fx)\text{sech}(e+fx)}{2f}-\frac{\arctan(\sinh(e+fx))}{2f}\right)-\frac{\tanh^3(e+fx)\text{sech}(e+fx)}{4f}\right)}{\sqrt{a\cosh^2(e+fx)}}$$

input
```
Int[Tanh[e + f*x]^4/Sqrt[a + a*Sinh[e + f*x]^2],x]
```

output
```
(Cosh[e + f*x]*(-1/4*(Sech[e + f*x]*Tanh[e + f*x]^3)/f - (3*(-1/2*ArcTan[S
inh[e + f*x]]/f + (Sech[e + f*x]*Tanh[e + f*x])/(2*f)))/4))/Sqrt[a*Cosh[e
+ f*x]^2]
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3091
```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[b*(a*Sec[e + f*x])^m*((b*Tan[e + f*x])^(n - 1)/(f*(m
 + n - 1))), x] - Simp[b^2*((n - 1)/(m + n - 1))   Int[(a*Sec[e + f*x])^m*(
b*Tan[e + f*x])^(n - 2), x], x] /; FreeQ[{a, b, e, f, m}, x] && GtQ[n, 1] &
& NeQ[m + n - 1, 0] && IntegersQ[2*m, 2*n]
```

rule 3655
```
Int[(u_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> Int[A
ctivateTrig[u*(a*cos[e + f*x]^2)^p], x] /; FreeQ[{a, b, e, f, p}, x] && EqQ
[a + b, 0]
```

rule 3686
```
Int[(u_.)*((b_.)*sin[(e_.) + (f_.)*(x_)]^(n_))^(p_), x_Symbol] :> With[{ff
= FreeFactors[Sin[e + f*x], x]}, Simp[(b*ff^n)^IntPart[p]*((b*Sin[e + f*x]^
n)^FracPart[p]/(Sin[e + f*x]/ff)^(n*FracPart[p]))   Int[ActivateTrig[u]*(Si
n[e + f*x]/ff)^(n*p), x], x]] /; FreeQ[{b, e, f, n, p}, x] &&  !IntegerQ[p]
 && IntegerQ[n] && (EqQ[u, 1] || MatchQ[u, ((d_.)*(trig_)[e + f*x])^(m_.) /
; FreeQ[{d, m}, x] && MemberQ[{sin, cos, tan, cot, sec, csc}, trig]])
```

rule 4257
```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Maple [A] (verified)

Time = 0.48 (sec) , antiderivative size = 145, normalized size of antiderivative = 1.59

method	result
default	$-\frac{\sqrt{a \sinh(fx+e)^2}\left(3\ln\left(\frac{2\sqrt{-a}\,\sqrt{a\sinh(fx+e)^2}-2a}{\cosh(fx+e)}\right)a\cosh(fx+e)^4+5\sqrt{a\sinh(fx+e)^2}\,\cosh(fx+e)^2\sqrt{-a}-2\sqrt{-a}\,\sqrt{a\sinh(fx+e}\right.}{8\cosh(fx+e)^3 a\sqrt{-a}\,\sinh(fx+e)\sqrt{a\cosh(fx+e)^2}\,f}$
risch	$-\frac{5\,e^{6fx+6e}-3\,e^{4fx+4e}+3\,e^{2fx+2e}-5}{4\sqrt{\left(e^{2fx+2e}+1\right)^2 a\,e^{-2fx-2e}}\,\left(e^{2fx+2e}+1\right)^3 f}+\frac{3i\ln\left(e^{fx}+ie^{-e}\right)\left(e^{2fx+2e}+1\right)e^{-fx-e}}{8f\sqrt{\left(e^{2fx+2e}+1\right)^2 a\,e^{-2fx-2e}}}-\frac{3i\ln\left(e^{fx}-ie^{-e}\right)\left(e^{2fx+2e}+1\right)e^{-fx}}{8f\sqrt{\left(e^{2fx+2e}+1\right)^2 a\,e^{-2fx-2e}}}$

input
```
int(tanh(f*x+e)^4/(a+a*sinh(f*x+e)^2)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-1/8/cosh(f*x+e)^3*(a*sinh(f*x+e)^2)^(1/2)*(3*ln(2/cosh(f*x+e)*((-a)^(1/2)
*(a*sinh(f*x+e)^2)^(1/2)-a))*a*cosh(f*x+e)^4+5*(a*sinh(f*x+e)^2)^(1/2)*cos
h(f*x+e)^2*(-a)^(1/2)-2*(-a)^(1/2)*(a*sinh(f*x+e)^2)^(1/2))/a/(-a)^(1/2)/s
inh(f*x+e)/(a*cosh(f*x+e)^2)^(1/2)/f
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1328 vs. 2(79) = 158.

Time = 0.12 (sec) , antiderivative size = 1328, normalized size of antiderivative = 14.59

$$\int \frac{\tanh^4(e+fx)}{\sqrt{a+a\sinh^2(e+fx)}}\,dx = \text{Too large to display}$$

input
```
integrate(tanh(f*x+e)^4/(a+a*sinh(f*x+e)^2)^(1/2),x, algorithm="fricas")
```

output
```
-1/4*(35*cosh(f*x + e)*e^(f*x + e)*sinh(f*x + e)^6 + 5*e^(f*x + e)*sinh(f*
x + e)^7 + 3*(35*cosh(f*x + e)^2 - 1)*e^(f*x + e)*sinh(f*x + e)^5 + 5*(35*
cosh(f*x + e)^3 - 3*cosh(f*x + e))*e^(f*x + e)*sinh(f*x + e)^4 + (175*cosh
(f*x + e)^4 - 30*cosh(f*x + e)^2 + 3)*e^(f*x + e)*sinh(f*x + e)^3 + 3*(35*
cosh(f*x + e)^5 - 10*cosh(f*x + e)^3 + 3*cosh(f*x + e))*e^(f*x + e)*sinh(f
*x + e)^2 + (35*cosh(f*x + e)^6 - 15*cosh(f*x + e)^4 + 9*cosh(f*x + e)^2 -
 5)*e^(f*x + e)*sinh(f*x + e) - 3*(8*cosh(f*x + e)*e^(f*x + e)*sinh(f*x +
e)^7 + e^(f*x + e)*sinh(f*x + e)^8 + 4*(7*cosh(f*x + e)^2 + 1)*e^(f*x + e)
*sinh(f*x + e)^6 + 8*(7*cosh(f*x + e)^3 + 3*cosh(f*x + e))*e^(f*x + e)*sin
h(f*x + e)^5 + 2*(35*cosh(f*x + e)^4 + 30*cosh(f*x + e)^2 + 3)*e^(f*x + e)
*sinh(f*x + e)^4 + 8*(7*cosh(f*x + e)^5 + 10*cosh(f*x + e)^3 + 3*cosh(f*x
+ e))*e^(f*x + e)*sinh(f*x + e)^3 + 4*(7*cosh(f*x + e)^6 + 15*cosh(f*x + e
)^4 + 9*cosh(f*x + e)^2 + 1)*e^(f*x + e)*sinh(f*x + e)^2 + 8*(cosh(f*x + e
)^7 + 3*cosh(f*x + e)^5 + 3*cosh(f*x + e)^3 + cosh(f*x + e))*e^(f*x + e)*s
inh(f*x + e) + (cosh(f*x + e)^8 + 4*cosh(f*x + e)^6 + 6*cosh(f*x + e)^4 +
4*cosh(f*x + e)^2 + 1)*e^(f*x + e))*arctan(cosh(f*x + e) + sinh(f*x + e))
+ (5*cosh(f*x + e)^7 - 3*cosh(f*x + e)^5 + 3*cosh(f*x + e)^3 - 5*cosh(f*x
+ e))*e^(f*x + e))*sqrt(a*e^(4*f*x + 4*e) + 2*a*e^(2*f*x + 2*e) + a)*e^(-f
*x - e)/(a*f*cosh(f*x + e)^8 + (a*f*e^(2*f*x + 2*e) + a*f)*sinh(f*x + e)^8
 + 4*a*f*cosh(f*x + e)^6 + 8*(a*f*cosh(f*x + e)*e^(2*f*x + 2*e) + a*f*c...
```

Sympy [F]

$$\int \frac{\tanh^4(e+fx)}{\sqrt{a+a\sinh^2(e+fx)}}\,dx = \int \frac{\tanh^4{(e+fx)}}{\sqrt{a\left(\sinh^2{(e+fx)}+1\right)}}\,dx$$

input
```
integrate(tanh(f*x+e)**4/(a+a*sinh(f*x+e)**2)**(1/2),x)
```

output
```
Integral(tanh(e + f*x)**4/sqrt(a*(sinh(e + f*x)**2 + 1)), x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 628 vs. $2(79) = 158$.

Time = 0.18 (sec) , antiderivative size = 628, normalized size of antiderivative = 6.90

$$\int \frac{\tanh^4(e+fx)}{\sqrt{a+a\sinh^2(e+fx)}}\,dx = \text{Too large to display}$$

input
```
integrate(tanh(f*x+e)^4/(a+a*sinh(f*x+e)^2)^(1/2),x, algorithm="maxima")
```

output
```
1/48*(15*arctan(e^(-f*x - e))/sqrt(a) - (15*e^(-f*x - e) + 55*e^(-3*f*x -
3*e) + 73*e^(-5*f*x - 5*e) - 15*e^(-7*f*x - 7*e))/(4*sqrt(a)*e^(-2*f*x - 2
*e) + 6*sqrt(a)*e^(-4*f*x - 4*e) + 4*sqrt(a)*e^(-6*f*x - 6*e) + sqrt(a)*e^
(-8*f*x - 8*e) + sqrt(a)))/f + 1/48*(15*arctan(e^(-f*x - e))/sqrt(a) - (15
*e^(-f*x - e) - 73*e^(-3*f*x - 3*e) - 55*e^(-5*f*x - 5*e) - 15*e^(-7*f*x -
 7*e))/(4*sqrt(a)*e^(-2*f*x - 2*e) + 6*sqrt(a)*e^(-4*f*x - 4*e) + 4*sqrt(a
)*e^(-6*f*x - 6*e) + sqrt(a)*e^(-8*f*x - 8*e) + sqrt(a)))/f - 3/32*(3*arct
an(e^(-f*x - e))/sqrt(a) - (3*e^(-f*x - e) + 11*e^(-3*f*x - 3*e) - 11*e^(-
5*f*x - 5*e) - 3*e^(-7*f*x - 7*e))/(4*sqrt(a)*e^(-2*f*x - 2*e) + 6*sqrt(a)
*e^(-4*f*x - 4*e) + 4*sqrt(a)*e^(-6*f*x - 6*e) + sqrt(a)*e^(-8*f*x - 8*e)
+ sqrt(a)))/f - 35/32*arctan(e^(-f*x - e))/(sqrt(a)*f) - 1/192*(279*e^(-f*
x - e) + 511*e^(-3*f*x - 3*e) + 385*e^(-5*f*x - 5*e) + 105*e^(-7*f*x - 7*e
))/((4*sqrt(a)*e^(-2*f*x - 2*e) + 6*sqrt(a)*e^(-4*f*x - 4*e) + 4*sqrt(a)*e
^(-6*f*x - 6*e) + sqrt(a)*e^(-8*f*x - 8*e) + sqrt(a))*f) + 1/192*(105*e^(-
f*x - e) + 385*e^(-3*f*x - 3*e) + 511*e^(-5*f*x - 5*e) + 279*e^(-7*f*x - 7
*e))/((4*sqrt(a)*e^(-2*f*x - 2*e) + 6*sqrt(a)*e^(-4*f*x - 4*e) + 4*sqrt(a)
*e^(-6*f*x - 6*e) + sqrt(a)*e^(-8*f*x - 8*e) + sqrt(a))*f)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{\tanh^4(e+fx)}{\sqrt{a+a\sinh^2(e+fx)}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(tanh(f*x+e)^4/(a+a*sinh(f*x+e)^2)^(1/2),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\tanh^4(e+fx)}{\sqrt{a+a\sinh^2(e+fx)}}\,dx=\int \frac{\tanh(e+f\,x)^4}{\sqrt{a\sinh\left(e+f\,x\right)^2+a}}\,dx$$

input
```
int(tanh(e + f*x)^4/(a + a*sinh(e + f*x)^2)^(1/2),x)
```

output
```
int(tanh(e + f*x)^4/(a + a*sinh(e + f*x)^2)^(1/2), x)
```

Reduce [F]

$$\int \frac{\tanh^4(e+fx)}{\sqrt{a+a\sinh^2(e+fx)}}\,dx=\frac{\sqrt{a}\left(\int \frac{\sqrt{\sinh(fx+e)^2+1}\,\tanh(fx+e)^4}{\sinh(fx+e)^2+1}dx\right)}{a}$$

input
```
int(tanh(f*x+e)^4/(a+a*sinh(f*x+e)^2)^(1/2),x)
```

output
```
(sqrt(a)*int((sqrt(sinh(e + f*x)**2 + 1)*tanh(e + f*x)**4)/(sinh(e + f*x)*
*2 + 1),x))/a
```

3.407 $\int \frac{\tanh^2(e+fx)}{\sqrt{a+a\sinh^2(e+fx)}}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 62

$$\int \frac{\tanh^2(e+fx)}{\sqrt{a+a\sinh^2(e+fx)}}\,dx=\frac{\arctan(\sinh(e+fx))\cosh(e+fx)}{2f\sqrt{a\cosh^2(e+fx)}}-\frac{\tanh(e+fx)}{2f\sqrt{a\cosh^2(e+fx)}}$$

output
```
1/2*arctan(sinh(f*x+e))*cosh(f*x+e)/f/(a*cosh(f*x+e)^2)^(1/2)-1/2*tanh(f*x
+e)/f/(a*cosh(f*x+e)^2)^(1/2)
```

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.71

$$\int \frac{\tanh^2(e+fx)}{\sqrt{a+a\sinh^2(e+fx)}}\,dx=\frac{\arctan(\sinh(e+fx))\cosh(e+fx)-\tanh(e+fx)}{2f\sqrt{a\cosh^2(e+fx)}}$$

input
```
Integrate[Tanh[e + f*x]^2/Sqrt[a + a*Sinh[e + f*x]^2],x]
```

output
```
(ArcTan[Sinh[e + f*x]]*Cosh[e + f*x] - Tanh[e + f*x])/(2*f*Sqrt[a*Cosh[e +
 f*x]^2])
```

Rubi [A] (verified)

Time = 0.57 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.90, number of steps used = 13, number of rules used = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.520, Rules used = {3042, 25, 3655, 25, 3042, 25, 3686, 25, 3042, 25, 3091, 3042, 4257}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\tanh^2(e+fx)}{\sqrt{a\sinh^2(e+fx)+a}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int -\frac{\tan(ie+ifx)^2}{\sqrt{a-a\sin(ie+ifx)^2}}dx$$

$$\Big\downarrow \text{25}$$

$$-\int \frac{\tan(ie+ifx)^2}{\sqrt{a-a\sin(ie+ifx)^2}}dx$$

$$\Big\downarrow \text{3655}$$

$$-\int -\frac{\tanh^2(e+fx)}{\sqrt{a\cosh^2(e+fx)}}dx$$

$$\Big\downarrow \text{25}$$

$$\int \frac{\tanh^2(e+fx)}{\sqrt{a\cosh^2(e+fx)}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int -\frac{1}{\tan\left(ie+ifx+\frac{\pi}{2}\right)^2\sqrt{a\sin\left(ie+ifx+\frac{\pi}{2}\right)^2}}dx$$

$$\Big\downarrow \text{25}$$

$$-\int \frac{1}{\sqrt{a\sin\left(\frac{1}{2}(2ie+\pi)+ifx\right)^2}\tan\left(\frac{1}{2}(2ie+\pi)+ifx\right)^2}dx$$

$$\Big\downarrow \text{3686}$$

$$-\frac{\cosh(e+fx)\int -\text{sech}(e+fx)\tanh^2(e+fx)dx}{\sqrt{a\cosh^2(e+fx)}}$$

$$\Big\downarrow \ 25$$

$$\frac{\cosh(e+fx)\int \text{sech}(e+fx)\tanh^2(e+fx)dx}{\sqrt{a\cosh^2(e+fx)}}$$

$$\Big\downarrow \ 3042$$

$$\frac{\cosh(e+fx)\int -\sec(ie+ifx)\tan(ie+ifx)^2dx}{\sqrt{a\cosh^2(e+fx)}}$$

$$\Big\downarrow \ 25$$

$$-\frac{\cosh(e+fx)\int \sec(ie+ifx)\tan(ie+ifx)^2dx}{\sqrt{a\cosh^2(e+fx)}}$$

$$\Big\downarrow \ 3091$$

$$-\frac{\cosh(e+fx)\left(\frac{\tanh(e+fx)\text{sech}(e+fx)}{2f}-\frac{1}{2}\int \text{sech}(e+fx)dx\right)}{\sqrt{a\cosh^2(e+fx)}}$$

$$\Big\downarrow \ 3042$$

$$-\frac{\cosh(e+fx)\left(\frac{\tanh(e+fx)\text{sech}(e+fx)}{2f}-\frac{1}{2}\int \csc\left(ie+ifx+\frac{\pi}{2}\right)dx\right)}{\sqrt{a\cosh^2(e+fx)}}$$

$$\Big\downarrow \ 4257$$

$$-\frac{\cosh(e+fx)\left(\frac{\tanh(e+fx)\text{sech}(e+fx)}{2f}-\frac{\arctan(\sinh(e+fx))}{2f}\right)}{\sqrt{a\cosh^2(e+fx)}}$$

input
```
Int[Tanh[e + f*x]^2/Sqrt[a + a*Sinh[e + f*x]^2],x]
```

output
```
-((Cosh[e + f*x]*(-1/2*ArcTan[Sinh[e + f*x]]/f + (Sech[e + f*x]*Tanh[e + f
*x])/(2*f)))/Sqrt[a*Cosh[e + f*x]^2])
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3091
```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[b*(a*Sec[e + f*x])^m*((b*Tan[e + f*x])^(n - 1)/(f*(m
 + n - 1))), x] - Simp[b^2*((n - 1)/(m + n - 1))   Int[(a*Sec[e + f*x])^m*(
b*Tan[e + f*x])^(n - 2), x], x] /; FreeQ[{a, b, e, f, m}, x] && GtQ[n, 1] &
& NeQ[m + n - 1, 0] && IntegersQ[2*m, 2*n]
```

rule 3655
```
Int[(u_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> Int[A
ctivateTrig[u*(a*cos[e + f*x]^2)^p], x] /; FreeQ[{a, b, e, f, p}, x] && EqQ
[a + b, 0]
```

rule 3686
```
Int[(u_.)*((b_.)*sin[(e_.) + (f_.)*(x_)]^(n_))^(p_), x_Symbol] :> With[{ff
= FreeFactors[Sin[e + f*x], x]}, Simp[(b*ff^n)^IntPart[p]*((b*Sin[e + f*x]^
n)^FracPart[p]/(Sin[e + f*x]/ff)^(n*FracPart[p]))   Int[ActivateTrig[u]*(Si
n[e + f*x]/ff)^(n*p), x], x]] /; FreeQ[{b, e, f, n, p}, x] &&  !IntegerQ[p]
 && IntegerQ[n] && (EqQ[u, 1] || MatchQ[u, ((d_.)*(trig_)[e + f*x])^(m_.) /
; FreeQ[{d, m}, x] && MemberQ[{sin, cos, tan, cot, sec, csc}, trig]])
```

rule 4257
```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 115 vs. 2(54) = 108.

Time = 0.43 (sec) , antiderivative size = 116, normalized size of antiderivative = 1.87

method	result
default	$-\frac{\sqrt{a\sinh(fx+e)^2}\left(\ln\left(\frac{2\sqrt{-a}\sqrt{a\sinh(fx+e)^2}-2a}{\cosh(fx+e)}\right)\cosh(fx+e)^2 a+\sqrt{-a}\sqrt{a\sinh(fx+e)^2}\right)}{2\cosh(fx+e)\sqrt{-a}\, a\sinh(fx+e)\sqrt{a\cosh(fx+e)^2}\, f}$
risch	$-\frac{e^{2fx+2e}-1}{\sqrt{\left(e^{2fx+2e}+1\right)^2 a\, e^{-2fx-2e}}\left(e^{2fx+2e}+1\right)f}+\frac{i\ln\left(e^{fx}+ie^{-e}\right)\left(e^{2fx+2e}+1\right)e^{-fx-e}}{2f\sqrt{\left(e^{2fx+2e}+1\right)^2 a\, e^{-2fx-2e}}}-\frac{i\ln\left(e^{fx}-ie^{-e}\right)\left(e^{2fx+2e}+1\right)e^{-fx-e}}{2f\sqrt{\left(e^{2fx+2e}+1\right)^2 a\, e^{-2fx-2e}}}$

input
```
int(tanh(f*x+e)^2/(a+a*sinh(f*x+e)^2)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-1/2/cosh(f*x+e)*(a*sinh(f*x+e)^2)^(1/2)*(ln(2/cosh(f*x+e)*((-a)^(1/2)*(a*
sinh(f*x+e)^2)^(1/2)-a))*cosh(f*x+e)^2*a+(-a)^(1/2)*(a*sinh(f*x+e)^2)^(1/2
))/(-a)^(1/2)/a/sinh(f*x+e)/(a*cosh(f*x+e)^2)^(1/2)/f
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 504 vs. 2(54) = 108.

Time = 0.10 (sec) , antiderivative size = 504, normalized size of antiderivative = 8.13

$$\int \frac{\tanh^2(e+fx)}{\sqrt{a+a\sinh^2(e+fx)}}\,dx = -\frac{\left(3\cosh\left(fx+e\right)e^{(fx+e)}\sinh\left(fx+e\right)^2+e^{(fx+e)}\sinh\left(fx+e\right)^3+\left(3\cosh\left(fx+e\right)^2-1\right)e^{(fx+e)}\sinh\left(\right.\right.}{af\cosh\left(fx+e\right)^4+\left(afe^{(2fx+2e)}+af\right)\sinh\left(fx+e\right)^4+2af\cosh\left(fx+e\right)}$$

input
```
integrate(tanh(f*x+e)^2/(a+a*sinh(f*x+e)^2)^(1/2),x, algorithm="fricas")
```

output
```
-(3*cosh(f*x + e)*e^(f*x + e)*sinh(f*x + e)^2 + e^(f*x + e)*sinh(f*x + e)^
3 + (3*cosh(f*x + e)^2 - 1)*e^(f*x + e)*sinh(f*x + e) - (4*cosh(f*x + e)*e
^(f*x + e)*sinh(f*x + e)^3 + e^(f*x + e)*sinh(f*x + e)^4 + 2*(3*cosh(f*x +
 e)^2 + 1)*e^(f*x + e)*sinh(f*x + e)^2 + 4*(cosh(f*x + e)^3 + cosh(f*x + e
))*e^(f*x + e)*sinh(f*x + e) + (cosh(f*x + e)^4 + 2*cosh(f*x + e)^2 + 1)*e
^(f*x + e))*arctan(cosh(f*x + e) + sinh(f*x + e)) + (cosh(f*x + e)^3 - cos
h(f*x + e))*e^(f*x + e))*sqrt(a*e^(4*f*x + 4*e) + 2*a*e^(2*f*x + 2*e) + a)
*e^(-f*x - e)/(a*f*cosh(f*x + e)^4 + (a*f*e^(2*f*x + 2*e) + a*f)*sinh(f*x
+ e)^4 + 2*a*f*cosh(f*x + e)^2 + 4*(a*f*cosh(f*x + e)*e^(2*f*x + 2*e) + a*
f*cosh(f*x + e))*sinh(f*x + e)^3 + 2*(3*a*f*cosh(f*x + e)^2 + a*f + (3*a*f
*cosh(f*x + e)^2 + a*f)*e^(2*f*x + 2*e))*sinh(f*x + e)^2 + a*f + (a*f*cosh
(f*x + e)^4 + 2*a*f*cosh(f*x + e)^2 + a*f)*e^(2*f*x + 2*e) + 4*(a*f*cosh(f
*x + e)^3 + a*f*cosh(f*x + e) + (a*f*cosh(f*x + e)^3 + a*f*cosh(f*x + e))*
e^(2*f*x + 2*e))*sinh(f*x + e))
```

Sympy [F]

$$\int \frac{\tanh^2(e+fx)}{\sqrt{a+a\sinh^2(e+fx)}}\,dx=\int \frac{\tanh^2{(e+fx)}}{\sqrt{a\left(\sinh^2{(e+fx)}+1\right)}}\,dx$$

input
```
integrate(tanh(f*x+e)**2/(a+a*sinh(f*x+e)**2)**(1/2),x)
```

output
```
Integral(tanh(e + f*x)**2/sqrt(a*(sinh(e + f*x)**2 + 1)), x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 217 vs. $2(54) = 108$.

Time = 0.17 (sec) , antiderivative size = 217, normalized size of antiderivative = 3.50

$$\int \frac{\tanh^2(e+fx)}{\sqrt{a+a\sinh^2(e+fx)}}\,dx = \frac{\frac{\arctan\left(e^{(-fx-e)}\right)}{\sqrt{a}} - \frac{e^{(-fx-e)}-e^{(-3\,fx-3\,e)}}{2\,\sqrt{a}e^{(-2\,fx-2\,e)}+\sqrt{a}e^{(-4\,fx-4\,e)}+\sqrt{a}}}{2\,f} - \frac{3\,\arctan\left(e^{(-fx-e)}\right)}{2\,\sqrt{a}f} - \frac{5\,e^{(-fx-e)}+3\,e^{(-3\,fx-3\,e)}}{4\left(2\,\sqrt{a}e^{(-2\,fx-2\,e)}+\sqrt{a}e^{(-4\,fx-4\,e)}+\sqrt{a}\right)f} + \frac{3\,e^{(-fx-e)}+5\,e^{(-3\,fx-3\,e)}}{4\left(2\,\sqrt{a}e^{(-2\,fx-2\,e)}+\sqrt{a}e^{(-4\,fx-4\,e)}+\sqrt{a}\right)f}$$

input
```
integrate(tanh(f*x+e)^2/(a+a*sinh(f*x+e)^2)^(1/2),x, algorithm="maxima")
```

output
```
1/2*(arctan(e^(-f*x - e))/sqrt(a) - (e^(-f*x - e) - e^(-3*f*x - 3*e))/(2*s
qrt(a)*e^(-2*f*x - 2*e) + sqrt(a)*e^(-4*f*x - 4*e) + sqrt(a)))/f - 3/2*arc
tan(e^(-f*x - e))/(sqrt(a)*f) - 1/4*(5*e^(-f*x - e) + 3*e^(-3*f*x - 3*e))/
((2*sqrt(a)*e^(-2*f*x - 2*e) + sqrt(a)*e^(-4*f*x - 4*e) + sqrt(a))*f) + 1/
4*(3*e^(-f*x - e) + 5*e^(-3*f*x - 3*e))/((2*sqrt(a)*e^(-2*f*x - 2*e) + sqr
t(a)*e^(-4*f*x - 4*e) + sqrt(a))*f)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{\tanh^2(e+fx)}{\sqrt{a+a\sinh^2(e+fx)}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(tanh(f*x+e)^2/(a+a*sinh(f*x+e)^2)^(1/2),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\tanh^2(e+fx)}{\sqrt{a+a\sinh^2(e+fx)}}\,dx=\int \frac{\tanh(e+f\,x)^2}{\sqrt{a\sinh\left(e+f\,x\right)^2+a}}\,dx$$

input
```
int(tanh(e + f*x)^2/(a + a*sinh(e + f*x)^2)^(1/2),x)
```

output
```
int(tanh(e + f*x)^2/(a + a*sinh(e + f*x)^2)^(1/2), x)
```

Reduce [F]

$$\int \frac{\tanh^2(e+fx)}{\sqrt{a+a\sinh^2(e+fx)}}\,dx=\frac{\sqrt{a}\left(\int \frac{\sqrt{\sinh(fx+e)^2+1}\,\tanh(fx+e)^2}{\sinh(fx+e)^2+1}dx\right)}{a}$$

input
```
int(tanh(f*x+e)^2/(a+a*sinh(f*x+e)^2)^(1/2),x)
```

output
```
(sqrt(a)*int((sqrt(sinh(e + f*x)**2 + 1)*tanh(e + f*x)**2)/(sinh(e + f*x)*
*2 + 1),x))/a
```

3.408 $\int \frac{\coth^2(e+fx)}{\sqrt{a+a\sinh^2(e+fx)}}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 25

$$\int \frac{\coth^2(e+fx)}{\sqrt{a+a\sinh^2(e+fx)}}\,dx = -\frac{\coth(e+fx)}{f\sqrt{a\cosh^2(e+fx)}}$$

output
```
-coth(f*x+e)/f/(a*cosh(f*x+e)^2)^(1/2)
```

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00

$$\int \frac{\coth^2(e+fx)}{\sqrt{a+a\sinh^2(e+fx)}}\,dx = -\frac{\coth(e+fx)}{f\sqrt{a\cosh^2(e+fx)}}$$

input
```
Integrate[Coth[e + f*x]^2/Sqrt[a + a*Sinh[e + f*x]^2],x]
```

output
```
-(Coth[e + f*x]/(f*Sqrt[a*Cosh[e + f*x]^2]))
```

Rubi [A] (verified)

Time = 0.48 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.440, Rules used = {3042, 25, 3655, 25, 3042, 25, 3686, 25, 3042, 3086, 24}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\coth^2(e+fx)}{\sqrt{a\sinh^2(e+fx)+a}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int -\frac{1}{\tan(ie+ifx)^2\sqrt{a-a\sin(ie+ifx)^2}}dx$$

$$\Big\downarrow \text{25}$$

$$-\int \frac{1}{\sqrt{a-a\sin(ie+ifx)^2}\tan(ie+ifx)^2}dx$$

$$\Big\downarrow \text{3655}$$

$$-\int -\frac{\coth^2(e+fx)}{\sqrt{a\cosh^2(e+fx)}}dx$$

$$\Big\downarrow \text{25}$$

$$\int \frac{\coth^2(e+fx)}{\sqrt{a\cosh^2(e+fx)}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int -\frac{\tan\left(ie+ifx+\frac{\pi}{2}\right)^2}{\sqrt{a\sin\left(ie+ifx+\frac{\pi}{2}\right)^2}}dx$$

$$\Big\downarrow \text{25}$$

$$-\int \frac{\tan\left(\frac{1}{2}(2ie+\pi)+ifx\right)^2}{\sqrt{a\sin\left(\frac{1}{2}(2ie+\pi)+ifx\right)^2}}dx$$

$$\Big\downarrow \text{3686}$$

$$-\frac{\cosh(e+fx)\int -\coth(e+fx)\text{csch}(e+fx)dx}{\sqrt{a\cosh^2(e+fx)}}$$

$$\Big\downarrow \text{ 25}$$

$$\frac{\cosh(e+fx)\int \coth(e+fx)\text{csch}(e+fx)dx}{\sqrt{a\cosh^2(e+fx)}}$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{\cosh(e+fx)\int \sec\left(ie+ifx-\frac{\pi}{2}\right)\tan\left(ie+ifx-\frac{\pi}{2}\right)dx}{\sqrt{a\cosh^2(e+fx)}}$$

$$\Big\downarrow \text{ 3086}$$

$$-\frac{i\cosh(e+fx)\int 1d(-i\text{csch}(e+fx))}{f\sqrt{a\cosh^2(e+fx)}}$$

$$\Big\downarrow \text{ 24}$$

$$-\frac{\coth(e+fx)}{f\sqrt{a\cosh^2(e+fx)}}$$

input
```
Int[Coth[e + f*x]^2/Sqrt[a + a*Sinh[e + f*x]^2],x]
```

output
```
-(Coth[e + f*x]/(f*Sqrt[a*Cosh[e + f*x]^2]))
```

Defintions of rubi rules used

rule 24
```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3086
```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Simp[a/f   Subst[Int[(a*x)^(m - 1)*(-1 + x^2)^((n - 1)/2
), x], x, Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2
] &&  !(IntegerQ[m/2] && LtQ[0, m, n + 1])
```

rule 3655
```
Int[(u_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> Int[A
ctivateTrig[u*(a*cos[e + f*x]^2)^p], x] /; FreeQ[{a, b, e, f, p}, x] && EqQ
[a + b, 0]
```

rule 3686
```
Int[(u_.)*((b_.)*sin[(e_.) + (f_.)*(x_)]^(n_))^(p_), x_Symbol] :> With[{ff
= FreeFactors[Sin[e + f*x], x]}, Simp[(b*ff^n)^IntPart[p]*((b*Sin[e + f*x]^
n)^FracPart[p]/(Sin[e + f*x]/ff)^(n*FracPart[p]))   Int[ActivateTrig[u]*(Si
n[e + f*x]/ff)^(n*p), x], x]] /; FreeQ[{b, e, f, n, p}, x] &&  !IntegerQ[p]
 && IntegerQ[n] && (EqQ[u, 1] || MatchQ[u, ((d_.)*(trig_)[e + f*x])^(m_.) /
; FreeQ[{d, m}, x] && MemberQ[{sin, cos, tan, cot, sec, csc}, trig]])
```

Maple [A] (verified)

Time = 0.37 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.28

method	result	size
default	$-\frac{\cosh\left(fx+e\right)}{\sinh\left(fx+e\right)\sqrt{a\cosh\left(fx+e\right)^{2}}\, f}$	32
risch	$-\frac{2\left({\mathrm e}^{2fx+2e}+1\right)}{\sqrt{\left({\mathrm e}^{2fx+2e}+1\right)^{2} a\,{\mathrm e}^{-2fx-2e}}\, f\left({\mathrm e}^{2fx+2e}-1\right)}$	56

input
```
int(coth(f*x+e)^2/(a+a*sinh(f*x+e)^2)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-cosh(f*x+e)/sinh(f*x+e)/(a*cosh(f*x+e)^2)^(1/2)/f
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 170 vs. $2(23) = 46$.

Time = 0.09 (sec) , antiderivative size = 170, normalized size of antiderivative = 6.80

$$\int \frac{\coth^2(e+fx)}{\sqrt{a+a\sinh^2(e+fx)}}\,dx = -\frac{2\sqrt{ae^{(4\,fx+4\,e)}+2\,ae^{(2\,fx+2\,e)}+a}\left(\cosh\left(fx+e\right)e^{(fx+e)}+e^{(fx+e)}\right.}{af\cosh\left(fx+e\right)^2+\left(afe^{(2\,fx+2\,e)}+af\right)\sinh\left(fx+e\right)^2-af+\left(af\cosh\left(fx+e\right)^2-af\right)e^{(2\,fx+2\,e)}+2}$$

input
```
integrate(coth(f*x+e)^2/(a+a*sinh(f*x+e)^2)^(1/2),x, algorithm="fricas")
```

output
```
-2*sqrt(a*e^(4*f*x + 4*e) + 2*a*e^(2*f*x + 2*e) + a)*(cosh(f*x + e)*e^(f*x
 + e) + e^(f*x + e)*sinh(f*x + e))*e^(-f*x - e)/(a*f*cosh(f*x + e)^2 + (a*
f*e^(2*f*x + 2*e) + a*f)*sinh(f*x + e)^2 - a*f + (a*f*cosh(f*x + e)^2 - a*
f)*e^(2*f*x + 2*e) + 2*(a*f*cosh(f*x + e)*e^(2*f*x + 2*e) + a*f*cosh(f*x +
 e))*sinh(f*x + e))
```

Sympy [F]

$$\int \frac{\coth^2(e+fx)}{\sqrt{a+a\sinh^2(e+fx)}}\,dx = \int \frac{\coth^2\left(e+fx\right)}{\sqrt{a\left(\sinh^2\left(e+fx\right)+1\right)}}\,dx$$

input
```
integrate(coth(f*x+e)**2/(a+a*sinh(f*x+e)**2)**(1/2),x)
```

output
```
Integral(coth(e + f*x)**2/sqrt(a*(sinh(e + f*x)**2 + 1)), x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 101 vs. $2(23) = 46$.

Time = 0.15 (sec) , antiderivative size = 101, normalized size of antiderivative = 4.04

$$\int \frac{\coth^2(e+fx)}{\sqrt{a+a\sinh^2(e+fx)}}\,dx = \frac{\frac{\arctan\left(e^{(-fx-e)}\right)}{\sqrt{a}} + \frac{\sqrt{a}e^{(-fx-e)}}{ae^{(-2\,fx-2\,e)}-a}}{f} - \frac{\arctan\left(e^{(-fx-e)}\right)}{\sqrt{a}f} + \frac{\sqrt{a}e^{(-fx-e)}}{\left(ae^{(-2\,fx-2\,e)}-a\right)f}$$

input
```
integrate(coth(f*x+e)^2/(a+a*sinh(f*x+e)^2)^(1/2),x, algorithm="maxima")
```

output
```
(arctan(e^(-f*x - e))/sqrt(a) + sqrt(a)*e^(-f*x - e)/(a*e^(-2*f*x - 2*e) -
 a))/f - arctan(e^(-f*x - e))/(sqrt(a)*f) + sqrt(a)*e^(-f*x - e)/((a*e^(-2
*f*x - 2*e) - a)*f)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{\coth^2(e+fx)}{\sqrt{a+a\sinh^2(e+fx)}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(coth(f*x+e)^2/(a+a*sinh(f*x+e)^2)^(1/2),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [B] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 76, normalized size of antiderivative = 3.04

$$\int \frac{\coth^2(e+fx)}{\sqrt{a+a\sinh^2(e+fx)}}\,dx = -\frac{4\,e^{3\,e+3\,f\,x}\,\sqrt{a+a\,\left(\frac{e^{e+f\,x}}{2}-\frac{e^{-e-f\,x}}{2}\right)^2}}{a\,f\,\left(e^{2\,e+2\,f\,x}-1\right)\,\left(e^{e+f\,x}+e^{3\,e+3\,f\,x}\right)}$$

input
```
int(coth(e + f*x)^2/(a + a*sinh(e + f*x)^2)^(1/2),x)
```

output
```
-(4*exp(3*e + 3*f*x)*(a + a*(exp(e + f*x)/2 - exp(- e - f*x)/2)^2)^(1/2))/
(a*f*(exp(2*e + 2*f*x) - 1)*(exp(e + f*x) + exp(3*e + 3*f*x)))
```

Reduce [F]

$$\int \frac{\coth^2(e+fx)}{\sqrt{a+a\sinh^2(e+fx)}}\,dx = \frac{\sqrt{a}\left(\int \frac{\sqrt{\sinh(fx+e)^2+1}\,\coth(fx+e)^2}{\sinh(fx+e)^2+1}dx\right)}{a}$$

input
```
int(coth(f*x+e)^2/(a+a*sinh(f*x+e)^2)^(1/2),x)
```

output
```
(sqrt(a)*int((sqrt(sinh(e + f*x)**2 + 1)*coth(e + f*x)**2)/(sinh(e + f*x)*
*2 + 1),x))/a
```

3.409 $\int \frac{\coth^4(e+fx)}{\sqrt{a+a\sinh^2(e+fx)}}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 61

$$\int \frac{\coth^4(e+fx)}{\sqrt{a+a\sinh^2(e+fx)}}\,dx = -\frac{\coth(e+fx)}{f\sqrt{a\cosh^2(e+fx)}} - \frac{\coth(e+fx)\operatorname{csch}^2(e+fx)}{3f\sqrt{a\cosh^2(e+fx)}}$$

output
```
-coth(f*x+e)/f/(a*cosh(f*x+e)^2)^(1/2)-1/3*coth(f*x+e)*csch(f*x+e)^2/f/(a*
cosh(f*x+e)^2)^(1/2)
```

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.61

$$\int \frac{\coth^4(e+fx)}{\sqrt{a+a\sinh^2(e+fx)}}\,dx = -\frac{\coth(e+fx)\left(3+\operatorname{csch}^2(e+fx)\right)}{3f\sqrt{a\cosh^2(e+fx)}}$$

input
```
Integrate[Coth[e + f*x]^4/Sqrt[a + a*Sinh[e + f*x]^2],x]
```

output
```
-1/3*(Coth[e + f*x]*(3 + Csch[e + f*x]^2))/(f*Sqrt[a*Cosh[e + f*x]^2])
```

Rubi [C] (verified)

Result contains complex when optimal does not.

Time = 0.52 (sec) , antiderivative size = 52, normalized size of antiderivative = 0.85, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.320$, Rules used = {3042, 3655, 3042, 3686, 3042, 25, 3086, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\coth^4(e+fx)}{\sqrt{a\sinh^2(e+fx)+a}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{\tan(ie+ifx)^4\sqrt{a-a\sin(ie+ifx)^2}}dx$$

$$\Big\downarrow \text{3655}$$

$$\int \frac{\coth^4(e+fx)}{\sqrt{a\cosh^2(e+fx)}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\tan\left(ie+ifx+\frac{\pi}{2}\right)^4}{\sqrt{a\sin\left(ie+ifx+\frac{\pi}{2}\right)^2}}dx$$

$$\Big\downarrow \text{3686}$$

$$\frac{\cosh(e+fx)\int \coth^3(e+fx)\text{csch}(e+fx)dx}{\sqrt{a\cosh^2(e+fx)}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\cosh(e+fx)\int -\sec\left(ie+ifx-\frac{\pi}{2}\right)\tan\left(ie+ifx-\frac{\pi}{2}\right)^3dx}{\sqrt{a\cosh^2(e+fx)}}$$

$$\Big\downarrow \text{25}$$

$$-\frac{\cosh(e+fx)\int \sec\left(\frac{1}{2}(2ie-\pi)+ifx\right)\tan\left(\frac{1}{2}(2ie-\pi)+ifx\right)^3dx}{\sqrt{a\cosh^2(e+fx)}}$$

$$\Big\downarrow \text{3086}$$

$$\frac{i\cosh(e+fx)\int\left(-\text{csch}^2(e+fx)-1\right)d(-i\text{csch}(e+fx))}{f\sqrt{a\cosh^2(e+fx)}}$$

$$\Big\downarrow \text{2009}$$

$$\frac{i\cosh(e+fx)\left(\frac{1}{3}i\text{csch}^3(e+fx)+i\text{csch}(e+fx)\right)}{f\sqrt{a\cosh^2(e+fx)}}$$

input
```
Int[Coth[e + f*x]^4/Sqrt[a + a*Sinh[e + f*x]^2],x]
```

output
```
(I*Cosh[e + f*x]*(I*Csch[e + f*x] + (I/3)*Csch[e + f*x]^3))/(f*Sqrt[a*Cosh
[e + f*x]^2])
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3086
```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Simp[a/f   Subst[Int[(a*x)^(m - 1)*(-1 + x^2)^((n - 1)/2
), x], x, Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2
] &&  !(IntegerQ[m/2] && LtQ[0, m, n + 1])
```

rule 3655
```
Int[(u_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> Int[A
ctivateTrig[u*(a*cos[e + f*x]^2)^p], x] /; FreeQ[{a, b, e, f, p}, x] && EqQ
[a + b, 0]
```

rule 3686
```
Int[(u_.)*((b_.)*sin[(e_.) + (f_.)*(x_)]^(n_))^(p_), x_Symbol] :> With[{ff
= FreeFactors[Sin[e + f*x], x]}, Simp[(b*ff^n)^IntPart[p]*((b*Sin[e + f*x]^
n)^FracPart[p]/(Sin[e + f*x]/ff)^(n*FracPart[p]))   Int[ActivateTrig[u]*(Si
n[e + f*x]/ff)^(n*p), x], x]] /; FreeQ[{b, e, f, n, p}, x] &&  !IntegerQ[p]
 && IntegerQ[n] && (EqQ[u, 1] || MatchQ[u, ((d_.)*(trig_)[e + f*x])^(m_.) /
; FreeQ[{d, m}, x] && MemberQ[{sin, cos, tan, cot, sec, csc}, trig]])
```

Maple [A] (verified)

Time = 0.40 (sec) , antiderivative size = 64, normalized size of antiderivative = 1.05

method	result	size
default	$-\frac{\cosh(fx+e)\left(3\cosh(fx+e)^{2}-2\right)}{3(\cosh(fx+e)-1)(\cosh(fx+e)+1)\sinh(fx+e)\sqrt{a\cosh(fx+e)^{2}}\, f}$	64
risch	$-\frac{2\left(\mathrm{e}^{2fx+2e}+1\right)\left(3\,\mathrm{e}^{4fx+4e}-2\,\mathrm{e}^{2fx+2e}+3\right)}{3\sqrt{\left(\mathrm{e}^{2fx+2e}+1\right)^{2} a\,\mathrm{e}^{-2fx-2e}}\, f\left(\mathrm{e}^{2fx+2e}-1\right)^{3}}$	80

input
```
int(coth(f*x+e)^4/(a+a*sinh(f*x+e)^2)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-1/3*cosh(f*x+e)*(3*cosh(f*x+e)^2-2)/(cosh(f*x+e)-1)/(cosh(f*x+e)+1)/sinh(
f*x+e)/(a*cosh(f*x+e)^2)^(1/2)/f
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 647 vs. 2(55) = 110.

Time = 0.09 (sec) , antiderivative size = 647, normalized size of antiderivative = 10.61

$$\int \frac{\coth^4(e+fx)}{\sqrt{a+a\sinh^2(e+fx)}}\,dx = \text{Too large to display}$$

input
```
integrate(coth(f*x+e)^4/(a+a*sinh(f*x+e)^2)^(1/2),x, algorithm="fricas")
```

output
```
-2/3*(15*cosh(f*x + e)*e^(f*x + e)*sinh(f*x + e)^4 + 3*e^(f*x + e)*sinh(f*
x + e)^5 + 2*(15*cosh(f*x + e)^2 - 1)*e^(f*x + e)*sinh(f*x + e)^3 + 6*(5*c
osh(f*x + e)^3 - cosh(f*x + e))*e^(f*x + e)*sinh(f*x + e)^2 + 3*(5*cosh(f*
x + e)^4 - 2*cosh(f*x + e)^2 + 1)*e^(f*x + e)*sinh(f*x + e) + (3*cosh(f*x
+ e)^5 - 2*cosh(f*x + e)^3 + 3*cosh(f*x + e))*e^(f*x + e))*sqrt(a*e^(4*f*x
 + 4*e) + 2*a*e^(2*f*x + 2*e) + a)*e^(-f*x - e)/(a*f*cosh(f*x + e)^6 + (a*
f*e^(2*f*x + 2*e) + a*f)*sinh(f*x + e)^6 - 3*a*f*cosh(f*x + e)^4 + 6*(a*f*
cosh(f*x + e)*e^(2*f*x + 2*e) + a*f*cosh(f*x + e))*sinh(f*x + e)^5 + 3*(5*
a*f*cosh(f*x + e)^2 - a*f + (5*a*f*cosh(f*x + e)^2 - a*f)*e^(2*f*x + 2*e))
*sinh(f*x + e)^4 + 3*a*f*cosh(f*x + e)^2 + 4*(5*a*f*cosh(f*x + e)^3 - 3*a*
f*cosh(f*x + e) + (5*a*f*cosh(f*x + e)^3 - 3*a*f*cosh(f*x + e))*e^(2*f*x +
 2*e))*sinh(f*x + e)^3 + 3*(5*a*f*cosh(f*x + e)^4 - 6*a*f*cosh(f*x + e)^2
+ a*f + (5*a*f*cosh(f*x + e)^4 - 6*a*f*cosh(f*x + e)^2 + a*f)*e^(2*f*x + 2
*e))*sinh(f*x + e)^2 - a*f + (a*f*cosh(f*x + e)^6 - 3*a*f*cosh(f*x + e)^4
+ 3*a*f*cosh(f*x + e)^2 - a*f)*e^(2*f*x + 2*e) + 6*(a*f*cosh(f*x + e)^5 -
2*a*f*cosh(f*x + e)^3 + a*f*cosh(f*x + e) + (a*f*cosh(f*x + e)^5 - 2*a*f*c
osh(f*x + e)^3 + a*f*cosh(f*x + e))*e^(2*f*x + 2*e))*sinh(f*x + e))
```

Sympy [F]

$$\int \frac{\coth^4(e+fx)}{\sqrt{a+a\sinh^2(e+fx)}}\,dx = \int \frac{\coth^4{(e+fx)}}{\sqrt{a\left(\sinh^2{(e+fx)}+1\right)}}\,dx$$

input
```
integrate(coth(f*x+e)**4/(a+a*sinh(f*x+e)**2)**(1/2),x)
```

output
```
Integral(coth(e + f*x)**4/sqrt(a*(sinh(e + f*x)**2 + 1)), x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 556 vs. $2(55) = 110$.

Time = 0.19 (sec) , antiderivative size = 556, normalized size of antiderivative = 9.11

$$\int \frac{\coth^4(e+fx)}{\sqrt{a+a\sinh^2(e+fx)}}\,dx = \text{Too large to display}$$

input
```
integrate(coth(f*x+e)^4/(a+a*sinh(f*x+e)^2)^(1/2),x, algorithm="maxima")
```

output
```
1/12*(6*arctan(e^(-f*x - e))/sqrt(a) + 3*log(e^(-f*x - e) + 1)/sqrt(a) - 3
*log(e^(-f*x - e) - 1)/sqrt(a) + 4*(3*sqrt(a)*e^(-f*x - e) - sqrt(a)*e^(-3
*f*x - 3*e))/(3*a*e^(-2*f*x - 2*e) - 3*a*e^(-4*f*x - 4*e) + a*e^(-6*f*x -
6*e) - a))/f + 1/12*(6*arctan(e^(-f*x - e))/sqrt(a) - 3*log(e^(-f*x - e) +
 1)/sqrt(a) + 3*log(e^(-f*x - e) - 1)/sqrt(a) - 4*(sqrt(a)*e^(-3*f*x - 3*e
) - 3*sqrt(a)*e^(-5*f*x - 5*e))/(3*a*e^(-2*f*x - 2*e) - 3*a*e^(-4*f*x - 4*
e) + a*e^(-6*f*x - 6*e) - a))/f - 1/4*(3*arctan(e^(-f*x - e))/sqrt(a) + (3
*sqrt(a)*e^(-f*x - e) - 10*sqrt(a)*e^(-3*f*x - 3*e) + 3*sqrt(a)*e^(-5*f*x
- 5*e))/(3*a*e^(-2*f*x - 2*e) - 3*a*e^(-4*f*x - 4*e) + a*e^(-6*f*x - 6*e)
- a))/f - 1/4*arctan(e^(-f*x - e))/(sqrt(a)*f) + 1/24*(27*sqrt(a)*e^(-f*x
- e) - 38*sqrt(a)*e^(-3*f*x - 3*e) + 15*sqrt(a)*e^(-5*f*x - 5*e))/((3*a*e^
(-2*f*x - 2*e) - 3*a*e^(-4*f*x - 4*e) + a*e^(-6*f*x - 6*e) - a)*f) + 1/24*
(15*sqrt(a)*e^(-f*x - e) - 38*sqrt(a)*e^(-3*f*x - 3*e) + 27*sqrt(a)*e^(-5*
f*x - 5*e))/((3*a*e^(-2*f*x - 2*e) - 3*a*e^(-4*f*x - 4*e) + a*e^(-6*f*x -
6*e) - a)*f)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{\coth^4(e+fx)}{\sqrt{a+a\sinh^2(e+fx)}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(coth(f*x+e)^4/(a+a*sinh(f*x+e)^2)^(1/2),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [B] (verification not implemented)

Time = 1.84 (sec) , antiderivative size = 95, normalized size of antiderivative = 1.56

$$\int \frac{\coth^4(e+fx)}{\sqrt{a+a\sinh^2(e+fx)}}\,dx$$
$$= -\frac{4\,\mathrm{e}^{2\,e+2\,f\,x}\,\sqrt{a+a\,{\left(\frac{\mathrm{e}^{e+f\,x}}{2}-\frac{\mathrm{e}^{-e-f\,x}}{2}\right)}^2}\,\left(3\,\mathrm{e}^{4\,e+4\,f\,x}-2\,\mathrm{e}^{2\,e+2\,f\,x}+3\right)}{3\,a\,f\,{\left(\mathrm{e}^{2\,e+2\,f\,x}-1\right)}^3\,\left(\mathrm{e}^{2\,e+2\,f\,x}+1\right)}$$

input
```
int(coth(e + f*x)^4/(a + a*sinh(e + f*x)^2)^(1/2),x)
```

output
```
-(4*exp(2*e + 2*f*x)*(a + a*(exp(e + f*x)/2 - exp(- e - f*x)/2)^2)^(1/2)*(
3*exp(4*e + 4*f*x) - 2*exp(2*e + 2*f*x) + 3))/(3*a*f*(exp(2*e + 2*f*x) - 1
)^3*(exp(2*e + 2*f*x) + 1))
```

Reduce [F]

$$\int \frac{\coth^4(e+fx)}{\sqrt{a+a\sinh^2(e+fx)}}\,dx = \frac{\sqrt{a}\left(\int \frac{\sqrt{\sinh(fx+e)^2+1}\,\coth(fx+e)^4}{\sinh(fx+e)^2+1}dx\right)}{a}$$

input
```
int(coth(f*x+e)^4/(a+a*sinh(f*x+e)^2)^(1/2),x)
```

output
```
(sqrt(a)*int((sqrt(sinh(e + f*x)**2 + 1)*coth(e + f*x)**4)/(sinh(e + f*x)*
*2 + 1),x))/a
```

3.410 $\int \frac{\coth^6(e+fx)}{\sqrt{a+a\sinh^2(e+fx)}}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 96

$$\int \frac{\coth^6(e+fx)}{\sqrt{a+a\sinh^2(e+fx)}}\,dx = -\frac{\coth(e+fx)}{f\sqrt{a\cosh^2(e+fx)}}-\frac{2\coth(e+fx)\operatorname{csch}^2(e+fx)}{3f\sqrt{a\cosh^2(e+fx)}}-\frac{\coth(e+fx)\operatorname{csch}^4(e+fx)}{5f\sqrt{a\cosh^2(e+fx)}}$$

output
```
-coth(f*x+e)/f/(a*cosh(f*x+e)^2)^(1/2)-2/3*coth(f*x+e)*csch(f*x+e)^2/f/(a*
cosh(f*x+e)^2)^(1/2)-1/5*coth(f*x+e)*csch(f*x+e)^4/f/(a*cosh(f*x+e)^2)^(1/
2)
```

Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 49, normalized size of antiderivative = 0.51

$$\int \frac{\coth^6(e+fx)}{\sqrt{a+a\sinh^2(e+fx)}}\,dx = -\frac{\coth(e+fx)\left(15+10\operatorname{csch}^2(e+fx)+3\operatorname{csch}^4(e+fx)\right)}{15f\sqrt{a\cosh^2(e+fx)}}$$

input
```
Integrate[Coth[e + f*x]^6/Sqrt[a + a*Sinh[e + f*x]^2],x]
```

output
```
-1/15*(Coth[e + f*x]*(15 + 10*Csch[e + f*x]^2 + 3*Csch[e + f*x]^4))/(f*Sqr
t[a*Cosh[e + f*x]^2])
```

Rubi [C] (verified)

Result contains complex when optimal does not.

Time = 0.52 (sec) , antiderivative size = 66, normalized size of antiderivative = 0.69, number of steps used = 13, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.480$, Rules used = {3042, 25, 3655, 25, 3042, 25, 3686, 25, 3042, 3086, 210, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\coth^6(e+fx)}{\sqrt{a\sinh^2(e+fx)+a}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int -\frac{1}{\tan(ie+ifx)^6\sqrt{a-a\sin(ie+ifx)^2}}dx$$

$$\Big\downarrow \text{25}$$

$$-\int \frac{1}{\sqrt{a-a\sin(ie+ifx)^2}\tan(ie+ifx)^6}dx$$

$$\Big\downarrow \text{3655}$$

$$-\int -\frac{\coth^6(e+fx)}{\sqrt{a\cosh^2(e+fx)}}dx$$

$$\Big\downarrow \text{25}$$

$$\int \frac{\coth^6(e+fx)}{\sqrt{a\cosh^2(e+fx)}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int -\frac{\tan\left(ie+ifx+\frac{\pi}{2}\right)^6}{\sqrt{a\sin\left(ie+ifx+\frac{\pi}{2}\right)^2}}dx$$

$$\Big\downarrow \text{25}$$

$$-\int \frac{\tan \left(\frac{1}{2}(2ie+\pi)+ifx\right)^6}{\sqrt{a\sin \left(\frac{1}{2}(2ie+\pi)+ifx\right)^2}}dx$$

$$\Big\downarrow \; 3686$$

$$-\frac{\cosh(e+fx)\int -\coth^5(e+fx)\text{csch}(e+fx)dx}{\sqrt{a\cosh^2(e+fx)}}$$

$$\Big\downarrow \; 25$$

$$\frac{\cosh(e+fx)\int \coth^5(e+fx)\text{csch}(e+fx)dx}{\sqrt{a\cosh^2(e+fx)}}$$

$$\Big\downarrow \; 3042$$

$$\frac{\cosh(e+fx)\int \sec \left(ie+ifx-\frac{\pi}{2}\right)\tan \left(ie+ifx-\frac{\pi}{2}\right)^5dx}{\sqrt{a\cosh^2(e+fx)}}$$

$$\Big\downarrow \; 3086$$

$$-\frac{i\cosh(e+fx)\int \left(-\text{csch}^2(e+fx)-1\right)^2d(-i\text{csch}(e+fx))}{f\sqrt{a\cosh^2(e+fx)}}$$

$$\Big\downarrow \; 210$$

$$-\frac{i\cosh(e+fx)\int \left(\text{csch}^4(e+fx)+2\text{csch}^2(e+fx)+1\right)d(-i\text{csch}(e+fx))}{f\sqrt{a\cosh^2(e+fx)}}$$

$$\Big\downarrow \; 2009$$

$$-\frac{i\cosh(e+fx)\left(-\frac{1}{5}i\text{csch}^5(e+fx)-\frac{2}{3}i\text{csch}^3(e+fx)-i\text{csch}(e+fx)\right)}{f\sqrt{a\cosh^2(e+fx)}}$$

input
```
Int[Coth[e + f*x]^6/Sqrt[a + a*Sinh[e + f*x]^2],x]
```

output
```
((-I)*Cosh[e + f*x]*((-I)*Csch[e + f*x] - ((2*I)/3)*Csch[e + f*x]^3 - (I/5
)*Csch[e + f*x]^5))/(f*Sqrt[a*Cosh[e + f*x]^2])
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 210
```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Int[ExpandIntegrand[(a + b*x^2
)^p, x], x] /; FreeQ[{a, b}, x] && IGtQ[p, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3086
```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Simp[a/f   Subst[Int[(a*x)^(m - 1)*(-1 + x^2)^((n - 1)/2
), x], x, Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2
] &&  !(IntegerQ[m/2] && LtQ[0, m, n + 1])
```

rule 3655
```
Int[(u_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> Int[A
ctivateTrig[u*(a*cos[e + f*x]^2)^p], x] /; FreeQ[{a, b, e, f, p}, x] && EqQ
[a + b, 0]
```

rule 3686
```
Int[(u_.)*((b_.)*sin[(e_.) + (f_.)*(x_)]^(n_))^(p_), x_Symbol] :> With[{ff
= FreeFactors[Sin[e + f*x], x]}, Simp[(b*ff^n)^IntPart[p]*((b*Sin[e + f*x]^
n)^FracPart[p]/(Sin[e + f*x]/ff)^(n*FracPart[p]))   Int[ActivateTrig[u]*(Si
n[e + f*x]/ff)^(n*p), x], x]] /; FreeQ[{b, e, f, n, p}, x] &&  !IntegerQ[p]
 && IntegerQ[n] && (EqQ[u, 1] || MatchQ[u, ((d_.)*(trig_)[e + f*x])^(m_.) /
; FreeQ[{d, m}, x] && MemberQ[{sin, cos, tan, cot, sec, csc}, trig]])
```

Maple [A] (verified)

Time = 0.44 (sec) , antiderivative size = 74, normalized size of antiderivative = 0.77

method	result	size
default	$-\frac{\cosh(fx+e)\left(15\cosh(fx+e)^4-20\cosh(fx+e)^2+8\right)}{15(\cosh(fx+e)+1)^2(\cosh(fx+e)-1)^2\sinh(fx+e)\sqrt{a\cosh(fx+e)^2}\,f}$	74
risch	$-\frac{2(\mathrm{e}^{2fx+2e}+1)(15\,\mathrm{e}^{8fx+8e}-20\,\mathrm{e}^{6fx+6e}+58\,\mathrm{e}^{4fx+4e}-20\,\mathrm{e}^{2fx+2e}+15)}{15\sqrt{(\mathrm{e}^{2fx+2e}+1)^2 a\,\mathrm{e}^{-2fx-2e}}\,f(\mathrm{e}^{2fx+2e}-1)^5}$	102

input
```
int(coth(f*x+e)^6/(a+a*sinh(f*x+e)^2)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-1/15*cosh(f*x+e)*(15*cosh(f*x+e)^4-20*cosh(f*x+e)^2+8)/(cosh(f*x+e)+1)^2/
(cosh(f*x+e)-1)^2/sinh(f*x+e)/(a*cosh(f*x+e)^2)^(1/2)/f
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1399 vs. 2(86) = 172.

Time = 0.11 (sec) , antiderivative size = 1399, normalized size of antiderivative = 14.57

$$\int \frac{\coth^6(e+fx)}{\sqrt{a+a\sinh^2(e+fx)}}\,dx = \text{Too large to display}$$

input
```
integrate(coth(f*x+e)^6/(a+a*sinh(f*x+e)^2)^(1/2),x, algorithm="fricas")
```

output
```
-2/15*(135*cosh(f*x + e)*e^(f*x + e)*sinh(f*x + e)^8 + 15*e^(f*x + e)*sinh
(f*x + e)^9 + 20*(27*cosh(f*x + e)^2 - 1)*e^(f*x + e)*sinh(f*x + e)^7 + 14
0*(9*cosh(f*x + e)^3 - cosh(f*x + e))*e^(f*x + e)*sinh(f*x + e)^6 + 2*(945
*cosh(f*x + e)^4 - 210*cosh(f*x + e)^2 + 29)*e^(f*x + e)*sinh(f*x + e)^5 +
 10*(189*cosh(f*x + e)^5 - 70*cosh(f*x + e)^3 + 29*cosh(f*x + e))*e^(f*x +
 e)*sinh(f*x + e)^4 + 20*(63*cosh(f*x + e)^6 - 35*cosh(f*x + e)^4 + 29*cos
h(f*x + e)^2 - 1)*e^(f*x + e)*sinh(f*x + e)^3 + 20*(27*cosh(f*x + e)^7 - 2
1*cosh(f*x + e)^5 + 29*cosh(f*x + e)^3 - 3*cosh(f*x + e))*e^(f*x + e)*sinh
(f*x + e)^2 + 5*(27*cosh(f*x + e)^8 - 28*cosh(f*x + e)^6 + 58*cosh(f*x + e
)^4 - 12*cosh(f*x + e)^2 + 3)*e^(f*x + e)*sinh(f*x + e) + (15*cosh(f*x + e
)^9 - 20*cosh(f*x + e)^7 + 58*cosh(f*x + e)^5 - 20*cosh(f*x + e)^3 + 15*co
sh(f*x + e))*e^(f*x + e))*sqrt(a*e^(4*f*x + 4*e) + 2*a*e^(2*f*x + 2*e) + a
)*e^(-f*x - e)/(a*f*cosh(f*x + e)^10 + (a*f*e^(2*f*x + 2*e) + a*f)*sinh(f*
x + e)^10 - 5*a*f*cosh(f*x + e)^8 + 10*(a*f*cosh(f*x + e)*e^(2*f*x + 2*e)
+ a*f*cosh(f*x + e))*sinh(f*x + e)^9 + 5*(9*a*f*cosh(f*x + e)^2 - a*f + (9
*a*f*cosh(f*x + e)^2 - a*f)*e^(2*f*x + 2*e))*sinh(f*x + e)^8 + 10*a*f*cosh
(f*x + e)^6 + 40*(3*a*f*cosh(f*x + e)^3 - a*f*cosh(f*x + e) + (3*a*f*cosh(
f*x + e)^3 - a*f*cosh(f*x + e))*e^(2*f*x + 2*e))*sinh(f*x + e)^7 + 10*(21*
a*f*cosh(f*x + e)^4 - 14*a*f*cosh(f*x + e)^2 + a*f + (21*a*f*cosh(f*x + e)
^4 - 14*a*f*cosh(f*x + e)^2 + a*f)*e^(2*f*x + 2*e))*sinh(f*x + e)^6 - 1...
```

Sympy [F]

$$\int \frac{\coth^6(e+fx)}{\sqrt{a+a\sinh^2(e+fx)}}\,dx=\int \frac{\coth^6{(e+fx)}}{\sqrt{a\left(\sinh^2{(e+fx)}+1\right)}}\,dx$$

input
```
integrate(coth(f*x+e)**6/(a+a*sinh(f*x+e)**2)**(1/2),x)
```

output
```
Integral(coth(e + f*x)**6/sqrt(a*(sinh(e + f*x)**2 + 1)), x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1231 vs. $2(86) = 172$.

Time = 0.24 (sec) , antiderivative size = 1231, normalized size of antiderivative = 12.82

$$\int \frac{\coth^6(e+fx)}{\sqrt{a+a\sinh^2(e+fx)}}\,dx = \text{Too large to display}$$

input
```
integrate(coth(f*x+e)^6/(a+a*sinh(f*x+e)^2)^(1/2),x, algorithm="maxima")
```

output
```
-1/256*(120*arctan(e^(-f*x - e))/sqrt(a) + 45*log(e^(-f*x - e) + 1)/sqrt(a
) - 45*log(e^(-f*x - e) - 1)/sqrt(a) + 2*(105*sqrt(a)*e^(-f*x - e) - 530*s
qrt(a)*e^(-3*f*x - 3*e) + 328*sqrt(a)*e^(-5*f*x - 5*e) - 110*sqrt(a)*e^(-7
*f*x - 7*e) + 15*sqrt(a)*e^(-9*f*x - 9*e))/(5*a*e^(-2*f*x - 2*e) - 10*a*e^
(-4*f*x - 4*e) + 10*a*e^(-6*f*x - 6*e) - 5*a*e^(-8*f*x - 8*e) + a*e^(-10*f
*x - 10*e) - a))/f - 1/256*(120*arctan(e^(-f*x - e))/sqrt(a) - 45*log(e^(-
f*x - e) + 1)/sqrt(a) + 45*log(e^(-f*x - e) - 1)/sqrt(a) + 2*(15*sqrt(a)*e
^(-f*x - e) - 110*sqrt(a)*e^(-3*f*x - 3*e) + 328*sqrt(a)*e^(-5*f*x - 5*e)
 - 530*sqrt(a)*e^(-7*f*x - 7*e) + 105*sqrt(a)*e^(-9*f*x - 9*e))/(5*a*e^(-2*
f*x - 2*e) - 10*a*e^(-4*f*x - 4*e) + 10*a*e^(-6*f*x - 6*e) - 5*a*e^(-8*f*x
 - 8*e) + a*e^(-10*f*x - 10*e) - a))/f + 1/320*(60*arctan(e^(-f*x - e))/sq
rt(a) + 75*log(e^(-f*x - e) + 1)/sqrt(a) - 75*log(e^(-f*x - e) - 1)/sqrt(a
) + 2*(105*sqrt(a)*e^(-f*x - e) + 130*sqrt(a)*e^(-3*f*x - 3*e) - 284*sqrt(
a)*e^(-5*f*x - 5*e) + 190*sqrt(a)*e^(-7*f*x - 7*e) - 45*sqrt(a)*e^(-9*f*x
 - 9*e))/(5*a*e^(-2*f*x - 2*e) - 10*a*e^(-4*f*x - 4*e) + 10*a*e^(-6*f*x - 6
*e) - 5*a*e^(-8*f*x - 8*e) + a*e^(-10*f*x - 10*e) - a))/f + 1/320*(60*arct
an(e^(-f*x - e))/sqrt(a) - 75*log(e^(-f*x - e) + 1)/sqrt(a) + 75*log(e^(-f
*x - e) - 1)/sqrt(a) - 2*(45*sqrt(a)*e^(-f*x - e) - 190*sqrt(a)*e^(-3*f*x
 - 3*e) + 284*sqrt(a)*e^(-5*f*x - 5*e) - 130*sqrt(a)*e^(-7*f*x - 7*e) - 105
*sqrt(a)*e^(-9*f*x - 9*e))/(5*a*e^(-2*f*x - 2*e) - 10*a*e^(-4*f*x - 4*e...
```

Giac [F(-2)]

Exception generated.

$$\int \frac{\coth^6(e+fx)}{\sqrt{a+a\sinh^2(e+fx)}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(coth(f*x+e)^6/(a+a*sinh(f*x+e)^2)^(1/2),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [B] (verification not implemented)

Time = 1.85 (sec) , antiderivative size = 381, normalized size of antiderivative = 3.97

$$\int \frac{\coth^6(e+fx)}{\sqrt{a+a\sinh^2(e+fx)}}\,dx = -\frac{4\,{\mathrm{e}}^{3\,e+3\,f\,x}\,\sqrt{a+a\,{\left(\frac{{\mathrm{e}}^{e+f\,x}}{2}-\frac{{\mathrm{e}}^{-e-f\,x}}{2}\right)}^2}}{a\,f\,\left({\mathrm{e}}^{2\,e+2\,f\,x}-1\right)\,\left({\mathrm{e}}^{e+f\,x}+{\mathrm{e}}^{3\,e+3\,f\,x}\right)} - \frac{32\,{\mathrm{e}}^{3\,e+3\,f\,x}\,\sqrt{a+a\,{\left(\frac{{\mathrm{e}}^{e+f\,x}}{2}-\frac{{\mathrm{e}}^{-e-f\,x}}{2}\right)}^2}}{3\,a\,f\,{\left({\mathrm{e}}^{2\,e+2\,f\,x}-1\right)}^2\,\left({\mathrm{e}}^{e+f\,x}+{\mathrm{e}}^{3\,e+3\,f\,x}\right)} - \frac{352\,{\mathrm{e}}^{3\,e+3\,f\,x}\,\sqrt{a+a\,{\left(\frac{{\mathrm{e}}^{e+f\,x}}{2}-\frac{{\mathrm{e}}^{-e-f\,x}}{2}\right)}^2}}{15\,a\,f\,{\left({\mathrm{e}}^{2\,e+2\,f\,x}-1\right)}^3\,\left({\mathrm{e}}^{e+f\,x}+{\mathrm{e}}^{3\,e+3\,f\,x}\right)} - \frac{128\,{\mathrm{e}}^{3\,e+3\,f\,x}\,\sqrt{a+a\,{\left(\frac{{\mathrm{e}}^{e+f\,x}}{2}-\frac{{\mathrm{e}}^{-e-f\,x}}{2}\right)}^2}}{5\,a\,f\,{\left({\mathrm{e}}^{2\,e+2\,f\,x}-1\right)}^4\,\left({\mathrm{e}}^{e+f\,x}+{\mathrm{e}}^{3\,e+3\,f\,x}\right)} - \frac{64\,{\mathrm{e}}^{3\,e+3\,f\,x}\,\sqrt{a+a\,{\left(\frac{{\mathrm{e}}^{e+f\,x}}{2}-\frac{{\mathrm{e}}^{-e-f\,x}}{2}\right)}^2}}{5\,a\,f\,{\left({\mathrm{e}}^{2\,e+2\,f\,x}-1\right)}^5\,\left({\mathrm{e}}^{e+f\,x}+{\mathrm{e}}^{3\,e+3\,f\,x}\right)}$$

input
```
int(coth(e + f*x)^6/(a + a*sinh(e + f*x)^2)^(1/2),x)
```

output
```
- (4*exp(3*e + 3*f*x)*(a + a*(exp(e + f*x)/2 - exp(- e - f*x)/2)^2)^(1/2))
/(a*f*(exp(2*e + 2*f*x) - 1)*(exp(e + f*x) + exp(3*e + 3*f*x))) - (32*exp(
3*e + 3*f*x)*(a + a*(exp(e + f*x)/2 - exp(- e - f*x)/2)^2)^(1/2))/(3*a*f*(
exp(2*e + 2*f*x) - 1)^2*(exp(e + f*x) + exp(3*e + 3*f*x))) - (352*exp(3*e
+ 3*f*x)*(a + a*(exp(e + f*x)/2 - exp(- e - f*x)/2)^2)^(1/2))/(15*a*f*(exp
(2*e + 2*f*x) - 1)^3*(exp(e + f*x) + exp(3*e + 3*f*x))) - (128*exp(3*e + 3
*f*x)*(a + a*(exp(e + f*x)/2 - exp(- e - f*x)/2)^2)^(1/2))/(5*a*f*(exp(2*e
 + 2*f*x) - 1)^4*(exp(e + f*x) + exp(3*e + 3*f*x))) - (64*exp(3*e + 3*f*x)
*(a + a*(exp(e + f*x)/2 - exp(- e - f*x)/2)^2)^(1/2))/(5*a*f*(exp(2*e + 2*
f*x) - 1)^5*(exp(e + f*x) + exp(3*e + 3*f*x)))
```

Reduce [F]

$$\int \frac{\coth^6(e+fx)}{\sqrt{a+a\sinh^2(e+fx)}}\,dx=\frac{\sqrt{a}\left(\int \frac{\sqrt{\sinh(fx+e)^2+1}\,\coth(fx+e)^6}{\sinh(fx+e)^2+1}dx\right)}{a}$$

input
```
int(coth(f*x+e)^6/(a+a*sinh(f*x+e)^2)^(1/2),x)
```

output
```
(sqrt(a)*int((sqrt(sinh(e + f*x)**2 + 1)*coth(e + f*x)**6)/(sinh(e + f*x)*
*2 + 1),x))/a
```

3.411 $\int \frac{\tanh^5(e+fx)}{\left(a+a\sinh^2(e+fx)\right)^{3/2}}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 68

$$\int \frac{\tanh^5(e+fx)}{\left(a+a\sinh^2(e+fx)\right)^{3/2}}\,dx=-\frac{a^2}{7f\left(a\cosh^2(e+fx)\right)^{7/2}}+\frac{2a}{5f\left(a\cosh^2(e+fx)\right)^{5/2}}-\frac{1}{3f\left(a\cosh^2(e+fx)\right)^{3/2}}$$

output
```
-1/7*a^2/f/(a*cosh(f*x+e)^2)^(7/2)+2/5*a/f/(a*cosh(f*x+e)^2)^(5/2)-1/3/f/(
a*cosh(f*x+e)^2)^(3/2)
```

Mathematica [A] (verified)

Time = 0.19 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.75

$$\int \frac{\tanh^5(e+fx)}{\left(a+a\sinh^2(e+fx)\right)^{3/2}}\,dx=\frac{\left(-15+42\cosh^2(e+fx)-35\cosh^4(e+fx)\right)\operatorname{sech}^4(e+fx)}{105f\left(a\cosh^2(e+fx)\right)^{3/2}}$$

input
```
Integrate[Tanh[e + f*x]^5/(a + a*Sinh[e + f*x]^2)^(3/2),x]
```

output
```
((-15 + 42*Cosh[e + f*x]^2 - 35*Cosh[e + f*x]^4)*Sech[e + f*x]^4)/(105*f*(
a*Cosh[e + f*x]^2)^(3/2))
```

Rubi [A] (verified)

Time = 0.41 (sec) , antiderivative size = 74, normalized size of antiderivative = 1.09, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.400$, Rules used = {3042, 26, 3655, 26, 3042, 26, 3684, 8, 53, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\tanh^5(e+fx)}{\left(a\sinh^2(e+fx)+a\right)^{3/2}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int -\frac{i\tan(ie+ifx)^5}{\left(a-a\sin(ie+ifx)^2\right)^{3/2}}dx$$

$$\Big\downarrow \text{26}$$

$$-i\int \frac{\tan(ie+ifx)^5}{\left(a-a\sin(ie+ifx)^2\right)^{3/2}}dx$$

$$\Big\downarrow \text{3655}$$

$$-i\int \frac{i\tanh^5(e+fx)}{\left(a\cosh^2(e+fx)\right)^{3/2}}dx$$

$$\Big\downarrow \text{26}$$

$$\int \frac{\tanh^5(e+fx)}{\left(a\cosh^2(e+fx)\right)^{3/2}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{i}{\tan\left(ie+ifx+\frac{\pi}{2}\right)^5\left(a\sin\left(ie+ifx+\frac{\pi}{2}\right)^2\right)^{3/2}}dx$$

$$\Big\downarrow \text{26}$$

$$i\int \frac{1}{\left(a\sin\left(\frac{1}{2}(2ie+\pi)+ifx\right)^2\right)^{3/2}\tan\left(\frac{1}{2}(2ie+\pi)+ifx\right)^5}dx$$

$$\Big\downarrow \text{3684}$$

$$\frac{\int \frac{(1-\cosh^2(e+fx))^2\operatorname{sech}^6(e+fx)}{(a\cosh^2(e+fx))^{3/2}}d\cosh^2(e+fx)}{2f}$$

$$\Big\downarrow \text{8}$$

$$\frac{a^3\int \frac{(1-\cosh^2(e+fx))^2}{(a\cosh^2(e+fx))^{9/2}}d\cosh^2(e+fx)}{2f}$$

$$\Big\downarrow \text{53}$$

$$\frac{a^3\int \left(\frac{1}{(a\cosh^2(e+fx))^{9/2}}-\frac{2}{(a\cosh^2(e+fx))^{7/2}a}+\frac{1}{(a\cosh^2(e+fx))^{5/2}a^2}\right)d\cosh^2(e+fx)}{2f}$$

$$\Big\downarrow \text{2009}$$

$$\frac{a^3\left(-\frac{2}{3a^3(a\cosh^2(e+fx))^{3/2}}+\frac{4}{5a^2(a\cosh^2(e+fx))^{5/2}}-\frac{2}{7a(a\cosh^2(e+fx))^{7/2}}\right)}{2f}$$

input
```
Int[Tanh[e + f*x]^5/(a + a*Sinh[e + f*x]^2)^(3/2),x]
```

output
```
(a^3*(-2/(7*a*(a*Cosh[e + f*x]^2)^(7/2)) + 4/(5*a^2*(a*Cosh[e + f*x]^2)^(5
/2)) - 2/(3*a^3*(a*Cosh[e + f*x]^2)^(3/2))))/(2*f)
```

Defintions of rubi rules used

rule 8
```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_))^(p_), x_Symbol] :> Simp[1/a^m   Int[u*(a*
x)^(m + p), x], x] /; FreeQ[{a, m, p}, x] && IntegerQ[m]
```

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 53
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && LeQ[7*m + 4*n + 4, 0])
|| LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3655
```
Int[(u_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> Int[A
ctivateTrig[u*(a*cos[e + f*x]^2)^p], x] /; FreeQ[{a, b, e, f, p}, x] && EqQ
[a + b, 0]
```

rule 3684
```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*tan[(e_.) + (f_.)*(x_)]^(m_.
), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x]^2, x]}, Simp[ff^((m + 1
)/2)/(2*f)   Subst[Int[x^((m - 1)/2)*((b*ff^(n/2)*x^(n/2))^p/(1 - ff*x)^((m
 + 1)/2)), x], x, Sin[e + f*x]^2/ff], x]] /; FreeQ[{b, e, f, p}, x] && Inte
gerQ[(m - 1)/2] && IntegerQ[n/2]
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.45 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.65

method	result	size
default	$\frac{\text{`int/indef0`}\left(\frac{\sinh(fx+e)^5}{\cosh(fx+e)^8 a\sqrt{a\cosh(fx+e)^2}},\sinh(fx+e)\right)}{f}$	44
risch	$-\frac{8\left(35\,\mathrm{e}^{8fx+8e}-28\,\mathrm{e}^{6fx+6e}+114\,\mathrm{e}^{4fx+4e}-28\,\mathrm{e}^{2fx+2e}+35\right)\mathrm{e}^{2fx+2e}}{105f\sqrt{\left(\mathrm{e}^{2fx+2e}+1\right)^2 a\,\mathrm{e}^{-2fx-2e}}\left(\mathrm{e}^{2fx+2e}+1\right)^6 a}$	103

input
```
int(tanh(f*x+e)^5/(a+a*sinh(f*x+e)^2)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
`int/indef0`(sinh(f*x+e)^5/cosh(f*x+e)^8/a/(a*cosh(f*x+e)^2)^(1/2),sinh(f*
x+e))/f
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2507 vs. $2(56) = 112$.

Time = 0.14 (sec) , antiderivative size = 2507, normalized size of antiderivative = 36.87

$$\int \frac{\tanh^5(e+fx)}{\left(a+a\sinh^2(e+fx)\right)^{3/2}}\,dx = \text{Too large to display}$$

input
```
integrate(tanh(f*x+e)^5/(a+a*sinh(f*x+e)^2)^(3/2),x, algorithm="fricas")
```

output
```
-8/105*(385*cosh(f*x + e)*e^(f*x + e)*sinh(f*x + e)^10 + 35*e^(f*x + e)*si
nh(f*x + e)^11 + 7*(275*cosh(f*x + e)^2 - 4)*e^(f*x + e)*sinh(f*x + e)^9 +
 21*(275*cosh(f*x + e)^3 - 12*cosh(f*x + e))*e^(f*x + e)*sinh(f*x + e)^8 +
 6*(1925*cosh(f*x + e)^4 - 168*cosh(f*x + e)^2 + 19)*e^(f*x + e)*sinh(f*x
+ e)^7 + 42*(385*cosh(f*x + e)^5 - 56*cosh(f*x + e)^3 + 19*cosh(f*x + e))*
e^(f*x + e)*sinh(f*x + e)^6 + 14*(1155*cosh(f*x + e)^6 - 252*cosh(f*x + e)
^4 + 171*cosh(f*x + e)^2 - 2)*e^(f*x + e)*sinh(f*x + e)^5 + 14*(825*cosh(f
*x + e)^7 - 252*cosh(f*x + e)^5 + 285*cosh(f*x + e)^3 - 10*cosh(f*x + e))*
e^(f*x + e)*sinh(f*x + e)^4 + 7*(825*cosh(f*x + e)^8 - 336*cosh(f*x + e)^6
 + 570*cosh(f*x + e)^4 - 40*cosh(f*x + e)^2 + 5)*e^(f*x + e)*sinh(f*x + e)
^3 + 7*(275*cosh(f*x + e)^9 - 144*cosh(f*x + e)^7 + 342*cosh(f*x + e)^5 -
40*cosh(f*x + e)^3 + 15*cosh(f*x + e))*e^(f*x + e)*sinh(f*x + e)^2 + 7*(55
*cosh(f*x + e)^10 - 36*cosh(f*x + e)^8 + 114*cosh(f*x + e)^6 - 20*cosh(f*x
 + e)^4 + 15*cosh(f*x + e)^2)*e^(f*x + e)*sinh(f*x + e) + (35*cosh(f*x + e
)^11 - 28*cosh(f*x + e)^9 + 114*cosh(f*x + e)^7 - 28*cosh(f*x + e)^5 + 35*
cosh(f*x + e)^3)*e^(f*x + e))*sqrt(a*e^(4*f*x + 4*e) + 2*a*e^(2*f*x + 2*e)
 + a)*e^(-f*x - e)/(a^2*f*cosh(f*x + e)^14 + 7*a^2*f*cosh(f*x + e)^12 + (a
^2*f*e^(2*f*x + 2*e) + a^2*f)*sinh(f*x + e)^14 + 14*(a^2*f*cosh(f*x + e)*e
^(2*f*x + 2*e) + a^2*f*cosh(f*x + e))*sinh(f*x + e)^13 + 21*a^2*f*cosh(f*x
 + e)^10 + 7*(13*a^2*f*cosh(f*x + e)^2 + a^2*f + (13*a^2*f*cosh(f*x + e...
```

Sympy [F]

$$\int \frac{\tanh^5(e+fx)}{\left(a+a\sinh^2(e+fx)\right)^{3/2}}\,dx=\int \frac{\tanh^5{(e+fx)}}{\left(a\left(\sinh^2{(e+fx)}+1\right)\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate(tanh(f*x+e)**5/(a+a*sinh(f*x+e)**2)**(3/2),x)
```

output
```
Integral(tanh(e + f*x)**5/(a*(sinh(e + f*x)**2 + 1))**(3/2), x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 586 vs. 2(56) = 112.

Time = 0.25 (sec) , antiderivative size = 586, normalized size of antiderivative = 8.62

$$\int \frac{\tanh^5(e+fx)}{\left(a+a\sinh^2(e+fx)\right)^{3/2}}\,dx=\text{Too large to display}$$

input
```
integrate(tanh(f*x+e)^5/(a+a*sinh(f*x+e)^2)^(3/2),x, algorithm="maxima")
```

output

```
-8/3*e^(-3*f*x - 3*e)/((7*a^(3/2)*e^(-2*f*x - 2*e) + 21*a^(3/2)*e^(-4*f*x
- 4*e) + 35*a^(3/2)*e^(-6*f*x - 6*e) + 35*a^(3/2)*e^(-8*f*x - 8*e) + 21*a^
(3/2)*e^(-10*f*x - 10*e) + 7*a^(3/2)*e^(-12*f*x - 12*e) + a^(3/2)*e^(-14*f
*x - 14*e) + a^(3/2))*f) + 32/15*e^(-5*f*x - 5*e)/((7*a^(3/2)*e^(-2*f*x -
2*e) + 21*a^(3/2)*e^(-4*f*x - 4*e) + 35*a^(3/2)*e^(-6*f*x - 6*e) + 35*a^(3
/2)*e^(-8*f*x - 8*e) + 21*a^(3/2)*e^(-10*f*x - 10*e) + 7*a^(3/2)*e^(-12*f*
x - 12*e) + a^(3/2)*e^(-14*f*x - 14*e) + a^(3/2))*f) - 304/35*e^(-7*f*x -
7*e)/((7*a^(3/2)*e^(-2*f*x - 2*e) + 21*a^(3/2)*e^(-4*f*x - 4*e) + 35*a^(3/
2)*e^(-6*f*x - 6*e) + 35*a^(3/2)*e^(-8*f*x - 8*e) + 21*a^(3/2)*e^(-10*f*x
- 10*e) + 7*a^(3/2)*e^(-12*f*x - 12*e) + a^(3/2)*e^(-14*f*x - 14*e) + a^(3
/2))*f) + 32/15*e^(-9*f*x - 9*e)/((7*a^(3/2)*e^(-2*f*x - 2*e) + 21*a^(3/2)
*e^(-4*f*x - 4*e) + 35*a^(3/2)*e^(-6*f*x - 6*e) + 35*a^(3/2)*e^(-8*f*x - 8
*e) + 21*a^(3/2)*e^(-10*f*x - 10*e) + 7*a^(3/2)*e^(-12*f*x - 12*e) + a^(3/
2)*e^(-14*f*x - 14*e) + a^(3/2))*f) - 8/3*e^(-11*f*x - 11*e)/((7*a^(3/2)*e
^(-2*f*x - 2*e) + 21*a^(3/2)*e^(-4*f*x - 4*e) + 35*a^(3/2)*e^(-6*f*x - 6*e
) + 35*a^(3/2)*e^(-8*f*x - 8*e) + 21*a^(3/2)*e^(-10*f*x - 10*e) + 7*a^(3/2
)*e^(-12*f*x - 12*e) + a^(3/2)*e^(-14*f*x - 14*e) + a^(3/2))*f)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{\tanh^5(e+fx)}{\left(a+a\sinh^2(e+fx)\right)^{3/2}}\,dx = \text{Exception raised: TypeError}$$

input

```
integrate(tanh(f*x+e)^5/(a+a*sinh(f*x+e)^2)^(3/2),x, algorithm="giac")
```

output

```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 457, normalized size of antiderivative = 6.72

$$\int \frac{\tanh^5(e+fx)}{\left(a+a\sinh^2(e+fx)\right)^{3/2}}\,dx = \frac{464\,e^{3\,e+3\,f\,x}\sqrt{a+a\left(\frac{e^{e+f\,x}}{2}-\frac{e^{-e-f\,x}}{2}\right)^2}}{15\,a^2\,f\,\left(e^{2\,e+2\,f\,x}+1\right)^3\,\left(e^{e+f\,x}+e^{3\,e+3\,f\,x}\right)} - \frac{16\,e^{3\,e+3\,f\,x}\sqrt{a+a\left(\frac{e^{e+f\,x}}{2}-\frac{e^{-e-f\,x}}{2}\right)^2}}{3\,a^2\,f\,\left(e^{2\,e+2\,f\,x}+1\right)^2\,\left(e^{e+f\,x}+e^{3\,e+3\,f\,x}\right)} - \frac{3072\,e^{3\,e+3\,f\,x}\sqrt{a+a\left(\frac{e^{e+f\,x}}{2}-\frac{e^{-e-f\,x}}{2}\right)^2}}{35\,a^2\,f\,\left(e^{2\,e+2\,f\,x}+1\right)^4\,\left(e^{e+f\,x}+e^{3\,e+3\,f\,x}\right)} + \frac{4736\,e^{3\,e+3\,f\,x}\sqrt{a+a\left(\frac{e^{e+f\,x}}{2}-\frac{e^{-e-f\,x}}{2}\right)^2}}{35\,a^2\,f\,\left(e^{2\,e+2\,f\,x}+1\right)^5\,\left(e^{e+f\,x}+e^{3\,e+3\,f\,x}\right)} - \frac{768\,e^{3\,e+3\,f\,x}\sqrt{a+a\left(\frac{e^{e+f\,x}}{2}-\frac{e^{-e-f\,x}}{2}\right)^2}}{7\,a^2\,f\,\left(e^{2\,e+2\,f\,x}+1\right)^6\,\left(e^{e+f\,x}+e^{3\,e+3\,f\,x}\right)} + \frac{256\,e^{3\,e+3\,f\,x}\sqrt{a+a\left(\frac{e^{e+f\,x}}{2}-\frac{e^{-e-f\,x}}{2}\right)^2}}{7\,a^2\,f\,\left(e^{2\,e+2\,f\,x}+1\right)^7\,\left(e^{e+f\,x}+e^{3\,e+3\,f\,x}\right)}$$

input
```
int(tanh(e + f*x)^5/(a + a*sinh(e + f*x)^2)^(3/2),x)
```

output
```
(464*exp(3*e + 3*f*x)*(a + a*(exp(e + f*x)/2 - exp(- e - f*x)/2)^2)^(1/2))
/(15*a^2*f*(exp(2*e + 2*f*x) + 1)^3*(exp(e + f*x) + exp(3*e + 3*f*x))) - (
16*exp(3*e + 3*f*x)*(a + a*(exp(e + f*x)/2 - exp(- e - f*x)/2)^2)^(1/2))/(
3*a^2*f*(exp(2*e + 2*f*x) + 1)^2*(exp(e + f*x) + exp(3*e + 3*f*x))) - (307
2*exp(3*e + 3*f*x)*(a + a*(exp(e + f*x)/2 - exp(- e - f*x)/2)^2)^(1/2))/(3
5*a^2*f*(exp(2*e + 2*f*x) + 1)^4*(exp(e + f*x) + exp(3*e + 3*f*x))) + (473
6*exp(3*e + 3*f*x)*(a + a*(exp(e + f*x)/2 - exp(- e - f*x)/2)^2)^(1/2))/(3
5*a^2*f*(exp(2*e + 2*f*x) + 1)^5*(exp(e + f*x) + exp(3*e + 3*f*x))) - (768
*exp(3*e + 3*f*x)*(a + a*(exp(e + f*x)/2 - exp(- e - f*x)/2)^2)^(1/2))/(7*
a^2*f*(exp(2*e + 2*f*x) + 1)^6*(exp(e + f*x) + exp(3*e + 3*f*x))) + (256*e
xp(3*e + 3*f*x)*(a + a*(exp(e + f*x)/2 - exp(- e - f*x)/2)^2)^(1/2))/(7*a^
2*f*(exp(2*e + 2*f*x) + 1)^7*(exp(e + f*x) + exp(3*e + 3*f*x)))
```

Reduce [F]

$$\int \frac{\tanh^5(e+fx)}{\left(a+a\sinh^2(e+fx)\right)^{3/2}}\,dx = \frac{\sqrt{a}\left(\int \frac{\sqrt{\sinh(fx+e)^2+1}\,\tanh(fx+e)^5}{\sinh(fx+e)^4+2\sinh(fx+e)^2+1}dx\right)}{a^2}$$

input
```
int(tanh(f*x+e)^5/(a+a*sinh(f*x+e)^2)^(3/2),x)
```

output
```
(sqrt(a)*int((sqrt(sinh(e + f*x)**2 + 1)*tanh(e + f*x)**5)/(sinh(e + f*x)*
*4 + 2*sinh(e + f*x)**2 + 1),x))/a**2
```

3.412 $\int \frac{\tanh^3(e+fx)}{(a+a\sinh^2(e+fx))^{3/2}}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 44

$$\int \frac{\tanh^3(e+fx)}{(a+a\sinh^2(e+fx))^{3/2}}\,dx = \frac{a}{5f\left(a\cosh^2(e+fx)\right)^{5/2}} - \frac{1}{3f\left(a\cosh^2(e+fx)\right)^{3/2}}$$

output
```
1/5*a/f/(a*cosh(f*x+e)^2)^(5/2)-1/3/f/(a*cosh(f*x+e)^2)^(3/2)
```

Mathematica [A] (verified)

Time = 0.17 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.77

$$\int \frac{\tanh^3(e+fx)}{(a+a\sinh^2(e+fx))^{3/2}}\,dx = \frac{a(3-5\cosh^2(e+fx))}{15f\left(a\cosh^2(e+fx)\right)^{5/2}}$$

input
```
Integrate[Tanh[e + f*x]^3/(a + a*Sinh[e + f*x]^2)^(3/2),x]
```

output
```
(a*(3 - 5*Cosh[e + f*x]^2))/(15*f*(a*Cosh[e + f*x]^2)^(5/2))
```

Rubi [A] (verified)

Time = 0.39 (sec) , antiderivative size = 53, normalized size of antiderivative = 1.20, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.400$, Rules used = {3042, 26, 3655, 26, 3042, 26, 3684, 8, 53, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\tanh^3(e+fx)}{\left(a\sinh^2(e+fx)+a\right)^{3/2}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{i\tan(ie+ifx)^3}{(a-a\sin(ie+ifx)^2)^{3/2}}dx$$

$$\Big\downarrow \text{26}$$

$$i\int \frac{\tan(ie+ifx)^3}{(a-a\sin(ie+ifx)^2)^{3/2}}dx$$

$$\Big\downarrow \text{3655}$$

$$i\int -\frac{i\tanh^3(e+fx)}{\left(a\cosh^2(e+fx)\right)^{3/2}}dx$$

$$\Big\downarrow \text{26}$$

$$\int \frac{\tanh^3(e+fx)}{\left(a\cosh^2(e+fx)\right)^{3/2}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int -\frac{i}{\tan\left(ie+ifx+\frac{\pi}{2}\right)^3\left(a\sin\left(ie+ifx+\frac{\pi}{2}\right)^2\right)^{3/2}}dx$$

$$\Big\downarrow \text{26}$$

$$-i\int \frac{1}{\left(a\sin\left(\frac{1}{2}(2ie+\pi)+ifx\right)^2\right)^{3/2}\tan\left(\frac{1}{2}(2ie+\pi)+ifx\right)^3}dx$$

$$\Big\downarrow \text{3684}$$

$$-\frac{\int \frac{(1-\cosh^2(e+fx))\text{sech}^4(e+fx)}{(a\cosh^2(e+fx))^{3/2}}d\cosh^2(e+fx)}{2f}$$

$$\Big\downarrow \text{ 8}$$

$$-\frac{a^2\int \frac{1-\cosh^2(e+fx)}{(a\cosh^2(e+fx))^{7/2}}d\cosh^2(e+fx)}{2f}$$

$$\Big\downarrow \text{ 53}$$

$$-\frac{a^2\int \left(\frac{1}{(a\cosh^2(e+fx))^{7/2}}-\frac{1}{a(a\cosh^2(e+fx))^{5/2}}\right)d\cosh^2(e+fx)}{2f}$$

$$\Big\downarrow \text{ 2009}$$

$$-\frac{a^2\left(\frac{2}{3a^2(a\cosh^2(e+fx))^{3/2}}-\frac{2}{5a(a\cosh^2(e+fx))^{5/2}}\right)}{2f}$$

input
```
Int[Tanh[e + f*x]^3/(a + a*Sinh[e + f*x]^2)^(3/2),x]
```

output
```
-1/2*(a^2*(-2/(5*a*(a*Cosh[e + f*x]^2)^(5/2)) + 2/(3*a^2*(a*Cosh[e + f*x]^
2)^(3/2))))/f
```

Defintions of rubi rules used

rule 8
```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_))^(p_), x_Symbol] :> Simp[1/a^m   Int[u*(a*
x)^(m + p), x], x] /; FreeQ[{a, m, p}, x] && IntegerQ[m]
```

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 53
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && LeQ[7*m + 4*n + 4, 0])
|| LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3655
```
Int[(u_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> Int[A
ctivateTrig[u*(a*cos[e + f*x]^2)^p], x] /; FreeQ[{a, b, e, f, p}, x] && EqQ
[a + b, 0]
```

rule 3684
```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*tan[(e_.) + (f_.)*(x_)]^(m_.
), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x]^2, x]}, Simp[ff^((m + 1
)/2)/(2*f)   Subst[Int[x^((m - 1)/2)*((b*ff^(n/2)*x^(n/2))^p/(1 - ff*x)^((m
 + 1)/2)), x], x, Sin[e + f*x]^2/ff], x]] /; FreeQ[{b, e, f, p}, x] && Inte
gerQ[(m - 1)/2] && IntegerQ[n/2]
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.40 (sec) , antiderivative size = 44, normalized size of antiderivative = 1.00

method	result	size
default	$\frac{\text{`int/indef0`}\left(\frac{\sinh(fx+e)^3}{\cosh(fx+e)^6 a\sqrt{a\cosh(fx+e)^2}},\sinh(fx+e)\right)}{f}$	44
risch	$-\frac{8\left(5\,{e}^{4fx+4e}-2\,{e}^{2fx+2e}+5\right){e}^{2fx+2e}}{15f\sqrt{\left({e}^{2fx+2e}+1\right)^2 a\,{e}^{-2fx-2e}}\,\left({e}^{2fx+2e}+1\right)^4 a}$	81

input
```
int(tanh(f*x+e)^3/(a+a*sinh(f*x+e)^2)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
`int/indef0`(sinh(f*x+e)^3/cosh(f*x+e)^6/a/(a*cosh(f*x+e)^2)^(1/2),sinh(f*
x+e))/f
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1400 vs. $2(36) = 72$.

Time = 0.11 (sec) , antiderivative size = 1400, normalized size of antiderivative = 31.82

$$\int \frac{\tanh^3(e+fx)}{\left(a+a\sinh^2(e+fx)\right)^{3/2}}\,dx = \text{Too large to display}$$

input
```
integrate(tanh(f*x+e)^3/(a+a*sinh(f*x+e)^2)^(3/2),x, algorithm="fricas")
```

output
```
-8/15*(35*cosh(f*x + e)*e^(f*x + e)*sinh(f*x + e)^6 + 5*e^(f*x + e)*sinh(f
*x + e)^7 + (105*cosh(f*x + e)^2 - 2)*e^(f*x + e)*sinh(f*x + e)^5 + 5*(35*
cosh(f*x + e)^3 - 2*cosh(f*x + e))*e^(f*x + e)*sinh(f*x + e)^4 + 5*(35*cos
h(f*x + e)^4 - 4*cosh(f*x + e)^2 + 1)*e^(f*x + e)*sinh(f*x + e)^3 + 5*(21*
cosh(f*x + e)^5 - 4*cosh(f*x + e)^3 + 3*cosh(f*x + e))*e^(f*x + e)*sinh(f*
x + e)^2 + 5*(7*cosh(f*x + e)^6 - 2*cosh(f*x + e)^4 + 3*cosh(f*x + e)^2)*e
^(f*x + e)*sinh(f*x + e) + (5*cosh(f*x + e)^7 - 2*cosh(f*x + e)^5 + 5*cosh
(f*x + e)^3)*e^(f*x + e))*sqrt(a*e^(4*f*x + 4*e) + 2*a*e^(2*f*x + 2*e) + a
)*e^(-f*x - e)/(a^2*f*cosh(f*x + e)^10 + 5*a^2*f*cosh(f*x + e)^8 + (a^2*f*
e^(2*f*x + 2*e) + a^2*f)*sinh(f*x + e)^10 + 10*(a^2*f*cosh(f*x + e)*e^(2*f
*x + 2*e) + a^2*f*cosh(f*x + e))*sinh(f*x + e)^9 + 10*a^2*f*cosh(f*x + e)^
6 + 5*(9*a^2*f*cosh(f*x + e)^2 + a^2*f + (9*a^2*f*cosh(f*x + e)^2 + a^2*f)
*e^(2*f*x + 2*e))*sinh(f*x + e)^8 + 40*(3*a^2*f*cosh(f*x + e)^3 + a^2*f*co
sh(f*x + e) + (3*a^2*f*cosh(f*x + e)^3 + a^2*f*cosh(f*x + e))*e^(2*f*x + 2
*e))*sinh(f*x + e)^7 + 10*a^2*f*cosh(f*x + e)^4 + 10*(21*a^2*f*cosh(f*x +
e)^4 + 14*a^2*f*cosh(f*x + e)^2 + a^2*f + (21*a^2*f*cosh(f*x + e)^4 + 14*a
^2*f*cosh(f*x + e)^2 + a^2*f)*e^(2*f*x + 2*e))*sinh(f*x + e)^6 + 4*(63*a^2
*f*cosh(f*x + e)^5 + 70*a^2*f*cosh(f*x + e)^3 + 15*a^2*f*cosh(f*x + e) + (
63*a^2*f*cosh(f*x + e)^5 + 70*a^2*f*cosh(f*x + e)^3 + 15*a^2*f*cosh(f*x +
e))*e^(2*f*x + 2*e))*sinh(f*x + e)^5 + 5*a^2*f*cosh(f*x + e)^2 + 10*(21...
```

Sympy [F]

$$\int \frac{\tanh^3(e+fx)}{\left(a+a\sinh^2(e+fx)\right)^{3/2}}\,dx=\int \frac{\tanh^3\left(e+fx\right)}{\left(a\left(\sinh^2\left(e+fx\right)+1\right)\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate(tanh(f*x+e)**3/(a+a*sinh(f*x+e)**2)**(3/2),x)
```

output
```
Integral(tanh(e + f*x)**3/(a*(sinh(e + f*x)**2 + 1))**(3/2), x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 268 vs. 2(36) = 72.

Time = 0.18 (sec) , antiderivative size = 268, normalized size of antiderivative = 6.09

$$\int \frac{\tanh^3(e+fx)}{\left(a+a\sinh^2(e+fx)\right)^{3/2}}\,dx=$$
$$-\frac{8\,e^{(-3\,fx-3\,e)}}{3\left(5\,a^{\frac{3}{2}}e^{(-2\,fx-2\,e)}+10\,a^{\frac{3}{2}}e^{(-4\,fx-4\,e)}+10\,a^{\frac{3}{2}}e^{(-6\,fx-6\,e)}+5\,a^{\frac{3}{2}}e^{(-8\,fx-8\,e)}+a^{\frac{3}{2}}e^{(-10\,fx-10\,e)}+a^{\frac{3}{2}}\right)f}$$
$$+\frac{16\,e^{(-5\,fx-5\,e)}}{15\left(5\,a^{\frac{3}{2}}e^{(-2\,fx-2\,e)}+10\,a^{\frac{3}{2}}e^{(-4\,fx-4\,e)}+10\,a^{\frac{3}{2}}e^{(-6\,fx-6\,e)}+5\,a^{\frac{3}{2}}e^{(-8\,fx-8\,e)}+a^{\frac{3}{2}}e^{(-10\,fx-10\,e)}+a^{\frac{3}{2}}\right)f}$$
$$-\frac{8\,e^{(-7\,fx-7\,e)}}{3\left(5\,a^{\frac{3}{2}}e^{(-2\,fx-2\,e)}+10\,a^{\frac{3}{2}}e^{(-4\,fx-4\,e)}+10\,a^{\frac{3}{2}}e^{(-6\,fx-6\,e)}+5\,a^{\frac{3}{2}}e^{(-8\,fx-8\,e)}+a^{\frac{3}{2}}e^{(-10\,fx-10\,e)}+a^{\frac{3}{2}}\right)f}$$

input
```
integrate(tanh(f*x+e)^3/(a+a*sinh(f*x+e)^2)^(3/2),x, algorithm="maxima")
```

output
```
-8/3*e^(-3*f*x - 3*e)/((5*a^(3/2)*e^(-2*f*x - 2*e) + 10*a^(3/2)*e^(-4*f*x
- 4*e) + 10*a^(3/2)*e^(-6*f*x - 6*e) + 5*a^(3/2)*e^(-8*f*x - 8*e) + a^(3/2
)*e^(-10*f*x - 10*e) + a^(3/2))*f) + 16/15*e^(-5*f*x - 5*e)/((5*a^(3/2)*e^
(-2*f*x - 2*e) + 10*a^(3/2)*e^(-4*f*x - 4*e) + 10*a^(3/2)*e^(-6*f*x - 6*e)
 + 5*a^(3/2)*e^(-8*f*x - 8*e) + a^(3/2)*e^(-10*f*x - 10*e) + a^(3/2))*f) -
 8/3*e^(-7*f*x - 7*e)/((5*a^(3/2)*e^(-2*f*x - 2*e) + 10*a^(3/2)*e^(-4*f*x
- 4*e) + 10*a^(3/2)*e^(-6*f*x - 6*e) + 5*a^(3/2)*e^(-8*f*x - 8*e) + a^(3/2
)*e^(-10*f*x - 10*e) + a^(3/2))*f)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{\tanh^3(e+fx)}{(a+a\sinh^2(e+fx))^{3/2}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(tanh(f*x+e)^3/(a+a*sinh(f*x+e)^2)^(3/2),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [B] (verification not implemented)

Time = 1.89 (sec) , antiderivative size = 305, normalized size of antiderivative = 6.93

$$\int \frac{\tanh^3(e+fx)}{(a+a\sinh^2(e+fx))^{3/2}}\,dx = \frac{272\,e^{3\,e+3\,f\,x}\,\sqrt{a+a\,\left(\frac{e^{e+f\,x}}{2}-\frac{e^{-e-f\,x}}{2}\right)^2}}{15\,a^2\,f\,\left(e^{2\,e+2\,f\,x}+1\right)^3\,\left(e^{e+f\,x}+e^{3\,e+3\,f\,x}\right)} - \frac{16\,e^{3\,e+3\,f\,x}\,\sqrt{a+a\,\left(\frac{e^{e+f\,x}}{2}-\frac{e^{-e-f\,x}}{2}\right)^2}}{3\,a^2\,f\,\left(e^{2\,e+2\,f\,x}+1\right)^2\,\left(e^{e+f\,x}+e^{3\,e+3\,f\,x}\right)} - \frac{128\,e^{3\,e+3\,f\,x}\,\sqrt{a+a\,\left(\frac{e^{e+f\,x}}{2}-\frac{e^{-e-f\,x}}{2}\right)^2}}{5\,a^2\,f\,\left(e^{2\,e+2\,f\,x}+1\right)^4\,\left(e^{e+f\,x}+e^{3\,e+3\,f\,x}\right)} + \frac{64\,e^{3\,e+3\,f\,x}\,\sqrt{a+a\,\left(\frac{e^{e+f\,x}}{2}-\frac{e^{-e-f\,x}}{2}\right)^2}}{5\,a^2\,f\,\left(e^{2\,e+2\,f\,x}+1\right)^5\,\left(e^{e+f\,x}+e^{3\,e+3\,f\,x}\right)}$$

input
```
int(tanh(e + f*x)^3/(a + a*sinh(e + f*x)^2)^(3/2),x)
```

output
```
(272*exp(3*e + 3*f*x)*(a + a*(exp(e + f*x)/2 - exp(- e - f*x)/2)^2)^(1/2))
/(15*a^2*f*(exp(2*e + 2*f*x) + 1)^3*(exp(e + f*x) + exp(3*e + 3*f*x))) - (
16*exp(3*e + 3*f*x)*(a + a*(exp(e + f*x)/2 - exp(- e - f*x)/2)^2)^(1/2))/(
3*a^2*f*(exp(2*e + 2*f*x) + 1)^2*(exp(e + f*x) + exp(3*e + 3*f*x))) - (128
*exp(3*e + 3*f*x)*(a + a*(exp(e + f*x)/2 - exp(- e - f*x)/2)^2)^(1/2))/(5*
a^2*f*(exp(2*e + 2*f*x) + 1)^4*(exp(e + f*x) + exp(3*e + 3*f*x))) + (64*ex
p(3*e + 3*f*x)*(a + a*(exp(e + f*x)/2 - exp(- e - f*x)/2)^2)^(1/2))/(5*a^2
*f*(exp(2*e + 2*f*x) + 1)^5*(exp(e + f*x) + exp(3*e + 3*f*x)))
```

Reduce [F]

$$\int \frac{\tanh^3(e+fx)}{\left(a+a\sinh^2(e+fx)\right)^{3/2}}\,dx = \frac{\sqrt{a}\left(\int \frac{\sqrt{\sinh(fx+e)^2+1}\,\tanh(fx+e)^3}{\sinh(fx+e)^4+2\sinh(fx+e)^2+1}dx\right)}{a^2}$$

input
```
int(tanh(f*x+e)^3/(a+a*sinh(f*x+e)^2)^(3/2),x)
```

output
```
(sqrt(a)*int((sqrt(sinh(e + f*x)**2 + 1)*tanh(e + f*x)**3)/(sinh(e + f*x)*
*4 + 2*sinh(e + f*x)**2 + 1),x))/a**2
```

3.413 $\int \frac{\tanh(e+fx)}{\left(a+a\sinh^2(e+fx)\right)^{3/2}}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 21

$$\int \frac{\tanh(e+fx)}{\left(a+a\sinh^2(e+fx)\right)^{3/2}}\,dx=-\frac{1}{3f\left(a\cosh^2(e+fx)\right)^{3/2}}$$

output
```
-1/3/f/(a*cosh(f*x+e)^2)^(3/2)
```

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.00

$$\int \frac{\tanh(e+fx)}{\left(a+a\sinh^2(e+fx)\right)^{3/2}}\,dx=-\frac{1}{3f\left(a\cosh^2(e+fx)\right)^{3/2}}$$

input
```
Integrate[Tanh[e + f*x]/(a + a*Sinh[e + f*x]^2)^(3/2),x]
```

output
```
-1/3*1/(f*(a*Cosh[e + f*x]^2)^(3/2))
```

Rubi [A] (verified)

Time = 0.32 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.391, Rules used = {3042, 26, 3655, 26, 3042, 26, 3684, 8, 17}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\tanh(e+fx)}{\left(a\sinh^2(e+fx)+a\right)^{3/2}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int -\frac{i\tan(ie+ifx)}{(a-a\sin(ie+ifx)^2)^{3/2}}dx$$

$$\Big\downarrow \text{26}$$

$$-i\int \frac{\tan(ie+ifx)}{(a-a\sin(ie+ifx)^2)^{3/2}}dx$$

$$\Big\downarrow \text{3655}$$

$$-i\int \frac{i\tanh(e+fx)}{\left(a\cosh^2(e+fx)\right)^{3/2}}dx$$

$$\Big\downarrow \text{26}$$

$$\int \frac{\tanh(e+fx)}{\left(a\cosh^2(e+fx)\right)^{3/2}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{i}{\tan\left(ie+ifx+\frac{\pi}{2}\right)\left(a\sin\left(ie+ifx+\frac{\pi}{2}\right)^2\right)^{3/2}}dx$$

$$\Big\downarrow \text{26}$$

$$i\int \frac{1}{\left(a\sin\left(\frac{1}{2}(2ie+\pi)+ifx\right)^2\right)^{3/2}\tan\left(\frac{1}{2}(2ie+\pi)+ifx\right)}dx$$

$$\Big\downarrow \text{3684}$$

$$\frac{\int \frac{\text{sech}^2(e+fx)}{\left(a\cosh^2(e+fx)\right)^{3/2}}d\cosh^2(e+fx)}{2f}$$

$$\Big\downarrow \; 8$$

$$\frac{a\int \frac{1}{\left(a\cosh^2(e+fx)\right)^{5/2}}d\cosh^2(e+fx)}{2f}$$

$$\Big\downarrow \; 17$$

$$-\frac{1}{3f\left(a\cosh^2(e+fx)\right)^{3/2}}$$

input
```
Int[Tanh[e + f*x]/(a + a*Sinh[e + f*x]^2)^(3/2),x]
```

output
```
-1/3*1/(f*(a*Cosh[e + f*x]^2)^(3/2))
```

Defintions of rubi rules used

rule 8
```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_))^(p_), x_Symbol] :> Simp[1/a^m   Int[u*(a*
x)^(m + p), x], x] /; FreeQ[{a, m, p}, x] && IntegerQ[m]
```

rule 17
```
Int[(c_.)*((a_.) + (b_.)*(x_))^(m_.), x_Symbol] :> Simp[c*((a + b*x)^(m + 1
)/(b*(m + 1))), x] /; FreeQ[{a, b, c, m}, x] && NeQ[m, -1]
```

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3655
```
Int[(u_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> Int[A
ctivateTrig[u*(a*cos[e + f*x]^2)^p], x] /; FreeQ[{a, b, e, f, p}, x] && EqQ
[a + b, 0]
```

rule 3684
```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*tan[(e_.) + (f_.)*(x_)]^(m_.
), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x]^2, x]}, Simp[ff^((m + 1
)/2)/(2*f)   Subst[Int[x^((m - 1)/2)*((b*ff^(n/2)*x^(n/2))^p/(1 - ff*x)^((m
 + 1)/2)), x], x, Sin[e + f*x]^2/ff], x]] /; FreeQ[{b, e, f, p}, x] && Inte
gerQ[(m - 1)/2] && IntegerQ[n/2]
```

Maple [A] (verified)

Time = 0.32 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.95

method	result	size
derivativedivides	$-\frac{1}{3f\left(a+a\sinh\left(fx+e\right)^{2}\right)^{\frac{3}{2}}}$	20
default	$-\frac{1}{3f\left(a+a\sinh\left(fx+e\right)^{2}\right)^{\frac{3}{2}}}$	20
risch	$-\frac{8\,\mathrm{e}^{2fx+2e}}{3f\sqrt{\left(\mathrm{e}^{2fx+2e}+1\right)^{2}a\,\mathrm{e}^{-2fx-2e}}\,\left(\mathrm{e}^{2fx+2e}+1\right)^{2}a}$	57

input
```
int(tanh(f*x+e)/(a+a*sinh(f*x+e)^2)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-1/3/f/(a+a*sinh(f*x+e)^2)^(3/2)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 608 vs. 2(17) = 34.

Time = 0.11 (sec) , antiderivative size = 608, normalized size of antiderivative = 28.95

$$\int \frac{\tanh(e+fx)}{\left(a+a\sinh^2(e+fx)\right)^{3/2}}\,dx = \text{Too large to display}$$

input
```
integrate(tanh(f*x+e)/(a+a*sinh(f*x+e)^2)^(3/2),x, algorithm="fricas")
```

output
```
-8/3*(cosh(f*x + e)^3*e^(f*x + e) + 3*cosh(f*x + e)^2*e^(f*x + e)*sinh(f*x
 + e) + 3*cosh(f*x + e)*e^(f*x + e)*sinh(f*x + e)^2 + e^(f*x + e)*sinh(f*x
 + e)^3)*sqrt(a*e^(4*f*x + 4*e) + 2*a*e^(2*f*x + 2*e) + a)*e^(-f*x - e)/(a
^2*f*cosh(f*x + e)^6 + 3*a^2*f*cosh(f*x + e)^4 + (a^2*f*e^(2*f*x + 2*e) +
a^2*f)*sinh(f*x + e)^6 + 6*(a^2*f*cosh(f*x + e)*e^(2*f*x + 2*e) + a^2*f*co
sh(f*x + e))*sinh(f*x + e)^5 + 3*a^2*f*cosh(f*x + e)^2 + 3*(5*a^2*f*cosh(f
*x + e)^2 + a^2*f + (5*a^2*f*cosh(f*x + e)^2 + a^2*f)*e^(2*f*x + 2*e))*sin
h(f*x + e)^4 + 4*(5*a^2*f*cosh(f*x + e)^3 + 3*a^2*f*cosh(f*x + e) + (5*a^2
*f*cosh(f*x + e)^3 + 3*a^2*f*cosh(f*x + e))*e^(2*f*x + 2*e))*sinh(f*x + e)
^3 + a^2*f + 3*(5*a^2*f*cosh(f*x + e)^4 + 6*a^2*f*cosh(f*x + e)^2 + a^2*f
+ (5*a^2*f*cosh(f*x + e)^4 + 6*a^2*f*cosh(f*x + e)^2 + a^2*f)*e^(2*f*x + 2
*e))*sinh(f*x + e)^2 + (a^2*f*cosh(f*x + e)^6 + 3*a^2*f*cosh(f*x + e)^4 +
3*a^2*f*cosh(f*x + e)^2 + a^2*f)*e^(2*f*x + 2*e) + 6*(a^2*f*cosh(f*x + e)^
5 + 2*a^2*f*cosh(f*x + e)^3 + a^2*f*cosh(f*x + e) + (a^2*f*cosh(f*x + e)^5
 + 2*a^2*f*cosh(f*x + e)^3 + a^2*f*cosh(f*x + e))*e^(2*f*x + 2*e))*sinh(f*
x + e))
```

Sympy [F]

$$\int \frac{\tanh(e+fx)}{(a+a\sinh^2(e+fx))^{3/2}}\,dx = \int \frac{\tanh{(e+fx)}}{\left(a\left(\sinh^2{(e+fx)}+1\right)\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate(tanh(f*x+e)/(a+a*sinh(f*x+e)**2)**(3/2),x)
```

output
```
Integral(tanh(e + f*x)/(a*(sinh(e + f*x)**2 + 1))**(3/2), x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 61 vs. $2(17) = 34$.

Time = 0.19 (sec) , antiderivative size = 61, normalized size of antiderivative = 2.90

$$\int \frac{\tanh(e+fx)}{(a+a\sinh^2(e+fx))^{3/2}}\,dx = -\frac{8\,e^{(-3\,fx-3\,e)}}{3\left(3\,a^{\frac{3}{2}}e^{(-2\,fx-2\,e)}+3\,a^{\frac{3}{2}}e^{(-4\,fx-4\,e)}+a^{\frac{3}{2}}e^{(-6\,fx-6\,e)}+a^{\frac{3}{2}}\right)f}$$

input
```
integrate(tanh(f*x+e)/(a+a*sinh(f*x+e)^2)^(3/2),x, algorithm="maxima")
```

output
```
-8/3*e^(-3*f*x - 3*e)/((3*a^(3/2)*e^(-2*f*x - 2*e) + 3*a^(3/2)*e^(-4*f*x -
 4*e) + a^(3/2)*e^(-6*f*x - 6*e) + a^(3/2))*f)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{\tanh(e+fx)}{\left(a+a\sinh^2(e+fx)\right)^{3/2}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(tanh(f*x+e)/(a+a*sinh(f*x+e)^2)^(3/2),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [B] (verification not implemented)

Time = 1.86 (sec) , antiderivative size = 58, normalized size of antiderivative = 2.76

$$\int \frac{\tanh(e+fx)}{\left(a+a\sinh^2(e+fx)\right)^{3/2}}\,dx = -\frac{16\,e^{4\,e+4\,f\,x}\,\sqrt{a+a\,\left(\frac{e^{e+f\,x}}{2}-\frac{e^{-e-f\,x}}{2}\right)^2}}{3\,a^2\,f\,\left(e^{2\,e+2\,f\,x}+1\right)^4}$$

input
```
int(tanh(e + f*x)/(a + a*sinh(e + f*x)^2)^(3/2),x)
```

output
```
-(16*exp(4*e + 4*f*x)*(a + a*(exp(e + f*x)/2 - exp(- e - f*x)/2)^2)^(1/2))
/(3*a^2*f*(exp(2*e + 2*f*x) + 1)^4)
```

Reduce [F]

$$\int \frac{\tanh(e+fx)}{\left(a+a\sinh^2(e+fx)\right)^{3/2}}\,dx = \frac{\sqrt{a}\left(\int \frac{\sqrt{\sinh(fx+e)^2+1}\,\tanh(fx+e)}{\sinh(fx+e)^4+2\sinh(fx+e)^2+1}dx\right)}{a^2}$$

input
```
int(tanh(f*x+e)/(a+a*sinh(f*x+e)^2)^(3/2),x)
```

output
```
(sqrt(a)*int((sqrt(sinh(e + f*x)**2 + 1)*tanh(e + f*x))/(sinh(e + f*x)**4
+ 2*sinh(e + f*x)**2 + 1),x))/a**2
```

3.414 $\int \frac{\coth(e+fx)}{\left(a+a\sinh^2(e+fx)\right)^{3/2}}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 53

$$\int \frac{\coth(e+fx)}{\left(a+a\sinh^2(e+fx)\right)^{3/2}}\,dx=-\frac{\operatorname{arctanh}\left(\frac{\sqrt{a\cosh^2(e+fx)}}{\sqrt{a}}\right)}{a^{3/2}f}+\frac{1}{af\sqrt{a\cosh^2(e+fx)}}$$

output
```
-arctanh((a*cosh(f*x+e)^2)^(1/2)/a^(1/2))/a^(3/2)/f+1/a/f/(a*cosh(f*x+e)^2
)^(1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 0.08 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.70

$$\int \frac{\coth(e+fx)}{\left(a+a\sinh^2(e+fx)\right)^{3/2}}\,dx=\frac{\text{Hypergeometric2F1}\left(-\frac{1}{2},1,\frac{1}{2},\cosh^2(e+fx)\right)}{af\sqrt{a\cosh^2(e+fx)}}$$

input
```
Integrate[Coth[e + f*x]/(a + a*Sinh[e + f*x]^2)^(3/2),x]
```

output
```
Hypergeometric2F1[-1/2, 1, 1/2, Cosh[e + f*x]^2]/(a*f*Sqrt[a*Cosh[e + f*x]
^2])
```

Rubi [A] (verified)

Time = 0.37 (sec) , antiderivative size = 55, normalized size of antiderivative = 1.04, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.435, Rules used = {3042, 26, 3655, 26, 3042, 26, 3684, 61, 73, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\coth(e+fx)}{\left(a\sinh^2(e+fx)+a\right)^{3/2}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{i}{\tan(ie+ifx)\left(a-a\sin(ie+ifx)^2\right)^{3/2}}dx$$

$$\Big\downarrow \text{26}$$

$$i\int \frac{1}{\left(a-a\sin(ie+ifx)^2\right)^{3/2}\tan(ie+ifx)}dx$$

$$\Big\downarrow \text{3655}$$

$$i\int -\frac{i\coth(e+fx)}{\left(a\cosh^2(e+fx)\right)^{3/2}}dx$$

$$\Big\downarrow \text{26}$$

$$\int \frac{\coth(e+fx)}{\left(a\cosh^2(e+fx)\right)^{3/2}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int -\frac{i\tan\left(ie+ifx+\frac{\pi}{2}\right)}{\left(a\sin\left(ie+ifx+\frac{\pi}{2}\right)^2\right)^{3/2}}dx$$

$$\Big\downarrow \text{26}$$

$$-i\int \frac{\tan \left(\frac{1}{2}(2ie+\pi)+ifx\right)}{\left(a\sin \left(\frac{1}{2}(2ie+\pi)+ifx\right)^2\right)^{3/2}}dx$$

$$\Big\downarrow \text{ 3684}$$

$$-\frac{\int \frac{1}{\left(a\cosh^2(e+fx)\right)^{3/2}\left(1-\cosh^2(e+fx)\right)}d\cosh^2(e+fx)}{2f}$$

$$\Big\downarrow \text{ 61}$$

$$-\frac{\frac{\int \frac{1}{\sqrt{a\cosh^2(e+fx)}\left(1-\cosh^2(e+fx)\right)}d\cosh^2(e+fx)}{a}-\frac{2}{a\sqrt{a\cosh^2(e+fx)}}}{2f}$$

$$\Big\downarrow \text{ 73}$$

$$-\frac{\frac{2\int \frac{1}{1-\frac{\cosh^4(e+fx)}{a}}d\sqrt{a\cosh^2(e+fx)}}{a^2}-\frac{2}{a\sqrt{a\cosh^2(e+fx)}}}{2f}$$

$$\Big\downarrow \text{ 219}$$

$$-\frac{\frac{2\text{arctanh}\left(\frac{\sqrt{a\cosh^2(e+fx)}}{\sqrt{a}}\right)}{a^{3/2}}-\frac{2}{a\sqrt{a\cosh^2(e+fx)}}}{2f}$$

input
```
Int[Coth[e + f*x]/(a + a*Sinh[e + f*x]^2)^(3/2),x]
```

output
```
-1/2*((2*ArcTanh[Sqrt[a*Cosh[e + f*x]^2]/Sqrt[a]])/a^(3/2) - 2/(a*Sqrt[a*C
osh[e + f*x]^2]))/f
```

Defintions of rubi rules used

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 61
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Simp[d*((
m + n + 2)/((b*c - a*d)*(m + 1)))   Int[(a + b*x)^(m + 1)*(c + d*x)^n, x],
x] /; FreeQ[{a, b, c, d, n}, x] && LtQ[m, -1] &&  !(LtQ[n, -1] && (EqQ[a, 0
] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && IntLinearQ[a, b, c, d
, m, n, x]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3655
```
Int[(u_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> Int[A
ctivateTrig[u*(a*cos[e + f*x]^2)^p], x] /; FreeQ[{a, b, e, f, p}, x] && EqQ
[a + b, 0]
```

rule 3684
```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*tan[(e_.) + (f_.)*(x_)]^(m_.
), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x]^2, x]}, Simp[ff^((m + 1
)/2)/(2*f)   Subst[Int[x^((m - 1)/2)*((b*ff^(n/2)*x^(n/2))^p/(1 - ff*x)^((m
 + 1)/2)), x], x, Sin[e + f*x]^2/ff], x]] /; FreeQ[{b, e, f, p}, x] && Inte
gerQ[(m - 1)/2] && IntegerQ[n/2]
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.40 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.83

method	result	size
default	$\frac{\text{` int/indef0`}\left(\frac{1}{\cosh(fx+e)^2\sinh(fx+e)a\sqrt{a\cosh(fx+e)^2}},\sinh(fx+e)\right)}{f}$	44
risch	$-\frac{\ln(\mathrm{e}^{fx}+\mathrm{e}^{-e})\mathrm{e}^{fx+e}-\ln(\mathrm{e}^{fx}-\mathrm{e}^{-e})\mathrm{e}^{fx+e}+\ln(\mathrm{e}^{fx}+\mathrm{e}^{-e})\mathrm{e}^{-fx-e}-\ln(\mathrm{e}^{fx}-\mathrm{e}^{-e})\mathrm{e}^{-fx-e}-2}{a\sqrt{(\mathrm{e}^{2fx+2e}+1)^2 a\,\mathrm{e}^{-2fx-2e}}\,f}$	117

input

```
int(coth(f*x+e)/(a+a*sinh(f*x+e)^2)^(3/2),x,method=_RETURNVERBOSE)
```

output

```
`int/indef0`(1/cosh(f*x+e)^2/sinh(f*x+e)/a/(a*cosh(f*x+e)^2)^(1/2),sinh(f*
x+e))/f
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 271 vs. 2(45) = 90.

Time = 0.10 (sec) , antiderivative size = 271, normalized size of antiderivative = 5.11

$$\int \frac{\coth(e+fx)}{\left(a+a\sinh^2(e+fx)\right)^{3/2}}\,dx = \frac{\sqrt{ae^{(4\,fx+4\,e)}+2\,ae^{(2\,fx+2\,e)}+a}\left(2\,\cosh\left(fx+e\right)e^{(fx+e)}+\left(2\,\cosh\left(fx+\right.\right.\right.}{a^2f\cosh\left(fx+e\right)^2+a^2f+\left(a^2fe^{(2\,fx+2\,e)}+a^2f\right)\sin}$$

input

```
integrate(coth(f*x+e)/(a+a*sinh(f*x+e)^2)^(3/2),x, algorithm="fricas")
```

output

```
sqrt(a*e^(4*f*x + 4*e) + 2*a*e^(2*f*x + 2*e) + a)*(2*cosh(f*x + e)*e^(f*x
+ e) + (2*cosh(f*x + e)*e^(f*x + e)*sinh(f*x + e) + e^(f*x + e)*sinh(f*x +
 e)^2 + (cosh(f*x + e)^2 + 1)*e^(f*x + e))*log((cosh(f*x + e) + sinh(f*x +
 e) - 1)/(cosh(f*x + e) + sinh(f*x + e) + 1)) + 2*e^(f*x + e)*sinh(f*x + e
))*e^(-f*x - e)/(a^2*f*cosh(f*x + e)^2 + a^2*f + (a^2*f*e^(2*f*x + 2*e) +
a^2*f)*sinh(f*x + e)^2 + (a^2*f*cosh(f*x + e)^2 + a^2*f)*e^(2*f*x + 2*e) +
 2*(a^2*f*cosh(f*x + e)*e^(2*f*x + 2*e) + a^2*f*cosh(f*x + e))*sinh(f*x +
e))
```

Sympy [F]

$$\int \frac{\coth(e+fx)}{(a+a\sinh^2(e+fx))^{3/2}}\,dx = \int \frac{\coth(e+fx)}{(a(\sinh^2(e+fx)+1))^{\frac{3}{2}}}\,dx$$

input
```
integrate(coth(f*x+e)/(a+a*sinh(f*x+e)**2)**(3/2),x)
```

output
```
Integral(coth(e + f*x)/(a*(sinh(e + f*x)**2 + 1))**(3/2), x)
```

Maxima [A] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 76, normalized size of antiderivative = 1.43

$$\int \frac{\coth(e+fx)}{(a+a\sinh^2(e+fx))^{3/2}}\,dx = \frac{2\sqrt{a}e^{(-fx-e)}}{(a^2e^{(-2\,fx-2\,e)}+a^2)f} - \frac{\log\left(e^{(-fx-e)}+1\right)}{a^{\frac{3}{2}}f} + \frac{\log\left(e^{(-fx-e)}-1\right)}{a^{\frac{3}{2}}f}$$

input
```
integrate(coth(f*x+e)/(a+a*sinh(f*x+e)^2)^(3/2),x, algorithm="maxima")
```

output
```
2*sqrt(a)*e^(-f*x - e)/((a^2*e^(-2*f*x - 2*e) + a^2)*f) - log(e^(-f*x - e)
 + 1)/(a^(3/2)*f) + log(e^(-f*x - e) - 1)/(a^(3/2)*f)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{\coth(e+fx)}{(a+a\sinh^2(e+fx))^{3/2}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(coth(f*x+e)/(a+a*sinh(f*x+e)^2)^(3/2),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\coth(e+fx)}{\left(a+a\sinh^2(e+fx)\right)^{3/2}}\,dx=\int \frac{\coth(e+f\,x)}{\left(a\sinh\left(e+f\,x\right)^2+a\right)^{3/2}}\,dx$$

input
```
int(coth(e + f*x)/(a + a*sinh(e + f*x)^2)^(3/2),x)
```

output
```
int(coth(e + f*x)/(a + a*sinh(e + f*x)^2)^(3/2), x)
```

Reduce [F]

$$\int \frac{\coth(e+fx)}{\left(a+a\sinh^2(e+fx)\right)^{3/2}}\,dx=\frac{\sqrt{a}\left(\int \frac{\sqrt{\sinh(fx+e)^2+1}\,\coth(fx+e)}{\sinh(fx+e)^4+2\sinh(fx+e)^2+1}dx\right)}{a^2}$$

input
```
int(coth(f*x+e)/(a+a*sinh(f*x+e)^2)^(3/2),x)
```

output
```
(sqrt(a)*int((sqrt(sinh(e + f*x)**2 + 1)*coth(e + f*x))/(sinh(e + f*x)**4
+ 2*sinh(e + f*x)**2 + 1),x))/a**2
```

3.415 $\int \frac{\coth^3(e+fx)}{\left(a+a\sinh^2(e+fx)\right)^{3/2}}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 66

$$\int \frac{\coth^3(e+fx)}{\left(a+a\sinh^2(e+fx)\right)^{3/2}}\,dx=\frac{\operatorname{arctanh}\left(\frac{\sqrt{a\cosh^2(e+fx)}}{\sqrt{a}}\right)}{2a^{3/2}f}-\frac{\sqrt{a\cosh^2(e+fx)}\operatorname{csch}^2(e+fx)}{2a^2f}$$

output
```
1/2*arctanh((a*cosh(f*x+e)^2)^(1/2)/a^(1/2))/a^(3/2)/f-1/2*(a*cosh(f*x+e)^
2)^(1/2)*csch(f*x+e)^2/a^2/f
```

Mathematica [A] (verified)

Time = 0.11 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.92

$$\int \frac{\coth^3(e+fx)}{\left(a+a\sinh^2(e+fx)\right)^{3/2}}\,dx=\frac{\sqrt{a\cosh^2(e+fx)}\left(\frac{\operatorname{arctanh}\left(\sqrt{\cosh^2(e+fx)}\right)}{\sqrt{\cosh^2(e+fx)}}-\operatorname{csch}^2(e+fx)\right)}{2a^2f}$$

input
```
Integrate[Coth[e + f*x]^3/(a + a*Sinh[e + f*x]^2)^(3/2),x]
```

output
```
(Sqrt[a*Cosh[e + f*x]^2]*(ArcTanh[Sqrt[Cosh[e + f*x]^2]]/Sqrt[Cosh[e + f*x
]^2] - Csch[e + f*x]^2))/(2*a^2*f)
```

Rubi [A] (verified)

Time = 0.39 (sec) , antiderivative size = 70, normalized size of antiderivative = 1.06, number of steps used = 12, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.440$, Rules used = {3042, 26, 3655, 26, 3042, 26, 3684, 8, 52, 73, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\coth^3(e+fx)}{\left(a\sinh^2(e+fx)+a\right)^{3/2}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int -\frac{i}{\tan(ie+ifx)^3\left(a-a\sin(ie+ifx)^2\right)^{3/2}}dx$$

$$\Big\downarrow \text{26}$$

$$-i\int \frac{1}{\left(a-a\sin(ie+ifx)^2\right)^{3/2}\tan(ie+ifx)^3}dx$$

$$\Big\downarrow \text{3655}$$

$$-i\int \frac{i\coth^3(e+fx)}{\left(a\cosh^2(e+fx)\right)^{3/2}}dx$$

$$\Big\downarrow \text{26}$$

$$\int \frac{\coth^3(e+fx)}{\left(a\cosh^2(e+fx)\right)^{3/2}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{i\tan\left(ie+ifx+\frac{\pi}{2}\right)^3}{\left(a\sin\left(ie+ifx+\frac{\pi}{2}\right)^2\right)^{3/2}}dx$$

$$\Big\downarrow \; 26$$

$$i\int \frac{\tan\left(\frac{1}{2}(2ie+\pi)+ifx\right)^3}{\left(a\sin\left(\frac{1}{2}(2ie+\pi)+ifx\right)^2\right)^{3/2}}dx$$

$$\Big\downarrow \; 3684$$

$$\frac{\int \frac{\cosh^2(e+fx)}{(a\cosh^2(e+fx))^{3/2}(1-\cosh^2(e+fx))^2}d\cosh^2(e+fx)}{2f}$$

$$\Big\downarrow \; 8$$

$$\frac{\int \frac{1}{\sqrt{a\cosh^2(e+fx)}(1-\cosh^2(e+fx))^2}d\cosh^2(e+fx)}{2af}$$

$$\Big\downarrow \; 52$$

$$\frac{\frac{1}{2}\int \frac{1}{\sqrt{a\cosh^2(e+fx)}(1-\cosh^2(e+fx))}d\cosh^2(e+fx)+\frac{\sqrt{a\cosh^2(e+fx)}}{a(1-\cosh^2(e+fx))}}{2af}$$

$$\Big\downarrow \; 73$$

$$\frac{\frac{\int \frac{1}{1-\frac{\cosh^4(e+fx)}{a}}d\sqrt{a\cosh^2(e+fx)}}{a}+\frac{\sqrt{a\cosh^2(e+fx)}}{a(1-\cosh^2(e+fx))}}{2af}$$

$$\Big\downarrow \; 219$$

$$\frac{\frac{\text{arctanh}\left(\frac{\sqrt{a\cosh^2(e+fx)}}{\sqrt{a}}\right)}{\sqrt{a}}+\frac{\sqrt{a\cosh^2(e+fx)}}{a(1-\cosh^2(e+fx))}}{2af}$$

input

```
Int[Coth[e + f*x]^3/(a + a*Sinh[e + f*x]^2)^(3/2),x]
```

output

```
(ArcTanh[Sqrt[a*Cosh[e + f*x]^2]/Sqrt[a]]/Sqrt[a] + Sqrt[a*Cosh[e + f*x]^2
]/(a*(1 - Cosh[e + f*x]^2)))/(2*a*f)
```

Defintions of rubi rules used

rule 8
```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_))^(p_), x_Symbol] :> Simp[1/a^m   Int[u*(a*
x)^(m + p), x], x] /; FreeQ[{a, m, p}, x] && IntegerQ[m]
```

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 52
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Simp[d*((
m + n + 2)/((b*c - a*d)*(m + 1)))   Int[(a + b*x)^(m + 1)*(c + d*x)^n, x],
x] /; FreeQ[{a, b, c, d, n}, x] && ILtQ[m, -1] && FractionQ[n] && LtQ[n, 0]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3655
```
Int[(u_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> Int[A
ctivateTrig[u*(a*cos[e + f*x]^2)^p], x] /; FreeQ[{a, b, e, f, p}, x] && EqQ
[a + b, 0]
```

rule 3684
```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*tan[(e_.) + (f_.)*(x_)]^(m_.
), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x]^2, x]}, Simp[ff^((m + 1
)/2)/(2*f)   Subst[Int[x^((m - 1)/2)*((b*ff^(n/2)*x^(n/2))^p/(1 - ff*x)^((m
 + 1)/2)), x], x, Sin[e + f*x]^2/ff], x]] /; FreeQ[{b, e, f, p}, x] && Inte
gerQ[(m - 1)/2] && IntegerQ[n/2]
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.36 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.55

method	result
default	$\frac{\text{`int/indef0`}\left(\frac{1}{\sinh(fx+e)^3 a\sqrt{a\cosh(fx+e)^2}}, \sinh(fx+e)\right)}{f}$
risch	$-\frac{\left(e^{2fx+2e}+1\right)^2}{a\sqrt{\left(e^{2fx+2e}+1\right)^2 a\, e^{-2fx-2e}}\, f\left(e^{2fx+2e}-1\right)^2} + \frac{\ln\left(e^{fx}+e^{-e}\right)\left(e^{2fx+2e}+1\right)e^{-fx-e}}{2f\sqrt{\left(e^{2fx+2e}+1\right)^2 a\, e^{-2fx-2e}}\, a} - \frac{\ln\left(e^{fx}-e^{-e}\right)\left(e^{2fx+2e}+1\right)e^{-fx-e}}{2f\sqrt{\left(e^{2fx+2e}+1\right)^2 a\, e^{-2fx-2e}}\, a}$

input
```
int(coth(f*x+e)^3/(a+a*sinh(f*x+e)^2)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
`int/indef0`(1/sinh(f*x+e)^3/a/(a*cosh(f*x+e)^2)^(1/2),sinh(f*x+e))/f
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 565 vs. 2(54) = 108.

Time = 0.09 (sec) , antiderivative size = 565, normalized size of antiderivative = 8.56

$$\int \frac{\coth^3(e+fx)}{\left(a+a\sinh^2(e+fx)\right)^{3/2}}\,dx = \text{Too large to display}$$

input
```
integrate(coth(f*x+e)^3/(a+a*sinh(f*x+e)^2)^(3/2),x, algorithm="fricas")
```

output
```
-1/2*(6*cosh(f*x + e)*e^(f*x + e)*sinh(f*x + e)^2 + 2*e^(f*x + e)*sinh(f*x
 + e)^3 + 2*(3*cosh(f*x + e)^2 + 1)*e^(f*x + e)*sinh(f*x + e) + 2*(cosh(f*
x + e)^3 + cosh(f*x + e))*e^(f*x + e) - (4*cosh(f*x + e)*e^(f*x + e)*sinh(
f*x + e)^3 + e^(f*x + e)*sinh(f*x + e)^4 + 2*(3*cosh(f*x + e)^2 - 1)*e^(f*
x + e)*sinh(f*x + e)^2 + 4*(cosh(f*x + e)^3 - cosh(f*x + e))*e^(f*x + e)*s
inh(f*x + e) + (cosh(f*x + e)^4 - 2*cosh(f*x + e)^2 + 1)*e^(f*x + e))*log(
(cosh(f*x + e) + sinh(f*x + e) + 1)/(cosh(f*x + e) + sinh(f*x + e) - 1)))*
sqrt(a*e^(4*f*x + 4*e) + 2*a*e^(2*f*x + 2*e) + a)*e^(-f*x - e)/(a^2*f*cosh
(f*x + e)^4 - 2*a^2*f*cosh(f*x + e)^2 + (a^2*f*e^(2*f*x + 2*e) + a^2*f)*si
nh(f*x + e)^4 + 4*(a^2*f*cosh(f*x + e)*e^(2*f*x + 2*e) + a^2*f*cosh(f*x +
e))*sinh(f*x + e)^3 + a^2*f + 2*(3*a^2*f*cosh(f*x + e)^2 - a^2*f + (3*a^2*
f*cosh(f*x + e)^2 - a^2*f)*e^(2*f*x + 2*e))*sinh(f*x + e)^2 + (a^2*f*cosh(
f*x + e)^4 - 2*a^2*f*cosh(f*x + e)^2 + a^2*f)*e^(2*f*x + 2*e) + 4*(a^2*f*c
osh(f*x + e)^3 - a^2*f*cosh(f*x + e) + (a^2*f*cosh(f*x + e)^3 - a^2*f*cosh
(f*x + e))*e^(2*f*x + 2*e))*sinh(f*x + e))
```

Sympy [F]

$$\int \frac{\coth^3(e+fx)}{\left(a+a\sinh^2(e+fx)\right)^{3/2}}\,dx=\int \frac{\coth^3\left(e+fx\right)}{\left(a\left(\sinh^2\left(e+fx\right)+1\right)\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate(coth(f*x+e)**3/(a+a*sinh(f*x+e)**2)**(3/2),x)
```

output
```
Integral(coth(e + f*x)**3/(a*(sinh(e + f*x)**2 + 1))**(3/2), x)
```

Maxima [A] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 100, normalized size of antiderivative = 1.52

$$\int \frac{\coth^3(e+fx)}{\left(a+a\sinh^2(e+fx)\right)^{3/2}}\,dx=\frac{e^{(-fx-e)}+e^{(-3\,fx-3\,e)}}{\left(2\,a^{\frac{3}{2}}e^{(-2\,fx-2\,e)}-a^{\frac{3}{2}}e^{(-4\,fx-4\,e)}-a^{\frac{3}{2}}\right)f}+\frac{\log\left(e^{(-fx-e)}+1\right)}{2\,a^{\frac{3}{2}}f}-\frac{\log\left(e^{(-fx-e)}-1\right)}{2\,a^{\frac{3}{2}}f}$$

input
```
integrate(coth(f*x+e)^3/(a+a*sinh(f*x+e)^2)^(3/2),x, algorithm="maxima")
```

output
```
(e^(-f*x - e) + e^(-3*f*x - 3*e))/((2*a^(3/2)*e^(-2*f*x - 2*e) - a^(3/2)*e
^(-4*f*x - 4*e) - a^(3/2))*f) + 1/2*log(e^(-f*x - e) + 1)/(a^(3/2)*f) - 1/
2*log(e^(-f*x - e) - 1)/(a^(3/2)*f)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{\coth^3(e+fx)}{(a+a\sinh^2(e+fx))^{3/2}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(coth(f*x+e)^3/(a+a*sinh(f*x+e)^2)^(3/2),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\coth^3(e+fx)}{(a+a\sinh^2(e+fx))^{3/2}}\,dx = \int \frac{\coth(e+f\,x)^3}{\left(a\sinh\left(e+f\,x\right)^2+a\right)^{3/2}}\,dx$$

input
```
int(coth(e + f*x)^3/(a + a*sinh(e + f*x)^2)^(3/2),x)
```

output
```
int(coth(e + f*x)^3/(a + a*sinh(e + f*x)^2)^(3/2), x)
```

Reduce [F]

$$\int \frac{\coth^3(e+fx)}{\left(a+a\sinh^2(e+fx)\right)^{3/2}}\,dx = \frac{\sqrt{a}\left(\int \frac{\sqrt{\sinh(fx+e)^2+1}\,\coth(fx+e)^3}{\sinh(fx+e)^4+2\sinh(fx+e)^2+1}dx\right)}{a^2}$$

input
```
int(coth(f*x+e)^3/(a+a*sinh(f*x+e)^2)^(3/2),x)
```

output
```
(sqrt(a)*int((sqrt(sinh(e + f*x)**2 + 1)*coth(e + f*x)**3)/(sinh(e + f*x)*
*4 + 2*sinh(e + f*x)**2 + 1),x))/a**2
```

3.416 $\int \frac{\tanh^2(e+fx)}{(a+a\sinh^2(e+fx))^{3/2}}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 106

$$\int \frac{\tanh^2(e+fx)}{(a+a\sinh^2(e+fx))^{3/2}}\,dx=\frac{\arctan(\sinh(e+fx))\cosh(e+fx)}{8af\sqrt{a\cosh^2(e+fx)}}+\frac{\tanh(e+fx)}{8af\sqrt{a\cosh^2(e+fx)}}-\frac{\operatorname{sech}^2(e+fx)\tanh(e+fx)}{4af\sqrt{a\cosh^2(e+fx)}}$$

output
```
1/8*arctan(sinh(f*x+e))*cosh(f*x+e)/a/f/(a*cosh(f*x+e)^2)^(1/2)+1/8*tanh(f
*x+e)/a/f/(a*cosh(f*x+e)^2)^(1/2)-1/4*sech(f*x+e)^2*tanh(f*x+e)/a/f/(a*cos
h(f*x+e)^2)^(1/2)
```

Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 58, normalized size of antiderivative = 0.55

$$\int \frac{\tanh^2(e+fx)}{(a+a\sinh^2(e+fx))^{3/2}}\,dx=\frac{\arctan(\sinh(e+fx))\cosh(e+fx)+\left(1-2\operatorname{sech}^2(e+fx)\right)\tanh(e+fx)}{8af\sqrt{a\cosh^2(e+fx)}}$$

input
```
Integrate[Tanh[e + f*x]^2/(a + a*Sinh[e + f*x]^2)^(3/2),x]
```

output
```
(ArcTan[Sinh[e + f*x]]*Cosh[e + f*x] + (1 - 2*Sech[e + f*x]^2)*Tanh[e + f*
x])/(8*a*f*Sqrt[a*Cosh[e + f*x]^2])
```

Rubi [A] (verified)

Time = 0.66 (sec) , antiderivative size = 85, normalized size of antiderivative = 0.80, number of steps used = 15, number of rules used = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.600$, Rules used = {3042, 25, 3655, 25, 3042, 25, 3686, 25, 3042, 25, 3091, 3042, 4255, 3042, 4257}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\tanh^2(e+fx)}{\left(a\sinh^2(e+fx)+a\right)^{3/2}}dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int -\frac{\tan(ie+ifx)^2}{\left(a-a\sin(ie+ifx)^2\right)^{3/2}}dx$$

$$\Big\downarrow \text{ 25}$$

$$-\int \frac{\tan(ie+ifx)^2}{\left(a-a\sin(ie+ifx)^2\right)^{3/2}}dx$$

$$\Big\downarrow \text{ 3655}$$

$$-\int -\frac{\tanh^2(e+fx)}{\left(a\cosh^2(e+fx)\right)^{3/2}}dx$$

$$\Big\downarrow \text{ 25}$$

$$\int \frac{\tanh^2(e+fx)}{\left(a\cosh^2(e+fx)\right)^{3/2}}dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int -\frac{1}{\tan\left(ie+ifx+\frac{\pi}{2}\right)^2\left(a\sin\left(ie+ifx+\frac{\pi}{2}\right)^2\right)^{3/2}}dx$$

$$\Big\downarrow \text{ 25}$$

$$-\int \frac{1}{\left(a\sin\left(\frac{1}{2}(2ie+\pi)+ifx\right)^2\right)^{3/2}\tan\left(\frac{1}{2}(2ie+\pi)+ifx\right)^2}dx$$

$$\Big\downarrow \ 3686$$

$$-\frac{\cosh(e+fx)\int -\text{sech}^3(e+fx)\tanh^2(e+fx)dx}{a\sqrt{a\cosh^2(e+fx)}}$$

$$\Big\downarrow \ 25$$

$$\frac{\cosh(e+fx)\int \text{sech}^3(e+fx)\tanh^2(e+fx)dx}{a\sqrt{a\cosh^2(e+fx)}}$$

$$\Big\downarrow \ 3042$$

$$\frac{\cosh(e+fx)\int -\sec(ie+ifx)^3\tan(ie+ifx)^2dx}{a\sqrt{a\cosh^2(e+fx)}}$$

$$\Big\downarrow \ 25$$

$$-\frac{\cosh(e+fx)\int \sec(ie+ifx)^3\tan(ie+ifx)^2dx}{a\sqrt{a\cosh^2(e+fx)}}$$

$$\Big\downarrow \ 3091$$

$$-\frac{\cosh(e+fx)\left(\frac{\tanh(e+fx)\text{sech}^3(e+fx)}{4f}-\frac{1}{4}\int \text{sech}^3(e+fx)dx\right)}{a\sqrt{a\cosh^2(e+fx)}}$$

$$\Big\downarrow \ 3042$$

$$-\frac{\cosh(e+fx)\left(\frac{\tanh(e+fx)\text{sech}^3(e+fx)}{4f}-\frac{1}{4}\int \csc\left(ie+ifx+\frac{\pi}{2}\right)^3dx\right)}{a\sqrt{a\cosh^2(e+fx)}}$$

$$\Big\downarrow \ 4255$$

$$-\frac{\cosh(e+fx)\left(\frac{1}{4}\left(-\frac{1}{2}\int \text{sech}(e+fx)dx-\frac{\tanh(e+fx)\text{sech}(e+fx)}{2f}\right)+\frac{\tanh(e+fx)\text{sech}^3(e+fx)}{4f}\right)}{a\sqrt{a\cosh^2(e+fx)}}$$

$$\Big\downarrow \ 3042$$

$$-\frac{\cosh(e+fx)\left(\frac{\tanh(e+fx)\text{sech}^3(e+fx)}{4f}+\frac{1}{4}\left(-\frac{\tanh(e+fx)\text{sech}(e+fx)}{2f}-\frac{1}{2}\int\csc\left(ie+ifx+\frac{\pi}{2}\right)dx\right)\right)}{a\sqrt{a\cosh^2(e+fx)}}$$

$$\Big\downarrow \text{4257}$$

$$-\frac{\cosh(e+fx)\left(\frac{1}{4}\left(-\frac{\arctan(\sinh(e+fx))}{2f}-\frac{\tanh(e+fx)\text{sech}(e+fx)}{2f}\right)+\frac{\tanh(e+fx)\text{sech}^3(e+fx)}{4f}\right)}{a\sqrt{a\cosh^2(e+fx)}}$$

input
```
Int[Tanh[e + f*x]^2/(a + a*Sinh[e + f*x]^2)^(3/2),x]
```

output
```
-((Cosh[e + f*x]*((Sech[e + f*x]^3*Tanh[e + f*x])/(4*f) + (-1/2*ArcTan[Sin
h[e + f*x]]/f - (Sech[e + f*x]*Tanh[e + f*x])/(2*f))/4))/(a*Sqrt[a*Cosh[e
+ f*x]^2]))
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3091
```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[b*(a*Sec[e + f*x])^m*((b*Tan[e + f*x])^(n - 1)/(f*(m
 + n - 1))), x] - Simp[b^2*((n - 1)/(m + n - 1))   Int[(a*Sec[e + f*x])^m*(
b*Tan[e + f*x])^(n - 2), x], x] /; FreeQ[{a, b, e, f, m}, x] && GtQ[n, 1] &
& NeQ[m + n - 1, 0] && IntegersQ[2*m, 2*n]
```

rule 3655
```
Int[(u_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> Int[A
ctivateTrig[u*(a*cos[e + f*x]^2)^p], x] /; FreeQ[{a, b, e, f, p}, x] && EqQ
[a + b, 0]
```

rule 3686
```
Int[(u_.)*((b_.)*sin[(e_.) + (f_.)*(x_)]^(n_))^(p_), x_Symbol] :> With[{ff
= FreeFactors[Sin[e + f*x], x]}, Simp[(b*ff^n)^IntPart[p]*((b*Sin[e + f*x]^
n)^FracPart[p]/(Sin[e + f*x]/ff)^(n*FracPart[p]))   Int[ActivateTrig[u]*(Si
n[e + f*x]/ff)^(n*p), x], x]] /; FreeQ[{b, e, f, n, p}, x] &&  !IntegerQ[p]
 && IntegerQ[n] && (EqQ[u, 1] || MatchQ[u, ((d_.)*(trig_)[e + f*x])^(m_.) /
; FreeQ[{d, m}, x] && MemberQ[{sin, cos, tan, cot, sec, csc}, trig]])
```

rule 4255
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Csc[c + d*x])^(n - 1)/(d*(n - 1))), x] + Simp[b^2*((n - 2)/(n - 1))
  Int[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
&& IntegerQ[2*n]
```

rule 4257
```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Maple [A] (verified)

Time = 0.46 (sec) , antiderivative size = 69, normalized size of antiderivative = 0.65

method	result
default	$\frac{\arctan(\sinh(fx+e))\cosh(fx+e)^4+\cosh(fx+e)^2\sinh(fx+e)-2\sinh(fx+e)}{8a\cosh(fx+e)^3\sqrt{a\cosh(fx+e)^2}\,f}$
risch	$\frac{e^{6fx+6e}-7\,e^{4fx+4e}+7\,e^{2fx+2e}-1}{4a\left(e^{2fx+2e}+1\right)^3\sqrt{\left(e^{2fx+2e}+1\right)^2a\,e^{-2fx-2e}}\,f}+\frac{i\ln\left(e^{fx}+ie^{-e}\right)\left(e^{2fx+2e}+1\right)e^{-fx-e}}{8f\sqrt{\left(e^{2fx+2e}+1\right)^2a\,e^{-2fx-2e}}\,a}-\frac{i\ln\left(e^{fx}-ie^{-e}\right)\left(e^{2fx+2e}+1\right)e^{-fx-e}}{8f\sqrt{\left(e^{2fx+2e}+1\right)^2a\,e^{-2fx-2e}}\,a}$

input
```
int(tanh(f*x+e)^2/(a+a*sinh(f*x+e)^2)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
1/8/a*(arctan(sinh(f*x+e))*cosh(f*x+e)^4+cosh(f*x+e)^2*sinh(f*x+e)-2*sinh(
f*x+e))/cosh(f*x+e)^3/(a*cosh(f*x+e)^2)^(1/2)/f
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1423 vs. $2(94) = 188$.

Time = 0.12 (sec) , antiderivative size = 1423, normalized size of antiderivative = 13.42

$$\int \frac{\tanh^2(e+fx)}{(a+a\sinh^2(e+fx))^{3/2}}\,dx = \text{Too large to display}$$

input
```
integrate(tanh(f*x+e)^2/(a+a*sinh(f*x+e)^2)^(3/2),x, algorithm="fricas")
```

output
```
1/4*(7*cosh(f*x + e)*e^(f*x + e)*sinh(f*x + e)^6 + e^(f*x + e)*sinh(f*x +
e)^7 + 7*(3*cosh(f*x + e)^2 - 1)*e^(f*x + e)*sinh(f*x + e)^5 + 35*(cosh(f*
x + e)^3 - cosh(f*x + e))*e^(f*x + e)*sinh(f*x + e)^4 + 7*(5*cosh(f*x + e)
^4 - 10*cosh(f*x + e)^2 + 1)*e^(f*x + e)*sinh(f*x + e)^3 + 7*(3*cosh(f*x +
 e)^5 - 10*cosh(f*x + e)^3 + 3*cosh(f*x + e))*e^(f*x + e)*sinh(f*x + e)^2
+ (7*cosh(f*x + e)^6 - 35*cosh(f*x + e)^4 + 21*cosh(f*x + e)^2 - 1)*e^(f*x
 + e)*sinh(f*x + e) + (8*cosh(f*x + e)*e^(f*x + e)*sinh(f*x + e)^7 + e^(f*
x + e)*sinh(f*x + e)^8 + 4*(7*cosh(f*x + e)^2 + 1)*e^(f*x + e)*sinh(f*x +
e)^6 + 8*(7*cosh(f*x + e)^3 + 3*cosh(f*x + e))*e^(f*x + e)*sinh(f*x + e)^5
 + 2*(35*cosh(f*x + e)^4 + 30*cosh(f*x + e)^2 + 3)*e^(f*x + e)*sinh(f*x +
e)^4 + 8*(7*cosh(f*x + e)^5 + 10*cosh(f*x + e)^3 + 3*cosh(f*x + e))*e^(f*x
 + e)*sinh(f*x + e)^3 + 4*(7*cosh(f*x + e)^6 + 15*cosh(f*x + e)^4 + 9*cosh
(f*x + e)^2 + 1)*e^(f*x + e)*sinh(f*x + e)^2 + 8*(cosh(f*x + e)^7 + 3*cosh
(f*x + e)^5 + 3*cosh(f*x + e)^3 + cosh(f*x + e))*e^(f*x + e)*sinh(f*x + e)
 + (cosh(f*x + e)^8 + 4*cosh(f*x + e)^6 + 6*cosh(f*x + e)^4 + 4*cosh(f*x +
 e)^2 + 1)*e^(f*x + e))*arctan(cosh(f*x + e) + sinh(f*x + e)) + (cosh(f*x
+ e)^7 - 7*cosh(f*x + e)^5 + 7*cosh(f*x + e)^3 - cosh(f*x + e))*e^(f*x + e
))*sqrt(a*e^(4*f*x + 4*e) + 2*a*e^(2*f*x + 2*e) + a)*e^(-f*x - e)/(a^2*f*c
osh(f*x + e)^8 + 4*a^2*f*cosh(f*x + e)^6 + (a^2*f*e^(2*f*x + 2*e) + a^2*f)
*sinh(f*x + e)^8 + 8*(a^2*f*cosh(f*x + e)*e^(2*f*x + 2*e) + a^2*f*cosh(...
```

Sympy [F]

$$\int \frac{\tanh^2(e+fx)}{(a+a\sinh^2(e+fx))^{3/2}}\,dx = \int \frac{\tanh^2(e+fx)}{\left(a\left(\sinh^2(e+fx)+1\right)\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate(tanh(f*x+e)**2/(a+a*sinh(f*x+e)**2)**(3/2),x)
```

output
```
Integral(tanh(e + f*x)**2/(a*(sinh(e + f*x)**2 + 1))**(3/2), x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 369 vs. $2(94) = 188$.

Time = 0.15 (sec) , antiderivative size = 369, normalized size of antiderivative = 3.48

$$\int \frac{\tanh^2(e+fx)}{(a+a\sinh^2(e+fx))^{3/2}}\,dx = -\frac{\frac{3\,e^{(-fx-e)}+11\,e^{(-3\,fx-3\,e)}-11\,e^{(-5\,fx-5\,e)}-3\,e^{(-7\,fx-7\,e)}}{4\,a^{\frac{3}{2}}e^{(-2\,fx-2\,e)}+6\,a^{\frac{3}{2}}e^{(-4\,fx-4\,e)}+4\,a^{\frac{3}{2}}e^{(-6\,fx-6\,e)}+a^{\frac{3}{2}}e^{(-8\,fx-8\,e)}+a^{\frac{3}{2}}} - \frac{3\arctan\left(e^{(-fx-e)}\right)}{a^{\frac{3}{2}}}}{8\,f} + \frac{15\,e^{(-fx-e)}+55\,e^{(-3\,fx-3\,e)}+73\,e^{(-5\,fx-5\,e)}-15\,e^{(-7\,fx-7\,e)}}{48\left(4\,a^{\frac{3}{2}}e^{(-2\,fx-2\,e)}+6\,a^{\frac{3}{2}}e^{(-4\,fx-4\,e)}+4\,a^{\frac{3}{2}}e^{(-6\,fx-6\,e)}+a^{\frac{3}{2}}e^{(-8\,fx-8\,e)}+a^{\frac{3}{2}}\right)f} + \frac{15\,e^{(-fx-e)}-73\,e^{(-3\,fx-3\,e)}-55\,e^{(-5\,fx-5\,e)}-15\,e^{(-7\,fx-7\,e)}}{48\left(4\,a^{\frac{3}{2}}e^{(-2\,fx-2\,e)}+6\,a^{\frac{3}{2}}e^{(-4\,fx-4\,e)}+4\,a^{\frac{3}{2}}e^{(-6\,fx-6\,e)}+a^{\frac{3}{2}}e^{(-8\,fx-8\,e)}+a^{\frac{3}{2}}\right)f} - \frac{5\arctan\left(e^{(-fx-e)}\right)}{8\,a^{\frac{3}{2}}f}$$

input
```
integrate(tanh(f*x+e)^2/(a+a*sinh(f*x+e)^2)^(3/2),x, algorithm="maxima")
```

output
```
-1/8*((3*e^(-f*x - e) + 11*e^(-3*f*x - 3*e) - 11*e^(-5*f*x - 5*e) - 3*e^(-
7*f*x - 7*e))/(4*a^(3/2)*e^(-2*f*x - 2*e) + 6*a^(3/2)*e^(-4*f*x - 4*e) + 4
*a^(3/2)*e^(-6*f*x - 6*e) + a^(3/2)*e^(-8*f*x - 8*e) + a^(3/2)) - 3*arctan
(e^(-f*x - e))/a^(3/2))/f + 1/48*(15*e^(-f*x - e) + 55*e^(-3*f*x - 3*e) +
73*e^(-5*f*x - 5*e) - 15*e^(-7*f*x - 7*e))/((4*a^(3/2)*e^(-2*f*x - 2*e) +
6*a^(3/2)*e^(-4*f*x - 4*e) + 4*a^(3/2)*e^(-6*f*x - 6*e) + a^(3/2)*e^(-8*f*
x - 8*e) + a^(3/2))*f) + 1/48*(15*e^(-f*x - e) - 73*e^(-3*f*x - 3*e) - 55*
e^(-5*f*x - 5*e) - 15*e^(-7*f*x - 7*e))/((4*a^(3/2)*e^(-2*f*x - 2*e) + 6*a
^(3/2)*e^(-4*f*x - 4*e) + 4*a^(3/2)*e^(-6*f*x - 6*e) + a^(3/2)*e^(-8*f*x -
 8*e) + a^(3/2))*f) - 5/8*arctan(e^(-f*x - e))/(a^(3/2)*f)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{\tanh^2(e+fx)}{(a+a\sinh^2(e+fx))^{3/2}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(tanh(f*x+e)^2/(a+a*sinh(f*x+e)^2)^(3/2),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\tanh^2(e+fx)}{(a+a\sinh^2(e+fx))^{3/2}}\,dx = \int \frac{\tanh(e+f\,x)^2}{\left(a\sinh\left(e+f\,x\right)^2+a\right)^{3/2}}\,dx$$

input
```
int(tanh(e + f*x)^2/(a + a*sinh(e + f*x)^2)^(3/2),x)
```

output
```
int(tanh(e + f*x)^2/(a + a*sinh(e + f*x)^2)^(3/2), x)
```

Reduce [F]

$$\int \frac{\tanh^2(e+fx)}{\left(a+a\sinh^2(e+fx)\right)^{3/2}}\,dx = \frac{\sqrt{a}\left(\int \frac{\sqrt{\sinh(fx+e)^2+1}\,\tanh(fx+e)^2}{\sinh(fx+e)^4+2\sinh(fx+e)^2+1}dx\right)}{a^2}$$

input
```
int(tanh(f*x+e)^2/(a+a*sinh(f*x+e)^2)^(3/2),x)
```

output
```
(sqrt(a)*int((sqrt(sinh(e + f*x)**2 + 1)*tanh(e + f*x)**2)/(sinh(e + f*x)*
*4 + 2*sinh(e + f*x)**2 + 1),x))/a**2
```

3.417 $\int \frac{\coth^2(e+fx)}{(a+a\sinh^2(e+fx))^{3/2}}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 64

$$\int \frac{\coth^2(e+fx)}{(a+a\sinh^2(e+fx))^{3/2}}\,dx = -\frac{\arctan(\sinh(e+fx))\cosh(e+fx)}{af\sqrt{a\cosh^2(e+fx)}} - \frac{\coth(e+fx)}{af\sqrt{a\cosh^2(e+fx)}}$$

output
```
-arctan(sinh(f*x+e))*cosh(f*x+e)/a/f/(a*cosh(f*x+e)^2)^(1/2)-coth(f*x+e)/a
/f/(a*cosh(f*x+e)^2)^(1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 0.06 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.72

$$\int \frac{\coth^2(e+fx)}{(a+a\sinh^2(e+fx))^{3/2}}\,dx = -\frac{\coth(e+fx)\,\text{Hypergeometric2F1}\left(-\frac{1}{2},1,\frac{1}{2},-\sinh^2(e+fx)\right)}{af\sqrt{a\cosh^2(e+fx)}}$$

input
```
Integrate[Coth[e + f*x]^2/(a + a*Sinh[e + f*x]^2)^(3/2),x]
```

output
```
-((Coth[e + f*x]*Hypergeometric2F1[-1/2, 1, 1/2, -Sinh[e + f*x]^2])/(a*f*S
qrt[a*Cosh[e + f*x]^2]))
```

Rubi [C] (verified)

Result contains complex when optimal does not.

Time = 0.45 (sec) , antiderivative size = 52, normalized size of antiderivative = 0.81, number of steps used = 15, number of rules used = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.560$, Rules used = {3042, 25, 3655, 25, 3042, 25, 3686, 25, 3042, 25, 3101, 25, 262, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\coth^2(e+fx)}{\left(a\sinh^2(e+fx)+a\right)^{3/2}}dx$$

$$\Big\downarrow \; 3042$$

$$\int -\frac{1}{\tan(ie+ifx)^2\left(a-a\sin(ie+ifx)^2\right)^{3/2}}dx$$

$$\Big\downarrow \; 25$$

$$-\int \frac{1}{\left(a-a\sin(ie+ifx)^2\right)^{3/2}\tan(ie+ifx)^2}dx$$

$$\Big\downarrow \; 3655$$

$$-\int -\frac{\coth^2(e+fx)}{\left(a\cosh^2(e+fx)\right)^{3/2}}dx$$

$$\Big\downarrow \; 25$$

$$\int \frac{\coth^2(e+fx)}{\left(a\cosh^2(e+fx)\right)^{3/2}}dx$$

$$\Big\downarrow \; 3042$$

$$\int -\frac{\tan\left(ie+ifx+\frac{\pi}{2}\right)^2}{\left(a\sin\left(ie+ifx+\frac{\pi}{2}\right)^2\right)^{3/2}}dx$$

$$\Big\downarrow \text{ 25}$$

$$-\int \frac{\tan\left(\frac{1}{2}(2ie+\pi)+ifx\right)^2}{\left(a\sin\left(\frac{1}{2}(2ie+\pi)+ifx\right)^2\right)^{3/2}}dx$$

$$\Big\downarrow \text{ 3686}$$

$$-\frac{\cosh(e+fx)\int -\text{csch}^2(e+fx)\text{sech}(e+fx)dx}{a\sqrt{a\cosh^2(e+fx)}}$$

$$\Big\downarrow \text{ 25}$$

$$\frac{\cosh(e+fx)\int \text{csch}^2(e+fx)\text{sech}(e+fx)dx}{a\sqrt{a\cosh^2(e+fx)}}$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{\cosh(e+fx)\int -\csc(ie+ifx)^2\sec(ie+ifx)dx}{a\sqrt{a\cosh^2(e+fx)}}$$

$$\Big\downarrow \text{ 25}$$

$$-\frac{\cosh(e+fx)\int \csc(ie+ifx)^2\sec(ie+ifx)dx}{a\sqrt{a\cosh^2(e+fx)}}$$

$$\Big\downarrow \text{ 3101}$$

$$-\frac{i\cosh(e+fx)\int \frac{\text{csch}^2(e+fx)}{\text{csch}^2(e+fx)+1}d(-i\text{csch}(e+fx))}{af\sqrt{a\cosh^2(e+fx)}}$$

$$\Big\downarrow \text{ 25}$$

$$\frac{i\cosh(e+fx)\int -\frac{\text{csch}^2(e+fx)}{\text{csch}^2(e+fx)+1}d(-i\text{csch}(e+fx))}{af\sqrt{a\cosh^2(e+fx)}}$$

$$\Big\downarrow \text{ 262}$$

$$-\frac{i\cosh(e+fx)\left(-\int \frac{1}{\text{csch}^2(e+fx)+1}d(-i\text{csch}(e+fx))-i\text{csch}(e+fx)\right)}{af\sqrt{a\cosh^2(e+fx)}}$$

$$\Big\downarrow \text{219}$$

$$-\frac{i\cosh(e+fx)(i\arctan(\operatorname{csch}(e+fx))-i\operatorname{csch}(e+fx))}{af\sqrt{a\cosh^2(e+fx)}}$$

input
```
Int[Coth[e + f*x]^2/(a + a*Sinh[e + f*x]^2)^(3/2),x]
```

output
```
((-I)*Cosh[e + f*x]*(I*ArcTan[Csch[e + f*x]] - I*Csch[e + f*x]))/(a*f*Sqrt
[a*Cosh[e + f*x]^2])
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 262
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[c*(c*x)
^(m - 1)*((a + b*x^2)^(p + 1)/(b*(m + 2*p + 1))), x] - Simp[a*c^2*((m - 1)/
(b*(m + 2*p + 1)))   Int[(c*x)^(m - 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b
, c, p}, x] && GtQ[m, 2 - 1] && NeQ[m + 2*p + 1, 0] && IntBinomialQ[a, b, c
, 2, m, p, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3101
```
Int[(csc[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*sec[(e_.) + (f_.)*(x_)]^(n_.), x_S
ymbol] :> Simp[-(f*a^n)^(-1)   Subst[Int[x^(m + n - 1)/(-1 + x^2/a^2)^((n +
 1)/2), x], x, a*Csc[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n
 + 1)/2] && !(IntegerQ[(m + 1)/2] && LtQ[0, m, n])
```

rule 3655
```
Int[(u_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> Int[A
ctivateTrig[u*(a*cos[e + f*x]^2)^p], x] /; FreeQ[{a, b, e, f, p}, x] && EqQ
[a + b, 0]
```

rule 3686
```
Int[(u_.)*((b_.)*sin[(e_.) + (f_.)*(x_)]^(n_))^(p_), x_Symbol] :> With[{ff
= FreeFactors[Sin[e + f*x], x]}, Simp[(b*ff^n)^IntPart[p]*((b*Sin[e + f*x]^
n)^FracPart[p]/(Sin[e + f*x]/ff)^(n*FracPart[p]))   Int[ActivateTrig[u]*(Si
n[e + f*x]/ff)^(n*p), x], x]] /; FreeQ[{b, e, f, n, p}, x] &&  !IntegerQ[p]
 && IntegerQ[n] && (EqQ[u, 1] || MatchQ[u, ((d_.)*(trig_)[e + f*x])^(m_.) /
; FreeQ[{d, m}, x] && MemberQ[{sin, cos, tan, cot, sec, csc}, trig]])
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 134 vs. $2(60) = 120$.

Time = 0.48 (sec) , antiderivative size = 135, normalized size of antiderivative = 2.11

method	result	size
default	$-\frac{\cosh(fx+e)\sqrt{a\sinh(fx+e)^2}\left(-\ln\left(\frac{2\sqrt{-a}\,\sqrt{a\sinh(fx+e)^2}-2a}{\cosh(fx+e)}\right)a\sinh(fx+e)^2+\sqrt{-a}\,\sqrt{a\sinh(fx+e)^2}\right)}{a^2\sqrt{-a}\,(\cosh(fx+e)-1)(\cosh(fx+e)+1)\sinh(fx+e)\sqrt{a\cosh(fx+e)^2}\,f}$	135
risch	$\frac{i\ln(e^{fx}-ie^{-e})e^{3fx+3e}-i\ln(e^{fx}+ie^{-e})e^{3fx+3e}-i\ln(e^{fx}-ie^{-e})e^{-fx-e}+i\ln(e^{fx}+ie^{-e})e^{-fx-e}-2e^{2fx+2e}-2}{a\sqrt{(e^{2fx+2e}+1)^2a\,e^{-2fx-2e}}\,f\,(e^{2fx+2e}-1)}$	160

input
```
int(coth(f*x+e)^2/(a+a*sinh(f*x+e)^2)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-1/a^2*cosh(f*x+e)*(a*sinh(f*x+e)^2)^(1/2)*(-ln(2/cosh(f*x+e)*((-a)^(1/2)*
(a*sinh(f*x+e)^2)^(1/2)-a))*a*sinh(f*x+e)^2+(-a)^(1/2)*(a*sinh(f*x+e)^2)^(
1/2))/(-a)^(1/2)/(cosh(f*x+e)-1)/(cosh(f*x+e)+1)/sinh(f*x+e)/(a*cosh(f*x+e
)^2)^(1/2)/f
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 254 vs. $2(60) = 120$.

Time = 0.09 (sec) , antiderivative size = 254, normalized size of antiderivative = 3.97

$$\int \frac{\coth^2(e+fx)}{\left(a+a\sinh^2(e+fx)\right)^{3/2}}\,dx = -\frac{2\left(\left(2\cosh\left(fx+e\right)e^{(fx+e)}\sinh\left(fx+e\right)+e^{(fx+e)}\sinh\left(fx+e\right)^2+\left(\cosh\left(fx+e\right)^2-1\right)e^{(fx+e)}\right)\arctan\left(\right.}{a^2f\cosh\left(fx+e\right)^2-a^2f+\left(a^2fe^{(2fx+2e)}+a^2f\right)\sinh\left(fx+e\right)^2+\left(a^2f\cosh\left(f\right.\right.}$$

input
```
integrate(coth(f*x+e)^2/(a+a*sinh(f*x+e)^2)^(3/2),x, algorithm="fricas")
```

output
```
-2*((2*cosh(f*x + e)*e^(f*x + e)*sinh(f*x + e) + e^(f*x + e)*sinh(f*x + e)
^2 + (cosh(f*x + e)^2 - 1)*e^(f*x + e))*arctan(cosh(f*x + e) + sinh(f*x +
e)) + cosh(f*x + e)*e^(f*x + e) + e^(f*x + e)*sinh(f*x + e))*sqrt(a*e^(4*f
*x + 4*e) + 2*a*e^(2*f*x + 2*e) + a)*e^(-f*x - e)/(a^2*f*cosh(f*x + e)^2 -
 a^2*f + (a^2*f*e^(2*f*x + 2*e) + a^2*f)*sinh(f*x + e)^2 + (a^2*f*cosh(f*x
 + e)^2 - a^2*f)*e^(2*f*x + 2*e) + 2*(a^2*f*cosh(f*x + e)*e^(2*f*x + 2*e)
+ a^2*f*cosh(f*x + e))*sinh(f*x + e))
```

Sympy [F]

$$\int \frac{\coth^2(e+fx)}{\left(a+a\sinh^2(e+fx)\right)^{3/2}}\,dx = \int \frac{\coth^2\left(e+fx\right)}{\left(a\left(\sinh^2\left(e+fx\right)+1\right)\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate(coth(f*x+e)**2/(a+a*sinh(f*x+e)**2)**(3/2),x)
```

output
```
Integral(coth(e + f*x)**2/(a*(sinh(e + f*x)**2 + 1))**(3/2), x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 321 vs. $2(60) = 120$.

Time = 0.15 (sec) , antiderivative size = 321, normalized size of antiderivative = 5.02

$$\int \frac{\coth^2(e+fx)}{\left(a+a\sinh^2(e+fx)\right)^{3/2}}\,dx = -\frac{\frac{3\sqrt{a}e^{(-fx-e)}+2\sqrt{a}e^{(-3fx-3e)}+3\sqrt{a}e^{(-5fx-5e)}}{a^2e^{(-2fx-2e)}-a^2e^{(-4fx-4e)}-a^2e^{(-6fx-6e)}+a^2}-\frac{3\arctan\left(e^{(-fx-e)}\right)}{a^{\frac{3}{2}}}}{2f} - \frac{5\sqrt{a}e^{(-fx-e)}+6\sqrt{a}e^{(-3fx-3e)}-3\sqrt{a}e^{(-5fx-5e)}}{4\left(a^2e^{(-2fx-2e)}-a^2e^{(-4fx-4e)}-a^2e^{(-6fx-6e)}+a^2\right)f} + \frac{3\sqrt{a}e^{(-fx-e)}-6\sqrt{a}e^{(-3fx-3e)}-5\sqrt{a}e^{(-5fx-5e)}}{4\left(a^2e^{(-2fx-2e)}-a^2e^{(-4fx-4e)}-a^2e^{(-6fx-6e)}+a^2\right)f} + \frac{\arctan\left(e^{(-fx-e)}\right)}{2a^{\frac{3}{2}}f}$$

input
```
integrate(coth(f*x+e)^2/(a+a*sinh(f*x+e)^2)^(3/2),x, algorithm="maxima")
```

output
```
-1/2*((3*sqrt(a)*e^(-f*x - e) + 2*sqrt(a)*e^(-3*f*x - 3*e) + 3*sqrt(a)*e^(
-5*f*x - 5*e))/(a^2*e^(-2*f*x - 2*e) - a^2*e^(-4*f*x - 4*e) - a^2*e^(-6*f*
x - 6*e) + a^2) - 3*arctan(e^(-f*x - e))/a^(3/2))/f - 1/4*(5*sqrt(a)*e^(-f
*x - e) + 6*sqrt(a)*e^(-3*f*x - 3*e) - 3*sqrt(a)*e^(-5*f*x - 5*e))/((a^2*e
^(-2*f*x - 2*e) - a^2*e^(-4*f*x - 4*e) - a^2*e^(-6*f*x - 6*e) + a^2)*f) +
1/4*(3*sqrt(a)*e^(-f*x - e) - 6*sqrt(a)*e^(-3*f*x - 3*e) - 5*sqrt(a)*e^(-5
*f*x - 5*e))/((a^2*e^(-2*f*x - 2*e) - a^2*e^(-4*f*x - 4*e) - a^2*e^(-6*f*x
 - 6*e) + a^2)*f) + 1/2*arctan(e^(-f*x - e))/(a^(3/2)*f)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{\coth^2(e+fx)}{\left(a+a\sinh^2(e+fx)\right)^{3/2}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(coth(f*x+e)^2/(a+a*sinh(f*x+e)^2)^(3/2),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\coth^2(e+fx)}{(a+a\sinh^2(e+fx))^{3/2}}\,dx = \int \frac{\coth(e+f\,x)^2}{\left(a\sinh\left(e+f\,x\right)^2+a\right)^{3/2}}\,dx$$

input
```
int(coth(e + f*x)^2/(a + a*sinh(e + f*x)^2)^(3/2),x)
```

output
```
int(coth(e + f*x)^2/(a + a*sinh(e + f*x)^2)^(3/2), x)
```

Reduce [F]

$$\int \frac{\coth^2(e+fx)}{(a+a\sinh^2(e+fx))^{3/2}}\,dx = \frac{\sqrt{a}\left(\int \frac{\sqrt{\sinh(fx+e)^2+1}\,\coth(fx+e)^2}{\sinh(fx+e)^4+2\sinh(fx+e)^2+1}dx\right)}{a^2}$$

input
```
int(coth(f*x+e)^2/(a+a*sinh(f*x+e)^2)^(3/2),x)
```

output
```
(sqrt(a)*int((sqrt(sinh(e + f*x)**2 + 1)*coth(e + f*x)**2)/(sinh(e + f*x)*
*4 + 2*sinh(e + f*x)**2 + 1),x))/a**2
```

3.418 $\int \frac{\coth^4(e+fx)}{(a+a\sinh^2(e+fx))^{3/2}}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 38

$$\int \frac{\coth^4(e+fx)}{(a+a\sinh^2(e+fx))^{3/2}}\,dx = -\frac{\coth(e+fx)\text{csch}^2(e+fx)}{3af\sqrt{a\cosh^2(e+fx)}}$$

output
```
-1/3*coth(f*x+e)*csch(f*x+e)^2/a/f/(a*cosh(f*x+e)^2)^(1/2)
```

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.76

$$\int \frac{\coth^4(e+fx)}{(a+a\sinh^2(e+fx))^{3/2}}\,dx = -\frac{\coth^3(e+fx)}{3f\left(a\cosh^2(e+fx)\right)^{3/2}}$$

input
```
Integrate[Coth[e + f*x]^4/(a + a*Sinh[e + f*x]^2)^(3/2),x]
```

output
```
-1/3*Coth[e + f*x]^3/(f*(a*Cosh[e + f*x]^2)^(3/2))
```

Rubi [A] (verified)

Time = 0.43 (sec) , antiderivative size = 38, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.320$, Rules used = {3042, 3655, 3042, 3686, 3042, 25, 3086, 15}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\coth^4(e+fx)}{\left(a\sinh^2(e+fx)+a\right)^{3/2}}dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \frac{1}{\tan(ie+ifx)^4\left(a-a\sin(ie+ifx)^2\right)^{3/2}}dx$$

$$\Big\downarrow \text{ 3655}$$

$$\int \frac{\coth^4(e+fx)}{\left(a\cosh^2(e+fx)\right)^{3/2}}dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \frac{\tan\left(ie+ifx+\frac{\pi}{2}\right)^4}{\left(a\sin\left(ie+ifx+\frac{\pi}{2}\right)^2\right)^{3/2}}dx$$

$$\Big\downarrow \text{ 3686}$$

$$\frac{\cosh(e+fx)\int \coth(e+fx)\text{csch}^3(e+fx)dx}{a\sqrt{a\cosh^2(e+fx)}}$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{\cosh(e+fx)\int -\sec\left(ie+ifx-\frac{\pi}{2}\right)^3\tan\left(ie+ifx-\frac{\pi}{2}\right)dx}{a\sqrt{a\cosh^2(e+fx)}}$$

$$\Big\downarrow \text{ 25}$$

$$-\frac{\cosh(e+fx)\int \sec\left(\frac{1}{2}(2ie-\pi)+ifx\right)^3\tan\left(\frac{1}{2}(2ie-\pi)+ifx\right)dx}{a\sqrt{a\cosh^2(e+fx)}}$$

$$\Big\downarrow \text{ 3086}$$

$$\frac{i\cosh(e+fx)\int -\text{csch}^2(e+fx)d(-i\text{csch}(e+fx))}{af\sqrt{a\cosh^2(e+fx)}}$$

$$\Big\downarrow \; 15$$

$$-\frac{\coth(e+fx)\text{csch}^2(e+fx)}{3af\sqrt{a\cosh^2(e+fx)}}$$

input
```
Int[Coth[e + f*x]^4/(a + a*Sinh[e + f*x]^2)^(3/2),x]
```

output
```
-1/3*(Coth[e + f*x]*Csch[e + f*x]^2)/(a*f*Sqrt[a*Cosh[e + f*x]^2])
```

Defintions of rubi rules used

rule 15
```
Int[(a_.)*(x_)^(m_.), x_Symbol] :> Simp[a*(x^(m + 1)/(m + 1)), x] /; FreeQ[
{a, m}, x] && NeQ[m, -1]
```

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3086
```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Simp[a/f   Subst[Int[(a*x)^(m - 1)*(-1 + x^2)^((n - 1)/2
), x], x, Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2
] &&  !(IntegerQ[m/2] && LtQ[0, m, n + 1])
```

rule 3655
```
Int[(u_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> Int[A
ctivateTrig[u*(a*cos[e + f*x]^2)^p], x] /; FreeQ[{a, b, e, f, p}, x] && EqQ
[a + b, 0]
```

rule 3686
```
Int[(u_.)*((b_.)*sin[(e_.) + (f_.)*(x_)]^(n_))^(p_), x_Symbol] :> With[{ff
= FreeFactors[Sin[e + f*x], x]}, Simp[(b*ff^n)^IntPart[p]*((b*Sin[e + f*x]^
n)^FracPart[p]/(Sin[e + f*x]/ff)^(n*FracPart[p]))   Int[ActivateTrig[u]*(Si
n[e + f*x]/ff)^(n*p), x], x]] /; FreeQ[{b, e, f, n, p}, x] &&  !IntegerQ[p]
 && IntegerQ[n] && (EqQ[u, 1] || MatchQ[u, ((d_.)*(trig_)[e + f*x])^(m_.) /
; FreeQ[{d, m}, x] && MemberQ[{sin, cos, tan, cot, sec, csc}, trig]])
```

Maple [A] (verified)

Time = 0.41 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.92

method	result	size
default	$-\frac{\cosh(fx+e)}{3a\sinh(fx+e)^{3}\sqrt{a\cosh(fx+e)^{2}}\,f}$	35
risch	$-\frac{8(\mathrm{e}^{2fx+2e}+1)\mathrm{e}^{2fx+2e}}{3(\mathrm{e}^{2fx+2e}-1)^{3}f\sqrt{(\mathrm{e}^{2fx+2e}+1)^{2}a\,\mathrm{e}^{-2fx-2e}}\,a}$	68

input
```
int(coth(f*x+e)^4/(a+a*sinh(f*x+e)^2)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-1/3*cosh(f*x+e)/a/sinh(f*x+e)^3/(a*cosh(f*x+e)^2)^(1/2)/f
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 612 vs. 2(34) = 68.

Time = 0.09 (sec) , antiderivative size = 612, normalized size of antiderivative = 16.11

$$\int \frac{\coth^4(e+fx)}{(a+a\sinh^2(e+fx))^{3/2}}\,dx = \text{Too large to display}$$

input
```
integrate(coth(f*x+e)^4/(a+a*sinh(f*x+e)^2)^(3/2),x, algorithm="fricas")
```

output
```
-8/3*(cosh(f*x + e)^3*e^(f*x + e) + 3*cosh(f*x + e)^2*e^(f*x + e)*sinh(f*x
 + e) + 3*cosh(f*x + e)*e^(f*x + e)*sinh(f*x + e)^2 + e^(f*x + e)*sinh(f*x
 + e)^3)*sqrt(a*e^(4*f*x + 4*e) + 2*a*e^(2*f*x + 2*e) + a)*e^(-f*x - e)/(a
^2*f*cosh(f*x + e)^6 - 3*a^2*f*cosh(f*x + e)^4 + (a^2*f*e^(2*f*x + 2*e) +
a^2*f)*sinh(f*x + e)^6 + 6*(a^2*f*cosh(f*x + e)*e^(2*f*x + 2*e) + a^2*f*co
sh(f*x + e))*sinh(f*x + e)^5 + 3*a^2*f*cosh(f*x + e)^2 + 3*(5*a^2*f*cosh(f
*x + e)^2 - a^2*f + (5*a^2*f*cosh(f*x + e)^2 - a^2*f)*e^(2*f*x + 2*e))*sin
h(f*x + e)^4 + 4*(5*a^2*f*cosh(f*x + e)^3 - 3*a^2*f*cosh(f*x + e) + (5*a^2
*f*cosh(f*x + e)^3 - 3*a^2*f*cosh(f*x + e))*e^(2*f*x + 2*e))*sinh(f*x + e)
^3 - a^2*f + 3*(5*a^2*f*cosh(f*x + e)^4 - 6*a^2*f*cosh(f*x + e)^2 + a^2*f
+ (5*a^2*f*cosh(f*x + e)^4 - 6*a^2*f*cosh(f*x + e)^2 + a^2*f)*e^(2*f*x + 2
*e))*sinh(f*x + e)^2 + (a^2*f*cosh(f*x + e)^6 - 3*a^2*f*cosh(f*x + e)^4 +
3*a^2*f*cosh(f*x + e)^2 - a^2*f)*e^(2*f*x + 2*e) + 6*(a^2*f*cosh(f*x + e)^
5 - 2*a^2*f*cosh(f*x + e)^3 + a^2*f*cosh(f*x + e) + (a^2*f*cosh(f*x + e)^5
 - 2*a^2*f*cosh(f*x + e)^3 + a^2*f*cosh(f*x + e))*e^(2*f*x + 2*e))*sinh(f*
x + e))
```

Sympy [F]

$$\int \frac{\coth^4(e+fx)}{(a+a\sinh^2(e+fx))^{3/2}}\,dx = \int \frac{\coth^4{(e+fx)}}{(a(\sinh^2{(e+fx)}+1))^{\frac{3}{2}}}\,dx$$

input
```
integrate(coth(f*x+e)**4/(a+a*sinh(f*x+e)**2)**(3/2),x)
```

output
```
Integral(coth(e + f*x)**4/(a*(sinh(e + f*x)**2 + 1))**(3/2), x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 823 vs. 2(34) = 68.

Time = 0.19 (sec) , antiderivative size = 823, normalized size of antiderivative = 21.66

$$\int \frac{\coth^4(e+fx)}{(a+a\sinh^2(e+fx))^{3/2}}\,dx = \text{Too large to display}$$

input
```
integrate(coth(f*x+e)^4/(a+a*sinh(f*x+e)^2)^(3/2),x, algorithm="maxima")
```

output
```
1/12*((21*e^(-f*x - e) - 16*e^(-3*f*x - 3*e) + 34*e^(-5*f*x - 5*e) + 8*e^(
-7*f*x - 7*e) - 15*e^(-9*f*x - 9*e))/(a^(3/2)*e^(-2*f*x - 2*e) + 2*a^(3/2)
*e^(-4*f*x - 4*e) - 2*a^(3/2)*e^(-6*f*x - 6*e) - a^(3/2)*e^(-8*f*x - 8*e)
+ a^(3/2)*e^(-10*f*x - 10*e) - a^(3/2)) + 3*arctan(e^(-f*x - e))/a^(3/2) +
 9*log(e^(-f*x - e) + 1)/a^(3/2) - 9*log(e^(-f*x - e) - 1)/a^(3/2))/f - 1/
12*((15*e^(-f*x - e) - 8*e^(-3*f*x - 3*e) - 34*e^(-5*f*x - 5*e) + 16*e^(-7
*f*x - 7*e) - 21*e^(-9*f*x - 9*e))/(a^(3/2)*e^(-2*f*x - 2*e) + 2*a^(3/2)*e
^(-4*f*x - 4*e) - 2*a^(3/2)*e^(-6*f*x - 6*e) - a^(3/2)*e^(-8*f*x - 8*e) +
a^(3/2)*e^(-10*f*x - 10*e) - a^(3/2)) - 3*arctan(e^(-f*x - e))/a^(3/2) + 9
*log(e^(-f*x - e) + 1)/a^(3/2) - 9*log(e^(-f*x - e) - 1)/a^(3/2))/f - 1/8*
((15*e^(-f*x - e) - 20*e^(-3*f*x - 3*e) - 22*e^(-5*f*x - 5*e) - 20*e^(-7*f
*x - 7*e) + 15*e^(-9*f*x - 9*e))/(a^(3/2)*e^(-2*f*x - 2*e) + 2*a^(3/2)*e^(
-4*f*x - 4*e) - 2*a^(3/2)*e^(-6*f*x - 6*e) - a^(3/2)*e^(-8*f*x - 8*e) + a^
(3/2)*e^(-10*f*x - 10*e) - a^(3/2)) + 15*arctan(e^(-f*x - e))/a^(3/2))/f +
 1/48*(45*e^(-f*x - e) - 52*e^(-3*f*x - 3*e) - 74*e^(-5*f*x - 5*e) + 92*e^
(-7*f*x - 7*e) + 21*e^(-9*f*x - 9*e))/((a^(3/2)*e^(-2*f*x - 2*e) + 2*a^(3/
2)*e^(-4*f*x - 4*e) - 2*a^(3/2)*e^(-6*f*x - 6*e) - a^(3/2)*e^(-8*f*x - 8*e
) + a^(3/2)*e^(-10*f*x - 10*e) - a^(3/2))*f) + 1/48*(21*e^(-f*x - e) + 92*
e^(-3*f*x - 3*e) - 74*e^(-5*f*x - 5*e) - 52*e^(-7*f*x - 7*e) + 45*e^(-9*f*
x - 9*e))/((a^(3/2)*e^(-2*f*x - 2*e) + 2*a^(3/2)*e^(-4*f*x - 4*e) - 2*a...
```

Giac [F(-2)]

Exception generated.

$$\int \frac{\coth^4(e+fx)}{(a+a\sinh^2(e+fx))^{3/2}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(coth(f*x+e)^4/(a+a*sinh(f*x+e)^2)^(3/2),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [B] (verification not implemented)

Time = 1.88 (sec) , antiderivative size = 71, normalized size of antiderivative = 1.87

$$\int \frac{\coth^4(e+fx)}{\left(a+a\sinh^2(e+fx)\right)^{3/2}}\,dx = -\frac{16\,\mathrm{e}^{4\,e+4\,f\,x}\,\sqrt{a+a\,{\left(\frac{\mathrm{e}^{e+f\,x}}{2}-\frac{\mathrm{e}^{-e-f\,x}}{2}\right)}^2}}{3\,a^2\,f\,{\left(\mathrm{e}^{2\,e+2\,f\,x}-1\right)}^3\,\left(\mathrm{e}^{2\,e+2\,f\,x}+1\right)}$$

input
```
int(coth(e + f*x)^4/(a + a*sinh(e + f*x)^2)^(3/2),x)
```

output
```
-(16*exp(4*e + 4*f*x)*(a + a*(exp(e + f*x)/2 - exp(- e - f*x)/2)^2)^(1/2))
/(3*a^2*f*(exp(2*e + 2*f*x) - 1)^3*(exp(2*e + 2*f*x) + 1))
```

Reduce [F]

$$\int \frac{\coth^4(e+fx)}{\left(a+a\sinh^2(e+fx)\right)^{3/2}}\,dx = \frac{\sqrt{a}\,\left(\int \frac{\sqrt{\sinh(fx+e)^2+1}\,\coth(fx+e)^4}{\sinh(fx+e)^4+2\sinh(fx+e)^2+1}dx\right)}{a^2}$$

input
```
int(coth(f*x+e)^4/(a+a*sinh(f*x+e)^2)^(3/2),x)
```

output
```
(sqrt(a)*int((sqrt(sinh(e + f*x)**2 + 1)*coth(e + f*x)**4)/(sinh(e + f*x)*
*4 + 2*sinh(e + f*x)**2 + 1),x))/a**2
```

3.419 $\int \frac{\coth^6(e+fx)}{\left(a+a\sinh^2(e+fx)\right)^{3/2}}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 77

$$\int \frac{\coth^6(e+fx)}{\left(a+a\sinh^2(e+fx)\right)^{3/2}}\,dx = -\frac{\coth(e+fx)\text{csch}^2(e+fx)}{3af\sqrt{a\cosh^2(e+fx)}}-\frac{\coth(e+fx)\text{csch}^4(e+fx)}{5af\sqrt{a\cosh^2(e+fx)}}$$

output
```
-1/3*coth(f*x+e)*csch(f*x+e)^2/a/f/(a*cosh(f*x+e)^2)^(1/2)-1/5*coth(f*x+e)
*csch(f*x+e)^4/a/f/(a*cosh(f*x+e)^2)^(1/2)
```

Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.53

$$\int \frac{\coth^6(e+fx)}{\left(a+a\sinh^2(e+fx)\right)^{3/2}}\,dx = -\frac{\coth^3(e+fx)\left(5+3\text{csch}^2(e+fx)\right)}{15f\left(a\cosh^2(e+fx)\right)^{3/2}}$$

input
```
Integrate[Coth[e + f*x]^6/(a + a*Sinh[e + f*x]^2)^(3/2),x]
```

output
```
-1/15*(Coth[e + f*x]^3*(5 + 3*Csch[e + f*x]^2))/(f*(a*Cosh[e + f*x]^2)^(3/
2))
```

Rubi [C] (verified)

Result contains complex when optimal does not.

Time = 0.48 (sec) , antiderivative size = 59, normalized size of antiderivative = 0.77, number of steps used = 14, number of rules used = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.520$, Rules used = {3042, 25, 3655, 25, 3042, 25, 3686, 25, 3042, 3086, 25, 244, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\coth^6(e+fx)}{\left(a\sinh^2(e+fx)+a\right)^{3/2}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int -\frac{1}{\tan(ie+ifx)^6\left(a-a\sin(ie+ifx)^2\right)^{3/2}}dx$$

$$\Big\downarrow \text{25}$$

$$-\int \frac{1}{\left(a-a\sin(ie+ifx)^2\right)^{3/2}\tan(ie+ifx)^6}dx$$

$$\Big\downarrow \text{3655}$$

$$-\int -\frac{\coth^6(e+fx)}{\left(a\cosh^2(e+fx)\right)^{3/2}}dx$$

$$\Big\downarrow \text{25}$$

$$\int \frac{\coth^6(e+fx)}{\left(a\cosh^2(e+fx)\right)^{3/2}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int -\frac{\tan\left(ie+ifx+\frac{\pi}{2}\right)^6}{\left(a\sin\left(ie+ifx+\frac{\pi}{2}\right)^2\right)^{3/2}}dx$$

$$\Big\downarrow \text{25}$$

$$-\int \frac{\tan \left(\frac{1}{2}(2ie+\pi)+ifx\right)^6}{\left(a \sin \left(\frac{1}{2}(2ie+\pi)+ifx\right)^2\right)^{3/2}}dx$$

$$\Big\downarrow \; 3686$$

$$-\frac{\cosh(e+fx)\int -\coth^3(e+fx)\text{csch}^3(e+fx)dx}{a\sqrt{a\cosh^2(e+fx)}}$$

$$\Big\downarrow \; 25$$

$$\frac{\cosh(e+fx)\int \coth^3(e+fx)\text{csch}^3(e+fx)dx}{a\sqrt{a\cosh^2(e+fx)}}$$

$$\Big\downarrow \; 3042$$

$$\frac{\cosh(e+fx)\int \sec \left(ie+ifx-\frac{\pi}{2}\right)^3 \tan \left(ie+ifx-\frac{\pi}{2}\right)^3 dx}{a\sqrt{a\cosh^2(e+fx)}}$$

$$\Big\downarrow \; 3086$$

$$-\frac{i\cosh(e+fx)\int \text{csch}^2(e+fx)\left(\text{csch}^2(e+fx)+1\right)d(-i\text{csch}(e+fx))}{af\sqrt{a\cosh^2(e+fx)}}$$

$$\Big\downarrow \; 25$$

$$\frac{i\cosh(e+fx)\int -\text{csch}^2(e+fx)\left(\text{csch}^2(e+fx)+1\right)d(-i\text{csch}(e+fx))}{af\sqrt{a\cosh^2(e+fx)}}$$

$$\Big\downarrow \; 244$$

$$\frac{i\cosh(e+fx)\int \left(-\text{csch}^4(e+fx)-\text{csch}^2(e+fx)\right)d(-i\text{csch}(e+fx))}{af\sqrt{a\cosh^2(e+fx)}}$$

$$\Big\downarrow \; 2009$$

$$-\frac{i\cosh(e+fx)\left(-\frac{1}{5}i\text{csch}^5(e+fx)-\frac{1}{3}i\text{csch}^3(e+fx)\right)}{af\sqrt{a\cosh^2(e+fx)}}$$

input
```
Int[Coth[e + f*x]^6/(a + a*Sinh[e + f*x]^2)^(3/2),x]
```

output
```
((-I)*Cosh[e + f*x]*((-1/3*I)*Csch[e + f*x]^3 - (I/5)*Csch[e + f*x]^5))/(a
*f*Sqrt[a*Cosh[e + f*x]^2])
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 244
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[Expand
Integrand[(c*x)^m*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x] && IGtQ[p
, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3086
```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Simp[a/f   Subst[Int[(a*x)^(m - 1)*(-1 + x^2)^((n - 1)/2
), x], x, Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2
] &&  !(IntegerQ[m/2] && LtQ[0, m, n + 1])
```

rule 3655
```
Int[(u_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> Int[A
ctivateTrig[u*(a*cos[e + f*x]^2)^p], x] /; FreeQ[{a, b, e, f, p}, x] && EqQ
[a + b, 0]
```

rule 3686
```
Int[(u_.)*((b_.)*sin[(e_.) + (f_.)*(x_)]^(n_))^(p_), x_Symbol] :> With[{ff
= FreeFactors[Sin[e + f*x], x]}, Simp[(b*ff^n)^IntPart[p]*((b*Sin[e + f*x]^
n)^FracPart[p]/(Sin[e + f*x]/ff)^(n*FracPart[p]))   Int[ActivateTrig[u]*(Si
n[e + f*x]/ff)^(n*p), x], x]] /; FreeQ[{b, e, f, n, p}, x] &&  !IntegerQ[p]
 && IntegerQ[n] && (EqQ[u, 1] || MatchQ[u, ((d_.)*(trig_)[e + f*x])^(m_.) /
; FreeQ[{d, m}, x] && MemberQ[{sin, cos, tan, cot, sec, csc}, trig]])
```

Maple [A] (verified)

Time = 0.42 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.61

method	result	size
default	$-\frac{\cosh(fx+e)\left(5\sinh(fx+e)^{2}+3\right)}{15a\sinh(fx+e)^{5}\sqrt{a\cosh(fx+e)^{2}}\,f}$	47
risch	$-\frac{8\left(5\,\mathrm{e}^{4fx+4e}+2\,\mathrm{e}^{2fx+2e}+5\right)\left(\mathrm{e}^{2fx+2e}+1\right)\mathrm{e}^{2fx+2e}}{15\left(\mathrm{e}^{2fx+2e}-1\right)^{5}f\sqrt{\left(\mathrm{e}^{2fx+2e}+1\right)^{2}a\,\mathrm{e}^{-2fx-2e}}\,a}$	92

input
```
int(coth(f*x+e)^6/(a+a*sinh(f*x+e)^2)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-1/15*cosh(f*x+e)*(5*sinh(f*x+e)^2+3)/a/sinh(f*x+e)^5/(a*cosh(f*x+e)^2)^(1
/2)/f
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1410 vs. 2(69) = 138.

Time = 0.10 (sec) , antiderivative size = 1410, normalized size of antiderivative = 18.31

$$\int \frac{\coth^{6}(e+fx)}{\left(a+a\sinh^{2}(e+fx)\right)^{3/2}}\,dx = \text{Too large to display}$$

input
```
integrate(coth(f*x+e)^6/(a+a*sinh(f*x+e)^2)^(3/2),x, algorithm="fricas")
```

output
```
-8/15*(35*cosh(f*x + e)*e^(f*x + e)*sinh(f*x + e)^6 + 5*e^(f*x + e)*sinh(f
*x + e)^7 + (105*cosh(f*x + e)^2 + 2)*e^(f*x + e)*sinh(f*x + e)^5 + 5*(35*
cosh(f*x + e)^3 + 2*cosh(f*x + e))*e^(f*x + e)*sinh(f*x + e)^4 + 5*(35*cos
h(f*x + e)^4 + 4*cosh(f*x + e)^2 + 1)*e^(f*x + e)*sinh(f*x + e)^3 + 5*(21*
cosh(f*x + e)^5 + 4*cosh(f*x + e)^3 + 3*cosh(f*x + e))*e^(f*x + e)*sinh(f*
x + e)^2 + 5*(7*cosh(f*x + e)^6 + 2*cosh(f*x + e)^4 + 3*cosh(f*x + e)^2)*e
^(f*x + e)*sinh(f*x + e) + (5*cosh(f*x + e)^7 + 2*cosh(f*x + e)^5 + 5*cosh
(f*x + e)^3)*e^(f*x + e))*sqrt(a*e^(4*f*x + 4*e) + 2*a*e^(2*f*x + 2*e) + a
)*e^(-f*x - e)/(a^2*f*cosh(f*x + e)^10 - 5*a^2*f*cosh(f*x + e)^8 + (a^2*f*
e^(2*f*x + 2*e) + a^2*f)*sinh(f*x + e)^10 + 10*(a^2*f*cosh(f*x + e)*e^(2*f
*x + 2*e) + a^2*f*cosh(f*x + e))*sinh(f*x + e)^9 + 10*a^2*f*cosh(f*x + e)^
6 + 5*(9*a^2*f*cosh(f*x + e)^2 - a^2*f + (9*a^2*f*cosh(f*x + e)^2 - a^2*f)
*e^(2*f*x + 2*e))*sinh(f*x + e)^8 + 40*(3*a^2*f*cosh(f*x + e)^3 - a^2*f*co
sh(f*x + e) + (3*a^2*f*cosh(f*x + e)^3 - a^2*f*cosh(f*x + e))*e^(2*f*x + 2
*e))*sinh(f*x + e)^7 - 10*a^2*f*cosh(f*x + e)^4 + 10*(21*a^2*f*cosh(f*x +
e)^4 - 14*a^2*f*cosh(f*x + e)^2 + a^2*f + (21*a^2*f*cosh(f*x + e)^4 - 14*a
^2*f*cosh(f*x + e)^2 + a^2*f)*e^(2*f*x + 2*e))*sinh(f*x + e)^6 + 4*(63*a^2
*f*cosh(f*x + e)^5 - 70*a^2*f*cosh(f*x + e)^3 + 15*a^2*f*cosh(f*x + e) + (
63*a^2*f*cosh(f*x + e)^5 - 70*a^2*f*cosh(f*x + e)^3 + 15*a^2*f*cosh(f*x +
e))*e^(2*f*x + 2*e))*sinh(f*x + e)^5 + 5*a^2*f*cosh(f*x + e)^2 + 10*(21...
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\coth^6(e+fx)}{\left(a+a\sinh^2(e+fx)\right)^{3/2}}\,dx=\text{Timed out}$$

input
```
integrate(coth(f*x+e)**6/(a+a*sinh(f*x+e)**2)**(3/2),x)
```

output
```
Timed out
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1531 vs. $2(69) = 138$.

Time = 0.27 (sec) , antiderivative size = 1531, normalized size of antiderivative = 19.88

$$\int \frac{\coth^6(e+fx)}{\left(a+a\sinh^2(e+fx)\right)^{3/2}}\,dx = \text{Too large to display}$$

input
```
integrate(coth(f*x+e)^6/(a+a*sinh(f*x+e)^2)^(3/2),x, algorithm="maxima")
```

output
```
-3/256*(2*(105*e^(-f*x - e) - 300*e^(-3*f*x - 3*e) + 81*e^(-5*f*x - 5*e) -
 248*e^(-7*f*x - 7*e) + 51*e^(-9*f*x - 9*e) + 100*e^(-11*f*x - 11*e) - 45*
e^(-13*f*x - 13*e))/(3*a^(3/2)*e^(-2*f*x - 2*e) - a^(3/2)*e^(-4*f*x - 4*e)
 - 5*a^(3/2)*e^(-6*f*x - 6*e) + 5*a^(3/2)*e^(-8*f*x - 8*e) + a^(3/2)*e^(-1
0*f*x - 10*e) - 3*a^(3/2)*e^(-12*f*x - 12*e) + a^(3/2)*e^(-14*f*x - 14*e)
- a^(3/2)) + 60*arctan(e^(-f*x - e))/a^(3/2) + 75*log(e^(-f*x - e) + 1)/a^
(3/2) - 75*log(e^(-f*x - e) - 1)/a^(3/2))/f + 1/48*((105*e^(-f*x - e) - 35
0*e^(-3*f*x - 3*e) + 231*e^(-5*f*x - 5*e) + 412*e^(-7*f*x - 7*e) + 231*e^(
-9*f*x - 9*e) - 350*e^(-11*f*x - 11*e) + 105*e^(-13*f*x - 13*e))/(3*a^(3/2
)*e^(-2*f*x - 2*e) - a^(3/2)*e^(-4*f*x - 4*e) - 5*a^(3/2)*e^(-6*f*x - 6*e)
 + 5*a^(3/2)*e^(-8*f*x - 8*e) + a^(3/2)*e^(-10*f*x - 10*e) - 3*a^(3/2)*e^(
-12*f*x - 12*e) + a^(3/2)*e^(-14*f*x - 14*e) - a^(3/2)) + 105*arctan(e^(-f
*x - e))/a^(3/2))/f + 3/256*(2*(45*e^(-f*x - e) - 100*e^(-3*f*x - 3*e) - 5
1*e^(-5*f*x - 5*e) + 248*e^(-7*f*x - 7*e) - 81*e^(-9*f*x - 9*e) + 300*e^(-
11*f*x - 11*e) - 105*e^(-13*f*x - 13*e))/(3*a^(3/2)*e^(-2*f*x - 2*e) - a^(
3/2)*e^(-4*f*x - 4*e) - 5*a^(3/2)*e^(-6*f*x - 6*e) + 5*a^(3/2)*e^(-8*f*x -
 8*e) + a^(3/2)*e^(-10*f*x - 10*e) - 3*a^(3/2)*e^(-12*f*x - 12*e) + a^(3/2
)*e^(-14*f*x - 14*e) - a^(3/2)) - 60*arctan(e^(-f*x - e))/a^(3/2) + 75*log
(e^(-f*x - e) + 1)/a^(3/2) - 75*log(e^(-f*x - e) - 1)/a^(3/2))/f - 3/320*(
4*(45*e^(-f*x - e) - 135*e^(-3*f*x - 3*e) + 54*e^(-5*f*x - 5*e) + 198*e...
```

Giac [F(-2)]

Exception generated.

$$\int \frac{\coth^6(e+fx)}{(a+a\sinh^2(e+fx))^{3/2}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(coth(f*x+e)^6/(a+a*sinh(f*x+e)^2)^(3/2),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 305, normalized size of antiderivative = 3.96

$$\int \frac{\coth^6(e+fx)}{(a+a\sinh^2(e+fx))^{3/2}}\,dx = -\frac{16\,e^{3\,e+3\,f\,x}\,\sqrt{a+a\,\left(\frac{e^{e+f\,x}}{2}-\frac{e^{-e-f\,x}}{2}\right)^2}}{3\,a^2\,f\,\left(e^{2\,e+2\,f\,x}-1\right)^2\,\left(e^{e+f\,x}+e^{3\,e+3\,f\,x}\right)} - \frac{272\,e^{3\,e+3\,f\,x}\,\sqrt{a+a\,\left(\frac{e^{e+f\,x}}{2}-\frac{e^{-e-f\,x}}{2}\right)^2}}{15\,a^2\,f\,\left(e^{2\,e+2\,f\,x}-1\right)^3\,\left(e^{e+f\,x}+e^{3\,e+3\,f\,x}\right)} - \frac{128\,e^{3\,e+3\,f\,x}\,\sqrt{a+a\,\left(\frac{e^{e+f\,x}}{2}-\frac{e^{-e-f\,x}}{2}\right)^2}}{5\,a^2\,f\,\left(e^{2\,e+2\,f\,x}-1\right)^4\,\left(e^{e+f\,x}+e^{3\,e+3\,f\,x}\right)} - \frac{64\,e^{3\,e+3\,f\,x}\,\sqrt{a+a\,\left(\frac{e^{e+f\,x}}{2}-\frac{e^{-e-f\,x}}{2}\right)^2}}{5\,a^2\,f\,\left(e^{2\,e+2\,f\,x}-1\right)^5\,\left(e^{e+f\,x}+e^{3\,e+3\,f\,x}\right)}$$

input
```
int(coth(e + f*x)^6/(a + a*sinh(e + f*x)^2)^(3/2),x)
```

output
```
- (16*exp(3*e + 3*f*x)*(a + a*(exp(e + f*x)/2 - exp(- e - f*x)/2)^2)^(1/2)
)/(3*a^2*f*(exp(2*e + 2*f*x) - 1)^2*(exp(e + f*x) + exp(3*e + 3*f*x))) - (
272*exp(3*e + 3*f*x)*(a + a*(exp(e + f*x)/2 - exp(- e - f*x)/2)^2)^(1/2))/
(15*a^2*f*(exp(2*e + 2*f*x) - 1)^3*(exp(e + f*x) + exp(3*e + 3*f*x))) - (1
28*exp(3*e + 3*f*x)*(a + a*(exp(e + f*x)/2 - exp(- e - f*x)/2)^2)^(1/2))/(
5*a^2*f*(exp(2*e + 2*f*x) - 1)^4*(exp(e + f*x) + exp(3*e + 3*f*x))) - (64*
exp(3*e + 3*f*x)*(a + a*(exp(e + f*x)/2 - exp(- e - f*x)/2)^2)^(1/2))/(5*a
^2*f*(exp(2*e + 2*f*x) - 1)^5*(exp(e + f*x) + exp(3*e + 3*f*x)))
```

Reduce [F]

$$\int \frac{\coth^6(e+fx)}{\left(a+a\sinh^2(e+fx)\right)^{3/2}}\,dx = \frac{\sqrt{a}\left(\int \frac{\sqrt{\sinh(fx+e)^2+1}\,\coth(fx+e)^6}{\sinh(fx+e)^4+2\sinh(fx+e)^2+1}dx\right)}{a^2}$$

input
```
int(coth(f*x+e)^6/(a+a*sinh(f*x+e)^2)^(3/2),x)
```

output
```
(sqrt(a)*int((sqrt(sinh(e + f*x)**2 + 1)*coth(e + f*x)**6)/(sinh(e + f*x)*
*4 + 2*sinh(e + f*x)**2 + 1),x))/a**2
```

3.420 $\int \frac{\coth^8(e+fx)}{(a+a\sinh^2(e+fx))^{3/2}}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 115

$$\int \frac{\coth^8(e+fx)}{(a+a\sinh^2(e+fx))^{3/2}}\,dx = -\frac{\coth(e+fx)\text{csch}^2(e+fx)}{3af\sqrt{a\cosh^2(e+fx)}} - \frac{2\coth(e+fx)\text{csch}^4(e+fx)}{5af\sqrt{a\cosh^2(e+fx)}} - \frac{\coth(e+fx)\text{csch}^6(e+fx)}{7af\sqrt{a\cosh^2(e+fx)}}$$

output
```
-1/3*coth(f*x+e)*csch(f*x+e)^2/a/f/(a*cosh(f*x+e)^2)^(1/2)-2/5*coth(f*x+e)
*csch(f*x+e)^4/a/f/(a*cosh(f*x+e)^2)^(1/2)-1/7*coth(f*x+e)*csch(f*x+e)^6/a
/f/(a*cosh(f*x+e)^2)^(1/2)
```

Mathematica [A] (verified)

Time = 0.10 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.44

$$\int \frac{\coth^8(e+fx)}{(a+a\sinh^2(e+fx))^{3/2}}\,dx = -\frac{\coth^3(e+fx)\left(35+42\text{csch}^2(e+fx)+15\text{csch}^4(e+fx)\right)}{105f\left(a\cosh^2(e+fx)\right)^{3/2}}$$

input
```
Integrate[Coth[e + f*x]^8/(a + a*Sinh[e + f*x]^2)^(3/2),x]
```

output
```
-1/105*(Coth[e + f*x]^3*(35 + 42*Csch[e + f*x]^2 + 15*Csch[e + f*x]^4))/(f
*(a*Cosh[e + f*x]^2)^(3/2))
```

Rubi [C] (verified)

Result contains complex when optimal does not.

Time = 0.47 (sec) , antiderivative size = 73, normalized size of antiderivative = 0.63, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.360$, Rules used = {3042, 3655, 3042, 3686, 3042, 25, 3086, 244, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\coth^8(e+fx)}{\left(a\sinh^2(e+fx)+a\right)^{3/2}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{\tan(ie+ifx)^8\left(a-a\sin(ie+ifx)^2\right)^{3/2}}dx$$

$$\Big\downarrow \text{3655}$$

$$\int \frac{\coth^8(e+fx)}{\left(a\cosh^2(e+fx)\right)^{3/2}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\tan\left(ie+ifx+\frac{\pi}{2}\right)^8}{\left(a\sin\left(ie+ifx+\frac{\pi}{2}\right)^2\right)^{3/2}}dx$$

$$\Big\downarrow \text{3686}$$

$$\frac{\cosh(e+fx)\int \coth^5(e+fx)\text{csch}^3(e+fx)dx}{a\sqrt{a\cosh^2(e+fx)}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\cosh(e+fx)\int -\sec\left(ie+ifx-\frac{\pi}{2}\right)^3\tan\left(ie+ifx-\frac{\pi}{2}\right)^5dx}{a\sqrt{a\cosh^2(e+fx)}}$$

$$\Big\downarrow \text{ 25}$$

$$-\frac{\cosh(e+fx)\int \sec\left(\frac{1}{2}(2ie-\pi)+ifx\right)^3\tan\left(\frac{1}{2}(2ie-\pi)+ifx\right)^5dx}{a\sqrt{a\cosh^2(e+fx)}}$$

$$\Big\downarrow \text{ 3086}$$

$$\frac{i\cosh(e+fx)\int -\text{csch}^2(e+fx)\left(\text{csch}^2(e+fx)+1\right)^2d(-i\text{csch}(e+fx))}{af\sqrt{a\cosh^2(e+fx)}}$$

$$\Big\downarrow \text{ 244}$$

$$\frac{i\cosh(e+fx)\int \left(-\text{csch}^6(e+fx)-2\text{csch}^4(e+fx)-\text{csch}^2(e+fx)\right)d(-i\text{csch}(e+fx))}{af\sqrt{a\cosh^2(e+fx)}}$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{i\cosh(e+fx)\left(\frac{1}{7}i\text{csch}^7(e+fx)+\frac{2}{5}i\text{csch}^5(e+fx)+\frac{1}{3}i\text{csch}^3(e+fx)\right)}{af\sqrt{a\cosh^2(e+fx)}}$$

input
```
Int[Coth[e + f*x]^8/(a + a*Sinh[e + f*x]^2)^(3/2),x]
```

output
```
(I*Cosh[e + f*x]*((I/3)*Csch[e + f*x]^3 + ((2*I)/5)*Csch[e + f*x]^5 + (I/7
)*Csch[e + f*x]^7))/(a*f*Sqrt[a*Cosh[e + f*x]^2])
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 244
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[Expand
Integrand[(c*x)^m*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x] && IGtQ[p
, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3086
```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Simp[a/f   Subst[Int[(a*x)^(m - 1)*(-1 + x^2)^((n - 1)/2
), x], x, Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2
] &&  !(IntegerQ[m/2] && LtQ[0, m, n + 1])
```

rule 3655
```
Int[(u_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> Int[A
ctivateTrig[u*(a*cos[e + f*x]^2)^p], x] /; FreeQ[{a, b, e, f, p}, x] && EqQ
[a + b, 0]
```

rule 3686
```
Int[(u_.)*((b_.)*sin[(e_.) + (f_.)*(x_)]^(n_))^(p_), x_Symbol] :> With[{ff
= FreeFactors[Sin[e + f*x], x]}, Simp[(b*ff^n)^IntPart[p]*((b*Sin[e + f*x]^
n)^FracPart[p]/(Sin[e + f*x]/ff)^(n*FracPart[p]))   Int[ActivateTrig[u]*(Si
n[e + f*x]/ff)^(n*p), x], x]] /; FreeQ[{b, e, f, n, p}, x] &&  !IntegerQ[p]
 && IntegerQ[n] && (EqQ[u, 1] || MatchQ[u, ((d_.)*(trig_)[e + f*x])^(m_.) /
; FreeQ[{d, m}, x] && MemberQ[{sin, cos, tan, cot, sec, csc}, trig]])
```

Maple [A] (verified)

Time = 0.46 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.50

method	result	size
default	$-\frac{\cosh(fx+e)\left(35\cosh(fx+e)^{4}-28\cosh(fx+e)^{2}+8\right)}{105a\sinh(fx+e)^{7}\sqrt{a\cosh(fx+e)^{2}}\,f}$	57
risch	$-\frac{8\left(35\,e^{8fx+8e}+28\,e^{6fx+6e}+114\,e^{4fx+4e}+28\,e^{2fx+2e}+35\right)\left(e^{2fx+2e}+1\right)e^{2fx+2e}}{105\left(e^{2fx+2e}-1\right)^{7}f\sqrt{\left(e^{2fx+2e}+1\right)^{2}a\,e^{-2fx-2e}}\,a}$	114

input
```
int(coth(f*x+e)^8/(a+a*sinh(f*x+e)^2)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-1/105*cosh(f*x+e)*(35*cosh(f*x+e)^4-28*cosh(f*x+e)^2+8)/a/sinh(f*x+e)^7/(
a*cosh(f*x+e)^2)^(1/2)/f
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2511 vs. $2(103) = 206$.

Time = 0.13 (sec) , antiderivative size = 2511, normalized size of antiderivative = 21.83

$$\int \frac{\coth^8(e+fx)}{\left(a+a\sinh^2(e+fx)\right)^{3/2}}\,dx = \text{Too large to display}$$

input
```
integrate(coth(f*x+e)^8/(a+a*sinh(f*x+e)^2)^(3/2),x, algorithm="fricas")
```

output
```
-8/105*(385*cosh(f*x + e)*e^(f*x + e)*sinh(f*x + e)^10 + 35*e^(f*x + e)*si
nh(f*x + e)^11 + 7*(275*cosh(f*x + e)^2 + 4)*e^(f*x + e)*sinh(f*x + e)^9 +
 21*(275*cosh(f*x + e)^3 + 12*cosh(f*x + e))*e^(f*x + e)*sinh(f*x + e)^8 +
 6*(1925*cosh(f*x + e)^4 + 168*cosh(f*x + e)^2 + 19)*e^(f*x + e)*sinh(f*x
+ e)^7 + 42*(385*cosh(f*x + e)^5 + 56*cosh(f*x + e)^3 + 19*cosh(f*x + e))*
e^(f*x + e)*sinh(f*x + e)^6 + 14*(1155*cosh(f*x + e)^6 + 252*cosh(f*x + e)
^4 + 171*cosh(f*x + e)^2 + 2)*e^(f*x + e)*sinh(f*x + e)^5 + 14*(825*cosh(f
*x + e)^7 + 252*cosh(f*x + e)^5 + 285*cosh(f*x + e)^3 + 10*cosh(f*x + e))*
e^(f*x + e)*sinh(f*x + e)^4 + 7*(825*cosh(f*x + e)^8 + 336*cosh(f*x + e)^6
 + 570*cosh(f*x + e)^4 + 40*cosh(f*x + e)^2 + 5)*e^(f*x + e)*sinh(f*x + e)
^3 + 7*(275*cosh(f*x + e)^9 + 144*cosh(f*x + e)^7 + 342*cosh(f*x + e)^5 +
40*cosh(f*x + e)^3 + 15*cosh(f*x + e))*e^(f*x + e)*sinh(f*x + e)^2 + 7*(55
*cosh(f*x + e)^10 + 36*cosh(f*x + e)^8 + 114*cosh(f*x + e)^6 + 20*cosh(f*x
 + e)^4 + 15*cosh(f*x + e)^2)*e^(f*x + e)*sinh(f*x + e) + (35*cosh(f*x + e
)^11 + 28*cosh(f*x + e)^9 + 114*cosh(f*x + e)^7 + 28*cosh(f*x + e)^5 + 35*
cosh(f*x + e)^3)*e^(f*x + e))*sqrt(a*e^(4*f*x + 4*e) + 2*a*e^(2*f*x + 2*e)
 + a)*e^(-f*x - e)/(a^2*f*cosh(f*x + e)^14 - 7*a^2*f*cosh(f*x + e)^12 + (a
^2*f*e^(2*f*x + 2*e) + a^2*f)*sinh(f*x + e)^14 + 14*(a^2*f*cosh(f*x + e)*e
^(2*f*x + 2*e) + a^2*f*cosh(f*x + e))*sinh(f*x + e)^13 + 21*a^2*f*cosh(f*x
 + e)^10 + 7*(13*a^2*f*cosh(f*x + e)^2 - a^2*f + (13*a^2*f*cosh(f*x + e...
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\coth^8(e+fx)}{\left(a+a\sinh^2(e+fx)\right)^{3/2}}\,dx = \text{Timed out}$$

input
```
integrate(coth(f*x+e)**8/(a+a*sinh(f*x+e)**2)**(3/2),x)
```

output
```
Timed out
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2216 vs. 2(103) = 206.

Time = 0.32 (sec) , antiderivative size = 2216, normalized size of antiderivative = 19.27

$$\int \frac{\coth^8(e+fx)}{\left(a+a\sinh^2(e+fx)\right)^{3/2}}\,dx = \text{Too large to display}$$

input
```
integrate(coth(f*x+e)^8/(a+a*sinh(f*x+e)^2)^(3/2),x, algorithm="maxima")
```

output
```
1/3840*(2*(4095*e^(-f*x - e) - 20090*e^(-3*f*x - 3*e) + 31654*e^(-5*f*x -
5*e) - 850*e^(-7*f*x - 7*e) - 51148*e^(-9*f*x - 9*e) + 51090*e^(-11*f*x -
11*e) - 2646*e^(-13*f*x - 13*e) + 4410*e^(-15*f*x - 15*e) - 1155*e^(-17*f*
x - 17*e))/(5*a^(3/2)*e^(-2*f*x - 2*e) - 8*a^(3/2)*e^(-4*f*x - 4*e) + 14*a
^(3/2)*e^(-8*f*x - 8*e) - 14*a^(3/2)*e^(-10*f*x - 10*e) + 8*a^(3/2)*e^(-14
*f*x - 14*e) - 5*a^(3/2)*e^(-16*f*x - 16*e) + a^(3/2)*e^(-18*f*x - 18*e) -
 a^(3/2)) + 2940*arctan(e^(-f*x - e))/a^(3/2) + 2625*log(e^(-f*x - e) + 1)
/a^(3/2) - 2625*log(e^(-f*x - e) - 1)/a^(3/2))/f - 1/8960*(2*(4095*e^(-f*x
 - e) - 21630*e^(-3*f*x - 3*e) + 39354*e^(-5*f*x - 5*e) - 13830*e^(-7*f*x
- 7*e) - 47848*e^(-9*f*x - 9*e) + 66950*e^(-11*f*x - 11*e) - 22106*e^(-13*
f*x - 13*e) - 18690*e^(-15*f*x - 15*e) + 3465*e^(-17*f*x - 17*e))/(5*a^(3/
2)*e^(-2*f*x - 2*e) - 8*a^(3/2)*e^(-4*f*x - 4*e) + 14*a^(3/2)*e^(-8*f*x -
8*e) - 14*a^(3/2)*e^(-10*f*x - 10*e) + 8*a^(3/2)*e^(-14*f*x - 14*e) - 5*a^
(3/2)*e^(-16*f*x - 16*e) + a^(3/2)*e^(-18*f*x - 18*e) - a^(3/2)) + 7560*ar
ctan(e^(-f*x - e))/a^(3/2) + 315*log(e^(-f*x - e) + 1)/a^(3/2) - 315*log(e
^(-f*x - e) - 1)/a^(3/2))/f - 1/8960*(2*(3465*e^(-f*x - e) - 18690*e^(-3*f
*x - 3*e) - 22106*e^(-5*f*x - 5*e) + 66950*e^(-7*f*x - 7*e) - 47848*e^(-9*
f*x - 9*e) - 13830*e^(-11*f*x - 11*e) + 39354*e^(-13*f*x - 13*e) - 21630*e
^(-15*f*x - 15*e) + 4095*e^(-17*f*x - 17*e))/(5*a^(3/2)*e^(-2*f*x - 2*e) -
 8*a^(3/2)*e^(-4*f*x - 4*e) + 14*a^(3/2)*e^(-8*f*x - 8*e) - 14*a^(3/2)*...
```

Giac [F(-2)]

Exception generated.

$$\int \frac{\coth^8(e+fx)}{(a+a\sinh^2(e+fx))^{3/2}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(coth(f*x+e)^8/(a+a*sinh(f*x+e)^2)^(3/2),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [B] (verification not implemented)

Time = 2.07 (sec) , antiderivative size = 457, normalized size of antiderivative = 3.97

$$\int \frac{\coth^8(e+fx)}{(a+a\sinh^2(e+fx))^{3/2}}\,dx = -\frac{16\,e^{3\,e+3\,f\,x}\,\sqrt{a+a\,\left(\frac{e^{e+f\,x}}{2}-\frac{e^{-e-f\,x}}{2}\right)^2}}{3\,a^2\,f\,\left(e^{2\,e+2\,f\,x}-1\right)^2\,\left(e^{e+f\,x}+e^{3\,e+3\,f\,x}\right)} - \frac{464\,e^{3\,e+3\,f\,x}\,\sqrt{a+a\,\left(\frac{e^{e+f\,x}}{2}-\frac{e^{-e-f\,x}}{2}\right)^2}}{15\,a^2\,f\,\left(e^{2\,e+2\,f\,x}-1\right)^3\,\left(e^{e+f\,x}+e^{3\,e+3\,f\,x}\right)} - \frac{3072\,e^{3\,e+3\,f\,x}\,\sqrt{a+a\,\left(\frac{e^{e+f\,x}}{2}-\frac{e^{-e-f\,x}}{2}\right)^2}}{35\,a^2\,f\,\left(e^{2\,e+2\,f\,x}-1\right)^4\,\left(e^{e+f\,x}+e^{3\,e+3\,f\,x}\right)} - \frac{4736\,e^{3\,e+3\,f\,x}\,\sqrt{a+a\,\left(\frac{e^{e+f\,x}}{2}-\frac{e^{-e-f\,x}}{2}\right)^2}}{35\,a^2\,f\,\left(e^{2\,e+2\,f\,x}-1\right)^5\,\left(e^{e+f\,x}+e^{3\,e+3\,f\,x}\right)} - \frac{768\,e^{3\,e+3\,f\,x}\,\sqrt{a+a\,\left(\frac{e^{e+f\,x}}{2}-\frac{e^{-e-f\,x}}{2}\right)^2}}{7\,a^2\,f\,\left(e^{2\,e+2\,f\,x}-1\right)^6\,\left(e^{e+f\,x}+e^{3\,e+3\,f\,x}\right)} - \frac{256\,e^{3\,e+3\,f\,x}\,\sqrt{a+a\,\left(\frac{e^{e+f\,x}}{2}-\frac{e^{-e-f\,x}}{2}\right)^2}}{7\,a^2\,f\,\left(e^{2\,e+2\,f\,x}-1\right)^7\,\left(e^{e+f\,x}+e^{3\,e+3\,f\,x}\right)}$$

input
```
int(coth(e + f*x)^8/(a + a*sinh(e + f*x)^2)^(3/2),x)
```

output
```
- (16*exp(3*e + 3*f*x)*(a + a*(exp(e + f*x)/2 - exp(- e - f*x)/2)^2)^(1/2)
)/(3*a^2*f*(exp(2*e + 2*f*x) - 1)^2*(exp(e + f*x) + exp(3*e + 3*f*x))) - (
464*exp(3*e + 3*f*x)*(a + a*(exp(e + f*x)/2 - exp(- e - f*x)/2)^2)^(1/2))/
(15*a^2*f*(exp(2*e + 2*f*x) - 1)^3*(exp(e + f*x) + exp(3*e + 3*f*x))) - (3
072*exp(3*e + 3*f*x)*(a + a*(exp(e + f*x)/2 - exp(- e - f*x)/2)^2)^(1/2))/
(35*a^2*f*(exp(2*e + 2*f*x) - 1)^4*(exp(e + f*x) + exp(3*e + 3*f*x))) - (4
736*exp(3*e + 3*f*x)*(a + a*(exp(e + f*x)/2 - exp(- e - f*x)/2)^2)^(1/2))/
(35*a^2*f*(exp(2*e + 2*f*x) - 1)^5*(exp(e + f*x) + exp(3*e + 3*f*x))) - (7
68*exp(3*e + 3*f*x)*(a + a*(exp(e + f*x)/2 - exp(- e - f*x)/2)^2)^(1/2))/(
7*a^2*f*(exp(2*e + 2*f*x) - 1)^6*(exp(e + f*x) + exp(3*e + 3*f*x))) - (256
*exp(3*e + 3*f*x)*(a + a*(exp(e + f*x)/2 - exp(- e - f*x)/2)^2)^(1/2))/(7*
a^2*f*(exp(2*e + 2*f*x) - 1)^7*(exp(e + f*x) + exp(3*e + 3*f*x)))
```

Reduce [F]

$$\int \frac{\coth^8(e+fx)}{\left(a+a\sinh^2(e+fx)\right)^{3/2}}\,dx = \frac{\sqrt{a}\left(\int \frac{\sqrt{\sinh(fx+e)^2+1}\,\coth(fx+e)^8}{\sinh(fx+e)^4+2\sinh(fx+e)^2+1}dx\right)}{a^2}$$

input
```
int(coth(f*x+e)^8/(a+a*sinh(f*x+e)^2)^(3/2),x)
```

output
```
(sqrt(a)*int((sqrt(sinh(e + f*x)**2 + 1)*coth(e + f*x)**8)/(sinh(e + f*x)*
*4 + 2*sinh(e + f*x)**2 + 1),x))/a**2
```

3.421 $\int \sqrt{a+b\sinh^2(e+fx)}\tanh^5(e+fx)\,dx$

Optimal result

Integrand size = 25, antiderivative size = 187

$$\int \sqrt{a+b\sinh^2(e+fx)}\tanh^5(e+fx)\,dx = -\frac{(8a^2-24ab+15b^2)\operatorname{arctanh}\left(\frac{\sqrt{a+b\sinh^2(e+fx)}}{\sqrt{a-b}}\right)}{8(a-b)^{3/2}f} + \frac{(8a^2-24ab+15b^2)\sqrt{a+b\sinh^2(e+fx)}}{8(a-b)^2f} + \frac{(8a-7b)\operatorname{sech}^2(e+fx)\left(a+b\sinh^2(e+fx)\right)^{3/2}}{8(a-b)^2f} - \frac{\operatorname{sech}^4(e+fx)\left(a+b\sinh^2(e+fx)\right)^{3/2}}{4(a-b)f}$$

output
```
-1/8*(8*a^2-24*a*b+15*b^2)*arctanh((a+b*sinh(f*x+e)^2)^(1/2)/(a-b)^(1/2))/
(a-b)^(3/2)/f+1/8*(8*a^2-24*a*b+15*b^2)*(a+b*sinh(f*x+e)^2)^(1/2)/(a-b)^2/
f+1/8*(8*a-7*b)*sech(f*x+e)^2*(a+b*sinh(f*x+e)^2)^(3/2)/(a-b)^2/f-1/4*sech
(f*x+e)^4*(a+b*sinh(f*x+e)^2)^(3/2)/(a-b)/f
```

Mathematica [A] (verified)

Time = 0.40 (sec) , antiderivative size = 151, normalized size of antiderivative = 0.81

$$\int \sqrt{a+b\sinh^2(e+fx)}\tanh^5(e+fx)\,dx =$$

$$-\frac{-\left((8a-7b)\text{sech}^2(e+fx)\left(a+b\sinh^2(e+fx)\right)^{3/2}\right)+2(a-b)\text{sech}^4(e+fx)\left(a+b\sinh^2(e+fx)\right)^{3/}}{8(a-b)^2 f}$$

input
```
Integrate[Sqrt[a + b*Sinh[e + f*x]^2]*Tanh[e + f*x]^5,x]
```

output
```
-1/8*(-((8*a - 7*b)*Sech[e + f*x]^2*(a + b*Sinh[e + f*x]^2)^(3/2)) + 2*(a
- b)*Sech[e + f*x]^4*(a + b*Sinh[e + f*x]^2)^(3/2) + (8*a^2 - 24*a*b + 15*
b^2)*(Sqrt[a - b]*ArcTanh[Sqrt[a + b*Sinh[e + f*x]^2]/Sqrt[a - b]] - Sqrt[
a + b*Sinh[e + f*x]^2]))/((a - b)^2*f)
```

Rubi [A] (verified)

Time = 0.40 (sec) , antiderivative size = 186, normalized size of antiderivative = 0.99, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.360$, Rules used = {3042, 26, 3673, 100, 27, 87, 60, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \tanh^5(e+fx)\sqrt{a+b\sinh^2(e+fx)}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int -i\tan(ie+ifx)^5\sqrt{a-b\sin(ie+ifx)^2}dx$$

$$\Big\downarrow \text{26}$$

$$-i\int \sqrt{a-b\sin(ie+ifx)^2}\tan(ie+ifx)^5dx$$

$$\Big\downarrow \text{3673}$$

$$\frac{\int \frac{\sinh^4(e+fx)\sqrt{b\sinh^2(e+fx)+a}}{\left(\sinh^2(e+fx)+1\right)^3}d\sinh^2(e+fx)}{2f}$$

$$\Big\downarrow \; 100$$

$$\frac{\frac{\int -\frac{\left(-4(a-b)\sinh^2(e+fx)+4a-3b\right)\sqrt{b\sinh^2(e+fx)+a}}{2\left(\sinh^2(e+fx)+1\right)^2}d\sinh^2(e+fx)}{2(a-b)}-\frac{\left(a+b\sinh^2(e+fx)\right)^{3/2}}{2(a-b)\left(\sinh^2(e+fx)+1\right)^2}}{2f}$$

$$\Big\downarrow \; 27$$

$$\frac{-\frac{\int \frac{\left(-4(a-b)\sinh^2(e+fx)+4a-3b\right)\sqrt{b\sinh^2(e+fx)+a}}{\left(\sinh^2(e+fx)+1\right)^2}d\sinh^2(e+fx)}{4(a-b)}-\frac{\left(a+b\sinh^2(e+fx)\right)^{3/2}}{2(a-b)\left(\sinh^2(e+fx)+1\right)^2}}{2f}$$

$$\Big\downarrow \; 87$$

$$\frac{-\frac{-\frac{\left(8a^2-24ab+15b^2\right)\int \frac{\sqrt{b\sinh^2(e+fx)+a}}{\sinh^2(e+fx)+1}d\sinh^2(e+fx)}{2(a-b)}-\frac{(8a-7b)\left(a+b\sinh^2(e+fx)\right)^{3/2}}{(a-b)\left(\sinh^2(e+fx)+1\right)}}{4(a-b)}-\frac{\left(a+b\sinh^2(e+fx)\right)^{3/2}}{2(a-b)\left(\sinh^2(e+fx)+1\right)^2}}{2f}$$

$$\Big\downarrow \; 60$$

$$\frac{-\frac{-\frac{\left(8a^2-24ab+15b^2\right)\left((a-b)\int \frac{1}{\left(\sinh^2(e+fx)+1\right)\sqrt{b\sinh^2(e+fx)+a}}d\sinh^2(e+fx)+2\sqrt{a+b\sinh^2(e+fx)}\right)}{2(a-b)}-\frac{(8a-7b)\left(a+b\sinh^2(e+fx)\right)^{3/2}}{(a-b)\left(\sinh^2(e+fx)+1\right)}}{4(a-b)}-\frac{\left(a+b\sinh^2(e+fx)\right)^{3/2}}{2(a-b)\left(\sinh^2(e+fx)+1\right)^2}}{2f}$$

$$\Big\downarrow \; 73$$

$$\frac{-\frac{-\frac{\left(8a^2-24ab+15b^2\right)\left(\frac{2(a-b)\int \frac{1}{\frac{\sinh^4(e+fx)}{b}-\frac{a}{b}+1}d\sqrt{b\sinh^2(e+fx)+a}}{b}+2\sqrt{a+b\sinh^2(e+fx)}\right)}{2(a-b)}-\frac{(8a-7b)\left(a+b\sinh^2(e+fx)\right)^{3/2}}{(a-b)\left(\sinh^2(e+fx)+1\right)}}{4(a-b)}-\frac{\left(a+b\sinh^2(e+fx)\right)^{3/2}}{2(a-b)\left(\sinh^2(e+fx)+1\right)^2}}{2f}$$

$$\Big\downarrow \; 221$$

$$-\frac{-\frac{-\frac{\left(8a^2-24ab+15b^2\right)\left(2\sqrt{a+b\sinh^2(e+fx)}-2\sqrt{a-b}\operatorname{arctanh}\left(\frac{\sqrt{a+b\sinh^2(e+fx)}}{\sqrt{a-b}}\right)\right)}{2(a-b)}-\frac{(8a-7b)\left(a+b\sinh^2(e+fx)\right)^{3/2}}{(a-b)\left(\sinh^2(e+fx)+1\right)}}{4(a-b)}-\frac{\left(a+b\sinh^2(e+fx)\right)^{3/2}}{2(a-b)\left(\sinh^2(e+fx)+1\right)^2}}{2f}$$

input
```
Int[Sqrt[a + b*Sinh[e + f*x]^2]*Tanh[e + f*x]^5,x]
```

output
```
(-1/2*(a + b*Sinh[e + f*x]^2)^(3/2)/((a - b)*(1 + Sinh[e + f*x]^2)^2) - (-
(((8*a - 7*b)*(a + b*Sinh[e + f*x]^2)^(3/2))/((a - b)*(1 + Sinh[e + f*x]^2
))) - ((8*a^2 - 24*a*b + 15*b^2)*(-2*Sqrt[a - b]*ArcTanh[Sqrt[a + b*Sinh[e
 + f*x]^2]/Sqrt[a - b]] + 2*Sqrt[a + b*Sinh[e + f*x]^2]))/(2*(a - b)))/(4*
(a - b)))/(2*f)
```

Defintions of rubi rules used

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 60
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Simp[n*((b*c - a*d)/(
b*(m + n + 1)))   Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
 c, d}, x] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ[m, 0] && ( !Integer
Q[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n + 2, 0] && IntLinear
Q[a, b, c, d, m, n, x]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 87
```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_] :> Simp[(-(b*e - a*f))*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(f*(p
+ 1)*(c*f - d*e))), x] - Simp[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p
+ 1)))/(f*(p + 1)*(c*f - d*e))   Int[(c + d*x)^n*(e + f*x)^(p + 1), x], x]
/; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || Intege
rQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))))
```

rule 100
```
Int[((a_.) + (b_.)*(x_))^2*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))^(
p_), x_] :> Simp[(b*c - a*d)^2*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(d^2*(d
*e - c*f)*(n + 1))), x] - Simp[1/(d^2*(d*e - c*f)*(n + 1))   Int[(c + d*x)^
(n + 1)*(e + f*x)^p*Simp[a^2*d^2*f*(n + p + 2) + b^2*c*(d*e*(n + 1) + c*f*(
p + 1)) - 2*a*b*d*(d*e*(n + 1) + c*f*(p + 1)) - b^2*d*(d*e - c*f)*(n + 1)*x
, x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && (LtQ[n, -1] || (EqQ[n
 + p + 3, 0] && NeQ[n, -1] && (SumSimplerQ[n, 1] ||  !SumSimplerQ[p, 1])))
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3673
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_.)*tan[(e_.) + (f_.)*(x_)]^
(m_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x]^2, x]}, Simp[ff^((m
 + 1)/2)/(2*f)   Subst[Int[x^((m - 1)/2)*((a + b*ff*x)^p/(1 - ff*x)^((m + 1
)/2)), x], x, Sin[e + f*x]^2/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && Integ
erQ[(m - 1)/2]
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.57 (sec) , antiderivative size = 43, normalized size of antiderivative = 0.23

method	result	size
default	$\frac{\text{`int/indef0`}\left(\frac{\sqrt{a+b\sinh(fx+e)^2}\,\sinh(fx+e)^5}{\cosh(fx+e)^6},\sinh(fx+e)\right)}{f}$	43

input
```
int((a+b*sinh(f*x+e)^2)^(1/2)*tanh(f*x+e)^5,x,method=_RETURNVERBOSE)
```

output
```
`int/indef0`((a+b*sinh(f*x+e)^2)^(1/2)*sinh(f*x+e)^5/cosh(f*x+e)^6,sinh(f*
x+e))/f
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2359 vs. $2(167) = 334$.

Time = 0.64 (sec) , antiderivative size = 4809, normalized size of antiderivative = 25.72

$$\int \sqrt{a+b\sinh^2(e+fx)}\tanh^5(e+fx)\,dx = \text{Too large to display}$$

input
```
integrate((a+b*sinh(f*x+e)^2)^(1/2)*tanh(f*x+e)^5,x, algorithm="fricas")
```

output
```
Too large to include
```

Sympy [F]

$$\int \sqrt{a+b\sinh^2(e+fx)}\tanh^5(e+fx)\,dx = \int \sqrt{a+b\sinh^2{(e+fx)}}\tanh^5{(e+fx)}\,dx$$

input
```
integrate((a+b*sinh(f*x+e)**2)**(1/2)*tanh(f*x+e)**5,x)
```

output
```
Integral(sqrt(a + b*sinh(e + f*x)**2)*tanh(e + f*x)**5, x)
```

Maxima [F]

$$\int \sqrt{a+b\sinh^2(e+fx)}\tanh^5(e+fx)\,dx=\int \sqrt{b\sinh\left(fx+e\right)^2+a}\tanh\left(fx+e\right)^5\,dx$$

input
```
integrate((a+b*sinh(f*x+e)^2)^(1/2)*tanh(f*x+e)^5,x, algorithm="maxima")
```

output
```
integrate(sqrt(b*sinh(f*x + e)^2 + a)*tanh(f*x + e)^5, x)
```

Giac [F]

$$\int \sqrt{a+b\sinh^2(e+fx)}\tanh^5(e+fx)\,dx=\int \sqrt{b\sinh\left(fx+e\right)^2+a}\tanh\left(fx+e\right)^5\,dx$$

input
```
integrate((a+b*sinh(f*x+e)^2)^(1/2)*tanh(f*x+e)^5,x, algorithm="giac")
```

output
```
integrate(sqrt(b*sinh(f*x + e)^2 + a)*tanh(f*x + e)^5, x)
```

Mupad [F(-1)]

Timed out.

$$\int \sqrt{a+b\sinh^2(e+fx)}\tanh^5(e+fx)\,dx=\int \tanh(e+f\,x)^5\,\sqrt{b\sinh\left(e+f\,x\right)^2+a}\,dx$$

input
```
int(tanh(e + f*x)^5*(a + b*sinh(e + f*x)^2)^(1/2),x)
```

output
```
int(tanh(e + f*x)^5*(a + b*sinh(e + f*x)^2)^(1/2), x)
```

Reduce [F]

$$\int \sqrt{a+b\sinh^2(e+fx)}\tanh^5(e+fx)\,dx = \int \sqrt{\sinh(fx+e)^2 b+a}\,\tanh(fx+e)^5\,dx$$

input
```
int((a+b*sinh(f*x+e)^2)^(1/2)*tanh(f*x+e)^5,x)
```

output
```
int(sqrt(sinh(e + f*x)**2*b + a)*tanh(e + f*x)**5,x)
```

3.422 $\int \sqrt{a+b\sinh^2(e+fx)}\tanh^3(e+fx)\,dx$

Optimal result

Integrand size = 25, antiderivative size = 126

$$\int \sqrt{a+b\sinh^2(e+fx)}\tanh^3(e+fx)\,dx = -\frac{(2a-3b)\operatorname{arctanh}\left(\frac{\sqrt{a+b\sinh^2(e+fx)}}{\sqrt{a-b}}\right)}{2\sqrt{a-b}f} + \frac{(2a-3b)\sqrt{a+b\sinh^2(e+fx)}}{2(a-b)f} + \frac{\operatorname{sech}^2(e+fx)\left(a+b\sinh^2(e+fx)\right)^{3/2}}{2(a-b)f}$$

output
```
-1/2*(2*a-3*b)*arctanh((a+b*sinh(f*x+e)^2)^(1/2)/(a-b)^(1/2))/(a-b)^(1/2)/
f+1/2*(2*a-3*b)*(a+b*sinh(f*x+e)^2)^(1/2)/(a-b)/f+1/2*sech(f*x+e)^2*(a+b*s
inh(f*x+e)^2)^(3/2)/(a-b)/f
```

Mathematica [A] (verified)

Time = 0.29 (sec) , antiderivative size = 88, normalized size of antiderivative = 0.70

$$\int \sqrt{a+b\sinh^2(e+fx)}\tanh^3(e+fx)\,dx$$
$$=\frac{-\frac{(2a-3b)\text{arctanh}\left(\frac{\sqrt{a+b\sinh^2(e+fx)}}{\sqrt{a-b}}\right)}{\sqrt{a-b}}+(2+\cosh(2(e+fx)))\text{sech}^2(e+fx)\sqrt{a+b\sinh^2(e+fx)}}{2f}$$

input
```
Integrate[Sqrt[a + b*Sinh[e + f*x]^2]*Tanh[e + f*x]^3,x]
```

output
```
(-(((2*a - 3*b)*ArcTanh[Sqrt[a + b*Sinh[e + f*x]^2]/Sqrt[a - b]])/Sqrt[a -
 b]) + (2 + Cosh[2*(e + f*x)])*Sech[e + f*x]^2*Sqrt[a + b*Sinh[e + f*x]^2]
)/(2*f)
```

Rubi [A] (verified)

Time = 0.31 (sec) , antiderivative size = 119, normalized size of antiderivative = 0.94, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.280, Rules used = {3042, 26, 3673, 87, 60, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \tanh^3(e+fx)\sqrt{a+b\sinh^2(e+fx)}\,dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int i\tan(ie+ifx)^3\sqrt{a-b\sin(ie+ifx)^2}dx$$

$$\Big\downarrow \text{ 26}$$

$$i\int \sqrt{a-b\sin(ie+ifx)^2}\tan(ie+ifx)^3dx$$

$$\Big\downarrow \text{ 3673}$$

$$\frac{\int \frac{\sinh^2(e+fx)\sqrt{b\sinh^2(e+fx)+a}}{(\sinh^2(e+fx)+1)^2}d\sinh^2(e+fx)}{2f}$$

$\Big\downarrow$ 87

$$\frac{\frac{(2a-3b)\int \frac{\sqrt{b\sinh^2(e+fx)+a}}{\sinh^2(e+fx)+1}d\sinh^2(e+fx)}{2(a-b)}+\frac{(a+b\sinh^2(e+fx))^{3/2}}{(a-b)(\sinh^2(e+fx)+1)}}{2f}$$

$\Big\downarrow$ 60

$$\frac{\frac{(2a-3b)\left((a-b)\int \frac{1}{(\sinh^2(e+fx)+1)\sqrt{b\sinh^2(e+fx)+a}}d\sinh^2(e+fx)+2\sqrt{a+b\sinh^2(e+fx)}\right)}{2(a-b)}+\frac{(a+b\sinh^2(e+fx))^{3/2}}{(a-b)(\sinh^2(e+fx)+1)}}{2f}$$

$\Big\downarrow$ 73

$$\frac{\frac{(2a-3b)\left(\frac{2(a-b)\int \frac{1}{\frac{\sinh^4(e+fx)}{b}-\frac{a}{b}+1}d\sqrt{b\sinh^2(e+fx)+a}}{b}+2\sqrt{a+b\sinh^2(e+fx)}\right)}{2(a-b)}+\frac{(a+b\sinh^2(e+fx))^{3/2}}{(a-b)(\sinh^2(e+fx)+1)}}{2f}$$

$\Big\downarrow$ 221

$$\frac{\frac{(2a-3b)\left(2\sqrt{a+b\sinh^2(e+fx)}-2\sqrt{a-b}\,\text{arctanh}\left(\frac{\sqrt{a+b\sinh^2(e+fx)}}{\sqrt{a-b}}\right)\right)}{2(a-b)}+\frac{(a+b\sinh^2(e+fx))^{3/2}}{(a-b)(\sinh^2(e+fx)+1)}}{2f}$$

input

```
Int[Sqrt[a + b*Sinh[e + f*x]^2]*Tanh[e + f*x]^3,x]
```

output

```
((a + b*Sinh[e + f*x]^2)^(3/2)/((a - b)*(1 + Sinh[e + f*x]^2)) + ((2*a - 3
*b)*(-2*Sqrt[a - b]*ArcTanh[Sqrt[a + b*Sinh[e + f*x]^2]/Sqrt[a - b]] + 2*S
qrt[a + b*Sinh[e + f*x]^2]))/(2*(a - b)))/(2*f)
```

Defintions of rubi rules used

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 60
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Simp[n*((b*c - a*d)/(
b*(m + n + 1)))   Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
 c, d}, x] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ[m, 0] && ( !Integer
Q[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n + 2, 0] && IntLinear
Q[a, b, c, d, m, n, x]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 87
```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_] :> Simp[(-(b*e - a*f))*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(f*(p
+ 1)*(c*f - d*e))), x] - Simp[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p
+ 1)))/(f*(p + 1)*(c*f - d*e))   Int[(c + d*x)^n*(e + f*x)^(p + 1), x], x]
/; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || Intege
rQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))))
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3673
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_.)*tan[(e_.) + (f_.)*(x_)]^
(m_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x]^2, x]}, Simp[ff^((m
 + 1)/2)/(2*f)   Subst[Int[x^((m - 1)/2)*((a + b*ff*x)^p/(1 - ff*x)^((m + 1
)/2)), x], x, Sin[e + f*x]^2/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && Integ
erQ[(m - 1)/2]
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.42 (sec) , antiderivative size = 43, normalized size of antiderivative = 0.34

method	result	size
default	$\frac{\text{`int/indef0`}\left(\frac{\sqrt{a+b\sinh(fx+e)^2}\,\sinh(fx+e)^3}{\cosh(fx+e)^4},\sinh(fx+e)\right)}{f}$	43

input
```
int((a+b*sinh(f*x+e)^2)^(1/2)*tanh(f*x+e)^3,x,method=_RETURNVERBOSE)
```

output
```
`int/indef0`((a+b*sinh(f*x+e)^2)^(1/2)*sinh(f*x+e)^3/cosh(f*x+e)^4,sinh(f*
x+e))/f
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 841 vs. 2(110) = 220.

Time = 0.51 (sec) , antiderivative size = 1774, normalized size of antiderivative = 14.08

$$\int \sqrt{a+b\sinh^2(e+fx)}\tanh^3(e+fx)\,dx = \text{Too large to display}$$

input
```
integrate((a+b*sinh(f*x+e)^2)^(1/2)*tanh(f*x+e)^3,x, algorithm="fricas")
```

output
```
[-1/4*(((2*a - 3*b)*cosh(f*x + e)^5 + 5*(2*a - 3*b)*cosh(f*x + e)*sinh(f*x
 + e)^4 + (2*a - 3*b)*sinh(f*x + e)^5 + 2*(2*a - 3*b)*cosh(f*x + e)^3 + 2*
(5*(2*a - 3*b)*cosh(f*x + e)^2 + 2*a - 3*b)*sinh(f*x + e)^3 + 2*(5*(2*a -
3*b)*cosh(f*x + e)^3 + 3*(2*a - 3*b)*cosh(f*x + e))*sinh(f*x + e)^2 + (2*a
 - 3*b)*cosh(f*x + e) + (5*(2*a - 3*b)*cosh(f*x + e)^4 + 6*(2*a - 3*b)*cos
h(f*x + e)^2 + 2*a - 3*b)*sinh(f*x + e))*sqrt(a - b)*log((b*cosh(f*x + e)^
4 + 4*b*cosh(f*x + e)*sinh(f*x + e)^3 + b*sinh(f*x + e)^4 + 2*(4*a - 3*b)*
cosh(f*x + e)^2 + 2*(3*b*cosh(f*x + e)^2 + 4*a - 3*b)*sinh(f*x + e)^2 + 4*
sqrt(2)*sqrt(a - b)*sqrt((b*cosh(f*x + e)^2 + b*sinh(f*x + e)^2 + 2*a - b)
/(cosh(f*x + e)^2 - 2*cosh(f*x + e)*sinh(f*x + e) + sinh(f*x + e)^2))*(cos
h(f*x + e) + sinh(f*x + e)) + 4*(b*cosh(f*x + e)^3 + (4*a - 3*b)*cosh(f*x
+ e))*sinh(f*x + e) + b)/(cosh(f*x + e)^4 + 4*cosh(f*x + e)*sinh(f*x + e)^
3 + sinh(f*x + e)^4 + 2*(3*cosh(f*x + e)^2 + 1)*sinh(f*x + e)^2 + 2*cosh(f
*x + e)^2 + 4*(cosh(f*x + e)^3 + cosh(f*x + e))*sinh(f*x + e) + 1)) - 2*sq
rt(2)*((a - b)*cosh(f*x + e)^4 + 4*(a - b)*cosh(f*x + e)*sinh(f*x + e)^3 +
 (a - b)*sinh(f*x + e)^4 + 4*(a - b)*cosh(f*x + e)^2 + 2*(3*(a - b)*cosh(f
*x + e)^2 + 2*a - 2*b)*sinh(f*x + e)^2 + 4*((a - b)*cosh(f*x + e)^3 + 2*(a
 - b)*cosh(f*x + e))*sinh(f*x + e) + a - b)*sqrt((b*cosh(f*x + e)^2 + b*si
nh(f*x + e)^2 + 2*a - b)/(cosh(f*x + e)^2 - 2*cosh(f*x + e)*sinh(f*x + e)
+ sinh(f*x + e)^2)))/((a - b)*f*cosh(f*x + e)^5 + 5*(a - b)*f*cosh(f*x ...
```

Sympy [F]

$$\int \sqrt{a+b\sinh^2(e+fx)}\tanh^3(e+fx)\,dx=\int \sqrt{a+b\sinh^2{(e+fx)}}\tanh^3{(e+fx)}\,dx$$

input
```
integrate((a+b*sinh(f*x+e)**2)**(1/2)*tanh(f*x+e)**3,x)
```

output
```
Integral(sqrt(a + b*sinh(e + f*x)**2)*tanh(e + f*x)**3, x)
```

Maxima [F]

$$\int \sqrt{a+b\sinh^2(e+fx)}\tanh^3(e+fx)\,dx=\int \sqrt{b\sinh\left(fx+e\right)^2+a}\tanh\left(fx+e\right)^3\,dx$$

input
```
integrate((a+b*sinh(f*x+e)^2)^(1/2)*tanh(f*x+e)^3,x, algorithm="maxima")
```

output
```
integrate(sqrt(b*sinh(f*x + e)^2 + a)*tanh(f*x + e)^3, x)
```

Giac [F]

$$\int \sqrt{a+b\sinh^2(e+fx)}\tanh^3(e+fx)\,dx=\int \sqrt{b\sinh\left(fx+e\right)^2+a}\tanh\left(fx+e\right)^3\,dx$$

input
```
integrate((a+b*sinh(f*x+e)^2)^(1/2)*tanh(f*x+e)^3,x, algorithm="giac")
```

output
```
integrate(sqrt(b*sinh(f*x + e)^2 + a)*tanh(f*x + e)^3, x)
```

Mupad [F(-1)]

Timed out.

$$\int \sqrt{a+b\sinh^2(e+fx)}\tanh^3(e+fx)\,dx=\int \tanh(e+f\,x)^3\,\sqrt{b\sinh\left(e+f\,x\right)^2+a}\,dx$$

input
```
int(tanh(e + f*x)^3*(a + b*sinh(e + f*x)^2)^(1/2),x)
```

output
```
int(tanh(e + f*x)^3*(a + b*sinh(e + f*x)^2)^(1/2), x)
```

Reduce [F]

$$\int \sqrt{a+b\sinh^2(e+fx)}\tanh^3(e+fx)\,dx = \int \sqrt{\sinh(fx+e)^2 b+a}\,\tanh(fx+e)^3\,dx$$

input
```
int((a+b*sinh(f*x+e)^2)^(1/2)*tanh(f*x+e)^3,x)
```

output
```
int(sqrt(sinh(e + f*x)**2*b + a)*tanh(e + f*x)**3,x)
```

3.423 $\int \sqrt{a+b\sinh^2(e+fx)}\tanh(e+fx)\,dx$

Optimal result

Integrand size = 23, antiderivative size = 62

$$\int \sqrt{a+b\sinh^2(e+fx)}\tanh(e+fx)\,dx=-\frac{\sqrt{a-b}\operatorname{arctanh}\left(\frac{\sqrt{a+b\sinh^2(e+fx)}}{\sqrt{a-b}}\right)}{f}+\frac{\sqrt{a+b\sinh^2(e+fx)}}{f}$$

output
```
-(a-b)^(1/2)*arctanh((a+b*sinh(f*x+e)^2)^(1/2)/(a-b)^(1/2))/f+(a+b*sinh(f*
x+e)^2)^(1/2)/f
```

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 65, normalized size of antiderivative = 1.05

$$\int \sqrt{a+b\sinh^2(e+fx)}\tanh(e+fx)\,dx=\frac{-\sqrt{a-b}\operatorname{arctanh}\left(\frac{\sqrt{a-b+b\cosh^2(e+fx)}}{\sqrt{a-b}}\right)+\sqrt{a-b+b\cosh^2(e+fx)}}{f}$$

input
```
Integrate[Sqrt[a + b*Sinh[e + f*x]^2]*Tanh[e + f*x],x]
```

output
```
(-(Sqrt[a - b]*ArcTanh[Sqrt[a - b + b*Cosh[e + f*x]^2]/Sqrt[a - b]]) + Sqr
t[a - b + b*Cosh[e + f*x]^2])/f
```

Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 64, normalized size of antiderivative = 1.03, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.261, Rules used = {3042, 26, 3673, 60, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \tanh(e+fx)\sqrt{a+b\sinh^2(e+fx)}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int -i\tan(ie+ifx)\sqrt{a-b\sin(ie+ifx)^2}dx$$

$$\Big\downarrow \text{26}$$

$$-i\int \sqrt{a-b\sin(ie+ifx)^2}\tan(ie+ifx)dx$$

$$\Big\downarrow \text{3673}$$

$$\frac{\int \frac{\sqrt{b\sinh^2(e+fx)+a}}{\sinh^2(e+fx)+1}d\sinh^2(e+fx)}{2f}$$

$$\Big\downarrow \text{60}$$

$$\frac{(a-b)\int \frac{1}{(\sinh^2(e+fx)+1)\sqrt{b\sinh^2(e+fx)+a}}d\sinh^2(e+fx)+2\sqrt{a+b\sinh^2(e+fx)}}{2f}$$

$$\Big\downarrow \text{73}$$

$$\frac{\frac{2(a-b)\int \frac{1}{\frac{\sinh^4(e+fx)}{b}-\frac{a}{b}+1}d\sqrt{b\sinh^2(e+fx)+a}}{b}+2\sqrt{a+b\sinh^2(e+fx)}}{2f}$$

$$\Big\downarrow \text{221}$$

$$\frac{2\sqrt{a+b\sinh^2(e+fx)}-2\sqrt{a-b}\text{arctanh}\left(\frac{\sqrt{a+b\sinh^2(e+fx)}}{\sqrt{a-b}}\right)}{2f}$$

input
```
Int[Sqrt[a + b*Sinh[e + f*x]^2]*Tanh[e + f*x],x]
```

output
```
(-2*Sqrt[a - b]*ArcTanh[Sqrt[a + b*Sinh[e + f*x]^2]/Sqrt[a - b]] + 2*Sqrt[
a + b*Sinh[e + f*x]^2])/(2*f)
```

Defintions of rubi rules used

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 60
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Simp[n*((b*c - a*d)/(
b*(m + n + 1)))   Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
 c, d}, x] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ[m, 0] && ( !Integer
Q[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n + 2, 0] && IntLinear
Q[a, b, c, d, m, n, x]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3673
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_.)*tan[(e_.) + (f_.)*(x_)]^
(m_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x]^2, x]}, Simp[ff^((m
 + 1)/2)/(2*f)   Subst[Int[x^((m - 1)/2)*((a + b*ff*x)^p/(1 - ff*x)^((m + 1
)/2)), x], x, Sin[e + f*x]^2/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && Integ
erQ[(m - 1)/2]
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.34 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.66

method	result	size
default	$\frac{\text{`int/indef0`}\left(\frac{\sqrt{a+b\sinh(fx+e)^{2}}\,\sinh(fx+e)}{\cosh(fx+e)^{2}},\sinh(fx+e)\right)}{f}$	41

input
```
int((a+b*sinh(f*x+e)^2)^(1/2)*tanh(f*x+e),x,method=_RETURNVERBOSE)
```

output
```
`int/indef0`((a+b*sinh(f*x+e)^2)^(1/2)*sinh(f*x+e)/cosh(f*x+e)^2,sinh(f*x+
e))/f
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 319 vs. 2(54) = 108.

Time = 0.47 (sec) , antiderivative size = 728, normalized size of antiderivative = 11.74

$$\int\sqrt{a+b\sinh^2(e+fx)}\tanh(e+fx)\,dx=\text{Too large to display}$$

input
```
integrate((a+b*sinh(f*x+e)^2)^(1/2)*tanh(f*x+e),x, algorithm="fricas")
```

output
```
[1/2*(sqrt(a - b)*(cosh(f*x + e) + sinh(f*x + e))*log((b*cosh(f*x + e)^4 +
 4*b*cosh(f*x + e)*sinh(f*x + e)^3 + b*sinh(f*x + e)^4 + 2*(4*a - 3*b)*cos
h(f*x + e)^2 + 2*(3*b*cosh(f*x + e)^2 + 4*a - 3*b)*sinh(f*x + e)^2 - 4*sqr
t(2)*sqrt(a - b)*sqrt((b*cosh(f*x + e)^2 + b*sinh(f*x + e)^2 + 2*a - b)/(c
osh(f*x + e)^2 - 2*cosh(f*x + e)*sinh(f*x + e) + sinh(f*x + e)^2))*(cosh(f
*x + e) + sinh(f*x + e)) + 4*(b*cosh(f*x + e)^3 + (4*a - 3*b)*cosh(f*x + e
))*sinh(f*x + e) + b)/(cosh(f*x + e)^4 + 4*cosh(f*x + e)*sinh(f*x + e)^3 +
 sinh(f*x + e)^4 + 2*(3*cosh(f*x + e)^2 + 1)*sinh(f*x + e)^2 + 2*cosh(f*x
+ e)^2 + 4*(cosh(f*x + e)^3 + cosh(f*x + e))*sinh(f*x + e) + 1)) + sqrt(2)
*sqrt((b*cosh(f*x + e)^2 + b*sinh(f*x + e)^2 + 2*a - b)/(cosh(f*x + e)^2 -
 2*cosh(f*x + e)*sinh(f*x + e) + sinh(f*x + e)^2)))/(f*cosh(f*x + e) + f*s
inh(f*x + e)), 1/2*(2*sqrt(-a + b)*(cosh(f*x + e) + sinh(f*x + e))*arctan(
2*sqrt(2)*sqrt(-a + b)*sqrt((b*cosh(f*x + e)^2 + b*sinh(f*x + e)^2 + 2*a -
 b)/(cosh(f*x + e)^2 - 2*cosh(f*x + e)*sinh(f*x + e) + sinh(f*x + e)^2))*(
cosh(f*x + e) + sinh(f*x + e))/(b*cosh(f*x + e)^4 + 4*b*cosh(f*x + e)*sinh
(f*x + e)^3 + b*sinh(f*x + e)^4 + 2*(2*a - b)*cosh(f*x + e)^2 + 2*(3*b*cos
h(f*x + e)^2 + 2*a - b)*sinh(f*x + e)^2 + 4*(b*cosh(f*x + e)^3 + (2*a - b)
*cosh(f*x + e))*sinh(f*x + e) + b)) + sqrt(2)*sqrt((b*cosh(f*x + e)^2 + b*
sinh(f*x + e)^2 + 2*a - b)/(cosh(f*x + e)^2 - 2*cosh(f*x + e)*sinh(f*x + e
) + sinh(f*x + e)^2)))/(f*cosh(f*x + e) + f*sinh(f*x + e))]
```

Sympy [F]

$$\int \sqrt{a + b \sinh^2(e + fx)} \tanh(e + fx) \, dx = \int \sqrt{a + b \sinh^2{(e + fx)}} \tanh{(e + fx)} \, dx$$

input
```
integrate((a+b*sinh(f*x+e)**2)**(1/2)*tanh(f*x+e),x)
```

output
```
Integral(sqrt(a + b*sinh(e + f*x)**2)*tanh(e + f*x), x)
```

Maxima [F]

$$\int \sqrt{a+b\sinh^2(e+fx)}\tanh(e+fx)\,dx=\int \sqrt{b\sinh\left(fx+e\right)^2+a}\tanh\left(fx+e\right)\,dx$$

input
```
integrate((a+b*sinh(f*x+e)^2)^(1/2)*tanh(f*x+e),x, algorithm="maxima")
```

output
```
integrate(sqrt(b*sinh(f*x + e)^2 + a)*tanh(f*x + e), x)
```

Giac [F]

$$\int \sqrt{a+b\sinh^2(e+fx)}\tanh(e+fx)\,dx=\int \sqrt{b\sinh\left(fx+e\right)^2+a}\tanh\left(fx+e\right)\,dx$$

input
```
integrate((a+b*sinh(f*x+e)^2)^(1/2)*tanh(f*x+e),x, algorithm="giac")
```

output
```
integrate(sqrt(b*sinh(f*x + e)^2 + a)*tanh(f*x + e), x)
```

Mupad [F(-1)]

Timed out.

$$\int \sqrt{a+b\sinh^2(e+fx)}\tanh(e+fx)\,dx=\int \tanh(e+f\,x)\,\sqrt{b\,{\sinh\left(e+f\,x\right)}^2+a}\,dx$$

input
```
int(tanh(e + f*x)*(a + b*sinh(e + f*x)^2)^(1/2),x)
```

output
```
int(tanh(e + f*x)*(a + b*sinh(e + f*x)^2)^(1/2), x)
```

Reduce [F]

$$\int \sqrt{a+b\sinh^2(e+fx)}\tanh(e+fx)\,dx=\int \sqrt{\sinh\left(fx+e\right)^2b+a}\,\tanh\left(fx+e\right)dx$$

input
```
int((a+b*sinh(f*x+e)^2)^(1/2)*tanh(f*x+e),x)
```

output
```
int(sqrt(sinh(e + f*x)**2*b + a)*tanh(e + f*x),x)
```

3.424 $\int \coth(e+fx)\sqrt{a+b\sinh^2(e+fx)}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 54

$$\int \coth(e+fx)\sqrt{a+b\sinh^2(e+fx)}\,dx=-\frac{\sqrt{a}\operatorname{arctanh}\left(\frac{\sqrt{a+b\sinh^2(e+fx)}}{\sqrt{a}}\right)}{f}+\frac{\sqrt{a+b\sinh^2(e+fx)}}{f}$$

output
```
-a^(1/2)*arctanh((a+b*sinh(f*x+e)^2)^(1/2)/a^(1/2))/f+(a+b*sinh(f*x+e)^2)^
(1/2)/f
```

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.98

$$\int \coth(e+fx)\sqrt{a+b\sinh^2(e+fx)}\,dx=-\frac{\sqrt{a}\operatorname{arctanh}\left(\frac{\sqrt{a+b\sinh^2(e+fx)}}{\sqrt{a}}\right)-\sqrt{a+b\sinh^2(e+fx)}}{f}$$

input
```
Integrate[Coth[e + f*x]*Sqrt[a + b*Sinh[e + f*x]^2],x]
```

output
```
-((Sqrt[a]*ArcTanh[Sqrt[a + b*Sinh[e + f*x]^2]/Sqrt[a]] - Sqrt[a + b*Sinh[
e + f*x]^2])/f)
```

Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 56, normalized size of antiderivative = 1.04, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.261, Rules used = {3042, 26, 3673, 60, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \coth(e+fx)\sqrt{a+b\sinh^2(e+fx)}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{i\sqrt{a-b\sin(ie+ifx)^2}}{\tan(ie+ifx)}dx$$

$$\Big\downarrow \text{26}$$

$$i\int \frac{\sqrt{a-b\sin(ie+ifx)^2}}{\tan(ie+ifx)}dx$$

$$\Big\downarrow \text{3673}$$

$$\frac{\int \text{csch}^2(e+fx)\sqrt{b\sinh^2(e+fx)+a}d\sinh^2(e+fx)}{2f}$$

$$\Big\downarrow \text{60}$$

$$\frac{a\int \frac{\text{csch}^2(e+fx)}{\sqrt{b\sinh^2(e+fx)+a}}d\sinh^2(e+fx)+2\sqrt{a+b\sinh^2(e+fx)}}{2f}$$

$$\Big\downarrow \text{73}$$

$$\frac{\frac{2a\int \frac{1}{\frac{\sinh^4(e+fx)}{b}-\frac{a}{b}}d\sqrt{b\sinh^2(e+fx)+a}}{b}+2\sqrt{a+b\sinh^2(e+fx)}}{2f}$$

$$\Big\downarrow \text{221}$$

$$\frac{2\sqrt{a+b\sinh^2(e+fx)}-2\sqrt{a}\text{arctanh}\left(\frac{\sqrt{a+b\sinh^2(e+fx)}}{\sqrt{a}}\right)}{2f}$$

input
```
Int[Coth[e + f*x]*Sqrt[a + b*Sinh[e + f*x]^2],x]
```

output
```
(-2*Sqrt[a]*ArcTanh[Sqrt[a + b*Sinh[e + f*x]^2]/Sqrt[a]] + 2*Sqrt[a + b*Si
nh[e + f*x]^2])/(2*f)
```

Defintions of rubi rules used

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 60
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Simp[n*((b*c - a*d)/(
b*(m + n + 1)))   Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
 c, d}, x] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ[m, 0] && ( !Integer
Q[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n + 2, 0] && IntLinear
Q[a, b, c, d, m, n, x]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3673
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_.)*tan[(e_.) + (f_.)*(x_)]^
(m_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x]^2, x]}, Simp[ff^((m
 + 1)/2)/(2*f)   Subst[Int[x^((m - 1)/2)*((a + b*ff*x)^p/(1 - ff*x)^((m + 1
)/2)), x], x, Sin[e + f*x]^2/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && Integ
erQ[(m - 1)/2]
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.33 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.85

method	result	size
default	$\frac{\text{`int/indef0`}\left(\frac{\sinh(fx+e)b+\frac{a}{\sinh(fx+e)}}{\sqrt{a+b\sinh(fx+e)^2}},\sinh(fx+e)\right)}{f}$	46

input
```
int(coth(f*x+e)*(a+b*sinh(f*x+e)^2)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
`int/indef0`((sinh(f*x+e)*b+a/sinh(f*x+e))/(a+b*sinh(f*x+e)^2)^(1/2),sinh(
f*x+e))/f
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 315 vs. 2(46) = 92.

Time = 0.29 (sec) , antiderivative size = 718, normalized size of antiderivative = 13.30

$$\int \coth(e+fx)\sqrt{a+b\sinh^2(e+fx)}\,dx = \text{Too large to display}$$

input
```
integrate(coth(f*x+e)*(a+b*sinh(f*x+e)^2)^(1/2),x, algorithm="fricas")
```

output
```
[1/2*(sqrt(a)*(cosh(f*x + e) + sinh(f*x + e))*log((b*cosh(f*x + e)^4 + 4*b
*cosh(f*x + e)*sinh(f*x + e)^3 + b*sinh(f*x + e)^4 + 2*(4*a - b)*cosh(f*x
+ e)^2 + 2*(3*b*cosh(f*x + e)^2 + 4*a - b)*sinh(f*x + e)^2 - 4*sqrt(2)*sqr
t(a)*sqrt((b*cosh(f*x + e)^2 + b*sinh(f*x + e)^2 + 2*a - b)/(cosh(f*x + e)
^2 - 2*cosh(f*x + e)*sinh(f*x + e) + sinh(f*x + e)^2))*(cosh(f*x + e) + si
nh(f*x + e)) + 4*(b*cosh(f*x + e)^3 + (4*a - b)*cosh(f*x + e))*sinh(f*x +
e) + b)/(cosh(f*x + e)^4 + 4*cosh(f*x + e)*sinh(f*x + e)^3 + sinh(f*x + e)
^4 + 2*(3*cosh(f*x + e)^2 - 1)*sinh(f*x + e)^2 - 2*cosh(f*x + e)^2 + 4*(co
sh(f*x + e)^3 - cosh(f*x + e))*sinh(f*x + e) + 1)) + sqrt(2)*sqrt((b*cosh(
f*x + e)^2 + b*sinh(f*x + e)^2 + 2*a - b)/(cosh(f*x + e)^2 - 2*cosh(f*x +
e)*sinh(f*x + e) + sinh(f*x + e)^2)))/(f*cosh(f*x + e) + f*sinh(f*x + e)),
 1/2*(2*sqrt(-a)*(cosh(f*x + e) + sinh(f*x + e))*arctan(2*sqrt(2)*sqrt(-a)
*sqrt((b*cosh(f*x + e)^2 + b*sinh(f*x + e)^2 + 2*a - b)/(cosh(f*x + e)^2 -
 2*cosh(f*x + e)*sinh(f*x + e) + sinh(f*x + e)^2))*(cosh(f*x + e) + sinh(f
*x + e))/(b*cosh(f*x + e)^4 + 4*b*cosh(f*x + e)*sinh(f*x + e)^3 + b*sinh(f
*x + e)^4 + 2*(2*a - b)*cosh(f*x + e)^2 + 2*(3*b*cosh(f*x + e)^2 + 2*a - b
)*sinh(f*x + e)^2 + 4*(b*cosh(f*x + e)^3 + (2*a - b)*cosh(f*x + e))*sinh(f
*x + e) + b)) + sqrt(2)*sqrt((b*cosh(f*x + e)^2 + b*sinh(f*x + e)^2 + 2*a
- b)/(cosh(f*x + e)^2 - 2*cosh(f*x + e)*sinh(f*x + e) + sinh(f*x + e)^2)))
/(f*cosh(f*x + e) + f*sinh(f*x + e))]
```

Sympy [F]

$$\int \coth(e+fx)\sqrt{a+b\sinh^2(e+fx)}\,dx=\int \sqrt{a+b\sinh^2{(e+fx)}}\coth{(e+fx)}\,dx$$

input
```
integrate(coth(f*x+e)*(a+b*sinh(f*x+e)**2)**(1/2),x)
```

output
```
Integral(sqrt(a + b*sinh(e + f*x)**2)*coth(e + f*x), x)
```

Maxima [F]

$$\int \coth(e+fx)\sqrt{a+b\sinh^2(e+fx)}\,dx = \int \sqrt{b\sinh\left(fx+e\right)^2+a}\coth\left(fx+e\right)\,dx$$

input
```
integrate(coth(f*x+e)*(a+b*sinh(f*x+e)^2)^(1/2),x, algorithm="maxima")
```

output
```
integrate(sqrt(b*sinh(f*x + e)^2 + a)*coth(f*x + e), x)
```

Giac [F]

$$\int \coth(e+fx)\sqrt{a+b\sinh^2(e+fx)}\,dx = \int \sqrt{b\sinh\left(fx+e\right)^2+a}\coth\left(fx+e\right)\,dx$$

input
```
integrate(coth(f*x+e)*(a+b*sinh(f*x+e)^2)^(1/2),x, algorithm="giac")
```

output
```
integrate(sqrt(b*sinh(f*x + e)^2 + a)*coth(f*x + e), x)
```

Mupad [F(-1)]

Timed out.

$$\int \coth(e+fx)\sqrt{a+b\sinh^2(e+fx)}\,dx = \int \coth(e+f\,x)\,\sqrt{b\sinh\left(e+f\,x\right)^2+a}\,dx$$

input
```
int(coth(e + f*x)*(a + b*sinh(e + f*x)^2)^(1/2),x)
```

output
```
int(coth(e + f*x)*(a + b*sinh(e + f*x)^2)^(1/2), x)
```

Reduce [F]

$$\int \coth(e+fx)\sqrt{a+b\sinh^2(e+fx)}\,dx = \int \sqrt{\sinh(fx+e)^2 b+a}\,\coth(fx+e)\,dx$$

input
```
int(coth(f*x+e)*(a+b*sinh(f*x+e)^2)^(1/2),x)
```

output
```
int(sqrt(sinh(e + f*x)**2*b + a)*coth(e + f*x),x)
```

3.425 $\int \coth^3(e+fx)\sqrt{a+b\sinh^2(e+fx)}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 106

$$\int \coth^3(e+fx)\sqrt{a+b\sinh^2(e+fx)}\,dx=-\frac{(2a+b)\operatorname{arctanh}\left(\frac{\sqrt{a+b\sinh^2(e+fx)}}{\sqrt{a}}\right)}{2\sqrt{a}f}+\frac{(2a+b)\sqrt{a+b\sinh^2(e+fx)}}{2af}-\frac{\operatorname{csch}^2(e+fx)\left(a+b\sinh^2(e+fx)\right)^{3/2}}{2af}$$

output

```
-1/2*(2*a+b)*arctanh((a+b*sinh(f*x+e)^2)^(1/2)/a^(1/2))/a^(1/2)/f+1/2*(2*a
+b)*(a+b*sinh(f*x+e)^2)^(1/2)/a/f-1/2*csch(f*x+e)^2*(a+b*sinh(f*x+e)^2)^(3
/2)/a/f
```

Mathematica [A] (verified)

Time = 0.25 (sec) , antiderivative size = 69, normalized size of antiderivative = 0.65

$$\int \coth^3(e+fx)\sqrt{a+b\sinh^2(e+fx)}\,dx = -\frac{\frac{(2a+b)\text{arctanh}\left(\frac{\sqrt{a+b\sinh^2(e+fx)}}{\sqrt{a}}\right)}{\sqrt{a}}+\left(-2+\text{csch}^2(e+fx)\right)\sqrt{a+b\sinh^2(e+fx)}}{2f}$$

input
```
Integrate[Coth[e + f*x]^3*Sqrt[a + b*Sinh[e + f*x]^2],x]
```

output
```
-1/2*(((2*a + b)*ArcTanh[Sqrt[a + b*Sinh[e + f*x]^2]/Sqrt[a]])/Sqrt[a] + (
-2 + Csch[e + f*x]^2)*Sqrt[a + b*Sinh[e + f*x]^2])/f
```

Rubi [A] (verified)

Time = 0.33 (sec) , antiderivative size = 98, normalized size of antiderivative = 0.92, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.280$, Rules used = {3042, 26, 3673, 87, 60, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \coth^3(e+fx)\sqrt{a+b\sinh^2(e+fx)}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int -\frac{i\sqrt{a-b\sin(ie+ifx)^2}}{\tan(ie+ifx)^3}dx$$

$$\Big\downarrow \text{26}$$

$$-i\int \frac{\sqrt{a-b\sin(ie+ifx)^2}}{\tan(ie+ifx)^3}dx$$

$$\Big\downarrow \text{3673}$$

$$\frac{\int \operatorname{csch}^4(e+fx)\left(\sinh^2(e+fx)+1\right)\sqrt{b\sinh^2(e+fx)+a}d\sinh^2(e+fx)}{2f}$$

$$\Big\downarrow \; 87$$

$$\frac{\frac{(2a+b)\int \operatorname{csch}^2(e+fx)\sqrt{b\sinh^2(e+fx)+a}d\sinh^2(e+fx)}{2a}-\frac{\operatorname{csch}^2(e+fx)\left(a+b\sinh^2(e+fx)\right)^{3/2}}{a}}{2f}$$

$$\Big\downarrow \; 60$$

$$\frac{\frac{(2a+b)\left(a\int \frac{\operatorname{csch}^2(e+fx)}{\sqrt{b\sinh^2(e+fx)+a}}d\sinh^2(e+fx)+2\sqrt{a+b\sinh^2(e+fx)}\right)}{2a}-\frac{\operatorname{csch}^2(e+fx)\left(a+b\sinh^2(e+fx)\right)^{3/2}}{a}}{2f}$$

$$\Big\downarrow \; 73$$

$$\frac{\frac{(2a+b)\left(\frac{2a\int \frac{1}{\frac{\sinh^4(e+fx)}{b}-\frac{a}{b}}d\sqrt{b\sinh^2(e+fx)+a}}{b}+2\sqrt{a+b\sinh^2(e+fx)}\right)}{2a}-\frac{\operatorname{csch}^2(e+fx)\left(a+b\sinh^2(e+fx)\right)^{3/2}}{a}}{2f}$$

$$\Big\downarrow \; 221$$

$$\frac{\frac{(2a+b)\left(2\sqrt{a+b\sinh^2(e+fx)}-2\sqrt{a}\operatorname{arctanh}\left(\frac{\sqrt{a+b\sinh^2(e+fx)}}{\sqrt{a}}\right)\right)}{2a}-\frac{\operatorname{csch}^2(e+fx)\left(a+b\sinh^2(e+fx)\right)^{3/2}}{a}}{2f}$$

input
```
Int[Coth[e + f*x]^3*Sqrt[a + b*Sinh[e + f*x]^2],x]
```

output
```
(-((Csch[e + f*x]^2*(a + b*Sinh[e + f*x]^2)^(3/2))/a) + ((2*a + b)*(-2*Sqr
t[a]*ArcTanh[Sqrt[a + b*Sinh[e + f*x]^2]/Sqrt[a]] + 2*Sqrt[a + b*Sinh[e +
f*x]^2]))/(2*a))/(2*f)
```

Defintions of rubi rules used

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 60
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Simp[n*((b*c - a*d)/(
b*(m + n + 1)))   Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
 c, d}, x] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ[m, 0] && ( !Integer
Q[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n + 2, 0] && IntLinear
Q[a, b, c, d, m, n, x]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 87
```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_] :> Simp[(-(b*e - a*f))*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(f*(p
+ 1)*(c*f - d*e))), x] - Simp[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p
+ 1)))/(f*(p + 1)*(c*f - d*e))   Int[(c + d*x)^n*(e + f*x)^(p + 1), x], x]
/; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || Intege
rQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))))
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3673
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_.)*tan[(e_.) + (f_.)*(x_)]^
(m_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x]^2, x]}, Simp[ff^((m
 + 1)/2)/(2*f)   Subst[Int[x^((m - 1)/2)*((a + b*ff*x)^p/(1 - ff*x)^((m + 1
)/2)), x], x, Sin[e + f*x]^2/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && Integ
erQ[(m - 1)/2]
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.44 (sec) , antiderivative size = 58, normalized size of antiderivative = 0.55

method	result	size
default	$\frac{\text{`int/indef0`}\left(\frac{\sinh(fx+e)b+\frac{a+b}{\sinh(fx+e)}+\frac{a}{\sinh(fx+e)^3}}{\sqrt{a+b\sinh(fx+e)^2}},\sinh(fx+e)\right)}{f}$	58

input
```
int(coth(f*x+e)^3*(a+b*sinh(f*x+e)^2)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
`int/indef0`((sinh(f*x+e)*b+(a+b)/sinh(f*x+e)+a/sinh(f*x+e)^3)/(a+b*sinh(f
*x+e)^2)^(1/2),sinh(f*x+e))/f
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 734 vs. $2(90) = 180$.

Time = 0.35 (sec) , antiderivative size = 1558, normalized size of antiderivative = 14.70

$$\int \coth^3(e+fx)\sqrt{a+b\sinh^2(e+fx)}\,dx = \text{Too large to display}$$

input
```
integrate(coth(f*x+e)^3*(a+b*sinh(f*x+e)^2)^(1/2),x, algorithm="fricas")
```

output
```
[1/4*(((2*a + b)*cosh(f*x + e)^5 + 5*(2*a + b)*cosh(f*x + e)*sinh(f*x + e)
^4 + (2*a + b)*sinh(f*x + e)^5 - 2*(2*a + b)*cosh(f*x + e)^3 + 2*(5*(2*a +
 b)*cosh(f*x + e)^2 - 2*a - b)*sinh(f*x + e)^3 + 2*(5*(2*a + b)*cosh(f*x +
 e)^3 - 3*(2*a + b)*cosh(f*x + e))*sinh(f*x + e)^2 + (2*a + b)*cosh(f*x +
e) + (5*(2*a + b)*cosh(f*x + e)^4 - 6*(2*a + b)*cosh(f*x + e)^2 + 2*a + b)
*sinh(f*x + e))*sqrt(a)*log((b*cosh(f*x + e)^4 + 4*b*cosh(f*x + e)*sinh(f*
x + e)^3 + b*sinh(f*x + e)^4 + 2*(4*a - b)*cosh(f*x + e)^2 + 2*(3*b*cosh(f
*x + e)^2 + 4*a - b)*sinh(f*x + e)^2 - 4*sqrt(2)*sqrt(a)*sqrt((b*cosh(f*x
+ e)^2 + b*sinh(f*x + e)^2 + 2*a - b)/(cosh(f*x + e)^2 - 2*cosh(f*x + e)*s
inh(f*x + e) + sinh(f*x + e)^2))*(cosh(f*x + e) + sinh(f*x + e)) + 4*(b*co
sh(f*x + e)^3 + (4*a - b)*cosh(f*x + e))*sinh(f*x + e) + b)/(cosh(f*x + e)
^4 + 4*cosh(f*x + e)*sinh(f*x + e)^3 + sinh(f*x + e)^4 + 2*(3*cosh(f*x + e
)^2 - 1)*sinh(f*x + e)^2 - 2*cosh(f*x + e)^2 + 4*(cosh(f*x + e)^3 - cosh(f
*x + e))*sinh(f*x + e) + 1)) + 2*sqrt(2)*(a*cosh(f*x + e)^4 + 4*a*cosh(f*x
 + e)*sinh(f*x + e)^3 + a*sinh(f*x + e)^4 - 4*a*cosh(f*x + e)^2 + 2*(3*a*c
osh(f*x + e)^2 - 2*a)*sinh(f*x + e)^2 + 4*(a*cosh(f*x + e)^3 - 2*a*cosh(f*
x + e))*sinh(f*x + e) + a)*sqrt((b*cosh(f*x + e)^2 + b*sinh(f*x + e)^2 + 2
*a - b)/(cosh(f*x + e)^2 - 2*cosh(f*x + e)*sinh(f*x + e) + sinh(f*x + e)^2
)))/(a*f*cosh(f*x + e)^5 + 5*a*f*cosh(f*x + e)*sinh(f*x + e)^4 + a*f*sinh(
f*x + e)^5 - 2*a*f*cosh(f*x + e)^3 + 2*(5*a*f*cosh(f*x + e)^2 - a*f)*si...
```

Sympy [F]

$$\int \coth^3(e+fx)\sqrt{a+b\sinh^2(e+fx)}\,dx=\int \sqrt{a+b\sinh^2{(e+fx)}}\coth^3{(e+fx)}\,dx$$

input
```
integrate(coth(f*x+e)**3*(a+b*sinh(f*x+e)**2)**(1/2),x)
```

output
```
Integral(sqrt(a + b*sinh(e + f*x)**2)*coth(e + f*x)**3, x)
```

Maxima [F]

$$\int \coth^3(e+fx)\sqrt{a+b\sinh^2(e+fx)}\,dx = \int \sqrt{b\sinh{(fx+e)}^2+a}\coth{(fx+e)}^3\,dx$$

input
```
integrate(coth(f*x+e)^3*(a+b*sinh(f*x+e)^2)^(1/2),x, algorithm="maxima")
```

output
```
integrate(sqrt(b*sinh(f*x + e)^2 + a)*coth(f*x + e)^3, x)
```

Giac [F]

$$\int \coth^3(e+fx)\sqrt{a+b\sinh^2(e+fx)}\,dx = \int \sqrt{b\sinh{(fx+e)}^2+a}\coth{(fx+e)}^3\,dx$$

input
```
integrate(coth(f*x+e)^3*(a+b*sinh(f*x+e)^2)^(1/2),x, algorithm="giac")
```

output
```
integrate(sqrt(b*sinh(f*x + e)^2 + a)*coth(f*x + e)^3, x)
```

Mupad [F(-1)]

Timed out.

$$\int \coth^3(e+fx)\sqrt{a+b\sinh^2(e+fx)}\,dx = \int \coth(e+f\,x)^3\,\sqrt{b\sinh{(e+f\,x)}^2+a}\,dx$$

input
```
int(coth(e + f*x)^3*(a + b*sinh(e + f*x)^2)^(1/2),x)
```

output
```
int(coth(e + f*x)^3*(a + b*sinh(e + f*x)^2)^(1/2), x)
```

Reduce [F]

$$\int \coth^3(e+fx)\sqrt{a+b\sinh^2(e+fx)}\,dx=\int \sqrt{\sinh\left(fx+e\right)^2b+a}\,\coth\left(fx+e\right)^3dx$$

input
```
int(coth(f*x+e)^3*(a+b*sinh(f*x+e)^2)^(1/2),x)
```

output
```
int(sqrt(sinh(e + f*x)**2*b + a)*coth(e + f*x)**3,x)
```

3.426 $\int \coth^5(e+fx)\sqrt{a+b\sinh^2(e+fx)}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 167

$$\int \coth^5(e+fx)\sqrt{a+b\sinh^2(e+fx)}\,dx = -\frac{(8a^2+8ab-b^2)\,\text{arctanh}\left(\frac{\sqrt{a+b\sinh^2(e+fx)}}{\sqrt{a}}\right)}{8a^{3/2}f} + \frac{(8a^2+8ab-b^2)\sqrt{a+b\sinh^2(e+fx)}}{8a^2f} - \frac{(8a-b)\text{csch}^2(e+fx)\left(a+b\sinh^2(e+fx)\right)^{3/2}}{8a^2f} - \frac{\text{csch}^4(e+fx)\left(a+b\sinh^2(e+fx)\right)^{3/2}}{4af}$$

output
```
-1/8*(8*a^2+8*a*b-b^2)*arctanh((a+b*sinh(f*x+e)^2)^(1/2)/a^(1/2))/a^(3/2)/
f+1/8*(8*a^2+8*a*b-b^2)*(a+b*sinh(f*x+e)^2)^(1/2)/a^2/f-1/8*(8*a-b)*csch(f
*x+e)^2*(a+b*sinh(f*x+e)^2)^(3/2)/a^2/f-1/4*csch(f*x+e)^4*(a+b*sinh(f*x+e)
^2)^(3/2)/a/f
```

Mathematica [A] (verified)

Time = 0.39 (sec) , antiderivative size = 102, normalized size of antiderivative = 0.61

$$\int \coth^5(e+fx)\sqrt{a+b\sinh^2(e+fx)}\,dx$$
$$=\frac{(-8a^2-8ab+b^2)\,\text{arctanh}\left(\frac{\sqrt{a+b\sinh^2(e+fx)}}{\sqrt{a}}\right)-\sqrt{a}(-8a+(8a+b)\text{csch}^2(e+fx)+2a\text{csch}^4(e+fx))\sqrt{}}{8a^{3/2}f}$$

input
```
Integrate[Coth[e + f*x]^5*Sqrt[a + b*Sinh[e + f*x]^2],x]
```

output
```
((-8*a^2 - 8*a*b + b^2)*ArcTanh[Sqrt[a + b*Sinh[e + f*x]^2]/Sqrt[a]] - Sqr
t[a]*(-8*a + (8*a + b)*Csch[e + f*x]^2 + 2*a*Csch[e + f*x]^4)*Sqrt[a + b*S
inh[e + f*x]^2])/(8*a^(3/2)*f)
```

Rubi [A] (verified)

Time = 0.40 (sec) , antiderivative size = 154, normalized size of antiderivative = 0.92, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.360$, Rules used = {3042, 26, 3673, 100, 27, 87, 60, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \coth^5(e+fx)\sqrt{a+b\sinh^2(e+fx)}\,dx$$
$$\Big\downarrow \text{3042}$$
$$\int \frac{i\sqrt{a-b\sin(ie+ifx)^2}}{\tan(ie+ifx)^5}dx$$
$$\Big\downarrow \text{26}$$
$$i\int \frac{\sqrt{a-b\sin(ie+ifx)^2}}{\tan(ie+ifx)^5}dx$$
$$\Big\downarrow \text{3673}$$

$$\frac{\int \operatorname{csch}^6(e+fx)\left(\sinh^2(e+fx)+1\right)^2\sqrt{b\sinh^2(e+fx)+a}d\sinh^2(e+fx)}{2f}$$

$\Big\downarrow$ 100

$$\frac{\frac{\int \frac{1}{2}\operatorname{csch}^4(e+fx)(4a\sinh^2(e+fx)+8a-b)\sqrt{b\sinh^2(e+fx)+a}d\sinh^2(e+fx)}{2a}-\frac{\operatorname{csch}^4(e+fx)(a+b\sinh^2(e+fx))^{3/2}}{2a}}{2f}$$

$\Big\downarrow$ 27

$$\frac{\frac{\int \operatorname{csch}^4(e+fx)(4a\sinh^2(e+fx)+8a-b)\sqrt{b\sinh^2(e+fx)+a}d\sinh^2(e+fx)}{4a}-\frac{\operatorname{csch}^4(e+fx)(a+b\sinh^2(e+fx))^{3/2}}{2a}}{2f}$$

$\Big\downarrow$ 87

$$\frac{\frac{\frac{(8a^2+8ab-b^2)\int \operatorname{csch}^2(e+fx)\sqrt{b\sinh^2(e+fx)+a}d\sinh^2(e+fx)}{2a}-\frac{(8a-b)\operatorname{csch}^2(e+fx)\left(a+b\sinh^2(e+fx)\right)^{3/2}}{a}}{4a}-\frac{\operatorname{csch}^4(e+fx)(a+b\sinh^2(e+fx))^{3/2}}{2a}}{2f}$$

$\Big\downarrow$ 60

$$\frac{\frac{\frac{(8a^2+8ab-b^2)\left(a\int \frac{\operatorname{csch}^2(e+fx)}{\sqrt{b\sinh^2(e+fx)+a}}d\sinh^2(e+fx)+2\sqrt{a+b\sinh^2(e+fx)}\right)}{2a}-\frac{(8a-b)\operatorname{csch}^2(e+fx)\left(a+b\sinh^2(e+fx)\right)^{3/2}}{a}}{4a}-\frac{\operatorname{csch}^4(e+fx)(a+b\sinh^2(}{2a}}{2f}$$

$\Big\downarrow$ 73

$$\frac{\frac{\frac{(8a^2+8ab-b^2)\left(\frac{2a\int \frac{1}{\frac{\sinh^4(e+fx)}{b}-\frac{a}{b}}d\sqrt{b\sinh^2(e+fx)+a}}{b}+2\sqrt{a+b\sinh^2(e+fx)}\right)}{2a}-\frac{(8a-b)\operatorname{csch}^2(e+fx)\left(a+b\sinh^2(e+fx)\right)^{3/2}}{a}}{4a}-\frac{\operatorname{csch}^4(e+fx)(a+b\,s}{2a}}{2f}$$

$\Big\downarrow$ 221

$$\frac{\frac{\frac{(8a^2+8ab-b^2)\left(2\sqrt{a+b\sinh^2(e+fx)}-2\sqrt{a}\operatorname{arctanh}\left(\frac{\sqrt{a+b\sinh^2(e+fx)}}{\sqrt{a}}\right)\right)}{2a}-\frac{(8a-b)\operatorname{csch}^2(e+fx)\left(a+b\sinh^2(e+fx)\right)^{3/2}}{a}}{4a}-\frac{\operatorname{csch}^4(e+fx)(a+b\sinh^2}{2a}}{2f}$$

input
```
Int[Coth[e + f*x]^5*Sqrt[a + b*Sinh[e + f*x]^2],x]
```

output
```
(-1/2*(Csch[e + f*x]^4*(a + b*Sinh[e + f*x]^2)^(3/2))/a + (-(((8*a - b)*Cs
ch[e + f*x]^2*(a + b*Sinh[e + f*x]^2)^(3/2))/a) + ((8*a^2 + 8*a*b - b^2)*(
-2*Sqrt[a]*ArcTanh[Sqrt[a + b*Sinh[e + f*x]^2]/Sqrt[a]] + 2*Sqrt[a + b*Sin
h[e + f*x]^2]))/(2*a))/(4*a))/(2*f)
```

Defintions of rubi rules used

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 60
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Simp[n*((b*c - a*d)/(
b*(m + n + 1)))   Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
 c, d}, x] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ[m, 0] && ( !Integer
Q[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n + 2, 0] && IntLinear
Q[a, b, c, d, m, n, x]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 87
```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_] :> Simp[(-(b*e - a*f))*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(f*(p
+ 1)*(c*f - d*e))), x] - Simp[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p
+ 1)))/(f*(p + 1)*(c*f - d*e))   Int[(c + d*x)^n*(e + f*x)^(p + 1), x], x]
/; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || Intege
rQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))))
```

rule 100
```
Int[((a_.) + (b_.)*(x_))^2*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))^(
p_), x_] :> Simp[(b*c - a*d)^2*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(d^2*(d
*e - c*f)*(n + 1))), x] - Simp[1/(d^2*(d*e - c*f)*(n + 1))   Int[(c + d*x)^
(n + 1)*(e + f*x)^p*Simp[a^2*d^2*f*(n + p + 2) + b^2*c*(d*e*(n + 1) + c*f*(
p + 1)) - 2*a*b*d*(d*e*(n + 1) + c*f*(p + 1)) - b^2*d*(d*e - c*f)*(n + 1)*x
, x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && (LtQ[n, -1] || (EqQ[n
 + p + 3, 0] && NeQ[n, -1] && (SumSimplerQ[n, 1] ||  !SumSimplerQ[p, 1])))
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3673
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_.)*tan[(e_.) + (f_.)*(x_)]^
(m_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x]^2, x]}, Simp[ff^((m
 + 1)/2)/(2*f)   Subst[Int[x^((m - 1)/2)*((a + b*ff*x)^p/(1 - ff*x)^((m + 1
)/2)), x], x, Sin[e + f*x]^2/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && Integ
erQ[(m - 1)/2]
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.53 (sec) , antiderivative size = 80, normalized size of antiderivative = 0.48

method	result	size
default	$\frac{\text{`int/indef0`}\left(\frac{\cosh(fx+e)^4\left(a-b+b\cosh(fx+e)^2\right)}{\sinh(fx+e)\left(\cosh(fx+e)^4-2\cosh(fx+e)^2+1\right)\sqrt{a+b\sinh(fx+e)^2}},\sinh(fx+e)\right)}{f}$	80

input
```
int(coth(f*x+e)^5*(a+b*sinh(f*x+e)^2)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
`int/indef0`(1/sinh(f*x+e)/(cosh(f*x+e)^4-2*cosh(f*x+e)^2+1)*cosh(f*x+e)^4
*(a-b+b*cosh(f*x+e)^2)/(a+b*sinh(f*x+e)^2)^(1/2),sinh(f*x+e))/f
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1952 vs. $2(147) = 294$.

Time = 0.44 (sec) , antiderivative size = 3993, normalized size of antiderivative = 23.91

$$\int \coth^5(e+fx)\sqrt{a+b\sinh^2(e+fx)}\,dx = \text{Too large to display}$$

input
```
integrate(coth(f*x+e)^5*(a+b*sinh(f*x+e)^2)^(1/2),x, algorithm="fricas")
```

output
```
Too large to include
```

Sympy [F(-1)]

Timed out.

$$\int \coth^5(e+fx)\sqrt{a+b\sinh^2(e+fx)}\,dx = \text{Timed out}$$

input
```
integrate(coth(f*x+e)**5*(a+b*sinh(f*x+e)**2)**(1/2),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \coth^5(e+fx)\sqrt{a+b\sinh^2(e+fx)}\,dx = \int \sqrt{b\sinh(fx+e)^2+a}\coth(fx+e)^5\,dx$$

input
```
integrate(coth(f*x+e)^5*(a+b*sinh(f*x+e)^2)^(1/2),x, algorithm="maxima")
```

output
```
integrate(sqrt(b*sinh(f*x + e)^2 + a)*coth(f*x + e)^5, x)
```

Giac [F]

$$\int \coth^5(e+fx)\sqrt{a+b\sinh^2(e+fx)}\,dx = \int \sqrt{b\sinh(fx+e)^2+a}\coth(fx+e)^5\,dx$$

input
```
integrate(coth(f*x+e)^5*(a+b*sinh(f*x+e)^2)^(1/2),x, algorithm="giac")
```

output
```
integrate(sqrt(b*sinh(f*x + e)^2 + a)*coth(f*x + e)^5, x)
```

Mupad [F(-1)]

Timed out.

$$\int \coth^5(e+fx)\sqrt{a+b\sinh^2(e+fx)}\,dx = \int \coth(e+f\,x)^5\,\sqrt{b\sinh(e+f\,x)^2+a}\,dx$$

input
```
int(coth(e + f*x)^5*(a + b*sinh(e + f*x)^2)^(1/2),x)
```

output
```
int(coth(e + f*x)^5*(a + b*sinh(e + f*x)^2)^(1/2), x)
```

Reduce [F]

$$\int \coth^5(e+fx)\sqrt{a+b\sinh^2(e+fx)}\,dx = \int \sqrt{\sinh(fx+e)^2b+a}\,\coth(fx+e)^5\,dx$$

input
```
int(coth(f*x+e)^5*(a+b*sinh(f*x+e)^2)^(1/2),x)
```

output
```
int(sqrt(sinh(e + f*x)**2*b + a)*coth(e + f*x)**5,x)
```

3.427 $\int \sqrt{a+b\sinh^2(e+fx)}\tanh^4(e+fx)\,dx$

Optimal result

Integrand size = 25, antiderivative size = 235

$$\begin{aligned}
&\int \sqrt{a+b\sinh^2(e+fx)}\tanh^4(e+fx)\,dx \\
&= -\frac{(7a-8b)E\left(\arctan(\sinh(e+fx))\left|1-\frac{b}{a}\right.\right)\operatorname{sech}(e+fx)\sqrt{a+b\sinh^2(e+fx)}}{3(a-b)f\sqrt{\frac{\operatorname{sech}^2(e+fx)(a+b\sinh^2(e+fx))}{a}}} \\
&\quad+\frac{(3a-4b)\operatorname{EllipticF}\left(\arctan(\sinh(e+fx)),1-\frac{b}{a}\right)\operatorname{sech}(e+fx)\sqrt{a+b\sinh^2(e+fx)}}{3(a-b)f\sqrt{\frac{\operatorname{sech}^2(e+fx)(a+b\sinh^2(e+fx))}{a}}} \\
&\quad+\frac{4\sqrt{a+b\sinh^2(e+fx)}\tanh(e+fx)}{3f}-\frac{\sqrt{a+b\sinh^2(e+fx)}\tanh^3(e+fx)}{3f}
\end{aligned}$$

output

```
-1/3*(7*a-8*b)*EllipticE(sinh(f*x+e)/(1+sinh(f*x+e)^2)^(1/2),(1-b/a)^(1/2)
)*sech(f*x+e)*(a+b*sinh(f*x+e)^2)^(1/2)/(a-b)/f/(sech(f*x+e)^2*(a+b*sinh(f
*x+e)^2)/a)^(1/2)+1/3*(3*a-4*b)*InverseJacobiAM(arctan(sinh(f*x+e)),(1-b/a
)^(1/2))*sech(f*x+e)*(a+b*sinh(f*x+e)^2)^(1/2)/(a-b)/f/(sech(f*x+e)^2*(a+b
*sinh(f*x+e)^2)/a)^(1/2)+4/3*(a+b*sinh(f*x+e)^2)^(1/2)*tanh(f*x+e)/f-1/3*(
a+b*sinh(f*x+e)^2)^(1/2)*tanh(f*x+e)^3/f
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 1.64 (sec) , antiderivative size = 214, normalized size of antiderivative = 0.91

$$\int \sqrt{a+b\sinh^2(e+fx)}\tanh^4(e+fx)\,dx$$
$$= \frac{-2ia(7a-8b)\sqrt{\frac{2a-b+b\cosh(2(e+fx))}{a}}E\left(i(e+fx)\left|\frac{b}{a}\right.\right)+8ia(a-b)\sqrt{\frac{2a-b+b\cosh(2(e+fx))}{a}}\text{EllipticF}\left(i(e+fx\right.}{6(a-b)f\sqrt{2a-b+b\cosh(2(}}$$

input
```
Integrate[Sqrt[a + b*Sinh[e + f*x]^2]*Tanh[e + f*x]^4,x]
```

output
```
((-2*I)*a*(7*a - 8*b)*Sqrt[(2*a - b + b*Cosh[2*(e + f*x)])/a]*EllipticE[I*
(e + f*x), b/a] + (8*I)*a*(a - b)*Sqrt[(2*a - b + b*Cosh[2*(e + f*x)])/a]*
EllipticF[I*(e + f*x), b/a] - ((8*a^2 - 12*a*b + b^2 + 4*(4*a^2 - 6*a*b +
b^2)*Cosh[2*(e + f*x)] + (4*a - 5*b)*b*Cosh[4*(e + f*x)])*Sech[e + f*x]^2*
Tanh[e + f*x])/(2*Sqrt[2]))/(6*(a - b)*f*Sqrt[2*a - b + b*Cosh[2*(e + f*x)
]])
```

Rubi [A] (verified)

Time = 0.59 (sec) , antiderivative size = 350, normalized size of antiderivative = 1.49, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.320$, Rules used = {3042, 3675, 369, 440, 406, 320, 388, 313}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \tanh^4(e+fx)\sqrt{a+b\sinh^2(e+fx)}\,dx$$
$$\Big\downarrow \text{ 3042}$$
$$\int \tan(ie+ifx)^4\sqrt{a-b\sin(ie+ifx)^2}dx$$
$$\Big\downarrow \text{ 3675}$$

$$\frac{\sqrt{\cosh^2(e+fx)}\operatorname{sech}(e+fx)\int \frac{\sinh^4(e+fx)\sqrt{b\sinh^2(e+fx)+a}}{\left(\sinh^2(e+fx)+1\right)^{5/2}}d\sinh(e+fx)}{f}$$

$$\Big\downarrow \; 369$$

$$\frac{\sqrt{\cosh^2(e+fx)}\operatorname{sech}(e+fx)\left(\frac{1}{3}\int \frac{\sinh^2(e+fx)\left(4b\sinh^2(e+fx)+3a\right)}{\left(\sinh^2(e+fx)+1\right)^{3/2}\sqrt{b\sinh^2(e+fx)+a}}d\sinh(e+fx)-\frac{\sinh^3(e+fx)\sqrt{a+b\sinh^2(e+fx)}}{3\left(\sinh^2(e+fx)+1\right)^{3/2}}\right)}{f}$$

$$\Big\downarrow \; 440$$

$$\frac{\sqrt{\cosh^2(e+fx)}\operatorname{sech}(e+fx)\left(\frac{1}{3}\left(\frac{\int \frac{(7a-8b)b\sinh^2(e+fx)+a(3a-4b)}{\sqrt{\sinh^2(e+fx)+1}\sqrt{b\sinh^2(e+fx)+a}}d\sinh(e+fx)}{a-b}-\frac{(3a-4b)\sinh(e+fx)\sqrt{a+b\sinh^2(e+fx)}}{(a-b)\sqrt{\sinh^2(e+fx)+1}}\right)-\cdots}{f}$$

$$\Big\downarrow \; 406$$

$$\frac{\sqrt{\cosh^2(e+fx)}\operatorname{sech}(e+fx)\left(\frac{1}{3}\left(\frac{a(3a-4b)\int \frac{1}{\sqrt{\sinh^2(e+fx)+1}\sqrt{b\sinh^2(e+fx)+a}}d\sinh(e+fx)+b(7a-8b)\int \frac{\sinh^2(e+fx)}{\sqrt{\sinh^2(e+fx)+1}\sqrt{b\sinh^2(e-\cdots}}\cdots}{a-b}\cdots\right.\right.}{f}$$

$$\Big\downarrow \; 320$$

$$\frac{\sqrt{\cosh^2(e+fx)}\operatorname{sech}(e+fx)\left(\frac{1}{3}\left(\frac{b(7a-8b)\int \frac{\sinh^2(e+fx)}{\sqrt{\sinh^2(e+fx)+1}\sqrt{b\sinh^2(e+fx)+a}}d\sinh(e+fx)+\frac{(3a-4b)\sqrt{a+b\sinh^2(e+fx)}\,\text{EllipticF}\left(\text{arcta}\cdots\right.}{\sqrt{\sinh^2(e+fx)+1}\sqrt{\frac{a+b\sinh^2\cdots}{a\left(\sinh^2(e\cdots\right.}}}}{a-b}\cdots\right.\right.}{f}$$

$$\Big\downarrow \; 388$$

$$\frac{\sqrt{\cosh^2(e+fx)}\operatorname{sech}(e+fx)\left(\frac{1}{3}\left(\frac{b(7a-8b)\left(\frac{\sinh(e+fx)\sqrt{a+b\sinh^2(e+fx)}}{b\sqrt{\sinh^2(e+fx)+1}}-\frac{\int \frac{\sqrt{b\sinh^2(e+fx)+a}}{\left(\sinh^2(e+fx)+1\right)^{3/2}}d\sinh(e+fx)}{b}\right)+\frac{(3a-4b)\sqrt{a+b\sinh^2(e\cdots}}{\sqrt{\sinh^2(e\cdots}}\cdots}{a-b}\cdots\right.\right.}{f}$$

$$\downarrow 313$$

$$\frac{\sqrt{\cosh^2(e+fx)}\operatorname{sech}(e+fx)\left(\frac{1}{3}\left(\frac{\frac{(3a-4b)\sqrt{a+b\sinh^2(e+fx)}\operatorname{EllipticF}\left(\arctan(\sinh(e+fx)),1-\frac{b}{a}\right)}{\sqrt{\sinh^2(e+fx)+1}\sqrt{\frac{a+b\sinh^2(e+fx)}{a\left(\sinh^2(e+fx)+1\right)}}}+b(7a-8b)\left(\frac{\sinh(e+fx)\sqrt{a+b\sinh^2(e+fx)}}{b\sqrt{\sinh^2(e+fx)+1}}\right.}{a-b}\right.\right.}{f}$$

input

```
Int[Sqrt[a + b*Sinh[e + f*x]^2]*Tanh[e + f*x]^4,x]
```

output

```
(Sqrt[Cosh[e + f*x]^2]*Sech[e + f*x]*(-1/3*(Sinh[e + f*x]^3*Sqrt[a + b*Sin
h[e + f*x]^2])/(1 + Sinh[e + f*x]^2)^(3/2) + (-(((3*a - 4*b)*Sinh[e + f*x]
*Sqrt[a + b*Sinh[e + f*x]^2])/((a - b)*Sqrt[1 + Sinh[e + f*x]^2])) + (((3*
a - 4*b)*EllipticF[ArcTan[Sinh[e + f*x]], 1 - b/a]*Sqrt[a + b*Sinh[e + f*x
]^2])/(Sqrt[1 + Sinh[e + f*x]^2]*Sqrt[(a + b*Sinh[e + f*x]^2)/(a*(1 + Sinh
[e + f*x]^2))]) + (7*a - 8*b)*b*((Sinh[e + f*x]*Sqrt[a + b*Sinh[e + f*x]^2
])/(b*Sqrt[1 + Sinh[e + f*x]^2]) - (EllipticE[ArcTan[Sinh[e + f*x]], 1 - b
/a]*Sqrt[a + b*Sinh[e + f*x]^2])/(b*Sqrt[1 + Sinh[e + f*x]^2]*Sqrt[(a + b*
Sinh[e + f*x]^2)/(a*(1 + Sinh[e + f*x]^2))])))/(a - b))/3))/f
```

Defintions of rubi rules used

rule 313

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/((c_) + (d_.)*(x_)^2)^(3/2), x_Symbol] :> Sim
p[(Sqrt[a + b*x^2]/(c*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(c
+ d*x^2)))]))*EllipticE[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; FreeQ
[{a, b, c, d}, x] && PosQ[b/a] && PosQ[d/c]
```

rule 320

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(Sqrt[a + b*x^2]/(a*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(
c + d*x^2)))]))*EllipticF[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; Fre
eQ[{a, b, c, d}, x] && PosQ[d/c] && PosQ[b/a] &&  !SimplerSqrtQ[b/a, d/c]
```

rule 369
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_
), x_Symbol] :> Simp[e*(e*x)^(m - 1)*(a + b*x^2)^(p + 1)*((c + d*x^2)^q/(2*
b*(p + 1))), x] - Simp[e^2/(2*b*(p + 1))   Int[(e*x)^(m - 2)*(a + b*x^2)^(p
 + 1)*(c + d*x^2)^(q - 1)*Simp[c*(m - 1) + d*(m + 2*q - 1)*x^2, x], x], x]
/; FreeQ[{a, b, c, d, e}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && GtQ[q, 0
] && GtQ[m, 1] && IntBinomialQ[a, b, c, d, e, m, 2, p, q, x]
```

rule 388
```
Int[(x_)^2/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol]
 :> Simp[x*(Sqrt[a + b*x^2]/(b*Sqrt[c + d*x^2])), x] - Simp[c/b   Int[Sqrt[
a + b*x^2]/(c + d*x^2)^(3/2), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
 a*d, 0] && PosQ[b/a] && PosQ[d/c] &&  !SimplerSqrtQ[b/a, d/c]
```

rule 406
```
Int[((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.)*((e_) + (f_.)*(
x_)^2), x_Symbol] :> Simp[e   Int[(a + b*x^2)^p*(c + d*x^2)^q, x], x] + Sim
p[f   Int[x^2*(a + b*x^2)^p*(c + d*x^2)^q, x], x] /; FreeQ[{a, b, c, d, e,
f, p, q}, x]
```

rule 440
```
Int[((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_
)*((e_) + (f_.)*(x_)^2), x_Symbol] :> Simp[g*(b*e - a*f)*(g*x)^(m - 1)*(a +
 b*x^2)^(p + 1)*((c + d*x^2)^(q + 1)/(2*b*(b*c - a*d)*(p + 1))), x] - Simp[
g^2/(2*b*(b*c - a*d)*(p + 1))   Int[(g*x)^(m - 2)*(a + b*x^2)^(p + 1)*(c +
d*x^2)^q*Simp[c*(b*e - a*f)*(m - 1) + (d*(b*e - a*f)*(m + 2*q + 1) - b*2*(c
*f - d*e)*(p + 1))*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, q}, x] &&
 LtQ[p, -1] && GtQ[m, 1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3675
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_.)*tan[(e_.) + (f_.)*(x_)]^
(m_), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Simp[ff^(m + 1
)*(Sqrt[Cos[e + f*x]^2]/(f*Cos[e + f*x]))   Subst[Int[x^m*((a + b*ff^2*x^2)
^p/(1 - ff^2*x^2)^((m + 1)/2)), x], x, Sin[e + f*x]/ff], x]] /; FreeQ[{a, b
, e, f, p}, x] && IntegerQ[m/2] &&  !IntegerQ[p]
```

Maple [A] (verified)

Time = 5.81 (sec) , antiderivative size = 366, normalized size of antiderivative = 1.56

method	result
default	$\frac{\left(-4\sqrt{-\frac{b}{a}}\,ab+5\sqrt{-\frac{b}{a}}\,b^2\right)\cosh(fx+e)^4\sinh(fx+e)+\left(-4\sqrt{-\frac{b}{a}}\,a^2+10\sqrt{-\frac{b}{a}}\,ab-6\sqrt{-\frac{b}{a}}\,b^2\right)\cosh(fx+e)^2\sinh(fx+e)+\sqrt{\frac{b\cosh(\ldots}{\ldots}}}{}$

input

```
int((a+b*sinh(f*x+e)^2)^(1/2)*tanh(f*x+e)^4,x,method=_RETURNVERBOSE)
```

output

```
1/3*((-4*(-b/a)^(1/2)*a*b+5*(-b/a)^(1/2)*b^2)*cosh(f*x+e)^4*sinh(f*x+e)+(-
4*(-b/a)^(1/2)*a^2+10*(-b/a)^(1/2)*a*b-6*(-b/a)^(1/2)*b^2)*cosh(f*x+e)^2*s
inh(f*x+e)+(b/a*cosh(f*x+e)^2+(a-b)/a)^(1/2)*(cosh(f*x+e)^2)^(1/2)*(3*Elli
pticF(sinh(f*x+e)*(-b/a)^(1/2),(1/b*a)^(1/2))*a^2-11*EllipticF(sinh(f*x+e)
*(-b/a)^(1/2),(1/b*a)^(1/2))*a*b+8*EllipticF(sinh(f*x+e)*(-b/a)^(1/2),(1/b
*a)^(1/2))*b^2+7*EllipticE(sinh(f*x+e)*(-b/a)^(1/2),(1/b*a)^(1/2))*a*b-8*E
llipticE(sinh(f*x+e)*(-b/a)^(1/2),(1/b*a)^(1/2))*b^2)*cosh(f*x+e)^2+((-b/a
)^(1/2)*a^2-2*(-b/a)^(1/2)*a*b+(-b/a)^(1/2)*b^2)*sinh(f*x+e))/cosh(f*x+e)^
3/(a-b)/(-b/a)^(1/2)/(a+b*sinh(f*x+e)^2)^(1/2)/f
```

Fricas [F]

$$\int \sqrt{a+b\sinh^2(e+fx)}\tanh^4(e+fx)\,dx = \int \sqrt{b\sinh\left(fx+e\right)^2+a}\tanh\left(fx+e\right)^4\,dx$$

input

```
integrate((a+b*sinh(f*x+e)^2)^(1/2)*tanh(f*x+e)^4,x, algorithm="fricas")
```

output

```
integral(sqrt(b*sinh(f*x + e)^2 + a)*tanh(f*x + e)^4, x)
```

Sympy [F]

$$\int \sqrt{a+b\sinh^2(e+fx)}\tanh^4(e+fx)\,dx = \int \sqrt{a+b\sinh^2{(e+fx)}}\tanh^4{(e+fx)}\,dx$$

input
```
integrate((a+b*sinh(f*x+e)**2)**(1/2)*tanh(f*x+e)**4,x)
```

output
```
Integral(sqrt(a + b*sinh(e + f*x)**2)*tanh(e + f*x)**4, x)
```

Maxima [F]

$$\int \sqrt{a+b\sinh^2(e+fx)}\tanh^4(e+fx)\,dx = \int \sqrt{b\sinh{(fx+e)}^2+a}\tanh{(fx+e)}^4\,dx$$

input
```
integrate((a+b*sinh(f*x+e)^2)^(1/2)*tanh(f*x+e)^4,x, algorithm="maxima")
```

output
```
integrate(sqrt(b*sinh(f*x + e)^2 + a)*tanh(f*x + e)^4, x)
```

Giac [F]

$$\int \sqrt{a+b\sinh^2(e+fx)}\tanh^4(e+fx)\,dx = \int \sqrt{b\sinh{(fx+e)}^2+a}\tanh{(fx+e)}^4\,dx$$

input
```
integrate((a+b*sinh(f*x+e)^2)^(1/2)*tanh(f*x+e)^4,x, algorithm="giac")
```

output
```
integrate(sqrt(b*sinh(f*x + e)^2 + a)*tanh(f*x + e)^4, x)
```

Mupad [F(-1)]

Timed out.

$$\int \sqrt{a+b\sinh^2(e+fx)}\tanh^4(e+fx)\,dx = \int \tanh(e+f\,x)^4\,\sqrt{b\sinh\left(e+f\,x\right)^2+a}\,dx$$

input
```
int(tanh(e + f*x)^4*(a + b*sinh(e + f*x)^2)^(1/2),x)
```

output
```
int(tanh(e + f*x)^4*(a + b*sinh(e + f*x)^2)^(1/2), x)
```

Reduce [F]

$$\int \sqrt{a+b\sinh^2(e+fx)}\tanh^4(e+fx)\,dx = \int \sqrt{\sinh\left(fx+e\right)^2b+a}\,\tanh\left(fx+e\right)^4dx$$

input
```
int((a+b*sinh(f*x+e)^2)^(1/2)*tanh(f*x+e)^4,x)
```

output
```
int(sqrt(sinh(e + f*x)**2*b + a)*tanh(e + f*x)**4,x)
```

3.428 $\int \sqrt{a+b\sinh^2(e+fx)}\tanh^2(e+fx)\,dx$

Optimal result

Integrand size = 25, antiderivative size = 168

$$\int \sqrt{a+b\sinh^2(e+fx)}\tanh^2(e+fx)\,dx = -\frac{2E\left(\arctan(\sinh(e+fx))\left|1-\frac{b}{a}\right.\right)\operatorname{sech}(e+fx)\sqrt{a+b\sinh^2(e+fx)}}{f\sqrt{\frac{\operatorname{sech}^2(e+fx)(a+b\sinh^2(e+fx))}{a}}} + \frac{\operatorname{EllipticF}\left(\arctan(\sinh(e+fx)),1-\frac{b}{a}\right)\operatorname{sech}(e+fx)\sqrt{a+b\sinh^2(e+fx)}}{f\sqrt{\frac{\operatorname{sech}^2(e+fx)(a+b\sinh^2(e+fx))}{a}}} + \frac{\sqrt{a+b\sinh^2(e+fx)}\tanh(e+fx)}{f}$$

output
```
-2*EllipticE(sinh(f*x+e)/(1+sinh(f*x+e)^2)^(1/2),(1-b/a)^(1/2))*sech(f*x+e
)*(a+b*sinh(f*x+e)^2)^(1/2)/f/(sech(f*x+e)^2*(a+b*sinh(f*x+e)^2)/a)^(1/2)+
InverseJacobiAM(arctan(sinh(f*x+e)),(1-b/a)^(1/2))*sech(f*x+e)*(a+b*sinh(f
*x+e)^2)^(1/2)/f/(sech(f*x+e)^2*(a+b*sinh(f*x+e)^2)/a)^(1/2)+(a+b*sinh(f*x
+e)^2)^(1/2)*tanh(f*x+e)/f
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.66 (sec) , antiderivative size = 150, normalized size of antiderivative = 0.89

$$\int \sqrt{a+b\sinh^2(e+fx)}\tanh^2(e+fx)\,dx$$
$$= \frac{-2i\sqrt{2}a\sqrt{\frac{2a-b+b\cosh(2(e+fx))}{a}}E\left(i(e+fx)\,\middle|\,\frac{b}{a}\right)+i\sqrt{2}a\sqrt{\frac{2a-b+b\cosh(2(e+fx))}{a}}\,\text{EllipticF}\left(i(e+fx),\frac{b}{a}\right)+(-2a}{f\sqrt{4a-2b+2b\cosh(2(e+fx))}}$$

input
```
Integrate[Sqrt[a + b*Sinh[e + f*x]^2]*Tanh[e + f*x]^2,x]
```

output
```
((-2*I)*Sqrt[2]*a*Sqrt[(2*a - b + b*Cosh[2*(e + f*x)])/a]*EllipticE[I*(e +
 f*x), b/a] + I*Sqrt[2]*a*Sqrt[(2*a - b + b*Cosh[2*(e + f*x)])/a]*Elliptic
F[I*(e + f*x), b/a] + (-2*a + b - b*Cosh[2*(e + f*x)])*Tanh[e + f*x])/(f*S
qrt[4*a - 2*b + 2*b*Cosh[2*(e + f*x)]])
```

Rubi [A] (verified)

Time = 0.50 (sec) , antiderivative size = 267, normalized size of antiderivative = 1.59, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.320$, Rules used = {3042, 25, 3675, 369, 406, 320, 388, 313}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \tanh^2(e+fx)\sqrt{a+b\sinh^2(e+fx)}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \tan(ie+ifx)^2\left(-\sqrt{a-b\sin(ie+ifx)^2}\right)dx$$

$$\Big\downarrow \text{25}$$

$$-\int \sqrt{a-b\sin(ie+ifx)^2}\tan(ie+ifx)^2dx$$

$$\Big\downarrow \text{3675}$$

$$\frac{\sqrt{\cosh^2(e+fx)}\operatorname{sech}(e+fx)\int \frac{\sinh^2(e+fx)\sqrt{b\sinh^2(e+fx)+a}}{\left(\sinh^2(e+fx)+1\right)^{3/2}}d\sinh(e+fx)}{f}$$

$\Big\downarrow$ 369

$$\frac{\sqrt{\cosh^2(e+fx)}\operatorname{sech}(e+fx)\left(\int \frac{2b\sinh^2(e+fx)+a}{\sqrt{\sinh^2(e+fx)+1}\sqrt{b\sinh^2(e+fx)+a}}d\sinh(e+fx)-\frac{\sinh(e+fx)\sqrt{a+b\sinh^2(e+fx)}}{\sqrt{\sinh^2(e+fx)+1}}\right)}{f}$$

$\Big\downarrow$ 406

$$\frac{\sqrt{\cosh^2(e+fx)}\operatorname{sech}(e+fx)\left(a\int \frac{1}{\sqrt{\sinh^2(e+fx)+1}\sqrt{b\sinh^2(e+fx)+a}}d\sinh(e+fx)+2b\int \frac{\sinh^2(e+fx)}{\sqrt{\sinh^2(e+fx)+1}\sqrt{b\sinh^2(e+f}}\right.}{f}$$

$\Big\downarrow$ 320

$$\frac{\sqrt{\cosh^2(e+fx)}\operatorname{sech}(e+fx)\left(2b\int \frac{\sinh^2(e+fx)}{\sqrt{\sinh^2(e+fx)+1}\sqrt{b\sinh^2(e+fx)+a}}d\sinh(e+fx)+\frac{\sqrt{a+b\sinh^2(e+fx)}\operatorname{EllipticF}\left(\arctan\right.}{\sqrt{\sinh^2(e+fx)+1}\sqrt{\frac{a+b\sin}{a\left(\sinh\right.}}}\right.}{f}$$

$\Big\downarrow$ 388

$$\frac{\sqrt{\cosh^2(e+fx)}\operatorname{sech}(e+fx)\left(2b\left(\frac{\sinh(e+fx)\sqrt{a+b\sinh^2(e+fx)}}{b\sqrt{\sinh^2(e+fx)+1}}-\frac{\int \frac{\sqrt{b\sinh^2(e+fx)+a}}{\left(\sinh^2(e+fx)+1\right)^{3/2}}d\sinh(e+fx)}{b}\right)+\frac{\sqrt{a+b\sinh^2(e+fx)}}{\sqrt{\sinh^2(e\right.}}\right.}{f}$$

$\Big\downarrow$ 313

$$\frac{\sqrt{\cosh^2(e+fx)}\operatorname{sech}(e+fx)\left(\frac{\sqrt{a+b\sinh^2(e+fx)}\operatorname{EllipticF}\left(\arctan(\sinh(e+fx)),1-\frac{b}{a}\right)}{\sqrt{\sinh^2(e+fx)+1}\sqrt{\frac{a+b\sinh^2(e+fx)}{a\left(\sinh^2(e+fx)+1\right)}}}+2b\left(\frac{\sinh(e+fx)\sqrt{a+b\sinh^2(e+fx)}}{b\sqrt{\sinh^2(e+fx)+1}}-\right.\right.}{f}$$

input
```
Int[Sqrt[a + b*Sinh[e + f*x]^2]*Tanh[e + f*x]^2,x]
```

output
```
(Sqrt[Cosh[e + f*x]^2]*Sech[e + f*x]*(-((Sinh[e + f*x]*Sqrt[a + b*Sinh[e +
 f*x]^2])/Sqrt[1 + Sinh[e + f*x]^2]) + (EllipticF[ArcTan[Sinh[e + f*x]], 1
 - b/a]*Sqrt[a + b*Sinh[e + f*x]^2])/(Sqrt[1 + Sinh[e + f*x]^2]*Sqrt[(a +
b*Sinh[e + f*x]^2)/(a*(1 + Sinh[e + f*x]^2))]) + 2*b*((Sinh[e + f*x]*Sqrt[
a + b*Sinh[e + f*x]^2])/(b*Sqrt[1 + Sinh[e + f*x]^2]) - (EllipticE[ArcTan[
Sinh[e + f*x]], 1 - b/a]*Sqrt[a + b*Sinh[e + f*x]^2])/(b*Sqrt[1 + Sinh[e +
 f*x]^2]*Sqrt[(a + b*Sinh[e + f*x]^2)/(a*(1 + Sinh[e + f*x]^2))]))))/f
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 313
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/((c_) + (d_.)*(x_)^2)^(3/2), x_Symbol] :> Sim
p[(Sqrt[a + b*x^2]/(c*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(c
+ d*x^2)))]))*EllipticE[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; FreeQ
[{a, b, c, d}, x] && PosQ[b/a] && PosQ[d/c]
```

rule 320
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(Sqrt[a + b*x^2]/(a*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(
c + d*x^2)))]))*EllipticF[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; Fre
eQ[{a, b, c, d}, x] && PosQ[d/c] && PosQ[b/a] &&  !SimplerSqrtQ[b/a, d/c]
```

rule 369
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_
), x_Symbol] :> Simp[e*(e*x)^(m - 1)*(a + b*x^2)^(p + 1)*((c + d*x^2)^q/(2*
b*(p + 1))), x] - Simp[e^2/(2*b*(p + 1))   Int[(e*x)^(m - 2)*(a + b*x^2)^(p
 + 1)*(c + d*x^2)^(q - 1)*Simp[c*(m - 1) + d*(m + 2*q - 1)*x^2, x], x], x]
/; FreeQ[{a, b, c, d, e}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && GtQ[q, 0
] && GtQ[m, 1] && IntBinomialQ[a, b, c, d, e, m, 2, p, q, x]
```

rule 388
```
Int[(x_)^2/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol]
 :> Simp[x*(Sqrt[a + b*x^2]/(b*Sqrt[c + d*x^2])), x] - Simp[c/b   Int[Sqrt[
a + b*x^2]/(c + d*x^2)^(3/2), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
 a*d, 0] && PosQ[b/a] && PosQ[d/c] &&  !SimplerSqrtQ[b/a, d/c]
```

rule 406
```
Int[((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.)*((e_) + (f_.)*(
x_)^2), x_Symbol] :> Simp[e   Int[(a + b*x^2)^p*(c + d*x^2)^q, x], x] + Sim
p[f   Int[x^2*(a + b*x^2)^p*(c + d*x^2)^q, x], x] /; FreeQ[{a, b, c, d, e,
f, p, q}, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3675
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_.)*tan[(e_.) + (f_.)*(x_)]^
(m_), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Simp[ff^(m + 1
)*(Sqrt[Cos[e + f*x]^2]/(f*Cos[e + f*x]))   Subst[Int[x^m*((a + b*ff^2*x^2)
^p/(1 - ff^2*x^2)^((m + 1)/2)), x], x, Sin[e + f*x]/ff], x]] /; FreeQ[{a, b
, e, f, p}, x] && IntegerQ[m/2] && !IntegerQ[p]
```

Maple [A] (verified)

Time = 3.84 (sec) , antiderivative size = 233, normalized size of antiderivative = 1.39

method	result
default	$\frac{-\sqrt{-\frac{b}{a}}\, b \sinh(fx+e)^3+a\sqrt{\frac{a+b\sinh(fx+e)^2}{a}}\,\sqrt{\frac{\cosh(2fx+2e)}{2}+\frac{1}{2}}\ \text{EllipticF}\left(\sinh(fx+e)\sqrt{-\frac{b}{a}},\sqrt{\frac{a}{b}}\right)-2b\sqrt{\frac{a+b\sinh(fx+e)^2}{a}}\,\sqrt{\frac{\cosh(2}{}}}{\sqrt{-\frac{b}{a}}\ \cosh(}$

input
```
int((a+b*sinh(f*x+e)^2)^(1/2)*tanh(f*x+e)^2,x,method=_RETURNVERBOSE)
```

output
```
(-(-b/a)^(1/2)*b*sinh(f*x+e)^3+a*((a+b*sinh(f*x+e)^2)/a)^(1/2)*(cosh(f*x+e
)^2)^(1/2)*EllipticF(sinh(f*x+e)*(-b/a)^(1/2),(1/b*a)^(1/2))-2*b*((a+b*sin
h(f*x+e)^2)/a)^(1/2)*(cosh(f*x+e)^2)^(1/2)*EllipticF(sinh(f*x+e)*(-b/a)^(1
/2),(1/b*a)^(1/2))+2*b*((a+b*sinh(f*x+e)^2)/a)^(1/2)*(cosh(f*x+e)^2)^(1/2)
*EllipticE(sinh(f*x+e)*(-b/a)^(1/2),(1/b*a)^(1/2))-(-b/a)^(1/2)*a*sinh(f*x
+e))/(-b/a)^(1/2)/cosh(f*x+e)/(a+b*sinh(f*x+e)^2)^(1/2)/f
```

Fricas [F]

$$\int \sqrt{a+b\sinh^2(e+fx)}\tanh^2(e+fx)\,dx = \int \sqrt{b\sinh\left(fx+e\right)^2+a}\tanh\left(fx+e\right)^2\,dx$$

input
```
integrate((a+b*sinh(f*x+e)^2)^(1/2)*tanh(f*x+e)^2,x, algorithm="fricas")
```

output
```
integral(sqrt(b*sinh(f*x + e)^2 + a)*tanh(f*x + e)^2, x)
```

Sympy [F]

$$\int \sqrt{a+b\sinh^2(e+fx)}\tanh^2(e+fx)\,dx = \int \sqrt{a+b\sinh^2\left(e+fx\right)}\tanh^2\left(e+fx\right)dx$$

input
```
integrate((a+b*sinh(f*x+e)**2)**(1/2)*tanh(f*x+e)**2,x)
```

output
```
Integral(sqrt(a + b*sinh(e + f*x)**2)*tanh(e + f*x)**2, x)
```

Maxima [F]

$$\int \sqrt{a+b\sinh^2(e+fx)}\tanh^2(e+fx)\,dx = \int \sqrt{b\sinh\left(fx+e\right)^2+a}\tanh\left(fx+e\right)^2\,dx$$

input
```
integrate((a+b*sinh(f*x+e)^2)^(1/2)*tanh(f*x+e)^2,x, algorithm="maxima")
```

output
```
integrate(sqrt(b*sinh(f*x + e)^2 + a)*tanh(f*x + e)^2, x)
```

Giac [F]

$$\int \sqrt{a+b\sinh^2(e+fx)}\tanh^2(e+fx)\,dx=\int \sqrt{b\sinh(fx+e)^2+a}\tanh(fx+e)^2\,dx$$

input
```
integrate((a+b*sinh(f*x+e)^2)^(1/2)*tanh(f*x+e)^2,x, algorithm="giac")
```

output
```
integrate(sqrt(b*sinh(f*x + e)^2 + a)*tanh(f*x + e)^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int \sqrt{a+b\sinh^2(e+fx)}\tanh^2(e+fx)\,dx=\int \tanh(e+f\,x)^2\,\sqrt{b\sinh(e+f\,x)^2+a}\,dx$$

input
```
int(tanh(e + f*x)^2*(a + b*sinh(e + f*x)^2)^(1/2),x)
```

output
```
int(tanh(e + f*x)^2*(a + b*sinh(e + f*x)^2)^(1/2), x)
```

Reduce [F]

$$\int \sqrt{a+b\sinh^2(e+fx)}\tanh^2(e+fx)\,dx=\int \sqrt{\sinh(fx+e)^2b+a}\,\tanh(fx+e)^2\,dx$$

input
```
int((a+b*sinh(f*x+e)^2)^(1/2)*tanh(f*x+e)^2,x)
```

output
```
int(sqrt(sinh(e + f*x)**2*b + a)*tanh(e + f*x)**2,x)
```

3.429 $\int \sqrt{a+b\sinh^2(e+fx)}\,dx$

Optimal result

Integrand size = 16, antiderivative size = 61

$$\int \sqrt{a+b\sinh^2(e+fx)}\,dx=-\frac{iE\left(ie+ifx\middle|\frac{b}{a}\right)\sqrt{a+b\sinh^2(e+fx)}}{f\sqrt{\frac{a+b\sinh^2(e+fx)}{a}}}$$

output
```
-I*EllipticE(sin(I*e+I*f*x),(b/a)^(1/2))*(a+b*sinh(f*x+e)^2)^(1/2)/f/((a+b
*sinh(f*x+e)^2)/a)^(1/2)
```

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 69, normalized size of antiderivative = 1.13

$$\int \sqrt{a+b\sinh^2(e+fx)}\,dx=-\frac{ia\sqrt{\frac{2a-b+b\cosh(2(e+fx))}{a}}E\left(i(e+fx)\middle|\frac{b}{a}\right)}{f\sqrt{2a-b+b\cosh(2(e+fx))}}$$

input
```
Integrate[Sqrt[a + b*Sinh[e + f*x]^2],x]
```

output
```
((-I)*a*Sqrt[(2*a - b + b*Cosh[2*(e + f*x)])/a]*EllipticE[I*(e + f*x), b/a
])/(f*Sqrt[2*a - b + b*Cosh[2*(e + f*x)]])
```

Rubi [A] (verified)

Time = 0.35 (sec) , antiderivative size = 60, normalized size of antiderivative = 0.98, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {3042, 3657, 3042, 3656}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{a+b\sinh^2(e+fx)}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \sqrt{a-b\sin(ie+ifx)^2}dx$$

$$\Big\downarrow \text{3657}$$

$$\frac{\sqrt{a+b\sinh^2(e+fx)}\int \sqrt{\frac{b\sinh^2(e+fx)}{a}+1}dx}{\sqrt{\frac{b\sinh^2(e+fx)}{a}+1}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\sqrt{a+b\sinh^2(e+fx)}\int \sqrt{1-\frac{b\sin(ie+ifx)^2}{a}}dx}{\sqrt{\frac{b\sinh^2(e+fx)}{a}+1}}$$

$$\Big\downarrow \text{3656}$$

$$-\frac{i\sqrt{a+b\sinh^2(e+fx)}E\left(ie+ifx|\frac{b}{a}\right)}{f\sqrt{\frac{b\sinh^2(e+fx)}{a}+1}}$$

input
```
Int[Sqrt[a + b*Sinh[e + f*x]^2],x]
```

output
```
((-I)*EllipticE[I*e + I*f*x, b/a]*Sqrt[a + b*Sinh[e + f*x]^2])/(f*Sqrt[1 +
 (b*Sinh[e + f*x]^2)/a])
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3656
```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2], x_Symbol] :> Simp[(Sqrt[a
]/f)*EllipticE[e + f*x, -b/a], x] /; FreeQ[{a, b, e, f}, x] && GtQ[a, 0]
```

rule 3657
```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2], x_Symbol] :> Simp[Sqrt[a
+ b*Sin[e + f*x]^2]/Sqrt[1 + b*(Sin[e + f*x]^2/a)]   Int[Sqrt[1 + (b*Sin[e
+ f*x]^2)/a], x], x] /; FreeQ[{a, b, e, f}, x] &&  !GtQ[a, 0]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 139 vs. $2(57) = 114$.

Time = 2.06 (sec) , antiderivative size = 140, normalized size of antiderivative = 2.30

method	result
default	$\frac{\sqrt{\frac{a+b\sinh(fx+e)^2}{a}}\,\sqrt{\frac{\cosh(2fx+2e)}{2}+\frac{1}{2}}\,\Big(a\,\mathrm{EllipticF}\left(\sinh(fx+e)\sqrt{-\frac{b}{a}},\sqrt{\frac{a}{b}}\right)-b\,\mathrm{EllipticF}\left(\sinh(fx+e)\sqrt{-\frac{b}{a}},\sqrt{\frac{a}{b}}\right)+b\,\mathrm{EllipticE}\Big(\mathrm{si}}{\sqrt{-\frac{b}{a}}\,\cosh(fx+e)\sqrt{a+b\sinh(fx+e)^2}\,f}$

input
```
int((a+b*sinh(f*x+e)^2)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
((a+b*sinh(f*x+e)^2)/a)^(1/2)*(cosh(f*x+e)^2)^(1/2)*(a*EllipticF(sinh(f*x+
e)*(-b/a)^(1/2),(1/b*a)^(1/2))-b*EllipticF(sinh(f*x+e)*(-b/a)^(1/2),(1/b*a
)^(1/2))+b*EllipticE(sinh(f*x+e)*(-b/a)^(1/2),(1/b*a)^(1/2)))/(-b/a)^(1/2)
/cosh(f*x+e)/(a+b*sinh(f*x+e)^2)^(1/2)/f
```

Fricas [F]

$$\int \sqrt{a+b\sinh^2(e+fx)}\,dx=\int \sqrt{b\sinh\left(fx+e\right)^2+a}\,dx$$

input
```
integrate((a+b*sinh(f*x+e)^2)^(1/2),x, algorithm="fricas")
```

output
```
integral(sqrt(b*sinh(f*x + e)^2 + a), x)
```

Sympy [F]

$$\int \sqrt{a+b\sinh^2(e+fx)}\,dx=\int \sqrt{a+b\sinh^2\left(e+fx\right)}\,dx$$

input
```
integrate((a+b*sinh(f*x+e)**2)**(1/2),x)
```

output
```
Integral(sqrt(a + b*sinh(e + f*x)**2), x)
```

Maxima [F]

$$\int \sqrt{a+b\sinh^2(e+fx)}\,dx=\int \sqrt{b\sinh\left(fx+e\right)^2+a}\,dx$$

input
```
integrate((a+b*sinh(f*x+e)^2)^(1/2),x, algorithm="maxima")
```

output
```
integrate(sqrt(b*sinh(f*x + e)^2 + a), x)
```

Giac [F]

$$\int \sqrt{a+b\sinh^2(e+fx)}\,dx=\int \sqrt{b\sinh{(fx+e)}^2+a}\,dx$$

input
```
integrate((a+b*sinh(f*x+e)^2)^(1/2),x, algorithm="giac")
```

output
```
integrate(sqrt(b*sinh(f*x + e)^2 + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int \sqrt{a+b\sinh^2(e+fx)}\,dx=\int \sqrt{b\,{\sinh\left(e+f\,x\right)}^2+a}\,dx$$

input
```
int((a + b*sinh(e + f*x)^2)^(1/2),x)
```

output
```
int((a + b*sinh(e + f*x)^2)^(1/2), x)
```

Reduce [F]

$$\int \sqrt{a+b\sinh^2(e+fx)}\,dx=\int \sqrt{\sinh{(fx+e)}^2 b+a}\,dx$$

input
```
int((a+b*sinh(f*x+e)^2)^(1/2),x)
```

output
```
int(sqrt(sinh(e + f*x)**2*b + a),x)
```

3.430 $\int \coth^2(e+fx)\sqrt{a+b\sinh^2(e+fx)}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 202

$$\begin{aligned}
&\int \coth^2(e+fx)\sqrt{a+b\sinh^2(e+fx)}\,dx \\
&= -\frac{\coth(e+fx)\sqrt{a+b\sinh^2(e+fx)}}{f} \\
&\quad -\frac{2E\left(\arctan(\sinh(e+fx))\,\middle|\,1-\frac{b}{a}\right)\operatorname{sech}(e+fx)\sqrt{a+b\sinh^2(e+fx)}}{f\sqrt{\frac{\operatorname{sech}^2(e+fx)(a+b\sinh^2(e+fx))}{a}}} \\
&\quad +\frac{(a+b)\operatorname{EllipticF}\left(\arctan(\sinh(e+fx)),1-\frac{b}{a}\right)\operatorname{sech}(e+fx)\sqrt{a+b\sinh^2(e+fx)}}{af\sqrt{\frac{\operatorname{sech}^2(e+fx)(a+b\sinh^2(e+fx))}{a}}} \\
&\quad +\frac{2\sqrt{a+b\sinh^2(e+fx)}\tanh(e+fx)}{f}
\end{aligned}$$

output
```
-coth(f*x+e)*(a+b*sinh(f*x+e)^2)^(1/2)/f-2*EllipticE(sinh(f*x+e)/(1+sinh(f
*x+e)^2)^(1/2),(1-b/a)^(1/2))*sech(f*x+e)*(a+b*sinh(f*x+e)^2)^(1/2)/f/(sec
h(f*x+e)^2*(a+b*sinh(f*x+e)^2)/a)^(1/2)+(a+b)*InverseJacobiAM(arctan(sinh(
f*x+e)),(1-b/a)^(1/2))*sech(f*x+e)*(a+b*sinh(f*x+e)^2)^(1/2)/a/f/(sech(f*x
+e)^2*(a+b*sinh(f*x+e)^2)/a)^(1/2)+2*(a+b*sinh(f*x+e)^2)^(1/2)*tanh(f*x+e)
/f
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.71 (sec) , antiderivative size = 154, normalized size of antiderivative = 0.76

$$\int \coth^2(e+fx)\sqrt{a+b\sinh^2(e+fx)}\,dx$$
$$= \frac{(-2a+b-b\cosh(2(e+fx)))\coth(e+fx) - 2i\sqrt{2}a\sqrt{\frac{2a-b+b\cosh(2(e+fx))}{a}}E\left(i(e+fx)\,\middle|\,\frac{b}{a}\right) + i\sqrt{2}(a-b)}{f\sqrt{4a-2b+2b\cosh(2(e+fx))}}$$

input
```
Integrate[Coth[e + f*x]^2*Sqrt[a + b*Sinh[e + f*x]^2],x]
```

output
```
((-2*a + b - b*Cosh[2*(e + f*x)])*Coth[e + f*x] - (2*I)*Sqrt[2]*a*Sqrt[(2*
a - b + b*Cosh[2*(e + f*x)])/a]*EllipticE[I*(e + f*x), b/a] + I*Sqrt[2]*(a
 - b)*Sqrt[(2*a - b + b*Cosh[2*(e + f*x)])/a]*EllipticF[I*(e + f*x), b/a])
/(f*Sqrt[4*a - 2*b + 2*b*Cosh[2*(e + f*x)]])
```

Rubi [A] (verified)

Time = 0.51 (sec) , antiderivative size = 273, normalized size of antiderivative = 1.35, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.360$, Rules used = {3042, 25, 3675, 375, 27, 406, 320, 388, 313}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \coth^2(e+fx)\sqrt{a+b\sinh^2(e+fx)}\,dx$$
$$\Big\downarrow \; 3042$$
$$\int -\frac{\sqrt{a-b\sin(ie+ifx)^2}}{\tan(ie+ifx)^2}dx$$
$$\Big\downarrow \; 25$$
$$-\int \frac{\sqrt{a-b\sin(ie+ifx)^2}}{\tan(ie+ifx)^2}dx$$

$\Big\downarrow$ 3675

$$\frac{\sqrt{\cosh^2(e+fx)}\operatorname{sech}(e+fx)\int \operatorname{csch}^2(e+fx)\sqrt{\sinh^2(e+fx)+1}\sqrt{b\sinh^2(e+fx)+a}d\sinh(e+fx)}{f}$$

$\Big\downarrow$ 375

$$\frac{\sqrt{\cosh^2(e+fx)}\operatorname{sech}(e+fx)\left(2\int \frac{2b\sinh^2(e+fx)+a+b}{2\sqrt{\sinh^2(e+fx)+1}\sqrt{b\sinh^2(e+fx)+a}}d\sinh(e+fx)-\sqrt{\sinh^2(e+fx)+1}\operatorname{csch}(e+\right.}{f}$$

$\Big\downarrow$ 27

$$\frac{\sqrt{\cosh^2(e+fx)}\operatorname{sech}(e+fx)\left(\int \frac{2b\sinh^2(e+fx)+a+b}{\sqrt{\sinh^2(e+fx)+1}\sqrt{b\sinh^2(e+fx)+a}}d\sinh(e+fx)-\sqrt{\sinh^2(e+fx)+1}\operatorname{csch}(e+f\right.}{f}$$

$\Big\downarrow$ 406

$$\frac{\sqrt{\cosh^2(e+fx)}\operatorname{sech}(e+fx)\left((a+b)\int \frac{1}{\sqrt{\sinh^2(e+fx)+1}\sqrt{b\sinh^2(e+fx)+a}}d\sinh(e+fx)+2b\int \frac{\sinh^2(e+fx)}{\sqrt{\sinh^2(e+fx)+1}\sqrt{b\sin}}\right.}{f}$$

$\Big\downarrow$ 320

$$\frac{\sqrt{\cosh^2(e+fx)}\operatorname{sech}(e+fx)\left(2b\int \frac{\sinh^2(e+fx)}{\sqrt{\sinh^2(e+fx)+1}\sqrt{b\sinh^2(e+fx)+a}}d\sinh(e+fx)+\frac{(a+b)\sqrt{a+b\sinh^2(e+fx)}\operatorname{EllipticF}\left(\right.}{a\sqrt{\sinh^2(e+fx)+1}\sqrt{\frac{a}{a(}}}\right.}{f}$$

$\Big\downarrow$ 388

$$\frac{\sqrt{\cosh^2(e+fx)}\operatorname{sech}(e+fx)\left(2b\left(\frac{\sinh(e+fx)\sqrt{a+b\sinh^2(e+fx)}}{b\sqrt{\sinh^2(e+fx)+1}}-\frac{\int \frac{\sqrt{b\sinh^2(e+fx)+a}}{\left(\sinh^2(e+fx)+1\right)^{3/2}}d\sinh(e+fx)}{b}\right)+\frac{(a+b)\sqrt{a+b\sinh^2(e}}{a\sqrt{\sinh}}\right.}{f}$$

$\Big\downarrow$ 313

$$\frac{\sqrt{\cosh^2(e+fx)}\operatorname{sech}(e+fx)\left(\frac{(a+b)\sqrt{a+b\sinh^2(e+fx)}\operatorname{EllipticF}\left(\arctan(\sinh(e+fx)),1-\frac{b}{a}\right)}{a\sqrt{\sinh^2(e+fx)+1}\sqrt{\frac{a+b\sinh^2(e+fx)}{a\left(\sinh^2(e+fx)+1\right)}}}+2b\left(\frac{\sinh(e+fx)\sqrt{a+b\sinh^2(e+fx)}}{b\sqrt{\sinh^2(e+fx)+1}}\right.\right.}{f}$$

input
```
Int[Coth[e + f*x]^2*Sqrt[a + b*Sinh[e + f*x]^2],x]
```

output
```
(Sqrt[Cosh[e + f*x]^2]*Sech[e + f*x]*(-(Csch[e + f*x]*Sqrt[1 + Sinh[e + f*
x]^2]*Sqrt[a + b*Sinh[e + f*x]^2]) + ((a + b)*EllipticF[ArcTan[Sinh[e + f*
x]], 1 - b/a]*Sqrt[a + b*Sinh[e + f*x]^2])/(a*Sqrt[1 + Sinh[e + f*x]^2]*Sq
rt[(a + b*Sinh[e + f*x]^2)/(a*(1 + Sinh[e + f*x]^2))]) + 2*b*((Sinh[e + f*
x]*Sqrt[a + b*Sinh[e + f*x]^2])/(b*Sqrt[1 + Sinh[e + f*x]^2]) - (EllipticE
[ArcTan[Sinh[e + f*x]], 1 - b/a]*Sqrt[a + b*Sinh[e + f*x]^2])/(b*Sqrt[1 +
Sinh[e + f*x]^2]*Sqrt[(a + b*Sinh[e + f*x]^2)/(a*(1 + Sinh[e + f*x]^2))]))
))/f
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 313
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/((c_) + (d_.)*(x_)^2)^(3/2), x_Symbol] :> Sim
p[(Sqrt[a + b*x^2]/(c*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(c
 + d*x^2)))]))*EllipticE[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; FreeQ
[{a, b, c, d}, x] && PosQ[b/a] && PosQ[d/c]
```

rule 320
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(Sqrt[a + b*x^2]/(a*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(
c + d*x^2)))]))*EllipticF[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; Fre
eQ[{a, b, c, d}, x] && PosQ[d/c] && PosQ[b/a] &&  !SimplerSqrtQ[b/a, d/c]
```

rule 375
```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_)
, x_Symbol] :> Simp[(e*x)^(m + 1)*(a + b*x^2)^p*((c + d*x^2)^q/(e*(m + 1)))
, x] - Simp[2/(e^2*(m + 1))   Int[(e*x)^(m + 2)*(a + b*x^2)^(p - 1)*(c + d*
x^2)^(q - 1)*Simp[b*c*p + a*d*q + b*d*(p + q)*x^2, x], x], x] /; FreeQ[{a,
b, c, d, e}, x] && NeQ[b*c - a*d, 0] && GtQ[q, 0] && LtQ[m, -1] && GtQ[p, 0
] && IntBinomialQ[a, b, c, d, e, m, 2, p, q, x]
```

rule 388
```
Int[(x_)^2/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol]
 :> Simp[x*(Sqrt[a + b*x^2]/(b*Sqrt[c + d*x^2])), x] - Simp[c/b   Int[Sqrt[
a + b*x^2]/(c + d*x^2)^(3/2), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
 a*d, 0] && PosQ[b/a] && PosQ[d/c] &&  !SimplerSqrtQ[b/a, d/c]
```

rule 406
```
Int[((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.)*((e_) + (f_.)*(
x_)^2), x_Symbol] :> Simp[e   Int[(a + b*x^2)^p*(c + d*x^2)^q, x], x] + Sim
p[f   Int[x^2*(a + b*x^2)^p*(c + d*x^2)^q, x], x] /; FreeQ[{a, b, c, d, e,
f, p, q}, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3675
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_.)*tan[(e_.) + (f_.)*(x_)]^
(m_), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Simp[ff^(m + 1
)*(Sqrt[Cos[e + f*x]^2]/(f*Cos[e + f*x]))   Subst[Int[x^m*((a + b*ff^2*x^2)
^p/(1 - ff^2*x^2)^((m + 1)/2)), x], x, Sin[e + f*x]/ff], x]] /; FreeQ[{a, b
, e, f, p}, x] && IntegerQ[m/2] &&  !IntegerQ[p]
```

Maple [A] (verified)

Time = 3.63 (sec) , antiderivative size = 214, normalized size of antiderivative = 1.06

method	result
default	$\frac{-\sqrt{-\frac{b}{a}}\, b\cosh(fx+e)^4+\left(-\sqrt{-\frac{b}{a}}\, a+\sqrt{-\frac{b}{a}}\, b\right)\cosh(fx+e)^2+\sinh(fx+e)\sqrt{\frac{b\cosh(fx+e)^2}{a}+\frac{a-b}{a}}\,\sqrt{\frac{\cosh(2fx+2e)}{2}+\frac{1}{2}}\,\Big(a\,\mathrm{EllipticF}\Big(s}{\sinh(fx+e)\sqrt{-\frac{b}{a}}\,\cosh(fx+e)\sqrt{a+b\sin}}$

input
```
int(coth(f*x+e)^2*(a+b*sinh(f*x+e)^2)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
(-(-b/a)^(1/2)*b*cosh(f*x+e)^4+(-(-b/a)^(1/2)*a+(-b/a)^(1/2)*b)*cosh(f*x+e
)^2+sinh(f*x+e)*(b/a*cosh(f*x+e)^2+(a-b)/a)^(1/2)*(cosh(f*x+e)^2)^(1/2)*(a
*EllipticF(sinh(f*x+e)*(-b/a)^(1/2),(1/b*a)^(1/2))-b*EllipticF(sinh(f*x+e)
*(-b/a)^(1/2),(1/b*a)^(1/2))+2*b*EllipticE(sinh(f*x+e)*(-b/a)^(1/2),(1/b*a
)^(1/2))))/sinh(f*x+e)/(-b/a)^(1/2)/cosh(f*x+e)/(a+b*sinh(f*x+e)^2)^(1/2)/
f
```

Fricas [F]

$$\int \coth^2(e+fx)\sqrt{a+b\sinh^2(e+fx)}\,dx = \int \sqrt{b\sinh\left(fx+e\right)^2+a}\coth\left(fx+e\right)^2\,dx$$

input
```
integrate(coth(f*x+e)^2*(a+b*sinh(f*x+e)^2)^(1/2),x, algorithm="fricas")
```

output
```
integral(sqrt(b*sinh(f*x + e)^2 + a)*coth(f*x + e)^2, x)
```

Sympy [F]

$$\int \coth^2(e+fx)\sqrt{a+b\sinh^2(e+fx)}\,dx = \int \sqrt{a+b\sinh^2\left(e+fx\right)}\coth^2\left(e+fx\right)dx$$

input
```
integrate(coth(f*x+e)**2*(a+b*sinh(f*x+e)**2)**(1/2),x)
```

output
```
Integral(sqrt(a + b*sinh(e + f*x)**2)*coth(e + f*x)**2, x)
```

Maxima [F]

$$\int \coth^2(e+fx)\sqrt{a+b\sinh^2(e+fx)}\,dx = \int \sqrt{b\sinh{(fx+e)}^2+a}\coth{(fx+e)}^2\,dx$$

input
```
integrate(coth(f*x+e)^2*(a+b*sinh(f*x+e)^2)^(1/2),x, algorithm="maxima")
```

output
```
integrate(sqrt(b*sinh(f*x + e)^2 + a)*coth(f*x + e)^2, x)
```

Giac [F]

$$\int \coth^2(e+fx)\sqrt{a+b\sinh^2(e+fx)}\,dx = \int \sqrt{b\sinh{(fx+e)}^2+a}\coth{(fx+e)}^2\,dx$$

input
```
integrate(coth(f*x+e)^2*(a+b*sinh(f*x+e)^2)^(1/2),x, algorithm="giac")
```

output
```
integrate(sqrt(b*sinh(f*x + e)^2 + a)*coth(f*x + e)^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int \coth^2(e+fx)\sqrt{a+b\sinh^2(e+fx)}\,dx = \int \coth(e+f\,x)^2\,\sqrt{b\sinh{(e+f\,x)}^2+a}\,dx$$

input
```
int(coth(e + f*x)^2*(a + b*sinh(e + f*x)^2)^(1/2),x)
```

output
```
int(coth(e + f*x)^2*(a + b*sinh(e + f*x)^2)^(1/2), x)
```

Reduce [F]

$$\int \coth^2(e+fx)\sqrt{a+b\sinh^2(e+fx)}\,dx = \int \sqrt{\sinh\left(fx+e\right)^2 b+a}\,\coth\left(fx+e\right)^2 dx$$

input
```
int(coth(f*x+e)^2*(a+b*sinh(f*x+e)^2)^(1/2),x)
```

output
```
int(sqrt(sinh(e + f*x)**2*b + a)*coth(e + f*x)**2,x)
```

3.431 $\int \coth^4(e+fx)\sqrt{a+b\sinh^2(e+fx)}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 270

$$\begin{aligned}\int \coth^4(e+fx)\sqrt{a+b\sinh^2(e+fx)}\,dx &= -\frac{(3a+b)\coth(e+fx)\sqrt{a+b\sinh^2(e+fx)}}{3af}-\frac{\coth^3(e+fx)\sqrt{a+b\sinh^2(e+fx)}}{3f}\\ &\quad-\frac{(7a+b)E\left(\arctan(\sinh(e+fx))\,\middle|\,1-\frac{b}{a}\right)\operatorname{sech}(e+fx)\sqrt{a+b\sinh^2(e+fx)}}{3af\sqrt{\frac{\operatorname{sech}^2(e+fx)(a+b\sinh^2(e+fx))}{a}}}\\ &\quad+\frac{(3a+5b)\operatorname{EllipticF}\left(\arctan(\sinh(e+fx)),1-\frac{b}{a}\right)\operatorname{sech}(e+fx)\sqrt{a+b\sinh^2(e+fx)}}{3af\sqrt{\frac{\operatorname{sech}^2(e+fx)(a+b\sinh^2(e+fx))}{a}}}\\ &\quad+\frac{(7a+b)\sqrt{a+b\sinh^2(e+fx)}\tanh(e+fx)}{3af}\end{aligned}$$

output
```
-1/3*(3*a+b)*coth(f*x+e)*(a+b*sinh(f*x+e)^2)^(1/2)/a/f-1/3*coth(f*x+e)^3*(
a+b*sinh(f*x+e)^2)^(1/2)/f-1/3*(7*a+b)*EllipticE(sinh(f*x+e)/(1+sinh(f*x+e
)^2)^(1/2),(1-b/a)^(1/2))*sech(f*x+e)*(a+b*sinh(f*x+e)^2)^(1/2)/a/f/(sech(
f*x+e)^2*(a+b*sinh(f*x+e)^2)/a)^(1/2)+1/3*(3*a+5*b)*InverseJacobiAM(arctan
(sinh(f*x+e)),(1-b/a)^(1/2))*sech(f*x+e)*(a+b*sinh(f*x+e)^2)^(1/2)/a/f/(se
ch(f*x+e)^2*(a+b*sinh(f*x+e)^2)/a)^(1/2)+1/3*(7*a+b)*(a+b*sinh(f*x+e)^2)^(
1/2)*tanh(f*x+e)/a/f
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 2.31 (sec) , antiderivative size = 210, normalized size of antiderivative = 0.78

$$\int \coth^4(e+fx)\sqrt{a+b\sinh^2(e+fx)}\,dx$$
$$= \frac{-\frac{(-8a^2+4ab+3b^2+4(4a^2-2ab-b^2)\cosh(2(e+fx))+b(4a+b)\cosh(4(e+fx)))\coth(e+fx)\operatorname{csch}^2(e+fx)}{2\sqrt{2}} - 2ia(7a+b)\sqrt{\frac{2a-b+b\cosh}{a}}}{6af\sqrt{2a-b+b\cosh(2(e+}}$$

input
```
Integrate[Coth[e + f*x]^4*Sqrt[a + b*Sinh[e + f*x]^2],x]
```

output
```
(-1/2*((-8*a^2 + 4*a*b + 3*b^2 + 4*(4*a^2 - 2*a*b - b^2)*Cosh[2*(e + f*x)]
 + b*(4*a + b)*Cosh[4*(e + f*x)])*Coth[e + f*x]*Csch[e + f*x]^2)/Sqrt[2] -
 (2*I)*a*(7*a + b)*Sqrt[(2*a - b + b*Cosh[2*(e + f*x)])/a]*EllipticE[I*(e
+ f*x), b/a] + (8*I)*a*(a - b)*Sqrt[(2*a - b + b*Cosh[2*(e + f*x)])/a]*Ell
ipticF[I*(e + f*x), b/a])/(6*a*f*Sqrt[2*a - b + b*Cosh[2*(e + f*x)]])
```

Rubi [A] (verified)

Time = 0.51 (sec) , antiderivative size = 338, normalized size of antiderivative = 1.25, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.360$, Rules used = {3042, 3675, 375, 27, 442, 406, 320, 388, 313}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \coth^4(e+fx)\sqrt{a+b\sinh^2(e+fx)}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\sqrt{a-b\sin(ie+ifx)^2}}{\tan(ie+ifx)^4}dx$$

$$\Big\downarrow \text{3675}$$

$$\frac{\sqrt{\cosh^2(e+fx)}\text{sech}(e+fx)\int \text{csch}^4(e+fx)\left(\sinh^2(e+fx)+1\right)^{3/2}\sqrt{b\sinh^2(e+fx)+a}d\sinh(e+fx)}{f}$$

$$\Big\downarrow \text{375}$$

$$\frac{\sqrt{\cosh^2(e+fx)}\text{sech}(e+fx)\left(\frac{2}{3}\int \frac{\text{csch}^2(e+fx)\sqrt{\sinh^2(e+fx)+1}\left(4b\sinh^2(e+fx)+3a+b\right)}{2\sqrt{b\sinh^2(e+fx)+a}}d\sinh(e+fx)-\frac{1}{3}\left(\sinh^2(e+fx\right.\right.}{f}$$

$$\Big\downarrow \text{27}$$

$$\frac{\sqrt{\cosh^2(e+fx)}\text{sech}(e+fx)\left(\frac{1}{3}\int \frac{\text{csch}^2(e+fx)\sqrt{\sinh^2(e+fx)+1}\left(4b\sinh^2(e+fx)+3a+b\right)}{\sqrt{b\sinh^2(e+fx)+a}}d\sinh(e+fx)-\frac{1}{3}\left(\sinh^2(e+fx\right.\right.}{f}$$

$$\Big\downarrow \text{442}$$

$$\frac{\sqrt{\cosh^2(e+fx)}\text{sech}(e+fx)\left(\frac{1}{3}\left(\frac{\int \frac{b(7a+b)\sinh^2(e+fx)+a(3a+5b)}{\sqrt{\sinh^2(e+fx)+1}\sqrt{b\sinh^2(e+fx)+a}}d\sinh(e+fx)}{a}-\frac{(3a+b)\sqrt{\sinh^2(e+fx)+1}\text{csch}(e+fx)\sqrt{a+b\sinh\ldots}}{a}\right.\right.}{f}$$

$$\Big\downarrow \text{406}$$

$$\frac{\sqrt{\cosh^2(e+fx)}\text{sech}(e+fx)\left(\frac{1}{3}\left(\frac{a(3a+5b)\int \frac{1}{\sqrt{\sinh^2(e+fx)+1}\sqrt{b\sinh^2(e+fx)+a}}d\sinh(e+fx)+b(7a+b)\int \frac{\sinh^2(e+fx)}{\sqrt{\sinh^2(e+fx)+1}\sqrt{b\sinh^2(e+\ldots}}}{a}\right.\right.}{}$$

$$\Big\downarrow \text{320}$$

$$\sqrt{\cosh^2(e+fx)}\text{sech}(e+fx)\left(\frac{1}{3}\left(\frac{b(7a+b)\int\frac{\sinh^2(e+fx)}{\sqrt{\sinh^2(e+fx)+1}\sqrt{b\sinh^2(e+fx)+a}}d\sinh(e+fx)+\frac{(3a+5b)\sqrt{a+b\sinh^2(e+fx)}\text{EllipticF}\left(\arctan\right.}{\sqrt{\sinh^2(e+fx)+1}\sqrt{\frac{a+b\sinh^2(}{a\left(\sinh^2(e+\right.}}}}{a}\right.\right.$$

$\downarrow$ 388

$$\sqrt{\cosh^2(e+fx)}\text{sech}(e+fx)\left(\frac{1}{3}\left(\frac{b(7a+b)\left(\frac{\sinh(e+fx)\sqrt{a+b\sinh^2(e+fx)}}{b\sqrt{\sinh^2(e+fx)+1}}-\frac{\int\frac{\sqrt{b\sinh^2(e+fx)+a}}{\left(\sinh^2(e+fx)+1\right)^{3/2}}d\sinh(e+fx)}{b}\right)+\frac{(3a+5b)\sqrt{a+b\sinh^2(e+}}{\sqrt{\sinh^2(e+}}}{a}\right.\right.$$

$\downarrow$ 313

$$\sqrt{\cosh^2(e+fx)}\text{sech}(e+fx)\left(\frac{1}{3}\left(\frac{\frac{(3a+5b)\sqrt{a+b\sinh^2(e+fx)}\text{EllipticF}\left(\arctan(\sinh(e+fx)),1-\frac{b}{a}\right)}{\sqrt{\sinh^2(e+fx)+1}\sqrt{\frac{a+b\sinh^2(e+fx)}{a\left(\sinh^2(e+fx)+1\right)}}}+b(7a+b)\left(\frac{\sinh(e+fx)\sqrt{a+b\sinh^2(e+fx)}}{b\sqrt{\sinh^2(e+fx)+1}}-\right.}{a}\right.\right.$$

input
```
Int[Coth[e + f*x]^4*Sqrt[a + b*Sinh[e + f*x]^2],x]
```

output
```
(Sqrt[Cosh[e + f*x]^2]*Sech[e + f*x]*(-1/3*(Csch[e + f*x]^3*(1 + Sinh[e +
f*x]^2)^(3/2)*Sqrt[a + b*Sinh[e + f*x]^2]) + (-(((3*a + b)*Csch[e + f*x]*S
qrt[1 + Sinh[e + f*x]^2]*Sqrt[a + b*Sinh[e + f*x]^2])/a) + (((3*a + 5*b)*E
llipticF[ArcTan[Sinh[e + f*x]], 1 - b/a]*Sqrt[a + b*Sinh[e + f*x]^2])/(Sqr
t[1 + Sinh[e + f*x]^2]*Sqrt[(a + b*Sinh[e + f*x]^2)/(a*(1 + Sinh[e + f*x]^
2))]) + b*(7*a + b)*((Sinh[e + f*x]*Sqrt[a + b*Sinh[e + f*x]^2])/(b*Sqrt[1
 + Sinh[e + f*x]^2]) - (EllipticE[ArcTan[Sinh[e + f*x]], 1 - b/a]*Sqrt[a +
 b*Sinh[e + f*x]^2])/(b*Sqrt[1 + Sinh[e + f*x]^2]*Sqrt[(a + b*Sinh[e + f*x
]^2)/(a*(1 + Sinh[e + f*x]^2))]))))/a)/3))/f
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 313
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/((c_) + (d_.)*(x_)^2)^(3/2), x_Symbol] :> Sim
p[(Sqrt[a + b*x^2]/(c*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(c
+ d*x^2)))]))*EllipticE[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; FreeQ
[{a, b, c, d}, x] && PosQ[b/a] && PosQ[d/c]
```

rule 320
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(Sqrt[a + b*x^2]/(a*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(
c + d*x^2)))]))*EllipticF[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; Fre
eQ[{a, b, c, d}, x] && PosQ[d/c] && PosQ[b/a] &&  !SimplerSqrtQ[b/a, d/c]
```

rule 375
```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_)
, x_Symbol] :> Simp[(e*x)^(m + 1)*(a + b*x^2)^p*((c + d*x^2)^q/(e*(m + 1)))
, x] - Simp[2/(e^2*(m + 1))   Int[(e*x)^(m + 2)*(a + b*x^2)^(p - 1)*(c + d*
x^2)^(q - 1)*Simp[b*c*p + a*d*q + b*d*(p + q)*x^2, x], x], x] /; FreeQ[{a,
b, c, d, e}, x] && NeQ[b*c - a*d, 0] && GtQ[q, 0] && LtQ[m, -1] && GtQ[p, 0
] && IntBinomialQ[a, b, c, d, e, m, 2, p, q, x]
```

rule 388
```
Int[(x_)^2/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol]
 :> Simp[x*(Sqrt[a + b*x^2]/(b*Sqrt[c + d*x^2])), x] - Simp[c/b   Int[Sqrt[
a + b*x^2]/(c + d*x^2)^(3/2), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
 a*d, 0] && PosQ[b/a] && PosQ[d/c] &&  !SimplerSqrtQ[b/a, d/c]
```

rule 406
```
Int[((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.)*((e_) + (f_.)*(
x_)^2), x_Symbol] :> Simp[e   Int[(a + b*x^2)^p*(c + d*x^2)^q, x], x] + Sim
p[f   Int[x^2*(a + b*x^2)^p*(c + d*x^2)^q, x], x] /; FreeQ[{a, b, c, d, e,
f, p, q}, x]
```

rule 442
```
Int[((g_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_
.)*((e_) + (f_.)*(x_)^2), x_Symbol] :> Simp[e*(g*x)^(m + 1)*(a + b*x^2)^(p
+ 1)*((c + d*x^2)^q/(a*g*(m + 1))), x] - Simp[1/(a*g^2*(m + 1))   Int[(g*x)
^(m + 2)*(a + b*x^2)^p*(c + d*x^2)^(q - 1)*Simp[c*(b*e - a*f)*(m + 1) + e*2
*(b*c*(p + 1) + a*d*q) + d*((b*e - a*f)*(m + 1) + b*e*2*(p + q + 1))*x^2, x
], x], x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] && GtQ[q, 0] && LtQ[m, -1]
&&  !(EqQ[q, 1] && SimplerQ[e + f*x^2, c + d*x^2])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3675
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_.)*tan[(e_.) + (f_.)*(x_)]^
(m_), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Simp[ff^(m + 1
)*(Sqrt[Cos[e + f*x]^2]/(f*Cos[e + f*x]))   Subst[Int[x^m*((a + b*ff^2*x^2)
^p/(1 - ff^2*x^2)^((m + 1)/2)), x], x, Sin[e + f*x]/ff], x]] /; FreeQ[{a, b
, e, f, p}, x] && IntegerQ[m/2] &&  !IntegerQ[p]
```

Maple [A] (verified)

Time = 5.00 (sec) , antiderivative size = 522, normalized size of antiderivative = 1.93

method	result
default	$\frac{-4\sqrt{-\frac{b}{a}}\,ab\sinh(fx+e)^6-\sqrt{-\frac{b}{a}}\,b^2\sinh(fx+e)^6+3a^2\sqrt{\frac{a+b\sinh(fx+e)^2}{a}}\,\sqrt{\frac{\cosh(2fx+2e)}{2}+\frac{1}{2}}\ \text{EllipticF}\left(\sinh(fx+e)\sqrt{-\frac{b}{a}},\sqrt{\frac{a}{b}}\right)\sin}{}$

input
```
int(coth(f*x+e)^4*(a+b*sinh(f*x+e)^2)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/3*(-4*(-b/a)^(1/2)*a*b*sinh(f*x+e)^6-(-b/a)^(1/2)*b^2*sinh(f*x+e)^6+3*a^
2*((a+b*sinh(f*x+e)^2)/a)^(1/2)*(cosh(f*x+e)^2)^(1/2)*EllipticF(sinh(f*x+e
)*(-b/a)^(1/2),(1/b*a)^(1/2))*sinh(f*x+e)^3-2*b*((a+b*sinh(f*x+e)^2)/a)^(1
/2)*(cosh(f*x+e)^2)^(1/2)*EllipticF(sinh(f*x+e)*(-b/a)^(1/2),(1/b*a)^(1/2)
)*a*sinh(f*x+e)^3-((a+b*sinh(f*x+e)^2)/a)^(1/2)*(cosh(f*x+e)^2)^(1/2)*Elli
pticF(sinh(f*x+e)*(-b/a)^(1/2),(1/b*a)^(1/2))*b^2*sinh(f*x+e)^3+7*((a+b*si
nh(f*x+e)^2)/a)^(1/2)*(cosh(f*x+e)^2)^(1/2)*EllipticE(sinh(f*x+e)*(-b/a)^(
1/2),(1/b*a)^(1/2))*a*b*sinh(f*x+e)^3+((a+b*sinh(f*x+e)^2)/a)^(1/2)*(cosh(
f*x+e)^2)^(1/2)*EllipticE(sinh(f*x+e)*(-b/a)^(1/2),(1/b*a)^(1/2))*b^2*sinh
(f*x+e)^3-4*(-b/a)^(1/2)*a^2*sinh(f*x+e)^4-6*(-b/a)^(1/2)*a*b*sinh(f*x+e)^
4-(-b/a)^(1/2)*b^2*sinh(f*x+e)^4-5*(-b/a)^(1/2)*a^2*sinh(f*x+e)^2-2*(-b/a)
^(1/2)*a*b*sinh(f*x+e)^2-(-b/a)^(1/2)*a^2)/a/sinh(f*x+e)^3/(-b/a)^(1/2)/co
sh(f*x+e)/(a+b*sinh(f*x+e)^2)^(1/2)/f
```

Fricas [F]

$$\int \coth^4(e+fx)\sqrt{a+b\sinh^2(e+fx)}\,dx = \int \sqrt{b\sinh\left(fx+e\right)^2+a}\coth\left(fx+e\right)^4\,dx$$

input
```
integrate(coth(f*x+e)^4*(a+b*sinh(f*x+e)^2)^(1/2),x, algorithm="fricas")
```

output
```
integral(sqrt(b*sinh(f*x + e)^2 + a)*coth(f*x + e)^4, x)
```

Sympy [F]

$$\int \coth^4(e+fx)\sqrt{a+b\sinh^2(e+fx)}\,dx = \int \sqrt{a+b\sinh^2\left(e+fx\right)}\coth^4\left(e+fx\right)dx$$

input
```
integrate(coth(f*x+e)**4*(a+b*sinh(f*x+e)**2)**(1/2),x)
```

output
```
Integral(sqrt(a + b*sinh(e + f*x)**2)*coth(e + f*x)**4, x)
```

Maxima [F]

$$\int \coth^4(e+fx)\sqrt{a+b\sinh^2(e+fx)}\,dx = \int \sqrt{b\sinh(fx+e)^2+a}\coth(fx+e)^4\,dx$$

input
```
integrate(coth(f*x+e)^4*(a+b*sinh(f*x+e)^2)^(1/2),x, algorithm="maxima")
```

output
```
integrate(sqrt(b*sinh(f*x + e)^2 + a)*coth(f*x + e)^4, x)
```

Giac [F]

$$\int \coth^4(e+fx)\sqrt{a+b\sinh^2(e+fx)}\,dx = \int \sqrt{b\sinh(fx+e)^2+a}\coth(fx+e)^4\,dx$$

input
```
integrate(coth(f*x+e)^4*(a+b*sinh(f*x+e)^2)^(1/2),x, algorithm="giac")
```

output
```
integrate(sqrt(b*sinh(f*x + e)^2 + a)*coth(f*x + e)^4, x)
```

Mupad [F(-1)]

Timed out.

$$\int \coth^4(e+fx)\sqrt{a+b\sinh^2(e+fx)}\,dx = \int \coth(e+f\,x)^4\,\sqrt{b\sinh(e+f\,x)^2+a}\,dx$$

input
```
int(coth(e + f*x)^4*(a + b*sinh(e + f*x)^2)^(1/2),x)
```

output
```
int(coth(e + f*x)^4*(a + b*sinh(e + f*x)^2)^(1/2), x)
```

Reduce [F]

$$\int \coth^4(e+fx)\sqrt{a+b\sinh^2(e+fx)}\,dx = \int \sqrt{\sinh(fx+e)^2 b+a}\,\coth(fx+e)^4\,dx$$

input

```
int(coth(f*x+e)^4*(a+b*sinh(f*x+e)^2)^(1/2),x)
```

output

```
int(sqrt(sinh(e + f*x)**2*b + a)*coth(e + f*x)**4,x)
```

3.432 $\int \left(a+b\sinh^2(e+fx)\right)^{3/2}\tanh^5(e+fx)\,dx$

Optimal result

Integrand size = 25, antiderivative size = 232

$$\int \left(a+b\sinh^2(e+fx)\right)^{3/2}\tanh^5(e+fx)\,dx = -\frac{(8a^2-40ab+35b^2)\,\text{arctanh}\left(\frac{\sqrt{a+b\sinh^2(e+fx)}}{\sqrt{a-b}}\right)}{8\sqrt{a-b}f} + \frac{(8a^2-40ab+35b^2)\sqrt{a+b\sinh^2(e+fx)}}{8(a-b)f} + \frac{(8a^2-40ab+35b^2)\left(a+b\sinh^2(e+fx)\right)^{3/2}}{24(a-b)^2f} + \frac{(8a-9b)\text{sech}^2(e+fx)\left(a+b\sinh^2(e+fx)\right)^{5/2}}{8(a-b)^2f} - \frac{\text{sech}^4(e+fx)\left(a+b\sinh^2(e+fx)\right)^{5/2}}{4(a-b)f}$$

output
```
-1/8*(8*a^2-40*a*b+35*b^2)*arctanh((a+b*sinh(f*x+e)^2)^(1/2)/(a-b)^(1/2))/
(a-b)^(1/2)/f+1/8*(8*a^2-40*a*b+35*b^2)*(a+b*sinh(f*x+e)^2)^(1/2)/(a-b)/f+
1/24*(8*a^2-40*a*b+35*b^2)*(a+b*sinh(f*x+e)^2)^(3/2)/(a-b)^2/f+1/8*(8*a-9*
b)*sech(f*x+e)^2*(a+b*sinh(f*x+e)^2)^(5/2)/(a-b)^2/f-1/4*sech(f*x+e)^4*(a+
b*sinh(f*x+e)^2)^(5/2)/(a-b)/f
```

Mathematica [A] (verified)

Time = 1.09 (sec) , antiderivative size = 169, normalized size of antiderivative = 0.73

$$\int \left(a+b\sinh^2(e+fx)\right)^{3/2}\tanh^5(e+fx)\,dx =$$

$$-\frac{-3(8a-9b)\text{sech}^2(e+fx)\left(a+b\sinh^2(e+fx)\right)^{5/2}+6(a-b)\text{sech}^4(e+fx)\left(a+b\sinh^2(e+fx)\right)^{5/2}-(\cdots}{2\cdots}$$

input
```
Integrate[(a + b*Sinh[e + f*x]^2)^(3/2)*Tanh[e + f*x]^5,x]
```

output
```
-1/24*(-3*(8*a - 9*b)*Sech[e + f*x]^2*(a + b*Sinh[e + f*x]^2)^(5/2) + 6*(a
 - b)*Sech[e + f*x]^4*(a + b*Sinh[e + f*x]^2)^(5/2) - (8*a^2 - 40*a*b + 35
*b^2)*(-3*(a - b)^(3/2)*ArcTanh[Sqrt[a + b*Sinh[e + f*x]^2]/Sqrt[a - b]] +
 Sqrt[a + b*Sinh[e + f*x]^2]*(4*a - 3*b + b*Sinh[e + f*x]^2)))/((a - b)^2*
f)
```

Rubi [A] (verified)

Time = 0.44 (sec) , antiderivative size = 213, normalized size of antiderivative = 0.92, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.400$, Rules used = {3042, 26, 3673, 100, 27, 87, 60, 60, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \tanh^5(e+fx)\left(a+b\sinh^2(e+fx)\right)^{3/2}dx$$

$$\Big\downarrow \text{3042}$$

$$\int -i\tan(ie+ifx)^5\left(a-b\sin(ie+ifx)^2\right)^{3/2}dx$$

$$\Big\downarrow \text{26}$$

$$-i\int \left(a-b\sin(ie+ifx)^2\right)^{3/2}\tan(ie+ifx)^5dx$$

$$\Big\downarrow \text{3673}$$

$$\frac{\int \frac{\sinh^4(e+fx)\left(b\sinh^2(e+fx)+a\right)^{3/2}}{\left(\sinh^2(e+fx)+1\right)^3}d\sinh^2(e+fx)}{2f}$$

$$\Big\downarrow \; 100$$

$$\frac{\frac{\int -\frac{\left(-4(a-b)\sinh^2(e+fx)+4a-5b\right)\left(b\sinh^2(e+fx)+a\right)^{3/2}}{2\left(\sinh^2(e+fx)+1\right)^2}d\sinh^2(e+fx)}{2(a-b)}-\frac{\left(a+b\sinh^2(e+fx)\right)^{5/2}}{2(a-b)\left(\sinh^2(e+fx)+1\right)^2}}{2f}$$

$$\Big\downarrow \; 27$$

$$\frac{-\frac{\int \frac{\left(-4(a-b)\sinh^2(e+fx)+4a-5b\right)\left(b\sinh^2(e+fx)+a\right)^{3/2}}{\left(\sinh^2(e+fx)+1\right)^2}d\sinh^2(e+fx)}{4(a-b)}-\frac{\left(a+b\sinh^2(e+fx)\right)^{5/2}}{2(a-b)\left(\sinh^2(e+fx)+1\right)^2}}{2f}$$

$$\Big\downarrow \; 87$$

$$\frac{-\frac{-\frac{\left(8a^2-40ab+35b^2\right)\int \frac{\left(b\sinh^2(e+fx)+a\right)^{3/2}}{\sinh^2(e+fx)+1}d\sinh^2(e+fx)}{2(a-b)}-\frac{(8a-9b)\left(a+b\sinh^2(e+fx)\right)^{5/2}}{(a-b)\left(\sinh^2(e+fx)+1\right)}}{4(a-b)}-\frac{\left(a+b\sinh^2(e+fx)\right)^{5/2}}{2(a-b)\left(\sinh^2(e+fx)+1\right)^2}}{2f}$$

$$\Big\downarrow \; 60$$

$$\frac{-\frac{-\frac{\left(8a^2-40ab+35b^2\right)\left((a-b)\int \frac{\sqrt{b\sinh^2(e+fx)+a}}{\sinh^2(e+fx)+1}d\sinh^2(e+fx)+\frac{2}{3}\left(a+b\sinh^2(e+fx)\right)^{3/2}\right)}{2(a-b)}-\frac{(8a-9b)\left(a+b\sinh^2(e+fx)\right)^{5/2}}{(a-b)\left(\sinh^2(e+fx)+1\right)}}{4(a-b)}-\frac{\left(a+b\sinh^2(e+fx)\right)^{5/2}}{2(a-b)\left(\sinh^2(e+fx)+1\right)^2}}{2f}$$

$$\Big\downarrow \; 60$$

$$\frac{-\frac{-\frac{\left(8a^2-40ab+35b^2\right)\left((a-b)\left((a-b)\int \frac{1}{\left(\sinh^2(e+fx)+1\right)\sqrt{b\sinh^2(e+fx)+a}}d\sinh^2(e+fx)+2\sqrt{a+b\sinh^2(e+fx)}\right)+\frac{2}{3}\left(a+b\sinh^2(e+fx)\right)^{3/2}\right)}{2(a-b)}-\frac{(8a-9b)\left(a+b\sinh^2(e+fx)\right)^{5/2}}{(a-b)\left(\sinh^2(e+fx)+1\right)}}{4(a-b)}-\frac{\left(a+b\sinh^2(e+fx)\right)^{5/2}}{2(a-b)\left(\sinh^2(e+fx)+1\right)^2}}{2f}$$

$$\Big\downarrow \; 73$$

$$-\frac{-\frac{\left(8a^2-40ab+35b^2\right)\left((a-b)\left(\frac{2(a-b)\int \frac{1}{\frac{\sinh^4(e+fx)}{b}-\frac{a}{b}+1}d\sqrt{b\sinh^2(e+fx)+a}}{b}+2\sqrt{a+b\sinh^2(e+fx)}\right)+\frac{2}{3}\left(a+b\sinh^2(e+fx)\right)^{3/2}\right)}{2(a-b)}-\frac{(8a-9b)\left(a+b\sinh^2(e\right.}{(a-b)\left(\sinh^2(e+f\right.}}{4(a-b)}}{2f}$$

$\downarrow$ 221

$$-\frac{-\frac{\left(8a^2-40ab+35b^2\right)\left((a-b)\left(2\sqrt{a+b\sinh^2(e+fx)}-2\sqrt{a-b}\,\text{arctanh}\left(\frac{\sqrt{a+b\sinh^2(e+fx)}}{\sqrt{a-b}}\right)\right)+\frac{2}{3}\left(a+b\sinh^2(e+fx)\right)^{3/2}\right)}{2(a-b)}-\frac{(8a-9b)\left(a+b\sinh^2(e+fx)\right)^{5/}}{(a-b)\left(\sinh^2(e+fx)+1\right)}}{4(a-b)}}{2f}$$

input
```
Int[(a + b*Sinh[e + f*x]^2)^(3/2)*Tanh[e + f*x]^5,x]
```

output
```
(-1/2*(a + b*Sinh[e + f*x]^2)^(5/2)/((a - b)*(1 + Sinh[e + f*x]^2)^2) - (-
(((8*a - 9*b)*(a + b*Sinh[e + f*x]^2)^(5/2))/((a - b)*(1 + Sinh[e + f*x]^2
))) - ((8*a^2 - 40*a*b + 35*b^2)*((2*(a + b*Sinh[e + f*x]^2)^(3/2))/3 + (a
 - b)*(-2*Sqrt[a - b]*ArcTanh[Sqrt[a + b*Sinh[e + f*x]^2]/Sqrt[a - b]] + 2
*Sqrt[a + b*Sinh[e + f*x]^2])))/(2*(a - b)))/(4*(a - b)))/(2*f)
```

Defintions of rubi rules used

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 60
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Simp[n*((b*c - a*d)/(
b*(m + n + 1)))   Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
 c, d}, x] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ[m, 0] && ( !Integer
Q[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n + 2, 0] && IntLinear
Q[a, b, c, d, m, n, x]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 87
```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_] :> Simp[(-(b*e - a*f))*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(f*(p
+ 1)*(c*f - d*e))), x] - Simp[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p
+ 1)))/(f*(p + 1)*(c*f - d*e))   Int[(c + d*x)^n*(e + f*x)^(p + 1), x], x]
/; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || Intege
rQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))))
```

rule 100
```
Int[((a_.) + (b_.)*(x_))^2*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))^(
p_), x_] :> Simp[(b*c - a*d)^2*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(d^2*(d
*e - c*f)*(n + 1))), x] - Simp[1/(d^2*(d*e - c*f)*(n + 1))   Int[(c + d*x)^
(n + 1)*(e + f*x)^p*Simp[a^2*d^2*f*(n + p + 2) + b^2*c*(d*e*(n + 1) + c*f*(
p + 1)) - 2*a*b*d*(d*e*(n + 1) + c*f*(p + 1)) - b^2*d*(d*e - c*f)*(n + 1)*x
, x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && (LtQ[n, -1] || (EqQ[n
 + p + 3, 0] && NeQ[n, -1] && (SumSimplerQ[n, 1] ||  !SumSimplerQ[p, 1])))
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3673
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_.)*tan[(e_.) + (f_.)*(x_)]^
(m_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x]^2, x]}, Simp[ff^((m
 + 1)/2)/(2*f)   Subst[Int[x^((m - 1)/2)*((a + b*ff*x)^p/(1 - ff*x)^((m + 1
)/2)), x], x, Sin[e + f*x]^2/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && Integ
erQ[(m - 1)/2]
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.62 (sec) , antiderivative size = 71, normalized size of antiderivative = 0.31

method	result	size
default	$\frac{\text{`int/indef0`}\left(\frac{\sinh(fx+e)^5\left(b^2\sinh(fx+e)^4+2\sinh(fx+e)^2ab+a^2\right)}{\cosh(fx+e)^6\sqrt{a+b\sinh(fx+e)^2}},\sinh(fx+e)\right)}{f}$	71

input
```
int((a+b*sinh(f*x+e)^2)^(3/2)*tanh(f*x+e)^5,x,method=_RETURNVERBOSE)
```

output
```
`int/indef0`(sinh(f*x+e)^5*(b^2*sinh(f*x+e)^4+2*sinh(f*x+e)^2*a*b+a^2)/cos
h(f*x+e)^6/(a+b*sinh(f*x+e)^2)^(1/2),sinh(f*x+e))/f
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 3196 vs. 2(208) = 416.

Time = 0.66 (sec) , antiderivative size = 6484, normalized size of antiderivative = 27.95

$$\int \left(a+b\sinh^2(e+fx)\right)^{3/2}\tanh^5(e+fx)\,dx=\text{Too large to display}$$

input
```
integrate((a+b*sinh(f*x+e)^2)^(3/2)*tanh(f*x+e)^5,x, algorithm="fricas")
```

output
```
Too large to include
```

Sympy [F(-1)]

Timed out.

$$\int \left(a+b\sinh^2(e+fx)\right)^{3/2}\tanh^5(e+fx)\,dx = \text{Timed out}$$

input
```
integrate((a+b*sinh(f*x+e)**2)**(3/2)*tanh(f*x+e)**5,x)
```

output
```
Timed out
```

Maxima [F]

$$\int \left(a+b\sinh^2(e+fx)\right)^{3/2}\tanh^5(e+fx)\,dx = \int \left(b\sinh\left(fx+e\right)^2+a\right)^{\frac{3}{2}}\tanh\left(fx+e\right)^5\,dx$$

input
```
integrate((a+b*sinh(f*x+e)^2)^(3/2)*tanh(f*x+e)^5,x, algorithm="maxima")
```

output
```
integrate((b*sinh(f*x + e)^2 + a)^(3/2)*tanh(f*x + e)^5, x)
```

Giac [F]

$$\int \left(a+b\sinh^2(e+fx)\right)^{3/2}\tanh^5(e+fx)\,dx = \int \left(b\sinh\left(fx+e\right)^2+a\right)^{\frac{3}{2}}\tanh\left(fx+e\right)^5\,dx$$

input
```
integrate((a+b*sinh(f*x+e)^2)^(3/2)*tanh(f*x+e)^5,x, algorithm="giac")
```

output
```
integrate((b*sinh(f*x + e)^2 + a)^(3/2)*tanh(f*x + e)^5, x)
```

Mupad [F(-1)]

Timed out.

$$\int (a+b\sinh^2(e+fx))^{3/2}\tanh^5(e+fx)\,dx = \int \tanh(e+f\,x)^5\,\left(b\sinh(e+f\,x)^2+a\right)^{3/2}\,dx$$

input
```
int(tanh(e + f*x)^5*(a + b*sinh(e + f*x)^2)^(3/2),x)
```

output
```
int(tanh(e + f*x)^5*(a + b*sinh(e + f*x)^2)^(3/2), x)
```

Reduce [F]

$$\int (a+b\sinh^2(e+fx))^{3/2}\tanh^5(e+fx)\,dx = \left(\int \sqrt{\sinh(fx+e)^2 b+a}\,\sinh(fx+e)^2\tanh(fx+e)^5\,dx\right) b + \left(\int \sqrt{\sinh(fx+e)^2 b+a}\,\tanh(fx+e)^5\,dx\right) a$$

input
```
int((a+b*sinh(f*x+e)^2)^(3/2)*tanh(f*x+e)^5,x)
```

output
```
int(sqrt(sinh(e + f*x)**2*b + a)*sinh(e + f*x)**2*tanh(e + f*x)**5,x)*b +
int(sqrt(sinh(e + f*x)**2*b + a)*tanh(e + f*x)**5,x)*a
```

3.433 $\int \left(a+b\sinh^2(e+fx)\right)^{3/2}\tanh^3(e+fx)\,dx$

Optimal result

Integrand size = 25, antiderivative size = 156

$$\int \left(a+b\sinh^2(e+fx)\right)^{3/2}\tanh^3(e+fx)\,dx = -\frac{(2a-5b)\sqrt{a-b}\,\text{arctanh}\left(\frac{\sqrt{a+b\sinh^2(e+fx)}}{\sqrt{a-b}}\right)}{2f}+\frac{(2a-5b)\sqrt{a+b\sinh^2(e+fx)}}{2f}+\frac{(2a-5b)\left(a+b\sinh^2(e+fx)\right)^{3/2}}{6(a-b)f}+\frac{\text{sech}^2(e+fx)\left(a+b\sinh^2(e+fx)\right)^{5/2}}{2(a-b)f}$$

output
```
-1/2*(2*a-5*b)*(a-b)^(1/2)*arctanh((a+b*sinh(f*x+e)^2)^(1/2)/(a-b)^(1/2))/
f+1/2*(2*a-5*b)*(a+b*sinh(f*x+e)^2)^(1/2)/f+1/6*(2*a-5*b)*(a+b*sinh(f*x+e)
^2)^(3/2)/(a-b)/f+1/2*sech(f*x+e)^2*(a+b*sinh(f*x+e)^2)^(5/2)/(a-b)/f
```

Mathematica [A] (verified)

Time = 0.37 (sec) , antiderivative size = 122, normalized size of antiderivative = 0.78

$$\int \left(a+b\sinh^2(e+fx)\right)^{3/2}\tanh^3(e+fx)\,dx=\frac{3\text{sech}^2(e+fx)\left(a+b\sinh^2(e+fx)\right)^{5/2}+(2a-5b)\left(-3(a-b)^{3/2}\text{arctanh}\left(\frac{\sqrt{a+b\sinh^2(e+fx)}}{\sqrt{a-b}}\right)+\right.}{6(a-b)f}$$

input
```
Integrate[(a + b*Sinh[e + f*x]^2)^(3/2)*Tanh[e + f*x]^3,x]
```

output
```
(3*Sech[e + f*x]^2*(a + b*Sinh[e + f*x]^2)^(5/2) + (2*a - 5*b)*(-3*(a - b)
^(3/2)*ArcTanh[Sqrt[a + b*Sinh[e + f*x]^2]/Sqrt[a - b]] + Sqrt[a + b*Sinh[
e + f*x]^2]*(4*a - 3*b + b*Sinh[e + f*x]^2)))/(6*(a - b)*f)
```

Rubi [A] (verified)

Time = 0.34 (sec) , antiderivative size = 146, normalized size of antiderivative = 0.94, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.320, Rules used = {3042, 26, 3673, 87, 60, 60, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \tanh^3(e+fx)\left(a+b\sinh^2(e+fx)\right)^{3/2}dx$$

$$\Big\downarrow \text{3042}$$

$$\int i\tan(ie+ifx)^3\left(a-b\sin(ie+ifx)^2\right)^{3/2}dx$$

$$\Big\downarrow \text{26}$$

$$i\int \left(a-b\sin(ie+ifx)^2\right)^{3/2}\tan(ie+ifx)^3dx$$

$$\Big\downarrow \text{3673}$$

$$\frac{\int \frac{\sinh^2(e+fx)\left(b\sinh^2(e+fx)+a\right)^{3/2}}{\left(\sinh^2(e+fx)+1\right)^2}d\sinh^2(e+fx)}{2f}$$

$$\Big\downarrow \; 87$$

$$\frac{\frac{(2a-5b)\int \frac{\left(b\sinh^2(e+fx)+a\right)^{3/2}}{\sinh^2(e+fx)+1}d\sinh^2(e+fx)}{2(a-b)}+\frac{\left(a+b\sinh^2(e+fx)\right)^{5/2}}{(a-b)\left(\sinh^2(e+fx)+1\right)}}{2f}$$

$$\Big\downarrow \; 60$$

$$\frac{\frac{(2a-5b)\left((a-b)\int \frac{\sqrt{b\sinh^2(e+fx)+a}}{\sinh^2(e+fx)+1}d\sinh^2(e+fx)+\frac{2}{3}\left(a+b\sinh^2(e+fx)\right)^{3/2}\right)}{2(a-b)}+\frac{\left(a+b\sinh^2(e+fx)\right)^{5/2}}{(a-b)\left(\sinh^2(e+fx)+1\right)}}{2f}$$

$$\Big\downarrow \; 60$$

$$\frac{\frac{(2a-5b)\left((a-b)\left((a-b)\int \frac{1}{\left(\sinh^2(e+fx)+1\right)\sqrt{b\sinh^2(e+fx)+a}}d\sinh^2(e+fx)+2\sqrt{a+b\sinh^2(e+fx)}\right)+\frac{2}{3}\left(a+b\sinh^2(e+fx)\right)^{3/2}\right)}{2(a-b)}+\frac{(a+b\sinh^2(}{(a-b)(\sinh^2}}{2f}$$

$$\Big\downarrow \; 73$$

$$\frac{\frac{(2a-5b)\left((a-b)\left(\frac{2(a-b)\int \frac{1}{\frac{\sinh^4(e+fx)}{b}-\frac{a}{b}+1}d\sqrt{b\sinh^2(e+fx)+a}}{b}+2\sqrt{a+b\sinh^2(e+fx)}\right)+\frac{2}{3}\left(a+b\sinh^2(e+fx)\right)^{3/2}\right)}{2(a-b)}+\frac{\left(a+b\sinh^2(e+fx)\right)^{5/}}{(a-b)\left(\sinh^2(e+fx)+1\right.}}{2f}$$

$$\Big\downarrow \; 221$$

$$\frac{\frac{(2a-5b)\left((a-b)\left(2\sqrt{a+b\sinh^2(e+fx)}-2\sqrt{a-b}\,\text{arctanh}\left(\frac{\sqrt{a+b\sinh^2(e+fx)}}{\sqrt{a-b}}\right)\right)+\frac{2}{3}\left(a+b\sinh^2(e+fx)\right)^{3/2}\right)}{2(a-b)}+\frac{\left(a+b\sinh^2(e+fx)\right)^{5/2}}{(a-b)\left(\sinh^2(e+fx)+1\right)}}{2f}$$

input
```
Int[(a + b*Sinh[e + f*x]^2)^(3/2)*Tanh[e + f*x]^3,x]
```

output
```
((a + b*Sinh[e + f*x]^2)^(5/2)/((a - b)*(1 + Sinh[e + f*x]^2)) + ((2*a - 5
*b)*((2*(a + b*Sinh[e + f*x]^2)^(3/2))/3 + (a - b)*(-2*Sqrt[a - b]*ArcTanh
[Sqrt[a + b*Sinh[e + f*x]^2]/Sqrt[a - b]] + 2*Sqrt[a + b*Sinh[e + f*x]^2])
))/(2*(a - b)))/(2*f)
```

Defintions of rubi rules used

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 60
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Simp[n*((b*c - a*d)/(
b*(m + n + 1)))   Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
 c, d}, x] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ[m, 0] && ( !Integer
Q[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n + 2, 0] && IntLinear
Q[a, b, c, d, m, n, x]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 87
```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_] :> Simp[(-(b*e - a*f))*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(f*(p
+ 1)*(c*f - d*e))), x] - Simp[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p
+ 1)))/(f*(p + 1)*(c*f - d*e))   Int[(c + d*x)^n*(e + f*x)^(p + 1), x], x]
/; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || Intege
rQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))))
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3673
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_.)*tan[(e_.) + (f_.)*(x_)]^
(m_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x]^2, x]}, Simp[ff^((m
 + 1)/2)/(2*f)   Subst[Int[x^((m - 1)/2)*((a + b*ff*x)^p/(1 - ff*x)^((m + 1
)/2)), x], x, Sin[e + f*x]^2/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && Integ
erQ[(m - 1)/2]
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.47 (sec) , antiderivative size = 71, normalized size of antiderivative = 0.46

method	result	size
default	$\frac{\text{`int/indef0`}\left(\frac{\sinh(fx+e)^3\left(b^2\sinh(fx+e)^4+2\sinh(fx+e)^2ab+a^2\right)}{\cosh(fx+e)^4\sqrt{a+b\sinh(fx+e)^2}},\sinh(fx+e)\right)}{f}$	71

input
```
int((a+b*sinh(f*x+e)^2)^(3/2)*tanh(f*x+e)^3,x,method=_RETURNVERBOSE)
```

output
```
`int/indef0`(sinh(f*x+e)^3*(b^2*sinh(f*x+e)^4+2*sinh(f*x+e)^2*a*b+a^2)/cos
h(f*x+e)^4/(a+b*sinh(f*x+e)^2)^(1/2),sinh(f*x+e))/f
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1233 vs. 2(136) = 272.

Time = 0.54 (sec) , antiderivative size = 2558, normalized size of antiderivative = 16.40

$$\int \left(a+b\sinh^2(e+fx)\right)^{3/2}\tanh^3(e+fx)\,dx=\text{Too large to display}$$

input
```
integrate((a+b*sinh(f*x+e)^2)^(3/2)*tanh(f*x+e)^3,x, algorithm="fricas")
```

output
```
[-1/24*(6*((2*a - 5*b)*cosh(f*x + e)^7 + 7*(2*a - 5*b)*cosh(f*x + e)*sinh(
f*x + e)^6 + (2*a - 5*b)*sinh(f*x + e)^7 + 2*(2*a - 5*b)*cosh(f*x + e)^5 +
 (21*(2*a - 5*b)*cosh(f*x + e)^2 + 4*a - 10*b)*sinh(f*x + e)^5 + 5*(7*(2*a
 - 5*b)*cosh(f*x + e)^3 + 2*(2*a - 5*b)*cosh(f*x + e))*sinh(f*x + e)^4 + (
2*a - 5*b)*cosh(f*x + e)^3 + (35*(2*a - 5*b)*cosh(f*x + e)^4 + 20*(2*a - 5
*b)*cosh(f*x + e)^2 + 2*a - 5*b)*sinh(f*x + e)^3 + (21*(2*a - 5*b)*cosh(f*
x + e)^5 + 20*(2*a - 5*b)*cosh(f*x + e)^3 + 3*(2*a - 5*b)*cosh(f*x + e))*s
inh(f*x + e)^2 + (7*(2*a - 5*b)*cosh(f*x + e)^6 + 10*(2*a - 5*b)*cosh(f*x
+ e)^4 + 3*(2*a - 5*b)*cosh(f*x + e)^2)*sinh(f*x + e))*sqrt(a - b)*log((b*
cosh(f*x + e)^4 + 4*b*cosh(f*x + e)*sinh(f*x + e)^3 + b*sinh(f*x + e)^4 +
2*(4*a - 3*b)*cosh(f*x + e)^2 + 2*(3*b*cosh(f*x + e)^2 + 4*a - 3*b)*sinh(f
*x + e)^2 + 4*sqrt(2)*sqrt(a - b)*sqrt((b*cosh(f*x + e)^2 + b*sinh(f*x + e
)^2 + 2*a - b)/(cosh(f*x + e)^2 - 2*cosh(f*x + e)*sinh(f*x + e) + sinh(f*x
 + e)^2))*(cosh(f*x + e) + sinh(f*x + e)) + 4*(b*cosh(f*x + e)^3 + (4*a -
3*b)*cosh(f*x + e))*sinh(f*x + e) + b)/(cosh(f*x + e)^4 + 4*cosh(f*x + e)*
sinh(f*x + e)^3 + sinh(f*x + e)^4 + 2*(3*cosh(f*x + e)^2 + 1)*sinh(f*x + e
)^2 + 2*cosh(f*x + e)^2 + 4*(cosh(f*x + e)^3 + cosh(f*x + e))*sinh(f*x + e
) + 1)) - sqrt(2)*(b*cosh(f*x + e)^8 + 8*b*cosh(f*x + e)*sinh(f*x + e)^7 +
 b*sinh(f*x + e)^8 + 8*(2*a - 3*b)*cosh(f*x + e)^6 + 4*(7*b*cosh(f*x + e)^
2 + 4*a - 6*b)*sinh(f*x + e)^6 + 8*(7*b*cosh(f*x + e)^3 + 6*(2*a - 3*b)...
```

Sympy [F(-1)]

Timed out.

$$\int \left(a+b\sinh^2(e+fx)\right)^{3/2}\tanh^3(e+fx)\,dx=\text{Timed out}$$

input
```
integrate((a+b*sinh(f*x+e)**2)**(3/2)*tanh(f*x+e)**3,x)
```

output
```
Timed out
```

Maxima [F]

$$\int (a+b\sinh^2(e+fx))^{3/2}\tanh^3(e+fx)\,dx = \int \left(b\sinh\left(fx+e\right)^2+a\right)^{\frac{3}{2}}\tanh\left(fx+e\right)^3\,dx$$

input
```
integrate((a+b*sinh(f*x+e)^2)^(3/2)*tanh(f*x+e)^3,x, algorithm="maxima")
```

output
```
integrate((b*sinh(f*x + e)^2 + a)^(3/2)*tanh(f*x + e)^3, x)
```

Giac [F]

$$\int (a+b\sinh^2(e+fx))^{3/2}\tanh^3(e+fx)\,dx = \int \left(b\sinh\left(fx+e\right)^2+a\right)^{\frac{3}{2}}\tanh\left(fx+e\right)^3\,dx$$

input
```
integrate((a+b*sinh(f*x+e)^2)^(3/2)*tanh(f*x+e)^3,x, algorithm="giac")
```

output
```
integrate((b*sinh(f*x + e)^2 + a)^(3/2)*tanh(f*x + e)^3, x)
```

Mupad [F(-1)]

Timed out.

$$\int (a+b\sinh^2(e+fx))^{3/2}\tanh^3(e+fx)\,dx = \int \tanh(e+f\,x)^3\,\left(b\sinh(e+f\,x)^2+a\right)^{3/2}\,dx$$

input
```
int(tanh(e + f*x)^3*(a + b*sinh(e + f*x)^2)^(3/2),x)
```

output
```
int(tanh(e + f*x)^3*(a + b*sinh(e + f*x)^2)^(3/2), x)
```

Reduce [F]

$$\int (a+b\sinh^2(e+fx))^{3/2}\tanh^3(e+fx)\,dx = \left(\int \sqrt{\sinh(fx+e)^2 b+a}\,\sinh(fx+e)^2\tanh(fx+e)^3\,dx\right)b + \left(\int \sqrt{\sinh(fx+e)^2 b+a}\,\tanh(fx+e)^3\,dx\right)a$$

input
```
int((a+b*sinh(f*x+e)^2)^(3/2)*tanh(f*x+e)^3,x)
```

output
```
int(sqrt(sinh(e + f*x)**2*b + a)*sinh(e + f*x)**2*tanh(e + f*x)**3,x)*b +
int(sqrt(sinh(e + f*x)**2*b + a)*tanh(e + f*x)**3,x)*a
```

3.434 $\int \left(a+b\sinh^2(e+fx)\right)^{3/2}\tanh(e+fx)\,dx$

Optimal result

Integrand size = 23, antiderivative size = 90

$$\int \left(a+b\sinh^2(e+fx)\right)^{3/2}\tanh(e+fx)\,dx=-\frac{(a-b)^{3/2}\text{arctanh}\left(\frac{\sqrt{a+b\sinh^2(e+fx)}}{\sqrt{a-b}}\right)}{f}+\frac{(a-b)\sqrt{a+b\sinh^2(e+fx)}}{f}+\frac{\left(a+b\sinh^2(e+fx)\right)^{3/2}}{3f}$$

output
```
-(a-b)^(3/2)*arctanh((a+b*sinh(f*x+e)^2)^(1/2)/(a-b)^(1/2))/f+(a-b)*(a+b*s
inh(f*x+e)^2)^(1/2)/f+1/3*(a+b*sinh(f*x+e)^2)^(3/2)/f
```

Mathematica [A] (verified)

Time = 0.11 (sec) , antiderivative size = 86, normalized size of antiderivative = 0.96

$$\int \left(a+b\sinh^2(e+fx)\right)^{3/2}\tanh(e+fx)\,dx=\frac{-3(a-b)^{3/2}\text{arctanh}\left(\frac{\sqrt{a-b+b\cosh^2(e+fx)}}{\sqrt{a-b}}\right)+\left(4a-4b+b\cosh^2(e+fx)\right)\sqrt{a-b+b\cosh^2(e+}}{3f}$$

input
```
Integrate[(a + b*Sinh[e + f*x]^2)^(3/2)*Tanh[e + f*x],x]
```

output
```
(-3*(a - b)^(3/2)*ArcTanh[Sqrt[a - b + b*Cosh[e + f*x]^2]/Sqrt[a - b]] + (
4*a - 4*b + b*Cosh[e + f*x]^2)*Sqrt[a - b + b*Cosh[e + f*x]^2])/(3*f)
```

Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 91, normalized size of antiderivative = 1.01, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.304$, Rules used = {3042, 26, 3673, 60, 60, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \tanh(e+fx)\left(a+b\sinh^2(e+fx)\right)^{3/2}dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int -i\tan(ie+ifx)\left(a-b\sin(ie+ifx)^2\right)^{3/2}dx$$

$$\Big\downarrow \text{ 26}$$

$$-i\int \left(a-b\sin(ie+ifx)^2\right)^{3/2}\tan(ie+ifx)dx$$

$$\Big\downarrow \text{ 3673}$$

$$\frac{\int \frac{\left(b\sinh^2(e+fx)+a\right)^{3/2}}{\sinh^2(e+fx)+1}d\sinh^2(e+fx)}{2f}$$

$$\Big\downarrow \text{ 60}$$

$$\frac{(a-b)\int \frac{\sqrt{b\sinh^2(e+fx)+a}}{\sinh^2(e+fx)+1}d\sinh^2(e+fx)+\frac{2}{3}\left(a+b\sinh^2(e+fx)\right)^{3/2}}{2f}$$

$$\Big\downarrow \text{ 60}$$

$$\frac{(a-b)\left((a-b)\int \frac{1}{\left(\sinh^2(e+fx)+1\right)\sqrt{b\sinh^2(e+fx)+a}}d\sinh^2(e+fx)+2\sqrt{a+b\sinh^2(e+fx)}\right)+\frac{2}{3}\left(a+b\sinh^2(e+\right.}{2f}$$

$$\Big\downarrow \text{73}$$

$$\frac{(a-b)\left(\frac{2(a-b)\int \frac{1}{\frac{\sinh^4(e+fx)}{b}-\frac{a}{b}+1}d\sqrt{b\sinh^2(e+fx)+a}}{b}+2\sqrt{a+b\sinh^2(e+fx)}\right)+\frac{2}{3}\left(a+b\sinh^2(e+fx)\right)^{3/2}}{2f}$$

$$\Big\downarrow \text{221}$$

$$\frac{(a-b)\left(2\sqrt{a+b\sinh^2(e+fx)}-2\sqrt{a-b}\text{arctanh}\left(\frac{\sqrt{a+b\sinh^2(e+fx)}}{\sqrt{a-b}}\right)\right)+\frac{2}{3}\left(a+b\sinh^2(e+fx)\right)^{3/2}}{2f}$$

input
```
Int[(a + b*Sinh[e + f*x]^2)^(3/2)*Tanh[e + f*x],x]
```

output
```
((2*(a + b*Sinh[e + f*x]^2)^(3/2))/3 + (a - b)*(-2*Sqrt[a - b]*ArcTanh[Sqr
t[a + b*Sinh[e + f*x]^2]/Sqrt[a - b]] + 2*Sqrt[a + b*Sinh[e + f*x]^2]))/(2
*f)
```

Defintions of rubi rules used

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 60
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Simp[n*((b*c - a*d)/(
b*(m + n + 1)))   Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
 c, d}, x] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ[m, 0] && ( !Integer
Q[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n + 2, 0] && IntLinear
Q[a, b, c, d, m, n, x]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3673
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_.)*tan[(e_.) + (f_.)*(x_)]^
(m_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x]^2, x]}, Simp[ff^((m
 + 1)/2)/(2*f)   Subst[Int[x^((m - 1)/2)*((a + b*ff*x)^p/(1 - ff*x)^((m + 1
)/2)), x], x, Sin[e + f*x]^2/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && Integ
erQ[(m - 1)/2]
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.37 (sec) , antiderivative size = 69, normalized size of antiderivative = 0.77

method	result	size
default	$\frac{\text{`int/indef0`}\left(\frac{\sinh(fx+e)\left(b^{2}\sinh(fx+e)^{4}+2\sinh(fx+e)^{2}ab+a^{2}\right)}{\cosh(fx+e)^{2}\sqrt{a+b\sinh(fx+e)^{2}}},\sinh(fx+e)\right)}{f}$	69

input
```
int((a+b*sinh(f*x+e)^2)^(3/2)*tanh(f*x+e),x,method=_RETURNVERBOSE)
```

output
```
`int/indef0`(sinh(f*x+e)*(b^2*sinh(f*x+e)^4+2*sinh(f*x+e)^2*a*b+a^2)/cosh(
f*x+e)^2/(a+b*sinh(f*x+e)^2)^(1/2),sinh(f*x+e))/f
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 532 vs. $2(78) = 156$.

Time = 0.48 (sec) , antiderivative size = 1156, normalized size of antiderivative = 12.84

$$\int \left(a + b\sinh^2(e + fx)\right)^{3/2} \tanh(e + fx)\, dx = \text{Too large to display}$$

input
```
integrate((a+b*sinh(f*x+e)^2)^(3/2)*tanh(f*x+e),x, algorithm="fricas")
```

output
```
[-1/24*(12*((a - b)*cosh(f*x + e)^3 + 3*(a - b)*cosh(f*x + e)^2*sinh(f*x +
 e) + 3*(a - b)*cosh(f*x + e)*sinh(f*x + e)^2 + (a - b)*sinh(f*x + e)^3)*s
qrt(a - b)*log((b*cosh(f*x + e)^4 + 4*b*cosh(f*x + e)*sinh(f*x + e)^3 + b*
sinh(f*x + e)^4 + 2*(4*a - 3*b)*cosh(f*x + e)^2 + 2*(3*b*cosh(f*x + e)^2 +
 4*a - 3*b)*sinh(f*x + e)^2 + 4*sqrt(2)*sqrt(a - b)*sqrt((b*cosh(f*x + e)^
2 + b*sinh(f*x + e)^2 + 2*a - b)/(cosh(f*x + e)^2 - 2*cosh(f*x + e)*sinh(f
*x + e) + sinh(f*x + e)^2))*(cosh(f*x + e) + sinh(f*x + e)) + 4*(b*cosh(f*
x + e)^3 + (4*a - 3*b)*cosh(f*x + e))*sinh(f*x + e) + b)/(cosh(f*x + e)^4
+ 4*cosh(f*x + e)*sinh(f*x + e)^3 + sinh(f*x + e)^4 + 2*(3*cosh(f*x + e)^2
 + 1)*sinh(f*x + e)^2 + 2*cosh(f*x + e)^2 + 4*(cosh(f*x + e)^3 + cosh(f*x
+ e))*sinh(f*x + e) + 1)) - sqrt(2)*(b*cosh(f*x + e)^4 + 4*b*cosh(f*x + e)
*sinh(f*x + e)^3 + b*sinh(f*x + e)^4 + 2*(8*a - 7*b)*cosh(f*x + e)^2 + 2*(
3*b*cosh(f*x + e)^2 + 8*a - 7*b)*sinh(f*x + e)^2 + 4*(b*cosh(f*x + e)^3 +
(8*a - 7*b)*cosh(f*x + e))*sinh(f*x + e) + b)*sqrt((b*cosh(f*x + e)^2 + b*
sinh(f*x + e)^2 + 2*a - b)/(cosh(f*x + e)^2 - 2*cosh(f*x + e)*sinh(f*x + e
) + sinh(f*x + e)^2)))/(f*cosh(f*x + e)^3 + 3*f*cosh(f*x + e)^2*sinh(f*x +
 e) + 3*f*cosh(f*x + e)*sinh(f*x + e)^2 + f*sinh(f*x + e)^3), 1/24*(24*((a
 - b)*cosh(f*x + e)^3 + 3*(a - b)*cosh(f*x + e)^2*sinh(f*x + e) + 3*(a - b
)*cosh(f*x + e)*sinh(f*x + e)^2 + (a - b)*sinh(f*x + e)^3)*sqrt(-a + b)*ar
ctan(2*sqrt(2)*sqrt(-a + b)*sqrt((b*cosh(f*x + e)^2 + b*sinh(f*x + e)^2...
```

Sympy [F]

$$\int \left(a+b\sinh^2(e+fx)\right)^{3/2}\tanh(e+fx)\,dx=\int \left(a+b\sinh^2\left(e+fx\right)\right)^{\frac{3}{2}}\tanh\left(e+fx\right)dx$$

input
```
integrate((a+b*sinh(f*x+e)**2)**(3/2)*tanh(f*x+e),x)
```

output
```
Integral((a + b*sinh(e + f*x)**2)**(3/2)*tanh(e + f*x), x)
```

Maxima [F]

$$\int \left(a+b\sinh^2(e+fx)\right)^{3/2}\tanh(e+fx)\,dx=\int \left(b\sinh\left(fx+e\right)^2+a\right)^{\frac{3}{2}}\tanh\left(fx+e\right)\,dx$$

input
```
integrate((a+b*sinh(f*x+e)^2)^(3/2)*tanh(f*x+e),x, algorithm="maxima")
```

output
```
integrate((b*sinh(f*x + e)^2 + a)^(3/2)*tanh(f*x + e), x)
```

Giac [F]

$$\int \left(a+b\sinh^2(e+fx)\right)^{3/2}\tanh(e+fx)\,dx=\int \left(b\sinh\left(fx+e\right)^2+a\right)^{\frac{3}{2}}\tanh\left(fx+e\right)\,dx$$

input
```
integrate((a+b*sinh(f*x+e)^2)^(3/2)*tanh(f*x+e),x, algorithm="giac")
```

output
```
integrate((b*sinh(f*x + e)^2 + a)^(3/2)*tanh(f*x + e), x)
```

Mupad [F(-1)]

Timed out.

$$\int \left(a+b\sinh^2(e+fx)\right)^{3/2}\tanh(e+fx)\,dx=\int \tanh(e+f\,x)\,\left(b\sinh(e+f\,x)^2+a\right)^{3/2}\,dx$$

input
```
int(tanh(e + f*x)*(a + b*sinh(e + f*x)^2)^(3/2),x)
```

output
```
int(tanh(e + f*x)*(a + b*sinh(e + f*x)^2)^(3/2), x)
```

Reduce [F]

$$\int \left(a+b\sinh^2(e+fx)\right)^{3/2}\tanh(e+fx)\,dx=\left(\int \sqrt{\sinh(fx+e)^2 b+a}\,\sinh(fx+e)^2\tanh(fx+e)\,dx\right)b+\left(\int \sqrt{\sinh(fx+e)^2 b+a}\,\tanh(fx+e)\,dx\right)a$$

input
```
int((a+b*sinh(f*x+e)^2)^(3/2)*tanh(f*x+e),x)
```

output
```
int(sqrt(sinh(e + f*x)**2*b + a)*sinh(e + f*x)**2*tanh(e + f*x),x)*b + int
(sqrt(sinh(e + f*x)**2*b + a)*tanh(e + f*x),x)*a
```

3.435 $\int \coth(e+fx) \left(a+b\sinh^2(e+fx)\right)^{3/2} dx$

Optimal result

Integrand size = 23, antiderivative size = 78

$$\int \coth(e+fx) \left(a+b\sinh^2(e+fx)\right)^{3/2} dx = -\frac{a^{3/2}\text{arctanh}\left(\frac{\sqrt{a+b\sinh^2(e+fx)}}{\sqrt{a}}\right)}{f} + \frac{a\sqrt{a+b\sinh^2(e+fx)}}{f} + \frac{\left(a+b\sinh^2(e+fx)\right)^{3/2}}{3f}$$

output
```
-a^(3/2)*arctanh((a+b*sinh(f*x+e)^2)^(1/2)/a^(1/2))/f+a*(a+b*sinh(f*x+e)^2
)^(1/2)/f+1/3*(a+b*sinh(f*x+e)^2)^(3/2)/f
```

Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 69, normalized size of antiderivative = 0.88

$$\int \coth(e+fx) \left(a+b\sinh^2(e+fx)\right)^{3/2} dx = \frac{-3a^{3/2}\text{arctanh}\left(\frac{\sqrt{a+b\sinh^2(e+fx)}}{\sqrt{a}}\right) + \sqrt{a+b\sinh^2(e+fx)}\left(4a+b\sinh^2(e+fx)\right)}{3f}$$

input
```
Integrate[Coth[e + f*x]*(a + b*Sinh[e + f*x]^2)^(3/2),x]
```

output
```
(-3*a^(3/2)*ArcTanh[Sqrt[a + b*Sinh[e + f*x]^2]/Sqrt[a]] + Sqrt[a + b*Sinh
[e + f*x]^2]*(4*a + b*Sinh[e + f*x]^2))/(3*f)
```

Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 79, normalized size of antiderivative = 1.01, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.304$, Rules used = {3042, 26, 3673, 60, 60, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \coth(e+fx)\left(a+b\sinh^2(e+fx)\right)^{3/2}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{i\left(a-b\sin(ie+ifx)^2\right)^{3/2}}{\tan(ie+ifx)}dx$$

$$\Big\downarrow \text{26}$$

$$i\int \frac{\left(a-b\sin(ie+ifx)^2\right)^{3/2}}{\tan(ie+ifx)}dx$$

$$\Big\downarrow \text{3673}$$

$$\frac{\int \text{csch}^2(e+fx)\left(b\sinh^2(e+fx)+a\right)^{3/2}d\sinh^2(e+fx)}{2f}$$

$$\Big\downarrow \text{60}$$

$$\frac{a\int \text{csch}^2(e+fx)\sqrt{b\sinh^2(e+fx)+a}d\sinh^2(e+fx)+\frac{2}{3}\left(a+b\sinh^2(e+fx)\right)^{3/2}}{2f}$$

$$\Big\downarrow \text{60}$$

$$\frac{a\left(a\int \frac{\text{csch}^2(e+fx)}{\sqrt{b\sinh^2(e+fx)+a}}d\sinh^2(e+fx)+2\sqrt{a+b\sinh^2(e+fx)}\right)+\frac{2}{3}\left(a+b\sinh^2(e+fx)\right)^{3/2}}{2f}$$

$$\Big\downarrow \text{73}$$

$$\frac{a\left(\frac{2a\int \frac{1}{\frac{\sinh^4(e+fx)}{b}-\frac{a}{b}}d\sqrt{b\sinh^2(e+fx)+a}}{b}+2\sqrt{a+b\sinh^2(e+fx)}\right)+\frac{2}{3}\left(a+b\sinh^2(e+fx)\right)^{3/2}}{2f}$$

$$\Big\downarrow \text{221}$$

$$\frac{a\left(2\sqrt{a+b\sinh^2(e+fx)}-2\sqrt{a}\text{arctanh}\left(\frac{\sqrt{a+b\sinh^2(e+fx)}}{\sqrt{a}}\right)\right)+\frac{2}{3}\left(a+b\sinh^2(e+fx)\right)^{3/2}}{2f}$$

input
```
Int[Coth[e + f*x]*(a + b*Sinh[e + f*x]^2)^(3/2),x]
```

output
```
((2*(a + b*Sinh[e + f*x]^2)^(3/2))/3 + a*(-2*Sqrt[a]*ArcTanh[Sqrt[a + b*Si
nh[e + f*x]^2]/Sqrt[a]] + 2*Sqrt[a + b*Sinh[e + f*x]^2]))/(2*f)
```

Defintions of rubi rules used

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 60
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Simp[n*((b*c - a*d)/(
b*(m + n + 1)))   Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
 c, d}, x] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ[m, 0] && ( !Integer
Q[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n + 2, 0] && IntLinear
Q[a, b, c, d, m, n, x]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3673
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_.)*tan[(e_.) + (f_.)*(x_)]^
(m_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x]^2, x]}, Simp[ff^((m
 + 1)/2)/(2*f)   Subst[Int[x^((m - 1)/2)*((a + b*ff*x)^p/(1 - ff*x)^((m + 1
)/2)), x], x, Sin[e + f*x]^2/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && Integ
erQ[(m - 1)/2]
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.36 (sec) , antiderivative size = 62, normalized size of antiderivative = 0.79

method	result	size
default	$\frac{\text{`int/indef0`}\left(\frac{\sinh(fx+e)^{3}b^{2}+2\sinh(fx+e)ab+\frac{a^{2}}{\sinh(fx+e)}}{\sqrt{a+b\sinh(fx+e)^{2}}},\sinh(fx+e)\right)}{f}$	62

input
```
int(coth(f*x+e)*(a+b*sinh(f*x+e)^2)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
`int/indef0`((sinh(f*x+e)^3*b^2+2*sinh(f*x+e)*a*b+a^2/sinh(f*x+e))/(a+b*si
nh(f*x+e)^2)^(1/2),sinh(f*x+e))/f
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 512 vs. $2(66) = 132$.

Time = 0.30 (sec) , antiderivative size = 1113, normalized size of antiderivative = 14.27

$$\int \coth(e + fx) \left(a + b\sinh^2(e + fx)\right)^{3/2} dx = \text{Too large to display}$$

input
```
integrate(coth(f*x+e)*(a+b*sinh(f*x+e)^2)^(3/2),x, algorithm="fricas")
```

output
```
[1/24*(12*(a*cosh(f*x + e)^3 + 3*a*cosh(f*x + e)^2*sinh(f*x + e) + 3*a*cos
h(f*x + e)*sinh(f*x + e)^2 + a*sinh(f*x + e)^3)*sqrt(a)*log((b*cosh(f*x +
e)^4 + 4*b*cosh(f*x + e)*sinh(f*x + e)^3 + b*sinh(f*x + e)^4 + 2*(4*a - b)
*cosh(f*x + e)^2 + 2*(3*b*cosh(f*x + e)^2 + 4*a - b)*sinh(f*x + e)^2 - 4*s
qrt(2)*sqrt(a)*sqrt((b*cosh(f*x + e)^2 + b*sinh(f*x + e)^2 + 2*a - b)/(cos
h(f*x + e)^2 - 2*cosh(f*x + e)*sinh(f*x + e) + sinh(f*x + e)^2))*(cosh(f*x
 + e) + sinh(f*x + e)) + 4*(b*cosh(f*x + e)^3 + (4*a - b)*cosh(f*x + e))*s
inh(f*x + e) + b)/(cosh(f*x + e)^4 + 4*cosh(f*x + e)*sinh(f*x + e)^3 + sin
h(f*x + e)^4 + 2*(3*cosh(f*x + e)^2 - 1)*sinh(f*x + e)^2 - 2*cosh(f*x + e)
^2 + 4*(cosh(f*x + e)^3 - cosh(f*x + e))*sinh(f*x + e) + 1)) + sqrt(2)*(b*
cosh(f*x + e)^4 + 4*b*cosh(f*x + e)*sinh(f*x + e)^3 + b*sinh(f*x + e)^4 +
2*(8*a - b)*cosh(f*x + e)^2 + 2*(3*b*cosh(f*x + e)^2 + 8*a - b)*sinh(f*x +
 e)^2 + 4*(b*cosh(f*x + e)^3 + (8*a - b)*cosh(f*x + e))*sinh(f*x + e) + b)
*sqrt((b*cosh(f*x + e)^2 + b*sinh(f*x + e)^2 + 2*a - b)/(cosh(f*x + e)^2 -
 2*cosh(f*x + e)*sinh(f*x + e) + sinh(f*x + e)^2)))/(f*cosh(f*x + e)^3 + 3
*f*cosh(f*x + e)^2*sinh(f*x + e) + 3*f*cosh(f*x + e)*sinh(f*x + e)^2 + f*s
inh(f*x + e)^3), 1/24*(24*(a*cosh(f*x + e)^3 + 3*a*cosh(f*x + e)^2*sinh(f*
x + e) + 3*a*cosh(f*x + e)*sinh(f*x + e)^2 + a*sinh(f*x + e)^3)*sqrt(-a)*a
rctan(2*sqrt(2)*sqrt(-a)*sqrt((b*cosh(f*x + e)^2 + b*sinh(f*x + e)^2 + 2*a
 - b)/(cosh(f*x + e)^2 - 2*cosh(f*x + e)*sinh(f*x + e) + sinh(f*x + e)^...
```

Sympy [F(-1)]

Timed out.

$$\int \coth(e+fx)\left(a+b\sinh^2(e+fx)\right)^{3/2}\,dx = \text{Timed out}$$

input
```
integrate(coth(f*x+e)*(a+b*sinh(f*x+e)**2)**(3/2),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \coth(e+fx)\left(a+b\sinh^2(e+fx)\right)^{3/2}\,dx = \int \left(b\sinh\left(fx+e\right)^2+a\right)^{\frac{3}{2}}\coth\left(fx+e\right)\,dx$$

input
```
integrate(coth(f*x+e)*(a+b*sinh(f*x+e)^2)^(3/2),x, algorithm="maxima")
```

output
```
integrate((b*sinh(f*x + e)^2 + a)^(3/2)*coth(f*x + e), x)
```

Giac [F]

$$\int \coth(e+fx)\left(a+b\sinh^2(e+fx)\right)^{3/2}\,dx = \int \left(b\sinh\left(fx+e\right)^2+a\right)^{\frac{3}{2}}\coth\left(fx+e\right)\,dx$$

input
```
integrate(coth(f*x+e)*(a+b*sinh(f*x+e)^2)^(3/2),x, algorithm="giac")
```

output
```
integrate((b*sinh(f*x + e)^2 + a)^(3/2)*coth(f*x + e), x)
```

Mupad [F(-1)]

Timed out.

$$\int \coth(e+fx) \left(a+b\sinh^2(e+fx)\right)^{3/2} dx = \int \coth(e+f\,x)\, \left(b\sinh(e+f\,x)^2+a\right)^{3/2} dx$$

input
```
int(coth(e + f*x)*(a + b*sinh(e + f*x)^2)^(3/2),x)
```

output
```
int(coth(e + f*x)*(a + b*sinh(e + f*x)^2)^(3/2), x)
```

Reduce [F]

$$\int \coth(e+fx) \left(a+b\sinh^2(e+fx)\right)^{3/2} dx = \left(\int \sqrt{\sinh(fx+e)^2 b+a}\, \coth(fx+e)\sinh(fx+e)^2 dx\right) b + \left(\int \sqrt{\sinh(fx+e)^2 b+a}\, \coth(fx+e)\, dx\right) a$$

input
```
int(coth(f*x+e)*(a+b*sinh(f*x+e)^2)^(3/2),x)
```

output
```
int(sqrt(sinh(e + f*x)**2*b + a)*coth(e + f*x)*sinh(e + f*x)**2,x)*b + int
(sqrt(sinh(e + f*x)**2*b + a)*coth(e + f*x),x)*a
```

3.436 $\int \coth^3(e+fx) \left(a+b\sinh^2(e+fx)\right)^{3/2} dx$

Optimal result

Integrand size = 25, antiderivative size = 140

$$\int \coth^3(e+fx) \left(a+b\sinh^2(e+fx)\right)^{3/2} dx = -\frac{\sqrt{a}(2a+3b)\operatorname{arctanh}\left(\frac{\sqrt{a+b\sinh^2(e+fx)}}{\sqrt{a}}\right)}{2f} + \frac{(2a+3b)\sqrt{a+b\sinh^2(e+fx)}}{2f} + \frac{(2a+3b)\left(a+b\sinh^2(e+fx)\right)^{3/2}}{6af} - \frac{\operatorname{csch}^2(e+fx)\left(a+b\sinh^2(e+fx)\right)^{5/2}}{2af}$$

output
```
-1/2*a^(1/2)*(2*a+3*b)*arctanh((a+b*sinh(f*x+e)^2)^(1/2)/a^(1/2))/f+1/2*(2
*a+3*b)*(a+b*sinh(f*x+e)^2)^(1/2)/f+1/6*(2*a+3*b)*(a+b*sinh(f*x+e)^2)^(3/2
)/a/f-1/2*csch(f*x+e)^2*(a+b*sinh(f*x+e)^2)^(5/2)/a/f
```

Mathematica [A] (verified)

Time = 0.33 (sec) , antiderivative size = 90, normalized size of antiderivative = 0.64

$$\int \coth^3(e+fx)\left(a+b\sinh^2(e+fx)\right)^{3/2}\,dx=\frac{-3\sqrt{a}(2a+3b)\text{arctanh}\left(\frac{\sqrt{a+b\sinh^2(e+fx)}}{\sqrt{a}}\right)+\left(8a+5b+b\cosh(2(e+fx))-3a\text{csch}^2(e+f\ldots\right.}{6f}$$

input

```
Integrate[Coth[e + f*x]^3*(a + b*Sinh[e + f*x]^2)^(3/2),x]
```

output

```
(-3*Sqrt[a]*(2*a + 3*b)*ArcTanh[Sqrt[a + b*Sinh[e + f*x]^2]/Sqrt[a]] + (8*
a + 5*b + b*Cosh[2*(e + f*x)] - 3*a*Csch[e + f*x]^2)*Sqrt[a + b*Sinh[e + f
*x]^2])/(6*f)
```

Rubi [A] (verified)

Time = 0.32 (sec) , antiderivative size = 123, normalized size of antiderivative = 0.88, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.320, Rules used = {3042, 26, 3673, 87, 60, 60, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \coth^3(e+fx)\left(a+b\sinh^2(e+fx)\right)^{3/2}dx$$

$$\Big\downarrow \text{3042}$$

$$\int -\frac{i\left(a-b\sin(ie+ifx)^2\right)^{3/2}}{\tan(ie+ifx)^3}dx$$

$$\Big\downarrow \text{26}$$

$$-i\int \frac{\left(a-b\sin(ie+ifx)^2\right)^{3/2}}{\tan(ie+ifx)^3}dx$$

$$\Big\downarrow \text{3673}$$

$$\frac{\int \operatorname{csch}^4(e+fx)\left(\sinh^2(e+fx)+1\right)\left(b\sinh^2(e+fx)+a\right)^{3/2}d\sinh^2(e+fx)}{2f}$$

$$\Big\downarrow \text{ 87}$$

$$\frac{\frac{(2a+3b)\int \operatorname{csch}^2(e+fx)\left(b\sinh^2(e+fx)+a\right)^{3/2}d\sinh^2(e+fx)}{2a}-\frac{\operatorname{csch}^2(e+fx)\left(a+b\sinh^2(e+fx)\right)^{5/2}}{a}}{2f}$$

$$\Big\downarrow \text{ 60}$$

$$\frac{\frac{(2a+3b)\left(a\int \operatorname{csch}^2(e+fx)\sqrt{b\sinh^2(e+fx)+a}d\sinh^2(e+fx)+\frac{2}{3}\left(a+b\sinh^2(e+fx)\right)^{3/2}\right)}{2a}-\frac{\operatorname{csch}^2(e+fx)\left(a+b\sinh^2(e+fx)\right)^{5/2}}{a}}{2f}$$

$$\Big\downarrow \text{ 60}$$

$$\frac{\frac{(2a+3b)\left(a\left(a\int \frac{\operatorname{csch}^2(e+fx)}{\sqrt{b\sinh^2(e+fx)+a}}d\sinh^2(e+fx)+2\sqrt{a+b\sinh^2(e+fx)}\right)+\frac{2}{3}\left(a+b\sinh^2(e+fx)\right)^{3/2}\right)}{2a}-\frac{\operatorname{csch}^2(e+fx)\left(a+b\sinh^2(e+fx)\right)^{5/2}}{a}}{2f}$$

$$\Big\downarrow \text{ 73}$$

$$\frac{\frac{(2a+3b)\left(a\left(\frac{2a\int \frac{1}{\frac{\sinh^4(e+fx)}{b}-\frac{a}{b}}d\sqrt{b\sinh^2(e+fx)+a}}{b}+2\sqrt{a+b\sinh^2(e+fx)}\right)+\frac{2}{3}\left(a+b\sinh^2(e+fx)\right)^{3/2}\right)}{2a}-\frac{\operatorname{csch}^2(e+fx)\left(a+b\sinh^2(e+fx)\right)^{5/2}}{a}}{2f}$$

$$\Big\downarrow \text{ 221}$$

$$\frac{\frac{(2a+3b)\left(a\left(2\sqrt{a+b\sinh^2(e+fx)}-2\sqrt{a}\operatorname{arctanh}\left(\frac{\sqrt{a+b\sinh^2(e+fx)}}{\sqrt{a}}\right)\right)+\frac{2}{3}\left(a+b\sinh^2(e+fx)\right)^{3/2}\right)}{2a}-\frac{\operatorname{csch}^2(e+fx)\left(a+b\sinh^2(e+fx)\right)^{5/2}}{a}}{2f}$$

input
```
Int[Coth[e + f*x]^3*(a + b*Sinh[e + f*x]^2)^(3/2),x]
```

output
```
(-((Csch[e + f*x]^2*(a + b*Sinh[e + f*x]^2)^(5/2))/a) + ((2*a + 3*b)*((2*(
a + b*Sinh[e + f*x]^2)^(3/2))/3 + a*(-2*Sqrt[a]*ArcTanh[Sqrt[a + b*Sinh[e
+ f*x]^2]/Sqrt[a]] + 2*Sqrt[a + b*Sinh[e + f*x]^2])))/(2*a))/(2*f)
```

Defintions of rubi rules used

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 60
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Simp[n*((b*c - a*d)/(
b*(m + n + 1)))   Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
 c, d}, x] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ[m, 0] && ( !Integer
Q[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n + 2, 0] && IntLinear
Q[a, b, c, d, m, n, x]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 87
```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_] :> Simp[(-(b*e - a*f))*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(f*(p
+ 1)*(c*f - d*e))), x] - Simp[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p
+ 1)))/(f*(p + 1)*(c*f - d*e))   Int[(c + d*x)^n*(e + f*x)^(p + 1), x], x]
/; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || Intege
rQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))))
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3673
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_.)*tan[(e_.) + (f_.)*(x_)]^
(m_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x]^2, x]}, Simp[ff^((m
 + 1)/2)/(2*f)   Subst[Int[x^((m - 1)/2)*((a + b*ff*x)^p/(1 - ff*x)^((m + 1
)/2)), x], x, Sin[e + f*x]^2/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && Integ
erQ[(m - 1)/2]
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.45 (sec) , antiderivative size = 84, normalized size of antiderivative = 0.60

method	result	size
default	$\frac{\text{` int/indef0`}\left(\frac{\sinh(fx+e)^3 b^2+\left(2ab+b^2\right)\sinh(fx+e)+\frac{a^2+2ab}{\sinh(fx+e)}+\frac{a^2}{\sinh(fx+e)^3}}{\sqrt{a+b\sinh(fx+e)^2}},\sinh(fx+e)\right)}{f}$	84

input
```
int(coth(f*x+e)^3*(a+b*sinh(f*x+e)^2)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
`int/indef0`((sinh(f*x+e)^3*b^2+(2*a*b+b^2)*sinh(f*x+e)+(a^2+2*a*b)/sinh(f
*x+e)+a^2/sinh(f*x+e)^3)/(a+b*sinh(f*x+e)^2)^(1/2),sinh(f*x+e))/f
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1215 vs. 2(120) = 240.

Time = 0.38 (sec) , antiderivative size = 2519, normalized size of antiderivative = 17.99

$$\int \coth^3(e+fx)\left(a+b\sinh^2(e+fx)\right)^{3/2}\,dx = \text{Too large to display}$$

input
```
integrate(coth(f*x+e)^3*(a+b*sinh(f*x+e)^2)^(3/2),x, algorithm="fricas")
```

output
```
[1/24*(6*((2*a + 3*b)*cosh(f*x + e)^7 + 7*(2*a + 3*b)*cosh(f*x + e)*sinh(f
*x + e)^6 + (2*a + 3*b)*sinh(f*x + e)^7 - 2*(2*a + 3*b)*cosh(f*x + e)^5 +
(21*(2*a + 3*b)*cosh(f*x + e)^2 - 4*a - 6*b)*sinh(f*x + e)^5 + 5*(7*(2*a +
 3*b)*cosh(f*x + e)^3 - 2*(2*a + 3*b)*cosh(f*x + e))*sinh(f*x + e)^4 + (2*
a + 3*b)*cosh(f*x + e)^3 + (35*(2*a + 3*b)*cosh(f*x + e)^4 - 20*(2*a + 3*b
)*cosh(f*x + e)^2 + 2*a + 3*b)*sinh(f*x + e)^3 + (21*(2*a + 3*b)*cosh(f*x
+ e)^5 - 20*(2*a + 3*b)*cosh(f*x + e)^3 + 3*(2*a + 3*b)*cosh(f*x + e))*sin
h(f*x + e)^2 + (7*(2*a + 3*b)*cosh(f*x + e)^6 - 10*(2*a + 3*b)*cosh(f*x +
e)^4 + 3*(2*a + 3*b)*cosh(f*x + e)^2)*sinh(f*x + e))*sqrt(a)*log((b*cosh(f
*x + e)^4 + 4*b*cosh(f*x + e)*sinh(f*x + e)^3 + b*sinh(f*x + e)^4 + 2*(4*a
 - b)*cosh(f*x + e)^2 + 2*(3*b*cosh(f*x + e)^2 + 4*a - b)*sinh(f*x + e)^2
- 4*sqrt(2)*sqrt(a)*sqrt((b*cosh(f*x + e)^2 + b*sinh(f*x + e)^2 + 2*a - b)
/(cosh(f*x + e)^2 - 2*cosh(f*x + e)*sinh(f*x + e) + sinh(f*x + e)^2))*(cos
h(f*x + e) + sinh(f*x + e)) + 4*(b*cosh(f*x + e)^3 + (4*a - b)*cosh(f*x +
e))*sinh(f*x + e) + b)/(cosh(f*x + e)^4 + 4*cosh(f*x + e)*sinh(f*x + e)^3
+ sinh(f*x + e)^4 + 2*(3*cosh(f*x + e)^2 - 1)*sinh(f*x + e)^2 - 2*cosh(f*x
 + e)^2 + 4*(cosh(f*x + e)^3 - cosh(f*x + e))*sinh(f*x + e) + 1)) + sqrt(2
)*(b*cosh(f*x + e)^8 + 8*b*cosh(f*x + e)*sinh(f*x + e)^7 + b*sinh(f*x + e)
^8 + 8*(2*a + b)*cosh(f*x + e)^6 + 4*(7*b*cosh(f*x + e)^2 + 4*a + 2*b)*sin
h(f*x + e)^6 + 8*(7*b*cosh(f*x + e)^3 + 6*(2*a + b)*cosh(f*x + e))*sinh...
```

Sympy [F(-1)]

Timed out.

$$\int \coth^3(e+fx)\left(a+b\sinh^2(e+fx)\right)^{3/2}\,dx=\text{Timed out}$$

input
```
integrate(coth(f*x+e)**3*(a+b*sinh(f*x+e)**2)**(3/2),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \coth^3(e+fx)\left(a+b\sinh^2(e+fx)\right)^{3/2}\,dx = \int \left(b\sinh\left(fx+e\right)^2+a\right)^{\frac{3}{2}}\coth\left(fx+e\right)^3\,dx$$

input
```
integrate(coth(f*x+e)^3*(a+b*sinh(f*x+e)^2)^(3/2),x, algorithm="maxima")
```

output
```
integrate((b*sinh(f*x + e)^2 + a)^(3/2)*coth(f*x + e)^3, x)
```

Giac [F]

$$\int \coth^3(e+fx)\left(a+b\sinh^2(e+fx)\right)^{3/2}\,dx = \int \left(b\sinh\left(fx+e\right)^2+a\right)^{\frac{3}{2}}\coth\left(fx+e\right)^3\,dx$$

input
```
integrate(coth(f*x+e)^3*(a+b*sinh(f*x+e)^2)^(3/2),x, algorithm="giac")
```

output
```
integrate((b*sinh(f*x + e)^2 + a)^(3/2)*coth(f*x + e)^3, x)
```

Mupad [F(-1)]

Timed out.

$$\int \coth^3(e+fx)\left(a+b\sinh^2(e+fx)\right)^{3/2}\,dx = \int \coth(e+f\,x)^3\,\left(b\sinh(e+f\,x)^2+a\right)^{3/2}\,dx$$

input
```
int(coth(e + f*x)^3*(a + b*sinh(e + f*x)^2)^(3/2),x)
```

output
```
int(coth(e + f*x)^3*(a + b*sinh(e + f*x)^2)^(3/2), x)
```

Reduce [F]

$$\int \coth^3(e+fx)\left(a+b\sinh^2(e+fx)\right)^{3/2}\,dx = \left(\int \sqrt{\sinh(fx+e)^2 b+a}\,\coth(fx+e)^3\sinh(fx+e)^2\,dx\right)b + \left(\int \sqrt{\sinh(fx+e)^2 b+a}\,\coth(fx+e)^3\,dx\right)a$$

input
```
int(coth(f*x+e)^3*(a+b*sinh(f*x+e)^2)^(3/2),x)
```

output
```
int(sqrt(sinh(e + f*x)**2*b + a)*coth(e + f*x)**3*sinh(e + f*x)**2,x)*b +
int(sqrt(sinh(e + f*x)**2*b + a)*coth(e + f*x)**3,x)*a
```

3.437 $\int \coth^5(e+fx)\left(a+b\sinh^2(e+fx)\right)^{3/2}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 203

$$\int \coth^5(e+fx)\left(a+b\sinh^2(e+fx)\right)^{3/2}\,dx = -\frac{(8a^2+3b(8a+b))\,\text{arctanh}\left(\frac{\sqrt{a+b\sinh^2(e+fx)}}{\sqrt{a}}\right)}{8\sqrt{a}f} + \frac{(8a^2+3b(8a+b))\sqrt{a+b\sinh^2(e+fx)}}{8af} + \frac{(8a^2+3b(8a+b))\left(a+b\sinh^2(e+fx)\right)^{3/2}}{24a^2f} - \frac{(8a+b)\text{csch}^2(e+fx)\left(a+b\sinh^2(e+fx)\right)^{5/2}}{8a^2f} - \frac{\text{csch}^4(e+fx)\left(a+b\sinh^2(e+fx)\right)^{5/2}}{4af}$$

output

```
-1/8*(8*a^2+3*b*(8*a+b))*arctanh((a+b*sinh(f*x+e)^2)^(1/2)/a^(1/2))/a^(1/2
)/f+1/8*(8*a^2+3*b*(8*a+b))*(a+b*sinh(f*x+e)^2)^(1/2)/a/f+1/24*(8*a^2+3*b*
(8*a+b))*(a+b*sinh(f*x+e)^2)^(3/2)/a^2/f-1/8*(8*a+b)*csch(f*x+e)^2*(a+b*si
nh(f*x+e)^2)^(5/2)/a^2/f-1/4*csch(f*x+e)^4*(a+b*sinh(f*x+e)^2)^(5/2)/a/f
```

Mathematica [A] (verified)

Time = 0.54 (sec) , antiderivative size = 123, normalized size of antiderivative = 0.61

$$\int \coth^5(e+fx)\left(a+b\sinh^2(e+fx)\right)^{3/2}dx = \frac{-3(8a^2+24ab+3b^2)\operatorname{arctanh}\left(\frac{\sqrt{a+b\sinh^2(e+fx)}}{\sqrt{a}}\right)+\sqrt{a}\sqrt{a+b\sinh^2(e+fx)}(-3(8a+5b)\mathrm{c}}{24\sqrt{a}f}$$

input
```
Integrate[Coth[e + f*x]^5*(a + b*Sinh[e + f*x]^2)^(3/2),x]
```

output
```
(-3*(8*a^2 + 24*a*b + 3*b^2)*ArcTanh[Sqrt[a + b*Sinh[e + f*x]^2]/Sqrt[a]]
+ Sqrt[a]*Sqrt[a + b*Sinh[e + f*x]^2]*(-3*(8*a + 5*b)*Csch[e + f*x]^2 - 6*
a*Csch[e + f*x]^4 + 8*(4*a + 6*b + b*Sinh[e + f*x]^2)))/(24*Sqrt[a]*f)
```

Rubi [A] (verified)

Time = 0.36 (sec) , antiderivative size = 174, normalized size of antiderivative = 0.86, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.400$, Rules used = {3042, 26, 3673, 100, 27, 87, 60, 60, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \coth^5(e+fx)\left(a+b\sinh^2(e+fx)\right)^{3/2}dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \frac{i\left(a-b\sin(ie+ifx)^2\right)^{3/2}}{\tan(ie+ifx)^5}dx$$

$$\Big\downarrow \text{ 26}$$

$$i\int \frac{\left(a-b\sin(ie+ifx)^2\right)^{3/2}}{\tan(ie+ifx)^5}dx$$

$$\Big\downarrow \text{ 3673}$$

$$\frac{\int \text{csch}^6(e+fx)\left(\sinh^2(e+fx)+1\right)^2\left(b\sinh^2(e+fx)+a\right)^{3/2}d\sinh^2(e+fx)}{2f}$$

$\downarrow$ 100

$$\frac{\frac{\int \frac{1}{2}\text{csch}^4(e+fx)\left(4a\sinh^2(e+fx)+8a+b\right)\left(b\sinh^2(e+fx)+a\right)^{3/2}d\sinh^2(e+fx)}{2a}-\frac{\text{csch}^4(e+fx)\left(a+b\sinh^2(e+fx)\right)^{5/2}}{2a}}{2f}$$

$\downarrow$ 27

$$\frac{\frac{\int \text{csch}^4(e+fx)\left(4a\sinh^2(e+fx)+8a+b\right)\left(b\sinh^2(e+fx)+a\right)^{3/2}d\sinh^2(e+fx)}{4a}-\frac{\text{csch}^4(e+fx)\left(a+b\sinh^2(e+fx)\right)^{5/2}}{2a}}{2f}$$

$\downarrow$ 87

$$\frac{\frac{\frac{\left(8a^2+3b(8a+b)\right)\int \text{csch}^2(e+fx)\left(b\sinh^2(e+fx)+a\right)^{3/2}d\sinh^2(e+fx)}{2a}-\frac{(8a+b)\text{csch}^2(e+fx)\left(a+b\sinh^2(e+fx)\right)^{5/2}}{a}}{4a}-\frac{\text{csch}^4(e+fx)(a+b\sinh^2(e+fx)}{2a}}{2f}$$

$\downarrow$ 60

$$\frac{\frac{\frac{\left(8a^2+3b(8a+b)\right)\left(a\int \text{csch}^2(e+fx)\sqrt{b\sinh^2(e+fx)+a}\,d\sinh^2(e+fx)+\frac{2}{3}\left(a+b\sinh^2(e+fx)\right)^{3/2}\right)}{2a}-\frac{(8a+b)\text{csch}^2(e+fx)\left(a+b\sinh^2(e+fx)\right)^{5/2}}{a}}{4a}-\text{csch}^4(e}{2f}$$

$\downarrow$ 60

$$\frac{\frac{\frac{\left(8a^2+3b(8a+b)\right)\left(a\left(a\int \frac{\text{csch}^2(e+fx)}{\sqrt{b\sinh^2(e+fx)+a}}d\sinh^2(e+fx)+2\sqrt{a+b\sinh^2(e+fx)}\right)+\frac{2}{3}\left(a+b\sinh^2(e+fx)\right)^{3/2}\right)}{2a}-\frac{(8a+b)\text{csch}^2(e+fx)\left(a+b\sinh^2(e+fx)\right)^{5/2}}{a}}{4a}}{2f}$$

$\downarrow$ 73

$$\frac{\frac{\frac{\left(8a^2+3b(8a+b)\right)\left(a\left(\frac{2a\int \frac{1}{\frac{\sinh^4(e+fx)}{b}-\frac{a}{b}}d\sqrt{b\sinh^2(e+fx)+a}}{b}+2\sqrt{a+b\sinh^2(e+fx)}\right)+\frac{2}{3}\left(a+b\sinh^2(e+fx)\right)^{3/2}\right)}{2a}-\frac{(8a+b)\text{csch}^2(e+fx)(a+b\sinh^2(e+fx)}{a}}{4a}}{2f}$$

$\downarrow$ 221

$$\frac{\frac{\frac{\left(8a^2+3b(8a+b)\right)\left(a\left(2\sqrt{a+b\sinh^2(e+fx)}-2\sqrt{a}\operatorname{arctanh}\left(\frac{\sqrt{a+b\sinh^2(e+fx)}}{\sqrt{a}}\right)\right)+\frac{2}{3}\left(a+b\sinh^2(e+fx)\right)^{3/2}\right)}{2a}-\frac{(8a+b)\operatorname{csch}^2(e+fx)\left(a+b\sinh^2(e+fx)\right)^{5/2}}{a}}{4a}}{2f}$$

input
```
Int[Coth[e + f*x]^5*(a + b*Sinh[e + f*x]^2)^(3/2),x]
```

output
```
(-1/2*(Csch[e + f*x]^4*(a + b*Sinh[e + f*x]^2)^(5/2))/a + (-(((8*a + b)*Cs
ch[e + f*x]^2*(a + b*Sinh[e + f*x]^2)^(5/2))/a) + ((8*a^2 + 3*b*(8*a + b))
*((2*(a + b*Sinh[e + f*x]^2)^(3/2))/3 + a*(-2*Sqrt[a]*ArcTanh[Sqrt[a + b*S
inh[e + f*x]^2]/Sqrt[a]] + 2*Sqrt[a + b*Sinh[e + f*x]^2])))/(2*a))/(4*a))/
(2*f)
```

Defintions of rubi rules used

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 60
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Simp[n*((b*c - a*d)/(
b*(m + n + 1)))   Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
 c, d}, x] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ[m, 0] && ( !Integer
Q[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n + 2, 0] && IntLinear
Q[a, b, c, d, m, n, x]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 87
```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_] :> Simp[(-(b*e - a*f))*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(f*(p
+ 1)*(c*f - d*e))), x] - Simp[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p
+ 1)))/(f*(p + 1)*(c*f - d*e))   Int[(c + d*x)^n*(e + f*x)^(p + 1), x], x]
/; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || Intege
rQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))))
```

rule 100
```
Int[((a_.) + (b_.)*(x_))^2*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))^(
p_), x_] :> Simp[(b*c - a*d)^2*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(d^2*(d
*e - c*f)*(n + 1))), x] - Simp[1/(d^2*(d*e - c*f)*(n + 1))   Int[(c + d*x)^
(n + 1)*(e + f*x)^p*Simp[a^2*d^2*f*(n + p + 2) + b^2*c*(d*e*(n + 1) + c*f*(
p + 1)) - 2*a*b*d*(d*e*(n + 1) + c*f*(p + 1)) - b^2*d*(d*e - c*f)*(n + 1)*x
, x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && (LtQ[n, -1] || (EqQ[n
 + p + 3, 0] && NeQ[n, -1] && (SumSimplerQ[n, 1] ||  !SumSimplerQ[p, 1])))
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3673
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_.)*tan[(e_.) + (f_.)*(x_)]^
(m_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x]^2, x]}, Simp[ff^((m
 + 1)/2)/(2*f)   Subst[Int[x^((m - 1)/2)*((a + b*ff*x)^p/(1 - ff*x)^((m + 1
)/2)), x], x, Sin[e + f*x]^2/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && Integ
erQ[(m - 1)/2]
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.55 (sec) , antiderivative size = 113, normalized size of antiderivative = 0.56

method	result	size
default	$\frac{\text{`int/indef0`}\left(\frac{\cosh(fx+e)^4\left(b^2\cosh(fx+e)^4+2\cosh(fx+e)^2ab-2b^2\cosh(fx+e)^2+a^2-2ab+b^2\right)}{\sinh(fx+e)\left(\cosh(fx+e)^4-2\cosh(fx+e)^2+1\right)\sqrt{a+b\sinh(fx+e)^2}},\sinh(fx+e)\right)}{f}$	113

input
```
int(coth(f*x+e)^5*(a+b*sinh(f*x+e)^2)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
`int/indef0`(1/sinh(f*x+e)/(cosh(f*x+e)^4-2*cosh(f*x+e)^2+1)*cosh(f*x+e)^4
*(b^2*cosh(f*x+e)^4+2*cosh(f*x+e)^2*a*b-2*b^2*cosh(f*x+e)^2+a^2-2*a*b+b^2)
/(a+b*sinh(f*x+e)^2)^(1/2),sinh(f*x+e))/f
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2766 vs. 2(179) = 358.

Time = 0.51 (sec) , antiderivative size = 5622, normalized size of antiderivative = 27.69

$$\int \coth^5(e+fx)\left(a+b\sinh^2(e+fx)\right)^{3/2}\,dx=\text{Too large to display}$$

input
```
integrate(coth(f*x+e)^5*(a+b*sinh(f*x+e)^2)^(3/2),x, algorithm="fricas")
```

output
```
Too large to include
```

Sympy [F(-1)]

Timed out.

$$\int \coth^5(e+fx)\left(a+b\sinh^2(e+fx)\right)^{3/2}\,dx=\text{Timed out}$$

input
```
integrate(coth(f*x+e)**5*(a+b*sinh(f*x+e)**2)**(3/2),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \coth^5(e+fx)\left(a+b\sinh^2(e+fx)\right)^{3/2}\,dx = \int \left(b\sinh\left(fx+e\right)^2+a\right)^{\frac{3}{2}}\coth\left(fx+e\right)^5\,dx$$

input
```
integrate(coth(f*x+e)^5*(a+b*sinh(f*x+e)^2)^(3/2),x, algorithm="maxima")
```

output
```
integrate((b*sinh(f*x + e)^2 + a)^(3/2)*coth(f*x + e)^5, x)
```

Giac [F]

$$\int \coth^5(e+fx)\left(a+b\sinh^2(e+fx)\right)^{3/2}\,dx = \int \left(b\sinh\left(fx+e\right)^2+a\right)^{\frac{3}{2}}\coth\left(fx+e\right)^5\,dx$$

input
```
integrate(coth(f*x+e)^5*(a+b*sinh(f*x+e)^2)^(3/2),x, algorithm="giac")
```

output
```
integrate((b*sinh(f*x + e)^2 + a)^(3/2)*coth(f*x + e)^5, x)
```

Mupad [F(-1)]

Timed out.

$$\int \coth^5(e+fx)\left(a+b\sinh^2(e+fx)\right)^{3/2}\,dx = \int \coth(e+f\,x)^5\,\left(b\sinh(e+f\,x)^2+a\right)^{3/2}\,dx$$

input
```
int(coth(e + f*x)^5*(a + b*sinh(e + f*x)^2)^(3/2),x)
```

output
```
int(coth(e + f*x)^5*(a + b*sinh(e + f*x)^2)^(3/2), x)
```

Reduce [F]

$$\int \coth^5(e+fx)\left(a+b\sinh^2(e+fx)\right)^{3/2}\,dx=\left(\int \sqrt{\sinh\left(fx+e\right)^2b+a}\,\coth\left(fx+e\right)^5\sinh\left(fx+e\right)^2dx\right)b+\left(\int \sqrt{\sinh\left(fx+e\right)^2b+a}\,\coth\left(fx+e\right)^5dx\right)a$$

input

```
int(coth(f*x+e)^5*(a+b*sinh(f*x+e)^2)^(3/2),x)
```

output

```
int(sqrt(sinh(e + f*x)**2*b + a)*coth(e + f*x)**5*sinh(e + f*x)**2,x)*b +
int(sqrt(sinh(e + f*x)**2*b + a)*coth(e + f*x)**5,x)*a
```

3.438 $\int \left(a+b\sinh^2(e+fx)\right)^{3/2}\tanh^4(e+fx)\,dx$

Optimal result

Integrand size = 25, antiderivative size = 264

$$\int \left(a+b\sinh^2(e+fx)\right)^{3/2}\tanh^4(e+fx)\,dx = -\frac{8(a-2b)E\left(\arctan(\sinh(e+fx))\left|1-\frac{b}{a}\right.\right)\operatorname{sech}(e+fx)\sqrt{a+b\sinh^2(e+fx)}}{3f\sqrt{\frac{\operatorname{sech}^2(e+fx)\left(a+b\sinh^2(e+fx)\right)}{a}}}+\frac{(3a-8b)\operatorname{EllipticF}\left(\arctan(\sinh(e+fx)),1-\frac{b}{a}\right)\operatorname{sech}(e+fx)\sqrt{a+b\sinh^2(e+fx)}}{3f\sqrt{\frac{\operatorname{sech}^2(e+fx)\left(a+b\sinh^2(e+fx)\right)}{a}}}+\frac{(5a-8b)\sqrt{a+b\sinh^2(e+fx)}\tanh(e+fx)}{3f}+\frac{2b\sinh^2(e+fx)\sqrt{a+b\sinh^2(e+fx)}\tanh(e+fx)}{3f}-\frac{\left(a+b\sinh^2(e+fx)\right)^{3/2}\tanh^3(e+fx)}{3f}$$

output
```
-8/3*(a-2*b)*EllipticE(sinh(f*x+e)/(1+sinh(f*x+e)^2)^(1/2),(1-b/a)^(1/2))*
sech(f*x+e)*(a+b*sinh(f*x+e)^2)^(1/2)/f/(sech(f*x+e)^2*(a+b*sinh(f*x+e)^2)
/a)^(1/2)+1/3*(3*a-8*b)*InverseJacobiAM(arctan(sinh(f*x+e)),(1-b/a)^(1/2))
*sech(f*x+e)*(a+b*sinh(f*x+e)^2)^(1/2)/f/(sech(f*x+e)^2*(a+b*sinh(f*x+e)^2
)/a)^(1/2)+1/3*(5*a-8*b)*(a+b*sinh(f*x+e)^2)^(1/2)*tanh(f*x+e)/f+2/3*b*sin
h(f*x+e)^2*(a+b*sinh(f*x+e)^2)^(1/2)*tanh(f*x+e)/f-1/3*(a+b*sinh(f*x+e)^2)
^(3/2)*tanh(f*x+e)^3/f
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 2.20 (sec) , antiderivative size = 224, normalized size of antiderivative = 0.85

$$\int \left(a+b\sinh^2(e+fx)\right)^{3/2}\tanh^4(e+fx)\,dx=\frac{-32ia(a-2b)\sqrt{\frac{2a-b+b\cosh(2(e+fx))}{a}}E\left(i(e+fx)\left|\frac{b}{a}\right.\right)+4ia(5a-8b)\sqrt{\frac{2a-b+b\cosh(2(e+fx))}{a}}\text{EllipticF}}{12f\sqrt{}}$$

input
```
Integrate[(a + b*Sinh[e + f*x]^2)^(3/2)*Tanh[e + f*x]^4,x]
```

output
```
((-32*I)*a*(a - 2*b)*Sqrt[(2*a - b + b*Cosh[2*(e + f*x)])/a]*EllipticE[I*(
e + f*x), b/a] + (4*I)*a*(5*a - 8*b)*Sqrt[(2*a - b + b*Cosh[2*(e + f*x)])/
a]*EllipticF[I*(e + f*x), b/a] - ((32*a^2 - 108*a*b + 18*b^2 + (64*a^2 - 1
60*a*b + 17*b^2)*Cosh[2*(e + f*x)] + 2*(6*a - 17*b)*b*Cosh[4*(e + f*x)] -
b^2*Cosh[6*(e + f*x)])*Sech[e + f*x]^2*Tanh[e + f*x])/(4*Sqrt[2]))/(12*f*S
qrt[2*a - b + b*Cosh[2*(e + f*x)]])
```

Rubi [A] (verified)

Time = 0.61 (sec) , antiderivative size = 379, normalized size of antiderivative = 1.44, number of steps used = 12, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.440$, Rules used = {3042, 3675, 369, 27, 439, 444, 27, 406, 320, 388, 313}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \tanh^4(e+fx)\left(a+b\sinh^2(e+fx)\right)^{3/2}dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \tan(ie+ifx)^4\left(a-b\sin(ie+ifx)^2\right)^{3/2}dx$$

$$\Big\downarrow \text{ 3675}$$

$$\frac{\sqrt{\cosh^2(e+fx)}\text{sech}(e+fx)\int \frac{\sinh^4(e+fx)\left(b\sinh^2(e+fx)+a\right)^{3/2}}{\left(\sinh^2(e+fx)+1\right)^{5/2}}d\sinh(e+fx)}{f}$$

$$\Big\downarrow \text{ 369}$$

$$\frac{\sqrt{\cosh^2(e+fx)}\text{sech}(e+fx)\left(\frac{1}{3}\int \frac{3\sinh^2(e+fx)\sqrt{b\sinh^2(e+fx)+a}\left(2b\sinh^2(e+fx)+a\right)}{\left(\sinh^2(e+fx)+1\right)^{3/2}}d\sinh(e+fx)-\frac{\sinh^3(e+fx)\left(a+b\sin\right.}{3\left(\sinh^2(e+fx\right.}\right.}{f}$$

$$\Big\downarrow \text{ 27}$$

$$\frac{\sqrt{\cosh^2(e+fx)}\text{sech}(e+fx)\left(\int \frac{\sinh^2(e+fx)\sqrt{b\sinh^2(e+fx)+a}\left(2b\sinh^2(e+fx)+a\right)}{\left(\sinh^2(e+fx)+1\right)^{3/2}}d\sinh(e+fx)-\frac{\sinh^3(e+fx)\left(a+b\sinh^2(\right.}{3\left(\sinh^2(e+fx)+1\right.}\right.}{f}$$

$$\Big\downarrow \text{ 439}$$

$$\frac{\sqrt{\cosh^2(e+fx)}\text{sech}(e+fx)\left(-\int \frac{\sinh^2(e+fx)\left((3a-8b)b\sinh^2(e+fx)+2a(a-3b)\right)}{\sqrt{\sinh^2(e+fx)+1}\sqrt{b\sinh^2(e+fx)+a}}d\sinh(e+fx)-\frac{\sinh^3(e+fx)\left(a+b\sinh^2(e\right.}{3\left(\sinh^2(e+fx)+1\right)^{3}}\right.}{f}$$

$$\Big\downarrow \text{ 444}$$

$$\frac{\sqrt{\cosh^2(e+fx)}\text{sech}(e+fx)\left(\frac{\int \frac{b\left(8(a-2b)b\sinh^2(e+fx)+a(3a-8b)\right)}{\sqrt{\sinh^2(e+fx)+1}\sqrt{b\sinh^2(e+fx)+a}}d\sinh(e+fx)}{3b}-\frac{1}{3}(3a-8b)\sqrt{\sinh^2(e+fx)+1}\sinh(e-\right.}{f}$$

$$\Big\downarrow \text{ 27}$$

$$\frac{\sqrt{\cosh^2(e+fx)}\text{sech}(e+fx)\left(\frac{1}{3}\int \frac{8(a-2b)b\sinh^2(e+fx)+a(3a-8b)}{\sqrt{\sinh^2(e+fx)+1}\sqrt{b\sinh^2(e+fx)+a}}d\sinh(e+fx)-\frac{1}{3}(3a-8b)\sqrt{\sinh^2(e+fx)+\right.}{f}$$

$\Big\downarrow$ 406

$$\sqrt{\cosh^2(e+fx)}\text{sech}(e+fx)\left(\frac{1}{3}\left(a(3a-8b)\int\frac{1}{\sqrt{\sinh^2(e+fx)+1}\sqrt{b\sinh^2(e+fx)+a}}d\sinh(e+fx)+8b(a-2b)\int\frac{}{\sqrt{\sin}}\right.\right.$$

$\Big\downarrow$ 320

$$\sqrt{\cosh^2(e+fx)}\text{sech}(e+fx)\left(\frac{1}{3}\left(8b(a-2b)\int\frac{\sinh^2(e+fx)}{\sqrt{\sinh^2(e+fx)+1}\sqrt{b\sinh^2(e+fx)+a}}d\sinh(e+fx)+\frac{(3a-8b)\sqrt{a+b\sinh^2(}}{\sqrt{\sinh^2}}\right.\right.$$

$\Big\downarrow$ 388

$$\sqrt{\cosh^2(e+fx)}\text{sech}(e+fx)\left(\frac{1}{3}\left(8b(a-2b)\left(\frac{\sinh(e+fx)\sqrt{a+b\sinh^2(e+fx)}}{b\sqrt{\sinh^2(e+fx)+1}}-\frac{\int\frac{\sqrt{b\sinh^2(e+fx)+a}}{\left(\sinh^2(e+fx)+1\right)^{3/2}}d\sinh(e+fx)}{b}\right)+\frac{(3a-}{}\right.\right.$$

$\Big\downarrow$ 313

$$\sqrt{\cosh^2(e+fx)}\text{sech}(e+fx)\left(\frac{1}{3}\left(\frac{(3a-8b)\sqrt{a+b\sinh^2(e+fx)}\,\text{EllipticF}\left(\arctan(\sinh(e+fx)),1-\frac{b}{a}\right)}{\sqrt{\sinh^2(e+fx)+1}\sqrt{\frac{a+b\sinh^2(e+fx)}{a\left(\sinh^2(e+fx)+1\right)}}}+8b(a-2b)\left(\frac{\sinh(e+fx)}{b\sqrt{\sin}}\right.\right.\right.$$

input
```
Int[(a + b*Sinh[e + f*x]^2)^(3/2)*Tanh[e + f*x]^4,x]
```

output
```
(Sqrt[Cosh[e + f*x]^2]*Sech[e + f*x]*(((a - 2*b)*Sinh[e + f*x]^3*Sqrt[a +
b*Sinh[e + f*x]^2])/Sqrt[1 + Sinh[e + f*x]^2] - ((3*a - 8*b)*Sinh[e + f*x]
*Sqrt[1 + Sinh[e + f*x]^2]*Sqrt[a + b*Sinh[e + f*x]^2])/3 - (Sinh[e + f*x]
^3*(a + b*Sinh[e + f*x]^2)^(3/2))/(3*(1 + Sinh[e + f*x]^2)^(3/2)) + (((3*a
 - 8*b)*EllipticF[ArcTan[Sinh[e + f*x]], 1 - b/a]*Sqrt[a + b*Sinh[e + f*x]
^2])/(Sqrt[1 + Sinh[e + f*x]^2]*Sqrt[(a + b*Sinh[e + f*x]^2)/(a*(1 + Sinh[
e + f*x]^2))]) + 8*(a - 2*b)*b*((Sinh[e + f*x]*Sqrt[a + b*Sinh[e + f*x]^2]
)/(b*Sqrt[1 + Sinh[e + f*x]^2]) - (EllipticE[ArcTan[Sinh[e + f*x]], 1 - b/
a]*Sqrt[a + b*Sinh[e + f*x]^2])/(b*Sqrt[1 + Sinh[e + f*x]^2]*Sqrt[(a + b*S
inh[e + f*x]^2)/(a*(1 + Sinh[e + f*x]^2))]))))/3))/f
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 313
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/((c_) + (d_.)*(x_)^2)^(3/2), x_Symbol] :> Sim
p[(Sqrt[a + b*x^2]/(c*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(c
+ d*x^2)))]))*EllipticE[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; FreeQ
[{a, b, c, d}, x] && PosQ[b/a] && PosQ[d/c]
```

rule 320
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(Sqrt[a + b*x^2]/(a*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(
c + d*x^2)))]))*EllipticF[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; Fre
eQ[{a, b, c, d}, x] && PosQ[d/c] && PosQ[b/a] &&  !SimplerSqrtQ[b/a, d/c]
```

rule 369
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_
), x_Symbol] :> Simp[e*(e*x)^(m - 1)*(a + b*x^2)^(p + 1)*((c + d*x^2)^q/(2*
b*(p + 1))), x] - Simp[e^2/(2*b*(p + 1))   Int[(e*x)^(m - 2)*(a + b*x^2)^(p
 + 1)*(c + d*x^2)^(q - 1)*Simp[c*(m - 1) + d*(m + 2*q - 1)*x^2, x], x], x]
/; FreeQ[{a, b, c, d, e}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && GtQ[q, 0
] && GtQ[m, 1] && IntBinomialQ[a, b, c, d, e, m, 2, p, q, x]
```

rule 388
```
Int[(x_)^2/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol]
 :> Simp[x*(Sqrt[a + b*x^2]/(b*Sqrt[c + d*x^2])), x] - Simp[c/b   Int[Sqrt[
a + b*x^2]/(c + d*x^2)^(3/2), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
 a*d, 0] && PosQ[b/a] && PosQ[d/c] &&  !SimplerSqrtQ[b/a, d/c]
```

rule 406
```
Int[((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.)*((e_) + (f_.)*(
x_)^2), x_Symbol] :> Simp[e   Int[(a + b*x^2)^p*(c + d*x^2)^q, x], x] + Sim
p[f   Int[x^2*(a + b*x^2)^p*(c + d*x^2)^q, x], x] /; FreeQ[{a, b, c, d, e,
f, p, q}, x]
```

rule 439
```
Int[((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_
.)*((e_) + (f_.)*(x_)^2), x_Symbol] :> Simp[(-(b*e - a*f))*(g*x)^(m + 1)*(a
 + b*x^2)^(p + 1)*((c + d*x^2)^q/(2*a*b*g*(p + 1))), x] + Simp[1/(2*a*b*(p
+ 1))   Int[(g*x)^m*(a + b*x^2)^(p + 1)*(c + d*x^2)^(q - 1)*Simp[c*(2*b*e*(
p + 1) + (b*e - a*f)*(m + 1)) + d*(2*b*e*(p + 1) + (b*e - a*f)*(m + 2*q + 1
))*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, m}, x] && LtQ[p, -1] && G
tQ[q, 0] &&  !(EqQ[q, 1] && SimplerQ[b*c - a*d, b*e - a*f])
```

rule 444
```
Int[((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q
_.)*((e_) + (f_.)*(x_)^2), x_Symbol] :> Simp[f*g*(g*x)^(m - 1)*(a + b*x^2)^
(p + 1)*((c + d*x^2)^(q + 1)/(b*d*(m + 2*(p + q + 1) + 1))), x] - Simp[g^2/
(b*d*(m + 2*(p + q + 1) + 1))   Int[(g*x)^(m - 2)*(a + b*x^2)^p*(c + d*x^2)
^q*Simp[a*f*c*(m - 1) + (a*f*d*(m + 2*q + 1) + b*(f*c*(m + 2*p + 1) - e*d*(
m + 2*(p + q + 1) + 1)))*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p,
q}, x] && GtQ[m, 1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3675
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_.)*tan[(e_.) + (f_.)*(x_)]^
(m_), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Simp[ff^(m + 1
)*(Sqrt[Cos[e + f*x]^2]/(f*Cos[e + f*x]))   Subst[Int[x^m*((a + b*ff^2*x^2)
^p/(1 - ff^2*x^2)^((m + 1)/2)), x], x, Sin[e + f*x]/ff], x]] /; FreeQ[{a, b
, e, f, p}, x] && IntegerQ[m/2] &&  !IntegerQ[p]
```

Maple [A] (verified)

Time = 7.04 (sec) , antiderivative size = 389, normalized size of antiderivative = 1.47

method	result
default	$-\frac{-\sqrt{-\frac{b}{a}}\, b^{2} \cosh(fx+e)^{6} \sinh(fx+e)+\left(3\sqrt{-\frac{b}{a}}\, ab-7\sqrt{-\frac{b}{a}}\, b^{2}\right) \cosh(fx+e)^{4} \sinh(fx+e)+\left(4\sqrt{-\frac{b}{a}}\, a^{2}-13\sqrt{-\frac{b}{a}}\, ab+9\sqrt{-\frac{b}{a}}\, b^{2}\right)}{}$

input
```
int((a+b*sinh(f*x+e)^2)^(3/2)*tanh(f*x+e)^4,x,method=_RETURNVERBOSE)
```

output
```
-1/3*(-(-b/a)^(1/2)*b^2*cosh(f*x+e)^6*sinh(f*x+e)+(3*(-b/a)^(1/2)*a*b-7*(-
b/a)^(1/2)*b^2)*cosh(f*x+e)^4*sinh(f*x+e)+(4*(-b/a)^(1/2)*a^2-13*(-b/a)^(1
/2)*a*b+9*(-b/a)^(1/2)*b^2)*cosh(f*x+e)^2*sinh(f*x+e)-(b/a*cosh(f*x+e)^2+(
a-b)/a)^(1/2)*(cosh(f*x+e)^2)^(1/2)*(3*EllipticF(sinh(f*x+e)*(-b/a)^(1/2),
(1/b*a)^(1/2))*a^2-16*EllipticF(sinh(f*x+e)*(-b/a)^(1/2),(1/b*a)^(1/2))*a*
b+16*EllipticF(sinh(f*x+e)*(-b/a)^(1/2),(1/b*a)^(1/2))*b^2+8*EllipticE(sin
h(f*x+e)*(-b/a)^(1/2),(1/b*a)^(1/2))*a*b-16*EllipticE(sinh(f*x+e)*(-b/a)^(
1/2),(1/b*a)^(1/2))*b^2)*cosh(f*x+e)^2+(-(-b/a)^(1/2)*a^2+2*(-b/a)^(1/2)*a
*b-(-b/a)^(1/2)*b^2)*sinh(f*x+e))/(-b/a)^(1/2)/cosh(f*x+e)^3/(a+b*sinh(f*x
+e)^2)^(1/2)/f
```

Fricas [F]

$$\int \left(a+b\sinh^2(e+fx)\right)^{3/2}\tanh^4(e+fx)\,dx=\int \left(b\sinh\left(fx+e\right)^2+a\right)^{\frac{3}{2}}\tanh\left(fx+e\right)^4\,dx$$

input
```
integrate((a+b*sinh(f*x+e)^2)^(3/2)*tanh(f*x+e)^4,x, algorithm="fricas")
```

output
```
integral((b*sinh(f*x + e)^2 + a)^(3/2)*tanh(f*x + e)^4, x)
```

Sympy [F(-1)]

Timed out.

$$\int \left(a+b\sinh^2(e+fx)\right)^{3/2}\tanh^4(e+fx)\,dx = \text{Timed out}$$

input
```
integrate((a+b*sinh(f*x+e)**2)**(3/2)*tanh(f*x+e)**4,x)
```

output
```
Timed out
```

Maxima [F]

$$\int \left(a+b\sinh^2(e+fx)\right)^{3/2}\tanh^4(e+fx)\,dx = \int \left(b\sinh\left(fx+e\right)^2+a\right)^{\frac{3}{2}}\tanh\left(fx+e\right)^4\,dx$$

input
```
integrate((a+b*sinh(f*x+e)^2)^(3/2)*tanh(f*x+e)^4,x, algorithm="maxima")
```

output
```
integrate((b*sinh(f*x + e)^2 + a)^(3/2)*tanh(f*x + e)^4, x)
```

Giac [F]

$$\int \left(a+b\sinh^2(e+fx)\right)^{3/2}\tanh^4(e+fx)\,dx = \int \left(b\sinh\left(fx+e\right)^2+a\right)^{\frac{3}{2}}\tanh\left(fx+e\right)^4\,dx$$

input
```
integrate((a+b*sinh(f*x+e)^2)^(3/2)*tanh(f*x+e)^4,x, algorithm="giac")
```

output
```
integrate((b*sinh(f*x + e)^2 + a)^(3/2)*tanh(f*x + e)^4, x)
```

Mupad [F(-1)]

Timed out.

$$\int (a + b\sinh^2(e+fx))^{3/2}\tanh^4(e+fx)\,dx = \int \tanh(e+f\,x)^4\,\left(b\sinh(e+f\,x)^2+a\right)^{3/2}\,dx$$

input
```
int(tanh(e + f*x)^4*(a + b*sinh(e + f*x)^2)^(3/2),x)
```

output
```
int(tanh(e + f*x)^4*(a + b*sinh(e + f*x)^2)^(3/2), x)
```

Reduce [F]

$$\int (a+b\sinh^2(e+fx))^{3/2}\tanh^4(e+fx)\,dx = \left(\int \sqrt{\sinh(fx+e)^2 b+a}\,\sinh(fx+e)^2\tanh(fx+e)^4\,dx\right)b + \left(\int \sqrt{\sinh(fx+e)^2 b+a}\,\tanh(fx+e)^4\,dx\right)a$$

input
```
int((a+b*sinh(f*x+e)^2)^(3/2)*tanh(f*x+e)^4,x)
```

output
```
int(sqrt(sinh(e + f*x)**2*b + a)*sinh(e + f*x)**2*tanh(e + f*x)**4,x)*b +
int(sqrt(sinh(e + f*x)**2*b + a)*tanh(e + f*x)**4,x)*a
```

3.439 $\int \left(a+b\sinh^2(e+fx)\right)^{3/2}\tanh^2(e+fx)\,dx$

Optimal result

Integrand size = 25, antiderivative size = 260

$$\int \left(a+b\sinh^2(e+fx)\right)^{3/2}\tanh^2(e+fx)\,dx = \frac{4b\cosh(e+fx)\sinh(e+fx)\sqrt{a+b\sinh^2(e+fx)}}{3f} - \frac{(7a-8b)E\left(\arctan(\sinh(e+fx))\middle|1-\frac{b}{a}\right)\operatorname{sech}(e+fx)\sqrt{a+b\sinh^2(e+fx)}}{3f\sqrt{\frac{\operatorname{sech}^2(e+fx)\left(a+b\sinh^2(e+fx)\right)}{a}}} + \frac{(3a-4b)\operatorname{EllipticF}\left(\arctan(\sinh(e+fx)),1-\frac{b}{a}\right)\operatorname{sech}(e+fx)\sqrt{a+b\sinh^2(e+fx)}}{3f\sqrt{\frac{\operatorname{sech}^2(e+fx)\left(a+b\sinh^2(e+fx)\right)}{a}}} + \frac{(7a-8b)\sqrt{a+b\sinh^2(e+fx)}\tanh(e+fx)}{3f} - \frac{\left(a+b\sinh^2(e+fx)\right)^{3/2}\tanh(e+fx)}{f}$$

output
```
4/3*b*cosh(f*x+e)*sinh(f*x+e)*(a+b*sinh(f*x+e)^2)^(1/2)/f-1/3*(7*a-8*b)*El
lipticE(sinh(f*x+e)/(1+sinh(f*x+e)^2)^(1/2),(1-b/a)^(1/2))*sech(f*x+e)*(a+
b*sinh(f*x+e)^2)^(1/2)/f/(sech(f*x+e)^2*(a+b*sinh(f*x+e)^2)/a)^(1/2)+1/3*(
3*a-4*b)*InverseJacobiAM(arctan(sinh(f*x+e)),(1-b/a)^(1/2))*sech(f*x+e)*(a
+b*sinh(f*x+e)^2)^(1/2)/f/(sech(f*x+e)^2*(a+b*sinh(f*x+e)^2)/a)^(1/2)+1/3*
(7*a-8*b)*(a+b*sinh(f*x+e)^2)^(1/2)*tanh(f*x+e)/f-(a+b*sinh(f*x+e)^2)^(3/2
)*tanh(f*x+e)/f
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 2.33 (sec) , antiderivative size = 188, normalized size of antiderivative = 0.72

$$\int \left(a+b\sinh^2(e+fx)\right)^{3/2}\tanh^2(e+fx)\,dx = \frac{-8ia(7a-8b)\sqrt{\frac{2a-b+b\cosh(2(e+fx))}{a}}E\left(i(e+fx)\middle|\frac{b}{a}\right)+32ia(a-b)\sqrt{\frac{2a-b+b\cosh(2(e+fx))}{a}}\,\text{EllipticF}}{24f\sqrt{2a}}$$

input
```
Integrate[(a + b*Sinh[e + f*x]^2)^(3/2)*Tanh[e + f*x]^2,x]
```

output
```
((-8*I)*a*(7*a - 8*b)*Sqrt[(2*a - b + b*Cosh[2*(e + f*x)])/a]*EllipticE[I*
(e + f*x), b/a] + (32*I)*a*(a - b)*Sqrt[(2*a - b + b*Cosh[2*(e + f*x)])/a]
*EllipticF[I*(e + f*x), b/a] + Sqrt[2]*(-24*a^2 + 40*a*b - 13*b^2 - 4*(2*a
 - 3*b)*b*Cosh[2*(e + f*x)] + b^2*Cosh[4*(e + f*x)])*Tanh[e + f*x])/(24*f*
Sqrt[2*a - b + b*Cosh[2*(e + f*x)]])
```

Rubi [A] (verified)

Time = 0.48 (sec) , antiderivative size = 326, normalized size of antiderivative = 1.25, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.360$, Rules used = {3042, 25, 3675, 369, 403, 406, 320, 388, 313}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \tanh^2(e+fx)\left(a+b\sinh^2(e+fx)\right)^{3/2}dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \tan(ie+ifx)^2\left(-\left(a-b\sin(ie+ifx)^2\right)^{3/2}\right)dx$$

$$\Big\downarrow \text{ 25}$$

$$-\int \left(a-b\sin(ie+ifx)^2\right)^{3/2}\tan(ie+ifx)^2dx$$

$$\Big\downarrow \text{ 3675}$$

$$\frac{\sqrt{\cosh^2(e+fx)}\text{sech}(e+fx)\int \frac{\sinh^2(e+fx)\left(b\sinh^2(e+fx)+a\right)^{3/2}}{\left(\sinh^2(e+fx)+1\right)^{3/2}}d\sinh(e+fx)}{f}$$

$$\Big\downarrow \text{ 369}$$

$$\frac{\sqrt{\cosh^2(e+fx)}\text{sech}(e+fx)\left(\int \frac{\sqrt{b\sinh^2(e+fx)+a}\left(4b\sinh^2(e+fx)+a\right)}{\sqrt{\sinh^2(e+fx)+1}}d\sinh(e+fx)-\frac{\sinh(e+fx)\left(a+b\sinh^2(e+fx)\right)^{3/2}}{\sqrt{\sinh^2(e+fx)+1}}\right)}{f}$$

$$\Big\downarrow \text{ 403}$$

$$\frac{\sqrt{\cosh^2(e+fx)}\text{sech}(e+fx)\left(\frac{1}{3}\int \frac{(7a-8b)b\sinh^2(e+fx)+a(3a-4b)}{\sqrt{\sinh^2(e+fx)+1}\sqrt{b\sinh^2(e+fx)+a}}d\sinh(e+fx)-\frac{\sinh(e+fx)\left(a+b\sinh^2(e+fx)\right)^{3/2}}{\sqrt{\sinh^2(e+fx)+1}}+\right.}{f}$$

$$\Big\downarrow \text{ 406}$$

$$\frac{\sqrt{\cosh^2(e+fx)}\text{sech}(e+fx)\left(\frac{1}{3}\left(a(3a-4b)\int \frac{1}{\sqrt{\sinh^2(e+fx)+1}\sqrt{b\sinh^2(e+fx)+a}}d\sinh(e+fx)+b(7a-8b)\int \frac{}{\sqrt{\sin}}\right.\right.}{}$$

$$\Big\downarrow \text{ 320}$$

$$\frac{\sqrt{\cosh^2(e+fx)}\text{sech}(e+fx)\left(\frac{1}{3}\left(b(7a-8b)\int \frac{\sinh^2(e+fx)}{\sqrt{\sinh^2(e+fx)+1}\sqrt{b\sinh^2(e+fx)+a}}d\sinh(e+fx)+\frac{(3a-4b)\sqrt{a+b\sinh^2(}}{\sqrt{\sinh^2}}\right.\right.}{}$$

$$\Big\downarrow \text{ 388}$$

$$\sqrt{\cosh^2(e+fx)}\text{sech}(e+fx)\left(\frac{1}{3}\left(b(7a-8b)\left(\frac{\sinh(e+fx)\sqrt{a+b\sinh^2(e+fx)}}{b\sqrt{\sinh^2(e+fx)+1}}-\frac{\int\frac{\sqrt{b\sinh^2(e+fx)+a}}{\left(\sinh^2(e+fx)+1\right)^{3/2}}d\sinh(e+fx)}{b}\right)+\frac{(3a-\right.\right.$$

$$\Big\downarrow \text{313}$$

$$\sqrt{\cosh^2(e+fx)}\text{sech}(e+fx)\left(\frac{1}{3}\left(\frac{(3a-4b)\sqrt{a+b\sinh^2(e+fx)}\,\text{EllipticF}\left(\arctan(\sinh(e+fx)),1-\frac{b}{a}\right)}{\sqrt{\sinh^2(e+fx)+1}\sqrt{\frac{a+b\sinh^2(e+fx)}{a\left(\sinh^2(e+fx)+1\right)}}}+b(7a-8b)\left(\frac{\sinh(e+fx)}{b\sqrt{\sin}}\right.\right.\right.$$

input
```
Int[(a + b*Sinh[e + f*x]^2)^(3/2)*Tanh[e + f*x]^2,x]
```

output
```
(Sqrt[Cosh[e + f*x]^2]*Sech[e + f*x]*((4*b*Sinh[e + f*x]*Sqrt[1 + Sinh[e +
 f*x]^2]*Sqrt[a + b*Sinh[e + f*x]^2])/3 - (Sinh[e + f*x]*(a + b*Sinh[e + f
*x]^2)^(3/2))/Sqrt[1 + Sinh[e + f*x]^2] + (((3*a - 4*b)*EllipticF[ArcTan[S
inh[e + f*x]], 1 - b/a]*Sqrt[a + b*Sinh[e + f*x]^2])/(Sqrt[1 + Sinh[e + f*
x]^2]*Sqrt[(a + b*Sinh[e + f*x]^2)/(a*(1 + Sinh[e + f*x]^2))]) + (7*a - 8*
b)*b*((Sinh[e + f*x]*Sqrt[a + b*Sinh[e + f*x]^2])/(b*Sqrt[1 + Sinh[e + f*x
]^2]) - (EllipticE[ArcTan[Sinh[e + f*x]], 1 - b/a]*Sqrt[a + b*Sinh[e + f*x
]^2])/(b*Sqrt[1 + Sinh[e + f*x]^2]*Sqrt[(a + b*Sinh[e + f*x]^2)/(a*(1 + Si
nh[e + f*x]^2))]))))/3))/f
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 313
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/((c_) + (d_.)*(x_)^2)^(3/2), x_Symbol] :> Sim
p[(Sqrt[a + b*x^2]/(c*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(c
 + d*x^2)))]))*EllipticE[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; FreeQ
[{a, b, c, d}, x] && PosQ[b/a] && PosQ[d/c]
```

rule 320
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(Sqrt[a + b*x^2]/(a*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(
c + d*x^2)))]))*EllipticF[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; Fre
eQ[{a, b, c, d}, x] && PosQ[d/c] && PosQ[b/a] &&  !SimplerSqrtQ[b/a, d/c]
```

rule 369
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_
), x_Symbol] :> Simp[e*(e*x)^(m - 1)*(a + b*x^2)^(p + 1)*((c + d*x^2)^q/(2*
b*(p + 1))), x] - Simp[e^2/(2*b*(p + 1))   Int[(e*x)^(m - 2)*(a + b*x^2)^(p
 + 1)*(c + d*x^2)^(q - 1)*Simp[c*(m - 1) + d*(m + 2*q - 1)*x^2, x], x], x]
/; FreeQ[{a, b, c, d, e}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && GtQ[q, 0
] && GtQ[m, 1] && IntBinomialQ[a, b, c, d, e, m, 2, p, q, x]
```

rule 388
```
Int[(x_)^2/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol]
 :> Simp[x*(Sqrt[a + b*x^2]/(b*Sqrt[c + d*x^2])), x] - Simp[c/b   Int[Sqrt[
a + b*x^2]/(c + d*x^2)^(3/2), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
 a*d, 0] && PosQ[b/a] && PosQ[d/c] &&  !SimplerSqrtQ[b/a, d/c]
```

rule 403
```
Int[((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.)*((e_) + (f_.)*(
x_)^2), x_Symbol] :> Simp[f*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^q/(b*(2*(p +
 q + 1) + 1))), x] + Simp[1/(b*(2*(p + q + 1) + 1))   Int[(a + b*x^2)^p*(c
+ d*x^2)^(q - 1)*Simp[c*(b*e - a*f + b*e*2*(p + q + 1)) + (d*(b*e - a*f) +
f*2*q*(b*c - a*d) + b*d*e*2*(p + q + 1))*x^2, x], x], x] /; FreeQ[{a, b, c,
 d, e, f, p}, x] && GtQ[q, 0] && NeQ[2*(p + q + 1) + 1, 0]
```

rule 406
```
Int[((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.)*((e_) + (f_.)*(
x_)^2), x_Symbol] :> Simp[e   Int[(a + b*x^2)^p*(c + d*x^2)^q, x], x] + Sim
p[f   Int[x^2*(a + b*x^2)^p*(c + d*x^2)^q, x], x] /; FreeQ[{a, b, c, d, e,
f, p, q}, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3675
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_.)*tan[(e_.) + (f_.)*(x_)]^
(m_), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Simp[ff^(m + 1
)*(Sqrt[Cos[e + f*x]^2]/(f*Cos[e + f*x]))   Subst[Int[x^m*((a + b*ff^2*x^2)
^p/(1 - ff^2*x^2)^((m + 1)/2)), x], x, Sin[e + f*x]/ff], x]] /; FreeQ[{a, b
, e, f, p}, x] && IntegerQ[m/2] &&  !IntegerQ[p]
```

Maple [A] (verified)

Time = 5.14 (sec) , antiderivative size = 414, normalized size of antiderivative = 1.59

method	result
default	$-\frac{-\sqrt{-\frac{b}{a}}\, b^{2} \sinh(fx+e)^{5}+2\sqrt{-\frac{b}{a}}\, ab \sinh(fx+e)^{3}-4\sqrt{-\frac{b}{a}}\, b^{2} \sinh(fx+e)^{3}-3a^{2}\sqrt{\frac{a+b\sinh(fx+e)^{2}}{a}}\,\sqrt{\frac{\cosh(2fx+2e)}{2}+\frac{1}{2}}\ \mathrm{EllipticF}\left(\cdots\right.}{\cdots}$

input
```
int((a+b*sinh(f*x+e)^2)^(3/2)*tanh(f*x+e)^2,x,method=_RETURNVERBOSE)
```

output
```
-1/3*(-(-b/a)^(1/2)*b^2*sinh(f*x+e)^5+2*(-b/a)^(1/2)*a*b*sinh(f*x+e)^3-4*(
-b/a)^(1/2)*b^2*sinh(f*x+e)^3-3*a^2*((a+b*sinh(f*x+e)^2)/a)^(1/2)*(cosh(f*
x+e)^2)^(1/2)*EllipticF(sinh(f*x+e)*(-b/a)^(1/2),(1/b*a)^(1/2))+11*a*((a+b
*sinh(f*x+e)^2)/a)^(1/2)*(cosh(f*x+e)^2)^(1/2)*EllipticF(sinh(f*x+e)*(-b/a
)^(1/2),(1/b*a)^(1/2))*b-8*((a+b*sinh(f*x+e)^2)/a)^(1/2)*(cosh(f*x+e)^2)^(
1/2)*EllipticF(sinh(f*x+e)*(-b/a)^(1/2),(1/b*a)^(1/2))*b^2-7*((a+b*sinh(f*
x+e)^2)/a)^(1/2)*(cosh(f*x+e)^2)^(1/2)*EllipticE(sinh(f*x+e)*(-b/a)^(1/2),
(1/b*a)^(1/2))*a*b+8*((a+b*sinh(f*x+e)^2)/a)^(1/2)*(cosh(f*x+e)^2)^(1/2)*E
llipticE(sinh(f*x+e)*(-b/a)^(1/2),(1/b*a)^(1/2))*b^2+3*(-b/a)^(1/2)*a^2*si
nh(f*x+e)-4*(-b/a)^(1/2)*a*b*sinh(f*x+e))/(-b/a)^(1/2)/cosh(f*x+e)/(a+b*si
nh(f*x+e)^2)^(1/2)/f
```

Fricas [F]

$$\int \left(a+b\sinh^2(e+fx)\right)^{3/2}\tanh^2(e+fx)\,dx=\int \left(b\sinh\left(fx+e\right)^2+a\right)^{\frac{3}{2}}\tanh\left(fx+e\right)^2\,dx$$

input
```
integrate((a+b*sinh(f*x+e)^2)^(3/2)*tanh(f*x+e)^2,x, algorithm="fricas")
```

output
```
integral((b*sinh(f*x + e)^2 + a)^(3/2)*tanh(f*x + e)^2, x)
```

Sympy [F]

$$\int \left(a+b\sinh^2(e+fx)\right)^{3/2}\tanh^2(e+fx)\,dx=\int \left(a+b\sinh^2\left(e+fx\right)\right)^{\frac{3}{2}}\tanh^2\left(e+fx\right)dx$$

input
```
integrate((a+b*sinh(f*x+e)**2)**(3/2)*tanh(f*x+e)**2,x)
```

output
```
Integral((a + b*sinh(e + f*x)**2)**(3/2)*tanh(e + f*x)**2, x)
```

Maxima [F]

$$\int \left(a+b\sinh^2(e+fx)\right)^{3/2}\tanh^2(e+fx)\,dx=\int \left(b\sinh\left(fx+e\right)^2+a\right)^{\frac{3}{2}}\tanh\left(fx+e\right)^2\,dx$$

input
```
integrate((a+b*sinh(f*x+e)^2)^(3/2)*tanh(f*x+e)^2,x, algorithm="maxima")
```

output
```
integrate((b*sinh(f*x + e)^2 + a)^(3/2)*tanh(f*x + e)^2, x)
```

Giac [F]

$$\int \left(a+b\sinh^2(e+fx)\right)^{3/2}\tanh^2(e+fx)\,dx=\int \left(b\sinh\left(fx+e\right)^2+a\right)^{\frac{3}{2}}\tanh\left(fx+e\right)^2\,dx$$

input
```
integrate((a+b*sinh(f*x+e)^2)^(3/2)*tanh(f*x+e)^2,x, algorithm="giac")
```

output
```
integrate((b*sinh(f*x + e)^2 + a)^(3/2)*tanh(f*x + e)^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int (a+b\sinh^2(e+fx))^{3/2}\tanh^2(e+fx)\,dx=\int \tanh(e+f\,x)^2\,\left(b\sinh(e+f\,x)^2+a\right)^{3/2}\,dx$$

input
```
int(tanh(e + f*x)^2*(a + b*sinh(e + f*x)^2)^(3/2),x)
```

output
```
int(tanh(e + f*x)^2*(a + b*sinh(e + f*x)^2)^(3/2), x)
```

Reduce [F]

$$\int \left(a+b\sinh^2(e+fx)\right)^{3/2}\tanh^2(e+fx)\,dx=\left(\int \sqrt{\sinh\left(fx+e\right)^2b+a}\,\sinh\left(fx+e\right)^2\tanh\left(fx+e\right)^2dx\right)b+\left(\int \sqrt{\sinh\left(fx+e\right)^2b+a}\,\tanh\left(fx+e\right)^2dx\right)a$$

input
```
int((a+b*sinh(f*x+e)^2)^(3/2)*tanh(f*x+e)^2,x)
```

output
```
int(sqrt(sinh(e + f*x)**2*b + a)*sinh(e + f*x)**2*tanh(e + f*x)**2,x)*b +
int(sqrt(sinh(e + f*x)**2*b + a)*tanh(e + f*x)**2,x)*a
```

3.440 $\int \left(a+b\sinh^2(e+fx)\right)^{3/2}\,dx$

Optimal result

Integrand size = 16, antiderivative size = 176

$$\int \left(a+b\sinh^2(e+fx)\right)^{3/2}\,dx=\frac{b\cosh(e+fx)\sinh(e+fx)\sqrt{a+b\sinh^2(e+fx)}}{3f}-\frac{2i(2a-b)E\left(ie+ifx\middle|\frac{b}{a}\right)\sqrt{a+b\sinh^2(e+fx)}}{3f\sqrt{\frac{a+b\sinh^2(e+fx)}{a}}}+\frac{ia(a-b)\,\mathrm{EllipticF}\left(ie+ifx,\frac{b}{a}\right)\sqrt{\frac{a+b\sinh^2(e+fx)}{a}}}{3f\sqrt{a+b\sinh^2(e+fx)}}$$

output
```
1/3*b*cosh(f*x+e)*sinh(f*x+e)*(a+b*sinh(f*x+e)^2)^(1/2)/f-2/3*I*(2*a-b)*El
lipticE(sin(I*e+I*f*x),(b/a)^(1/2))*(a+b*sinh(f*x+e)^2)^(1/2)/f/((a+b*sinh
(f*x+e)^2)/a)^(1/2)+1/3*I*a*(a-b)*InverseJacobiAM(I*e+I*f*x,(b/a)^(1/2))*(
(a+b*sinh(f*x+e)^2)/a)^(1/2)/f/(a+b*sinh(f*x+e)^2)^(1/2)
```

Mathematica [A] (verified)

Time = 0.51 (sec) , antiderivative size = 169, normalized size of antiderivative = 0.96

$$\int (a + b\sinh^2(e + fx))^{3/2} \, dx = \frac{-4i\sqrt{2}a(2a-b)\sqrt{\frac{2a-b+b\cosh(2(e+fx))}{a}}E\left(i(e+fx)\,\middle|\,\frac{b}{a}\right) + 2i\sqrt{2}a(a-b)\sqrt{\frac{2a-b+b\cosh(2(e+fx))}{a}}\text{E}}{6f\sqrt{4a-2b+2b\cosh(2(e}}$$

input
```
Integrate[(a + b*Sinh[e + f*x]^2)^(3/2),x]
```

output
```
((-4*I)*Sqrt[2]*a*(2*a - b)*Sqrt[(2*a - b + b*Cosh[2*(e + f*x)])/a]*Ellipt
icE[I*(e + f*x), b/a] + (2*I)*Sqrt[2]*a*(a - b)*Sqrt[(2*a - b + b*Cosh[2*(
e + f*x)])/a]*EllipticF[I*(e + f*x), b/a] + b*(2*a - b + b*Cosh[2*(e + f*x
)])*Sinh[2*(e + f*x)])/(6*f*Sqrt[4*a - 2*b + 2*b*Cosh[2*(e + f*x)]])
```

Rubi [A] (verified)

Time = 0.90 (sec) , antiderivative size = 175, normalized size of antiderivative = 0.99, number of steps used = 11, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.688$, Rules used = {3042, 3659, 3042, 3651, 3042, 3657, 3042, 3656, 3662, 3042, 3661}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a + b\sinh^2(e + fx))^{3/2} \, dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int (a - b\sin(ie + ifx)^2)^{3/2} \, dx$$

$$\Big\downarrow \text{ 3659}$$

$$\frac{1}{3}\int \frac{2(2a-b)b\sinh^2(e+fx) + a(3a-b)}{\sqrt{b\sinh^2(e+fx)+a}}dx + \frac{b\sinh(e+fx)\cosh(e+fx)\sqrt{a+b\sinh^2(e+fx)}}{3f}$$

$$\Big\downarrow \text{3042}$$

$$\frac{b\sinh(e+fx)\cosh(e+fx)\sqrt{a+b\sinh^2(e+fx)}}{3f}+\frac{1}{3}\int\frac{a(3a-b)-2(2a-b)b\sin(ie+ifx)^2}{\sqrt{a-b\sin(ie+ifx)^2}}dx$$

$$\Big\downarrow \text{3651}$$

$$\frac{1}{3}\left(2(2a-b)\int\sqrt{b\sinh^2(e+fx)+a}dx-a(a-b)\int\frac{1}{\sqrt{b\sinh^2(e+fx)+a}}dx\right)+\frac{b\sinh(e+fx)\cosh(e+fx)\sqrt{a+b\sinh^2(e+fx)}}{3f}$$

$$\Big\downarrow \text{3042}$$

$$\frac{b\sinh(e+fx)\cosh(e+fx)\sqrt{a+b\sinh^2(e+fx)}}{3f}+\frac{1}{3}\left(2(2a-b)\int\sqrt{a-b\sin(ie+ifx)^2}dx-a(a-b)\int\frac{1}{\sqrt{a-b\sin(ie+ifx)^2}}dx\right)$$

$$\Big\downarrow \text{3657}$$

$$\frac{b\sinh(e+fx)\cosh(e+fx)\sqrt{a+b\sinh^2(e+fx)}}{3f}+\frac{1}{3}\left(\frac{2(2a-b)\sqrt{a+b\sinh^2(e+fx)}\int\sqrt{\frac{b\sinh^2(e+fx)}{a}+1}dx}{\sqrt{\frac{b\sinh^2(e+fx)}{a}+1}}-a(a-b)\int\frac{1}{\sqrt{a-b\sin(ie+ifx)^2}}dx\right)$$

$$\Big\downarrow \text{3042}$$

$$\frac{b\sinh(e+fx)\cosh(e+fx)\sqrt{a+b\sinh^2(e+fx)}}{3f}+\frac{1}{3}\left(\frac{2(2a-b)\sqrt{a+b\sinh^2(e+fx)}\int\sqrt{1-\frac{b\sin(ie+ifx)^2}{a}}dx}{\sqrt{\frac{b\sinh^2(e+fx)}{a}+1}}-a(a-b)\int\frac{1}{\sqrt{a-b\sin(ie+ifx)^2}}dx\right)$$

$$\Big\downarrow \text{3656}$$

$$\frac{b\sinh(e+fx)\cosh(e+fx)\sqrt{a+b\sinh^2(e+fx)}}{3f}+\frac{1}{3}\left(-a(a-b)\int\frac{1}{\sqrt{a-b\sin(ie+ifx)^2}}dx-\frac{2i(2a-b)\sqrt{a+b\sinh^2(e+fx)}E\left(ie+ifx\middle|\frac{b}{a}\right)}{f\sqrt{\frac{b\sinh^2(e+fx)}{a}+1}}\right)$$

$\big\downarrow$ 3662

$$\frac{b\sinh(e+fx)\cosh(e+fx)\sqrt{a+b\sinh^2(e+fx)}}{3f}+\frac{1}{3}\left(-\frac{a(a-b)\sqrt{\frac{b\sinh^2(e+fx)}{a}+1}\int\frac{1}{\sqrt{\frac{b\sinh^2(e+fx)}{a}+1}}dx}{\sqrt{a+b\sinh^2(e+fx)}}-\frac{2i(2a-b)\sqrt{a+b\sinh^2(e+fx)}E\left(ie+ifx\middle|\frac{b}{a}\right)}{f\sqrt{\frac{b\sinh^2(e+fx)}{a}+1}}\right)$$

$\big\downarrow$ 3042

$$\frac{b\sinh(e+fx)\cosh(e+fx)\sqrt{a+b\sinh^2(e+fx)}}{3f}+\frac{1}{3}\left(-\frac{a(a-b)\sqrt{\frac{b\sinh^2(e+fx)}{a}+1}\int\frac{1}{\sqrt{1-\frac{b\sin(ie+ifx)^2}{a}}}dx}{\sqrt{a+b\sinh^2(e+fx)}}-\frac{2i(2a-b)\sqrt{a+b\sinh^2(e+fx)}E\left(ie+ifx\middle|\frac{b}{a}\right)}{f\sqrt{\frac{b\sinh^2(e+fx)}{a}+1}}\right)$$

$\big\downarrow$ 3661

$$\frac{b\sinh(e+fx)\cosh(e+fx)\sqrt{a+b\sinh^2(e+fx)}}{3f}+\frac{1}{3}\left(\frac{ia(a-b)\sqrt{\frac{b\sinh^2(e+fx)}{a}+1}\,\text{EllipticF}\left(ie+ifx,\frac{b}{a}\right)}{f\sqrt{a+b\sinh^2(e+fx)}}-\frac{2i(2a-b)\sqrt{a+b\sinh^2(e+fx)}E\left(ie+ifx\middle|\frac{b}{a}\right)}{f\sqrt{\frac{b\sinh^2(e+fx)}{a}+1}}\right)$$

input
```
Int[(a + b*Sinh[e + f*x]^2)^(3/2),x]
```

output
```
(b*Cosh[e + f*x]*Sinh[e + f*x]*Sqrt[a + b*Sinh[e + f*x]^2])/(3*f) + (((-2*
I)*(2*a - b)*EllipticE[I*e + I*f*x, b/a]*Sqrt[a + b*Sinh[e + f*x]^2])/(f*S
qrt[1 + (b*Sinh[e + f*x]^2)/a]) + (I*a*(a - b)*EllipticF[I*e + I*f*x, b/a]
*Sqrt[1 + (b*Sinh[e + f*x]^2)/a])/(f*Sqrt[a + b*Sinh[e + f*x]^2]))/3
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3651
```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)]^2)/Sqrt[(a_) + (b_.)*sin[(e_.) +
 (f_.)*(x_)]^2], x_Symbol] :> Simp[B/b   Int[Sqrt[a + b*Sin[e + f*x]^2], x]
, x] + Simp[(A*b - a*B)/b   Int[1/Sqrt[a + b*Sin[e + f*x]^2], x], x] /; Fre
eQ[{a, b, e, f, A, B}, x]
```

rule 3656
```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2], x_Symbol] :> Simp[(Sqrt[a
]/f)*EllipticE[e + f*x, -b/a], x] /; FreeQ[{a, b, e, f}, x] && GtQ[a, 0]
```

rule 3657
```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2], x_Symbol] :> Simp[Sqrt[a
+ b*Sin[e + f*x]^2]/Sqrt[1 + b*(Sin[e + f*x]^2/a)]   Int[Sqrt[1 + (b*Sin[e
+ f*x]^2)/a], x], x] /; FreeQ[{a, b, e, f}, x] &&  !GtQ[a, 0]
```

rule 3659
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> Simp[(-b)*C
os[e + f*x]*Sin[e + f*x]*((a + b*Sin[e + f*x]^2)^(p - 1)/(2*f*p)), x] + Sim
p[1/(2*p)   Int[(a + b*Sin[e + f*x]^2)^(p - 2)*Simp[a*(b + 2*a*p) + b*(2*a
+ b)*(2*p - 1)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, e, f}, x] && NeQ[
a + b, 0] && GtQ[p, 1]
```

rule 3661
```
Int[1/Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2], x_Symbol] :> Simp[(1/(S
qrt[a]*f))*EllipticF[e + f*x, -b/a], x] /; FreeQ[{a, b, e, f}, x] && GtQ[a,
 0]
```

rule 3662
```
Int[1/Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2], x_Symbol] :> Simp[Sqrt[
1 + b*(Sin[e + f*x]^2/a)]/Sqrt[a + b*Sin[e + f*x]^2]   Int[1/Sqrt[1 + (b*Si
n[e + f*x]^2)/a], x], x] /; FreeQ[{a, b, e, f}, x] &&  !GtQ[a, 0]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 427 vs. $2(159) = 318$.

Time = 3.73 (sec) , antiderivative size = 428, normalized size of antiderivative = 2.43

method	result
default	$\frac{\sqrt{-\frac{b}{a}}\, b^{2} \cosh(fx+e)^{4} \sinh(fx+e)+\sqrt{-\frac{b}{a}}\, ab \cosh(fx+e)^{2} \sinh(fx+e)-\sqrt{-\frac{b}{a}}\, b^{2} \cosh(fx+e)^{2} \sinh(fx+e)+3a^{2}\sqrt{\frac{b\cosh(fx+e)^{2}}{a}+\cdots}}{}$

input
```
int((a+b*sinh(f*x+e)^2)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
1/3*((-b/a)^(1/2)*b^2*cosh(f*x+e)^4*sinh(f*x+e)+(-b/a)^(1/2)*a*b*cosh(f*x+
e)^2*sinh(f*x+e)-(-b/a)^(1/2)*b^2*cosh(f*x+e)^2*sinh(f*x+e)+3*a^2*(b/a*cos
h(f*x+e)^2+(a-b)/a)^(1/2)*(cosh(f*x+e)^2)^(1/2)*EllipticF(sinh(f*x+e)*(-b/
a)^(1/2),(1/b*a)^(1/2))-5*a*(b/a*cosh(f*x+e)^2+(a-b)/a)^(1/2)*(cosh(f*x+e)
^2)^(1/2)*EllipticF(sinh(f*x+e)*(-b/a)^(1/2),(1/b*a)^(1/2))*b+2*(b/a*cosh(
f*x+e)^2+(a-b)/a)^(1/2)*(cosh(f*x+e)^2)^(1/2)*EllipticF(sinh(f*x+e)*(-b/a)
^(1/2),(1/b*a)^(1/2))*b^2+4*(b/a*cosh(f*x+e)^2+(a-b)/a)^(1/2)*(cosh(f*x+e)
^2)^(1/2)*EllipticE(sinh(f*x+e)*(-b/a)^(1/2),(1/b*a)^(1/2))*a*b-2*(b/a*cos
h(f*x+e)^2+(a-b)/a)^(1/2)*(cosh(f*x+e)^2)^(1/2)*EllipticE(sinh(f*x+e)*(-b/
a)^(1/2),(1/b*a)^(1/2))*b^2)/(-b/a)^(1/2)/cosh(f*x+e)/(a+b*sinh(f*x+e)^2)^
(1/2)/f
```

Fricas [F]

$$\int \left(a+b\sinh^2(e+fx)\right)^{3/2}\,dx=\int \left(b\sinh\left(fx+e\right)^{2}+a\right)^{\frac{3}{2}}\,dx$$

input
```
integrate((a+b*sinh(f*x+e)^2)^(3/2),x, algorithm="fricas")
```

output
```
integral((b*sinh(f*x + e)^2 + a)^(3/2), x)
```

Sympy [F]

$$\int \left(a+b\sinh^2(e+fx)\right)^{3/2}\,dx=\int \left(a+b\sinh^2\left(e+fx\right)\right)^{\frac{3}{2}}\,dx$$

input
```
integrate((a+b*sinh(f*x+e)**2)**(3/2),x)
```

output
```
Integral((a + b*sinh(e + f*x)**2)**(3/2), x)
```

Maxima [F]

$$\int \left(a+b\sinh^2(e+fx)\right)^{3/2}\,dx=\int \left(b\sinh\left(fx+e\right)^2+a\right)^{\frac{3}{2}}\,dx$$

input
```
integrate((a+b*sinh(f*x+e)^2)^(3/2),x, algorithm="maxima")
```

output
```
integrate((b*sinh(f*x + e)^2 + a)^(3/2), x)
```

Giac [F]

$$\int \left(a+b\sinh^2(e+fx)\right)^{3/2}\,dx=\int \left(b\sinh\left(fx+e\right)^2+a\right)^{\frac{3}{2}}\,dx$$

input
```
integrate((a+b*sinh(f*x+e)^2)^(3/2),x, algorithm="giac")
```

output
```
integrate((b*sinh(f*x + e)^2 + a)^(3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \left(a+b\sinh^2(e+fx)\right)^{3/2}\,dx=\int \left(b\sinh(e+f\,x)^2+a\right)^{3/2}\,dx$$

input
```
int((a + b*sinh(e + f*x)^2)^(3/2),x)
```

output
```
int((a + b*sinh(e + f*x)^2)^(3/2), x)
```

Reduce [F]

$$\int \left(a+b\sinh^2(e+fx)\right)^{3/2}\,dx=\left(\int \sqrt{\sinh\left(fx+e\right)^2b+a}dx\right)a+\left(\int \sqrt{\sinh\left(fx+e\right)^2b+a}\,\sinh\left(fx+e\right)^2dx\right)b$$

input
```
int((a+b*sinh(f*x+e)^2)^(3/2),x)
```

output
```
int(sqrt(sinh(e + f*x)**2*b + a),x)*a + int(sqrt(sinh(e + f*x)**2*b + a)*s
inh(e + f*x)**2,x)*b
```

3.441 $\int \coth^2(e+fx)\left(a+b\sinh^2(e+fx)\right)^{3/2}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 256

$$\int \coth^2(e+fx)\left(a+b\sinh^2(e+fx)\right)^{3/2}\,dx = \frac{4b\cosh(e+fx)\sinh(e+fx)\sqrt{a+b\sinh^2(e+fx)}}{3f} - \frac{\coth(e+fx)\left(a+b\sinh^2(e+fx)\right)^{3/2}}{f} - \frac{(7a+b)E\left(\arctan(\sinh(e+fx))\left|1-\frac{b}{a}\right.\right)\operatorname{sech}(e+fx)\sqrt{a+b\sinh^2(e+fx)}}{3f\sqrt{\frac{\operatorname{sech}^2(e+fx)(a+b\sinh^2(e+fx))}{a}}} + \frac{(3a+5b)\operatorname{EllipticF}\left(\arctan(\sinh(e+fx)),1-\frac{b}{a}\right)\operatorname{sech}(e+fx)\sqrt{a+b\sinh^2(e+fx)}}{3f\sqrt{\frac{\operatorname{sech}^2(e+fx)(a+b\sinh^2(e+fx))}{a}}} + \frac{(7a+b)\sqrt{a+b\sinh^2(e+fx)}\tanh(e+fx)}{3f}$$

output
```
4/3*b*cosh(f*x+e)*sinh(f*x+e)*(a+b*sinh(f*x+e)^2)^(1/2)/f-coth(f*x+e)*(a+b
*sinh(f*x+e)^2)^(3/2)/f-1/3*(7*a+b)*EllipticE(sinh(f*x+e)/(1+sinh(f*x+e)^2
)^(1/2),(1-b/a)^(1/2))*sech(f*x+e)*(a+b*sinh(f*x+e)^2)^(1/2)/f/(sech(f*x+e
)^2*(a+b*sinh(f*x+e)^2)/a)^(1/2)+1/3*(3*a+5*b)*InverseJacobiAM(arctan(sinh
(f*x+e)),(1-b/a)^(1/2))*sech(f*x+e)*(a+b*sinh(f*x+e)^2)^(1/2)/f/(sech(f*x+
e)^2*(a+b*sinh(f*x+e)^2)/a)^(1/2)+1/3*(7*a+b)*(a+b*sinh(f*x+e)^2)^(1/2)*ta
nh(f*x+e)/f
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 1.85 (sec) , antiderivative size = 184, normalized size of antiderivative = 0.72

$$\int \coth^2(e+fx)\left(a+b\sinh^2(e+fx)\right)^{3/2}\,dx=\frac{\sqrt{2}(-24a^2+8ab+3b^2-4b(2a+b)\cosh(2(e+fx))+b^2\cosh(4(e+fx)))\coth(e+fx)-}{24f\sqrt{2a}}$$

input
```
Integrate[Coth[e + f*x]^2*(a + b*Sinh[e + f*x]^2)^(3/2),x]
```

output
```
(Sqrt[2]*(-24*a^2 + 8*a*b + 3*b^2 - 4*b*(2*a + b)*Cosh[2*(e + f*x)] + b^2*
Cosh[4*(e + f*x)])*Coth[e + f*x] - (8*I)*a*(7*a + b)*Sqrt[(2*a - b + b*Cos
h[2*(e + f*x)])/a]*EllipticE[I*(e + f*x), b/a] + (32*I)*a*(a - b)*Sqrt[(2*
a - b + b*Cosh[2*(e + f*x)])/a]*EllipticF[I*(e + f*x), b/a])/(24*f*Sqrt[2*
a - b + b*Cosh[2*(e + f*x)]])
```

Rubi [A] (verified)

Time = 0.55 (sec) , antiderivative size = 324, normalized size of antiderivative = 1.27, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.400$, Rules used = {3042, 25, 3675, 375, 27, 403, 406, 320, 388, 313}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \coth^2(e+fx) \left(a+b\sinh^2(e+fx)\right)^{3/2} dx$$

$\big\downarrow$ 3042

$$\int -\frac{\left(a-b\sin(ie+ifx)^2\right)^{3/2}}{\tan(ie+ifx)^2}dx$$

$\big\downarrow$ 25

$$-\int \frac{\left(a-b\sin(ie+ifx)^2\right)^{3/2}}{\tan(ie+ifx)^2}dx$$

$\big\downarrow$ 3675

$$\frac{\sqrt{\cosh^2(e+fx)}\text{sech}(e+fx)\int \text{csch}^2(e+fx)\sqrt{\sinh^2(e+fx)+1}\left(b\sinh^2(e+fx)+a\right)^{3/2}d\sinh(e+fx)}{f}$$

$\big\downarrow$ 375

$$\frac{\sqrt{\cosh^2(e+fx)}\text{sech}(e+fx)\left(2\int \frac{\sqrt{b\sinh^2(e+fx)+a}\left(4b\sinh^2(e+fx)+a+3b\right)}{2\sqrt{\sinh^2(e+fx)+1}}d\sinh(e+fx)-\sqrt{\sinh^2(e+fx)+1}\text{csch}\right.}{f}$$

$\big\downarrow$ 27

$$\frac{\sqrt{\cosh^2(e+fx)}\text{sech}(e+fx)\left(\int \frac{\sqrt{b\sinh^2(e+fx)+a}\left(4b\sinh^2(e+fx)+a+3b\right)}{\sqrt{\sinh^2(e+fx)+1}}d\sinh(e+fx)-\sqrt{\sinh^2(e+fx)+1}\text{csch}(e\right.}{f}$$

$\big\downarrow$ 403

$$\frac{\sqrt{\cosh^2(e+fx)}\text{sech}(e+fx)\left(\frac{1}{3}\int \frac{b(7a+b)\sinh^2(e+fx)+a(3a+5b)}{\sqrt{\sinh^2(e+fx)+1}\sqrt{b\sinh^2(e+fx)+a}}d\sinh(e+fx)+\frac{4}{3}b\sinh(e+fx)\sqrt{\sinh^2(e+f\right.}{f}$$

$\big\downarrow$ 406

$$\sqrt{\cosh^2(e+fx)}\text{sech}(e+fx)\left(\frac{1}{3}\left(a(3a+5b)\int \frac{1}{\sqrt{\sinh^2(e+fx)+1}\sqrt{b\sinh^2(e+fx)+a}}d\sinh(e+fx)+b(7a+b)\int \frac{}{\sqrt{\sinh}}\right.\right.$$

$\big\downarrow$ 320

$$\sqrt{\cosh^2(e+fx)}\text{sech}(e+fx)\left(\frac{1}{3}\left(b(7a+b)\int \frac{\sinh^2(e+fx)}{\sqrt{\sinh^2(e+fx)+1}\sqrt{b\sinh^2(e+fx)+a}}d\sinh(e+fx)+\frac{(3a+5b)\sqrt{a+b\sinh^2(e\cdots}}{\sqrt{\sinh^2(e\cdots}}\right.\right.$$

$$\Big\downarrow \text{388}$$

$$\sqrt{\cosh^2(e+fx)}\text{sech}(e+fx)\left(\frac{1}{3}\left(b(7a+b)\left(\frac{\sinh(e+fx)\sqrt{a+b\sinh^2(e+fx)}}{b\sqrt{\sinh^2(e+fx)+1}}-\frac{\int \frac{\sqrt{b\sinh^2(e+fx)+a}}{\left(\sinh^2(e+fx)+1\right)^{3/2}}d\sinh(e+fx)}{b}\right)+\frac{(3a+5\cdots}{\cdots}\right.\right.$$

$$\Big\downarrow \text{313}$$

$$\sqrt{\cosh^2(e+fx)}\text{sech}(e+fx)\left(\frac{1}{3}\left(\frac{(3a+5b)\sqrt{a+b\sinh^2(e+fx)}\,\text{EllipticF}\left(\arctan(\sinh(e+fx)),1-\frac{b}{a}\right)}{\sqrt{\sinh^2(e+fx)+1}\sqrt{\frac{a+b\sinh^2(e+fx)}{a\left(\sinh^2(e+fx)+1\right)}}}+b(7a+b)\left(\frac{\sinh(e+fx)\sqrt{\cdots}}{b\sqrt{\sinh\cdots}}\right.\right.\right.$$

input
```
Int[Coth[e + f*x]^2*(a + b*Sinh[e + f*x]^2)^(3/2),x]
```

output
```
(Sqrt[Cosh[e + f*x]^2]*Sech[e + f*x]*((4*b*Sinh[e + f*x]*Sqrt[1 + Sinh[e +
 f*x]^2]*Sqrt[a + b*Sinh[e + f*x]^2])/3 - Csch[e + f*x]*Sqrt[1 + Sinh[e +
f*x]^2]*(a + b*Sinh[e + f*x]^2)^(3/2) + (((3*a + 5*b)*EllipticF[ArcTan[Sin
h[e + f*x]], 1 - b/a]*Sqrt[a + b*Sinh[e + f*x]^2])/(Sqrt[1 + Sinh[e + f*x]
^2]*Sqrt[(a + b*Sinh[e + f*x]^2)/(a*(1 + Sinh[e + f*x]^2))]) + b*(7*a + b)
*((Sinh[e + f*x]*Sqrt[a + b*Sinh[e + f*x]^2])/(b*Sqrt[1 + Sinh[e + f*x]^2]
) - (EllipticE[ArcTan[Sinh[e + f*x]], 1 - b/a]*Sqrt[a + b*Sinh[e + f*x]^2]
)/(b*Sqrt[1 + Sinh[e + f*x]^2]*Sqrt[(a + b*Sinh[e + f*x]^2)/(a*(1 + Sinh[e
 + f*x]^2))]))))/3))/f
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 313
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/((c_) + (d_.)*(x_)^2)^(3/2), x_Symbol] :> Sim
p[(Sqrt[a + b*x^2]/(c*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(c
+ d*x^2)))]))*EllipticE[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; FreeQ
[{a, b, c, d}, x] && PosQ[b/a] && PosQ[d/c]
```

rule 320
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(Sqrt[a + b*x^2]/(a*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(
c + d*x^2)))]))*EllipticF[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; Fre
eQ[{a, b, c, d}, x] && PosQ[d/c] && PosQ[b/a] &&  !SimplerSqrtQ[b/a, d/c]
```

rule 375
```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_)
, x_Symbol] :> Simp[(e*x)^(m + 1)*(a + b*x^2)^p*((c + d*x^2)^q/(e*(m + 1)))
, x] - Simp[2/(e^2*(m + 1))   Int[(e*x)^(m + 2)*(a + b*x^2)^(p - 1)*(c + d*
x^2)^(q - 1)*Simp[b*c*p + a*d*q + b*d*(p + q)*x^2, x], x], x] /; FreeQ[{a,
b, c, d, e}, x] && NeQ[b*c - a*d, 0] && GtQ[q, 0] && LtQ[m, -1] && GtQ[p, 0
] && IntBinomialQ[a, b, c, d, e, m, 2, p, q, x]
```

rule 388
```
Int[(x_)^2/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol]
 :> Simp[x*(Sqrt[a + b*x^2]/(b*Sqrt[c + d*x^2])), x] - Simp[c/b   Int[Sqrt[
a + b*x^2]/(c + d*x^2)^(3/2), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
 a*d, 0] && PosQ[b/a] && PosQ[d/c] &&  !SimplerSqrtQ[b/a, d/c]
```

rule 403
```
Int[((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.)*((e_) + (f_.)*(
x_)^2), x_Symbol] :> Simp[f*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^q/(b*(2*(p +
 q + 1) + 1))), x] + Simp[1/(b*(2*(p + q + 1) + 1))   Int[(a + b*x^2)^p*(c
+ d*x^2)^(q - 1)*Simp[c*(b*e - a*f + b*e*2*(p + q + 1)) + (d*(b*e - a*f) +
f*2*q*(b*c - a*d) + b*d*e*2*(p + q + 1))*x^2, x], x], x] /; FreeQ[{a, b, c,
 d, e, f, p}, x] && GtQ[q, 0] && NeQ[2*(p + q + 1) + 1, 0]
```

rule 406
```
Int[((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.)*((e_) + (f_.)*(
x_)^2), x_Symbol] :> Simp[e   Int[(a + b*x^2)^p*(c + d*x^2)^q, x], x] + Sim
p[f   Int[x^2*(a + b*x^2)^p*(c + d*x^2)^q, x], x] /; FreeQ[{a, b, c, d, e,
f, p, q}, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3675
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_.)*tan[(e_.) + (f_.)*(x_)]^
(m_), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Simp[ff^(m + 1
)*(Sqrt[Cos[e + f*x]^2]/(f*Cos[e + f*x]))   Subst[Int[x^m*((a + b*ff^2*x^2)
^p/(1 - ff^2*x^2)^((m + 1)/2)), x], x, Sin[e + f*x]/ff], x]] /; FreeQ[{a, b
, e, f, p}, x] && IntegerQ[m/2] && !IntegerQ[p]
```

Maple [A] (verified)

Time = 5.32 (sec) , antiderivative size = 327, normalized size of antiderivative = 1.28

method	result
default	$\frac{\sqrt{-\frac{b}{a}}\, b^{2} \cosh(fx+e)^{6}+\left(-2\sqrt{-\frac{b}{a}}\, ab-2\sqrt{-\frac{b}{a}}\, b^{2}\right) \cosh(fx+e)^{4}+\left(-3\sqrt{-\frac{b}{a}}\, a^{2}+2\sqrt{-\frac{b}{a}}\, ab+\sqrt{-\frac{b}{a}}\, b^{2}\right) \cosh(fx+e)^{2}+\sinh(fx+e)}{}$

input
```
int(coth(f*x+e)^2*(a+b*sinh(f*x+e)^2)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
1/3*((-b/a)^(1/2)*b^2*cosh(f*x+e)^6+(-2*(-b/a)^(1/2)*a*b-2*(-b/a)^(1/2)*b^
2)*cosh(f*x+e)^4+(-3*(-b/a)^(1/2)*a^2+2*(-b/a)^(1/2)*a*b+(-b/a)^(1/2)*b^2)
*cosh(f*x+e)^2+sinh(f*x+e)*(b/a*cosh(f*x+e)^2+(a-b)/a)^(1/2)*(cosh(f*x+e)^
2)^(1/2)*(3*EllipticF(sinh(f*x+e)*(-b/a)^(1/2),(1/b*a)^(1/2))*a^2-2*Ellipt
icF(sinh(f*x+e)*(-b/a)^(1/2),(1/b*a)^(1/2))*a*b-EllipticF(sinh(f*x+e)*(-b/
a)^(1/2),(1/b*a)^(1/2))*b^2+7*EllipticE(sinh(f*x+e)*(-b/a)^(1/2),(1/b*a)^(
1/2))*a*b+EllipticE(sinh(f*x+e)*(-b/a)^(1/2),(1/b*a)^(1/2))*b^2))/sinh(f*x
+e)/(-b/a)^(1/2)/cosh(f*x+e)/(a+b*sinh(f*x+e)^2)^(1/2)/f
```

Fricas [F]

$$\int \coth^2(e+fx)\left(a+b\sinh^2(e+fx)\right)^{3/2}\,dx=\int \left(b\sinh\left(fx+e\right)^2+a\right)^{\frac{3}{2}}\coth\left(fx+e\right)^2\,dx$$

input
```
integrate(coth(f*x+e)^2*(a+b*sinh(f*x+e)^2)^(3/2),x, algorithm="fricas")
```

output
```
integral((b*coth(f*x + e)^2*sinh(f*x + e)^2 + a*coth(f*x + e)^2)*sqrt(b*si
nh(f*x + e)^2 + a), x)
```

Sympy [F(-1)]

Timed out.

$$\int \coth^2(e+fx)\left(a+b\sinh^2(e+fx)\right)^{3/2}\,dx=\text{Timed out}$$

input
```
integrate(coth(f*x+e)**2*(a+b*sinh(f*x+e)**2)**(3/2),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \coth^2(e+fx)\left(a+b\sinh^2(e+fx)\right)^{3/2}\,dx=\int \left(b\sinh\left(fx+e\right)^2+a\right)^{\frac{3}{2}}\coth\left(fx+e\right)^2\,dx$$

input
```
integrate(coth(f*x+e)^2*(a+b*sinh(f*x+e)^2)^(3/2),x, algorithm="maxima")
```

output
```
integrate((b*sinh(f*x + e)^2 + a)^(3/2)*coth(f*x + e)^2, x)
```

Giac [F]

$$\int \coth^2(e+fx)\left(a+b\sinh^2(e+fx)\right)^{3/2}\,dx=\int \left(b\sinh\left(fx+e\right)^2+a\right)^{\frac{3}{2}}\coth\left(fx+e\right)^2\,dx$$

input
```
integrate(coth(f*x+e)^2*(a+b*sinh(f*x+e)^2)^(3/2),x, algorithm="giac")
```

output
```
integrate((b*sinh(f*x + e)^2 + a)^(3/2)*coth(f*x + e)^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int \coth^2(e+fx)\left(a+b\sinh^2(e+fx)\right)^{3/2}\,dx=\int \mathrm{coth}(e+f\,x)^2\,\left(b\,\mathrm{sinh}(e+f\,x)^2+a\right)^{3/2}\,dx$$

input
```
int(coth(e + f*x)^2*(a + b*sinh(e + f*x)^2)^(3/2),x)
```

output
```
int(coth(e + f*x)^2*(a + b*sinh(e + f*x)^2)^(3/2), x)
```

Reduce [F]

$$\int \coth^2(e+fx)\left(a+b\sinh^2(e+fx)\right)^{3/2}\,dx=\left(\int \sqrt{\sinh(fx+e)^2b+a}\,\coth(fx+e)^2\sinh(fx+e)^2\,dx\right)b+\left(\int \sqrt{\sinh(fx+e)^2b+a}\,\coth(fx+e)^2\,dx\right)a$$

input
```
int(coth(f*x+e)^2*(a+b*sinh(f*x+e)^2)^(3/2),x)
```

output
```
int(sqrt(sinh(e + f*x)**2*b + a)*coth(e + f*x)**2*sinh(e + f*x)**2,x)*b +
int(sqrt(sinh(e + f*x)**2*b + a)*coth(e + f*x)**2,x)*a
```

3.442 $\int \coth^4(e+fx)\left(a+b\sinh^2(e+fx)\right)^{3/2}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 306

$$\int \coth^4(e+fx)\left(a+b\sinh^2(e+fx)\right)^{3/2}\,dx = -\frac{(a+b)\cosh^2(e+fx)\coth(e+fx)\sqrt{a+b\sinh^2(e+fx)}}{f} + \frac{(3a+5b)\cosh(e+fx)\sinh(e+fx)\sqrt{a+b\sinh^2(e+fx)}}{3f} - \frac{\coth^3(e+fx)\left(a+b\sinh^2(e+fx)\right)^{3/2}}{3f} - \frac{8(a+b)E\left(\arctan(\sinh(e+fx))\,\middle|\,1-\frac{b}{a}\right)\operatorname{sech}(e+fx)\sqrt{a+b\sinh^2(e+fx)}}{3f\sqrt{\frac{\operatorname{sech}^2(e+fx)(a+b\sinh^2(e+fx))}{a}}} + \frac{(3a+b)(a+3b)\operatorname{EllipticF}\left(\arctan(\sinh(e+fx)),1-\frac{b}{a}\right)\operatorname{sech}(e+fx)\sqrt{a+b\sinh^2(e+fx)}}{3af\sqrt{\frac{\operatorname{sech}^2(e+fx)(a+b\sinh^2(e+fx))}{a}}} + \frac{8(a+b)\sqrt{a+b\sinh^2(e+fx)}\tanh(e+fx)}{3f}$$

output
```
-(a+b)*cosh(f*x+e)^2*coth(f*x+e)*(a+b*sinh(f*x+e)^2)^(1/2)/f+1/3*(3*a+5*b)
*cosh(f*x+e)*sinh(f*x+e)*(a+b*sinh(f*x+e)^2)^(1/2)/f-1/3*coth(f*x+e)^3*(a+
b*sinh(f*x+e)^2)^(3/2)/f-8/3*(a+b)*EllipticE(sinh(f*x+e)/(1+sinh(f*x+e)^2)
^(1/2),(1-b/a)^(1/2))*sech(f*x+e)*(a+b*sinh(f*x+e)^2)^(1/2)/f/(sech(f*x+e)
^2*(a+b*sinh(f*x+e)^2)/a)^(1/2)+1/3*(3*a+b)*(a+3*b)*InverseJacobiAM(arctan
(sinh(f*x+e)),(1-b/a)^(1/2))*sech(f*x+e)*(a+b*sinh(f*x+e)^2)^(1/2)/a/f/(se
ch(f*x+e)^2*(a+b*sinh(f*x+e)^2)/a)^(1/2)+8/3*(a+b)*(a+b*sinh(f*x+e)^2)^(1/
2)*tanh(f*x+e)/f
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 3.51 (sec) , antiderivative size = 229, normalized size of antiderivative = 0.75

$$\int \coth^4(e+fx)\left(a+b\sinh^2(e+fx)\right)^{3/2}\,dx = -\frac{\left(-32a^2-44ab+58b^2+\left(64a^2+32ab-79b^2\right)\cosh(2(e+fx))+2b(6a+11b)\cosh(4(e+fx))-b^2\cosh(6(e+fx))\right)\coth(e+fx)\text{csc}}{4\sqrt{2}}$$

input
```
Integrate[Coth[e + f*x]^4*(a + b*Sinh[e + f*x]^2)^(3/2),x]
```

output
```
(-1/4*((-32*a^2 - 44*a*b + 58*b^2 + (64*a^2 + 32*a*b - 79*b^2)*Cosh[2*(e +
 f*x)] + 2*b*(6*a + 11*b)*Cosh[4*(e + f*x)] - b^2*Cosh[6*(e + f*x)])*Coth[
e + f*x]*Csch[e + f*x]^2)/Sqrt[2] - (32*I)*a*(a + b)*Sqrt[(2*a - b + b*Cos
h[2*(e + f*x)])/a]*EllipticE[I*(e + f*x), b/a] + (4*I)*(5*a^2 - 2*a*b - 3*
b^2)*Sqrt[(2*a - b + b*Cosh[2*(e + f*x)])/a]*EllipticF[I*(e + f*x), b/a])/
(12*f*Sqrt[2*a - b + b*Cosh[2*(e + f*x)]])
```

Rubi [A] (verified)

Time = 0.65 (sec) , antiderivative size = 380, normalized size of antiderivative = 1.24, number of steps used = 12, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.440, Rules used = {3042, 3675, 375, 27, 442, 403, 27, 406, 320, 388, 313}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \coth^4(e+fx)\left(a+b\sinh^2(e+fx)\right)^{3/2}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\left(a-b\sin(ie+ifx)^2\right)^{3/2}}{\tan(ie+ifx)^4}dx$$

$$\Big\downarrow \text{3675}$$

$$\frac{\sqrt{\cosh^2(e+fx)}\text{sech}(e+fx)\int \text{csch}^4(e+fx)\left(\sinh^2(e+fx)+1\right)^{3/2}\left(b\sinh^2(e+fx)+a\right)^{3/2}d\sinh(e+fx)}{f}$$

$$\Big\downarrow \text{375}$$

$$\frac{\sqrt{\cosh^2(e+fx)}\text{sech}(e+fx)\left(\frac{2}{3}\int \frac{3}{2}\text{csch}^2(e+fx)\sqrt{\sinh^2(e+fx)+1}\sqrt{b\sinh^2(e+fx)+a}(2b\sinh^2(e+fx)+\right.}{f}$$

$$\Big\downarrow \text{27}$$

$$\frac{\sqrt{\cosh^2(e+fx)}\text{sech}(e+fx)\left(\int \text{csch}^2(e+fx)\sqrt{\sinh^2(e+fx)+1}\sqrt{b\sinh^2(e+fx)+a}(2b\sinh^2(e+fx)+a-\right.}{f}$$

$$\Big\downarrow \text{442}$$

$$\frac{\sqrt{\cosh^2(e+fx)}\text{sech}(e+fx)\left(\int \frac{\sqrt{\sinh^2(e+fx)+1}(2a^2+5ba+b^2+b(3a+5b)\sinh^2(e+fx))}{\sqrt{b\sinh^2(e+fx)+a}}d\sinh(e+fx)-\frac{1}{3}(\sinh^2(e+fx)\right.}{}$$

$$\Big\downarrow \text{403}$$

$$\sqrt{\cosh^2(e+fx)}\text{sech}(e+fx)\left(\frac{\int\frac{b\left(8b(a+b)\sinh^2(e+fx)+(3a+b)(a+3b)\right)}{\sqrt{\sinh^2(e+fx)+1}\sqrt{b\sinh^2(e+fx)+a}}d\sinh(e+fx)}{3b}+\frac{1}{3}(3a+5b)\sinh(e+fx)\sqrt{\sinh^2(e+\right.$$

$\downarrow$ 27

$$\sqrt{\cosh^2(e+fx)}\text{sech}(e+fx)\left(\frac{1}{3}\int\frac{8b(a+b)\sinh^2(e+fx)+(3a+b)(a+3b)}{\sqrt{\sinh^2(e+fx)+1}\sqrt{b\sinh^2(e+fx)+a}}d\sinh(e+fx)+\frac{1}{3}(3a+5b)\sinh(e+fx)\sqrt{\sinh\right.$$

$\downarrow$ 406

$$\sqrt{\cosh^2(e+fx)}\text{sech}(e+fx)\left(\frac{1}{3}\left((3a+b)(a+3b)\int\frac{1}{\sqrt{\sinh^2(e+fx)+1}\sqrt{b\sinh^2(e+fx)+a}}d\sinh(e+fx)+8b(a+b)\int\right.\right.$$

$\downarrow$ 320

$$\sqrt{\cosh^2(e+fx)}\text{sech}(e+fx)\left(\frac{1}{3}\left(8b(a+b)\int\frac{\sinh^2(e+fx)}{\sqrt{\sinh^2(e+fx)+1}\sqrt{b\sinh^2(e+fx)+a}}d\sinh(e+fx)+\frac{(3a+b)(a+3b)\sqrt{a+b\sin}}{a\sqrt{\sinh}}\right.\right.$$

$\downarrow$ 388

$$\sqrt{\cosh^2(e+fx)}\text{sech}(e+fx)\left(\frac{1}{3}\left(8b(a+b)\left(\frac{\sinh(e+fx)\sqrt{a+b\sinh^2(e+fx)}}{b\sqrt{\sinh^2(e+fx)+1}}-\frac{\int\frac{\sqrt{b\sinh^2(e+fx)+a}}{\left(\sinh^2(e+fx)+1\right)^{3/2}}d\sinh(e+fx)}{b}\right)+\frac{(3a+}{}\right.\right.$$

$\downarrow$ 313

$$\sqrt{\cosh^2(e+fx)}\text{sech}(e+fx)\left(\frac{1}{3}\left(\frac{(3a+b)(a+3b)\sqrt{a+b\sinh^2(e+fx)}\text{EllipticF}\left(\arctan(\sinh(e+fx)),1-\frac{b}{a}\right)}{a\sqrt{\sinh^2(e+fx)+1}\sqrt{\frac{a+b\sinh^2(e+fx)}{a\left(\sinh^2(e+fx)+1\right)}}}+8b(a+b)\left(\frac{\sinh(e+}{b}\right.\right.\right.$$

input
```
Int[Coth[e + f*x]^4*(a + b*Sinh[e + f*x]^2)^(3/2),x]
```

output
```
(Sqrt[Cosh[e + f*x]^2]*Sech[e + f*x]*(((3*a + 5*b)*Sinh[e + f*x]*Sqrt[1 +
Sinh[e + f*x]^2]*Sqrt[a + b*Sinh[e + f*x]^2])/3 - (a + b)*Csch[e + f*x]*(1
 + Sinh[e + f*x]^2)^(3/2)*Sqrt[a + b*Sinh[e + f*x]^2] - (Csch[e + f*x]^3*(
1 + Sinh[e + f*x]^2)^(3/2)*(a + b*Sinh[e + f*x]^2)^(3/2))/3 + (((3*a + b)*
(a + 3*b)*EllipticF[ArcTan[Sinh[e + f*x]], 1 - b/a]*Sqrt[a + b*Sinh[e + f*
x]^2])/(a*Sqrt[1 + Sinh[e + f*x]^2]*Sqrt[(a + b*Sinh[e + f*x]^2)/(a*(1 + S
inh[e + f*x]^2))]) + 8*b*(a + b)*((Sinh[e + f*x]*Sqrt[a + b*Sinh[e + f*x]^
2])/(b*Sqrt[1 + Sinh[e + f*x]^2]) - (EllipticE[ArcTan[Sinh[e + f*x]], 1 -
b/a]*Sqrt[a + b*Sinh[e + f*x]^2])/(b*Sqrt[1 + Sinh[e + f*x]^2]*Sqrt[(a + b
*Sinh[e + f*x]^2)/(a*(1 + Sinh[e + f*x]^2))]))))/3))/f
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 313
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/((c_) + (d_.)*(x_)^2)^(3/2), x_Symbol] :> Sim
p[(Sqrt[a + b*x^2]/(c*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(c
+ d*x^2)))]))*EllipticE[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; FreeQ
[{a, b, c, d}, x] && PosQ[b/a] && PosQ[d/c]
```

rule 320
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(Sqrt[a + b*x^2]/(a*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(
c + d*x^2)))]))*EllipticF[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; Fre
eQ[{a, b, c, d}, x] && PosQ[d/c] && PosQ[b/a] &&  !SimplerSqrtQ[b/a, d/c]
```

rule 375
```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_)
, x_Symbol] :> Simp[(e*x)^(m + 1)*(a + b*x^2)^p*((c + d*x^2)^q/(e*(m + 1)))
, x] - Simp[2/(e^2*(m + 1))   Int[(e*x)^(m + 2)*(a + b*x^2)^(p - 1)*(c + d*
x^2)^(q - 1)*Simp[b*c*p + a*d*q + b*d*(p + q)*x^2, x], x], x] /; FreeQ[{a,
b, c, d, e}, x] && NeQ[b*c - a*d, 0] && GtQ[q, 0] && LtQ[m, -1] && GtQ[p, 0
] && IntBinomialQ[a, b, c, d, e, m, 2, p, q, x]
```

rule 388
```
Int[(x_)^2/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol]
 :> Simp[x*(Sqrt[a + b*x^2]/(b*Sqrt[c + d*x^2])), x] - Simp[c/b   Int[Sqrt[
a + b*x^2]/(c + d*x^2)^(3/2), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
 a*d, 0] && PosQ[b/a] && PosQ[d/c] &&  !SimplerSqrtQ[b/a, d/c]
```

rule 403
```
Int[((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.)*((e_) + (f_.)*(
x_)^2), x_Symbol] :> Simp[f*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^q/(b*(2*(p +
 q + 1) + 1))), x] + Simp[1/(b*(2*(p + q + 1) + 1))   Int[(a + b*x^2)^p*(c
+ d*x^2)^(q - 1)*Simp[c*(b*e - a*f + b*e*2*(p + q + 1)) + (d*(b*e - a*f) +
f*2*q*(b*c - a*d) + b*d*e*2*(p + q + 1))*x^2, x], x], x] /; FreeQ[{a, b, c,
 d, e, f, p}, x] && GtQ[q, 0] && NeQ[2*(p + q + 1) + 1, 0]
```

rule 406
```
Int[((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.)*((e_) + (f_.)*(
x_)^2), x_Symbol] :> Simp[e   Int[(a + b*x^2)^p*(c + d*x^2)^q, x], x] + Sim
p[f   Int[x^2*(a + b*x^2)^p*(c + d*x^2)^q, x], x] /; FreeQ[{a, b, c, d, e,
f, p, q}, x]
```

rule 442
```
Int[((g_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_
.)*((e_) + (f_.)*(x_)^2), x_Symbol] :> Simp[e*(g*x)^(m + 1)*(a + b*x^2)^(p
+ 1)*((c + d*x^2)^q/(a*g*(m + 1))), x] - Simp[1/(a*g^2*(m + 1))   Int[(g*x)
^(m + 2)*(a + b*x^2)^p*(c + d*x^2)^(q - 1)*Simp[c*(b*e - a*f)*(m + 1) + e*2
*(b*c*(p + 1) + a*d*q) + d*((b*e - a*f)*(m + 1) + b*e*2*(p + q + 1))*x^2, x
], x], x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] && GtQ[q, 0] && LtQ[m, -1]
&&  !(EqQ[q, 1] && SimplerQ[e + f*x^2, c + d*x^2])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3675
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_.)*tan[(e_.) + (f_.)*(x_)]^
(m_), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Simp[ff^(m + 1
)*(Sqrt[Cos[e + f*x]^2]/(f*Cos[e + f*x]))   Subst[Int[x^m*((a + b*ff^2*x^2)
^p/(1 - ff^2*x^2)^((m + 1)/2)), x], x, Sin[e + f*x]/ff], x]] /; FreeQ[{a, b
, e, f, p}, x] && IntegerQ[m/2] &&  !IntegerQ[p]
```

Maple [A] (verified)

Time = 6.22 (sec) , antiderivative size = 540, normalized size of antiderivative = 1.76

method	result
default	$\frac{\sqrt{-\frac{b}{a}}\, b^2 \sinh(fx+e)^8-3\sqrt{-\frac{b}{a}}\, ab \sinh(fx+e)^6-3\sqrt{-\frac{b}{a}}\, b^2 \sinh(fx+e)^6+3a^2\sqrt{\frac{a+b\sinh(fx+e)^2}{a}}\,\sqrt{\frac{\cosh(2fx+2e)}{2}+\frac{1}{2}}\ \mathrm{EllipticF}\Big(\sinh}{}$

input
```
int(coth(f*x+e)^4*(a+b*sinh(f*x+e)^2)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
1/3*((-b/a)^(1/2)*b^2*sinh(f*x+e)^8-3*(-b/a)^(1/2)*a*b*sinh(f*x+e)^6-3*(-b
/a)^(1/2)*b^2*sinh(f*x+e)^6+3*a^2*((a+b*sinh(f*x+e)^2)/a)^(1/2)*(cosh(f*x+
e)^2)^(1/2)*EllipticF(sinh(f*x+e)*(-b/a)^(1/2),(1/b*a)^(1/2))*sinh(f*x+e)^
3+2*b*((a+b*sinh(f*x+e)^2)/a)^(1/2)*(cosh(f*x+e)^2)^(1/2)*EllipticF(sinh(f
*x+e)*(-b/a)^(1/2),(1/b*a)^(1/2))*a*sinh(f*x+e)^3-5*((a+b*sinh(f*x+e)^2)/a
)^(1/2)*(cosh(f*x+e)^2)^(1/2)*EllipticF(sinh(f*x+e)*(-b/a)^(1/2),(1/b*a)^(
1/2))*b^2*sinh(f*x+e)^3+8*((a+b*sinh(f*x+e)^2)/a)^(1/2)*(cosh(f*x+e)^2)^(1
/2)*EllipticE(sinh(f*x+e)*(-b/a)^(1/2),(1/b*a)^(1/2))*a*b*sinh(f*x+e)^3+8*
((a+b*sinh(f*x+e)^2)/a)^(1/2)*(cosh(f*x+e)^2)^(1/2)*EllipticE(sinh(f*x+e)*
(-b/a)^(1/2),(1/b*a)^(1/2))*b^2*sinh(f*x+e)^3-4*(-b/a)^(1/2)*a^2*sinh(f*x+
e)^4-8*(-b/a)^(1/2)*a*b*sinh(f*x+e)^4-4*(-b/a)^(1/2)*b^2*sinh(f*x+e)^4-5*(
-b/a)^(1/2)*a^2*sinh(f*x+e)^2-5*(-b/a)^(1/2)*a*b*sinh(f*x+e)^2-(-b/a)^(1/2
)*a^2)/(-b/a)^(1/2)/sinh(f*x+e)^3/cosh(f*x+e)/(a+b*sinh(f*x+e)^2)^(1/2)/f
```

Fricas [F]

$$\int \coth^4(e+fx)\left(a+b\sinh^2(e+fx)\right)^{3/2}\,dx=\int \left(b\sinh\left(fx+e\right)^2+a\right)^{\frac{3}{2}}\coth\left(fx+e\right)^4\,dx$$

input
```
integrate(coth(f*x+e)^4*(a+b*sinh(f*x+e)^2)^(3/2),x, algorithm="fricas")
```

output
```
integral((b*coth(f*x + e)^4*sinh(f*x + e)^2 + a*coth(f*x + e)^4)*sqrt(b*si
nh(f*x + e)^2 + a), x)
```

Sympy [F(-1)]

Timed out.

$$\int \coth^4(e+fx)\left(a+b\sinh^2(e+fx)\right)^{3/2}\,dx=\text{Timed out}$$

input
```
integrate(coth(f*x+e)**4*(a+b*sinh(f*x+e)**2)**(3/2),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \coth^4(e+fx)\left(a+b\sinh^2(e+fx)\right)^{3/2}\,dx=\int \left(b\sinh\left(fx+e\right)^2+a\right)^{\frac{3}{2}}\coth\left(fx+e\right)^4\,dx$$

input
```
integrate(coth(f*x+e)^4*(a+b*sinh(f*x+e)^2)^(3/2),x, algorithm="maxima")
```

output
```
integrate((b*sinh(f*x + e)^2 + a)^(3/2)*coth(f*x + e)^4, x)
```

Giac [F]

$$\int \coth^4(e+fx)\left(a+b\sinh^2(e+fx)\right)^{3/2}\,dx=\int \left(b\sinh\left(fx+e\right)^2+a\right)^{\frac{3}{2}}\coth\left(fx+e\right)^4\,dx$$

input
```
integrate(coth(f*x+e)^4*(a+b*sinh(f*x+e)^2)^(3/2),x, algorithm="giac")
```

output
```
integrate((b*sinh(f*x + e)^2 + a)^(3/2)*coth(f*x + e)^4, x)
```

Mupad [F(-1)]

Timed out.

$$\int \coth^4(e+fx)\left(a+b\sinh^2(e+fx)\right)^{3/2}\,dx=\int \coth(e+f\,x)^4\left(b\sinh(e+f\,x)^2+a\right)^{3/2}\,dx$$

input
```
int(coth(e + f*x)^4*(a + b*sinh(e + f*x)^2)^(3/2),x)
```

output
```
int(coth(e + f*x)^4*(a + b*sinh(e + f*x)^2)^(3/2), x)
```

Reduce [F]

$$\int \coth^4(e+fx)\left(a+b\sinh^2(e+fx)\right)^{3/2}\,dx=\left(\int \sqrt{\sinh\left(fx+e\right)^2b+a}\,\coth\left(fx+e\right)^4\sinh\left(fx+e\right)^2dx\right)b+\left(\int \sqrt{\sinh\left(fx+e\right)^2b+a}\,\coth\left(fx+e\right)^4dx\right)a$$

input
```
int(coth(f*x+e)^4*(a+b*sinh(f*x+e)^2)^(3/2),x)
```

output
```
int(sqrt(sinh(e + f*x)**2*b + a)*coth(e + f*x)**4*sinh(e + f*x)**2,x)*b +
int(sqrt(sinh(e + f*x)**2*b + a)*coth(e + f*x)**4,x)*a
```

3.443 $\int \frac{\tanh^5(e+fx)}{\sqrt{a+b\sinh^2(e+fx)}}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 142

$$\int \frac{\tanh^5(e+fx)}{\sqrt{a+b\sinh^2(e+fx)}}\,dx = -\frac{(8a^2-8ab+3b^2)\,\text{arctanh}\left(\frac{\sqrt{a+b\sinh^2(e+fx)}}{\sqrt{a-b}}\right)}{8(a-b)^{5/2}f} + \frac{(8a-5b)\text{sech}^2(e+fx)\sqrt{a+b\sinh^2(e+fx)}}{8(a-b)^2f} - \frac{\text{sech}^4(e+fx)\sqrt{a+b\sinh^2(e+fx)}}{4(a-b)f}$$

output
```
-1/8*(8*a^2-8*a*b+3*b^2)*arctanh((a+b*sinh(f*x+e)^2)^(1/2)/(a-b)^(1/2))/(a
-b)^(5/2)/f+1/8*(8*a-5*b)*sech(f*x+e)^2*(a+b*sinh(f*x+e)^2)^(1/2)/(a-b)^2/
f-1/4*sech(f*x+e)^4*(a+b*sinh(f*x+e)^2)^(1/2)/(a-b)/f
```

Mathematica [A] (verified)

Time = 0.34 (sec) , antiderivative size = 116, normalized size of antiderivative = 0.82

$$\int \frac{\tanh^5(e+fx)}{\sqrt{a+b\sinh^2(e+fx)}}\,dx$$
$$= \frac{(-8a^2+8ab-3b^2)\operatorname{arctanh}\left(\frac{\sqrt{a+b\sinh^2(e+fx)}}{\sqrt{a-b}}\right)+\sqrt{a-b}\operatorname{sech}^2(e+fx)\left(8a-5b-2(a-b)\operatorname{sech}^2(e+fx\right.}{8(a-b)^{5/2}f}$$

input
```
Integrate[Tanh[e + f*x]^5/Sqrt[a + b*Sinh[e + f*x]^2],x]
```

output
```
((-8*a^2 + 8*a*b - 3*b^2)*ArcTanh[Sqrt[a + b*Sinh[e + f*x]^2]/Sqrt[a - b]]
 + Sqrt[a - b]*Sech[e + f*x]^2*(8*a - 5*b - 2*(a - b)*Sech[e + f*x]^2)*Sqr
t[a + b*Sinh[e + f*x]^2])/(8*(a - b)^(5/2)*f)
```

Rubi [A] (verified)

Time = 0.43 (sec) , antiderivative size = 155, normalized size of antiderivative = 1.09, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.320, Rules used = {3042, 26, 3673, 100, 27, 87, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\tanh^5(e+fx)}{\sqrt{a+b\sinh^2(e+fx)}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int -\frac{i\tan(ie+ifx)^5}{\sqrt{a-b\sin(ie+ifx)^2}}dx$$

$$\Big\downarrow \text{26}$$

$$-i\int \frac{\tan(ie+ifx)^5}{\sqrt{a-b\sin(ie+ifx)^2}}dx$$

$$\Big\downarrow \textbf{3673}$$

$$\frac{\int \frac{\sinh^4(e+fx)}{(\sinh^2(e+fx)+1)^3\sqrt{b\sinh^2(e+fx)+a}}d\sinh^2(e+fx)}{2f}$$

$$\Big\downarrow \textbf{100}$$

$$\frac{\frac{\int -\frac{-4(a-b)\sinh^2(e+fx)+4a-b}{2\left(\sinh^2(e+fx)+1\right)^2\sqrt{b\sinh^2(e+fx)+a}}d\sinh^2(e+fx)}{2(a-b)}-\frac{\sqrt{a+b\sinh^2(e+fx)}}{2(a-b)(\sinh^2(e+fx)+1)^2}}{2f}$$

$$\Big\downarrow \textbf{27}$$

$$\frac{-\frac{\int \frac{-4(a-b)\sinh^2(e+fx)+4a-b}{\left(\sinh^2(e+fx)+1\right)^2\sqrt{b\sinh^2(e+fx)+a}}d\sinh^2(e+fx)}{4(a-b)}-\frac{\sqrt{a+b\sinh^2(e+fx)}}{2(a-b)(\sinh^2(e+fx)+1)^2}}{2f}$$

$$\Big\downarrow \textbf{87}$$

$$\frac{-\frac{-\frac{\left(8a^2-8ab+3b^2\right)\int \frac{1}{\left(\sinh^2(e+fx)+1\right)\sqrt{b\sinh^2(e+fx)+a}}d\sinh^2(e+fx)}{2(a-b)}-\frac{(8a-5b)\sqrt{a+b\sinh^2(e+fx)}}{(a-b)\left(\sinh^2(e+fx)+1\right)}}{4(a-b)}-\frac{\sqrt{a+b\sinh^2(e+fx)}}{2(a-b)(\sinh^2(e+fx)+1)^2}}{2f}$$

$$\Big\downarrow \textbf{73}$$

$$\frac{-\frac{-\frac{\left(8a^2-8ab+3b^2\right)\int \frac{1}{\frac{\sinh^4(e+fx)}{b}-\frac{a}{b}+1}d\sqrt{b\sinh^2(e+fx)+a}}{b(a-b)}-\frac{(8a-5b)\sqrt{a+b\sinh^2(e+fx)}}{(a-b)\left(\sinh^2(e+fx)+1\right)}}{4(a-b)}-\frac{\sqrt{a+b\sinh^2(e+fx)}}{2(a-b)(\sinh^2(e+fx)+1)^2}}{2f}$$

$$\Big\downarrow \textbf{221}$$

$$\frac{-\frac{\frac{\left(8a^2-8ab+3b^2\right)\text{arctanh}\left(\frac{\sqrt{a+b\sinh^2(e+fx)}}{\sqrt{a-b}}\right)}{(a-b)^{3/2}}-\frac{(8a-5b)\sqrt{a+b\sinh^2(e+fx)}}{(a-b)\left(\sinh^2(e+fx)+1\right)}}{4(a-b)}-\frac{\sqrt{a+b\sinh^2(e+fx)}}{2(a-b)(\sinh^2(e+fx)+1)^2}}{2f}$$

input

```
Int[Tanh[e + f*x]^5/Sqrt[a + b*Sinh[e + f*x]^2],x]
```

output
```
(-1/2*Sqrt[a + b*Sinh[e + f*x]^2]/((a - b)*(1 + Sinh[e + f*x]^2)^2) - (((8
*a^2 - 8*a*b + 3*b^2)*ArcTanh[Sqrt[a + b*Sinh[e + f*x]^2]/Sqrt[a - b]])/(a
 - b)^(3/2) - ((8*a - 5*b)*Sqrt[a + b*Sinh[e + f*x]^2])/((a - b)*(1 + Sinh
[e + f*x]^2)))/(4*(a - b)))/(2*f)
```

Defintions of rubi rules used

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 87
```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_] :> Simp[(-(b*e - a*f))*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(f*(p
+ 1)*(c*f - d*e))), x] - Simp[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p
+ 1)))/(f*(p + 1)*(c*f - d*e))   Int[(c + d*x)^n*(e + f*x)^(p + 1), x], x]
/; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || Intege
rQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))))
```

rule 100
```
Int[((a_.) + (b_.)*(x_))^2*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))^(
p_), x_] :> Simp[(b*c - a*d)^2*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(d^2*(d
*e - c*f)*(n + 1))), x] - Simp[1/(d^2*(d*e - c*f)*(n + 1))   Int[(c + d*x)^
(n + 1)*(e + f*x)^p*Simp[a^2*d^2*f*(n + p + 2) + b^2*c*(d*e*(n + 1) + c*f*(
p + 1)) - 2*a*b*d*(d*e*(n + 1) + c*f*(p + 1)) - b^2*d*(d*e - c*f)*(n + 1)*x
, x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && (LtQ[n, -1] || (EqQ[n
 + p + 3, 0] && NeQ[n, -1] && (SumSimplerQ[n, 1] ||  !SumSimplerQ[p, 1])))
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3673
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_.)*tan[(e_.) + (f_.)*(x_)]^
(m_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x]^2, x]}, Simp[ff^((m
 + 1)/2)/(2*f)   Subst[Int[x^((m - 1)/2)*((a + b*ff*x)^p/(1 - ff*x)^((m + 1
)/2)), x], x, Sin[e + f*x]^2/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && Integ
erQ[(m - 1)/2]
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.62 (sec) , antiderivative size = 43, normalized size of antiderivative = 0.30

method	result	size
default	$\frac{\text{`int/indef0`}\left(\frac{\sinh(fx+e)^{5}}{\cosh(fx+e)^{6}\sqrt{a+b\sinh(fx+e)^{2}}},\sinh(fx+e)\right)}{f}$	43

input
```
int(tanh(f*x+e)^5/(a+b*sinh(f*x+e)^2)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
`int/indef0`(sinh(f*x+e)^5/cosh(f*x+e)^6/(a+b*sinh(f*x+e)^2)^(1/2),sinh(f*
x+e))/f
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2057 vs. $2(126) = 252$.

Time = 0.26 (sec) , antiderivative size = 4205, normalized size of antiderivative = 29.61

$$\int \frac{\tanh^5(e+fx)}{\sqrt{a+b\sinh^2(e+fx)}}\,dx = \text{Too large to display}$$

input
```
integrate(tanh(f*x+e)^5/(a+b*sinh(f*x+e)^2)^(1/2),x, algorithm="fricas")
```

output
```
Too large to include
```

Sympy [F]

$$\int \frac{\tanh^5(e+fx)}{\sqrt{a+b\sinh^2(e+fx)}}\,dx = \int \frac{\tanh^5(e+fx)}{\sqrt{a+b\sinh^2(e+fx)}}\,dx$$

input
```
integrate(tanh(f*x+e)**5/(a+b*sinh(f*x+e)**2)**(1/2),x)
```

output
```
Integral(tanh(e + f*x)**5/sqrt(a + b*sinh(e + f*x)**2), x)
```

Maxima [F]

$$\int \frac{\tanh^5(e+fx)}{\sqrt{a+b\sinh^2(e+fx)}}\,dx = \int \frac{\tanh(fx+e)^5}{\sqrt{b\sinh(fx+e)^2+a}}\,dx$$

input
```
integrate(tanh(f*x+e)^5/(a+b*sinh(f*x+e)^2)^(1/2),x, algorithm="maxima")
```

output
```
integrate(tanh(f*x + e)^5/sqrt(b*sinh(f*x + e)^2 + a), x)
```

Giac [F]

$$\int \frac{\tanh^5(e+fx)}{\sqrt{a+b\sinh^2(e+fx)}}\,dx = \int \frac{\tanh(fx+e)^5}{\sqrt{b\sinh(fx+e)^2+a}}\,dx$$

input
```
integrate(tanh(f*x+e)^5/(a+b*sinh(f*x+e)^2)^(1/2),x, algorithm="giac")
```

output
```
integrate(tanh(f*x + e)^5/sqrt(b*sinh(f*x + e)^2 + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\tanh^5(e+fx)}{\sqrt{a+b\sinh^2(e+fx)}}\,dx = \int \frac{\tanh(e+f\,x)^5}{\sqrt{b\sinh(e+f\,x)^2+a}}\,dx$$

input
```
int(tanh(e + f*x)^5/(a + b*sinh(e + f*x)^2)^(1/2),x)
```

output
```
int(tanh(e + f*x)^5/(a + b*sinh(e + f*x)^2)^(1/2), x)
```

Reduce [F]

$$\int \frac{\tanh^5(e+fx)}{\sqrt{a+b\sinh^2(e+fx)}}\,dx = \int \frac{\sqrt{\sinh(fx+e)^2b+a}\,\tanh(fx+e)^5}{\sinh(fx+e)^2b+a}\,dx$$

input
```
int(tanh(f*x+e)^5/(a+b*sinh(f*x+e)^2)^(1/2),x)
```

output
```
int((sqrt(sinh(e + f*x)**2*b + a)*tanh(e + f*x)**5)/(sinh(e + f*x)**2*b +
a),x)
```

3.444 $\int \frac{\tanh^3(e+fx)}{\sqrt{a+b\sinh^2(e+fx)}}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 89

$$\int \frac{\tanh^3(e+fx)}{\sqrt{a+b\sinh^2(e+fx)}}\,dx=-\frac{(2a-b)\operatorname{arctanh}\left(\frac{\sqrt{a+b\sinh^2(e+fx)}}{\sqrt{a-b}}\right)}{2(a-b)^{3/2}f}+\frac{\operatorname{sech}^2(e+fx)\sqrt{a+b\sinh^2(e+fx)}}{2(a-b)f}$$

output
```
-1/2*(2*a-b)*arctanh((a+b*sinh(f*x+e)^2)^(1/2)/(a-b)^(1/2))/(a-b)^(3/2)/f+
1/2*sech(f*x+e)^2*(a+b*sinh(f*x+e)^2)^(1/2)/(a-b)/f
```

Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 85, normalized size of antiderivative = 0.96

$$\int \frac{\tanh^3(e+fx)}{\sqrt{a+b\sinh^2(e+fx)}}\,dx=-\frac{\frac{(2a-b)\operatorname{arctanh}\left(\frac{\sqrt{a+b\sinh^2(e+fx)}}{\sqrt{a-b}}\right)}{(a-b)^{3/2}}-\frac{\operatorname{sech}^2(e+fx)\sqrt{a+b\sinh^2(e+fx)}}{a-b}}{2f}$$

input
```
Integrate[Tanh[e + f*x]^3/Sqrt[a + b*Sinh[e + f*x]^2],x]
```

output
```
-1/2*(((2*a - b)*ArcTanh[Sqrt[a + b*Sinh[e + f*x]^2]/Sqrt[a - b]])/(a - b)
^(3/2) - (Sech[e + f*x]^2*Sqrt[a + b*Sinh[e + f*x]^2])/(a - b))/f
```

Rubi [A] (verified)

Time = 0.34 (sec) , antiderivative size = 89, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.240, Rules used = {3042, 26, 3673, 87, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\tanh^3(e+fx)}{\sqrt{a+b\sinh^2(e+fx)}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{i\tan(ie+ifx)^3}{\sqrt{a-b\sin(ie+ifx)^2}}dx$$

$$\Big\downarrow \text{26}$$

$$i\int \frac{\tan(ie+ifx)^3}{\sqrt{a-b\sin(ie+ifx)^2}}dx$$

$$\Big\downarrow \text{3673}$$

$$\frac{\int \frac{\sinh^2(e+fx)}{\left(\sinh^2(e+fx)+1\right)^2\sqrt{b\sinh^2(e+fx)+a}}d\sinh^2(e+fx)}{2f}$$

$$\Big\downarrow \text{87}$$

$$\frac{\frac{(2a-b)\int \frac{1}{\left(\sinh^2(e+fx)+1\right)\sqrt{b\sinh^2(e+fx)+a}}d\sinh^2(e+fx)}{2(a-b)}+\frac{\sqrt{a+b\sinh^2(e+fx)}}{(a-b)\left(\sinh^2(e+fx)+1\right)}}{2f}$$

$$\Big\downarrow \text{73}$$

$$\frac{\frac{(2a-b)\int \frac{1}{\frac{\sinh^4(e+fx)}{b}-\frac{a}{b}+1}d\sqrt{b\sinh^2(e+fx)+a}}{b(a-b)}+\frac{\sqrt{a+b\sinh^2(e+fx)}}{(a-b)(\sinh^2(e+fx)+1)}}{2f}$$

$$\Big\downarrow \; 221$$

$$\frac{\frac{\sqrt{a+b\sinh^2(e+fx)}}{(a-b)(\sinh^2(e+fx)+1)}-\frac{(2a-b)\text{arctanh}\left(\frac{\sqrt{a+b\sinh^2(e+fx)}}{\sqrt{a-b}}\right)}{(a-b)^{3/2}}}{2f}$$

input
```
Int[Tanh[e + f*x]^3/Sqrt[a + b*Sinh[e + f*x]^2],x]
```

output
```
(-(((2*a - b)*ArcTanh[Sqrt[a + b*Sinh[e + f*x]^2]/Sqrt[a - b]])/(a - b)^(3
/2)) + Sqrt[a + b*Sinh[e + f*x]^2]/((a - b)*(1 + Sinh[e + f*x]^2)))/(2*f)
```

Defintions of rubi rules used

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 87
```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_] :> Simp[(-(b*e - a*f))*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(f*(p
+ 1)*(c*f - d*e))), x] - Simp[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p
+ 1)))/(f*(p + 1)*(c*f - d*e))   Int[(c + d*x)^n*(e + f*x)^(p + 1), x], x]
/; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || Intege
rQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))))
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3673
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_.)*tan[(e_.) + (f_.)*(x_)]^
(m_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x]^2, x]}, Simp[ff^((m
 + 1)/2)/(2*f)   Subst[Int[x^((m - 1)/2)*((a + b*ff*x)^p/(1 - ff*x)^((m + 1
)/2)), x], x, Sin[e + f*x]^2/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && Integ
erQ[(m - 1)/2]
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.55 (sec) , antiderivative size = 43, normalized size of antiderivative = 0.48

method	result	size
default	$\frac{\text{`int/indef0`}\left(\frac{\sinh(fx+e)^3}{\cosh(fx+e)^4\sqrt{a+b\sinh(fx+e)^2}},\sinh(fx+e)\right)}{f}$	43

input
```
int(tanh(f*x+e)^3/(a+b*sinh(f*x+e)^2)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
`int/indef0`(sinh(f*x+e)^3/cosh(f*x+e)^4/(a+b*sinh(f*x+e)^2)^(1/2),sinh(f*
x+e))/f
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 667 vs. $2(77) = 154$.

Time = 0.18 (sec) , antiderivative size = 1425, normalized size of antiderivative = 16.01

$$\int \frac{\tanh^3(e+fx)}{\sqrt{a+b\sinh^2(e+fx)}}\,dx = \text{Too large to display}$$

input
```
integrate(tanh(f*x+e)^3/(a+b*sinh(f*x+e)^2)^(1/2),x, algorithm="fricas")
```

output
```
[1/4*(((2*a - b)*cosh(f*x + e)^4 + 4*(2*a - b)*cosh(f*x + e)*sinh(f*x + e)
^3 + (2*a - b)*sinh(f*x + e)^4 + 2*(2*a - b)*cosh(f*x + e)^2 + 2*(3*(2*a -
 b)*cosh(f*x + e)^2 + 2*a - b)*sinh(f*x + e)^2 + 4*((2*a - b)*cosh(f*x + e
)^3 + (2*a - b)*cosh(f*x + e))*sinh(f*x + e) + 2*a - b)*sqrt(a - b)*log((b
*cosh(f*x + e)^4 + 4*b*cosh(f*x + e)*sinh(f*x + e)^3 + b*sinh(f*x + e)^4 +
 2*(4*a - 3*b)*cosh(f*x + e)^2 + 2*(3*b*cosh(f*x + e)^2 + 4*a - 3*b)*sinh(
f*x + e)^2 - 4*sqrt(2)*sqrt(a - b)*sqrt((b*cosh(f*x + e)^2 + b*sinh(f*x +
e)^2 + 2*a - b)/(cosh(f*x + e)^2 - 2*cosh(f*x + e)*sinh(f*x + e) + sinh(f*
x + e)^2))*(cosh(f*x + e) + sinh(f*x + e)) + 4*(b*cosh(f*x + e)^3 + (4*a -
 3*b)*cosh(f*x + e))*sinh(f*x + e) + b)/(cosh(f*x + e)^4 + 4*cosh(f*x + e)
*sinh(f*x + e)^3 + sinh(f*x + e)^4 + 2*(3*cosh(f*x + e)^2 + 1)*sinh(f*x +
e)^2 + 2*cosh(f*x + e)^2 + 4*(cosh(f*x + e)^3 + cosh(f*x + e))*sinh(f*x +
e) + 1)) + 4*sqrt(2)*((a - b)*cosh(f*x + e) + (a - b)*sinh(f*x + e))*sqrt(
(b*cosh(f*x + e)^2 + b*sinh(f*x + e)^2 + 2*a - b)/(cosh(f*x + e)^2 - 2*cos
h(f*x + e)*sinh(f*x + e) + sinh(f*x + e)^2)))/((a^2 - 2*a*b + b^2)*f*cosh(
f*x + e)^4 + 4*(a^2 - 2*a*b + b^2)*f*cosh(f*x + e)*sinh(f*x + e)^3 + (a^2
- 2*a*b + b^2)*f*sinh(f*x + e)^4 + 2*(a^2 - 2*a*b + b^2)*f*cosh(f*x + e)^2
 + 2*(3*(a^2 - 2*a*b + b^2)*f*cosh(f*x + e)^2 + (a^2 - 2*a*b + b^2)*f)*sin
h(f*x + e)^2 + (a^2 - 2*a*b + b^2)*f + 4*((a^2 - 2*a*b + b^2)*f*cosh(f*x +
 e)^3 + (a^2 - 2*a*b + b^2)*f*cosh(f*x + e))*sinh(f*x + e)), 1/2*(((2*a...
```

Sympy [F]

$$\int \frac{\tanh^3(e+fx)}{\sqrt{a+b\sinh^2(e+fx)}}\,dx = \int \frac{\tanh^3{(e+fx)}}{\sqrt{a+b\sinh^2{(e+fx)}}}\,dx$$

input
```
integrate(tanh(f*x+e)**3/(a+b*sinh(f*x+e)**2)**(1/2),x)
```

output
```
Integral(tanh(e + f*x)**3/sqrt(a + b*sinh(e + f*x)**2), x)
```

Maxima [F]

$$\int \frac{\tanh^3(e+fx)}{\sqrt{a+b\sinh^2(e+fx)}}\,dx = \int \frac{\tanh{(fx+e)}^3}{\sqrt{b\sinh{(fx+e)}^2+a}}\,dx$$

input
```
integrate(tanh(f*x+e)^3/(a+b*sinh(f*x+e)^2)^(1/2),x, algorithm="maxima")
```

output
```
integrate(tanh(f*x + e)^3/sqrt(b*sinh(f*x + e)^2 + a), x)
```

Giac [F]

$$\int \frac{\tanh^3(e+fx)}{\sqrt{a+b\sinh^2(e+fx)}}\,dx = \int \frac{\tanh{(fx+e)}^3}{\sqrt{b\sinh{(fx+e)}^2+a}}\,dx$$

input
```
integrate(tanh(f*x+e)^3/(a+b*sinh(f*x+e)^2)^(1/2),x, algorithm="giac")
```

output
```
integrate(tanh(f*x + e)^3/sqrt(b*sinh(f*x + e)^2 + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\tanh^3(e+fx)}{\sqrt{a+b\sinh^2(e+fx)}}\,dx=\int \frac{\tanh(e+f\,x)^3}{\sqrt{b\sinh\left(e+f\,x\right)^2+a}}\,dx$$

input
```
int(tanh(e + f*x)^3/(a + b*sinh(e + f*x)^2)^(1/2),x)
```

output
```
int(tanh(e + f*x)^3/(a + b*sinh(e + f*x)^2)^(1/2), x)
```

Reduce [F]

$$\int \frac{\tanh^3(e+fx)}{\sqrt{a+b\sinh^2(e+fx)}}\,dx=\int \frac{\sqrt{\sinh\left(fx+e\right)^2 b+a}\,\tanh\left(fx+e\right)^3}{\sinh\left(fx+e\right)^2 b+a}dx$$

input
```
int(tanh(f*x+e)^3/(a+b*sinh(f*x+e)^2)^(1/2),x)
```

output
```
int((sqrt(sinh(e + f*x)**2*b + a)*tanh(e + f*x)**3)/(sinh(e + f*x)**2*b +
a),x)
```

3.445 $\int \frac{\tanh(e+fx)}{\sqrt{a+b\sinh^2(e+fx)}}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 41

$$\int \frac{\tanh(e+fx)}{\sqrt{a+b\sinh^2(e+fx)}}\,dx = -\frac{\operatorname{arctanh}\left(\frac{\sqrt{a+b\sinh^2(e+fx)}}{\sqrt{a-b}}\right)}{\sqrt{a-b}f}$$

output
```
-arctanh((a+b*sinh(f*x+e)^2)^(1/2)/(a-b)^(1/2))/(a-b)^(1/2)/f
```

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 44, normalized size of antiderivative = 1.07

$$\int \frac{\tanh(e+fx)}{\sqrt{a+b\sinh^2(e+fx)}}\,dx = -\frac{\operatorname{arctanh}\left(\frac{\sqrt{a-b+b\cosh^2(e+fx)}}{\sqrt{a-b}}\right)}{\sqrt{a-b}f}$$

input
```
Integrate[Tanh[e + f*x]/Sqrt[a + b*Sinh[e + f*x]^2],x]
```

output
```
-(ArcTanh[Sqrt[a - b + b*Cosh[e + f*x]^2]/Sqrt[a - b]]/(Sqrt[a - b]*f))
```

Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 41, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.217, Rules used = {3042, 26, 3673, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\tanh(e+fx)}{\sqrt{a+b\sinh^2(e+fx)}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int -\frac{i\tan(ie+ifx)}{\sqrt{a-b\sin(ie+ifx)^2}}dx$$

$$\Big\downarrow \text{26}$$

$$-i\int \frac{\tan(ie+ifx)}{\sqrt{a-b\sin(ie+ifx)^2}}dx$$

$$\Big\downarrow \text{3673}$$

$$\frac{\int \frac{1}{(\sinh^2(e+fx)+1)\sqrt{b\sinh^2(e+fx)+a}}d\sinh^2(e+fx)}{2f}$$

$$\Big\downarrow \text{73}$$

$$\frac{\int \frac{1}{\frac{\sinh^4(e+fx)}{b}-\frac{a}{b}+1}d\sqrt{b\sinh^2(e+fx)+a}}{bf}$$

$$\Big\downarrow \text{221}$$

$$-\frac{\text{arctanh}\left(\frac{\sqrt{a+b\sinh^2(e+fx)}}{\sqrt{a-b}}\right)}{f\sqrt{a-b}}$$

input
```
Int[Tanh[e + f*x]/Sqrt[a + b*Sinh[e + f*x]^2],x]
```

output
```
-(ArcTanh[Sqrt[a + b*Sinh[e + f*x]^2]/Sqrt[a - b]]/(Sqrt[a - b]*f))
```

Defintions of rubi rules used

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3673
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_.)*tan[(e_.) + (f_.)*(x_)]^
(m_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x]^2, x]}, Simp[ff^((m
 + 1)/2)/(2*f)   Subst[Int[x^((m - 1)/2)*((a + b*ff*x)^p/(1 - ff*x)^((m + 1
)/2)), x], x, Sin[e + f*x]^2/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && Integ
erQ[(m - 1)/2]
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.43 (sec) , antiderivative size = 41, normalized size of antiderivative = 1.00

method	result	size
default	$\frac{\text{`int/indef0`}\left(\frac{\sinh(fx+e)}{\cosh(fx+e)^2\sqrt{a+b\sinh(fx+e)^2}},\sinh(fx+e)\right)}{f}$	41

input
```
int(tanh(f*x+e)/(a+b*sinh(f*x+e)^2)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
`int/indef0`(sinh(f*x+e)/cosh(f*x+e)^2/(a+b*sinh(f*x+e)^2)^(1/2),sinh(f*x+
e))/f
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 226 vs. $2(35) = 70$.

Time = 0.15 (sec) , antiderivative size = 537, normalized size of antiderivative = 13.10

$$\int \frac{\tanh(e+fx)}{\sqrt{a+b\sinh^2(e+fx)}}\,dx = \text{Too large to display}$$

input
```
integrate(tanh(f*x+e)/(a+b*sinh(f*x+e)^2)^(1/2),x, algorithm="fricas")
```

output
```
[1/2*log((b*cosh(f*x + e)^4 + 4*b*cosh(f*x + e)*sinh(f*x + e)^3 + b*sinh(f
*x + e)^4 + 2*(4*a - 3*b)*cosh(f*x + e)^2 + 2*(3*b*cosh(f*x + e)^2 + 4*a -
 3*b)*sinh(f*x + e)^2 - 4*sqrt(2)*sqrt(a - b)*sqrt((b*cosh(f*x + e)^2 + b*
sinh(f*x + e)^2 + 2*a - b)/(cosh(f*x + e)^2 - 2*cosh(f*x + e)*sinh(f*x + e
) + sinh(f*x + e)^2))*(cosh(f*x + e) + sinh(f*x + e)) + 4*(b*cosh(f*x + e)
^3 + (4*a - 3*b)*cosh(f*x + e))*sinh(f*x + e) + b)/(cosh(f*x + e)^4 + 4*co
sh(f*x + e)*sinh(f*x + e)^3 + sinh(f*x + e)^4 + 2*(3*cosh(f*x + e)^2 + 1)*
sinh(f*x + e)^2 + 2*cosh(f*x + e)^2 + 4*(cosh(f*x + e)^3 + cosh(f*x + e))*
sinh(f*x + e) + 1))/(sqrt(a - b)*f), sqrt(-a + b)*arctan(2*sqrt(2)*sqrt(-a
 + b)*sqrt((b*cosh(f*x + e)^2 + b*sinh(f*x + e)^2 + 2*a - b)/(cosh(f*x + e
)^2 - 2*cosh(f*x + e)*sinh(f*x + e) + sinh(f*x + e)^2))*(cosh(f*x + e) + s
inh(f*x + e))/(b*cosh(f*x + e)^4 + 4*b*cosh(f*x + e)*sinh(f*x + e)^3 + b*s
inh(f*x + e)^4 + 2*(2*a - b)*cosh(f*x + e)^2 + 2*(3*b*cosh(f*x + e)^2 + 2*
a - b)*sinh(f*x + e)^2 + 4*(b*cosh(f*x + e)^3 + (2*a - b)*cosh(f*x + e))*s
inh(f*x + e) + b))/((a - b)*f)]
```

Sympy [F]

$$\int \frac{\tanh(e+fx)}{\sqrt{a+b\sinh^2(e+fx)}}\,dx = \int \frac{\tanh(e+fx)}{\sqrt{a+b\sinh^2(e+fx)}}\,dx$$

input
```
integrate(tanh(f*x+e)/(a+b*sinh(f*x+e)**2)**(1/2),x)
```

output
```
Integral(tanh(e + f*x)/sqrt(a + b*sinh(e + f*x)**2), x)
```

Maxima [F]

$$\int \frac{\tanh(e+fx)}{\sqrt{a+b\sinh^2(e+fx)}}\,dx = \int \frac{\tanh(fx+e)}{\sqrt{b\sinh(fx+e)^2+a}}\,dx$$

input
```
integrate(tanh(f*x+e)/(a+b*sinh(f*x+e)^2)^(1/2),x, algorithm="maxima")
```

output
```
integrate(tanh(f*x + e)/sqrt(b*sinh(f*x + e)^2 + a), x)
```

Giac [F]

$$\int \frac{\tanh(e+fx)}{\sqrt{a+b\sinh^2(e+fx)}}\,dx = \int \frac{\tanh(fx+e)}{\sqrt{b\sinh(fx+e)^2+a}}\,dx$$

input
```
integrate(tanh(f*x+e)/(a+b*sinh(f*x+e)^2)^(1/2),x, algorithm="giac")
```

output
```
integrate(tanh(f*x + e)/sqrt(b*sinh(f*x + e)^2 + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\tanh(e + fx)}{\sqrt{a + b\sinh^2(e + fx)}}\, dx = \int \frac{\tanh(e + f\,x)}{\sqrt{b\sinh\left(e + f\,x\right)^2 + a}}\, dx$$

input
```
int(tanh(e + f*x)/(a + b*sinh(e + f*x)^2)^(1/2),x)
```

output
```
int(tanh(e + f*x)/(a + b*sinh(e + f*x)^2)^(1/2), x)
```

Reduce [F]

$$\int \frac{\tanh(e + fx)}{\sqrt{a + b\sinh^2(e + fx)}}\, dx = \int \frac{\sqrt{\sinh\left(fx + e\right)^2 b + a}\,\tanh\left(fx + e\right)}{\sinh\left(fx + e\right)^2 b + a}dx$$

input
```
int(tanh(f*x+e)/(a+b*sinh(f*x+e)^2)^(1/2),x)
```

output
```
int((sqrt(sinh(e + f*x)**2*b + a)*tanh(e + f*x))/(sinh(e + f*x)**2*b + a),
x)
```

3.446 $\int \frac{\coth(e+fx)}{\sqrt{a+b\sinh^2(e+fx)}}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 33

$$\int \frac{\coth(e+fx)}{\sqrt{a+b\sinh^2(e+fx)}}\,dx = -\frac{\operatorname{arctanh}\left(\frac{\sqrt{a+b\sinh^2(e+fx)}}{\sqrt{a}}\right)}{\sqrt{a}f}$$

output
```
-arctanh((a+b*sinh(f*x+e)^2)^(1/2)/a^(1/2))/a^(1/2)/f
```

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 33, normalized size of antiderivative = 1.00

$$\int \frac{\coth(e+fx)}{\sqrt{a+b\sinh^2(e+fx)}}\,dx = -\frac{\operatorname{arctanh}\left(\frac{\sqrt{a+b\sinh^2(e+fx)}}{\sqrt{a}}\right)}{\sqrt{a}f}$$

input
```
Integrate[Coth[e + f*x]/Sqrt[a + b*Sinh[e + f*x]^2],x]
```

output
```
-(ArcTanh[Sqrt[a + b*Sinh[e + f*x]^2]/Sqrt[a]]/(Sqrt[a]*f))
```

Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 33, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.217, Rules used = {3042, 26, 3673, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\coth(e+fx)}{\sqrt{a+b\sinh^2(e+fx)}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{i}{\tan(ie+ifx)\sqrt{a-b\sin(ie+ifx)^2}}dx$$

$$\Big\downarrow \text{26}$$

$$i\int \frac{1}{\sqrt{a-b\sin(ie+ifx)^2}\tan(ie+ifx)}dx$$

$$\Big\downarrow \text{3673}$$

$$\frac{\int \frac{\text{csch}^2(e+fx)}{\sqrt{b\sinh^2(e+fx)+a}}d\sinh^2(e+fx)}{2f}$$

$$\Big\downarrow \text{73}$$

$$\frac{\int \frac{1}{\frac{\sinh^4(e+fx)}{b}-\frac{a}{b}}d\sqrt{b\sinh^2(e+fx)+a}}{bf}$$

$$\Big\downarrow \text{221}$$

$$-\frac{\text{arctanh}\left(\frac{\sqrt{a+b\sinh^2(e+fx)}}{\sqrt{a}}\right)}{\sqrt{a}f}$$

input
```
Int[Coth[e + f*x]/Sqrt[a + b*Sinh[e + f*x]^2],x]
```

output
```
-(ArcTanh[Sqrt[a + b*Sinh[e + f*x]^2]/Sqrt[a]]/(Sqrt[a]*f))
```

Defintions of rubi rules used

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3673
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_.)*tan[(e_.) + (f_.)*(x_)]^
(m_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x]^2, x]}, Simp[ff^((m
 + 1)/2)/(2*f)   Subst[Int[x^((m - 1)/2)*((a + b*ff*x)^p/(1 - ff*x)^((m + 1
)/2)), x], x, Sin[e + f*x]^2/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && Integ
erQ[(m - 1)/2]
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.43 (sec) , antiderivative size = 35, normalized size of antiderivative = 1.06

method	result	size
default	$\frac{\text{` int/indef0`}\left(\frac{1}{\sinh(fx+e)\sqrt{a+b\sinh(fx+e)^{2}}},\sinh(fx+e)\right)}{f}$	35

input
```
int(coth(f*x+e)/(a+b*sinh(f*x+e)^2)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
`int/indef0`(1/sinh(f*x+e)/(a+b*sinh(f*x+e)^2)^(1/2),sinh(f*x+e))/f
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 218 vs. $2(27) = 54$.

Time = 0.14 (sec) , antiderivative size = 523, normalized size of antiderivative = 15.85

$$\int \frac{\coth(e+fx)}{\sqrt{a+b\sinh^2(e+fx)}}\,dx$$

$$= \left[\frac{\log\left(\frac{b\cosh(fx+e)^4+4\,b\cosh(fx+e)\sinh(fx+e)^3+b\sinh(fx+e)^4+2\,(4\,a-b)\cosh(fx+e)^2+2\left(3\,b\cosh(fx+e)^2+4\,a-b\right)\sinh(fx+e)^2-4\,\sqrt{}\cdots}{\cosh(fx+e)^4+4\,\cosh(fx+e)\sinh(fx+e)^3+\sinh(fx+e)^4+2\left(3\,\cosh(fx+e)\cdots\right.}\right.}{2\cdots} \right.$$

input
```
integrate(coth(f*x+e)/(a+b*sinh(f*x+e)^2)^(1/2),x, algorithm="fricas")
```

output
```
[1/2*log((b*cosh(f*x + e)^4 + 4*b*cosh(f*x + e)*sinh(f*x + e)^3 + b*sinh(f
*x + e)^4 + 2*(4*a - b)*cosh(f*x + e)^2 + 2*(3*b*cosh(f*x + e)^2 + 4*a - b
)*sinh(f*x + e)^2 - 4*sqrt(2)*sqrt(a)*sqrt((b*cosh(f*x + e)^2 + b*sinh(f*x
 + e)^2 + 2*a - b)/(cosh(f*x + e)^2 - 2*cosh(f*x + e)*sinh(f*x + e) + sinh
(f*x + e)^2))*(cosh(f*x + e) + sinh(f*x + e)) + 4*(b*cosh(f*x + e)^3 + (4*
a - b)*cosh(f*x + e))*sinh(f*x + e) + b)/(cosh(f*x + e)^4 + 4*cosh(f*x + e
)*sinh(f*x + e)^3 + sinh(f*x + e)^4 + 2*(3*cosh(f*x + e)^2 - 1)*sinh(f*x +
 e)^2 - 2*cosh(f*x + e)^2 + 4*(cosh(f*x + e)^3 - cosh(f*x + e))*sinh(f*x +
 e) + 1))/(sqrt(a)*f), sqrt(-a)*arctan(2*sqrt(2)*sqrt(-a)*sqrt((b*cosh(f*x
 + e)^2 + b*sinh(f*x + e)^2 + 2*a - b)/(cosh(f*x + e)^2 - 2*cosh(f*x + e)*
sinh(f*x + e) + sinh(f*x + e)^2))*(cosh(f*x + e) + sinh(f*x + e))/(b*cosh(
f*x + e)^4 + 4*b*cosh(f*x + e)*sinh(f*x + e)^3 + b*sinh(f*x + e)^4 + 2*(2*
a - b)*cosh(f*x + e)^2 + 2*(3*b*cosh(f*x + e)^2 + 2*a - b)*sinh(f*x + e)^2
 + 4*(b*cosh(f*x + e)^3 + (2*a - b)*cosh(f*x + e))*sinh(f*x + e) + b))/(a*
f)]
```

Sympy [F]

$$\int \frac{\coth(e+fx)}{\sqrt{a+b\sinh^2(e+fx)}}\,dx = \int \frac{\coth\left(e+fx\right)}{\sqrt{a+b\sinh^2\left(e+fx\right)}}\,dx$$

input
```
integrate(coth(f*x+e)/(a+b*sinh(f*x+e)**2)**(1/2),x)
```

output
```
Integral(coth(e + f*x)/sqrt(a + b*sinh(e + f*x)**2), x)
```

Maxima [F]

$$\int \frac{\coth(e+fx)}{\sqrt{a+b\sinh^2(e+fx)}}\,dx = \int \frac{\coth\left(fx+e\right)}{\sqrt{b\sinh\left(fx+e\right)^2+a}}\,dx$$

input
```
integrate(coth(f*x+e)/(a+b*sinh(f*x+e)^2)^(1/2),x, algorithm="maxima")
```

output
```
integrate(coth(f*x + e)/sqrt(b*sinh(f*x + e)^2 + a), x)
```

Giac [F]

$$\int \frac{\coth(e+fx)}{\sqrt{a+b\sinh^2(e+fx)}}\,dx = \int \frac{\coth\left(fx+e\right)}{\sqrt{b\sinh\left(fx+e\right)^2+a}}\,dx$$

input
```
integrate(coth(f*x+e)/(a+b*sinh(f*x+e)^2)^(1/2),x, algorithm="giac")
```

output
```
integrate(coth(f*x + e)/sqrt(b*sinh(f*x + e)^2 + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\coth(e+fx)}{\sqrt{a+b\sinh^2(e+fx)}}\,dx=\int \frac{\coth(e+f\,x)}{\sqrt{b\sinh\left(e+f\,x\right)^2+a}}\,dx$$

input
```
int(coth(e + f*x)/(a + b*sinh(e + f*x)^2)^(1/2),x)
```

output
```
int(coth(e + f*x)/(a + b*sinh(e + f*x)^2)^(1/2), x)
```

Reduce [F]

$$\int \frac{\coth(e+fx)}{\sqrt{a+b\sinh^2(e+fx)}}\,dx=\int \frac{\sqrt{\sinh\left(fx+e\right)^2 b+a}\,\coth\left(fx+e\right)}{\sinh\left(fx+e\right)^2 b+a}dx$$

input
```
int(coth(f*x+e)/(a+b*sinh(f*x+e)^2)^(1/2),x)
```

output
```
int((sqrt(sinh(e + f*x)**2*b + a)*coth(e + f*x))/(sinh(e + f*x)**2*b + a),
x)
```

3.447 $\int \frac{\coth^3(e+fx)}{\sqrt{a+b\sinh^2(e+fx)}}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 77

$$\int \frac{\coth^3(e+fx)}{\sqrt{a+b\sinh^2(e+fx)}}\,dx = -\frac{(2a-b)\operatorname{arctanh}\left(\frac{\sqrt{a+b\sinh^2(e+fx)}}{\sqrt{a}}\right)}{2a^{3/2}f} - \frac{\operatorname{csch}^2(e+fx)\sqrt{a+b\sinh^2(e+fx)}}{2af}$$

output
```
-1/2*(2*a-b)*arctanh((a+b*sinh(f*x+e)^2)^(1/2)/a^(1/2))/a^(3/2)/f-1/2*csch
(f*x+e)^2*(a+b*sinh(f*x+e)^2)^(1/2)/a/f
```

Mathematica [A] (verified)

Time = 0.13 (sec) , antiderivative size = 72, normalized size of antiderivative = 0.94

$$\int \frac{\coth^3(e+fx)}{\sqrt{a+b\sinh^2(e+fx)}}\,dx = -\frac{\frac{(2a-b)\operatorname{arctanh}\left(\frac{\sqrt{a+b\sinh^2(e+fx)}}{\sqrt{a}}\right)}{a^{3/2}} + \frac{\operatorname{csch}^2(e+fx)\sqrt{a+b\sinh^2(e+fx)}}{a}}{2f}$$

input
```
Integrate[Coth[e + f*x]^3/Sqrt[a + b*Sinh[e + f*x]^2],x]
```

output
```
-1/2*(((2*a - b)*ArcTanh[Sqrt[a + b*Sinh[e + f*x]^2]/Sqrt[a]])/a^(3/2) + (
Csch[e + f*x]^2*Sqrt[a + b*Sinh[e + f*x]^2])/a)/f
```

Rubi [A] (verified)

Time = 0.36 (sec) , antiderivative size = 74, normalized size of antiderivative = 0.96, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.240$, Rules used = {3042, 26, 3673, 87, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\coth^3(e+fx)}{\sqrt{a+b\sinh^2(e+fx)}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int -\frac{i}{\tan(ie+ifx)^3\sqrt{a-b\sin(ie+ifx)^2}}dx$$

$$\Big\downarrow \text{26}$$

$$-i\int \frac{1}{\sqrt{a-b\sin(ie+ifx)^2}\tan(ie+ifx)^3}dx$$

$$\Big\downarrow \text{3673}$$

$$\frac{\int \frac{\text{csch}^4(e+fx)\left(\sinh^2(e+fx)+1\right)}{\sqrt{b\sinh^2(e+fx)+a}}d\sinh^2(e+fx)}{2f}$$

$$\Big\downarrow \text{87}$$

$$\frac{\frac{(2a-b)\int \frac{\text{csch}^2(e+fx)}{\sqrt{b\sinh^2(e+fx)+a}}d\sinh^2(e+fx)}{2a}-\frac{\text{csch}^2(e+fx)\sqrt{a+b\sinh^2(e+fx)}}{a}}{2f}$$

$$\Big\downarrow \text{73}$$

$$\frac{\frac{(2a-b)\int \frac{1}{\frac{\sinh^4(e+fx)}{b}-\frac{a}{b}}d\sqrt{b\sinh^2(e+fx)+a}}{ab}-\frac{\operatorname{csch}^2(e+fx)\sqrt{a+b\sinh^2(e+fx)}}{a}}{2f}$$

$$\Big\downarrow \; 221$$

$$\frac{-\frac{(2a-b)\operatorname{arctanh}\left(\frac{\sqrt{a+b\sinh^2(e+fx)}}{\sqrt{a}}\right)}{a^{3/2}}-\frac{\operatorname{csch}^2(e+fx)\sqrt{a+b\sinh^2(e+fx)}}{a}}{2f}$$

input
```
Int[Coth[e + f*x]^3/Sqrt[a + b*Sinh[e + f*x]^2],x]
```

output
```
(-(((2*a - b)*ArcTanh[Sqrt[a + b*Sinh[e + f*x]^2]/Sqrt[a]])/a^(3/2)) - (Cs
ch[e + f*x]^2*Sqrt[a + b*Sinh[e + f*x]^2])/a)/(2*f)
```

Defintions of rubi rules used

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 87
```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_] :> Simp[(-(b*e - a*f))*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(f*(p
+ 1)*(c*f - d*e))), x] - Simp[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p
+ 1)))/(f*(p + 1)*(c*f - d*e))   Int[(c + d*x)^n*(e + f*x)^(p + 1), x], x]
/; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || Intege
rQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))))
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3673
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_.)*tan[(e_.) + (f_.)*(x_)]^
(m_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x]^2, x]}, Simp[ff^((m
 + 1)/2)/(2*f)   Subst[Int[x^((m - 1)/2)*((a + b*ff*x)^p/(1 - ff*x)^((m + 1
)/2)), x], x, Sin[e + f*x]^2/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && Integ
erQ[(m - 1)/2]
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.53 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.57

method	result	size
default	$\frac{\text{`int/indef0`}\left(\frac{\frac{1}{\sinh(fx+e)}+\frac{1}{\sinh(fx+e)^{3}}}{\sqrt{a+b\sinh(fx+e)^{2}}}, \sinh(fx+e)\right)}{f}$	44

input
```
int(coth(f*x+e)^3/(a+b*sinh(f*x+e)^2)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
`int/indef0`((1/sinh(f*x+e)+1/sinh(f*x+e)^3)/(a+b*sinh(f*x+e)^2)^(1/2),sin
h(f*x+e))/f
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 584 vs. 2(65) = 130.

Time = 0.15 (sec) , antiderivative size = 1257, normalized size of antiderivative = 16.32

$$\int \frac{\coth^3(e+fx)}{\sqrt{a+b\sinh^2(e+fx)}}\,dx = \text{Too large to display}$$

input
```
integrate(coth(f*x+e)^3/(a+b*sinh(f*x+e)^2)^(1/2),x, algorithm="fricas")
```

output
```
[-1/4*(((2*a - b)*cosh(f*x + e)^4 + 4*(2*a - b)*cosh(f*x + e)*sinh(f*x + e
)^3 + (2*a - b)*sinh(f*x + e)^4 - 2*(2*a - b)*cosh(f*x + e)^2 + 2*(3*(2*a
- b)*cosh(f*x + e)^2 - 2*a + b)*sinh(f*x + e)^2 + 4*((2*a - b)*cosh(f*x +
e)^3 - (2*a - b)*cosh(f*x + e))*sinh(f*x + e) + 2*a - b)*sqrt(a)*log((b*co
sh(f*x + e)^4 + 4*b*cosh(f*x + e)*sinh(f*x + e)^3 + b*sinh(f*x + e)^4 + 2*
(4*a - b)*cosh(f*x + e)^2 + 2*(3*b*cosh(f*x + e)^2 + 4*a - b)*sinh(f*x + e
)^2 + 4*sqrt(2)*sqrt(a)*sqrt((b*cosh(f*x + e)^2 + b*sinh(f*x + e)^2 + 2*a
- b)/(cosh(f*x + e)^2 - 2*cosh(f*x + e)*sinh(f*x + e) + sinh(f*x + e)^2))*
(cosh(f*x + e) + sinh(f*x + e)) + 4*(b*cosh(f*x + e)^3 + (4*a - b)*cosh(f*
x + e))*sinh(f*x + e) + b)/(cosh(f*x + e)^4 + 4*cosh(f*x + e)*sinh(f*x + e
)^3 + sinh(f*x + e)^4 + 2*(3*cosh(f*x + e)^2 - 1)*sinh(f*x + e)^2 - 2*cosh
(f*x + e)^2 + 4*(cosh(f*x + e)^3 - cosh(f*x + e))*sinh(f*x + e) + 1)) + 4*
sqrt(2)*(a*cosh(f*x + e) + a*sinh(f*x + e))*sqrt((b*cosh(f*x + e)^2 + b*si
nh(f*x + e)^2 + 2*a - b)/(cosh(f*x + e)^2 - 2*cosh(f*x + e)*sinh(f*x + e)
+ sinh(f*x + e)^2)))/(a^2*f*cosh(f*x + e)^4 + 4*a^2*f*cosh(f*x + e)*sinh(f
*x + e)^3 + a^2*f*sinh(f*x + e)^4 - 2*a^2*f*cosh(f*x + e)^2 + a^2*f + 2*(3
*a^2*f*cosh(f*x + e)^2 - a^2*f)*sinh(f*x + e)^2 + 4*(a^2*f*cosh(f*x + e)^3
 - a^2*f*cosh(f*x + e))*sinh(f*x + e)), 1/2*(((2*a - b)*cosh(f*x + e)^4 +
4*(2*a - b)*cosh(f*x + e)*sinh(f*x + e)^3 + (2*a - b)*sinh(f*x + e)^4 - 2*
(2*a - b)*cosh(f*x + e)^2 + 2*(3*(2*a - b)*cosh(f*x + e)^2 - 2*a + b)*s...
```

Sympy [F]

$$\int \frac{\coth^3(e+fx)}{\sqrt{a+b\sinh^2(e+fx)}}\,dx=\int \frac{\coth^3{(e+fx)}}{\sqrt{a+b\sinh^2{(e+fx)}}}\,dx$$

input
```
integrate(coth(f*x+e)**3/(a+b*sinh(f*x+e)**2)**(1/2),x)
```

output
```
Integral(coth(e + f*x)**3/sqrt(a + b*sinh(e + f*x)**2), x)
```

Maxima [F]

$$\int \frac{\coth^3(e+fx)}{\sqrt{a+b\sinh^2(e+fx)}}\,dx = \int \frac{\coth\left(fx+e\right)^3}{\sqrt{b\sinh\left(fx+e\right)^2+a}}\,dx$$

input
```
integrate(coth(f*x+e)^3/(a+b*sinh(f*x+e)^2)^(1/2),x, algorithm="maxima")
```

output
```
integrate(coth(f*x + e)^3/sqrt(b*sinh(f*x + e)^2 + a), x)
```

Giac [F]

$$\int \frac{\coth^3(e+fx)}{\sqrt{a+b\sinh^2(e+fx)}}\,dx = \int \frac{\coth\left(fx+e\right)^3}{\sqrt{b\sinh\left(fx+e\right)^2+a}}\,dx$$

input
```
integrate(coth(f*x+e)^3/(a+b*sinh(f*x+e)^2)^(1/2),x, algorithm="giac")
```

output
```
integrate(coth(f*x + e)^3/sqrt(b*sinh(f*x + e)^2 + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\coth^3(e+fx)}{\sqrt{a+b\sinh^2(e+fx)}}\,dx = \int \frac{\coth(e+f\,x)^3}{\sqrt{b\sinh\left(e+f\,x\right)^2+a}}\,dx$$

input
```
int(coth(e + f*x)^3/(a + b*sinh(e + f*x)^2)^(1/2),x)
```

output
```
int(coth(e + f*x)^3/(a + b*sinh(e + f*x)^2)^(1/2), x)
```

Reduce [F]

$$\int \frac{\coth^3(e+fx)}{\sqrt{a+b\sinh^2(e+fx)}}\,dx=\int \frac{\sqrt{\sinh(fx+e)^2 b+a}\,\coth(fx+e)^3}{\sinh(fx+e)^2 b+a}dx$$

input
```
int(coth(f*x+e)^3/(a+b*sinh(f*x+e)^2)^(1/2),x)
```

output
```
int((sqrt(sinh(e + f*x)**2*b + a)*coth(e + f*x)**3)/(sinh(e + f*x)**2*b +
a),x)
```

3.448 $\int \frac{\coth^5(e+fx)}{\sqrt{a+b\sinh^2(e+fx)}}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 126

$$\int \frac{\coth^5(e+fx)}{\sqrt{a+b\sinh^2(e+fx)}}\,dx = -\frac{(8a^2-8ab+3b^2)\,\text{arctanh}\left(\frac{\sqrt{a+b\sinh^2(e+fx)}}{\sqrt{a}}\right)}{8a^{5/2}f} - \frac{(8a-3b)\text{csch}^2(e+fx)\sqrt{a+b\sinh^2(e+fx)}}{8a^2f} - \frac{\text{csch}^4(e+fx)\sqrt{a+b\sinh^2(e+fx)}}{4af}$$

output
```
-1/8*(8*a^2-8*a*b+3*b^2)*arctanh((a+b*sinh(f*x+e)^2)^(1/2)/a^(1/2))/a^(5/2
)/f-1/8*(8*a-3*b)*csch(f*x+e)^2*(a+b*sinh(f*x+e)^2)^(1/2)/a^2/f-1/4*csch(f
*x+e)^4*(a+b*sinh(f*x+e)^2)^(1/2)/a/f
```

Mathematica [A] (verified)

Time = 0.25 (sec) , antiderivative size = 100, normalized size of antiderivative = 0.79

$$\int \frac{\coth^5(e+fx)}{\sqrt{a+b\sinh^2(e+fx)}}\,dx$$
$$= \frac{(-8a^2+8ab-3b^2)\,\text{arctanh}\left(\frac{\sqrt{a+b\sinh^2(e+fx)}}{\sqrt{a}}\right)+\sqrt{a}\text{csch}^2(e+fx)\left(-8a+3b-2a\text{csch}^2(e+fx)\right)\sqrt{a+}}{8a^{5/2}f}$$

input
```
Integrate[Coth[e + f*x]^5/Sqrt[a + b*Sinh[e + f*x]^2],x]
```

output
```
((-8*a^2 + 8*a*b - 3*b^2)*ArcTanh[Sqrt[a + b*Sinh[e + f*x]^2]/Sqrt[a]] + S
qrt[a]*Csch[e + f*x]^2*(-8*a + 3*b - 2*a*Csch[e + f*x]^2)*Sqrt[a + b*Sinh[
e + f*x]^2])/(8*a^(5/2)*f)
```

Rubi [A] (verified)

Time = 0.38 (sec) , antiderivative size = 128, normalized size of antiderivative = 1.02, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.320$, Rules used = {3042, 26, 3673, 100, 27, 87, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\coth^5(e+fx)}{\sqrt{a+b\sinh^2(e+fx)}}\,dx$$
$$\Big\downarrow \; 3042$$
$$\int \frac{i}{\tan(ie+ifx)^5\sqrt{a-b\sin(ie+ifx)^2}}dx$$
$$\Big\downarrow \; 26$$
$$i\int \frac{1}{\sqrt{a-b\sin(ie+ifx)^2}\tan(ie+ifx)^5}dx$$
$$\Big\downarrow \; 3673$$

$$\frac{\int \frac{\operatorname{csch}^6(e+fx)(\sinh^2(e+fx)+1)^2}{\sqrt{b\sinh^2(e+fx)+a}}d\sinh^2(e+fx)}{2f}$$

$$\Big\downarrow \text{100}$$

$$\frac{\frac{\int \frac{\operatorname{csch}^4(e+fx)\left(4a\sinh^2(e+fx)+8a-3b\right)}{2\sqrt{b\sinh^2(e+fx)+a}}d\sinh^2(e+fx)}{2a}-\frac{\operatorname{csch}^4(e+fx)\sqrt{a+b\sinh^2(e+fx)}}{2a}}{2f}$$

$$\Big\downarrow \text{27}$$

$$\frac{\frac{\int \frac{\operatorname{csch}^4(e+fx)\left(4a\sinh^2(e+fx)+8a-3b\right)}{\sqrt{b\sinh^2(e+fx)+a}}d\sinh^2(e+fx)}{4a}-\frac{\operatorname{csch}^4(e+fx)\sqrt{a+b\sinh^2(e+fx)}}{2a}}{2f}$$

$$\Big\downarrow \text{87}$$

$$\frac{\frac{\frac{\left(8a^2-8ab+3b^2\right)\int \frac{\operatorname{csch}^2(e+fx)}{\sqrt{b\sinh^2(e+fx)+a}}d\sinh^2(e+fx)}{2a}-\frac{(8a-3b)\operatorname{csch}^2(e+fx)\sqrt{a+b\sinh^2(e+fx)}}{a}}{4a}-\frac{\operatorname{csch}^4(e+fx)\sqrt{a+b\sinh^2(e+fx)}}{2a}}{2f}$$

$$\Big\downarrow \text{73}$$

$$\frac{\frac{\frac{\left(8a^2-8ab+3b^2\right)\int \frac{1}{\frac{\sinh^4(e+fx)}{b}-\frac{a}{b}}d\sqrt{b\sinh^2(e+fx)+a}}{ab}-\frac{(8a-3b)\operatorname{csch}^2(e+fx)\sqrt{a+b\sinh^2(e+fx)}}{a}}{4a}-\frac{\operatorname{csch}^4(e+fx)\sqrt{a+b\sinh^2(e+fx)}}{2a}}{2f}$$

$$\Big\downarrow \text{221}$$

$$\frac{\frac{-\frac{\left(8a^2-8ab+3b^2\right)\operatorname{arctanh}\left(\frac{\sqrt{a+b\sinh^2(e+fx)}}{\sqrt{a}}\right)}{a^{3/2}}-\frac{(8a-3b)\operatorname{csch}^2(e+fx)\sqrt{a+b\sinh^2(e+fx)}}{a}}{4a}-\frac{\operatorname{csch}^4(e+fx)\sqrt{a+b\sinh^2(e+fx)}}{2a}}{2f}$$

input

```
Int[Coth[e + f*x]^5/Sqrt[a + b*Sinh[e + f*x]^2],x]
```

output

```
(-1/2*(Csch[e + f*x]^4*Sqrt[a + b*Sinh[e + f*x]^2])/a + (-(((8*a^2 - 8*a*b
 + 3*b^2)*ArcTanh[Sqrt[a + b*Sinh[e + f*x]^2]/Sqrt[a]])/a^(3/2)) - ((8*a -
 3*b)*Csch[e + f*x]^2*Sqrt[a + b*Sinh[e + f*x]^2])/a)/(4*a))/(2*f)
```

Defintions of rubi rules used

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 87
```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_] :> Simp[(-(b*e - a*f))*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(f*(p
+ 1)*(c*f - d*e))), x] - Simp[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p
+ 1)))/(f*(p + 1)*(c*f - d*e))   Int[(c + d*x)^n*(e + f*x)^(p + 1), x], x]
/; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || Intege
rQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))))
```

rule 100
```
Int[((a_.) + (b_.)*(x_))^2*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))^(
p_), x_] :> Simp[(b*c - a*d)^2*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(d^2*(d
*e - c*f)*(n + 1))), x] - Simp[1/(d^2*(d*e - c*f)*(n + 1))   Int[(c + d*x)^
(n + 1)*(e + f*x)^p*Simp[a^2*d^2*f*(n + p + 2) + b^2*c*(d*e*(n + 1) + c*f*(
p + 1)) - 2*a*b*d*(d*e*(n + 1) + c*f*(p + 1)) - b^2*d*(d*e - c*f)*(n + 1)*x
, x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && (LtQ[n, -1] || (EqQ[n
 + p + 3, 0] && NeQ[n, -1] && (SumSimplerQ[n, 1] ||  !SumSimplerQ[p, 1])))
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3673
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_.)*tan[(e_.) + (f_.)*(x_)]^
(m_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x]^2, x]}, Simp[ff^((m
 + 1)/2)/(2*f)   Subst[Int[x^((m - 1)/2)*((a + b*ff*x)^p/(1 - ff*x)^((m + 1
)/2)), x], x, Sin[e + f*x]^2/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && Integ
erQ[(m - 1)/2]
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.60 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.43

method	result	size
default	$\frac{\text{`int/indef0`}\left(\frac{\frac{1}{\sinh(fx+e)}+\frac{2}{\sinh(fx+e)^{3}}+\frac{1}{\sinh(fx+e)^{5}}}{\sqrt{a+b\sinh(fx+e)^{2}}},\sinh(fx+e)\right)}{f}$	54

input
```
int(coth(f*x+e)^5/(a+b*sinh(f*x+e)^2)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
`int/indef0`((1/sinh(f*x+e)+2/sinh(f*x+e)^3+1/sinh(f*x+e)^5)/(a+b*sinh(f*x
+e)^2)^(1/2),sinh(f*x+e))/f
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1555 vs. 2(110) = 220.

Time = 0.19 (sec) , antiderivative size = 3199, normalized size of antiderivative = 25.39

$$\int \frac{\coth^5(e+fx)}{\sqrt{a+b\sinh^2(e+fx)}}\,dx = \text{Too large to display}$$

input
```
integrate(coth(f*x+e)^5/(a+b*sinh(f*x+e)^2)^(1/2),x, algorithm="fricas")
```

output
```
Too large to include
```

Sympy [F]

$$\int \frac{\coth^5(e+fx)}{\sqrt{a+b\sinh^2(e+fx)}}\,dx=\int \frac{\coth^5(e+fx)}{\sqrt{a+b\sinh^2(e+fx)}}\,dx$$

input
```
integrate(coth(f*x+e)**5/(a+b*sinh(f*x+e)**2)**(1/2),x)
```

output
```
Integral(coth(e + f*x)**5/sqrt(a + b*sinh(e + f*x)**2), x)
```

Maxima [F]

$$\int \frac{\coth^5(e+fx)}{\sqrt{a+b\sinh^2(e+fx)}}\,dx=\int \frac{\coth(fx+e)^5}{\sqrt{b\sinh(fx+e)^2+a}}\,dx$$

input
```
integrate(coth(f*x+e)^5/(a+b*sinh(f*x+e)^2)^(1/2),x, algorithm="maxima")
```

output
```
integrate(coth(f*x + e)^5/sqrt(b*sinh(f*x + e)^2 + a), x)
```

Giac [F]

$$\int \frac{\coth^5(e+fx)}{\sqrt{a+b\sinh^2(e+fx)}}\,dx=\int \frac{\coth(fx+e)^5}{\sqrt{b\sinh(fx+e)^2+a}}\,dx$$

input
```
integrate(coth(f*x+e)^5/(a+b*sinh(f*x+e)^2)^(1/2),x, algorithm="giac")
```

output
```
integrate(coth(f*x + e)^5/sqrt(b*sinh(f*x + e)^2 + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\coth^5(e+fx)}{\sqrt{a+b\sinh^2(e+fx)}}\,dx=\int \frac{\mathrm{coth}(e+f\,x)^5}{\sqrt{b\,\mathrm{sinh}\left(e+f\,x\right)^2+a}}\,dx$$

input
```
int(coth(e + f*x)^5/(a + b*sinh(e + f*x)^2)^(1/2),x)
```

output
```
int(coth(e + f*x)^5/(a + b*sinh(e + f*x)^2)^(1/2), x)
```

Reduce [F]

$$\int \frac{\coth^5(e+fx)}{\sqrt{a+b\sinh^2(e+fx)}}\,dx=\int \frac{\sqrt{\sinh\left(fx+e\right)^2 b+a}\,\coth\left(fx+e\right)^5}{\sinh\left(fx+e\right)^2 b+a}dx$$

input
```
int(coth(f*x+e)^5/(a+b*sinh(f*x+e)^2)^(1/2),x)
```

output
```
int((sqrt(sinh(e + f*x)**2*b + a)*coth(e + f*x)**5)/(sinh(e + f*x)**2*b +
a),x)
```

3.449 $\int \frac{\tanh^4(e+fx)}{\sqrt{a+b\sinh^2(e+fx)}}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 219

$$\begin{aligned}\int \frac{\tanh^4(e+fx)}{\sqrt{a+b\sinh^2(e+fx)}}\,dx &= -\frac{2(2a-b)E\left(\arctan(\sinh(e+fx))\left|1-\frac{b}{a}\right.\right)\operatorname{sech}(e+fx)\sqrt{a+b\sinh^2(e+fx)}}{3(a-b)^2f\sqrt{\frac{\operatorname{sech}^2(e+fx)(a+b\sinh^2(e+fx))}{a}}}\\&\quad+\frac{(3a-b)\operatorname{EllipticF}\left(\arctan(\sinh(e+fx)),1-\frac{b}{a}\right)\operatorname{sech}(e+fx)\sqrt{a+b\sinh^2(e+fx)}}{3(a-b)^2f\sqrt{\frac{\operatorname{sech}^2(e+fx)(a+b\sinh^2(e+fx))}{a}}}\\&\quad+\frac{\operatorname{sech}^2(e+fx)\sqrt{a+b\sinh^2(e+fx)}\tanh(e+fx)}{3(a-b)f}\end{aligned}$$

output
```
-2/3*(2*a-b)*EllipticE(sinh(f*x+e)/(1+sinh(f*x+e)^2)^(1/2),(1-b/a)^(1/2))*
sech(f*x+e)*(a+b*sinh(f*x+e)^2)^(1/2)/(a-b)^2/f/(sech(f*x+e)^2*(a+b*sinh(f
*x+e)^2)/a)^(1/2)+1/3*(3*a-b)*InverseJacobiAM(arctan(sinh(f*x+e)),(1-b/a)^
(1/2))*sech(f*x+e)*(a+b*sinh(f*x+e)^2)^(1/2)/(a-b)^2/f/(sech(f*x+e)^2*(a+b
*sinh(f*x+e)^2)/a)^(1/2)+1/3*sech(f*x+e)^2*(a+b*sinh(f*x+e)^2)^(1/2)*tanh(
f*x+e)/(a-b)/f
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 1.68 (sec) , antiderivative size = 206, normalized size of antiderivative = 0.94

$$\int \frac{\tanh^4(e+fx)}{\sqrt{a+b\sinh^2(e+fx)}}\,dx$$
$$= \frac{-4ia(2a-b)\sqrt{\frac{2a-b+b\cosh(2(e+fx))}{a}}E\left(i(e+fx)\,\middle|\,\frac{b}{a}\right)+2ia(a-b)\sqrt{\frac{2a-b+b\cosh(2(e+fx))}{a}}\,\text{EllipticF}\left(i(e+fx),\right.}{6(a-b)^2f\sqrt{2a-b+b\cosh(2(e+}}$$

input
```
Integrate[Tanh[e + f*x]^4/Sqrt[a + b*Sinh[e + f*x]^2],x]
```

output
```
((-4*I)*a*(2*a - b)*Sqrt[(2*a - b + b*Cosh[2*(e + f*x)])/a]*EllipticE[I*(e
 + f*x), b/a] + (2*I)*a*(a - b)*Sqrt[(2*a - b + b*Cosh[2*(e + f*x)])/a]*El
lipticF[I*(e + f*x), b/a] - ((2*(4*a^2 - 3*a*b + b^2)*Cosh[2*(e + f*x)] +
(2*a - b)*(2*a + b + b*Cosh[4*(e + f*x)]))*Sech[e + f*x]^2*Tanh[e + f*x])/
Sqrt[2])/(6*(a - b)^2*f*Sqrt[2*a - b + b*Cosh[2*(e + f*x)]])
```

Rubi [A] (verified)

Time = 0.44 (sec) , antiderivative size = 270, normalized size of antiderivative = 1.23, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.240$, Rules used = {3042, 3675, 372, 400, 313, 320}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\tanh^4(e+fx)}{\sqrt{a+b\sinh^2(e+fx)}}\,dx$$
$$\Big\downarrow \text{ 3042}$$
$$\int \frac{\tan(ie+ifx)^4}{\sqrt{a-b\sin(ie+ifx)^2}}dx$$
$$\Big\downarrow \text{ 3675}$$

$$\frac{\sqrt{\cosh^2(e+fx)}\operatorname{sech}(e+fx)\int \frac{\sinh^4(e+fx)}{\left(\sinh^2(e+fx)+1\right)^{5/2}\sqrt{b\sinh^2(e+fx)+a}}d\sinh(e+fx)}{f}$$

$$\Big\downarrow \textbf{372}$$

$$\frac{\sqrt{\cosh^2(e+fx)}\operatorname{sech}(e+fx)\left(\frac{\sinh(e+fx)\sqrt{a+b\sinh^2(e+fx)}}{3(a-b)\left(\sinh^2(e+fx)+1\right)^{3/2}}-\frac{\int \frac{a-(3a-2b)\sinh^2(e+fx)}{\left(\sinh^2(e+fx)+1\right)^{3/2}\sqrt{b\sinh^2(e+fx)+a}}d\sinh(e+fx)}{3(a-b)}\right)}{f}$$

$$\Big\downarrow \textbf{400}$$

$$\frac{\sqrt{\cosh^2(e+fx)}\operatorname{sech}(e+fx)\left(\frac{\sinh(e+fx)\sqrt{a+b\sinh^2(e+fx)}}{3(a-b)\left(\sinh^2(e+fx)+1\right)^{3/2}}-\frac{\frac{2(2a-b)\int \frac{\sqrt{b\sinh^2(e+fx)+a}}{\left(\sinh^2(e+fx)+1\right)^{3/2}}d\sinh(e+fx)}{a-b}-\frac{a(3a-b)\int \frac{}{\sqrt{\sinh^2(e+fx)+}}}{}}{3(a-b)}\right.}{f}$$

$$\Big\downarrow \textbf{313}$$

$$\frac{\sqrt{\cosh^2(e+fx)}\operatorname{sech}(e+fx)\left(\frac{\sinh(e+fx)\sqrt{a+b\sinh^2(e+fx)}}{3(a-b)\left(\sinh^2(e+fx)+1\right)^{3/2}}-\frac{\frac{2(2a-b)\sqrt{a+b\sinh^2(e+fx)}E\left(\arctan(\sinh(e+fx))\middle|1-\frac{b}{a}\right)}{(a-b)\sqrt{\sinh^2(e+fx)+1}\sqrt{\frac{a+b\sinh^2(e+fx)}{a\left(\sinh^2(e+fx)+1\right)}}}-\frac{a(3a-b)\int}{\sqrt{\sin}}}{3(a-b)}\right.}{f}$$

$$\Big\downarrow \textbf{320}$$

$$\frac{\sqrt{\cosh^2(e+fx)}\operatorname{sech}(e+fx)\left(\frac{\sinh(e+fx)\sqrt{a+b\sinh^2(e+fx)}}{3(a-b)\left(\sinh^2(e+fx)+1\right)^{3/2}}-\frac{\frac{2(2a-b)\sqrt{a+b\sinh^2(e+fx)}E\left(\arctan(\sinh(e+fx))\middle|1-\frac{b}{a}\right)}{(a-b)\sqrt{\sinh^2(e+fx)+1}\sqrt{\frac{a+b\sinh^2(e+fx)}{a\left(\sinh^2(e+fx)+1\right)}}}-\frac{(3a-b)\sqrt{a+b\,\mathrm{si}}}{(a-b)}}{3(a-b)}\right.}{f}$$

input
```
Int[Tanh[e + f*x]^4/Sqrt[a + b*Sinh[e + f*x]^2],x]
```

output
```
(Sqrt[Cosh[e + f*x]^2]*Sech[e + f*x]*((Sinh[e + f*x]*Sqrt[a + b*Sinh[e + f
*x]^2])/(3*(a - b)*(1 + Sinh[e + f*x]^2)^(3/2)) - ((2*(2*a - b)*EllipticE[
ArcTan[Sinh[e + f*x]], 1 - b/a]*Sqrt[a + b*Sinh[e + f*x]^2])/((a - b)*Sqrt
[1 + Sinh[e + f*x]^2]*Sqrt[(a + b*Sinh[e + f*x]^2)/(a*(1 + Sinh[e + f*x]^2
))]) - ((3*a - b)*EllipticF[ArcTan[Sinh[e + f*x]], 1 - b/a]*Sqrt[a + b*Sin
h[e + f*x]^2])/((a - b)*Sqrt[1 + Sinh[e + f*x]^2]*Sqrt[(a + b*Sinh[e + f*x
]^2)/(a*(1 + Sinh[e + f*x]^2))]))/(3*(a - b))))/f
```

Defintions of rubi rules used

rule 313
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/((c_) + (d_.)*(x_)^2)^(3/2), x_Symbol] :> Sim
p[(Sqrt[a + b*x^2]/(c*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(c
 + d*x^2)))]))*EllipticE[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; FreeQ
[{a, b, c, d}, x] && PosQ[b/a] && PosQ[d/c]
```

rule 320
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(Sqrt[a + b*x^2]/(a*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(
c + d*x^2)))]))*EllipticF[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; Fre
eQ[{a, b, c, d}, x] && PosQ[d/c] && PosQ[b/a] &&  !SimplerSqrtQ[b/a, d/c]
```

rule 372
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_
), x_Symbol] :> Simp[(-a)*e^3*(e*x)^(m - 3)*(a + b*x^2)^(p + 1)*((c + d*x^2
)^(q + 1)/(2*b*(b*c - a*d)*(p + 1))), x] + Simp[e^4/(2*b*(b*c - a*d)*(p + 1
))   Int[(e*x)^(m - 4)*(a + b*x^2)^(p + 1)*(c + d*x^2)^q*Simp[a*c*(m - 3) +
 (a*d*(m + 2*q - 1) + 2*b*c*(p + 1))*x^2, x], x], x] /; FreeQ[{a, b, c, d,
e, q}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && GtQ[m, 3] && IntBinomialQ[a
, b, c, d, e, m, 2, p, q, x]
```

rule 400
```
Int[((e_) + (f_.)*(x_)^2)/(Sqrt[(a_) + (b_.)*(x_)^2]*((c_) + (d_.)*(x_)^2)^
(3/2)), x_Symbol] :> Simp[(b*e - a*f)/(b*c - a*d)   Int[1/(Sqrt[a + b*x^2]*
Sqrt[c + d*x^2]), x], x] - Simp[(d*e - c*f)/(b*c - a*d)   Int[Sqrt[a + b*x^
2]/(c + d*x^2)^(3/2), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && PosQ[b/a] &
& PosQ[d/c]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3675
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_.)*tan[(e_.) + (f_.)*(x_)]^
(m_), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Simp[ff^(m + 1
)*(Sqrt[Cos[e + f*x]^2]/(f*Cos[e + f*x]))   Subst[Int[x^m*((a + b*ff^2*x^2)
^p/(1 - ff^2*x^2)^((m + 1)/2)), x], x, Sin[e + f*x]/ff], x]] /; FreeQ[{a, b
, e, f, p}, x] && IntegerQ[m/2] &&  !IntegerQ[p]
```

Maple [A] (verified)

Time = 3.38 (sec) , antiderivative size = 369, normalized size of antiderivative = 1.68

method	result
default	$-\frac{\left(4\sqrt{-\frac{b}{a}}\,ab-2\sqrt{-\frac{b}{a}}\,b^{2}\right)\cosh(fx+e)^{4}\sinh(fx+e)+\left(4\sqrt{-\frac{b}{a}}\,a^{2}-7\sqrt{-\frac{b}{a}}\,ab+3\sqrt{-\frac{b}{a}}\,b^{2}\right)\cosh(fx+e)^{2}\sinh(fx+e)-\sqrt{\frac{b\cosh(fx}{a}}}{}$

input
```
int(tanh(f*x+e)^4/(a+b*sinh(f*x+e)^2)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-1/3*((4*(-b/a)^(1/2)*a*b-2*(-b/a)^(1/2)*b^2)*cosh(f*x+e)^4*sinh(f*x+e)+(4
*(-b/a)^(1/2)*a^2-7*(-b/a)^(1/2)*a*b+3*(-b/a)^(1/2)*b^2)*cosh(f*x+e)^2*sin
h(f*x+e)-(b/a*cosh(f*x+e)^2+(a-b)/a)^(1/2)*(cosh(f*x+e)^2)^(1/2)*(3*Ellipt
icF(sinh(f*x+e)*(-b/a)^(1/2),(1/b*a)^(1/2))*a^2-5*EllipticF(sinh(f*x+e)*(-
b/a)^(1/2),(1/b*a)^(1/2))*a*b+2*EllipticF(sinh(f*x+e)*(-b/a)^(1/2),(1/b*a)
^(1/2))*b^2+4*EllipticE(sinh(f*x+e)*(-b/a)^(1/2),(1/b*a)^(1/2))*a*b-2*Elli
pticE(sinh(f*x+e)*(-b/a)^(1/2),(1/b*a)^(1/2))*b^2)*cosh(f*x+e)^2+(-(-b/a)^
(1/2)*a^2+2*(-b/a)^(1/2)*a*b-(-b/a)^(1/2)*b^2)*sinh(f*x+e))/cosh(f*x+e)^3/
(a-b)^2/(-b/a)^(1/2)/(a+b*sinh(f*x+e)^2)^(1/2)/f
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2755 vs. $2(219) = 438$.

Time = 0.13 (sec) , antiderivative size = 2755, normalized size of antiderivative = 12.58

$$\int \frac{\tanh^4(e+fx)}{\sqrt{a+b\sinh^2(e+fx)}}\,dx = \text{Too large to display}$$

input
```
integrate(tanh(f*x+e)^4/(a+b*sinh(f*x+e)^2)^(1/2),x, algorithm="fricas")
```

output
```
2/3*(((4*a^2*b - 4*a*b^2 + b^3)*cosh(f*x + e)^6 + 6*(4*a^2*b - 4*a*b^2 + b
^3)*cosh(f*x + e)*sinh(f*x + e)^5 + (4*a^2*b - 4*a*b^2 + b^3)*sinh(f*x + e
)^6 + 3*(4*a^2*b - 4*a*b^2 + b^3)*cosh(f*x + e)^4 + 3*(4*a^2*b - 4*a*b^2 +
 b^3 + 5*(4*a^2*b - 4*a*b^2 + b^3)*cosh(f*x + e)^2)*sinh(f*x + e)^4 + 4*(5
*(4*a^2*b - 4*a*b^2 + b^3)*cosh(f*x + e)^3 + 3*(4*a^2*b - 4*a*b^2 + b^3)*c
osh(f*x + e))*sinh(f*x + e)^3 + 4*a^2*b - 4*a*b^2 + b^3 + 3*(4*a^2*b - 4*a
*b^2 + b^3)*cosh(f*x + e)^2 + 3*(5*(4*a^2*b - 4*a*b^2 + b^3)*cosh(f*x + e)
^4 + 4*a^2*b - 4*a*b^2 + b^3 + 6*(4*a^2*b - 4*a*b^2 + b^3)*cosh(f*x + e)^2
)*sinh(f*x + e)^2 + 6*((4*a^2*b - 4*a*b^2 + b^3)*cosh(f*x + e)^5 + 2*(4*a^
2*b - 4*a*b^2 + b^3)*cosh(f*x + e)^3 + (4*a^2*b - 4*a*b^2 + b^3)*cosh(f*x
+ e))*sinh(f*x + e) - 2*((2*a*b^2 - b^3)*cosh(f*x + e)^6 + 6*(2*a*b^2 - b^
3)*cosh(f*x + e)*sinh(f*x + e)^5 + (2*a*b^2 - b^3)*sinh(f*x + e)^6 + 3*(2*
a*b^2 - b^3)*cosh(f*x + e)^4 + 3*(2*a*b^2 - b^3 + 5*(2*a*b^2 - b^3)*cosh(f
*x + e)^2)*sinh(f*x + e)^4 + 4*(5*(2*a*b^2 - b^3)*cosh(f*x + e)^3 + 3*(2*a
*b^2 - b^3)*cosh(f*x + e))*sinh(f*x + e)^3 + 2*a*b^2 - b^3 + 3*(2*a*b^2 -
b^3)*cosh(f*x + e)^2 + 3*(5*(2*a*b^2 - b^3)*cosh(f*x + e)^4 + 2*a*b^2 - b^
3 + 6*(2*a*b^2 - b^3)*cosh(f*x + e)^2)*sinh(f*x + e)^2 + 6*((2*a*b^2 - b^3
)*cosh(f*x + e)^5 + 2*(2*a*b^2 - b^3)*cosh(f*x + e)^3 + (2*a*b^2 - b^3)*co
sh(f*x + e))*sinh(f*x + e))*sqrt((a^2 - a*b)/b^2))*sqrt(b)*sqrt((2*b*sqrt(
(a^2 - a*b)/b^2) - 2*a + b)/b)*elliptic_e(arcsin(sqrt((2*b*sqrt((a^2 - ...
```

Sympy [F]

$$\int \frac{\tanh^4(e+fx)}{\sqrt{a+b\sinh^2(e+fx)}}\,dx = \int \frac{\tanh^4{(e+fx)}}{\sqrt{a+b\sinh^2{(e+fx)}}}\,dx$$

input
```
integrate(tanh(f*x+e)**4/(a+b*sinh(f*x+e)**2)**(1/2),x)
```

output
```
Integral(tanh(e + f*x)**4/sqrt(a + b*sinh(e + f*x)**2), x)
```

Maxima [F]

$$\int \frac{\tanh^4(e+fx)}{\sqrt{a+b\sinh^2(e+fx)}}\,dx = \int \frac{\tanh{(fx+e)}^4}{\sqrt{b\sinh{(fx+e)}^2+a}}\,dx$$

input
```
integrate(tanh(f*x+e)^4/(a+b*sinh(f*x+e)^2)^(1/2),x, algorithm="maxima")
```

output
```
integrate(tanh(f*x + e)^4/sqrt(b*sinh(f*x + e)^2 + a), x)
```

Giac [F]

$$\int \frac{\tanh^4(e+fx)}{\sqrt{a+b\sinh^2(e+fx)}}\,dx = \int \frac{\tanh{(fx+e)}^4}{\sqrt{b\sinh{(fx+e)}^2+a}}\,dx$$

input
```
integrate(tanh(f*x+e)^4/(a+b*sinh(f*x+e)^2)^(1/2),x, algorithm="giac")
```

output
```
integrate(tanh(f*x + e)^4/sqrt(b*sinh(f*x + e)^2 + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\tanh^4(e+fx)}{\sqrt{a+b\sinh^2(e+fx)}}\,dx = \int \frac{\tanh(e+f\,x)^4}{\sqrt{b\sinh\left(e+f\,x\right)^2+a}}\,dx$$

input
```
int(tanh(e + f*x)^4/(a + b*sinh(e + f*x)^2)^(1/2),x)
```

output
```
int(tanh(e + f*x)^4/(a + b*sinh(e + f*x)^2)^(1/2), x)
```

Reduce [F]

$$\int \frac{\tanh^4(e+fx)}{\sqrt{a+b\sinh^2(e+fx)}}\,dx = \int \frac{\sqrt{\sinh\left(fx+e\right)^2b+a}\,\tanh\left(fx+e\right)^4}{\sinh\left(fx+e\right)^2b+a}dx$$

input
```
int(tanh(f*x+e)^4/(a+b*sinh(f*x+e)^2)^(1/2),x)
```

output
```
int((sqrt(sinh(e + f*x)**2*b + a)*tanh(e + f*x)**4)/(sinh(e + f*x)**2*b +
a),x)
```

3.450 $\int \frac{\tanh^2(e+fx)}{\sqrt{a+b\sinh^2(e+fx)}}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 156

$$\int \frac{\tanh^2(e+fx)}{\sqrt{a+b\sinh^2(e+fx)}}\,dx = -\frac{E\left(\arctan(\sinh(e+fx))\left|1-\frac{b}{a}\right.\right)\operatorname{sech}(e+fx)\sqrt{a+b\sinh^2(e+fx)}}{(a-b)f\sqrt{\frac{\operatorname{sech}^2(e+fx)(a+b\sinh^2(e+fx))}{a}}} + \frac{\operatorname{EllipticF}\left(\arctan(\sinh(e+fx)),1-\frac{b}{a}\right)\operatorname{sech}(e+fx)\sqrt{a+b\sinh^2(e+fx)}}{(a-b)f\sqrt{\frac{\operatorname{sech}^2(e+fx)(a+b\sinh^2(e+fx))}{a}}}$$

output
```
-EllipticE(sinh(f*x+e)/(1+sinh(f*x+e)^2)^(1/2),(1-b/a)^(1/2))*sech(f*x+e)*
(a+b*sinh(f*x+e)^2)^(1/2)/(a-b)/f/(sech(f*x+e)^2*(a+b*sinh(f*x+e)^2)/a)^(1
/2)+InverseJacobiAM(arctan(sinh(f*x+e)),(1-b/a)^(1/2))*sech(f*x+e)*(a+b*si
nh(f*x+e)^2)^(1/2)/(a-b)/f/(sech(f*x+e)^2*(a+b*sinh(f*x+e)^2)/a)^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.57 (sec) , antiderivative size = 109, normalized size of antiderivative = 0.70

$$\int \frac{\tanh^2(e+fx)}{\sqrt{a+b\sinh^2(e+fx)}}\,dx = \frac{-2ia\sqrt{\frac{2a-b+b\cosh(2(e+fx))}{a}}E\left(i(e+fx)\left|\frac{b}{a}\right.\right)+\sqrt{2}(-2a+b-b\cosh(2(e+fx)))\tanh(e+fx)}{2(a-b)f\sqrt{2a-b+b\cosh(2(e+fx))}}$$

input
```
Integrate[Tanh[e + f*x]^2/Sqrt[a + b*Sinh[e + f*x]^2],x]
```

output
```
((-2*I)*a*Sqrt[(2*a - b + b*Cosh[2*(e + f*x)])/a]*EllipticE[I*(e + f*x), b
/a] + Sqrt[2]*(-2*a + b - b*Cosh[2*(e + f*x)])*Tanh[e + f*x])/(2*(a - b)*f
*Sqrt[2*a - b + b*Cosh[2*(e + f*x)]])
```

Rubi [A] (verified)

Time = 0.45 (sec) , antiderivative size = 282, normalized size of antiderivative = 1.81, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.320, Rules used = {3042, 25, 3675, 373, 324, 320, 388, 313}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\tanh^2(e+fx)}{\sqrt{a+b\sinh^2(e+fx)}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int -\frac{\tan(ie+ifx)^2}{\sqrt{a-b\sin(ie+ifx)^2}}dx$$

$$\Big\downarrow \text{25}$$

$$-\int \frac{\tan(ie+ifx)^2}{\sqrt{a-b\sin(ie+ifx)^2}}dx$$

$$\Big\downarrow \text{3675}$$

$$\frac{\sqrt{\cosh^2(e+fx)}\text{sech}(e+fx)\int \frac{\sinh^2(e+fx)}{\left(\sinh^2(e+fx)+1\right)^{3/2}\sqrt{b\sinh^2(e+fx)+a}}d\sinh(e+fx)}{f}$$

$$\Big\downarrow \text{373}$$

$$\frac{\sqrt{\cosh^2(e+fx)}\text{sech}(e+fx)\left(\frac{\int \frac{\sqrt{b\sinh^2(e+fx)+a}}{\sqrt{\sinh^2(e+fx)+1}}d\sinh(e+fx)}{a-b}-\frac{\sinh(e+fx)\sqrt{a+b\sinh^2(e+fx)}}{(a-b)\sqrt{\sinh^2(e+fx)+1}}\right)}{f}$$

$$\Big\downarrow \text{324}$$

$$\frac{\sqrt{\cosh^2(e+fx)}\text{sech}(e+fx)\left(\frac{a\int \frac{1}{\sqrt{\sinh^2(e+fx)+1}\sqrt{b\sinh^2(e+fx)+a}}d\sinh(e+fx)+b\int \frac{\sinh^2(e+fx)}{\sqrt{\sinh^2(e+fx)+1}\sqrt{b\sinh^2(e+fx)+a}}d\sinh(e+fx)}{a-b}\right.}{f}$$

$$\Big\downarrow \text{320}$$

$$\frac{\sqrt{\cosh^2(e+fx)}\text{sech}(e+fx)\left(\frac{b\int \frac{\sinh^2(e+fx)}{\sqrt{\sinh^2(e+fx)+1}\sqrt{b\sinh^2(e+fx)+a}}d\sinh(e+fx)+\frac{\sqrt{a+b\sinh^2(e+fx)}\,\text{EllipticF}\left(\arctan(\sinh(e+fx)),1-\frac{b}{a}\right)}{\sqrt{\sinh^2(e+fx)+1}\sqrt{\frac{a+b\sinh^2(e+fx)}{a\left(\sinh^2(e+fx)+1\right)}}}}{a-b}\right.}{f}$$

$$\Big\downarrow \text{388}$$

$$\frac{\sqrt{\cosh^2(e+fx)}\text{sech}(e+fx)\left(\frac{b\left(\frac{\sinh(e+fx)\sqrt{a+b\sinh^2(e+fx)}}{b\sqrt{\sinh^2(e+fx)+1}}-\frac{\int \frac{\sqrt{b\sinh^2(e+fx)+a}}{\left(\sinh^2(e+fx)+1\right)^{3/2}}d\sinh(e+fx)}{b}\right)+\frac{\sqrt{a+b\sinh^2(e+fx)}\,\text{EllipticF}\left(\arctan\right.}{\sqrt{\sinh^2(e+fx)+1}\sqrt{\frac{a+b\sin}{a\left(\sinh^2\right.}}}}{a-b}\right.}{f}$$

$$\Big\downarrow \text{313}$$

$$\frac{\sqrt{\cosh^2(e+fx)}\operatorname{sech}(e+fx)\left(\frac{\frac{\sqrt{a+b\sinh^2(e+fx)}\operatorname{EllipticF}\left(\arctan(\sinh(e+fx)),1-\frac{b}{a}\right)}{\sqrt{\sinh^2(e+fx)+1}\sqrt{\frac{a+b\sinh^2(e+fx)}{a\left(\sinh^2(e+fx)+1\right)}}}+b\left(\frac{\sinh(e+fx)\sqrt{a+b\sinh^2(e+fx)}}{b\sqrt{\sinh^2(e+fx)+1}}-\frac{\sqrt{a+b\sinh^2(e+fx)}}{b\sqrt{\sinh^2(e+fx)}}\right.}{a-b}\right.}{f}$$

input
```
Int[Tanh[e + f*x]^2/Sqrt[a + b*Sinh[e + f*x]^2],x]
```

output
```
(Sqrt[Cosh[e + f*x]^2]*Sech[e + f*x]*(-((Sinh[e + f*x]*Sqrt[a + b*Sinh[e +
 f*x]^2])/((a - b)*Sqrt[1 + Sinh[e + f*x]^2])) + ((EllipticF[ArcTan[Sinh[e
 + f*x]], 1 - b/a]*Sqrt[a + b*Sinh[e + f*x]^2])/(Sqrt[1 + Sinh[e + f*x]^2]
*Sqrt[(a + b*Sinh[e + f*x]^2)/(a*(1 + Sinh[e + f*x]^2))]) + b*((Sinh[e + f
*x]*Sqrt[a + b*Sinh[e + f*x]^2])/(b*Sqrt[1 + Sinh[e + f*x]^2]) - (Elliptic
E[ArcTan[Sinh[e + f*x]], 1 - b/a]*Sqrt[a + b*Sinh[e + f*x]^2])/(b*Sqrt[1 +
 Sinh[e + f*x]^2]*Sqrt[(a + b*Sinh[e + f*x]^2)/(a*(1 + Sinh[e + f*x]^2))])
))/(a - b)))/f
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 313
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/((c_) + (d_.)*(x_)^2)^(3/2), x_Symbol] :> Sim
p[(Sqrt[a + b*x^2]/(c*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(c
+ d*x^2)))]))*EllipticE[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; FreeQ
[{a, b, c, d}, x] && PosQ[b/a] && PosQ[d/c]
```

rule 320
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(Sqrt[a + b*x^2]/(a*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(
c + d*x^2)))]))*EllipticF[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; Fre
eQ[{a, b, c, d}, x] && PosQ[d/c] && PosQ[b/a] &&  !SimplerSqrtQ[b/a, d/c]
```

rule 324
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
a   Int[1/(Sqrt[a + b*x^2]*Sqrt[c + d*x^2]), x], x] + Simp[b   Int[x^2/(Sqr
t[a + b*x^2]*Sqrt[c + d*x^2]), x], x] /; FreeQ[{a, b, c, d}, x] && PosQ[d/c
] && PosQ[b/a]
```

rule 373
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_
), x_Symbol] :> Simp[e*(e*x)^(m - 1)*(a + b*x^2)^(p + 1)*((c + d*x^2)^(q +
1)/(2*(b*c - a*d)*(p + 1))), x] - Simp[e^2/(2*(b*c - a*d)*(p + 1))   Int[(e
*x)^(m - 2)*(a + b*x^2)^(p + 1)*(c + d*x^2)^q*Simp[c*(m - 1) + d*(m + 2*p +
 2*q + 3)*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, q}, x] && NeQ[b*c - a*d,
 0] && LtQ[p, -1] && GtQ[m, 1] && LeQ[m, 3] && IntBinomialQ[a, b, c, d, e,
m, 2, p, q, x]
```

rule 388
```
Int[(x_)^2/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol]
 :> Simp[x*(Sqrt[a + b*x^2]/(b*Sqrt[c + d*x^2])), x] - Simp[c/b   Int[Sqrt[
a + b*x^2]/(c + d*x^2)^(3/2), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
 a*d, 0] && PosQ[b/a] && PosQ[d/c] &&  !SimplerSqrtQ[b/a, d/c]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3675
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_.)*tan[(e_.) + (f_.)*(x_)]^
(m_), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Simp[ff^(m + 1
)*(Sqrt[Cos[e + f*x]^2]/(f*Cos[e + f*x]))   Subst[Int[x^m*((a + b*ff^2*x^2)
^p/(1 - ff^2*x^2)^((m + 1)/2)), x], x, Sin[e + f*x]/ff], x]] /; FreeQ[{a, b
, e, f, p}, x] && IntegerQ[m/2] &&  !IntegerQ[p]
```

Maple [A] (verified)

Time = 2.33 (sec) , antiderivative size = 239, normalized size of antiderivative = 1.53

method	result
default	$-\frac{\sqrt{-\frac{b}{a}}\, b \sinh(fx+e)^3 - a\sqrt{\frac{a+b\sinh(fx+e)^2}{a}}\, \sqrt{\frac{\cosh(2fx+2e)}{2}+\frac{1}{2}}\ \mathrm{EllipticF}\left(\sinh(fx+e)\sqrt{-\frac{b}{a}}, \sqrt{\frac{a}{b}}\right) + b\sqrt{\frac{a+b\sinh(fx+e)^2}{a}}\, \sqrt{\frac{\cosh(2}{2}}}{(a-b)\sqrt{-\frac{b}{a}}\ \mathrm{co}}$

input
```
int(tanh(f*x+e)^2/(a+b*sinh(f*x+e)^2)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-((-b/a)^(1/2)*b*sinh(f*x+e)^3-a*((a+b*sinh(f*x+e)^2)/a)^(1/2)*(cosh(f*x+e
)^2)^(1/2)*EllipticF(sinh(f*x+e)*(-b/a)^(1/2),(1/b*a)^(1/2))+b*((a+b*sinh(
f*x+e)^2)/a)^(1/2)*(cosh(f*x+e)^2)^(1/2)*EllipticF(sinh(f*x+e)*(-b/a)^(1/2
),(1/b*a)^(1/2))-b*((a+b*sinh(f*x+e)^2)/a)^(1/2)*(cosh(f*x+e)^2)^(1/2)*Ell
ipticE(sinh(f*x+e)*(-b/a)^(1/2),(1/b*a)^(1/2))+(-b/a)^(1/2)*a*sinh(f*x+e))
/(a-b)/(-b/a)^(1/2)/cosh(f*x+e)/(a+b*sinh(f*x+e)^2)^(1/2)/f
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 703 vs. $2(164) = 328$.

Time = 0.11 (sec) , antiderivative size = 703, normalized size of antiderivative = 4.51

$$\int \frac{\tanh^2(e+fx)}{\sqrt{a+b\sinh^2(e+fx)}}\,dx = \text{Too large to display}$$

input
```
integrate(tanh(f*x+e)^2/(a+b*sinh(f*x+e)^2)^(1/2),x, algorithm="fricas")
```

output
```
(((2*a*b - b^2)*cosh(f*x + e)^2 + 2*(2*a*b - b^2)*cosh(f*x + e)*sinh(f*x +
 e) + (2*a*b - b^2)*sinh(f*x + e)^2 + 2*a*b - b^2 - 2*(b^2*cosh(f*x + e)^2
 + 2*b^2*cosh(f*x + e)*sinh(f*x + e) + b^2*sinh(f*x + e)^2 + b^2)*sqrt((a^
2 - a*b)/b^2))*sqrt(b)*sqrt((2*b*sqrt((a^2 - a*b)/b^2) - 2*a + b)/b)*ellip
tic_e(arcsin(sqrt((2*b*sqrt((a^2 - a*b)/b^2) - 2*a + b)/b)*(cosh(f*x + e)
+ sinh(f*x + e))), (8*a^2 - 8*a*b + b^2 + 4*(2*a*b - b^2)*sqrt((a^2 - a*b)
/b^2))/b^2) - 2*((2*a^2 - a*b)*cosh(f*x + e)^2 + 2*(2*a^2 - a*b)*cosh(f*x
+ e)*sinh(f*x + e) + (2*a^2 - a*b)*sinh(f*x + e)^2 + 2*a^2 - a*b + 2*((a*b
 - b^2)*cosh(f*x + e)^2 + 2*(a*b - b^2)*cosh(f*x + e)*sinh(f*x + e) + (a*b
 - b^2)*sinh(f*x + e)^2 + a*b - b^2)*sqrt((a^2 - a*b)/b^2))*sqrt(b)*sqrt((
2*b*sqrt((a^2 - a*b)/b^2) - 2*a + b)/b)*elliptic_f(arcsin(sqrt((2*b*sqrt((
a^2 - a*b)/b^2) - 2*a + b)/b)*(cosh(f*x + e) + sinh(f*x + e))), (8*a^2 - 8
*a*b + b^2 + 4*(2*a*b - b^2)*sqrt((a^2 - a*b)/b^2))/b^2) - sqrt(2)*(b^2*co
sh(f*x + e) + b^2*sinh(f*x + e))*sqrt((b*cosh(f*x + e)^2 + b*sinh(f*x + e)
^2 + 2*a - b)/(cosh(f*x + e)^2 - 2*cosh(f*x + e)*sinh(f*x + e) + sinh(f*x
+ e)^2)))/((a*b^2 - b^3)*f*cosh(f*x + e)^2 + 2*(a*b^2 - b^3)*f*cosh(f*x +
e)*sinh(f*x + e) + (a*b^2 - b^3)*f*sinh(f*x + e)^2 + (a*b^2 - b^3)*f)
```

Sympy [F]

$$\int \frac{\tanh^2(e+fx)}{\sqrt{a+b\sinh^2(e+fx)}}\,dx = \int \frac{\tanh^2(e+fx)}{\sqrt{a+b\sinh^2(e+fx)}}\,dx$$

input
```
integrate(tanh(f*x+e)**2/(a+b*sinh(f*x+e)**2)**(1/2),x)
```

output
```
Integral(tanh(e + f*x)**2/sqrt(a + b*sinh(e + f*x)**2), x)
```

Maxima [F]

$$\int \frac{\tanh^2(e+fx)}{\sqrt{a+b\sinh^2(e+fx)}}\,dx = \int \frac{\tanh(fx+e)^2}{\sqrt{b\sinh(fx+e)^2+a}}\,dx$$

input
```
integrate(tanh(f*x+e)^2/(a+b*sinh(f*x+e)^2)^(1/2),x, algorithm="maxima")
```

output
```
integrate(tanh(f*x + e)^2/sqrt(b*sinh(f*x + e)^2 + a), x)
```

Giac [F]

$$\int \frac{\tanh^2(e+fx)}{\sqrt{a+b\sinh^2(e+fx)}}\,dx = \int \frac{\tanh(fx+e)^2}{\sqrt{b\sinh(fx+e)^2+a}}\,dx$$

input
```
integrate(tanh(f*x+e)^2/(a+b*sinh(f*x+e)^2)^(1/2),x, algorithm="giac")
```

output
```
integrate(tanh(f*x + e)^2/sqrt(b*sinh(f*x + e)^2 + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\tanh^2(e+fx)}{\sqrt{a+b\sinh^2(e+fx)}}\,dx = \int \frac{\tanh(e+f\,x)^2}{\sqrt{b\sinh\left(e+f\,x\right)^2+a}}\,dx$$

input
```
int(tanh(e + f*x)^2/(a + b*sinh(e + f*x)^2)^(1/2),x)
```

output
```
int(tanh(e + f*x)^2/(a + b*sinh(e + f*x)^2)^(1/2), x)
```

Reduce [F]

$$\int \frac{\tanh^2(e+fx)}{\sqrt{a+b\sinh^2(e+fx)}}\,dx = \int \frac{\sqrt{\sinh\left(fx+e\right)^2 b+a}\,\tanh\left(fx+e\right)^2}{\sinh\left(fx+e\right)^2 b+a}dx$$

input
```
int(tanh(f*x+e)^2/(a+b*sinh(f*x+e)^2)^(1/2),x)
```

output
```
int((sqrt(sinh(e + f*x)**2*b + a)*tanh(e + f*x)**2)/(sinh(e + f*x)**2*b +
a),x)
```

3.451 $\int \frac{1}{\sqrt{a+b\sinh^2(e+fx)}}\,dx$

Optimal result

Integrand size = 16, antiderivative size = 61

$$\int \frac{1}{\sqrt{a+b\sinh^2(e+fx)}}\,dx = -\frac{i\,\text{EllipticF}\left(ie+ifx,\frac{b}{a}\right)\sqrt{\frac{a+b\sinh^2(e+fx)}{a}}}{f\sqrt{a+b\sinh^2(e+fx)}}$$

output
```
-I*InverseJacobiAM(I*e+I*f*x,(b/a)^(1/2))*((a+b*sinh(f*x+e)^2)/a)^(1/2)/f/
(a+b*sinh(f*x+e)^2)^(1/2)
```

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 68, normalized size of antiderivative = 1.11

$$\int \frac{1}{\sqrt{a+b\sinh^2(e+fx)}}\,dx = -\frac{i\sqrt{\frac{2a-b+b\cosh(2(e+fx))}{a}}\,\text{EllipticF}\left(i(e+fx),\frac{b}{a}\right)}{f\sqrt{2a-b+b\cosh(2(e+fx))}}$$

input
```
Integrate[1/Sqrt[a + b*Sinh[e + f*x]^2],x]
```

output
```
((-I)*Sqrt[(2*a - b + b*Cosh[2*(e + f*x)])/a]*EllipticF[I*(e + f*x), b/a])
/(f*Sqrt[2*a - b + b*Cosh[2*(e + f*x)]])
```

Rubi [A] (verified)

Time = 0.32 (sec) , antiderivative size = 60, normalized size of antiderivative = 0.98, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {3042, 3662, 3042, 3661}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt{a+b\sinh^2(e+fx)}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{\sqrt{a-b\sin(ie+ifx)^2}}dx$$

$$\Big\downarrow \text{3662}$$

$$\frac{\sqrt{\frac{b\sinh^2(e+fx)}{a}+1}\int \frac{1}{\sqrt{\frac{b\sinh^2(e+fx)}{a}+1}}dx}{\sqrt{a+b\sinh^2(e+fx)}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\sqrt{\frac{b\sinh^2(e+fx)}{a}+1}\int \frac{1}{\sqrt{1-\frac{b\sin(ie+ifx)^2}{a}}}dx}{\sqrt{a+b\sinh^2(e+fx)}}$$

$$\Big\downarrow \text{3661}$$

$$-\frac{i\sqrt{\frac{b\sinh^2(e+fx)}{a}+1}\operatorname{EllipticF}\left(ie+ifx,\frac{b}{a}\right)}{f\sqrt{a+b\sinh^2(e+fx)}}$$

input
```
Int[1/Sqrt[a + b*Sinh[e + f*x]^2],x]
```

output
```
((-I)*EllipticF[I*e + I*f*x, b/a]*Sqrt[1 + (b*Sinh[e + f*x]^2)/a])/(f*Sqrt
[a + b*Sinh[e + f*x]^2])
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3661
```
Int[1/Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2], x_Symbol] :> Simp[(1/(S
qrt[a]*f))*EllipticF[e + f*x, -b/a], x] /; FreeQ[{a, b, e, f}, x] && GtQ[a,
 0]
```

rule 3662
```
Int[1/Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2], x_Symbol] :> Simp[Sqrt[
1 + b*(Sin[e + f*x]^2/a)]/Sqrt[a + b*Sin[e + f*x]^2]   Int[1/Sqrt[1 + (b*Si
n[e + f*x]^2)/a], x], x] /; FreeQ[{a, b, e, f}, x] &&  !GtQ[a, 0]
```

Maple [A] (verified)

Time = 0.65 (sec) , antiderivative size = 86, normalized size of antiderivative = 1.41

method	result	size
default	$\frac{\sqrt{\frac{a+b\sinh(fx+e)^2}{a}}\,\sqrt{\frac{\cosh(2fx+2e)}{2}+\frac{1}{2}}\,\operatorname{EllipticF}\left(\sinh(fx+e)\sqrt{-\frac{b}{a}},\sqrt{\frac{a}{b}}\right)}{\sqrt{-\frac{b}{a}}\,\cosh(fx+e)\sqrt{a+b\sinh(fx+e)^2}\,f}$	86

input
```
int(1/(a+b*sinh(f*x+e)^2)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/(-b/a)^(1/2)*((a+b*sinh(f*x+e)^2)/a)^(1/2)*(cosh(f*x+e)^2)^(1/2)*Ellipti
cF(sinh(f*x+e)*(-b/a)^(1/2),(1/b*a)^(1/2))/cosh(f*x+e)/(a+b*sinh(f*x+e)^2)
^(1/2)/f
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 147 vs. $2(53) = 106$.

Time = 0.09 (sec) , antiderivative size = 147, normalized size of antiderivative = 2.41

$$\int \frac{1}{\sqrt{a+b\sinh^2(e+fx)}}\,dx = -\frac{2\left(2\,b\sqrt{\frac{a^2-ab}{b^2}}+2\,a-b\right)\sqrt{\frac{2\,b\sqrt{\frac{a^2-ab}{b^2}}-2\,a+b}{b}}F\left(\arcsin\left(\sqrt{\frac{2\,b\sqrt{\frac{a^2-ab}{b^2}}-2\,a+b}{b}}(\cosh\left(fx+e\right)+\sinh\left(fx+e\right)\right)\right)}{b^{\frac{3}{2}}f}$$

input
```
integrate(1/(a+b*sinh(f*x+e)^2)^(1/2),x, algorithm="fricas")
```

output
```
-2*(2*b*sqrt((a^2 - a*b)/b^2) + 2*a - b)*sqrt((2*b*sqrt((a^2 - a*b)/b^2) -
 2*a + b)/b)*elliptic_f(arcsin(sqrt((2*b*sqrt((a^2 - a*b)/b^2) - 2*a + b)/
b)*(cosh(f*x + e) + sinh(f*x + e))), (8*a^2 - 8*a*b + b^2 + 4*(2*a*b - b^2
)*sqrt((a^2 - a*b)/b^2))/b^2)/(b^(3/2)*f)
```

Sympy [F]

$$\int \frac{1}{\sqrt{a+b\sinh^2(e+fx)}}\,dx = \int \frac{1}{\sqrt{a+b\sinh^2{(e+fx)}}}\,dx$$

input
```
integrate(1/(a+b*sinh(f*x+e)**2)**(1/2),x)
```

output
```
Integral(1/sqrt(a + b*sinh(e + f*x)**2), x)
```

Maxima [F]

$$\int \frac{1}{\sqrt{a+b\sinh^2(e+fx)}}\,dx=\int \frac{1}{\sqrt{b\sinh\left(fx+e\right)^2+a}}\,dx$$

input
```
integrate(1/(a+b*sinh(f*x+e)^2)^(1/2),x, algorithm="maxima")
```

output
```
integrate(1/sqrt(b*sinh(f*x + e)^2 + a), x)
```

Giac [F]

$$\int \frac{1}{\sqrt{a+b\sinh^2(e+fx)}}\,dx=\int \frac{1}{\sqrt{b\sinh\left(fx+e\right)^2+a}}\,dx$$

input
```
integrate(1/(a+b*sinh(f*x+e)^2)^(1/2),x, algorithm="giac")
```

output
```
integrate(1/sqrt(b*sinh(f*x + e)^2 + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt{a+b\sinh^2(e+fx)}}\,dx=\int \frac{1}{\sqrt{b\,{\mathrm{sinh}\left(e+f\,x\right)}^2+a}}\,dx$$

input
```
int(1/(a + b*sinh(e + f*x)^2)^(1/2),x)
```

output
```
int(1/(a + b*sinh(e + f*x)^2)^(1/2), x)
```

Reduce [F]

$$\int \frac{1}{\sqrt{a+b\sinh^2(e+fx)}}\,dx=\int \frac{\sqrt{\sinh\left(fx+e\right)^2b+a}}{\sinh\left(fx+e\right)^2b+a}dx$$

input
```
int(1/(a+b*sinh(f*x+e)^2)^(1/2),x)
```

output
```
int(sqrt(sinh(e + f*x)**2*b + a)/(sinh(e + f*x)**2*b + a),x)
```

3.452 $\int \frac{\coth^2(e+fx)}{\sqrt{a+b\sinh^2(e+fx)}}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 211

$$\int \frac{\coth^2(e+fx)}{\sqrt{a+b\sinh^2(e+fx)}}\,dx = -\frac{\coth(e+fx)}{f\sqrt{a+b\sinh^2(e+fx)}} - \frac{\sqrt{b}\cosh(e+fx)E\left(\arctan\left(\frac{\sqrt{b}\sinh(e+fx)}{\sqrt{a}}\right)|1-\frac{a}{b}\right)}{\sqrt{a}f\sqrt{\frac{a\cosh^2(e+fx)}{a+b\sinh^2(e+fx)}}\sqrt{a+b\sinh^2(e+fx)}} + \frac{\sqrt{a}\cosh(e+fx)\,\mathrm{EllipticF}\left(\arctan\left(\frac{\sqrt{b}\sinh(e+fx)}{\sqrt{a}}\right),1-\frac{a}{b}\right)}{\sqrt{b}f\sqrt{\frac{a\cosh^2(e+fx)}{a+b\sinh^2(e+fx)}}\sqrt{a+b\sinh^2(e+fx)}}$$

output
```
-coth(f*x+e)/f/(a+b*sinh(f*x+e)^2)^(1/2)-b^(1/2)*cosh(f*x+e)*EllipticE(b^(
1/2)*sinh(f*x+e)/a^(1/2)/(1+b*sinh(f*x+e)^2/a)^(1/2),(1-a/b)^(1/2))/a^(1/2
)/f/(a*cosh(f*x+e)^2/(a+b*sinh(f*x+e)^2))^(1/2)/(a+b*sinh(f*x+e)^2)^(1/2)+
a^(1/2)*cosh(f*x+e)*InverseJacobiAM(arctan(b^(1/2)*sinh(f*x+e)/a^(1/2)),(1
-a/b)^(1/2))/b^(1/2)/f/(a*cosh(f*x+e)^2/(a+b*sinh(f*x+e)^2))^(1/2)/(a+b*si
nh(f*x+e)^2)^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.55 (sec) , antiderivative size = 105, normalized size of antiderivative = 0.50

$$\int \frac{\coth^2(e+fx)}{\sqrt{a+b\sinh^2(e+fx)}}\,dx = \frac{\sqrt{2}(-2a+b-b\cosh(2(e+fx)))\coth(e+fx)-2ia\sqrt{\frac{2a-b+b\cosh(2(e+fx))}{a}}E\left(i(e+fx)\left|\frac{b}{a}\right.\right)}{2af\sqrt{2a-b+b\cosh(2(e+fx))}}$$

input
```
Integrate[Coth[e + f*x]^2/Sqrt[a + b*Sinh[e + f*x]^2],x]
```

output
```
(Sqrt[2]*(-2*a + b - b*Cosh[2*(e + f*x)])*Coth[e + f*x] - (2*I)*a*Sqrt[(2*
a - b + b*Cosh[2*(e + f*x)])/a]*EllipticE[I*(e + f*x), b/a])/(2*a*f*Sqrt[2
*a - b + b*Cosh[2*(e + f*x)]])
```

Rubi [A] (verified)

Time = 0.44 (sec) , antiderivative size = 274, normalized size of antiderivative = 1.30, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.320$, Rules used = {3042, 25, 3675, 377, 324, 320, 388, 313}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\coth^2(e+fx)}{\sqrt{a+b\sinh^2(e+fx)}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int -\frac{1}{\tan(ie+ifx)^2\sqrt{a-b\sin(ie+ifx)^2}}dx$$

$$\Big\downarrow \text{25}$$

$$-\int \frac{1}{\sqrt{a-b\sin(ie+ifx)^2}\tan(ie+ifx)^2}dx$$

$\Big\downarrow$ 3675

$$\frac{\sqrt{\cosh^2(e+fx)}\operatorname{sech}(e+fx)\int \frac{\operatorname{csch}^2(e+fx)\sqrt{\sinh^2(e+fx)+1}}{\sqrt{b\sinh^2(e+fx)+a}}d\sinh(e+fx)}{f}$$

$\Big\downarrow$ 377

$$\frac{\sqrt{\cosh^2(e+fx)}\operatorname{sech}(e+fx)\left(\frac{\int \frac{\sqrt{b\sinh^2(e+fx)+a}}{\sqrt{\sinh^2(e+fx)+1}}d\sinh(e+fx)}{a}-\frac{\sqrt{\sinh^2(e+fx)+1}\operatorname{csch}(e+fx)\sqrt{a+b\sinh^2(e+fx)}}{a}\right)}{f}$$

$\Big\downarrow$ 324

$$\frac{\sqrt{\cosh^2(e+fx)}\operatorname{sech}(e+fx)\left(\frac{a\int \frac{1}{\sqrt{\sinh^2(e+fx)+1}\sqrt{b\sinh^2(e+fx)+a}}d\sinh(e+fx)+b\int \frac{\sinh^2(e+fx)}{\sqrt{\sinh^2(e+fx)+1}\sqrt{b\sinh^2(e+fx)+a}}d\sinh(e+fx)}{a}\cdots\right.}{f}$$

$\Big\downarrow$ 320

$$\frac{\sqrt{\cosh^2(e+fx)}\operatorname{sech}(e+fx)\left(\frac{b\int \frac{\sinh^2(e+fx)}{\sqrt{\sinh^2(e+fx)+1}\sqrt{b\sinh^2(e+fx)+a}}d\sinh(e+fx)+\frac{\sqrt{a+b\sinh^2(e+fx)}\operatorname{EllipticF}\left(\arctan(\sinh(e+fx)),1-\frac{b}{a}\right)}{\sqrt{\sinh^2(e+fx)+1}\sqrt{\frac{a+b\sinh^2(e+fx)}{a\left(\sinh^2(e+fx)+1\right)}}}}{a}\cdots\right.}{f}$$

$\Big\downarrow$ 388

$$\frac{\sqrt{\cosh^2(e+fx)}\operatorname{sech}(e+fx)\left(\frac{b\left(\frac{\sinh(e+fx)\sqrt{a+b\sinh^2(e+fx)}}{b\sqrt{\sinh^2(e+fx)+1}}-\frac{\int \frac{\sqrt{b\sinh^2(e+fx)+a}}{\left(\sinh^2(e+fx)+1\right)^{3/2}}d\sinh(e+fx)}{b}\right)+\frac{\sqrt{a+b\sinh^2(e+fx)}\operatorname{EllipticF}\left(\arctan\cdots\right.}{\sqrt{\sinh^2(e+fx)+1}\sqrt{\frac{a+b\sin\cdots}{a\left(\sinh^2\cdots\right.}}}}{a}\cdots\right.}{f}$$

$\Big\downarrow$ 313

$$\frac{\sqrt{\cosh^2(e+fx)}\operatorname{sech}(e+fx)\left(\frac{\frac{\sqrt{a+b\sinh^2(e+fx)}\operatorname{EllipticF}\left(\arctan(\sinh(e+fx)),1-\frac{b}{a}\right)}{\sqrt{\sinh^2(e+fx)+1}\sqrt{\frac{a+b\sinh^2(e+fx)}{a\left(\sinh^2(e+fx)+1\right)}}}+b\left(\frac{\sinh(e+fx)\sqrt{a+b\sinh^2(e+fx)}}{b\sqrt{\sinh^2(e+fx)+1}}-\frac{\sqrt{a+b\sinh^2(e+fx)}}{b\sqrt{\sinh^2(e+fx)}}\right.}{a}\right.}{f}$$

input
```
Int[Coth[e + f*x]^2/Sqrt[a + b*Sinh[e + f*x]^2],x]
```

output
```
(Sqrt[Cosh[e + f*x]^2]*Sech[e + f*x]*(-((Csch[e + f*x]*Sqrt[1 + Sinh[e + f
*x]^2]*Sqrt[a + b*Sinh[e + f*x]^2])/a) + ((EllipticF[ArcTan[Sinh[e + f*x]]
, 1 - b/a]*Sqrt[a + b*Sinh[e + f*x]^2])/(Sqrt[1 + Sinh[e + f*x]^2]*Sqrt[(a
 + b*Sinh[e + f*x]^2)/(a*(1 + Sinh[e + f*x]^2))]) + b*((Sinh[e + f*x]*Sqrt
[a + b*Sinh[e + f*x]^2])/(b*Sqrt[1 + Sinh[e + f*x]^2]) - (EllipticE[ArcTan
[Sinh[e + f*x]], 1 - b/a]*Sqrt[a + b*Sinh[e + f*x]^2])/(b*Sqrt[1 + Sinh[e
+ f*x]^2]*Sqrt[(a + b*Sinh[e + f*x]^2)/(a*(1 + Sinh[e + f*x]^2))])))/a))/f
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 313
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/((c_) + (d_.)*(x_)^2)^(3/2), x_Symbol] :> Sim
p[(Sqrt[a + b*x^2]/(c*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(c
+ d*x^2)))]))*EllipticE[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; FreeQ
[{a, b, c, d}, x] && PosQ[b/a] && PosQ[d/c]
```

rule 320
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(Sqrt[a + b*x^2]/(a*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(
c + d*x^2)))]))*EllipticF[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; Fre
eQ[{a, b, c, d}, x] && PosQ[d/c] && PosQ[b/a] &&  !SimplerSqrtQ[b/a, d/c]
```

rule 324
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
a   Int[1/(Sqrt[a + b*x^2]*Sqrt[c + d*x^2]), x], x] + Simp[b   Int[x^2/(Sqr
t[a + b*x^2]*Sqrt[c + d*x^2]), x], x] /; FreeQ[{a, b, c, d}, x] && PosQ[d/c
] && PosQ[b/a]
```

rule 377
```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_)
, x_Symbol] :> Simp[(e*x)^(m + 1)*(a + b*x^2)^(p + 1)*((c + d*x^2)^q/(a*e*(
m + 1))), x] - Simp[1/(a*e^2*(m + 1))   Int[(e*x)^(m + 2)*(a + b*x^2)^p*(c
+ d*x^2)^(q - 1)*Simp[b*c*(m + 1) + 2*(b*c*(p + 1) + a*d*q) + d*(b*(m + 1)
+ 2*b*(p + q + 1))*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b
*c - a*d, 0] && LtQ[0, q, 1] && LtQ[m, -1] && IntBinomialQ[a, b, c, d, e, m
, 2, p, q, x]
```

rule 388
```
Int[(x_)^2/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol]
 :> Simp[x*(Sqrt[a + b*x^2]/(b*Sqrt[c + d*x^2])), x] - Simp[c/b   Int[Sqrt[
a + b*x^2]/(c + d*x^2)^(3/2), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
 a*d, 0] && PosQ[b/a] && PosQ[d/c] &&  !SimplerSqrtQ[b/a, d/c]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3675
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_.)*tan[(e_.) + (f_.)*(x_)]^
(m_), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Simp[ff^(m + 1
)*(Sqrt[Cos[e + f*x]^2]/(f*Cos[e + f*x]))   Subst[Int[x^m*((a + b*ff^2*x^2)
^p/(1 - ff^2*x^2)^((m + 1)/2)), x], x, Sin[e + f*x]/ff], x]] /; FreeQ[{a, b
, e, f, p}, x] && IntegerQ[m/2] &&  !IntegerQ[p]
```

Maple [A] (verified)

Time = 1.82 (sec) , antiderivative size = 216, normalized size of antiderivative = 1.02

method	result
default	$\frac{-\sqrt{-\frac{b}{a}}\, b\cosh(fx+e)^4+\left(-\sqrt{-\frac{b}{a}}\, a+\sqrt{-\frac{b}{a}}\, b\right)\cosh(fx+e)^2+\sinh(fx+e)\sqrt{\frac{b\cosh(fx+e)^2}{a}+\frac{a-b}{a}}\,\sqrt{\frac{\cosh(2fx+2e)}{2}+\frac{1}{2}}\left(a\,\mathrm{EllipticF}\left(\mathrm{s}\right.\right.}{\sqrt{-\frac{b}{a}}\, a\sinh(fx+e)\cosh(fx+e)\sqrt{a+b\sin}}$

input
```
int(coth(f*x+e)^2/(a+b*sinh(f*x+e)^2)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
(-(-b/a)^(1/2)*b*cosh(f*x+e)^4+(-(-b/a)^(1/2)*a+(-b/a)^(1/2)*b)*cosh(f*x+e
)^2+sinh(f*x+e)*(b/a*cosh(f*x+e)^2+(a-b)/a)^(1/2)*(cosh(f*x+e)^2)^(1/2)*(a
*EllipticF(sinh(f*x+e)*(-b/a)^(1/2),(1/b*a)^(1/2))-b*EllipticF(sinh(f*x+e)
*(-b/a)^(1/2),(1/b*a)^(1/2))+b*EllipticE(sinh(f*x+e)*(-b/a)^(1/2),(1/b*a)^
(1/2))))/(-b/a)^(1/2)/a/sinh(f*x+e)/cosh(f*x+e)/(a+b*sinh(f*x+e)^2)^(1/2)/
f
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 674 vs. $2(205) = 410$.

Time = 0.10 (sec) , antiderivative size = 674, normalized size of antiderivative = 3.19

$$\int \frac{\coth^2(e+fx)}{\sqrt{a+b\sinh^2(e+fx)}}\,dx = \text{Too large to display}$$

input
```
integrate(coth(f*x+e)^2/(a+b*sinh(f*x+e)^2)^(1/2),x, algorithm="fricas")
```

output
```
(((2*a*b - b^2)*cosh(f*x + e)^2 + 2*(2*a*b - b^2)*cosh(f*x + e)*sinh(f*x +
 e) + (2*a*b - b^2)*sinh(f*x + e)^2 - 2*a*b + b^2 - 2*(b^2*cosh(f*x + e)^2
 + 2*b^2*cosh(f*x + e)*sinh(f*x + e) + b^2*sinh(f*x + e)^2 - b^2)*sqrt((a^
2 - a*b)/b^2))*sqrt(b)*sqrt((2*b*sqrt((a^2 - a*b)/b^2) - 2*a + b)/b)*ellip
tic_e(arcsin(sqrt((2*b*sqrt((a^2 - a*b)/b^2) - 2*a + b)/b)*(cosh(f*x + e)
+ sinh(f*x + e))), (8*a^2 - 8*a*b + b^2 + 4*(2*a*b - b^2)*sqrt((a^2 - a*b)
/b^2))/b^2) - 2*((2*a^2 - a*b)*cosh(f*x + e)^2 + 2*(2*a^2 - a*b)*cosh(f*x
+ e)*sinh(f*x + e) + (2*a^2 - a*b)*sinh(f*x + e)^2 - 2*a^2 + a*b + 2*((a*b
 - b^2)*cosh(f*x + e)^2 + 2*(a*b - b^2)*cosh(f*x + e)*sinh(f*x + e) + (a*b
 - b^2)*sinh(f*x + e)^2 - a*b + b^2)*sqrt((a^2 - a*b)/b^2))*sqrt(b)*sqrt((
2*b*sqrt((a^2 - a*b)/b^2) - 2*a + b)/b)*elliptic_f(arcsin(sqrt((2*b*sqrt((
a^2 - a*b)/b^2) - 2*a + b)/b)*(cosh(f*x + e) + sinh(f*x + e))), (8*a^2 - 8
*a*b + b^2 + 4*(2*a*b - b^2)*sqrt((a^2 - a*b)/b^2))/b^2) - sqrt(2)*(b^2*co
sh(f*x + e) + b^2*sinh(f*x + e))*sqrt((b*cosh(f*x + e)^2 + b*sinh(f*x + e)
^2 + 2*a - b)/(cosh(f*x + e)^2 - 2*cosh(f*x + e)*sinh(f*x + e) + sinh(f*x
+ e)^2)))/(a*b^2*f*cosh(f*x + e)^2 + 2*a*b^2*f*cosh(f*x + e)*sinh(f*x + e)
 + a*b^2*f*sinh(f*x + e)^2 - a*b^2*f)
```

Sympy [F]

$$\int \frac{\coth^2(e+fx)}{\sqrt{a+b\sinh^2(e+fx)}}\,dx = \int \frac{\coth^2(e+fx)}{\sqrt{a+b\sinh^2(e+fx)}}\,dx$$

input
```
integrate(coth(f*x+e)**2/(a+b*sinh(f*x+e)**2)**(1/2),x)
```

output
```
Integral(coth(e + f*x)**2/sqrt(a + b*sinh(e + f*x)**2), x)
```

Maxima [F]

$$\int \frac{\coth^2(e+fx)}{\sqrt{a+b\sinh^2(e+fx)}}\,dx = \int \frac{\coth(fx+e)^2}{\sqrt{b\sinh(fx+e)^2+a}}\,dx$$

input
```
integrate(coth(f*x+e)^2/(a+b*sinh(f*x+e)^2)^(1/2),x, algorithm="maxima")
```

output
```
integrate(coth(f*x + e)^2/sqrt(b*sinh(f*x + e)^2 + a), x)
```

Giac [F]

$$\int \frac{\coth^2(e+fx)}{\sqrt{a+b\sinh^2(e+fx)}}\,dx = \int \frac{\coth(fx+e)^2}{\sqrt{b\sinh(fx+e)^2+a}}\,dx$$

input
```
integrate(coth(f*x+e)^2/(a+b*sinh(f*x+e)^2)^(1/2),x, algorithm="giac")
```

output
```
integrate(coth(f*x + e)^2/sqrt(b*sinh(f*x + e)^2 + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\coth^2(e+fx)}{\sqrt{a+b\sinh^2(e+fx)}}\,dx=\int \frac{\coth(e+f\,x)^2}{\sqrt{b\sinh\left(e+f\,x\right)^2+a}}\,dx$$

input
```
int(coth(e + f*x)^2/(a + b*sinh(e + f*x)^2)^(1/2),x)
```

output
```
int(coth(e + f*x)^2/(a + b*sinh(e + f*x)^2)^(1/2), x)
```

Reduce [F]

$$\int \frac{\coth^2(e+fx)}{\sqrt{a+b\sinh^2(e+fx)}}\,dx=\int \frac{\sqrt{\sinh\left(fx+e\right)^2b+a}\,\coth\left(fx+e\right)^2}{\sinh\left(fx+e\right)^2b+a}dx$$

input
```
int(coth(f*x+e)^2/(a+b*sinh(f*x+e)^2)^(1/2),x)
```

output
```
int((sqrt(sinh(e + f*x)**2*b + a)*coth(e + f*x)**2)/(sinh(e + f*x)**2*b +
a),x)
```

3.453 $\int \frac{\coth^4(e+fx)}{\sqrt{a+b\sinh^2(e+fx)}}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 252

$$\int \frac{\coth^4(e+fx)}{\sqrt{a+b\sinh^2(e+fx)}}\,dx = -\frac{\coth(e+fx)\operatorname{csch}^2(e+fx)\sqrt{a+b\sinh^2(e+fx)}}{3af} - \frac{2(2a-b)\operatorname{csch}(e+fx)\operatorname{sech}(e+fx)\sqrt{a+b\sinh^2(e+fx)}}{3a^2f} - \frac{2(2a-b)E\left(\arctan(\sinh(e+fx))\,\middle|\,1-\frac{b}{a}\right)\operatorname{sech}(e+fx)\sqrt{a+b\sinh^2(e+fx)}}{3a^2f\sqrt{\frac{\operatorname{sech}^2(e+fx)(a+b\sinh^2(e+fx))}{a}}} + \frac{(3a-b)\operatorname{EllipticF}\left(\arctan(\sinh(e+fx)),1-\frac{b}{a}\right)\operatorname{sech}(e+fx)\sqrt{a+b\sinh^2(e+fx)}}{3a^2f\sqrt{\frac{\operatorname{sech}^2(e+fx)(a+b\sinh^2(e+fx))}{a}}}$$

output
```
-1/3*coth(f*x+e)*csch(f*x+e)^2*(a+b*sinh(f*x+e)^2)^(1/2)/a/f-2/3*(2*a-b)*c
sch(f*x+e)*sech(f*x+e)*(a+b*sinh(f*x+e)^2)^(1/2)/a^2/f-2/3*(2*a-b)*Ellipti
cE(sinh(f*x+e)/(1+sinh(f*x+e)^2)^(1/2),(1-b/a)^(1/2))*sech(f*x+e)*(a+b*sin
h(f*x+e)^2)^(1/2)/a^2/f/(sech(f*x+e)^2*(a+b*sinh(f*x+e)^2)/a)^(1/2)+1/3*(3
*a-b)*InverseJacobiAM(arctan(sinh(f*x+e)),(1-b/a)^(1/2))*sech(f*x+e)*(a+b*
sinh(f*x+e)^2)^(1/2)/a^2/f/(sech(f*x+e)^2*(a+b*sinh(f*x+e)^2)/a)^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 2.66 (sec) , antiderivative size = 208, normalized size of antiderivative = 0.83

$$\int \frac{\coth^4(e+fx)}{\sqrt{a+b\sinh^2(e+fx)}}\,dx = \frac{-\frac{(2(4a^2-5ab+2b^2)\cosh(2(e+fx))-(2a-b)(2a-3b-b\cosh(4(e+fx))))\coth(e+fx)\operatorname{csch}^2(e+fx)}{\sqrt{2}} - 4ia(2a-b)\sqrt{\frac{2a-b+b\cosh(2(e+\cdots}{a}}\cdots}{6a^2f\sqrt{2a-b+b\cosh(2(e+f\cdots}}$$

input
```
Integrate[Coth[e + f*x]^4/Sqrt[a + b*Sinh[e + f*x]^2],x]
```

output
```
(-(((2*(4*a^2 - 5*a*b + 2*b^2)*Cosh[2*(e + f*x)] - (2*a - b)*(2*a - 3*b -
b*Cosh[4*(e + f*x)]))*Coth[e + f*x]*Csch[e + f*x]^2)/Sqrt[2]) - (4*I)*a*(2
*a - b)*Sqrt[(2*a - b + b*Cosh[2*(e + f*x)])/a]*EllipticE[I*(e + f*x), b/a
] + (2*I)*a*(a - b)*Sqrt[(2*a - b + b*Cosh[2*(e + f*x)])/a]*EllipticF[I*(e
 + f*x), b/a])/(6*a^2*f*Sqrt[2*a - b + b*Cosh[2*(e + f*x)]])
```

Rubi [A] (verified)

Time = 0.53 (sec) , antiderivative size = 349, normalized size of antiderivative = 1.38, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.360$, Rules used = {3042, 3675, 376, 445, 25, 406, 320, 388, 313}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\coth^4(e+fx)}{\sqrt{a+b\sinh^2(e+fx)}}\,dx$$

$$\Big\downarrow \; 3042$$

$$\int \frac{1}{\tan(ie+ifx)^4\sqrt{a-b\sin(ie+ifx)^2}}dx$$

$$\Big\downarrow \; 3675$$

$$\frac{\sqrt{\cosh^2(e+fx)}\operatorname{sech}(e+fx)\int \frac{\operatorname{csch}^4(e+fx)\left(\sinh^2(e+fx)+1\right)^{3/2}}{\sqrt{b\sinh^2(e+fx)+a}}d\sinh(e+fx)}{f}$$

$\downarrow$ 376

$$\frac{\sqrt{\cosh^2(e+fx)}\operatorname{sech}(e+fx)\left(\frac{\int \frac{\operatorname{csch}^2(e+fx)\left((3a-b)\sinh^2(e+fx)+2(2a-b)\right)}{\sqrt{\sinh^2(e+fx)+1}\sqrt{b\sinh^2(e+fx)+a}}d\sinh(e+fx)}{3a}-\frac{\sqrt{\sinh^2(e+fx)+1}\operatorname{csch}^3(e+fx)\sqrt{a+b\sin}}{3a}\right.}{f}$$

$\downarrow$ 445

$$\frac{\sqrt{\cosh^2(e+fx)}\operatorname{sech}(e+fx)\left(\frac{-\frac{\int -\frac{2(2a-b)b\sinh^2(e+fx)+a(3a-b)}{\sqrt{\sinh^2(e+fx)+1}\sqrt{b\sinh^2(e+fx)+a}}d\sinh(e+fx)}{a}-\frac{2(2a-b)\sqrt{\sinh^2(e+fx)+1}\operatorname{csch}(e+fx)\sqrt{a+b\sinh^2(e+f}}{a}}{3a}\right.}{f}$$

$\downarrow$ 25

$$\frac{\sqrt{\cosh^2(e+fx)}\operatorname{sech}(e+fx)\left(\frac{\frac{\int \frac{2(2a-b)b\sinh^2(e+fx)+a(3a-b)}{\sqrt{\sinh^2(e+fx)+1}\sqrt{b\sinh^2(e+fx)+a}}d\sinh(e+fx)}{a}-\frac{2(2a-b)\sqrt{\sinh^2(e+fx)+1}\operatorname{csch}(e+fx)\sqrt{a+b\sinh^2(e+fx)}}{a}}{3a}\right.}{f}$$

$\downarrow$ 406

$$\frac{\sqrt{\cosh^2(e+fx)}\operatorname{sech}(e+fx)\left(\frac{\frac{a(3a-b)\int \frac{1}{\sqrt{\sinh^2(e+fx)+1}\sqrt{b\sinh^2(e+fx)+a}}d\sinh(e+fx)+2b(2a-b)\int \frac{\sinh^2(e+fx)}{\sqrt{\sinh^2(e+fx)+1}\sqrt{b\sinh^2(e+fx)+a}}d\sin}{a}}{3a}\right.}{f}$$

$\downarrow$ 320

$$\frac{\sqrt{\cosh^2(e+fx)}\operatorname{sech}(e+fx)\left(\frac{\frac{2b(2a-b)\int \frac{\sinh^2(e+fx)}{\sqrt{\sinh^2(e+fx)+1}\sqrt{b\sinh^2(e+fx)+a}}d\sinh(e+fx)+\frac{(3a-b)\sqrt{a+b\sinh^2(e+fx)}\operatorname{EllipticF}\left(\arctan(\sinh(e+\right.}{\sqrt{\sinh^2(e+fx)+1}\sqrt{\frac{a+b\sinh^2(e+fx)}{a\left(\sinh^2(e+fx)+1\right)}}}}{a}}{3a}\right.}{f}$$

$\downarrow$ 388

$$\frac{\sqrt{\cosh^2(e+fx)}\text{sech}(e+fx)\left(\frac{\frac{2b(2a-b)\left(\frac{\sinh(e+fx)\sqrt{a+b\sinh^2(e+fx)}}{b\sqrt{\sinh^2(e+fx)+1}}-\frac{\int\frac{\sqrt{b\sinh^2(e+fx)+a}}{\left(\sinh^2(e+fx)+1\right)^{3/2}}d\sinh(e+fx)}{b}\right)+\frac{(3a-b)\sqrt{a+b\sinh^2(e+fx)}\,\text{El}}{\sqrt{\sinh^2(e+fx)+1}}}{a}}{3a}\right.}{}$$

$\downarrow$ 313

$$\frac{\sqrt{\cosh^2(e+fx)}\text{sech}(e+fx)\left(\frac{\frac{\frac{(3a-b)\sqrt{a+b\sinh^2(e+fx)}\,\text{EllipticF}\left(\arctan(\sinh(e+fx)),1-\frac{b}{a}\right)}{\sqrt{\sinh^2(e+fx)+1}\sqrt{\frac{a+b\sinh^2(e+fx)}{a\left(\sinh^2(e+fx)+1\right)}}}+2b(2a-b)\left(\frac{\sinh(e+fx)\sqrt{a+b\sinh^2(e+fx)}}{b\sqrt{\sinh^2(e+fx)+1}}-\frac{\sqrt{a+}}{b}\right.}{a}}{3a}\right.}{}$$

input
```
Int[Coth[e + f*x]^4/Sqrt[a + b*Sinh[e + f*x]^2],x]
```

output
```
(Sqrt[Cosh[e + f*x]^2]*Sech[e + f*x]*(-1/3*(Csch[e + f*x]^3*Sqrt[1 + Sinh[
e + f*x]^2]*Sqrt[a + b*Sinh[e + f*x]^2])/a + ((-2*(2*a - b)*Csch[e + f*x]*
Sqrt[1 + Sinh[e + f*x]^2]*Sqrt[a + b*Sinh[e + f*x]^2])/a + (((3*a - b)*Ell
ipticF[ArcTan[Sinh[e + f*x]], 1 - b/a]*Sqrt[a + b*Sinh[e + f*x]^2])/(Sqrt[
1 + Sinh[e + f*x]^2]*Sqrt[(a + b*Sinh[e + f*x]^2)/(a*(1 + Sinh[e + f*x]^2)
)]) + 2*(2*a - b)*b*((Sinh[e + f*x]*Sqrt[a + b*Sinh[e + f*x]^2])/(b*Sqrt[1
 + Sinh[e + f*x]^2]) - (EllipticE[ArcTan[Sinh[e + f*x]], 1 - b/a]*Sqrt[a +
 b*Sinh[e + f*x]^2])/(b*Sqrt[1 + Sinh[e + f*x]^2]*Sqrt[(a + b*Sinh[e + f*x
]^2)/(a*(1 + Sinh[e + f*x]^2))]))))/a)/(3*a)))/f
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 313
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/((c_) + (d_.)*(x_)^2)^(3/2), x_Symbol] :> Sim
p[(Sqrt[a + b*x^2]/(c*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(c
+ d*x^2)))]))*EllipticE[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; FreeQ
[{a, b, c, d}, x] && PosQ[b/a] && PosQ[d/c]
```

rule 320
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(Sqrt[a + b*x^2]/(a*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(
c + d*x^2)))]))*EllipticF[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; Fre
eQ[{a, b, c, d}, x] && PosQ[d/c] && PosQ[b/a] &&  !SimplerSqrtQ[b/a, d/c]
```

rule 376
```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_)
, x_Symbol] :> Simp[c*(e*x)^(m + 1)*(a + b*x^2)^(p + 1)*((c + d*x^2)^(q - 1
)/(a*e*(m + 1))), x] - Simp[1/(a*e^2*(m + 1))   Int[(e*x)^(m + 2)*(a + b*x^
2)^p*(c + d*x^2)^(q - 2)*Simp[c*(b*c - a*d)*(m + 1) + 2*c*(b*c*(p + 1) + a*
d*(q - 1)) + d*((b*c - a*d)*(m + 1) + 2*b*c*(p + q))*x^2, x], x], x] /; Fre
eQ[{a, b, c, d, e, p}, x] && NeQ[b*c - a*d, 0] && GtQ[q, 1] && LtQ[m, -1] &
& IntBinomialQ[a, b, c, d, e, m, 2, p, q, x]
```

rule 388
```
Int[(x_)^2/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol]
 :> Simp[x*(Sqrt[a + b*x^2]/(b*Sqrt[c + d*x^2])), x] - Simp[c/b   Int[Sqrt[
a + b*x^2]/(c + d*x^2)^(3/2), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
 a*d, 0] && PosQ[b/a] && PosQ[d/c] &&  !SimplerSqrtQ[b/a, d/c]
```

rule 406
```
Int[((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.)*((e_) + (f_.)*(
x_)^2), x_Symbol] :> Simp[e   Int[(a + b*x^2)^p*(c + d*x^2)^q, x], x] + Sim
p[f   Int[x^2*(a + b*x^2)^p*(c + d*x^2)^q, x], x] /; FreeQ[{a, b, c, d, e,
f, p, q}, x]
```

rule 445
```
Int[((g_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_
.)*((e_) + (f_.)*(x_)^2), x_Symbol] :> Simp[e*(g*x)^(m + 1)*(a + b*x^2)^(p
+ 1)*((c + d*x^2)^(q + 1)/(a*c*g*(m + 1))), x] + Simp[1/(a*c*g^2*(m + 1))
 Int[(g*x)^(m + 2)*(a + b*x^2)^p*(c + d*x^2)^q*Simp[a*f*c*(m + 1) - e*(b*c
+ a*d)*(m + 2 + 1) - e*2*(b*c*p + a*d*q) - b*e*d*(m + 2*(p + q + 2) + 1)*x^
2, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p, q}, x] && LtQ[m, -1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3675
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_.)*tan[(e_.) + (f_.)*(x_)]^
(m_), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Simp[ff^(m + 1
)*(Sqrt[Cos[e + f*x]^2]/(f*Cos[e + f*x]))   Subst[Int[x^m*((a + b*ff^2*x^2)
^p/(1 - ff^2*x^2)^((m + 1)/2)), x], x, Sin[e + f*x]/ff], x]] /; FreeQ[{a, b
, e, f, p}, x] && IntegerQ[m/2] &&  !IntegerQ[p]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 521 vs. $2(248) = 496$.

Time = 3.34 (sec) , antiderivative size = 522, normalized size of antiderivative = 2.07

method	result
default	$-\frac{4\sqrt{-\frac{b}{a}}\, ab\sinh(fx+e)^{6}-2\sqrt{-\frac{b}{a}}\, b^{2}\sinh(fx+e)^{6}-3a^{2}\sqrt{\frac{a+b\sinh(fx+e)^{2}}{a}}\, \sqrt{\frac{\cosh(2fx+2e)}{2}+\frac{1}{2}}\, \operatorname{EllipticF}\left(\sinh(fx+e)\sqrt{-\frac{b}{a}}, \sqrt{\frac{a}{b}}\right) \mathrm{s}}{}$

input
```
int(coth(f*x+e)^4/(a+b*sinh(f*x+e)^2)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-1/3*(4*(-b/a)^(1/2)*a*b*sinh(f*x+e)^6-2*(-b/a)^(1/2)*b^2*sinh(f*x+e)^6-3*
a^2*((a+b*sinh(f*x+e)^2)/a)^(1/2)*(cosh(f*x+e)^2)^(1/2)*EllipticF(sinh(f*x
+e)*(-b/a)^(1/2),(1/b*a)^(1/2))*sinh(f*x+e)^3+5*b*((a+b*sinh(f*x+e)^2)/a)^
(1/2)*(cosh(f*x+e)^2)^(1/2)*EllipticF(sinh(f*x+e)*(-b/a)^(1/2),(1/b*a)^(1/
2))*a*sinh(f*x+e)^3-2*((a+b*sinh(f*x+e)^2)/a)^(1/2)*(cosh(f*x+e)^2)^(1/2)*
EllipticF(sinh(f*x+e)*(-b/a)^(1/2),(1/b*a)^(1/2))*b^2*sinh(f*x+e)^3-4*((a+
b*sinh(f*x+e)^2)/a)^(1/2)*(cosh(f*x+e)^2)^(1/2)*EllipticE(sinh(f*x+e)*(-b/
a)^(1/2),(1/b*a)^(1/2))*a*b*sinh(f*x+e)^3+2*((a+b*sinh(f*x+e)^2)/a)^(1/2)*
(cosh(f*x+e)^2)^(1/2)*EllipticE(sinh(f*x+e)*(-b/a)^(1/2),(1/b*a)^(1/2))*b^
2*sinh(f*x+e)^3+4*(-b/a)^(1/2)*a^2*sinh(f*x+e)^4+3*(-b/a)^(1/2)*a*b*sinh(f
*x+e)^4-2*(-b/a)^(1/2)*b^2*sinh(f*x+e)^4+5*(-b/a)^(1/2)*a^2*sinh(f*x+e)^2-
(-b/a)^(1/2)*a*b*sinh(f*x+e)^2+(-b/a)^(1/2)*a^2)/(-b/a)^(1/2)/a^2/sinh(f*x
+e)^3/cosh(f*x+e)/(a+b*sinh(f*x+e)^2)^(1/2)/f
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2584 vs. $2(248) = 496$.

Time = 0.14 (sec) , antiderivative size = 2584, normalized size of antiderivative = 10.25

$$\int \frac{\coth^4(e+fx)}{\sqrt{a+b\sinh^2(e+fx)}}\,dx = \text{Too large to display}$$

input
```
integrate(coth(f*x+e)^4/(a+b*sinh(f*x+e)^2)^(1/2),x, algorithm="fricas")
```

output
```
2/3*(((4*a^2*b - 4*a*b^2 + b^3)*cosh(f*x + e)^6 + 6*(4*a^2*b - 4*a*b^2 + b
^3)*cosh(f*x + e)*sinh(f*x + e)^5 + (4*a^2*b - 4*a*b^2 + b^3)*sinh(f*x + e
)^6 - 3*(4*a^2*b - 4*a*b^2 + b^3)*cosh(f*x + e)^4 - 3*(4*a^2*b - 4*a*b^2 +
 b^3 - 5*(4*a^2*b - 4*a*b^2 + b^3)*cosh(f*x + e)^2)*sinh(f*x + e)^4 + 4*(5
*(4*a^2*b - 4*a*b^2 + b^3)*cosh(f*x + e)^3 - 3*(4*a^2*b - 4*a*b^2 + b^3)*c
osh(f*x + e))*sinh(f*x + e)^3 - 4*a^2*b + 4*a*b^2 - b^3 + 3*(4*a^2*b - 4*a
*b^2 + b^3)*cosh(f*x + e)^2 + 3*(5*(4*a^2*b - 4*a*b^2 + b^3)*cosh(f*x + e)
^4 + 4*a^2*b - 4*a*b^2 + b^3 - 6*(4*a^2*b - 4*a*b^2 + b^3)*cosh(f*x + e)^2
)*sinh(f*x + e)^2 + 6*((4*a^2*b - 4*a*b^2 + b^3)*cosh(f*x + e)^5 - 2*(4*a^
2*b - 4*a*b^2 + b^3)*cosh(f*x + e)^3 + (4*a^2*b - 4*a*b^2 + b^3)*cosh(f*x
+ e))*sinh(f*x + e) - 2*((2*a*b^2 - b^3)*cosh(f*x + e)^6 + 6*(2*a*b^2 - b^
3)*cosh(f*x + e)*sinh(f*x + e)^5 + (2*a*b^2 - b^3)*sinh(f*x + e)^6 - 3*(2*
a*b^2 - b^3)*cosh(f*x + e)^4 - 3*(2*a*b^2 - b^3 - 5*(2*a*b^2 - b^3)*cosh(f
*x + e)^2)*sinh(f*x + e)^4 + 4*(5*(2*a*b^2 - b^3)*cosh(f*x + e)^3 - 3*(2*a
*b^2 - b^3)*cosh(f*x + e))*sinh(f*x + e)^3 - 2*a*b^2 + b^3 + 3*(2*a*b^2 -
b^3)*cosh(f*x + e)^2 + 3*(5*(2*a*b^2 - b^3)*cosh(f*x + e)^4 + 2*a*b^2 - b^
3 - 6*(2*a*b^2 - b^3)*cosh(f*x + e)^2)*sinh(f*x + e)^2 + 6*((2*a*b^2 - b^3
)*cosh(f*x + e)^5 - 2*(2*a*b^2 - b^3)*cosh(f*x + e)^3 + (2*a*b^2 - b^3)*co
sh(f*x + e))*sinh(f*x + e))*sqrt((a^2 - a*b)/b^2))*sqrt(b)*sqrt((2*b*sqrt(
(a^2 - a*b)/b^2) - 2*a + b)/b)*elliptic_e(arcsin(sqrt((2*b*sqrt((a^2 - ...
```

Sympy [F]

$$\int \frac{\coth^4(e+fx)}{\sqrt{a+b\sinh^2(e+fx)}}\,dx = \int \frac{\coth^4{(e+fx)}}{\sqrt{a+b\sinh^2{(e+fx)}}}\,dx$$

input
```
integrate(coth(f*x+e)**4/(a+b*sinh(f*x+e)**2)**(1/2),x)
```

output
```
Integral(coth(e + f*x)**4/sqrt(a + b*sinh(e + f*x)**2), x)
```

Maxima [F]

$$\int \frac{\coth^4(e+fx)}{\sqrt{a+b\sinh^2(e+fx)}}\,dx = \int \frac{\coth(fx+e)^4}{\sqrt{b\sinh(fx+e)^2+a}}\,dx$$

input
```
integrate(coth(f*x+e)^4/(a+b*sinh(f*x+e)^2)^(1/2),x, algorithm="maxima")
```

output
```
integrate(coth(f*x + e)^4/sqrt(b*sinh(f*x + e)^2 + a), x)
```

Giac [F]

$$\int \frac{\coth^4(e+fx)}{\sqrt{a+b\sinh^2(e+fx)}}\,dx = \int \frac{\coth(fx+e)^4}{\sqrt{b\sinh(fx+e)^2+a}}\,dx$$

input
```
integrate(coth(f*x+e)^4/(a+b*sinh(f*x+e)^2)^(1/2),x, algorithm="giac")
```

output
```
integrate(coth(f*x + e)^4/sqrt(b*sinh(f*x + e)^2 + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\coth^4(e+fx)}{\sqrt{a+b\sinh^2(e+fx)}}\,dx = \int \frac{\coth(e+f\,x)^4}{\sqrt{b\sinh(e+f\,x)^2+a}}\,dx$$

input
```
int(coth(e + f*x)^4/(a + b*sinh(e + f*x)^2)^(1/2),x)
```

output
```
int(coth(e + f*x)^4/(a + b*sinh(e + f*x)^2)^(1/2), x)
```

Reduce [F]

$$\int \frac{\coth^4(e+fx)}{\sqrt{a+b\sinh^2(e+fx)}}\,dx = \int \frac{\sqrt{\sinh(fx+e)^2 b+a}\,\coth(fx+e)^4}{\sinh(fx+e)^2 b+a}dx$$

input
```
int(coth(f*x+e)^4/(a+b*sinh(f*x+e)^2)^(1/2),x)
```

output
```
int((sqrt(sinh(e + f*x)**2*b + a)*coth(e + f*x)**4)/(sinh(e + f*x)**2*b +
a),x)
```

3.454 $\int \frac{\tanh^5(e+fx)}{(a+b\sinh^2(e+fx))^{3/2}}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 187

$$\int \frac{\tanh^5(e+fx)}{(a+b\sinh^2(e+fx))^{3/2}}\,dx = -\frac{(8a^2+8ab-b^2)\operatorname{arctanh}\left(\frac{\sqrt{a+b\sinh^2(e+fx)}}{\sqrt{a-b}}\right)}{8(a-b)^{7/2}f} + \frac{8a^2+8ab-b^2}{8(a-b)^3 f\sqrt{a+b\sinh^2(e+fx)}} + \frac{(8a-3b)\operatorname{sech}^2(e+fx)}{8(a-b)^2 f\sqrt{a+b\sinh^2(e+fx)}} - \frac{\operatorname{sech}^4(e+fx)}{4(a-b)f\sqrt{a+b\sinh^2(e+fx)}}$$

output

```
-1/8*(8*a^2+8*a*b-b^2)*arctanh((a+b*sinh(f*x+e)^2)^(1/2)/(a-b)^(1/2))/(a-b
)^(7/2)/f+1/8*(8*a^2+8*a*b-b^2)/(a-b)^3/f/(a+b*sinh(f*x+e)^2)^(1/2)+1/8*(8
*a-3*b)*sech(f*x+e)^2/(a-b)^2/f/(a+b*sinh(f*x+e)^2)^(1/2)-1/4*sech(f*x+e)^
4/(a-b)/f/(a+b*sinh(f*x+e)^2)^(1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 0.34 (sec) , antiderivative size = 113, normalized size of antiderivative = 0.60

$$\int \frac{\tanh^5(e+fx)}{\left(a+b\sinh^2(e+fx)\right)^{3/2}}\,dx = \frac{(8a^2+8ab-b^2)\,\text{Hypergeometric2F1}\left(-\frac{1}{2},1,\frac{1}{2},\frac{a+b\sinh^2(e+fx)}{a-b}\right)+\frac{1}{2}(a-b}{8(a-b)^3f\sqrt{a+b\sinh^2(e}$$

input
```
Integrate[Tanh[e + f*x]^5/(a + b*Sinh[e + f*x]^2)^(3/2),x]
```

output
```
((8*a^2 + 8*a*b - b^2)*Hypergeometric2F1[-1/2, 1, 1/2, (a + b*Sinh[e + f*x
]^2)/(a - b)] + ((a - b)*(4*a + b + (8*a - 3*b)*Cosh[2*(e + f*x)])*Sech[e
+ f*x]^4)/2)/(8*(a - b)^3*f*Sqrt[a + b*Sinh[e + f*x]^2])
```

Rubi [A] (verified)

Time = 0.42 (sec) , antiderivative size = 193, normalized size of antiderivative = 1.03, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.360$, Rules used = {3042, 26, 3673, 100, 27, 87, 61, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\tanh^5(e+fx)}{\left(a+b\sinh^2(e+fx)\right)^{3/2}}dx$$

$$\Big\downarrow \ 3042$$

$$\int -\frac{i\tan(ie+ifx)^5}{\left(a-b\sin(ie+ifx)^2\right)^{3/2}}dx$$

$$\Big\downarrow \ 26$$

$$-i\int \frac{\tan(ie+ifx)^5}{\left(a-b\sin(ie+ifx)^2\right)^{3/2}}dx$$

$$\Big\downarrow \ 3673$$

$$\frac{\int \frac{\sinh^4(e+fx)}{(\sinh^2(e+fx)+1)^3(b\sinh^2(e+fx)+a)^{3/2}}d\sinh^2(e+fx)}{2f}$$

$\downarrow$ 100

$$\frac{\frac{\int -\frac{-4(a-b)\sinh^2(e+fx)+4a+b}{2\left(\sinh^2(e+fx)+1\right)^2\left(b\sinh^2(e+fx)+a\right)^{3/2}}d\sinh^2(e+fx)}{2(a-b)}-\frac{1}{2(a-b)(\sinh^2(e+fx)+1)^2\sqrt{a+b\sinh^2(e+fx)}}}{2f}$$

$\downarrow$ 27

$$\frac{-\frac{\int \frac{-4(a-b)\sinh^2(e+fx)+4a+b}{\left(\sinh^2(e+fx)+1\right)^2\left(b\sinh^2(e+fx)+a\right)^{3/2}}d\sinh^2(e+fx)}{4(a-b)}-\frac{1}{2(a-b)(\sinh^2(e+fx)+1)^2\sqrt{a+b\sinh^2(e+fx)}}}{2f}$$

$\downarrow$ 87

$$\frac{-\frac{-\frac{\left(8a^2+8ab-b^2\right)\int \frac{1}{\left(\sinh^2(e+fx)+1\right)\left(b\sinh^2(e+fx)+a\right)^{3/2}}d\sinh^2(e+fx)}{2(a-b)}-\frac{8a-3b}{(a-b)\left(\sinh^2(e+fx)+1\right)\sqrt{a+b\sinh^2(e+fx)}}}{4(a-b)}-\frac{1}{2(a-b)(\sinh^2(e+fx)+1)^2\sqrt{}}}{2f}$$

$\downarrow$ 61

$$\frac{-\frac{-\frac{\left(8a^2+8ab-b^2\right)\left(\frac{\int \frac{1}{\left(\sinh^2(e+fx)+1\right)\sqrt{b\sinh^2(e+fx)+a}}d\sinh^2(e+fx)}{a-b}+\frac{2}{(a-b)\sqrt{a+b\sinh^2(e+fx)}}\right)}{2(a-b)}-\frac{8a-3b}{(a-b)\left(\sinh^2(e+fx)+1\right)\sqrt{a+b\sinh^2(e+fx)}}}{4(a-b)}-\frac{}{2(a}}{2f}$$

$\downarrow$ 73

$$\frac{-\frac{-\frac{\left(8a^2+8ab-b^2\right)\left(\frac{2\int \frac{1}{\frac{\sinh^4(e+fx)}{b}-\frac{a}{b}+1}d\sqrt{b\sinh^2(e+fx)+a}}{b(a-b)}+\frac{2}{(a-b)\sqrt{a+b\sinh^2(e+fx)}}\right)}{2(a-b)}-\frac{8a-3b}{(a-b)\left(\sinh^2(e+fx)+1\right)\sqrt{a+b\sinh^2(e+fx)}}}{4(a-b)}-\frac{}{2(a-b)(\sinh}}{2f}$$

$\downarrow$ 221

$$-\frac{-\frac{-\frac{\left(8a^2+8ab-b^2\right)\left(\frac{2}{(a-b)\sqrt{a+b\sinh^2(e+fx)}}-\frac{2\text{arctanh}\left(\frac{\sqrt{a+b\sinh^2(e+fx)}}{\sqrt{a-b}}\right)}{(a-b)^{3/2}}\right)}{2(a-b)}-\frac{8a-3b}{(a-b)\left(\sinh^2(e+fx)+1\right)\sqrt{a+b\sinh^2(e+fx)}}}{4(a-b)}-\frac{}{2(a-b)\left(\sinh^2(e+f\right.}}{2f}$$

input
```
Int[Tanh[e + f*x]^5/(a + b*Sinh[e + f*x]^2)^(3/2),x]
```

output
```
(-1/2*1/((a - b)*(1 + Sinh[e + f*x]^2)^2*Sqrt[a + b*Sinh[e + f*x]^2]) - (-
((8*a - 3*b)/((a - b)*(1 + Sinh[e + f*x]^2)*Sqrt[a + b*Sinh[e + f*x]^2]))
- ((8*a^2 + 8*a*b - b^2)*((-2*ArcTanh[Sqrt[a + b*Sinh[e + f*x]^2]/Sqrt[a -
 b]])/(a - b)^(3/2) + 2/((a - b)*Sqrt[a + b*Sinh[e + f*x]^2])))/(2*(a - b)
))/(4*(a - b)))/(2*f)
```

Defintions of rubi rules used

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 61
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Simp[d*((
m + n + 2)/((b*c - a*d)*(m + 1)))   Int[(a + b*x)^(m + 1)*(c + d*x)^n, x],
x] /; FreeQ[{a, b, c, d, n}, x] && LtQ[m, -1] &&  !(LtQ[n, -1] && (EqQ[a, 0
] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && IntLinearQ[a, b, c, d
, m, n, x]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 87
```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_] :> Simp[(-(b*e - a*f))*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(f*(p
+ 1)*(c*f - d*e))), x] - Simp[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p
+ 1)))/(f*(p + 1)*(c*f - d*e))   Int[(c + d*x)^n*(e + f*x)^(p + 1), x], x]
/; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || Intege
rQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))))
```

rule 100
```
Int[((a_.) + (b_.)*(x_))^2*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))^(
p_), x_] :> Simp[(b*c - a*d)^2*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(d^2*(d
*e - c*f)*(n + 1))), x] - Simp[1/(d^2*(d*e - c*f)*(n + 1))   Int[(c + d*x)^
(n + 1)*(e + f*x)^p*Simp[a^2*d^2*f*(n + p + 2) + b^2*c*(d*e*(n + 1) + c*f*(
p + 1)) - 2*a*b*d*(d*e*(n + 1) + c*f*(p + 1)) - b^2*d*(d*e - c*f)*(n + 1)*x
, x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && (LtQ[n, -1] || (EqQ[n
 + p + 3, 0] && NeQ[n, -1] && (SumSimplerQ[n, 1] ||  !SumSimplerQ[p, 1])))
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3673
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_.)*tan[(e_.) + (f_.)*(x_)]^
(m_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x]^2, x]}, Simp[ff^((m
 + 1)/2)/(2*f)   Subst[Int[x^((m - 1)/2)*((a + b*ff*x)^p/(1 - ff*x)^((m + 1
)/2)), x], x, Sin[e + f*x]^2/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && Integ
erQ[(m - 1)/2]
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 12.20 (sec) , antiderivative size = 103, normalized size of antiderivative = 0.55

method	result	size
default	$\frac{\text{`int/indef0`}\left(-\frac{\sinh(fx+e)^5\sqrt{a+b\sinh(fx+e)^2}\,\cosh(fx+e)^4}{-b^2\cosh(fx+e)^{14}+\left(-2ab+2b^2\right)\cosh(fx+e)^{12}+\left(-a^2+2ab-b^2\right)\cosh(fx+e)^{10}},\sinh(fx+e)\right)}{f}$	103
risch	Expression too large to display	2587315

input
```
int(tanh(f*x+e)^5/(a+b*sinh(f*x+e)^2)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
`int/indef0`(-sinh(f*x+e)^5*(a+b*sinh(f*x+e)^2)^(1/2)*cosh(f*x+e)^4/(-b^2*
cosh(f*x+e)^14+(-2*a*b+2*b^2)*cosh(f*x+e)^12+(-a^2+2*a*b-b^2)*cosh(f*x+e)^
10),sinh(f*x+e))/f
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 5091 vs. 2(167) = 334.

Time = 0.59 (sec) , antiderivative size = 10273, normalized size of antiderivative = 54.94

$$\int \frac{\tanh^5(e+fx)}{\left(a+b\sinh^2(e+fx)\right)^{3/2}}\,dx = \text{Too large to display}$$

input
```
integrate(tanh(f*x+e)^5/(a+b*sinh(f*x+e)^2)^(3/2),x, algorithm="fricas")
```

output
```
Too large to include
```

Sympy [F]

$$\int \frac{\tanh^5(e+fx)}{(a+b\sinh^2(e+fx))^{3/2}}\,dx = \int \frac{\tanh^5(e+fx)}{(a+b\sinh^2(e+fx))^{\frac{3}{2}}}\,dx$$

input
```
integrate(tanh(f*x+e)**5/(a+b*sinh(f*x+e)**2)**(3/2),x)
```

output
```
Integral(tanh(e + f*x)**5/(a + b*sinh(e + f*x)**2)**(3/2), x)
```

Maxima [F]

$$\int \frac{\tanh^5(e+fx)}{(a+b\sinh^2(e+fx))^{3/2}}\,dx = \int \frac{\tanh(fx+e)^5}{(b\sinh(fx+e)^2+a)^{\frac{3}{2}}}\,dx$$

input
```
integrate(tanh(f*x+e)^5/(a+b*sinh(f*x+e)^2)^(3/2),x, algorithm="maxima")
```

output
```
integrate(tanh(f*x + e)^5/(b*sinh(f*x + e)^2 + a)^(3/2), x)
```

Giac [F]

$$\int \frac{\tanh^5(e+fx)}{(a+b\sinh^2(e+fx))^{3/2}}\,dx = \int \frac{\tanh(fx+e)^5}{(b\sinh(fx+e)^2+a)^{\frac{3}{2}}}\,dx$$

input
```
integrate(tanh(f*x+e)^5/(a+b*sinh(f*x+e)^2)^(3/2),x, algorithm="giac")
```

output
```
integrate(tanh(f*x + e)^5/(b*sinh(f*x + e)^2 + a)^(3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\tanh^5(e+fx)}{\left(a+b\sinh^2(e+fx)\right)^{3/2}}\,dx = \text{Hanged}$$

input
```
int(tanh(e + f*x)^5/(a + b*sinh(e + f*x)^2)^(3/2),x)
```

output
```
\text{Hanged}
```

Reduce [F]

$$\int \frac{\tanh^5(e+fx)}{\left(a+b\sinh^2(e+fx)\right)^{3/2}}\,dx = \int \frac{\sqrt{\sinh\left(fx+e\right)^2 b+a}\,\tanh\left(fx+e\right)^5}{\sinh\left(fx+e\right)^4 b^2+2\sinh\left(fx+e\right)^2 ab+a^2}dx$$

input
```
int(tanh(f*x+e)^5/(a+b*sinh(f*x+e)^2)^(3/2),x)
```

output
```
int((sqrt(sinh(e + f*x)**2*b + a)*tanh(e + f*x)**5)/(sinh(e + f*x)**4*b**2
 + 2*sinh(e + f*x)**2*a*b + a**2),x)
```

3.455 $\int \frac{\tanh^3(e+fx)}{\left(a+b\sinh^2(e+fx)\right)^{3/2}}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 122

$$\int \frac{\tanh^3(e+fx)}{\left(a+b\sinh^2(e+fx)\right)^{3/2}}\,dx = -\frac{(2a+b)\operatorname{arctanh}\left(\frac{\sqrt{a+b\sinh^2(e+fx)}}{\sqrt{a-b}}\right)}{2(a-b)^{5/2}f} + \frac{2a+b}{2(a-b)^2f\sqrt{a+b\sinh^2(e+fx)}} + \frac{\operatorname{sech}^2(e+fx)}{2(a-b)f\sqrt{a+b\sinh^2(e+fx)}}$$

output
```
-1/2*(2*a+b)*arctanh((a+b*sinh(f*x+e)^2)^(1/2)/(a-b)^(1/2))/(a-b)^(5/2)/f+
1/2*(2*a+b)/(a-b)^2/f/(a+b*sinh(f*x+e)^2)^(1/2)+1/2*sech(f*x+e)^2/(a-b)/f/
(a+b*sinh(f*x+e)^2)^(1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 0.08 (sec) , antiderivative size = 79, normalized size of antiderivative = 0.65

$$\int \frac{\tanh^3(e+fx)}{\left(a+b\sinh^2(e+fx)\right)^{3/2}}\,dx = \frac{(2a+b)\,\text{Hypergeometric2F1}\left(-\frac{1}{2},1,\frac{1}{2},\frac{a+b\sinh^2(e+fx)}{a-b}\right)+(a-b)\operatorname{sech}^2(e+}{2(a-b)^2f\sqrt{a+b\sinh^2(e+fx)}}$$

input
```
Integrate[Tanh[e + f*x]^3/(a + b*Sinh[e + f*x]^2)^(3/2),x]
```

output
```
((2*a + b)*Hypergeometric2F1[-1/2, 1, 1/2, (a + b*Sinh[e + f*x]^2)/(a - b)
] + (a - b)*Sech[e + f*x]^2)/(2*(a - b)^2*f*Sqrt[a + b*Sinh[e + f*x]^2])
```

Rubi [A] (verified)

Time = 0.33 (sec) , antiderivative size = 124, normalized size of antiderivative = 1.02, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.280$, Rules used = {3042, 26, 3673, 87, 61, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\tanh^3(e+fx)}{\left(a+b\sinh^2(e+fx)\right)^{3/2}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{i\tan(ie+ifx)^3}{(a-b\sin(ie+ifx)^2)^{3/2}}dx$$

$$\Big\downarrow \text{26}$$

$$i\int \frac{\tan(ie+ifx)^3}{(a-b\sin(ie+ifx)^2)^{3/2}}dx$$

$$\Big\downarrow \text{3673}$$

$$\frac{\int \frac{\sinh^2(e+fx)}{\left(\sinh^2(e+fx)+1\right)^2\left(b\sinh^2(e+fx)+a\right)^{3/2}}d\sinh^2(e+fx)}{2f}$$

$$\Big\downarrow \text{87}$$

$$\frac{\frac{(2a+b)\int \frac{1}{\left(\sinh^2(e+fx)+1\right)\left(b\sinh^2(e+fx)+a\right)^{3/2}}d\sinh^2(e+fx)}{2(a-b)}+\frac{1}{(a-b)\left(\sinh^2(e+fx)+1\right)\sqrt{a+b\sinh^2(e+fx)}}}{2f}$$

$$\Big\downarrow \text{61}$$

$$\frac{\frac{(2a+b)\left(\frac{\int \frac{1}{\left(\sinh^2(e+fx)+1\right)\sqrt{b\sinh^2(e+fx)+a}}d\sinh^2(e+fx)}{a-b}+\frac{2}{(a-b)\sqrt{a+b\sinh^2(e+fx)}}\right)}{2(a-b)}+\frac{1}{(a-b)\left(\sinh^2(e+fx)+1\right)\sqrt{a+b\sinh^2(e+fx)}}}{2f}$$

$$\Big\downarrow \text{73}$$

$$\frac{\frac{(2a+b)\left(\frac{2\int \frac{1}{\frac{\sinh^4(e+fx)}{b}-\frac{a}{b}+1}d\sqrt{b\sinh^2(e+fx)+a}}{b(a-b)}+\frac{2}{(a-b)\sqrt{a+b\sinh^2(e+fx)}}\right)}{2(a-b)}+\frac{1}{(a-b)\left(\sinh^2(e+fx)+1\right)\sqrt{a+b\sinh^2(e+fx)}}}{2f}$$

$$\Big\downarrow \text{221}$$

$$\frac{\frac{(2a+b)\left(\frac{2}{(a-b)\sqrt{a+b\sinh^2(e+fx)}}-\frac{2\text{arctanh}\left(\frac{\sqrt{a+b\sinh^2(e+fx)}}{\sqrt{a-b}}\right)}{(a-b)^{3/2}}\right)}{2(a-b)}+\frac{1}{(a-b)\left(\sinh^2(e+fx)+1\right)\sqrt{a+b\sinh^2(e+fx)}}}{2f}$$

input
```
Int[Tanh[e + f*x]^3/(a + b*Sinh[e + f*x]^2)^(3/2),x]
```

output
```
(1/((a - b)*(1 + Sinh[e + f*x]^2)*Sqrt[a + b*Sinh[e + f*x]^2]) + ((2*a + b
)*((-2*ArcTanh[Sqrt[a + b*Sinh[e + f*x]^2]/Sqrt[a - b]])/(a - b)^(3/2) + 2
/((a - b)*Sqrt[a + b*Sinh[e + f*x]^2])))/(2*(a - b)))/(2*f)
```

Defintions of rubi rules used

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 61
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Simp[d*((
m + n + 2)/((b*c - a*d)*(m + 1)))   Int[(a + b*x)^(m + 1)*(c + d*x)^n, x],
x] /; FreeQ[{a, b, c, d, n}, x] && LtQ[m, -1] &&  !(LtQ[n, -1] && (EqQ[a, 0
] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && IntLinearQ[a, b, c, d
, m, n, x]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 87
```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_] :> Simp[(-(b*e - a*f))*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(f*(p
+ 1)*(c*f - d*e))), x] - Simp[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p
+ 1)))/(f*(p + 1)*(c*f - d*e))   Int[(c + d*x)^n*(e + f*x)^(p + 1), x], x]
/; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || Intege
rQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))))
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3673
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_.)*tan[(e_.) + (f_.)*(x_)]^
(m_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x]^2, x]}, Simp[ff^((m
 + 1)/2)/(2*f)   Subst[Int[x^((m - 1)/2)*((a + b*ff*x)^p/(1 - ff*x)^((m + 1
)/2)), x], x, Sin[e + f*x]^2/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && Integ
erQ[(m - 1)/2]
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 1.23 (sec) , antiderivative size = 103, normalized size of antiderivative = 0.84

method	result	size
default	$\frac{\text{`int/indef0`}\left(-\frac{\sinh(fx+e)^{3}\sqrt{a+b\sinh(fx+e)^{2}}\,\cosh(fx+e)^{2}}{-b^{2}\cosh(fx+e)^{10}+\left(-2ab+2b^{2}\right)\cosh(fx+e)^{8}+\left(-a^{2}+2ab-b^{2}\right)\cosh(fx+e)^{6}},\sinh(fx+e)\right)}{f}$	103
risch	Expression too large to display	289421

input
```
int(tanh(f*x+e)^3/(a+b*sinh(f*x+e)^2)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
`int/indef0`(-sinh(f*x+e)^3*(a+b*sinh(f*x+e)^2)^(1/2)*cosh(f*x+e)^2/(-b^2*
cosh(f*x+e)^10+(-2*a*b+2*b^2)*cosh(f*x+e)^8+(-a^2+2*a*b-b^2)*cosh(f*x+e)^6
),sinh(f*x+e))/f
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2032 vs. 2(106) = 212.

Time = 0.27 (sec) , antiderivative size = 4155, normalized size of antiderivative = 34.06

$$\int \frac{\tanh^3(e+fx)}{\left(a+b\sinh^2(e+fx)\right)^{3/2}}\,dx = \text{Too large to display}$$

input
```
integrate(tanh(f*x+e)^3/(a+b*sinh(f*x+e)^2)^(3/2),x, algorithm="fricas")
```

output
```
Too large to include
```

Sympy [F]

$$\int \frac{\tanh^3(e+fx)}{(a+b\sinh^2(e+fx))^{3/2}}\,dx = \int \frac{\tanh^3(e+fx)}{(a+b\sinh^2(e+fx))^{\frac{3}{2}}}\,dx$$

input
```
integrate(tanh(f*x+e)**3/(a+b*sinh(f*x+e)**2)**(3/2),x)
```

output
```
Integral(tanh(e + f*x)**3/(a + b*sinh(e + f*x)**2)**(3/2), x)
```

Maxima [F]

$$\int \frac{\tanh^3(e+fx)}{(a+b\sinh^2(e+fx))^{3/2}}\,dx = \int \frac{\tanh(fx+e)^3}{(b\sinh(fx+e)^2+a)^{\frac{3}{2}}}\,dx$$

input
```
integrate(tanh(f*x+e)^3/(a+b*sinh(f*x+e)^2)^(3/2),x, algorithm="maxima")
```

output
```
integrate(tanh(f*x + e)^3/(b*sinh(f*x + e)^2 + a)^(3/2), x)
```

Giac [F]

$$\int \frac{\tanh^3(e+fx)}{(a+b\sinh^2(e+fx))^{3/2}}\,dx = \int \frac{\tanh(fx+e)^3}{(b\sinh(fx+e)^2+a)^{\frac{3}{2}}}\,dx$$

input
```
integrate(tanh(f*x+e)^3/(a+b*sinh(f*x+e)^2)^(3/2),x, algorithm="giac")
```

output
```
integrate(tanh(f*x + e)^3/(b*sinh(f*x + e)^2 + a)^(3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\tanh^3(e+fx)}{\left(a+b\sinh^2(e+fx)\right)^{3/2}}\,dx = \int \frac{\tanh(e+f\,x)^3}{\left(b\sinh\left(e+f\,x\right)^2+a\right)^{3/2}}\,dx$$

input
```
int(tanh(e + f*x)^3/(a + b*sinh(e + f*x)^2)^(3/2),x)
```

output
```
int(tanh(e + f*x)^3/(a + b*sinh(e + f*x)^2)^(3/2), x)
```

Reduce [F]

$$\int \frac{\tanh^3(e+fx)}{\left(a+b\sinh^2(e+fx)\right)^{3/2}}\,dx = \int \frac{\sqrt{\sinh\left(fx+e\right)^2 b+a}\,\tanh\left(fx+e\right)^3}{\sinh\left(fx+e\right)^4 b^2+2\sinh\left(fx+e\right)^2 ab+a^2}dx$$

input
```
int(tanh(f*x+e)^3/(a+b*sinh(f*x+e)^2)^(3/2),x)
```

output
```
int((sqrt(sinh(e + f*x)**2*b + a)*tanh(e + f*x)**3)/(sinh(e + f*x)**4*b**2
 + 2*sinh(e + f*x)**2*a*b + a**2),x)
```

3.456 $\int \frac{\tanh(e+fx)}{(a+b\sinh^2(e+fx))^{3/2}}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 69

$$\int \frac{\tanh(e+fx)}{(a+b\sinh^2(e+fx))^{3/2}}\,dx = -\frac{\operatorname{arctanh}\left(\frac{\sqrt{a+b\sinh^2(e+fx)}}{\sqrt{a-b}}\right)}{(a-b)^{3/2}f} + \frac{1}{(a-b)f\sqrt{a+b\sinh^2(e+fx)}}$$

output
```
-arctanh((a+b*sinh(f*x+e)^2)^(1/2)/(a-b)^(1/2))/(a-b)^(3/2)/f+1/(a-b)/f/(a
+b*sinh(f*x+e)^2)^(1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 0.06 (sec) , antiderivative size = 58, normalized size of antiderivative = 0.84

$$\int \frac{\tanh(e+fx)}{(a+b\sinh^2(e+fx))^{3/2}}\,dx = -\frac{\text{Hypergeometric2F1}\left(-\frac{1}{2},1,\frac{1}{2},1+\frac{b\cosh^2(e+fx)}{a-b}\right)}{(-a+b)f\sqrt{a-b+b\cosh^2(e+fx)}}$$

input
```
Integrate[Tanh[e + f*x]/(a + b*Sinh[e + f*x]^2)^(3/2),x]
```

output
```
-(Hypergeometric2F1[-1/2, 1, 1/2, 1 + (b*Cosh[e + f*x]^2)/(a - b)]/((-a +
b)*f*Sqrt[a - b + b*Cosh[e + f*x]^2]))
```

Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 71, normalized size of antiderivative = 1.03, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.261$, Rules used = {3042, 26, 3673, 61, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\tanh(e+fx)}{\left(a+b\sinh^2(e+fx)\right)^{3/2}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int -\frac{i\tan(ie+ifx)}{\left(a-b\sin(ie+ifx)^2\right)^{3/2}}dx$$

$$\Big\downarrow \text{26}$$

$$-i\int \frac{\tan(ie+ifx)}{\left(a-b\sin(ie+ifx)^2\right)^{3/2}}dx$$

$$\Big\downarrow \text{3673}$$

$$\frac{\int \frac{1}{\left(\sinh^2(e+fx)+1\right)\left(b\sinh^2(e+fx)+a\right)^{3/2}}d\sinh^2(e+fx)}{2f}$$

$$\Big\downarrow \text{61}$$

$$\frac{\frac{\int \frac{1}{\left(\sinh^2(e+fx)+1\right)\sqrt{b\sinh^2(e+fx)+a}}d\sinh^2(e+fx)}{a-b}+\frac{2}{(a-b)\sqrt{a+b\sinh^2(e+fx)}}}{2f}$$

$$\Big\downarrow \text{73}$$

$$\frac{\frac{2\int \frac{1}{\frac{\sinh^4(e+fx)}{b}-\frac{a}{b}+1}d\sqrt{b\sinh^2(e+fx)+a}}{b(a-b)}+\frac{2}{(a-b)\sqrt{a+b\sinh^2(e+fx)}}}{2f}$$

$$\Big\downarrow 221$$

$$\frac{\frac{2}{(a-b)\sqrt{a+b\sinh^2(e+fx)}}-\frac{2\text{arctanh}\left(\frac{\sqrt{a+b\sinh^2(e+fx)}}{\sqrt{a-b}}\right)}{(a-b)^{3/2}}}{2f}$$

input
```
Int[Tanh[e + f*x]/(a + b*Sinh[e + f*x]^2)^(3/2),x]
```

output
```
((-2*ArcTanh[Sqrt[a + b*Sinh[e + f*x]^2]/Sqrt[a - b]])/(a - b)^(3/2) + 2/(
(a - b)*Sqrt[a + b*Sinh[e + f*x]^2]))/(2*f)
```

Defintions of rubi rules used

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 61
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Simp[d*((
m + n + 2)/((b*c - a*d)*(m + 1)))   Int[(a + b*x)^(m + 1)*(c + d*x)^n, x],
x] /; FreeQ[{a, b, c, d, n}, x] && LtQ[m, -1] &&  !(LtQ[n, -1] && (EqQ[a, 0
] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && IntLinearQ[a, b, c, d
, m, n, x]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3673
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_.)*tan[(e_.) + (f_.)*(x_)]^
(m_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x]^2, x]}, Simp[ff^((m
 + 1)/2)/(2*f)   Subst[Int[x^((m - 1)/2)*((a + b*ff*x)^p/(1 - ff*x)^((m + 1
)/2)), x], x, Sin[e + f*x]^2/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && Integ
erQ[(m - 1)/2]
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.45 (sec) , antiderivative size = 93, normalized size of antiderivative = 1.35

method	result	size
default	$\frac{\text{`int/indef0`}\left(-\frac{\sinh(fx+e)\sqrt{a+b\sinh(fx+e)^{2}}}{-b^{2}\sinh(fx+e)^{6}+\left(-2ab-b^{2}\right)\sinh(fx+e)^{4}+\left(-a^{2}-2ab\right)\sinh(fx+e)^{2}-a^{2}},\sinh(fx+e)\right)}{f}$	93

input
```
int(tanh(f*x+e)/(a+b*sinh(f*x+e)^2)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
`int/indef0`(-sinh(f*x+e)*(a+b*sinh(f*x+e)^2)^(1/2)/(-b^2*sinh(f*x+e)^6+(-
2*a*b-b^2)*sinh(f*x+e)^4+(-a^2-2*a*b)*sinh(f*x+e)^2-a^2),sinh(f*x+e))/f
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 691 vs. $2(61) = 122$.

Time = 0.16 (sec) , antiderivative size = 1474, normalized size of antiderivative = 21.36

$$\int \frac{\tanh(e+fx)}{(a+b\sinh^2(e+fx))^{3/2}}\,dx = \text{Too large to display}$$

input
```
integrate(tanh(f*x+e)/(a+b*sinh(f*x+e)^2)^(3/2),x, algorithm="fricas")
```

output
```
[-1/2*((b*cosh(f*x + e)^4 + 4*b*cosh(f*x + e)*sinh(f*x + e)^3 + b*sinh(f*x
 + e)^4 + 2*(2*a - b)*cosh(f*x + e)^2 + 2*(3*b*cosh(f*x + e)^2 + 2*a - b)*
sinh(f*x + e)^2 + 4*(b*cosh(f*x + e)^3 + (2*a - b)*cosh(f*x + e))*sinh(f*x
 + e) + b)*sqrt(a - b)*log((b*cosh(f*x + e)^4 + 4*b*cosh(f*x + e)*sinh(f*x
 + e)^3 + b*sinh(f*x + e)^4 + 2*(4*a - 3*b)*cosh(f*x + e)^2 + 2*(3*b*cosh(
f*x + e)^2 + 4*a - 3*b)*sinh(f*x + e)^2 + 4*sqrt(2)*sqrt(a - b)*sqrt((b*co
sh(f*x + e)^2 + b*sinh(f*x + e)^2 + 2*a - b)/(cosh(f*x + e)^2 - 2*cosh(f*x
 + e)*sinh(f*x + e) + sinh(f*x + e)^2))*(cosh(f*x + e) + sinh(f*x + e)) +
4*(b*cosh(f*x + e)^3 + (4*a - 3*b)*cosh(f*x + e))*sinh(f*x + e) + b)/(cosh
(f*x + e)^4 + 4*cosh(f*x + e)*sinh(f*x + e)^3 + sinh(f*x + e)^4 + 2*(3*cos
h(f*x + e)^2 + 1)*sinh(f*x + e)^2 + 2*cosh(f*x + e)^2 + 4*(cosh(f*x + e)^3
 + cosh(f*x + e))*sinh(f*x + e) + 1)) - 4*sqrt(2)*((a - b)*cosh(f*x + e) +
 (a - b)*sinh(f*x + e))*sqrt((b*cosh(f*x + e)^2 + b*sinh(f*x + e)^2 + 2*a
- b)/(cosh(f*x + e)^2 - 2*cosh(f*x + e)*sinh(f*x + e) + sinh(f*x + e)^2)))
/((a^2*b - 2*a*b^2 + b^3)*f*cosh(f*x + e)^4 + 4*(a^2*b - 2*a*b^2 + b^3)*f*
cosh(f*x + e)*sinh(f*x + e)^3 + (a^2*b - 2*a*b^2 + b^3)*f*sinh(f*x + e)^4
+ 2*(2*a^3 - 5*a^2*b + 4*a*b^2 - b^3)*f*cosh(f*x + e)^2 + 2*(3*(a^2*b - 2*
a*b^2 + b^3)*f*cosh(f*x + e)^2 + (2*a^3 - 5*a^2*b + 4*a*b^2 - b^3)*f)*sinh
(f*x + e)^2 + (a^2*b - 2*a*b^2 + b^3)*f + 4*((a^2*b - 2*a*b^2 + b^3)*f*cos
h(f*x + e)^3 + (2*a^3 - 5*a^2*b + 4*a*b^2 - b^3)*f*cosh(f*x + e))*sinh(...
```

Sympy [F]

$$\int \frac{\tanh(e+fx)}{(a+b\sinh^2(e+fx))^{3/2}}\,dx = \int \frac{\tanh{(e+fx)}}{\left(a+b\sinh^2{(e+fx)}\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate(tanh(f*x+e)/(a+b*sinh(f*x+e)**2)**(3/2),x)
```

output
```
Integral(tanh(e + f*x)/(a + b*sinh(e + f*x)**2)**(3/2), x)
```

Maxima [F]

$$\int \frac{\tanh(e+fx)}{(a+b\sinh^2(e+fx))^{3/2}}\,dx = \int \frac{\tanh{(fx+e)}}{\left(b\sinh{(fx+e)}^2+a\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate(tanh(f*x+e)/(a+b*sinh(f*x+e)^2)^(3/2),x, algorithm="maxima")
```

output
```
integrate(tanh(f*x + e)/(b*sinh(f*x + e)^2 + a)^(3/2), x)
```

Giac [F]

$$\int \frac{\tanh(e+fx)}{(a+b\sinh^2(e+fx))^{3/2}}\,dx = \int \frac{\tanh{(fx+e)}}{\left(b\sinh{(fx+e)}^2+a\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate(tanh(f*x+e)/(a+b*sinh(f*x+e)^2)^(3/2),x, algorithm="giac")
```

output
```
integrate(tanh(f*x + e)/(b*sinh(f*x + e)^2 + a)^(3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\tanh(e+fx)}{\left(a+b\sinh^2(e+fx)\right)^{3/2}}\,dx=\int \frac{\tanh(e+f\,x)}{\left(b\sinh\left(e+f\,x\right)^2+a\right)^{3/2}}\,dx$$

input
```
int(tanh(e + f*x)/(a + b*sinh(e + f*x)^2)^(3/2),x)
```

output
```
int(tanh(e + f*x)/(a + b*sinh(e + f*x)^2)^(3/2), x)
```

Reduce [F]

$$\int \frac{\tanh(e+fx)}{\left(a+b\sinh^2(e+fx)\right)^{3/2}}\,dx=\int \frac{\sqrt{\sinh\left(fx+e\right)^2 b+a}\,\tanh\left(fx+e\right)}{\sinh\left(fx+e\right)^4 b^2+2\sinh\left(fx+e\right)^2 ab+a^2}dx$$

input
```
int(tanh(f*x+e)/(a+b*sinh(f*x+e)^2)^(3/2),x)
```

output
```
int((sqrt(sinh(e + f*x)**2*b + a)*tanh(e + f*x))/(sinh(e + f*x)**4*b**2 +
2*sinh(e + f*x)**2*a*b + a**2),x)
```

3.457 $\int \frac{\coth(e+fx)}{(a+b\sinh^2(e+fx))^{3/2}}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 57

$$\int \frac{\coth(e+fx)}{(a+b\sinh^2(e+fx))^{3/2}}\,dx = -\frac{\operatorname{arctanh}\left(\frac{\sqrt{a+b\sinh^2(e+fx)}}{\sqrt{a}}\right)}{a^{3/2}f}+\frac{1}{af\sqrt{a+b\sinh^2(e+fx)}}$$

output
```
-arctanh((a+b*sinh(f*x+e)^2)^(1/2)/a^(1/2))/a^(3/2)/f+1/a/f/(a+b*sinh(f*x+
e)^2)^(1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 0.05 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.81

$$\int \frac{\coth(e+fx)}{(a+b\sinh^2(e+fx))^{3/2}}\,dx = \frac{\text{Hypergeometric2F1}\left(-\frac{1}{2},1,\frac{1}{2},1+\frac{b\sinh^2(e+fx)}{a}\right)}{af\sqrt{a+b\sinh^2(e+fx)}}$$

input
```
Integrate[Coth[e + f*x]/(a + b*Sinh[e + f*x]^2)^(3/2),x]
```

output
```
Hypergeometric2F1[-1/2, 1, 1/2, 1 + (b*Sinh[e + f*x]^2)/a]/(a*f*Sqrt[a + b
*Sinh[e + f*x]^2])
```

Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 59, normalized size of antiderivative = 1.04, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.261$, Rules used = {3042, 26, 3673, 61, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\coth(e+fx)}{\left(a+b\sinh^2(e+fx)\right)^{3/2}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{i}{\tan(ie+ifx)\left(a-b\sin(ie+ifx)^2\right)^{3/2}}dx$$

$$\Big\downarrow \text{26}$$

$$i\int \frac{1}{\left(a-b\sin(ie+ifx)^2\right)^{3/2}\tan(ie+ifx)}dx$$

$$\Big\downarrow \text{3673}$$

$$\frac{\int \frac{\operatorname{csch}^2(e+fx)}{\left(b\sinh^2(e+fx)+a\right)^{3/2}}d\sinh^2(e+fx)}{2f}$$

$$\Big\downarrow \text{61}$$

$$\frac{\frac{\int \frac{\operatorname{csch}^2(e+fx)}{\sqrt{b\sinh^2(e+fx)+a}}d\sinh^2(e+fx)}{a}+\frac{2}{a\sqrt{a+b\sinh^2(e+fx)}}}{2f}$$

$$\Big\downarrow \text{73}$$

$$\frac{\frac{2\int \frac{1}{\frac{\sinh^4(e+fx)}{b}-\frac{a}{b}}d\sqrt{b\sinh^2(e+fx)+a}}{ab}+\frac{2}{a\sqrt{a+b\sinh^2(e+fx)}}}{2f}$$

$$\downarrow \text{221}$$

$$\frac{\frac{2}{a\sqrt{a+b\sinh^2(e+fx)}}-\frac{2\text{arctanh}\left(\frac{\sqrt{a+b\sinh^2(e+fx)}}{\sqrt{a}}\right)}{a^{3/2}}}{2f}$$

input
```
Int[Coth[e + f*x]/(a + b*Sinh[e + f*x]^2)^(3/2),x]
```

output
```
((-2*ArcTanh[Sqrt[a + b*Sinh[e + f*x]^2]/Sqrt[a]])/a^(3/2) + 2/(a*Sqrt[a +
 b*Sinh[e + f*x]^2]))/(2*f)
```

Defintions of rubi rules used

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 61
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Simp[d*((
m + n + 2)/((b*c - a*d)*(m + 1)))   Int[(a + b*x)^(m + 1)*(c + d*x)^n, x],
x] /; FreeQ[{a, b, c, d, n}, x] && LtQ[m, -1] &&  !(LtQ[n, -1] && (EqQ[a, 0
] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && IntLinearQ[a, b, c, d
, m, n, x]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3673
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_.)*tan[(e_.) + (f_.)*(x_)]^
(m_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x]^2, x]}, Simp[ff^((m
 + 1)/2)/(2*f)   Subst[Int[x^((m - 1)/2)*((a + b*ff*x)^p/(1 - ff*x)^((m + 1
)/2)), x], x, Sin[e + f*x]^2/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && Integ
erQ[(m - 1)/2]
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.43 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.61

method	result	size
default	$\frac{\text{`int/indef0`}\left(\frac{1}{\sinh(fx+e)\left(a+b\sinh(fx+e)^{2}\right)^{\frac{3}{2}}},\sinh(fx+e)\right)}{f}$	35

input
```
int(coth(f*x+e)/(a+b*sinh(f*x+e)^2)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
`int/indef0`(1/sinh(f*x+e)/(a+b*sinh(f*x+e)^2)^(3/2),sinh(f*x+e))/f
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 580 vs. 2(49) = 98.

Time = 0.15 (sec) , antiderivative size = 1250, normalized size of antiderivative = 21.93

$$\int \frac{\coth(e+fx)}{\left(a+b\sinh^2(e+fx)\right)^{3/2}}\,dx = \text{Too large to display}$$

input
```
integrate(coth(f*x+e)/(a+b*sinh(f*x+e)^2)^(3/2),x, algorithm="fricas")
```

output
```
[1/2*((b*cosh(f*x + e)^4 + 4*b*cosh(f*x + e)*sinh(f*x + e)^3 + b*sinh(f*x
+ e)^4 + 2*(2*a - b)*cosh(f*x + e)^2 + 2*(3*b*cosh(f*x + e)^2 + 2*a - b)*s
inh(f*x + e)^2 + 4*(b*cosh(f*x + e)^3 + (2*a - b)*cosh(f*x + e))*sinh(f*x
+ e) + b)*sqrt(a)*log((b*cosh(f*x + e)^4 + 4*b*cosh(f*x + e)*sinh(f*x + e)
^3 + b*sinh(f*x + e)^4 + 2*(4*a - b)*cosh(f*x + e)^2 + 2*(3*b*cosh(f*x + e
)^2 + 4*a - b)*sinh(f*x + e)^2 - 4*sqrt(2)*sqrt(a)*sqrt((b*cosh(f*x + e)^2
 + b*sinh(f*x + e)^2 + 2*a - b)/(cosh(f*x + e)^2 - 2*cosh(f*x + e)*sinh(f*
x + e) + sinh(f*x + e)^2))*(cosh(f*x + e) + sinh(f*x + e)) + 4*(b*cosh(f*x
 + e)^3 + (4*a - b)*cosh(f*x + e))*sinh(f*x + e) + b)/(cosh(f*x + e)^4 + 4
*cosh(f*x + e)*sinh(f*x + e)^3 + sinh(f*x + e)^4 + 2*(3*cosh(f*x + e)^2 -
1)*sinh(f*x + e)^2 - 2*cosh(f*x + e)^2 + 4*(cosh(f*x + e)^3 - cosh(f*x + e
))*sinh(f*x + e) + 1)) + 4*sqrt(2)*(a*cosh(f*x + e) + a*sinh(f*x + e))*sqr
t((b*cosh(f*x + e)^2 + b*sinh(f*x + e)^2 + 2*a - b)/(cosh(f*x + e)^2 - 2*c
osh(f*x + e)*sinh(f*x + e) + sinh(f*x + e)^2)))/(a^2*b*f*cosh(f*x + e)^4 +
 4*a^2*b*f*cosh(f*x + e)*sinh(f*x + e)^3 + a^2*b*f*sinh(f*x + e)^4 + a^2*b
*f + 2*(2*a^3 - a^2*b)*f*cosh(f*x + e)^2 + 2*(3*a^2*b*f*cosh(f*x + e)^2 +
(2*a^3 - a^2*b)*f)*sinh(f*x + e)^2 + 4*(a^2*b*f*cosh(f*x + e)^3 + (2*a^3 -
 a^2*b)*f*cosh(f*x + e))*sinh(f*x + e)), ((b*cosh(f*x + e)^4 + 4*b*cosh(f*
x + e)*sinh(f*x + e)^3 + b*sinh(f*x + e)^4 + 2*(2*a - b)*cosh(f*x + e)^2 +
 2*(3*b*cosh(f*x + e)^2 + 2*a - b)*sinh(f*x + e)^2 + 4*(b*cosh(f*x + e)...
```

Sympy [F]

$$\int \frac{\coth(e+fx)}{\left(a+b\sinh^2(e+fx)\right)^{3/2}}\,dx=\int \frac{\coth\left(e+fx\right)}{\left(a+b\sinh^2\left(e+fx\right)\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate(coth(f*x+e)/(a+b*sinh(f*x+e)**2)**(3/2),x)
```

output
```
Integral(coth(e + f*x)/(a + b*sinh(e + f*x)**2)**(3/2), x)
```

Maxima [F]

$$\int \frac{\coth(e+fx)}{\left(a+b\sinh^2(e+fx)\right)^{3/2}}\,dx = \int \frac{\coth\left(fx+e\right)}{\left(b\sinh\left(fx+e\right)^2+a\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate(coth(f*x+e)/(a+b*sinh(f*x+e)^2)^(3/2),x, algorithm="maxima")
```

output
```
integrate(coth(f*x + e)/(b*sinh(f*x + e)^2 + a)^(3/2), x)
```

Giac [F]

$$\int \frac{\coth(e+fx)}{\left(a+b\sinh^2(e+fx)\right)^{3/2}}\,dx = \int \frac{\coth\left(fx+e\right)}{\left(b\sinh\left(fx+e\right)^2+a\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate(coth(f*x+e)/(a+b*sinh(f*x+e)^2)^(3/2),x, algorithm="giac")
```

output
```
integrate(coth(f*x + e)/(b*sinh(f*x + e)^2 + a)^(3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\coth(e+fx)}{\left(a+b\sinh^2(e+fx)\right)^{3/2}}\,dx = \int \frac{\coth(e+f\,x)}{{\left(b\,{\mathrm{sinh}\left(e+f\,x\right)}^2+a\right)}^{3/2}}\,dx$$

input
```
int(coth(e + f*x)/(a + b*sinh(e + f*x)^2)^(3/2),x)
```

output
```
int(coth(e + f*x)/(a + b*sinh(e + f*x)^2)^(3/2), x)
```

Reduce [F]

$$\int \frac{\coth(e+fx)}{\left(a+b\sinh^2(e+fx)\right)^{3/2}}\,dx = \int \frac{\sqrt{\sinh\left(fx+e\right)^2 b+a}\,\coth\left(fx+e\right)}{\sinh\left(fx+e\right)^4 b^2+2\sinh\left(fx+e\right)^2 ab+a^2}dx$$

input
```
int(coth(f*x+e)/(a+b*sinh(f*x+e)^2)^(3/2),x)
```

output
```
int((sqrt(sinh(e + f*x)**2*b + a)*coth(e + f*x))/(sinh(e + f*x)**4*b**2 +
2*sinh(e + f*x)**2*a*b + a**2),x)
```

3.458 $\int \frac{\coth^3(e+fx)}{\left(a+b\sinh^2(e+fx)\right)^{3/2}}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 110

$$\int \frac{\coth^3(e+fx)}{\left(a+b\sinh^2(e+fx)\right)^{3/2}}\,dx = -\frac{(2a-3b)\operatorname{arctanh}\left(\frac{\sqrt{a+b\sinh^2(e+fx)}}{\sqrt{a}}\right)}{2a^{5/2}f} + \frac{2a-3b}{2a^2f\sqrt{a+b\sinh^2(e+fx)}} - \frac{\operatorname{csch}^2(e+fx)}{2af\sqrt{a+b\sinh^2(e+fx)}}$$

output
```
-1/2*(2*a-3*b)*arctanh((a+b*sinh(f*x+e)^2)^(1/2)/a^(1/2))/a^(5/2)/f+1/2*(2
*a-3*b)/a^2/f/(a+b*sinh(f*x+e)^2)^(1/2)-1/2*csch(f*x+e)^2/a/f/(a+b*sinh(f*
x+e)^2)^(1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 0.07 (sec) , antiderivative size = 69, normalized size of antiderivative = 0.63

$$\int \frac{\coth^3(e+fx)}{\left(a+b\sinh^2(e+fx)\right)^{3/2}}\,dx = \frac{-a\operatorname{csch}^2(e+fx) + (2a-3b)\,\text{Hypergeometric2F1}\left(-\frac{1}{2},1,\frac{1}{2},1+\frac{b\sinh^2(e+}{a}\right.}{2a^2f\sqrt{a+b\sinh^2(e+fx)}}$$

input
```
Integrate[Coth[e + f*x]^3/(a + b*Sinh[e + f*x]^2)^(3/2),x]
```

output
```
(-(a*Csch[e + f*x]^2) + (2*a - 3*b)*Hypergeometric2F1[-1/2, 1, 1/2, 1 + (b
*Sinh[e + f*x]^2)/a])/(2*a^2*f*Sqrt[a + b*Sinh[e + f*x]^2])
```

Rubi [A] (verified)

Time = 0.34 (sec) , antiderivative size = 103, normalized size of antiderivative = 0.94, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.280$, Rules used = {3042, 26, 3673, 87, 61, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\coth^3(e+fx)}{\left(a+b\sinh^2(e+fx)\right)^{3/2}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int -\frac{i}{\tan(ie+ifx)^3\left(a-b\sin(ie+ifx)^2\right)^{3/2}}dx$$

$$\Big\downarrow \text{26}$$

$$-i\int \frac{1}{\left(a-b\sin(ie+ifx)^2\right)^{3/2}\tan(ie+ifx)^3}dx$$

$$\Big\downarrow \text{3673}$$

$$\frac{\int \frac{\text{csch}^4(e+fx)\left(\sinh^2(e+fx)+1\right)}{\left(b\sinh^2(e+fx)+a\right)^{3/2}}d\sinh^2(e+fx)}{2f}$$

$$\Big\downarrow \text{87}$$

$$\frac{\frac{(2a-3b)\int \frac{\text{csch}^2(e+fx)}{\left(b\sinh^2(e+fx)+a\right)^{3/2}}d\sinh^2(e+fx)}{2a}-\frac{\text{csch}^2(e+fx)}{a\sqrt{a+b\sinh^2(e+fx)}}}{2f}$$

$$\Big\downarrow \text{61}$$

$$\frac{\frac{(2a-3b)\left(\frac{\int \frac{\text{csch}^2(e+fx)}{\sqrt{b\sinh^2(e+fx)+a}}d\sinh^2(e+fx)}{a}+\frac{2}{a\sqrt{a+b\sinh^2(e+fx)}}\right)}{2a}-\frac{\text{csch}^2(e+fx)}{a\sqrt{a+b\sinh^2(e+fx)}}}{2f}$$

$$\Big\downarrow \text{73}$$

$$\frac{\frac{(2a-3b)\left(\frac{2\int \frac{1}{\frac{\sinh^4(e+fx)}{b}-\frac{a}{b}}d\sqrt{b\sinh^2(e+fx)+a}}{ab}+\frac{2}{a\sqrt{a+b\sinh^2(e+fx)}}\right)}{2a}-\frac{\text{csch}^2(e+fx)}{a\sqrt{a+b\sinh^2(e+fx)}}}{2f}$$

$$\Big\downarrow \text{221}$$

$$\frac{\frac{(2a-3b)\left(\frac{2}{a\sqrt{a+b\sinh^2(e+fx)}}-\frac{2\text{arctanh}\left(\frac{\sqrt{a+b\sinh^2(e+fx)}}{\sqrt{a}}\right)}{a^{3/2}}\right)}{2a}-\frac{\text{csch}^2(e+fx)}{a\sqrt{a+b\sinh^2(e+fx)}}}{2f}$$

input
```
Int[Coth[e + f*x]^3/(a + b*Sinh[e + f*x]^2)^(3/2),x]
```

output
```
(-(Csch[e + f*x]^2/(a*Sqrt[a + b*Sinh[e + f*x]^2])) + ((2*a - 3*b)*((-2*Ar
cTanh[Sqrt[a + b*Sinh[e + f*x]^2]/Sqrt[a]])/a^(3/2) + 2/(a*Sqrt[a + b*Sinh
[e + f*x]^2])))/(2*a))/(2*f)
```

Defintions of rubi rules used

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 61
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Simp[d*((
m + n + 2)/((b*c - a*d)*(m + 1)))   Int[(a + b*x)^(m + 1)*(c + d*x)^n, x],
x] /; FreeQ[{a, b, c, d, n}, x] && LtQ[m, -1] &&  !(LtQ[n, -1] && (EqQ[a, 0
] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && IntLinearQ[a, b, c, d
, m, n, x]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 87
```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_] :> Simp[(-(b*e - a*f))*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(f*(p
+ 1)*(c*f - d*e))), x] - Simp[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p
+ 1)))/(f*(p + 1)*(c*f - d*e))   Int[(c + d*x)^n*(e + f*x)^(p + 1), x], x]
/; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || Intege
rQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))))
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3673
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_.)*tan[(e_.) + (f_.)*(x_)]^
(m_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x]^2, x]}, Simp[ff^((m
 + 1)/2)/(2*f)   Subst[Int[x^((m - 1)/2)*((a + b*ff*x)^p/(1 - ff*x)^((m + 1
)/2)), x], x, Sin[e + f*x]^2/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && Integ
erQ[(m - 1)/2]
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 1.20 (sec) , antiderivative size = 43, normalized size of antiderivative = 0.39

method	result	size
default	$\frac{\text{`int/indef0`}\left(\frac{\cosh(fx+e)^{2}}{\sinh(fx+e)^{3}\left(a+b\sinh(fx+e)^{2}\right)^{\frac{3}{2}}},\sinh(fx+e)\right)}{f}$	43
risch	Expression too large to display	289430

input
```
int(coth(f*x+e)^3/(a+b*sinh(f*x+e)^2)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
`int/indef0`(cosh(f*x+e)^2/sinh(f*x+e)^3/(a+b*sinh(f*x+e)^2)^(3/2),sinh(f*
x+e))/f
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1626 vs. 2(94) = 188.

Time = 0.20 (sec) , antiderivative size = 3341, normalized size of antiderivative = 30.37

$$\int \frac{\coth^3(e+fx)}{\left(a+b\sinh^2(e+fx)\right)^{3/2}}\,dx = \text{Too large to display}$$

input
```
integrate(coth(f*x+e)^3/(a+b*sinh(f*x+e)^2)^(3/2),x, algorithm="fricas")
```

output
```
Too large to include
```

Sympy [F]

$$\int \frac{\coth^3(e+fx)}{(a+b\sinh^2(e+fx))^{3/2}}\,dx = \int \frac{\coth^3(e+fx)}{(a+b\sinh^2(e+fx))^{\frac{3}{2}}}\,dx$$

input
```
integrate(coth(f*x+e)**3/(a+b*sinh(f*x+e)**2)**(3/2),x)
```

output
```
Integral(coth(e + f*x)**3/(a + b*sinh(e + f*x)**2)**(3/2), x)
```

Maxima [F]

$$\int \frac{\coth^3(e+fx)}{(a+b\sinh^2(e+fx))^{3/2}}\,dx = \int \frac{\coth(fx+e)^3}{(b\sinh(fx+e)^2+a)^{\frac{3}{2}}}\,dx$$

input
```
integrate(coth(f*x+e)^3/(a+b*sinh(f*x+e)^2)^(3/2),x, algorithm="maxima")
```

output
```
integrate(coth(f*x + e)^3/(b*sinh(f*x + e)^2 + a)^(3/2), x)
```

Giac [F]

$$\int \frac{\coth^3(e+fx)}{(a+b\sinh^2(e+fx))^{3/2}}\,dx = \int \frac{\coth(fx+e)^3}{(b\sinh(fx+e)^2+a)^{\frac{3}{2}}}\,dx$$

input
```
integrate(coth(f*x+e)^3/(a+b*sinh(f*x+e)^2)^(3/2),x, algorithm="giac")
```

output
```
integrate(coth(f*x + e)^3/(b*sinh(f*x + e)^2 + a)^(3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\coth^3(e+fx)}{\left(a+b\sinh^2(e+fx)\right)^{3/2}}\,dx=\int \frac{\coth(e+f\,x)^3}{\left(b\sinh\left(e+f\,x\right)^2+a\right)^{3/2}}\,dx$$

input
```
int(coth(e + f*x)^3/(a + b*sinh(e + f*x)^2)^(3/2),x)
```

output
```
int(coth(e + f*x)^3/(a + b*sinh(e + f*x)^2)^(3/2), x)
```

Reduce [F]

$$\int \frac{\coth^3(e+fx)}{\left(a+b\sinh^2(e+fx)\right)^{3/2}}\,dx=\int \frac{\sqrt{\sinh\left(fx+e\right)^2b+a}\,\coth\left(fx+e\right)^3}{\sinh\left(fx+e\right)^4b^2+2\sinh\left(fx+e\right)^2ab+a^2}dx$$

input
```
int(coth(f*x+e)^3/(a+b*sinh(f*x+e)^2)^(3/2),x)
```

output
```
int((sqrt(sinh(e + f*x)**2*b + a)*coth(e + f*x)**3)/(sinh(e + f*x)**4*b**2
 + 2*sinh(e + f*x)**2*a*b + a**2),x)
```

3.459 $\int \frac{\coth^5(e+fx)}{\left(a+b\sinh^2(e+fx)\right)^{3/2}}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 167

$$\int \frac{\coth^5(e+fx)}{\left(a+b\sinh^2(e+fx)\right)^{3/2}}\,dx = -\frac{(8a^2-24ab+15b^2)\,\mathrm{arctanh}\left(\frac{\sqrt{a+b\sinh^2(e+fx)}}{\sqrt{a}}\right)}{8a^{7/2}f}+\frac{8a^2-24ab+15b^2}{8a^3f\sqrt{a+b\sinh^2(e+fx)}}-\frac{(8a-5b)\mathrm{csch}^2(e+fx)}{8a^2f\sqrt{a+b\sinh^2(e+fx)}}-\frac{\mathrm{csch}^4(e+fx)}{4af\sqrt{a+b\sinh^2(e+fx)}}$$

output
```
-1/8*(8*a^2-24*a*b+15*b^2)*arctanh((a+b*sinh(f*x+e)^2)^(1/2)/a^(1/2))/a^(7
/2)/f+1/8*(8*a^2-24*a*b+15*b^2)/a^3/f/(a+b*sinh(f*x+e)^2)^(1/2)-1/8*(8*a-5
*b)*csch(f*x+e)^2/a^2/f/(a+b*sinh(f*x+e)^2)^(1/2)-1/4*csch(f*x+e)^4/a/f/(a
+b*sinh(f*x+e)^2)^(1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 0.23 (sec) , antiderivative size = 94, normalized size of antiderivative = 0.56

$$\int \frac{\coth^5(e+fx)}{\left(a+b\sinh^2(e+fx)\right)^{3/2}}\,dx = \frac{a\operatorname{csch}^2(e+fx)\left(-8a+5b-2a\operatorname{csch}^2(e+fx)\right)+\left(8a^2-24ab+15b^2\right)\operatorname{Hy}}{8a^3f\sqrt{a+b\sinh^2(e+fx)}}$$

input
```
Integrate[Coth[e + f*x]^5/(a + b*Sinh[e + f*x]^2)^(3/2),x]
```

output
```
(a*Csch[e + f*x]^2*(-8*a + 5*b - 2*a*Csch[e + f*x]^2) + (8*a^2 - 24*a*b +
15*b^2)*Hypergeometric2F1[-1/2, 1, 1/2, 1 + (b*Sinh[e + f*x]^2)/a])/(8*a^3
*f*Sqrt[a + b*Sinh[e + f*x]^2])
```

Rubi [A] (verified)

Time = 0.36 (sec) , antiderivative size = 157, normalized size of antiderivative = 0.94, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.360$, Rules used = {3042, 26, 3673, 100, 27, 87, 61, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\coth^5(e+fx)}{\left(a+b\sinh^2(e+fx)\right)^{3/2}}dx$$

$$\Big\downarrow \ 3042$$

$$\int \frac{i}{\tan(ie+ifx)^5\left(a-b\sin(ie+ifx)^2\right)^{3/2}}dx$$

$$\Big\downarrow \ 26$$

$$i\int \frac{1}{\left(a-b\sin(ie+ifx)^2\right)^{3/2}\tan(ie+ifx)^5}dx$$

$$\Big\downarrow \ 3673$$

$$\frac{\int \frac{\operatorname{csch}^6(e+fx)(\sinh^2(e+fx)+1)^2}{(b\sinh^2(e+fx)+a)^{3/2}}d\sinh^2(e+fx)}{2f}$$

$\downarrow$ 100

$$\frac{\frac{\int \frac{\operatorname{csch}^4(e+fx)\left(4a\sinh^2(e+fx)+8a-5b\right)}{2\left(b\sinh^2(e+fx)+a\right)^{3/2}}d\sinh^2(e+fx)}{2a}-\frac{\operatorname{csch}^4(e+fx)}{2a\sqrt{a+b\sinh^2(e+fx)}}}{2f}$$

$\downarrow$ 27

$$\frac{\frac{\int \frac{\operatorname{csch}^4(e+fx)\left(4a\sinh^2(e+fx)+8a-5b\right)}{\left(b\sinh^2(e+fx)+a\right)^{3/2}}d\sinh^2(e+fx)}{4a}-\frac{\operatorname{csch}^4(e+fx)}{2a\sqrt{a+b\sinh^2(e+fx)}}}{2f}$$

$\downarrow$ 87

$$\frac{\frac{\frac{\left(8a^2-24ab+15b^2\right)\int \frac{\operatorname{csch}^2(e+fx)}{\left(b\sinh^2(e+fx)+a\right)^{3/2}}d\sinh^2(e+fx)}{2a}-\frac{(8a-5b)\operatorname{csch}^2(e+fx)}{a\sqrt{a+b\sinh^2(e+fx)}}}{4a}-\frac{\operatorname{csch}^4(e+fx)}{2a\sqrt{a+b\sinh^2(e+fx)}}}{2f}$$

$\downarrow$ 61

$$\frac{\frac{\frac{\left(8a^2-24ab+15b^2\right)\left(\frac{\int \frac{\operatorname{csch}^2(e+fx)}{\sqrt{b\sinh^2(e+fx)+a}}d\sinh^2(e+fx)}{a}+\frac{2}{a\sqrt{a+b\sinh^2(e+fx)}}\right)}{2a}-\frac{(8a-5b)\operatorname{csch}^2(e+fx)}{a\sqrt{a+b\sinh^2(e+fx)}}}{4a}-\frac{\operatorname{csch}^4(e+fx)}{2a\sqrt{a+b\sinh^2(e+fx)}}}{2f}$$

$\downarrow$ 73

$$\frac{\frac{\frac{\left(8a^2-24ab+15b^2\right)\left(\frac{2\int \frac{1}{\frac{\sinh^4(e+fx)}{b}-\frac{a}{b}}d\sqrt{b\sinh^2(e+fx)+a}}{ab}+\frac{2}{a\sqrt{a+b\sinh^2(e+fx)}}\right)}{2a}-\frac{(8a-5b)\operatorname{csch}^2(e+fx)}{a\sqrt{a+b\sinh^2(e+fx)}}}{4a}-\frac{\operatorname{csch}^4(e+fx)}{2a\sqrt{a+b\sinh^2(e+fx)}}}{2f}$$

$\downarrow$ 221

$$\frac{\frac{\frac{\left(8a^2-24ab+15b^2\right)\left(\frac{2}{a\sqrt{a+b\sinh^2(e+fx)}}-\frac{2\text{arctanh}\left(\frac{\sqrt{a+b\sinh^2(e+fx)}}{\sqrt{a}}\right)}{a^{3/2}}\right)}{2a}-\frac{(8a-5b)\text{csch}^2(e+fx)}{a\sqrt{a+b\sinh^2(e+fx)}}}{4a}-\frac{\text{csch}^4(e+fx)}{2a\sqrt{a+b\sinh^2(e+fx)}}}{2f}$$

input
```
Int[Coth[e + f*x]^5/(a + b*Sinh[e + f*x]^2)^(3/2),x]
```

output
```
(-1/2*Csch[e + f*x]^4/(a*Sqrt[a + b*Sinh[e + f*x]^2]) + (-(((8*a - 5*b)*Cs
ch[e + f*x]^2)/(a*Sqrt[a + b*Sinh[e + f*x]^2])) + ((8*a^2 - 24*a*b + 15*b^
2)*((-2*ArcTanh[Sqrt[a + b*Sinh[e + f*x]^2]/Sqrt[a]])/a^(3/2) + 2/(a*Sqrt[
a + b*Sinh[e + f*x]^2])))/(2*a))/(4*a))/(2*f)
```

Defintions of rubi rules used

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 61
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Simp[d*((
m + n + 2)/((b*c - a*d)*(m + 1)))   Int[(a + b*x)^(m + 1)*(c + d*x)^n, x],
x] /; FreeQ[{a, b, c, d, n}, x] && LtQ[m, -1] &&  !(LtQ[n, -1] && (EqQ[a, 0
] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && IntLinearQ[a, b, c, d
, m, n, x]
```

rule 73

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 87

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_] :> Simp[(-(b*e - a*f))*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(f*(p
+ 1)*(c*f - d*e))), x] - Simp[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p
+ 1)))/(f*(p + 1)*(c*f - d*e))   Int[(c + d*x)^n*(e + f*x)^(p + 1), x], x]
/; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || Intege
rQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))))
```

rule 100

```
Int[((a_.) + (b_.)*(x_))^2*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))^(
p_), x_] :> Simp[(b*c - a*d)^2*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(d^2*(d
*e - c*f)*(n + 1))), x] - Simp[1/(d^2*(d*e - c*f)*(n + 1))   Int[(c + d*x)^
(n + 1)*(e + f*x)^p*Simp[a^2*d^2*f*(n + p + 2) + b^2*c*(d*e*(n + 1) + c*f*(
p + 1)) - 2*a*b*d*(d*e*(n + 1) + c*f*(p + 1)) - b^2*d*(d*e - c*f)*(n + 1)*x
, x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && (LtQ[n, -1] || (EqQ[n
 + p + 3, 0] && NeQ[n, -1] && (SumSimplerQ[n, 1] ||  !SumSimplerQ[p, 1])))
```

rule 221

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 3042

```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3673

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_.)*tan[(e_.) + (f_.)*(x_)]^
(m_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x]^2, x]}, Simp[ff^((m
 + 1)/2)/(2*f)   Subst[Int[x^((m - 1)/2)*((a + b*ff*x)^p/(1 - ff*x)^((m + 1
)/2)), x], x, Sin[e + f*x]^2/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && Integ
erQ[(m - 1)/2]
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 8.22 (sec) , antiderivative size = 43, normalized size of antiderivative = 0.26

method	result	size
default	$\frac{\text{`int/indef0`}\left(\frac{\cosh(fx+e)^4}{\sinh(fx+e)^5\left(a+b\sinh(fx+e)^2\right)^{\frac{3}{2}}},\sinh(fx+e)\right)}{f}$	43
risch	Expression too large to display	2586426

input
```
int(coth(f*x+e)^5/(a+b*sinh(f*x+e)^2)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
`int/indef0`(cosh(f*x+e)^4/sinh(f*x+e)^5/(a+b*sinh(f*x+e)^2)^(3/2),sinh(f*
x+e))/f
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 3793 vs. 2(147) = 294.

Time = 0.41 (sec) , antiderivative size = 7675, normalized size of antiderivative = 45.96

$$\int \frac{\coth^5(e+fx)}{\left(a+b\sinh^2(e+fx)\right)^{3/2}}\,dx = \text{Too large to display}$$

input
```
integrate(coth(f*x+e)^5/(a+b*sinh(f*x+e)^2)^(3/2),x, algorithm="fricas")
```

output
```
Too large to include
```

Sympy [F]

$$\int \frac{\coth^5(e+fx)}{\left(a+b\sinh^2(e+fx)\right)^{3/2}}\,dx=\int \frac{\coth^5\left(e+fx\right)}{\left(a+b\sinh^2\left(e+fx\right)\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate(coth(f*x+e)**5/(a+b*sinh(f*x+e)**2)**(3/2),x)
```

output
```
Integral(coth(e + f*x)**5/(a + b*sinh(e + f*x)**2)**(3/2), x)
```

Maxima [F]

$$\int \frac{\coth^5(e+fx)}{\left(a+b\sinh^2(e+fx)\right)^{3/2}}\,dx=\int \frac{\coth\left(fx+e\right)^5}{\left(b\sinh\left(fx+e\right)^2+a\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate(coth(f*x+e)^5/(a+b*sinh(f*x+e)^2)^(3/2),x, algorithm="maxima")
```

output
```
integrate(coth(f*x + e)^5/(b*sinh(f*x + e)^2 + a)^(3/2), x)
```

Giac [F]

$$\int \frac{\coth^5(e+fx)}{\left(a+b\sinh^2(e+fx)\right)^{3/2}}\,dx=\int \frac{\coth\left(fx+e\right)^5}{\left(b\sinh\left(fx+e\right)^2+a\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate(coth(f*x+e)^5/(a+b*sinh(f*x+e)^2)^(3/2),x, algorithm="giac")
```

output
```
integrate(coth(f*x + e)^5/(b*sinh(f*x + e)^2 + a)^(3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\coth^5(e+fx)}{\left(a+b\sinh^2(e+fx)\right)^{3/2}}\,dx=\text{Hanged}$$

input
```
int(coth(e + f*x)^5/(a + b*sinh(e + f*x)^2)^(3/2),x)
```

output
```
\text{Hanged}
```

Reduce [F]

$$\int \frac{\coth^5(e+fx)}{\left(a+b\sinh^2(e+fx)\right)^{3/2}}\,dx=\int \frac{\sqrt{\sinh(fx+e)^2 b+a}\,\coth(fx+e)^5}{\sinh(fx+e)^4 b^2+2\sinh(fx+e)^2 ab+a^2}dx$$

input
```
int(coth(f*x+e)^5/(a+b*sinh(f*x+e)^2)^(3/2),x)
```

output
```
int((sqrt(sinh(e + f*x)**2*b + a)*coth(e + f*x)**5)/(sinh(e + f*x)**4*b**2
 + 2*sinh(e + f*x)**2*a*b + a**2),x)
```

3.460 $\int \frac{\tanh^4(e+fx)}{\left(a+b\sinh^2(e+fx)\right)^{3/2}}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 296

$$\int \frac{\tanh^4(e+fx)}{\left(a+b\sinh^2(e+fx)\right)^{3/2}}\,dx = -\frac{\sqrt{a}\sqrt{b}(7a+b)\cosh(e+fx)E\left(\arctan\left(\frac{\sqrt{b}\sinh(e+fx)}{\sqrt{a}}\right)|1-\frac{a}{b}\right)}{3(a-b)^3 f\sqrt{\frac{a\cosh^2(e+fx)}{a+b\sinh^2(e+fx)}}\sqrt{a+b\sinh^2(e+fx)}} + \frac{a^{3/2}(3a+5b)\cosh(e+fx)\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{b}\sinh(e+fx)}{\sqrt{a}}\right),1-\frac{a}{b}\right)}{3(a-b)^3\sqrt{b}f\sqrt{\frac{a\cosh^2(e+fx)}{a+b\sinh^2(e+fx)}}\sqrt{a+b\sinh^2(e+fx)}} - \frac{4a\tanh(e+fx)}{3(a-b)^2 f\sqrt{a+b\sinh^2(e+fx)}} + \frac{\text{sech}^2(e+fx)\tanh(e+fx)}{3(a-b)f\sqrt{a+b\sinh^2(e+fx)}}$$

output
```
-1/3*a^(1/2)*b^(1/2)*(7*a+b)*cosh(f*x+e)*EllipticE(b^(1/2)*sinh(f*x+e)/a^(
1/2)/(1+b*sinh(f*x+e)^2/a)^(1/2),(1-a/b)^(1/2))/(a-b)^3/f/(a*cosh(f*x+e)^2
/(a+b*sinh(f*x+e)^2))^(1/2)/(a+b*sinh(f*x+e)^2)^(1/2)+1/3*a^(3/2)*(3*a+5*b
)*cosh(f*x+e)*InverseJacobiAM(arctan(b^(1/2)*sinh(f*x+e)/a^(1/2)),(1-a/b)^
(1/2))/(a-b)^3/b^(1/2)/f/(a*cosh(f*x+e)^2/(a+b*sinh(f*x+e)^2))^(1/2)/(a+b*
sinh(f*x+e)^2)^(1/2)-4/3*a*tanh(f*x+e)/(a-b)^2/f/(a+b*sinh(f*x+e)^2)^(1/2)
+1/3*sech(f*x+e)^2*tanh(f*x+e)/(a-b)/f/(a+b*sinh(f*x+e)^2)^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 1.85 (sec) , antiderivative size = 212, normalized size of antiderivative = 0.72

$$\int \frac{\tanh^4(e+fx)}{\left(a+b\sinh^2(e+fx)\right)^{3/2}}\,dx = \frac{-2ia(7a+b)\sqrt{\frac{2a-b+b\cosh(2(e+fx))}{a}}E\left(i(e+fx)\left|\frac{b}{a}\right.\right)+8ia(a-b)\sqrt{\frac{2a-b+b\cosh(2(e+fx))}{a}}\cdots}{\cdots}$$

input
```
Integrate[Tanh[e + f*x]^4/(a + b*Sinh[e + f*x]^2)^(3/2),x]
```

output
```
((-2*I)*a*(7*a + b)*Sqrt[(2*a - b + b*Cosh[2*(e + f*x)])/a]*EllipticE[I*(e
 + f*x), b/a] + (8*I)*a*(a - b)*Sqrt[(2*a - b + b*Cosh[2*(e + f*x)])/a]*El
lipticF[I*(e + f*x), b/a] - ((8*a^2 + 21*a*b - 5*b^2 + 4*(4*a^2 + 3*a*b +
b^2)*Cosh[2*(e + f*x)] + b*(7*a + b)*Cosh[4*(e + f*x)])*Sech[e + f*x]^2*Ta
nh[e + f*x])/(2*Sqrt[2]))/(6*(a - b)^3*f*Sqrt[2*a - b + b*Cosh[2*(e + f*x)
]])
```

Rubi [A] (verified)

Time = 0.48 (sec) , antiderivative size = 345, normalized size of antiderivative = 1.17, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.320$, Rules used = {3042, 3675, 372, 27, 402, 400, 313, 320}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\tanh^4(e+fx)}{\left(a+b\sinh^2(e+fx)\right)^{3/2}}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\tan(ie+ifx)^4}{(a-b\sin(ie+ifx)^2)^{3/2}}dx$$

$$\Big\downarrow \text{3675}$$

$$\frac{\sqrt{\cosh^2(e+fx)}\text{sech}(e+fx)\int \frac{\sinh^4(e+fx)}{(\sinh^2(e+fx)+1)^{5/2}(b\sinh^2(e+fx)+a)^{3/2}}d\sinh(e+fx)}{f}$$

$\downarrow$ 372

$$\frac{\sqrt{\cosh^2(e+fx)}\text{sech}(e+fx)\left(\frac{\sinh(e+fx)}{3(a-b)(\sinh^2(e+fx)+1)^{3/2}\sqrt{a+b\sinh^2(e+fx)}}-\frac{\int \frac{a(1-3\sinh^2(e+fx))}{(\sinh^2(e+fx)+1)^{3/2}(b\sinh^2(e+fx)+a)^{3/2}}d\sinh(e}{3(a-b)}\right.}{f}$$

$\downarrow$ 27

$$\frac{\sqrt{\cosh^2(e+fx)}\text{sech}(e+fx)\left(\frac{\sinh(e+fx)}{3(a-b)(\sinh^2(e+fx)+1)^{3/2}\sqrt{a+b\sinh^2(e+fx)}}-\frac{a\int \frac{1-3\sinh^2(e+fx)}{(\sinh^2(e+fx)+1)^{3/2}(b\sinh^2(e+fx)+a)^{3/2}}d\sinh(}{3(a-b)}\right.}{f}$$

$\downarrow$ 402

$$\frac{\sqrt{\cosh^2(e+fx)}\text{sech}(e+fx)\left(\frac{\sinh(e+fx)}{3(a-b)(\sinh^2(e+fx)+1)^{3/2}\sqrt{a+b\sinh^2(e+fx)}}-\frac{a\left(\frac{4\sinh(e+fx)}{(a-b)\sqrt{\sinh^2(e+fx)+1}\sqrt{a+b\sinh^2(e+fx)}}-\frac{\int}{\sqrt{\sin}}\right.}{3(a-b}\right.}{f}$$

$\downarrow$ 400

$$\frac{\sqrt{\cosh^2(e+fx)}\text{sech}(e+fx)\left(\frac{\sinh(e+fx)}{3(a-b)(\sinh^2(e+fx)+1)^{3/2}\sqrt{a+b\sinh^2(e+fx)}}-\frac{a\left(\frac{4\sinh(e+fx)}{(a-b)\sqrt{\sinh^2(e+fx)+1}\sqrt{a+b\sinh^2(e+fx)}}-\frac{(3a+5b}{}\right.}{}\right.}{f}$$

$\downarrow$ 313

$$\frac{\sqrt{\cosh^2(e+fx)}\operatorname{sech}(e+fx)\left(\frac{\sinh(e+fx)}{3(a-b)\left(\sinh^2(e+fx)+1\right)^{3/2}\sqrt{a+b\sinh^2(e+fx)}}-\frac{a\left(\frac{4\sinh(e+fx)}{(a-b)\sqrt{\sinh^2(e+fx)+1}\sqrt{a+b\sinh^2(e+fx)}}-\frac{(3a+5b\cdots}{\cdots}\right.}{\cdots}\right.}{f}$$

$\big\downarrow$ 320

$$\frac{\sqrt{\cosh^2(e+fx)}\operatorname{sech}(e+fx)\left(\frac{\sinh(e+fx)}{3(a-b)\left(\sinh^2(e+fx)+1\right)^{3/2}\sqrt{a+b\sinh^2(e+fx)}}-\frac{a\left(\frac{4\sinh(e+fx)}{(a-b)\sqrt{\sinh^2(e+fx)+1}\sqrt{a+b\sinh^2(e+fx)}}-\frac{(3a+5b\cdots}{\cdots}\right.}{\cdots}\right.}{f}$$

input
```
Int[Tanh[e + f*x]^4/(a + b*Sinh[e + f*x]^2)^(3/2),x]
```

output
```
(Sqrt[Cosh[e + f*x]^2]*Sech[e + f*x]*(Sinh[e + f*x]/(3*(a - b)*(1 + Sinh[e
 + f*x]^2)^(3/2)*Sqrt[a + b*Sinh[e + f*x]^2]) - (a*((4*Sinh[e + f*x])/((a
- b)*Sqrt[1 + Sinh[e + f*x]^2]*Sqrt[a + b*Sinh[e + f*x]^2]) - (-((Sqrt[b]*
(7*a + b)*EllipticE[ArcTan[(Sqrt[b]*Sinh[e + f*x])/Sqrt[a]], 1 - a/b]*Sqrt
[1 + Sinh[e + f*x]^2])/(Sqrt[a]*(a - b)*Sqrt[(a*(1 + Sinh[e + f*x]^2))/(a
+ b*Sinh[e + f*x]^2)]*Sqrt[a + b*Sinh[e + f*x]^2])) + ((3*a + 5*b)*Ellipti
cF[ArcTan[Sinh[e + f*x]], 1 - b/a]*Sqrt[a + b*Sinh[e + f*x]^2])/(a*(a - b)
*Sqrt[1 + Sinh[e + f*x]^2]*Sqrt[(a + b*Sinh[e + f*x]^2)/(a*(1 + Sinh[e + f
*x]^2))]))/(a - b)))/(3*(a - b))))/f
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 313
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/((c_) + (d_.)*(x_)^2)^(3/2), x_Symbol] :> Sim
p[(Sqrt[a + b*x^2]/(c*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(c
+ d*x^2)))]))*EllipticE[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; FreeQ
[{a, b, c, d}, x] && PosQ[b/a] && PosQ[d/c]
```

rule 320
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(Sqrt[a + b*x^2]/(a*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(
c + d*x^2)))]))*EllipticF[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; Fre
eQ[{a, b, c, d}, x] && PosQ[d/c] && PosQ[b/a] &&  !SimplerSqrtQ[b/a, d/c]
```

rule 372
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_
), x_Symbol] :> Simp[(-a)*e^3*(e*x)^(m - 3)*(a + b*x^2)^(p + 1)*((c + d*x^2
)^(q + 1)/(2*b*(b*c - a*d)*(p + 1))), x] + Simp[e^4/(2*b*(b*c - a*d)*(p + 1
))   Int[(e*x)^(m - 4)*(a + b*x^2)^(p + 1)*(c + d*x^2)^q*Simp[a*c*(m - 3) +
 (a*d*(m + 2*q - 1) + 2*b*c*(p + 1))*x^2, x], x], x] /; FreeQ[{a, b, c, d,
e, q}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && GtQ[m, 3] && IntBinomialQ[a
, b, c, d, e, m, 2, p, q, x]
```

rule 400
```
Int[((e_) + (f_.)*(x_)^2)/(Sqrt[(a_) + (b_.)*(x_)^2]*((c_) + (d_.)*(x_)^2)^
(3/2)), x_Symbol] :> Simp[(b*e - a*f)/(b*c - a*d)   Int[1/(Sqrt[a + b*x^2]*
Sqrt[c + d*x^2]), x], x] - Simp[(d*e - c*f)/(b*c - a*d)   Int[Sqrt[a + b*x^
2]/(c + d*x^2)^(3/2), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && PosQ[b/a] &
& PosQ[d/c]
```

rule 402
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_.)*((e_) + (f_.)*(x
_)^2), x_Symbol] :> Simp[(-(b*e - a*f))*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^
(q + 1)/(a*2*(b*c - a*d)*(p + 1))), x] + Simp[1/(a*2*(b*c - a*d)*(p + 1))
 Int[(a + b*x^2)^(p + 1)*(c + d*x^2)^q*Simp[c*(b*e - a*f) + e*2*(b*c - a*d)
*(p + 1) + d*(b*e - a*f)*(2*(p + q + 2) + 1)*x^2, x], x], x] /; FreeQ[{a, b
, c, d, e, f, q}, x] && LtQ[p, -1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3675
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_.)*tan[(e_.) + (f_.)*(x_)]^
(m_), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Simp[ff^(m + 1
)*(Sqrt[Cos[e + f*x]^2]/(f*Cos[e + f*x]))   Subst[Int[x^m*((a + b*ff^2*x^2)
^p/(1 - ff^2*x^2)^((m + 1)/2)), x], x, Sin[e + f*x]/ff], x]] /; FreeQ[{a, b
, e, f, p}, x] && IntegerQ[m/2] &&  !IntegerQ[p]
```

Maple [A] (verified)

Time = 10.21 (sec) , antiderivative size = 354, normalized size of antiderivative = 1.20

method	result
default	$-\frac{\left(7\sqrt{-\frac{b}{a}}\,ab+\sqrt{-\frac{b}{a}}\,b^2\right)\cosh(fx+e)^4\sinh(fx+e)+\left(4\sqrt{-\frac{b}{a}}\,a^2-4\sqrt{-\frac{b}{a}}\,ab\right)\cosh(fx+e)^2\sinh(fx+e)-\sqrt{\frac{b\cosh(fx+e)^2}{a}+\frac{a-b}{a}}\,\sqrt{}$
risch	Expression too large to display

input
```
int(tanh(f*x+e)^4/(a+b*sinh(f*x+e)^2)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-1/3*((7*(-b/a)^(1/2)*a*b+(-b/a)^(1/2)*b^2)*cosh(f*x+e)^4*sinh(f*x+e)+(4*(
-b/a)^(1/2)*a^2-4*(-b/a)^(1/2)*a*b)*cosh(f*x+e)^2*sinh(f*x+e)-(b/a*cosh(f*
x+e)^2+(a-b)/a)^(1/2)*(cosh(f*x+e)^2)^(1/2)*(3*EllipticF(sinh(f*x+e)*(-b/a
)^(1/2),(1/b*a)^(1/2))*a^2-2*EllipticF(sinh(f*x+e)*(-b/a)^(1/2),(1/b*a)^(1
/2))*a*b-EllipticF(sinh(f*x+e)*(-b/a)^(1/2),(1/b*a)^(1/2))*b^2+7*EllipticE
(sinh(f*x+e)*(-b/a)^(1/2),(1/b*a)^(1/2))*a*b+EllipticE(sinh(f*x+e)*(-b/a)^
(1/2),(1/b*a)^(1/2))*b^2)*cosh(f*x+e)^2+(-(-b/a)^(1/2)*a^2+2*(-b/a)^(1/2)*
a*b-(-b/a)^(1/2)*b^2)*sinh(f*x+e))/(-b/a)^(1/2)/cosh(f*x+e)^3/(a-b)^3/(a+b
*sinh(f*x+e)^2)^(1/2)/f
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 7400 vs. 2(280) = 560.

Time = 0.31 (sec) , antiderivative size = 7400, normalized size of antiderivative = 25.00

$$\int \frac{\tanh^4(e+fx)}{(a+b\sinh^2(e+fx))^{3/2}}\,dx = \text{Too large to display}$$

input
```
integrate(tanh(f*x+e)^4/(a+b*sinh(f*x+e)^2)^(3/2),x, algorithm="fricas")
```

output
```
Too large to include
```

Sympy [F]

$$\int \frac{\tanh^4(e+fx)}{(a+b\sinh^2(e+fx))^{3/2}}\,dx = \int \frac{\tanh^4(e+fx)}{(a+b\sinh^2(e+fx))^{\frac{3}{2}}}\,dx$$

input
```
integrate(tanh(f*x+e)**4/(a+b*sinh(f*x+e)**2)**(3/2),x)
```

output
```
Integral(tanh(e + f*x)**4/(a + b*sinh(e + f*x)**2)**(3/2), x)
```

Maxima [F]

$$\int \frac{\tanh^4(e+fx)}{(a+b\sinh^2(e+fx))^{3/2}}\,dx = \int \frac{\tanh(fx+e)^4}{(b\sinh(fx+e)^2+a)^{\frac{3}{2}}}\,dx$$

input
```
integrate(tanh(f*x+e)^4/(a+b*sinh(f*x+e)^2)^(3/2),x, algorithm="maxima")
```

output
```
integrate(tanh(f*x + e)^4/(b*sinh(f*x + e)^2 + a)^(3/2), x)
```

Giac [F]

$$\int \frac{\tanh^4(e+fx)}{(a+b\sinh^2(e+fx))^{3/2}}\,dx = \int \frac{\tanh(fx+e)^4}{(b\sinh(fx+e)^2+a)^{\frac{3}{2}}}\,dx$$

input
```
integrate(tanh(f*x+e)^4/(a+b*sinh(f*x+e)^2)^(3/2),x, algorithm="giac")
```

output
```
integrate(tanh(f*x + e)^4/(b*sinh(f*x + e)^2 + a)^(3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\tanh^4(e+fx)}{(a+b\sinh^2(e+fx))^{3/2}}\,dx = \int \frac{\tanh(e+f\,x)^4}{(b\sinh(e+f\,x)^2+a)^{3/2}}\,dx$$

input
```
int(tanh(e + f*x)^4/(a + b*sinh(e + f*x)^2)^(3/2),x)
```

output
```
int(tanh(e + f*x)^4/(a + b*sinh(e + f*x)^2)^(3/2), x)
```

Reduce [F]

$$\int \frac{\tanh^4(e+fx)}{(a+b\sinh^2(e+fx))^{3/2}}\,dx = \int \frac{\sqrt{\sinh(fx+e)^2b+a}\,\tanh(fx+e)^4}{\sinh(fx+e)^4b^2+2\sinh(fx+e)^2ab+a^2}dx$$

input
```
int(tanh(f*x+e)^4/(a+b*sinh(f*x+e)^2)^(3/2),x)
```

output
```
int((sqrt(sinh(e + f*x)**2*b + a)*tanh(e + f*x)**4)/(sinh(e + f*x)**4*b**2
 + 2*sinh(e + f*x)**2*a*b + a**2),x)
```

3.461 $\int \frac{\tanh^2(e+fx)}{(a+b\sinh^2(e+fx))^{3/2}}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 235

$$\int \frac{\tanh^2(e+fx)}{(a+b\sinh^2(e+fx))^{3/2}}\,dx = -\frac{2\sqrt{a}\sqrt{b}\cosh(e+fx)E\left(\arctan\left(\frac{\sqrt{b}\sinh(e+fx)}{\sqrt{a}}\right)|1-\frac{a}{b}\right)}{(a-b)^2 f\sqrt{\frac{a\cosh^2(e+fx)}{a+b\sinh^2(e+fx)}}\sqrt{a+b\sinh^2(e+fx)}} + \frac{\sqrt{a}(a+b)\cosh(e+fx)\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{b}\sinh(e+fx)}{\sqrt{a}}\right),1-\frac{a}{b}\right)}{(a-b)^2\sqrt{b}f\sqrt{\frac{a\cosh^2(e+fx)}{a+b\sinh^2(e+fx)}}\sqrt{a+b\sinh^2(e+fx)}} - \frac{\tanh(e+fx)}{(a-b)f\sqrt{a+b\sinh^2(e+fx)}}$$

output

```
-2*a^(1/2)*b^(1/2)*cosh(f*x+e)*EllipticE(b^(1/2)*sinh(f*x+e)/a^(1/2)/(1+b*
sinh(f*x+e)^2/a)^(1/2),(1-a/b)^(1/2))/(a-b)^2/f/(a*cosh(f*x+e)^2/(a+b*sinh
(f*x+e)^2))^(1/2)/(a+b*sinh(f*x+e)^2)^(1/2)+a^(1/2)*(a+b)*cosh(f*x+e)*Inve
rseJacobiAM(arctan(b^(1/2)*sinh(f*x+e)/a^(1/2)),(1-a/b)^(1/2))/(a-b)^2/b^(
1/2)/f/(a*cosh(f*x+e)^2/(a+b*sinh(f*x+e)^2))^(1/2)/(a+b*sinh(f*x+e)^2)^(1/
2)-tanh(f*x+e)/(a-b)/f/(a+b*sinh(f*x+e)^2)^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 1.22 (sec) , antiderivative size = 158, normalized size of antiderivative = 0.67

$$\int \frac{\tanh^2(e+fx)}{(a+b\sinh^2(e+fx))^{3/2}}\,dx = \frac{-2i\sqrt{2}a\sqrt{\frac{2a-b+b\cosh(2(e+fx))}{a}}E\left(i(e+fx)\,\middle|\,\frac{b}{a}\right)+i\sqrt{2}(a-b)\sqrt{\frac{2a-b+b\cosh(2(}{a}}}{(a-b)^2f\sqrt{4a-2b+}}$$

input
```
Integrate[Tanh[e + f*x]^2/(a + b*Sinh[e + f*x]^2)^(3/2),x]
```

output
```
((-2*I)*Sqrt[2]*a*Sqrt[(2*a - b + b*Cosh[2*(e + f*x)])/a]*EllipticE[I*(e +
 f*x), b/a] + I*Sqrt[2]*(a - b)*Sqrt[(2*a - b + b*Cosh[2*(e + f*x)])/a]*El
lipticF[I*(e + f*x), b/a] - 2*(a + b*Cosh[2*(e + f*x)])*Tanh[e + f*x])/((a
 - b)^2*f*Sqrt[4*a - 2*b + 2*b*Cosh[2*(e + f*x)]])
```

Rubi [A] (verified)

Time = 0.43 (sec) , antiderivative size = 275, normalized size of antiderivative = 1.17, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.280$, Rules used = {3042, 25, 3675, 373, 400, 313, 320}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\tanh^2(e+fx)}{(a+b\sinh^2(e+fx))^{3/2}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int -\frac{\tan(ie+ifx)^2}{(a-b\sin(ie+ifx)^2)^{3/2}}dx$$

$$\Big\downarrow \text{25}$$

$$-\int \frac{\tan(ie+ifx)^2}{(a-b\sin(ie+ifx)^2)^{3/2}}dx$$

$$\Big\downarrow \text{3675}$$

$$\frac{\sqrt{\cosh^2(e+fx)}\operatorname{sech}(e+fx)\int \frac{\sinh^2(e+fx)}{(\sinh^2(e+fx)+1)^{3/2}(b\sinh^2(e+fx)+a)^{3/2}}d\sinh(e+fx)}{f}$$

$\big\downarrow$ 373

$$\frac{\sqrt{\cosh^2(e+fx)}\operatorname{sech}(e+fx)\left(\frac{\int \frac{a-b\sinh^2(e+fx)}{\sqrt{\sinh^2(e+fx)+1}\left(b\sinh^2(e+fx)+a\right)^{3/2}}d\sinh(e+fx)}{a-b}-\frac{\sinh(e+fx)}{(a-b)\sqrt{\sinh^2(e+fx)+1}\sqrt{a+b\sinh^2(e+fx)}}\right)}{f}$$

$\big\downarrow$ 400

$$\frac{\sqrt{\cosh^2(e+fx)}\operatorname{sech}(e+fx)\left(\frac{\frac{(a+b)\int \frac{1}{\sqrt{\sinh^2(e+fx)+1}\sqrt{b\sinh^2(e+fx)+a}}d\sinh(e+fx)}{a-b}-\frac{2ab\int \frac{\sqrt{\sinh^2(e+fx)+1}}{\left(b\sinh^2(e+fx)+a\right)^{3/2}}d\sinh(e+fx)}{a-b}}{a-b}-\frac{}{(a-b)}\right.}{f}$$

$\big\downarrow$ 313

$$\frac{\sqrt{\cosh^2(e+fx)}\operatorname{sech}(e+fx)\left(\frac{\frac{(a+b)\int \frac{1}{\sqrt{\sinh^2(e+fx)+1}\sqrt{b\sinh^2(e+fx)+a}}d\sinh(e+fx)}{a-b}-\frac{2\sqrt{a}\sqrt{b}\sqrt{\sinh^2(e+fx)+1}E\left(\arctan\left(\frac{\sqrt{b}\sinh(e+fx)}{\sqrt{a}}\right)\right|}{(a-b)\sqrt{\frac{a\left(\sinh^2(e+fx)+1\right)}{a+b\sinh^2(e+fx)}}\sqrt{a+b\sinh^2(e+fx)}}}{a-b}\right.}{f}$$

$\big\downarrow$ 320

$$\frac{\sqrt{\cosh^2(e+fx)}\operatorname{sech}(e+fx)\left(\frac{\frac{(a+b)\sqrt{a+b\sinh^2(e+fx)}\operatorname{EllipticF}\left(\arctan(\sinh(e+fx)),1-\frac{b}{a}\right)}{a(a-b)\sqrt{\sinh^2(e+fx)+1}\sqrt{\frac{a+b\sinh^2(e+fx)}{a\left(\sinh^2(e+fx)+1\right)}}}-\frac{2\sqrt{a}\sqrt{b}\sqrt{\sinh^2(e+fx)+1}E\left(\arctan\left(\frac{\sqrt{b}\sinh(e+fx)}{\sqrt{a}}\right.\right.}{(a-b)\sqrt{\frac{a\left(\sinh^2(e+fx)+1\right)}{a+b\sinh^2(e+fx)}}\sqrt{a+b\sinh^2(e+}}}{a-b}\right.}{f}$$

input

```
Int[Tanh[e + f*x]^2/(a + b*Sinh[e + f*x]^2)^(3/2),x]
```

output
```
(Sqrt[Cosh[e + f*x]^2]*Sech[e + f*x]*(-(Sinh[e + f*x]/((a - b)*Sqrt[1 + Si
nh[e + f*x]^2]*Sqrt[a + b*Sinh[e + f*x]^2])) + ((-2*Sqrt[a]*Sqrt[b]*Ellipt
icE[ArcTan[(Sqrt[b]*Sinh[e + f*x])/Sqrt[a]], 1 - a/b]*Sqrt[1 + Sinh[e + f*
x]^2])/((a - b)*Sqrt[(a*(1 + Sinh[e + f*x]^2))/(a + b*Sinh[e + f*x]^2)]*Sq
rt[a + b*Sinh[e + f*x]^2]) + ((a + b)*EllipticF[ArcTan[Sinh[e + f*x]], 1 -
 b/a]*Sqrt[a + b*Sinh[e + f*x]^2])/(a*(a - b)*Sqrt[1 + Sinh[e + f*x]^2]*Sq
rt[(a + b*Sinh[e + f*x]^2)/(a*(1 + Sinh[e + f*x]^2))]))/(a - b)))/f
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 313
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/((c_) + (d_.)*(x_)^2)^(3/2), x_Symbol] :> Sim
p[(Sqrt[a + b*x^2]/(c*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(c
+ d*x^2)))]))*EllipticE[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; FreeQ
[{a, b, c, d}, x] && PosQ[b/a] && PosQ[d/c]
```

rule 320
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(Sqrt[a + b*x^2]/(a*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(
c + d*x^2)))]))*EllipticF[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; Fre
eQ[{a, b, c, d}, x] && PosQ[d/c] && PosQ[b/a] &&  !SimplerSqrtQ[b/a, d/c]
```

rule 373
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_
), x_Symbol] :> Simp[e*(e*x)^(m - 1)*(a + b*x^2)^(p + 1)*((c + d*x^2)^(q +
1)/(2*(b*c - a*d)*(p + 1))), x] - Simp[e^2/(2*(b*c - a*d)*(p + 1))   Int[(e
*x)^(m - 2)*(a + b*x^2)^(p + 1)*(c + d*x^2)^q*Simp[c*(m - 1) + d*(m + 2*p +
 2*q + 3)*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, q}, x] && NeQ[b*c - a*d,
 0] && LtQ[p, -1] && GtQ[m, 1] && LeQ[m, 3] && IntBinomialQ[a, b, c, d, e,
m, 2, p, q, x]
```

rule 400
```
Int[((e_) + (f_.)*(x_)^2)/(Sqrt[(a_) + (b_.)*(x_)^2]*((c_) + (d_.)*(x_)^2)^
(3/2)), x_Symbol] :> Simp[(b*e - a*f)/(b*c - a*d)   Int[1/(Sqrt[a + b*x^2]*
Sqrt[c + d*x^2]), x], x] - Simp[(d*e - c*f)/(b*c - a*d)   Int[Sqrt[a + b*x^
2]/(c + d*x^2)^(3/2), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && PosQ[b/a] &
& PosQ[d/c]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3675
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_.)*tan[(e_.) + (f_.)*(x_)]^
(m_), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Simp[ff^(m + 1
)*(Sqrt[Cos[e + f*x]^2]/(f*Cos[e + f*x]))   Subst[Int[x^m*((a + b*ff^2*x^2)
^p/(1 - ff^2*x^2)^((m + 1)/2)), x], x, Sin[e + f*x]/ff], x]] /; FreeQ[{a, b
, e, f, p}, x] && IntegerQ[m/2] &&  !IntegerQ[p]
```

Maple [A] (verified)

Time = 4.44 (sec) , antiderivative size = 256, normalized size of antiderivative = 1.09

method	result
default	$-\frac{2\sqrt{-\frac{b}{a}}\, b\sinh(fx+e)^3-a\sqrt{\frac{a+b\sinh(fx+e)^2}{a}}\,\sqrt{\frac{\cosh(2fx+2e)}{2}+\frac{1}{2}}\ \mathrm{EllipticF}\left(\sinh(fx+e)\sqrt{-\frac{b}{a}},\sqrt{\frac{a}{b}}\right)+b\sqrt{\frac{a+b\sinh(fx+e)^2}{a}}\,\sqrt{\frac{\cosh(2}{}}}{(a-}$
risch	Expression too large to display

input
```
int(tanh(f*x+e)^2/(a+b*sinh(f*x+e)^2)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-(2*(-b/a)^(1/2)*b*sinh(f*x+e)^3-a*((a+b*sinh(f*x+e)^2)/a)^(1/2)*(cosh(f*x
+e)^2)^(1/2)*EllipticF(sinh(f*x+e)*(-b/a)^(1/2),(1/b*a)^(1/2))+b*((a+b*sin
h(f*x+e)^2)/a)^(1/2)*(cosh(f*x+e)^2)^(1/2)*EllipticF(sinh(f*x+e)*(-b/a)^(1
/2),(1/b*a)^(1/2))-2*b*((a+b*sinh(f*x+e)^2)/a)^(1/2)*(cosh(f*x+e)^2)^(1/2)
*EllipticE(sinh(f*x+e)*(-b/a)^(1/2),(1/b*a)^(1/2))+(-b/a)^(1/2)*a*sinh(f*x
+e)+b*sinh(f*x+e)*(-b/a)^(1/2))/(a-b)^2/(-b/a)^(1/2)/cosh(f*x+e)/(a+b*sinh
(f*x+e)^2)^(1/2)/f
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2612 vs. $2(229) = 458$.

Time = 0.13 (sec) , antiderivative size = 2612, normalized size of antiderivative = 11.11

$$\int \frac{\tanh^2(e+fx)}{\left(a+b\sinh^2(e+fx)\right)^{3/2}}\,dx = \text{Too large to display}$$

input
```
integrate(tanh(f*x+e)^2/(a+b*sinh(f*x+e)^2)^(3/2),x, algorithm="fricas")
```

output
```
2*(((2*a*b^2 - b^3)*cosh(f*x + e)^6 + 6*(2*a*b^2 - b^3)*cosh(f*x + e)*sinh
(f*x + e)^5 + (2*a*b^2 - b^3)*sinh(f*x + e)^6 + (8*a^2*b - 6*a*b^2 + b^3)*
cosh(f*x + e)^4 + (8*a^2*b - 6*a*b^2 + b^3 + 15*(2*a*b^2 - b^3)*cosh(f*x +
 e)^2)*sinh(f*x + e)^4 + 4*(5*(2*a*b^2 - b^3)*cosh(f*x + e)^3 + (8*a^2*b -
 6*a*b^2 + b^3)*cosh(f*x + e))*sinh(f*x + e)^3 + 2*a*b^2 - b^3 + (8*a^2*b
- 6*a*b^2 + b^3)*cosh(f*x + e)^2 + (15*(2*a*b^2 - b^3)*cosh(f*x + e)^4 + 8
*a^2*b - 6*a*b^2 + b^3 + 6*(8*a^2*b - 6*a*b^2 + b^3)*cosh(f*x + e)^2)*sinh
(f*x + e)^2 + 2*(3*(2*a*b^2 - b^3)*cosh(f*x + e)^5 + 2*(8*a^2*b - 6*a*b^2
+ b^3)*cosh(f*x + e)^3 + (8*a^2*b - 6*a*b^2 + b^3)*cosh(f*x + e))*sinh(f*x
 + e) - 2*(b^3*cosh(f*x + e)^6 + 6*b^3*cosh(f*x + e)*sinh(f*x + e)^5 + b^3
*sinh(f*x + e)^6 + (4*a*b^2 - b^3)*cosh(f*x + e)^4 + (15*b^3*cosh(f*x + e)
^2 + 4*a*b^2 - b^3)*sinh(f*x + e)^4 + 4*(5*b^3*cosh(f*x + e)^3 + (4*a*b^2
- b^3)*cosh(f*x + e))*sinh(f*x + e)^3 + b^3 + (4*a*b^2 - b^3)*cosh(f*x + e
)^2 + (15*b^3*cosh(f*x + e)^4 + 4*a*b^2 - b^3 + 6*(4*a*b^2 - b^3)*cosh(f*x
 + e)^2)*sinh(f*x + e)^2 + 2*(3*b^3*cosh(f*x + e)^5 + 2*(4*a*b^2 - b^3)*co
sh(f*x + e)^3 + (4*a*b^2 - b^3)*cosh(f*x + e))*sinh(f*x + e))*sqrt((a^2 -
a*b)/b^2))*sqrt(b)*sqrt((2*b*sqrt((a^2 - a*b)/b^2) - 2*a + b)/b)*elliptic_
e(arcsin(sqrt((2*b*sqrt((a^2 - a*b)/b^2) - 2*a + b)/b)*(cosh(f*x + e) + si
nh(f*x + e))), (8*a^2 - 8*a*b + b^2 + 4*(2*a*b - b^2)*sqrt((a^2 - a*b)/b^2
))/b^2) - ((2*a^2*b + a*b^2 - b^3)*cosh(f*x + e)^6 + 6*(2*a^2*b + a*b^2...
```

Sympy [F]

$$\int \frac{\tanh^2(e+fx)}{(a+b\sinh^2(e+fx))^{3/2}}\,dx = \int \frac{\tanh^2{(e+fx)}}{\left(a+b\sinh^2{(e+fx)}\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate(tanh(f*x+e)**2/(a+b*sinh(f*x+e)**2)**(3/2),x)
```

output
```
Integral(tanh(e + f*x)**2/(a + b*sinh(e + f*x)**2)**(3/2), x)
```

Maxima [F]

$$\int \frac{\tanh^2(e+fx)}{(a+b\sinh^2(e+fx))^{3/2}}\,dx = \int \frac{\tanh{(fx+e)}^2}{\left(b\sinh{(fx+e)}^2+a\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate(tanh(f*x+e)^2/(a+b*sinh(f*x+e)^2)^(3/2),x, algorithm="maxima")
```

output
```
integrate(tanh(f*x + e)^2/(b*sinh(f*x + e)^2 + a)^(3/2), x)
```

Giac [F]

$$\int \frac{\tanh^2(e+fx)}{(a+b\sinh^2(e+fx))^{3/2}}\,dx = \int \frac{\tanh{(fx+e)}^2}{\left(b\sinh{(fx+e)}^2+a\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate(tanh(f*x+e)^2/(a+b*sinh(f*x+e)^2)^(3/2),x, algorithm="giac")
```

output
```
integrate(tanh(f*x + e)^2/(b*sinh(f*x + e)^2 + a)^(3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\tanh^2(e+fx)}{\left(a+b\sinh^2(e+fx)\right)^{3/2}}\,dx=\int \frac{\tanh(e+f\,x)^2}{\left(b\sinh\left(e+f\,x\right)^2+a\right)^{3/2}}\,dx$$

input
```
int(tanh(e + f*x)^2/(a + b*sinh(e + f*x)^2)^(3/2),x)
```

output
```
int(tanh(e + f*x)^2/(a + b*sinh(e + f*x)^2)^(3/2), x)
```

Reduce [F]

$$\int \frac{\tanh^2(e+fx)}{\left(a+b\sinh^2(e+fx)\right)^{3/2}}\,dx=\int \frac{\sqrt{\sinh\left(fx+e\right)^2 b+a}\,\tanh\left(fx+e\right)^2}{\sinh\left(fx+e\right)^4 b^2+2\sinh\left(fx+e\right)^2 ab+a^2}dx$$

input
```
int(tanh(f*x+e)^2/(a+b*sinh(f*x+e)^2)^(3/2),x)
```

output
```
int((sqrt(sinh(e + f*x)**2*b + a)*tanh(e + f*x)**2)/(sinh(e + f*x)**4*b**2
 + 2*sinh(e + f*x)**2*a*b + a**2),x)
```

3.462 $\int \frac{1}{\left(a+b\sinh^2(e+fx)\right)^{3/2}}\,dx$

Optimal result

Integrand size = 16, antiderivative size = 116

$$\int \frac{1}{\left(a+b\sinh^2(e+fx)\right)^{3/2}}\,dx = -\frac{b\cosh(e+fx)\sinh(e+fx)}{a(a-b)f\sqrt{a+b\sinh^2(e+fx)}} - \frac{iE\left(ie+ifx\middle|\frac{b}{a}\right)\sqrt{a+b\sinh^2(e+fx)}}{a(a-b)f\sqrt{\frac{a+b\sinh^2(e+fx)}{a}}}$$

output
```
-b*cosh(f*x+e)*sinh(f*x+e)/a/(a-b)/f/(a+b*sinh(f*x+e)^2)^(1/2)-I*EllipticE
(sin(I*e+I*f*x),(b/a)^(1/2))*(a+b*sinh(f*x+e)^2)^(1/2)/a/(a-b)/f/((a+b*sin
h(f*x+e)^2)/a)^(1/2)
```

Mathematica [A] (verified)

Time = 0.10 (sec) , antiderivative size = 100, normalized size of antiderivative = 0.86

$$\int \frac{1}{\left(a+b\sinh^2(e+fx)\right)^{3/2}}\,dx = \frac{-2ia\sqrt{\frac{2a-b+b\cosh(2(e+fx))}{a}}E\left(i(e+fx)\middle|\frac{b}{a}\right)-\sqrt{2}b\sinh(2(e+fx))}{2a(a-b)f\sqrt{2a-b+b\cosh(2(e+fx))}}$$

input
```
Integrate[(a + b*Sinh[e + f*x]^2)^(-3/2),x]
```

output
```
((-2*I)*a*Sqrt[(2*a - b + b*Cosh[2*(e + f*x)])/a]*EllipticE[I*(e + f*x), b
/a] - Sqrt[2]*b*Sinh[2*(e + f*x)])/(2*a*(a - b)*f*Sqrt[2*a - b + b*Cosh[2*
(e + f*x)]])
```

Rubi [A] (verified)

Time = 0.57 (sec) , antiderivative size = 115, normalized size of antiderivative = 0.99, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.438, Rules used = {3042, 3663, 25, 3042, 3657, 3042, 3656}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\left(a+b\sinh^2(e+fx)\right)^{3/2}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{\left(a-b\sin(ie+ifx)^2\right)^{3/2}}dx$$

$$\Big\downarrow \text{3663}$$

$$-\frac{\int -\sqrt{b\sinh^2(e+fx)+a}dx}{a(a-b)}-\frac{b\sinh(e+fx)\cosh(e+fx)}{af(a-b)\sqrt{a+b\sinh^2(e+fx)}}$$

$$\Big\downarrow \text{25}$$

$$\frac{\int \sqrt{b\sinh^2(e+fx)+a}dx}{a(a-b)}-\frac{b\sinh(e+fx)\cosh(e+fx)}{af(a-b)\sqrt{a+b\sinh^2(e+fx)}}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{b\sinh(e+fx)\cosh(e+fx)}{af(a-b)\sqrt{a+b\sinh^2(e+fx)}}+\frac{\int \sqrt{a-b\sin(ie+ifx)^2}dx}{a(a-b)}$$

$$\Big\downarrow \text{3657}$$

$$\frac{\sqrt{a+b\sinh^2(e+fx)}\int \sqrt{\frac{b\sinh^2(e+fx)}{a}+1}dx}{a(a-b)\sqrt{\frac{b\sinh^2(e+fx)}{a}+1}}-\frac{b\sinh(e+fx)\cosh(e+fx)}{af(a-b)\sqrt{a+b\sinh^2(e+fx)}}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{b\sinh(e+fx)\cosh(e+fx)}{af(a-b)\sqrt{a+b\sinh^2(e+fx)}}+\frac{\sqrt{a+b\sinh^2(e+fx)}\int\sqrt{1-\frac{b\sin(ie+ifx)^2}{a}}dx}{a(a-b)\sqrt{\frac{b\sinh^2(e+fx)}{a}+1}}$$

$$\Big\downarrow \text{3656}$$

$$-\frac{b\sinh(e+fx)\cosh(e+fx)}{af(a-b)\sqrt{a+b\sinh^2(e+fx)}}-\frac{i\sqrt{a+b\sinh^2(e+fx)}E\left(ie+ifx|\frac{b}{a}\right)}{af(a-b)\sqrt{\frac{b\sinh^2(e+fx)}{a}+1}}$$

input
```
Int[(a + b*Sinh[e + f*x]^2)^(-3/2),x]
```

output
```
-((b*Cosh[e + f*x]*Sinh[e + f*x])/(a*(a - b)*f*Sqrt[a + b*Sinh[e + f*x]^2]
)) - (I*EllipticE[I*e + I*f*x, b/a]*Sqrt[a + b*Sinh[e + f*x]^2])/(a*(a - b
)*f*Sqrt[1 + (b*Sinh[e + f*x]^2)/a])
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3656
```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2], x_Symbol] :> Simp[(Sqrt[a
]/f)*EllipticE[e + f*x, -b/a], x] /; FreeQ[{a, b, e, f}, x] && GtQ[a, 0]
```

rule 3657
```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2], x_Symbol] :> Simp[Sqrt[a
+ b*Sin[e + f*x]^2]/Sqrt[1 + b*(Sin[e + f*x]^2/a)]   Int[Sqrt[1 + (b*Sin[e
+ f*x]^2)/a], x], x] /; FreeQ[{a, b, e, f}, x] &&  !GtQ[a, 0]
```

rule 3663
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> Simp[(-b)*C
os[e + f*x]*Sin[e + f*x]*((a + b*Sin[e + f*x]^2)^(p + 1)/(2*a*f*(p + 1)*(a
+ b))), x] + Simp[1/(2*a*(p + 1)*(a + b))   Int[(a + b*Sin[e + f*x]^2)^(p +
 1)*Simp[2*a*(p + 1) + b*(2*p + 3) - 2*b*(p + 2)*Sin[e + f*x]^2, x], x], x]
 /; FreeQ[{a, b, e, f}, x] && NeQ[a + b, 0] && LtQ[p, -1]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 251 vs. $2(110) = 220$.

Time = 1.04 (sec) , antiderivative size = 252, normalized size of antiderivative = 2.17

method	result
default	$\frac{-\sqrt{-\frac{b}{a}}\, b \cosh(fx+e)^2 \sinh(fx+e)+a\sqrt{\frac{b\cosh(fx+e)^2}{a}+\frac{a-b}{a}}\, \sqrt{\frac{\cosh(2fx+2e)}{2}+\frac{1}{2}}\, \mathrm{EllipticF}\left(\sinh(fx+e)\sqrt{-\frac{b}{a}}, \sqrt{\frac{a}{b}}\right)-\sqrt{\frac{b\cosh(fx+e}{a}}}{a(a-b)\sqrt{-\frac{b}{a}}\, \cos}$

input
```
int(1/(a+b*sinh(f*x+e)^2)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
(-(-b/a)^(1/2)*b*cosh(f*x+e)^2*sinh(f*x+e)+a*(b/a*cosh(f*x+e)^2+(a-b)/a)^(
1/2)*(cosh(f*x+e)^2)^(1/2)*EllipticF(sinh(f*x+e)*(-b/a)^(1/2),(1/b*a)^(1/2
))-(b/a*cosh(f*x+e)^2+(a-b)/a)^(1/2)*(cosh(f*x+e)^2)^(1/2)*EllipticF(sinh(
f*x+e)*(-b/a)^(1/2),(1/b*a)^(1/2))*b+(b/a*cosh(f*x+e)^2+(a-b)/a)^(1/2)*(co
sh(f*x+e)^2)^(1/2)*EllipticE(sinh(f*x+e)*(-b/a)^(1/2),(1/b*a)^(1/2))*b)/a/
(a-b)/(-b/a)^(1/2)/cosh(f*x+e)/(a+b*sinh(f*x+e)^2)^(1/2)/f
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1464 vs. $2(106) = 212$.

Time = 0.13 (sec) , antiderivative size = 1464, normalized size of antiderivative = 12.62

$$\int \frac{1}{\left(a+b\sinh^2(e+fx)\right)^{3/2}}\, dx = \text{Too large to display}$$

input
```
integrate(1/(a+b*sinh(f*x+e)^2)^(3/2),x, algorithm="fricas")
```

output
```
(((2*a*b^2 - b^3)*cosh(f*x + e)^4 + 4*(2*a*b^2 - b^3)*cosh(f*x + e)*sinh(f
*x + e)^3 + (2*a*b^2 - b^3)*sinh(f*x + e)^4 + 2*a*b^2 - b^3 + 2*(4*a^2*b -
 4*a*b^2 + b^3)*cosh(f*x + e)^2 + 2*(4*a^2*b - 4*a*b^2 + b^3 + 3*(2*a*b^2
- b^3)*cosh(f*x + e)^2)*sinh(f*x + e)^2 + 4*((2*a*b^2 - b^3)*cosh(f*x + e)
^3 + (4*a^2*b - 4*a*b^2 + b^3)*cosh(f*x + e))*sinh(f*x + e) - 2*(b^3*cosh(
f*x + e)^4 + 4*b^3*cosh(f*x + e)*sinh(f*x + e)^3 + b^3*sinh(f*x + e)^4 + b
^3 + 2*(2*a*b^2 - b^3)*cosh(f*x + e)^2 + 2*(3*b^3*cosh(f*x + e)^2 + 2*a*b^
2 - b^3)*sinh(f*x + e)^2 + 4*(b^3*cosh(f*x + e)^3 + (2*a*b^2 - b^3)*cosh(f
*x + e))*sinh(f*x + e))*sqrt((a^2 - a*b)/b^2))*sqrt(b)*sqrt((2*b*sqrt((a^2
 - a*b)/b^2) - 2*a + b)/b)*elliptic_e(arcsin(sqrt((2*b*sqrt((a^2 - a*b)/b^
2) - 2*a + b)/b)*(cosh(f*x + e) + sinh(f*x + e))), (8*a^2 - 8*a*b + b^2 +
4*(2*a*b - b^2)*sqrt((a^2 - a*b)/b^2))/b^2) - 2*((2*a^2*b - a*b^2)*cosh(f*
x + e)^4 + 4*(2*a^2*b - a*b^2)*cosh(f*x + e)*sinh(f*x + e)^3 + (2*a^2*b -
a*b^2)*sinh(f*x + e)^4 + 2*a^2*b - a*b^2 + 2*(4*a^3 - 4*a^2*b + a*b^2)*cos
h(f*x + e)^2 + 2*(4*a^3 - 4*a^2*b + a*b^2 + 3*(2*a^2*b - a*b^2)*cosh(f*x +
 e)^2)*sinh(f*x + e)^2 + 4*((2*a^2*b - a*b^2)*cosh(f*x + e)^3 + (4*a^3 - 4
*a^2*b + a*b^2)*cosh(f*x + e))*sinh(f*x + e) + 2*((a*b^2 - b^3)*cosh(f*x +
 e)^4 + 4*(a*b^2 - b^3)*cosh(f*x + e)*sinh(f*x + e)^3 + (a*b^2 - b^3)*sinh
(f*x + e)^4 + a*b^2 - b^3 + 2*(2*a^2*b - 3*a*b^2 + b^3)*cosh(f*x + e)^2 +
2*(2*a^2*b - 3*a*b^2 + b^3 + 3*(a*b^2 - b^3)*cosh(f*x + e)^2)*sinh(f*x ...
```

Sympy [F]

$$\int \frac{1}{\left(a+b\sinh^2(e+fx)\right)^{3/2}}\,dx=\int \frac{1}{\left(a+b\sinh^{2}\left(e+fx\right)\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/(a+b*sinh(f*x+e)**2)**(3/2),x)
```

output
```
Integral((a + b*sinh(e + f*x)**2)**(-3/2), x)
```

Maxima [F]

$$\int \frac{1}{\left(a+b\sinh^2(e+fx)\right)^{3/2}}\,dx=\int \frac{1}{\left(b\sinh\left(fx+e\right)^2+a\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/(a+b*sinh(f*x+e)^2)^(3/2),x, algorithm="maxima")
```

output
```
integrate((b*sinh(f*x + e)^2 + a)^(-3/2), x)
```

Giac [F]

$$\int \frac{1}{\left(a+b\sinh^2(e+fx)\right)^{3/2}}\,dx=\int \frac{1}{\left(b\sinh\left(fx+e\right)^2+a\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/(a+b*sinh(f*x+e)^2)^(3/2),x, algorithm="giac")
```

output
```
integrate((b*sinh(f*x + e)^2 + a)^(-3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\left(a+b\sinh^2(e+fx)\right)^{3/2}}\,dx=\int \frac{1}{\left(b\sinh\left(e+f\,x\right)^2+a\right)^{3/2}}\,dx$$

input
```
int(1/(a + b*sinh(e + f*x)^2)^(3/2),x)
```

output
```
int(1/(a + b*sinh(e + f*x)^2)^(3/2), x)
```

Reduce [F]

$$\int \frac{1}{\left(a+b\sinh^2(e+fx)\right)^{3/2}}\,dx=\int \frac{\sqrt{\sinh\left(fx+e\right)^{2}b+a}}{\sinh\left(fx+e\right)^{4}b^{2}+2\sinh\left(fx+e\right)^{2}ab+a^{2}}dx$$

input
```
int(1/(a+b*sinh(f*x+e)^2)^(3/2),x)
```

output
```
int(sqrt(sinh(e + f*x)**2*b + a)/(sinh(e + f*x)**4*b**2 + 2*sinh(e + f*x)*
*2*a*b + a**2),x)
```

3.463 $\int \frac{\coth^2(e+fx)}{(a+b\sinh^2(e+fx))^{3/2}}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 213

$$\begin{aligned}\int \frac{\coth^2(e+fx)}{(a+b\sinh^2(e+fx))^{3/2}}\,dx &= \frac{\coth(e+fx)}{af\sqrt{a+b\sinh^2(e+fx)}} \\ &- \frac{2\text{csch}(e+fx)\text{sech}(e+fx)\sqrt{a+b\sinh^2(e+fx)}}{a^2f} \\ &- \frac{2E\left(\arctan(\sinh(e+fx))\left|1-\frac{b}{a}\right.\right)\text{sech}(e+fx)\sqrt{a+b\sinh^2(e+fx)}}{a^2f\sqrt{\frac{\text{sech}^2(e+fx)(a+b\sinh^2(e+fx))}{a}}} \\ &+ \frac{\text{EllipticF}\left(\arctan(\sinh(e+fx)),1-\frac{b}{a}\right)\text{sech}(e+fx)\sqrt{a+b\sinh^2(e+fx)}}{a^2f\sqrt{\frac{\text{sech}^2(e+fx)(a+b\sinh^2(e+fx))}{a}}}\end{aligned}$$

output
```
coth(f*x+e)/a/f/(a+b*sinh(f*x+e)^2)^(1/2)-2*csch(f*x+e)*sech(f*x+e)*(a+b*s
inh(f*x+e)^2)^(1/2)/a^2/f-2*EllipticE(sinh(f*x+e)/(1+sinh(f*x+e)^2)^(1/2),
(1-b/a)^(1/2))*sech(f*x+e)*(a+b*sinh(f*x+e)^2)^(1/2)/a^2/f/(sech(f*x+e)^2*
(a+b*sinh(f*x+e)^2)/a)^(1/2)+InverseJacobiAM(arctan(sinh(f*x+e)),(1-b/a)^(
1/2))*sech(f*x+e)*(a+b*sinh(f*x+e)^2)^(1/2)/a^2/f/(sech(f*x+e)^2*(a+b*sinh
(f*x+e)^2)/a)^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.90 (sec) , antiderivative size = 153, normalized size of antiderivative = 0.72

$$\int \frac{\coth^2(e+fx)}{\left(a+b\sinh^2(e+fx)\right)^{3/2}}\,dx = \frac{-2(a-b+b\cosh(2(e+fx)))\coth(e+fx)-2i\sqrt{2}a\sqrt{\frac{2a-b+b\cosh(2(e+fx))}{a}}}{a^2f\sqrt{4a-2b+2b\,c}}$$

input
```
Integrate[Coth[e + f*x]^2/(a + b*Sinh[e + f*x]^2)^(3/2),x]
```

output
```
(-2*(a - b + b*Cosh[2*(e + f*x)])*Coth[e + f*x] - (2*I)*Sqrt[2]*a*Sqrt[(2*
a - b + b*Cosh[2*(e + f*x)])/a]*EllipticE[I*(e + f*x), b/a] + I*Sqrt[2]*a*
Sqrt[(2*a - b + b*Cosh[2*(e + f*x)])/a]*EllipticF[I*(e + f*x), b/a])/(a^2*
f*Sqrt[4*a - 2*b + 2*b*Cosh[2*(e + f*x)]])
```

Rubi [A] (verified)

Time = 0.57 (sec) , antiderivative size = 320, normalized size of antiderivative = 1.50, number of steps used = 12, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.440, Rules used = {3042, 25, 3675, 371, 25, 445, 25, 406, 320, 388, 313}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\coth^2(e+fx)}{\left(a+b\sinh^2(e+fx)\right)^{3/2}}dx$$

$$\Big\downarrow \; 3042$$

$$\int -\frac{1}{\tan(ie+ifx)^2\left(a-b\sin(ie+ifx)^2\right)^{3/2}}dx$$

$$\Big\downarrow \; 25$$

$$-\int \frac{1}{\left(a-b\sin(ie+ifx)^2\right)^{3/2}\tan(ie+ifx)^2}dx$$

$$\Big\downarrow \; 3675$$

$$\frac{\sqrt{\cosh^2(e+fx)}\text{sech}(e+fx)\int \frac{\text{csch}^2(e+fx)\sqrt{\sinh^2(e+fx)+1}}{\left(b\sinh^2(e+fx)+a\right)^{3/2}}d\sinh(e+fx)}{f}$$

$$\Big\downarrow \; 371$$

$$\frac{\sqrt{\cosh^2(e+fx)}\text{sech}(e+fx)\left(\frac{\sqrt{\sinh^2(e+fx)+1}\text{csch}(e+fx)}{a\sqrt{a+b\sinh^2(e+fx)}}-\frac{\int -\frac{\text{csch}^2(e+fx)\left(\sinh^2(e+fx)+2\right)}{\sqrt{\sinh^2(e+fx)+1}\sqrt{b\sinh^2(e+fx)+a}}d\sinh(e+fx)}{a}\right)}{f}$$

$$\Big\downarrow \; 25$$

$$\frac{\sqrt{\cosh^2(e+fx)}\text{sech}(e+fx)\left(\frac{\int \frac{\text{csch}^2(e+fx)\left(\sinh^2(e+fx)+2\right)}{\sqrt{\sinh^2(e+fx)+1}\sqrt{b\sinh^2(e+fx)+a}}d\sinh(e+fx)}{a}+\frac{\sqrt{\sinh^2(e+fx)+1}\text{csch}(e+fx)}{a\sqrt{a+b\sinh^2(e+fx)}}\right)}{f}$$

$$\Big\downarrow \; 445$$

$$\frac{\sqrt{\cosh^2(e+fx)}\text{sech}(e+fx)\left(\frac{-\frac{\int -\frac{2b\sinh^2(e+fx)+a}{\sqrt{\sinh^2(e+fx)+1}\sqrt{b\sinh^2(e+fx)+a}}d\sinh(e+fx)}{a}-\frac{2\sqrt{\sinh^2(e+fx)+1}\text{csch}(e+fx)\sqrt{a+b\sinh^2(e+fx)}}{a}}{a}+\cdots\right.}{f}$$

$$\Big\downarrow \; 25$$

$$\frac{\sqrt{\cosh^2(e+fx)}\text{sech}(e+fx)\left(\frac{\frac{\int \frac{2b\sinh^2(e+fx)+a}{\sqrt{\sinh^2(e+fx)+1}\sqrt{b\sinh^2(e+fx)+a}}d\sinh(e+fx)}{a}-\frac{2\sqrt{\sinh^2(e+fx)+1}\text{csch}(e+fx)\sqrt{a+b\sinh^2(e+fx)}}{a}}{a}+\frac{\sqrt{\text{s}\cdots}}{\cdots}\right.}{f}$$

$$\Big\downarrow \; 406$$

$$\frac{\sqrt{\cosh^2(e+fx)}\text{sech}(e+fx)\left(\frac{\frac{a\int \frac{1}{\sqrt{\sinh^2(e+fx)+1}\sqrt{b\sinh^2(e+fx)+a}}d\sinh(e+fx)+2b\int \frac{\sinh^2(e+fx)}{\sqrt{\sinh^2(e+fx)+1}\sqrt{b\sinh^2(e+fx)+a}}d\sinh(e+fx)}{a}-\frac{2\cdots}{\cdots}}{a}\cdots\right.}{f}$$

$$\Big\downarrow \; 320$$

$$\frac{\sqrt{\cosh^2(e+fx)}\operatorname{sech}(e+fx)\left(\frac{\frac{2b\int \frac{\sinh^2(e+fx)}{\sqrt{\sinh^2(e+fx)+1}\sqrt{b\sinh^2(e+fx)+a}}d\sinh(e+fx)+\frac{\sqrt{a+b\sinh^2(e+fx)}\,\text{EllipticF}\left(\arctan(\sinh(e+fx)),1-\frac{b}{a}\right)}{\sqrt{\sinh^2(e+fx)+1}\sqrt{\frac{a+b\sinh^2(e+fx)}{a\left(\sinh^2(e+fx)+1\right)}}}}{a} -}{a}\right.}{f}$$

$\big\downarrow$ 388

$$\frac{\sqrt{\cosh^2(e+fx)}\operatorname{sech}(e+fx)\left(\frac{\frac{2b\left(\frac{\sinh(e+fx)\sqrt{a+b\sinh^2(e+fx)}}{b\sqrt{\sinh^2(e+fx)+1}}-\frac{\int\frac{\sqrt{b\sinh^2(e+fx)+a}}{\left(\sinh^2(e+fx)+1\right)^{3/2}}d\sinh(e+fx)}{b}\right)+\frac{\sqrt{a+b\sinh^2(e+fx)}\,\text{EllipticF}\left(\arctan\right.}{\sqrt{\sinh^2(e+fx)+1}\sqrt{\frac{a+b\sin}{a\left(\sinh^2\right.}}}}{a}}{a}\right.}{f}$$

$\big\downarrow$ 313

$$\frac{\sqrt{\cosh^2(e+fx)}\operatorname{sech}(e+fx)\left(\frac{\frac{\frac{\sqrt{a+b\sinh^2(e+fx)}\,\text{EllipticF}\left(\arctan(\sinh(e+fx)),1-\frac{b}{a}\right)}{\sqrt{\sinh^2(e+fx)+1}\sqrt{\frac{a+b\sinh^2(e+fx)}{a\left(\sinh^2(e+fx)+1\right)}}}+2b\left(\frac{\sinh(e+fx)\sqrt{a+b\sinh^2(e+fx)}}{b\sqrt{\sinh^2(e+fx)+1}}-\frac{\sqrt{a+b\sinh^2(e+fx)}}{b\sqrt{\sinh^2(e+fx}}\right.}{a}}{a}\right.}{f}$$

input

```
Int[Coth[e + f*x]^2/(a + b*Sinh[e + f*x]^2)^(3/2),x]
```

output
```
(Sqrt[Cosh[e + f*x]^2]*Sech[e + f*x]*((Csch[e + f*x]*Sqrt[1 + Sinh[e + f*x
]^2])/(a*Sqrt[a + b*Sinh[e + f*x]^2]) + ((-2*Csch[e + f*x]*Sqrt[1 + Sinh[e
 + f*x]^2]*Sqrt[a + b*Sinh[e + f*x]^2])/a + ((EllipticF[ArcTan[Sinh[e + f*
x]], 1 - b/a]*Sqrt[a + b*Sinh[e + f*x]^2])/(Sqrt[1 + Sinh[e + f*x]^2]*Sqrt
[(a + b*Sinh[e + f*x]^2)/(a*(1 + Sinh[e + f*x]^2))]) + 2*b*((Sinh[e + f*x]
*Sqrt[a + b*Sinh[e + f*x]^2])/(b*Sqrt[1 + Sinh[e + f*x]^2]) - (EllipticE[A
rcTan[Sinh[e + f*x]], 1 - b/a]*Sqrt[a + b*Sinh[e + f*x]^2])/(b*Sqrt[1 + Si
nh[e + f*x]^2]*Sqrt[(a + b*Sinh[e + f*x]^2)/(a*(1 + Sinh[e + f*x]^2))])))/
a)/a))/f
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 313
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/((c_) + (d_.)*(x_)^2)^(3/2), x_Symbol] :> Sim
p[(Sqrt[a + b*x^2]/(c*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(c
+ d*x^2)))]))*EllipticE[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; FreeQ
[{a, b, c, d}, x] && PosQ[b/a] && PosQ[d/c]
```

rule 320
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(Sqrt[a + b*x^2]/(a*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(
c + d*x^2)))]))*EllipticF[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; Fre
eQ[{a, b, c, d}, x] && PosQ[d/c] && PosQ[b/a] &&  !SimplerSqrtQ[b/a, d/c]
```

rule 371
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_
), x_Symbol] :> Simp[(-(e*x)^(m + 1))*(a + b*x^2)^(p + 1)*((c + d*x^2)^q/(a
*e*2*(p + 1))), x] + Simp[1/(a*2*(p + 1))   Int[(e*x)^m*(a + b*x^2)^(p + 1)
*(c + d*x^2)^(q - 1)*Simp[c*(m + 2*(p + 1) + 1) + d*(m + 2*(p + q + 1) + 1)
*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b*c - a*d, 0] && Lt
Q[p, -1] && LtQ[0, q, 1] && IntBinomialQ[a, b, c, d, e, m, 2, p, q, x]
```

rule 388
```
Int[(x_)^2/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol]
 :> Simp[x*(Sqrt[a + b*x^2]/(b*Sqrt[c + d*x^2])), x] - Simp[c/b   Int[Sqrt[
a + b*x^2]/(c + d*x^2)^(3/2), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
 a*d, 0] && PosQ[b/a] && PosQ[d/c] &&  !SimplerSqrtQ[b/a, d/c]
```

rule 406
```
Int[((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.)*((e_) + (f_.)*(
x_)^2), x_Symbol] :> Simp[e   Int[(a + b*x^2)^p*(c + d*x^2)^q, x], x] + Sim
p[f   Int[x^2*(a + b*x^2)^p*(c + d*x^2)^q, x], x] /; FreeQ[{a, b, c, d, e,
f, p, q}, x]
```

rule 445
```
Int[((g_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_
.)*((e_) + (f_.)*(x_)^2), x_Symbol] :> Simp[e*(g*x)^(m + 1)*(a + b*x^2)^(p
+ 1)*((c + d*x^2)^(q + 1)/(a*c*g*(m + 1))), x] + Simp[1/(a*c*g^2*(m + 1))
 Int[(g*x)^(m + 2)*(a + b*x^2)^p*(c + d*x^2)^q*Simp[a*f*c*(m + 1) - e*(b*c
+ a*d)*(m + 2 + 1) - e*2*(b*c*p + a*d*q) - b*e*d*(m + 2*(p + q + 2) + 1)*x^
2, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p, q}, x] && LtQ[m, -1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3675
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_.)*tan[(e_.) + (f_.)*(x_)]^
(m_), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Simp[ff^(m + 1
)*(Sqrt[Cos[e + f*x]^2]/(f*Cos[e + f*x]))   Subst[Int[x^m*((a + b*ff^2*x^2)
^p/(1 - ff^2*x^2)^((m + 1)/2)), x], x, Sin[e + f*x]/ff], x]] /; FreeQ[{a, b
, e, f, p}, x] && IntegerQ[m/2] &&  !IntegerQ[p]
```

Maple [A] (verified)

Time = 4.13 (sec) , antiderivative size = 218, normalized size of antiderivative = 1.02

method	result
default	$\frac{-2\sqrt{-\frac{b}{a}}\,b\cosh(fx+e)^4+\left(-\sqrt{-\frac{b}{a}}\,a+2\sqrt{-\frac{b}{a}}\,b\right)\cosh(fx+e)^2+\sinh(fx+e)\sqrt{\frac{\cosh(2fx+2e)}{2}+\frac{1}{2}}\,\sqrt{\frac{b\cosh(fx+e)^2}{a}+\frac{a-b}{a}}\,\left(a\,\mathrm{EllipticF}\right.}{\sqrt{-\frac{b}{a}}\,\sinh(fx+e)a^2\cosh(fx+e)\sqrt{a+b}}$
risch	Expression too large to display

input
```
int(coth(f*x+e)^2/(a+b*sinh(f*x+e)^2)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
(-2*(-b/a)^(1/2)*b*cosh(f*x+e)^4+(-(-b/a)^(1/2)*a+2*(-b/a)^(1/2)*b)*cosh(f
*x+e)^2+sinh(f*x+e)*(cosh(f*x+e)^2)^(1/2)*(b/a*cosh(f*x+e)^2+(a-b)/a)^(1/2
)*(a*EllipticF(sinh(f*x+e)*(-b/a)^(1/2),(1/b*a)^(1/2))-2*b*EllipticF(sinh(
f*x+e)*(-b/a)^(1/2),(1/b*a)^(1/2))+2*b*EllipticE(sinh(f*x+e)*(-b/a)^(1/2),
(1/b*a)^(1/2))))/(-b/a)^(1/2)/sinh(f*x+e)/a^2/cosh(f*x+e)/(a+b*sinh(f*x+e)
^2)^(1/2)/f
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2436 vs. $2(217) = 434$.

Time = 0.12 (sec) , antiderivative size = 2436, normalized size of antiderivative = 11.44

$$\int \frac{\coth^2(e+fx)}{\left(a+b\sinh^2(e+fx)\right)^{3/2}}\,dx = \text{Too large to display}$$

input
```
integrate(coth(f*x+e)^2/(a+b*sinh(f*x+e)^2)^(3/2),x, algorithm="fricas")
```

output
```
2*(((2*a*b^2 - b^3)*cosh(f*x + e)^6 + 6*(2*a*b^2 - b^3)*cosh(f*x + e)*sinh
(f*x + e)^5 + (2*a*b^2 - b^3)*sinh(f*x + e)^6 + (8*a^2*b - 10*a*b^2 + 3*b^
3)*cosh(f*x + e)^4 + (8*a^2*b - 10*a*b^2 + 3*b^3 + 15*(2*a*b^2 - b^3)*cosh
(f*x + e)^2)*sinh(f*x + e)^4 + 4*(5*(2*a*b^2 - b^3)*cosh(f*x + e)^3 + (8*a
^2*b - 10*a*b^2 + 3*b^3)*cosh(f*x + e))*sinh(f*x + e)^3 - 2*a*b^2 + b^3 -
(8*a^2*b - 10*a*b^2 + 3*b^3)*cosh(f*x + e)^2 + (15*(2*a*b^2 - b^3)*cosh(f*
x + e)^4 - 8*a^2*b + 10*a*b^2 - 3*b^3 + 6*(8*a^2*b - 10*a*b^2 + 3*b^3)*cos
h(f*x + e)^2)*sinh(f*x + e)^2 + 2*(3*(2*a*b^2 - b^3)*cosh(f*x + e)^5 + 2*(
8*a^2*b - 10*a*b^2 + 3*b^3)*cosh(f*x + e)^3 - (8*a^2*b - 10*a*b^2 + 3*b^3)
*cosh(f*x + e))*sinh(f*x + e) - 2*(b^3*cosh(f*x + e)^6 + 6*b^3*cosh(f*x +
e)*sinh(f*x + e)^5 + b^3*sinh(f*x + e)^6 + (4*a*b^2 - 3*b^3)*cosh(f*x + e)
^4 + (15*b^3*cosh(f*x + e)^2 + 4*a*b^2 - 3*b^3)*sinh(f*x + e)^4 + 4*(5*b^3
*cosh(f*x + e)^3 + (4*a*b^2 - 3*b^3)*cosh(f*x + e))*sinh(f*x + e)^3 - b^3
- (4*a*b^2 - 3*b^3)*cosh(f*x + e)^2 + (15*b^3*cosh(f*x + e)^4 - 4*a*b^2 +
3*b^3 + 6*(4*a*b^2 - 3*b^3)*cosh(f*x + e)^2)*sinh(f*x + e)^2 + 2*(3*b^3*co
sh(f*x + e)^5 + 2*(4*a*b^2 - 3*b^3)*cosh(f*x + e)^3 - (4*a*b^2 - 3*b^3)*co
sh(f*x + e))*sinh(f*x + e))*sqrt((a^2 - a*b)/b^2))*sqrt(b)*sqrt((2*b*sqrt(
(a^2 - a*b)/b^2) - 2*a + b)/b)*elliptic_e(arcsin(sqrt((2*b*sqrt((a^2 - a*b
)/b^2) - 2*a + b)/b)*(cosh(f*x + e) + sinh(f*x + e))), (8*a^2 - 8*a*b + b^
2 + 4*(2*a*b - b^2)*sqrt((a^2 - a*b)/b^2))/b^2) - ((2*a^2*b - a*b^2)*co...
```

Sympy [F]

$$\int \frac{\coth^2(e+fx)}{\left(a+b\sinh^2(e+fx)\right)^{3/2}}\,dx=\int \frac{\coth^2{\left(e+fx\right)}}{\left(a+b\sinh^2{\left(e+fx\right)}\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate(coth(f*x+e)**2/(a+b*sinh(f*x+e)**2)**(3/2),x)
```

output
```
Integral(coth(e + f*x)**2/(a + b*sinh(e + f*x)**2)**(3/2), x)
```

Maxima [F]

$$\int \frac{\coth^2(e+fx)}{(a+b\sinh^2(e+fx))^{3/2}}\,dx = \int \frac{\coth(fx+e)^2}{(b\sinh(fx+e)^2+a)^{\frac{3}{2}}}\,dx$$

input
```
integrate(coth(f*x+e)^2/(a+b*sinh(f*x+e)^2)^(3/2),x, algorithm="maxima")
```

output
```
integrate(coth(f*x + e)^2/(b*sinh(f*x + e)^2 + a)^(3/2), x)
```

Giac [F]

$$\int \frac{\coth^2(e+fx)}{(a+b\sinh^2(e+fx))^{3/2}}\,dx = \int \frac{\coth(fx+e)^2}{(b\sinh(fx+e)^2+a)^{\frac{3}{2}}}\,dx$$

input
```
integrate(coth(f*x+e)^2/(a+b*sinh(f*x+e)^2)^(3/2),x, algorithm="giac")
```

output
```
integrate(coth(f*x + e)^2/(b*sinh(f*x + e)^2 + a)^(3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\coth^2(e+fx)}{(a+b\sinh^2(e+fx))^{3/2}}\,dx = \int \frac{\coth(e+f\,x)^2}{(b\sinh(e+f\,x)^2+a)^{3/2}}\,dx$$

input
```
int(coth(e + f*x)^2/(a + b*sinh(e + f*x)^2)^(3/2),x)
```

output
```
int(coth(e + f*x)^2/(a + b*sinh(e + f*x)^2)^(3/2), x)
```

Reduce [F]

$$\int \frac{\coth^2(e+fx)}{\left(a+b\sinh^2(e+fx)\right)^{3/2}}\,dx = \int \frac{\sqrt{\sinh\left(fx+e\right)^2 b+a}\,\coth\left(fx+e\right)^2}{\sinh\left(fx+e\right)^4 b^2+2\sinh\left(fx+e\right)^2 ab+a^2}dx$$

input
```
int(coth(f*x+e)^2/(a+b*sinh(f*x+e)^2)^(3/2),x)
```

output
```
int((sqrt(sinh(e + f*x)**2*b + a)*coth(e + f*x)**2)/(sinh(e + f*x)**4*b**2
 + 2*sinh(e + f*x)**2*a*b + a**2),x)
```

3.464 $\int \frac{\coth^4(e+fx)}{(a+b\sinh^2(e+fx))^{3/2}}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 308

$$\int \frac{\coth^4(e+fx)}{\left(a+b\sinh^2(e+fx)\right)^{3/2}}\,dx = -\frac{(a-b)\coth(e+fx)\operatorname{csch}^2(e+fx)}{abf\sqrt{a+b\sinh^2(e+fx)}}$$
$$+\frac{(3a-4b)\coth(e+fx)\operatorname{csch}^2(e+fx)\sqrt{a+b\sinh^2(e+fx)}}{3a^2bf}$$
$$-\frac{(7a-8b)\operatorname{csch}(e+fx)\operatorname{sech}(e+fx)\sqrt{a+b\sinh^2(e+fx)}}{3a^3f}$$
$$-\frac{(7a-8b)E\left(\arctan(\sinh(e+fx))\left|1-\frac{b}{a}\right.\right)\operatorname{sech}(e+fx)\sqrt{a+b\sinh^2(e+fx)}}{3a^3f\sqrt{\frac{\operatorname{sech}^2(e+fx)(a+b\sinh^2(e+fx))}{a}}}$$
$$+\frac{(3a-4b)\operatorname{EllipticF}\left(\arctan(\sinh(e+fx)),1-\frac{b}{a}\right)\operatorname{sech}(e+fx)\sqrt{a+b\sinh^2(e+fx)}}{3a^3f\sqrt{\frac{\operatorname{sech}^2(e+fx)(a+b\sinh^2(e+fx))}{a}}}$$

output
```
-(a-b)*coth(f*x+e)*csch(f*x+e)^2/a/b/f/(a+b*sinh(f*x+e)^2)^(1/2)+1/3*(3*a-
4*b)*coth(f*x+e)*csch(f*x+e)^2*(a+b*sinh(f*x+e)^2)^(1/2)/a^2/b/f-1/3*(7*a-
8*b)*csch(f*x+e)*sech(f*x+e)*(a+b*sinh(f*x+e)^2)^(1/2)/a^3/f-1/3*(7*a-8*b)
*EllipticE(sinh(f*x+e)/(1+sinh(f*x+e)^2)^(1/2),(1-b/a)^(1/2))*sech(f*x+e)*
(a+b*sinh(f*x+e)^2)^(1/2)/a^3/f/(sech(f*x+e)^2*(a+b*sinh(f*x+e)^2)/a)^(1/2
)+1/3*(3*a-4*b)*InverseJacobiAM(arctan(sinh(f*x+e)),(1-b/a)^(1/2))*sech(f*
x+e)*(a+b*sinh(f*x+e)^2)^(1/2)/a^3/f/(sech(f*x+e)^2*(a+b*sinh(f*x+e)^2)/a)
^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 2.47 (sec) , antiderivative size = 214, normalized size of antiderivative = 0.69

$$\int \frac{\coth^4(e+fx)}{\left(a+b\sinh^2(e+fx)\right)^{3/2}}\,dx = \frac{-\frac{\left(-8a^2+37ab-24b^2+4\left(4a^2-11ab+8b^2\right)\cosh(2(e+fx))+(7a-8b)b\cosh(4(e+fx))\right)\coth(e+fx)\text{cs}}{2\sqrt{2}}}{}$$

input
```
Integrate[Coth[e + f*x]^4/(a + b*Sinh[e + f*x]^2)^(3/2),x]
```

output
```
(-1/2*((-8*a^2 + 37*a*b - 24*b^2 + 4*(4*a^2 - 11*a*b + 8*b^2)*Cosh[2*(e +
f*x)] + (7*a - 8*b)*b*Cosh[4*(e + f*x)])*Coth[e + f*x]*Csch[e + f*x]^2)/Sq
rt[2] - (2*I)*a*(7*a - 8*b)*Sqrt[(2*a - b + b*Cosh[2*(e + f*x)])/a]*Ellipt
icE[I*(e + f*x), b/a] + (8*I)*a*(a - b)*Sqrt[(2*a - b + b*Cosh[2*(e + f*x)
])/a]*EllipticF[I*(e + f*x), b/a])/(6*a^3*f*Sqrt[2*a - b + b*Cosh[2*(e + f
*x)]])
```

Rubi [A] (verified)

Time = 0.73 (sec) , antiderivative size = 416, normalized size of antiderivative = 1.35, number of steps used = 12, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.440$, Rules used = {3042, 3675, 370, 445, 27, 445, 25, 406, 320, 388, 313}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\coth^4(e+fx)}{\left(a+b\sinh^2(e+fx)\right)^{3/2}}dx$$

$\Big\downarrow$ 3042

$$\int \frac{1}{\tan(ie+ifx)^4\left(a-b\sin(ie+ifx)^2\right)^{3/2}}dx$$

$\Big\downarrow$ 3675

$$\frac{\sqrt{\cosh^2(e+fx)}\text{sech}(e+fx)\int \frac{\text{csch}^4(e+fx)\left(\sinh^2(e+fx)+1\right)^{3/2}}{\left(b\sinh^2(e+fx)+a\right)^{3/2}}d\sinh(e+fx)}{f}$$

$\Big\downarrow$ 370

$$\frac{\sqrt{\cosh^2(e+fx)}\text{sech}(e+fx)\left(-\frac{\int \frac{\text{csch}^4(e+fx)\left((2a-3b)\sinh^2(e+fx)+3a-4b\right)}{\sqrt{\sinh^2(e+fx)+1}\sqrt{b\sinh^2(e+fx)+a}}d\sinh(e+fx)}{ab}-\frac{(a-b)\sqrt{\sinh^2(e+fx)+1}\text{csch}^3(e+fx)}{ab\sqrt{a+b\sinh^2(e+fx)}}\right.}{f}$$

$\Big\downarrow$ 445

$$\frac{\sqrt{\cosh^2(e+fx)}\text{sech}(e+fx)\left(-\frac{-\frac{\int \frac{b\text{csch}^2(e+fx)\left((3a-4b)\sinh^2(e+fx)+7a-8b\right)}{\sqrt{\sinh^2(e+fx)+1}\sqrt{b\sinh^2(e+fx)+a}}d\sinh(e+fx)}{3a}-\frac{(3a-4b)\sqrt{\sinh^2(e+fx)+1}\text{csch}^3(e+fx)\sqrt{a-}}{3a}}{ab}\right.}{f}$$

$\Big\downarrow$ 27

$$\frac{\sqrt{\cosh^2(e+fx)}\text{sech}(e+fx)\left(-\frac{-\frac{b\int \frac{\text{csch}^2(e+fx)\left((3a-4b)\sinh^2(e+fx)+7a-8b\right)}{\sqrt{\sinh^2(e+fx)+1}\sqrt{b\sinh^2(e+fx)+a}}d\sinh(e+fx)}{3a}-\frac{(3a-4b)\sqrt{\sinh^2(e+fx)+1}\text{csch}^3(e+fx)\sqrt{a-}}{3a}}{ab}\right.}{f}$$

$\Big\downarrow$ 445

$$\frac{\sqrt{\cosh^2(e+fx)}\operatorname{sech}(e+fx)\left(-\frac{-\frac{b\left(-\frac{\int -\frac{(7a-8b)b\sinh^2(e+fx)+a(3a-4b)}{\sqrt{\sinh^2(e+fx)+1}\sqrt{b\sinh^2(e+fx)+a}}d\sinh(e+fx)}{a}-\frac{(7a-8b)\sqrt{\sinh^2(e+fx)+1}\operatorname{csch}(e+fx)\sqrt{a+b\sinh^2\cdots}}{a}\right.}{3a}}{ab}\right.}{f}$$

$\downarrow$ 25

$$\frac{\sqrt{\cosh^2(e+fx)}\operatorname{sech}(e+fx)\left(-\frac{-\frac{b\left(\frac{\int \frac{(7a-8b)b\sinh^2(e+fx)+a(3a-4b)}{\sqrt{\sinh^2(e+fx)+1}\sqrt{b\sinh^2(e+fx)+a}}d\sinh(e+fx)}{a}-\frac{(7a-8b)\sqrt{\sinh^2(e+fx)+1}\operatorname{csch}(e+fx)\sqrt{a+b\sinh^2(\cdots}}{a}\right.}{3a}}{ab}\right.}{f}$$

$\downarrow$ 406

$$\sqrt{\cosh^2(e+fx)}\operatorname{sech}(e+fx)\left(-\frac{-\frac{b\left(\frac{a(3a-4b)\int \frac{1}{\sqrt{\sinh^2(e+fx)+1}\sqrt{b\sinh^2(e+fx)+a}}d\sinh(e+fx)+b(7a-8b)\int \frac{\sinh^2(e+fx)}{\sqrt{\sinh^2(e+fx)+1}\sqrt{b\sinh^2(e+f\cdots}}}{a}\right.}{3a}}{\cdots}\right.$$

$\downarrow$ 320

$$\sqrt{\cosh^2(e+fx)}\operatorname{sech}(e+fx)\left(-\frac{-\frac{b\left(\frac{b(7a-8b)\int \frac{\sinh^2(e+fx)}{\sqrt{\sinh^2(e+fx)+1}\sqrt{b\sinh^2(e+fx)+a}}d\sinh(e+fx)+\frac{(3a-4b)\sqrt{a+b\sinh^2(e+fx)}\text{EllipticF}\left(\text{arctan}\ldots\right.}{\sqrt{\sinh^2(e+fx)+1}\sqrt{\frac{a+b\sinh^2\ldots}{a\left(\sinh^2(e+\ldots\right.}}}}{a}\right.}{3a}\ldots\right.}{\ldots}$$

$\downarrow$ 388

$$\sqrt{\cosh^2(e+fx)}\operatorname{sech}(e+fx)\left(-\frac{-\frac{b\left(\frac{b(7a-8b)\left(\frac{\sinh(e+fx)\sqrt{a+b\sinh^2(e+fx)}}{b\sqrt{\sinh^2(e+fx)+1}}-\frac{\int \frac{\sqrt{b\sinh^2(e+fx)+a}}{\left(\sinh^2(e+fx)+1\right)^{3/2}}d\sinh(e+fx)}{b}\right)}{a}+\frac{(3a-4b)\sqrt{a+b\sinh^2(\ldots}}{\sqrt{\sinh^2(e\ldots}}\right.}{3a}\ldots\right.}{\ldots}$$

$\downarrow$ 313

$$\frac{\sqrt{\cosh^2(e+fx)}\operatorname{sech}(e+fx)\left(-\frac{-\frac{b\left(\frac{(3a-4b)\sqrt{a+b\sinh^2(e+fx)}\operatorname{EllipticF}\left(\arctan(\sinh(e+fx)),1-\frac{b}{a}\right)}{\sqrt{\sinh^2(e+fx)+1}\sqrt{\frac{a+b\sinh^2(e+fx)}{a\left(\sinh^2(e+fx)+1\right)}}}+b(7a-8b)\left(\frac{\sinh(e+fx)\sqrt{a+b\sinh^2(e+fx}}{b\sqrt{\sinh^2(e+fx)+1}}\right.\right.}{a}}{}\right.}{}$$

input
```
Int[Coth[e + f*x]^4/(a + b*Sinh[e + f*x]^2)^(3/2),x]
```

output
```
(Sqrt[Cosh[e + f*x]^2]*Sech[e + f*x]*(-(((a - b)*Csch[e + f*x]^3*Sqrt[1 +
Sinh[e + f*x]^2])/(a*b*Sqrt[a + b*Sinh[e + f*x]^2])) - (-1/3*((3*a - 4*b)*
Csch[e + f*x]^3*Sqrt[1 + Sinh[e + f*x]^2]*Sqrt[a + b*Sinh[e + f*x]^2])/a -
 (b*(-(((7*a - 8*b)*Csch[e + f*x]*Sqrt[1 + Sinh[e + f*x]^2]*Sqrt[a + b*Sin
h[e + f*x]^2])/a) + (((3*a - 4*b)*EllipticF[ArcTan[Sinh[e + f*x]], 1 - b/a
]*Sqrt[a + b*Sinh[e + f*x]^2])/(Sqrt[1 + Sinh[e + f*x]^2]*Sqrt[(a + b*Sinh
[e + f*x]^2)/(a*(1 + Sinh[e + f*x]^2))]) + (7*a - 8*b)*b*((Sinh[e + f*x]*S
qrt[a + b*Sinh[e + f*x]^2])/(b*Sqrt[1 + Sinh[e + f*x]^2]) - (EllipticE[Arc
Tan[Sinh[e + f*x]], 1 - b/a]*Sqrt[a + b*Sinh[e + f*x]^2])/(b*Sqrt[1 + Sinh
[e + f*x]^2]*Sqrt[(a + b*Sinh[e + f*x]^2)/(a*(1 + Sinh[e + f*x]^2))]))))/a)
)/(3*a))/(a*b)))/f
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 313
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/((c_) + (d_.)*(x_)^2)^(3/2), x_Symbol] :> Sim
p[(Sqrt[a + b*x^2]/(c*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(c
+ d*x^2)))]))*EllipticE[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; FreeQ
[{a, b, c, d}, x] && PosQ[b/a] && PosQ[d/c]
```

rule 320
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(Sqrt[a + b*x^2]/(a*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(
c + d*x^2)))]))*EllipticF[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; Fre
eQ[{a, b, c, d}, x] && PosQ[d/c] && PosQ[b/a] &&  !SimplerSqrtQ[b/a, d/c]
```

rule 370
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_
), x_Symbol] :> Simp[(-(b*c - a*d))*(e*x)^(m + 1)*(a + b*x^2)^(p + 1)*((c +
 d*x^2)^(q - 1)/(a*b*e*2*(p + 1))), x] + Simp[1/(a*b*2*(p + 1))   Int[(e*x)
^m*(a + b*x^2)^(p + 1)*(c + d*x^2)^(q - 2)*Simp[c*(b*c*2*(p + 1) + (b*c - a
*d)*(m + 1)) + d*(b*c*2*(p + 1) + (b*c - a*d)*(m + 2*(q - 1) + 1))*x^2, x],
 x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1]
&& GtQ[q, 1] && IntBinomialQ[a, b, c, d, e, m, 2, p, q, x]
```

rule 388
```
Int[(x_)^2/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol]
 :> Simp[x*(Sqrt[a + b*x^2]/(b*Sqrt[c + d*x^2])), x] - Simp[c/b   Int[Sqrt[
a + b*x^2]/(c + d*x^2)^(3/2), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
 a*d, 0] && PosQ[b/a] && PosQ[d/c] &&  !SimplerSqrtQ[b/a, d/c]
```

rule 406
```
Int[((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.)*((e_) + (f_.)*(
x_)^2), x_Symbol] :> Simp[e   Int[(a + b*x^2)^p*(c + d*x^2)^q, x], x] + Sim
p[f   Int[x^2*(a + b*x^2)^p*(c + d*x^2)^q, x], x] /; FreeQ[{a, b, c, d, e,
f, p, q}, x]
```

rule 445
```
Int[((g_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_
.)*((e_) + (f_.)*(x_)^2), x_Symbol] :> Simp[e*(g*x)^(m + 1)*(a + b*x^2)^(p
+ 1)*((c + d*x^2)^(q + 1)/(a*c*g*(m + 1))), x] + Simp[1/(a*c*g^2*(m + 1))
 Int[(g*x)^(m + 2)*(a + b*x^2)^p*(c + d*x^2)^q*Simp[a*f*c*(m + 1) - e*(b*c
+ a*d)*(m + 2 + 1) - e*2*(b*c*p + a*d*q) - b*e*d*(m + 2*(p + q + 2) + 1)*x^
2, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p, q}, x] && LtQ[m, -1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3675
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_.)*tan[(e_.) + (f_.)*(x_)]^
(m_), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Simp[ff^(m + 1
)*(Sqrt[Cos[e + f*x]^2]/(f*Cos[e + f*x]))   Subst[Int[x^m*((a + b*ff^2*x^2)
^p/(1 - ff^2*x^2)^((m + 1)/2)), x], x, Sin[e + f*x]/ff], x]] /; FreeQ[{a, b
, e, f, p}, x] && IntegerQ[m/2] &&  !IntegerQ[p]
```

Maple [A] (verified)

Time = 8.05 (sec) , antiderivative size = 522, normalized size of antiderivative = 1.69

method	result
default	$-\frac{7\sqrt{-\frac{b}{a}}\,ab\sinh(fx+e)^6-8\sqrt{-\frac{b}{a}}\,b^2\sinh(fx+e)^6-3a^2\sqrt{\frac{a+b\sinh(fx+e)^2}{a}}\,\sqrt{\frac{\cosh(2fx+2e)}{2}+\frac{1}{2}}\ \operatorname{EllipticF}\left(\sinh(fx+e)\sqrt{-\frac{b}{a}},\sqrt{\frac{a}{b}}\right)\ s}{\ }$
risch	Expression too large to display

input
```
int(coth(f*x+e)^4/(a+b*sinh(f*x+e)^2)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-1/3*(7*(-b/a)^(1/2)*a*b*sinh(f*x+e)^6-8*(-b/a)^(1/2)*b^2*sinh(f*x+e)^6-3*
a^2*((a+b*sinh(f*x+e)^2)/a)^(1/2)*(cosh(f*x+e)^2)^(1/2)*EllipticF(sinh(f*x
+e)*(-b/a)^(1/2),(1/b*a)^(1/2))*sinh(f*x+e)^3+11*b*((a+b*sinh(f*x+e)^2)/a)
^(1/2)*(cosh(f*x+e)^2)^(1/2)*EllipticF(sinh(f*x+e)*(-b/a)^(1/2),(1/b*a)^(1
/2))*a*sinh(f*x+e)^3-8*((a+b*sinh(f*x+e)^2)/a)^(1/2)*(cosh(f*x+e)^2)^(1/2)
*EllipticF(sinh(f*x+e)*(-b/a)^(1/2),(1/b*a)^(1/2))*b^2*sinh(f*x+e)^3-7*((a
+b*sinh(f*x+e)^2)/a)^(1/2)*(cosh(f*x+e)^2)^(1/2)*EllipticE(sinh(f*x+e)*(-b
/a)^(1/2),(1/b*a)^(1/2))*a*b*sinh(f*x+e)^3+8*((a+b*sinh(f*x+e)^2)/a)^(1/2)
*(cosh(f*x+e)^2)^(1/2)*EllipticE(sinh(f*x+e)*(-b/a)^(1/2),(1/b*a)^(1/2))*b
^2*sinh(f*x+e)^3+4*(-b/a)^(1/2)*a^2*sinh(f*x+e)^4+3*(-b/a)^(1/2)*a*b*sinh(
f*x+e)^4-8*(-b/a)^(1/2)*b^2*sinh(f*x+e)^4+5*(-b/a)^(1/2)*a^2*sinh(f*x+e)^2
-4*(-b/a)^(1/2)*a*b*sinh(f*x+e)^2+(-b/a)^(1/2)*a^2)/(-b/a)^(1/2)/a^3/sinh(
f*x+e)^3/cosh(f*x+e)/(a+b*sinh(f*x+e)^2)^(1/2)/f
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 6862 vs. $2(302) = 604$.

Time = 0.23 (sec) , antiderivative size = 6862, normalized size of antiderivative = 22.28

$$\int \frac{\coth^4(e+fx)}{(a+b\sinh^2(e+fx))^{3/2}}\,dx = \text{Too large to display}$$

input
```
integrate(coth(f*x+e)^4/(a+b*sinh(f*x+e)^2)^(3/2),x, algorithm="fricas")
```

output
```
Too large to include
```

Sympy [F]

$$\int \frac{\coth^4(e+fx)}{(a+b\sinh^2(e+fx))^{3/2}}\,dx = \int \frac{\coth^4(e+fx)}{(a+b\sinh^2(e+fx))^{\frac{3}{2}}}\,dx$$

input
```
integrate(coth(f*x+e)**4/(a+b*sinh(f*x+e)**2)**(3/2),x)
```

output
```
Integral(coth(e + f*x)**4/(a + b*sinh(e + f*x)**2)**(3/2), x)
```

Maxima [F]

$$\int \frac{\coth^4(e+fx)}{(a+b\sinh^2(e+fx))^{3/2}}\,dx = \int \frac{\coth(fx+e)^4}{(b\sinh(fx+e)^2+a)^{\frac{3}{2}}}\,dx$$

input
```
integrate(coth(f*x+e)^4/(a+b*sinh(f*x+e)^2)^(3/2),x, algorithm="maxima")
```

output
```
integrate(coth(f*x + e)^4/(b*sinh(f*x + e)^2 + a)^(3/2), x)
```

Giac [F]

$$\int \frac{\coth^4(e+fx)}{\left(a+b\sinh^2(e+fx)\right)^{3/2}}\,dx = \int \frac{\coth\left(fx+e\right)^4}{\left(b\sinh\left(fx+e\right)^2+a\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate(coth(f*x+e)^4/(a+b*sinh(f*x+e)^2)^(3/2),x, algorithm="giac")
```

output
```
integrate(coth(f*x + e)^4/(b*sinh(f*x + e)^2 + a)^(3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\coth^4(e+fx)}{\left(a+b\sinh^2(e+fx)\right)^{3/2}}\,dx = \int \frac{\coth(e+f\,x)^4}{\left(b\sinh\left(e+f\,x\right)^2+a\right)^{3/2}}\,dx$$

input
```
int(coth(e + f*x)^4/(a + b*sinh(e + f*x)^2)^(3/2),x)
```

output
```
int(coth(e + f*x)^4/(a + b*sinh(e + f*x)^2)^(3/2), x)
```

Reduce [F]

$$\int \frac{\coth^4(e+fx)}{\left(a+b\sinh^2(e+fx)\right)^{3/2}}\,dx = \int \frac{\sqrt{\sinh\left(fx+e\right)^2 b+a}\,\coth\left(fx+e\right)^4}{\sinh\left(fx+e\right)^4 b^2+2\sinh\left(fx+e\right)^2 ab+a^2}dx$$

input
```
int(coth(f*x+e)^4/(a+b*sinh(f*x+e)^2)^(3/2),x)
```

output
```
int((sqrt(sinh(e + f*x)**2*b + a)*coth(e + f*x)**4)/(sinh(e + f*x)**4*b**2
 + 2*sinh(e + f*x)**2*a*b + a**2),x)
```

3.465 $\int \frac{\tanh^5(e+fx)}{(a+b\sinh^2(e+fx))^{5/2}}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 232

$$\int \frac{\tanh^5(e+fx)}{(a+b\sinh^2(e+fx))^{5/2}}\,dx = -\frac{(8a^2+24ab+3b^2)\operatorname{arctanh}\left(\frac{\sqrt{a+b\sinh^2(e+fx)}}{\sqrt{a-b}}\right)}{8(a-b)^{9/2}f} + \frac{8a^2+24ab+3b^2}{24(a-b)^3f\,(a+b\sinh^2(e+fx))^{3/2}} + \frac{(8a-b)\operatorname{sech}^2(e+fx)}{8(a-b)^2f\,(a+b\sinh^2(e+fx))^{3/2}} - \frac{\operatorname{sech}^4(e+fx)}{4(a-b)f\,(a+b\sinh^2(e+fx))^{3/2}} + \frac{8a^2+24ab+3b^2}{8(a-b)^4f\sqrt{a+b\sinh^2(e+fx)}}$$

output
```
-1/8*(8*a^2+24*a*b+3*b^2)*arctanh((a+b*sinh(f*x+e)^2)^(1/2)/(a-b)^(1/2))/(
a-b)^(9/2)/f+1/24*(8*a^2+24*a*b+3*b^2)/(a-b)^3/f/(a+b*sinh(f*x+e)^2)^(3/2)
+1/8*(8*a-b)*sech(f*x+e)^2/(a-b)^2/f/(a+b*sinh(f*x+e)^2)^(3/2)-1/4*sech(f*
x+e)^4/(a-b)/f/(a+b*sinh(f*x+e)^2)^(3/2)+1/8*(8*a^2+24*a*b+3*b^2)/(a-b)^4/
f/(a+b*sinh(f*x+e)^2)^(1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 0.40 (sec) , antiderivative size = 114, normalized size of antiderivative = 0.49

$$\int \frac{\tanh^5(e+fx)}{\left(a+b\sinh^2(e+fx)\right)^{5/2}}\,dx = \frac{2(8a^2+24ab+3b^2)\,\text{Hypergeometric2F1}\left(-\frac{3}{2},1,-\frac{1}{2},\frac{a+b\sinh^2(e+fx)}{a-b}\right)+3(}{48(a-b)^3f\left(a+b\sinh^2(e\right.}$$

input
```
Integrate[Tanh[e + f*x]^5/(a + b*Sinh[e + f*x]^2)^(5/2),x]
```

output
```
(2*(8*a^2 + 24*a*b + 3*b^2)*Hypergeometric2F1[-3/2, 1, -1/2, (a + b*Sinh[e
 + f*x]^2)/(a - b)] + 3*(a - b)*(4*a + 3*b + (8*a - b)*Cosh[2*(e + f*x)])*
Sech[e + f*x]^4)/(48*(a - b)^3*f*(a + b*Sinh[e + f*x]^2)^(3/2))
```

Rubi [A] (verified)

Time = 0.50 (sec) , antiderivative size = 229, normalized size of antiderivative = 0.99, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.400$, Rules used = {3042, 26, 3673, 100, 27, 87, 61, 61, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\tanh^5(e+fx)}{\left(a+b\sinh^2(e+fx)\right)^{5/2}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int -\frac{i\tan(ie+ifx)^5}{(a-b\sin(ie+ifx)^2)^{5/2}}dx$$

$$\Big\downarrow \text{26}$$

$$-i\int \frac{\tan(ie+ifx)^5}{(a-b\sin(ie+ifx)^2)^{5/2}}dx$$

$$\Big\downarrow \text{3673}$$

$$\frac{\int \frac{\sinh^4(e+fx)}{\left(\sinh^2(e+fx)+1\right)^3\left(b\sinh^2(e+fx)+a\right)^{5/2}}d\sinh^2(e+fx)}{2f}$$

$\Big\downarrow$ 100

$$\frac{\frac{\int -\frac{-4(a-b)\sinh^2(e+fx)+4a+3b}{2\left(\sinh^2(e+fx)+1\right)^2\left(b\sinh^2(e+fx)+a\right)^{5/2}}d\sinh^2(e+fx)}{2(a-b)}-\frac{1}{2(a-b)\left(\sinh^2(e+fx)+1\right)^2\left(a+b\sinh^2(e+fx)\right)^{3/2}}}{2f}$$

$\Big\downarrow$ 27

$$\frac{-\frac{\int \frac{-4(a-b)\sinh^2(e+fx)+4a+3b}{\left(\sinh^2(e+fx)+1\right)^2\left(b\sinh^2(e+fx)+a\right)^{5/2}}d\sinh^2(e+fx)}{4(a-b)}-\frac{1}{2(a-b)\left(\sinh^2(e+fx)+1\right)^2\left(a+b\sinh^2(e+fx)\right)^{3/2}}}{2f}$$

$\Big\downarrow$ 87

$$\frac{-\frac{-\frac{\left(8a^2+24ab+3b^2\right)\int \frac{1}{\left(\sinh^2(e+fx)+1\right)\left(b\sinh^2(e+fx)+a\right)^{5/2}}d\sinh^2(e+fx)}{2(a-b)}-\frac{8a-b}{(a-b)\left(\sinh^2(e+fx)+1\right)\left(a+b\sinh^2(e+fx)\right)^{3/2}}}{4(a-b)}-\frac{}{2(a-b)\left(\sinh^2(e+fx)+\right.}}{2f}$$

$\Big\downarrow$ 61

$$\frac{-\frac{-\frac{\left(8a^2+24ab+3b^2\right)\left(\frac{\int \frac{1}{\left(\sinh^2(e+fx)+1\right)\left(b\sinh^2(e+fx)+a\right)^{3/2}}d\sinh^2(e+fx)}{a-b}+\frac{2}{3(a-b)\left(a+b\sinh^2(e+fx)\right)^{3/2}}\right)}{2(a-b)}-\frac{8a-b}{(a-b)\left(\sinh^2(e+fx)+1\right)\left(a+b\sinh^2(e+f\right.}}{4(a-b)}}{2f}$$

$\Big\downarrow$ 61

$$\frac{-\frac{-\frac{\left(8a^2+24ab+3b^2\right)\left(\frac{\frac{\int \frac{1}{\left(\sinh^2(e+fx)+1\right)\sqrt{b\sinh^2(e+fx)+a}}d\sinh^2(e+fx)}{a-b}+\frac{2}{(a-b)\sqrt{a+b\sinh^2(e+fx)}}}{a-b}+\frac{2}{3(a-b)\left(a+b\sinh^2(e+fx)\right)^{3/2}}\right)}{2(a-b)}-\frac{}{(a-b)\left(\sinh^2(e+\right.}}{4(a-b)}}{2f}$$

$\Big\downarrow$ 73

$$-\frac{-\frac{\left(8a^2+24ab+3b^2\right)\left(\frac{\frac{2\int \frac{1}{\frac{\sinh^4(e+fx)}{b}-\frac{a}{b}+1}d\sqrt{b\sinh^2(e+fx)+a}}{b(a-b)}+\frac{2}{(a-b)\sqrt{a+b\sinh^2(e+fx)}}}{a-b}+\frac{2}{3(a-b)\left(a+b\sinh^2(e+fx)\right)^{3/2}}\right)}{2(a-b)}-\frac{8a-b}{(a-b)\left(\sinh^2(e+fx)+1\right)\left(\cdots\right.}}{4(a-b)}}{2f}$$

$\Big\downarrow$ 221

$$-\frac{-\frac{\left(8a^2+24ab+3b^2\right)\left(\frac{\frac{2}{(a-b)\sqrt{a+b\sinh^2(e+fx)}}-\frac{2\text{arctanh}\left(\frac{\sqrt{a+b\sinh^2(e+fx)}}{\sqrt{a-b}}\right)}{(a-b)^{3/2}}}{a-b}+\frac{2}{3(a-b)\left(a+b\sinh^2(e+fx)\right)^{3/2}}\right)}{2(a-b)}-\frac{8a-b}{(a-b)\left(\sinh^2(e+fx)+1\right)\left(a+b\sinh\cdots\right.}}{4(a-b)}}{2f}$$

input
```
Int[Tanh[e + f*x]^5/(a + b*Sinh[e + f*x]^2)^(5/2),x]
```

output
```
(-1/2*1/((a - b)*(1 + Sinh[e + f*x]^2)^2*(a + b*Sinh[e + f*x]^2)^(3/2)) -
(-((8*a - b)/((a - b)*(1 + Sinh[e + f*x]^2)*(a + b*Sinh[e + f*x]^2)^(3/2))
) - ((8*a^2 + 24*a*b + 3*b^2)*(2/(3*(a - b)*(a + b*Sinh[e + f*x]^2)^(3/2))
 + ((-2*ArcTanh[Sqrt[a + b*Sinh[e + f*x]^2]/Sqrt[a - b]])/(a - b)^(3/2) +
2/((a - b)*Sqrt[a + b*Sinh[e + f*x]^2]))/(a - b)))/(2*(a - b)))/(4*(a - b)
))/(2*f)
```

Defintions of rubi rules used

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 61
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Simp[d*((
m + n + 2)/((b*c - a*d)*(m + 1)))   Int[(a + b*x)^(m + 1)*(c + d*x)^n, x],
x] /; FreeQ[{a, b, c, d, n}, x] && LtQ[m, -1] &&  !(LtQ[n, -1] && (EqQ[a, 0
] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && IntLinearQ[a, b, c, d
, m, n, x]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 87
```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_] :> Simp[(-(b*e - a*f))*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(f*(p
+ 1)*(c*f - d*e))), x] - Simp[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p
+ 1)))/(f*(p + 1)*(c*f - d*e))   Int[(c + d*x)^n*(e + f*x)^(p + 1), x], x]
/; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || Intege
rQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))))
```

rule 100
```
Int[((a_.) + (b_.)*(x_))^2*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))^(
p_), x_] :> Simp[(b*c - a*d)^2*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(d^2*(d
*e - c*f)*(n + 1))), x] - Simp[1/(d^2*(d*e - c*f)*(n + 1))   Int[(c + d*x)^
(n + 1)*(e + f*x)^p*Simp[a^2*d^2*f*(n + p + 2) + b^2*c*(d*e*(n + 1) + c*f*(
p + 1)) - 2*a*b*d*(d*e*(n + 1) + c*f*(p + 1)) - b^2*d*(d*e - c*f)*(n + 1)*x
, x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && (LtQ[n, -1] || (EqQ[n
 + p + 3, 0] && NeQ[n, -1] && (SumSimplerQ[n, 1] ||  !SumSimplerQ[p, 1])))
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3673
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_.)*tan[(e_.) + (f_.)*(x_)]^
(m_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x]^2, x]}, Simp[ff^((m
 + 1)/2)/(2*f)   Subst[Int[x^((m - 1)/2)*((a + b*ff*x)^p/(1 - ff*x)^((m + 1
)/2)), x], x, Sin[e + f*x]^2/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && Integ
erQ[(m - 1)/2]
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 2.85 (sec) , antiderivative size = 213, normalized size of antiderivative = 0.92

method	result
default	$\frac{\text{`int/indef0`}\left(-\frac{\sinh(fx+e)^5\left(b^2\sinh(fx+e)^4+2\sinh(fx+e)^2ab+a^2\right)\cosh}{\left(-b^4\cosh(fx+e)^{18}+\left(-4a\,b^3+4b^4\right)\cosh(fx+e)^{16}+\left(-6a^2b^2+12a\,b^3-6b^4\right)\cosh(fx+e)^{14}+\left(-4a^3b+12a^2b^2-12a\,b^3+4b^4\right)\cosh}\right.}{f}$
risch	Expression too large to display

input
```
int(tanh(f*x+e)^5/(a+b*sinh(f*x+e)^2)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
`int/indef0`(-sinh(f*x+e)^5*(b^2*sinh(f*x+e)^4+2*sinh(f*x+e)^2*a*b+a^2)*co
sh(f*x+e)^4/(-b^4*cosh(f*x+e)^18+(-4*a*b^3+4*b^4)*cosh(f*x+e)^16+(-6*a^2*b
^2+12*a*b^3-6*b^4)*cosh(f*x+e)^14+(-4*a^3*b+12*a^2*b^2-12*a*b^3+4*b^4)*cos
h(f*x+e)^12+(-a^4+4*a^3*b-6*a^2*b^2+4*a*b^3-b^4)*cosh(f*x+e)^10)/(a+b*sinh
(f*x+e)^2)^(1/2),sinh(f*x+e))/f
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 10156 vs. 2(208) = 416.

Time = 1.76 (sec) , antiderivative size = 20403, normalized size of antiderivative = 87.94

$$\int \frac{\tanh^5(e+fx)}{\left(a+b\sinh^2(e+fx)\right)^{5/2}}\,dx = \text{Too large to display}$$

input
```
integrate(tanh(f*x+e)^5/(a+b*sinh(f*x+e)^2)^(5/2),x, algorithm="fricas")
```

output
```
Too large to include
```

Sympy [F]

$$\int \frac{\tanh^5(e+fx)}{\left(a+b\sinh^2(e+fx)\right)^{5/2}}\,dx = \int \frac{\tanh^5{(e+fx)}}{\left(a+b\sinh^2{(e+fx)}\right)^{\frac{5}{2}}}\,dx$$

input
```
integrate(tanh(f*x+e)**5/(a+b*sinh(f*x+e)**2)**(5/2),x)
```

output
```
Integral(tanh(e + f*x)**5/(a + b*sinh(e + f*x)**2)**(5/2), x)
```

Maxima [F]

$$\int \frac{\tanh^5(e+fx)}{\left(a+b\sinh^2(e+fx)\right)^{5/2}}\,dx = \int \frac{\tanh\left(fx+e\right)^5}{\left(b\sinh\left(fx+e\right)^2+a\right)^{\frac{5}{2}}}\,dx$$

input
```
integrate(tanh(f*x+e)^5/(a+b*sinh(f*x+e)^2)^(5/2),x, algorithm="maxima")
```

output
```
integrate(tanh(f*x + e)^5/(b*sinh(f*x + e)^2 + a)^(5/2), x)
```

Giac [F]

$$\int \frac{\tanh^5(e+fx)}{\left(a+b\sinh^2(e+fx)\right)^{5/2}}\,dx = \int \frac{\tanh\left(fx+e\right)^5}{\left(b\sinh\left(fx+e\right)^2+a\right)^{\frac{5}{2}}}\,dx$$

input
```
integrate(tanh(f*x+e)^5/(a+b*sinh(f*x+e)^2)^(5/2),x, algorithm="giac")
```

output
```
integrate(tanh(f*x + e)^5/(b*sinh(f*x + e)^2 + a)^(5/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\tanh^5(e+fx)}{\left(a+b\sinh^2(e+fx)\right)^{5/2}}\,dx = \text{Hanged}$$

input
```
int(tanh(e + f*x)^5/(a + b*sinh(e + f*x)^2)^(5/2),x)
```

output
```
\text{Hanged}
```

Reduce [F]

$$\int \frac{\tanh^5(e+fx)}{\left(a+b\sinh^2(e+fx)\right)^{5/2}}\,dx = \int \frac{\sqrt{\sinh\left(fx+e\right)^2 b+a}\,\tanh\left(fx+e\right)^5}{\sinh\left(fx+e\right)^6 b^3+3\sinh\left(fx+e\right)^4 a\,b^2+3\sinh\left(fx+e\right)^2 a^2b+a^3}dx$$

input
```
int(tanh(f*x+e)^5/(a+b*sinh(f*x+e)^2)^(5/2),x)
```

output
```
int((sqrt(sinh(e + f*x)**2*b + a)*tanh(e + f*x)**5)/(sinh(e + f*x)**6*b**3
 + 3*sinh(e + f*x)**4*a*b**2 + 3*sinh(e + f*x)**2*a**2*b + a**3),x)
```

3.466 $\int \frac{\tanh^3(e+fx)}{\left(a+b\sinh^2(e+fx)\right)^{5/2}}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 163

$$\int \frac{\tanh^3(e+fx)}{\left(a+b\sinh^2(e+fx)\right)^{5/2}}\,dx = -\frac{(2a+3b)\operatorname{arctanh}\left(\frac{\sqrt{a+b\sinh^2(e+fx)}}{\sqrt{a-b}}\right)}{2(a-b)^{7/2}f} + \frac{2a+3b}{6(a-b)^2f\left(a+b\sinh^2(e+fx)\right)^{3/2}} + \frac{\operatorname{sech}^2(e+fx)}{2(a-b)f\left(a+b\sinh^2(e+fx)\right)^{3/2}} + \frac{2a+3b}{2(a-b)^3f\sqrt{a+b\sinh^2(e+fx)}}$$

output
```
-1/2*(2*a+3*b)*arctanh((a+b*sinh(f*x+e)^2)^(1/2)/(a-b)^(1/2))/(a-b)^(7/2)/
f+1/6*(2*a+3*b)/(a-b)^2/f/(a+b*sinh(f*x+e)^2)^(3/2)+1/2*sech(f*x+e)^2/(a-b
)/f/(a+b*sinh(f*x+e)^2)^(3/2)+1/2*(2*a+3*b)/(a-b)^3/f/(a+b*sinh(f*x+e)^2)^
(1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 0.09 (sec) , antiderivative size = 82, normalized size of antiderivative = 0.50

$$\int \frac{\tanh^3(e+fx)}{\left(a+b\sinh^2(e+fx)\right)^{5/2}}\,dx = \frac{(2a+3b)\,\text{Hypergeometric2F1}\left(-\frac{3}{2},1,-\frac{1}{2},\frac{a+b\sinh^2(e+fx)}{a-b}\right)+3(a-b)\text{sech}^2}{6(a-b)^2f\left(a+b\sinh^2(e+fx)\right)^{3/2}}$$

input
```
Integrate[Tanh[e + f*x]^3/(a + b*Sinh[e + f*x]^2)^(5/2),x]
```

output
```
((2*a + 3*b)*Hypergeometric2F1[-3/2, 1, -1/2, (a + b*Sinh[e + f*x]^2)/(a -
 b)] + 3*(a - b)*Sech[e + f*x]^2)/(6*(a - b)^2*f*(a + b*Sinh[e + f*x]^2)^(
3/2))
```

Rubi [A] (verified)

Time = 0.39 (sec) , antiderivative size = 162, normalized size of antiderivative = 0.99, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.320$, Rules used = {3042, 26, 3673, 87, 61, 61, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\tanh^3(e+fx)}{\left(a+b\sinh^2(e+fx)\right)^{5/2}}dx$$

$$\Big\downarrow \; 3042$$

$$\int \frac{i\tan(ie+ifx)^3}{(a-b\sin(ie+ifx)^2)^{5/2}}dx$$

$$\Big\downarrow \; 26$$

$$i\int \frac{\tan(ie+ifx)^3}{(a-b\sin(ie+ifx)^2)^{5/2}}dx$$

$$\Big\downarrow \; 3673$$

$$\frac{\int \frac{\sinh^2(e+fx)}{\left(\sinh^2(e+fx)+1\right)^2\left(b\sinh^2(e+fx)+a\right)^{5/2}}d\sinh^2(e+fx)}{2f}$$

$\downarrow$ 87

$$\frac{\frac{(2a+3b)\int \frac{1}{\left(\sinh^2(e+fx)+1\right)\left(b\sinh^2(e+fx)+a\right)^{5/2}}d\sinh^2(e+fx)}{2(a-b)}+\frac{1}{(a-b)\left(\sinh^2(e+fx)+1\right)\left(a+b\sinh^2(e+fx)\right)^{3/2}}}{2f}$$

$\downarrow$ 61

$$\frac{\frac{(2a+3b)\left(\frac{\int \frac{1}{\left(\sinh^2(e+fx)+1\right)\left(b\sinh^2(e+fx)+a\right)^{3/2}}d\sinh^2(e+fx)}{a-b}+\frac{2}{3(a-b)\left(a+b\sinh^2(e+fx)\right)^{3/2}}\right)}{2(a-b)}+\frac{1}{(a-b)\left(\sinh^2(e+fx)+1\right)\left(a+b\sinh^2(e+fx)\right)^{3/}}}{2f}$$

$\downarrow$ 61

$$\frac{\frac{(2a+3b)\left(\frac{\frac{\int \frac{1}{\left(\sinh^2(e+fx)+1\right)\sqrt{b\sinh^2(e+fx)+a}}d\sinh^2(e+fx)}{a-b}+\frac{2}{(a-b)\sqrt{a+b\sinh^2(e+fx)}}}{a-b}+\frac{2}{3(a-b)\left(a+b\sinh^2(e+fx)\right)^{3/2}}\right)}{2(a-b)}+\frac{1}{(a-b)\left(\sinh^2(e+fx)+1\right)}}{2f}$$

$\downarrow$ 73

$$\frac{\frac{(2a+3b)\left(\frac{\frac{2\int \frac{1}{\frac{\sinh^4(e+fx)}{b}-\frac{a}{b}+1}d\sqrt{b\sinh^2(e+fx)+a}}{b(a-b)}+\frac{2}{(a-b)\sqrt{a+b\sinh^2(e+fx)}}}{a-b}+\frac{2}{3(a-b)\left(a+b\sinh^2(e+fx)\right)^{3/2}}\right)}{2(a-b)}+\frac{1}{(a-b)\left(\sinh^2(e+fx)+1\right)\left(a+b\sin\right.}}{2f}$$

$\downarrow$ 221

$$\frac{\frac{(2a+3b)\left(\frac{\frac{2}{(a-b)\sqrt{a+b\sinh^2(e+fx)}}-\frac{2\text{arctanh}\left(\frac{\sqrt{a+b\sinh^2(e+fx)}}{\sqrt{a-b}}\right)}{(a-b)^{3/2}}}{a-b}+\frac{2}{3(a-b)\left(a+b\sinh^2(e+fx)\right)^{3/2}}\right)}{2(a-b)}+\frac{1}{(a-b)\left(\sinh^2(e+fx)+1\right)\left(a+b\sinh^2(e+f\right.}}{2f}$$

input
```
Int[Tanh[e + f*x]^3/(a + b*Sinh[e + f*x]^2)^(5/2),x]
```

output
```
(1/((a - b)*(1 + Sinh[e + f*x]^2)*(a + b*Sinh[e + f*x]^2)^(3/2)) + ((2*a +
 3*b)*(2/(3*(a - b)*(a + b*Sinh[e + f*x]^2)^(3/2)) + ((-2*ArcTanh[Sqrt[a +
 b*Sinh[e + f*x]^2]/Sqrt[a - b]])/(a - b)^(3/2) + 2/((a - b)*Sqrt[a + b*Si
nh[e + f*x]^2]))/(a - b)))/(2*(a - b)))/(2*f)
```

Defintions of rubi rules used

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 61
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Simp[d*((
m + n + 2)/((b*c - a*d)*(m + 1)))   Int[(a + b*x)^(m + 1)*(c + d*x)^n, x],
x] /; FreeQ[{a, b, c, d, n}, x] && LtQ[m, -1] &&  !(LtQ[n, -1] && (EqQ[a, 0
] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && IntLinearQ[a, b, c, d
, m, n, x]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 87
```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_] :> Simp[(-(b*e - a*f))*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(f*(p
+ 1)*(c*f - d*e))), x] - Simp[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p
+ 1)))/(f*(p + 1)*(c*f - d*e))   Int[(c + d*x)^n*(e + f*x)^(p + 1), x], x]
/; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || Intege
rQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))))
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3673
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_.)*tan[(e_.) + (f_.)*(x_)]^
(m_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x]^2, x]}, Simp[ff^((m
 + 1)/2)/(2*f)   Subst[Int[x^((m - 1)/2)*((a + b*ff*x)^p/(1 - ff*x)^((m + 1
)/2)), x], x, Sin[e + f*x]^2/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && Integ
erQ[(m - 1)/2]
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 1.32 (sec) , antiderivative size = 213, normalized size of antiderivative = 1.31

method	result
default	$\frac{\text{`int/indef0`}\left(-\frac{\sinh(fx+e)^3\left(b^2\sinh(fx+e)^4+2\sinh(fx+e)^2ab+a^2\right)\cosh(}{\left(-b^4\cosh(fx+e)^{14}+\left(-4a\,b^3+4b^4\right)\cosh(fx+e)^{12}+\left(-6a^2b^2+12a\,b^3-6b^4\right)\cosh(fx+e)^{10}+\left(-4a^3b+12a^2b^2-12a\,b^3+4b^4\right)\cosh}\right.}{f}$
risch	Expression too large to display

input
```
int(tanh(f*x+e)^3/(a+b*sinh(f*x+e)^2)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
`int/indef0`(-sinh(f*x+e)^3*(b^2*sinh(f*x+e)^4+2*sinh(f*x+e)^2*a*b+a^2)*co
sh(f*x+e)^2/(-b^4*cosh(f*x+e)^14+(-4*a*b^3+4*b^4)*cosh(f*x+e)^12+(-6*a^2*b
^2+12*a*b^3-6*b^4)*cosh(f*x+e)^10+(-4*a^3*b+12*a^2*b^2-12*a*b^3+4*b^4)*cos
h(f*x+e)^8+(-a^4+4*a^3*b-6*a^2*b^2+4*a*b^3-b^4)*cosh(f*x+e)^6)/(a+b*sinh(f
*x+e)^2)^(1/2),sinh(f*x+e))/f
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 5260 vs. $2(143) = 286$.

Time = 0.67 (sec) , antiderivative size = 10611, normalized size of antiderivative = 65.10

$$\int \frac{\tanh^3(e+fx)}{\left(a+b\sinh^2(e+fx)\right)^{5/2}}\,dx = \text{Too large to display}$$

input
```
integrate(tanh(f*x+e)^3/(a+b*sinh(f*x+e)^2)^(5/2),x, algorithm="fricas")
```

output
```
Too large to include
```

Sympy [F]

$$\int \frac{\tanh^3(e+fx)}{\left(a+b\sinh^2(e+fx)\right)^{5/2}}\,dx = \int \frac{\tanh^3{(e+fx)}}{\left(a+b\sinh^2{(e+fx)}\right)^{\frac{5}{2}}}\,dx$$

input
```
integrate(tanh(f*x+e)**3/(a+b*sinh(f*x+e)**2)**(5/2),x)
```

output
```
Integral(tanh(e + f*x)**3/(a + b*sinh(e + f*x)**2)**(5/2), x)
```

Maxima [F]

$$\int \frac{\tanh^3(e+fx)}{(a+b\sinh^2(e+fx))^{5/2}}\,dx = \int \frac{\tanh(fx+e)^3}{(b\sinh(fx+e)^2+a)^{\frac{5}{2}}}\,dx$$

input
```
integrate(tanh(f*x+e)^3/(a+b*sinh(f*x+e)^2)^(5/2),x, algorithm="maxima")
```

output
```
integrate(tanh(f*x + e)^3/(b*sinh(f*x + e)^2 + a)^(5/2), x)
```

Giac [F]

$$\int \frac{\tanh^3(e+fx)}{(a+b\sinh^2(e+fx))^{5/2}}\,dx = \int \frac{\tanh(fx+e)^3}{(b\sinh(fx+e)^2+a)^{\frac{5}{2}}}\,dx$$

input
```
integrate(tanh(f*x+e)^3/(a+b*sinh(f*x+e)^2)^(5/2),x, algorithm="giac")
```

output
```
integrate(tanh(f*x + e)^3/(b*sinh(f*x + e)^2 + a)^(5/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\tanh^3(e+fx)}{(a+b\sinh^2(e+fx))^{5/2}}\,dx = \int \frac{\tanh(e+f\,x)^3}{(b\sinh(e+f\,x)^2+a)^{5/2}}\,dx$$

input
```
int(tanh(e + f*x)^3/(a + b*sinh(e + f*x)^2)^(5/2),x)
```

output
```
int(tanh(e + f*x)^3/(a + b*sinh(e + f*x)^2)^(5/2), x)
```

Reduce [F]

$$\int \frac{\tanh^3(e+fx)}{\left(a+b\sinh^2(e+fx)\right)^{5/2}}\,dx = \int \frac{\sqrt{\sinh(fx+e)^2 b+a}\,\tanh(fx+e)^3}{\sinh(fx+e)^6 b^3+3\sinh(fx+e)^4 a b^2+3\sinh(fx+e)^2 a^2 b+a^3}dx$$

input
```
int(tanh(f*x+e)^3/(a+b*sinh(f*x+e)^2)^(5/2),x)
```

output
```
int((sqrt(sinh(e + f*x)**2*b + a)*tanh(e + f*x)**3)/(sinh(e + f*x)**6*b**3
 + 3*sinh(e + f*x)**4*a*b**2 + 3*sinh(e + f*x)**2*a**2*b + a**3),x)
```

3.467 $\int \frac{\tanh(e+fx)}{(a+b\sinh^2(e+fx))^{5/2}}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 99

$$\int \frac{\tanh(e+fx)}{(a+b\sinh^2(e+fx))^{5/2}}\,dx = -\frac{\operatorname{arctanh}\left(\frac{\sqrt{a+b\sinh^2(e+fx)}}{\sqrt{a-b}}\right)}{(a-b)^{5/2}f} + \frac{1}{3(a-b)f\left(a+b\sinh^2(e+fx)\right)^{3/2}} + \frac{1}{(a-b)^2f\sqrt{a+b\sinh^2(e+fx)}}$$

output
```
-arctanh((a+b*sinh(f*x+e)^2)^(1/2)/(a-b)^(1/2))/(a-b)^(5/2)/f+1/3/(a-b)/f/
(a+b*sinh(f*x+e)^2)^(3/2)+1/(a-b)^2/f/(a+b*sinh(f*x+e)^2)^(1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 0.05 (sec) , antiderivative size = 60, normalized size of antiderivative = 0.61

$$\int \frac{\tanh(e+fx)}{(a+b\sinh^2(e+fx))^{5/2}}\,dx = \frac{\text{Hypergeometric2F1}\left(-\frac{3}{2},1,-\frac{1}{2},1+\frac{b\cosh^2(e+fx)}{a-b}\right)}{3(a-b)f\left(a-b+b\cosh^2(e+fx)\right)^{3/2}}$$

input
```
Integrate[Tanh[e + f*x]/(a + b*Sinh[e + f*x]^2)^(5/2),x]
```

output
```
Hypergeometric2F1[-3/2, 1, -1/2, 1 + (b*Cosh[e + f*x]^2)/(a - b)]/(3*(a -
b)*f*(a - b + b*Cosh[e + f*x]^2)^(3/2))
```

Rubi [A] (verified)

Time = 0.33 (sec) , antiderivative size = 107, normalized size of antiderivative = 1.08, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.304$, Rules used = {3042, 26, 3673, 61, 61, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\tanh(e+fx)}{\left(a+b\sinh^2(e+fx)\right)^{5/2}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int -\frac{i\tan(ie+ifx)}{(a-b\sin(ie+ifx)^2)^{5/2}}dx$$

$$\Big\downarrow \text{26}$$

$$-i\int \frac{\tan(ie+ifx)}{(a-b\sin(ie+ifx)^2)^{5/2}}dx$$

$$\Big\downarrow \text{3673}$$

$$\frac{\int \frac{1}{(\sinh^2(e+fx)+1)\left(b\sinh^2(e+fx)+a\right)^{5/2}}d\sinh^2(e+fx)}{2f}$$

$$\Big\downarrow \text{61}$$

$$\frac{\frac{\int \frac{1}{\left(\sinh^2(e+fx)+1\right)\left(b\sinh^2(e+fx)+a\right)^{3/2}}d\sinh^2(e+fx)}{a-b}+\frac{2}{3(a-b)\left(a+b\sinh^2(e+fx)\right)^{3/2}}}{2f}$$

$$\Big\downarrow \text{61}$$

$$\frac{\frac{\frac{\int \frac{1}{\left(\sinh^2(e+fx)+1\right)\sqrt{b\sinh^2(e+fx)+a}}d\sinh^2(e+fx)}{a-b}+\frac{2}{(a-b)\sqrt{a+b\sinh^2(e+fx)}}}{a-b}+\frac{2}{3(a-b)\left(a+b\sinh^2(e+fx)\right)^{3/2}}}{2f}$$

$$\Big\downarrow \ 73$$

$$\frac{\frac{\frac{2\int \frac{1}{\frac{\sinh^4(e+fx)}{b}-\frac{a}{b}+1}d\sqrt{b\sinh^2(e+fx)+a}}{b(a-b)}+\frac{2}{(a-b)\sqrt{a+b\sinh^2(e+fx)}}}{a-b}+\frac{2}{3(a-b)\left(a+b\sinh^2(e+fx)\right)^{3/2}}}{2f}$$

$$\Big\downarrow \ 221$$

$$\frac{\frac{\frac{2}{(a-b)\sqrt{a+b\sinh^2(e+fx)}}-\frac{2\text{arctanh}\left(\frac{\sqrt{a+b\sinh^2(e+fx)}}{\sqrt{a-b}}\right)}{(a-b)^{3/2}}}{a-b}+\frac{2}{3(a-b)\left(a+b\sinh^2(e+fx)\right)^{3/2}}}{2f}$$

input
```
Int[Tanh[e + f*x]/(a + b*Sinh[e + f*x]^2)^(5/2),x]
```

output
```
(2/(3*(a - b)*(a + b*Sinh[e + f*x]^2)^(3/2)) + ((-2*ArcTanh[Sqrt[a + b*Sin
h[e + f*x]^2]/Sqrt[a - b]])/(a - b)^(3/2) + 2/((a - b)*Sqrt[a + b*Sinh[e +
 f*x]^2]))/(a - b))/(2*f)
```

Defintions of rubi rules used

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 61
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Simp[d*((
m + n + 2)/((b*c - a*d)*(m + 1)))   Int[(a + b*x)^(m + 1)*(c + d*x)^n, x],
x] /; FreeQ[{a, b, c, d, n}, x] && LtQ[m, -1] && !(LtQ[n, -1] && (EqQ[a, 0
] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && IntLinearQ[a, b, c, d
, m, n, x]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3673
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_.)*tan[(e_.) + (f_.)*(x_)]^
(m_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x]^2, x]}, Simp[ff^((m
 + 1)/2)/(2*f)   Subst[Int[x^((m - 1)/2)*((a + b*ff*x)^p/(1 - ff*x)^((m + 1
)/2)), x], x, Sin[e + f*x]^2/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && Integ
erQ[(m - 1)/2]
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.84 (sec) , antiderivative size = 173, normalized size of antiderivative = 1.75

method	result
default	$\frac{`int/indef0`\left(-\frac{\sinh(fx+e)\left(b^2\sinh(fx+e)^4+2\sinh(fx+e)^2ab+a^2\right)}{\left(-b^4\sinh(fx+e)^{10}+\left(-4a\,b^3-b^4\right)\sinh(fx+e)^8+\left(-6a^2b^2-4a\,b^3\right)\sinh(fx+e)^6+\left(-4a^3b-6a^2b^2\right)\sinh(fx+e)^4+\left(-a^4-4a^3b\right)\right.}\right.}{f}$
risch	Expression too large to display

input
```
int(tanh(f*x+e)/(a+b*sinh(f*x+e)^2)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
`int/indef0`(-sinh(f*x+e)*(b^2*sinh(f*x+e)^4+2*sinh(f*x+e)^2*a*b+a^2)/(-b^
4*sinh(f*x+e)^10+(-4*a*b^3-b^4)*sinh(f*x+e)^8+(-6*a^2*b^2-4*a*b^3)*sinh(f*
x+e)^6+(-4*a^3*b-6*a^2*b^2)*sinh(f*x+e)^4+(-a^4-4*a^3*b)*sinh(f*x+e)^2-a^4
)/(a+b*sinh(f*x+e)^2)^(1/2),sinh(f*x+e))/f
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2107 vs. $2(87) = 174$.

Time = 0.26 (sec) , antiderivative size = 4305, normalized size of antiderivative = 43.48

$$\int \frac{\tanh(e+fx)}{\left(a+b\sinh^2(e+fx)\right)^{5/2}}\,dx = \text{Too large to display}$$

input
```
integrate(tanh(f*x+e)/(a+b*sinh(f*x+e)^2)^(5/2),x, algorithm="fricas")
```

output
```
Too large to include
```

Sympy [F]

$$\int \frac{\tanh(e+fx)}{\left(a+b\sinh^2(e+fx)\right)^{5/2}}\,dx = \int \frac{\tanh{\left(e+fx \right)}}{\left(a+b\sinh^2{\left(e+fx \right)}\right)^{\frac{5}{2}}}\,dx$$

input
```
integrate(tanh(f*x+e)/(a+b*sinh(f*x+e)**2)**(5/2),x)
```

output
```
Integral(tanh(e + f*x)/(a + b*sinh(e + f*x)**2)**(5/2), x)
```

Maxima [F]

$$\int \frac{\tanh(e + fx)}{(a + b\sinh^2(e + fx))^{5/2}}\, dx = \int \frac{\tanh(fx + e)}{(b\sinh(fx + e)^2 + a)^{\frac{5}{2}}}\, dx$$

input
```
integrate(tanh(f*x+e)/(a+b*sinh(f*x+e)^2)^(5/2),x, algorithm="maxima")
```

output
```
integrate(tanh(f*x + e)/(b*sinh(f*x + e)^2 + a)^(5/2), x)
```

Giac [F]

$$\int \frac{\tanh(e + fx)}{(a + b\sinh^2(e + fx))^{5/2}}\, dx = \int \frac{\tanh(fx + e)}{(b\sinh(fx + e)^2 + a)^{\frac{5}{2}}}\, dx$$

input
```
integrate(tanh(f*x+e)/(a+b*sinh(f*x+e)^2)^(5/2),x, algorithm="giac")
```

output
```
integrate(tanh(f*x + e)/(b*sinh(f*x + e)^2 + a)^(5/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\tanh(e + fx)}{(a + b\sinh^2(e + fx))^{5/2}}\, dx = \int \frac{\tanh(e + f\,x)}{(b\sinh(e + f\,x)^2 + a)^{5/2}}\, dx$$

input
```
int(tanh(e + f*x)/(a + b*sinh(e + f*x)^2)^(5/2),x)
```

output
```
int(tanh(e + f*x)/(a + b*sinh(e + f*x)^2)^(5/2), x)
```

Reduce [F]

$$\int \frac{\tanh(e+fx)}{\left(a+b\sinh^2(e+fx)\right)^{5/2}}\,dx = \int \frac{\sqrt{\sinh\left(fx+e\right)^2 b+a}\,\tanh\left(fx+e\right)}{\sinh\left(fx+e\right)^6 b^3+3\sinh\left(fx+e\right)^4 a\,b^2+3\sinh\left(fx+e\right)^2 a^2b+a^3}dx$$

input

```
int(tanh(f*x+e)/(a+b*sinh(f*x+e)^2)^(5/2),x)
```

output

```
int((sqrt(sinh(e + f*x)**2*b + a)*tanh(e + f*x))/(sinh(e + f*x)**6*b**3 +
3*sinh(e + f*x)**4*a*b**2 + 3*sinh(e + f*x)**2*a**2*b + a**3),x)
```

3.468 $\int \frac{\coth(e+fx)}{\left(a+b\sinh^2(e+fx)\right)^{5/2}}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 83

$$\int \frac{\coth(e+fx)}{\left(a+b\sinh^2(e+fx)\right)^{5/2}}\,dx = -\frac{\operatorname{arctanh}\left(\frac{\sqrt{a+b\sinh^2(e+fx)}}{\sqrt{a}}\right)}{a^{5/2}f} + \frac{1}{3af\left(a+b\sinh^2(e+fx)\right)^{3/2}} + \frac{1}{a^2f\sqrt{a+b\sinh^2(e+fx)}}$$

output
```
-arctanh((a+b*sinh(f*x+e)^2)^(1/2)/a^(1/2))/a^(5/2)/f+1/3/a/f/(a+b*sinh(f*
x+e)^2)^(3/2)+1/a^2/f/(a+b*sinh(f*x+e)^2)^(1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 0.04 (sec) , antiderivative size = 49, normalized size of antiderivative = 0.59

$$\int \frac{\coth(e+fx)}{\left(a+b\sinh^2(e+fx)\right)^{5/2}}\,dx = \frac{\text{Hypergeometric2F1}\left(-\frac{3}{2},1,-\frac{1}{2},1+\frac{b\sinh^2(e+fx)}{a}\right)}{3af\left(a+b\sinh^2(e+fx)\right)^{3/2}}$$

input
```
Integrate[Coth[e + f*x]/(a + b*Sinh[e + f*x]^2)^(5/2),x]
```

output
```
Hypergeometric2F1[-3/2, 1, -1/2, 1 + (b*Sinh[e + f*x]^2)/a]/(3*a*f*(a + b*
Sinh[e + f*x]^2)^(3/2))
```

Rubi [A] (verified)

Time = 0.32 (sec) , antiderivative size = 87, normalized size of antiderivative = 1.05, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.304$, Rules used = {3042, 26, 3673, 61, 61, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\coth(e+fx)}{\left(a+b\sinh^2(e+fx)\right)^{5/2}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{i}{\tan(ie+ifx)\left(a-b\sin(ie+ifx)^2\right)^{5/2}}dx$$

$$\Big\downarrow \text{26}$$

$$i\int \frac{1}{\left(a-b\sin(ie+ifx)^2\right)^{5/2}\tan(ie+ifx)}dx$$

$$\Big\downarrow \text{3673}$$

$$\frac{\int \frac{\text{csch}^2(e+fx)}{\left(b\sinh^2(e+fx)+a\right)^{5/2}}d\sinh^2(e+fx)}{2f}$$

$$\Big\downarrow \text{61}$$

$$\frac{\frac{\int \frac{\text{csch}^2(e+fx)}{\left(b\sinh^2(e+fx)+a\right)^{3/2}}d\sinh^2(e+fx)}{a}+\frac{2}{3a\left(a+b\sinh^2(e+fx)\right)^{3/2}}}{2f}$$

$$\Big\downarrow \text{61}$$

$$\frac{\frac{\frac{\int \frac{\text{csch}^2(e+fx)}{\sqrt{b\sinh^2(e+fx)+a}}d\sinh^2(e+fx)}{a}+\frac{2}{a\sqrt{a+b\sinh^2(e+fx)}}}{a}+\frac{2}{3a\left(a+b\sinh^2(e+fx)\right)^{3/2}}}{2f}$$

$$\Big\downarrow \; 73$$

$$\frac{\frac{\frac{2\int \frac{1}{\frac{\sinh^4(e+fx)}{b}-\frac{a}{b}}d\sqrt{b\sinh^2(e+fx)+a}}{ab}+\frac{2}{a\sqrt{a+b\sinh^2(e+fx)}}}{a}+\frac{2}{3a\left(a+b\sinh^2(e+fx)\right)^{3/2}}}{2f}$$

$$\Big\downarrow \; 221$$

$$\frac{\frac{\frac{2}{a\sqrt{a+b\sinh^2(e+fx)}}-\frac{2\text{arctanh}\left(\frac{\sqrt{a+b\sinh^2(e+fx)}}{\sqrt{a}}\right)}{a^{3/2}}}{a}+\frac{2}{3a\left(a+b\sinh^2(e+fx)\right)^{3/2}}}{2f}$$

input
```
Int[Coth[e + f*x]/(a + b*Sinh[e + f*x]^2)^(5/2),x]
```

output
```
(2/(3*a*(a + b*Sinh[e + f*x]^2)^(3/2)) + ((-2*ArcTanh[Sqrt[a + b*Sinh[e +
f*x]^2]/Sqrt[a]])/a^(3/2) + 2/(a*Sqrt[a + b*Sinh[e + f*x]^2]))/a)/(2*f)
```

Defintions of rubi rules used

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 61
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Simp[d*((
m + n + 2)/((b*c - a*d)*(m + 1)))   Int[(a + b*x)^(m + 1)*(c + d*x)^n, x],
x] /; FreeQ[{a, b, c, d, n}, x] && LtQ[m, -1] &&  !(LtQ[n, -1] && (EqQ[a, 0
] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && IntLinearQ[a, b, c, d
, m, n, x]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3673
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_.)*tan[(e_.) + (f_.)*(x_)]^
(m_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x]^2, x]}, Simp[ff^((m
 + 1)/2)/(2*f)   Subst[Int[x^((m - 1)/2)*((a + b*ff*x)^p/(1 - ff*x)^((m + 1
)/2)), x], x, Sin[e + f*x]^2/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && Integ
erQ[(m - 1)/2]
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.85 (sec) , antiderivative size = 65, normalized size of antiderivative = 0.78

method	result	size
default	$\frac{\text{`int/indef0`}\left(\frac{1}{\left(b^{2} \sinh \left(f x +e \right)^{4}+2 \sinh \left(f x +e \right)^{2} a b +a^{2}\right) \sinh \left(f x +e \right) \sqrt{a +b \sinh \left(f x +e \right)^{2}}}, \sinh \left(f x +e \right)\right)}{f}$	65
risch	Expression too large to display	47165

input
```
int(coth(f*x+e)/(a+b*sinh(f*x+e)^2)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
`int/indef0`(1/(b^2*sinh(f*x+e)^4+2*sinh(f*x+e)^2*a*b+a^2)/sinh(f*x+e)/(a+
b*sinh(f*x+e)^2)^(1/2),sinh(f*x+e))/f
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1554 vs. $2(71) = 142$.

Time = 0.21 (sec) , antiderivative size = 3197, normalized size of antiderivative = 38.52

$$\int \frac{\coth(e+fx)}{\left(a+b\sinh^2(e+fx)\right)^{5/2}}\,dx = \text{Too large to display}$$

input
```
integrate(coth(f*x+e)/(a+b*sinh(f*x+e)^2)^(5/2),x, algorithm="fricas")
```

output
```
Too large to include
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\coth(e+fx)}{\left(a+b\sinh^2(e+fx)\right)^{5/2}}\,dx = \text{Timed out}$$

input
```
integrate(coth(f*x+e)/(a+b*sinh(f*x+e)**2)**(5/2),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{\coth(e+fx)}{\left(a+b\sinh^2(e+fx)\right)^{5/2}}\,dx = \int \frac{\coth\left(fx+e\right)}{\left(b\sinh\left(fx+e\right)^2+a\right)^{\frac{5}{2}}}\,dx$$

input
```
integrate(coth(f*x+e)/(a+b*sinh(f*x+e)^2)^(5/2),x, algorithm="maxima")
```

output
```
integrate(coth(f*x + e)/(b*sinh(f*x + e)^2 + a)^(5/2), x)
```

Giac [F]

$$\int \frac{\coth(e+fx)}{\left(a+b\sinh^2(e+fx)\right)^{5/2}}\,dx = \int \frac{\coth\left(fx+e\right)}{\left(b\sinh\left(fx+e\right)^2+a\right)^{\frac{5}{2}}}\,dx$$

input
```
integrate(coth(f*x+e)/(a+b*sinh(f*x+e)^2)^(5/2),x, algorithm="giac")
```

output
```
integrate(coth(f*x + e)/(b*sinh(f*x + e)^2 + a)^(5/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\coth(e+fx)}{\left(a+b\sinh^2(e+fx)\right)^{5/2}}\,dx = \int \frac{\coth(e+f\,x)}{\left(b\sinh\left(e+f\,x\right)^2+a\right)^{5/2}}\,dx$$

input
```
int(coth(e + f*x)/(a + b*sinh(e + f*x)^2)^(5/2),x)
```

output
```
int(coth(e + f*x)/(a + b*sinh(e + f*x)^2)^(5/2), x)
```

Reduce [F]

$$\int \frac{\coth(e+fx)}{\left(a+b\sinh^2(e+fx)\right)^{5/2}}\,dx = \int \frac{\sqrt{\sinh\left(fx+e\right)^2 b+a}\,\coth\left(fx+e\right)}{\sinh\left(fx+e\right)^6 b^3+3\sinh\left(fx+e\right)^4 a\,b^2+3\sinh\left(fx+e\right)^2 a^2b+a^3}dx$$

input
```
int(coth(f*x+e)/(a+b*sinh(f*x+e)^2)^(5/2),x)
```

output
```
int((sqrt(sinh(e + f*x)**2*b + a)*coth(e + f*x))/(sinh(e + f*x)**6*b**3 +
3*sinh(e + f*x)**4*a*b**2 + 3*sinh(e + f*x)**2*a**2*b + a**3),x)
```

3.469 $\int \frac{\coth^3(e+fx)}{\left(a+b\sinh^2(e+fx)\right)^{5/2}}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 143

$$\int \frac{\coth^3(e+fx)}{\left(a+b\sinh^2(e+fx)\right)^{5/2}}\,dx = -\frac{(2a-5b)\operatorname{arctanh}\left(\frac{\sqrt{a+b\sinh^2(e+fx)}}{\sqrt{a}}\right)}{2a^{7/2}f} + \frac{2a-5b}{6a^2f\left(a+b\sinh^2(e+fx)\right)^{3/2}} - \frac{\operatorname{csch}^2(e+fx)}{2af\left(a+b\sinh^2(e+fx)\right)^{3/2}} + \frac{2a-5b}{2a^3f\sqrt{a+b\sinh^2(e+fx)}}$$

output
```
-1/2*(2*a-5*b)*arctanh((a+b*sinh(f*x+e)^2)^(1/2)/a^(1/2))/a^(7/2)/f+1/6*(2
*a-5*b)/a^2/f/(a+b*sinh(f*x+e)^2)^(3/2)-1/2*csch(f*x+e)^2/a/f/(a+b*sinh(f*
x+e)^2)^(3/2)+1/2*(2*a-5*b)/a^3/f/(a+b*sinh(f*x+e)^2)^(1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 0.19 (sec) , antiderivative size = 69, normalized size of antiderivative = 0.48

$$\int \frac{\coth^3(e+fx)}{\left(a+b\sinh^2(e+fx)\right)^{5/2}}\,dx = -\frac{3a\text{csch}^2(e+fx)+(-2a+5b)\,\text{Hypergeometric2F1}\left(-\frac{3}{2},1,-\frac{1}{2},1+\frac{b\sinh^2(e+fx)}{a}\right)}{6a^2f\left(a+b\sinh^2(e+fx)\right)^{3/2}}$$

input
```
Integrate[Coth[e + f*x]^3/(a + b*Sinh[e + f*x]^2)^(5/2),x]
```

output
```
-1/6*(3*a*Csch[e + f*x]^2 + (-2*a + 5*b)*Hypergeometric2F1[-3/2, 1, -1/2,
1 + (b*Sinh[e + f*x]^2)/a])/(a^2*f*(a + b*Sinh[e + f*x]^2)^(3/2))
```

Rubi [A] (verified)

Time = 0.34 (sec) , antiderivative size = 131, normalized size of antiderivative = 0.92, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.320$, Rules used = {3042, 26, 3673, 87, 61, 61, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\coth^3(e+fx)}{\left(a+b\sinh^2(e+fx)\right)^{5/2}}\,dx$$

$$\Big\downarrow \ 3042$$

$$\int -\frac{i}{\tan(ie+ifx)^3\left(a-b\sin(ie+ifx)^2\right)^{5/2}}dx$$

$$\Big\downarrow \ 26$$

$$-i\int \frac{1}{\left(a-b\sin(ie+ifx)^2\right)^{5/2}\tan(ie+ifx)^3}dx$$

$$\Big\downarrow \ 3673$$

$$\frac{\int \frac{\operatorname{csch}^4(e+fx)\left(\sinh^2(e+fx)+1\right)}{\left(b\sinh^2(e+fx)+a\right)^{5/2}}d\sinh^2(e+fx)}{2f}$$

$$\Big\downarrow \; 87$$

$$\frac{\frac{(2a-5b)\int \frac{\operatorname{csch}^2(e+fx)}{\left(b\sinh^2(e+fx)+a\right)^{5/2}}d\sinh^2(e+fx)}{2a}-\frac{\operatorname{csch}^2(e+fx)}{a\left(a+b\sinh^2(e+fx)\right)^{3/2}}}{2f}$$

$$\Big\downarrow \; 61$$

$$\frac{\frac{(2a-5b)\left(\frac{\int \frac{\operatorname{csch}^2(e+fx)}{\left(b\sinh^2(e+fx)+a\right)^{3/2}}d\sinh^2(e+fx)}{a}+\frac{2}{3a\left(a+b\sinh^2(e+fx)\right)^{3/2}}\right)}{2a}-\frac{\operatorname{csch}^2(e+fx)}{a\left(a+b\sinh^2(e+fx)\right)^{3/2}}}{2f}$$

$$\Big\downarrow \; 61$$

$$\frac{\frac{(2a-5b)\left(\frac{\frac{\int \frac{\operatorname{csch}^2(e+fx)}{\sqrt{b\sinh^2(e+fx)+a}}d\sinh^2(e+fx)}{a}+\frac{2}{a\sqrt{a+b\sinh^2(e+fx)}}}{a}+\frac{2}{3a\left(a+b\sinh^2(e+fx)\right)^{3/2}}\right)}{2a}-\frac{\operatorname{csch}^2(e+fx)}{a\left(a+b\sinh^2(e+fx)\right)^{3/2}}}{2f}$$

$$\Big\downarrow \; 73$$

$$\frac{\frac{(2a-5b)\left(\frac{\frac{2\int \frac{1}{\frac{\sinh^4(e+fx)}{b}-\frac{a}{b}}d\sqrt{b\sinh^2(e+fx)+a}}{ab}+\frac{2}{a\sqrt{a+b\sinh^2(e+fx)}}}{a}+\frac{2}{3a\left(a+b\sinh^2(e+fx)\right)^{3/2}}\right)}{2a}-\frac{\operatorname{csch}^2(e+fx)}{a\left(a+b\sinh^2(e+fx)\right)^{3/2}}}{2f}$$

$$\Big\downarrow \; 221$$

$$\frac{\frac{(2a-5b)\left(\frac{\frac{2}{a\sqrt{a+b\sinh^2(e+fx)}}-\frac{2\operatorname{arctanh}\left(\frac{\sqrt{a+b\sinh^2(e+fx)}}{\sqrt{a}}\right)}{a^{3/2}}}{a}+\frac{2}{3a\left(a+b\sinh^2(e+fx)\right)^{3/2}}\right)}{2a}-\frac{\operatorname{csch}^2(e+fx)}{a\left(a+b\sinh^2(e+fx)\right)^{3/2}}}{2f}$$

input
```
Int[Coth[e + f*x]^3/(a + b*Sinh[e + f*x]^2)^(5/2),x]
```

output
```
(-(Csch[e + f*x]^2/(a*(a + b*Sinh[e + f*x]^2)^(3/2))) + ((2*a - 5*b)*(2/(3
*a*(a + b*Sinh[e + f*x]^2)^(3/2)) + ((-2*ArcTanh[Sqrt[a + b*Sinh[e + f*x]^
2]/Sqrt[a]])/a^(3/2) + 2/(a*Sqrt[a + b*Sinh[e + f*x]^2]))/a))/(2*a))/(2*f)
```

Defintions of rubi rules used

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 61
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Simp[d*((
m + n + 2)/((b*c - a*d)*(m + 1)))   Int[(a + b*x)^(m + 1)*(c + d*x)^n, x],
x] /; FreeQ[{a, b, c, d, n}, x] && LtQ[m, -1] &&  !(LtQ[n, -1] && (EqQ[a, 0
] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && IntLinearQ[a, b, c, d
, m, n, x]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 87
```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_] :> Simp[(-(b*e - a*f))*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(f*(p
+ 1)*(c*f - d*e))), x] - Simp[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p
+ 1)))/(f*(p + 1)*(c*f - d*e))   Int[(c + d*x)^n*(e + f*x)^(p + 1), x], x]
/; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || Intege
rQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))))
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3673
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_.)*tan[(e_.) + (f_.)*(x_)]^
(m_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x]^2, x]}, Simp[ff^((m
 + 1)/2)/(2*f)   Subst[Int[x^((m - 1)/2)*((a + b*ff*x)^p/(1 - ff*x)^((m + 1
)/2)), x], x, Sin[e + f*x]^2/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && Integ
erQ[(m - 1)/2]
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 1.23 (sec) , antiderivative size = 73, normalized size of antiderivative = 0.51

method	result	size
default	$\frac{\text{`int/indef0`}\left(\frac{\cosh(fx+e)^2}{\left(b^2\sinh(fx+e)^4+2\sinh(fx+e)^2ab+a^2\right)\sinh(fx+e)^3\sqrt{a+b\sinh(fx+e)^2}},\sinh(fx+e)\right)}{f}$	73
risch	Expression too large to display	309511

input
```
int(coth(f*x+e)^3/(a+b*sinh(f*x+e)^2)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
`int/indef0`(cosh(f*x+e)^2/(b^2*sinh(f*x+e)^4+2*sinh(f*x+e)^2*a*b+a^2)/sin
h(f*x+e)^3/(a+b*sinh(f*x+e)^2)^(1/2),sinh(f*x+e))/f
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 3809 vs. 2(123) = 246.

Time = 0.44 (sec) , antiderivative size = 7707, normalized size of antiderivative = 53.90

$$\int \frac{\coth^3(e+fx)}{\left(a+b\sinh^2(e+fx)\right)^{5/2}}\,dx = \text{Too large to display}$$

input
```
integrate(coth(f*x+e)^3/(a+b*sinh(f*x+e)^2)^(5/2),x, algorithm="fricas")
```

output
```
Too large to include
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\coth^3(e+fx)}{\left(a+b\sinh^2(e+fx)\right)^{5/2}}\,dx = \text{Timed out}$$

input
```
integrate(coth(f*x+e)**3/(a+b*sinh(f*x+e)**2)**(5/2),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{\coth^3(e+fx)}{\left(a+b\sinh^2(e+fx)\right)^{5/2}}\,dx = \int \frac{\coth\left(fx+e\right)^3}{\left(b\sinh\left(fx+e\right)^2+a\right)^{\frac{5}{2}}}\,dx$$

input
```
integrate(coth(f*x+e)^3/(a+b*sinh(f*x+e)^2)^(5/2),x, algorithm="maxima")
```

output
```
integrate(coth(f*x + e)^3/(b*sinh(f*x + e)^2 + a)^(5/2), x)
```

Giac [F]

$$\int \frac{\coth^3(e+fx)}{\left(a+b\sinh^2(e+fx)\right)^{5/2}}\,dx = \int \frac{\coth\left(fx+e\right)^3}{\left(b\sinh\left(fx+e\right)^2+a\right)^{\frac{5}{2}}}\,dx$$

input
```
integrate(coth(f*x+e)^3/(a+b*sinh(f*x+e)^2)^(5/2),x, algorithm="giac")
```

output
```
integrate(coth(f*x + e)^3/(b*sinh(f*x + e)^2 + a)^(5/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\coth^3(e+fx)}{\left(a+b\sinh^2(e+fx)\right)^{5/2}}\,dx=\int \frac{\coth(e+f\,x)^3}{\left(b\sinh\left(e+f\,x\right)^2+a\right)^{5/2}}\,dx$$

input
```
int(coth(e + f*x)^3/(a + b*sinh(e + f*x)^2)^(5/2),x)
```

output
```
int(coth(e + f*x)^3/(a + b*sinh(e + f*x)^2)^(5/2), x)
```

Reduce [F]

$$\int \frac{\coth^3(e+fx)}{\left(a+b\sinh^2(e+fx)\right)^{5/2}}\,dx=\int \frac{\sqrt{\sinh\left(fx+e\right)^2b+a}\,\coth\left(fx+e\right)^3}{\sinh\left(fx+e\right)^6b^3+3\sinh\left(fx+e\right)^4ab^2+3\sinh\left(fx+e\right)^2a^2b+a^3}dx$$

input
```
int(coth(f*x+e)^3/(a+b*sinh(f*x+e)^2)^(5/2),x)
```

output
```
int((sqrt(sinh(e + f*x)**2*b + a)*coth(e + f*x)**3)/(sinh(e + f*x)**6*b**3
 + 3*sinh(e + f*x)**4*a*b**2 + 3*sinh(e + f*x)**2*a**2*b + a**3),x)
```

3.470 $\int \frac{\coth^5(e+fx)}{(a+b\sinh^2(e+fx))^{5/2}}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 208

$$\begin{aligned}\int \frac{\coth^5(e+fx)}{\left(a+b\sinh^2(e+fx)\right)^{5/2}}\,dx = &-\frac{(8a^2-40ab+35b^2)\operatorname{arctanh}\left(\frac{\sqrt{a+b\sinh^2(e+fx)}}{\sqrt{a}}\right)}{8a^{9/2}f}\\ &+\frac{8a^2-40ab+35b^2}{24a^3f\left(a+b\sinh^2(e+fx)\right)^{3/2}}-\frac{(8a-7b)\operatorname{csch}^2(e+fx)}{8a^2f\left(a+b\sinh^2(e+fx)\right)^{3/2}}\\ &-\frac{\operatorname{csch}^4(e+fx)}{4af\left(a+b\sinh^2(e+fx)\right)^{3/2}}+\frac{8a^2-40ab+35b^2}{8a^4f\sqrt{a+b\sinh^2(e+fx)}}\end{aligned}$$

output

```
-1/8*(8*a^2-40*a*b+35*b^2)*arctanh((a+b*sinh(f*x+e)^2)^(1/2)/a^(1/2))/a^(9
/2)/f+1/24*(8*a^2-40*a*b+35*b^2)/a^3/f/(a+b*sinh(f*x+e)^2)^(3/2)-1/8*(8*a-
7*b)*csch(f*x+e)^2/a^2/f/(a+b*sinh(f*x+e)^2)^(3/2)-1/4*csch(f*x+e)^4/a/f/(
a+b*sinh(f*x+e)^2)^(3/2)+1/8*(8*a^2-40*a*b+35*b^2)/a^4/f/(a+b*sinh(f*x+e)^
2)^(1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 0.28 (sec) , antiderivative size = 117, normalized size of antiderivative = 0.56

$$\int \frac{\coth^5(e+fx)}{(a+b\sinh^2(e+fx))^{5/2}}\,dx =$$
$$-\frac{\operatorname{csch}^2(e+fx)\left(3a\operatorname{csch}^2(e+fx)\left(8a-7b+2a\operatorname{csch}^2(e+fx)\right)+(-8a^2+40ab-35b^2)\text{Hypergeometric2}\right.}{24a^3f\left(b+a\operatorname{csch}^2(e+fx)\right)\sqrt{a+b\sinh^2(e+fx)}}$$

input
```
Integrate[Coth[e + f*x]^5/(a + b*Sinh[e + f*x]^2)^(5/2),x]
```

output
```
-1/24*(Csch[e + f*x]^2*(3*a*Csch[e + f*x]^2*(8*a - 7*b + 2*a*Csch[e + f*x]
^2) + (-8*a^2 + 40*a*b - 35*b^2)*Hypergeometric2F1[-3/2, 1, -1/2, 1 + (b*S
inh[e + f*x]^2)/a]))/(a^3*f*(b + a*Csch[e + f*x]^2)*Sqrt[a + b*Sinh[e + f*
x]^2])
```

Rubi [A] (verified)

Time = 0.40 (sec) , antiderivative size = 185, normalized size of antiderivative = 0.89, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.400$, Rules used = {3042, 26, 3673, 100, 27, 87, 61, 61, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\coth^5(e+fx)}{(a+b\sinh^2(e+fx))^{5/2}}dx$$

$$\Big\downarrow \ 3042$$

$$\int \frac{i}{\tan(ie+ifx)^5\left(a-b\sin(ie+ifx)^2\right)^{5/2}}dx$$

$$\Big\downarrow \ 26$$

$$i\int \frac{1}{\left(a-b\sin(ie+ifx)^2\right)^{5/2}\tan(ie+ifx)^5}dx$$

$$\Big\downarrow \; 3673$$

$$\frac{\int \frac{\operatorname{csch}^6(e+fx)\left(\sinh^2(e+fx)+1\right)^2}{\left(b\sinh^2(e+fx)+a\right)^{5/2}}d\sinh^2(e+fx)}{2f}$$

$$\Big\downarrow \; 100$$

$$\frac{\frac{\int \frac{\operatorname{csch}^4(e+fx)\left(4a\sinh^2(e+fx)+8a-7b\right)}{2\left(b\sinh^2(e+fx)+a\right)^{5/2}}d\sinh^2(e+fx)}{2a}-\frac{\operatorname{csch}^4(e+fx)}{2a\left(a+b\sinh^2(e+fx)\right)^{3/2}}}{2f}$$

$$\Big\downarrow \; 27$$

$$\frac{\frac{\int \frac{\operatorname{csch}^4(e+fx)\left(4a\sinh^2(e+fx)+8a-7b\right)}{\left(b\sinh^2(e+fx)+a\right)^{5/2}}d\sinh^2(e+fx)}{4a}-\frac{\operatorname{csch}^4(e+fx)}{2a\left(a+b\sinh^2(e+fx)\right)^{3/2}}}{2f}$$

$$\Big\downarrow \; 87$$

$$\frac{\frac{\frac{\left(8a^2-40ab+35b^2\right)\int \frac{\operatorname{csch}^2(e+fx)}{\left(b\sinh^2(e+fx)+a\right)^{5/2}}d\sinh^2(e+fx)}{2a}-\frac{(8a-7b)\operatorname{csch}^2(e+fx)}{a\left(a+b\sinh^2(e+fx)\right)^{3/2}}}{4a}-\frac{\operatorname{csch}^4(e+fx)}{2a\left(a+b\sinh^2(e+fx)\right)^{3/2}}}{2f}$$

$$\Big\downarrow \; 61$$

$$\frac{\frac{\frac{\left(8a^2-40ab+35b^2\right)\left(\frac{\int \frac{\operatorname{csch}^2(e+fx)}{\left(b\sinh^2(e+fx)+a\right)^{3/2}}d\sinh^2(e+fx)}{a}+\frac{2}{3a\left(a+b\sinh^2(e+fx)\right)^{3/2}}\right)}{2a}-\frac{(8a-7b)\operatorname{csch}^2(e+fx)}{a\left(a+b\sinh^2(e+fx)\right)^{3/2}}}{4a}-\frac{\operatorname{csch}^4(e+fx)}{2a\left(a+b\sinh^2(e+fx)\right)^{3/2}}}{2f}$$

$$\Big\downarrow \; 61$$

$$\frac{\frac{\frac{\left(8a^2-40ab+35b^2\right)\left(\frac{\frac{\int \frac{\text{csch}^2(e+fx)}{\sqrt{b\sinh^2(e+fx)+a}}d\sinh^2(e+fx)}{a}+\frac{2}{a\sqrt{a+b\sinh^2(e+fx)}}}{a}+\frac{2}{3a\left(a+b\sinh^2(e+fx)\right)^{3/2}}\right)}{2a}-\frac{(8a-7b)\text{csch}^2(e+fx)}{a\left(a+b\sinh^2(e+fx)\right)^{3/2}}}{4a}-\frac{\text{csch}^4(e+\cdots}{2a(a+b\sinh^2(e\cdots}}{2f}$$

$$\Big\downarrow \; 73$$

$$\frac{\frac{\frac{\left(8a^2-40ab+35b^2\right)\left(\frac{\frac{2\int \frac{1}{\frac{\sinh^4(e+fx)}{b}-\frac{a}{b}}d\sqrt{b\sinh^2(e+fx)+a}}{ab}+\frac{2}{a\sqrt{a+b\sinh^2(e+fx)}}}{a}+\frac{2}{3a\left(a+b\sinh^2(e+fx)\right)^{3/2}}\right)}{2a}-\frac{(8a-7b)\text{csch}^2(e+fx)}{a\left(a+b\sinh^2(e+fx)\right)^{3/2}}}{4a}-\frac{\text{csch}\cdots}{2a(a+b\sin\cdots}}{2f}$$

$$\Big\downarrow \; 221$$

$$\frac{\frac{\frac{\left(8a^2-40ab+35b^2\right)\left(\frac{\frac{2}{a\sqrt{a+b\sinh^2(e+fx)}}-\frac{2\text{arctanh}\left(\frac{\sqrt{a+b\sinh^2(e+fx)}}{\sqrt{a}}\right)}{a^{3/2}}}{a}+\frac{2}{3a\left(a+b\sinh^2(e+fx)\right)^{3/2}}\right)}{2a}-\frac{(8a-7b)\text{csch}^2(e+fx)}{a\left(a+b\sinh^2(e+fx)\right)^{3/2}}}{4a}-\frac{\text{csch}^4(e+\cdots}{2a(a+b\sinh^2(e\cdots}}{2f}$$

input

```
Int[Coth[e + f*x]^5/(a + b*Sinh[e + f*x]^2)^(5/2),x]
```

output

```
(-1/2*Csch[e + f*x]^4/(a*(a + b*Sinh[e + f*x]^2)^(3/2)) + (-(((8*a - 7*b)*
Csch[e + f*x]^2)/(a*(a + b*Sinh[e + f*x]^2)^(3/2))) + ((8*a^2 - 40*a*b + 3
5*b^2)*(2/(3*a*(a + b*Sinh[e + f*x]^2)^(3/2)) + ((-2*ArcTanh[Sqrt[a + b*Si
nh[e + f*x]^2]/Sqrt[a]])/a^(3/2) + 2/(a*Sqrt[a + b*Sinh[e + f*x]^2]))/a))/
(2*a))/(4*a))/(2*f)
```

Defintions of rubi rules used

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 61
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Simp[d*((
m + n + 2)/((b*c - a*d)*(m + 1)))   Int[(a + b*x)^(m + 1)*(c + d*x)^n, x],
x] /; FreeQ[{a, b, c, d, n}, x] && LtQ[m, -1] &&  !(LtQ[n, -1] && (EqQ[a, 0
] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && IntLinearQ[a, b, c, d
, m, n, x]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 87
```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_] :> Simp[(-(b*e - a*f))*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(f*(p
+ 1)*(c*f - d*e))), x] - Simp[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p
+ 1)))/(f*(p + 1)*(c*f - d*e))   Int[(c + d*x)^n*(e + f*x)^(p + 1), x], x]
/; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || Intege
rQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))))
```

rule 100
```
Int[((a_.) + (b_.)*(x_))^2*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))^(
p_), x_] :> Simp[(b*c - a*d)^2*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(d^2*(d
*e - c*f)*(n + 1))), x] - Simp[1/(d^2*(d*e - c*f)*(n + 1))   Int[(c + d*x)^
(n + 1)*(e + f*x)^p*Simp[a^2*d^2*f*(n + p + 2) + b^2*c*(d*e*(n + 1) + c*f*(
p + 1)) - 2*a*b*d*(d*e*(n + 1) + c*f*(p + 1)) - b^2*d*(d*e - c*f)*(n + 1)*x
, x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && (LtQ[n, -1] || (EqQ[n
 + p + 3, 0] && NeQ[n, -1] && (SumSimplerQ[n, 1] ||  !SumSimplerQ[p, 1])))
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3673
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_.)*tan[(e_.) + (f_.)*(x_)]^
(m_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x]^2, x]}, Simp[ff^((m
 + 1)/2)/(2*f)   Subst[Int[x^((m - 1)/2)*((a + b*ff*x)^p/(1 - ff*x)^((m + 1
)/2)), x], x, Sin[e + f*x]^2/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && Integ
erQ[(m - 1)/2]
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 2.80 (sec) , antiderivative size = 73, normalized size of antiderivative = 0.35

method	result	size
default	$\frac{\text{`int/indef0`}\left(\frac{\cosh(fx+e)^4}{\left(b^2\sinh(fx+e)^4+2\sinh(fx+e)^2ab+a^2\right)\sinh(fx+e)^5\sqrt{a+b\sinh(fx+e)^2}},\sinh(fx+e)\right)}{f}$	73
risch	Expression too large to display	2630726

input
```
int(coth(f*x+e)^5/(a+b*sinh(f*x+e)^2)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
`int/indef0`(cosh(f*x+e)^4/(b^2*sinh(f*x+e)^4+2*sinh(f*x+e)^2*a*b+a^2)/sin
h(f*x+e)^5/(a+b*sinh(f*x+e)^2)^(1/2),sinh(f*x+e))/f
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 7563 vs. $2(184) = 368$.

Time = 1.21 (sec) , antiderivative size = 15215, normalized size of antiderivative = 73.15

$$\int \frac{\coth^5(e+fx)}{\left(a+b\sinh^2(e+fx)\right)^{5/2}}\,dx = \text{Too large to display}$$

input
```
integrate(coth(f*x+e)^5/(a+b*sinh(f*x+e)^2)^(5/2),x, algorithm="fricas")
```

output
```
Too large to include
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\coth^5(e+fx)}{\left(a+b\sinh^2(e+fx)\right)^{5/2}}\,dx = \text{Timed out}$$

input
```
integrate(coth(f*x+e)**5/(a+b*sinh(f*x+e)**2)**(5/2),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{\coth^5(e+fx)}{\left(a+b\sinh^2(e+fx)\right)^{5/2}}\,dx = \int \frac{\coth\left(fx+e\right)^5}{\left(b\sinh\left(fx+e\right)^2+a\right)^{\frac{5}{2}}}\,dx$$

input
```
integrate(coth(f*x+e)^5/(a+b*sinh(f*x+e)^2)^(5/2),x, algorithm="maxima")
```

output
```
integrate(coth(f*x + e)^5/(b*sinh(f*x + e)^2 + a)^(5/2), x)
```

Giac [F]

$$\int \frac{\coth^5(e+fx)}{\left(a+b\sinh^2(e+fx)\right)^{5/2}}\,dx = \int \frac{\coth\left(fx+e\right)^5}{\left(b\sinh\left(fx+e\right)^2+a\right)^{\frac{5}{2}}}\,dx$$

input
```
integrate(coth(f*x+e)^5/(a+b*sinh(f*x+e)^2)^(5/2),x, algorithm="giac")
```

output
```
integrate(coth(f*x + e)^5/(b*sinh(f*x + e)^2 + a)^(5/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\coth^5(e+fx)}{\left(a+b\sinh^2(e+fx)\right)^{5/2}}\,dx = \text{Hanged}$$

input
```
int(coth(e + f*x)^5/(a + b*sinh(e + f*x)^2)^(5/2),x)
```

output
```
\text{Hanged}
```

Reduce [F]

$$\int \frac{\coth^5(e+fx)}{\left(a+b\sinh^2(e+fx)\right)^{5/2}}\,dx = \int \frac{\sqrt{\sinh\left(fx+e\right)^2 b+a}\,\coth\left(fx+e\right)^5}{\sinh\left(fx+e\right)^6 b^3+3\sinh\left(fx+e\right)^4 a\,b^2+3\sinh\left(fx+e\right)^2 a^2b+a^3}dx$$

input
```
int(coth(f*x+e)^5/(a+b*sinh(f*x+e)^2)^(5/2),x)
```

output
```
int((sqrt(sinh(e + f*x)**2*b + a)*coth(e + f*x)**5)/(sinh(e + f*x)**6*b**3
 + 3*sinh(e + f*x)**4*a*b**2 + 3*sinh(e + f*x)**2*a**2*b + a**3),x)
```

3.471 $\int \frac{\tanh^4(e+fx)}{\left(a+b\sinh^2(e+fx)\right)^{5/2}}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 356

$$\begin{aligned}\int \frac{\tanh^4(e+fx)}{\left(a+b\sinh^2(e+fx)\right)^{5/2}}\,dx &= -\frac{b(5a+3b)\cosh(e+fx)\sinh(e+fx)}{3(a-b)^3f\left(a+b\sinh^2(e+fx)\right)^{3/2}}\\
&-\frac{8\sqrt{a}\sqrt{b}(a+b)\cosh(e+fx)E\left(\arctan\left(\frac{\sqrt{b}\sinh(e+fx)}{\sqrt{a}}\right)|1-\frac{a}{b}\right)}{3(a-b)^4f\sqrt{\frac{a\cosh^2(e+fx)}{a+b\sinh^2(e+fx)}}\sqrt{a+b\sinh^2(e+fx)}}\\
&+\frac{\sqrt{a}(3a^2+10ab+3b^2)\cosh(e+fx)\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{b}\sinh(e+fx)}{\sqrt{a}}\right),1-\frac{a}{b}\right)}{3(a-b)^4\sqrt{b}f\sqrt{\frac{a\cosh^2(e+fx)}{a+b\sinh^2(e+fx)}}\sqrt{a+b\sinh^2(e+fx)}}\\
&-\frac{2(2a+b)\tanh(e+fx)}{3(a-b)^2f\left(a+b\sinh^2(e+fx)\right)^{3/2}}+\frac{\text{sech}^2(e+fx)\tanh(e+fx)}{3(a-b)f\left(a+b\sinh^2(e+fx)\right)^{3/2}}\end{aligned}$$

output
```
-1/3*b*(5*a+3*b)*cosh(f*x+e)*sinh(f*x+e)/(a-b)^3/f/(a+b*sinh(f*x+e)^2)^(3/
2)-8/3*a^(1/2)*b^(1/2)*(a+b)*cosh(f*x+e)*EllipticE(b^(1/2)*sinh(f*x+e)/a^(
1/2)/(1+b*sinh(f*x+e)^2/a)^(1/2),(1-a/b)^(1/2))/(a-b)^4/f/(a*cosh(f*x+e)^2
/(a+b*sinh(f*x+e)^2))^(1/2)/(a+b*sinh(f*x+e)^2)^(1/2)+1/3*a^(1/2)*(3*a^2+1
0*a*b+3*b^2)*cosh(f*x+e)*InverseJacobiAM(arctan(b^(1/2)*sinh(f*x+e)/a^(1/2
)),(1-a/b)^(1/2))/(a-b)^4/b^(1/2)/f/(a*cosh(f*x+e)^2/(a+b*sinh(f*x+e)^2))^
(1/2)/(a+b*sinh(f*x+e)^2)^(1/2)-2/3*(2*a+b)*tanh(f*x+e)/(a-b)^2/f/(a+b*sin
h(f*x+e)^2)^(3/2)+1/3*sech(f*x+e)^2*tanh(f*x+e)/(a-b)/f/(a+b*sinh(f*x+e)^2
)^(3/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 2.74 (sec) , antiderivative size = 252, normalized size of antiderivative = 0.71

$$\int \frac{\tanh^4(e+fx)}{\left(a+b\sinh^2(e+fx)\right)^{5/2}}\,dx =$$

$$-\frac{i\left(2ab\left(\frac{2a-b+b\cosh(2(e+fx))}{a}\right)^{3/2}\left(8a(a+b)E\left(i(e+fx)\,\middle|\,\frac{b}{a}\right)+\left(-5a^2+2ab+3b^2\right)\text{EllipticF}\left(i(e+fx),\frac{b}{a}\right)\right)\right.}{}$$

input
```
Integrate[Tanh[e + f*x]^4/(a + b*Sinh[e + f*x]^2)^(5/2),x]
```

output
```
((-1/6*I)*(2*a*b*((2*a - b + b*Cosh[2*(e + f*x)])/a)^(3/2)*(8*a*(a + b)*El
lipticE[I*(e + f*x), b/a] + (-5*a^2 + 2*a*b + 3*b^2)*EllipticF[I*(e + f*x)
, b/a]) - I*Sqrt[2]*b*(2*a*(a - b)*b*Sinh[2*(e + f*x)] + 4*b*(a + b)*(2*a
- b + b*Cosh[2*(e + f*x)])*Sinh[2*(e + f*x)] + 4*(a + b)*(2*a - b + b*Cosh
[2*(e + f*x)])^2*Tanh[e + f*x] - (a - b)*(2*a - b + b*Cosh[2*(e + f*x)])^2
*Sech[e + f*x]^2*Tanh[e + f*x])))/((a - b)^4*b*f*(2*a - b + b*Cosh[2*(e +
f*x)])^(3/2))
```

Rubi [A] (verified)

Time = 0.58 (sec) , antiderivative size = 417, normalized size of antiderivative = 1.17, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.400$, Rules used = {3042, 3675, 372, 402, 27, 402, 27, 400, 313, 320}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\tanh^4(e+fx)}{\left(a+b\sinh^2(e+fx)\right)^{5/2}}dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \frac{\tan(ie+ifx)^4}{\left(a-b\sin(ie+ifx)^2\right)^{5/2}}dx$$

$$\Big\downarrow \text{ 3675}$$

$$\frac{\sqrt{\cosh^2(e+fx)}\text{sech}(e+fx)\int \frac{\sinh^4(e+fx)}{\left(\sinh^2(e+fx)+1\right)^{5/2}\left(b\sinh^2(e+fx)+a\right)^{5/2}}d\sinh(e+fx)}{f}$$

$$\Big\downarrow \text{ 372}$$

$$\frac{\sqrt{\cosh^2(e+fx)}\text{sech}(e+fx)\left(\frac{\sinh(e+fx)}{3(a-b)\left(\sinh^2(e+fx)+1\right)^{3/2}\left(a+b\sinh^2(e+fx)\right)^{3/2}}-\frac{\int \frac{a-(3a+2b)\sinh^2(e+fx)}{\left(\sinh^2(e+fx)+1\right)^{3/2}\left(b\sinh^2(e+fx)+a\right)^{5/2}}d\sinh}{3(a-b)}\right.}{f}$$

$$\Big\downarrow \text{ 402}$$

$$\frac{\sqrt{\cosh^2(e+fx)}\text{sech}(e+fx)\left(\frac{\sinh(e+fx)}{3(a-b)\left(\sinh^2(e+fx)+1\right)^{3/2}\left(a+b\sinh^2(e+fx)\right)^{3/2}}-\frac{\frac{2(2a+b)\sinh(e+fx)}{(a-b)\sqrt{\sinh^2(e+fx)+1}\left(a+b\sinh^2(e+fx)\right)^{3/2}}-\frac{\int -}{}}{3(a}\right.}{f}$$

$$\Big\downarrow \text{ 27}$$

$$\frac{\sqrt{\cosh^2(e+fx)}\operatorname{sech}(e+fx)\left(\frac{\sinh(e+fx)}{3(a-b)\left(\sinh^2(e+fx)+1\right)^{3/2}\left(a+b\sinh^2(e+fx)\right)^{3/2}}-\frac{\frac{2(2a+b)\sinh(e+fx)}{(a-b)\sqrt{\sinh^2(e+fx)+1}\left(a+b\sinh^2(e+fx)\right)^{3/2}}-\frac{3\int \cdots}{\cdots}}{3(a \cdots}\right.}{f}$$

$\downarrow$ 402

$$\frac{\sqrt{\cosh^2(e+fx)}\operatorname{sech}(e+fx)\left(\frac{\sinh(e+fx)}{3(a-b)\left(\sinh^2(e+fx)+1\right)^{3/2}\left(a+b\sinh^2(e+fx)\right)^{3/2}}-\frac{\frac{2(2a+b)\sinh(e+fx)}{(a-b)\sqrt{\sinh^2(e+fx)+1}\left(a+b\sinh^2(e+fx)\right)^{3/2}}-\frac{3\left(\cdots\right)}{\cdots}}{\cdots}\right.}{f}$$

$\downarrow$ 27

$$\frac{\sqrt{\cosh^2(e+fx)}\operatorname{sech}(e+fx)\left(\frac{\sinh(e+fx)}{3(a-b)\left(\sinh^2(e+fx)+1\right)^{3/2}\left(a+b\sinh^2(e+fx)\right)^{3/2}}-\frac{\frac{2(2a+b)\sinh(e+fx)}{(a-b)\sqrt{\sinh^2(e+fx)+1}\left(a+b\sinh^2(e+fx)\right)^{3/2}}-\frac{3\left(\cdots\right)}{\cdots}}{\cdots}\right.}{f}$$

$\downarrow$ 400

$$\sqrt{\cosh^2(e+fx)}\operatorname{sech}(e+fx)\left(\frac{\sinh(e+fx)}{3(a-b)\left(\sinh^2(e+fx)+1\right)^{3/2}\left(a+b\sinh^2(e+fx)\right)^{3/2}}-\frac{\frac{2(2a+b)\sinh(e+fx)}{(a-b)\sqrt{\sinh^2(e+fx)+1}\left(a+b\sinh^2(e+fx)\right)^{3/2}}-\frac{3\left(\right.}{}}{}\right.$$

$\downarrow$ 313

$$\sqrt{\cosh^2(e+fx)}\operatorname{sech}(e+fx)\left(\frac{\sinh(e+fx)}{3(a-b)\left(\sinh^2(e+fx)+1\right)^{3/2}\left(a+b\sinh^2(e+fx)\right)^{3/2}}-\frac{\frac{2(2a+b)\sinh(e+fx)}{(a-b)\sqrt{\sinh^2(e+fx)+1}\left(a+b\sinh^2(e+fx)\right)^{3/2}}-\frac{3\left(\right.}{}}{}\right.$$

$\downarrow$ 320

$$\sqrt{\cosh^2(e+fx)}\operatorname{sech}(e+fx)\left(\frac{\sinh(e+fx)}{3(a-b)\left(\sinh^2(e+fx)+1\right)^{3/2}\left(a+b\sinh^2(e+fx)\right)^{3/2}}-\frac{\frac{2(2a+b)\sinh(e+fx)}{(a-b)\sqrt{\sinh^2(e+fx)+1}\left(a+b\sinh^2(e+fx)\right)^{3/2}}-\frac{3\left(\cdots\right.}{\cdots}}{\cdots}\right.$$

input
```
Int[Tanh[e + f*x]^4/(a + b*Sinh[e + f*x]^2)^(5/2),x]
```

output
```
(Sqrt[Cosh[e + f*x]^2]*Sech[e + f*x]*(Sinh[e + f*x]/(3*(a - b)*(1 + Sinh[e
 + f*x]^2)^(3/2)*(a + b*Sinh[e + f*x]^2)^(3/2)) - ((2*(2*a + b)*Sinh[e + f
*x])/((a - b)*Sqrt[1 + Sinh[e + f*x]^2]*(a + b*Sinh[e + f*x]^2)^(3/2)) - (
3*(-1/3*(b*(5*a + 3*b)*Sinh[e + f*x]*Sqrt[1 + Sinh[e + f*x]^2])/((a - b)*(
a + b*Sinh[e + f*x]^2)^(3/2)) + ((-8*Sqrt[a]*Sqrt[b]*(a + b)*EllipticE[Arc
Tan[(Sqrt[b]*Sinh[e + f*x])/Sqrt[a]], 1 - a/b]*Sqrt[1 + Sinh[e + f*x]^2])/
((a - b)*Sqrt[(a*(1 + Sinh[e + f*x]^2))/(a + b*Sinh[e + f*x]^2)]*Sqrt[a +
b*Sinh[e + f*x]^2]) + ((3*a + b)*(a + 3*b)*EllipticF[ArcTan[Sinh[e + f*x]]
, 1 - b/a]*Sqrt[a + b*Sinh[e + f*x]^2])/(a*(a - b)*Sqrt[1 + Sinh[e + f*x]^
2]*Sqrt[(a + b*Sinh[e + f*x]^2)/(a*(1 + Sinh[e + f*x]^2))]))/(3*(a - b))))
/(a - b))/(3*(a - b))))/f
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 313
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/((c_) + (d_.)*(x_)^2)^(3/2), x_Symbol] :> Sim
p[(Sqrt[a + b*x^2]/(c*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(c
+ d*x^2)))]))*EllipticE[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; FreeQ
[{a, b, c, d}, x] && PosQ[b/a] && PosQ[d/c]
```

rule 320
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(Sqrt[a + b*x^2]/(a*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(
c + d*x^2)))]))*EllipticF[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; Fre
eQ[{a, b, c, d}, x] && PosQ[d/c] && PosQ[b/a] &&  !SimplerSqrtQ[b/a, d/c]
```

rule 372
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_
), x_Symbol] :> Simp[(-a)*e^3*(e*x)^(m - 3)*(a + b*x^2)^(p + 1)*((c + d*x^2
)^(q + 1)/(2*b*(b*c - a*d)*(p + 1))), x] + Simp[e^4/(2*b*(b*c - a*d)*(p + 1
))   Int[(e*x)^(m - 4)*(a + b*x^2)^(p + 1)*(c + d*x^2)^q*Simp[a*c*(m - 3) +
 (a*d*(m + 2*q - 1) + 2*b*c*(p + 1))*x^2, x], x], x] /; FreeQ[{a, b, c, d,
e, q}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && GtQ[m, 3] && IntBinomialQ[a
, b, c, d, e, m, 2, p, q, x]
```

rule 400
```
Int[((e_) + (f_.)*(x_)^2)/(Sqrt[(a_) + (b_.)*(x_)^2]*((c_) + (d_.)*(x_)^2)^
(3/2)), x_Symbol] :> Simp[(b*e - a*f)/(b*c - a*d)   Int[1/(Sqrt[a + b*x^2]*
Sqrt[c + d*x^2]), x], x] - Simp[(d*e - c*f)/(b*c - a*d)   Int[Sqrt[a + b*x^
2]/(c + d*x^2)^(3/2), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && PosQ[b/a] &
& PosQ[d/c]
```

rule 402
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_.)*((e_) + (f_.)*(x
_)^2), x_Symbol] :> Simp[(-(b*e - a*f))*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^
(q + 1)/(a*2*(b*c - a*d)*(p + 1))), x] + Simp[1/(a*2*(b*c - a*d)*(p + 1))
 Int[(a + b*x^2)^(p + 1)*(c + d*x^2)^q*Simp[c*(b*e - a*f) + e*2*(b*c - a*d)
*(p + 1) + d*(b*e - a*f)*(2*(p + q + 2) + 1)*x^2, x], x], x] /; FreeQ[{a, b
, c, d, e, f, q}, x] && LtQ[p, -1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3675
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_.)*tan[(e_.) + (f_.)*(x_)]^
(m_), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Simp[ff^(m + 1
)*(Sqrt[Cos[e + f*x]^2]/(f*Cos[e + f*x]))   Subst[Int[x^m*((a + b*ff^2*x^2)
^p/(1 - ff^2*x^2)^((m + 1)/2)), x], x, Sin[e + f*x]/ff], x]] /; FreeQ[{a, b
, e, f, p}, x] && IntegerQ[m/2] &&  !IntegerQ[p]
```

Maple [A] (verified)

Time = 9.51 (sec) , antiderivative size = 661, normalized size of antiderivative = 1.86

method	result
default	$\frac{\left(-8\sqrt{-\frac{b}{a}}\,a\,b^2-8\sqrt{-\frac{b}{a}}\,b^3\right)\cosh(fx+e)^6\sinh(fx+e)+\left(-13\sqrt{-\frac{b}{a}}\,a^2b+2\sqrt{-\frac{b}{a}}\,a\,b^2+11\sqrt{-\frac{b}{a}}\,b^3\right)\cosh(fx+e)^4\sinh(fx+e)+\sqrt{\cdots}}{}$
risch	Expression too large to display

input
```
int(tanh(f*x+e)^4/(a+b*sinh(f*x+e)^2)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
1/3*((-8*(-b/a)^(1/2)*a*b^2-8*(-b/a)^(1/2)*b^3)*cosh(f*x+e)^6*sinh(f*x+e)+
(-13*(-b/a)^(1/2)*a^2*b+2*(-b/a)^(1/2)*a*b^2+11*(-b/a)^(1/2)*b^3)*cosh(f*x
+e)^4*sinh(f*x+e)+(cosh(f*x+e)^2)^(1/2)*(b/a*cosh(f*x+e)^2+(a-b)/a)^(1/2)*
b*(3*EllipticF(sinh(f*x+e)*(-b/a)^(1/2),(1/b*a)^(1/2))*a^2+2*EllipticF(sin
h(f*x+e)*(-b/a)^(1/2),(1/b*a)^(1/2))*a*b-5*EllipticF(sinh(f*x+e)*(-b/a)^(1
/2),(1/b*a)^(1/2))*b^2+8*EllipticE(sinh(f*x+e)*(-b/a)^(1/2),(1/b*a)^(1/2))
*a*b+8*EllipticE(sinh(f*x+e)*(-b/a)^(1/2),(1/b*a)^(1/2))*b^2)*cosh(f*x+e)^
4+(-4*(-b/a)^(1/2)*a^3+6*(-b/a)^(1/2)*a^2*b-2*(-b/a)^(1/2)*b^3)*cosh(f*x+e
)^2*sinh(f*x+e)+(cosh(f*x+e)^2)^(1/2)*(b/a*cosh(f*x+e)^2+(a-b)/a)^(1/2)*(3
*EllipticF(sinh(f*x+e)*(-b/a)^(1/2),(1/b*a)^(1/2))*a^3-EllipticF(sinh(f*x+
e)*(-b/a)^(1/2),(1/b*a)^(1/2))*a^2*b-7*EllipticF(sinh(f*x+e)*(-b/a)^(1/2),
(1/b*a)^(1/2))*a*b^2+5*EllipticF(sinh(f*x+e)*(-b/a)^(1/2),(1/b*a)^(1/2))*b
^3+8*EllipticE(sinh(f*x+e)*(-b/a)^(1/2),(1/b*a)^(1/2))*a^2*b-8*EllipticE(s
inh(f*x+e)*(-b/a)^(1/2),(1/b*a)^(1/2))*b^3)*cosh(f*x+e)^2+((-b/a)^(1/2)*a^
3-3*(-b/a)^(1/2)*a^2*b+3*(-b/a)^(1/2)*a*b^2-(-b/a)^(1/2)*b^3)*sinh(f*x+e))
/(a+b*sinh(f*x+e)^2)^(3/2)/(-b/a)^(1/2)/cosh(f*x+e)^3/(a-b)^4/f
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 15718 vs. $2(336) = 672$.

Time = 0.84 (sec) , antiderivative size = 15718, normalized size of antiderivative = 44.15

$$\int \frac{\tanh^4(e+fx)}{\left(a+b\sinh^2(e+fx)\right)^{5/2}}\,dx = \text{Too large to display}$$

input
```
integrate(tanh(f*x+e)^4/(a+b*sinh(f*x+e)^2)^(5/2),x, algorithm="fricas")
```

output
```
Too large to include
```

Sympy [F]

$$\int \frac{\tanh^4(e+fx)}{\left(a+b\sinh^2(e+fx)\right)^{5/2}}\,dx=\int \frac{\tanh^4\left(e+fx\right)}{\left(a+b\sinh^2\left(e+fx\right)\right)^{\frac{5}{2}}}\,dx$$

input
```
integrate(tanh(f*x+e)**4/(a+b*sinh(f*x+e)**2)**(5/2),x)
```

output
```
Integral(tanh(e + f*x)**4/(a + b*sinh(e + f*x)**2)**(5/2), x)
```

Maxima [F]

$$\int \frac{\tanh^4(e+fx)}{\left(a+b\sinh^2(e+fx)\right)^{5/2}}\,dx=\int \frac{\tanh\left(fx+e\right)^4}{\left(b\sinh\left(fx+e\right)^2+a\right)^{\frac{5}{2}}}\,dx$$

input
```
integrate(tanh(f*x+e)^4/(a+b*sinh(f*x+e)^2)^(5/2),x, algorithm="maxima")
```

output
```
integrate(tanh(f*x + e)^4/(b*sinh(f*x + e)^2 + a)^(5/2), x)
```

Giac [F]

$$\int \frac{\tanh^4(e+fx)}{\left(a+b\sinh^2(e+fx)\right)^{5/2}}\,dx=\int \frac{\tanh\left(fx+e\right)^4}{\left(b\sinh\left(fx+e\right)^2+a\right)^{\frac{5}{2}}}\,dx$$

input
```
integrate(tanh(f*x+e)^4/(a+b*sinh(f*x+e)^2)^(5/2),x, algorithm="giac")
```

output
```
integrate(tanh(f*x + e)^4/(b*sinh(f*x + e)^2 + a)^(5/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\tanh^4(e+fx)}{\left(a+b\sinh^2(e+fx)\right)^{5/2}}\,dx=\int \frac{\tanh(e+f\,x)^4}{\left(b\sinh\left(e+f\,x\right)^2+a\right)^{5/2}}\,dx$$

input
```
int(tanh(e + f*x)^4/(a + b*sinh(e + f*x)^2)^(5/2),x)
```

output
```
int(tanh(e + f*x)^4/(a + b*sinh(e + f*x)^2)^(5/2), x)
```

Reduce [F]

$$\int \frac{\tanh^4(e+fx)}{\left(a+b\sinh^2(e+fx)\right)^{5/2}}\,dx=\int \frac{\sqrt{\sinh\left(fx+e\right)^2b+a}\,\tanh\left(fx+e\right)^4}{\sinh\left(fx+e\right)^6b^3+3\sinh\left(fx+e\right)^4ab^2+3\sinh\left(fx+e\right)^2a^2b+a^3}dx$$

input
```
int(tanh(f*x+e)^4/(a+b*sinh(f*x+e)^2)^(5/2),x)
```

output
```
int((sqrt(sinh(e + f*x)**2*b + a)*tanh(e + f*x)**4)/(sinh(e + f*x)**6*b**3
 + 3*sinh(e + f*x)**4*a*b**2 + 3*sinh(e + f*x)**2*a**2*b + a**3),x)
```

3.472 $\int \frac{\tanh^2(e+fx)}{(a+b\sinh^2(e+fx))^{5/2}}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 292

$$\int \frac{\tanh^2(e+fx)}{(a+b\sinh^2(e+fx))^{5/2}}\,dx = -\frac{4b\cosh(e+fx)\sinh(e+fx)}{3(a-b)^2f\,(a+b\sinh^2(e+fx))^{3/2}} - \frac{\sqrt{b}(7a+b)\cosh(e+fx)E\left(\arctan\left(\frac{\sqrt{b}\sinh(e+fx)}{\sqrt{a}}\right)|1-\frac{a}{b}\right)}{3\sqrt{a}(a-b)^3f\sqrt{\frac{a\cosh^2(e+fx)}{a+b\sinh^2(e+fx)}}\sqrt{a+b\sinh^2(e+fx)}} + \frac{\sqrt{a}(3a+5b)\cosh(e+fx)\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{b}\sinh(e+fx)}{\sqrt{a}}\right),1-\frac{a}{b}\right)}{3(a-b)^3\sqrt{b}f\sqrt{\frac{a\cosh^2(e+fx)}{a+b\sinh^2(e+fx)}}\sqrt{a+b\sinh^2(e+fx)}} - \frac{\tanh(e+fx)}{(a-b)f\,(a+b\sinh^2(e+fx))^{3/2}}$$

output
```
-4/3*b*cosh(f*x+e)*sinh(f*x+e)/(a-b)^2/f/(a+b*sinh(f*x+e)^2)^(3/2)-1/3*b^(
1/2)*(7*a+b)*cosh(f*x+e)*EllipticE(b^(1/2)*sinh(f*x+e)/a^(1/2)/(1+b*sinh(f
*x+e)^2/a)^(1/2),(1-a/b)^(1/2))/a^(1/2)/(a-b)^3/f/(a*cosh(f*x+e)^2/(a+b*si
nh(f*x+e)^2))^(1/2)/(a+b*sinh(f*x+e)^2)^(1/2)+1/3*a^(1/2)*(3*a+5*b)*cosh(f
*x+e)*InverseJacobiAM(arctan(b^(1/2)*sinh(f*x+e)/a^(1/2)),(1-a/b)^(1/2))/(
a-b)^3/b^(1/2)/f/(a*cosh(f*x+e)^2/(a+b*sinh(f*x+e)^2))^(1/2)/(a+b*sinh(f*x
+e)^2)^(1/2)-tanh(f*x+e)/(a-b)/f/(a+b*sinh(f*x+e)^2)^(3/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 2.16 (sec) , antiderivative size = 215, normalized size of antiderivative = 0.74

$$\int \frac{\tanh^2(e+fx)}{\left(a+b\sinh^2(e+fx)\right)^{5/2}}\,dx = \frac{-2ia^2(7a+b)\left(\frac{2a-b+b\cosh(2(e+fx))}{a}\right)^{3/2}E\left(i(e+fx)\,\middle|\,\frac{b}{a}\right)+8ia^2(a-b)\left(\frac{2a\cdots}{}\right.}{6\cdots}$$

input
```
Integrate[Tanh[e + f*x]^2/(a + b*Sinh[e + f*x]^2)^(5/2),x]
```

output
```
((-2*I)*a^2*(7*a + b)*((2*a - b + b*Cosh[2*(e + f*x)])/a)^(3/2)*EllipticE[
I*(e + f*x), b/a] + (8*I)*a^2*(a - b)*((2*a - b + b*Cosh[2*(e + f*x)])/a)^
(3/2)*EllipticF[I*(e + f*x), b/a] - ((24*a^3 - 4*a^2*b + 5*a*b^2 - b^3 + 4
*a*(11*a - 3*b)*b*Cosh[2*(e + f*x)] + b^2*(7*a + b)*Cosh[4*(e + f*x)])*Tan
h[e + f*x])/Sqrt[2])/(6*a*(a - b)^3*f*(2*a - b + b*Cosh[2*(e + f*x)])^(3/2
))
```

Rubi [A] (verified)

Time = 0.51 (sec) , antiderivative size = 344, normalized size of antiderivative = 1.18, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.360$, Rules used = {3042, 25, 3675, 373, 402, 27, 400, 313, 320}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\tanh^2(e+fx)}{\left(a+b\sinh^2(e+fx)\right)^{5/2}}\,dx$$

$$\Big\downarrow \; 3042$$

$$\int -\frac{\tan(ie+ifx)^2}{\left(a-b\sin(ie+ifx)^2\right)^{5/2}}dx$$

$$\Big\downarrow \; 25$$

$$-\int \frac{\tan(ie+ifx)^2}{\left(a-b\sin(ie+ifx)^2\right)^{5/2}}dx$$

$\Big\downarrow$ 3675

$$\frac{\sqrt{\cosh^2(e+fx)}\operatorname{sech}(e+fx)\int \frac{\sinh^2(e+fx)}{(\sinh^2(e+fx)+1)^{3/2}(b\sinh^2(e+fx)+a)^{5/2}}d\sinh(e+fx)}{f}$$

$\Big\downarrow$ 373

$$\frac{\sqrt{\cosh^2(e+fx)}\operatorname{sech}(e+fx)\left(\frac{\int \frac{a-3b\sinh^2(e+fx)}{\sqrt{\sinh^2(e+fx)+1}\left(b\sinh^2(e+fx)+a\right)^{5/2}}d\sinh(e+fx)}{a-b}-\frac{\sinh(e+fx)}{(a-b)\sqrt{\sinh^2(e+fx)+1}\left(a+b\sinh^2(e+fx)\right)^{3/2}}\right.}{f}$$

$\Big\downarrow$ 402

$$\frac{\sqrt{\cosh^2(e+fx)}\operatorname{sech}(e+fx)\left(\frac{\frac{\int \frac{a\left(-4b\sinh^2(e+fx)+3a+b\right)}{\sqrt{\sinh^2(e+fx)+1}\left(b\sinh^2(e+fx)+a\right)^{3/2}}d\sinh(e+fx)}{3a(a-b)}-\frac{4b\sinh(e+fx)\sqrt{\sinh^2(e+fx)+1}}{3(a-b)\left(a+b\sinh^2(e+fx)\right)^{3/2}}}{a-b}-\frac{}{(a-b)\sqrt{\sinh^2(e+}}\right.}{f}$$

$\Big\downarrow$ 27

$$\frac{\sqrt{\cosh^2(e+fx)}\operatorname{sech}(e+fx)\left(\frac{\frac{\int \frac{-4b\sinh^2(e+fx)+3a+b}{\sqrt{\sinh^2(e+fx)+1}\left(b\sinh^2(e+fx)+a\right)^{3/2}}d\sinh(e+fx)}{3(a-b)}-\frac{4b\sinh(e+fx)\sqrt{\sinh^2(e+fx)+1}}{3(a-b)\left(a+b\sinh^2(e+fx)\right)^{3/2}}}{a-b}-\frac{}{(a-b)\sqrt{\sinh^2(e+}}\right.}{f}$$

$\Big\downarrow$ 400

$$\frac{\sqrt{\cosh^2(e+fx)}\operatorname{sech}(e+fx)\left(\frac{\frac{\frac{(3a+5b)\int \frac{1}{\sqrt{\sinh^2(e+fx)+1}\sqrt{b\sinh^2(e+fx)+a}}d\sinh(e+fx)}{a-b}-\frac{b(7a+b)\int \frac{\sqrt{\sinh^2(e+fx)+1}}{\left(b\sinh^2(e+fx)+a\right)^{3/2}}d\sinh(e+fx)}{a-b}}{3(a-b)}-\frac{4}{3}}{a-b}\right.}{f}$$

$\Big\downarrow$ 313

$$\frac{\sqrt{\cosh^2(e+fx)}\operatorname{sech}(e+fx)\left(\frac{\frac{\frac{(3a+5b)\int \frac{1}{\sqrt{\sinh^2(e+fx)+1}\sqrt{b\sinh^2(e+fx)+a}}d\sinh(e+fx)}{a-b}-\frac{\sqrt{b}(7a+b)\sqrt{\sinh^2(e+fx)+1}E\left(\arctan\left(\frac{\sqrt{b}\sinh(e+f}{\sqrt{a}}\right.\right.}{\sqrt{a}(a-b)\sqrt{\frac{a\left(\sinh^2(e+fx)+1\right)}{a+b\sinh^2(e+fx)}}\sqrt{a+b\sinh^2(e}}}{3(a-b)}}{a-b}\right.}{f}$$

$\downarrow$ 320

$$\frac{\sqrt{\cosh^2(e+fx)}\operatorname{sech}(e+fx)\left(\frac{\frac{\frac{(3a+5b)\sqrt{a+b\sinh^2(e+fx)}\,\text{EllipticF}\left(\arctan(\sinh(e+fx)),1-\frac{b}{a}\right)}{a(a-b)\sqrt{\sinh^2(e+fx)+1}\sqrt{\frac{a+b\sinh^2(e+fx)}{a\left(\sinh^2(e+fx)+1\right)}}}-\frac{\sqrt{b}(7a+b)\sqrt{\sinh^2(e+fx)+1}E\left(\arctan\left(\frac{\sqrt{b}\sinh(e}{\sqrt{a}}\right.\right.}{\sqrt{a}(a-b)\sqrt{\frac{a\left(\sinh^2(e+fx)+1\right)}{a+b\sinh^2(e+fx)}}\sqrt{a+b\sinh}}}{3(a-b)}}{a-b}\right.}{f}$$

input
```
Int[Tanh[e + f*x]^2/(a + b*Sinh[e + f*x]^2)^(5/2),x]
```

output
```
(Sqrt[Cosh[e + f*x]^2]*Sech[e + f*x]*(-(Sinh[e + f*x]/((a - b)*Sqrt[1 + Si
nh[e + f*x]^2]*(a + b*Sinh[e + f*x]^2)^(3/2))) + ((-4*b*Sinh[e + f*x]*Sqrt
[1 + Sinh[e + f*x]^2])/(3*(a - b)*(a + b*Sinh[e + f*x]^2)^(3/2)) + (-((Sqr
t[b]*(7*a + b)*EllipticE[ArcTan[(Sqrt[b]*Sinh[e + f*x])/Sqrt[a]], 1 - a/b]
*Sqrt[1 + Sinh[e + f*x]^2])/(Sqrt[a]*(a - b)*Sqrt[(a*(1 + Sinh[e + f*x]^2)
)/(a + b*Sinh[e + f*x]^2)]*Sqrt[a + b*Sinh[e + f*x]^2])) + ((3*a + 5*b)*El
lipticF[ArcTan[Sinh[e + f*x]], 1 - b/a]*Sqrt[a + b*Sinh[e + f*x]^2])/(a*(a
 - b)*Sqrt[1 + Sinh[e + f*x]^2]*Sqrt[(a + b*Sinh[e + f*x]^2)/(a*(1 + Sinh[
e + f*x]^2))]))/(3*(a - b)))/(a - b)))/f
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 313
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/((c_) + (d_.)*(x_)^2)^(3/2), x_Symbol] :> Sim
p[(Sqrt[a + b*x^2]/(c*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(c
+ d*x^2)))]))*EllipticE[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; FreeQ
[{a, b, c, d}, x] && PosQ[b/a] && PosQ[d/c]
```

rule 320
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(Sqrt[a + b*x^2]/(a*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(
c + d*x^2)))]))*EllipticF[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; Fre
eQ[{a, b, c, d}, x] && PosQ[d/c] && PosQ[b/a] &&  !SimplerSqrtQ[b/a, d/c]
```

rule 373
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_
), x_Symbol] :> Simp[e*(e*x)^(m - 1)*(a + b*x^2)^(p + 1)*((c + d*x^2)^(q +
1)/(2*(b*c - a*d)*(p + 1))), x] - Simp[e^2/(2*(b*c - a*d)*(p + 1))   Int[(e
*x)^(m - 2)*(a + b*x^2)^(p + 1)*(c + d*x^2)^q*Simp[c*(m - 1) + d*(m + 2*p +
 2*q + 3)*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, q}, x] && NeQ[b*c - a*d,
 0] && LtQ[p, -1] && GtQ[m, 1] && LeQ[m, 3] && IntBinomialQ[a, b, c, d, e,
m, 2, p, q, x]
```

rule 400
```
Int[((e_) + (f_.)*(x_)^2)/(Sqrt[(a_) + (b_.)*(x_)^2]*((c_) + (d_.)*(x_)^2)^
(3/2)), x_Symbol] :> Simp[(b*e - a*f)/(b*c - a*d)   Int[1/(Sqrt[a + b*x^2]*
Sqrt[c + d*x^2]), x], x] - Simp[(d*e - c*f)/(b*c - a*d)   Int[Sqrt[a + b*x^
2]/(c + d*x^2)^(3/2), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && PosQ[b/a] &
& PosQ[d/c]
```

rule 402
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_.)*((e_) + (f_.)*(x
_)^2), x_Symbol] :> Simp[(-(b*e - a*f))*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^
(q + 1)/(a*2*(b*c - a*d)*(p + 1))), x] + Simp[1/(a*2*(b*c - a*d)*(p + 1))
 Int[(a + b*x^2)^(p + 1)*(c + d*x^2)^q*Simp[c*(b*e - a*f) + e*2*(b*c - a*d)
*(p + 1) + d*(b*e - a*f)*(2*(p + q + 2) + 1)*x^2, x], x], x] /; FreeQ[{a, b
, c, d, e, f, q}, x] && LtQ[p, -1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3675
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_.)*tan[(e_.) + (f_.)*(x_)]^
(m_), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Simp[ff^(m + 1
)*(Sqrt[Cos[e + f*x]^2]/(f*Cos[e + f*x]))   Subst[Int[x^m*((a + b*ff^2*x^2)
^p/(1 - ff^2*x^2)^((m + 1)/2)), x], x, Sin[e + f*x]/ff], x]] /; FreeQ[{a, b
, e, f, p}, x] && IntegerQ[m/2] &&  !IntegerQ[p]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 796 vs. $2(278) = 556$.

Time = 6.42 (sec) , antiderivative size = 797, normalized size of antiderivative = 2.73

method	result
default	$-\frac{7\sqrt{-\frac{b}{a}}\,a\,b^{2}\sinh(fx+e)^{5}+\sqrt{-\frac{b}{a}}\,b^{3}\sinh(fx+e)^{5}-3\sqrt{\frac{a+b\sinh(fx+e)^{2}}{a}}\,\sqrt{\frac{\cosh(2fx+2e)}{2}+\frac{1}{2}}\,\operatorname{EllipticF}\left(\sinh(fx+e)\sqrt{-\frac{b}{a}},\sqrt{\frac{a}{b}}\right)a^{2}b}{}$
risch	Expression too large to display

input
```
int(tanh(f*x+e)^2/(a+b*sinh(f*x+e)^2)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
-1/3*(7*(-b/a)^(1/2)*a*b^2*sinh(f*x+e)^5+(-b/a)^(1/2)*b^3*sinh(f*x+e)^5-3*
((a+b*sinh(f*x+e)^2)/a)^(1/2)*(cosh(f*x+e)^2)^(1/2)*EllipticF(sinh(f*x+e)*
(-b/a)^(1/2),(1/b*a)^(1/2))*a^2*b*sinh(f*x+e)^2+2*((a+b*sinh(f*x+e)^2)/a)^
(1/2)*(cosh(f*x+e)^2)^(1/2)*EllipticF(sinh(f*x+e)*(-b/a)^(1/2),(1/b*a)^(1/
2))*a*b^2*sinh(f*x+e)^2+((a+b*sinh(f*x+e)^2)/a)^(1/2)*(cosh(f*x+e)^2)^(1/2
)*EllipticF(sinh(f*x+e)*(-b/a)^(1/2),(1/b*a)^(1/2))*b^3*sinh(f*x+e)^2-7*((
a+b*sinh(f*x+e)^2)/a)^(1/2)*(cosh(f*x+e)^2)^(1/2)*EllipticE(sinh(f*x+e)*(-
b/a)^(1/2),(1/b*a)^(1/2))*a*b^2*sinh(f*x+e)^2-((a+b*sinh(f*x+e)^2)/a)^(1/2
)*(cosh(f*x+e)^2)^(1/2)*EllipticE(sinh(f*x+e)*(-b/a)^(1/2),(1/b*a)^(1/2))*
b^3*sinh(f*x+e)^2+11*(-b/a)^(1/2)*a^2*b*sinh(f*x+e)^3+4*(-b/a)^(1/2)*a*b^2
*sinh(f*x+e)^3+(-b/a)^(1/2)*b^3*sinh(f*x+e)^3-3*((a+b*sinh(f*x+e)^2)/a)^(1
/2)*(cosh(f*x+e)^2)^(1/2)*EllipticF(sinh(f*x+e)*(-b/a)^(1/2),(1/b*a)^(1/2)
)*a^3+2*((a+b*sinh(f*x+e)^2)/a)^(1/2)*(cosh(f*x+e)^2)^(1/2)*EllipticF(sinh
(f*x+e)*(-b/a)^(1/2),(1/b*a)^(1/2))*a^2*b+((a+b*sinh(f*x+e)^2)/a)^(1/2)*(c
osh(f*x+e)^2)^(1/2)*EllipticF(sinh(f*x+e)*(-b/a)^(1/2),(1/b*a)^(1/2))*a*b^
2-7*((a+b*sinh(f*x+e)^2)/a)^(1/2)*(cosh(f*x+e)^2)^(1/2)*EllipticE(sinh(f*x
+e)*(-b/a)^(1/2),(1/b*a)^(1/2))*a^2*b-((a+b*sinh(f*x+e)^2)/a)^(1/2)*(cosh(
f*x+e)^2)^(1/2)*EllipticE(sinh(f*x+e)*(-b/a)^(1/2),(1/b*a)^(1/2))*a*b^2+3*
(-b/a)^(1/2)*a^3*sinh(f*x+e)+5*sinh(f*x+e)*b*a^2*(-b/a)^(1/2))/(-b/a)^(1/2
)/(a+b*sinh(f*x+e)^2)^(3/2)/(a-b)^3/a/cosh(f*x+e)/f
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 8226 vs. $2(278) = 556$.

Time = 0.36 (sec) , antiderivative size = 8226, normalized size of antiderivative = 28.17

$$\int \frac{\tanh^2(e+fx)}{\left(a+b\sinh^2(e+fx)\right)^{5/2}}\,dx = \text{Too large to display}$$

input
```
integrate(tanh(f*x+e)^2/(a+b*sinh(f*x+e)^2)^(5/2),x, algorithm="fricas")
```

output
```
Too large to include
```

Sympy [F]

$$\int \frac{\tanh^2(e+fx)}{(a+b\sinh^2(e+fx))^{5/2}}\,dx = \int \frac{\tanh^2(e+fx)}{(a+b\sinh^2(e+fx))^{\frac{5}{2}}}\,dx$$

input
```
integrate(tanh(f*x+e)**2/(a+b*sinh(f*x+e)**2)**(5/2),x)
```

output
```
Integral(tanh(e + f*x)**2/(a + b*sinh(e + f*x)**2)**(5/2), x)
```

Maxima [F]

$$\int \frac{\tanh^2(e+fx)}{(a+b\sinh^2(e+fx))^{5/2}}\,dx = \int \frac{\tanh(fx+e)^2}{(b\sinh(fx+e)^2+a)^{\frac{5}{2}}}\,dx$$

input
```
integrate(tanh(f*x+e)^2/(a+b*sinh(f*x+e)^2)^(5/2),x, algorithm="maxima")
```

output
```
integrate(tanh(f*x + e)^2/(b*sinh(f*x + e)^2 + a)^(5/2), x)
```

Giac [F]

$$\int \frac{\tanh^2(e+fx)}{(a+b\sinh^2(e+fx))^{5/2}}\,dx = \int \frac{\tanh(fx+e)^2}{(b\sinh(fx+e)^2+a)^{\frac{5}{2}}}\,dx$$

input
```
integrate(tanh(f*x+e)^2/(a+b*sinh(f*x+e)^2)^(5/2),x, algorithm="giac")
```

output
```
integrate(tanh(f*x + e)^2/(b*sinh(f*x + e)^2 + a)^(5/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\tanh^2(e+fx)}{\left(a+b\sinh^2(e+fx)\right)^{5/2}}\,dx=\int \frac{\tanh(e+f\,x)^2}{\left(b\sinh\left(e+f\,x\right)^2+a\right)^{5/2}}\,dx$$

input
```
int(tanh(e + f*x)^2/(a + b*sinh(e + f*x)^2)^(5/2),x)
```

output
```
int(tanh(e + f*x)^2/(a + b*sinh(e + f*x)^2)^(5/2), x)
```

Reduce [F]

$$\int \frac{\tanh^2(e+fx)}{\left(a+b\sinh^2(e+fx)\right)^{5/2}}\,dx=\int \frac{\sqrt{\sinh\left(fx+e\right)^2b+a}\,\tanh\left(fx+e\right)^2}{\sinh\left(fx+e\right)^6b^3+3\sinh\left(fx+e\right)^4a\,b^2+3\sinh\left(fx+e\right)^2a^2b+a^3}dx$$

input
```
int(tanh(f*x+e)^2/(a+b*sinh(f*x+e)^2)^(5/2),x)
```

output
```
int((sqrt(sinh(e + f*x)**2*b + a)*tanh(e + f*x)**2)/(sinh(e + f*x)**6*b**3
 + 3*sinh(e + f*x)**4*a*b**2 + 3*sinh(e + f*x)**2*a**2*b + a**3),x)
```

3.473 $\int \frac{1}{\left(a+b\sinh^2(e+fx)\right)^{5/2}}\,dx$

Optimal result

Integrand size = 16, antiderivative size = 253

$$\begin{aligned}\int \frac{1}{\left(a+b\sinh^2(e+fx)\right)^{5/2}}\,dx &= -\frac{b\cosh(e+fx)\sinh(e+fx)}{3a(a-b)f\left(a+b\sinh^2(e+fx)\right)^{3/2}}\\ &\quad-\frac{2(2a-b)b\cosh(e+fx)\sinh(e+fx)}{3a^2(a-b)^2f\sqrt{a+b\sinh^2(e+fx)}}\\ &\quad-\frac{2i(2a-b)E\left(ie+ifx\middle|\frac{b}{a}\right)\sqrt{a+b\sinh^2(e+fx)}}{3a^2(a-b)^2f\sqrt{\frac{a+b\sinh^2(e+fx)}{a}}}\\ &\quad+\frac{i\,\text{EllipticF}\left(ie+ifx,\frac{b}{a}\right)\sqrt{\frac{a+b\sinh^2(e+fx)}{a}}}{3a(a-b)f\sqrt{a+b\sinh^2(e+fx)}}\end{aligned}$$

output
```
-1/3*b*cosh(f*x+e)*sinh(f*x+e)/a/(a-b)/f/(a+b*sinh(f*x+e)^2)^(3/2)-2/3*(2*
a-b)*b*cosh(f*x+e)*sinh(f*x+e)/a^2/(a-b)^2/f/(a+b*sinh(f*x+e)^2)^(1/2)-2/3
*I*(2*a-b)*EllipticE(sin(I*e+I*f*x),(b/a)^(1/2))*(a+b*sinh(f*x+e)^2)^(1/2)
/a^2/(a-b)^2/f/((a+b*sinh(f*x+e)^2)/a)^(1/2)+1/3*I*InverseJacobiAM(I*e+I*f
*x,(b/a)^(1/2))*((a+b*sinh(f*x+e)^2)/a)^(1/2)/a/(a-b)/f/(a+b*sinh(f*x+e)^2
)^(1/2)
```

Mathematica [A] (verified)

Time = 0.93 (sec) , antiderivative size = 190, normalized size of antiderivative = 0.75

$$\int \frac{1}{\left(a+b\sinh^2(e+fx)\right)^{5/2}}\,dx = \frac{-2ia^2(2a-b)\left(\frac{2a-b+b\cosh(2(e+fx))}{a}\right)^{3/2}E\left(i(e+fx)\,\middle|\,\frac{b}{a}\right)+ia^2(a-b)\left(\frac{2a-\cdots}{}\right.\cdots}{3\cdots}$$

input
```
Integrate[(a + b*Sinh[e + f*x]^2)^(-5/2),x]
```

output
```
((-2*I)*a^2*(2*a - b)*((2*a - b + b*Cosh[2*(e + f*x)])/a)^(3/2)*EllipticE[
I*(e + f*x), b/a] + I*a^2*(a - b)*((2*a - b + b*Cosh[2*(e + f*x)])/a)^(3/2
)*EllipticF[I*(e + f*x), b/a] + Sqrt[2]*b*(-5*a^2 + 5*a*b - b^2 + b*(-2*a
+ b)*Cosh[2*(e + f*x)])*Sinh[2*(e + f*x)])/(3*a^2*(a - b)^2*f*(2*a - b + b
*Cosh[2*(e + f*x)])^(3/2))
```

Rubi [A] (verified)

Time = 1.32 (sec) , antiderivative size = 258, normalized size of antiderivative = 1.02, number of steps used = 14, number of rules used = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.875$, Rules used = {3042, 3663, 25, 3042, 3652, 3042, 3651, 3042, 3657, 3042, 3656, 3662, 3042, 3661}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\left(a+b\sinh^2(e+fx)\right)^{5/2}}\,dx$$

$$\Big\downarrow \; 3042$$

$$\int \frac{1}{(a-b\sin(ie+ifx)^2)^{5/2}}dx$$

$$\Big\downarrow \; 3663$$

$$-\frac{\int -\frac{-b\sinh^2(e+fx)+3a-2b}{(b\sinh^2(e+fx)+a)^{3/2}}dx}{3a(a-b)}-\frac{b\sinh(e+fx)\cosh(e+fx)}{3af(a-b)\left(a+b\sinh^2(e+fx)\right)^{3/2}}$$

$\downarrow$ 25

$$\frac{\int \frac{-b\sinh^2(e+fx)+3a-2b}{\left(b\sinh^2(e+fx)+a\right)^{3/2}}dx}{3a(a-b)}-\frac{b\sinh(e+fx)\cosh(e+fx)}{3af(a-b)\left(a+b\sinh^2(e+fx)\right)^{3/2}}$$

$\downarrow$ 3042

$$-\frac{b\sinh(e+fx)\cosh(e+fx)}{3af(a-b)\left(a+b\sinh^2(e+fx)\right)^{3/2}}+\frac{\int \frac{b\sin(ie+ifx)^2+3a-2b}{\left(a-b\sin(ie+ifx)^2\right)^{3/2}}dx}{3a(a-b)}$$

$\downarrow$ 3652

$$\frac{\frac{\int \frac{2(2a-b)b\sinh^2(e+fx)+a(3a-b)}{\sqrt{b\sinh^2(e+fx)+a}}dx}{a(a-b)}-\frac{2b(2a-b)\sinh(e+fx)\cosh(e+fx)}{af(a-b)\sqrt{a+b\sinh^2(e+fx)}}}{3a(a-b)}-\frac{b\sinh(e+fx)\cosh(e+fx)}{3af(a-b)\left(a+b\sinh^2(e+fx)\right)^{3/2}}$$

$\downarrow$ 3042

$$-\frac{b\sinh(e+fx)\cosh(e+fx)}{3af(a-b)\left(a+b\sinh^2(e+fx)\right)^{3/2}}+\frac{-\frac{2b(2a-b)\sinh(e+fx)\cosh(e+fx)}{af(a-b)\sqrt{a+b\sinh^2(e+fx)}}+\frac{\int \frac{a(3a-b)-2(2a-b)b\sin(ie+ifx)^2}{\sqrt{a-b\sin(ie+ifx)^2}}dx}{a(a-b)}}{3a(a-b)}$$

$\downarrow$ 3651

$$\frac{\frac{2(2a-b)\int \sqrt{b\sinh^2(e+fx)+a}dx-a(a-b)\int \frac{1}{\sqrt{b\sinh^2(e+fx)+a}}dx}{a(a-b)}-\frac{2b(2a-b)\sinh(e+fx)\cosh(e+fx)}{af(a-b)\sqrt{a+b\sinh^2(e+fx)}}}{3a(a-b)}-\frac{b\sinh(e+fx)\cosh(e+fx)}{3af(a-b)\left(a+b\sinh^2(e+fx)\right)^{3/2}}$$

$\downarrow$ 3042

$$-\frac{b\sinh(e+fx)\cosh(e+fx)}{3af(a-b)\left(a+b\sinh^2(e+fx)\right)^{3/2}}+\frac{-\frac{2b(2a-b)\sinh(e+fx)\cosh(e+fx)}{af(a-b)\sqrt{a+b\sinh^2(e+fx)}}+\frac{2(2a-b)\int \sqrt{a-b\sin(ie+ifx)^2}dx-a(a-b)\int \frac{1}{\sqrt{a-b\sin(ie+ifx)^2}}dx}{a(a-b)}}{3a(a-b)}$$

$\downarrow$ 3657

$$\frac{-\frac{b\sinh(e+fx)\cosh(e+fx)}{3af(a-b)\left(a+b\sinh^2(e+fx)\right)^{3/2}}+}{}$$

$$\frac{-\frac{2b(2a-b)\sinh(e+fx)\cosh(e+fx)}{af(a-b)\sqrt{a+b\sinh^2(e+fx)}}+\frac{\frac{2(2a-b)\sqrt{a+b\sinh^2(e+fx)}\int\sqrt{\frac{b\sinh^2(e+fx)}{a}+1}dx}{\sqrt{\frac{b\sinh^2(e+fx)}{a}+1}}-a(a-b)\int\frac{1}{\sqrt{a-b\sin(ie+ifx)^2}}dx}{a(a-b)}}{3a(a-b)}$$

$\downarrow$ 3042

$$-\frac{b\sinh(e+fx)\cosh(e+fx)}{3af(a-b)\left(a+b\sinh^2(e+fx)\right)^{3/2}}+$$

$$\frac{-\frac{2b(2a-b)\sinh(e+fx)\cosh(e+fx)}{af(a-b)\sqrt{a+b\sinh^2(e+fx)}}+\frac{\frac{2(2a-b)\sqrt{a+b\sinh^2(e+fx)}\int\sqrt{1-\frac{b\sin(ie+ifx)^2}{a}}dx}{\sqrt{\frac{b\sinh^2(e+fx)}{a}+1}}-a(a-b)\int\frac{1}{\sqrt{a-b\sin(ie+ifx)^2}}dx}{a(a-b)}}{3a(a-b)}$$

$\downarrow$ 3656

$$-\frac{b\sinh(e+fx)\cosh(e+fx)}{3af(a-b)\left(a+b\sinh^2(e+fx)\right)^{3/2}}+$$

$$\frac{-\frac{2b(2a-b)\sinh(e+fx)\cosh(e+fx)}{af(a-b)\sqrt{a+b\sinh^2(e+fx)}}+\frac{-a(a-b)\int\frac{1}{\sqrt{a-b\sin(ie+ifx)^2}}dx-\frac{2i(2a-b)\sqrt{a+b\sinh^2(e+fx)}E\left(ie+ifx\middle|\frac{b}{a}\right)}{f\sqrt{\frac{b\sinh^2(e+fx)}{a}+1}}}{a(a-b)}}{3a(a-b)}$$

$\downarrow$ 3662

$$-\frac{b\sinh(e+fx)\cosh(e+fx)}{3af(a-b)\left(a+b\sinh^2(e+fx)\right)^{3/2}}+$$

$$\frac{-\frac{2b(2a-b)\sinh(e+fx)\cosh(e+fx)}{af(a-b)\sqrt{a+b\sinh^2(e+fx)}}+\frac{-\frac{a(a-b)\sqrt{\frac{b\sinh^2(e+fx)}{a}+1}\int\frac{1}{\sqrt{\frac{b\sinh^2(e+fx)}{a}+1}}dx}{\sqrt{a+b\sinh^2(e+fx)}}-\frac{2i(2a-b)\sqrt{a+b\sinh^2(e+fx)}E\left(ie+ifx\middle|\frac{b}{a}\right)}{f\sqrt{\frac{b\sinh^2(e+fx)}{a}+1}}}{a(a-b)}}{3a(a-b)}$$

$\downarrow$ 3042

$$-\frac{b\sinh(e+fx)\cosh(e+fx)}{3af(a-b)\left(a+b\sinh^2(e+fx)\right)^{3/2}}+$$

$$\frac{-\frac{2b(2a-b)\sinh(e+fx)\cosh(e+fx)}{af(a-b)\sqrt{a+b\sinh^2(e+fx)}}+\frac{-\frac{a(a-b)\sqrt{\frac{b\sinh^2(e+fx)}{a}+1}\int\frac{1}{\sqrt{1-\frac{b\sin(ie+ifx)^2}{a}}}dx}{\sqrt{a+b\sinh^2(e+fx)}}-\frac{2i(2a-b)\sqrt{a+b\sinh^2(e+fx)}E\left(ie+ifx\middle|\frac{b}{a}\right)}{f\sqrt{\frac{b\sinh^2(e+fx)}{a}+1}}}{a(a-b)}}{3a(a-b)}$$

$$\Big\downarrow \text{3661}$$

$$-\frac{b\sinh(e+fx)\cosh(e+fx)}{3af(a-b)\left(a+b\sinh^2(e+fx)\right)^{3/2}}+\frac{-\frac{2b(2a-b)\sinh(e+fx)\cosh(e+fx)}{af(a-b)\sqrt{a+b\sinh^2(e+fx)}}+\frac{\frac{ia(a-b)\sqrt{\frac{b\sinh^2(e+fx)}{a}+1}\operatorname{EllipticF}\left(ie+ifx,\frac{b}{a}\right)}{f\sqrt{a+b\sinh^2(e+fx)}}-\frac{2i(2a-b)\sqrt{a+b\sinh^2(e+fx)}E\left(ie+ifx\middle|\frac{b}{a}\right)}{f\sqrt{\frac{b\sinh^2(e+fx)}{a}+1}}}{a(a-b)}}{3a(a-b)}$$

input
```
Int[(a + b*Sinh[e + f*x]^2)^(-5/2),x]
```

output
```
-1/3*(b*Cosh[e + f*x]*Sinh[e + f*x])/(a*(a - b)*f*(a + b*Sinh[e + f*x]^2)^
(3/2)) + ((-2*(2*a - b)*b*Cosh[e + f*x]*Sinh[e + f*x])/(a*(a - b)*f*Sqrt[a
 + b*Sinh[e + f*x]^2]) + (((-2*I)*(2*a - b)*EllipticE[I*e + I*f*x, b/a]*Sq
rt[a + b*Sinh[e + f*x]^2])/(f*Sqrt[1 + (b*Sinh[e + f*x]^2)/a]) + (I*a*(a -
 b)*EllipticF[I*e + I*f*x, b/a]*Sqrt[1 + (b*Sinh[e + f*x]^2)/a])/(f*Sqrt[a
 + b*Sinh[e + f*x]^2]))/(a*(a - b)))/(3*a*(a - b))
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3651
```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)]^2)/Sqrt[(a_) + (b_.)*sin[(e_.) +
 (f_.)*(x_)]^2], x_Symbol] :> Simp[B/b   Int[Sqrt[a + b*Sin[e + f*x]^2], x]
, x] + Simp[(A*b - a*B)/b   Int[1/Sqrt[a + b*Sin[e + f*x]^2], x], x] /; Fre
eQ[{a, b, e, f, A, B}, x]
```

rule 3652
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)]^2), x_Symbol] :> Simp[(-(A*b - a*B))*Cos[e + f*x]*Sin[e + f*x
]*((a + b*Sin[e + f*x]^2)^(p + 1)/(2*a*f*(a + b)*(p + 1))), x] - Simp[1/(2*
a*(a + b)*(p + 1))   Int[(a + b*Sin[e + f*x]^2)^(p + 1)*Simp[a*B - A*(2*a*(
p + 1) + b*(2*p + 3)) + 2*(A*b - a*B)*(p + 2)*Sin[e + f*x]^2, x], x], x] /;
 FreeQ[{a, b, e, f, A, B}, x] && LtQ[p, -1] && NeQ[a + b, 0]
```

rule 3656
```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2], x_Symbol] :> Simp[(Sqrt[a
]/f)*EllipticE[e + f*x, -b/a], x] /; FreeQ[{a, b, e, f}, x] && GtQ[a, 0]
```

rule 3657
```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2], x_Symbol] :> Simp[Sqrt[a
+ b*Sin[e + f*x]^2]/Sqrt[1 + b*(Sin[e + f*x]^2/a)]   Int[Sqrt[1 + (b*Sin[e
+ f*x]^2)/a], x], x] /; FreeQ[{a, b, e, f}, x] &&  !GtQ[a, 0]
```

rule 3661
```
Int[1/Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2], x_Symbol] :> Simp[(1/(S
qrt[a]*f))*EllipticF[e + f*x, -b/a], x] /; FreeQ[{a, b, e, f}, x] && GtQ[a,
 0]
```

rule 3662
```
Int[1/Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2], x_Symbol] :> Simp[Sqrt[
1 + b*(Sin[e + f*x]^2/a)]/Sqrt[a + b*Sin[e + f*x]^2]   Int[1/Sqrt[1 + (b*Si
n[e + f*x]^2)/a], x], x] /; FreeQ[{a, b, e, f}, x] &&  !GtQ[a, 0]
```

rule 3663
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> Simp[(-b)*C
os[e + f*x]*Sin[e + f*x]*((a + b*Sin[e + f*x]^2)^(p + 1)/(2*a*f*(p + 1)*(a
+ b))), x] + Simp[1/(2*a*(p + 1)*(a + b))   Int[(a + b*Sin[e + f*x]^2)^(p +
 1)*Simp[2*a*(p + 1) + b*(2*p + 3) - 2*b*(p + 2)*Sin[e + f*x]^2, x], x], x]
 /; FreeQ[{a, b, e, f}, x] && NeQ[a + b, 0] && LtQ[p, -1]
```

Maple [A] (verified)

Time = 1.52 (sec) , antiderivative size = 406, normalized size of antiderivative = 1.60

method	result
default	$\frac{\sqrt{\left(a+b\sinh(fx+e)^2\right)\cosh(fx+e)^2}\left(-\frac{\sinh(fx+e)\sqrt{\left(a+b\sinh(fx+e)^2\right)\cosh(fx+e)^2}}{3ba(a-b)\left(\sinh(fx+e)^2+\frac{a}{b}\right)^2}-\frac{2b\cosh(fx+e)^2\sinh(fx+e)(2a-b)}{3a^2(a-b)^2\sqrt{\left(a+b\sinh(fx+e)^2\right)\cosh(fx+e)^2}}+\frac{(3}{}\right.}{}$
risch	Expression too large to display

input
```
int(1/(a+b*sinh(f*x+e)^2)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
((a+b*sinh(f*x+e)^2)*cosh(f*x+e)^2)^(1/2)*(-1/3/b/a/(a-b)*sinh(f*x+e)*((a+
b*sinh(f*x+e)^2)*cosh(f*x+e)^2)^(1/2)/(sinh(f*x+e)^2+1/b*a)^2-2/3*b*cosh(f
*x+e)^2/a^2/(a-b)^2*sinh(f*x+e)*(2*a-b)/((a+b*sinh(f*x+e)^2)*cosh(f*x+e)^2
)^(1/2)+(3*a-b)/(3*a^3-6*a^2*b+3*a*b^2)/(-b/a)^(1/2)*((a+b*sinh(f*x+e)^2)/
a)^(1/2)*(cosh(f*x+e)^2)^(1/2)/((a+b*sinh(f*x+e)^2)*cosh(f*x+e)^2)^(1/2)*E
llipticF(sinh(f*x+e)*(-b/a)^(1/2),(1/b*a)^(1/2))-2/3*b*(2*a-b)/a^2/(a-b)^2
/(-b/a)^(1/2)*((a+b*sinh(f*x+e)^2)/a)^(1/2)*(cosh(f*x+e)^2)^(1/2)/((a+b*si
nh(f*x+e)^2)*cosh(f*x+e)^2)^(1/2)*(EllipticF(sinh(f*x+e)*(-b/a)^(1/2),(1/b
*a)^(1/2))-EllipticE(sinh(f*x+e)*(-b/a)^(1/2),(1/b*a)^(1/2))))/cosh(f*x+e)
/(a+b*sinh(f*x+e)^2)^(1/2)/f
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 5442 vs. $2(225) = 450$.

Time = 0.22 (sec) , antiderivative size = 5442, normalized size of antiderivative = 21.51

$$\int \frac{1}{\left(a+b\sinh^2(e+fx)\right)^{5/2}}\,dx = \text{Too large to display}$$

input
```
integrate(1/(a+b*sinh(f*x+e)^2)^(5/2),x, algorithm="fricas")
```

output
```
Too large to include
```

Sympy [F]

$$\int \frac{1}{\left(a+b\sinh^2(e+fx)\right)^{5/2}}\,dx=\int \frac{1}{\left(a+b\sinh^{2}\left(e+fx\right)\right)^{\frac{5}{2}}}\,dx$$

input
```
integrate(1/(a+b*sinh(f*x+e)**2)**(5/2),x)
```

output
```
Integral((a + b*sinh(e + f*x)**2)**(-5/2), x)
```

Maxima [F]

$$\int \frac{1}{\left(a+b\sinh^2(e+fx)\right)^{5/2}}\,dx=\int \frac{1}{\left(b\sinh\left(fx+e\right)^{2}+a\right)^{\frac{5}{2}}}\,dx$$

input
```
integrate(1/(a+b*sinh(f*x+e)^2)^(5/2),x, algorithm="maxima")
```

output
```
integrate((b*sinh(f*x + e)^2 + a)^(-5/2), x)
```

Giac [F]

$$\int \frac{1}{\left(a+b\sinh^2(e+fx)\right)^{5/2}}\,dx=\int \frac{1}{\left(b\sinh\left(fx+e\right)^{2}+a\right)^{\frac{5}{2}}}\,dx$$

input
```
integrate(1/(a+b*sinh(f*x+e)^2)^(5/2),x, algorithm="giac")
```

output
```
integrate((b*sinh(f*x + e)^2 + a)^(-5/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\left(a+b\sinh^2(e+fx)\right)^{5/2}}\,dx=\int \frac{1}{\left(b\sinh\left(e+f\,x\right)^2+a\right)^{5/2}}\,dx$$

input
```
int(1/(a + b*sinh(e + f*x)^2)^(5/2),x)
```

output
```
int(1/(a + b*sinh(e + f*x)^2)^(5/2), x)
```

Reduce [F]

$$\int \frac{1}{\left(a+b\sinh^2(e+fx)\right)^{5/2}}\,dx=\int \frac{\sqrt{\sinh\left(fx+e\right)^2b+a}}{\sinh\left(fx+e\right)^6b^3+3\sinh\left(fx+e\right)^4a\,b^2+3\sinh\left(fx+e\right)^2a^2b+a^3}dx$$

input
```
int(1/(a+b*sinh(f*x+e)^2)^(5/2),x)
```

output
```
int(sqrt(sinh(e + f*x)**2*b + a)/(sinh(e + f*x)**6*b**3 + 3*sinh(e + f*x)*
*4*a*b**2 + 3*sinh(e + f*x)**2*a**2*b + a**3),x)
```

3.474 $\int \frac{\coth^2(e+fx)}{(a+b\sinh^2(e+fx))^{5/2}}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 281

$$\begin{aligned}\int \frac{\coth^2(e+fx)}{(a+b\sinh^2(e+fx))^{5/2}}\,dx &= \frac{\coth(e+fx)}{3af\,(a+b\sinh^2(e+fx))^{3/2}} \\ &- \frac{4\coth(e+fx)}{3a^2f\sqrt{a+b\sinh^2(e+fx)}} \\ &- \frac{(7a-8b)\sqrt{b}\cosh(e+fx)E\left(\arctan\left(\frac{\sqrt{b}\sinh(e+fx)}{\sqrt{a}}\right)|1-\frac{a}{b}\right)}{3a^{5/2}(a-b)f\sqrt{\frac{a\cosh^2(e+fx)}{a+b\sinh^2(e+fx)}}\sqrt{a+b\sinh^2(e+fx)}} \\ &+ \frac{(3a-4b)\cosh(e+fx)\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{b}\sinh(e+fx)}{\sqrt{a}}\right),1-\frac{a}{b}\right)}{3a^{3/2}(a-b)\sqrt{b}f\sqrt{\frac{a\cosh^2(e+fx)}{a+b\sinh^2(e+fx)}}\sqrt{a+b\sinh^2(e+fx)}}\end{aligned}$$

output
```
1/3*coth(f*x+e)/a/f/(a+b*sinh(f*x+e)^2)^(3/2)-4/3*coth(f*x+e)/a^2/f/(a+b*s
inh(f*x+e)^2)^(1/2)-1/3*(7*a-8*b)*b^(1/2)*cosh(f*x+e)*EllipticE(b^(1/2)*si
nh(f*x+e)/a^(1/2)/(1+b*sinh(f*x+e)^2/a)^(1/2),(1-a/b)^(1/2))/a^(5/2)/(a-b)
/f/(a*cosh(f*x+e)^2/(a+b*sinh(f*x+e)^2))^(1/2)/(a+b*sinh(f*x+e)^2)^(1/2)+1
/3*(3*a-4*b)*cosh(f*x+e)*InverseJacobiAM(arctan(b^(1/2)*sinh(f*x+e)/a^(1/2
)),(1-a/b)^(1/2))/a^(3/2)/(a-b)/b^(1/2)/f/(a*cosh(f*x+e)^2/(a+b*sinh(f*x+e
)^2))^(1/2)/(a+b*sinh(f*x+e)^2)^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 2.30 (sec) , antiderivative size = 226, normalized size of antiderivative = 0.80

$$\int \frac{\coth^2(e+fx)}{\left(a+b\sinh^2(e+fx)\right)^{5/2}}\,dx = \frac{-\frac{(24a^3-68a^2b+69ab^2-24b^3+4b(11a^2-19ab+8b^2)\cosh(2(e+fx))+(7a-8b)b^2\cosh(4(e+fx)))\operatorname{cot}}{\sqrt{2}}}{}$$

input
```
Integrate[Coth[e + f*x]^2/(a + b*Sinh[e + f*x]^2)^(5/2),x]
```

output
```
(-(((24*a^3 - 68*a^2*b + 69*a*b^2 - 24*b^3 + 4*b*(11*a^2 - 19*a*b + 8*b^2)
*Cosh[2*(e + f*x)] + (7*a - 8*b)*b^2*Cosh[4*(e + f*x)])*Coth[e + f*x])/Sqr
t[2]) - (2*I)*a^2*(7*a - 8*b)*((2*a - b + b*Cosh[2*(e + f*x)])/a)^(3/2)*El
lipticE[I*(e + f*x), b/a] + (8*I)*a^2*(a - b)*((2*a - b + b*Cosh[2*(e + f*
x)])/a)^(3/2)*EllipticF[I*(e + f*x), b/a])/(6*a^3*(a - b)*f*(2*a - b + b*C
osh[2*(e + f*x)])^(3/2))
```

Rubi [A] (verified)

Time = 0.62 (sec) , antiderivative size = 412, normalized size of antiderivative = 1.47, number of steps used = 13, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.480$, Rules used = {3042, 25, 3675, 371, 25, 441, 445, 25, 406, 320, 388, 313}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\coth^2(e+fx)}{\left(a+b\sinh^2(e+fx)\right)^{5/2}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int -\frac{1}{\tan(ie+ifx)^2\left(a-b\sin(ie+ifx)^2\right)^{5/2}}dx$$

$$\Big\downarrow \text{25}$$

$$-\int \frac{1}{\left(a-b\sin(ie+ifx)^2\right)^{5/2}\tan(ie+ifx)^2}dx$$

$$\Big\downarrow \text{3675}$$

$$\frac{\sqrt{\cosh^2(e+fx)}\operatorname{sech}(e+fx)\int \frac{\operatorname{csch}^2(e+fx)\sqrt{\sinh^2(e+fx)+1}}{\left(b\sinh^2(e+fx)+a\right)^{5/2}}d\sinh(e+fx)}{f}$$

$$\Big\downarrow \text{371}$$

$$\frac{\sqrt{\cosh^2(e+fx)}\operatorname{sech}(e+fx)\left(\frac{\sqrt{\sinh^2(e+fx)+1}\operatorname{csch}(e+fx)}{3a\left(a+b\sinh^2(e+fx)\right)^{3/2}}-\frac{\int -\frac{\operatorname{csch}^2(e+fx)\left(3\sinh^2(e+fx)+4\right)}{\sqrt{\sinh^2(e+fx)+1}\left(b\sinh^2(e+fx)+a\right)^{3/2}}d\sinh(e+fx)}{3a}\right)}{f}$$

$$\Big\downarrow \text{25}$$

$$\frac{\sqrt{\cosh^2(e+fx)}\operatorname{sech}(e+fx)\left(\frac{\int \frac{\operatorname{csch}^2(e+fx)\left(3\sinh^2(e+fx)+4\right)}{\sqrt{\sinh^2(e+fx)+1}\left(b\sinh^2(e+fx)+a\right)^{3/2}}d\sinh(e+fx)}{3a}+\frac{\sqrt{\sinh^2(e+fx)+1}\operatorname{csch}(e+fx)}{3a\left(a+b\sinh^2(e+fx)\right)^{3/2}}\right)}{f}$$

$$\Big\downarrow \text{441}$$

$$\frac{\sqrt{\cosh^2(e+fx)}\operatorname{sech}(e+fx)\left(\frac{\frac{\int \frac{\operatorname{csch}^2(e+fx)\left((3a-4b)\sinh^2(e+fx)+7a-8b\right)}{\sqrt{\sinh^2(e+fx)+1}\sqrt{b\sinh^2(e+fx)+a}}d\sinh(e+fx)}{a(a-b)}+\frac{(3a-4b)\sqrt{\sinh^2(e+fx)+1}\operatorname{csch}(e+fx)}{a(a-b)\sqrt{a+b\sinh^2(e+fx)}}}{3a}+\frac{\sqrt{\sin}}{3a}\right.}{f}$$

$$\Big\downarrow \text{445}$$

$$\frac{\sqrt{\cosh^2(e+fx)}\operatorname{sech}(e+fx)\left(\frac{\frac{-\frac{\int -\frac{(7a-8b)b\sinh^2(e+fx)+a(3a-4b)}{\sqrt{\sinh^2(e+fx)+1}\sqrt{b\sinh^2(e+fx)+a}}d\sinh(e+fx)}{a}-\frac{(7a-8b)\sqrt{\sinh^2(e+fx)+1}\operatorname{csch}(e+fx)\sqrt{a+b\sinh^2(e+f}}{a}}{a(a-b)}}{3a}\right.}{f}$$

$$\Big\downarrow \text{25}$$

$$\frac{\sqrt{\cosh^2(e+fx)}\text{sech}(e+fx)\left(\frac{\frac{\frac{\int \frac{(7a-8b)b\sinh^2(e+fx)+a(3a-4b)}{\sqrt{\sinh^2(e+fx)+1}\sqrt{b\sinh^2(e+fx)+a}}d\sinh(e+fx)}{a}-\frac{(7a-8b)\sqrt{\sinh^2(e+fx)+1}\text{csch}(e+fx)\sqrt{a+b\sinh^2(e+fx)}}{a}}{a(a-b)}-\cdots}{3a}\right.\cdots}{f}$$

$\downarrow$ 406

$$\frac{\sqrt{\cosh^2(e+fx)}\text{sech}(e+fx)\left(\frac{\frac{\frac{a(3a-4b)\int \frac{1}{\sqrt{\sinh^2(e+fx)+1}\sqrt{b\sinh^2(e+fx)+a}}d\sinh(e+fx)+b(7a-8b)\int \frac{\sinh^2(e+fx)}{\sqrt{\sinh^2(e+fx)+1}\sqrt{b\sinh^2(e+fx)+a}}d\cdots}{a}\cdots}{a(a-b)}\cdots}{3a\cdots}\right.\cdots}{f}$$

$\downarrow$ 320

$$\frac{\sqrt{\cosh^2(e+fx)}\text{sech}(e+fx)\left(\frac{\frac{\frac{b(7a-8b)\int \frac{\sinh^2(e+fx)}{\sqrt{\sinh^2(e+fx)+1}\sqrt{b\sinh^2(e+fx)+a}}d\sinh(e+fx)+\frac{(3a-4b)\sqrt{a+b\sinh^2(e+fx)}\,\text{EllipticF}\left(\arctan(\sinh(e\cdots\right.}{\sqrt{\sinh^2(e+fx)+1}\sqrt{\frac{a+b\sinh^2(e+fx)}{a\left(\sinh^2(e+fx)+1\right)}}\cdots}}{a}\cdots}{a(a-b)}\cdots}{\cdots}\right.\cdots}{f}$$

$\downarrow$ 388

$$\frac{\sqrt{\cosh^2(e+fx)}\text{sech}(e+fx)\left(\frac{\frac{\frac{b(7a-8b)\left(\frac{\sinh(e+fx)\sqrt{a+b\sinh^2(e+fx)}}{b\sqrt{\sinh^2(e+fx)+1}}-\frac{\int \frac{\sqrt{b\sinh^2(e+fx)+a}}{\left(\sinh^2(e+fx)+1\right)^{3/2}}d\sinh(e+fx)}{b}\right)+\frac{(3a-4b)\sqrt{a+b\sinh^2(e+fx)}\,\text{E}\cdots}{\sqrt{\sinh^2(e+fx)+\cdots}}}{a}\cdots}{a(a-b)}\cdots}{\cdots}\right.\cdots}{\cdots}$$

$\downarrow$ 313

$$\frac{\sqrt{\cosh^2(e+fx)}\operatorname{sech}(e+fx)\left(\frac{\frac{\frac{(3a-4b)\sqrt{a+b\sinh^2(e+fx)}\operatorname{EllipticF}\left(\arctan(\sinh(e+fx)),1-\frac{b}{a}\right)}{\sqrt{\sinh^2(e+fx)+1}\sqrt{\frac{a+b\sinh^2(e+fx)}{a\left(\sinh^2(e+fx)+1\right)}}}+b(7a-8b)\left(\frac{\sinh(e+fx)\sqrt{a+b\sinh^2(e+fx)}}{b\sqrt{\sinh^2(e+fx)+1}}-\frac{\sqrt{a\cdots}}{\cdots}\right.}{a}}{a(a-b\cdots}\cdots\right.}{\cdots}$$

input
```
Int[Coth[e + f*x]^2/(a + b*Sinh[e + f*x]^2)^(5/2),x]
```

output
```
(Sqrt[Cosh[e + f*x]^2]*Sech[e + f*x]*((Csch[e + f*x]*Sqrt[1 + Sinh[e + f*x
]^2])/(3*a*(a + b*Sinh[e + f*x]^2)^(3/2)) + (((3*a - 4*b)*Csch[e + f*x]*Sq
rt[1 + Sinh[e + f*x]^2])/(a*(a - b)*Sqrt[a + b*Sinh[e + f*x]^2]) + (-(((7*
a - 8*b)*Csch[e + f*x]*Sqrt[1 + Sinh[e + f*x]^2]*Sqrt[a + b*Sinh[e + f*x]^
2])/a) + (((3*a - 4*b)*EllipticF[ArcTan[Sinh[e + f*x]], 1 - b/a]*Sqrt[a +
b*Sinh[e + f*x]^2])/(Sqrt[1 + Sinh[e + f*x]^2]*Sqrt[(a + b*Sinh[e + f*x]^2
)/(a*(1 + Sinh[e + f*x]^2))]) + (7*a - 8*b)*b*((Sinh[e + f*x]*Sqrt[a + b*S
inh[e + f*x]^2])/(b*Sqrt[1 + Sinh[e + f*x]^2]) - (EllipticE[ArcTan[Sinh[e
+ f*x]], 1 - b/a]*Sqrt[a + b*Sinh[e + f*x]^2])/(b*Sqrt[1 + Sinh[e + f*x]^2
]*Sqrt[(a + b*Sinh[e + f*x]^2)/(a*(1 + Sinh[e + f*x]^2))])))/a)/(a*(a - b)
))/(3*a)))/f
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 313
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/((c_) + (d_.)*(x_)^2)^(3/2), x_Symbol] :> Sim
p[(Sqrt[a + b*x^2]/(c*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(c
 + d*x^2)))]))*EllipticE[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; FreeQ
[{a, b, c, d}, x] && PosQ[b/a] && PosQ[d/c]
```

rule 320
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(Sqrt[a + b*x^2]/(a*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(
c + d*x^2)))]))*EllipticF[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; Fre
eQ[{a, b, c, d}, x] && PosQ[d/c] && PosQ[b/a] &&  !SimplerSqrtQ[b/a, d/c]
```

rule 371
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_
), x_Symbol] :> Simp[(-(e*x)^(m + 1))*(a + b*x^2)^(p + 1)*((c + d*x^2)^q/(a
*e*2*(p + 1))), x] + Simp[1/(a*2*(p + 1))   Int[(e*x)^m*(a + b*x^2)^(p + 1)
*(c + d*x^2)^(q - 1)*Simp[c*(m + 2*(p + 1) + 1) + d*(m + 2*(p + q + 1) + 1)
*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b*c - a*d, 0] && Lt
Q[p, -1] && LtQ[0, q, 1] && IntBinomialQ[a, b, c, d, e, m, 2, p, q, x]
```

rule 388
```
Int[(x_)^2/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol]
 :> Simp[x*(Sqrt[a + b*x^2]/(b*Sqrt[c + d*x^2])), x] - Simp[c/b   Int[Sqrt[
a + b*x^2]/(c + d*x^2)^(3/2), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
 a*d, 0] && PosQ[b/a] && PosQ[d/c] &&  !SimplerSqrtQ[b/a, d/c]
```

rule 406
```
Int[((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.)*((e_) + (f_.)*(
x_)^2), x_Symbol] :> Simp[e   Int[(a + b*x^2)^p*(c + d*x^2)^q, x], x] + Sim
p[f   Int[x^2*(a + b*x^2)^p*(c + d*x^2)^q, x], x] /; FreeQ[{a, b, c, d, e,
f, p, q}, x]
```

rule 441
```
Int[((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_
)*((e_) + (f_.)*(x_)^2), x_Symbol] :> Simp[(-(b*e - a*f))*(g*x)^(m + 1)*(a
+ b*x^2)^(p + 1)*((c + d*x^2)^(q + 1)/(a*g*2*(b*c - a*d)*(p + 1))), x] + Si
mp[1/(a*2*(b*c - a*d)*(p + 1))   Int[(g*x)^m*(a + b*x^2)^(p + 1)*(c + d*x^2
)^q*Simp[c*(b*e - a*f)*(m + 1) + e*2*(b*c - a*d)*(p + 1) + d*(b*e - a*f)*(m
 + 2*(p + q + 2) + 1)*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, m, q},
 x] && LtQ[p, -1]
```

rule 445
```
Int[((g_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_
.)*((e_) + (f_.)*(x_)^2), x_Symbol] :> Simp[e*(g*x)^(m + 1)*(a + b*x^2)^(p
+ 1)*((c + d*x^2)^(q + 1)/(a*c*g*(m + 1))), x] + Simp[1/(a*c*g^2*(m + 1))
 Int[(g*x)^(m + 2)*(a + b*x^2)^p*(c + d*x^2)^q*Simp[a*f*c*(m + 1) - e*(b*c
+ a*d)*(m + 2 + 1) - e*2*(b*c*p + a*d*q) - b*e*d*(m + 2*(p + q + 2) + 1)*x^
2, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p, q}, x] && LtQ[m, -1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3675
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_.)*tan[(e_.) + (f_.)*(x_)]^
(m_), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Simp[ff^(m + 1
)*(Sqrt[Cos[e + f*x]^2]/(f*Cos[e + f*x]))   Subst[Int[x^m*((a + b*ff^2*x^2)
^p/(1 - ff^2*x^2)^((m + 1)/2)), x], x, Sin[e + f*x]/ff], x]] /; FreeQ[{a, b
, e, f, p}, x] && IntegerQ[m/2] &&  !IntegerQ[p]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 641 vs. $2(265) = 530$.

Time = 6.31 (sec) , antiderivative size = 642, normalized size of antiderivative = 2.28

method	result
default	$-\frac{\left(7\sqrt{-\frac{b}{a}}\, a\, b^{2}-8\sqrt{-\frac{b}{a}}\, b^{3}\right)\cosh(fx+e)^{6}+\left(11\sqrt{-\frac{b}{a}}\, a^{2} b-26\sqrt{-\frac{b}{a}}\, a\, b^{2}+16\sqrt{-\frac{b}{a}}\, b^{3}\right)\cosh(fx+e)^{4}-\sqrt{\frac{b\cosh(fx+e)^{2}}{a}+\frac{a-b}{a}}\,\sqrt{\cos}}{}$
risch	Expression too large to display

input
```
int(coth(f*x+e)^2/(a+b*sinh(f*x+e)^2)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
-1/3*((7*(-b/a)^(1/2)*a*b^2-8*(-b/a)^(1/2)*b^3)*cosh(f*x+e)^6+(11*(-b/a)^(
1/2)*a^2*b-26*(-b/a)^(1/2)*a*b^2+16*(-b/a)^(1/2)*b^3)*cosh(f*x+e)^4-(b/a*c
osh(f*x+e)^2+(a-b)/a)^(1/2)*(cosh(f*x+e)^2)^(1/2)*b*(3*EllipticF(sinh(f*x+
e)*(-b/a)^(1/2),(1/b*a)^(1/2))*a^2-11*EllipticF(sinh(f*x+e)*(-b/a)^(1/2),(
1/b*a)^(1/2))*a*b+8*EllipticF(sinh(f*x+e)*(-b/a)^(1/2),(1/b*a)^(1/2))*b^2+
7*EllipticE(sinh(f*x+e)*(-b/a)^(1/2),(1/b*a)^(1/2))*a*b-8*EllipticE(sinh(f
*x+e)*(-b/a)^(1/2),(1/b*a)^(1/2))*b^2)*cosh(f*x+e)^2*sinh(f*x+e)+(3*(-b/a)
^(1/2)*a^3-14*(-b/a)^(1/2)*a^2*b+19*(-b/a)^(1/2)*a*b^2-8*(-b/a)^(1/2)*b^3)
*cosh(f*x+e)^2-(b/a*cosh(f*x+e)^2+(a-b)/a)^(1/2)*(cosh(f*x+e)^2)^(1/2)*(3*
EllipticF(sinh(f*x+e)*(-b/a)^(1/2),(1/b*a)^(1/2))*a^3-14*EllipticF(sinh(f*
x+e)*(-b/a)^(1/2),(1/b*a)^(1/2))*a^2*b+19*EllipticF(sinh(f*x+e)*(-b/a)^(1/
2),(1/b*a)^(1/2))*a*b^2-8*EllipticF(sinh(f*x+e)*(-b/a)^(1/2),(1/b*a)^(1/2)
)*b^3+7*EllipticE(sinh(f*x+e)*(-b/a)^(1/2),(1/b*a)^(1/2))*a^2*b-15*Ellipti
cE(sinh(f*x+e)*(-b/a)^(1/2),(1/b*a)^(1/2))*a*b^2+8*EllipticE(sinh(f*x+e)*(
-b/a)^(1/2),(1/b*a)^(1/2))*b^3)*sinh(f*x+e))/(-b/a)^(1/2)/sinh(f*x+e)/a^3/
(a-b)/(a+b*sinh(f*x+e)^2)^(3/2)/cosh(f*x+e)/f
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 7847 vs. $2(265) = 530$.

Time = 0.31 (sec) , antiderivative size = 7847, normalized size of antiderivative = 27.93

$$\int \frac{\coth^2(e+fx)}{\left(a+b\sinh^2(e+fx)\right)^{5/2}}\,dx = \text{Too large to display}$$

input
```
integrate(coth(f*x+e)^2/(a+b*sinh(f*x+e)^2)^(5/2),x, algorithm="fricas")
```

output
```
Too large to include
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\coth^2(e+fx)}{\left(a+b\sinh^2(e+fx)\right)^{5/2}}\,dx=\text{Timed out}$$

input
```
integrate(coth(f*x+e)**2/(a+b*sinh(f*x+e)**2)**(5/2),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{\coth^2(e+fx)}{\left(a+b\sinh^2(e+fx)\right)^{5/2}}\,dx=\int \frac{\coth\left(fx+e\right)^2}{\left(b\sinh\left(fx+e\right)^2+a\right)^{\frac{5}{2}}}\,dx$$

input
```
integrate(coth(f*x+e)^2/(a+b*sinh(f*x+e)^2)^(5/2),x, algorithm="maxima")
```

output
```
integrate(coth(f*x + e)^2/(b*sinh(f*x + e)^2 + a)^(5/2), x)
```

Giac [F]

$$\int \frac{\coth^2(e+fx)}{\left(a+b\sinh^2(e+fx)\right)^{5/2}}\,dx=\int \frac{\coth\left(fx+e\right)^2}{\left(b\sinh\left(fx+e\right)^2+a\right)^{\frac{5}{2}}}\,dx$$

input
```
integrate(coth(f*x+e)^2/(a+b*sinh(f*x+e)^2)^(5/2),x, algorithm="giac")
```

output
```
integrate(coth(f*x + e)^2/(b*sinh(f*x + e)^2 + a)^(5/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\coth^2(e+fx)}{\left(a+b\sinh^2(e+fx)\right)^{5/2}}\,dx = \int \frac{\coth(e+f\,x)^2}{\left(b\sinh\left(e+f\,x\right)^2+a\right)^{5/2}}\,dx$$

input
```
int(coth(e + f*x)^2/(a + b*sinh(e + f*x)^2)^(5/2),x)
```

output
```
int(coth(e + f*x)^2/(a + b*sinh(e + f*x)^2)^(5/2), x)
```

Reduce [F]

$$\int \frac{\coth^2(e+fx)}{\left(a+b\sinh^2(e+fx)\right)^{5/2}}\,dx = \int \frac{\sqrt{\sinh\left(fx+e\right)^2 b+a}\,\coth\left(fx+e\right)^2}{\sinh\left(fx+e\right)^6 b^3+3\sinh\left(fx+e\right)^4 a\,b^2+3\sinh\left(fx+e\right)^2 a^2b+a^3}dx$$

input
```
int(coth(f*x+e)^2/(a+b*sinh(f*x+e)^2)^(5/2),x)
```

output
```
int((sqrt(sinh(e + f*x)**2*b + a)*coth(e + f*x)**2)/(sinh(e + f*x)**6*b**3
 + 3*sinh(e + f*x)**4*a*b**2 + 3*sinh(e + f*x)**2*a**2*b + a**3),x)
```

3.475 $\int \frac{\coth^4(e+fx)}{(a+b\sinh^2(e+fx))^{5/2}}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 336

$$\begin{aligned}\int \frac{\coth^4(e+fx)}{(a+b\sinh^2(e+fx))^{5/2}}\,dx &= -\frac{(a-b)\coth(e+fx)\mathrm{csch}^2(e+fx)}{3abf\,(a+b\sinh^2(e+fx))^{3/2}} \\ &\quad -\frac{(5a-8b)\coth(e+fx)}{3a^3f\sqrt{a+b\sinh^2(e+fx)}}+\frac{(a-2b)\coth(e+fx)\mathrm{csch}^2(e+fx)}{3a^2bf\sqrt{a+b\sinh^2(e+fx)}} \\ &\quad -\frac{8(a-2b)\sqrt{b}\cosh(e+fx)E\left(\arctan\left(\frac{\sqrt{b}\sinh(e+fx)}{\sqrt{a}}\right)|1-\frac{a}{b}\right)}{3a^{7/2}f\sqrt{\frac{a\cosh^2(e+fx)}{a+b\sinh^2(e+fx)}}\sqrt{a+b\sinh^2(e+fx)}} \\ &\quad +\frac{(3a-8b)\cosh(e+fx)\,\mathrm{EllipticF}\left(\arctan\left(\frac{\sqrt{b}\sinh(e+fx)}{\sqrt{a}}\right),1-\frac{a}{b}\right)}{3a^{5/2}\sqrt{b}f\sqrt{\frac{a\cosh^2(e+fx)}{a+b\sinh^2(e+fx)}}\sqrt{a+b\sinh^2(e+fx)}}\end{aligned}$$

output
```
-1/3*(a-b)*coth(f*x+e)*csch(f*x+e)^2/a/b/f/(a+b*sinh(f*x+e)^2)^(3/2)-1/3*(
5*a-8*b)*coth(f*x+e)/a^3/f/(a+b*sinh(f*x+e)^2)^(1/2)+1/3*(a-2*b)*coth(f*x+
e)*csch(f*x+e)^2/a^2/b/f/(a+b*sinh(f*x+e)^2)^(1/2)-8/3*(a-2*b)*b^(1/2)*cos
h(f*x+e)*EllipticE(b^(1/2)*sinh(f*x+e)/a^(1/2)/(1+b*sinh(f*x+e)^2/a)^(1/2)
,(1-a/b)^(1/2))/a^(7/2)/f/(a*cosh(f*x+e)^2/(a+b*sinh(f*x+e)^2))^(1/2)/(a+b
*sinh(f*x+e)^2)^(1/2)+1/3*(3*a-8*b)*cosh(f*x+e)*InverseJacobiAM(arctan(b^(
1/2)*sinh(f*x+e)/a^(1/2)),(1-a/b)^(1/2))/a^(5/2)/b^(1/2)/f/(a*cosh(f*x+e)^
2/(a+b*sinh(f*x+e)^2))^(1/2)/(a+b*sinh(f*x+e)^2)^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 2.28 (sec) , antiderivative size = 247, normalized size of antiderivative = 0.74

$$\int \frac{\coth^4(e+fx)}{\left(a+b\sinh^2(e+fx)\right)^{5/2}}\,dx =$$

$$-\frac{i\left(\frac{ib(8a^3-63a^2b+92ab^2-40b^3-2(8a^3-38a^2b+63ab^2-30b^3)\cosh(2(e+fx))-b(13a^2-36ab+24b^2)\cosh(4(e+fx))-2ab^2\cosh(6(e+fx))+4b^3 c}{\sqrt{2}}\right.}{6a^4bf}$$

input
```
Integrate[Coth[e + f*x]^4/(a + b*Sinh[e + f*x]^2)^(5/2),x]
```

output
```
((-1/6*I)*((I*b*(8*a^3 - 63*a^2*b + 92*a*b^2 - 40*b^3 - 2*(8*a^3 - 38*a^2*
b + 63*a*b^2 - 30*b^3)*Cosh[2*(e + f*x)] - b*(13*a^2 - 36*a*b + 24*b^2)*Co
sh[4*(e + f*x)] - 2*a*b^2*Cosh[6*(e + f*x)] + 4*b^3*Cosh[6*(e + f*x)])*Cot
h[e + f*x]*Csch[e + f*x]^2)/Sqrt[2] + 2*a^2*b*((2*a - b + b*Cosh[2*(e + f*
x)])/a)^(3/2)*(8*(a - 2*b)*EllipticE[I*(e + f*x), b/a] + (-5*a + 8*b)*Elli
pticF[I*(e + f*x), b/a])))/(a^4*b*f*(2*a - b + b*Cosh[2*(e + f*x)])^(3/2))
```

Rubi [A] (verified)

Time = 0.71 (sec) , antiderivative size = 471, normalized size of antiderivative = 1.40, number of steps used = 14, number of rules used = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.520$, Rules used = {3042, 3675, 370, 441, 27, 445, 27, 445, 25, 406, 320, 388, 313}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\coth^4(e+fx)}{\left(a+b\sinh^2(e+fx)\right)^{5/2}}dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \frac{1}{\tan(ie+ifx)^4\left(a-b\sin(ie+ifx)^2\right)^{5/2}}dx$$

$$\Big\downarrow \text{ 3675}$$

$$\frac{\sqrt{\cosh^2(e+fx)}\operatorname{sech}(e+fx)\int \frac{\operatorname{csch}^4(e+fx)\left(\sinh^2(e+fx)+1\right)^{3/2}}{\left(b\sinh^2(e+fx)+a\right)^{5/2}}d\sinh(e+fx)}{f}$$

$$\Big\downarrow \ 370$$

$$\frac{\sqrt{\cosh^2(e+fx)}\operatorname{sech}(e+fx)\left(-\frac{\int \frac{\operatorname{csch}^4(e+fx)\left((2a-5b)\sinh^2(e+fx)+3(a-2b)\right)}{\sqrt{\sinh^2(e+fx)+1}\left(b\sinh^2(e+fx)+a\right)^{3/2}}d\sinh(e+fx)}{3ab}-\frac{(a-b)\sqrt{\sinh^2(e+fx)+1}\operatorname{csch}^3(e+fx}{3ab\left(a+b\sinh^2(e+fx)\right)^{3/2}}\right.}{f}$$

$$\Big\downarrow \ 441$$

$$\frac{\sqrt{\cosh^2(e+fx)}\operatorname{sech}(e+fx)\left(-\frac{\frac{\int \frac{3(a-b)\operatorname{csch}^4(e+fx)\left(2(a-3b)\sinh^2(e+fx)+3a-8b\right)}{\sqrt{\sinh^2(e+fx)+1}\sqrt{b\sinh^2(e+fx)+a}}d\sinh(e+fx)}{a(a-b)}+\frac{2(a-3b)\sqrt{\sinh^2(e+fx)+1}\operatorname{csch}^3(e+fx)}{a\sqrt{a+b\sinh^2(e+fx)}}}{3ab}\right.}{f}$$

$$\Big\downarrow \ 27$$

$$\frac{\sqrt{\cosh^2(e+fx)}\operatorname{sech}(e+fx)\left(-\frac{\frac{3\int \frac{\operatorname{csch}^4(e+fx)\left(2(a-3b)\sinh^2(e+fx)+3a-8b\right)}{\sqrt{\sinh^2(e+fx)+1}\sqrt{b\sinh^2(e+fx)+a}}d\sinh(e+fx)}{a}+\frac{2(a-3b)\sqrt{\sinh^2(e+fx)+1}\operatorname{csch}^3(e+fx)}{a\sqrt{a+b\sinh^2(e+fx)}}}{3ab}-\right.}{f}$$

$$\Big\downarrow \ 445$$

$$\frac{\sqrt{\cosh^2(e+fx)}\operatorname{sech}(e+fx)\left(-\frac{\frac{3\left(-\frac{\int \frac{b\operatorname{csch}^2(e+fx)\left((3a-8b)\sinh^2(e+fx)+8(a-2b)\right)}{\sqrt{\sinh^2(e+fx)+1}\sqrt{b\sinh^2(e+fx)+a}}d\sinh(e+fx)}{3a}-\frac{(3a-8b)\sqrt{\sinh^2(e+fx)+1}\operatorname{csch}^3(e+fx}{3a}\right.}{a}}{3ab}\right.}{f}$$

$$\Big\downarrow \ 27$$

$$\frac{\sqrt{\cosh^2(e+fx)}\operatorname{sech}(e+fx)\left(-\frac{3\left(-\frac{b\int\frac{\operatorname{csch}^2(e+fx)\left((3a-8b)\sinh^2(e+fx)+8(a-2b)\right)}{\sqrt{\sinh^2(e+fx)+1}\sqrt{b\sinh^2(e+fx)+a}}d\sinh(e+fx)}{3a}-\frac{(3a-8b)\sqrt{\sinh^2(e+fx)+1}\operatorname{csch}^3(e+fx}{3a}\right.}{a}}{3ab}\right.}{f}$$

$\downarrow$ 445

$$\frac{\sqrt{\cosh^2(e+fx)}\operatorname{sech}(e+fx)\left(-\frac{3\left(-\frac{b\left(-\frac{\int-\frac{8(a-2b)b\sinh^2(e+fx)+a(3a-8b)}{\sqrt{\sinh^2(e+fx)+1}\sqrt{b\sinh^2(e+fx)+a}}d\sinh(e+fx)}{a}-\frac{8(a-2b)\sqrt{\sinh^2(e+fx)+1}\operatorname{csch}(e+fx)\sqrt{a+b}}{a}\right.}{3a}\right.}{a}\right.}{}$$

$\downarrow$ 25

$$\sqrt{\cosh^2(e+fx)}\text{sech}(e+fx)\left(-\frac{3\left(-\frac{b\left(\frac{\int \frac{8(a-2b)b\sinh^2(e+fx)+a(3a-8b)}{\sqrt{\sinh^2(e+fx)+1}\sqrt{b\sinh^2(e+fx)+a}}d\sinh(e+fx)}{a}-\frac{8(a-2b)\sqrt{\sinh^2(e+fx)+1}\text{csch}(e+fx)\sqrt{a+b\sinh \cdots}}{a}\right.}{3a}\right.}{a}\right.$$

$\downarrow$ 406

$$\sqrt{\cosh^2(e+fx)}\text{sech}(e+fx)\left(-\frac{3\left(-\frac{b\left(\frac{a(3a-8b)\int \frac{1}{\sqrt{\sinh^2(e+fx)+1}\sqrt{b\sinh^2(e+fx)+a}}d\sinh(e+fx)+8b(a-2b)\int \frac{\sinh^2(e+fx)}{\sqrt{\sinh^2(e+fx)+1}\sqrt{b\sinh^2(e \cdots}}}{a}\right.}{3a}\right.}{\cdots}\right.$$

$\downarrow$ 320

$$\sqrt{\cosh^2(e+fx)}\operatorname{sech}(e+fx)\left(-\frac{3\left(-\frac{b\left(\frac{8b(a-2b)\int\frac{\sinh^2(e+fx)}{\sqrt{\sinh^2(e+fx)+1}\sqrt{b\sinh^2(e+fx)+a}}d\sinh(e+fx)+\frac{(3a-8b)\sqrt{a+b\sinh^2(e+fx)}\,\text{EllipticF}\left(\text{arc}\ldots\right.}{\sqrt{\sinh^2(e+fx)+1}\sqrt{\frac{a+b\sin\ldots}{a\left(\sinh^2\ldots\right.}}}}{a}\right)}{3a}\ldots\right)}{\ldots}\ldots\right.}{\ldots}$$

$\downarrow$ 388

$$\sqrt{\cosh^2(e+fx)}\operatorname{sech}(e+fx)\left(-\frac{3-\frac{b\left(\frac{8b(a-2b)\left(\frac{\sinh(e+fx)\sqrt{a+b\sinh^2(e+fx)}}{b\sqrt{\sinh^2(e+fx)+1}}-\frac{\int\frac{\sqrt{b\sinh^2(e+fx)+a}}{\left(\sinh^2(e+fx)+1\right)^{3/2}}d\sinh(e+fx)}{b}\right)}{a}+\frac{(3a-8b)\sqrt{a+b\sinh\cdots}}{\sqrt{\sinh\cdots}}\cdots\right)}{\cdots}}{\cdots}\cdots\right)$$

$\downarrow$ 313

$$\sqrt{\cosh^2(e+fx)}\operatorname{sech}(e+fx)\left(-\frac{3\left(-\frac{b\left(\frac{(3a-8b)\sqrt{a+b\sinh^2(e+fx)}\operatorname{EllipticF}\left(\arctan(\sinh(e+fx)),1-\frac{b}{a}\right)}{\sqrt{\sinh^2(e+fx)+1}\sqrt{\frac{a+b\sinh^2(e+fx)}{a\left(\sinh^2(e+fx)+1\right)}}}+8b(a-2b)\left(\frac{\sinh(e+fx)\sqrt{a+b\sinh^2(e-}{b\sqrt{\sinh^2(e+fx)+1}}\right.}{a}\right.}{}\right.}{}\right.$$

input `Int[Coth[e + f*x]^4/(a + b*Sinh[e + f*x]^2)^(5/2),x]`

output
```
(Sqrt[Cosh[e + f*x]^2]*Sech[e + f*x]*(-1/3*((a - b)*Csch[e + f*x]^3*Sqrt[1
 + Sinh[e + f*x]^2])/(a*b*(a + b*Sinh[e + f*x]^2)^(3/2)) - ((2*(a - 3*b)*C
sch[e + f*x]^3*Sqrt[1 + Sinh[e + f*x]^2])/(a*Sqrt[a + b*Sinh[e + f*x]^2])
+ (3*(-1/3*((3*a - 8*b)*Csch[e + f*x]^3*Sqrt[1 + Sinh[e + f*x]^2]*Sqrt[a +
 b*Sinh[e + f*x]^2])/a - (b*((-8*(a - 2*b)*Csch[e + f*x]*Sqrt[1 + Sinh[e +
 f*x]^2]*Sqrt[a + b*Sinh[e + f*x]^2])/a + (((3*a - 8*b)*EllipticF[ArcTan[S
inh[e + f*x]], 1 - b/a]*Sqrt[a + b*Sinh[e + f*x]^2])/(Sqrt[1 + Sinh[e + f*
x]^2]*Sqrt[(a + b*Sinh[e + f*x]^2)/(a*(1 + Sinh[e + f*x]^2))]) + 8*(a - 2*
b)*b*((Sinh[e + f*x]*Sqrt[a + b*Sinh[e + f*x]^2])/(b*Sqrt[1 + Sinh[e + f*x
]^2]) - (EllipticE[ArcTan[Sinh[e + f*x]], 1 - b/a]*Sqrt[a + b*Sinh[e + f*x
]^2])/(b*Sqrt[1 + Sinh[e + f*x]^2]*Sqrt[(a + b*Sinh[e + f*x]^2)/(a*(1 + Si
nh[e + f*x]^2))]))))/a))/(3*a)))/a)/(3*a*b)))/f
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 313
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/((c_) + (d_.)*(x_)^2)^(3/2), x_Symbol] :> Sim
p[(Sqrt[a + b*x^2]/(c*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(c
+ d*x^2)))]))*EllipticE[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; FreeQ
[{a, b, c, d}, x] && PosQ[b/a] && PosQ[d/c]
```

rule 320
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(Sqrt[a + b*x^2]/(a*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(
c + d*x^2)))]))*EllipticF[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; Fre
eQ[{a, b, c, d}, x] && PosQ[d/c] && PosQ[b/a] &&  !SimplerSqrtQ[b/a, d/c]
```

rule 370
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_
), x_Symbol] :> Simp[(-(b*c - a*d))*(e*x)^(m + 1)*(a + b*x^2)^(p + 1)*((c +
 d*x^2)^(q - 1)/(a*b*e*2*(p + 1))), x] + Simp[1/(a*b*2*(p + 1))   Int[(e*x)
^m*(a + b*x^2)^(p + 1)*(c + d*x^2)^(q - 2)*Simp[c*(b*c*2*(p + 1) + (b*c - a
*d)*(m + 1)) + d*(b*c*2*(p + 1) + (b*c - a*d)*(m + 2*(q - 1) + 1))*x^2, x],
 x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1]
&& GtQ[q, 1] && IntBinomialQ[a, b, c, d, e, m, 2, p, q, x]
```

rule 388
```
Int[(x_)^2/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol]
 :> Simp[x*(Sqrt[a + b*x^2]/(b*Sqrt[c + d*x^2])), x] - Simp[c/b   Int[Sqrt[
a + b*x^2]/(c + d*x^2)^(3/2), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
 a*d, 0] && PosQ[b/a] && PosQ[d/c] &&  !SimplerSqrtQ[b/a, d/c]
```

rule 406
```
Int[((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.)*((e_) + (f_.)*(
x_)^2), x_Symbol] :> Simp[e   Int[(a + b*x^2)^p*(c + d*x^2)^q, x], x] + Sim
p[f   Int[x^2*(a + b*x^2)^p*(c + d*x^2)^q, x], x] /; FreeQ[{a, b, c, d, e,
f, p, q}, x]
```

rule 441
```
Int[((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_
)*((e_) + (f_.)*(x_)^2), x_Symbol] :> Simp[(-(b*e - a*f))*(g*x)^(m + 1)*(a
+ b*x^2)^(p + 1)*((c + d*x^2)^(q + 1)/(a*g*2*(b*c - a*d)*(p + 1))), x] + Si
mp[1/(a*2*(b*c - a*d)*(p + 1))   Int[(g*x)^m*(a + b*x^2)^(p + 1)*(c + d*x^2
)^q*Simp[c*(b*e - a*f)*(m + 1) + e*2*(b*c - a*d)*(p + 1) + d*(b*e - a*f)*(m
 + 2*(p + q + 2) + 1)*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, m, q},
 x] && LtQ[p, -1]
```

rule 445
```
Int[((g_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_
.)*((e_) + (f_.)*(x_)^2), x_Symbol] :> Simp[e*(g*x)^(m + 1)*(a + b*x^2)^(p
+ 1)*((c + d*x^2)^(q + 1)/(a*c*g*(m + 1))), x] + Simp[1/(a*c*g^2*(m + 1))
 Int[(g*x)^(m + 2)*(a + b*x^2)^p*(c + d*x^2)^q*Simp[a*f*c*(m + 1) - e*(b*c
+ a*d)*(m + 2 + 1) - e*2*(b*c*p + a*d*q) - b*e*d*(m + 2*(p + q + 2) + 1)*x^
2, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p, q}, x] && LtQ[m, -1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3675
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_.)*tan[(e_.) + (f_.)*(x_)]^
(m_), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Simp[ff^(m + 1
)*(Sqrt[Cos[e + f*x]^2]/(f*Cos[e + f*x]))   Subst[Int[x^m*((a + b*ff^2*x^2)
^p/(1 - ff^2*x^2)^((m + 1)/2)), x], x, Sin[e + f*x]/ff], x]] /; FreeQ[{a, b
, e, f, p}, x] && IntegerQ[m/2] &&  !IntegerQ[p]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 922 vs. 2(316) = 632.

Time = 8.57 (sec) , antiderivative size = 923, normalized size of antiderivative = 2.75

method	result	size
default	Expression too large to display	923
risch	Expression too large to display	1122996

input
```
int(coth(f*x+e)^4/(a+b*sinh(f*x+e)^2)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
-1/3*(8*(-b/a)^(1/2)*a*b^2*sinh(f*x+e)^8-16*(-b/a)^(1/2)*b^3*sinh(f*x+e)^8
-3*((a+b*sinh(f*x+e)^2)/a)^(1/2)*(cosh(f*x+e)^2)^(1/2)*EllipticF(sinh(f*x+
e)*(-b/a)^(1/2),(1/b*a)^(1/2))*a^2*b*sinh(f*x+e)^5+16*((a+b*sinh(f*x+e)^2)
/a)^(1/2)*(cosh(f*x+e)^2)^(1/2)*EllipticF(sinh(f*x+e)*(-b/a)^(1/2),(1/b*a)
^(1/2))*a*b^2*sinh(f*x+e)^5-16*((a+b*sinh(f*x+e)^2)/a)^(1/2)*(cosh(f*x+e)^
2)^(1/2)*EllipticF(sinh(f*x+e)*(-b/a)^(1/2),(1/b*a)^(1/2))*b^3*sinh(f*x+e)
^5-8*((a+b*sinh(f*x+e)^2)/a)^(1/2)*(cosh(f*x+e)^2)^(1/2)*EllipticE(sinh(f*
x+e)*(-b/a)^(1/2),(1/b*a)^(1/2))*a*b^2*sinh(f*x+e)^5+16*((a+b*sinh(f*x+e)^
2)/a)^(1/2)*(cosh(f*x+e)^2)^(1/2)*EllipticE(sinh(f*x+e)*(-b/a)^(1/2),(1/b*
a)^(1/2))*b^3*sinh(f*x+e)^5+13*(-b/a)^(1/2)*a^2*b*sinh(f*x+e)^6-16*(-b/a)^
(1/2)*a*b^2*sinh(f*x+e)^6-16*(-b/a)^(1/2)*b^3*sinh(f*x+e)^6-3*((a+b*sinh(f
*x+e)^2)/a)^(1/2)*(cosh(f*x+e)^2)^(1/2)*EllipticF(sinh(f*x+e)*(-b/a)^(1/2)
,(1/b*a)^(1/2))*a^3*sinh(f*x+e)^3+16*((a+b*sinh(f*x+e)^2)/a)^(1/2)*(cosh(f
*x+e)^2)^(1/2)*EllipticF(sinh(f*x+e)*(-b/a)^(1/2),(1/b*a)^(1/2))*a^2*b*sin
h(f*x+e)^3-16*((a+b*sinh(f*x+e)^2)/a)^(1/2)*(cosh(f*x+e)^2)^(1/2)*Elliptic
F(sinh(f*x+e)*(-b/a)^(1/2),(1/b*a)^(1/2))*a*b^2*sinh(f*x+e)^3-8*((a+b*sinh
(f*x+e)^2)/a)^(1/2)*(cosh(f*x+e)^2)^(1/2)*EllipticE(sinh(f*x+e)*(-b/a)^(1/
2),(1/b*a)^(1/2))*a^2*b*sinh(f*x+e)^3+16*((a+b*sinh(f*x+e)^2)/a)^(1/2)*(co
sh(f*x+e)^2)^(1/2)*EllipticE(sinh(f*x+e)*(-b/a)^(1/2),(1/b*a)^(1/2))*a*b^2
*sinh(f*x+e)^3+4*(-b/a)^(1/2)*a^3*sinh(f*x+e)^4+7*(-b/a)^(1/2)*a^2*b*si...
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 13823 vs. $2(316) = 632$.

Time = 0.61 (sec) , antiderivative size = 13823, normalized size of antiderivative = 41.14

$$\int \frac{\coth^4(e+fx)}{(a+b\sinh^2(e+fx))^{5/2}}\,dx = \text{Too large to display}$$

input
```
integrate(coth(f*x+e)^4/(a+b*sinh(f*x+e)^2)^(5/2),x, algorithm="fricas")
```

output
```
Too large to include
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\coth^4(e+fx)}{(a+b\sinh^2(e+fx))^{5/2}}\,dx = \text{Timed out}$$

input
```
integrate(coth(f*x+e)**4/(a+b*sinh(f*x+e)**2)**(5/2),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{\coth^4(e+fx)}{(a+b\sinh^2(e+fx))^{5/2}}\,dx = \int \frac{\coth(fx+e)^4}{(b\sinh(fx+e)^2+a)^{\frac{5}{2}}}\,dx$$

input
```
integrate(coth(f*x+e)^4/(a+b*sinh(f*x+e)^2)^(5/2),x, algorithm="maxima")
```

output
```
integrate(coth(f*x + e)^4/(b*sinh(f*x + e)^2 + a)^(5/2), x)
```

Giac [F]

$$\int \frac{\coth^4(e+fx)}{(a+b\sinh^2(e+fx))^{5/2}}\,dx = \int \frac{\coth(fx+e)^4}{(b\sinh(fx+e)^2+a)^{\frac{5}{2}}}\,dx$$

input
```
integrate(coth(f*x+e)^4/(a+b*sinh(f*x+e)^2)^(5/2),x, algorithm="giac")
```

output
```
integrate(coth(f*x + e)^4/(b*sinh(f*x + e)^2 + a)^(5/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\coth^4(e+fx)}{(a+b\sinh^2(e+fx))^{5/2}}\,dx = \int \frac{\coth(e+f\,x)^4}{(b\sinh(e+f\,x)^2+a)^{5/2}}\,dx$$

input
```
int(coth(e + f*x)^4/(a + b*sinh(e + f*x)^2)^(5/2),x)
```

output
```
int(coth(e + f*x)^4/(a + b*sinh(e + f*x)^2)^(5/2), x)
```

Reduce [F]

$$\int \frac{\coth^4(e+fx)}{(a+b\sinh^2(e+fx))^{5/2}}\,dx = \int \frac{\sqrt{\sinh(fx+e)^2 b+a}\,\coth(fx+e)^4}{\sinh(fx+e)^6 b^3+3\sinh(fx+e)^4 a b^2+3\sinh(fx+e)^2 a^2b+a^3}dx$$

input
```
int(coth(f*x+e)^4/(a+b*sinh(f*x+e)^2)^(5/2),x)
```

output
```
int((sqrt(sinh(e + f*x)**2*b + a)*coth(e + f*x)**4)/(sinh(e + f*x)**6*b**3
 + 3*sinh(e + f*x)**4*a*b**2 + 3*sinh(e + f*x)**2*a**2*b + a**3),x)
```

3.476 $\int (a+b\sinh^2(e+fx))^p (d\tanh(e+fx))^m\,dx$

Optimal result

Integrand size = 25, antiderivative size = 122

$$\int (a+b\sinh^2(e+fx))^p (d\tanh(e+fx))^m\,dx = \frac{\text{AppellF1}\left(\frac{1+m}{2},\frac{1+m}{2},-p,\frac{3+m}{2},-\sinh^2(e+fx),-\frac{b\sinh^2(e+fx)}{a}\right)\cosh^2(e+fx)^{\frac{1+m}{2}}(a+b\sinh^2(e+fx))^p}{df(1+m)}$$

output
```
AppellF1(1/2+1/2*m,1/2+1/2*m,-p,3/2+1/2*m,-sinh(f*x+e)^2,-b*sinh(f*x+e)^2/
a)*(cosh(f*x+e)^2)^(1/2+1/2*m)*(a+b*sinh(f*x+e)^2)^p*(d*tanh(f*x+e))^(1+m)
/d/f/(1+m)/((1+b*sinh(f*x+e)^2/a)^p)
```

Mathematica [F]

$$\int (a+b\sinh^2(e+fx))^p (d\tanh(e+fx))^m\,dx = \int (a+b\sinh^2(e+fx))^p (d\tanh(e+fx))^m\,dx$$

input
```
Integrate[(a + b*Sinh[e + f*x]^2)^p*(d*Tanh[e + f*x])^m,x]
```

output
```
Integrate[(a + b*Sinh[e + f*x]^2)^p*(d*Tanh[e + f*x])^m, x]
```

Rubi [A] (verified)

Time = 0.47 (sec) , antiderivative size = 149, normalized size of antiderivative = 1.22, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {3042, 3676, 393, 152, 150}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (d\tanh(e+fx))^m \left(a+b\sinh^2(e+fx)\right)^p dx$$

$$\Big\downarrow \; 3042$$

$$\int (-id\tan(ie+ifx))^m \left(a-b\sin(ie+ifx)^2\right)^p dx$$

$$\Big\downarrow \; 3676$$

$$\frac{i(i\sinh(e+fx))^{-m-1}\cosh^2(e+fx)^{\frac{m+1}{2}}(d\tanh(e+fx))^{m+1}\int (i\sinh(e+fx))^m\left(\sinh^2(e+fx)+1\right)^{\frac{1}{2}(-m-1)}(b$$ $$df$$

$$\Big\downarrow \; 393$$

$$\frac{\sinh^2(e+fx)^{\frac{1-m}{2}-1}\cosh^2(e+fx)^{\frac{m+1}{2}}(d\tanh(e+fx))^{m+1}\int \sinh^2(e+fx)^{\frac{m-1}{2}}\left(\sinh^2(e+fx)+1\right)^{\frac{1}{2}(-m-1)}(b s$$ $$2df$$

$$\Big\downarrow \; 152$$

$$\frac{\sinh^2(e+fx)^{\frac{1-m}{2}-1}\cosh^2(e+fx)^{\frac{m+1}{2}}(d\tanh(e+fx))^{m+1}\left(a+b\sinh^2(e+fx)\right)^p\left(\frac{b\sinh^2(e+fx)}{a}+1\right)^{-p}\int \sinh^2($$ $$2df$$

$$\Big\downarrow \; 150$$

$$\frac{\sinh^2(e+fx)^{\frac{1-m}{2}+\frac{m+1}{2}-1}\cosh^2(e+fx)^{\frac{m+1}{2}}(d\tanh(e+fx))^{m+1}\left(a+b\sinh^2(e+fx)\right)^p\left(\frac{b\sinh^2(e+fx)}{a}+1\right)^{-p}\text{Ap}$$ $$df(m+1)$$

input
```
Int[(a + b*Sinh[e + f*x]^2)^p*(d*Tanh[e + f*x])^m,x]
```

output
```
(AppellF1[(1 + m)/2, (1 + m)/2, -p, (3 + m)/2, -Sinh[e + f*x]^2, -((b*Sinh
[e + f*x]^2)/a)]*(Cosh[e + f*x]^2)^((1 + m)/2)*(Sinh[e + f*x]^2)^(-1 + (1
- m)/2 + (1 + m)/2)*(a + b*Sinh[e + f*x]^2)^p*(d*Tanh[e + f*x])^(1 + m))/(
d*f*(1 + m)*(1 + (b*Sinh[e + f*x]^2)/a)^p)
```

Defintions of rubi rules used

rule 150
```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_)*((e_) + (f_.)*(x_))^(p_), x_
] :> Simp[c^n*e^p*((b*x)^(m + 1)/(b*(m + 1)))*AppellF1[m + 1, -n, -p, m + 2
, (-d)*(x/c), (-f)*(x/e)], x] /; FreeQ[{b, c, d, e, f, m, n, p}, x] && !In
tegerQ[m] && !IntegerQ[n] && GtQ[c, 0] && (IntegerQ[p] || GtQ[e, 0])
```

rule 152
```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_)*((e_) + (f_.)*(x_))^(p_), x_
] :> Simp[c^IntPart[n]*((c + d*x)^FracPart[n]/(1 + d*(x/c))^FracPart[n])
Int[(b*x)^m*(1 + d*(x/c))^n*(e + f*x)^p, x], x] /; FreeQ[{b, c, d, e, f, m,
 n, p}, x] && !IntegerQ[m] && !IntegerQ[n] && !GtQ[c, 0]
```

rule 393
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q
_.), x_Symbol] :> Simp[(e*x)^m/(2*x*(x^2)^(Simplify[(m + 1)/2] - 1))   Subs
t[Int[x^(Simplify[(m + 1)/2] - 1)*(a + b*x)^p*(c + d*x)^q, x], x, x^2], x]
/; FreeQ[{a, b, c, d, e, m, p, q}, x] && NeQ[b*c - a*d, 0] && IntegerQ[Simp
lify[m + 2*p]] && !IntegerQ[m]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3676
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_.)*((d_.)*tan[(e_.) + (f_.)
*(x_)])^(m_), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Simp[f
f*(d*Tan[e + f*x])^(m + 1)*((Cos[e + f*x]^2)^((m + 1)/2)/(d*f*Sin[e + f*x]^
(m + 1)))   Subst[Int[(ff*x)^m*((a + b*ff^2*x^2)^p/(1 - ff^2*x^2)^((m + 1)/
2)), x], x, Sin[e + f*x]/ff], x]] /; FreeQ[{a, b, d, e, f, m, p}, x] &&  !I
ntegerQ[m]
```

Maple [F]

$$\int \left(a+b\sinh \left(fx+e\right)^{2}\right)^{p} \left(d\tanh \left(fx+e\right)\right)^{m} dx$$

input
```
int((a+b*sinh(f*x+e)^2)^p*(d*tanh(f*x+e))^m,x)
```

output
```
int((a+b*sinh(f*x+e)^2)^p*(d*tanh(f*x+e))^m,x)
```

Fricas [F]

$$\int \left(a+b\sinh^2(e+fx)\right)^p (d\tanh(e+fx))^m dx$$
$$= \int \left(b\sinh \left(fx+e\right)^{2}+a\right)^{p}\left(d\tanh \left(fx+e\right)\right)^{m} dx$$

input
```
integrate((a+b*sinh(f*x+e)^2)^p*(d*tanh(f*x+e))^m,x, algorithm="fricas")
```

output
```
integral((b*sinh(f*x + e)^2 + a)^p*(d*tanh(f*x + e))^m, x)
```

Sympy [F(-1)]

Timed out.

$$\int \left(a+b\sinh^2(e+fx)\right)^p (d\tanh(e+fx))^m\,dx = \text{Timed out}$$

input
```
integrate((a+b*sinh(f*x+e)**2)**p*(d*tanh(f*x+e))**m,x)
```

output
```
Timed out
```

Maxima [F]

$$\int \left(a+b\sinh^2(e+fx)\right)^p (d\tanh(e+fx))^m\,dx$$
$$= \int \left(b\sinh{(fx+e)}^2+a\right)^p (d\tanh{(fx+e)})^m\,dx$$

input
```
integrate((a+b*sinh(f*x+e)^2)^p*(d*tanh(f*x+e))^m,x, algorithm="maxima")
```

output
```
integrate((b*sinh(f*x + e)^2 + a)^p*(d*tanh(f*x + e))^m, x)
```

Giac [F]

$$\int \left(a+b\sinh^2(e+fx)\right)^p (d\tanh(e+fx))^m\,dx$$
$$= \int \left(b\sinh{(fx+e)}^2+a\right)^p (d\tanh{(fx+e)})^m\,dx$$

input
```
integrate((a+b*sinh(f*x+e)^2)^p*(d*tanh(f*x+e))^m,x, algorithm="giac")
```

output
```
integrate((b*sinh(f*x + e)^2 + a)^p*(d*tanh(f*x + e))^m, x)
```

Mupad [F(-1)]

Timed out.

$$\int \left(a+b\sinh^2(e+fx)\right)^p (d\tanh(e+fx))^m\,dx$$
$$= \int (d\tanh(e+f\,x))^m \left(b\sinh(e+f\,x)^2+a\right)^p\,dx$$

input
```
int((d*tanh(e + f*x))^m*(a + b*sinh(e + f*x)^2)^p,x)
```

output
```
int((d*tanh(e + f*x))^m*(a + b*sinh(e + f*x)^2)^p, x)
```

Reduce [F]

$$\int \left(a+b\sinh^2(e+fx)\right)^p (d\tanh(e+fx))^m\,dx$$
$$= d^m\left(\int \tanh(fx+e)^m \left(\sinh(fx+e)^2 b+a\right)^p dx\right)$$

input
```
int((a+b*sinh(f*x+e)^2)^p*(d*tanh(f*x+e))^m,x)
```

output
```
d**m*int(tanh(e + f*x)**m*(sinh(e + f*x)**2*b + a)**p,x)
```

3.477 $\int \left(a+b\sinh^2(c+dx)\right)^p \tanh^3(c+dx)\,dx$

Optimal result

Integrand size = 23, antiderivative size = 110

$$\int \left(a+b\sinh^2(c+dx)\right)^p \tanh^3(c+dx)\,dx = -\frac{(a-b(1+p))\,\text{Hypergeometric2F1}\left(1,1+p,2+p,\frac{a+b\sinh^2(c+dx)}{a-b}\right)\left(a+b\sinh^2(c+dx)\right)^{1+p}}{2(a-b)^2d(1+p)} + \frac{\text{sech}^2(c+dx)\left(a+b\sinh^2(c+dx)\right)^{1+p}}{2(a-b)d}$$

output
```
-1/2*(a-b*(p+1))*hypergeom([1, p+1],[2+p],(a+b*sinh(d*x+c)^2)/(a-b))*(a+b*
sinh(d*x+c)^2)^(p+1)/(a-b)^2/d/(p+1)+1/2*sech(d*x+c)^2*(a+b*sinh(d*x+c)^2)
^(p+1)/(a-b)/d
```

Mathematica [A] (verified)

Time = 0.18 (sec) , antiderivative size = 90, normalized size of antiderivative = 0.82

$$\int \left(a+b\sinh^2(c+dx)\right)^p \tanh^3(c+dx)\,dx$$

$$= \frac{\left((-a+b+bp)\,\text{Hypergeometric2F1}\left(1,1+p,2+p,\frac{a+b\sinh^2(c+dx)}{a-b}\right)+(a-b)(1+p)\text{sech}^2(c+dx)\right)(a+}{2(a-b)^2d(1+p)}$$

input
```
Integrate[(a + b*Sinh[c + d*x]^2)^p*Tanh[c + d*x]^3,x]
```

output
```
(((-a + b + b*p)*Hypergeometric2F1[1, 1 + p, 2 + p, (a + b*Sinh[c + d*x]^2
)/(a - b)] + (a - b)*(1 + p)*Sech[c + d*x]^2)*(a + b*Sinh[c + d*x]^2)^(1 +
 p))/(2*(a - b)^2*d*(1 + p))
```

Rubi [A] (verified)

Time = 0.37 (sec) , antiderivative size = 110, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.217$, Rules used = {3042, 26, 3673, 87, 78}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \tanh^3(c+dx)\left(a+b\sinh^2(c+dx)\right)^p\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int i\tan(ic+idx)^3\left(a-b\sin(ic+idx)^2\right)^p\,dx$$

$$\Big\downarrow \text{26}$$

$$i\int \left(a-b\sin(ic+idx)^2\right)^p\tan(ic+idx)^3dx$$

$$\Big\downarrow \text{3673}$$

$$\frac{\int \frac{\sinh^2(c+dx)\left(b\sinh^2(c+dx)+a\right)^p}{\left(\sinh^2(c+dx)+1\right)^2}d\sinh^2(c+dx)}{2d}$$

$$\Big\downarrow \text{87}$$

$$\frac{\frac{(a-b(p+1))\int \frac{\left(b\sinh^2(c+dx)+a\right)^p}{\sinh^2(c+dx)+1}d\sinh^2(c+dx)}{a-b}+\frac{\left(a+b\sinh^2(c+dx)\right)^{p+1}}{(a-b)\left(\sinh^2(c+dx)+1\right)}}{2d}$$

$$\Big\downarrow \text{78}$$

$$\frac{\frac{\left(a+b\sinh^2(c+dx)\right)^{p+1}}{(a-b)\left(\sinh^2(c+dx)+1\right)}-\frac{(a-b(p+1))\left(a+b\sinh^2(c+dx)\right)^{p+1}\text{Hypergeometric2F1}\left(1,p+1,p+2,\frac{b\sinh^2(c+dx)+a}{a-b}\right)}{(p+1)(a-b)^2}}{2d}$$

input
```
Int[(a + b*Sinh[c + d*x]^2)^p*Tanh[c + d*x]^3,x]
```

output
```
(-(((a - b*(1 + p))*Hypergeometric2F1[1, 1 + p, 2 + p, (a + b*Sinh[c + d*x
]^2)/(a - b)]*(a + b*Sinh[c + d*x]^2)^(1 + p))/((a - b)^2*(1 + p))) + (a +
 b*Sinh[c + d*x]^2)^(1 + p)/((a - b)*(1 + Sinh[c + d*x]^2)))/(2*d)
```

Defintions of rubi rules used

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 78
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[(b
*c - a*d)^n*((a + b*x)^(m + 1)/(b^(n + 1)*(m + 1)))*Hypergeometric2F1[-n, m
 + 1, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m}, x]
 &&  !IntegerQ[m] && IntegerQ[n]
```

rule 87
```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_] :> Simp[(-(b*e - a*f))*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(f*(p
+ 1)*(c*f - d*e))), x] - Simp[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p
+ 1)))/(f*(p + 1)*(c*f - d*e))   Int[(c + d*x)^n*(e + f*x)^(p + 1), x], x]
/; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || Intege
rQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))))
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3673
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_.)*tan[(e_.) + (f_.)*(x_)]^
(m_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x]^2, x]}, Simp[ff^((m
 + 1)/2)/(2*f)   Subst[Int[x^((m - 1)/2)*((a + b*ff*x)^p/(1 - ff*x)^((m + 1
)/2)), x], x, Sin[e + f*x]^2/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && Integ
erQ[(m - 1)/2]
```

Maple [F]

$$\int \left(a + b\sinh \left(dx + c\right)^2\right)^p \tanh \left(dx + c\right)^3 dx$$

input
```
int((a+b*sinh(d*x+c)^2)^p*tanh(d*x+c)^3,x)
```

output
```
int((a+b*sinh(d*x+c)^2)^p*tanh(d*x+c)^3,x)
```

Fricas [F]

$$\int \left(a + b\sinh^2(c + dx)\right)^p \tanh^3(c + dx)\, dx = \int \left(b\sinh \left(dx + c\right)^2 + a\right)^p \tanh \left(dx + c\right)^3\, dx$$

input
```
integrate((a+b*sinh(d*x+c)^2)^p*tanh(d*x+c)^3,x, algorithm="fricas")
```

output
```
integral((b*sinh(d*x + c)^2 + a)^p*tanh(d*x + c)^3, x)
```

Sympy [F(-1)]

Timed out.

$$\int \left(a + b\sinh^2(c + dx)\right)^p \tanh^3(c + dx)\, dx = \text{Timed out}$$

input
```
integrate((a+b*sinh(d*x+c)**2)**p*tanh(d*x+c)**3,x)
```

output
```
Timed out
```

Maxima [F]

$$\int \left(a+b\sinh^2(c+dx)\right)^p \tanh^3(c+dx)\,dx=\int \left(b\sinh\left(dx+c\right)^2+a\right)^p \tanh\left(dx+c\right)^3\,dx$$

input
```
integrate((a+b*sinh(d*x+c)^2)^p*tanh(d*x+c)^3,x, algorithm="maxima")
```

output
```
integrate((b*sinh(d*x + c)^2 + a)^p*tanh(d*x + c)^3, x)
```

Giac [F]

$$\int \left(a+b\sinh^2(c+dx)\right)^p \tanh^3(c+dx)\,dx=\int \left(b\sinh\left(dx+c\right)^2+a\right)^p \tanh\left(dx+c\right)^3\,dx$$

input
```
integrate((a+b*sinh(d*x+c)^2)^p*tanh(d*x+c)^3,x, algorithm="giac")
```

output
```
integrate((b*sinh(d*x + c)^2 + a)^p*tanh(d*x + c)^3, x)
```

Mupad [F(-1)]

Timed out.

$$\int \left(a+b\sinh^2(c+dx)\right)^p \tanh^3(c+dx)\,dx=\int \tanh(c+d\,x)^3\,\left(b\sinh(c+d\,x)^2+a\right)^p\,dx$$

input
```
int(tanh(c + d*x)^3*(a + b*sinh(c + d*x)^2)^p,x)
```

output
```
int(tanh(c + d*x)^3*(a + b*sinh(c + d*x)^2)^p, x)
```

Reduce [F]

$$\int \left(a+b\sinh^2(c+dx)\right)^p \tanh^3(c+dx)\,dx = \int \left(\sinh\left(dx+c\right)^2 b+a\right)^p \tanh\left(dx+c\right)^3 dx$$

input
```
int((a+b*sinh(d*x+c)^2)^p*tanh(d*x+c)^3,x)
```

output
```
int((a+b*sinh(d*x+c)^2)^p*tanh(d*x+c)^3,x)
```

3.478 $\int \left(a+b\sinh^2(c+dx)\right)^p \tanh(c+dx)\,dx$

Optimal result

Integrand size = 21, antiderivative size = 63

$$\int \left(a+b\sinh^2(c+dx)\right)^p \tanh(c+dx)\,dx = -\frac{\text{Hypergeometric2F1}\left(1,1+p,2+p,\frac{a+b\sinh^2(c+dx)}{a-b}\right)\left(a+b\sinh^2(c+dx)\right)^{1+p}}{2(a-b)d(1+p)}$$

output
```
-1/2*hypergeom([1, p+1],[2+p],(a+b*sinh(d*x+c)^2)/(a-b))*(a+b*sinh(d*x+c)^
2)^(p+1)/(a-b)/d/(p+1)
```

Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 65, normalized size of antiderivative = 1.03

$$\int \left(a+b\sinh^2(c+dx)\right)^p \tanh(c+dx)\,dx = -\frac{\left(a-b+b\cosh^2(c+dx)\right)^{1+p}\text{Hypergeometric2F1}\left(1,1+p,2+p,1+\frac{b\cosh^2(c+dx)}{a-b}\right)}{2(a-b)d(1+p)}$$

input
```
Integrate[(a + b*Sinh[c + d*x]^2)^p*Tanh[c + d*x],x]
```

output
```
-1/2*((a - b + b*Cosh[c + d*x]^2)^(1 + p)*Hypergeometric2F1[1, 1 + p, 2 +
p, 1 + (b*Cosh[c + d*x]^2)/(a - b)])/((a - b)*d*(1 + p))
```

Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 63, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.190, Rules used = {3042, 26, 3673, 78}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \tanh(c+dx)\left(a+b\sinh^2(c+dx)\right)^p\,dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int -i\tan(ic+idx)\left(a-b\sin(ic+idx)^2\right)^p dx$$

$$\Big\downarrow \text{ 26}$$

$$-i\int \left(a-b\sin(ic+idx)^2\right)^p\tan(ic+idx)dx$$

$$\Big\downarrow \text{ 3673}$$

$$\frac{\int \frac{\left(b\sinh^2(c+dx)+a\right)^p}{\sinh^2(c+dx)+1}d\sinh^2(c+dx)}{2d}$$

$$\Big\downarrow \text{ 78}$$

$$-\frac{\left(a+b\sinh^2(c+dx)\right)^{p+1}\text{Hypergeometric2F1}\left(1,p+1,p+2,\frac{b\sinh^2(c+dx)+a}{a-b}\right)}{2d(p+1)(a-b)}$$

input
```
Int[(a + b*Sinh[c + d*x]^2)^p*Tanh[c + d*x],x]
```

output
```
-1/2*(Hypergeometric2F1[1, 1 + p, 2 + p, (a + b*Sinh[c + d*x]^2)/(a - b)]*
(a + b*Sinh[c + d*x]^2)^(1 + p))/((a - b)*d*(1 + p))
```

Defintions of rubi rules used

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 78
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[(b
*c - a*d)^n*((a + b*x)^(m + 1)/(b^(n + 1)*(m + 1)))*Hypergeometric2F1[-n, m
 + 1, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m}, x]
 &&  !IntegerQ[m] && IntegerQ[n]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3673
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_.)*tan[(e_.) + (f_.)*(x_)]^
(m_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x]^2, x]}, Simp[ff^((m
 + 1)/2)/(2*f)   Subst[Int[x^((m - 1)/2)*((a + b*ff*x)^p/(1 - ff*x)^((m + 1
)/2)), x], x, Sin[e + f*x]^2/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && Integ
erQ[(m - 1)/2]
```

Maple [F]

$$\int \left(a+b\sinh \left(dx+c\right)^{2}\right)^{p}\tanh \left(dx+c\right)dx$$

input
```
int((a+b*sinh(d*x+c)^2)^p*tanh(d*x+c),x)
```

output
```
int((a+b*sinh(d*x+c)^2)^p*tanh(d*x+c),x)
```

Fricas [F]

$$\int \left(a+b\sinh^2(c+dx)\right)^p \tanh(c+dx)\,dx = \int \left(b\sinh\left(dx+c\right)^2+a\right)^p \tanh\left(dx+c\right)\,dx$$

input
```
integrate((a+b*sinh(d*x+c)^2)^p*tanh(d*x+c),x, algorithm="fricas")
```

output
```
integral((b*sinh(d*x + c)^2 + a)^p*tanh(d*x + c), x)
```

Sympy [F]

$$\int \left(a+b\sinh^2(c+dx)\right)^p \tanh(c+dx)\,dx = \int \left(a+b\sinh^2\left(c+dx\right)\right)^p \tanh\left(c+dx\right)dx$$

input
```
integrate((a+b*sinh(d*x+c)**2)**p*tanh(d*x+c),x)
```

output
```
Integral((a + b*sinh(c + d*x)**2)**p*tanh(c + d*x), x)
```

Maxima [F]

$$\int \left(a+b\sinh^2(c+dx)\right)^p \tanh(c+dx)\,dx = \int \left(b\sinh\left(dx+c\right)^2+a\right)^p \tanh\left(dx+c\right)\,dx$$

input
```
integrate((a+b*sinh(d*x+c)^2)^p*tanh(d*x+c),x, algorithm="maxima")
```

output
```
integrate((b*sinh(d*x + c)^2 + a)^p*tanh(d*x + c), x)
```

Giac [F]

$$\int \left(a+b\sinh^2(c+dx)\right)^p\tanh(c+dx)\,dx=\int \left(b\sinh\left(dx+c\right)^2+a\right)^p\tanh\left(dx+c\right)\,dx$$

input
```
integrate((a+b*sinh(d*x+c)^2)^p*tanh(d*x+c),x, algorithm="giac")
```

output
```
integrate((b*sinh(d*x + c)^2 + a)^p*tanh(d*x + c), x)
```

Mupad [F(-1)]

Timed out.

$$\int \left(a+b\sinh^2(c+dx)\right)^p\tanh(c+dx)\,dx=\int \tanh(c+d\,x)\,\left(b\sinh(c+d\,x)^2+a\right)^p\,dx$$

input
```
int(tanh(c + d*x)*(a + b*sinh(c + d*x)^2)^p,x)
```

output
```
int(tanh(c + d*x)*(a + b*sinh(c + d*x)^2)^p, x)
```

Reduce [F]

$$\int \left(a+b\sinh^2(c+dx)\right)^p\tanh(c+dx)\,dx$$
$$=\frac{4\left(e^{4dx+4c}b+4e^{2dx+2c}a-2e^{2dx+2c}b+b\right)^p a-3\left(e^{4dx+4c}b+4e^{2dx+2c}a-2e^{2dx+2c}b+b\right)^p b-8e^{2dpx+2cp+6c}\left(\right.}{}$$

input
```
int((a+b*sinh(d*x+c)^2)^p*tanh(d*x+c),x)
```

output
```
(4*(e**(4*c + 4*d*x)*b + 4*e**(2*c + 2*d*x)*a - 2*e**(2*c + 2*d*x)*b + b)*
*p*a - 3*(e**(4*c + 4*d*x)*b + 4*e**(2*c + 2*d*x)*a - 2*e**(2*c + 2*d*x)*b
 + b)**p*b - 8*e**(2*c*p + 6*c + 2*d*p*x)*int((e**(6*d*x)*(e**(4*c + 4*d*x
)*b + 4*e**(2*c + 2*d*x)*a - 2*e**(2*c + 2*d*x)*b + b)**p)/(e**(2*c*p + 6*
c + 2*d*p*x + 6*d*x)*b + 4*e**(2*c*p + 4*c + 2*d*p*x + 4*d*x)*a - e**(2*c*
p + 4*c + 2*d*p*x + 4*d*x)*b + 4*e**(2*c*p + 2*c + 2*d*p*x + 2*d*x)*a - e*
*(2*c*p + 2*c + 2*d*p*x + 2*d*x)*b + e**(2*c*p + 2*d*p*x)*b),x)*a*b*d*p +
8*e**(2*c*p + 6*c + 2*d*p*x)*int((e**(6*d*x)*(e**(4*c + 4*d*x)*b + 4*e**(2
*c + 2*d*x)*a - 2*e**(2*c + 2*d*x)*b + b)**p)/(e**(2*c*p + 6*c + 2*d*p*x +
 6*d*x)*b + 4*e**(2*c*p + 4*c + 2*d*p*x + 4*d*x)*a - e**(2*c*p + 4*c + 2*d
*p*x + 4*d*x)*b + 4*e**(2*c*p + 2*c + 2*d*p*x + 2*d*x)*a - e**(2*c*p + 2*c
 + 2*d*p*x + 2*d*x)*b + e**(2*c*p + 2*d*p*x)*b),x)*b**2*d*p + 8*e**(2*c*p
+ 2*d*p*x)*int((e**(4*c + 4*d*x)*b + 4*e**(2*c + 2*d*x)*a - 2*e**(2*c + 2*
d*x)*b + b)**p/(e**(2*c*p + 6*c + 2*d*p*x + 6*d*x)*b + 4*e**(2*c*p + 4*c +
 2*d*p*x + 4*d*x)*a - e**(2*c*p + 4*c + 2*d*p*x + 4*d*x)*b + 4*e**(2*c*p +
 2*c + 2*d*p*x + 2*d*x)*a - e**(2*c*p + 2*c + 2*d*p*x + 2*d*x)*b + e**(2*c
*p + 2*d*p*x)*b),x)*a*b*d*p - 8*e**(2*c*p + 2*d*p*x)*int((e**(4*c + 4*d*x)
*b + 4*e**(2*c + 2*d*x)*a - 2*e**(2*c + 2*d*x)*b + b)**p/(e**(2*c*p + 6*c
+ 2*d*p*x + 6*d*x)*b + 4*e**(2*c*p + 4*c + 2*d*p*x + 4*d*x)*a - e**(2*c*p
+ 4*c + 2*d*p*x + 4*d*x)*b + 4*e**(2*c*p + 2*c + 2*d*p*x + 2*d*x)*a - e...
```

3.479 $\int \coth(c+dx) \left(a+b\sinh^2(c+dx)\right)^p \, dx$

Optimal result

Integrand size = 21, antiderivative size = 54

$$\int \coth(c+dx) \left(a+b\sinh^2(c+dx)\right)^p \, dx = -\frac{\text{Hypergeometric2F1}\left(1,1+p,2+p,1+\frac{b\sinh^2(c+dx)}{a}\right)\left(a+b\sinh^2(c+dx)\right)^{1+p}}{2ad(1+p)}$$

output
```
-1/2*hypergeom([1, p+1],[2+p],1+b*sinh(d*x+c)^2/a)*(a+b*sinh(d*x+c)^2)^(p+
1)/a/d/(p+1)
```

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 54, normalized size of antiderivative = 1.00

$$\int \coth(c+dx) \left(a+b\sinh^2(c+dx)\right)^p \, dx = -\frac{\text{Hypergeometric2F1}\left(1,1+p,2+p,1+\frac{b\sinh^2(c+dx)}{a}\right)\left(a+b\sinh^2(c+dx)\right)^{1+p}}{2ad(1+p)}$$

input
```
Integrate[Coth[c + d*x]*(a + b*Sinh[c + d*x]^2)^p,x]
```

output
```
-1/2*(Hypergeometric2F1[1, 1 + p, 2 + p, 1 + (b*Sinh[c + d*x]^2)/a]*(a + b
*Sinh[c + d*x]^2)^(1 + p))/(a*d*(1 + p))
```

Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 54, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.190$, Rules used = {3042, 26, 3673, 75}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \coth(c+dx)\left(a+b\sinh^2(c+dx)\right)^p\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{i\left(a-b\sin(ic+idx)^2\right)^p}{\tan(ic+idx)}dx$$

$$\Big\downarrow \text{26}$$

$$i\int \frac{\left(a-b\sin(ic+idx)^2\right)^p}{\tan(ic+idx)}dx$$

$$\Big\downarrow \text{3673}$$

$$\frac{\int \operatorname{csch}^2(c+dx)\left(b\sinh^2(c+dx)+a\right)^p d\sinh^2(c+dx)}{2d}$$

$$\Big\downarrow \text{75}$$

$$-\frac{\left(a+b\sinh^2(c+dx)\right)^{p+1}\text{Hypergeometric2F1}\left(1,p+1,p+2,\frac{b\sinh^2(c+dx)}{a}+1\right)}{2ad(p+1)}$$

input
```
Int[Coth[c + d*x]*(a + b*Sinh[c + d*x]^2)^p,x]
```

output

```
-1/2*(Hypergeometric2F1[1, 1 + p, 2 + p, 1 + (b*Sinh[c + d*x]^2)/a]*(a + b
*Sinh[c + d*x]^2)^(1 + p))/(a*d*(1 + p))
```

Defintions of rubi rules used

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 75
```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((c + d*x
)^(n + 1)/(d*(n + 1)*(-d/(b*c))^m))*Hypergeometric2F1[-m, n + 1, n + 2, 1 +
 d*(x/c)], x] /; FreeQ[{b, c, d, m, n}, x] &&  !IntegerQ[n] && (IntegerQ[m]
 || GtQ[-d/(b*c), 0])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3673
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_.)*tan[(e_.) + (f_.)*(x_)]^
(m_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x]^2, x]}, Simp[ff^((m
 + 1)/2)/(2*f)   Subst[Int[x^((m - 1)/2)*((a + b*ff*x)^p/(1 - ff*x)^((m + 1
)/2)), x], x, Sin[e + f*x]^2/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && Integ
erQ[(m - 1)/2]
```

Maple [F]

$$\int \coth(dx+c)\left(a+b\sinh(dx+c)^2\right)^p dx$$

input
```
int(coth(d*x+c)*(a+b*sinh(d*x+c)^2)^p,x)
```

output
```
int(coth(d*x+c)*(a+b*sinh(d*x+c)^2)^p,x)
```

Fricas [F]

$$\int \coth(c+dx) \left(a+b\sinh^2(c+dx)\right)^p \, dx = \int \left(b\sinh\left(dx+c\right)^2+a\right)^p \coth\left(dx+c\right) \, dx$$

input
```
integrate(coth(d*x+c)*(a+b*sinh(d*x+c)^2)^p,x, algorithm="fricas")
```

output
```
integral((b*sinh(d*x + c)^2 + a)^p*coth(d*x + c), x)
```

Sympy [F(-1)]

Timed out.

$$\int \coth(c+dx) \left(a+b\sinh^2(c+dx)\right)^p \, dx = \text{Timed out}$$

input
```
integrate(coth(d*x+c)*(a+b*sinh(d*x+c)**2)**p,x)
```

output
```
Timed out
```

Maxima [F]

$$\int \coth(c+dx) \left(a+b\sinh^2(c+dx)\right)^p \, dx = \int \left(b\sinh\left(dx+c\right)^2+a\right)^p \coth\left(dx+c\right) \, dx$$

input
```
integrate(coth(d*x+c)*(a+b*sinh(d*x+c)^2)^p,x, algorithm="maxima")
```

output
```
integrate((b*sinh(d*x + c)^2 + a)^p*coth(d*x + c), x)
```

Giac [F]

$$\int \coth(c+dx)\left(a+b\sinh^2(c+dx)\right)^p\,dx=\int \left(b\sinh\left(dx+c\right)^2+a\right)^p\coth\left(dx+c\right)\,dx$$

input
```
integrate(coth(d*x+c)*(a+b*sinh(d*x+c)^2)^p,x, algorithm="giac")
```

output
```
integrate((b*sinh(d*x + c)^2 + a)^p*coth(d*x + c), x)
```

Mupad [F(-1)]

Timed out.

$$\int \coth(c+dx)\left(a+b\sinh^2(c+dx)\right)^p\,dx=\int \coth(c+d\,x)\,\left(b\sinh(c+d\,x)^2+a\right)^p\,dx$$

input
```
int(coth(c + d*x)*(a + b*sinh(c + d*x)^2)^p,x)
```

output
```
int(coth(c + d*x)*(a + b*sinh(c + d*x)^2)^p, x)
```

Reduce [F]

$$\int \coth(c+dx)\left(a+b\sinh^2(c+dx)\right)^p\,dx$$
$$=\frac{-4\left(e^{4dx+4c}b+4e^{2dx+2c}a-2e^{2dx+2c}b+b\right)^p a+\left(e^{4dx+4c}b+4e^{2dx+2c}a-2e^{2dx+2c}b+b\right)^p b+8e^{2dpx+2cp+6c}\Big(}{}$$

input
```
int(coth(d*x+c)*(a+b*sinh(d*x+c)^2)^p,x)
```

output
```
( - 4*(e**(4*c + 4*d*x)*b + 4*e**(2*c + 2*d*x)*a - 2*e**(2*c + 2*d*x)*b +
b)**p*a + (e**(4*c + 4*d*x)*b + 4*e**(2*c + 2*d*x)*a - 2*e**(2*c + 2*d*x)*
b + b)**p*b + 8*e**(2*c*p + 6*c + 2*d*p*x)*int((e**(6*d*x)*(e**(4*c + 4*d*
x)*b + 4*e**(2*c + 2*d*x)*a - 2*e**(2*c + 2*d*x)*b + b)**p)/(e**(2*c*p + 6
*c + 2*d*p*x + 6*d*x)*b + 4*e**(2*c*p + 4*c + 2*d*p*x + 4*d*x)*a - 3*e**(2
*c*p + 4*c + 2*d*p*x + 4*d*x)*b - 4*e**(2*c*p + 2*c + 2*d*p*x + 2*d*x)*a +
 3*e**(2*c*p + 2*c + 2*d*p*x + 2*d*x)*b - e**(2*c*p + 2*d*p*x)*b),x)*a*b*d
*p + 8*e**(2*c*p + 2*d*p*x)*int((e**(4*c + 4*d*x)*b + 4*e**(2*c + 2*d*x)*a
 - 2*e**(2*c + 2*d*x)*b + b)**p/(e**(2*c*p + 6*c + 2*d*p*x + 6*d*x)*b + 4*
e**(2*c*p + 4*c + 2*d*p*x + 4*d*x)*a - 3*e**(2*c*p + 4*c + 2*d*p*x + 4*d*x
)*b - 4*e**(2*c*p + 2*c + 2*d*p*x + 2*d*x)*a + 3*e**(2*c*p + 2*c + 2*d*p*x
 + 2*d*x)*b - e**(2*c*p + 2*d*p*x)*b),x)*a*b*d*p)/(2*e**(2*c*p + 2*d*p*x)*
4**p*b*d*p)
```

3.480 $\int \coth^3(c+dx)\left(a+b\sinh^2(c+dx)\right)^p\,dx$

Optimal result

Integrand size = 23, antiderivative size = 94

$$\int \coth^3(c+dx)\left(a+b\sinh^2(c+dx)\right)^p\,dx = -\frac{\operatorname{csch}^2(c+dx)\left(a+b\sinh^2(c+dx)\right)^{1+p}}{2ad} - \frac{(a+bp)\,\text{Hypergeometric2F1}\left(1,1+p,2+p,1+\frac{b\sinh^2(c+dx)}{a}\right)\left(a+b\sinh^2(c+dx)\right)^{1+p}}{2a^2d(1+p)}$$

output
```
-1/2*csch(d*x+c)^2*(a+b*sinh(d*x+c)^2)^(p+1)/a/d-1/2*(b*p+a)*hypergeom([1,
p+1],[2+p],1+b*sinh(d*x+c)^2/a)*(a+b*sinh(d*x+c)^2)^(p+1)/a^2/d/(p+1)
```

Mathematica [A] (verified)

Time = 0.27 (sec) , antiderivative size = 71, normalized size of antiderivative = 0.76

$$\int \coth^3(c+dx)\left(a+b\sinh^2(c+dx)\right)^p\,dx = -\frac{\left(a\operatorname{csch}^2(c+dx)+\frac{(a+bp)\,\text{Hypergeometric2F1}\left(1,1+p,2+p,1+\frac{b\sinh^2(c+dx)}{a}\right)}{1+p}\right)\left(a+b\sinh^2(c+dx)\right)^{1+p}}{2a^2d}$$

input
```
Integrate[Coth[c + d*x]^3*(a + b*Sinh[c + d*x]^2)^p,x]
```

output
```
-1/2*((a*Csch[c + d*x]^2 + ((a + b*p)*Hypergeometric2F1[1, 1 + p, 2 + p, 1
 + (b*Sinh[c + d*x]^2)/a])/(1 + p))*(a + b*Sinh[c + d*x]^2)^(1 + p))/(a^2*
d)
```

Rubi [A] (verified)

Time = 0.31 (sec) , antiderivative size = 91, normalized size of antiderivative = 0.97, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.217$, Rules used = {3042, 26, 3673, 87, 75}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \coth^3(c+dx)\left(a+b\sinh^2(c+dx)\right)^p\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int -\frac{i\left(a-b\sin(ic+idx)^2\right)^p}{\tan(ic+idx)^3}dx$$

$$\Big\downarrow \text{26}$$

$$-i\int \frac{\left(a-b\sin(ic+idx)^2\right)^p}{\tan(ic+idx)^3}dx$$

$$\Big\downarrow \text{3673}$$

$$\frac{\int \operatorname{csch}^4(c+dx)\left(\sinh^2(c+dx)+1\right)\left(b\sinh^2(c+dx)+a\right)^p d\sinh^2(c+dx)}{2d}$$

$$\Big\downarrow \text{87}$$

$$\frac{\frac{(a+bp)\int \operatorname{csch}^2(c+dx)\left(b\sinh^2(c+dx)+a\right)^p d\sinh^2(c+dx)}{a}-\frac{\operatorname{csch}^2(c+dx)\left(a+b\sinh^2(c+dx)\right)^{p+1}}{a}}{2d}$$

$$\Big\downarrow \text{75}$$

$$\frac{-\frac{(a+bp)\left(a+b\sinh^2(c+dx)\right)^{p+1}\text{Hypergeometric2F1}\left(1,p+1,p+2,\frac{b\sinh^2(c+dx)}{a}+1\right)}{a^2(p+1)}-\frac{\operatorname{csch}^2(c+dx)\left(a+b\sinh^2(c+dx)\right)^{p+1}}{a}}{2d}$$

input
```
Int[Coth[c + d*x]^3*(a + b*Sinh[c + d*x]^2)^p,x]
```

output
```
(-((Csch[c + d*x]^2*(a + b*Sinh[c + d*x]^2)^(1 + p))/a) - ((a + b*p)*Hyper
geometric2F1[1, 1 + p, 2 + p, 1 + (b*Sinh[c + d*x]^2)/a]*(a + b*Sinh[c + d
*x]^2)^(1 + p))/(a^2*(1 + p)))/(2*d)
```

Defintions of rubi rules used

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 75
```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((c + d*x
)^(n + 1)/(d*(n + 1)*(-d/(b*c))^m))*Hypergeometric2F1[-m, n + 1, n + 2, 1 +
 d*(x/c)], x] /; FreeQ[{b, c, d, m, n}, x] &&  !IntegerQ[n] && (IntegerQ[m]
 || GtQ[-d/(b*c), 0])
```

rule 87
```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_] :> Simp[(-(b*e - a*f))*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(f*(p
+ 1)*(c*f - d*e))), x] - Simp[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p
+ 1)))/(f*(p + 1)*(c*f - d*e))   Int[(c + d*x)^n*(e + f*x)^(p + 1), x], x]
/; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || Intege
rQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))))
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3673
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_.)*tan[(e_.) + (f_.)*(x_)]^
(m_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x]^2, x]}, Simp[ff^((m
 + 1)/2)/(2*f)   Subst[Int[x^((m - 1)/2)*((a + b*ff*x)^p/(1 - ff*x)^((m + 1
)/2)), x], x, Sin[e + f*x]^2/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && Integ
erQ[(m - 1)/2]
```

Maple [F]

$$\int \coth(dx+c)^3 \left(a+b\sinh(dx+c)^2\right)^p dx$$

input
```
int(coth(d*x+c)^3*(a+b*sinh(d*x+c)^2)^p,x)
```

output
```
int(coth(d*x+c)^3*(a+b*sinh(d*x+c)^2)^p,x)
```

Fricas [F]

$$\int \coth^3(c+dx)\left(a+b\sinh^2(c+dx)\right)^p\, dx = \int \left(b\sinh(dx+c)^2+a\right)^p \coth(dx+c)^3\, dx$$

input
```
integrate(coth(d*x+c)^3*(a+b*sinh(d*x+c)^2)^p,x, algorithm="fricas")
```

output
```
integral((b*sinh(d*x + c)^2 + a)^p*coth(d*x + c)^3, x)
```

Sympy [F(-1)]

Timed out.

$$\int \coth^3(c+dx)\left(a+b\sinh^2(c+dx)\right)^p\, dx = \text{Timed out}$$

input
```
integrate(coth(d*x+c)**3*(a+b*sinh(d*x+c)**2)**p,x)
```

output
```
Timed out
```

Maxima [F]

$$\int \coth^3(c+dx)\left(a+b\sinh^2(c+dx)\right)^p\,dx = \int \left(b\sinh\left(dx+c\right)^2+a\right)^p\coth\left(dx+c\right)^3\,dx$$

input
```
integrate(coth(d*x+c)^3*(a+b*sinh(d*x+c)^2)^p,x, algorithm="maxima")
```

output
```
integrate((b*sinh(d*x + c)^2 + a)^p*coth(d*x + c)^3, x)
```

Giac [F]

$$\int \coth^3(c+dx)\left(a+b\sinh^2(c+dx)\right)^p\,dx = \int \left(b\sinh\left(dx+c\right)^2+a\right)^p\coth\left(dx+c\right)^3\,dx$$

input
```
integrate(coth(d*x+c)^3*(a+b*sinh(d*x+c)^2)^p,x, algorithm="giac")
```

output
```
integrate((b*sinh(d*x + c)^2 + a)^p*coth(d*x + c)^3, x)
```

Mupad [F(-1)]

Timed out.

$$\int \coth^3(c+dx)\left(a+b\sinh^2(c+dx)\right)^p\,dx = \int \coth(c+d\,x)^3\left(b\sinh(c+d\,x)^2+a\right)^p\,dx$$

input
```
int(coth(c + d*x)^3*(a + b*sinh(c + d*x)^2)^p,x)
```

output
```
int(coth(c + d*x)^3*(a + b*sinh(c + d*x)^2)^p, x)
```

Reduce [F]

$$\int \coth^3(c+dx)\left(a+b\sinh^2(c+dx)\right)^p\,dx=\int \coth\left(dx+c\right)^3\left(\sinh\left(dx+c\right)^2b+a\right)^p dx$$

input
```
int(coth(d*x+c)^3*(a+b*sinh(d*x+c)^2)^p,x)
```

output
```
int(coth(d*x+c)^3*(a+b*sinh(d*x+c)^2)^p,x)
```

3.481 $\int \left(a+b\sinh^2(c+dx)\right)^p \tanh^4(c+dx)\,dx$

Optimal result

Integrand size = 23, antiderivative size = 103

$$\int \left(a+b\sinh^2(c+dx)\right)^p \tanh^4(c+dx)\,dx = \frac{\text{AppellF1}\left(\frac{5}{2},\frac{5}{2},-p,\frac{7}{2},-\sinh^2(c+dx),-\frac{b\sinh^2(c+dx)}{a}\right)\sqrt{\cosh^2(c+dx)}\sinh^4(c+dx)\left(a+b\sinh^2(c+dx\right.}{5d}$$

output
```
1/5*AppellF1(5/2,5/2,-p,7/2,-sinh(d*x+c)^2,-b*sinh(d*x+c)^2/a)*(cosh(d*x+c
)^2)^(1/2)*sinh(d*x+c)^4*(a+b*sinh(d*x+c)^2)^p*tanh(d*x+c)/d/((1+b*sinh(d*
x+c)^2/a)^p)
```

Mathematica [F]

$$\int \left(a+b\sinh^2(c+dx)\right)^p \tanh^4(c+dx)\,dx = \int \left(a+b\sinh^2(c+dx)\right)^p \tanh^4(c+dx)\,dx$$

input
```
Integrate[(a + b*Sinh[c + d*x]^2)^p*Tanh[c + d*x]^4,x]
```

output
```
Integrate[(a + b*Sinh[c + d*x]^2)^p*Tanh[c + d*x]^4, x]
```

Rubi [A] (verified)

Time = 0.38 (sec) , antiderivative size = 103, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.174$, Rules used = {3042, 3675, 395, 394}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \tanh^4(c+dx)\left(a+b\sinh^2(c+dx)\right)^p\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \tan(ic+idx)^4\left(a-b\sin(ic+idx)^2\right)^p dx$$

$$\Big\downarrow \text{3675}$$

$$\frac{\sqrt{\cosh^2(c+dx)}\text{sech}(c+dx)\int \frac{\sinh^4(c+dx)(b\sinh^2(c+dx)+a)^p}{(\sinh^2(c+dx)+1)^{5/2}}d\sinh(c+dx)}{d}$$

$$\Big\downarrow \text{395}$$

$$\frac{\sqrt{\cosh^2(c+dx)}\text{sech}(c+dx)\left(a+b\sinh^2(c+dx)\right)^p\left(\frac{b\sinh^2(c+dx)}{a}+1\right)^{-p}\int \frac{\sinh^4(c+dx)\left(\frac{b\sinh^2(c+dx)}{a}+1\right)^p}{(\sinh^2(c+dx)+1)^{5/2}}d\sinh(c+dx)}{d}$$

$$\Big\downarrow \text{394}$$

$$\frac{\sinh^4(c+dx)\sqrt{\cosh^2(c+dx)}\tanh(c+dx)\left(a+b\sinh^2(c+dx)\right)^p\left(\frac{b\sinh^2(c+dx)}{a}+1\right)^{-p}\text{AppellF1}\left(\frac{5}{2},\frac{5}{2},-p,\frac{7}{2},-\sinh^2(c+dx),-\frac{b\sinh^2(c+dx)}{a}\right)}{5d}$$

input
```
Int[(a + b*Sinh[c + d*x]^2)^p*Tanh[c + d*x]^4,x]
```

output
```
(AppellF1[5/2, 5/2, -p, 7/2, -Sinh[c + d*x]^2, -((b*Sinh[c + d*x]^2)/a)]*S
qrt[Cosh[c + d*x]^2]*Sinh[c + d*x]^4*(a + b*Sinh[c + d*x]^2)^p*Tanh[c + d*
x])/(5*d*(1 + (b*Sinh[c + d*x]^2)/a)^p)
```

Defintions of rubi rules used

rule 394
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_
), x_Symbol] :> Simp[a^p*c^q*((e*x)^(m + 1)/(e*(m + 1)))*AppellF1[(m + 1)/2
, -p, -q, 1 + (m + 1)/2, (-b)*(x^2/a), (-d)*(x^2/c)], x] /; FreeQ[{a, b, c,
 d, e, m, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, 1] && (Int
egerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

rule 395
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_
), x_Symbol] :> Simp[a^IntPart[p]*((a + b*x^2)^FracPart[p]/(1 + b*(x^2/a))^
FracPart[p])   Int[(e*x)^m*(1 + b*(x^2/a))^p*(c + d*x^2)^q, x], x] /; FreeQ
[{a, b, c, d, e, m, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m,
1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3675
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_.)*tan[(e_.) + (f_.)*(x_)]^
(m_), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Simp[ff^(m + 1
)*(Sqrt[Cos[e + f*x]^2]/(f*Cos[e + f*x]))   Subst[Int[x^m*((a + b*ff^2*x^2)
^p/(1 - ff^2*x^2)^((m + 1)/2)), x], x, Sin[e + f*x]/ff], x]] /; FreeQ[{a, b
, e, f, p}, x] && IntegerQ[m/2] &&  !IntegerQ[p]
```

Maple [F]

$$\int \left(a+b\sinh \left(dx+c\right)^{2}\right)^{p} \tanh \left(dx+c\right)^{4}dx$$

input
```
int((a+b*sinh(d*x+c)^2)^p*tanh(d*x+c)^4,x)
```

output
```
int((a+b*sinh(d*x+c)^2)^p*tanh(d*x+c)^4,x)
```

Fricas [F]

$$\int \left(a+b\sinh^2(c+dx)\right)^p \tanh^4(c+dx)\,dx = \int \left(b\sinh\left(dx+c\right)^2+a\right)^p \tanh\left(dx+c\right)^4\,dx$$

input
```
integrate((a+b*sinh(d*x+c)^2)^p*tanh(d*x+c)^4,x, algorithm="fricas")
```

output
```
integral((b*sinh(d*x + c)^2 + a)^p*tanh(d*x + c)^4, x)
```

Sympy [F(-1)]

Timed out.

$$\int \left(a+b\sinh^2(c+dx)\right)^p \tanh^4(c+dx)\,dx = \text{Timed out}$$

input
```
integrate((a+b*sinh(d*x+c)**2)**p*tanh(d*x+c)**4,x)
```

output
```
Timed out
```

Maxima [F]

$$\int \left(a+b\sinh^2(c+dx)\right)^p \tanh^4(c+dx)\,dx = \int \left(b\sinh\left(dx+c\right)^2+a\right)^p \tanh\left(dx+c\right)^4\,dx$$

input
```
integrate((a+b*sinh(d*x+c)^2)^p*tanh(d*x+c)^4,x, algorithm="maxima")
```

output
```
integrate((b*sinh(d*x + c)^2 + a)^p*tanh(d*x + c)^4, x)
```

Giac [F]

$$\int \left(a+b\sinh^2(c+dx)\right)^p \tanh^4(c+dx)\,dx = \int \left(b\sinh\left(dx+c\right)^2+a\right)^p \tanh\left(dx+c\right)^4\,dx$$

input
```
integrate((a+b*sinh(d*x+c)^2)^p*tanh(d*x+c)^4,x, algorithm="giac")
```

output
```
integrate((b*sinh(d*x + c)^2 + a)^p*tanh(d*x + c)^4, x)
```

Mupad [F(-1)]

Timed out.

$$\int \left(a+b\sinh^2(c+dx)\right)^p \tanh^4(c+dx)\,dx = \int \tanh(c+d\,x)^4\,\left(b\sinh(c+d\,x)^2+a\right)^p\,dx$$

input
```
int(tanh(c + d*x)^4*(a + b*sinh(c + d*x)^2)^p,x)
```

output
```
int(tanh(c + d*x)^4*(a + b*sinh(c + d*x)^2)^p, x)
```

Reduce [F]

$$\int \left(a+b\sinh^2(c+dx)\right)^p \tanh^4(c+dx)\,dx = \int \left(\sinh\left(dx+c\right)^2 b+a\right)^p \tanh\left(dx+c\right)^4 dx$$

input
```
int((a+b*sinh(d*x+c)^2)^p*tanh(d*x+c)^4,x)
```

output
```
int((a+b*sinh(d*x+c)^2)^p*tanh(d*x+c)^4,x)
```

3.482 $\int (a+b\sinh^2(c+dx))^p \tanh^2(c+dx)\,dx$

Optimal result

Integrand size = 23, antiderivative size = 103

$$\int (a+b\sinh^2(c+dx))^p \tanh^2(c+dx)\,dx = \frac{\operatorname{AppellF1}\left(\frac{3}{2},\frac{3}{2},-p,\frac{5}{2},-\sinh^2(c+dx),-\frac{b\sinh^2(c+dx)}{a}\right)\sqrt{\cosh^2(c+dx)}\sinh^2(c+dx)\,(a+b\sinh^2(c+dx}{3d}$$

output
```
1/3*AppellF1(3/2,3/2,-p,5/2,-sinh(d*x+c)^2,-b*sinh(d*x+c)^2/a)*(cosh(d*x+c
)^2)^(1/2)*sinh(d*x+c)^2*(a+b*sinh(d*x+c)^2)^p*tanh(d*x+c)/d/((1+b*sinh(d*
x+c)^2/a)^p)
```

Mathematica [F]

$$\int (a+b\sinh^2(c+dx))^p \tanh^2(c+dx)\,dx = \int (a+b\sinh^2(c+dx))^p \tanh^2(c+dx)\,dx$$

input
```
Integrate[(a + b*Sinh[c + d*x]^2)^p*Tanh[c + d*x]^2,x]
```

output
```
Integrate[(a + b*Sinh[c + d*x]^2)^p*Tanh[c + d*x]^2, x]
```

Rubi [A] (verified)

Time = 0.35 (sec) , antiderivative size = 103, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.217, Rules used = {3042, 25, 3675, 395, 394}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \tanh^2(c+dx)\left(a+b\sinh^2(c+dx)\right)^p dx$$

$$\Big\downarrow \text{3042}$$

$$\int \tan(ic+idx)^2\left(-\left(a-b\sin(ic+idx)^2\right)^p\right)dx$$

$$\Big\downarrow \text{25}$$

$$-\int \left(a-b\sin(ic+idx)^2\right)^p\tan(ic+idx)^2dx$$

$$\Big\downarrow \text{3675}$$

$$\frac{\sqrt{\cosh^2(c+dx)}\operatorname{sech}(c+dx)\int \frac{\sinh^2(c+dx)\left(b\sinh^2(c+dx)+a\right)^p}{\left(\sinh^2(c+dx)+1\right)^{3/2}}d\sinh(c+dx)}{d}$$

$$\Big\downarrow \text{395}$$

$$\frac{\sqrt{\cosh^2(c+dx)}\operatorname{sech}(c+dx)\left(a+b\sinh^2(c+dx)\right)^p\left(\frac{b\sinh^2(c+dx)}{a}+1\right)^{-p}\int \frac{\sinh^2(c+dx)\left(\frac{b\sinh^2(c+dx)}{a}+1\right)^p}{\left(\sinh^2(c+dx)+1\right)^{3/2}}d\sinh(c+$$

$$\Big\downarrow \text{394}$$

$$\frac{\sinh^2(c+dx)\sqrt{\cosh^2(c+dx)}\tanh(c+dx)\left(a+b\sinh^2(c+dx)\right)^p\left(\frac{b\sinh^2(c+dx)}{a}+1\right)^{-p}\operatorname{AppellF1}\left(\frac{3}{2},\frac{3}{2},-p,\frac{5}{2},-\right.}{3d}$$

input
```
Int[(a + b*Sinh[c + d*x]^2)^p*Tanh[c + d*x]^2,x]
```

output
```
(AppellF1[3/2, 3/2, -p, 5/2, -Sinh[c + d*x]^2, -((b*Sinh[c + d*x]^2)/a)]*S
qrt[Cosh[c + d*x]^2]*Sinh[c + d*x]^2*(a + b*Sinh[c + d*x]^2)^p*Tanh[c + d*
x])/(3*d*(1 + (b*Sinh[c + d*x]^2)/a)^p)
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 394
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_
), x_Symbol] :> Simp[a^p*c^q*((e*x)^(m + 1)/(e*(m + 1)))*AppellF1[(m + 1)/2
, -p, -q, 1 + (m + 1)/2, (-b)*(x^2/a), (-d)*(x^2/c)], x] /; FreeQ[{a, b, c,
 d, e, m, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, 1] && (Int
egerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

rule 395
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_
), x_Symbol] :> Simp[a^IntPart[p]*((a + b*x^2)^FracPart[p]/(1 + b*(x^2/a))^
FracPart[p])   Int[(e*x)^m*(1 + b*(x^2/a))^p*(c + d*x^2)^q, x], x] /; FreeQ
[{a, b, c, d, e, m, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m,
1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3675
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_.)*tan[(e_.) + (f_.)*(x_)]^
(m_), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Simp[ff^(m + 1
)*(Sqrt[Cos[e + f*x]^2]/(f*Cos[e + f*x]))   Subst[Int[x^m*((a + b*ff^2*x^2)
^p/(1 - ff^2*x^2)^((m + 1)/2)), x], x, Sin[e + f*x]/ff], x]] /; FreeQ[{a, b
, e, f, p}, x] && IntegerQ[m/2] &&  !IntegerQ[p]
```

Maple [F]

$$\int \left(a+b\sinh\left(dx+c\right)^{2}\right)^{p} \tanh\left(dx+c\right)^{2} dx$$

input
```
int((a+b*sinh(d*x+c)^2)^p*tanh(d*x+c)^2,x)
```

output
```
int((a+b*sinh(d*x+c)^2)^p*tanh(d*x+c)^2,x)
```

Fricas [F]

$$\int \left(a+b\sinh^{2}(c+dx)\right)^{p} \tanh^{2}(c+dx)\,dx = \int \left(b\sinh\left(dx+c\right)^{2}+a\right)^{p} \tanh\left(dx+c\right)^{2} dx$$

input
```
integrate((a+b*sinh(d*x+c)^2)^p*tanh(d*x+c)^2,x, algorithm="fricas")
```

output
```
integral((b*sinh(d*x + c)^2 + a)^p*tanh(d*x + c)^2, x)
```

Sympy [F(-1)]

Timed out.

$$\int \left(a+b\sinh^{2}(c+dx)\right)^{p} \tanh^{2}(c+dx)\,dx = \text{Timed out}$$

input
```
integrate((a+b*sinh(d*x+c)**2)**p*tanh(d*x+c)**2,x)
```

output
```
Timed out
```

Maxima [F]

$$\int \left(a+b\sinh^2(c+dx)\right)^p \tanh^2(c+dx)\,dx=\int \left(b\sinh\left(dx+c\right)^2+a\right)^p\tanh\left(dx+c\right)^2\,dx$$

input
```
integrate((a+b*sinh(d*x+c)^2)^p*tanh(d*x+c)^2,x, algorithm="maxima")
```

output
```
integrate((b*sinh(d*x + c)^2 + a)^p*tanh(d*x + c)^2, x)
```

Giac [F]

$$\int \left(a+b\sinh^2(c+dx)\right)^p \tanh^2(c+dx)\,dx=\int \left(b\sinh\left(dx+c\right)^2+a\right)^p\tanh\left(dx+c\right)^2\,dx$$

input
```
integrate((a+b*sinh(d*x+c)^2)^p*tanh(d*x+c)^2,x, algorithm="giac")
```

output
```
integrate((b*sinh(d*x + c)^2 + a)^p*tanh(d*x + c)^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int \left(a+b\sinh^2(c+dx)\right)^p \tanh^2(c+dx)\,dx=\int \tanh(c+d\,x)^2\,\left(b\sinh(c+d\,x)^2+a\right)^p\,dx$$

input
```
int(tanh(c + d*x)^2*(a + b*sinh(c + d*x)^2)^p,x)
```

output
```
int(tanh(c + d*x)^2*(a + b*sinh(c + d*x)^2)^p, x)
```

Reduce [F]

$$\int \left(a+b\sinh^2(c+dx)\right)^p \tanh^2(c+dx)\,dx=\int \left(\sinh\left(dx+c\right)^2 b+a\right)^p \tanh\left(dx+c\right)^2 dx$$

input
```
int((a+b*sinh(d*x+c)^2)^p*tanh(d*x+c)^2,x)
```

output
```
int((a+b*sinh(d*x+c)^2)^p*tanh(d*x+c)^2,x)
```

3.483 $\int \coth^2(c+dx) \left(a+b\sinh^2(c+dx)\right)^p \, dx$

Optimal result

Integrand size = 23, antiderivative size = 99

$$\int \coth^2(c+dx) \left(a+b\sinh^2(c+dx)\right)^p \, dx =$$

$$-\frac{\text{AppellF1}\left(-\frac{1}{2},-\frac{1}{2},-p,\frac{1}{2},-\sinh^2(c+dx),-\frac{b\sinh^2(c+dx)}{a}\right)\sqrt{\cosh^2(c+dx)}\text{csch}(c+dx)\text{sech}(c+dx)\,(a}{d}$$

output
```
-AppellF1(-1/2,-1/2,-p,1/2,-sinh(d*x+c)^2,-b*sinh(d*x+c)^2/a)*(cosh(d*x+c)
^2)^(1/2)*csch(d*x+c)*sech(d*x+c)*(a+b*sinh(d*x+c)^2)^p/d/((1+b*sinh(d*x+c
)^2/a)^p)
```

Mathematica [F]

$$\int \coth^2(c+dx) \left(a+b\sinh^2(c+dx)\right)^p \, dx = \int \coth^2(c+dx) \left(a+b\sinh^2(c+dx)\right)^p \, dx$$

input
```
Integrate[Coth[c + d*x]^2*(a + b*Sinh[c + d*x]^2)^p,x]
```

output
```
Integrate[Coth[c + d*x]^2*(a + b*Sinh[c + d*x]^2)^p, x]
```

Rubi [A] (verified)

Time = 0.35 (sec) , antiderivative size = 99, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.217$, Rules used = {3042, 25, 3675, 395, 394}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \coth^2(c+dx)\left(a+b\sinh^2(c+dx)\right)^p\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int -\frac{\left(a-b\sin(ic+idx)^2\right)^p}{\tan(ic+idx)^2}dx$$

$$\Big\downarrow \text{25}$$

$$-\int \frac{\left(a-b\sin(ic+idx)^2\right)^p}{\tan(ic+idx)^2}dx$$

$$\Big\downarrow \text{3675}$$

$$\frac{\sqrt{\cosh^2(c+dx)}\operatorname{sech}(c+dx)\int \operatorname{csch}^2(c+dx)\sqrt{\sinh^2(c+dx)+1}\left(b\sinh^2(c+dx)+a\right)^p d\sinh(c+dx)}{d}$$

$$\Big\downarrow \text{395}$$

$$\frac{\sqrt{\cosh^2(c+dx)}\operatorname{sech}(c+dx)\left(a+b\sinh^2(c+dx)\right)^p\left(\frac{b\sinh^2(c+dx)}{a}+1\right)^{-p}\int \operatorname{csch}^2(c+dx)\sqrt{\sinh^2(c+dx)+1}\Big(\frac{b s}{}$$

$$\Big\downarrow \text{394}$$

$$-\frac{\sqrt{\cosh^2(c+dx)}\operatorname{csch}(c+dx)\operatorname{sech}(c+dx)\left(a+b\sinh^2(c+dx)\right)^p\left(\frac{b\sinh^2(c+dx)}{a}+1\right)^{-p}\operatorname{AppellF1}\left(-\frac{1}{2},-\frac{1}{2},-p,\frac{1}{2}\right.}{d}$$

input

```
Int[Coth[c + d*x]^2*(a + b*Sinh[c + d*x]^2)^p,x]
```

output
```
-((AppellF1[-1/2, -1/2, -p, 1/2, -Sinh[c + d*x]^2, -((b*Sinh[c + d*x]^2)/a
)]*Sqrt[Cosh[c + d*x]^2]*Csch[c + d*x]*Sech[c + d*x]*(a + b*Sinh[c + d*x]^
2)^p)/(d*(1 + (b*Sinh[c + d*x]^2)/a)^p))
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 394
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_
), x_Symbol] :> Simp[a^p*c^q*((e*x)^(m + 1)/(e*(m + 1)))*AppellF1[(m + 1)/2
, -p, -q, 1 + (m + 1)/2, (-b)*(x^2/a), (-d)*(x^2/c)], x] /; FreeQ[{a, b, c,
 d, e, m, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, 1] && (Int
egerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

rule 395
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_
), x_Symbol] :> Simp[a^IntPart[p]*((a + b*x^2)^FracPart[p]/(1 + b*(x^2/a))^
FracPart[p])   Int[(e*x)^m*(1 + b*(x^2/a))^p*(c + d*x^2)^q, x], x] /; FreeQ
[{a, b, c, d, e, m, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m,
1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3675
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_.)*tan[(e_.) + (f_.)*(x_)]^
(m_), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Simp[ff^(m + 1
)*(Sqrt[Cos[e + f*x]^2]/(f*Cos[e + f*x]))   Subst[Int[x^m*((a + b*ff^2*x^2)
^p/(1 - ff^2*x^2)^((m + 1)/2)), x], x, Sin[e + f*x]/ff], x]] /; FreeQ[{a, b
, e, f, p}, x] && IntegerQ[m/2] &&  !IntegerQ[p]
```

Maple [F]

$$\int \coth(dx+c)^2 \left(a+b\sinh(dx+c)^2\right)^p dx$$

input
```
int(coth(d*x+c)^2*(a+b*sinh(d*x+c)^2)^p,x)
```

output
```
int(coth(d*x+c)^2*(a+b*sinh(d*x+c)^2)^p,x)
```

Fricas [F]

$$\int \coth^2(c+dx) \left(a+b\sinh^2(c+dx)\right)^p \, dx = \int \left(b\sinh(dx+c)^2+a\right)^p \coth(dx+c)^2 \, dx$$

input
```
integrate(coth(d*x+c)^2*(a+b*sinh(d*x+c)^2)^p,x, algorithm="fricas")
```

output
```
integral((b*sinh(d*x + c)^2 + a)^p*coth(d*x + c)^2, x)
```

Sympy [F(-1)]

Timed out.

$$\int \coth^2(c+dx) \left(a+b\sinh^2(c+dx)\right)^p \, dx = \text{Timed out}$$

input
```
integrate(coth(d*x+c)**2*(a+b*sinh(d*x+c)**2)**p,x)
```

output
```
Timed out
```

Maxima [F]

$$\int \coth^2(c+dx)\left(a+b\sinh^2(c+dx)\right)^p\,dx=\int \left(b\sinh\left(dx+c\right)^2+a\right)^p\coth\left(dx+c\right)^2\,dx$$

input
```
integrate(coth(d*x+c)^2*(a+b*sinh(d*x+c)^2)^p,x, algorithm="maxima")
```

output
```
integrate((b*sinh(d*x + c)^2 + a)^p*coth(d*x + c)^2, x)
```

Giac [F]

$$\int \coth^2(c+dx)\left(a+b\sinh^2(c+dx)\right)^p\,dx=\int \left(b\sinh\left(dx+c\right)^2+a\right)^p\coth\left(dx+c\right)^2\,dx$$

input
```
integrate(coth(d*x+c)^2*(a+b*sinh(d*x+c)^2)^p,x, algorithm="giac")
```

output
```
integrate((b*sinh(d*x + c)^2 + a)^p*coth(d*x + c)^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int \coth^2(c+dx)\left(a+b\sinh^2(c+dx)\right)^p\,dx=\int \coth(c+d\,x)^2\,\left(b\sinh(c+d\,x)^2+a\right)^p\,dx$$

input
```
int(coth(c + d*x)^2*(a + b*sinh(c + d*x)^2)^p,x)
```

output
```
int(coth(c + d*x)^2*(a + b*sinh(c + d*x)^2)^p, x)
```

Reduce [F]

$$\int \coth^2(c+dx)\left(a+b\sinh^2(c+dx)\right)^p\,dx=\int \coth\left(dx+c\right)^2\left(\sinh\left(dx+c\right)^2b+a\right)^p dx$$

input
```
int(coth(d*x+c)^2*(a+b*sinh(d*x+c)^2)^p,x)
```

output
```
int(coth(d*x+c)^2*(a+b*sinh(d*x+c)^2)^p,x)
```

3.484 $\int \coth^4(c+dx) \left(a+b\sinh^2(c+dx)\right)^p \, dx$

Optimal result

Integrand size = 23, antiderivative size = 103

$$\int \coth^4(c+dx) \left(a+b\sinh^2(c+dx)\right)^p \, dx = -\frac{\text{AppellF1}\left(-\frac{3}{2},-\frac{3}{2},-p,-\frac{1}{2},-\sinh^2(c+dx),-\frac{b\sinh^2(c+dx)}{a}\right)\sqrt{\cosh^2(c+dx)}\text{csch}^3(c+dx)\text{sech}(c+dx}{3d}$$

output
```
-1/3*AppellF1(-3/2,-3/2,-p,-1/2,-sinh(d*x+c)^2,-b*sinh(d*x+c)^2/a)*(cosh(d
*x+c)^2)^(1/2)*csch(d*x+c)^3*sech(d*x+c)*(a+b*sinh(d*x+c)^2)^p/d/((1+b*sin
h(d*x+c)^2/a)^p)
```

Mathematica [F]

$$\int \coth^4(c+dx) \left(a+b\sinh^2(c+dx)\right)^p \, dx = \int \coth^4(c+dx) \left(a+b\sinh^2(c+dx)\right)^p \, dx$$

input
```
Integrate[Coth[c + d*x]^4*(a + b*Sinh[c + d*x]^2)^p,x]
```

output
```
Integrate[Coth[c + d*x]^4*(a + b*Sinh[c + d*x]^2)^p, x]
```

Rubi [A] (verified)

Time = 0.35 (sec) , antiderivative size = 103, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.174$, Rules used = {3042, 3675, 395, 394}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \coth^4(c+dx)\left(a+b\sinh^2(c+dx)\right)^p\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\left(a-b\sin(ic+idx)^2\right)^p}{\tan(ic+idx)^4}dx$$

$$\Big\downarrow \text{3675}$$

$$\frac{\sqrt{\cosh^2(c+dx)}\text{sech}(c+dx)\int \text{csch}^4(c+dx)\left(\sinh^2(c+dx)+1\right)^{3/2}\left(b\sinh^2(c+dx)+a\right)^p d\sinh(c+dx)}{d}$$

$$\Big\downarrow \text{395}$$

$$\frac{\sqrt{\cosh^2(c+dx)}\text{sech}(c+dx)\left(a+b\sinh^2(c+dx)\right)^p\left(\frac{b\sinh^2(c+dx)}{a}+1\right)^{-p}\int \text{csch}^4(c+dx)\left(\sinh^2(c+dx)+1\right)^{3/2}$$

$$\Big\downarrow \text{394}$$

$$-\frac{\sqrt{\cosh^2(c+dx)}\text{csch}^3(c+dx)\text{sech}(c+dx)\left(a+b\sinh^2(c+dx)\right)^p\left(\frac{b\sinh^2(c+dx)}{a}+1\right)^{-p}\text{AppellF1}\left(-\frac{3}{2},-\frac{3}{2},-p,\right.}{3d}$$

input
```
Int[Coth[c + d*x]^4*(a + b*Sinh[c + d*x]^2)^p,x]
```

output
```
-1/3*(AppellF1[-3/2, -3/2, -p, -1/2, -Sinh[c + d*x]^2, -((b*Sinh[c + d*x]^
2)/a)]*Sqrt[Cosh[c + d*x]^2]*Csch[c + d*x]^3*Sech[c + d*x]*(a + b*Sinh[c +
 d*x]^2)^p)/(d*(1 + (b*Sinh[c + d*x]^2)/a)^p)
```

Defintions of rubi rules used

rule 394
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_
), x_Symbol] :> Simp[a^p*c^q*((e*x)^(m + 1)/(e*(m + 1)))*AppellF1[(m + 1)/2
, -p, -q, 1 + (m + 1)/2, (-b)*(x^2/a), (-d)*(x^2/c)], x] /; FreeQ[{a, b, c,
 d, e, m, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, 1] && (Int
egerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

rule 395
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_
), x_Symbol] :> Simp[a^IntPart[p]*((a + b*x^2)^FracPart[p]/(1 + b*(x^2/a))^
FracPart[p])   Int[(e*x)^m*(1 + b*(x^2/a))^p*(c + d*x^2)^q, x], x] /; FreeQ
[{a, b, c, d, e, m, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m,
1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3675
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_.)*tan[(e_.) + (f_.)*(x_)]^
(m_), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Simp[ff^(m + 1
)*(Sqrt[Cos[e + f*x]^2]/(f*Cos[e + f*x]))   Subst[Int[x^m*((a + b*ff^2*x^2)
^p/(1 - ff^2*x^2)^((m + 1)/2)), x], x, Sin[e + f*x]/ff], x]] /; FreeQ[{a, b
, e, f, p}, x] && IntegerQ[m/2] &&  !IntegerQ[p]
```

Maple [F]

$$\int \coth(dx+c)^4 \left(a+b\sinh(dx+c)^2\right)^p dx$$

input
```
int(coth(d*x+c)^4*(a+b*sinh(d*x+c)^2)^p,x)
```

output
```
int(coth(d*x+c)^4*(a+b*sinh(d*x+c)^2)^p,x)
```

Fricas [F]

$$\int \coth^4(c+dx)\left(a+b\sinh^2(c+dx)\right)^p\,dx=\int \left(b\sinh\left(dx+c\right)^2+a\right)^p\coth\left(dx+c\right)^4\,dx$$

input
```
integrate(coth(d*x+c)^4*(a+b*sinh(d*x+c)^2)^p,x, algorithm="fricas")
```

output
```
integral((b*sinh(d*x + c)^2 + a)^p*coth(d*x + c)^4, x)
```

Sympy [F(-1)]

Timed out.

$$\int \coth^4(c+dx)\left(a+b\sinh^2(c+dx)\right)^p\,dx=\text{Timed out}$$

input
```
integrate(coth(d*x+c)**4*(a+b*sinh(d*x+c)**2)**p,x)
```

output
```
Timed out
```

Maxima [F]

$$\int \coth^4(c+dx)\left(a+b\sinh^2(c+dx)\right)^p\,dx=\int \left(b\sinh\left(dx+c\right)^2+a\right)^p\coth\left(dx+c\right)^4\,dx$$

input
```
integrate(coth(d*x+c)^4*(a+b*sinh(d*x+c)^2)^p,x, algorithm="maxima")
```

output
```
integrate((b*sinh(d*x + c)^2 + a)^p*coth(d*x + c)^4, x)
```

Giac [F]

$$\int \coth^4(c+dx)\left(a+b\sinh^2(c+dx)\right)^p\,dx=\int \left(b\sinh\left(dx+c\right)^2+a\right)^p\coth\left(dx+c\right)^4\,dx$$

input
```
integrate(coth(d*x+c)^4*(a+b*sinh(d*x+c)^2)^p,x, algorithm="giac")
```

output
```
integrate((b*sinh(d*x + c)^2 + a)^p*coth(d*x + c)^4, x)
```

Mupad [F(-1)]

Timed out.

$$\int \coth^4(c+dx)\left(a+b\sinh^2(c+dx)\right)^p\,dx=\int \coth(c+d\,x)^4\left(b\sinh(c+d\,x)^2+a\right)^p\,dx$$

input
```
int(coth(c + d*x)^4*(a + b*sinh(c + d*x)^2)^p,x)
```

output
```
int(coth(c + d*x)^4*(a + b*sinh(c + d*x)^2)^p, x)
```

Reduce [F]

$$\int \coth^4(c+dx)\left(a+b\sinh^2(c+dx)\right)^p\,dx=\int \coth\left(dx+c\right)^4\left(\sinh\left(dx+c\right)^2b+a\right)^p\,dx$$

input
```
int(coth(d*x+c)^4*(a+b*sinh(d*x+c)^2)^p,x)
```

output
```
int(coth(d*x+c)^4*(a+b*sinh(d*x+c)^2)^p,x)
```

3.485 $\int \frac{\coth^3(x)}{a+b\sinh^3(x)}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 152

$$\int \frac{\coth^3(x)}{a+b\sinh^3(x)}\,dx = \frac{b^{2/3}\arctan\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}\sinh(x)}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}a^{5/3}}-\frac{\operatorname{csch}^2(x)}{2a}+\frac{\log(\sinh(x))}{a}-\frac{b^{2/3}\log\left(\sqrt[3]{a}+\sqrt[3]{b}\sinh(x)\right)}{3a^{5/3}}+\frac{b^{2/3}\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}\sinh(x)+b^{2/3}\sinh^2(x)\right)}{6a^{5/3}}-\frac{\log\left(a+b\sinh^3(x)\right)}{3a}$$

output
```
1/3*b^(2/3)*arctan(1/3*(a^(1/3)-2*b^(1/3)*sinh(x))*3^(1/2)/a^(1/3))*3^(1/2
)/a^(5/3)-1/2*csch(x)^2/a+ln(sinh(x))/a-1/3*b^(2/3)*ln(a^(1/3)+b^(1/3)*sin
h(x))/a^(5/3)+1/6*b^(2/3)*ln(a^(2/3)-a^(1/3)*b^(1/3)*sinh(x)+b^(2/3)*sinh(
x)^2)/a^(5/3)-1/3*ln(a+b*sinh(x)^3)/a
```

Mathematica [A] (verified)

Time = 0.23 (sec) , antiderivative size = 136, normalized size of antiderivative = 0.89

$$\int \frac{\coth^3(x)}{a+b\sinh^3(x)}\,dx = -\frac{\operatorname{csch}^2(x)}{2a}+\frac{\log(\sinh(x))}{a}$$
$$-\frac{\left(a^{2/3}+(-1)^{2/3}b^{2/3}\right)\log\left(-(-1)^{2/3}\sqrt[3]{a}-\sqrt[3]{b}\sinh(x)\right)+\left(a^{2/3}+b^{2/3}\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}\sinh(x)\right)+\left(a^{2/3}\right.}{3a^{5/3}}$$

input
```
Integrate[Coth[x]^3/(a + b*Sinh[x]^3),x]
```

output
```
-1/2*Csch[x]^2/a + Log[Sinh[x]]/a - ((a^(2/3) + (-1)^(2/3)*b^(2/3))*Log[-(
(-1)^(2/3)*a^(1/3)) - b^(1/3)*Sinh[x]] + (a^(2/3) + b^(2/3))*Log[a^(1/3) +
 b^(1/3)*Sinh[x]] + (a^(2/3) - (-1)^(1/3)*b^(2/3))*Log[a^(1/3) + (-1)^(2/3
)*b^(1/3)*Sinh[x]])/(3*a^(5/3))
```

Rubi [A] (verified)

Time = 0.52 (sec) , antiderivative size = 152, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {3042, 26, 3709, 2373, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\coth^3(x)}{a+b\sinh^3(x)}dx$$
$$\Big\downarrow \text{ 3042}$$
$$\int -\frac{i}{\tan(ix)^3\left(a+ib\sin(ix)^3\right)}dx$$
$$\Big\downarrow \text{ 26}$$
$$-i\int \frac{1}{\left(ib\sin(ix)^3+a\right)\tan(ix)^3}dx$$
$$\Big\downarrow \text{ 3709}$$

$$\int \frac{(\sinh^2(x)+1)\operatorname{csch}^3(x)}{a+b\sinh^3(x)}d\sinh(x)$$

$$\Big\downarrow \text{2373}$$

$$\int \left(\frac{-b\sinh^2(x)-b}{a\left(a+b\sinh^3(x)\right)}+\frac{\operatorname{csch}^3(x)}{a}+\frac{\operatorname{csch}(x)}{a}\right)d\sinh(x)$$

$$\Big\downarrow \text{2009}$$

$$\frac{b^{2/3}\arctan\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}\sinh(x)}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}a^{5/3}}+\frac{b^{2/3}\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}\sinh(x)+b^{2/3}\sinh^2(x)\right)}{6a^{5/3}}-\frac{b^{2/3}\log\left(\sqrt[3]{a}+\sqrt[3]{b}\sinh(x)\right)}{3a^{5/3}}-\frac{\log\left(a+b\sinh^3(x)\right)}{3a}-\frac{\operatorname{csch}^2(x)}{2a}+\frac{\log(\sinh(x))}{a}$$

input
```
Int[Coth[x]^3/(a + b*Sinh[x]^3),x]
```

output
```
(b^(2/3)*ArcTan[(a^(1/3) - 2*b^(1/3)*Sinh[x])/(Sqrt[3]*a^(1/3))])/(Sqrt[3]
*a^(5/3)) - Csch[x]^2/(2*a) + Log[Sinh[x]]/a - (b^(2/3)*Log[a^(1/3) + b^(1
/3)*Sinh[x]])/(3*a^(5/3)) + (b^(2/3)*Log[a^(2/3) - a^(1/3)*b^(1/3)*Sinh[x]
 + b^(2/3)*Sinh[x]^2])/(6*a^(5/3)) - Log[a + b*Sinh[x]^3]/(3*a)
```

Defintions of rubi rules used

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2373
```
Int[((Pq_)*((c_.)*(x_))^(m_.))/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[E
xpandIntegrand[(c*x)^m*(Pq/(a + b*x^n)), x], x] /; FreeQ[{a, b, c, m}, x] &
& PolyQ[Pq, x] && IntegerQ[n] &&  !IGtQ[m, 0]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3709
```
Int[((a_) + (b_.)*((c_.)*sin[(e_.) + (f_.)*(x_)])^(n_))^(p_.)*tan[(e_.) + (
f_.)*(x_)]^(m_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Si
mp[ff^(m + 1)/f   Subst[Int[x^m*((a + b*(c*ff*x)^n)^p/(1 - ff^2*x^2)^((m +
1)/2)), x], x, Sin[e + f*x]/ff], x]] /; FreeQ[{a, b, c, e, f, n, p}, x] &&
ILtQ[(m - 1)/2, 0]
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 8.80 (sec) , antiderivative size = 91, normalized size of antiderivative = 0.60

method	result
risch	$-\frac{2\,\mathrm{e}^{2x}}{(\mathrm{e}^{2x}-1)^2 a}+\left(\sum_{\_R=\mathrm{RootOf}(27a^5\_Z^3+27a^4\_Z^2+9a^3\_Z+a^2+b^2)}\_R\ln\left(\mathrm{e}^{2x}+\left(-\frac{6a^2\_R}{b}-\frac{2a}{b}\right)\mathrm{e}^{x}-1\right)\right)$ -
default	$-\frac{\tanh\left(\frac{x}{2}\right)^2}{8a}+\frac{\sum_{\_R=\mathrm{RootOf}\left(a\_Z^6-3a\_Z^4-8b\_Z^3+3a\_Z^2-a\right)}\frac{\left(-\_R^5 a-\_R^4 b+2\_R^3 a+4\_R^2 b-\_R a+b\right)\ln\left(\tanh\left(\frac{x}{2}\right)-\_\right.}{\_R^5 a-2\_R^3 a-4\_R^2 b+\_R a}}{3a}$

input
```
int(coth(x)^3/(a+b*sinh(x)^3),x,method=_RETURNVERBOSE)
```

output
```
-2*exp(2*x)/(exp(2*x)-1)^2/a+sum(_R*ln(exp(2*x)+(-6/b*a^2*_R-2/b*a)*exp(x)
-1),_R=RootOf(27*_Z^3*a^5+27*_Z^2*a^4+9*_Z*a^3+a^2+b^2))+1/a*ln(exp(2*x)-1
)
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.88 (sec) , antiderivative size = 1432, normalized size of antiderivative = 9.42

$$\int \frac{\coth^3(x)}{a+b\sinh^3(x)}\,dx = \text{Too large to display}$$

input
```
integrate(coth(x)^3/(a+b*sinh(x)^3),x, algorithm="fricas")
```

output
```
-1/12*(2*(a*cosh(x)^4 + 4*a*cosh(x)*sinh(x)^3 + a*sinh(x)^4 - 2*a*cosh(x)^
2 + 2*(3*a*cosh(x)^2 - a)*sinh(x)^2 + 4*(a*cosh(x)^3 - a*cosh(x))*sinh(x)
+ a)*((1/2)^(1/3)*(I*sqrt(3) + 1)*(1/a^3 + b^2/a^5 - (a^2 + b^2)/a^5)^(1/3
) + 2/a)*log(b*cosh(x)^2 + b*sinh(x)^2 + (a^2*cosh(x) + a^2*sinh(x))*((1/2
)^(1/3)*(I*sqrt(3) + 1)*(1/a^3 + b^2/a^5 - (a^2 + b^2)/a^5)^(1/3) + 2/a) -
 2*a*cosh(x) + 2*(b*cosh(x) - a)*sinh(x) - b) + 24*cosh(x)^2 + (6*cosh(x)^
4 + 24*cosh(x)*sinh(x)^3 + 6*sinh(x)^4 + 12*(3*cosh(x)^2 - 1)*sinh(x)^2 -
(a*cosh(x)^4 + 4*a*cosh(x)*sinh(x)^3 + a*sinh(x)^4 - 2*a*cosh(x)^2 + 2*(3*
a*cosh(x)^2 - a)*sinh(x)^2 + 4*(a*cosh(x)^3 - a*cosh(x))*sinh(x) + a)*((1/
2)^(1/3)*(I*sqrt(3) + 1)*(1/a^3 + b^2/a^5 - (a^2 + b^2)/a^5)^(1/3) + 2/a)
+ 3*sqrt(1/3)*(a*cosh(x)^4 + 4*a*cosh(x)*sinh(x)^3 + a*sinh(x)^4 - 2*a*cos
h(x)^2 + 2*(3*a*cosh(x)^2 - a)*sinh(x)^2 + 4*(a*cosh(x)^3 - a*cosh(x))*sin
h(x) + a)*sqrt(-(((1/2)^(1/3)*(I*sqrt(3) + 1)*(1/a^3 + b^2/a^5 - (a^2 + b^
2)/a^5)^(1/3) + 2/a)^2*a^2 - 4*((1/2)^(1/3)*(I*sqrt(3) + 1)*(1/a^3 + b^2/a
^5 - (a^2 + b^2)/a^5)^(1/3) + 2/a)*a + 4)/a^2) - 12*cosh(x)^2 + 24*(cosh(x
)^3 - cosh(x))*sinh(x) + 6)*log(b*cosh(x)^2 + b*sinh(x)^2 - 1/2*(a^2*cosh(
x) + a^2*sinh(x))*((1/2)^(1/3)*(I*sqrt(3) + 1)*(1/a^3 + b^2/a^5 - (a^2 + b
^2)/a^5)^(1/3) + 2/a) + 3/2*sqrt(1/3)*(a^2*cosh(x) + a^2*sinh(x))*sqrt(-((
(1/2)^(1/3)*(I*sqrt(3) + 1)*(1/a^3 + b^2/a^5 - (a^2 + b^2)/a^5)^(1/3) + 2/
a)^2*a^2 - 4*((1/2)^(1/3)*(I*sqrt(3) + 1)*(1/a^3 + b^2/a^5 - (a^2 + b^2...
```

Sympy [F]

$$\int \frac{\coth^3(x)}{a+b\sinh^3(x)}\,dx=\int \frac{\coth^3{(x)}}{a+b\sinh^3{(x)}}\,dx$$

input
```
integrate(coth(x)**3/(a+b*sinh(x)**3),x)
```

output
```
Integral(coth(x)**3/(a + b*sinh(x)**3), x)
```

Maxima [F]

$$\int \frac{\coth^3(x)}{a+b\sinh^3(x)}\,dx=\int \frac{\coth(x)^3}{b\sinh(x)^3+a}\,dx$$

input
```
integrate(coth(x)^3/(a+b*sinh(x)^3),x, algorithm="maxima")
```

output
```
2*b*(x/(a*b) - integrate((b*e^(5*x) - 3*b*e^(3*x) + 8*a*e^(2*x) + 3*b*e^x)
*e^x/(b*e^(6*x) - 3*b*e^(4*x) + 8*a*e^(3*x) + 3*b*e^(2*x) - b), x)/(a*b))
- 6*b*integrate(e^(4*x)/(b*e^(6*x) - 3*b*e^(4*x) + 8*a*e^(3*x) + 3*b*e^(2*
x) - b), x)/a - 2*(x*e^(4*x) - (2*x - 1)*e^(2*x) + x)/(a*e^(4*x) - 2*a*e^(
2*x) + a) + log(e^x + 1)/a + log(e^x - 1)/a + 8*integrate(e^(3*x)/(b*e^(6*
x) - 3*b*e^(4*x) + 8*a*e^(3*x) + 3*b*e^(2*x) - b), x)
```

Giac [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 209, normalized size of antiderivative = 1.38

$$\int \frac{\coth^3(x)}{a+b\sinh^3(x)}\,dx=\frac{b\left(-\frac{a}{b}\right)^{\frac{1}{3}}\log\left(\left|-2\left(-\frac{a}{b}\right)^{\frac{1}{3}}-e^{(-x)}+e^{x}\right|\right)}{3\,a^2} - \frac{\log\left(\left|-b\left(e^{(-x)}-e^{x}\right)^3+8\,a\right|\right)}{3\,a}+\frac{\log\left(\left|-e^{(-x)}+e^{x}\right|\right)}{a} - \frac{\sqrt{3}\left(-ab^2\right)^{\frac{1}{3}}\arctan\left(\frac{\sqrt{3}\left(\left(-\frac{a}{b}\right)^{\frac{1}{3}}-e^{(-x)}+e^{x}\right)}{3\left(-\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{3\,a^2} - \frac{\left(-ab^2\right)^{\frac{1}{3}}\log\left(\left(e^{(-x)}-e^{x}\right)^2-2\left(-\frac{a}{b}\right)^{\frac{1}{3}}\left(e^{(-x)}-e^{x}\right)+4\left(-\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6\,a^2} - \frac{3\left(e^{(-x)}-e^{x}\right)^2+4}{2\,a\left(e^{(-x)}-e^{x}\right)^2}$$

input
```
integrate(coth(x)^3/(a+b*sinh(x)^3),x, algorithm="giac")
```

output
```
1/3*b*(-a/b)^(1/3)*log(abs(-2*(-a/b)^(1/3) - e^(-x) + e^x))/a^2 - 1/3*log(
abs(-b*(e^(-x) - e^x)^3 + 8*a))/a + log(abs(-e^(-x) + e^x))/a - 1/3*sqrt(3
)*(-a*b^2)^(1/3)*arctan(1/3*sqrt(3)*((-a/b)^(1/3) - e^(-x) + e^x)/(-a/b)^(
1/3))/a^2 - 1/6*(-a*b^2)^(1/3)*log((e^(-x) - e^x)^2 - 2*(-a/b)^(1/3)*(e^(-
x) - e^x) + 4*(-a/b)^(2/3))/a^2 - 1/2*(3*(e^(-x) - e^x)^2 + 4)/(a*(e^(-x)
- e^x)^2)
```

Mupad [B] (verification not implemented)

Time = 0.95 (sec) , antiderivative size = 1129, normalized size of antiderivative = 7.43

$$\int \frac{\coth^3(x)}{a+b\sinh^3(x)}\,dx = \text{Too large to display}$$

input
```
int(coth(x)^3/(a + b*sinh(x)^3),x)
```

output
```
2/(a - a*exp(2*x)) - 2/(a - 2*a*exp(2*x) + a*exp(4*x)) + symsum(log((50331
648*a^6*exp(2*x) + 786432*b^6*exp(2*x) - 452984832*root(27*a^5*z^3 + 27*a^
4*z^2 + 9*a^3*z + b^2 + a^2, z, k)*a^7 - 50331648*a^6 - 786432*b^6 - 13589
54496*root(27*a^5*z^3 + 27*a^4*z^2 + 9*a^3*z + b^2 + a^2, z, k)^2*a^8 - 13
58954496*root(27*a^5*z^3 + 27*a^4*z^2 + 9*a^3*z + b^2 + a^2, z, k)^3*a^9 -
 50593792*a^2*b^4 - 102498304*a^4*b^2 + 1358954496*root(27*a^5*z^3 + 27*a^
4*z^2 + 9*a^3*z + b^2 + a^2, z, k)^2*a^8*exp(2*x) + 1358954496*root(27*a^5
*z^3 + 27*a^4*z^2 + 9*a^3*z + b^2 + a^2, z, k)^3*a^9*exp(2*x) + 50593792*a
^2*b^4*exp(2*x) + 102498304*a^4*b^2*exp(2*x) - 7602176*root(27*a^5*z^3 + 2
7*a^4*z^2 + 9*a^3*z + b^2 + a^2, z, k)*a^3*b^4 - 465305600*root(27*a^5*z^3
 + 27*a^4*z^2 + 9*a^3*z + b^2 + a^2, z, k)*a^5*b^2 + 524288*a*b^5*exp(x) -
 24379392*root(27*a^5*z^3 + 27*a^4*z^2 + 9*a^3*z + b^2 + a^2, z, k)^2*a^4*
b^4 - 1383333888*root(27*a^5*z^3 + 27*a^4*z^2 + 9*a^3*z + b^2 + a^2, z, k)
^2*a^6*b^2 - 18874368*root(27*a^5*z^3 + 27*a^4*z^2 + 9*a^3*z + b^2 + a^2,
z, k)^3*a^5*b^4 - 1370750976*root(27*a^5*z^3 + 27*a^4*z^2 + 9*a^3*z + b^2
+ a^2, z, k)^3*a^7*b^2 + 452984832*root(27*a^5*z^3 + 27*a^4*z^2 + 9*a^3*z
+ b^2 + a^2, z, k)*a^7*exp(2*x) + 5242880*a^3*b^3*exp(x) - 524288*root(27*
a^5*z^3 + 27*a^4*z^2 + 9*a^3*z + b^2 + a^2, z, k)*a^2*b^5*exp(x) + 8912896
*root(27*a^5*z^3 + 27*a^4*z^2 + 9*a^3*z + b^2 + a^2, z, k)*a^4*b^3*exp(x)
+ 7602176*root(27*a^5*z^3 + 27*a^4*z^2 + 9*a^3*z + b^2 + a^2, z, k)*a^3...
```

Reduce [F]

$$\int \frac{\coth^3(x)}{a + b\sinh^3(x)}\,dx = \text{too large to display}$$

input

```
int(coth(x)^3/(a+b*sinh(x)^3),x)
```

output

```
(2304*e**(4*x)*int(e**(4*x)/(e**(12*x)*b - 6*e**(10*x)*b + 8*e**(9*x)*a +
15*e**(8*x)*b - 24*e**(7*x)*a - 20*e**(6*x)*b + 24*e**(5*x)*a + 15*e**(4*x
)*b - 8*e**(3*x)*a - 6*e**(2*x)*b + b),x)*a**2*b - 4096*e**(4*x)*int(e**(3
*x)/(e**(12*x)*b - 6*e**(10*x)*b + 8*e**(9*x)*a + 15*e**(8*x)*b - 24*e**(7
*x)*a - 20*e**(6*x)*b + 24*e**(5*x)*a + 15*e**(4*x)*b - 8*e**(3*x)*a - 6*e
**(2*x)*b + b),x)*a**3 + 160*e**(4*x)*int(e**(3*x)/(e**(12*x)*b - 6*e**(10
*x)*b + 8*e**(9*x)*a + 15*e**(8*x)*b - 24*e**(7*x)*a - 20*e**(6*x)*b + 24*
e**(5*x)*a + 15*e**(4*x)*b - 8*e**(3*x)*a - 6*e**(2*x)*b + b),x)*a*b**2 -
1536*e**(4*x)*int(e**(2*x)/(e**(12*x)*b - 6*e**(10*x)*b + 8*e**(9*x)*a + 1
5*e**(8*x)*b - 24*e**(7*x)*a - 20*e**(6*x)*b + 24*e**(5*x)*a + 15*e**(4*x)
*b - 8*e**(3*x)*a - 6*e**(2*x)*b + b),x)*a**2*b - 96*e**(4*x)*int(e**x/(e*
*(12*x)*b - 6*e**(10*x)*b + 8*e**(9*x)*a + 15*e**(8*x)*b - 24*e**(7*x)*a -
 20*e**(6*x)*b + 24*e**(5*x)*a + 15*e**(4*x)*b - 8*e**(3*x)*a - 6*e**(2*x)
*b + b),x)*a*b**2 + 512*e**(4*x)*int(1/(e**(12*x)*b - 6*e**(10*x)*b + 8*e*
*(9*x)*a + 15*e**(8*x)*b - 24*e**(7*x)*a - 20*e**(6*x)*b + 24*e**(5*x)*a +
 15*e**(4*x)*b - 8*e**(3*x)*a - 6*e**(2*x)*b + b),x)*a**2*b + 256*e**(4*x)
*log(e**x - 1)*a**2 - 14*e**(4*x)*log(e**x - 1)*a*b + 9*e**(4*x)*log(e**x
- 1)*b**2 + 256*e**(4*x)*log(e**x + 1)*a**2 + 14*e**(4*x)*log(e**x + 1)*a*
b + 9*e**(4*x)*log(e**x + 1)*b**2 - 3*e**(4*x)*log(e**(6*x)*b - 3*e**(4*x)
*b + 8*e**(3*x)*a + 3*e**(2*x)*b - b)*b**2 - 512*e**(4*x)*a**2*x + 128*...
```

3.486 $\int \frac{\coth(x)}{\sqrt{a+b\sinh^3(x)}}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 28

$$\int \frac{\coth(x)}{\sqrt{a+b\sinh^3(x)}}\,dx = -\frac{2\,\mathrm{arctanh}\left(\frac{\sqrt{a+b\sinh^3(x)}}{\sqrt{a}}\right)}{3\sqrt{a}}$$

output
```
-2/3*arctanh((a+b*sinh(x)^3)^(1/2)/a^(1/2))/a^(1/2)
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.00

$$\int \frac{\coth(x)}{\sqrt{a+b\sinh^3(x)}}\,dx = -\frac{2\,\mathrm{arctanh}\left(\frac{\sqrt{a+b\sinh^3(x)}}{\sqrt{a}}\right)}{3\sqrt{a}}$$

input
```
Integrate[Coth[x]/Sqrt[a + b*Sinh[x]^3],x]
```

output
```
(-2*ArcTanh[Sqrt[a + b*Sinh[x]^3]/Sqrt[a]])/(3*Sqrt[a])
```

Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.400$, Rules used = {3042, 26, 3709, 798, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\coth(x)}{\sqrt{a+b\sinh^3(x)}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{i}{\tan(ix)\sqrt{a+ib\sin(ix)^3}}dx$$

$$\Big\downarrow \text{26}$$

$$i\int \frac{1}{\sqrt{ib\sin(ix)^3+a}\tan(ix)}dx$$

$$\Big\downarrow \text{3709}$$

$$\int \frac{\operatorname{csch}(x)}{\sqrt{a+b\sinh^3(x)}}d\sinh(x)$$

$$\Big\downarrow \text{798}$$

$$\frac{1}{3}\int \frac{\operatorname{csch}(x)}{\sqrt{b\sinh^3(x)+a}}d\sinh^3(x)$$

$$\Big\downarrow \text{73}$$

$$\frac{2\int \frac{1}{\frac{\sinh^6(x)}{b}-\frac{a}{b}}d\sqrt{b\sinh^3(x)+a}}{3b}$$

$$\Big\downarrow \text{221}$$

$$-\frac{2\text{arctanh}\left(\frac{\sqrt{a+b\sinh^3(x)}}{\sqrt{a}}\right)}{3\sqrt{a}}$$

input
```
Int[Coth[x]/Sqrt[a + b*Sinh[x]^3],x]
```

output
```
(-2*ArcTanh[Sqrt[a + b*Sinh[x]^3]/Sqrt[a]])/(3*Sqrt[a])
```

Defintions of rubi rules used

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 798
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[1/n   Subst
[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a,
b, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3709
```
Int[((a_) + (b_.)*((c_.)*sin[(e_.) + (f_.)*(x_)])^(n_))^(p_.)*tan[(e_.) + (
f_.)*(x_)]^(m_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Si
mp[ff^(m + 1)/f   Subst[Int[x^m*((a + b*(c*ff*x)^n)^p/(1 - ff^2*x^2)^((m +
1)/2)), x], x, Sin[e + f*x]/ff], x]] /; FreeQ[{a, b, c, e, f, n, p}, x] &&
ILtQ[(m - 1)/2, 0]
```

Maple [F]

$$\int \frac{\coth(x)}{\sqrt{a+b\sinh(x)^{3}}}dx$$

input
```
int(coth(x)/(a+b*sinh(x)^3)^(1/2),x)
```

output
```
int(coth(x)/(a+b*sinh(x)^3)^(1/2),x)
```

Fricas [F(-2)]

Exception generated.

$$\int \frac{\coth(x)}{\sqrt{a+b\sinh^3(x)}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(coth(x)/(a+b*sinh(x)^3)^(1/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   fail
ed of mode Union(SparseUnivariatePolynomial(Expression(Integer)),failed) c
annot be coerced to mode SparseUnivariatePolynomial(Expression(Integer))
```

Sympy [F]

$$\int \frac{\coth(x)}{\sqrt{a+b\sinh^3(x)}}\,dx = \int \frac{\coth(x)}{\sqrt{a+b\sinh^3(x)}}\,dx$$

input
```
integrate(coth(x)/(a+b*sinh(x)**3)**(1/2),x)
```

output
```
Integral(coth(x)/sqrt(a + b*sinh(x)**3), x)
```

Maxima [F]

$$\int \frac{\coth(x)}{\sqrt{a+b\sinh^3(x)}}\,dx = \int \frac{\coth(x)}{\sqrt{b\sinh(x)^3+a}}\,dx$$

input
```
integrate(coth(x)/(a+b*sinh(x)^3)^(1/2),x, algorithm="maxima")
```

output
```
integrate(coth(x)/sqrt(b*sinh(x)^3 + a), x)
```

Giac [F]

$$\int \frac{\coth(x)}{\sqrt{a+b\sinh^3(x)}}\,dx = \int \frac{\coth(x)}{\sqrt{b\sinh(x)^3+a}}\,dx$$

input
```
integrate(coth(x)/(a+b*sinh(x)^3)^(1/2),x, algorithm="giac")
```

output
```
integrate(coth(x)/sqrt(b*sinh(x)^3 + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\coth(x)}{\sqrt{a+b\sinh^3(x)}}\,dx = \int \frac{\coth(x)}{\sqrt{b\sinh(x)^3+a}}\,dx$$

input
```
int(coth(x)/(a + b*sinh(x)^3)^(1/2),x)
```

output
```
int(coth(x)/(a + b*sinh(x)^3)^(1/2), x)
```

Reduce [F]

$$\int \frac{\coth(x)}{\sqrt{a+b\sinh^3(x)}}\,dx = \int \frac{\sqrt{\sinh{(x)}^3 b+a}\,\coth{(x)}}{\sinh{(x)}^3 b+a}dx$$

input
```
int(coth(x)/(a+b*sinh(x)^3)^(1/2),x)
```

output
```
int((sqrt(sinh(x)**3*b + a)*coth(x))/(sinh(x)**3*b + a),x)
```

3.487 $\int \coth(x)\sqrt{a+b\sinh^3(x)}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 45

$$\int \coth(x)\sqrt{a+b\sinh^3(x)}\,dx = -\frac{2}{3}\sqrt{a}\text{arctanh}\left(\frac{\sqrt{a+b\sinh^3(x)}}{\sqrt{a}}\right)+\frac{2}{3}\sqrt{a+b\sinh^3(x)}$$

output
```
-2/3*a^(1/2)*arctanh((a+b*sinh(x)^3)^(1/2)/a^(1/2))+2/3*(a+b*sinh(x)^3)^(1
/2)
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 45, normalized size of antiderivative = 1.00

$$\int \coth(x)\sqrt{a+b\sinh^3(x)}\,dx = -\frac{2}{3}\sqrt{a}\text{arctanh}\left(\frac{\sqrt{a+b\sinh^3(x)}}{\sqrt{a}}\right)+\frac{2}{3}\sqrt{a+b\sinh^3(x)}$$

input
```
Integrate[Coth[x]*Sqrt[a + b*Sinh[x]^3],x]
```

output
```
(-2*Sqrt[a]*ArcTanh[Sqrt[a + b*Sinh[x]^3]/Sqrt[a]])/3 + (2*Sqrt[a + b*Sinh
[x]^3])/3
```

Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 45, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.467, Rules used = {3042, 26, 3709, 798, 60, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \coth(x)\sqrt{a+b\sinh^3(x)}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{i\sqrt{a+ib\sin(ix)^3}}{\tan(ix)}dx$$

$$\Big\downarrow \text{26}$$

$$i\int \frac{\sqrt{ib\sin(ix)^3+a}}{\tan(ix)}dx$$

$$\Big\downarrow \text{3709}$$

$$\int \operatorname{csch}(x)\sqrt{a+b\sinh^3(x)}d\sinh(x)$$

$$\Big\downarrow \text{798}$$

$$\frac{1}{3}\int \operatorname{csch}(x)\sqrt{b\sinh^3(x)+a}d\sinh^3(x)$$

$$\Big\downarrow \text{60}$$

$$\frac{1}{3}\left(a\int \frac{\operatorname{csch}(x)}{\sqrt{b\sinh^3(x)+a}}d\sinh^3(x)+2\sqrt{a+b\sinh^3(x)}\right)$$

$$\Big\downarrow \text{73}$$

$$\frac{1}{3}\left(\frac{2a\int \frac{1}{\frac{\sinh^6(x)}{b}-\frac{a}{b}}d\sqrt{b\sinh^3(x)+a}}{b}+2\sqrt{a+b\sinh^3(x)}\right)$$

$$\Big\downarrow \text{221}$$

$$\frac{1}{3}\left(2\sqrt{a+b\sinh^3(x)}-2\sqrt{a}\text{arctanh}\left(\frac{\sqrt{a+b\sinh^3(x)}}{\sqrt{a}}\right)\right)$$

input
```
Int[Coth[x]*Sqrt[a + b*Sinh[x]^3],x]
```

output
```
(-2*Sqrt[a]*ArcTanh[Sqrt[a + b*Sinh[x]^3]/Sqrt[a]] + 2*Sqrt[a + b*Sinh[x]^
3])/3
```

Defintions of rubi rules used

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 60
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Simp[n*((b*c - a*d)/(
b*(m + n + 1)))   Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
 c, d}, x] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ[m, 0] && ( !Integer
Q[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n + 2, 0] && IntLinear
Q[a, b, c, d, m, n, x]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 798
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[1/n   Subst
[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a,
b, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3709
```
Int[((a_) + (b_.)*((c_.)*sin[(e_.) + (f_.)*(x_)])^(n_))^(p_.)*tan[(e_.) + (
f_.)*(x_)]^(m_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Si
mp[ff^(m + 1)/f   Subst[Int[x^m*((a + b*(c*ff*x)^n)^p/(1 - ff^2*x^2)^((m +
1)/2)), x], x, Sin[e + f*x]/ff], x]] /; FreeQ[{a, b, c, e, f, n, p}, x] &&
ILtQ[(m - 1)/2, 0]
```

Maple [F]

$$\int \coth(x)\sqrt{a+b\sinh(x)^{3}}dx$$

input
```
int(coth(x)*(a+b*sinh(x)^3)^(1/2),x)
```

output
```
int(coth(x)*(a+b*sinh(x)^3)^(1/2),x)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 472 vs. $2(33) = 66$.

Time = 1.41 (sec) , antiderivative size = 1860, normalized size of antiderivative = 41.33

$$\int \coth(x)\sqrt{a+b\sinh^3(x)}\,dx = \text{Too large to display}$$

input
```
integrate(coth(x)*(a+b*sinh(x)^3)^(1/2),x, algorithm="fricas")
```

output
```
[1/6*(sqrt(a)*(cosh(x) + sinh(x))*log(-(b^2*cosh(x)^12 + 12*b^2*cosh(x)*si
nh(x)^11 + b^2*sinh(x)^12 - 6*b^2*cosh(x)^10 + 64*a*b*cosh(x)^9 + 6*(11*b^
2*cosh(x)^2 - b^2)*sinh(x)^10 + 15*b^2*cosh(x)^8 + 4*(55*b^2*cosh(x)^3 - 1
5*b^2*cosh(x) + 16*a*b)*sinh(x)^9 - 192*a*b*cosh(x)^7 + 3*(165*b^2*cosh(x)
^4 - 90*b^2*cosh(x)^2 + 192*a*b*cosh(x) + 5*b^2)*sinh(x)^8 + 24*(33*b^2*co
sh(x)^5 - 30*b^2*cosh(x)^3 + 96*a*b*cosh(x)^2 + 5*b^2*cosh(x) - 8*a*b)*sin
h(x)^7 + 192*a*b*cosh(x)^5 + 4*(128*a^2 - 5*b^2)*cosh(x)^6 + 4*(231*b^2*co
sh(x)^6 - 315*b^2*cosh(x)^4 + 1344*a*b*cosh(x)^3 + 105*b^2*cosh(x)^2 - 336
*a*b*cosh(x) + 128*a^2 - 5*b^2)*sinh(x)^6 + 15*b^2*cosh(x)^4 + 24*(33*b^2*
cosh(x)^7 - 63*b^2*cosh(x)^5 + 336*a*b*cosh(x)^4 + 35*b^2*cosh(x)^3 - 168*
a*b*cosh(x)^2 + 8*a*b + (128*a^2 - 5*b^2)*cosh(x))*sinh(x)^5 - 64*a*b*cosh
(x)^3 + 3*(165*b^2*cosh(x)^8 - 420*b^2*cosh(x)^6 + 2688*a*b*cosh(x)^5 + 35
0*b^2*cosh(x)^4 - 2240*a*b*cosh(x)^3 + 320*a*b*cosh(x) + 20*(128*a^2 - 5*b
^2)*cosh(x)^2 + 5*b^2)*sinh(x)^4 - 6*b^2*cosh(x)^2 + 4*(55*b^2*cosh(x)^9 -
 180*b^2*cosh(x)^7 + 1344*a*b*cosh(x)^6 + 210*b^2*cosh(x)^5 - 1680*a*b*cos
h(x)^4 + 480*a*b*cosh(x)^2 + 20*(128*a^2 - 5*b^2)*cosh(x)^3 + 15*b^2*cosh(
x) - 16*a*b)*sinh(x)^3 + 6*(11*b^2*cosh(x)^10 - 45*b^2*cosh(x)^8 + 384*a*b
*cosh(x)^7 + 70*b^2*cosh(x)^6 - 672*a*b*cosh(x)^5 + 320*a*b*cosh(x)^3 + 10
*(128*a^2 - 5*b^2)*cosh(x)^4 + 15*b^2*cosh(x)^2 - 32*a*b*cosh(x) - b^2)*si
nh(x)^2 + b^2 - 16*(b*cosh(x)^8 + 8*b*cosh(x)*sinh(x)^7 + b*sinh(x)^8 -...
```

Sympy [F]

$$\int \coth(x)\sqrt{a+b\sinh^3(x)}\,dx = \int \sqrt{a+b\sinh^3(x)}\coth(x)\,dx$$

input
```
integrate(coth(x)*(a+b*sinh(x)**3)**(1/2),x)
```

output
```
Integral(sqrt(a + b*sinh(x)**3)*coth(x), x)
```

Maxima [F]

$$\int \coth(x)\sqrt{a+b\sinh^3(x)}\,dx = \int \sqrt{b\sinh{(x)}^3+a}\coth{(x)}\,dx$$

input
```
integrate(coth(x)*(a+b*sinh(x)^3)^(1/2),x, algorithm="maxima")
```

output
```
integrate(sqrt(b*sinh(x)^3 + a)*coth(x), x)
```

Giac [F]

$$\int \coth(x)\sqrt{a+b\sinh^3(x)}\,dx = \int \sqrt{b\sinh{(x)}^3+a}\coth{(x)}\,dx$$

input
```
integrate(coth(x)*(a+b*sinh(x)^3)^(1/2),x, algorithm="giac")
```

output
```
integrate(sqrt(b*sinh(x)^3 + a)*coth(x), x)
```

Mupad [F(-1)]

Timed out.

$$\int \coth(x)\sqrt{a+b\sinh^3(x)}\,dx = \int \coth(x)\,\sqrt{b\sinh{(x)}^3+a}\,dx$$

input
```
int(coth(x)*(a + b*sinh(x)^3)^(1/2),x)
```

output
```
int(coth(x)*(a + b*sinh(x)^3)^(1/2), x)
```

Reduce [F]

$$\int \coth(x)\sqrt{a+b\sinh^3(x)}\,dx=\int \sqrt{\sinh(x)^3 b+a}\,\coth(x)\,dx$$

input
```
int(coth(x)*(a+b*sinh(x)^3)^(1/2),x)
```

output
```
int(sqrt(sinh(x)**3*b + a)*coth(x),x)
```

3.488 $\int \frac{\coth(x)}{\sqrt{a+b\sinh^n(x)}}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 29

$$\int \frac{\coth(x)}{\sqrt{a+b\sinh^n(x)}}\,dx = -\frac{2\operatorname{arctanh}\left(\frac{\sqrt{a+b\sinh^n(x)}}{\sqrt{a}}\right)}{\sqrt{a}n}$$

output
```
-2*arctanh((a+b*sinh(x)^n)^(1/2)/a^(1/2))/a^(1/2)/n
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.00

$$\int \frac{\coth(x)}{\sqrt{a+b\sinh^n(x)}}\,dx = -\frac{2\operatorname{arctanh}\left(\frac{\sqrt{a+b\sinh^n(x)}}{\sqrt{a}}\right)}{\sqrt{a}n}$$

input
```
Integrate[Coth[x]/Sqrt[a + b*Sinh[x]^n],x]
```

output
```
(-2*ArcTanh[Sqrt[a + b*Sinh[x]^n]/Sqrt[a]])/(Sqrt[a]*n)
```

Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.400$, Rules used = {3042, 26, 3709, 798, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\coth(x)}{\sqrt{a+b\sinh^n(x)}}dx$$
$$\Big\downarrow \text{3042}$$
$$\int \frac{i}{\tan(ix)\sqrt{a+b(-i\sin(ix))^n}}dx$$
$$\Big\downarrow \text{26}$$
$$i\int \frac{1}{\sqrt{b(-i\sin(ix))^n+a}\tan(ix)}dx$$
$$\Big\downarrow \text{3709}$$
$$\int \frac{\text{csch}(x)}{\sqrt{a+b\sinh^n(x)}}d\sinh(x)$$
$$\Big\downarrow \text{798}$$
$$\frac{\int \frac{\text{csch}(x)}{\sqrt{b\sinh^n(x)+a}}d\sinh^n(x)}{n}$$
$$\Big\downarrow \text{73}$$
$$\frac{2\int \frac{1}{\frac{\sinh^{2n}(x)}{b}-\frac{a}{b}}d\sqrt{b\sinh^n(x)+a}}{bn}$$
$$\Big\downarrow \text{221}$$
$$-\frac{2\text{arctanh}\left(\frac{\sqrt{a+b\sinh^n(x)}}{\sqrt{a}}\right)}{\sqrt{a}n}$$

input
```
Int[Coth[x]/Sqrt[a + b*Sinh[x]^n],x]
```

output
```
(-2*ArcTanh[Sqrt[a + b*Sinh[x]^n]/Sqrt[a]])/(Sqrt[a]*n)
```

Defintions of rubi rules used

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 798
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[1/n   Subst
[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a,
b, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3709
```
Int[((a_) + (b_.)*((c_.)*sin[(e_.) + (f_.)*(x_)])^(n_))^(p_.)*tan[(e_.) + (
f_.)*(x_)]^(m_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Si
mp[ff^(m + 1)/f   Subst[Int[x^m*((a + b*(c*ff*x)^n)^p/(1 - ff^2*x^2)^((m +
1)/2)), x], x, Sin[e + f*x]/ff], x]] /; FreeQ[{a, b, c, e, f, n, p}, x] &&
ILtQ[(m - 1)/2, 0]
```

Maple [A] (verified)

Time = 0.37 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.83

method	result	size
derivativedivides	$-\frac{2\,\mathrm{arctanh}\left(\frac{\sqrt{a+b\sinh(x)^n}}{\sqrt{a}}\right)}{\sqrt{a}\,n}$	24
default	$-\frac{2\,\mathrm{arctanh}\left(\frac{\sqrt{a+b\sinh(x)^n}}{\sqrt{a}}\right)}{\sqrt{a}\,n}$	24

input
```
int(coth(x)/(a+b*sinh(x)^n)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-2*arctanh((a+b*sinh(x)^n)^(1/2)/a^(1/2))/a^(1/2)/n
```

Fricas [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 110, normalized size of antiderivative = 3.79

$$\int \frac{\coth(x)}{\sqrt{a+b\sinh^n(x)}}\,dx = \left[\frac{\log\left(\frac{b\cosh(n\log(\sinh(x)))+b\sinh(n\log(\sinh(x)))-2\sqrt{b\cosh(n\log(\sinh(x)))+b\sinh(n\log(\sinh(x)))+a}\sqrt{a}+2a}{\cosh(n\log(\sinh(x)))+\sinh(n\log(\sinh(x)))}\right)}{\sqrt{a}n}, \frac{2\sqrt{-a}\arctan\left(\cdot\right.}{}\right.$$

input
```
integrate(coth(x)/(a+b*sinh(x)^n)^(1/2),x, algorithm="fricas")
```

output
```
[log((b*cosh(n*log(sinh(x))) + b*sinh(n*log(sinh(x))) - 2*sqrt(b*cosh(n*lo
g(sinh(x))) + b*sinh(n*log(sinh(x))) + a)*sqrt(a) + 2*a)/(cosh(n*log(sinh(
x))) + sinh(n*log(sinh(x)))))/(sqrt(a)*n), 2*sqrt(-a)*arctan(sqrt(-a)/sqrt
(b*cosh(n*log(sinh(x))) + b*sinh(n*log(sinh(x))) + a))/(a*n)]
```

Sympy [F]

$$\int \frac{\coth(x)}{\sqrt{a+b\sinh^n(x)}}\,dx=\int \frac{\coth{(x)}}{\sqrt{a+b\sinh^{n}{(x)}}}\,dx$$

input
```
integrate(coth(x)/(a+b*sinh(x)**n)**(1/2),x)
```

output
```
Integral(coth(x)/sqrt(a + b*sinh(x)**n), x)
```

Maxima [F]

$$\int \frac{\coth(x)}{\sqrt{a+b\sinh^n(x)}}\,dx=\int \frac{\coth{(x)}}{\sqrt{b\sinh{(x)}^{n}+a}}\,dx$$

input
```
integrate(coth(x)/(a+b*sinh(x)^n)^(1/2),x, algorithm="maxima")
```

output
```
integrate(coth(x)/sqrt(b*sinh(x)^n + a), x)
```

Giac [F]

$$\int \frac{\coth(x)}{\sqrt{a+b\sinh^n(x)}}\,dx=\int \frac{\coth{(x)}}{\sqrt{b\sinh{(x)}^{n}+a}}\,dx$$

input
```
integrate(coth(x)/(a+b*sinh(x)^n)^(1/2),x, algorithm="giac")
```

output
```
integrate(coth(x)/sqrt(b*sinh(x)^n + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\coth(x)}{\sqrt{a+b\sinh^n(x)}}\,dx=\int \frac{\coth(x)}{\sqrt{a+b\sinh\left(x\right)^n}}\,dx$$

input
```
int(coth(x)/(a + b*sinh(x)^n)^(1/2),x)
```

output
```
int(coth(x)/(a + b*sinh(x)^n)^(1/2), x)
```

Reduce [F]

$$\int \frac{\coth(x)}{\sqrt{a+b\sinh^n(x)}}\,dx=\int \frac{\sqrt{\sinh\left(x\right)^n b+a}\,\coth\left(x\right)}{\sinh\left(x\right)^n b+a}dx$$

input
```
int(coth(x)/(a+b*sinh(x)^n)^(1/2),x)
```

output
```
int((sqrt(sinh(x)**n*b + a)*coth(x))/(sinh(x)**n*b + a),x)
```

3.489 $\int \coth(x)\sqrt{a+b\sinh^n(x)}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 47

$$\int \coth(x)\sqrt{a+b\sinh^n(x)}\,dx=-\frac{2\sqrt{a}\,\text{arctanh}\left(\frac{\sqrt{a+b\sinh^n(x)}}{\sqrt{a}}\right)}{n}+\frac{2\sqrt{a+b\sinh^n(x)}}{n}$$

output
```
-2*a^(1/2)*arctanh((a+b*sinh(x)^n)^(1/2)/a^(1/2))/n+2*(a+b*sinh(x)^n)^(1/2
)/n
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 45, normalized size of antiderivative = 0.96

$$\int \coth(x)\sqrt{a+b\sinh^n(x)}\,dx=\frac{-2\sqrt{a}\,\text{arctanh}\left(\frac{\sqrt{a+b\sinh^n(x)}}{\sqrt{a}}\right)+2\sqrt{a+b\sinh^n(x)}}{n}$$

input
```
Integrate[Coth[x]*Sqrt[a + b*Sinh[x]^n],x]
```

output
```
(-2*Sqrt[a]*ArcTanh[Sqrt[a + b*Sinh[x]^n]/Sqrt[a]] + 2*Sqrt[a + b*Sinh[x]^
n])/n
```

Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 45, normalized size of antiderivative = 0.96, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.467, Rules used = {3042, 26, 3709, 798, 60, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \coth(x)\sqrt{a+b\sinh^n(x)}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{i\sqrt{a+b(-i\sin(ix))^n}}{\tan(ix)}dx$$

$$\Big\downarrow \text{26}$$

$$i\int \frac{\sqrt{b(-i\sin(ix))^n+a}}{\tan(ix)}dx$$

$$\Big\downarrow \text{3709}$$

$$\int \operatorname{csch}(x)\sqrt{a+b\sinh^n(x)}d\sinh(x)$$

$$\Big\downarrow \text{798}$$

$$\frac{\int \operatorname{csch}(x)\sqrt{b\sinh^n(x)+a}d\sinh^n(x)}{n}$$

$$\Big\downarrow \text{60}$$

$$\frac{a\int \frac{\operatorname{csch}(x)}{\sqrt{b\sinh^n(x)+a}}d\sinh^n(x)+2\sqrt{a+b\sinh^n(x)}}{n}$$

$$\Big\downarrow \text{73}$$

$$\frac{\frac{2a\int \frac{1}{\frac{\sinh^{2n}(x)}{b}-\frac{a}{b}}d\sqrt{b\sinh^n(x)+a}}{b}+2\sqrt{a+b\sinh^n(x)}}{n}$$

$$\Big\downarrow \text{221}$$

$$\frac{2\sqrt{a+b\sinh^n(x)}-2\sqrt{a}\text{arctanh}\left(\frac{\sqrt{a+b\sinh^n(x)}}{\sqrt{a}}\right)}{n}$$

input
```
Int[Coth[x]*Sqrt[a + b*Sinh[x]^n],x]
```

output
```
(-2*Sqrt[a]*ArcTanh[Sqrt[a + b*Sinh[x]^n]/Sqrt[a]] + 2*Sqrt[a + b*Sinh[x]^
n])/n
```

Defintions of rubi rules used

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 60
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Simp[n*((b*c - a*d)/(
b*(m + n + 1)))   Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
 c, d}, x] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ[m, 0] && ( !Integer
Q[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n + 2, 0] && IntLinear
Q[a, b, c, d, m, n, x]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 798
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[1/n   Subst
[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a,
b, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3709
```
Int[((a_) + (b_.)*((c_.)*sin[(e_.) + (f_.)*(x_)])^(n_))^(p_.)*tan[(e_.) + (
f_.)*(x_)]^(m_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Si
mp[ff^(m + 1)/f   Subst[Int[x^m*((a + b*(c*ff*x)^n)^p/(1 - ff^2*x^2)^((m +
1)/2)), x], x, Sin[e + f*x]/ff], x]] /; FreeQ[{a, b, c, e, f, n, p}, x] &&
ILtQ[(m - 1)/2, 0]
```

Maple [A] (verified)

Time = 0.29 (sec) , antiderivative size = 38, normalized size of antiderivative = 0.81

method	result	size
derivativedivides	$\frac{2\sqrt{a+b\sinh(x)^{n}}-2\sqrt{a}\,\operatorname{arctanh}\left(\frac{\sqrt{a+b\sinh(x)^{n}}}{\sqrt{a}}\right)}{n}$	38
default	$\frac{2\sqrt{a+b\sinh(x)^{n}}-2\sqrt{a}\,\operatorname{arctanh}\left(\frac{\sqrt{a+b\sinh(x)^{n}}}{\sqrt{a}}\right)}{n}$	38

input
```
int(coth(x)*(a+b*sinh(x)^n)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/n*(2*(a+b*sinh(x)^n)^(1/2)-2*a^(1/2)*arctanh((a+b*sinh(x)^n)^(1/2)/a^(1/
2)))
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 153, normalized size of antiderivative = 3.26

$$\int \coth(x)\sqrt{a+b\sinh^{n}(x)}\,dx$$
$$= \left[\frac{\sqrt{a}\log\left(\frac{b\cosh(n\log(\sinh(x)))+b\sinh(n\log(\sinh(x)))-2\sqrt{b\cosh(n\log(\sinh(x)))+b\sinh(n\log(\sinh(x)))+a}\sqrt{a}+2a}{\cosh(n\log(\sinh(x)))+\sinh(n\log(\sinh(x)))}\right)+2\sqrt{b\cosh(}{n}\right.$$

input
```
integrate(coth(x)*(a+b*sinh(x)^n)^(1/2),x, algorithm="fricas")
```

output
```
[(sqrt(a)*log((b*cosh(n*log(sinh(x))) + b*sinh(n*log(sinh(x))) - 2*sqrt(b*
cosh(n*log(sinh(x))) + b*sinh(n*log(sinh(x))) + a)*sqrt(a) + 2*a)/(cosh(n*
log(sinh(x))) + sinh(n*log(sinh(x))))) + 2*sqrt(b*cosh(n*log(sinh(x))) + b
*sinh(n*log(sinh(x))) + a))/n, 2*(sqrt(-a)*arctan(sqrt(-a)/sqrt(b*cosh(n*l
og(sinh(x))) + b*sinh(n*log(sinh(x))) + a)) + sqrt(b*cosh(n*log(sinh(x)))
+ b*sinh(n*log(sinh(x))) + a))/n]
```

Sympy [F]

$$\int \coth(x)\sqrt{a+b\sinh^n(x)}\,dx = \int \sqrt{a+b\sinh^n{(x)}}\coth{(x)}\,dx$$

input
```
integrate(coth(x)*(a+b*sinh(x)**n)**(1/2),x)
```

output
```
Integral(sqrt(a + b*sinh(x)**n)*coth(x), x)
```

Maxima [F]

$$\int \coth(x)\sqrt{a+b\sinh^n(x)}\,dx = \int \sqrt{b\sinh{(x)}^n+a}\coth{(x)}\,dx$$

input
```
integrate(coth(x)*(a+b*sinh(x)^n)^(1/2),x, algorithm="maxima")
```

output
```
integrate(sqrt(b*sinh(x)^n + a)*coth(x), x)
```

Giac [F]

$$\int \coth(x)\sqrt{a+b\sinh^n(x)}\,dx = \int \sqrt{b\sinh{(x)}^n + a}\coth{(x)}\ dx$$

input
```
integrate(coth(x)*(a+b*sinh(x)^n)^(1/2),x, algorithm="giac")
```

output
```
integrate(sqrt(b*sinh(x)^n + a)*coth(x), x)
```

Mupad [F(-1)]

Timed out.

$$\int \coth(x)\sqrt{a+b\sinh^n(x)}\,dx = \int \coth(x)\ \sqrt{a+b\sinh{(x)}^n}\,dx$$

input
```
int(coth(x)*(a + b*sinh(x)^n)^(1/2),x)
```

output
```
int(coth(x)*(a + b*sinh(x)^n)^(1/2), x)
```

Reduce [F]

$$\int \coth(x)\sqrt{a+b\sinh^n(x)}\,dx = \int \sqrt{\sinh{(x)}^n b + a}\ \coth{(x)}\,dx$$

input
```
int(coth(x)*(a+b*sinh(x)^n)^(1/2),x)
```

output
```
int(sqrt(sinh(x)**n*b + a)*coth(x),x)
```

CHAPTER 4

Appendix

4.1 Listing of Grading functions

The following are the current version of the grading functions used for grading the quality of the antiderivative with reference to the optimal antiderivative included in the test suite.

There is a version for Maple and for Mathematica/Rubi. There is a version for grading Sympy and version for use with Sagemath.

The following are links to the current source code.

The following are the listings of source code of the grading functions.

Mathematica and Rubi grading function

```
(* Original version thanks to Albert Rich emailed on 03/21/2017 *)
(* ::Package:: *)

(* Nasser: April 7,2022. add second output which gives reason for the grade *)
(*                       Small rewrite of logic in main function to make it*)
(*                       match Maple's logic. No change in functionality otherwise*)

(* ::Subsection:: *)
(*GradeAntiderivative[result,optimal]*)
```

```
(* ::Text:: *)
(*If result and optimal are mathematical expressions, *)
(*          GradeAntiderivative[result,optimal] returns*)
(*  "F" if the result fails to integrate an expression that*)
(*      is integrable*)
(*  "C" if result involves higher level functions than necessary*)
(*  "B" if result is more than twice the size of the optimal*)
(*      antiderivative*)
(*  "A" if result can be considered optimal*)


GradeAntiderivative[result_,optimal_] := Module[{expnResult,expnOptimal,leafCountResult,leaf
    expnResult  = ExpnType[result];
    expnOptimal = ExpnType[optimal];
    leafCountResult = LeafCount[result];
    leafCountOptimal = LeafCount[optimal];

    (*Print["expnResult=",expnResult," expnOptimal=",expnOptimal];*)
    If[expnResult<=expnOptimal,
        If[Not[FreeQ[result,Complex]], (*result contains complex*)
            If[Not[FreeQ[optimal,Complex]], (*optimal contains complex*)
                If[leafCountResult<=2*leafCountOptimal,
                    finalresult={"A",""}
                    ,(*ELSE*)
                    finalresult={"B","Both result and optimal contain complex but leaf count
                ]
            ,(*ELSE*)
                finalresult={"C","Result contains complex when optimal does not."}
            ]
        ,(*ELSE*)(*result does not contains complex*)
            If[leafCountResult<=2*leafCountOptimal,
               finalresult={"A",""}
            ,(*ELSE*)
               finalresult={"B","Leaf count is larger than twice the leaf count of optimal.
              ]
        ]
    ,(*ELSE*) (*expnResult>expnOptimal*)
        If[FreeQ[result,Integrate] && FreeQ[result,Int],
            finalresult={"C","Result contains higher order function than in optimal. Order "
            ,
```

```
            finalresult={"F","Contains unresolved integral."}
        ]
    ];

    finalresult
]

(* ::Text:: *)
(*The following summarizes the type number assigned an *)
(*expression based on the functions it involves*)
(*1 = rational function*)
(*2 = algebraic function*)
(*3 = elementary function*)
(*4 = special function*)
(*5 = hyperpergeometric function*)
(*6 = appell function*)
(*7 = rootsum function*)
(*8 = integrate function*)
(*9 = unknown function*)


ExpnType[expn_] :=
  If[AtomQ[expn],
    1,
  If[ListQ[expn],
    Max[Map[ExpnType,expn]],
  If[Head[expn]===Power,
    If[IntegerQ[expn[[2]]],
      ExpnType[expn[[1]]],
    If[Head[expn[[2]]]===Rational,
      If[IntegerQ[expn[[1]]] || Head[expn[[1]]]===Rational,
        1,
      Max[ExpnType[expn[[1]]],2]],
    Max[ExpnType[expn[[1]]],ExpnType[expn[[2]]],3]]],
  If[Head[expn]===Plus || Head[expn]===Times,
    Max[ExpnType[First[expn]],ExpnType[Rest[expn]]],
  If[ElementaryFunctionQ[Head[expn]],
    Max[3,ExpnType[expn[[1]]]],
  If[SpecialFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],4]],
  If[HypergeometricFunctionQ[Head[expn]],
```

```
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],5]],
  If[AppellFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],6]],
  If[Head[expn]===RootSum,
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],7]],
  If[Head[expn]===Integrate || Head[expn]===Int,
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],8]],
  9]]]]]]]]]]


ElementaryFunctionQ[func_] :=
  MemberQ[{
  Exp,Log,
  Sin,Cos,Tan,Cot,Sec,Csc,
  ArcSin,ArcCos,ArcTan,ArcCot,ArcSec,ArcCsc,
  Sinh,Cosh,Tanh,Coth,Sech,Csch,
  ArcSinh,ArcCosh,ArcTanh,ArcCoth,ArcSech,ArcCsch
},func]


SpecialFunctionQ[func_] :=
  MemberQ[{
  Erf, Erfc, Erfi,
  FresnelS, FresnelC,
  ExpIntegralE, ExpIntegralEi, LogIntegral,
  SinIntegral, CosIntegral, SinhIntegral, CoshIntegral,
  Gamma, LogGamma, PolyGamma,
  Zeta, PolyLog, ProductLog,
  EllipticF, EllipticE, EllipticPi
},func]


HypergeometricFunctionQ[func_] :=
  MemberQ[{Hypergeometric1F1,Hypergeometric2F1,HypergeometricPFQ},func]


AppellFunctionQ[func_] :=
  MemberQ[{AppellF1},func]
```

Maple grading function

```
# File: GradeAntiderivative.mpl
# Original version thanks to Albert Rich emailed on 03/21/2017

#Nasser 03/22/2017  Use Maple leaf count instead since buildin
#Nasser 03/23/2017  missing 'ln' for ElementaryFunctionQ added
#Nasser 03/24/2017  corrected the check for complex result
#Nasser 10/27/2017  check for leafsize and do not call ExpnType()
#                   if leaf size is "too large". Set at 500,000
#Nasser 12/22/2019  Added debug flag, added 'dilog' to special functions
#                   see problem 156, file Apostol_Problems
#Nasser 4/07/2022   add second output which gives reason for the grade

GradeAntiderivative := proc(result,optimal)
local leaf_count_result,
        leaf_count_optimal,
        ExpnType_result,
        ExpnType_optimal,
        debug:=false;

        leaf_count_result:=leafcount(result);
        #do NOT call ExpnType() if leaf size is too large. Recursion problem
        if leaf_count_result > 500000 then
            return "B","result has leaf size over 500,000. Avoiding possible recursion issue
        fi;

        leaf_count_optimal := leafcount(optimal);
        ExpnType_result  := ExpnType(result);
        ExpnType_optimal := ExpnType(optimal);

        if debug then
            print("ExpnType_result",ExpnType_result," ExpnType_optimal=",ExpnType_optimal);
        fi;

# If result and optimal are mathematical expressions,
#  GradeAntiderivative[result,optimal] returns
#   "F" if the result fails to integrate an expression that
#       is integrable
#   "C" if result involves higher level functions than necessary
#   "B" if result is more than twice the size of the optimal
```

```
#       antiderivative
#   "A" if result can be considered optimal

    #This check below actually is not needed, since I only
    #call this grading only for passed integrals. i.e. I check
    #for "F" before calling this. But no harm of keeping it here.
    #just in case.


    if not type(result,freeof('int')) then
        return "F","Result contains unresolved integral";
    fi;


    if ExpnType_result<=ExpnType_optimal then
        if debug then
             print("ExpnType_result<=ExpnType_optimal");
        fi;
        if is_contains_complex(result) then
            if is_contains_complex(optimal) then
                if debug then
                        print("both result and optimal complex");
                fi;
                if leaf_count_result<=2*leaf_count_optimal then
                    return "A"," ";
                else
                   return "B",cat("Both result and optimal contain complex but leaf count of
                                   convert(leaf_count_result,string)," vs. $2 (",
                                   convert(leaf_count_optimal,string)," ) = ",convert(2*leaf
                end if
            else #result contains complex but optimal is not
                if debug then
                        print("result contains complex but optimal is not");
                fi;
                return "C","Result contains complex when optimal does not.";
            fi;
        else # result do not contain complex
             # this assumes optimal do not as well. No check is needed here.
            if debug then
                   print("result do not contain complex, this assumes optimal do not as well
            fi;
```

```
            if leaf_count_result<=2*leaf_count_optimal then
                if debug then
                    print("leaf_count_result<=2*leaf_count_optimal");
                fi;
                return "A"," ";
            else
                if debug then
                    print("leaf_count_result>2*leaf_count_optimal");
                fi;
                return "B",cat("Leaf count of result is larger than twice the leaf count of
                                 convert(leaf_count_result,string),"$ vs. $2(",
                                 convert(leaf_count_optimal,string),")=",convert(2*leaf_co
            fi;
        fi;
    else #ExpnType(result) > ExpnType(optimal)
        if debug then
            print("ExpnType(result) > ExpnType(optimal)");
        fi;
        return "C",cat("Result contains higher order function than in optimal. Order ",
                       convert(ExpnType_result,string)," vs. order ",
                       convert(ExpnType_optimal,string),".");
    fi;

end proc:

#
# is_contains_complex(result)
# takes expressions and returns true if it contains "I" else false
#
#Nasser 032417
is_contains_complex:= proc(expression)
  return (has(expression,I));
end proc:

# The following summarizes the type number assigned an expression
# based on the functions it involves
# 1 = rational function
# 2 = algebraic function
# 3 = elementary function
# 4 = special function
# 5 = hyperpergeometric function
```

```
# 6 = appell function
# 7 = rootsum function
# 8 = integrate function
# 9 = unknown function

ExpnType := proc(expn)
  if type(expn,'atomic') then
    1
  elif type(expn,'list') then
    apply(max,map(ExpnType,expn))
  elif type(expn,'sqrt') then
    if type(op(1,expn),'rational') then
       1
    else
       max(2,ExpnType(op(1,expn)))
    end if
  elif type(expn,'`^`') then
    if type(op(2,expn),'integer') then
      ExpnType(op(1,expn))
    elif type(op(2,expn),'rational') then
      if type(op(1,expn),'rational') then
         1
      else
         max(2,ExpnType(op(1,expn)))
      end if
    else
         max(3,ExpnType(op(1,expn)),ExpnType(op(2,expn)))
    end if
  elif type(expn,'`+`') or type(expn,'`*`') then
    max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
  elif ElementaryFunctionQ(op(0,expn)) then
    max(3,ExpnType(op(1,expn)))
  elif SpecialFunctionQ(op(0,expn)) then
    max(4,apply(max,map(ExpnType,[op(expn)])))
  elif HypergeometricFunctionQ(op(0,expn)) then
    max(5,apply(max,map(ExpnType,[op(expn)])))
  elif AppellFunctionQ(op(0,expn)) then
    max(6,apply(max,map(ExpnType,[op(expn)])))
  elif op(0,expn)='int' then
    max(8,apply(max,map(ExpnType,[op(expn)]))) else
  9
```

```
  end if
end proc:


ElementaryFunctionQ := proc(func)
  member(func,[
        exp,log,ln,
        sin,cos,tan,cot,sec,csc,
        arcsin,arccos,arctan,arccot,arcsec,arccsc,
        sinh,cosh,tanh,coth,sech,csch,
        arcsinh,arccosh,arctanh,arccoth,arcsech,arccsch])
end proc:

SpecialFunctionQ := proc(func)
  member(func,[
        erf,erfc,erfi,
        FresnelS,FresnelC,
        Ei,Ei,Li,Si,Ci,Shi,Chi,
        GAMMA,lnGAMMA,Psi,Zeta,polylog,dilog,LambertW,
        EllipticF,EllipticE,EllipticPi])
end proc:

HypergeometricFunctionQ := proc(func)
  member(func,[Hypergeometric1F1,hypergeom,HypergeometricPFQ])
end proc:

AppellFunctionQ := proc(func)
  member(func,[AppellF1])
end proc:

# u is a sum or product.  rest(u) returns all but the
# first term or factor of u.
rest := proc(u) local v;
  if nops(u)=2 then
     op(2,u)
  else
     apply(op(0,u),op(2..nops(u),u))
  end if
end proc:

#leafcount(u) returns the number of nodes in u.
```

```
#Nasser 3/23/17  Replaced by build-in leafCount from package in Maple
leafcount := proc(u)
   MmaTranslator[Mma][LeafCount](u);
end proc:
```

Sympy grading function

```
#Dec 24, 2019. Nasser M. Abbasi:
#              Port of original Maple grading function by
#              Albert Rich to use with Sympy/Python
#Dec 27, 2019  Nasser. Added `RootSum`. See problem 177, Timofeev file
#              added 'exp_polar'
from sympy import *

def leaf_count(expr):
   #sympy do not have leaf count function. This is approximation
   return round(1.7*count_ops(expr))

def is_sqrt(expr):
   if isinstance(expr,Pow):
      if expr.args[1] == Rational(1,2):
         return True
      else:
         return False
   else:
      return False

def is_elementary_function(func):
   return func in [exp,log,ln,sin,cos,tan,cot,sec,csc,
         asin,acos,atan,acot,asec,acsc,sinh,cosh,tanh,coth,sech,csch,
         asinh,acosh,atanh,acoth,asech,acsch
      ]

def is_special_function(func):
   return func in [ erf,erfc,erfi,
          fresnels,fresnelc,Ei,Ei,Li,Si,Ci,Shi,Chi,
          gamma,loggamma,digamma,zeta,polylog,LambertW,
          elliptic_f,elliptic_e,elliptic_pi,exp_polar
       ]
```

```
def is_hypergeometric_function(func):
   return func in [hyper]

def is_appell_function(func):
   return func in [appellf1]

def is_atom(expn):
   try:
      if expn.isAtom or isinstance(expn,int) or isinstance(expn,float):
        return True
      else:
        return False

   except AttributeError as error:
      return False

def expnType(expn):
   debug=False
   if debug:
      print("expn=",expn,"type(expn)=",type(expn))

   if is_atom(expn):
     return 1
   elif isinstance(expn,list):
      return max(map(expnType, expn))    #apply(max,map(ExpnType,expn))
   elif  is_sqrt(expn):
      if isinstance(expn.args[0],Rational): #type(op(1,expn),'rational')
         return 1
      else:
         return max(2,expnType(expn.args[0]))   #max(2,ExpnType(op(1,expn)))
   elif isinstance(expn,Pow):    #type(expn,'`^`')
      if isinstance(expn.args[1],Integer):  #type(op(2,expn),'integer')
         return expnType(expn.args[0])    #ExpnType(op(1,expn))
      elif isinstance(expn.args[1],Rational):  #type(op(2,expn),'rational')
         if isinstance(expn.args[0],Rational): #type(op(1,expn),'rational')
            return 1
         else:
            return max(2,expnType(expn.args[0]))   #max(2,ExpnType(op(1,expn)))
      else:
         return max(3,expnType(expn.args[0]),expnType(expn.args[1])) #max(3,ExpnType(op(1,expn)),Expr
   elif isinstance(expn,Add) or isinstance(expn,Mul): #type(expn,'`+`') or type(expn,'`*`')
```

```
        m1 = expnType(expn.args[0])
        m2 = expnType(list(expn.args[1:]))
        return max(m1,m2)  #max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
    elif is_elementary_function(expn.func):  #ElementaryFunctionQ(op(0,expn))
        return max(3,expnType(expn.args[0]))  #max(3,ExpnType(op(1,expn)))
    elif is_special_function(expn.func): #SpecialFunctionQ(op(0,expn))
        m1 = max(map(expnType, list(expn.args)))
        return max(4,m1)  #max(4,apply(max,map(ExpnType,[op(expn)])))
    elif is_hypergeometric_function(expn.func): #HypergeometricFunctionQ(op(0,expn))
        m1 = max(map(expnType, list(expn.args)))
        return max(5,m1)  #max(5,apply(max,map(ExpnType,[op(expn)])))
    elif is_appell_function(expn.func):
        m1 = max(map(expnType, list(expn.args)))
        return max(6,m1)  #max(5,apply(max,map(ExpnType,[op(expn)])))
    elif isinstance(expn,RootSum):
        m1 = max(map(expnType, list(expn.args))) #Apply[Max,Append[Map[ExpnType,Apply[List,expn]],7]],
        return max(7,m1)
    elif str(expn).find("Integral") != -1:
        m1 = max(map(expnType, list(expn.args)))
        return max(8,m1)  #max(5,apply(max,map(ExpnType,[op(expn)])))
    else:
        return 9

#main function
def grade_antiderivative(result,optimal):

    #print ("Enter grade_antiderivative for sagemath")
    #print("Enter grade_antiderivative, result=",result," optimal=",optimal)

    leaf_count_result  = leaf_count(result)
    leaf_count_optimal = leaf_count(optimal)

    #print("leaf_count_result=",leaf_count_result)
    #print("leaf_count_optimal=",leaf_count_optimal)

    expnType_result  = expnType(result)
    expnType_optimal = expnType(optimal)

    if str(result).find("Integral") != -1:
        grade = "F"
        grade_annotation =""
```

```
else:
    if expnType_result <= expnType_optimal:
        if result.has(I):
            if optimal.has(I): #both result and optimal complex
                if leaf_count_result <= 2*leaf_count_optimal:
                    grade = "A"
                    grade_annotation =""
                else:
                    grade = "B"
                    grade_annotation ="Both result and optimal contain complex but leaf count of result is lar
            else: #result contains complex but optimal is not
                grade = "C"
                grade_annotation ="Result contains complex when optimal does not."
        else: # result do not contain complex, this assumes optimal do not as well
            if leaf_count_result <= 2*leaf_count_optimal:
                grade = "A"
                grade_annotation =""
            else:
                grade = "B"
                grade_annotation ="Leaf count of result is larger than twice the leaf count of optimal. "+str(
    else:
        grade = "C"
        grade_annotation ="Result contains higher order function than in optimal. Order "+str(ExpnType


#print("Before returning. grade=",grade, " grade_annotation=",grade_annotation)

return grade, grade_annotation
```

SageMath grading function

```
#Dec 24, 2019. Nasser: Ported original Maple grading function by
#            Albert Rich to use with Sagemath. This is used to
#            grade Fricas, Giac and Maxima results.
#Dec 24, 2019. Nasser: Added 'exp_integral_e' and 'sng', 'sin_integral'
#            'arctan2','floor','abs','log_integral'
#June 4, 2022  Made default grade_annotation "none" instead of "" due
#            issue later when reading the file.
#July 14, 2022. Added ellipticF. This is until they fix sagemath, then remove it.
```

```
from sage.all import *
from sage.symbolic.operators import add_vararg, mul_vararg

debug=False;

def tree_size(expr):
    r"""
    Return the tree size of this expression.
    """
    #print("Enter tree_size, expr is ",expr)

    if expr not in SR:
        # deal with lists, tuples, vectors
        return 1 + sum(tree_size(a) for a in expr)
    expr = SR(expr)
    x, aa = expr.operator(), expr.operands()
    if x is None:
        return 1
    else:
        return 1 + sum(tree_size(a) for a in aa)

def is_sqrt(expr):
    if expr.operator() == operator.pow:   #isinstance(expr,Pow):
        if expr.operands()[1]==1/2: #expr.args[1] == Rational(1,2):
            if debug: print ("expr is sqrt")
            return True
        else:
            return False
    else:
        return False

def is_elementary_function(func):
    #debug=False
    m = func.name() in ['exp','log','ln',
            'sin','cos','tan','cot','sec','csc',
            'arcsin','arccos','arctan','arccot','arcsec','arccsc',
            'sinh','cosh','tanh','coth','sech','csch',
            'arcsinh','arccosh','arctanh','arccoth','arcsech','arccsch','sgn',
        'arctan2','floor','abs'
        ]
    if debug:
```

```
        if m:
            print ("func ", func , " is elementary_function")
        else:
            print ("func ", func , " is NOT elementary_function")


    return m

def is_special_function(func):
    #debug=False
    if debug:
        print ("type(func)=", type(func))

    m= func.name() in ['erf','erfc','erfi','fresnel_sin','fresnel_cos','Ei',
           'Ei','Li','Si','sin_integral','Ci','cos_integral','Shi','sinh_integral'
           'Chi','cosh_integral','gamma','log_gamma','psi','zeta',
           'polylog','lambert_w','elliptic_f','elliptic_e','ellipticF',
           'elliptic_pi','exp_integral_e','log_integral',
           'weierstrassPInverse','weierstrass','weierstrassP','weierstrassZeta',
           'weierstrassPPrime','weierstrassSigma']

    if debug:
        print ("m=",m)
        if m:
            print ("func ", func ," is special_function")
        else:
            print ("func ", func ," is NOT special_function")


    return m


def is_hypergeometric_function(func):
    return func.name() in ['hypergeometric','hypergeometric_M','hypergeometric_U']

def is_appell_function(func):
    return func.name() in ['hypergeometric']    #[appellf1] can't find this in sagemath

def is_atom(expn):

    #debug=False
```

```
    if debug:
         print ("Enter is_atom, expn=",expn)

    if not hasattr(expn, 'parent'):
        return False


    #thanks to answer at https://ask.sagemath.org/question/49179/what-is-sagemath-equivalent-to-atomic
    try:
        if expn.parent() is SR:
            return expn.operator() is None
        if expn.parent() in (ZZ, QQ, AA, QQbar):
            return expn in expn.parent() # Should always return True
        if hasattr(expn.parent(),"base_ring") and hasattr(expn.parent(),"gens"):
            return expn in expn.parent().base_ring() or expn in expn.parent().gens()

        return False

    except AttributeError as error:
        print("Exception,AttributeError in  is_atom")
        print ("cought exception" , type(error).__name__ )
        return False


def expnType(expn):

    if debug:
        print (">>>>>Enter expnType, expn=", expn)
        print (">>>>>is_atom(expn)=", is_atom(expn))

    if is_atom(expn):
        return 1
    elif type(expn)==list:   #isinstance(expn,list):
        return max(map(expnType, expn))    #apply(max,map(ExpnType,expn))
    elif  is_sqrt(expn):
        if  type(expn.operands()[0])==Rational:  #type(isinstance(expn.args[0],Rational):
            return 1
        else:
            return max(2,expnType(expn.operands()[0]))   #max(2,expnType(expn.args[0]))
    elif expn.operator() == operator.pow:    #isinstance(expn,Pow)
        if type(expn.operands()[1])==Integer:   #isinstance(expn.args[1],Integer)
```

```
        return expnType(expn.operands()[0])   #expnType(expn.args[0])
    elif type(expn.operands()[1])==Rational:  #isinstance(expn.args[1],Rational)
        if type(expn.operands()[0])==Rational: #isinstance(expn.args[0],Rational)
            return 1
        else:
            return max(2,expnType(expn.operands()[0]))  #max(2,expnType(expn.args[0]))
    else:
        return max(3,expnType(expn.operands()[0]),expnType(expn.operands()[1])) #max(3,expnType(expr
elif expn.operator() == add_vararg or expn.operator() == mul_vararg: #isinstance(expn,Add) or isins
    m1 = expnType(expn.operands()[0]) #expnType(expn.args[0])
    m2 = expnType(expn.operands()[1:]) #expnType(list(expn.args[1:]))
    return max(m1,m2)  #max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
elif is_elementary_function(expn.operator()):  #is_elementary_function(expn.func)
    return max(3,expnType(expn.operands()[0]))
elif is_special_function(expn.operator()): #is_special_function(expn.func)
    m1 = max(map(expnType, expn.operands()))      #max(map(expnType, list(expn.args)))
    return max(4,m1)   #max(4,m1)
elif is_hypergeometric_function(expn.operator()): #is_hypergeometric_function(expn.func)
    m1 = max(map(expnType, expn.operands()))      #max(map(expnType, list(expn.args)))
    return max(5,m1)   #max(5,m1)
elif is_appell_function(expn.operator()):
    m1 = max(map(expnType, expn.operands()))      #max(map(expnType, list(expn.args)))
    return max(6,m1)   #max(6,m1)
elif str(expn).find("Integral") != -1: #this will never happen, since it
        #is checked before calling the grading function that is passed.
        #but kept it here.
    m1 = max(map(expnType, expn.operands()))      #max(map(expnType, list(expn.args)))
    return max(8,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
else:
    return 9

#main function
def grade_antiderivative(result,optimal):


    if debug:
        print ("Enter grade_antiderivative for sagemath")
        print("Enter grade_antiderivative, result=",result)
        print("Enter grade_antiderivative, optimal=",optimal)
        print("type(anti)=",type(result))
        print("type(optimal)=",type(optimal))
```

```
leaf_count_result  = tree_size(result) #leaf_count(result)
leaf_count_optimal = tree_size(optimal) #leaf_count(optimal)

#if debug: print ("leaf_count_result=", leaf_count_result, "leaf_count_optimal=",leaf_count_optimal)


expnType_result  = expnType(result)
expnType_optimal = expnType(optimal)

if debug: print ("expnType_result=", expnType_result, "expnType_optimal=",expnType_optimal)

if expnType_result <= expnType_optimal:
    if result.has(I):
        if optimal.has(I): #both result and optimal complex
            if leaf_count_result <= 2*leaf_count_optimal:
                grade = "A"
                grade_annotation ="none"
            else:
                grade = "B"
                grade_annotation ="Both result and optimal contain complex but leaf count of result is larger
        else: #result contains complex but optimal is not
            grade = "C"
            grade_annotation ="Result contains complex when optimal does not."
    else: # result do not contain complex, this assumes optimal do not as well
        if leaf_count_result <= 2*leaf_count_optimal:
            grade = "A"
            grade_annotation ="none"
        else:
            grade = "B"
            grade_annotation ="Leaf count of result is larger than twice the leaf count of optimal. "+str(lea
else:
    grade = "C"
    grade_annotation ="Result contains higher order function than in optimal. Order "+str(expnType_re


print("Before returning. grade=",grade, " grade_annotation=",grade_annotation)

return grade, grade_annotation
```

4.2 Links to plain text integration problems used in this report for each CAS

1. Mathematica integration problems as .m file
2. Maple integration problems as .txt file
3. Sagemath integration problems as .sage file
4. Reduce integration problems as .txt file
5. Mupad integration problems as .txt file
6. Sympy integration problems as .py file